Modeling and compensation of hysteresis in piezoelectric actuators

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ABSTRACT

Piezoelectric actuator has the advantages of high rigidity, wide bandwidth, fast response and high resolution. Therefore, they are widely used in many micro and nano positioning applications. However, the hysteresis characteristic in the piezoelectric actuator (PEA) seriously affects its positioning accuracy and even causes instability. In this paper, a modified Prandtl-Ishlinskii (MPI) model, which can describe the rate asymmetric hysteresis of piezoelectric actuator, is studied. The hysteresis compensation is realized by using the rate dependent Prandtl-Ishlinskii model based on the improved Prandtl-Ishlinskii hysteresis model and the hysteresis characteristics of the driver measured in the laboratory under the frequency input of up to 100 Hz. In order to further reduce the error of feedforward compensation, a sliding mode controller is designed. The stability of the control system is proved by Lyapunov theory. The experimental results show that the linear error of the system is reduced from 10% to less than 1%, and the tracking error can also be reduced by 90%.

1. Introduction

Piezoelectric actuator has the advantages of large output force, wide frequency band and fast frequency response. Piezoelectric actuators play an important role in micro nano applications [1]. However, as a kind of polar material, piezoelectric actuator often shows the nonlinear hysteresis between input current and output displacement. At high input rate, the hysteresis nonlinearity is often more significant, which will not only directly affect the accuracy of high-precision positioning, but also lead to response oscillation and error, resulting in poor tracking performance of the closed-loop system [2]. Generally, it is necessary to identify the hysteresis model and the compensation controller based on the inverse hysteresis model to realize the high-precision positioning control of pea, and the establishment of the hysteresis model and parameter identification play a fundamental and important role in the establishment of the accurate hysteresis model.

In order to solve this problem, many hysteresis models describing the hysteresis nonlinearity of piezoelectric actuator are proposed in the literature, such as Preisach model [3], Krasnosel’skii-Pokrovskii model [4], Prandtl-Ishlinskii model and differential equation based hysteresis model, such as backlash model, Duhem model, Bouc-Wen model [5]. However, few studies show that these hysteretic models can describe symmetric and rate independent hysteresis, but in practical applications, hysteretic models show asymmetry and dependence.

An improved sliding mode motion tracking control method for piezoelectric actuator is proposed. A control method for parameter uncertainty, nonlinearity (including hysteresis effect) and other unmodelled disturbances is proposed, and the asymmetric hysteresis characteristics of generalized play operator are described by using the improved PI model (MPI). This MPI has been used in model-based compensation schemes. A feedforward control method based on MPI model is proposed. In order to further reduce the influence of compensation error, parameter uncertainty and external interference, based on the identification model of piezoelectric actuator, the proposed sliding mode control method is established, and the proposed control method is analyzed. The stability of the proposed control method is proved theoretically, and the performance of the proposed scheme is verified by the experimental results. The scheme can compensate the hysteresis effect.

2. Hysteresis mathematical model

In this section, we mainly discuss how to use generalized hysteresis operator to describe MPI model and how to design feedforward controller to compensate the actuator’s hysteresis nonlinearity.
where $\tau$ is determined with the given threshold $\zeta$.

Based on the generalized play operator, the MPI model can be given by

$$y(t) = h(u) + \int_0^\infty p(t)P[u](t)\,dt$$

(4)

where $h(u) = a_1u^3 + a_2u^2 + a_3u$, and $a_1, a_2, a_3$ are polynomial coefficients.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = a_1 \begin{bmatrix} u_1^3 \\ u_2^3 \\ \vdots \\ u_n^3 \end{bmatrix} + a_2 \begin{bmatrix} u_1^2 \\ u_2^2 \\ \vdots \\ u_n^2 \end{bmatrix} + a_3 \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} + \sum_{i=1}^n \rho_i P[u_i]$$

(5)

2.2. Parameter identification

Because the MPI model is non-linear and non-differentiable, the parameter identification method of traditional linear model can't be applied directly. In this paper, particle swarm optimization (PSO) is used, which is a popular optimization algorithm. The objective function $f(X)$ is expressed as follows:

$$f(X) = \frac{1}{N} \sum_{i=1}^N (y_i - \tilde{y}_i)$$

(6)

where $N$ is the size of the data, $X = [a_1, a_2, a_3, a_4, b_1, b_2, c, c, \ldots, \rho_1, \ldots, \rho_n]$ is parameters to be identified, $y_i$ and $\tilde{y}_i$ are the output predicted by the MPI model and the actual system, respectively. The basic PSO formulas can be expressed by the velocity and position as follows:

$$\begin{bmatrix} V_{1i}^{t+1} \\ V_{2i}^{t+1} \\ \vdots \\ V_{ni}^{t+1} \end{bmatrix} = \omega V_{Pi}^{t} + c_1 r_1(t) \left( P_{max} - X_i^{t} \right) + c_2 r_2(t) \left( G_{max} - X_i^{t} \right)$$

(7)

$$X_{i}^{t+1} = X_i^{t} + V_{Pi}^{t+1}$$

### Table 1. Model parameters.

| Number | $r_i$ | $b$ | $a_i$ |
|--------|-------|-----|-------|
| 1      | 0.1   | 0.2054 | 0.2547 |
| 2      | 0.2   | 0.1985 | -0.00358 |
| 3      | 0.3   | 0.0237 | 0.7123 |
| 4      | 0.4   | 0.0148 |       |
| 5      | 0.5   | 0.0456 |       |
| 6      | 0.6   | 0.0645 |       |
| 7      | 0.7   | 0.0051 |       |
| 8      | 0.8   | 0.0014 |       |
| 9      | 0.9   | 0.0033 |       |

| Name   | Value  |
|--------|--------|
| $K_r$  | 20     |
| $K_b$  | 7.6    |
| $\rho$ | 7700   |
| $h$    | 0.005  |
| $r$    | 0.0064 |
| $T_e$  | 4.6    |
| C      | 0.0000072 |
| $R$    | 10     |
| $b$    | 61     |
| $k$    | 428155 |
Figure 4. Model validation on sinusoidal control signals of 10 Hz with variable amplitude (Normalization): (a) Expected input and actual output curves with variable amplitude; (b) Experimental and fitting curves with variable amplitude; (c) Model fitting error with variable amplitude.

Table 2. Model fitting maximum error and mean square error.

| Control signals (Hz)            | Max error (%) | Mean square error (%) |
|--------------------------------|---------------|-----------------------|
| 2Hz constant amplitude         | 0.31          | 0.21                  |
| 5Hz constant amplitude         | 0.43          | 0.25                  |
| 10Hz constant amplitude        | 0.49          | 0.18                  |
| 20Hz constant amplitude        | 0.69          | 0.35                  |
| 50Hz constant amplitude        | 1.29          | 0.36                  |
| 100Hz constant amplitude       | 0.92          | 0.36                  |
| 5Hz variable amplitude         | 0.47          | 0.21                  |
| 10Hz variable amplitude        | 0.64          | 0.22                  |
| 100Hz variable amplitude       | 1.64          | 0.62                  |

Figure 5. Feedforward compensation experiment with 10 Hz variable amplitude sine input: (a) Feedforward compensation before and after hysteresis curve with variable amplitude; (b) Linearity error with variable amplitude.
where \( r_i = \text{rand}(i), i = 1, 2, \ldots, \) are random numbers between 0 and 1. \( \omega \) is the inertia weight, and \( c_1 \) and \( c_2 \) are local search acceleration coefficients and global search acceleration coefficient, respectively. If the particle is in the historical optimal position for a long time, the second and third terms in Eq. (7) tend to 0, which means \( X_t \rightarrow P_{\text{best}_i} \). This will lead to a local optimization, rather than the global optimization. In order to avoid this situation, an improved rate update method is proposed in this paper. The swarm not only follows the historical best position and global best position, but also adds the information of the optimal position of a particle which is selected randomly. In addition, since the inertia weight affects the search area, the large inertia value and the small inertia value are conducive to global search and local search, so \( \omega \) is selected as the inertial weight of dynamic nonlinear attenuation and it can be expressed by:

\[
\omega_t = \omega_{\text{min}} \exp\left(\frac{\omega_{\text{max}} - \omega_{\text{min}}}{T} (T - t)\right), i = 1, 2, \ldots, T
\]

where \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) are the maximum and minimum inertia weights respectively; \( T \) represents the time of the \( i \)th particle and \( T \) is the maximum number of iterations. Based on the above design, the modify PSO (MPSO) with velocity update formula is expressed as follows:

\[
V_{t+1} = \omega_{\min} \exp\left(\frac{\omega_{\max} - \omega_{\min}}{T} (T - t)\right) V_t + c_1 r_1 (P_{\text{best}_i} - X_t) + c_2 r_2 (G_{\text{best}} - X_t) + c_3 r_3 (Q_{\text{best}_i} - X_t)
\]

where \( c_3 \) is the acceleration coefficient, \( r_3 = \text{rand}(i) \) is a random number between 0 and 1; \( Q_{\text{best}_i} \) is the optimal position information of a particle which is selected randomly.

Step 1: Set the number of operators.
Step 2: Initialize the parameters of particle swarm such as particle swarm size, maximum number of iterations, various acceleration coefficients, maximum and minimum weight values, velocity and position boundary values, etc.
Step 3: Initialize the velocity and position of each particle with the following formula:

\[
\begin{cases}
X_t = X_{\text{max}} + \text{rand}((X_{\text{max}} - X_{\text{min}})) \\
V_t = V_{\text{max}} + \text{rand}((V_{\text{max}} - V_{\text{min}}))
\end{cases}
\]

Step 4: Calculate the fitness of each particle according to (7).
Step 5: Search for individual optimal position and global optimal position.
### 3. Controller design

In order to improve the dynamic positioning accuracy of the piezoelectric actuator, reduce the parameter uncertainty, compensation error and external interference of the piezoelectric actuator, the driving system of the piezo-electric actuator is modeled, and an adaptive backstep sliding mode controller is designed. The stability of the control system is proved by choosing appropriate Lyapunov function and intermediate dummy variable (see Figure 1).

#### 3.1. Model of piezoelectric actuator

The driving system of the piezoelectric actuator is mainly composed of mechanical and electrical parts. Due to the complex structure of piezoelectric drive system, this paper establishes the hysteresis model of piezoelectric drive from mechanical and electrical aspects, which is convenient for subsequent modeling and analysis.

This chapter introduces the drive circuit and control circuit of piezoelectric driver in detail. The dynamic equation from the input stage to the output stage can be expressed by the following three transfer functions:

\[
G_1(s) = \frac{u_d(s)}{u_C(s)} = K_1
\]

\[
G_2(s) = \frac{G_1(s)}{RC_s + 1}
\]

\[
G_3(s) = K_3
\]

where \(u_d\) is the input; \(u_C\) is the voltage due to this effect; \(u_C\) is the voltage; \(P[\cdot]\) is the hysteresis effect; \(C_{\text{voltage}}\) the total capacitance and resistance connected in parallel with the electromechanical transformer, which have a ratio of \(T_c\); \(K_1\) and \(K_3\) represent voltage and power amplification of the driving circuit. According to Newton’s law, the mechanical model can be written as follows:

\[
F_x = F_i = m\ddot{x} + b\dot{x} + kx = F_e + F_i
\]

\[
m\ddot{x} + b\dot{x} + kx = F_e
\]

where \(F_i\) and \(F_e\) are the external force and the driving force, respectively; \(m\) is the mass of the piezoelectric actuator. Due to the intrinsic hysteretic nonlinearity and tracking performance of piezoelectric actuator is mainly focused in this paper, the external force on the piezoelectric actuator is temporarily ignored that is \(F_e = 0\).

\[
m\ddot{x} + b\dot{x} + kx = T_x u
\]

After Laplace transformation, the transfer function can be expressed as

\[
G_4(s) = \frac{T_x}{ms^2 + bs + k}
\]

Combining Eq. (18) with Eq. (21), we can obtain

\[
G(s) = G_4(s)G_2(s)G_3(s)
\]

\[
= K_1 \frac{T_x}{RCs + 1} \frac{T_x}{K_3 K_2 T^2_x (RCs + 1)(b^2 + ks^2 + bs + k)}
\]

In this experiment, the parameters are listed in Table 1.
3.2. Modeling uncertainties

The system model is as follows:

\[
\begin{aligned}
    x_1 &= \dot{x} \\
    x_2 &= \dot{x}_1 \\
    x_3 &= \dot{x}_2
\end{aligned}
\]

according to equation (19), equation (20) can be written as follows:

\[
\begin{aligned}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= x_3 \\
    \dot{x}_3 &= a_1x_1 + a_2x_2 + a_3x_3 + u + d_n + d
\end{aligned}
\]

where \( x_1 = y \), \( d_n \) is the feedforward compensation error, \( d \) is the external disturbance. Both of them are bounded disturbances.

\[
\begin{aligned}
    |d_n| &\leq \tilde{N} \\
    |d| &\leq D
\end{aligned}
\]

where \( \tilde{N} \) and \( D \) are the infinites of nonlinear error and interference.

3.3. Sliding mode controller

The coefficients \( a_1, a_2, a_3 \) can be obtained as follows:

\[
\begin{aligned}
    a_1 &= RCP + \rho x^2 \\
    a_2 &= RCb + \rho x^2 \\
    a_3 &= RCk + b
\end{aligned}
\]

The system tracking error is defined by

\[
\begin{aligned}
    z_1 &= x_1 - x_d \\
    z_2 &= x_2 - a_1 \\
    \dot{z}_3 &= x_3 - a_2
\end{aligned}
\]

where \( x_d \) is the expected value, \( a_1 \) and \( a_2 \) are the control laws to be selected, and the derivative of \( z_1 \) can be written as follows:

\[
\dot{z}_1 = \dot{x}_1 - \dot{x}_d = x_2 - a_1 - \dot{x}_d
\]

Choosing the Lyapunov function:

\[ V_1 = \frac{1}{2} z_1^2 \]

the derivative of Eq. (26) can be obtained as follows:

\[ \dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 + a_2 - \dot{x}_d) \]

Choosing \( a_1 = -c_1z_1 + \dot{x}_d \), \( c_1 > 0 \).

\[ \dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 + c_1z_1 + \dot{x}_d - \dot{x}_d) = -c_1z_1^2 + z_1z_2 \]

the derivative of \( z_2 \) can be expressed as:

\[ \dot{z}_2 = z_2 - a_1 = x_1 - a_1 \]

choosing the Lyapunov function:

\[ V_2 = \frac{1}{2} z_2^2 \]

the derivative of Eq. (30) can be obtained as follows:

\[ \dot{V}_2 = z_2 \dot{z}_2 = z_2 (x_1 - \dot{a}_1) = z_2 (z_3 + a_2 - \dot{x}_d) \]

Let \( a_2 = -c_2z_2 - z_1 + \dot{a}_1 \), \( c_2 > 0 \). Eq. (31) can be rewritten as:

\[ \dot{V}_2 = z_2 (a_2 - \dot{a}_1) = -c_2z_2^2 - z_1z_2 + z_2z_3 \]

the derivative of \( z_3 \) can be expressed as:

\[ \dot{z}_3 = \dot{x}_1 - \dot{a}_2 = a_1x_1 + a_2x_2 + a_3x_3 + u + d_n + d - \dot{a}_2 \]

Define the sliding surface of the system:

\[ s = k_1z_1 + k_2z_1 + \dot{z}_3 \]

where \( k_1 > 0 \) and \( k_2 > 0 \) are constants. The condition that the system reaches the sliding surface \( \dot{s} = 0 \), choose the Lyapunov function:

\[ V_3 = \frac{1}{2} \dot{s}^2 \]

The derivative of Eq. (35) can be obtained

\[ \dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - k_2z_2^2 + s(k_1x_1 - k_2\dot{a}_1 + \dot{z}_3 - \dot{a}_2 + x_2 - a_1) = -c_1z_1^2 - c_2z_2^2 - k_2z_2^2 + s(k_1x_1 - k_2\dot{a}_1 + a_1x_1 + a_2x_2 + \ldots + a_3x_3 + u + d_n + d - \dot{a}_2 + x_2 - a_1) \]

3.4. Proof of stability

Define \( a_1, a_2, a_3 \) as the estimation errors

\[
\begin{aligned}
    \tilde{a}_1 &= a_1 - \hat{a}_1 \\
    \tilde{a}_2 &= a_2 - \hat{a}_2 \\
    \tilde{a}_3 &= a_3 - \hat{a}_3
\end{aligned}
\]

where \( \hat{a}_1, \hat{a}_2, \hat{a}_3 \) are estimates of the coefficients respec-tively \( a_1, a_2, a_3 \) are estimation errors. All coefficients are bounded estimates. Choose the Lyapunov function:

\[ V = V_1 + \frac{1}{2a_1\tilde{a}_1^2} + \frac{1}{2a_1\tilde{a}_2^2} + \frac{1}{2a_1\tilde{a}_3^2} \]

the derivative of Eq. (38) can be expressed as:

\[ \dot{V} = -c_1\tilde{z}_1^2 - (c_2 + k_1)\tilde{z}_2^2 + s[(a_1 - \hat{a}_1)x_1 + (a_2 + k_1)x_2 + (a_3 + k_1)x_3 + \ldots + u + d_n + d - \dot{a}_2 - k_1\dot{a}_1] + \frac{1}{a_1\tilde{a}_1}[\dot{\tilde{a}}_1(\tilde{a}_1 - \hat{a}_1) + \frac{1}{a_1\tilde{a}_2}][\dot{\tilde{a}}_2(\tilde{a}_2 - \hat{a}_2) + \frac{1}{a_1\tilde{a}_3}][\dot{\tilde{a}}_3(\tilde{a}_3 - \hat{a}_3)] \]

define \( k_{SS} = S\text{sign}(s) - \text{Dsign}(s) + Y \) then have:

\[ Y = \hat{a}_2 + k_1\hat{a}_1 + a_1 \]

\[ \dot{V} = -c_1\tilde{z}_1^2 - (c_2 + k_1)\tilde{z}_2^2 + s[(a_1 - \hat{a}_1)x_1 + (a_2 + k_1)x_2 + (a_3 + k_1)x_3 + \ldots + k_1\text{sign}(s) - \text{Dsign}(s) + s + d + \dot{d}] + \frac{1}{a_1\tilde{a}_1}[\dot{\tilde{a}}_1(\tilde{a}_1 - \hat{a}_1) + \frac{1}{a_1\tilde{a}_2}][\dot{\tilde{a}}_2(\tilde{a}_2 - \hat{a}_2) + \frac{1}{a_1\tilde{a}_3}][\dot{\tilde{a}}_3(\tilde{a}_3 - \hat{a}_3)] \]

selecting the adaptive control law:

\[
\begin{aligned}
    \dot{\tilde{a}}_1 &= s\dot{x}_1x_1 - \hat{a}_1 \\
    \dot{\tilde{a}}_2 &= s\dot{x}_2x_2 + \hat{a}_2 \\
    \dot{\tilde{a}}_3 &= s\dot{x}_3x_3 + \hat{a}_3
\end{aligned}
\]

where \( \dot{N}|s| \geq d_n s \) and Eq. (40) can be rewritten as:

\[ \dot{V} = -c_1\tilde{z}_1^2 - (c_2 + k_1)\tilde{z}_2^2 + k_1\dot{z}^2 + \hat{a}_2 (s\dot{x}_1 - \frac{\dot{a}_1}{a_1\tilde{a}_1}) + \ldots + \hat{a}_2 (s\dot{x}_2 - \frac{\dot{a}_2}{a_1\tilde{a}_2}) + \hat{a}_3 (s\dot{x}_3 - \frac{\dot{a}_3}{a_1\tilde{a}_3}) \]

\[ \dot{\bar{N}}|s| - \dot{\tilde{a}}_1|s| - \dot{\tilde{a}}_2|s| + \dot{d}_n s + d \]

\[ \leq -c_1\tilde{z}_1^2 - (c_2 + k_1)\tilde{z}_2^2 - k_1\dot{z}^2 \leq 0 \]
4. Experiment

4.1. Experiment setup

Piezoelectric ceramic driver is S-330.2SL piezoelectric ceramic driver produced by Physik instrument company. It has the characteristics of maximum angle, equivalent capacity of each axis 3μF and fast response time. The power supply voltage is 100V, the control voltage is 0–100V, and the feedback structure is resistance strain sensor. The voltage corresponding to the deformation position of the current piezoelectric driver is feedback to the circuit. Figure 2 shows the piezoelectric driver control system, mainly divided into control circuit and drive circuit. The control circuit processes the input signal and the drive circuit amplifies the control signal to drive the piezoelectric driver.

4.2. Experiment results

Firstly, the error of the fitting model is tested in the experiment as shown in Figure 3, where \(v(t)\) is expected control input, \(y_c(t)\) is the experimental output, \(y_f(t)\) is output of the model, and \(e(t)\) is error.

The constant amplitude and variable amplitude sinusoidal signals of 10Hz are applied to the system, and the proposed model fitting method is verified. The results are shown in Figure 4. The hysteresis codes used in this work can be seen in the supplementary file code.rar.

In this paper, in addition to the experiment of 10 Hz sinusoidal control signal, other frequency control signals are tested and analyzed. The maximum error and mean square error of the fitting model at other frequencies are shown in Table 2.

Secondly, the constant amplitude and variable amplitude sinusoidal signals of 10Hz are applied to the system to verify the effectiveness of the feedforward linearization compensation method (see Figure 5).

The constant amplitude and variable amplitude sinusoids with 80Hz are applied on the system for evaluating the effectiveness of the proposed control strategy (see Figures 6 and 7 and Tables 3 and 4).

5. Conclusion

In this paper, a MPI model with generalized clearance operator is proposed, and a feedforward compensation controller is designed to reduce the asymmetric hysteresis effect. In order to reduce the compensation error, an adaptive backstepping sliding mode controller is designed. By choosing appropriate Lyapunov function and intermediate dummy variable, the stability of the system is proved. The experimental results show that the linearity error of the system is reduced from 10% to less than 1%, and the tracking error is reduced by 90%.

Declarations

Author contribution statement

Zhiliang Yu: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data.

Yue Wu: Analyzed and interpreted the data; Wrote the paper.

Zhiyi Fang & Hailin Sun: Contributed reagents, materials, analysis tools or data.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

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