Fermion localization in generalised Randall Sundrum model

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A generalized two-brane Randall-Sundrum warped braneworld model admits of solutions of the warp factor for both positive or negative cosmological constant on the visible 3-brane which can resolve the naturalness problem in connection with the fine tuning of Higgs mass in the standard model of elementary particles. To explore the location of the standard model fermions in such a generalised warped model, we, in this work, determine the dependence of the localization profile of a bulk fermion on the brane cosmological constant brane tension and the bulk fermion mass. Our results reveal that for a positive and small value of the induced cosmological constant a bulk fermion is localized close to the brane. On the other hand for a visible brane with negative cosmological constant and positive tension , the fermions are localized inside the bulk leading to phenomenologically interesting possibilities.

I. INTRODUCTION

Ever since the Kaluza-Klein proposal of possible existence of extra spatial dimension(s), there have been intense activities to explore the role of such dimensions in various observable phenomena in our four dimensional universe. Emergence of string theory in early eighties enhanced this interest further by theoretically predicting the inevitable presence of such extra dimensions. Furthermore some phenomenological extra dimensional models were proposed in the context of the so called ‘naturalness problem’ in the standard model of elementary particles.

Standard model has been extremely successful in explaining physical phenomena up to TeV scale. It however encounters the well-known fine tuning or naturalness problem in connection with the Higgs scalar mass. ADD model [1] introduced the notion of large extra dimensions to bring the effective Planck scale down to TeV which removed the fine tuning problem at the expense of introducing intermediate mass scales corresponding to the large extra dimensions i.e. which in turn brings back the hierarchy problem in a new guise. Alternatively, Randall-Sundrum warped braneworld model [2] has been particularly successful in resolving the fine tuning problem without bringing in any arbitrary intermediate scale between Planck and TeV. The two-brane RS model has the following features : TeV brane cosmological constant is zero which is close but not equal to the presently observed value of $\sim 10^{-48} \text{ GeV}^4$. In addition the TeV brane representing our universe is possessed with negative brane tension.

Such braneworld scenario subsequently has been generalized such that the brane cosmological constant (cc) as well as the brane tension of the visible 3-brane can be both positive or negative [3]. This model also resolves the fine tuning problem but now the values of brane-bulk parameter in the warp factor takes depends on the values of the induced cc on the visible brane [4]. Such generalised scenario also admits of positive tension brane.

An important question in the context of the original RS model has been ‘Can the standard model fermions also propagate in the bulk just as gravity?’ From the string theory point of view the fermions being open string mode are naturally confined to the 3-branes. However if one relaxes the string theory constraint, the fermions indeed can propagate in the bulk. In that case , using a localizing field in the bulk ( say a scalar field ) , such bulk fermion can be localized at different region in the bulk by appropriately choosing the interaction potential between the fermion and the scalar [5, 6, 7, 9, 10]. Various phenomenological implications of having the standard model fermions inside the bulk have been studied extensively. It has been shown that appropriate choice for the masses of the bulk fermions determine their localization in extra dimension which in turn leads to various interesting phenomenological consequences including a possible explanation for the fermion mass hierarchy in the standard model derived as an effective theory on the visible 3-brane [3, 6, 7, 9]. Schemes of electroweak symmetry breaking as well as chiral symmetry breaking have also been suggested by introducing a fourth generation fermion in the bulk [8]. It was also shown that a small Dirac neutrino mass can be obtained without invoking see-saw mechanism for certain bulk fermion models [9]. Numerous works have been done in recent years with fermions ( both massless and massive ) in the bulk of such warped geometric model to explore the consequent phenomenological implications [11].

In this work, we consider a massless as well as a massive bulk fermion and study the localization profile of the fermionic wave function in the generalised RS model as a function of the visible brane cosmological constant for both dS and AdS cases. Our result demonstrates that the localization region of the bulk fermion crucially depends on the value as well as the sign of the induced

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brane cosmological constant. As the overlap of a fermion wave function on the TeV brane is shown to depend on the brane cosmological constant therefore various parameters in the effective four dimensional action also in turn depends on the choice of the brane cosmological constant. Interesting phenomenological implications are therefore expected in such a scenario.

II. THE MODEL

In the RS warped braneworld model \[2\] the cosmological constant (cc) on the visible brane is zero and also the visible brane tension is negative. The modulus stability of such a brane world was achieved by introducing a bulk scalar \[12\]. However, the zero cc of the visible brane is not consistent with the observed small value. It has been demonstrated in \[4\] that this condition can be relaxed for a more general warp factor which include branes with non-zero cc \[3\]. This warp factor has been obtained by extremising the following the action:

\[
S = \int d^3x \sqrt{-G(M^3 R - \Lambda)} + \int d^4x \sqrt{-g_i} \mathcal{V}_i \tag{2.1}
\]

where \( \Lambda \) is the bulk cc, \( R \) is the bulk Ricci scalar and \( \mathcal{V}_i \) is the tension of the \( i \)th brane. The warped geometry has been obtained by considering a constant curvature brane spacetime, as opposed to a flat spacetime. The general form of the warped metric is

\[
ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r^2 dy^2 \tag{2.2}
\]

where \( r \) is the modulus associated with the extra dimension and \( \mu, \nu \) are brane coordinate indices. The scalar mass warping is achieved through the warp factor \( e^{-A(kr\pi)} = m_n = 10^{-n} \) where \( k = \sqrt{\frac{\Lambda}{4\pi M^3}} \sim \text{Planck Mass} \). ‘\( n \)’ the warp factor index must be set to 16 to achieve the desired warping. Let us consider that the induced brane cc is \( \Omega \). The brane metric \( g_{\mu\nu} \) may correspond to dS-Schwarzschild and AdS-Schwarzschild spacetimes for the induced brane cc, \( \Omega > 0 \) and \( \Omega < 0 \) respectively \[12\]. The magnitude of the induced cc on the brane in the generalized RS \[4\] case is non-vanishing in general and is given by \( \omega^2 = 10^{-N} \) (in Planck units) where \( \omega^2 \equiv \Omega/3k^2 \) for AdS and dS brane respectively \[15\]. A careful analysis reveals that for a negative brane cosmological constant \( N \) has a minimum value given by \( N_{min} = 2n \) leading to an upper bound on the magnitude of the cosmological constant while there is no such upper bound for the induced positive cosmological constant \[3\].

For AdS bulk (\( \Lambda < 0 \)) considering the induced cosmological constant on the visible brane to be negative the following solution for the warp factor is obtained:

\[
e^{-A} = \omega \cosh \left( \ln \frac{\omega}{c_1} + k y \right) \tag{2.3}
\]

where \( \omega^2 \equiv -\Omega/3k^2 \) and \( c_1 = 1 + \sqrt{1 + \omega^2} \) for the warp factor normalized to unity at \( y = 0 \). One can show that real solution for the warp factor exists if and only if \( \omega^2 \leq 10^{-2n} \) which leads to an upper bound for the magnitude of the cosmological constant (\( \sim 10^{-N} \)) given by \( N = N_{min} = 2n \). So, for \( n = 16 \) one obtains \( \omega_{max}^2 = 10^{-32} \). Also for \( N = N_{min} \) there is a degenerate solution for \( k r \pi = n \ln 10 + \ln 2 \). The two solutions of \( k r \pi \) for \( (N - 2n) > 1 \) are given by \( k r_1 \pi = n \ln 10 + 10^{-N-2n} \) and \( k r_2 \pi = (N - n) \ln 10 + \ln 4 \) which for \( n = 16 \) and \( N = 124 \) becomes

\[
k r_1 \pi \sim 36.84 + 10^{-93}, \quad k r_2 \pi = 250.07 \tag{2.4}\]

The hierarchy problem has been solved for the above solutions. Interestingly one can obtain the visible brane tension to be positive for the second solution. Also the visible brane tension may be zero for \( N = N_{min} = 2n \) and it is negative for \( r = r_1 \). For the present observed value of the cosmological constant \( \sim 10^{-48} \text{GeV}^4 \) the first solution \( k r_1 \pi \) corresponds to RS value plus a minute correction with negative tension visible brane whereas the other solution leads to a positive visible brane tension.

![FIG. 1: Variation of krπ](image)

III. FERMION LOCALIZATION

Now we study the fermion localization scenario in the above mentioned cases. For the general metric given in \[22\] we first find out the localization conditions and then study the different cases explicitly. The Lagrangian for the 5D massive Dirac fermions is given by

\[
\sqrt{-g_5} \mathcal{L}_{\text{Dirac}} = \sqrt{-g_5} \bar{\Psi} i\Gamma^a D_a \Psi - m_5 \bar{\Psi} \Psi \tag{3.1}\]
where \( g_5 = \text{det}(g_{ab}) \) is the determinant of 5D metric and \( m_5 \) is the 5D fermion mass. The curved space gamma matrices are given by \( \Gamma^a = (\epsilon^A(y)\gamma^\mu, -i\gamma^5) \) where \( \gamma^\mu, \gamma^5 \) represent 4D gamma matrices in chiral representation. The Clifford algebra \( \{\Gamma^a, \Gamma^b\} = 2g^{ab} \) is obeyed by curved gamma matrices \( \tilde{\epsilon} \). The covariant derivative can be calculated using the metric in (2.2) and is given by,

\[
D_\mu = \partial_\mu - \frac{1}{2} \Gamma_\mu \Gamma^4 A'(y)e^{-A(y)}; \quad D_5 = \partial_y \quad (3.2)
\]

In this set up the Dirac Lagrangian, \( \sqrt{-g_5}L_{\text{Dirac}} \), turns out to be

\[
e^{-4A(y)}\Psi \left[ ie^{A(y)}\gamma^\mu \partial_\mu + \gamma^5(\partial_y - 2A'(y)) - m_5 \right] \Psi \quad (3.3)
\]

We decompose the five-dimensional spinor as \( \Psi(x^\mu, y) = \psi_\mu(x^\mu)\xi(y) \). In the massless case the definite chiral states \( \psi_L(x^\mu) \) and \( \psi_R(x^\mu) \) correspond to left and right chiral states in four dimension. The \( \psi_L \) and \( \psi_R \) are constructed by, \( \psi_{L,R} = \frac{1}{2}(1 \pm \gamma^5)\psi \). Here \( \xi \) denote the extra dimensional component of the fermion wave function. We then can decompose five-dimensional spinor in the following way \( \tilde{\epsilon}_L \) \( \tilde{\epsilon}_R \)

\[
\Psi(x^\mu, y) = \psi_L(x^\mu)\xi_L(y) + \psi_R(x^\mu)\xi_R(y) \quad (3.4)
\]

Substituting the above decomposition in the Dirac Lagrangian we obtain the following equations for the fermions,

\[
e^{-A(y)}[\pm(\partial_y - 2A'(y)) - m_5]\xi_{R,L}(y) = -m_n \xi_{L,R}(y) \quad (3.5)
\]

The 4D fermions obey the canonical equation of motion, \( \psi_\mu^\dagger \partial_\mu \psi_{L,R} = m_n \psi_{L,R} \). Because of the extremely tiny value of the observed brane cosmological constant we have used near-flat 3-brane metric while evaluating the fermion equations of motion and have kept terms only up to \( \omega^2 \) in the resulting expressions. Moreover the equation \( (3.5) \) will be obtained provided the following normalization conditions are satisfied.

\[
\int_0^\pi e^{-3A(y)}\xi_{L_m,R_m}\xi_{L_n,R_n} dy = \delta_{mn} \quad (3.6)
\]

\[
\int_0^\pi e^{-3A(y)}\xi_{L_n} \xi_{R_n} dy = 0 \quad (3.7)
\]

Now we study the fermion zero mode (i.e. \( m_n = 0 \)) localization scenario of both massless and massive 5D bulk fermions in the generalized RS model \( [4] \) with both positive or negative brane cosmological constant. Consider a massless bulk fermion in the AdS 3-brane model. For the warp factor given in Eq. (2.3), the left and right chiral massless modes turn out to be

\[
\xi_{L,R}(y) = N_1 \text{sech}^2 \left( \ln \frac{\omega}{\epsilon_1} + ky \right) \quad (3.8)
\]

As discussed earlier, in the AdS case there are two solutions for the modulus for a given value of brane cosmological constant which solves the hierarchy problem.
brane cosmological constant. This feature is expected to have interesting phenomenological implications as has been discussed previously. Moreover a crucial point to note is that the present model exhibits the possibility of localizing the fermion in different region inside bulk for different choices of the brane cosmological constant only when the brane tension is positive. Such a situation is known to be favoured from the point of view of stability of the brane as opposed to the branes with negative tension.

![FIG. 4: Chiral massless modes with different 5D mass terms are plotted along extra dimension for a negative tension visible brane with negative induced cosmological constant.](image4)

We have also obtained interesting results considering 5D fermion mass to be non-zero. The profile of fermion zeromode gets further scaled by the bulk mass term, $m_5$, as follows

$$\xi_{L,R}(y) = N_3 \operatorname{sech}^2 \left( \ln \frac{\omega}{c_1} + k y \right) e^{\pm m_5 y} \quad (3.9)$$

Note that the degeneracy between the two chiral modes has been lifted by the mass term. For a given small value of $cc$ as the bulk fermion becomes more and more massive the left chiral mode has higher peak values on the visible brane (Fig. 4) whereas the right chiral mode shows exactly the reverse nature. If we consider the 5D mass term to be $c k$ then the fermion wavefunction becomes

$$\xi_{L,R}(y) \sim e^{(\pm cc + \frac{1}{2}) k y} \sqrt{1 + \frac{\omega^2}{c_1^2}}.$$  

For $c > \frac{1}{2}$ the right handed zero mode has a very small wavefunction on the visible brane (this property helps one to obtain small neutrino masses) and these are typically localized near the UV brane. As the present value of cosmological constant is very small it does have much significant contribution to the overlap of the wave function. We further notice that for smaller magnitude of the cosmological constant the wave function shifts towards the Planck brane. It is also worthwhile to mention that the orbifold symmetry allows only one solution to be present.

We now discuss the localization scenario with massless 5D fermions on branes with positive brane cosmological constant. The corresponding warp factor in Eq. 2.5 yields the following solutions for the left and right chiral massless modes of the fermions.

$$\xi_{L,R}(y) = N_2 \operatorname{cosech}^2 \left( \ln \frac{\omega_2}{\omega} + k y \right) \quad (3.10)$$

The zeromodes are plotted in Fig. 5 for different values of the cosmological constant on the brane. As the value of the cosmological constant becomes smaller the fermions become more sharply localized on the visible brane. Thus the fact that our visible universe has a very small and positive cosmological constant indicates that the bulk fermions are likely to get localized near the visible brane just as predicted in string theory. This in some sense, is an anthropic explanation of the smallness of the cosmological constant $[14]$ in connection with the brane localization of the fermion in string based model. However in this region of positive brane cosmological constant the brane tension is always negative and the fermion wave function peak shifts more and more inside the bulk as the value of the positive cosmological constant increase. This is depicted in the figure 7. If we turn on the 5D bulk fermion mass then the fermion zero modes become

$$\xi_{L,R}(y) = N_2 \operatorname{cosech}^2 \left( \ln \frac{\omega_2}{\omega} + k y \right) e^{\pm m_5 y}.$$  

For the smaller values of $cc$ as the 5D mass term increases the left modes have higher peak value on the visible brane whereas the right mode gets more and more supressed. If we consider the mass term as $c k$ then we find that the right handed mode gets extremely supressed at the visible brane while getting localized near the Planck brane. This feature can give rise to the fermion mass hierarchies on the brane.

**IV. CONCLUSION**

The localization of bulk fermions in warped brane world models have always been an interesting area of study specially from phenomenological point of view. While a string-based braneworld model prefers to have
the standard model fermions being tied to the visible brane as an open string excitation, the prospects of the presence of standard model fermions in the bulk in much wider class of braneworld models have drawn special attention because of their important phenomenological implications. All these models essentially deals with zero cosmological constant on the visible brane i.e with flat 3-branes. In this work we have determined the role of the induced cosmological constant on the profile of a massless fermion wave function and have explored how does it depend on the magnitude as well as signature of the induced cosmological constant on the visible brane in a generalised RS model. Our results indicate that both for de-Sitter as well as anti-de-Sitter visible brane, the localization of the fermion wave function in the bulk becomes sharply peaked near the brane as the magnitude of the induced cosmological constant becomes smaller. This corresponds to a string-based braneworld picture where the fermions are attached to the branes as open string modes. On the other hand for anti-de Sitter brane, the region where brane tension is positive, we find that the fermions are localized inside the bulk. For de-Sitter brane on the other hand, we find that cosmological constant can be arbitrarily large ( unlike the anti deSitter case) and the fermion wave function peaks inside the bulk for a large cosmological constant. In both the cases the overlap of the fermion wave function with the visible brane depends on the value of the brane cosmological constant, bulk fermion mass and all the other parameters in the effective 3+1 dimensional theory on the visible brane. Thus apart from the bulk fermion mass, the brane cosmological constant also controls the effective theory on the visible 3-brane This implies that the brane cosmological constant has an important role in determining various phenomenological parameters as perceived in a 3-brane slice, located at an orbifold fixed point.

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