Shafting stiffness calculation of satellite antenna scanning mechanism

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Abstract. As a critical rotating joint component of space mechanism, the shafting of satellite antenna scanning mechanism needs high stiffness and reliability. In this paper, the axial stiffness and radial stiffness of the shafting are calculated based on theoretical analysis, and the stiffness performance of the shafting is simulated and verified by finite element method. The results show that the relative error of radial stiffness and axial stiffness is 8.70% and 8.14% respectively. The calculation method adopted can effectively evaluate the stiffness performance of precision shafting and guide the optimal design of the shafting structure.

1. Introduction

Scanning mechanism is the executive mechanism to realize the scanning function of the truss antenna. It is used to change the beam scanning of the antenna in orbit, so as to adjust the service area of the antenna system. Scanning mechanism should work in orbit for a long time, which is the crucial component affecting the lifetime and reliability of the antenna subsystem. Therefore, the adaptability of the mechanism shafting to the space environment is highly demanded.

The stiffness of shafting is an essential factor affecting the operation accuracy, vibration noise and service life of scanning mechanism. Shafting stiffness is affected by structural characteristics, such as bearing stiffness, assembly mode. Many domestic and foreign scholars have done a lot of research on the stiffness analysis of bearings. Jones proposed a relatively complete quasi-static analysis and raceway control theory of rolling bearings [1]. In this theory, the concept of stiffness matrix of rolling bearing is first put forward, and the influence of centrifugal force and gyroscopic moment on stiffness is considered, which can reflect the stiffness of rolling bearing more accurately [2,3]. Chen calculated the stiffness of angular contact ball bearings under external loads. Compared with the experimental results, the consistency is higher [4]. Harris [5] and Deng Sier [6] carried out in-depth theoretical research on the rolling bearing, and widely used in the bearing design and analysis. Tiwari [7], Horie M [8-11], Frisoli A [12] studied the modeling of clearance mechanism, the influence of joint clearance on static accuracy, dynamic characteristics, stiffness, stability of mechanism.

The precision of satellite antenna scanning mechanism will directly affect the scanning precision of antenna beam in orbit. It is required that the mechanism shafting has high support stiffness. In this paper, the shafting stiffness of the antenna’s three-axis scanning mechanism is theoretically calculated and simulated. The radial and axial stiffness of paired 71918AC angular contact ball bearings are studied, which provides a theoretical basis for the design and optimization of the antenna scanning mechanism.
2. Scanning mechanism shafting
The three-axis scanning mechanism of satellite antenna studied is an executive component located at the end of the antenna driving mechanism, which consists of three rotating units whose axes intersect at one point as shown in Figure 1. The antenna reflector is driven to rotate by coupling of three rotating units, and finally the antenna beam is scanned at a predetermined position. To optimize the antenna structure, the shafting layout is penetrating. This design makes the structure of the antenna driving shafting more compact, which saves the space of the driving mechanism and achieves weight reduction. The rotating unit includes servo motor, harmonic reducer, motor output shaft, precise angular contact ball bearing and so on, as shown in Figure 2. The output power of the servo motor is decelerated and the torque is raised by the harmonic reducer. As an important intermediate transmission link, the shafting unit completes the motion transmission between the output of the harmonic reducer and the antenna.

Figure 1. Three-axis scanning mechanism. Figure 2. Scanning mechanism shafting.

3. Theoretical calculation
Due to vibration that the three axes form a straight line with different planes, the frame reflector can not rotate around the focus of the feed, and the rotational accuracy decrease. Therefore, it is necessary to calculate the stiffness of the shafting system.

3.1. Radial stiffness analysis of shafting
The rotating unit of the mechanism bears the radial load through the frame, and the shafting frame produces radial deformation. Ignoring the deflection of the motor output shafting and its local structure, the radial stiffness analysis model is established as shown in Figure 3. $K_1$ and $K_2$ are the radial support stiffness of single-row bearings, $K_3$ is the radial stiffness of harmonic reducer, and $K_4$ is the radial stiffness of the motor output end.

When the long axis of flexible wheel is perpendicular to the direction of radial load, that is $K_3 = 0$, the radial stiffness of the shafting is the lowest, which can be expressed as

$$K = K_1 + K_2 + K_4 \quad (1)$$

According to the loading condition of the mechanism, the pre-tightening force is applied to the matching bearing. The equation for calculating the radial stiffness of rolling bearings is

$$k = 0.118 \times 10^5 (DF Z^2 \cos^5 \alpha)^{1/3} \quad (2)$$

where $F_r$ is the radial load on the bearing, $Z$ is the number of ball bearings, $\alpha$ is the contact angle under load.

The radial load-stiffness curve of the shafting is obtained, as shown in Figure 4.
3.2. Analysis of axial stiffness of shafting

Since the structure frame, the upper cover and the over-wire ring are connected, and there is no axial connection between the servo motor and the base, as shown in Figure 5. Ignoring the meshing axial friction between the rigid wheel and the flexible wheel of the harmonic reducer, the axial stiffness of the shafting depends entirely on the axial stiffness of the preloaded bearing group.

According to Hertz contact theory, for angular contact ball bearings with point contact, the relationship between contact load $Q$ and deformation $\delta_n$ is as follows

$$Q = K_n \delta_n^{1.5}$$  \hspace{1cm} (5)

where $K_n$ is the normal load-displacement coefficient.

Assuming that the axial load $F_a$ acts on all rolling bodies of ball bearings with the same load, as shown in Figure 5, then
\[ Q = \frac{F_a}{Z \sin \alpha} \]  

Equation (7) can be obtained by combining the above equations,

\[ \frac{F_a}{ZK_0(BD)^{1.5}} = \sin \alpha \left( \frac{\cos \alpha^0}{\cos \alpha} - 1 \right)^{1.5} \]  

External loads are applied along the axial direction. If the ball and the groove are not separated, the total axial displacement of the bearing under the combined action of preload and external load is as follows

\[ \delta_1 = \delta_p + \delta_a \]  

where \( \delta_1 \) is the total axial displacement of bearing 1, \( \delta_p \) is the axial displacement of bearing under preload.

At the same time, the total displacement on bearing 2 is

\[ \delta_2 = \begin{cases} \delta_p - \delta_a & (\delta > \delta_a) \\ 0 & (\delta \leq \delta_a) \end{cases} \]  

where \( \delta_2 \) is the total axial displacement of bearing 2, \( \delta \) is the relative displacement of the inner and outer rings of the bearing under axial load.

For the convenience of analysis, the axial load is acted on the shafting at the center of the bearing, and the relationship is as follows

\[ \frac{F_a}{ZK_0(BD)^{1.5}} = \sin \alpha_1 \left( \frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{1.5} - \sin \alpha_2 \left( \frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{1.5} \]  

where \( \alpha_1 \) is the contact angle of bearing 1 under axial load. \( \alpha_2 \) is the contact angle of bearing 1 under axial load. \( K \) is the axial displacement constant related to the total curvature of ball bearings.

By combining Equation (8), (9) and (10), we can get

\[ \frac{\sin(\alpha_1 - \alpha_0)}{\cos \alpha_1} + \frac{\sin(\alpha_2 - \alpha_0)}{\cos \alpha_2} = \frac{2\delta_p}{BD} \]  

According to Equation (10) and Equation (11), the contact angles \( \alpha_1, \alpha_2 \) of two bearings under axial load can be calculated by Newton-Raphson method, and the axial displacement \( \delta_1 \) of shafting under axial load can be determined by Equation (3).

The load-displacement relationship of angular contact ball bearings under axial load is determined by the above calculation method, so that the relationship between the axial stiffness of the shafting and the load on the bearing can be further obtained.

4. Simulation analysis of shafting stiffness

By analyzing the structure of the shafting, it can be seen that the axial stiffness of the shafting is related to the motor, bearing, inner and outer sleeves, frame shell, etc. The spring element is used to simulate bearing stiffness and preload in ANSYS. The mesh size is 2 mm. The number of meshes is 200730, and the number of nodes is 541799. The load constraints are shown in Figure 6.

Fixed constraints are imposed on the mounting base of the unit: 50N load is applied to the shafting in the radial direction by the spring element, 100N load is applied to the end cover of the unit. The radial and axial deformations are shown in Figure 7 and 8, respectively.

The theoretical calculation and simulation analysis of the radial and axial stiffness of the shafting are shown in Table 1.
Figure 6. Load constraints.

Figure 7. Radial deformation nephogram.

Figure 8. Axial deformation nephogram.

Table 1. Stiffness of shafting.

| Stiffness         | Theoretical value (N/mm) | Simulation value (N/mm) | Relative error |
|-------------------|--------------------------|-------------------------|----------------|
| Radial stiffness  | $2.99 \times 10^5$       | $2.73 \times 10^5$      | 8.70%          |
| Axial stiffness   | $2.21 \times 10^5$       | $2.39 \times 10^5$      | 8.14%          |

The results show that the relative errors of radial stiffness and axial stiffness are 8.70% and 8.14% respectively, which verifies the validity of theoretical analysis of shafting.

5. Conclusions
In this paper, the working principle of the satellite antenna scanning mechanism and the structure of the precision shafting assembly are studied. The radial and axial stiffness of the shafting are calculated by using the theory of load-displacement relationship. The simulation results show that the relative errors between the two systems are 8.70% and 8.14%, respectively. The positioning preloading shafting shown in the example has higher stiffness and can meet the application requirements. The calculation method adopted in this paper has specific guiding significance for evaluating the stiffness performance of precision shafting and optimum design of shafting structure.

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