Boson stars, primary photons and phase transitions

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Abstract. We analyse the possibility that the dark matter candidate is from the approximate scale symmetry theory of the hidden scalar sector. The study includes the warm dark matter scenario and the Bose-Einstein condensation which may lead to massive dark scalar boson stars giving rise to direct detection through observation of the primary (direct) photons. The dynamical system of the scalar particles, the dilatons, at finite temperature and chemical potential is considered. The fluctuation of the particle density increases sharply within the increasing of the temperature. When the phase transition approaches, the fluctuation of the particle density has the non-monotonous rising when the ground state of the relative chemical potential tends to one with the infinite number of the particles. Our results suggest that the phase transition in the boson star may be identified through the fluctuation in yield of primary photons induced directly by the conformal anomaly. The fluctuation rate of the primary photons grows up intensively in the infra-red to become very large at the phase transition.

1. Introduction

The spontaneous breaking symmetry has a major role in particle physics and cosmology where the phase transitions (PT) can occur at some extreme conditions (e.g., high enough temperature, the high value of the chemical potential, etc.). It is well-known that the thermal corrections to the Higgs potential restore the electroweak (EW) gauge symmetry in the early Universe. The PT corresponds to an initial thermal state (hidden sector) which is invariant under the conformal group $G$. This state has a nontrivial couplings to the other states being the physical matter fields. The latter do not possess the invariance under $G$, but they are invariant under the chiral symmetry group triggered by $G$. For the physical mass of the Higgs boson the standard model (SM) symmetry restoration PT is a smooth cross-over rather than the true PT [1]. In the early Universe at high temperatures, the particles exhibit an asymptotic freedom that means there is the gas of almost free light massive and/or massless particles, the dark matter (DM). All scales in particle physics and cosmology are the subjects of the light scalar fields which are the DM candidates. If the weak and the EW scales are embedded in the SM, one can apply the complete formalism of the high-temperature weak and EW transition between
the low-energy broken phase where the scalar field has a non-vanishing expectation value, and a high-temperature symmetric phase. In theories beyond the SM, the introduction of the additional scalar fields can result in the potential having the different temperature-dependent behaviour from that of the SM. When the temperature of the hidden scalar sector is the order of the strong coupling scale $\sim O(\Lambda)$ the light particles with spin 0 are expected to be emerged. In particular, the scalar particles can lock up into color spin 0 neutral states, the ”glueballs”, whose mass is of the order of $\Lambda$ [2]. There are expectations in gluon-gluon bound states existence as hadronic (colourless) states in their own right as simpler structures than the more conventional hadronic states involving quarks [3-5]. The lightest hidden scalar fields with the mass $<10^4$ TeV could be the candidate for DM. These scalars could be cosmologically long lived with the lifetime $\sim 10^{17}$ sec (approximately the age of the Universe) giving by the decay width of the scalar into two gravitons [6].

The critical phenomena if occurred may be considered through the quantum PT with the Bose-Einstein condensation (BEC) of the scalar field. In this case, the condensation takes place in a single zero mode that suggests the breaking of conformal symmetry. The $SU(3)$ gauge theory studied on the lattice with $N_f = 8$ Dirac fermions in fundamental representation show an evidence of a light scalar singlet [7]. The latter might be a dilaton in the effective field theory [8-10] well before the recent activities in which the dilaton, a pseudo-Goldstone boson of the spontaneous breaking of conformal symmetry, provide, in particular, a portal between the dark matter and the visible sector of matter [11-13]. The dynamics and the properties of the dilatons (and their decays) at the temperature and the chemical potential around their critical values may be the keys to understanding the evolution of the Universe. One can imply the existence of the Goldstone-like modes: the scale symmetry is broken explicitly resulting with an appearance of the dilaton in the spectrum, accompanying by $\pi$-mesons as a consequence of the chiral symmetry breaking.

It is well-known that the quantum effects, e.g. the gluon fluctuations, break the conformal (scale) invariance. There is the anomaly in the trace $\theta_{\mu}^{\mu}$ of the energy-momentum tensor $\theta_{\mu\nu}$, $\theta_{\mu}^{\mu} = \partial_{\mu} S_{\mu}^{\mu} \neq 0$, where the dilatation current $S_{\mu}^{\mu} = \theta_{\mu}^{\mu} x_{\mu}$ does not conserved with respect to the scale transformations of the coordinates $x_{\mu} \rightarrow \omega x_{\mu}$ ($\omega$ is an arbitrary constant). If the explicit breaking scale symmetry does not play the dominant role, the spontaneous breaking of chiral symmetry may imply the spontaneous breaking of the approximate scale symmetry. The dilaton appeared as a pseudo-Goldstone boson is associated with the chiral condensate occurred in the region where the gauge coupling constants are slowly running and an effective fermion coupling constant has reached the critical value [9].

The theory of strong interactions is a very good example for the theory that possesses the conformal invariance at the classical level because there are no dimensional parameters associated with the classical formulation of the theory. There is a direct connection between $\theta_{\mu}^{\mu}$ and the gluon field-strength tensor $G_{\mu\nu}^{a}$, $\theta_{\mu}^{\mu} = [\beta(\alpha_s)/(4 \alpha_s)] G_{\mu\nu}^{a} G^{a, \mu\nu}$, where $\alpha_s = \alpha_s(M)$ is the renormalised gauge coupling defined at
the scale $M$; $\beta(\alpha_s)$ is the renormalisation group $\beta$-function. The scale $M$ is generated within the quantum effects when the vacuum is disordered and the scale invariance is destroyed, $M = M_{UV} \exp [-2\pi/(b_0 \alpha_s)]$, with $M_{UV}$ being the ultra-violet (UV) scale and $b_0$ is the first coefficient in the $\beta$-function. The breaking of the conformal invariance assumes that all the processes are governed by the conformal anomaly (CA) resulting from the running coupling constant $\alpha_s$. The derivative of $S_{\mu}$ is proportional to $\beta$-function and to the quark masses. For $SU(N_c)$ gauge theory with $N_c$ number of colours and $N_f$ number of flavours in the fundamental representation, the $\beta$-function is well-known

$$\beta(\alpha_s) \equiv M \frac{\partial \alpha_s(M)}{\partial M} \equiv -\frac{b_0}{2\pi} \alpha_s^2 - \frac{b_1}{(2\pi)^2} \alpha_s^3 + ..., $$

where $b_1$ is the coefficient. There can be an approximate scale (dilatation) symmetry if $\beta(\alpha_s)$ is small enough and $\alpha_s(M)$ is slowly running with $M$. The theory becomes conformal in the infra-red (IR) with the non-trivial solution $\alpha_s^* = -2\pi b_0/b_1$ (the IR fixed point (IRFP) or the Banks-Zaks [14] conformal point) in the perturbative domain if $b_0 = (11N_c - 2N_f)/3$ is small. The latter is happened when $(N_c/N_f) = 2/11$. Once $N_f$ decreases near the PT (the value of $\alpha_s^*$ increases) the latter is characterised by $\alpha_s^* < \alpha_c^*$ at which the spontaneous breaking of the chiral symmetry is emerged. In the neighborhood of the IRFP, the $\beta$-function is approximated by $\beta(\alpha_s) \simeq -\Delta\alpha_s(M/\Lambda)^{\delta}$, where $0 < \Delta\alpha_s = \delta(\alpha_s^* - \alpha_c^*) << \alpha_c^*, \delta \leq O(1)$ [15].

The dilaton couples to photons (even before running any SM particles in the loop) through the trace containing the gauge invariant operator of the photon strength tensor. They will cause the dilaton to decay into the photons. It means the CA acts as a source of the primary (direct) photons (or soft photons in heavy-ion collisions [16]) not produced in decays of light hadrons. The operator associated with the primary photon defines the highest weight of representation of conformal symmetry and this operator obeys the unitarity condition $d \geq j_1 + j_2 + 2 - \delta_{j_1,j_2,0}$ for the scaling dimension $d$, where $j_1$ and $j_2$ are the Lorentz spin operators (the "primary" operator means not a derivative of another operator).

There is an importance of the primary photons emission for the study of evolution pattern of heavy ion collisions, especially in the very early phase. For example, the correlations of direct photons can shed the light to the space-time distribution of the hot matter prior to freeze-out [17]. However, the investigation of direct photons meets the considerable difficulties mainly due to very small yield of photons emitted directly from the hot dense zone in comparison to the huge background of photons produced due to the standard decay of the hadrons. If the PT is situated in some regions accessible to heavy ion collisions experiments one should identified it through some observables. The signature of the PT is the non-monotonous behaviour of observable fluctuation where the latter increases very crucially.

We investigate the possible evidence of the DM candidate from the approximate scale symmetry, the theory of the hidden scalar sector. The DM is the lightest hidden scalar field which is likely a dilaton. We suggest the novel approach to the
phase transition and to the approximate scale symmetry breaking with the challenge phenomenology where the primary photons induced by CA are in fluctuating regime. The rate of the correlation length related to the mass of the dilaton plays an important role as an indicator of the PT achievement if this length grows very sharply. So, the primary photons effects start to play a significant role when the correlation length grows up intensively in the proximity of the PT or the critical point. The scalar dilaton could be warm and may have the novel feature of BEC into compact massive boson stars [18] leading to direct detection through the observation of primary photons. In the particle physics experiments it could also be tested if there exist the interactions of the dilaton field $\sigma$ with the Higgs boson field operator $|h|^2, \sim const|h(x)|^2 \sum_{k=1}^{N} c_k \chi(x)$ in the sliced-operator form (the tower of hidden dilatons) with $c_k$ being the coefficients and $N$ is the number of particles.

2. The toy dilaton model

We start with the effective model where the Lagrangian density (LD)

$$L = \frac{1}{2} f^2 \sigma (\partial_\mu e^\sigma)^2 + \frac{1}{2} f^2 \pi e^{2\sigma} (\partial_\mu \pi)^2 + ... \quad (1)$$

is invariant under the scale transformations $x_\mu \rightarrow \omega x_\mu$; the $\pi$-meson field $\pi(x) = f^{-1}_\pi \pi(x)$ is found within transformation $\pi(x) \rightarrow \pi(xe^\omega)$ and the dilaton field $\sigma$ transforms non-linearly

$$\sigma(x) \rightarrow \sigma(xe^\omega) + \omega. \quad (2)$$

The LD (1) has the more habitual form

$$L = \frac{1}{2} (\partial \chi)^2 + \left(\frac{f_\pi}{f_\chi}\right)^2 \chi^2 (\partial \pi)^2 + ..., \quad (3)$$

if the dilaton field is parameterised as $\sigma(x) \rightarrow \chi(x) = f_\chi e^{\sigma(x)}$, which transforms non-linearly under (2). In the LD (3), $f_\chi = \langle \chi \rangle$ is the vacuum expectation value (vev) of the order parameter $\chi$ for the scale symmetry breaking determined by the dynamics of the underlying strong sector. The constant $f_\chi$ is defined from

$$\langle 0 | S^{\mu} \chi(p) \rangle = ip^\mu f_\chi, \quad \partial_\mu \langle 0 | S^{\mu}(x) \chi(p) \rangle = -f_\chi m_\chi^2 e^{-ipx},$$

where

$$\langle 0 | \theta^{\mu\nu}(x) \chi(p) \rangle = f_\chi (p^\mu p^\nu - g^{\mu\nu} p^2) e^{-ipx}, \quad p^2 = m_\chi^2.$$ 

Here, $m_\chi$ is the mass of the dilaton, $p_\mu$ is the momentum conjugate to $x_\mu$ and $|0\rangle$ is the vacuum state corresponding to spontaneously broken chiral and dilatation symmetries.

Actually, the dilaton is naturally seen in field theory given by the LD (see, e.g., [19])

$$L = \sum_i g_i(\zeta) O_i(x) \quad (4)$$
where the local operator $O_i(x)$ has the scaling dimension $d_i$. The LD (4) is invariant under dilatation transformations $x^\mu \to e^{\alpha x^\mu}$ with $O_i(x) \to e^{\omega d_i} O_i(e^{\alpha}x)$ if $g_i(\zeta)$ is replaced by $g_i(\zeta) \to g_i[(\zeta \epsilon^\sigma)] e^{\sigma(4-d_i)}$, where $\sigma(x)$ as the conformal compensator introduces a flat direction. Obviously, the theory is invariant under scaling transformation if $d_i = 4$. We suppose that at the scale $\geq \Lambda$ the dilaton is formed as the bound state of two gluons, the glueball $\chi = O^{++}$, with the mass $m_\chi \sim O(\Lambda)$. These dilaton fields interact directly each other in the effective theory, which is a consequence of the microscopic hidden gluon interactions. The $\chi$ representing the gluon composition $\langle G_{\mu\nu} G^{\mu\nu} \rangle$ responsible for the QCD trace anomaly [20] may be understood as the string ring solution so that the ends of the string meet each other to form a circle with some finite radius. The glueball becomes massive via the non-vanishing gluon condensation $\langle \chi \rangle \neq 0$ [21]. The characteristic feature of the PT is the very sharp increasing of the correlation length $\xi$. The latter describes the fluctuations of the order parameter $\chi$ and acts as a regulator in the IR with $\xi \sim m_\chi^{-1}$. If we assume that $m_\chi$ is the continuous function of the temperature $T$, then there might be a phase transition at the critical temperature $T_c$ with $m_\chi(T_c) = 0$. The correlation length $\xi$ is not measured directly, however, it influences the fluctuations of the observables, e.g., the primary photons to which the critical mode couples. Actually, in the vicinity of the PT, $\xi$ is much larger than that of the size of the particle interacting region at early times. The dilaton is a mediator between the conformal sector and the Standard Model (SM). However, at high enough temperatures the role of the mediator is smeared, the dilaton becomes massless in the limit in which the conformal symmetry is recovered, $\theta_\mu^\nu = 0$, and the PT is approached.

3. The warm DM Scenario

We consider the model containing dilatons as almost ideal weakly interacting gas (e.g., the glueball gas) at finite temperature. In case of statistical equilibrium at temperature $T = \beta^{-1}$ the partition function for $N$ quantum states is

$$ Z_N = Sp e^{-H\beta}, $$

where $H$ is the Hamiltonian $H = \sum_{1 \leq j \leq N} H(j)$ and $\beta$ in (5) differs from those of the $\beta$-function. For the system of the dilaton functions $\chi_f(x)$ which are regular functions in $f$ representation, one has the equation $H(j) \chi_f(x_j) = F(f) \chi_f(x_j)$, where

$$ H = \sum_f F(f) b^+_f b_f = \sum_f F(f) n_f $$

in terms of the creation and the annihilation operators, $b^+_f$ and $b_f$, respectively; $n_f$ is the occupation number. Here, $F(f) = E(f) - \mu Q(f)$ with $E(f)$ being the energy, $\mu$ is the chemical potential of the dilaton associated with $Q$, which is the operator $N_f$ of particles of the type $f$ with the mean value $Tr\{\rho N_f\} = ˆn_f \Omega$. The $\rho$ is the statistical operator, $n_f$ is the density of particles of the type $f$ in the volume $\Omega$ of the boson star. Note that in the model with a complex scalar dilaton field the chemical potential $\mu$ looks like the
temporal component of a gauge field. The interactions between the dilatons should lead to the thermal equilibrium, and in the case of large \( n_f \) - to the formation of BEC. In principle, the operators \( b_f \) in \( (6) \) can be distorted by the random quantum fluctuations (e.g., by gluons) through the operator \( r_f, b_f \rightarrow b_f = a_f + r_f \), where \( a_f \) is the bare operator. In terms of \( F(f) \) and \( n_f \) the function \( Z_N \) \( (5) \) has the form \( [17] \)

\[
Z_N = \sum_{n_f} e^{-\beta \sum f F(f)n_f}.
\]  

(7)

Since all the operators \( \ldots n_f \ldots \) commute to each other, they may be indentified as the observables. The calculation of \( (7) \) meets the difficulties because of the condition \( \sum_f n_f = N \) in the limit \( N \rightarrow \infty \) in the final stage calculations. This is important because of the particle decays: the dilatons (glueballs) are unstable, they may decay into two (primary) photons which, in case of the photons showers, are registered as a signal that the proximity of PT is approached.

The PT manifests itself through the critical chemical potential \( \mu_c \) and the critical temperature \( T_c \). Let us consider the following power series

\[
P(\bar{\mu}) = \sum_{N=1}^{\infty} Z_N \bar{\mu}^N,
\]  

(8)

which is the function on \( \mu \) and \( \beta \), where \( \bar{\mu} = \mu/\mu_c \). Having in mind \( (7) \) one has

\[
P(\bar{\mu}) = \sum_{n_f} e^{-\beta \sum f F(f)n_f} \bar{\mu} \sum_f n_f = \prod_f \frac{1}{1 - \bar{\mu} e^{-F(f)} \beta}.
\]  

(9)

We consider for simplicity that \( F(f) \geq 0 \) in \( (9) \). Actually, the convergence radius \( R \) of the series \( (8) \) will not be less than 1. In the vicinity of PT \( (\bar{\mu} \simeq 1) \) one has \( \mu_c < E(f)/Q(f) \).

Let us consider \( (8) \) in the form

\[
\frac{P(\bar{\mu})}{\bar{\mu}^N} = \sum_{N'=0}^{\infty} \frac{Z_{N'} \bar{\mu}^{N'}}{\bar{\mu}^N},
\]  

(10)

on the real axis \( 0 < \bar{\mu} < R \). Because of positive \( Z_{N'} \), the function \( (10) \) has the only one minimum on \((0, R)\)

\[
\frac{d^2}{d\bar{\mu}^2} \left[ \frac{P(\bar{\mu})}{\bar{\mu}^N} \right] = \sum_{N'=0}^{\infty} (N' - N)(N' - N - 1) Z_{N'} \bar{\mu}^{N' - N - 2} > 0.
\]

The function \( (10) \) tends to infinity when \( \bar{\mu} \rightarrow 0 \) and when \( \bar{\mu} \rightarrow R \). In the interval \((0, R)\) there is a point \( \bar{\mu} = \bar{\mu}_0 \) at which \( (10) \) has a single minimum, i.e.

\[
\frac{d}{d\bar{\mu}} \left[ \frac{P(\bar{\mu})}{\bar{\mu}^N} \right]_{\bar{\mu} = \bar{\mu}_0} = \sum_{N'=0}^{\infty} Z_{N'} (N' - N) \bar{\mu}_0^{N' - N - 1} = 0.
\]  

(11)

If one goes alone the vertical axis, the ratio \( (10) \) has a maximum at \( \bar{\mu}_0 \). As long as \( \bar{\mu} < \bar{\mu}_0 \), no state with \( Q \neq 0 \) can compete with the vacuum state \((E = 0, Q = 0)\) for the role of the ground state. In case when \( \bar{\mu} > \bar{\mu}_0 \), the point \( \bar{\mu}_0 \) is the ground state at given \( \mu \).
In the phase space the spectrum of "quasi-momenta" $f$ is almost continuous, and there will be an exact continuous spectrum in the limit $\Omega \rightarrow \infty$. The number $\Delta N$ of different $\Delta f$ in volume $\Omega$ is $(\Delta N/\Delta f) = \text{const} \cdot \Omega$. Having in mind that (see (9))

$$P(\bar{\mu}) = \exp \left\{ - \sum_f \ln \left[ 1 - \bar{\mu} e^{-F(f)\beta} \right] \right\},$$

one can find the asymptotic equality for large $N$

$$\sum f \ln \left[ 1 - \bar{\mu} e^{-F(f)\beta} \right] = N \Phi(\bar{\mu}),$$

where

$$\Phi(\bar{\mu}) = \text{const} \cdot v \beta K(\bar{\mu}), \quad v = \frac{\Omega}{N},$$

$K(\bar{\mu})$ is the thermochemical potential of the dilaton $\chi$

$$K(\bar{\mu}) = \beta^{-1} \int \ln \left[ 1 - \bar{\mu} e^{-F(f)\beta} \right] df,$$

which gives the contribution to the thermodynamic potential

$$K = K_\chi + V_\chi + \lambda \left( \frac{f_\chi}{2} \right)^4.$$

Here, $V_\chi$ is the potential term in the LD of the dilaton

$$L_\chi = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V_\chi,$$

$$V_\chi = \frac{\lambda}{4} \chi^4 \left( \frac{\ln \chi}{f_\chi} - \frac{1}{4} \right),$$

where $\lambda = m_{g}^2/f_{\chi}^2$. The term $\lambda(f_{\chi}/2)^4$ in (13) is added so that $K = 0$ at $T = 0$ and $\chi = \langle \chi \rangle = f_\chi$.

In terms of the dilaton (glueball) and the gluon degrees of freedom (d.o.f.) the potential (13) looks like

$$K = \theta(\beta - \beta_c) K_\chi(\bar{\mu}) + \theta(\beta_c - \beta) K_{gH},$$

where $K_{gH} = K_g + K_H$, is an effective gluon thermodynamic potential with the quasi-gluon energy $E_g = \sqrt{[\vec{p}]^2 + m_g^2}$. The effective gluon mass $m_g$ is introduced for the phenomenological reason. $K_H$ is the model-dependent Haar measure contribution (see for details [22]) The potential $K_g$ is the model-dependent function

$$K_g \simeq \frac{m_g^2}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \frac{C_n}{n} K_2(n \beta m_g),$$

where the temperature-dependent gluon mass $m_g = m_g(\beta) = g(\beta)/\beta$, $g(\beta)$ is the effective gauge coupling; the color coefficients $C_n$ are given in [22]; $K_2(x)$ is the Bessel function. The quark contribution in the potential (14) at $T > T_c$ has the negative sign compared to that of the gluon part. The physical result can restrict the number
of quark flavours. At zero chemical potential the combined potential $K(T > T_c)$ will be positive for 3 light quark flavours if the "constituent" mass of the "quasi-gluon", 0.85 GeV and the "constituent" mass of the quark, 0.3 GeV, are used [23]. The second term in (14) yields the first-order PT at the critical point as found in the SU(3) lattice calculations [24,25]. The model potential (14) indicates that the gluons are forbidden below the critical temperature as the coloured degrees of freedom. Both forms (12) and (15) match each other at the PT. For further study where the PT is the well-defined singularity with $T$ and $\mu$ we use the function

$$P(\bar{\mu}) \bar{\mu}^{-N} = \left[\frac{1}{\bar{\mu}^{-1} e^{-\Phi(\bar{\mu})}}\right]^N.$$  (16)

It does not concern the phase diagram to scan the critical point (in QCD) where the position of the latter is not clear from the theoretical side. In our case, the PT is a well-defined singularity with $T$ and $\mu$. In order to find the PT we need to calculate (16) where the relevant singularity will be visible. Let us consider the circle $C$ with the radius $r = \bar{\mu}_0$ with the origin being at zero. To this end, the partition function is

$$Z_N = \frac{1}{2\pi i} \int_C \frac{P(\bar{\mu})}{\bar{\mu}^{N+1}} d\bar{\mu} \rightarrow \frac{1}{2\pi} \int_{-\pi}^{+\pi} \frac{P(\rho e^{i\varphi})}{\rho^N e^{iN\varphi}} d\varphi.$$  (17)

Since $r \neq 0$ the maximum of the function under the integration in (17) is expected at $\varphi = 0$ taking into account the number of particles $N$ in the exponential function in (16). Hence, the calculation of (16) depends on the behaviour of the function under the integration in (17) at $\varphi = 0$. Taking into account the minimum condition (11) we have

$$\frac{\partial}{\partial \varphi} \left[\frac{P(\rho e^{i\varphi})}{\rho^N e^{iN\varphi}}\right]_{\varphi=0} = 0.$$  

The asymptotic form of the partition function is

$$\ln Z_N \simeq -\sum_f \ln \left[1 - \frac{1}{\bar{\mu}_0 e^{-F(\mu)\beta}}\right] - N \ln \bar{\mu}_0 - \ln 2\sqrt{\kappa \pi N},$$

where $\kappa = -\Phi'(\bar{\mu}_0) - \Phi''(\bar{\mu}_0)$. Considering Eq. (11) at $\bar{\mu} = \bar{\mu}_0$, one can find

$$\sum_f \hat{n}_f = \sum_f \frac{1}{\bar{\mu}_0^{-1} e^{F(f)\beta} - 1} = N$$  (18)

and hence, $\bar{\mu}_0$ can be extracted from (18). One can calculate the sum of the quantum states up to the singular point defined by the relation between $\bar{\mu}_0$ and $F\beta$, and large $N$. We assume the large number $N$ in (18) which is correct if the dilatons are light. This is important in the sense to the proposal of condensed dark matter bosons in the early stage after the heavy ion collisions. The latter in some sense corresponds to Bose star formation as the lamps of BEC bounded by self-gravity [26].

4. The condensation and the fluctuations

The hidden scalar (glueball) particles could have the correct relic density and be non-relativistic enough. As the temperature of the hidden scalar sector lowers below $\sim \Lambda$,
the dilatons can act as a classical form of dark matter in the late Universe. In this case, the dilaton becomes the warm DM with the density \( \sim \exp(-\Lambda \beta) \) which follows down very rapidly. The dilatons are unstable and they decay to the primary (direct) photons or/and to the dark photons. There are almost negligible interactions between the dilatons and their annihilations weakly influence the final number of DM particles. Consider the model where the dilatons are produced in the volume \( \Omega \) as a cube with the side of the length \( L = \Omega^{1/3} \). We assume the wave function of the dilaton in the form \( \phi_p(q) = \Omega^{-1/2} e^{i q p} \), where \( p^\alpha = (2\pi/L) n^\alpha, \alpha = 1, 2, 3; n^\alpha = 0, \pm 1, ... \) and the energy is \( E_p = |p|^2/(2m_\chi) \) (in the units with the Planck constant \( h = 1 \)). We consider two cases: the high temperature case A), where \( \bar{\mu}_0 e^{\mu Q_3} < 1 \), and the low temperature case B), where \( \bar{\mu}_0 e^{\mu Q_3} \sim 1 \).

In the case A), the function \( \bar{n}_f \) is regular on \( f \), and the sum \( \sum_f \bar{n}_f \) is replaced by the integral
\[
\frac{1}{v} = \frac{1}{\Omega} \sum_f \bar{n}_f \to \frac{1}{(2\pi)^3} \int \bar{n}(f) d^3f.
\]
Using (18) we arrive at the equality:
\[
\int_0^\infty \frac{x^2 dx}{\bar{\mu}_0^{-1} e^{-\mu Q_3 x^2} - 1} = \frac{2\pi^2}{v} \left( \frac{\beta}{2m_\chi} \right)^{3/2}.
\]
(19)

In the case A), the integral in the l.h.s. of (26) increases if \( \bar{\mu}_0 e^{\mu Q_3} \) increases as well. The Eq. (26) has the solution in terms of \( \mu Q_3 \) and \( \bar{\mu}_0 \) when
\[
\frac{2\pi^2}{v} \left( \frac{\beta}{2m_\chi} \right)^{3/2} < \frac{\sqrt{\pi}}{4} B,
\]
(20)

where the r.h.s. of inequality (20) is the result of calculation of the integral in the l.h.s. of (26) under the condition \( T \to \mu Q \ln^{-1}(\bar{\mu}_0^{-1}) \), and \( B = 2, 612... \) is the Riemann’s zeta-function, \( \zeta(3/2) \). The case A) is realised when the temperature \( T \) exceeds the critical one, \( T > T_c \), where the critical temperature is defined by the dilaton mass and the inverse density \( v = \Omega/N \)
\[
T_c = \frac{2\pi}{m_\chi} (v B)^{-2/3}, \quad m_\chi \neq 0.
\]

One can easily find that the correlation length \( \xi \) is defined by \( \mu \) and has the dependence of \( N \) (through \( v \)). The singular behaviour of \( \xi \) is governed by the ground state \( \bar{\mu}_0 \):
\[
\xi \sim \frac{\mu Q}{2\pi} (v B)^{2/3} \ln^{-1} \left( \frac{1}{\bar{\mu}_0} \right).
\]
(21)
The fluctuation of the dilaton mass become more correlated, and the length scale increases toward infinity when the ground state \( \bar{\mu}_0 \to 1 \). The fluctuations at \( T << T_c \) have short correlation length (\( \bar{\mu}_0 < 1 \)). For finite and small \( v \) the correlation length \( \xi \) can not be definitely developed to cause the PT. Under the thermal influence the non-monotonic behaviour of \( \xi \) is assumed to be as an indicator of the PT (see also [27], [28] for the case of the QCD phase diagram with the critical point).
In the case B) our interest is in small $|p| \leq \delta$ (maximal $N$), where the CP is approached. Here,
\[
\frac{1}{\Omega} \sum_{|p| \leq \delta} \bar{n}_p = \frac{1}{v} - \frac{1}{\Omega} \sum_{|p| \geq \delta} \bar{n}_p,
\]
where
\[
\lim_{\delta \to 0, \ N \to \infty} \frac{1}{\Omega} \sum_{|p| \leq \delta} \bar{n}_p = \frac{1}{v} - \frac{1}{\Omega} \sum_{|p| \geq \delta} \bar{n}_p = \frac{1}{\Omega} \left[ 1 - \left( \frac{\beta_c}{\beta} \right)^{3/2} \right].
\]
Hence, the case B) takes place for $T < T_c$, where the only part of the total number of particles proportional to $\sim \left( \frac{\beta_c}{\beta} \right)^{3/2}$ is distributed on all the spectrum of momenta. The rest one $\sim \left[ 1 - \left( \frac{\beta_c}{\beta} \right)^{3/2} \right]$ is the scalar condensate. As the result, in the case of high temperatures, the condensates stay close to almost zero, while at low temperatures the condensate obtains a large value.

Now, one can connect the results for the fluctuations of $\chi$ to the fluctuations of observables. To this, we suppose that the particles are in the local volume $V$ of the boson star which is less than $\Omega$. The number of particles $n_V$ in $V$ is $\sum_{1 \leq j \leq N} \hat{n}_V(q_j)$, where $\hat{n}_V(q) = 1$ if $q \in V$, and $\hat{n}_V(q) = 0$ otherwise. The volume $V$ is defined by the geometry of the boson star. The event-by-event fluctuation of the particle density $\langle (n_V - \langle n_V \rangle)^2 \rangle$ with temperature is
\[
\langle (n_V - \langle n_V \rangle)^2 \rangle = 1 = \frac{\sqrt{2}}{\pi^2} (m_{\chi} T)^{3/2} \int_0^\infty \frac{x^2 dx}{(\bar{\mu}_0 e^{-\mu Q} e^{x^2} - 1)^2},
\]
where $\langle n_V \rangle = V/\Omega$. Obviously, (22) increases sharply when the temperature $T \to \mu Q/\ln(1/\bar{\mu}_0)$. When the PT approaches, the fluctuation (22) at the CP is
\[
\langle (n_V - \langle n_V \rangle)^2 \rangle = 1 = 4 \sqrt{\frac{\pi B}{z_c}} \int_0^\infty \frac{x^2 dx}{(z_c e^{x^2} - 1)^2},
\]
where $z_c = \bar{\mu}_0^{-1} e^{-a_c}$, $a_c \simeq \mu_c Q \Lambda (vB)^{2/3}/(2 \pi)$. One can expect the non-monotonous rising of the fluctuation (22) when the ground state $\bar{\mu}_0 \to 1$ with $v \to 0$ at infinite number $N$ of particles. There are no free parameters in (23) when the CP is approached.

5. The primary (direct) photons

In the exact scale symmetry, $\chi$ couples to the SM particles via the trace of $\theta_{\mu\nu}$
\[
L = \frac{X}{f_X} \left( \theta^\mu_{\mu_{tree}} + \theta^\mu_{\mu_{anom}} \right),
\]
where the first term in (24) is (the contributions from heavy quarks and heavy gauge bosons are neglected)
\[
\theta^\mu_{\mu_{tree}} = - \sum_q [m_q + \gamma_m(g)] \bar{q} q - \frac{1}{2} m_{\chi}^2 \chi^2 + \partial_\mu \chi \partial^\mu \chi.
\]
Here, $q$ is the quark field with the mass $m_q$; $\gamma_m$ are the corresponding anomalous dimensions. In contrast to the SM, the dilaton couples to massless gauge bosons even
before running any SM particles in the loop, through the trace anomaly. The latter has the following term for photons and gluons:

$$\theta_{\mu}^{\text{anom}} = -\frac{\alpha}{8\pi} b_{\text{EM}} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha_s}{8\pi} \sum_i b_{0 i} G_{\mu\nu}^a G^{\mu\nu a}, \quad (25)$$

where $\alpha$ is the fine coupling constant, $b_{\text{EM}}$ and $b_{0 i}$ are the coefficients of the electromagnetic (EM) and the QCD $\beta$-functions, respectively. If the strong and the EM interactions are embedded in the conformal sector the following relation for light and heavy particles sectors is established above the scale $\Lambda$ (in UV): $\sum_{\text{light}} b_{0} = -\sum_{\text{heavy}} b_{0}$, where the mass of $\chi$ splits the light and heavy states. The anomaly term for gluons in $(25)$ is evident, where the only $n_L$ particles lighter than $\chi$ are included in the $\beta$ - function, $\beta(g) = b_{0}^{\text{light}} g^3/(16\pi^2)$, $g^2 = 4\pi\alpha_s$. For $m_\chi \sim O(\Lambda)$ one has $n_L = 3$ that indicates about 14 times increase of the dilaton-gluon-gluon coupling strength compared to that of the SM Higgs boson.

The light dilaton operates with the low invariant masses, where two photons are induced effectively by gluon operators. In the low-energy effective theory, valid below the conformal scale $\sim 4\pi f_\chi$, at small transfer-momentum $q$, $\langle \gamma\gamma|\theta_{\mu}^{\text{anom}}(q)|0\rangle \simeq 0 \ [29]$ and $\langle \gamma\gamma| b_{0}^{\text{light}} G_{\mu\nu}^a G^{\mu\nu a}|0\rangle = -\langle \gamma\gamma| b_{\text{EM}}^{\text{light}} F_{\mu\nu} F^{\mu\nu}|0\rangle, \bar{q} = 0$.

The partial decay width $\chi \rightarrow \gamma\gamma$ is

$$\Gamma(\chi \rightarrow \gamma\gamma) \simeq \left(\frac{\alpha}{4\pi}\right) \frac{m_\chi^3}{16\pi f_\chi^2}, \quad (26)$$

where the only CA contributes through

$$F_{\text{anom}} = -(2n_L/3)(b_{\text{EM}}/b_{0}^{\text{light}}), \quad b_{\text{EM}} = -4 \sum_{u,d,s} e_q^2 = -8/3,$$

eq$ is the charge of the light quark. When one approaches the PT (the first-order transition) the absolute value of $F_{\text{anom}}$ decreases due to increasing on $b_{0}^{\text{light}}$ as $n_L \rightarrow 0$. There are fluctuations of the dilaton field with the finite mass $\sim O(\Lambda)$ which is model-dependent. In the proximity to the IRFP the dilaton mass is $m_\chi \simeq \sqrt{1 - N_f/N_c^2}\Lambda$ [15], where $N_f$ is the critical value of $N_f$ corresponding to $\alpha_s^c$ at which the chiral symmetry is breaking and the confinement arises. On the other hand, from lattice calculations [30,31], the lightest glueball masses approach a constant at large number $N$ of colours in the hidden $SU(N)$ sector, and can be well defined as $m = (\alpha + \beta/N^2)\Lambda$. Using these two parameterisations, one can conclude that $\alpha$ should be less than 1, and $\beta$ is order one parameter. The constant $\lambda \simeq 113$ in the thermodynamic potential [13] is fixed using the glueball mass $m_\chi = 1.7 \text{ GeV}$ [23] and the vacuum energy density $E = \lambda(f_\chi/2)^4 = 0.6 \text{ GeV \ fm}^{-3}$ [32]. On the other hand, $\lambda \sim O(1)$, $\lambda \simeq (1 - N_f/N_c^2) \simeq (\alpha + \beta/N^2)^2$, if $m_\chi \sim O(\Lambda)$ and $f_\chi \simeq \Lambda$. 

Because of the strong couplings of the dilatons to gluons and photons one can expect the abundant production of the dilatons (glueballs) due to gluon-gluon fusion and the decays of the dilatons (glueballs) to primary photons. In the early Universe, the scalar glueballs are unstable and it is expected the showers of primary photons including the dark photons [33] due to conformal gluon and electromagnetic anomalies. The measurement of the photons escape in the early stage of heavy-ion or proton-proton (pp) collisions provides a decisive way to observe and to differentiate the direct (primary) photons and the ordinary photons in the decays of the secondary produced light hadrons, e.g., $\pi^0 \rightarrow \gamma \gamma$. The trace-anomaly term (25) contributes for dilatons but not for light hadrons. It is only necessary to count the event numbers of $\gamma \gamma$ in the heavy-ion or pp collisions at different energies. In the confinement stage, the effects of the explicit breaking the scale symmetry are dominant, and it is assumed that the confinement triggers the spontaneous breaking the chiral symmetry for light quarks. One can estimate the fluctuation rate of the photon production in the approximate conformal sector (the proximity of the PT) via the rate

$$r_{\gamma\gamma} = 1 + \frac{BR(\pi^0 \rightarrow \gamma \gamma)}{BR(\chi \rightarrow \gamma \gamma)} = 1 + m_\pi^3 \left( \frac{6}{F_{anom}} \right)^2 \xi^3,$$

(27)

where $BR(P \rightarrow \gamma \gamma)$ is the branching ratio in the process $P \rightarrow \gamma \gamma$ ($P : \pi^0, \chi$). At the PT one can expect the sharp rising of the fluctuation rate $r_\chi$ in the IR ($\alpha_s^* > \alpha_s^c$) relevant to direct photons where at large distances we use the effective d.o.f. in terms of neutral $\pi^0$-mesons with the mass $m_\pi$. The abundant escape of the photons will be as $\xi(T \rightarrow T_c) \rightarrow \infty$ and $N_f \rightarrow N_f^c$. The critical value $N_f = N_f^c$ separates the conformal phase from the one with confinement and massless quark formation in the frame to the chiral gauge theory. The method is independent of the values of the model parameters where for an order of magnitude one can take $f_\chi \simeq \Lambda$ and the pion constant $f_\pi \simeq 0.3 \Lambda$.

The result (27) is consistent with the physical pattern where the dilaton is emerged at the scales $\geq \Lambda$ as well as $\pi^0$'s and other light quark bound states. It is easily to find that at the PT, $r_\chi \rightarrow \infty$ when the number of light quarks $n_L \rightarrow 0$. Thus, one can expect to find the non-monotonous raising on the fluctuations of primary photons once is going away from UV to IR. The contribution with (27) is counting by the detection of (primary) photons which indicate the region of the PT and the critical point where the escape and detection of the photons are maximal. The measurement of the photon fluctuations can be used to determine whether the quantum system is in the vicinity of the PT or not.

6. Boson stars

In the beginning of this section let us note that the scalar dilaton in the form of the glueball is a well-motivated example coming from a pure Yang-Mills hidden sector, which locks up into the bound state of two gluons in the early Universe. The glueball DM may condense into the boson stars and be observed by gravitational lensing effect. In the paper [6], there was presented the direct coupling of the hidden sector associated
with the scalar glueball field $\chi$ in the boson star to the photons in the hidden sector of the SU($N$) gauge theory with an unspecified value of $N$

$$\frac{1}{M_{\text{cut}}^4} H_{\mu\nu} H^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \rightarrow \frac{N m^3}{M_{\text{cut}}^4} \chi F_{\alpha\beta} F^{\alpha\beta},$$

where $H_{\mu\nu}$ and $F_{\alpha\beta}$ are the strength tensors of the hidden gauge field of the group SU($N$) and the photon, respectively; $M_{\text{cut}}$ is the cutoff scale; $m$ is the mass of the glueball. The decay rate of $\chi$ into two photons in the point-like (direct) interaction in the star is [6]

$$\Gamma(\chi \rightarrow \gamma\gamma) = \frac{1}{4\pi} m N^2 \left(\frac{m}{M_{\text{cut}}}\right)^8,$$

(28)

where the value $N$ making $\chi$ a self-interacting hidden matter is $N \approx \text{Max}[0.1\text{GeV}/m]^{3/4}, 2]$. The combined result in the conformal anomaly (26) and the direct interaction (28) between the scalar glueball and the photons gives the strongest constraints on the scale $M_{\text{cut}}$ with the DM mass in the MeV's scale. We find that for the DM mass $m \sim \mathcal{O}(\Lambda)$ the cutoff $M_{\text{cut}}$ is allowed to be as low as the weak scale: $M_{\text{cut}} \geq 3.4$ GeV and $M_{\text{cut}} \geq 5.2$ GeV for $\Lambda = 330$ MeV and $\Lambda = 500$ MeV, respectively.

7. Conclusions

To conclude, we investigated the possible evidence of the DM candidate from the approximate scale symmetry, the theory of the hidden scalar sector. The DM is the lightest hidden scalar field which is likely the dilaton. The latter could be warm and may have the novel feature of BEC into compact massive boson stars. We proposed the combined method which matches the effective model of the dilaton in terms of the glueball to the one of free gluons at the PT. We find the PT is achieved at high chemical potential $\mu$ (the case B) with smaller particle momentum (and, hence, the energy). There is the sharp increasing of the fluctuation of the particle density at high ratio $T/\mu \sim \ln^{-1}(\bar{\mu}/0)$ (22).

The boson stars are unstable and the showers of the photons can be registered because of unstable dilatons (glueballs) decaying to primary photons. These decays could contribute to new sources of cosmic rays. In the laboratory experiments, the PT can be found as those followed by IRFP where the primary photons are detected. The origin of these photons is the CA via the decays of the dilatons. When the incident energy scans from high to low values, one can find the non-monotonous behaviour in fluctuations of primary photons: the fluctuations rate $r_{\gamma\gamma}$ grow up in the IR to become very large at the PT (27). The PT and the CP have the very clear signature: the shower increasing of the photons flow in the detector compared to that produced by light hadrons. The information about the event with the PT for the given experimental conditions can be obtained by measuring the ratios of $\gamma$-quanta yields and compared (fitting) to known model with $T$ and $\mu$. 


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