Higgs Production in Two-Photon Process and Transition Form Factor

Norihisa Watanabe\textsuperscript{a}, Yoshimasu Kurihara\textsuperscript{a}, Ken Sasaki\textsuperscript{b}, Tsuneo Uematsu\textsuperscript{c}

\textsuperscript{a}High Energy Accelerator Research Organization (KEK)
Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{b} Dept. of Physics, Faculty of Engineering
Yokohama National University, Yokohama 240-8501, Japan
\textsuperscript{c} Institute for Liberal Arts and Sciences, Kyoto University, Kyoto 606-8501, Japan
and Maskwa Institute, Kyoto Sangyo University, Kyoto 603-8555, Japan

Abstract
The Higgs production in the two-photon fusion process is investigated where one of the photons is off-shell while the other one is on-shell. This process is realized in either electron-positron collision or electron-photon collision where the scattered electron or positron is detected (single tagging) and described by the transition form factor. We calculate the contributions to the transition form factor of the Higgs boson coming from top-quark loops and W-boson loops. We then study the $Q^2$ dependence of each contribution to the total transition form factor and also of the differential cross section for the Higgs production.

Keywords: Higgs production, two-photon fusion, transition form factor, linear collider

1. Introduction
There has been much interest in the diphoton decay of the Higgs boson discovered at LHC experiments \cite{1}, since its coupling to the photon is connected with the question whether it is really a Standard Model (SM) Higgs boson or the one beyond SM, such as in the minimal supersymmetric standard model (MSSM) or in composite models. It would be intriguing to investigate the properties of the SM Higgs boson through the production process in the two-photon fusion: $2\gamma \rightarrow H$, which might be realized at ILC \cite{2}, and is just the opposite reaction of the diphoton decay mode of the Higgs boson: $H \rightarrow 2\gamma$. The Higgs diphoton decay goes through charged fermion loops and W-boson loops as discussed in Refs.\cite{3, 4, 5, 6, 7, 8, 9} and the references therein.

Figure 1: $e^+ e^-$ two-photon fusion process for the Higgs production.

Here we particularly interested in the virtual and real two-photon processes (i) the electron-positron collision in Fig.\textsuperscript{1} where one of the scattered electron (or positron) is detected (single tagging), and (ii) the electron-photon collision $e\gamma \rightarrow eH$ shown in Fig.\textsuperscript{2} where we observe the scattered electron. From these processes we can measure the so-called “transition form factor” of the Higgs boson as a function of the virtual-photon mass squared\cite{11}.

In this paper we investigate the SM Higgs boson production in the virtual and real two-photon fusion $\gamma^*\gamma \rightarrow H$ shown in Figs.\textsuperscript{1} and \textsuperscript{2} and calculate the transition form factor of the Higgs boson at one-loop level. First we examine the tensor structure of the transition amplitude for $\gamma^*\gamma \rightarrow H$ which respects gauge invariance. We then evaluate the contributions to the amplitude from charged fermion loops and W-boson loops.

2. Higgs production and transition amplitude

The transition amplitude for $\gamma^*\gamma \rightarrow H$ extracted from the process in Fig.\textsuperscript{2} is given by

$$M \equiv \langle H(T|\gamma^*\gamma)\gamma(k_2)\rangle = e^l(k_1)e^r(k_2) A_{\mu\nu}(k_1, k_2),$$

(1)

\textsuperscript{1} The $\gamma^*\gamma \rightarrow \not{\ell}\not{\ell}$ transition form factor was first investigated in QCD \cite{11}. The recent experimental data were given in Refs.\cite{11} [2].
where $\epsilon^{\mu}(k_1) (\epsilon^\nu(k_2))$ is the polarization vector of the incident virtual (real) photon, $k_1^2 = -Q^2 < 0$ and $k_2^2 = 0$. Due to the electromagnetic gauge invariance, the tensor $A_{\mu\nu}$ can be decomposed as

\[
A_{\mu\nu}(k_1, k_2) = \left( g_{\mu\nu}(k_1 \cdot k_2) - k_2 \gamma_{\mu} k_1 \gamma_{\nu} \right) S_1(m^2, Q^2, m_H^2) \\
+ \left( k_1 \gamma_\mu k_2 - \gamma^\mu k_1 k_2 \right) S_2(m^2, Q^2, m_H^2),
\]

where $m_H$ is the Higgs boson mass satisfying $(k_1 + k_2)^2 = m_H^2$ and the intermediate particle masses in the loop are collectively denoted by $m$. Since $k_2^2 \epsilon_\nu(k_2) = 0$, the transition amplitude reads

\[
M = \left[ g^\mu\nu(k_1 \cdot k_2) - k_2^\mu k_1^\nu \right] S_1(m^2, Q^2, m_H^2) \epsilon_\mu(k_1) \epsilon_\nu(k_2).
\]

### 3. Transition form factor

For a virtual and real two-photon process, we define the transition form factors $F_{\text{total}}, F_{1/2}$ and $F_1$ as follows:

\[
S_1(m^2, Q^2, m_H^2) \left( \frac{g^2}{(4\pi)^2} \right) = F_{\text{total}}(Q^2, m_H^2) = \sum_f N_f e_f^2 F_{1/2}(\rho_f, \tau_f) + F_1(\rho_w, \tau_w),
\]

where $e$ and $g$ are the electromagnetic and weak gauge couplings, respectively, and $m_w$ is the W boson mass. $F_{1/2}$ and $F_1$ are contributions from fermion loops and W boson loops, respectively, $N_f$ is a color factor (1 for leptons and 3 for quarks), $e_f$ is the electromagnetic charge of fermion in the unit of proton charge and

\[
\rho_f \equiv \frac{Q^2}{4m_f^2}, \quad \tau_f \equiv \frac{4m_f^2}{m_H^2}, \quad \rho_w \equiv \frac{Q^2}{4m_w^2}, \quad \tau_w \equiv \frac{4m_w^2}{m_H^2}.
\]

#### 3.1. Fermion-loop contribution

We calculate the charged fermion triangle-loop diagrams shown in Fig.3 and obtain

\[
F_{1/2}(\rho, \tau) = -\frac{2\tau}{1 + \rho \tau} \left( 1 + \frac{1 - \tau}{1 + \rho \tau} \right) \left( f(\tau) + \frac{1}{4} g(\rho) \right)
\\
+ \frac{\tau}{1 + \rho \tau} \left( 2\rho \sqrt{\frac{\tau}{1 - \sqrt{1 - \tau}}} - \sqrt{\frac{\tau}{1 + \rho \tau}} \right) \left( f(\tau) - \sqrt{\frac{\tau}{1 + \rho}} \sqrt{g(\rho)} \right),
\]

where

\[
f(\tau) = \begin{cases}
\sin^{-1} \sqrt{\tau}, & \text{for } \tau \geq 1, \\
-\frac{1}{4} \log \left[ \frac{1}{1 - \sqrt{1 - \tau}} - \left( \frac{\tau}{1 + \rho \tau} \right) \right], & \text{for } \tau < 1,
\end{cases}
\]

\[
g(\rho) = \frac{\log \left( \sqrt{\rho} + 1 + \sqrt{\rho} \right)^2}{\sqrt{\rho} + 1 - \sqrt{\rho}}.
\]

Eq.(6) shows that the fermion loop contribution $F_{1/2}$ is proportional to $\tau_f$, i.e., the fermion mass squared $m_f^2$. Thus the contributions to the transition form factor from leptons and light-flavour $(u, d, s, c$ and $b$) quarks are negligibly small compared to the one from top quark. Therefore, from now on, we consider only the top quark loop contribution for $F_{1/2}$.

#### 3.2. W-boson loop contribution

Next we calculate the W-boson loop diagrams in unitary gauge shown in Fig.4 and obtain

\[
F_1(\rho, \tau) = \frac{1}{1 + \rho \tau} \left( \frac{\tau}{1 + \rho \tau} \right) \left( 4\rho + 8\rho^2 \tau + 6(1 + \rho \tau) - 3\tau \right) f(\tau) + \frac{1}{4} g(\rho)
\\
+ (4\rho + 2(1 + \rho \tau) + 3\tau)
\\
\times \left( 1 + \frac{2\rho \tau}{1 + \rho \tau} \right) \sqrt{\frac{\tau}{1 - \sqrt{1 - \tau}}} - \frac{\tau}{1 + \rho \tau} \sqrt{\rho(\rho + 1) \sqrt{g(\rho)}} \right),
\]

where the expressions of $f(\tau)$ and $g(\rho)$ are given in Eqs.(7) and (9), respectively.

It is noted that $f(\tau)$ and $g(\rho)$ appear in the expressions of $F_{1/2}(\rho, \tau)$ in Eq.(6) and $F_1(\rho, \tau)$ in Eq.(10) as the following combinations,

\[
f(\tau) + \frac{1}{4} g(\rho) \quad \text{and} \quad 2\rho \sqrt{\frac{\tau}{1 - \sqrt{1 - \tau}}} - \sqrt{\rho(\rho + 1) \sqrt{g(\rho)}}.
\]

which arise from the two- and three-point scalar integrals by Passarino and Veltman [13], as will be discussed in a separate paper [14]. Similar combinations for the time-like virtual mass, which are different from our space-like case, appear in the Higgs decay processes $H \to \gamma'\gamma$ and $H \to Z^*\gamma$ in Ref.[15] (see also Ref. [9] for on-shell decays, $H \to \gamma\gamma$ and $H \to Z\gamma$).
4. Numerical analysis

4.1. Transition form factor

In the limit $Q^2 \to 0$ (or $\rho \to 0$), $F_{1/2}(\rho, \tau)$ and $F_1(\rho, \tau)$ reduce, respectively, to

$$F_{1/2}(\rho, \tau) = -2\tau[1 + (1 - \tau)f(\tau)], \quad \text{(12)}$$

$$F_1(\rho, \tau) = 2 + 3\tau W + 3\tau W(2 - \tau)f(\tau), \quad \text{(13)}$$

which coincide with the results for the top-quark and W-boson loop contributions to $H \to 2\gamma$ decay amplitude.

On the other hand, for large $Q^2$, $F_{1/2}(\rho, \tau)$ and $F_1(\rho, \tau)$ show quite different behaviours:

$$F_{1/2}(\rho, \tau) \to \infty, \quad \text{(14)}$$

$$F_1(\rho, \tau) \to 0, \quad \text{(15)}$$

where we note that $g(\rho) \to \log^2(4\rho)$ as $\rho \to \infty$. Thus $F_{1/2}(\rho, \tau)$ is decreasing to zero while $F_1(\rho, \tau)$ is increasing as $Q^2$ becomes large.

**Figure 5:** Transition form factors as a function of $Q^2/m_H^2$. Red, green and blue curves correspond to top-quark, W-boson and total contributions, respectively. We choose mass parameters as $m_H = 126$ GeV, $m_t = 173$ GeV and $m_W = 80$ GeV.

We plot, in Fig. 5, $N_c e^2 F_{1/2}(\rho, \tau)$, $F_1(\rho, \tau)$ and $F_{\text{total}}$ as a function of $Q^2/m_H^2$. We take mass parameters as $m_H = 126$ GeV, $m_t = 173$ GeV and $m_W = 80$ GeV. We have $\tau_\ell = 7.54$ and $\tau_W = 1.61$. We see that $W$-loop contribution $F_1(\rho, \tau)$ is positive, much larger in magnitude than top-quark loop contribution $F_{1/2}(\rho, \tau)$ and grows with $Q^2$. In contrast, $F_{1/2}(\rho, \tau)$ is negative and does not vary much with $Q^2$ and stays almost constant. Thus, $F_{\text{total}}$, the sum of top-quark and $W$-boson loop contributions, grows with $Q^2$. Actually it grows as $\log^2(Q^2/m_W)$ for large $Q^2$.

4.2. Differential cross section

The differential cross section for the Higgs production via $\gamma\gamma$ fusion in $e\gamma \to eH$ shown in Fig. 6 is given by

$$\frac{d\sigma_{\text{\gamma\gamma fusion}}}{dQ^2} = \frac{\alpha^4_{\text{em}}}{64 \pi^2} \left(1 + \frac{u^2}{s^2}\right) \frac{1}{m_W^2} |F_{\text{total}}(Q^2)|^2, \quad \text{(16)}$$

where $s = (l + k)^2$, $u = (k - l)^2$ and $\alpha_{\text{em}} = e^2/(4\pi)$.

Now a question may be posed about feasibility of extracting the transition form factor from the differential cross section. A possible competing process for $e\gamma \to eH$ with single electron tagging is $Z\gamma$ fusion process which is obtained in Fig. 2 by replacing the virtual photon $\gamma^*$ with the Z-boson.

**Figure 6:** The differential cross section for the Higgs production as a function of $Q^2/m_H^2$ with $m_H = 126$ GeV. Blue and red curves correspond to the contributions from $\gamma\gamma$ fusion and $Z\gamma$ fusion, respectively.

We plot, in Fig. 6, the differential cross section $d\sigma/dQ^2$ for $e\gamma \to eH$ which originates from $\gamma\gamma$ fusion in blue (red) line as a function of $Q^2/m_H^2$ for $\sqrt{s} = 200$GeV. Because of the mass squared term in the Z-boson propagator as well as $Q^2$ which appears as an overall factor, we see that the $Z\gamma$ fusion gives much less contribution to $d\sigma/dQ^2$ than the $\gamma\gamma$ fusion in the forward region where $Q^2/m_H^2$ is smaller than 1. Therefore, the transition form factor of the Higgs boson via $\gamma\gamma$ fusion is measurable once a $e\gamma$ colliding machine is constructed. In addition, regarding the total cross section, the $\gamma\gamma$ fusion contribution is dominant over that of $Z\gamma$ fusion for $\sqrt{s} \leq 400$ GeV, which will be discussed in the ref. [14].

5. Summary and Discussion

In this paper we have studied the transition form factor of the SM Higgs boson which arises from top-quark loops and W-boson loops. Its $Q^2$ dependence is summarized in Fig. 5. The main contribution comes from $W$-boson loops and $F_{\text{total}}(Q^2, m_H^2)$ grows with $Q^2$. This is a prediction based on the SM about the behavior form factor of the Higgs boson. Any deviation of $Q^2$ dependence from the SM prediction may suggest a possible signature of the new physics beyond SM, such as MSSM [16] or composite models [17, 18].
The transition form factor of the Higgs boson may also be measured in the electron-positron collision, where the dominant processes for the Higgs production are the Higgs-strahlung via s-channel Z-boson and ZZ or WW fusion at tree level. Consider the case in which one of the scattered electron (or positron) is detected (single tagging) and the untagged lepton is scattered into a small angle emitting an almost-real photon shown in Fig.1. Single tagging eliminates the Higgs-strahlung and WW fusion contributions, only leaving ZZ fusion process. However, if the kinematical region is restricted to the forward directions, ZZ fusion is expected to be insignificant compared to $\gamma^*\gamma$ fusion. Then using the equivalent-photon approximation [19], the corresponding Higgs production cross section can be written in terms of the transition form factor given in Eq.(4).

As for the future subject we should include the higher-order effects due to QCD and electroweak interactions. Also it may be interesting to see if the notion of transition form factor is applicable to the Higgs physics at the photon collider as discussed in Ref. [20] and the references therein.

Finally in the case of MSSM or two-Higgs doublet model, there exist the charged Higgs bosons $H^\pm$. We present the result on a charged scalar contribution to the transition form factor of the Higgs boson. Taking the coupling of the charged Higgs to the neutral Higgs to be $-gm_{H^\pm}/m_W$ as in Ref.[9], we obtain

$$F_0(\rho, \tau) = \frac{\tau}{1 + \rho \tau} \left[ 1 - \frac{\tau}{(1 + \rho \tau)} \left( f(\tau) + \frac{1}{4} g(\rho) \right) \right] + \frac{\tau}{1 + \rho \tau} \left( 2 \rho \sqrt{1 + \sqrt{f(\tau) - \sqrt{g(\rho) + 1}}} \sqrt{g(\rho)} \right), \quad (17)$$

where $\rho = \frac{\alpha^2}{m_{H^\pm}^2}$ and $\tau = \frac{m_{H^\pm}^2}{m_W^2}$. In the limit $Q^2 \to 0$, we get

$$F_0(\rho \to 0, \tau) = \tau [1 - f(\tau)], \quad (18)$$

which coincides with the last equation of (2.17) of Ref.[9].

Numerical analysis of $F_0(\rho, \tau)$ by varying parameters $\rho$ and $\tau$ shows that the scalar loop contribution is very small compared to that of $W$ loop.

References

[1] ATLAS Collaboration, Phys. Lett. B 716 (2012) 1-29; CMS Collaboration, Phys. Lett. B 716 (2012) 30-61.
[2] http://www.linearcollider.org/cms
[3] J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B 106 (1976) 292.
[4] B. L. Ioffe and V. A. Khvoie, Fiz. Elem.-Chastits At. Yadra 9 (1978) 118 [Sov. J. Part. Nucl. 9 (1978) 50].
[5] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. 30 (1979) 711 [Sov. J. Nucl. Phys. 30, (1979) 711]; Phys. Rev. D 85 (2012) 013015.
[6] T. G. Rizzo, Phys. Rev. D 22 (1980) 178.
[7] M. B. Gavela, G. Girardi, C. Malleville and P. Sorba, Nucl. Phys. B 193 (1981) 257.
[8] W. J. Marciano, C. Zhang and S. Willenbrock, Phys. Rev. D 85 (2012) 013002.
[9] J. F. Gunion, H. E. Haber, G. Kane and S. Dawson, “The Higgs Hunter’s Guide” (Addison-Wesley, 1990).
[10] G. P. Lepage and S. J. Brodsky, Phys. Rev. D 9 (1980) 2157.
[11] B. Aubert et al., (The BaBar Collaboration), Phys. Rev. D 80 (2009) 052002.