Negative result measurements in mesoscopic systems

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Abstract

We investigate measurement of electron transport in quantum dot systems by using single-electron transistor as a noninvasive detector. It is demonstrated that such a detector can operate in the “negative-result measurement” regime. In this case the measured current is not distorted, providing that it is a non-coherent one. For a coherent transport, however, the possibility of observing a particular state out of coherent superposition leads to distortion of a measured current even in the “negative-result measurement” regime. The corresponding decoherence rate is obtained in the framework of quantum rate equations.

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Rapid progress in nanoscale devices made it possible to produce new type of detectors like the quantum point-contact and the single electron transistor (SET), which have been already used in different quantum measurements [1–4]. These devices have been considered also as possible detectors for a single two level system (q-bit) [5–7]. It is of a great advantage that these detectors can be treated entirely quantum mechanically, so that the related measurement process can be investigated in great details. In particular, one can study quantum mechanical mechanism of decoherence and its influence on a measured system.

In this letter we consider a measurement of electron current in quantum dots by using SET in close proximity of a measured system, so it monitors the movement of single electrons inside the system [2,4]. We demonstrate that varying parameters of SET one can put
it in the “negative result measurement” regime [8]. In this case the detector becomes a non-distractive if a measured current is incoherent one. However, in the case of coherent measured current, the negative result measurement distorts it via the decoherence. We evaluate the decoherence rate for this process and demonstrate that it is directly related to a possibility of observation of a particular quantum state of the measured system out of the linear superposition. Otherwise the negative result measurement would not affect the measured current, even if the latter is a coherent one. This phenomenon produces a peculiar effect in the current which can be observed experimentally.

We start with a description of measurement of resonant tunneling currents in quantum dots by using SET. The system is shown schematically in Fig. 1 [2]. The SET, represented by the upper dot, is in close proximity to the lower dot (the measured system). Both dots are coupled to two separate reservoirs at zero temperature. The resonant levels $E_0$ and $E_1$ are taken between the Fermi levels in the corresponding reservoirs, $\tilde{E}_F^L > E_0 > \tilde{E}_F^R$ and $\tilde{E}_F^L > E_1 > E_F^R$. In the absence of electrostatic interaction between electrons the dc resonant currents in the detector and the measured system are respectively [9]

$$I_{D}^{(0)} = e \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R}, \quad I_{S}^{(0)} = e \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R},$$

where $\gamma_{L,R}$ and $\Gamma_{L,R}$ are the tunneling partial widths of the levels $E_0$ and $E_1$ due to coupling with left and right reservoirs. The situation is different in the presence of electron-electron interaction between the dots, $H_{\text{int}} = U n_0 n_1$, where $n_{0,1}$ are the occupancies of the upper and the lower dots and $U$ is the Coulomb repulsion energy. If $E_0 + U > \tilde{E}_F^L$, an electron from the left reservoir cannot enter the upper dot when the lower dot is occupied [Fig. 1 (b)]. On the other hand, if an electron occupies the upper dot [Fig. 1 (a′,b′)], the displacement of the level $E_1$ of the lower dot is less important, since it remains below the Fermi level, $E_1 + U < \tilde{E}_F^L$. Thus, the upper dot can be considered as a detector registering charging of the lower dot via variation of its current [2]. For instance, by measuring the variation of the average detector current ($\Delta I_D$) due to the measurement, one can determine the average current in the lower dot, $I_S$. If $\Delta I_D \gg I_S$, the detector represents an amplifier, which can
measure very small currents [4].

Fig. 1: Measurement of resonant current in a single-dot structure by another, nearby dot. All possible electron states of the detector (the upper well) and the measured system (the lower well) are shown. Also indicated are the tunneling rates (γ and Γ) of the detector and the measured system respectively.

In fact, the described detector affects the measured system. Indeed, if the detector is occupied [Fig. 1 (a’,b’)], an electron enters the lower dot with the energy \( E_1 + U \). As a result, the corresponding tunneling rates are modified (\( \Gamma_{L,R} \rightarrow \Gamma'_{L,R} \)), and therefore the measured current is distorted. One finds, however, that the states with empty detector, \(|a\rangle\) and \(|b\rangle\) [Fig. 1 (a,b)], do not distort the measured system. Nevertheless, the measurement process does take place: the detector current is interrupted whenever an electron occupies the measured system, but it flows freely when the measured system is empty. Such a
measurement is in fact the negative result measurement [8]. Therefore, in order to put the above detector in the negative result regime, we need to diminish the role of the states $|a'\rangle$ and $|b'\rangle$ in the measurement process. It can be done by varying the penetrability of the detector barriers, so that $\gamma_R \gg \gamma_L$. In this case the dwelling time of an electron inside the detector can be strongly diminished. Indeed, the average charge inside a double-barrier structure [9] $\frac{e\gamma_L}{(\gamma_L + \gamma_R)} \to 0$ for $\gamma_L/\gamma_R \to 0$. Then an electron entering the detector leaves it immediately, remaining the measured system undistorted.

Now we evaluate the measured current explicitly in order to confirm that the above measurement is a non-distractive one. The currents through the detector and the measured system are determined by the density-matrix for the entire system $\rho(t)$, which obeys the Schrödinger equation $i\dot{\rho}(t) = [\mathcal{H}, \rho]$ for $\mathcal{H} = H_D + H_S + H_{int}$, where $H_{D,S}$ are the tunneling Hamiltonians of the detector and the measured system, respectively, and $H_{int} = U\sigma_n n_1$. The current in the detector (or in the measured system) is the time derivative of the total average charge $Q(t)$ accumulated in the corresponding right reservoir (collector): $I(t) = \dot{Q}(t)$, where $Q(t) = e\text{Tr}[\rho^R(t)]$ and $\rho^R(t)$ is the density-matrix of the collector. It was shown [10] that $I(t)$ is directly related to the density-matrix of the multi-dot system $\sigma_{ij}(t)$ with $i, j = \{a, a', b, b'\}$, obtained from the total density-matrix $\rho(t)$ by tracing out the reservoir states. One finds that the current in the detector or in the measured system is given by

$$I(t) = e\sum_j \sigma_{jj}(t)\Gamma^{(j)}_R,$$  \hspace{1cm} (2)

where the sum is taken over the states $|j\rangle$ in which the well adjacent to the corresponding collector is occupied, and $\Gamma^{(j)}_R$ is the partial width of the state $|j\rangle$ due to tunneling to the collector ($\gamma_R$ or $\Gamma_R$). In turn, $\sigma(t)$ obeys the following system of the rate equations [10]

$$\begin{align*}
\dot{\sigma}_{aa} &= -(\gamma_L + \Gamma_L)\sigma_{aa} + \gamma_R\sigma_{a'a'} + \Gamma_R\sigma_{bb} \\
\dot{\sigma}_{bb} &= -\Gamma_R\sigma_{bb} + \Gamma_L\sigma_{aa} + (\gamma'_L + \gamma'_R)\sigma_{b'b'} \\
\dot{\sigma}_{a'a'} &= -(\gamma_R + \Gamma'_L)\sigma_{a'a'} + \gamma_{L}\sigma_{aa} + \Gamma'_{R}\sigma_{b'b'} \\
\dot{\sigma}_{b'b'} &= -(\gamma'_L + \Gamma'_R)\sigma_{b'b'} + \Gamma'_L\sigma_{a'a'},
\end{align*}$$  \hspace{1cm} (3)
where the states \( |a\rangle, |b\rangle, |a'\rangle, |b'\rangle \) are the available states of the entire system, Fig. 1. Note the off-diagonal density-matrix elements (coherencies) do not enter in Eqs. (3), and therefore these equations describe a non-coherent transport. The reason is that there is no transitions between the isolate states in this system, which characterize the coherent transport [10].

In the case of measurement, Fig. 1, the currents in the detector and in the lower dot are 
\[
I_D^{(1)}(t) = e[\gamma_R \sigma_{a'a'}(t) + \gamma'_R \sigma_{b'b'}(t)] \quad \text{and} \quad I_S^{(1)}(t) = e[\Gamma_R \sigma_{bb}(t) + \Gamma'_R \sigma_{bb'}(t)],
\]
respectively, Eq. (2). The stationary (dc) current corresponds to \( I = I(t \to \infty) \). Solving Eqs. (3) in the limit \( \gamma_R, \gamma'_R \gg \gamma_L, \gamma'_L \) we find
\[
\frac{\Delta I_D}{I_S^{(1)}} = \frac{\gamma_L}{\Gamma_R}, \quad I_S^{(1)} = e \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} = I_S^{(0)}, \tag{4}
\]
where \( \Delta I_D = I_D^{(0)} - I_D^{(1)} \) is a variation of the detector current with respect to the case of no measurement. The first equation shows that the SET amplifies quantum signals if the ratio \( \gamma_L/\Gamma_R \gg 1 \). Thus, one can measure small current \( I_S \) by measuring variation of the detector current \( \Delta I_D \) [1]. On the other hand the measured current \( I_S \) is not distorted by the detector, as follows from the second equation.

Consider now a measurement of resonant transport in a coupled-dot structure [11], Fig. 2. In this case the electron transport is a coherent one, since an electron inside the double dot appears in the linear superposition of two states (\( E_1 \) and \( E_2 \)). The SET detector, represented by the upper dot, is taken in close proximity to the second dot of the double-dot system, thus measuring charging of that dot. For simplicity, we assume strong Coulomb repulsion between two electrons inside the coupled-dot, so only one electron can occupy the measured system [12]. Fig. 2 shows all possible electron configurations of the double-dot when the detector is empty. Similar to the previous case (Fig. 1) each of the states, \( |a\rangle, |b\rangle, |c\rangle \) has its counterpart \( |a'\rangle, |b'\rangle, |c'\rangle \) corresponding to the occupied detector. \( U_{1,2} \) is the Coulomb repulsion energy between the detector and the measured system for the electron occupying the first or the second dot. We consider \( E_0 + U_2 > \tilde{E}_F \), but \( E_0 + U_1 < \tilde{E}_F \). Therefore the detector is blocked only when the second dot is occupied.
Consider first the case of no measurement (no interaction with the upper dot). The available states of the double-dot system, $|a\rangle$, $|b\rangle$ and $|c\rangle$, are those as shown in Fig. 2. The resonant current through this system is described by the Bloch-type rate equations, derived from the microscopic Schrödinger equation [10]

\[
\begin{align*}
\dot{\sigma}_{aa} &= -\Gamma_L \sigma_{aa} + \Gamma_R \sigma_{cc} \\
\dot{\sigma}_{bb} &= \Gamma_L \sigma_{aa} + i\Omega (\sigma_{bc} - \sigma_{cb}) \\
\dot{\sigma}_{cc} &= -\Gamma_R \sigma_{cc} - i\Omega (\sigma_{bc} - \sigma_{cb}) \\
\dot{\sigma}_{bc} &= i\epsilon \sigma_{bc} + i\Omega (\sigma_{bb} - \sigma_{cc}) - \frac{1}{2}\Gamma_R \sigma_{bc},
\end{align*}
\]

where $\epsilon = E_2 - E_1$ and $\sigma_{cb} = \sigma_{bc}^\dagger$. The diagonal density-matrix elements $\sigma_{ii}$ are the probabilities of finding the system in one of the states, $|a\rangle$, $|b\rangle$ and $|c\rangle$. In the distinction with the
dephasing rate is proportional to $\Gamma_R$ on reservoirs. (In our case the state $|I\rangle$ between two isolated states, $E_I$ proportional to the half of decay rates of the states $|I\rangle$ generated by $\sigma_I$ system, given by Eq. (2): $S(t \to \infty)$, Eq. (2). Solving Eqs. (5) one obtains [12]

$$I_S^{(0)} = e\frac{\Gamma_R \Omega^2}{\epsilon^2 + \Gamma_R^2/4 + \Omega^2(2 + \Gamma_R/\Gamma_L)}$$ (6)

Note that the dissipation of the “coherences”, $\sigma_{bc}$, is generated by the last term in Eq. (5d), proportional to the half of decay rates of the states $|b\rangle$ and $|c\rangle$ due to their coupling with the reservoirs. (In our case the state $|b\rangle$ cannot decay, but only the state $|c\rangle$, so the corresponding dephasing rate is proportional to $\Gamma_R$). Since the resonant current proceeds via hopping between two dots, generated by $\sigma_{bc}$, it decreases with $\Gamma_R$, Eq. (6).

Now we “switch on”the detector. The available states of the entire system are $|a\rangle$, $|b\rangle$, $|c\rangle$, Fig. 2, and $|a'\rangle$, $|b'\rangle$, $|c'\rangle$, corresponding to empty and occupied detector, respectively. For simplicity we assume that all transition tunneling amplitudes are weakly dependent on energy, so $\Gamma, \gamma, \Omega = \Gamma', \gamma', \Omega'$. The rate equations describing the transport in the entire system are [10]

$$\dot{\sigma}_{aa} = -(\Gamma_L + \gamma_L)\sigma_{aa} + \gamma_R\sigma_{a'a'} + \Gamma_R\sigma_{cc}$$ (7a)
$$\dot{\sigma}_{a'a'} = -(\Gamma_L + \gamma_R)\sigma_{a'a'} + \gamma_L\sigma_{aa} + \Gamma_R\sigma_{c'c'}$$ (7b)
$$\dot{\sigma}_{bb} = \Gamma_L\sigma_{aa} + i\Omega(\sigma_{bc} - \sigma_{cb}) - \gamma_L\sigma_{bb} + \gamma_R\sigma_{b'b'}$$ (7c)
$$\dot{\sigma}_{b'b'} = \Gamma_L\sigma_{a'a'} + i\Omega(\sigma_{b'c' - \sigma_{c'b'}}) + \gamma_L\sigma_{bb} - \gamma_R\sigma_{b'b'}$$ (7d)
$$\dot{\sigma}_{cc} = -\Gamma_R\sigma_{cc} - i\Omega(\sigma_{bc} - \sigma_{cb}) + (\gamma_L + \gamma_R)\sigma_{c'c'}$$ (7e)
$$\dot{\sigma}_{c'c'} = -\Gamma_R\sigma_{c'c'} - i\Omega(\sigma_{b'c' - \sigma_{c'b'}}) - (\gamma_L + \gamma_R)\sigma_{c'c'}$$ (7f)
$$\dot{\sigma}_{bc} = i\epsilon\sigma_{bc} + i\Omega(\sigma_{bb} - \sigma_{cc}) - \frac{1}{2}(\Gamma_R + \gamma_L)\sigma_{bc} + \gamma_R\sigma_{b'b'}$$ (7g)
$$\dot{\sigma}_{b'b'} = i(\epsilon - U_1 + U_2)\sigma_{b'b'} + i\Omega(\sigma_{b'b'} - \sigma_{c'c'}) - \frac{1}{2}(\gamma_L + 2\gamma_R + \Gamma_R)\sigma_{b'b'}.$$ (7h)

Solving these equations one finds the average current in the detector and the coupled-dot system, given by Eq. (2): $I_D^{(1)} = e\gamma_R(\sigma_{a'a'} + \sigma_{b'b'} + \sigma_{c'c'})$, and $I_S^{(1)} = e\Gamma_R(\sigma_{cc} + \sigma_{c'c'})$. 

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Let us take again the limit of the negative result measurement, $\gamma_R \gg \gamma_L$, in which the detector is not expected to affect the measured system. If so, the density-matrix of the entire system, traced over the detector states would coincide with the density-matrix for the double-dot system without detector, Eqs. (5). However, this is not the case. Indeed, by introducing the reduced density matrix of the measured system, $\bar{\sigma}_{ij} = \sigma_{ij} + \sigma_{i'j'}$, one finds from Eqs. (7) that in the above limit of the negative result measurement $\bar{\sigma}_{ij}$ obeys the following equations

\begin{align}
\dot{\bar{\sigma}}_{aa} &= -\Gamma_L \bar{\sigma}_{aa} + \Gamma_R \bar{\sigma}_{cc} \quad (8a) \\
\dot{\bar{\sigma}}_{bb} &= \Gamma_L \bar{\sigma}_{aa} + i\Omega(\bar{\sigma}_{bc} - \bar{\sigma}_{cb}) \quad (8b) \\
\dot{\bar{\sigma}}_{cc} &= -\Gamma_R \bar{\sigma}_{cc} - i\Omega(\bar{\sigma}_{bc} - \bar{\sigma}_{cb}) \quad (8c) \\
\dot{\bar{\sigma}}_{bc} &= i\epsilon \bar{\sigma}_{bc} + i\Omega(\bar{\sigma}_{bb} - \bar{\sigma}_{cc}) - \frac{1}{2}(\Gamma_R + \gamma_L)\bar{\sigma}_{bc}, \quad (8d)
\end{align}

where the resonant current flowing through this system is respectively $I^{(1)}_S = e\Gamma_R \bar{\sigma}_{cc}(t \to \infty)$, Eq. (2). Note that Eqs. (8) obtained for the SET detector in the negative result regime, coincide with the rate equations for the point-contact detector [5], although the both detectors operate in a different way.

Let us compare Eqs. (8) with Eqs. (5). One finds that equations for the diagonal matrix elements are the same. Yet it is not so for the off-diagonal matrix elements. The difference is in the additional dephasing rate, $\gamma_L/2$, generated by the detector. It is easy to trace its origin. In accordance with the Bloch equations the dissipation of the nondiagonal density-matrix elements $\bar{\sigma}_{bc}$ is the half of all possible decay rates of each of the states ($|b\rangle$ and $|c\rangle$). In the presence of the detector, the state $|b\rangle$, Fig. 2, has an additional decay channel, corresponding to the possibility for an electron to enter the detector. Thus, despite of the dwelling time of an electron in the detector tends to zero and therefore the related detector state does not distort the measured system, the possibility for an electron to enter the detector influences the measured current very drastically. Indeed, solving Eqs. (7), (8) one obtains

\begin{align}
\frac{\Delta I_D}{I^{(1)}_S} &= \frac{\gamma_L}{\Gamma_R}, \quad I^{(1)}_S = e\frac{\Gamma_R\Omega^2}{e^2/\eta + \eta\Gamma_R^2/4 + \Omega^2(2 + \Gamma_R/\Gamma_L)} \neq I^{(0)}_S, \quad (9)
\end{align}
where $\eta = 1 + (\gamma_L/\Gamma_R)$. If we compare Eqs. (9) with Eqs. (4) we find that SET can measure the resonant current in a coupled-dot structure precisely in the same way as in the previous case of a single dot. However, the measured system is distorted now. For instance, if $\gamma_L \Gamma_R \gg \Omega^2$ and $\epsilon = 0$, the measured current $I_S^{(1)} \simeq I_S^{(0)}/\eta \ll I_S^{(0)}$.

The additional decoherence rate $\gamma_L$ appears in Eq. (8d) only when the detector can distinguish a particular dot occupied by an electron. Yet, such an “observation” effect disappears if $\tilde{E}_L^F < E_0 + U_1$. In this case an electron cannot enter the detector no matter which of the dots of the measured system is occupied. Then the additional decay channel for the state $|c\rangle$ is blocked and Eq. (8d) coincides with Eq. (5d), i.e. the measured average current remains undistorted, $I_S^{(1)} = I_S^{(0)}$ although the detector still interacts with the measured system ($\Delta I_D \neq 0$). Such a peculiar dependence of the average current $I_S$ on $\tilde{E}_F^L$ is shown in Fig. 3. This “measurement” effect can be observed experimentally by varying the detector voltage, or by moving the resonance level $E_0$.

![Fig. 3: The average current in the double-dot structure with aligned level ($E_1 = E_2$) as a function of the Fermi energy of the left reservoir adjacent to the detector.](image)

In conclusion, by using the quantum rate equations method we demonstrated that SET detector can work in the negative result measurement regime. In this case the SET detector does not distort an observed system, if its motion is determined by classical rate equations, i.e. involving only the diagonal density-matrix elements. Such a situation is realised in the
resonant transport through a single level. However, in the case of coherent transport, as in the resonant tunneling through coupled dots, the negative result measurement always distorts a measured system, providing that the detector can measure the charging of a particular dot. This measurement effect is accounted by the quantum rate equation, which allows us to evaluate the corresponding decoherence rate. However, if the detector cannot distinguish the charging of a particular dot it becomes a non-distractive again.

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