Microwave-induced Resonant Reflection and Localization of Ballistic Electrons in Quantum Microchannels

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Abstract

We show that electron transport in a ballistic microchannel supporting both propagating and reflected modes can be completely blocked by applying a microwave electromagnetic field. The effect is due to resonant reflection caused by multiple coherent electron-photon scattering involving at least two spatially localized scattering centers in the channel. With many such scattering centers present the conductance is shown to have an irregular dependence on bias voltage, gate voltage and frequency with irregularly spaced dips corresponding to resonant reflection. When averaged over bias, gate voltage or frequency the conductance will decay exponentially with channel length in full analogy with the localization of 1D electrons caused by impurity scattering.

The dynamics of a mesoscopic system subject to a time dependent field depends crucially on the relation between the phase breaking time $\tau_\phi$ and the time $t_0$ needed for electrons to pass through the system. In the absence of phase breaking processes, $\tau_\phi \gg t_0$, one could expect phase coherent dynamic phenomena to occur in mesoscopic systems in close analogy with the well known static phenomena. They should be highly tunable by for instance a magnetic field or by electric fields due to applied bias- or gate voltages. The photoconductance of a ballistic microchannel created in a gated AlGaAs heterostructure is an example. In a channel with a cross section that varies so slowly that the adiabatic approximation applies, coherent electron-photon scattering corresponding to transitions between different transverse modes has been shown to be localized in space and to lead to transitions between propagating and reflected modes [1]. This type of indirect backscattering has many features in common with impurity scattering and has been suggested [2] to give rise to quantum interference effects in the electron transport properties. In particular the photoconductance caused by single photon scattering (absorption) was shown to be an oscillating function of an applied gate voltage in a quasi-one dimensional channel containing a microwidening [3]. At large enough electromagnetic fields, multiple coherent electron-photon scattering results in a coherent resonant coupling of two different transverse modes in the channel. Because the electron-photon interaction is localized in space, this resonant coupling can be expressed in terms of a Landau-Zener breakdown [3]. In this paper we pursue this line of argument and show that resonant electron-photon scattering strongly modifies the electron transport and that it may lead to resonant (total) reflection of electrons and hence block the electron transport through a quasi-1D channel completely.

The channel geometry we have in mind is shown in Fig. 1a. We consider the simplest case where only a single transverse mode is ballistically propagating through the channel (schematically illustrated by the bold lines in Fig. 1a). The microwave-induced resonant coupling of this propagating mode with localized modes in the smooth widenings of the

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Figure 1: (a) Schematic view of a quasi-1D channel with one propagating mode (thick arrows) and several microwidenings with trapped modes (thin arrows). (b) Diagram describing scattering between propagating and trapped modes in a microwidening (see text)

channel (thin lines in Fig. 1a) is the phenomenon of interest here. Still referring to Fig. 1a, the longitudinal coordinates $x_1(n)$ and $x_2(n)$ give the positions where the resonant condition $\Delta U \equiv U_2(x) - U_1(x) = \hbar \omega$ is satisfied and Landau-Zener scattering takes place. Here $U_n(x)$ is the effective potential barrier for the quasi-one dimensional motion of electrons in the $n$:th transverse mode [1]. The total transmission through the channel is determined by the sum of the probability amplitudes for all electron paths. In the straight parts of the channel shown in Fig. 1a the electron states are superpositions of left- and right moving plane waves of amplitudes $A_1(n)$ and $A_2(n)$ respectively. Each widening is characterized by a scattering matrix $S_n$, which couples the amplitudes of incoming and outgoing waves,

$$
\begin{pmatrix}
A_1(n+1) \\
A_2(n)
\end{pmatrix}
= S_n
\begin{pmatrix}
A_1(n) \\
A_2(n+1)
\end{pmatrix},
\quad S_n \equiv e^{i \xi_n} \begin{pmatrix}
R_n & R_n^* \\
R_n^* & -i \Re_n
\end{pmatrix}.
$$

(1)

We will first calculate the scattering matrix $S_n$ and then later use the result to calculate the total transmission probability of the channel, which determines the conductance through the Landauer formula.

The internal structure of the scattering matrix $S_n$ is schematically shown in Fig. 1b (where the index $n$ is suppressed for convenience). Scattering is localized in space and may occur at $x_1$ or $x_2$. At each site scattering of right moving electrons (upper part of Fig. 1b) and left moving electrons (lower part) must be considered separately. The closed curve in the middle of Fig. 1b represents the path of an electron state trapped in the widening. The scattering matrices $S_{ic}$ and $S_{id}$ that appear in Fig. 1b describe Landau-Zener transitions at point $x_i$ for the case when the transverse energy levels are converging ($c$) and diverging ($d$) in the direction of longitudinal motion. They are related as $S_{id} = \sigma_y S_{ic} \sigma_y$, where $\sigma_y$ is a Pauli spin matrix, and can be expressed in terms of two phases $\theta_i, \vartheta_i$ — related to the amplitudes of the two resonantly coupled modes — and the probability $r_i^2$ for an intermode transition as

$$
S_{ic} \equiv e^{i \theta_i} \begin{pmatrix}
t_i & i r_i e^{i \vartheta_i} \\
ir_i e^{-i \vartheta_i} & t_i
\end{pmatrix},
\quad t_i^2 + r_i^2 = 1, \quad r_i = \left(1 - e^{-a_i}\right)^{1/2}.
$$

(2)

The quantity $r_i$ is determined by the single parameter $a_i = e^{2 E^2/p \omega |\Delta U'(x_i)|}$ (as are $\theta_i, \vartheta_i$). $E$ and $\omega$ is the amplitude and frequency of the electromagnetic field polarized in the transverse direction, while $p$ is the longitudinal momentum of the electron at the scattering sites.
There is a useful formal analogy between our scattering problem and the familiar case of electron scattering by (tunneling through) a double barrier structure. If we interpret $A_1(n)$ and $A_1(n+1)$ as the amplitudes of an incident and a reflected wave, then $S_{1c}$ and $S_{2d}$ correspond to the first and $S_{1d}$ and $S_{2c}$ to the second barrier in the analogous double barrier tunneling problem.

Note that the two barriers are identical, since each has one part corresponding to scattering at site $x_1$ and another part due to scattering at $x_2$. This immediately tells us that there is a possibility for reflectionless transmission from $A_1(n)$ through the double barrier structure to $A_2(n)$ for certain values of the electron energy, which we can readily calculate. In the original problem, however, $A_2(n)$ is the amplitude of the reflected rather than the transmitted wave (cf. Fig. 2). Hence resonant coupling of two transverse modes by an electromagnetic field can lead to resonant reflection of electrons at a microwidening in an otherwise straight channel.

Straightforward calculations lead to the following result for the transmission probability through the microwidening:

$$T^2(E) = \frac{4T^2 \sin^2 (\phi_L(E) - \eta(\Delta \varphi(E)))}{(1 - T^2)^2 + 4T^2 \sin^2 (\phi_L(E) - \eta(\Delta \varphi(E)))} \tag{3}$$

Here the phase $\phi_L$ is the total phase gained by the electron as it propagates around the closed trajectory (middle part of Fig. 1b), while $\Delta \varphi$ is the difference in phase gained along the two different trajectories going from scatterer $S_{1c}$ to $S_{2d}$. These two phases depend on two independent geometric parameters, say the length and width of the microwidening. In an experiment that implies they depend on applied gate voltages and on radiation frequency. While the phase $\eta(\varphi)$ in (3) is always less than $\pi/2$, the phase $\phi_L$ is a linear function of energy in the quasiclassical limit so that a discrete set of electron energies $\{E_m\}$ exists for which $\sin(\phi_L(E_m) - \eta(\Delta \varphi(E_m))) = 0$. At these energies the transmission probability $T^2$ goes to zero and resonant reflection takes place. The width of the resonance is determined by the parameter $T^2$, which can be expressed in terms of the probability $r^2$ for inter-mode Landau-Zener transitions. In the simple case of a symmetric widening ($r_1 = r_2 = r$), we find

$$T^2 = 1 - 4t^2 r^2 \sin^2 (\Delta \varphi/2) \tag{4}$$

It is interesting to note that $T^2$ is close to unity and the effect of the electromagnetic field is small for the two limiting cases of weak ($t \ll 1$) and strong ($r \ll 1$) field as long as we are outside the resonant regions (which are narrow in both cases). Corrections are due to the fact that $1 - T^2 \neq 0$ and coincide with those calculated in the single photon approximation [2]. This approximation breaks down at the resonant energies, however, where the transmission is suppressed to zero irrespective of the field strength. For a single microwidening in an otherwise straight channel the conductance $G = (2e^2/h)T^2(E = E_F)$ should therefore show a series of resonance “dips” as the gate voltage is varied. This resonant structure becomes richer and less regular if several microwidenings are present. The transmission amplitude can in this case be obtained from a product of transfer matrices describing the individual widenings. If the number of widenings is large — and if there are no correlations between the parameters describing their geometries — the theory of disordered one-dimensional systems...
can be used to find the result \[4\]

\[ G = \frac{2e^2}{h} \exp \left( \sum_{n=1}^{N} \ln |T_n| \right). \]

Here \( T_n \) is the transmission amplitude for the \( n \)th widening as given by \[2\]. The criterion \( T_n = 0 \) for at least one \( n \) determines the gate voltages for which electron transport is completely blocked by the electromagnetic field. Taking an ensemble average in the standard theory of disordered systems corresponds here to averaging over gate voltage. Hence we get for our problem the same exponential decay of conductance found for the electron localization problem in disordered (equilibrium) conductors, i.e.

\[ G = \frac{2e^2}{h} \exp(-N/R_L), \quad R_L^{-1} = \langle \ln \text{Max}(r_n, t_n) \rangle, \quad \langle f_n \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f_n \]  

(6)

Note that the localization radius \( R_L \) depends on the probability for Landau-Zener transitions.

The prospects for observing the effects discussed above in an experiment depend on the probability for intermode scattering caused by other mechanisms, such as electron-phonon or impurity scattering. Two different criteria apply for the possibilities to observe resonant reflection and the exponential suppression of the averaged current, respectively. In the first case the rate \( \nu \) of non-radiative inter-mode transitions should be smaller than the width of the discrete electron spectrum of trapped (non-propagating) modes in a widening: \( \nu \ll r^2t^2\omega(d/L_W) \). Here \( L_W \) is the characteristic length of the widening and we have used the relation \( v_F \sim \omega d \) between the Fermi velocity and channel width.

If we assume that \( \nu \) is due to phonon emission we have \( \nu \sim \gamma \omega^3/\omega_D^2 \) — \( \omega_D \) is the Debye frequency and \( \gamma \) the electron-phonon coupling constant — and conclude that the inequality

\[ \gamma (\omega/\omega_D)^2 \ll r^2t^2 (d/L_W) \]

(7)

must be fulfilled for resonant reflection to be observable. This should be possible to achieve experimentally. For the exponential decay of the averaged conductance to be observable, it is necessary to maintain phase coherence along the entire channel (not only within one microwidening). Therefore the stronger inequality \( \gamma (\omega/\omega_D)^2 \ll (d/L) \) must be fulfilled. Here \( L \) is the total length of the channel and we have assumed \( r \sim t \sim 1 \).

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