Azimuthal asymmetries from $\theta$ Vacuum

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We present the complete azimuthal asymmetries at leading twist in terms of fragmentation functions in di-hadron production semi-inclusive electron positron annihilation process. When the non-trivial $\theta$ vacuum is taken into consideration, the parity symmetry of quantum chromodynamics is violated. As a consequence of the local $\mathcal{P}$-odd effects, $\mathcal{P}$-odd fragmentation functions would contribute to the azimuthal asymmetries. Azimuthal asymmetry coming from two interference terms with opposite signs vanishes when sum over many events. This symmetry only survives on the event-by-event basis. Azimuthal asymmetry coming from two interference terms with same signs survives and can be measured to extract the $\mathcal{P}$-odd fragmentation functions. We also present the hadron polarizations.

I. INTRODUCTION

Parton distribution functions (PDFs) and fragmentation functions (FFs) are two important quantities in describing the high energy reactions. Both of them are long distance non-perturbative quantities which cannot be calculated with perturbative theory, they are mainly calculated by phenomenological models and parametrized by experiment data [1–15]. When transverse momentum dependent (TMD) PDFs and FFs are considered, the sensitive quantities studied in experiments are often different azimuthal asymmetries. In the quantum field theoretical formulation, PDFs and/or FFs are given by the quark-quark correlators (correlation functions) that are defined as $4 \times 4$ matrices in Dirac space depending on the hadron state. Thus, the correlators can be written as the products of the Dirac matrices and coefficients. The corresponding coefficients can be further decomposed by the Lorentz covariants and scalar functions which are known as PDFs and FFs.

The quark-quark correlator satisfies two constraints, Hermiticity and parity conservation. Hermiticity ensures that all the scalar functions are real while parity conservation strictly limits the numbers of scalar functions. Though parity violating effect is not expected in perturbative quantum chromodynamics (QCD), the non-trivial $\theta$ vacuum leads to the constraint breakdown [16–19]. The $\theta$ term enters in the lagrangian as,

$$L_\theta = \frac{\theta}{16\pi^2} \text{Tr} \left[ F_{\mu\nu} F^{\mu\nu} \right],$$

where $F_{\mu\nu}$ is the gluon field strength tensor and $F^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2$. This $\theta$ term in lagrangian violates the parity ($\mathcal{P}$) and/or charge-parity ($\mathcal{CP}$) symmetries. Even though the measurements of electric dipole moment of neutron indicate that the parity ($\mathcal{P}$) violation is local [20], Kharzeev, Pisarski and Tytgat have shown it can be directly observed [21]. In heavy ion collisions, the $\theta$ term also leads to the famous chiral magnetic effect [22–24]. Efremov, Kharzeev and Kang also discussed the local $\mathcal{P}$-odd effect in fragmentation process. In this case, the $\mathcal{P}$-odd FFs emerge and they argued that the effect can be detected in experiments via physical observables, e.g., handedness correlation and azimuthal asymmetries [25, 26].

Electron positron annihilation process is regarded as the cleanest place to study the FFs, because there are no hadronic effects in initial states. As mentioned before the TMD FFs can be extracted from the measurement of azimuthal symmetries. In ref. [26], Kang and Kharzeev derived the general form of the FF for quarks fragmenting into spin-0 hadrons (pseudoscalars) at leading twist. Without the parity constraint they found two more $\mathcal{P}$-odd FFs which are related to two azimuthal asymmetries, i.e., $\cos 2\theta$ and $\sin 2\theta$. They presented a estimation of the magnitude of the $\sin 2\theta$ asymmetry and found $I(\tilde{\theta}, z_1, z_2) \sim 1.5\%$. This is a significant asymmetry and can be measured in experiment. In this paper we extend the calculation to spin-1/2 hadron production. We calculate the unpolarized azimuthal asymmetries, single-spin asymmetries and double spin asymmetries, respectively. We also calculate the hadron polarizations.

This paper is organized as follows. In Sect. II we present the kinematics of the semi-inclusive electron positron annihilation (SIA) process and the hadronic tensor. The calculations and results of azimuthal asymmetries and hadron polarizations are shown in Sect. III. A brief summary will be given in Sect. IV.

II. KINEMATICS AND HADRONIC TENSOR

First of all we present the kinematics of the SIA process in which two hadron are produced in two back-to-back jets, see Fig. 1. In this paper we only consider the electromagnetic interaction for distinguishing the $\mathcal{P}$-odd effect in QCD from that in weak interaction theory. The differential cross section of SIA process can be written as a contraction of the leptonic tensor and hadronic tensor, i.e.,

$$\frac{2E_1 E_2 d\sigma}{d^3 p_1 d^3 p_2} = \frac{N_c \alpha_{em}^2 e_q^2}{s Q^4} L_{\mu\nu}(l_1, l_2) W^{\mu\nu}(p_1, p_2),$$

where $N_c$ denotes the color factor, $\alpha_{em}$ is the fine structure constant, $e_q$ is the electric charge of quark $q$ and $s = Q^2 = q^2 = (l_1 + l_2)^2$. The leptonic tensor is given by

$$L_{\mu\nu}(l_1, l_2) = l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - g_{\mu\nu} (l_1 \cdot l_2).$$
We choose the lepton center-of-mass frame where $p_2$ lies in the positive $z$ direction, see Fig. 2. In this frame we use the conventions and notation given in ref. [27]. On the perpendicular basis the hadronic tensor can be parameterized as,

$$L_{\mu\nu} = Q^2 \left[ -A(y)g_\perp^{\mu\nu} + 2B(y)\xi'^{\mu}x'^{\nu} - 2B(y)(\xi'^{\mu}x^{\nu} + \frac{1}{2}g_\perp^{\mu\nu}) - C(y)D(y)\xi^{\mu}x^{\nu} \right]. \quad (2.3)$$

where $\hat{s}_\perp = \hat{p}_\perp/D(y)Q$, $\hat{s}_\perp^{\mu} \equiv \frac{2\hat{p}_\perp^\mu}{\hat{Q}} - \frac{\hat{q}^\mu}{\hat{Q}}$, $A(y) = \frac{1}{2} - y + \sqrt{y(1-y)}$, $B(y) = y(1-y), C(y) = 1 - 2y, D(y) = \sqrt{y(1-y)}g_\perp^{\mu\nu} \equiv g_\perp^{\mu\nu} - \hat{p}_\perp^{\mu}\hat{p}_\perp^{\nu}$, where $\hat{p}_\perp \equiv \frac{\hat{q}}{\hat{Q}}$. Equation. (2.3) can be used to take the cross section by contracting with the following hadronic tensor:

$$W^{\mu\nu}(p_1, p_2) = \Delta \sum_X \langle 0|J^{\mu}(0)p_1, p_2; X\rangle\langle p_1, p_2; X|J^{\nu}(0)0\rangle \tag{2.4}$$

where $\Delta = \delta(y - p_1 - p_2 - X)$. Besides the Lorentz covariance, the hadronic tensor $W^{\mu\nu}$ satisfies the general constraints imposed by hermiticity, current conservation, and parity conservation in the electromagnetic process, i.e.,

$$W^{\mu\nu}(p_1, S_1; p_2, S_2) = W^{\mu\nu}(p_1, S_1; p_2, S_2), \tag{2.5}$$

$$q_\mu W^{\mu\nu}(p_1, S_1; p_2, S_2) = q_\nu W^{\mu\nu}(p_1, S_1; p_2, S_2) = 0, \tag{2.6}$$

$$W^{\mu\nu}(p_1, S_1; p_2, S_2) = W_{\mu\nu}(\hat{p}_\perp^\mu, \hat{p}_\perp^\nu; \hat{p}_\perp^2, \hat{p}_\perp^2), \tag{2.7}$$

where a vector with the superscript $\mathcal{P}$ denotes the result after space reflection such as $\hat{p}_\perp^\mu = p_\perp^\mu$. The hadronic tensor can not be calculated perturbatively because it contains the nonperturbative hadronization process. To obtain the cross section, we can decompose the hadronic tensor with the basic Lorentz tensors as shown in ref. [28, 29]. We do not repeat the decomposition in this paper. Though hadronic tensor can not be calculated in perturative theory, it can be calculated in the parton model [30–32]. In QCD parton model, at leading twist the hadronic tensor can be rewritten as,

$$W^{\mu\nu}(p_1, p_2) = \int d^4k T d^2k T d^2k T \delta^2(q T - \hat{k}_1 T - \hat{k}_2 T) \times \text{Tr}\left(\hat{\Lambda}(p_2, k_2 T)\gamma^\mu\Delta(p_1, k_1 T)\gamma^\nu\right), \tag{2.8}$$

where $\Delta(z_1, k_1 T)$ and $\hat{\Delta}(z_2, k_2 T)$ are correlation functions. They are not gauge invariant. The physical gauge invariant correlation functions must include the gauge link, $\mathcal{L}(0, \infty) = \mathcal{P}\text{exp}\left[i\int_0^{\infty} dz\Delta A(y + z)\right]$. In this case, the gauge invariant correlation functions can be rewritten as,

$$\hat{E}(p_1, k_1 T) = \frac{1}{2\pi} \int d^2\xi T d^2\xi T e^{-i\xi T}\sum_X \langle 0|\psi(0)\mathcal{L}(0, \infty) \times |p_1, X\rangle\langle p_1, X|\psi(\xi T)\mathcal{L}(\xi T, \infty)0\rangle_{\xi T=0}, \tag{2.9}$$

$$\hat{E}(p_2, k_2 T) = \frac{1}{2\pi} \int d^2\xi T d^2\xi T e^{-i\xi T}\sum_X \langle 0|\psi(0)\mathcal{L}(0, \infty) \times |p_2, X\rangle\langle p_2, X|\psi(\xi T)\mathcal{L}(\xi T, \infty)0\rangle_{\xi T=0}. \tag{2.10}$$

The semi-inclusive electron positron annihilation process. The momenta of particles are shown in the parentheses.
\[\varepsilon_{\alpha}^{P} = \hat{\eta}_{\alpha} [\frac{k_{T}}{M} H_{i}^{l} + \varepsilon_{T S_{l}} H_{i T} + \varepsilon_{T h_{0}} \Gamma_{l}^{1}], \quad (2.17)\]

where the superscript \( P \) denote the parity violating and \( H_{i}^{l} = \lambda H_{i}^{l} + \varepsilon_{S_{l}} H_{i T} \). Among the total 16 scalar and pseudoscalar functions obtained in Eqs. (2.12)-(2.17), 8 of them are parity conserved and the others are parity violated. They are known as fragment functions. For every parity conserved FF there is a parity violated one corresponding to it, e.g., \( D_{1} \) and \( D_{1} \) have the same forms and \( G_{1L} \) and \( G_{1L} \) have the same forms. The notation used here takes the following convention: \( D(D), G(G) \) and \( H(H) \) are FFs which represent the unpolarized, longitudinally polarized and transversely polarized quarks fragmenting into hadrons, respectively. We will also use different symbols, \( L \) and \( T \), in the subscripts to denote the polarization of the produced hadron in the polarization case. A \( \perp \) in the superscript denotes that the corresponding basic Lorentz covariant is \( k_{T} \)-dependent [33].

If we decompose the quark field in Eqs. (2.9)-(2.10) into the sum of the right-and left-handed parts, i.e., \( \psi = \psi_{R} + \psi_{L} \) with \( \psi_{R/L} = \frac{1}{2}(1 \pm \gamma_{5}) \psi \). It can be seen that for \( \Gamma = I \), \( I \gamma_{5} \) and \( i\sigma_{\alpha\beta} \gamma_{5} \), \( \hat{\psi}_{R} \gamma_{5} \psi_{L} \) and \( \hat{\psi}_{R} \gamma_{5} \psi_{R} \) are non-zero. Therefore, the terms related to them (i.e., the \( H \)'s) stand for helicity-flipped quark structure and are called chiral-odd (\( \chi \)-odd). Similarly, for \( \Gamma = \gamma^{5} \) and \( \gamma^{5}\gamma^{5} \), \( \hat{\psi}_{R} \gamma_{5} \psi_{L} \) and \( \hat{\psi}_{R} \gamma_{5} \psi_{R} \) are non-zero. Therefore, the terms related to them (i.e. the \( D \)'s and the \( G \)'s) do not flip the quark helicity and are \( \chi \)-even. We also recall the properties of the fermion bilinears under time-reversal \( \tilde{T} \), i.e.,

\[\tilde{T} \{ \hat{\psi}_{R} \gamma_{5} \psi_{R}, \hat{\psi}_{R} \gamma_{a} \psi_{L}, \hat{\psi}_{R} \gamma_{5} \gamma_{a} \psi_{L}, \hat{\psi}_{R} \gamma_{5} \gamma_{a} \gamma_{5} \psi_{L} \} \Rightarrow \{ \hat{\psi}_{R}, -\hat{\psi}_{R} \gamma_{5} \psi_{R}, \hat{\psi}_{R} \gamma_{5} \psi_{L}, \hat{\psi}_{R} \gamma_{5} \gamma_{5} \psi_{L} \}. \quad (2.18)\]

Applying the time-reversal behavior of the corresponding basic Lorentz covariants with Eq. (2.18), we can determine whether a FF defined via quark-quark correlators given by Eqs. (2.12)-(2.17) is time reversal even (T-even) or odd (T-odd). In this way, we find out that \( D_{1T}, G_{1T}, H_{1T}, H_{1T}^{*}, H_{1T}^{*} \) and \( H_{1T}^{*} \) are T-odd while all the others are T-even. We also note that they are usually referred as “naive T-odd” or “naive T-even” because the interactions between the produced hadron and the rest debris from the collision can destroy simple regularities and all of them can exist in a practical hadronization process.

With the complete decompositions of the correlation functions we can calculate the hadronic tensor. By substituting Eqs. (2.12)-(2.17) into Eq. (2.8), we obtain the complete hadronic tensor in SIA process at leading twist:

\[ W_{\mu \nu} = \frac{4}{\pi i z_{1}, 2} \int d^{2} k_{1 T} d^{2} k_{2 T} \left( \frac{2\pi^{2}}{2\pi^{2}} \right) \delta^{2}(\vec{q}_{T} - k_{1 T} - k_{2 T}) \times \left\{ d_{T}^{\mu \nu}[\left[ D_{1} + \lambda_{1} G_{1L} + \frac{\varepsilon_{T S_{1}}}{M} D_{1 T} + \frac{1}{M} \left( \frac{k_{1 T}}{M} S_{1} \right) D_{1 T}] - \left[ D_{1} + \lambda_{1} G_{1L} + \frac{\varepsilon_{T S_{1}}}{M} G_{1} + \frac{1}{M} \left( \frac{k_{1 T}}{M} S_{1} \right) G_{1 T}] \right] - \frac{d_{T}^{\mu \nu}}{M_{1} M_{2}} \left( H_{1}^{T} + H_{15}^{T} \right) \left( H_{1}^{T} + H_{15}^{T} \right) - \frac{d_{T}^{\mu \nu}}{M_{1} M_{2}} \left( H_{1}^{T} + H_{15}^{T} \right) \left( H_{1}^{T} + H_{15}^{T} \right) \right\} \]

(2.19)
know that the hadronic tensor here is given in different frame as used in Eq. (2.3), see ref. [27]. Equation (2.19) is the complete hadronic tensor which is given in terms of both the \( \mathcal{P} \)-even and \( \mathcal{P} \)-odd FFs in SIA process. It can be easily checked that the hadronic tensor given in Eq. (2.19) satisfies the current conservation constraint, i.e., \( q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0 \). Substituting Eq. (2.19) and Eq. (2.3) into Eq. (2.1), we can obtain the complete cross section in SIA process at leading twist.

III. AZIMUTHAL ASYMMETRIES AND HADRON POLARIZATIONS

In the previous section, we show the complete hadronic tensor at leading twist in SIA process in terms of \( \mathcal{P} \)-even and \( \mathcal{P} \)-odd FFs. In this section, we present the results in three parts based on the polarization of the produced dihadron, i.e., unpolarized part, single hadron polarized part and double hadron polarized part, respectively.

A. Unpolarized cross section and asymmetries

First of all we consider the unpolarized case where the hadrons produced in final state are spin-0 particles. The differential cross section is given by,

\[
\frac{2E_1E_2d\sigma}{d^3p_1d^3p_2} = \frac{8N_c\alpha_s^2e^2}{Q^4z_1z_2} \left[ A(y) \left( C[D_1\bar{D}_1] - C[D_1\bar{D}_1] \right) - B(y) \cos 2\phi C \left[ w_{12}(H_1^+\bar{H}_1^+ + \mathcal{H}_1^+\bar{\mathcal{H}}_1^+) \right] \right. \\
\left. - B(y) \sin 2\phi C \left[ w_{12}(H_1^+\bar{H}_1^+ - \mathcal{H}_1^+\bar{\mathcal{H}}_1^+) \right] \right],
\]

(3.1)

where \( \phi \) denotes the angle between the lepton-hadron plane and hadron-hadron plane, see Fig. 2. The notation \( C[D_1\bar{D}_1] \) represents a convolution,

\[
C[D_1\bar{D}_1] = \int \frac{d^3k_{1T}d^2k_{2T}}{(2\pi)^3} \delta^2(\vec{q}_T - \vec{k}_{1T} - \vec{k}_{2T}) D_1D_1.
\]

(3.2)

The weight function \( w_{12} = (\hat{h} \cdot \hat{k}_{1T} \bar{h} \cdot \hat{k}_{2T} - \hat{k}_{1T} \cdot \hat{k}_{2T})/M_1M_2 \). In Eq. (3.1), the \( \cos 2\phi \) term corresponds to the Collins effect which is a reflection of the Collins function \( H_1^+(\bar{H}_1^+) \) [34]. Apart from the Collins function, there is also a contribution of \( \mathcal{P} \)-odd Collins-type function \( \mathcal{H}_1^+(\bar{\mathcal{H}}_1^+) \). This azimuthal asymmetry be written as,

\[
A_{UU}^{\cos 2\phi} = \frac{B(y) C[w_{12}(H_1^+\bar{H}_1^+ + \mathcal{H}_1^+\bar{\mathcal{H}}_1^+)]}{A(y) \left( C[D_1\bar{D}_1] - C[D_1\bar{D}_1] \right)},
\]

(3.3)

where the subscript \( UU \) denotes unpolarization of hadron 1 and hadron 2, respectively. The following weighted functions are often used in the measurements of FFs,

\[
D_1(z) = \int \frac{d^3k_T}{(2\pi)^3} D_1(z,k_T),
\]

(3.4)

\[
H_1^+(z) = \int \frac{d^3k_T}{(2\pi)^3} \frac{|k_T|}{M} H_1^+(z,k_T).
\]

(3.5)

Then we have

\[
A_{UU}^{\sin 2\phi} = \frac{B(y) [H_1^+\bar{H}_1^+ + \mathcal{H}_1^+\bar{\mathcal{H}}_1^+]}{A(y) \left( D_1\bar{D}_1 - D_1\bar{D}_1 \right)}.
\]

(3.6)

We notice that the two terms in the numerator have the same signs, which means both the Collins function and Collins-type function contribute to the \( \cos 2\phi \) azimuthal asymmetry. In other words, \( \cos 2\phi \) have alternative origin except for the Collins function as indicated in ref. [26]. The \( \mathcal{P} \)-odd FFs that company with the Collins function make it difficult to extract the Collins effect from measurements experimentally.

Apart from the \( \cos 2\phi \) azimuthal asymmetry, we also obtain another azimuthal asymmetry at leading twist, i.e., \( \sin 2\phi \) asymmetry.

\[
A_{UU}^{\sin 2\phi} = \frac{B(y) [H_1^+\bar{H}_1^+ - \mathcal{H}_1^+\bar{\mathcal{H}}_1^+]}{A(y) \left( D_1\bar{D}_1 - D_1\bar{D}_1 \right)}.
\]

(3.7)

\( A_{UU}^{\sin 2\phi} \) vanishes if only \( \mathcal{P} \)-even FFs are taken into consideration. The first term in the numerator stands for the interference between Collins function \( H_1^+ \) and \( \mathcal{P} \)-odd (anti) Collins-type FF \( \mathcal{H}_1^+ \) while the second terms stands for the interference between (anti) Collins function \( \mathcal{H}_1^+ \) and \( \mathcal{P} \)-odd Collins-type FF \( \mathcal{H}_1^+ \). We also notice that the signs before these two terms are opposite, which means that the \( \sin 2\phi \) asymmetry vanishes when sum over many events. Thus, this \( \mathcal{P} \)-odd effect only survives on the event-by-event basis.

B. Single hadron polarized cross section and asymmetries

Single spin asymmetries are very good candidates to study the PDFs and/or FFs. To calculate the single spin asymmetries, in this section we assume that hadron 1 is spin-\( 1/2 \) particle while hadron 2 is spin-0 particle. The differential cross section is given by,

\[
\frac{2E_1E_2d\sigma}{d^3p_1d^3p_2} = \frac{8N_c\alpha_s^2e^2}{Q^4z_1z_2} \left[ A(y) \left( C[D_1\bar{D}_1] - C[D_1\bar{D}_1] \right) \right. \\
\left. + A(y) S_{1T} \left[ \cos(\phi - \phi_S) C[w_1(D_1\bar{D}_1 - G_{1T}\bar{D}_1)] \right. \\
\left. + \sin(\phi - \phi_S) C[w_1(G_{1T}\bar{D}_1 - D_{1T}\bar{D}_1)] \right] \right. \\
\left. - B(y) S_{1T} \left[ \cos(\phi + \phi_S) C[w_2(H_1\bar{H}_1 + \mathcal{H}_1\bar{\mathcal{H}}_1)] \right. \\
\left. + \sin(\phi + \phi_S) C[w_2(H_1\bar{H}_1 - \mathcal{H}_1\bar{\mathcal{H}}_1)] \right] \right],
\]

(3.8)

where the weighted functions \( w_1 = \hat{h} \cdot \hat{k}_{1T}/M_1, w_2 = \hat{h} \cdot \hat{k}_{2T}/M_2, \phi_S \) denotes the azimuthal angle of hadron 1 polarization, which is the angle between the polarization vector \( S_1 \) and the lepton-hadron plane. For \( \phi_S \), we have the similar definition in the following context. For simplicity, we only present terms which are relevant to our discussions in Eq. (3.8).
transverse polarization parts vanish if the transverse momentum is integrated. For the unintegrated case, we first consider the following two asymmetries,

\[
A_{TU}^{\sin(\phi_{D_1})} = \frac{C[w_1(G_{T_1}^{\perp}D_1) - D_1^{\perp}T_{D_1})]}{C[D_1^{\perp}D_1] - C[D_{T_1}^{\perp}D_1]},
\]

(3.9)

\[
A_{TU}^{\cos(\phi_{D_1})} = \frac{C[w_1(G_{T_1}^{\perp}D_1) - D_1^{\perp}T_{D_1})]}{C[D_1^{\perp}D_1] - C[D_{T_1}^{\perp}D_1]},
\]

(3.10)

From Eq. (3.9) it can be seen that \(A_{TU}^{\sin(\phi_{D_1})}\) corresponds to the Sivers effect \((D_{T_1}^{\perp})\) in fragmentation process. \(D_1^{\perp}\) is the transverse momentum dependent FF which describes the unpolarized quark into transversely polarized hadron. It is not clear if the \(P\)-odd term, \(\vec{G}_{T_1}^{\perp}D_1\), reduces the contribution of \(D_1^{\perp}T_{D_1}\). It depends on the absolute values of these terms. If the absolute values of these two terms have opposite signs, the asymmetry shall be enhanced by the \(P\)-odd term. If the absolute values of these two terms have same signs, the asymmetry shall be reduced by the \(P\)-odd term. For the other azimuthal asymmetry, \(A_{TU}^{\cos(\phi_{D_1})}\) which comes from the interferences of the \(P\)-odd FF \((\vec{D}_1)\) and \(P\)-even FF \((\vec{G}_{T_1})\) only survives on the even-by-even basis as we have discussed for Eq. (3.6).

Besides, we calculate two azimuthal asymmetries which are generated from the chiral-odd FFs. They are given by,

\[
A_{TU}^{\sin(\phi_{D_1})} = \frac{C[w_2(H_{T_1}^{\perp}H_1^{\perp} - H_1^{\perp}\bar{H}_1^{\perp})]}{C[D_1^{\perp}D_1] - C[D_{T_1}^{\perp}D_1]},
\]

(3.11)

\[
A_{TU}^{\cos(\phi_{D_1})} = \frac{C[w_2(H_{T_1}^{\perp}H_1^{\perp} + H_1^{\perp}\bar{H}_1^{\perp})]}{C[D_1^{\perp}D_1] - C[D_{T_1}^{\perp}D_1]},
\]

(3.12)

We can see these two asymmetries are related to the transverse polarization transformation function \(H_{T_1}\) and Collins function \(H_1^{\perp}\). It can be seen that \(A_{TU}^{\cos(\phi_{D_1})}\) vanishes if parity constraint is imposed because it comes from the interference between \(P\)-even and \(P\)-odd FFs. However, these two terms in the numerator in Eq. (3.12) survive when the \textit{local} parity violating effect is considered. We also note that these two terms have the same signs, which means \(A_{TU}^{\cos(\phi_{D_1})}\) does not vanish when sum over many events. Thus, \(A_{TU}^{\cos(\phi_{D_1})}\) is important for determining the \(P\)-odd FFs by measuring the azimuthal asymmetry at leading twist.

C. Double hadron polarized cross section and asymmetries

For the double hadron polarized case, we consider two spin-\(1/2\) hadrons in the final states. There are 54 terms in the cross section and we do not present them in this paper. For simplicity, we only consider the integrated differential cross section:

\[
\frac{d\sigma}{dz_1dz_2dy} = \frac{4\pi N_c^2\alpha^2_{em}}{Q^2} [A(y)(D_1\bar{D}_1 - D_1\bar{D}_1)]

\quad - B(y)|S_{T_1}|[\cos(\phi_{S_1} + \phi_{S_2})(H_{T_1}\bar{H}_{1T} + H_{T_1}\bar{H}_{1T})

\quad - |\sin(\phi_{S_1} + \phi_{S_2})(H_{T_1}\bar{H}_{1T} - H_{T_1}\bar{H}_{1T})].
\]

(3.13)

To obtain Eq. (3.13), we have used

\[
d^3p_1d^3p_2/E_1E_2 = (dz_1(z_2Q^2dz_2/4\pi)p_{1L}d\Omega_2

\quad = \pi Q^2z_1z_2dz_1dz_2d\Omega_2,\quad (3.14)
\]

There are two azimuthal asymmetries which correspond to the double spin asymmetries,

\[
\tilde{A}_{UU}^{\cos(\phi_{D_1} + \phi_{D_2})} = \frac{\bar{B}(y)H_{T_1}\bar{H}_{1T} + H_{T_1}\bar{H}_{1T}}{A(y)(D_1\bar{D}_1 - D_1\bar{D}_1)},
\]

(3.15)

\[
\tilde{A}_{UU}^{\sin(\phi_{D_1} + \phi_{D_2})} = \frac{\bar{B}(y)H_{T_1}\bar{H}_{1T} - H_{T_1}\bar{H}_{1T}}{A(y)(D_1\bar{D}_1 - D_1\bar{D}_1)}.
\]

(3.16)

For \(A_{UU}^{\cos(\phi_{D_1} + \phi_{D_2})}\), it can be seen that both \(H_{T_1}\bar{H}_{1T}\) and \(H_{T_1}\bar{H}_{1T}\) contribute to this asymmetry which means \(A_{UU}^{\cos(\phi_{D_1} + \phi_{D_2})}\) has two origins. In this case the \(P\)-odd FFs complicate the extraction of the transverse polarization transformation function \(H_1\). This is the same to Collins asymmetry, \(A_{UU}^{2\phi}\). The asymmetry \(A_{UU}^{\sin(\phi_{D_1} + \phi_{D_2})}\), similar to \(A_{UU}^{2\phi}\), arises from the two interference terms with opposite signs. So it can only survive on event-by-event basis.

So far we calculate eight kinds of azimuthal asymmetries and divide them into four parts.

- \(A_{UU}^{\cos(\phi_{D_1})}\) and \(A_{TU}^{\cos(\phi_{D_1})}\) In this part, both the \(P\)-odd FFs and \(P\)-even FFs have positive contributions which means these azimuthal asymmetry has two origins.

- \(A_{UU}^{\sin(\phi_{D_1})}\) and \(A_{TU}^{\cos(\phi_{D_1})}\) In this part, asymmetry comes from two interferences between the \(P\)-odd FFs and \(P\)-even FFs. These two terms have opposite signs. Therefore, these azimuthal asymmetries only survive on event-by-event basis.

- \(A_{TU}^{\cos(\phi_{D_1})}\) and \(A_{TU}^{\sin(\phi_{D_1})}\). Asymmetry arises from both the \(P\)-odd FFs and \(P\)-even FFs. The \(P\)-odd term and the \(P\)-even term have opposite signs.

- \(A_{TU}^{\cos(\phi_{D_1})}\). Two interference terms contributing to azimuthal asymmetry \(A_{TU}^{\cos(\phi_{D_1})}\) have same signs and they can survive when sum over many events. This is important for determining the \(P\)-odd FFs by measuring the azimuthal asymmetry at leading twist.

D. Hadron polarizations

In addition to the azimuthal asymmetries discussed in the previous context, hadron polarizations also can be measured in SIA experiments [35]. In this subsection we assume that hadron 1 is a spin-\(1/2\) particle while hadron 2 is a spin-0 particle. In the following we take \(\langle S_T^2\rangle\) for example to illustrate calculations.

Hadron polarization \(\langle S_T^2\rangle\) denotes the probability of the hadrons on the spin \(S_T^2\) state. For a system of hadrons, we


\[
\begin{array}{c|cccccc}
\lambda & |\psi_{z+}\rangle & |\psi_{z-}\rangle & |\psi_{y+}\rangle & |\psi_{y-}\rangle & |\psi_{x+}\rangle & |\psi_{x-}\rangle \\
\hline
S_T^z & 1 & -1 & 0 & 0 & 0 & 0 \\
S_T^y & 0 & 0 & 1 & -1 & 0 & 0 \\
S_T^x & 0 & 0 & 0 & 0 & 1 & -1 \\
\end{array}
\]

TABLE I: Eigenvalues of the system on eigenstates of \(\lambda, S_T^z, S_T^y\).

The polarization of a system can be calculated with the following equation,

\[
O = \langle O \rangle = \text{Tr}[\rho O],
\]

where \(O\) denotes any spin operator. Using Eqs. (3.17)-(3.19), we obtain

\[
\langle S_T^z \rangle = P(1, \frac{\pi}{2}, 0) - P(-1, \frac{\pi}{2}, 0),
\]

\[
\langle S_T^y \rangle = P(1, 0, 0) - P(-1, 0, 0),
\]

\[
\langle S_T^x \rangle = P(1, \frac{\pi}{2}, \frac{\pi}{2}) - P(-1, \frac{\pi}{2}, \frac{\pi}{2}).
\]

For the eigenvalues, we need to solve the Schrodinger equation to obtain them. They are given in Table I. Substituting the eigenvalues into cross section and using Eqs. (3.20)-(3.22), we can obtain the hadron polarizations.

First of all we consider the longitudinal polarization. By applying Eq. (3.21) and eigenvalues shown in Table I, we obtain the longitudinal polarization of hadron 1,

\[
\langle \lambda_1 \rangle = \frac{C[G_{1L} D_1] - C[G_{1L} D_1]}{C[D_1 D_1] - C[D_1 D_1]}. \tag{3.23}
\]

It can be seen from Eq. (3.23) that the longitudinal polarization arises form the interference between the \(P\)-odd FF(s) \(G_{1L}\) (\(D_1\)) and the \(P\)-even FF(s) \(D_1\) (\(G_{1L}\)). It vanishes if only the \(P\)-even FFs are considered. The signs of these two terms in the numerator are opposite which means that \(\langle \lambda_1 \rangle\) only survives on event-by-event basis because the averaging cross section of many event vanishes.

In Fig. 2, we defined the lepton-hadron plane which is determined by momenta \(l_1, l_2\) and \(p_2\). \(p_2\) lies in the z-direction while the x-direction is determined by the transverse momenta of leptons. In this frame the transverse polarization is defined with respect to the lepton-hadron plane. Chen et al. argued in ref. [29, 36], the transverse hadron polarization \(\langle S_T^y \rangle\) corresponds to \(|S_T| \cos \phi_1\), while \(\langle S_T^x \rangle\) corresponds to \(|S_T| \sin \phi_3\).

They vanish at leading twist but survive at twist-3 level. However, Chen et al. also discussed the transverse polarizations with respect to hadron-hadron plane which is defined by the momenta of the two hadrons as shown in Fig. 2. In this case we use unit vectors, \(\vec{e}_n\) and \(\vec{e}_t\), to define the transverse directions. Here \(\vec{e}_n = \vec{p}_1 \times \vec{p}_2|/|\vec{p}_1 \times \vec{p}_2| = (-\sin \phi, \cos \phi, 0)\) and \(\vec{e}_t = \vec{p}_{1T} / |\vec{p}_{1T}| = (\cos \phi, \sin \phi, 0)\), i.e., the normal and tangent of the hadron-hadron plane, respectively. Thus, the transverse polarization \(\langle S_T^y \rangle\) corresponds to \(|S_T| \sin (\phi_2 - \phi)\) while \(\langle S_T^x \rangle\) corresponds to \(|S_T| \cos (\phi_2 - \phi)\). Therefore, we obtain the transverse polarizations with respect to hadron-hadron plane,

\[
\langle S_T^y \rangle = -\frac{C'[w_1(G_{1T}^2 D_1 - D_{1T}^2 D_1)]}{C[D_1 D_1] - C[D_1 D_1]}, \tag{3.24}
\]

\[
\langle S_T^x \rangle = \frac{C'[w_1(G_{1T}^2 D_1 - G_{1T}^2 D_1)]}{C[D_1 D_1] - C[D_1 D_1]}. \tag{3.25}
\]

For \(\langle S_T^y \rangle\), it reflects the effect of Sivers-type FF \(D_{1T}^⊥\) and can be measured at leading twist level. The transverse polarization \(\langle S_T^x \rangle\) comes from the interference of the \(P\)-odd FF(s) and \(P\)-even FF(s) and it only survives on event-by-event basis.

**IV. SUMMARY**

When the non-trivial \(\theta\) vacuum is taken into consideration, parity symmetry can be violated in QCD. The \(\theta\) term can lead to the physical observables in fragmentation process, e.g., handedness correlation and azimuthal asymmetries. In this paper, we perform the azimuthal asymmetries and hadron polarizations calculations with both the \(P\)-odd and the \(P\)-even FFs. By measuring the asymmetries both the \(\theta\) vacuum and \(P\)-odd FFs can be determined for azimuthal asymmetries being sensitive quantities. We calculate eight azimuthal asymmetries, \(A_{UU}^{\cos 2\phi}\) and \(A_{TT}^{\cos \phi}A_{TT}^{\sin \phi}\), both the \(P\)-odd FFs and \(P\)-even FFs have positive contributions and can be studied simultaneously. For \(A_{UU}^{\cos 2\phi}, A_{TU}^{\cos \phi}\) and \(A_{TU}^{\sin \phi}\), each of these asymmetries comes from two interferences between the \(P\)-odd FFs and \(P\)-even FFs. These two terms have opposite signs. Therefore, these azimuthal asymmetries only survive on event-by-event basis. For \(A_{TU}^{\sin 2\phi} A_{TU}^{\cos \phi} A_{TU}^{\sin \phi}\), each of these asymmetries arises from both the \(P\)-odd FFs and \(P\)-even FFs. The \(P\)-odd term and the \(P\)-even term have opposite signs. Those two interference terms contributing to \(A_{TU}^{\sin 2\phi}\) have same signs and \(A_{TU}^{\cos \phi}\) can survive when sum over many events. We also calculate the hadron polarizations. We find that the longitudinal polarization in SIA process arising form the interference between the \(P\)-odd FF(s) \(G_{1L}\) (\(D_1\)) and the \(P\)-even FF(s) \(D_1\) (\(G_{1L}\)). For \(\langle S_T^y \rangle\), it reflects the effect of Sivers-type FF \(D_{1T}^⊥\) and \(P\)-odd FF \(G_{1T}^T\). The transverse polar-
ization $\langle S_1^t \rangle$ comes from the interference of the $P$-odd FF(s) and $P$-even FF(s) and it only survives on event-by-event basis.

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