Theoretical research of impact of the changed elastic and damping parameters of vehicle tyres and loading on the wheels breakaway time from the cobblestone road

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Abstract. The article is devoted to the theoretical research of impact of the changed tyre stiffness and damping on the wheels breakaway time from the cobblestone road when driving at different speeds. The spacial four-support five-mass mathematical model of GAZ 322132 two-axle vehicle dynamics has been developed considering the breakaway. There are the results of the mathematical model theoretical research in the form of dependences of the front left wheel breakaway time from different combinations of tyre stiffness and damping at different vehicle speeds.

1. Introduction
The issue of the increased safety of the vehicle movement based on reasonable selection of some tyre and spring parameters was considered by the authors of this article in the works [1-5] where the conditions for tyre breakaway from the bearing surface have been determined.

The authors of the article have developed the vehicle tyres with reduced stiffness and increasing damping properties [6-8]. However, the impact of the changed elastic and damping parameters of vehicle tyres and loading on the wheels breakaway time from the road has not been researched and is of interest.

2. Mathematical model of the vehicle
The GAZ 322132 two-axle vehicle that is very popular in the roads of the Russian Federation is used as an item of research. The vehicle general view and basic technical characteristics used as initial data for calculations in the mathematical model are given in Figure 1 and Table 1, respectively.

Figure 1. General view (a) and overall dimensions of the vehicle GAZ 322132 (b).
| Parameter                                      | Designation | Value | Dimension |
|-----------------------------------------------|-------------|-------|-----------|
| Sprung mass:                                   |             |       |           |
| full weight                                   |             | 3350  | kg        |
| curb weight                                   |             | 2380  | kg        |
| Wheelbase                                     |             | 2900  | kg        |
| Moment of inertia of the sprung mass relative to the \(x\) axis | \(J_x\)    | 6250  | kg·m\(^2\) |
| Moment of inertia of the sprung mass relative to the \(y\) axis | \(J_y\)    | 2000  | kg·m\(^2\) |
| Front unsprung mass                           | \(m_1\)    | 70    | kg        |
| Rear unsprung mass                            | \(m_2\)    | 150   | kg        |
| Front suspension damping ratio                 | \(k_1\)    | 4206  | N·s/m     |
| Rear suspension damping ratio                  | \(k_2\)    | 6845  | N·s/m     |
| Stiffness of front suspension springs          | \(c_1\)    | 56627 | N/m       |
| Stiffness of rear suspension springs           | \(c_2\)    | 92157 | N/m       |

The calculation scheme of the spacial 4-support 5-mass mathematical model that is equivalent to the GAZ 322132 oscillatory system is given in Figure 2.

**Figure 2.** The calculation scheme of the spacial 4-support 5-mass mathematical model of the two-axle vehicle: 1 – sprung mass; 2 – front elastic element (spring); 3 – rear elastic element (spring); 4 – front damper (shock absorber); 5 – rear damper (shock absorber); 6 – front unsprung mass; 7 – rear unsprung mass; 8 and 10 – front tyre with elastic and damping properties; 9 and 11 – rear tyre with elastic and damping properties.
The calculation scheme has the following symbols: \( M \) – sprung mass; \( J_1 \) and \( J_2 \) – moments of inertia of the sprung mass about the relevant axes; \( c_1 \) and \( c_2 \) – spring stiffness of the front and rear suspensions; \( k_1 \) and \( k_2 \) – damping rates of shock-absorbers of the front and rear suspensions; \( m_1 \) and \( m_2 \) – front and rear unsprung masses; \( c_1t \) and \( c_2t \) – tyre stiffness of the front and rear wheels; \( k_1t \) and \( k_2t \) – damping rates of the front and rear tyres; \( l_1 \) and \( l_2 \) – distances from the centre of weights to vertical axes of the front and rear suspensions; \( B \) – distance between the wheels of the same axle (track); \( x, y \) and \( z \) – longitudinal, transverse and vertical axes of sprung mass in static position; \( q_1t \) and \( q_2t \) – kinematic perturbance under the front left and rear left wheels; \( q_1r \) and \( q_2r \) – kinematic perturbance under the front right and rear right wheels; \( z \) – vertical shift of the centre of sprung mass; \( \psi, \varphi \) and \( \chi \) – angular shift of sprung mass, respectively, relative to the axes \( x, y \) and \( z \); \( \zeta_1t \) and \( \zeta_2t \) – vertical shift of the front left and rear left unsprung masses; \( \zeta_1r \) and \( \zeta_2r \) – vertical shift of the front right and rear right unsprung masses.

5-mass vehicle model dynamics illustrated in Figure 2 considering the tyre breakaway is described by the following differential equation system:

\[
M\ddot{z} + k_1(2\dot{z} + 2\dot{\phi}_1 - \dot{\zeta}_1t) + k_2(2\dot{z} + 2\dot{\phi}_2 - \dot{\zeta}_2t) + c_1(2\dot{z} + 2\dot{\phi}_1 - \dot{\zeta}_1t) + c_2(2\dot{z} + 2\dot{\phi}_2 - \dot{\zeta}_2t) = 0; \\
J_1\ddot{\psi} + k_1(2\dot{\psi} + 2\dot{\psi}_1 - \dot{\zeta}_1t) \phi_1 - k_2(2\dot{\psi} + 2\dot{\psi}_2 - \dot{\zeta}_2t) \phi_2 + c_1(2\dot{\psi} + 2\dot{\psi}_1 - \dot{\zeta}_1t) \phi_1 - c_2(2\dot{\psi} + 2\dot{\psi}_2 - \dot{\zeta}_2t) \phi_2 = 0; \\
J_2\ddot{\phi}_2 + k_1(2\dot{\phi}_1 - \dot{\zeta}_1t) + k_2(2\dot{\phi}_2 - \dot{\zeta}_2t) = 0;
\]

\[m_1\ddot{\zeta}_1t / 2 + k_1(\dot{\zeta}_1t - \dot{q}_1t) + F_{1u} - k_1(\dot{\phi}_1 + \psi B / 2 - \dot{\zeta}_1t) - c_1(z + \phi l_1 + \psi B / 2 - \dot{\zeta}_1t) = 0; \\
m_1\ddot{\zeta}_1r / 2 + k_1(\dot{\zeta}_1r - \dot{q}_1r) + F_{1u} - k_1(\dot{\phi}_1 + \psi B / 2 - \dot{\zeta}_1r) - c_1(z + \phi l_1 + \psi B / 2 - \dot{\zeta}_1r) = 0; \\
m_2\ddot{\zeta}_2t / 2 + k_2(\dot{\zeta}_2t - \dot{q}_2t) + F_{2u} - k_2(\dot{\phi}_2 + \psi B / 2 - \dot{\zeta}_2t) - c_2(z + \phi l_2 + \psi B / 2 - \dot{\zeta}_2t) = 0; \\
m_2\ddot{\zeta}_2r / 2 + k_2(\dot{\zeta}_2r - \dot{q}_2r) + F_{2u} - k_2(\dot{\phi}_2 + \psi B / 2 - \dot{\zeta}_2r) - c_2(z + \phi l_2 + \psi B / 2 - \dot{\zeta}_2r) = 0.
\]

Here, the vehicle tyre elastic forces considering the wheel breakaway are determined on the basis of the following:

\[
F_{1u} = \begin{cases} 
   c_1(\zeta_{1u} - q_{1u}) & \text{if } c_1(\zeta_{1u} - q_{1u}) \leq \frac{M_1 + m_1}{2} \cdot g; \\
   \frac{M_1 + m_1}{2} \cdot g & \text{if } c_1(\zeta_{1u} - q_{1u}) > \frac{M_1 + m_1}{2} \cdot g,
\end{cases}
\]

\[
F_{1r} = \begin{cases} 
   c_1(\zeta_{1r} - q_{1r}) & \text{if } c_1(\zeta_{1r} - q_{1r}) \leq \frac{M_1 + m_1}{2} \cdot g; \\
   \frac{M_1 + m_1}{2} \cdot g & \text{if } c_1(\zeta_{1r} - q_{1r}) > \frac{M_1 + m_1}{2} \cdot g,
\end{cases}
\]

\[
F_{2u} = \begin{cases} 
   c_1(\zeta_{2u} - q_{2u}) & \text{if } c_1(\zeta_{2u} - q_{2u}) \leq \frac{M_2 + m_2}{2} \cdot g; \\
   \frac{M_2 + m_2}{2} \cdot g & \text{if } c_1(\zeta_{2u} - q_{2u}) > \frac{M_2 + m_2}{2} \cdot g,
\end{cases}
\]

\[
F_{2r} = \begin{cases} 
   c_1(\zeta_{2r} - q_{2r}) & \text{if } c_1(\zeta_{2r} - q_{2r}) \leq \frac{M_2 + m_2}{2} \cdot g; \\
   \frac{M_2 + m_2}{2} \cdot g & \text{if } c_1(\zeta_{2r} - q_{2r}) > \frac{M_2 + m_2}{2} \cdot g,
\end{cases}
\]
\[ F_{2z} = \begin{cases} c_1(\zeta_{2z} - q_{2z}) & \text{if } c_1(\zeta_{2z} - q_{2z}) \leq \frac{M_z + m_z}{2} \cdot g, \\ \frac{M_z + m_z}{2} \cdot g & \text{if } c_1(\zeta_{2z} - q_{2z}) > \frac{M_z + m_z}{2} \cdot g, \end{cases} \]  

(5)

\[ M_1 = M \cdot \frac{l_1}{l_1 + l_2} ; \; M_2 = M \cdot \frac{l_2}{l_1 + l_2}, \]  

(6)

where \( M_1 \) and \( M_2 \) – parts of the sprung mass \( M \) on the front and rear vehicle axles, respectively.

3. Theoretical research results

The broken cobblestone road was used as a microprofile in the calculations. The vehicle movement interval during which the breakaway time was determined was selected as 2.5 s as such interval is sufficient for loss of vehicle control due to breakaway and reduces the calculation time. The mathematical model of the vehicle was solved by simulation modelling in the SIMULINK application of the MATLAB software suite.

As a result of the conducted theoretical research, the dependences of the front left wheel breakaway duration from different tyre stiffness values has been found within 100...800 kN/m with a pitch of 100 kN/m and damping rate in the tyre within 0...10 kNs/m with a pitch of 1 kNs/m at the vehicle speeds of 40, 60, 90 and 110 km/h, respectively. These dependences are given in Figure 3 in the form of surfaces.

As can be seen from Figure 3, with the tyre serial parameters \( c_{t1} = 400 \text{ kN/m} \) and \( k_{t1} = 0 \text{ kNs/m} \), the total duration of the front left wheel breakaway \( \Delta T \) for full and curb weights of the vehicle is 6.6% and 17.9% at 40 km/h; 13.9% and 29.7% at 60 km/h; 19.6% and 32.9% at 90 km/h; 27.0% and 37.0% at 110 km/h, respectively. Thus, with the vehicle speed increase from 40 to 110 km/h, the total duration of the front left wheel breakaway increases by 4 and 2 times, respectively.

Based on the surfaces shown in Figure 3, the increased tyre stiffness without damping leads to significant increase in total duration of breakaway. Insertion of a damping unit into the vehicle tyre and subsequent increase of the inelastic resistance rate in the tyre provides for sharp decrease in the total duration of breakaway.

Figure 4 illustrates the comparative dependence graphs of the total duration of the front left wheel breakaway and vehicle speed with full and curb weights.

When the tyre stiffness doubles (curves 2 and 6) that corresponds to the use of low-profile tyres, \( \Delta T \) increases by 1.5-3 times. The total duration of the front left wheel breakaway \( \Delta T \) for full and curb weights of the vehicle is 19.1% and 34% at 40 km/h; 33.3% and 47.7% at 60 km/h; 39.2% and 51.0% at 90 km/h; 44.2% and 55.6% at 110 km/h, respectively.

When the tyre stiffness decreases two times (curves 3 and 7), \( \Delta T \) decreases by 1.7-4 times. The total duration of the front left wheel breakaway \( \Delta T \) for full and curb weights of the vehicle is 2.8% and 7.2% at 40 km/h; 3.9% and 14.2% at 60 km/h; 4.8% and 16.1% at 90 km/h; 9.4% and 21% at 110 km/h, respectively.

When the tyre stiffness decreases two times and the damping rate increases to 5 kNs/m (curves 4 and 8), \( \Delta T \) decreases almost to zero. The total duration of the front left breakaway \( \Delta T \) for full and curb weights of the vehicle is 0% and 1.6% at 40 km/h; 0.5% and 3.4% at 60 km/h; 0.7% and 2.5% at 90 km/h; 0.6% and 3.4% at 110 km/h, respectively.

4. Conclusions

Therefore, the conducted theoretical research has established that when driving on the broken cobblestone road the decrease of the tyre stiffness leads to significant decrease in total duration of wheel breakaway, and insignificant tyre damping reduces the wheel breakaway from the bearing surface almost to zero.
Figure 3. The dependences of the total duration $\Delta T$ of the front left wheel breakaway from different tyre stiffness and damping for full and curb weights and different vehicle speed: a, c, e, g – $M = 3350$ kg; b, d, f, h – $M = 2380$ kg; a, b – $v_v = 40$ km/h; c, d – $v_v = 60$ km/h; e, f – $v_v = 90$ km/h; g, h – $v_v = 110$ km/h.
Figure 4. The dependences of the total duration $\Delta T$ of the front left wheel breakaway from vehicle speed for full and curb weights and different combinations of tyre stiffness and damping: 1, 2, 3, 4 – $M = 3350$ kg; 5, 6, 7, 8 – $M = 2380$ kg and $k_{c1} = 0$ kNs/m; 2, 6 – $c_{t1} = 800$ kN/m and $k_{t1} = 0$ kNs/m; 3, 7 – $c_{t1} = 200$ kN/m and $k_{t1} = 0$ kNs/m; 4, 8 – $c_{t1} = 200$ kN/m and $k_{t1} = 5$ kNs/m.

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