Abstract

The nonabelian Berry phase is computed in the $T$ dualized quantum mechanics obtained from the $USp(2k)$ matrix model. Integrating the fermions, we find that each of the spacetime points $X^{(i)}_{\nu}$ is equipped with a pair of $su(2)$ Lie algebra valued pointlike singularities located at a distance $m_{(f)}$ from the orientifold surface. On a four dimensional paraboloid embedded in the five dimensional Euclidean space, these singularities are recognized as the BPST instantons.
Roles of instanton configurations in nonabelian gauge theory have been thoroughly investigated in the literature in various circumstances. More recently, the Yang-Mills instanton has turned out to play an interesting role in (super)string theory as well with the advent of D branes \[1\]. In ref. \[2\], Witten has argued that the zero size limit of the Yang-Mills instanton is identified with the five brane of type I superstring theory. Equivalence of branes and their bound states with gauge theory instantons appears to be a general feature of superstrings \[3\] and should be demonstrated fully with ample examples. In this letter, we will find that a particular formulation of unified superstring theory via the matrix model also exhibits the BPST instanton \[4\]. A notable feature is that this configuration emerges naturally after the integrations of fermions.

There are several matrix models \[5, 6, 7, 8, 9\] proposed for unified superstring theory up to now, which embody the notion of noncommuting coordinates \[10\]. The model we will adopt below is the reduced (i.e. zero-dimensional) model based on the USp Lie algebra descending from Type I superstrings \[8, 9\]. We find that the effective action which depends on the path in the space of spacetime points \(X_M^{(i)}\) (which are dynamical variables) sitting at the diagonal entries of the bosonic matrices contains a coupling to the instanton background. This background emerges from integrations of the fermionic degrees of freedom \[2\]. The nonabelian Berry phase \[13\] (see also \[14, 15\]) plays a decisive role in this phenomenon. The degrees of freedom belonging to the fundamental representation will turn out to give surviving contributions after the cancellations due to the symmetry of the roots and that of the weights are taken into account.

Let us briefly review the construction of the reduced USp(2k) matrix model. The action consists of the three parts and can be written by borrowing the \(d = 4\) superfield notation and dropping the spacetime dependence. We denote by \(S_{\text{vec}}\) the action for the vector multiplet of \(d = 4, \mathcal{N} = 2\) USp(2k) supersymmetric gauge theory and by \(S_{\text{asym,(fund)}}\) that of the hypermultiplet belonging to the antisymmetric (fundamental) representation. (See \[9\] for detail.) The action, which is denoted by \(S_0\) in the absence of \(n_f\) of the fundamental hypermultiplets, is expressible as

\[
S_0 = S_{d=10}^{d=10} \left( \hat{\rho}_{\mu} , \hat{\Psi} , \hat{\Psi} \right) ,
\]

\[
S_{d=10}^{d=10} \left( \hat{\rho}_{\mu} , \hat{\Psi} , \hat{\Psi} \right) = \frac{1}{g^2} Tr \left( \frac{1}{4} \left[ \hat{\rho}_{\mu} , \hat{\rho}_{\nu} \right] \left[ \hat{\rho}_{\mu} , \hat{\rho}_{\nu} \right] - \frac{1}{2} \hat{\Psi} \Gamma^M \left[ \hat{\rho}_{\mu} , \hat{\Psi} \right] \right) .
\]

This action can be understood as the projection from the type IIB matrix model. The attendant projector

\[
\hat{\rho}_{\mu} \equiv \frac{1}{2} \left( \bullet \mp F^{-1} \bullet F \right)
\]

takes any \(U(2k)\) matrix (denoted by a symbol with an underline) into the matrix lying in the adjoint representation of USp(2k) and that in the antisymmetric representation respectively. The symbol \(\hat{\rho}_{\mu}\) is a matrix with Lorentz indices and \(\hat{\rho}_{\mu}\) is a matrix with spinor indices:

\[
\hat{\rho}_{\mu} = \text{diag}(\hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu}, \hat{\rho}_{\mu})
\]

\(^2\) See \[11, 12\] for similar computation.
\[ \hat{\rho}_{fT} = \hat{\rho}_{-1(4)} \otimes \begin{pmatrix} 1(2) & 0 \\ 0 & 1(2) \end{pmatrix} + \hat{\rho}_{+1(4)} \otimes \begin{pmatrix} 0 & 1(2) \\ 1(2) & 0 \end{pmatrix} . \] (4)

The third part \( S_{\text{fund}} \) reads

\[ S_{\text{fund}} = \frac{1}{g^2} \sum_{f=1}^{n_f} \left[ \int d^2 \theta d^2 \bar{\theta} \left( Q_i^v (e^{2\nu})^v_i Q_{(f)j}^i + \bar{Q}_i^{(f)j} (e^{-2\nu})^i_j \bar{Q}_i^{*} \right) \\
+ \left\{ \int d^2 \theta \left( m_i (\bar{Q}_i^{(f)j} Q_{(f)j}^i + \sqrt{2} \bar{Q}_i^{(f)j} (\Phi)^i_j Q_{(f)j}^i \right) + h.c. \right\} \right] , \] (5)

\[ Q_i = Q_i + \sqrt{2} \bar{\theta} \psi_i + \theta \theta F_{Q_i} . \] (6)

This part is designed to create an open string sector.

It has been demonstrated in [8, 9] that the model is uniquely selected by the three requirements: 1) having eight dynamical and eight kinematical supercharges, 2) obtained by an appropriate projection from the \( IIB \) matrix model and an addition of the degrees of freedom corresponding to open strings, 3) nonorientable. The last requirement selects the \( usp \) Lie algebra both from the planar diagram analysis [8] and more clearly from the structure of the large \( k \) Schwinger-Dyson equations or equivalently that of the closed and open loop equations [10]. This latter analysis brings at the same time the \( SO(2n_f) \) Chan-Paton factor coming from the flavor symmetry. Reflecting eq. (4), the classical vacuum is broken by \( Z_2 \) in the six adjoint directions through orientifold projections. The mass \( m_i (\nu) \) denotes the distance of \( D \)-objects from orientifold surfaces, which is the case in the representation of strings as effective worldvolume gauge theory. This will be seen to hold in the present discussion based on matrices as well. Letting \( k \) to infinity enables us to construct matrix \( T \) duality transformation via the recipe of [17]. Via this transformation, the perturbative properties of strings built from matrices can be represented as the worldvolume \( USp \) gauge theories in various dimensions. The consistency with the literature has been checked in some cases [3].

Let us construct the above mentioned effective action which depends on the paths \( \{ \Gamma_A^{(R)} \} \) in the parameter space labelled by the five sets of the adjoint spacetime points

\[ X_\nu = diag(X_\nu^{(1)}, \ldots X_\nu^{(k)}, -X_\nu^{(1)}, \ldots -X_\nu^{(k)}) \quad \nu = 1, 2, 3, 4, 7. \] This can be accomplished by utilizing the representation of the model as \( T \) dualized quantum mechanics [15]. ( We choose \( \nu_0 = 0 \) gauge):

\[ Z \left[ X_\nu; m_i (\nu), \{ \Gamma_A^{(R)} \}, \{ \sigma_i^{(R)} \}, \{ \sigma_j^{(R)} \} \right] = \int [D\bar{v}_M] \prod_{f=1}^{n_f} [DQ_{(f)}] [DQ^*_{(f)}] [D\bar{Q}_{(f)}] [D\bar{Q}^*_{(f)}] e^{iS_B} \]

\[ \lim_{T \to \infty} \langle \Psi; \{ \sigma_i^{(R)} \} | P \exp \left[ -i \int_0^T dt H_{\text{fermion}} (t) \right] | \Psi; \{ \sigma_j^{(R)} \} \rangle . \] (7)

Here \( v_M = X_M + \bar{v}_M \) and \( S^R \) is the path of \( S \) which does not contain any fermion. For simplicity, we have set the dependence of the remaining antisymmetric spacetime points \( X_I = diag(X_I^{(1)}, \ldots X_I^{(k)}, X_I^{(1)}, \ldots X_I^{(k)}) \quad I = 5, 6, 8, 9 \) to zero [8]. Due to this choice, the

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Footnote:

3 This appears to be a valid approximation as worldvolume gauge theories in various dimensions obtained as matrix \( T \) duals from this model have string interpretation on their Coulomb phase. See, for example, [19].
operator $H_{\text{fermion}}$ becomes a direct sum of the three Hamiltonians $H_{\text{fund}}, H_{\text{adj}}$ and $H_{\text{asym}}$ respectively obtained from the fermionic part of $S_{\text{fund}}, S_{\text{adj}}$ and $S_{\text{asym}}$ after T duality. Their $t$ dependence comes from that of $X_\nu$ which acts as external parameters on the Hilbert space of fermions. The quantity (7) was studied in [12], ignoring the degeneracy of the ket vector $|\Psi\rangle$ which is an adiabatic eigenstate of the system. Here we consider a set of degenerate adiabatic eigenstates. The degeneracy of the initial state and that of the final one are respectively specified by a set of labels $\{\sigma_{iA}\}$ and $\{\sigma_{fA}\}$, where the indices $A$ and $(R)$ specify the species of fermions.

Let $R = \text{fund, adj, antisym}$. We denote by $e^{(A)}_{(R)}$ the standard eigenbases belonging to the roots of $sp(2k)$ and the weights of the fundamental representation and those of the antisymmetric representation respectively. Let us expand the two component fermions as

$$\psi^{(R)}_\ell = \sum_A \tilde{b}^{(R)}_A e^{(A)}_{(R)} / \sqrt{2}, \quad \bar{\psi}^{(R)}_\ell = \sum_A \bar{b}^{(R)}_A e^{(A)\dagger}_{(R)} / \sqrt{2},$$

where $N_{(\text{adj})} = 2k^2 + k$, $N_{(\text{antisym})} = 2k^2 - k$ and $N_{(\text{fund})} = 2k$. We find that all of the three Hamiltonians $H_{\text{fund}}, H_{\text{adj}}$ and $H_{\text{asym}}$ are expressible in terms of a generic one

$$g^2 H_0 (X_\ell, \Phi, \Phi^*; (R), A) = -\bar{b}^{(R)}_{A\alpha} \sigma^{\alpha\beta} X_\beta b^{(R)}_{A\beta} - d^{(R)\alpha}_A d^{(R)\beta}_A X_\alpha^\beta + \sqrt{2} \Phi b^{(R)}_A d^{(R)\dagger}_A + \sqrt{2} \Phi^* \bar{b}^{(R)\dagger}_A d^{(R)}_A$$

provided we replace the five parameters

$$X_\ell, \quad \Phi = \frac{X_4 + iX_7}{\sqrt{2}}, \quad \Phi^* = \frac{X_4 - iX_7}{\sqrt{2}} \quad \ell = 1, 2, 3$$

by the appropriate ones. (See argument of $A$ in eq. (31) below.)

The Berry connection appears in one or three particle state of $H_0$ [12] with respect to the Clifford vacuum $|\Omega\rangle$; $b^a |\Omega\rangle = \bar{d}_a |\Omega\rangle = 0$. We suppress the labels $A$ and $(R)$ seen in eqs. (8),(9) for a while. Let us write $|\Psi\rangle = \left(h_\alpha d^\alpha + \bar{h}^\dot{\alpha} \bar{b}_\dot{\alpha}\right) |\Omega\rangle$ and $\Psi \equiv (h_\alpha, \bar{h}^\dot{\alpha})^t$. The transition amplitude of an adiabatic process reads

$$\lim_{T \to \infty} \langle \Psi | P \exp \left[-i \int_0^T dt H_0(t) \right] | \Psi\rangle = \psi^\dagger P \exp \left[-i \int_0^\infty E(t) dt + i \int_\Gamma d\gamma (X_\nu, \Phi, \Phi^*) \right] \psi.$$

Here $\Gamma$ is a closed path in the parameter space. The connection one-form is

$$id\gamma(t) = -\psi^\dagger(t) d\psi(t) \equiv -iA,$$

which is in general matrix-valued. Let us consider for definiteness a set of two degenerate adiabatic eigenstates with positive energy, which is specified by an index $\sigma = 1, 4$. Using the completeness $\sum_{\sigma = 1, 4} \psi_\sigma \psi^\dagger_\sigma = 1_{(2)}$, we find

$$\sum_{\sigma = 1, 4} \lim_{T \to \infty} \langle \Psi_\sigma | P \exp \left[-i \int_0^T dt H_0(t) \right] | \Psi_\sigma\rangle = tr P \exp \left[-i \int_0^\infty E(t) dt - i \int_\Gamma A(X_\nu) \right]$$

4 Complete analysis which includes the antisymmetric spacetime points is now in progress by us [20]. This will further clarify the physical picture of the nonabelian Berry phase in the $USp(2k)$ matrix model.

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from eq. (11). Here the trace is taken with respect to the two-dimensional subspace.

Now the problem is to obtain the nonabelian (su(2) Lie algebra valued) Berry connection associated with the first quantized hamiltonian

\[ H = \frac{R}{g^2} \sum_{\nu=1,2,3,4,7} N^\nu \Gamma_\nu, \ R \equiv \sqrt{(X^1)^2 + (X^2)^2 + (X^3)^2 + (X^4)^2 + (X^7)^2}, \ N^\nu \equiv \frac{X^\nu}{R}, \]  

(14)

where \( \Gamma_\nu \) are the five dimensional gamma matrices obeying the Clifford algebra and the explicit representation can be read off from eq. (9). The projection operators are

\[ P_\pm = \frac{1}{2} (1 \pm N^\nu \Gamma_\nu) , \quad P_\pm^2 = P_\pm , \quad P_\mp = P_\pm , \]  

(15)

which satisfy

\[ H P_\pm = \pm \frac{R}{g^2} P_\pm . \]  

(16)

Denoting by \( \mathbf{e}_i (i = 1, 2, 3, 4) \) the unit vector in the \( i \)-th direction, we write a set of normalized eigenvectors belonging to the plus eigenvalue as

\[ \psi_i = \frac{1}{\mathcal{N}_i} P_+ \mathbf{e}_i . \]  

(17)

Here, \( i = 1, 4 \) refer to the sections around the north pole \( X^3 = R \) while \( i = 2, 3 \) to the ones around the south pole \( X^3 = -R \). The \( \mathcal{N}_i \) are the normalization factors:

\[ \mathcal{N} \equiv \mathcal{N}_1 = \mathcal{N}_4 = \sqrt{\frac{1 + N^3}{2}} , \quad \mathcal{N}' \equiv \mathcal{N}_2 = \mathcal{N}_3 = \sqrt{\frac{1 - N^3}{2}} . \]  

(18)

We focus our attention on the sections near the north pole. The Berry connection is

\[ iA = \left( \begin{array}{c} \psi_1 \\ \psi_4 \end{array} \right) d(\psi_1, \psi_4) = \left( \begin{array}{c} \mathbf{e}_1 \\ \mathbf{e}_4 \end{array} \right) \mathcal{M} (\mathbf{e}_1 \mathbf{e}_4) , \]  

(19)

where \( \mathcal{M} \equiv \frac{1}{\mathcal{N}} P_+^\dagger d\frac{1}{\mathcal{N}} P_+ \). Introducing \( C^{\mu\nu} \equiv (N^\mu dN^\nu - N^\nu dN^\mu) \), we obtain

\[ \mathcal{M} = \frac{1}{1 + N^3} \left( \frac{1}{2} dN^\mu \Gamma_\mu + \frac{1}{4} C^{\mu\nu} \Gamma_\mu \Gamma_\nu - \frac{1}{1 + N^3} dN^3 P_+ \right) . \]  

(20)

Working out each entry of the \( 2 \times 2 \) matrix \( A \), we find, after some calculation,

\[ A(X^i) = \frac{R}{2(R + X^3)} \frac{1}{R^2} \mathbf{B} \cdot \sigma , \]  

(21)

where

\[ \mathbf{B} \equiv \begin{bmatrix} B^1 \\ B^2 \\ B^3 \end{bmatrix} = \begin{bmatrix} X^7 dX^1 - X^1 dX^7 - X^2 dX^4 + X^4 dX^2 \\ X^1 dX^4 - X^4 dX^1 - X^2 dX^7 + X^7 dX^2 \\ X^4 dX^7 - X^7 dX^4 - X^2 dX^1 + X^1 dX^2 \end{bmatrix} . \]  

(22)

Observe that \( R \) and \( X^3 \) appear only in the overall scale factor.
To proceed further, we parametrize \( S^3 \) of unit radius by the coordinates

\[
Y^\nu \equiv \frac{1}{\sqrt{R^2 - (X^3)^2}} X^\nu, \quad (\nu = 1, 2, 4, 7) \tag{23}
\]

\[
(Y^1)^2 + (Y^2)^2 + (Y^4)^2 + (Y^7)^2 = 1 . \tag{24}
\]

We find

\[
\frac{1}{R^2 - (X^3)^2} B = Y \times dY + Y dY^2 - Y^2 dY , \tag{25}
\]

where \( Y \equiv (Y^4, Y^7, Y^1)^t \). The \( Y \) coordinates parametrize the \( SU(2) \) group element as well:

\[
T \equiv Y^2 1_2 + i Y \cdot \sigma , \tag{26}
\]

from which we can make the pure gauge configuration

\[
dTT^{-1} = (dY^2 1_2 + iY \cdot \sigma)(Y^2 1_2 - iY \cdot \sigma) = i(dY \times Y + Y^2 dY - Y dY^2) \cdot \sigma. \tag{27}
\]

Eq. (21) and eq. (25) tell us

\[
A(X^\nu) = p(R, X^3)dTT^{-1} . \tag{28}
\]

The prefactor \( p(R, X^3) \) is of interest and can be written as

\[
p(R, X^3) = \frac{\tau^2}{\tau^2 + \lambda^2} , \quad \tau = \sqrt{R^2 - (X^3)^2} , \quad \lambda = R + X^3. \tag{29}
\]

The nonabelian connection \( A \) is in fact the instanton configuration of Belavin et. al \[4\]. The size of the instanton \( \lambda \) is not a \textit{bonafide} parameter in the model but is chosen to be the fifth coordinate in the five dimensional Euclidean space. For fixed \( \lambda \), the four dimensional subspace embedded into the \( R^5 \) is a paraboloid wrapping the singularity. An observer on this recognizes the pointlike singularity as BPST instanton. As \( \lambda \) goes to zero, this paraboloid gets degenerated into an \( SU(2) \) counterpart of the Dirac string connecting the origin and the infinity. Note also that the prefactor is written in terms of the angle measured from the north pole as

\[
p(\theta) = \frac{1}{2}(1 - \cos \theta) , \quad N^3 \equiv R \cos \theta . \tag{30}
\]

Returning to the expression (4) and taking a sum over the labels \( \sigma^{(R)}_{fA} = \sigma^{(R)}_{iA} \), we find that the second line is expressible as the product of the factors

\[
TrP \exp \left(-i \sum_{f=1}^{n_f} \sum_{A=1}^{2k} 1 \otimes \cdots \int_{A(f)}^{A(f,und)} A \left[ w^A \cdot X, \frac{m(f)}{\sqrt{2}} + w^A \cdot \Phi, \frac{m(f)}{\sqrt{2}} + w^A \cdot \Phi^\dagger \right] \cdots \otimes 1 \right)
\]

\[
TrP \exp \left(-i \sum_{A=1}^{2k^2-2k} 1 \otimes \cdots \int_{A}^{A(adj)} A \left[ R^A \cdot X, iR^A \cdot \Phi, iR^A \cdot \Phi^\dagger \right] \cdots \otimes 1 \right)
\]

\[
TrP \exp \left(-i \sum_{A=1}^{2k^2-2k} 1 \otimes \cdots \int_{A}^{A(adj)} A \left[ w^A_{asym} \cdot X, w^A_{asym} \cdot \Phi, w^A_{asym} \cdot \Phi^\dagger \right] \cdots \otimes 1 \right) . \tag{31}
\]
We have included the energy dependence seen in eq. (13) in $S_B$ as this is perturbatively cancelled by the contribution from bosonic integration. The symbols seen in the arguments are

$$\begin{align*}
\{w^A | 1 \leq A \leq 2k\} &= \{\pm e^{(i)}, 1 \leq i \leq k\} \\
\{R^A | 1 \leq A \leq 2k^2\} &= \{\pm 2e^{(i)}, e^{(i)} - e^{(j)}, \pm (e^{(i)} + e^{(j)}) | 1 \leq i, j, \leq k\} \\
\{w^A_{\text{asym}} | 1 \leq A \leq 2k^2 - 2k\} &= \{\pm (e^{(i)} + e^{(j)}), e^{(i)} - e^{(j)}, 1 \leq i, j, \leq k\}.
\end{align*}$$

The second and the third lines are respectively the nonzero roots and the weights in the antisymmetric representation of $usp(2k)$. We have denoted by $e^{(i)} (1 \leq i \leq k)$ the orthonormal basis vectors of $k$-dimensional Euclidean space and

$$X_\ell = \sum_{i=1}^k e^{(i)} X_\ell^{(i)}, \quad \Phi = \sum_{i=1}^k e^{(i)} \frac{X_\ell^{(i)} + iX_\ell^{(i)}}{\sqrt{2}}, \quad \Phi^\dagger = \sum_{i=1}^k e^{(i)} \frac{X_\ell^{(i)} - iX_\ell^{(i)}}{\sqrt{2}}. \quad (33)$$

Unlike the case of the abelian Berry phase examined in [12], the second or the third line of eq. (31) do not quite collapse to unity. Let us, however, exploit the symmetry of the roots and the weights under $e^{(i)} \leftrightarrow -e^{(i)}$. Observe that, due to this symmetry, we can pair the two-dimensional vector space associated with $R^A$ (or $w^A_{\text{asym}}$) and that with $-R^A$ (or $-w^A_{\text{asym}}$). Let us symmetrize the tensor product of these two two-dimensional vector spaces. On this, the nonabelian Berry phase is reduced to the pure gauge configuration

$$\mathcal{A}(X^\nu)_{\{i \overline{\delta} \ell \}}^{\{j \delta \ell \}} + \mathcal{A}(-X^\nu)_{\{i \overline{\delta} \ell \}}^{\{j \delta \ell \}} = (dTT^{-1})_{\{i \overline{\delta} \ell \}}^{\{j \delta \ell \}}, \quad (34)$$

and this can be gauged away. As for the first line of eq. (31), the mass terms prevent this from happening.

After all these operations, eq. (31) becomes

$$TrP \exp \left( -i \sum_{f=1}^{n_f} \sum_{A=1}^{2k} 1 \otimes \cdots \int_{X_{unid}} A \left[ w^A \cdot X_\ell, \frac{m(f)}{\sqrt{2}} + w^A \cdot \Phi, \frac{m(f)}{\sqrt{2}} + w^A \cdot \Phi^\dagger \right] \cdots \otimes 1 \right). \quad (35)$$

Note that in the antisymmetrized part of eq. (31) the nonabelian Berry phase is present generically in all three lines. It is present in the $IIB$ matrix case as well.

We have a collection of pairs of Yang-Mills instanton configurations which preserve a fraction of supersymmetries and which will presumably take their worldvolumes in four of the antisymmetric directions representing $R^4$ and time. The natural interpretation along the line of [2] and [3] is that we have a collection of pairs of $D0 - D4$ bound state system. This point will become clear in the full-fledged calculation which includes the antisymmetric spacetime points [20]. Each pair belongs to a different spacetime point $X_\ell^{(i)}$ whose quantum mechanical average over all $i$ predict spacetime properties such as the size of the universe. In the classical consideration, they are located at a distance $m(f)$ away from the orientifold plane. This of course conforms to the picture already developed in [2, 19]. A difference from the case of [2] is that we have an observer dependent size of the instanton. It is more appropriate, however, to view this configuration as a pointlike nonabelian Yang monopole [21] in the five dimensional space. This will be an eventual picture identified with D-branes.
and their bound states. Another notable feature of our instanton/Yang monopole is that the $su(2)$ index originates from the spinor index of the fermions. These points certainly deserve further studies.

The authors thank Toshihiro Matsuo, Toshio Nakatsu and Asato Tsuchiya for helpful discussion on this subject.
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