Combination of LMS Algorithm in Adaptive Noise Cancellation System

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Abstract. In the adaptive noise cancellation system, the traditional fixed step size LMS algorithm is used to filter the noise signal, but the noise signal cannot be completely cancelled out. In order to realize the two-stage filtering, this paper proposes to combine two fixed step size LMS algorithms and to combine a fixed step size LMS algorithm with a variable step size LMS algorithm based on Bessel function. The combinations of different LMS algorithms are applied to the adaptive noise cancellation system, and the MATLAB simulation experiment is performed on the noisy signal. Theoretical analysis and simulation results show that the two-stage filtering realized by combination of different LMS algorithms is obviously better than the one-stage filtering realized by a single fixed step LMS algorithm, and the performance in terms of fast convergence speed, tracking speed and steady state error is improved.

1. Introduction
The most commonly used algorithm in adaptive filters is the fixed step size least mean square (LMS) algorithm, which was proposed by Hoff and Widrow in 1960 [1]. The fixed step size LMS algorithm is based on the minimum mean square error criterion and the gradient algorithm. It improves the estimation calculation of the mean square error gradient and takes the gradient of a single error sample square as the estimation value of the mean square error gradient [1]. The LMS algorithm has the advantages of low computational complexity and easy implementation, while the disadvantages of this algorithm is that the convergence speed is slow, related to the statistical characteristics of the input signal. The step size of the algorithm is determined, but the step size selection is uncertain. If the step size is too large, the convergence speed becomes faster, however, the stability is not good at this time; if the step size is too small, the convergence speed is slow, while the stability is good at this time. Therefore, the selection of step size in the fixed step size LMS algorithm is a very difficult problem [1,2]. In order to solve the contradiction among the step size factor, convergence speed, tracking speed and steady state error, various variable step size LMS algorithms have been proposed [3,4]. However, these variable step size LMS algorithms are only applied to the one-stage adaptive filter, and the noise in the adaptive noise cancellation system cannot be completely cancelled out. Therefore, this paper proposes to combine different LMS algorithms to achieve two-stage filtering, which include a combination of two fixed step size LMS algorithms as well as a combination of a fixed step size LMS algorithm and a variable-step-length LMS algorithm based on Bessel function to form a two-stage
adaptive filter. The signal polluted by noise in the adaptive noise cancellation system is calculated with reference signal to eliminate the noise signal, thereby obtaining a useful signal. Simulation results show that the two-stage filtering realized by combination of different LMS algorithms is obviously better than the one-stage filtering realized by a single fixed step LMS algorithm. It can achieve faster convergence speed, less misalignment and better tracking performance.

2. Basic principle of secondary filter
The two-stage filter is a filter bank to combine adaptive filters of different algorithms, so as to perform some functions required [5,6]. Assume that the filter bank is composed of two adaptive filters F1 and F2, each filter can use different algorithms or the same algorithms to set different parameters according to actual needs. The schematic diagram of the two-stage adaptive filter composed of different LMS algorithms is shown in Fig. 1.

The basic idea of multi-stage filter combination design can be obtained from Fig. 1, that is, the output signal of the previous filter can be used as the input signal of the next filter, and different filters can select different algorithms or the same algorithm. In this paper, the combination of two fixed step size LMS algorithms as well as the combination of a fixed step size LMS algorithm and a variable step size LMS algorithm based on Bessel function is adopted to realize the adaptive filtering of noise signals. The results show that the two-stage filter can get better filtering effect than the single filter.

3. Adaptive Algorithm
Least mean square algorithm [7~9]:

Iterative formula of the fixed step size LMS algorithm:
Filtering output:

\[ y(n) = w^T(n) * x(n) \]

Error signal:

\[ e(n) = d(n) - y(n) \]

Formula of weight vector update:

\[ w(n+1) = w(n) + 2 \mu e(n) * x(n) \]

Where: \( w(n) \) is the weight vector of the adaptive filter at time n, \( x(n) \) is the input signal vector at time n, \( d(n) \) is the desired output value, \( e(n) \) is the error signal, and \( \mu \) is the constant that controls the
convergence rate, called the step factor. The condition for the convergence of the LMS algorithm is
\[ 0 < \mu < \frac{1}{\lambda_{\text{max}}} \] and \( \lambda_{\text{max}} \) is the maximum eigenvalue of the input signal’s autocorrelation matrix.

**Variable Step Size LMS Algorithm Based on Bessel Function**

Iterative formula of variable step size LMS algorithm based on Bessel function:

**Filtering output:**

\[
y(n) = w^T(n)x(n)
\]

**Error signal:**

\[
e(n) = d(n) - y(n)
\]

**Formula of step size update:**

\[
\mu(n) = - \alpha \cdot (\text{besselj}(0, \text{abs}(e(n)) \cdot \pi \cdot 0.5))^3 - \alpha
\]

**Formula of weight vector update:**

\[
w(n+1) = w(n) + 2\mu(n) \cdot e(n) \cdot x(n)
\]

Where: \( w(n) \) is the weight vector of the adaptive filter at time \( n \), \( x(n) \) is the input signal vector at time \( n \), \( d(n) \) is the desired output value, \( y(n) \) is the output signal of filter, and \( e(n) \) is the error signal. The parameter \( \alpha > 0 \) determines the convergence speed, \( b \) determines the convergence of graph, and \( \text{besselj}(0, \text{abs}(e(n)) \cdot \pi \cdot 0.5) \) represents the first type zero-order Bessel function when the variable is \( \text{abs}(e(n)) \).

**Fig. 2** Relationship between step size and error

It can be seen from Fig. 2 that when \( a = 0.00001 \) and \( b \) takes different values, the bottom characteristics of three curves are different, where \( b=11 \) is sharper than \( b=8 \), and \( b=8 \) is sharper than \( b=5 \), that is, when \( e(n) \) near zero, the \( b=8 \) curve is more gradual than the \( b=11 \) curve; while the \( b=5 \) curve has a more gradual shape than the \( b=8 \) curve, but the convergence speed is not fast enough. When \( b=8 \), a small \( e(n) \) corresponds to a small \( \mu(n) \), and when \( b=11 \), a small \( e(n) \) corresponds to a larger \( \mu(n) \). Therefore, when \( a = 0.00001 \), \( b=8 \), the steady-state misalignment noise is smaller than \( a = 0.00001 \), \( b=5 \) and \( a = 0.00001 \), and \( b=11 \). Therefore, the variable step size LMS algorithm based on Bessel function is obtained as follows:

\[
\mu(n+1) = -0.00001 \cdot (\text{besselj}(0, \text{abs}(e(n)) \cdot \pi \cdot 0.5)^3 + 0.00001
\]
4. Simulation of adaptive noise cancellation system

In this paper, the fixed-step LMS algorithm, the combination of two fixed-step LMS algorithms as well as the combination of a fixed-step LMS algorithm and a variable step size LMS algorithm based on Bessel function are used respectively to filter the sinusoidal plus noise signal of the two-order weighted adaptive filter [10~12] under the same conditions. Finally, these results are compared.

![Simulation of adaptive noise cancellation system](image)

**Fig. 3** The effect of the fixed step size LMS algorithm to achieve filtering

In Fig.3, an adaptive filter is adopted to filter noise signals with a fixed step size LMS algorithm. Fig.4 and Fig.5 respectively show the effect of the adaptive filter on the primary and secondary filtering of the noise signal by the combination of two fixed step size LMS algorithms. Obviously, the combination of two fixed step size LMS algorithms realizes better secondary filtering effect of adaptive filter on noise signal than that of the primary filter, which can effectively cancel the noise signal, accelerate the convergence speed and reduce the error. Fig.6 and Fig.7 respectively show the effect of the adaptive filter on the primary and secondary filtering of the noise signal by the fixed step size with the variable step size LMS algorithm. The conclusion is that the secondary filtering effect of the adaptive filter is better than that of the primary filter. It is illustrated that the combination of different LMS algorithms can effectively cancel out the noise signal in the adaptive noise cancellation system.

![Simulation of adaptive noise cancellation system](image)

**Fig. 4** The effect of two fixed step size LMS algorithms to achieve the primary filtering  
**Fig. 5** The effect of two fixed step size LMS algorithms to achieve the secondary filtering
Fig. 6 The effect of fixed and variable step size LMS algorithms to achieve the primary filtering

Fig. 7 The effect of fixed and variable step size LMS algorithms to achieve the secondary filtering

5. Conclusion
This paper introduces the basic principle of the two-stage adaptive filter combination design, and mainly proposes the design idea of combining different LMS algorithms to realize the adaptive filter to filter the noise signal. In addition, the fixed step size LMS algorithm and the variable step size LMS algorithm based on Bessel function are introduced. Finally, in the MATLAB environment, the fixed step size LMS algorithm and two different LMS algorithms are combined to implement the simulation experiment of the adaptive noise cancellation system. The results show that, on the one hand, the secondary adaptive filter can effectively eliminate the interference of uncorrelated noise, accelerate the convergence speed, reduce the steady-state offset, and improve the tracking ability in the time-domain system, effectively resisting the interference in the input signal. On the other hand, compared with the first-stage filtering, it has better cancellation of the noise signal and better suppression of the input-side uncorrelated noise interference. Consequently, the combination of LMS algorithms plays a very important role in the adaptive noise cancellation system.

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