Possible new resonance at the $D^*\bar{D}^*$ threshold in $e^+e^-$ annihilation

S. Dubynskiy
School of Physics and Astronomy, University of Minnesota,
Minneapolis, MN 55455
and
M.B. Voloshin
William I. Fine Theoretical Physics Institute, University of Minnesota,
Minneapolis, MN 55455
and
Institute of Theoretical and Experimental Physics, Moscow, 117218

Abstract
We argue that the recent CLEO-c data on $e^+e^-$ annihilation into pairs of charmed mesons at c.m. energy around 4.0 GeV are not well described by a single resonance $\psi(4040)$, but can be better understood if there is an additional narrow resonance with mass within few MeV from the $D^*\bar{D}^*$ threshold.
The strong dynamics of heavy and light quarks gives rise to rich structures near the thresholds of open heavy flavor states. In particular, the cross section of the $e^+e^-$ annihilation is well known to display an intricate behavior in the region of the thresholds for production of various pairs of $D$ ($D^*$) mesons. The most prominent peak in the annihilation cross section is conventionally labeled as the $\psi(4040)$ resonance. It has been argued long ago that this resonance (then called $\psi(4028)$) might be of a ‘molecular’ type, i.e. essentially made of a meson pair $D^*\bar{D}^*$, on the basis of the observed great enhancement of its coupling to $D^*\bar{D}$ and $D\bar{D}^*$ in comparison with its interaction with the other open charmed meson channels: $D\bar{D}$ and $D\bar{D}^*$. However it was later pointed out that a similar pattern of couplings to meson pairs can arise for a pure charmonium state due to a kinematic cancellation in overlap integrals at the momentum corresponding to the meson states $D\bar{D}$ and $D\bar{D}^*$. In other words, the latter scheme suggests a suppression of these lighter channels rather than an enhancement of the coupling to $D^*\bar{D}^*$. This issue has not been resolved so far due to lack of more detailed data on the production of the charmed mesons in $e^+e^-$ annihilation in the vicinity of the peak. Recently the CLEO-c experiment has produced a greatly improved set of data from the scan of the $e^+e^-$ production cross section separately for each open channel with charmed meson pairs. The data display a highly nontrivial behavior in each of the channels in all the scanned energy range from 3970 MeV to 4260 MeV, and in particular at energies around the $\psi(4040)$ peak. The purpose of this letter is to point out that the CLEO-c data in the immediate vicinity of the threshold for the $D^*\bar{D}^*$ meson pairs, specifically in the energy range from 3970 MeV to 4060 MeV, cannot be described with a reasonable statistical significance by a single resonance and a non-resonant background. Rather we find that the data suggest that in addition to a resonance with parameters close to those of $\psi(4040)$ there exists a narrow structure, with a width of not more than few MeV, at a c.m. energy very close to $4010 \pm 0.15$ MeV, i.e. very close to the thresholds for the neutral vector meson pairs $D^{*0}\bar{D}^{*0}$ ($4013 \pm 1$ MeV) and for the charged ones $D^{*+}\bar{D}^{*-}$ ($4020 \pm 1$ MeV). The available CLEO-c data however are yet not detailed enough to fit the parameters of the resonance suggested by this structure. We were only able to make a very preliminary estimate of the $e^+e^-$ branching fraction of such resonance as $B_{ee} \sim 10^{-7}$.

If confirmed by a more detailed experimental study the suggested resonance would tantalizingly invite a ‘molecular’ interpretation and an analogy with the now well known resonance $X(3872)$ at the threshold of the meson pair $D^{*0}\bar{D}^{*0}$. However, unlike the $X(3872)$ resonance, which corresponds to an $S$-wave meson pair, the suggested resonance would cor-
respond to a $P$-wave charmed meson pair. The appearance of threshold singularities in different partial waves can imply a quite nontrivial dynamics of the strong interaction between heavy-light mesons.

In what follows we use the CLEO-c scan data\[5\] for the $e^+e^-$ annihilation cross section at six values of the c.m. energy (in MeV): 3070, 3090, 4010, 4015, 4030 and 4060. The cross section data are available separately for the channels $\bar{D}D$, $D_s\bar{D}_s$, $D\bar{D}^*$ and $D^*\bar{D}^*$. Where appropriate, these data are for the sum over the pairs of charged and neutral mesons, and we assume that the mesons are produced by the isotopically singlet electromagnetic current of the charmed quarks, and the isotopic asymmetry, relevant for the channel $D^*\bar{D}^*$, due to the isotopic mass difference and due to the Coulomb interaction is taken into account. We attempt a fit of the data to a model of a smooth background and a resonance, taking into account the interference effects and the effects due to the onset of the $D^*\bar{D}^*$ channel at the threshold and the channel mixing. In the actual calculation we use, instead of the cross sections, the dimensionless production rate coefficients $R_k$ defined as

$$\sigma(e^+e^- \to D\bar{D}) = \sigma_0(s) 2v_D^3 R_1, \quad \sigma(e^+e^- \to D_s\bar{D}_s) = \sigma_0(s) v_{D_s}^3 R_2,$$

$$\sigma(e^+e^- \to D\bar{D}^* + D^*\bar{D}) = \sigma_0(s) 6\left(\frac{2p}{\sqrt{s}}\right)^3 R_3, \quad \sigma(e^+e^- \to D^*\bar{D}^*) = \sigma_0(s) 7(v_0^3 + v_+^3) R_4,$$

with $\sigma_0 = \pi\alpha^2/(3s)$. Here $v_D$, $v_{D_s}$, $v_0$ and $v_+$ stand for the c.m. velocities of each of the mesons in respectively the channels $D\bar{D}$, $D_s\bar{D}_s$, $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$, while for the channel $D\bar{D}^* + D^*\bar{D}$ with mesons of unequal mass the velocity factor is replaced by $(2p/\sqrt{s})$ with $p$ being the c.m. momentum. The extra factors 1, 3 and 7 in respectively the pseudoscalar-pseudoscalar, pseudoscalar-vector and the vector-vector channels correspond to the ratio of the corresponding production cross section in the simplest model\[2, 7\] of independent quark spins, so that the inequality between $R_1$, $R_3$ and $R_4$ also illustrates a conspicuous deviation from this model.

The values of the coefficients $R_i$ corresponding to the CLEO-c data at the discussed energies are given in the Table 1. One can readily see from the table that the rate coefficient $R_4$ is very large near the threshold of the $D^*\bar{D}^*$ pairs, a behavior noticed long ago\[2\], and which behavior can make sense only if the production of the vector meson pairs is dominated by a resonance, while for the other channels both a resonant and a non-resonant production amplitudes are generally present. Accordingly, we parametrize the rate coefficients as\[8\]

$$R_k(E) = \left| a_k + \frac{b_k + i c_k}{D(E)} \right|^2$$

(2)
Table 1: The dimensionless production rate coefficients $R_k$ defined by Eq. (1) calculated from the CLEO-c scan data\[5\].

| E(MeV) | 3070  | 3090  | 4010  | 4015  | 4030  | 4060  |
|--------|-------|-------|-------|-------|-------|-------|
| $R_1$  | 1.89 ± 0.27 | 1.18 ± 0.23 | 0.61 ± 0.15 | 0.28 ± 0.24 | 2.92 ± 0.27 | 3.69 ± 0.25 |
| $R_2$  | 34.2 ± 8.7  | 22.6 ± 5.3  | 28.9 ± 3.2  | 23.0 ± 5.4  | 14.0 ± 2.9  | 2.7 ± 1.5  |
| $R_3$  | 49.8 ± 1.4  | 45.6 ± 1.2  | 45.4 ± 0.8  | 47.9 ± 1.6  | 36.8 ± 0.9  | 17.5 ± 0.6  |
| $R_4$  | —     | —     | —     | —     | 709 ± 161 | 357 ± 14  | 81.8 ± 3.1 |

for $k=1, 2, \text{and } 3,$ and

$$R_4(E) = \frac{b^2}{|D(E)|^2},$$ \hspace{1cm} (3)

where $a_k, b_k, c_k, \text{and } b$ are real numbers\(^1\), and $D(E)$ is the resonant denominator, which takes into account the threshold effects due to the strong coupling of the resonance to the $D^*\bar{D}^*$ channel. The expression for the latter factor at $E$ above the $D^{*+}D^{*-}$ threshold has the form

$$D = E - W_0 + \frac{i}{4} \left[ 2 \Gamma_0 + \frac{(E - 2 M_0)^{3/2}}{w^{1/2}} + \frac{(E - 2 M_+)^{3/2}}{w^{1/2}} \right],$$ \hspace{1cm} (4)

where $M_0 (M_+)$ is the mass of the $D^{*0}$ ($D^{*+}$) meson, $W_0$ is the ‘nominal’ mass of the resonance, and $\Gamma_0$ is the width of the resonance decay into all channels except for $D^*\bar{D}^*$. The decay rate into the latter final states rapidly changes near the threshold and is accounted by the last two terms in the square braces in Eq. (4) with $w$ being a parameter with dimension of energy, describing the coupling of the discussed resonance to $D^*\bar{D}^*$. At energy $E$ below one or both of the $D^*\bar{D}^*$ thresholds, the resonance denominator $D(E)$ is found from Eq. (4) by analytical continuation. Thus at the energy between the thresholds for the neutral and the charged $D^*$ mesons, $2 M_0 < E < 2 M_+$, one finds

$$D = \frac{1}{4} \left[ \frac{(2 M_+ - E)^{3/2}}{w^{1/2}} + E - W_0 + \frac{i}{4} \left[ 2 \Gamma_0 + \frac{(E - 2 M_0)^{3/2}}{w^{1/2}} \right] \right],$$ \hspace{1cm} (5)

and below both thresholds, i.e. at $E < 2 M_0$,

$$D = \frac{1}{4} \left[ \frac{(2 M_+ - E)^{3/2}}{w^{1/2}} + \frac{(2 M_0 - E)^{3/2}}{w^{1/2}} \right] + E - W_0 + \frac{i}{2} \Gamma_0 .$$ \hspace{1cm} (6)

\(^1\)The insensitivity of the cross section to the overall phases of the production amplitudes allows one to set e.g. the fit parameters $a_k$ and the factor $b$ for the $D^*\bar{D}^*$ channel as real.
Using the described model of a resonance plus background production of the charmed mesons we have made a fit to the data in Table 1. Although the resulting central values of the fit parameters ($W_0 = 4023 \text{ MeV}, \Gamma_0 = 67 \text{ MeV}$) for the resonance are in a reasonable agreement with the data listed in the PDG Tables[1] for $\psi(4040)$, the statistical significance of the fit is quite poor: $\chi^2/N_{\text{dof}} = 17.6/8$. We also have not found any improvement in the statistical significance by introducing a linear in the momentum form-factor $(1 - p^2/\mu^2)$ in the amplitudes. Another modification of the described model by introducing a non-resonant background for the $D^*$ pair production amplitude is not technically feasible right now due to lack of data. Indeed, there are in fact two $P$-wave amplitudes describing the production of the $D^*\bar{D}^*$ pairs near the threshold: one with the total spin of the meson pair $S = 0$ and the other with $S = 2^2$. The resonant production amplitude in the considered model corresponds to a fixed composition of these two amplitudes, which composition is generally different in the non-resonant part, which thus introduces at least four new parameters. It should be however noticed that introducing a non-resonant production of the $D^*$ mesons at the threshold is neither physically palatable, nor it is likely to improve agreement with the data. Indeed, in order to significantly affect the cross section the non-resonant amplitude should be comparable with the resonant one, which is very big, as can be seen from the $R_4$ entries in Table 1. We are not aware of any reasonable mechanism that would explain such an enhancement of the non-resonant amplitude. On the other hand the rather large width of the $\psi(4040)$ resonance implies only a smooth variation of the cross section on the energy scale considered here. The data however apparently display a small but sharp ‘wiggle’ in the $D\bar{D}$ data at 4010 and 4015 MeV and a possible steep rise towards the threshold in the $D^*\bar{D}^*$ cross section at 4015 MeV. In fact this feature in the data is the main contributor to a large $\chi^2$ in our fit. Thus introducing a smooth non-resonant term in the $D^*\bar{D}^*$ production amplitude is very unlikely to improve the situation.

An obvious effect, which is significantly enhanced in the immediate vicinity of the threshold for the charged mesons, $D^{*+}D^{*-}$, is the Coulomb interaction between the mesons. Therefore one can probe this effect as a possible reason for the steeper behavior of the cross sections near the threshold indicated by the data. However, we have not found any improvement in the statistical significance of the fit by including the Coulomb interaction. In other words at the available scan points the Coulomb effect is too weak to be visible in the data. We

\[^{2}\text{It looks reasonable to neglect in the threshold region the third possible amplitude, corresponding to an } F\text{-wave production with } S = 2.\]
nevertheless present here the relevant formulas, which can be helpful if more detailed data immediately near the threshold become available.

Generally, the effect of the Coulomb interaction can be complicated in the resonance region by the interference with the strong phase. However due to the relatively large width of $\psi(4040)$ the variation of the strong phase on the scale of few MeV around the threshold (where the Coulomb effect is most essential) can be neglected, and one can neglect such interference and estimate the effect of the Coulomb interaction by considering the slow $D^*\pm$ mesons as point charges. In this approximation the Coulomb effect in the $D^{*+}D^{*-}$ channel can be found by adapting the textbook formulas. The corrected expression for $R_4$ takes the form

$$R_4(E) = \frac{b^2}{|D_c(E)|^2} \left[ \frac{v_0^3 + v_+^3 F_c(E)}{v_0^3 + v_+^3} \right],$$  

(7)

and in the expressions for rest of the rate coefficients, as well as in the latter formula, the resonant denominator $D(E)$ is replaced by the Coulomb-corrected $D_c(E)$, which is described as follows. At $E > 2M_+$, i.e. above the charged meson threshold the term $(E - 2M_+)^{3/2}/w_{1/2}$ in Eq.(4) is replaced as

$$(E - 2M_+)^{3/2}/w_{1/2} \rightarrow (E - 2M_+)^{3/2}/F_c(E),$$  

(8)

while below the charged meson threshold the corresponding term in the equations (5) and (6) is replaced as

$$(2M_+ - E)^{3/2}/w_{1/2} \rightarrow (2M_+ - E)^{3/2}/\tilde{F}_c(E),$$  

(9)

where $F_c$ ($\tilde{F}_c$) is the Coulomb factor in the $P$ wave above (below) the threshold. Above the threshold this factor is conveniently expressed in terms of the Coulomb parameter $\lambda = M_+\alpha/(2k)$, where $k = \sqrt{M_+(E - 2M_+)}$ is the c.m. momentum of the $D^*$ meson:

$$F_c = \frac{2\pi \lambda (1 + \lambda^2)}{1 - \exp(-2\pi \lambda)}. $$  

(10)

The analytical continuation of this expression below the threshold can be written in terms of the real quantity $\nu = M_+\alpha/(2\kappa)$ with $\kappa = \sqrt{M_+(2M_+ - E)}$:

$$\tilde{F}_c = (1 - \nu^2) \nu \left( \pi \cot \pi \nu - 2 \ln \frac{\kappa_0}{\kappa} \right). $$  

(11)

The parameter $\kappa_0$ in the latter formula can be chosen arbitrarily, and any variation in such choice can be absorbed in a shift in $W_0$ and rescaling the resonant amplitude coefficients $b_k$, $c_k$, and $b$.  

5
The only hypothesis that we have found to be in a statistically significant agreement with the data is that the cross section at 4010 and 4015 MeV for the $D\bar{D}$ channel and at 4015 MeV for the $D^*\bar{D}^*$ channel is contributed by a new resonance. The resonance has to be quite narrow, with a width of at most few MeV, in order to not affect the cross section at the rest of the scan energies. Within this hypothesis the 4010 and 4015 MeV data for these channels are to be removed from the fit, and the remaining data are to be fit within the previously described model. At this point we cannot assess from the available data if the suggested resonance has any significant coupling to the $D^*\bar{D}$ or $D_s\bar{D}_s$ channels. In the fit we have made the simplest assumption that such coupling can be neglected and retained all the data for the latter channels in the fit. In this way we find that the quality of the fit to the described subset of data is quite acceptable: $\chi^2/N_{\text{dof}} = 3.0/5$, with the central values of the resonance parameters: $W_0 = 4019$ MeV, $\Gamma_0 = 65$ MeV and $w = 9.85$ MeV. The plots corresponding to this fit are shown in Fig. 1.

![Figure 1: The plots of the rate coefficients $R$ corresponding to the fit with excluded data points at energy 4010 and 4015 MeV for the $D\bar{D}$ channel and at 4015 MeV for the $D^*\bar{D}^*$ channel. The excluded points are shown by filled circles.](image-url)
If the deviation of the data points at 4010 and 4015 MeV indeed implies an existence of relatively narrow resonance in addition to a broader and more prominent resonance, conventionally associated with \(\psi(4040)\)\(^3\), an interesting problem arises of explaining the coexistence of the two resonances with the same quantum numbers \(J^{PC} = 1^{--}\) but apparently with strikingly different decay properties.

One such difference, in the strength of the coupling to \(e^+e^-\), can in fact be estimated from the data. Indeed, if the production of \(D^*\bar{D}^*\) at \(E = 4030\) MeV is dominated by the resonance \(\psi(4040)\), the \(e^+e^-\) width of the resonance, \(\Gamma_{ee}(4040)\), can be found from the well-known formula

\[
\Gamma_{ee} = \frac{W_0^2 \sigma(e^+e^- \rightarrow D^*\bar{D}^*)}{3\pi \Gamma_{D^*\bar{D}^*}} \left[ (E - W_0)^2 + \frac{1}{4} (\Gamma_0 + \Gamma_{D^*\bar{D}^*})^2 \right],
\]

where

\[
\Gamma_{D^*\bar{D}^*} = \frac{1}{2} \left[ \frac{(E - 2M_0)^{3/2}}{w^{1/2}} + \frac{(E - 2M_+)^{3/2}}{w^{1/2}} \right]
\]

is the (energy-dependent) width of the resonance decay into \(D^*\bar{D}^*\) pairs. Using the results of the fit for the parameters \(W_0, \Gamma_0\), and \(w\), one can readily estimate \(\Gamma_{D^*\bar{D}^*}(4030) \approx 16\) MeV, and \(\Gamma_{ee} \approx 1.7\) KeV.

The coupling to the \(e^+e^-\) channel of the suggested new resonance \(X\) can be very approximately estimated from a formula, similar to Eq. (12) applied at the maximum of the resonance:

\[
\mathcal{B}_{ee}(X) = \frac{M_X^2 \sigma(e^+e^- \rightarrow X)_{\text{max}}}{12\pi}.
\]

An estimate of the cross section, associated with the hypothetical new resonance is naturally uncertain. The deviation of the data point at 4015 MeV in the \(D^*\bar{D}^*\) channel is comparable to the error, and is about 0.15 of the already small (due to very small phase space) cross section \(\sigma(e^+e^- \rightarrow D^*\bar{D}^*) \approx 0.15\) nb. Therefore only about 0.02 of the \(e^+e^-\) annihilation cross section at this energy can be associated with the \(D^{*0}\bar{D}^{*0}\) channel. On the other hand in the \(D\bar{D}\) channel the cross section has a minimum at 4015 MeV, apparently due to a destructive interference with the background. Thus, if the ‘would be cross section’ \(\sigma(e^+e^- \rightarrow X \rightarrow D\bar{D})\) is estimated as the depth of this minimum (i.e. assuming maximal negative interference): \(\sigma(e^+e^- \rightarrow X \rightarrow D\bar{D}) \sim 0.1\) nb, one can arrive at the estimate \(\mathcal{B}_{ee}(X) \sim 10^{-7}\). Since, as can

\(^3\)We use the PDG notation \(\psi(4040)\) for the dominant resonance, even though its ‘nominal’ position is given at 4019 MeV by the fit.
be seen from the plot for $R_1$ in Figure 1, the total width of the $X$ resonance is likely to be few MeV, one can estimate the $e^+e^-$ width of $X$ in the ballpark of few tenths of eV.

Clearly, such large ratio of the $e^+e^-$ widths (of order $10^4$) of two closely spaced resonances implies that either the mixing between them is finely tuned and results in a great cancellation of the coupling of $X$ to $e^+e^-$, or the mixing is very small: not larger than $O(0.01)$ in the amplitude. A small mixing can be due to a completely different structure of the two states. Indeed the dominant relatively broad resonance can well be a $3^3S_1$ charmonium state. The estimated $\Gamma_{ee}$ is close to that predicted for such a state in a charmonium model\cite{7} ($\Gamma_{ee} = 1.5$ KeV), although the position is somewhat off that in the model ($M(3S) = 4.11$ GeV). It can be also noted that our estimate of $\Gamma_{ee}$ as well as the model prediction is somewhat higher than the PDG value $\Gamma_{ee} = (0.75 \pm 0.15)$ KeV for $\psi(4040)$. However, given the uncertainties and that the data and their analyses are still in flux, we believe that the disagreement is not dramatic. In either case it is quite clear that the $e^+e^-$ width of $\psi(4040)$ is in the KeV range, i.e. that typical of a $3^3S_1$ charmonium state. On the other hand, the new hypothetical resonance $X$ can be naturally explained as a $D^*\bar{D}^*$ $P$-wave molecule, similar to the $S$-wave $D^{*0}\bar{D}^0$ molecule $X(3872)$. In particular the small width $\Gamma_{ee}(X)$ then agrees with the expected\cite{3} $e^+e^-$ width of a $P$-wave molecule with typical size $O(1\text{ fm})$.

The considered here interpretation of the peculiar behavior of the CLEO-c scan data in the immediate vicinity of the $D^*\bar{D}^*$ threshold in terms of existence of a narrow resonance $X$ in addition to $\psi(4040)$ can be either confirmed or rejected if more detailed $e^+e^-$ data in this energy region become available. We would like to point out in this connection that it would also be extremely instructive if angular data for the $D^*\bar{D}^*$ channel were available. Namely, as previously mentioned, the vector meson pairs can be produced in two $P$-wave states with different total spin: $S = 0$ and $S = 2$. If the picture of two resonances considered here is correct, then it is highly likely that the states $X$ and $\psi(4040)$ are coupled to quite different combinations of these two spin states. The interplay between the two resonances should then result in a rapid variation with energy of a measurable angular correlation, e.g. the angular distribution of the pions from the decays $D^* \to D\pi$ with respect to the direction of the initial beams, which distribution is sensitive to both the relative absolute value and the phase of the two spin amplitudes\cite{11}.

While this work was finalized there appeared the paper\cite{12} presenting the Belle data on the cross section of the $e^+e^-$ annihilation to the $D^*\bar{D}^*$ and $D^*\bar{D}$ final states near and above 4.0 GeV. This encourages one to expect that more detailed results will become available soon,
including possibly the data on the $D\bar{D}$ annihilation channel.

This work is supported in part by the DOE grant DE-FG02-94ER40823.

References

[1] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592, 1 (2004).

[2] A. de Rujula, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 38, 317 (1977).

[3] M. B. Voloshin and L. B. Okun, JETP Lett. 23, 333 (1976).

[4] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B 71, 397 (1977).

[5] R. Poling, Invited talk at 4th Flavor Physics and CP Violation Conference (FPCP 2006), Vancouver, British Columbia, Canada, 9-12 Apr 2006. arXiv:hep-ex/0606016

[6] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) arXiv:hep-ex/0309032.

[7] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys.Rev.Lett. 36, 500 (1976); K. D. Lane and E. Eichten, Phys.Rev.Lett. 37, 477 (1976) [Erratum-ibid. 37, 1105 (1976)]. E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys.Rev.D 17, 3090 (1978) [Erratum-ibid. D 21, 313 (1980)]; E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys.Rev.D 21, 203 (1980).

[8] M. B. Voloshin, Minnesota FTPI report TPI-MINN-06-22-T, UMN-TH-2508-06, Jan 2006. arXiv:hep-ph/0602233.

[9] M. B. Voloshin, Mod. Phys. Lett. A 18, 1783 (2003) arXiv:hep-ph/0301076. M. B. Voloshin, Phys. Atom. Nucl. 68, 771 (2005) [Yad. Fiz. 68, 804 (2005)] arXiv:hep-ph/0402171.

[10] L. D. Landau and E. M. Lifshits, Quantum Mechanics (Non-relativistic Theory), Third Edition, Pergamon, Oxford, 1977. Sect. 36.

[11] M. B. Voloshin, Invited talk at 4th Flavor Physics and CP Violation Conference (FPCP 2006), Vancouver, British Columbia, Canada, 9-12 Apr 2006. arXiv:hep-ph/0605063.

[12] K. Abe et.al. [Belle Collaboration], August 2006, arXiv:hep-ex/0608018