Effect of beam-beam kick on electron beam quality in first bunched electron cooler

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Abstract. The low energy RHIC electron cooler (LEReC) currently under commissioning at BNL is going to be the first non-magnetized bunched electron cooler (EC). For successful cooling LEReC requires that the electrons in the cooling section (CS) have small angles with respect to co-propagating ions. Since there is no strong magnetic field in the CS, the effects of ions on both the trajectory and focusing of the e-bunches is critical. In this paper we consider the ion beam kick on the electron bunches and derive requirements to the respective alignment of electron and ion beams in non-magnetized coolers.

1. Introduction
The low energy RHIC electron cooler [1] is a crucial part of the RHIC run dedicated to the search of the QCD critical point in the nuclear matter phase diagram [2].

![LEReC cooling sections](image)

**Figure 1.** LEReC cooling sections.

The LEReC, which is presently under commissioning at BNL, will be the first EC employing RF acceleration of electron beam. Traditional electron coolers [3] are utilizing the DC e-beam. Since the future expansion of the ECs to the energies exceeding several MeV must be based on the RF acceleration, the accelerator physics lessons learned from the LEReC design, commissioning and operation are of utter importance to the field.

The LEReC accelerator consists of a 400 keV photo-gun followed by the SRF accelerating cavity, which accelerates the beam to 1.6-2.6 MeV. The transport beamline and the merger bring the electron beam to the two cooling sections in the Yellow and in the Blue RHIC rings separated by the 180° bending magnet. The ‘blue’ CS is followed by the extraction to the beam dump.
In the LEReC CS (schematically shown in Fig. 1) ‘cold’ e-bunches overlap and co-propagate with ‘hot’ Au ions reducing their emittance and energy spread, thus ‘cooling’. There are 30 electron bunches overlapping with each ion bunch (see Fig. 2). The nominal ion bunch has Gaussian longitudinal distribution with RMS length $\sigma_{zi} = 3 \text{ m}$ (for the LEReC operation of 1.6MeV) and the peak current of 0.2 A. The RMS length of e-bunches in the CS is 3 cm and the distance between electron bunches is 43 cm.

![Figure 2. Ion and electron bunches in LEReC CS. Electron bunches are color coded based on their longitudinal position with respect to ion bunch.](image)

Each LEReC CS contains 8 solenoids combined with trajectory correctors and the BPMs located downstream of each solenoid. The distance between solenoid centers $L_{s2s} = 3 \text{ m}$.

The LEReC is a non-magnetized EC - it doesn’t utilize strong magnetic field in the cooling section. The short CS solenoids are used only for correction of the beam envelope. Under some commissioning scenarios the CS solenoids could be switched off completely. A similar example where only a weak magnetic field was employed in the cooling section, is relativistic DC electron cooler at FNAL [4, 5].

Successful cooling of the ions in non-magnetized EC requires small transverse electron angles in the CS (in LEReC case angles must be $\leq 100 \mu \text{rad}$). Among the reasons for the possible large electron angles with respect to ions are the CS ambient magnetic field [6], the mismatch of e-beam envelope to the cooling section [7], the misalignment of the CS solenoids and the angular misalignment of electron and ion trajectories [8]. Yet another possible reason for relative electron-ion angles is the beam-beam kick (BBK) experienced by the e-bunch transversely displaced with respect to the center of the ion bunch. This effect also limits the applicability of some cooling optimization techniques used in FNAL EC.

In this paper we study the requirements set by the ion-electron BBK to the accuracy of transverse overlap of the ion and electron bunches.

2. Theoretical Considerations
Let us consider the dynamics of a single electron probing longitudinal slice of the ion bunch with current $I_i$.

If the electron is radially displaced from the center of the axially symmetric ion beam by $r$, then (from Gauss and Ampere laws) the force acting on the electron towards the center of the ion beam is given by:

$$F = \frac{e\lambda(r)}{2\pi\varepsilon_0 r^2\gamma^2}$$

(1)
Here \( e \) is the elementary charge, \( \lambda(r) \) is the linear density of the ion charge encircled by radius \( r \), \( \varepsilon_0 \) is the vacuum permittivity and \( \gamma \) is the relativistic factor.

Introducing the pathlength \( s \) along the beamline as an independent variable we get the equation of motion:

\[
\frac{d^2r}{ds^2} = -\frac{2\beta c}{I_A(\beta \gamma)^3} \frac{\lambda(r)}{r} \tag{2}
\]

Here \( \beta \) is the ratio of electron velocity and speed of light \( c \) and \( I_A \) is Alfven current.

For the ion bunch with Gaussian transverse distribution with RMS size \( \sigma \) (in the LEReC CS \( \sigma = 4 \) mm): \( \lambda(r) = I_i(1 - e^{-r^2/2\sigma^2})/\beta c \). Denoting the differentiation with respect to \( s \) by an apostrophe, we obtain:

\[
\begin{align*}
  r' &= \theta \\
  \theta' &= -K \frac{1 - e^{-r^2/2\sigma^2}}{r}
\end{align*} \tag{3}
\]

Here \( \theta \) is the trajectory angle and \( K = \frac{2I_i}{I_A(\beta \gamma)^3} \) is the generalized perveance of the longitudinal slice of the ion bunch probed by the electron.

While (3) must be solved numerically, it is useful to consider the uniform transverse ion distribution with the equivalent radius \( R = \sqrt{2}\sigma \). For uniform distribution \( \lambda(r) = I_i r^2/(R^2 \beta c) \) and from (2):

\[
\begin{align*}
  r' &= \theta \\
  \theta' &= -\frac{K}{R^2} r
\end{align*} \tag{4}
\]

Assuming that at the entrance to the cooling section \( r(0) = r_0 \) and \( \theta(0) = 0 \), the solution of (4) becomes:

\[
r = r_0 \cos \left( \sqrt{K} s/R \right), \quad \theta = -r_0 \sqrt{K} \sin \left( \sqrt{K} s/R \right) \tag{5}
\]

Switching from the single particle dynamics to the motion of the center of mass (COM) of the e-beam we shall notice that (4) and (5) are derived for the linear ‘focusing’ force. Hence, in the approximation of a transversely uniform ion bunch the COM dynamics is described by (5).

The e-beam inside the ion bunch with Gaussian transverse distribution experiences nonlinear force. Nonetheless, as we discuss below, the correction from this effect to the COM motion is small and (3) can still be used for studying the requirements to the relative transverse displacement of electron and ion bunches.

3. Alignment Requirements

Equation (5) provides a simple way to estimate the maximum tolerable electron-ion displacement in the CS.

Let us assume that the electron-ion angles due to the BBK shall not exceed one third of the total ‘angular’ budget in the CS (\( \theta_{BBK} = 30 \) \( \mu \)rad). Then the limit on the tolerable transverse displacement of the electron and ion bunch COMs is:

\[
r \leq \sqrt{I_A/I_i(\beta \gamma)^3/2} \sigma \cdot \theta_{BBK} \tag{6}
\]

For the LEReC parameters at operational energy of 1.6 MeV the maximum tolerable displacement is 0.28 mm.

Next, we integrate (3) numerically with an explicit, exactly simplectic, second order method [9]. Results of simulations are shown in Fig. 3. The simulations are performed for the e-bunch probing the central longitudinal slice of the ion bunch (\( I_i = 0.2 \) A), where the force on the COM of the displaced e-bunch is the largest. For comparison we also show the analytic approximation (5). As one can see, for the small displacements the analytic formulas (5) and (6) are in perfect agreement with numerical simulations.
To estimate whether single particle equation (3) is suitable for describing the motion of electron beam COM we simulated the trajectories of the ensemble of electrons with various distributions. We found that for the submillimetre initial relative electron-ion COM displacements the correction to the solution of (3) is negligible.

Finally, we consider the presence of solenoids in the CS. We simulate short LEReC CS solenoids as instantaneous kicks occurring every three meters:

$$\theta_s = -k_s L_s r = \frac{B_s^2 L_s}{4(B\rho)^2} r$$

Here $L_s$ and $B_s$ are respectively the magnetic length and the field of the solenoid, and for 1.6 MeV energy $B\rho = 68$ G·m. For typical LEReC parameters $k_s = 0.2$ m$^{-2}$.

For analytical formula we can approximate lumped LEReC focusing in the CS as continuous solenoidal field with $\tilde{k} = k_s L_s / L_{s2s}$. Then (6) becomes:

$$r \leq \frac{\sqrt{I_i(\beta\gamma)^{3/2}a}}{\sqrt{I_i + \tilde{k} I_A(\beta\gamma)^{3/2}a^2}} \theta_{BBK}$$

As a matter fact, (8) represents the exact analytical solution for FNAL EC since its CS, unlike the LEReC CS, is immersed in a continuous weak solenoid.

From (8) we find that the transverse electron-ion COM displacement must not exceed 0.22 mm.

Figure 4 shows results of simulations and an analytical solution for the e-beam COM trajectory in the field of ion bunch and in the presence of solenoidal focusing in the LEReC CS for initial displacement of 0.22 mm.

4. Discussion of Results
A weak magnetic field in the CS results in strict requirement to electron-ion alignment. For the discussed LEReC parameters the tolerable relative transverse displacement of electron and ion bunch centers of mass is 200 µm.
Figure 4. Simulated and analytically calculated trajectories of the electron bunch COM displaced by 0.22 mm with respect to the center of the ion bunch in the LEReC CS in the presence of nominal solenoidal focusing.

Each of 30 electron bunches is probing the individual longitudinal slice of the ion bunch and the trajectory of each e-bunch is different for the same initial displacement. To understand how the trajectories of 30 electron bunches overlapped with a single ion bunch are detected by the CS BPMs, we simulate and average the trajectories of these 30 e-bunches (see Fig. 5).

Figure 5. Simulated trajectories and angles of 30 e-bunches overlapped with a single ion bunch. Colors of traces correspond to e-bunch longitudinal position shown in Fig. 2. Red x-trace on trajectory plot represents expected BPM readings.

Finally, we notice that the e-beam enters the blue CS with dispersion $D = 0.7$ m due to
the 180° bend. Hence, our requirement to the electron-ion trajectories alignment transfers to the requirement of average energy stability of about $3 \cdot 10^{-4}$. Our nominal requirement to the energy stability is $2 \cdot 10^{-4}$ [1, 10].

5. Conclusion

We considered the effect of beam-beam kick on the angles of electron bunches in the cooling section in the LEReC - the bunched electron cooler, which is currently under commissioning at BNL.

Simple analytical formulas for estimating the maximum tolerable relative transverse displacement of the center of masses of electron and ion beams were derived. These formulas are applicable to both bunched and DC non-magnetized electron coolers. We found that the derived formulas are in a perfect agreement with the simulations of electron beam motion in the nonlinear field of the ion beam.

For the LEReC it was found that the relative transverse displacement of the ion and electron bunches must be kept within 200 µm.

Acknowledgments

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