Abstract

We study $R^2$ gravity in $(2 + \epsilon)$-dimensional quantum gravity. Taking care of the oversubtraction problem in the conformal mode dynamics, we perform a full order calculation of string susceptibility in the $\epsilon \to 0$ limit. The result is consistent with that obtained through Liouville approach.
Two-dimensional quantum gravity [1] has been studied intensively through Liouville theory [2] and the matrix model [3] these several years, and indeed its progress has provided us with much insight into quantum gravity and string theory. There are two ways to proceed further; one is to go beyond the so-called $c = 1$ barrier and the other is to explore higher dimensional quantum gravity. As for the former, $R^2$ gravity, which has been studied for years [4, 5], can be considered as a possible trial. Recently Kawai and Nakayama [6] have succeeded in treating it in the framework of Liouville theory. It has been discovered that the surface becomes locally smooth allowing a central charge greater than one. Although $R^2$ gravity is not acceptable as a realistic string theory due to loss of positivity, it is still worth studying as a new type of universality class of two-dimensional quantum gravity. In contrast to ordinary gravity, $R^2$ gravity is difficult to study in the matrix model and so far we do not have any other results which can be compared with theirs. It seems, therefore, desirable to investigate the theory in other approaches.

As for exploring higher-dimensional quantum gravity, numerical simulations provide a powerful tool; yet we should develop analytic approaches at the same time. One of the possibilities is $(2 + \epsilon)$–dimensional quantum gravity [7, 8, 9]. The formalism, however, contains some subtlety concerning the oversubtraction problem in the conformal mode dynamics, as was discovered by Kawai, Kitazawa and Ninomiya [8]. Considering the situation, we feel that it is worth while acquiring a deeper insight into the formalism by applying it to other theories. In this letter, therefore, we study $R^2$ gravity in $(2 + \epsilon)$–dimensional quantum gravity. We obtain results consistent with ref. [6] by taking the $\epsilon \to 0$ limit in the strong coupling regime. This provides another success in treating $R^2$ gravity.

We define a $(2 + \epsilon)$–dimensional system corresponding to $R^2$ gravity by the following action,

$$
S = \frac{\mu^\epsilon}{G} \int d^D x \sqrt{g} R + \frac{\mu^\epsilon}{4m^2} \int d^D x \sqrt{g} R^2 + \Lambda \mu^\epsilon \int d^D x \sqrt{g} + \mu^\epsilon \sum_{i=1}^c \int d^D x \sqrt{g} \frac{1}{2} g^{\mu \nu} \partial_\mu f_i \partial_\nu f_i,
$$

(1)

where $G$ is the gravitational constant, $\Lambda$ is the cosmological constant, $f_i$ is the matter field, and $D = 2 + \epsilon$. $m$ is a parameter with mass dimension, which corresponds to the inverse of the range controlled by the $R^2$ term. Since the above action contains higher derivatives, which is difficult to deal with, we introduce an auxiliary field $\chi$ and replace the $R^2$ term
with
\[ \mu^\epsilon \int d^D x \sqrt{g} (-i R \chi + m^2 \chi^2). \tag{2} \]

In the Appendix, we calculate the one-loop counterterms for the generalized action
\[ S = \mu^\epsilon \int d^D x \sqrt{g} \left( \frac{1}{2} K(\chi) g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + L(\chi) R + M(\chi) \right) + \text{(matters)}, \tag{3} \]
which reduces to the action considered by setting \( K(\chi) = 0 \), \( L(\chi) = \frac{1}{G} - i \chi \) and \( M(\chi) = \Lambda + m^2 \chi^2 \). As can be seen in (25) and (26), the counterterm for the \( \chi \)-kinetic term can be set to 0 by choosing appropriately the function \( f \), which comes from the freedom of gauge fixing (19). Note also that the renormalization of \( M(\chi) \) is self-contained, which enables us to treat it separately as an inserted operator. Thus the action including the one-loop counterterm reads
\[ S + S_{\text{c.t.}} = \mu^\epsilon \int d^D x \sqrt{g} \left( \frac{1}{G} - i \chi - \frac{1}{2\pi \epsilon} \frac{24 - c}{12} \right) R + \text{(matters)}. \tag{4} \]

Special care should be taken for the counterterm
\[ - \frac{24 - c}{24\pi \epsilon} \mu^\epsilon \int d^D x \sqrt{g} R. \tag{5} \]
Let us expand the metric as
\[ g_{\mu\nu} = \delta_{\mu\rho} (e^h)^\rho_\nu e^{-\phi}, \]
where \( \phi \) is the conformal mode and \( h \) is the traceless symmetric tensor field. The counterterm (5) then yields the one for the kinetic term of the conformal mode \( \frac{24 - c}{24\pi \epsilon} \frac{1}{4} \partial_\mu \phi \partial_\nu \phi \), which is \( O(1) \), while the divergent one-loop diagram for the \( \phi \) two-point function gives \( O(\epsilon) \) quantity. The situation is just the same as in ref. [8]. The counterterm (5) is therefore an oversubtraction for the conformal mode, which forces us to incorporate this counterterm in the tree-level action and redo the perturbative expansion with the effective action
\[ S_{\text{eff}} = \mu^\epsilon \int d^D x \sqrt{g} \left( \frac{1}{G} - i \chi - \frac{1}{2\pi \epsilon} \frac{24 - c}{12} \right) R + \text{(matters)}. \tag{6} \]
This amounts to redefining the \( L(\chi) \) as \( L(\chi) = \frac{1}{G} - i \chi - \frac{24 - c}{24\pi \epsilon} \).

We show in the following that one can obtain results consistent with ref. [6] in the \( \epsilon \to 0 \) limit in the strong coupling regime, i.e. \( G \gg \epsilon \). This is to be expected, since in the infrared limit \( R^2 \) gravity reduces to ordinary gravity without \( R^2 \) term, which was reproduced in ref. [8] also in the strong coupling regime.
Let us consider the renormalization of the operators $\int d^D x \sqrt{g}$ and $\int d^D x \sqrt{g} \chi^2$. We first show, up to two-loop level, that the divergent parts coming from the diagrams with $h_{\mu\nu}$ line cancel as a whole and therefore do not contribute to the renormalization of the operators considered.

In order to diagonalize the kinetic terms in the action after gauge fixing, we introduce the new quantum fields $\Phi, X$ and $H_{\mu\nu}$ through

$$
\phi = \tilde{F} \Phi + \frac{2L'}{\epsilon L} \tilde{I} X \\
\chi = \tilde{I} X \\
h_{\mu\nu} = L^{-1/2} H_{\mu\nu},
$$

where $\tilde{I}$ and $\tilde{F}$ are given through

$$
\frac{1}{\tilde{I}^2} = \frac{L^2}{L} \left( 1 + \frac{D}{\epsilon} \right) \\
\tilde{F}^2 = -\frac{4}{\epsilon D L}.
$$

After this field redefinition, the kinetic term reduces to the following standard form

$$
\int d^D x \sqrt{\hat{g}} \left\{ \frac{1}{4} H_{\nu,\rho}^\mu H_{\mu,\rho}^{-} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu X \partial_\nu X \right\}.
$$

The interaction vertices including $H_{\mu\nu}$ are

$$
\int d^D x \sqrt{\hat{g}} \left\{ \frac{\epsilon}{4} (D-1) L^{1/2} \tilde{F}^2 H_{\mu\rho}^\nu \partial_\mu \Phi \partial_\rho \Phi - \frac{1}{\epsilon} (D-1) L^{-3/2} L^2 \tilde{I}^2 H_{\mu\rho}^\nu \partial_\mu X \partial_\rho X \\
- \frac{\epsilon}{8} \tilde{F}^2 H_{\mu\rho}^\nu H_{\rho,\epsilon}^\mu \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2\epsilon} (D-1) L^{-2} L^2 \tilde{F}^2 H_{\mu\rho}^\nu \partial_\mu X \partial_\nu X + \cdots \right\}.
$$

The operators can be written in terms of the new quantum fields as

$$
\int d^D x \sqrt{\hat{g}} = \int d^D x \sqrt{\hat{g}} e^{-\frac{\nu}{2} (\tilde{F} \Phi + \frac{2L'}{\epsilon L} \tilde{I} X)} \\
\int d^D x \sqrt{\hat{g}} \chi^2 = \int d^D x \sqrt{\hat{g}} e^{-\frac{\nu}{2} (\tilde{F} \Phi + \frac{2L'}{\epsilon L} \tilde{I} X)} (\tilde{\chi} + \tilde{I} X)^2.
$$

The Figure shows the list of the diagrams with $H_{\mu\nu}$ line we have to consider when we evaluate the one-point functions of the above operators up to two-loop level. (a) and (b) correspond to $\langle \Phi^2 \rangle$, while (c) and (d) correspond to $\langle X^2 \rangle$. Although each diagram has $O(\frac{1}{\epsilon})$ divergence (Note that $L \sim O(\frac{1}{\epsilon})$), an explicit calculation shows that the divergent parts of (a) and (b), as well as (c) and (d), cancel each other. One can also check that the contribution of the ghosts and the matters is finite, due to the suppression factors of $\epsilon$ and $L^{-1} \sim O(\epsilon)$ in
the action. Thus we have shown that the diagrams containing $h_{\mu \nu}$, ghosts or matters do not affect the renormalization of the operators at least up to two-loop level. We expect that this holds true to all orders of the loop expansion and that two-dimensional $R^2$ gravity is completely governed by the dynamics of the conformal mode $\phi$ and the auxiliary field $\chi$.

Dropping the $h_{\mu \nu}$ field, the ghosts and the matters, the effective action reads

$$\int d^D x \sqrt{g} \left( \frac{1}{G} - i\chi - \frac{24 - c}{24\pi \epsilon} \right) R$$

$$\sim \int d^D x \sqrt{\hat{g}} \left\{ -ie^{-\frac{4\psi}{G}} \left( \hat{R} - (D - 1)\hat{g}^{\mu \nu} \nabla_\mu \partial_\nu \phi + \frac{1}{4}(D - 1)\hat{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \right) (\chi + \hat{\chi}) 
+ \left( \frac{1}{G} - \frac{24 - c}{24\pi \epsilon} \right) e^{-\frac{4\psi}{G}} \left( \hat{R} - \frac{1}{4}(D - 1)\hat{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \right) \right\} .$$

(10)

Introducing new variables $\psi$ and $\xi$ through

$$e^{-\frac{4\psi}{G}} \chi = \xi$$
$$e^{-\frac{4\psi}{G}} = 1 + \frac{\epsilon}{4} \psi,$$

(11)

the terms relevant to the renormalization of the operators considered are

$$\sim \int d^D x \sqrt{\hat{g}} \left\{ i(D - 1)\hat{g}^{\mu \nu} \partial_\mu \psi \partial_\nu \xi + \frac{24 - c}{96\pi} (D - 1)\hat{g}^{\mu \nu} \partial_\mu \psi \partial_\nu \psi \right\},$$

(12)

which means that the problem is reduced to a free field theory with the propagators

$$\langle \psi(p)\psi(-p) \rangle = 0$$
$$\langle \psi(p)\xi(-p) \rangle = \frac{-i}{D - 1} \frac{1}{p^2}$$
$$\langle \xi(p)\xi(-p) \rangle = \frac{24 - c}{48\pi} \frac{1}{D - 1} \frac{1}{p^2}.$$

(13)

Let us evaluate the divergence of the one-point functions of the operators. As for the cosmological term, one gets

$$\langle \int d^D x \sqrt{g} \rangle = \int d^D x \sqrt{\hat{g}} \langle e^{\frac{4\psi}{G} \log(1 + \frac{\epsilon}{4} \psi)} \rangle$$
$$= \int d^D x \sqrt{\hat{g}},$$

due to $\langle \psi(p)\psi(-p) \rangle = 0$. Thus one finds that the cosmological term is not renormalized. As for the mass term, one gets

$$\langle \int d^D x \sqrt{g} \chi^2 \rangle$$
Rescaling the metric as

\[ z \rightarrow \lambda z \]

Strictly speaking, one should have taken care of the \( O(1) \) contributions to the term proportional to \( \hat{\chi} \) in the last step of the equality. One can check, however, that starting from the action with \( \chi \)-linear term and adopting the minimal subtraction scheme is equivalent to the above manipulation.

The bare operators, therefore, can be written as

\[
m_0^2 \int d^Dx \sqrt{g} \chi_0^2 + \Lambda_0 \int d^Dx \sqrt{g}
\]

\[ = m^2 \mu^\epsilon \int d^Dx \sqrt{g} \left\{ \left( \chi - i \frac{1}{2\pi \epsilon} \right)^2 + \frac{18 - c}{48\pi} \frac{1}{2\pi \epsilon} \right\} + \Lambda \mu^\epsilon \int d^Dx \sqrt{g}, \quad (14)\]

from which one can read off the relations between the bare parameters and the renormalized ones as

\[
\begin{align*}
\chi_0 &= \chi - i \frac{1}{2\pi \epsilon} \\
m_0^2 &= m^2 \mu^\epsilon \\
\Lambda_0 &= \Lambda \mu^\epsilon + \frac{18 - c}{48\pi} \frac{1}{2\pi \epsilon} m^2 \mu^\epsilon.
\end{align*}\]

Using the above relations, one can evaluate the area dependence of the partition function in the \( \epsilon \rightarrow 0 \) limit as follows.

\[
Z(A) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\chi_0 \exp \left[ -\mu^\epsilon \int d^Dx \sqrt{g} \left( \frac{1}{G} - i\chi - \frac{24 - c}{24\pi \epsilon} \right) R - m_0^2 \int d^Dx \sqrt{g} \chi_0^2 - \Lambda_0 \int d^Dx \sqrt{g} \right] \cdot \delta \left( \mu^\epsilon \int d^Dx \sqrt{g} \left| \mu - A \right. \right).
\]

Rescaling the metric as \( g_{\mu\nu} \rightarrow \lambda g_{\mu\nu} \),

\[
Z(A) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\chi_0 \exp \left[ -\lambda^{D/2} \mu^\epsilon \int d^Dx \sqrt{g} \left( \frac{1}{G} - i\chi - \frac{12 - c}{24\pi \epsilon} \right) R - \lambda^{D/2} m^2 \mu^\epsilon \int d^Dx \sqrt{g} \chi_0^2 \right. \\
- \lambda^{D/2} \left( \Lambda \mu^\epsilon + \frac{18 - c}{48\pi} \frac{1}{2\pi \epsilon} m^2 \mu^\epsilon \right) \int d^Dx \sqrt{g} \right] \cdot \delta \left( \lambda^{D/2} \mu^\epsilon \int d^Dx \sqrt{g} \left| \lambda^{D/2} \mu^\epsilon - A \right. \right) \]

\[ = \int \mathcal{D}g_{\mu\nu} \exp \left[ \frac{\epsilon}{2} \log \lambda \frac{12 - c}{24\pi \epsilon} \mu^\epsilon \int d^Dx \sqrt{g} R - \frac{\epsilon}{2} \log \lambda \frac{18 - c}{48\pi} \frac{1}{2\pi \epsilon} m^2 \mu^\epsilon \int d^Dx \sqrt{g} \right] \]

\[ \cdot \exp \left[ -\mu^\epsilon \int d^Dx \sqrt{g} \left( \frac{1}{G} R - \frac{12 - c}{24\pi \epsilon} R + \frac{1}{4m^2 \lambda^2} R^2 \right) - \Lambda_0 \lambda \int d^Dx \sqrt{g} \right] \cdot \delta \left( \lambda^{D/2} \mu^\epsilon \int d^Dx \sqrt{g} \left| \mu - A \right. \right). \]
Setting \( \lambda = A \),

\[
Z(A) = A^{\gamma_{\text{str}} - 3} e^{-\Lambda_0 A} \int \mathcal{D}g_{\mu\nu} \exp \left[ -\mu^\epsilon \int d^Dx \sqrt{g} \left( \frac{1}{G} - \frac{12 - c}{24\pi\epsilon} R + \frac{1}{4m^2 A} R^2 \right) \right] \cdot \delta \left( \mu^\epsilon \int d^Dx \sqrt{g} \right) - 1 \right),
\]

where \( \gamma_{\text{str}} \) is the string susceptibility given by

\[
\gamma_{\text{str}} = 2 + \frac{c - 12}{6} (1 - h) + \frac{c - 18}{192\pi^2} m^2 A. \tag{16}
\]

For \( m^2 A \ll 1 \), the classical solution dominates in the path integral. One gets, after taking account of the fluctuation around the classical solution, the area dependence of the partition function as,

\[
Z(A) \sim A^{\gamma_{\text{str}} - 3} e^{-\Lambda A} \exp \left( -\frac{16\pi^2 (1 - h)^2}{m^2 A} \right). \tag{17}
\]

One should note here that our \( m^2 \) corresponds to \( 8\pi \) times the \( m^2 \) of ref. [6]. Comparing our result with that of ref. [6], the only discrepancy is the \( c \)-independent coefficient of \( m^2 A \) in eq. (16), which is subtraction scheme dependent. We can, therefore, conclude that the two results are consistent.

In this letter we have studied \( R^2 \) gravity in the formalism of \( (2 + \epsilon) \)-dimensional quantum gravity. After taking care of the oversubtraction problem and dropping the \( h \)-field, the ghosts and the matters, the theory reduces to a free field theory, which enables a full order calculation of the string susceptibility. The result is consistent with that of ref [6]. In our calculation, the peculiar \( (c - 12) \) factor comes from the shift of the \( \chi \)-field and the \( A \)-dependent term comes from the fact that the \( \chi^2 \) operator generates a cosmological term after renormalization.

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Appendix

In this appendix, we calculate the one-loop counterterms for the most general renormalizable action with a scalar field $\chi$ and $c$ species of conformal matter,

$$S = \mu^4 \int d^D x \sqrt{g} \left( \frac{1}{2} K(\chi) g^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi + L(\chi) R + M(\chi) \right) + \text{(matters)}. \quad (18)$$

Adopting the background field method, we replace $\chi$ with $\hat{\chi} + \chi$ and parametrize $g_{\mu \nu}$ as

$$g_{\mu \nu} = \hat{g}_{\mu \rho}(e^h)^\rho_\nu e^{-\phi},$$

where $\hat{\chi}$ and $\hat{g}_{\mu \nu}$ are the background fields, $\phi$ is the conformal mode, and $h$ is the traceless symmetric tensor field. We expand the action up to the second order of $\chi$, $\phi$ and $h$, and drop the first order terms following the prescription of the background field method. We can choose the gauge fixing term so that the mixing terms between $h$ and the other fields may be cancelled,

$$S_{\text{g.f.}} = \mu^4 \int d^D x \sqrt{\hat{g}} \left\{ \frac{1}{2} \left( K(\chi) + \frac{L'(\chi)}{L(\chi)} \right) \hat{g}^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi + \frac{1}{4} L(\chi) h^{\mu \nu} h_{\mu \nu} \right\} \hat{g}^{-1} \partial_{\sigma} \phi \partial^\sigma \phi,$$

where the comma represents the covariant derivative with respect to $\hat{g}_{\mu \nu}$. Note that the function $f$ can be taken arbitrary. The ghost action can be determined from the gauge fixing term as

$$S_{\text{ghost}} = \mu^4 \int d^D x \sqrt{\hat{g}} \left\{ -\partial_{\nu} \hat{g}^{\mu \nu} \partial_{\mu} \chi - \hat{g}^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi + \frac{1}{4} \left( L'(\chi) - \frac{L''(\chi)}{L(\chi)} \right) \hat{g}^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi \right\}. \quad (20)$$

The kinetic terms of $\chi$, $h$ and $\phi$, including those from the gauge fixing term, thus read

$$\mu^4 \int d^D x \sqrt{\hat{g}} \left\{ \frac{1}{2} \left( K(\chi) + \frac{L'(\chi)}{L(\chi)} \right) \hat{g}^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi + \frac{1}{4} L(\chi) h^{\mu \nu} h_{\mu \nu} \right\} \hat{g}^{-1} \partial_{\sigma} \phi \partial^\sigma \phi,$$

Note that the kinematical pole which appears in the case of ordinary gravity does not show up here due to the $\phi-\chi$ coupling. We introduce the new quantum fields $X$, $\Phi$ and $H^\mu_\nu$, through

$$\begin{align*}
\chi &= F(\chi) X - \frac{D}{2} F(\chi)^2 L'(\chi) I(\chi) \Phi \\
\phi &= I(\chi) \Phi \\
h^\mu_\nu &= L(\chi)^{-1/2} H^\mu_\nu, \quad (22)
\end{align*}$$
where $F(\hat{\chi})$ and $I(\hat{\chi})$ are defined through
\[
\frac{1}{F(\hat{\chi})^2} = K(\hat{\chi}) + \frac{L'(\hat{\chi})^2}{L(\hat{\chi})} \\
\frac{1}{I(\hat{\chi})^2} = \frac{1}{4} \left( \epsilon DL(\hat{\chi}) + D^2 F(\hat{\chi})^2 L'(\hat{\chi})^2 \right).
\] (23)

After this field redefinition, the kinetic term reduces to the standard form
\[
\mu^\epsilon \int d^D x \sqrt{\tilde{g}} \left\{ \frac{1}{4} H_{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu X \partial_\nu X \right\}.
\] (24)

Taking account of the other terms coming from $S$, $S_{\text{g.f.}}$, and $S_{\text{ghost}}$, and calculating the one-loop counterterms through the ’t Hooft-Veltman formalism [10], one obtains the final result for the action with the counterterms as
\[
S + S_{\text{c.t.}} = \mu^\epsilon \int d^D x \sqrt{g} \left[ \frac{1}{2} \left\{ K(\chi) - \frac{1}{2\pi \epsilon} P(\chi) \right\} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right.
\]
\[
+ \left\{ L(\chi) - \frac{1}{2\pi \epsilon} \frac{24 - c}{12} \right\} R
\]
\[
+ \left\{ M(\chi) - \frac{1}{2\pi \epsilon} \left( \frac{1}{2} M(\chi) I(\chi)^2 + M'(\chi) \frac{1}{L'(\chi)} \right) \right\}
\]
\[
+ \text{(matters)},
\] (25)

where $P(\chi)$ is given by
\[
P = -2L'^{-2} L'' + 2L^{-1} L'' + 2F^2 L^{-1} L'' (K' + 2L^{-1} L' L'' - L^{-2} L'^3)
\]
\[
- \frac{1}{2} F^4 (K' + 2L^{-1} L' L'' - L^{-2} L'^3)^2
\]
\[
+ 3L^{-1} K - 2L^{-1} F^{-2} + \frac{9}{2} L^{-2} L'^2 - f.
\] (26)
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The diagrams with $h_{\mu\nu}$ line we have to consider at two-loop level. The solid line, the dash line and the wavy line represent the propagators of the $\Phi$–field, the $X$–field and the $H$–field respectively. (a) and (b) correspond to $\langle \Phi^2 \rangle$, while (c) and (d) correspond to $\langle X^2 \rangle$. 

Figure caption
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