The Classification of Methods of Defining of Calculated Mechanical and Physical Characteristics of Composite Materials

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Abstract. A short description with the analysis of the choice of the equivalent characteristics of the material with the further classification of the casting method is represented in this article. The choice and definition of elastic characteristics of composite materials depend on a number of factors: they are the geometry of the estimated element of the construction, the stress-strained state and the mathematical model of this state, the calculation method, the final precision of the solution. The technical methods of calculation of such elements as rods, shells and plates are analytically connected with the resolving equation set-up. One of the most widely used approaches to the solution of practical tasks is to transform heterogeneous materials to homogeneous ones, either isotropic or orthotropic. The most common variants of the choice of physical and mechanical characteristics of the composite material are discussed in this article. One limitation is introduced, that is the work in the elastic strain range and the fulfillment of the conditions of the strain compatibility of the reinforced matrix. These limitations are in accordance with the security of the work of the constructions. A high-quality formulae analysis depending on the final task is given in the article. The given formulae analysis for defining the effectiveness of physical and mathematical characteristics of composite rods, plates and shells enables to make calculations for more complicated tasks. A structural scheme of methods of defining of characteristics of composite materials is given.

1. Introduction
Composite materials are a heterogeneous system. The material consists of two or more components, which possess different mechanical and physical properties. The distinguishing feature of these materials from homogeneous ones is the ability to manipulate their qualities while creating them.

The creation of new composite materials is on the increase. The spheres of science and production, which use composite materials are developing in the most dynamic way. A detailed classification of these materials is necessary to make the right choice of the composite material for every particular case.

It is known from the mechanics of deformed elastic systems that their statics and dynamics can be described as a system in the matrix form:

\[ A \frac{\partial^2 u}{\partial t^2} + B \frac{\partial u}{\partial t} + Cu + F = 0, \]

where, \( u \) – is a displacement vector, \( A \)–a mass matrix, \( B \) - a deformation matrix, \( C \) – a rigidity matrix, \( F \)– an external load vector. One of the types of calculations for composite systems is turning a heterogeneous system into a homogeneous one, which is described by the system (1).

The calculation of constructions made from composite materials using both analytical and numerical methods is connected with the defining of the physical and mechanical characteristics of the material.

The choice and definition of elastic characteristics of composite materials depend on a number of factors: they are the geometry of the estimated element of the construction, the stress-strained state and the mathematical model of this state, the calculation method, the final precision of the solution.
2. Materials and Method

The variants of the structure of composite materials based on the matrix type and the reinforcement type are given in the following reference materials [1,2]. Laminates, disperse-reinforced and fibrous composite materials are most widely used. The disperse reinforcement of the matrix can be achieved with the help of the powder which is represented as macroscopic and microscopic particles. The main advantage of fibrous structures is their ability to produce fibers as monocrystals, which possess high strength. Composite materials, which have the gradient distribution of the powder according to the given law constitute disperse-reinforced composite materials. For example, cermet is a structure, which represents the reciprocal penetration of one material into another one. The reinforcement of the matrix of the dimensional shape (figure 1) provides the dimensional rigidity of the composite material. The matrix material additionally enlarges the stability of the compression elements of the matrix. Plate-rod reinforcement allows to optimize the distribution of the material for plates and shells working in the conditions of simple and combined stress.

![Figure 1. Dimensional reinforcement of composite materials](image)

A considerable number of variants of turning a heterogeneous system to the homogeneous one can be represented in the structural scheme of the calculated model (figure 2).

Averaging elastic constants of the material. In 1887 Voigt introduced the idea of averaging by volume taking into consideration the homogeneity of the generalized displacement, and in 1929 Reiss introduced the idea of averaging according to the homogeneity of the united strains area [1].

Nemirovskiy Y.V. and Reznikov B.S. [3] applied a structural approach while solving the task of the mechanics of deformable solids in order to define the elasticity component and temperature rigidity of the material. Tensor components are expressed with the help of mechanical characteristics of the elements of the composition, the reinforcement structure and other macroscopic parameters. It is postulated that longitudinal forces are accepted by reinforcing fibers and shear forces are accepted by the matrix. The strains in the matrix and reinforcement elements follow Duhamel-Neumann’s law (subject to postulation):

\[
\sigma^c_{\alpha\beta} = \frac{E_c}{1-\nu_c}(\varepsilon_{\alpha\alpha} + \nu_c \varepsilon_{\beta\beta}) - \frac{a_c E_c}{1-\nu_c} \theta, \quad \alpha \neq \beta, \quad (2)
\]

\[
\sigma^c_{12} = \frac{E_c}{1+\nu_c} \varepsilon_{12}, \quad \sigma^c_{33} = 2G_c \varepsilon^c_{33}, \quad \sigma_{ak} = E_{ak} \varepsilon_{ak} - a_{ak} E_{ak} \theta, \quad (3)
\]
where $\sigma, e, E, G, \nu$ — strain, deformation, Young’s modulus, shear modulus, Poisson’s ratio; $c, a$ — indices for the bonding material (matrix) and reinforcing elements; $k$ — a set of fibers; 1, 2, 3 — Cartesian coordinates; $\alpha, \beta$ — Gaussian coordinates; $\theta$ — temperature changes.

Figure 2. The structural scheme of the calculated element

In micromechanics the defining of the effective elasticity modules is achieved with the help of averaging by volume the values of components of the tensors of strains and deformations under definite boundary conditions [4]. Effective modules $C_{ijkl}$ are found from the equations of the generalized Hooke’s law which connect strains $\sigma$ and linear elastic deformations $\varepsilon$:

$$
\sigma_{ij} = C_{ijkl} \cdot \varepsilon_{kl}
$$

The simplest variant of the macromechanical approach is averaging by the cross-section area $A$ or by volume $V$:

$$
E_k = \frac{E_m A_m + E_a A_a}{A_m + A_a}
$$

where indices «м» and «в» correspond to the matrix and the reinforcing fiber of the composite material. In [5] the formulas of the longitudinal elasticity for the unidirectional continuous fiber composite (orthogonal directions - axes $X, Y$) are given:

$$
E_{XX} = E_a V_a + E_m (1 - V_a),
$$

$$
E_{YY} = \frac{E_a E_m}{E_m V_a' + E_a (1 - V_a')}
$$

where $V_a, V_a'$ — is the ratio of the fiber cross-section areas to the composite and the ratio of the fiber length to the length of the matrix of the studied element.
We got the formula (6) from the condition of the strain compatibility, Hooke’s law and the representation of the longitudinal force as a sum of strains in the matrix and fibers. We got the formula (7) when studying the singled out element from the force equation in the fiber and the matrix, the implementation of Hooke’s law, the representation of the relative deformation of the element as a sum of deformations of the fiber and the matrix.

In the works [6,7] they took into account the position of fibers at the angle $\alpha$ (for unidirectional reinforcement, $\alpha = 1$; for orthogonal reinforcement, $\alpha = 0.5$ and for random reinforcement $\alpha = 3/8$):

$$E_k = \alpha E_k V_k + E_{v_k} V_k$$

(8)

This formula goes well with the experiment [1].

The simultaneous averaging by strains and deformations is described in the work [8].

The averaging by rigidity parameters with the plates and shells bending is suggested in the work [1].

For this variant the cylindrical rigidity which we obtained by singling out a beam-strip looks as

$$D_{1p} = \frac{E_M \alpha}{1 - \nu_M^2} \frac{z_0^3}{6} + \frac{E_M \alpha}{1 - \nu_M^2} \frac{h^3}{6} + \frac{z_0 h^2}{2} + \frac{\pi a d^2}{4 t_B} \{ (z_0 - \frac{d}{2})^2 + (z_0 - k_B - \frac{3d}{2})^2 \} \left[ \frac{E_B}{1 - \nu_B^2} - \frac{E_M}{1 - \nu_M^2} \right] \left[ (h - \frac{3d}{2} - k_B - z_0)^2 + (h - \frac{d}{2} - z_0)^2 \right] \left( \frac{E_B}{1 - \nu_B^2} - \frac{E_M}{1 - \nu_M^2} \right)$$

(9)

where $E_M, E_M^c, E_B$ – are longitudinal elasticity modules for the matrix (a stretched area or compressed areas for multimodulus materials) and fibers; $\nu_M, \nu_B$ – are coefficients of the transversal strain deformation of the matrix and the fibers; $h$ – the thickness of the plate (the shell) $\alpha$ – the width of the beam-strip; $d$ – the diameter of the fibers; $t_B, k_B$– the distance of the fibers in the plate surface and in the thickness correspondingly; $z_0$– the position of the neutral axis of the cross-section of the beam-strip with the equivalent area which is found from the condition of equaling zero relatively the neutral axis of the statical moment of the equivalent area of the cross-section.

Having singled out a beam-strip in the orthogonal direction, we get the cylindrical rigidity in the same way $D_{2p}$.

Studying the torque of the beam-strip we get the rigidity which takes into consideration the torque:

$$D_{3p} = G_M \frac{a h^3}{12} + \frac{\pi a d^2}{2 t_B} \left[ \left( \frac{h}{2} - \frac{d}{2} \right)^2 + \left( \frac{h}{2} - k_B - \frac{3d}{2} \right)^2 \right] (G_B - G_M),$$

(10)

where $G_M, G_B$ – are shear modules of the matrix and the fibers.

If necessary the equivalent mechanical characteristics can be defined using the known formulas for the equivalent orthotropic plates:

$$D_{1p} = \frac{E_h h^3}{12(1 - \nu_1^2 \nu_2^m)}, D_{2p} = \frac{E_2^p h^3}{12(1 - \nu_2^m \nu_2)}, D_{3p} = \frac{G_h h^3}{12}, D_{1p}^2 = D_{2p}^2, D_{1p}^p E_{1p} = D_{2p}^p E_{1p},$$

(11)

$$D_{1p}^p V_{1p} = D_{2p}^p V_{2p}.$$

It is suggested to integrate the averaging by rigidity parameters with the multilayer plates and shells bending over the thickness of the packet [9,10].

In the monograph [11] G.L. Gorynin and Y.V. Neirovskiy when studying bending and torque of laminated rods and plates base on averaging the displacement dots along the cross-section area $F$:

$$u(z) = \frac{1}{F} \sum F_i u_i dF.$$

(12)

Averaged constants depending on the position of the $x$-layer and its elastic modulus $E_i$ are included into the resolving equations:

$$c_0 = \sum E_i x dF / \sum E_i dF.$$

(13)
After we have chosen the model of the material, it is advised to choose the casting method of a heterogeneous system to a homogeneous one in order to calculate the elements of a composite system. An analytical approach to solving the task (1) is reduced to the preliminary defining of equivalent matrixes A, B, C through elastic constants of the components of the composite system.

3. Results and Discussion

A structural scheme of defining methods of physical and mechanical characteristics of the material depending on the different qualities of the material is shown in figure 3.

Calculation methods of composite constructions and the corresponding formulas are described in details in the works listed in the reference list of the given article.

![Diagram](image)

Figure 3. A structural scheme of methods of defining physical and mechanical characteristics of composite materials
4. Conclusion

According to many researchers the experimental check of the suggested methodologies and formulas is impeded by the fact that as being different from homogeneous materials, the heterogeneity of composite materials and especially their manufacturing technology makes the results of strain and deformation calculations different. Besides, multimodulus qualities of materials from which they are produced influence greatly the calculations of composite materials. Depending on the aim, it is essential to have a high-quality analysis of formulas. Averaging by volume is well-suited for the central compression or stretching of the elements for disperse-reinforced materials with the homogeneous distribution of the reinforcement. For bending elements formulas [1] are more suitable which take into consideration the position of the reinforcing fibers, multimodulus qualities and the position of the neutral axis and the plane. The given analysis of formulas defining the effectiveness of physical and mechanical properties of composite rods, plates and shells enables to make calculations for more complex problems.

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