Properties of heavy flavoured mesons using NRQCD formalism

Ajay Kumar Rai
Applied Physics Department, Sardar Vallabh National Institute of Technology, Surat, Gujarat, India, 395 007
E-mail: raiajayk@gmail.com, akr@ashdsvnit.ac.in

Abstract. The mass spectrum of the $c\bar{c}$ meson is reviewed in non-relativistic phenomenological quark anti-quark potential of the type $V(r) = -\frac{C}{r} + Ar^n$, with $n$ varying from 0.5 to 2. The spin hyperfine and spin-orbit interactions are employed to obtain the masses of the pseudoscalar and vector mesons. The decay constants with QCD corrections are computed in this model. The di-gamma and di-leptonic decays of $c\bar{c}$ meson are investigated using some of the known potential models by incorporating radiative corrections up to the lowest order, bound state effects and contribution from quark/antiquark propagator. The decay rates of $c\bar{c}$ meson are studied in the NRQCD formalism, and also with the relativistic corrections of order $v^4$. These decay widths are also computed using the pNRQCD for comparison.

1. Introduction

Heavy flavoured mesons have a rich spectroscopy with many narrow states lying under the threshold of open flavour production. The investigation of the various properties of these mesons gives very important insight into heavy quark dynamics. Recently, there have been renewed interest in the spectroscopy of the heavy flavoured hadrons due to number of experimental facilities (CLEO at the Cornell Electron Storage Ring, DELPHI, Belle at KEK, BaBar, LHCb at the CERN, Panda at the GSI etc) which have been continuously providing and expected to provide more accurate and new information about the hadrons from low flavour to heavy flavour sector [1].

Among many theoretical attempts or approaches to explain the hadron properties based on its quark structure very few were successful in predicting the hadronic properties starting from mass spectra to decay widths. For the mass predictions, the non-relativistic potential models with Buchmüller and Tye [2], Martin [3, 4, 5], Log [6, 7], Cornell [8] etc., were successful at the heavy flavour sectors. There exist relativistic approaches for the study of the different hadronic properties [9, 10]. Some potential models have also predicted the masses and various decays of the heavy-heavy mesons which are in fair agreement with the experimental results [11, 12, 13, 14, 15, 16, 17, 18]. A comprehensive review of developments in heavy quarkonium physics is available in ref [19].

The new role of the heavy flavour studies as the testing ground for the non-perturbative aspects of QCD, demands extension of earlier phenomenological potential model studies on quarkonium masses to their predictions of decay widths with the non-perturbative approaches like NRQCD [20]. It is in terms of their short distance and long distance coefficients. The long distance
coefficients are obtained through phenomenological potential model description of the mesons. The mass spectrum of $c\bar{c}$, meson in the potential scheme of coulomb plus power potential ($CPP_{\nu}$) with the power index ($\nu$), varying from 0.5 to 2. Spin hyperfine interaction is introduced to get the S-wave masses of the pseudoscalar and vector mesons. We present details of the non-relativistic treatment of the heavy quarks along with the computed results in section-2. The decay constants $f_{P,V}$ of these mesons incorporating QCD corrections up to $O(\alpha_s)$ are also presented in this section. In section-3, we present the details of the computations of the di-gamma decays of pseudoscalar state and the leptonic decay widths of the vector states of the $c\bar{c}$ quarkonium in the frame work of the NRQCD formalism as well as other treatments incorporating different correction terms to the respective decay widths. Though the NRQCD formalism takes advantage of the fact that heavy quark mass is much larger than the other energy scales such as the binding energy scale, $\Lambda_{QCD}$ and $|\vec{p}|$, the energy fluctuations of the heavy quarks of the order of the light energy scale are implemented in pNRQCD [21, 22, 23]. Thus we also compute the decay widths in the pNRQCD formalism for comparison. Conclusions are drawn in section-4.

2. Non-relativistic Treatment for $Q\bar{Q}$ systems

For the study of heavy-heavy flavoured bound state system such as $c\bar{c}$ we consider a non-relativistic Hamiltonian given by [16, 17, 18]

$$H = M + \frac{p^2}{2m} + V(r)$$

where $M = m_Q + m_{\bar{Q}}$ is the total mass of quark and antiquark, m is reduced mass, p is the relative momentum of each quark and $V(r)$ is the quark antiquark potential. Recently, we have considered a general power potential with colour coulomb term of the form

$$V(r) = \frac{-\alpha_c}{r} + Ar^\nu$$

as the static quark-antiquark interaction potential ($CPP_{\nu}$) [24, 25]. Here, for the study of mesons, $\alpha_c = \frac{4}{3}\alpha_s$, $\alpha_s$ being the strong running coupling constant, A is the potential parameter and $\nu$ a general power. For the present study of heavy-heavy flavour mesons, we employ the exponential trial wave function of the hydrogenic type to generate the Schrödinger mass spectra. Within the Ritz variational scheme using the trial radial wave function we obtain the expectation values of the Hamiltonian as $\langle H \rangle = E(\mu, \nu)$, Which gives the spin average mass of the ground state. For excited states the trial wave function is multiplied by an appropriate orthogonal polynomial function such that the excited trial wave function gets ortho-normalized (For more details see Refs. [26, 27, 28, 29, 30]). Parameters used in calculations are $\alpha_c(c\bar{c}) = 0.40$, $m_c = 1.31$ GeV and A(taken from ref[26]). The spin-spin interaction among the constituent quarks provides the mass splitting of $J = 0^{-+}$ and $1^{-+}$ states. Accordingly, the spin-spin interactions are taken as [31]

$$V_{S_Q \cdot S_{\bar{Q}}}(r) = \frac{8}{9} \frac{\alpha_s}{m_Qm_{\bar{Q}}} \vec{S}_Q \cdot \vec{S}_{\bar{Q}} 4\pi \delta(r)$$

The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes. The decay constants of pseudoscalar ($f_P$) and vector ($f_V$) mesons are obtained by incorporating a first order QCD correction factor, we compute them using the relation,

$$f_P^2 = \frac{12}{M_P/V} \left| \psi_P/V(0) \right|^2 \left( 1 - \frac{\alpha_s}{\pi} \left( 2 - \frac{m_Q - m_{\bar{Q}}}{m_Q + m_{\bar{Q}}} \ln \frac{m_{\bar{Q}}}{m_Q} \right) \right)$$

$\psi_P/V(0)$ is the wave function at origin for pseudoscalar and vector mesons respectively.
Table 1. Mass Spectra (in GeV) of $c\bar{c}$ meson (S states).

| State | Potential Index $\nu$ | \[1\] | \[33\] |
|-------|----------------------|-------|-------|
| $1^1S_0$ | 0.5   | 3.000 | 2.980 | 2.960 | 2.950 | 2.942 | 2.926 | 2.922 | 2.915 | 2.912 | 2.909 | 2.906 |
|       | 0.7   | 3.092 | 3.100 | 3.109 | 3.112 | 3.116 | 3.123 | 3.129 | 3.134 | 3.139 | 3.144 | 3.149 |
|       | 0.9   | 3.352 | 3.427 | 3.495 | 3.552 | 3.549 | 3.579 | 3.636 | 3.668 | 3.696 | 3.708 | 3.720 |
| $2^1S_0$ | 0.7   | 3.375 | 3.464 | 3.547 | 3.583 | 3.619 | 3.683 | 3.739 | 3.788 | 3.832 | 3.852 | 3.868 |
|       | 0.9   | 3.541 | 3.697 | 3.846 | 3.912 | 3.979 | 4.042 | 4.122 | 4.209 | 4.396 | 4.436 | 4.491 |
| $3^1S_0$ | 0.7   | 3.553 | 3.717 | 3.878 | 3.95  | 4.024 | 4.161 | 4.285 | 4.395 | 4.500 | 4.547 | 4.602 |
|       | 0.9   | 3.684 | 3.912 | 4.140 | 4.246 | 4.354 | 4.558 | 4.748 | 4.920 | 5.082 | 5.158 | 4.250 |
| $1^3S_1$ | 0.7   | 3.309 | 3.427 | 3.495 | 3.552 | 3.549 | 3.597 | 3.628 | 3.668 | 3.696 | 3.708 | 3.720 |
|       | 0.9   | 3.375 | 3.464 | 3.547 | 3.583 | 3.619 | 3.683 | 3.739 | 3.788 | 3.832 | 3.852 | 3.868 |
| $2^3S_1$ | 0.7   | 3.541 | 3.697 | 3.846 | 3.912 | 3.979 | 4.042 | 4.122 | 4.209 | 4.396 | 4.436 | 4.491 |
|       | 0.9   | 3.684 | 3.912 | 4.140 | 4.246 | 4.354 | 4.558 | 4.748 | 4.920 | 5.082 | 5.158 | 4.250 |
| $3^3S_1$ | 0.7   | 3.309 | 3.427 | 3.495 | 3.552 | 3.549 | 3.597 | 3.628 | 3.668 | 3.696 | 3.708 | 3.720 |
|       | 0.9   | 3.375 | 3.464 | 3.547 | 3.583 | 3.619 | 3.683 | 3.739 | 3.788 | 3.832 | 3.852 | 3.868 |

Table 2. Pseudoscalar and Vector decay constants($f_P$ and $f_V$) in MeV of $c\bar{c}$ States

| $\nu$ | 1S | 2S | 3S | 1S | 2S | 3S |
|-------|----|----|----|----|----|----|
| $f_P$ | 430 | 348 | 204 | 165 | 140 | 113 |
| $f_{P\text{cor}}$ | 437 | 353 | 205 | 170 | 153 | 127 |
| $f_P$ | 476 | 419 | 263 | 224 | 198 | 172 |
| $f_{P\text{cor}}$ | 495 | 438 | 282 | 242 | 216 | 190 |
| $f_V$ | 507 | 451 | 295 | 239 | 213 | 187 |
| $f_{V\text{cor}}$ | 532 | 486 | 305 | 255 | 229 | 203 |

Where quantity in bracket is the QCD correction factor given by [32]. The computed $f_P$ and $f_V$ for $c\bar{c}$ system using Eqn(4) and results are tabulated in Table(2).

3. Decay rates of quarkonia

Along with the mass spectrum, successful predictions of various decay widths of heavy flavoured systems have remained as testing ground for the success of phenomenological models. As an attempt to improve the theoretical predictions involving the phenomenological description of the meson, using the radial wave functions and other model parameters of the different potential models we study the decay of $^1S_0$ quarkonium into gamma pairs and the decay of $^3S_1$ into leptons pairs based on NRQCD formalism [20]. It is expected that the NRQCD formalism has all the corrective contributions for the right predictions of the decay rates. NRQCD factorization expressions for the decay rates of quarkonium is given by [35]

\[
\Gamma(^1S_0 \rightarrow \gamma\gamma) = \frac{F_{\gamma\gamma}(^1S_0)}{m^2} X + \frac{G_{\gamma\gamma}(^1S_0)}{m^4} Y
\]

\[
\Gamma(^3S_1 \rightarrow e^+e^-) = \frac{F_{ee}(^3S_1)}{m^2} X + \frac{G_{ee}(^3S_1)}{m^4} Y
\]
### Table 3. Decay rates of $0^{-+} \rightarrow \gamma \gamma$ and the relevant correction terms of $\eta_c$ meson.

| Models      | $\Gamma_0$ (keV) | $\Gamma_B$ (keV) | $\Gamma_C$ (keV) | $\Gamma_R$ (keV) | $\Gamma_T$ (keV) | $\Gamma_{NRQCD}$ (keV) | $\Gamma_{wrc}$ (keV) | $\Gamma_{pNRQCD}$ (keV) | $\Gamma_{Others}$ (keV) |
|-------------|------------------|------------------|------------------|------------------|------------------|------------------------|-----------------------|------------------------|------------------------|
| ERHM        | 7.460            | 0.645            | -0.860           | -2.855           | 4.391            | 4.005                  | 7.956                 | 8.450                  |                        |
| BT          | 10.870           | 0.146            | -0.195           | -4.206           | 6.615            | 6.554                  | 7.406                 | 11.291                 | 7.4                    |
| PL(Martin)  | 13.406           | -5.578           | 7.438            | -6.196           | 9.069            | 8.432                  | 5.007                 | 8.542                  | $\pm 1.4$[1]          |
| Log         | 10.937           | -0.147           | 0.196            | -4.349           | 6.636            | 6.309                  | 7.210                 | 10.965                 | 8.5[33]               |
| Cornell     | 19.512           | -9.167           | 12.222           | -6.581           | 16.066           | 13.787                 | 4.792                 | 12.957                 | 7.5 [36]              |
| $CPP_r=0.5$ | 8.173            | 2.070            | -2.761           | -2.635           | 4.847            | 9.622                  | 10.535                | 11.968                 | 9.02                  |
| 0.7         | 13.465           | 3.093            | -4.124           | -4.342           | 8.092            | 16.990                 | 9.910                 | 18.602                 | 9.32[34]              |
| 1.0         | 14.649           | 3.277            | -4.370           | -4.724           | 8.833            | 18.700                 | 9.804                 | 20.093                 | 9.684[25]             |
| 1.1         | 15.812           | 3.461            | -5.615           | -5.099           | 9.560            | 20.425                 | 9.706                 | 21.558                 | 9.02                  |
| 1.3         | 17.987           | 3.767            | -5.022           | -5.800           | 10.931           | 23.710                 | 9.552                 | 24.245                 | 9.02                  |
| 1.5         | 19.971           | 4.005            | -5.340           | -6.440           | 12.196           | 26.768                 | 9.431                 | 26.677                 | 9.02                  |
| 1.7         | 22.788           | 4.190            | -5.586           | -7.026           | 13.363           | 29.615                 | 9.338                 | 28.838                 | 9.02                  |
| 2.0         | 23.502           | 4.347            | -5.796           | -7.578           | 14.474           | 32.353                 | 9.265                 | 30.836                 | 9.02                  |

ERHM [14, 15], BT [2], PL (Martin) [3], Log [6], Cornell [8]

The long distance coefficients $X$ and $Y$ are the NRQCD matrix elements for the decay. The vacuum saturation approximations allowed the matrix elements of the four fermion operators to be expressed in terms of renormalized wave function parameters as [20]

\[
X = \langle 2S+1 L_J | O_1(h) | 2S+1 L_J \rangle \simeq \frac{2}{3\pi} |R_{cw}(0)|^2 \left[ 1 + O(v^4) \right] \tag{7}
\]

\[
Y = \langle 2S+1 L_J | P_1(h) | 2S+1 L_J \rangle \simeq -\frac{2}{3\pi} |R_{cw} \nabla^2 R_{cw}| \left[ 1 + O(v^4) \right] \tag{8}
\]

The short distance coefficients $F$’s and $G$’s of the order of $\alpha_s^2$ and $\alpha_s^3$ are given by [35]

\[
F_{\gamma\gamma}(1S_0) = 2\pi Q^4 \alpha_s^2 \left[ 1 + \left( \frac{\pi^2}{4} - 5 \right) C_F \frac{\alpha_s}{\pi} \right], \quad G_{\gamma\gamma}(1S_0) = \frac{-8\pi Q^4}{3} \alpha_s^2 \tag{9}
\]

\[
F_{ee}(3S_1) = \frac{2\pi Q^2 \alpha_s^2}{3} \left\{ 1 - 4C_F \frac{\alpha_s(m)}{\pi} + \left[ -117.46 + 0.82n_f + \frac{140\pi^2}{27} \ln \left( \frac{2m}{\mu_A} \right) \right] \right\} \left( \frac{\alpha_s}{\pi} \right)^2 \tag{10}
\]

\[
G_{ee}(3S_1) = -\frac{8\pi Q^2}{9} \alpha_s^2 \tag{11}
\]

Where $C_F = (N_c^2 - 1)/(2N_c) = 4/3$, and $C_A = N_c = 3$ is the colour factor.

### 3.1. Decay rates with relativistic corrections

We compute the decay widths of quarkonia in NRQCD formalism incorporating the relativistic correction through $O(v^4)$ as [35] $\Gamma_{wrc}(1S_0 \rightarrow \gamma\gamma)$

\[
= 2 \left[ \left( \frac{1}{m^2} \right) F_{\gamma\gamma}(1S_0) + \left( \frac{p^2}{m^4} \right) G_{\gamma\gamma}(1S_0) + \left( \frac{p^4}{m^6} \right) H_{\gamma\gamma}(1S_0) + \left( \frac{p^4}{m^6} \right) H_{\gamma\gamma}(1S_0) \right] \tag{12}
\]
Table 4. Decay rates of $1^- \rightarrow l^+ l^-$ and the relevant correction terms of $J/\psi$ meson.

| Systems | Models | $\Gamma_{VW}$ (keV) | $\Gamma_{cor}$ (keV) | $\Gamma_{rad}$ (keV) | $\Gamma_T$ (keV) | $\Gamma_{NRQCD}$ (keV) | $\Gamma_{wrc}$ (keV) | $\Gamma_{pNRQCD}$ (keV) | $\Gamma_{EXP}$ [1] (keV) |
|---------|--------|----------------------|----------------------|----------------------|-----------------|------------------------|----------------------|------------------------|----------------------|
| $J/\psi$ | ERHM   | 5.595                | -0.289               | -3.381               | 1.925           | 2.542                  | 6.040                | 4.360                  |                      |
|         | BT     | 8.152                | -0.240               | -4.982               | 2.930           | 2.539                  | 5.623                | 5.810                  | 5.55                 |
|         | PL (Martin) | 10.055             | -1.089               | -7.341               | 3.803           | 3.312                  | 3.801                | 3.473                  | $\pm 14$             |
|         | Log    | 8.203                | -0.171               | -5.152               | 2.879           | 1.956                  | 5.474                | 5.515                  | $\pm 0.02$           |
|         | Cornell | 14.634              | -1.837               | -7.701               | 8.769           | 7.925                  | 3.638                | 7.457                  | 6.60[33]             |
|         | $CPP_\nu = 0.5$ | 6.130              | -3.122               | -0.624               | 2.384           | 4.212                  | 7.927                | 6.693                  | 7.37[34]             |
|         | 0.7     | 8.189                | -4.171               | -0.845               | 2.384           | 4.212                  | 7.927                | 6.693                  | 7.37[34]             |
|         | 0.9     | 10.153               | -5.171               | -1.065               | 3.917           | 8.246                  | 7.485                | 11.198                 |                      |
|         | 1.0     | 11.053               | -5.629               | -1.165               | 4.259           | 9.353                  | 7.412                | 12.210                 |                      |
|         | 1.1     | 11.946               | -6.084               | -1.268               | 4.594           | 10.430                 | 7.345                | 13.242                 |                      |
|         | 1.3     | 13.621               | -6.937               | -1.463               | 5.221           | 12.558                 | 7.485                | 15.150                 |                      |
|         | 1.5     | 15.165               | -7.723               | -1.645               | 5.797           | 14.636                 | 7.162                | 16.938                 |                      |
|         | 1.7     | 16.582               | -8.445               | -1.813               | 6.324           | 16.643                 | 7.103                | 18.572                 |                      |
|         | 1.9     | 17.920               | -9.126               | -1.982               | 6.812           | 18.659                 | 7.060                | 20.163                 |                      |
|         | 2.0     | 18.549               | -9.447               | -2.061               | 7.041           | 19.634                 | 7.043                | 20.908                 |                      |

ERHM [14, 15], BT [2], PL (Martin) [3], Log [6], Cornell [8]

where the short-distance coefficients at leading order in $\alpha_s^0$ are

$$F_{\gamma\gamma}^{(1S_0)} = 2\pi Q^4 \alpha^2, \quad G_{\gamma\gamma}^{(1S_0)} = -\frac{8\pi}{3} Q^4 \alpha^2, \quad H_{\gamma\gamma}^{(1S_0)} + H_{\gamma\gamma}^{(2S_0)} = \frac{136\pi}{45} Q^4 \alpha^2$$ (13)

We see that, in NRQCD through relativistic corrections of $O(\nu^4)$, the decay width for a $^3S_1$ state in to an $e^+e^-$ pair is given by $\Gamma_{wrc}^\nu (^3S_1 \rightarrow e^+e^-)$

$$= 2 \left[ \left( \frac{1}{m^2} \right) F_{ee}^{(3S_1)} + \left( \frac{p^2}{m^4} \right) G_{ee}^{(3S_1)} + \left( \frac{p^4}{m^6} \right) H_{ee}^{(3S_1)} + \left( \frac{p^4}{m^6} \right) H_{ee}^{(3S_1)} \right]$$ (14)

where the short-distance coefficients at leading order in $\alpha_s^0$ are

$$F_{ee}^{(3S_1)} = \frac{2\pi}{3} Q^2 \alpha^2, \quad G_{ee}^{(3S_1)} = -\frac{8\pi}{9} Q^2 \alpha^2, \quad H_{ee}^{(3S_1)} + H_{ee}^{(3S_1)} = \frac{58\pi}{54} Q^2 \alpha^2$$ (15)

where $p$ and $m$ are the relative momentum and mass of the quarks. The relative momentum in our computations using $CPP_\nu$ models are obtained from the Fermi momentum parameter $\bar{\mu}$. For comparison, we also compute these decay widths using pNRQCD formalism [23] as well as based on a systematically improved gauge invariant description of all hard processes involving the heavy quark anti-quark systems of pseudoscalar and vector mesons given by [37].

Accordingly, the two photon decay width of the pseudoscalar meson is given by [17]

$$\Gamma_T (0^{-+}\rightarrow2\gamma) = \Gamma_0 + \Gamma_B + \Gamma_C + \Gamma_R$$ (16)
Here $\Gamma_0$ is the conventional Van Royen-Weisskopf term for the $0^{-+} \to \gamma\gamma$ decays [38], while $\Gamma_B$, $\Gamma_c$ and $\Gamma_R$ are the corrections due to bound state effects, correction coming from the quark propagator and the radiative corrections respectively for this decay. Where the different terms of Eqn.(17) are computed as [37]

$$\Gamma_0 = \frac{12\alpha_e^2e^4}{M_P^2} R^2(0), \quad \Gamma_B = -\frac{2\varepsilon_B}{M_P} \Gamma_0, \quad \Gamma_c = \frac{16\nabla^2 R(0)}{3M_P^2 R(0)} \Gamma_0, \quad \Gamma_R = \frac{\alpha_s}{\pi} \left( \frac{\pi^2 - 20}{3} \right) \Gamma_0 \quad (17)$$

Here the quark antiquark binding energy $\varepsilon_B$ is defined in terms of the $m_1$ and $m_2$ of the quark and antiquark masses as $\varepsilon_B = m_1 + m_2 - M_P$. Similarly, the leptonic decay width of the vector meson with correction terms is computed as

$$\Gamma_T(1^{-}\to l^+l^-) = \Gamma_{VW} + \Gamma_{cor} + \Gamma_{rad} \quad (18)$$

where

$$\Gamma_{VW} = \frac{4\alpha_e^2e^2}{M_P^2} R^2(0), \quad \Gamma_{rad} = -\frac{16}{3\pi} \alpha_s \Gamma_{VW} \quad and \quad \Gamma_{cor} = \frac{4}{3M_P^2} \frac{\nabla^2 R(0)}{R(0)} \Gamma_{VW} \quad (19)$$

$\Gamma_{rad}$, $\Gamma_{cor}$ are the radiative correction and the correction term due to the quark propagator within the vector meson. It is obvious to note that the computations of the decay rates and the various correction terms described here require the right description of the meson state through its radial wave function at the origin, $R(0)$ and its mass $M$ along with other model parameters like $\alpha_s$ and quark masses. Generally, due to lack of exact solutions for colour dynamics, $R(0)$ and $M$ can be considered as free parameters of the theory [37]. However, we found appropriate to employ phenomenological model predictions of the mesonic mass and corresponding wave function to calculate the decay rates.

### 4. Discussion and conclusion

Here we have studied the mass spectrum (S-Wave), decay constants with and without QCD corrections and decay rates in $CPP_\nu$ and other potential models. The potential model parameters and the masses of the charmed quark obtained from the respective potential model predictions have been employed to study their decay properties in the conventional Van Royen-Weisskopf formula with various corrections, in NRQCD with and without relativistic corrections and also in pNRQCD formalism.

It is interesting to note that the ERHM[15] predictions of the di-gamma decay widths of $\eta_c$ and leptonic decay width of $J/\psi$ are in good agreement with the respective experimental results with out any corrections. The decay properties studied here shown similar behaviour with NRQCD and pNRQCD formalism. While it is found that the relativistic correction of $O(\nu^4)$ contribute considerably to the decay widths.

The $\Gamma_{NRQCD}$, widths for $\eta_c \to \gamma\gamma$ computed using the parameters of the phenomenological models, BT, PL and Log are in excellent agreement with the respective experimental values (see Tables 3). Their predictions along with $CPP_\nu$ using the conventional formula $\Gamma_0$ with corrections ($\Gamma_T$) are also close to the experimental values at $\nu = 0.7$. With the relativistic corrections ERHM, BT and Log potential results are close to experimental as well as other theoretical values but in case of pNRQCD only ERHM and PL results are in order rest are overestimating. In case di-leptonic decay of $J/\psi$ in conventional formula, NRQCD and $CPP_\nu$ at 1.3 and 0.7 are close to experimental values. But in case of relativistic corrections and pNRQCD formalism BT and Log predictions are better. We have calculated the decay rates in various potential models using their input parameters like quark masses and running coupling constant and predicted
meson mass and wave function at origin. All these parameters and predictions are different in different potential models. Therefore the decay rates calculated in these models are not in mutual agreement with each other.

The present study of the decay rates of quarkonia clearly indicates the relative importance of QCD related corrections on the phenomenological potential models. The success of CPP in the determination of the $S$ wave masses and decay rates of $c\bar{c}$ system provide future scopes to study various transition rate and excited states of these mesonic system. With the masses and wave functions of the heavy flavour mesons at hand, it would be rather simple to compute various transition rates such as $E1$ and $M1$ in these mesons.

4.1. Acknowledgments

Author would like to thanks Prof. P. C. Vinodkumar of Sardar Patel University, Vallabh Vidyanagar, Anand, Gujarat, for the valuable discussion.

5. References

[1] Particle Data Group, K. Nakamura et al. 2010 Jnl. Phys G. 37 075021
[2] Buchmuller and Tye 1981 Phys.Rev D24 132
[3] A. Martin 1980 Phys. Lett. 93 B 338
[4] A. Martin 1979 Phys. Lett. 82 B 272
[5] J. L. Richardson 1979 Phys. Lett. 82 B 272
[6] C. Quigg and J. L. Rosner 1977 Phys. Lett. 71 B 153
[7] C. Quigg and J. L. Rosner 1979 Phys. Rept. 56 167
[8] E. Eichten et. al. 1978 Phys. Rev. D 17 3090
[9] Altarelli G, Cabilibo N, Corbo G, Maiani L and Martinelli G 1982 Nucl. Phys. B 208. 365
[10] D. Ebert R.N. Faustov and V. O. Galkin 2003 Phys. Rev. D 67 014027
[11] S. N. Gupta, J. M. Johnson and W. W. Repko 1996 Phys. Rev. D 54 2075
[12] S. Godfrey 1986 Phys. Rev. D 33 1391
[13] Hwang D S, Kim C S and Wuk Namgung 1996 Phys. Rev. D 53 4951
[14] J N Pandya and P C Vinodkumar 2001 Pramana J. Phys. 57 821
[15] P. C. Vinodkumar, J. N. Pandya, V. M. Baner and S. B. Khadikkar 1999 Eur. Phys. J. A 4 83
[16] Ajay Kumar Rai, R H Parmar and P C Vinodkumar 2002 Jnl. Phys G. 28, 2275
[17] Ajay Kumar Rai, J N Pandya and P C Vinodkumar 2005 Jnl. Phys G. 31 1453
[18] Ajay Kumar Rai and P C Vinodkumar 2006 Pramana J. Phys. 66 953
[19] N. Barnbilla, et al. 2004 CERN Yellow Report, CERN-2005-005, Geneva: arXive:hep-ph/0412158
[20] Bodwin G T, Braaten Erich and Lepage G P 1995 Phys. Rev. D 51, 1125; 55 (E) 1997
[21] A. Pineda and J. Soto 1998 Nucl. Phys. B, Proc. Suppl, 64 428
[22] Nora Brambilla et. al. 2003 Phys. Rev. D 67 034018
[23] Nora Brambilla, Antonia Pineda and Joan Soto, Antonio Vairo 2005 Rev. Mod. Phys. 77 1423
[24] Naynesh Deviani and Ajay Kumar Rai and P C Vinodkumar 2011 Phys. Rev. D 84 074030
[25] A. Parmar Bhavin Patel and P C Vinodkumar 2010 Nucl. Phys. A 848 299
[26] Ajay Kumar Rai, B. Patel and P.C. Vinodkumar 2008 Phys. Rev. C 78 055202
[27] Ajay Kumar Rai, J.N. Pandya and P.C. Vinodkumar 2008 Eur. Phys. J. A 38 77
[28] Ajay Kumar Rai and P.C. Vinodkumar 2010 AIP Conf. Proc. 1257 316
[29] Ajay Kumar Rai 2011 AIP Conf. Proc. 1343 415
[30] J.N. Pandya, Ajay Kumar Rai and P.C. Vinodkumar 2007 hep-ph/0701026
[31] Gershtein S S, Kiselev V V, Likhoded A K and Tkhabadze A V 1995 Phys. Rev. D 51, 3613
[32] E. Braaten and S. Fleming 1995 Phys. Rev. D 52 181
[33] Bai-Quing Li and Kuang-Ta Chao 2009 Phys. Rev. D 79 094004
[34] Bhavin Patel and P C Vinodkumar 2009 J. Phys. G 36 035003
[35] Bodwin G T and Petrelli A 2002 Phys. Rev. D 66 094011
[36] Braaten Erich and Lee J 2003 Phys. Rev. D 67 054007
[37] Hafsakhan and Pervez Hoodbhoy 1996 Phys. Rev. D53, 2534
[38] R. Van Royen and V. F. Weisskopf 1967 Nuovo Cimento 50, 617

5th DAE-BRNS Workshop on Hadron Physics (Hadron 2011) IOP Publishing
Journal of Physics: Conference Series 374 (2012) 012017 doi:10.1088/1742-6596/374/1/012017