Generalized Heavy-to-Light Form Factors in Light-Cone Sum Rules

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We study the form factors for a heavy meson into the S-wave $K\pi/\pi\pi$ system with an invariant mass below 1 GeV. The mesonic final state interactions are described in terms of the scalar form factors, which are obtained from unitarized chiral perturbation theory. Employing generalized light-cone distribution amplitudes, we compute the heavy-to-light transition using light-cone sum rules. Our approach simultaneously respects constraints from analyticity and unitarity, and also takes advantage of the power expansion in the $1/m_b$ and the strong coupling constant.

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Introduction – $B$ decays into a light vector meson are of particular interest as they can provide valuable information to extract the Standard Model (SM) parameters and therefore test the SM. In the case that large deviations from the SM calculations are found, these will shed light on new physics scenarios. Examples for such type of decays include e.g. the process $B \to \rho(\to \pi\pi)l\bar{\nu}$ for the extraction of the CKM matrix element $|V_{ub}|$, the reaction $B \to K^*(\to K\pi)l^+l^-$ to test the chirality structure in weak interaction, and the decay $B_s \to J/\psi\phi(\to K\bar{K})$ to determine the $B_s - \bar{B}_s$ mixing phase. Recent experimental data on these channels can be found in Refs. [1–4].

Due to the short lifetime, the light vector meson can not be directly detected by experiments and must be reconstructed from the two or three pseudo-scalars $\pi/K$ final state. Thus these decay modes are at least four-body processes and the semi-leptonic ones are refereed to as $B_{4\ell}$ decays in the literature [5] (for a recent dispersion theoretical approach to this reaction, see Ref. [6]). To select candidate events and suppress the combinatorial background, experimentalists often implement kinematic cuts on the invariant mass. During this procedure various partial waves of the $K\pi/\pi\pi$ system may get entangled and bring dilutions to physical observables. Particularly it is very likely the S-wave contributions are of great importance [7–29]. Therefore it is mandatory to have reliable and accurate predictions considering the high precision achieved or to be achieved by experiments.

Decay amplitudes for semi-leptonic $B$ decays into two light-pseudoscalar mesons show two distinctive features. On the one hand, the final state interaction of the two pseudo-scalars should satisfy unitarity and analyticity. On the other hand, the $b$ mass scale is much higher than the hadronic scale, which allows an expansion of the hard-scattering kernels in terms of the strong coupling constant and the dimensionless power-scaling parameter $\Lambda_{QCD}/m_b$. In this paper, we aim to develop a formalism that makes use of both these advantages. It simultaneously combines the perturbation theory at the $m_b$ scale based on the operator product expansion and the low-energy effective theory inspired by the chiral symmetry to describe the S-wave $\pi\pi$ and $K\pi$ scattering. For concreteness, we will choose the $B \to K\pi$ matrix elements with the $K\pi$ invariant mass below
1 GeV as an example in the following, while other processes including the charm meson decay can be treated in an analogous way. If the factorisation can be proved, these form factors will also play an important role in the study of charmless three-body $B$ decays \cite{30–33}.

**Generalized form factor** – The matrix elements

\[
\langle (K\pi)_0(p_{K\pi})|\bar{s}\gamma_\mu\gamma_5 b|B(p_B)\rangle = -i \frac{1}{m_{K\pi}} \left\{ P_\mu - \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu \right\} F_{i1}^{B\rightarrow K\pi}(m_{K\pi}^2, q^2)
+ \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu F_{i0}^{B\rightarrow K\pi}(m_{K\pi}^2, q^2),
\]

\[
\langle (K\pi)_0(p_{K\pi})|\bar{s}\sigma_{\mu\nu} q^\nu b|B(p_B)\rangle = -\frac{F_{iT}^{B\rightarrow K\pi}(m_{K\pi}^2, q^2)}{m_{K\pi}(m_B + m_{K\pi})} \left[ q^2 P_\mu - (m_B^2 - m_{K\pi}^2) q_\mu \right],
\]

(1)

define the S-wave generalized form factors $F_i$ \cite{16}. Here, $P = p_B + p_{K\pi}$ and $q = p_B - p_{K\pi}$.

The $K\pi$ system with invariant mass below 1 GeV can be treated as a light hadron and more explicitly in the kinematics region we are considering, the $m_{K\pi}$ is small and the $K\pi$ system moves very fast, the soft-collinear effective theory (SCET) is applicable \cite{34–37}. As shown later this $K\pi$ system has similar light-cone distribution amplitudes with the ones for a light hadron. The transition matrix elements for $B \rightarrow K\pi$ may be factorized in the same way as the ordinary $B$-to-light ones like the $B \rightarrow \pi$ transition. It has been demonstrated in SCET that, in the soft contribution limit, the form factors obey factorization \cite{37–39}:

\[
F_i = C_i \xi(q^2) + \Delta F_i,
\]

(2)

where $C_i$ are the short-distance and calculable functions, and $\xi$ is a universal soft form factor from the large recoil symmetry in the heavy quark $m_b \rightarrow \infty$ and large energy $E \rightarrow \infty$ limit \cite{10}. Symmetry breaking terms, starting at order $\alpha_s$, can be encoded into $\Delta F_i$, and can be expressed as a convolution in terms of the LCDA \cite{37–39, 41, 42}.

Watson’s theorem implies that phases measured in the $K\pi$ elastic scattering and in a decay channel where the $K\pi$ system decouple with other hadrons are equal (modulo $\pi$ radians). This leads to

\[
\langle (K\pi)_0|\bar{s}\Gamma b|B\rangle \propto F_{K\pi}(m_{K\pi}^2),
\]

(3)

where the strangeness-changing scalar form factors are defined by

\[
\langle 0|\bar{s}d|K\pi\rangle = C_X B_0 F_{K\pi}(m_{K\pi}^2) .
\]

(4)

$C_X$ is an isospin factor and $B_0$ is proportional to the QCD condensate parameter. For the $K^{-}\pi^+$, $C_X = 1$. Below the $K + 3\pi$ threshold, about 911 MeV, the $K\pi$ scattering is strictly elastic. The inelastic contributions in the $K\pi$ scattering comes from the $K + 3\pi$ or $K\eta$. In the region from 911 MeV to 1 GeV, the $K + 3\pi$ channel has a limited phase space, and thus is generically suppressed. Moreover, as a process-dependent study, it has been demonstrated the states with two additional pions will not give sizeable contributions to physical observables \cite{43}. Though differences may be expected, some similarities might be shared. We leave the $K + 3\pi$ contributions for future work.
The $K\eta$ coupled-channel effects can be included in the unitarized approach of chiral perturbation theory \cite{44–48}.

In the following we will choose the light-cone sum rules (LCSR) to calculate the $F_i$. An analysis in other approaches like the $k_T$ factorisation \cite{49–53} would be similar, and for recent developments in this approach see Refs. \cite{54–62}. As a reconciliation of the original QCD sum rule approach \cite{63,64} and the application of perturbation theory to hard processes, LCSR exhibit several advantages in the calculation of quantities like the meson form factors \cite{65–69}. In the hard scattering region the operator product expansion (OPE) near the light-cone is applicable. Based on the light-cone OPE, form factors are expressed as a convolution of light-cone distribution amplitudes (LCDA) with a perturbatively calculable hard kernel. The leading twist and a few sub-leading twist LCDA give the dominant contribution, while higher twist terms are suppressed.

The calculation begins with the correlation function:

$$\Pi(p_{K\pi}, q) = i \int d^4 x e^{i q \cdot x} \langle (K\pi)_0(p_{K\pi}) | T \{ j_{\Gamma_1}(x), j_{\Gamma_2}(0) \} | 0 \rangle,$$

where $j_{\Gamma_1}$ is one of the currents in Eq. (1) defining the form factors: $j_{\Gamma_1} = \bar{s} \gamma_\mu \gamma_5 b$ for $F_1$ and $F_0$, and $j_{\Gamma_1} = \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b$ for $F_T$. We choose $j_{\Gamma_2} = b i \gamma_5 d$ to interpolate the $B$ meson, whose matrix element gives the decay constant $f_B$:

$$\langle B(p_B) | b i \gamma_5 d | 0 \rangle = \frac{m_B^2}{m_B + m_d} f_B.$$

The hadronic representation of the correlation function consists in the contribution of the $B$ meson and of the higher resonances and the continuum state:

$$\Pi^{\text{HAD}}(p_{K\pi}, q) = \frac{\langle (K\pi)_0(p_{K\pi}) | j_{\Gamma_1} | B(p_{B} + q) \rangle \langle B(p_{B} + q) | j_{\Gamma_2} | 0 \rangle}{m_B^2 - (p_{K\pi} + q)^2}$$

$$+ \int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_{K\pi} + q)^2},$$

where higher resonances and the continuum of states are described in terms of the spectral function $\rho^h(s, q^2)$ and start from the threshold $s_0$.

The correlation function in Eq. (5) can also be evaluated in the deep Euclidean region in QCD at the quark level. The quark-hadron duality guarantees the equality of the two calculations and thus we obtain the sum rules

$$\langle (K\pi)_0(p_{K\pi}) | j_{\Gamma_1} | B(p_B) \rangle \langle B(p_B) | j_{\Gamma_2} | 0 \rangle \exp \left[ -\frac{m_B^2}{M^2} \right] = \frac{1}{\pi} \int_{(m_b + m_d)^2}^{s_0} ds \exp[-s/M^2] \text{Im}\Pi^{\text{QCD}}(s, q^2).$$

In the above, a Borel transformation has been performed to improve the convergence of the OPE series, and to enhance the contribution of the low-lying states to the correlation function for suitably chosen values of $M^2$.

The calculation of $\Pi^{\text{QCD}}$ is based on the expansion of the T-product in the correlation function near the light-cone, which produces matrix elements of non-local quark-gluon operators. These
quantities are in terms of the generalized LCDA of increasing twist [71–74]:

\[ \langle (K\pi)_0 | \bar s(x) \gamma_\mu d(0) | 0 \rangle = N p_{K\pi} \frac{1}{m_{K\pi}} \int_0^1 du e^{ip_{K\pi} x} \Phi_{K\pi}(u), \]

\[ \langle (K\pi)_0 | \bar s(x) d(0) | 0 \rangle = N \int_0^1 du e^{ip_{K\pi} x} \Phi_{K\pi}^s(u), \]

\[ \langle (K\pi)_0 | \bar s(x) \sigma_{\mu\nu} d(0) | 0 \rangle = -N \frac{1}{6} (p_{K\pi} \mu x_\nu - p_{K\pi} \nu x_\mu) \int_0^1 du e^{ip_{K\pi} x} \Phi_{K\pi}^\sigma(u), \]  

where \( N = C X B_0 F_{K\pi} \). Due to the Watson’s theorem, the above matrix elements are proportional to the \( K\pi \) scalar form factors which have been absorbed into the normalisation constant \( N \). As a result, the distribution amplitudes, \( \Phi_{K\pi} \) and \( \Phi_{K\pi}^s, \Phi_{K\pi}^\sigma \), are real.

The LCDA \( \Phi_{K\pi} \) is twist-2, and the other two are twist-3. Their normalisations are given as

\[ \int_0^1 du \Phi_{K\pi}(u) = \frac{m_s - m_d}{m_{K\pi}}, \]

\[ \int_0^1 du \Phi_{K\pi}^s(u) = \int_0^1 du \Phi_{K\pi}^\sigma(u) = 1. \]  

The use of conformal symmetry in QCD [70] indicates that the twist-3 LCDA have the asymptotic form [71–74]:

\[ \Phi_{K\pi}^s(u) = 1, \]

\[ \Phi_{K\pi}^\sigma(u) = 6u(1-u), \]  

and the twist-2 LCDA can be expanded in terms of Gegenbauer moments:

\[ \Phi_{K\pi}(u) = 6u(1-u) \sum_n a_n C_{3/2}^n(2u - 1). \]  

It is worthwhile to stress that these generalized LCDA for a two-hadron system have the same form as the ones for a light meson [71–74].

Results – For the sake of presentation, we define

\[ F_i(q^2, m_{K\pi}^2) = C X B_0 m_{K\pi} F_{K\pi}(m_{K\pi}^2) \bar F_i(m_{K\pi}^2, q^2), \]  

with the expressions

\[ \bar F_+ = N_F \left\{ \int_0^1 \frac{du}{u} \exp \left[ -\frac{m_b^2 + u m_{K\pi}^2 - \bar u q^2}{u M^2} \right] \left[ -m_b \Phi_{K\pi}(u) + u m_{K\pi} \Phi_{K\pi}^s(u) + \frac{1}{3} m_{K\pi} \Phi_{K\pi}^\sigma(u) \right] + \exp \left[ -s_0/M^2 \right] \frac{m_{K\pi} \Phi_{K\pi}(u)}{6} \frac{m_b^2 - u_0^2 m_{K\pi}^2 + q^2}{m_b^2 + u_0^2 m_{K\pi}^2 - q^2} \right\}, \]  

(14)
\[
\bar{F}_- = N_F \left\{ \int_{u_0}^1 \frac{du}{u} \exp \left[ \frac{-m_b^2 + u\bar{u}m_K^2 - \bar{u}q^2}{m_K^2} \right] \left[ m_b \Phi_{K\pi}(u) + (2-u)m_K^2 \Phi_{K\pi}^*(u) \right] + \frac{1-u}{3u} m_K^2 \Phi_{K\pi}(u) \right. \\
- \frac{u_0(m_b^2 + q^2 - u_0^2m_K^2)}{u_0(m_b^2 + u_0^2m_K^2 - q^2)} \left. \exp\left[\frac{-s_0/M^2}{m_K^2}\Phi_{K\pi}(u_0)\right] \right\}, \\
\]

\[
\bar{F}_T = 2N_F(m_B + m_K) \left\{ \int_{u_0}^1 \frac{du}{u} \exp \left[ \frac{-m_b^2 + u\bar{u}m_K^2}{m_K^2} \right] \left[ -\frac{\Phi_{K\pi}(u)}{2} + m_b \frac{m_K^2 \Phi_{K\pi}^*(u)}{6uM^2} \right] \right. \\
+ m_b \frac{m_K^2 \Phi_{K\pi}^*(u_0)}{6} \exp\left[-\frac{s_0}{M^2}\right] \right\},
\]

where

\[
N_F = \frac{m_b + m_s}{2m_Bf_B} \exp\left[\frac{m_B^2}{M^2}\right], \\
u_0 = \frac{m_K^2 + q^2 - s_0 + \sqrt{(m_K^2 + q^2 - s_0)^2 + 4m_K^2(m_b^2 - q^2)}}{2m_K^2}. 
\]

Our formulae can be compared to the results for the $B$ to a scalar $\bar{q}q$ meson transition. Quantities including the invariant mass and LCDA for the $K\pi$ system will be replaced by those for the scalar $\bar{q}q$ resonance as in Ref. [10, 75].

![Graph](image)

**FIG. 1:** Scalar $K\pi$ form factors calculated in unitarized chiral perturbation theory. Solid, dashed and dotted lines correspond to the magnitude, the real and the imaginary part, in order.

The scalar form factor $F_{K\pi}$ has been calculated in the unitarized approach embedded in the chiral perturbation theory, and we refer the reader to Ref. [16] for details. We quote these results displayed in Fig. [1] where the solid, dashed and dotted lines correspond to the magnitude, the real and the imaginary part of $F_{K\pi}$, respectively. From this figure, we can see the imaginary part shows...
an approximate linear dependence on $m_{K\pi}^2$. Such behaviour can be derived from the calculation in chiral perturbation theory and we quote the next-to-leading order results [76]:

$$F_{K\pi}^\chi(s) = 1 + \frac{4L_5^r s}{f^2} + \frac{s}{4\Delta_{K\pi}} (5\mu_\pi - 2\mu_K - 3\mu_{\eta_8}) + \bar{J}_{K\pi,K\pi} + \frac{1}{3} \bar{J}_{K\eta_8,KK\eta_8},$$  \hspace{1cm} (18)

where $L_5^r$ is a low energy constant, and

$$K_{K\pi,K\pi} = -\frac{1}{8f^2} \left( 2\Sigma - 5s + \frac{3\Delta_{K\pi}^2}{s} \right), \quad K_{K\eta_8,K\pi} = -\frac{1}{8f^2} \left( 3s - 2\Sigma - \frac{\Delta_{K\pi}^2}{s} \right),$$

$$\mu_i = \frac{M_i^2}{32\pi^2 f^2} \log \left( \frac{M_i^2}{\mu^2} \right),$$

$$\bar{J} = \frac{1}{32\pi^2} \left[ 2 + \left( \frac{M_1^2 - M_2^2}{s} - \frac{M_1^2 + M_2^2}{M_1^2 - M_2^2} \right) \log \frac{M_2^2}{M_1^2} 
- \lambda(s) \left( \log(s + \lambda(s) + M_1^2 - M_2^2) + \log(s + \lambda(s) - M_1^2 + M_2^2) 
- \log(-s + \lambda(s) - M_1^2 + M_2^2) - \log(-s + \lambda(s) + M_1^2 - M_2^2) \right) \right],$$  \hspace{1cm} (19)

and $\Sigma = M_1^2 + M_2^2$, $\Delta_{K\pi} = M_{K\pi}^2 - M_2^2$. $f$ is the pion decay constant, $f = 92.4$ MeV, $\lambda^2(s) = [s - (M_1 + M_2)^2][s - (M_1 + M_2)^2]$, and $s \equiv s + i\epsilon$ ensures that the correct sheet of the logarithm is set. The imaginary part of the scalar form factor $F_{K\pi}^\chi$ arises from the function $\bar{J}$:

$$\text{Im}[\bar{J}] = \frac{1}{16\pi} \frac{\lambda(s)}{s},$$  \hspace{1cm} (20)

which leads to an approximate linear dependence on the $m_{K\pi}^2$ below 1 GeV$^2$. However, this linear dependence disappears in the region with large $m_{K\pi}^2$ since higher-order contributions become important and are taken into account in the unitarized approach. This has been discussed in detail in Ref. [16].

The $B$ meson decay constant is taken from the Lattice QCD calculation of ref. [77]: $f_B = (196.9 \pm 8.9)$ MeV. As demonstrated above, one of the most key inputs is the two-hadron LCDA. We will use asymptotic forms for the twist-3 ones, but no knowledge on the twist-2 is available at present. In Ref. [78], the authors have studied the LCDA for the light scalar mesons below 1 GeV in the $q\bar{q}$ scenario. We shall use these results in our numerical calculation, bearing in mind large uncertainties that may be introduced by this approximation. To the best of our knowledge, there is no available results on non-asymptotic twist-3 LCDA for scalar mesons below 1 GeV. Studies of LCDA for scalar mesons above 1 GeV can be found in Refs. [79][81], but these results are not applicable here due to the large differences in the (invariant) mass. In the future, we hope the situation can be improved using nonperturbative QCD tools including Lattice QCD simulations.

It is interesting to notice that $F_i$ can also be evaluated with other interpolating currents. One example is the chiral current [81][82], which has the advantage of isolating different contributions by twists. In this framework, choosing the suitable current, one can completely smear out the uncertain twist-2 LCDA in the QCD calculation, with the price of the complex hadronic representation since the parity partner of the $B$ meson also contributes to the same correlation function.
The criteria in LCSR to find sets of parameters $M^2$ (the Borel parameter) and $s_0$ (the continuum threshold) is that the resulting form factor does not depend much on the precise values of these parameters; additionally both the continuum contribution, the dispersive integral from $s_0$ to $\infty$ in Eq. (7), and the higher power corrections, arising from the neglected higher twist LCDA, should not be significant. One more requirement on the $s_0$ is that it should not be too much away from the “reasonable” value: $s_0$ is to separate the ground state from higher mass contributions, and thus should be below the next known resonance, in this case, $B_1$ with $J^P = 1^+$. Thus approximately this parameter should be close to $33$ GeV$^2$ [83]. Studies of ordinary heavy-to-light form factors in LCSR, see for instance Ref. [84], also suggested a similar result, ranging from $33$ GeV$^2$ to $36$ GeV$^2$, while some bigger values are derived in the recent update of $B \to \pi$ form factor in LCSR [85].

Numerical results based on LCSR for the auxiliary function $F_1$ at the $K\pi$ threshold $m_{K\pi} = m_K + m_\pi$ are given in Fig. 2, where the dependence of the form factor $F_1$ (left panel) and the continuum/total ratio (right panel) on the Borel parameter are shown. The continuum contribution to the form factors is obtained by invoking the quark-hadron duality above the threshold $s_0$ and calculating the correlation function on the QCD side. Solid lines denote the central value while the dashed curves correspond to variations of threshold parameter: $s_0 = (34 \pm 2)$ GeV$^2$. From this figure, we can see that results for $F_1$ are stable against the variation of $M^2$ when $M^2 > 6$ GeV$^2$, and meanwhile the continuum contribution is typically smaller than 30%. Unfortunately, due to the lack of knowledge on the 3-particle twist-3 and higher twist generalized LCDA, we are unable to estimate the power corrections due to these LCDA, and we hope this situation can be improved with more dedicated studies in the future.
Choosing the value $M^2 = 8\text{GeV}^2$, we show the results in Fig. 3 for the dependence on the squared invariant mass of the $K\pi$ system and the squared momentum transfer $q^2$. As we can see, the results increase with the $q^2$. This behaviour is similar to the $B \to \pi$ [85] and $B \to \rho$ [84] form factors. More results and phenomenological consequences will be published elsewhere.

Conclusions – We have formulated an approach to explore the S-wave generalized form factors for the heavy meson transitions into the $\pi\pi, K\pi$ final state. We have adopted unitarized chiral perturbation theory to account for the final state interactions, and include these effects in the scalar form factors and generalized light-cone distribution amplitudes. The heavy-to-light transition is calculated within QCD sum rules on the light-cone. Our approach simultaneously respects constraints from unitarity and analyticity, and also takes advantage of the power expansion in the $1/m_b$ and the strong coupling constant. With these form factors at hand based on improved results on the generalized LCDA, one may reliably explore the S-wave effects in semi-leptonic heavy meson decays and further non-leptonic charmless three-body $B$ processes if the factorization holds.

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