Aspects of warped braneworld models

Soumitra SenGupta 1
Department of Theoretical Physics,
Indian Association for the Cultivation of Science
Jadavpur, Kolkata- 700 032, India

Abstract

We review various key issues in connection with the warped braneworld models which provide us with new insights and explanations of physical phenomena through interesting geometrical features of such extra dimensional theories. Starting from the original Randall-Sundrum two brane models, we have discussed the stability, hierarchy and other important issues in connection with such braneworld. The role of higher derivative terms in the bulk for modulus stabilization has been explained. Implications of the existence of various bulk fields have been discussed and it has been shown how a warped braneworld model can explain the invisibility of all antisymmetric bulk tensor fields on our brane. We have also generalised the model for more than one warped dimensions in the form of a multiply warped spacetime. It is shown that such model can offer an explanation to the mass hierarchy among the standard model fermions and the localization of fermions on the standard model brane with a definite chirality.

1 E-mail: tpssg@iacs.res.in
1 Background

Standard model of elementary particles has been extremely successful in explaining physical phenomena up to scale close to Tev. The background gauge theory which refers to the vast disparity between the electroweak scale and the Planck scale however gives rise to the well known hierarchy/fine tuning problem in connection with the mass of the only scalar particle in the theory namely Higgs [1]. Higgs mass receives large quadratic quantum correction as,

$$\delta m^2_H = \alpha \Lambda^2$$

where \(\Lambda\) is the cutoff of the theory i. Planck scale or GUT scale. To keep \(m_H\) within it’s allowed value \(\sim\) Tev, \(\alpha\) must be unnaturally fine tuned to \(\sim 10^{-32}\), leading to the ‘naturalness problem’. Supersymmetry [1] may remove this large quadratic divergence at the expense of bringing in the a large number of superpartners (called sparticles ) in the theory, all of which are so far undetected. This indicates a broken supersymmetry at the present energy scale which in turn generates large cosmological constant which is not consistent with it’s presently observed small value. To circumvent this, the local version of supersymmetry called supergravity [2] was introduced to have broken SUSY with zero cosmological constant. But the resulting theory is not renormalizable unless one embeds the supergravity model in a more fundamental theory like string theory [3].

Moreover, what will happen if the signature of supersymmetry is not found near Tev scale in the forthcoming experiments? A high scale SUSY (as allowed in a Stringy scenario) will certainly not resolve the hierarchy or the fine tuning problem. So unless one subscribes to the exotic ideas like landscape or anthropic principle [4] in favour of fine tuning , we will have to look for some alternative paths to resolve this longstanding issue.

Theories with extra spatial dimension(s) is one such possible alternatives in this direction. Such theories have attracted a lot of attention because of the new geometric approach to solve the hierarchy problem. The two most prominent models in this context are 1) ADD model , proposed by Arkani-hamed, Dimopoulos and Dvali [5], and 2) RS model, proposed by Randall and Sundrum [6].

In ADD model the extra spatial dimension(s) are large and compactified on circles of radii \(R\). The large radius of the extra dimension pulls down the effective higher dimensional Planck scale i.e the quantum gravity scale. For two or more extra dimensions, the large radius \(R\) can be chosen consistently so that the quantum gravity scale \(M_d\) at d-dimension and hence the cut-off of the theory \(\Lambda\) has the desired value \(\sim\) Tev. However in this process it introduces a new hierarchy of length and therefore mass scale in the theory in the form of the large radius \(R\) ( much larger than the Planck length).

In an alternative scenario, considering one extra spatial dimension Randall and Sundrum proposed a 5 dimensional warped geometric model in an anti-deSitter (ADS) bulk spacetime which we describe now.
2 Randall-Sundrum Model

In Randall-Sundrum scenario the extra coordinate $y = r\phi$ is compactified on a $S_1/Z_2$ orbifold with two 3-branes placed at the two orbifold fixed points $\phi = 0, \pi$, where $r$ is the radius of $S_1$. Using $M_{Pl(5)} \equiv M$ the five dimensional action can be written as,

$$ S = S_{Gravity} + S_{vis} + S_{hid} $$

where,

$$ S_{Gravity} = \int d^4x \sqrt{-G} [2M^3R - \Lambda] $$

$$ S_{vis} = \int d^4x \sqrt{-g_{vis}} [L_{vis} - V_{vis}] $$

$$ S_{hid} = \int d^4x \sqrt{-g_{hid}} [L_{hid} - V_{hid}] $$

(1)

Metric ansatz:

$$ ds^2 = e^{-A} \eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\phi^2 $$

(2)

Warp factor $A(y)$ and the brane tensions are found by solving the 5 dimensional Einstein’s equation with orbifolded boundary conditions

$$ A = 2kr\phi $$

$$ V_{hid} = -V_{vis} = 24M^3k $$

$$ \left[ k^2 = \frac{-\Lambda}{24M^2} \right] $$

(3)

The bulk space-time is taken to be anti-desitter with a negative cosmological constant $\Lambda$.

$$ \left( \frac{m_H}{m_0} \right)^2 = e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \approx (10^{-16})^2 $$

$$ \Rightarrow kr = \frac{16}{\pi} \ln(10) = 11.6279 \ldots \leftarrow \text{RS value} $$

with

$k \sim M_P$ and $r \sim l_P$

So hierarchy problem is resolved without introducing any new scale.

A large hierarchy therefore emerges naturally from a small conformal factor.

In this scenario the five dimensional Planck mass $M_5$ is almost equal to the four dimensional Planck mass $M_4$ for the value of $kr \sim 11.5$ which is required to resolve the hierarchy problem.

3 Modulus stabilization

The tiny value ($\sim$ near Planck length) of the modulus $r$ which measures the separation of the two branes is associated with the vacuum expectation value (VEV) of a massless four-dimensional
scalar field (modulus field) which has zero potential, so that \( r \) is not determined by the dynamics of the model. Goldberger and Wise (GW) [7] proposed to generate such a potential classically by introducing a bulk massive scalar field with quartic interaction terms localized at the two 3-branes and finally obtained a value \( kr \sim 12 \) by minimizing the potential, without any fine-tuning of parameters. In this analysis however the back-reaction of the scalar field on the background geometry was neglected. Before examining this modulus stabilization issue more critically, we begin with a brief description of Goldberger-Wise analysis, which begins with an action of the form:

\[
S = S_{\text{Gravity}} + S_{\text{vis}} + S_{\text{hid}} + S_{\text{scalar}},
\]

where,

\[
S_{\text{Gravity}} = \int d^4x \, r \, d\phi \sqrt{G}[2M^3 \, R + \Lambda],
\]

\[
S_{\text{vis}} = \int d^4x \sqrt{-g_s}[L_s - V_s],
\]

\[
S_{\text{hid}} = \int d^4x \sqrt{-g_p}[L_p - V_p],
\]

\[
S_{\text{scalar}} = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} \sqrt{G}(G^{AB} \partial_A \Phi \partial_B \Phi - m^2 \Phi^2) - \int \sqrt{-g_p} \lambda_p(\Phi^2 - v_p^2) - \int d^4x \sqrt{-g_s} \lambda_s(\Phi^2 - v_s^2).
\]

Here \( \Lambda \) is the five dimensional cosmological constant, \( V_s, V_p \) are the visible and hidden brane tensions. \( \Phi \) develops a \( \phi \)-dependent vacuum expectation value, which has to be determined from the classical equation of motion,

\[
\partial_\phi(e^{-4\sigma} \partial_\phi \Phi) = m^2 r^2 e^{-4\sigma} \Phi + 4^{-4\sigma} \lambda_v r \Phi(\Phi^2 - v_s^2) \delta(\phi - \pi) + 4e^{-4\sigma} \lambda_p r \Phi(\Phi^2 - v_p^2) \delta(\phi)
\]

where \( \sigma = kr|\phi| \). In the bulk (where the delta functions are not relevant), the equation has a general solution:

\[
\Phi(\phi) = e^{2\sigma}[Ae^{\nu\phi} + Be^{-\nu\phi}]
\]

where \( \nu = \sqrt{4 + m^2/k^2} \). The solution is now plugged back into the original scalar field action and integrated over \( \phi \) to obtain an effective 4-dimensional potential for \( r \), of the form:

\[
V_\Phi(r) = k(\nu + 2)A^2(e^{2\nu kr\pi} - 1) + k(\nu - 2)B^2(1 - e^{-2\nu kr\pi}) + \lambda_s e^{-4\nu kr\pi}(\Phi(\pi)^2 - v_s^2)^2 + \lambda_p \Phi(0)^2 - v_p^2].
\]

The coefficients should be determined by matching the delta-function terms at the boundaries. The resultant condition on \( A \) and \( B \) is obtained as follows:

\[
k[(2 + \nu)A + (2 - \nu)B] - 2\lambda_p \Phi(0)[\Phi(0)^2 - v_p^2] = 0.
\]

\[
k e^{2\nu kr\pi}[(2 + \nu)e^{\nu kr\pi} A + (2 - \nu)e^{-\nu kr\pi} B] + 2\lambda_s \Phi(\pi)[\Phi(\pi)^2 - v_s^2] = 0.
\]
According to [7] if one considers infinite $\lambda$ limit, one can choose $\Phi(0) = v_p$ and $\Phi(\pi) = v_s$ as the minimum energy configuration.

Before re-examining the analysis of [7] for arbitrary value of $\lambda$, we first calculate the first and second derivative of the potential to determine the exact extremization condition for the modulus without resorting to any approximation [8]. A long but straightforward calculation yields, the first derivative for the potential (assuming explicit $r$ dependence of $A$ and $B$) as,

$$V'_\Phi(r) = -4k\pi\lambda_se^{-4kr\pi}(\Phi(\pi)^2 - v_s^2) - 4k^2\pi[(\nu + 2)e^{2\nu kr\pi}A^2 + (2 - \nu)e^{-2\nu kr\pi}B^2 + (4 - \nu^2)AB].$$

where prime denotes differentiation with respect to $r$.

Using the extremization condition for the potential ($V'_\Phi(r) = 0$) an exact form for the second derivative of the potential may be obtained as

$$V''_\Phi(r) = 4k^2\pi\nu[(2 + \nu)AB' + (\nu - 2)BA']$$

The sign of R.H.S. of the above equation determines whether the stationary value of the modulus $r$ is a stable value or not.

Assume that for arbitrary value of $\lambda$ (not infinity) the boundary value of the scalar field at the two orbifold fixed points are $\Phi(\phi = 0) = Q_p(r)$ and $\Phi(\phi = \pi) = Q_s(r)$. Now the undetermined constants $A$ and $B$ in terms of these quantities, $Q_p$ and $Q_s$ are given as,

$$A = \frac{Q_s(r)e^{-2\sigma} - Q_p(r)e^{-\nu\sigma}}{2\sinh(\nu\sigma)}$$

$$B = \frac{Q_p(r)e^{\nu\sigma} - Q_s(r)e^{-2\sigma}}{2\sinh(\nu\sigma)}$$

Substituting the expressions for $A$ and $B$ in Eq.45 and equ.14 under stability condition we arrive at (for $\lambda_s \neq 0$),

$$x^2 \left( 1 + \frac{k}{\lambda_s Q_s^2} \right) = \hat{C}^2,$$

where

$$x = \frac{Q_p}{Q_s} - \frac{2 + \nu}{2\nu}e^{(\nu-2)\sigma} - \frac{\nu - 2}{2\nu}e^{-(\nu+2)\sigma}, \quad \hat{C} = \left\{ \frac{2 + \nu}{2\nu}e^{(\nu-2)\sigma} + \frac{\nu - 2}{2\nu}e^{-(\nu+2)\sigma} \right\} C$$

Now, it is easy to manipulate the expression given below from the Eq.18 that is

$$kr = \frac{1}{\pi(\nu-2)} \ln \left[ \frac{\frac{2 + \nu}{2\nu} + \frac{\nu - 2}{2\nu}e^{-2\nu\sigma}}{Q_p(r)\frac{Q_s(r)}{\sqrt{k + \lambda_s Q_s^2(r)}}} \right]$$

for the stationary condition i.e $V'_\Phi(r) = 0$. Here

$$C = \sqrt{1 - \frac{1}{\nu^2}\left\{ (2 + \nu)e^{2(\nu-2)\sigma} - e^{-4\sigma}(4 - \nu^2) + (2 - \nu)e^{-2(\nu+2)\sigma} \right\}}$$
It may be noted that in the large $kr$ limit, $C \sim \sqrt{\frac{\nu - 2}{\nu + 2}}$. The expression for $kr$ in equ. 20), is an exact expression for the stationary value of the modulus $r$ and is valid for any value of the brane coupling constants $\lambda$.

Using the above results, the expression for $V''$ becomes

$$V''(r) = -\frac{4k\pi \nu e^{-2\sigma}}{\sinh(\nu \sigma)} \left[ \lambda_p (Q^2_p - v^2_p) Q_p' Q_p' + \lambda_s (Q^2_s - v^2_s) Q_s Q_s' + \pi \lambda_p \lambda_s (Q^2_s - v^2_s)(Q^2_p - v^2_p) Q_p Q_s \right]$$

(22)

where 'prime' denotes derivative with respect to $r$.

At this point, we want to reiterate that no approximation has been made so far and all the results are exact.

**Stability Analysis**

We now re-examine the stability of the modulus $r$.

**Case I:** $\lambda \to \infty$

From the expression for the potential Eq.11, the minimum energy configuration leads to the

$$Q_s \to v_s ; \quad Q_p \to v_p$$

(23)

which are constants. This completely agrees with the result of [7] along with the following new features:

In this limit the expressions for the stationary points become

$$kr = \frac{1}{\pi(\nu - 2)} \ln \left[ \frac{2\nu}{2 + \nu} \frac{v_p(r)}{v_s(r)} + \frac{1}{1 + \sqrt{\frac{\nu - 2}{\nu + 2}}} \right]$$

(24)

In the infinity limit of the coupling constant

$$Q'_s = 0, \quad ; \quad Q'_p = 0$$

(25)

and

$$V''(r) = -\frac{4k\pi \nu e^{-2\sigma}}{\sinh(\nu \sigma)} \left[ \pi \lambda_p \lambda_s (Q^2_s - v^2_s)(Q^2_p - v^2_p) Q_p Q_s \right]$$

(26)

Clearly for a wide range of parameter values, the expression $\lambda_p (Q^2_p - v^2_p)$ is negative. In general it can be both positive or negative according to the value of $kr$. However, for

$$kr_+ = \frac{1}{\pi(\nu - 2)} \ln \left[ \frac{2\nu}{2 + \nu} \frac{v_p(r)}{v_s(r)} + \frac{1}{1 + \sqrt{\frac{\nu - 2}{\nu + 2}}} \right]$$

(27)

$V''(r) > 0$, that means $kr_+$ is a stable point for the minimum of the potential. So, for the other value of $kr_-$, the potential will be maximum. Clearly no extreme fine tuning of the parameters is required to get the right magnitude for $kr$. It is worthwhile to note that the expression for the value of $kr$ is different from that of [7]. This is why if we calculate the value of $kr$ corresponding to the values of parameters $m/k = 0.2$ and $v_p/v_s = 1.5$ given in [7] we get $kr = 10.846$ which is less
than what have been predicted in [7]. For instance, \(v_p/v_s = 2.3\) and \(\nu = 2.02\) yields \(k_r = 12.2504\) for the minimum of the potential. The value of \(k_r = 14.4993\) corresponds to maximum of the potential. Clearly these two values are very close to each other.

Case II: \(\lambda\) is finite but very large,

From the Eq. 45, it is clear that if the brane coupling constant is large but finite, the value of the scalar field \(Q_p\) on the Planck brane is lower than \(v_p\) and approaches \(v_p\) as the value of the brane coupling constant tends to infinity. We calculate the corrections to the boundary scalar field values in the leading \(1/\lambda\) order correction. These become

\[
Q_p(r) = v_p + \frac{k v_s}{\lambda_p v_p^2} \frac{\nu e^{-2\sigma}}{4 \sinh (\nu \sigma)} \left\{ \frac{v_s}{v_p} - \left\{ \frac{2 + \nu}{2\nu} e^{(2-\nu)\sigma} + \frac{2 - \nu}{2\nu} e^{(\nu+2)\sigma} \right\} \right\}
\]

\[
Q_s(r) = v_s + \frac{k v_p}{\lambda_s v_s^2} \frac{\nu e^{2\sigma}}{4 \sinh (\nu \sigma)} \left\{ \frac{v_p}{v_s} - \left\{ \frac{2 + \nu}{2\nu} e^{(\nu-2)\sigma} + \frac{2 - \nu}{2\nu} e^{-(\nu+2)\sigma} \right\} \right\}
\]

(28)

In this case the modified expression for \(k_r\) becomes (in the large \(k_r\) limit),

\[
k_r = \frac{1}{\pi (\nu - 2)} \ln \left[ \frac{2\nu}{2 + \nu} \pm \frac{n}{(1 - \frac{1}{2}q)} \{ 1 + \frac{\nu - 2}{2\nu} e^{(2-\nu)k_r \pi} \} \right]
\]

(29)

where, \(\nu, n = v_p/v_s, t = k/(\lambda_p v_p^2)\) and \(q = k/(\lambda_s v_s^2)\) are four parameters of the model. Here also as in the previous case we obtain one minimum as well as one maximum for the potential. The positive sign corresponds to the value of \(k_r\) which minimizes the radion potential.

Our analysis therefore shows that the modulus \(r\) may be stabilized even for finite but large value of \(\lambda\). The stable value of the modulus in this case once again solves the hierarchy problem without any unnatural fine tuning of the parameters although the stable value for the modulus differs marginally from that estimated by GW. We have determined the modified value of \(k_r\) because of the the finiteness of \(\lambda\), to the leading order correction around the infinite value i.e. in terms of inverse of \(\lambda\). As stated earlier our exact calculation indicates that there exists simultaneously a very closely spaced (\(\sim\) Planck length \(l_p))\) maximum along with the minimum. This may have interesting consequences in a quantum mechanical version of such a model leading to the possibility of tunneling from one radion vev to the other.

The minimum of this effective potential gives us the stabilized value of the compactification radius \(r_c\). Several other works have been done in this direction [9, 10, 11, 12, 13, 14, 15]. However, in the calculation of GW [7], the back-reaction of the scalar field on the background metric was ignored. Such back-reaction was included later in ref. [16] and exact solutions for the background metric and the scalar field have been given for some specific class of potentials, motivated by five-dimensional gauged supergravity analysis [17]. Assigning appropriate values of the scalar field on the two branes, a stable value of the modulus \(r\) was estimated from the solution of the scalar field. Another interesting work is in the context of a scalar field potential with a supersymmetric form,
where it has been shown that the resulting model is stable [18].

We now examine the modulus stabilization by resorting to the usual modulus potential calculation and its subsequent minimization for a very general class of bulk scalar action. Keeping the non-canonical as well as higher derivative term in the bulk scalar action we find the general condition for the modulus stabilization for back-reacted RS model [6]. We exhibit the role of higher derivative terms in stabilizing the modulus as well as resolving the gauge hierarchy problem.

4 Higher derivative terms and stability

We start with a general action similar to that in our earlier work [19]. In the last few years there have been many models where the presence of a bulk scalar field is shown to have an important role in the context of stability issue of brane-world scenario, bulk-brane cosmological dynamics, higher dimensional black hole solutions and also in many other phenomenological issues in particle physics [7, 16, 20, 21, 22, 23]. Here we resort to a somewhat general type of self interacting scalar field along with the gravity in the bulk in order to analyze the stability of the RS type two-brane model. We consider the following 5-dimensional bulk action

\[
S = \int d^5x \sqrt{-g} \left[ -M^3 R + F(\phi, X) - V(\phi) \right] - \int d^4x \int dy \sqrt{-g_4} \delta(y - y_a) \lambda_a(\phi). \tag{30}
\]

where \(X = \partial_\phi \partial^4 \phi\), with ‘\(A\)’ spanning the whole 5-dimensional bulk spacetime. The index ‘\(a\)’ runs over the brane locations and the corresponding brane potentials are denoted by \(\lambda_a\). The scalar field is assumed to be only the function of extra spatial coordinate \(y\).

Taking the line element in the form

\[
ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \tag{31}
\]

where \(\{y\}\) is the extra compact coordinate with radius \(r_c\) such that \(dy^2 = r_c^2 d\theta^2\), \(\theta\) being the angular coordinate. The field equations turn out to be

\[
F_X \phi'' - 2F_{XX} \phi'^2 \phi'' = 4F_X \phi' A' - \frac{\partial F_X}{\partial \phi} \phi'^2 - \frac{1}{2} \left( \frac{\partial F}{\partial \phi} - \frac{\partial V}{\partial \phi} \right) + \frac{1}{2} \sum_a \frac{\partial \lambda_a(\phi)}{\partial \phi} \delta(y - y_a) \tag{32}
\]

\[
A'^2 = 4CF_X \phi'^2 + 2C \left[ F(X, \phi) - V(\phi) \right] \tag{33}
\]

\[
A'' = 8CF_X \phi'^2 + 4C \sum_a \lambda_a(\phi) \delta(y - y_a) \tag{34}
\]

where

\[
C = \frac{1}{24M^3}; \quad F_X = \frac{\partial F(X, \phi)}{\partial X}; \quad F_{XX} = \frac{\partial^2 F(X, \phi)}{\partial X^2}. \tag{35}
\]

and prime ‘\(\prime\)’ denotes partial differentiation with respect to \(y\). Two of the above equations are independent and the other one automatically follows from the energy conservation in the bulk.
The boundary conditions are
\begin{align}
2 \left( F_X \phi' \right)_{y=0} &= \frac{1}{2} \frac{\partial \lambda_0 (\phi_0)}{\partial \phi} ; \quad -2 \left( F_X \phi' \right)_{y=\pi r_c} = \frac{1}{2} \frac{\partial \lambda_\pi (\phi_\pi)}{\partial \phi} \\
2 A'(y)_{y=0} &= 4 \lambda_0 (\phi_0) ; \quad -2 A'(y)_{y=\pi r_c} = 4 \lambda_\pi (\phi_\pi)
\end{align}
(36)

Now, without knowing the solutions of the above equations explicitly, we may analyze the stability of the modulus \( r_c \), following the mechanism developed by Goldberger and Wise [7]. The brane separation \( r_c \) in general is a dynamical variable associated with the metric component \( g_{55} \). Integrating out the scalar field action over the extra coordinate \( y \) in the background of the back-reacted five dimensional metric, the 4-dimensional effective potential for the brane separation \( r_c \) is obtained as
\[ V_{\text{eff}}(r_c) = -2 \int_0^{y_c \pi} dy \, e^{-4A(y)} \left[ -M^3 R + F(X, \phi) - V(\phi) \right] + e^{-4A(0)} \lambda_0 (\phi_0) + e^{-4A(\pi r_c)} \lambda_\pi (\phi_\pi) \]
(38)

It may be noted that the effective potential is calculated with the warp factor \( A(y) \), which takes care of the full back reaction of the scalar field on the 5D metric through the equations of motion. Therefore the effective potential for the modulus \( r_c \) is calculated by integrating out the full action in the five dimensional background back-reacted metric. The modulus \( r_c \) in general can be a dynamical variable and the minimum of its effective potential determines the corresponding stable value. The role of bulk scalar field here is to stabilize the modulus associated with \( g_{55} \) in the bulk five dimensional background spacetime.

Now, using the above two boundary conditions and expression for the potential from the above equation of motion
\[ V(\phi) = -\frac{1}{2C} A'^2 + 2F_X \phi'^2 + F(X, \phi), \]
(39)
and also the expression for the Ricci scalar \( R = 20A'(y)^2 - 8A''(y) \), one gets the expression for the effective potential as
\[ V_{\text{eff}} = -16M^3 \left[ A'(0) - A'(\pi r_c) e^{-4A(\pi r_c)} \right]. \]
(40)

Now, taking derivative with respect \( y_c = \pi r_c \), one finds, by the use of the equations of motion, the following algebraic equation
\[ \frac{\partial V_{\text{eff}}(r_c)}{\partial (\pi r_c)} = 16M^3 e^{-4A(y)} \left[ A''(y) - 4A'(y)^2 \right]_{\pi r_c} \]
(41)

In order to have an extremum for \( V_{\text{eff}} \) at some value of \( r_c \), the right hand side of Eq.(41) must vanish at that (stable) value of \( r_c \). This immediately implies that the value \( A''(y) \) must be positive and equals to the value of \( 4A'(y)^2 \) at \( y = r_c \). Thus for different solutions of the warp factor for different bulk scalar actions, the above condition determines the corresponding stable value of the modulus \( r_c \).

Let us now resort to the general solutions of the full set of field equations.

We start with the case of a bulk scalar action with a simple non-canonical kinetic term, without any higher derivatives:
\[ F(X, \phi) = f(\phi) X, \]
(42)
where \( f(\phi) \) is any well-behaved explicit function of the scalar field \( \phi \). Let us assume that 
\[
 f(\phi) = \frac{\partial g(\phi)}{\partial \phi},
\]
where \( g(\phi) \) is another explicit function of \( \phi \). Then for a specific form of the potential
\[
 V(\phi) = \frac{1}{16} \left( \frac{\partial g}{\partial \phi} \right)^2 \left( \frac{\partial W}{\partial g} \right)^2 - 2C W(g(\phi))^2
\]
(43)
it is straightforward to verify, for some \( W(g(\phi)) \) and \( g(\phi) \), that a solution to
\[
 \phi' = \frac{1}{4} \frac{\partial W}{\partial g} ; \quad A' = 2C W(g(\phi)).
\]
(44)
are valid solutions provided
\[
 [g(\phi) \phi']_a = \frac{1}{2} \frac{\partial \lambda_a}{\partial \phi_a} (\phi_a) ; \quad [A']_a = 4C \lambda_a(\phi_a).
\]
(45)

Now, let us consider a more general case to include higher derivative term such as,
\[
 F(X, \phi) = K(\phi) X + L(\phi) X^2.
\]
(46)

One of the motivations to consider this type of term in the Lagrangian originates from string theory [24]. The low-energy effective string action contains higher-order derivative terms coming from \( \alpha' \) and loop corrections, where \( \alpha' \) is related to the string length scale \( \lambda_s \) via the relation \( \alpha' = \lambda_s / 2\pi \).

Thus the 4-dimensional Lagrangian involves a non-canonical kinetic term for the scalar field. With appropriate redefinition of the scalar field we can recast such a term in the Lagrangian as
\[
 F(X, \phi) = f(\phi)[X - \beta X^2]
\]
(47)
where \( \beta \) is a constant parameter (in this case it is equal to unity) and \( \{X, \phi\} \) are new variables in terms of old variables. This type of action is common in K-essence cosmological inflationary models [25], which is equivalent to a scalar action in the 5-dimensional bulk with an appropriate potential function. The scalar field is assumed to depend only on the extra (fifth) dimension \( y \).

Now, assuming \( f(\phi) = \partial g(\phi) / \partial \phi \), for a specific form of the potential
\[
 V(\phi) = \frac{1}{16} \left( \frac{\partial g}{\partial \phi} \right)^2 \left( \frac{\partial W}{\partial h} \right)^2 + \frac{3\beta}{256} \left( \frac{\partial g}{\partial \phi} \right)^4 \left( \frac{\partial W}{\partial h} \right)^4 - 2CW^2
\]
(48)
and for some arbitrary \( W(h(g(\phi))) \) and \( g(\phi) \), it is straightforward to verify that a solution to
\[
 \phi' = \frac{1}{4} \frac{\partial W}{\partial h} ; \quad A' = 2C W(h(g(\phi))),
\]
(49)
with the constraint relation
\[
 \frac{dh}{dg} = 1 + \frac{\beta}{8} \left( \frac{\partial W}{\partial h} \right)^2,
\]
(50)
is also a solution provided we have
\[
 [g(\phi)(1 - 2\beta X) \phi']_a = \frac{1}{2} \frac{\partial \lambda_a}{\partial \phi_a} (\phi_a) ; \quad [A']_a = 4C \lambda_a(\phi_a).
\]
(51)
In the \( \beta \to 0 \) limit we at once get back the system of equations dealing with only the simple non-canonical kinetic term (42) in the scalar action, as discussed in the first part of this section. If
in addition, \( f(\phi) \rightarrow 1/2 \), one deals with the usual canonical kinetic term which has been discussed in detail in ref.[16].

Considering now
\[
W(h) = \frac{k}{2C} - u h^2(g(\phi)) \quad ; \quad g(\phi) = \alpha \phi,
\]
where \( k, u \) and \( \alpha \) are the initial constant parameters of our model with their appropriate dimensions, Eq.(50) gives the solution for \( h(g(\phi)) \):
\[
h(g(\phi)) = \frac{1}{u \sqrt{\beta/2}} \tan \left( u \sqrt{\beta/2} \alpha \phi \right)
\]
Clearly, in the limit \( \beta \rightarrow 0 \) we have \( h = g \), which corresponds to what only the simple non-canonical kinetic term gives us [Eqs.(43, 44)].

From Eq.(49) the solution for \( \phi \) becomes
\[
\sin \left( u \sqrt{\beta/2} \alpha \phi \right) = A_0 e^{-u\alpha y/2}
\]
where \( A_0 = \sin \left( u \sqrt{\beta/2} \alpha \phi_0 \right) \) with \( \phi|_{y=0} \equiv \phi_0 \). The solution for the warp factor \( A(y) \) takes the form
\[
A(y) = ky - \frac{4C}{\beta u^2 \alpha} \ln \left( 1 - A_0^2 e^{-u\alpha y} \right).
\]
This is the exact form of the back-reacted warp factor where the first term on the right hand side is same as that obtained by RS in absence of any scalar field and the second term is the result of back-reaction due to the bulk scalar.

In the limit \( \beta \rightarrow 0 \) and \( \alpha = \text{constant} \),
\[
A(y) = ky + 2C \alpha \phi_0^2 e^{-u\alpha y}
\]
which is the exactly what has been discussed in ref.[16].

Following the Goldberger-Wise mechanism [7], we now analyze the stability of our specific model, with \( F(X, \phi) \) given by Eq.(47) in the preceding section. From Eq.(41), we have the extremal condition in a generic situation:
\[
4A' - A'' = 0.
\]
Now, putting the expressions for the various derivatives of the metric solution \( A \) in the above Eq.57 one gets,
\[
QY^2 + PY - \frac{\beta u^2 \alpha k^2}{C} = 0,
\]
where,
\[
Q = u^2 \alpha^2 \left[ 1 - \frac{16C}{\beta u^2 \alpha} \right] \quad ; \quad P = u\alpha(8k + u\alpha) \quad ; \quad Y = \frac{A_0^2 e^{-u\alpha y}}{1 - A_0^2 e^{-u\alpha y}}
\]
From the above equation clearly, we can have two different cases.
Case i) $Q > 0$, then the above equation has only one real solution considering the fact that $\mathcal{Y} > 0$. The corresponding root is

$$\mathcal{Y} = \frac{1}{2Q} \left( \sqrt{p^2 + \frac{4Q\beta u^2\alpha k^2}{C}} - p \right)$$ \hspace{1cm} (60)$$

which gives us the maximum of the potential. So, the point we get is unstable.

Case ii) $Q < 0$, which in turn says $\beta << 1$ provided $u \sim$ Plank scale, then we have two solutions

$$\tilde{\mathcal{Y}}_{\pm} = \frac{1}{2|Q|} \left( p \pm \sqrt{p^2 - \frac{4|Q|\beta u^2\alpha k^2}{C}} \right)$$ \hspace{1cm} (61)$$

where $|Q|$ is the absolute value of $Q$. As we have checked that the larger value of $\mathcal{Y} = \tilde{\mathcal{Y}}_+$ gives us the stable point for the modulus. So, naturally, the lower value $\mathcal{Y} = \tilde{\mathcal{Y}}_-$ gives the unstable point.

For these minimum $\tilde{\mathcal{Y}}_+$ and maximum $\tilde{\mathcal{Y}}_-$ of the effective radion potential, one gets respective distance moduli $y^\pm_{\pi}$ as

$$y^\pm_{\pi} = \frac{1}{u\alpha} \log \left[ \frac{\left( p + 2|Q| \right) \pm \sqrt{p^2 - \frac{4|Q|\beta u^2\alpha k^2}{C}} A_0^2}{p \pm \sqrt{p^2 - \frac{4|Q|\beta u^2\alpha k^2}{C}}} \right]$$ \hspace{1cm} (62)$$

So, by using the above expression for the stable modulus, we get the expression for stable value of $A(r_s)$ as

$$A(r_s) = m \ln \left[ \frac{(1 + \tilde{\mathcal{Y}}_+) A_0^2}{\tilde{\mathcal{Y}}_+} \right] - n \ln \left[ \frac{1}{1 + \tilde{\mathcal{Y}}_+} \right]$$ \hspace{1cm} (63)$$

where,

$$m = \frac{k}{(u\alpha)} \quad ; \quad n = \frac{4C}{(u^2\beta\alpha)}$$ \hspace{1cm} (64)$$

For proper choice of the values of the parameter, this warp factor can produce the desired warping from the Planck scale to Tev scale without any unnatural fine tuning.

5 Bulk tensor fields

In the brane world model, the standard model (SM) fields are assumed to lie on a 3-brane, while gravity propagates in a 5-dimensional anti-de Sitter bulk spacetime [26]. A natural explanation of such a description comes from string theory, with the SM fields arising as excitation modes of an open string whose ends lie on the brane. The graviton, on the other hand, is a closed string excitation and hence can propagate into the bulk. The massless graviton mode has a coupling $\sim 1/M_P$ with all matter, while the massive modes have enhanced coupling through the warp factor. It not only accounts for the observed impact of gravity in our universe but also raises...
hopes for new signals in accelerator experiments [27]. However, there are various antisymmetric tensor fields which are excitations of a closed string and therefore can be expected to lie in the bulk similarly as gravity. The question is why are the effects of their massless modes less perceptible than the force of gravitation?

Bulk fields other than gravitons have been studied earlier in RS scenarios, starting from bulk scalars which have been claimed to be required for stabilisation of the modulus [7]. Bulk gauge fields and fermions have been considered with various phenomenological implications [28]. While some of such scenarios are testable in accelerator experiments [29] or observations in the neutrino sector [30], in general they do not cause any contradiction with our observations so far.

However, the situation with tensor fields of various ranks (higher than 1) is slightly different. It has been already shown a closed string excitation rank-2 antisymmetric tensor field such as the Kalb-Ramond excitation [31] can be in the bulk as just as the graviton, has similar coupling to matter as gravity. Such a field is equivalent to torsion in spacetime [32], on which the experimental limits are quite severe [33]. This apparent contradiction has been resolved in [34] where it has been shown that the zero mode of the antisymmetric tensor field gets an additional exponential suppression compared to the graviton on the visible brane. This could well be an explanation of why we see the effect of curvature but not of torsion in the evolution of the universe. Can we similarly address the effects of other, higher rank, antisymmetric fields which occur in the NS-NS or RR sector of closed string excitations [3]? We explore it now [35].

For a rank-n antisymmetric tensor gauge field $X_{a_1a_2...a_n}$, the corresponding field strength tensor has rank-$(n+1)$.

$$Y_{a_1a_2...a_{n+1}} = \partial_{[a_{n+1}} X_{a_1a_2...a_n]}$$

(65)

Since a spacetime of dimension $D$ admits of a maximum rank $D$ for an antisymmetric tensor, one can at most have $(n+1) = D$. Thus any antisymmetric tensor field $X$ can have a maximum rank $D-1$, beyond which it will all have either zero components or will become an auxiliary field with the field strength tensor vanishing identically. In absence of any mass term such an auxiliary field can be eliminated via the equations of motion due to gauge invariance.

For rank-5 field strength tensor $Y_{ABCMN}$ of a rank-4 field, one gets two kinds of terms, namely:

$$Y_{ABCMN} = \partial_{[\mu} X_{\nu\alpha\beta\gamma]}$$

(66)

and

$$Y_{ABCMN} = \partial_{[\mu} X_{\nu\alpha\beta\gamma]}$$

(67)

The Latin indices denote bulk co-ordinates, the Greek indices run over the $(3+1)$ Minkowski co-ordinates and $y$ stands for the compact dimension coordinate. The first class of terms can be removed using the gauge freedom

$$\delta X_{ABCM} = \partial_{[A} A_{BCM]}$$

(68)
which allows the use of 10 gauge-fixing conditions for an antisymmetric \( \Lambda_{BCM} \). As a result one can use

\[
X_{\nu \alpha \beta y} = 0
\]  

(69)

The second class of terms do not yield any kinetic energy for \( X_{\mu \nu \alpha \beta} \) on the visible brane, and they can thus be removed using the equation of motion. In principle, such an auxiliary field can have an interaction term of the form \( X_{\mu \nu \alpha \beta} B^{\mu \nu} B^{\alpha \beta} \) with second rank antisymmetric tensor fields. If such terms at all exist, they will at most result in quartic self-couplings of the rank-2 field. Thus the rank-4 antisymmetric tensor fields (and of course those of all higher ranks) have no role to play in the four-dimensional world in the RS scenario.

Thus all that can matter are the lower rank antisymmetric tensor fields. The case of a rank-2 field in the bulk (known as the Kalb-Ramond field) has been already investigated.

Let us now consider the only antisymmetric tensor field of a higher rank, surviving on the 3-brane. This is a rank three tensor \( X_{MNA} \), with the corresponding field strength \( Y_{MNA} \). The action for such a field in 5-dimensions is

\[
S = \int d^5x \sqrt{-G} Y_{MNA} Y^{MNA}
\]

(70)

where \( G \) is the determinant of the 5-dimensional metric. Using the explicit form of the RS metric and taking into account the gauge fixing condition \( X_{\mu \nu y} = 0 \), one obtains

\[
S_x = \int d^4x \int d\phi \left[ e^{4\sigma} \eta^{\mu \lambda} \eta^{\nu \rho} \eta^{\alpha \gamma} \eta^{\beta \delta} Y_{\mu \nu \alpha \beta} Y_{\lambda \rho \gamma \delta} + 4 \frac{e^{2\sigma}}{r_c^2} \eta^{\mu \lambda} \eta^{\nu \rho} \eta^{\alpha \gamma} \partial_\phi X_{\mu \nu \alpha} \partial_\phi X_{\lambda \rho \gamma} \right]
\]

(71)

The Kaluza-Klein decomposition of the field \( X \) is,

\[
X_{\mu \nu \alpha}(x, \phi) = \sum_{n=0}^\infty X_{\mu \nu \alpha}^n(x) \frac{\chi^m(\phi)}{\sqrt{r_c}}
\]

(72)

an effective action of the following form can be obtained in terms of the projections \( X_{\mu \nu \alpha}^n \) on the visible brane:

\[
S_X = \int d^4x \sum_n \left[ \eta^{\mu \lambda} \eta^{\nu \rho} \eta^{\alpha \gamma} \eta^{\beta \delta} Y_{\mu \nu \alpha \beta} X_{\lambda \rho \gamma \delta}^n + 4 m_n^2 \eta^{\mu \lambda} \eta^{\nu \rho} \eta^{\alpha \gamma} X_{\mu \nu \alpha}^n X_{\lambda \rho \gamma}^n \right]
\]

(73)

where \( m_n^2 \) is defined through the relation

\[
- \frac{1}{r_c} \frac{d}{d\phi} \left( e^{2\sigma} \frac{d}{d\phi} \chi^n \right) = m_n^2 \chi^n e^{4\sigma}
\]

(74)

and \( \chi^n \) satisfies the orthonormal condition

\[
\int e^{4\sigma} \chi^m(\phi) \chi^n(\phi) d\phi = \delta_{mn}
\]

(75)

In terms of \( z_n = \frac{m_n}{k^2} e^{\sigma} \) and \( x_n = z_n(\pi) = \frac{m_n}{k^2} e^{kr_c \pi} \), the solution for the massive modes can be obtained from the equation as,
The values of the first few massive modes of the rank-3 antisymmetric tensor field are listed in Table 1, where we have also shown the masses of the graviton as well as the rank-2 antisymmetric Kaluza-Klein modes. It may be noted that the rank-3 field has higher mass than the remaining two at every order, and, while the Kalb-Ramond massive modes can have some signature at, say, the Large hadron collider (LHC), that of the rank-3 massive tensor field is likely to be more elusive.

| $n$ | 1    | 2    | 3    | 4    |
|-----|------|------|------|------|
| $m_n^{grav}$ (TeV) | 1.66 | 3.04 | 4.40 | 5.77 |
| $m_n^{KR}$ (TeV)   | 2.87 | 5.26 | 7.62 | 9.99 |
| $m_n^X$ (TeV)      | 4.44 | 7.28 | 10.05| 12.79|

Table 1: The masses of a few low-lying modes of the graviton, Kalb-Ramond (KR) and rank-3 antisymmetric tensor (X) fields, for $kr_c = 12$ and $k = 10^{19}$Gev.

Finally we examine the massless mode, whose strength on the brane needs to be compared to that of the graviton and the rank-2 field. The solution for this mode is given by

$$\chi^0 = \frac{C_1}{2kr_c} e^{-2\sigma} + C_2$$

(77)

Requiring the continuity of $\frac{d\chi^0}{d\phi}$ at the orbifold fixed point $\phi = \pi$, one obtains $C_1 = 0$. The normalisation condition then gives

$$\chi^0 = \sqrt{2kr_c} e^{-2kr_c \pi}$$

(78)

Thus the zero mode of the rank-3 antisymmetric tensor field is suppressed by an additional exponential factor relative to the corresponding rank-2 field which already has an exponential suppression compared to the zero mode of the graviton. Using the same argument as in reference [34], one can extend this result into the coupling of the field X to matter, and show that the interaction with, say, spin-1/2 fields is suppressed by a factor $e^{-2kr_c \pi}$. Thus the higher rank antisymmetric field excitations have progressively insignificant roles to play on the visible brane, with the fields vanishing identically beyond rank 3 [35].

Thus the graviton has a unique role among the various closed string excitations in a warped geometry. This is because the intensity of its massless mode on the 3-brane leads to coupling $\sim 1/M_P$ with matter fields, which is consistent with the part played by gravity (or more precisely the curvature of spacetime) observed in our universe. On the contrary the bulk antisymmetric tensor fields upto rank-3 can still have non-vanishing zero modes in four-dimensional spacetime. However their strength is progressively diminished for ranks-2 and 3. This may well serve as an explanation of why their role in the observable universe on the Tev 3-brane is imperceptible. The masses of the higher modes also tend to increase with rank, making them less and less relevant to accelerator experiments.
6 Fine tuning problem in presence of bulk antisymmetric tensor field

It has been discussed in the previous section that apart from graviton there are other massless closed string excitations also which are free to enter the bulk. One of such field is the two form of antisymmetric tensor field namely the Kalb-Ramond (KR) field \(B_{MN}\) with the corresponding third rand antisymmetric tensor field strength \(H_{MNL}\) such that \(H_{MNL} = \partial_{[M}B_{NLM]}\). Apart from [34] the implications of the presence of such a bulk field in a RS braneworld has already been analyzed in various contexts [36]. Here we re-examine the fine tuning problem in connection with the Higgs mass, when such a field exists in the bulk [15, 37]

We begin with the action,

\[
S_{Gravity} = \int d^4x \ d\phi \sqrt{-G} \ [2M^3 R - \frac{\Lambda}{5-d}]
\]

\[
S_{vis} = \int d^4x \sqrt{-g_{vis}} \ [L_{vis} - V_{vis}]
\]

\[
S_{hid} = \int d^4x \sqrt{-g_{hid}} \ [L_{hid} - V_{hid}]
\]

\[
S_{KR} = \int d^4x \ d\phi \sqrt{-G} \ [H_{MNL}H^{MNL}]
\]

(79)

We also consider the warped metric ansatz: \(ds^2 = e^{-A} \eta_{\mu\nu}dx^\mu dx^\nu + r^2 d\phi^2 \leftarrow \text{extra dim}\)

Solving the five dimensional Einstein’s equation, the solution for the warp factor turns out to be,

\[
e^{-A} = \frac{\sqrt{b}}{2kr} \cosh(2kr\phi + 2kr\tilde{c})
\]

\[
\frac{2kr}{\sqrt{b}} = \cosh(2kr\tilde{c}) \text{ , such that } A(0) = 1
\]

\[
c = -\frac{1}{2kr} \tanh^{-1} \left( \frac{V_{hid}}{24M^3k} \right) = -\pi + \frac{1}{2kr} \tanh^{-1} \left( \frac{V_{vis}}{24M^3k} \right)
\]

(80)

Here \(b\) measures the KR Energy density.

The scalar mass warping is now given by,

\[
\left( \frac{m_H}{m_0} \right)^2 = e^{-2A}_{|\phi=\pi} = \frac{\sqrt{b}}{2kr} \cosh \left[ 2kr\pi + \cosh^{-1} \frac{2kr}{\sqrt{b}} \right]
\]
\[
= \left[ \cosh (2kr\pi) - \sinh (2kr\pi) \sqrt{1 - \frac{b}{(2kr)^2}} \right] \\
\approx (10^{-16})^2
\]

Inverting the above expression we obtain,

\[
b = (2kr)^2 \left[ 1 - \left( \coth(2kr\pi) - \left( \frac{m_H}{m_0} \right)^2 \coth(2kr\pi) \right)^2 \right]
\]

How large \( b \) can be to obtain the desired warping from Planck scale to Tev scale?

\[
\log |b| \text{ vs } kr, \text{ for } \frac{m_H}{m_0} = 10^{-16}
\]

We thus have,

For \( kr \) same as RS value, \( b = 0 \)

For \( kr < \) RS value, \( b < 0 \)

and for \( kr > \) RS value, \( b > 0 \)

But the maximum possible value for \( b \) in this entire range turns out to be,

\[
b_{\text{max}} = 10^{-62}
\]

This indicates extreme fine tuning of \( b \) ! Thus in order to avoid the fine tuning \( \sim 10^{-32} \) of the scalar (Higgs) mass the warped geometry model was proposed to achieve the desired warping from Planck scale to Tev scale geometrically. But our result indicates that such geometric warping can be achieved only if the energy density of the bulk KR field is fine tuned \( \sim 10^{-62} \).

We have generalized our work by including the dilaton field (another massless string excitation) also in the bulk [38]. Once again our result revealed that an unnatural fine tuning of the energy density parameter of the KR field is necessary to achieve the desired warping from the Planck scale to the Tev scale.

Thus in the backdrop of a string inspired model the fine tuning problem reappears in a new guise.

7 Generalization to six dimension: Fermion mass splitting

We now generalize the Randall-Sundrum warped brane model to a spacetime with more than one warped dimensions [39, 40]. In a six dimensional warped space-time we consider a doubly warped space-time as \( M^{1,5} \rightarrow [M^{1,3} \times S^1/Z_2] \times S^1/Z_2 \) [40]. The non-compact directions would be denoted by \( x^\mu (\mu = 0..3) \) and the orbifolded directions by the angular coordinates \( y \) and \( z \) with \( R_y \) and \( r_z \) as respective moduli. Four 4-branes are placed at the orbifold fixed points: \( y = 0, \pi \) and \( z = 0, \pi \) with appropriate brane tensions. Four 3-branes appear at the four intersection
region of these 4-branes. We also consider an ADS bulk with a negative cosmological constant $\Lambda$. We thus have a brane-box like space-time.

The six dimensional warped metric ansatz:

$$ds^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2dy^2] + r_z^2dz^2$$

The total bulk-brane action is thus given by,

$$S = S_6 + S_5 + S_4$$

$$S_6 = \int d^4x\,dy\,dz\,\sqrt{-g_6}\,(R_6 - \Lambda)$$

$$S_5 = \int d^4x\,dy\,dz\,[V_1\,\delta(y) + V_2\,\delta(y - \pi)]$$

$$+ \int d^4x\,dy\,dz\,[V_3\,\delta(z) + V_4\,\delta(z - \pi)]$$

$$S_4 = \int d^4x\,\sqrt{-g_{vis}}[L - \hat{V}]$$  \hspace{1cm} (83)

In general $V_1, V_2$ are functions of $z$ while $V_3, V_4$ are functions of $y$. The intersecting 4-branes give rise to 3-branes located at, $(y, z) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$.

Substituting the metric in six dimensional Einstein’s equation the solutions are:

$$a(y) = \exp(-cy)$$

$$b(z) = \frac{\cosh( kz)}{\cosh(k \pi)}$$  \hspace{1cm} (84)

Minimum warping at the 3-brane located at $y = 0, z = \pi$. Maximum warping at the 3-brane located at $y = \pi, z = 0$.

Here

$$c \equiv \frac{R_y k}{r_z \cosh(k \pi)}$$

$$k \equiv r_z \sqrt{-\frac{\Lambda}{10 M^4}}$$  \hspace{1cm} (85)

Using the orbifolded boundary condition the full metric thus takes the form,

$$ds^2 = \frac{\cosh^2(kz)}{\cosh^2(k \pi)}\left[\exp(-2c\,|y|)\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + R_y^2\,dy^2\right]$$

$$+ \,r_z^2\,dz^2$$  \hspace{1cm} (86)

Also the $4 + 1$ brane tensions become coordinate dependent and are given as,

$$V_1(z) = -V_2(z) = 8M^2\sqrt{-\frac{\Lambda}{10}}\,\text{sech}(kz)$$

$$V_3(y) = 0$$

$$V_4(y) = -\frac{8M^4k}{r_z}\tanh(k\pi)$$  \hspace{1cm} (87)
We therefore observe that, the 3 branes appear at the intersection of the various 4 branes. There are 4 such 3 branes located at \((y, z) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)\). The metric on the 3-brane located at \((y = 0, z = \pi)\) suffers no warping. So it is identified with the Planck brane. Similarly we identify the SM brane with the one at \(y = \pi, z = 0\). Planck scale mass \(m_0\) is warped to

\[
m = m_0 \frac{r_z c}{R_y k} \exp(-\pi c) = m_0 \frac{\exp(-\pi c)}{\cosh(k \pi)}
\]

(88)

An interesting picture emerges from this. To have substantial warping in the \(z\)-direction (from \(z = 0\) to \(z = \pi\)), \(k \pi\) must be substantial, i.e. of same order of magnitude as in the usual RS case. But \(c\) is given by

\[
c \equiv \frac{R_y k}{r_z \cosh(k \pi)}
\]

(89)

This immediately means that \(c\) must be small for \(r_z \sim R_y\). Thus we cannot have a large warping in \(y\)-direction as well if we want to avoid a new and undesirable hierarchy between the moduli. Thus of the two branes located at \(y, z = 0, 0\) and \(y, z = \pi, \pi\), one must have a natural mass scale close to the Planck scale, while for the other it is close to the TeV scale. This resembles to the fine structure splitting of energy levels.

If we repeat this calculation for a seven dimensional warped space-time, we similarly find, eight 3-branes appearing from the intersection of the hyper-surfaces. Four of these are close to TeV scale and four others are close to Planck scale. Thus increasing the number of warping in space-time results into two clusters of branes around Planck and TeV scale.

An interesting phenomenological consequences like mass splitting of the standard model fermions on the brane follows from this. The SM-like fields in each of these 3-branes will have apparent mass-scales (on each brane) close to TeV with some splitting between them. To understand this, imagine the SM fermions being defined by 5-dimensional fields with \(x^\mu\) and \(y\) dependence, restricted to the 4-brane say at \(z = 0\) which now defines the “bulk” for these fields. This 4-brane also intersects two other 4-branes at \(y = 0\) and \(y = \pi\) respectively. If the major warping has occurred in the \(z\)-direction, then the natural mass scale of these fields is still \(O(\text{TeV})\). The presence of a \(y\)-dependence leads to a non-trivial bulk wave-function. This, in turn, changes the overlap of the fermion wave-function with that of a scalar located on the 3-brane and thus the effective Yukawa coupling. The slightly differing interactions on the distant 3-brane located at two different values of \(y\) would result in a hierarchy amongst the effective Yukawa couplings and the fermion masses on the 3-brane. Adjusting the parameters suitably the observed mass splitting among the different generations of fermions can be explained [41]. It has also been shown that in this model the coordinate dependent brane tension, which is equivalent to a scalar field distribution on the 4-brane localizes the left chiral fermionic mode on our 3-brane. Thus the fermion localization problem is automatically resolved in such a multiply warped spacetime[42].
8 Conclusions

The RS model is known to solve the hierarchy problem with gravity in the bulk. In this review we have discussed various implications and possible extensions of such warped braneworld model. The issue of modulus stabilization has been explored in details with higher derivative terms present in the bulk. The role of various bulk antisymmetric tensor fields have been discussed. Such studies are specially relevant in the context of string inspired models. It is shown that RS model offers a natural explanation of invisibility of all the antisymmetric tensor fields on our brane and thereby making the status of the closed string symmetric tensor excitation namely gravity quite distinct from the antisymmetric closed string excitations. Finally a multiply warped geometric model is proposed as a natural extension to RS model in six and higher dimensions. Such model is shown to offer a possible explanation of the standard model fermion mass hierarchy, a proper resolution of which has been eluding us for a long time. Moreover the consistency requirement of such a model leads to the localization of fermions on Tev brane with definite chirality. Thus a multiply warped spacetime offers a mechanism to obtain chiral fermions in our universe.

References

[1] See for example M.Drees, hep-ph/9611409; S.P.Martin, hep-ph/9709356.

[2] See for example H.P. Nilles, Phys.Rept.110 1,1984.

[3] See for example M.B.Green, J.H. Schwarz and E.Witten, Superstring Theory, Vol.1 and 2, CUP.

[4] S.Weinberg, Phys. Rev. Lett. 59, 2607(1987); R.Bousso and J.Polchinski, JHEP 0006, 006 (2000); L.Susskind, hep-th/0302219; M. Douglas, JHEP 0305,046 (2003)

[5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 257 (1998).

[6] L. Randall and R. Sundrum, Phys. Rev. Lett Phys. Rev. Lett. 83, 4690 (1999); ibid Phys. Rev. Lett. 83, 3370 (1999).

[7] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. 83 4922 (1999); ibid Phys.Rev. Lett. B475 275 (2000).

[8] A.Dey, D.Maity, S.SenGupta, Phys.Rev.D 75, 107901 (2007).

[9] M. Luty and R. Sundrum, Phys. Rev. D64 065012 (2001); F. Brummer, A. Hebecker and E. Trincherini, Nucl. Phys. B738 283 (2006); G.L. Alberghi et al, Phys. Rev. D72 025005 (2005); T. Kobayashi and K. Yoshioka, JHEP 0411 024 (2004); S. Ichinose and A. Murayama,
Phys. Lett. B625 106 (2005); S. Kalyana Rama, Phys. Lett. B495 176 (2000); H. K. Jassal, hep-th/0312253.

[10] N. Maru, N. Okada, hep-th/0508113; M. Eto, N. Maru and N. Sakai, Phys. Rev. D70 086002 (2004); N. Maru and N. Okada, Phys. Rev. D70 025002 (2004);

[11] I. I. Kogan et al, Nucl. Phys. B584 313 (2000); D. Choudhury et al, JHEP 09 021 (2000); S. Forste, hep-th/0110055.

[12] P. C. Ferreira and P. V. Moniz, hep-th/0601070; ibid hep-th/0601086; G. L. Alberghi et al, Phys. Rev. D72 025005 (2005); G. L. Alberghi and A. Tronconi, Phys. Rev. D73 027702 (2006); A. A. Saharian and M. R. Setare, Phys. Lett. B552 119 (2003).

[13] C. Csaki et al, Phys. Lett. B462, 34 (1999); ibid Phys. Rev. D62, 045015 (2000)

[14] C. Csaki et al, Phys. Rev. D63 065002, 2001.

[15] S. Das, A. Dey, S. SenGupta, Class. Quant. Grav. 23 L67 (2006); H. Yoshiguchi et al, JCAP 0603 018 (2006); E. E. Boos et al, hep-th/0511185.

[16] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, Phys. Rev. D62, 046008 (2000)[hep-th/9909134].

[17] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, Adv. Theor. Math. Phys. 3, 363 (1999).

[18] J.L. Lehners, P. Smyth, K.S. Stelle, Class. Quant. Grav. 22, 2589 (2005).

[19] D. Maity and S. SenGupta, Phys. Lett. B 643, 348 (2006); D. Maity, S. SenGupta and S. Sur, [hep-th/0604195].

[20] W. D. Goldberger and M. B. Wise, Phys. Rev. D60 107505 (1999); S. Kachru, M. B. Schulz and E. Silverstein, Phys. Rev. D62 045021 (2000); H. A. Chamblin and H. S. Reall, Nucl. Phys. B562 133 (1999); C. Csaki, [hep-ph/0404096]; R. Neves, TSPU Vestnik 44N7 94 (2004); E. Dudas and M. Quiros, Nucl. Phys. B721 309 (2005); J. Lesgourgues and L. Sorbo, Phys. Rev. D69, 084010 (2004); B. Grzadkowski and J. F. Gunion, Phys. Rev. D68, 055002 (2003).

[21] R. Koley and S. Kar Phys. Lett. B623 244 (2005); M. Pospelov, [hep-ph/0412280]; M. Parry and S. Pichler, JCAP 0411, 005 (2004).

[22] A. Knapman and D. J. Toms, Phys. Rev. D69, 044023 (2004); J. E. Kim, B. Kyae and Q. Shafi, Phys. Rev. D70, 064039 (2004); Y. Himemoto and M. Sasaki, Prog. Theor. Phys. Suppl. 148, 235 (2003); S. Kobayashi and K. Koyama, JHEP, 0212, 058 (2002); P. Kanti, S. Lee and K. A. Olive, Phys. Rev. D67, 024037 (2003); Y. Himemoto, T. Tanaka and M. Sasaki, Phys. Rev. D65, 104020 (2002); S. C. Davis, JHEP 0203, 058 (2002); S. C. Davis, JHEP
0203, 054 (2002); D. Langlois and M. Rodriguez-Martinez, Phys. Rev. D64, 123507 (2001); A. Flachi and D. J. Toms, Nucl. Phys. B610, 144 (2001); J. M. Cline and H. Firouzjaahi, Phys. Lett. B495, 271 (2000); J. M. Cline and H. Firouzjaahi Phys. Rev. D64, 023505 (2001); R.N. Mohapatra, A. Perez-Lorenzana and C. A. de Sousa Pires Phys. Rev. D62, 105030 (2000); P. Kanti, K. A. Olive and M. Pospelov, Phys. Lett. B481, 386 (2000).

[23] P. Brax, C. van de Bruck, A. C. Davis and C.S. Rhodes Phys. Lett. B531, 135 (2002); A. Flachi and D.J. Toms, Nucl. Phys. B610, 144 (2001)

[24] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000); C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, Phys. Rev. D63, 103510 (2001); C. Armendariz-Picon, T. Damour and V. Mukhanov, Phys. Lett. B458, 209 (1999); J. Garriga and V. Mukhanov, Phys. Lett. B458, 219 (1999).

[25] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D62, 023511 (2000);

[26] For a general discussion see, for example, Y. A. Kubyshin, arXiv: hep-ph/0111027; V.A. Rubakov, Phys.Usp. 44,(2001); C. Csaki, arXiv:hep-ph/0404096

[27] See, for example, D. K. Ghosh and S. Raychaudhuri, Phys.Lett. B495, 114, (2000); John F. Gunion, arXiv: hep-ph/0410379; T. Rizzo, Phys.Lett.B647 43, (2007), M. Arai et al., Phys. Rev. D75,095008 (2007); CDF Collaboration (T. Aaltonen et al.), arXiv:0707.2294 [hep-ex]

[28] H. Davoudiasl, J.L. Hewett and T.G. Rizzo ,Phys.Rev.Lett.84,2080,(2000); Phys.Lett.B473 43,(2000).

[29] See, for example, S. Chang et al., Phys.Rev.D62,084025 (2000); R. Kitano, Phys.Lett. B481. 39 (2000); J.-P. Lee, Eur. Phys. J. C34, 237 (2004); M. Guchait, F. Mahmoudi, K. Sridhar, JHEP 0705, 103 (2007); T. Rizzo, Phys.Lett. B647 43 (2007).

[30] Y.Grossman, M. Neubert, Phys.Lett.B474, 361 (2000).

[31] M. Kalb and P. Ramond , Phys.Rev.D9, 2273 (1974).

[32] .P. Majumdar , S. SenGupta, Class.Quant.Grav.16, L89 ,(1999).

[33] R.T. Hammond, Phys.Rev.D52,6918 (1995); S.SenGupta and S.Sur, Eur.Phys.Lett.65, 601 (2004) and references therein.

[34] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys.Rev.Lett.89, 121101,(2002; Erratum-ibid.89, 259902,(2002)

[35] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys.Rev.D 76, 121501 (2007).
[36] S.SenGupta and S.Sur, Europhys.Lett.\textbf{65} 601 (2004); D.Maity and S.SenGupta, Class.Quant.Grav \textbf{21} 3379 (2004); D.Maity, P.Majumdar and S.SenGupta, JCAP \textbf{0406} 005 (2004); B.Mukhopadhyaya, S.Sen, S.Sen and S.SenGupta, Phys.Rev.\textbf{D70} 066009 (2004); D.Maity, S.SenGupta and S.Sur, Phys.Rev.\textbf{D72} 066012 (2005).

[37] S.Das, A.Dey, S.SenGupta, J.Phys.Conf.Ser.\textbf{68} 012009,(2007).

[38] S.Das, A.Dey, S.SenGupta, arXiv/0704.3119 [hep-th] ( To appear in Euro.Phys. Lett.).

[39] S. Randjbar-Daemi and M.E. Shaposhnikov, Phys.Lett.\textbf{B491} 329 (2000); P. Kanti, R. Madden and K.A. Olive, Phys.Rev.\textbf{D64} 044021 (2001).

[40] D.Choudhury, S.SenGupta, Phys.Rev.\textbf{D76},064030 (2007).

[41] B.S. Balakrishna, Phys.Rev.Lett \textbf{60} 1602 (1988); S.M. Barr, Phys. Rev. \textbf{D21} 1424(1980); H. Naoyuki and Y. Shimizu, hep-ph/0210146.;B.A. Dobrescu and E. Poppitz, Phys.Rev.Lett. \textbf{47} 031801 (2001).

[42] R.Koley, J. Mitra, S. SenGupta, arXiv:0804.1019 [hep-th], ( To appear in Phys.Rev.D ).