A New Approach to
Axial Vector Model Calculations

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Abstract

We consider the one-loop effective action due to a spinor loop coupled to an abelian
vector and axial vector field background. After rewriting this effective action in terms
of an auxiliary non-abelian gauge connection, we use the De Witt expansion to analyze
both its anomalous and non-anomalous content. The same transformation allows us
to obtain a novel worldline path integral representation for this effective action which
avoids the usual separation into the real and imaginary parts of the Euclidean effective
action, as well as the introduction of auxiliary dimensions.

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1 Introduction

It is well known that if a spinor field $\psi$ is coupled to background fields $A_\mu$ and $A_{5\mu}$ so that
\begin{equation}
L = \overline{\psi} (\gamma_\mu \partial^\mu + \gamma_5 A_5 - m) \psi \quad (p = -i\partial)
\end{equation}
then the axial current $j_5^\mu = \overline{\psi} \gamma_\mu \gamma_5 \psi$ has an anomalous divergence \[1\]. One-loop processes can be analyzed in terms of the functional determinant
\begin{equation}
iW^{(1)}_{\text{eff}} = \ln \text{Det}(\gamma_\mu \partial^\mu + \gamma_5 A_5 - m).
\end{equation}
In Euclidean space, the corresponding operator $H = \gamma_\mu \partial^\mu + \gamma_5 A_5 - im$ is not Hermitian, and the anomaly can be attributed to the phase of the functional determinant appearing in \[1\]. In this paper, we avoid the separation of the effective action into its real and imaginary part. Instead, we rewrite it in terms of the effective action for a scalar loop in a certain non-abelian background, and use the De Witt expansion \[4\] to discuss its anomalous and non-anomalous content. (We could also work directly in Minkowski space.)

The same transformation allows us to derive a novel worldline path integral representation for the effective action. While various such representations have been known and used for decades in the case of pure vector amplitudes \[1,2,3,7\], generalizations to amplitudes involving axial vectors and/or pseudoscalars have been constructed only quite recently \[8,9,10,11\]. Those were motivated by the discovery of an efficient way of evaluating this type of path integral \[12\] which allows one, in particular, to recover many of the calculational improvements which have been achieved by representing field theory amplitudes as the infinite string tension limits of appropriate string amplitudes \[13\] (see also \[14\] for the closely related “Quantum Mechanical Path Integral Formalism”). However the previous approaches to axial vector models implied a separate treatment of the parity - even and - odd parts of the effective action \[1\], and, in the latter case, the use of insertion operators in analogy to superstring theory. Here we will show that, for the special case where the background consists only of a vector and axial vector field, there is a much simpler solution for this problem which allows one to treat both parts of the effective action on the same footing.

2 The One-Loop Euclidean Effective Action

It is easily established that
\begin{equation}
(p + A + \gamma_5 A_5)^2 = (p_\mu + A_\mu - \gamma_5 \sigma_{\mu\nu} A_5^\nu)^2 + (D - 2) A_5^2 + i A_5^\nu \gamma_5 - \frac{i}{2} \sigma_{\mu\nu} (\partial^\mu A_\nu - \partial^\nu A_\mu)
\end{equation}

1In \[10\] a way was found to combine both parts into a single expression, however at the cost of introducing an additional parameter integration.

2In Euclidean space we use $\gamma_{Ej} \equiv i\gamma_j, \gamma_{E4} \equiv \gamma_0, \gamma_{E5} \equiv \gamma_5$, so that $\{\gamma_{Ea}, \gamma_{Eb}\} = 2\delta_{ab}, a, b = \mu, 5$. The subscript “E” is omitted in the following. The Euclidean $\varepsilon$ - tensor is defined by $\varepsilon_{1234} = 1$. 

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\[ \sigma_{\mu\nu} \equiv \frac{1}{2} [\gamma_{\mu}, \gamma_{\nu}] \]. Here we used the Dirac algebra in \( D = 4 \) dimensions, dimensionally continued with an anticommuting \( \gamma_5 \). We can write (2.1) in the form

\[ (p + A + \gamma_5 A_5)^2 = -(\partial_\mu + iA_\mu)^2 + a \]  

where

\[ A_\mu \equiv A_\mu - \gamma_5 \sigma_{\mu\nu} A_\nu \]  

(2.3)

\[ a \equiv -\frac{i}{2} \sigma_{\mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu) + i\gamma_5 A_{5,\mu} + (D - 2)A_5^2 \]  

(2.4)

Using the usual argument that

\[ \text{Det}[(p + A + \gamma_5 A_5) - im] = \text{Det}[(p + A + \gamma_5 A_5) + im] = \text{Det}^{1/2}[(p + A + \gamma_5 A_5)^2 + m^2] \]  

(2.5)

we can then write the Euclidean effective action in the following form,

\[ W^{(1)}_{\text{eff}} = -\frac{1}{2} \text{Tr} \int_0^\infty \frac{dT}{T} \exp \left\{ -T \left[ -(\partial_\mu + iA_\mu)^2 + a + m^2 \right] \right\} \]  

(2.6)

Up to the global sign, this is formally identical with the effective action for a scalar loop in a background containing a non-abelian gauge field \( A \) and a potential \( a \). The operator in (2.6) is such that \( W^{(1)}_{\text{eff}}[A_\mu, a] \) is invariant under the transformations

\[ A_\mu \rightarrow A_\mu + \partial_\mu \Lambda + i[A_\mu, \Lambda] \]
\[ a \rightarrow a + i[a, \Lambda] \]  

(2.7)

where the gauge parameter \( \Lambda \) has values in the Clifford algebra. However it should be noted that those transformations are not identical with the transformations

\[ A_\mu \rightarrow A_\mu + \partial_\mu \theta \]
\[ A_5 \rightarrow A_5 + \partial_\mu \theta_5 \]  

(2.8)

(2.9)

(unless \( A_5 = 0 \)) as would be expected naively from (1.1) in the case where \( m = 0 \). To see the reason for this discrepancy, we examine carefully the steps used in deriving (2.1). This involves making the replacements
\[
(p \cdot \gamma + m \cdot \gamma)^2 = p^2 + \frac{1}{2} [p_\mu (\gamma_\mu m \cdot \gamma) + (\gamma_\mu m \cdot \gamma)p_\mu] - \frac{i}{2} (\gamma_\mu m \cdot \gamma)_{\mu} \\
+ \frac{1}{2} [p_\mu (m \cdot \gamma_\mu) + (m \cdot \gamma_\mu)p_\mu] + \frac{i}{2} (m \cdot \gamma_\mu)_{\mu} + (m \cdot \gamma)^2
\]  
(2.10)

and subsequently obtaining

\[
= (p_\mu + \frac{1}{2} \gamma_\mu m \cdot \gamma + \frac{1}{2} m \cdot \gamma_\mu)^2 \\
- \frac{1}{4} (\gamma_\mu m \cdot \gamma + m \cdot \gamma_\mu)^2 - \frac{i}{2} (\gamma_\mu m \cdot \gamma - m \cdot \gamma_\mu)_{\mu} + (m \cdot \gamma)^2
\]  
(2.11)

On the left side of eq. (2.10), the shift \( p \rightarrow p + \partial \Lambda \) can be compensated for by the replacement of \( m \) by \( m - \partial \Lambda \). This is not true on the right side of eq. (2.10) unless \( (\gamma_\mu \partial_\Lambda \cdot \gamma - \partial_\Lambda \cdot \gamma_\mu)_{\mu} = 0 \) which holds when \( \partial_\mu \Lambda \) is identified with \( \partial_\mu \omega \) where \( \omega \) is a scalar (which is what occurs if \( m \) is the vector field \( A_\mu \)), but does not hold when \( \partial_\mu \Lambda \) is identified with \( \partial_\mu \omega \gamma_5 \) (which is what occurs if \( m \) is the axial vector field \( A_5^\mu \)). Consequently (2.7) and (2.9) are not equivalent.

The De Witt expansion \[4\]

\[
\text{Tr} e^{-HT} = \frac{1}{(4\pi T)^2} \sum_{n=0}^{\infty} \int dx a_n(x, x) T^n
\]  
(2.12)

can now be applied to the effective action in (2.6), and the known results \[16, 17, 18\] for the non-abelian heat-kernel coefficients used. The first few coefficients appearing in (2.12) are given by

\[
a_1 = -\text{tr}[a] \\
a_2 = \text{tr} \left[ -\frac{1}{12} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} a^2 \right] \\
a_3 = -\frac{1}{12} \text{tr} \left[ 2a^3 + S_\mu S^\mu - a F_{\mu \nu} F^{\mu \nu} - \frac{4}{15} i F_{\kappa \lambda \mu} F_{\lambda \mu} + \frac{1}{10} F_{\kappa \lambda \mu} F_{\kappa \lambda \mu} \right]
\]  
(2.13, 2.14, 2.15)

\( F_{\mu \nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu], \quad S_\mu \equiv [\partial_\mu + iA_\mu, a], \quad F_{\alpha \beta \gamma} \equiv [\partial_\alpha + iA_\alpha, F_{\beta \gamma}] \).

Performing the Dirac traces one obtains

\[
a_1 = -4(D - 2) A_5^2 \\
a_2 = \left( \frac{2}{3} F_{\mu \nu} F_{\mu \nu} + \frac{4}{3} \left[ (\partial_\mu A_{5 \nu})(\partial^\mu A_5^{\nu}) - (\partial \cdot A_5)^2 \right] \right) \\
= \frac{2}{3} \left( F_{\mu \nu} F_{\mu \nu} + F_5^\mu F_5^\mu \right) + \text{total derivative terms}
\]  
(2.16, 2.17)

\( F_{\mu \nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu; \quad F_5^\mu \equiv \partial_\mu A_5^\nu - \partial_\nu A_5^\mu \). Thus the vector field strength tensor appears automatically, but the axial vector field strength tensor only after the addition of suitable total

\(^3a_1 \) and \( a_2 \) have already been calculated in the context of quantum gravity with torsion \[19\] (we thank I.L. Shapiro for this information).
derivative terms to the effective Lagrangian. Since the ultraviolet ($T = 0$) divergence is controlled by $a_2$, we see from (2.14) that the divergent part of the effective action can be removed by counter terms which are consistent with invariance under the transformations (2.8), (2.9).

Similarly, the contribution to $a_3$ from the three-point function $< AAA_5 >$ is given by

$$a^3_{AAA_5} = i e^{\alpha \beta \gamma \delta} \left( F_{\alpha \beta} \left[ \frac{1}{2} F_{\gamma \delta} A_{5,\lambda} + \frac{2}{3} F_{\lambda \gamma} A_{5,\lambda} \right] - \frac{2}{3} F_{\alpha \beta,\lambda} F_{\delta \lambda} A_{5}^2 \right)$$ (2.18)

We see from (2.18) that invariance under the transformation of (2.8) is retained in the $< AAA_5 >$ process while that of (2.9) is lost. Furthermore, there is no divergence in this anomalous three-point process at one-loop order, which is consistent with [20]. In the abelian case considered here all higher $a_n$’s must be non-anomalous (for a discussion of the non-abelian effective action see [21] and refs. therein).

Alternatively, eqs. (2.2), (2.5) could also be used for writing down a set of second-order Feynman rules generalizing the ones of [22], and thus for scattering amplitude calculations.

We also note that by (2.2) and (2.15) the following transformation can be made in the full generating functional for a $U(1)$ axial gauge theory,

$$\int dA_5^\mu d\psi^+ d\psi \exp \left\{ - \int dx \left( \frac{1}{4} F_{\mu \nu}^5 F_{\mu \nu}^5 + \psi^+ (\not{p} + \gamma_5 A_5) \psi \right) \right\}$$

$$= \int dA_5^\mu d\chi \exp \left\{ - \int dx \left[ \frac{3}{8} \text{tr} \left( - \frac{1}{12} F_{\mu \nu} F_{\mu \nu} + \frac{1}{2} a^2 \right) + \chi^+ \left( (p_\mu + A_\mu)^2 + a \right) \chi \right] \right\}$$ (2.19)

($\chi^+ = \chi^T$). The action now has the form of a Spin(4) gauge theory with a real fermionic scalar field $\chi$.

3 Worldline Path Integral Representation of the Effective Action

In this section we will transform eq. (2.6) into a first-quantized worldline path integral representation for the effective action. The standard procedure for this transformation is the coherent state method [23]. It was applied to the present problem already by D’Hoker and Gagné [10], albeit in a formalism based on six-dimensional Dirac matrices, and with a separate treatment for the real and the imaginary part of the effective action.

Note that the representation (2.6) yields the complete effective action, real and imaginary part. The price which we have to pay for this property is the non-hermiticity of the kinetic operator in the

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4The identification of this result with the result of the heat kernel expansion in standard field theory requires the use of the identity $e^{\alpha \beta \gamma \delta} \left( F_{\alpha \beta,\lambda} F_{\gamma \lambda} V_5 + F_{\alpha \beta} F_{\gamma \lambda,\lambda} V_5 + F_{\alpha \beta} F_{\gamma \lambda} V_5 \right) = 0$, which holds true for an arbitrary field strength tensor $F$ and vector $V$.
exponent. However we still have positivity of this operator for sufficiently weak background fields, which is sufficient for perturbative purposes. We refer the reader to [10] for a detailed account of the coherent state method, and just present the final result of the transformation,

\[
W_{\text{eff}}^{(1)} = -2 \int_0^\infty \frac{dT}{T} e^{-m^2T} \int \mathcal{D}x \int \mathcal{D}\psi e^{-\int_0^T d\tau L(\tau)}
\]

\[
L(\tau) = \frac{1}{4} \dot{x}^2 + \frac{1}{2} \psi_\mu \dot{\psi}^\mu + i \dot{x}^\mu A_\mu - i \psi^\mu F_{\mu\nu} \psi^\nu - 2i \gamma_5 \dot{x}^\mu \psi_\nu A_\nu^\mu + i \gamma_5 \partial_\mu A_\mu^5 + (D - 2) A_5^2
\]

(3.1)

Here \( \int \mathcal{D}x \) denotes the coordinate path integral over the space of all closed loops with fixed proper-time length \( T \), and \( \int \mathcal{D}\psi \) a path integral over Grassmann-valued functions. The periodicity properties of \( \int \mathcal{D}\psi \) are determined by the operator \( \gamma_5 \); after expansion of the interaction exponential a given term in the integrand will have to be evaluated using antiperiodic (periodic) boundary conditions on \( \psi \), \( \psi(T) = -(+) \psi(0) \), if it contains \( \gamma_5 \) at an even (odd) power. After the boundary conditions are determined \( \gamma_5 \) can be replaced by unity.

The perturbative evaluation of the double path integral can then be done as usual (see, e.g., [24]) using worldline Green’s functions adapted to the periodicity conditions. For the coordinate path integral one must first remove the zero mode contained in the path integral, which may be done by fixing the average position \( x_0^\mu \equiv \frac{1}{T} \int_0^T d\tau x^\mu(\tau) \) of the loop. The reduced path integral over the variable \( y^\mu \equiv x^\mu - x_0^\mu \) is then evaluated with a correlator

\[
\langle y^\mu(\tau_1) y^\nu(\tau_2) \rangle = -g^{\mu\nu} \left( |\tau_1 - \tau_2| - \frac{(\tau_1 - \tau_2)^2}{T} \right) \equiv -g^{\mu\nu} G_B(\tau_1, \tau_2)
\]

(3.2)

(other choices are also possible [14]). With our conventions, the free Gaussian coordinate path integral is, in dimensional regularization, equal to

\[
\int \mathcal{D}x \ e^{-\int_0^T d\tau \frac{1}{4} \dot{x}^2} = (4\pi T)^{-\frac{D}{2}} \int dx_0
\]

(3.3)

For the Grassmann path integral one has to proceed differently depending on the boundary conditions. In the antiperiodic case (“A”) there is no zero mode, so that \( \int_A \mathcal{D}\psi \) can be executed straightforwardly with a worldline correlator

\[
\langle \psi^\mu(\tau_1) \psi^\nu(\tau_2) \rangle = \frac{1}{2} g^{\mu\nu} \text{sign}(\tau_1 - \tau_2) \equiv \frac{1}{2} g^{\mu\nu} G_F(\tau_1, \tau_2)
\]

(3.4)

In the periodic case (“P”) one has again a zero mode which must be separated off, so that

\[
\int_P \mathcal{D}\psi = \int d\psi_0 \int \mathcal{D}\xi \\
\psi^\mu(\tau) = \psi_0^\mu + \xi^\mu(\tau)
\]

5
\[ \int_0^T d\tau \xi(\tau) = 0 \]  
(3.5)

The zero mode integration produces an \( \varepsilon \) - tensor via

\[ \int d^4\psi_0^\mu \psi_0^\nu \psi_0^\kappa \psi_0^\lambda = \varepsilon^{\mu\nu\kappa\lambda} \]  
(3.6)

and the \( \xi \) - path integral can be performed using the correlator

\[ \langle \xi^\mu(\tau_1) \xi^\nu(\tau_2) \rangle = g^{\mu\nu} \left( \frac{1}{2} \text{sign}(\tau_1 - \tau_2) - \frac{\tau_1 - \tau_2}{T} \right) = \frac{1}{2} g^{\mu\nu} \dot{G}_B(\tau_1, \tau_2) \]  
(3.7)

(a “dot” always refers to a derivative with respect to the first variable). The free Grassmann path integrals are normalized to unity in the antiperiodic, and to \( \frac{1}{T} \) in the periodic case.

We have verified for a number of cases that the path integral representation (3.1) indeed reproduces the correct field theory results, for example the coefficient \( a_{AA_A} \) of eq. (2.18). We remark that for the calculation of divergent amplitudes it is essential to keep the coefficient of the \( A_5^2 \) term in the worldline Lagrangian \( D \) - dependent, if one wishes to use this formalism together with dimensional regularization. This can be seen already in the case of the (massive) \( \langle A_5 A_5 \rangle \) amplitude, where this term produces a tadpole contribution which is necessary to obtain the same result as is reached in the corresponding field theory calculation (performed with a naive anticommuting \( \gamma_5 \)).

### 4 Worldline Path Integral Calculation of the ABJ Anomaly

Finally, we apply the vector - axial vector path integral to yet another recalculation of the chiral anomaly, this time in momentum space. The usefulness of first-quantized path integrals for the calculation of anomalies was already established in [23, 26] (see also the \( D = 2 \) calculation in [9] which is more closely related to the following one). Thus we would like to calculate the anomalous divergence of the axial current in the \( \langle A_5 A_5 \rangle \) amplitude, which in field theory would be given in terms of a sum of two triangle diagrams. To extract this amplitude from the effective action, as usual we must specialize the background fields to plane waves,

\[
A^\mu(x) = \varepsilon_1^\mu e^{ik_1 \cdot x} + \varepsilon_2^\mu e^{ik_2 \cdot x} \\
A_5^\mu(x) = \varepsilon_3^\mu e^{ik_3 \cdot x}
\]  
(4.1)

and then keep the part of the effective action which is linear in all polarization vectors. In particular the \( A_5^2 \) - piece in the worldline Lagrangian will not yet contribute at this level. Thus we find the following representation for the three-point function,
\[
\langle A^\mu [k_1] A^\nu [k_2] A^\rho_0 [k_3] \rangle = -2i \int_0^\infty \frac{dT}{T} e^{-m^2 T} \int D\xi \int D\psi \exp \left\{- \int_0^T d\tau \left( \frac{1}{4} \dot{\xi}^2 + \frac{1}{2} \psi \cdot \dot{\psi} \right) \right\} \times \int_0^T d\tau_1 (\dot{x}_1^\mu + 2i \psi_0^\mu k_1 \cdot \psi_1) e^{ik_1 \cdot x_1} \int_0^T d\tau_2 (\dot{x}_2^\nu + 2i \psi_2^\nu k_2 \cdot \psi_2) e^{ik_2 \cdot x_2} \times \int_0^T d\tau_3 (ik_3^\rho + 2 \psi_3^\rho \dot{x}_3 \cdot \psi_3) e^{ik_3 \cdot x_3}
\tag{4.2}
\]

where the Grassmann path integral is periodic in this case. It is important to note here the following two facts. Firstly, this expression represents not a single triangle diagram but the sum of the two. Secondly, this expression is already manifestly gauge invariant, i.e. transversal in the vector current indices. If one multiplies the right hand side by, say, \( k_1^\mu \), then the photon vertex operator representing leg 1 becomes the integral of a total derivative, which vanishes due to periodicity. This mechanism is, of course, well-known from string theory. Nothing analogous holds true for the axial-vector vertex operator. Thus the structure of the path integral eq.(4.4) already forces the divergence of the vector current to vanish, and we clearly have to look at the axial vector current to find the anomalous divergence. We are interested in this divergence only, rather than in a calculation of the complete amplitude, so that we can simplify by contracting eq.(4.2) with \( k_3^\rho \). Also we can put legs 1 and 2 on-shell, \( k_1^2 = k_2^2 = 0 \), and restrict ourselves to the massless case.

Since for this amplitude the Grassmann path integral has periodic boundary conditions, according to the above we have to rewrite \( \psi_i^\alpha (\tau) = \psi_0^\alpha + \xi_i^\alpha (\tau) \), and then to keep only those terms which contain four factors of the zero mode piece \( \psi_0 \). Using eqs.(3.2),(3.6),(3.7) we obtain (deleting the energy-momentum conservation factor)

\[
k_3^\rho \langle A^\mu A^\nu A^\rho_0 \rangle = -2i \varepsilon^{\mu \nu \lambda} k_1^\mu k_2^\nu \int_0^\infty \frac{dT}{T} (4\pi T)^{-2} \prod_{i=1}^3 \int_0^T d\tau_i \exp \left\{ (G_{B12} - G_{B13} - G_{B23}) k_1 \cdot k_2 \right\} \times \left\{ 2 + (\dot{G}_{B12} + \dot{G}_{B23} + \dot{G}_{B31})(\ddot{G}_{B13} - \ddot{G}_{B23}) \right\}
\tag{4.3}
\]

Here momentum conservation has been used to eliminate \( k_3 \), and we abbreviate \( G_{ Bij} \equiv G_B (\tau_i, \tau_j) \) etc. We remove the second derivatives \( \dot{G}_{B13} (\dot{G}_{B23}) \) by a partial integration in \( \tau_1 (\tau_2) \). The expression in brackets then turns into

\[
k_1 \cdot k_2 \left\{ 2 - (\dot{G}_{B12} + \dot{G}_{B23} + \dot{G}_{B31})^2 + \ddot{G}_{B12} - \ddot{G}_{B13} - \ddot{G}_{B23}^2 \right\} = 4 \frac{k_1 \cdot k_2}{T} (G_{B13} + G_{B23} - G_{B12}) \tag{4.4}
\]

(Here the identities \( \dot{G}_{Bij} + \dot{G}_{Bjk} + \dot{G}_{Bki} = - \text{sign}(\tau_i - \tau_j) \text{sign}(\tau_j - \tau_k) \text{sign}(\tau_k - \tau_i) \) and \( \ddot{G}_{Bij} = 1 - 4 \frac{G_{B13}}{T} \) are useful). But this is precisely the same expression which appears also in the exponential factor
After a rescaling to the unit circle, and performance of the trivial $T$-integral, we find therefore a complete cancellation between the Feynman numerator and denominator polynomials \[5\]. Thus without further integration we obtain already the desired result for the anomalous divergence,

\[
k_\rho^\mu \langle A^\mu A^\nu A^\rho_5 \rangle = -\frac{8}{(4\pi)^2} \varepsilon^{\mu\nu\kappa\lambda} k_1^\kappa k_2^\lambda
\]

(4.5)

5 Discussion

We have shown that the effective action for a spinor loop coupled to a vector and axial vector background can be reexpressed in terms of an auxiliary nonabelian gauge field and potential. This has allowed us to use known results for the DeWitt expansion, and also to discuss the chiral anomaly from a novel point of view. Indeed, by the form of eq. (2.1), it is evident that an anomaly will be generated in the divergence of any symmetry corresponding to invariance under a chiral gauge transformation in the original action.

Moreover, we have derived a worldline path integral representation for this effective action which, while similar to previous proposals, has some obvious advantages. In field theory terms it corresponds to a naive anticommuting treatment of $\gamma_5$ in dimensional regularisation. However in contrast to the NDR scheme in field theory it fixes the ABJ anomaly in such a way that the anomalous currents are confined to the axial vectors.

The method employed generalizes in an obvious way to the inclusion of an additional scalar field, as well as to the non-abelian case. This will be discussed in a separate publication, where we will also present some more extensive calculations.

It would be interesting to pursue this approach in conjunction with anomalies in higher dimensions [27] and the diffeomorphism anomalies [2].

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\[5\]This cancellation occurs even if one does not put legs 1 and 2 on-shell.
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