Stable bimaximal neutrino mixing pattern

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Abstract

The neutrino oscillation experiments increasingly point towards a mixing pattern that can be parametrised with two near maximal and one small mixing angle. We investigate whether such a mixing pattern can be generated as a fixed point of renormalisation group evolution.

I. INTRODUCTION

The data from the various neutrino experiments indicate a bimaximal mixing among three neutrino generations. The atmospheric neutrino puzzle requires a near maximal mixing between a pair of neutrinos [1]. The solar neutrino puzzle has various possible solutions but the large angle MSW solution is favoured [2]. As the mass squared difference required for these two solutions differ by about few orders of magnitude, we must have two different pairs with large mixing. The CHOOZ experiment constrain one of the mixing angles to be small [3]. So out of three mixing angles two are large and one is small indicating a bimaximal structure. The large mixing in neutrino is in sharp contrast to the very small mixing in the quark sector. Flavor symmetries may generate bimaximal mixing structure in neutrino masses [4]. But such structure generated at the high scale may not be stable under renormalisation group evolution to low scale if neutrino masses are quasi degenerate [5,6]. Alternatively renormalisation Group evolution from high scale to the present scale have been proposed as a mechanism to generate large mixing from a small mixing [7]. But in these mechanism enhancement of the mixing angle is scale dependent and hence may be unstable and fine tuned. [8].

We do a three generation analysis and find that large mixing can be a stable fixed point of renormalisation group evolution in certain region of experimentally allowed parameter space. We consider a quasi degenerate mass structure with one of the state in an opposite CP phase to the other two.

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Existence of large mixing angle as a stable fixed point of renormalisation group evolution has the advantage that once the mixing attains the stable large value, it doesn’t change much under renormalisation group evolution and hence there is no fine tuning. So one doesn’t have to explicitly evolve the mixing matrix under renormalisation group. The fixed point value of the mixing can be obtained after a small perturbation on the neutrino mass matrix at the high scale due to radiative correction.

One may take the view that unlike the quark and charged lepton mixing, all the three neutrino mixing angle can be large at the high scale. This is because quark and charged lepton masses show a hierarchical structure while neutrinos may not. In this case we investigate whether one of the mixing angle attain small value as fixed point due to radiative correction while the other two remain unchanged. If so then this mechanism can also produce the required bimaximal mixing pattern. In the next section we present the analysis to generate large stable mixing due to perturbation from radiative correction. In section 3 we investigate whether a stable small mixing can be obtained from all large mixing at high scale. In section 4 we present the conclusion.

II. RADIATIVE ENHANCEMENT OF MIXING

Consider a Majorana mass structure with three almost degenerate masses. The CP phase of one of the species is opposite to that of the other two.

\[ M_D = (m, m, -m) \] (1)

Such a degenerate mass pattern can be obtained from a seesaw mechanism \cite{9} with non-diagonal structure in right handed Majorana masses \cite{10}. In these models a pair of opposite CP states are maximally mixed and they form the pseudo Dirac neutrinos \cite{11}. The other two mixing angles are small. Let us parametrise the mixing matrix \( U \) as follows

\[ U(\psi, \phi, \omega) = R_{23}(\psi)R_{13}(\phi)R_{12}(\omega) \] (2)

The mixing matrix can contain two physical Majorana phases. We ignore these phases in our calculations. A detailed analysis including these phases and different types of hierarchy amongst the masses have been done by Haba et. al. \cite{12} in MSSM. The interesting cases of the stability of maximal and zero mixing occurs when the difference of these physical phases is 0 or \( \pi/2 \) and all the masses are degenerate. We do a model independent analysis through a perturbation of the mass matrix at the high scale and investigate whether a bimaximal mixing can be a fixed point of the evolution of the mass matrix under radiative correction.

Let the (2,3) states be the pseudo Dirac pair in \( M_D \). So \( \psi \) is maximal and \( \phi \) and \( \omega \) are small. In this parametrisation and the mass spectrum eq.(1), the mass matrix in the flavor basis will be
\[ M = U M_D U^\dagger \]
\[ = m \begin{pmatrix} C_2\phi & -S_2\phi S_\psi & -S_2\phi C_\psi \\ -S_2\phi S_\psi & (1 - 2C_2^2S_\psi^2) & -S_2\psi C_\phi^2 \\ -S_2\phi C_\psi & -S_2\psi C_\phi^2 & 1 - 2C_2^2C_\psi^2 \end{pmatrix} \]

Due to radiative correction the effective mass matrix at the low scale \( M_Z \) is
\[ M(M_Z) = \begin{pmatrix} 1 + \delta_e & 0 & 0 \\ 0 & 1 + \delta_\mu & 0 \\ 0 & 0 & 1 + \delta_\tau \end{pmatrix} M \begin{pmatrix} 1 + \delta_e & 0 & 0 \\ 0 & 1 + \delta_\mu & 0 \\ 0 & 0 & 1 + \delta_\tau \end{pmatrix} \]

Let
\[ \delta_\mu = \delta_e + \delta_{e\mu} \text{ and } \delta_\tau = \delta_e + \delta_{e\tau} \]

To the first order in \( \delta_e, \delta_{e\mu} \) and \( \delta_{e\tau} \), \( M(M_Z) \) can be written as
\[ M(M_Z) = (1 + 2\delta_e)M + \delta M \]

where
\[ \delta M = m \begin{pmatrix} 0 & -S_2\phi S_\psi \delta_{e\mu} & -S_2\phi C_\psi \delta_{e\tau} \\ -S_2\phi S_\psi \delta_{e\mu} & (1 - 2C_2^2S_\psi^2)2\delta_{e\mu} & -S_2\psi C_\phi^2(\delta_{e\mu} + \delta_{e\tau}) \\ -S_2\phi C_\psi \delta_{e\tau} & -S_2\psi C_\phi^2(\delta_{e\mu} + \delta_{e\tau}) & 1 - 2C_2^2C_\psi^22\delta_{e\tau} \end{pmatrix} \]

Here we see that due to degeneracy in the (1,2) sector, and the suitable parametrisation the angle \( \omega \) doesn’t enter the mass matrix \( M \). So \( \omega \) can be arbitrary at the high scale.

The first part of \( M(M_Z) \) in (6) \( (1 + 2\delta_e)M \) is diagonalised by the same mixing matrix \( U(\psi, \phi, \omega) \). The three eigenvalues are just \( 1 + 2\delta_e \) times the eigenvalues of \( M \). So the degeneracy is not lifted in this part. Treating the second term \( \delta M \) as a perturbation over \( (1 + \delta_e)M \) we get the following split in the degenerate mass eigenvalues
\[ \delta m_{1,2} = (M_{22} \pm \sqrt{M_{22}^2 + M_{12}^2})\delta_{e\mu} \]

The corresponding mass eigenvectors in this sector are given by a rotation in the (1,2) sector \( R_{12}(\omega') \) where \( \omega' \) is given by
\[ \tan \omega' = \frac{M_{12}}{M_{22} + \sqrt{M_{22}^2 + M_{12}^2}} \]

The changes in the state 3 is small and from the parametrisation of the mixing matrix in eq.(2) we see that the angles \( \psi \) and \( \phi \) do not change much. So the mixing after the perturbation is given by
\[ U(\omega', \psi', \phi') = R_{23}(\psi')R_{13}(\phi')R_{12}(\omega') \] (10)

where \( \psi' \approx \psi \) and \( \phi' \approx \phi \).

Eq. (9) says that \( \omega' \) is independent of radiative corrections (\( \delta_e, \delta_\mu \) and \( \delta_\tau \)) at a given scale. So \( \omega' \) achieves the stable value independent of renormalisation Group Evolution.

The stable value of \( \omega' \) depends on the high scale values of \( \phi \) and \( \psi \). From eq. (9) we see that when \( M_{22} \ll M_{12} \) \( \omega' \) is near maximal. This happens for small \( \phi \) and near maximal \( \psi \). In figure 1 we plot \( \omega' \) verses \( \psi \) for various values of \( \phi \).

The interesting point is that large \( \omega' \) is obtained for small values of \( \phi \) and very near maximal value of \( \psi \). This is the region of interest suggested by the CHOOZ constraint and the atmospheric neutrino data. If the high scale mass matrix is generated by a seesaw mechanism that produces degenerate neutrino masses with a pseudo Dirac pair, it is natural to have \( \phi \) and \( \omega \) small at the high scale while \( \psi \) is maximal [10]. Then due to quantum correction \( \omega \) stabilises to a large value by radiative correction. This mechanism is model independent. It will work in any theory where the radiative corrections can be expressed as in eq. (9). The drawback of this mechanism is that the angle \( \psi \) is constrained to be in a very small window around 45° for small values of \( \phi \) if we want large \( \omega' \). For example if we demand \( \omega \) to be above 30° then \( \psi \) has to be between 44° and 45° for \( \phi = 1° \). For \( \phi = 10° \) the region of \( \psi \) is constrained to be between 42° - 45°. If \( \psi \) lies below 40° this mechanism cannot explain the large angle solar neutrino solution. However it may be possible that \( \psi \) may be almost exactly maximal as suggested by the current atmospheric neutrino data. To make a simple estimate about how much \( \psi \) can deviate from maximal mixing due to radiative correction, we evaluate the small perturbative change in the third eigenstate with \( CP = -1 \). As we are interested in small \( \phi \) region we put \( \phi = 0 \). So after radiative correction the third mass state with eigenvalue \( -m \) is

\[
|3'\rangle = \begin{pmatrix}
0 \\
S_\psi \\
C_\psi
\end{pmatrix} + \frac{S_2 \psi C_\phi^2 S_\phi' \delta_{\epsilon \mu} + \delta_{\epsilon \tau}}{2} \begin{pmatrix}
0 \\
C_\psi \\
- S_\psi
\end{pmatrix}
\] (11)

This shows that the change in the state \( |3\rangle \) indicates a change in the angle \( \psi \) by \( \frac{S_2 \psi S_\phi \delta_\phi}{2} \) in radians, i.e. \( \left( \frac{S_2 \psi S_\phi \delta_\phi}{2} \frac{180}{\pi} \right) \)°. If \( \delta_\tau \approx 10^{-2} \) then the change in the angle \( \psi \) is less than 0.3° around maximal \( \psi \). So the angle \( \psi \) seems to be stable enough to stay near maximal within few degrees.

### III. A VARIANT OF THE MECHANISM

In the above section we considered generation of a large mixing from a small mixing at the high scale. However we may have the view that having large mixing between the mass
states can be as natural as having small mixing. Then radiative correction may produce a small mixing between a pair with same CP parity while not affecting the other two large mixing. Then this can be an alternative mechanism to generate a stable bimaximal mixing pattern. Here we see if this can be done.

We consider the following Majorana mass spectrum at the high scale which is different from eq.(11):

\[ M_D = (m, -m, m) \] (12)

Let us parametrise the mixing matrix \( U \) as follows

\[ U(\omega, \psi, \phi) = R_{23}(\psi)R_{12}(\omega)R_{13}(\phi) \] (13)

Note that this parametrisation of mixing matrix is different from that in the standard parametrisation given in eq.(2). This parametrisation is useful here as we wish to find the stable value of the (1,3) mixing angle \( \phi \) after radiative correction as these two states have the same CP parity. As long as \( \phi \) is small this parametrisation will have nearly the same prediction for the mixing angles \( \omega \) and \( \psi \) as with the standard parametrisation in eq.(2). This is because for small values of \( \phi \), \( R_{13}(\phi) \) is very nearly identity and hence it commutes with all other matrices. Hence its position doesn’t change the predictions as long as the relative order of the large mixing matrices \( R_{12}(\omega) \) and \( R_{23}(\psi) \) are not changed.

The mass matrix in the flavor basis is

\[ M = U M_D U^\dagger \] (14)

\[ = m \begin{pmatrix}
C_{2\omega} & -S_{2\omega}C_\psi & S_{2\omega}S_\psi \\
S_{2\omega}C_\psi & (1 - 2C^2_\omega C^2_\psi) & S_{2\psi}C^2_\omega \\
S_{2\omega}S_\psi & S_{2\psi}C^2_\omega & 1 - 2C^2_\omega S^2_\psi
\end{pmatrix} \] (15)

Note that the angle \( \phi \) doesn’t appear in \( M \) due to the degeneracy in the (1,3) states.

With this parametrisation and the radiative correction given by eq.(5), the stable value of the mixing angle \( \phi \) is

\[ \tan \phi' = \frac{M_{13}}{M_{33} + \sqrt{M_{33}^2 + M_{13}^2}} \] (16)

We see from eq.(15) that \( \phi' \) is not small unless \( \omega \) and \( \psi \) both are small. In fig.2 we show a plot of the fixed point value of \( \phi \) as a function of \( \psi \) for various values of \( \omega \). For large values of \( \omega (> 30^o) \) we see that \( \phi \) is not small near large or maximal \( \psi \). So we cannot produce a bimaximal mixing in this way.
IV. CONCLUSION

Bimaximal mixing pattern can be obtained from small mixing at high scale. We present an analysis based on finding stable fixed point of mixing angle after small perturbation to the mass matrix due to radiative correction. This has the advantage that the large stable mixing can be evaluated without requiring to evolve the mixing from high scale down to the present scale. This is possible because the mixing doesn’t change with scale further down after attaining the fixed point value. The large mixing being a fixed point this solution is not fine tuned as is the case with some of the earlier analysis. We present a variant of the mechanism to see whether one small mixing can be obtained from all large mixing at high scale by radiative correction. We find that this is not possible in the region of interest indicated by experiments.
FIG. 1. Fixed point values of $\omega'$ as a function of $\psi$ for various values of $\phi = 1^\circ, 5^\circ, 10^\circ$ and $20^\circ$. 
FIG. 2. Fixed point values of $\phi'$ as a function of $\psi$ for various values of $\omega = 30^\circ$, $40^\circ$ and $45^\circ$. 
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