Multicomponent dark matter particles in a two-loop neutrino model

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Abstract

We construct a loop induced seesaw model in a TeV scale theory with gauged $U(1)_{B−L}$ symmetry. Light neutrino masses are generated at two-loop level and right-handed neutrinos also obtain their masses by one-loop effect. Multi-component Dark Matters (DMs) are included in our model due to the remnant discrete symmetry after the $B−L$ symmetry breaking and the $Z_2$ parity which is originally imposed to the model. We investigate the multi-component DM properties, in which we have two fermionic DMs with different mass scales, $\mathcal{O}(10)$ GeV and $\mathcal{O}(100-1000)$ GeV. The former mass corresponds to the lightest right-handed neutrino mass induced by the loop effect, although the latter one to the SM gauge singlet fermion. We show each of the DM annihilation processes and compare to the observation of relic abundance, together with the constraints of Lepton Flavor Violation (LFV) and active neutrino masses. Moreover we show that our model has some parameter region allowed by the direct detection result reported by XENON100, and it is possible to search the region by the future XENON experiment.

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I. INTRODUCTION

It has been verified that neutrinos have tiny masses by neutrino oscillation experiments [1–7]. Unfortunately the finite neutrino masses are not explained in the framework of the Standard Model (SM). A lot of models have been proposed to extend this point [8–11]. On the other hand the existence of non-baryonic Dark Matter (DM), which dominates about 23% of the Universe from the CMB observation by WMAP [12], is also shown by the cosmological observations in our Universe [13, 14]. Moreover direct detection experiments of DM are performed around the world such as XENON100 [16], CRESSTII [17], CoGeNT [18], DAMA [19] and TEXONO [20]. Especially XENON100 experiment gives the most severe limit for elastic scattering cross section between DM and nucleon [16]. This implies that DM in the Universe interacts very weakly with quarks. It would be that DM has no interaction with quarks. DM is required in the Universe, however a candidate particle of DM is also not included in the SM. Although the property of neutrino in the SM is similar with that of DM, neutrinos are too light to be DM candidate. Thus in order to improve this problem, it is necessary to add new particles as DM candidate in the SM. Therefore, these current experiments about neutrinos and DM suggest serious verifications that the SM should be modified in order to accommodate the existence of DM as well as non-vanishing neutrino masses.

Radiative seesaw models are known as attractive frameworks for new physics at TeV scale that can provide an elegant solution to explain these two matters of grave concern simultaneously [21–24]. This kind of model correlates the finite neutrino masses with the existence of DM since neutrino masses are generated by radiative effect and DM runs inside the loop. In particular, the radiative seesaw model proposed by Ernest Ma [21] is one of the simplest models. Subsequently there are a lot of recent works in terms of the model [25–27] and the extended models [28–41]. The other models of radiative neutrino mass are studied in Refs. [42–51].

In this paper, we propose a new model of two-loop induced neutrino masses with local $B − L$ symmetry. Due to the two remnant Abelian symmetries ($Z_2$ and $Z_6$) even after the $B − L$ and electroweak symmetry breaking, our model has multi-component DMs. Two or three particles of them can be DMs simultaneously depending on the mass hierarchy. Moreover since one of DMs obtains the mass at one loop, we expect it to be rather light with mass of $O(10)$ GeV. We check whether they can satisfy the correct relic density of DM observed by WMAP, and also the upper bound of elastic cross section with nucleon by

\[^{1}\text{Very recently, new result of the DM relic density was given by Planck measurement as } \Omega_{\text{DM}} h^2 = 0.1196 \pm 0.0031.\]
This paper is organized as follows. In Section 2, we show our model and discuss the Higgs sector including the Higgs potential, stationary condition, S-T parameters and neutrino mass in lepton sector. In Section 3, we analyze DM phenomenologies. We summarize and conclude in Section 4.

II. THE TWO-LOOP RADIATIVE SEESAW MODEL

A. Model setup

| Particle | $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $N^c$ | $S$ | $\bar{S}$ |
|----------|-----|-------|-------|-----|-------|-------|-----|---------|
| $(SU(2)_L, U(1)_Y)$ | $(2, 1/6)$ | $(1, -2/3)$ | $(1, 1/3)$ | $(2, -1/2)$ | $(1, 1)$ | $(1, 0)$ | $(1, 0)$ | $(1, 0)$ |

| $Y_{B-L}$ | $1/3$ | $-1/3$ | $-1/3$ | $-1$ | $1$ | $1$ | $-1/2$ | $1/2$ |
| $Z_2$ | $+$ | $+$ | $+$ | $+$ | $-$ | $-$ | $+$ | $+$ |
| $Z_6$ | $2$ | $-2$ | $-2$ | $0$ | $0$ | $0$ | $-3$ | $3$ |

TABLE I: The particle contents and the charges for fermions. Notice that the $Z_6$ is the remnant symmetry obtained after $B-L$ symmetry breaking as we will discuss later.

| Particle | $\Sigma$ | $\Phi$ | $\eta$ | $\chi$ |
|----------|---------|-------|-------|-------|
| $(SU(2)_L, U(1)_Y)$ | $(1, 0)$ | $(2, 1/2)$ | $(2, 1/2)$ | $(1, 0)$ |
| $Y_{B-L}$ | $1$ | $0$ | $0$ | $-1/2$ |
| $Z_2$ | $+$ | $+$ | $-$ | $+$ |
| $Z_6$ | $0$ | $0$ | $0$ | $-3$ |

TABLE II: The particle contents and the charges for bosons. Notice that the $Z_6$ is the remnant symmetry obtained after $B-L$ symmetry breaking as we will discuss later.

We propose a two-loop radiative seesaw model with $U(1)_{B-L}$ which is an extended model of the seesaw model of Ma [21]. The particle contents are shown in Tabs. I and II. We add three right-handed neutrinos $N^c$, three SM gauge singlet fermions $S$ and $\bar{S}$, a $SU(2)_L$ doublet scalar $\eta$ and $B-L$ charged scalars $\chi$ and $\Sigma$ to the SM content, where $\eta$ and $\chi$ are assumed not to have vacuum expectation value (VEV). The $B-L$ charged scalar $\Sigma$ is the source of the spontaneous $B-L$ breaking by its VEV of $\langle \Sigma \rangle = v'/\sqrt{2} \sim O(10)$ TeV. The $Z_2$ parity is also imposed so as to stabilize DM candidates. The right-handed neutrinos $N^c$ do not have masses at tree level. As a result, the neutrino mass is obtained not through the one-loop level (just like Ma-model [21]) but through the two-loop level.
The renormalizable Lagrangian for Yukawa sector and Higgs potential are given by

\[ -\mathcal{L}_{\text{Yukawa}} = y_{\ell} \Phi^\dagger e^L + y_{\nu} \eta^\dagger LN^c + y_N N^c \chi S + y_S \Sigma S^S + y_{\tilde{S}} \Sigma^S + \text{h.c.}, \]  

\[ -\mathcal{L}_{\text{Higgs}} = m_\Phi^2 \Phi^\dagger \Phi + m_\Sigma^2 \Sigma^\dagger \Sigma + m_\chi^2 \chi^\dagger \chi + m_5 [\chi^2 \Sigma + \text{h.c.}] \]

\[ + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \lambda_5 [(\Phi^\dagger \eta)^2 + \text{h.c.}] \]

\[ + \lambda_6 (\Sigma^\dagger \Sigma)^2 + \lambda_7 (\Sigma^\dagger \Sigma)(\Phi^\dagger \Phi) + \lambda_8 (\Sigma^\dagger \Sigma)(\eta^\dagger \eta) + \lambda_9 (\chi^\dagger \chi)^2 \]

\[ + \lambda_{10} (\chi^\dagger \chi)(\Phi^\dagger \Phi) + \lambda_{11} (\chi^\dagger \chi)(\eta^\dagger \eta) + \lambda_{12} (\chi \Sigma)^2, \]  

where \( \lambda_5 \) has been chosen real without any loss of generality. The couplings \( \lambda_1, \lambda_2, \lambda_6 \) and \( \lambda_9 \) have to be positive to stabilize the Higgs potential. Inserting the tadpole conditions: \( m_1^2 = -\lambda_1 v^2 - \lambda_7 v'^2 / 2 \) and \( m_3^2 = -\lambda_6 v^2 - \lambda_7 v'^2 / 2 \), the resulting mass matrix of the neutral component of \( \Phi \) and \( \Sigma \) defined as

\[ \Phi^0 = \frac{v + \phi^0(x)}{\sqrt{2}}, \quad \Sigma = \frac{v' + \sigma(x)}{\sqrt{2}}, \]  

is given by

\[ m^2(\phi^0, \sigma) = \begin{pmatrix} 2\lambda_1 v^2 & \lambda_7 v v' \\ \lambda_7 v v' & 2\lambda_6 v'^2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \]  

where \( h \) implies SM-like Higgs and \( H \) is an additional Higgs mass eigenstate. The mixing angle \( \alpha \) is given by

\[ \tan 2\alpha = \frac{\lambda_7 v v'}{\lambda_1 v^2 - \lambda_6 v'^2}. \]  

The Higgs bosons \( \phi^0 \) and \( \sigma \) are rewritten in terms of the mass eigenstates \( h \) and \( H \) as

\[ \phi^0 = h \cos \alpha + H \sin \alpha, \]

\[ \sigma = -h \sin \alpha + H \cos \alpha. \]  

The other scalar masses are found as

\[ m_2^2 = m_1^2(\eta^\pm) = m_2^2 + \frac{1}{2} \lambda_3 v^2 + \frac{1}{2} \lambda_8 v'^2, \]

\[ m_2^2 = m_2^2(\text{Re} \, \eta^0) = m_2^2 + \frac{1}{2} \lambda_8 v'^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + 2\lambda_5) v^2, \]

\[ m_2^2 = m_2^2(\text{Im} \, \eta^0) = m_2^2 + \frac{1}{2} \lambda_8 v'^2 + \frac{1}{2} (\lambda_3 + \lambda_4 - 2\lambda_5) v^2, \]

\[ m_{\chi_R}^2 = m_4^2 + \frac{1}{2} \lambda_{10} v^2 + \frac{1}{2} \lambda_{12} v'^2 + \sqrt{2} m_5 v', \]

\[ m_{\chi_I}^2 = m_4^2 + \frac{1}{2} \lambda_{10} v^2 + \frac{1}{2} \lambda_{12} v'^2 - \sqrt{2} m_5 v'. \]
The tadpole conditions for $\eta$ and $\chi$, which are given by \( \frac{\partial V}{\partial \eta} \bigg|_{\text{VEV}} = 0 \), \( \frac{\partial V}{\partial \chi} \bigg|_{\text{VEV}} = 0 \), and \( 0 < \frac{\partial^2 V}{\partial \eta^2} \bigg|_{\text{VEV}} \) and \( 0 < \frac{\partial^2 V}{\partial \chi^2} \bigg|_{\text{VEV}} \) tell us that

\[
0 < m_2^2 + \frac{v^2}{2} (\lambda_3 + \lambda_4 + 2\lambda_5) + \frac{v^2}{2} \lambda_8, \quad 0 < m_4^2 + \frac{v^2}{2} \lambda_{10} + \sqrt{2} m_5 v' + \frac{v^2}{2} \lambda_{12}, \tag{II.12}
\]

in order to satisfy the condition $v_\eta = 0$ and $v_\chi = 0$ at tree level, respectively. In order to avoid that $\langle \Phi \rangle = \langle \Sigma \rangle = 0$ be a local minimum, we require the following condition:

\[
\lambda_7 - \frac{2}{3} \sqrt{\lambda_1 \lambda_6} < 0. \tag{II.13}
\]

To achieve the global minimum at $\langle \eta \rangle = \langle \chi \rangle = 0$, we find the following condition

\[
0 < \lambda_{11} - \frac{2}{3} \sqrt{\lambda_2 \lambda_9}. \tag{II.14}
\]

Finally, if the following conditions

\[
0 < \lambda_3 + \frac{2}{3} \sqrt{\lambda_1 \lambda_2}, \quad 0 < \lambda_7 + \frac{2}{3} \sqrt{\lambda_1 \lambda_6}, \quad 0 < \lambda_{10} + \frac{2}{3} \sqrt{\lambda_1 \lambda_9}, \quad 0 < \lambda_8 + \frac{2}{3} \sqrt{\lambda_2 \lambda_6}, \quad 0 < \lambda_{11} + \frac{2}{3} \sqrt{\lambda_2 \lambda_9}, \quad 0 < \lambda_{12} + \frac{2}{3} \sqrt{\lambda_6 \lambda_9}, \tag{II.15}
\]

are satisfied, the Higgs potential Eq.(II.2) is bounded from below.

### B. S and T parameters

It is worth mentioning the new contributions to the S and T parameters due to the new scalar boson $\eta$, which are given in Refs. [52, 53] as

\[
S_{\text{new}} = \frac{1}{2\pi} \int_0^1 dx (1-x) x \ln \left[ \frac{x m_{\eta R}^2 + (1-x)m_{\eta I}^2}{m_{\eta}^2} \right], \tag{II.16}
\]

\[
T_{\text{new}} = \frac{1}{32\pi^2 \alpha_{\text{em}} v^2} \left[ F(m_\eta, m_{\eta R}) + F(m_\eta, m_{\eta I}) - F(m_{\eta I}, m_{\eta R}) \right], \tag{II.17}
\]

\[
F(m_1, m_2) = \frac{m_1^2 + m_2^2}{2} - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right), \tag{II.18}
\]

where $\alpha_{\text{em}} = 1/137$ is the fine structure constant. The experimental deviations from the SM predictions, under $m_{h_{SM}} = 126$ GeV, are given by [54]

\[
S_{\text{new}} = 0.03 \pm 0.10, \quad T_{\text{new}} = 0.05 \pm 0.12, \tag{II.19}
\]

When the masses are $1 \leq m_1/m_2 \lesssim 3$, the function $F(m_1, m_2)$ is approximated to

\[
F(m_1, m_2) \approx \frac{2}{3} (m_1 - m_2)^2, \tag{II.20}
\]

as in Ref. [52]. From Eq (II.19), we get the following constraint for $\eta$ masses,

\[
(m_\eta - m_{\eta R}) (m_\eta - m_{\eta I}) \lesssim 133 \text{ GeV}. \tag{II.21}
\]
C. Neutrino mass matrix and LFV processes

The active neutrino mass matrix at the two-loop level as depicted in Fig. 1 is given by

\[
(m_\nu)_{\alpha\beta} = \left( y_\nu y_\nu^\dagger \right)_{\alpha\beta} \frac{m_S}{4(4\pi)^4} \int_0^1 dx \int_{0}^{1-x} dy \frac{1}{x(1-x)} \left[ I\left( m_{S}^2, m_{RR}^2, m_{RI}^2 \right) - I\left( m_{S}^2, m_{IR}^2, m_{II}^2 \right) \right],
\]

where

\[
I(m_1^2, m_2^2, m_3^2) = \frac{m_1^2 m_2^2 \log \left( \frac{m_2^2}{m_1^2} \right) + m_2^2 m_3^2 \log \left( \frac{m_3^2}{m_2^2} \right) + m_3^2 m_1^2 \log \left( \frac{m_1^2}{m_3^2} \right)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)},
\]

and \( m_S \) is the mass of \( S \), abbreviating generation index of \( S \). Since the neutrino mass scale should be roughly \( m_\nu \sim 10^{-1} \) eV, the product of \( y_\nu^2 N_y \) and the integral by \( x \) and \( y \) of order \( 10^{-8} \) is required when \( m_S \sim 1 \) TeV. If \( y_\nu^2 N_y \sim 1 \), \( m_{\eta_{R}} \simeq m_{\eta_{I}} \) is required, which is realized by small \( \lambda_5 \).

The Branching Ratio (Br) of charged Lepton Flavor Violation (LFV) processes \( \ell_{\alpha} \to \ell_{\beta}\gamma \) (\( \alpha, \beta = e, \mu, \tau \)) is given by

\[
Br(\ell_{\alpha} \to \ell_{\beta}\gamma) = \frac{\alpha_{em} \left| (y_\nu y_\nu^\dagger)_{\alpha\beta} \right|^2}{768\pi G_F^2 m_q^2} Br(\ell_{\alpha} \to \ell_{\beta}\nu_{a}\nu_{b}),
\]

where right-handed neutrino mass is neglected here. The latest limit for \( \mu \to e\gamma \) is given by MEG experiment \(^{55}\) as

\[
Br(\mu \to e\gamma) < 5.7 \times 10^{-13}.
\]

For sum of active neutrino masses, the limit of \( \sum m_\nu < 0.933 \) eV is imposed from the cosmological observation \(^{15}\). In the next section, we take into account these constraints of S-T parameters, LFV and the neutrino mass in the discussion of DM.
FIG. 2: The annihilation channels of fermionic DM $N$ (upper row) and $\bar{S}$ (lower row).

III. DARK MATTERS

We discuss the DM properties in this section. The $\mathbb{Z}_2$ parity imposed to the model stabilizes DM. In addition to the $\mathbb{Z}_2$ parity, we have a remnant $\mathbb{Z}_6$ symmetry after $B - L$ symmetry breaking which stabilizes particles charged under the $\mathbb{Z}_6$ symmetry as well.\(^2\) As a result, our model has two or three DM candidates simultaneously, and the number of DMs depends on mass hierarchy of DM candidates included in the model. In general we have five DM candidates which are $N^c$, $S$, $\bar{S}$ as fermionic DMs and $\chi_{R(I)}$, $\eta_{R(I)}$ as bosonic DMs (For bosonic part, the DM property of the imaginary part is almost same as that of the real one). Of these, three particles can be DMs when decay of a charged particle under the $\mathbb{Z}_2$ and/or $\mathbb{Z}_6$ symmetry is kinematically forbidden, otherwise we have two DMs. The mass of $N^c$ is expected to be somewhat light ($\simeq \mathcal{O}(10)$ GeV) because its mass is generated at the one-loop level. The other particles have typically the mass of $B - L$ symmetry breaking scale. Therefore it is natural to choose $N^c$ as the lightest DM. In this case, $\eta_R$ and $\eta_I$ cannot be DM since they have the same charge with $N^c$ under $\mathbb{Z}_2$ and $\mathbb{Z}_6$ symmetry. The remaining DM candidates are $S$, $\bar{S}$, $\chi_{R}$ and $\chi_{I}$. The interactions of $S$ and $\bar{S}$ are almost same except for Yukawa interaction $y_{N} N^{c} \chi S$. This Yukawa interaction leads DM-exchanging scattering like $SS \rightarrow N^c N^c$. The DM candidates $\chi_{R}$ and $\chi_{I}$ have also similar DM-exchanging scattering $\chi_{R(I)} \chi_{R(I)} \rightarrow N^c N^c$ via Yukawa interaction $y_{N} N^{c} \chi S$ as same as the case of $S$. Thus as the simplest case, we consider two-component DMs ($N^c$ and $\bar{S}$) in the following since there is

\(^2\) One straightforwardly finds the remnant $\mathbb{Z}_6$ symmetry is derived and the charges are obtained by multiplying 6 to the $B - L$ charges of all the particles so as to being the minimal integers.
no exchange scattering between $N^c$ and $\bar{S}$ at the tree level.$^3$

The mass matrix of three right-handed neutrinos $N^c$ is radiatively induced and the expression is found as

$$(m_{N^c})_{ij} = \frac{3}{4(\pi)^2} \sum_{k=1}^{3} \frac{(y_N)_{ik}(y_N)_{jk}m_{S_k}}{m_{\chi_R}^2 - m_{\chi_R}^2} \ln \left( \frac{m_{\chi_R}^2}{m_{\chi_R}^2 - m_{S_k}^2} \right) - \frac{m_{\chi_I}^2}{m_{\chi_I}^2 - m_{S_k}^2} \ln \left( \frac{m_{\chi_I}^2}{m_{S_k}^2} \right).$$  (III.1)

We need multi-pair of $S$ and $\bar{S}$ to generate three non-zero right-handed neutrino masses, otherwise only one non-zero mass is generated. An interesting feature of the model is the connection between the light neutrino masses Eq. (II.22) and the right-handed neutrino masses Eq. (III.1). The right-handed neutrino masses tend to be small corresponding to the tiny light neutrino masses. The mass matrix can be diagonalized by a unitary matrix and the lightest particle, which is called as simply $N$ below, is DM. We choose the diagonal basis of right-handed neutrinos. The annihilation channels of the right-handed neutrino DM into the SM particles are shown in the upper part of Fig. 2, where the annihilation channel via $B - L$ gauge boson is omitted since the contribution is small enough due to exchange of heavy $B - L$ gauge boson. We obtain the annihilation cross sections as

$$\sigma_{v_{rel}}(NN \rightarrow \ell_L\ell_L) = \frac{[\langle y_{\nu}^\dagger y_{\nu} \rangle_{11}]^2}{48 \pi m_N^2} G(\alpha_\eta, \alpha_\eta) v_{rel}^2,$$  (III.2)

$$\sigma_{v_{rel}}(NN \rightarrow \nu_L\nu_L) = \frac{[\langle y_{\nu}^\dagger y_{\nu} \rangle_{11}]^2}{48 m_N^2} \sum_{a,b=R,I} G(\alpha_a, \alpha_b) v_{rel}^2,$$  (III.3)

$$\sigma_{v_{rel}}(NN \rightarrow \nu_L\nu_L) = \frac{[\langle y_{\nu}^\dagger y_{\nu} \rangle_{11}]^2}{64 \pi m_N^2} (\alpha_R - \alpha_I)^2 \left[ 1 + \frac{v_{rel}^2}{6} \left( 3 - 10 (\alpha_R + \alpha_I) + 6 (\alpha_R + \alpha_I)^2 - 4 \alpha_R \alpha_I \right) \right]$$  (III.4)

where $G(x, y) = xy (1 - x - y + 2xy)$, $\alpha_\eta = m_{\eta_R}^2 / (m_{\eta_R}^2 + m_{\eta_I}^2)$ and $\alpha_a = m_{S_a}^2 / (m_{S_a}^2 + m_{m_a}^2)$, $a = R, I$. In the last expression, the symmetric factor 1/2 should be multiplied when the flavor of the final state $\nu_L\nu_L$ is the same. The chiral suppression is not effective for the pair of (anti-)neutrino final state channels if the mass splitting between $\eta_R$ and $\eta_I$ is not negligible.

We analyze the relic abundance of the lightest right-handed neutrino $N$ with the constraints from S-T parameters, the neutrino mass and LFV Eq. (II.26). We sweep the pa-

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$^3$ Regarding the pair of $(N^c, S)$ or $(N^c, \chi_{R(I)})$ as DMs, it would be the simplest if Yukawa coupling $y_S$ is large enough because most of $S$ or $\chi_{R(I)}$ annihilates into $N^c N^c$. In this case, one can consider that only $N^c$ is substantial DM.
parameters in the following range:

\[ 10^2 \text{ GeV} < m_S < 10^4 \text{ GeV}, \quad 10^{-4} < (y_\nu)_{e1} \approx (y_\nu)_{\mu 1} < 1, \]  \quad (III.5)

\[ 10^2 \text{ GeV} < m_{\eta_R(I)} < 10^3 \text{ GeV}, \quad 10^2 \text{ GeV} < m_{\chi_R(I)} < 10^4 \text{ GeV}. \]  \quad (III.6)

The mass of \( \eta \) must satisfy the constraint of S-T parameters Eq. (II.21), and the typical mass scale of \( S, \chi \) and \( \eta \) is of \( \mathcal{O}(1) \) TeV since we assume the \( B-L \) breaking scale is several TeV. Only the Yukawa couplings \( (y_\nu)_{e1} \) and \( y_N \) are fixed to \( (y_\nu)_{\tau 1} = 1.0 \) and \( (y_N)_{ik} = \mathcal{O}(1) \) in order to have a proper annihilation cross section, being consistent with \( \mu \to e\gamma \). As mentioned before, even if some elements of \( y_\nu \) and \( y_N \) are of \( \mathcal{O}(1) \), one can obtain correct neutrino masses and mixings by taking small value of \( \lambda_5 \) and by choosing appropriate texture of Yukawa couplings. As concerning above factors, the right-handed neutrino masses shown in Eq. (III.1) induced by the one-loop effect become \( \mathcal{O}(10) \) GeV, otherwise the right-handed neutrino masses become much lighter and the annihilation cross section will be too small because it is proportional to \( \alpha_{\eta,R,I} \).

The result in the case of \( N \) DM is shown in Fig. 3 and each point satisfies the constraint from S-T parameters, appropriate light neutrino mass scale \( \sim 10^{-1} \) eV, the LFV constraint \( \text{Br}(\mu \to e\gamma) < 5.7 \times 10^{-13} \) and thermal DM relic density which corresponds to \( \sigma v_{\text{rel}} \gtrsim 3 \times 10^{-26} \text{ cm}^3/\text{s} \). The reason of the inequality in the cross section is that we have two DM candidates. One should note that total relic density is supplied by \( N \) and the other DM \( \bar{S} \). Therefore even if the annihilation cross section is larger than \( 3 \times 10^{-26} \text{ cm}^3/\text{s} \), the lack of the amount of DM is compensated by the amount of \( \bar{S} \). One can see from the upper left panel that more than around 300 GeV of \( m_S \) can satisfy the current upper bound of the branching ratio for \( \mu \to e\gamma \). At the same time, Yukawa couplings relating with \( \mu \to e\gamma \) should be less than \( 10^{-2} \) as shown in the upper right panel. The right-handed neutrino mass are of \( 10 \sim 80 \) GeV in this case. The magnitude of Yukawa couplings is also important to obtain the proper light neutrino mass scale (See Eq. (II.22)). In addition to the Yukawa coupling, a small mass splitting between \( \eta_R \) and \( \eta_I \) is required to obtain correct light neutrino masses as the left lower panel shows. As a result, our scenario of DMs is still p-wave dominant since Eq. (III.4) is suppressed. Due to the small \( \eta \) mass splitting, we can get tiny neutrino masses because of a cancellation among the integrands of Eq. (II.22). The masses of \( \eta_R \) and \( \eta_I \) are restricted to be roughly \( 10^2 \) GeV. Thus charged \( \eta^+ \) mass is less than around 300 GeV from the constraint of S-T parameters. In addition, charged Higgs search at LHC would give a stringent bound for \( \eta^+ \). Some mass regions of charged Higgs for SUSY models are excluded by the decay of the charged Higgs into slepton plus missing energy \[ ^{56,58}. \] According to the CMS result \[ ^{58}, \] \( 120 \lesssim m_\tilde{\ell} \lesssim 270 \) GeV is excluded for slepton mass. This model shows a similar signal via \( \eta^+ \to \ell_\alpha N \), thus almost same mass region of \( \eta^+ \) can be expected to be excluded. The lower bound of charged Higgs is also obtained from LEP experiment, and
FIG. 3: The parameter spaces of satisfying LFV, the light neutrino mass scale, the thermal DM relic density. The red, green, blue points imply the DM relic density fraction of $N$ as $0.7 < \Omega_N/\Omega < 1.0$, $0.3 < \Omega_N/\Omega < 0.7$, $\Omega_N/\Omega < 0.3$ respectively.

the bound is around 70 GeV under some conditions [59, 60]. Regarding masses of $\chi_R$ and $\chi_I$ as shown in the right-lower panel, mass hierarchy between $\chi_R$ and $\chi_I$ is necessary to get the scale of the right-handed neutrino masses which is connected with the size of the annihilation cross section.

Next, we move on discussion of $\bar{S}$ DM. This DM does not have any interactions with $N$ at tree level. Hence we can consider these two DMs separately. The annihilation cross
sections of $\bar{S}$ shown in the lower part of Fig. 2 are given as
\begin{align}
\sigma_{\text{rel}}(\bar{S}\bar{S} \to f f) &= \sum_f \frac{c_f y_S^2 y_f^2}{8\pi} m_S^2 \left| \frac{1}{D} \right|^2 \left( 1 - \frac{m_f^2}{m_S^2} \right)^{3/2} v_{\text{rel}}^2, \quad (\text{III.7}) \\
\sigma_{\text{rel}}(\bar{S}\bar{S} \to W^+W^-) &= \frac{y_S^2 (g^2 + g_2^2)}{128\pi} m_S^2 \left| \frac{1}{D} \right|^2 \left( 3 - 4 \frac{m_S^2}{m_W^2} + 4 \frac{m_S^2}{m_Z^2} \right) \left( 1 - \frac{m_Z^2}{m_S^2} \right)^{1/2} v_{\text{rel}}^2, \quad (\text{III.8}) \\
\sigma_{\text{rel}}(\bar{S}\bar{S} \to ZZ) &= \frac{y_S^2 (g^2 + g_2^2)}{128\pi} m_S^2 \left| \frac{1}{D} \right|^2 \left( 3 - 4 \frac{m_S^2}{m_Z^2} + 4 \frac{m_S^2}{m_Z^2} \right) \left( 1 - \frac{m_Z^2}{m_S^2} \right)^{1/2} v_{\text{rel}}^2, \quad (\text{III.9}) \\
\sigma_{\text{rel}}(\bar{S}\bar{S} \to hh) &= \frac{y_S^4 \sin^4 \alpha}{8\pi m_S^2} \left( 1 - \frac{m_h^2}{m_S^2} \right)^{1/2} \beta_h^2 \left[ 1 - \beta_h \frac{3}{12} \left( 1 - \frac{m_h^2}{m_S^2} \right) + \frac{\beta_h^2}{12} \left( 1 - \frac{m_h^2}{m_S^2} \right)^2 \right] v_{\text{rel}}^2. \quad (\text{III.10})
\end{align}

where $m_S$ is the mass of $\bar{S}$ and the color factor $c_f$ is 1 for leptons and 3 for quarks. The parameter $\beta_h$ is defined as $\beta_h = m_h^2 / (2m_S^2 - m_h^2)$ and $D$ is the propagator of the SM-like Higgs $h$ and an extra Higgs $H$.

\begin{equation}
\frac{1}{D} = \frac{\sin \alpha \cos \alpha}{4m_S^2 - m_h^2 + i m_h \Gamma_h} - \frac{\sin \alpha \cos \alpha}{4m_S^2 - m_H^2 + i m_H \Gamma_H}. \quad (\text{III.11})
\end{equation}

The $HH$ final state process can be obtained by replacing $m_h \to m_H$ and $\sin \alpha \to \cos \alpha$ in Eq. (III.10).

We have $m_S, y_S, \sin \alpha, m_H$ as parameters and sweep in the following range:

\begin{align}
200 \text{ GeV} < m_S(H) < 5000 \text{ GeV}, \quad 10^{-3} < y_S < 1, \quad 10^{-3} < \sin \alpha < 1, \quad (\text{III.12})
\end{align}

satisfying $\sigma v \gtrsim 3 \times 10^{-26}$ cm$^3$/s. The result is shown in Fig. 3. The left panel shows that the $m_S < m_H$ roughly holds when $m_S$ is larger than around 1 TeV. Moreover the couplings have to be of order one to get correct relic density of $\bar{S}$ (the right panel). It causes the elastic cross section with nucleis to be larger than the upper bound by XENON100 as we will discuss below.

Let us move on to the discussion of direct detection of DMs. Although our DM consists of two components $N$ and $\bar{S}$, $N$ does not have any interactions with quarks at tree level since it is a right-handed neutrino. The other DM $\bar{S}$ interacts with quarks via Higgs exchange. Thus it is possible to explore the DM in direct detection experiments like XENON100 [16].

The Spin Independent (SI) elastic cross section $\sigma_{\text{SI}}$ with proton $p$ is given by

\begin{equation}
\sigma_{\text{SI}} = \frac{4 \mu_S^2 y_S^2 \sin^2 \alpha \cos^2 \alpha m_p^2}{\pi 2 v^2} \left( \frac{1}{m_H^2} - \frac{1}{m_H^2} \right)^2 \left( \sum_q f_q^p \right)^2, \quad (\text{III.13})
\end{equation}

\footnote{The DM $N$ can interact with quarks at loop level through electromagnetic interactions [25, 61] and it would be large as same as being detected by the XENON experiment.}
where $\mu_{\tilde{S}} = \left( m_{\tilde{S}}^{-1} + m_p^{-1} \right)^{-1}$ is the DM-proton reduced mass. The parameters $f_q^p$ which imply the contribution of each quark to proton mass are calculated by the lattice simulation \[62, 63\] as

\begin{align}
    f_u^p &= 0.023, \\
    f_d^p &= 0.032, \\
    f_s^p &= 0.020,
\end{align}

(III.14)

for the light quarks and $f_Q^p = 2/27 \left( 1 - \sum_{q \leq 3} f_q^p \right)$ for the heavy quarks $Q$ where $q \leq 3$ implies the summation of the light quarks. The recent another calculation is performed in Ref. \[64\]. Fig. 5 is the comparison with XENON100 upper bound \[16\] with the same parameter obtained from the analysis of the relic density. In the case that $\tilde{S}$ DM to be dominant, most of parameter region allowed by the relic density is excluded by the XENON100 upper bound due to the large Yukawa coupling $y_{\tilde{S}}$. Despite of such a strong constraint, some allowed parameter region certainly exists. These parameters imply that Yukawa coupling $y_{\tilde{S}}$ (with large mixing of $\alpha$) is rather small and the mass of $m_{\tilde{S}}$ is close to a resonance for the annihilation cross section in Eq. (III.7)-(III.9). Needless to say, we can easily relax such a situation because of multi-component DM scenario. Since we have two DMs in the model, the fraction parameter of relic density $\xi$, which stands for the fraction of relic abundance of $\tilde{S}$ to the total abundance, makes the XENON100 limit looser, and wide allowed parameters appear.
FIG. 5: The comparison of the elastic cross section of $\tilde{S}$ with nucleon where the XENON100 upper bound is also drawn together where the parameter $\xi$ stands for the fraction of relic abundance of $\tilde{S}$ to the total abundance.

IV. CONCLUSIONS

We have constructed a two-loop radiative seesaw model with local $B-L$ symmetry at the TeV scale that provides neutrino masses. We have also studied the multi-component DM properties, in which we have two fermionic DMs with different mass scale; $\mathcal{O}(10)$ GeV for $N$ and $100 \sim 1000$ GeV for $\tilde{S}$. Although $N$ is right-handed neutrino, its mass is generated by the one-loop effect. We have shown that the allowed mass regions of the particles $\eta_{R(I)}$, $\chi_{R(I)}$, an extra Higgs $H$, $S$ and Yukawa couplings constrained by S-T parameters, $\text{Br}(\mu \to e \gamma) < 5.7 \times 10^{-13}$, $m_\nu \sim 0.1$ eV, and annihilation cross section of DMs, in which for example we found the mass of $\eta_R$ and $\eta_I$ should be degenerate: $10^{-8} \lesssim |m_{\eta_R} - m_{\eta_I}| \lesssim 10^{-6}$ GeV. The upper bound of $m_N$ is around 80 GeV due to the loop-induced mechanism of $m_N$. Too light $m_N$ leads too much relic density of $N$ because of the small annihilation cross section. We have investigated allowed parameter region from direct detection of $\tilde{S}$ DM through the interaction with Higgses. Moreover, we have found that the region of the large mixing $\sin \alpha$ will be testable by the exposure of the future XENON experiment. Our model would be revealed by the other characteristic evidences such as two gamma line signals in cosmic ray coming from the annihilation of two component DMs.
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