What is unsharp quantum reality?
A conceptual analysis

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Abstract. In their paper Unsharp Quantum Reality, Paul Busch and Gregg Jaeger explain the conception of unsharp reality in terms of Positive Operator Valued (POV) measures, potentiality, and single-case probabilities. The present paper discusses their conception from a philosophical point of view, focusing on the ontological interpretation of POV measures employed in it and on its relation to Peter Mittelstaedt’s work on the semantics and ontology of quantum mechanics. Busch and Jaeger extend Mittelstaedt’s approach by adding a realistic interpretation of unsharp quantum properties to it. Their account of unsharp quantum reality generalizes Heisenberg’s potentiality interpretation of quantum states. In order to explain the transition from potential to actual quantum properties, they suggest a Ghirardi-Rimini-Weber (GRW) collapse theory. Their approach comes close to recent work on quantum propensities. However, the question remains what unsharp quantum reality means when no measurement takes place, as for example claimed in the quark model of the subatomic constitution of matter.

1. Introduction
In 1994, I first met Paul Busch as a member of Peter Mittelstaedt’s research group in Cologne. Since then, I had intriguing discussions with them on the semantics and ontology of quantum mechanics and their conception of unsharp reality. Beyond the formal quantum theory of measurement and its extension in terms of Positive Operator Valued (POV) measures, the question remained in which philosophical terms one may conceive of unsharp quantum reality. Busch’s joint paper with Gregg Jaeger Unsharp Quantum Reality [1] provides an answer in terms of potentiality, the causal power of unsharp quantum properties, and the transition from potential to actual quantum properties by a spontaneous collapse as described in the Ghirardi-Rimini-Weber (GRW) approach. Busch and Jaeger’s conception of unsharp quantum reality is a promising step towards a realistic interpretation of quantum theory without recourse to many-worlds or hidden variables. However, due to the operational approach to quantum theory on which it is based [2] it has certain limitations, as concerns a realistic account of the constitution of matter.

2. The question
The background of Busch’s and Jaeger’s account of unsharp quantum reality is the quantum theory of measurement [3, 4] and Mittelstaedt’s work on the semantics and ontology of quantum theory [5, 6]. Mittelstaedt always claimed that the principles of quantum theory are fundamental and universally valid, whereas the theories of classical physics have a restricted domain in which
they only approximately hold. The assumption that quantum theory is universally valid gives rise to the quest for a quantum ontology. The unresolved measurement problem, however, precludes to establish it in a straightforward way. In order to spell it out, Mittelstaedt attributed an object semantics to the world of classical physics, but a (weaker) process semantics to the quantum domain [5]. In face of the quantum measurement problem, the crucial question is then: To what extent are both kinds of semantics compatible?

In order to clarify the relation of both kinds of theory and their respective ontologies, Mittelstaedt followed Busch, taking up the conception of unsharp properties which may be attributed to single quantum systems [7]. It is based on Heisenberg’s uncertainty relation and the possibility of joint unsharp measurements of position and momentum [8] or other pairs of non-commuting quantum observables (which Bohr called complementary [9]). According to the probabilistic interpretation of quantum mechanics, the uncertainty relation $\Delta x \Delta p \sim \hbar$ expresses the widths of the probability distributions of the observables position and momentum in a given state [10]. Hence, the corresponding unsharp properties of position and momentum have probabilistic meaning. They are properties only attributed with a certain probability to a quantum system. However, no ignorance of probability is tenable for them, according to the standard interpretation of quantum mechanics. Therefore, the question arises: In which sense may such unsharp properties give rise to a quantum ontology? A convincing ontology should not describe mere possibilities but the actual world. So, what is the reality of these probabilistic properties?

Mittelstaedt attempted to spell the unsharp quantum ontology out [6] relying on the formal approach to unsharp observables in terms of Positive Operator Valued (POV) measures [2, 3]. Based on his earlier work on the semantics and ontology of quantum theory [5], he showed on the basis of Quantum Logic how the classical ontology has to be relaxed to a Quantum Ontology $O(Q)$ restricted to the value definite propositions that still remain in the quantum domain. Then, he emphasized that this is not sufficient, given that the resulting assumptions about value definite propositions are still too strong:

In order to re-establish consistency [...], $O(Q)$ must be further relaxed, in particular with respect to the value definiteness of elementary properties. [...] As the envisaged result we expect self-consistency between quantum mechanics and a new weak ontology $O(Q^U)$ of unsharp properties. [6, p 82]

According to his previous investigations concerning universality, quantum mechanics is semantically consistent at the probabilistic level, but not at the level of single quantum systems [11, 12]. Hence the need for a further ontological relaxation. The resulting unsharp quantum ontology $O(Q^U)$ is obviously expressed in terms of probabilities. It deals with possibilities rather than with the actual facts of physical reality. Is this really an ontology? Or, to put it in other words, what is unsharp reality?

3. Potentiality

To shed light on this intriguing question, Busch and Jaeger wrote the paper entitled Unsharp Quantum Reality as a contribution to the Festschrift for P Mittelstaedt on his 80th birthday [1]. In it, they address Mittelstaedt’s claim that quantum mechanics is semantically consistent at the probabilistic level. The proof of this claim is based on considering an ensemble of $N$ identically prepared quantum systems as one composite system [4, 11, 12]. Busch and Jaeger comment on this approach as follows [1]:

However, the entity that carries the probability value as an (approximately) actual property is an ensemble of equally prepared quantum systems. Since this system is composed of independent subsystems, the question remains how one could then explain that such independent objects act together to cause the emergence of an approximately
actual eigenvalue of the respective frequency operators of the ensemble. After all, the
description of the ensemble as a compound system is hardly more than a conceptual
construct. (p 1350)

In spite of this conceptual construct, the ensemble exactly behaves as to be expected:

The answer is therefore to be sought in the fact that each constituent has been treated—
prepared—in an identical way. […] Through its preparation, each individual system
is constrained to ‘respond’ to a measurement so as to induce a specific outcome in
accordance with the probability law specified by its quantum state. (pp 1350–1)

Then, Busch and Jaeger add their own claim concerning unsharp reality:

Each possible value of the observable to be measured has a limited degree of reality,
quantified by the associated quantum probability. Accordingly, the individual has a
limited capacity to cause the actualization of each of the corresponding pointer values
(and, in a repeatable measurement, the values of the observable). On many repetitions
(or in many parallel runs) of the measurement, the degrees of reality of the different
values are then manifested as the frequencies of the outcomes. (p 1351)

Sharp properties of quantum systems correspond to spectral projections on eigenstates, and
hence to definite values of the respective observable, according to the eigenstate-eigenvalue link
of standard quantum theory. Sharp properties are actual, they have probability 1 and belong
to physical reality. In contrast, unsharp properties of quantum systems correspond to effects
of POV measures which are no projections and give rise to fuzzy values. Unsharp properties
obey uncertainty relations in Heisenberg’s sense and express probabilities < 1. Busch and Jaeger
suggest to interpret them in terms of single case probabilities as potential properties with a degree
of reality < 1. In particular, they consider the effects \( E \) of an isolated quantum system in a
pure state \( \psi \), represented by a projection \( P = P_\psi \). The time evolution of \( \psi \) is determined by the
Schrödinger equation, resulting in a continuous evolution of single case probabilities or degrees
of reality \( f_\psi(E) = \text{tr}[PE] = \langle \psi | E \psi \rangle \). They conclude that “all the effects \( E \) of the system […]
are simultaneously real to a degree (actualization tendency) given by \( f_\psi(E) \).” [1, p 1363]

The background of their approach is Heisenberg’s potentiality interpretation of quantum
mechanics, according to which the wave function of a quantum state expresses ‘an objective
tendency or possibility, a ‘potentia’ in the sense of Aristotelian philosophy’ [13]. The wave
function expresses the tendency to become actual by the measurement of an observable. The
measurement transforms this potentiality into actuality. The result of this process is described
by the collapse of the wave function to an eigenstate of the respective observable. Busch and
Jaeger generalize Heisenberg’s approach to all states and effects of quantum systems, ascribing
physical reality with a degree \( f_\psi(E) < 1 \) to the quantum states \( \psi \) and their effects \( E \). And
they add an important causal aspect to it, namely that the quantum states, respectively their
degrees of reality, are probabilistic causes for the actualization of certain measurement results.

4. Propensities, dispositions and GRW

Busch and Jaeger’s account of unsharp reality stands in the tradition of the propensity
interpretations of quantum theory proposed by Margenau [14] and Popper [15] in the 1950s.
These approaches first provoked substantial criticism, but now there are new prospects for
propensities [16, 17].

A propensity is a quantified tendency to become actual. According to this definition, the
degrees of reality < 1 of unsharp properties determined by \( f_\psi(E) = \text{tr}[PE] = \langle \psi | E \psi \rangle \) are
propensities. In philosophy of science, propensities or probabilistic causes are also discussed
in terms of dispositions. Busch and Jaeger emphasize that their conception of unsharp reality
comes close to Esfeld’s account of quantum dispositions with causal powers [18].
One of the problems of Popper’s approach was to apply his propensity conception to probability in general as well as to quantum theory [16]. It has indeed been argued ‘that any broadly dispositional analysis of probability will either fail to give an adequate explication of probability, or else will fail to provide an explication that can be gainfully employed elsewhere (for instance, in empirical science or in the regulation of credence)’, with the conclusion [19] that this ‘does place quite strong constraints on what type of propensity interpretation can be maintained.’

The recent approaches to quantum propensities and dispositions are more successful. On the one hand, they do not identify propensities and probabilities. In particular, it has been shown that quantum propensities substantially differ from (classical) conditional Kolmogorov probabilities [20]. On the other hand, the recent approaches emphasize that quantum propensities or dispositions require a GRW type collapse theory in order to contribute to a genuinely metaphysical, or realistic, interpretation of quantum mechanics, beyond instrumentalist views [21]. Busch and Jaeger indeed claim that in order to explain the objectification of measurement results (that is, the transition from potential to actual quantum properties), a GRW type theory is needed [1, p 1359-61]. Vice versa, it has been claimed that the GRW approach is a good candidate for an ontology of quantum dispositions [22]. As to be expected, such an ontology of quantum propensities or dispositions and the associated GRW collapse theory also take the non-locality of quantum states into account [1, 22].

In recent papers, Jaeger enhanced the conception of unsharp quantum reality [1] to clarify the relation between Heisenberg’s potentiality interpretation of quantum mechanics and the Aristotelian concept of potentia [23], and attempted to make it more precise in terms of the distinction of actuality vs. reality [24]. He argues that Heisenberg’s account of Aristotle’s distinction of potentiality ($\delta$υνομικ) and actuality ($\epsilonνεργεια$) does not fall short of Aristotle’s concepts, given that the spontaneous transition from potential to actual properties occurring in quantum measurements correspond to the Aristotelian concept of chance, which means a spontaneous event or process in nature devoid of any teleological connotation [23]. Indeed Aristotle’s concept of chance refers to events occurring with objective probability. Applied to probabilistic quantum transitions, Aristotelian chances hence amount to quantum propensities and the associated spontaneous transitions from potential to actual quantum properties. In addition, Jaeger refines Heisenberg’s distinctions of potentiality and actuality by distinguishing furthermore reality from actuality, and he suggests that unsharp quantum properties with an actualization tendency or degree of reality $\frac{1}{2} < f_E(P) < 1$ may be called real, in spite of not being actual [24].

Jaeger emphasizes that Heisenberg himself did not distinguish the concepts of reality and actuality. However, this matter of fact may partially be due to a translation problem raised by the English original version of Physics and Philosophy. In German, and in particular in the Kantian tradition to which Heisenberg belonged, one may distinguish reality (Realiität) and actuality (Wirklichkeit). In the German translation of Physics and Philosophy, Heisenberg accordingly uses the term Wirklichkeit (actuality) as opposed to potentiality (Möglichkeit) [25]. But in the English original version, he had used the term reality rather than the term actuality, which would have been the correct opposite of potentiality.

Jaeger’s way of refining Heisenberg’s distinction of potentiality and actuality in terms of a non-Heisenbergian distinction of reality and actuality is in two regards problematic. On the one hand, it does not conform to Kant’s distinction of reality and actuality [26], a conceptual distinction that is however more important for philosophers than for physicists. On the other hand, it introduces an arbitrary distinction between (approximately) real quantum properties with probability $\frac{1}{2} < f_E(P) < 1$ and unsharp quantum properties with a degree of reality $0 < f_E(P) \leq \frac{1}{2}$, referring to an early paper of Busch [26]. But unsharp quantum reality comes in continuous degrees. There is no reason to attribute significantly more reality to quantum
properties with probability $\frac{1}{2} + \epsilon$ than to properties with $0 < f_E(P) \leq \frac{1}{2}$. Quantum events with a very low probability $0 < f_E(P) \ll 1$ happen. I resist to say that they happen quite often, but they happen. They do in the case of large angle Rutherford scattering, to mention just a famous example. The methods of data analysis in particle physics show that in the quantum field theory of scattering such examples indeed are ubiquitous. They cannot be neglected in the reconstruction of particle tracks and quantum data probability distributions [27, 28]. Hence, a convincing account of unsharp quantum reality should include them, and with them, any degrees of reality $f_E(P) \ll 1$. As long as the quantum probabilities are non-zero, the corresponding quantum states belong to unsharp quantum reality, as Jaeger indeed claims, attributing causal efficacy to them. But his approach still misses to emphasize and discuss the physical significance of unsharp quantum reality in the range of $0 < f(E(P)) < \frac{1}{2}$.

5. Conclusions
Busch’s and Jaeger’s account of unsharp quantum reality [1] offers a realistic interpretation of quantum states in terms of quantum actualization tendencies, or quantum propensities. Their approach is based on a formal generalization of quantum mechanics in Hilbert space by POV measures which make it possible to express joint measurements of non-commuting (complementary) quantum observables and their joint probabilities, expressed by (generalized) Heisenberg uncertainty relations. They express unsharp quantum reality in terms of the degrees of reality $0 < f_E(P) < 1$ of quantum effects $f_E(P)$, including quantum effects of low but non-zero probabilities. In order to explain the transition from quantum superpositions, or potential quantum properties, to actual measurement outcomes, they suggest that the realistic interpretation of unsharp quantum properties requires a GRW type collapse theory.

The advantage of this new realistic interpretation of quantum mechanics is that it makes no recourse to speculative metaphysical assumptions such as many-world or hidden variables, which have to be added to quantum mechanics and its probabilistic standard interpretation. It is in accordance with the recent discussion of quantum propensities, dispositions, and the corresponding ontology of a GRW approach to the quantum measurement problem. And it even admits degrees of reality $0 < f(E(P)) < \frac{1}{2}$.

However, due to its foundation in the operational account of quantum physics [2] Busch’s and Jaeger’s account of unsharp quantum reality is limited. In order to be in accordance with the practice of physics, any realistic interpretation of quantum theory should be able to explain the current constituent model of matter which is based on the standard model of particle physics. According to this model, quarks and leptons are the parts of matter. In particle physics, the existence of these quantum parts of matter is established in terms of sum rules for momentum-energy, spin, parity, and so on, which are based on the scattering experiments of high energy physics [2]. The quarks within matter make up the protons and neutrons, which in turn make up the atomic nuclei. Within the protons and neutrons, however, the quarks only come in superpositions, given that their mass eigenstates differ from their flavour eigenstates. In addition, due to quark confinement no single quarks can be measured. Hence, any merely operational interpretation of the unsharp properties of quarks misses the role of quarks as actual matter constituents, which are well-established parts of physical reality. In particular with regard to compound quantum systems, additional concepts are needed in order to conceive of unsharp quantum reality beyond an operational quantum physics.

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