On the noise induced by the measurement of the THz electrical current in quantum devices

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Abstract. From a quantum point of view, it is mandatory to include the measurement process when predicting the time evolution of a quantum system. In this paper, a model to treat the measurement of the terahertz (THz) electrical current in quantum devices is presented. The explicit interaction of a quantum system with an external measuring apparatus is analyzed through the unambiguous notion of the Bohmian conditional wave function, the wave function of a subsystem. It is shown that such a THz quantum measurement process can be modeled as a weak measurement: the systems suffer a small perturbation due to the apparatus, but the current is measured with a great uncertainty. This uncertainty implies that a new source of noise appears at THz frequencies. Numerical (quantum Monte Carlo) experiments are performed confirming the weak character of this measurement. This work also indicates that at low frequencies this noise is negligible and it can be ignored. From a classical point of view, the origin of this noise due to the measurement at THz frequencies can be attributed to the plasmonic effect of those electrons at the contacts (by interpreting the contacts themselves as part of the measuring apparatus).

Keywords: mesoscopic systems (theory), current fluctuations, quantum transport
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1. Introduction

What does measuring the electrical current at terahertz (THz) frequency mean? Answering this question is not easy from an experimental or theoretical point of view. At such frequencies, the displacement current (related to time-dependent variations of the electric field) becomes even more important than the conduction current (related to the number of particles crossing a surface). In general, for semi-classical electron device simulations, it is usually assumed that the interaction with an external measuring apparatus does not alter the properties of the system itself. In contrast, for quantum device simulations, it is mandatory to take into account the effect of the apparatus on the measured system. The quantum device evolves differently if the system is measured or not.

In typical quantum devices, see figure 1, the whole experimental setup is divided into two parts: the system (also known as the active region of the device), of which we want to get information, and the measuring apparatus, composed of the probe and meter, which is responsible for extracting the information from the system. In principle, one can envision three options for considering the interaction of the system with an external apparatus in quantum device simulations:
The first option is to not consider the measurement apparatus and take the information directly from the simulated non-measured quantum system.

(ii) The second option is to look for an operator which encapsulates the effect/perturbation of the apparatus on the wave function of the measured system and take the information directly from the evolution of the system including the operator in the equation of motion.

(iii) The third option, which will be investigated in the present paper, is to include the system and apparatus in the simulations and get information from the simulated system + apparatus. We will use the Bohmian trajectories which provide a privileged framework to pursue this option.

Hereafter we elaborate more on options (i), (ii) and (iii) briefly exposed, trying to underline the advantages and disadvantages of each one.

1.1. THz current without modeling the measurement apparatus

The option of not including the apparatus in the simulations seems a very bad choice in order to model the measurement of the electrical current, but actually many electron device simulations are carried out in this way. In fact, as a byproduct of this work, we will show that in the DC regime (direct current), and at very low frequency, it is not necessary to include the apparatus to get accurate values of the current. We give a simple argument for this. We can reasonably assume that an electron device is ergodic, i.e. the mean value obtained from the ensemble is equal to the mean value in time of a certain quantity (a detailed discussion about the ergodic assumption is far from the scope of this paper). This implies that one has to measure the system only once to get

![Figure 1. Schematic representation of the studied system. We have separated the problem in three parts, in the middle there is the system we are interested in, which interacts through Coulomb interaction (red dashed line) with the electrons in the metal cable (probe). Finally the probe interacts with the ammeter (meter) which gives the final result of the measurement. The surfaces $S_m$ and $S_A$ (green dashed dotted line) used in the text are indicated.](image)
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the DC current and the subsequent evolution of the measured system does not matter. So, in this way the problem of including the measuring apparatus can be avoided, and it is possible to obtain reliable results from the simulated system at DC. However, at THz frequency measurements, there is no proper argument that justifies the non-inclusion of the apparatus.

1.2. THz current modeling the measurement apparatus with operators

This option is based on the traditional quantum mechanical procedure to describe the interaction between the quantum system and the measuring apparatus (the cables, the environment, etc) by encapsulating the latter into a non-unitary operator. This has the great practical advantage of reducing the computational burden of the simulation: only the degrees of freedom of the system are simulated. However, at THz frequencies, many questions about the properties of such an operator arise: which is the operator that determines the (non-unitary) evolution of the wave function when measuring the electrical THz current? Is it ‘continuous’ or ‘instantaneous’, with a ‘weak’ or ‘strong’ perturbation of the wave function? [1] To the best of our knowledge, no such THz current operator has been presented. So, it seems that this option, although feasible in principle, is not easily pursued.

1.3. THz current modeling the measurement apparatus with quantum trajectories

In this work, we follow the third option. We will discuss a model for an ammeter that measures the total (conduction plus displacement) current at THz frequency [2, 3]. We will consider the interaction between the electrons in a metal surface (probe), working as a sensing electrode, and the electrons in the device active region (system). In the following we will explain in details the model that we have developed. We will show that the measurement of the electrical current in a large metallic surface implies an unavoidable source of noise [2], which is generally ignored in most high-frequency quantum simulations, and a small perturbation of the quantum system. These two properties, in the context of quantum measurements, mean that the THz measurement of the current can be interpreted as a weak measurement [4].

2. Model development

As said in the introduction, it is not easy at all to model the measurement of the THz current [5, 6], and before entering into the details of our model let us describe the situation we have to face when addressing this problem. In principle one has to consider the setup depicted in figure 1, where there is a typical two-terminal device contacted by two cables and with two ammeters. Ideally one wants to solve the complete problem quantum mechanically, but unfortunately this is not accessible with up-to-date computer capabilities. This is the well known many-body problem: only three, four or five degrees of freedom can be treated fully quantum mechanically [7]. Thus an approximation, to tackle our measurement problem in THz quantum devices, is required.
As a first approximation one would desire a method able to tackle as many degrees of freedom as possible and that provides the output results (in our case the measured current). Our idea is to find an appropriate formalism which is able to include the measuring apparatus (or at least a part of it) in the simulations. For this aim an alternative version of quantum mechanics named Bohmian mechanics [8–11] turns out to be very useful. This theory uses the standard quantum mechanical wave function, which evolves according to the usual Schrödinger equation, and attributes definite positions for the particles at each time.

Let us briefly review the two basic laws of Bohmian mechanics. The first law says that the many particle wave function is a solution of the well-known Schrödinger equation:

\[
\frac{i\hbar}{\partial t} \Psi(x_1, x_2, \ldots, x_N, t) = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x_i^2} + V(x_1, x_2, \ldots, x_N, t) \Psi(x_1, x_2, \ldots, x_N, t).
\]

The second law is the guidance equation for each particle, which gives its evolution in time:

\[
\frac{dX_k(t)}{dt} = \frac{\hbar}{m_k} \text{Im} \left( \frac{\nabla_k \Psi(x_1, x_2, \ldots, x_N, t)}{\Psi(x_1, x_2, \ldots, x_N, t)} \right) \bigg|_{x_i = X_i(t), \ldots, x_i = X_i(t), \ldots, x_N = X_N(t)},
\]

where we denote the actual positions of the particles with capital letters, i.e. $X_k(t)$ means a trajectory, while $x_i$ is a degree of freedom of the problem.

Along with the many particle wave function in equation (1), together with the particle trajectories in equation (2), the theory provides an unambiguous definition of the wave function of a subsystem called the conditional wave function [7–9, 12]. The latter is simply defined from the many particle wave function $\Psi$ where all the degrees of freedom are substituted by the actual position of the particles, i.e. $x_i \rightarrow X_i(t)$, except for one particle. For example the conditional wave function of particle 1 is simply given by:

\[
\psi_1(x_1, t) \equiv \Psi(x_1, X_2(t), \ldots, X_N(t), t).
\]

The fundamental point that justifies the relevance of the conditional wave function is that the trajectories obtained from the many particle wave function $\Psi$ are exactly the same as the trajectories computed from the conditional wave function $\psi_k$:

\[
\psi_k(x_k, t) \equiv \Psi(x_k, X_{\neq k}(t), \ldots, x_N(t), t),
\]

\[
\frac{dX_k(t)}{dt} = \frac{\hbar}{m_k} \text{Im} \left( \frac{\nabla_k \psi_k(x_k, t)}{\psi_k(x_k, t)} \right) \bigg|_{x_i = X_i(t), \ldots, x_i = X_i(t), \ldots, x_N = X_N(t)} = \frac{\hbar}{m_k} \text{Im} \left( \frac{\nabla_k \psi_k(x_k, t)}{\psi_k(x_k, t)} \right) \bigg|_{x_i = X_i(t)}.
\]

Thus we can obtain the same trajectory, for example for particle $k$, either from the many particle wave function $\Psi$ or from the conditional wave function $\psi_k$. The

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remarkable fact is that the conditional wave function defined in equation (3) has its
own equation of motion:

\[ i\hbar \frac{\partial \psi_i(x_1, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x_1^2} + V(x_1, x_2(t), ..., x_N(t), t) \right. \]

\[ \left. + A(x_1, x_2(t), ..., x_N(t), t) + iB(x_1, x_2(t), ..., x_N(t), t) \right] \psi_i(x_1, t), \]

where \( V \) is called the conditional potential, i.e. the potential felt by particle 1 because
of all the other particles, while the real and imaginary potentials \( A \) and \( B \) are defined
from the many particle wave function \( \Psi \) (for a detailed derivation of these potentials
see [7, 12]). Let us mention that the interaction between the quantum system and
the measuring apparatus, studied through quantum (Bohmian) trajectories, provides a
microscopic definition of the interaction with the apparatus, without the need of post-
tulating an operator [2, 13, 14].

2.1. How to calculate the output results

Before entering into the details of the model let us mention how the measurement of
the electrical current is performed in terms of positions \( X_i(t) \) of the (Bohmian) elec-
trons and conditional wave functions \( \psi_i \). Let us specify that, although we only consider
the potential \( V \) in equation (1) (a quasi-static approximation), we implicitly assume
that the dynamics of electrons are compatible with Maxwell’s equations so that there
is an electromagnetic propagation of the total current along the cable (that connects
the quantum system and the ammeter in figure 1). The total current on \( S_m \) is equal
to the current on the surface, \( S_A \), far from the active region. This equivalence (due to
the divergencelessness of the total current) is exact for the sum of the particle plus the
displacement currents, but not for the particle current alone [15, 16]. Once we
have considered such propagation, the ammeter transforms the total current \( S_A \)
to a pointer value [17]. So, as reported in figure 1, there are three main parts involved
in this measurement. First, the system or the device active region. Second, the probe
which is responsible to translate the current until the meter. And, third, the meter itself
that actually translates the value of the current into a pointer position in the ammeter.

The total current, \( I_T(t) = I_{p}(t) + I_{d}(t) \), is composed by the displacement component
\( I_d(t) \), defined as the surface integral of the temporal derivative of the electric field,
plus the particle component, \( I_{p}(t) \), defined as the net number of electrons crossing the
surface \( S_A \) [13]. For simplicity, we shall focus only on the displacement component
of the total current (no electrons crossing the surface when the current is measured).
So, \( I_d(t) \) can be computed as the time derivative of the flux \( \Phi \) of the electric field
\( \mathbf{E} \equiv \mathbf{E}(X_1(t), ..., X_N(t), t) \) produced by all (system plus apparatus) \( N \) electrons, described
by positions \( X_i(t) \) and conditional wave functions \( \psi_i \), on the surface \( S_A \) using the relation [2, 11]:

\[ I_d(t) = \int_{S_A} \epsilon(r) \frac{d\mathbf{E}}{dt} \cdot ds = \sum_{i=1}^{N} \nabla \Phi(X_i(t)) \cdot \mathbf{v}_i, \]

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where the flux $\Phi$ explicitly defined in the appendix depends on each electron position and $v$ is the Bohmian velocity, which is obtained from $\psi_i$ using equation (4).

For a detailed derivation of equation (6) the reader is guided to [17]. We underline that equation (6) provides the measured current by the ammeter as a function only of the position of the particles, $X_i(t)$, and the velocity of the particles $v$. We underline that equation (6) provides the current carried by all the particles (system, probe and meter) and not only the current generated by the particles of the system.

In the rest of the paper we focus on the situation in which the electrical current is measured in a large metallic surface, large in the sense that the squared distance from the particle to the surface, say $L_x$ of figure 1, is much smaller than the surface where the current is recollected, say $S_A$. We consider also the simplest case where there is only one particle in the active region of the device, say particle 1. In this particular case equation (6) becomes:

$$I_d(t) \propto \frac{d\Phi(E)}{dt} = \frac{d}{dt}\left(\alpha X_1(t) + \sum_{j=2}^{N} \Phi(X_j(t))\right) \propto v_{x_1} + \sum_{j=2}^{N} \nabla \Phi(X_j(t)) \cdot v_j,$$

(7)

where $v_{x_1}$ is the $x$-component of the Bohmian velocity of particle 1. These results, equations (6) and (7), have been obtained in [17], nevertheless we reported a brief summary in the appendix. Equation (7) will be used extensively in the rest of the text. We emphasize that equation (7) provides a relation between (i) the total current of the quantum system itself which is proportional to $v_{x_1}$ and (ii) the current effectively measured by the ammeter which we denote by $I_d(t)$. Notice that Bohmian mechanics is a quantum theory without observers. Therefore, it permits us to talk about the current of the quantum system even though it is not the measured value.

2.2. System-probe interaction

In principle, we would need to consider all the particles, described by $X_i(t)$ and $\psi_i$, of figure 1 in order to simulate exactly the whole system, but as said above this is not possible for the well known quantum many-body problem [7, 18]. If we focus on the dynamics of the particle in the active region of the device, with position $X_i(t)$ and conditional wave function $\psi_i(x_i, t)$, we easily realize that its interaction with all other particles can be divided according to its effect on $X_i(t)$. The first type of particles are those $N-1$ particles close to $X_i(t)$ where the full Coulomb interaction with this particle is relevant. We simulated explicitly how the $N-1$ particles affect $X_i(t)$ and, very importantly, also how $X_i(t)$ affect the $N-1$ particles. The second type of particles are those far enough from $X_i(t)$ so that we assume that they slightly affect $X_i(t)$ and the effect of this particle on them is negligible. Then, the global effect of these particles on $X_i(t)$ is computed by the (mean field) quasi-electrostatic boundary conditions. Clearly this distinction between the two types of particles depends on the distance to (and the energies of) the electron in the active region of the device. In summary, we only consider the particle, described by $X_i(t)$ and $\psi_i(x_i, t)$, belonging to the quantum system, and the nearest electrons, described by $\{X_2(t), \psi_2(x_2, t)\}, ..., \{X_N(t), \psi_N(x_N, t)\}$, in the metal surface $S_A$ (see figure 2). The effect of the rest of the electrons are included in the (quasi-)electrostatic boundary conditions of the problem. In addition, for simplicity,
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we are considering that the particle in the active region of the device is moving only in the transport direction \(x_1 \equiv (x_1, 0, 0)\). Thus, we are explicitly neglecting the part of the ammeter’s (meter), where the current is actually measured and we concentrate only on the system–probe interaction.

In principle the exact solution of the conditional wave function is provided by equation (5), but as already briefly explained, the potentials \(A\) and \(B\) are exactly known only if the many particle wave function \(\Psi\) is known everywhere in the configuration space, which is not possible. Fortunately, we can provide a suitable approximation for the problem studied here. We can use the approximation, reported in [7, 12], where the spatial dependence of the potentials \(A\) and \(B\) is neglected: \(A(x_1, X_2(t), ..., X_N(t), t) \approx A(X_1(t), X_2(t), ..., X_N(t), t)\) and \(B(x_1, X_2(t), ..., X_N(t), t) \approx B(X_1(t), X_2(t), ..., X_N(t), t)\), while keeping the spatial dependence of \(V(x_1, X_2(t), ..., X_N(t), t)\). Thus the evolution of the conditional wave function of the electron in the device active region is given by the following equation:

\[
i\hbar \frac{\partial \psi(x_1, t)}{\partial t} = [H_0 + V] \psi(x_1, t),
\]

where \(V = V(x_1, X_2(t), ..., X_N(t))\) is the conditional Coulomb potential felt by the system and \(H_0\) is its free Hamiltonian. Obviously, the electron in the active region of the device is still affected by the other particles composing the metal surface through \(V\), this point is crucial for including the back action of the measuring apparatus on the quantum system, i.e. the actual effect of the measuring apparatus on the measured system.

On the other hand, the electrons in the metal surface (the probe) are simulated as follows. Each electron \(X_i(t)\) interacts with the other electrons in the metal \(X_2(t), ..., X_{k-1}(t), X_{k+1}(t), ..., X_N(t)\) plus with the electron in the active region of the device \(X_1(t)\). The simulations reported hereafter in section 4 are performed considering

\[\text{Figure 2.}\text{ Schematically depicted is the Coulomb interaction (red dashed lines) and the conditional wave function (black solid line) solution of equation (8). This is the actual system (together with the analogous right probe) used in the numerical simulations in section 4.}\]
approximately 1000 electrons in the metal surfaces. For simulating such trajectories \( \mathbf{X}_k(t) \) we can take the time derivative of equation (4), obtaining:

\[
\frac{d^2 \mathbf{X}_k(t)}{dt^2} = -\frac{1}{m_k} \nabla_k (V + Q)|_{x_1=\mathbf{x}_1(t), \ldots, x_N=\mathbf{x}_N(t)} ,
\]

where \( Q = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{\nabla^2 |\Psi|}{|\Psi|} \) is called quantum potential and where \( m_i \) is the electron’s mass in the metal. Because of the large number of electrons, the contribution \( \nabla^2 |\Psi| \) to the quantum potential is somehow randomized and it becomes small compared to \( V \) so we can approximate equation (9) as follows:

\[
\frac{d^2 \mathbf{X}_k(t)}{dt^2} = \frac{\mathbf{F}_k}{m_k} .
\]

The force \( \mathbf{F}_k \) consists of two contributions, the Coulomb force \( \mathbf{F}_{k\text{Coulomb}} = -\nabla_k V \) and a viscosity term (in order to simulate the interaction with phonons):

\[
\mathbf{F}_k = \mathbf{F}_{k\text{Coulomb}} - \gamma \mathbf{v}_k
\]

where \( \gamma = 3.374 \cdot 10^{-17} \text{ Kg s}^{-1} \). A more realistic treatment of the irreversible dynamics due to the electron–phonon interaction, beyond expression (11), will certainly provide quantitative (but not qualitative) differences in the results of section 4. The number of electrons in the metal (3D) surface is chosen roughly as the density of copper \( (n_{Cu} = 8.43 \cdot 10^{28} \text{ m}^{-3}) \). In the simulations reported hereafter, the surface where the electrons in the metal are simulated is \( S_m = 2.5 \cdot 10^{-17} \text{ m}^2 \) with a width of \( 5 \cdot 10^{-9} \text{ m} \) and the time step is \( \Delta t = 4 \cdot 10^{-17} \text{ s} \). The type of probe (metal) used in each experiment (number of particles, geometry, etc) obviously affects quantitatively (not qualitatively) the effects reported in section 4. The measured values depend not only on the system, but also on the type of measuring apparatus. In summary, we have presented a microscopic model for studying the interaction between a (measured) quantum system and a measuring apparatus with the Bohmian trajectories formalism (through particle positions \( \mathbf{X}_i(t) \) and conditional wave functions \( \psi_i(\mathbf{x}) \)). We have been able to include the back action of the apparatus on the measured system and we have provided an explicit equation to calculate the measured output current. We remark that, in the model just presented, not only the particle \( x_1 \) in the active region of the device is affected by the electrons in the metal surface, but also vice versa.

3. Does an unavoidable source of noise due to the measurement of the THz current exist?

In the previous section we have developed an equation (7) for the measurement of the electrical current at THz frequency plus a model to determine the equation of motion for all the particles \( \mathbf{X}_i(t), \ i = 1, \ldots, N \). Now we want to clarify the following question: Does an unavoidable source of noise due to the measurement of the THz current exist? Our answer will be supported by numerical results in next section. Here, we provide...
some qualitative arguments on the physics of the type of measurement we are explain-
ing in this work. We focus the attention on equation (7), which we report here again:

\[ I_{th}(t) \propto v_{x_1} + \sum_{j=2}^{N} \nabla \Phi(X_j(t)) \cdot v_j. \]  

We observe that the current is composed by two terms. The first term is propor-
tional to the velocity of the electron in the active region of the device, \( v_{x_1} \). This corre-
sponds to the signal that we want to get from the measured system. The second term
in equation (12), which depends on all of the rest of the electrons composing the probe,
provides the noise of the measurement process. We can see that the instantaneous
current, \( I_{th}(t) \), in equation (12) is affected by this term. In fact the random movement
of the electrons in the probe produces a random current output. So, we can interpret
equation (12) as the sum of the signal, first term, plus the additional source of noise,
the second term. The signal to noise ratio will be discussed in the next section. But is it
actually a new source of noise? This movement of electrons is also known as plasmons,
i.e. collective motions of the electrons composing the metal surface. We assume that
the contacts (responsible for the transmission of the current from the quantum system
to the ammeter) are an unavoidable part of any measuring ammeter.

The question here is whether this additional source of noise has to be taken into
account when performing quantum device simulations or not. We can step back to the
three options we have enumerated in the introduction. Option 1.1, i.e. not modeling
the measurement apparatus, needs obviously to include this source of noise because
option 1.1 alone does not contain any information on the apparatus. Instead, working
with option 1.2, one must take care that the operator chosen for the measurement of
the electrical current at THz frequencies fits with this source of noise we have found.
On the other hand, option 1.3, which is the one used in the present paper, naturally
includes this additional source of noise. In addition, the novelty here is the way we
have achieved this result: the model we have proposed in the previous section, through
the notion of the conditional wave function and Bohmian trajectories, has permitted
us to interpret, in a still purely quantum mechanical way, the interaction between the
measured system and the probe. So, thanks to the theory and the model, we have been
able to deduce this additional and unavoidable source of noise. As a byproduct of this
work, we will also obtain that in the DC regime (direct current), and at very low fre-
quency, it is not necessary to include the apparatus (or its noise) to get accurate values
of the DC current. In the next section we provide some numerical results supporting
the argument just exposed here.

4. Numerical results

Here we analyze the results obtained from numerical experiments performed with the
model presented in section 2. In particular in section 4.1 we show how the measurement
of the electrical current in a large surface at THz frequencies provides an additional
source of noise. In section 4.2 we show how this noise can be interpreted as a weak
measurement. The reader can find in [17] how this weak measurement can be used to
reconstruct the Bohmian trajectory of an electron in a multi-terminal device. In section 4.2 the dependency on frequency of the presented model is also briefly addressed.

4.1. Additional source of noise

In figure 3, we report the instantaneous value of the displacement current measured in the surface $S_A$ when considering all the electrons of the system and the probe (red solid line), and when considering only the electron of the system (green dashed line).

One can see that the instantaneous current calculated from equation (12), i.e. when considering the contribution of all the electrons in the metal (including the probe), differs considerably from the instantaneous current when considering only the electron in the device active region, i.e. without including the probe. The difference is due to the second term appearing in equation (12). As already discussed in section 3, the random movement of the electrons in the probe produces a random current output, as can be clearly seen in figure 3.

The large fluctuations in the instantaneous value of the displacement current reported in figure 3, when considering the system and the probe, means an additional source of noise due also to the interaction of the electrons in the metal with the particle $x_1$ in the active region of the device.

4.2. Weak measurement

From the numerical simulations, reported in figure 4, we observe that the second term in equation (12) has large fluctuations but it is constant when evaluated over an ensemble of identically prepared experiments, $\sum_{j=2}^{N} (\nabla \Phi(X_j(t)) \cdot v_j) \approx \text{const}$. So, it is possible to write the ensemble value of equation (12) as:

$$\langle I_0(t) \rangle_{S_A} \propto \langle p_x \rangle.$$ (13)

Equation (13) shows that the mean value of the total electrical current in a large metallic surface is proportional to the mean value of the momentum ($x$-component, i.e. the component perpendicular to the surface) of the quantum particle in the device. In figure 4 the mean value of the (weak) measured total current computed from equation (12) is reported, which is equal to the value obtained without considering the ammeter, confirming thus equation (13).

In addition, see [2, 17], the measurement scheme just presented also implies that when the information of the measured current is very noisy, the quantum system is only slightly perturbed, and vice versa. This fact is completely in agreement with the fundamental properties of quantum measurement: if one looks for precise information, one has to pay the price of perturbing the system significantly (the so-called collapse of the wave function or strong measurement). On the other hand if one does not require such precise information (e.g. the instantaneous value of the displacement current seen in figure 3) one can leave the wave function of the system almost unaltered (known in the literature as weak measurement [4]).

So, in the context of quantum measurements, the measurement scheme of the total current in a large surface just presented in this work is a weak measurement [4]. Thus adding the result obtained in equation (13), the weak measurement of the total current seems to be able to be approximated, in the language of Gaussian measurement Kraus operators, by:
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\[ \dot{I}_w = C_w \int dp e^{-\frac{(p-p_0)^2}{2\sigma^2}} |p\rangle \langle p|, \]  

(14)

where \( p \) is the momentum (x-component) of the particle in the device and \( C_w \) is a suitable constant. We underline that the Gaussian operator defined in equation (14) works as an approximation for the output results (of the displacement current) but does not
include properly the effect on the quantum system. In fact, the Gaussian approximation of the measuring operator does not capture all the fluctuations reported in the inset of figure 4(a). In fact, the apparently random oscillations of the inset of figure 4(a) really show the difficulties, discussed in option 1.2, in properly developing a quantum operator for such a type of measurement (small variations on the oscillatory behavior of the operator imply dramatic changes in the dynamics of the system wave packet).

Finally, we discuss how the width $\sigma_w$ of the weak measurement changes with the frequency of the measurement. It has been seen that depending on the frequency, the information about the measured total current changes (see [2]). In figure 5 it is reported how $\sigma_w$ varies with the frequency of the measurement. It can be seen that lowering the frequency yields more precise information about the system (the width of the Gaussian decreases). In this sense the source of noise described in this article is unavoidable at high-frequency (THz) regimes, while for lower frequencies, the option 1.1 of not including the apparatus in the simulation still works because such noise becomes negligible when integrated over a large time interval to provide the DC value.

5. Conclusions and discussion

We have presented a novel model to include how the measuring apparatus affects the value of the measured current at high frequencies (THz) in quantum devices. In particular, we have studied the effect of the collective motion of electrons in the metals (the contacts of the device that first collect the total current) on the measurement of the THz electrical current of electronic devices from a quantum point of view. This scheme of the measurement process allows us to differentiate between (i) the total current of the quantum system itself and (ii) the current effectively measured by the ammeter. According to our analysis, when a large fluctuation in the current appears (i.e. the

Figure 5. Red solid line: probability distribution of the measured displacement current at a frequency of $f = 500$ THz. Blue dashed line: probability distribution of the measured displacement current at $f = 50$ THz.
current of the quantum system is very different from the measured one), the measurement of the THz current implies a slight perturbation of the quantum system, and vice versa. Additionally, we have also shown that the mean value of the total current, measured by the apparatus obtained by repeating the same experiment many times, provides the strong measured value of the current of the system. Therefore, we conclude:

- The measured current contains an unavoidable source of noise (that corresponds to the plasmonic motion in the contacts) plus a signal (that corresponds to the current of the quantum system itself).

- The signal-to-noise ratio of the measured current depends (on many physical parameters of the contacts, i.e. on the concrete measuring apparatus, and) on the frequency. For DC values the noise is so small that it can be neglected, but for high (THz) frequencies it cannot be dismissed.

- In the context of quantum measurements, the THz measurement of the current can be interpreted as a weak measurement.

Finally, it should also be mentioned that the weak measurement of the total current at high frequency opens a new path for envisioning experiments for reconstructing (Bohmian) trajectories and wave function of electrons in solid state systems, similar to those already performed for photons [19, 20]. The authors have presented a recent work [17] which pursues this idea.

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Appendix. Displacement current on a large surface

In this appendix we provide a demonstration of equation (7) in the text, the reader can find the same development in [17].

According to our discussion in the main text, the capital letters \( \{X_i(t), Y_i(t), Z_i(t)\} \) denote the actual (Bohmian) positions of the particles, where \( i \) identifies the \( i \)-th particle. The flux of the electric field through a general ideal surface \( S_A \) in figure 2 (defined as a plane of area \( L_y \cdot L_z \) perpendicular to the \( \hat{x} \) direction and placed in \( x = x_A \), i.e. defined by the points \( \{x_A, 0 \leq y' \leq L_y, 0 \leq z' \leq L_z\} \), generated by a particle in position \( \{X, Y, Z\} \)) can be calculated as:

\[
\Phi(X, Y, Z) = \int_{S_i} \mathbf{E}(X, Y, Z, x_A, y', z') \cdot ds,
\]

(A.1)
where we have eliminated the subindex \( i \) and time \( t \) to simplify the notation. The electric field \( \mathbf{E} \) is just computed from the Coulomb force of the electron in the mentioned surface. In the simple case in which the particle is located in \( \{ X, L_y/2, L_z/2 \} \), it moves only in the \( \hat{x} \) direction and \( L_y = L_z \equiv L (S_A = L^2) \), equation (A.1) becomes:

\[
\Phi(X) = \frac{q}{\pi \epsilon} \tan^{-1}\left( \frac{S_A}{4(x_A - X)\sqrt{(x_A - X)^2 + \frac{S_A}{4}}} \right). \tag{A.2}
\]

Let us evaluate equation (A.2) in the situation in which \( S_A \gg (x_A - X)^2 \). This means that the maximum distance (squared) between the electron inside the device active region and the surface is much smaller than the surface itself. In order to work out an approximate form for equation (A.2) in this regime, the change of variable \( \chi = (x_A - X) \) can be considered. For simplicity, we assume that the electron is located on the left of the surface (i.e. \( X < x_A \Rightarrow \chi > 0 \)), then:

\[
\Phi(\chi) = \frac{q}{\pi \epsilon} \tan^{-1}\left( \frac{S_A}{4\chi^2\sqrt{1 + \frac{S_A}{2\chi^2}}} \right). \tag{A.3}
\]

Then, calling \( \xi^2 = \frac{2\chi^2}{S_A} \), equation (A.3) becomes

\[
\Phi(\xi) = \frac{q}{\pi \epsilon} \tan^{-1}\left( \frac{1}{2\sqrt{\xi^2(1 + \xi^2)}} \right), \tag{A.4}
\]

such that the condition \( S_A \gg \chi^2 \) becomes equivalent to \( \xi \ll 1 \). So equation (A.4) becomes simply:

\[
\Phi(\xi)_{\xi^2 \ll 1} = \frac{q}{\pi \epsilon} \tan^{-1}\left( \frac{1}{2\sqrt{\xi^2}} \right). \tag{A.5}
\]

Remembering that \( \tan^{-1}(\alpha \xi) + \tan^{-1}(\frac{1}{\alpha \xi}) = \frac{\pi}{2} \) for \( \xi > 0 \) then one has:

\[
\Phi(\xi) = \frac{q}{\pi \epsilon} \left[ \frac{\pi}{2} - \tan^{-1}(2\xi) \right]. \tag{A.6}
\]

In equation (A.6) the term \( \tan^{-1}(2\xi) \) can be expanded obtaining:

\[
\Phi(\xi) = \frac{q}{\pi \epsilon} \left[ \frac{\pi}{2} - 2\xi + \frac{(2\xi)^3}{3} - ... \right]. \tag{A.7}
\]

This last expression, equation (A.7), can be truncated at first order of \( \xi \) for our large surface. Thus recalling the original variables one arrives at:

\[
\Phi(x) = \frac{q}{\pi \epsilon} \left[ \frac{\pi}{2} - 2\sqrt{\frac{2}{S_A}} (x_A - X) \right] \propto X. \tag{A.8}
\]
Equation (A.8) is an important result, it demonstrates that the flux of the electric field generated by a particle in a very large surface is proportional to the position of the particle.

Now the general problem considered here can be discussed, i.e. to derive a microscopic analysis of the measurement of the total electrical current in a large metallic surface. In order to do that, one has to ‘enlarge’ the system considering also all the electrons composing the metallic surface, as described in the main text.

Without assuming anything about the dynamics of the electrons in the metal, one can say that they contribute to the flux of the total electric field as described by equation (A.1) by the superposition principle. One obtains, suppressing the dependence on $x_A$ and making reference to the position of the electron in the device as $X_1$, the following expression:

$$\Phi(X_1, X_2, \ldots, X_N) = \alpha X_1 + \sum_{j=2}^{N} \Phi(X_j),$$

(A.9)

where the actual Bohmian positions of the particles $X_k$ have been used and $\alpha$ is a suitable constant. In equation (A.9) one can clearly see that the total electric flux is due to a contribution from the electron in the system $\propto X_1$ and another due to all the other electrons in the metal.

So far, it has been considered that the electron in the active region of the device is not crossing the surface and therefore one gets that the total electric current is due only to the displacement current contribution. So, the displacement current becomes:

$$I_d \propto \frac{d\Phi}{dt} = \frac{d}{dt} \left(\alpha X_1 + \sum_{j=2}^{N} \Phi(X_j)\right) \propto v_{x_1} + \sum_{j=2}^{N} \nabla \Phi(X_j) \cdot v_j,$$

(A.10)

which is the result, equation (7), used in the main text of the article.

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