Nonlinear Numerical and Analytical Assessment of the Shear Strength of RC and SFRC Beams Externally Strengthened with CFRP Sheets

Sabiha Barour,1 Abdesselam Zergua,1 Farid Bouziadi,2 Mosbeh R. Kaloo3,4 and Waleed E. El-Demerdash5

1Department of Civil Engineering, Université des Frères Mentouri, Constantine, Algeria
2Department of Civil Engineering, Laboratory of Materials Sciences and Environment, Hassiba Benbouali University of Chlef, Chlef, Algeria
3Department of Public Works Engineering, Mansoura University, Mansoura 35516, Egypt
4Department of Civil and Environmental Engineering, Incheon National University, Incheon 22012, Republic of Korea
5Department of Civil Engineering, Misr Higher Institute for Engineering and Technology, Mansoura 35516, Egypt

Correspondence should be addressed to Mosbeh R. Kaloo; mosbeh@mans.edu.eg

Received 8 January 2022; Revised 7 April 2022; Accepted 20 April 2022; Published 11 May 2022

Academic Editor: Marco Filippo Ferrotto

Copyright © 2022 Sabiha Barour et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This study investigates numerically utilizing nonlinear finite element (ANSYS software) and analytically the shear response of the Reinforced Concrete (RC) beams. Different beams are considered in the current study, such as RC, steel fibre reinforced concrete (SFRC) without web reinforcement, and RC externally reinforced in the shear zone with carbon fibre reinforced polymer (CFRP) sheets. Nonlinear finite element model (FEM) is designed to simulate the performance of the designed beams. The results of FEM are compared to experimental measurements and standard design codes (ACI 440.2R-17, FIB 14, CNR-DT200, and ACI 318-19). According to the experimental approach and nonlinear finite element, the enhancement in the load carrying capacity of SFRC beam due to CFRP strengthening decreases with a volume fraction of steel fibres of 2%. However, the effect of CFRP strengthening on the shear behaviour of RC beams was observed in increased load carrying and ultimate deflection capacities as a result of the CFRP strengthening. The results show that CFRP has a significant contribution to shear strength. At each load increment, the created model accurately reproduced the initial and progressive crack patterns. A comparison of nonlinear finite element and analytical models was conducted using the codes ACI 440.2R-17, FIB 14, CNR-DT200, and ACI 318-19. Numerically, the FEM results showed a high agreement with ACI 440.2R-17 standard code, with correlation approach to 99%. The comparison experimental load capacity of beams to FEM and ACI 440.2R-17 shows that the FEM can be significantly used to estimate the shear strength of beams in the X-Y directions with simulating different scenarios of CFRP and SFRC characteristics. The discrepancy between the nonlinear FEA and the theoretical predictions from the ACI 440.2R-17 code is less than 1%, from the FIB14 code is less than 2%, from the CNR-DT200 code is less than 15%, and from the ACI 318-19 code is less than 30%. The ultimate load capacity evaluated based on ACI 440.2R-17 code provision shows a good agreement with the experimental data as compared to the others’ code provision. The results of the finite element analysis and analytical models were in good agreement with the experimental results. The most significant advantage of finite element analysis over experimental approaches was that it can aid in the investigation of different output results that cannot be measured experimentally, such as shear stress in the XY direction throughout the beam strengthened in shear with different CFRP properties and steel fibre reinforced concrete (SFRC).
1. Introduction

Although concrete has a low tensile strength compared to that performance under compressive loads, additional materials, such as steel and carbon fibres, can significantly improve the mechanical properties in both tensile and compressive strengths [1, 2]. Fibres are used to resist microcrack development, as well as the ductility and tensile strength of concrete characteristics [3]. The experimental tests are commonly used to assess the performance of concrete properties. Many researchers implemented different tests in evaluating the performance of beams that contain steel fibre reinforced concrete (SFRC) or/and RC externally reinforced in the shear zone with fibre reinforced polymer (FRP) sheets beams [4–6].

Cho and Kim [7] used SFRC to improve stiffness, ultimate strength, and ductility of beams that failed in shear–flexural failure. Biolozi and Cattaneo [8] evaluated the shear-flexural response of SFRC beams with longitudinal and transverse reinforcement in the form of monotonic behaviour under a four-point bending test. They found that the SFRC can be significantly used to improve the shear and bending strengths of RC beams. Also, SFRC improves ductility and stiffness of RC beams. Ranjan Sahoo and Sharma [9] evaluated the impact of SFRC in improving the flexural and shear deficiency and ductility of the flexural members; and the results showed that behaviour is high in improving these characteristics. On the other hand, narrow cracks were detected when using additional steel [10]. Tiberti [10] also concluded that the fibres significantly improve the tension stiffening of RC beams. Zhang [11] performed threepoint bending tests on SFRC notched beams. The results showed that as the loading rate increased, the fracture energy and peak load increased. Despite the improved mechanical properties of concrete when using the steel fibres, more investigations are still required for further understanding of the characteristics of mechanical properties and cracks development of RC beams. In the current study, the performance of RC beams under different load factors is evaluated. The beams’ conditions due to structures purposes are proposed. The advantages of using fibre reinforced polymers (FRP) can be summarized as follows: improving the tensile strength, decreasing the weight of concrete, easy and quick application even in confined spaces, etc. Here, externally bonded (EB) laminates, near-surface mounted (NSM) bars/strips, mechanical anchorage systems, or grooving methods with or without adhesives are commonly used to reinforce the RC beams [12, 13]. However, previous CFRP experimental studies demonstrated the effectiveness of using externally bonded CFRP composites to improve the shear strength of RC structural beams. CFRP composite materials increase the strength and ductility of the structural elements [14–16].

Adhikary and Mutsuyoshi [17] evaluated the effect of CFRP on shear strength of RC beams; various wrapping schemes were used and evaluated. The most effectiveness on shear values was vertical U-wrap of the sheet. Furthermore, the shear capacity of beams increases compared a control beam. Belarbi [18] investigated the behaviour of full-scale bridge beams using external reinforcement with FRP sheets; the results showed that the ultimate load of the beam increases. U-shape wrapping and full wrapping beam strengthening were compared; in addition, CFRP strips perpendicular to the specimen’s axis and those replaced with strips placed at an angle of 45° were evaluated. The results showed that the full wrapping technique increased load capacity of beams; furthermore, the failure mechanism changed from brittle shear failure to the desirable flexural failure [19].

Despite the experimental works commonly used in the evaluation of RC members, the cost and time consumption affect the quality and processing evaluation of materials and elements of RC; for that, the studies that focused on using high-advanced materials are almost limited. Nowadays, FEM are applied to overcome these problems. Bangash [20] evaluated structural behaviour using a numerical approach based on FEM. Wolanski [21] simulated the flexural failure of RC and prestressed concrete beams through FEM. Islam [22] studied the strength and load-deflection relationship of the SFRC beams. Three types of beams were modelled, i.e., single shear, double shear, and flexural shear; the results showed that the failure mechanism of a reinforced concrete beam is well modelled using FEA, and the predicted failure load is very close to the measured failure load during experimental testing. Dahmani [23] designed a FEM for detecting the crack propagation in RC beams. They discovered a high level of agreement between experimental and numerical results. The FEM has also been used in recent decades to determine the overall behaviour of structures, and the use of numerical models aids in developing a good understanding of the behaviour and carrying out parametric studies at a lower cost [24–27].

Meanwhile, FEM are applied in modelling the performance of beams strengthened by fibres materials. For instance, Sasimal [28] used FE ANSYS software in modelling the performance of RC beams externally strengthened with a single layer of FRP sheet; they suggested that using the FRP sheets with the shell element (SHELL181) can improve the performance of RC beams to resist the shear behaviour. Also, using multilayered FRP could improve the results. The epoxy matrix was modelled using linear solid elements (SOLID185) using FEA to investigate the behaviour of RC beams reinforced with FRP laminates by Jayajothi [29]. In their study, flexural reinforcement was achieved by attaching FRP laminates to the bottom of the RC beam and shear reinforcement was attained at the support section using U-wraps [29]. They studied the initial cracking of the RC beam, the yielding of steel reinforcement, and the strength limit state of the beam. They found a difference in behaviour between the RC beams strengthened with and without CFRP layers. In addition, Harirar and Kulkarni [30] used the ANSYS software to simulate the performances of reinforced RC beams with CFRP sheets, and their results were highly correlated with their experimental results. Elaysian [31] and Barbour and Zergua [32] investigated numerically the shear zone behaviour of RC beams externally reinforced with FRP, and the results were consistent with the previous published results. Hawileh [33] created 3D finite element models to
simulate the response and performance of RC beams externally reinforced with short-length CFRP plates, and the simulation results highly agreed with the measured experimental data.

Therefore, the present investigation aims to study numerically the shear behaviour of reinforced concrete and steel fibre reinforced concrete beams. Without stirrups and externally strengthened are used in the shear zone with carbon fibre reinforced polymer (CFRP) sheets. Static three-point bending is used in the current study. The obtained results are compared with Keskin [34] results. The main parameters investigated are the effect of concrete compressive strength, CFRP sheets, and volume fraction of steel fibres on the shear strength of beams. Crack patterns of RC beams are also presented and analysed. The comparative study between numerical and analytical models ACI440 [35], FIB14 [36], CNR-DT200 [37], and ACI 318 [38] for RC beam is also investigated.

2. Modelling and Validation of the Experimental Studies

2.1. Summary of Experimental Program. According to Keskin [34], the experimental work is designed and developed in the current study. Three-point bending is used to study the shear response of reinforced concrete (RC) beams. Different types of beams are considered: RC beams, steel fibre reinforced concrete (SFRC) beams without shear reinforcement, and RC externally strengthened in the shear zone using carbon fibre reinforced polymer (CFRP) sheets.

Four RC beams with rectangular cross section dimensions of 150 × 230 mm and a total length of 1400 mm are used. Figure 1 depicts the geometry, longitudinal reinforcement arrangement, and configuration of CFRP wraps of one of the beams. The beam span is 1000 mm from support to support. Table 1 summarizes the properties of the test beams studied in the current study.

2.2. Three-Dimension (3D) Nonlinear FEM. The performance of RC and SFRC beams without shear reinforcement (stirrups or ties) and RC externally strengthened in shear zone by CFRP is investigated using the nonlinear ANSYS FEM [39]. The model is designed to estimate the performance of beams in the experimental testing. A 3D nonlinear FEM is also applied to predict the complete response of RC beams (displacements, strains and stresses, ultimate shear loads, failure modes, cracking pattern, and so on). The FEM of beam A2.5RC10/10 is demonstrated in Figure 2. The design theory of the material used is presented as follows.

2.2.1. Concrete. In ANSYS program [39], the concrete solid element SOLID65 is commonly used to model the 3D behaviour of concrete with or without reinforcing bars (rebars), as presented in Figure 3. The element is defined by eight nodes; each comprises three translational degrees of freedom ($u_x$, $u_y$, and $u_z$). The solid element is capable of cracking in tension and crushing in compression [39, 40].

The material properties of concrete are determined through experimental testing and presented in Table 2. In the current study, two compressive strengths are considered: 39 MPa and 21.43 MPa, and the Poisson ratio is assumed 0.2. The input of the used material is considered according to American Concrete Institute (ACI) [41]. The elastic modulus, ultimate uniaxial compressive strength $f_c$, and ultimate uniaxial tensile strength (modulus of rupture, $f_r = 0.62 \sqrt{f_c}$) are determined based on ACI.

To simulate the nonlinear behaviour of the designed elements, elastic modulus, uniaxial compressive strength, uniaxial tensile strength, Poisson’s ratio, and two required shear transfer coefficients for an open crack $\beta_1$ and a closed crack $\beta_2$ are considered as input values. These coefficients range from 0 for a smooth crack to 1 for a rough crack. The values used for $\beta_1$ and $\beta_2$ are 0.3 and 0.9, respectively. Figure 3 depicts the steel fibres as rebars. They are represented by a smeared model, which assumes that reinforcement is distributed throughout the concrete elements in a defined region of the FEM meshes [42].

This model requires the volume ratio (VR) and the orientation angles (THETA, PHI), which correspond to the volume fraction of steel fibres (VF) and the orientation factor (α). Three cases are defined for the orientation factor [42]:

(i) One-dimensional (1D) case: In this case, all fibres are aligned in one direction. The orientation coefficient is 1s in the plane perpendicular to the fibre alignment direction and 0 in the other two orthogonal planes.

(ii) Two-dimensional (2D) case: The fibres are all aligned in parallel planes in this case. Two of the three orthogonal planes should have the same number of fibres, but the third does not. Its value in this case is 0.637.

(iii) Three-dimensional (3D) case: The orientation of fibres is restricted. The number of cut fibres in three orthogonal planes is the same. The value ranges from 0.405 to 0.5.

2.2.2. Steel Reinforcement. Figure 4 illustrates the LINK180 element, which is used to model the longitudinal reinforcements. Each node has three degrees of freedom (DOF): translations in the nodal X, Y, and Z directions. Plasticity, creep, rotation, large deflection, and large strain are all supported.

The steel bar is considered to be an elastic-plastic behaviour with strain hardening and with identical tensile and compressive behaviour [39, 44, 45]. The LINK180 element is assumed to be bilinear isotropic and identical in tension and compression. Bilinear isotropic material is assumed based on the Von-Misses failure criteria. For bilinear isotropic hardening model of LINK180 element, with the specified yield stress, the stress-strain curve of reinforcement continues along the second slope defined by the tangent modulus, as presented in Table 3. Also, the real constants for this model are shown in Table 4.
2.2.3. Loading and Supporting Steel Plates. In order to avoid stress concentration problems, the SOLID185 element is used for modelling the loading and supporting steel plates as elastic linear isotropic material, as presented in Figure 5. Eight nodes are assumed for the SOLID185; in the current study, each is designed with three DOF and translations of each are in the X, Y, and Z directions. SOLID185 has plasticity, hyperelasticity, constraint stiffness, creep, significant deflection, and large strain capabilities [39, 46].

Elastic modulus and Poisson’s ratio of steel plates are 200 GPa and 0.3, respectively. The material parameters of steel plates utilized for numerical simulation using the ANSYS FEA programme are shown in Table 5.

Elastic modulus and Poisson’s ratio of steel plates are 200 GPa and 0.3, respectively. The material parameters of steel plates utilized for numerical simulation using the ANSYS FEA programme are shown in Table 5.

The surface interaction between SOLID185 (steel plates at supports and at the point of application of load) and SOLID65 (concrete) is created using CONTA174 and TARGET170 elements. A surface-to-surface contact is created by considering the applied load and support as a rigid element [47].

2.2.4. Carbon Fibre-Reinforced Polymer (CFRP). The CFRP sheets are modelled using the SHELL181 element; see Figure 6. It is a four-node element with six DOFs at each node, with translations in the X, Y, and Z directions and rotations about the X, Y, and Z. SHELL181 is well-suited for linear and large rotation applications, as well as nonlinear applications with high strain [38].

Tensile strength, elasticity modulus, shear modulus, and Poisson’s ratio are the material properties used in the numerical study for CFRP wraps, and the determined values for these parameters are 4900 MPa, 230 GPa, 89.84 GPa, and 0.28, respectively. CFRP sheets have a layer thickness of 0.166 mm. The various parameters of CFRP sheets that are used into the NLFE ANSYS software are the number of layers (one layer) and the fibre orientation layer (0°). Tensile behaviour of FRP materials composed of a single fibre material is characterised by a linearly elastic stress-strain relationship until failure. Figure 7 depicts this relationship by Al-Zaid [49].

According to previous investigations [29, 33], a perfect bond is considered to model the interaction between the concrete surface and CFRP. Since debonding is not observed between the concrete and CFRP (during the experimental study), a perfect bond approach is adopted for this numerical study, which is the same approach used for contact between the concrete and steel reinforcement. The epoxy adhesive is also modelled using the SOLID185 element. The perfect bonding is assumed to model the interface between the CFRP and the adhesive material. The concrete-epoxy adhesive interface is also assumed to be perfectly bonded [47].

3. FEM Results and Discussion

3.1. Validation of the FE Model. Figure 8 depicts a comparison of the load-deflection behaviour of control and SFRC beams (see Table 1) without web reinforcement externally bonded without and with CFRP sheets. Here, the midspan deflection of FEM and experimental results in nonlinear evaluation is presented to verify the results of the numerical model. From the figure, it can be seen that the agreement between FEM and experimental measurements is high until the failure mode. In addition, the additional steel fibres in the RC beam (A2.5F2.0) increase the shear strength of the beams when compared to the control beam (A2.5 R). This is because the inclusion of steel fibres in concrete increases the bond in the concrete matrix.
Furthermore, it is shown that the ultimate load in RC beams reinforced with FRP sheets (A2.5RC10/10) increases when compared to SFRC beams without web reinforcement externally bonded without and with CFRP sheets beam (A2.5 F 2.0C10/10). The agreement of these results can be seen in both experimental and FEM results.
Table 2: Material properties of concrete in the proposed FEM.

| Material model number | Element type | Material properties |
|-----------------------|--------------|---------------------|
|                       |              | Linear isotropic    |
|                       |              | EX 28251 MPa        |
|                       |              | PRXY 0.2            |
|                       |              | Multilinear isotropic |
|                       |              | Point strain stress (MPa) |
| 1                     | Solid65      | 0                    |
|                       |              | 0.0005244           |
|                       |              | 0.00065             |
|                       |              | 0.0008              |
|                       |              | 0.001               |
|                       |              | 0.0012              |
|                       |              | 0.0014              |
|                       |              | 0.0016              |
|                       |              | 0.0018              |
|                       |              | 0.002               |
|                       |              | 0.0022              |
|                       |              | 0.0026574           |
|                       |              | 0.003               |
|                       |              | Open shear Coef. transfer 0.3 |
|                       |              | Closed shear Transfer coef. 0.9 |
|                       |              | Uniaxial cracking Stress 3.87 |
|                       |              | Uniaxial crushing Stress 39 |
|                       |              | Biaxial crushing Stress 0 |
|                       |              | Hydostatic Pressure 0 |
|                       |              | Hydro biax Crushing stress 0 |
|                       |              | Tensile crack factor 0 |

Figure 4: LINK180-3D element bars [39, 43].

Table 3: Material properties of steel reinforcement FEM.

| Material model number | Element type | Material properties |
|-----------------------|--------------|---------------------|
|                       |              | Linear isotropic    |
|                       |              | Elasticity models, EX 200 000 MPa |
|                       |              | Poisson’s ratio, PRXY 0.3 |
| 2                     | LINK180      | Bilinear isotropic  |
|                       |              | Yield stress 420 MPa |
|                       |              | Ultimate stress 550 MPa |

Table 4: Element type and real constants used for FEM of steel reinforcement.

| Real constant set | Element type | Real constants |
|-------------------|--------------|----------------|
| 2                 | LINK180      | Cross-sectional area (mm²) 200.96 |
Because the bond between the concrete and reinforcement steel is assumed to be acceptable in the FEM, this means the FEM can be used for nonlinear behaviour of beams. Table 6 illustrates the comparison of the differences between experimental measurements and FEM results for the ultimate load and corresponding midspan deflection. The results show that the percentage difference between predicted failure loads and obtained experimental data ranges from 15.47% to 0.43%. The percentage difference between predicted midspan deflection at failure load and obtained experimental data ranges from 9.90% to 0.049%. The failure load is predicted by the FEMs to be less than 16% different, while the midspan deflection is predicted to be less than 10% different. Thus, the nonlinear finite element method using ANSYS software is capable of precisely predicting the shear response of the reinforced concrete and steel fibre reinforced concrete beams without any web reinforcement, which is externally strengthened in the shear zone using carbon fibre reinforced polymer sheets and subjected to static three-point bending. This indicates the validity and reliability of the proposed FEMs are acceptable and can be used in nonlinear evaluation of beam’s behaviours.

Figure 9 presents the comparison of the ultimate experimental and ultimate numerical midspan load; meanwhile Figure 10 shows the comparison of the ultimate experimental and ultimate numerical midspan deflection for all beams. From both figures, it can be seen that the agreement between experimental and numerical results for maximum deflections and maximum loads is high. The statistical analysis using $t$-test depicted that there is no significant ($P < 0.05$) difference between experimental and numerical values.

Furthermore, Tables 7 and 8 present the change in failure and deflection of beams. Here, the change is determined between the experimental measurements and FEMs results. All of the strengthened RC beams behave stiffer than the unstrengthened RC beams, resulting in a change in the deflection and failure load values shown in Tables 7 and 8.

### 3.2. Evolution of FE Crack Patterns

The FE ANSYS software can record the configuration of cracks in concrete at each loading step until the failure mode of specimens. The cracks appear when the concrete reaches its ultimate tensile strength. As shown in Figure 11(a), fine vertical flexural cracks appear around the midspan of all beams during the early stages of loading. However, as load increases, new flexural cracks are observed far from the midspan area, and these vertical flexural cracks appearing around the midspan begin to extend towards the loading point, as shown in Figures 11(b) and 11(c).

Figures 12 and 13 show the crack patterns of beam A2.5RC10/10 for the experimental and numerical approaches, respectively. Also, Figures 14 and 15 show the crack patterns of beam A2.5RC10/10 for the experimental and numerical approaches, respectively. The failure pattern obtained from FEM clearly agrees with the experimental data. The nonlinear FEM by ANSYS programme displays circles at cracking or crushing locations in concrete elements. The first crack at the integration point is indicated by a red circle outline, the second by a green outline, and the

---

**Table 5: Material properties used for FE of steel plates.**

| Material model number | Element type | Material properties |
|-----------------------|--------------|---------------------|
| 3                     | SOLID185     | Linear isotropic EX 200000 MPa PRXY 0.3 |

**Figure 5: SOLID185 3D element [39, 44].**

**Figure 6: SHELL181 element [39, 48].**

**Figure 7: Stress-strain relationships for steel reinforcement and FRP composite materials.**

Because the bond between the concrete and reinforcement steel is assumed to be acceptable in the FEM, this means the FEM can be used for nonlinear behaviour of
In general, the mode of failure and crack pattern evolution observed from the analysis agrees well with that of laboratory tested beams.

3.3. FEM Behaviour. Nonlinear FEMs were proven to be trustworthy [39], based on experimental measurements. Number of strain gauges or linear variable differential transducers (LVDTs) at a few points in the beam is usually used to measure the experimental strain and deflection. However, the advantage of FEM over experimental approaches is that FEM provides displacement throughout the beam for the entire loading. Figure 16 illustrates the displacement at failure in the Y direction for beams A2.5RC10/10 and A2.5F2.0C10/10. In addition, for beams A2.5RC10/10 and A2.5F2.0C10/10, Figures 17(a) and 17(b)

| Beams          | Ultimate load (kN) | Difference (%) | Deflection at failure load (mm) | Difference (%) |
|----------------|-------------------|---------------|--------------------------------|---------------|
|                | Exp. | Num. | (Exp. − Num.)/Exp. | Exp. | Num. | (Exp. − Num.)/Exp. |               |
| A2.5 R         | 82.64 | 82.28 | +0.43               | 2.011 | 2.010 | +0.049              |               |
| A2.5F2.0       | 100.37 | 96.04 | +4.31               | 5.017 | 5     | +0.29               |               |
| A2.5RC10/10    | 178.51 | 181.28 | −1.55               | 17.09 | 16.73 | +2.10               |               |
| A2.5F2.0C10/10 | 135.77 | 114.75 | +15.47              | 6.24  | 5.62  | +9.90               |               |

Table 6: Numerical model validation.
Figure 9: Comparison between ultimate experimental and ultimate numerical midspan load for all beams.

Figure 10: Comparison between ultimate experimental and ultimate numerical midspan deflection for all beams.

Table 7: The percentage change in failure load values obtained from the experimental approach and numerical simulation.

| Beam          | Change of failure load in the experiment (%) | Change of failure load in the numerical (%) |
|---------------|---------------------------------------------|--------------------------------------------|
| A2.5 R        | —                                           | —                                          |
| A2.5F2.0      | 17.66                                       | 14.32                                      |
| A2.5RC10/10   | 53.70                                       | 54.31                                      |
| A2.5F2.0C10/10| 39.13                                       | 28.29                                      |

Table 8: The percentage change in deflection values obtained from the experimental approach and numerical simulation.

| Beam          | Change of deflection in the experiment (%) | Change of deflection load in the numerical (%) |
|---------------|--------------------------------------------|-----------------------------------------------|
| A2.5 R        | —                                           | —                                            |
| A2.5F2.0      | 59.91                                       | 59.8                                         |
| A2.5RC10/10   | 88.23                                       | 87.98                                        |
| A2.5F2.0C10/10| 67.77                                       | 64.23                                        |
depict the history of deflections throughout loading steps from starting to failure.

Figure 18 depicts the shear stress in the XY direction throughout the beam A2.5RC10/10. The value of maximum shear stress in the XY direction can be found in the CFRP sheets part of this diagram. As a result, the FEM demonstrates the efficacy of CFR composites in improving the shear strength of RC beams.

4. Prediction of Ultimate Load Using Analytical Models

In this section, the estimated ultimate load of an RC externally bonded with CFRP beam (A2.5RC10/10) subjected to three-point bending is numerically evaluated in this section utilizing theoretical models from standard codes ACI440 [35], FIB14 [36], and CNR-DT200 [37]. This step is applied to verify the FEMs results. Table 9 shows the mechanical properties of concrete and CFRP materials sheets used in the analytical approach.

4.1. ACI440 [35]. The American Code for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures [35] is a reference for the design and construction of externally bonded FRP systems for strengthening concrete structures. The equations algorithm below is used to predict the ultimate load \( P_n \) of RC beams strengthened in shear zone with CFRP strips using code of (ACI 440, 2017) for beam A2.5RC10/10. The recommended additional reduction factors for FRP shear reinforcement \( \psi_f \) equals 0.95 for completely wrapped members:

\[
V_n = V_c + V_s + \psi_f V_f. \tag{1}
\]

Nominal shear strength provided by concrete with steel flexural reinforcement,

\[
V_c = 0.17 \sqrt{f'_c b_w d}.
\]

Area of FRP shear reinforcement with spacing,

\[
A_{f_s} = 2n f_w w_f = 33.2 \text{ mm}^2.
\]

Modification factor applied to \( K_r \) to account for concrete strength \( K_i \).
Figure 16: FE model deflections throughout the beams. (a) A2.5RC10/10. (b) A2.5F2.0C10/10.

Figure 17: History of deflections throughout loading steps from start to failure. (a) A2.5RC10/10. (b) A2.5F2.0C10/10.

\[
K_1 = \left( \frac{f_u^C}{27} \right)^{2/3} = 1.27.
\]

Active bond length of FRP laminate \( L_e \)

\[
= \frac{23300}{\left( m_t E_f \right)^{0.5\eta}} = 51.27 \text{mm}.
\]  

Modification factor applied to \( K_v \) to account for wrapping scheme, \( K_2 \)

\[
K_2 = \frac{d_{fu} - L_e}{d_{fu}} = 0.743,
\]

\[
K_v = \frac{K_1 K_2 L_e}{11900 \epsilon_{fu}} = 0.190.
\]
Effective strain level in FRP reinforcement attained at failure $\varepsilon_{fe}$

\[ \varepsilon_{fe} = K_{ve} \varepsilon_{fu} = 0.0040, \]

where $K_{ve} = 200 / (1000 + 1.3 \cdot 0.9 d) \leq 0.15$.

Nominal shear strength provided by FRP stirrups $V_f$

\[ V_f = \frac{A_{vf} \varepsilon_{fe} E_f}{S_f} = 61.08 \text{kN}, \]

where $A_{vf} = \frac{2 t_f}{b_f S_f}$.

\[ V_n = V_c + V_s + \Psi_f V_f = 89.86 \text{kN}. \]

4.2. FIB14 [35]. The FIB14 code [36] is used to predict the ultimate load ($P_n$) of an RC beam that is externally bonded with FRP materials. According to code of FIB14, the following design guidelines are recommended to calculate the ultimate load ($P_n$) of RC beam strengthened with CFRP in shear zone:

The shear capacity of a strengthened element, $V_{Rd}$

\[ V_{Rd} = V_{c,d} + V_{w,d} + V_{f,d}. \]

Table 9: Mechanical properties for concrete and CFRP composites materials.

| Property                  | Value       |
|---------------------------|-------------|
| Specified compressive strength of concrete $f'_c$ | 39 MPa       |
| Elastic modulus of CFRP $E_f$ | 230 GPa     |
| Thickness of CFRP layer $t_f$ | 0.166 mm    |

4.2.1. Concrete Contribution to Shear Capacity.

\[ V_{Rd,c} = V_{c,d} = k_v \cdot \frac{\sqrt{f'_{ck} b_w}}{y_c} \cdot 0.9 \cdot d, \]

where $\psi = 0.00221$.

4.2.2. FRP Contribution to Shear Capacity.

\[ V_{f,d} = 0.9 \varepsilon_{fe} E_f \rho_f \frac{b_w d (\cot \theta + \cot \alpha) \sin \alpha}{S_f}, \]

where $\theta$ is angle of diagonal crack with respect to the member axis, assumed equal to 45°; $\alpha$ is angle between principal fibre orientation and longitudinal axis of member.

For FRP reinforcement in the form of strips or sheets of width $b_w$ at spacing $S_f$:

\[ \rho_f = \left( \frac{2 t_f}{b_w} \right) \left( \frac{S_f}{b_f} \right), \]

where $\rho_f = 0.00221$.

The design value of the effective FRP strain in the principal material direction is given as in the preceding analysis of shear for the case of fully wrapped FRP:
\[
\varepsilon_{fd,e} = \varepsilon_{fke} \frac{Y_f}{Y_f}.
\] (11)

Fully wrapped (or properly anchored) CFRP fracture controls
\[
\varepsilon_{fke} = 0.17 \left( \frac{f_{cm}^{2/3}}{E_{f}\rho_f} \right)^{0.3} \varepsilon_{fu},
\] (12)

where \( f_{cm} \) is the cylindrical compressive strength of concrete in MPa.

\( \varepsilon_{fke} \) is \( K \varepsilon_{fke} = 0.007376, \)

\[
\varepsilon_{fd,e} = \varepsilon_{fke} = \frac{0.007376}{1.30} = 0.0056,
\] (13)

\[
V_{fd} = 0.9 \varepsilon_{fd,e} E_{fd}\rho_f b_w (\cot \theta + \cot \alpha)\sin \alpha,
\]

\[\therefore V_{fd} = 76.85 \text{kN}.\]

4.3. CNR-DT200 [37]. The following equations algorithm is proposed by the Italian code CNR-DT [37] to determine the ultimate load \( (P_u) \) of the RC beams externally strengthened with FRP. The ultimate load should be calculated as follows:

\[
V_{R,d} = \min \left\{ V_{R,d,c} + V_{R,d,s} + V_{R,d,f} : V_{R,d,max} \right\},
\] (14)

where \( V_{R,d,c} \) is the concrete contribution to the shear capacity, calculated as follows:

\[
V_{R,d,c} = 0.6 f_{c,d} b d \delta,
\] (15)

where \( f_{c,d} \) is the design tensile strength of the concrete, and \( \delta = 1, b = 150 \text{mm} \) and \( d = 200 \text{mm} \).

\( V_{R,d,s} \) is the FRP contribution to the shear capacity to be evaluated as indicated in the following. In the same case of a RC member with a rectangular cross section completely wrapped configurations, the FRP contribution to the shear capacity shall be calculated as follows:

\[
V_{R,d,f} = \frac{1}{Y_{R,d}} 0.9 f_{fe,d} d t_f (\cot \theta + \cot \beta) \frac{W_f}{P_f},
\] (16)

where \( f_{fe,d} \) is the completely wrapped configurations:

\[
f_{fe,d} = f_{f dd} \left[ 1 - \frac{1}{6} \min \left\{ \frac{l_c \sin \beta}{0.9 d_{hw}} \right\} \right] + \frac{1}{2} (\phi_R f_{fd} - f_{fd}) \left[ 1 - \frac{l_c \sin \beta}{\min \{0.9 d_{hw}\}} \right],
\] (17)

where \( f_{fd} \) is the FRP design strength and to be evaluated:

\[
\phi_R = 0.2 + 1.6 \frac{r_c}{b_w}.
\] (18)

\[0 \leq \frac{r_c}{b_w} \leq 0.5.\]

\( r_c \) is the corner radius of the section to be wrapped and is equal to 30 mm,

\[
\phi_R = 0.52.\]

\( f_{f dd} \) is the bond strength of CFRP beam and calculated from the following equation:

\[
f_{f dd} = \frac{1}{Y_{f,d} Y_c} \frac{2 E_f \Gamma_{fk}}{n f_{f,1}},
\] (20)

where \( n_f \) is the number of CFRP plies. \( l_c \) is the optimal bonded length

\[
l_c = \frac{E_f t_f}{2 f_{cm}}.
\]

\( f_{cm} = 0.30 f_{ck}^{2/3} = 2.95 \text{MPa}, \)

\( E_f = 230000 \text{MPa}, \)

\( t_f = 0.166 \text{mm}, \)

\( l_c = 80.44, \)

\( \Gamma_{fk} \) is the specific fracture energy of the FRP concrete interface.
\( \Gamma_{Fk} = 0.03 \, K_b \sqrt{f_{ck} f_{ctm}}, \)

\( f_{ck} = 31 \text{MPa}, \)

\( f_{ctm} = 2.95 \text{MPa}, \)

\( K_b = \sqrt{\frac{2 - b_f/l_b}{1 + b_f/400}} \geq 1, \)

\( b_f = 150 \text{mm}, \)

\( b = 150 \text{mm}, \)

\( K_b = 0.84, \)

\( \Gamma_{Fk} = 0.28, \)

\( f_{ld} = \frac{1}{\gamma_{ld} \sqrt{\gamma_c}} \sqrt{\frac{2 \cdot E_f \cdot \Gamma_{Fk}}{n_f \cdot A_{ld}}}, \)

\( n_f = 1, \)

\( \gamma_{ld} = 1.20, \)

\( \gamma_c = 1.5, \)

\( f_{ld} = 601.41, \)

\( f_{fsd} = 601.41 \left[ 1 - \frac{1}{6} \frac{80.44 \cdot 1}{180} \right] + \frac{1}{2} \left( 0.52 \cdot 4900 - 601.41 \right) \left[ 1 - \frac{80.44 \cdot 1}{180} \right], \)

\( f_{fsd} = 1094.84, \)

\( V_{Rd} = \frac{1}{2} \gamma_{Rd} \cdot 0.9 \cdot f_{fsd} \cdot 2t_f \cdot (cot \theta + cot \beta) \cdot \frac{W_f}{P_f}, \)

\( W_f = 100 \text{mm}, \)

\( P_f = 100 \text{mm}, \)

\( \therefore V_{Rd} = 79.33 \text{kN}. \)

### 5. Verification Results

Table 10 and Figure 19 compare the FEM results with the codes predictions of the ultimate load at failure for beam A2.5RC10/10. The results show that the percentage difference is less than 1%, 2%, and 15% relative to ACI 440, FIB 14 [36], and CNR-DT, respectively. There is clear agreement between the nonlinear FEM results and the various expected CFRP codes.

### 6. Prediction of Ultimate Load for RC Beam

The estimated ultimate load of beam A2.5 R subjected to three-point bending is numerically calculated in this section utilizing theoretical models from the current ACI 318 code. Using the ACI 318 code for beam A2.5 R, the following step-by-step algorithm is used to anticipate the behaviour of RC beams.

**Step 1.** Preliminary calculations:

Area of reinforcing steel:

\[
A_s = 401.92 \text{mm}^2. \tag{23}
\]

**Step 2.** The depth of neutral axis (N.A):

\[
a = \frac{A_s f_s}{0.85 f'_c b}, \tag{24}
\]

where

\[
d = 200 \text{mm}, \]

\[
b = 150 \text{mm}, \]

\[
a = 33.94 \text{mm}. \tag{25}
\]

**Step 3.** Flexural strength components contribution of steel in flexure can be given as follows:

\[
M_{ns} = A_s f_s \left( d - \frac{a}{2} \right), \tag{26}
\]

\[
M_{ns} = 30.89 \text{kN.m}. \]

Table 11 shows the comparisons between the FEM results and the theoretical code prediction of the ultimate load at failure for beam A2.5 R.
7. Conclusion

Four nonlinear 3D FEMs are designed in this study to simulate the shear behaviour of RC and SFRC beams exposed to three-point bending without web reinforcement externally strengthened with CFRP sheet in the shear zone. Furthermore, a comparative study is implemented between the analytical models proposed by the current code of ACI 440.2R-17, FIB 14, CNR-DT2004, and ACI 318 and nonlinear FEMs. The following conclusions can be summarized:

(1) At all stages of loading until failure, there is a high correlation between the experimental and nonlinear FEM results.

(2) The developed FEMs can be used as a numerical platform to overcome the drawback of experimental testing. The developed FEMs can be used to investigate and predict the behaviour of RC and SFRC beams strengthened in shear with CFRP sheets of varying properties.

(3) The differences in nonlinear FEM and experimentally measured one range from 0.43% to 15.47% and 0.049% to 9.90% for the ultimate load carrying capacity and midspan deflection, respectively.

(4) The using of CFRP only can improve the shear behaviour of RC beams.

(5) The discrepancy between the nonlinear FEM and the theoretical predictions of standard codes shows the nonlinear FEM can be used to predict the ultimate load with an accuracy approaching 70%.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning (2019R1I1A1A01062202).

References

[1] J. Susetyo, P. Gauvreau, and F. Vecchio, "Effectiveness of steel fiber as minimum shear reinforcement," *ACI Structural Journal*, vol. 108, no. 4, pp. 488–496, 2011.

[2] J. J. Beaudoin, *Handbook of Fiber-Reinforced Concrete Principles, Properties, Developments and Applications*, Noyes Publications, Park Ridge, NJ, 1990.

[3] D. A. Fanella and A. E. Naaman, "Stress-strain properties of fibre reinforced mortar in compression," *Journal Proceedings*, vol. 82, no. 4, pp. 475–483, 1985.

[4] D. M. Özcan, A. Bayraktar, A. Şahin, T. Haktanir, and T. Türker, "Experimental and finite element analysis on the steel fibre-reinforced concrete (SFRC) beams ultimate behaviour," *Construction and Building Materials*, vol. 23, no. 2, pp. 1064–1077, 2009.
[21] A. J. Wolanski, "Flexural behaviour of reinforced and prestressed concrete beams using finite element analysis," Doctoral dissertation, Market University, Wisconsin, United States, 2004.
[22] M. M. Islam, M. S. Khatun, M. R. U. Islam, J. F. Dola, M. Hussian, and A. Siddique, "Finite element analysis of steel fiber reinforced concrete (SFRC): Validation of experimental shear capacities of beams," Procedia Engineering, vol. 90, pp. 89–95, 2014.
[23] P. Fanning, "Nonlinear models of reinforced and post-tensioned concrete beams," Electronic Journal of Structural Engineering, vol. 1, no. 2, pp. 111–119, 2001.
[24] A. M. Ibrahim and M. S. Mahmood, "Finite element modelling of reinforced concrete beams strengthened with FRP laminates," European Journal of Scientific Research, vol. 30, no. 4, pp. 526–541, 2009.
[25] P. Parandaman and M. Jayaraman, "Finite element analysis of reinforced concrete beam retrofitted with different fiber composites," Middle-East Journal of Scientific Research, vol. 22, no. 7, pp. 948–953, 2014.
[26] M. A. Musmar, M. I. Rjoub, and M. A. Hadi, "Nonlinear finite element analysis of shallow reinforced concrete beams using Solid65 element," ARPN Journal of Engineering and Applied Sciences, vol. 9, no. 2, pp. 85–89, 2014.
[27] L. Dahmani, A. Khemane, and S. Kaci, "Crack identification in reinforced concrete beams using ANSYS software," Strength of Materials, vol. 42, no. 2, pp. 232–240, 2010.
[28] S. Sasmal, S. Kalidoss, and V. Srinivas, "Nonlinear finite element analysis of FRP strengthened reinforced concrete beams," Journal of The Institution of Engineers (India): Series A, vol. 93, no. 4, pp. 241–249, 2012.
[29] P. Jayaiothi, R. Kumutha, and K. Viji, "Finite element analysis of FRP strengthened RC beams using ANSYS," Asian Journal of Civil Engineering, vol. 14, no. 4, pp. 631–642, 2013.
[30] A. S. Harihar and D. K. Kulkarni, "Finite element analysis of reinforced concrete beam strengthened with CFRP sheets," Bonfiring International Journal of Man Machine Interface, vol. 4, no. Special Issue, pp. 206–209, 2016.
[31] I. Elyasian, N. Abdoli, and H. R. Roumagh, "Evaluation of parameters effective in FRP shear strengthening of RC beams using FE method," Asian Journal of Civil Engineering (Building and Housing), vol. 7, no. 3, pp. 249–257, 2006.
[32] S. Barour, A. Zergua, F. Bouziadi, and W. Abed Jasim, "Finite element analysis of CFRP externally strengthened reinforced concrete beams subjected to three-point bending," World Journal of Engineering, vol. 17, no. 2, pp. 183–202, 2019.
[33] R. A. Hawileh, M. Z. Naser, and I. A. Abdalla, "Finite element simulation of reinforced concrete beams externally strengthened with short-length CFRP plates," Composites Part B: Engineering, vol. 45, no. 1, pp. 1722–1730, 2013.
[34] R. S. O. Keskin, G. Arslan, and K. Sengun, "Influence of CFRP on the shear strength of RC and SFRC beams," Construction and Building Materials, vol. 151, pp. 16–24, 2017.
[35] Aci, Aci 440.2-17, Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures, Farmington Hills, Michigan, US, 2017.
[36] Fibi4, "Externally bonded FRP reinforcement for RC structures. The international federation for structural concrete (CEB-FIB)," technical report bulletin 14, Sika Services AG, Switzerland, 2001.
[37] K. Soudki and T. Alkhrdaji, "Guide for the design and construction of externally bonded FRP systems for strengthening existing structures," Structures Congress, 2005.
[38] Aci (American Concrete Institute), ACI 318–19: Building Code Requirements for Structural Concrete and Commentary, American Concrete Institute, Farmington Hills, MI, 2019.

[39] Ansys release version 12, A finite element computer software and user manual for nonlinear structural analysis, ANSYS Canonsburg, Pennsylvania, 2007.

[40] N. Lahmar, F. Bouziadi, B. Boulekbache et al., “Experimental and finite element analysis of shrinkage of concrete made with recycled coarse aggregates subjected to thermal loading,” Construction and Building Materials, vol. 247, Article ID 118564, 2020.

[41] American Concrete Institute (Aci), “Building code requirements for structural concrete,” ACI 318-05ACI 318-05, Farmington Hills, Michigan, 2005.

[42] J. Wuest, “Comportement structural des bétons de fibres ultra performants en traction dans des éléments composés,” THESIS, EPFL, Lausanne, 2007.

[43] T. Tahenni, F. Bouziadi, B. Boulekbache, and S. Amziane, “Experimental and nonlinear finite element analysis of shear behaviour of reinforced concrete beams,” Structures, vol. 29, pp. 1582–1596, 2021.

[44] F. Bouziadi, B. Boulekbache, A. Haddi, C. Djelal, and M. Hamrat, “Numerical analysis of shrinkage of steel fiber reinforced high-strength concrete subjected to thermal loading,” Construction and Building Materials, vol. 181, pp. 381–393, 2018.

[45] M. Hamrat, F. Bouziadi, B. Boulekbache et al., “Experimental and numerical investigation on the deflection behavior of pre-cracked and repaired reinforced concrete beams with fiber-reinforced polymer,” Construction and Building Materials, vol. 249, Article ID 118745, 2020.

[46] F. Bouziadi, B. Boulekbache, A. Haddi, M. Hamrat, and C. Djelal, “Finite element modelling of creep behaviour of FRP-externally strengthened reinforced concrete beams,” Engineering Structures, vol. 204, Article ID 109908, 2020.

[47] N. K. Banjara, K. Ramanjaneyulu, and K. Ramanjaneyulu, “Experimental and numerical investigations on the performance evaluation of shear deficient and GFRP strengthened reinforced concrete beams,” Construction and Building Materials, vol. 137, pp. 520–534, 2017.

[48] S. Barour and A. Zergua, “Numerical analysis of reinforced concrete beams strengthened in shear using carbon fiber reinforced polymer materials,” Journal of Engineering, Design and Technology, vol. 19, no. 2, pp. 339–357, 2020.

[49] R. Z. Al-Zaid, A. I. Al-Negheimish, M. A. Al-Saawani, and A. K. El-Sayed, “Analytical study on RC beams strengthened for flexure with externally bonded FRP reinforcement,” Composites Part B: Engineering, vol. 43, no. 2, pp. 129–141, 2012.