Production of intense episodic Alfvén pulses: GRMHD simulation of black hole accretion disks

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ABSTRACT

The episodic dynamics of the magnetic eruption of a spinning black hole (BH) accretion disks and its associated intense shapeup of their jets is studied via three-dimensional general-relativistic magnetohydrodynamics (GRMHD). The embedded magnetic fields in the disk get amplified by the magnetorotational instability (MRI) so large as to cause an eruption of magnetic field (reconnection) and large chunks of matter episodically accrete toward the roots of the jets upon such an event. We also find that the eruption events produce intensive Alfvén pulses, which propagate through the jets. After the eruption, the disk backs to the weakly magnetic states. Such disk activities cause short time variabilities in mass accretion rate at the event horizon as well as electromagnetic luminosity inside the jet. Since the dimensionless strength parameter $a_0 = eE/m_e\omega_c$ of these Alfvén wave pulses is extremely high for a substantial fraction of Eddington accretion rate accretion flow onto a supermassive black hole, the Alfvén shocks turn into ultrarelativistic ($a_0 \gg 1$) bow wake acceleration, manifesting into the ultra-high energy cosmic rays and electrons which finally emit gamma-rays. Since our GRMHD model has universality in its spatial and temporal scales, it is applicable to a wide range of astrophysical objects ranging from those of AGN (which is the primary target of this research), to micro-quasars. Properties such as time variabilities of blazar gamma-ray flares and spectrum observed by Fermi Gamma-ray Observatory are well explained by linear acceleration of electrons by the bow wake.

Key words: (magnetohydrodynamics) MHD – accretion discs – (galaxies:) quasars: supermassive black holes — galaxies: jets

1 INTRODUCTION

Active galactic Nuclei (AGNs) are high energy astronomical objects, so that they emit non-thermal radiation in any frequency ranges of radiation, in other words, radio, infrared, visible lights, ultraviolet, X-rays, and gamma-rays (Begelman et al. 1984; Hughes 1991; Burgarella et al. 1993; Tsiganos 1996; Ferrari 1998). These central engines are believed to be accreting supermassive black holes, with relativistic jet whose bulk Lorentz factor is $\sim 10$ (Biretta et al. 1999; Asada et al. 2014; Boccardi et al. 2016). Their jets show strong time variabilities in the timescales from days to years (Fossati et al. 1998; Abdo et al. 2009, 2010a,b,c, 2011; Ackermann et al. 2010; Chen et al. 2013; Edelson et al. 2013). In the extreme cases, blazars show bursts of hours (Ackermann et al. 2016; Britto et al. 2016). These radia-

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the large number of scatterings necessary to reach highest energies, 2) energy losses through the synchrotron emission at the bending associated with scatterings, and 3) difficulty in the escape of particles which are initially magnetically confined in the acceleration domain (e.g., Kotera & Olinto (2011)).

On the other hand, Tajima & Dawson (1979) proposed that particles can be accelerated by the wakefield induced by an intense laser pulse, see review by Tajima et al. (2017). This long lasting energy elevated state of wakefield may be regarded a Higgs state (Higgs 1964) of plasma. In particular, ponderomotive force which is proportional to gradient of $E^2$ works to accelerate the charged particles, effectively, where $E$ is electric field of the electromagnetic wave. The acceleration towards relativistic regime by the ponderomotive force is confirmed by recent experiments by ultra intense lasers for electrons (Leemans et al. 2006; Nakamura et al. 2007). Positron acceleration driven by wakefields by electrons is also reported by Corde et al. (2015).

Takahashi et al. (2000); Chen et al. (2002); Chang et al. (2009) applied this mechanism to magnetotowave induced plasma wakefield acceleration for ultra high energy cosmic rays. Recently, Ebisuzaki & Tajima (2014a,b) applied this wakefield acceleration theory to the relativistic jets launched from an accreting black hole. In such astrophysical context going far beyond the laboratory scales of wakefields (see, for example, Tajima et al. (2017)), the relativistic factors that characterize the dynamics $a_0 = E/m_e c$ becomes far greater than unity. This has not been achieved in the laboratory yet, though simulations began to peek into it. In this regime (Ebisuzaki & Tajima 2014a,b) the ponderomotive acceleration has advantages over the Fermi mechanism. In close scrutiny, the wakefields are composed with two parts the frontal bow part and the following stern (wake) part. Here we may call simply the bow wave and stern wave. In the work of high $a_0$ simulation, it was shown in Lau et al. (2015) that the bow wave (it is driven directly by the ponderomotive force) is dominant over the stern wake. The advantages of this bow wake acceleration over the Fermi mechanism are:

(i) The ponderomotive force provides an extremely high accelerating field (including the wakefield).

(ii) It does not require particle bending, which would cause strong synchrotron radiation losses in extreme energies.

(iii) The accelerating fields and particles move in the collinear direction at the same velocity, the speed of light, so that the acceleration has a built-in coherence called “relativistic coherence” (Tajima 2010); in contrast, the Fermi acceleration mechanism, based on multiple scatterings, is intrinsically incoherent and stochastic.

(iv) No escape problem (Kotera & Olinto 2011) exists. Particles can escape from the acceleration region since the accelerating fields naturally decay out.

They found that protons can be accelerated even above $Z eV \sim 10^{22} eV$ in the bow wake of a burst of Alfven waves emitted by an accretion disk around a black hole with the mass of $10^8 M_\odot$. Ebisuzaki & Tajima (2014a) used three major assumptions based on the standard $\alpha$-disk model (Shakura & Sunyaev 1973).

- Assumption A: the magnetic field energy $E_B$ included in an Alfven wave burst is assumed as:

$$E_B = (B_D^2/4\pi)\pi(10R_0)^2ZD = 1.6 \times 10^{48}(m/0.1)(m/10^8)^2 \ erg.$$  

where $B_D$ is the magnetic field stored in the inner most regions of the accretion disk, $R_0 = 2R_\bullet$ is the Schwarzschild radius of the black hole, $Zp$ is the thickness of the disk, $R_\bullet$ is gravitational radius, $m$ is the accretion rate normalized by the Eddington luminosity, and $m$ is the mass of the black hole in the unit of solar mass.

- Assumption B: they assumed that the angular frequency $\omega_A$ of the Alfven wave corresponds to that excited by magnetorotational instability (MRI (Velikhov 1959; Chandrasekhar 1960; Balbus & Hawley 1991; Matsumoto & Tajima 1995)), which takes place in a magnetized accretion disk, in other words:

$$\omega_A = 2\pi c_{AD}/\lambda_A \sim 2.6 \times 10^{-5}(m/10^8)^{1/2} Hz.$$  

where $\lambda_A$ is the wavelength of the Alfven wave, and $c_{AD}$ is speed of Alfven wave. Ebisuzaki & Tajima (2014a) showed that the Alfven shock gives rise to electromagnetic wave pulse with $\omega = \omega_A$ along the propagation in the jets through the mode conversion, as the density and magnetic fields in the jets decrease during the jet propagation.

- Assumption C: the recurrence rate $\nu_A$ of the Alfven burst is evaluated as:

$$\nu_A = \eta\nu_{AD}/ZD \ Hz,$$  

where $\eta$ is the episode-dependent parameter of the order of unity.

They found that the non-dimensional strength parameter $a_0 = E/m_e c$ is as high as $10^{10}$ for the case of $m = 0.1$ and $m = 10^8$, where $e$ is electric charge, $E$ is the intensity of the electric field, $m_e$ is mass of electron, and $c$ is speed of light. The ponderomotive force of this extremely relativistic waves co-linearly accelerate to the jet particle up to the maximum energy:

$$W_{\text{max}} = 2.9 \times 10^{22}\sqrt{\Gamma/20}/(m/0.1)^{1/3}(m/10^8)^{2/3} \ eV.$$  

where $q$ is the charge of the particle and $\Gamma$ is the bulk Lorentz factor of the jet. Recent one-dimensional particle in cell (PIC) simulation shows maximum energy gain via a ponderomotive force in the bow wake and the maximum energy is almost proportional to $a_0^2$ (Lau et al. 2015). Based on the above estimation, Ebisuzaki & Tajima (2014a) concluded that the accreting supermassive black hole is the ZeV ($10^{22} eV$) linear accelerator.

GRMHD simulations of accretion flows onto the black hole have been done since early works by Koide et al. (1999, 2000). Some improvements in the numerical method for solving GRMHD equations made it possible to follow the long-term dynamics of magnetized accretion flows (De Villiers et al. 2003; Gammie et al. 2003). 2D axisymmetric and full 3D simulations have been done to study properties of accretion disk, Blandford-Znajek efficiency, jet and so on.

McKinney & Gammie (2004) have studied outward going electromagnetic power through the event horizon, i.e, Blandford-Znajek process by 2D axis-symmetric simulations. McKinney (2006) studied long term magnetized jet
2 GENERAL RELATIVISTIC MAGNETOHYDRODYNAMIC SIMULATION METHOD

2.1 Basic Equations

We numerically solve general relativistic magnetohydrodynamic equations, assuming a fixed metric around a black hole. The unit in which \( G M_{\text{BH}} \) and \( c \) are unity is adopted, where \( G \) is gravitational constant, \( M_{\text{BH}} \) is mass of the central black hole. The scales of length and time are \( R_{g} = \sqrt{G M_{\text{BH}} / c^{2}} \) and \( G M_{\text{BH}} c^{-2} \), respectively. The mass and energy is scale free. The achieved mass accretion rate at the event horizon is used to scale of the mass and energy, for example. The metric around a rotating black hole whose dimensionless spin parameter is \( a \) can be described by Boyer-Lindquist (BL) coordinate or Kerr-Schott (KS) coordinate.

The line element in BL coordinate is

\[
d s_{\text{BL}}^{2} = g_{tt} d t^{2} + g_{rr} d r^{2} + g_{\theta \theta} d \theta^{2} + g_{\phi \phi} d \phi^{2} + 2 g_{\theta \phi} d \theta d \phi, \tag{5}
\]

where \( g_{tt} = -(1 - 2 r / \Sigma) \), \( g_{rr} = \Sigma / \partial r \), \( g_{\theta \theta} = \Sigma \), \( g_{\phi \phi} = A \sin^{2} \theta / \Sigma \), \( g_{\theta \phi} = -2 a \sin \theta \cos \theta \), \( \Delta = r^{2} - 2 r + a^{2} \), and \( A = (r^{2} + a^{2})^{2} - a^{2} \Delta \sin^{2} \theta \). We follow standard notation used in Misner et al. (1973), i.e., metric tensor for Minkowski space is \( \delta_{\mu \nu} = (1, 1, 1, 1) \).

The line element in Kerr-Schott (KS) coordinate is

\[
d s_{\text{KS}}^{2} = g_{\tau \tau} d \tau^{2} + g_{r r} d r^{2} + g_{\theta \theta} d \theta^{2} + g_{\phi \phi} d \phi^{2} + 2 g_{\theta \phi} d \theta d \phi, \tag{6}
\]

where \( g_{\tau \tau} = -(1 - 2 r / \Sigma) \), \( g_{r r} = \Sigma / \partial r \), \( g_{\theta \theta} = \Sigma \), \( g_{\phi \phi} = A \sin^{2} \theta / \Sigma \), \( g_{\theta \phi} = -2 a \sin \theta \cos \theta \), and \( \phi = \theta + \alpha \). We also use so-called modified Kerr-Schott (mKS) coordinate \((x_{0}, x_{1}, x_{2}, x_{3})\) so that the numerical grids are fine near the event horizon and the equator. The transformation between Kerr-Schott coordinate and modified Kerr-Schott coordinate is described as \( z = x_{0}, \ r = \exp(\chi_{1}, \ \theta = \pi x_{2} + \frac{\chi(1 - h)}{2} \sin(2 \pi x_{2}), \ \phi = x_{3} \), where \( h \) is a parameter which controls how the grids are concentrated around the equator. We have done three cases of resolution in the polar and azimuthal angle grid. Constant grid for polar angle, i.e., \( h = 1 \) are used for two lower resolution cases. The grid numbers are \( N_{1} = 124, \ N_{2} = 124, \) and \( N_{3} = 60, \) and \( N_{1} = 124, \ N_{2} = 252, \) and \( N_{3} = 28 \) which are uniformly spaced for both cases. In another case we set \( h = 0.2 \) so that the polar grids concentrates around the equator. The grid numbers are \( N_{1} = 124, \ N_{2} = 252, \) and \( N_{3} = 60 \) which are uniformly spaced. Since the resolution of polar grid with \( h = 0.2 \) is about 10 and 5 times better than that with \( h = 1 \) at the equator for the cases with \( N_{1} = 124, \ N_{3} = 252, \) respectively, we can capture shorter wavelength and faster growing mode of MRI for poloidal direction by the highest resolution case. The highest resolution is comparable to those used in recent 3D GRMHD simulations (McKinney et al. 2012; Penna et al. 2010). Computational domain covers from inside the event horizon to \( r = 30000R_{g} \), \( [0.01 \pi, 0.99 \pi] \) in polar angle, and \([0, 2 \pi]\) in azimuthal angle. Contravariant vectors in Boyer-Lindquist coordinate and Kerr-Schott coordinate are related with \( u^{\mu}_{\text{KS}} = u^{\mu}_{\text{BL}} + (2 r / a) \partial u_{\text{BL}}, \ u^{0}_{\text{BL}} = u^{0}_{\text{KS}}, \) \( u^{\theta}_{\text{KS}} = u^{\theta}_{\text{BL}}, \) \( u^{\phi}_{\text{KS}} = (a / \Lambda) u^{\phi}_{\text{BL}} \) and \( \phi = \phi_{\text{BL}}. \) Contravariant vectors in Kerr-Schott coordinate and modified Kerr-Schott coordinate are related with \( u^{0}_{\text{KS}} = u^{0}_{\text{mKS}}, \ u^{\theta}_{\text{mKS}} = (\sigma(1 - (1 - h) / \cos(2 \pi x_{2}))) u^{\theta}_{\text{mKS}}, \) and \( \phi_{\text{mKS}} = u^{\phi}_{\text{mKS}}. \) Mass and energy-momentum conservation laws are,

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} \rho u^{\mu} = 0, \tag{7}
\]

\[
\partial_{\mu} \sqrt{-g} T_{\mu \nu}^{\text{EM}} = \epsilon / \sqrt{T_{\mu \nu}^{\text{EM}}}, \tag{8}
\]

where \( T_{\mu \nu}^{\text{EM}} \) is energy momentum tensor, \( \rho \) is rest mass density, \( u^{\mu} \) is fluid 4-velocity, \( g \) is determinant of metric tensor, i.e., \( g_{\text{BL}} = g_{\text{KS}} = -2^{2} / \sin^{2} \theta, \) and \( g_{\text{mKS}} = -\pi^{2} / (1 - (1 - h) / \cos(2 \pi x_{2}))^{2} / \sin^{2} \theta, \) and \( \Gamma_{ij}^{k} = \text{Christoffel symbol which is defined as } \Gamma_{ij}^{k} = (1/2) g^{kl}(\partial g_{lj} / \partial x^{i} + \partial g_{lj} / \partial x^{i} - \partial g_{ij} / \partial x^{l}). \) Energy momentum tensor which includes matter and electromagnetic parts is defined as follows

\[
T_{\mu \nu}^{\text{EM}} = T_{\mu \nu}^{\text{MA}} + T_{\mu \nu}^{\text{EM}}, \tag{9}
\]

\[
T_{\mu \nu}^{\text{MA}} = \rho \mu u^{\mu} u^{\nu} + \mu u_{\nu}, \tag{10}
\]

\[
T_{\mu \nu}^{\text{EM}} = F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} \epsilon^{\mu \nu \rho \sigma} F_{\rho \sigma} F_{\mu \nu}, \tag{11}
\]

where \( h(\equiv 1 + U / \rho + \rho_{h} / \rho) \) is specific enthalpy, \( U \) is thermal energy density, \( \rho_{h} \) is thermal pressure, and \( \epsilon_{\mu \nu \rho \sigma} \) is the Faraday tensor and a factor of \( \sqrt{-g} \) is absorbed into the definition of \( F_{\mu \nu} \). The dual of the Faraday tensor is

\[
\epsilon_{\mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} F_{\rho \sigma}, \tag{12}
\]

where \( \epsilon^{\mu \nu \rho \sigma} = \sqrt{-g} \epsilon^{\rho \sigma \mu \nu} \) and \( \epsilon^{\mu \nu \rho \sigma} \) is the completely antisymmetric Levi-Civita symbol \( (\epsilon^{012} = -\epsilon_{012} = -1). \) The magnetic field observed by normal observer is

\[
g^{\mu \nu} = -F^{\mu \nu} n_{\nu} = \nonumber \epsilon^{\mu \nu}, \tag{13}
\]

where \( n_{\nu} = (-a, 0, 0, 0) \) is the normal observer’s four-velocity and \( \epsilon = \sqrt{1 / g^{\mu \nu}} \) is the lapse. Note the time component of \( \mathbf{B} \) is zero, since \( \mathbf{B} = \mathbf{a} \hat{t} = 0 \). Here we introduce another magnetic field which is used in Gammie et al. (2003); Noble et al. (2006); Nagataki (2009) as

\[
B^{\mu} = \epsilon^{\mu \nu} F^{\nu} = \frac{B_{B}}{a}. \tag{14}
\]
The time component of $B^\mu$ is also zero. We also introduce four magnetic field $b^\mu$ which is measured by an observer at rest in the fluid,

$$b^\mu = -\ast F^{\mu \nu} u_\nu. \quad (15)$$

$B^i$ and $b^\mu$ are related with

$$b^\mu = B^\mu u_\mu, \quad (16)$$

$$b^i = (B^i + u^i b^j)/u^j. \quad (17)$$

$B^\mu u_\mu = 0$ is satisfied. By using this magnetic four vector electromagnetic component of energy momentum tensor and the dual of Faraday tensor can be written as

$$T^{\mu \nu}_{EM} = \frac{1}{2} b^\mu b^\nu + p_b b^\mu b^\nu, \quad (18)$$

$$\ast F^{\mu \nu} = b^\mu b^\nu - b^i u^i = \frac{B^\mu b^\nu - B^\nu b^\mu}{u^\mu}, \quad (19)$$

where the magnetic pressure is $p_b = b^\mu b_\mu/2 = b^2/2$. The Maxwell equations are written as

$$\ast F^{\mu \nu \nu} = 0. \quad (20)$$

By using Eqs. (19) these equations give

$$\partial_t \left( \sqrt{-g} B^i \right) = 0, \quad (21)$$

$$\partial_\nu \left( \sqrt{-g} \sqrt{g} B^i \right) + \partial_j \left( \sqrt{-g} (b^i u^j - b^j u^i) \right) = 0. \quad (22)$$

These are non-monopole constraint equation and time evolution of spacial magnetic field equations, i.e., the induction equations, respectively.

In order to close the equations, ideal gas equation of state $p_b = (\gamma - 1)u^i u_\nu$ is adopted, where $\gamma$ is the specific heat ratio which is assumed to be constant ($\gamma = 4/3$) $^1$. We ignore self-gravity of the gas around the black hole and any radiative processes, assuming radiatively inefficient accretion flow (RIAF) in the disk (Narayan & Yi 1994), although the effects of radiation have been discussed by Ryan et al. (2017). We numerically solve these equations by GRMHD code developed by one of authors (Nagataki 2009, 2011). Magnetohydrodynamic equations are solved by using shock capturing method (HLL method), applying 2nd order interpolation to reconstruct of physical quantities at the cell surfaces and 2nd order time integration by using TVD (total variation diminishing) Runge-Kutta method, see also Gammie et al. (2003); Noble et al. (2006). The boundary conditions are zero gradient for $x_1$ and periodic one for $x_3$.

### 2.2 Initial Condition

We adopt the Fishbone-Moncrief solution as an initial condition for hydrodynamic quantities as adopted for Recent GRMHD simulations (McKinney & Gammie 2004; McKinney 2006; McKinney et al. 2012; Shiokawa et al. 2012). This solution describes the hydro-static torus solution around a rotating black hole. The gravitational force by the central black hole, the centrifugal force and the pressure gradient force balance each other. There are some free parameters to give a solution. We assume the disk inner edge at the equator is at $r = 6.0 R_g$ and a constant specific angular momentum ($l^2 \equiv |u^i u_\phi| = 4.45$). The 4-velocity is firstly given at Boyer Lindquist coordinate then transformed to Kerr-Schild one and to modified Kerr-Schild one. The dimensionless spin parameter is assumed to be $a = 0.9$. The radii of event horizon and the innermost stable circular orbits (ISCO) at the equator are at $r_h(a = 0.9) \sim 1.4 R_g$ and $r_{ISCO}(a = 0.9) \sim 2.32 R_g$, respectively. The initial disk profile is on the plane including polar axis shown in Fig. 1 which shows mass density contour. The disk is geometrically thick and is different from standard accretion disk (Shakura & Sunyaev 1973) in which the geometrically thin disk is assumed.

As we discussed in Sec. 1 magnetic fields play an important role not only for the dynamics of the accretion flows but also for the dynamics of the outflows. We impose initially weak magnetic field inside the disk as a seed. This weak magnetic field violates the initial static situation and is expected to be amplified by winding and MRI.

Here we introduce the four-vector potential $A$ of the electromagnetic field. The Faraday tensor is defined by this vector potential as follows.

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (23)$$

In this study we assume initially closed poloidal magnetic field, i.e., the toroidal component of the initial vector potential is,

$$A_\phi = \max(p/p_{\text{max}} - 0.2, 0). \quad (24)$$

where $p_{\text{max}}$ is maximum mass density in the initial torus. Other spacial components are zero, i.e., $A_r = A_\theta = 0$. The minimal plasma $B$ which is the ratio of the thermal pressure to the magnetic pressure is 100 in the disk. Since the vector potential has only toroidal components, the poloidal magnetic field is imposed. To violate axis-symmetry maximally 5% amplitude random perturbation is imposed in the thermal pressure i.e., thermal pressure is

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$^1$ The calculation with $\gamma = 5/3$ shows some minor differences compared to that with $\gamma = 4/3$ as discussed by McKinney & Gammie (2004); Mignone & McKinney (2007). Similar time variabilities in the inflows and outflows discussed below are observed by adopting two different specific heat ratio in the equation of state.
\( \rho_{\text{th}} = \rho_0 (0.95 + 0.1C) \), where \( \rho_0 \) is equilibrium thermal pressure derived by Fishbone-Moncrief solution and \( C \) is random number in the range of \( 0 \leq C \leq 1 \). This perturbation violates axis-symmetry of the system and triggers generation of non axisymmetric mode.

3 EPISODIC ERUPTION OF DISKS AND JETS

At first we discuss the results based on the highest resolution calculation. The resolution effect, i.e., comparison with the results by lower resolution calculations is briefly discussed later. The main properties, such as the amplification and the dissipation of the magnetic field inside the disk, Alfvén wave emission from the disk, and these time variabilities, which will be discussed below are common for all calculations by different resolutions, although characteristic timescales are different each other.

3.1 B-field amplification and mass accretion

The magnetic field in the disk is amplified by winding effect and MRI, as follows. The initially imposed poloidal magnetic field is stretched in the toroidal direction, generating toroidal components, since there is a differential rotation inside the accretion disk. Although initially imposed magnetic field is weak, i.e., the minimum plasma \( \beta \) is 100 inside the disk, the strength of the magnetic field quickly increases by the winding effect (Duez et al. 2006) and MRI. Soon stratified filaments which is parallel to the equatorial plane appear around the equator in the magnetic pressure contour. Fig. 2(a) shows the volume averaged strength of the magnetic field \( \langle B^2 \rangle \) at the equator i.e., averaged \( \langle B^2 \rangle \) over \( r_{\text{ISCO}} \leq r \leq 10 R_g \), \( \theta = \pi/2 \), and \( 0 \leq \phi \leq 2\pi \). Since the magnetic field is stretched by the differential rotation in the disk, toroidal component dominates over the poloidal component after \( t \sim 100 \, \text{GM}_\text{BH} c^{-3} \), though both components show strong time variability. Fig. 2(b) shows the poloidal accretion rate at the event horizon \( \dot{M}(\eta_L,t) \), where the mass accretion rate at a radius is defined as

\[
\dot{M}(\eta_L,t) = -\int -\mathbf{k} \cdot \mathbf{u} \, dA_{\text{mKS}} \text{d}t,
\]

where \( dA \) is area element, for example, \( dA_{\text{mKS}} = \Delta \eta^2 \Delta \lambda^3 \), and the sign is chosen so that the case of mass inflow is positive mass accretion. The mass accretion rate \( \dot{M}(\eta_L,t) \) also shows strong time variability, and synchronized with magnetic field strength near the ISCO.

Bottom two panels in Fig. 3 show the contours of \( 1/\beta \) at the equator at \( t = 7550 \, \text{GM}_\text{BH} c^{-3} \) and \( t = 7640 \, \text{GM}_\text{BH} c^{-3} \). Between these two figures the state of the disk near the disk inner edge \( (r \sim 6 R_g) \) transitioned from high \( \beta \) state \( (\beta^{-1} \sim 10^{-2}) \) to low \( \beta \) state \( (\beta^{-1} \sim 1) \). Since we sometimes observe that the plasma \( \beta \) in the disk is more than 100, we use “low” \( \beta \) state for the disk with a plasma \( \beta \) of order of unity at which the disk is still gas pressure supported. It should be noted that “low \( \beta \) state” is defined as a magnetically supported disk with \( \beta \lesssim 1 \), for example, Mineshige et al. (1995). The characteristics of two states presented in Mineshige et al. (1995) are listed in the top of Fig. 3. Middle two panels in Fig. 3 show the toroidal magnetic field lines of two disks from MHD simulations by Tajima & Gilden (1987). At the low \( \beta \) state (right panels in Fig. 3) the toroidal magnetic field is stretched to the limit and magnetic field energy is stored. Since some field lines are almost anti-parallel and very close to each other, the reconnection will happen eventually. After magnetic field energy is dissipated via reconnection, the system becomes high \( \beta \) state (left panels in Fig. 3).

Bar structures near the disk inner edge can be seen (bottom panels in Fig. 3). The non-axisymmetric mode is excited, as shown in global hydrodynamic and magnetohydrodynamic simulations of accretion disks, for example Tajima & Gilden (1987); Machida & Matsumoto (2003); Kiuchi et al. (2011); McKinney et al. (2012). Figure 4 shows the contours of \( 1/\beta \) (at the equator), mass density \( (\rho - \rho_g) \), and magnetic pressure \( (\rho - \rho_g) \) at two different times as shown in Fig. 3. Along the polar axis low density and highly magnetized region appears. It corresponds to the Poynting flux dominated jet. Thus baryless and highly magnetized jet is formed along the polar axis. In this region the Alfvén speed is almost speed of light \( \sim c \). A disk wind blows between the jet and the accretion disk. Filamentary structures which are excited by MRI can be seen in the magnetic pressure contours. The thickness of filaments is \( \sim 0.1 R_g \), as shown in Fig. 4.

Both the averaged strength of the magnetic field near the ISCO at the equator and the mass accretion rate at the event horizon show synchronous time variability. This is because that the mass accretion rate at the event horizon is strongly affected by the activities of magnetic field amplification near the disk inner edge. The magnetic field amplification via MRI enhances the specific angular momentum transfer inside the accretion disk, resulting in the increase of the accretion rate at the event horizon. This means that the amplification of magnetic fields acts as a viscosity which is introduced as \( \alpha \)-viscosity (Shakura & Sunyaev 1973). Typical increase timescales are \( 20 - 60 \, \text{GM}_\text{BH} c^{-3} \). While the magnetic field is amplified, the mass accretion rate at the event horizon rapidly increases. As we will show later, the outflow properties are also show intense time variability which is strongly related with particle acceleration via wakefield acceleration.

The mass accretion rate repeatedly shows sharp rises followed by gradual falling down. The rising timescale for the quick increase of mass accretion rate corresponds to the value of the growth timescale of MRI

\[
\tau_1 \sim f_{\text{MRI}} \Omega_{\text{MRI}}^{-1}.
\]

where \( f_{\text{MRI}} \) is order of unity, \( \Omega_{\text{MRI}} \) is growth rate of MRI, and \( \Omega_{\text{MRI}} = 3 \Omega_K/4 \) for the fastest growing mode and \( \Omega_K(r) = r^{-3/2} \) is Newtonian Keplerian angular velocity. The timescale for the fastest growing MRI mode is

\[
\tau_1 \sim 4.7 f_{\text{MRI}} \left( \frac{r}{\text{ISCO}} \right)^{3/2} [\text{GM}_\text{BH} c^{-3}]^{-1}.
\]

This timescale at around \( r = 6 - 8 \, R_g \) is almost as same as the timescale of increase of the strength of magnetic field in the disk. By the analysis of MRI for Newtonian MHD, the angular frequency of the mode is \( k_c \sim \sqrt{15}/16 \Omega_K \approx \Omega_K \) for the fastest growing mode for \( z \) (parallel to polar axis) direction in Keplerian accretion disk, where \( k \) is wave number.
and $c_A$ is Alfvén speed of $z$ component. The volume averaged Alfvén speed ($c_A$)~$\sqrt{\nabla^2 b_0/(\rho \vec{v} + \vec{b}^2)}$ near the ISCO ($r_{\text{ISCO}} < r < 10R_g$) at the equator is typically $3 \times 10^{-3}c$. Thus the wavelength of this mode is $\lambda = 2\pi/k_z = 2\pi(c_A)/\Omega_K \sim 0.022(r/r_{\text{ISCO}})^{1.3}R_g$ with a grid size near the ISCO and at the equator $r\Delta\theta = 0.0057(r/r_{\text{ISCO}})R_g$ for higher resolution case. Our simulation shows that the typical rising timescale in poloidal magnetic field amplification $\sim 30GM_{\text{BH}}c^{-3}$. Corresponding wavelength of MRI is estimated $\sim 0.14R_g$ at $r = 8R_g$. This structure is well resolved by more than 8 grids and is consistent with thickness of the filamentary structure near the equator shown in magnetic pressure panel in Fig. 4.

Since the episodic period of this quick increase of strength of poloidal magnetic field, i.e., large peak to peak, is $\tau_2 \sim 100 - 400GM_{\text{BH}}c^{-3}$ which is about 2-6 times longer than the Keplerian orbital period at $r = 6R_g$ near the ISCO ($\sim 22(r/r_{\text{ISCO}})GM_{\text{BH}}c^{-3}$). This timescale for the recurrence is roughly consistent with the analysis by local shearing box (Stone et al. 1996; Suzuki & Inutsuka 2009; Shi et al. 2010; O’Neill et al. 2011) in which about 10 times orbital period at the radius of magnetic field amplification is observed as repeat timescale.

Along the polar axis funnel nozzles appear (Fig. 4). The outward going electromagnetic luminosity, opening angle of this jet are also time variable like as the mass accretion rate. The radial velocity just above the black hole and becomes positive at typically $10 < r < 20R_g$, i.e., stagnation surfaces.

Figure 5(b) shows radial electromagnetic luminosity calculated by the area integration only around polar region ($0 < \theta < 20^\circ$) at the radius $r = 15R_g$. Electromagnetic luminosity shows similar short time variability with the magnetic field amplification near the disk inner edge. We can see typical rising timescale of the flares is as same as that for rising timescale of magnetic field in the disk, i.e., $\tau_1 \sim 30GM_{\text{BH}}c^{-3}$ and the typical cycle of flares is also as same as repeat cycle of magnetic field amplification $\tau_2 \sim 100GM_{\text{BH}}c^{-3}$. We have observed some active phases in electromagnetic luminosity in the jet. In these active phase, the averaged radial electromagnetic flux increases and becomes about a few tens percent of the averaged disk Alfvén flux at the equator at around $t = 1300, 3000, 4000$, and $8200GM_{\text{BH}}c^{-3}$, as shown in the Fig. 5(a). The disk Alfvén flux at the equator is evaluated as an average of $z$ component of Alfvén energy flux at the equator ($\langle E_{EM}/dV \rangle$) times half of Alfvén speed ($c_A/2$) inside the disk $(r_{\text{ISCO}} < r < 10R_g)$. A large fraction of emitted Alfvén waves goes into the jet, when the level of the electromagnetic luminosity in the jet becomes a few tens percent of Alfvén flux in the disk. This is almost consistent with the assumption A as we discuss in the next section.

Figure 6 shows time evolution and the vertical structure of the averaged toroidal magnetic field ($\langle |B^\phi|^2 \rangle^{1/2}$) as shown in Shi et al. (2010); Machida et al. (2013), i.e, butterfly diagram. The average is taken at $0 < \phi < 2\pi$ and $3R_g \leq R \leq 3.2R_g$, where $R$ is distance from the polar axis. Although initially no magnetic field is imposed at $R = 3R_g$, soon the magnetic field is transported to this site with accreting gas due to MRI growth and angular momentum exchange. After that the magnetic fields quasi-periodically goes up to the north and goes down to the south from above and below the equator due to the magnetic buoyancy, i.e., Parker instability (Parker 1966), as shown by Suzuki & Inutsuka (2009); Shi et al. (2010); Machida et al. (2013). Each episode corresponds to the one cycle of the disk state transitions. The magnetic fields sometimes changes its sign, although it happens much less than that observed in
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| Parameter | High $\beta$ disk | Low $\beta$ disk |
|-----------|-------------------|-----------------|
| $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$ | $\beta > 1$ ($P_{\text{gas}}$ supported) | $\beta \lesssim 1$ ($P_{\text{mag}}$ supported) |
| Configuration | Optically thick disk (cooling-dominated) + corona | Optically thin disk (advection-dominated) consisting of blobs |
| Dissipation of magnetic fields | Escape via buoyancy and reconnection | Reconnection |
| Dissipation of energy | Continuous | Sporadic |
| Spectrum | Soft + hard tail | Hard power law |
| Fluctuations | Small | Large |

**Figure 3.** Table: Properties of high and low $\beta$ states of the disks taken from Mineshige et al. (1995) (©AAS. Reproduced with permission). Middle panels: Toroidal magnetic field lines at high $\beta$ state (left) and low $\beta$ state taken from Tajima & Gilden (1987) (©AAS. Reproduced with permission). Bottom panels: Inverse of plasma beta ($\beta^{-1}$) contours at the equator shown by logarithmic scales at $t = 7550GM_{\text{BH}} c^{-3}$ (low $\beta$ state, left) and at $t = 7640GM_{\text{BH}} c^{-3}$ (low $\beta$ state, right). The shadowed regions in the circle indicate inside the event horizon.

**Figure 4.** Contours of inverse of the plasma $\beta$ (at the equator), mass density ($y-z$ plane), and magnetic pressure ($x-z$ plane) at two different times as shown in Fig. 3. The domain shows $-80GM_{\text{BH}} c^{-3} < x < 0$, $-80GM_{\text{BH}} c^{-3} < y < 0$, and $0 < z < 70GM_{\text{BH}} c^{-2}$. 

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Figure 5. The jet activities. (a): time evolution of area averaged radial electromagnetic flux at \( r = 15R_g \) and \( 0 < \theta < 20^\circ \) (purple). Time evolution of averaged electromagnetic flux inside the disk calculated by \( \langle (E_{EM})/(dV) \rangle (c_A \rangle )/2 \) at the equator and at \( r_{ISCO} < r < 10R_g \) (green). (b) Time evolution of electromagnetic (Poynting) power in the jet at \( r = 15R_g \) calculated by the area integration of electromagnetic flux at \( 0 < \theta < 20^\circ \). (c) same as (b) but the period from \( t = 4000GM_{BH}/c^3 \) to \( t = 6000GM_{BH}/c^3 \) is shown. Rising and repeat timescales of flares are presented in the figure.

Shi et al. (2010) who did high resolution simulations in local hearing box. Around the equator magnetic fields rise up with a speed \( \sim 10^{-3}c \) which corresponds to the averaged Alfvén speed of \( z \) component \( (c_A) \) there. Strong magnetic fields sometimes goes up or goes down from the equator to outside of the disk. The appearance of the flare in the Poynting luminosity (Fig. 5 (b) and (c)) in the jet corresponds to this strong magnetic field escape from the disk.

The magnetic field lines in the jets are connected not with the disks but with the middle and high latitude of the central black hole. The outward going electric magnetic luminosity from the middle and high latitude of the central black hole is not so high as compared with that in the jet. The Alfvén waves emitted from the disks do not directly goes into the jets as assumed in Ebisuzaki & Tajima (2014a). As shown above the time variabilities of the Poynting flux in the jet is as same as that of the magnetic fields strength in the disk and shows strong correlation. The amplification of Poynting flux above the black hole and the disks occurs by the Alfvén fluxes from the disk. Another possibility is that the blobs falling onto the black hall interact with magnetic fields which are connected with those in the jets.

3.2 resolution effect

In this subsection we discuss the resolution effect. We have performed calculations by three different grid types in polar and azimuthal angle as described in sec. 2.1. The highest resolution case for which the results are shown above is by non-uniform grids in polar angle for which the grids concentrates around the equator, resulting in about 10 or 5 times better in the polar angle around the equator than that for other two cases in which constant polar angle grids are adopted with around a half or same number of polar grids. In all cases we have observed properties discussed above, such as, time variable magnetic field amplification in the disk, disk state transition between low and high plasma \( \beta \) states, time variable mass accretion onto black holes, and Poynting flux dominated jet with some flares. The timescale of the fastest growing mode \( (30GM_{BH}/c^3) \) is observed in the amplification of magnetic fields in the disk for highest resolution case, whereas longer timescales (typically \( 50GM_{BH}/c^3 \) for 2nd highest resolution case and \( 80GM_{BH}/c^3 \) or longer time scale for the lowest resolution case) are observed. The thinnest and multiple filaments in magnetic pressure con-
tour around the equator are observed for the highest resolution case. These results mean the longer wavelength mode of MRI is observed in the lower resolution cases. The recurrence timescale for higher resolution case is also faster than that by low resolution cases.

4 PARTICLE ACCELERATION

As shown in Fig. 5 (b) flares in electromagnetic power in the jet are observed, where the Alfvén speed is almost speed of light because of the low mass density. Large amplitude Alfvén waves become electromagnetic waves by mode conversion of strongly relativistic waves (Daniel & Tajima 1997, 1998; Ebisuzaki & Tajima 2014a). The interaction of the electromagnetic waves and the plasma can result in the acceleration of the charged particles by the ponderomotive force i.e., wakefield acceleration (Tajima & Dawson 1979). The key for the efficient wakefield acceleration is the Lorentz invariant dimensionless strength parameter of the wave (Esarey et al. 2009),

$$a_0 = \frac{eE}{m_e\omega_c}. \tag{28}$$

The velocity of the oscillation motion of the charged particles via electric field becomes speed of light, when $a_0 \sim 1$. If the strength parameter $a_0$ highly exceeds unity, the ponderomotive force works to accelerate the charged particles to relativistic regime to wave propagating direction.

4.1 Comparison with Ebisuzaki & Tajima (2014a)

In order to evaluate $a_0$, Ebisuzaki & Tajima (2014a) used three assumptions A, B and C. Based on the results of numerical simulation, we intend to confirm three assumptions. First, assumption A tells us that the Alfvén flux in the jet is equal to that in the disk. As shown in Fig. 5(a), electromagnetic flux in the jet becomes a few tens percent of Alfvén flux in the accretion disk at the some active phases of the electric magnetic luminosity in the jet. Thus most Alfvén waves emitted from the disk via Alfvén burst goes to the jet as assumed in Ebisuzaki & Tajima (2014a) at this epoch, in other words, assumption A, in which all Alfvén waves are assumed to go into the jet.

Second, Ebisuzaki & Tajima (2014a) assumed that magnetic field amplification occurs at $R = 10R_s = 20R_g$ for the standard disk model (Shakura & Sunyaev 1973). In Ebisuzaki & Tajima (2014a) the timescale of the magnetic field amplification ($\tau_1$) and frequency of the Alfvén wave are determined by the MRI growth rate (Eq. (2)). Although they evaluated it around $10 R_s$, the magnetic field amplification occurs at any radius in the disk. Since magnetic field amplification which affects to the mass accretion rate and the time variabilities in the jet mainly occurs inside compared with the assumption by Ebisuzaki & Tajima (2014a), the timescales are shorter than those of them due to faster rotation period. If we apply $R = 6.4R_g = 3.2R_s$ instead of $R = 20R_g = 10R_s$, for Ebisuzaki & Tajima (2014a) model, the timescales are close to our numerical results as shown in table 1. The reason why the magnetic field amplification at smaller radius may be due to the high spinning of the black hole, i.e., $a = 0.9$ for which both the event horizon and the radius of ISCO is smaller than those for non-spinning black hole case ($r_{ISCO}(a = 0) = 6 R_g$). This timescale is well consistent to the rising timescales of blazar flares observed for 3C454.3 which will be discussed in next subsection. In other words, assumption B is OK in qualitatively, but Eq. (2) must to be $\omega_A \sim 1.0 \times 10^{-4} (m/10^8) \times Hz$ (see also table 2).

Finally, Ebisuzaki & Tajima (2014a) estimated the repeat timescale as the crossing time of the Alfvén wave in the disk, i.e, $Z_D/c_A$ (assumption C) for the standard disk model (Shakura & Sunyaev 1973). When we apply the radius $R = 6.4R_g = 3.2R_s$ instead of $R = 20R_g = 10R_s$ in Eq. (3), we obtain $Z_D/c_A = 354GM_{BH}c^{-3}$, ignoring factor $\eta$ which is order of unity. This value is close to our typical $\tau_2 = 100GM_{BH}c^{-3}$. The case of $R = 20R_g = 10R_s$ is also listed in the table 1.

We can reevaluate the strength parameter $a_0$ as

$$a_0 = \frac{eE}{m_e\omega_c} = 1.4 \times 10^{11} \left( \frac{M_{BH}}{10^8 M_\odot} \right)^{1/2} \left( \frac{M_{e}\omega_c^2}{0.1L_{Ed}} \right)^{1/2}. \tag{29}$$

Here electric field is estimated as $E = ((c_{AD})/c)^{1/2} (B_D)$. The angular frequency of the pulsed electromagnetic wave originating from the Alfvén shock (see Ebisuzaki & Tajima (2014a)) $\omega_D = 2\pi c_{AD}/\lambda_{AD}$, where $\lambda_{AD}$ is assumed to be 0.14 $R_g$. We use the values at $t = 7900GM_{BH}c^{-3}$ and the time averaged mass accretion rate $7750GM_{BH}c^{-3} \leq t \leq 8300GM_{BH}c^{-3}$) at the event horizon $M_{e} = 55.8$ is used as a normalization to 10% Eddington luminosity for $M_{BH} = 10^8$ solar masses. The estimated strength parameter highly ex-
ceeds unity as discussed in Ebisuzaki & Tajima (2014a). This suggests efficient particle acceleration via wakefield acceleration may occur in the jet. Since both electrons and protons are accelerated at the large amplitude electromagnetic flares via ponderomotive forces and these particle move with the waves, high energy non-thermal electrons are concentrated at these waves.

If we apply the radius for the magnetic field amplification at \( R = 6.4 R_g = 3.2 R_A \) instead of \( R = 20 R_g = 10 R_A \) for Eq. (3), the estimated values such as the angular frequency of Alfvén wave in the jet \( \omega_J \), recurrence rate of the burst \( 1/\tau_A \), acceleration time \( D_2/c \), maximum energy of accelerated particle \( W_{\text{max}} \), total accretion power \( L_{\text{tot}} \), Alfvén luminosity \( L_A \) in the jet, gamma-ray luminosity \( L_Y \), and UHECR luminosity \( L_{\text{UHECR}} \) in Ebisuzaki & Tajima (2014a) are revised. Table 2 is revised version of Table 1 in Ebisuzaki & Tajima (2014a). Figure 7 is the revised version of Fig. 4 in Ebisuzaki & Tajima (2014a) which shows the relation of the maximum energy of accelerated particle \( W_{\text{max}} \) as a function of mass of central black hole and accretion power \( L_{\text{tot}} = M \gamma_e^2 \). Since we do not consider any radiative processes, i.e., RIAF \( L_{\text{tot}} \leq 10% \) Eddington luminosity), the gray shadowed region shows the objects for the UHECR \( W_{\text{max}} \geq 10^{20} \) eV accelerators.

### Figure 7

Revised version of Fig. 4 in Ebisuzaki & Tajima (2014a) applying the radius \( R = 6.4R_g = 3.2R_A \) instead of \( R = 20R_g = 10R_A \) as the magnetic field amplification site for the standard disk model (Shakura & Sunyaev 1973). Solid lines represent maximum energy of accelerated particle for the energies \( W_{\text{max}} = 10^{18}, 10^{20}, 10^{22} \) and \( 10^{24} \) eV via ponderomotive acceleration on the plane of the central black hole mass and accretion power \( L_{\text{tot}} = MC^2 \), assuming charge of the accelerated particle \( q = 1 \), and bulk Lorentz factor of the jet \( \Gamma = 20 \). Dashed lines show 10%, 0.1% and 0.001% accretion rate to the Eddington luminosity. Since we do not consider any radiative processes, i.e., RIAF \( L_{\text{tot}} \leq 10% \) Eddington luminosity), the gray shadowed region shows the objects for the UHECR \( W_{\text{max}} \geq 10^{20} \) eV accelerators by our models.

### 4.2 gamma-rays

Blazar is a subclass of AGNs for which the jets are close to our line of sight. A blazar jet is very bright due to relativistic beaming effect, including high energy gamma-ray bands. They are observed in multi-wavelength from radio to TeV gamma rays. Short time variabilities and polarization are observed, including recent \( AGILE \) and Fermi observations for high energy gamma ray bands (Abdo et al. 2009, 2010a,b,c; Ackermann et al. 2010; Striani et al. 2010; Abdo et al. 2011; Bonnoli et al. 2011; Ackermann et al. 2012; Chen et al. 2013; Ackermann et al. 2016; Britto et al. 2016).

These non-thermal emissions are usually explained by the internal shock model (Rees 1978), i.e., two shell collisions (Spada et al. 2001; Kino et al. 2004; Mimica & Aloy 2010; Pe’er et al. 2017) for which a rapid shell catches up a slow shell, forming relativistic shocks. At the shocks particle accelerations, generating non-thermal particles, are expected by Fermi acceleration mechanism. Finally non-thermal emission is produced via synchrotron emission and inverse Compton emission. Other gamma ray emission from accretion disks and related processes have been suggested by Holcomb & Tajima (1991); Haswell et al. (1992).

Our model can naturally explain properties of observed active gamma-ray flares, i.e., spectrum and timescales of flares. For electrons energy loss via synchrotron radiation causes a cutoff around PeV regime (Ebisuzaki & Tajima 2014a), although heavier particles, such as protons and heavier nuclei are accelerated up to ultra high energy cosmic ray regime \( (\sim 10^{20} eV) \) and beyond. The accelerated electrons emit radiation from radio to high energy gamma-rays via synchrotron radiation and inverse Compton emission mechanism. The distribution of accelerated non-thermal particles becomes power law with a power law index \( \sim -2 \) (Mima et al. 1991), which is consistent with the observed blazar spectrum with the power law index close to \( -2 \). The photon index becomes close to \( -2 \) when the light curve of gamma-rays becomes active phase (Abdo et al. 2010a, 2011; Britto et al. 2016). This anti-correlation between the gamma-ray light curves and the photon index also supports our results.

From our numerical simulations the rising timescale of electromagnetic flares in the jet is as same as rising timescale of magnetic field amplification in the disk, i.e., typically \( \tau_1 \sim 30GM_BHc^{-3} \) and timescales of peak to peak in the flares are as same as timescale of repeat cycle of magnetic field amplification in the disk, i.e., typically \( \tau_2 \sim 100GM_BHc^{-3} \).

As for comparison with observed gamma-ray flares of blazars the rising timescale of flares and timescales of the cycle of the flares are normalized by the \((1+z)GM_BHc^{-3}\), where \( z \) is cosmological redshift of the object. The redshifts for two objects are \( z_{3C454.3} = 0.86 \) (Lynden 1967) and \( z_{AO0235+164} = 0.94 \). From the observations of line widths of H\(\beta \) in the broad line region (BLR), the mass of central black hole in 3C454.3 is estimated from \( 5 \times 10^8 M_\odot \) (Bonnoli et al. 2011) to \( 4 \times 10^9 M_\odot \) (Gu et al. 2001). In this paper we adopt \( 5 \times 10^8 M_\odot \) as a mass of central black hole in 3C454.3, since the estimation of the BLR was done by using C\(IV \) line with less contamination by the non-thermal continuum (Bonnoli et al. 2011). The mass of central black hole in AO0235+164 is also
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Table 1. Comparison of the timescales unit in $GM_{\text{BH}}c^{-3}$ of rising flares and repeat cycle of flares between our numerical results, blazar observations, and Ebisuzaki & Tajima (2014a) (ET14), Black hole masses $M_{\text{BH}}$ (M$_{\odot}$) = 5 x 10$^8$ (Bonnoli et al., 2011), and $M_{\text{BH}}$ (AO2025+164) = 5.85 x 10$^8$M$_{\odot}$ (Liu et al., 2006), are used. For Ebisuzaki & Tajima (2014a) model, two different radii ($R = 6R_g$ (their original assumption)) at which magnetic field amplification occurs are considered.

|                      | 3C454.3 | AO0235+164 | ET14 | ET14 |
|----------------------|---------|------------|------|------|
| rising timescale of flares ($\tau_1$) | 30      | 57$^a$     | 325$^b$ | 128  |
| repeat cycle of flares ($\tau_2$)       | 100     | 132$^a$    | 433$^b$ | 354  |

Table 2. Revised version of the Table 1 in Ebisuzaki & Tajima (2014a), where angular frequency in the jet is $\omega_A$, recurrence rate of the burst is $v_A$, acceleration length is $D_A$, maximum energy of accelerated particle is $W_{\text{max}}$, $\bar{\tau}$ is the charge of the particle, $m$ is bulk Lorentz factor of the jet, total accretion power is $L_{\text{tot}}$, Alfvén luminosity in the jet is $L_A$, gamma-ray luminosity is $L_{\text{HECR}}$, and the magnetic field amplification is $\bar{\tau}$. Black hole masses $M_{\text{BH}}$ (AO2025+164) = 5.85 x 10$^8$M$_{\odot}$ (Liu et al., 2006) from the line width of H$\beta$ in the BLR.

5 DISCUSSIONS AND SUMMARY

We have performed 3D GRMHD simulation of accretion flows around a spinning black hole ($a = 0.9$) in order to study the AGN jets from the system of supermassive black hole and surrounding accretion disk as an ultra high energy cosmic ray accelerator via wake field acceleration mechanism.

We start our simulation from a hydrostatic disk, i.e., Fishbone-Moncrief solution with a weak magnetic field and random perturbation in thermal pressure which violate the hydrostatic state and axis-symmetry. We follow the time evolution of the system until 8300GM$_{\text{BH}}c^{-3}$. Initially imposed magnetic field is well amplified, due to differential rotation of the disk. Non axis-symmetric mode, i.e., bar mode near the disk edge grows up. For highest resolution calculation case, the typical timescale of the magnetic field growth near the disk edge is $\bar{\tau} = 30GM_{\text{BH}}c^{-3}$ which corresponds to inverse of the growth rate of the MRI for the almost fastest growing mode. For lower resolution calculation case, the time scale of the magnetic field growth near the disk edge becomes longer one. And the thickness of the filamental structure in the magnetic pressure around the equator increases by the lower resolution calculations. Amplified magnetic field once drops then grows up again. The typical repeat timescale is $\bar{\tau}_2 = 100GM_{\text{BH}}c^{-3}$ which corresponds to the analysis by high resolution local shearing box simulations. The transition between low $\beta$ state and high $\beta$ state repeats. This short time variability for the growth of poloidal magnetic field near the disk edge also can be seen in the mass accretion rate at the event horizon which means the mass accretion seen in our numerical simulation triggered by angular momentum transfer by the growth of the magnetic fields.

We have two types outflows as shown in Fig. 3. One is the low density, magnetized, and collimated outflow, i.e. jets. The other is disk winds which are dense gas flow and between the disk surface and jets. In the jet short time variabilities of the electromagnetic luminosity are observed. The timescales are similar with those seen in the mass accretion rate at the event horizon and poloidal Alfvén energy flux in the disk near the ISCO, i.e., typical rising timescales of flares and typical repeat cycle are as same as the rising timescales of the magnetic field amplification and repeat cycle of the magnetic field amplification, $\bar{\tau}_1 = 30GM_{\text{BH}}c^{-3}$ and $\bar{\tau}_2 = 100GM_{\text{BH}}c^{-3}$, respectively. Thus short pulsed relativistic Alfvén waves are emitted from the accretion disk, when a part of stored magnetic field energy is released. Since the strength parameter of these waves extremely high as $\sim 10^{11}$ for the 10$^8$ solar masses central black hole and 10% Edding-

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**Table 1.** Comparison of the timescales unit in $GM_{\text{BH}}c^{-3}$ of rising flares and repeat cycle of flares between our numerical results, blazar observations, and Ebisuzaki & Tajima (2014a) (ET14), Black hole masses $M_{\text{BH}}$ (M$_{\odot}$) = 5 x 10$^8$ (Bonnoli et al., 2011), and $M_{\text{BH}}$ (AO2025+164) = 5.85 x 10$^8$M$_{\odot}$ (Liu et al., 2006), are used. For Ebisuzaki & Tajima (2014a) model, two different radii ($R = 6R_g$ (their original assumption)) at which magnetic field amplification occurs are considered.

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We adopted 7 days as repeat time and 3 days as rise time for 3C454.3, though various timescales with different times are observed from 3C454.3. Among them, the seven days flare observed by Abdo et al. (2011) is the most energetic. The estimated apparent isotropic gamma-ray energy in the seven days flare is 4 times or more higher than those of flares reported in Abdo et al. (2010a); Britto et al. (2016). Sub-energetic and shorter timescale flares reported in Striani et al. (2010); Ackermann et al. (2012); Britto et al. (2016) can be explained the result of the magnetic eruption via reconnection at the smaller region in the accretion flows.

For AO0235+164 we compare the flare reported in Abdo et al. (2010a), since the flare is the most energetic one in the apparent isotropic energy compared with other flares of AO0235+164, for example Ackermann et al. (2012). The rising timescale is three weeks and repeat timescale is four weeks. Table 1 summarize of the comparison between our results, theoretical model by Ebisuzaki & Tajima (2014a) and observations. Both timescales for 3C454.3 are good agreement with our results. For AO0235+164 the timescales are longer than those for our results which suggests the magnetic field amplification may occur outward where the timescale of MRI growth becomes longer.

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**Table 2.** Revised version of the Table 1 in Ebisuzaki & Tajima (2014a), where angular frequency in the jet is $\omega_A$, recurrence rate of the burst is $v_A$, acceleration length is $D_A$, maximum energy of accelerated particle is $W_{\text{max}}$, $\bar{\tau}$ is the charge of the particle, $m$ is bulk Lorentz factor of the jet, total accretion power is $L_{\text{tot}}$, Alfvén luminosity in the jet is $L_A$, gamma-ray luminosity is $L_{\text{HECR}}$, and the magnetic field amplification is $\bar{\tau}$. Black hole masses $M_{\text{BH}}$ (AO2025+164) = 5.85 x 10$^8$M$_{\odot}$ (Liu et al., 2006) from the line width of H$\beta$ in the BLR.

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ton accretion rate accretion flows, the wakefield acceleration proposed by Tajima & Dawson (1979) can be applied in the jet after mode conversion from Alfvén waves to electromagnetic waves. There are some advantages against Fermi acceleration mechanism (Fermi 1954). When we apply this mechanism to the cosmic ray acceleration, the highest energy of cosmic ray reaches $10^{23}$ eV which is enough high to explain the ultra high energy cosmic rays. We observe magnetic field amplification occurs inner radius compared with the assumption by Ebisuzaki & Tajima (2014a), i.e., $R = 20R_g$. If we apply the model by Ebisuzaki & Tajima (2014a) assuming that magnetic field amplification occurs much inside the disk. The two timescales are consistent with our numeral results.

Since both protons and electrons are accelerated via ponderomotive force in the jet, High energy gamma-ray emission are observed if we see the jet almost on-axis, i.e., blazars. The observed gamma-ray flare timescales such as rising timescales of flares and repeat cycle of flares for 3C454.3 by Fermi Gamma-ray Observatory (Abdo et al. 2010a) are well explained by our bow wave acceleration model. Telescope Array experiment reported that there is hotspot for the cosmic ray with the energy higher than 57EeV (Abbasi et al. 2014). This observation supports that AGN jet is the origin of cosmic ray.

Lastly, the consequences from our present work include the following implication on the gravitational observation. Since non-axisymmetric mode grows in the disk, mass accretion onto the black hole causes the emission of gravitational waves (Kinchi et al. 2011). We estimate the levels of the signal of these gravitational waves, assuming the black hole and accretion disk system. The dimensionless amplitude of the gravitational wave at the coalescing phase can be estimated as Matsubayashi et al. (2004):

$$h_{\text{coal}} = 5.45 \times 10^{-21} \left( \frac{c_{\text{GW}}}{0.01} \right) \left( \frac{4 \text{Gpc}}{R} \right) \left( \frac{\mu}{\sqrt{2} \times 10^3 M_\odot} \right),$$

(30)

where $c_{\text{GW}}$ is efficiency and we here assume as 1%, $\mu$ is reduced mass of the unit of solar mass for the black hole and the blob ~ mass of the blob. The mass of the blob is estimated by $M_f$. Figure 8 (a) shows time evolution of estimated amplitude of the gravitational waves from our mass accretion rate for the blazar 3C454.3. The mass accretion history from $t = 6500GM_{\text{BH}}c^{-3}$ to $t = 8300GM_{\text{BH}}c^{-3}$ is used for this plot.

Figure 8 (b) shows the estimated amplitude of the gravitational wave for some objects, such as gamma-ray active blazars AO0235+164 ($z = 0.94$) and 3C454.3 ($z = 0.86$), nearby AGN jet M87, and famous stellar black hole Cygnus X-1. We assume 1% Eddington mass accretion rate and typical frequency of the gravitational wave signal is $f_{\text{GW}}^{-1}/(1+z)$, where $z$ is redshift of the object. Approximated sensitivity curves of the KAGRA (Nakano et al. 2015) proposed space gravitational wave detectors such as eLISA (Klein et al. 2016), preDECIGO Nakamura et al. (2016), DECIGO (Kawamura et al. 2006) and BBO (Yagi & Seto 2011) are also presented. The signal level is so far small compared to the limit of the presently operating or proposed gravitational antennas.

Our model can be applied to magnetic accretion flows onto the central objects, arising from other events such as black hole or neutron star collisions. For example, recently the gravitational waves from neutron star merger have been detected by LIGO and Virgo gravitational wave detectors (Abbott et al. 2017a). Short gamma-ray burst followed this event just 1.7 s later of the two neutron stars merger (Abbott et al. 2017b). If the jets which emit gamma-rays are powered by magnetic accreting flows onto the merged object, strong and relativistic pulses of Alfvén waves would be emitted like our analysis of accretion disks and then charged particles in the jet are accelerated by the electromagnetic waves as Takahashi et al. (2000) discussed. We see the present acceleration mechanism and its signature of gamma-ray bursts in an ubiquitous range of phenomena.

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