Accuracy in Measuring the Neutron Star Mass in the Gravitational Wave Parameter Estimation for Black Hole-Neutron Star Binaries

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(Received 25 July 2016, in final form 28 July 2016)

Recently, two gravitational wave (GW) signals, named as GW150914 and GW151226, have been detected by the two LIGO detectors. Although both signals were identified as originating from merging black hole (BH) binaries, GWs from systems containing neutron stars (NSs) are also expected to be detected in the near future by the advanced detector network. In this work, we assess the accuracy in measuring the NS mass ($M_{\text{NS}}$) for the GWs from BH-NS binaries adopting the Advanced LIGO sensitivity with a signal-to-noise ratio of 10. By using the Fisher matrix method, we calculate the measurement errors ($\sigma$) in $M_{\text{NS}}$ assuming a NS mass of $1 \leq M_{\text{NS}}/M_\odot \leq 2$ and low-mass BHs with masses in the range of $4 \leq M_{\text{BH}}/M_\odot \leq 10$. We use the TaylorF2 waveform model in which the spins are aligned with the orbital angular momentum, but here we only consider the BH spins. We find that the fractional errors ($\sigma/M_{\text{NS}} \times 100$) are in the range of $10\% - 50\%$ in our mass region for a given dimensionless BH spin $\chi_{\text{BH}} = 0$. The errors tend to increase as the BH spin increases, and this tendency is stronger for higher NS masses (or higher total masses). In particular, for the highest mass NSs ($M_{\text{NS}} = 2 M_\odot$), the errors $\sigma$ can be larger than the true value of $M_{\text{NS}}$ if the dimensionless BH spin exceeds $\sim 0.6$.

PACS numbers: 04.30.-w, 04.80.Nn, 95.55.Ym
Keywords: Gravitational waves, Parameter estimation, Fisher matrix, Black hole, Neutron star
DOI: 10.3938/jkps.69.884

I. INTRODUCTION

During the first observational run of the Advanced LIGO detectors, the first gravitational wave (GW) signals were detected, and further detailed analyses showed that those GWs were emitted from merging binary black holes (BBHs) [1–4]. These observations indicate that future observation runs of the advanced detector network will yield many more BBH merger signals [5–7]. On the other hand, GW signals from black hole (BH)-neutron star (NS) or NS-NS binaries are expected to be captured a few times per year in the near future [5,6,8], although those signals will not be as many as the BBH signals.

Detection of GWs from BH-NS or NS-NS binaries is very important because those signals can tell us about the nature of NSs, particularly the NS masses. Determining the upper limit of the NS mass is one of the most challenging issues in modern astrophysics. In various theoretical models, the NS masses are expected to range between $1 M_\odot$ and $3 M_\odot$ [9]. On the other hand, except for the two higher mass NSs whose masses are $\sim 2 M_\odot$ [11,12], in most of the well-measured NS-NS or NS-white dwarf binaries, the NS masses seem to be clustered around $\sim 1.4 M_\odot$ [10]. However, more observations are still necessary to robustly confirm the NS mass limit, and GWs from BH-NS or NS-NS binaries should provide a tighter constraint on the distribution of NS masses.

In GW data analysis for compact binary coalescences, once a detection is made in the search pipeline, a detailed analysis is performed in the parameter estimation pipeline to identify the physical parameters of the binary source [2,13,14]. The result of the parameter estimation is given by a probability density distribution in parameter space. Typically, a very long computation time is required to complete the parameter estimation procedure. However, the measurement errors can be easily approximated by using the Fisher matrix (FM) method if the signal is strong enough and the noise is Gaussian.

In our previous work [15], we showed the measurement accuracy of the NS mass for various companion masses in a nonspinning binary system. In this work, we extend the previous work to a more generic system where the binaries can have spins. Particularly, we only consider the BH spin because the contribution of the NS spin to the binary evolution is negligible compared to that of the BH spin. Furthermore, we also assume that the BH spin angular momentum is aligned with the orbital angular momentum so that the binary does not precess during the evolution. As in the previous work, we adopt a simple Fourier-domain waveform model and use the

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FM method to predict the measurement errors of the NS mass with the Advanced LIGO detector sensitivity.

II. WAVEFORM MODEL FOR ALIGNED-SPIN BINARIES

In past studies on GW data analyses, the most commonly used waveform model is the TaylorF2, which is a Fourier-domain model obtained from a time-domain post-Newtonian model via the stationary phase approximation [16–18]. The waveform function of the TaylorF2 is expressed as

\[ h(f) = Af^{-\gamma/6}e^{i\Psi(f)}, \quad (1) \]

where \( A \) is the wave amplitude that consists of the binary masses and the extrinsic parameters. The amplitude simply sets a scale for the matched filter output, so does not affect our analysis. The wave phase is defined as

\[ \Psi(f) = 2\pi ft_c - 2\phi_c - \frac{\pi}{4} + \frac{3}{128\eta}v^2 \phi(f), \quad (2) \]

where \( t_c \) and \( \phi_c \) are the coalescence time and the coalescence phase, respectively, \( \eta = m_1m_2/M^2 \) is the symmetric mass ratio with \( M = m_1 + m_2 \), and \( \phi(f) \) can be expressed by using the post-Newtonian expansion as [19]

\[
\phi(f) = 1 + \left( \frac{3715}{756} + \frac{55}{9} \eta \right) v^2 + (4\beta - 16\pi) v^3 + \left( \frac{15293365}{504032} + \frac{27145}{504} \eta + \frac{3085}{72} \eta^2 - 10\sigma \right) v^4 \\
+ \left( \frac{38645\pi}{756} - \frac{65\pi}{9} \eta - \gamma \right) (1 + 3 \log v) v^5 + \left( \frac{11583231236531}{4694215680} - \frac{640}{3} \pi^2 - \frac{684\gamma_c}{21} - \frac{684\log(4v)}{21} \right) v^6 \\
+ \left( \frac{2255\pi^2}{12} - \frac{15737765635}{3048192} \right) \eta + \frac{76055}{1728} \eta^2 - \frac{127285}{1296} \eta^3 + \frac{77096675\pi}{254016} + \frac{378515\pi}{1512} \eta - \frac{74045\pi}{756} \eta^2 \right) v^7,
\]

with \( v = [\pi f M]^{1/3}, \gamma_c = 0.577216... \) being the Euler constant, and the terms \( \beta, \sigma, \) and \( \gamma \) denoting the leading-order spin-orbit coupling, leading-order spin-spin coupling, and next-to-leading-order spin-orbit coupling, respectively. For an aligned-spin system, these can be expressed as

\[
\beta = \sum_{i=1}^{2} \left[ \frac{113}{12} \left( \frac{m_i}{M} \right)^2 + \frac{25\eta}{4} \right] \chi_i, \quad \sigma = \frac{474\eta}{48} \chi_1 \chi_2 + \sum_{i=1}^{2} \frac{81}{16} \left( \frac{m_i}{M} \right)^2 \chi_i^2 \\
\gamma = \sum_{i=1}^{2} \left[ \left( \frac{732985}{2268} + \frac{140\eta}{9} \right) \left( \frac{m_i}{M} \right)^2 + \eta \left( \frac{13915}{84} - \frac{10\eta}{3} \right) \right] \chi_i, \quad (4)
\]

where \( \chi_i \equiv S_i/m_i^2 \) is a dimensionless BH spin, \( S_i \) being the spin angular momentum of the \( i \)th compact object. For a nonspinning system, the above spin terms are simply set to 0, so we need only to consider the four physical parameters \( m_1, m_2, t_c, \) and \( \phi_c \). On the other hand, for an aligned-spin system the two parameters \( \chi_1 \) and \( \chi_2 \) should also be included. However, since we decided to consider only the BH spin in this work, we remove the second spin component by choosing \( \chi_2 \equiv \chi_{NS} = 0 \). Thus, we will deal with the five parameters \( m_1, m_2, \chi_1, t_c, \) and \( \phi_c \) in our analysis.

III. MEASUREMENT ERROR IN THE PARAMETER ESTIMATION

A match between a detector data stream \( x(t) \) and a model waveform \( h(t) \) can be obtained by using the inner product \( \langle x|h \rangle \) as

\[
\langle x|h \rangle = 4\text{Re} \int_{f_{\text{low}}}^{\infty} \frac{\bar{x}(f)\bar{h}^*(f)}{S_n(f)} df, \quad (5)
\]

where \( \bar{x}(f) \) and \( \bar{h}(f) \) represent the Fourier transforms of \( x(t) \) and \( h(t) \), respectively, \( S_n(f) \) is the noise power spectral density for the detector and \( f_{\text{low}} \) is the low-frequency
where $\Gamma^{-1}$ is the FM and $\rho_{M}$ is the measurement error. We take into account the zero-detuned, high-power noise power spectral density of Advanced LIGO [20] and choose $f_{\text{low}}$ to be 10 Hz.

The parameter estimation algorithm performs the above match computations iteratively until the algorithm recovers the true values of the parameters. Thus, the efficiency of the parameter estimation is subject to how fast the algorithm can find the true values, and its computation time mainly depends on the waveform model and the dimension of the parameter space. Generally, this procedure is a time-consuming task because the parameter estimation explores the whole parameter space without any information about the true parameters except for the coalescence time [2,4]. On the other hand, if an incident GW signal is buried in Gaussian noise and strong enough, the posterior probability density function of the parameter estimation is given by a Gaussian distribution of the form [21]

$$p(\Delta \lambda^i) \propto \exp\left[-\frac{1}{2} \Gamma_{ij} \Delta \lambda^i \Delta \lambda^j\right],$$

where $\Gamma_{ij}$ is the FM defined as [22–24]

$$\Gamma_{ij} = -\rho^2 \left\langle \frac{\partial \tilde{h}}{\partial \lambda_i} \frac{\partial \tilde{h}}{\partial \lambda_j} \right\rangle_{\lambda = \lambda_{\text{true}}},$$

with $\rho \equiv \langle h | h \rangle^{1/2}$ being the signal-to-noise ratio (SNR) and $\tilde{h} \equiv h / \sqrt{\langle h | h \rangle}$ being the normalized waveform. The inverse of $\Gamma_{ij}$ represents the covariance matrix of the parameter errors, and the error ($\sigma_i$) of each parameter is determined by $\sigma_i = \sqrt{(\Gamma^{-1})_{ii}}$. Thus, $\sigma_i$ is inversely proportional to the SNR.

For our BH-NS system, by applying the phase function in Eq. (2) to the FM formalism in Eq. (7), we can calculate a $5 \times 5$ matrix for the parameters $\{M_{\text{BH}}, M_{\text{NS}}, \chi_{\text{BH}}, c, \phi_i\}$ with true values given and obtain the measurement error $\sigma_i$ of each parameter. On the other hand, the parameters $c$ and $\phi_i$ are arbitrary constants, so unimportant in the analysis. However, unlike the extrinsic parameters incorporated in the waveform amplitude, these are, in general, strongly correlated with the other parameters. Therefore, they should be taken into account simultaneously with the other parameters in the construction of the FM.

In order for the FM approach to be valid, two conditions are required: Gaussian noise and high SNR. If non-Gaussian noises are mixed in the data, the posterior probability density function can show a non-Gaussian distribution, and the recovered parameter values can be biased. However, such non-Gaussian noises are unlikely to have a significant impact on the parameter estimation result if the SNR is high enough [25]. In this work, we assume a moderately high SNR of 10, as in the previous work [15]. One should note that whether the FM approach is valid or not at this SNR level is uncertain. Here, we choose the SNR arbitrarily just to give the fiducial values, since the measurement error in the parameter estimation is simply inversely proportional to the SNR, for a real SNR ($\rho_{\text{real}}$) with which the validity of the FM approach is guaranteed, a real error ($\sigma_{\text{real}}$) can be obtained as $\sigma_{\text{real}} = \rho / \rho_{\text{real}}$, where $\rho$ denotes the error given in our result. On the other hand, it has been noted that due to the abrupt cutoff of the waveforms at the innermost stable circular orbit ($\pi M f_{\text{isco}} = 6^{-3/2}$), the FM result obtained by using the TaylorF2 model cannot be trusted in the high-mass region beyond $\sim 10 M_\odot$. Therefore, we only consider the BH mass up to $10 M_\odot$.

IV. RESULT: MEASUREMENT ERROR FOR THE NEUTRON STAR MASS

Here, we present our results for the measurement errors in $M_{\text{NS}}$ for various BH masses and spins. As in the previous work [15], we assume the mass range of the fiducial NS to be $1 \leq M_{\text{NS}} / M_\odot \leq 2$ and the BH mass range to be $4 \leq M_{\text{BH}} / M_\odot \leq 10$. We only consider a single detector, and we assume the SNR to be 10. On the other hand, while we considered a nonspinning system and hence used only the two mass parameters in the previous work, we consider an aligned-spin system here, and the BH spin parameter is also taken into account in this work.

In Fig. 1, we show the measurement errors in $M_{\text{NS}}$ in the $M_{\text{BH}} - M_{\text{NS}}$ plane, where the values indicate the fractional errors defined by $\sigma / M_{\text{NS}} \times 100$. Here, the BH spin is assumed to be 0. We find that the fractional error is the largest with the heaviest NS and the lightest BH binary (upper left corner) and tends to decrease with increasing BH mass or decreasing NS mass. This trend is similar to the result for the nonspinning system given in [15]. However, the measurement accuracy is significantly lower.
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V. DISCUSSION

In this work, we investigated the measurement accuracy of the NS mass in the parameter estimation of GWs from BH-NS binaries. We adopted the 3.5 pN aligned-spin TaylorF2 waveform model in which the spin terms are included up to 2.5 pN and applied this model to the FM method. In our BH-NS binaries, we assumed the BH mass to be lower than 10 $M_\odot$ and the spin to be aligned with the orbital angular momentum. The NS mass is assumed to be $1 \leq M_{\text{NS}}/M_\odot \leq 2$, and the NS spin is not considered in the analysis. The result shows that the fractional errors ($\sigma/M_{\text{NS}} \times 100$) are in the range of 10%–50% in our mass region for a given BH spin $\chi_{\text{BH}} = 0$ and that these errors become larger as the BH spin increases. In particular, by comparing our result with that of our previous work [15] where we considered a nonspinning system, we confirmed that the mass ratio is strongly correlated with the spin in an aligned-spin system. The errors are overall about 20 times larger than those for the nonspinning system in the same mass region.

Typically, the evolution of merging binaries containing BHs is divided into three phases: inspiral, merger, and ringdown. In the low-mass region considered in this work, the merger-ringdown phases are outside the detector sensitivity band; thus, only the inspiral waveform models such as TaylorF2 can be used in the GW data analysis. However, for more massive systems, full inspiral-merger-ringdown waveform models should be used so as not to lose the merger-ringdown portions. For this purpose, various models have been developed over the past years, and the phenomenological models have been commonly used [34–40] (for a brief description of these models, see [41]). Since the phenomenological models are constructed as analytic functions in the frequency domain, those are also applicable to the FM method. Therefore, our work can be easily extended to higher mass BBHs.

Finally, it is worth noting that the orbital plane can precess if the spin is misaligned with the orbital angular momentum. For space-based detectors such as LISA, several works have shown that the precession effect can break the mass-spin degeneracy, thus improving the mea-

Fig. 2. Measurement errors ($\sigma/M_{\text{NS}} \times 100$) calculated by changing the BH spin ($\chi_{\text{BH}}$). We assume a fixed mass ratio of $M_{\text{BH}}/M_{\text{NS}} = 4$ and a SNR of $\rho = 10$.

decreased compared to that of the nonspinning system. The errors in Fig. 1 are in the range of about 10%–50%, and those are overall about 20 times larger than the errors for the nonspinning system given in [15]. The main cause of this difference is that the parameter space to be explored is extended from a four parameter system to a five parameter one by taking into account the spin parameter additionally. In general, if a spinning binary system is considered in the parameter estimation, a degeneracy between the components’ mass ratio and their spins significantly degrades our ability to measure the individual component masses [17,18]. Therefore, in the posterior distribution of the parameter estimation for an aligned-spin system, the mass parameters are strongly correlated with the spin parameter; thus, the range of error in $M_{\text{NS}}$ can be significantly increased compared to that for a nonspinning system (for example, see [29,30]). Simply speaking, for the case of a nonspinning system, since we already know that the system has no spin, we do not need to consider the spin parameter, and the measurement error comes from the uncertainties of the two mass parameters. However, for the case of an aligned-spin system, since the system can have a spin, but we do not know the true value of the spin, we should incorporate the additional uncertainty inherent in the spin parameter, which is strongly correlated with the mass parameters.

In Fig. 2, we show the dependence of the fractional errors on the BH spin. Here, we calculate the errors only by varying the BH spin from 0 to 0.9 for three NS masses of $2 M_\odot$, $1.5 M_\odot$, and $1 M_\odot$, assuming a fixed mass ratio of $M_{\text{BH}}/M_{\text{NS}} = 4$. Generally, the spin-mass correlation tends to be stronger as the BH spin increases; thus, a high BH spin lowers the measurement accuracy of the masses. We, therefore, find that the errors become larger as the BH spin increases. We also find that the increasing tendency is more pronounced for higher NS masses (or higher total masses). The fractional errors can increase from 23% to 93% and from 12% to 30% for $M_{\text{NS}} = 1.5 M_\odot$ and $1 M_\odot$, respectively. Especially, for the case of $M_{\text{NS}} = 2 M_\odot$, we found that the fractional errors increased from 36% to 182%, and those are larger than the true value of $M_{\text{NS}}$ if $\chi_{\text{BH}} > 0.6$. Most interestingly, in certain cases, we were unable to distinguish between BHs and NSs. For example, if the true values of a GW signal are $M_{\text{NS}}$, $M_{\text{BH}} = 2.8 M_\odot$ and $\chi_{\text{BH}} = 0.9$, the recovered NS mass is in the range of $0 < M_{\text{NS}} \leq 5.6 M_\odot$; then, we cannot determine whether this is a NS or a light BH (for more examples, see [31–33]).
measurement accuracy of the mass and the spin parameters [42, 43]. The authors of [24, 44, 45] showed that such improvement can also be achieved for ground-based detectors. One of the phenomenological models, i.e., PhenomP [40], was designed to model the precessing BBH waveforms; thus, with this model, our analysis can be extended to a precessing system.

ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (Ministry of Science, ICT & Future Planning) (No. 2016R1C1B2010064). This work used the computing resources at the KISTI Global Science Experimental Data Hub Center (GSDC).

REFERENCES

[1] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016).
[2] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 241102 (2016).
[3] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 241103 (2016).
[4] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), arXiv:1606.01210 (2016).
[5] J. Abadie et al. (LIGO Scientific Collaboration and Virgo Collaboration), Class. Quantum Grav. 27, 173001 (2010).
[6] M. Dominik, E. Berti, R. O’Shaughnessy, I. Mandel, K. Belczynski, C. Fryer, D. Holz, T. Bulik and F. Pannarale, Astrophys. J. 806, 263 (2015).
[7] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), arXiv:1606.04856 (2016).
[8] C. Kim, B. P. P. Perera and M. A. McLaughlin, MNRAS 448, 928 (2015).
[9] J. M. Lattimer and M. Prakash, Phy. Repts. 442, 109 (2007).
[10] M. Prakash, arXiv:1307.0397 (2013).
[11] J. Aasi et al., Science 340, 448 (2013).
[12] J. Antoniadis et al., (LIGO Scientific Collaboration, Virgo Collaboration), Phys. Rev. D 88, 062001 (2013).
[13] J. Aasi et al., (LIGO Scientific Collaboration, Virgo Collaboration), Phys. Rev. D 88, 104023 (2013).
[14] H.-S. Cho, J. Korean Phys. Soc. 66, 1637 (2015).
[15] H.-S. Cho, J. Korean Phys. Soc. 67, 960 (2015).
[16] B. S. Sathyaprakash and S. V. Dhurandhar, Phys. Rev. D 44, 3819 (1991).
[17] C. Cutler and E. É. Flanagan, Phys. Rev. D 49, 2658 (1994).
[18] E. Poisson and C. M. Will, Phys. Rev. D 52, 848 (1995).
[19] K. G. Arun, A. Buonanno, G. Faye and E. Ochsner, Phys. Rev. D 79, 104023 (2009).
[20] Advanced LIGO anticipated sensitivity curves, https://dcc.ligo.org/LIGO-T0900288/public.
[21] L. S. Finn, Phys. Rev. D 46, 5236 (1992).
[22] M. Vallisneri, Phys. Rev. D 77, 042001 (2008).
[23] P. Jaranowski and A. Królak, Phys. Rev. D 49, 1723 (1994).
[24] H.-S. Cho, E. Ochsner, R. O’Shaughnessy, C. Kim and C.-H. Lee, Phys. Rev. D 87, 024004 (2013).
[25] C. P. L. Berry, et al., Astrophys. J. 804, 114 (2015).
[26] C. L. Rodriguez, B. Farr, W. Farr and I. Mandel, Phys. Rev. D 88, 084013 (2013).
[27] I. Mandel, C. Berry, F. Ohme, S. Fairhurst and W. M. Farr, Class. Quantum Grav. 31, 155005 (2014).
[28] H.-S. Cho and C.-H. Lee, Class. Quantum Grav. 31, 235009 (2014).
[29] R. O’Shaughnessy, B. Farr, H.-S. Cho, C. Kim and C.-H. Lee, Phys. Rev. D 89, 064048 (2014).
[30] H.-S. Cho, Class. Quantum Grav. 32, 235007 (2015).
[31] M. Hannam, D. A. Brown, S. Fairhurst, C. L. Fryer and I. W. Harry, Astrophys. J. 766, L14 (2013).
[32] T. B. Littenberg, B. Farr, C. Coughlin, V. Kalogera and D. E. Holz, Astrophys. J. 807, L24 (2015).
[33] I. Mandel, C. Haster, M. Dominik and K. Belczynski, MNRAS 450, L85 (2015).
[34] P. Ajith et al., Class. Quantum Grav. 24, S689 (2007).
[35] P. Ajith et al., Phys. Rev. D 77, 104017 (2008); 79, 129901(E) (2009).
[36] P. Ajith, Class. Quantum Grav. 25, 114033 (2008).
[37] P. Ajith et al., Phys. Rev. Lett. 106, 241101 (2011).
[38] L. Santamaria et al., Phys. Rev. D 82, 064016 (2010).
[39] S. Khan et al., Phys. Rev. D 77, 044007 (2016).
[40] M. Hannam et al., Phys. Rev. Lett. 113, 151101 (2014).
[41] H.-S. Cho, Class. Quantum Grav. 32, 215023 (2015).
[42] R. N. Lang and S. A. Hughes, Phys. Rev. D 74, 122001 (2006).
[43] A. Klein, P. Jetzer and M. Sereno, Phys. Rev. D 80, 064027 (2009).
[44] R. O’Shaughnessy, B. Farr, E. Ochsner, H.-S. Cho, V. Raymond, C. Kim and C.-H. Lee, Phys. Rev. D 89, 102005 (2014).
[45] K. Chatziioannou, N. Cornish, A. Klein and N. Yunes, Astrophys. J. 798, L17 (2014).