A VIRIAL CORE IN THE SCULPTOR DWARF SPHEROIDAL GALAXY

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ABSTRACT

The projected virial theorem is applied to the case of multiple stellar populations in the nearby dwarf spheroidal galaxies. As each population must reside in the same gravitational potential, this provides strong constraints on the nature of the dark matter halo. We derive necessary conditions for two populations with Plummer or exponential surface brightnesses to reside in a cusped Navarro–Frenk–White (NFW) halo. We apply our methods to the Sculptor dwarf spheroidal, and show that there is no NFW halo compatible with the energetics of the two populations. The dark halo must possess a core radius of ~120 pc for the virial solutions for the two populations to be consistent. This conclusion remains true, even if the effects of flattening or self-gravity of the stellar populations are included.

Key words: dark matter – galaxies: dwarf – galaxies: individual (Sculptor) – galaxies: kinematics and dynamics

1. INTRODUCTION

Using high-resolution numerical simulations of structure formation, Navarro et al. (1995) first proposed that dark halos have the universal form

\[ \rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}, \]

where \( r_s \) is the scale radius and \( \rho_0 \) is the density normalization. Sometimes it is useful to define a halo radius \( r_h \), which is the radius within which the mean density is a factor of \( \Delta_h = 200 \) times greater than the critical density. Then, the enclosed mass of the Navarro–Frenk–White (NFW) profile is

\[ M(r) = 4\pi \rho_0 r_s^3 \left[ \log(1 + cx) - \frac{cx}{1 + cx} \right], \]

where \( x = r/r_h \) and \( c = r_h/r_s \). The two parameters of the NFW halo can be alternatively chosen as the concentration \( c \) and the mass \( M = M(r_h) \). The NFW model remains the best two-parameter fit to simulation data, though Navarro et al. (2004) provided evidence that the three-parameter Einasto profile gives a better match.

Despite an extensive body of research, it is unclear whether the shapes of dark halos inferred from observations are consistent with cosmological predictions. In galaxies like the Milky Way and M31, the dark halo profiles have been significantly affected by the cooling of baryons, which makes them unattractive candidates for testing the prediction. Therefore, research has concentrated on two very different classes of objects, namely, galaxy clusters and dwarf galaxies. In clusters, the high temperature and low density of the intracluster gas prevents efficient cooling, and so the baryons do not significantly affect the potential of the dark matter. There is reasonably good evidence from cluster lensing that the profiles follow the NFW law, although with higher concentrations than predicted (e.g., Broadhurst et al. 2008).

In dwarf galaxies at the present epoch, the baryonic component is feeble in comparison with the dark matter. Low surface brightness galaxies with H I and Hα rotation curves have been studied extensively by de Blok & Bosma (2002), who conclude that cored rather than cusped halos are favored. The dwarf spheroidal galaxies have no gas, and are pressure-supported rather than rotation-supported, and so are more difficult to assess. It was quickly realized that the radial velocities of thousands of bright giant stars could be gathered with modern multi-object spectrographs (Kleyna et al. 2002; Walker et al. 2007). Dynamical modeling with either the Jeans equations or phase space distribution functions then yields constraints on the dark matter profile. However, this line of inquiry seemed to have stalled with the realization of the degeneracies inherent in the Jeans equations, making the data sets compatible with both cored and cusped dark halo models depending on the underlying assumptions (Evans et al. 2009; Walker et al. 2009).

The problem has received new impetus with the growing realization that dwarf spheroidals often contain two or more stellar populations (Battaglia et al. 2006; Koch et al. 2006). This significantly strengthens any constraints from dynamical modeling, as now each sub-population must separately be in equilibrium in the same potential. There have been three recent and thought-provoking results. First, Battaglia et al. (2008) used the Jeans equations to model the velocity dispersion profiles of the two populations in the Sculptor dSph, finding that a cored halo is preferred to an NFW halo, though the latter is still statistically consistent with the observations. Second, using the same data set but more rigorous methods based on distribution functions, Amorisco & Evans (2012a) showed that NFW halos are rejected at high significance, performing substantially poorer than cored models. Finally, Walker & Peñarrubia (2011) used the half-light radii of the two populations in the Fornax and Sculptor dSphs to estimate the enclosed mass. This exploited earlier results showing that the mass within the half-light radius is robust against changes in the anisotropy (Walker et al. 2009; Wolf et al. 2010). Given the enclosed mass at two radii, the slope of density is inferred and shown to be in contradiction with NFW halos.

Here, we provide a simple line of reasoning based on the energies of the sub-populations. This brings the earlier arguments of Battaglia et al. (2008), Walker & Peñarrubia (2011), and Amorisco & Evans (2012a) into sharp focus. We show that the energetics of the sub-populations in one of the best-studied dSphs, Sculptor, cannot be made consistent with an NFW halo.

2. THE PROJECTED VIRIAL THEOREM

Let \( z \) be the line-of-sight direction. Then, the projected virial theorem relates the projected component of the pressure and
potential energy tensors through the equation $2K_{\text{los}} + W_{\text{los}} = 0$, where

$$K_{\text{los}} = \frac{Y_*}{2} \int_0^\infty \nu(r^2) d^3x, \quad W_{\text{los}} = -\int_0^\infty \nu z \partial_z \Phi d^3x.$$  \hfill (3)

Here, $\nu$ is the luminosity density of a population with stellar mass-to-light ratio $Y_*$, moving in the gravity field $\Phi$. Under the assumption of spherical symmetry, the tensors become (e.g., Merrifield & Kent 1990)

$$K_{\text{los}} = \pi Y_* \int_0^\infty \mu \nu \sigma^2 R dR,$$  \hfill (4)

$$W_{\text{los}} = -\frac{4\pi G}{3} \int_0^\infty \nu M(r) r dr,$$  \hfill (5)

where $M(r)$ is the mass enclosed within radius $r$. These formulae involve the line-of-sight velocity second moment $\langle v_r^2 \rangle$ and the surface brightness $\mu$, which are directly accessible to observation. Note that the velocity anisotropy of the stellar population does not enter, which gives the virial equations an immediate advantage over the Jeans equations. Note also that virial quantities, which are gross volume integrals, can be computed more robustly than the gradients of observables required by the Jeans equations. Furthermore, there is no problem in dealing with rotation, as the integral $K_{\text{los}}$ requires just the sum of square velocities in ordered and random motions along the line of sight.

The only unknown in the projected virial equation is the total gravitational potential $\Phi(r)$ in which the population moves, or equivalently, the enclosed mass $M(r)$. If preferred, $W_{\text{los}}$ can be recast in terms of the total mass density $\rho(r)$ which generates $\Phi$, namely,

$$W_{\text{los}} = -\frac{16\pi G}{3} Y_* \int_0^\infty \rho \mu(r) \int_0^R \rho(r) \frac{r^2 dr}{\sqrt{R^2 - r^2}} dR.$$  \hfill (6)

Here, we have eliminated the luminosity density in terms of the surface brightness, which is an observable.

The projected virial equation by itself is of course insufficient to determine the potential or mass distribution directly. Suppose now there are two populations in equilibrium in the same total potential (e.g., Agnello & Evans 2012). In the dSphs, a younger metal-rich and an older metal-poor population with associated velocity dispersion profiles $\sigma_r$ and $\sigma_p$, and surface brightnesses $\mu_r$ and $\mu_p$ (with corresponding luminosity densities $\nu_r$ and $\nu_p$) are often present. For example, there is good evidence that Sculptor (Battaglia et al. 2008; Walker & Peñarrubia 2011), Carina (Bono et al. 2010), and Fornax (Amorisco & Evans 2012b) contain at least two distinct populations. Then, the projected virial theorem must be satisfied for each population, so we have

$$K_{\text{los},p} = \frac{W_{\text{los},p}}{W_{\text{los},r}} = \frac{\int_0^\infty \mu_p \sigma_p^2 R dR}{\int_0^\infty \mu_r \sigma_r^2 R dR} = \frac{\int_0^\infty \nu_p M(r) r dr}{\int_0^\infty \nu_r M(r) r dr}.$$  \hfill (7)

The only unknown is the potential, or equivalently, the enclosed mass $M(r)$. This result is powerful enough to rule out some dark matter halos. The physical reason is that a stellar population with velocity dispersion $\langle v_r^2 \rangle$ offers the best constraint on the potential at radii given by $\Phi(r) \approx \langle v_r^2 \rangle$. With more than one population, the potential is restricted at more than one location.

3. APPLICATION TO SCULTOR

3.1. Simple Results

If the potential is heavily dominated by dark matter, we can neglect the self-gravity of the luminous components in a first approximation (relaxed in the next subsection). In this case, both the kinetic energy tensor $K_{\text{los}}$ and the potential energy tensor $W_{\text{los}}$ scale linearly with the stellar mass-to-light ratio, which can then be eliminated.

The surface brightness profiles of dSphs are well fit by Plummer laws:

$$\mu(R) = \frac{\mu_0}{(1 + R^2/R_h^2)^{2}}.$$  \hfill (8)

This assumption is not critical to our argument, but it does provide a useful illustrative model. We shall in any case sketch the extension to another commonly used profile, namely, the exponential law. Fitting the Plummer law to the profiles of the metal-rich and metal-poor populations in Sculptor gives $R_{h,r} = 230 \pm 10$ pc for the metal-rich and $R_{h,p} = 350 \pm 10$ pc for the metal-poor (consistent with the values in Battaglia et al. 2008 and Amorisco & Evans 2012a). A convenient fitting formula for the velocity dispersion profile is

$$\langle v_r^2 \rangle = \frac{\sigma_0^2}{(1 + a^2/R_h^2)},$$  \hfill (9)

which gives a flattish profile with amplitude $\sigma_0$ out to a characteristic length scale $a$. This is typical of the velocity dispersion profiles in, for example, Walker et al. (2007) or Battaglia et al. (2008). Again, this assumption is not necessary for our argument, as the relevant virial integrals can always be computed numerically. For Sculptor, fits provide $\sigma_{0,r} = 8.7 \pm 1.0$ km s$^{-1}$ and $a_r = 240 \pm 55$ pc for the metal-rich population and $\sigma_{0,p} = 10.9\pm0.8$ km s$^{-1}$ and $a_p = 1920\pm850$ pc for the metal-poor population.

With these laws, the kinetic energy integral is analytic with

$$K_{\text{los}} = \frac{\pi Y_* \mu_0 \sigma_0^2 a^2}{R_h^2} - 1 - \frac{2 \log(a/R_h)}{(1 - a^2/R_h^2)^3}.$$  \hfill (10)

The metal-rich limit is given by $a_r \approx R_{h,r}$, the metal-poor limit is given by $a_p \gg R_{h,p}$. This gives

$$\frac{K_{\text{los},p}}{K_{\text{los},r}} = 2 \left( \frac{\mu_{0,p}}{\mu_{0,r}} \right) \left( \frac{R_{h,p}}{R_{h,r}} \right)^2 \left( \frac{\sigma_{0,p}}{\sigma_{0,r}} \right)^2.$$  \hfill (11)

The projected potential energy of Plummer profiles embedded in NFW halos is also exact, as

$$W_{\text{los}} = -\frac{8\pi^2 G Y_* \mu_0 \rho_0 R_h^3}{3\Delta^{3/2}} \left[ (R_h^2 + 3r_s R_h - 2 R_h^2) \log \left( \frac{R_h (\Delta + R_h)}{r_s (\Delta - r_s)} \right) \right],$$  \hfill (12)

with $\Delta^2 = R_h^2 + r_s^2$. With the values of $R_h$ listed above, the virial ratio $W_{\text{los},p}/W_{\text{los},r}$ as a function of $r_s$ is increasing and bound from above. In particular, we have

$$\frac{W_{\text{los},p}}{W_{\text{los},r}} < \left( \frac{\mu_{0,p}}{\mu_{0,r}} \right) \left( \frac{R_{h,p}}{R_{h,r}} \right)^3.$$  \hfill (13)
Hence, a necessary condition for an NFW halo to support two stellar populations with Plummer profiles is

\[
\left( \frac{\sigma_{0,t}}{\sigma_{0,p}} \right)^2 > 2 \left( \frac{R_{h,t}}{R_{h,p}} \right).
\]  

(14)

This is identical to Equation (22) of Amorisco & Evans (2012a), derived under different assumptions. If, instead of Plummer profiles, exponential laws are used to fit the surface brightness profiles, then the numerical factor becomes 1.9 instead of 2 in Equation (11). The analog of Equation (13) is unchanged, so that the necessary condition for an NFW halo to support two stellar populations with exponential surface brightness profiles is

\[
\left( \frac{\sigma_{0,t}}{\sigma_{0,p}} \right)^2 > 1.9 \left( \frac{R_{h,t}}{R_{h,p}} \right).
\]  

(15)

Using the best-fitting values provided above for Sculptor, we can immediately check whether the NFW potential is ruled out. Note that the constraints are simply the requirement that there is an NFW model with \( r_c < \infty \). This is a much looser constraint than requiring consistency with an NFW model with a concentration \( c \approx 20 \), as predicted by cold dark matter theories.

### 3.2. The Virial Stripes

The simple results already suggest that the energetics of the two populations are inconsistent with an NFW profile. However, it is prudent to confirm this result numerically, discarding some of the simplifying assumptions made above.

Since the measured profiles come with errors, we operate in the following manner. For each value of \( r_s \), we compute \( \rho_0 \) separately for the two populations for many different photometric (Equation (8)) and kinematic (Equation (9)) profiles. We weight each result with the likelihood of the fit. Then, varying \( r_s \) produces a virial stripe for each population in the \((\rho_0, r_s)\) plane. If the two stripes overlap at 2\( \sigma \) at a particular \( r_s \), then the model for the potential is plausible at the 2\( \sigma \) level. Nothing prevents us from including the contribution of the luminous tracers to the potential as well. The virial relations then also depend on the stellar mass-to-light ratio \( \Upsilon_* \), which may be different for the two populations. The projected potential energy \( W_{\text{dm}} \) has a contribution \( W_{\text{dm}} \) from the dark component and a correction \( W_{\text{self}} \) from the two luminous ones. For Plummer profiles, we have for the \( i \)th population

\[
W_{\text{self},i} = \pi^2 G \mu_{0,i} R_{h,i} \sum_j \Upsilon_{*,j} \mu_{0,j} R_{h,j}^2 w(R_{h,i}/R_{h,j}),
\]  

(16)

with

\[
w(x) = \frac{x^3 \left[(5x^2 + 3)K(1 - x^2) - (x^2 + 7)E(1 - x^2)\right]}{3(1 - x^2)^3},
\]  

(17)

where \( K, E \) are complete elliptic integrals and \( \Upsilon_{*,j} \) is the luminous mass-to-light ratio of the \( j \)th population. As the \( \mu_{0,j} \) are given by number counts and not directly by luminosities, a common rescaling is applied to both populations such that the total luminosity is fixed at the observed value (taken from Table 6 in Irwin & Hatzidimitriou 1995).

The upper panel of Figure 1 shows the virial stripes for the two populations in Sculptor, including the effects of self-gravity for the luminous component. The two stripes never overlap at the 2\( \sigma \) level. This confirms the result deduced from our simple argument in the previous section: there is no NFW halo compatible with the kinetic energies of the two stellar components in Sculptor. The middle and lower panels of Figure 1 show the virial stripes when the dark halo density is a cusped NFW potential, including the self-gravity of the stellar populations (\( \Upsilon_* = 8 \)). Purple shows the metal-rich population and blue the metal-poor population. In each stripe, the central line is the mean value of log10

\[
\log_{10}(W_{\text{DM}}/M_\odot\,\text{pc}^2) = \frac{\rho_0}{\left(e^2 + r_s^2/r_c^2\right)^{3/2}} \left(1 + r_s^2/r_c^2\right).
\]  

(18)

In this case, the stripes do overlap at the 2\( \sigma \) level provided that the core radius \( r_c \equiv r_c, e \) is at least 150 pc, if the self-gravity of the stellar populations is neglected. Incorporating
self-gravity causes the core radius to increase somewhat, as we see in Table 1. This shows how the minimum \( r_s \) for 2\( \sigma \) overlap varies with changing \( \Upsilon_\star \) for different models. The first column (\( \Upsilon_\star \rightarrow 0 \)) stands for models in which the self-gravity of the stars is omitted. Since both half-light radii are smaller or equal to \( r_s \) in the dark matter only case, adding self-gravity is expected to yield larger cores, as in fact is confirmed by the results in the table.

| \( \epsilon = r_c/r_s \) | \( r_s \) (in kpc) | \( r_s \) (in kpc) | \( r_s \) (in kpc) |
|-------------------|------------------|------------------|------------------|
| \( \Upsilon_\star \rightarrow 0 \) | \( \Upsilon_\star = 4 \) | \( \Upsilon_\star = 8 \) |
| 1                | 0.72             | 1.06             | 1.23             |
| 0.5              | 0.94             | 1.40             | 1.54             |
| 0.25             | 1.2              | 1.92             | 2.20             |
| 0.125            | 1.6              | 2.88             | 3.28             |
| 0.0625           | 2.4              | 4.48             | 4.96             |

4. DISCUSSION AND CONCLUSIONS

The arguments in this Letter show that the kinematics of multiple populations in dSphs provide a substantial challenge to the predictions of cold dark matter cosmogonies. In the case of one of the best-studied dSphs, Sculptor, there is no NFW dark halo that is compatible with the available photometric and kinematics data. The problem is very basic—the energetics of the metal-rich and metal-poor populations do not permit them to reside in the same NFW halo. Are there any loopholes?

4.1. Reliability of Separation of Sub-populations?

This Letter has used the separation proposed by Battaglia et al. (2008), who used a hard cut in metallicity to define the metal-poor ([Fe/H] < −1.7) and metal-rich populations ([Fe/H] > −1.5). This is open to the objection that the kinematics and metallicity are interlocked, and so the cut may be subject to unperceived biases. The sub-populations can also be separated using a maximum likelihood approach with metallicity and kinematics treated jointly. We have checked our calculations using such a method on the sample of stars in Tolstoy et al. (2004; also used by Battaglia et al. 2008). Walker & Penarrubia (2011) have also tackled this problem, using a Bayesian likelihood estimator based on positional, kinematical, and chemical data to separate the sub-populations. Their algorithm also makes assumptions that may or may not be warranted (for example, the velocity distributions are assumed isothermal). Nonetheless, although the details of the separation are different, Walker & Penarrubia (2011) come to the same conclusion that the multiple populations of Sculptor are inconsistent with an NFW halo. Different methods of separation, using different algorithms, arrive at the same answer, which offers comfort to those concerned about reliability.

4.2. Steady State, Well-mixed Populations?

An assumption in the virial theorem is that the stellar populations are in a steady state and well mixed. Recently, de Boer et al. (2012) have provided a star formation history for Sculptor. There was a peak in star formation ~13–14 Gyr ago, providing the bulk of the old, metal-poor population. Star formation continued until \( t_\circ \approx 7 \) Gyr ago. With the typical dark matter densities found from the virial stripes, the dynamical timescale \( (4\pi G\rho_0)^{-1/2} \) is at least one order of magnitude smaller than \( t_\circ \). It seems that the two populations have indeed reached a steady state and that each of them is internally well mixed. From the Hubble Space Telescope proper motions, the orbital period of Sculptor is estimated to be ~2.2 Gyr (Piatek et al. 2006). Its pericentric distance is ~68 kpc, and the time of the last pericentric passage was ~1 Gyr ago. As the dSph always remains in the outer parts of the Milky Way halo, it seems very improbable that tidal stirring can overturn the steady state hypothesis.

4.3. Flattening?

A drawback of all previous models of dSphs is that they are restricted to spherical symmetry. This is not the case for our virial arguments.

The isophotes of the Sculptor dSph have a measured ellipticity of 0.3 (Irwin & Hatzidimitriou 1995). In fact, the flattening of the dark matter potential must be rounder than the flattening of the luminosity density, so spherical models are probably a good approximation. As a check, we generalize our virial arguments to flattened stellar populations (with axis ratio \( q \)) in a flattened NFW dark matter halo (with axis ratio \( g \)). Let us assume that Sculptor is viewed edge-on, and the density and kinematics of both stellar populations are stratified on surfaces \( x^2 + y^2 = q^2 z^2 \). We also make the natural assumption that the minor axes of the dark matter and the luminous matter are aligned. Then, each kinetic energy term in Equation (7) scales by \( q/g \). Let \( q/g = 0.7 \) and \( \epsilon = 0 \). The virial arguments are particularly robust and emphasize the important point that the energetics of the two populations in Sculptor are completely incompatible with a 1/r dark matter density cusp. For at least this dwarf galaxy, we can...
securely state that it either never had a $1/r$ density cusp, or if it did have one, feedback processes have now removed it.

Of course, there has been much recent interest in converting cusped halos into cored halos. A number of promising possibilities have been suggested, including impulsive mass loss from supernovae (Read & Gilmore 2005), winds and gas flows driven by supernovae (Mashchenko et al. 2006), and infalling baryonic clumps (Cole et al. 2011). Which of these, if any, were responsible for the core in Sculptor is an important question for future work.

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