Radiative decay of the X(3872) as a mixed molecule-charmonium state in QCD Sum Rules

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I. INTRUDUCTION

The X(3872) state has been first observed by the Belle collaboration in the decay $B^+ \rightarrow X(3872)K^+$, and was later confirmed by CDF, D0 and BaBar [2]. The current world average mass is $m_X = (3871.4 \pm 0.6)$ MeV, and the width is $\Gamma < 2.3$ MeV at 90% confidence level. Babar collaborations reported the radiative decay mode $X(3872) \rightarrow \gamma J/\psi$ [3, 4], which determines $C = +$. Belle Collaboration reported the branching ratio:

$$\frac{\Gamma(X \rightarrow J/\psi \gamma)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.14 \pm 0.05.$$  \hspace{1cm} (1)

Further studies from Belle and CDF that combine angular information and kinematic properties of the $\pi^+ \pi^-$ pair, strongly favors the quantum numbers $J^{PC} = 1^{++}$ or $2^{+}$ [3, 5, 6]. Between these quantum numbers, a recent BaBar measurement favors the $J^{PC} = 2^{-+}$ assignment [6]. However, established properties of the X(3872) are in conflict with this assignment [7, 8]. In this work we assume the quantum numbers of the X(3872) to be $J^{PC} = 1^{++}$.

The interest in this new state has been increasing, since the mass of the X(3872) could not be related to any charmonium state with the quantum numbers $J^{PC} = 1^{++}$ in the constituent quark models [10], indicating that the conventional quark-antiquark structure should by abandoned in this case. Another interesting experimental finding is the fact that the decay rates of the processes $X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0$ and $X(3872) \rightarrow J/\psi \pi^+ \pi^-$ are comparable [3]:

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \pm 0.3.$$  \hspace{1cm} (2)

This ratio indicates a strong isospin and G parity violation, which is incompatible with a $c\bar{c}$ structure for X(3872). The decay $X \rightarrow J/\psi \omega$ was also observed by BaBar Collaboration [7] at a rate:

$$\frac{B(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{B(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.8 \pm 0.3,$$  \hspace{1cm} (3)

which is consistent with the result in Eq. (2).

The isospin violation problem can be easily avoided in a multiquark approach. In this context the molecular picture has gained attention. The observation of the above mentioned decays, plus the coincidence between the X mass and the $D^{*0}D^{0}$ threshold: $M(D^{*0}D^{0}) = (3781.81 \pm 0.36)$ MeV [11], inspired the proposal that the X(3872) could be a molecular ($D^{*0}D^{0} - D^{0}D^{*0}$) bound state with small binding energy [12, 13]. The $D^{*0}D^{0}$ molecule is not an isospin eigenstate and the rate in Eq. (2) could be explained in a very natural way in this model.

Although the molecular picture is gaining attention with studies indicating that it can be a suitable description for the X(3872) structure [14], there are also some experimental data that seem to indicate the existence of a $c\bar{c}$ component in its structure. In ref. [15], a simulation for the production of a bound $D^{0}D^{*0}$ state with binding energy as small as 0.25 MeV, reported a production cross section that is an order of magnitude smaller than the cross section obtained from the CDF data. A similar result was obtained in ref. [16] in a more phenomenological analysis. However, as pointed out in ref. [17], a consistent analysis of the $D^{0}D^{*0}$ molecule production requires taking into account the effect of final state interactions of the $D$ and $D^{*}$ mesons.

Besides this debate, the recent observation, reported by BaBar [18], of the decay $X(3872) \rightarrow \psi(2S)\gamma$ at a rate:

$$\frac{B(X \rightarrow \psi(2S)\gamma)}{B(X \rightarrow \psi\gamma)} = 3.4 \pm 1.4,$$  \hspace{1cm} (4)

is much bigger than the molecular prediction [19]:

$$\frac{\Gamma(X \rightarrow \psi(2S)\gamma)}{\Gamma(X \rightarrow \psi\gamma)} \sim 4 \times 10^{-3}.$$  \hspace{1cm} (5)

Another interesting interpretation for the X(3872) is that it could be a compact tetraquark state [20, 21]. In
particular, Terasaki argues that with a tetraquark interpretation the ratio in Eq. [11] could be easily explained.

In Ref. [24] the QCDSR approach was used to study the $X$ structure including the possibility of the mixing between two and four-quark states. This was implemented following the prescription suggested in [25] for the light sector. The mixing is done at the level of the currents and is extended to the charm sector. In a different context (not in QCDSR), a similar mixing was suggested already some time ago by Suzuki [10]. Physically, this corresponds to a fluctuation of the $c\bar{c}$ state where a gluon is emitted and subsequently splits into a light quark-antiquark pair, which lives for some time and behaves like a molecule-like state. The possibility that the $X(3872)$ is the mixing of two-quarks and molecular states was also considered to investigate the radiative decay in the effective Lagrangian approach [26], and to explain the data from BaBar [18] and Belle [27] using a Flatté analysis [28].

In this work we will focus on the radiative decay $X(3872) \rightarrow J/\psi \gamma$. We use the mixed two-quark and four-quark prescription of Ref. [24] to perform a QCD sum rule analysis of the radiative decay $X(3872) \rightarrow J/\psi \gamma$.

II. THE MIXED TWO-QUARK / FOUR QUARK OPERATOR

The mixed charmonium-molecular current proposed in Ref. [24] will be used to study radiative decay of the $X(3872)$ in the QCD sum rules framework.

For the charmonium part we use the conventional axial current:

$$ j^{(2)}_\mu (x) = \bar{c}_a (x) \gamma_\mu \gamma_5 c_a (x). \tag{6} $$

The $D \ D^*$ molecule is interpolated by [29, 31]:

$$ j^{(4q)}_\mu (x) = \frac{1}{\sqrt{2}} \left[ (\bar{q}_a (x) \gamma_5 c_a (x) \bar{c}_b (x) \gamma_\mu q_b (x)) - (\bar{q}_a (x) \gamma_5 c_a (x) \bar{c}_b (x) \gamma_\mu q_b (x)) \right], \tag{7} $$

As in Ref. [25] we define the normalized two-quark current as

$$ j^{(2q)}_\mu = \frac{1}{\sqrt{2}} \langle \bar{u}u \rangle j^{(2)}_\mu, \tag{8} $$

and from these two currents we build the following mixed charmonium-molecular current for the $X(3872)$:

$$ J^{\alpha \mu}_\mu (x) = \sin (\theta) j^{(4q)}_\mu (x) + \cos (\theta) j^{(2q)}_\mu (x). \tag{9} $$

Following Ref. [24] we will consider a $D^0 \bar{D}^{*0}$ molecular state with a small admixture of $D^+ \bar{D}^{*-}$ and $D^- \bar{D}^{*+}$ components:

$$ j^{\alpha}_\mu (x) = \cos \theta J^{\alpha}_\mu (x) + \sin \theta J^{\alpha}_\mu (x), \tag{10} $$

with $J^{\alpha}_\mu (x), \ (q = u, d)$, given by the mixed two-quark/four-quark current in Eq. [9].

III. THE THREE POINT CORRELATOR

In this section we use QCD sum rules to study the vertex associated to the decay $X(3872) \rightarrow J/\psi \gamma$. The QCD sum rules approach [32–34] is based on the principle of duality. It consists in the assumption that a correlation function may be described at both quark and hadron levels. At the hadronic level (the phenomenological side) the correlation function is calculated introducing hadron characteristics such as masses and coupling constants. At the quark level, the correlation function is written in terms of quark and gluon fields and a Wilson’s operator product expansion (OPE) is used to deal with the complex structure of the QCD vacuum.

The QCD sum rule calculation for the vertex $X(3872) J/\psi \gamma$ is centered around the three-point function given by

$$ \Pi_{\mu \nu \alpha} (p, p', q) = \int d^4 x d^4 y \ e^{i p' \cdot x} e^{i q \cdot y} \Pi_{\mu \nu \alpha} (x, y), \tag{11} $$

with

$$ \Pi_{\mu \nu \alpha} (x, y) = \langle 0 | i T [ j^{\psi}_\mu (x) j^{\gamma}_\nu (y) j^{(0)}_{\alpha} (0)] | 0 \rangle, \tag{12} $$

where $p = p' + q$ and the interpolating fields are given by:

$$ j^{\psi}_\mu = \bar{c}_a \gamma_\mu c_a, \tag{13} $$

$$ j^{\gamma}_\mu = \sum_{q=u,d,c} e_q \bar{q} q \gamma_\mu q, \tag{14} $$

with $e_q = \frac{2}{3} e$ for quarks $u$ and $c$, and $e_q = - \frac{1}{3} e$ for quark $d$ ($e$ is the modulus of the electron charge). The current $J^{\alpha}_\mu$ is given by the mixed charmonium-molecule current in Eq. [10].

In our analysis, we consider the quarks $u$ and $d$ to be degenerate, i.e., $m_u = m_d$ and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{q}q \rangle$, then by inserting the mixed current [10] in Eq. [12], we arrive at the following relation for the correlator

$$ \Pi_{\mu \nu \alpha} (x, y) = \frac{e \sin \theta}{3} \left( 2 \cos \alpha - \sin \alpha \right) \Pi_{\mu \nu \alpha}^{\text{mol}} (x, y) + \frac{e \langle \bar{q}q \rangle}{6 \sqrt{2}} \cos \theta (\cos \alpha + \sin \alpha) \Pi_{\mu \nu \alpha}^{\text{cc}} (x, y). \tag{15} $$

The relation for the correlator is written in terms of the charmonium and molecule contributions. For the charmonium term we have

$$ \Pi_{\mu \nu \alpha}^{\text{cc}} (x, y) = \langle 0 | i T [ j^{\psi}_\mu (x) j^{\gamma}_\nu (y) j^{(0)}_{\alpha} (0)] | 0 \rangle, \tag{16} $$

and the molecular term is given by

$$ \Pi_{\mu \nu \alpha}^{\text{mol}} (x, y) = \langle 0 | i T [ j^{\psi}_\mu (x) j^{\gamma}_\nu (y) j^{(4q)}_{\alpha} (0)] | 0 \rangle, \tag{17} $$

with $j^{(2q)}_\mu$ and $j^{(4q)}_\mu$ given by Eqs. [8] and [7] respectively.
We now proceed to the calculation of both charmonium and molecular contributions in the OPE side. By inserting the currents of the two-quark component, \( J/\psi \), and photon, respectively defined in Eqs. (3), (13) and (14), in Eq. (16), we obtain for the charmonium contribution the following relation:

\[
\Pi_{\mu \nu}^{\text{OPE}}(x, y) = \frac{2}{3} \text{Tr} \left[ \gamma_\mu S_{ab}(x - y) \gamma_\nu S_{bc}(y) \gamma_\alpha \gamma_5 S_{ca}( - x) \right] + \gamma_\mu S_{ab}(x) \gamma_\alpha \gamma_5 S_{bc}(y) \gamma_\nu S_{ca}( - x + y) \right],
\]

where \( S_{ab}^2(x - y) = \langle 0 | T[q_a(x) \bar{q}_b(y)] | 0 \rangle \) is the full propagator of the quark \( q \) (here \( a, b, c \) are color indices).

For the molecular contribution we use the four-quark current defined in Eq. (17), as well as the currents for the \( J/\psi \) and the photon. Inserting these currents in Eq. (17), we get

\[
\Pi_{\mu \nu}^{\text{OPE}}(x, y) = \frac{1}{\sqrt{2}} \text{Tr} \left[ \gamma_\mu S_{ab}(x) \gamma_\nu S_{bc}(y) \gamma_\alpha \gamma_5 S_{ca}( - x) \gamma_\alpha \gamma_5 S_{ca}( - x + y) \right].
\]

(19)

To evaluate the phenomenological side of the sum rule we insert, in Eq. (11), intermediate states for \( X \) and \( J/\psi \). We use the following definitions:

\[
(0 | j_\mu^X | \psi(p')) = m_\psi f_\psi \epsilon_\mu(p');
\]

(20)

\[
(0 | j_\mu^X | 0) = (\cos \alpha + \sin \alpha) \lambda_\phi \epsilon_\phi^\alpha(p),
\]

(21)

where the meson-current coupling parameter is extracted from the two-point function, and its value was obtained in Ref. 24: \( \lambda_\phi = (3.6 \pm 0.9) \times 10^{-3} \text{ GeV}^{-1} \). We obtain the following expression:

\[
\Pi_{\mu \nu}^{\text{phen}}(p, p', q) = -\frac{(\cos \alpha + \sin \alpha) \lambda_\phi m_\psi f_\psi \epsilon_\mu(p')}{(p^2 - m_\psi^2)(p'^2 - m_\psi^2)} \times \langle \psi(p') | j_\mu^X | X(p) \rangle.
\]

(22)

The remaining matrix element can be related to the one that describes the decay \( X \to \gamma J/\psi \):

\[
\langle \psi(p') | j_\mu^X(q) | X(p) \rangle = i \epsilon_\mu(q) \mathcal{M}(X(p) \to \gamma(q) J/\psi(p')), \]

(23)

and we can define 24

\[
\mathcal{M}(X(p) \to \gamma(q) J/\psi(p')) = e^{i \epsilon_\mu(q) \epsilon_\mu(p')} \epsilon_\mu(q) \epsilon_\mu(p') \times \frac{q_\mu}{m_X^2} (A g_{\mu \lambda} g_{\alpha \nu} p \cdot q + B g_{\mu \lambda} p_\alpha q_\nu + C g_{\alpha \nu} p_\lambda q_\mu).
\]

(24)

where \( A, B, C \) are dimensionless couplings. Using this relation in Eq. (22), we can write the phenomenological side of the sum rule as:

\[
\Pi_{\mu \nu}^{\text{phen}}(p, p', q) = \frac{ie(\cos \alpha + \sin \alpha) \lambda_\phi m_\psi f_\psi}{m_X^2 (p^2 - m_\psi^2)} \times \left( e^{\alpha \mu \nu} q_\mu p \cdot q A + e^{\alpha \nu \lambda} p_\lambda q_\mu B - e^{\alpha \mu \lambda} q_\mu p_\nu C + e^{\alpha \nu \lambda} p_\lambda p_\nu (C - A) \frac{p \cdot q}{m_\psi^2} \right).
\]

(25)

In the OPE side we work in leading order in \( \alpha_s \) and we consider condensates up to dimension five, as shown in Fig. 1. In the phenomenological side, as we can see in Eq. (25), there are five independent structures. We choose one convenient structure to determine each one of the couplings \( A, B, C \) in Eq. (24). Taking the limit \( p^2 = p'^2 = -P^2 \) and doing a single Borel transform to \( P^2 \to M^2 \), we arrive at a general formula for the sum rule for each structure \( i \):

\[
G_i(Q^2) \left( e^{-m_\phi^2/M^2} - e^{-m_\psi^2/M^2} \right) + H_i(Q^2) e^{-s_0/M^2} = \Pi_i^{\text{OPE}}(M^2, Q^2),
\]

(26)

where \( Q^2 = -q^2 \) and \( H_i(Q^2) \) gives the contribution of the pole-continuum transitions [35–37]. In the following, we show the expression of the sum rules for the three structures that we have chosen to work.

1. Structure 1: \( e^{\alpha \mu \nu} q_\mu \)

The RHS of the sum rule for the structure \( e^{\alpha \mu \nu} q_\mu \) (structure 1) have both charmonium and molecule contributions:

\[
\Pi_1^{\text{OPE}}(M^2, Q^2) = -\langle \bar{q} q \rangle \left[ \frac{\sin \theta (2 \cos \alpha - \sin \alpha)}{3Q^4} \times \langle \bar{q} q \rangle \frac{\cos \theta}{2Q^2} (\cos \alpha + \sin \alpha) \Pi_i^{\text{OPE}}(M^2, Q^2) \right],
\]

(27)

where the molecular contribution is given by

\[
\Pi_1^{\text{mol}}(M^2, Q^2) = \left( 1 - \frac{m_\phi^2}{3Q^2} \right) \int_{m_\phi^2}^{\infty} du \ e^{-u/M^2} \times \frac{1}{\sqrt{1 - \frac{4m_\phi^2}{u}}} \left( \frac{1 + \frac{m_\psi^2}{u}}{2} \right) + \frac{m_\phi^2 m_\psi^2}{16} \int_0^1 d \alpha \left( 1 + 3\alpha \right) e^{-\frac{m_\phi^2}{u(1 - \alpha)} M^4}.
\]

(28)

and the charmonium contribution is

\[
\Pi_1^{\text{phen}}(M^2, Q^2) = -\int_{m_\phi^2}^{\infty} ds \int_{m_\phi^2}^{\infty} du \ e^{-\frac{s}{3Q^2}} \frac{2}{\sqrt{\lambda}} \times \left( \frac{m_\phi^2 + t u(t - u)}{\lambda} \right),
\]

(29)
where $\lambda = \lambda(s, t, u) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$, and $t = -Q^2 < 0$.

In the above expressions the parameters $s_0 = (m_X + \Delta s)^2$ and $u_0 = (m_\psi + \Delta u)^2$ are the continuum thresholds for $X$ and $J/\psi$ respectively. The limits of the integral in $u$ are:

$$u_\pm = s + t + \frac{1}{2m_c^2} ( -st \pm \sqrt{st(s - 4m_c^2)(t - 4m_c^2)}).$$

The integrals in $s$ and $u$ also obey the following conditions:

$$t < u, \ 4m_c^2 \leq s_0. \tag{31}$$

Since the photon is off-shell in the vertex $X J/\psi \gamma$ it is required the introduction of form factors. Then in the left hand side of the sum rule, we define the function $G_1(Q^2)$, which is related to the form factor $A(Q^2)$ as:

$$G_1(Q^2) = \frac{3\sqrt{2}\pi^2 (\cos \alpha + \sin \alpha)\lambda q m_\psi f_\psi}{m_X^2 (m_X^2 - m_\psi^2)} A(Q^2). \tag{32}$$

2. Structure 2: $\epsilon^{\mu \nu \sigma \lambda} p_\mu p_\nu q_\lambda$

The RHS of the sum rule for the structure $\epsilon^{\mu \nu \sigma \lambda} p_\mu p_\nu q_\lambda$ (structure 2) has only molecular contribution:

$$\Pi_2^{\text{OPE}}(M^2, Q^2) = \frac{m_0^2(q \bar{q})}{Q^4} \int_0^1 da \frac{1 - \alpha}{\alpha} e^{-\frac{m_0^2}{m(1-\alpha)\lambda}}. \tag{33}$$

In the left hand side of the sum rule we define the function $G_2(Q^2)$, which is related to the sum of form factor $A(Q^2) + B(Q^2)$ as:

$$G_2(Q^2) = \frac{3\sqrt{2}\pi^2 (\cos \alpha + \sin \alpha)\lambda q m_\psi f_\psi (A(Q^2) + B(Q^2))}{\sin \theta (2\cos \alpha - \sin \alpha) m_X^2 (m_X^2 - m_\psi^2)} \sin \theta^2 \tag{34}.$$ 

3. Structure 3: $\epsilon^{\mu \nu \lambda \sigma} p_\mu q_\lambda q_\sigma$

The RHS of the sum rule for the structure $\epsilon^{\mu \nu \lambda \sigma} p_\mu q_\lambda q_\sigma$ (structure 3) has only charmonium contribution:

$$\Pi_3^{\text{OPE}}(M^2, Q^2) = (q \bar{q}) \int_0^{s_0} ds \int_{u_0}^{a_+} du e^{-\frac{m_0^2}{m(1-\alpha)\lambda}} \times \left[ tu + m_0^2 (-s + t + u) \right] \frac{3}{\lambda^3/2}. \tag{35}$$

The integrals in this equation obey the same relations and conditions defined for the Eq. (29).

In the left hand side of the sum rule we define the function $G_3(Q^2)$, which is related to the form factor $C(Q^2)$ as:

$$G_3(Q^2) = \frac{6\sqrt{2}\pi^2 (\cos \alpha + \sin \alpha)\lambda q m_\psi f_\psi}{\cos \theta m_X^2 (m_X^2 - m_\psi^2)} C(Q^2). \tag{36}$$

IV. NUMERICAL ANALYSIS

The sum rules are analysed numerically using the following values for quark masses and QCD condensates.
and for meson masses e decay constants:
\[ m_c(m_c) = (1.23 \pm 0.05) \text{ GeV}, \]
\[ \langle q\bar{q} \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3, \]
\[ m_0^2 = 0.8 \text{ GeV}^2, \]
\[ m_\psi = 3.1 \text{ GeV} \]
\[ m_X = 3.87 \text{ GeV} \]
\[ f_\psi = 0.405 \text{ GeV} \] (37)

The value of the angle \( \alpha \) that defines the mixing between the \( D^0 D^{*0} \), \( \bar{D}^0 D^{*0} \) and \( D^+ D^{-} D^+ D^{-} \) has been obtained previously in Ref. [24, 24, 35]:
\[ \alpha = 20^\circ \] (38)

For the mixing angle of two and four quark states, \( \theta \), we use the values that were obtained in the QCD sum rules analysis of the mass of the \( X \) and the decay mode \( X \rightarrow J/\psi(n\pi) \) [24]:
\[ \theta = (9 \pm 4)^\circ. \] (39)

In the LHS of Eq. (20), the unknown functions \( G_i(Q^2) \) and \( H_i(Q^2) \) have to be determined by matching both sides of the sum rule. In Fig. 2, we show the points obtained if we isolate the functions \( G_i(Q^2) \) in Eq. (20) and vary both \( Q^2 \) and \( M^2 \). The functions \( G_i(Q^2) \) (and consequently \( A(Q^2), B(Q^2), C(Q^2) \)) should not depend on \( M^2 \), so we limit our fit to a region where the function is clearly stable in \( M^2 \) to all values of \( Q^2 \). We can see in Fig. 2 that the regions of stability in \( M^2 \) for \( G_1(Q^2) \) is 7.0 GeV^2 \( \leq M^2 \leq 8.5 \) GeV^2, for \( G_2(Q^2) \) is 6.5 GeV^2 \( \leq M^2 \leq 7.5 \) GeV^2, and for \( G_3(Q^2) \) is 8.0 GeV^2 \( \leq M^2 \leq 9.0 \) GeV^2.
In Fig. (3) we show, through the dots, the QCDSR results for the functions $G_i(Q^2)$ as a function of $Q^2$. The form factors $A(Q^2), B(Q^2), \text{ and } C(Q^2)$ can be easily obtained by using Eqs. (32), (34), and (36). Since the coupling constants, appearing in Eq. (24), are defined as the value of the form factors at the photon pole: $Q^2 = 0$, to determine the couplings $A, B \text{ and } C$ we have to extrapolate $A(Q^2), B(Q^2), \text{ and } C(Q^2)$ to a region where the sum rules are no longer valid (since the QCDSR results are valid at the deep Euclidean region). To do that we fit the QCDSR results, shown in Fig. (3), as exponential functions:

$$G_i(Q^2) = g_1 e^{-g_2 Q^2}.$$  \hspace{1cm} (40)

We do the fitting for $s_0^{1/2} = 4.4 \text{ GeV} \text{ and } u_0^{1/2} = 3.6 \text{ GeV}$ as the results do not depend much on this parameters. The numerical values of the fitting parameters are shown in the Table I.

From Fig. (3) we can see that the $Q^2$ dependence of the QCDSR results for the functions $G_i(Q^2)$ are well reproduced by the chosen parametrization, in the interval $2.5 \text{ GeV}^2 \leq Q^2 \leq 4.5 \text{ GeV}^2$, where the QCDSR are valid.

Using Eqs. (32), (34), (36) and (40) and varying $\theta$ in the range $5^\circ \leq \theta \leq 13^\circ$ we get:

$$A = A(Q^2 = 0) = 18.65 \pm 0.94;$$
$$A + B = (A + B)(Q^2 = 0) = -0.24 \pm 0.11;$$
$$C = C(Q^2 = 0) = -0.843 \pm 0.008.$$  \hspace{1cm} (41)

| $G_1$ | $G_2$ | $G_3$ |
|-------|-------|-------|
| $g_1$ | 0.056 GeV$^3$ | -0.0069 GeV | -0.013 GeV$^3$ |
| $g_2$ | 0.25 GeV$^{-2}$ | 0.365 GeV$^{-2}$ | 0.41 GeV$^{-2}$ |

TABLE I. Results for the fitting parameters.
The decay width is given in terms of these couplings through [20]:

\[ \Gamma(X \to J/\psi \gamma) = \frac{\alpha}{3} \frac{p^*}{m_X^3} \left( (A + B)^2 + \frac{m_X^2}{m_{\psi}^2} (A + C)^2 \right) \]

(42)

where \( p^* = (m_X^2 - m_{\psi}^2)/(2m_X) \) is the three-momentum of the photon in the \( X \) rest frame. To compare our results with the experimental data shown in Eq. (1) we use the same range of the mixing angle \( \theta \) obtained in the Ref. [24], which was computed in the same range of the mixing angle \( \theta \) and with the same angle \( \alpha = 20^\circ \): \( \Gamma(X \to J/\psi \pi^+\pi^-) = 9.3 \pm 6.9 \text{ MeV} \). We get

\[ \frac{\Gamma(X \to J/\psi \gamma)}{\Gamma(X \to J/\psi \pi^+\pi^-)} = 0.19 \pm 0.13 \]

(43)

which is in complete agreement with the experimental result.

V. CONCLUSIONS

We have presented a QCDSR analysis of the three-point function of the radiative decay of the \( X(3872) \) meson by considering a mixed charmonium-molecular current. We find that the sum rules results in Eqs. (13) are compatible with experimental data. These results were obtained by considering the mixing angles in Eq. (10) and (14) with the values \( \alpha = 20^\circ \) and \( 5^\circ \leq \theta \leq 13^\circ \). The present result is also compatible with previous analysis of the mass of the \( X \) state and the decays into \( J/\psi \pi^0\pi^+\pi^- \) and \( J/\psi\pi^+\pi^- \) [24], since the values of the mixing angles used in both calculations are the same. It is important to mention that there is no free parameter in the present analysis and, therefore, the result presented here strengthens the conclusion reached in Ref. [24] that the \( X(3872) \) is probably a state with charmonium and molecular components.

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