Parametric electromagnetic transfer function estimation at USArray Site MNF34

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Abstract. We propose a new parametric approach to electromagnetic transfer function (EMTF) estimation which has attributes not found in the nonparametric approach most extensively used by the magnetotelluric (MT) community. Firstly, parametric EMTFs are smooth by construction, which is consistent with the underlying physics. Secondly, fewer parameters are generally required to represent an EMTF by parametric means. Thirdly, our parametric approach can simplify data quality control and editing of time series because of the direct operation in the time domain. Our approach is based on the mature discipline of system identification which is concerned with parametric transfer function determination given system input and output. In this study, MT data from one USArray site are analyzed, showing a statistical advantage in reproducing measured geoelectric field time series using a parametric model EMTF versus the standard EMTF approach.

1. Introduction
An electromagnetic transfer function (EMTF) relates the surface magnetic and geoelectric fields through an impedance tensor. Given spatial samples of this primary datum over an area, the process of magnetotelluric (MT) inversion produces volumetric images of Earth's interior conductivity structure to depths of hundreds of meters to hundreds of kilometers. MT reconstructions provide invaluable information both for commercial mineral and hydrocarbon exploration and investigations of fundamental geologic processes. The importance of this technique is exemplified by the widespread deployment of MT-dedicated instrumentation, including continent-wide surveys such as the SinoProbe in China. EMTFs also play a key role in modeling and ultimately forecasting geomagnetically induced currents (GICs) that, can, during the most intense geomagnetic disturbances, pose a national-security-level natural hazard to power-grid system integrity.

EMTF accuracy and precision determines lithospheric conductivity reconstruction fidelity in MT inversion and geoelectric field retrieval accuracy in GIC modeling. Given the importance of EMTFs and challenging noise environment encountered in MT data acquisition, researchers continue to investigate more robust and efficient EMTF estimation methods, e.g., [1-4], building from the foundations established by the robust statistical methods developed in the 1980s [5], [6]. The underlying statistical problem is to determine the frequency-dependent impedance tensor that best describes the linear relation between surface measurements of magnetic and geoelectric fields. Almost all EMTF estimation methods to date are nonparametric, meaning each tensor component is
determined independently in frequency and, consequently, there is no preference for EMTF smoothness.

However, smoothness in the EMTF impedance is physically justified because lithospheric electromagnetic induction is diffusive. EMTF smoothness is powerful prior knowledge that can, for example, help to mitigate the impact of electrically noisy environments in proximity to DC electrified railways [7]. MT surveys in China face increasing challenges as the railway expands beyond the 2016 network of 80,000 km of electrified and 22,000 km of high-speed track. Our efforts will provide a new class of EMTF estimation methodology better-suited to areas with severe electrical noise contamination such as the footprint of the SinoProbe intersecting with the growing electrified rail network.

Though the MT community has long and successfully focused on the nonparametric approach, the potential that parametric methods are more statistically efficient has been acknowledged by a leading export in the field [8]. Our methods derive from the mature discipline of system identification, e.g., see [9], which utilizes statistically rigorous approaches to construct dynamic system models given data measured at the system input and output. The system identification approach is nascent in EMTF estimation, with, to the best of our knowledge, only two published investigations: [10], which considers only simulated data, and [11], a conference paper that reports on an error-in-variables [12] study, to first order ($n_\alpha = 1$ in the notation of Section 3.2.2) only, in a frequency range (5 to 300 Hz) outside the scope of MT inversion and GIC modeling. Our research is based on real USArray data and focuses on a very low frequency range ($10^{-5}$ to $10^{-2}$ Hz). System identification offers a rich set of models, algorithms, evaluation criteria, and firm theoretical foundations to characterize linear-system transfer functions [13], [14] that, with research effort, can yield a new path to estimate the cross-disciplinary and fundamental datum provided by EMTFs.

The remainder of this paper is organized in the following manner. First, in Section 2, we describe the MT data analyzed in the study. Next, in Section 3, we define both the standard nonparametric approach and our proposed parametric approach for estimating an MT tensor given measured, surface geomagnetic and geoelectric time series. Experimental results are given in Section 4, where we compare the estimated MT tensors and geoelectric field time series reproduced using the MT tensors found with both approaches. Finally, conclusions are given in Section 5.

2. Material

2.1. USArray

The data considered in this work originates from the USArray project. With the purpose to understanding the structure, character, evolution, and activities from the surface to the deep interior of the North American landscape, the Earth Scope has been active since 2001 [15]. The USArray, one part of this plan, intends to instrument the continental United States by hundreds of array sites (using seismometers and sensors). In this study, we consider data from the so-called Transportable Array (MT TA) of the USArray.

Figure 1. USArray Transportable and Flexible Array deployment locations (up to 2019).
The MT TA is a large-coverage-area and long-term array plan. In order to accomplish the overall coverage goal of at least 2000 array sites in United States with only 400 measuring devices, the observation sites are moved every two years. The completed MT TA sites locations are shown in Figure 1.

2.2. Data
To determine the magnetotelluric (MT) tensors for use in inversion, MT surveys collect surface magnetic and geoelectric field measurements at a number of spatially distributed instrumentation sites. We obtain the required measured data from the Incorporated Research Institutions for Seismology Research (IRIS) Data Management Center (http://ds.iris.edu). We denote the northward and eastward directed surface magnetic field time series by $B_n(t_n)$ and $B_e(t_n)$, respectively, and the geoelectric field time series by $E_n(t_n)$ and $E_e(t_n)$ using the same directional convention where north and east are aligned with the magnetic pole. In this work, it is necessary to also consider the measurements aligned to the geographic pole. After the appropriate coordinate rotation, the original $B_n(t_n)$, $B_e(t_n)$, $E_n(t_n)$ and $E_e(t_n)$ are replaced by $B_n(t_n)$, $B_e(t_n)$, $E_n(t_n)$ and $E_e(t_n)$, respectively.

Impedance is an important concept in the magnetotelluric method, defined as the ratio of the horizontal electric field component to the vertical magnetic field, which is described as a tensor in the general case. Expressed in the frequency domain and under the standard assumptions of MT [16], the tensor impedance relationship between the surface magnetic and geoelectric fields is given by

$$Z(f) = \begin{bmatrix} Z_{xx}(f) & Z_{xy}(f) \\ Z_{yx}(f) & Z_{yy}(f) \end{bmatrix} = \begin{bmatrix} Z_{00}(f) & Z_{01}(f) \\ Z_{10}(f) & Z_{11}(f) \end{bmatrix}$$

where $\mu_0$ is the permeability of free space (situations with significant permeability are not considered here) and $Z_{ij}(f)$ is the MT tensor component relating the $i$th to the $j$th spatial coordinate.

Example data consisting of 1-second sampled data are shown in Figure 2. The depicted data were measured at USArray MT TA site MNF34 located at 45.99 N and -95.18 E (see Figure 1) with a length of 2000 samples over the time range from UTC 18:03:21 to 18:36:40 on July 14, 2012. This site was selected for study for its position in the geologically complex region of central Minnesota in the United States and being scrutinized in prior analysis and required no manual data quality control, e.g., outlier rejection, jump discontinuity correction, or data gap mitigation.

![Figure 2](image)

(a) $E_n(t_n)$ and $E_x(t_n)$.
(b) $B_n(t_n)$ and $B_x(t_n)$.
(c) $B_e(t_n)$ and $B_y(t_n)$.

Figure 2. MT survey data measured at USArray station MNF34 on July 14, 2012 UTC. The time series data are sampled with a period of 1s and plotted with the mean subtracted.

3. Methodology
This study focuses on the statistical determination of the frequency-domain impedance tensor (1) that relates the measured surface magnetic field to the measured geoelectric field, i.e., data such as shown in Figure 2. To give context, we outline the established EMTF determination method [5], [6] and show that it is a nonparametric approach. The parametric approach, based on system identification methodology, is then described.
3.1. Nonparametric estimation

The goal in EMTF estimation is to determine the transfer functions $Z_{xx}(f)$ and $Z_{xy}(f)$, namely the first row $Z_1(f)$ of the EMTF tensor $Z(f)$ in (1), relating the northward geoelectric field component time series measurement $E_x(t_n)$ to the surface magnetic field time series measurements $B_x(t_n)$ and $B_y(t_n)$. The second row of $Z(f)$, i.e., the transfer function components $Z_{yx}(f)$ and $Z_{yy}(f)$ comprising $Z_2(f)$, is determined similarly by estimating the relation between the eastward geoelectric field time series measurement $E_y(t_n)$ and the surface magnetic field measurements. For convenience, the following discussion focuses only on $Z_1(f)$.

First, the measured time series are divided into $I$ equal-length segments. Each segment is transformed to the frequency domain with the fast Fourier transform (FFT) [17]. To accommodate FFT analysis, the segments will have a length in the hundreds to thousands of samples. Next, for each frequency $f_k$, the model relating the input to output is

$$
\begin{bmatrix}
E^1_x(f_k) \\
\vdots \\
E^I_x(f_k)
\end{bmatrix} = \frac{1}{\mu_0} \begin{bmatrix}
B^1_x(f_k) & B^1_y(f_k) \\
B^I_x(f_k) & B^I_y(f_k)
\end{bmatrix} \begin{bmatrix}
Z_{xx}(f_k) \\
Z_{xy}(f_k)
\end{bmatrix} + \begin{bmatrix}
\epsilon^1_x(f_k) \\
\vdots \\
\epsilon^I_x(f_k)
\end{bmatrix}
$$

where $B^i_x(f_k)$, $B^i_y(f_k)$ and $E^i_x(f_k)$ are determined from the $i$th data segment and $\epsilon^i_x(f_k)$ accounts for all errors. The relation (2) can be expressed in condensed form using vector notation with

$$
E_x(f_k) = \mu_0^{-1} B(f_k) Z_x(f_k) + \epsilon_x(f_k)
$$

However, in practice, MT data are found to be non-Gaussian, non-stationary, and to contain outliers. Given these complications, the prevailing approach has been to employ the M-estimator with a Huber loss function as in

$$
Z_{1M} = \arg \min_{Z_{xx}(f_k), Z_{xy}(f_k)} \sum_{i=1}^{I} \rho \left( \left| \frac{E^i_x(f_k) - \mu_0^{-1} [B^i_x(f_k) Z_{xx}(f_k) + B^i_y(f_k) Z_{xy}(f_k)]}{\sigma_0} \right| \right)
$$

where

$$
\rho(r) = \begin{cases} 
  r^2/2, & |r| < r_0 \\
  r_0^2/2 - r_0^2/2, & |r| \geq r_0.
\end{cases}
$$

![Figure 3. Flow chart for system identification.](image)

3.2. Parametric estimation

The proposed parametric MT tensor determination approach is based on system identification principles. The essence of system identification is to select a model by an optimization criterion from a given set of models which can best fit the characteristics of the actual process to be studied given the system input and output data [17]. The procedure of system identification is shown in Figure 3.
3.2.1. Identification objective. This step includes various preparation substeps necessary for system identification, including determining the anticipated objective, training and validating data set selection (the training data set is used for training and obtaining a system model while the validating data set is to test the trained model), data preprocessing and quality control, and so on. In this study, the objective is the MT impedance tensor \( Z(f) \), and related input/output data are \( B_x(t_n), B_y(t_n) \) and \( E_x(t_n) \), respectively.

3.2.2. Model selection. Considering the non-stationarity of the input/output time-domain data series, the well-known auto-regressive with exogenous inputs (ARX) system structure was found to be appropriate in this EMTF estimation scenario. The general form of the single input/output ARX model is

\[
y(t_n) + a_1 y(t_n - 1) + \cdots + a_{n_a} y(t_n - n_a) = b_0 u(t_n) + \cdots + b_{n_b-1} u(t_n - n_b - 1) + e(t_n).
\] (6)

Extending it to our study as a DISO (double input and single output) model to fit the situation encountered in EMTF estimation yields the general form

\[
y(t) = \frac{B(z)}{A_1(z)} u_1(t) + \frac{C(z)}{A_2(z)} u_2(t) + e(t)
\] (7)

where

\[
\begin{align*}
A_1(z) &= 1 + a_{11} z^{-1} + \cdots + a_{1n_a} z^{-n_a1} \\
A_2(z) &= 1 + a_{22} z^{-1} + \cdots + a_{2n_a} z^{-n_a2} \\
B(z) &= b_0 + b_{1} z^{-1} + \cdots + b_{n_b-1} z^{-(n_b-1)} \\
C(z) &= c_0 + c_{1} z^{-1} + \cdots + c_{n_b-1} z^{-(n_b-1)}
\end{align*}
\] (8)

The functions above are expressed using the \( z \)-transform [17] where the symbol \( z \) is an operator indicating a time delay, e.g., \( a_{11} z^{-1} u_1(t) = a_{11} u_1(t - 1) \), and higher powers of \( z \) denote correspondingly longer time delays, e.g., \( z^{-n} u_1(t) = u_1(t - n) \). In the following, we define the data set, noise term, and model parameters as

\[
\begin{align*}
\text{parameters } \theta &\in \{a_{11}, \ldots, a_{1n_a}, a_{21}, \ldots, a_{2n_a}, b_0, \ldots, b_{n_b-1}, c_0, \ldots, c_{n_b-1}\} \\
\text{current input: } u(t_n), \text{ current response: } y(t_n) \\
\text{current stochastic noise term: } e(t_n)
\end{align*}
\]

The ARX model guarantees EMTF smoothness which is powerful prior knowledge that can, for example, help to mitigate the impact of electrically noisy environments in proximity to DC electrified railways [7]. Besides, it is based on the time domain and very flexible in analysis, prediction, fault diagnosis, classification and control [18].

3.2.3. Parameter estimation. Two parts are considered in this step: model order selection and fit method selection. First, motivated by the underlying physics that the geoelectric field response is instantaneous for MT survey data sample intervals, the EMTF model order can be more than one step which is typically applied in system identification. In this case, we consider the final prediction error criterion (FPE) [19]:

\[
FPE(p) = (1 + \frac{p}{N}) \left(1 - \frac{p}{N}\right)^{-1} \left(\bar{\gamma}_0 - \sum_{i=1}^{p} a_i \bar{\gamma}_i\right)
\] (9)

where \(\bar{\gamma}_i\) are the autocovariance functions of each order of the sequence, \(N\) is the length of the data series, and \(p\) is the model order. In practice, we considered different models from low-order to high-order and calculated the corresponding FPEs to determine the order \(p\) that minimizes the FPE. Then we fit the ARX system model by the prediction error method (PEM) [20], [21] which is commonly employed in system identification. PEM operates by positing the optimization problem

\[
\hat{\theta}_N = \arg\min_{\theta} \sum_{n=1}^{N} \mathbb{E}[y(t_n) - \hat{y}(t_n|\theta)]
\] (10)

where \(\hat{y}(t_n|\theta)\) is the dynamic system response, i.e., the modeled geoelectric field \(\hat{E}_x(t_n)\) in this context, given the system model parameters \(\theta\).
We emphasize, in contrast to the nonparametric approach that must analyze data segments with lengths of hundreds to thousands of samples, that the sliding parametric data window is of length max(na, nb - 1) + 1 and typically has less than 10 data samples.

3.2.4. Model validation. This step aims to choose an appropriate cost and computational optimization algorithm to train the system model to agree with the data. Typically, the cost function is based on a residual error metric and, possibly, an additional regularizing cost element. In practice, this step is typically combined into the parameter estimation.

After system identification, the frequency domain expression of the MT impedance tensor can be determined by

\[
\begin{align*}
Z_{xx}(z) &= \frac{B(z)}{A_1(z)} \\
Z_{xy}(z) &= \frac{C(z)}{A_2(z)}
\end{align*}
\]  

(11)

The complex expression as a function of frequency is found by replacing the variable z with \(e^{-j2\pi f}\) [17] where \(j = \sqrt{-1}\).

The overall procedure is iterative, requiring systematic evaluation and testing of different parameter orders, model structures, and data segments to build sufficient confidence that the model fit captures the underlying, true system dynamics. Fortunately, the system identification discipline is mature and software systems such as the Matlab System Identification Toolbox provide the means for rapid and systematic model fitting and evaluation.

4. Experiment

Two EMTFs are now compared: the nonparametric version found with the procedure described in Section 3.1 and the parametric version obtained using the approach outlined in Section 3.2.

The nonparametric MNF34 EMTF [22] is available at the Searchable Product Depository (SPUD, http://ds.iris.edu/spud) and is derived from the full, nearly month-long and freely available magnetic, i.e., \(B_x(t_n)\) and \(B_y(t_n)\), and geoelectric, i.e., \(E_x(t_n)\) and \(E_y(t_n)\), field time series measured at MNF34. The impedance tensor here are given in the frequency domain and described with 30 specific frequencies between approximately \(10^{-5}\) to \(10^{-1}\) Hz.

The parametric EMTF was estimated using the Matlab System Identification Toolbox. The training set, shown in Figure 2, consists of 1-second sampled \(B_x(t_n)\), \(B_y(t_n)\), and \(E_x(t_n)\) time series with a length of 2000 samples over the time range from UTC 18:03:21 to 18:36:40 on July 14, 2012. After 20 iterations, it was found that the OE model with \(n_{a1} = n_{a2} = n_b = n_c = 5\) best fit the data and with a fit accuracy 90.49% in terms of minimizing the residual error and the FPE criteria.

In this case, the estimated parameters, FPE and MSE are given in Table 1. MSE is the mean square error, which in this study, is equal to variance. It is clear that \(\{c_i\}_{i=1}^4\) is much larger than \(\{b_i\}_{i=1}^4\), which is expected given that \(E_x\) is more strongly determined by \(B_y\), and the presence of \(B_x\) in this equation results from unevenly-distributed interior conductivity regions.

| \(a_{11}, a_{12}, a_{13}, a_{14}, a_{15}\) | -2.397, 1.321, 1.111 - 1.585, 0.5496 |
| \(a_{21}, a_{22}, a_{23}, a_{24}, a_{25}\) | -1.824, 1.149, -0.4093, -0.02921, 0.1172 |
| \(b_1, b_2, b_3, b_4\) | -0.35, 1.924, -2.792, 1.218 |
| \(c_1, c_2, c_3, c_4\) | 27.46, -64.89, 47.78, -10.36 |
| FPE | 2.957 |
| MSE | 2.875 |

In this study, we considered two means for comparing the nonparametric and parametric estimated EMTFs: 1) directly in the frequency domain, i.e., the apparent resistivity and phase, and 2) electric field response reproduction given magnetic field input.
4.1. Apparent resistivity and phase

Apparent resistivity is a parameter reflecting changes in rock and ore conductivity. It is determined in a similar way to determining uniform horizontal resistivity, but they are not equal. The two values are the same only in the case when the underground rock is evenly distributed (two or more types of rock or ore with different conductivities) or the surface is flat.

The apparent resistivity \[ \rho_{a,ij}(f) = \frac{1}{2\pi f \mu_0} |Z_{ij}(f)|^2 \] (12)

and the phase angle is given by

\[ \phi = \frac{180}{\pi} \arctan \left( \frac{\Im(Z_{ij}(f))}{\Re(Z_{ij}(f))} \right) \] (13)

where \( \Re(\cdot) \) and \( \Im(\cdot) \) are the real and imaginary component of each corresponding argument, respectively.

The apparent resistivity and phase are plotted in Figure 4. We emphasize that the response at both extremes of the period scale are important: the longest periods penetrate the deepest and are of greater interest to MT studies while the shorter periods that are associated with larger apparent resistivity and hence larger frequency-domain input to output response, are more important to GIC modeling efforts. The general morphological similarities and striking magnitude differences are both noteworthy. Smoothness provides a powerful and physically-motivated constraint to guide the statistical procedure that determines an EMTF given surface magnetic and geoelectric field measurements. As expected, the agreement between the geographically oriented parametric EMTF (plotted in green and labelled as parametric (geo)) agrees better with the geographically oriented nonparametric EMTF (shown in red) than the geomagnetically oriented parametric EMTF (show in blue).

![Figure 4](image)

**Figure 4.** Nonparametric and parametric EMTF frequency-domain characteristics (the abscissa is transformed from frequency to time).
4.2. Electric field response estimation

As both the nonparametric and parametric EMTF estimation approaches are found by minimizing the residual error, determining which EMTF is the more faithful model begins by comparing the estimated geoelectric field response $\mathbf{E}_x(t_n)$ to the geoelectric field measurements $\mathbf{E}_x(t)$. The nonparametric EMTF estimated geoelectric field time series $\mathbf{E}_x(t_n)$ is calculated using the MNF34 EMTF impedances given by the points in Figure 4. The parametric version of $\mathbf{E}_x(t_n)$ is found by calculating the OE system model response (11) with the estimated parameters $\{a_i\}_{i=1}^5$, $\{a_{Ii}\}_{i=1}^5$, $\{b_i\}_{i=1}^4$, and $\{c_{Ii}\}_{i=1}^4$.

The simulated $\mathbf{E}_x(t_n)$ of the two methods, the measured $\mathbf{E}_x(t)$ and the fit residuals for 18:45:00 to 19:00:00 UTC (represented as a validating data set, a time interval beginning after the training-set time interval which ends at 18:36:40) are shown in Figure 5. Several general features are clear. First, the nonparametric determined geoelectric field $\mathbf{E}_x(t_n)$ differs more significantly and erratically with the measured $\mathbf{E}_x(t)$. Second, the parametric determined $\mathbf{E}_x(t_n)$ exhibits a general negative bias with respect to the measured $\mathbf{E}_x(t)$, an issue possibly explained by the fact that this initial study considered only an approximately 30-minute training interval and longer period information is hence unavailable to guide the parameter optimization.

The power spectral density (PSD) plot for the measured $\mathbf{E}_x(t)$ and residuals of two estimated geoelectric field time series $\mathbf{E}_x(t_n)$ using the whole validating data set of 900 seconds is shown in Figure 6. We find the nonparametric method performs well at low frequency, but it is clear that the parametric method performs better overall. This is to be expected given the absence of longer period information in the training dataset used to fit the parametric EMTF model.

The residual statistics are summarized in Table 2. Our initial study illustrates the potential of parametric EMTF determination. We conclude from the MSE reported in Table 2 that the parametric EMTF more faithfully reproduces the measured geoelectric field $\mathbf{E}_x(t_n)$. On this basis, the parametric apparent conductivity shown in Figure 4 is the better model and its use in MT studies would yield a more faithful reconstruction of the interior ground conductivity. GIC studies using the parametric EMTF to retrieve the geoelectric field would also benefit. However, the relatively short training interval and negative bias may indicate that the longer periods, hence information pertaining to deeper ground penetration, requires additional scrutiny to be conducted in future work.

![Figure 5](image1.png)

(a) Measured $E_x(t_n)$ (mean subtracted) and nonparametric and parametric determined $\mathbf{E}_x(t_n)$.

![Figure 6](image2.png)

(b) Nonparametric and parametric residuals.

Figure 5. A reconstruction of $E_x(t_n)$ in 15-minutes interval (18:45 to 19:00).

Figure 6. PSD of the measured $E_x(t_n)$ to the residual errors of $\mathbf{E}_x(t_n)$.
Table 2. Electric field reproduction summary statistics: mean square error (MSE), standard deviation (Std.) and coefficient of determination ($R^2$). The units are $\text{mV/km}$ (and MSE is $[\text{mV/km}]^2$).

| Model     | N  | Mean | Std. | MSE  | $R^2$ | Min. | Max.    |
|-----------|----|------|------|------|-------|------|---------|
| Nonparametric | 900 | -0.32 | 8.67 | 3.96 | 0.93  | -23.10 | 22.10   |
| Parametric  | 900 | -0.15 | 7.23 | 2.29 | 0.96  | -15.54 | 18.28   |
| Measured   | 900 | 0.49  | 7.26 | /    | /     | -18.59 | 18.47   |

5. Conclusion

Our study investigates a new class of parametric EMTFs found using system identification methodology and founded on the physically justified constraint of smoothness. From the theoretical foundations and experiment results, the parametric approach offers clear potential advantages:

**Smoothness.** Smoothness in the EMTF impedance is physically justified because lithospheric electromagnetic induction is diffusive. The prevailing nonparametric approaches generally estimate the impedance independently at each frequency. Our parametric approach has the advantage that it yields EMTFs that are smooth by construction.

**Fewer data and parameter requirements.** In contrast to the nonparametric approach that must analyze data segments with length in the hundreds or thousands of samples, the parametric approach requires typically less than 10 data samples for its sliding data window. This means we can determine results with smaller size training datasets. Furthermore, fewer model fit parameters are required for the parametric approach and the net gain is a more statistically efficient procedure that yields a more faithful EMTF given the same amount of data.

**Analysis in time-domain only.** Unlike the nonparametric approach which operates in the frequency-domain, our parametric approach operates in the time-domain where data quality control and editing of time series is simplified.

This study has investigated but one measurement site of one MT survey, demonstrating an advantage to the parametric approach. Future work will investigate a multitude of sites and extensively compare nonparametric and parametric EMTF estimates. The aforementioned advantages are of great significance as MT surveys face noise contamination challenges originating from ever increasing societal electrification demands. We expect to develop EMTFs of even higher fidelity, extending the project in scope to numerous sites across different MT survey data sets. EMTFs are the fundamental datum in both MT inversion and GIC modeling. As such, our proposed efforts will translate into higher fidelity MT reconstructions of interior conductivity and modeling systems for GIC hazard forecasting and mitigation.

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