Block Sparse Memory Improved Proportionate Affine Projection Sign Algorithm

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A block sparse memory improved proportionate affine projection sign algorithm (BS-MIP-APSA) is proposed for block sparse system identification under impulsive noise. The new BS-MIP-APSA not only inherits the performance improvement for block-sparse system identification, but also achieves robustness to impulsive noise and the efficiency of the memory improved proportionate affine projection sign algorithm (MIP-APSA). Simulations indicate that it can provide both faster convergence rate and better tracking ability under impulsive interference for block sparse system identification as compared to APSA and MIP-APSA.

Introduction: Adaptive filters have been widely used in various applications of system identification in which the normalized least mean square (NLMS) algorithm is well-known due to its simplicity, but suffers from slow convergence for colored input [1]. The affine projection algorithm (APA) provides better convergence for colored input compared with NLMS [2]. Meanwhile, the family of affine projection sign algorithm (APSA) has been proposed to improve the performance of APA under impulsive noise together with lower complexity [3]. In order to exploit the sparsity of some echo paths, the real-coefficient improved proportionate APSA (RIP-APSA) was proposed [4], and a memory improved proportionate APSA (MIP-APSA) was further proposed to achieve improved steady-state misalignment with similar computational complexity compared with RIP-APSA [5]. Recently, the block-sparse improved proportionate NLMS (BS-IPNLMS) algorithm was proposed to improve the performance of IPNLMS for identifying block-sparse systems [7]. In this Letter, motivated by both BS-APNLMS and MIP-APSA, we will propose a block sparse memory improved proportionate APSA (BS-MIP-APSA) algorithm, which not only inherits the performance improvement for block-sparse system identification, but also achieves robustness to impulsive noise and the efficiency of MIP-APSA.

Review of MIP-APSA: For echo cancellation, the far-end signal $x(n)$ is filtered through the echo path $h(n)$ to get the desired signal $y(n)$.

$$y(n) = x^T(n)h(n) + v(n)$$  \hspace{1cm} (1)

$$x(n) = \left[ x(n)x(n-1)\cdots x(n-L+1) \right]^T$$  \hspace{1cm} (2)

$$h(n) = \left[ h_0(n) h_1(n) \cdots h_{L-1}(n) \right]$$  \hspace{1cm} (3)

super-script $T$ denotes transposition, $L$ is the filter length, $n$ is the time index, and $v(n)$ is the background noise plus near-end signals. Let $h(n)$ be the $L \times 1$ adaptive filter coefficient vector which estimates the true echo path vector $h(n)$ at iteration $n$, and group the $M$ most recent input vectors together:

$$X(n) = \left[ x(n)x(n-1)\cdots x(n-M+1) \right]$$  \hspace{1cm} (4)

$$\delta(n) = y(n) - X^T(n)h(n-1)$$  \hspace{1cm} (5)

$$y(n) = \left[ y(n)y(n-1)\cdots y(n-M+1) \right]$$  \hspace{1cm} (6)

where $M$ is called the projection order. In [5], MIP-APSA proposed the following weight update:

$$g(n) = \left[ g_0(n), g_1(n), \cdots, g_{L-1}(n) \right]$$  \hspace{1cm} (7)

$$g_i(n) = \frac{(1-\alpha)}{2L} + \frac{(1+\alpha)}{2L} \left[ h_i(n) \right] + \epsilon$$  \hspace{1cm} (8)

$$P(n) = \left[ g(n) \otimes x(n), P_{-1}(n-1) \right]$$  \hspace{1cm} (9)

$$x_{\delta}(n) = P(n) \operatorname{sgn}(e(n))$$  \hspace{1cm} (10)

$$h(n+1) = h(n) + \frac{\mu x_{\delta}(n)}{\sqrt{\delta + x_{\delta}^T(n)x_{\delta}(n)}}$$  \hspace{1cm} (11)

where $-1 \leq \alpha < 1$, $l = 0, 1, \cdots, L-1$, $\epsilon$ is a small positive constant that avoids division by zero, the operation $\otimes$ denotes the Hadamard product, $P_{-1}(n-1)$ contains the first $M-1$ columns of $P(n-1)$.

$$\operatorname{sgn}(\cdot)$$ takes the sign of each element of a vector, and $\epsilon$ is a small positive constant. Compared with RIP-PAPSA, MIP-PAPSA takes into account the ‘proportionate history’ from the last $M$ moments of time. More details can be found in [5]-[6].

Algorithm design: In network echo cancellation, the network echo path is typically characterized by a bulk delay dependent on network loading, encoding, and jitter buffer delays and an “active” dispersive region in the range of $\delta-\delta$ duration [1]. Meanwhile, it is well-known that NLMS is preferred over PNLMs for dispersive system. Therefore, considering the block-sparse characteristic of the network impulse response, the BS-PNLMS algorithm was proposed to improve the PNLMs algorithm by exploiting this special block-sparse characteristic, in which BS-PNLMS used the same step-size within each block and the step-sizes for each block were proportionate to their relative magnitude [7]. We propose to take in account the block-sparse characteristic and partition the MIP-APSA adaptive filter coefficients into $N$ groups with group-length $P$. Let $N = P \times P$,

$$\hat{h}(n) = [\hat{h}_0(n), \hat{h}_1(n), \cdots, \hat{h}_{N-1}(n)]$$  \hspace{1cm} (12)

then the control matrix $g(n)$ in (7)-(8) is be replaced by

$$\bar{g}(n) = \left[ \bar{g}_0(n) I_P, \bar{g}_1(n) I_P, \cdots, \bar{g}_{N-1}(n) I_P \right]$$  \hspace{1cm} (13)

$$\bar{g}_k(n) = \frac{(1-\alpha)}{2L} + \frac{(1+\alpha)}{2L} \left[ h_k(n) \right] + \epsilon$$  \hspace{1cm} (14)

in which $I_P$ is a $P$-length column vector of all ones, and $\left\| h_k(n) \right\| = \sqrt{\sum_{i=0}^{P-1} h_{k,i}^2(n)}$, $k = 0, 1, \cdots, N-1$. The weight update equation for BS-MIP-APSA is

$$\hat{P}(n) = \left[ \bar{g}(n) \otimes x(n), \hat{P}_{-1}(n-1) \right]$$  \hspace{1cm} (15)

$$\hat{x}_{\delta}(n) = \hat{P}(n) \operatorname{sgn}(e(n))$$  \hspace{1cm} (16)

$$\hat{h}(n) = \hat{h}(n-1) + \frac{\mu \hat{x}_{\delta}(n)}{\sqrt{\delta + \hat{x}_{\delta}^T(n)\hat{x}_{\delta}(n)}}$$  \hspace{1cm} (17)

where $\hat{P}_{-1}(n-1)$ also contains the first $M-1$ columns of $\hat{P}(n-1)$.

It should be noted that the proposed BS-MIP-APSA includes both APSA and MIP-APSA. The MIP-APSA algorithm is a special case of proposed BS-MIP-APSA with group length $P = 1$. Meanwhile, when $P$ is chosen as $L$, the BS-MIP-APSA algorithm degenerates to APSA.
Complexity: Compared with traditional RIP-APSA and MIP-APSA, the extra computational complexity of the BS-MIP-APSA arises from the computation of the $l_1$ norm in (14), which requires $L$ multiplications and $N$ square roots. The complexity of the square root could be reduced through a look up table or Taylor series [7]. Meanwhile, the increase in complexity can be offset by the performance improvement as shown in the simulation results.

Simulation results: In our simulation, the echo path is a $L = 512$ finite impulse response (FIR) filter, and the adaptive filter is the same length. We generated colored input signals by filtering white Gaussian noise through a first order system with a pole at 0.8. Independent white Gaussian noise is added to the system background with a signal-to-noise ratio (SNR) of 40 dB. The impulsive noise with signal-to-interference ratio (SIR) of 0 dB is generated as a Bernoulli-Gaussian (BG) distribution. BG is a product of a Bernoulli process and a Gaussian process, and the probability for Bernoulli process is 0.1. The performance was evaluated through the normalized misalignment: $10\log_{10}\left(\frac{|\hat{h}_e - h_0\|}{\|h_0\|}\right)$. In order to evaluate the tracking ability, we switch the echo path from the one-cluster block-sparse system of Fig. 1(a) to the two-cluster block-sparse system of Fig. 1(b).

![Fig. 1 Two block-sparse systems used in the simulations: (a) one-cluster block-sparse system, (b) two-cluster block-sparse system.](image)

The APSA and MIP-APSA algorithms are compared with BS-MIP-APSA. The parameters are $\mu = 0.001$, $\alpha = 0.01$, $\delta = 0.01$, $\alpha = 0$, $M = 2$, and $P = 4$. In the first case, we show the normalized misalignment for colored input in Fig. 2. We could see that the proposed BS-MIP-APSA achieves both faster convergence rate and better tracking ability. In Fig. 3, the performance of BS-MIP-APSA is compared with APSA and MIP-APSA for speech input signal, and we found that our proposed algorithm demonstrates better performance too.

Conclusion: We have proposed a block-sparse memory improved affine projection sign algorithm to improve the performance of block-sparse system identification. Simulations demonstrate the proposed algorithm has both faster convergence speed and tracking ability for block-sparse system identification compared with APSA and MIP-APSA algorithms.

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References

1. Benesty, J., Gänslner, T., Morgan, D.R., Sondhi, M.M. and Gay, S.L., Advances in network and acoustic echo cancellation. Springer, 2001.
2. Ozeki, K., and Umeda, T. ‘An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties’, Electron. Commun. Jpn., 1984, 67-A, (5), pp. 19-27.
3. Shao, T., Zheng, Y.R., and Benesty, J.: ‘An affine projection sign algorithm robust against impulsive interferences’, IEEE Signal Process. Lett., 2010, 17, (4), pp. 327–330.
4. Yang, Z., Zheng, Y.R., and Grant, S.L.: ‘Proportionate Affine Projection Sign Algorithms for Network Echo Cancellation’, IEEE Trans. Audio, Speech, Lang. Process., 19, (8), pp. 2273-2284.
5. Albú, F., and Kwan, H.K.: ‘Memory improved proportionate affine projection sign algorithm’, Electron. Lett., 2012, 48, (20), pp. 1279-1281.
6. Paleologu, C., Ciochina, S., and Benesty, J.: ‘An efficient proportionate affine projection algorithm for echo cancellation’, IEEE Signal Process. Lett., 2010, 17, (2), pp. 165–168.
7. Liu, J., and Grant, S.L.: ‘Proportionate adaptive filtering for block-sparse system identification’, arXiv preprint arXiv:1508.04172, 2015.