$N = (4, 2)$ Chiral Supergravity in Six Dimensions and Solvable Lie Algebras

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ABSTRACT

Decomposition of the solvable Lie algebras of maximal supergravities in $D = 4, 5$ and 6 indicates, at least at the geometrical level, the existence of an $N = (4, 2)$ chiral supergravity theory in $D = 6$ dimensions. This theory, with 24 supercharges, reduces to the known $N = 6$ supergravity after a toroidal compactification to $D = 5$ and $D = 4$. Evidence for this theory was given long ago by B. Julia. We show that this theory suffers from a gravitational anomaly equal to $4/7$ of the pure $N = (4, 0)$ supergravity anomaly. However, unlike the latter, the absence of $N = (4, 2)$ matter to cancel the anomaly presumably makes this theory inconsistent. We discuss the obstruction in defining this theory in $D = 6$, starting from an $N = 6$ five-dimensional string model in the decompactification limit. The set of massless states necessary for the anomaly cancellation appears in this limit; as a result the $N = (4, 2)$ supergravity in $D = 6$ is extended to $N = (4, 4)$ maximal supergravity theory.

CERN-TH/97-303
November 1997

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1 Introduction

Supersymmetry algebras in any dimension and their representations are closely related to properties of spinors. Spinors of $O(D-1,1)$ are real for $D = 2, 3, 9 \mod 8$, pseudoreal for $D = 5, 6, 7 \mod 8$, and complex in $D = 4, 8 \mod 8$. The above reality properties refer to spinors of a given chirality when the dimension is even. The corresponding supersymmetry algebras have automorphism groups $H$ (sometimes called R-symmetry), which fall in three categories [1]:

i) $O(N)$, ii) $USp(N)$ and iii) $U(N)$,

$N$ being the number of spinorial charges in the appropriate dimension. We note that in $D = 2$ and $6 \mod 8$ algebras with different numbers of chiral charges $N_L, N_R$ exist with $O(N_L) \times O(N_R)$ and $USp(N_L) \times USp(N_R)$ as $H$-groups respectively. If one considers compactifications of higher-dimensional theories on smooth manifolds the number of supersymmetry charges $N$ in lower dimensions is always $2^n$ for some $n$. This leaves aside certain theories, which can be constructed in lower dimensions, where the number of supercharge components is [2], [3]:

- In $D = 4$: 12, 20 and 24, corresponding to $N = 3, 5$ and 6 supergravity with $H = U(3), U(5)$ and $U(6)$ respectively.
- In $D = 5$ and $D = 6 : 24$, corresponding to $N= 6$ with $H = USp(6)$ and $N = (4,2)$ with $H = USp(4) \times USp(2)$ respectively.\footnote{Instead of the notation, $N = 1$ for the minimal and $N = 2$ for the maximal supersymmetry in $D = 6$, we are using the chiral notation $N = (2,0)$ for the minimal and $N = (4,4)$ for the maximal supersymmetry; this notation is more convenient in $D = 6$ since the chiral spinors in $D = 6$ are Majorana symplectic [3].}

Theories in $D = 4$ and 5, have been shown to exist as consistent type II string compactifications on asymmetric orbifolds [3], [4]. The $N = 6$ supergravity is special because it is the only one that can be lifted to six dimensions where it becomes chiral [5]. In $D = 4$ the $N = 6$ supergravity is also special when is defined in anti de Sitter space. In fact $Osp(6|4)$ gauge supergravity is the only one with a zero-center representation. [3]

In section 2 of this work we explore some properties of $N = (4,2)$ supermultiplets in $D = 6$ by means of the chain decomposition $N = (4,4) \rightarrow N = (4,2) \rightarrow N = (0,2)$. This allow a derivation of the structure of the $N = (4,2)$ supergravity, and in particular a relation of its scalar manifold to the decomposition of the maximal supergravity into $N = 2$ sub-theories. We show that this theory suffers from a gravitational anomaly, which is $4/7$ of the $N = (4,0)$ pure supergravity anomaly.

In section 3 we derive the $D = 5$, $N = (4,2)$ string ground state and show why the four- and five- dimensional descendants of the anomalous six-dimensional theory are not affected by this anomaly when one includes massive degrees of freedom dictated by string theory.

In section 4 we analyse the two decompactification limits $R \rightarrow \infty, R \rightarrow 0$ and we show the restoration of $N = (4,4)$ in $D = 6$. Our conclusions are in section 5.
2 \( N = 2 \) and \( N = 6 \) decomposition of maximal supergravity in \( D = 4, 5 \) and 6 dimensions.

Let us first consider the general decomposition of maximal supergravities in terms of \( N = 2 \) multiplets \( \mathbf{2}, \mathbf{3}, \mathbf{4} \). The important fact in this decomposition is that the spin 3/2 multiplet does not contain scalars. If we associate to the scalar manifold \( \mathcal{M} = G/H \) a solvable group with a Lie algebra \( \mathcal{S} \), then we find that the \( \mathcal{S} \) vector space decomposes into \( \mathcal{S}_V + \mathcal{S}_H \), where \( \mathcal{S}_V \) and \( \mathcal{S}_H \) are solvable Lie algebras associated to \( J_{\text{max}} = 1 \) and \( J_{\text{max}} = \frac{1}{2} \) respectively. The \( J_{\text{max}} = \frac{1}{2} \) \( \mathbf{8}, \mathbf{4} \), multiplets are hyper-multiplets and \( \mathcal{S}_H \) is thus related to a quaternionic space with holonomy \( SU(2) \times H \); \( H \) is the automorphism group of the supersymmetry algebra in the corresponding dimension.

Note that in \( D = 6 \), since the vector multiplet has no scalars, the \( J_{\text{max}} = 1 \) multiplet is a tensor multiplet. Our starting point is to decompose the maximal \( D = 6 \) supermultiplet into \( N = 2 \) ones. This amounts to decomposing the \( N = (4,4) \) representation into \( N = (0,2) \) massless representations.

The \( N = (0,2) \) representations are:

- **graviton**: \( G \to (1_L, 1_R) + 2(1_L, \frac{1}{2}R) + (1_L, 0_R) \)
- **gravitino**: \( g_1 \to 2(1_L, \frac{1}{2}R) + 4(1_L, 0_R) \)
- **gravitino**: \( g_2 \to 2(\frac{1}{2}L, 1_R) + 4(\frac{1}{2}L, \frac{1}{2}R) + 2(\frac{1}{2}L, 0_R) \)
- **vector**: \( V \to (\frac{1}{2}L, \frac{1}{2}R) + 2(\frac{3}{2}L, 0_R) \)
- **tensor**: \( T \to (0_L, 1_R) + 2(0_L, \frac{1}{2}R) + (0_L, 0_R) \)
- **hyper**: \( H \to 2(0_L, \frac{1}{2}R) + 4(0_L, 0_R) \)

All the above multiplets are obtained by tensoring Clifford algebra states \( (0_L, \frac{1}{2}R), 2(0_L, 0_R) \) by a \( (J_L, J_R), SU(2)_L \times SU(2)_R \) representation, with a doubling (when required) by CPT.

The \( N = (4,4)_G \) graviton multiplet decomposes into \( N = (0,2) \) as follows:

\[
[N = (4,4)]_G = (1_L, 1_R) + 4(1_L, \frac{1}{2}R) + 4(\frac{1}{2}L, 1_R) + 16(\frac{1}{2}L, \frac{1}{2}R) + 5(1_L, 0_R) + 5(0_L, 1_R) + 20(\frac{3}{2}L, 0_R) + 20(0_L, \frac{1}{2}R) + 25(0_L, 0_R).
\]

\[
= 1G + 1g_1 + 2g_2 + 8V + 5T + 5H
\]

On the other hand we may decompose \( N = (4,4) \to N = (0,2) \) through the chain \( N = (4,4) \to N = (4,2) \to N(0,2) \).

We first get \([N = (4,4)]_G \to [N = (4,2)]_G + [N = (4,2)]_g \), \( (256 \to 128 + 128 \) states). Then we can further decompose \([N = (4,2)]_G \) and \([N = (4,2)]_g \) into \( N = (0,2) \) multiplets to finally obtain:

- \([N = (4,2)]_G = (1_L, 1_R) + 8(\frac{1}{2}L, \frac{1}{2}R) + 8(1_L, 0_R) + 5(0_L, 1_R) + 5(0_L, 0_R)\)
\[ + 4(\frac{1}{2}L, 1_R) + 2(1_L, \frac{1}{2}R) + 10(0_L, \frac{1}{2}R) + 4(\frac{1}{2}L, 0_R) \]
\[ = [N = (0, 2)]_G + 2[N = (0, 2)]_{g_2} + 5[N = (0, 2)]_T \]
\[ + [N = (4, 2)]_g = 2(1_L, \frac{1}{2}R) + 8(\frac{1}{2}L, \frac{1}{2}R) + 10(0_L, \frac{1}{2}R) + 
+ 4(1_L, 0_R) + 20(0_L, 0_R) \]
\[ = [N = (0, 2)]_{g_1} + 8[N = (0, 2)]_V + 5[N = (0, 2)]_H \]

This implies, in particular, the solvable Lie algebra decomposition

\[ S \left[ \frac{O(5, 5)}{O(5) \times O(5)} \right] = S \left[ \frac{O(1, 5)}{O(5)} \right] + S \left[ \frac{O(4, 5)}{O(4) \times O(5)} \right] \]

i.e. algebras with a rank-1 tensor multiplet coset and a rank-4 quaternionic manifold with holonomy \( SU(2) \times H \), \( H = USp(2) \times USp(4) \). This statement shows that the decomposition of the maximal solvable Lie algebra into \( N = 2 \) solvable Lie algebras corresponds to the scalar sector of the two \( N = (4, 2) \) multiplets into which the \( [N = (4, 4)]_g \) multiplet decomposes. If we delete the \( [N = (4, 2)]_g \) multiplet we obtain a pure \( N = (4, 2) \) supergravity theory with scalar manifold \( O(1, 5)/O(5) \). This result was obtained by Julia in [5].

Note that a similar procedure in \( D = 5 \) and \( D = 4 \) gives analogous relations among solvable Lie algebras [8]–[7]:

\[ S \left[ \frac{E_{6(6)}}{USp(8)} \right] = S \left[ \frac{SU^*(6)}{USp(6)} \right] + S \left[ \frac{F_{4(4)}}{USp(2) \times USp(6)} \right] \quad \text{in } D = 5 \]
\[ S \left[ \frac{E_{7(7)}}{SU(8)} \right] = S \left[ \frac{SO^*(12)}{U(6)} \right] + S \left[ \frac{E_{6(2)}}{SU(2) \times SU(6)} \right] \quad \text{in } D = 4. \]

The rank of the above cosets in the various dimensions decomposes as follows:

- In \( D = 6 \): \( 5 = 1V + 4H \)
- In \( D = 5 \): \( 6 = 2V + 4H \)
- In \( D = 4 \): \( 7 = 3V + 4H \)

This shows that the fields that correspond to the compactification radii are vector multiplets; the rank of the hypermultiplet manifold does not change with the dimensions. The rank-4 quaternionic spaces at \( D = 6, 5 \) and \( 4 \), correspond to the following maximal decompositions of the corresponding isometry groups:

\[ F_{4(4)} \to O(5, 4), \quad 52 \to 36 + 16. \]
\[ E_{6(2)} \to F_{4(4)}, \quad 78 \to 52 + 26. \]

The total number of states of the \( N = (4, 2) \) supergravity in six dimensions is \( 128 = 64 \) bosons + 64 fermions as in \( D = 4 \) and \( D = 5 \). However the theory in \( D = 6 \) is anomalous since the gravitational anomaly is \(-360I_\pm^2\), where \( I_\pm \) stands for the anomaly contribution of one Weyl fermion in six dimensions. The gravitational
anomaly contribution of the pure $N = (4, 0)$ supergravity multiplet is $-630I_{2}$. In the case of the $(4, 0)$ theory the anomaly is cancelled by adding twenty-one $(4, 0)$ tensor multiplets, giving a contribution to the anomaly $+21 \times 30I_{2}$ [1]. In the present case, owing to the absence of matter multiplets with $J_{\text{max}}$ less than $\frac{3}{2}$, the $N = (4, 2)$ cannot be anomaly-free. If one adds a $[N = (4, 2)]_{g_{1}}$ multiplet then the supersymmetry is extended to $N = (4, 4)$, which is an anomaly-free theory. It seems that this is the only consistent way to cancel the anomaly. Another possibility to cancel the anomaly would be to add twelve $(4, 0)$ tensor multiplets; this solution, however, explicitly violates $N = (4, 2)$ supersymmetry.

It is of interest to understand the reason why the six-dimensional theory with 24 supercharges is anomalous while, in lower dimensions, theories with the same number of supercharges can be constructed in type II string theories [3].

In the next sections we will show the restoration of $N = (4, 4)$ in six dimensions, starting from a five-dimensional $N = 6$ string theory and decompactifying one of the dimensions considering either $R_{6} \rightarrow \infty$ or $R_{6} \rightarrow 0$.

3 $N = 6 \text{ construction in five dimensions}$

In order to construct in string theory a ground state with $N = (4_{L} + 2_{R})$ it is necessary to consider the type II theory with maximal supersymmetry $N = (4_{L} + 4_{R})$; then one has to reduce the supersymmetry asymmetrically by projecting out half of the right-moving supercharges. The heterotic or type I construction of $N = (4_{L} + 2_{R})$ ground states is impossible, since the maximal supersymmetry is $N = (4_{L} + 0_{R})$ in the heterotic theory and $N = 4$ in that of type I.

We start by presenting the type II $N = (4_{L} + 2_{R})$ string models and write down their partition function. Models with the same number of supercharges in $D = 4$ have been obtained by applying projections to the maximal-supersymmetry string theories [3], which preserve modular invariance as well as the conformal symmetries on the string world-sheet. The maximal-supersymmetry type II model is described in the light-cone gauge, by 8 world-sheet left/right-moving bosonic and fermionic coordinates. In our notation, in $D = 5$, the coordinates $\psi_{\mu}^{L,R}$ and $X_{\mu}^{L,R}$ ($\mu = 3, 4, 5$) represent the space-time degrees of freedom (in the light-cone gauge), whereas the remaining ones correspond to the fermionic $\chi_{I}^{L,R}$ and bosonic $\phi_{I}^{L,R}$, $I = 1, 2, 3, 4, 5$ internal degrees of freedom. This description will be appropriate for the asymmetric orbifold construction [4], where four of the right-moving internal coordinates are twisted in order to reduce by a factor of 2 the right-moving supersymmetries:

$$Z_{2} : (x_{R}^{I}, \phi_{R}^{I}) \rightarrow -(x_{R}^{I}, \phi_{R}^{I}), \quad I = 2, 3, 4, 5 \quad (3.4)$$

In the fermionic construction [12], [13], $\phi_{I}^{L,R}$ ($I = 1, 2, 3, 4, 5$) are replaced by a pair of Majorana–Weyl spinors $y_{I}^{L,R}$ and $\omega_{I}^{L,R}$ ($I = 1, 2, 3, 4, 5$), by means of the 2d-boson-


fermion equivalence:

\[ J_{I}^{L,R} = \partial \phi_{I}^{L,R} = y_{I}^{L,R} \omega_{I}^{L,R}. \] (3.5)

The construction of string models amounts to a choice of boundary conditions for the 2d fermions \( \chi_{I}^{L,R}, y_{I}^{L,R}, \omega_{I}^{L,R} \), which satisfies local and global consistency requirements [12]. The \( N = (4L + 4R) \) maximal supersymmetry model, constructed in this manner, have four space-time supercharges originating from the left-moving sector and another four from the right-moving sector. In the language of the fermionic construction [12], this model is defined by introducing three basis sets, \( F, S \) and \( \bar{S} \). The first one contains all the left- and the right-moving fermions:

\[ F = [\psi_{\mu}^{L}, \chi_{I}^{L}, y_{I}^{L}, \omega_{I}^{L} | \psi_{\mu}^{R}, \chi_{I}^{R}, y_{I}^{R}, \omega_{I}^{R}] \quad (\mu = 3, 4, 5; I = 1, \ldots, 5), \] (3.6)

and the basis sets \( S \) and \( \bar{S} \), which contain only eight left- or right-moving fermions and generate the GSO projections of the maximal supersymmetry [3]:

\[ S = [\psi_{\mu}^{L}, \chi_{I}^{L}] \quad \bar{S} = [\psi_{\mu}^{R}, \chi_{I}^{R}]. \] (3.7)

Four of the gravitinos of the \( N = (4L + 4R) \) model belong to the \( S \)-sector and the other four to the \( \bar{S} \)-sector. Then, by applying the appropriate projection one obtains the \( N = (4L + 2R) \) superstring model.

A possible projection for constructing \( N = (4L + 2R) \) is specified by a choice of fermion basis \( b \) such that

\[ b = [\psi_{\mu}^{R}, \chi_{I}^{R}, y_{I}^{L}, y_{1}^{L}, \omega_{1}^{L}, y_{1}^{R}, \omega_{1}^{R}]. \] (3.8)

Then \( b \) generates the desire left–right-asymmetric \( Z_{2} \) projection. Namely, it acts as a twist on \( \chi_{I}^{R} \) and \( J_{I}^{R} \) when \( I = 2, 3, 4, 5 \). Also it acts non-trivially on the set of fermions

\[ T = [y_{I}^{L}, \omega_{I}^{L}, y_{I}^{R}, \omega_{I}^{R}]. \] (3.9)

This action however keeps the currents \( J_{I}^{L,R} = \partial \phi_{I}^{L,R} \) invariant. In the bosonic \( \phi_{I}^{L,R} \) language, the non-trivial action on \( T \) implies a shifting by 1/2 unit on the \( \phi_{I} \) lattice; \( \phi_{I}^{L,R} \to \phi_{I}^{L,R} + \pi \). Therefore, the \( Z_{2} \) defined by \( b \) acts as a shift on \( \phi_{I} \) and as an asymmetric twist on the remaining four internal supercoordinates \( \chi_{I}^{R}, \phi_{I}^{R}, I = 2, 3, 4, 5 \). This asymmetric \( Z_{2} \) breaks half of the right-moving supersymmetries by projecting out two of the gravitinos of the \( \bar{S} \)-sector. The resulting model has the desired \( N = (4L + 2R) \) supersymmetry. The presence of \( T \) fermions in \( b \) makes the states coming from the \( b \)-twisted sector massive. Indeed, the lowest state in the \( b \)-twisted sector, has right-moving conformal dimensions \( \Delta_{R} = \frac{10}{16} \), which corresponds to the conformal dimension of a spin field constructed with the ten right-moving fermions \( spin[\psi_{\mu}^{R}, \chi_{I}^{R}, y_{I}^{R}, y_{1}^{R}, \omega_{1}^{R}]. \) Thus, the lower right-moving mass level in this sector is \( [m_{R}(b)]^{2} = \frac{10}{16} - \frac{1}{2} = \frac{1}{8}. \)
The absence of massless twisted sectors in any consistent orbifold construction is expected. Indeed, the the fact that the massless spectrum of $N = (4_L + 2_R)$ in $D = 5$ is fully determined by the graviton supermultiplet implies that all the $N = (4_L + 2_R)$ massless states must be the $Z_2$-invariant states of $N = (4_L + 4_R)$ supergravity. These states are all present in the untwisted sector and therefore all extra states have to be massive.

Using for instance,
\[
\tilde{b} = \left[ \psi^R_\mu, \chi^R_1, y^R_{2,3,4,5} \right]
\] (3.10)

instead of $b$, then the $\tilde{b}$ asymmetric projection still projects out the two right-moving gravitinos from the $\tilde{S}$-sector. In this case, however, the $\tilde{b}$-twisted sector contains some extra massless states, since the conformal dimension of $spin[ \psi^R_\mu, \chi^R_1, y^R_{2,3,4,5} ]$ is precisely 1/2. Among the extra massless states of the $\tilde{b}$-twisted sector, there exist two extra gravitinos with vertex operator
\[
V^b_{3/2} = \Psi^L_\mu \, spin[ \psi^R_\mu, \chi^R_1, y^R_{2,3,4,5} ]_+,
\] (3.11)

and so the $N = (4_L + 2_R)$ supersymmetry is extended again to the maximal $N = (4_L + 4_R)$.

Already at this point one can guess the difficulties that will arise in constructing a six-dimensional string model with $N = (4_L + 2_R)$ supersymmetry. Indeed, in six dimensions the coordinate $\phi_1$ is non-compact and thus, the only available asymmetric projection is the one that is based on $\tilde{b}$; this projection, however, gives rise to massless $b$ twisted sectors, which contain two extra right-moving gravitinos.

In the following, we will discuss in more detail the $b$ fermionic model in $D = 5$. Then this model will be generalized in order to include the compactification radius $R_1$ modulus associated to $\phi_1$. Finally, in the next section, we study the extension of supersymmetry in the two decompactification limits $R_1 \to \infty$ and $R_1 \to 0$. We will show that the helicity supertraces $B_{2n} = \text{str}[s]^{2n}$ vanish for $n = 0, 1, 2, 3$ in both decompactification limits, indicating the extension of $N = (4_L + 2_R) \to N = (4_L + 4_R)$.

We start with the partition function of $N = (4_L + 2_R)$ based on $b$:
\[
Z_b^{D=5} = \frac{1}{\text{Im}\tau^2} \frac{1}{\eta^3\bar{\eta}^3} \frac{1}{4} \sum_{a,b,a,b=0} \frac{(-)^{a+b+ab}}{\eta^4} \frac{(-)^{a+b+\bar{a}b}}{\bar{\eta}^4} \\
\times \frac{1}{2} \sum_{h,g} \theta^{[a]}_{\tilde{b}} \{ \theta_{\tilde{b}}^{[a-h]} \theta_{\tilde{b}}^{[a-h]} \} \theta_{\tilde{b}}^{[a-h]} Z_{5,5}[i\eta].
\] (3.12)

In the above expression $Z_{5,5}[i\eta]$ denotes the contribution of the five compactified coordinates $\phi^{L,R}_I$, $I = 1, \ldots, 5$ (fermionized); four of them are $(h, g)$-twisted while $\phi^{L,R}_1$ is $(h, g)$-shifted. In terms of fermionic characters, the analytic expression of $Z_{5,5}[i\eta]$ is:
\[
Z_{5,5}[i\eta] = \frac{1}{2} \sum_{\gamma, \delta} \left[ \frac{\theta^{[\gamma-h]}_{\tilde{b}+g} \theta^{[\gamma-h]}_{\tilde{b}+g}}{\eta \bar{\eta}} (-)^{\delta h+\gamma g+h_g} \right] \times \left[ \frac{\theta^{[\beta]}_{\tilde{b}} \theta^{[\gamma]}_{\tilde{b}} \theta^{[\gamma]}_{\tilde{b}} \theta^{[\gamma]}_{\tilde{b}}}{\eta \bar{\eta}} (-)^{\gamma g} \right].
\] (3.13)
where the term in the first square bracket corresponds to the contribution of the compactified coordinate $\phi_1$ written in terms of the fermionic characters of $y_1^{L,R}, \omega_1^{L,R}$. In terms of $\phi_1$-lattice characters, it corresponds to a fixed $S^1$ radius at the fermionic point $R_1 = 1/\sqrt{2}$. The term in the second square bracket corresponds to the characters of the $Z_2$ asymmetric orbifold, where four right-moving coordinates $J_I \rightarrow -J_I$, $I = 2, 3, 4, 5$ are twisted. The phase factors $(-)^{b\gamma + g + h}, (-)^{h}$ are dictated by modular invariance.

As we have already explained, we would like to examine the behaviour of the $N = (4L + 2R)$ model in the decompactification limits $R_1 \rightarrow \infty$ and $R_1 \rightarrow 0$. For this purpose we must deform the above model and move away from the fermionic point by switching on the marginal deformation that is associated to the modulus $R_1$.

Before doing that, let us first examine the moduli space of the above model, namely all possible $J^L, J^R$ (1,1)-deformations which we are able to switch on simultaneously. Due to the left–right asymmetry the only possible deformations are:

$$J^L_1 \cdot J^R_1 \rightarrow R_1 \quad \text{and} \quad J^L_1 \cdot J^R_1 \rightarrow Y_I, \ I = 2, 3, 4, 5.$$ 

The first deformation corresponds to the $R_1$ modulus while the remaining four correspond to the Wilson lines $Y_I$ moduli. Altogether they form the perturbative moduli space constructed from the $\text{NS-scalars } M_p = O(5, 1)$.

The perturbative moduli space is extended to $M_{np} = SU^*(6)/USp(6)$ once we include the dilaton moduli (singlet under $O(5)$) and the remaining $\text{RR-scalars (in 15, 10, and 10'}$ representations of $O(5)$). The perturbative string states form the charge lattice $\Gamma_{15,10}$ associated to the moduli space $\mathcal{M}_p$.

The absence of continuous deformations associated to the twisted coordinates $\phi^R_I, \ I = 2, 3, 4, 5$ implies that all radii $R_I, \ I = 2, 3, 4, 5$, of the “twisted” four-dimensional space must be frozen to some special values of the moduli space (the $SO(8)$ fermionic point in our case). This also means that the asymmetric orbifold projection is well defined only for special values of the moduli of the initially untwisted $\Gamma_{4,4}$ lattice, e.g. the fermionic $SO(8)$ symmetric point.

Keeping this point in mind we can generalize the model to include the radius and Wilson line deformations by replacing $Z_{5,5,1}^{[h]}$ by $Z_{5,5}(R_1,Y_I)$; since the non-vanishing Wilson lines do not affect the decompactification limits we will restrict ourselves to $Y_I = 0$, keeping the modulus $R_1$ arbitrary:

$$Z_{5,5,1}^{[h]}(R_1, Y_I = 0) = \frac{\Gamma_{1,1,1}^{[h]}(R_1)}{\eta \bar{\eta}} Z_{4,4,1}^{[h]}. \quad (3.15)$$

where $Z_{4,4,1}^{[h]}$ is the asymmetric orbifold contribution of the four internal coordinates $\phi_I, \ I = 2, 3, 4, 5$

$$Z_{4,4,1}^{[h]} = \frac{1}{2} \sum_{\gamma, \delta} \left[ \frac{\theta^{[\gamma]} \bar{\theta}^{[\gamma+ h]} \bar{\theta}^{[\gamma - h]} \theta^{[\gamma - h]}}{\eta^4} \right] (\eta \bar{\eta})^4 \quad (3.16)$$
\[ \Gamma_{1,1}[\phi](R_1) = \sum_{m,n} \exp \left[ i\pi gm + i2\pi \tau P_L^2 - i2\pi \bar{\tau} P_R^2 \right] \]

with

\[ P_L = \frac{1}{2} \left[ \frac{m}{R_1} + \frac{(2n + h)R_1}{2} \right], \quad P_R = \frac{1}{2} \left[ \frac{m}{R_1} - \frac{(2n + h)R_1}{2} \right] \]

(3.17)

is the shifted \( \phi_1 \)-lattice \[15], \[16], \[17].

The \( N = (4L+2R) \) model constructed above is an asymmetric freely acting orbifold. Following refs. \[15], \[16], \[17\] it corresponds to a partial spontaneous breaking \[15], \[16\] of supersymmetry \( N = (4L+2R) \rightarrow N = (4L+2R) \). The massless states of this model consist only of \( N = (4L+2R) \) gravitational multiplet. The remaining massless states of an \( N = (4L+4R) \) (two 3/2-multiplets) become massive, with a common mass equal to:

\[ (m_{3/2})_{7,8} = \frac{1}{R_1^2} \quad (h = 0, \ |m| = 1, \ n = 0). \]

These states correspond to the “untwisted” sector states with momentum \( |m| = 1 \) and \( 2n + h = 0 \). The “odd” winding states \( h = 1 \) correspond to the twisted sector. The lowest twisted states correspond to two massive 3/2-multiplets with masses proportional to:

\[ (m_{3/2})_{7,8'} = \frac{R_2^2}{4} \quad (h = 1, \ m = 0, \ |2n + h| = 1) \]

4 Six-dimensional decompactification of the five-dimensional \( N = (4L + 2R) \) string model

A six-dimensional model in string theory can be defined from a five-dimensional one in two different ways: either by sending the compactification radius \( R_1 \rightarrow \infty \) or \( R_1 \rightarrow 0 \). In the first case the Kaluza-Klein (KK) momenta become the six-dimensional continuous momentum \( P_6^2 \sim m^2/R_1^2 \). In the limit \( R_1 \rightarrow 0 \), however, the KK states become superheavy while the string windings give rise to the six-dimensional continuous momentum \( \tilde{P}_6^2 \sim n^2R^2 \). Starting with the \( D = 5, \ N = (4L + 2R) \) string defined in the previous section, we would like to examine the six-dimensional theories obtained in the two stringy decompactification limits.

The five-dimensional spectrum of this model always contains 6 massless gravitinos. There is also a tower of massive gravitinos with the same \( R \) symmetry charges having even KK-momentum \( m \) and even winding charge \( n' = 2n + h \) (\( h = 0 \)). When \( R_1 \rightarrow \infty \) or 0 one obtains 6 massless gravitinos with continuous momenta, either \( P_6 \) or \( \tilde{P}_6 \). However, there exist two more extra towers of massive gravitinos with different \( R \)-symmetry charges, which can become massless in the two decompactification limits, namely:
In the limit $R_1 \to \infty$ the (two) massive gravitinos with odd KK-momentum $m$ become massless, $(m_{3/2}^2)_{7,8} \to 0$, while the “winding” gravitinos $(m_{3/2}^2)_{7',8'} \to \infty$ become superheavy and decouple from the spectrum. The two extra massless gravitinos $(m_{3/2}^2)_{7,8} = 0$ together with the six massless gravitinos of $N = (4L + 2R)$ restore in six dimensions the $N = (4L + 4R)$ supersymmetry.

In the other limit, $R_1 \to 0$, the two gravitinos with odd KK momentum $m$, $(m_{3/2}^2)_{7,8} \to \infty$ become superheavy and decouple while the gravitinos with winding $n' = 2n + h$ odd ($h = 1$) become massless $(m_{3/2}^2)_{7',8'} \to 0$.

The “field theory” analogue of the above model can be obtained by a Scherk-Schwarz \cite{18} supersymmetry breaking mechanism \cite{19}, \cite{15}, \cite{16} starting either from $N = 2$ in $D = 10$ or from $N = 1$ in $D = 11$. The difference from the string model is the absence of the winding states, $n' = 2n + h = 0$ ($n = 0$, $h = 0$). Therefore, in field theory, the first decompactification limit, $R_1 \to \infty$, gives the same effective supergravity theory as in string theory. On the other hand the second limit, $R_1 \to 0$, does not correspond any more to a decompactification but rather to a “dimensional reduction” where all KK states are truncated out from the spectrum. In the limit $R_1 \to 0$ the field theory remains five-dimensional.

The restoration of the maximal supersymmetry in $D = 6$ can be checked in the two decompactification limits by constructing the helicity supertrace \cite{20}, \cite{21}, \cite{14}, \cite{16}, \cite{17} as a function of the radius $R_1$:

$$B_{2n}(R_1; t) = \text{tr} \left( (-)^{2n} [s]^{2n} e^{-\pi M_s^2(R_1)} \right) = \text{str} \left( [s]^{2n} e^{-\pi M_s^2(R_1)} \right). \quad (4.18)$$

The little group of massless particle in in $D=5$ is $SO(3)$. By “helicity” $s$ we mean $U(1)$ charge of a $U(1)$ sub-group of $SO(3)$. $M_s(R_1)$ denotes the mass of the state.

If there is a restoration of $N = (4L + 4R)$, $B_6$ has to vanish in both decompactification limits due to the $N = 8$ supertrace identities:

$$B_{2n} = 0, \quad n = 0, 1, 2, 3. \quad \text{in} \quad N = 8.$$ 

In $N = 6$ $B_6$ is not trivial since the supertrace identities for $N = 6$ are:

$$B_{2n} = 0, \quad n = 0, 1, 2. \quad \text{in} \quad N = 6.$$ 

The supergravity massless sector of the $N = 6$ supergravity gives a non-trivial $B_6$(SUGRA) = 45/2. If in the decompactification limits the $N = 6$ supersymmetry is extended to $N = 8$ due to the presence of extra massless states, then $B_6 = 0$ must be found in both limits.

In order to derive $B_{2n}$ in string theory it is convenient to define the helicity generating partition function \cite{27}, \cite{13}, \cite{10}, \cite{25}, \cite{24}:

$$Z_{\text{string}}(v, \bar{v}) = \text{Tr} \ q^{L_0} \bar{q}^{\bar{L}_0} \ e^{2\pi i s L_0 - 2\pi i \bar{s} \bar{L}_0}, \quad q = e^{2\pi i v}, \quad \bar{q} = e^{2\pi i \bar{v}}, \quad t = \text{Im} \tau, \quad (4.19)$$
where \( s_L \) and \( s_R \) denote the left- and right-moving helicities. The physical helicity is given by \( s = s_L + s_R \). Once \( Z^{\text{string}}(v, \bar{v}) \) is defined, then \( B_{2n} \) can be derived from \( Z^{\text{string}}(v, \bar{v}) \) by the action of the left- and right-helicity operators

\[
Q = \frac{1}{2\pi i} \frac{\partial}{\partial v}, \quad \bar{Q} = -\frac{1}{2\pi i} \frac{\partial}{\partial \bar{v}}, \quad \text{so that} \quad B_{2n}^{\text{string}} = (Q + \bar{Q})^{2n} Z^{\text{string}}(v, \bar{v}) \big|_{v=\bar{v}=0}.
\]

The \( v, \bar{v} \) helicity modifications for the partition function have been studied earlier, in order to obtain exact solutions of string theory in the background of magnetic fields and to investigate the associated phase-transition phenomena \cite{22}, \cite{23}, \cite{21}. In that context the quantities \( v \) and \( \bar{v} \) play the role of the background magnetic field.

An explicit expression for \( Z^{\text{string}}(v, \bar{v}) \) for the \( N = (4, 2) \) string model of the previous section is given by an expression that is similar to the \( v = \bar{v} = 0 \) partition function:

\[
Z^{\text{string}}(v, \bar{v}) = \frac{\xi(v) \bar{\xi}(\bar{v})}{\eta^3 \bar{\eta}^3} \frac{1}{4} \sum_{a,b,a,b=0} (-)^{a+b+ab} \frac{(-)^{\bar{a}+\bar{b}+\bar{a} \bar{b}}}{\eta^4 \bar{\eta}^4} \times \frac{1}{2} \sum_{h,g} \theta[a][v] \bar{\theta}[\bar{a}][\bar{v}] \theta[b][\bar{b}] \bar{\theta}[\bar{b}][v] \theta[\bar{a}][\bar{b}] \bar{\theta}[\bar{a} \bar{b}][v] \bar{\theta}[\bar{a} \bar{b}][\bar{v}] Z_{5,5}^{h}[\theta](R_1),
\]

where the \( \xi(v) \) and \( \bar{\xi}(\bar{v}) \) modifications is due to the helicity charge of the 2d bosonic oscillators. The \( v, \bar{v} \) modifications due to the 2d fermionic degrees of freedom give rise to non-zero characteristics to the fermionic \( \theta[a][v] \) and \( \bar{\theta}[\bar{a}][\bar{v}] \) functions \cite{22}, \cite{13}, \cite{14}, \cite{25}, \cite{24}. The analytic expression of \( \xi(v) \) is:

\[
\xi(v) = \frac{\prod_{1}^{\infty} (1 - q^n e^{2\pi iv})(1 - q^n e^{-2\pi iv})}{\sin \pi v} \frac{\theta[1]}{\theta[1]} \frac{\eta^2}{\eta^2} = \frac{\sin \pi v}{\pi} \frac{\theta[1]}{\theta[1]},
\]

\[
\xi(v) = \xi(-v), \quad \xi(0) = 1.
\]

In the expression for \( Z^{\text{string}}(v, \bar{v}) \) we can sum over the indices \( (a, b) \) and \( (\bar{a}, \bar{b}) \) using the Riemann identity of theta functions

\[
\frac{1}{2} \sum_{a,b} (-)^{a+b+ab} \eta^4 \theta[a][v] \bar{\theta}[\bar{a}][\bar{v}] \theta[b][\bar{b}] \bar{\theta}[\bar{b}][v] \theta[\bar{a}][\bar{b}] \bar{\theta}[\bar{a} \bar{b}][\bar{v}] = \frac{1}{4} \theta[1] \left( \frac{v}{2} \right) \theta[1] \left( \frac{v}{2} \right) \theta[1] \left( \frac{v}{2} \right) \theta[1] \left( \frac{v}{2} \right),
\]

and

\[
\frac{1}{2} \sum_{a,b} (-)^{\bar{a}+\bar{b}+\bar{a} \bar{b}} \bar{\eta}^4 \theta[\bar{a}][\bar{v}] \bar{\theta}[\bar{a}][\bar{v}] \bar{\theta}[\bar{a} \bar{b}][\bar{v}] \theta[\bar{a} \bar{b}][\bar{v}] = \frac{1}{4} \bar{\theta}[1] \left( \frac{\bar{v}}{2} \right) \bar{\theta}[1] \left( \frac{\bar{v}}{2} \right) \bar{\theta}[1] \left( \frac{\bar{v}}{2} \right) \bar{\theta}[1] \left( \frac{\bar{v}}{2} \right),
\]

(4.24)

The vanishing of \( B_0, B_1 \) and \( B_2 \) follows automatically from the vanishing of \( \theta[1](v/2) \) and \( \bar{\theta}[1](\bar{v}/2) \) for \( v = 0 \) and \( \bar{v} = 0 \): Indeed, \( Z^{\text{string}}(v, \bar{v}) \) vanishes like \( v^4 \bar{v}^2 \) in the
sector with \((h, g) \neq (0, 0)\) \((N = 6\) sector\), and it vanishes like \(v^4 \bar{v}^2\) in the sector with \((h, g) = (0, 0)\) \((N = 8\) sector\). Therefore, the only non-trivial helicity supertrace is
\[
B_6 = (Q + Q)^6 Z_{\text{string}}(v, \bar{v})|_{v = \bar{v} = 0} = 15Q^4 Q^2 Z_{\text{string}}(v, \bar{v})|_{v = \bar{v} = 0},
\]
receiving non-vanishing contribution from the \(N = 6\) sector with \((h, g) \neq (0, 0)\).

Furthermore, using the identities
\[
4iQ \theta_1^{[1]}(v/2) = \theta_0^{[0]} \theta_1^{[1]} = 2\eta^3,
\]
the expression for \(B_6\), as a function of \(R_1\) \([24], [25]\), simplifies to:
\[
B_6 = \frac{45}{4} \sum_{(h, g) \neq (0, 0)} \chi_{[g]}^{[h]} \Gamma_{1,1}^{[h]} |_{R_1},
\]
where \(\Gamma_{1,1}^{[h]} |_{R_1}\) is the radius-dependent shifted lattice and \(\chi_{[g]}^{[h]}\) \([24], [25]\) is a holomorphic function coming from the four left-moving invariant coordinates \(\phi_I^L, I = 2, 3, 4, 5:\)
\[
\chi_{[g]}^{[h]} = \frac{1}{2} \sum_{\gamma, \delta} \theta_1^{[\gamma]} \left[ e^{i\pi(h + g\gamma)} - e^{i\pi(g + h\delta)} \right].
\]

In the infrared limit \(\text{Im} \tau \to \infty\) only the massless states give a non-zero contribution for any finite \(R_1\). In this limit
\[
B_6(t \to \infty) = 45/2 = -360 \ s_{1/2}, \quad t \equiv \text{Im} \ \tau
\]
\(s_{1/2} = -1/16\) denotes the contribution to \(\text{str}[s] \) of a massless spin-1/2 state. Thus, \(B_6(t \to \infty)\) matches the contribution of the massless fields of the \(N = 6\) supergravity multiplet \(B^{N=6}_{6}\) (SUGRA) = \(-360 \ s_{1/2}\). Therefore for any finite \(R_1\) the massless degrees of freedom are precisely those of the \(N = (4L + 2R)\) supergravity multiplet in \(D = 5\).

Having obtained \(B_6\) as a function of \(R_1\), we can examine the limits \(R_1 \to \infty\) and \(R_1 \to 0\). In the first limit the winding states are superheavy and give exponentially suppressed contributions to \(B_6\). Only the zero winding sector survives:
\[
B_6(R_1 \to \infty) = \frac{45}{4} \chi_0^{[1]} \Gamma_0^{[1]} = \frac{45}{4} \left[ \theta_3^0(\tau) + \theta_4^0(\tau) \right] \theta_4 \left[ \frac{it}{R_1^2} \right] = \frac{45}{2} \theta_4 \left[ \frac{it}{R_1^2} \right]
\]
where the last equality follows from the left/right mass-level matching condition in the zero winding sector. Performing a Poisson resummation of the last expression we find
\[
\left( B_6 \right)_{D=5} = R^{-\frac{3}{2}} \left( B_6 \right)_{D=6},
\]
with
\[
\left( B_6 \right)_{D=6} = \frac{45}{2} \theta_2 \left[ \frac{iR_1^2}{t} \right] \sim \frac{45}{2} \theta_2 \left( 2\exp \left( -\pi R_1^2/4t \right) \right).
\]
In the above equation we renormalize the six-dimensional $(B_6)_{D=6}$ by dividing out the $S^1$ volume $R_1$ and the contribution of the zero modes of a non-compact coordinate $t^{−1/2}$. Thus we show that $B_6$ vanishes exponentially in the decompactification limit $R_1 \to \infty$ due to the restoration of $N = 8$ in $D = 6$.

When $R_1 \to 0$ the KK momenta become superheavy, so that only the sector with zero KK-momenta ($m = 0$) survives:

$$B_6(R_1 \to 0) = \frac{45}{4} \sum_{(h,g) \neq (0,0)} \chi_{[h]}^{[0]} \Gamma^{[0]}_{[g]} |_{m=0}$$

$$= \frac{45}{4} \left( \chi_{[0]}^{[0]} + \chi_{[1]}^{[1]} \right) \theta_2 \left[ itR_1^2 \right] + \frac{45}{4} \chi_{[1]}^{[0]} \theta_3 \left[ itR_1^2 \right]$$

$$= \frac{45}{2} \left( \theta_3 \left[ itR_1^2 \right] - \theta_2 \left[ itR_1^2 \right] \right), \quad (4.33)$$

where we have used the identity $\chi_{[0]}^{[0]} + \chi_{[0]}^{[1]} + \chi_{[1]}^{[1]} = 0$, and the left/right-mass level matching conditions are valid in the $m = 0$ sector. Performing a Poisson resummation of the above expression for $B_6$ we obtain:

$$(B_6)_{D=5} = \frac{t^{−1/2}}{R_1} (B_6)_{D=6} \quad (4.34)$$

with

$$(B_6)_{D=6} = \frac{45}{2} \left( \theta_3 \left[ \frac{i}{tR_1^2} \right] - \theta_4 \left[ \frac{i}{tR_1^2} \right] \right) \sim \frac{45}{2} \mathcal{O} \left( 2 \exp \left( \frac{-\pi}{tR_1^2} \right) \right), \quad (4.35)$$

where the normalization factor from $D = 5$ to $D = 6$ now involves the dual $S^1$ volume $1/R_1$. As in the previous decompactification limit, $B_6(R_1 \to 0)$ vanishes exponentially in the dual decompactification limit ($R_1 \to 0$) due to the restoration of $N = 8$ in $D = 6$.

5 Conclusions

In this paper we have given evidence for the existence of a chiral $N = (4, 2)$ supergravity in six dimensions, which reduces to known $N = 6$ supergravity upon dimensional reduction at $D = 5$ and $D = 4$.

This theory is based on an automorphism group of the supersymmetry algebra, which is $H = USp(4) \times USp(2)$ and a “duality group”, which is $O(1, 5)$. Therefore the fermions of this theory are in $H$-representations, the vectors are in the $O(1, 5)$ spinor representation, the two-forms are in the vector representation of $O(1, 5)$ and the scalars are coordinates of the $O(1, 5)/O(5)$ coset. This is a consequence of the $(0, 2)$ decomposition of the $(4, 2)$ supergravity multiplet, which reveals that the scalars are in five tensor multiplets [20].
This theory, like $(2,0)$ and $(4,0) D = 6$ supergravities, is a non-Lagrangian theory, due to the presence of self-dual 2-forms, but it can presumably be defined through a consistent set of equations of motion, by using supersymmetry. However, unlike the other chiral theories, this theory has a gravitational anomaly that cannot be cancelled by adding matter, since there are no $N = (4,2)$ multiplets with spin $J < 3/2$.

As a consequence the $N = (4,2)$ theory is inconsistent at the quantum level. A way the anomaly cancellation can be achieved is, either by adding another gravitino multiplet, which enlarges the theory to $N = (4,4)$, or by adding twelve $N = (4,0)$ matter multiplets, which are expected to explicitly break two supersymmetries and then make the theory inconsistent.

It is interesting to understand why the analogous theories at $D = 4$ and $D = 5$ are consistent string backgrounds while the $D = 6$ case is not. If we define the $D = 6$ theory by decompactifying one dimension from the $D = 5$ case, then some extra massive states become massless in the decompactification limits $R_6 \to \infty$ or 0. In both limits the extra massless states correspond to an extra $N = (4,2)$ gravitino multiplet, which cancels precisely the six-dimensional anomaly and the $B_6$-helicity supertrace at the same time. The supersymmetry is extended to the maximal one and the theory becomes left/right-symmetric.

**Acknowledgements**

We would like to thank L. Andrianopoli, M. Flato and C. Fronsdal for interesting discussions.

This work is supported in part by EEC under TMR contracts ERB-4061-PL-95-0789, ERBFMRX-CT96-0045 and by DOE grant DE-GG03-91ER40662.
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