Charmonium decay in the $C^3P_0$ Model

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In this work, we use the $C^3P_0$ model to calculate the decay widths of the low lying charmonium $J^{PC} = 1^{--}$ states, nominally $J/\psi(1S)$ and $\psi(2S)$, in the following common channels: $\rho \pi$, $\omega \eta$, $\omega \eta'$, $K^{*+} K^-$, $K^{*0} \bar{K}^0$, $\phi \eta$, $\phi \eta'$.

KEYWORDS:
Charmonium decay; $C^3P_0$ model; Fock-Tani formalism.

1 | INTRODUCTION

One of the main challenges of hadron physics today is the search for exotic excitations. In this direction the PANDA Experiment will be one of the future experiments to probe hadron matter with this goal. Located at the Facility for Antiproton and Ion Research (FAIR), PANDA is an acronym of antiProton ANnihilation at DArmstadt in Darmstadt, Germany [FAIR-PANDA 2020].

Many of the recent observations have been made in the charmonium energy regime. For example, the $\chi_{c1}(3872)$, $\psi(4260)$ and $Z_c(4430)$ states exhibit unexpected exotic properties. While the $\chi_{c1}(3872)$ and $Z_c(4430)$ could be multiquark states, the $\psi(4260)$ is a candidate for being a genuine $c\bar{c}g$ charmonium hybrid [Godfrey & Olsen 2008]. Concerning open charm and charm strange systems for many of the recent observations like e.g. the $D_{s0}^*(2317)\pm$ or $D_{s1}^*(2460)\pm$ the internal structure is still unknown.

The charmonium system, since its discovery in 1974 (Aubert & et al. 1974; Augustin & et al. 1974), has become a reference in the study of meson spectroscopy. The experimentally clear spectrum of relatively narrow states below the open-charm $DD$ threshold of 3.73 GeV can be identified with the $1S$, $1P$, and $2S$ $c\bar{c}$ levels predicted by potential models, which incorporate a color Coulomb term at short distances and a linear scalar confining term at large distances (Barnes, Godfrey, & Swanson 2005).

In the present work, we shall concentrate on the low lying charmonium mesons, which are the $J/\psi(1S)$ and $\psi(2S)$ states. We employ a mapping technique in order to obtain an effective interaction for meson decay. A particular mapping technique long used in atomic physics, the Fock-Tani formalism (FTf), has been applied to hadron-hadron scattering interactions with constituent interchange (Hadjimichef, Krein, Szpigel, & da Veiga 1998). This technique has been extended in order to include meson decay (da Silva, da Silva, de Quadros, & Hadjimichef 2008).

In summary, starting with a microscopic $q\bar{q}$ pair-creation interaction, in the leading order, the $^3P_0$ results are reproduced. In the NLO corrections appear due to the bound-state nature of the mesons that modifies $q\bar{q}$ interaction strength, which is the $^1S_0$ model.

2 | FOCK-TANI FORMALISM AND THE $C^3P_0$ MODEL

This section reviews the formal aspects of the mapping procedure and how it is implemented to quark-antiquark meson states (Hadjimichef, Krein, Szpigel, & da Veiga 1998; Hadjimichef et al. 1998). In the Fock-Tani formalism one starts with the Fock representation of the system using field operators of elementary constituents which satisfy canonical
(anti) commutation relations. Composite-particle field operators are linear combinations of the elementary-particle operators and do not generally satisfy canonical (anti) commutation relations. “Ideal” field operators acting on an enlarged Fock space are then introduced in close correspondence with the composite ones. Next, a given unitary transformation, which transforms the single composite states into single ideal states, is introduced. Application of the unitary operator on the microscopic Hamiltonian, or on other hermitian operators expressed in terms of the elementary constituent field operators, gives equivalent operators which contain the ideal field operators. The effective Hamiltonian in the new representation has a clear microscopic Hamiltonian, or on other hermitian operators expressed in terms of the quark operators. Therefore, in order to obtain the operators in the new representation, one writes expressed in terms of the quark operators. In one involves only ideal meson operators, and where the first term involves only quark operators, the second term involves only antiquark operators, the third term describes the quark and antiquark operators q̄q̄ and q̄q̄ are kinematically independent of the m̄q and m̄q

\[
\{ q_\mu, m_\beta \} = [ q_\mu, m_\beta ] = [ \bar{q}_\mu, m_\alpha ] = [ \bar{q}_\mu, m_\alpha^+ ] = 0.
\]

The unitary operator U of the transformation is

\[
U(t) = \exp \{ i \vec{F} \},
\]

where F is the generator of the transformation and t is a parameter which is set to \(-\pi/2\) to implement the mapping. The generator F of the transformation is

\[
\vec{F} = m_\alpha^+ \vec{\hat{M}}_\alpha - \vec{\hat{M}}_\alpha^+ m_\alpha
\]

with

\[
[ \vec{M}_\alpha, \vec{M}_\beta^+ ] = \delta_{\alpha\beta} + \mathcal{O}(\Phi^{\nu+1}),
\]

\[
[ \vec{M}_\alpha, \vec{M}_\beta ] = [ \vec{M}_\alpha^+, \vec{M}_\beta^+ ] = 0.
\]

It is easy to see from (10) that \( F^+ = -F \) which ensures that U is unitary. The index i in (11) represents the order of the expansion in powers of the wave function \( \Phi \). The \( \vec{M}_\alpha \) operator is determined up to a specific order n consistent with (12).

The next step is to obtain the transformed operators in the new representation. The basic operators of the model are expressed in terms of the quark operators. Therefore, in order to obtain the operators in the new representation, one writes

\[
q(t) = U^{-1} q U, \quad \bar{q}(t) = U^{-1} \bar{q} U.
\]

Once a microscopic interaction Hamiltonian \( H_I \) is defined, at the quark level, a new transformed Hamiltonian can be obtained. This effective interaction, the "Fock-Tani Hamiltonian (\( H_{FT} \)), is obtained by the application of the unitary operator U on the microscopic Hamiltonian \( H_I \), i.e., \( H_{FT} = U^{-1} \hat{H}_I U \).

The transformed Hamiltonian describes all possible processes involving mesons and quarks. The general structure of \( H_{FT} \) is of the following form

\[
H_{FT} = H_q + H_m + H_{mq},
\]

where the first term involves only quark operators, the second one involves only ideal meson operators, and \( H_{mq} \) involves quark and meson operators. In \( H_{FT} \) there are higher order terms that provide bound-state corrections to the lower order ones.
The basic quantity for these corrections is the bound-state kernel (BSK) $\Delta$ defined as
\[
\Delta(\rho; \lambda) = \Phi^\rho \Phi^\lambda.
\] (15)
The physical meaning of the $\Delta$ kernel becomes evident, in the sense that it modifies the quark-antiquark interaction strength (da Silva et al., 2008, Hadjimichef et al., 1998).

In the present calculation, the microscopic interaction Hamiltonian is a pair creation Hamiltonian $H_{q\bar{q}}$ defined as
\[
H_{q\bar{q}} = V_{\mu\nu} \bar{q}_\mu \gamma_\nu q_\nu,
\] (16)
where in (16) a sum/integration is again implied over repeated indexes (da Silva et al., 2008). The pair creation potential $V_{\mu\nu}$ is given by
\[
V_{\mu\nu} = g \delta_{c,\gamma} \delta_{\mu,\gamma} \delta(\vec{p}_\mu + \vec{p}_\nu) \vec{u}_{\nu}(\vec{p}_\mu) \vec{v}_{\mu}(\vec{p}_\nu),
\] (17)
with $g = 2 m_\gamma$ a usual choice for the decay amplitude is combined with the pair production strength and the indexes $c_\mu, \gamma_\mu$ are of color, flavor and spin. The pair production is obtained from the non-relativistic limit of $H_{q\bar{q}}$, involving Dirac quark fields (Ackleh, Barnes, & Swanson, 1996). Applying the Fock-Tani transformation to $H_{q\bar{q}}$ one obtains the effective Hamiltonian that describes a decay process. In the FTI perspective a new aspect is introduced to meson decay: bound-state corrections. The lowest order correction is one that involves only one bound state kernel $\Delta$. The bound-state corrected, $C^3 P_0$ Hamiltonian, is
\[
H_{C^3P^0} = -\Phi^\rho \Phi^\lambda \Phi^\rho \Phi^\lambda V_{C^3P^0} m^3_{\rho} m^3_{\lambda},
\] (18)
where $V_{C^3P^0}$ is a condensed notation for
\[
V_{C^3P^0} = \left( \vec{\delta} + \vec{\Delta} \right) V_{\mu\nu},
\] (19)
where
\[
\vec{\delta} = \delta_{\mu,\gamma} \delta_{\nu,\gamma} \delta_{\rho,\mu} \delta_{\lambda,\nu}
\]
\[
\vec{\Delta} = \frac{1}{4} \left[ \delta_{\nu,\gamma} \delta_{\mu,\lambda} \Delta(\rho; \lambda) + \delta_{\nu,\mu} \delta_{\lambda,\gamma} \Delta(\rho; \mu) + \delta_{\mu,\gamma} \delta_{\lambda,\nu} \Delta(\rho; \lambda) + \delta_{\mu,\nu} \delta_{\lambda,\gamma} \Delta(\rho; \lambda) + \delta_{\rho,\nu} \delta_{\lambda,\mu} \Delta(\rho; \lambda) - 2 \delta_{\rho,\gamma} \delta_{\lambda,\mu} \Delta(\rho; \lambda) \right].
\] (20)
The first term of (19), involving $\vec{\delta}$ is the usual $3 P_0$ decay potential. The following $\vec{\Delta}$ term, containing three $\Delta$‘s, is the bound-state correction to the potential. In the ideal meson space the initial and final states involve only ideal meson operators $|\rho\rangle = m^3_{\rho}|0\rangle$ and $|\phi\rangle = m^3_{\lambda}|0\rangle$. The $C^3 P_0$ amplitude is obtained by the following matrix element,
\[
\langle \phi | H_{C^3P^0} | \rho \rangle = \delta(\vec{P}_\rho - \vec{P}_\phi) H_{C^3P^0}^{\rho\phi}
\] (21)
shown in Fig. 1. The $H_{C^3P^0}$ decay amplitude is combined with relativistic phase space, resulting in the decay width
\[
\Gamma_{\rho \rightarrow \phi} = 2 \pi P \frac{E_a E_b}{M_f} \int d\Omega |H_{C^3P^0}|^2
\] (22)
which, after integration in the solid angle $\Omega$, a usual choice for the meson momenta is made: $\vec{P}_\lambda = 0$ ($P = |\vec{P}_a| = |\vec{P}_b|$).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Diagrams representing the decay amplitude of the $C^3 P_0$ model, $h_{C^3P^0}$. The diagram (a) is the usual $3 P_0$; diagrams (b), (c) and (d) correspond, respectively, to the three bound-state correction terms in Eq. (20), where the dark blue bubbles represent the bound-state kernel $\Delta(\rho; \lambda)$ insertion.}
\end{figure}

3 1 RESULTS AND CONCLUSIONS

As a testing ground for the $C^3 P_0$ model in the charm sector, we shall calculate the decay widths of the lowest charmonium $1^{--}$ states, nominally $J/\psi(1S)$ and $\psi(2S)$ in the following common channels: $\rho \pi$, $\omega \eta$, $\omega \eta'$, $K^+ K^-$, $K^{*0} K^0$, $\phi \eta$, $\phi \eta'$.

It is well known that the $c\bar{c}$ content for these decay channels are strongly suppressed by the OZI rule (Iizuka, 1966; Okubo, 1963; Zweig, 1964a, 1964b). For the $C^3 P_0$ model to be applicable to these channels we shall introduce a flavor mixing scheme. The $J/\psi(1S)$, $\phi$ and $\omega$ mesons shall be considered as the following mixture (Feldmann, Kroll, & Stecker, 1998)
\[
|\omega\rangle = |n\bar{n}\rangle - 0.06 |s\bar{s}\rangle - 1.5 \times 10^{-3} |c\bar{c}\rangle
\]
\[
|\phi\rangle = 0.06 |n\bar{n}\rangle + |s\bar{s}\rangle - 0.9 \times 10^{-3} |c\bar{c}\rangle
\]
\[
|J/\psi\rangle = 1.5 \times 10^{-3} |n\bar{n}\rangle + 0.9 |s\bar{s}\rangle + |c\bar{c}\rangle.
\] (23)

A similar mixture occurs for pseudoscalar mesons $\eta$, $\eta'$ and $\eta_c$ (Feldmann, Kroll, & Stecker, 1998)
\[
|\eta\rangle = 0.77 |n\bar{n}\rangle - 0.63 |s\bar{s}\rangle - 0.006 |c\bar{c}\rangle
\]
\[
|\eta'\rangle = 0.63 |n\bar{n}\rangle + 0.77 |s\bar{s}\rangle - 0.016 |c\bar{c}\rangle
\]
\[
|\eta_c\rangle = 0.015 |n\bar{n}\rangle + 0.008 |s\bar{s}\rangle + |c\bar{c}\rangle.
\] (24)

We shall assume that the $\psi(2S)$ has the mixing as $J/\psi(1S)$.

The spatial wave functions used are of type harmonic oscillator. Therefore, we have as free parameters, beyond the coupling constant $\gamma$, the widths $\beta$’s of the Gaussians. The mesons...
This work is intended as an initial and modest guide for future experimental studies of charmed meson spectroscopy, that may become possible with the advent of PANDA Experiment, that will probe the $c\bar{c}$ sector in search of exotic states such as the $c\bar{c}g$ charmonium hybrid.

The approach presents improvements when compared to the leading order $^3P_0$ model. A detailed comparison and discussion of the $^3P_0$ model with $C^3P_0$ can be found in (de Quadros, da Silva, da Silva, & Hadjimichef, 2020). It was shown that for $\phi$ mesons, which are strange quarkonia $s\bar{s}$ states, the inclusion of the correction terms reduced the $R$ value as a clear demonstration that the bound-state correction globally improves the fit. The average difference between the predictions of $^3P_0$ and $C^3P_0$, in each individual channel, ranged from 1% to 14%. The higher differences were in the channels with lighter mesons in the final state. In the heavier channels, the leading order $^3P_0$ is dominant and the bound-state corrections represent an actual next to leading order correction. A similar effect is expected in the higher $c\bar{c}$ sector and for exotic charmonium hybrid $c\bar{c}g$ states.

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**TABLE 1** Decay width of $J/\psi(1S)$, where the experimental data is from PDG (Zyla & et al., 2020).

| $\Gamma$ | $\Gamma_{C3P0}$ (keV) | $\Gamma_{Exp}$ (keV) |
|----------|---------------------|---------------------|
| $\rho \pi$ | 1.55 | 1.57 |
| $\omega \eta$ | 0.16 | 0.16 |
| $\omega \eta'$ | 0.17 | 0.02 |
| $K^{+}K^{-}$ | 0.47 | 0.46 |
| $K^{0}\bar{K}^{0}$ | 0.48 | 0.39 |
| $\phi \eta$ | 0.08 | 0.07 |
| $\phi \eta'$ | 0.04 | 0.04 |

$R = 0.41$

**TABLE 2** Decay width of $\psi(2S)$, again the experimental data is from PDG (Zyla & et al., 2020).

| $\Gamma$ | $\Gamma_{C3P0}$ (eV) | $\Gamma_{Exp}$ (eV) |
|----------|---------------------|---------------------|
| $\rho \pi$ | 9.47 | 9.40 |
| $\omega \eta$ | 3.67 | 3.23 |
| $\omega \eta'$ | 2.23 | 9.40 |
| $K^{+}K^{-}$ | 5.06 | 5.12 |
| $K^{0}\bar{K}^{0}$ | 5.09 | 32.04 |
| $\phi \eta$ | 1.51 | 9.11 |
| $\phi \eta'$ | 3.28 | 4.52 |

$R = 1.44$

used in the bound-state kernel were the $\phi$ and $\omega$ and are allowed to mix as in Eq. (23). A particular notation is introduced for the Gaussian widths for the BSK meson: $\beta_{\phi_s}$, $\beta_{\omega_s}$, $\beta_{1/\rho_s}$, and $\beta_{3/\rho_s}$. The model was adjusted in order to minimize $R$ defined by

$$R^2 = \sum_{i=1}^{n} \left| a_i(\gamma, \beta) - 1 \right|^2$$

(25)

with $a_i(\gamma, \beta) = \Gamma_i^{th} / \Gamma_i^{exp}$.

After the simulation the best $J/\psi(1S)$ fit to the experimental data is achieved with $\gamma = 0.33$, $\beta_{\rho} = 0.182$ GeV, $\beta_{\rho} = 0.55$ GeV, $\beta_{\omega} = 0.55$ GeV, $\beta_{\omega} = 0.328$ GeV, $\beta_{\phi} = 0.348$ GeV, $\beta_{\phi} = 0.170$ GeV, $\beta_{K} = 0.4$ GeV, $\beta_{K} = 0.315$ GeV, $\beta_{\omega} = 0.340$ GeV, $\beta_{\omega} = 0.3$ GeV, $\beta_{\omega} = 0.6$ GeV and $\beta_{1/\rho} = 0.3$ GeV. These results and the experimental values are shown in table 1.

The best $\psi(2S)$ fit to the experimental data is achieved with $\gamma = 0.33$, $\beta_{\rho} = 0.042$ GeV, $\beta_{\rho} = 0.55$ GeV, $\beta_{\omega} = 0.7$ GeV, $\beta_{\omega} = 0.7$ GeV, $\beta_{\omega} = 0.8$ GeV, $\beta_{\phi} = 0.8$ GeV, $\beta_{K} = 0.47$ GeV, $\beta_{K} = 0.8$ GeV, $\beta_{\phi} = 0.9$ GeV, $\beta_{\omega} = 0.6$ GeV and $\beta_{1/\rho} = 0.042$ GeV. These results and the experimental values are shown in table 2.