Level Correlations for Metal-Insulator Transition

Pragya Shukla*

Department of Physics, Indian Institute of Technology, Kharagpur, India.

Abstract

We study the level-statistics of a disordered system undergoing the Anderson type metal-insulator transition. The disordered Hamiltonian is a sparse random matrix in the site representation and the statistics is obtained by taking an ensemble of such matrices. It is shown that the transition of levels due to change of various parameters e.g. disorder, system size, hopping rate can be mapped to the perturbation driven evolution of the eigenvalues of an ensemble subjected to Wigner-Dyson type perturbation with an initial state given by a Poisson ensemble; the mapping is then used to obtain desired level-correlations.

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The degree of disorder and dimensionality of a disordered system plays a significant role in the metal-insulator (MI) transition [1]. The level statistics during the transition is governed by a parameter which is a function of both, disorder as well as dimensionality [2]. The purpose of this paper is to identify the parameter and study the effect of its variation on the spectral fluctuations.

The statistical behaviour of the levels of a disordered Hamiltonian depends on their number \( r \) within the energy ranges of interest and the dimensionless conductance \( g \). The parameter \( g \) is the ratio of two energy scales, namely, Thouless energy \( E_c \) and mean level spacing \( \Delta \). The energy scale \( E_c = \hbar D/L^2 \), with \( D \) as diffusion coefficient and \( L \) system size, corresponds to ergodic time-scales (ergodic classical limit) above which the wavefunction has fully diffused in the entire volume \( L_d \) of the system and therefore does not feel the space dimensionality \( d \). The eigenfunctions associated with levels within energy ranges smaller than \( E_c \) are extended structureless objects leaving no signature on the level correlations. The number \( r < g \), therefore, results in a universality of level correlations which can be well-modeled by the invariant ensembles of random matrix theory (RMT) [3]. For \( r > g \), the time scales associated with the dynamics are such that the wavefunction is not yet fully diffused in the whole volume. The eigenfunctions in the corresponding energy-range are localized, either partially or fully depending on the dimensionality, and their spatial correlations can significantly affect the level correlations.

The metallic phase of a disordered system corresponds to \( g \to \infty \) in the thermodynamic limit \( L \to \infty \). The states, therefore, in any large but finite interval are always extended with a Wigner-Dyson (WD) level-statistics, characteristic of strong level-repulsion [3]. In the insulator phase \( g \to 0 \) in the thermodynamic limit, implying no overlap between eigenfunctions. The corresponding level statistics can be modeled by the statistics of independent random numbers, that is, Poisson statistics. A change in the degree of disorder results in the variation of \( g \) (for fixed \( r \)) which can induce a transformation of extended states into localized states thereby causing a transition of spectral fluctuations. However the MI transition is very sensitive to the dimensions of the system. For \( d = 1,2 \) and, in thermodynamic limit,
the system always remains an insulator even for very weak disorder. For $d > 2$, the increase of disorder results in decreasing $g$, leading to an extended to localized state transition at a critical value of disorder. In the region near the critical point of the transition, known as Anderson transition, the eigenstates are neither localized nor extended but form self-similar multifractal structures with strong fluctuations at all length scales [4]. This results in a new type of universal level-statistics, known as critical spectral statistics (CSS), which is neither Poissonian nor Wigner-Dyson but is a hybrid of both [2,5]. For finite system size also, a crossover from Poisson to WD statistics can be seen among levels as a function of the parameter $\zeta/L$ with $\zeta$ as the correlation length. However the detailed analytical information about various level correlations in the critical regime for infinite systems as well as for the intermediate stages, during transition in finite systems, is still missing. Our study attempts to bridge this gap.

The level statistics for various transition stages is significantly affected by the nature of the localization of the eigenfunctions and can suitably be modeled by an ensemble containing eigenvalue-eigenfunction correlations (therefore basis-dependent). Recently a new technique has been developed to deal with such cases where the eigenvalue distribution for an ensemble, non-invariant under a change of basis, is mapped to the one for an ensemble invariant under change of basis and subjected to a random perturbation [6]. The mapping is achieved by identifying a basis-dependent parameter in the former to the perturbation (e.g symmetry-breaking) parameter in the latter. The distribution for the invariant case can subsequently be mapped to the non-stationary states of the particles governed by Calogero-Sutherland-Moser (CSM) Hamiltonian [3]. The known particle correlations can therefore be used to determine level-correlations of the non-invariant ensembles. The present study uses the technique to analyze the spectral properties of the levels undergoing MI transition by taking an ensemble of Anderson Hamiltonians in the site representation. As expected, the basis dependent parameter, governing the transition in this case, is a function of disorder, dimensionality and hopping rate and is related to $g$.

The Anderson model for a disordered system is described by a d-dimensional disordered
lattice, of size $L$, with a Hamiltonian $H = \sum_n \epsilon_n a_n^+ a_n + \sum_{n \neq m} t (a_n^+ a_m + a_m a_n^+)$ in tight-binding approximation \[1\]. Here $a_n^+$ and $a_n$ are the creation and annihilation operators of an electron at a site $n$ in a lattice and $m$ refers to the nearest neighbors of the site $n$. The site energies $\epsilon_n$, measured in units of the overlap integral between adjacent sites correspond to the random potential. The hopping is generally assumed to connect only the $z$ nearest-neighbors with amplitude $t$ so that the electron kinetic energy spread or bandwidth is $zt$. In the configuration space representation the Hamiltonian $H$ turns out to be a sparse matrix of size $N = L^d$ with diagonal matrix elements as the site-energies $\epsilon_i$. The level-statistics can therefore be studied by analyzing the properties of an ensemble of (i) sparse real symmetric matrices in presence of a time-reversal symmetry and (ii) sparse complex Hermitian matrices in absence of a time-reversal symmetry.

We first consider the case of MI transition brought about by decreasing the diagonal disorder. In this case, site-energies $\epsilon_i$ are taken to be independent random variables with probability-density $p(\epsilon_i)$. In the original Anderson model \[1\] $p(\epsilon)$ was taken to be a constant $W^{-1}$ between $-W/2$ to $W/2$. Various physical arguments and approximations used in this case led to conclusion that all the states are localized for $W > 4Kt \ln(W^2/t)$ with $K$ as a function of $z$ and $d$. However, as well-known now, MI transition does not depend on the nature of $p(\epsilon)$ and latter can also be chosen as Gaussian; the type of $p(\epsilon)$ affects only the critical point of the transition \[1\]. The ensemble measure $\rho(H)$, for any intermediate state of MI transition brought about by diagonal disorder, can therefore be described by

\[ \rho(H, y, b) \propto \exp\left( -\sum_{s=1}^{2} \sum_{k \leq l} \alpha_{kl;s} (H_{kk;s} - b_{kk;s})^2 \right) \prod_{k \leq l; s=1,2} \delta(H_{kl;s} - b_{kl;s}) \]

with subscript ”$s$” referring to real and imaginary parts and the Kronecker delta function implying the non-randomness of off-diagonal matrix element $(H_{kl} = b_{kl;1} + ib_{kl;2})$. However $\rho(H)$ can be expressed in a more general form:

\[ \rho(H, y, b) = C \exp\left( -\sum_{s=1}^{2} \sum_{k \leq l} \alpha_{kl;s} (H_{kl;s} - b_{kl;s})^2 \right) \]

with $C = \prod_{k \leq l} \prod_{s=1}^{2} \sqrt{\frac{\alpha_{kl;s}}{\pi}}$ as the normalization constant, $y$ as the set of the coefficients $y_{kl;s} = \alpha_{kl;s} g_{kl} = \frac{\alpha_{kl;s}}{2 <H^2_{kl;s}>}$ and $b$ as the set of all $b_{kl;s}$ with $g_{kl} = 1 + \delta_{kl}$. As obvious, in limit
\(\alpha_{kl;1}, \alpha_{kl;2} \rightarrow \infty\), eq.(1) corresponds to the non-random nature of the off-diagonal part of \(H\). However note, for later reference, that a same mean value for all the matrix elements, that is \(b_{kl;s} = \epsilon\) for all \(k, l\) (irrespective of \(\alpha_{kl}\) values) implies just a shift of the origin in the matrix space and does not affect the eigenvalue distribution.

The MI transition is brought about by a competitive variation of the disorder and hopping rate which can be mimicked by a change of the distribution parameters of \(\rho(H)\). The changed eigenvalue distribution as a result can be obtained by integrating \(\rho(H)\) over associated eigenvector space. Let \(P(\mu, y, b)\) be the probability of finding eigenvalues \(\lambda_i\) of \(H\) between \(\mu_i\) and \(\mu_i + \text{d}\mu_i\) at a given \(y\) and \(b\), it can be expressed as

\[
P(\mu, y, b) = \int\prod_{i=1}^{N} \delta(\mu_i - \lambda_i)\rho(H, y, b)dH.
\]

As discussed in ref. [6], a particular combination of the evolution of \(P\) with respect to various distribution parameters, namely,

\[
DP = \sum_n \frac{\partial}{\partial \mu_n} \left[ \frac{\partial}{\partial \mu_n} + \sum_{m \neq n} \frac{\beta}{\mu_m - \mu_n} + \gamma \mu_n \right] P
\]

with \(\beta\) depending on the symmetry class of the ensemble (\(\beta = 1\) for real symmetric matrices and \(\beta = 2\) for complex Hermitian ones). The multi-parametric evolution of \(P\) can further be reduced in terms of the evolution with respect to a single parameter \(Y\), that is,

\[
DP = \frac{\partial P}{\partial Y}
\]

where \(Y\) is a function of various \(y_{kl;s}\) and \(b_{kl;s}\).

\[
Y = \frac{1}{2N^2} \sum_{k \leq l} \sum_{s=1}^{2} \left[ \frac{1}{2} \ln \frac{y_{kl;s}}{|y_{kl;s} - \gamma|} - \frac{1}{\gamma} \ln |b_{kl;s}| \right] + C
\]

and can therefore be referred as the 'complexity parameter'. Here \(C\) is an arbitrary constant. The eq.(2) can now be rewritten as follows,

\[
\frac{\partial P}{\partial Y} = \sum_n \frac{\partial}{\partial \mu_n} \left[ \frac{\partial}{\partial \mu_n} + \sum_{m \neq n} \frac{\beta}{\mu_m - \mu_n} + \gamma \mu_n \right] P
\]

For \(y_{kl,s} \rightarrow \gamma\) and \(b_{kl,s} \rightarrow \epsilon\) (for almost all \(k, l\)), \(DP \rightarrow 0\) and the evolution reaches the steady state given by a WD distribution \(P(\mu) = \prod_{i < j} |\mu_i - \mu_j|^{\beta_2} e^{-\frac{1}{2} \sum_k \mu_k^2}\). (Note, in this limit, \(\rho(H) \propto e^{-\gamma \text{Tr}H^2 - \epsilon \text{Tr}H}\) which indeed has a WD distribution and therefore agrees with
the solution of eq.(4) in limit $DP \to 0)$. The eq.(4) is similar to the equation governing a perturbation driven transition of the eigenvalues, of a Hamiltonian $H = H_0 + \tau V$, between various universality classes of WD ensembles. Here $H_0$ is an initial ensemble with perturbation $V$ causing a transition to another (or same) universality class. The transition to equilibrium, with perturbation strength $\tau$ as the evolution parameter, is rapid, discontinuous for infinite dimensions of matrices (analogous to Anderson transition). But for small-$\tau$ and large $N$, a smooth transition can be seen in terms of a rescaled parameter $\Lambda$ which measures locally the mean-square perturbation matrix element in units of the average level spacing $[7]$. The intermediate states of this transition have been well-studied in past and many results for their spectral fluctuations are already known $[8,9]$. By replacing the perturbation parameter by complexity parameter, these results can directly be used for the corresponding measures for the MI transition. Note this is equivalent to say that if the initial Anderson Hamiltonian is described by $H_0$, the changing complexity (the combined effect of changing disorder and hopping rate) can be treated as a perturbation $V$ of strength $Y$, taken from WD ensemble, with Hamiltonian $H$ of the system represented by $H = H_0 + YV$.

The $n^{th}$ order level density correlation $R_n$ is defined as $R_n = \langle \nu(\mu_1, Y) \dots \nu(\mu_n, Y) \rangle$ with $\nu(\mu, Y) = N^{-1} \sum_i \delta(\mu - \mu_i)$ as the density of eigenvalues and $\langle \ldots \rangle$ implying the ensemble average. The $R_n$ can also be expressed in terms of $P$: $R_n = \frac{N!}{(N-n)!} \int P(\mu, Y) d\mu_{n+1} \ldots d\mu_N$. A solution of eq.(4), averaged over initial conditions, can therefore be used to obtain various correlations for a given set of distribution parameters. In practice, however, the involved integrals could only be solved for the cases when $H$ is complex Hermitian in nature $[8,9]$. A direct integration of F-P equation (4) leads to the BBGKY hierarchic relations among the unfolded correlators $R_n(r_1, \ldots, r_n; \Lambda) = \text{Lim}_N \rightarrow \infty; \frac{R_n(\mu_1, \ldots, \mu_n, Y)}{R_1(\mu_1; Y) \ldots R_1(\mu_n; Y)}$ with $r = \int^r R_1(\mu; Y) dY$ and $\Lambda = (Y - Y_0)^2 / \Delta^2$ ($\Delta$ as the mean level spacing = $R_1^{-1} = (N < \nu >)^{-1}$ $[8,9]$).

$$\frac{\partial R_n}{\partial \Lambda} = \sum_j \frac{\partial^2 R_n}{\partial r^2_j} - \beta \sum_{j \neq k} \frac{\partial}{\partial r_j} \left( \frac{R_n}{r_j - r_k} \right) - \beta \sum_j \frac{\partial}{\partial r_j} \int_{-\infty}^{\infty} \frac{R_{n+1}}{r_j - r_k}$$ (5)

Here note a perturbation of strength $\Lambda$ mixes levels in a spectrum span of $\sqrt{\Lambda}$ only, due to level repulsion $[9]$. This results in a sensitivity of the correlations within a given energy
range $r$ to the ratio $r/\sqrt{\Lambda}$. In the case of MI transition, therefore, $\Lambda$ can be identified with $g^2$.

For $n = 2$ and small values of $r$, the integral term in eq.(5) makes a negligible contribution thus leading to following approximated closed form equation for $R_2$

$$\frac{\partial R_2}{\partial \Lambda} = 2 \frac{\partial^2 R_2}{\partial r^2} - 2 \beta \frac{\partial R_2}{\partial r} \frac{r}{r}$$  \hspace{1cm} (6)

Similarly for large-$r$ behaviour, $R_2$ can be obtained from the following relation

$$R_2(r, \Lambda) = R_2(r, \infty) + 2\beta \Lambda \int_{-\infty}^{\infty} ds \frac{R_2(r - s; 0) - R_2(r - s; \infty)}{s^2 + 4\pi^2 \beta^2 \Lambda^2}.$$  \hspace{1cm} (7)

The hierarchic equation can then be used to obtain an approximate form of the higher order correlations. For example, the approximate information about $R_3$ can be extracted by a substitution of large and small $r$ behaviour of $R_2$ in eq.(5) with $n = 2$.

For studies of the level-correlations during MI transition, the system can initially be considered in an insulator regime where all the eigenvectors become localized on individual sites of the lattice (strong disorder limit). This results in a diagonal form of the matrix $H$ with the eigenvalues independent from each other. The insulator limit can therefore be modeled by ensemble (1) with $\alpha_{kl} \to \infty$ for $k \neq l$, $\alpha_{kk} = \alpha_I$ (for all $k$-values) and $b_{kl} \to \epsilon$ (for all $k, l$), giving, $Y_0 = \frac{1}{4N} \ln \left[ \frac{2\alpha_0}{2\alpha_0 - \gamma} \right] - \frac{1}{\gamma} \ln \epsilon + C$. (Here the origin of the energy-axis is chosen at $\epsilon$ so as to avoid the undefined nature of $\ln b_{kl}$ near $b_{kl} \to 0$). The decrease of the diagonal disorder, that is, an increase of $\alpha_{kk}$ from $\alpha_0$ to some finite values (while $\alpha_{kl}, k \neq l$, remains infinite throughout the transition) will ultimately lead to metal regime with fully delocalized wavefunctions. A change in the hopping rate of the connected sites from $\epsilon$ to $\epsilon + t$ ($b_{kl} \to \epsilon + t$ for only those $k, l$ values which are connected, $b_{kl} = \epsilon$ for others) will also affect the transition. As mentioned above, the eigenvalue distribution of $H$ in the metal regime can be well-modeled by the WD ensemble; let it be described by all $\alpha_{kl} \to \alpha_M(> \alpha_0)$. Thus for the study of transition in this case we can choose $\gamma = 2\alpha_M$. For any intermediate state of the transition with hopping rate $t$ and disorder coefficient $\alpha$, the complexity parameter $Y = \frac{1}{2N^2} \left[ \frac{N}{2\alpha_0} \ln \frac{2\alpha_0}{|2\alpha_0 - \gamma|} - \frac{K}{\gamma} \ln(t + \epsilon) - \frac{2N^2 K}{\gamma} \ln \epsilon \right] + C$. Here $K = \kappa N$ is
the total number of the sites connected by hopping and depends on the dimensionality $d$ of
the system (as $N = L^d$). The transition parameter can now be given as follows, with the
mean level spacing $\Delta = \langle \nu \rangle^{-1}$: $\Lambda = 2\alpha_M (Y - Y_0)/\Delta^2 = \alpha_M < \nu >^2 Na(\alpha, t)/2$ with
$a(\alpha, t) \equiv \left[ \ln \frac{\alpha}{\alpha_0} - \frac{\kappa}{\alpha_M} \ln \frac{t+\epsilon}{\epsilon} \right]$. As obvious from the above, the transition is governed
by relative values of the disorder and the hopping. Here $\Lambda \rightarrow 0$ leads to a fully localized
regime which, corresponds to the condition $a(\alpha, t) < \frac{1}{N}$. Similarly the condition $a(\alpha, t) > \frac{1}{N}$
corresponds to $\Lambda \rightarrow \infty$ for large $N$ and therefore extended states. For $a(\alpha, t) = \frac{\mu}{N}$ ($\mu$ as
a $N$-independent function), $\Lambda = \alpha_M < \nu >^2$ $\mu \equiv \Lambda^*$ is independent of $N$ which therefore
remains same for infinite systems ($< \nu >$ being independent of $N$). This $\Lambda$-value corresponds
to the critical point of the transition in infinite systems, the statistics being Poisson and WD
below and above $\Lambda^*$, respectively. The condition for the critical region or mobility edge can
therefore be given as $\ln \frac{\alpha}{\alpha_0} + \frac{\alpha - \alpha_0}{\alpha_M} \approx \frac{\kappa}{\alpha_M} \ln (t+1)$ (taking $\epsilon = 1$ without any loss of generality).
As $\frac{\alpha - \alpha_0}{\alpha_M} << 1$ even for large $\alpha$-values, the condition is always satisfied if $\frac{\kappa}{\alpha_M} \rightarrow 0$. This
explains the localization of all the states in infinitely long wires (or strictly 1-d systems where
$z$ is very small) even for very weak disorder. With increasing dimensionality $d$, connectivity
$K$ of the lattice and thereby the possibility of $\Lambda >> 0$ and the delocalized states increases.
As reflected by its form, $\Lambda$ also plays the role of the finite size scaling parameter. Note that
the $\Lambda$-dependence of various correlators indicates an interaction of levels different from that
of Poisson or WD if $\Lambda$ remains finite in limit $N \rightarrow \infty$. This implies the presence of a new
type of universal statistics near the critical point $\Lambda^*$ of the transition. For finite systems,
however, there is a continuous family of intermediate statistics between Poisson and WD,
described by $\Lambda$ and the symmetry of the Hamiltonian.

The fluctuation measures for various stages of the MI transition can be obtained from
those of the perturbation driven Poisson $\rightarrow$ WD ensemble transition, by replacing perturbation
parameter by $Y$. The two level density correlator $R_2(r; \Lambda)$ for the Anderson transition
in finite systems, in presence of a magnetic field, can therefore be given by the $R_2$ for Poisson
$\rightarrow$ GUE transition driven by the perturbation $[8,10]$:
\[ R_2(r; \Lambda) - R_2(r; \infty) = \frac{4}{\pi} \int_0^\infty dx \int_{-1}^1 dz \cos(2\pi rx) \exp \left[ -8\pi^2 \Lambda x(1 + x + 2z\sqrt{x}) \right] \left( \frac{\sqrt{1 - z^2}(1 + 2z\sqrt{x})}{1 + x + 2z\sqrt{x}} \right) \]

where \( R_2(r, \infty) = 1 - \frac{\sin^2(\pi r)}{\pi^2 r^2} \) and \( R_2(r, 0) = 1 \) corresponding to metal and insulator regime respectively. A substitution of \( \Lambda = \Lambda^* \) in eq.(8) will thus give the two level correlation for critical regime of Anderson transition.

An important characteristic of CSS is the level compressibility \( \chi \) which is basically a measure of the level repulsion; \( \chi = 0 \) in the metallic phase and \( \chi = 1 \) in the insulator phase and takes an intermediate value between 0 and 1 at the hybrid phase near the critical point. The eq.(4.15) of [10] can be used to obtain \( \chi(\Lambda) \approx 1 - \int_{-\infty}^{\infty} Y_2(r; \Lambda) dr \) with \( Y_2 = 1 - R_2 \), \( \chi(\Lambda) = 1 + \int_0^\infty du \frac{\Delta_1}{2} I_1(z) - \sqrt{8z/\pi} I_2(z) \exp[-(\Lambda/2)u^2 - \pi\sqrt{\Lambda/2u}] \) with \( z = \sqrt{2 * \pi \Lambda^2 u^4} \) and \( I_n \) as the \( n \)th Bessel function. (Note eq.(8) can not directly be used to calculate \( \chi \) due to technical difficulties). It can be checked that \( \chi \to 1 \) for \( \Lambda \to 0 \) (insulator limit) and \( \chi \to 0 \) for \( \Lambda \to \infty \) (metal limit). As obvious from the smooth nature of the functional form of \( \chi \), any non-zero finite \( \Lambda \)-value will correspond to a value of \( \chi \) intermediate between zero and one. The critical region will thus have a finite level compressibility different from both metal and insulator regimes and therefore multifractal nature of the eigenvectors [4].

The statistical measures for the Anderson transition in presence of a time-reversal symmetry can similarly be obtained by using their equivalence to those for the Poisson \( \to \) GOE transition. Although much information is not available about the latter (due to the technical difficulties in integrating over associated orthogonal space of eigenvectors) however some approximate results are known. For example, the \( R_2 \) for small \( r \) can be obtained by solving the eq.(6) for \( \beta = 1 \), \( R_2(r, \Lambda) \approx (2r - 1)^{1/2} J_1(2r - 1)^{3/2}/3\Lambda \) \( e^{r/2\Lambda} \) with \( J_1(z) \) as the Bessel function. For \( r << \sqrt{\Lambda} \), \( R_2 \propto r \) implying level-repulsion of the same degree as for the delocalized regime; this is consistent with description of \( \Lambda \) as \( g^2 \). For large-\( r \), \( R_2 \) can be obtained from eq.(7) by taking \( R_2(r; 0) = 1 \) (Poisson limit), \( \beta = 1 \) and \( R_2(r, \infty) = 1 - \frac{\sin^2(\pi r)}{\pi^2 r^2} - \left( \int_r^\infty dx \frac{\sin\pi x}{\pi x} \right) \left( \frac{d}{dr} \frac{\sin\pi x}{\pi x} \right) \) (GOE limit); \( R_2(r, \Lambda) \approx R_2(r, \infty) + \frac{4\Lambda}{r^2 + 4\pi^2 \beta^2 \Lambda^2} \).

The exact formulation for the level compressibility \( \chi \) for this case can not be obtained
due to lack of the knowledge of \( R_2(r, \Lambda) \) for entire energy-range. However the presence of a non-zero, non-unity compressibility can also be seen from the nearest-neighbour spacing distribution \( P(s) \) for large \( s \) values. This is because the compressibility of the spectrum is basically a reflection of the correlations of levels at large distances. For WD case, \( P(s) \propto s e^{-\pi s^2/4} \) decays as a Gaussian near the tail indicating no compressibility of levels. As for Poisson limit \( P(s) \propto e^{-s} \), the unity coefficient of \( s \) in the exponent implies a very high degree of compressibility. An exponential tail of \( P(s) \) with a non-unity coefficient will thus correspond to a a compressibility less than Poisson case; if \( P(s) \propto e^{-ks} \) for large \( s \), \( \chi \approx 1/2k \). The \( P(s) \) for MI transition can be given by using the one for the Poisson \( \rightarrow \) GOE transition \[12\], \( P(s, \Lambda) = \frac{su(\Lambda)}{\Lambda} e^{-u^2(\Lambda)s^2/4\Lambda^2} \int_0^\infty d\zeta e^{-\zeta^2-2\Lambda^2I_0(s\zeta u(\Lambda)/\Lambda)} \) with \( u(\Lambda) = \sqrt{\pi}U(-1/2, 0, \Lambda^2) \), \( U \) being Kummer function and \( I_0 \) Bessel function. Although this result is rigorous for \( 2 \times 2 \) matrix space but is proved reliable for systems with many levels. (The dominance of nearest-neighbour interaction is also supported by the mapping of the eigenvalues dynamics to the particle dynamics of calogero Hamiltonian with inverse-square interaction). As can be checked, for \( s >> \Lambda \), \( P(s, \Lambda) \propto e^{-ks} \) with \( k = \sqrt{\pi}\Lambda \). Near the critical point, therefore, \( k = \sqrt{\pi}\Lambda^* > 1 \) and \( \chi \approx 1/2\sqrt{\pi}\Lambda^* \). For small separations, that is, \( s << \Delta \), \( P(s, \Lambda^*) \) has a linear dependence on \( s \), indicating a WD type behaviour. The spacing distribution for CSS is thus a hybrid of Poisson and WD type distributions; this result is in agreement with numerical study of \[12\].

The case of the MI transition due to off-diagonal Gaussian disorder can similarly be studied by taking finite \( \alpha_{kl} \) values for all \( k, l \); the \( Y \) for any intermediate stage of the transition brought about by a most general type of diagonal and off-diagonal disorder is given by eq.\( (3) \). As obvious from the above, the presence or absence of the off-diagonal disorder will only affect the form of \( Y \), leaving, therefore, the qualitative behaviour of various correlators unaffected. The presence of different boundary conditions (e.g. periodic) or different topology for disordered lattice will also result in different \( Y \)-values (as various \( y_{kl}s \) will be affected), leading to different level-statistics even if degree of disorder, hopping rate and dimensionality is same; this is in agreement with numerical observations \[13\].
It should be noticed here that the expressions of $\chi(\Lambda)$ as well as $P(s, \Lambda)$ are also valid for Rosenzweig-Porter (RP) ensemble ($< H_{ii}^2 >= v_1$ for all $i$, $< H_{ij}^2 >= v_2$ for all $i \neq j$ and $v_1, v_2$ different in general), thus implying a non-zero compressibility for finite $\Lambda$-values in RP model too contrary to earlier claims [11]. Our result is in agreement with Dyson’s conjecture for Brownian ensembles [7] according to which the parameter $\Lambda$ governs a smooth transition in fluctuation properties; the large-$s$ behaviour too, of $P(s, \Lambda)$, is therefore expected to vary smoothly from $e^{-a}$ (Poisson) to $e^{-\beta s^2}$ (Wigner).

To summarize, our analytical study shows that the statistical transition of levels of a disordered system due to change of the parameters e.g. disorder, and system size can be reduced to the transition with respect to a single scaling parameter. Our results also confirm the presence of just three type of universal statistics for infinite system, namely Poisson above critical value of the disorder, WD below critical value of disorder and a new type of statistics for the critical region. The critical region statistics is a hybrid of the Poisson and WD and shows a finite level compressibility. The universality of CSS is restricted in the sense that, although system-independent, it depends on the critical value $\Lambda = \Lambda^*$ too besides the underlying symmetry of the system. We have discussed here only the 2$^{nd}$ order level-statistics for Anderson transition. The information about the higher order correlations can be obtained from the hierarchical equation (5) for the correlators or by analyzing particle correlators of CSM Hamiltonian. Further the mapping of the level-dynamics during Anderson transition to particle-dynamics of CSM Hamiltonian once more indicates the dominance of two body interaction in nature.
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[14] * E-Mail: Shukla@phy.iitkgp.ernet.in