Integrated Waveform Design for Centralized MIMO-OFDM-BPSK-LFM Radar Communication

Qicheng Ma¹, Jianbin Lu¹, Duo Liu¹ and Yunfan Kong¹

¹College of Electronic Engineering, Naval University of Engineering, Wuhan, Hubei, 430030, China
3208864544@qq.com, 13297945169@sina.cn, liuduo52890@sina.com, 394891768@qq.com

Abstract. Because the signal amplitude corresponding to the phase of BPSK modulation is ±1, it’s autocorrelation function is shaped like a mountain peak. The fuzzy function of BPSK-LFM is multiplied by two parts, the left part is the fuzzy function of the standard LFM signal, and the right part is the autocorrelation function of the BPSK communication symbol, which won’t change with the random change of the communication symbol. Therefore, the fuzzy function performance of BPSK-LFM signal is better than that of LFM signal, but the transmission rate of communication is low. The shape of the fuzzy function will change with the change of random communication symbol in QPSK-LFM and 8PSK-LFM modulation. OFDM multi-carrier modulation can improve spectrum efficiency and communication transmission rate. MIMO technology can multiply channel capacity and spectrum efficiency without increasing signal bandwidth B. As a result, the integrated radar communication signal designed in this paper is a centralized MIMO-OFDM-BPSK-LFM signal. The distance between receiving antennas and transmitting antennas is very small in centralized MIMO radars, which is easy to place together.

1. Introduction

A centralized MIMO-OFDM radar echo model is introduced in detail in Reference [1], and a centralized radar GLRT detector and a AMF detector are designed. Reference [2] analyzes the fuzzy function of the multi-symbol OFDM radar communication sharing signal, simulates and analyzes the influence of signal modulation mode, carries number and OFDM symbol number on the performance of the fuzzy function, and studies the resolution of the multi-symbol sharing signal. The simulation result shows that different modulation modes affect the fuzzy performance of the signal, the larger the carrier number, the better the distance resolution of the signal, the greater the symbol number, the better the speed resolution of the signal [2]. Reference [3] analyzes the error rate (BER) mathematical relationship considering the signal-to-noise ratio (SNR) and verified by using MATLAB simulation. At the same time, the BPSK modulation scheme is considered. For each channel of the OFDM-MIMO system, BER performance will be evaluated, and the number of transmitter and receiver antenna will be changed. It has been proved that the probability of BER and detection increases relative to the SNR per bit [3] because diversity is used at the transmitter or receiver. Reference [4] by MIMO modeling and simulation of the system, we study and compare the output constellation and system error rate under the same channel condition. The simulation results show that the anti-noise performance of BPSK is the best and the QAM is the worst, and the higher the coding rate is, the greater the error rate is [4].
2. MPSK Modulation and Demodulation Principle
The form of carrier does not affect the theoretical bit error rate.

2.1. BPSK Modulation and Demodulation Principle

![Figure 1 BPSK Constellation](image)

The time domain expression [5] BPSK modulation with amplitude 1 is shown in formula 1. In order to simplify the expression, the amplitude of the modulated signal in this paper is 1.

\[
S_B(t) = \cos(2\pi f_c t + \phi_0 + \theta(t))
\]  

In formula 1, \(\theta(t)\) takes 0 or \(\pi\). The phase corresponding to the symbol "1" is 0, and the phase corresponding to the symbol "0" is \(\pi\). For simplifying the formula, make \(\phi_0 = 0\).

Low pass filter solution:

\[
S_B \cos(2\pi f_c t) = \left(\cos(2\pi f_c t)\cos(\theta(t)) - \sin(2\pi f_c t)\sin(\theta(t))\right)\cos(2\pi f_c t)
\]
\[
= \frac{1 + \cos(4\pi f_c t)}{2} \cos(\theta(t)) - \frac{\sin(4\pi f_c t)}{2} \sin(\theta(t))
\]
\[
= \frac{\cos(\theta(t))}{2} + \frac{\cos(4\pi f_c t)\cos(\theta(t)) - \sin(4\pi f_c t)\sin(\theta(t))}{2}
\]
\[
\approx \frac{\cos(\theta(t))}{2}
\]  

Without considering the noise, under the ideal low pass filter, the demodulation result of the input signal is shown in formula 2, and the amplitude of the demodulated signal is \(\pm 1\). Considering the noise, the formula can be deduced by using the integral solution method as shown in formula 3.

\[
\int_0^{T_s} (S_B + n(t))\cos(2\pi f_c t)dt
\]
\[
= \int_0^{T_s} \left(\frac{\cos(\theta(t))}{2} + \frac{\sin(4\pi f_c t)\sin(\theta(t))}{2} + n(t)\cos(2\pi f_c t)\right)dt
\]
\[
= \frac{\cos(\theta(t))}{4f_c} + \int_0^{T_s} n(t)\cos(2\pi f_c t)dt
\]
\[
= \frac{\cos(\theta(t))}{4f_c} + \sum_{i=1}^{N_t} n(t_i)\cos(2\pi f_c t_i) = r_i
\]  

The sending time of each symbol in Formula 3 is \(T_s\). In order to ensure that the integral of \(\cos(4\pi f_c t)\) is 0, an integer multiple of \(T_s\) to \(1/(2f_c)\) can be obtained. In order to simplify the solution result, let \(T_s = 1/(2f_c)\). If \(r_i > 0\), the judgment result is symbol "1". If \(r_i < 0\), the judgment symbol is "0".

The receiver's receiving signal is \(S_B(t) + n(t)\), and the bit error rate in this paper is determined by integral method. Since the amplitude difference corresponding to its phase difference can reach \(2\) maximum, its bit error rate is the lowest among all MPSK modulations. At the best receiver BPSK the average bit error rate of modulation is shown in formula 4.

\[
P_{BB} = Q\left(\frac{2E_B}{N_0}\right)
\]  

BPSK each symbol in modulation includes only 1 bit of information, so the bit error rate is the same as the bit error rate in BPSK modulation. The binary communication system outputs \(s_i(t)\) and
$s_2(t)$ with equal probability[6], where $s_1(t) = \sqrt{E_b}f_1(t), s_2(t) = -\sqrt{E_b}f_1(t), f_1(t) = \cos(2\pi f_c t)$. The probability density functions corresponding to $s_1(t)$ and $s_2(t)$ are shown in Formula 5.

$$p(r_1/s_1) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(r_1 - \sqrt{E_b})^2}{N_0}\right]$$

$$p(r_1/s_2) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(r_1 + \sqrt{E_b})^2}{N_0}\right]$$

\[ (5) \]

$$P_b = P(s_1) \int_0^\infty p(r_1/s_1)dr_1 + P(s_2) \int_0^\infty p(r_1/s_2)dr_1$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

\[ (6) \]

2.2. QPSK Modulation and Demodulation Principle

QPSK modulation is to use different phases in 4 to represent different types of input symbols in 4, the principle of which is shown in 1.

| Input symbol | Phase |
|--------------|-------|
| 1 Road       | Q Road| Output |
| 1            | 1     | $\pi/4$ |
| 0            | 1     | $3\pi/4$ |
| 0            | 0     | $5\pi/4$ |
| 1            | 0     | $7\pi/4$ |

QPSK signal is expressed as shown in formula 7[7].

$$s_i = A \cos(\omega_c t + \phi_i)$$

\[ (7) \]

$\phi_i$ is the phase of cosine carrier, whose phase has four states QPSK modulation principle is shown in Figure 3[8].

String / parallel conversion: odd characters in strings are divided into I paths and even digits into Q paths. Unipolar / bipolar transformation: symbol 1 indicates that the amplitude of the carrier is 1, and
symbol 0 indicates that the amplitude of the carrier is -1. Q reason for adding a symbol to the carrier of the line is to ensure that the phase corresponding to the 11 symbol is. (Under cosine decomposition formula \( \cos(a + b) + b \cos(a) \sin(b) \)).

The demodulation schematic is shown in Figure 4\[9\].

![Figure 4 QPSK Demodulation Principle](image)

Suppose the bit of the sign is \( a \) on the I path, and the bit of the sign is \( a, b \in (-1, 1) \) on the Q path.

\[
s = \cos(\omega_c t) - b \sin(\omega_c t) + n(t)
\]

After I low pass filtering:

\[
x^* \cos(\omega_c t) = (a \cos(\omega_c t) - b \sin(\omega_c t) + n(t))^* \cos(\omega_c t)
\]

\[
= a \frac{1 + \cos(2\omega_c t)}{2} - b \sin(2\omega_c t) + n(t) \cos(\omega_c t)
\]

(9)

After Q low pass filtering:

\[
x^* - \sin(\omega_c t) = (a \cos(\omega_c t) - b \sin(\omega_c t) + n(t))^* - \sin(\omega_c t)
\]

\[
= b \frac{1 - \sin(2\omega_c t)}{2} - a \sin(2\omega_c t) - n(t) \sin(\omega_c t)
\]

(10)

The filtering results are doubled after I and Q filtering, and the residual noise is doubled because the noise cannot be completely filtered. QPSK modulation theory BER is shown in formula 11.

\[
Q(x) = \int_{\frac{1}{\sqrt{2\pi}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} t^2) dt
\]

\[
Q(x) = \frac{1}{2} \text{erfc}(\frac{1}{\sqrt{2}} x)
\]

\[
p_e = \frac{1}{2} \text{erfc}(\frac{\text{snr}}{\sqrt{2}}) = Q(\sqrt{\text{snr}})
\]

(11)

![Figure 5 BPSK BER simulation](image)

![Figure 6 QPSK Error rate simulation](image)
2.3. 8PSK Modulation and Demodulation Principle

Figure 7 8PSK Modulation Principle

Figure 8 8PSK Demodulation Principle

Figure 9 8PSK constellation

Figure 10 8PSK BER simulation

Figure 7 shows that every 3 communication symbols correspond to a phase in 8 PSK modulation. Figure 9 details the modulation phase corresponding to different communication symbols.

2.4. Summary
Comparing figures 10, 6 and 2, it is found that the BPSK has the lowest bit error rate and the 8 PSK has the highest bit error rate. The whole symbol or phase is wrong as long as one of the bits contains more than 1 for QPSK and 8 PSK, from the point of view of transmission rate, \( V(8PSK):V(QPSK):(BPSK)=3:2:1 \).

3. MPSK-LFM Fuzzy Function

3.1. BPSK-LFM Fuzzy functions
BPSK-LFM the form of the signal is shown in formula 12.

\[
S_{y-L} = \cos(2\pi f_c t + \phi_i + \theta_j), f_c = f_0 + \frac{1}{2}kt, t \in [-\frac{t_p}{2}, \frac{t_p}{2}]
\]  

Suppose the complex envelope of m symbol male 1 is transmitted in time \([-T/2, T/2]\) as shown in formula 13.
\[ u(t) = \frac{1}{\sqrt{t_p}} \exp[-j\frac{k t^2}{2} + \theta_j] \]

\[ = \frac{1}{\sqrt{t_p}} \phi(t) \exp[-j\frac{k t^2}{2}], t \in \left[ \frac{-t_p}{2} \cdot \frac{t_p}{2} \right] \]

(13)

In Formula 13, \( \phi(t) \) is the discrete variable determined by the BPSK constellation chart (1). The calculation formula is shown in Formula 2.

\[ \phi(t) = a_o(t) + j a_o(t), a_o(t) = \pm 1, a_e(t) = 0 \]

(14)

A fuzzy function of a BPSK-LFM signal is shown in formula 15.

\[ \chi(t, \xi) = \int_{-\infty}^{\infty} u(t) u^*(t + \tau) e^{j2\pi \xi t} dt \]

\[ = \frac{1}{T_p} \int_{-\infty}^{\infty} \exp[j(-\frac{k}{2} t^2 + \frac{k}{2} (t + \tau)^2 + 2\pi \xi t)] dt \cdot E[\phi(t)\phi^*(t + \tau)] \]

(15)

In Male 15, the left side of " has the same form as the fuzzy function of LFM signal, while the right side is the auto correlation function of \( \phi(t) \). The solution of autocorrelation function is shown in Formula 16 [11].

\[ R_f(\tau) = \int_{-\infty}^{\infty} f(t)f^*(t - \tau)dt \]

(16)

In BPSK modulation, \( a_e(t) = 0 \) is always true, \( a_o(t) = 1 \) or -1. The solution of autocorrelation function of \( \phi(t) \) is shown in formula 17.

\[ E[\phi(t)\phi^*(t - \tau)] \]

\[ = \int_{-\infty}^{\infty} \phi(t)\phi^*(t - \tau)dt \]

\[ = \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} a_o(t)a_o^*(t)dt \]

\[ = \frac{T_p}{2}, |\tau| \leq T_p \]

\[ = 0, \text{others} \]

(17)

Observe formula 15 and formula 17. In BPSK modulation, \( 2\phi(t) = 0 \) or \( 2\pi \), because of \( e^{j0} = e^{j2\pi} \), the radar fuzzy function of BPSK-LFM signal and LFM signal is the same. Therefore, further simplification can obtain the fuzzy function of BPSK-LFM signal as shown in Formula 18.

\[ \chi_{B-L}(\tau, \xi) = \begin{cases} e^{-j\mu\tau} e^{j\omega_d\frac{\mu}{2}(T_p - |\tau|)} \sin \frac{\mu\tau + \omega_d(T_p - |\tau|)}{2}, & |\tau| \leq T_p \\ 0, & |\tau| > T_p \end{cases} 

(18)
Observing and comparing figs. 12 and 16,13 and 14, it is found that when the communication symbols change randomly, the fuzzy function characteristics of the BPSK-LFM signal are better than the LFM signal, the energy is more concentrated, and the same distance and speed resolution are higher. But its disadvantages are low transmission rate and low spectrum efficiency.

3.2. BPSK-LFM Fuzzy functions

In QPSK modulation, one phase is determined for every two communication symbols. QPSK modulation signals correspond to 4 phases, which are \( \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \) respectively. In the solution of the fuzzy function, the phase corresponding to the communication symbol is \( \frac{\pi}{2} \).

If \( \Re(t) = \exp[j(-\frac{\mu}{2}t^2 + \frac{\mu}{2}(t + \tau)^2 + 2\pi \xi t + 2\phi(t))] \), a simplified QPSK-LFM signal is shown in formula 19.

\[
\chi_{Q-L}(\tau, \xi) = \frac{1}{T_p} \left( \int_{-T_p/2}^{T_p/2} \Re(t)dt + \ldots + \int_{-T_p/2}^{T_p/2} \Re(t)dt \right)
\]

The observation formula 19, because of the different time period, the integral result of each section is different. With the increase of the number of communication symbols, the computer will calculate more and more, and the simulation time will be too long. Therefore, QPSK-LFM average fuzzy function is obtained as shown in formula 20.
\[
\chi_{Q-L}(\tau, \xi) = \begin{cases} 
\frac{\sum_{i=1}^{m/2} e^{j2\phi_i}}{m/2} e^{-j\tau\frac{\mu_d}{2} (T_p - |\tau|)} & \text{if } |\tau| < T_p \\
0 & \text{if } |\tau| > T_p
\end{cases}
\]

Formula 20 requires \(m\) to be a multiple of 2. When the four phases in QPSK modulation appear randomly with an equal probability of 0.25, the phase \(2\phi(t)\) corresponding to the communication symbol will become \(\frac{\pi}{2} - \frac{3\pi}{2}\) randomly with a probability of 0.5, at this moment, \(\frac{\sum_{i=1}^{m/2} e^{j2\phi_i}}{m/2}\) is approximately 0, and \(\chi_{Q-L}(\tau, \xi)\) is approximately 0.

By observing figure 15, it is found that the amplitude of fuzzy function of QPSK-LFM signal will be greatly reduced due to the influence of random communication symbols. In figure 15, the communication symbols "1" and "0" are randomly generated, the probability of production is 0.5, and the amplitude of the fuzzy function in figure 15 is particularly low. In order to make the fuzzy function performance of QPSK-LFM signal the same as that of LFM signal, it is required that \(2\phi(t)\) is always equal to, and there are two corresponding combinations, namely \(\sigma/4, 3\pi/4\) and \(3\pi/4, 7\pi/4\). According to FIG.3, it can be seen that the corresponding communication transmission symbols of these two combinations are ["11", "00"] and ["10", "01"] respectively. In QPSK modulation, there is \(2^m\) symbol combination corresponding to \(m\) symbols transmitted without considering the radar fuzzy function. In order to ensure that the performance of the radar fuzzy function remains unchanged, there is \(2^m/2\) symbol combination corresponding to \(m\) symbols transmitted, so the total quantity of information that can be transmitted changes to the performance of the original transmission symbol due to communication, which is greatly affected.

### 3.3 8PSK-LFM Fuzzy Function

In 8PSK modulation, one phase is determined for every three communication symbols. According to Figure 8, 8PSK modulation signal corresponds to 8 phases, which are \(0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{4}\), respectively. In the solution of fuzzy function, the phase corresponding to communication symbol is the average PSK-LFM paste number shown in Formula 21.

\[
\chi_{8-L}(\tau, \xi) = \begin{cases} 
\frac{\sum_{i=1}^{m/3} e^{j2\phi_i}}{m/3} e^{-j\tau\frac{\mu_d}{2} (T_p - |\tau|)} & \text{if } |\tau| < T_p \\
0 & \text{if } |\tau| > T_p
\end{cases}
\]

When 8 phases in 8PSK modulation appear randomly with an equal probability of 0.125, \(2\phi(t)\) corresponding to the communication symbol will become randomly with a probability of 0.25, which is equal to 0, approximately 0, and its fuzzy function is basically the same as that shown in FIG. 15. In order for the performance of 8psk-lfm fuzzy function to be the same as that of LFM fuzzy function, it
is required that the communication symbol is always one of $0, \frac{\pi}{2}, \frac{3\pi}{2}$, and the phase for is one of $[0, \pi], [\pi/4.5\pi/4], [\pi/2, 3\pi/2], [3\pi/4.7\pi/4]$, so the total amount of communication information transmitted will be the original.

3.4. Summary
To sum up, for MPSK modulation, with the increase of $M$, in order to ensure the performance of radar fuzzy function remains unchanged, the total amount of communication information that can be transmitted gradually decreases to $\frac{m^2}{2m^2} = \frac{m^2-m}{2m^2}$. Although with the increase of $M$, the information transmission rate of MPSK modulation gradually increases, which gradually increases to $log_2 M$ times that of BPSK modulation, the total amount of communication information transmitted reduces too much, so it is not a good way to improve the modulation order while ensuring the performance of radar fuzzy function remains unchanged. BPSK-LFM modulation can realize the transmission of communication information without affecting the total class of communication transmission information under the condition that the performance of radar fuzzy function is invariant.

4. Integrated MIMO-OFDM-BPSK-LFM Radar Communication Signal

4.1. Centralized MIMO-OFDM Radar Signal Model
The multi-carrier modulation technology in OFDM technology can effectively solve the problems of low BPSK modulation signal transmission rate and low spectral efficiency. Under the condition that the performance of radar fuzzy function remains unchanged, OFDM-BPSK-LFM signal can greatly improve the communication transmission rate and spectral efficiency. Assuming that the carrier number of OFDM signal is $m_1$, the communication transmission rate of OFDM signal is doubled $m_1$ compared with BPSK-LFM signal.

\[ S_{M0-B-L}(t) = \exp(j2\pi(f_0t + \frac{1}{2}kt^2 + (M-1)f_p t + \phi(t))) \quad (22) \]

In Formula 22, $f_p$ is the frequency difference between two adjacent elements. The OFDM-BPSK-LFM signal is shown in Formula 23.

\[ S_{0-B-L}(t) = \sum_{M=1}^{m_1} S_{M0-B-L}(t) \quad (23) \]

The orthogonality of formula 22 is shown in formula 24.
\[
\int_{0}^{T_p} S_{MO-B-L}(t) S_{NO-B-L}^*(t) dt = \int_{0}^{T_p} e^{i2\pi(M-N)f_p t} dt
\]

\begin{align*}
&= \left\{ \begin{array}{ll}
A, & M = N \\
0, & M \neq N
\end{array} \right.
\end{align*}

(24)

In formula 24, A is a constant. When \( T_p = k f_p \), where \( K \) is a positive integer, and signal SMO-B-L (t) is orthogonal within [0, \( T_p \)], the mixed signal of multi-carrier frequency can be completely separated during demodulation. If the bandwidth of a subcarrier is \( B_1 \), the total bandwidth of all signals is \( B_1 + (M-1)f_p \). The OFDM-BPSK-LFM fuzzy function expression of the signal is shown in formula 25.

\[
\chi_{0-B-L} = \int_{-\infty}^{\infty} S_0-B-L S_0^* e^{i2\pi\xi t} dt
\]

\[
= \int_{0}^{T_p} \sum_{i=1}^{M} e^{i(2\pi f_0 + (M-1)f_p + \frac{1}{2}k(t+\xi))} \sum_{i=1}^{M} e^{-j(2\pi f_0 + (M-1)f_p + \frac{1}{2}k(t+\tau))}(t+\tau) + a_i) e^{j2\pi \xi t} dt
\]

(25)

In Formula 25, \( a_i \) is the random communication symbol, \( a_i \in \{0, 1\} \). Observation formula 25, that due to the random variation of \( a_i \), SO-B-L and in the event of a random changes, so the fuzzy function also is random changes, so the fuzzy function to ensure that the radar performance unchanged, data communication transmission will reduce amount of class, communication performance degradation, it is also the OFDM - LFM radar signal communication integration as a signal of faults.

The MIMO radar designed in this paper includes \( M \) transmitting antenna and \( N \) receiving antenna. One signal is transmitted MIMO-OFDM each antenna in the transmitter, each antenna in the receiver receives all the transmitted signals, and then the signal of the \( i \) transmitting antenna is obtained by orthogonal separation of the mixed waveform at the receiving end through the orthogonal separation period[14]. A centralized MIMO radar can be placed centrally because of the small spacing between the transmitting and receiving elements and between the receiving and receiving elements[15].

4.2. Performance Analysis of Fuzzy Function of 4.2 Signal

A fuzzy function of a MIMO-OFDM-BPSK-LFM signal is shown in formula 26.

\[
\chi_{M-O-B-L}(t) = \int_{-\infty}^{\infty} S_m(t) S_m^*(t + \tau) e^{i2\pi f_p t} dt
\]

(26)

The equivalent formula of formula 26 is shown in formula 27.
\[
X_{M-O-B-L}(\tau; (m-n)f_p; f_d) \\
= \int_{-\infty}^{\infty} S_m(t) S_n(t + \tau) e^{j2\pi f_d t} dt \\
= \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} \text{rect}(t) \text{rect}(\frac{t + \tau}{T_p}) e^{j2\pi (f_0 + 0.5k(t + \tau)) t + \phi(t)} e^{-j2\pi (f_0 + 0.5k(t + \tau)) (t + \tau) + \phi(t + \tau)} e^{j2\pi f_d t} dt \\
= \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} e^{j2\pi (f_0 + 0.5k(t + \tau)) t} e^{-j2\pi (f_0 + 0.5k(t + \tau)) (t + \tau)} e^{j2\pi f_d t} dt \\
= \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} e^{j2\pi (f_0 + 0.5k(t + \tau)) t} \cdot E[\phi(t)\phi^*(t - \tau)] \\
= \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} \frac{1}{T_p^2} e^{j2\pi (f_0 + 0.5k(t + \tau)) t} \cdot \left( T_p - |\tau| \right) \\
\tag{27}
\]

By simplifying formula 27, formula 28 can be obtained.

\[
X_{M-O-B-L}(\tau; (m-n)f_p; f_d) \\
= (T_p - |\tau|) \cdot e^{j2\pi (f_0 + n(f_0 + k\tau)) + \frac{1}{2}k^2\tau^2} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} e^{j2\pi f_p + k\tau + f_d t} dt, 0 \leq \tau \leq T_p \\
\tag{28}
\]

According to the symmetry of the fuzzy function, equation 29 can be obtained by setting \( \alpha = (m - n)f_p + k\tau + f_d \).

\[
X_{M-O-B-L}(\tau; (m-n)f_p; f_d) \\
= \begin{cases} 
(T_p - |\tau|) \cdot e^{j2\pi (f_0 + n(f_0 + k\tau)) + \frac{1}{2}k^2\tau^2} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} e^{j2\pi f_p - k\tau + f_d t} dt, & 0 \leq |\tau| \leq T_p \\
0, & \text{other}
\end{cases} \\
\tag{29}
\]

Simplify formula 29 to obtain formula 30.

\[
|X_{M-O-B-L}(\tau; (m-n)f_p; f_d)|^2 \\
= \left| \frac{\sin \pi \alpha(T_p - |\tau|)}{\pi \alpha(T_p - |\tau|)} \right|^2 \cdot \left| 1 - \frac{|\tau|}{T_p} \right|^2, |\tau| \leq T_p \\
\tag{30}
\]

As \( m=n \), formula 30 becomes a self-fuzzy function, as shown in formula 31.

\[
|X_{M-O-B-L}(\tau; 0; f_d)|^2 = \left| \frac{\sin \pi \alpha(T_p - |\tau|)}{\pi \alpha(T_p - |\tau|)} \right|^2 \cdot \left| \frac{T_p - |\tau|}{T_p} \right|^3, |\tau| \leq T_p \\
\tag{31}
\]

The first half of Formula 31 is divided into standard sinc function. When \( |\tau| = k \cdot k \), formula 31 is divided into standard hown in formula 31. ned value, so when the received signal appears doppler frequency mismatch, that is, when \( |\tau| = d \), the use of the fuzzy function will be shifted, which will affect the radar's accurate positioning of the target.

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Figure 20 Self-fuzzy Function of Figure 1
Figure 21 Self-fuzzy Function of Figure 2

Compared with figures 20 and 21, it is found that with the increase of bandwidth \( B \), the negative effect of Doppler shift on fuzzy function becomes smaller and smaller, and the higher the accuracy of
radar positioning to target. Let $\tau=0$, the zero-time delay response of the self-fuzzy function in the Doppler axis of Formula 31 is shown in Formula 32. The first zero of the fuzzy function appears at $f_d=\pm 1/T_p$.

$$|\chi_{M-0-B-L}(0; 0; f_d)|^2 = \left| \frac{\sin \pi f_d T_p}{\pi f_d T_p} \right|^2 T_p$$

$$= |\text{sinc}(\pi f_d T_p)|^2 T_p$$

(32)

By comparing Figure 22 and 23, it is found that pulse compression is not sensitive to Doppler fuzzy function, which indicates that MIMO-OFDM-BPSK-LFM signal has a larger doppler shift tolerance. Let $f_d = 0$, the zero-time delay response of the self-fuzzy function of Formula 31 on the Doppler axis is shown in Formula 33.

$$|\chi_{M-0-B-L}(\tau; 0; 0)|^2 = |\text{sinc}(\pi k \tau (T_p - |\tau| T_p^2))|^2 \cdot |1 - |\tau|/T_p|^2$$

(33)

By comparing Figure 24 and 25, it is found that pulse compression is sensitive to zero doppler fuzzy function, so the resolution of radar can be improved by increasing the bandwidth. When $m \neq n, f_d = 0$, the cross fuzzy function of formula 31 can be obtained as shown in Formula 34.

$$|\chi_{M-0-B-L}(\tau; (m - n) f_p; 0)|^2$$

$$= |\text{sinc}(\pi (m - n) f_p + k \tau) (T_p - |\tau|)|^2 \cdot |T_p - |\tau| T_p^2|, |\tau| \leq T_p$$

(34)

When $T_p = |\tau|$, the fuzzy function of formula 34 is 0, which indicates that the two radar pulses have no overlap and the signals are completely orthogonal. When $(m-n)f_p = -k \tau$, it indicates that the two radar pulses have overlapped parts, and the location of the staggered peak is shown in Formula 35.
\[
\tau = \frac{-(m-n)f_p}{k} = \frac{-(m-n)}{kT_p} = \frac{-(m-n)}{B}
\] (35)

And the properties of the sinc function, the larger the \(|m-n|\) is, the smaller the sinc function is, the smaller the cross fuzzy function is. Therefore, in order to reduce the false alarm probability, the frequency interval of MIMO-OFDM-BPSK-LFM signal should be larger.

4.3. Summary
In this paper, the fuzzy function performance of MIMO-OFDM-BPSK-LFM signal is analyzed in detail. Under the condition of a certain pulse width of \(T_p\), the peak value of the cross-fuzzy function can be effectively reduced by increasing the signal bandwidth \(B\) and increasing the frequency interval of adjacent channels \(f_p\). In order to achieve the ideal mutually fuzzy function shape, the radar needs to have a large bandwidth \(B\).

Acknowledgments
Thanks for the fund support of national natural science foundation of China (61501486), research on multi-function digital array radar resource scheduling technology.

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