Inclined hydromagnetic impact on tangent hyperbolic fluid flow over a vertical stretched sheet

Cite as: AIP Advances 9, 125022 (2019); https://doi.org/10.1063/1.5123188
Submitted: 06 August 2019 . Accepted: 20 November 2019 . Published Online: 17 December 2019

A. Ali, R. Hussain, and Misbah Maroof
Inclined hydromagnetic impact on tangent hyperbolic fluid flow over a vertical stretched sheet

Cite as: AIP Advances 9, 125022 (2019); doi: 10.1063/1.5123188
Submitted: 6 August 2019 • Accepted: 20 November 2019 •
Published Online: 17 December 2019

A. Ali, R. Hussain, and Misbah Maroof

AFFILIATIONS
Mirpur University of Science and Technology (MUST), Mirpur 10250, AJK, Pakistan

© 2019 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). https://doi.org/10.1063/1.5123188

INTRODUCTION

The tangent hyperbolic fluid model is one of the most important fluid models in the group of non-Newtonian fluids. From laboratory experiments, it is found that this model predicts the shear thinning phenomenon very precisely. Tangent hyperbolic fluids are being used mostly in laboratory experiments and industries. Whipped cream, blood, solutions, melts, paint, polymers, and ketchup are the main examples of the tangent hyperbolic model in the fields of industry and biology. Kumar et al. probed heat transfer in dusty hyperbolic tangent fluid with respect to magnetic field and thermal radiation toward a deformable sheet. The main focus of this study was to analyze the MHD flow and radiative heat transfer of the tangent hyperbolic fluid model with fluid particle suspension. Akbar et al. investigated the numerical solutions of the tangent hyperbolic model toward a deformable surface in the existence of MHD and discussed the behavior of parameters that occurred in the modeled equations. The Runge-Kutta method is used to solve the flow equations. They found that the Weissenberg number is counter positive for fluid momentum. Hayat et al. examined the tangent hyperbolic fluid for the progression of thermal and momentum boundary layers. They concluded that with large quantities of the Weissenberg number and power-law index, the profile of momentum was shortened. Salahuddin et al. examined the tangent hyperbolic fluid model with a flow of stagnation point toward a deformable cylinder. Kumar et al. examined the tangent hyperbolic squeezed flow with a sensor surface along variable thermal conductivity. Rehman et al. discussed the tangent hyperbolic fluid flow toward the inclined cylindrical surfaces and the surfaces that are deformable.

The transformation of heat developing nanofluids is among the hot fields of analysis due to their encouraging heat transfer characteristics. In this field, the most recent published work can be studied in Refs. 7–10. Nasir et al. scrutinized the Darcy Forchheimer nanofluid thin film flow of single-walled carbon nanotubes and heat transfer analysis over an unsteady stretching sheet. Shah et al. studied radiative heat and mass transfer analysis of the micropolar nanofluid flow of a Casson fluid.
between two rotating parallel plates with effects of the Hall current. Sheikholeslami et al. scrutinized the applications of electric field for the augmentation of ferrofluid heat transfer in an enclosure including double moving walls. Chaim has performed a noteworthy work on variable thermal conductivity toward a deformable surface. In his analysis, it was investigated that the fluid’s temperature increases as the variable thermal conductivity rises, but, at the same time, the wall gradient declines. Reddy C et al. illustrated MHD and the heat transfer flow along variable thermal conductivity and variable thickness toward a deformable sheet. Shokouhmand et al. explored variable thermal conductivity with two-dimensional porous fins. Sreenivasulu et al. carried out the study of variable thermal conductivity on the MHD flow for a deformable surface with a thermally stratified medium.

Magneto hydrodynamics is the analysis of highly electrically conducted fluids with magnetic properties. It performs a vital role in different fields such as geophysics, agriculture, meteorology, solar physics, petroleum industries, and astrophysics. Plasmas, electrolytes, metals, liquids, and salt water are some examples of magneto fluids. Rashidi et al. investigated the mixed convection of heat transfer of the nanofluid in a channel with sinusoidal walls under the MHD effect. Ahmad et al. analyzed the Darcy–Forchheimer MHD couple stress 3D nanofluid over an exponentially stretching sheet through Cattaneo–Christov convective heat flux. Shah et al. investigated the transient process in a finned triple tube during phase changing of aluminum oxide enhanced pulsed-code modulation (PCM). Again, Shah et al. investigated the Darcy-Forchheimer 3D micropolar rotational nanofluid flow of single wall and multiwall carbon nanotubes based on fluids (water, engine oil, ethylene glycol, and kerosene oil). A uniform MHD effect on water based nanofluid thermal behavior in a porous enclosure with an ellipse shaped obstacle has been studied by Sheikholeslami et al. Shah et al. analyzed the electrical MHD and Hall current impact on the micropolar nanofluid flow between rotating parallel plates. They examined the combined effect of magnetic and electric fields on micropolar nanofluids between two parallel plates in a rotating system. The heat transfer and MHD flow toward an exponentially deformable sheet with radiation and viscous dissipation have been examined by Sungu. Ruslan and Yaroslav studied the MHD numerical direct simulation of heat transfer with the combined influences of the thermo-gravitational and longitudinal magnetic field.

In very-high-power output devices, forced convection alone is not enough to dissipate all the heat. In such cases, combining natural convection with forced convection (mixed convection) will often give required results. Mixed convection mainly occurs in many technical and industrial applications. A heat exchanger placed in a low-velocity environment, solar collectors, electronic devices cooled by fans, and cooling of nuclear reactors during an emergency shutdown are some of the examples of the mixed convection phenomenon. Yang and Wu carried out the effect of the aspect ratio and assisted buoyancy on flow reversal for mixed convection with an imposed flow rate in a vertical 3D rectangular duct. They obtained the results for the mixed convection flow with an imposed inlet flow rate in a heated duct with a uniform wall temperature. Thermal patterns are presented and investigated for different buoyancy parameters and aspect ratios. Khan et al. examined mixed convective heat transfer to the Sisko fluid over a stretching surface with convective boundary conditions. Ahmad et al. discussed an MHD mixed convection Jeffrey fluid and heat transfer toward an exponentially stretching surface. The mixed convection flow of the Eyring-Powell nanofluid toward a plate and cone was examined by Khan et al. Izadi et al. presented the mixed convection heat transfer and entropy generation of a nanofluid containing carbon nanotubes, flowing in a three dimensional rectangular channel. They investigated that with an increase in the opposed buoyancy parameter, the nanofluid velocity near the channel wall reduces and, therefore, causes a reduction in the Nusselt number.

In the glance of the aforesaid literature survey, it has to be noticed that no work has been done to investigate the output of mixed convection of the tangent hyperbolic fluid flow in the existence of MHD and variable thermal conductivity. Therefore, the present endeavor is concentrated on this direction. The exclusive intention of this paper is to examine the effect of mixed convection of the hyperbolic tangent fluid flow with MHD and variable thermal conductivity toward a deformable sheet. The configuration of the present article is derived in such a way that partial differential equations (PDEs) can be converted into ordinary differential equations (ODEs) and then solved by BVP4C (MATLAB package). The behavior of different parameters, i.e., mixed convection, power law index, Hartmann number, aligned angle, Prandtl number, and Weissenberg number, has been examined for velocity and temperature profiles. The obtained results are expressed through graphs and tables in detail.

**MATHEMATICAL FORMULATION**

For a steady, two dimensional, incompressible, and electrically conducted flow of the tangent hyperbolic fluid, consider a deformable sheet coexisting along the plane $y = 0$, and the flow is being limited to $y > 0$, as shown in Fig. 1.

Here, the $y$-axis is the direction of the flow along the sheet, and the $x$-axis is perpendicular to the $y$-axis. Variable thermal conductivity mixed convection and MHD effects are also taken into account. A uniform magnetic force is applied in an inclined direction. The tensor of the tangent hyperbolic fluid model is:

$$\tau = [\mu_\infty + (\mu_0 + \mu_\infty)\tanh(1\gamma)^n]A_1,$$

where $\mu_0$ and $\mu_\infty$ represent the zero and infinite shear rate viscosities, respectively, $\tau$ is the stress tensor, $\Gamma$ represents the power law index, $\Gamma$ symbolizes the time dependent material constant, and $A_1$ is the first Rivlin-Erickson tensor. $\Gamma$ is defined as

$$\Gamma = \sqrt{\frac{1}{2} \sum_j \sum_i \overline{T}_{ij} \overline{T}_{ji} = \sqrt{\frac{1}{2} \Pi},}$$

where $\Pi = \frac{1}{2} \text{tr}(\mathbf{V}) + (\mathbf{V})^T \mathbf{V}$. It is not possible to consider the problem by taking infinite shear rate viscosity, so we take $\mu_\infty = 0$, and since a hyperbolic tangent fluid model has shear thinning behavior ($I(\overline{\gamma}) < 1$), then, from Eq. (1),

$$\tau = \mu_0 [n(1\gamma - 1)]A_1,$$

After applying the technique of the boundary layer, the governing equations of temperature and momentum are
In these expressions, we take the form by using the following:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}
\]

\[
u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} = v(1 - n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} u \phi^2(x) \sin(\phi) + \lambda(T - T_{\infty}), \tag{5}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha^*}{\sqrt{\pi}} \frac{\partial^2 T}{\partial y^2} - \left( \frac{\alpha^*}{\sqrt{\pi}} \right)^2 \frac{\partial T}{\partial y}. \tag{6}
\]

where \(u\) and \(v\) are the velocity components along the \(x\) and \(y\) directions, respectively, \(v\) represents the kinematic viscosity, \(\rho\) is the density of the fluid, \(\phi\) symbolizes the aligned angle, \(\lambda\) is the mixed convection parameter, \(\sigma\) denotes thermal diffusivity, \(\beta\) is the magnetic field, and \(T\) represents temperature. The boundary conditions are

\[
\begin{align*}
    u &= u_w(x) = ax, v = 0, T \rightarrow T_w \text{ at } y = 0, \\
    u &= 0, T \rightarrow T_{\infty} \text{ as } y \rightarrow \infty.
\end{align*} \tag{7}
\]

where \(u_w\) is the velocity of the fluid surface along the wall.

The following similarity transformations have been used:

\[
\begin{align*}
    u &= axf(\eta), v = -\sqrt{\pi} v f(\eta), \eta = \sqrt{T}y, \\
    \theta(\eta) &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, \alpha^* = \alpha (1 + \epsilon \theta),
\end{align*} \tag{8}
\]

where \(\epsilon\) denotes the parameter of variable thermal conductivity, \(\alpha^*\) is the parameter of thermal diffusivity, \(T\) represents fluids temperature, \(T_w\) is the wall temperature, and \(T_{\infty}\) is the surrounding fluid temperature. Equations (4)–(7) take the form by using the following equation:

\[
(1 - n)f''' + ff'' - f'^2 + nWef''f''' - Mf' \sin^2(\phi) + \lambda \theta = 0, \tag{9}
\]

\[
\theta''(1 + \epsilon \theta) + \epsilon f'^2 + Pr \theta f' = 0. \tag{10}
\]

In these expressions, \(W_e = \sqrt{\pi} \phi(a)_w\) is the Weissenberg number, \(M = \phi^2\rho_{\infty}a\) is the Hartmann number, \(\lambda = \frac{1}{\sqrt{\pi}}(T_w - T_{\infty})\) is the mixed convection parameter, and \(Pr = \frac{\lambda}{\alpha}\) is the Prandtl number.

Then, the boundary conditions become

\[
f = 0, f' = 1, \text{ at } \eta = 0, f'' \rightarrow 0 \text{ at } \eta \rightarrow \infty. \tag{11}
\]

The coefficient of skin friction can be defined as

\[
C_f = \frac{\tau_w}{\rho v^2}, \tag{12}
\]

\[
\tau_w = (1 - n) \mu C f' \left( \frac{\partial f}{\partial y} \right)^2. \tag{13}
\]

Using the values of \(\tau_w\) and \(\mu\) in Eq. (12), we get

\[
C_f (Rex)^{\frac{1}{2}} = (1 - n)f'(0) + \frac{n}{2} W_e (f''(0))^2. \tag{14}
\]

Also, the local Nusselt number is

\[
\frac{Nu_x}{(Rex)^{\frac{1}{2}}} = -\theta'(0). \tag{15}
\]

**NUMERICAL SOLUTION**

The solution of the nonlinear ordinary differential Eqs. (9) and (10) with the boundary conditions as per (11) is obtained by using the technique of BVP4c (MATLAB package) (see Refs. 31 and 32.) since Eqs. (9) and (10) are 3rd and 2nd order nonlinear ODEs. In order to reduce these two equations into first order ODEs, we consider

\[
\begin{align*}
    f &= y(1), \\
    f' &= y(2), \\
    f'' &= y(3), \\
    \theta &= y(4), \\
    \theta' &= y(5),
\end{align*} \tag{16}
\]

with boundary conditions

\[
y_1(2) = 1, y_2(2) = 0, y_3(4) = 1, y_4(4) = 0.
\]
RESULTS AND DISCUSSION

Graphical interpretation is used to epitomize the repercussions of different expedient parameters for velocity and temperature profiles, as shown in Figs. 2–7. Figure 2 describes the demeanor of the Weissenberg number $W_e$ for a velocity distribution $f'(\eta)$. It can be noticed that the velocity distribution $f'(\eta)$ dwindles owing to the growth in the Weissenberg number $W_e$. The Weissenberg number $W_e$ can be defined as the relation between a particular procedure time and the relaxation time of fluid. Growth in the Weissenberg number $W_e$ enhances the relaxation time due to which the viscosity of fluid particles increases. The ascendancy of viscosity creates

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Influence of $W_e$ on $f'(\eta)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Influence of $M$ on $f'(\eta)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Influence of $\phi$ on $f'(\eta)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Influence of $n$ on $f'(\eta)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Influence of $Pr$ on $\theta'(\eta)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Influence of $\varepsilon$ on $\theta'(\eta)$.}
\end{figure}
hindrance to the fluid and, consequently, the velocity distribution dwindles. Figure 3 shows the behavior of the Hartmann number $M$ for a velocity profile $f'(\eta)$. It is an apparent fact that the velocity profile $f'(\eta)$ decreases for greater values of the Hartmann number $M$. The physical reasoning behind this phenomenon is that the Lorentz force is strengthened due to greater values of the Hartmann number, as a result of which it creates resistance in the fluid flow. For incrementing values of the inclined angle $\phi$, there is a decline in the velocity distribution $f'(\eta)$, as shown in Fig. 4. It is just because of the fact that when there is escalation in the aligned angle $\phi$, the magnetic field enhances. The demeanor of the power-law index $n$ for velocity distribution has been shown in Fig. 5. It is depicted that for every incrementing value of the power law index $n$, the velocity distribution declines. Figure 6 elucidates the demeanor of the Prandtl number $Pr$ on the temperature profile $\theta(\eta)$. It is interpreted that the Prandtl number is elucidated as the relation between momentum diffusivity and thermal diffusivity. From this figure, it is palpable certitude that due to exceeding quantities of the Prandtl number $Pr$, there is growth in temperature distribution. The logic of this phenomenon is that due to exceeding quantities of the Prandtl number $Pr$, the thermal diffusivity of the fluid declines. The demeanor of the variable thermal conductivity $\varepsilon$ for temperature distribution is depicted in Fig. 7. It is seen that augmentation in the variable thermal conductivity $\varepsilon$ leads to escalation in the temperature distribution $\theta(\eta)$. As far as skin friction is concerned, there is a decline due to the escalation of the power law index $n$. The demeanor of skin friction is
The velocity distribution $j'(n)$ dwindles for the Hartmann number ($M$), Weissenberg number ($We$), power law index ($n$), and inclined angle ($\phi$). It is perceived that these parameters resist the fluid flow, whereas velocity distribution increases for greater values of the mixed convection parameter $\lambda$.

- Temperature distribution increases against the small parameter ($\epsilon$), whereas for greater values of the Prandtl number ($Pr$), the temperature profile decreases.
- The coefficient of skin friction declines due to growth in the power law index ($n$).
- The Nusselt number increases with the Prandtl number, whereas for incrementing values of the small parameter ($\epsilon$) and power law index ($n$), it declines.

**NOMENCLATURE**

- $A_1$ first Rivlin Erickson tensor
- $We$ Weissenberg number
- $u$ component of velocity along x-axis
- $v$ component of velocity along y-axis
- $n$ power law index
- $T$ temperature
- $T_w$ wall temperature
- $T_\infty$ surrounding fluid temperature
- $M$ Hartmann number

**Greek symbols**

- $\mu_0$ zero shear rate viscosity
- $\mu_\infty$ infinite shear rate viscosity
- $\nu$ kinematic viscosity
- $\phi$ aligned angle
- $\rho$ density of fluid
- $\alpha^*$ thermal diffusivity
- $\beta$ magnetic field
- $\Gamma$ time dependent material constant
- $\epsilon$ variable thermal conductivity parameter
- $Pr$ Prandtl number
- $\dot{q}$ heat flux

**REFERENCES**

1. K. G. Kumar, B. J. Gireesha, and R. S. R. Gorla, “Flow and heat transfer of dusty hyperbolic tangential fluid over a stretching sheet in the presence of thermal radiation and magnetic field,” Int. J. Mech. Mater. Eng. 13(2), 1–11 (2018).
2. N. S. Akbar, S. Nadeem, R. U. Haq, and Z. H. Khan, “Numerical solutions of magnetohydrodynamics boundary layer flow of tangent hyperbolic fluid towards a stretching sheet,” Indian J. Phys. 87, 1121–1124 (2013).
3. T. Hayat, M. I. Khan, M. Waqas, and A. Alsaedi, “Radiative flow of hyperbolic tangent liquid subject to Joule heating,” Results Phys. 7, 2197–2203 (2017).
4. T. Salahuddin, M. Y. Malik, A. Hussain, M. Awais, I. Khan, and M. Khan, “Analysis of tangent hyperbolic fluid impinging on a stretching cylinder near the stagnation point,” Results Phys. 7, 426–434 (2017).
9. G. Kumar, B. J. Giresha, M. R. Krishnamurthy, and N. G. Rudraswamy, “An unsteady squeezed flow of a tangent hyperbolic fluid over a sensor surface in the presence of variable thermal conductivity,” Results Phys. 7, 3031–3036 (2017).

10. K. U. Rehman, A. A. Malik, M. Y. Malik, and N. U. Saba, “Mutual effects of thermal radiation and thermal stratification on tangent hyperbolic fluid flow yields by both cylindrical and flat surfaces,” Case Stud. Therm. Eng. 10, 244–254 (2017).

11. Z. Shah, A. Dawar, E. O. Alzahrani, P. Kamum, A. J. Khan, and S. Islam, “Hall effect on couple stress 3D nanofluid flow over an exponentially stretching surface with Cattaneo–Christov heat flux model,” IEEE Access 7, 64844 (2019).

12. I. Ameen, Z. Shah, S. Islam, S. Nasir, W. Khan, P. Kamum, and P. Thounthong, “Hall and ion-slip effect on CNTS nanofluid over a porous extending surface through heat generation and absorption,” Entropy 21(8), 801 (2019).

13. N. Ahmed, S. T. M. Din, and S. M. Hassan, “Flow and heat transfer of nanofluid in an asymmetric channel with expanding and contracting walls suspended by carbon nanotubes: A numerical investigation,” Aerosp. Sci. Technol. 48, 53–60 (2016).

14. M. Jawad, Z. Shah, A. Khan, W. Khan, P. Kamum, and S. Islam, “Entropy generation and heat transfer analysis in MHD unsteady rotating flow for aqueous suspensions of carbon nanotubes with nonlinear thermal radiation and viscous dissipation effect,” Entropy 21, 492 (2019).

15. S. Nasir, Z. Shah, S. Islam, E. Bonyah, and T. Gul, “Darcy Forchheimer nanofluid thin film flow of SWCNTs and heat transfer analysis over an unsteady stretching sheet,” AIP Adv. 9, 015223 (2019).

16. Z. Shah, S. Islam, H. Ayaz, and S. Khan, “Radiative heat and mass transfer analysis of micropolar nanofluid flow of Casson fluid between two rotating parallel plates with effects of Hall current,” J. Heat Transfer 141(2), 022401 (2019).

17. M. Sheikhholeslami, Z. Shah, A. Tassaddiq, A. Shafee, and I. Khan, “Application of electric field for augmentation of ferrofluid heat transfer in an enclosure including double moving walls,” IEEE Access 7, 21048–21056 (2019).

18. T. C. Chaim, “Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet,” Acta Mech. 129, 63–72 (1998).

19. S. Reddy C, K. Naikoti, and M. M. Rashidi, “MHD flow and heat transfer characteristics of Williamson nanofluid over a stretching sheet with variable thickness and variable thermal conductivity,” Trans. A. Razmadze Math. Inst. 171, 195–211 (2017).

20. I. Shokouhmand, A. Sattari, S. E. H. Doost, and A. Maghbooli, “Analysis of two-dimensional porous fins with variable thermal conductivity,” Heat Trans. Asian Res. 47(2), 404–419 (2018).

21. P. Sreenivasulu, T. Poornimaband, and N. B. Reddy, “Variable thermal conductivity influence on hydromantic flow past a stretching cylinder in a thermally stratified medium with heat source/sink,” Front. Heat Mass Transfer 9, 1 (2017).

22. M. M. Rashidi, M. Nasiri, M. Khezerloo, and N. Laarqi, “Numerical investigation of magnetic field effect on mixed convection heat transfer of nanofluid in a channel with sinusoidal walls,” J. Magn. Magn. Mater. 401, 159 (2016).

23. M. W. Ahmad, P. Kamum, Z. Shah, A. A. Farooq, R. Nawaz, A. Dawar, S. Islam, and P. Thounthong, “Darcy–Forchheimer MHD couple stress 3D nanofluid over an exponentially stretching sheet through Cattaneo–Christov convective heat flux with zero nanoparticles mass flux conditions,” Entropy 21, 867 (2019).

24. Z. Shah, A. Shafee, A. R. A. Qawasmi, and I. Tlili, “Transient process in a finned triplex tube during phase changing of aluminum oxide enhanced PCM,” Eur. Phys. J. Plus 134, 173 (2019).

25. Z. Shah, A. Dawar, S. Islam, I. Khan, and D. L. C. Ching, “Darcy-Forchheimer flow of radiative carbon nanotubes with microstructure and inertial characteristics in the rotating frame,” Case Stud. Therm. Eng. 12, 823–832 (2018).

26. M. Sheikhholeslami, Z. Shah, A. Shafee, I. Khan, and I. Tlili, “Uniform magnetic force impact on water based nanofluid thermal behavior in a porous enclosure with ellipse shaped obstacle,” Sci. Rep. 9, 1196 (2019).

27. Z. Shah, S. Islam, T. Gul, E. Bonyah, and M. A. Khan, “The electrical MHD and Hall current impact on micropolar nanofluid flow between rotating parallel plates,” Results Phys. 9, 1201–1214 (2018).

28. T. C. Sungu, “Numerical investigation on MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation effects,” ITM Web Conf. 13, 01025 (2017).

29. A. Ruslanand and L. Yaroslav, “Direct numerical simulation of MHD heat transfer of the liquid metal in a horizontal pipe with the joint effect of the longitudinal magnetic field and thermo-gravitational convection,” MATEC Web Conf. 115, 02016 (2017).

30. G. Yang and J. Y. Wu, “Effect of aspect ratio and assisted buoyancy on flow reversal for mixed convection with imposed flow rate in a vertical three dimensional rectangular duct,” Int. J. Heat Mass Transfer 77, 335–343 (2014).

31. M. Khan, R. Malik, and A. Munir, “Mixed convective heat transfer to Sisko fluid over a radially stretching sheet in the presence of convective boundary conditions,” AIP Adv. 5, 087178 (2015).

32. K. Ahmad, Z. Hanouf, and A. Ishak, “Mixed convection Jeffrey fluid flow over an exponentially stretching sheet with magnetohydrodynamic effect,” AIP Adv. 6, 035024 (2016).

33. I. Khan, M. Khan, M. Y. Malik, T. Salahuddin, and Shaqfautullah, “Mixed convection flow of Eyring–Powell nanofluid over a cone and plate with chemical reactive species,” Results Phys. 7, 3716–3722 (2017).

34. M. Izadi, S. M. R. H. Pour, A. K. Yasuri, and A. J. Chamkha, “Mixed convection of a nanofluid in a three-dimensional channel,” J. Therm. Anal. Calorim. 136, 2461–2475 (2019).

35. W. Ibrahim, “Magneto-hydrodynamics (MHD) flow of tangent hyperbolic fluid with nanoparticles past a stretching sheet with second order slip and convective boundary conditions,” Results Phys. 7, 3723–3731 (2017).

36. W. Ibrahim, “MHD boundary layer flow and heat transfer of micropolar fluid past a stretching sheet with second order slip,” J. Brazilian Soc. Mech. Sci. Eng. 39, 791–799 (2017).

37. M. Y. Malik, S. Bilal, T. Salahuddin, and K. U. Rehman, “Three-dimensional Williamson fluid flow over a linear stretching surface,” Math. Sci. Lett. 6(1), 53–61 (2017).

38. T. Salahuddin, M. Y. Malik, A. Hussain, S. Bilal, and M. Awais, “Combined effects of variable thermal conductivity and MHD flow on pseudoplastic fluid over a stretching cylinder by using Keller box method,” Inf. Sci. Lett. 5, 11 (2016).