\(PT\)-symmetric cavity magnon polaritons

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Avoided level crossing is often regarded as the hallmark of strong magnon-photon coupling, with the anti-crossing gap quantifying the coupling strength. A recent report on the level attraction of dissipative magnon-photon interaction however came as a surprise. In this work, we propose an input-output theory for describing cavity magnon polaritons with the peculiar parity-time (\(PT\)) symmetry. In the exact \(PT\) phase, we predict a "Z"-shape spectrum including two side-band modes and a dark-state branch with an ultra-narrow linewidth. The spectrum evolves to a step function when the polariton touches the third-order exceptional point, accompanied by an enhanced sensitivity with respect to the detuning. The predicted magnetic sensitivity is one or two orders of magnitude higher than that of the state-of-the-art magnetoelectric sensor. Purcell-like effect is observed when the \(PT\) symmetry is broken. A wave-scattering calculation of a realistic magnetic bilayer inside a microwave cavity is implemented to compare with the results from the input-output formalism, establishing the one-to-one correspondence of parameters in the two approaches.

Strong light-matter interaction lies in the heart of cavity quantum electrodynamics and quantum information science. It allows the Rabi splitting and polaritonic eigenmode of hybrid systems. The subject has been extensively studied in the hybridized cavity and two-level system, including atoms [1], molecules [2, 3], superconducting qubits [4], and quantum dots [5, 6]. In recent years, cavity spintronics—the emerging interdiscipline of cavity quantum electrodynamics and spintronics—has been rapidly developing. A central issue in the community is to observe the cavity magnon polariton (CMP) [7–15]. Thanks to the extremely low damping and high spin density in ferrimagnetic insulators yttrium iron garnet (YIG) [16], the strong coupling between magnons and microwave photons has been realized. The effective coupling strength is typically identified from the anti-crossing gap of the transmission spectra. The intriguing strong-coupling and level repulsion physics can be either described by a quantum mechanical model, i.e., Jaynes-Cummings model [17] for single excitations and Tavis-Cummings model [18] for collective ones, or a classical model based on the Landau-Lifshitz-Gilbert (LLG) equation [19, 20]. Very recently, an exotic level attraction of non-Hermitian magnon-photon coupling was reported [21, 22], which opens a new avenue for exploring cavity spintronics.

Non-Hermitian phenomena are ubiquitous in nature, among which the most exciting one are those respecting parity-time (\(PT\)) symmetry. A non-Hermitian Hamiltonian satisfying the \(PT\) symmetry could exhibit entirely real spectra below the exceptional point (EP) [23, 24]. The order of the EP is determined by the number of eigenvalues and eigenstates that simultaneously coalesce. Currently, \(PT\) symmetry has been investigated in a broad field of quantum mechanics [23, 24], optics [25–28], acoustics [29, 30], electronics [31], and very recently in spintronics [32, 33] and magnetics and magnonics [34–37], with tantalizing physics, such as fast quantum state evolution [38], single-mode \(PT\) laser [25], unidirectional light transport [39], unidirectionally transparent acoustic cloak [29], enhanced sensor telemetry [40], and tunable antiferromagnetic to ferromagnetic phase transition [37], being reported. However, the property of \(PT\)-symmetric magnons coupled to cavity photons is yet to be addressed.

In this work, we theoretically study the coherent coupling between a cavity photon and two magnons with balanced gain and loss (see Fig. 1). The direct exchange coupling between magnons is assumed to be absent in the present model. The Hamiltonian under consideration is thus non-Hermitian, but \(PT\)–symmetric. From the input-output formalism, we derive the analytical formula of the transmission coefficient \(S_{21}\) and identify a novel "Z"-shape spectrum in the exact \(PT\) phase. An exceptional sensitivity around the third-order EP is predicted. We implement a wave-scattering calculation on a real system consisting of YIG bilayers in a microwave cavity to verify the predictions from the input-output formalism.

We start with the following bosonic Hamiltonian

\[
\mathcal{H} = \hbar \omega_0 \hat{a}^\dagger \hat{a} + \hbar (\omega_0 + i\beta) \hat{s}_1^\dagger \hat{s}_1 + \hbar \omega_s - i\beta \hat{s}_1^\dagger \hat{s}_2^\dagger \hat{s}_2 + \hbar c. \tag{1}
\]

where \(\hat{a}^\dagger (\hat{a})\) and \(\hat{s}_1^\dagger (\hat{s}_1, \hat{s}_2)\) are the photon and magnon creation (annihilation) operators, respectively, \(\omega_0\) is the cavity resonant frequency, \(\omega_s\) denotes the Zeeman splitting, \(\beta > 0\) describes the energy dissipation/amplification rate with environments, and \(g\) represents the magnon-photon coupling strength. Under a combined operation of parity \(P\) (\(\hat{s}_1 \leftrightarrow \hat{s}_2\)) and time reversal \(T\) (\(i \rightarrow -i, \hat{s}_{1(2)} \rightarrow -\hat{s}_{1(2)}, \text{ and } \hat{a} \rightarrow -\hat{a}\)), it is straightforward to find that Eq. (1) is invariant and thus respects the

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**Fig. 1:** Schematic illustration of photon scatterings by two magnons with balanced gain and loss in a microwave cavity.
$\mathcal{PT}$ symmetry. We consider single particle processes, so that three states $|\hat{a}^\dagger 0\rangle, |\hat{s}_1^\dagger 0\rangle, |\hat{s}_2^\dagger 0\rangle$ constitute the complete basis, where $|0\rangle$ represents the vacuum state. The Hamiltonian can therefore be expressed in the following matrix form (set $h = 1$),

$$\mathcal{H} = \begin{pmatrix} \omega_c + i\beta & 0 & g \\ 0 & \omega_c - i\beta & g \\ g & g & \omega_c \end{pmatrix}.$$  \hspace{1cm} (2)

By solving $\mathcal{H}|\phi\rangle = \omega|\phi\rangle$, we obtain the following cubic equation for the eigenvalues,

$$\left(\Omega^2 + P^2\right)\left(\Omega + \Delta\right) - 2\Omega = 0,$$  \hspace{1cm} (3)

with $\Omega = (\omega - \omega_s)/g, \Delta = (\omega_c - \omega_c)/g$ the frequency detuning, and $P = \beta/g$ being the balanced gain-loss parameter. Figure 2(a) shows the roots of (3) with a detuning parameter $\Delta = -0.3$. There are three real solutions at a small $P$, which corresponds to the unbroken $\mathcal{PT}$ phase. By increasing $P$, one pair of eigenvalues coalesce at $P_{\text{EP2}}$ and then bifurcate into the complex plane when $P > P_{\text{EP2}}$. Here $\text{EPN}$ represents the $N$th-order EP, with $N$ an integer. For a zero detuning ($\Delta = 0$), the closed-form solutions of the three eigenvalues are $\omega = \omega_c \pm \sqrt{2g^2 - \beta^2}$ for side modes and $\omega = \omega_c$ for the central mode, as shown in Fig. 2(b). The third-order exceptional point EP3 appears when $\beta = \sqrt{2g}$ (or $P = P_{\text{EP3}} = \sqrt{2}$), with the unique coalesced eigenstate being $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1)^T$. For $P < P_{\text{EP3}}$, the side modes represent an abnormal Rabi splitting with the Rabi frequency depending not only on the coupling strength, but also on the gain-loss parameter. This is a sharp contrast to their Hermitian counterpart. Interestingly, we note that the flat central mode (real for all $P$) actually corresponds to a dark-state polariton [41–43] (see analysis below).

The phase diagram [plotted in Fig. 2(c)] is determined by the sign of the discriminant

$$\Lambda = P^2\Delta^4 + (2P^4 + 10P^2 - 1)\Delta^2 + (P^2 - 2)^3,$$  \hspace{1cm} (4)

of (3). $\Lambda < 0$ gives the exact (or unbroken) $\mathcal{PT}$ phase, in which all three eigenvalues are real and the eigenvectors satisfy the so-called biorthogonal relation $\langle \phi|\phi\rangle = \delta_{ij}$ with $i, j = 1, 2, 3$ [44]. For $\Lambda > 0$, only one real eigenvalue survives and the other two become complex conjugated, which corresponds to the broken $\mathcal{PT}$ phase. EP2 happens along the critical curve $\Lambda = 0$ but with $\Delta \neq 0$ [see the grey curve in Fig. 2(c)]. EP3 emerges when both $P > P_{\text{EP3}} = \sqrt{2}$ and $\Delta = 0$ are simultaneously satisfied [see the red star in Fig. 2(c)].

We next derive the transmission coefficient of the hybridized system. To this end, we assume that the cavity is interacting with a harmonic bosonic bath (environment). By introducing the noise and dissipation functions into the Heisenberg equations of operators, we obtain the following quantum Langevin equations [45, 46],

$$\dot{\hat{a}} = (-\omega_c - \kappa_c)\hat{a} - ig(\hat{s}_1 + \hat{s}_2) + \hat{b}_\text{in},$$  \hspace{1cm} (5a)

$$\dot{\hat{s}}_1 = (-\omega_c + \beta)\hat{s}_1 - ig\hat{a},$$  \hspace{1cm} (gain) (5b)

$$\dot{\hat{s}}_2 = (-\omega_c - \beta)\hat{s}_2 - ig\hat{a},$$  \hspace{1cm} (loss) (5c)

where $\kappa_c$ represents the leakage rate of a photon to the environment (the internal loss of the cavity is assumed to be negligibly small), and $\hat{b}_\text{in}$ is the input/output field from the thermal bath, satisfying the input-output formula $\hat{b}_\text{in} + \hat{b}_\text{out} = 2\kappa_c\hat{a}$ [17, 46]. After some algebra, we obtain the frequency-resolved transmission coefficient (see Supplemental Material Sec. A [47] for detailed derivations),

$$S_{21} = -\frac{\kappa_c}{\omega - \omega_c - \kappa_c + \Sigma(\omega)},$$  \hspace{1cm} (6)

where the total self-energy $\Sigma(\omega) = \Sigma^{\text{gain}}(\omega) + \Sigma^{\text{loss}}(\omega)$ caused by the magnon-photon coupling includes two parts: $\Sigma^{\text{gain/loss}}(\omega) = g^2/[i(\omega - \omega_s) \pm \beta]$ for gain (+) and loss (−), respectively. We note that $\Sigma(\omega)$ now is purely imaginary, leading to a fully transparent transmission at resonance.

FIG. 2: Evolution of eigenvalues as the gain-loss parameter $P$, with the solid and dashed curves respectively representing the real and imaginary part of eigenfrequencies. The detuning parameters are chosen to be (a) $\Delta = -0.3$ and (b) $\Delta = 0$. The cavity frequency is set as $\omega_c/g = 5$. (c) $\mathcal{PT}$-symmetric phase transition diagram.
It has been shown that the non-Hermitian degeneracy can provide an enhancement of sensitivity $\propto |\Delta|^{1/2}$ at the $N$-th order EP. The sensitivity is conventionally defined as the splitting of eigenfrequencies perturbed around the EP. However, it becomes unfeasible due to the significant spectrum broadening near the EP as shown in Fig. 3(b). Further, due to the complex nature of the frequency bifurcation in the vicinity of EP, the view of exceptional precision of exceptional-point sensors has been challenged [52, 53] by arguing that the sensitivity of EP2 is limited by quantum fluctuations [52] and/or statistical noises [53]. In the present model, there always exists a real central mode no matter whether the $\mathcal{PT}$ symmetry is broken or not. We therefore suggest a more appropriate definition of the sensitivity as the separation between the always-real central mode and the constant cavity mode. At $P = P_{\text{EP3}}$, we find $\Delta\kappa_{\text{EP3}}/g = -\text{sgn}(\Delta)\Delta^{1/2} |\Delta|^{1/3}$, excellently consistent with numerical results in the small detuning regime, as plotted in Fig. 3(f). Following the idea in Ref. [53], we obtain the condition for the noise-less sensing performance $\Delta/\sigma \gtrsim 2$ with $\sigma$ representing the noise/fluctuation (see Supplemental Material Sec. B [47] for details). Such features near the 3rd order exceptional point can be utilized for designing magnetic sensor with high precision. Considering the frequency resolution for a cavity $\Delta \kappa_{\text{EP3}} \sim \kappa_c$, we obtain the sensitivity for detecting magnetic fields $\frac{\Delta B}{B} \approx \frac{\sqrt{2\sigma}}{\gamma}$, where $\gamma$ is the gyromagnetic ratio and $C \sim g^2/\kappa_c^2$ is the strong coupling cooperativity ranging from $10^{-2}$ to $10^0$ [7, 10, 11]. For a microwave cavity working at GHz with a MHz resolution and a sub-MHz noise, we can therefore estimate the magnetic sensitivity $\sim 10^{-14}$ T/√Hz, which is one or two orders of magnitude higher than that of the state-of-the-art magnetoelectric sensors [54].

Nonlinear effects have been completely ignored in the above calculation, which is justified only when the average magnon number is negligibly small. At a mean-field level, the nonlinear correction can be taken into account by re-writing the magnon part of Eqs. (5) as,

\begin{align}
\hat{s}_1 &= -i\omega_\delta - i\eta (\hat{s}_1^\dagger \hat{s}_1 + \beta) \hat{s}_1 - i\gamma \hat{a}, \quad \text{(gain)} \tag{8a} \\
\hat{s}_2 &= -i\omega_\delta - i\eta (\hat{s}_2^\dagger \hat{s}_2 - \beta) \hat{s}_2 - i\gamma \hat{a}, \quad \text{(loss)} \tag{8b}
\end{align}

where $\eta$ is the nonlinear coefficient possibly caused by the magnetic anisotropy [55, 56]. Solving these equations, we obtain $\langle \hat{s}_1^\dagger \hat{s}_1 \rangle = \langle \hat{s}_2^\dagger \hat{s}_2 \rangle = \frac{\eta_0}{\beta + \alpha^2}$, with $\eta_0 = \langle \hat{a}^\dagger \hat{a} \rangle$ the average photon number in the cavity. For a small $\eta$, the EP3 is slightly shifted to $\Delta = -\frac{\eta_0 \kappa c}{\gamma}$ with $P = \sqrt{2}$. So far we illustrated the essence of $\mathcal{PT}$-symmetric CMPs only through a toy model Hamiltonian (1). A physical realization is necessary to be sought to testify the theoretical predictions. To this end, we follow the 1-dimensional scattering method in Ref. [19], and consider a ferromagnetic bilayer placed in a microwave cavity [as shown in Fig. 4(a)]. The cavity wall is modelled by a delta permeability function $\mu = \mu_0 (1 + 2\delta(x + L/2) + 2\ell\delta(x - L/2))$, where $L$ is the cavity width and $\ell$ is the wall opacity. The dynamics of magneti-
zation $\mathbf{M}$ is governed by the LLG equation,
\[ \frac{d\mathbf{M}_j}{dt} = -\gamma_0 \mathbf{M}_j \times \mathbf{H}_{\text{eff}} + \frac{\alpha_j}{M_s} \mathbf{M}_j \times \frac{d\mathbf{M}_j}{dt}, \] (9)
where $\mu_0$ is the vacuum permeability. The effective magnetic field $\mathbf{H}_{\text{eff}} = H_z + h$ consists of the external and rf magnetic fields. $\mathbf{M}_j$ with $j = 1, 2$ labels the left and right magnets with balanced magnetic gain and loss $\alpha_1 = -\alpha$ and $\alpha_2 = \alpha (\alpha > 0)$, respectively.

Considering small-amplitude magnetization oscillations $\mathbf{M}_j = M_0 \mathbf{e}_z + \mathbf{m}_j$ with $|\mathbf{m}_j| \ll M_0$ and $M_0$ being the saturation magnetization, $\mathbf{m}_j$ is driven by the rf magnetic field $\mathbf{h}$ satisfying the Maxwell’s equation
\[ (\nabla^2 + k^2_q) \mathbf{h} = \nabla (\nabla \cdot \mathbf{h}) - k^2 \mathbf{m}, \] (10)
where $k^2_q = \varepsilon_0 \mu_0 \varepsilon_0^2 - \varepsilon_0^2 q^2$, $q$ is the vacuum light wave-vector, $\varepsilon_0 = \varepsilon / \varepsilon_0$ is the relative permittivity of ferromagnets, and $\mathbf{m} = \mathbf{m}_{02}$ for $-d/2 < x < 0$ ($0 < x < d/2$).

Assuming a linearly polarized microwave field $h_x(x, t) = \psi(x)e^{-i \omega t}$ traveling along the $x$-direction, the wave vector in magnetic bilayer takes the form $k_j = q \sqrt{\varepsilon_0 \mu_0} \varepsilon_0^2$ [19] for a given frequency $\omega$, where $\mu_0 = \frac{2\varepsilon_0^2 \varepsilon_0^2 - \varepsilon_0^2 q^2}{\omega^2 + 2 i \omega_0^2} \varepsilon_0$ is the Voigt permeability with $\omega_0 = \omega - i \omega_1 \varepsilon_1$, $\omega_1 = \gamma_0 \mu_0 H$, and $\omega_M = \gamma_0 M_s$. The microwave field $\psi(x)$ in different regimes [see Fig. 4(a)] can be expressed as
\[
\begin{align*}
\psi_1 &= e^{i q x} + re^{-i q x}, & \psi_2 &= a_1 e^{i q x} + a_2 e^{-i q x}, \\
\psi_3 &= b_1 e^{i k_M x} + b_2 e^{-i k_M x}, & \psi_4 &= b_3 e^{i k_M x} + b_4 e^{-i k_M x}, \\
\psi_5 &= a_3 e^{i q x} + a_4 e^{-i q x}, & \psi_6 &= t e^{i q x},
\end{align*}
\] (11c)
where coefficients $\{r, t, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4\}$ are determined by the electromagnetic boundary conditions at the interfaces.

We adopt the magnetic material parameters of YIG in the calculations, e.g., $\varepsilon_i = 15$ [57], $\mu_0 M_s = 175$ mT [58], and $\gamma/(2\pi) = 28$ GHz/T. Figure 4(b) shows the transmission spectrum for a small damping $\alpha = 0.002$, which exhibits a similar “Z”-shape with Fig. 3(a). From the wave-scattering calculation, we establish the following correspondence of parameters in the toy model and the present one
\[ \omega_s = \sqrt{\omega_h(\omega_M + \omega_H)}, \quad \beta = \frac{\alpha}{2} \sqrt{\omega_H^2 + 4\omega_0^2}, \quad g = \frac{g_{\text{eff}}}{\sqrt{2}}, \] (12)
where $g_{\text{eff}}$ is the effective coupling strength represented by the anti-crossing gap of the conventional strong-coupling spectrum [19] [see also Fig. 4(e)]. For a ferromagnetic bilayer of the thickness $d = 5 \mu$m, its value can be found from the spectrum $g_{\text{eff}} \approx 0.012 \omega_M$. We therefore deduce the critical Gilbert-type gain-loss parameter $\alpha_{\text{EP3}} \approx 0.0057$. The transmission spectrum at the EP3 is plotted in Fig. 4(c), which demonstrates similar dependence on the frequency and the detuning as Fig. 3(b). For a large $\alpha$, the system goes into the $PT$ symmetry broken phase, with a Purcell-like effect shown in Fig. 4(d).

From the experimental point of view, negative magnetic damping is necessary for observing the $PT$-symmetric CMP. It can be realized, for instance, by parametric drivings, spin transfer torques, and heterostructuring ferroelectric/ferromagnet [34–36, 59].

To conclude, we have presented both the input-output theory and the realistic wave-scattering calculation for describing $PT$-symmetric cavity magnon polaritons. We derived the analytical formula of the transmission coefficient and found a novel “Z”-shape spectrum in the exact $PT$ phase with an ultra-narrow linewidth for the output fields. An enhanced sensitivity near the third-order EP was predicted to be one or two orders of magnitude higher than that of the state-of-the-art magnetoelectric sensor, not limited by the quantum or statistical noise under proper conditions. Purcell-like effect was identified when the $PT$ is broken. This study provides the theoretical framework for the emerging $PT$-symmetric cavity spintronics, and offers a new pathway for designing ultra-sensitive magnetometers.

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Supplemental Material: $\mathcal{PT}$–symmetric cavity magnon polaritons

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A. Input-output formalism

An input-output theory [S1] is derived for a cavity interacting with a thermal bath. Our starting point is the total Hamiltonian

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{sys}} + \mathcal{H}_{\text{bath}} + \mathcal{H}_{\text{int}},$$

(A.1)

where $\mathcal{H}_{\text{sys}}$ describes the intracavity dynamics, the same as Eq. (1) in the main text, $\mathcal{H}_{\text{bath}}$ is the bath Hamiltonian

$$\mathcal{H}_{\text{bath}} = \hbar \sum_k \omega_k \hat{b}_k^{\dagger} \hat{b}_k,$$

(A.2)

with the bosonic creation (annihilation) operator $\hat{b}_k^{\dagger}$ ($\hat{b}_k$). They are coupled by the interaction term

$$\mathcal{H}_{\text{int}} = \hbar \sum_k \left( f_k \hat{\alpha}^{\dagger} \hat{b}_k + f_k^{*} \hat{b}_k^{\dagger} \hat{\alpha} \right),$$

(A.3)

with the commutation relation $[\hat{\alpha}, \hat{\alpha}^{\dagger}] = 1$ and $[\hat{b}_k, \hat{b}_k^{\dagger}] = \delta_{kk}$. $f_k$ is the coupling strength.

In Heisenberg picture, the time-dependent operator $\hat{O}(t) := e^{i\mathcal{H}_t} \hat{O} e^{-i\mathcal{H}_t}$ satisfies the following equations

$$\dot{\hat{\alpha}}(t) = \frac{i}{\hbar} \left[ \mathcal{H}_{\text{sys}}, \hat{\alpha}(t) \right] - i \sum_k f_k \dot{\hat{b}}_k(t),$$

(A.4a)

$$\dot{\hat{b}}_k(t) = -i \omega_k \hat{b}_k(t) - i f_k^{*} \hat{\alpha}(t).$$

(A.4b)
We find that the input and output fields satisfy the commutation relations from which we obtain the input-output formula $[S_2, S_3]$, then obtain the quantum Langevin equations.

For a two-port cavity, the input and output fields are connected by a scattering matrix, the input and output fields are defined as

$$b_{in}(t) = -i \sum_k f_k \hat{b}_k(t_0) e^{-i \omega_k (t-t_0)} + i \sum_k |\Delta_k|^2 \int_{t_0}^{t} d\tau e^{-i \omega_k (t-\tau)} \hat{a}(\tau),$$

$$b_{out}(t) = i \sum_k f_k \hat{b}_k(t_1) e^{-i \omega_k (t-t_1)} + i \sum_k |\Delta_k|^2 \int_{t_1}^{t} d\tau e^{-i \omega_k (t-\tau)} \hat{a}(\tau).$$

We thus have

$$\hat{a}(t) = \frac{i}{\hbar} [H_{sys}, \hat{a}(t)] - i \sum_k f_k \hat{b}_k(t_0) e^{-i \omega_k (t-t_0)} - \sum_k |\Delta_k|^2 \int_{t_0}^{t} d\tau e^{-i \omega_k (t-\tau)} \hat{a}(\tau), \quad (t < t_0)$$

$$= \frac{i}{\hbar} [H_{sys}, \hat{a}(t)] - i \sum_k f_k \hat{b}_k(t_1) e^{-i \omega_k (t-t_1)} + \sum_k |\Delta_k|^2 \int_{t_0}^{t} d\tau e^{-i \omega_k (t-\tau)} \hat{a}(\tau), \quad (t < t_1)$$

The input and output fields are defined as

$$\hat{b}_{in}(t) = -i \sum_k f_k \hat{b}_k(t_0) e^{-i \omega_k (t-t_0)}, \quad \hat{b}_{out}(t) = i \sum_k f_k \hat{b}_k(t_1) e^{-i \omega_k (t-t_1)}.$$

We then aim to convert the summation to the integral by introducing the mode density $\rho_k$. Assuming that both the mode density $\rho_k$ and the coupling strength $f_k$ are mode independent, i.e., $\rho_k = \rho$ and $f_k = f$, we obtain the following relation

$$\sum_k \rightarrow \int d\omega_k, \quad \kappa_c = 2\pi \rho |f|^2, \quad \int_{-\infty}^{\infty} d\omega_k e^{-i \omega_k (t-t')} = 2\pi \delta(t-t'),$$

$$\int_{t_0}^{t} d\tau \delta(t-\tau) \hat{a}(\tau) = \int_{t_0}^{t} d\tau \delta(t-\tau) \hat{a}(\tau) = \frac{1}{2} \hat{a}(t).$$

Then, Eq. (A.4a) can be simplified to

$$\dot{\hat{a}}(t) = \frac{i}{\hbar} [H_{sys}, \hat{a}(t)] + \hat{b}_{in}(t) - \frac{\kappa_c}{2} \hat{a}(t)$$

$$= \frac{i}{\hbar} [H_{sys}, \hat{a}(t)] - \hat{b}_{out}(t) + \frac{\kappa_c}{2} \hat{a}(t),$$

from which we obtain the input-output formula $[S_2, S_3]$.

$$\hat{b}_{in}(t) + \hat{b}_{out}(t) = \kappa_c \hat{a}(t).$$

We find that the input and output fields satisfy the commutation relations

$$[\hat{b}_{in}(t), \hat{b}_{out}(t')] = [\hat{b}_{out}(t), \hat{b}_{in}(t')] = \kappa_c \delta(t-t').$$

For a two-port cavity, the input and output fields are connected by a scattering matrix,

$$\begin{pmatrix} \hat{b}_{out}^{(1)} \\ \hat{b}_{out}^{(2)} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} \hat{b}_{in}^{(1)} \\ \hat{b}_{in}^{(2)} \end{pmatrix}.$$

So, every port satisfies the input-output relation $\hat{b}_{out}^{(1,2)} + \hat{b}_{in}^{(1,2)} = \kappa_c \hat{a}$, while the total field satisfies $\hat{b}_{out} + \hat{b}_{in} = 2\kappa_c \hat{a}$ with total input field $\hat{b}_{in} = \hat{b}_{in}^{(1)} + \hat{b}_{in}^{(2)}$, and total output field $\hat{b}_{out} = \hat{b}_{out}^{(1)} + \hat{b}_{out}^{(2)}$. Considering only one input field from port 1, i.e., $\hat{b}_{in}^{(2)} = 0$, we then obtain the quantum Langevin equations,

$$\dot{\hat{a}} = (-i \omega_k - \kappa_c) \hat{a} - ig(\delta_1 + \delta_2) + \hat{b}_{in},$$

$$\dot{\delta}_1 = (-i \omega_k + \beta) \delta_1 - ig \delta_2, \quad \text{(gain)}$$

$$\dot{\delta}_2 = (-i \omega_k - \beta) \delta_2 - ig \delta_1, \quad \text{(loss)}$$

Solving the above equations in frequency space, we obtain,

$$s_1(\omega) = \frac{ig \alpha(\omega)}{i(\omega - \omega_c) + \beta}, \quad s_2(\omega) = \frac{ig \alpha(\omega)}{i(\omega - \omega_c) - \beta}, \quad \alpha(\omega) = \frac{-b_{in}(\omega)}{i(\omega - \omega_c) - \kappa_c + \Sigma(\omega)}.$$
averaged sensitivity is then with the target detecting signal at

\[ \Delta \]

By substituting the above relations into the input-output formula, we have

\[ b_{\text{out}}^{(1)} + b_{\text{in}}^{(1)} = -\frac{\kappa_c b_{\text{in}}^{(1)}(\omega)}{i(\omega - \omega_c) - \kappa_c + \Sigma(\omega)}, \quad b_{\text{out}}^{(2)} = -\frac{\kappa_c b_{\text{in}}^{(1)}(\omega)}{i(\omega - \omega_c) - \kappa_c + \Sigma(\omega)}. \] (A.19)

One therefore obtains the frequency-resolved reflection and transmission coefficients,

\[ S_{11} = \frac{b_{\text{out}}^{(1)}}{b_{\text{in}}^{(1)}} = -1 - \frac{\kappa_c}{i(\omega - \omega_c) - \kappa_c + \Sigma(\omega)}, \quad S_{21} = \frac{b_{\text{out}}^{(2)}}{b_{\text{in}}^{(1)}} = -\frac{\kappa_c}{i(\omega - \omega_c) - \kappa_c + \Sigma(\omega)}. \] (A.20)

### B. Sensitivity of fluctuating sensors

Non-Hermitian degeneracy can provide an enhancement of sensitivity. In our context, the sensitivity is defined as the separation between the always-real central mode and the constant cavity mode at \( P_{\text{EP3}} \),

\[ \delta \omega_{\text{EP3}}/g = -\text{sgn}(\Delta)\delta \theta, \quad \text{with} \quad \delta \theta = 2^{1/3}\Delta^{1/3}. \] (B.1)

We now assume an ensemble of CMP systems at EP3 with a Gaussian distribution of the detuning parameter \( \Delta \),

\[ G(\Delta - \Delta_0) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{1}{2} (\Delta - \Delta_0)^2 / \sigma^2 \right], \] (B.2)

with the target detecting signal at \( \Delta_0 \) and \( \sigma \) being the noise fluctuation. For simplicity, we assume \( \Delta_0 \gg 0. \) The ensemble-averaged sensitivity is then

\[ \langle \delta \theta \rangle = \frac{\sigma^{1/3}}{2^{1/6}\sqrt{\pi}} I_0(x_0), \] (B.3)

with

\[ I_0(x_0) = \int_{-\infty}^{\infty} |x + x_0|^{1/3} e^{-\frac{1}{2} x^2} dx, \] (B.4)

where \( x_0 = \Delta_0/\sigma. \) The ensemble-averaged sensitivity in the small and large signal/noise ratio limits is

\[ \langle \delta \theta \rangle \approx \begin{cases} \sqrt{\frac{2}{\pi}} \Gamma \left( \frac{3}{2} \right) \sigma^{1/3}, & x_0 \ll 1, \\ 2^{1/3} \Delta_0^{1/3}, & x_0 \gg 1. \end{cases} \] (B.5)

The sensitivity is found to be free from noise in a large signal/noise ratio, i.e., \( x_0 \gg 1 \), while it is completely determined by the noise/fluctuation in the opposite limit. To see how fast the sensitivity is recovered, we introduce a sensitivity-diminution factor \( F_0 = 2^{-1/3}\Delta_0^{-1/3}\langle \delta \theta \rangle \), so that \( F_0 = 1 \) represents the noise-free sensitivity regime. Figure S1 clearly shows that the noise-less sensing is well performed for \( x_0 \gg 1 \) (see the red curve shown in Fig. S1).

In Ref. [S4], a different definition of the sensitivity however is introduced

\[ \frac{\partial \langle \delta \theta \rangle}{\partial \Delta_0} = \frac{1}{2^{1/6}\sqrt{\pi} \sigma^{2/3}} I_1(x_0), \] (B.6)

with

\[ I_1(x_0) = \int_{-\infty}^{\infty} x|x + x_0|^{1/3} e^{-\frac{1}{2} x^2} dx. \] (B.7)
FIG. S5: Sensitivity-diminution factor as a function of $x_0$.

The sensitivity then takes the following asymptotic form:

$$\frac{\partial \langle \delta \theta \rangle}{\partial \Delta_0} \approx \begin{cases} \frac{\Gamma(\frac{1}{3})}{\sqrt{2\pi} \sigma^{\frac{1}{3}}} \Delta_0, & x_0 \ll 1, \\ \frac{2^{1/3}}{3} \Delta_0^{-2/3}, & x_0 \gg 1. \end{cases} \quad (B.8)$$

For a large signal/noise ratio, i.e., $x_0 \gg 1$, the sensitivity is also found to be free from noise fluctuations. Similarly, we introduce another sensitivity-diminution function $F_1 = \frac{3}{\pi^{1/3}} \Delta_0^{2/3} \frac{\partial \langle \delta \theta \rangle}{\partial \Delta_0}$ (see the blue curve shown in Fig. S1). $F_1 = 1$ represents the noise-less response. There is a clear diminution of the sensitivity to the noise fluctuations for $x_0 \ll 1$, while the noise-less sensing performance is recovered for $x_0 \geq 2$.

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