Abstract
In the framework of an algebraic approach, we consider a quantum teleportation procedure. It turns out that using the quantum measurement nonlocality hypothesis is unnecessary for describing this procedure. We study the question of what material objects are information carriers for quantum teleportation.

1 Introduction
The term "teleportation" in the title of this paper can provoke totally justified suspicion in a reader of a scientific journal. This term is so closely associated with science fiction and pseudoscientific literature. Nevertheless, this term is widely used in the scientific field often called "quantum information physics" (see, e.g., [1]). This field has developed intensively in recent years, and great hopes of very interesting practical applications are connected with it.

A specific method for transferring information is called "teleportation." This method has some mysterious features even in the purely scientific literature because the material information carrier is not clearly indicated. Instead, nonlocality, which is supposedly inherent in quantum measurements, is cited.

Here, we try to give a grounded, visual picture of teleportation, indicating what material carrier transfers information of one or another type. We use a special version of the algebraic approach to quantum theory. A detailed description of this approach can be found in [2], and a condensed one, in [3]. In those papers, we used an inductive method to construct the theory. First, we studied physical phenomena, then noted the physical patterns inherent in them, and finally described these patterns in the form of mathematical axioms. Here, we assume that the reader already has a knowledge of the phenomenological justification of the axioms and restrict ourself to formulating them. Also, we do not describe how the standard mathematical apparatus of quantum mechanics can be obtained using these axioms. We again refer the interested reader to [2, 3].

2 Main statements of the approach
The central notion used in this approach is an "observable." An observable is an attribute of a physical system whose numerical value can be obtained using a certain measuring procedure. All the observables are assumed to be dimensionless. All the measurements are divided into reproducible and nonreproducible ones and also into compatible and incompatible ones.
Compatible measurements are conducted using compatible measuring devices. If there exist compatible measuring devices for a group of observables, then such observables are said to be compatible (simultaneously measurable).

**Postulate 1.** Observables $\hat{A}$ of a physical system are Hermitian elements of some $C^*$-algebra $\mathfrak{A}$, $(\hat{A} \in \mathfrak{A}, \hat{A}^* = \hat{A})$.

We let $\mathfrak{A}_+$ ($\mathfrak{A}_+ \subset \mathfrak{A}$) denote the set of observables.

**Postulate 2.** The set of compatible observables is a maximal real associative commutative subalgebra $\mathfrak{Q}_\xi$ of the algebra $\mathfrak{A}$ ($\mathfrak{Q}_\xi \subset \mathfrak{A}_+$).

The index $\xi$ ranging a set $\Xi$ distinguishes one such subalgebra from another. There is only one such subalgebra for a classical system; there are infinitely many for a quantum system [2].

We regard the set $\mathfrak{A}_+$ as a mathematical representation of the physical system under study and the sets $\mathfrak{Q}_\xi$ as mathematical representations of the corresponding classical subsystems of the physical system. These classical subsystems are open (interacting between themselves) and do not have their own dynamics. The state of a classical system is its attribute that uniquely predetermines the results of measurements of all the observables of the system. Therefore, we formulate the following postulate.

**Postulate 3.** The state of a classical subsystem whose observables are elements of the subalgebra $\mathfrak{Q}_\xi$ is described by a character of this subalgebra.

We recall that a homomorphic map of the associative commutative algebra to the set of numbers is called the character $\varphi_\xi(\cdot)$ of this algebra: $\hat{A} \xrightarrow{\varphi_\xi} \varphi_\xi(\hat{A}), \hat{A} \in \mathfrak{Q}_\xi$ (see, e.g., [4]).

Because the observables belonging to the subalgebra $\mathfrak{Q}_\xi$ are compatible, there exists a set of measuring devices designed for compatible measurements of these observables. We say that these devices belong to the $\xi$-type.

The set $\mathfrak{A}_+$ of observables of a quantum system can be regarded as a collection of subsets $\mathfrak{Q}_\xi$. Therefore, the quantum system can be regarded as a set of corresponding open classical subsystems. Each observable of the quantum system belongs to a certain subset $\mathfrak{Q}_\xi$. Accordingly, if the states of all classical subsystems were known, then we could predict the result of measuring any observable of the quantum system. Based on this, we call the set $\varphi = [\varphi_\xi]$ ($\xi \in \Xi$) of functionals $\varphi_\xi(\cdot)$ each of which is the character of the corresponding subalgebra $\mathfrak{Q}_\xi$, the elementary state of a physical system. The following postulate is central in the described approach.

**Postulate 4.** The result of each individual measurement of the observables of a physical system is determined by the elementary state of this system.

This postulate does not say that the result is uniquely determined. The point is that the same observable $A$ can simultaneously belong to several subalgebras $\mathfrak{Q}_\xi$. Therefore, we can use devices of different types to measure it. We are accustomed to the fact that if the device is "good," then the measurement result is independent of the device. But this holds only in the case where all the devices can be calibrated in the same way. Such a calibration is, in
principle, possible in the classical case. But such a calibration, as shown in [2], cannot be
implemented for quantum systems because of the presence of incompatible measurements.
Therefore, measurement results in the quantum case depend on two factors: the elementary
state and the type of device used for measuring. In the general case, we say that measuring
devices belong to the ξ-type if $\hat{A}$ is obtained as a result of measurements of the observable
$\varphi_\xi(\hat{A})$.

In some cases, measurement results can be independent of the type of measuring device,
i.e., $\varphi_\xi(\hat{A}) = \varphi_\xi'(\hat{A})$ for all $\Omega_\xi, \Omega_\xi'$, containing $\hat{A}$. In this case, we say that the elementary
state $\varphi$ is stable for the observable $\hat{A}$.

We note that in the majority of proofs demonstrating that the existence of local physical
reality determining measurement results is impossible, it is tacitly assumed that this result
is independent of the type of measuring device (see, e.g., [5, 6]).

There is one more hindrance to the unique prediction of measurement results in the
quantum case. The point is that it is impossible to determine the elementary state of a
system uniquely in experiments. Only compatible measuring devices can be used to fix it.
Using such devices, we can determine the functional $\varphi_\xi(\cdot)$ only for one value of $\xi$ ($\xi = \eta$).
All the other functionals $\varphi_\xi(\cdot)$ contained in the elementary state $[\varphi_\xi]$, remain undetermined.
Figuratively speaking, we can say that the elementary state is a holographic image of a
physical system. Using classical measuring devices, we can find only a plane image. In this
case, each measurement changes an original holographic pattern. Therefore, it is impossible
to obtain a complete holographic image.

We unite all the elementary states $[\varphi_\xi]$ having the same restriction to the subalgebra
$\Omega_\eta$, i.e., the same functional $\varphi_\eta$, into the equivalence class $\{\varphi\}_\eta$. Thus, it is possible in
experiments to uniquely fix only the equivalence class to which the elementary state of
interest belongs. If we know that some elementary state $\varphi = [\varphi_\xi]$ belongs to the equivalence
class $\{\varphi\}_\eta$, then we can uniquely predict what result can be obtained in the measurement
of the observable $\hat{A} \in \Omega_\eta$ this result is $\varphi_\eta(\hat{A})$. But if $\hat{A} \notin \Omega_\eta$, then it is impossible to say
anything definite about the measurement result. For different elementary states belonging
to $\{\varphi\}_\eta$, the measurement results are different. The quantum state fixed by certain values of
the observable $\hat{A}$ from the subalgebra has such physical properties in the standard quantum
mechanics $\Omega_\eta$.

For the observables $\hat{A} \notin \Omega_\eta$ only the probabilities of the determination of different results
can be predicted. It is postulated in the standard mathematical apparatus of quantum
mechanics that these probabilities are determined by either a vector of some Hilbert space
or a statistical operator (the density matrix). This postulate holds in practice, but it is
impossible to understand its justification at the intuitive level. Using the approach given
here, we can justify this postulate. It is natural to assume that the probability distribution
of measurement results is determined by the probability distribution of different elementary
states in the equivalence class $\{\varphi\}_\eta$. Therefore, we formulate the following postulate.

**Postulate 5.** The equivalence class $\{\varphi\}_\eta$ can be endowed with the structure of a
probability space.

We note that an elementary state satisfies the requirements that the classical (Kol-
mogorov [7]) probability theory imposes on an elementary event: one and only one ele-
mentary event occurs in each trial, i.e., elementary events exclude each other. This allows
endowing $\{\varphi\}_\eta$ with the standard (classical) structure of a probability space $(\Omega, \mathcal{F}, P)$ (see, e.g., [7, 8]). Here, $\Omega$ is the set of elementary events (states), $\mathcal{F}$ is the Boolean $\sigma$-algebra of events $F$, $P$ is the probability that an event $F \in \mathcal{F}$ occurs. The event $F$ is a subset of the set $\Omega$. It is assumed that the event $F$, occurs in a trial if one of the elementary events belonging to this subset occurs.

Thus, constructing a special artificial quantum probability theory for quantum systems is unnecessary; indeed, the standard probability theory can be used. It is only necessary to take into account that not all possible Boolean algebras $\mathcal{F}$ are admissible in the quantum case [2, 3]. If this feature of quantum systems is not taken into account, then it is easy to obtain the Bell inequalities [9, 10, 11], which contradict experimental data.

If $\{\varphi\}_\eta$ is endowed with the structure of a probability space, then we can use standard probability theory methods to easily construct [2] the functional $\Psi_\eta(\hat{A})$, that specifies the mean of an observable $\hat{A}$ in the equivalence class $\{\varphi\}_\eta$. To identify this functional with the quantum mean of the observable $\hat{A}$, this functional must be linear. Therefore, we must formulate the following postulate.

**Postulate 6.** The probability structure of the equivalence class $\{\varphi\}_\eta$ is such that the functional $\Psi_\eta(\hat{A})$ is linear on the algebra $\mathfrak{A}$.

In this case, we can assume that the functional $\Psi_\eta(\hat{A})$ describes the corresponding quantum state. In other words, we can formulate the following definition in this approach.

**Definition.** We call the class $\{\varphi\}_\eta$ $\varphi_\eta$-equivalent elementary states that are stable on the subalgebra $\Omega_\eta$ the quantum state $\Psi_\eta(\hat{A})$. This equivalence class must admit the described structure of a probability space.

The quantum state thus defined is pure [2].

We have previously noted that the result of a particular measurement of an observable can depend on the type of measuring device used to measure it. At the same time, experiments show that the following assertion holds.

**Postulate 7.** The mean of an observable in a fixed quantum state is independent of the type of measuring device used to determine it.

Having the $C^*$-algebra $\mathfrak{A}$ and the linear functional $\Psi(\cdot)$ on this algebra and using the canonical Gelfand-Naimark-Segal construction, we can realize the representation of this algebra (see, e.g., [12]) by bounded linear operators in a Hilbert space $\mathfrak{H}$:

$$\hat{A} \leftrightarrow \Pi(\hat{A}), \quad \hat{A} \in \mathfrak{A}, \quad \Pi(\hat{A}) \in \mathfrak{B}(\mathfrak{H}),$$

where $\mathfrak{B}(\mathfrak{H})$ is the set of bounded linear operators in $\mathfrak{H}$. In this case, the mean $\langle \hat{A} \rangle$ of the observable $\hat{A}$ in the quantum state $\Psi$ can be expressed as the mathematical expectation of the operator $\Pi(\hat{A})$:

$$\langle \hat{A} \rangle = \langle \Psi | \Pi(\hat{A}) | \Psi \rangle,$$

where $|\Psi\rangle \in \mathfrak{H}$ is the corresponding vector of the Hilbert space.
We call the set of physical systems whose elementary states form the equivalence class \( \{ \varphi \}_\eta \) the quantum ensemble. Thus, the standard mathematical apparatus of quantum mechanics can be used to describe quantum ensembles but not an individual quantum system. An elementary state is the adequate mathematical characteristic of an individual quantum system.

The same elementary state can be considered as an element of different sets. The probability characteristics that are associated with a particular physical system significantly depend on these sets. Therefore, in the proposed approach, we must reformulate the assertion "the physical system under study is in a given quantum state" (this assertion is usual in the standard approach) as "the physical system under consideration is in an elementary state that belongs to a given quantum state." We can assume that the same elementary state can simultaneously belong to another quantum state. Thus, a quantum state is not quite an objective characteristic of an individual physical system.

3 Entangled states

So-called entangled states play the central role in the quantum teleportation procedure. In the literature, entangled states typical of a two-particle system in which each of the particles can be in two orthogonal quantum states \(|\pm\rangle\) are most often considered:

\[
|\Psi^{(-)}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2),
\]

\[
|\Psi^{(+)}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |-\rangle_2 + |-\rangle_1 |+\rangle_2),
\]

\[
|\Phi^{(-)}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |+\rangle_2 - |-\rangle_1 |-\rangle_2),
\]

\[
|\Phi^{(+)}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |+\rangle_2 + |-\rangle_1 |-\rangle_2).
\]

These states are often called the Bell states. Their characteristic feature is as follows. Any of these states can be selected using measuring devices, but after such a selection, it is impossible to uniquely predict in which of the two possible quantum states \(|+\rangle\) or \(|-\rangle\) we find each of the particles in the subsequent measurement. At the same time, after the quantum state of one of the particles is measured, the state of the other particle is uniquely predicted.

The state \( |\Psi^{(-)}\rangle_{12} \) is usually considered in the discussion of the Einstein-Podolsky-Rosen paradox \cite{13} and is therefore often called the EPR state. Thus, the system consisting of two particles with spin 1/2 was considered in the version proposed by Bohm \cite{14}. Then \(|+\rangle\) is the quantum state with the spin projection on the \( z \) axis equal to \(+1/2\), and \(|-\rangle\) is the state with the projection equal to \(-1/2\).

In the state \(|\Psi^{(-)}\rangle_{12}\) the total spin \( S = S_1 + S_2 \) is zero. The state \(|\Psi^{(+)}\rangle_{12}\) is the state with total spin 1 and the projection of the total spin on the \( z \) axis equal to 0, \( \frac{1}{\sqrt{2}} (|\Phi^{(+)}\rangle_{12} + |\Phi^{(-)}\rangle_{12}) \) is the state with total spin 1 and the projection +1, and the state \( \frac{1}{\sqrt{2}} (|\Phi^{(+)}\rangle_{12} - |\Phi^{(-)}\rangle_{12}) \) is the state with total spin 1 and the projection −1. It is also convenient to use a similar terminology for other two-level systems, assuming that the states \(|\Psi^{(\pm)}\rangle_{12}\), \(|\Phi^{(\pm)}\rangle_{12}\) are characterized by the corresponding values of observables, which we call quasispins.
The characteristic feature of the state $|\Psi(-\rangle_{12}$ is its spherical symmetry. Therefore, it preserves its form if the projections on the $x$ axis or, generally, on an arbitrary direction $n$ are considered instead of the projections on the $z$ axis. In this state, for any $n$, the relation $S_{n1} + S_{n2} = 0$

holds, where $S_{n1}(S_{n2})$ is the (quasi)spin projection of the $i$-th particle on the direction $n$. For such a system, the EPR paradox consists in the following. According to the standard ideology of quantum mechanics, none of the particles in the state $|\Psi(-\rangle_{12}$ has a definite value of the (quasi)spin projection on the direction $n$. That the first particle takes a certain value of the projection as a result of the action of the measuring device seems quite probable. But it is very difficult to understand how the measuring device could affect the second particle if this particle and this device are located in regions separated by a spacelike interval.

In the framework of the scheme described in the preceding section, the EPR paradox is trivially solved. The quantum state $|\Psi(-\rangle_{12}$ describes not one two-particle system but an ensemble of such systems. Let the index $\alpha$ distinguish one element of the ensemble from another. Each of the particles in the $\alpha$-th system is in a quite definite elementary state $[\varphi^{\alpha}_{11}]$ for the first particle and in $[\varphi^{\alpha}_{22}]$ the second. In the case under consideration, it is convenient to take the three-dimensional unit vector $n$ as the index $\xi$ fixing the commutative subalgebra. In this case, we must assume that the same subalgebra corresponds to the vectors $n$ and $-n$. Then the characters $\varphi^{\alpha}_{11}$ and $\varphi^{\alpha}_{22}$ the elementary states describe the (quasi)spin projections on the direction $n$ for the first and the second particle. They take the values $[2])$: $\varphi^{\alpha}_{ni} = S^{\alpha}_{ni}$ ($i = 1, 2$), where $S^{\alpha}_{ni} = -S^{\alpha}_{-ni}$, and $S^{\alpha}_{n1} = -S^{\alpha}_{n2}$, and $S^{\alpha}_{n1}$ is equal to either $+1/2$ or $-1/2$ depending on $\alpha$ and $n$. Under such conditions, the elementary state of one particle of the EPR pair is the negative copy of the elementary state of the other particle. Therefore, a measurement of the value of an observable of the first particle is automatically a measurement of the corresponding observable of the second particle irrespective of the location of this particle. Such measurement is said to be indirect. In the case of such a measurement, the measured object is not subjected to the action of the measuring device. Therefore, its elementary state remains unchanged. The strict correlation between the elementary states of the particles in the EPR pair is its characteristic feature. This correlation develops in the prearrangement of the EPR pair rather than in the subsequent measurement. This correlation is not a consequence of the projection postulate; just the opposite, the statements in the projection postulate are a consequence of such a correlation. Similar correlations are also inherent in other entangled states.

4 Some devices of quantum physics

To understand the quantum teleportation procedure, it is necessary to understand the operation principles of the devices used. We give a very brief description of three such devices in this section.

In the preceding section, the EPR paradox was studied using an example of a system consisting of two particles with spins 1/2. But it is technically very difficult to create the required EPR pair for such a system. It is considerably simpler to obtain such a pair optically. The corresponding device is called a type-II parametric frequency down-converter. In the Russian literature, this device is often called a source of spontaneous parametric scattering (SPR source). A nonlinear optical crystal irradiated by a UV laser is the basis of the SPR
source. Laser photons are scattered in the crystal. For such scattering, one photon at the input as a rule yields one photon at the output. But one photon sometimes generates two photons with much lower (many orders lower) intensities. For SPRs of the second type, these photons turn out to be polarized in two mutually orthogonal directions $H$ (horizontal) and $V$ (vertical). We can choose conditions such that these photons form an EPR pair, i.e., their quantum state can be described by the vector $\ket{\Psi^(-)_{12}}$ (see formula (1)) where $\ket{+}$ and $\ket{-}$ correspond to the respective horizontal and vertical polarizations.

The second device is a (simple) beam splitter. It is designed to mix two photon beams without changing their intrinsic characteristics. We can visualize this device as a semitransparent plate (see Fig. 1) with two photon beams are incident at equal angles from above and below in one plane. If photons from different beams are incident on the plate at different instants, then each of them passes through the plate independently with probability 1/2 or reflects from it with probability 1/2. But if two photons are simultaneously incident, then they interfere according to the formula

\begin{align*}
|H, V\rangle_{in}^u & \rightarrow \frac{1}{\sqrt{2}} \left[ |H, V\rangle_{out}^u + |H, V\rangle_{out}^d \right], \\
|H, V\rangle_{in}^d & \rightarrow \frac{1}{\sqrt{2}} \left[ |H, V\rangle_{out}^u - |H, V\rangle_{out}^d \right],
\end{align*}

where the subscripts $u$ and $d$ denote the upper and lower beams and $|H, V\rangle$ means either $|H\rangle$ or $|V\rangle$. For definiteness, Fig. 1 shows the case where the upper incident beam is horizontally polarized (it is described by the vector $|H\rangle_{in}^u$) and the lower beam is vertically polarized $|V\rangle_{in}^d$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{beam_splitter.png}
\caption{Simple beam splitter.}
\end{figure}

It follows from formulas (1) that as a result of passing through the beam splitter, the Bell states (see formulas (2)) transform as

\begin{align*}
|\Psi^(-)\rangle_{in}^{ud} & \rightarrow \frac{1}{\sqrt{2}} \left[ |H\rangle_{out}^u |V\rangle_{out}^d - |V\rangle_{out}^u |H\rangle_{out}^d \right] \equiv -|\Psi^(-)\rangle_{out}^{ud}, \\
|\Psi^+\rangle_{in}^{ud} & \rightarrow \frac{1}{\sqrt{2}} \left[ |H\rangle_{out}^u |V\rangle_{out}^d - |H\rangle_{out}^d |V\rangle_{out}^d \right],
\end{align*}
|Φ(−)⟩_{ud}^{in} \rightarrow \frac{1}{\sqrt{2}} \left[ |H⟩_{u}^{out} |H⟩_{d}^{out} - |V⟩_{u}^{out} |V⟩_{d}^{out} - |H⟩_{d}^{out} |H⟩_{u}^{out} + |V⟩_{d}^{out} |V⟩_{u}^{out} \right],
|Φ(+)⟩_{ud}^{in} \rightarrow \frac{1}{\sqrt{2}} \left[ |H⟩_{u}^{out} |H⟩_{d}^{out} + |V⟩_{u}^{out} |V⟩_{d}^{out} - |H⟩_{d}^{out} |H⟩_{u}^{out} - |V⟩_{d}^{out} |V⟩_{u}^{out} \right].

In other words, after the beam splitter, the photons in each simultaneous pair turn out to be on different sides of the plate if they are incident in the state |Ψ(−)⟩_{ud}^{in} (quasispin 0) and on the same side for the states |Ψ(+)⟩_{ud}^{in}, |Φ(−)⟩_{ud}^{in}, and |Φ(+)⟩_{ud}^{in} (quasispin 1).

The third device is a polarization beam splitter (PBS). The geometry of this device determines the so-called (orthogonal) polarization basis. A photon beam incident on the PBS is directed along the first basis vector. The other two basis vectors determine the vertical and horizontal directions. The beam passes through the PBS if it is polarized in the horizontal direction and is reflected if it is polarized in the vertical direction. The PBS is often used in combination with detectors located in the propagation directions of the secondary beams. Using such a device, we can determine photon polarization.

If the PBS is rotated through an angle about the first basis vector, then we obtain a new polarization basis. After a beam with certain polarization in the original basis passes through the PBS, it decomposes into two subbeams each of which has a certain (horizontal or vertical) polarization in the new basis. If the detectors used in experiments are one-photon detectors (sensitive to separate photons), then we can determine which polarization a separate photon has in the new basis. According to the standard ideology of quantum mechanics, a separate photon of the beam polarized in the direction of one of the vectors of the original basis has no definite polarization with respect to the vectors of the rotated basis. Passing through the PBS, it acquires such a polarization. In this case, such a selection process has no objective reason. It is purely random in this sense.

According to the ideology in Sec. 2, each separate photon has a particular (vertical or horizontal) polarization in any polarization basis. All these polarizations depend on the elementary state of this photon. But we initially know only the polarization in the original basis. A rotated PBS classifies photons of the original beam according to their polarizations in its polarization basis. At the same time, the PBS perturbs the previously known polarization in the original basis. Thus, using PBSs and one-photon detectors, we can determine the polarization of each photon in any direction but only one at a time.

## 5 Quantum teleportation

Figure 2 shows a scheme of quantum teleportation.

Here, S is the source of the initial state, EPR is the source of EPR pairs, A is the analyzer of the Bell states (Alice), B is the unitary converter (Bob), {C} is the classical communication channel, {1} is the carrier of the initial teleported state, {2} – {3} is the EPR pair, and {4} is the carrier of the final teleported state.

We give the standard description of the teleportation scheme (see, e.g., [15]). Each of the particles {1},{2},{3}, and {4} participating in the process can be in one of the quantum levels |+⟩ or |−⟩. The source S emits particle {1} in the quantum state |Ψ⟩_{1} = α|+⟩ + β|−⟩, where |α|^2 + |β|^2 = 1. In the general case, α and β can be unknown. The EPR source emits particles {2} and {3} in the quantum state |Ψ⟩_{23} (see formula (1)). The quantum state of the three-particle system consisting of particles {1},{2}, and {3} is described by the vector |Ψ⟩_{123} = |Ψ⟩_{1} ⊗ |Ψ⟩_{23} which can be decomposed in terms of the Bell states of particles.
Along which the quasispin projection of each particle \( z/2 \). Let the state \( S \) be the state \(|\alpha\rangle_{3} - |\beta\rangle_{3} \) in the experiments. The numbers \( \alpha \) and \( S \) specify a direction \( n \), along which the quasispin projection of each particle \( \{1\} \) of the beam is definitely equal to \( 1/2 \). Let the \( z \) axis be along the direction \( n \). Then, for the quasispin projection, the equality \( S_{z} = +1/2 \) holds in the quantum state \(-\alpha|+\rangle_{3} - \beta|\rangle_{3} \), the equality \( S_{z} = -1/2 \) holds in the state \(-\alpha|+\rangle_{3} + \beta|\rangle_{3} \), the equality \( S_{x} = +1/2 \) holds in the state \( \alpha|\rangle_{3} + \beta|+\rangle_{3} \), and the equality \( S_{x} = -1/2 \) holds in the state \( |\alpha\rangle_{3} - \beta|+\rangle_{3} \).

We now regard the analyzer \( A \) in combination with particle \( \{1\} \) as a measuring device. The action of this combined measuring device on the beam of particles \( \{2\} \) can be interpreted.
two ways (see formula (4)). On one hand, this device divides the beam of particles \{2\} into four subbeams, in each of which particles \{2\} (in combination with particles \{1\}) are in one of the Bell states. This result is fixed by the analyzer \textit{A}. On the other hand, the particles \{2\} in each of these four subbeams have definite values of the quasispin projections either on the \textit{z} axis or on the \textit{x} axis. Because of the strict correlation between the elementary states of particles \{2\} and \{3\}, the beam of particles \{3\} automatically divides into four subbeams in each of which particles \{3\} have certain quasispin projections. That is, we have a typical example of an indirect measurement of the quasispin projection for particle \{3\} in this case. Using the classical communication channel, Alice reports the result of such an indirect measurement to Bob. Bob applies the corresponding unitary transformation to particles \{3\}. As a result of this measurement, only some information about this elementary state needed for Bob’s subsequent actions is obtained.

We call attention to the fact that the elementary state of particle \{3\} does not become the same as that of particle \{1\} after all the described manipulations. These particles only turn out to be in the same equivalence class. Thus, particle \{3\} does not become an exact copy of particle \{1\}; therefore, the term "teleportation" used to describe this procedure is not especially appropriate.

We now discuss an actual experiment in which teleportation was observed [16]. The schematic diagram of the experimental setup is shown in Fig. 3.

Figure 3: Schematic diagram of the quantum teleportation experiment.

Here, \textit{UF} is the laser (the source of ultraviolet pulses), \textit{EPR} is the EPR source; \textit{M}, \textit{M}_1, and \textit{M}_2 are totally reflecting mirrors, \textit{BS} is the simple beam splitter, \textit{PBS} is the polarization beam splitter, \textit{S} is the coder of the initial state of photon \{1\}, \textit{D}_0, \textit{D}_1, \textit{D}_2, \textit{D}_{+3}, and \textit{D}_{−3}
are the detectors, and \( \{C\} \) is the classical communication channel (the coincidence circuit for detector combinations).

For simplicity of argument, we assume that all photons under consideration propagate in the same plane and that the horizontal direction of the original polarization basis is in this plane.

A laser UV pulse is incident on a nonlinear crystal (EPR). The EPR pair \( \{0\}-\{1\} \) is produced in this crystal. After the pulse is transmitted through the crystal, it is reflected from the mirror \( M \) and again enters the crystal, where it produces the second EPR pair \( \{2\}-\{3\} \). Photon \( \{1\} \) reflecting from the mirror \( M_1 \), enters the coder \( S \), where it acquires a particular polarization. Two versions were studied in the actual experiment: polarization at a 45\(^0\) angle and at a 90\(^0\) angle. After this, photon \( \{1\} \) enters the beam splitter \( BS \).

Photon \( \{2\} \) reflecting from the mirror \( M_2 \) enters the same beam splitter \( BS \) but on the other side. Moving the mirror \( M \), we can ensure that photons \( \{1\} \) and \( \{2\} \) are inserted in the beam splitter \( BS \) simultaneously. Then they interfere in the beam splitter; after passing through it, either they both enter one of the detectors \( D_1 \) or \( D_2 \) or one of them enters the detector \( D_1 \) and the other enters \( D_2 \).

Photon \( \{3\} \) is directed to the polarization beam splitter and then enters either the detector \( D_{+3} \) or \( D_{-3} \) depending on the polarization. Photon \( \{0\} \) is immediately directed to the detector \( D_0 \). The coincidence circuit \( \{C\} \) registers events in which either the detectors \( D_0, D_1, D_2, \) and \( D_{+3} \) or the detectors \( D_0, D_1, D_2, \) and \( D_{-3} \) operate simultaneously.

It was taken into account in the experiment that the creation of an EPR pair is a very rare event compared with the creation of single photons, and the creation of two EPR pairs is a much rarer event. Therefore, it is very important to separate random coincidences in the detector operation from significant coincidences. In this regard, the detector \( D_0 \) is required to separate the events containing photon \( \{1\} \), whose state must be teleported. The experimenters interpreted coincidence in the operation of the detectors \( D_1 \) and \( D_2 \) as evidence that photons \( \{1\} \) and \( \{2\} \) turn out to be in the quantum state \( \Psi^{(-)}_{12} \) after such a measurement. Accordingly, photon \( \{3\} \) acquires the same quantum state as photon \( \{1\} \) after passing through the coder \( S \). To verify this, the polarization beam splitter \( PBS \) was oriented such that if photon \( \{3\} \) had the same polarization as encoded in photon \( \{1\} \), then photon \( \{3\} \) must enter the detector \( D_{+3} \). Therefore, the situation in which the coincidence \( D_0 - D_1 - D_2 - D_{+3} \) is observed and there is no coincidence \( D_0 - D_1 - D_2 - D_{-3} \) must correspond to the case of teleportation.

Actually, the experimenters studied the rate of coincidences of such type depending on the position of the mirror \( M \), i.e., depending on how photons \( \{1\} \) and \( \{2\} \) simultaneously enter the beam splitter \( BS \); in other words, whether they interfere in the beam splitter or are independently scattered. For total interference, the number of coincidences \( D_0 - D_1 - D_2 - D_{-3} \) must decrease to zero. If there is no interference, both types of coincidences turn out to be random, and their rates become the same. Indeed, a considerable dip (of approximately one order) corresponding to the interference of photons \( \{1\} \) and \( \{2\} \) was observed in the graph of the dependence of the rate coincidences \( D_0 - D_1 - D_2 - D_{-3} \) on the position of the mirror \( M \). This was confirmed in subsequent similar experiments [17].

But a new result not easily interpreted was also obtained. The point is that by changing the lengths of the paths of photons \( \{1\} \) and \( \{2\} \), we can ensure that the described effect is attained in the case where the time delay for the detectors \( D_1 \) and \( D_2 \) is inserted in the coincidence circuit. In this case, the teleportation registered by the detectors \( D_{+3} \) and \( D_{-3} \) is obtained earlier than that registered by the detectors \( D_1 \) and \( D_2 \). Such a result was indeed
observed. Moreover, we can in principle ensure that the teleportation is fixed before photon \( \{1\} \) acquires a particular polarization. Strictly speaking, all this does not formally contradict the standard mathematical apparatus of quantum mechanics, but it is hard to believe that the future can affect the past.

We now consider how the same experiment can be interpreted in the framework of the scheme described in Sec. 2. We first consider the case where photon \( \{1\} \) passing through the coder is vertically polarized in the original basis. Let the polarization beam splitter \( PBS \) be rotated through a 90° angle with respect to the original basis. Then photon \( \{3\} \) enters the detector \( D_{+3} \) if it is polarized vertically in the original basis but enters the detector \( D_{-3} \) if it is horizontally polarized.

We consider the case where the detector \( D_{-3} \) operates. Because photons \( \{2\} \) and \( \{3\} \) form an EPR pair, photon \( \{2\} \) is vertically polarized in this case. Hence, both photons \( \{1\} \) and \( \{2\} \) entering the beam splitter \( BS \) have the same polarization in this case. Therefore, if they turn out to be in the beam splitter \( BS \) simultaneously, then they both enter the same detector (either \( D_1 \) or \( D_2 \)) after the beam splitter (see formula (3)). That is, the probability of the coincidence \( D_0 - D_1 - D_2 - D_{-3} \) is theoretically zero. In fact, this probability is nonzero at least because the EPR source does not produce ideal pairs.

In contrast to the standard quantum mechanics, a strict time correlation between the events is predicted in the given scheme. Within a small error related to the duration of the laser pulse and to the fact that photons \( \{2\} \) and \( \{3\} \) can be emitted at different instants, the positive effect can be reached if the coincidence circuit is adjusted as follows. The instants \( t_0 \) (the time when photon \( \{0\} \) attains the detector \( D_0 \)), \( t_{12} \) (the time when photons \( \{1\} \) and \( \{2\} \) attain the detectors \( D_1 \) and \( D_2 \)) and \( t_3 \) (the time when photon \( \{3\} \) attain the detector \( D_{-3} \) or \( D_{+3} \)) must be assumed to be simultaneous for it. The times \( t_0, t_{12}, \) and \( t_3 \) can be in any ratio astronomically.

We now consider the case where photon \( \{1\} \) is polarized at a 45° angle in the original polarization basis. Let the detector \( D_{-3} \) register a photon polarized at a −45° angle. Then photon \( \{2\} \) is polarized at a 45° angle. The subsequent discussion is easier in terms of quasispin than in terms of polarization. We associate the quasispin orientation along the \( x \) axis with the polarization at 45°, i.e., \( |45^0\rangle \to |S_x = 1/2\rangle \). Hence, in the case under consideration, the elementary state of two-particle system \( \{12\} \) consisting of photons \( \{1\} \) and \( \{2\} \) belongs to the quantum state \( |S_x = 1/2\rangle_1|S_x = 1/2\rangle_2 \). According to the standard rules of quantum mechanics, the total quasispin is equal to 1 in this quantum state. Therefore, according to formulas (3) both photons passing through the beam splitter \( BS \) must enter the same detector (\( D_1 \) or \( D_2 \)). As a result, the coincidence circuit does not register this case, i.e., the result is the same as for the photon polarization at a 90° angle.

But the situation can turn out to be more complicated in actuality. The point is that two-particle system \( \{12\} \) was not prearranged as a quantum system with the value of the squared total quasispin \( S^2 \) equal to 2. The elementary states of two-particle system \( \{12\} \) were not selected using the value of the observable \( \hat{S}^2 \). Therefore, there is no guarantee what such a value will necessarily be in each elementary event. At best, if we can believe Postulate 7, we can only state that the mean of the observable \( \hat{S}^2 \) in an infinite number of trials is equal to 2.

In the actual experiment, the number of cases where the presence of coincidences \( D_0 - D_1 - D_2 - D_{+3} \) and the absence of coincidences \( D_0 - D_1 - D_2 - D_{-3} \) are registered is very small. Therefore, in each trial, the probability of the deviation of the value of the observable \( \hat{S}^2 \) from the mean is sufficiently large. This probability must decrease to improve
the statistics. It seems to follow from the preceding that it is useless to improve the statistics in an attempt to discover the deviation from the predictions of standard quantum mechanics. But this is not so. We must only treat these deviations as the effect under study rather than as measurement errors. For example, we can study the quantity

$$\rho = \frac{N_- (45)}{N_- (90)} \quad N_- (45) + N_+ (45) = N_- (90) + N_+ (90),$$

where $N_- (45)$ is the number of coincidences $D_0-D_1-D_2-D_{-3}$ in the absence of coincidences $D_0-D_1-D_2-D_{+3}$, $N_+ (45)$ is the number of coincidences $D_0-D_1-D_2-D_{-3}$ in the absence of coincidences $D_0-D_1-D_2-D_{+3}$ for the polarization of photon $\{1\}$ at a $45^0$ angle, and $N_- (90)$ and $N_+ (90)$ are the same numbers for the polarization at a $90^0$ angle. If the standard quantum mechanics is used in the case under consideration, then it must be expected that the value of $\rho$ must approach unity to improve the statistics. But if we use the approach proposed here, then it must be expected that the value of $\rho$ is always greater than unity. We note that a similar result was already discovered experimentally [17]). But the experimenters were prone to think that this result was an experimental error. In such a case, a special experiment should be conducted to study this effect more thoroughly.

6 Teleportation of entanglement

The phenomenon that obtained the name "teleportation of entanglement" is often considered the strongest argument in favor of nonlocality inherent in quantum measurements. This phenomena can be understood from Fig. 4, where $EPR \, 1$ and $EPR \, 2$ are the sources of independent EPR pairs, $A$ is the analyzer of the Bell states (Alice), $B$ is the unitary converter (Bob), $\{C\}$ is the classical communication channel (the coincidence circuit), $\{0\}-\{1\}$ is the first EPR pair, and $\{2\}-\{3\}$ is the second EPR pair.

![Figure 4: Scheme for teleportation of entanglement.](image)

As before, we first give the standard description of the scheme for teleportation of entanglement [18]. The source $EPR \, 1$ emits the pair $\{0\}-\{1\}$ in the quantum state $|\Psi^{(-)}_{01}\rangle$. The source $EPR \, 2$ independently emits the pair $\{2\}-\{3\}$ in the quantum state $|\Psi^{(-)}_{23}\rangle$. The
vector of the four-particle quantum state $|\Psi\rangle_{0123} = |\Psi(-)\rangle_{01} \otimes |\Psi(-)\rangle_{23}$ can be decomposed in terms of the Bell states of particles \{1\}-%2-\{2\}:

$$|\Psi\rangle_{0123} = \frac{1}{2} \left\{ |\Psi(+)\rangle_{03} |\Psi(+)\rangle_{12} - |\Psi(-)\rangle_{03} |\Psi(-)\rangle_{12} - |\Phi(+)^{\dagger}\rangle_{03} |\Phi(+)^{\dagger}\rangle_{12} + |\Phi(-)^{\dagger}\rangle_{03} |\Phi(-)^{\dagger}\rangle_{12} \right\}. $$

This formula is simultaneously a decomposition of the vector $|\Psi\rangle_{0123}$ in terms of the Bell states of particles \{0\}-%3-\{3\}. Using the analyzer $A$, Alice determines the Bell state from the four possibilities for the pair \{1\}-%2-\{2\} and reports the result to Bob using the classical communication channel. For example, let $|\Psi(-)\rangle_{12}$ be such a state. According to the projection postulate, the state $|\Psi\rangle_{0123}$ then collapses to the state $|\Psi(-)\rangle_{03} |\Psi(-)\rangle_{12}$. Therefore, Bob can now state that the pair \{0\}-%3-\{3\} is in the quantum entangled state $|\Psi(-)\rangle_{03}$.

Particles \{0\} and \{3\} were initially completely independent. Further, they were not subjected to any physical action. All manipulations were performed with particles \{1\} and \{2\}. Nevertheless, after these manipulations, particles \{0\} and \{3\} somehow mysteriously turned out to be in the entangled state. We can give a much more obvious description of the correlation between particles \{0\} and \{3\} in the framework of the approach assumed here.

Using the analyzer $A$, Alice divides the beam of particles \{1\} and \{2\} into four subbeams by using a criterion for the presence of a particular correlation between the particles in each subbeam. These correlations are not produced by the analyzer $A$; the latter only serves to select pairs in which such correlations appear randomly when these particles were emitted by the sources EPR 1 and EPR 2.

The elementary state of particle \{0\} is strictly correlated with the elementary state of particle \{1\}, and there is a similar case for particles \{2\} and \{3\}. Therefore, dividing the beam of particles \{1\} and \{2\} into subbeams automatically divides the beam of particles \{0\} and \{3\} into the corresponding subbeams. In each pair \{0\}-\{3\}, there is a correlation that is the (negative) copy of the correlation in the pair \{1\}-\{2\}. Using the classical communication channel \{C\}, we can establish which subbeam the particular pair \{0\}-\{3\} belongs to. Here, we must keep in mind that the correlation in the pair \{0\}-\{3\} is the copy of the correlation in the pair \{1\}-\{2\} that existed before the particles passed through the analyzer $A$. At the same time, the analyzer $A$ can perform the function of not only a measuring device but also a device rearranging a new elementary state. Therefore, the correlation in the pair \{0\}-\{3\} is generally not an exact copy of the correlation in the pair \{1\}-\{2\} after the pair passes through the analyzer $A$.

Further, we discuss actual experiments in which the phenomenon of teleportation of entanglement was studied \[18, 17\]. The schematic diagram of the experimental setup is shown in Fig. 5. For the most part, it is similar to the experimental setup shown in Fig. 3. The difference is that there is no coder of the initial state in Fig. 5; on the other hand, there are two beam splitters $PBS_0$ and $PBS_3$ instead of one polarization beam splitter $PBS$ and two detectors $D_{-0}$ and $D_{+0}$ instead of one detector $D_0$.

The detector $D_{+0}$ operates when photon \{0\} is horizontally polarized in the polarization basis of the beam splitter $PBS_0$. The detector $D_{-0}$ operates when photon \{0\} is vertically polarized. The detectors $D_{+3}$ and $D_{-3}$ operate similarly, but their response is connected with the polarization basis of the beam splitter $PBS_3$.

The experimenters studied the versions of the coincidences $D_1 - D_2 - D_{+0} - D_{-3}$ for different orientations of the polarization beam splitters with respect to the original polarization basis. They assumed that in the case where the detectors $D_1$ and $D_2$ operate simultaneously, photons \{1\} and \{2\} turn out to be in the entangled two-particle state $|\Psi(-)\rangle_{12}$. Therefore,
the quantum two-particle state of photons \{0\} and \{3\} is \(\Psi^{(-)}_{03}\) according to the projection postulate. Hence, the entangled state of photons \{1\} and \{2\} produced using the beam splitter \(BS\) and detectors \(D_1\) and \(D_2\) is transferred to photons \{0\} and \{3\} without any physical action. These photons were completely independent before the manipulations with photons \{1\} and \{2\}.

The following facts are considered to prove that photons \{0\} and \{3\} really turn out to be in the quantum state \(\Psi^{(-)}_{03}\). The fourfold coincidences \(D_1 - D_2 - D_{+0} - D_{-3}\) and \(D_1 - D_2 - D_{-0} - D_{+3}\) reach their maximums and the coincidences \(D_1 - D_2 - D_{+0} - D_{-3}\) and \(D_1 - D_2 - D_{-0} - D_{+3}\) reach their minimums for the same orientation of the polarization beam splitters \(PBS_0\) and \(PBS_3\). Conversely, if the beam splitters \(PBS_0\) and \(PBS_3\) are orthogonal to each other, then the coincidences \(D_1 - D_2 - D_{+0} - D_{-3}\) and \(D_1 - D_2 - D_{-0} - D_{+3}\) reach their maximums and the coincidences \(D_1 - D_2 - D_{+0} - D_{-3}\) and \(D_1 - D_2 - D_{-0} - D_{+3}\) reach their minimums. Under simultaneous rotation of the polarization beam splitters through the same angle, the maximums remain maximums and the minimums remain minimums. This fact was interpreted as evidence of the spherical symmetry of the two-particle quantum state of photons \{0\} and \{3\}, which is typical of the state \(\Psi^{(-)}_{03}\).

But the actual experiment did not show complete spherical symmetry. The point is that the depths of the maximums and minimums change under the described rotation of the polarization beam splitter axes, reaching their largest values when the axes of the polarization beam splitters \(PBS_0\) and \(PBS_3\) are rotated through either \(0^\circ\) or \(90^\circ\) with respect to the original polarization basis and their least values for \(45^\circ\).

In the framework of the approach proposed here, the results of similar experiments can be interpreted by a method similar to that described in Sec. 5. The simple beam splitter \(BS\)
and the detectors \( D_1 \) and \( D_2 \) select those pairs from the set of different pairs of photons \{1\} and \{2\} whose photons have a mutually orthogonal polarization in the original polarization basis. In addition, the mean of the total quasispin of these pairs is zero. Because the elementary state of photon \{0\} is the (negative) copy of the elementary state of photon \{1\} and the elementary state of photon \{2\} is the (negative) copy of the elementary state of photon \{3\}, the pairs of photons \{0\} and \{3\} with the same properties are simultaneously selected. Therefore, in the case where the polarization beam splitters \( PBS_0 \) and \( PBS_3 \) are oriented at an angle of 0° or 90° with respect to the original basis, the polarizations of photons \{0\} and \{3\} must be orthogonal to each other in the polarization bases related to the beam splitters \( PBS_0 \) and \( PBS_3 \). Accordingly, the versions of the coincidences observed in the experiment must indeed be observed. In the case where the polarization beam splitters \( PBS_0 \) and \( PBS_3 \) are oriented at a 45° angle with respect to the original basis, we can advance the same arguments used in Sec. 5. As a result, we conclude that for an infinite number of trials, the polarizations of photons \{0\} and \{3\} measured using the beam splitters \( PBS_0 \) and \( PBS_3 \) are orthogonal on the average. For a small number of trials, we can observe considerable deviations from this rule.

Thus, in the experiment described above, we have the selection of events involving the approximate desired correlation and not teleportation of entanglement.

7 Conclusions

There is a widespread opinion that teleportation is a special quantum method for transferring information and that this method can turn out to be very effective. But it follows from our considerations in this paper that there need not be a close relation between teleportation and the quantum features. We can easily give a classical analogue of quantum teleportation that has long been used in practice. Before setting out to sea, the captain of a ship obtains a sealed envelope from his headquarters containing several numbered variants of the instruction prescribing subsequent actions. When the ship is at sea, the captain receives an order by radio to open the envelope and act according to the instruction bearing a certain number. In quantum teleportation, a particle from the EPR pair plays the role of the envelope, and a classical communication channel plays the role of the radio.

Teleportation changes the traditional method of the action slightly. Instead of one envelope with an instruction, an automatic device prepares two identical envelopes with instructions. In these instructions, the action variants are numbered identically but randomly. One envelope is sent to the captain on the ship, and the other is sent to his headquarters on shore. Before the envelopes are opened, nobody knows the number assigned to each variant. Further, the chief opens his envelope, chooses the necessary variant, and sends its number to the captain. Such a method of action has a higher degree of secrecy compared with the traditional one. But a higher price must be paid for this advantage. In the case of the traditional method of action, the envelope could be delivered to the captain before the departure. In the case of quantum teleportation, the ”envelope” is sent while the captain is at sea. This can turn out to be a very complicated technical problem. Therefore, one should not cherish vain hopes to use the potential of quantum teleportation.
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