Analysis of the vector hidden-charm tetraquark states without explicit P-waves via the QCD sum rules

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Abstract

In the present work, we adopt the scalar, pseudoscalar, vector, axialvector and tensor (anti)diquark operators as the elementary building blocks to construct vector and tensor local four-quark currents without introducing explicit P-waves, and explore the mass spectrum of the vector hidden-charm tetraquark states via the QCD sum rules comprehensively, and revisit the interpretations of the existing Y states in the scenario of vector tetraquark states. We resort to the energy scale formula to enhance the pole contributions and improve the convergent behaviors of the operator product expansion, and we should bear in mind that the predictions are rather sensitive to the particular energy scales which obey the uniform/same constraint. The predicted vector hidden-charm tetraquark states can be confronted to the experimental data in the future.

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Key words: Tetraquark state, QCD sum rules

1 Introduction

In recent years, several vector charmonium-like states have been observed, such as the Y(4008), Y(4160), Y(4220), Y(4260), Y(4320), Y(4360), Y(4390), Y(4630), Y(4660), etc, they cannot be accommodated suitably in the conventional two quark model, we have to introduce additional quark or gluon degrees of freedom to explore their properties. For example, the Y(4260) observed by the BaBar collaboration, the Y(4220), Y(4390) and Y(4320) observed by the BESIII collaboration, and the Y(4360), Y(4660), Y(4630) observed by the Belle collaboration are excellent candidates for the vector tetraquark states. Considering the analogous masses and decay widths, we can take the Y(4260), Y(4220) and Y(4230) as the same meson, the Y(4360) and Y(4320) as the same meson, and the Y(4660) and Y(4630) as the same meson.

The QCD sum rules method is a vigorous theoretical tool in exploring the properties of the exotic X, Y and Z states, there have been several possible interpretations of the existing Y states according to the investigations via the QCD sum rules, such as the vector tetraquark states, the tetraquark molecular states (color-singlet-color-singlet type tetraquark states), the charmonium-tetraquark mixing states.

In the scenario of tetraquark states, we can construct the color-antitriplet-color-triplet type, color-sextet-color-antisextet type, color-singlet-color-singlet type and color-octet-color-octet type four-quark configurations to explore the tetraquark properties. All those four-quark configurations are compact objects as we choose the local currents to interpolate them in the QCD sum rules. While we usually prefer the diquarks in the color-antitriplet to other configurations in selecting the basic building blocks, because the attractive interactions induced by one-gluon exchange favor forming diquarks in the color-antitriplet.

The diquark operators $\varepsilon^{ijk} q_i^c T q_j^c$ in the color-antitriplet have five structures in Dirac spinor space, where the $i$, $j$ and $k$ are color indexes, $CT = C \gamma_5$, $C$, $C \gamma_\mu \gamma_5$, $C \gamma_\mu$ and $C \sigma_{\mu\nu}$ for the scalar, pseudoscalar, vector, axialvector and tensor diquarks, respectively. In the non-relativistic quark model, an additional P-wave can change the parity by contributing a factor $(-)^L = -$, where the angular momentum $L = 1$. The $C \gamma_5$ and $C \gamma_\mu$ diquark states have the spin-parity $J^P = 0^+$ and $1^+$, respectively, the corresponding $C$ and $C \gamma_\mu \gamma_5$ diquark states have the spin-parity $J^P = 0^-$ and $1^-$, respectively, the net effects of the P-waves are embodied in the underlined $\gamma_5$ in the $C \gamma_5$.
and $C\gamma_\mu\gamma_5$. The tensor diquark states have both the $J^P = 1^+$ and $1^-$ components, we project the $1^+$ and $1^-$ components explicitly in one way or the other, and introduce the symbols $\tilde{A}$ and $\tilde{V}$ to represent the corresponding axialvector and vector diquark operators, respectively. We can also introduce the P-wave explicitly in the $C\gamma_5$ and $C\gamma_\mu$ diquark operators and acquire the vector diquark operators $\varepsilon^{ijk}\bar{q}_j^T C\gamma_5 \partial_\mu q_k^T$ or tensor diquark operators $\varepsilon^{ijk}\bar{q}_j^T C\gamma_\mu \partial_\nu q_k^T$, where the derivative $\tilde{\partial}_\mu = \partial_\mu - \gamma_5 \gamma_\mu \partial_\mu$ embodies the P-wave effects.

To explore the ground state hadron masses and make possible interpretations of the $Y$ states, we can adopt an S-wave diquark (antidiquark) and a P-wave antidiquark (diquark) pair as the basic building blocks to construct the vector hidden-charm tetraquark states without introducing an explicit P-wave, or introduce an explicit P-wave between the S-wave diquark and antidiquark constituents to construct the vector hidden-charm tetraquark states, and investigate them with the QCD sum rules in a comprehensive way to avoid possible biased analyses.

The predictions depend heavily on the input parameters chosen at the QCD side of the QCD sum rules, even in the same scenario of the tetraquark states, the same interpolating currents lead to quite different interpretations of the $Y$ states [8, 9, 10]. A comprehensive analysis with the same input parameters and same (or uniform) treatments is necessary, although all the $X$, $Y$ and $Z$ states have been investigated with the QCD sum rules in one way or the other.

In Refs. [15, 16], we introduce an explicit P-wave between the diquark and antidiquark constituents to construct the vector tetraquark states via the QCD sum rules systematically by calculating the vacuum condensates up to dimension 10 in a consistent way, and prefer the modified energy scale formula $\mu = \sqrt{M_x^2 - (2M_c + 0.5\text{GeV})^2}$ with the effective $c$-quark mass $M_c$ to select the pertinent energy scales of the QCD spectral densities considering the explicit P-wave, and acquire the lowest masses of the vector tetraquark states up to now and reexamine the possible interpretations of all the existing $Y$ states. The predictions support identifying the $Y(4220/4260)$, $Y(4320/4360)$, $Y(4390)$ and $Z(4250)$ as the vector tetraquark states with a relative P-wave between the diquark and antidiquark pair, see Table I. In Table I we show the quantum numbers of the $Y$ states in the non-relativistic diquark-antidiquark model explicitly, where the $L$ is the angular momentum between the diquark and antidiquark constituents, the total spin $\vec{S} = \vec{S}_{qc} + \vec{S}_{\bar{q}\bar{c}}$, and the total angular momentum $\vec{J} = \vec{S} + \vec{L}$.

In the phenomenological type-II diquark-antidiquark model [23], L. Maiani et al identify the $Y(4008)$, $Y(4260)$, $Y(4290/4220)$ and $Y(4630)$ as the four ground states with $L = 1$ based on the effective Hamiltonian with the spin-spin and spin-orbit interactions but neglect the spin-spin interactions between the quarks and antiquarks. In Ref. [24], A. Ali et al incorporate the dominant spin-spin, spin-orbit and tensor interactions, and observe that the preferred interpretations of the ground states with $L = 1$ are the $Y(4220)$, $Y(4330)$, $Y(4390)$, $Y(4660)$, which are also shown explicitly in Table I.

From Table I we can see clearly that the $Y(4660)$ cannot be identified as the diquark-antidiquark type vector tetraquark state with a relative P-wave between the diquark and antidiquark constituents according to the calculations based on the QCD sum rules [10], which differs from Ref. [24] remarkably. More works on the $Y(4660)$ based on the QCD sum rules are still needed.

Now let us go back to the vector tetraquark states without introducing the explicit P-waves. In Ref. [10], we construct the $C \otimes \gamma_\mu C$ and $C\gamma_5 \otimes \gamma_\mu C\gamma_5$ type four-quark currents with the $J^{PC} = 1^{--}$ to interpolate the vector tetraquark states, and acquire four QCD sum rules. We use the energy scale formula $\mu = \sqrt{M_y^2 - (2M_c)^2}$ to select the pertinent energy scales of the QCD spectral densities considering the P-wave is embodied implicitly in the (anti)diquarks, and adopt the experimental masses of the $Y(4260/4220)$, $Y(4360/4320)$, $Y(4390)$ and $Y(4660/4630)$ as input parameters and make great efforts to fit the pole residues to reproduce the QCD sum rules at the quark level. The numerical results support identifying the $Y(4660/4630)$ as the $C \otimes \gamma_\mu C$ type vector tetraquark state $c\bar{c}ss$, identifying the $Y(4360/4320)$ as $C\gamma_5 \otimes \gamma_5 \gamma_\mu C$ type vector tetraquark state $c\bar{c}g\bar{q}$, and
Table 1: The masses of the vector tetraquark states from the QCD sum rules [16], and possible interpretations of the $Y$ states based on the QCD sum rules [16] and diquark-antidiquark model [24].

| $|S_{PQ}; S, L; J\rangle$ | $M_Y$(GeV) | [16] | [24] |
|--------------------------|------------|-------|-------|
| $[0, 0; 0, 1; 1]$        | 4.24 ± 0.10 | $Y(4220)$ | $Y(4220)$ |
| $|1, 0; 1, 1; 1\rangle + |0, 1; 1, 1; 1\rangle$ | 4.31 ± 0.10 | $Y(4320/4390)$ | $Y(4330)$ |
| $[1, 1; 0, 1; 1]$        | 4.28 ± 0.10 | $Y(4220/4320)$ | $Y(4390)$ |
| $[1, 1; 2, 1; 1]$        | 4.33 ± 0.10 | $Y(4320/4390)$ | $Y(4660)$ |

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the vector hidden-charm tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.
2 QCD sum rules for the vector hidden-charm tetraquark states

Firstly let us write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$,

\[
\Pi_{\mu\nu}(p) = i \int d^4xe^{ipx}(0|T\left\{J_{\mu}(x)J_{\nu}^+(0)\right\}|0),
\]
\[
\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4xe^{ipx}(0|T\left\{J_{\mu\nu}(x)J_{\alpha\beta}^+(0)\right\}|0),
\]

where the currents

\[
J_{\mu}(x) = J_{-\mu}^{PA}(x), J_{+\mu}^{PA}(x), J_{-\mu}^{SV}(x), J_{+\mu}^{SV}(x), J_{-\mu}^{\tilde{V}A}(x), J_{+\mu}^{\tilde{V}A}(x), J_{-\mu}^{\tilde{A}V}(x), J_{+\mu}^{\tilde{A}V}(x), J_{\mu\nu}(x) = J_{-\mu\nu}^{SV}(x), J_{+\mu\nu}^{SV}(x), J_{-\mu\nu}^{\tilde{V}A}(x), J_{+\mu\nu}^{\tilde{V}A}(x), J_{-\mu\nu}^{\tilde{A}V}(x), J_{+\mu\nu}^{\tilde{A}V}(x),
\]

\[
J_{-\mu}^{PA}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\epsilon^k(x)d^m(x)\gamma_\mu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\epsilon^k(x)d^m(x)C\tilde{c}^Tn(x)\right],
\]
\[
J_{+\mu}^{PA}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)C\tilde{c}^Tn(x)\right],
\]
\[
J_{-\mu}^{SV}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) + u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[
J_{+\mu}^{SV}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[
J_{-\mu}^{\tilde{V}A}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[
J_{+\mu}^{\tilde{V}A}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) + u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[
J_{-\mu}^{\tilde{A}V}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[
J_{+\mu}^{\tilde{A}V}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}\left[u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x) - u^T_j(x)C\gamma_\mu\gamma_\nu\epsilon^k(x)d^m(x)\gamma_\nu C\tilde{c}^Tn(x)\right],
\]
\[ J^{AA}_{\mu\nu}(x) = \frac{\varepsilon^{ijk} \varepsilon^{imn}}{\sqrt{2}} \left[ u^{Tj}(x) C \gamma_\mu c^k(x) \bar{d}^m(x) \gamma_\nu C \bar{c}^T n(x) - u^{Tj}(x) C \gamma_\nu c^k(x) \bar{d}^m(x) \gamma_\mu C \bar{c}^T n(x) \right], \]  

the \( i, j, k, m, n \) are color indexes, the \( C \) is the charge conjugation matrix, the subscripts \( \pm \) stand for the positive and negative charge-conjugation, respectively, the superscripts \( P, S, V(V) \) and \( A \bar{A} \) stand for the pseudoscalar, scalar, vector and axialvector diquark and antidiquark operators, respectively. The current \( J^{PA}_{\mu}(x) \) was studied in Refs. [10, 11], the current \( J^{AA}_{\mu}(x) \) was studied in Ref. [11], the current \( J^{AA}_{\mu\nu}(x) \) was studied in Ref. [12], while the current \( J^{SV}_{\mu\nu}(x) \) was studied in Ref. [17]. In the present work, we update the old calculations and perform new analysis in a comprehensive way, and make great efforts to exhaust all the possible vector tetraquark configurations without introducing an explicit P-wave. There are thirteen currents, we acquire original results for the eight currents, \( J^{SV}_{\mu\nu}(x), J^{VA}_{\mu\nu}(x), J^{AV}_{\mu\nu}(x), J^{AV}_{0\nu}(x), J^{SA}_{\mu\nu}(x), J^{AP}_{\mu\nu}(x) \) and \( J^{PA}_{0\nu}(x) \).

Under the parity transform \( \hat{P} \), the four-quark current operators \( J_{\mu}(x) \) and \( J_{\mu\nu}(x) \) have the properties,

\[
\begin{align*}
\hat{P} J_{\mu}(x) \hat{P}^{-1} &= +J_{\mu}(\hat{x}), \\
\hat{P} J_{\mu\nu}(x) \hat{P}^{-1} &= -J_{\mu\nu}(\hat{x}) , \\
\hat{P} J^{AA}_{\mu\nu}(x) \hat{P}^{-1} &= +J^{AA}_{\mu\nu}(\hat{x}),
\end{align*}
\]  

(7)

according to the properties of the diquark constituents,

\[
\begin{align*}
\hat{P} e^{ijk} q^T_j(x) C T c_k(x) \hat{P}^{-1} &= -e^{ijk} q^T_j(\hat{x}) C \gamma^0 \gamma^0 c_k(\hat{x}), \\
\hat{P} e^{ijk} \bar{q}_j(x) C C \bar{c}_k(x) \hat{P}^{-1} &= -e^{ijk} \bar{q}_j(\hat{x}) C \gamma^0 \gamma^0 \bar{c}_k(\hat{x}),
\end{align*}
\]  

(8)

where \( \hat{J}_{\mu\nu}(x) = J^{SV}_{\mu\nu}(x), J^{SV}_{\mu\nu}(x), J^{PA}_{\mu\nu}(x), J^{AP}_{\mu\nu}(x) \), the coordinates \( x^\mu = (t, \vec{x}) \) and \( \hat{x}^\mu = (t, -\vec{x}) \). For \( \Gamma = 1, \gamma_5, \gamma_\mu \gamma_5, \sigma_{\mu\nu}, \sigma_{\mu\nu} \gamma_5 \), we acquire \( \gamma^0 \Gamma \gamma^0 = 1, -\gamma_5, \gamma_\mu, -\gamma_\mu \gamma_5, \sigma_{\mu\nu}, -\sigma_{\mu\nu} \gamma_5 \). We rewrite Eq. (7) in more explicit form,

\[
\begin{align*}
\hat{P} J_1(x) \hat{P}^{-1} &= -J_1(\hat{x}), \\
\hat{P} J_{ij}(x) \hat{P}^{-1} &= -J_{ij}(\hat{x}), \\
\hat{P} J^{AA}_{0\mu}(x) \hat{P}^{-1} &= -J^{AA}_{0\mu}(\hat{x}),
\end{align*}
\]  

(9)

\[
\begin{align*}
\hat{P} J_0(x) \hat{P}^{-1} &= +J_0(\hat{x}), \\
\hat{P} J_{i0}(x) \hat{P}^{-1} &= +J_{i0}(\hat{x}), \\
\hat{P} J^{AA}_{ij}(x) \hat{P}^{-1} &= +J^{AA}_{ij}(\hat{x}),
\end{align*}
\]  

(10)

where \( i, j = 1, 2, 3 \). Now we can see clearly that the currents \( J_{\mu}(x) \) and \( J_{\mu\nu}(x) \) have both negative-parity and positive-parity components, which couple potentially to the hidden-charm tetraquark states with the negative-parity and positive-parity, respectively, we separate their contributions explicitly by introducing the suitable projectors in practical calculations.

Under the charge-conjugation transform \( \hat{C} \), the four-quark currents \( J_{\mu}(x) \) and \( J_{\mu\nu}(x) \) have the properties,

\[
\begin{align*}
\hat{C} J_{\pm,\mu}(x) \hat{C}^{-1} &= \pm J_{\pm,\mu}(x) |_{u+d}, \\
\hat{C} J_{\pm,\mu\nu}(x) \hat{C}^{-1} &= \pm J_{\pm,\mu\nu}(x) |_{u+d}.
\end{align*}
\]  

(11)
The four-quark currents $J_\mu(x)$ and $J_{\mu\nu}(x)$ with hidden-charm have the symbolic structure $\bar{c}c\bar{u}d$ and the isospin $(I, I_3) = (1, 1)$, we can construct other currents in the isospin multiplets in a similar way. In the isospin limit $m_u = m_d = m_c$, the four-quark currents with the symbolic structures,

$$\bar{c}c\bar{u}d, \bar{c}c\bar{d}u, \bar{c}c\bar{u} - \bar{d}d \sqrt{2}, \bar{c}c\bar{u} + \bar{d}d \sqrt{2},$$

(12)
couple potentially to the vector hidden-charm tetraquark states with almost degenerated masses, the currents with the isospin $I = 1$ and 0 result in the same expressions of the QCD sum rules. Only the four-quark currents with the symbolic structures $\bar{c}c\bar{u} - \bar{d}d \sqrt{2}$ and $\bar{c}c\bar{u} + \bar{d}d \sqrt{2}$ have definite charge-conjugation, and we take it for granted that the vector tetraquark states $\bar{c}c\bar{u}$ have the same charge-conjugation as their charge-neutral cousins.

At the phenomenological side, we insert a perfect set of intermediate hadronic states with the same quantum numbers (spin, parity, charge-conjugation) as the local four-quark currents $J_\mu(x)$ and $J_{\mu\nu}(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ to acquire the hadronic spectral representation [27] [28] [29], and separate the contributions of the lowest vector hidden-charm tetraquark states,

$$\Pi_{\mu\nu}(p) = \frac{\lambda_y^2}{M_{Y_+}^2 - p^2} \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2} \right) + \cdots,$$

$$\bar{\Pi}_{\mu\nu\alpha\beta}(p) = \frac{\lambda_y^2}{M_{Y_-}^2 - p^2} \left( p^2 g_{\mu\alpha}g_{\nu\beta} - p^2 g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}p_{\nu}p_{\beta} - g_{\mu\beta}p_{\nu}p_{\alpha} + g_{\nu\alpha}p_{\mu}p_{\beta} + g_{\nu\beta}p_{\mu}p_{\alpha} \right)$$

$$+ \frac{\lambda_z^2}{M_{Z_+}^2 - p^2} \left( -g_{\mu\alpha}p_{\nu}p_{\beta} - g_{\mu\beta}p_{\nu}p_{\alpha} + g_{\nu\alpha}p_{\mu}p_{\beta} + g_{\nu\beta}p_{\mu}p_{\alpha} \right) + \cdots,$$

$$\Pi_{\mu\nu\alpha\beta}^{AA}(p) = \frac{\lambda_y^2}{M_{Z_-}^2 - p^2} \left( p^2 g_{\mu\alpha}g_{\nu\beta} - p^2 g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}p_{\nu}p_{\beta} - g_{\mu\beta}p_{\nu}p_{\alpha} + g_{\nu\alpha}p_{\mu}p_{\beta} + g_{\nu\beta}p_{\mu}p_{\alpha} \right)$$

$$+ \frac{\lambda_z^2}{M_{Z_-}^2 - p^2} \left( -g_{\mu\alpha}p_{\nu}p_{\beta} - g_{\mu\beta}p_{\nu}p_{\alpha} + g_{\nu\alpha}p_{\mu}p_{\beta} + g_{\nu\beta}p_{\mu}p_{\alpha} \right) + \cdots,$$

(13)

where the pole residues $\lambda_Y$ and $\lambda_Z$ are defined by

$$\langle 0 | J_\mu(0) | Y_+^- (p) \rangle = \lambda_y \varepsilon_\mu,$$

$$\langle 0 | J_{\mu\nu}(0) | Y_-^- (p) \rangle = \lambda_{\mu\nu\alpha\beta} \varepsilon^\alpha p^\beta,$$

$$\langle 0 | \bar{J}_{\mu\nu}(0) | Z_+^+ (p) \rangle = \frac{\lambda_z}{M_{Z_+}} \left( \varepsilon_{\mu \nu} - \varepsilon_\nu p_\mu \right),$$

$$\langle 0 | J^{AA}_{\mu\nu}(0) | Z_-^- (p) \rangle = \frac{\lambda_z}{M_{Z_-}} \left( \varepsilon_{\mu \nu} - \varepsilon_\nu p_\mu \right),$$

$$\langle 0 | J^{AA}_{\mu\nu}(0) | Y_+^- (p) \rangle = \frac{\lambda_y}{M_{Y_-}} \left( \varepsilon_{\mu \nu} - \varepsilon_\nu p_\mu \right),$$

(14)
the $\varepsilon_{\mu\nu}$ and $\varepsilon_{\mu\nu}$ are the polarization vectors, we add the superscripts/subscripts $\pm$ to indicate the positive and negative parity, respectively. We choose the components $\Pi_-(p^2)$ and $p^2 \bar{\Pi}_-(p^2)$ to explore the negative-parity hidden-charm tetraquark states with the angular momentum $J = 1$.

At the QCD side of the correlation functions $\Pi_{\mu
u}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$, there are two heavy quark propagators and two light quark propagators. Supposing that each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we acquire a quark-gluon operator $g_s G_q G \bar{u} \bar{d} dd$ of dimension 10, therefore we should calculate the vacuum condensates at least up to dimension 10 to evaluate the convergent behavior of the operator product expansion, and take into account the vacuum condensates $(\bar{q}q)$, $(\alpha^2 \pi)$, $(\bar{q}g_s \sigma Gq)$, $(\bar{q}q)^2$, $g_s^2 \bar{q}q)^2$, $(\bar{q}q)(\alpha^2 \pi)$, $(\bar{q}q)(\bar{q}g_s \sigma Gq)$, $(\bar{q}g_s \sigma Gq)^2$ and $(\bar{q}q)^2(\alpha^2 \pi)$, which are vacuum expectations of the quark-gluon operators of the order $O(\alpha_s^k)$ with $k \leq 1$ [11, 30, 31]. The highest vacuum condensates $(\bar{q}g_s \sigma Gq)^2$ and $(\bar{q}q)^2(\alpha^2 \pi)$ are associated with $\frac{1}{\Lambda}$, $\frac{1}{\Lambda}$ and $\frac{1}{\Lambda}$, and play an important role in acquiring the Borel windows, although they are of minor importance or play a tiny role in the Borel windows. We recalculate the contributions having their origins in the higher dimensional vacuum condensates according to the identity $t_{ij}^a t_{mn}^a = -\frac{1}{2}\delta_{ij}\delta_{mn} + \frac{1}{2}\delta_{jm}\delta_{in}$, and acquire slightly different analytical expressions compared with the old calculations for the three currents $J_{\mu}^{PA}(x)$, $J_{+,\mu}^{PA}(x)$ and $J_{+,\mu}^{AA}(x)$ [10, 11, 12], where $t^a = \lambda^a/2$, the $\lambda^a$ is the Gell-Mann matrix. The four-quark condensate $g^2(\bar{q}q)^2$ has its origins in the vacuum expectations $(\bar{q}g_s \tau^a \bar{q}g_s \bar{q}g_s \tau^a \bar{q}g_s \bar{q})$, $\bar{q}j J^\mu \bar{q}j J^\nu \bar{q}j J^\nu \bar{q}j J^\nu$ combined with the vacuum saturation assumption, rather than has its origins in the radiative corrections for the four-quark condensate $(\bar{q}q)^2$, where $D_\alpha = \delta_\alpha - ig_s G^\alpha t^a$, its contributions are tiny and neglected in most of the QCD sum rules. The strong fine-structure-constant/coupling-constant $\alpha_s(\mu) = \frac{\alpha_s(1\text{GeV})}{\mu}$ appears at the tree level, which is energy scale dependent and manifests the necessity of applying the energy scale formula $\mu = \sqrt{M_{X_{Y/Z}}^2 + (2M_c)^2}$.

We match the phenomenological side with the QCD side of the components $\Pi_-(p^2)$ and $p^2 \bar{\Pi}_-(p^2)$ below the continuum thresholds $s_0$ with the help of spectral representation, and perform Borel transform in regard to the variable $P^2 = -p^2$ to acquire the QCD sum rules:

$$\frac{\lambda^2}{\nu} \exp \left( -\frac{M_Y^2}{T^2} \right) = \int_{4M_c^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right),$$

the explicit expressions of the QCD spectral densities $\rho(s)$ are neglected for simplicity.

We differentiate Eq. (15) in regard to the variable $\tau = \frac{1}{T^2}$, and acquire the QCD sum rules for the masses of the vector hidden-charm tetraquark states $Y_c$,

$$M_Y^2 = \frac{\int_{4M_c^2}^{s_0} ds \frac{d}{d\tau} \rho(s) \exp(-\tau s)}{\int_{4M_c^2}^{s_0} d\tau \rho(s) \exp(-\tau s)}.$$  

3 Numerical results and discussions

The quark masses and vacuum condensates depend on the energy scale, we write down the energy-scale dependence of the input parameters at the QCD side,

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}},$$

$$\langle \bar{q}g_s \sigma Gq \rangle(\mu) = \langle \bar{q}g_s \sigma Gq \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}},$$

$$m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right].$$ (17)
from the renormalization group equation, where \( t = \log(\frac{\mu^2}{\Lambda_{QCD}^2}}), \( b_0 = \frac{33-2n_f}{12\pi}, \ b_1 = \frac{153-19n_f}{24\pi^2}, \ b_2 = \frac{2857 - 5033n_f + 325n_f^2}{1296}, \ \Lambda_{QCD} = 210 \text{ MeV}, 292 \text{ MeV} \) and 332 MeV for the flavors \( n_f = 5, 4 \) and 3, respectively.\[^{[32, [33]}\] In this work, as the up, down and charm quarks are concerned, we adopt the flavor number \( n_f = 4 \) and neglect the small \( u \) and \( d \) quark masses.

At the beginning points, we adopt the standard-values/conventional-values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3, \langle \bar{q}q, Gq \rangle = m_0^2 \langle \bar{q}q \rangle, \ m_0^2 = (0.8 \pm 0.1 \text{ GeV})^2, \ \langle \frac{\pi G_{FF}}{\pi} \rangle = (0.012 \pm 0.004) \text{ GeV}^3 \) at the typical energy scale \( \mu = 1 \text{ GeV} \)[27, [28, [29, [34]]] and adopt the modified minimal subtracted mass \( m_c(m_c) = (1.275 \pm 0.025) \text{ GeV} \) from the Particle Data Group [32].

In this work, we resort to the energy scale formula \( \mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2} \) with the updated value \( M_c = 1.82 \text{ GeV} \) to determine the pertinent energy scales of the QCD spectral densities in the QCD sum rules [11, [12]. We can rewrite the energy scale formula in the form,

\[
M_{X/Y/Z}^2 = \mu^2 + \text{Constants},
\]

where the Constants have the value \( 4M_c^2 \) and are fitted phenomenologically by the QCD sum rules, the predicted tetraquark masses and the pertinent energy scales of the QCD spectral densities have a Regge-trajectory-like relation. The constraint shown in Eq. (18) plays an important role in enhancing the pole contributions and improving the convergent behaviors of the operator product expansion so as to acquire the lowest tetraquark (molecular) masses, which are calculated with the QCD sum rules in Eq. (16).

Now let us take a short digression to discuss why the QCD sum rules for the hidden-charm tetraquark (molecular) states depend on the energy scales. We can write the correlation functions \( \Pi(p^2) \) for the hidden-charm four-quark currents in the form,

\[
\Pi(p^2) = \int_{4m_c^2}^{s_0} ds \frac{\rho_{QCD}(s, \mu)}{s - p^2} + \int_{s_0}^{\infty} ds \frac{\rho_{QCD}(s, \mu)}{s - p^2},
\]

through dispersion relation, and they are energy scale independent or independent on the energy scale we select to carry out the operator product expansion (up to a factor of the anomalous dimensions of the currents \( \gamma_1 \)), but which does not ensure the contributions of the ground states,

\[
\frac{d}{d\mu} \int_{4m_c^2}^{s_0} ds \frac{\rho_{QCD}(s, \mu)}{s - p^2} \rightarrow 0.
\]

In carrying out the operator product expansion, we usually neglect the radiative corrections, even in the QCD sum rules for the traditional/conventional mesons, we fail to take account of the radiative corrections up to arbitrary orders; we factorize the higher dimensional vacuum condensates into lower dimensional ones according to vacuum saturation, consequently we modify the energy scale dependence of the higher dimensional vacuum condensates more or less. Furthermore, we introduce the truncations \( s_0 \) to exclude the contaminations from the continuum states and higher resonances, and we have no knowledge of the possible correlation between the threshold \( 4m_c^2(\mu) \) and continuum threshold \( s_0 \). After performing the Borel transform, we acquire the integrals,

\[
\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s, \mu) \exp \left( -\frac{s}{T^2} \right),
\]

which are sensitive to the energy scale \( \mu \). Variations of the energy scale \( \mu \) can result in variations of the integral ranges \( 4m_c^2(\mu) - s_0 \) of the variable \( ds \) besides the QCD spectral densities \( \rho_{QCD}(s, \mu) \), consequently, they result in variations of the Borel windows and predicted tetraquark (molecule) masses and pole residues. We resort to the energy scale formula to determine the pertinent energy scales consistently.

We usually consult the experimental data on the mass gaps between the ground states (1S) and first radial excited states (2S) to acquire the continuum threshold parameters \( s_0 \). According to the
(possible) quantum numbers (such as spin, parity, charge-conjugation, etc), decay channels and mass gaps, if we prefer the scenarios of tetraquark states to other interpretations, we can tentatively identify the $X(3915)$ and $X(4500)$ as the 1S and 2S hidden-charm tetraquark states with the quantum numbers $J^{PC} = 0^{++}$, identify the $Z_c(3900)$ and $Z_c(4430)$ as the 1S and 2S hidden-charm tetraquark states with the quantum numbers $J^{PC} = 1^{+-}$, respectively, identify the $Z_c(4020)$ and $Z_c(4600)$ as the 1S and 2S hidden-charm tetraquark states with the quantum numbers $J^{PC} = 1^{+-}$, respectively, and identify the $X(4140)$ and $X(4685)$ as the 1S and 2S hidden-charm tetraquark states with the quantum numbers $J^{PC} = 1^{++}$, respectively. The mass gaps between the 1S and 2S hidden-charm tetraquark states are about $0.57 \sim 0.59\text{GeV}$. In this work, we can set the continuum threshold parameters as $\sqrt{s_0} = M_Y + 0.4 \sim 0.6\text{GeV}$.

Compared with previous works [10] [11] [12] [17], we adopt the uniform constraint between the ground state masses $M_Y$ and continuum threshold parameters $s_0$, $\sqrt{s_0} = M_Y + 0.5 \pm 0.1\text{GeV}$, for all the vector tetraquark states, and perform a consistent analysis by including additional eight novel currents (or QCD sum rules). Furthermore, we correct some minor errors in the numerical calculations in Ref.[11].

The pole or ground state dominance at the phenomenological side and convergence of the operator product expansion at the QCD side are two elementary criteria, we should satisfy the two elementary criteria to acquire reliable/robust QCD sum rules. Now we write down explicitly the definitions for the pole contributions (PC),

$$PC = \frac{\int_{4m_c^2}^{s_0} ds \rho(s) \exp \left(-\frac{s}{T}\right)}{\int_{4m_c^2}^{\infty} ds \rho(s) \exp \left(-\frac{s}{T}\right)},$$

and the contributions of the vacuum condensates of dimension $n$,

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_n(s) \exp \left(-\frac{s}{T}\right)}{\int_{4m_c^2}^{\infty} ds \rho(s) \exp \left(-\frac{s}{T}\right)}.$$ 

In the present work, we require the contributions $|D(10)| \sim 1\%$ or $<1\%$ at the Borel windows.

We search for the best Borel parameters and continuum threshold parameters via trial and error. At last, we acquire the Borel windows, continuum threshold parameters, suitable energy scales of the QCD spectral densities and pole contributions, which are shown explicitly in Table 2. From the Table, we can see plainly that the pole contributions are about $(40 - 60)\%$ at the phenomenological side, while the central values are larger than 50%, the pole dominance criterion is satisfied very good. In Fig.1 we plot the absolute contributions of the vacuum condensates $|D(n)|$ with the central values of the input parameters shown in Table 2. From the figure, we can see clearly that the main contributions have their origins in the perturbative terms, the higher dimensional vacuum condensates play a minor important role (or they are of tiny importance). For example, the contributions of the vacuum condensates of dimension 10 are $|D(10)| \ll 1\%$, just like our expectation, the convergent behavior of the operator product expansion is very good.

We take account of all the uncertainties from the relevant parameters and acquire the masses and pole residues of the vector hidden-charm tetraquark states with both the positive and negative charge-conjugation from the coupled equations (15) and (10), and we also present them clearly in Table 2. From Table 2 we can see clearly that the energy scale formula $\mu = \sqrt{M_X^2 + 2M_c^2}$ is satisfied very good. The energy scale formula plays an important role (or is of crucial importance) in enhancing the pole contributions and improving the convergent behaviors of the operator product expansion in the QCD sum rules for the hidden-charm (or hidden-bottom) and doubly charmed (or doubly bottom) tetraquark and pentaquark (molecular) states [11] [19] [25] [26] [31] [38]. In Fig.2 we plot the predicted masses of the $|uc\overline{c}\overline{d}\overline{A}| + |uc\overline{A}\overline{c}|$ and $|uc\overline{c}\overline{d}\overline{A}| + |uc\overline{A}\overline{c}|$ tetraquark states with the quantum numbers $J^{PC} = 1^{--}$ and $1^{+-}$ respectively via variations of the Borel parameters at much larger ranges than the Borel widows as a typical example. From the figure, we
can see explicitly that there appear very flat platforms in the Borel windows as a matter of fact, which can exclude additional uncertainties coming from the Borel parameters.

If we abandon the energy scale formula $\mu = \sqrt{M_X^2 - (2M_y)^2}$ and choose the particular energy scale $\mu = 1 \text{ GeV}$, we can perform the same procedure by searching for the best Borel parameters and continuum threshold parameters via trial and error. At last, we acquire the Borel windows, continuum threshold parameters and pole contributions, which are presented plainly in Table 3. From Table 3 we can see clearly that the pole contributions are about $(40 - 60)\%$, just like the corresponding ones shown in Table 2; again the pole dominance criterion is satisfied very good. At the QCD side, the dominant contributions have their origins in the perturbative terms, the operator product expansion converges very well. Again, we take account of all the uncertainties from the relevant parameters and reach the masses and pole residues of the vector hidden-charm tetraquark states, which are also presented plainly in Table 3.

In Table 4 we present the possible interpretations of the ground state vector hidden-charm tetraquark states, the present predictions support identifying the $Y(4360/4390)$ as the $[uc]_A[dc]_A - [uc]_A[dc]_A$ hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{--}$ and identifying the $Y(4600)$ as the $[uc]_A[dc]_A - [uc]_A[dc]_A$ or $[uc]_A[dc]_A$ hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{--}$.

In Table 4 we also present the masses of the ground state vector hidden-charm tetraquark states extracted at the particular energy scale $\mu = 1 \text{ GeV}$, the predicted tetraquark masses $M_y$ are much larger than the corresponding ones $M_y$, which disfavors identifying all the existing $Y$ states as the vector tetraquark states due to the large masses.

In Ref. [16], the predicted mass $M_Y = 4.24 \pm 0.10 \text{ GeV}$ for the $[0, 0; 0, 1; 1]$ tetraquark state supports identifying the $Y(4260/4220)$ as the $C_{\gamma^5}\hat{\sigma}_\mu \hat{\gamma}_\mu C$ type vector hidden-charm tetraquark state; the predicted mass $M_Y = 4.28 \pm 0.10 \text{ GeV}$ for the $[1, 1; 0, 1; 1]$ tetraquark state supports identifying the $Y(4260/4220)$ or $Y(4360/4320)$ as the $C_{\gamma^5}\hat{\gamma}_\mu \hat{\gamma}_\mu C$ type vector hidden-charm tetraquark state; the predicted masses $M_Y = 4.31 \pm 0.10 \text{ GeV}$ for the $[\frac{1}{2}, 0; 1, 1; 1] + [0, 1; 1, 1; 1]$ tetraquark state and $M_Y = 4.33 \pm 0.10 \text{ GeV}$ for the $[1, 1; 2, 1; 1]$ tetraquark state supports identifying the $Y(4360/4320)$ or $Y(4390)$ as the $C_{\gamma^5}\hat{\gamma}_\mu \hat{\gamma}_\mu C + C_{\gamma^5}\hat{\gamma}_\mu \hat{\gamma}_\mu C$ type or $C_{\gamma^5}\hat{\gamma}_\mu \hat{\gamma}_\mu C$ type.
Figure 2: The masses of the \([uc]_P[dc]_A - [uc]_A[dc]_P(I)\) and \([uc]_P[dc]_A + [uc]_A[dc]_P(II)\) vector tetraquark states via variations of the Borel parameters \(T^2\), where the regions between the two vertical lines are the Borel windows.

\[ \otimes \gamma_\mu C + C \gamma_\mu \otimes \bar{\partial}_\mu \otimes \gamma_5 C - C \gamma_\mu \otimes \bar{\partial}_\nu \otimes \gamma_\mu C - C \gamma_\nu \otimes \bar{\partial}_\mu \otimes \gamma_5 C \] type vector hidden-charm tetraquark states, see Table [1].

In short, there are enough rooms to accommodate the existing \(Y\) states above 4.2 GeV. From Table [1] and Table [4], we can draw the conclusion tentatively that the \(Y(4320/4360)\) and \(Y(4390)\) maybe have more than one Fock components, which have an explicit P-wave between the diquark and antidiquark constituents or have an implicit P-wave in the diquark (or antidiquark) constituent; while the \(Y(4660)\) maybe have more than one Fock components, which have an implicit P-wave in the diquark (or antidiquark) constituent. Otherwise, there maybe exist several \(Y\) states with almost degenerated masses but quite different quark structures.

In Table [5], we compare the present predictions with the old calculations [10, 11, 12, 17]. From the Table, we can see explicitly that they are compatible with each other within uncertainties, or they have overlaps within uncertainties. In this work, we recalculate the contributions of the higher dimensional vacuum condensates and acquire slightly different analytical expressions compared with the old calculations in Refs. [10, 11, 12], and correct some minor errors in the numerical calculations in Ref. [11]. Most importantly, we adopt the uniform/same constraint between the ground state masses \(M_Y\) and continuum threshold parameters \(s_0\), \(\sqrt{s_0} = M_Y + 0.5 \pm 0.1\) GeV, for all the thirteen vector tetraquark states, and expect to acquire more robust/reliable QCD sum rules.

We can confront the vector hidden-charm tetraquark states predicted in this work to the experimental data in the future at the BESIII, LHCb, Belle II, CEPC, FCC, ILC, which maybe shed light on the nature of the exotic \(X, Y, Z\) particles. We can investigate or search for the neutral \(Y_c\) tetraquark states with the quantum numbers \(J^{PC} = 1^{--}\) and \(1^{--}\) through the two-body or three-body strong decays,

\[
\begin{align*}
Y_c(1^{--}) &\rightarrow \chi_c\rho/\omega, J/\psi\pi^+\pi^- , J/\psi K\bar{K}, \eta_c\rho/\omega, \chi_c\rho/\omega, \\
Y_c(1^{--}) &\rightarrow J/\psi\rho/\omega, h_c\rho/\omega.
\end{align*}
\] (24)

4 Conclusion

In the present work, we adopt the scalar, pseudoscalar, vector, axialvector and tensor (anti)diquark operators as the elementary building blocks to construct vector and tensor four-quark currents as
| $Y_c$                  | $J^{PC}$ | $\sqrt{s_0}(\text{GeV})$ | $\mu(\text{GeV})$ | pole | $M_Y(\text{GeV})$ | $\lambda_Y(\text{GeV}^2)$ |
|-----------------------|----------|---------------------------|-------------------|------|------------------|-----------------------------|
| $[uc]_P^A dc - [uc]_A dc P$ | 1−−      | 3.7−4.1                   | 5.15±0.10         | 2.9  | 4.66±0.07        | (7.19±0.84)×10^{-2}         |
| $[uc]_P^A dc + [uc]_A dc P$ | 1++      | 3.7−4.1                   | 5.10±0.10         | 2.8  | 4.61±0.07        | (6.69±0.80)×10^{-2}         |
| $[uc]_S^V dc + [uc]_V dc S$ | 1−−      | 3.2−3.6                   | 4.85±0.10         | 2.4  | 4.35±0.08        | (4.32±0.61)×10^{-2}         |
| $[uc]_S^V dc - [uc]_V dc S$ | 1++      | 3.7−4.1                   | 5.15±0.10         | 2.9  | 4.66±0.09        | (6.67±0.82)×10^{-2}         |
| $[uc]_V^A dc - [uc]_A dc V$ | 1−−      | 3.6−4.0                   | 5.05±0.10         | 2.7  | 4.53±0.07        | (1.03±0.14)×10^{-1}         |
| $[uc]_V^A dc + [uc]_A dc V$ | 1++      | 3.7−4.1                   | 5.15±0.10         | 2.9  | 4.65±0.08        | (1.13±0.15)×10^{-1}         |
| $[uc]_V^A dc - [uc]_A dc V$ | 1−−      | 3.5−3.9                   | 5.00±0.10         | 2.6  | 4.48±0.08        | (9.47±1.27)×10^{-2}         |
| $[uc]_V^A dc + [uc]_A dc V$ | 1++      | 3.6−4.0                   | 5.05±0.10         | 2.7  | 4.55±0.07        | (1.06±0.14)×10^{-1}         |
| $[uc]_V^A dc - [uc]_A dc V$ | 1−−      | 3.4−3.8                   | 5.00±0.10         | 2.6  | 4.50±0.09        | (4.78±0.66)×10^{-2}         |
| $[uc]_V^A dc + [uc]_A dc V$ | 1++      | 3.4−3.8                   | 5.00±0.10         | 2.6  | 4.50±0.09        | (4.79±0.66)×10^{-2}         |
| $[uc]_P^A dc - [uc]_A dc P$ | 1−−      | 3.7−4.1                   | 5.10±0.10         | 2.8  | 4.60±0.07        | (6.32±0.74)×10^{-2}         |
| $[uc]_P^A dc + [uc]_A dc P$ | 1++      | 3.7−4.1                   | 5.10±0.10         | 2.8  | 4.61±0.08        | (6.36±0.74)×10^{-2}         |

Table 2: The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities, pole contributions, masses and pole residues for the ground state vector hidden-charm tetraquark states, where the constraint in Eq. (18) is satisfied.
Table 3: The Borel parameters, continuum threshold parameters, pole contributions, masses and pole residues for the ground state vector hidden-charm tetraquark states with the energy scales $\mu = 1\text{ GeV}$, where the constraint in Eq. [18] is not satisfied.
### Table 4: The possible interpretations of the ground state vector hidden-charm tetraquark states, the isospin limit is implied, the vector tetraquark states $\bar{c}c\bar{u}u, \bar{c}c\bar{u}u$, $\bar{c}c\bar{u}u$ and $\bar{c}c\bar{u}u$ have almost degenerated masses.

| $Y_c$                                                                 | $J^{PC}$ | $M_Y$(GeV) | Interpretations            | $M_Y$(GeV) |
|------------------------------------------------------------------------|----------|------------|-----------------------------|------------|
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_P$                                    | 1--      | 4.66 ± 0.07| $? Y(4660)$                 | 5.20 ± 0.07|
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_P$                                    | 1+       | 4.61 ± 0.07|                             | 5.14 ± 0.08|
| $[uc]_S[dc]_V + [uc]_V[dc]_S$                                         | 1--      | 4.35 ± 0.08| $Y(4360/4390)$              | 4.75 ± 0.09|
| $[uc]_S[dc]_V - [uc]_V[dc]_S$                                         | 1+       | 4.66 ± 0.09|                             | 5.00 ± 0.10|
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.53 ± 0.07|                             | 4.99 ± 0.09|
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1+       | 4.65 ± 0.08|                             | 4.99 ± 0.09|
| $[uc]_\gamma[dc]_V + [uc]_V[dc]_\bar{\gamma}$                        | 1--      | 4.48 ± 0.08|                             | 4.90 ± 0.08|
| $[uc]_\gamma[dc]_V - [uc]_V[dc]_\bar{\gamma}$                        | 1+       | 4.55 ± 0.07|                             | 5.05 ± 0.08|
| $[uc]_S[dc]_\bar{\gamma} - [uc]_\bar{\gamma}[dc]_S$                  | 1--      | 4.50 ± 0.09|                             | 4.75 ± 0.11|
| $[uc]_S[dc]_\bar{\gamma} + [uc]_\bar{\gamma}[dc]_S$                  | 1+       | 4.50 ± 0.09|                             | 4.75 ± 0.11|
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.60 ± 0.07|                             | 5.15 ± 0.07|
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1+       | 4.61 ± 0.08|                             | 5.16 ± 0.07|
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.69 ± 0.08| $Y(4660)$                   | 5.05 ± 0.09|

### Table 5: The masses of the ground state vector hidden-charm tetraquark states compared with the old calculations.

| $Y_c$                                                                 | $J^{PC}$ | $M_Y$(GeV) | $M_Y$(GeV) (Old Work) |
|------------------------------------------------------------------------|----------|------------|-----------------------|
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_P$                                    | 1--      | 4.66 ± 0.07| 4.59 ± 0.08 [10]; 4.66±0.11 [11] |
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_P$                                    | 1+       | 4.61 ± 0.07| 4.57±0.12 [11] |
| $[uc]_S[dc]_V + [uc]_V[dc]_S$                                         | 1--      | 4.35 ± 0.08| 4.34 ± 0.08 [10] |
| $[uc]_S[dc]_V - [uc]_V[dc]_S$                                         | 1+       | 4.66 ± 0.09| |
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.53 ± 0.07| |
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1+       | 4.65 ± 0.08| |
| $[uc]_\gamma[dc]_V + [uc]_V[dc]_\bar{\gamma}$                        | 1--      | 4.48 ± 0.08| |
| $[uc]_\gamma[dc]_V - [uc]_V[dc]_\bar{\gamma}$                        | 1+       | 4.55 ± 0.07| |
| $[uc]_S[dc]_\bar{\gamma} - [uc]_\bar{\gamma}[dc]_S$                  | 1--      | 4.50 ± 0.09| 4.61 ± 0.08 [17] |
| $[uc]_S[dc]_\bar{\gamma} + [uc]_\bar{\gamma}[dc]_S$                  | 1+       | 4.50 ± 0.09| |
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.60 ± 0.07| |
| $[uc]_\gamma[dc]_A + [uc]_A[dc]_\bar{\gamma}$                        | 1+       | 4.61 ± 0.08| |
| $[uc]_\gamma[dc]_A - [uc]_A[dc]_\bar{\gamma}$                        | 1--      | 4.69 ± 0.08| 4.66 ± 0.09 [12] |
many as possible without introducing an explicit P-wave (in other words, the negative-parity of the (anti)diquarks embodies the P-wave effect in an implicit way), and explore the mass spectrum of the vector hidden-charm tetraquark states via the QCD sum rules comprehensively, and revisit the interpretations of the existing $Y$ states in the scenario of vector tetraquark states. We acquire eight original QCD sum rules and update the calculations of the five old QCD sum rules. In addition, we resort to the energy scale formula to enhance the pole contributions and improve the convergent behaviors of the operator product expansion, and we should bear in mind that the predictions are rather sensitive to the particular energy scales which obey the uniform/same constraint. The present predictions support identifying the $Y(4360/4390)$ as the $[uc]_S[dc]_V + [uc]_V[dc]_S$ hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{--}$ and identifying the $Y(4660)$ as the $[uc]_P[dc]_A - [uc]_A[dc]_P$ or $[uc]_A[dc]_A$ hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{--}$.

We take account of our previous works on the vector tetraquark states with an explicit P-wave between the diquark and antidiquark constituents, and acquire the conclusion tentatively that the $Y(4320/4360)$, $Y(4390)$ and $Y(4660)$ maybe have more than one Fock components. The $Y(4320/4360)$ and $Y(4390)$ maybe have an explicit P-wave between the diquark and antidiquark constituents or an implicit P-wave in the diquark (or antidiquark) constituent; the $Y(4660)$ maybe have an implicit P-wave in the diquark (or antidiquark) constituent. Otherwise there maybe exist more than one $Y$ states with almost degenerated masses but quite different quark configurations. All in all, we can accommodate all the exotic $Y$ states above 4.2 GeV in a consistent way. We can confront the predicted vector hidden-charm tetraquark states to the experimental data at the BESIII, LHCb, Belle II, CEPC, FCC, ILC in the future.

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