A 1993 Look at the Lower Bound on the Top Quark Mass from CP Violation

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Abstract

We point out that the lower bound on $m_t$ from the CP violation parameter $\epsilon_K$ has increased considerably. Using Wolfenstein parametrization of the CKM matrix we derive an analytic expression for this bound as a function of $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and the non-perturbative parameter $B_K$. For $B_K \leq 0.80$, $|V_{cb}| \leq 0.040$ and $|V_{ub}/V_{cb}| \leq 0.10$ we find $m_t > 130 GeV$. However, for $B_K \leq 0.70$, $|V_{cb}| \leq 0.038$ and $|V_{ub}/V_{cb}| \leq 0.08$ the bound is raised to $m_t > 205 GeV$. The lower bound on $m_t$ from $B^0 - \bar{B}^0$ mixing is also reconsidered.
Ten years ago Ginsparg, Glashow and Wise \[1\] calculated a lower bound on the top quark mass \(m_t\) by analysing the CP violation parameter \(\epsilon_K\) in the standard model. Subsequently a more detailed analysis has been done by Slominski, Steger and the present author \[2\]. The lower bound on \(m_t\) considered in \[1, 2\] takes the general form
\[
m_t \geq F(|V_{cb}|, |V_{ub}/V_{cb}|, B_K)
\]
(1)

where \(V_{ij}\) are the elements of the Cabibbo-Kobayashi-Maskawa matrix and \(B_K\) is a non-perturbative parameter to be specified below. The function \(F\) increases with decreasing \(|V_{cb}|\), \(|V_{ub}/V_{cb}|\) and \(B_K\).

In 1983 the values of \(|V_{cb}|\), \(|V_{ub}/V_{cb}|\) and of \(B_K\) have been poorly known and no useful bound on \(m_t\) could be obtained. During the last ten years the experimental determinations of \(|V_{cb}|\) and \(|V_{ub}/V_{cb}|\) have been considerably improved and some progress has been made in calculating \(B_K\). It is of interest then to reanalyse the bound in view of these developments and in view of the possibility that the top quark could be discovered soon.

In fact the present “best” values for \(|V_{cb}|\), \(|V_{ub}/V_{cb}|\) and \(B_K\) are substantially lower than the corresponding values of 1983. Consequently the lower bound on \(m_t\) from \(\epsilon_K\) can be considerably improved.

We note:

- The new experimental results on exclusive semi-leptonic \(B\)-decays \[3\] combined with the increased \(B\)-meson life-time measured at LEP and by CDF at Tevatron \(\tau_B = 1.49 \pm 0.03\,\text{ps}\) \[4\] give with the help of the Heavy Quark Effective Theory \[5, 6\]

  \[
  \kappa \equiv |V_{cb}| \left( \frac{\tau_B}{1.5\,\text{ps}} \right)^{1/2} = 0.038 \pm 0.006
  \]

  (2)

  Similar results can be found in \[7, 8\]. This value should be compared with \(|V_{cb}| \approx 0.05\) of 1983.

- The most recent determinations of \(|V_{ub}/V_{cb}|\) give typically \[9, 10\]

  \[
  \frac{|V_{ub}|}{|V_{cb}|} = 0.08 \pm 0.02
  \]

  (3)
The extensive calculations of $B_K$ using lattice gauge theories, $1/N$ expansion and QCD sum rules of various sorts, give consistently $B_K < 1$. The range

$$B_K = 0.7 \pm 0.2$$  \hspace{1cm} (4)$$

is a good representation of the $1/N$ [11], lattice results [12] and QCD sum rules results [13]. The QCD hadron duality approach gives somewhat lower values $B_K = 0.4 \pm 0.1$ [14]. Finally the leading term in the chiral perturbation theory in the strict $SU(3)$ limit gives $B_K = 1/3$ [15].

The next-to-leading QCD corrections to the QCD factor $\eta_2$ (see below) [16] decrease its value from 0.62 to 0.57 making the lower bound on $m_t$ somewhat stronger.

In the coming years further improvements on $|V_{cb}|$, $|V_{ud}/V_{cb}|$ and $B_K$ are to be expected. It is then of interest to reanalyse the bound (1) as a function of these three parameters. As a byproduct of our analysis we will derive an approximate analytic expression for the function $F$. Our analysis complements the numerous analyses of the unitarity triangle where the lower bound on $m_t$ has not been addressed.

Formulated in terms of the Wolfenstein parameters $\lambda$, $A$, $\rho$ and $\eta$ the lower bound on $m_t$ arises from the measured value of $\epsilon_K$ roughly as follows. The usual box diagram calculation together with the experimental value for $\epsilon_K$ specifies the following hyperbola in the $(\rho, \eta > 0)$ plane [17, 18]

$$\eta \left[ (1 - \rho) A^2 \eta_2 S(x_t) + \lambda^{-4} P_0 \right] A^2 B_K = 0.223$$  \hspace{1cm} (5)$$

Here

$$A \equiv |V_{cb}| / \lambda^2$$ \hspace{1cm} (6)$$

$$P_0 = \eta_3 S(x_c, x_t) - \eta_1 x_c$$ \hspace{1cm} (7)$$

$$S(x_c, x_t) = x_c \left[ \ln \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} \left( 1 + \frac{x_t}{1-x_t} \ln x_t \right) \right]$$ \hspace{1cm} (8)$$

$$S(x_t) = x_t \left[ \frac{1}{4} + \frac{9}{4} \left( \frac{1}{1-x_t} - \frac{3}{2} \frac{1}{(1-x_t)^2} \right) + \frac{3}{2} \left( \frac{x_t}{|x_t-1|} \right)^3 \ln x_t \right]$$ \hspace{1cm} (9)$$

where $x_t = m_t^2/M_W^2$. $B_K$ is the renormalization group invariant non-perturbative parameter describing the size of $< \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} | K^0 >$ and $\eta_i$ represent QCD corrections to the box diagrams.
On the other hand the experimental value of $|V_{ub}/V_{cb}|$ determines a circle of the radius

$$R_b = \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = \left[ \rho^2 + \eta^2 \right]^{1/2} \tag{10}$$

centered at $(\rho, \eta) = (0, 0)$. The hyperbola (5) intersects the circle (10) in two points, one for $\rho < 0$ and the other for $\rho > 0$. With decreasing $A$ (or $|V_{cb}|$), $B_K$ and $m_t$, the hyperbola (5) moves away from the origin of the $(\rho, \eta)$ plane. For sufficiently low $A$, $B_K$, $m_t$ and $R_b$ the hyperbola and the circle only touch each other. This way a lower bound for $m_t$ as a function of $B_K$, $V_{cb}$ and $|V_{ub}/V_{cb}|$ can be found.

For the numerical evaluation of this bound it is better to use the exact standard parametrization of the CKM matrix \[19\] and a more accurate formula for $\epsilon_K$ given e.g. in (5.22) of ref.\[18\]. However, the formulation given above allows to find a simple analytic expression for $(m_t)_{min}$ which to an accuracy better than 2% reproduces the results of a more elaborate analysis.

In order to find a formula for $(m_t)_{min}$ which is simple and accurate we proceed as follows. We note first that in the “exact” numerical analysis which uses the standard parametrization of the CKM matrix the lower bound on $m_t$ corresponds to $\sin \delta = 1$ i.e. to $\rho = 0$. Equations (5) and (10) being approximations give generally $(m_t)_{min}$ at a small negative value of $\rho$ rather than at $\rho = 0$. In spite of this the most efficient strategy is to set $\rho = 0$ in (5) and (10) to find the following condition for $(m_t)_{min}$:

$$S(x_t) = 1 \quad \rho^2 - \frac{0.223}{A^2 B_K R_b} - \frac{P_0}{\lambda^4}$$

\[11\]

We use next \[18\]

$$S(x_t) = 0.784 \quad x_t^{0.76} \tag{12}$$

which is an excellent approximation of (9) in the range $100 \leq m_t \leq 300 \text{ GeV}$. In the same range of $m_t$, $P_0$ takes the values $6.25 \times 10^{-4} \leq P_0 \leq 7.59 \times 10^{-4}$ where we have used $\eta_1 = 0.85$, $\eta_3 = 0.36$ and $m_c = 1.4 \text{ GeV}$. In view of this weak $m_t$-dependence compared to (12), $P_0$ can be approximated by a constant. This constant can be chosen in such a way that the analytic bound reproduces the results of a more accurate numerical analysis as good as possible. $P_0 = 6.25 \times 10^{-4}$ turns out to be a good choice.

From (11) and (12) we finally obtain the analytic lower bound on $m_t$:  

$$m_t \geq (m_t)_{min} = M_W \left[ 1 - \left( \frac{P_0}{\lambda^4} \right) \right]^{0.658} \tag{13}$$
which is the main formula of this letter. To this end we have set $\eta_2 = 0.57$ [16]. This QCD factor is so defined that the resulting $m_t$ is the current top quark mass normalized at $\mu = m_t$. The $m_t$ dependence of $\eta_2$ can be neglected. Formula (13) together with (6) and (10) gives $(m_t)_{min}$ as a function of $|V_{cb}|, |V_{ub}/V_{cb}|$ and $B_K$. It is evident from (13) that this bound increases with decreasing $A$ (or $|V_{cb}|$), $B_K$ and $R_b$ (or $|V_{ub}/V_{cb}|$). We note a very strong dependence on $A$. We have checked that in the ranges of $|V_{cb}|, |V_{ub}/V_{cb}|$ and $B_K$ used below, our analytic bound gives a very good representation (generally to better than 2% accuracy) of a more elaborate numerical analysis which uses accurate formulae of ref.[18].

In order to get better acquainted with the bound (13) we note that for $0.035 \leq |V_{cb}| \leq 0.043$ and $0.06 \leq |V_{ub}/V_{cb}| \leq 0.10$ we have $0.72 \leq A \leq 0.89$ and $0.27 \leq R_b \leq 0.45$ respectively. Consequently for $B_K$ given in (4) we find $3.1 \leq 1/(A^2 B_K R_b) \leq 14.3$. For central values $B_K = 0.7, |V_{ub}/V_{cb}| = 0.08$ and $|V_{cb}| = 0.038$ we have $1/(A^2 B_K R_b) = 6.5$.

The bound in (13) is very interesting as it results from a different sector of the standard model than the corresponding bound on $m_t$ obtained from high precision electroweak studies at LEP and Tevatron. Whereas the fate of the latter bounds depends sensitively on the precise measurements of $M_W, \Gamma_Z, \sin^2\Theta_W$, etc., the lower bound in (13) is subject to our knowledge of $|V_{cb}|, |V_{ub}/V_{cb}|$ and $B_K$.

In figs. 1-3 we plot $(m_t)_{min}$ as a function of $|V_{cb}|$ for different values of $B_K$ and $|V_{ub}/V_{cb}|$. We have set $M_W = 80. GeV$. These plots are self explanatory. Therefore we only make a few comments.

We note that for $B_K \leq 0.5$ the lowest value of $m_t$ consistent with the observed CP violation is generally substantially larger than 200 GeV. For $B_K$ in the range (4), the values of $(m_t)_{min}$ are compatible with the restrictions on $m_t$ (150 ± 40 GeV) coming from electroweak studies, although for $|V_{ub}/V_{cb}| \leq 0.08$, $|V_{cb}| \leq 0.038$ and $B_K \leq 0.7$ again $(m_t)_{min} > 200$ GeV is favoured. In such a situation a discovery of the top quark with $m_t \approx 150$ GeV would certainly require new positive contributions to $\epsilon_K$ in order to lower the bound on $m_t$ coming from the observed CP violation. Charged Higgs exchanges and/or various supersymmetry contributions to $\epsilon_K$ would be the prime candidates but such an analysis is beyond the scope of this letter.

It is interesting to compare these bounds with those which can be obtained from $B_0 - \bar{B}_0$ mixing alone. The quark model bound and the $B_0 - \bar{B}_0$ mixing bound
by the parameter \( x_d = \Delta M/\Gamma_B \) determines in the \((\rho, \eta)\) plane a circle centered at \((\rho, \eta) = (1, 0)\) and having the radius

\[
R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| = \left[ (1 - \rho)^2 + \eta^2 \right]^{1/2} \tag{14}
\]

Using the usual formulae for box diagrams with top quark exchanges [18] it is straightforward to find the value of \( m_t \) consistent with \( x_d \) as a function of \( R_t \), the \( B \)-meson decay constant \( F_B \), the parameter \( B_B \) analogous to \( B_K \) and \( \kappa \) of (2). Defining

\[
r \equiv \left[ \frac{x_d}{0.67} \left( \frac{200 \text{MeV}}{F_B \sqrt{B_B}} \right) \left( \frac{0.038}{\kappa} \right) \sqrt{\frac{0.55}{\eta_B}} \right]^{1.32} \tag{15}
\]

we find

\[
m_t = r \ R_t^{-1.32} \ 179.4 \ \text{GeV} \tag{16}
\]

Here \( \eta_B \) is the QCD factor analogous to \( \eta_2 \) and calculated to be \( \eta_B = 0.55 \) [16].

Since (10) and (14) must be consistent with each other the lower bound on \( m_t \) from \( B^o - \bar{B}^o \) mixing without any constraint from \( \epsilon_K \) is found by setting \( \eta = 0 \) and \( \rho = -R_b \) which implies \( R_t = 1 + R_b \). This gives then

\[
(m_t)_{\text{min}} = \begin{cases} 
  r \cdot 110 \ \text{GeV} & |V_{ub}/V_{cb}| = 0.10 \\
  r \cdot 120 \ \text{GeV} & |V_{ub}/V_{cb}| = 0.08 \\
  r \cdot 131 \ \text{GeV} & |V_{ub}/V_{cb}| = 0.06 
\end{cases} \tag{17}
\]

We observe that this bound depends much weaker on \( |V_{ub}/V_{cb}| \) than the bound on \( \epsilon_K \) given in (13).

The fate of the lower bound from \( B^o - \bar{B}^o \) mixing depends on the ratio \( r \) in (15). With \( x_d = 0.67 \pm 0.10 \) [20], \( \kappa = 0.038 \pm 0.006 \) and \( F_B \sqrt{B_B} = 200 \pm 40 \text{MeV} \) [21], one has the range \( 0.58 \leq r \leq 1.85 \) i.e. a large uncertainty. For central values of the parameters the lower bound from \( B^o - \bar{B}^o \) mixing is generally weaker than from \( \epsilon_K \). It should however be emphasized that the lower bound from \( B^o - \bar{B}^o \) mixing in 1993 is substantially higher than the bounds found in the 80’s [22]. This is primarily due to the values of \( \kappa \) and \( \eta_B \) which were substantially higher in those days. For instance with \( V_{cb} \approx 0.05 \) and \( \tau_B \approx 1.2 \text{ps} \) one had \( \kappa \approx 0.046 \). Taking in addition \( \eta_B \approx 0.85 \) used in older analyses decreases the lower bound on \( m_t \) by a factor of 1.7. This factor is somewhat reduced by the higher values of \( F_B \) found at present.

In this letter we have emphasized that the lower bounds on \( m_t \) coming from \( \epsilon_K \) and \( B^o - \bar{B}^o \) mixing increased considerably during the last years. In particular we...
\( \epsilon_K \) appears to be higher than the estimates of \( m_t \) from the precision electroweak studies at LEP. Are these some hints for the physics beyond the standard model? In order to answer this question continuous efforts should be made to decrease the uncertainties in \( |V_{cb}|, |V_{ub}/V_{cb}|, B_K \) and \( F_B \). It will be exciting to watch in the coming years the developments in \( B \)-decays, in electroweak studies and in particular in top quark searches. We hope that figs. 1-3 and the analytic lower bounds on \( m_t \) derived here will help to see quickly where we stand.

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**Figure Captions**

Fig.1 The lower bound on \( m_t \) from \( \epsilon_K \) as a function of \( |V_{cb}| \) for different \( B_K \) and \( |V_{ub}/V_{cb}| = 0.06 \).

Fig.2 Same as fig.1 but for \( |V_{ub}/V_{cb}| = 0.08 \).

Fig.3 Same as fig.1 but for \( |V_{ub}/V_{cb}| = 0.10 \).