The shapes of galaxies in the Sloan Digital Sky Survey

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ABSTRACT

We determine the underlying shapes of spiral and elliptical galaxies in the Sloan Digital Sky Survey Data Release 6 (SDSS DR6) from the observed distribution of projected galaxy shapes, taking into account the effects of dust extinction and reddening. We assume that the underlying shapes of spirals and ellipticals are well approximated by triaxial ellipsoids. The elliptical galaxy data are consistent with oblate spheroids, with a correlation between luminosity and ellipticity: the mean values of minor to middle axis ratios are $0.41 \pm 0.03$ for $M_e \approx -18$ ellipticals and $0.76 \pm 0.04$ for $M_e \approx -22.5$ ellipticals. Ellipticals show almost no dependence of axial ratio on galaxy colour, implying a negligible dust optical depth.

There is a strong variation of spiral galaxy shapes with colour indicating the presence of dust. The intrinsic shapes of spiral galaxies in the SDSS DR6 are consistent with flat discs with a mean and dispersion of thickness to diameter ratio of $(21 \pm 2)$ per cent, and a face-on ellipticity, $e$, of $\ln(e) = -2.33 \pm 0.79$. Not including the effects of dust in the model leads to discs that are systematically rounder by up to 60 per cent. More luminous spiral galaxies tend to have thicker and rounder discs than lower luminosity spirals. Both elliptical and spiral galaxies tend to be rounder for larger galaxies.

The marginalized value of the edge-on r-band dust extinction $E_0$ in spiral galaxies is $E_0 \simeq 0.45$ mag for galaxies of median colours, increasing to $E_0 = 1$ mag for $g - r > 0.9$ and $E_0 = 1.9$ for the luminous and most compact galaxies, with half-light radii $< 2 h^{-1}$ kpc.

Key words: Surveys -- galaxies: fundamental parameters -- galaxies: general -- galaxies: structure

1 INTRODUCTION

The quantitative study of intrinsic galaxy shapes started with Hubble (1930), who measured the projected axial ratios of elliptical galaxies when classifying them into what would later become the Hubble sequence. Using the projected axial ratios measured from photographic plates of 254 spiral galaxies from the Reference Catalogue of Bright Galaxies (de Vaucouleurs & de Vaucouleurs 1964), Sandage, Freeman & Stokes (1970) concluded that the discs of spiral galaxies were circular, with a disc thickness (defined as the ratio of disc height to diameter) of $\gamma = 0.25$. Later estimates from photographic plate surveys were performed by Binggeli (1980), Benacchio & Galletta (1980) and Binney & de Vaucouleurs (1981), who concluded that galactic discs were consistent with almost circular ellipses, with a mean ellipticity of $\epsilon = 0.1$. These results, based on small samples of galaxies, have been superseded in recent years by much larger studies from CCD imaging and scans of wide-field photographic surveys (Fasano & Vio 1991). Lambas, Maddox & Loveday (1992) analysed a sample of $\sim 13,000$ APM galaxies, and found that the distribution of ellipticities was well fitted by a one-sided Gaussian distribution centred on $\epsilon = 0$ with a dispersion of $\sigma_\epsilon = 0.13$ and a mean of $\langle \epsilon \rangle = 0.1$. Rix & Zaritsky (1995) studied a sample of kinematically selected face-on spiral galaxies in more detail, finding a typical ellipticity of $\epsilon = 0.045$ in the galactic disc potential.

Spatially resolved observations of internal kinematics can sort out the three-dimensional shape of a galaxy (Binney 1985; Franx, Illingworth & de Zeeuw 1991; Statler 1994a,b; Statler & Fry 1994; Bak & Statler 2000; Statler, Lambright & Bak 2001). Andersen et al. (2001) and Andersen & Bershady (2003) applied this method to 24 largely face-on spirals and found a mean ellipticity of $\langle \epsilon \rangle = 0.076$, similar to that of Rix & Zaritsky (1995). However, in both cases the selection of face-on objects may have introduced systematic biases in the sample. Future work with the SAURON spectrograph (Bacon et al. 2001; de Zeeuw et al. 2002) will allow detailed three-dimensional models to be created for a much larger number of galaxies.

Taking advantage of the large number of galaxies with high-quality photometry and shape measurements in the Sloan Digital Sky Survey (SDSS; York et al. 2000), Ryden (2004) selected a sample of spiral galaxies from the SDSS Data Release 1 (DR1;
Abazajian et al. 2003), chosen to minimize systematics due to seeing. She found that the distribution of galactic disc ellipticities can be well fitted by a Gaussian distribution in lnε with a mean of −1.85 and a standard deviation of 0.89. Vincent & Ryden (2005) extended this work using the SDSS DR3 (Abazajian et al. 2005), and fit the distribution of axis ratios of both ellipticals and spirals to triaxial models. Assuming a uniform triaxiality (i.e. all galaxies are either prolate, triaxial or oblate), they found that both spiral and elliptical distributions are consistent with oblate spheroids. Moreover, high-luminosity elliptical galaxies show rounder shapes than do lower luminosity ellipticals.

Elliptical galaxies were once believed to be axisymmetric oblate spheroids, until it was discovered that their rotation velocities were insufficient to support such a geometry (Bertola & Capaccioli 1975). Binney (1976) suggested that ellipticals could be well described by a triaxial ellipsoid but Davies et al. (1983) found that small ellipticals are better fitted by oblate spheroids. This variety of intrinsic shapes for elliptical galaxies makes it difficult to obtain their intrinsic shapes using only their apparent images; when this approach is used on large numbers of elliptical galaxies, it is often necessary to assume a triaxiality as in Vincent & Ryden (2005), or to use the misalignment between the internal isophotes of individual elliptical galaxies as suggested by Binney & Merrifield (1998). The study of intrinsic shapes of spheroids has been recently extended to bulges in spiral galaxies by Méndez-Abreu et al. (2008); bulge shapes are found to be consistent with a mean axial ratio in the equatorial plane of \(B/A = 0.85\).

Astronomers as early as Holmberg (1958) realized that the shape distribution of spiral galaxies is affected by the presence of dust. Optically thick dust obscuration aligned in the rotational plane of spirals will cause edge-on objects to appear systematically fainter, and thus they will be underrepresented in magnitude-limited samples, biasing the estimates of intrinsic galaxy shapes. The dust extinction of galaxies is important for understanding the true luminosities of galaxies, the distribution of interstellar medium (ISM) in galaxies, and the relationship between optical and infrared emission from galaxies (for reviews, see Davies & Burstein 1995 and Calzetti 2001), and studies of the brightness of galaxies as a function of axial ratio should allow the effects of dust to be quantified. Valentijn (1990) studied the shapes and surface brightness of 16 000 galaxies from digitized photographic plates, and interpreted the data as indicating an optically thick component in disc galaxies, extending well beyond the apparent optical extent of the galaxy. Burstein, Haynes & Faber (1991) and Choloniewski (1991) however showed that Valentijn’s results were due in part to selection effects, and found that the diameters of galaxies were independent of inclination; see Davies et al. (1993) and Valentijn (1994) for further discussion of these issues. Peletier & Willner (1992) expanded on the effects of selection biases with inclination, and emphasized that the dust opacity may depend on galaxy luminosity. Tully et al. (1998), for example, found a 1.3 mag difference in the R band between face-on and edge-on luminous galaxies, but found a negligible effect for intrinsically faint galaxies. Holwerda et al. (2005a,b) used a more direct method for obtaining the opacities of spiral discs, consisting of measuring the number of field galaxies seen through galactic discs using images from the Hubble Space Telescope WFPC2 archival data. This method had previously been applied to ground-based data by many other authors, including Zaritsky (1994), Nelson, Zaritsky & Cutri (1998) and Keel & White (2001). Valotto & Giovanelli (2004) followed a different approach to derive the dust extinction in galaxies, using the inner part of the rotation curves of spiral galaxies.

More recently, a number of groups have studied the variation of galaxy properties with the inclination angle with respect to the line of sight, or more directly with projected galaxy shapes, to draw conclusions regarding dust extinction in spiral galaxies. Shao et al. (2007) measured dust extinction in spiral SDSS DR2 (Abazajian et al. 2004) galaxies by studying the luminosity function (LF) of galaxies with different inclination angles, and using the intrinsic galaxy shapes as inferred from the distribution of projected axis ratios. They interpret the decrease in characteristic LF luminosity \((L^*)\) with increasing inclination as an effect of dust extinction, where the disc optical depth is roughly proportional to the cosine of the inclination angle. However, they did not take into account the influence of dust on the projected shapes of galaxies. Unterborn & Ryden (2008) also study the variation of the LF with inclination using a subsample of \(\sim 78 000\) galaxies from the SDSS DR6 (Adelman-McCarthy et al. 2008), finding similar results for the dependence of extinction on projected shape. They use this to define an extinction-unbiased sample of spiral galaxies for which they estimate intrinsic shapes. Even though their results indicate that these galaxies are consistent with flattened discs as was found by previous authors (e.g. Ryden 2004), the definition of the sample makes it difficult to compare their results with previous estimates. Maller et al. (2008) study the variations of galaxy properties with inclination and derive extinction corrections using the NYU-VAGC (Blanton et al. 2005), which combines data from SDSS and the Two Micron All Sky Survey (Skrutskie et al. 2006). The median extinction over their whole sample (all morphological types) is 0.3 mag in the g band. Finally, Driver et al. (2007) also study the dependence of the LF with inclination, by decomposing their sample of galaxies in the Millennium Galaxy Catalogue (Liske et al. 2003; Driver et al. 2005) into bulge and disc components, and are able to deduce the residual face-on attenuation.

Dust has also been found in elliptical galaxies. Ebneter, Davis & Djorgovski (1988) used colour maps to find evidence of dust in more than 30 per cent of their sample of elliptical galaxies; \(\sim 2.5\) per cent of the galaxies showed evidence for a dusty disc. However, the amount of dust in ellipticals is rather smaller than that in spirals. For instance, Knapp et al. (1989) found that an elliptical galaxy contains between 1 and 10 per cent of the dust content present in a spiral galaxy of similar luminosity (see also Goudfrooij 2000; Krause et al. 2003; Leeuw et al. 2004). Far-infrared observations of elliptical galaxies by Tenini et al. (2004) also place constraints on the mass of dust in ellipticals in the range \(M_{\text{dust}} = 10^5 - 10^7 h^{-1} M_{\odot}\), where \(h\) is the Hubble constant in units of \(100\) km s\(^{-1}\) Mpc\(^{-1}\). This mass is \(10^{-4}\) of the stellar mass, a much smaller fraction than that seen in spiral galaxies, where the fraction is of the order of \(5 \times 10^{-3}\) (see e.g. Stevens, Amure & Gear 2005).

This paper will use the SDSS DR6 to derive the intrinsic three-dimensional shapes of spiral and elliptical galaxies using the apparent photometric shapes of galaxies. We will include an in-depth analysis of the effects of dust on the distribution of apparent shapes of spiral galaxies, which will explain most of the trends seen in the distribution of spiral shapes with luminosity and colour. Our analysis weights galaxies by the inverse of the volume out to which they can be seen, thus simulating a volume-limited catalogue, and allowing us to use the full sample of galaxies available in the SDSS DR6. Furthermore, the large number of galaxies present in this sample allows us to study the dependence of the intrinsic shapes of galaxies with luminosity, colour and physical size. Throughout this paper we assume a standard ΛCDM cosmology, with matter density parameter \(\Omega_m = 0.3\) and a cosmological constant corresponding to \(\Omega_\Lambda = 0.7\).
This paper is organized as follows. In Section 2 we will briefly describe the SDSS DR6 galaxies, and the parameters we consider when measuring the distribution of shapes. Section 3 explains our methodology, including our model for the effects of dust on the observed distribution of axis ratios. Section 4 shows our results, and Section 5 summarizes the main conclusions drawn from this work.

2 THE SDSS GALAXY SAMPLE

We select the ∼585 000 galaxies from the r < 17.77 magnitude-limited main spectroscopic galaxy sample of the SDSS (Strauss et al. 2002) from the SDSS DR6 (Adelman-McCarthy et al. 2008). The SDSS imaging data consist of CCD imaging data in five photometric bands (ugriz; Fukugita et al. 1996), taken with a drift-scan camera (Gunn et al. 1998) on a dedicated wide-field 2.5-m telescope (Gunn et al. 2006). The properties of all detected objects in the images are measured (Lupton, Gunn & Szalay 1999; Stoughton et al. 2002), and are calibrated astrometrically (Pier et al. 2003) and photometrically (Smith et al. 2002; Ivezić et al. 2004; Tucker et al. 2006). We K-correct the galaxy magnitudes using V3.2 of the code described in Blanton & Roweis (2007).

The image of each galaxy in the SDSS sample is fitted to two-dimensional models of a de Vaucouleurs (1948) surface profile and an exponential profile, each convolved with the point spread function of the image. This fitting procedure provides a measurement of the model axial ratios (b/a) of each galaxy image in a way robust to seeing, as well as the effective radius and position angle of the galaxy. The SDSS image pipeline also fits an ellipse to the 25 mag arcsec−2 isophote of each galaxy, and determines the so-called adaptive moments (Bernstein & Jarvis 2002); while the axis ratios via these statistics are generally in good agreement with those from the model fits, they are affected by seeing, and thus tend to yield systematically rounder shapes than do the model fits.

We will infer the three-dimensional shapes of spiral and elliptical galaxies separately. Park & Choi (2005) presented a very accurate way to determine SDSS galaxy morphologies using colour gradients that could be used to separate the DR6 catalogue into spiral and elliptical galaxies, but this would require analysing the images of each individual galaxy separately. We use an alternative method: in fitting the exponential and de Vaucouleurs models, the SDSS imaging pipeline also asks for the best linear combination of these models (Abazajian et al. 2004), as quantified by the parameter fracDeV. We use this parameter to distinguish spiral galaxies (fracDeV < 0.8) from ellipticals (fracDeV ≥ 0.8). It is not possible using these techniques to separate out lenticular or S0 galaxies; the definition of this morphological type is difficult using photometric data, out to the redshifts explored in this work. The axis ratios from the exponential and de Vaucouleurs models are in excellent agreement, independent of the value of fracDeV, with a scatter of about 0.05 around the identity line, but we adopt the axis ratios from the exponential fit when fracDeV < 0.8, and the de Vaucouleurs parameters otherwise. All our analyses are carried out using model DeVaucouleurs or exponential r-band magnitudes and g − r colours, depending on the galaxy type.

3 THE INTRINSIC SHAPES OF GALAXIES

In this section we will measure the distribution of projected axis ratios and present the model that will allow us to infer their intrinsic shapes.

3.1 Distributions of projected axis ratios of elliptical and spiral galaxies

Fig. 1 shows distributions of spiral galaxies on the left-hand panels, and of elliptical galaxies on the right-hand panels. The top panels show distributions of galaxy redshifts, and the middle and lower panels the distributions of projected axis ratios. In addition, the thick solid lines in the top and middle panels show the distributions of redshifts and projected axis ratios for the full sample of galaxies in the SDSS DR6. In these panels, the thin lines illustrate the variation of redshift and projected axis ratio distributions as the typical galaxy luminosity of the sample is increased. More luminous galaxies tend to show rounder apparent shapes (axial ratios closer to unity), suggesting that the intrinsic shapes of galaxies are a function of luminosity. This is a flux-limited sample, and thus has a strong correlation between luminosity and redshift, as shown in the top panel. With this in mind, we simulate a volume-limited measurement of apparent shapes by simply weighting each galaxy by 1/V_{max}, where V_{max} is the volume corresponding to the maximum distance out to which a galaxy of a given apparent magnitude enters the flux-limited catalogue, taking into account K-corrections. As a result of these procedures, our final samples of spirals and ellipticals contain a total of 282 203 and 303 390 galaxies, respectively. There is an additional effect for spiral galaxies, whereby the internal extinction will cause edge-on objects to appear fainter than equally luminous face-on objects. We thus first volume-limited ignoring this effect, determined the inclination dependence of the extinction as described in Section 3.2 below and then redefined our volume-limited sample taking this extinction into account and repeated our analysis. We found that the inferred extinction differed by only 0.1 mag between the two analyses, so we did not iterate further. The solid lines in the bottom panels of Fig. 1 show the resulting distributions of axial ratios for spiral and elliptical galaxies. Not surprisingly, the spiral galaxy distribution is skewed towards lower axial ratios (b/a values) than are the ellipticals, which are rounder with b/a closer to 1.

The distribution becomes flatter when using the 1/V_{max} weighting. In particular, the weighted distribution of b/a values for spiral galaxies is qualitatively similar to that of Ryden (2004), who found a flat distribution over a wide range of b/a values from b/a = 0.2 to 0.7 for a volume-limited sample of SDSS DR2 spiral galaxies. We use 1/V_{max} weighting in all the analyses that follow. Galaxies fainter than M_v = −5 log_{10}(h) = −17 have very small values of V_{max} and thus tend to dominate the noise of the estimate of the distribution function; we therefore drop such low-luminosity objects in what follows. Throughout this paper, error bars were calculated using the jackknife method.

We model galaxies as triaxial ellipsoids of major axis A, middle axis B and minor axis C, parametrized by two axis ratios, C/B and B/A, and we will determine the distribution of axis ratios of the spiral and elliptical populations separately. Following Ryden (2004), we assume that the distribution of 1 − C/B of the three-dimensional structure can be approximated by a Gaussian with mean γ (this parameter is related to μ_γ in Ryden 2004, via γ = 1 − μ_γ) and standard deviation σ_γ. We also assume that there is a lognormal distribution in the quantity ε = log(1 −B/A), with mean μ and

1 The advantage of using the 1/V_{max} weight is the larger sample size and the increased range of luminosities that can be explored; using a true volume-limited sample would restrict our analysis to a sample almost ∼10 times smaller composed only of intrinsically bright galaxies.
Figure 1. Left-hand panels show distributions obtained for spiral galaxies, right-hand panels for elliptical galaxies. Top panels: distribution of galaxy redshifts, normalized so that the area under each curve is unity. The thick solid line corresponds to all the galaxies in the SDSS DR6. The thin lines show the distribution of axis ratios for galaxies in different bins of absolute magnitude as indicated in the key, and for spiral and elliptical galaxies separately. Middle panels: normalized distribution of axis ratios for the same samples of galaxies as in the top panels; the thick line corresponds to the full sample of galaxies. Bottom panels: distribution of axis ratios, summed over all luminosities. The solid lines show the results when a $1/V_{\text{max}}$ weight is applied to each galaxy. The dotted lines show the results with no weighting. Errors are calculated using the jackknife technique.

dispersion $\sigma$. Larger values of $\gamma$ and $\mu$ correspond to more elliptical objects in the $BC$ and $AB$ planes, respectively. Given values of axis ratios drawn from these distributions, and a random viewing angle ($\theta$, $\phi$), we compute the resulting apparent axis ratio, $b/a$ (Binney 1985), using equations (12)–(15) from Ryden (2004). Repeating this multiple times gives a model distribution $[N_{\text{model}}(b/a)]$ which can be compared directly to our measured volume-weighted distributions $[N(b/a)]$.

This model assumes that the dust extinction in galaxies is independent of the viewing angle. However, we can test for the effects of dust by exploring the dependence of the shape distribution on absolute magnitude and colour. The top left-hand panel of Fig. 2 shows the median $b/a$ of spiral and elliptical galaxies as a function of $r$-band absolute magnitude. Both spiral and elliptical galaxies tend to be rounder at higher luminosities. Spiral galaxies show a change of $\Delta(b/a) \simeq 0.2$ between absolute magnitude values of $M_r - 5 \log_{10}(h) = -18.5$ and $-22.5$. The variation in $b/a$ for ellipticals is smaller, $\Delta(b/a) \simeq 0.1$.

The top right-hand panel shows the median $b/a$ as a function of $g-r$ colour. Red spiral galaxies show systematically larger axis ratios than do blue spirals. We interpret the colour and luminosity dependence of spirals as due to the reddening and dimming effects of dust, which becomes more prominent for edge-on systems. Elliptical galaxies show only a mild variation in shape with absolute magnitude, and no significant effect in colour. Thus, not unexpectedly, we see no evidence for a dust layer aligned with the principal plane of elliptical galaxies, and assume that the trend in the apparent shape of ellipticals with luminosity corresponds to a real variation of their intrinsic shapes. We study the dependence of projected and intrinsic properties of spiral and elliptical galaxies in more detail in Section 4.

The middle panels of Fig. 2 show the luminosity and colour distribution functions (left- and right-hand panels, respectively), both estimated using the $1/V_{\text{max}}$ estimator, for spiral and elliptical galaxies. The filled circles in the middle left-hand panel give the $r$-band LF estimate from Blanton et al. (2003a), with Schechter parameters $M^* - 5 \log_{10}(h) = -20.44$ and $\alpha = -1.05$; our results for the full sample (black solid lines) are in excellent agreement. As the effects of dust are more severe for edge-on objects, we also calculate the luminosity and colour functions for face-on objects ($b/a > 0.8$) only. These are shown as solid lines for the spiral galaxies in the bottom panels of this figure; the dashed lines show the luminosity and colour functions for all the spiral galaxies for comparison. The break in the face-on LF is shifted towards higher luminosities, as expected for the sample of galaxies least affected by dust. This measurement of the “unextincted” LF is in agreement with results from Shao et al. (2007), Unterborn & Ryden (2008) and, taking into account the range allowed by the analysis, with Maller et al. (2008). On the other hand, the colour function of spiral galaxies is shifted to the red due to the effects of dust. These estimates of unextincted luminosity and unreddened colour functions will be needed when we model the effects of dust, a subject to which we now turn.
In this section, we develop a simple model for the impact of a planar distribution of dust on the distribution of apparent axis ratios of spiral galaxies. We assume no correlation between the dust column and the physical diameter of the galaxy, an assumption we will justify posteriori. While Unterborn & Ryden (2008) use the inclination dependence of the LF to define a sample of galaxies not affected by dust (i.e. not biased towards face-on objects), our shape fitting solves for the dust effects self-consistently.

We follow the following steps to produce the predicted distribution of projected axis ratios, given our assumed distribution of axis ratios and our measured luminosity and colour distributions.

(i) We assume that the amount of extinction and reddening are roughly proportional to the path-length of the light through the galaxy. Therefore, we expect a minimum extinction when a galaxy is seen face-on, and an increasing extinction as the line of sight approaches the plane of the galactic disc. Similarly, Shao et al. (2007), Unterborn & Ryden (2008) and Maller et al. (2008) all find that the optical depth increases monotonically with the inclination angle. The following parametrization of the angle dependence of dust extinction is not intended as a physical model, but as a heuristic guess for the scaling. Consider an oblate triaxial galaxy with axis ratios given by $x = B/A$ and $y = C/B$. The total dust extinction as a function of inclination $\theta$ in our model is

$$E(\theta) = \begin{cases} E_0(1 + y - \cos \theta), & \text{if } \cos \theta > y, \\ E_0, & \text{if } \cos \theta < y, \end{cases}$$

where $E_0$ is the edge-on extinction in magnitudes in a given band and $y$ is the galaxy height to diameter ratio extracted from a distribution of mean $\gamma$ and width $\sigma_\gamma$. The same can be assumed for the dust reddening,

$$R(\theta) = \begin{cases} R_0(1 + y - \cos \theta), & \text{if } \cos \theta > y, \\ R_0, & \text{if } \cos \theta < y, \end{cases}$$

where $R_0$ is the edge-on reddening in magnitudes. In the optically thin case, we can tie $R_0$ to the extinction via $E_0 = 2.77R_0$, as is appropriate for the $r$ band and $g - r$ colour. We find that our model results are not strongly dependent on this parameter and therefore can be applied to either the optically thin or thick case.

(ii) We produce an extincted LF defined by

$$\phi_E(M, \theta) = \phi(M + E(\theta)),$$

where $\phi(M)$ is the unextincted LF calculated using only face-on galaxies.

We define the ratio, $f_E$, between the number of observed (extincted) and intrinsic galaxies of a given luminosity $f_E(M) = \phi_E(M)/\phi(M)$. Similarly, we define $f_R(g - r)$ as the ratio between the underlying and reddened distributions of galaxy colours.

(iii) We calculate the ratio of the number of galaxies seen at inclination $\theta$ to the number expected without extinction, by multiplying the effects of reddening and extinction together,

$$\psi(\theta) = \frac{\int_{-\infty}^{\theta} \int_{-\infty}^{\infty} f_E(M) \phi_E(M) \phi_E(C) W(C, M) dC dM}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(M) \phi(C) W(C, M) dC dM},$$

where $C = g - r$, and the function $W$ contains the correlation between colour and $M_c$. We assume that $W$ is Gaussian with mean and dispersion extracted directly from the data; this correlation is compatible with the results shown in figs 11 and 12 of Blanton et al. (2003b). The subindex $s$ indicates that the luminosity and colour functions correspond to a particular subsample of galaxies; these subsamples are defined using sharp cuts in the allowed luminosity and colour ranges. This indicates that $\psi$ depends not only on the amount of extinction and reddening, but also on the range of luminosities and colours present in each subsample of galaxies. Note that equation (3) ignores the presence of large-scale structure, which is justified given the large solid angle of the SDSS sample.

(iv) We then construct the model distribution of apparent axis ratios $N_{model}(b/a)$ as described in Section 3. Rather than selecting the cosine of the viewing angle from a flat distribution, we select it from $\psi(\theta)$ as given in equation (3). In the case where $E_0 = 0$ and $R_0 = 0$, $\psi(\theta) = 1$.

In general, the effect of dust extinction and reddening is to decrease the number of galaxies seen edge-on relative to those that are face-on. This decrease depends strongly on the luminosity and colour functions, as well as on the selected range of luminosities and colours. The above process gives a prediction for the observed projected axis ratio distribution for a given set of triaxial galaxy parameter distribution functions and dust properties. Note that for a given viewing angle, the model states that the dust affects the likelihood that a galaxy would enter the sample at that viewing angle, but does not affect the observed projected axis ratio. This is a good approximation as long as the dust is smoothly distributed within individual galaxies.

In the following section, we will constrain these parameters by fitting these predictions to the observed axis ratio distribution.
3.3 Parameter fitting

We aim to constrain the parameters \( \mu, \sigma, \gamma \) and \( \sigma_y \), which describe the intrinsic shapes of galaxies by fitting the observed axis ratio distribution. Spirals and ellipticals have intrinsically different shapes, and we fit to the two separately. For spirals, we also include the effects of dust via the extinction parameter \( E_0 \) which in turn defines the reddening \( R_0 \).

We define a grid in parameter space \( p \) (four parameters for ellipticals, five for spirals). Using the parameters of each grid point \( p_i \), we generate random three-dimensional axis ratios from the assumed distribution, observed at a random orientation (modulated by the effects of dust, as described above). We then generate 10 independent model distributions of projected axis ratios, each containing as many galaxies as the sample of galaxies under analysis. We take the average of these 10 distributions as the final model distribution, \( N_{\text{model}}(b/a, \, \{p\}_i) \), and use the jackknife errors, \( \sigma_{\text{jackknife}}(b/a) \), obtained from the observed distribution to define a \( \chi^2 \) between the real data, \( N(b/a) \), and the model,

\[
\chi^2(p_i) = \sum_{b/a \text{ bins}} \left[ \frac{N_{\text{model}}(b/a, \, \{p\}_i) - N(b/a)}{\sigma_{\text{jackknife}}(b/a)} \right]^2.
\]

The best-fitting parameters correspond to the minimum value of \( \chi^2 \) throughout the parameter grid. Throughout this analysis, we use a bin size of \( \Delta(b/a) = 0.1 \) in presenting the observed and model distributions of \( b/a \) and when calculating \( \chi^2 \).

4 RESULTS

4.1 Elliptical galaxies

The grid of parameters we used for elliptical galaxies is shown in Table 1. The grids include the full range of values suggested for these parameters in the literature. The number of steps for each parameter, shown in the fourth column, corresponds to a coarse initial grid; once the best-fitting parameters are found we redo the analysis on a finer grid with twice the resolution centred on the best-fitting parameters. We repeat this refinement twice. Given the lack of colour dependence of the axis ratio distribution for ellipticals, we have assumed no dust. In order to explore the dependence of galaxy shape on luminosity, we divide the ellipticals by \( r \)-band absolute magnitude in four bins, with boundaries given by \( M_r - 5 \log_{10}(h) = -24, -21, -20, -19 \) and \( -17 \).

We calculate the marginalized one-parameter likelihoods (normalized to the maximum likelihood) resulting from fitting the observed \( N(b/a) \). The resulting best-fitting parameters are shown in Fig. 3. The ‘error bars’ correspond to the best-fitting widths, \( \sigma \) and \( \sigma_y \), of the distributions of \( \log(1 - B/A) \) and \( 1 - C/B \). As we guessed from the projected axis ratios, more luminous elliptical galaxies are consistent with a rounder underlying shape: the mean axis ratio \( \gamma \) changes from 0.6 to 0.2 with increasing luminosity (see Table 2 for values of \( \gamma \) and estimated errors not shown in the figure). The quantity \( \mu \) is much more constant, with the implied

| Parameter | Minimum value | Maximum value | Number of steps |
|-----------|---------------|---------------|-----------------|
| \( \mu \) | -4.05         | -0.05         | 21              |
| \( \sigma \) | 0.4           | 3             | 18              |
| \( \gamma \) | 0.09          | 0.99          | 19              |
| \( \sigma_y \) | 0.01          | 0.36          | 12              |

Figure 3. Dependence of the best-fitting parameters on galaxy luminosity, for elliptical galaxies. Squares represent the best-fitting parameters from the marginalized, one-parameter probabilities. Circles correspond to the best-fitting parameters in the four-dimensional parameter space. Dotted lines and shaded areas indicate the best-fitting parameters for the full sample of elliptical galaxies. Top panel: variations in the typical \( \mu = \text{median} \times \log (1 - B/A) \) axis ratio; the error bars and shaded areas do not correspond to uncertainties in the parameters but indicate the best-fitting width \( \sigma \) for the Gaussian distribution of \( \mu \) values used in the model. Bottom panel: same as the top panel, but for \( \gamma = \text{median} \times (1 - C/B) \).

\( B/A \) three-dimensional axis ratio varying only slightly, from \( \approx 0.95 \) to \( \approx 0.92 \).

In this figure, squares show the best-fitting parameters corresponding to the marginalized one-parameter maximum likelihoods. The open symbols show the best-fitting parameters as obtained from the full parameter space. In most cases, the two estimates agree reasonably well. Table 2 shows the values of the best-fitting parameters for the four subsamples of elliptical galaxies, as well as the maximum likelihood value. The reduced \( \chi^2 \) is \( \leq 2 \), indicative of a good fit, except for the highest luminosity subsample and the total sample. In these cases, our simple model does not give a statistically rigorous good fit, but it still follows the shape of the observed distribution quite well. Fig. 4 directly compares the observed axis ratio distribution with the model fits for the full sample of elliptical galaxies. Fig. 5 shows the comparison in each bin of absolute magnitude; the agreement between the model and observed distributions, while not perfect in every case, is impressive, and thus the four-parameter model is adequate to describe the real distributions of axis ratios for the galaxy luminosities explored here.

Vincent & Ryden (2005) used slightly different analysis techniques and a sample a quarter of the size the one used here, but they found results in very good agreement with our own, namely that low-luminosity elliptical galaxies are more consistent with prolate spheroids than are high-luminosity ellipticals.

The analysis of the marginalized two-parameter likelihoods indicates that there are little or no degeneracies between the parameters used to fit the projected shapes of elliptical galaxies.
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Table 2. Best-fitting model parameters for elliptical galaxies.

| Parameter | Sample 1 $-17 > M_r > -19$ | Sample 2 $-19 > M_r > -20$ | Sample 3 $-20 > M_r > -21$ | Sample 4 $-21 > M_r > -24$ | All |
|-----------|----------------------------|----------------------------|----------------------------|----------------------------|-----|
| $\mu$     | $-2.85 \pm 0.30$           | $-3.05 \pm 0.20$           | $-2.75 \pm 0.10$           | $-3.85 \pm 0.15$           | $-2.2 \pm 0.1$   |
| $\sigma$  | $1.15 \pm 0.35$            | $1.00 \pm 0.05$            | $2.60 \pm 0.15$            | $2.35 \pm 0.20$            | $1.4 \pm 0.1$    |
| $\gamma$  | $0.41 \pm 0.03$            | $0.36 \pm 0.06$            | $0.56 \pm 0.02$            | $0.76 \pm 0.04$            | $0.57 \pm 0.06$  |
| $\sigma_\gamma$ | $0.17 \pm 0.03$ | $0.21 \pm 0.02$ | $0.25 \pm 0.02$ | $0.17 \pm 0.01$ | $0.21 \pm 0.02$ |
| $\chi^2$/d.o.f. | $0.41$ | $2.0$ | $1.72$ | $7.2$ | $6.8$ |

4.2 Spiral galaxies

The distribution of spiral galaxy axis ratios depends both on absolute magnitude and colour (Fig. 2), which we interpret as the effect of dust in the rotational plane of the galaxies. We model this as described in Section 3.2, giving a five-parameter model. We use the grid of parameters from Table 3 to find the model quantities that best reproduce the distribution of projected spiral galaxy shapes.

The location of the minimum $\chi^2$ is given in Table 4; covariance between the parameters in our model explains the discrepancies with the marginalized values. As can be seen, in most cases the $\chi^2$ values are small, which indicates an excellent agreement between the model and the data.

This corresponds to a likelihood more than two orders of magnitude higher than the best fit found by Ryden (2004) for the spiral galaxies in the DR1 of the SDSS. Our inclusion of a dust model is partly responsible for the improvement in the agreement between model and data; the best-fitting model with no dust to the full SDSS DR6 sample of spiral galaxies is characterized by $\chi^2$/d.o.f. $= 1.14$, somewhat higher than that for the model with dust, for which $\chi^2$/d.o.f. $= 0.41$. The sample used by Ryden (2004) is also a factor of $\sim 25$ smaller than our full sample of spiral galaxies, and is therefore prone to higher sample variance.

It should be noted that dust extinction and the parameter $\mu$ are somewhat degenerate (none of the other parameter pairs show appreciable degeneracy). Fig. 6 shows the marginalized $\mu$ versus $E_0$ likelihood contours for the full sample of spiral galaxies. Interestingly, the axis ratio distribution alone allows the detection of extinction at only slightly better than 1$\sigma$. However, the strong relationship between colour and axis ratios shown in Fig. 2, and the results by Shao et al. (2007), Unterborn & Ryden (2008) and Maller et al. (2008), strongly indicate the presence of dust extinction in spirals.

Fig. 7 compares the observed spiral axis ratio distribution with that from our model; the two are in excellent agreement. This figure uses the full spiral sample; we have assumed that the dust and shape properties of spirals are independent of luminosity. We test that assumption in Fig. 8, which shows the observed $b/a$ distribution for spirals in different luminosity ranges, as well as the model prediction (dashed lines). The luminosity dependence in this model comes about solely from the angular selection function from...
Table 4. Best-fitting model parameters for spiral galaxies: full sample and dependence on luminosity.

| Parameter | Sample 1 $-17 > M_r > -19$ | Sample 2 $-19 > M_r > -20$ | Sample 3 $-20 > M_r > -21$ | Sample 4 $-21 > M_r > -24$ | All |
|-----------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----|
| $E_0$     | 0.20 ± 0.61                 | 0.72 ± 0.68                 | 0.8 ± 0.7                   | 0.72 ± 0.49                 | 0.44 ± 0.24 |
| $\mu$     | $-2.13 ± 0.38$              | $-2.41 ± 0.48$              | $-2.17 ± 0.41$              | $-2.17 ± 0.34$              | $-2.33 ± 0.13$ |
| $\sigma$  | 0.73 ± 0.19                 | 0.76 ± 0.25                 | 0.70 ± 0.35                 | 0.79 ± 0.31                 | 0.79 ± 0.16 |
| $\gamma$  | 0.79 ± 0.02                 | 0.79 ± 0.03                 | 0.74 ± 0.03                 | 0.62 ± 0.04                 | 0.79 ± 0.02 |
| $\sigma_\gamma$ | 0.048 ± 0.007             | 0.051 ± 0.007              | 0.06 ± 0.01                 | 0.11 ± 0.02                 | 0.050 ± 0.015 |
| $\chi^2$/d.o.f. | 0.21                        | 0.49                        | 1.31                        | 0.43                        | 0.41 |

Figure 6. Marginalized likelihood contours in the $\mu$–$E_0$ plane, for the full sample of spiral galaxies, corresponding to 1, 2 and 3σ confidence levels (shown as solid, dashed and dotted lines, respectively).

Figure 7. Comparison between the best-fitting model axis ratio distributions (solid line) and the actual measured distributions from the full sample of spiral galaxies in the SDSS DR6 (open symbols with error bars). Errors are calculated using the jackknife method.

Equation (3) through the different ranges of luminosity that define each sample. There is excellent agreement between the low-luminosity spirals and the model; indeed, the low-luminosity subsample has a $b/a$ distribution very close to that of the full sample, which is a consequence of our $1/V_{max}$ weighting. This agreement progressively degrades as we go towards higher luminosity. Indeed, as we saw for ellipticals, the high-luminosity spirals tend to have larger axis ratios (i.e. to be more round) than do low-luminosity objects.

There are two ways we might model this effect. A higher dust column for more luminous galaxies could preferentially remove edge-on objects from the high-luminosity bin, as suggested by Huizinga & van Albada (1992). Alternatively, as in ellipticals, there could be a direct correlation between three-dimensional shape and luminosity for spirals (e.g. Giovanelli et al. 1995). Indeed, there is a strong correlation between Hubble type and luminosity whereby high-luminosity spirals tend to be early-types with large bulges (Roberts & Haynes 1994; Tasca & White 2005). This can explain the low number of small $b/a$ objects at such luminosities.

We test these two options by fitting our model separately to galaxies in each range of absolute magnitude shown in Fig. 8; the solid lines show the best-fitting distributions with $E_0$ as a free parameter, and the dashed lines the best fits without dust ($E_0 = 0$). Fig. 9 shows the dependence of the best-fitting parameters as a function of absolute magnitude. The marginalized estimates are in
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Figure 9. Dependence of the best-fitting parameters on galaxy luminosity for spiral galaxies. Circles correspond to the best-fitting parameters in the five-dimensional parameter space. Open symbols show results when including dust and filled circles show the results with no dust. The horizontal lines indicate the best-fitting parameters obtained from the full sample of spiral galaxies (dotted lines show the best fit when including dust, thick-dashed lines with no dust). Shaded areas and thin-dashed lines show the variance in the parameters that best fit the observed distributions of projected axis ratios. Top panel: variations in the best-fitting values of extinction (filled symbols, dashed line). Error bars show the ranges of extinction values such that the likelihood is above $\Delta L = 0.36$. Middle panel: variations in the typical $\mu = \log (1 - B/A)$ axis ratio; the error bars indicate the best-fitting width $\sigma_\gamma$ for the Gaussian distribution of $\gamma$ values used in the model. The corresponding distributions for each subsample are shown on the right-hand side in individual subpanels. Bottom panel: same as middle panel for $\gamma = 1 - C/B$ (thick-dashed lines are not shown to improve clarity).

We now explore the different intrinsic shapes of galaxies according to their $g - r$ colour. This analysis may provide better constraints on the dust extinction in spiral galaxies. Fig. 10 shows the axis ratio distribution in bins of $g - r$. The dotted lines in this figure show the model using a fixed galaxy shape and dust corresponding to the best fit to the full spiral galaxy sample. The agreement is reasonable for all except the reddest subsample. Therefore, we allow the parameters to vary (solid lines), and the fit to the different
disappears for large sizes. Between the recovered values of $\mu$ and $\sigma$ when including dust.

Up to this point we had assumed that there is no dependence of shape and dust extinction with the physical size of the galaxy. The best-fitting parameter values are those appropriate for galaxies of median size within each subsample. However, the intrinsic shape and the amount of dust extinction of galaxies might depend on their sizes; galaxies of different sizes but similar luminosities might have different dynamical histories, and the dust column might reasonably be larger in larger galaxies.

The top panel of Fig. 12 shows the dependence of the median $b/a$ on projected galaxy size as given by the photometric model scale-length $r_{gal}$ (exponential or de Vaucouleurs depending on the value of the fracDeV parameter). The median $b/a$ decreases significantly for larger galaxies for both spirals and ellipticals. This effect is present at all galaxy luminosities.

With this dependence in mind, Fig. 13 shows the results of fitting our model to samples of galaxies in bins of physical size at constant luminosity, where the left-hand panels show the results for elliptical galaxies (parameters $\mu$ and $\gamma$), and the right-hand panels for spiral galaxies (parameters $\mu$, $\gamma$ and $E_0$). The best-fitting parameters are presented in Table 6 for faint, spiral and elliptical galaxies, and in Table 7 for bright galaxies. At a given luminosity, small elliptical galaxies have similar model parameters as those of the full sample of ellipticals, although at high luminosity, they tend to show slightly more elongated shapes (as reflected in the $\mu$ parameter). Large elliptical galaxies, on the other hand, are more elongated than the full sample. In particular, the $r_{gal} > 7 h^{-1}$ kpc sample can be identified with elongated prolate shapes. Quantitatively, at low luminosities ($M_r = -18$), the median axis ratios are $B/A = 0.94$ and $C/B = 0.42$ for small galaxies, and $B/A = 0.85$ and $C/B = 0.1$ for large galaxies. We examined the SDSS images and colours of ellipticals of the most extreme axis ratio ($b/a < 0.1$); all appeared to be correctly classified.

Low-luminosity spiral galaxies, on the other hand, go from median axis ratios of $B/A = 0.9$ and $C/B = 0.3$ for small sizes to $B/A = 0.36$ and $C/B = 0.12$ for large galaxies. Thus, the disc thickness decreases with increasing size. Shao et al. (2007) assumed that $\mu$ and $\sigma$ are independent of size, whereas we found that most of the changes in the shape of galaxies with size are actually absorbed by variations in these parameters. However, the conclusions of Shao et al. are robust to this detail and indicate, as our results do, that larger spiral galaxies tend to have flatter discs.

5 DISCUSSION AND CONCLUSIONS

In this paper we have addressed the problem of reproducing the observed distributions of projected axis ratios of galaxies from the SDSS. We have introduced a number of improvements over previous works including: (i) the use of larger samples of galaxies made possible by the introduction of an iterative $1/V_{max}$ weighting scheme, (ii) the inclusion of the effects of dust extinction on the distribution of apparent $b/a$ axis ratios for spiral galaxies, (iii) the analysis of dependence of galaxy shapes on galaxy luminosity, colour and physical size.

We developed a simple model for the effects of dust extinction on the distribution of apparent axis ratios and used it to constrain the intrinsic shapes of spiral galaxies. We characterize a given galaxy as a triaxial ellipsoid of axes $A,B$ and $C$ from major to minor. The full sample of spiral galaxies is characterized by the mean and standard deviation of $1-C/B$ of $\mu = -2.33 \pm 0.13$ and $\sigma = 0.79 \pm 0.16$;

Table 5. Best-fitting model parameters for spiral galaxies: dependence on galaxy colour.

| Parameter | Sample 1 | Sample 2 | Sample 3 | Sample 4 |
|-----------|----------|----------|----------|----------|
|           | $-0.2 < g - r < 0.6$ | $0.6 < g - r < 0.75$ | $0.75 < g - r < 0.85$ | $0.85 < g - r < 1.1$ |
| $E_0$     | $0.4 \pm 0.3$ | $0.6 \pm 0.6$ | $0.7 \pm 0.5$ | $1.0 \pm 0.5$ |
| $\mu$     | $-2.13 \pm 0.41$ | $-2.77 \pm 0.45$ | $-2.45 \pm 0.38$ | $-2.41 \pm 0.33$ |
| $\sigma$  | $0.7 \pm 0.4$ | $0.61 \pm 0.31$ | $0.91 \pm 0.36$ | $0.73 \pm 0.32$ |
| $\gamma$  | $0.80 \pm 0.03$ | $0.80 \pm 0.03$ | $0.80 \pm 0.02$ | $0.79 \pm 0.02$ |
| $\sigma_\gamma$ | $0.054 \pm 0.012$ | $0.054 \pm 0.008$ | $0.052 \pm 0.006$ | $0.050 \pm 0.005$ |
| $\chi^2$/d.o.f. | 0.33 | 0.17 | 0.15 | 0.20 |
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Figure 11. Dependence of the best-fitting parameters on galaxy colour for spiral galaxies. Panels, lines and symbols are as in Fig. 9.

the distribution of \( \log(1 - B/A) \) is modelled as a lognormal with mean \( \gamma = 0.79 \pm 0.02 \) and standard deviation \( \sigma_\gamma = 0.050 \pm 0.015 \). These values are in good agreement with the thickness of discs derived by Ryden (2004), who finds \( \gamma = 0.216 \), although her face-on discs are more elliptic, \( \mu = -1.85 \), and the distribution widths are somewhat different from ours. More recently, Unterborn & Ryden (2008) analyse the shapes of a sample of spiral galaxies which has been corrected for the biases introduced by dust on the distribution of projected shapes. The resulting spiral shapes are characterized by \( \mu = -2.56, \sigma = 0.91, \gamma = 0.216 \) and \( \sigma_\gamma = 0.067 \), which are roughly consistent with the results we present here; however, their sample selection is explicitly dependent on their model for the dependence of dust on inclination angle, which makes it difficult to make a more quantitative comparison.

We also studied variations in the intrinsic shapes of galaxies with galaxy luminosity (Table 4), colour (Table 5) and physical size (Tables 6 and 7). As luminosity and colour are correlated, and in particular more luminous spiral galaxies show larger bulges, we find rounder spiral galaxies at larger luminosities and \( g - r \) colours. At a given luminosity, larger galaxies tend to be flatter.

Our quoted parameters are obtained by marginalizing over the extinction parameter, \( E_0 \). The extinction is not well constrained by the axis ratio distribution alone, but for our full sample, we find a value \( E_0 = 0.44 \pm 0.24 \) after marginalizing over other parameters, in good agreement with results from the literature.

Using our results from Table 4, we determine the relation between the observed projected axis ratios of spiral galaxies and the inclination angles of their discs for four different luminosity ranges. In order to do this, we simply calculate \( (b/a)_{\text{max}} \) at which the distribution of projected axis ratio peaks, as well as the 10th and 90th percentiles, for a given narrow range of viewing angles. We use the percentiles to infer the range of angles that correspond to a given \( b/a \) ratio that can be measured from a spiral galaxy. Fig. 14 and Table 8 show the relation between projected axis ratio \( b/a \) and polar viewing angle \( \theta \) for spiral galaxies of different luminosities. The shaded area shows the ranges of polar viewing angles that correspond to a given value of \( b/a \) for the fainter sample. As brighter galaxies tend to have thicker discs, the value of \( b/a \) corresponding to edge-on bright galaxies is higher than for fainter galaxies. Note that these results take into account the effects of dust extinction.

The axis ratio distribution of elliptical galaxies shows no dependence on colour, suggesting that dust extinction is not important for this sample, and we do not include it in our modelling. The full sample of elliptical galaxies are characterized by parameters \( \mu = -2.2 \pm 0.2, \sigma = 1.4 \pm 0.10, \gamma = 0.57 \pm 0.06 \) and \( \sigma_\gamma = 0.21 \pm 0.02 \), which correspond to slightly oblate spheroids in agreement with...
Figure 12. Variation of projected axis ratio $b/a$ with the physical size of galaxies. Dashed lines correspond to spiral galaxies and solid lines to elliptical galaxies. Each panel corresponds to a different luminosity bin.

Table 6. Best-fitting model parameters: dependence on galaxy size for galaxies with $M_r > -19$. Small galaxies satisfy $r_{gal} < 2 h^{-1}$ kpc, medium galaxies, $2 < r_{gal} < 7 h^{-1}$ kpc and large galaxies, $r_{gal} > 7 h^{-1}$ kpc.

| Parameter | Spirals small | Spirals medium | Spirals large | Ellipticals small | Ellipticals medium | Ellipticals large |
|-----------|---------------|---------------|--------------|------------------|-------------------|------------------|
| $E_0$     | 0.6           | 0.16          | 0.16         | -1.18            | -1.18             | -1.18            |
| $\mu$     | -2.29         | -1.73         | -0.45        | -3.18            | -2.03             | -1.18            |
| $\sigma$  | 0.76          | 0.64          | 1.54         | 0.75             | 1.6               | 1.6              |
| $\gamma$  | 0.71          | 0.83          | 0.89         | 0.445            | 0.325             | 0.19             |
| $\sigma_\gamma$ | 0.05 | 0.02 | 0.01 | 0.17 | 0.17 | 0.05 |

Figure 13. Best-fitting parameters as a function of galaxy luminosity for different galaxy sizes (in different symbols as shown in the figure key). Left-hand panels correspond to elliptical galaxies and right-hand panels to spiral galaxies. The top panel shows the variation of the extinction, $E_0$, middle panels show the mean value of $\mu$ and bottom panels show the variation of the $\gamma$ parameter.

with results by Vincent & Ryden (2005). More luminous ellipticals tend to be rounder, although ellipticals are oblate at all luminosities. Although Vincent & Ryden (2005) found that the lowest luminosity ellipticals are best fitted by prolate spheroids, their inferred $C/A$ axis ratio is consistent with our estimates.

Table 7. Best-fitting model parameters: dependence on galaxy size for galaxies with $M_r < -21$. Small, medium and large galaxies are selected as in Table 6.

| Parameter | Spirals small | Spirals medium | Spirals large | Ellipticals small | Ellipticals medium | Ellipticals large |
|-----------|---------------|---------------|--------------|------------------|-------------------|------------------|
| $E_0$     | 1.90          | 0.72          | 0.59         | -3.37            | -1.12             | -3.37            |
| $\mu$     | -3.17         | -2.49         | -2.13        | -2.52            | -1.12             | -2.52            |
| $\sigma$  | 0.91          | 0.58          | 0.79         | 2.7              | 2.6               | 2.6              |
| $\gamma$  | 0.31          | 0.48          | 0.66         | 0.795            | 0.545             | 0.695            |
| $\sigma_\gamma$ | 0.04 | 0.14 | 0.09 | 0.22 | 0.13 | 0.17 |

Figure 14. Relation between the polar viewing angle, $\theta$, and projected axis ratios, $b/a$, for subsamples of spiral galaxies of different absolute magnitudes. The shaded area encloses 80 per cent of the possible viewing angles associated with a given value of $b/a$.

This paper presented the most detailed statistical study of shapes of galaxies separated in spirals and ellipticals to date, and includes the effects of dust on the distribution of projected shapes of spirals in a self-consistent way. It is remarkable that dust can be inferred from the distribution of projected axis ratios alone, although only at a low statistical significance. The dependence of intrinsic galaxy shapes and extinction with luminosity, colour and size expands the range of tests that galaxy formation models need to satisfy, to continue improving the modelling of the processes that drive the evolution of galaxies.

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Table 8. Relation between bla and polar viewing angle, $\theta$, for subsamples of spiral galaxies corresponding to different ranges of absolute magnitude.

| $\theta$ (°) | $<M_r>$ = -18.0 | $<M_r>$ = -20.1 | $<M_r>$ = -21.2 | $<M_r>$ = -22.7 |
|-------------|----------------|----------------|----------------|----------------|
| 3           | 0.8815         | 0.9104         | 0.886          | 0.8864         |
| 7           | 0.8812         | 0.91           | 0.8858         | 0.8863         |
| 11          | 0.88           | 0.9083         | 0.8847         | 0.8855         |
| 15          | 0.8783         | 0.9059         | 0.8833         | 0.8831         |
| 19          | 0.8724         | 0.898           | 0.8778         | 0.8777         |
| 23          | 0.8616         | 0.8844         | 0.8673         | 0.8731         |
| 27          | 0.8487         | 0.8687         | 0.8513         | 0.8603         |
| 31          | 0.8265         | 0.8429         | 0.8343         | 0.842           |
| 35          | 0.7981         | 0.8115         | 0.807          | 0.8188         |
| 39          | 0.7687         | 0.7797         | 0.7745         | 0.7911         |
| 43          | 0.73           | 0.7388         | 0.7373         | 0.7596         |
| 47          | 0.6871         | 0.694           | 0.6963         | 0.7308         |
| 51          | 0.6407         | 0.6458         | 0.6566         | 0.6929         |
| 55          | 0.5946         | 0.5983         | 0.6091         | 0.6531         |
| 59          | 0.5424         | 0.5449         | 0.5597         | 0.6124         |
| 63          | 0.4885         | 0.4899         | 0.5092         | 0.5721         |
| 67          | 0.4339         | 0.4344         | 0.4587         | 0.5332         |
| 71          | 0.3801         | 0.3797         | 0.4099         | 0.4968         |
| 75          | 0.3291         | 0.3279         | 0.3647         | 0.4647         |
| 79          | 0.2838         | 0.2816         | 0.3258         | 0.4383         |
| 83          | 0.2484         | 0.2454         | 0.2964         | 0.4195         |
| 87          | 0.2284         | 0.2248         | 0.2805         | 0.4095         |

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REFERENCES

Abazajian K. et al., 2003, AJ, 126, 2081
Abazajian K. et al., 2004, AJ, 128, 502
Abazajian K. et al., 2005, AJ, 129, 1755

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Park C., Choi Y., 2005, ApJ, 635, L29
Peletier R. F., Willner S. P., 1992, AJ, 103, 1761
Pier J., Munn J., Hindsley R., Hennessy G., Kent S., Lupton R., Ivezić Ž., 2003, AJ, 125, 1559
Rix H., Zaritsky D., 1995, ApJ, 447, 82
Roberts M. S., Haynes M., 1994, ARA&A, 32, 115
Ryden B., 2004, ApJ, 601, 214
Sandage A., Freeman K., Stokes N. R., 1970, ApJ, 160, 831
Shao Z., Xiao Q., Shen S., Mo H., 2007, ApJ, 659, 1159
Skrutskie M. F. et al., 2006, AJ, 131, 1163
Smith J. et al., 2002, AJ, 123, 2121
Statler T. S., 1994a, ApJ, 425, 458
Statler T. S., 1994b, ApJ, 425, 500
Statler T. S., Fry A. M., 1994, ApJ, 425, 481
Statler T. S., Lambright H., Bak J., 2001, ApJ, 549, 871
Stevens J. A., Amure M., Gear W. K., 2005, MNRAS, 357, 361
Stoughton C. et al., 2002, AJ, 123, 485

Strauss M. et al., 2002, AJ, 124, 1810
Tasca L. A. M., White S. D. M., 2005, preprint (astro-ph/0507249)
Temi P., Brighenti F., Mathews W., Gregman J., 2004, ApJS, 151, 237
Tucker D. et al., 2006, Astron. Nachr., 328, 821
Tully R. B., Pierce M. J., Huang J.-S., Saunders W., Verheijen M. A. W., Witchalls P. L., 1998, AJ, 115, 2264
Unterborn C. T., Ryden B. S., 2008, ApJ, preprint (arXiv:0801.2400)
Valentijn E., 1990, Nat, 346, 153
Valentijn E., 1994, MNRAS, 266, 614
Valotto C., Giovanelli R., 2004, AJ, 128, 115
Vincent R. A., Ryden B. S., 2005, ApJ, 623, 137
York D. et al., 2000, AJ, 120, 1579
Zaritsky D., 1994, AJ, 108, 1619

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