Gravitational Dephasing in Optical Lattice Clocks

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Optical lattice clocks can now resolve the height difference below 1 cm by means of the gravitational redshift. Further improvement in the stability enables the clocks to resolve the height difference of subsystems within an atomic ensemble that is conventionally interrogated as a single coherent spin state, resulting in the dephasing of the coherent spin state. This effect is observable with a clock of the stability of $\sim 10^{-21}$ by introducing single layer resolved imaging system for three dimensional optical lattice, and limits the 1 s stability of the clock at $10^{-19}$ without proper compensations. This can be the first observation of the decoherence of a quantum state by a gravitational effect, and the suppression of other systematic shifts to observe this gravitational dephasing seems feasible.

Atomic clocks witnessed a significant improvement in their stability in past decades [1, 2]. Multiple groups demonstrate the relative stability in the order of $10^{-18}$−$10^{-19}$ with different atomic species[3–7], leading to the relative accuracy of $10^{-15}$. Recent comparisons of the stability between multiple atomic systems with a common laser systems reach the level of $10^{-19}$[8–10], and clocks with the $10^{-19}$ accuracy are appearing [11]. Among them, optical lattice clocks [3–5, 7–10] have an advantage of interrogating many, e.g. thousands of, atoms simultaneously, resulting in high stability in a fixed amount of time.

The long-lasting researches triggered several variations of the optical lattice clocks. The most conventional one traps single atomic species in an one-dimensional optical lattice, and the atomic state is read out by collecting scattered light from the atomic ensemble with a single-channel photodetector such as a photomultiplier tube [3, 5]. Currently, some clocks trap atoms in a two [12] or three-dimensional [13] optical lattice, or an array of optical tweezers [8]. High resolution imaging system [14] and single-atom-resolvable imaging system [8] brought a better understanding of the internal structure of the atomic ensemble. Some optical clock systems can trap multiple atomic species in the same vacuum chamber [15], and the control and the understanding of the environment, such as temperature [16], electric field [17], and magnetic field, has improved significantly. Also, combining an optical cavity and an optical lattice clock opened a way for the further improvement in the stability of the state-of-the-art optical lattice clock with a help of quantum metrology [12].

Many phenomena became detectable with these high precision clocks. One example of them is a bunch of different systematic shifts and uncertainties [16–23]. Improved clock stability allows more precise measurements of the energy shift of the clock transition, and this leads to better understandings of the systematic uncertainties. So far, these are followed by the implementation of more sophisticated ways of cancelling the systematic shifts and uncertainties, contributing to the improvement of the stability of the clocks.

One of the such systematic shifts is the gravitational redshift. The high clock stability has achieved a resolution to the gravitational redshift induced by the Earth’s gravity in the lab. The first report had a resolution of tens of centimeters [24], and recent reports have the precision of 1 cm height difference consistent with the conventional geodesy [25], and the measurement outside the lab also demonstrated the similar resolution [26]. On one hand, this is a good progress towards the precise measurement of the height on Earth by detecting the delay of the time. On the other hand, even more precise measurement in the height can resolve the height difference of atoms in different part of an atomic ensemble currently regarded as a single coherent spin state (CSS). This can potentially limit the stability of the optical lattice clock, unless proper measures are taken. Unfortunately, proper measures are not as trivial as other external fields such as magnetic and electric fields, due to the inability of applying compensation gravitational field or making a shield for the environmental gravitational field. In this paper, the effect of the gravitational redshift on an atomic ensemble is first analyzed, and then the required suppression of other systematic effects to detect the effect is discussed.

The amount of the gravitational redshift $\Delta \nu$ of an atomic transition of frequency $\nu$ between two places on Earth with a height difference of $\Delta h$ is

$$\frac{\Delta \nu}{\nu} = \frac{g \Delta h}{c^2}, \quad (1)$$

assuming that the gravitational acceleration $g$ is locally uniform around the two places, where $c$ is the speed of light. Assuming $g = 9.80665 \text{ m/s}^2$, the amount is $\Delta \nu/\nu = 1.09 \times 10^{-18}$ for $\Delta h = 1 \text{ cm}$, consistent with the 1 cm height resolution for the currently reported best stability around $10^{-18}$ [3–7, 11]. Naively, if the stability increases up to $10^{-23}$, the resolution in the height is $\sim 100 \text{ nm}$, and the height difference of the each lattice site is resolvable.

The fundamental limit of the stability of the optical lattice clock comes from the quantum projection noise (QPN) at the state detection. The relative stability of
the clock due to the QPN is described as
\[ \sigma_{\text{QPN}} = \frac{1}{\omega_0 \tau_R} \sqrt{\frac{T_C}{\tau}} \sqrt{\frac{\xi^2_W}{N}}, \] 
(2)
where \( \omega_0 \) is the resonant angular frequency of the clock transition, \( \tau_R \) is the interrogation time, \( T_C \) is the time for one cycle of the clock operation, \( \tau \) is the overall integration time, \( \xi^2_W \) is the Wineland parameter, and \( N \) is the number of atoms in the system [12]. The amount of the QPN for a CSS is called the standard quantum limit (SQL), where \( \xi^2_W = 1 \), and with a spin squeezed state and other entangled states, \( \xi^2_W \) can be smaller than 1. To detect the small effect of the gravitational redshift, the SQL needs to be smaller than the amount of the gravitational redshift.

Current state-of-the-art optical lattice clocks and clocks based on neutral atoms has \( \tau_R = 30 \text{ s} \) [8]. Because the typical time for loading atoms and post-interrogation measurement is at most 1 s in total, when \( \tau_R = 30 \text{ s} \), \( \frac{T_C}{\tau_R} \approx 1 \). To set the distance between the atoms in each lattice site and \( \omega_0 \), an optical lattice clock with ytterbium (Yb) is assumed, where \( \omega_0 = 2\pi \times 5.18295 \times 10^{14} \text{ Hz} \) and \( \lambda = 759.356 \text{ nm} \) for the optical lattice [19].

To estimate parameters where the effect of the gravitational redshift in the single atomic ensemble starts to be visible, the atomic system is assumed to consist of a cube of \( n_{\text{site}} \) sites along the \( x \) and \( y \) axes and \( n_{\text{site}} + 1 \) sites along the \( z \) axis, each of which holds a single atom, resulting in \( N = n_{\text{site}}^2(n_{\text{site}} + 1) \). Because the effect of the gravitational redshift appears along the \( z \) axis, it is convenient to assume that \( n_{\text{site}}^2 \) atoms in a single layer that is perpendicular to the \( z \) axis form a single CSS. The criterion for the effect of the gravitational redshift to be visible is set as the phase shift over the interrogation time (\( \tau = 30 \text{ s} \)) between the top and the bottom layer as large as the SQL of each layer. The analysis is made for a single interrogation as the time evolution of a quantum state, resulting in \( \tau = \tau_0 \). These assumptions reduces Eq. 2 to \( \sigma_{\text{QPN}} = 1/\omega_0 \tau_0 n_{\text{site}} \) for each layer, and the amount of the gravitational redshift between the top and the bottom layer is \( gn_{\text{site}} \lambda/2c^2 \). Equating these two gives \( n_{\text{site}} = 497 \), leading to \( \sigma_{\text{QPN}} = 2.06 \times 10^{-20} \) for a single layer and \( \sigma_{\text{QPN}} = 9.23 \times 10^{-22} \) for whole \( N \) atoms. With these conditions, the top layer is incoherent with the bottom layer after the time evolution over 30 s, and the whole atomic ensemble that was initially coherent is no longer a single CSS. This is a kind of decoherence that is induced by an effect of the gravity, meaning that a high-precision optical lattice clock can be a tool to observe a decoherence of a quantum state induced by gravity for the first time. Note that this is a different phenomenon from the conventional gravitational decoherence, where quantum mechanical fluctuation of the spacetime decoheres a quantum state [27, 28]. Also, some previous reports on the decoherence of clocks by gravitational effects regard the source of decoherence as the interaction between atoms through gravity [29, 30], which is also different perspective from the analysis here. Rather, the form of the decoherence resembles the dephasing in the context of NMR induced by an external gravitational field, and thus this phenomenon is called gravitational dephasing in this article.

The usage of the entangled state enhances the sensitivity to the gravitational dephasing. Naively, Eq. 2 shows the reduction of the QPN by \( s \) factor of \( \xi_W \). In addition, entangled states are more sensitive to the decoherence than a CSS. Unless the inter-layer entanglement is negligible, the dephasing between the layers breaks the coherence, which distorts the distribution of the state on the Bloch sphere. Such a change can be detected by a state tomography.

The total atom number of \( N = 1.23 \times 10^8 \) in the system of \( n_{\text{site}} = 497 \) is orders of magnitude larger than the largest number mentioned in Ref. [13]. Also, the length of the one edge of the cube is slightly larger the beam size of 170 \( \mu \text{m} \) for a one-dimensional lattice [10]. From these perspectives, decent amount of technical development is necessary to observe the gravitational dephasing in the optical lattice clock. However, the dephasing can be described not between different layers but between larger portion of the atomic ensemble. For example, when the coherence is defined between upper half and the lower half, the decoherence is observable at \( n_{\text{site}} = 165 \), corresponding to \( N = 4.49 \times 10^6 \). This increases the feasibility substantially.

To quantify the effect more quantitatively, the behavior of the whole system is calculated by summing up the behavior of each layer. Specifically, atoms in a single layer of the same \( z \) do not experience any relative phase shift due to the gravitational redshift, and therefore each layer can be regarded as a CSS. The overall atomic ensemble can be described as the sum of the \( n_{\text{site}} + 1 \) layers perpendicular to \( z \) axis. Because the relative phase shift of the upper half is the same amount with the opposite sign compared to the lower half, the phase drift between the atomic system and the local oscillator observable in the Ramsey sequence does not change even with the existence of the significant gravitational redshift. However, relative shift between each layer shortens the effective length of the overall Bloch vector under the gravitational redshift.

This shortening of the overall Bloch vector is numerically estimated by the sum of the Bloch vector for each layer:
\[ \left( \begin{array}{c} S_x \\ S_y \\ S_z \end{array} \right) = \sum_{k=-n_{\text{site}}/2}^{n_{\text{site}}/2} \left( \begin{array}{c} \cos(\phi_1 + k\phi_2)t \\ \sin(\phi_1 + k\phi_2)t \end{array} \right), \] 
(3)
where \( S_x(y) \) is the \( x \) (\( y \)) component of the overall Bloch vector, \( \phi_1 \) is the phase shift per unit time due to the phase drift of the laser, and \( \phi_2 \) is the phase drift per unit time due to the gravitational redshift per layer. The length
of the Bloch vector for a single layer is normalized to 1. The nominal phase shift due to the laser drift without the gravitational redshift is \( \phi_l \), and the measured phase shift \( \phi_{\text{eff}} \) is calculated as \( \phi_{\text{eff}} = \sin^{-1}(S_l/n_{\text{site}} + 1) \). Figure 1 shows \( \phi_{\text{eff}} / \phi_l \) for different \( n_{\text{site}} \). At \( n_{\text{site}} = 100 \), no significant degradation in the phase measurement is observed even for \( \tau = 200 \) s, but for \( n_{\text{site}} = 500 \), the measured phase drift is underestimating the actual phase drift by 40 \% at \( \tau = 100 \) s. This clearly shows that the stability of the atomic clock is limited by the gravitational dephasing. The shift more than the SQL, which is \( 1/n_{\text{site}} \) for the Bloch vector for a single layer normalized to 1, is certainly not allowed for an accurate operation of an optical lattice clock.

The maximally allowed interrogation time is defined as the time when the systematic shift due to the dephasing is as large as the SQL. Fig. 2 shows the SQL limited stability at such maximum interrogation time for different \( n_{\text{site}} \) and \( \phi_l \). At small \( n_{\text{site}} \), the stability scales to \( n_{\text{site}}^{-1} \). This is when the stability is limited by the coherence of the laser for small \( n_{\text{site}} \), and \( \tau \) is basically more or less constant. In fact, for the smallest \( n_{\text{site}} = 2 \) and \( \phi_l = 10^{-6} \) s\(^{-1} \), to obtain the stability shown in the plot, \( \tau \) has to be as large as \( 1.97 \times 10^6 \), which satisfies \( \phi_l \tau \sim 1 \). This \( \tau \) is unrealistically large given the current technology on the stabilization of the laser and atomic system. Assuming the laser is infinitely stable, the plot is dominated by the \( \sim n_{\text{site}}^{0.25} \) scaling due to nonzero \( \phi_g \). The minimum of the stability is determined by the compromise between these two factors. This is, for instance, for \( \phi_l = 10^{-2} \) s\(^{-1} \), \( 2 \times 10^{-19} \) with \( n_{\text{site}} \simeq 200 \). Because the scaling of the stability to \( \phi_g \) is not large, regardless of the improvement in the laser stability, the stability of the optical lattice clock is limited to \( \sim 10^{-19} \) per 1 s due to the gravitational redshift. Note that this is consistent with the naive estimate that 100 \( \mu \)m shift of the height can be detected with \( 10^{-20} \) stability clock; the interrogation time required to reach \( 2 \times 10^{-19} \) with \( n_{\text{site}} = 200 \) is \( \tau = 60 \) s, and therefore actual stability in a single sequence is \( 2.58 \times 10^{-20} \). This is larger than the stability limit \( 1.30 \times 10^{-21} \) of the relativistic limit on a trapped single atom discussed in Ref. [31].

The suppression of the dephasing is impossible, as far as the optical lattice clock is on Earth, unlike the dephasing due to the magnetic field or electric field that can be compensated by applying a bias field. The spin echo sequence retains the coherence, but the cancellation of the phase shift of the laser against the atomic system makes it impractical to implement it to the optical lattice clock system. A way to circumvent the limitation to the coherence is to implement the single-atom-resolved detection, which is developed recently in the field of quantum degenerate gas [32, 33]. The state-of-the-art single-atom-resolved detection for 2D optical lattice in a single plane is not ideal for trapping a large number of atoms. Currently available best resolution for the detection of a 3D optical lattice is 1.1 \( \mu \)m [14]. If this improves further down to the level that each layer can be detected separately, the systematic compensation after the measurement is possible. Note that as far as the relative shift within an atomic ensemble is concerned, the uncertainty in the absolute height from the geodesy and any local deviation of the gravitational acceleration from the standard value is not a problem. The variation of \( g \) in the order of 0.1% on Earth is not significant, either, as far as the lower order compensation by \( g \) is discussed.

To obtain this tiny effect of the gravitational redshift, other systematic shifts needs to be suppressed. As Fig. 2 shows, to observe the gravitational redshift with realistic number of atoms in the trap, the linewidth of the laser of \( \sim 1 \) mHz or smaller, which is slightly better than currently available most stable laser [18], is required.

Some systematic shifts for optical lattice clocks are not
problems in the current analysis as far as the goal is to observe the relative frequency shift induced by the gravitational redshift, not the absolute shift from the true value. The density shift is one of such systematic shifts, because a three-dimensional lattice with single atom in each site has uniform density. The probe AC Stark shift does not matter, either. Typically, the probe beam is set at a much larger beam size than the trap beam, where the position difference of the intensity is suppressed better than the lattice laser. The background gas collision [10] is also negligible, because it happens uniformly over the atomic ensemble.

Other shifts needs to be carefully estimated. Here, the assumption is to detect the gravitational redshift with \( \frac{1}{3} \) corresponding to \( \Delta z = 39.68 \) \( \mu \)m, which gives the absolute shift of \( \Delta \nu = 2.145 \times 10^{-6} \) Hz between the top and the bottom layer. The allowed field gradient by the coefficient for the first order Zeeman shift of 199.516(2) Hz/G [10] is \( 2.69 \times 10^{-4} \) G/m. The precise measurement of the local magnetic field down to this level can be performed with transitions that are sensitive to magnetic field. For example, the \( ^1S_0 \rightarrow ^3P_2 \) transition has the Zeeman splitting of 2.1 MHz/G with 14 s lifetime. The resulting frequency shift of 0.0214 Hz between the top and bottom layers is detectable and therefore proper compensation based on the measurement is possible. Also, in the typical operation of the optical lattice clock, the first order Zeeman shift is cancelled by alternatingly measuring the transition frequency of different circular polarizations. Similar cancellation can also be applied to the observation of the gravitational redshift. With the field gradient limited by the first order Zeeman shift, the second order Zeeman shift of -0.06095(7) Hz/G^2 [10] induces a negligible amount of the frequency shift. The allowed electric field gradient of \( < 3.04 \times 10^4 \) (V/m)/m based on the coefficient for the DC Stark shift of 3.626 \( \times 10^{-6} \) Hz/(V/m)^2 [34] is achievable by a metal-lic shield in the vacuum chamber and indium tin oxide coating on the viewports.

The state-of-the-art understanding of the lattice AC Stark shift is that with proper choice of the polarization and frequency, the power dependence of the frequency shift is quadratic, and staying at the proper combination of the power and frequency cancels the linear term [19]. To estimate the residual shift quadratic to the power, the trap beam of \( w = 170 \) \( \mu \)m [10] is assumed. The ratio of the power between the top and bottom layers is proportional to that of the beam area \( w^2(z)/w^2(z+100\lambda) \), which has maxima at \( z/2R = \pm \sqrt{(100\lambda/z)^2 + 4}/2 \) away from the waist, resulting in the maximum power change of 0.0846 %. Because 10 % power change generates \( 1 \times 10^{-19} \) shift [19], this is smaller than the gravitational redshift. Proper alignment to the waist would reduce the shift further.

Note that power difference of the \( x \) and \( y \) beams in the \( z \) direction due to the Gaussian profile is significantly larger than the shift estimated above for a three-dimensional lattice. However, the profile of the power is not monotonic around the center of the beam, and thus the frequency shift does not have the same behavior as the gravitational redshift. This provides a way to discriminate the lattice AC Stark shift from the gravitational redshift. For the accurate operation of the optical lattice clock, the vector AC Stark shift is another concern. However, because the relative shift is concerned, the vector AC Stark shift generating the common shift of the transition frequency does not affect the relative shift.

The difficulties in estimating black body radiation (BBR) shift in the accurate operation of the optical lattice clock is eased to the relative shift between the different part of the atomic ensemble with respect to the gravitational redshift detection. The main concern here is that the nonuniform temperature of the surrounding structure can generate the transition frequency shift linear to the position, similar to the gravitational redshift. To estimate this, suppose the two opposite chamber walls have temperature of \( T_1 = 293 \) K and \( T_2 = 294 \) K. Based on the typical size of the vacuum chamber for the optical lattice clock, the chamber wall is assumed to be 5 cm away. This gives the BBR shift ratio of \( (T_2^3 d\Omega_+ + T_1^3 d\Omega_-)/(T_2^3 d\Omega_+ + T_1^3 d\Omega_-) \), where \( d\Omega_+(-) \) is the solid angle for the nearest (farthest) layer from the wall. This gives 2.73 \( \times 10^{-5} \) difference in the BBR field, resulting in 6.46 \( \times 10^{-20} \) shift in the BBR shift with the overall shift of \(-2.37 \times 10^{-15} \) [10]. This means that to observe the gravitational redshift, uniformity of the temperature over the chamber in the level of 10 mK is desired. Note that this estimate is an upper limit for the one hemisphere being \( T_1 \) and the other being \( T_2 \), and system-specific estimates can give smaller amount of the difference in the BBR field, leading to less stringent requirement for the temperature uniformity. Also, designing the vacuum chamber in a way to suppress the temperature nonuniformity in the vertical direction is possible, e.g., by locating the oven at the same height as the atomic ensemble. If such suppressions are not good enough, another transition that has a different sensitivity to the BBR shift can be utilized to evaluate the shift in the amount of BBR field [15].

Overall, the suppression of the systematic background to the gravitational redshift looks achievable, and therefore the observation of the gravitational dephasing seems possible.

The high precision of the optical lattice clock is reaching to the level of the gravitational redshift observable within an atomic ensemble. This can be visible at the relative stability of \( 10^{-21} \), and would limit the 1 s stability of the clock to \( 10^{-19} \). The observation of this effect can be the first observation of the decoherence of a quantum state induced by gravity. The observation is only possible when other systematic shifts are removed.
which seems reasonably feasible.

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