A comment on bouncing and cyclic branes in more than one extra-dimension

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We argue that bouncing branes occur naturally when there is more than one extra-dimension.

We consider three-branes embedded in space-times with a horizon and an isometry group $SO(6)$. As soon as the brane angular momentum is large enough, a repulsive barrier prevents the branes from reaching the horizon. We illustrate this phenomenon with the case of D3-branes in an AdS$_5$-Schwarzschild $\times S_5$ background and asymptotically flat space-time.

Recently there has been renewed interest in the study of oscillating (cyclic) and bouncing universes\textsuperscript{[1, 2]} motivated in part by different brane world scenarios\textsuperscript{[3]}. In these studies only one extra dimension is typically considered, and matter on the brane is neglected so that the motion is solely induced from the bulk. When there is one extra dimension (or equivalently, when the brane is only free to move in one extra dimension), a number of different bulk space-times have been shown to lead to bouncing and oscillating universes, for instance in the case of a charged AdS$_5$ blackhole\textsuperscript{[4]} and the Klebanov-Strassler background\textsuperscript{[5]}.

The purpose of this short note is simply to comment on the generic occurrence of bouncing branes when there is more than one extra dimension in which the brane can move. To illustrate this point we consider the example of a brane moving in AdS$_7$-Schwarzschild $\times S_5$. The dynamics are parametrised by a conserved angular momentum on $S_5$\textsuperscript{[6]} reminiscent of a point particle moving in a central well. If the angular momentum is large enough a centrifugal potential develops at small scales off which the brane universe ‘bounces’. Furthermore if the spatial sections of the brane have positive curvature then, as we illustrate below, a repulsive potential also develops at large scales. The brane may then be bound between the two ‘bumps’ leading to an oscillating or cyclic universe\textsuperscript{[7]}. We also consider the case of asymptotically flat universes where the same bouncing behaviour is generic.

Consider a D3-brane moving in Sch-AdS$_7$ $\times S_5$ spacetime which is the near horizon limit of a BPS D3-black-brane. The bulk metric in the Einstein frame is given by

\begin{equation}
\begin{aligned}
    ds^2 &= g_{00}(r)dt^2 + g_{ij}(r)dx^i dx^j + g_{rr}(r)dr^2 + g_s(r)d\Omega^2_5 \\
    &= \frac{y^2}{L^2} (-f(r)dt^2 + \chi_{ij} dx^i dx^j) + \frac{L^2 dr^2}{f(r)r^2} + L^2 d\Omega^2_5
\end{aligned}
\end{equation}

where $L$ is the AdS curvature, $\chi_{ij}$ ($i = 1, 2, 3$) is the metric on a space of constant curvature $k = 1, 0, -1$ (closed, flat, open respectively), and

\begin{equation}
f(r) = k \frac{L^2}{r^2} + 1 - \left( \frac{rH}{r} \right)^4.
\end{equation}

The metric (1) is a solution of the ten dimensional supergravity equations with corresponding bulk 4-form field

\begin{equation}
C_{0123} = \sqrt{\chi} \left( - \left( \frac{r}{L} \right)^4 + \frac{r_H^4}{2L^4} \right)
\end{equation}

and a constant dilaton $\Phi$ (which we set to zero). It will be convenient to work with the dimensionless variables

\begin{equation}
y = \frac{r}{L}, \quad r_0 = \frac{r_H}{L}
\end{equation}

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so that for \( k = 0 \) the horizon is at \( y = r_0 \). When \( k = \pm 1 \) the horizon is at \( 2y = -k + (1 + 4r_0^2)^{1/2} \). Later we will comment briefly on the case \( r_0 = 0, k = -1 \), namely open \( \text{AdS}_5 \times S_5 \) which has a horizon at \( y = 1 \).

The D3-brane dynamics are governed by the Dirac-Born-Infeld action

\[
S = -T \int d^4x e^{-\Phi} \sqrt{-\det(\gamma_{\mu\nu})} - qT \int d^4x \hat{C}_4
\]

(5)

where \( T \) is the brane tension, \( q = (-)1 \) for a BPS (anti-) brane, and \( \gamma_{\mu\nu}, \mu = 0 \ldots 3 \) is the induced brane metric in the string frame. As mentioned above we set all matter on the brane to zero so that the pull-back of the Neveu-Schwarz anti-symmetric two-form \( B_{\mu\nu} \) as well as the worldvolume anti-symmetric gauge fields \( F_{\mu\nu} \) are assumed to vanish. In the static gauge, the induced metric is given by

\[
\gamma_{00} = e^{\Phi/2} \left( g_{00} + g_{rr} \dot{t}^2 + g_r \dot{\phi}^2 \right), \quad \gamma_{ij} = e^{\Phi/2} g_d \chi_{ij}
\]

where \( \dot{} = d/dt, \phi^2 = h_{pq} \dot{\phi}^p \dot{\phi}^q \) with \( h_{pq} \) (\( p = 1, \ldots, 5 \)) the metric on the 5-sphere, and \( r(t), \phi(t) \) are the brane position at time \( t \). These can therefore be determined through the Lagrangian \( \mathcal{L} \) defined by

\[
S = TV_3 \int dt \mathcal{L}, \quad \mathcal{L} = -\sqrt{a + b\dot{t}^2 + c\dot{\phi}^2 + e}
\]

where \( V_3 = \int d^3x \sqrt{-\chi} \) is the spatial volume of the probe, and

\[
a \equiv -g_{00}g_3^3 = y^8f(y), \quad b \equiv -g_{rr}g_3^3 = -\frac{y^4}{f}, \quad c \equiv -g_3^3g_s = -y^6L^2, \quad e \equiv -q\dot{C}_4 = q(y^4 - \frac{r_0^4}{2}). \quad (8)
\]

(The backreaction of the brane on the bulk is neglected \[9\].) As discussed in \[9\], it follows from \[7\] that there is a corresponding conserved positive energy \( E \) and angular momentum \( \ell \) of the brane around the \( S^5 \) from which

\[
\dot{\phi}^2 = \frac{a^2\ell^2}{c^2(E + e)^2}, \quad \dot{t}^2 = -\frac{a}{b} \left[ 1 + \frac{a}{c} (\ell^2 - c) \right].
\]

(9)

For an observer living on the brane, however, the brane time \( \tau \) and the scale factor \( S(\tau) \) are given by

\[
dS_3^2 = -dr^2 + S^2(\tau) \chi_{ij} dx^i dx^j \longmapsto dr^2 = -\gamma_{00}(t)dt^2, \quad S^2 = g_4(t) = y^2 = (r/L)^2
\]

(10)

From Eqs. \(8\) and \(10\) we find

\[
\frac{dr}{d\tau} = \frac{a^2dt^2/(E + e)^2g_3^3}{S^2} \text{ so that}
\]

\[
\left( \frac{dr}{d\tau} \right)^2 = \frac{1}{abc} g_3^3 [c(E + e)^2 + a(\ell^2 - c)].
\]

(11)

Thus the Friedman equation on the brane, \( H = \frac{1}{3} \frac{dS}{d\tau} \), is given by

\[
H^2 = \frac{q^2}{L^2} - \frac{k}{a^2} + \frac{r_0^2L^2}{a^4} + L^6 \left( \frac{\dot{E}}{a^2} + \frac{2q}{L^4} \right) + \ell^2 \frac{L^4}{a^6} \left( -\frac{k}{a^2} + \frac{r_0^2L^2}{a^4} - \frac{1}{L^2} \right)
\]

(12)

where \( a(\tau) = S(\tau)L = r(\tau) \) is the dimensionful scale factor (not to be confused with the parameter \( a \) in \[8\]) and \( \dot{E} = E - qr_0^2/2 \). The \( \ell \)-independent terms are familiar from non-\( Z_2 \)-symmetric brane world cosmology \[11\] with \( \dot{E} \) being a measure of \( Z_2 \) symmetry breaking \[11\]. Note in \[12\] the new \( \ell \) dependent terms coming from the \( S_5 \) whose effect we want to study. Observe also that the effective cosmological constant on the brane vanishes for BPS branes \( q = \pm 1 \). In the context of brane-worlds, this is the Randall-Sundrum fine tuning condition for the bulk and brane cosmological constants. As in four dimensional cosmology there is a curvature term. Moreover there is also a “dark radiation” term arising from the existence of a horizon in ten dimensions. From now on we focus only on BPS branes.

Rather than solving directly for \( a(\tau) \) from \[12\] it is simpler to consider the effective potential for the brane motion, \( V^r = E - \frac{1}{2}(dr/d\tau)^2 \), which is given by

\[
V_{\text{eff}}(k, y, \ell, E) = E + \frac{1}{2y^6} \left[ fy^2 \left( \ell^2 + y^6 \right) - \left( \dot{E} + qy^4 \right)^2 \right]
\]

(13)

where \( f = f(k, y) \) is given in \[2\] and we have defined the dimensionless angular momentum

\[
\hat{\ell} = \frac{\ell}{L}
\]

(14)
reaches a circular orbit with corresponding constant scale factor $y$ as well as the definition of $a$. Not reach infinity due to (15). The minimum value of $S$ for $k = 0$ and in figure 1b for $k = 0$, so that from (13) there is an upper bound on the brane scale factor $y$. Therefore for $k = 0$ a bounce can occur. Let $\ell = 0$ the brane may reach infinity since $E^r V^r_{\text{eff}} = 1$, an incoming (contracting) universe will bounce off the centrifugal potential if $\ell = \ell_c$, where $\ell_c$ is the critical angular momentum for which $V^r_{\text{eff}} = E$ independently of $k$ meaning that the vicinity of the horizon is an allowed region. Finally, notice that the coefficient of the $\hat{\ell}^2$ is positive and given by $f y^2/2$. Thus for sufficiently large $\hat{\ell}^2$ and in some intermediate range of $y$, we expect $V^r_{\text{eff}} > E$ which is the centrifugal barrier. This behaviour is shown in figure 1a for $k = 0$ and in figure 1b for $k = 1$.

Therefore for $k = 0, -1$, an incoming (contracting) universe will bounce off the centrifugal potential if $\ell > \ell_c$, where $\ell_c$ is the critical angular momentum for which $V^r_{\text{eff}} = E$ (see figure [a] and below). The bounce occurs at an $\ell$-dependent minimum value of $S_m(\tau) = y_m(\tau)$ after which the brane moves radially outwards and expands. For $\ell = \ell_c$, the brane reaches a circular orbit with corresponding constant scale factor $y_c$, and for $\ell < \ell_c$ the brane falls into the horizon. For $k = 1$, on the contrary, the bounce turns into a cyclic or oscillating universe (see figure [b]) since the brane may not reach infinity due to (13).

In some cases the equations of motion are integrable. Consider the brane trajectory $r(\phi)$. Using equations (3), as well as the definition of $a, b, c$ and $e$ in [3], leads to (for $q^2 = 1$)

$$\phi = \frac{\pm \hat{\ell}}{2} \int_{z(\phi)}^{z_0} \frac{dz}{\left[-k z^4 + 2q E z^3 - \hat{\ell}^2 z^2 + (E^2 - k \hat{\ell}^2) z + \hat{\ell}^2 r_0^4\right]^{1/2}}, \quad (16)$$

where $z = y^2$, we have set $\phi(z_0) = 0$ where $z_0$ is the initial position of the brane, and the $+/−$ sign corresponds to an initially ingoing/outgoing brane. (These are initial conditions.)

**Flat universe: $k = 0$**

When $k = 0$ a bounce can occur. Let $z_0 \to \infty$ and consider an incoming brane. Then a straightforward change of variables allows equation (14) to be written in the form $\phi = \int_0^\infty dx (4 x^3 - g_2 x - g_3)^{-1/2}$ where $\wp(\phi, g_2, g_3)$ is the Weierstrass function (10). Thus

$$\frac{r^2}{L^2} = y^2 = \frac{\hat{\ell}^2}{2qE} \left[\frac{1}{3} + \wp(\phi, g_2, g_3)\right], \quad (17)$$

where

$$g_2 = -4qE \left(\frac{\hat{E}^2}{\hat{\ell}^4} - \frac{1}{6qE}\right), \quad g_3 = -4qE \left(\frac{2r_0^3 E^2}{\hat{\ell}^4} + \frac{\hat{E}^2 q E}{\hat{\ell}^4} - \frac{1}{2\tau}\right). \quad (18)$$

**FIG. 1:** LH panel: $V^r_{\text{eff}}$ for a flat universe, $k = 0$ and $E = 20$. RH panel: $V^r_{\text{eff}}$ for a closed universe, $k = 1$ and $E = 10$. In each case $q = +1$ and the central curve (blue, labeled ii) has $\ell = \ell_c$ ($\sim 16$ for $k = 0$ and $\sim 8$ for $k = 1$), the lower curve (red, labeled iii) has $\ell < \ell_c$, and the upper curve (green, labeled i) has $\ell > \ell_c$. Note that in the LH panel $V^r_{\text{eff}} \to E$ as $r \to \infty$ as expected from (15). In the RH panel $V^r_{\text{eff}} \to E + 1/2$. Notice also the uppermost curve (brown, labeled iv) in the RH panel: if $\ell$ is sufficiently large then there is neither a bounce nor a cyclic universe.
Thus a bounce can only occur for \( \ell \) curve of figure 1b which has \( \ell > \ell_c \) for the nature of the denominator of (16). There may now be different pairs of repeated roots as can be seen from the upper curve of figure 1a. (Parametric plot of \( y(\tau) \) and \( \phi(\tau) \), with initial condition placing the ingoing brane at \((4, 3)\).) The central dark circle is the blackhole horizon at \( r_0 = 1 \). The trajectory of a brane with a large angular momentum (blue curve, labeled \( i \)) is only slightly deflected by the blackhole. As \( \ell_c \) decreases (green and then red curve, labeled \( ii \) and \( iii \) respectively) the trajectory is more and more eccentric. For \( \ell < \ell_c \), the brane falls into the blackhole horizon. This is represented by the black curve (labeled \( iv \)) which has \( \ell \leq \ell_c \). RH panel: corresponding plot of the brane scale factor \( y(\tau) \) clearly showing the bounce.

The properties of the elliptic function \( \varphi(\theta; g_2, g_3) \) depend on the sign of the discriminant

\[
\Delta = g_2^3 - 27g_3^2. \tag{19}
\]

Since \( g_2 \) and \( g_3 \) are both \( \ell \) and \( E \) dependent, imposing \( \Delta = 0 \) for a given \( E \) will determine a given angular momentum: this is the critical angular momentum \( \ell_c \) mentioned above and shown in figure 1. For large \( E \gg r_0 \) (or equivalently in flat \( \text{AdS}_5 \times S_5 \)) we find

\[
\Delta \sim (\ell^4 - 8E^3) \tag{20}
\]

so that \( \ell_c \sim 8^{1/4}E^{3/4} \) — the exact expression valid for all \( r_0 \) is given in the appendix of [8]. For \( \ell > \ell_c \), \( \Delta > 0 \) and \( \varphi(\theta; g_2, g_3) \) is a periodic function whose period determines the scattering angle \( \phi_s \) at which the outgoing brane is scattered off the centrifugal barrier. For \( \Delta < 0 \) the brane may reach the horizon. The parametric plot of \( r(\phi) \) given in (17) is shown in figure 3b where the blackhole horizon is the solid circular line. When \( \ell < \ell_c \) the orbiting brane is seen to spiral into the blackhole horizon in a finite \( \tau \)-time. The other curves are branes with \( \ell > \ell_c \) which follow hyperbolic trajectories with decreasing eccentricity as \( \ell \) increases. In the critical case \( \ell = \ell_c \) the brane reaches the circular orbit whose radius \( r_c \) can be determined from the properties of \( \varphi \) for \( \Delta = 0 \):

\[
\frac{r_c^2}{L^2} = \frac{\hat{\ell}_c^2}{2qE} \left[ \frac{1}{3} - \frac{3}{2} \frac{g_3}{g_2} \right] \quad \text{where} \quad g_2^3 - 27g_3^2 = 27g_3^2. \tag{21}
\]

Finally, one can use (3) and (17) to obtain \( \eta(\phi) \) where \( \eta \) is brane conformal time:

\[
\eta = \frac{1}{\ell} \int_0^\phi y^2(\phi) = \frac{\hat{\ell}}{2qE} \left[ \frac{\phi}{3} + \zeta(\phi; g_2, g_3) \right], \tag{22}
\]

where \( \zeta \) is the Weierstrass zeta function [11]. In figure 3b we have converted back to \( \tau \)-time and plotted the brane scale factor \( y(\tau) \) for the different trajectories of figure 3a. The bounce is clearly observed when \( \ell > \ell_c \).

Closed universe \( k = 1 \).

As noted above and seen in figure 3b, it is now possible to have a cyclic universe if \( \ell > \ell_c \). The corresponding brane trajectory and scale factor for such a cyclic universe is shown in figure 3 which has been determined numerically since the equations of motion are no longer integrable. Generally these cyclic trajectories do not close on themselves. A further important difference between this closed case and the \( k = 0 \) one discussed above comes from the quartic nature of the denominator of (16). There may now be different pairs of repeated roots as can be seen from the upper curve of figure 3b which has \( \ell > \ell_m \): in this regime \( V_{\text{eff}}^r > E \) for all \( y \) except in a very small vicinity of the horizon. Thus a bounce can only occur for \( \ell_c < \ell < \ell_m \).
FIG. 3: LH panel: The cyclic brane trajectory in a closed universe $k = 1$, corresponding to the potential of figure 1b. Here only one trajectory is drawn—in red, the trajectory from $\tau_0 = 0$ to $\tau_0 + 50$, in green the trajectory from $\tau_0 + 50$ to $\tau_0 + 100$. The trajectories are generally not closed. RH panel: $y(\tau)$ for the brane. The cyclic nature is clearly observed.

Comments on $AdS_5 \times S_5$.

An interesting limit to take is $r_0 = 0$, namely a brane moving in $AdS_5 \times S_5$. Let us focus in particular on the static limit $E = 0$. Then the effective potential $V^\tau_{\text{eff}}$ of (13) is given by

$$V^\tau_{\text{eff}} = \frac{1}{2} \left[ k + \hat{\ell}^2 y_6 (y^2 + k) \right].$$

(23)

Thus, as expected, the potential is flat when $\hat{\ell} = 0$. In that case, the brane has no kinetic energy (i.e. is static) when $k = 0$, and constant kinetic energy when $k = -1$. In the closed case $V^\tau_{\text{eff}} > E$ always.

When $k = -1$ there are non-trivial effects when $\hat{\ell} \neq 0$. From (2) the horizon is at $y_h = 1$ where $V^\tau_{\text{eff}} = -1/2$. Once more the coefficient of $\hat{\ell}^2$ is positive so that we expect a centrifugal barrier. Indeed the potential is similar to that of figure 1a and again we find a bouncing universe with $\hat{\ell}_c = \sqrt{27}/2$.

Asymptotically Flat Space-time

There is a large class of spacetimes for which bouncing branes generically occur. Consider the dynamics of a brane probe embedded in an asymptotically flat spacetime whose isometry group is assumed to contain $SO(6)$. Thus the background metric (in the Einstein frame) is

$$ds^2 = e^{2A(r)}(-dt^2 + d\bar{x}^2) + e^{2B(r)}(dr^2 + r^2 d\Omega_5^2),$$

(24)

where we also assume that the dilaton $\Phi(r)$ and the background RR field $C_{0123}(r) \equiv e^{\Lambda(r)}$ depend only on $r$ and vanish at infinity. We take the coordinate system to be valid with $r \in [r_H, \infty]$ where $r_H$ is an horizon such that $e^{2A(r_H)} = 0$. Then from (8)

$$a = e^{8A}, \quad b = -e^{2(3A+B)} = \frac{c}{r^2}, \quad e = -qe^\Lambda,$$

(25)

$r^2$ is given in (8) and the brane scale factor is $S = e^{\Phi/A} e^A$.

Rather than studying $V^\tau_{\text{eff}}$ as above, let us consider for simplicity $V^\ell_{\text{eff}}(r, \ell, E) = E - \dot{r}^2/2$ which contains the same information, though now as seen by a bulk observer. First of all notice that

$$V^\ell_{\text{eff}}(r_H, \ell, E) = E$$

i.e. the kinetic energy as seen from an observer at infinity vanishes at the horizon (the brane takes an infinite $t$-time to fall into the horizon). When the probe is very far from the horizon, $r \rightarrow \infty$, it is straightforward to obtain that

$$\dot{r}^2 \rightarrow 1 - \frac{1}{E^2}.$$
Hence the probe will be unable to escape to infinity if $E < 1$. This is very similar to the $k = 1$ case studied above in the explicit example of Sch-AdS$_5 \times S_5$. On the other hand, if $E > 1$ the probe will be able to escape. The limiting case is when $E = 1$, for which the kinetic energy of the probe vanishes at infinity where there is no force on the probe. Let us now consider a finite value of $r > r_H$. As $\ell$ goes to infinity, the term in $V^{\ell}_{\text{eff}}(r, \ell, E)$ proportional to $\ell^2$ dominates implying that the potential becomes large and positive over a finite interval in $r$ for a large enough value $\ell > \ell_c$, just as in the example of Sch-AdS$_5 \times S_5$. This implies that, for $E < 1$ and $\ell > \ell_c$, there will be cyclic solutions as in Fig. 2. On the other hand, for $E > 1$ and $\ell > \ell_c$ the brane motion possesses a branch of solutions where the brane is forced to be between a finite value $r_{\text{bounce}}$ and infinity, cf. Fig. 1. The value of $r_{\text{bounce}}$ is the largest positive root of $V^{\ell}_{\text{eff}}(r, \ell, E) = E$. It depends both on $\ell$ and $E$ but is guaranteed to exist as long as $E > 1$ and $\ell > \ell_c$. For a generic background the vanishing of $\dot{r}^2$ at the bounce is linear $\dot{r}^2 \sim (r - r_{\text{bounce}})$ implying that both in bulk time and proper brane time the bounce takes place in a finite amount of time. Notice that the bounce is intimately linked to the existence of a conserved quantity $\ell$ and therefore on the presence of more than one extra-dimension.

Finally let us notice that the bouncing branes that we have considered are not singular. Hence they should be amenable to a full treatment of the cosmological perturbations before and after the bounce.

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