Understanding $I=2\pi\pi$ Interaction

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Abstract. A correct understanding and description of the $I=2\pi\pi$ S-wave interaction is important for the extraction of the $I=0\pi\pi$ S-wave interaction from experimental data and for understanding the $I=0\pi\pi$ S-wave interaction theoretically. With t-channel $\rho$, $f_2(1270)$ exchange and the $\pi\pi\rightarrow\rho\rho\rightarrow\pi\pi$ box diagram contribution, we reproduce the $\pi\pi$ isotensor S-wave and D-wave scattering phase shifts and inelasticities up to 2.2 GeV quite well in a K-matrix formalism.

INTRODUCTION

Much attention has been paid to the isospin $I=0\pi\pi$ S-wave interaction due to its direct relation to the $\sigma$ particle and the scalar glueball candidates. However, to really understand the isoscalar $\pi\pi$ S-wave interaction, one must first understand the isospin $I=2\pi\pi$ S-wave interaction due to the following two reasons: (1) There are no known $s$-channel resonances and less coupled channels in $I=2\pi\pi$ system, so it is much simpler than the $I=0\pi\pi$ S-wave interaction; (2) To extract $I=0\pi\pi$ S-wave phase shifts from experimental data on $\pi^+\pi^-\rightarrow\pi^+\pi^-$ and $\pi^+\pi^-\rightarrow\pi^0\pi^0$ obtained by $\pi N\rightarrow\pi\pi N$ reactions, one needs an input of the $I=2\pi\pi$ S-wave interaction. While the $I=2\pi\pi$ S-wave interaction can be extracted from the pure $I=2\pi^\pm\pi^\mp\rightarrow\pi^\mp\pi^\mp$ reactions, the $I=0\pi\pi$ S-wave interaction can only be extracted from $\pi^+\pi^-\rightarrow\pi^+\pi^-$ and $\pi^+\pi^-\rightarrow\pi^0\pi^0$ reactions which are mixture of $I=0$ and $I=2$ contributions. The relation between the $\pi^+\pi^-\rightarrow\pi^+\pi^-$, $\pi^0\pi^0$ S-wave amplitudes and the isospin decoupled amplitudes is as the following:

$$T_s(+-,+-) = T_s^{I=0}/3 + T_s^{I=2}/6,$$

$$T_s(+-,00) = T_s^{I=0}/3 - T_s^{I=2}/3.$$  (1)

(2)

The $T_s^{I=0}$ was usually extracted from $T_s(+-,+-)$ and $T_s(+-,00)$ information by assuming some kind of $T_s^{I=2}$ amplitude.

Up to now, experimental information on the $I=2\pi\pi$ scattering mainly came from $\pi^+p\rightarrow\pi^+\pi^+n$ [1] and $\pi^-d\rightarrow\pi^-\pi^-pp$ [2, 3] reactions. The main features for the $I=2\pi\pi$ S-wave phase shifts $\delta_0^2$ and inelasticities $\eta_0^2$ are: (1) the $\delta_0^2$ goes down more and more negative as the $\pi\pi$ invariant mass increases from $\pi\pi$ threshold up to 1.1 GeV; (2) the $\delta_0^2$ starts to increase for energies above about 1.1 GeV; (3) the $\eta_0^2$ starts to deviate from 1 for energies above 1.1 GeV. The first feature can be well explained by the t-channel $\rho$ exchange force [4, 5, 6, 7] while the effect of t-channel scalar exchange is extremely small and can be neglected [7]. Due to the relative poor quality of the $I=2\pi\pi$ scattering...
data above 1.1 GeV, the other two features are usually overlooked. In this work \[8\], we show in a K-matrix formalism \[5, 9\] that these two features can be well reproduced by the t-channel $f_2(1270)$ exchange and the $\pi\pi$-$\rho\rho$ coupled-channel effect, respectively.

Recently, the $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering from the old $\pi N$ scattering experiments with both unpolarized \[10\] and polarized targets \[11\] has been re-analyzed \[12, 13\] in combination with new information from $p\bar{p}$ and other experiments. The $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering has also been studied by E852 \[14\], GAMS \[15\] Collaborations and analyzed \[16\]. In Refs. \[10, 12\], a scattering length formula for $I=2$ $\pi\pi$ S-wave was used; in Ref. \[16\] another empirical parametrization was used. These parametrizations give similar phase shifts up to 1.1 GeV, but differ at higher energies. All these previous analyses have ignored the inelastic effects in the $I=2$ channel. We will demonstrate that the correct description of the $I=2$ S-wave interaction has significant impact on the extraction of the $I=0$ $\pi\pi$ S-wave amplitude for energies above 1.1 GeV.

**THEORETICAL FRAMEWORK**

We follow the $K$-matrix formalism as in Refs. \[5, 9\]. For the two-channel case, the two-dimensional $K$ matrix and phase space $\rho(s)$ matrix are

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}, \quad \rho(s) = \begin{pmatrix} \rho_1(s) \\ 0 \\ 0 \\ \rho_2(s) \end{pmatrix},$$

with $i=1,2$ representing $\pi\pi$ and $\rho\rho$ channel, respectively.

$$T_{11} = \frac{K_{11} - i\rho_2(K_{11}K_{22} - K_{12}K_{21})}{1 - i\rho_1K_{11} - i\rho_2K_{22} - \rho_1\rho_2(K_{11}K_{22} - K_{12}K_{21})}.$$  \hspace{1cm} (4)

Ignoring the interaction between $\rho\rho$, we have $K_{22} = 0$; then

$$T_{11} = \frac{K_{11} + iK_{12}\rho_2K_{21}}{1 - i\rho_1(K_{11} + iK_{12}\rho_2K_{21})},$$

where $iK_{12}\rho_2K_{21}$ corresponds to the contribution of the $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$ box diagram.

In order to obtain $K_{11}$, we incorporate the $t$-channel $f_2(1270)$ contribution into the $t$-channel $\rho$ exchange term by the Dalitz-Tuan method \[5, 12\].

$$K_{11} = \frac{K_{\rho}(s) + K_{f_2}(s)}{1 - \rho_1^2(s)K_{\rho}(s)K_{f_2}(s)}.$$  \hspace{1cm} (6)

The amplitudes for the $t$-channel $\rho$ and $f_2(1270)$ meson exchange without considering the vertex form factor were given in Ref. \[8\] for studying $I=0$ $\pi\pi$ scattering. In this work, we include a form factor of conventional monopole type to take into account the off-shell behavior of the exchanged mesons: $F(q^2) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2)$ with $(m, q)$ the mass and four-vector momentum, respectively, of exchanged mesons, and $\Lambda$ the cutoff parameter to be determined by experimental data. The resulted $K_{\rho}$ and $K_{f_2}$ are given in our paper \[8\].
For energies above the $\rho\rho$ threshold, the inelastic effect should be taken into account in the $I=2$ $\pi\pi$ channel. Note that unlike $I=0$ $\pi\pi$ channel, the $I=2$ $\pi\pi$ channel does not couple to the $K\bar{K}$ and $\omega\omega$ channels due to isospin conservation. The $\pi\pi$ channel couples to the $\rho\rho$ channel by the t-channel $\pi$ exchange as shown in Fig.1.

Assuming the on-shell approximation [18] for the $\rho\rho$ intermediate state, the amplitude for the box diagrams are calculated and projected to the S- and D-wave [8]. It is related to the K-matrix by $T_{\text{box}}^{I=2} = iK_{12}\rho_2K_{12}$. For the phase space factor $\rho_2$, the width of the $\rho$ meson is taken into account in our calculation.

NUMERICAL RESULTS AND DISCUSSION

From the formalism given above, we get $I=2$ $\pi\pi$ S-wave and D-wave phase shifts and inelasticities as shown in Fig.2. The t-channel $\rho$ exchange alone (dashed lines) reproduces the phase shifts for energies up to 1.1 GeV very well with the form factor parameter $\Lambda_{\rho\pi\pi} = 1.5$ GeV [5], but underestimates the phase shifts at higher energies. The inclusion of the t-channel $f_2(1270)$ exchange (dot-dashed lines) increases the phase shifts especially for energies above 1 GeV and can reproduce the phase shift data very well with the form factor parameter $\Lambda_{f_2\pi\pi} = 1.7$ GeV. However they only contribute to the elastic scattering and cannot produce the inelasticities for energies above 1 GeV.

The experimental information on the inelasticities is scarce for the $I=2$ $\pi\pi$ scattering. Two data points were given by Ref.[3] for energies $1\sim 1.5$ GeV. For energies $1.5\sim 2$ GeV, Ref.[2] estimated to be $0.5 \pm 0.2$ for the $\eta_0^2$ in one solution and assumed $\eta_0^2 = 1.53 - 0.475m_{\pi\pi}$ (GeV/c$^2$) for another solution. The two solutions gave similar results for the $I=2$ $\pi\pi$ S-wave phase shifts. For the $I=2$ $\pi\pi$ D-wave scattering, the inelasticity could not be measured well and was assumed to be $\eta_2^2 = 1$ for the extraction of the $\delta_2^2$.

Although there are only three data points with large error bars for the inelasticity parameter $\eta_0^2$ of the $I=2$ $\pi\pi$ S-wave scattering, it is clear that the inelastic effect may be significant around 1.6 GeV. In order to reproduce this inelasticity, it is necessary to consider the $\pi\pi \leftrightarrow \rho\rho$ coupling channel effect. We find that including contribution from the $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$ box diagram in our K-matrix formalism, the $I=2$ S-wave inelasticity data can be very well reproduced without introducing any more free parameter as shown in Fig.2. The same diagram also predicts a broad shallow dip around 1.7 GeV for the inelasticity of the $I=2$ D-wave scattering. Assuming $\eta_2^2 = 1$ for energies around 1.7 GeV may bias the $\delta_2^2$ data around this energy. This may be the reason that the $\delta_2^2$ data around 1.7 GeV has the largest discrepancy with our theoretical result. The box diagram has little influence to the phase shifts although it produces large inelasticity.

Inspired by the new claim of a pentaquark state [19], we also explored the possibility of including an $I=2$ s-channel resonance to reproduce the $I=2$ $\pi\pi$ S-wave scattering.
FIGURE 2. The $I = 2 \pi\pi$ S-wave ($\delta_0^2$, $\eta_0^2$) and D-wave ($\delta_2^2$, $\eta_2^2$) phase shifts and inelasticities. Data are from Refs.[1, 2, 3]. The solid curves represent the total contribution of $\rho$, $f_2$ exchange and the box diagram; dot-dashed curves from $\rho$ and $f_2$ exchange; dashed curves from t-channel $\rho$ exchange only.

FIGURE 3. Full amplitudes squared of $\pi^+\pi^- \rightarrow \pi^+\pi^-$ (a) and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ (b). The lines correspond to using $T_s^{I=0}$ from Ref.[5] plus various input of $T_s^{I=2}$: $T_s^{I=2} = 0$ (dotted lines); $T_s^{I=2}$ from Refs.[5, 10] (dot-dashed lines); $T_s^{I=2}$ from Ref.[16] (dashed lines); $T_s^{I=2}$ from this work (solid line).

data instead of using the t-channel $f_2$ exchange and the $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$ box diagram, but failed to reproduce the data. While the $\eta_0^2$ needs the resonance with mass around 1.6GeV, the $\delta_0^2$ needs the mass above 2.3 GeV with a much broader width.

To demonstrate the significance of possible impact of the $I = 2$ input for the extraction of $I = 0 \pi\pi$ amplitude, we calculate the full S-wave amplitudes for $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ according to Eqs.(12) with $T_s^{I=0}$ from Ref.[5] plus various $T_s^{I=2}$ inputs. The corresponding full S-wave amplitudes squared ($|\rho_1 T|^2$) are shown in Fig.3. The dotted lines are the results with $T_s^{I=2} = 0$. The dot-dashed lines correspond to the scattering length formula used in Refs.[5, 10, 12], which is similar to the result by considering only the t-channel $\rho$ exchange contribution. The dashed lines use the new empirical $T_s^{I=2}$ formula of Ref.[16], which is similar to the result by considering t-channel $\rho$ and $f_2$ exchange contributions, but ignoring the inelasticity caused by the
$\pi\pi \leftrightarrow \rho\rho$ box diagram contribution. The solid lines are results with our $T_s^{I=2}$ including the t-channel $\rho, f_2$ exchange and the box diagram. It is clear that $T_s^{I=2}$ has significant contribution to the amplitudes for $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ processes, hence has significant impact on the extraction of the $T_s^{I=0}$ amplitude. Previous inputs of $T_s^{I=2}$ give similar results as our new $T_s^{I=2}$ for energies below 1.1 GeV, but differ from ours significantly for higher energies. The inclusion of both t-channel $f_2$ exchange and $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$ box diagram contribution is important.

In summary, the basic features of $I=2$ $\pi\pi$ scattering phase shifts and inelasticities can be well reproduced by the t-channel meson ($\rho, f_2$) exchange and the $\pi\pi \leftrightarrow \rho\rho$ coupled-channel effect in the K-matrix formalism. The t-channel $\rho$ exchange provides repulsive negative phase shifts while the t-channel $f_2(1270)$ gives an attractive force to increase the phase shifts for $\pi\pi$ scattering above 1 GeV, and the coupled-channel box diagram causes the inelasticities. A correct description of the $I=2$ $\pi\pi$ scattering has significant impact on the extraction of the $I=0$ scattering amplitudes from $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ data, especially for energies above 1.2 GeV. A re-analysis of these data with our new description of the $I=2$ $\pi\pi$ scattering will be carried out as our next step.

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