Determination of Cross layer interference in Heterogeneous network using stochastic geometry

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Abstract. This document describes the determination of the cross-layer interference for a heterogeneous network. HetNet Stands in the statistical parameters using the mathematical tool StoGeo stochastic geometry that is based on the poisson point process, PPP. A two-tiers network with differing power and distribution patterns is proposed, although defined as independent and identically distributed at each tiers in the low power nodes NBP, NHP respectively, and they are presented under the homogenous PPP. Power is considered at each constant tiers and fading and interference distributions are planned under approximation models for gamma distribution.

1. Introduction

The interference in a mobile network is the main limitation to maximize the use of the network, it is the tiers of the efficiency terms of the network with its spectral resources. For a heterogeneous network there are different sources, these are divided into two types which are described with the base stations of the same tiers and those that are at different tiers or external and different from the physical and electrical characteristics of the base stations.

In principle it is the description of a scenario that illustrates the growth of traffic and data usage in mobile networks and the solution that counteracts the inconveniences presented to cover the demand in traffic. For those who consider different scenarios [1], one of them based on the efficient use of the spectrum by aggregation of subcarriers, another based on the implementation and massification of the technologies in MIMO (Multiple In Multiple Out) antennas and finally a scenario where it is intended to optimize radio resources by implementing network topologies that increase the data traffic capacity, on which the research work is focused. To this, the available network elements present nonhomogeneous properties in terms of their electrical radio properties such as radiation patterns, frequency and power coverage. These proprietary characteristics define the network and are called heterogeneous networks of which for its acronym in English HetNet (Heterogeneous Network) present a solution to the demand for data traffic in mobile networks [1].

The operational technical foundation of heterogeneous networks is focused on the use and implementation of small cells. These are modeled and defined as network elements at different tiers.

Among the different effects caused in the implementation of heterogeneous networks is studied the interference effect of small cells at different tiers of the network. This interference called cross-layer interference or variation in literature also called cross-tier, which consists of degrading the power of the
transmitted signals because of the superposition of other signals transmitted by other network elements at different tiers [2].

The solution to mitigate this type of interference is focused on algorithms that make a more efficient use of the available frequencies, among which we will focus on choosing the most suitable ones in terms of optimization and viability in the implementation, for this it is necessary to consolidate the studies and ongoing analysis of existing and implemented techniques. These techniques are based on stochastic processes or random variable calculations in which they are found [3]:

**2. Modeling of the interference**

One of the most important metrics of network performance and user experience is the SINR interference signal to noise ratio, defined as the total power of the received signal in relation to the power received from all sources other than the desired transmitter, plus the thermal noise power in the receiver (which is always present, even in the absence of interference) [4]. In a given link between a service BS and a user, the SINR determines the rate of bit errors in the link, if you want to optimize the operation of a heterogeneous network, you need to know the spatial distribution of the SINR and its dependence on the deployment parameters of the network. On the (That is, a link for transmissions from a BS to a user) [5].

In the heterogeneous network scenario as in figure 1, the origins of the interference are described, from which it can be said that the total interference.

\[
I_T = \sum_{k=1}^{K} \zeta_k \tag{1}
\]

Where \( \zeta_k \) is the interference at each tiers of the network, \( k = 1,2,3 \ldots \) indicates each tiers of the network Generally, the definition of interference is like all disturbances to a source in a base-station or user equipment.

Furthermore, the fixed transmission power conditions of the base-stations and the channel model (Rayleigh) are considered to be identical and independent distributed (iid) for all the links in the network. Under these conditions of interference raised.

If the nodes are distributed according to a stochastic poisson point process (PPP) Figure 1 the interference function can be stipulated as:

\[
I(t) = \sum_{i=1}^{\infty} \ell(t - x_i) = \sum_{x \in \Phi} \ell(t - x) \tag{2}
\]
Figure 1. Distribution of the Base-stations under PPP

Where $\Phi = \{x_i\}_{i=1}^{\infty}$ is stationary Poisson process point in $\mathbb{R}$, and $\ell$ is a loss function and is given by the impulse response which implies that the losses are given by the distance and without taking into account in the fading of the channel. Under this consideration interference estimation lacks realistic and practical meaning for it is defined.

$$I(y) = \sum_{x \in \Phi} h_x \ell(y - x)$$  \hspace{1cm} (3)

Where $h_x$ represent the fading constant the model of the channel and is considered iid

For access channels, ALOHA is a complement for PPP since it maintains the distribution properties of the PPP and the intensity $\lambda$ for element (node) of the network. Therefore it retains the interference characteristics.

If we perform the derivation in brief instances of interference using Campbell's theorem, we obtain:

$$\mathbb{E}I = \sum_{x \in \Phi} h_x \ell(||x||) = \lambda \mathbb{E}(h) \int_{\mathbb{R}^d} \ell(||x||) dx$$  \hspace{1cm} (4)

$$= \lambda c_d \int_0^{\infty} \ell(r) r^{d-1} dr$$  \hspace{1cm} (5)

Where $c_d = |b(0,1)|$ is the volume of the d-dimension of the unit sphere.
In equation (5) the mean of the interference distribution is described. By using the function of the probability generator to characterize the statistical variables associated with the interference, we have

\[ \mathcal{L}_I(s) \triangleq \mathbb{E}[e^{-sI}] = \mathbb{E} \left[ \exp \left( -s \sum_{r \in \Phi} h_r r^{-\alpha} \right) \right] \tag{6} \]

This is equal to calculate the Laplace transform of the interference function and applying the mapping theorem [7].

\[ \mathcal{L}_I(s) \triangleq \exp \left( -\lambda c_d \mathbb{E}[h^\delta] \Gamma(1 - \delta)s^\delta \right) \tag{7} \]

Where \( \delta = \frac{d}{\alpha} \) is the symmetric difference between \( d \) (order of the dimension) and \( \alpha \) exponent of the loss function and \( \Gamma \) is the distribution gamma function.

\[ \Gamma \triangleq \int_0^\infty t^{\beta - 1} e^t dt \tag{8} \]

In the case of fading Rayleigh \( \mathbb{E}[h^\delta] = \Gamma(1 + \delta) \)

\[ \mathcal{L}_I(s) \triangleq \exp \left( -\lambda c_d \mathbb{E}[h^\delta] \frac{\pi \delta}{\sin(\pi \delta)} \right) \tag{9} \]

Considering a network under the distribution of the PPP with fading, one of the approaches that generalizes the characterization of the interference is the use of probability generating function. Where is given in equation 5 where the mementos to be generated are related to the mean, variance, symmetry and kurtosis in sequential order. That associated with its probability distribution function FDP and the accumulated probability function FPA. You determine the properties of the interference under the network with the PPP model [5] [7].

In a multi-tiers network, structure and topology are considered under different characteristics for the nodes. In essence, the low power nodes NBP small cell and the high power nodes NHP referring to the macro cellular network [8].

Among the characteristics to be taken into account for the analysis is the following:

Transmission power that is referring to the energy with which the signals are injected to the antennas, the radiation pattern that has a direct inference in the coverage of the cells, and frequency of operation of the carrier signals [9].
Under these premises the condition of a two-tiers network is established where the macro tiers refers to a homogeneous PPP distribution with intensity \( \lambda_m \) and the low power tiers refers to femto cells with a homogeneous PPP distribution with \( \lambda_f \) such that \( \lambda_f > \lambda_m \).

![Figure 2. Effect of interference by power and coverage (blue) and NHP (red) between a NBP](image)

For the proposed condition that each distribution of each tier is given by homogeneous PPP, the resulting interference will be parameterized by \( I_k = (\alpha_k, \beta_k, P_k, \lambda_k) \), Where \( k = 1,2,3..M \) refers to the tier of the network \( \alpha \) and \( \beta \) are the distribution parameters (exponential, gamma, chi square) \( P_k, \lambda_k \) are the transmission power parameter and the intensity of the PPP in each tier. Because the transmission power is different for each tier, the coverage region or Rg will be a function of the transmission power [6].

If we inscribe the center of the circle of radius Rg in the Voronoi cell condition as the principle of stochastic geometry we can analyze the performance of a particular cell is given. If you don’t wish to use the Word template provided, please set the margins of your Word document as follows.

\[
\frac{1}{2\sqrt{\sum_{k=1}^{M} \alpha^2 \lambda_k}}
\]

Where \( a = 1 + \frac{P_k}{P_f} \)

In the case of the heterogeneous network, the total interference would be given by dominant interfering; from the base stations outside the guard region of tier k, which is the firing noise field. So the total interference would be given by

\[
I_{tot} = \sum_{k=1}^{M} \sum_{x \in \Phi_k/B(Rc+Rg)} \frac{G_{kj}^x H_{kj}^x P_{kj}^x}{\rho(x_k)}
\]
Where Rc is the cell region smaller than \( G_k \) \( H_k \) Rg It is the fading for the small scale network and for the large scale respectively.

**Figure 3.** inscriptions of circumference Rg in cell with voronoi tessellation

Referring to figure 3 the interference caused by the nodes of different tiers is called cross layer and it is of special interest for this work in this situation is defined a region of excursion in the radio receiver \( R_{cross} \) and what we will call \( B_{cross} \).

And the cross layer interference is defined as:

\[
I_{cross} = \sum_{x \in \Phi_{cross} \cap B_{cross}} \frac{G_x H_x P_{cross}}{\ell(R_x)} (12)
\]

Where \( P_{cross} \) is the interference power.

### 3. Approximation Of Interference

The modeling of the cross layer interference has a strict and direct relationship with the distribution models of low power and high power base stations.

Among them, the distribution models under the specific processes that describe the location of the base stations or nodes define the behavior of the interference and in general the network.

For in the case of interference in heterogeneous networks is considered an approximation under the continuous distribution gamma [6] and the application of the first and second moments that coincide with the laplace transform as seen in [5]. For this we consider the parameterization exposed in section III and the substitution of equation 12 for \( Z = G_x H_x P_{cross} \).

\[
E[I(y)] = E[I] = E[p] \lambda \int_{\mathbb{R}^2} \frac{1}{\ell(ly)} dy = \int_0^{\infty} (1 - G(s)) ds 2\pi \lambda \int_0^{\infty} \frac{r}{\ell(r)} dr (13)
\]

From which it is concluded that the expected value and the variance for the interference are given by [6]:

\[ EI_0(\beta R_c) = \frac{1}{\lambda CR_c^3} \sum_{m=1}^{M} \lambda_m EZ^{(m)} a_m^2 \left( a_m^2 + \beta^2 \right) \left( \beta^2 - a_m^2 \right)^2 \]  

(14)

\[ EI_m(\beta R_c) = \frac{2\pi \lambda_m E Z^{(m)} a_m^2}{2CR_c(\beta^2 - a_m^2)^2} \]  

(15)

\[ VI_m(\beta R_c) = \frac{2\pi \lambda_m E (Z^{(m)})^2 a_m^2 (a_m^4 + 6a_m^2\beta^2 + 3\beta^4)}{6C^2 R_c^6 (a_m^2 - \beta^2)^6} \]  

(16)

What it indicates in equation 14 and 15 corresponds to the first and second moments of the gamma distribution function. The parameter \( C = \prod_{i=1}^{N} (\theta_{mn} / \theta_i) \), \( \theta \) is the argument of the gamma function.

4. Result

The interference at a point is given by two terms such that \( I = I_0 + I_1 \) which must be in function of the position of a point \( r \) where \( I_0 \) is the dominant interference in the guard region \( R_g \) and \( I_1 \) is the interference of the all stations base under the PPP outside the guard region [6].

We modelled a two-tier network by homogeneous PPP at each tier where \( \lambda_m \) for the high power tier or NHP is less than \( \lambda_f \) and the cell and guard region is a linear function of the transmission power [10].

As shown in figure 4, the heterogeneous network of two tiers with the indications of the guard region

![Figure 4. Two-tiered heterogeneous network with guard region](image-url)
For this condition the variation of the interference in the guard region as a function of time is given by equation 14 from which it can be seen that $I_0$ is the first moment of the guard interference.

![Figure 5](image_url)

**Figure 5.** First moment of interference in the guard region.

Of which in equation 12 the fading of the large-scale and small-scale tier are approximated by the gamma distribution as defined above given in its FDP as that in figure 6.

![Figure 6](image_url)

**Figure 6.** Distribution of fading at two tiers under the Gamma distribution

5. **Conclusions**

The models studied have maximum effectiveness depending on the problems faced, this problem fully chosen and identified limits the performance of the network in many areas or variable as coverage capacity number of users among others. The model of the generated interference is modeled by equation 12. Interference in the heterogeneous network of 2 tiers.

It is considered as the sum of the cross-references and is modeled under the parametrization of the intensity functions $\lambda$ at each tier, the transmission power and under the approximation of the fading at each tier according to the parameters of the Gamma distribution.

The interference indicates the performance limit of a wireless network for this reason is vitally important to raise models of analysis and study to characterize or measure it.
The stochastic geometry as a mathematical tool for modeling random functions is very effective to roughly represent the behavior of a heterogeneous network. And in future statements the characterization of the SINR function that combines environment and noise variables defined by Equation 17.

\[
SINR(y) = \frac{P_t(x_0)Ah_{x_0y} \|x_0 - y\|^\eta}{W + \sum_{x \in \zeta} P_t(x) Ah_{xy} \|x - y\|^\eta}
\]  

Where \( A \) is the gain of the channel, \( h \) represents the fade factor \( \|x_0 - y\|^\eta \), is the distance in terms of the model of slope-intercept channel [5], \( P_t \) refers to the transmitter power and \( W \) it's the noise in the channel.

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