HYPERNUCLEAR POTENTIALS AND THE
PSEUDOSCALAR MESON EXCHANGE CONTRIBUTION

Cesar Barbero(1)†, Dubravko Horvat(2)♭, Franjo Krmpotić(1)‡,
Zoran Narančić(2) and Dubravko Tadić(3)¶

(1)Departamento de Física, Facultad de Ciencias,
Universidad Nacional de La Plata, C. C. 67,
1900 La Plata, Argentina

(2)Department of Physics, Faculty of Electrical Engineering,
University of Zagreb, 10 000 Zagreb, Croatia

(3)Physics Department,
University of Zagreb, 10 000 Zagreb, Croatia

Abstract

The pieces of the hypernuclear strangeness violating potential due to the pseudoscalar meson exchanges are derived using methods which were successfully applied to hyperon nonleptonic decays. The estimates are tested by comparison with measured hyperon nonleptonic decay amplitudes. All isospin changes \( \Delta I = 1/2 \) and \( \Delta I = 3/2 \) are included in the derived potential. All calculational methods used are reviewed and described in detail.
1. Introduction

Hypernuclei $^A_Λ N$, heavier than $^5_Λ$He, decay, mainly through nonmesonic channels. In these channels the $Λ$ mass excess of 176 MeV is converted into kinetic energy of a final state nucleon. Such a large momentum transfer suggests that heavier mesons exchanges in a baryon-baryon potential could play a significant role. However this review is dealing with pseudoscalar meson exchanges, such as $π$, $K$ and $η$ only. The potential pieces due to the vector and/or axial-vector meson exchanges will be studied along the similar lines in the following paper [1]. The same goes for the calculations of the hypernuclear decay widths $Γ$, which involve detailed description of nuclear states [2]. In hypernuclear decays one can study parity conserving (PC) and parity violating (PV) part of the weak interaction. In that sense the nonmesonic decay

$$Λ + N \rightarrow N + N$$

or

$$Λ + p^+ \rightarrow p^+ + n^0$$

and

$$Λ + n^0 \rightarrow n^0 + n^0$$

(1-1)

are the $ΔS = 1$ analogy of the weak $NN \rightarrow NN$ nuclear PV reaction. However the weak PC $ΛN \rightarrow NN$ decay is also observable in experiments. Experimental hypernuclear programs within BNL, KEK, CEBAF and DUBNA promise enough data for exhaustive hypernuclear studies [3-7].

In this paper we will employ one pseudoscalar meson exchange mechanism to induce the effective baryon-baryon (hyperon) potential (Fig. 1.1).

Fig. 1.1 - One meson baryon-baryon exchange diagram. $N$ and $Λ$ are $S = 0$ and $S = −1$ baryons and $M$ is a non-strange pseudoscalar meson. $W$ and $S$ are the strong and weak vertices.
The weak vertex $W$ for pseudoscalar meson exchange is closely connected with the theory of hyperon nonleptonic decays [8-15]. These decays have been studied and analysed in detail. Here we will review a particular theoretical approach [8,13,14]. The related methods have already been applied [5] in estimates of the "new" $\Delta S = 1$ vertices, such as $NNK$ which appear in processes (1-1) and which are shown in Fig.1.1. Here we intend to include all known and relevant contributions, but for decuplet poles [16].

The strength of a weak vertex $W$

$$W = \overline{u}(p_N)(A + B\gamma_5)u(p_\Lambda)$$

(1-2)

can be parametrized by $A$ and $B$ amplitudes. For certain weak vertices such as $\Lambda \to N + \pi$ for example, the amplitudes $A$ and $B$ can be taken directly from the experiment. In the experimentaly measured decay all particles are on the mass shell. That is not necessarily so in the case of the exchange interaction Fig.1-1 inside the nuclear matter. However all possible corrections will be neglected in the following. Theoretical uncertainties, as explained in detail below, are so large, that such niceties have no practical importance.

The theoretical analysis will be carried out for some of directly measurable amplitudes also in order to check the theoretical accuracy. One has to rely entirely on the theory when dealing with weak vertex corresponding to $N \to N + K$ and $\Lambda \to N + \eta$ transitions. In that sense the investigation of the strangeness violating nuclear interaction is a wellcome test for the general theoretical understanding of the hyperon nonleptonic processes.

Use will be made of theoretical schemes which were developed for the hyperon nonleptonic decays [13, 14, 17, 18] and for the deduction of the weak parity violating nuclear potential [19,20].

For the strong meson vertices the standard $PC$ (parity coneserving) form and the flavour SU(3) symmetry is assumed.

The study of the hyperon nonleptonic decays has in the eighties reached certain level of success based on current algebra and pole dominance. Some results [14, 21] were elaborated in monographs and reviews [8, 11, 13].

The theoretically consistent and complete (as far as the mentioned approximations go) results are obtained by combining contributions which previously were used in either one or the other of discussed works. For example, in describing $p-$wave $(B)$ amplitudes Ref. [21] used baryon and meson poles. Ref.s [14, 17, 18] used baryon poles and separable (factorizable) contributions. It turns out, as discussed in detail below, that one has to combine all three contributions in a well defined way.
Separable contributions associated with operators $O_1 \ldots, O_4$ (see 2 below) contain an axial vector current matrix element
\[
\langle B_f | A^\alpha \mu | B_i \rangle \sim \bar{u}_{B_f} [\gamma_\mu \gamma_5 g_A + i q_\mu \gamma_5 g_P] u_{B_i}.
\] (1-3)

Here the $g_A$ contribution contains a formfactor which is associated with the axial vector meson exchanges [8, 22, 23] and thus it should be included as "an axial vector meson pole contribution". The term $g_P$ was not included in the earlier estimate [14, 21]. It is dominated by the pseudoscalar meson (i.e. kaon) pole, so that it’s contribution is included in the more general kaon pole contributions. Separable contributions from operators $O_5$ and $O_6$ are, as shown below, contained in the meson pole term.

The parity violating ($s$–wave) amplitudes were estimated by Ref. [21] using the current algebra terms (CAT) and the contributions coming from the commutators involving $O_5$ and $O_6$ operators. In the case of the $p$–wave amplitudes, the analogous terms are included in the kaon pole pieces [13, 21]. Ref.s [14, 17, 18] used CAT and factorizable contributions. Obviously the complete estimate, involving leading poles should contain everything.

The weak PV ($A$) amplitude contains current algebra and separable parts, whereas the PC ($B$) amplitude gets its contributions from pole terms and separable parts, i.e.
\[
A = A_{CA} + A_{SEP}
\]
\[
B = B_{POLE} + B_{SEP},
\] (1-4a)

or more precisely
\[
A = \sum_{a=\pi,K,\eta} \left[A_{CA/a} + A_{SEP/a}\right]
\]
\[
B = \sum_{a=\pi,K,\eta} \left[B_{POLE/a} + B_{SEP/a}\right].
\] (1-4b)

The following sections will be devoted to specific pion, kaon and $\eta$–contributions to PV and PC weak $\Lambda$–decay amplitudes. The separable pieces are given in Section 3 while the baryonic pole contributions are described in Section 4. Section 5 is devoted to so called current algebra contributions. The connection between separable contributions and the meson poles is discussed in Section 6. Section 7 compares the calculated weak $BBM$ amplitudes with the measured ones. The theoretically predicted $NNK$ and $NN\eta$ amplitudes are compared with some other theoretical predictions. The derivation of the nonrelativistic weak potentials is described in Section 8. The $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces are listed separably. The effective $\Delta S = 1$ nonrelativistic potential, which can serve as an input in nuclear calculations, are listed in Section 9. Various theoretical and calculational details can be found in numerous appendices.
Some attention will be paid to the so called ”double counting problem” [11, 18] by comparing various contributions to $B$ amplitudes.

Closely related theoretical methods were used in Ref. [5]. This approach investigates the importance (or unimportance) of additional contributions, such as separable terms and kaon poles, which are introduced here.

The weak $NNK$ interactions were investigated using heavy baryon chiral perturbation theory also [22]. Their results will be compared with results obtained by methods corresponding to the scheme outlined by formulae (1-4).
2. Weak And Strong Hamiltonian

The weak one pion exchange potential can be extracted from the Feynman amplitude shown in Fig.1.1. This diagram corresponds to a second order term in the $S$-matrix expansion. The details of that expansion, performed in an effective field theory, can be found in Section 8 below. Here we want to specify the strength of the weak (i.e. $A$, $B$) and of the strong vertex (i.e. $g_{B'B_M}$). The calculation of the weak strengths $A$ and $B$, which correspond to the hyperon nonleptonic decay amplitudes [13,14,17,18,21] will be the main topic of the following Sections 3.-6.

The weak vertices are determined by the effective weak $\Delta S = 1$ Hamiltonian which is in the quark basis given by [23,24]

$$H_{\text{eff.}}^{(W)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i O_i,$$

(2-1)

can be conveniently written as

$$H_{\text{eff.}}^{(W)} = -\sqrt{2} G_F \sin \theta_C \cos \theta_C \sum_i C_i O_i.$$

(2-2)

Here $C_i$'s are the QCD Wilson coefficients [23] calculated in the six-quark Standard model environment by solving the renormalization group equations to the one-loop QCD corrections, evaluated are the scale $\mu = 0.5 \text{ GeV}$. Their values are given in Table 2.1. $G_F$ is Fermi constant and $\sin \theta_C$ and $\cos \theta_C$ (later denoted by $s_C$ and $c_C$) are sine/cosine of the Cabibbo angle.

| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|-------|-------|-------|-------|-------|-------|
| -2.358 | 0.080 | 0.082 | 0.411 | -0.080 | -0.021 |

*Table 2.1 - Wilson coefficients evaluated are the scale $\mu = 0.5 \text{ GeV}$*

The four-quark ($V - A$) operators comprising the effective weak Hamiltonian (2-1)
are

\[ \mathcal{O}_1 = (\bar{d}_L \gamma_\mu s_L)(\bar{u}_L \gamma^\mu u_L) - (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu s_L) \tag{8, 1/2} \]

\[ \mathcal{O}_2 = (\bar{d}_L \gamma_\mu s_L)(\bar{u}_L \gamma^\mu u_L) + (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu s_L) 
+ 2(\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu d_L) + 2(\bar{d}_L \gamma_\mu s_L)(\bar{s}_L \gamma^\mu s_L) \tag{8, 1/2} \]

\[ \mathcal{O}_3 = (\bar{d}_L \gamma_\mu s_L)(\bar{u}_L \gamma^\mu u_L) + (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu s_L) 
+ 2(\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu d_L) - 3(\bar{d}_L \gamma_\mu s_L)(\bar{s}_L \gamma^\mu s_L) \tag{27, 1/2} \]

\[ \mathcal{O}_4 = (\bar{d}_L \gamma_\mu s_L)(\bar{u}_L \gamma^\mu u_L) + (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu s_L) 
- (\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu d_L) \tag{27, 3/2} \]

\[ \mathcal{O}_5 = (\bar{d}_L \gamma_\mu A s_L)(\bar{u}_R \gamma^\mu A u_R) + (\bar{d}_L \gamma_\mu A s_L)(\bar{d}_R \gamma^\mu A d_R) 
+ (\bar{d}_L \gamma_\mu A s_L)(\bar{s}_R \gamma^\mu A s_R) \tag{8, 1/2} \]

\[ \mathcal{O}_6 = (\bar{d}_L \gamma_\mu s_L)(\bar{u}_R \gamma^\mu u_R) + (\bar{d}_L \gamma_\mu s_L)(\bar{d}_R \gamma^\mu d_R) 
+ (\bar{d}_L \gamma_\mu s_L)(\bar{s}_R \gamma^\mu s_R) \tag{8, 1/2} \]

All the operators in (2-3) are normal ordered and their SU(3) (flavour representation, isospin) content is also given. In the above expressions the following notation is used

\[ u_L \equiv \frac{1}{2}(1 - \gamma_5)u; \quad u_R \equiv \frac{1}{2}(1 + \gamma_5)u \quad \text{so for instance} \]

\[ (\bar{d}_L \gamma_\mu s_L)(\bar{u}_L \gamma^\mu u_L) \equiv \left[ \bar{d}L \gamma^\mu (1 - \gamma_5)s \right] \left[ \bar{u}L \gamma_\mu (1 - \gamma_5)u \right] \]

\[ = \frac{1}{4} \left[ \bar{d} \gamma_\mu (1 - \gamma_5)s \right] \left[ \bar{u} \gamma^\mu (1 - \gamma_5)u \right] \]

\[ = \frac{1}{4} \left[ \bar{d}(s)_{(V-A)}(\bar{u}u)_{(V-A)} \right], \quad \text{and} \]

\[ (\bar{d}_R \gamma_\mu d_R) \equiv \left[ \bar{d}L \gamma^\mu (1 - \gamma_5)\gamma_\mu \frac{1}{2}(1 + \gamma_5)d \right] \]

\[ = \frac{1}{2} \bar{d} \gamma_\mu (1 + \gamma_5)d \]

The strong and weak vertices corresponding to effective baryon-baryon-meson couplings are given by

\[ \mathcal{H}^{(S)}_{NN\pi} = ig_{NN\pi}(\bar{\Psi}_N \gamma_5 \Psi_N \Phi_\pi) \tag{2-5} \]

and

\[ \mathcal{H}^{(W)}_{AN\pi} = -ig_{FN\pi}^2(\bar{\Psi}_N (A - B\gamma_5)(\Psi_A^S \Phi_\pi). \tag{2-6} \]

Here is \( G_F \) the Fermi coupling constant whose value is usually given in the following combination

\[ G_F m^2_{\pi^0} = 2.21 \times 10^{-7} \quad \text{i.e.} \quad G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}. \tag{2-7} \]
$\Psi^S_\Lambda$ is the isospin spurion $\binom{0}{1}$ included here to enforce the $\Delta I = 1/2$ rule observed in the $\Lambda$ decays. The weak amplitudes $A$ and $B$ are calculated below, see Table 8.1. They obtain contributions corresponding to current algebra forms, separable forms and various pole terms.

The strong coupling constants differ according to their isomultiplet content which can be inferred from their SU(3) coupling. The following results are obtained (see for instance [25-28]):

- $NN\pi$: $g_{pnn^+} = \sqrt{2}g_{NN\pi}$; $g_{nn^0} = -g_{NN\pi}$; $g_{pp^0} = g_{NN\pi}$;
- $N\Sigma K$: $g_{nK^+\Sigma} = \sqrt{2}g_{N\Sigma K}$; $g_{pK^0\Sigma} = \sqrt{2}g_{N\Sigma K}$; $g_{nK^0\Sigma} = -g_{N\Sigma K}$;
- $\Xi\Xi\pi$: $g_{\Xi^-\Xi^0\pi^+} = -\sqrt{2}g_{\Xi\Xi\pi}$; $g_{\Xi^-\Xi^-\pi^0} = -g_{\Xi\Xi\pi}$; $g_{\Xi^0\Xi^0\pi^0} = g_{\Xi\Xi\pi}$;
- $NK\Lambda$: $g_{pK^+\Lambda} = g_{pK^0\Lambda} = g_{N\Lambda\Lambda}$;
- $\Lambda\Sigma\pi$: $g_{\Lambda\Sigma^+\pi^-} = g_{\Lambda\Sigma^-\pi^+} = g_{\Lambda\Sigma^0\pi^0} = g_{\Lambda\Sigma\pi}$;

| $MBB'$ | $\pi NN$ | $\pi \Lambda \Sigma$ | $\pi \Sigma \Sigma$ | $\pi \Xi \Xi$ | $K \Lambda N$ | $K \Xi \Lambda$ | $K \Sigma N$ |
|--------|--------|----------------|----------------|-------------|-------------|-------------|-------------|
| $g_{MBB'}$ | 1 | $\frac{2(1-f)}{\sqrt{3}}$ | $2f$ | $(2f-1)$ | $-(1+2f)$ | $\frac{4f-1}{\sqrt{3}}$ | $1-2f$ |

Table 2.2 - Strong coupling constants in units of $g$; eg. $g_{\pi NN} = g$ etc.
3. Separable contributions

In this section we calculate the separable contributions to \( A \) and \( B \) amplitudes. In the quark basis they are symbolized by diagrams (a) and (b) in Fig.3.1.

![Fig. 3.1 - Separable ((a), (b)) and non-separable ((c), (d)) contributions in \( \Lambda \) decays in a proton \( p \) and neutron \( n \). \( \mathcal{H}_{\text{eff}}^{(W)} \) acts at the black dot.](image)

Four quarks emerging from the black dot in Fig.3.1 are produced by the four-quark operators in (2-1) and (2-2).

In order to account for all the possible processes one has to include contributions from the Fierz-transformed (FIT) operators \( O_i \) averaged over colours. This average brings in a factor of 1/3. The two relations govern all the Fierz transformations

\[
[\vec{q}_1 \gamma^\mu (1 - \gamma_5) q_2] \cdot [\vec{q}_3 \gamma^\mu (1 - \gamma_5) q_4] = [\vec{q}_1 \gamma^\mu (1 - \gamma_5) q_4] \cdot [\vec{q}_3 \gamma^\mu (1 - \gamma_5) q_2]
\]

\[
[\vec{q}_1 \gamma^\mu (1 - \gamma_5) q_2] \cdot [\vec{q}_3 \gamma^\mu (1 + \gamma_5) q_4] = -2[\vec{q}_1 (1 + \gamma_5) q_4] \cdot [\vec{q}_3 (1 - \gamma_5) q_2].
\]

(3-1)

In the above transformations the anticommutativity of fermion fields has been taken into account. The Fierz identities for eight Gell-Mann SU(3) \( \lambda \)-matrices which are necessary for the calculations are

\[
\lambda^a_b \cdot \lambda^c_d = \frac{16}{9} \delta^a_d \delta^c_b - \frac{1}{3} \lambda^a_d \lambda^c_b
\]

(3-2)

\[
\lambda^a_d \cdot \lambda^c_b = 2 \delta^a_b \delta^c_d - \frac{2}{3} \delta^a_d \delta^c_b.
\]
FIT operators could be expressed in terms of the original $O_i$ operators:

\[
O_1^{\text{FT}} = -O_1, \quad \text{so that} \quad O_1 \to O_1 + \frac{1}{3} O_1^{\text{FT}} = \left(1 - \frac{1}{3}\right) O_1
\]

\[
O_2^{\text{FT}} = O_2, \quad \text{so that} \quad O_2 \to O_2 + \frac{1}{3} O_2^{\text{FT}} = \left(1 + \frac{1}{3}\right) O_2
\]

\[
O_3^{\text{FT}} = O_3, \quad \text{so that} \quad O_3 \to O_3 + \frac{1}{3} O_3^{\text{FT}} = \left(1 + \frac{1}{3}\right) O_3
\]

\[
O_4^{\text{FT}} = O_4, \quad \text{so that} \quad O_4 \to O_4 + \frac{1}{3} O_4^{\text{FT}} = \left(1 + \frac{1}{3}\right) O_4
\]

\[
O_5^{\text{FT}} = -2(\bar{d}_L q^i_R) (\bar{q}^i_R s_L), \quad \text{so that} \quad O_5 \to O_5 + \frac{1}{3} O_5^{\text{FT}} = O_5 + \frac{3}{16} O_6
\]

\[
O_6^{\text{FT}} = -2(\bar{d}_L q^i_R) (\bar{q}^i_R s_L), \quad \text{so that} \quad O_6 \to O_6 + \frac{1}{3} O_6^{\text{FT}}
\]

\[
= O_6 - \frac{2}{3} (\bar{d}_L q^i_R) (\bar{q}^i_R s_L)
\]

\[
= O_6 - \frac{2}{3} (\bar{d}_L q^i_R) (\bar{q}^i_R s_L)
\]

\[
= O_6 - \frac{2}{3} (\bar{d}_L q^i_R) (\bar{q}^i_R s_L)
\]

\[
O_5^{\text{FT}} = \frac{3}{16} O_6 \quad \text{so that} \quad O_5 \to O_5 + O_5^{\text{FT}} = O_5 + \frac{3}{16} O_6
\]

There is also a simple relation between a matrix element of the operators $O_5$ and $O_6$ which is used frequently

\[
\langle B' | O_5 | B \rangle = \frac{16}{3} \langle B' | O_6 | B \rangle.
\] (3-4)

In the FIT operators $O_5, O_6$ the following quark combinations occur

\[
(d_L s_R)(d_R d_L) \equiv \bar{d} \frac{1}{2} (1 + \gamma_5) \frac{1}{2} (1 + \gamma_5) s \bar{d} \frac{1}{2} (1 - \gamma_5) \frac{1}{2} (1 - \gamma_5) d
\]

\[
= \frac{1}{4} \bar{d} (1 + \gamma_5) s \bar{d} (1 - \gamma_5) d.
\] (3-5)

From the Dirac equation for quark fields one gets useful relations which are necessary to calculate matrix elements of scalar and pseudo-scalar densities occurring in the FIT operators $O_5$ and $O_6$ (see (3-7), (3-8) and (3-4a)). For instance

\[
\overline{d} \gamma_5 u = (p_d - p'_u) \mu \overline{d} (p_d) \gamma^\mu \gamma_5 u (p'_u), \quad \frac{1}{m_d + m_u}
\]

\[
\overline{s} \gamma_5 u = (p_d - p'_u) \mu \overline{s} (p'_u) \gamma^\mu s (p'_u). \quad \frac{1}{m_u - m_s}
\] (3-6)

The effective Hamiltonian (2-1) is used to calculate the weak decay amplitudes which correspond to the weak vertex in Fig.1.1. The general procedure is briefly outlined: the invariant decay amplitude for the process (baryon $\rightarrow$ baryon + meson)

\[
B \to B' + M
\] (3-7)

is given by the factorization (separation) assumption as

\[
\langle M, B' | \mathcal{H}_{\text{eff}} | B \rangle = -\sqrt{2} G_F \sin \theta_C \cos \theta_C \sum_i C_i \langle M | (V - A)_i | 0 \rangle \langle B' | (V - A)_i | B \rangle
\] (3-8)
where the \((V - A)_i\) are currents (operator parts) which could account for the particular transition between the vacuum \(|0\rangle\) and a meson \(|M\rangle\) and between two (octet) baryons \(B\) and \(B'\). The character of the meson matrix element \(|M\rangle (V - A) |0\rangle\) depends on the meson state: if \(M\) belongs to the SU(3) pseudo-scalar octet, only \(A\) part of the four-quark operator contributes. (When a vector meson is considered only \(V\) part remains.) For the baryon matrix element both \(V\) and \(A\) parts occur giving PV or PC contributions. Baryon matrix elements are expressed in terms of vector/axial-vector form factors \(v_{BB'}/a_{BB'}\) which occur in semileptonic baryon decays. These form factors are expressible in terms of \(F\) and \(D\) constants. The form factors are given in Table 3.1.

| BB' | SU(3) content | \(v_{NN'}\) | \(a_{NN'}\) |
|-----|---------------|-------------|-------------|
| np  | \(\tilde{\pi}^-\) | 1           | \(F + D\)  |
| nn  | \(\tilde{\pi}^0\) | \(-1/\sqrt{2}\) | \(-1/\sqrt{2}\) (\(F + D\)) |
| pp  | \(\tilde{\pi}^0\) | \(1/\sqrt{2}\) | \(1/\sqrt{2}\) (\(F + D\)) |
| pp  | \(\tilde{\eta}_8\) | \(3/\sqrt{6}\) | \(1/\sqrt{6}\) (\(3F - D\)) |
| nn  | \(\tilde{\eta}_8\) | \(3/\sqrt{6}\) | \(1/\sqrt{6}\) (\(3F - D\)) |
| nn  | \(\bar{u}u\)     | 1           | \(F - D\)  |
| nn  | \(\bar{d}d\)     | 2           | \(2F\)      |
| nn  | \(\tilde{\eta}_1\) | \(3/\sqrt{3}\) | \(1/\sqrt{3}\) (\(3F - D\)) |
| pp  | \(\tilde{\eta}_1\) | \(3/\sqrt{3}\) | \(1/\sqrt{3}\) (\(3F - D\)) |
| \(\Lambda p\) | \(K^-\) | \(-3/\sqrt{6}\) | \(-1/\sqrt{6}\) (\(3F + D\)) |
| \(\Lambda n\) | \(-\bar{K}^0\) | \(-3/\sqrt{6}\) | \(-1/\sqrt{6}\) (\(3F + D\)) |

Table 3.1 - Nucleon vector and axial-vector form factors

One defines

\[
\langle B' | V^\mu_a | B \rangle = \bar{u}_{B'}(p') \gamma^\mu v^a_{BB'} u_B(p) \tag{3-9}
\]

and

\[
\langle B' | A^\mu_a | B \rangle = \bar{u}_{B'}(p') \gamma^\mu \gamma_5 a^a_{BB'} u_B(p) \tag{3-10a}
\]

or

\[
a^a_{BB'} = g_A \Lambda_a. \tag{3-10b}
\]

Here \(\Lambda_a = \lambda_a/2\) is the well known SU(3) matrix [9,26]. A very important ingredient for the above outlined calculation is the knowledge of a meson matrix element. It is calculated either by using the partially conserved axial-vector current (PCAC) hypothesis [8,9,11] (in
the case of pseudo scalar (PS) mesons $\pi$, $\eta$, $K$ or by using the current-field identity (CFI) [19,29] or the meson-nucleon $\sigma$–term [30-32] (in the case of vector mesons).

The PCAC hypothesis is based on the divergence of the axial-vector current

$$ A_\mu^a = \overline{\psi}\gamma_\mu\gamma_5\frac{\lambda^a}{2}\psi. \quad (3-11) $$

This is an operator relation and its divergence is applied to a (physical) state represented by a ket (or bra), i.e.

$$ \partial_\mu A_\mu^a |0\rangle = i f_\phi p_\mu p^\mu \phi^\dagger |0\rangle = i f_\phi m_\phi^2 |\phi\rangle. \quad (3-12) $$

Since we want to express the meson matrix element of the form given in (3-3), i.e. $\langle M | A_\mu | 0 \rangle$, the $\phi$ operator has to create a state

$$ M \sim \frac{q_\lambda + i b}{\sqrt{2}}. \quad (3-14) $$

which, for $\pi^0$ gives $\pi^0 = \frac{q}{\sqrt{2}}(\lambda_3), \gamma = (u, d, s)$ (here "T" means transposed), or for $\eta = \frac{q}{\sqrt{2}}(\lambda_8)$.

For the positive pion one gets that $a + i b$ from (3-14) equals $1 - i2$, so we have

$$ \partial_\mu A_{1-i2}^\mu = C f_\pi m_\pi^2 \pi^+ = C f_\pi m_\pi^2 d^\dagger b^\dagger. \quad (3-15) $$

Here $b_u$ anihilates a quark of the flavour $u$ whereas $d_d$ anihilates an antiquark of the flavour $d$. The constant $C$ is to be determined and it depends on the isospin content of the meson in question. In the above case the calculation gives $C = 1$. We can write now all the relevant PCAC relations

$$ \partial_\mu A_{3-i5}^\mu = \frac{\sqrt{2}}{2} f_K m_K^2 \pi^0 \quad \partial_\mu A_{1-i2}^\mu = f_\pi m_\pi^2 \pi^+ $$

$$ \partial_\mu A_{4-i5}^\mu = f_K m_K^2 \pi^0 \quad \partial_\mu A_{0-i7}^\mu = f_K m_K^2 \pi^+ $$

$$ \partial_\mu A_{6}^\mu = \frac{\sqrt{2}}{2} f_\eta m_\eta^2 \eta. \quad (3-16) $$

The above relations enable us to relate the following matrix elements

$$ \langle \pi^0 | \overline{u}\gamma_\mu \gamma_5 d | 0 \rangle = - \langle \pi^0 | \overline{d}\gamma_\mu \gamma_5 u | 0 \rangle = \frac{\sqrt{2}}{2} \langle \pi^- | \overline{d}\gamma_\mu \gamma_5 u | 0 \rangle \quad (3-17) $$

eq \frac{\sqrt{2}}{2} \langle \pi^- | \overline{d}\gamma_\mu \gamma_5 u | 0 \rangle \quad (3-17) $$

etc.
4. The Baryon Pole Contributions

The parity conserving (PC) $B$ amplitude gets part of its contributions from the baryon pole terms given for a particular case of the $\Lambda$ decay in the Fig.4.1.

![Fig.4.1 - Baryon pole terms contributions: (a) s-channel and (b) u-channel.]

We will consider one particular example of the baryon pole calculation in full detail. Two possible $\Lambda$–decays (which almost 100% saturate the $\Lambda$ width) are

$$A(\Lambda_0^0) \quad \Lambda^0 \rightarrow n + \pi^0$$

$$A(\Lambda^-) \quad \Lambda^0 \rightarrow p + \pi^-.$$  \hspace{1cm} (4-1)

By assigning the momenta to incoming and outgoing particle, i.e.

$$\Lambda(p_1) \rightarrow \pi^-(q) + \Sigma^+(p_1 - q) \rightarrow p(p_2)$$  \hspace{1cm} (4-2)

we calculate the Feynman amplitude, corresponding to the $u$–channel, Fig.4.1, using the effective Hamiltonians given in (2-5) and (2-6), as follows

$$\pi(p_2)(A - B\gamma_5) \frac{(p_1 - \phi) + m_{\Sigma}}{(p_1 - q)^2 - m_{\Sigma}^2} \gamma_5 g_{\Sigma\pi^-\Lambda} u(p_1).$$  \hspace{1cm} (4-3)

Here $g_{\Sigma\pi\Lambda}$ is the strong coupling constant listed in table 2.2.

Since $p_1 - q = p_2$ and $p_2^2 = m_p^2$ we get

$$\pi(p_2)(A - B\gamma_5) \frac{m_p + m_{\Sigma}}{(m_p - m_{\Sigma})(m_p + m_{\Sigma})} \gamma_5 g_{\Sigma\pi\Lambda} u(p_1).$$  \hspace{1cm} (4-4)

The result for $B(\Lambda_0^0)$ amplitude, Fig.4.1, is

$$B^{\text{POLE}}(\Lambda_0^0) = g_{\Lambda\Sigma^+\pi^-} \frac{a_{\Sigma^+\pi^-}}{\Sigma^+ - \Lambda} + g_{n\pi^-p} \frac{a_{\Lambda n}}{n - \Lambda}.$$  \hspace{1cm} (4-5)

However, in the soft pion limit, which is used to determine the $a_{ij}$ amplitude (see Section 5 below), $B$ amplitude has to vanish. That imposes the condition that is given by a subtraction of the soft amplitude $B^{\text{POLE}}(q^2 = 0)$, i.e.

$$B^{\text{POLE}}(q^2) - B^{\text{POLE}}(0) \equiv B^{\text{POLE}}(\text{eq.(4-7,8)}).$$  \hspace{1cm} (4-6)
Here \( q^2 \) is the pion (kaon, \( \eta \)) four-momentum. As ref. [3] uses \( B_{\text{POLE}}(q^2) \) only, that explains some numerical differences which will appear in Tables 7.1, 7.3 and 7.4. In detail one finds:

\[
B_{\pi}^{\text{POLE}}(\Lambda_0^-) = 2g(\Lambda + p) \left[ \frac{a_{\Sigma n}}{\sqrt{3}} \cdot \frac{d}{(\Sigma^+ - p)(\Lambda + \Sigma^+)} - \frac{a_{\Lambda n}}{\sqrt{2}} \cdot \frac{f + d}{(\Lambda - N)(N + P)} \right] \tag{4-7}
\]

\[
B_{\pi}^{\text{POLE}}(\Lambda_0^0) = -\frac{1}{\sqrt{2}} B_{\pi}^{\text{POLE}}(\Lambda_0^-). \tag{4-8}
\]

In the above expression the following notation for masses is used: \( \Lambda \equiv m_\Lambda \), or \( p \equiv m_p \), etc. Recall also that \( f + d = 1 \), so only \( f' \)'s appear in (4-8).

\[Fig.4.2\ - Baryon pole terms contributions: (a) s-channel and (b) u-channel, \( \pi^0 \) emission.\]
Kaon exchange contributions are determined by

$$B_K^{\text{POLE}}(p_{1}^{+}) = g(p + n) \left[ (1 - 2f) \frac{a_{\Sigma^0 n}}{(\Sigma^0 - n)(\Sigma^0 + p)} - \frac{1}{\sqrt{3}} (1 + 2f) \frac{a_{\Lambda^0 n}}{(\Lambda^0 - n)(\Lambda^0 + p)} \right]$$

$$B_K^{\text{POLE}}(n_{0}^{0}) = 2gn \left\{ \left[ -\frac{1}{\sqrt{3}} (1 + 2f) \right] \frac{a_{\Lambda^0 n}}{\Lambda^0 - n^2} - (1 - 2f) \frac{a_{\Sigma^0 n}}{\Sigma^0 - n^2} \right\}$$

$$B_K^{\text{POLE}}(p_{0}^{+}) = 4gp(1 - 2f)\sqrt{2} \frac{a_{\Sigma^0 + p}}{\Sigma^0 - p^2}$$

Here for instance is (see Sec. 5).

$$a_{\Sigma^0 n} = f_\pi \sqrt{2} A(\Sigma^-),$$

etc.

Fig. 4.3 - Pole diagrams for the proton nonleptonic decay: $p \rightarrow n K^+$

Fig. 4.4 - Pole diagrams for the neutron nonleptonic decay: $n \rightarrow n K^0$

Fig. 4.5 - Pole diagrams for the proton nonleptonic decay: $p \rightarrow p K^0$
$\eta$ exchange contributions are: $u$–channel contribution for the sub-process $\Lambda \rightarrow \eta_1 + n$ evaluated by the baryon-pole model contains $\Lambda^0$ as the intermediate hyperon whereas in the $t$–channel $n$ is the intermediate baryon, so that strong vertices contain $g_{\Lambda\Lambda\eta}$ and $g_{NN\eta}$ form-factors (see Fig.4.6).

$$B_{\eta_1}^{\text{POLE}}(\Lambda^0_{\eta_1}) = g \sqrt{\frac{2}{3}} (1 - f) \frac{\Lambda + n}{\Lambda - n} \left( \frac{1}{\Lambda} - \frac{2}{n} \right) \left( \frac{1}{\sqrt{2}} \cdot a_{\Lambda n} \right)$$

(4-13)

$$B_{\eta_8}^{\text{POLE}}(\Lambda^0_{\eta_8}) = \frac{a_{\Lambda^0 n}}{2} \frac{\Lambda + n}{\Lambda - n} \left( g_{\Lambda\Lambda\eta} \frac{\Lambda}{\Lambda} - g_{NN\eta} \frac{n}{n} \right) \left( \frac{1}{\sqrt{2}} \cdot a_{\Lambda n} \right)$$

(4-14)

Fig. 4.6 - Baryon pole $\eta_1$ emission: $s$–channel and $u$–channel contributions

In the numerical calculations (see 7. below) we will be using quantities $\tilde{a}_{i\ell}$ instead of $a_{i\ell}$ introduced above.
5. Current Algebra Contributions

Calculation of weak matrix elements by methods of current algebra (CA) consists of several approximation procedures which lead eventually to simple relations among transition amplitudes.

The $S$ matrix element is expressed in term of $in$ and $out$ field operators where \[33,34\]
\[
\phi(x, t) \simeq \phi_{in}(\vec{x}, t) \quad \text{for} \quad t \to -\infty \\
\phi(x, t) \simeq \phi_{out}(\vec{x}, t) \quad \text{for} \quad t \to +\infty.
\] (5-1)

(Here we ignore the renormalization constant appearing in front of the $in/out$-states.) The initial particle state is of the form

\[|k_1, \ldots, k_N; in\rangle = a_{in}^\dagger(k_N) \cdots a_{in}^\dagger(k_1)|0\rangle (5-2)\]

and the $S$-matrix serves to connect the $in$ and $out$ states

\[|k_1, \ldots, k_N; in\rangle = S|k_1, \ldots, k_N; out\rangle. (5-3)\]

The meson field operator $\phi_A(x)$, for which we are going to use more handy notation i.e. $\phi(x)_A \to M_A(x)$, could be expressed by the PCAC (3-16)

\[M_{(A=a+ib)} = \frac{1}{m^2_M f_M} \partial_\mu A^\mu_{a+ib}, (5-4)\]

and the meson field, being the Heisenberg field operator, annihilates a meson state with the normalization in accordance with the creation/annihilation operator commutation relations

\[\langle 0 | M_{a+ib}(x) | M_{a'+ib'}(k) \rangle = \frac{1}{(2\pi)^3 2\omega_k} e^{-ikx} (\delta_{aa'}\delta_{bb'}). (5-5)\]

Using the time-ordering operator $T$ one can calculate the matrix element as follows

\[
\langle B' M_A(k; out) | \mathcal{H}_W(0) | B \rangle = i \int d^4x \frac{e^{ikx}}{(2\pi)^3 2\omega_k} (\Box x + m^2_M) \langle B' | T[M_{a+ib}(x) \mathcal{H}_W(0)] | B \rangle \\
= i C_M \left(1 - \frac{k^2}{m^2_M}\right) \int d^4x e^{ikx} \langle B' | T[\partial_\mu A^\mu_{a+ib}(x) \mathcal{H}_W(0)] | B \rangle.
\] (5-6)

Here $C_M$ depends on the particular (PS-)meson. The time ordered product could be written in the following way $T[A(x)B(0)] = \theta(x^0)[A(x), B(0)] + B(0)A(x)$. From

\[
e^{ikx} \theta(x_0) \partial_\mu A^\mu_{a+ib}(x) = \partial_\mu \left[e^{ikx} \theta(x_0) A^\mu_{a+ib}(x)\right] - ik_\mu \theta(x_0) e^{ikx} A^\mu_{a+ib}(x) \\
- \left[\partial_\mu \theta(x_0)\right] e^{ikx} A^\mu_{a+ib}(x) (5-7)
\]
and using the derivative of the Heaviside function

\[ \frac{\partial}{\partial t} \theta(t) = - \frac{\partial}{\partial t} \theta(-t) = \delta(t) \] (5-8)

we find

\[
e^{ikx} \mathcal{T}[\partial_\mu A^\mu_{a+ib}(x)\mathcal{H}_W(0)] = -ik_\mu e^{ikx} \mathcal{T}[A^\mu_{a+ib}(x)\mathcal{H}_W(0)] - \delta(x_0)[A^0_{a+ib}(x), \mathcal{H}_W(0)]
\]

\[ + \partial_\mu \{ e^{ikx} \mathcal{T}[A^\mu_{a+ib}(x)\mathcal{H}_W(0)] \}. \] (5-9)

Hence we can write the above matrix element in the following way

\[
\langle B' M_A(k; \text{out}) | \mathcal{H}_W(0) | B \rangle = i C_M \int \frac{d^4 x}{m^2_M} e^{ikx} \{ -ik_\lambda \langle B | \mathcal{T}[A^\lambda_{a+ib}(x)\mathcal{H}_W(0) | B] - \delta(x_0) \langle B' | [A^0_{a+ib}(x), \mathcal{H}_W(0)] | B \rangle \}. \] (5-10)

By taking the limit \( k \to 0 \) (soft pion limit for off-shell pions) we get a typical current algebra (CA) relation

\[
\mathcal{M}(q \to 0) = -i C_M \langle B' | [F^5_{a+ib}(0), \mathcal{H}_W(0)] | B \rangle. \] (5-11)

Here the SU(3) (axial) charge is defined by

\[
F^5_{a+ib}(t) = \int d^3 x A^0_{a+ib}(t, \vec{x}). \] (5-12)

The following CA relations hold [9,13,24]

\[
[F^5_{a+ib}, \mathcal{H}_W] = [F_{a+ib}, \mathcal{H}^{PV}_W]
\]

\[
[F^5_{a+ib}, \mathcal{H}^{PC}_W] = [F_{a+ib}, \mathcal{H}^{PV}_W]
\]

\[
[F^5_{a+ib}, \mathcal{H}^{PV}_W] = [F_{a+ib}, \mathcal{H}^{PC}_W]. \] (5-13)

Here \( F_{a+ib} \)'s are (vector) charges defined in the similar way as \( F^5_{a+ib} \)'s, (5-12) i.e.

\[
F_{a+ib} = \int d^3 x V^0_{a+ib}(t, \vec{x}). \] (5-14)

In order to evaluate the above commutator we can use the SU(3) relations connecting the axial charges and field operators

\[
M_{a+ib} = \frac{1}{2}(\lambda_a + i\lambda_b \gamma_5)q = \sqrt{2} F^5_{a+ib} \quad \text{and} \quad M_A |B\rangle = i f_{ABC} |C\rangle \] (5-15)
where \( f_{ABC} \) are the SU(3) structure constants and \( q^T = (u, d, s) \) (\( T \) stands for transposed!).

For instance for the \( K^0 \) field we have

\[
M_{a+i0} \rightarrow \phi_{K^0} = \sqrt{2} F^+_e K^0 = \sqrt{2} F_{6-i7} = \frac{1}{2} (\lambda_6 - i\lambda_7) \gamma_5 q. \tag{5-16}
\]

The above field annihilates a \( K^0 \) in \(| \rangle \) or creates a \( K^0 \) in \(< | \). Also \( \phi_{K^+} = \sqrt{2} F_{4-i5} \) etc.

So by here displayed procedure we have extracted the current algebra contribution to the matrix element \( M \).

The above relations can be neatly expressed by the SU(3) baryon (antibaryon) \( B \) (\( \overline{B} \)) and meson \( P \) octet matrices

\[
E^a_b = \begin{pmatrix}
\Sigma^0 / \sqrt{2} + \Lambda^0 / \sqrt{6} & \Sigma^+ & p \\
\Sigma^- & -\Sigma^0 / \sqrt{2} + \Lambda^0 / \sqrt{6} & n \\
\Xi^- & \Xi^0 & -2\Lambda^0 / \sqrt{6}
\end{pmatrix},
\]

\[
P^a_b = \begin{pmatrix}
\pi^0 / \sqrt{2} + \eta^0 / \sqrt{6} & \pi^+ & K^+ \\
-\pi^0 / \sqrt{2} + \eta^0 / \sqrt{6} & n & K^0 \\
K^- & -\eta^0 / \sqrt{6} & -2\eta^0 / \sqrt{6}
\end{pmatrix}. \tag{5-17a}
\]

and

\[
P^a_b = \begin{pmatrix}
\pi^0 / \sqrt{2} + \eta^0 / \sqrt{6} & \pi^+ & K^+ \\
-\pi^0 / \sqrt{2} + \eta^0 / \sqrt{6} & n & K^0 \\
K^- & -\eta^0 / \sqrt{6} & -2\eta^0 / \sqrt{6}
\end{pmatrix}. \tag{5-17b}
\]

Alternatively one can write

\[
\begin{align*}
\Sigma^+ &= \frac{1}{\sqrt{2}} (B_1 - iB_2) & p &= \frac{1}{\sqrt{2}} (B_4 - iB_5) \\
\Sigma^- &= \frac{1}{\sqrt{2}} (B_1 + iB_2) & n &= \frac{1}{\sqrt{2}} (B_6 - iB_7) \\
\Sigma^0 &= B_3 & \Xi^- &= \frac{1}{\sqrt{2}} (B_4 + iB_5) \\
\Lambda^0 &= B_8 & \Xi^0 &= \frac{1}{\sqrt{2}} (B_6 + iB_7)
\end{align*} \tag{5-18a}
\]

and

\[
\begin{align*}
\pi^+ &= \frac{1}{\sqrt{2}} (P_1 - iP_2) & K^+ &= \frac{1}{\sqrt{2}} (P_4 - iP_5) \\
\pi^- &= \frac{1}{\sqrt{2}} (P_1 + iP_2) & K^0 &= \frac{1}{\sqrt{2}} (P_6 - iP_7) \\
\pi^0 &= P_3 & K^- &= \frac{1}{\sqrt{2}} (P_4 + iP_5) \\
\eta_8 &= P_8 & \overline{K^0} &= \frac{1}{\sqrt{2}} (P_6 + iP_7) \tag{5-18b}
\end{align*}
\]
The quark SU(3) flavour contents of the baryon and meson octet states are:

\[
\begin{align*}
\Sigma^+ &\sim uus & p &\sim uud \\
\Sigma^- &\sim dds & n &\sim udd \\
\Sigma^0 &\sim uds & \Xi^- &\sim dss \\
\Lambda^0 &\sim uds & \Xi^0 &\sim uss
\end{align*}
\]

(5-18c)

and

\[
\begin{align*}
\pi^+ &= \bar{d}u \\
\pi^- &= \bar{u}d \\
\pi^0 &= \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \\
\eta_8 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)
\end{align*}
\]

(5-18d)

The SU(3) meson states could be written in the standard basis \[9,35,36\] as

\[
\begin{align*}
\bar{u}u &= \bar{q} \left( \frac{\sqrt{3}}{6} \lambda^8 + \frac{\sqrt{6}}{6} \lambda^0 + \frac{1}{2} \lambda^3 \right) q \quad \text{corresponding to} \\
\bar{u}u &\rightarrow \frac{\sqrt{6}}{6} \eta_8 + \frac{\sqrt{3}}{3} \eta_1 + \frac{\sqrt{2}}{2} \pi^0 \\
\bar{d}d &= \bar{q} \left( \frac{\sqrt{3}}{6} \lambda^8 + \frac{\sqrt{6}}{6} \lambda^0 - \frac{1}{2} \lambda^3 \right) q \quad \text{corresponding to} \\
\bar{d}d &\rightarrow \frac{\sqrt{6}}{6} \eta_8 + \frac{\sqrt{3}}{3} \eta_1 - \frac{\sqrt{2}}{2} \pi^0 \\
\bar{s}s &= \bar{q} \left( -\frac{\sqrt{3}}{3} \lambda^8 + \frac{\sqrt{6}}{6} \lambda^0 \right) q \quad \text{corresponding to} \\
\bar{s}s &\rightarrow -\frac{\sqrt{6}}{3} \eta_8 + \frac{\sqrt{3}}{3} \eta_1
\end{align*}
\]

(5-18e)

The SU(3) meson states states could be written in the standard basis \[9,35,36\] as
\[ |M\rangle = |8; Y, I, I_3\rangle, \text{ with } Y = B + S. \text{ So} \]

\[ |\pi^+\rangle = \mathcal{P}_1^2|0\rangle = -|8; 0, 1, +1\rangle \]
\[ |\pi^-\rangle = \mathcal{P}_2^1|0\rangle = |8; 0, 1, -1\rangle \]
\[ |\pi^0\rangle = \frac{1}{\sqrt{2}}(\mathcal{P}_1^1 - \mathcal{P}_2^2)|0\rangle = |8; 0, 1, 0\rangle \]
\[ |K^+\rangle = \mathcal{P}_1^3|0\rangle = |8; 1, 1/2, 1/2\rangle \]
\[ |K^0\rangle = \mathcal{P}_2^3|0\rangle = |8; 1, 1/2, -1/2\rangle \]
\[ |\bar{K}^0\rangle = \mathcal{P}_3^2|0\rangle = |8; -1, 1/2, 1/2\rangle \]
\[ |K^-\rangle = \mathcal{P}_3^1|0\rangle = -|8; -1, 1/2, -1/2\rangle \]
\[ |\eta^0\rangle = -\frac{3}{\sqrt{6}}\mathcal{P}_3^3|0\rangle = |8; 0, 0, 0\rangle \]

The corresponding SU(2) (iso) multiplets are

\[ K^a = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad \bar{K}^a = \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix}, \quad \pi^a = \begin{pmatrix} \pi^0 \\ \pi^+ \end{pmatrix}, \quad \pi^a = \begin{pmatrix} \pi^- \\ \pi^0 \end{pmatrix}, \quad \Sigma^a = \begin{pmatrix} \Sigma^0 \\ \Sigma^+ \end{pmatrix}, \quad \Sigma^a = \begin{pmatrix} \Sigma^- \\ \Sigma^0 \end{pmatrix} \]

(5-18g)

When in (5-18a,b) defined operators act on an initial ket or final bra they annihilate or create corresponding states. From the SU(3) charges one can construct the well known operators \( F_+ \) or \( F_- \) for instance (together with \( F_3 \)) which have the standard SU(2) group properties

\[ \langle p|F_+ = \langle n| \quad \langle n|F_3 = -\frac{1}{2}\langle n| \]
\[ F_+|\Xi^-\rangle = -|\Xi^0\rangle \quad F_3|\Xi^0\rangle = \frac{1}{2}|\Xi^0\rangle \]
\[ F_+|\Sigma^-\rangle = \sqrt{2}|\Sigma^0\rangle \quad F_-|\Sigma^+\rangle = -\sqrt{2}|\Sigma^0\rangle \]

(5-19)

When the commutators appearing in the matrix elements \( M \) (5-15) are evaluated one gets the transition amplitudes which are the \textit{current algebra} contributions to \( A \).\footnote{One has to be very careful with the definitions of the initial and final states and with operators acting on the states. The usual definition which is adopted here as well is that operators \textit{annihilate} the corresponding particle state in the initial state. Careless application of this convention could lead to much confusion in a calculation to which many authors contribute by loosely following a given convention!}

\footnote{The PC amplitude \( B \) does not have the contribution coming from the above commutators, but only the pole diagrams contribute as we have shown in the preceding section.}
Therefore

\[
A(\Lambda^0_0) = -\frac{\sqrt{2}}{f_\pi} \frac{1}{\sqrt{2}} \langle n | [F_3, \mathcal{H}^{PC}_W(0)] | \Lambda \rangle = -\frac{1}{f_\pi} \langle n | \mathcal{H}^{PC}_W(0) | \Lambda \rangle
\]

\[
= -\frac{1}{f_\pi} a_{\Lambda n}
\]  

(5-20)

\[
A(\Lambda^0_0) = -\frac{\sqrt{2}}{f_\pi} \frac{1}{\sqrt{2}} \langle n | [F_3, \mathcal{H}^{PC}_W(0)] | \Lambda \rangle
\]

\[
= \frac{1}{\sqrt{2} f_\pi} a_{\Lambda n}
\]  

(5-21)

\[
A(\Xi^-) = -\frac{\sqrt{2}}{f_\pi} \frac{1}{\sqrt{2}} \langle \Lambda | [F_3, \mathcal{H}^{PC}_W(0)] | \Xi^- \rangle = -\frac{1}{f_\pi} \langle \Lambda | \mathcal{H}^{PC}_W | \Xi^- \rangle
\]

\[
= -\frac{1}{f_\pi} a_{\Xi^0 \Lambda}
\]  

(5-22)

\[
A(\Xi^0_0) = -\frac{1}{\sqrt{2} f_\pi} a_{\Xi^0 \Lambda}
\]  

(5-23)

\[
A(\Sigma^-) = \frac{\sqrt{2}}{f_\pi} a_{\Sigma^0 n}
\]  

(5-24)

\[
A(\Sigma^0_0) = -\frac{1}{\sqrt{2} f_\pi} \left[ a_{\Sigma^0 +} + \sqrt{2} a_{\Sigma^0 n} \right]
\]  

(5-25)

\[
A(\Sigma^+_0) = \frac{1}{\sqrt{2} f_\pi} a_{\Sigma^0 +}
\]  

(5-26)
6. Meson Poles and Separable Contributions

Some terms in separable contributions (Sec. 3) are also included in the meson pole contributions, Fig. 6.1. However, with some care, one can avoid the double counting. The separable contributions due to operators $O_1, O_2, O_3, O_4$ as calculated in Section 3, are not included in the meson (for example kaon) contributions. The meson (kaon) pole contributes to the parity conserving $B$ amplitudes only.

The separable $PC$ contributions (3-8) by operators $O_1, O_2, O_3, O_4$ were in our case calculated using the product of axial vector currents $A^\mu_a(x)$:

$$\langle B_\beta | A^\mu_a | B_\alpha \rangle \cdot \langle L_a | A^\mu_a | 0 \rangle = B(\text{sep.})$$  \hspace{1cm} (6-1)

Here, in accordance with (3-10),

$$\langle B_\beta | A^\mu_a(0) | B_\alpha \rangle = \gamma^\mu \gamma_5 g_A(q^2) \left[ 1 \frac{g_P(q^2)}{M_\alpha + M_\beta} q^\mu \gamma_5 \right] \Lambda_a u_\alpha(p)$$  \hspace{1cm} (6-2)

and $L_a$ is some meson.† The results presented in Sec. 3 were obtained by assuming that $g_P$ term (induced pseudoscalar form factor) can be neglected. The contribution due to the induced pseudoscalar $g_P$ is contained only in the kaon-pole term as described below. Some needed definitions and conventions are given in Appendix A.

The meson pole contribution to the process in which a meson $L$ is emitted

$$B_\alpha \rightarrow B_\beta + L_a$$  \hspace{1cm} (6-3)

is shown in Fig.6.1.

---

† For $B(\Lambda_0^-) L_a = \pi^-$. 

Fig. 6.1 - The strong vertex is $g_{\alpha \beta}$ and the weak Hamiltonian $H_w$ contains the current product (6-1). The particles in the process $\Lambda \rightarrow p + \pi$ are indicated in the parentheses.
The amplitude shown in Fig. 6.1 which corresponds to $\phi$ meson pole, is\textsuperscript{†}

$$B^{MP} = \langle L | H_w | \phi \rangle \frac{i}{q^2 - m_{\phi}^2 + i\epsilon} \bar{u}_\beta \gamma_5 \Lambda_a u_\alpha g_{\alpha\beta}. \quad (6-4)$$

When one approximates

$$\langle L | H_w | \phi \rangle \simeq \langle L | A^b_\mu | 0 \rangle \langle 0 | A^\mu_u | \phi \rangle, \quad (6-5)$$

then by using the relation from Appendix A

$$\frac{g_{P}(q^2)}{M_\alpha + M_\beta} \simeq (-g_{\alpha\beta} f_{\phi}(q^2)) \quad (6-6a)$$

and $i f_{\phi} q^\mu = \langle 0 | A^\mu_u | \phi \rangle$, one can show that

$$B^{MP} \simeq \langle L | A^b_\mu | 0 \rangle \cdot \bar{u}_\beta \frac{1}{M_\alpha + M_\beta} g_{P} q^\mu \gamma_5 \Lambda_a u_\alpha. \quad (6-6b)$$

Thus the induced pseudoscalar ($g_P$) contribution to the $B$(sep) is included in the $M$(pole) term. When calculating the separable contribution from operators $O_1, \ldots, O_4$, one should include only the $g_A$ form factors as it has been done in Sec. 3.

The operators $O_5$ and $O_6$ contribute pieces which look like

$$O_x \sim (\bar{\psi}_1 \gamma_5 \psi_2)(\bar{\psi}_3 \gamma_5 \psi_4)$$

$$\langle B_\beta | \bar{\psi}_1 \gamma_5 \psi_2 | B_\alpha \rangle \neq 0 \quad (6-7)$$

$$\langle L | \bar{\psi}_3 \gamma_5 \psi_4 | 0 \rangle \neq 0$$

Here $\psi_i$ symbolize quark fields appearing in $H_w$. The corresponding $B^{MP}$ contribution is obtained from (6-4) by the replacement

$$\langle L | H_w | \phi \rangle \rightarrow \langle L | O_x | \phi \rangle. \quad (6-8)$$

For meson pseudoscalar densities one uses identities

$$\bar{\psi}_i \gamma_5 \psi_j = \frac{1}{m_i + m_j} (-i)(\partial_\mu \bar{\psi}_i \gamma^\mu \gamma_5 \psi_j)$$

$$= \frac{1}{m_i + m_j} (-i) (i(p_i^\mu - p_j^\mu)) \bar{\psi}_i \gamma_\mu \gamma_5 \psi_j. \quad (6-9)$$

\textsuperscript{†} Here $B^{MB}$ is a contribution due to a meson pole, while $B^{POLE}$ in Ch.4 denotes a contribution connected with a baryon pole.
Thus
\[
\langle B_\beta | \bar{\psi}_1 \gamma_5 \psi_2 | B_\alpha \rangle = (-i) \langle B_\beta | \partial_\mu A_\mu^a | B_\alpha \rangle \frac{1}{(m_1 + m_2)} \\
= \frac{f_\phi \bar{q}^2}{q^2 - m_\phi^2} g_{\alpha \beta} \bar{\psi}_1 \gamma_5 \Lambda_a u_\alpha \cdot \frac{1}{m_1 + m_2}.
\]
(6-10)

Here the notation \( \bar{q}^2 \) was used instead of \( m_M^2 \).

A separable contribution corresponding to (3-8) thus have a generic form
\[
B_x^{(sep)} = \frac{f_\phi \bar{q}^2}{(m_1 + m_2)(q^2 - m_\phi^2)} g_{\alpha \beta} \bar{\psi}_1 \gamma_5 \Lambda_a u_\alpha \langle L | \bar{\psi}_3 \gamma_5 \psi_4 | 0 \rangle.
\]
(6-11)

It openly displays \( \phi \)-meson pole so one expects that it must be included in the \( B^{MP} \) contribution. In order to test that one again approximates (6-5) and writes (6-4)
\[
B_{x}^{MP} \approx \langle L | \bar{\psi}_3 \gamma_5 \psi_4 | 0 \rangle \langle 0 | \bar{\psi}_1 \gamma_5 \psi_2 | \phi \rangle \frac{i}{q^2 - m_\phi^2} g_{\alpha \beta} \bar{\psi}_1 \gamma_5 \Lambda_a u_\alpha.
\]
(6-12a)

The first mesonic matrix element is
\[
\langle 0 | \bar{\psi}_1 \gamma_5 \psi_2 | \phi \rangle = \frac{1}{m_1 + m_2} (-i) \langle 0 | \partial_\mu A_\mu^a | \phi_a \rangle = \frac{1}{m_1 + m_2} (-i) q^2 f_\phi.
\]
(6-13)

Thus
\[
B_{x}^{MP} \approx \frac{f_\phi \bar{q}^2}{(m_1 + m_2)(q^2 - m_\phi^2)} g_{\alpha \beta} \bar{\psi}_1 \gamma_5 \Lambda_a u_\alpha \langle L | \bar{\psi}_3 \gamma_5 \psi_4 | 0 \rangle.
\]
(6-12b)

The approximation (6-12) is exactly equal to \( B_x^{(sep)} \) (6-11) if one puts
\[
\bar{q}^2 \equiv q^2.
\]
(6-14)

Alternatively and formally one can compare the meson pole approximation (6-12a) with the expression (6-11) converting
\[
\frac{f_\phi m_\phi^2}{m_1 + m_2} = \langle 0 | \partial_\mu A_\mu^a | \phi \rangle \frac{m_1 + m_2}{m_1 + m_2} = i \langle 0 | \bar{\psi}_1 \gamma_5 \psi_2 | \phi \rangle \langle q^2 = m_\phi^2 \rangle.
\]
(6-15)

The insertion of (6-15) in (6-11) converts it into (6-12a). However in that formal procedure one has neglected the question about meson \( \phi \) being off-mass-shell. Some additional details can be found in Appendix A.

In the derivation of PCAC constant one has assumed that the meson \( \phi \) was on the mass shell. As that meson is not on the mass shell neither in the diagram in Fig.6.1, nor in the expression (6-11), the reading (6-14) seems to be amply justified. The separable contributions from operators \( O_5 \) and \( O_6 \) are included in the meson (for example kaon) pole.

For the process
\[
B_\alpha \rightarrow B_\beta + M_a.
\]
(6-16)
one can now formulate the rule for the calculation of $B$ amplitudes:

$$B = B_{\text{baryon pole}} + B_{\text{sep}} + B_{\text{MP}}. \quad (6-17)$$

Here $B_{\text{sep}}$ contains only the contribution from the $g_A$ terms associated with the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$. The $g_P$ terms for these operators and the separable contributions from $\mathcal{O}_5$ and $\mathcal{O}_6$ are included in $B_{\text{MP}}$ piece. The contributions $B_{\text{baryon-pole}} = B_{\text{POLE}}$ were calculated in Section 4 while $B_{\text{sep}}$ can be found in Sec. 3.
7. Weak Baryon-Baryon-Meson Amplitudes

It is very important to test the quality of the approximate procedure which was used to calculate $A$ and $B$ amplitudes. That can be done by:

(i) Comparison with other theoretical calculations,

(ii) Comparison with experimental results.

The first task can be easily accomplished by studying the amplitudes corresponding to $\eta$ and $K$ exchanges given in Tables 7.1-7.4. Ref. [5] did not distinguished $\eta_1$ (singlet) from $\eta_8$ (octet) exchanges. In their approach $\eta$ was supposed to belong to an SU(3) octet. Our results for the octet $\eta$ are fairly close to Ref. [5]. $A_{\eta_8}$ differs by about 8%, while the discrepancy for $B_{\eta_8}$ is 18%. The origin of those discrepancies is obviously connected with the separable and pole terms which appear in our calculational scheme. As discussed in Sections 3 and 4, they were not used or they were used in different forms by ref. [5].

In view of that, the agreement within 18% suggests that used calculational schemes are relatively stable and, hopefully, reliable to within about 20%. That is the accuracy that was hoped for in the theoretical descriptions of the hyperon nonleptonic decays [8-15].

The amplitudes appearing with the kaon strangeness violation vertices are compared in Table 7.3. The differences are small, below 29%, for $A(p_0^\pi)$, $A(p_0^\pi)$ and $A(n_0^\pi)$. They are much larger for the corresponding $B$ amplitudes. There the absolute value of $B(p_0^\pi)$ amplitudes, as calculated by [22], is about 3 times smaller then ours. However in all cases signs are the same. If $B_K(p_0^\pi)$ is excluded, the largest discrepancy is about 21%. Our result for $B_K(p_0^\pi)$ amplitude differs from Ref. [5] by about 35% only.

Comparison with experimental results is, and can be, performed in a limited sense only. The experimental $A_{\pi}$ amplitudes are used to find $\bar{a}(B'B)$ matrix elements via a substraction procedure described in Appendix H. Those $\bar{a}_{BB'}$ quantities are then used in baryon terms which were defined in Section 4. Together with separable contributions and kaon poles they lead to the theoretical prediction of $B_{\pi}$ amplitudes, which are shown in Tables 7.1 and 7.4. Reference [5] has used simple, unsubtracted pole terms as was discussed in Section 4. More complicated and hopefully better, approximation leads to somewhat better agreement with the experiment in most cases. According to Table 7.4 one finds that discrepancies with experiments for various amplitudes are as follows: $B_\pi(\Lambda^0_0)$, 6%; $B_\pi(\Lambda^0_0)$, 2%; $B_\pi(\Sigma^+_0)$, 14%; $B_\pi(\Sigma^+_0)$, 48%; $B_\pi(\Xi^0_0)$, 4%; and $B_\pi(\Xi^-)$, 6%. While $B_\pi(\Lambda)$ and $B_\pi(\Xi)$ amplitudes are reproduced within 6%, the theoretical prediction for $B_\pi(\Sigma)$ is poor. Our theoretical $B_\pi(\Sigma^-)$ amplitude is off by factor 5.2. However the corresponding numbers, which were found using a simplified approximation, analogous to the one applied by Ref. [5] are: $B_\pi(\Lambda^0_0)$ 44%; $B_\pi(\Lambda^0_0)$ 42%; $B_\pi(\Sigma^+_0)$ 35%; $B_\pi(\Sigma^+_0)$ 3 times too small; $B_\pi(\Xi^0_0)$ 1%; $B_\pi(\Xi^-)$ 1%, and $B_\pi(\Sigma^-)$ disagrees by factor 6.3. Thus for most amplitudes simplified approximation gives poorer agreement with the experimental values.

There were always some difficulties associated with the theoretical description of the $\Sigma$-hyperon nonleptonic decays. Attempted explanations involved additional $1/2^-(1/2^+)$ baryon resonance poles [37-40], instantons [41] etc.

Overall one feels that the first column in Table 7.3 might constitute a better approximation of the real physical results, than given by the last column. No theoretical comparison with Ref. [22] is feasible, as that reference uses a quite different method of evaluation. However it is encouraging that all methods give similar relative magnitudes and relative sign. The worst disagreement is for $B_K(p_0^\pi)$. However, $A_K(p_0^\pi)$ agrees within
If the agreement among various theoretical results is an acceptable indicator, one could say that $A_K(N)$ amplitudes are determined with better accuracy than the $B_K(N)$ ones. A sceptical observer would claim that the results presented in Table 7.3 are the order of magnitude estimates at best.

Our method has produced both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces in the effective potential (9-6). In the formula (9-11) those pieces are written separately, with isospin operators $\mathbf{1} \cdot \mathbf{1}$ and $\mathbf{\tau}_i \cdot \mathbf{\tau}_j$ for $\Delta I = 1/2$ and $\mathbf{T}_i \mathbf{T}_j$ for $\Delta I = 3/2$. However, $\Delta I = 3/2$ pieces are small and their magnitude is comparable with the theoretical errors which were discussed above. This can be starkly illustrated if one calculates the $\Delta I = 3/2$ piece, associated with $\Lambda \to N\pi$ weak amplitudes. One finds

$$B_\pi(\Lambda^0)_{\exp} + \sqrt{2}B_\pi(\Lambda^0)_{\exp} = 0.32 \times 10^{-7}. \quad (7-1)$$

This experimental $\Delta I = 3/2$ piece has a different sign than the predicted one:

$$B_\pi(\Lambda^0)_{\text{BHNT}} + \sqrt{2}B_\pi(\Lambda^0)_{\text{BHNT}} = -0.26 \times 10^{-7}. \quad (7-2)$$

Effect is due to subtraction of large (relatively speaking) numbers. Yet $B(\Lambda)_{\exp}$ agree within 6% with $B_\pi(\Lambda)_{\text{BHNT}}$. The same degree of agreement is shown by $B(\Xi)$ amplitudes. But in that case $\Delta I = 3/2$ pieces $(B\Xi_{\text{exp}} + \sqrt{2}B\Xi^0_{\text{exp}})$ are $0.013 \times 10^{-7}$ (exp.) and $0.018 \times 10^{-7}$ (theor.).

In the deduction of the nuclear potential (9-11) we have used, naturally, the experimental $B_\pi(\Lambda)$ value. Thus the whole discussion, involving the theoretical $B_\pi(\Lambda)$ and $B_\pi(\Xi)$ values, serves as an indication for theoretical uncertainties connected with the predicted $A_K(N)$ and $B_K(N)$ (Table 7.3) values. It is hard to quantify those uncertainties. While the theoretical $\Delta I = 3/2$ piece, connected with $B_\pi(\Lambda)$'s, has wrong sign (but its magnitude is comparable with the experimental value) the $B_\pi(\Xi)$ based $\Delta I = 3/2$ piece agrees with the experimental value within 40%.
Table 7.1 - Transition amplitudes: \([\text{BHKNT}]\equiv\) present work: (a) a complete amplitude; (b) an amplitude without the separable contribution (this work). The asterisk \((*)\) denotes that the experimental amplitude was used instead. \(\dagger\equiv\) experimental value assumed.
To facilitate a comparison between our results and those of ref. [5] we translate the

| Ref. [5] | Total Ampl. | Ampl. without SEP |
|----------|-------------|------------------|
| $A_\pi$ | 3.25        | 3.25             |
| $B_\pi$ | 22.35       | 22.27            |
| $A_{\eta_1}$ | --        | 5.60             |
| $A_{\eta_8}$ | 5.63       | 5.19             |
| $B_{\eta_1}$ | --        | 41.10            |
| $B_{\eta_8}$ | 31.60      | 38.41($m_\eta^2$) | 28.82 |
| $C_{K}^{PV}$ | 2.38       | $A_K(p_+^+)$ : 1.31 |
| $C_{K}^{PC}$ | 41.30      | $B_K(p_+^+)$ : 42.38 |
| $D_{K}^{PV}$ | 6.53       | $A_K(p_0^+)$ : 4.08 |
| $D_{K}^{PC}$ | -14.72     | $B_K(p_0^+)$ : -21.87 |
| $C_{K}^{PV} + D_{K}^{PV}$ | 6.33      | $A_K(n_0^0)$ : 6.25 |
| $C_{K}^{PC} + D_{K}^{PC}$ | 26.58     | $B_K(n_0^0)$ : 17.99 |

Table 7.2 - Comparison between transition amplitudes (in $10^{-7}$ units, w.o. dimension) as given in [5] and this work. A complete amplitude includes a separable contribution. Two different results in the Ref. [5] column correspond to two different sets of results as quoted there.
### Table 7.3 - Nonleptonic amplitudes ($\times 10^7$); † this work, compared with rescaled values of Ref. [22] and [5].

| Amplitude | Tot. ampl.† | Ref.[19] | Ref.[3] |
|-----------|-------------|----------|--------|
| $A(p_0^+)$ | 4.08        | 4.09     | 4.64   |
| $B(p_0^+)$ | -21.87      | -7.6     | -14.72 |
| $A(p_1^+)$ | 1.31        | 1.09     | 1.69   |
| $B(p_1^+)$ | 42.38       | 33.40    | 41.30  |
| $A(n_0^0)$ | 6.25        | 5.19     | 6.33   |
| $B(n_0^0)$ | 17.99       | 26.16    | 26.58  |

### Table 7.4 - Nonleptonic $p$–wave (parity conserving) amplitudes ($\times 10^7$); † this work. *Contributions from operators $O_1 - O_4$; ‡Contributions from operators $O_5, O_6$.

| Ampl. $\times 10^7$ | $B_{\exp}$ | $B^{(\text{POLE})}_{\Lambda}$ | Sep. (1−4)* | Mes. pole | Tot.† amp. | Pole | Sep. (5−6)‡ | Ref. [5] |
|---------------------|-------------|-------------------------------|--------------|-----------|------------|------|--------------|--------|
| $B_{\pi}(\Lambda_0^0)$ | 22.4        | 19.38                         | 4.79         | -1.9      | 22.27      | 23.03| 6.98         | 15.6   |
| $B_{\pi}(\Lambda_0^0)$ | -15.61      | -13.70                        | -1.03        | -1.2      | -15.93     | -16.28| -3.68        | -11.03 |
| $B_{\pi}(\Sigma_1^+)$ | 41.83       | 48.73                         |              |           | 48.73      | 37.68|              | 30.97  |
| $B_{\pi}(\Sigma_1^+)$ | 26.74       | 18.04                         | -0.45        | 0.5       | 18.09      | 15.85| -1.62        | 9.13   |
| $B_{\pi}(\Sigma_1^-)$ | -1.44       | -5.39                         | -2.17        | 0.1       | -7.46      | -8.83| -2.79        | -9.08  |
| $B_{\pi}(\Xi_0^0)$    | -12.13      | -12.67                        | -0.29        | 0.3       | -12.66     | -11.28| -1.05        | -12.21 |
| $B_{\pi}(\Xi_0^-)$    | 17.45       | 17.45                         | -1.40        | 0.4       | 16.45      | 15.97| -1.94        | 17.26  |
| $B_{\pi}(p_0^+)$      | -           | 34.38                         | -7.5         | 15.5      | 42.38      | 47.60| -138.6       | 41.3   |
| $B_{\pi}(p_0^-)$      | -           | -14.59                        | 0.12         | -7.4      | -21.87     | -12.99| -11.76       | -6.63  |
| $B_{\pi}(n_0^0)$      | -           | 21.56                         | -10.97       | 7.4       | 17.99      | 24.58| -57.03       | 26.71  |
| $B_{\eta}(\Lambda_0^0)$ | -           | 26.09                         | 1.16         | 13.85     | 41.10      | 28.16| -130.8       |        |
| $B_{\eta}(\Lambda_0^0)$ | -           | 28.82                         | -6.76        | 16.35     | 38.41      | 31.10| 762.36       | 31.60  |

References:

1. Ref. [19]
2. Ref. [3]
ref. [5] formulae to a more transparent form, i.e. we write

\[
\begin{align*}
C_{PV}^{K}\left|_{[5]} \right. &= A_K(p_+^+)\left|_{[BHNT]} \right. \\
C_{PC}^{PV}\left|_{[5]} \right. &= B_K(p_0^+)\left|_{[BHNT]} \right. \\
D_{K}^{PV}\left|_{[5]} \right. &= A_K(p_0^+)\left|_{[BHNT]} \right. \\
D_{K}^{PC}\left|_{[5]} \right. &= B_K(p_0^+)\left|_{[BHNT]} \right. \\
C_{PV}^{K} + D_{PV}^{M}\left|_{[5]} \right. &= A_K(n_0^0)\left|_{[BHNT]} \right. \\
C_{PC}^{K} + D_{PC}^{M}\left|_{[5]} \right. &= B_K(n_0^0)\left|_{[BHNT]} \right. 
\end{align*}
\]

The above equalities should be fulfilled only for "bare" amplitudes i.e. the amplitudes without the separable contributions! In some future study one has to consider the contribution of the SU(3) decuplet poles, which are successfully employed by ref. [37].
8. Effective Field Theory and The Weak $|\Delta S| = 1$ Potential

Instead of starting with quarks ($u, d, s$) we formulate formalism (an effective one) in which baryons ($N, \Lambda$) and mesons ($\pi, K, \eta$) appear. Such an approach facilitates the deduction of the effective potential presented in the next section. We will start with $\pi$—exchange contribution.

Process (transition, potential) under consideration is

$$\Lambda + N \rightarrow N + N; \quad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

(8-1)

In a typical diagram one has one weak and one strong vertex, as shown in Fig.1.1. In the perturbation calculation this diagram can be deduced from the effective interaction Hamiltonian (8-2)

$$H_{\text{eff}} = \int d^4x \bar{\psi}_n(x)G_F m_\pi^2(A + \gamma_5 B)\psi_\Lambda(x)\phi_{\pi^0}(x)$$

$$+ \int d^4x \bar{\psi}_p(x)G_F m_\pi^2(A + \gamma_5 B)\psi_\Lambda(x)\phi_{\pi^+}(x)$$

$$+ g_{\pi NN} \int d^4x \bar{\psi}_N(x)\gamma_5 \tau_i \psi_N(x)\phi_{\pi_i}(x)$$

$$= H_{\text{W } \pi^0} + H_{\text{W } \pi^+} + H_s$$

$$= \int d^4x \left( h_0(x) + h_+(x) + h_S(x) \right).$$

(8-2a)

The last term in (8-2a) is the strong nucleon-pion interaction, which can be written as

$$H_s = g_{\pi NN} \int d^4x \bar{\psi}_N(x)\gamma_5 \tau_i \psi_N(x)\phi_{\pi_i}(x)$$

$$= g_{\pi NN} \int d^4x \left\{ \left[ \bar{\psi}_p(x)\gamma_5 \psi_p(x) - \bar{\psi}_n(x)\gamma_5 \psi_n(x) \right] \phi_{\pi^0}(x) + \sqrt{2}\bar{\psi}_p(x)\gamma_5 \psi_n(x)\phi_{\pi^+}(x) \\
+ \sqrt{2}\bar{\psi}_n(x)\gamma_5 \psi_p(x)\phi_{\pi^-}(x) \right\}. $$

(8-2b)

Sometimes one also writes

$$g_{pp\pi^0} = -g_{nn\pi^0} = g_{\pi NN}$$

$$g_{pn\pi^+} = g_{np\pi^-} = \sqrt{2}g_{\pi NN}.$$

(8-2c)

The diagram Fig.1.1 is due to the second order contribution

$$H_{\text{eff}}H_{\text{eff}} = \int d^4x_1 d^4x_2 \left[ h_0(x_1) + h_+(x_1) + h_S(x_1) \right] \left[ h_0(x_1) + h_+(x_1) + h_S(x_1) \right]$$

$$= \int d^4x_1 d^4x_2 \left[ h_0(x_1)h_S(x_2) + h_S(x_1)h_0(x_2) + h_+(x_1)h_S(x_2) \\
+ h_S(x_1)h_+(x_2) \right] + R$$

(8-3)
Here only \(|\Delta S| = 1\) terms are shown. The part \(R\) contains the strong pion exchange such as
\[
\mathcal{H}_{\text{eff}}\mathcal{H}_{\text{eff}} = \int d^4x_1\,d^4x_2\,h_S(x_1)h_S(x_2). \tag{8-4}
\]
In the terms shown in (8-3) the pion field contraction must be carried out. That leaves us with an effective baryon-baryon (i.e. \(\Lambda + N \rightarrow N + N\)) interaction. For example (see Appendix E for more details)
\[
V_{\text{op}} = \int d^4x_1\,d^4x_2[\phi_{\pi^0}(x_1)\phi_{\pi^0}(x_2)]\bar{\psi}_n(x_1)G_Fm_\pi^2A(\Lambda_0^0)\psi_\Lambda(x_1)\cdot g_{\pi NN}\bar{\psi}_n(x_2)\gamma_5\psi_n(x_2)
+ \text{similar terms of the same type} \tag{8-5a}
\]
One can replace:
\[
\int d^4x_1\,d^4x_2\phi_{\pi^0}(x_1)\phi_{\pi^0}(x_2) \rightarrow \int d^3x_1\,d^3x_2\Delta(1, 2). \tag{8-5b}
\]
Here \(\Delta(1, 2)\) is the Yukawa function. Integration over time components goes into overall energy conservation. In order to do that one assumes that all baryon states are stationary states, i.e. that baryon operator is of the form
\[
\psi(x) = \sum_i a_i e^{-iE_i t/\hbar}\phi_i(\vec{r}). \tag{8-5c}
\]
Time dependence of the pion propagator can be also included explicitly or replaced by \(\delta(t_1 - t_2)\). More about that can be found in the Appendix G.

The final \(V_{\text{op}}\) is
\[
V_{\text{op}} = \int d^3x_1\,d^3x_2\bigg\{[-g_{\pi NN}\bar{\psi}_n(x_1)G_Fm_\pi^2[A(\Lambda_0^0) + B(\Lambda_0^0)\gamma_5]\psi_\Lambda(x_1)
\cdot \bar{\psi}_n(x_2)\gamma_5\psi_n(x_2):\Delta(1, 2)
\tag{8-6}
+ (1 \leftrightarrow 2)]

+ g_{\pi NN}\bar{\psi}_p(x_1)G_Fm_\pi^2[A(\Lambda_0^0) + B(\Lambda_0^0)\gamma_5]\psi_\Lambda(x_1)\cdot \bar{\psi}_p(x_2)\gamma_5\psi_p(x_2):\Delta(1, 2) + (1 \leftrightarrow 2)

+ g_{\pi NN}\sqrt{2}\bar{\psi}_p(x_1)G_Fm_\pi^2[A(\Lambda_0^0) + B(\Lambda_0^0)\gamma_5]\psi_\Lambda(x_1)\cdot \bar{\psi}_n(x_2)\gamma_5\psi_p(x_2):\Delta(1, 2)
\tag{8-6}
+ (1 \leftrightarrow 2)]\bigg\}
\]
Here the first two terms contribute to \(\Lambda + n \rightarrow n + n\) while the rest contributes to \(\Lambda + p \rightarrow n + p\). The notation \(::\) indicates the normal ordering.

In the nonrelativistic limit we have:
\[
\psi_a \simeq \left(\frac{\chi_a}{\vec{\sigma} \cdot \vec{p}_a\chi_a}\right) \tag{8-7}
\]
where $\chi_a$ a solution of a Schrödinger equation is a two-component spinor. From the expression (8-6) we can see that there are two contributions to the weak-strong mixing amplitude whose space-time properties are determined by the Lorentz structure of vertices. First amplitude (see Fig.8-1) has $[\gamma_5(S)] \otimes [1(W)]$ structure and the second one has $[\gamma_5(S)] \otimes [\gamma_5(W)]$ structure. Two different contributions are treated on the same footing: in the nonrelativistic limes of Dirac spinors given in (8-7) i.e. for slowly moving particles we can write for different combinations of spinors (to the leading terms in $1/m_N$) ($\vec{q} = \vec{p}_1 - \vec{p}_3$)

\[
\begin{align*}
\overline{\psi}_3 \gamma_5 \psi_1 & \approx \frac{1}{2m_N} \chi_3 (\vec{\sigma} \cdot \vec{q}) \chi_1; & \text{scalar (8-8a)} \\
\overline{\psi}_3 \gamma^i \gamma_5 \psi_1 & \approx \chi_3 (\sigma^i) \chi_1; & \text{space comp. of a axial vector (8-8c)} \\
\overline{\psi}_3 \gamma^0 \gamma_5 \psi_1 & \approx \frac{1}{2m_N} \chi_3 (\vec{\sigma} \cdot \vec{p}_1 + \vec{\sigma} \cdot \vec{p}_3) \chi_1; & \text{time comp. of a axial vector (8-8d)} \\
\overline{\psi}_3 \gamma^i \psi_1 & \approx \frac{1}{2m_N} \chi_3 (\sigma^i \vec{\sigma} \cdot \vec{p}_1 + \vec{\sigma} \cdot \vec{p}_3 \sigma^i) \chi_1; & \text{space comp. of a vector (8-8e)} \\
\overline{\psi}_3 \gamma_0 \psi_1 & \approx \chi_3 \chi_1; & \text{time comp. of a vector (8-8f)}
\end{align*}
\]

From
\[
(2\pi)^3 (q^2 + m_M^2)^{-1} = \frac{1}{4\pi} \int e^{iq \cdot r} e^{-m_M r} d^3 r
\]

one gets
\[
\Delta(|\vec{r}_1 - \vec{r}_2|) = V(r) = -\frac{1}{4\pi} \frac{e^{-m_M r}}{r}
\]

i.e. the known Yukawa potential. As discussed in Appendix G the mass dependence can be more complicated than given in (8-9).
For the vertices of the form \([\gamma_5(S)] \otimes [\gamma_5(W)]\) we get a baryon-baryon potential

\[
V(r) \sim (\vec{\sigma}_1 \nabla_1)(\vec{\sigma}_2 \nabla_2) \frac{e^{-m_M r}}{r},
\]

where \(r = |\vec{r}_1 - \vec{r}_2|\). By using some of the identities given in Appendix G we obtain four different contributions (in the coordinate space) for the effective potential

\[
V_S = \frac{1}{3} \left[ \frac{m^2}{4\pi r} e^{-m_M r} - \delta(r) \right],
\]

\[
V_T = \frac{1}{3} \frac{m^2}{4\pi r} e^{-m_M r} \left[ 1 + \frac{3}{m_M r} + \frac{3}{(m_M r)^2} \right],
\]

\[
V_{PV} = \frac{m M}{4\pi r} e^{-m_M r} \left( 1 + \frac{1}{m_M r} \right).
\]

If the weak decay \(\Lambda \to N + \pi\) obeys the isospin selection rule \(|\Delta I| = 1/2\) then it is possible to establish the connection

\[
\frac{F(\Lambda_0^0)}{F(\Lambda_0^0)} = -\sqrt{2}; \quad (F = A, B)
\]

\[
\Lambda_0^0 \sim \Lambda_0^0 \to p + \pi^-
\]

With \(f = G_F m^2_\pi (A + B \gamma_5)\) one can write

\[
V_{\text{op}} = \int d^3x_1 d^3x_2 \left\{ g_{\pi NN} \overline{\psi}_n(x_1) f \psi_\Lambda(x_1) \cdot \left[ -\overline{\psi}_n(x_2) \gamma_5 \psi_n(x_2) \right. \right.

\[
+ \overline{\psi}_p(x_2) \gamma_5 \psi_p(x_2) \left[ \Delta(1, 2) + (1 \leftrightarrow 2) \right]

\]

\[
- 2 g_{\pi NN} \overline{\psi}_p(x_1) f \psi_\Lambda(x_1) \cdot \overline{\psi}_n(x_2) \gamma_5 \psi_p(x_2) \cdot \Delta(1, 2) + (1 \leftrightarrow 2) \right\}
\]

The bilinear combinations which contain \(\gamma_5\) and which are associated with the strong vertex in Fig.1.1, can be written as

\[
\overline{\psi}_N \tau_3 \psi_N = \overline{\psi}_p \psi_p - \overline{\psi}_n \psi_n; \quad \overline{\psi}_N \tau_+ \psi_N = \overline{\psi}_p \psi_n
\]

The \(|\Delta I| = 1/2\) weak Hamiltonian transforms as the Pauli spinor

\[
\chi^{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]
Such spinor is sometime known as *spurion*. The bilinear combination containing $\Lambda$ field can be written as

\[
\overline{\psi}_N \tau^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\Lambda} = \left( \overline{\psi}_p, \overline{\psi}_n \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\Lambda} \\
= \left( - \right) \overline{\psi}_n \psi_{\Lambda}.
\]

\[
\overline{\psi}_N \tau^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\Lambda} = \left( \overline{\psi}_p, \overline{\psi}_n \right) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\Lambda} \\
= \overline{\psi}_p \psi_{\Lambda}.
\]

Formally one can assume that $\Lambda$–field is a quasi-spinor

\[
\psi_{\Lambda n} = \begin{pmatrix} 0 \\ \Psi_{\Lambda} \end{pmatrix}.
\]

(8-16b)

For future purpose we will also introduce

\[
\psi_{\Lambda p} = \tau^+ \psi_{\Lambda n} = \begin{pmatrix} \psi_{\Lambda} \\ 0 \end{pmatrix}
\]

(8-16c)

With conventions (3-7) one can put (3-5) into the form (only the part is kept in which $\Lambda$ is in $x_1$!)

\[
V_{\text{op}} = - \sum_i \int d^3x_1 d^3x_2 g_{\pi NN} \overline{\psi}_N(x_1) \tau_i f \psi_{\Lambda n}(x_1) \cdot \overline{\psi}_N(x_2) \tau_i \gamma_5 \psi_N(x_2)
\]

(8-17)

which has been used by [3].

With (8-14) and (8-16) one obtains

\[
\left( - \right) \left( \overline{\psi}_N \tau^3 \psi_{\Lambda n} \right) \left( \overline{\psi}_N \tau_3 \psi_N \right) = + \left( \overline{\psi} \psi_{\Lambda} \right) \left[ \overline{\psi}_p \psi_p - \overline{\psi}_n \psi_n \right]
\]

\[
\left( - \right) 2 \left( \overline{\psi}_N \tau^+ \psi_{\Lambda n} \right) \left( \overline{\psi}_N \tau^- \psi_N \right) = -2 \left( \overline{\psi} \psi_{\Lambda} \right) \left( \overline{\psi}_n \psi_p \right)
\]

(8-18)

We have also used

\[
\sum_i \tau^+_1 \tau^-_2 = \vec{\tau}_1 \cdot \vec{\tau}_2 = \tau_1^3 \tau_2^3 + 2(\tau_1^+ \tau^-_1 + \tau^-_1 \tau_2^+).
\]

(8-19)

Thus the $|\Delta I| = 1/2$ $V_{\text{op}}$ is given by (8-17).

One can add the $|\Delta I| = 3/2$ piece (see Appendix C)

\[
V_{\text{op}}(3/2) = \sum_h \int d^3x_1 d^3x_2 g_{\pi NN} \overline{\psi}_N(x_1) \cdot T^m \chi_{1/2}^m_{\pi} \psi_N(x_1) \cdot \tau_i \gamma_5 \psi_N(x_2)
\]

\[
h = \alpha \text{ or } \beta \gamma_5.
\]

(8-20a)
The magnitudes of \( \alpha \) (or \( \beta \)), as well as their relative signs with respect to \( A \) (or \( B \)) \((8-2)\) are calculated theoretically in Sec. 7. Here \( \psi_N \) are isospinors

\[
\psi_N(x) = \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix},
\]

and

\[
\chi_{1/2}^m \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

is an isospinor while

\[
\psi_\Lambda(x)
\]

is an isoscalar.

If one calculates with nuclear wave functions then the potential can be obtained from

\[
V_{op} = V_{op}(1/2) + V_{op}(3/2),
\]

by omitting the baryon fields \( \psi_N \) and \( \psi_\Lambda \). Thus \( V_{op} \rightarrow V \)

\[
V = (-) \sum_f g_{\pi NN}(f)_1(\gamma_5)_2 \vec{T}_1 \cdot \vec{T}_2 + \sum_h g_{\pi NN}(h)_1(\gamma_5)_2 \left( \vec{T}_1^m \cdot \chi_{1/2}^m(1) \right) \cdot \vec{T}_2.
\]

The notation

\[
\vec{T}_1^m \cdot \chi_{1/2}^m(1)
\]

means that in the actual calculation of that term one has to associate an isospinor \( \chi_{1/2}^m(1) \) with the \( \Lambda \) particle, which is also in the position \( x_1 \). (Thus \( 8-22 \) is not symmetrized.) In the first term in \((8-22a)\) the \( \Lambda \) particle, which is neutral like neutron, automatically has the spinor \( \chi_{1/2}^{-1/2}(1) \). The second term in \((8-22a)\) must be read as

\[
N^\dagger \vec{T}_1^m \chi_{1/2}^m \vec{T}_2 \rightarrow \begin{cases} \text{(proton:)} & \chi_{1/2}^m \vec{T}_1 \chi_{1/2}^m \cdot \vec{T}_2 \\
\text{(neutron:)} & \chi_{-1/2}^m \vec{T}_1 \chi_{-1/2}^m \cdot \vec{T}_2 \end{cases}
\]

with

\[
\chi_{1/2}^m \vec{T}_1 \chi_{1/2}^m \cdot \vec{T}_2 = C_{1-11/21/2}^3 \vec{T}_1 \chi_{1/2}^m \cdot \vec{T}_2
\]

\[
\chi_{-1/2}^m \vec{T}_1 \chi_{-1/2}^m \cdot \vec{T}_2 = C_{101/2-1/2}^3 \vec{T}_1 \chi_{-1/2}^m \cdot \vec{T}_2
\]

\[
\vec{T}_1^m \cdot \vec{T}_2 = \frac{1}{\sqrt{2}}(1, -i, 0)(\tau_1, \tau_2, \tau_3) = \sqrt{2}\tau_-
\]

\[
\vec{T}_1^m \cdot \vec{T}_2 = \tau_2^3.
\]
The notation in (8-1a) and (8-3b) can be understood without the explicit reference to isospin $\chi_{1/2}^m(1)$ (see also Ref. [42,43], formulae (2.29) and (2.43): their $T = T^\dagger$(ours.).) One can write

\[ \bar{\psi}_N(x_1)T_i\psi_A^{-1/2}(x_2) \]  

(8-24)

In isospin formalism that means

\[ \chi^{m_s}(1/2)^\dagger T_i \Delta^M (3/2). \]  

(8-25a)

Here $N$ and $\Delta$ are $I = 1/2$ and $I = 3/2$ spinors, with the projections $m_s$ and $M$. The actual values of these projections are

\[ m_s = \pm 1/2 \quad \text{for proton, neutron} \]
\[ M = -1/2 \quad \text{for weak transitions.} \]

The transition isospin $\vec{T}$ is defined by its matrix elements\(^\dagger\)

\[ (T_i)_{m_s M} = \sum_r C_{1/2 r 1/2 m_s}^{3/2 M} t_i^r. \]  

(8-25b)

The form (8-3), and analogous forms for $\eta$ and $K$ exchanges, automatically lead to a symmetric potential. Our final results, in the isospin dependent formalism, are displayed in Section 9. Some formal details are explained in Appendices C and D.

\[^\dagger\] Other details are given in Appendix C.
9. The Effective Potential

Methods described in Section 8. and Apendices A and G allow construction of the strangeness violating potential which corresponds to the exchange of the pseudoscalar mesons $\pi$, $K$ and $\eta$. Its radial dependence is described by

\[
V_C(M) = \frac{e^{-m_M r}}{4\pi r} \quad (9-1)
\]

\[
V_S(M) = \frac{1}{3} \left[ \frac{m_M^2}{4\pi r} e^{-m_M r} - \delta(r) \right] \quad (9-2)
\]

\[
V_T(M) = \frac{1}{3} \frac{m_M^2}{4\pi r} e^{-m_M r} \left[ 1 + \frac{3}{m_M r} + \frac{3}{(m_M r)^2} \right] \quad (9-3)
\]

\[
V_{PV}(M) = \frac{m_M}{4\pi r} e^{-m_M r} \left( 1 + \frac{1}{m_M r} \right) \quad (M = \{\pi, K, \eta\}). \quad (9-4a)
\]

According to Appendix G one can replace $m_M$ by $\epsilon_M = [m_M^2 - (m_\Lambda - m_N)^2/4]^{1/2}$. There is a simple derivative connection between different radial functions, i.e.:

\[
-\frac{\partial}{\partial r} V_C = V_{PV} \quad (9-4b)
\]

\[
\frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} V_C \right] = 3V_T = -3\frac{\partial}{\partial r} \left[ \frac{1}{r} V_{PV} \right] \quad (9-4b)
\]

The above radial parts get multiplied by spin-isospin components of the weak-strong vertices and corresponding amplitudes given in the former sections whose numerical values are given in Sec. 7. Therefore, written formally

\[
V_C \rightarrow V_C \cdot 0 \quad (9-5a)
\]

\[
V_S \rightarrow V_S(\{\pi, K, \eta\}) \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \cdot \{B_\pi, B_K, B_\eta\} \quad (9-5b)
\]

\[
V_T \rightarrow V_T(\{\pi, K, \eta\}) \cdot (\hat{S}_{12}) \cdot \{B_\pi, B_K, B_\eta\} \quad (9-5c)
\]

\[
V_{PV} \rightarrow V_{PV}(\{\pi, K, \eta\}) \cdot (\hat{r} \cdot \vec{\sigma}_2) \cdot \{A_\pi, A_K, A_\eta\}. \quad (9-5d)
\]

Here we introduced $\hat{S}_{12} = 3 \cdot (\vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2/3)$. The effective potential could be written in two different (but equivalent) ways: the first version displays the particle content of the different part of the potential, whereas the other version shows more explicitly its isospin character.

The particle content version of the effective potential has to be written in such a way that the particular channel of the $\Lambda - Nucleon$ interaction is specified. The isospin character version unifies both channels and enables us to write the effective potential as an operator in the isospace.

The particle content version is given first, for the $\Lambda + p \rightarrow p + n$ channel:
\[ V(\vec{r})_{p \rightarrow p n} = V_C(r) \cdot 0 \]
\[ + (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \cdot \left\{ V_S(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_\Lambda + m_N)} \left( 2\vec{a} + \sqrt{\frac{b}{3}} \right)^{\frac{1}{2}} \left[ \left( \vec{p}p_1 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_1 \right)(\vec{p}\Lambda)_1 \right] \]
\[ + V_S(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_\Lambda + m_N)} \left( -\vec{a} + \sqrt{\frac{b}{3}} \right)^{\frac{1}{2}} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \]
\[ + \frac{g_{KNN}}{2m_N(m_\Lambda + m_N)} \left( V_S(K^+)(\vec{c} + \vec{e}) \frac{1}{2} \left[ \left( \vec{p}p_1 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \right) \]
\[ + V_S(K^0)(\vec{d} + \vec{e}) \frac{1}{2} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \]
\[ + \frac{1}{2m_N(m_\Lambda + m_N)} \left( V_S(\eta_1) \cdot g_{\eta_1 NN} \vec{f}_1 \frac{1}{2} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \right) \]
\[ + V_S(\eta_2) \cdot g_{\eta_2 NN} \vec{g}_1 \frac{1}{2} \left( \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 \right) \right\} \]
\[ + S_{12} \cdot \left\{ V_T(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_\Lambda + m_N)} \left( 2\vec{a} + \sqrt{\frac{b}{3}} \right)^{\frac{1}{2}} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 \right] \]
\[ + V_T(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_\Lambda + m_N)} \left( -\vec{a} + \sqrt{\frac{b}{3}} \right)^{\frac{1}{2}} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 \right] \]
\[ + \frac{g_{KNN}}{2m_N(m_\Lambda + m_N)} \left( V_T(K^+)(\vec{c} + \vec{e}) \frac{1}{2} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \right) \]
\[ + V_T(K^0)(\vec{d} + \vec{e}) \frac{1}{2} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \]
\[ + \frac{1}{2m_N(m_\Lambda + m_N)} \left( \left[ V_T(\eta_1) \cdot g_{\eta_1 NN} \vec{f}_1 \frac{1}{2} \left[ \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 \right] \right) \right) \]
\[ + V_T(\eta_2) \cdot g_{\eta_2 NN} \vec{g}_1 \frac{1}{2} \left( \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_1 + \left( \vec{p}p_2 \right)(\vec{p}\Lambda)_2 \right) \right\} \]
\[ + \left\{ V_{PV}(\pi) \cdot g_{NN\pi} \right\}
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Here the quantities $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ etc. are vertices - weak (nonleptonic) amplitudes for the isoscalar and isotensor parts of the potential, and "bare" quantities like $a$, $b$, $c$ etc. are the corresponding parity-violating amplitudes. These quantities connect two different ways (versions, see above) in which the effective potential could be written. A particle exchanged could be inferred from the radial function $(V_k(\pi))$ for instance, for $k = T, S, PV$ multiplying the corresponding terms.

For the other channel the particle version is given

$$V(\tilde{r})|_{\Lambda \rightarrow n, n} = V_C(r) \cdot 0$$

$$+ (\sigma_1 \cdot \sigma_2) \cdot \left\{ \frac{g_{\pi NN}}{2m_N(m_\Lambda + m_N)} (V_S(\pi) \left( \tilde{a} - \sqrt{\frac{2}{3}} \tilde{b} \right) \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{g_{K\Lambda}}{2m_N(m_\Lambda + m_N)} V_S(K^0) (\tilde{c} + \tilde{d} - \tilde{e}) \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{1}{2m_N(m_\Lambda + m_N)} \left( g_{\eta_1 NN} V_S(\eta_1) \tilde{f} + g_{\eta_8 NN} V_S(\eta_8) \tilde{g} \right) 
\cdot \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] \right\}$$

$$+ \tilde{S}_{12} \cdot \left\{ \frac{g_{\pi NN}}{2m_N(m_\Lambda + m_N)} (V_{T}(\pi) \left( \tilde{a} - \sqrt{\frac{2}{3}} \tilde{b} \right) \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{g_{K\Lambda}}{2m_N(m_\Lambda + m_N)} V_T(K^0) (\tilde{c} + \tilde{d} - \tilde{e}) \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{1}{2m_N(m_\Lambda + m_N)} \left( g_{\eta_1 NN} V_T(\eta_1) \tilde{f} + g_{\eta_8 NN} V_T(\eta_8) \tilde{g} \right) 
\cdot \frac{1}{2} [((\bar{\pi}n)_2(\pi\Lambda)_1 + (\bar{\pi}n)_1(\pi\Lambda)_2)] \right\}$$

$$+ \left\{ \frac{g_{\pi NN}}{2m_N} V_{PV}(\pi) \left( a - \sqrt{\frac{2}{3}} b \right) \frac{1}{2} [- (\sigma_2 \tilde{r})(\bar{\pi}n)_2(\pi\Lambda)_1 + (\sigma_1 \tilde{r})(\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{g_{K\Lambda}}{2m_N} V_{PV}(K^0)(\tilde{c} + \tilde{d} - \tilde{e}) \frac{1}{2} [(\sigma_1 \tilde{r})(\bar{\pi}n)_2(\pi\Lambda)_1 + (\sigma_2 \tilde{r})(\bar{\pi}n)_1(\pi\Lambda)_2)] 
+ \frac{1}{2m_N} \left( g_{\eta_1 NN} V_{PV}(\eta_1) \tilde{f} + g_{\eta_8 NN} V_{PV}(\eta_8) \tilde{g} \right) 
\cdot \frac{1}{2} [- (\sigma_2 \tilde{r})(\bar{\pi}n)_2(\pi\Lambda)_1 + (\sigma_1 \tilde{r})(\bar{\pi}n)_1(\pi\Lambda)_2)] \right\}$$
To introduce the isospin formalism, as done already before in Sec. 8, one recalls the three different combinations which occur in the effective weak-strong interference Hamiltonian, i.e.

\[
\begin{align*}
\left( \mathbf{N} \mathbf{1} \Lambda \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right)_1 \left( \mathbf{N} \mathbf{1} \mathbf{N} \right)_2 &= \beta_1 \quad \Delta I = 1/2 \\
\left( \mathbf{N} \mathbf{1} \Lambda \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right)_1 \left( \mathbf{N} \mathbf{1} \mathbf{N} \right)_2 &= \beta_1 \quad \Delta I = 1/2 \\
\left( \mathbf{N} [\bar{T} \chi] \Lambda \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right)_1 \left( \mathbf{N} \mathbf{1} \mathbf{N} \right)_2 &= \beta_T \quad \Delta I = 3/2
\end{align*}
\]
Their particle content is

\[ \begin{align*}
\beta_1 &= (\pi\Lambda)_1[\bar{\nu}p_2 + (\nu n)_2] \\
\beta_r &= (\pi\Lambda)_1[(\bar{\nu}n)_2 - (\bar{\nu}p)_2] + 2(\nu\Lambda)_1(\nu p)_2 \\
\beta_T &= \sqrt{\frac{2}{3}} \{ (\pi\Lambda)_1(\pi p)_2 + (\pi\Lambda)_1[(\pi p)_2 - (\pi n)_2] \}.
\end{align*} \tag{9-9} \]

The *isospin explicit version* of the effective potential could now be written in such a way (for the PV exchange of the pions, for instance) by connecting both versions

\[ \begin{align*}
V_\pi &= a\beta_r + b\beta_T \\
&= a(\pi\Lambda)_1[(\pi p)_2 + (\pi n)_2] \\
&\quad + b\sqrt{\frac{2}{3}} \{ (\pi\Lambda)_1(\pi p)_2 + (\pi\Lambda)_1[(\pi p)_2 - (\pi n)_2] \} \\
&= (\pi\Lambda)_1(\pi n)_2 \left( a - \sqrt{\frac{2}{3}}b \right) \\
&\quad + (\pi\Lambda)_1(\pi p)_2 \left( -a + \sqrt{\frac{2}{3}}b \right) \\
&\quad + (\bar{\nu}\Lambda)_1(\bar{\nu}p)_2 \left( 2a + \sqrt{\frac{2}{3}}b \right). \tag{9-10} \end{align*} \]

Finally the *isospin explicit version* is given by
\begin{align}
\mathbf{V}(\vec{r}) &= V_C(\vec{r}) \cdot 0 \\
&+ (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \cdot \left\{ V_S(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_A + m_N)} \cdot \vec{a} \cdot \frac{1}{2}(\vec{r}_1 \cdot \vec{r}_2 + \vec{r}_1 \cdot \vec{r}_2) \\
&+ V_S(\pi) \cdot \frac{g_{NN\pi}}{2m_N(m_A + m_N)} \cdot \vec{b} \cdot \frac{1}{2}(\vec{T}_1 \cdot \vec{r}_2 + \vec{r}_1 \cdot \vec{T}_2) \\
&+ \frac{g_{KNN}}{2m_N(m_A + m_N)} V_S(K) \left( \frac{1}{2} \vec{c} + \vec{d} \right) \frac{1}{2} [(1_1 \cdot 1_2) + (1_1 \cdot 1_2)] \\
&+ V_S(K) \cdot \vec{c} \cdot \frac{1}{2}(\vec{T}_1 \cdot \vec{r}_2 + \vec{r}_1 \cdot \vec{T}_2) \\
&+ V_S(K) \cdot \vec{c} \cdot \frac{1}{2}(\vec{T}_1 \cdot \vec{r}_2 + \vec{r}_1 \cdot \vec{T}_2) \right\} \\
&+ \frac{1}{2m_N(m_A + m_N)} \left\{ V_T(\pi) \cdot g_{NNNN} \cdot \vec{\tau} + V_T(\pi) \cdot g_{NNNN} \cdot \vec{\tau} \right\} \\
&+ \frac{1}{2m_N(m_A + m_N)} \left( V_T(\eta_1) \cdot g_{NNNN} \cdot \vec{\tau} + V_T(\eta_1) \cdot g_{NNNN} \cdot \vec{\tau} \right) \\
&+ V_T(\pi) \cdot g_{NNNN} \cdot \vec{\tau} + V_T(\pi) \cdot g_{NNNN} \cdot \vec{\tau} \right\} \\
&+ \left\{ V_PV(\pi) \cdot \frac{g_{NN\pi}}{2m_N} \cdot \vec{a} \cdot \frac{1}{2}(-\vec{r}_2 \cdot \vec{r}_1) \cdot \vec{r}_2 + (\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{r}_2 \\
&+ V_P(\pi) \cdot \frac{g_{NN\pi}}{2m_N} \cdot \vec{b} \cdot \frac{1}{2}(-\vec{r}_2 \cdot \vec{r}_1) \cdot \vec{r}_2 + (\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{r}_2 \\
&+ \frac{g_{KNN}}{2m_N} V_P(K) \left( \frac{1}{2} \vec{c} + \vec{d} \right) \frac{1}{2} [(\vec{r}_1 \cdot \vec{r}_2) \cdot (1_1 \cdot 1_2) - (\vec{r}_2 \cdot \vec{r}_1) \cdot (1_1 \cdot 1_2)] \\
&+ V_P(K) \cdot \vec{c} \cdot \frac{1}{2}(\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{T}_2 + (\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{T}_2 \\
&+ V_P(K) \cdot \vec{c} \cdot \frac{1}{2}(\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{T}_2 + (\vec{r}_1 \cdot \vec{r}_2) \cdot \vec{T}_2 \right\} \\
&+ \frac{1}{2m_N} \left( V_S(\eta_1) \cdot g_{NNNN} \cdot \vec{\tau} + V_S(\eta_1) \cdot g_{NNNN} \cdot \vec{\tau} \right) \right\}
\end{align}
Appendix A: Meson Poles and Double Counting

Baryonic matrix element of a general axial vector current has the form (6-2), i.e.

\[
\langle B_\beta | A_\mu^a(0) | B_\alpha \rangle = \overline{u}(p') \left[ \gamma^\mu \gamma_5 g_A(q^2) + \frac{1}{M_\alpha + M_\beta} g_P(q^2) q^\mu \gamma_5 \right] \Lambda_a u_\alpha(p). \tag{A-1}
\]

Here \( \Lambda_a \) is some matrix (for example \( \lambda_a/2 \)) which describes the internal (for example SU(3) flavour) hadron symmetry. Its precise form is not needed here.

Meson matrix element is

\[
\langle 0 | A_\mu^a(0) | M_b \rangle = i \delta_{ab} M q^\mu. \tag{A-2}
\]

If the meson is on the mass shell, then

\[
q^2 = m_M^2. \tag{A-3a}
\]

The PCAC relation is

\[
\partial_\mu A_\mu^a = C \Phi_a. \tag{A-4}
\]

It leads to

\[
\langle 0 | \partial_\mu A_\mu^a(x) | M_a \rangle = \partial_\mu \left[ e^{-iqx} \langle 0 | A_\mu^a(0) | M_a \rangle \right] = (-i) e^{-iqx} q_\mu \langle 0 | A_\mu^a(0) | M \rangle
= q^2 f_M(q^2) e^{-iqx} = C(q^2) e^{-iqx}. \tag{A-5}
\]

If the meson is on the mass shell, one should select

\[
C(m_M^2) = m_M^2 f_M. \tag{A-3b}
\]
Also

\[
\partial_\mu \langle B_\beta | A_\alpha^\mu(x) | B_\alpha \rangle = \partial_\mu \left[ e^{i(p'-p)x} \langle B_\beta | A_\alpha^\mu(0) | B_\alpha \rangle \right]
\]

\[
= i e^{i(p'-p)x} \bar{u}_\beta(p') \left[ g_A(p - p') - \frac{q^2}{M_\alpha + M_\beta} g_P \right] \gamma_5 \Lambda_a u_\alpha(p)
\]

\[
= (-i) e^{i(p'-p)x} \bar{u}_\beta(p') \left[ (M_\alpha + M_\beta) g_A + \frac{q^2}{M_\alpha + M_\beta} g_P \right] \gamma_5 \Lambda_a u_\alpha(p)
\]

\[
= C \langle B_\beta | \Phi_a(x) | B_\alpha \rangle
\]

\[
= C e^{i(p'-p)x} \langle B_\beta | \Phi_a(0) | B_\alpha \rangle
\]

\[
= C e^{-iqx} \langle B_\beta | \Phi_a(0) | B_\alpha \rangle.
\] (A-6)

The matrix element \( \langle B_\beta | \Phi_a | B_\alpha \rangle \) can be calculated by using the equation of motion \([33]\)

\[
(\Box + m^2_M) \Phi_a(x) = -ig_{\alpha\beta} \bar{\psi}_\beta(x) \gamma_5 \Lambda_a \psi_\beta(x),
\] (A-7)

which leads to

\[
(-q^2 + m^2_M) \langle B_\beta | \Phi | B_\alpha \rangle = -ig_{\alpha\beta} \bar{u}_\beta \gamma_5 \Lambda_a u_\alpha.
\] (A-8)

Here \( g_A, g_P, g_{\alpha\beta} \) and \( C \) are in general some functions of \( q^2 \).

By inserting (A-8) in (A-6) and dropping unimportant factors, one finds a relation

\[
(M_\alpha + M_\beta) g_A(q^2) + \frac{q^2}{M_\alpha + M_\beta} g_P(q^2) = m^2_M f_M \frac{g_{\alpha\beta}(q^2)}{m^2_M - q^2}.
\] (A-9)

In the limit \( q^2 \to 0 \), assuming that

\[
g_{\alpha\beta}(0) = g_{\alpha\beta}(m^2_M) = g_{\alpha\beta}
\]

\[
g_A(0) = g_A = 1.25
\] (A-10)

one finds the famous Goldberger-Treiman (GT) relation (generalized):

\[
\frac{g_A}{f_M} = \frac{g_{\alpha\beta}}{M_\alpha + M_\beta}.
\] (A-11)
Fig. A.1 - Here $g_{\alpha\beta}$ appears in the strong vertex. The axial vector current acts on the intermediate meson $M$, as indicated.

The induced pseudoscalar formfactor $g_P$ is dominated by the pseudoscalar meson (for example pion) pole. This is described by the diagram shown in Fig. A.1.

The diagram in Fig. A.1 corresponds to

\[
(Fig. A.1)^\mu = \bar{u}_\beta(p')\gamma_5\Lambda_\alpha u_\alpha(p)g_{\alpha\beta}(q^2)\frac{i}{q^2 - m_M^2 + i\epsilon}\langle M_b| A^\mu_b |0\rangle. \tag{A-12}
\]

Inserting (A-2) and comparing the result with (A-1) one finds

\[
\frac{g_P(q^2)}{M_\alpha + M_\beta} \simeq (-)g_{\alpha\beta}(q^2)f_M(q^2)\frac{i}{q^2 - m_M^2}. \tag{A-13a}
\]

Inclusion of vertex and other corrections, such as multipion exchanges [6], turns that into

\[
\frac{g_P(q^2)}{M_\alpha + M_\beta} = H(q^2) + \frac{g_{\alpha\beta}(q^2)f_M(q^2)}{m_M^2 - q^2}F(q^2). \tag{A-13b}
\]

If one neglects $H$ and assumes

\[
g_{\alpha\beta}(q^2) \simeq g_{\alpha\beta}(m_M^2)
\]
\[
f_M(q^2) \simeq f_M(m_M^2) = f_M
\]
\[
F(q^2) \simeq 1,
\]

(A-13c)
one finds the well-known estimate

\[
\frac{g_P(q^2)}{M_\alpha + M_\beta} \simeq \frac{g_{\alpha\beta} f_M}{m_M^2 - q^2}.
\] (A-14)

For the nucleon (N)-pion (\(\pi\)) system one finds

\[
g_P \sim \frac{4M_N^2 g_A}{m_\pi^2 - q^2}.
\] (A-15)

In the case of the muon capture the effective pseudoscalar constant becomes

\[
\frac{g_P}{2M_N} \cdot m_\mu = \frac{2M_N m_\mu g_A}{m_\pi^2 - q^2(m_\mu)} \simeq \frac{2M_N m_\mu g_A}{m_\pi^2 + 0.88 \cdot m_\mu^2}.
\] (A-16)

Here the pion mass \(m_\mu\) comes from the lepton current

\[
q_\rho \bar{\psi}_\nu \gamma^\rho \gamma_5 \psi_\mu = (-)m_\mu \bar{\psi}_\nu \gamma_5 \psi_\mu.
\] (A-17)

The expression (A-16) corresponds to the formula (4.37) in [13]. (Their \(g_P\) is not equal to our \(g_P\) (A-15).)

The GT relation (A-11) and most of the following formulae were derived by using (A-3) convention. Some particular questions that might arise when the meson \(\Phi_a\) is not on the mass shell will be mentioned in the main text.
Appendix B: Commutators Involving Bilinear Quark Field Combinations

The canonical anti-commutation relations for Dirac (i.e. quark) fields are [34]

\[ \{ \psi_\alpha(t, x), \psi_\beta^\dagger(t, y) \} = \delta_{\alpha\beta}\delta(x - y). \]  \hspace{1cm} (B-1)

When applying current algebra (CA) on PCAC one encounters the commutators of the bilinear combinations of quark fields such as

\[ [\psi_\phi^\dagger(z)\Delta_\phi\psi_\epsilon(z), \psi_\alpha^\dagger(x)\Gamma_{\alpha\beta}\psi_\beta(x)]|_{z_0=x_0} = C. \]  \hspace{1cm} (B-2)

Here \( \Delta \) and \( \Gamma \) are some products of Dirac \( \gamma \)-matrices and the inner group operators corresponding to SU(3) flavor and color. By repeatedly using (B-1) one can write

\[ \psi_\phi^\dagger \psi_\epsilon = \psi_\phi^\dagger \psi_\epsilon + \delta_\epsilon_\alpha \psi_\phi^\dagger \psi_\beta \delta(x - z) \]

\[ - \delta_\beta_\alpha \psi_\phi^\dagger \psi_\epsilon \delta(x - z). \]  \hspace{1cm} (B-3)

This leads to

\[ C = [\psi_\phi^\dagger \Delta_\phi \Gamma_\epsilon \psi_\epsilon - \psi_\alpha^\dagger \Gamma_{\alpha\beta} \Delta_\beta \psi_\beta] \delta(x - z) \]

\[ = \psi^+(x)[\Delta, \Gamma]\psi(x)\delta(x - z). \]  \hspace{1cm} (B-4)

Let us take \( \Delta \) and \( \Gamma \) of the form

\[ \Delta = D t_i \]

\[ \Gamma = G t_j; \quad t_i = \lambda_i/2 \]  \hspace{1cm} (B-5a)

Here \( \lambda_i \) are SU(3)-flavour matrices while \( D \) and \( G \) are some combinations of Dirac matrices. One can write

\[ [t_i D, t_j G] = \frac{1}{2} [t_i, t_j] [D, G] + \frac{1}{2} [t_i, t_j] [D, G]. \]  \hspace{1cm} (B-5b)

It is useful to specify that for some cases:

(1)

\[ D = \gamma_5, \quad G = \gamma_0 \Gamma_{L,R}^\mu \]

\[ \Gamma_{L,R}^\mu = \gamma_\mu (1 \mp \gamma_5). \]  \hspace{1cm} (B-5c)

This gives

\[ [D, G] = \gamma_5 \gamma_0 \Gamma^\mu - \gamma_0 \Gamma^\mu \gamma_5 = 0 \]

\[ \{D, G\} = 2\gamma_5 \gamma_0 \Gamma^\mu \]  \hspace{1cm} (B-5d)

\[ [t_i, t_j] = if_{ijk} t_k \]
\[ D = 1, \quad G = \gamma_0 \Gamma_{L,R} \]

\[ [D, G] = \gamma_0 \Gamma^\mu - \gamma_0 \Gamma^\mu = 0 \]  \hspace{1cm} (B-5e)

\[ \{D, G\} = 2\gamma_0 \Gamma^\mu \]

\[ D = \gamma_5, \quad G_{L,R} = \gamma_0 (1 \mp \gamma_5) \]

\[ [D, G_{L,R}] = \gamma_5 \gamma_0 (1 \mp \gamma_5) - \gamma_0 (1 \mp \gamma_5) \]

\[ = 2\gamma_5 \gamma^0 (1 \mp \gamma_5) \]  \hspace{1cm} (B-6)

\[ \{D, G\} = \gamma_5 \gamma_0 (1 \mp \gamma_5) + \gamma_0 (1 \mp \gamma_5) \gamma_5 = 0. \]

The relations (B-4) and (B-5) can be used to derive CA \cite{44, 45}. With

\[ j_\mu^i = \bar{\psi} \gamma^\mu t_i \psi \]

\[ j_\mu^5 = \bar{\psi} \gamma^\mu \gamma_5 t_i \psi, \]  \hspace{1cm} (B-7a)

one finds

\[ [j_0^i(x), j_j^\mu(y)]_{x_0=y_0} = i f_{ijk} j_k^\mu(x) \delta(x - y) \]

\[ [j_0^i(x), j_j^5(y)]_{x_0=y_0} = i f_{ijk} j_k^5(x) \delta(x - y) \]  \hspace{1cm} (B-7b)

\[ [j_0^5(x), j_j^\mu(y)]_{x_0=y_0} = i f_{ijk} j_k^\mu(x) \delta(x - y) \]

\[ [j_0^5(x), j_j^5(y)]_{x_0=y_0} = i f_{ijk} j_k^5(x) \delta(x - y) \]

This can be easily transformed into the commutation rules for the SU(3) generators

\[ F_i = \int d^x j_0^i(x) \]  \hspace{1cm} (B-8a)

and the axial vector charges

\[ F_i^5 = \int d^x j_0^5(x). \]  \hspace{1cm} (B-8b)

Integration over \( x \) simply removes delta-functions in (B-7).

General structure of the operators \( O_1, O_2, O_3 \) and \( O_4 \) contains terms as

\[ j_{\mu\kappa}(x) j_{\epsilon}^{\mu}(x) + j_{\mu\kappa 5}(x) j_{\epsilon}^{\mu 5}(x) = b(PC) \]

\[ j_{\mu\kappa 5}(x) j_{\epsilon}^{\mu}(x) + j_{\mu\kappa 5}(x) j_{\epsilon}^{\mu 5}(x) = a(PV). \]  \hspace{1cm} (B-9)
Quark fields $\psi$ in those terms are not normally ordered. (Some details about that statement can be found in Appendix E.) Thus one can simply apply (B-7) and (B-8) in order to show

$$[F_i, O_A(PV)] = -[F_i, O_A(PC)] \quad (A = 1, 2, 3, 4). \quad (B-10a)$$

Operators $O_5$ and $O_6$ as shown in Appendix E contain VEV’s when written without normal ordering. Obviously

$$[F_i, \tilde{O}_B(PV)] = -[F_i, \tilde{O}_B(PC)] \quad (B = 1, 2, 3, 4). \quad (\tilde{O} \text{ is not NOP}) \quad (B-10b)$$

In addition one has to calculate the commutator with the term $\bar{d}(1 \pm \gamma_5)s$. One finds (with $\psi = (u, d, s)$):

$$[F_3^5, \bar{d}(1 - \gamma_5)s] = -\frac{1}{2}\bar{d}(1 - \gamma_5)s$$

$$[F_3^5, \bar{d}(1 + \gamma_5)s] = \frac{1}{2}\bar{d}(1 + \gamma_5)s$$

$$[F_3, \bar{d}(1 - \gamma_5)s] = -\frac{1}{2}\bar{d}(1 - \gamma_5)s$$

$$[F_3, \bar{d}(1 + \gamma_5)s] = -\frac{1}{2}\bar{d}(1 + \gamma_5)s$$

(B-11)

Thus one can write†

$$[F_3^5, \hat{O}_6] = [F_3^5, \tilde{O}_6] - \frac{2}{6}\langle 0 | \bar{d}d | 0 \rangle \bar{d}(1 - \gamma_5)s + \frac{2}{6}\langle 0 | \bar{s}s | 0 \rangle \bar{d}(1 + \gamma_5)s$$

$$[F_3, \hat{O}_6] = [F_3, \tilde{O}_6] - \frac{2}{6}\langle 0 | \bar{d}d | 0 \rangle \bar{d}(1 - \gamma_5)s - \frac{2}{6}\langle 0 | \bar{s}s | 0 \rangle \bar{d}(1 + \gamma_5)s.$$  

(B-12a)

With

$$[F_3^5, \tilde{O}_6] = -[F_3, \tilde{O}_6], \quad L_d = \langle 0 | \bar{d}d | 0 \rangle \bar{d}(1 - \gamma_5)s, \quad L_s = \langle 0 | \bar{s}s | 0 \rangle \bar{d}(1 + \gamma_5)s, \quad (B-12b)$$

one can write

$$[F_3^5, \hat{O}_6] = [F_3^5, \tilde{O}_6] - \frac{2}{6}L_d + \frac{2}{6}L_s$$

$$= -[F_3, \tilde{O}_6] - \frac{2}{6}L_d - \frac{2}{6}L_s - \frac{2}{6}L_d + \frac{2}{6}L_s$$

$$= -[F_3, \tilde{O}_6] - \frac{2}{3}\langle 0 | \bar{d}d | 0 \rangle \bar{d}(1 - \gamma_5)s.$$  

(B-12c)

† See Appendix E for the definition of $\hat{O}$. 

This is the formula (17) in ref. [21] (up to the different sign convention). The operator $\hat{O}_5$ satisfies the same relation with the replacement

$$L_d \rightarrow \frac{16}{3}L_d.$$  \hfill (B-12d)

Everything can be generalized for kaon emission by selecting appropriate indices in (B-11).
Appendix C: Transition Isospin $\vec{T}$

This formalism, suitable for isospin $I = 3/2$ (or spin $S = 3/2$) particles is described in Ref.s [42] and [43].

An isospin $I = 3/2$ object can be constructed by combining the isovectors$^\dagger$

$$t^1 = \frac{1}{\sqrt{2}}(1, i, 0), \quad t^{-1} = \frac{1}{\sqrt{2}}(1, -i, 0), \quad t^3 = (0, 0, 1),$$

(C-1)

with the isospin function $\chi_{1/2}^m$. One finds

$$Z_i^M = \sum_{r,m} C_{1r1/2m}^3 t^r_i \chi_{1/2}^m. \quad (i = 1, 2, 3)$$

(C-2)

A coupling with isospin $I = 1$ field (for example pion $\pi$) is now

$$N_{m_s}^+ \pi_i Z_i^M = \chi_{1/2}^{m_s} \sum_{r,m} C_{1r1/2m}^3 \chi_{1/2}^m t^r \vec{\pi}. \quad (C-3)$$

Here $N_{m_s}$ is either proton ($m_s = 1/2$) or neutron ($m_s = -1/2$), and $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$. One can introduce transition isospin $\vec{T}$ which is defined by its matrix element$^\ddagger$:

$$\langle \vec{T} \rangle_{m_s M} = C_{1r1/2m_s}^3 t^r \cdot \vec{\pi}. \quad (C-4)$$

Then (C-3) can be written as

$$N_{m_s}^+ \langle \vec{T} \cdot \vec{\pi} \rangle Z_i^M (3/2) = F(m_s M) = \chi_{1/2}^{m_s} \sum_{r,m} C_{1r1/2m}^3 \chi_{1/2}^m t^r \vec{\pi}.$$

(C-5a)

(Here $\chi^{m_s^\dagger} \chi^m = \delta_{m_s m}$.) As an example let us take an object with $M = -1/2$. Then for $m_s = -1/2$ one gets

$$\sum_{r} C_{1r1/21/2}^{3/2} = C_{101/21/2}^{3/2} = \sqrt{\frac{2}{3}}.$$

(C-5b)

$$F(-1/2, -1/2) = \sqrt{\frac{2}{3}} t^0 \cdot \vec{\pi} = \sqrt{\frac{2}{3}} \pi^0.$$

$^\dagger$ For spin Ref. [42] uses $\epsilon^+ = -(1, i, 0)/\sqrt{2}$. The $+$ sign here leads to the usual pion field definition $\pi^{\pm} = (\pi_1 \mp i\pi_2)/\sqrt{2}$.

$^\ddagger$ Here again we have small difference with the Ref. [42] convention: our $\vec{T} = \vec{T}^*$. Ref. [42]. In our case $I = 3/2$ object will always be on the R.H.S. in bilinear combination (C-3).
and
\[ \pi Z^{-1/2}(3/2) \cdot \pi^0 = \mathcal{F}(-1/2, -1/2) \] (C-6)

The expression (C-6) is the isospin conserving coupling between \( I = 1/2 \) \( \mathcal{N}(\bar{p}, \pi) \) field, \( I = 3/2 \mathcal{E} \) field and \( \pi \) (pion) field.

One also finds for \( m_2 = 1/2 \):
\[
\sum_r C_r^{3/2 - 1/2} = C_1^{3/2 - 1/2} = C_1^{1/2 - 1/2} = \frac{1}{\sqrt{3}}
\]
\[ F(1/2, -1/2) = \frac{1}{\sqrt{3}} \vec{t}^{-1} \cdot \vec{\pi} = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\pi_1 - i\pi_2) = \frac{1}{\sqrt{3}} \pi^+, \] (C-7a)

and
\[ \bar{p} Z^{-1/2}(3/2) \cdot \pi^+ = \mathcal{F}(1/2, -1/2) \]
\[ \mathcal{F}(m_s, M) = \chi^{m_s \dagger} Z^M \cdot F(m_s, M) \] (C-7b)
\[ m_s = 1/2 \quad \text{proton} \]
\[ m_s = -1/2 \quad \text{neutron} \]

Remark: the same formalism will be used for weak vertices \( \mathcal{N}\Lambda\pi \) where \( \Lambda \) will be given the quasi isospin \( I = 1/2 \) and \( I = 3/2 \).
Appendix D: $\Delta I = 3/2$ Isospin Change

The pion exchange contributions contain very small $\Delta I = 3/2$ pieces. This reflects the fact that the experimental $\Lambda \to N\pi$ decay amplitudes contain very very small $\Delta I = 3/2$ contributions. For example, from $B(\Lambda^0_0)_{\text{exp}} = 22.4$ and $B(\Lambda^0_0)_{\text{exp}} = -15.61$ (in units of $10^{-7}$) one can deduce

$$-B(\Lambda^0_0)/\sqrt{2} = -15.84$$

$$\left| \frac{B(\Lambda^0_0)/\sqrt{2} - B(\Lambda^0_0)}{B(\Lambda^0_0)} \right| = 0.02$$

Somewhat different picture emerges in the case of weak $NNK$ couplings, whose values are shown in Table D.1. Our results were obtained by including separable contributions (see Table 7.4) which contain $\Delta I = 3/2$ piece. Experimental values were used in pole terms, and they had also $\Delta I = 3/2$ pieces. The relative impotrance of the $\Delta I = 3/2$ terms can be directly seen from the general weak potential (9-11). One can obtain some interesting information from Table D.1 also by checking how well is the $\Delta I = 1/2$ sum rule (D-1) satisfied

$$F(p_0^+ + p_0^+) = \Sigma(F)$$

$$\Sigma(F) = F(n_0^0)$$

$$F = A, B.$$ (D-1)

A useful measure of the discrepancy is

$$D(F) = \left| \frac{\Sigma(F) - F(n_0^0)}{\Sigma(F)} \right|$$
In our case one finds (in $10^{-7}$ units)
\[
\Sigma(A) = 5.39 \\
A(n^0_0) = 6.25 \\
D(A) = 0.16 \\
\Sigma(B) = 25.73 \\
B(n^0_0) = 17.99 \\
D(B) = 0.30
\]

The amplitudes of Ref. [5] were obtained with the assumption that $\mathcal{H}_W$ contains only $\Delta I = 1/2$ piece. Thus from Table D.1 one finds
\[
D(A) = 0; \quad D(B) = 0. \quad (D-4)
\]

In the chiral Lagrangian approach of ref. [22] the $\Delta I = 3/2$ contributions are very small
\[
D(A) = 0.002, \quad D(B) = 0.014. \quad (D-5)
\]

They have also used the $\Delta I = 1/2$ Hamiltonian.

♠ ♠ ♠ ♠ ♠
Appendix E: Commutators and Normal-ordered Operators

The product of fields appearing in the four-quark operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\ldots$, $\mathcal{O}_6$ comprising the effective weak Hamiltonian are normal-ordered. Thus one has to be careful when calculating the commutators which appear in evaluation of the current algebra contributions (CAC). The current commutators, or more general, the commutators of bilinear quark-field forms, are best used if the operators $\mathcal{O}_i$ are written in the form which is no longer normally ordered. (Some details about commutators can be found in Section 5.)

The task of unscrambling the normally ordered product (NOP) can be achieved by using

(a) Wick’s theorem for normal-ordered products (WT) [34],

(b) Fierz transformation (FIT) which has been described in Sec.3.

The WT for a product of four quark fields $\psi_\alpha$ (here $\alpha$ denotes all indices: Dirac’s components, SU(3) flavour, SU(3) colour, etc.) is:

$$
\overline{\psi}_\alpha \psi_\beta \overline{\psi}_\gamma \psi_\delta = : \overline{\psi}_\alpha \psi_\beta \overline{\psi}_\gamma \psi_\delta : + \overline{\psi}_\alpha \psi_\beta : \psi_\gamma \psi_\delta : + \psi_\alpha \overline{\psi}_\beta : \psi_\gamma \psi_\delta + \cdots + \overline{\psi}_\alpha \psi_\delta : \psi_\beta \psi_\gamma : + \overline{\psi}_\beta \psi_\gamma : \overline{\psi}_\alpha \psi_\delta : 
$$

(E-1a)

Here $\mathcal{O}$ symbolizes the normal-ordering and the lower sign the vacuum expectation value (VEV). It means the following

$$
\overline{\psi}_\alpha \psi_\beta = \langle 0 | \overline{\psi}_\alpha \psi_\beta | 0 \rangle.
$$

(E-1b)

Only the first row is important for the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$. For the operators $\mathcal{O}_5$ and $\mathcal{O}_6$ one has to consider the first and the last row only.
This follows from a physical fact that VEV’s for non-scalar (be it in the Lorentz-space, or in the ”inner”-space, i.e. SU(3) etc.) quantities have to vanish.

This means

\[ \langle 0 | \overline{\psi}_\ell \Gamma^\mu_{L,R} \psi_k | 0 \rangle = 0 \]  \hspace{1cm} (E-2a)

\[ \Gamma^\mu_{L,R} = \gamma^\mu (1 \mp \gamma_5) \]

and

\[ \langle 0 | \overline{\psi}_m \Lambda^m \psi_p | 0 \rangle = 0 \]  \hspace{1cm} (E-2b)

Any quark combination appearing in the operators \( O_1 - O_4 \) of the form

\[ \overline{\psi}_\alpha (\Gamma^\mu_L)_{\alpha \beta} \psi_\beta \overline{\psi}_\gamma (\Gamma_\mu L)_{\gamma \delta} \psi_\delta \]  \hspace{1cm} (E-3a)

FIT produces an equality

\[ \left( \overline{\psi}_\alpha (\Gamma^\mu_L)_{\alpha \beta} \psi_\beta \right) \left( \overline{\psi}_\gamma (\Gamma_\mu L)_{\gamma \delta} \psi_\delta \right) = \left( \overline{\psi}_\alpha (\Gamma^\mu_L)_{\alpha \delta} \psi_\delta \right) \left( \overline{\psi}_\gamma (\Gamma_\mu L)_{\gamma \beta} \psi_\beta \right) . \]  \hspace{1cm} (E-3b)

Thus the last term in (E-1a), for example, can be written as

\[ \langle 0 | \overline{\psi}_\alpha (\Gamma^\mu_L)_{\alpha \delta} \psi_\delta | 0 \rangle \cdot \langle 0 | \overline{\psi}_\gamma (\Gamma_\mu L)_{\gamma \beta} \psi_\beta | 0 \rangle = 0. \]  \hspace{1cm} (E-4)

The same conclusion can be drawn for all other terms which contain VEV’s. When one calculates commutators involving the operators \( O_{1,2,3,4} \), NOP associated complications do not appear.

However FIT \( O_5 \) and \( O_6 \) contain scalar and pseudoscalar quantities so that their NOP is not trivial. From

\[ \left( \overline{\psi}_\alpha (\Gamma_\mu L)_{\alpha \beta} \psi_\beta \right) \left( \overline{\psi}_\gamma (\Gamma^\mu_L)_{\gamma \delta} \psi_\delta \right) = (-2)(\overline{\psi}_\alpha (1 + \gamma_5)_{\alpha \delta} \psi_\delta) (\overline{\psi}_\alpha (1 - \gamma_5)_{\gamma \beta} \psi_\beta) \]  \hspace{1cm} (E-5)

and from (E-1) one obtains

\[ : \left( \overline{\psi}_\alpha (\Gamma_\mu L)_{\alpha \beta} \psi_\beta \right) \left( \overline{\psi}_\gamma (\Gamma^\mu_L)_{\gamma \delta} \psi_\delta \right) : \hspace{1cm} (\overline{\psi}_\alpha (\Gamma_\mu L)_{\alpha \beta} \psi_\beta) (\overline{\psi}_\gamma (\Gamma^\mu_L)_{\gamma \delta} \psi_\delta) \]

\[ + 2 : \left( \overline{\psi}_\alpha (1 + \gamma_5)_{\alpha \delta} \psi_\delta \right) \langle 0 | \overline{\psi}_\gamma \psi_\beta | 0 \rangle \]  \hspace{1cm} (E-6)

\[ + 2 : \left( \overline{\psi}_\gamma (1 - \gamma_5)_{\gamma \beta} \psi_\beta \right) \langle 0 | \overline{\psi}_\alpha \psi_\delta | 0 \rangle . \]
Once this is specified for $\hat{O}_6^\sharp$, one has to take care of color indices too. The spinors were actually coupled in color sectors as follows
\[
(\bar{\psi}^i_\alpha \psi^j_\beta)(\bar{\psi}^j_\gamma \psi^i_\delta).
\] (E-7a)

The rearrangement (E-6) turns that into
\[
(\bar{\psi}^j_\alpha \psi^i_\delta)\langle 0 | \bar{\psi}^j_\gamma \psi^i_\beta | 0 \rangle
\] (E-7b)
as only the color singlet can have VEV, one obtains
\[
(\bar{\psi}^j_\alpha \psi^i_\delta)\frac{1}{3}\delta_{ij}\langle 0 | \bar{\psi}^k_\gamma \psi^k_\beta | 0 \rangle.
\] (E-7c)

Finally this gives
\[
\hat{O}_6 = \hat{O}_6 + \frac{2}{3}\langle 0 | \bar{d}d | 0 \rangle (1 - \gamma_5)s
\]
\[
+ \frac{2}{3}\langle 0 | \bar{s}s | 0 \rangle (1 + \gamma_5)s.
\] (E-8)

Here $\hat{O}_6^\♭$ is the $\hat{O}_6$ operator which is not normal-ordered. This result is in full qualitative agreement with Ref. [21], formula (33). (One has to take into account that $\gamma_5(\text{here}) = -\gamma_5([21])$. The analogous result can be obtained for $\hat{O}_5$ which contains SU(3) color matrices $\lambda_A$. They satisfy the equality (3-6)
\[
\sum_A (\lambda_A)_{ab}(\lambda_A)_{cd} = \frac{16}{9}\delta_{ad}\delta_{cb} - \frac{1}{3}(\lambda)_{ad}(\lambda)_{cb}.
\] (E-9)

FIT of $\hat{O}_5$ in the Lorentz-space has to be combined with (E-9). It means that in the Lorentz-space $\hat{O}_5$ must have the same form as $\hat{O}_6$ (E-7), but one has to introduce (E-9) into the second and third term on the right-hand-side (RHS) of (E-8). The equality (E-2b)
\[
\text{It is defined by } \hat{O}_6^\sharp =: (\overrightarrow{\bar{d}}^\mu \Gamma_L^\mu|\overrightarrow{s}\Gamma_R^\mu R u + \overrightarrow{\bar{d}}\Gamma_R^\mu d + \overrightarrow{s}\Gamma_R^\mu R s) :.
\]
\[
\text{Here we are using the definition of } \hat{O}_{5,6} \text{ which differs, with respect to Ref. [11] for instance, by a factor 4, i.e. } \hat{O}_{5,6}(\text{here}) = 4 \cdot \hat{O}_{5,6}(\text{Ref.[11]}).
\]
means that only the first RHS term in (E-9) contributes. Thus VEV terms in (E-8) are multiplied by 16/3.

\[
\hat{O}_5 = \hat{O}_5 + \frac{32}{9} \langle 0 | \bar{d}d | 0 \rangle d(1 - \gamma_5) s \\
+ \frac{32}{9} \langle 0 | \bar{s}s | 0 \rangle d(1 + \gamma_5) s.
\]  

(E-10)

The expression (E-10) agrees fully with the formula (33) of ref. [21].

Some additional formal manipulations are shown in Appendix B.
Appendix F: Isospin and/or Baryon Decomposition of The Weak Potential

The isospin decomposition of an effective weak two particle potential acting among baryons depends on the isospin decomposition of the weak vertices. The amplitudes corresponding to $\Lambda \rightarrow B + \pi$ transitions, can be parametrized for $\Delta I = 1/2$ transition as

$$\Lambda \rightarrow n + \pi^0; \quad f(\Lambda^0_n)$$

$$f(\Lambda^0_n) = \frac{1}{\sqrt{3}} \alpha \quad |\Delta I| = 1/2$$

$$\Lambda \rightarrow p + \pi^-; \quad f(\Lambda^-_p)$$

$$f(\Lambda^-_p) = -\sqrt{\frac{2}{3}} \alpha \quad (F-1)$$

Here we have displayed only particle content and isospin properties. The same relations (F-1) hold for both $s$–wave and $p$–wave contributions.

The $|\Delta I| = 3/2$ transitions are parametrized as

$$g(\Lambda^0_n) = \sqrt{\frac{2}{3}} \beta \quad |\Delta I| = 3/2 \quad (F-2)$$

$$g(\Lambda^0_p) = \sqrt{\frac{1}{3}} \beta.$$  

The weak amplitudes corresponding to $N \rightarrow N + K$ transitions can be parametrized as follows

$$n \rightarrow n + K^0; \quad f(n^0_n)$$

$$f(n^0_n) = \delta$$

$$p \rightarrow p + K^0; \quad f(p^+_0)$$

$$f(p^+_0) = \frac{1}{2}(\xi + \delta) \quad |\Delta I| = 3/2 \quad (F-3)$$

$$p \rightarrow n + K^+; \quad f(p^+_n)$$

$$f(p^+_n) = \frac{1}{2}(-\xi + \delta)$$
The only weak vertex from which $\eta$ is emitted corresponds to the process $\Lambda \rightarrow n + \eta$. Here only $|\Delta I| = 1/2$ piece of $\mathcal{H}_W$ can contribute. Thus the weak potential due to the $\eta$ exchange satisfies the same selection rule.

The potential due to a pion exchange, which corresponds to a diagram analogous to Fig.1.1, has the following particle (i.e. baryonic) contents:

$$
\begin{align*}
&(\bar{n}\Lambda)_1 \pi^0 \pi^0 (\bar{n}n)_2 S \cdot \frac{1}{\sqrt{3}} \alpha \cdot (-g_{NN\pi}) \\
&(\bar{n}\Lambda)_1 \pi^0 \pi^0 (\bar{p}p)_2 S \cdot \frac{1}{\sqrt{3}} \alpha \cdot g_{NN\pi} \\
&(\bar{p}\Lambda)_1 \pi^- \pi^- (\bar{n}p)_2 S \cdot (-1) \cdot \sqrt{\frac{2}{3}} \beta \cdot (\sqrt{2}g_{NN\pi})
\end{align*}
$$

Here $1, 2$ denotes the spin and coordinate dependence while $w$ and $S$ mean the weak and the strong vertex respectively. $g_{NN\pi}$ is the strong coupling constant. The combination $M\overline{M}$ symbolizes the meson propagator which appears as a Yukawa function in the weak potential.

The $|\Delta I| = 3/2$ piece is

$$
\begin{align*}
&(\bar{n}\Lambda)_1 \pi^0 \pi^0 (\bar{n}n)_2 S \cdot \sqrt{\frac{2}{3}} \beta \cdot (-g_{NN\pi}) \\
&(\bar{n}\Lambda)_1 \pi^0 \pi^0 (\bar{p}p)_2 S \cdot \sqrt{\frac{2}{3}} \beta \cdot g_{NN\pi} \\
&(\bar{p}\Lambda)_1 \pi^- \pi^- (\bar{n}p)_2 S \cdot \frac{1}{\sqrt{3}} \beta \cdot (\sqrt{2}g_{NN\pi})
\end{align*}
$$
The kaon exchange results in the combinations
\[
\begin{align*}
(\pi\Lambda)_1 S K^0 K^0 (\pi n)_{2w} \cdot \gamma g_{NNK} \\
(\pi\Lambda)_1 S K^0 K^0 (\bar{p} p)_{2w} \cdot \frac{1}{2} (\xi + \delta) \cdot g_{NNK} \\
(\bar{p}\Lambda)_1 S K^- K^- (\bar{p} p)_{2w} \cdot \frac{1}{2} (-\xi + \delta) \cdot g_{NNK}
\end{align*}
\]
|\Delta I| = 1/2

(F-7a)

The comparison with Ref. [3] is easier if one introduces the notation
\[
\begin{align*}
\frac{1}{2} (\xi + \delta) g_{NNK} &= d \\
\frac{1}{2} (-\xi + \delta) g_{NNK} &= d \\
\delta \cdot g_{NNK} &= c + d.
\end{align*}
\]
(F-7b)

In (F-5,6,7) and in the following, the \( \Lambda \) baryon is always in the vertex 1, irrespectively whether it is the strong or the weak vertex. (The complete potential, as discussed in Section 9, is symmetric in 1 \( \leftrightarrow \) 2.)

The \(|\Delta I| = 3/2 \) kaon exchange pieces are
\[
\begin{align*}
(\pi\Lambda)_1 S K^0 K^0 (\bar{p} p)_{2w} \cdot e, \\
(\pi\Lambda)_1 S K^0 K^0 (\bar{p} p)_{2w} \cdot \frac{1}{2} (\xi + \delta) \cdot (-e) \\
(\bar{p}\Lambda)_1 S K^- K^- (\bar{p} p)_{2w} \cdot (-e)
\end{align*}
\]
|\Delta I| = 3/2

(F-7a)

Here \( e \equiv \gamma g_{NNK} / 2 \).

The expressions (F-5-8) can be connected with (8-17) and (8-20) by using the following isospin dependent quantities
\[
\begin{align*}
\beta_1 &= \left( \mathcal{N}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Lambda \right)_1 (\mathcal{N} 1 N)_{2} \\
\Delta I &= 1/2 \\
\beta_T &= \left( \mathcal{N}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Lambda \right)_1 (\mathcal{N} T N)_{2} \\
\beta_T &= \left( \mathcal{N}_1 (\mathcal{T}^N_1 \Lambda) \right)_1 (\mathcal{N} T N)_{2} \\
\Delta I &= 3/2
\end{align*}
\]
(F-9a)
Here

\[ \mathcal{N} = (\bar{p}, \bar{n}) \]

\[ \mathcal{N}(\bar{T}_\chi) \vec{\tau} = \sum_{m,r} C_{1r1m}^{3/2-1/2} \vec{t}_1 \cdot \vec{\tau}. \]  

(F-10a)

The summation over \( m \) goes over two isospin states \( m = \pm 1/2 \) contained in \( N \). Thus

\[ \mathcal{N}(\bar{T}_\chi) \vec{\tau} = \sqrt{\frac{2}{3}}(\tau^- + \tau^3) \]  

(F-10b)

The symbol \( \Lambda \) carries strangeness and has no isospin dependence. With (F-10) one obtains

\[ \beta_1 = (\pi \Lambda)_1[(\bar{p}p)_2 + (\bar{n}n)_2] \]

\[ \beta_\tau = (\pi \Lambda)_1[(\bar{m}n)_2 - (\bar{p}p)_2] + 2(\bar{p} \Lambda)_1(\bar{p}p)_2 \]  

(F-9b)

\[ \beta_T = \sqrt{\frac{2}{3}}[(\bar{p} \Lambda)_1(\bar{p}p)_2 + (\pi \Lambda)_1[(\bar{p}p)_2 - (\bar{n}n)_2] \]

The isospin dependence of the pion exchange contribution can be written as

\[ V_\pi = \tilde{A} \vec{\tau}_1 \cdot \vec{\tau}_2 + \tilde{B} \vec{T}_1 \cdot \vec{\tau}_2. \]  

(F-11a)

(This is not symmetrized). The factors \( \tilde{A} \) and \( \tilde{B} \) contain all spatial and spin operators. In theoretical nuclear calculations the operator (F-10a) is to be sandwiched between multi-particle baryon states containing \( N \) and \( \Lambda \).

The correspondence with (F-5) and (F-6) is established if one "reads" (F-11a) as

\[ V_\pi \rightarrow \tilde{A} \beta_\tau + \tilde{B} \beta_T \]

\[ = (\pi \Lambda)_1(\bar{m}n)_2 \left[ \tilde{A} - \sqrt{\frac{2}{3}} \tilde{B} \right] \]

\[ + (\pi \Lambda)_1(\bar{p}p)_2 \left[ -\tilde{A} + \sqrt{\frac{2}{3}} \tilde{B} \right] \]  

(F-11b)

\[ + (\bar{p} \Lambda)_1(\bar{p}p)_2 \left[ 2\tilde{A} + \sqrt{\frac{2}{3}} \tilde{B} \right] \]

\[ + (\bar{p} \Lambda)_1(\bar{p}p)_2 \left[ 2\tilde{A} + \sqrt{\frac{2}{3}} \tilde{B} \right] \]
The kaon exchange contribution is

\[ V_K = \frac{1}{2} \tilde{C}(1 + \vec{\tau}_1 \cdot \vec{\tau}_2) + \tilde{D} + \tilde{E} \vec{T}_1 \cdot \vec{\tau}_2 \]

\[ \longrightarrow \left( \frac{1}{2} \tilde{C} + \tilde{D} \right) \eta_0 + \frac{1}{2} \tilde{C} \eta_\tau + \tilde{E} \eta_T \]

\[ = (\pi \Lambda) (\overline{m} n) \frac{1}{2} (\tilde{C} + \tilde{D} - \tilde{E}) \]

\[ + (\pi \Lambda) (\overline{p} p) \frac{1}{2} (\tilde{C} + \tilde{D}) \]

\[ + (\pi \Lambda) (\overline{p} n) \frac{1}{2} (\tilde{D} + \tilde{E}) \]  \hspace{1cm} (F-12)

Expressions (F-9b) can be inverted. By introducing the notation

\[ a = (\pi \Lambda) (\overline{p} p) \]

\[ b = (\pi \Lambda) (\overline{m} n) \]  \hspace{1cm} (F-13)

\[ c = (\overline{p} \Lambda) (\overline{p} p) \]

one finds

\[ a = \frac{1}{2} \beta_1 - \frac{1}{6} \beta_\tau + \frac{1}{\sqrt{6}} \beta_T \]

\[ b = \frac{1}{2} \beta_1 + \frac{1}{6} \beta_\tau - \frac{1}{\sqrt{6}} \beta_T \]  \hspace{1cm} (F-14)

\[ c = + \frac{1}{3} \beta_\tau + \frac{1}{\sqrt{6}} \beta_T. \]
Appendix G: Shifted Yukawa Function

It is usually stated [44,45] that when a force is transmitted by a particle then the range of that force depends on the mass of the intermediate particle. Ever since Yukawa’s seminal work [46] the Yukawa potential

\[ V_Y(r) = \frac{e^{-\mu r}}{r} \]  \hspace{1cm} (G-1)

was a textbook feature [45], describing nucleon-nucleon interaction, produced by one meson exchange. It turns out that the original statement (1) is somewhat modified in hypernuclei [5] where one has nucleons and strange particles.\(^2\) In that case the range of the force depends both on the intermediate meson mass and the baryon masses.

Moreover, in order to connect the more general procedure with the result (G-1) valid for the baryons which all have equal masses, one can alternatively use an interesting generalized definition of the delta-function [47].

A transitional amplitude \(A_{f \Lambda}\) corresponding to the one pion (or any meson) exchange has a generic form [45]

\[ A_{f \Lambda} = \frac{N}{2} \int d^4x \int d^4y \int d^4k \frac{e^{-ik(x-y)}}{k^2 - \mu^2 + i\epsilon} \cdot \langle f | T(S(x)W(y)) | \Lambda \rangle. \]  \hspace{1cm} (G-2a)

Here

\[ k^2 = k_0^2 - \vec{k}^2 \]  \hspace{1cm} (G-2b)

\[ kx = k_0x_0 - \vec{k} \cdot \vec{x}, \]

Here \(S(x)\) and \(W(x)\) are some baryon densities (scalar or pseudoscalar) which are sources of the meson (pion) field, which mass is \(\mu\). Detailed form of those quantities need not concern us here. If all baryons are nucleons \(N\), then the calculation can be found, for

\(^2\) For instance the \(^{12}_\Lambda\)C nucleus consists of 5 neutrons, 6 protons and the \(\Lambda\)-hyperon.
example, in ref. [45], p.213. In a more general case the density $S(x)$ can contain a strange baryon, for example Λ hyperon [5].

The time ordered product in (2a) can be written as

$$\langle f|T(S(x)W(y))|\Lambda\rangle = \theta(x-y)\sum_n \langle f|S(x)|n\rangle\langle n|W(y)|\Lambda\rangle$$

$$+ \theta(y-x)\sum_s \langle f|W(y)|s\rangle\langle s|S(x)|\Lambda\rangle. \tag{G-3}$$

Here the intermediate states

$$|n\rangle; \quad S = 0 \tag{G-4}$$

have no strangeness while the states

$$|s\rangle; \quad S = -1 \tag{G-5}$$

must contain a strange baryon.

In (G-2a) one can integrate over times $(x_0, y_0)$. A useful identity is

$$\langle f|S(x)|n\rangle = e^{i(E_f-E_n)x_0}\langle f|S(\vec{x})|n\rangle. \tag{G-6}$$

An analogous expression holds for other matrix elements. Also

$$dx^0 dy^0 = 2d\xi d\eta;$$

$$x^0 = \xi + \eta \tag{G-7}$$

$$y^0 = -\xi + \eta$$

The integration $d\eta$ can be immediately carried out which results in

$$A_{f\Lambda} = N \int d^3x d^3y d\xi \int d^4k \frac{e^{-2i\xi k_0 + i\vec{k}\cdot(\vec{x}-\vec{y})}}{k^2 - \mu^2 + i\epsilon}$$

$$\cdot 2\pi\delta(E_f - E_\Lambda)\left[\theta(\xi)\sum_n e^{i\Delta_n\xi}\alpha_n(\vec{x}, \vec{y}) + \theta(-\xi)\sum_s e^{-i\Delta_s\xi}\beta_s(\vec{y}, \vec{x})\right]. \tag{G-8a}$$
Here

\[ \Delta_i = E_f + E_\Lambda - 2E_i \]
\[ \alpha_n = \langle f | S(\vec{x}) | n \rangle \langle n | W(\vec{y}) | \Lambda \rangle \]
\[ \beta_s = \langle f | W(\vec{y}) | s \rangle \langle s | S(\vec{x}) | \Lambda \rangle, \]
and \( E_k \) are relativistic energies.

First one integrates over \( d\xi \) obtaining

\[ A_{f\Lambda} = (2\pi i) N \delta (E_f - E_\Lambda) \int d^3x d^3y \int d^4k \frac{e^{i\vec{k} \cdot (\vec{x} - \vec{y})}}{k^2 - \mu^2 + i\epsilon} \]
\[ \cdot \left[ \sum_n \alpha_n \frac{1}{\Delta_n - 2k_0 + i\epsilon} + \sum_s \beta_s \frac{1}{\Delta_s + 2k_0 + i\epsilon} \right]. \]

Integration in the complex \( k_0 \) plane over the contours shown in Fig.G.1 leads to

\[ A_{f\Lambda} = 2\pi^2 \delta (E_f - E_\Lambda) N \int d^3x d^3y \int d^3k \cdot e^{i\vec{k} \cdot (\vec{x} - \vec{y})} \]
\[ \cdot \left[ \sum_n \alpha_n (\vec{x}, \vec{y}) \frac{1}{\omega(\Delta_n - 2\omega)} + \sum_s \beta_s (\vec{y}, \vec{x}) \frac{1}{\omega(\Delta_s - 2\omega)} \right] \]
\[ \omega = \sqrt{\vec{k}^2 + \mu^2}. \]

**Fig.G.1 - The contours in the \( k_0 \) plane.**

\[ \omega^2 = \vec{k}^2 + \mu^2. \]
In the nonrelativistic limit the energy differences $\Delta_i$ can be approximated by the corresponding baryon mass differences. Schematically one can use the following baryonic contents:

| $|f\rangle$ | $K$ nucleons |
| $|\Lambda\rangle$ | $1$ $\Lambda + (K - 1)$ nucleons |
| $|s\rangle$ | $1$ $\Lambda + (K - 1)$ nucleons |
| $|n\rangle$ | $K$ nucleons. |

Thus

$$
E_f \rightarrow K \cdot m_N
$$

$$
E_\Lambda \rightarrow (K - 1) \cdot m_N + m_\Lambda
$$

$$
E_s \rightarrow (K - 1) \cdot m_N + m_\Lambda
$$

$$
E_n \rightarrow K \cdot m_N
$$

$$
\Delta_s = E_f + E_\Lambda - 2E_s \rightarrow -2\delta
$$

$$
\Delta_n = E_f + E_\Lambda - 2E_n \rightarrow 2\delta
$$

$$
\delta = (m_\Lambda - m_N)/2
$$

In such an approximation the factors depending on $\omega$ and $\Delta_i$ can be taken out of the summations in (G-10). Furthermore

$$
\sum_n \alpha_n(\vec{x}, \vec{y}) = S(\vec{x})W(\vec{y})
$$

$$
\sum_s \beta_n(\vec{x}, \vec{y}) = W(\vec{y})S(\vec{x})
$$

As in the nonrelativistic approach $W$ and $S$ become operators in the configuration space acting on (Schrödinger) wave functions \[45,48\], their order is immaterial. Thus one obtains

$$
A_{f\Lambda} = (2\pi^2)N\delta(E_f - E_\Lambda) \int d^3x \int d^3y \int d^3k \cdot e^{i\vec{k}(\vec{x} - \vec{y})} \langle f | O(\vec{x}, \vec{y}) |\Lambda\rangle
$$

$$
\cdot \frac{1}{2} \left[ \frac{1}{\omega(\delta + \omega)} + \frac{1}{\omega(\delta - \omega)} \right] \langle f | O(\vec{x}, \vec{y}) |\Lambda\rangle
$$

$$
O(\vec{x}, \vec{y}) = S(\vec{x})W(\vec{y})
$$
The integration over $\vec{k}$ gives the \textit{shifted} Yukawa function

$$\int d^3k \cdot e^{i\vec{k} \cdot (\vec{x} - \vec{y})} \frac{1}{2} \left[ \frac{(-1)}{\omega(\delta + \omega)} + \frac{1}{\omega(\delta - \omega)} \right] =$$

$$\int d^3k \frac{e^{i\vec{k} \cdot (\vec{x} - \vec{y})}}{\omega^2 - \delta^2} = (-2\pi^2) \frac{e^{-\epsilon r}}{r}$$

$$r = |\vec{x} - \vec{y}|$$

$$\epsilon = \sqrt{\mu^2 - \delta^2} = \sqrt{\mu^2 - \frac{1}{4}(m_\Lambda - m_N)^2}.$$  \hspace{1cm} (G-14)

Eventually one obtains a generic form

$$A_{f\Lambda} = (-4\pi^4) N \int d^3x \int d^3y \frac{e^{-\epsilon r}}{r} \langle f | O(\vec{x}, \vec{y}) | \Lambda \rangle$$

$$\text{in which the Yukawa potential (function) depends on the effective mass}$$

$$\epsilon < \mu.$$  \hspace{1cm} (G-15)

If one deals with nucleons only, then

$$\delta = 0$$

$$\epsilon \rightarrow \mu,$$  \hspace{1cm} (G-16)

and the standard textbook [45] form (G-1) is recovered.

This last result can be obtained also by starting from the expression (G-8). In the strong nucleon-nucleon interaction the variable $k$ corresponds to the invariant nucleon momentum transfer [45]

$$k^2 = (p - p')^2$$

$$p = (E_p, \vec{p}); \quad E_p = \sqrt{\vec{p}^2 + M_N^2}.$$  \hspace{1cm} (G-18)
In the nonrelativistic limit, when $|\vec{p}| \ll M_N$ one can approximate

$$E_p \simeq M_N$$

$$\delta \to E_p - E_p' \simeq M_N - M_N = 0$$

$$k^2 \sim -\vec{k}^2$$

$$\int d^4k \frac{e^{-ik(x-y)}}{k^2 - \mu^2 + i\epsilon} \to (-1) \int dk_0 \cdot d^3k \frac{e^{-2i\epsilon k_0 \xi e^{i\vec{k} \cdot \vec{r}}}}{k^2 + \mu^2}$$

$$(G-19)$$

$$= (-1)\pi \delta(\xi) \cdot \int d^3k \frac{e^{ik \cdot \vec{r}}}{\sqrt{k^2 + \mu^2}}$$

$$= (-1)\delta(\xi) 2\pi^3 e^{-\mu r} r.$$ $$r = |\vec{x} - \vec{y}|.$$ When this is introduced in (G-8) with $|\Lambda\rangle = |i\rangle$ (nucleon state), one finds

$$A_{fi} = N \int d^3x d^3y d\xi (-1)\delta(\xi) \frac{e^{-\mu r}}{r}$$

$$\cdot (2\pi)\delta(E_f - E_i) \left[ \theta(\xi) \sum_n e^{i\Delta_n \xi} \alpha_n(\vec{x}, \vec{y}) + \theta(-\xi) \sum_s e^{-i\Delta_s \xi} \beta_s(\vec{x}, \vec{y}) \right]$$

$$(G-20a)$$

with

$$\Delta_i = E_f - E_i \to 0.$$ $$(G-20b)$$

In order to integrate over $\xi$ one uses the identity [5]

$$\int d\xi \theta(\xi) \delta(\xi) \cdot e^{i\alpha \xi} = \frac{1}{2},$$

$$(G-21)$$

and eventually obtains

$$A_{fi} = A_{f \Lambda=i}(\epsilon \to \mu),$$

$$(G-22)$$

as already mentioned above (G-17). In a sense, that can be considered as an example for the novel delta-function identity derived by ref. [47].

Although interesting as a matter of principle the shifted Yukawa potential does not lead to some startling numerical differences. The range of the potential is somewhat
increased, as can be seen by plotting the ratio

\[ \frac{e^{-\epsilon r}}{e^{-\mu r}} = e^{(\mu - \epsilon)r} = X(r) \]  

which is shown in Fig.G.2 (a). The shifted potential has an increased range. With \( \mu = m_\pi = 0.70 \text{ fm}^{-1} \) and \( \delta = (m_\Lambda - m_N)/2 = 0.45 \text{ fm}^{-1} \) one finds: \( r = 0.5 \text{ fm} \) \( X = 1.08 \) and \( r = 1 \text{ fm} \) \( X = 1.17 \) for example.

\[ \begin{align*}
X(r) & \\
\tilde{X}(r) &
\end{align*} \]

\( (a) \quad (b) \)

Fig.G.2 - The ratio (23) (a) and (26) (b) is plotted as a function of \( r \).

In the actual calculation with hypernuclei one introduces a monopole form-factor at the each vertex \([5]\). The denominators in (G-14) and (G-19) are thus replaced by

\[ \frac{1}{k^2 + \phi^2} \rightarrow \frac{(\Lambda^2 - \mu^2)}{(k^2 + \phi^2)(k^2 + \Lambda^2)^2} = W(\phi) \quad (\phi = \epsilon, \mu). \]  

The Fourier transform of that is

\[ \mathcal{F}\{W(\phi)\} = 2\pi^2 \left[ \frac{\Lambda^2 - \mu^2}{\Lambda^2 - \phi^2} \left( \frac{e^{-\phi r}}{r} - \frac{e^{-\Lambda r}}{r} \right) - \frac{\Lambda^2 - \mu^2}{2\Lambda} \cdot e^{-\Lambda r} \right] \]
The ratio

\[ \tilde{X}(r) = \frac{\mathcal{F}\{W(e)\}}{\mathcal{F}\{W(\mu)\}} \]  

(G-26)

is also plotted in Fig.G.2 (b) for \( \Lambda = 1.3 \text{ GeV} \) [5]. One finds \( \tilde{X} = 1.09 \) at 0.5 fm for example.
Appendix H: Effective $\tilde{a}_{BB}$ Amplitude and Baryon Pole Terms

Parity violating $A$ amplitudes obtain here the following contributions

\[ A = A_{CA} + A_{\text{Sep}} \]  

(H-1)

The first corresponds to formulae (5-20) to (5-26) while the second one is calculated as described in (3-20). The contributions $A$ (H-1) should be compared with the experimental result $A_{\text{exp}}$, i.e.

\[ A_{\text{exp}} \sim A = A_{CA} + A_{\text{Sep}}. \]  

(H-2)

The weak vertices in the baryon pole terms (see Fig.4.1) are determined by the $A_{CA}$ which has a generic form

\[ A_{CA} \sim \frac{1}{f_\pi} (B'| H_{\Pi}^{PC} | B) = \frac{1}{f_\pi} \tilde{a}_{B'B}. \]  

(H-3)

One actually uses

\[ \lim_{q \to 0} A = A_{CA}. \]  

(H-4)

Thus in order to find $\tilde{a}_{B'B}$ one must use an approximate expression whose a generic form is

\[ \frac{1}{f_\pi} \tilde{a}_{B'B} \simeq [A_{B'B/\text{exp}} - A_{B'B/\text{SEP}}]. \]  

(H-5)
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