Spirit of Mathematics Critical Thinking Skills (CTS)

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Abstract: The mathematical critical-thinking skill is a process of thinking systematically to develop logical and critical thinking on mathematical problems, which characterize and demand to learn in the 21st century. This conceptual paper aims to analyze the spirit of critical thinking skill, and various approaches that can be applied in mathematics learning. Based on the analysis of several theories and research findings from various countries in the world, it can be concluded that the mathematical critical-thinking skill is very important for students, too; (i) help rational thinking in making decisions to express an idea, (ii) dare to make conclusions with alternative logical thinking, and (iii) able to examine and disregard various complex problems in learning Mathematics. Indeed, mathematics learning does not occur, if the learning process has not demonstrated the spirit of developing mathematical critical thinking skills.

Keywords: Critical thinking skill, Develop logical, Mathematical problems, Learning in the 21st century, Rational thinking

1. Introduction

Critical thinking is the careful application of reason in the determination of whether a claim is true. Notice that it is not so much coming up with claims, true or otherwise, that constitutes critical thinking, it is the evaluation of claims [1]. Critical thinking is accuracy in analytical thinking [2]. Critical thinking is a way of thinking about any subject, content, or problem, where the thinker improves the quality of his thinking by skilfully using intellectuals [3].

Critical thinking is defined as an awareness of own’s thinking (self-reflection) and the ability (foundation skills) and willingness (willingness to question) to clarify and improve understanding which aids in drawing appropriate conclusions and making the best decisions possible within a context (knowledge base) [4]. Critical thinking is a way of thinking that is reflective which is directed to reach conclusions based on evidence and consideration [5]. The term critical thinking explained as in the following picture:

![Critical Thinking Skills Diagram](image.png)

**Figure 1. Critical Thinking Skills [5]**
Critical thinking is one of the thinking skills needed in the 21st century where information dynamically [6]. The critical-thinking skill can also be interpreted as a way of thinking that is reasonable or based on reason [7]. Critical thinking skill is one of the skills that students need to have in the present century [8]. Critical thinking skills can be referred to and evaluated through the process and assessment of students' thinking levels [9]. The critical-thinking skill includes several things: namely analysis, interpretation, inference, synthesis, and evaluation [10], [11]. Developing critical thinking skills can diagnose the thinking abilities of high-level students. By providing feedback to see the level of student thinking [12]. Also, the critical-thinking skills a collective experience [13]. For this reason, collaboration is needed between all teachers in implementing the learning process [14]. Agreeing with that, this critical thinking ability can be developed cumulatively as student progress through an appropriate learning process [15].

Syntax in developing critical thinking skills, namely: 1) elementary clarification, 2) the basis for the decision, 3) inference, 4) advanced clarification, dan 5) supposition and integration [16].

One expert suggested that critical thinking ability is a mathematical ability that is essential and is a goal in learning mathematics [17]. Agree with that, the mathematical critical-thinking skill is an ability possessed by students used in solving mathematical problems [18]. The critical-thinking skill in mathematics can also be interpreted as the ability to include prior knowledge [19], cognitive strategies to generalize, prove or evaluate mathematical situations which include: testing, questioning, connecting every aspect that exists in mathematical problems [20], [21]. Mathematical material and critical thinking are two things that have coherence and cannot be separated [22]. Mathematical material can be understood through critical thinking, and critical thinking is trained through a series of mathematics learning processes [23]. The uniqueness and complexity of the elements in mathematics require learners to be able to think critically in learning them [24], [25]. Also, it was explained that the mathematical critical-thinking skill is a high-level thinking process that can be used in the formation of systems or students' conceptual understanding [26].

Developing critical thinking skills requires an effective, constructive, and cognitive assessment to make it easier for teachers to uncover students' abilities [27], [28]. Based on this, the teacher needs to apply the right variety of methods in developing students' critical thinking skills [29]. One of them is by building new curriculum models and approaches and teaching techniques that lead to critical thinking and mathematical problem solving [30].

Indicators of mathematical critical thinking ability consist of 1) focusing on one question, problem and theme, 2) checking the truth of arguments, statements and solution processes 3) asking and answering with reasons, 4) observing with criteria, identifying assumptions, understanding well, and
identify relevant and irrelevant data, 5) reduce and induce, 6) make judgments, assess thoroughly, 7) find alternatives [31]. Ennis defines critical thinking as thinking to decide what to believe or do [32]. His research investigates the feasibility and consequences of developing critical thinking among students through an instruction program in probability [33].

Based on the results of previous studies, research has been conducted to analyze the critical-thinking skill mathematically, as was done by [34] by using the Missouri Mathematics Project (MMP) learning model, then [35] offers cooperative learning-based learning models. Norris revealed that the importance of applying problem-based learning or the term Problem Based Learning approach as an effort to function the brain's work system in responding to students' critical attitudes [36].

Based on these definitions and presentations, it can be concluded that mathematical critical-thinking skill is very important for students [17]. Because with this ability, students can be rational and logical [37], able to provide appropriate alternative conclusions [38], and able to examine various problems that occur in everyday life [39]. The importance of mathematical critical thinking skills is to help students to think analytically, logically, and systematically and be able to examine various problems [40].

Mathematical thinking skills can be classified into two types based on the level of complexity, namely: high level (high order critical thinking skills (HOCTS) or high-level mathematical thinking) and low level (low order critical thinking skills (LOCTS) or low-level mathematical thinking)[41]. Ennis's opinion only talks about high and low. Meanwhile, according to the study of researchers in learning mathematics, it is not enough to just talk high and low. But the keyword is critical thinking. So, the researchers offer as a refinement of the concept, is Critical Thinking Skills (CTS). For this reason, the author tries to explain the purpose of this conceptual article, which is to find out the important role of mathematical critical thinking skills and offer various approaches that can be applied in mathematics learning.

2. The Importance of Ability to Think Math Critical Learning in Mathematics Learning

The importance of the critical-thinking skill is to be able to develop and communicate ideas in learning [42], can check the truth of information, can produce rational decisions, and can support increased learning achievement [43]–[45]. By having the ability to think critically, a person can regulate, adjust, change, or improve his mind [46], so he can decide to act more precisely [47]. Critical thinking skill is an important and vital topic in modern education. All educators are interested in teaching critical thinking to their students [48].

The critical-thinking skill is an important element which is an inseparable unity, students must have in analyzing mathematical problems [49], and needs to be embedded in the learning process. For this reason, students need to be able to construct their understanding and knowledge to be able to express ideas or ideas in solving problems [50]. The following illustration is a unity about the importance of developing mathematical critical thinking skills:
Also, the critical-thinking skill in learning mathematics can minimize the occurrence of mistakes when solving problems, can train students to participate actively to gain and feel experiences that are meaningful in the learning process [51]. Therefore, the critical-thinking skill in learning mathematics can be developed by presenting a particular situation so that students can construct their understanding through the given situation [52]. So that the final result will be obtained a settlement with the right conclusions and students will get used to facing challenges [53].

Activities that demonstrate critical thinking skills in mathematics learning include: (i) the ability to identify problems; (ii) ability to find fault; (iii) ability to analyze a statement; and (iv) the ability to apply concepts to new problems. Here is an example of critical thinking in learning mathematics:

**Critical Thinking**

Discuss with your friends. By utilizing the distributive nature of multiplication for addition and the distributive nature of multiplication for reduction, prove the multiplication of the following algebraic forms.

$$(ax + b)(ax - b) = a^2x^2 - b^2$$

$$(ax + b)^2 = a^2x^2 + 2abx + b^2$$

$$(ax - b)^2 = a^2x^2 - 2abx + b^2$$

Some positive impacts experienced by students from critical thinking in mathematics learning include (1) Practicing problem-solving skills, (2) The emergence of innovative questions and designing appropriate solutions, (3) Actively building arguments by showing accurate and logical evidence [47]. Greenwald stated that the critical-thinking skill in mathematics learning includes several indicators including (1) interpretation, namely the ability to understand or express the meaning of data or situations presented in a mathematical problem; (2) analysis, namely the ability to identify the relationship between the data given and reasoning given arguments; (3) evaluation, namely the ability to find and prove errors in a mathematical problem; (4) decision, namely the ability to make conclusions from a mathematical problem [20].

The steps that can be taken by teachers in improving students' critical thinking skills are: (i) using learning strategies that demonstrate student activity, (ii) focusing instruction rather than content, and (iii) using assessment techniques that give students intellectual challenges rather than recalling memory. Some things that can hinder a teacher in critical thinking: lack of training, limited resources, biased bias and time constraints conspiring to negate the learning environment that promotes critical thinking [54]. However, actively involving students in collaborative activities can encourage students' critical thinking. In this way, it is possible if the teacher models the thought process using effective
questioning techniques, and guides the students' critical thinking processes [55]. These examples provide a challenge for teachers in considering students as users of information and not recipients of information [56].

Contrary to the importance of critical thinking skills in learning, several facts show that this ability is still relatively low [57]. Based on several studies, obtained information that most students experience problems in terms of low critical thinking skills. Most students are accustomed to doing learning activities in the form of memorizing concepts, formulas, and solving problems mathematically, without being accompanied by the development of critical thinking skills of a problem they face in real life [58]. Also, there are still very few schools that teach students to think critically [59]. The same thing was expressed by Stupple that teachers still enjoy teaching with conventional learning patterns and see very few opportunities to do more innovative activities [60]. Also, it was mentioned that high school students are unable to complete tasks that demonstrate critical thinking competence. The situation that has been described previously suggests that the selection of learning methods by teachers is a very important thing that needs to be considered to foster critical thinking skills [61].

3. Some Approaches in the Development of Math Critical Thinking Ability in Mathematics Learning

Learning approaches that can improve students' critical thinking skills are metacognitive learning models, reciprocal teaching approaches [62], Metacognitive Instruction approach [63], [64], RME model learning approach (Realistic Mathematics Education) [65], [66]. Besides, scientific-based mathematics teaching materials are very effective in increasing mathematical critical thinking skills [67]. Furthermore, one of the abilities that are thought to affect the mathematical critical-thinking skill is Adversity Quotient (AQ)[68]. This is in line with Saido's opinion in his research that the importance of Adversity Quotient (AQ) to hone students' critical thinking skills on mathematics learning achievement [69]. This means that the level of AQ influences students' mathematical critical thinking skills even though the contribution is not too large.

Besides the learning process requires a model and approach, the importance of the teacher's empathy which must also be proclaimed in handling the learning process [70]. It can also be done training activities for teachers [55] so that the learning process that is a vision and mission in developing critical thinking skills can also be done well. Not only that but improving the quality of teachers [56] to develop students' critical thinking skills can also be done through media-based learning and technology. Thus, the importance of collaboration between teacher quality and the ability to apply learning approaches to achieving the development of students' mathematical critical thinking abilities.

4. Conclusion

The importance of mathematical critical thinking skills that must be possessed by each individual in carrying out the learning process. This is so the learning process is more systematic, structured, and tricks the brain to think logically. The results of this conceptual article description conclude that the mathematical critical-thinking skill in learning mathematics can be done with a variety of approaches. This shows the importance of mathematical critical thinking skills, which can be easily done using a variety of approaches in practical ways.

One effort that can be done is to improve the quality of education that focuses on developing students' critical thinking skills. Critical thinking can be developed through classroom learning, especially in learning mathematics. The development of critical thinking skills enables students to become accustomed to facing challenges and solving problems by analyzing their thinking to make choices and draw conclusions.

References
[1] J. D. Trifone 1987 The Best of Logical Thingking Am. Biol. Teach 49 8.
[2] S. Carson 2015 Targeting Critical Thinking Skills in a First-Year Undergraduate Research Course J. Microbiol. Biol. Educ 16 2

[3] M. L. Styers, P. A. Van Zandt, and K. L. Hayden 2018 Active Learning in Flipped Life Science Courses Promotes Development of Critical Thinking Skills CBE Life Sci. Educ 17 3

[4] P. Chambers 2018 Teaching Mathematics Developing As a Reflective Secondary Teacher. (UK: Sage Publications)

[5] V. R. Ruggerio 2012 Beyond Feelings: A Guide to Critical Thinking, 9th ed. (New York: McGraw-Hill)

[6] E. Aizikovitsh-Udi and D. Cheng 2015 Developing Critical Thinking Skills From Dispositions to Abilities: Mathematics Education From Early Childhood to High School Creat. Educ 6 1,

[7] N. G. Holmes, C. E. Wieman, and D. A. Bonn 2015 Teaching Critical Thinking,” in Proceedings of the National Academy of Sciences USA

[8] J. E. Mcpeck 2017 Critical Thinking And Education (New York: Routledge Taylor & Franch Group Publisher)

[9] C. Bereiter and M. Scardamalia 1987 An Attainable Versionof High Literacy: Approaches to Teaching Higher-Order Thinking Skills in Reading and Writing Curric. Inq 17

[10] Wartono, M. N. Hudha, and J. R. Batlolona 2018 How Are The Physics Critical Thinking Skills of The Students Taught by Using Inquiry-Discovery Through Empirical and Theorethical Overview? EURASIA J. Math. Sci. Technol. Educ 14 2

[11] S. A. Hadi, E. Susantini, and R. Agustini, “Training of Students’ Critical Thinking Skills Through the implementation of a Modified Free Inquiry Model Journal of Physics: Conf. Series 947 1

[12] R. Benjamin 2015 The Case for Comparative Institutional Assessment of Higher-Order Thinking Skills Change 40 6

[13] Y. Abosalem 2016 Assessment Techniques and Students’ Higher-Order Thinking Skills Int. J. Second. Educ 4 1

[14] C. Sereni 2015 Teaching Strategies for Critical Thinking Skills Acad. Exch. Q 19 3

[15] P. Crenshaw, E. Hale, and S. L. Harper 2016 Producing Intellectual Labor in The Classroom: The Utilization of a Critical Thinking Model to Help Students Take Command of Their Thinking J. Coll. Teach. Learn 8 7

[16] A. L. Costa and R. Ennis 1985 Developing Minds ‘Goals For Critical Thinking Curriculum (Alexandria: The Educational Resources Information Center (ERIC))

[17] C. R. Huber and N. R. Kuncel 2016 Does College Teach Critical Thinking: A Meta-Analysis J. Educ. Res 86 2

[18] O. L. Liu, L. Mao, L. Frankel, and J. Xu 2016 Assessing Critical Thinking in Higher Education: The Heighen Approach and Preliminary Validity Evidence Assess. Eval. High. Educ 41

[19] A. H. Schoenfeld 1992 Learning to Think Mathematically: Problem Solving, Metacognition and Sense of Mathematics,” Handbook of Reasearch on Mathematics Teaching and Learning. (New York: Macmillan)

[20] R. R. Greenwald and I. J. Quitadamo 2014 A Mind of Their Own: Using Inquiry-Based Teaching to Build Critical Thinking Skills and Intellectual Engagement in An Undergraduate Neuroanatomy Course J. Undergrad. Neurosci. Educ.

[21] B. Beyer 1983 Common Sense About Teaching Thinking,” Educ. Leadersh. 41 3

[22] O. Kormaz 2016 The Effect of Scratch- and Lego Mindstorms Ev3-Based Programming Activities on Academic Achievement, Problem-Solving Skills and Logical-Mathematical Thinking Skills of Students,” Malaysian Online J. Educ. Sci 4 3

[23] M. Ransdell 2016 Design Process Rubrics: Identifying and Enhancing Critical Thinking in Creative Problem Solving in Proceedings of the Interior Design Educators Council Conference

[24] P. Baybutt 2016 A Framework for Critical Thinking in Process Safety Management (America: American Institute of Chemical Engineers)
[25] R. Bell and M. Loon 2015 The Impact of Critical Thinking Disposition on Learning Using Business Simulations International Journal of Management Education 13 2

[26] K. T. Taube 1997 Critical Thinking Ability And Disposition As Factors Of Performance On A Written Critical Thinking Test J. JSTOR 46 2

[27] L. Mutakinati, I. Anwari, and K. Yoshihiseke 2018 Analysis of Students Critical Thinking Skill of Middle School Through STEM Education Project-Based Learning J. Pendidik. IPA Indonesia 7 1

[28] S. A. Rodzalan and M. M. Saat 2015 The Perception of Critical Thinking and Problem Solving Skill Among Malaysian Undergraduate Students Procedia-Social Behav. Sci 172

[29] I. W. Widana, I. M. Y. Parwata, N. N. Parmithi, I. G. A. T. Jayantika, K. Sukendra, and I. W. Sumandy 2018 Higher Order Thinking Skills Assessment Towards Critical Thinking on Mathematics Lesson Int. J. Soc. Sci. Humanit. 2 1

[30] R. H. Ennis and J. Wheary 1995 Gender Bias in Critical Thinking: Continuing the Dialogue,” Educ. Theory 45 2

[31] J. B. Baron and R. J. Sternberg 1987 The Teaching Thinking Skill (New York: W. H. Freeman and Company)

[32] R. H. Ennis 1987 A Taxonomy of Critical Thinking: Dispositions and Abilities (New York: Freeman)

[33] R. Swartz 1992 Critical Thinking, The Curriculum, and The Problem of Transfer in The Second International Conference

[34] J. T. Broadbear 2013 Essential Elements of Lessons Designed to Promote Critical Thinking African J. Math. Comput. Sci. Res. 3 3

[35] G. Nosich 2014 Learning to Think Things Through. A Guide to Critical Thinking Across The Curriculum, 4th ed.(Boston: MA: Pearson)

[36] S. P. Norris and R. H. Ennis 1989 Evaluating Critical Thinking (Pacific Grove: CA: Midwest Publications)

[37] Netriwati 2015 Meningkatkan Kemampuan Berfikir Logis Matematis Mahasiswa dengan Menggunakan Rangkaian Listrik pada Materi Logika Al-Jabar J. Pendidik. Mat 6 1

[38] C. Xia, X., Lu and B. Wang 2018 Research on Mathematics Instruction Experiment Based Problem Posing J. Math. Educ. 1 1

[39] D. F. Halpern 1999 Teaching for Critical Thinking: Helping College Students Develop The Skills and Dispositions of a Critical Thinker New Dir. Teach. Learn

[40] R. Fogarty and J. McTighe 1993 Critical Thinking Assessment J. Theory Pract. 32 3

[41] R. H. Ennis 1993 Critical Thinking Assessment Theory Pract. 32 3

[42] Syafrimen, N. M. Ishak, and N. Erlina 2017 Emotional Intelligence Profile of Prospective Teachers J. Eng. Appl. Sci. 12 7

[43] E. E. Peter 2014 Critical Thinking: Essence for Teaching Mathematics and Mathematics Problem Solving Skills African J. Math. Comput. Sci. Res. 5 3

[44] A. Lawson 1993 At What Levels of Education is the Teaching of Thinking Effective? Theory Pract. 32 3

[45] A. J. Bishop 1993 Significant Influences on Children’s Learning of Mathematics (France: UNESCO Paris)

[46] A. Fisher and A. Thompson 1993 Testing Reasoning Ability: Center for Research in Critical Thinking (UK: University of East Anglia)

[47] H. Liberna 2015 Peningkatan Kemampuan Berpikir Kritis Matematis Siswa melalui Penggunaan Metode Improve pada Materi Sistem Persamaan Linear Dua Variabel J. Form 2 3,

[48] S. D. Schaferman 2017 An Introduction to Critical Thinking,” Nursing 49 6

[49] R. Y. Sari, Netriwati, and F. I. Sari 2017 Pengaruh Model Pembelajaran Attention, Relevance, Confidence and Satisfaction (ARCS) terhadap Kemampuan Berpikir Matematis Berdasarkan Taksonomi Bloom Numer. J. Mat. dan Pendidik. Mat. 1 1

[50] A. F. Aini, A. Zafirah, Engkizar, F. Anwar, Z. Arifin, and S. Syafril 2018 The Implementation
of Character Values Toward Students Through Congkak Game For Mathematics Instruction Media *J. Penelit. Pendidik.* **35** 2

[51] L. W. Howard, T. L. P. Tang, and M. Jill Austin 2014 Teaching Critical Thinking Skills: Ability, Motivation, Intervention, and the Pygmalion Effect *J. Bus. Ethics,* **128** 1

[52] J. Kleinig 2016 Trust and Critical Thinking *Educ. Philos. Theory* **50** 2

[53] M. Duran and I. Dokme 2016 The Effect of The Inquiry-Based Learning Approach on Student’s Critical-Thinking Skills *Eurasia J. Math. Sci. Technol. Educ* **12** 12

[54] A. Pahrudin, Irwandani, E. Triyana, Y. Oktarisa, and C. Anwar 2019 The Analysis of Pre-Service Physics Teachers In Scientific Literacy: Focus on The Competence and Knowledge Aspects *J. Pendidik. IPA Indones.* **8** 1

[55] Y. Yusnita *et al.* 2018 The Effect of Professional Education and Training for Teachers (PLPG) in Improving Pedagogic Competence and Teacher Performance *Tadris J. Kegur. dan Ilmu Tarb.* **3** 2

[56] A. Khairatul, A. H. Tamuri, and S. Syafrimen 2018 Skills of Computer Utilization Among Islamic Education Teachers in The Higher Secondary School *Article Preprint*

[57] A. Khoiri 2017 Analisis Kemampuan Berfikir Kritis siswa Menggunakan Teams Assisted Individualization *Sci. Educ. J* **1** 2.

[58] E. A. Silver 1996 Posing Mathematical Problems: An Exploratory Study *J. Res. Math. Educ.*, **27** 3

[59] J. Moon 2018 *Critical Thinking: An Exploration of Theory and Practice.* (Madison Avenue: Routledge Taylor & Francis Group)

[60] E. J. N. Stupple *et al.* 2016 Development of the Critical Thinking Toolkit (CriTT): A Measure of Student Attitudes and Beliefs about Critical Thinking *Elsevier*

[61] P. T. Terenzini, L. Springer, E. T. Pascarella, and A. Nora 1995 Influences Affecting The Development Of Students Critical Thinking Skills *J. Res. High. Educ.* **36** 1

[62] M. Gholami *et al.* 2016 Comparing The Effects of Problem-Based Learning and The Traditional Lecture Method on Critical Thinking Skills *ELSEVIER*

[63] A. J. Canas, P. Reiska, and A. Mollits 2017 Developing Higher-Order Thinking Skills With Concept Mapping: A Case of Pedagogic Fraiity *Knowl. Manag. E-Learning* **9** 3

[64] Runisah, T. Herman, and J. A. Dahlan 2017 Using The 5E Learning Cycle With Metacognitive Technique to Enhance Students Mathematical Critical Thinking Skills *J. Emerg. Math. Educ.*, **1** 1

[65] B. Tanujaya, R. C. I. Prahanmana, and J. Mumu 2017 Mathematics Instruction, Problems, Challenges and Opportunities: A Case Study in Manokwari Regency, Indonesia *World Trans. Eng. Technol. Educ* **15** 3

[66] V. den Heuvel-Panhuizen and P. M. and Drijvers 2014 Realistic Mathematics Education,” *Encyclopedia of Mathematics Education* (Dordrecht: Springer Netherlands)

[67] I. Nurhikmayati and M. G. Jatisunda 2019 Pengembangan Bahan Ajar Matematika Berbasis Scientific yang Berorientasi pada Kemampuan Berpikir Kritis Matematis Siswa *J. Pendidik. Mat* **8** 1

[68] P. G. Stoltz 1997 *Adversity Quotient: Turning Obstacles Into Opportunities* (Canada: Simultaneously)

[69] G. M. Saido, S. Siraj, A. B. Bin Nordin, and O. S. Al_Amedy 2015 Higher Order Thinking Skills Among Secondary School Students in Science Learning *Malaysian Online J. Educ. Sci.*, **3** 3

[70] Syafrimen, N. Mohd.Ishak, and N. Erlina 2017 Six Ways to Develop Empathy of Educators *J. Eng. Appl. Sci* **12** 7