Electric power steering: an overview of dynamics equation and how it's developed for large vehicle

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Abstract. Development of steering system has begun with manual steering, hydraulic power steering, and electric power steering. At present, the market trend shows that electric power steering is preferred compared to the other two steering systems, namely manual steering and hydraulic power steering. Electric power steering was chosen because of its light operation and ease in the control system. Although for large vehicles each tends to use hydraulic power steering. In this paper, we will explain about the existing dynamic motion equation models in electric power steering. The booster position of the motor will affect the shape of the overall dynamic motion of the electric power steering system. Initially, research on the dynamic equation of the electric power steering system only involved the inertial component and the stiffness of the steering wheel and column steering components. In the next study, it has developed involving elements of damping and also forces of the wheels and roads. The components included in this movement are also increasing, namely, rack and pinion, tie rod, and drag link. This element is due to the more accurate calculation of the torque needed by the motor booster and its control system. In large cars, the difference between city cars is the distance from the steering rod, which is 2380 mm or almost 2-4 times that of the city car, so the similarity of the motion will also be different. Stiffness equation is needed for the drag link mechanism structure, which involves more stems combined between truss and frame. This mechanism is what distinguishes it later with the equation of the dynamic motion of the electric power steering system with the city car and its kind.

1. Background
University Indonesia has succeeded in developing a large type of electric car, the EV-Bus Molina UI. This bus uses a 200 kW main drive motor, for AC 15 kW, for a 4 kW compressor and 7.5 kW Power Steering. The Power Steering system used is Hydraulic Power Steering. Energy efficiency has reached a range of 60-80%. From the survey data about the use of electric power steering in large cars, neither...
ICE-BUS and EV-Bus systems have used electric power steering. Because of the superiority of Electric Power Steering, it will be converted from Hydraulic Power Steering to Electric Power Steering on the EV-Bus Molina UI [1].

Analysis of power consumption for this electric bus results power efficiency in intervals of 65% to 80%, torque efficiency in 75% to 82.5% intervals, and relatively constant and small steering power load on the main motor[2]. Another study that developed in UI Mechanical Engineering is a power transfer system, including coupling, transmission[3] and the other. Other research about EV-Bus results obtained from the proposed strategy shows a 0.8% fuel reduction in the New European Driving Cycle (NEDC)[4]. The current steering system is hydraulic power steering, where power from an electric motor is forwarded to the hydraulic pump and forwarded to valves and actuators. Then this steering system will be replaced with Electric Power Steering (EPS). This EPS is very profitable because it is the stage where steer by wire[5].

On large buses, the distance between the steering column and front overhang is almost 2-4 times that of the city car type. Therefore the steering structure must be made in such a way that the rotation of the wheel drive mechanism can guarantee the relationship between the steering wheel and the turning angle of the tire. Then in this paper, the types of steering will be presented first from manual steering to electric power steering. After that, reverse engineering of the EV-Bus Molina UI was carried out which is expected to produce a drag link structure. With this mechanism, it will open the way for further research such as kinematic and dynamic analysis as well as constructing the structural stiffness matrix.

2. Methodology
At this stage, several electric power steering models will be reviewed from several references where this model will be applied to the Universitas Indonesia's electric bus (EV-Bus). The first model reviewed is a steering structure consisting of a wheel drive and a steering column. That is connected to a pair of gears that connect it to an assist motor, such as figure 2[6]. The resulting control structure has three major loops. An inner-loop force with fast dynamics is used first to damp the steering shaft flexibility. An outer position loop is thus designed to ensure accurate tracking and good disturbance rejection properties. The position control is composed of feedback and a feed-forward loop. Concerning the figure, the equation that characterizes the system equation is:

$$J_m \ddot{\theta}_m = - \frac{k}{N} (\theta_{red} - \theta_v) + \frac{C_{red}}{N} + u \quad (1)$$

$$J_p \ddot{\theta}_v = C_v - k(\theta_v - \theta_{red}) \quad (2)$$

The second model to be reviewed in this paper is Electric Power Steering (EPS) that should be able to propose torque assistance. The model may be adapted for different kinds of driver population: young drivers, aged people, disabled people, etc. The control framework should also include observers that allow recovering signals which are not sensed and a control law that compensates for the column flexibilities. Regarding Figure 2[7], the equation that characterizes the system equation is

$$J_p \ddot{\theta}_v = \tau_v - k(\theta_v - \theta_s) - B_v \dot{\theta}_v \quad (3)$$

$$J_r \ddot{\theta}_s = -k(\theta_s - \theta_v) - N_2 B_m \theta_s + \frac{\tau_a}{N_1} + N_2 u \quad (4)$$

The next mechanical model of EPS is based on single-pinion architecture suitable for light vehicles and consists of following elements: a steering rack, a steering column coupled to the steering rack through a pinion gear, and the assist motor. Tie-rods connect the steering rack to the tires (Figure 1)[8]. The equation of motion of the steering wheel is expressed by:

$$J_p \ddot{\theta}_v = \tau_v - k(\theta_v - \theta) - B(\dot{\theta}_v - \dot{\theta}) \quad (5)$$

moreover, for the assist-motor equation:

$$J_m \ddot{\theta}_m = \tau_m - k_m (\theta_m - \theta N_2) - B_m (\dot{\theta}_m - \dot{\theta} N_2) \quad (6)$$
The steering column section determines the equation:
\[ j \dot{\theta} = F_c N_1 - k(\theta - \theta_v) - B(\dot{\theta} - \dot{\theta}_v) - k_m(\theta N_2 - \theta_m) - B_m(\dot{\theta} N_2 - \dot{\theta}_m) \]  
(7)

The equation of the tire load is given by:
\[ F_r = -k_r x_r \]  
(8)

Next model in this paper is electric power steering system by involving the equation of the electric motor and also the force on the wheel from the road. The dynamic model of the EPAS establishes the relation between steering mechanism, electric dynamics of the motor, and tire/road contact forces. Figure 1[9] shows the model of steering mechanism equipped with a brushed DC motor. According to Newton laws of motion, the equations of motion can be written as follows:
\[ J_c \ddot{\theta}_c = -K_c \dot{\theta}_c - B_c \dot{\theta}_c + K_c \frac{\theta_m}{N} - F_c \text{sign}(\dot{\theta}_c) + T_d \]  
(9)
\[ J_{eq} \ddot{\theta}_m = K_c \frac{\theta_m}{N} - \left( \frac{K_c}{N^2} + \frac{K_p R_p^2}{N^2} \right) \dot{\theta}_m - B_{eq} \dot{\theta}_m + K_t I_m - F_m \text{sign}(\dot{\theta}_m) - \frac{R_p}{N} F_r \]  
(10)

Electric motor dynamics are given by:
\[ \dot{I}_m = -R I_m - K_t \dot{\theta}_m + V \]  
(11)

During static steering, the area of contact between the front tires and the road surface does not change in the region of small steering-wheel angles. Since the torsion of the tires acts as a spring, the steering angle and reaction torque has a proportional relationship with nearly constant gain. Figure 1[10] shows a model of a steering mechanism equipped with EPS. Linearization of this model aims to conduct frequency analysis and explain the vibration mechanism during static steering. This model can be represented as a balance between the motor inertia, the steering torque applied by the driver \( T_{hdl} \), the assist torque produced by the motor \( \text{ASSIST} \) and the reaction torque of the steering mechanism \( T_{trans} \). The equation is below:
\[ j \ddot{\theta} = T_{\text{assist}} + T_{hdl} - T_{\text{trans}} \]  
(12)
\[ T_{\text{assist}} = G_{\text{gear}}(K_t I_m - T_{\text{fric}} \text{sgn}(\dot{\theta})) \]  
(13)
\[ T_{\text{hdl}} = K_{\text{tSEN}}(\theta_{\text{hdl}} - \theta_s) + C_{\text{tSEN}}(\dot{\theta}_{\text{hdl}} - \dot{\theta}_s) \]  
(14)

The last electric power steering model reviewed in this paper is an EPS control strategy to reduce steering vibration associated with a disturbance from road wheels. The steering mechanism is expressed by Figure 1[11]. Here, “\( T_{\text{hdl}} \)” denotes steering torque at the steering shaft, “\( T_d \)” denotes steering torque at the steering wheel and “\( T_{\text{trans}} \)” denotes the reaction torque at the steering shaft. \( T_{\text{trans}} \) is the sum of alignment torque from the road wheels “\( T_{\text{align}} \)”, disturbance torque from the road wheels “\( T_{\text{dist}} \)” and the frictional loss torque of the rack & pinion “\( T_{\text{fric}} \)”.
\[ j \ddot{\theta}_s = T_{\text{assist}} + T_{\text{hdl}} - T_{\text{trans}} \]  
(15)
\[ j \ddot{\theta}_{\text{hdl}} = T_d - T_{\text{hdl}} \]  
(16)

where:
\[ T_{\text{trans}} = T_{\text{align}} + T_{\text{dist}} + T_{\text{fric}} \text{sgn}(\dot{\theta}) \]  
(17)
\[ T_{\text{assist}} = G_{\text{gear}}(K_t I_m - T_{\text{fric}} \text{sgn}(\dot{\theta})) \]  
(18)
\[ T_{\text{hdl}} = K_{\text{tSEN}}(\theta_{\text{hdl}} - \theta_s) + C_{\text{tSEN}}(\dot{\theta}_{\text{hdl}} - \dot{\theta}_s) \]  
(19)

After reviewing several models of electric power steering, the next step is to collect data about the use of the steering system on large cars or buses. The use of electric power steering on vehicles is still on small vehicles, such as city cars. While for large-sized vehicles such as buses, it still uses a power...
steering hydraulic system. The following is the data on the steering on the bus for the type of internal combustion engine and electric vehicle. From table 1[12] can be seen almost all buses (ICE), using hydraulic power steering. While the bus (EV) is the same, only the type of Volvo 7900 Bus-EV has used electrically powered hydraulic steering, but not yet electric power steering. So there is still a big opportunity to research electric power steering on this large vehicle. The current condition of the steering system installed on the University of Indonesia (EV-Bus) electric bus is still Hydraulic Power Steering. In this study, the hydraulic system in Figure 10a.[12] will be replaced with Electric Power Steering (EPS), as shown in Figure 10b[12]. It is expected that this system can reduce energy use with no energy losses due to the many valves in the hydraulic system. Where energy from the battery it can be used for torsion motors which directly help to steer rod movements, as will be reviewed in literature studies. Besides that, the electrical system maintenance is cheaper than the 25% hydraulic system [13].

3. Result and Discussion
Dimensions of the power steering components will be obtained with reverse engineering[14] step must be carried out on the components of the Molina EV-Bus hydraulic power steering (Fig. 11a.[12]). This bus uses the Hino chassis (Fig. 11b[12]). The basic difference in this chassis is the distance of the front overhang, which is on average twice that of an ordinary car, which is around 2380 mm, as shown in Fig. 11c[12]. Reverse engineering is done by looking at and directly measuring the steering mechanism in the EV-Bus car. In Fig. 12[12], the photo results from the bottom view of the steering mechanism are shown (a). After that, cad modeling (b) and kinematic mechanism (c) of the steering structure installed on the bus were made.

There is a real difference between the mechanism and the ordinary manual steering car (Figure 1). Due to the relative distance between the steering wheel and axle beam, there is an additional drop arm structure ext and drag link ext.

![Figure 1. Schematic Diagram of Hydraulic Power Steering (EV-BUS Molina UI)](image)

From some models reviewed, none of them are by the steering construct on the Molina EV-Bus UI. Because electric power steering is currently still for small vehicles. Therefore it is necessary to further model this EPS so that it can precisely how much torque is needed by the motor booster to help the steering column movement. From the kinematic model that has been made, an analysis needs to be done to ensure the movement of the wheel drive and tire movements. For the dynamic equation, it is also necessary to make the equivalent rigidity of the kinematic structure. The mechanism certainly requires other quantities, including the dimensions of each stem and its elasticity. So further research is kinematic analysis and determines the stiffness matrix of the steering structure. Fig.1[11]. The identification of variables from each component involved in the electric power steering system from several models that have been reviewed can be seen in Table 1 below:
TABLE 1. Results of Overview of Dynamics Equation

| EPS Variable | Figure 2 [6] | Figure 2 [7] | Figure 1 [8] | Figure 1 [9] |
|--------------|-------------|-------------|-------------|-------------|
| Inertia      | ✓           | ✓           | ✓           | ✓           |
| Stiffness    | ✓           | ✓           | ✓           | ✓           |
| Damping      | ✓           | ✓           | ✓           | ✓           |
| Angle        | ✓           | ✓           | ✓           | ✓           |

A=Steering Wheel, B=Steering Column, C=Rack and Pinion, D=Motor Assist Shaft, E=Motor, F=Tire

TABLE 1. Results of Dynamics Equation (continue…)

| EPS Variable | Figure 2 [10] | Figure 1 [11] | New Model (EV-Bus) |
|--------------|---------------|---------------|-------------------|
| Inertia      | ✓             | ✓             | ✓                 |
| Stiffness    | ✓             | ✓             | ✓                 |
| Damping      | ✓             | ✓             | ✓                 |
| Angle        | ✓             | ✓             | ✓                 |

A=Steering Wheel, B=Steering Column, C=Rack and Pinion, D=Motor Assist Shaft, E=Motor, F=Tire, G=Drag Link Mechanism

New dynamic equation of EV-Bus UI that will be built for an electric power steering system. This equation is a combination of several of the previous models, which will involve many components involving the variables of inertia, stiffness, and attenuation. Because the EV-Bus system also has a drag link system consisting of several bars, the equivalent stiffness matrix will first be made[15].

The Drag-link structure consists of truss and frame structures; it must first be known the local and global stiffness matrix form and the transformation matrix of each of these structures. The structure of the kinematic mechanism system is the truss and frame structure, so to form the equivalent stiffness matrix of the truss matrix and frame as follows. The local stiffness matrix truss structure is as follows:

\[
k = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix} \tag{20}
\]

The transformation matrix for the truss structure is:

\[
\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} \tag{21}
\]

Where \( l = \cos \theta = \frac{x_2-x_1}{L} \); \( m = \sin \theta = \frac{y_2-y_1}{L} \); \( L = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \)

so the global rigidity matrix for the truss structure is
The stiffness matrix in the frame structure is as follows:

\[
k = \frac{EA}{L} \begin{bmatrix}
 l^2 & lm & -l^2 & -lm \\
 lm & l^2 & -lm & -l^2 \\
 -l^2 & -lm & l^2 & lm \\
 -lm & -l^2 & lm & l^2 \\
\end{bmatrix}
\] (22)

The stiffness matrix in the frame structure is as follows:

\[
\begin{bmatrix}
 f_1 \\
f_2 \\
f_3 \\
f_4 \\
f_5 \\
f_6 \\
\end{bmatrix} = \begin{bmatrix}
 \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \\
 -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{L}{L^3} & \frac{12EI}{L^2} & \frac{6EI}{L^2} \\
 0 & -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \\
\end{bmatrix} \begin{bmatrix}
 u_1 \\
u_2 \\
u_3 \\
u_4 \\
u_5 \\
u_6 \\
\end{bmatrix}
\] (23)

From equation (20-23), a local stiffness matrix for each bar can be made. By entering the boundary conditions of each stem, the global stiffness matrix of this mechanism system can be formed. These boundary conditions can ensure the input and output movements of this mechanism. This equivalent stiffness matrix is conducive in making overall dynamic movements in large-sized vehicles. It can help in estimating the amount of torque required by a motorist to change hydraulic powers steering to electric power steering in large-sized cars. The dynamic equation will discuss in further research on this University of Indonesia electric bus vehicle.

4. Conclusion
From a review of several equations of motion in the electric power steering system, several variables are quite important in developing these equations, including the inertia of the mass, stiffness, and damping of each component. The main elements in this system are wheel drive, column steering, pinion gear, and wheels. The unique mechanism for this large size car is the drag link mechanism, which consists of a truss and frame structure. There are four rods as a truss structure and two bars as a frame structure. The drag structure of this link will connect the pinion and gear movements to the knuckle rod on the front wheel. With this equivalent rigidity matrix, the process of forming electric power steering dynamic equations will be more straightforward.

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