Generalized Study with Isospin-phased Topological Approach
on the $B \to K\pi$ Puzzle

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Abstract

We study the decay processes $B \to K\pi$ by using generalized decay amplitudes with both the final state re-scattering strong phases and the topological quark diagrammatic strong phases included together as part of fitting parameters. Using a generalized approach, so called “isospin phased topological approach”, for all the currently available data of $B \to K\pi$ decays, we determine the allowed values of the relevant theoretical parameters, corresponding to the electroweak penguin, the color-suppressed tree contribution, strong phase differences, etc. In order to find the most likely values of the parameters in a statistically reliable way, we use the $\chi^2$ minimization technique. We find that the long distance final state re-scattering, when taken at proper value, can provide a reasonable fit to the standard model with the perturbative QCD estimated values, and therefore, it is premature to conclude that it requires new physics to explain the CP violating $B \to K\pi$ data.

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I. INTRODUCTION

There are four different decay channels (and their anti-particle decay channels) for $B \rightarrow K \pi$ processes, depending on the electric charge configuration: $B^+ \rightarrow K^0 \pi^+$, $B^+ \rightarrow K^+ \pi^0$, $B^0 \rightarrow K^+ \pi^-$, and $B^0 \rightarrow K^0 \pi^0$. All the $B \rightarrow K \pi$ modes have already been observed in experiments and their CP-averaged branching ratios have been measured within a few percent errors by the BaBar and Belle collaborations \[1, 2, 3, 4, 5, 6\]. The observations of the direct CP asymmetry in $B^0 \rightarrow K^+ \pi^-$ have also been recently achieved at the $5.7\sigma$ level by BaBar and Belle \[7, 8, 9, 10, 11\]. For the other $B \rightarrow K \pi$ modes, the experimental results of the direct CP asymmetries still include large errors. Certain experimental data (e.g., the branching ratios (BRs)) for $B \rightarrow K \pi$ are currently more precise than the theoretical model predictions based on QCD factorization (QCDF), perturbative QCD (pQCD), and so on. Thus, these decay modes can provide very useful information for improving the model calculations, and at the same time, the model-independent study becomes very important.

In the light of those new data, including the direct CP asymmetry in $B^0 \rightarrow K^+ \pi^-$, many works have been done to study the implications of the data \[12, 13, 14, 15, 16, 17, 18, 19, 20, 21\]. The quark level subprocesses for $B \rightarrow K \pi$ decays are $b \rightarrow sq\bar{q}$ ($q = u, d$) penguin processes, which are potentially very sensitive to any new physics effects beyond the standard model (SM). Thus, with the currently available precision data, it is very important to investigate these modes as generally and critically as possible.

At the bottom of this important problem, lies the fact that final state re-scattering phases by strong interaction play a crucial role in generating CP violation. Part of the strong interaction phases (as short-distance strong phases (sdSPs) related to topological quark diagrams) can be investigated by using various QCD models such as QCDF or pQCD, even though the results are quite model dependent. However, there are also long-distance final state strong phases (ldSPs), that are very difficult to calculate due to hadronic interactions at low energy scale (even thought some attempts had been done). Despite facing the fact that we do not understand strong phases very well and that there are already many theoretical claims of possible new physics from the data of $B \rightarrow K \pi$, here we are trying to be as model-independent as possible in fitting the $B \rightarrow K \pi$ data, by choosing both sdSPs and ldSPs as free fitting parameters, to see if new physics is still required by the recent data or not.

In this work, we study the decay processes $B \rightarrow K \pi$ by using generalized decay ampli-
tudes within the so-called “isospin-phased topological approach”, i.e. with both the strong phases of the final state re-scattering (ldSPs) and the strong phases of the topological quark diagrammatic origin (sdSPs). If we ignore topological strong phase differences, this general approach becomes the original isospin approach. Inversely, if we ignore the strong phase differences in isospin amplitudes, it reduces to the normal topological analysis [22], which is based on an exact flavor SU(3) symmetry in B meson decays [23].

We note that the importance of final state interactions (FSI) has recently been recognized again in hadronic B decays [24], in which the importance of the existence of the soft final state re-scattering effects has been pointed out, especially in a model-independent topological quark diagrammatic analysis of $B \to D\pi$ [25] and $B \to K\pi$ decays. It is certainly conceivable that FSI can modify the predictions based on the short distance diagrammatic analysis. Likewise, the branching ratios of certain decay modes $B_{s,d} \to \pi\pi$ are expected to be extremely small if re-scattering effects through FSI do not alter the predictions of the diagrammatic approach. We notice that the soft final state re-scattering effects can modify the exact flavor SU(3) symmetry even though the isospin symmetry will still hold with FSI, since FSI may rather be at low energy scale of light final particles. In other words, at the scale that FSI are activated, the flavor SU(3) is broken but the SU(2) isospin symmetry is still valid. And FSI can be parameterized as the isospin phases in the limit of the elastic re-scattering.

We note that including both sdSPs and ldSPs may amounts to a possible double counting: Since they typically involve physics at different scales with different symmetries like flavor SU(3) symmetry or SU(2) isospin symmetry, the double counting may not be as serious as one thinks. As is well known, the scattering of hadrons exhibits a two-component structure of “soft” and “hard”. We associate the high scale hard scattering components between pointlike constituents with the SU(3) strong phases of quark diagramatic origin, and the low scale soft components of FSI with the SU(2) isospin phases. In addition, since they are just taken as model independent fitting parameters, even if there are some double counting of physical effects contained in our fitting parameters, it is not going to affect our final physical conclusions regarding new physics.

Here we are mainly interested in investigating whether the conventional SM predictions are consistent with the current data even after we include FSI effects. Furthermore, if there are some deviations between the conventional estimates and the experimental results,
we intend to identify carefully the source of the deviations and estimate how large the
correction from the source can be. Then, by comparing our result with the conventional
SM predictions, we shall be able to verify whether the current data indicate any new physics
effects. In order to find the most likely values of the theoretical parameters in a statistically
reliable way, we will adopt the $\chi^2$ analysis.

We organize the paper as follows: In Section II, we introduce all the relevant formulas
for $B \to K\pi$ decays, step-by-step generalizing the decay amplitudes. We also give formulas
for BRs, direct CP violations and mixing-induced direct CP violation. In Section III, we
present the summary of the recent experimental results on $B \to K\pi$ modes. And we do $\chi^2$
analysis for all the experimental observables, and discuss its physical implication within our
generalized approach. In Section IV, we give conclusions.

II. THE DETAILED FORMULAS FOR $B \to K\pi$ DECAY MODES IN THE
ISOSPIN-PHASED TOPOLOGICAL APPROACH

We first introduce the decay amplitudes within the generalized “isospin-phased topologi-
cal approach”, then we summarize the formulas for the relevant decay amplitudes, BRs,
direct and indirect (mixing-induced) CP asymmetries of $B \to K\pi$ processes.

A. Fusion of Isospin and Topological Approaches

We write $B$-meson decay amplitude, e.g. $B^0 \to \pi^- K^+$, by including both the topological
strong phases and the isospin related strong phases, so called “isospin-phased topological
approach”, as:

$$ A = \sum_I A_I \exp(i\Delta_I) $$  \hspace{1cm} (1)

$$ = \sum_I [D_1 \exp(i\delta_1) + D_2 \exp(i\delta_2) + ...]_I \exp(i\Delta_I), $$  \hspace{1cm} (2)

where $A_I$ is an isospin amplitude with the isospin related (ldSPs) strong phase $\Delta_I$, and we
decompose each isospin amplitude into a sum of the topological diagrammatic amplitudes,
$D_i$, with the topological strong phases (sdSPs) $\delta_i$. As can be seen below, for $B^0 \to \pi^- K^+$
decay channel $A_I$’s are expressed with 3 isospin amplitudes $A_{3/2}$, $A_{1/2}$ and $B_{1/2}$, with strong
amplitude with a positive real value $|A_I|$ (except for the relevant weak phases). Inversely, if we ignore phase differences among isospin amplitudes, the framework reduces to the normal topological analysis.

As final states, we take $\pi^+ \equiv u\bar{d}$, $\pi^0 \equiv \frac{d\bar{d} - u\bar{u}}{\sqrt{2}}$, $\pi^- \equiv -d\bar{u}$, for pions and $K^+ \equiv u\bar{s}$, $K^0 \equiv d\bar{s}$, $\bar{K}^0 \equiv s\bar{d}$, $K^- \equiv -s\bar{u}$, for kaons, and for $B$ mesons $B^+ \equiv \bar{b}u$, $B^0 \equiv \bar{b}d$, $\bar{B}^0 \equiv b\bar{d}$, $B^- \equiv -b\bar{u}$. Isospin relations are

\[
A(B^0 \rightarrow \pi^- K^+) \equiv X = A_{3/2} + A_{1/2} - B_{1/2},
\]
\[
\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) \equiv Y = 2A_{3/2} - A_{1/2} + B_{1/2},
\]
\[
A(B^+ \rightarrow \pi^+ K^0) \equiv Z = A_{3/2} + A_{1/2} + B_{1/2},
\]
\[
\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) \equiv W = 2A_{3/2} - A_{1/2} - B_{1/2},
\]

where $A$ and $B$ represent $\Delta I = 1$ and $\Delta I = 0$ components, respectively, and subscripts denote the isospin of final states. These quantities satisfy the isospin relation,

\[
X + Y = Z + W.
\]

Decay amplitudes can be written in terms of topological contributions

\[
A(B^0 \rightarrow \pi^- K^+) \leftrightarrow -P - P_{EW}^C - T,
\]
\[
A(B^0 \rightarrow \pi^0 K^0) \leftrightarrow \frac{1}{\sqrt{2}}P - \frac{1}{\sqrt{2}}P_{EW} - \frac{1}{\sqrt{2}}C,
\]
\[
A(B^+ \rightarrow \pi^+ K^0) \leftrightarrow P + A,
\]
\[
A(B^+ \rightarrow \pi^0 K^+) \leftrightarrow -\frac{1}{\sqrt{2}}P - \frac{1}{\sqrt{2}}P_{EW} - \frac{1}{\sqrt{2}}P_{EW}^C - \frac{1}{\sqrt{2}}T - \frac{1}{\sqrt{2}}C - \frac{1}{\sqrt{2}}A,
\]

where $T, P, P_{EW}, C, P_{EW}^C, P_C, A$ are ($T$) tree, ($P$) penguin, ($P_{EW}$) electroweak (EW) penguin, ($C$) color suppressed tree, ($P_{EW}^C$) color-suppressed EW penguin, and ($A$) annihilation amplitude, with strong phases $\delta_T, \delta_P, \delta_{EW}, \delta_C$, etc., respectively. $P$ represents the combination $P = P' - \frac{1}{3}P_{EW}^C$.

Now we introduce isospin phase $\alpha'_{3/2}, \alpha'_{1/2}$ and $\beta'_{1/2}$ to each isospin component ($A_{3/2}, A_{1/2}, B_{1/2}$), we get:

\[
A_{3/2} = \frac{1}{3}(-T - P_{EW} - C - P_{EW}^C)e^{i\alpha'_{3/2}},
\]
\[ A_{1/2} = \frac{1}{6}(-T + 2P_{EW} + 2C - P_{EW}^C + 3A)e^{i\alpha'_{1/2}}, \quad (13) \]
\[ B_{1/2} = \frac{1}{2}(2P + T + P_{EW}^C + A)e^{i\beta'_{1/2}}, \quad (14) \]

where we ignored \( A \) (weak annihilation contribution) because it is estimated to be much smaller than others within the SM evaluations of pQCD and QCD factorization.

### B. Branching Ratios, Direct and Mixing-Induced CP Asymmetries

In this subsection we express branching ratios, direct and indirect CP asymmetries. We remark that there exists a (naive) conventional hierarchy within the SM among the topological diagrammatic contributions:

\[ 1 > r_T \sim r_{EW} > r_C \sim r_{EW}^C > r_A, \quad (15) \]

where

\[ r_T \equiv |T|/|P|, \quad r_{EW} \equiv |P_{EW}|/|P|, \quad r_C \equiv |C|/|P|, \quad r_{EW}^C \equiv |P_{EW}^C|/|P|, \quad r_A \equiv |A|/|P|. \]

For instance, in the pQCD approach, those ratios are roughly estimated as

\[ r_T \approx 0.21, \quad r_{EW} \approx 0.14, \quad r_C \approx 0.02, \quad r_{EW}^C \approx 0.01, \quad r_A \approx 0.005. \quad (16) \]

It is also known that within the SM under flavor SU(3) symmetry, the relation \( \delta^T \approx \delta^{EW} \) holds to a good approximation \[27\], which can be deduced from the fact that the topology of the color-allowed tree diagram is similar to that of the EW penguin diagram. Here we neglect the tiny quantities \( r_A \) and \( r_{EW}^C \). However, because recent studies on two-body hadronic \( B \) decays show that the color-suppressed tree contribution could be enhanced to a large amount through certain mechanisms \[24, 25, 28\], we keep \( r_C \), in order to take that possibility into account. This treatment differs from that in Refs. \[12, 29\], where all the linear terms for \( r_A \) and \( r_{EW}^C \) as well as \( r_C \) were simply neglected. We note that the physical strong phases, which appear in the branching ratio and CP asymmetries after taking out overall phases, are defined to be

\[ \delta_T \equiv \delta'_T - \delta'_P, \quad \delta_{EW} \equiv \delta'_{EW} - \delta'_P, \quad \delta_C \equiv \delta'_C - \delta'_P, \quad \delta_{EW}^C \equiv \delta_{EW}^C - \delta'_P, \]

and

\[ \alpha_{1/2} \equiv \alpha'_{1/2} - \beta'_{1/2} \quad \text{and} \quad \alpha_{3/2} \equiv \alpha'_{3/2} - \beta'_{1/2}. \]
Here for simplicity we have defined parameters listed below:

\[ C^X_{r_T} \equiv \text{Re} \left[ e^{i\delta_T} \Lambda_X \right], \quad C^X_{r_{EW}} \equiv \text{Re} \left[ e^{i\delta_{EW}} \Sigma_X \right], \]

\[ C^X_{r_C} \equiv \text{Re} \left[ ie^{i\delta_C} \Sigma_X \right], \]

\[ S^X_{r_T} \equiv \text{Re} \left[ ie^{i\delta_T} \Lambda_X \right], \quad S^X_{r_C} \equiv \text{Re} \left[ ie^{i\delta_C} \Sigma_X \right], \]

\[ C^X_{r_T r_{EW}} \equiv \text{Re} \left[ e^{i(\delta_T-\delta_{EW})} \Lambda_X \Sigma_X^* \right], \quad S^X_{r_T r_{EW}} \equiv \text{Re} \left[ ie^{i(\delta_T-\delta_{EW})} \Lambda_X \Sigma_X^* \right], \]

where

\[ \Lambda_X = e^{-i\beta_1'/2} \left( -\frac{1}{2} e^{i\beta_1'/2} - \frac{1}{3} e^{i\alpha_3'/2} - \frac{1}{6} e^{i\alpha_4'/2} \right), \quad \Sigma_X = e^{-i\beta_1'/2} \left( -\frac{1}{3} e^{i\alpha_3'/2} + \frac{1}{3} e^{i\alpha_4'/2} \right), \]

\[ \Lambda_Y = e^{-i\beta_1'/2} \left( \frac{1}{2} e^{i\beta_1'/2} - \frac{2}{3} e^{i\alpha_3'/2} + \frac{1}{6} e^{i\alpha_4'/2} \right), \quad \Sigma_Y = e^{-i\beta_1'/2} \left( -\frac{2}{3} e^{i\alpha_3'/2} - \frac{1}{3} e^{i\alpha_4'/2} \right), \]

\[ \Lambda_Z = e^{-i\beta_1'/2} \left( \frac{1}{2} e^{i\beta_1'/2} - \frac{1}{3} e^{i\alpha_3'/2} - \frac{1}{6} e^{i\alpha_4'/2} \right), \quad \Sigma_Z = \Sigma_X, \]

\[ \Lambda_W = e^{-i\beta_1'/2} \left( -\frac{1}{2} e^{i\beta_1'/2} - \frac{2}{3} e^{i\alpha_3'/2} + \frac{1}{6} e^{i\alpha_4'/2} \right), \quad \Sigma_W = \Sigma_Y. \]

The CKM elements are \( \lambda_t \equiv V^*_{tb} V_{ts}, \lambda_u \equiv V^*_{ub} V_{us} \) and \( \phi_3 \equiv \gamma \) is the angle of the unitarity triangle.

\[ A_{CP}^+ \equiv \frac{B(\bar{B}^0 \to K^+ \pi^-) - B(B^0 \to K^- \pi^+)}{B(\bar{B}^0 \to K^- \pi^+) + B(B^0 \to K^+ \pi^-)} = \frac{|\bar{X}|^2 - |X|^2}{|X|^2 + |X|^2} \]
for $S$ is defined as mass eigenstates. $B$-contributions are neglected for $A$-CP asymmetries, because the dominant contribution to them is identical. However, the current experimental data show that $A$ have mutually opposite signs, as shown in Table I.

Notice that considering the conventional hierarchy given in (15) and (16), the direct $c$. Mixing-induced CP asymmetry

The time-dependent CP asymmetry for $S_\pi^0$ (up to $\pi^0$) is defined as

\[
A_{CP}^{00} \equiv \frac{B(B^0 \to K^0\pi^-) - B(B^0 \to K^0\pi^+)}{B(B^- \to K^-\pi^0) + B(B^+ \to K^+\pi^0)} = \frac{|Z|^2 - |Z'|^2}{|Z|^2 + |Z'|^2}
\]

\[
\simeq 2r_T \sin \phi_3 r_T u_{\pi^0}^* + 2r_T^2 \sin(2\phi_3) r_T u_{\pi^0}^* C_{r_T}^0 (22)
\]

\[
\begin{align*}
A_{CP}^{00} & \equiv \frac{B(B^0 \to K^0\pi^-) - B(B^0 \to K^0\pi^+)}{B(B^- \to K^-\pi^0) + B(B^+ \to K^+\pi^0)} = \frac{|Z|^2 - |Z'|^2}{|Z|^2 + |Z'|^2} \\
& \simeq 2r_T \sin \phi_3 S_{r_T}^Z + 2r_T^2 \sin(2\phi_3) S_{r_T}^Z C_{r_T}^Z (23)
\end{align*}
\]

\[
\begin{align*}
A_{CP}^{00} & \equiv \frac{B(B^0 \to K^0\pi^-) - B(B^0 \to K^0\pi^+)}{B(B^- \to K^-\pi^0) + B(B^+ \to K^+\pi^0)} = \frac{|Z|^2 - |Z'|^2}{|Z|^2 + |Z'|^2} \\
& \simeq 2r_T \sin \phi_3 S_{r_T}^Y + 2r_T^2 \sin(2\phi_3) S_{r_T}^Y C_{r_T}^Y (24)
\end{align*}
\]

$c$. Mixing-induced CP asymmetry

The time-dependent CP asymmetry for $B^0 \to K_s\pi^0$ is defined as

\[
A_{K_s\pi^0}(t) \equiv \frac{\Gamma(B^0(t) \to K_s\pi^0) - \Gamma(B^0(t) \to K_s\pi^0)}{\Gamma(B^0(t) \to K_s\pi^0) + \Gamma(B^0(t) \to K_s\pi^0)} = \frac{S_{K_s\pi^0} \sin(\Delta m_d t) + C_{K_s\pi^0} \cos(\Delta m_d t), (25)}
\]

where $\Gamma$ denotes the relevant decay rate and $\Delta m_d$ is the mass difference between the two $B^0$ mass eigenstates. $S_{K_s\pi^0}$ and $C_{K_s\pi^0}$ are CP violating parameters. In the case that the tree contributions are neglected for $B^0 \to K_s\pi^0$, the mixing-induced CP violating parameter $S_{K_s\pi^0}$ is equal to $\sin(2\phi_1)$ [$\phi_1 \equiv \beta$ is the angle of the unitarity triangle]. The expression for $S_{K_s\pi^0}$ (up to $r$ order) is given by

\[
S_{K_s\pi^0} = \text{Im} \left( -e^{-2i\phi_1} \frac{Y_{e/\pi}^*}{Y_{e/\pi}} \right) \simeq \sin 2\phi_1 - 2r_T \sin \phi_3 \Re \left( e^{-2i\phi_1} e^{i\delta_T} |Y_{e/\pi}| - 2r_T \sin \phi_3 \Re \left(e^{-2i\phi_1} e^{i\delta_S} |Y_{e/\pi}| \right) \right). (26)
\]
The measured value of $S_{K_S \pi^0}$ (Table I) is different from the well-established value of $\sin(2\phi_1) = 0.725 \pm 0.037$ measured through $B \to J/\psi K^{(*)}$ [30]. It may indicate that the subleading terms including $r_C$ and $r_{EW}$ in Eq. (26) play an important role. In Ref. [21], the authors showed that as $S_{K_S \pi^0}$ varies, the allowed region for $r_C$ varies very sensitively, but that for $r_W$ not, as can be seen from Eq. (26).

III. THE $B \to K\pi$ PUZZLE AND ITS PHYSICAL IMPLICATIONS

A. Summary of Present Experimental Results

We first summarize the present status of the experimental results on $B \to K\pi$ modes in Table I, which includes the BRs, the direct CP asymmetries ($A_{CP}$), and the mixing-induced CP asymmetry ($S_{K_S \pi^0}$). We see that the averages of the current experimental values for the BRs include only a few percent errors. Furthermore, the direct CP asymmetry in $B^0 \to K^{\pm} \pi^{\mp}$ has been recently observed by the BaBar and Belle collaborations whose values are in good agreement with each other (Table I): the world average value is

$$A_{CP}^{+} = -0.109 \pm 0.020 \ .$$

The direct CP asymmetry data for the other $B \to K\pi$ modes involve large uncertainties.

We comment on the values of the ratios between the BRs for the $B \to K\pi$ modes, $R_1$, $R_c$, and $R_n$, which are obtained from the experimental results given in Table I:

$$R_1 = \frac{\tau^+ B^{+-}}{\tau^0 B^{0+}} = 0.82 \pm 0.06 \ ,$$

$$R_c = \frac{2 \bar{B}^{+0}}{\bar{B}^{0+}} = 1.00 \pm 0.09 \ ,$$

$$R_n = \frac{B^{+-}}{2 \bar{B}^{00}} = 0.79 \pm 0.08 \ .$$

It has been claimed that within the SM, $R_c - R_n \approx 0$ [14, 31]. From their definitions, it is indeed clear that $R_c \approx R_n$, if the $r^2$-order terms including $r_{EW}$ or $r_C$ are negligible. In other words, any difference between $R_c$ and $R_n$ would arise from the contributions from the subdominant $r^2$-order terms including $r_{EW}$ or $r_C$. The above experimental data show the pattern $R_c > R_n$ [14, 31], which would imply an enhancement of the electroweak penguin and/or the color-suppressed tree contributions.
TABLE I: Experimental data on the CP-averaged branching ratios ($\bar{B}$ in units of $10^{-6}$), the direct CP asymmetries ($A_{CP}$), and the mixing-induced CP asymmetry ($S_{K_s\pi^0}$) for $B \rightarrow K\pi$ modes. The $S_{K_s\pi^0}$ is equal to $\sin(2\phi_1)$ in the case that tree amplitudes are neglected for $B^0 \rightarrow K_s\pi^0$.

|                | CLEO         | Belle        | BaBar        | Average     |
|----------------|--------------|--------------|--------------|-------------|
| $B(B^\pm \rightarrow K^0\pi^\pm)$ | $18.8^{+3.7+2.1}_{-3.3-1.8}$ | $22.0 \pm 1.9 \pm 1.1$ | $26.0 \pm 1.3 \pm 1.0$ | $24.1 \pm 1.3$ |
| $B(B^\pm \rightarrow K^\pm\pi^0)$  | $12.9^{+2.4+1.2}_{-2.2-1.1}$  | $12.0 \pm 1.3^{+1.3}_{-0.9}$  | $12.0 \pm 0.7 \pm 0.6$  | $12.1 \pm 0.8$  |
| $B(B^0 \rightarrow K^\pm\pi^\mp)$  | $18.0^{+2.3+1.2}_{-2.1-0.9}$  | $18.5 \pm 1.0 \pm 0.7$  | $17.9 \pm 0.9 \pm 0.7$  | $18.2 \pm 0.8$  |
| $B(B^0 \rightarrow K^0\pi^0)$       | $12.8^{+4.0+1.7}_{-3.3-1.4}$  | $11.7 \pm 2.3^{+1.2}_{-1.3}$  | $11.4 \pm 0.9 \pm 0.6$  | $11.5 \pm 1.0$  |

We note that in this analysis we do not consider $B \rightarrow \pi\pi$ modes simultaneously with $B \rightarrow K\pi$ modes, though they can be connected to each other by using flavor SU(3) symmetry. The reason is that we do not want that our analysis to be spoiled by the unknown effects of the flavor SU(3) breaking relations after the inclusion of FSI effects.

We remind that assuming the conventional hierarchy as in Eqs. (13) and (16), $A_{CP}^0$ is expected to be almost the same as $A_{CP}^\pm$: in particular, they would have the same sign. However, the data show that $A_{CP}^0$ differs by 3.4σ from $A_{CP}^\pm$. This is a very interesting observation with the new measurements of $A_{CP}^\pm$ by BaBar and Belle, even though the measurements of $A_{CP}^0$ still include sizable errors. One may need to explain on the theoretical basis how this feature can arise.

B. Global $\chi^2$ Analysis and Theoretical Implications

Based on the current experimental data shown in Table I, we critically investigate their implications to the underlying theory on the $B \rightarrow K\pi$ processes. There are nine observables available for the $B \rightarrow K\pi$ modes, as shown in Table II. However, there are ten theoretical
parameters ($|P|$, $r_T$, $r_{EW}$, $r_C$, $\delta^T$, $\delta^{EW}$, $\delta^C$, $\alpha_{1/2}$, $\alpha_{3/2}$ and $\phi_3 (\equiv \gamma$ of the unitary triangle)) relevant to the above nine observables, neglecting the very small terms of the annihilation contribution\(^1\) $r_a$, and the color suppressed electroweak penguin contribution $r_{EW}$. We have fixed $\sin 2\phi_1 (\equiv \beta) = 0.726$ to its central value of the experimental measurements. Therefore, we have to fix at least one of the theoretical input parameters by assuming a model-calculated value or the previously measured central value in order to do $\chi^2$ analysis.

1. **Global $\chi^2$ Analysis within Isospin-phased Topological Approach**

TABLE II: $\chi^2$ fit for cases(a-d) and their combinations, including all sdSPs and ldSPs. Fixed parameters in each column appear in parentheses.

|                | (a)      | (b)      | (a+b)    | (c)      | (d)      | (c+d)    |
|----------------|----------|----------|----------|----------|----------|----------|
| $\chi^2_{\text{min}}/\text{d.o.f}$ | 0.0/0    | 0.0/0    | 0.0/1    | 0.1/0    | 2.02/0   | 2.04/1   |
| $p$            | 22.7±2.3 | 23.56±0.78| 22.9±0.80| 21.9±1.8 | 24.0±1.3 | 23.8±1.3 |
| $\alpha_{1/2}$ | −0.03±0.41| −0.10±0.25| −0.05±0.27| 0.52±0.39| 0.25±0.71| 0.35±0.25|
| $\alpha_{3/2}$ | 0.23±0.71 | 0.0±0.35  | 0.18±0.36 | −1.04±0.68| 3.04±0.095| 3.04±0.087|
| $\delta^T$     | 0.31±0.46 | 0.32±0.13  | 0.28±0.11 | 0.128±0.046| 0.20±0.10 | 0.19±0.11|
| $\delta^{EW}$  | 1.68±0.69 | 2.00±0.34  | 1.73±0.29 | ( = $\delta^T$) | 0.55±2.0 | ( = $\delta^T$) |
| $\delta^C$     | −3.01±0.43| −2.90±0.29 | −3.01±0.35| −3.92±0.65| ( = $\delta_p$) | ( = $\delta_p$) |
| $r_T$          | 0.19±0.24 | (0.21)     | (0.21)    | 0.58±0.18 | 0.33±0.14 | 0.31±0.11|
| $r_{EW}$       | 0.35±0.13 | 0.322±0.080| 0.347±0.087| 0.01±0.33| 0.16±0.20 | 0.142±0.040|
| $r_C$          | 0.28±0.24 | 0.30±0.19  | 0.29±0.21 | 0.54±0.28 | 0.00±0.15 | 0.00±0.15|
| $\phi_3(\gamma)$ | (60.0)   | 50.9±5.5  | (60.0)   | 73.3±8.4 | 64.0±10.0 | 64.8±10.4|

We do the $\chi^2$ analysis to investigate sources of physics beyond the SM step-by-step as follows: for 9 observables (4 branching ratios, 4 direct CP asymmetries and one indirect CP asymmetry) with 9 input parameters by fixing case(a): $\phi_3 = 60^0$ (but $p \propto \kappa |P|^2 |\lambda_1|^2$, $\alpha_{1/2}$, $\alpha_{3/2}$, $\delta_{T,EW,C}$ and $r_{T,EW,C}$ are free parameters.), or case(b): $r_T = 0.21$ (the pQCD central value) or case(c): $\delta_T = \delta_{EW}$ or case(d): $\delta_C = 0$ (i.e. $\delta^C = \delta^P$). We further exercise by fixing

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1 After the annihilation term $r_a$ is neglected, the observable $A_{CP}^{0+}$ still remains non-zero due to non-zero values of the isospin phases. In our approach, all nine observables still remain relevant.
case(e): the values of all sdSPs zero (typical isospin analysis), where we have three fewer parameters, or case(f): the values of all ldSPs zero (usual topological quark diagrammatic approach), in which we have two fewer parameters; and combinations of cases(a-f).

In Table III we show the results of $\chi^2$-fitting of 9 observables by parameters including both ldSPs and sdSPs. For the minimum of $\chi^2$, in cases(a, b, c, a+b) $\chi^2_{\text{min}}$ value are almost zero, whereas in cases(d) $\chi^2_{\text{min}}$ becomes larger than 2, which means that $\delta_C = \delta_P$ for sdSPs is not a good assumption. From now on, we do not consider the case(d) anymore, except the combination with case(c) which assumes $\delta_C = \delta_P$ as well as $\delta_T = \delta_{EW}$.

We searched the whole range of parameter spaces for global $\chi^2_{\text{min}}$, and we obtained the large ldSPs: $\alpha_{1/2}$ in the range $-0.9 < \alpha_{1/2} < 0.5$ (in the unit of radian) and $\alpha_{1/2} = 0$ is allowed within the range of 1$\sigma$ in cases(a, b, d, a+b); and for $\alpha_{3/2}$, in cases(a, b, a+b) $\alpha_{3/2} = 0$ is allowed within the 1$\sigma$ level; whereas in case(c) $\alpha_{3/2} = 0$ is not allowed in 1$\sigma$. Central values of $\delta_T$ are distributed in the range $0.7^\circ < \delta_T < 18^\circ$, and central values of $\delta_{EW}$ are large and about $96^\circ < \delta_{EW} < 115^\circ$ in cases(a, b, a+b); however, the assumption $\delta_T = \delta_{EW}$ is quite valid with larger $r_T$, much larger than typical pQCD estimates. Central values of $\delta_C$ are very large and almost $180^\circ$ in all cases with the exception of cases(d, c+d) in which $\delta_C = \delta_P$ is assumed. For $r_T$, in cases(a, b, a+b) central values are in good agreement with its pQCD value $r_T = 0.21$. As for $r_{EW}$ and $r_C$, in general large values are preferred for better $\chi^2$ fitting; central values of $r_{EW}$ are around 0.35, much larger than its typical pQCD value $r_{EW} = 0.14$, in cases(a, b, a+b). For $r_C$, cases(a, b, c, a+b) exclude the pQCD value $r_C = 0.018$ by more than 1$\sigma$.

| CP asym. (Exp.) | (a)                  | (b)                  | (a+b)                 | (c)                  | (d)                  | (e+d)                 |
|-----------------|----------------------|----------------------|-----------------------|----------------------|----------------------|-----------------------|
| $A_{CP}^0$      | $-0.109\pm0.28$      | $-0.109\pm0.067$    | $-0.109\pm0.080$      | $-0.107\pm0.33$      | $-0.106\pm0.100$      | $-0.106\pm0.038$      |
|                 | $0.04\pm0.04$        | $0.04\pm0.17$       | $0.04\pm0.21$         | $0.04\pm0.48$        | $0.026\pm0.085$       | $0.025\pm0.043$       |
| $A_{CP}^0$      | $-0.020\pm0.11$      | $-0.020\pm0.055$    | $-0.020\pm0.070$      | $-0.021\pm0.30$      | $-0.014\pm0.122$      | $-0.013\pm0.053$      |
| $A_{CP}^0$      | $-0.09\pm0.28$       | $-0.09\pm0.16$      | $-0.08\pm0.20$        | $-0.09\pm0.57$       | $-0.18\pm0.21$        | $-0.19\pm0.11$        |
| $S_{K^+\pi^0}$  | $0.34\pm0.38$        | $0.34\pm0.26$       | $0.33\pm0.32$         | $0.43\pm0.30$        | $0.09\pm0.28$         | $0.12\pm0.05$         |

In Table III we show the estimated CP asymmetries from the best fit parameters for each case in Table II to be compared with the experimental average values of Table I. We do
not show the estimated branching ratios because there are almost no differences between the experimental values and the theoretically estimated values from the best fit parameters.

A few comments are in order here: (i) Contrary to the previous finding within the topological approach, case(c) confirms $\delta_T = \delta_{EW}$ of Ref. 27. It can be understood by the fact that the topology of the tree diagram is quite similar to that of the EW penguin diagram. (ii) Case(c+d) is very interesting since it gives the pQCD estimated values on the parameters, $r_T$, $r_{EW}$, $r_C$ and $\phi_3$ (see Eq. (10)). That is, if we include FSI, we can fit the parameters within the SM without invoking any unknown new physics effects at all. (iii) The ldSPs from FSI can be quite large, and therefore, we cannot ignore the final state re-scattering effects even in $B$ meson decays. (iv) Indeed, it has been long advocated that charming-penguin contributions can increase significantly the $B \to K\pi$ rates and yield better agreement with experiment 32. Strong phase $\delta_c$ from charming-penguin has been also found substantially large, $\delta_c \sim 20^\circ$ 33. (v) Long distance strong phases has been modelled as re-scattering of some intermediate two-body states with one particle exchange in the $t$ channel and the absorptive part of the re-scattering amplitude via optical theorem. Large long distance contributions are found as $\delta_T - \delta_C \sim 90^\circ$ with FSI, too. (vi) It has been found that by using general features of soft strong interactions soft scattering does not decrease for large $m_B$, and inelastic processes are expected to be leading sources of strong phases. Please note that in this paper we only include the elastic FSI contributions through isospin phases.

2. Null Hypothesis Test by Statistical $p$-value

Here we make a hypothetical test by using a statistical $p$-value, which is defined by the integral $\int_{\chi^2_{\text{min}}}^{\infty} g(t; d) dt$, where $\chi^2_{\text{min}}$ is the minimum $\chi^2$ value obtained and $g(t; d)$ the probability density function for $\chi^2$ distribution with given $d$ degree of freedom. One hypothesis

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2 From the viewpoint of Regge theory, it is pointed out that even in the heavy quark limit there are non-vanishing inelastic non-perturbative FSI effect remaining on each individual channels. However, in authors showed the inelastic FSI effects will be cancelled by each other when all individual channels are summed up. Because in $B$ hadronic decays the $b$-quark mass is finite, therefore, such a cancellation might be incomplete and the non-perturbative inelastic effects could be sizable.

3 In the sense of statistics, the $p$-value is the probability that the observed value happens to be observed under the “null hypothesis”. When the $p$-value is smaller than a threshold (usually 5% is used), it can be said that the null hypothesis is disfavored and the “alternative hypothesis” is favored.
TABLE IV: The $\chi^2$-result with fixed $r$’s (i.e. $r_T = 0.21$, $r_{EW} = 0.14$ and $r_C = 0.018$). In “free” case, $p$, $\alpha_{1/2}$, $\alpha_{3/2}$, $\delta_{T,EW,C}$ and $\phi_3$ are all free parameters. In each cases 2 ldSPs, 4 branching ratio, 4 direct and 1 indirect CP assymmetries at the $\chi^2$-minimum are also shown.

| Strong Phases | both ldSPs and sdSPs | only sdSPs | only ldSPs |
|---------------|----------------------|------------|------------|
| (a) + free $\phi_3$ | 2.3/2 | 4.3/3 | 8.4/4 |
| (c+d) | 12.8/5| | 16.2/5 |
| $p$-value(%) | 32 | 23 | 7.8 |
| (f) | 2.5 | | |
| (f+c) | 0.6 | | |
| (e) | | | |

which we want to test is the assumption that the parameters $r_T$, $r_{EW}$ and $r_C$ take their pQCD estimated values, e.g. $r_T \sim 0.21$, $r_{EW} \sim 0.14$ and $r_C \sim 0.018$. We call this the “amplitude assumption”. The other hypothesis is the relation among sdSPs: $\delta_T = \delta_{EW}$ and $\delta_C = \delta_P$, which we call the “strong phase assumption”.

In Table IV we summarize the $\chi^2_{min}$ and its $p$-value with amplitude and strong phase assumptions for each case — case with only ldSPs, case with only sdSPs and case with both ldSPs and sdSPs. In the case with both ldSPs and sdSPs, $p$-value is about 32% with non-constrained sdSPs (free), and one with constrained sdSPs (case(c+d)) is still around 23%. In the case with only sdSPs (cases(f, f+c)), $p$-value in (f) is 7.8%, while in (f+c) the probability is reduced to 2.5% and this case is almost ruled out. In the case with only ldSPs (case(e)), the $p$-value is only 0.6% and this case must be completely ruled out. Therefore, this test confirms our previous claim that the generalized “isospin-phased topological approach” can fit the $B \to K\pi$ decays well with the SM values of the parameters, which are estimated from pQCD, without invoking any unknown new physics effects. We again conclude that we cannot ignore the final state re-scattering effects and ldSPs even in $B$ meson decays.

3. Dependence of $S_{K_S\pi}$ and $A^{+0}_{CP}$ on $r_{EW}$ and $r_C$

Now we would like to make a few comments on the sensitivity of the observable $S_{K_S\pi}$ to the parameter $r_C$. The experimental value of $S_{K_S\pi}$ is much smaller than that of $S_{J/\psi K_S}$. As implied by [26], the theoretical prediction of $S_{K_S\pi}$ can be very sensitive to the parameter
TABLE V: $\chi^2$ result for (A) varying $S_{K\pi^0} = 0.2, 0.4, 0.6, 0.7$, and, (B) varying $A_{CP}^{\pm 0} = +0.04, +0.02, -0.02, -0.04$. Error is assumed to be 20%. Concerning the input parameters, we choose case(a) (i.e. ldSPs and sdSPs are included, $\phi_3 = 60^\circ$ is fixed, and $\sin 2\beta = 0.726$ is used).

(A)

| $S_{K\pi^0}$ | 0.20±0.04 | 0.40±0.08 | 0.60±0.12 | 0.70±0.14 |
|---------------|-----------|-----------|-----------|-----------|
| $\chi^2_{\text{min}}$ | 0.0       | 0.0       | 0.0       | 0.0       |
| $r_{\text{EW}}$ | 0.34±0.16 | 0.36±0.12 | 0.36±0.10 | 0.36±0.10 |
| $r_{C}$ | 0.39±0.09 | 0.23±0.09 | 0.08±0.07 | 0.06±0.11 |

(B)

| $A_{CP}^{\pm 0}$ | +0.04 ± 0.008 | +0.02 ± 0.004 | −0.02 ± 0.004 | −0.04 ± 0.008 |
|------------------|----------------|----------------|----------------|----------------|
| $\chi^2_{\text{min}}$ | 0.0       | 0.0       | 0.0       | 0.0       |
| $r_{\text{EW}}$ | 0.35 ± 0.13 | 0.30 ± 0.16 | 0.38 ± 0.16 | 0.38 ± 0.10 |
| $r_{C}$ | 0.28 ± 0.25 | 0.28 ± 0.24 | 0.15 ± 0.09 | 0.26 ± 0.26 |

$r_{C}$. For illustration, in part (A) of Table V we vary $S_{K\pi^0}$ around the present experimental value, specifically, $S_{K\pi^0}$ is assumed to be $(0.20 \pm 0.04), (0.40 \pm 0.08), (0.60 \pm 0.12)$ and $(0.70 \pm 0.14)$, respectively. Here just for the illustrative purpose, we set 20% errors in each case. (Also, to be consistent, we set 20% errors to all the data whose current errors are larger than 20%, such as $A_{ij}^{\pm 0}$. ) It can be seen that as $S_{K\pi^0}$ varies, the allowed region for $r_{C}$ varies in a clearly correlated manner. Just for comparison, in (B) of Table V we also present the case when the value of $A_{CP}^{\pm 0}$ varies. We recall that the averaged experimental value of $A_{CP}^{\pm 0}$ is positive, in contrast to the value of $A_{CP}^{\mp 0}$. Again for the illustrative purpose, $A_{CP}^{\pm 0}$ is assumed to be $(+0.04 \pm 0.008), (+0.02 \pm 0.004)$, respectively. (To be consistent, we also set 20% errors to all the data whose current errors are larger than 20%, such as $S_{K\pi^0}$ and $A_{CP}^{\pm 0}$. ) In contrast to case (A), the dependence of $r_{\text{EW}}$ and $r_{C}$ on $A_{CP}^{\pm 0}$ is very obscure.

IV. CONCLUSIONS

All CP-averaged branching ratios of $B \to K\pi$ modes have been recently measured within a few percent errors by the BaBar and Belle collaborations. The observations of the direct
CP asymmetry in $B^0 \to K^{\pm}\pi^{\mp}$ have also been recently achieved at the $5.7\sigma$ level by BaBar and Belle. Certain experimental data (e.g., the branching ratios (BRs)) for $B \to K\pi$ are currently more precise than the theoretical model predictions based on QCD factorization or perturbative QCD. Thus, with the currently available precision data, it is very important to investigate these modes as generally and critically as possible. The importance of the soft final state re-scattering effects, especially, in a model-independent topological quark diagrammatic analysis of $B \to D\pi$, $B \to K\pi$ and $B \to \pi\pi$ decays, has also been recognized recently. It is certainly conceivable that final state interactions can modify the predictions based on the short distance diagrammatic analysis. In this work, we studied the decay processes $B \to K\pi$ by using generalized decay amplitudes within the so-called “isospin-phased topological approach”, i.e. with both the strong phases of the final state re-scattering and the strong phases of the topological quark diagrammatic origin. We are mainly interested in investigating whether the conventional SM predictions are consistent with the current data even after we include final state interaction effects. In order to find the most likely values of the parameters in a statically reliable way, we used the $\chi^2$ analysis.

Our result shows that: (i) Contrary to the previous finding within the topological approach, we confirmed $\delta_T = \delta_{EW}$ of Ref. 27, which can be understood by the fact that the topology of the tree diagram is quite similar to that of the EW penguin diagram. (ii) If we include final state interaction effects, we can fit theoretical input parameters within the SM without invoking any unknown new physics effects at all. (iii) The final state re-scattering phases can be quite large, and therefore, we cannot ignore the final state re-scattering effects even in $B$ meson decays. (iv) We also found that there are strong correlations between the parameter $r_C$ and the time dependent indirect CP observable $S_{K_s\pi^0}$: If $S_{K_s\pi^0} \approx S_{J/\psi K_s}$, then $r_C < 0.1$. However, the present experimental value, $S_{K_s\pi^0} \ll S_{J/\psi K_s}$, directly implies very large $r_C$ like $0.3 \sim 0.4$, if final state re-scattering phases are not considered.

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