Baryogenesis, Inflation and Superstrings

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Abstract

We study the conditions for successful Affleck–Dine baryogenesis in generic inflation and supergravity scenarios, finding powerful restrictions on them. String-based SUGRA models are especially interesting since they are surprisingly suitable for the implementation not only of AD baryogenesis but also inflation itself, presenting a nice solution to the $\eta$–problem.

1 Introduction

In the absence of better alternatives, the Affleck and Dine (AD) mechanism \cite{1,2} is a very attractive method for baryogenesis. It takes place if in the early universe some scalar “AD fields”, $\phi$, carrying baryon or lepton number ($B$ or $L$), get large initial vacuum expectation values (VEV’s), $\phi_{in}$. Then the equations of motion of $\phi$ (together with the presence of some baryon-violating operator involving the $\phi$ fields) lead to a net final baryon number \cite{1}. The mechanism is quite natural and efficient and does not need any particular tuning of parameters. The key point is therefore how to generate the large initial VEV’s, $\phi_{in}$. These have been commonly associated in the literature with the existence of (approximately) flat directions involving $\phi$. Then quantum fluctuations during inflation may yield a large $\phi_{in}$.

However, during inflation SUSY is necessarily spontaneously broken since the scalar potential $V$ gets a VEV, $\langle V \rangle \approx 3H^2 M_P^2$, and effective SUSY soft breaking terms, in particular effective soft masses of the order of the Hubble constant $H$ are generated, which clearly spoils any flat direction \cite{2}. Nevertheless, large initial VEV’s are still possible if these effective masses squared are negative, something that is possible in a generic SUGRA theory \cite{2}. The negative mass destabilizes the potential at the origin and, for large values of $\phi$, the potential is lifted, e.g. by $F$–terms coming from $\sim \phi^n$ terms in the superpotential $W$. Thus, a temporal minimum is generated. The $\phi$ field evolves rapidly towards the temporal minimum during this period. For the subsequent evolution to be succesful for baryogenesis, it is necessary that the operators that lift the potential are non-renormalizable (i.e. $n \geq 4$), which is perfectly possible \cite{2}.

To summarize, successful AD baryogenesis requires that the $\phi$ effective potential 

\begin{equation}
\end{equation}

\textit{during inflation contains}...
1. negative effective mass terms, $m_\phi^2 \leq 0$,

2. non-renormalizable terms to lift “flat directions” of the potential.

2 D–Inflation

The possibility of D–inflation [3, 4, 5] (i.e. inflation triggered by a non-vanishing D–term) in SUGRA scenarios is very attractive since, as has been often claimed in the literature, F–inflation seems to lead naturally to too large inflaton mass terms that disable the inflationary process (see end of sect.4).

In order to be concrete, it is convenient to suppose that inflation is mainly triggered by a single D–term (this does not reduce the generality of the analysis). Then, a suitable choice is to suppose that the relevant D–term is associated to one “anomalous” $U(1)$ [6, 7], which takes the form

$$V_D = \frac{1}{2} D^2 = \frac{1}{2} g^2 \left| \xi + \sum q_j |z_j|^2 K_{jj} \right|^2,$$  \hspace{1cm} (1)

where $g$ is the corresponding gauge coupling, $q_j$ are the charges of all the chiral fields, $z_j$, under the anomalous $U(1)$ and $K_{jj}$ is the Kähler metric (we are assuming here a basis for the $z_j$ fields where the Kähler metric is diagonal). Finally, the constant $\xi$ is related to the apparent anomaly, $\xi = g^2 M_P^2 (\sum q_j/192\pi^2)$. At low energy the D–term is cancelled by the VEV’s of some of the scalars entering Eq.(1), but initially $\langle D \rangle$ may be different from zero, thus triggering inflation. Let us also notice that the scenario is quite insensitive to the details of the Kähler potential $K$ (note in particular that $(K_{jj})^{1/2} z_j$ are simply the canonically normalized chiral fields).

Concerning AD baryogenesis, there are two main scenarios to consider, depending on $q_\phi \neq 0$ or $q_\phi = 0$. In the first case, it is clear that the D–term induces an effective mass term for $\phi$, which is negative provided

$$\text{sign}(\xi) \text{ sign}(q_\phi) = -1, \hspace{1cm} (2)$$

then the effective mass squared is negative and we expect $\langle \phi \rangle_{in} \neq 0$.

Nevertheless, this cannot be the whole story, since in the absence of additional $\phi$–dependent terms in $V$, $\langle \phi \rangle_{in}$ would adjust itself to cancel the D–term, thus disabling the inflationary process and breaking $B$ or $L$ at low energy. Thus, we need extra contributions yielding $\langle D \rangle_{in} \neq 0$, $\langle \phi \rangle_f = 0$. These may come from (a) low-energy soft breaking terms, (b) F–terms, (c) D–terms. We do not have space to review in detail the three possibilities (the interested reader is referred to ref.[8]). For our purposes, it is enough to say that no one of them works for the goal of AD baryogenesis. The reason is that either the extra contributions are too small (case (a)) or they lead to a potential which is lifted by renormalizable terms (contradicting condition 2 of Sect.1).

This leaves us just with the second scenario, namely, $q_\phi = 0$. Then, from (1), $m_\phi^2 = 0$ during inflation. So, there is a truly flat direction along $\phi$ and the AD mechanism can be implemented in the old-fashioned way after all! This argument is only exact at tree
level. Strictly speaking, there are small contributions to $m_\phi$ coming from higher loop corrections and the expected $O(\text{TeV})$ low-energy supersymmetry breaking effects. In any case, $\phi$ will acquire a large VEV during inflation due to quantum fluctuations if the correlation length for de Sitter fluctuations, $l_{\text{coh}} \approx H^{-1}\exp(3H^2/2m_\phi^2)$, is large compared to the horizon size. This translates into the condition $H^2/m_\phi^2 \gtrsim 40$, which is easily fulfilled in this context. So, the $q_\phi = 0$ scenario is really selected for AD baryogenesis, leading to very interesting physics.

### 3 F–Inflation

F–Inflation occurs when inflation is triggered by a non-vanishing F–term of the appropriate size. Concerning the implementation of the AD mechanism in F–inflationary scenarios, the main question (see Sect. 1) is whether it is possible to get an effective mass squared $m_\phi < 0$ or $m_\phi = 0$ for the AD field, $\phi$, during inflation. To answer this question, we need to examine the F–part of the effective potential $V$, in a SUGRA theory

$$V = e^G \left( G_j K^{ji} G_i - 3 \right) = F^j K_{ji} F^i - 3e^G. \quad (3)$$

Here $G = K + \log |W|^2$ where $W$ is the superpotential, $K^{ij}$ is the inverse of the Kähler metric and $F^i = e^{G/2} K^{ijk} G_{jk}$ are the corresponding auxiliary fields. During inflation, $\langle V \rangle_{\text{in}} = V_0 \simeq H^2 M_P^2$, which implies that some $F$ fields are different from zero, thus breaking SUSY. The effective gravitino mass squared during the inflationary epoch is given by $m^2_{3/2} = e^G = e^K |W|^2$ in $M_P$ units. The SUSY breakdown induces soft terms for all the scalars, in particular for $\phi$. More precisely, the value of the effective mass squared, $m_\phi^2$, is intimately related to the form of $K$. It is convenient to parametrize $K$ as

$$K = K_0(I) + K_{\phi\phi}\phi^2 + \cdots, \quad (4)$$

where $I$ represents generically the inflaton or inflatons. Plugging (4) in (3) it is straightforward to see that $|V|$ is an important result, that if there is no mixing between $\phi$ and $I$ in the quadratic term of $K$, i.e. if $K_{\phi\phi} = K_{\phi\phi}(I)$, then the effective mass squared for the canonically normalized field, $(K_{\phi\phi})^{-1/2}\phi$, is $m^2_\phi = m^2_{3/2} + V_0/M_P^2$. Hence $m^2_\phi$ is of $O(H^2)$ and positive and, therefore, the AD mechanism cannot be implemented. This excludes, for instance, minimal SUGRA.

A successful implementation of the AD mechanism thus requires a mixing in the quadratic term of $K$ of the inflaton $I$ and AD fields $\phi$, i.e. $K_{\phi\phi} = K_{\phi\phi}(I)$ in Eq. (4). This mixing should be remarkably strong and even so the possibility of a negative effective mass term is not guaranteed. For example, if we consider the following simple scenario $K = K_0(I) + |\phi|^2 + a|I|^2|\phi|^2$, where $a$ is some unspecified coupling, it is possible to see after some algebra that for $|I|^2 > 3/4$ (in Planck units) there is no value of $a$ for which $m^2_\phi \leq 0$. For smaller values of $I$, negative masses squared are possible if $a > 1/3$.

Finally, let us note that the possibility of a very small mass $m^2_\phi \sim 0$ (also welcome for a successful inflation itself, as we will discuss shortly) does not seem natural at first sight, since it would imply some conspiracy between the various contributions to $m^2_\phi$. The AD mechanism requires a mixing of $\phi$ with the inflaton $I$, which is not a priori guaranteed.
coming from \[ (3) \]. However, the study of the SUGRA scenarios coming from strings provides beautiful surprises in this sense, as we are about to see.

4 String Scenarios

The best motivated SUGRA scenarios are those coming from string theories. The corresponding Kähler potential, $K$, is greatly constrained and, therefore, the implementation of AD baryogenesis for the F-inflation framework is not trivial at all. In order to be concrete we will consider orbifold constructions, where the (tree–level) Kähler potential is given by \[ (11) \]

$$K = -\log(S + S) - 3 \log(T + T) + \sum_j (T + T)^{n_j} |z_j|^2.$$  \hspace{1cm} (5)

Here $S$ is the dilaton and $T$ denotes generically the moduli fields, $z_j$ are the chiral fields and $n_j$ the corresponding modular weights. The latter depend on the type of orbifold considered and the twisted sector to which the field belongs. The possible values of $n_j$ are $n_j = -1, -2, -3, -4, -5$. The discrete character of $n_j$ will play a relevant role later on.

Since a strong mixing between the inflaton and the AD field $\phi$ in the quadratic term ($\propto |\phi|^2$) of $K$ is required (see Sect. 3), our first conclusion is that $T$ is the natural inflaton candidate in string theories. S–dominated inflation cannot work, and this is a completely general result since the (tree-level) S–dependence of $K$ is universal in string theories. We should recall, however, that a strong mixing in $K$ is a necessary but not sufficient condition for $m_{\phi}^2 \leq 0$. We must then examine the precise value of $m_{\phi}^2$ in the presence of a non-vanishing cosmological constant $\langle V \rangle_m = V_0 > 0$. Restricting ourselves to the moduli-dominated case, $m_{\phi}^2$ is given by \[ (12) \]

$$m_{\phi}^2 = m_{3/2}^2 \left\{ (3 + n_{\phi}) C^2 - 2 \right\},$$  \hspace{1cm} (6)

where $m_{3/2}^2 = e^K |W|^2$, $C^2 = 1 + [V_0/(3 M_P^2 m_{3/2}^2)]$ and $n_{\phi}$ is the modular weight of the AD field $\phi$. Since $C^2 > 1$, it is clear that $n_{\phi} \leq -3$ is a sufficient condition to get $m_{\phi}^2 \leq 0$. States with $n_{\phi} \leq -3$ occur in all the orbifold constructions, so $m_{\phi}^2 \leq 0$ is perfectly natural, something that was not trivial at all \textit{a priori}. Actually, if $C^2 \leq 2$, then $m_{\phi}^2 \leq 0$ whenever $n_{\phi} \leq -2$, which is a very common case.

There is a particularly interesting limit of eq.(6) that could well be realized in practice. Namely, since $V_0 = K_{TT} |F|^2 - 3 m_{3/2}^2 = 3 H^2 M_P^2$, it may perfectly happen that $K_{TT} |F|^2 \gg m_{3/2}^2$, and thus $C^2 \gg 1$ (see definition of $C^2$ after eq.(3)). Then, from eq.(1)

$$m_{\phi}^2 \approx H^2 (3 + n_{\phi}).$$  \hspace{1cm} (7)

Hence for $n_{\phi} = -3$ we get $m_{\phi}^2 \approx 0$. So, we see that the possibility of a very small mass $m_{\phi}^2 \approx 0$ can occur in F–inflation, as it was the case in D–inflation. Notice that there is no fine-tuning here, since $n_{\phi}$ is a discrete number which can only take the values $n_{\phi} = -1, -2, -3, -4, -5$. 

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What is even more important: this is also good news for F–inflation itself. As has been pointed out in the literature, F–inflation has the problem that if the inflaton mass is \( O(H) \), as expected at first sight during inflation, then the necessary slow rollover is disabled (this is the so-called \( \eta \)–problem). We see here, however, that a hybrid–inflation scenario \([13]\) in which \( T \) is the field responsible for the large \( V_0 \) and a second field (any one with \( n = -3 \)) is responsible for the slow rollover is perfectly viable. This is a nice surprise since F-inflation is very difficult to implement in generic SUGRA theories, even with fine-tuning!

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