GMRT observation towards detecting the post-reionization 21-cm signal

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ABSTRACT
The redshifted 21-cm signal from neutral hydrogen (HI) is an important future probe of the high-redshift Universe. We have analysed 610 MHz Giant Metrewave Radio Telescope (GMRT) observations towards detecting this signal from \(z = 1.32\). The multi-frequency angular power spectrum \(C_\ell(\Delta \nu)\) is used to characterize the statistical properties of the background radiation across angular scales \(\sim 20\) arcsec to 10 arcmin, and a frequency bandwidth of 7.5 MHz with resolution 125 kHz. The measured \(C_\ell(\Delta \nu)\) which ranges from 7 to 18 mK\(^2\) is dominated by foregrounds, the expected HI signal \(C_\ell^\text{HI}(\Delta \nu)\) \(\sim 10^{-6}\) to \(10^{-7}\) mK\(^2\) is several orders of magnitude smaller and detecting this is a big challenge. The foregrounds, believed to originate from continuum sources, is expected to vary smoothly with \(\Delta \nu\) whereas the HI signal decorrelates within \(~0.5\) MHz, and this holds the promise of separating the two. For each \(\ell\), we use the interval \(0.5 \leq \Delta \nu \leq 7.5\) MHz to fit a fourth-order polynomial which is subtracted from the measured \(C_\ell(\Delta \nu)\) to remove any smoothly varying component across the entire bandwidth \(\Delta \nu \leq 7.5\) MHz. The residual \(C_\ell(\Delta \nu)\) we find, has an oscillatory pattern with amplitude and period, respectively, \(~0.1\) mK\(^2\) and \(\Delta \nu = 3\) MHz at the smallest \(\ell\) value of 1476, and the amplitude and period decreasing with increasing \(\ell\). Applying a suitably chosen high pass filter, we are able to remove the residual oscillatory pattern for \(\ell = 1476\) where the residual \(C_\ell(\Delta \nu)\) is now consistent with zero at the 3\(\sigma\) noise level. Based on this we conclude that we have successfully removed the foregrounds at \(\ell = 1476\) and the residuals are consistent with noise. We use this to place an upper limit on the HI signal whose amplitude is determined by \(\tilde{x}_\text{HI} b (C_\ell^\text{HI}(\Delta \nu) \propto [\tilde{x}_\text{HI} b]^2)\), where \(\tilde{x}_\text{HI}\) and \(b\) are the HI neutral fraction and the HI bias, respectively. A value of \(\tilde{x}_\text{HI} b\) greater than 7.95 would have been detected in our observation, and is therefore ruled out at the 3\(\sigma\) level. For comparison, studies of quasar absorption spectra indicate \(\tilde{x}_\text{HI} \sim 2.5 \times 10^{-2}\) which is \(~330\) times smaller than our upper limit. We have not succeeded in completely removing the residual oscillatory pattern, whose cause is presently unknown to us, for the larger \(\ell\) values.

Key words: cosmology: observations – diffuse radiation – large-scale structure of Universe.

1 INTRODUCTION
Detecting redshifted 21-cm radiation from neutral hydrogen (HI) at high redshifts is of considerable interest in cosmology (Furlanetto, Oh & Briggs 2006). At redshifts \(z \leq 6\), the bulk of the neutral gas is in clouds that have HI column densities in excess of \(2 \times 10^{20}\) atoms cm\(^{-2}\) (Lanzetta, Wolfe & Turnshek 1995; Storrie-Lombardi, McMahon & Irwin 1996; Péroux et al. 2003). These high column density clouds are observed as damped Lyman-\(\alpha\) absorption lines seen in quasar spectra. The analysis of quasar spectra indicates that the ratio of the density \(\rho_{\text{gas}}(z)\) of neutral gas to the present critical density \(\rho_{c0}\) of the Universe has a nearly constant value \(\Omega_{\text{gas}}(z) \sim \rho_{\text{gas}}(z)/\rho_{c0} \sim 10^{-3}\), over a large redshift range \(0.5 \leq z \leq 5.0\) (Storrie-Lombardi et al. 1996; Rao & Turnshek 2000; Péroux et al. 2003; Prochaska & Herbert-Fort 2004; Rao, Turnshek & Nestor 2006; Kanekar et al. 2009). The redshifted 21-cm radiation from the HI in this redshift range will be seen in emission. The emission from individual clouds (<10 \(\mu\) Jy) is too weak to be detected with existing instruments unless the image is significantly magnified by gravitational lensing (Saini, Bharadwaj & Sethi 2001). The collective emission from the undetected clouds is present as a very faint background in all radio
observations at frequencies below 1420 MHz. The fluctuations in this background radiation carry an imprint of the H$\alpha$ distribution at the redshift $z$ where the radiation originated. The possibility of detecting this holds the potential of providing us with a new observational probe of large-scale structures (Kumar, Padmanabhan & Subramanian 1995; Bagla, Nath & Padmanabhan 1997; Bharadwaj, Nath & Sethi 2001; Bharadwaj & Sethi 2001; Bharadwaj & Pandey 2003; Bharadwaj & Srikant 2004; Zaldarriaga, Furlanetto & Hernquist 2004; Ali, Bharadwaj & Pandey 2005; Wyithe & Loeb 2007, 2008; Bagla, Khandai & Datta 2010). In a recent paper, Pen et al. (2009) report a detection of the post-reionization H$\alpha$ signal through the cross-correlation between the HIPASS and the 6dGRS data.

Observations of redshifted 21-cm radiation can in principle be carried out over a large redshift range starting from the cosmological Dark Ages through the Epoch of Reionization to the present epoch (Bharadwaj & Ali 2005), allowing us to trace out both the evolution history of neutral hydrogen and the growth of structures in the Universe. Redshifted 21-cm observations also hold the potential of allowing us to probe the expansion history of the Universe (Mcquinn et al. 2006; Chang et al. 2008; Bharadwaj, Sethi & Saini 2009; Visbal, Loeb & Wyithe 2009).

The Giant Metrewave Radio Telescope (GMRT; Swarup et al. 1991), currently operating at several frequency bands in the frequency range 150 to 1420 MHz, is well suited for carrying out observations towards detecting the H$_{\alpha}$ signal over a large redshift range from $z \sim 0$ to $\sim 8.3$ and angular scales of $\sim 10$ arcsec to $\sim 1^\circ$. In this paper, we report results from the analysis of 610 MHz observations towards detecting the redshifted 21-cm signal from the cosmological H$_{\alpha}$ distribution at $z = 1.32$.

We have characterized, possibly for the first time, the statistical properties of the background radiation at 610 MHz across $\sim 20$ arcsec to $10$ arcmin angular scales and a frequency bandwidth of 7.5 MHz with a resolution of 125 kHz using the multifrequency angular power spectrum $C_{l}(\Delta\nu)$ (hereafter MAPS; Datta, Roy Choudhury & Bharadwaj 2007). This jointly characterizes the angular ($l$) and frequency ($\Delta\nu$) dependence of the fluctuations in the 610 MHz radiation in the field of view (FOV) of our observations. Foregrounds from different astrophysical sources are expected to be a few orders of magnitude larger than the predicted 21-cm signal (Shaver et al. 1999; Di Matteo et al. 2002; Oh & Mack 2003; Santos, Cooray & Knox 2005; Wang et al. 2006; Ali, Bharadwaj & Chengalur 2008) and our 610 MHz GMRT observations are expected to be nearly entirely dominated by foregrounds which are predicted to be at least a thousand times larger than the H$_{\alpha}$ signal. Separating the H$_{\alpha}$ signal from foregrounds is the most important challenge for cosmological redshifted 21-cm observations.

The foregrounds are believed to have a smooth continuum spectrum and the contribution to $C_{l}(\Delta\nu)$ is expected to vary very slowly with $\Delta\nu$ across the band (7.5 MHz) of our analysis. The contribution from the H$_{\alpha}$ signal decorrelates very rapidly with increasing $\Delta\nu$ and is expected to be uncorrelated beyond $\Delta\nu = 0.5$ MHz at the angular scales ($l = 10^3$ to $l = 3 \times 10^3$) of our analysis. This property of the signal holds the promise of allowing us to separate the signal from the foregrounds. In this paper we propose and implement a technique that uses polynomial fitting in $\Delta\nu$ to subtract out any smoothly varying component from the measured $C_{l}(\Delta\nu)$. The residuals are expected to contain only the H$_{\alpha}$ signal and noise. The target of the present work is to test if the polynomial subtraction successfully removes the foregrounds to a level such that the residuals are consistent with noise. The noise in the current observation is considerably larger than the H$_{\alpha}$signal and longer observations would be needed for detecting the H$_{\alpha}$ signal.

The present work closely follows an earlier paper (Ali et al. 2008) which analysed 150 MHz GMRT observations. We note that the prospect of detecting the redshifted 21-cm signal considerably increases at higher frequencies (e.g. 610 MHz) where the foreground contribution and noise are both smaller. Further, the problem of man-made radio frequency interference is considerably more severe at 150 MHz as compared to 610 MHz.

A brief outline of the paper follows. Section 2 describes the observation and data analysis; Section 3 presents the visibility correlation technique that we use to estimate $C_{l}(\Delta\nu)$ and also presents the estimated values; Sections 4 and 5 present model predictions for the H$_{\alpha}$ signal and foregrounds, respectively, while Section 6 describes our proposed technique of foreground removal and finally Section 7 contains results and conclusions.

2 GMRT OBSERVATIONS AND DATA ANALYSIS

The GMRT has 30 fixed antennas each of diameter 45 m. 14 of which are randomly distributed in a central square $1.1 \times 1.1$ km in extent, while the rest of the antennas are distributed approximately in a ‘Y’ shaped configuration. The shortest antenna separation (baseline) is around 60 m including projection effects while the largest separation can be as long as 26 km. The hybrid configuration of the GMRT gives reasonably good sensitivity to probe both compact and extended sources.

The observed FOV is centred on $\alpha_{2000} = 12^h36^m49^s$, $\delta_{2000} = 62^\circ17'57''$ which is situated near Hubble Deep Field North (HDF-N) ($\alpha_{2000} = 12^h36^m49^s$, $\delta_{2000} = 62^\circ12'58''$). The galactic coordinates of the observed field is $l = 125.87$, $b = 54.74$. The sky temperature determined at this location is $20$ K in the 408 MHz Haslam et al. (1982) map. The observations were carried out over three days from September 4 to 7, in 2002, and the total observation time was almost $30$ h (including calibration). The observation had a centre frequency of 618 MHz, and a total bandwidth of 16 MHz divided into 128 frequency channels, each 125 kHz wide. The integration time was $16$ s and visibilities were recorded for two orthogonal circular polarizations. The calibrator sources 3C147 and 3C286 were used for flux calibration and 1313+675 was used for phase calibration. The phase calibrator was observed every half hour to correct for temporal variations in the system gain. We have used the Astronomical Image Processing Software (AIPS) to analyse the recorded visibility data. The flux of these two flux calibrators was estimated by extending the Baars scale (Baars et al. 1977) to low frequencies using the AIPS task ‘ETSY’. Standard AIPS tasks were used to flag all data that could be visually identified as being bad. The entire lower sideband data were found to be bad and were discarded from the subsequent analysis. Data from different days were calibrated and flagged separately and then combined using the AIPS task ‘BACKCON’. We find that the channels near the edge of the band are relatively noisy and hence only the 100 central channels were used in the subsequent analysis.

An initial two-dimensional (2D) image of the FOV showed four bright sources with considerable imaging artefacts. To improve our image quality, initially we have subtracted out the clean components (CC) of these bright sources by moving them to the phase centre using appropriate RA-SHIFT and Dec-SHIFT within AIPS. Then, we add back the brightest source and use this for three rounds

1 http://www.gmrt.ncra.tifr.res.in

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of self-calibration with time intervals of 3 and 2 min for phase calibration and finally 20 min for amplitude and phase calibration and then subtract out the brightest source again. The same process is followed for rest of the bright sources. Subsequent to this, we have also subtracted out all the weaker sources from our FOV and used the AIPS task `TVFLG' to flag out any bad visibility. Finally, we have collapsed all frequency channels and clipped the resulting visibilities at 0.07 Jy. At each stage the same calibration and flag tables were also applied to the original 100 channel data which contain all the sources.

The large FOV ($\theta_{\text{FWHM}} = 43\,\text{arcmin}$) of the GMRT at 610 MHz leads to considerable errors if the non-planar nature of the GMRT antenna distribution is not taken into account. We use the three-dimensional (3D) imaging feature (Perley 1999) in the AIPS task `IMAGR' in which the entire FOV is divided into multiple sub-fields, each of which is imaged separately. Here, a $1.5 \times 1.5$ FOV was imaged using 163 facets. The presence of a large number of sources in the field allows us to do self-calibration loops to improve the image quality. We have applied four rounds of self-calibration, the first three only for the phase and the final round for both amplitude and phase. The time interval for the gain correction was chosen as 3, 2, 1 and 20 min for the successive self-calibration loops. At every stage, the calibration tables were applied to the original 100 channel data. The 100 channels were collapsed into 10 channels which were used to make a continuum image of the entire FOV. Fig. 1 shows our continuum image of bandwidth $12.5\,\text{MHz}$ centred at $617\,\text{MHz}$. The synthesized beam has full width a half-maximum (FWHM) $8.2 \times 5.8\,\text{arcsec}^2$. The off source rms noise in the image was $\sim 60\,\mu\text{Jy}\,\text{Beam}^{-1}$ and the image quality had improved considerably. The rms noise around the brighter sources is higher, using the AIPS task `TVSTAT' we notice that it is around $100\,\mu\text{Jy}\,\text{Beam}^{-1}$. The brighter sources are also found to be accompanied by a region of negative flux density; these are presumably the results of residual phase errors which were not corrected in our self-calibration process. The maximum and minimum flux densities in the final image are $\sim 250\,\text{mJy}\,\text{Beam}^{-1}$ and $\sim 2.8\,\text{mJy}\,\text{Beam}^{-1}$, respectively.

The subsequent analysis was done using the calibrated visibilities of the original 100 channel data with all the sources. The data contain 510,528 baselines, each of which has visibilities for 2 circular polarizations. The baselines are in the range $200\lambda$ to $20\,\lambda$. The visibilities from the two polarizations were combined ($V = (V_{RR} + V_{LL})/2$) for the subsequent analysis. The final calibrated data contain 33,233,698 visibilities. The real part of the visibilities has a mean of 0.68 mJy and rms of 0.25 Jy. Similarly, the imaginary part has a mean of 2.34 mJy and rms of 0.25 Jy.

It is often convenient to assume that the visibilities have a Gaussian distribution. The distribution of the real part of the visibilities is shown in a histogram (Fig. 2). We find that a Gaussian gives a reasonably good fit to the data within $3\sigma$, which contains the bulk of the data. The number counts predicted by the Gaussian falls much faster than the data at large visibility values $|Re(V)| > 0.75\,\text{Jy}$. The imaginary part of the visibilities shows a similar behaviour. Deviation from Gaussian statistics is expected to mainly affect the error estimate in the visibility correlation. We expect this effect to be small, since the discrepancy is for only small fraction of visibilities.

Figure 1. Our continuum image of bandwidth $12.5\,\text{MHz}$ centred at $618\,\text{MHz}$. The FOV was imaged using 163 facets which have been combined using the AIPS task FLATN. The rms off-source noise is $60\,\mu\text{Jy}\,\text{Beam}^{-1}$. Most of the extended features visible in the image are imaging artefacts around the four bright sources.

Figure 2. Histogram showing the distribution of the real parts of all the 33,233,698 measured visibilities. The same is shown on a linear scale (left-hand panel) and a log-linear scale (right-hand panel). A Gaussian with mean 0.683 mJy and rms 0.251 Jy, values calculated for the real parts of the measured visibilities, is plotted as a dotted line in both panels. While the Gaussian fits the data very well at small amplitudes, there is a discrepancy at high amplitudes ($\sim 0.75\,\text{Jy}$) which is visible only in the right-hand panel.
3 VISIBLE CORRELATIONS AND
THE ANGULAR POWER SPECTRUM

The aim here is to quantify the statistical properties, in angle and frequency, of the 610 MHz sky signal. For a frequency \( v \), the angular dependence of the brightness temperature distribution on the sky \( T(v, \hat{n}) \) may be expanded in spherical harmonics as

\[
T(v, \hat{n}) = \sum_{\ell, m} a_{\ell m}(v) Y_{\ell m}(\hat{n}).
\]

(1)

The MAPS (Datta et al. 2007), which jointly characterizes the dependence on angular scale and frequency separation, is defined as

\[
C_{\ell}(\Delta v) \equiv C_{\ell}(v, v + \Delta v) = \langle a_{\ell m}(v) a_{\ell m}^*(v + \Delta v) \rangle.
\]

(2)

Here \( \ell \) refers to the angular modes on the sky. The sky signal is assumed to be statistically isotropic. We also assume that for the relatively small bandwidth of our observation (\( \Delta v \ll v \)), the frequency dependence can be entirely characterized through \( \Delta v \) whereby we do not explicitly show \( v \) as an argument in equation (2).

We use the correlation between pairs of visibilities \( V(U, v) \) and \( V(U + \Delta U, v + \Delta v) \)

\[
V_2(U, v; U + \Delta U, v + \Delta v) \equiv \langle V(U, v) V^*(U + \Delta U, v + \Delta v) \rangle
\]

(3)
to estimate \( C_{\ell}(\Delta v) \). The correlation of a visibility with itself is excluded to avoid a positive noise bias in the estimator. Ali et al. (2008) as well as Dutta et al. (2009) contain detailed discussions of the estimator, and we highlight only a few salient features here.

The GMRT primary beam pattern is well approximated by a Gaussian \( A(\theta) = e^{-4\pi \theta^2 / \theta_{FWHM}^2} \) where \( \theta_{FWHM} \approx 0.6 \times 10^{-6} \) arcmin (\( \theta_{FWHM} = 43 \) arcmin) at 610 MHz. In a situation where \( \Delta U \) is small such that \( |\Delta U| < (\pi \theta_{FWHM})^{-1} = 42.4 \times 10^{-6} \) (\( \theta_{FWHM} \) in radians), the expected correlation \( V_2(U, v; U + \Delta U, v + \Delta v) \) in equation (3) does not depend on \( \Delta U \) whereby we may express it as \( V_2(U, \Delta v) \). Further, if \( U \gg \theta_{FWHM} \) we have

\[
V_2(U, \Delta v) = \frac{\pi \theta_{FWHM}^2}{2} \left( \frac{\partial B_\nu}{\partial T} \right)^2 C_{2\nu T}(\Delta v) Q(\Delta v).
\]

(4)

where \( B_\nu = 2 \pi k_B T / c^2 \) is the specific intensity of blackbody radiation in the Rayleigh–Jeans limit. Both \( \theta_{FWHM} \) and \( \left( \frac{\partial B_\nu}{\partial T} \right) \) depend on the frequency. In our analysis, we treat these as constants with the value being evaluated at 610 MHz. It is possible to incorporate the effect of the \( \Delta v \) dependence of \( \theta_{FWHM} \) and \( \left( \frac{\partial B_\nu}{\partial T} \right) \) through the function \( Q(\Delta v) \) in equation (4). This is expected to be a slowly varying function of \( \Delta v \) with a variation of \( \sim 1 \) per cent across the \( \Delta v \) range of our observation. We have not explicitly considered the function \( Q(\Delta v) \) in our present analysis. This is expected to introduce an extra, slowly varying \( \Delta v \) dependence in the estimated \( C_{\ell}(\Delta v) \). This slowly varying \( \Delta v \) dependence, as we shall discuss later, can be included in the foreground model and separated from the H i signal which varies rapidly with \( \Delta v \).

Equation (5) gives the final expression that we use to estimate the angular power spectrum (MAPS)

\[
C_{2\nu T}(\Delta v) = 87 \left( \frac{mK}{\rm Jy} \right)^2 \times V_2(U, \Delta v).
\]

(5)

In our analysis we have correlated only pairs of baselines which satisfy the condition \( |\Delta U| \leq 10 \lambda \). We have restricted the analysis to baselines 200 \( \lambda \leq U \leq 5000 \lambda \). To test if the visibility correlation is actually independent of \( \Delta U \) we have also considered \( |\Delta U| \leq 5 \lambda \) and 20 \( \lambda \). The results are unchanged for 20 \( \lambda \) and they are rather noisy for 5 \( \lambda \), there being very few baseline pairs to correlate.

The measured \( V_2(U, \Delta v) \) will, in general, have real and imaginary parts (Fig. 3). As seen in equation (5), the expectation value is predicted to be real, the expectation value of the imaginary part being zero. We use the real part of the measured \( V_2(U, \Delta v) \) to estimate \( C_{\ell}(\Delta v) \) through equation (5). A small imaginary part arises due to the noise in the individual visibilities. This introduces random fluctuations in both the real and imaginary parts of the measured \( V_2(U, \Delta v) \). Fig. 3 shows the measured \( V_2(U, \Delta v) \) and the inferred \( C_{\ell}(\Delta v) \) for \( \Delta v = 0 \). As expected, the imaginary part is much smaller than the real part of \( V_2(U, \Delta v) \). Note that we use the notation \( C_{\ell} \equiv C_{\ell}(\Delta v = 0) \).

We next consider the expected statistical fluctuations (error) in \( V_2(U, \Delta v) \). The total error has two parts, i.e. system noise and the cosmic variance. The total variance \( \langle \Delta V_2(U, \Delta v) \rangle^2 \) can be calculated as

\[
\langle \Delta V_2(U, \Delta v) \rangle^2 = \frac{\langle N^2 \rangle}{2N_P} + \frac{\langle V_2(U, \Delta v) \rangle^2}{N_B},
\]

(6)

where \( \langle N^2 \rangle = \langle N^N \rangle \) is the variance of the noise contribution \( N \) in the visibilities that we use in our analysis, \( N_P \) is the total number of baseline pairs that contribute to \( V_2(U, \Delta v) \) and \( N_B \) is the number of independent estimates of \( V_2(U, \Delta v) \). Here \( \langle N^N \rangle = \sigma^2 \) where \( \sigma \) is the rms noise, for a single polarization, in the real part (or equivalently the imaginary part) of a visibility. The value of \( \sigma \) is expected to be (Thompson, Moran & Swenson 1986)

\[
\sigma = \frac{\sqrt{3k_B T_{sys}}}{A_{eff} \sqrt{\Delta v \Delta t}},
\]

(7)

where \( T_{sys} \) is the total system temperature, \( k_B \) is the Boltzmann constant, \( A_{eff} \) is the effective collecting area of each antenna, \( \Delta v \) is the channel width and \( \Delta t \) is correlator integration time. For the parameters of our observations, \( T_{sys} \approx 100 \mathrm{K}, 2 T_{sys} k_B / A_{eff} = 300 \mathrm{Jy}, \Delta v = 0.125 \mathrm{MHz} \) and \( \Delta t = 16 \mathrm{s} \) we have \( \sigma^2 = 2.25 \times 10^{-2} \mathrm{Jy}^2 \).

In our analysis we have used \( \langle N^N \rangle = 1.25 \times 10^{-1} \mathrm{Jy}^2 \) which is the sum of the variance of the real and imaginary components of the measured visibilities. In our observation the total error is dominated by the cosmic variance which is a few orders of magnitude larger than the system noise in the entire \( U \) range that we have considered.

The \( \Delta v \) dependence of \( C_{\ell}(\Delta v) \) is shown in Fig. 4. We have considered \( U \) values below 1.25 \( \lambda \), where \( \ell = 2 \pi U \). As discussed later, the H i signal falls at \( U > 1 \lambda \) which is why we have not considered baselines larger than 1.25 \( \lambda \). We find that for nearly
all the values of $\ell$ shown in the figure the variation in $C_\ell(\Delta \nu)$ with $\Delta \nu$ is roughly between 0.2 to 0.6 mK$^2$ across the 7.5 MHz band. The fractional variation in $C_\ell(\Delta \nu)$ ranges from 1.5 to 3.6 per cent. We note that an oscillatory pattern is visible in $C_\ell(\Delta \nu)$ at nearly all values of $\ell$. The pattern is most pronounced at the lower $\ell$ values. The error bars shown in Fig. 4 include only the system noise contribution. The measured $C_\ell(\Delta \nu)$ is expected to be dominated by foregrounds which are believed to be largely independent of $\Delta \nu$. For a fixed $\ell$ the cosmic variance then is expected to introduce the same error (independent of $\Delta \nu$) across the entire band. As a consequence we do not consider the cosmic variance for the $\Delta \nu$ dependence shown in Fig. 4.

The 2D Fourier transform relation between the sky brightness and the visibilities assumed in deriving equation (4) is not strictly valid for GMRT’s FOV ($\Theta_{\text{FWHM}} = 43$ arcmin). In addition to $uv$ which are the components of the baseline in the plane normal to the direction of observation, it is also necessary to consider $w$ the component along the observing direction. To assess the impact of the $w$ term we have repeated the analysis using only a limited range of baselines for which $w \leq 0.5 \times U$. We find that limiting the maximum $w$ value does not make any qualitative change in our results.

4 THE EXPECTED REDSHELFED H I 21-CM SIGNAL

Our observing frequency $\nu = 610$ MHz corresponds to a redshift of $z = 1.32$ for the H I 21-cm radiation. Observations of Lyman $\alpha$ absorption lines seen in quasar spectra indicate that the ratio of the density $\rho_{\text{gas}}(z)$ of neutral gas to the present critical density $\rho_{\text{crit}}$ of the Universe has a nearly constant value $\rho_{\text{gas}}(z)/\rho_{\text{crit}} \sim 10^{-3}$, over a large redshift range $0 \leq z \leq 3.5$. This implies that the mean neutral fraction of the hydrogen gas is $x_{\text{HI}} = 50 \frac{\Omega_\text{b} h^2}{\Omega_{\Lambda} h^2} (0.02/\Omega_b h^2) = 2.45 \times 10^{-2}$ which we adopt for our analysis. The redshifted 21 cm radiation from the H I will be seen in emission as a very faint background in our observation. The fluctuations in this background with angle and frequency are a direct probe of the H I distribution at the redshift $z = 1.32$ where the radiation originated. We calculate the MAPS for the redshifted 21-cm signal (Datta et al. 2007) using

$$C_\ell(\Delta \nu) = \frac{T^2}{\nu r_s^2} \int_0^\infty dk_\parallel \cos(k_\parallel r_\perp \Delta \nu) P_{\text{HI}}(k),$$

where the 3D wave vector $k$ has been decomposed into components $k_\parallel$ and $l/r_s$, along the line of sight and in the plane of the sky, respectively. The comoving distance $r_s$ is the distance at which the H I radiation originated. Note that $(1+z)^{-1} r_s = d_A(z)$ is the angular diameter distance and $r_\perp = dr_\parallel/d \nu$. The temperature occurring in equation (8) is given by

$$T(z) = 4.0 \text{ mK} \left(1+z\right)^2 \left(\frac{\Omega_\text{b} h^2}{0.02}\right) \left(\frac{0.7}{H_0}\right) \frac{H_0}{H(z)},$$

and $P_{\text{HI}}(k)$ is the 3D power spectrum of the ‘21-cm radiation efficiency in redshift space’ (Bharadwaj & Ali 2005) which in this situation is given by

$$P_{\text{HI}}(k) = x_{\text{HI}}^2 b^2 \left(1 + \beta \mu^2\right)^2 P(k).$$

The term $(1 + \beta \mu^2)^2$ arises because of the H I peculiar velocities (Bharadwaj et al. 2001; Bharadwaj & Ali 2004), which we assume to be determined by the dark matter. This is the familiar redshift-space distortion seen in galaxy redshift surveys, where $\beta$ is the linear distortion parameter and $\mu = k_\parallel/k$. On the large scales of interest here, it is reasonable to assume that HI traces the dark matter with a possible linear bias $b$, whereby the 3D H I power spectrum is $b^2 P(k)$, where $P(k)$ is the dark matter power spectrum at the redshift where the H I signal originated. Unless mentioned otherwise, we
The signal would be larger at $\ell < 1000$, but the GMRT's FOV restricts us from accessing these $\ell$ values. The measured $C_\ell$ values are around $10^7$ to $10^8$ times larger than the predicted $\text{HI}$ signal. The predicted signal decorrelates rapidly with increasing $\Delta \nu$ and $\kappa_\ell(\Delta \nu)$ falls by 90 per cent or more [$\kappa_\ell(\Delta \nu) < 0.1$] at $\Delta \nu = 0.5$ MHz. The value of $\Delta \nu$ where $\kappa_\ell(\Delta \nu)$ falls by 90 per cent is smaller for larger values of $\ell$. Further, we find that the expected $\text{HI}$ signal is anti-correlated at large values of $\Delta \nu (\sim 0.8$ MHz) where $\kappa_\ell(\Delta \nu)$ has a small negative value.

5 FOREGROUND MODEL PREDICTIONS

The radiation coming from different astrophysical sources, other than the $\text{HI}$ signal, contributes to the foreground radiation. Here, we mainly focus on the two most dominant foreground components namely extragalactic point sources and the diffuse synchrotron radiation from our own Galaxy. The free–free emissions from our Galaxy and external galaxies (Shaver et al. 1999) make much smaller contributions though each of these is individually larger than the $\text{HI}$ signal. We have modelled the MAPS for each foreground component as

$$C_\ell(\Delta \nu) = A \left(\frac{1000}{\ell}\right)^{\gamma} \kappa_\ell(\Delta \nu), \quad (13)$$

where $A$, $\gamma$ and $\kappa_\ell(\Delta \nu)$ are the amplitude, the power-law index and frequency decorrelation function, respectively. The different foreground components considered here are all continuum radiation which are known to vary smoothly with frequency. For each component we denote the spectral index using $\alpha$, whereby the amplitude scales as $A \propto \nu^\alpha$. The value of $A$, whenever used in this paper, is at a fixed frequency of 610 MHz. The continuum nature of the foreground components also implies that we expect $\kappa_\ell(\Delta \nu)$ to be of the order of unity and vary smoothly with $\Delta \nu$. The foregrounds will remain correlated across the frequency band of our observation, unlike the $\text{HI}$ signal which decorrelates rapidly within $\Delta \nu = 0.5$ MHz. Given the absence of any direct observational constraints on $\kappa_\ell(\Delta \nu)$ for any of the foreground components at the angular scales and frequencies of our interest, we do not attempt to make any model predictions for this quantity beyond assuming that it varies smoothly with $\Delta \nu$ across the frequency band of observation. In the subsequent discussion we focus on model predictions of $A$ and $\gamma$ which are tabulated in Table 1 for the different foreground components.

Extra-galactic point sources are expected to dominate the sky at 610 MHz. We have estimated the point source contribution using the 610 MHz differential source count from Garn et al. (2008). This is the average differential source count of 610 MHz GMRT observations in three different fields of view, namely the Spitzer

| Foregrounds                  | $A$(mK$^2$) | $\alpha$ | $\gamma$ |
|-----------------------------|-------------|----------|----------|
| Point source (clustered part) | 20.03 x (2.07) | 0.32 | 2.07 | 0.9 |
| Point source (Poisson part)  | 8.38 x (2.07) | 1.16 | 2.07 | 0   |
| Galactic synchrotron         | 0.122       | 2.80     | 2.4      |
| Galactic free–free           | 1.14 x 10^{-4} | 2.15 | 3.0      |
| Extra Galactic free–free     | 2.11 x 10^{-5} | 2.1 | 1.0      |

Table 1. Values of the parameters used for characterizing different foreground contributions at 610 MHz. Here $S_\ell$ is the flux of the brightest source in the FOV.
extragalactic First Look, ELAIS-N1 and Lockman Hole surveys. The differential source count in the flux range of their observation ($\sim 0.3$ to 200 mJy) is well fitted by a single power law

$$\frac{dN}{dS} = \frac{1259}{\text{Jy}\cdot\text{Sr}} \left( \frac{S}{1\text{Jy}} \right)^{-1.84}.$$  \hspace{1cm} (14)

We have assumed that the same power law also holds for the fainter sources below the detection limit.

Point sources make two distinct contributions to the angular power spectrum; the first being the Poisson noise due to the discrete nature of the sources and the second arising from the clustering of the sources. The Poisson contribution, which is independent of $\ell$, is calculated using

$$C_{\ell} = \left( \frac{\partial B}{\partial T} \right)^{-2} \int_0^S S^2 \frac{dN}{dS} dS,$$  \hspace{1cm} (15)

where $S = 250\text{ mJy}$ is the flux of the brightest source in our FOV.

The uncertainty in the Poisson contribution involves the fourth moment $\int_0^S S^4 \frac{dN}{dS} dS$ of the source count and is given by

$$[\Delta C_{\ell}(0)]^2 = \left( \frac{S}{\text{Jy}} \right)^{2.32} \left( 69.63 - 133.15 \left( \frac{S}{\text{Jy}} \right)^{0.84} \right).$$  \hspace{1cm} (16)

The analysis of large samples of nearby radio-galaxies has shown that the point sources are clustered. Cress et al. (1996) have measured the angular two-point correlation function at 1.4 GHz (FIRST Radio Survey), across an angular scale of 0.02 to 2$^\circ$, equivalent to an $\ell$ range of $90 < \ell < 9000$. Throughout the entire angular scale the measured two-point correlation function can be well fitted with a single power law of the form $u(\theta) = (\theta/\theta_0)^{-\beta}$, where $\beta = 1.1$ and $\theta_0 = 17.4\text{ arcmin}$. This partly covers the range of angular scales ($\sim 10\text{ arcmin}$ to 20 arcsec or $\sim 1000 < \ell < 3 \times 10^3$) that we are interested in. We will assume that the clustering of the sources remains unchanged at our observing frequency. They have also reported that on small scales ($< 0.2$) the double and multi-component sources tend to have a larger clustering amplitude than that of the whole sample. They also found that the sources with flux densities below 2 mJy have a much shallower slope ($\sim 0.97$) for the measured correlation function. It seems that the amplitude and slope of the measured two-point correlation function changes with the angular scale and flux densities of the sources. For our present purpose, we have used $u(\theta) = (\theta/\theta_0)^{-1.1}$ which have been measured up to $\ell = 9000$. We have assumed that the slope of the two-point correlation function will remain unchanged beyond $\ell = 9000$. We then have

$$C_{\ell} = \left( \frac{\partial B}{\partial T} \right)^{-2} \left( \int_0^S S \frac{dN}{dS} dS \right)^2 w_{\ell},$$  \hspace{1cm} (17)

where $w_{\ell} \propto \ell^{\beta-2}$ is the angular power spectrum which is the Fourier transform of $u(\theta)$.

The Galactic diffuse synchrotron radiation is believed to be produced by cosmic ray electrons propagating in the magnetic field of the Galaxy (Ginzburg & Syrovatskii 1969). The angular power spectrum is predicted to scale as $\ell^{-\gamma}$ with $\gamma \approx 2.4$ (Tegmark et al. 2000) to angular scales as small as 4 arcmin, and the spectral index has a value of $\sim 2.8$ (Reich & Reich 1988; Jonas, Baart & Nicolson 1998). Here, we have extrapolated the parameters from the 130 MHz foreground model prediction of Santos et al. (2005). Recently, Bernardi et al. (2009) have characterized the power spectrum of the total diffuse radiation at 150 MHz at the angular scales of our interest. The $\gamma$ value that we adopt in our foreground model is consistent with that found by Bernardi et al. (2009). The total error in our model predictions is calculated by adding the variances from different contributions.

Fig. 7 shows the point source and synchrotron contributions along with the total measured signal. At large angular scales ($\ell < 10^3$), the foreground model prediction is dominated by the clustering of point sources, the point source Poisson contribution being the second largest component at these angular scales. This is reversed at smaller angular scales ($\ell > 10^3$) where the point source Poisson contribution dominates and the clustering component is the second largest contribution. The Galactic synchrotron contribution, also shown in Fig. 7, is much smaller at all the angular scales of our interest. The contributions from Galactic and extra-galactic free-free emission, whose parameters have been extrapolated from Santos et al. (2005), are also listed in Table 1. These are much smaller and hence are not shown in Fig. 7. The expected H I signal ($C_{\ell} \sim 10^{-6}$ to $10^{-7}\text{mK}^2$) is much smaller than all the foreground components mentioned here, and is not shown in the figure.

We find that the measured $C_{\ell}$ is within the 1$\sigma$ error bars of the model prediction, for $\ell \leq 2300$. The measured $C_{\ell}$ is around three times larger than the model predictions at smaller angular scales where the measured values do not lie within the 1$\sigma$ error bars of the model predictions. The source of this discrepancy is, at present, unknown to us. The model predictions require the source properties to be extrapolated to faint flux levels and small angular scales where direct observations are not available. It is possible that the model predictions have been underestimated. For the present work we assume that the measured $C_{\ell}$ is correct and that the model predictions have been underestimated at small angular scales.

For the subsequent analysis in this paper we shall assume that the measured $C_{\ell}(\Delta v)$ is a combination of contributions from foregrounds, the H I signal and noise. Further, the H I signal being several orders of magnitude smaller than the foregrounds, we may interpret the measured $C_{\ell}(\Delta v)$ as an estimate of the foregrounds actually present in our FOV.

6 FOREGROUND REMOVAL

Removing the foregrounds which, as we have seen, are several orders of magnitude larger than the H I signal is possibly the biggest challenge for detecting the H I signal. There have been quite a few...
earlier works on this, nearly all either theoretical or simulation. All attempts in this direction are based on the assumption that the foregrounds are continuum radiation which vary slowly with frequency whereas the H I is a line emission which varies rapidly with frequency.

A possible line of approach is to represent the sky signal as an image cube where in addition to the two angular coordinates on the sky we have the frequency as the third dimension. For each angular position, polynomial fitting is used to subtract out the component of the sky signal that varies slowly with frequency. The residual sky signal is expected to contain only the H I signal and noise (Morales, Bowman & Hewitt 2006; Jelíč et al. 2008; Bowman, Morales & Hewitt 2009; Liu, Tegmark & Zaldarriaga 2009b). Liu et al. (2009a) show that this method of foreground removal has problems which could be particularly severe at large baselines if the uv sampling is sparse. They propose an alternate method where the frequency dependence of the visibility data is fitted with a polynomial and this is used to subtract out the slowly varying component. The residuals are expected to contain only noise and the H I signal.

In this work, we have attempted to subtract out the brightest point sources from the image using standard AIPS tasks. We have used the AIPS task ‘UVSUB’ to subtract the CC of the brightest sources from the visibility data. Continuum images were used for this purpose. The resulting visibility data were used to make a new image. We find that this method fails to remove the point sources efficiently. Several imaging artefacts remain in the vicinity of bright sources even after the sources have been removed. Similar findings were reported in Ali et al. (2008) where the same technique was used to remove point sources from 150 MHz GMRT observations. Given the poor performance of this image-based technique, we have not pursued it any further. The visibility-based technique proposed by Liu et al. (2009a) requires the data to be gridded in uv plane. The estimator that we have used to determine \( C(\ell,\nu) \) (Section 3) works with the individual visibilities. Using the gridded data (Hobson & Maisinger 2002) would introduce a positive noise bias in \( C(\ell,\nu) \) and hence we do not adopt this technique here.

The foreground subtraction techniques discussed above all attempt to remove the foregrounds before determining the angular power spectrum. Here, we propose a different method where the foregrounds are subtracted after determining the angular power spectrum. The measured \( C(\ell,\nu) \) (Figs 3 and 4) is a sum of the foregrounds, noise and the H I signal. The H I signal decays rapidly with increasing \( \nu \). This contribution is less than 10 per cent for \( \Delta \nu \geq 0.5 \) MHz, and it is negligibly small for \( \Delta \nu > 1 \) MHz (Fig. 6). We assume that \( C(\ell,\nu) \) measured in the frequency interval \( 0.5 \) MHz \( \leq \Delta \nu \leq 7.5 \) MHz contains only foreground and noise. Further, we assume that the foreground contribution to \( C(\ell,\nu) \) has a slow \( \Delta \nu \) dependence which can be well fitted by a low-order polynomial. Note that, in addition to the intrinsic \( \Delta \nu \) dependence of the foreground, the measured \( C(\ell,\nu) \) has an additional \( \Delta \nu \) dependence arising from the factor \( Q(\Delta \nu) \) (equation 4). The latter is a slow, monotonic variation and we expect that both these effects can be adequately accounted for by a low-order polynomial. We use the interval \( 0.5 \) MHz \( \leq \Delta \nu \leq 7.5 \) MHz to estimate this polynomial, which is then used to subtract the foreground contribution from \( C(\ell,\nu) \) across the entire range of our measurement (\( 0 < \Delta \nu \leq 7.5 \) MHz). The residual \( C(\ell,\nu) \) is expected to be a sum of only the H I signal and noise.

In order to illustrate our technique of foreground subtraction and to demonstrate its efficacy, we first apply it to simulated data where a known H I signal has been put in by hand. Given the uncertainty in our current understanding of the foreground properties and of the effects that have possibly been introduced during the observation and the subsequent analysis, we are guided by the measured \( C(\ell,\nu) \) for our simulations. We find that the measured \( C(\ell,\nu) \) (Fig. 4) has a value around \( \sim 10 \) mK\(^2\), with \( \sim 5 \) per cent variation with \( \Delta \nu \) across the 7.5 MHz band. Further, the error has a typical value \( \sqrt{\Delta C^2(\ell,\nu)} \sim 0.01 \) mK\(^2\) (system noise only). We have simulated the measured MAPS using

\[
C(\ell,\nu) = \sum a_n (\nu)^n + \delta + \alpha C^{HI}(\Delta \nu),
\]

where the polynomial \( \sum a_n (\nu)^n \) represents the slowly varying \( \Delta \nu \) dependence which causes \( C(\ell,\nu) \) to vary by \( \sim 10 \) per cent across the 7.5 MHz band. Our \( C(\ell,\nu) \) estimator (equation 5) is even in \( \Delta \nu \), and hence we have only considered polynomials of even order. Our simulation was restricted to fourth-order polynomials where the coefficients \( c_0, c_2, c_4 \) are Gaussian random variables with mean 12, 0, 0 mK\(^2\) and rms 1, 10\(^{-1}\), 10\(^{-4}\)mK\(^2\), respectively. The term \( \delta \) is a Gaussian random variable of rms 0.01 mK\(^2\) which incorporates the error and \( C^{HI}(\Delta \nu) \) is the H I signal (equation 8). The noise in our observation is considerably larger than the H I signal, and it would not be possible to detect the signal even if the foregrounds were perfectly subtracted. The factor \( \alpha \) in our simulations amplifies the H I signal so that it lies above the noise. The value of \( \alpha \) has been chosen such that \( C(\ell,\nu) = 5 \times 0.01 \) mK\(^2\) (5\(\sigma\)) at the value of \( \Delta \nu \) where \( C(\ell,\nu) \) is 70 per cent of the peak value \( C(0) \). The simulations have exactly the same frequency bandwidth and channel width as the measured \( C(\ell,\nu) \). Though in this paper we have only considered fourth-order polynomials for our simulations, the same procedure can easily be repeated considering even polynomials of any order.

Fig. 8 shows the simulated \( C(\ell,\nu) \) for the different values of \( \ell \). Note that the polynomial coefficients \( c_n \) are different for each realization of the simulation. We have fitted the simulated data with a fourth-order polynomial using the interval \( 0.5 \) MHz \( \leq \Delta \nu \leq 7.5 \) MHz. The best-fitting polynomial is also shown in Fig. 8. The residuals, after the best-fitting polynomial is subtracted from the simulated \( C(\ell,\nu) \), are shown in Fig. 9. In the interval \( 0.5 \) MHz \( \leq \Delta \nu \leq 7.5 \) MHz, the residuals are within \( \pm 3 \)\(\sigma\) from zero which is consistent with noise. Fig. 10 shows the residuals in the range \( \Delta \nu \leq 1 \) MHz overlaid with the H I signal that had been added by hand. We find that our foreground subtraction technique successfully extracts the H I signal that had been added in the simulated data, despite it being buried in foregrounds which are \( \sim 200 \) times larger. We note that we have also tried a slightly different technique of foreground subtraction where we have used the entire \( \Delta \nu \) range (\( \leq 7.5 \) MHz) to estimate the polynomial. We find that the latter technique does not correctly recover the H I signal that had been put in by hand.

7 RESULT AND CONCLUSIONS

We have measured the statistical properties of the background radiation across angular scales 20 arcsec to 10 arcmin using the multi-frequency angular power spectrum \( C(\ell,\nu) \). Frequency channels 20 to 80 were used for the analysis. This corresponds to a total bandwidth of 7.5 MHz with a resolution of 125 kHz. The measured \( C(\ell,\nu) \) has values around 12 mK\(^2\). Considering first the \( \ell \) dependence of \( C(\ell,\nu) \) (Fig. 3), starting from \( \sim 18 \) mK\(^2\) at \( \ell \approx 1000 \), it drops to \( \sim 9 \) mK\(^2\) at \( \ell \approx 2000 \) and then rises to a nearly constant value of around 13 mK\(^2\). The uncertainty in \( C(\ell,\nu) \) is mainly due to the sample variance, i.e. the fact that we have observed a single \( \sim 1.5 \times 1.5 \) FOV which gives a limited number of independent estimates of \( C(\ell,\nu) \), the system noise makes a relatively smaller
contribution. We next consider the \( \Delta \nu \) dependence of \( C_\ell(\Delta \nu) \) for different values of \( \ell \) (Fig. 4). Assuming that the foreground contributions all have a smooth power-law \( \nu \) dependence, the expected \( \Delta \nu \) dependence may be estimated through a Taylor series expansion as

\[
C_\ell(\Delta \nu) = C_\ell \left[ 1 + B(\Delta \nu/\nu)^2 + \ldots \right]
\]

where \( B \) is constant of the order unity. The odd powers of \( \Delta \nu/\nu \) cancel out because the estimator averages positive and negative \( \Delta \nu \) values. The expected change in \( C_\ell(\Delta \nu) \) is \( \sim 1.5 \times 10^{-2} \) per cent for \( \Delta \nu = 7.5 \) MHz. The measured

\[\begin{align*}
\text{Figure 8.} & \quad \text{The simulated } C_\ell(\Delta \nu) \text{ with } 3\sigma \text{ error bars (system noise) is shown for different values of } \ell. \text{ The solid curve shows the best-fitting fourth-order polynomial determined using the interval } 0.5 \text{ MHz} \leq \Delta \nu \leq 7.5 \text{ MHz.} \\
\text{Figure 9.} & \quad \text{The residual, with } 3\sigma \text{ error bars, after subtracting the best-fitting fourth-order polynomial from the simulated } C_\ell(\Delta \nu). 
\end{align*}\]
$C(t(\Delta v))$ (Fig. 4) has a smooth variation of the order of a few per cent (1 to 4 per cent) across the 7.5 MHz bandwidth of our observation. In addition to the smooth $\Delta v$ dependence, we also notice a small oscillatory pattern in the measured $C(t(\Delta v))$. The expected H$\alpha$ contribution to $C(t(\Delta v))$ is $\sim 10^{-7}$ times smaller than the measured values, and we interpret the measured $C(t(\Delta v))$ as being nearly entirely foregrounds and noise.

We next consider results for foreground removal using the technique discussed in Section 6. For a fixed $\ell$, the frequency range $0.5$ MHz $\leq \Delta v \leq 7.5$ MHz was used to estimate a fourth-order polynomial fit to $C(t(\Delta v))$. The $C(t(-\Delta v)) = C(t(\Delta v))$ symmetry of the $C(t(\Delta v))$ estimator was applied in the fitting procedure. This fit was used to subtract out the foreground contribution from the entire frequency range $\Delta v \leq 7.5$ MHz. The performance of this foreground removal technique was assessed by visually inspecting the fit and the residuals across the entire band. We find that increasing the order of the polynomial does not result in any significant improvement, and hence we restrict our analysis to a fourth-order polynomial for which the fits have been shown in Fig. 4.

The residuals in $C(t(\Delta v))$, we find, typically have values within 0.1 mK$^2$ (Fig. 11). In all cases the residuals are not consistent with $C(t(\Delta v)) = 0$ (i.e. noise only). The residuals, we find, have a nearly sinusoidal oscillatory pattern. These oscillations are most pronounced for the lowest $\ell$ value where it has an amplitude of $\sim 0.1$ mK$^2$ and a period of $\Delta v \sim 3$ MHz. The period and amplitude both decrease with increasing $\ell$. The oscillations are possibly not very well resolved at the larger $\ell$ values due to the 0.125 MHz channel width. The oscillations would possibly be more distinctly visible in observations with higher frequency resolution. The oscillatory residual pattern is quite distinct from the expected H$\alpha$ signal and also from random noise, and in principle it should be possible to distinguish between these by considering the Fourier transform

$$\tilde{C}(t_\ell) = \sum_n e^{i2\pi n \Delta v_\ell} C(t_\ell),$$

(19)

where $n, m = -59, -58, ..., 0, ..., 58, 59, \Delta v_\ell = n \times 0.125$ MHz and $t_\ell = m(119 \times 0.125$ MHz)$^{-1}$. We expect the oscillatory pattern to manifest itself as a localized feature in $\tilde{C}(t)$ and it should be possible to remove the oscillatory feature by applying a suitable filter to $\tilde{C}(t)$. We find that for the smallest $\ell$ values the amplitude of $\tilde{C}(t)$ is peaked at a few $t_\ell$ values located within $|m| \leq 10$. Based on this we have chosen a filter

$$F(t_\ell) = \left\{ \begin{array} {l} m \leq m_c \rightarrow 1.0 - e^{-|m-m_c|^2/2} \quad |m| > m_c \end{array} \right.$$

(20)

such that $\tilde{F}(t_\ell)\tilde{C}(t_\ell)$ removes the Fourier components within $|m| \leq m_c$ from the residual $\tilde{C}(t_\ell)$. Calculating $C(t(\Delta v))$ after applying the filter, we find that for the smallest $\ell$ value the oscillatory pattern is removed if we use $m_c = 7$ or larger (Fig. 12). The oscillatory pattern is somewhat reduced for the next two $\ell$ values while the three largest $\ell$ values are not much affected by the filter with $m_c = 7$. It is possible to remove the oscillatory pattern from the second largest $\ell$ value by increasing the value of $m_c$ to $m_c = 14$, but the oscillatory pattern still persists for the larger $\ell$ values. Increasing $m_c$ will also reduce the H$\alpha$ signal, and hence we do not consider $m_c = 14$ in the subsequent discussion. The filter is also expected to affect the noise estimates, and the noise in the different $C(t(\Delta v))$ will be correlated as a consequence of the filter. For $m_c = 7$, we are filtering out $\sim 10$ per cent of the $C(t) \ell$ values, and hence we do not expect this to be a very severe effect. Thus, for the purpose of this paper, it is reasonable to assume that the noise is unaffected by the filter.

We find that for the smallest $\ell$ value ($\ell = 1476$) the residuals are consistent with zero at the 3$\sigma$ level. Based on this, we
conclude that we have successfully removed the foreground contribution from the measured $C_\ell(\Delta \nu)$ at this value of $\ell$. The residual oscillatory pattern persists at all the larger $\ell$ values where we are not successful in completely removing the foregrounds. The cause of this oscillatory residual, which at the moment is unknown to us, is an important issue which we plan to investigate in future.

We next use the measured $C_\ell(\Delta \nu)$ at $\ell = 1476$ to place an upper limit on the $\text{HI}$ signal. The amplitude of the expected $\text{HI}$ signal is determined by the factor $(\bar{x}_\text{HI}, b)^2$ (equations 8 and 10) where $\bar{x}_\text{HI}$ and...
The residuals from the measured $C_t(\Delta \nu)$ are also shown (triangles).

$b$ are the H$\text{I}$ neutral fraction and the H$\text{I}$ bias parameter, respectively. In the discussion till now we have used $\bar{x}_Hb = 2.45 \times 10^{-2}$ to estimate the expected H$\text{I}$ signal $C^H_t(\Delta \nu)$. We now consider $\bar{x}_Hb$ as a free parameter whose value is unknown, and ask if it is possible to use our observation to place an upper limit on the value of $\bar{x}_Hb$. Considering $\bar{x}_Hb$ as an unknown parameter, the expected H$\text{I}$ signal $C^H_t[\bar{x}_Hb](\Delta \nu)$ can be expressed as

$$C^H_t[\bar{x}_Hb](\Delta \nu) = \left( \frac{\bar{x}_Hb}{2.45 \times 10^{-2}} \right)^2 C^H_t(\Delta \nu).$$

The H$\text{I}$ signal would be detectable in our observation at the 3 $\sigma$ level if

$$C^H_t[\bar{x}_Hb](\Delta \nu) > 3 \sqrt{\left( C^H_t[\bar{x}_Hb](\Delta \nu) \right)^2 / N_E + \left\{ \Delta C_t(\Delta \nu) \right\}^2_{\text{sys}}},$$

where $N_E$ is the number of independent estimates of the signal, and the terms $\left( C^H_t[\bar{x}_Hb](\Delta \nu) \right)^2 / N_E$ and $\left\{ \Delta C_t(\Delta \nu) \right\}^2_{\text{sys}}$ are, respectively, the sample variance and system noise contributions to the total variance.

The fact that for $\ell = 1476$ the measured $C_t(\Delta \nu)$ is consistent with noise, and the signal is not detected, allows us to use equation (22) to place an upper limit on $\bar{x}_Hb$. The filter $\tilde{F}(\tau)$ that has been used to remove the oscillatory pattern in the residual also affects the signal. We have applied the same filter to $C^H_t[\bar{x}_Hb](\Delta \nu)$ (Fig. 13) and used this in equation (22). The filtered signal is maximum at $\Delta \nu = 0$, and we use this data point to place a 3 $\sigma$ upper limit on $\bar{x}_Hb$. A value of $\bar{x}_Hb$ greater than 7.95 would have been detected in our observation, and is therefore ruled out at the 3 $\sigma$ level. Our upper limit is around 330 times larger than the value that we have estimated based on results from quasar absorption spectra which imply $\bar{x}_H = 2.45 \times 10^{-2}$ and the assumption that $b = 1$. The H$\text{I}$ signal should, in principle, be detectable in observations that are a few hundred times more sensitive than the one that has been analysed here.

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