Impact of mechanism vibration characteristics by joint clearance and optimization design of its multi-objective robustness

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Abstract. Joint clearances and friction characteristics significantly influence the mechanism vibration characteristics; for example: as for joint clearances, the shaft and bearing of its clearance joint collide to bring about the dynamic normal contact force and tangential coulomb friction force while the mechanism works; thus, the whole system may vibrate; moreover, the mechanism is under contact-impact with impact force constraint from free movement under action of the above dynamic forces; in addition, the mechanism topology structure also changes. The constraint relationship between joints may be established by a repeated complex nonlinear dynamic process (idle stroke - contact-impact - elastic compression - rebound – impact relief - idle stroke movement - contact-impact). Analysis of vibration characteristics of joint parts is still a challenging open task by far. The dynamic equations for any mechanism with clearance is often a set of strong coupling, high-dimensional and complex time-varying nonlinear differential equations which are solved very difficultly. Moreover, complicated chaotic motions very sensitive to initial values in impact and vibration due to clearance let high-precision simulation and prediction of their dynamic behaviors be more difficult; on the other hand, their subsequent wearing necessarily leads to some certain fluctuation of structure clearance parameters, which acts as one primary factor for vibration of the mechanical system. A dynamic model was established to the device for opening the deepwater robot cabin door with joint clearance by utilizing the finite element method and analysis was carried out to its vibration characteristics in this study. Moreover, its response model was carried out by utilizing the DOE method and then the robust optimization design was performed to sizes of the joint clearance and the friction coefficient change range so that the optimization design results may be regarded as reference data for selecting bearings and controlling manufacturing process parameters for the opening mechanism. Several optimization objectives such as x/y/z accelerations for various measuring points and dynamic reaction forces of mounting brackets, and a few constraints including manufacturing process were taken into account in the optimization models, which were solved by utilizing the multi-objective genetic algorithm (NSGA-II). The vibration characteristics of the optimized opening
mechanism are superior to those of the original design. In addition, the numerical forecast results are in good agreement with the test results of the prototype.

1. Introduction
Mechanical vibration characteristics are one of the most important influencing factors for the operating performances of the system so that how to reduce to the greatest extent its negative impact to the system shall be a huge challenge for designers. The system intrinsic properties (mass, stiffness and damping), dynamic driving characteristics (such as load stiffness and speed) and joint clearances may influence importantly its vibration characteristics. Along with the development and maturity of the linear and nonlinear dynamics theories, higher accuracy simulation and analysis may be carried out to effects of intrinsic properties and dynamic driving characteristics to the system vibration characteristics by utilizing the multi-object dynamics method, and the corresponding mechanical vibration issues may also be solved systematically.

However, analysis of vibration characteristics of mechanisms including clearances is still a challenging tasks by far. By taking a clearance hinge as an example, its shaft and bearing may collide to result in the impact contact force as shown in Figure 1, which includes the normal contact force and the tangential coulomb friction force while this mechanism works; and this mechanism may starts its contact impact stage under the impact force constraint from the free movement state so that its topology structure may also changes. In general, the constraint relationship between joints may be established by a repeated complex nonlinear dynamic process (idle stroke - contact-impact - elastic compression - rebound – impact relief - idle stroke movement - contact-impact).

Strong nonlinear dynamic characteristics of joint clearances also depend on this effect. Thus, the dynamic equations of any mechanism with clearance are often a set of strong coupling, high dimensional and complex time-varying nonlinear differential equations which are very difficult to solve; moreover, its clearance impact and vibration include complex chaotic motions very sensitive to initial values so that high precision simulation and prediction to its dynamic behavior shall be very difficult.

![Figure 1. Clearance hinge impact force model](image)
2. Description of the mechanism model

2.1. Mechanism operating principle
As shown in Figure 3, this underwater mechanism is primarily made up of 4 parts (Cover Plate 1, Linkage 2, Rubbish Bin 3 and Base 4). While compressed solid wastes are put into the rubbish bin, the linkage is driven by the system to extend and open the cover plate; thus, solid wastes may be thrown away.

Figure 3. Schematic diagram of the underwater mechanism structure
2.2. Mechanism FEM model
For real simulation of dynamic performances of the underwater mechanism, the FEM method was utilized here to establish a real contact impact joint and complex mechanical processes of joints through the impact relations. Its FEM model of the overall underwater mechanism assembly and the corresponding clearance dynamic model are shown in Figures 4 and 5, respectively, where 3 contact pairs are used to simulate the axial and radial clearances at the both sides.

![Underwater mechanism assembly diagram](image1)

**Figure 4.** Underwater mechanism assembly diagram

Contact Joint 1 with lateral clearance  Contact Joint 3 with lateral clearance  Contact Joint 2 with lateral clearance

![Clearance dynamic model](image2)

**Figure 5.** Clearance dynamic model

3. Optimization processes

3.1. Determination of design variables
Based on significant effects of the system nonlinear vibration characteristics by clearances, the lateral and axial clearances and the clearance friction coefficient are selected as optimization parameters. By taking the actual processing conditions into account, the upper and lower limits of each parameter are shown in Table 1.
Table 1. Design variables

| Optimization parameter | Lower limit | Upper limit |
|------------------------|-------------|-------------|
| Lateral clearance (mm) | 0.05        | 2.5         |
| Radial clearance (mm)  | 0           | 0.35        |
| Friction coefficient   | 0.01        | 0.2         |

3.2. Selection of response variables

Frequency response analysis was carried out to the system. The unit harmonic excitation acts as the input to solve responses of some important output points of the system. The boundary conditions and output locations of the frequency response analysis are shown in Figure 6.

![Figure 6. Schematic diagram of boundary conditions and output locations of the frequency response analysis.](image)

The simulation data were processed to obtain the output parameters shown in Table 2.

Table 2 Simulation output parameters

| Name                     | Type     | Symbol   |
|--------------------------|----------|----------|
| Barrel Acceleration      | RMS      | $a_{R1}$ |
|                          | Amplitude| $a_{a1}$ |
| Cover plate Acceleration | RMS      | $a_{R2}$ |
|                          | Amplitude| $a_{a2}$ |
| Base Exciting force      | RMS      | $F_{RMS}$|
|                          | Amplitude| $F_{amp}$|
| Base Exciting torque     | RMS      | $M_{RMS}$|
|                          | Amplitude| $M_{amp}$|

For decreasing optimization and analysis difficulties and the number of response variables, the linear weighting method was utilized to construct a unified objective function for the cover plate and barrel accelerations and the corresponding weighting coefficients were given according to the importance of the cover plate and barrel vibrations. Because the cover plate vibration is directly transferred outwards by the sea water, the mechanism operating performances may be influenced more directly; thus, the
weighted factors for the cover plate and barrel accelerations are 0.6 and 0.4, respectively; and they are expressed as:

\[ a_R = 0.4a_{R1} + 0.6a_{R2} \]
\[ a_a = 0.4a_{a1} + 0.6a_{a2} \] (1)

Where: \(a_R\) and \(a_a\) represent the response RMS and amplitude values of the weighted accelerations, respectively.

3.3. Multi-response robust optimal mathematical model

By taking those issues such as actual processing conditions and operating wearing into account, the clearance size and friction coefficient may difficultly be controlled as a some certain constant value. These parameters always vary slightly near their setting values; at the same time, their corresponding response values may also change. Thus, optimization shall be necessarily carried out to not only design parameters but also the robustness of the clearance sizes and friction coefficient.

The so-called robustness is essentially the response value is not sensitive to any small change of a parameter and it changes slightly if the controllable factors slightly fluctuates in a certain range; and it may be regarded as that this point is stable. Thus, the tiny range of a response value near some certain point may be utilized to measure its robustness. A certain number of sample points are evenly generated by Latin hypercube sampling within a tiny range of each design point, the difference between whose response maximum value (\(\hat{y}_{\text{max}}\)) and minimum value (\(\hat{y}_{\text{min}}\)) may be used to approximately represent the response range (\(R\)), namely:

\[ R = (\max \hat{y}) - (\min \hat{y}) \] (2)

The desirability function method is a traditional method to solve the multi-response issues, which is simple and widely used. For performing optimality design and meeting the robust design requirements, an optimization desirability function for evaluation of optimality design and a robust desirability function for evaluation of robustness design were introduced here, respectively. The former is expressed by:

\[ d = \begin{cases} 1 & y \leq T \\ \left( \frac{U - y}{U - T} \right)^T & T < y < U \\ 0 & y \geq U \end{cases} \] (3)

The response variables are in line with the smaller-the-better type characteristic, the better the response value is the smaller it is. \(U\) represents the upper limit of a response variable and \(T\) represents the target value of a response variable in Equation (3). The upper limits and target values of the response variables depend on the operating performance requirements of the mechanism, and the specific parameters are given in Table 3. While \(y\) goes beyond its response upper limit (\(U\)), its satisfaction degree is 0; and while \(y\) is lowered, its satisfaction degree rises; and while \(y\) is less than its target value, its satisfaction degree is 1.
\[
    dr = \begin{cases} 
    0 & R \geq U - T \\
    \left(1 - \frac{R}{U - T}\right)^{t} & 0 < R < U - T \\
    1 & R = O 
    \end{cases}
\]  

(4)

Where \( t \) and \( s \) represent the importance of the response with reference to its target in Equations (3-3) and (3-4); and \( s = t = 1 \) is taken here in accordance with the mechanism operating characteristics.

Table 3. Upper and target values of response variables

| Response variable | Upper limit | Target value |
|-------------------|-------------|--------------|
| \( F_{RMS} / N \) | 12          | 3            |
| \( F_{amp} / N \) | 245         | 32           |
| \( M_{RMS} / N \cdot mm \) | 941        | 239          |
| \( M_{amp} / N \cdot mm \) | 18675       | 3009         |
| \( a_k / mm \cdot s^{-2} \) | 377       | 96           |
| \( a_a / mm \cdot s^{-2} \) | 7485       | 1207         |

Normally, optimality and robustness often cannot coexist; the robustness of the response point where the optimal solution exists may be poor not to similarly meet the actual demand. An issue on balancing the optimality and robustness design is involved. The geometric weighting method was utilized here to construct the robust optimal desirability function (Equation (5)). The weighting factors of optimality and robustness are set to evaluated the optimization robustness of the design point.

\[
    D = d^\omega_1 d_r^\omega_2
\]

(5)

Where: \( D \) represents the robust optimal satisfaction degree; \( \omega_1 \) and \( \omega_2 \) represent weighting factors of optimality and robustness satisfaction degrees, respectively, which are taken as \( \omega_1 = \omega_2 = 0.5 \) here. \( x_1, x_2 \), and \( x_3 \) are assumed to represent the lateral and radial clearances and the friction coefficient, respectively. The mathematical model for optimization of the multi-response robustness is expressed as:

**Design variables**
\[
    0.05 mm \leq x_1 \leq 2.5 mm \\
    0 mm \leq x_2 \leq 0.2 mm \\
    0.01 \leq x_3 \leq 0.2
\]

**Objective function for maximization**

\[
    D_{a_k}, D_{a_a}, D_{F_{RMS}}, D_{F_{amp}}, D_{M_{RMS}}, D_{M_{amp}}
\]

**Constraints**

\[
    y_i \leq U_i
\]

Where: \( D_{a_k}, D_{a_a}, D_{F_{RMS}}, D_{F_{amp}}, D_{M_{RMS}} \) and \( D_{M_{amp}} \) represent the corresponding robust optimal satisfaction degrees of the response variables \( (a_k, a_a, F_{RMS}, F_{amp}, M_{RMS} \text{ and } M_{amp}) \), respectively; and \( y_i \) and \( U_i \) represent each response variable and its corresponding response upper limit, respectively.
3.4. Establishment of the response surface model
The uniform design experiment method put forward by FANG Kaitai and WANG Yuan was utilized hereto generate 100 initial sample points and the corresponding response value of each sample point was calculated through the FEM model of the underwater mechanism. Test data were introduced into the multi-object and multidisciplinary optimization software (ModeFRONTIER) developed by ESTECO Corporation; moreover, various RS algorithms were selected in accordance with the actual situations to construct 6 response surfaces responding to 6 response variables, respectively, which are shown in Figure 7 (the friction coefficient is taken as a constant value (0.1)).

3.5. Verification of the response surface model accuracy
Prior to starting optimization, the accuracy inspection shall be first carried out to fitted response surface model to determine whether the optimization design may be carried out to hinge clearances based on this model.

5 sample points representing inspection points of the fitting model were selected randomly by utilizing RANDOM in DOE design here to compare true values with fitted values and solve their relative errors based on Equation (6); thus, the model accuracy may be verified.
Figure 7. Response variables vs. response surfaces

\[ R_{yi} = \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\% \]  

(6)

Where: \( R_{yi} \) represents the relative error between the true and fitted values of the ith sample point; \( y_i \) represents the true value of the ith sample point from the FEM model; and \( \hat{y}_i \) represents the fitted value of the ith sample point from the response surface model.

The accuracy inspection was carried out to the response surface model based on Equation (6) and the inspection results are given in Tables 4-6, whose analysis results indicate that the relative errors of \( M_{RMS} \) and \( M_{amp} \) are relatively large for the fitting model and the relative error of \( M_{RMS} \) of Sample Point101 is maximum (4.5%); on the other hand, all relative errors from the fitting and simulation
models are less than 5%; thus, the response surface accuracy meets the requirements and it may replace the simulation model for performance of the multi-response robust optimal design.

Table 4. Relative errors of fitting models for $F_{RMS}$ and $F_{amp}$

| Sample point ID | $F_{RMS} / N$          | $F_{amp} / N$          |
|-----------------|------------------------|------------------------|
| Response variable | True value | Fitted value | Relative error (%) | True value | Fitted value | Relative error (%) |
| 101             | 3.344      | 3.215      | 3.8        | 48.655     | 50.117      | 3.0                    |
| 102             | 26.799     | 26.238     | 2.1        | 589.886    | 574.548     | 2.5                    |
| 103             | 7.574      | 7.838      | 3.5        | 152.072    | 148.397     | 2.9                    |
| 104             | 16.259     | 15.971     | 1.8        | 324.072    | 309.816     | 4.4                    |
| 105             | 26.977     | 28.109     | 4.2        | 574.381    | 561.732     | 2.9                    |

Table 5. Relative errors of fitting models for $M_{RMS}$ and $M_{amp}$

| Sample point ID | $M_{RMS} / N \cdot mm$ | $M_{amp} / N \cdot mm$ |
|-----------------|-------------------------|-------------------------|
| Response variable | True value | Fitted value | Relative error (%) | True value | Fitted value | Relative error (%) |
| 101             | 255.559    | 267.069     | 4.5        | 3009.132   | 2879.032    | 4.3                    |
| 102             | 2708.572   | 2619.207    | 3.3        | 59628.377  | 62057.177   | 4.1                    |
| 103             | 589.728    | 614.495     | 4.2        | 11612.087  | 11169.62    | 3.8                    |
| 104             | 1074.328   | 1050.688    | 2.2        | 22221.376  | 22798.746   | 2.6                    |
| 105             | 2859.745   | 2748.959    | 3.9        | 61152.58   | 58889.376   | 3.7                    |

Table 6. Relative errors of fitting models for $a_R$ and $a_a$

| Sample point ID | $a_R / mm \cdot s^{-2}$ | $a_a / mm \cdot s^{-2}$ |
|-----------------|--------------------------|--------------------------|
| Response variable | True value | Fitted value | Relative error (%) | True value | Fitted value | Relative error (%) |
| 101             | 102.425      | 100.224      | 2.2        | 1206.572   | 1250.006    | 3.6                    |
| 102             | 1085.037     | 1047.035     | 3.5        | 23886.744  | 24841.440   | 4.0                    |
| 103             | 236.346      | 242.49       | 2.6        | 4653.959   | 4509.689    | 3.1                    |
| 104             | 430.707      | 412.617      | 4.2        | 8907.995   | 9076.25     | 1.9                    |
| 105             | 1145.286     | 1114.085     | 2.7        | 24495.495  | 25450.317   | 3.9                    |

3.6. Establishment of the optimization model
The optimization algorithm is NSGA-II (Non-dominated Sorting Genetic Algorithm II) which is an improved form of the conventional genetic algorithm and where the design operators (such as rapid non-dominated sorting operator, individual crowding distance operator and elite strategy selection operator) are introduced to well solve those issues (such as difficulty selecting parameters and low computational efficiency) for the conventional genetic algorithm and improve the calculation speed and robustness of the algorithm. In accordance with the research situations, 20 generations are selected as the evolution generations and the hybrid rate is assumed as 0.9. The ModeFRONTIER optimization model is shown in Figure 8.

4. Optimization results

4.1. Analysis of optimization results
2000 design points (including 1403 feasible solutions and 55 optimal solutions) are generated in total after 20 evolution generations. Figure 9 shows the multi history chart of each design target, where the design points in lighting green are optimal front points of the optimization results and which indicates that design points are gradually gathered to the optimal front along with performing the optimization iteration.

While there are more than 2 optimization targets, the common optimal front is no longer a curve but it is a 3D or more dimensional surface or hyper-surface. There are 6 optimization targets in total here and the optimal front is a 6D abstract hyper-surface, for whose presentation 6 optimization targets were split here to use a 2D scatter diagram (Figure 10) between $D_{aR}$ and $D_{aA}$, and a 4D bubble chart (Figure 11) between $D_{F_{RM}}$, $D_{F_{amu}}$ and $D_{M_{amu}}$, $D_{M_{RM}}$ to represent the optimal front of the optimization targets.

**Figure 9.** Multi history chart
Figure 10. Pareto front between maximizing $D_{ag}$ (on X) and $D_{aD}$ (on Y).

Figure 11. Pareto front between maximizing $D_{F\text{ext}}$ (on X), $D_{\text{amp}}$ (on Y) and $D_{M\text{amp}}$ (on color) and $D_{M\text{max}}$ (on diameter).
Figure 12 shows the 3D scatter diagram for all design points in the design space, where the optimal front points are in lighting green and which indicate that a lot of optimal design points are gathered in Region A but only a few optimal design points are scattered in Regions B and C along with performing the optimization iteration. During the design stage of the mechanism, clearance parameters shall be selected as can as possible in Design Spaces A, B and C in accordance with the actual situations.

4.2. Verification of optimization results
For verifying the optimization results, 3 optimal design points were selected in Regions A, B and C, respectively; and the error analysis was carried out by comparison of the fitted response values of response surfaces and simulation values from the FEM model to verify the optimization accuracy. Comparison was carried out to the response simulation values of optimization and initial points from the FEM model to verify the optimization effects.

| Sample point ID | Lateral clearance (mm) | Radial clearance (mm) | Friction coefficient |
|-----------------|------------------------|-----------------------|---------------------|
| 0               | 2.500                  | 0.500                 | 0.100               |
| 1132            | 1.943                  | 0.173                 | 0.085               |
| 1378            | 1.768                  | 0.036                 | 0.074               |
| 1402            | 0.966                  | 0.341                 | 0.053               |

| Sample point ID | True value | Fitted value | Relative error % | True value | Fitted value | Relative error (%) |
|-----------------|------------|--------------|------------------|------------|--------------|--------------------|
| 1132            | 0.611      | 0.594        | 2.9              | 15.63      | 14.97        | 4.2                |
| 1378            | 1.813      | 1.731        | 4.5              | 16.84      | 16.268       | 3.4                |
| 1402            | 0.957      | 0.923        | 3.6              | 11.09      | 11.61        | 4.7                |
Table 9. Relative errors from the fitted models for $M_{\text{RMS}}$ and $M_{\text{amp}}$

| Sample point ID | $M_{\text{RMS}}/N\cdot\text{mm}$ | $M_{\text{amp}}/N\cdot\text{mm}$ |
|-----------------|---------------------------------|---------------------------------|
|                 | True value                      | Fitted value                    | Relative error (%) | True value | Fitted value | Relative error (%) |
| 1132            | 1263                           | 1317                           | 4.2               | 10751      | 10977        | 2.1               |
| 1378            | 1996                           | 1934                           | 3.1               | 7146       | 6873         | 3.8               |
| 1402            | 1813                           | 1884                           | 3.9               | 9287       | 9607         | 3.4               |

Table 10. Relative errors from the fitted models for $a_R$ and $a_a$

| Sample point ID | $a_R/mm\cdot s^{-2}$ | $a_a/mm\cdot s^{-2}$ |
|-----------------|----------------------|----------------------|
|                 | True value           | Fitted value         | Relative error (%) | True value | Fitted value | Relative error (%) |
| 101             | 50.565               | 49.959               | 1.2               | 1190       | 1145         | 3.8               |
| 102             | 59.960               | 62.533               | 4.3               | 3168       | 3234         | 2.1               |
| 103             | 49.801               | 48.606               | 2.4               | 1986       | 1928         | 2.9               |

Table 11. Comparison of response variables of initial and optimization points

| Design point ID | $F_{\text{RMS}}/N$ | $F_{\text{amp}}/N$ | $M_{\text{RMS}}/N\cdot\text{mm}$ | $M_{\text{amp}}/N\cdot\text{mm}$ | $a_R/mm\cdot s^{-2}$ | $a_a/mm\cdot s^{-2}$ |
|-----------------|--------------------|--------------------|---------------------------------|---------------------------------|----------------------|----------------------|
| 0               | 4.074              | 65.059             | 3516                            | 60953                           | 300.704              | 6539                 |
| 1132            | 0.611              | 15.63              | 1263                            | 10751                           | 50.565               | 1190                 |
| 1378            | 1.813              | 16.84              | 1996                            | 7146                            | 59.960               | 3168                 |
| 1402            | 0.957              | 11.09              | 1813                            | 9287                            | 49.801               | 1986                 |

It is known from Tables 7-11 that the relative errors between the fitted values of optimization point response surfaces and the true values from the simulation model are less than 5% so that the optimization results shall be reliable. In comparison with those of the initial design points, the corresponding response values of optimization points are lowered to different extent and the optimized mechanism vibration characteristics are remarkably better than the initial mechanism ones.

5. Conclusion

A kind of multi-response robustness clearance parameter optimization method was put forward based on the response surface method in this study and it was applied to some certain underwater mechanism so that optimization results may indicate its vibration characteristics are improved significantly. Such method may be applied to a relatively complicated mechanical system with clearances and the parameter robustness issue which is difficult to solve for traditional numerical optimization techniques; moreover, the theoretical difficulty for analysis and prediction of a complicated nonlinear vibration system for the traditional dynamics may be avoided. Response surfaces may be constructed to simulate the internal relations between clearance parameters and vibration responses to perform the robustness optimization; thus, such method is of great importance in engineering practice.
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