One Approach on Derivation of the Schrödinger Equation of Free Particle

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The Schrödinger equation based on the de Broglie wave is the most fundamental equation of the quantum mechanics. There can be no doubt about its prediction validity. However, the probabilistic interpretation on the quantum mechanics has insoluble semantic interpretations like ‘reduction of wave packet’ on observations of physical values. Especially, it is not clear that the wave function \( \Psi \) which is described by complex function, is whether ‘formality’ or ‘reality’ to express the state of particle motion. On this paper, we interpret the wave nature of particle as not the inherency of particle itself, but the motional property of particle in fluctuated space-time due to the kinetic energy and momentum the belief that the kinetic energy and momentum fluctuates the microscopic space-time, and the particle move through the fluctuated space-time adversely. Then, through the particle motion in the Euclidean space-time, the particle will be recognized as if it has the wave nature. We estimate the governing equation of fluctuations of microscopic space-time based on the macroscopic law of motion. On this paper, the equivalence between the governing equation and the Schrödinger equation is indicated.

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\[ dX = \frac{\partial \phi}{\partial x} \, dx + \frac{\partial \phi}{\partial t} \, dt, \quad (1) \]
\[ dT = \frac{\partial \eta}{\partial x} \, dx + \frac{\partial \eta}{\partial t} \, dt, \quad (2) \]

And, as dynamic law, the following hypothesizes are considered.

**Hypothesis I:**

On the macroscopic space-time system, the speed of free particle under uniform motion is defined as follows using of the macroscopic space \( X \) and time \( T \):

\[ \frac{dX}{dT} = V \quad (3) \]

Further, the kinetic energy \( E \) is also defined as

\[ E = \frac{1}{2} m \mathbf{V}^2 = \frac{1}{2} \left( \frac{dX}{dT} \right)^2. \quad (4) \]

**Hypothesis II:**

On the macroscopic space-time system, the particle moves along the geodesic line due to the least action. The geodesic line is defined by a function which minimize the following variation:

\[ \int L \, dT = \int \left[ \frac{m}{2} \left( \frac{dX}{dT} \right)^2 \right] \, dT \quad (5) \]

The second hypothesis is the Lagrangian function \( L \) defined by (5) on the macroscopic space-time, has the minimum value under the microscopic space-time, i.e. the Lagrangian function \( L \) satisfies the Euler-Lagranj function equation defined by the microscopic space-time system. On the macroscopic system, (5) give the linear motion as the geodesic line of uniform motion. So, this hypothesis requires that the macroscopic geodesic line is also the one on the microscopic space-time.

First, the Hypothesis I is considered. From (1), (2)
and (3), we have
\[
\frac{dX}{dT} = \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial t} = V .
\]  
(6)

(6) leads to
\[
\frac{\partial \phi}{\partial t} \frac{dx}{dt} - \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial t} = V .
\]  
(7)

with help of \( \dot{x} = \frac{dx}{dt} \) which is the particle speed defined by the microscopic spacetime,
\[
\dot{x} = \frac{dx}{dt} \rightleftarrows \frac{\partial \phi}{\partial t} \frac{dx}{dt} - \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial t} = V .
\]  
(8)

Further, let us consider the following function \( \psi \) :
\[
\psi = \phi - V .
\]  
(9)

Then, from (8),
\[
\dot{\psi} = \frac{\partial \psi}{\partial x} \frac{dx}{dt} = \frac{\partial \psi}{\partial t} = 0 .
\]  
(10)

so that means
\[
\dot{x} = \frac{\partial \psi}{\partial x} \frac{dx}{dt} + \frac{\partial \psi}{\partial t} = 0 .
\]  
(11)

hence
\[
\frac{d^2 \psi}{dt^2} = \dot{x} \left( \frac{\partial \psi}{\partial x} \frac{dx}{dt} + \frac{\partial \psi}{\partial t} \right) + \frac{\partial \psi}{\partial \psi} = \frac{\partial \psi}{\partial \psi} = 0 .
\]  
(12)

This means that the function \( \psi \) which is defined in the microscopic spacetime satisfies the one dimensional classical wave equation.

Next, let us consider Hypothesis II. This is the Euler-Lagrangeian equation in the microscopic spacetime. The L is defined by the macroscopic spacetime as follows,
\[
L = \frac{m}{2} \left( \frac{dX}{dT} \right)^2 = \frac{m}{2} \left( \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial t} \right)^2 = \frac{m}{2} V^2 .
\]  
(14)

Equation (14) satisfies the following Euler-Lagrangeian equation,
\[
\frac{d}{dT} \left( \frac{\partial L}{\partial \psi} \right) - \frac{\partial L}{\partial \psi} = 0 .
\]  
(15)

From (14), the first term in the left-hand side of (15) is
\[
\frac{\partial L}{\partial \psi} = \frac{m}{2} V^2 \rightleftarrows m V \frac{\partial}{\partial \psi} .
\]  
(16)

Thus
\[
\frac{d}{dT} \left( \frac{\partial L}{\partial \psi} \right) = \frac{m}{2} V^2 \rightleftarrows m V \frac{\partial}{\partial \psi} .
\]  
(17)

Further, the second term in the left-hand side of (15) is
\[
\frac{\partial L}{\partial \psi} = \frac{m}{2} V^2 = m V \frac{\partial}{\partial \psi} .
\]  
(18)

Thus, (15) is
\[
\frac{d}{dT} \left( \frac{\partial L}{\partial \psi} \right) - \frac{\partial L}{\partial \psi} = 0 .
\]  
(19)

where, the temporal differentiation is defined like
\[
\frac{d}{dT} = \dot{x} \frac{\partial}{\partial \psi} + \frac{\partial}{\partial \psi} .
\]  
(20)

On (19), the first term of the left-hand side is estimated as,
\[
\frac{d}{dT} \left( \frac{\partial L}{\partial \psi} \right) = \dot{x} \frac{\partial}{\partial \psi} + \frac{\partial}{\partial \psi} .
\]  
(21)

where,
\[
\frac{d}{dT} \left( \frac{\partial L}{\partial \psi} \right) = \dot{x} \frac{\partial}{\partial \psi} + \frac{\partial}{\partial \psi} .
\]  
(22)

Finally, from (19), (21) and (22),
\[
\dot{x} \frac{\partial}{\partial \psi} + \frac{\partial}{\partial \psi} = 0 .
\]  
(23)

(23) can be rewritten as
\[
\frac{d}{dT} \left( \frac{dX}{dT} \right) = 0 .
\]  
(24)

with help of (9),
\[
\frac{d}{dT} \left( \frac{dX}{dT} \right) = \frac{\partial}{\partial \psi} .
\]  
(25)

From (24), (25)
with help of (9),
\[
\frac{\partial^2 \psi}{\partial t^2} = \frac{h^2 \lambda^2}{4m} \frac{\partial^2 \psi}{\partial \lambda^2}.
\]  
(27)

Newly, the solution \( \Psi \) satisfies the classical wave equation and vice versa. (13) certifies that the solution \( \Psi \) defined as the solution of (27) always minimizes the Lagrangian function (14). However, (27) is merely the kinetic condition which the microscopic spacetime should satisfy with help of (9) and (3) and (5). (30) To consistent with the macroscopic motion equation, the governing equation that describes the fluctuation of the microscopic spacetime should satisfy the following relationship between the kinetic energy and the momentum. It means that only the solution of (27) which satisfies the following relationship in addition, as a binding condition is consistent with the macroscopic motion equation, the microscopic spacetime should satisfy to consistent with (3) and (5). To make consideration of the collect microscopic space-time, the divergent \( \alpha \) and \( \beta \) are never acceptable physically, the constant \( C \) should be negative as follows,
\[
C = -\left(2\pi \nu \right)^2.
\]  
(31)

Thus,
\[
\frac{d^2 \alpha(x)}{dx^2} = \left(\frac{2\pi \nu}{x}\right)^2 \alpha(x),
\]  
(32)

\[
\frac{d^2 \beta(t)}{dt^2} = -\left(2\pi \nu \right)^2 \beta(t).
\]  
(33)

Multiplying (32) by \( \beta(t) \) and (33) by \( \alpha(x) \), the following relationships are obtained,
\[
\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{2\pi \nu}{x}\right)^2 \psi = \left(\frac{2\pi \nu}{\lambda}\right)^2 \psi,
\]  
(34)

\[
\frac{\partial^2 \psi}{\partial t^2} = -\left(2\pi \nu \right)^2 \psi.
\]  
(35)

Where, \( \lambda \) is the wavelength. Here we assume that the frequency \( \nu \) and the wavelength \( \lambda \) which is defined by (34) and (35) respectively satisfy the following Einstein-de Broglie’s formula,
\[
E = \hbar \nu, \quad p = \frac{\hbar}{\lambda}.
\]  
(36)

where \( \hbar \) is the Planck’s constant. We substitute (34)-(36) into (29). As a result, the solution \( \Psi \) satisfies the following wave equation,
\[
\frac{\partial^2 \psi}{\partial t^2} = \frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial \lambda^2}.
\]  
(37)

where,
\[
\hbar = \frac{\hbar}{2\pi}.
\]  
(38)

And, as might be expected, the relationship of \( \chi = \nu \) can be gotten. The (37) satisfies the binding condition (28) through the Einstein-de Broglie’s formula, and is just the wave equation of the dispersive wave which is common of the vibration problem of span. It is known that in 1926 Schrödinger considered the equation (37) is the wave equation of particle motion by letters to Lorentz and Planck from Schrödinger. As a general property, (37) is rewritten as,
\[
\left(\frac{\partial}{\partial t} - i\frac{\hbar}{2m} \frac{\partial^2}{\partial \lambda^2}\right) \left(\psi + i\frac{\hbar}{2m} \frac{\partial \psi}{\partial \lambda}\right) = 0,
\]  
(39)

or
\[
\left(\frac{\partial}{\partial t} + i\frac{\hbar}{2m} \frac{\partial^2}{\partial \lambda^2}\right) \left(\psi - i\frac{\hbar}{2m} \frac{\partial \psi}{\partial \lambda}\right) = 0.
\]  
(40)

Then, the \( \Psi \) satisfy the following equations.
\[
\frac{\partial \psi}{\partial t} + i\frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial \lambda^2} = 0,
\]  
(39)

\[
\frac{\partial \psi}{\partial t} - i\frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial \lambda^2} = 0.
\]  
(40)

These are just the Schrödinger equation of \( \Psi \) and \( \Psi^* \) which is the complex conjugate function of \( \Psi \). So, the general solution of (37) is given as a sum of the solutions of (39) and (40) as follows,
\[
\psi + \psi^* = \text{Re}(\psi).
\]

This means the solution \( \Psi \) is always the real function and is not the general solution of the Schrödinger equation that may describe by the complex function. Further, as (37) includes the second-order temporal differentiation, on the standard probabilistic interpretation (37) never certify the conservation law of the probability density. So, we can guess that Schrödinger abandoned (37) as the correct wave equation of particle motion. On this paper, (37) is re-evaluated using Dirac’s formula \[4\] that was used to the relativistic wave equation for the electron from the Klein-Gordon equation. First, we extend (37) to the three dimensional one as follows,
\[
\frac{\partial^2 \psi}{\partial t^2} = \left(\frac{\hbar}{2m}\right)^2 \left(\frac{\partial^2 \psi}{\partial \lambda^2} + \frac{\partial^2 \psi}{\partial \nu^2} + \frac{\partial^2 \psi}{\partial \lambda^2}\right).
\]  
(41)
To reduce the order of the temporal differentiation, (41) can be rewritten as follows:

\[
\hbar \frac{\partial \psi}{\partial t} + \pm i \frac{\hbar^2}{2m} \left( \sigma_x \frac{\partial^2 \psi}{\partial x^2} + \sigma_y \frac{\partial^2 \psi}{\partial y^2} + \sigma_z \frac{\partial^2 \psi}{\partial z^2} \right). \tag{42}
\]

This is the reasonable modification of (39) and (40). To duplicate (41) by multiplying (42) by the differential operator of both sides, the following conditions should be satisfied.

\[
\sigma_x, \sigma_y, \sigma_z = 1, \quad \sigma_x, \sigma_y, \sigma_z = -1 \quad (2\pi). \tag{43}
\]

If the \( \sigma_x, \sigma_y, \alpha \) are the following scheme of the Pauli matrices, then (42) can be duplicated to (41).

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{44}
\]

Then, the \( \Psi \) should be written as a vector, and (42) can be rewritten as follows,

\[
\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \pm \frac{\hbar^2}{2m} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \frac{\hbar^2}{2m} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{45}\]

\[
\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = -\frac{\hbar^2}{2m} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} - \frac{\hbar^2}{2m} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}. \tag{46}\]

As this paper is considered under the nonrelativistic approximation, the \( \Psi \) is the two-dimensional vector unlike the Dirac equation. (45), (46) can be rewritten with each vector component as follows,

\[
\hbar \frac{\partial}{\partial t} \psi_1 = \frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial y^2} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial z^2}, \tag{47}\]

\[
\hbar \frac{\partial}{\partial t} \psi_2 = \frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial x^2} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial y^2} - \frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial z^2}, \tag{48}\]

\[
\hbar \frac{\partial}{\partial t} \psi_3 = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_3}{\partial x^2} - \frac{\hbar^2}{2m} \frac{\partial^2 \psi_3}{\partial y^2} - \frac{\hbar^2}{2m} \frac{\partial^2 \psi_3}{\partial z^2}, \tag{49}\]

\[
\hbar \frac{\partial}{\partial t} \psi_4 = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_4}{\partial x^2} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi_4}{\partial y^2} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi_4}{\partial z^2}. \tag{50}\]

(47)-(50) are considered as the Schrödinger equations that include the spin angular momentum. To confirm it, the free particle motion along the \( z \) direction is considered by (45). Then, (45) is simplified as

\[
\hbar \frac{\partial}{\partial t} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{51}\]

As a solution of \((\Psi_1, \Psi_2)\), we have the following harmonic one,

\[
\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{-(p_0 - E)t}. \tag{52}\]

By substitution of (52) into (53),

\[
-\hbar E \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -i \frac{p^2}{2m} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \tag{53}\]

\[
\begin{pmatrix} E - \frac{p^2}{2m} \\ 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0. \tag{54}\]

As \((u_1, u_2)\) should have non zero solution, then

\[
\begin{pmatrix} E - \frac{p^2}{2m} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0. \tag{55}\]

Under the nonrelativistic approximation, the kinetic energy should be positive. Then,

\[
E = \frac{p^2}{2m}. \tag{56}\]

This means (54) satisfies the correct relationship between the kinetic energy and the momentum as an eigenvalue. From the eigenvalue (56), the eigenvector (52) is written as follows,

\[
\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{-i(p_0 - E)t} = u_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-(p_0 - E)t}. \tag{57}\]

Similarly, we consider the free particle motion along the \( z \) direction by (46). Then, (46) is

\[
\hbar \frac{\partial}{\partial t} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = i \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}. \tag{58}\]

Under the harmonic solution of \((\Psi_3, \Psi_4)\), (58) is

\[
\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} e^{i(p_0 - E)t} = u_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i(p_0 - E)t}. \tag{59}\]

Thus, the eigenvector \((\Psi_3, \Psi_4)\) is

\[
\begin{pmatrix} \Psi_3 \\ \Psi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i(p_0 - E)t}. \tag{60}\]

(57) and (60) can be written as follows,

\[
\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = u_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i(p_0 - E)t}. \tag{61}\]
These are just the wave functions that can describe the spin up, and the spin down. Actually, using the following Pauli's spin operator,

$$ S_{\pm} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. $$

Then, with the limiting behavior of between the Newton's law and the microscopic dynamic law,

$$ S_{+} \psi_+ = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i \frac{\hbar}{2} (p_\perp \cdot \xi)} = \frac{\hbar}{2} \psi_+, $$

$$ S_{-} \psi_+ = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i \frac{\hbar}{2} (p_\perp \cdot \xi)} = -\frac{\hbar}{2} \psi_-. $$

These describe the wave function with the spin angular momentum correctly. The reasoning of the fluctuated spacetime in the microscopic systems, can lead to the Schrödinger equation with the spin angular momentum naturally.

In addition, if (37) is separated into (39) and (40), the general solution is the real function. On the other hands, using (42) and (43) that is separated by the Pauli matrices, the other general solution of (41) is as follows,

$$ \psi_+ + \psi_- = \begin{pmatrix} u_+ \\ u_- \end{pmatrix} e^{i \frac{\hbar}{2} (p_\perp \cdot \xi)}. $$

This solution is not the real function and describes the superimposed state of the spin up and the spin down. Actually, the inner product of (66),

$$ |\psi_+ \cdot \psi_-|^2 = u_+ \cdot u_- + u_- \cdot u_-, $$

indicates the superposition of the spin up and the spin down.

§3. Discussions

Firstly, let us consider the meaning of the $\Psi$. The $\Psi$ is defined by (9) as follows,

$$ \Psi = \phi - V \eta = X - V \cdot T. $$

By the modification, (68) is rewritten as

$$ X = V \cdot T + \psi. $$

In the macroscopic dynamic law of the free particle, the $\Psi$ is constant that corresponds to the initial condition. On the other hand, because of the existence of the fluctuations of the microscopic space-time by the particle motion in the microscopic motion law, the $\Psi$ never becomes constant. That is, the $\Psi$ can be interpreted as the 'disparity' or 'deviation' between the Newton's law and the microscopic dynamic law. Then, with the limiting behavior of $\Psi \to 0$, (69) can be completely corresponding to the Newton's law whose initial position is assumed to be 0.

When the fluctuations of the microscopic space-time are negligible small, the microscopic space-time systems may be equivalent to the macroscopic ones. Then, it is extremely natural that the microscopic dynamical law can be close to the Newton's law defined on the macroscopic systems. The present concept of the microscopic dynamic law says that the wave function $\Psi$ and the macroscopic i.e. Newton's one can be related directly without using the Ehrenfest theory. The $\Psi$ defined by (9) has the dimension of distance. If the $\Psi$ is the complex function, the inner product of the $\Psi$ and $\Psi^*$, that is the complex conjugate function of $\Psi$, has the dimension of the distance squared. When the inner product of $(\Psi \cdot \Psi^*)$ is normalized to unity, the standard probability interpretation of the Copenhagen school can be applicable to understand the meaning of the $\Psi$.

On two particle systems, the wave function is defined as a six dimensional 'wave' that includes the coordinates of particle A and particle B. Even on this case, the $\Psi$ can be considered that it express the 'disparity' with the Newton's law. The disparity is caused by the fluctuations of the microscopic space-time through the motion of the particle A and B. So, only the $\Psi$ of the one particle system is wavelike exactly.

Secondarily, let us consider the difference between the macroscopic and microscopic motion of the particle. If the microscopic space-time system has the fluctuation for the macroscopic and Euclidean system where we can recognize and define the particle motion, we should accept the particle existence at the 'past' and the 'future' of the microscopic time at the 'macroscopic present'. Then, the particle may not have the 'deterministic' path at the macroscopic 'present', as the macroscopic 'present' includes the microscopic 'past' and 'future'. The relativity of the 'present' between the macroscopic and microscopic times clarifies the deference between the quantum mechanics and the classical ones. That is, the classical dynamic equation is determined along the unidirectional time axis from the past to the future. But, in the microscopic systems, the 'past' and 'future' of microscopic time at the macroscopic 'present' disturb to predict the 'deterministic' particle motion as the meaning of the macroscopic systems.

In that sense, our actions to observe the physical quantity of the particle motion may make the 'present' of the particle on the macroscopic systems. The observation is the frozen figure of the microscopic motion at the macroscopic 'present'. It means that the observation is the action to hypostatize the particle motion at the macroscopic 'present'. Then, the observation gives an unavoidable interaction to the particle motion. As well as the interaction is usually estimated by the Compton effects on the Heisenberg uncertainty principle, it may be considered as the interference between the double fluctuated space-time due to the particle motion and the observation.

From here onwards, even if the fluctuation of the microscopic space-time leads to the wave nature of the particle, the probability interpretation can have applicability to predict the particle motion as above mentioned. However, the most important difference with the probability interpretation is that the particle is just the 'particle' and the wave nature is caused by the fluctuation of the microscopic...
integral by Feynman is equivalent to the quantum mechanics based on the path concept of the existence of infinite paths of the particle motion. One might derive the Schrödinger equation using stochastic methods. This interpretation that advocates the existence of the particle paths may be eliminated naturally.

If the problem of the two-slit experiment is examined in terms of wave mechanics, the wave nature of the particle is considered as the attribution of particle itself. And the other, on this paper, the wave nature is the attribution of the microscopic spacet ime. The particle motion gives fluctuations to the microscopic spacet ime, and vice versa, the spacet ime determined the particle motion. Then, on the macroscopic and Euclidean space-time systems, the particle motion may be recognized as the wave nature. Using such a concept, the wave-particle duality is understood naturally. Further, the Schrödinger equation that includes the spin angular momentum is derived naturally. Even this concept, the probability interpretation can be applicable to predict the physical quantity. But, as the present concept never requires the reduction of wave packet on the observation, this is looked more rational than the concept based on the wave nature of particle.

§4. Conclusion

On the standard quantum mechanics, the wave nature of the particle is considered as the attribution of particle itself. And the other, on this paper, the wave nature is the attribution of the microscopic spacet ime. The particle motion gives fluctuations to the microscopic spacet ime, and vice versa, the spacet ime determined the particle motion. Then, on the macroscopic and Euclidean space-time systems, the particle motion may be recognized as the wave nature. Using such a concept, the wave-particle duality is understood naturally. Further, the Schrödinger equation that includes the spin angular momentum is derived naturally. Even this concept, the probability interpretation can be applicable to predict the physical quantity. But, as the present concept never requires the reduction of wave packet on the observation, this is looked more rational than the concept based on the wave nature of particle.

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