Scaling behaviors and novel creep motion of ac-driven flux lines in type II superconductor with random point pins

Wei-Ping Cao\textsuperscript{1,2}, Meng-Bo Luo\textsuperscript{1,2} and Xiao Hu\textsuperscript{1,3}

\textsuperscript{1} WPI Center for Materials Nanoarchitectonics, National Institute for Materials Science, Tsukuba 305-0044, Japan
\textsuperscript{2} Department of Physics, Zhejiang University, Hangzhou 310027, China
E-mail: Hu.Xiao@nims.go.jp

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\textbf{Abstract.} We performed Langevin dynamics simulations for the ac-driven flux lines in a type II superconductor with random point-like pinning centers. Scaling properties of flux-line velocity with respect to an instantaneous driving force of small frequency and around the critical dc depinning force are revealed successfully, which provides precise estimates on dynamic critical exponents. From the scaling function, we derive a creep law associated with activation by regular shaking. The effective energy barrier vanishes at the critical dc depinning point in a square-root way when the instantaneous driving force increases. The frequency plays a similar role to temperature in conventional creep motions, but in a nontrivial way governed by the critical exponents. We have also performed systematic finite-size scaling analysis for flux-line velocity in transient processes with dc driving, which provide estimates on critical exponents in good agreement with those derived with ac driving. The scaling law is checked successfully.

\textsuperscript{3} Author to whom any correspondence should be addressed.
1. Introduction

Dynamics of elastic manifolds driven through a random medium is one of the rich paradigms in condensed matter physics. Important examples range from ferromagnetic or ferroelectric domains, to charge-density waves, frontiers of fluids in porous media, Wigner crystal and flux lines in type II superconductors [1–3]. The notion that the depinning transition by a dc driving at zero temperature in these systems exhibits properties similar to the critical phenomena of a second-order phase transition at thermal equilibrium has been verified successfully in many systems in this class [4], while a possibility was raised recently on the absence of divergent correlation length below the depinning point [5].

Special attention has been paid to the dynamic phenomena of flux lines in bulk superconductors with randomly distributed defects [6]. It is because, first of all, the system is important for applications of superconductivity, such as the transport of a large current and the generation of a strong magnetic field. Flux lines are also unique with respect to the depinning transition due to the two-dimensional (2D) dynamic degrees of freedom under ac driving transverse to the direction of magnetic field, which hampers an analytic treatment based on the functional renormalization group [7]. Computer simulations based on Langevin dynamics [8–14] were performed in order to fill the gap.

While the critical depinning properties of flux lines under dc driving were addressed quite satisfactorily (see, e.g., [14]), behaviors under ac driving have not yet been fully explored so far, despite its importance and convenience as an experimental probe. Fortunately, a scaling theory for small frequencies around the critical dc depinning point has been formulated based on analysis on an elastic string embedded in a 2D random medium [15], which provides an insight into this problem and motivates the present study.

Here, we perform Langevin dynamics simulations for the ac-driven flux lines in the vortex glass state deformed plastically from a vortex lattice by quenched random point-like pins. We first check the scaling hypothesis on flux-line velocity with respect to an instantaneous driving force of small frequency and around the critical dc depinning force proposed by Nattermann et al [15]. In order to investigate possible effects from finite systems in simulations on the accuracy of critical exponents, we also simulate two transient dynamic processes under dc driving and perform a systematic finite-size scaling analysis, which permits us to evaluate the dynamic critical exponents separately. Finally we check whether the critical exponents satisfy the scaling law [2, 16].
2. Model and simulation technique

The model system is a stack of superconducting layers with a perpendicular magnetic field, similar to that used in [12, 14]. Three-dimensional flux lines are modeled as elastic lines connecting pancake vortices, which sit on a superconducting layer of thickness $d$ with period $s$. The overdamped equation of motion of the $i$th vortex at position $\mathbf{r}_i$ is

$$\eta \dot{\mathbf{r}}_i = - \sum_{j \neq i} \nabla_i U^{VV}(\mathbf{r}_{ij}) - \sum_p \nabla_i U^{VP}(\mathbf{r}_{ip}) + \mathbf{F} + \mathbf{F}_{\text{th}},$$

(1)

with $\eta$ being the viscosity coefficient. The intraplane vortex repulsion is given by the modified Bessel function $U^{VV}(\rho_{ij}, z_{ij} = 0) = d \epsilon_0 K_0(\rho_{ij}/\lambda_{ab})$, where $\epsilon_0 = \phi_0^2/2\pi \mu_0 \lambda_{ab}^2$ with $\lambda_{ab}$ being the magnetic penetration depth of the superconducting layer and $\mu_0$ the permeability of vacuum. The interplane vortex attraction is approximated by a spring potential between the magnetic penetration depth of the superconducting layer and $\mu_0$ the permeability of vacuum. The interplane vortex attraction is approximated by a spring potential between two vortices belonging to the same flux line and sitting on adjacent layers, $U^{VV}(\rho_{ij}, z_{ij} = s) = (s \epsilon_0/\pi) [1 + \ln(\lambda_{ab}/s)][(\rho_{ij}/2r_g)^2 - 1]$ for $\rho_{ij} \leq 2r_g$ and $U^{VV}(\rho_{ij}, z_{ij} = s) = (s \epsilon_0/\pi) [1 + \ln(\lambda_{ab}/s)](\rho_{ij}/r_g - 2)$ otherwise, where $r_g = \gamma s$ with $\gamma$ the anisotropy parameter. The pinning potential is $U^{VP}(\rho_{ip}) = -\alpha A_p \exp[-(\rho_{ip}/R_p)^2]$, where $A_p = (\epsilon_0 d/4) \ln[1 + (R_p^2/2\xi_{ab}^2)]$ with $\xi_{ab}$ the in-plane coherence length and $\alpha$ the dimensionless pinning strength. Pinning centers are distributed at random positions on layers but with a constant pinning range and dimensionless pinning strength $\alpha$. $\mathbf{F}$ is the Lorentz force uniform over the system but varies with time as $F(t) = A \sin(t)$. Finally, $\mathbf{F}_{\text{th}}$ is the thermal noise force with zero mean $\langle F_{\text{th}}(z, t) \rangle = 0$ and a correlator $\langle F_{\text{th}}^q(z, t) F_{\text{th}}^p(z', t') \rangle = 2\eta T \delta^{pq} \delta(z - z') \delta(t - t')$ with $p, q = x, y$. We treat explicitly the case of $\kappa = \lambda_{ab}/\xi_{ab} = 90$, $\gamma = 133$ and $d = 2.83 \times 10^{-3} \lambda_{ab}$, $s = 8.33 \times 10^{-3} \lambda_{ab}$. The present system is to be taken as a model of YBCO, although the anisotropy parameter is slightly large for the convenience of simulation. We note that the value of the anisotropy parameter does not change the universal properties addressed in the present study.

In this work, the units for length, energy, temperature, force, time and velocity are taken as $\lambda_{ab}$, $d \epsilon_0$, $d \epsilon_0/k_B$, $d \epsilon_0/\lambda_{ab}$, $\eta \lambda_{ab}^2/d \epsilon_0 (\equiv \tau_0)$ and $d \epsilon_0/\eta \lambda_{ab}$, respectively. The results will be given in dimensionless units hereafter. The dynamic equation (1) is integrated with the second-order Runge–Kutta algorithm with $\Delta t = 0.01–0.02$.

In simulations, the area density of flux lines is set as 0.2, while that of pins is 1. For example, there are totally $N_v = 180$ flux lines and $N_p = 900$ randomly distributed point pins on each layer for a system of lateral size $L \times L = 30 \times 30$ and 20 layers in the $c$-axis. Periodic boundary conditions in all directions are considered. Intra-plane vortex–vortex repulsions are cut off at $r_{\text{cut}} = 6$. The pinning strength is set as $\alpha = 0.2$, associated with a vortex glass at thermal equilibrium [14]. For different system sizes, the densities of the flux lines and the point pins are fixed. In the present work, we concentrate on zero temperature.

3. ac driving

We first investigate motions of flux lines under an ac driving $F(t) = A \sin(\omega t)$ with $A = 0.4$. During one period, flux lines travel over distances much larger than mean separations of pins and vortices, and in some cases comparable with the system size.

Double hysteresis loops [15] are observed as displayed in the inset of figure 1(a) for $\omega = 0.01 \pi$. It is clear that for a finite $\omega$ the velocity of the system is nonzero even when the
instantaneous driving force is below the critical dc depinning force $F_{c0}$ derived from motions of flux lines under dc driving [14]. We therefore refer to $F_{c0}$ simply as the critical depinning force hereafter.

As the frequency $\omega$ approaches zero, the double loops shrink to the depinning curve of the dc driving [14], as seen in the main panel of figure 1(a) where the instantaneous force dependence of the velocity is shown around $F_{c0}$ in the increasing branch. According to the scaling theory [15], the behavior of the system for $\omega < \omega_p = F_{c0}/\eta h \approx 0.08\pi$ ($h$ is taken as the

**Figure 1.** (a) Hysteretic loops of the velocity of flux lines under ac driving $F(t) = 0.4 \sin(0.01\pi t)$ (inset) and the part around $F_{c0}$ in the increasing branch (dashed rectangle in the inset) for several frequencies (main panel). (b) Scaling plot according to equation (2) (main panel) and asymptote of the scaling function $\phi(x)$ in equation (2) as $x \to -\infty$ (inset). Simulations were performed with $L = 30$ and the numbers of samples are 80–200 depending on the frequency.
mean separation of pins) is governed by the critical properties of the depinning fixed point

\[ v(t) = \omega^{\beta/\nu z} \phi \left[ \left( \frac{F(t)}{F_{c0}} - 1 \right)^{\omega^{-1/\nu z}} \right]; \]

(2)

here the critical exponents are defined by the onset of the velocity \( v \sim (F/F_{c0} - 1)^\beta \) under dc driving, the divergent correlation length \( \xi \sim |F/F_{c0} - 1|^{-\nu} \) and the growth of correlation length with time \( \xi \sim t^{1/z} \) at \( F = F_{c0} \). It is required that \( \phi(x) \sim x^\beta \) for \( x \to \infty \) in order to recover the steady depinning behavior.

The scaling plot according to equation (2) sees good collapsing with \( \beta/\nu z = 0.40 \pm 0.01 \), \( 1/\nu z = 0.53 \pm 0.02 \) and \( F_{c0} = 0.232 \pm 0.002 \) as displayed in the main panel of figure 1(b). The estimates on the exponent \( \beta = 0.76 \pm 0.02 \) and \( F_{c0} \) agree well with the previous results based on dc driving at low but finite temperatures (\( \beta = 0.75 \pm 0.01 \) and \( F_{c0} = 0.232 \pm 0.001 \)) [14].

A clear asymptotic behavior is observed for the scaling function \( \phi(x) \) as \( x \to -\infty \) in the main panel of figure 1(b). As replotted in the inset of figure 1(b), the asymptote is well described by

\[ v(t) \sim \omega^{\beta/\nu z} \exp \left[ -\frac{0.35 \sqrt{1 - F(t)/F_{c0}}}{\omega^{1/2\nu z}} \right], \]

(3)

for \( F(t) < F_{c0} \) with 0.35 a nonuniversal coefficient. The motion of flux lines is creep-like, with an effective energy barrier \( 0.35 \sqrt{1 - F(t)/F_{c0}} \) which vanishes when the instantaneous driving force approaches \( F_{c0} \) from below. The creep motions of flux lines here are caused by the regular shaking of the ac driving, instead of the thermal activations at finite temperature [14, 17]. Because of the random pinning potential landscape, regular shaking contributes to activation of flux lines over pinning barriers in a complex way governed by the critical properties, both steady and dynamic, of the system, which is captured by the frequency dependence in the creep law (3). A theory on the square-root suppression of the energy barrier is not available at this moment.

To end this section, we recall that the scaling form of ac driving predicted in [15] is for 1D displacements of unbreakable elastic manifolds, whereas the depinning phenomena of flux lines in the vortex glass state are associated with plastic 2D deformations and motions.

As seen in the scaling theory (2), the critical exponents \( \nu \) and \( z \) appear in the form of a product in the dynamics under ac driving, which leaves a full description of the critical dynamics unavailable. On the other hand, because of the hysteretic response of flux lines to ac driving, it is still hard to perform any serious analysis of finite-size effects on the accuracy of the critical exponents with the computing resources available at the moment. These issues are addressed in what follows.

4. Onset of collective pinning

Let us now investigate a process associated with the onset of collective pinning, which involves only the exponent \( \nu \). We lay at \( t = 0 \) a perfect triangular lattice of flux lines on the random medium, and start to drive the flux lines by a dc force. The flux-line lattice deforms during traveling when the individual flux lines adapt to the random potential landscape. As a result, the dragging caused by random pinnings becomes stronger due to the collective pinning mechanism [18]. When the dc driving force is below the critical depinning one, the flux lines should be stopped as a whole after traveling a certain distance. This traveling distance depends...
Figure 2. (a) Traveling distance of the flux lines after relaxing from a perfect triangular lattice for several finite systems (main panel) and scaling plot based on the finite-size scaling ansatz (5) (inset). (b) Asymptote of the scaling function \( S(x) \) in the scaling theory (5) for \( x \to -\infty \) and (c) for \( x \to x_c \). The numbers of samples are 10–50 depending on the system size. Error bars are given for the system of \( L = 30 \) and those for other system sizes are similar.

on how far the driving force is from the critical point, and is determined by the critical properties of the depinning fixed point.

We have simulated this process in several finite systems with different lateral sizes while keeping the same number of layers, and the results are displayed in the main panel of figure 2(a). It is clear that, for a given finite system, the traveling distance diverges (since a periodic boundary condition is adopted) as the driving force approaches a critical value given by the system size \( F_{c0}(L) \) (the critical depinning force referred to in the above section and that in [14]
is to be taken as \(F_{c0}(L = 30)\). The critical behavior for an infinite system is given by
\[
D \sim (1 - F / F_{c0}^\infty)^{-\nu_D},
\]
with an exponent \(\nu_D\) and the critical depinning force in an infinite system \(F_{c0}^\infty\), and the convergence from finite systems to the infinite system should be described by the finite-size scaling theory [19]:
\[
D = L^{\nu_D / \nu} S[(F / F_{c0}^\infty - 1) L^{1 / \nu}] .
\]
The scaling plot according to equation (5) is shown in the inset of figure 2(a). From the successful scaling plot, we obtain \(\nu_D / \nu = 1.00 \pm 0.05, \nu = 1.06 \pm 0.04\) and \(F_{c0}^\infty = 0.2365 \pm 0.0005\). The present estimate on the critical depinning force agrees with the above one derived for ac driving and that in the previous work for dc driving [14], but with the highest precision since finite-size effects are taken into account here.

Two features of the scaling function \(S(x)\) are observed as follows: (i) as shown in figure 2(b), \(S(x) \sim (-x)^{-\nu_D}\) for \(x \to -\infty\) as requested by the scaling theory, which achieves the critical behavior (4). (ii) As shown in figure 2(c), \(S(x) \simeq a(x_c - x)^{-\rho}\) for \(x \to x_c\), with \(a \simeq 0.53, x_c \simeq 0.89\) and \(\rho \simeq 1.86\). This relation gives the system-size dependence of the critical depinning force \(F_{c0}(L) / F_{c0}^\infty = 1 + x_c L^{-1 / \nu}\).

5. Critical slowing down

As a second transient process associated with dc driving, we study the critical slowing down of flux lines, which provides a way to estimate separately the exponent \(\zeta\). Here a steady state of flux lines under the driving force \(F = 0.6 > F_{c0}^\infty\) is prepared. At \(t = 0\) the driving force is reduced to \(F_{c0}\). The velocity of the flux lines then decreases with time according to \(v \sim t^{-\beta / \nu z}\) [20]. We have simulated the critical slowing down for several finite systems, and the results are depicted in figure 3(a). The velocity decreases more quickly in a smaller system since the corresponding critical force is larger, as seen in the onset process of collective pinning in figure 2(a). The finite-size scaling behavior at \(F = F_{c0}^\infty\) is described by [20, 21]
\[
v(t, L) = b^{\beta / \nu} v(b^{\nu} t, bL)
\]
with \(b\) being a rescaling factor. The scaling plot is performed successfully as shown in figure 3(b), and we obtain \(\beta / \nu = 0.72 \pm 0.02\) and \(z = 1.80 \pm 0.03\). The slope of the scaling function at small argument is given by \(\beta / \nu z \simeq 0.40\) as required by the scaling theory on short-time dynamics [20, 21].

Combing the results of the previous sections, we obtain \(\beta = 0.76 \pm 0.02, \nu = 1.06 \pm 0.04\) and \(z = 1.80 \pm 0.03\). The exponents satisfy the scaling law \(\beta + \nu(2 - z) = 1\) [2, 16]. The roughness exponent \(\zeta\) can be evaluated by the scaling relation \(\nu = 1 / (2 - \zeta)\) [16] as \(\zeta = 1.06 \pm 0.04\). It is noted that \(\zeta > 1\) implies that local relative displacements between vortices grow with distance and thus the lattice is elastically broken in the thermodynamic limit. While this is consistent with the vortex glass state under consideration, the issue needs further investigation since the value of \(\zeta\) is only marginally larger than unity, the threshold value.

6. Conclusion

We have performed systematic Langevin dynamics simulations for the current driven flux lines in a type II superconductor with random point-like pinning centers, focusing on the
zero-temperature depinning transition. For ac drivings, we observed a creep law associated with activation by regular shaking, with the effective energy barrier vanishing at the critical dc depinning point in a square-root way when the instantaneous driving force increases. The frequency plays a similar role to temperature in conventional creep motions, in a nontrivial way governed by the critical exponents. We have also performed a systematic finite-size scaling analysis for flux-line velocity in transient processes with dc driving, which, combined with the results for ac driving, permitted us to evaluate the dynamic critical exponents separately. These approaches converge consistently to a set of critical exponents: $\beta = 0.76 \pm 0.02$, $\nu = 1.06 \pm 0.04$ and $z = 1.80 \pm 0.03$, which satisfy the scaling law $\beta + \nu(2 - z) = 1$. The exponent $\beta$ is also in good agreement with that obtained in our previous study for dc driving at finite but low temperatures. A theory on the creep-like law caused by regular shaking and specifically an understanding of the frequency dependence and the square-root behavior of the energy barrier remains a future problem.

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References

[1] Kardar M 1998 Phys. Rep. 301 85
[2] Fisher D S 1998 Phys. Rep. 301 113
[3] Nattermann T and Scheidl S 2000 Adv. Phys. 49 607
  Brazovskii S and Nattermann T 2004 Adv. Phys. 53 177
[4] Fisher D S 1983 Phys. Rev. Lett. 50 1486
  Fisher D S 1985 Phys. Rev. B 31 1396
[5] Kolton A B, Rosso A, Giamarchi T and Krauth W 2006 Phys. Rev. Lett. 97 057001
  Kolton A B, Rosso A, Giamarchi T and Krauth W 2009 Phys. Rev. B 79 184207
[6] Blatter G, Feigel’man M V, Geshkenbein V B, Larkin A I and Vinokur V M 1994 Rev. Mod. Phys. 66 1125
[7] Fisher D S 1986 Phys. Rev. Lett. 56 1964
  Balents L and Fisher D S 1993 Phys. Rev. B 48 5949
[8] Brandt E H 1983 Phys. Rev. Lett. 50 1599
  Brandt E H 1983 J. Low Temp. Phys. 53 41
[9] Ryu S, Doniach S, Deutscher G and Kapitulnik A 1992 Phys. Rev. Lett. 68 710
[10] Reichhardt C, Olson C J and Nori F 1997 Phys. Rev. Lett. 78 2648
[11] van Otterlo A, Scalletar R T and Zimányi G T 1998 Phys. Rev. Lett. 81 1497
[12] Olive E, Soret J C, Le Doussal P and Giamarchi T 2003 Phys. Rev. Lett. 91 037005
[13] Bustingorry S, Cugliandolo L F and Domínguez D 2006 Phys. Rev. Lett. 96 027001
[14] Luo M B and Hu X 2007 Phys. Rev. Lett. 98 267002
[15] Glatz A, Nattermann T and Pokrovsky V 2003 Phys. Rev. Lett. 90 047201
[16] Nattermann T, Stepanow S, Tang L H and Leschhorn H 1992 J. Physique II 2 1483
[17] Müller M, Gorokhov D A and Blatter G 2001 Phys. Rev. B 63 184305
[18] Larkin A I and Ovchinnikov Yu N 1979 J. Low Temp. Phys. 34 409
[19] Roters L, Hucht A, Lübeck S, Nowak U and Usadel K D 1999 Phys. Rev. E 60 5202
[20] Janssen H K, Schaub B and Schmittmann B 1989 Z. Phys. B 73 539
[21] Li Z B, Schülke L and Zheng B 1996 Phys. Rev. E 53 2940