Pattern recognition with simple oscillating circuits

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Pattern recognition

http://niceteeth.wordpress.com/2008/11/05/mona-lise-with-braces/

http://scrapetv.com/News/News%20Pages/Business/images-4/mona-lisa.jpg
Neural Networks

Biological Neural Network

Artificial Neural Network

http://www.whatthebleep.com/download/

http://en.wikibooks.org/wiki/Cyberbotics'_Robot_Curriculum/
Oscillatory Neurocomputers with Dynamic Connectivity

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Our study of thalamo-cortical systems suggests a new architecture for a neurocomputer that consists of oscillators having different frequencies and that are connected weakly via a common medium forced by an external input. Even though such oscillators are all interconnected homogeneously, the external input imposes a dynamic connectivity. We use Kuramoto’s model to illustrate the idea and to prove that such a neurocomputer has oscillatory associative properties. Then we discuss a general case. The

Interaction through a common support

Oscillators with different frequency

Time-dependent input

A oscillatory neuro computer
The Mathematical Model

\[ \vartheta_i(t) = \Omega_i t + \varphi_i \]

\[ \dot{\vartheta}_i = \Omega_i + \varepsilon a(t) \sum_{j=1}^{n} \sin(\vartheta_j - \vartheta_i) \]

suitable choice of time-dependent coupling function \( a(t) \)

weak coupling strength: \( \varepsilon << 1 \)

\[ \dot{\varphi}_i = \varepsilon \sum_{j=1}^{n} w_{ij} \sin(\varphi_j - \varphi_i) \]

Associative memory properties, similar to Hopfield network
The Mathematical Model

\[ \dot{\phi}_i = \epsilon \sum_{j=1}^{n} w_{ij} \sin(\varphi_j - \varphi_i) \]

Steady states:
\[ \dot{\phi}_i = 0 \Rightarrow \varphi_j - \varphi_i = \left\{ \begin{array}{c} 0 \\ \pi \end{array} \right. \]

Stability:
\[ w_{ij} > 0 \Rightarrow \varphi_j - \varphi_i = 0 \text{ is stabilized} \]
\[ w_{ij} < 0 \Rightarrow \varphi_j - \varphi_i = \pi \text{ is stabilized} \]

Storage of a binary pattern \( \xi \)

\( \xi \in \{-1, +1\}, \)
\[ w_{ij} = \xi_i \xi_j \]

all \( \xi_i = -1 \Rightarrow \varphi_i \rightarrow \varphi_- \)
all \( \xi_i = +1 \Rightarrow \varphi_i \rightarrow \varphi_+ \)
Storing patterns

\[ a(t) = a_0 \sum_{i \neq j} \xi_i \xi_j \cos((\omega_i - \omega_j)t) \]
Recognizing patterns

\[ a(t) = a_0 \sum_{i \neq j} \xi_i \xi_j \cos((\omega_i - \omega_j)t) \]

\[ a(t) = a_0 + \sum_{k} \sum_{i \neq j} \xi_i^k \xi_j^k \cos((\omega_i - \omega_j)t) \]

\( \square = 1 \)

\( \blacksquare = -1 \)
Experiment requirements

\[ \dot{\mathcal{G}}_i = \Omega_i + \varepsilon a(t) \sum_{j=1}^{n} \sin(\mathcal{G}_j - \mathcal{G}_i) \]

• **Sinusoidal coupling** in \( \mathcal{G} \)
• **Sufficiently weak coupling**
• **Coupling mechanism that allows for arbitrary variations of the coupling function** \( a(t) \) **in time**, in particular \( a(t) < 0 \) and \( a(t) > 0 \)
Single oscillator circuit

tunnel diode with negative differential resistance

\[ U(t) \]

\[ U_{\text{offset}} \]

\[ I \text{[mA]} \]

\[ t \]

\[ U \text{[mV]} \]
Coupling two oscillators

\[ U_1(t) \]
\[ R \ll R_{ex} \rightarrow U_1(t) \approx U_{ex}(t) \approx U_2(t), \text{ strong coupling} \]

\[ U_{ex}(t) \]
\[ R \gg R_{ex} \rightarrow U_{ex}(t) \approx 0, \text{ weak coupling} \]

Graphs showing oscillations for \( U_1(t) \) and \( U_2(t) \) with and without coupling.
Weak coupling effects

frequencies will not change, but oscillations tend to get “in sync” at times when $R_{ex}$ is relatively high

$\Rightarrow$ phase shifts change

this effect is most pronounced if

$$\omega_R = \omega_1 - \omega_2$$
Controlling two oscillator phases

\[ \varphi \quad (\text{phase shift}) \]

\[ R_{\text{ex}} [\Omega] \]

\( R = 150 \Omega \)

23342 Hz

15590 Hz

7752 Hz
Coupling several oscillators

\[ C_i \dot{U}_i = C_i \dot{U}_i \big|_{R_{ex}=0} + \frac{R_{ex}}{R(R/N + R_{ex})} \langle U_i \rangle \]

coupling function:

\[ \varepsilon(R_{ex}) = \frac{R_{ex}}{R(R/N + R_{ex})} \]

coupling depends non-linearly on \( R_{ex} \)
Shape of $\varepsilon(R_{\text{ex}}(t))$ for higher $N$

$$\varepsilon(R_{\text{ex}}) = \frac{R_{\text{ex}}}{R(R / N + R_{\text{ex}})}$$

$\Rightarrow$ amplitude/offset ratio decreases with larger $N$:

 Ideal coupling function:
- zero offset
- zero distortion from sine wave

$R_{\text{ex}} \in [20\Omega, 80\Omega]$, $R = 150\Omega$
Better coupling with negative impedances

replace \( R_{\text{ex}}(t) \) by a subcircuit with negative impedance elements:

\[
\varepsilon(R_{\text{ex}}) = \frac{R_{\text{ex}}}{R(R/N + R_{\text{ex}})}
\]

choose:

\[
Z_1 = \langle -R_{\text{var}} \rangle, \quad Z_2 = -\frac{R}{N}
\]

\[
\Rightarrow \varepsilon(R_{\text{var}}) = \frac{R_{\text{var}} - \langle R_{\text{var}} \rangle}{R^2}
\]

=> offset and distortion can be corrected, the coupling strength can be tuned much more flexible now
Storing patterns

R. Hölzel and KK, New J. Phys. 13 (2011) 073031
Recognizing patterns

R. Hölzel and KK, New J. Phys. 13 (2011) 073031
Further recognition examples
Scalability issues

$$\Delta \omega < \min_{j \neq i} \left| \omega_i - \omega_j \right|$$

must be fulfilled for any number $N$ of oscillators

For near optimal Golomb rulers

$$\min_{j \neq i} \left| \omega_i - \omega_j \right| \geq \frac{\omega_N - \omega_1}{N^2}$$

$\omega_N$ : maximal frequency
$\omega_1$ : minimal frequency

Accuracy of a single oscillator:

$$\frac{\Delta \omega}{\omega} < \frac{1}{N^2}$$
Conclusions

- Experimental manipulations of phase shifts with a weak global coupling possible
- Information can be stored in the relative phase shifts
- Experimental proof of principle that a network of weakly coupled electrical oscillators with a time-dependent global coupling can perform pattern recognition

- But...
- No long term recognition
- Accuracy requirement constrains number of oscillators in a network.