A Method for System Reliability Evaluation Based on Inexact Interval Probability

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ABSTRACT: With a large number of new components and new technologies widely used in power system, the method of power equipment outage probability based on law of large number will no longer be applicable due to the lack of statistical data. This paper considers the high order uncertainty of component failure probability under the condition of small sample size and uses interval values to represent the reliability parameters of the component when carrying out the risk assessment of power system, and propose Belief Universal Generating Function by combining Universal Generating Function with Belief Function Theory. This method can effectively avoid over estimation in interval arithmetic and get a more accurate interval value of power system risk assessment index.

1. INTRODUCTION
In recent years, with the construction of UHV engineering, smart grid engineering, flexible transmission engineering and global energy Internet, the scale of power systems is expanding, and its safe and stable operation faces greater challenges. For the risk assessment of power systems, the reliability level is determined by the reliability level of each component. In the traditional power system risk assessment, the reliability parameters of the components are generally considered to be constant, but the reliability parameters of the components are subject to the impact of many factors.
Especially due to the lack of operational experience and fault data, there are many cognitive uncertainties in the new components. Therefore, the traditional reliability assessment based on the law of large numbers with accurate probability cannot be used for small sample cases lacking statistical data\(^1\)-\(^2\).

In the power system risk assessment, there are fuzzy set theory, evidence theory, Bayesian theory, interval analysis and other methods to quantify cognitive uncertainty\(^3\)-\(^4\). Literature \(^5\) proposes an algorithm for constructing fuzzy probabilities to represent or simulate mixed accidental and cognitive uncertainties in finite-scale sets. In \(^6\), the general generation function method is used to calculate the interval probability of multi-state system (MSS), and an affine algorithm is proposed for the overestimation phenomenon in interval operation. In \(^7\), the trust function theory is combined with the UGF method to calculate the reliability interval of the MSS in consideration of the common cause failure. Based on the uncertainty of the parameters, the literature \(^8\) solves the problem of system reliability optimization through interval analysis. At present, fuzzy numbers\(^9\)-\(^10\) and interval numbers\(^11\) are often used at home and abroad to represent high-order uncertainties in the field of power system risk assessment.

This paper draws on the existing research results and applies the uncertainty theory to the risk assessment of large power systems. When the reliability value of the power equipment is represented by the interval value in the case of a small sample, the interval value of the system risk assessment index is calculated by the trust general generation function method.

2. UNCERTAINTY THEORY OF POWER SYSTEM RISK ASSESSMENT

2.1 The expression and algorithm of interval probability

Interval probability is also known as Imprecise Probability\(^12\). The core idea is to use interval values to represent the probability of random events occurring. Its definition is as follows:

If a variable \(x\) is an element of the set \(X=\{x_1, x_2, \ldots, x_n\}\), the set of intervals \(W=\{[P_i, \overline{P}_i]\}\) is satisfied: for any \(i\), \(P_i \leq \overline{P}_i\). If for any \(P_i \in [P_{i-1}, P_{i+1}]\), there are \(P_{i-1} \in [P_{i-1}, P_{i+1}], \ldots, i \in [P_{n-1}, P_n]\), making

\[
\sum_{i=1}^{n} P_i = 1
\]

Then the interval \(P_i = [P_{i-1}, P_{i+1}], i = 1, \ldots, n\) becomes the interval probability of \(x_i\).

2.2 Trust Generic Generation Functions

The essence of the UGF method is discrete random. A description of the probability distribution of a variable. The Belief Function Theory is an important theory for expressing and dealing with the uncertainty of knowledge. The basic idea is to use the multi-value mapping method to find the upper and lower bounds of the probability.

Combine UGF and BFT to form the Belief Universal Generating Function (BUGF). According to the definition of UGF, if the probability distribution function of the random variable \(X\) is: \(x = (x_1, x_2, \ldots, x_k)\), \(p = (p_1, p_2, \ldots, p_k)\), then the z-transform of the random variable \(X\) is as follows:

\[
u(z) = \sum_{p}^{z^p} p
\]

Suppose component \(j\) has \(n_j\) states whose properties are elements in set \(g_j = \{g_{j1}, g_{j2}, \ldots, g_{nj}\}\), and the corresponding interval probabilities are \([P_{j1}, \overline{P}_{j1}], [P_{j2}, \overline{P}_{j2}], \ldots, [P_{nj}, \overline{P}_{nj}]\). The mass function is defined as follows.

\[
m(S) = \begin{cases} 
1 - \sum_{i=1}^{n} p_{x_n}(S = g_{p_n}, n = 1, 2, \ldots, n_j) & \text{if } S = \{g_{j1}, g_{j2}, \ldots, g_{nj}\} \\
0 & \text{else}
\end{cases}
\]
Where $S$ is a set containing all states of component $j$, indicating that the component has at least $P_{j,n}$, $n = 1,2,\ldots,n_j$ with a probability of $g_{j,n}$ state. Based on the mapping method of equation (3), the state performance of component $j$ can be expressed as $[g_j]$, $[g_j]_2$, $\ldots$, $[g_j]_k$. For a system with $n$ components, when the performance level $g_j$ of the component is a set and can be expressed as $[g_j](1 \leq j \leq n, [g_j] \subset g_j)$, the relationship of the system structure before and after mapping is:

$$
\phi([g_1], \ldots, [g_n]) = \{ \phi([g_1], \ldots, [g_n]) \mid g_j \in [g_j] \}
$$

(4)

Then the BUGF of component $j$ is

$$
U^g_j(z) = \sum_{l=1}^{k_j} \sum_{l=1}^{k_j} \prod_{j=1}^{m_j} m_{j,l} z^{g_{j,l}}
$$

(5)

Where $k_j$ is the number of states of element $j$.

According to the definition of BFT, the BUGF of MSS consisting of $n$ components is:

$$
\Omega_m(U^g_1(z), \ldots, U^g_n(z)) = \Omega_m(\sum_{l=1}^{k_1} \sum_{l=1}^{k_1} \prod_{j=1}^{m_j} m_{j,l} z^{g_{j,l}})
$$

(6)

$$
\Omega_m(\sum_{l=1}^{k_1} \sum_{l=1}^{k_1} \prod_{j=1}^{m_j} m_{j,l} z^{g_{j,l}}) = \Omega_m(\sum_{l=1}^{k_1} \sum_{l=1}^{k_1} \prod_{j=1}^{m_j} m_{j,l} z^{g_{j,l}})
$$

(7)

Where $\Omega_m$ is a special operator that implements a mapping of the probability distribution function of a system to the probability distribution function of each element in the system.

Then when the performance requirement of the system is $w$, define two operators $1^+_{\phi}$ and $1^-_{\phi}$ to obtain the likelihood function and the trust function:

$$
1^+_{\phi}(z) = \begin{cases} 1, & \phi([g_1], \ldots, [g_n]) \cap w \neq 0 \\ 0, & \text{else} \end{cases}
$$

(8)

$$
1^-_{\phi}(z) = \begin{cases} 1, & \phi([g_1], \ldots, [g_n]) \subseteq w \\ 0, & \text{else} \end{cases}
$$

(9)

In the formula, $\phi([g_1], \ldots, [g_n]) \cap w \neq 0$ means that at least one element in the interval value performance can meet the performance requirements of the system, $\phi([g_1], \ldots, [g_n]) \subseteq w$ indicates that all elements in the interval value performance can meet the performance requirements of the system, so that the likelihood function and the trust function can find the upper and lower bounds of the reliability according to equations (10) and (11).

$$
P_l(w) = 1^+_{\phi}(U(z))
$$

$$
Bel(w) = 1^-_{\phi}(U(z))
$$

(10)

(11)

3. BUGF-BASED POWER SYSTEM RISK ASSESSMENT

Assume that there are a total of $n$ generators and $r$ lines in the transmission and transmission system to be evaluated. For $n$ generators, assume that the state probability of the former $m$ generator sets due to insufficient statistical data is the interval probability; the remaining generators are traditional equipment, and the state probability is point value. Then, the UGF form of the reliability parameter of the $i$-th generator is:

$$
U_p = \sum_{i=1}^{n} p_i z^{n_i} + \sum_{j=1}^{m} \sum_{i=1}^{n} m_{j,i} z^{n_i} x = 1, \ldots, n
$$

(12)
Where \( P_{ij} \) represents the probability that the \( i \)-th generator is in state \( j_i \), \( k_i \) represents the number of states of generator \( i \), and \( x_{ij} \) represents the maximum processing of generator \( i \) in state \( j_i \). For conventional devices, the state probability is a point value, and \( [P_{ij}] = [P_{ij}, \bar{P}_{ij}] = P_{ij} \), so equation (12) is still true.

Convert the reliability parameters of the generator to the BUGF form

\[
U_n^i = \sum_{l=1}^{k_i} m_l z^l_{ij} = \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij}, j = 1, \ldots, n
\]  

(13)

For \( r \) lines, the first \( s \) lines (referred to as uncertain lines) have a probability of outage due to insufficient statistical data, etc.; the remaining lines are traditional devices, and the state probability is point value. The line uses a two-state model. Then, the UGF form of the reliability parameter of the \( i \)-th line is:

\[
U_i = \sum_{l=1}^{k_i} p_{ij} z^l_{ij} = \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij}, j = 1, \ldots, r
\]  

(14)

Where \( l_{ij} = 0 \) indicates that the \( i \)-th line is out of service, \( p_{ij} \) indicates its probability, \( l_{ij} = 1 \) indicates that the \( i \)-th line is running, and \( p_{ij} \) indicates its probability.

Convert the line reliability parameter to BUGF form

\[
U^i = \sum_{l=1}^{k_i} m_l z^l_{ij} = \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij}, j = 1, \ldots, r
\]  

(15)

In the formula, the last term indicates that the line has the probability that any one of the sets \( \{0, 1\} \) is taken.

For a power system with \( n \) components, assuming that the state probability of each component is a point value, the \( u \) function of component \( i \) is \( u_i \), and a special operator \( \Omega_\sigma \) is defined to obtain its risk assessment index:

\[
U_i(z) = \Omega_\sigma (u_i, u_1, \ldots, u_n)
\]  

(16)

\[
= \Omega_\sigma \left( \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij}, \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij}, \ldots, \sum_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^l_{ij} \right)
\]

\[
= \sum_{l=1}^{k_i} \sum_{l=1}^{k_i} \sum_{l=1}^{k_i} \prod_{l=1}^{k_i} [P_{ij}, \bar{P}_{ij}] z^{l_{ij} \in \{0,1\}}
\]

Where \( \sigma(g_{j_i}, g_{j_2}, \ldots, g_{j_n}) \) is the \( g_{ji} \) determined for the state performance of each component, \( j = 1,2,\ldots, n \).

If there are components in the system whose state probability is interval probability, then firstly establish the component reliability parameter in the form of BUGF according to equations (13)-(15), and then use the operator \( \Omega_\sigma \) to generate the BUGF of the system:

\[
U^i(z) = \Omega_\sigma (U^1(z), \ldots, U^n(z))
\]  

(17)

For the probability of loss of load [LOLP] of the system, the upper and lower limits are:

\[
Pr(LOLP) = f_{\sigma_0}(U^i(z))
\]  

(18)

\[
Bel(LOLP) = f_{\sigma_0}(U^i(z))
\]  

(19)
Where $f_{\text{lolp}}$, $f_{\text{halp}}$ are operators that compute the likelihood function and the trust function based on the current load level, which are defined in detail as:

$$f_{\text{halp}}(z^*) = \begin{cases} 
0, & \sigma([g_1],[g_n]) \subseteq ld \\
1, & \text{else} 
\end{cases} \quad (20)$$

$$f_{\text{lolp}}(z^*) = \begin{cases} 
0, & \sigma([g_1],[g_n]) \cap ld \neq 0 \\
1, & \text{else} 
\end{cases} \quad (21)$$

Where $ld$ represents the current load level, $\sigma([g_1],[g_n]) \subseteq ld$ indicates that all cases of the performance set meet the load requirements of the system, and the system can be guaranteed to lose load. $\sigma([g_1],[g_n]) \cap ld \neq 0$ means that at least one of the performance sets satisfies the load requirements of the system, and the system does not lose load.

For the expected power shortage of the system [EDNS], the upper and lower limits are:

$$P(\text{EDNS}) = f_{\text{lolp}}(U_s^*) \quad (22)$$

$$\text{Bel}(\text{EDNS}) = f_{\text{halp}}(U_s^*) \quad (23)$$

Where $ld$ is the current load level, $f_{\text{lolp}}$, $f_{\text{halp}}$ is the operator that computes the likelihood function and the trust function based on the current load level $ld$. $f_{\text{lolp}}(z^*) = \max \sum C_i$ (24)

In the formula, $\max(C)$ represents the maximum value of the optimal reduction load in all cases of the performance set. $f_{\text{halp}}(z^*) = \min \sum C_i$ (25)

Where $\min(C)$ represents the minimum value of the optimal load reduction in all cases of the system performance set.

### 4. CASE ANALYSIS

#### 4.1 Case parameters

Figure 1 shows a simple power system with two generators and a load. The output of generator 1 and generator 2 and their probability are shown in Tables 1 and 2. Regardless of the random failure of the line, the load is 150MW. This example will use the trust general generation function method, the interval operation method and the nonlinear optimization method to obtain the power system risk assessment index LOLP.

#### 4.2 Algorithm

Trust the generic generator function method:

Write the BUGF form of Unit 1 and Unit 2 as:

$$U_{G_1} = 0.096R^0 + 0.095R^{200} + 0.795R^{200} + 0.014z^{0,100,200}$$

$$U_{G_2} = 0.090R^0 + 0.195R^{200} + 0.695R^{200} + 0.02z^{0,150,300}$$

Then the BUGF form of the power generation system is:
So when the load demand is 150MW, we can find the trust function:

\[
U^*_1 = \phi(U^*_{11}, U^*_{12}) = 0.00864z^2 + 0.00855z^{100} + 0.01872z^{200} + 0.07155z^{300} + 0.018525z^{400} + 0.06672z^{500} + 0.155025z^{600} + 0.066025z^{700} + 0.552525z^{800} + 0.00273z^{900} + 0.00973z^{1000} + 0.0159z^{1100} + 0.00028z^{1200}
\]  

(28)

Find the likelihood function:

\[
P(Bel(150)) = Bel(150) + 0.00126 + 0.00192 + 0.0019 + 0.00028
\]

\[= 0.97745\]  

(29)

The system's LOLP indicator:

\[
LOLP = [1 - P(Bel(150)) - Bel(150)] = [0.01719, 0.02255]
\]  

(31)

Interval operation method:

According to the interval algorithm introduced in Section 2.1, the state probability interval of the power generation system is calculated as shown in Table 3.

Obviously, when the load is 150MW, the LOLP indicator of the system is [0, 0.05093].

Nonlinear optimization method:

The probability that the maximum output of unit 1 is 0,100,200 is \(P_{11}, P_{12}, P_{13}\), and the probability of unit 2's maximum output is 0,150,300 is \(P_{21}, P_{22}, P_{23}\) respectively, then the upper and lower bounds of the LOLP interval of the system risk assessment index are obtained. Optimization:

\[
\max R = p_{11} \cdot (p_{22} + p_{23}) + p_{12} \cdot (p_{21} + p_{22}) + p_{13} \cdot (p_{21} + p_{22} + p_{23})
\]

\[
0.096 \leq p_{11} \leq 0.102
\]

\[
0.095 \leq p_{12} \leq 0.105
\]

\[
0.795 \leq p_{13} \leq 0.805
\]

\[
0.090 \leq p_{21} \leq 0.110
\]

\[
0.195 \leq p_{22} \leq 0.205
\]

\[
0.695 \leq p_{23} \leq 0.705
\]

\[
p_{11} + p_{12} + p_{13} = 1
\]

\[
p_{21} + p_{22} + p_{23} = 1
\]

s.t.

(32)

\[
\min R = p_{11} \cdot (p_{22} + p_{23}) + p_{12} \cdot (p_{21} + p_{22}) + p_{13} \cdot (p_{21} + p_{22} + p_{23})
\]

\[
0.096 \leq p_{11} \leq 0.102
\]

\[
0.095 \leq p_{12} \leq 0.105
\]

\[
0.795 \leq p_{13} \leq 0.805
\]

\[
0.090 \leq p_{21} \leq 0.110
\]

\[
0.195 \leq p_{22} \leq 0.205
\]

\[
0.695 \leq p_{23} \leq 0.705
\]

\[
p_{11} + p_{12} + p_{13} = 1
\]

\[
p_{21} + p_{22} + p_{23} = 1
\]

s.t.

(33)

The range of LOLP can be determined as [0.01931, 0.02255].
Comparing the three methods, the nonlinear optimization method considers all the constraints, so it can be considered that the obtained interval is the real interval. Obviously, the interval obtained by the BUGF method is narrower than the interval obtained by the interval algorithm, and contains real intervals. However, when solving the reliability problem of the actual large-scale power system, the objective function is difficult to analyze and express, so the nonlinear optimization method cannot be directly applied to solving the reliability problem of the power system. At this time, the BUGF method can reduce the overestimation phenomenon in the interval calculation with respect to the interval calculation method, and has certain practical significance.

5. CONCLUSION
In the case of small samples, it is important to evaluate the impact of the uncertainty of power system component reliability parameters on power system risk assessment for the safe and stable operation of power systems. In this paper, the general generation function and the trust function method are introduced into the power system risk assessment which takes into account the high-order uncertainty, and a new algorithm for solving the power system risk assessment index when the power system component reliability parameter is the interval number is proposed. The algorithm first converts the component reliability parameter into the BUGF form, converts the state probability from the interval value to the point value, correspondingly calculates the probability of the system in each state, and then finds the likelihood function and trust function according to the given load demand. The function acts as the upper and lower bounds of the system risk assessment indicator. It can be seen from the results of the example that the proposed method can reduce the overestimation phenomenon in the interval operation and obtain a narrower risk assessment index. It also reflects the uncertainty of the system risk assessment index caused by the uncertainty of the component reliability index, taking into account the reliability of the component to be evaluated and the actual needs of the project site.

| Maximum output of the unit /MW | 0          | 100        | 200      |
|-------------------------------|------------|------------|----------|
| Unit 1 state probability     | [0.096,0.102] | [0.095,0.105] | [0.795,0.805] |

| Maximum output of the unit /MW | 0   | 100  | 200  |
|-------------------------------|-----|------|------|
| Unit 2 state probability     | [0.09,0.110] | [0.195,0.205] | [0.695,0.705] |

| Maximum output of power generation system | Interval probability |
|------------------------------------------|----------------------|
| 0                                        | [0.00864,0.01122]    |
| 100                                      | [0.00855,0.01155]    |
| 150                                      | [0.01872,0.02091]    |
| 200                                      | [0.07155,0.08855]    |
| 250                                      | [0.018525,0.21525]   |
| 300                                      | [0.06672,0.07191]    |
| 350                                      | [0.0155025,0.165025] |
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