Extra Dimensions, Isosinglet Charged Leptons and Neutrino Factories.

G.C. Branco,1,* D. Delépine,1,† B. Nobre,1,‡ and J. Santiago2,§

1Centro de Física das Interacções Fundamentais (CFIF), Departamento de Física, Instituto Superior Técnico, Av.Rovisco Pais, 1049-001 Lisboa, Portugal

2Departamento de Física Teórica y del Cosmos and Centro Andaluz de Física de Partículas Elementales (CAFPE), Universidad de Granada, E-18071 Granada, Spain

Abstract

Isosinglet fermions naturally arise in a variety of extensions of the Standard Model, in particular in models with extra dimensions. In this paper, we study the effect of the addition of a new isosinglet charged lepton to the standard spectrum, with special emphasis on implications for neutrino asymmetries to be measured at future neutrino factories. Lepton flavour violation in neutral current and lepton universality constraints are extensively discussed. We show that new physics effects in $\nu_e - \nu_\mu$ CP asymmetries are significantly enhanced due to leptonic maximal mixings but still too small to give a signature at future neutrino factories. A signal for CP asymmetries in $\nu_\mu - \nu_\tau$ channel due to new physics could be observed at $1 - 3 \sigma$ if lepton flavour violating $\tau$ decays are seen in a very close future in $B$-factories like BELLE experiment.
I. INTRODUCTION

Two of the most exciting recent developments in particle physics are, on one hand, the experimental evidence for neutrino oscillations [1, 2] pointing towards non-vanishing neutrino masses and mixings and, on the other hand, the suggestion of models with more than three spatial dimensions [3] which have the attractive feature of addressing, within a novel framework, long standing puzzles like, for example, the gauge hierarchy problem and the elementary fermion mass spectrum.

In this paper, we analyse some of the phenomenological consequences of isosinglet charged leptons which naturally arise in some of the models with extra-dimensions, discussing in particular the interplay between the study of lepton flavour violating rare processes and new physics effects observable in $CP$ asymmetries at neutrino factories[4].

In models with extra-dimensions, the use of new techniques is accompanied by a very interesting phenomenology, with a large number of new particles populating the spectrum at energies typically of the order of the compactification scale. Extra-dimensional models represent then a very interesting framework for physics beyond the SM, with the Kaluza-Klein (KK) excitations of the bulk fields being extra gauge bosons, fermions or scalars.

Arkani-Hamed and Schmaltz [5] have proposed in the context of these theories the split fermion idea, which stands for the use of the localization properties of bulk fermion zero modes to suppress coefficients without the need of symmetries protecting the corresponding operators. One particularly interesting application of this idea is a natural realisation of the observed pattern of fermion masses and mixing angles [6, 7]. Recently, it was shown in [7] that, under very general circumstances, it is possible to decouple the mass scale of the fermion KK excitations from the compactification scale by means of multi-localizing the fermion zero modes. In this way it was possible to naturally realise the observed spectrum of quark masses and mixing angles in a model with a multi-brane scalar background, a compactification scale $M_c \sim 100$ TeV $^1$ and still have a light KK excitation of the right-handed (RH) top quark with observable phenomenological consequences at present or future colliders.


\footnote{This value is consistent with the bounds on flavour changing neutral currents in these models [8].}
In the class of models described in [7], the RH component of the heaviest fermions are naturally multi-localized. Thus they typically have light (vector-like) KK excitations mixing mainly with them. One extra ingredient appearing in these models is that the SM fermion gauge couplings are modified due to mixing with their KK excitations, with corrections which are proportional to the masses of the SM fermions. We will apply the above scenario to the leptonic sector [6, 7], analysing in detail the phenomenological consequences of the isosinglet charged leptons which naturally arise in this class of models. The most natural situation is to have a multi-localized RH tau lepton with a light isosinglet KK excitation mixing mainly with it. We will see in the sequel that, for phenomenological reasons, it is crucial that the new vector-like charged lepton mixes mainly with the tau.

The analysis of new physics effects in the leptonic sector, is specially relevant in view of the various experimental projects designed to uncover the neutrino world, in particular the planned neutrino factories [4] which will measure neutrino masses and mixings with great precision and eventually $CP$ violation in the leptonic sector. Furthermore, neutrino factories have the potential to uncover new physics, which can arise in a variety of extensions of the Standard Model (SM), such as models with vector-like neutrinos [9]. In the class of models with vector-like charged leptons which we consider in this paper, we will find explicit realizations of the model independent results previously found [10, 11].

The paper is organised as follows. In the next section, we present a general description of models with extra isosinglet charged leptons, with special emphasis on lepton-flavour-violating processes and introduce the structure motivated by models with extra dimensions. The results of this section are model independent and therefore relevant for any extension of the SM with isosinglet charged leptons. In section III, we discuss the phenomenological implications of isosinglet charged leptons on neutrino oscillations and in particular on $CP$ asymmetries at neutrino factories. In section IV, we present our main results while our conclusions are summarised in section V. We end the paper with an appendix where the KK description of theories with extra dimensions is reviewed.
II. GENERAL DESCRIPTION

In this section, first we recall some well-known features of models with vector-like particles, in particular the structure of the neutral and charged flavour-changing leptonic currents. Secondly, the constraints coming from rare lepton-flavour violating decays and lepton universality are presented. Finally, the extra-dimensional framework for isosinglet charged lepton is described.

A. Charged lepton masses and mixings.

Let us consider the addition of a vector-like charged lepton, singlet under $SU(2)_L$, to the SM spectrum. The mass Lagrangian for the charged leptons can be written with complete generality in the current eigenstate basis as

$$\mathcal{L}_m = \left( \begin{array}{c} l_L' \\ L_L \end{array} \right) \left( \begin{array}{cc} m_l & J \\ 0 & M \end{array} \right) \left( \begin{array}{c} l_R \\ L_R \end{array} \right) + h.c.,$$  \hspace{1cm} (1)

where $m_l$ is a $3 \times 3$ mass matrix and $J$ describes the mixing between the first 3 families and the vector-like charged lepton. $L_{L,R}$ are the Left-Handed (LH) and Right-Handed (RH) components of the isosinglet charged lepton and $l_L', l_R$ the usual LH and RH charged lepton fields respectively ($e, \mu, \tau$). Note that the $(1 \times 3)$ zero matrix in eq.(1) corresponds to an allowed choice of weak-basis and does not imply any loss of generality. The matrices $m_l$ and $J$ are $\Delta I = 1/2$ mass terms, therefore proportional to the SM Higgs vacuum expectation value while $M$ is an $SU(2)_L \otimes U(1)_Y$ invariant mass terms, which can be arbitrarily large, since it is not protected by the low energy gauge symmetry.

Before diagonalising the charged lepton mass matrix, let us have a look at the neutrino mass sector. We shall assume that naturally small LH neutrino Majorana masses are generated through the breaking of lepton number at high energy. It is well known that the seesaw mechanism provides one of the most attractive scenarios for generating small LH neutrino masses [12]. However, for our discussion, the detailed origin of neutrino masses is not important. The Majorana mass term for the 3 light
LH neutrinos can be written as

\[ \mathcal{L}_{\nu}^{\Delta L=2} = \frac{1}{2} \overline{\nu}_{L} M_{\nu} \nu_{L} + h.c., \]  

(2)

with \( M_{\nu} \) a 3 × 3 symmetric matrix which can be diagonalised by a unitary transformation \( U_{\nu} \)

\[ U_{\nu}^{T} M_{\nu} U_{\nu} \equiv \text{diag}(m_{\nu_e}, m_{\nu_{\mu}}, m_{\nu_{\tau}}). \]  

(3)

We can go to the mass eigenstate basis for the LH neutrinos by making the corresponding unitary transformation on the lepton doublets

\[ \begin{pmatrix} \nu_{L}^{m} \\ l_{L} \end{pmatrix} = U_{\nu}^{\dagger} \begin{pmatrix} \nu_{L} \\ l'_{L} \end{pmatrix}. \]  

(4)

Of course, eq.(1) is not invariant under this transformation. In the neutrino mass eigenstate basis, the mass matrix for the charged leptons is given now by

\[ \mathcal{L}_{m} = \left( \mathbf{T}_{L} \mathbf{T}_{L} \right) \begin{pmatrix} U_{\nu}^{\dagger} m_{l} & U_{\nu}^{\dagger} J \\ 0 & M \end{pmatrix} \begin{pmatrix} l_{R} \\ L_{R} \end{pmatrix} + h.c. \equiv (\bar{\mathbf{T}}_{L}) M_{l}(l_{R}). \]  

(5)

It may seem not necessary to discuss the neutrino sector at this point. Indeed, without loss of generality, we could have started our discussion with eq. (1) in the physical basis for neutrinos. However, models where vector-like fermions naturally appear (e.g. models with extra-dimensions) often predict some textures for \( J, m_{l} \) or \( M_{\nu} \). In such cases, it is convenient to work with eq.(5) where the constraints on the structure of \( J, m_{l} \) and \( M_{\nu} \) can be implemented in a straightforward way.

The charged lepton mass matrix \( M_{l} \) is diagonalized by the unitary transformations \( U_{L} \) and \( U_{R} \)

\[ U_{L}^{\dagger} M_{l} U_{R} = \text{diag}(m_{e}, m_{\mu}, m_{\tau}, M_{D}), \]  

(6)

with the mass eigenstates \( l_{L,R}^{m} \) given by

\[ (l_{L,R}) = U_{L,R}(l_{L,R}^{m}). \]  

(7)

The left rotation \( U_{L} \) can be written as [13, 14]

\[ U_{L} = \begin{pmatrix} U_{\nu}^{\dagger} K & U_{\nu}^{\dagger} J/M \\ -J^{\dagger} K/M & 1 \end{pmatrix} + O \left( \frac{m_{l}}{M} \right)^{2}, \]  

(8)

where \( K \) is the unitary matrix which diagonalizes \( m_{l} m_{l}^{\dagger} \).
B. Z-mediated flavour changing neutral current interactions.

The leptonic neutral current gauge interaction reads, in the weak eigenstate basis,

\[ \mathcal{L}_Z = \frac{g}{\cos \theta_w} Z_\mu (J_3^\mu - \sin^2 \theta_w J_{em}^\mu), \]  

with

\[ J_3^\mu = \frac{1}{2} \bar{\nu} L \gamma^\mu \nu L - \frac{1}{2} \bar{\tau} L \gamma^\mu \tau L, \]  

\[ J_{em}^\mu = - (\bar{\tau} \gamma^\mu \tau + \bar{L} \gamma^\mu L). \]

In the mass eigenstate basis, for the light charged leptons, one gets

\[ \mathcal{L}_{Z}^{\text{light}} = - \frac{g}{2 \cos \theta_w} Z_\mu \tilde{m}_l^m \left[ \delta_{ik} (1 - 2 \sin^2 \theta_w) - \beta_{ik} \right] \gamma^\mu \tilde{m}_L^k, \]

with \( i, k = 1, 2, 3 \), \( \beta_{ik} \equiv U^*_{Lai} U_{Lak} \). The effect of the vector-like singlet is the appearance of the flavour changing neutral couplings (FCNC) \( \beta_{ik} \) at tree level. Note that at first order in \( m_l^2/M^2 \),

\[ \beta_{ij} = \frac{J_i K_{ik}^* J_k^* K_{kj}}{M^2}, \]

and thus these couplings are naturally suppressed if \( J_i \ll M \).

C. Higgs mediated FCNC.

The interaction between charged leptons and the neutral Higgs boson is given, in the weak eigenstate basis, by

\[ \mathcal{L}_{H} = - \frac{g}{2 M_W} \left[ \bar{l}_L m_l l_R + \bar{l}_L J l_R + \text{h.c.} \right] H^0. \]

After diagonalisation of the mass matrix, the Lagrangian for the light charged leptons becomes

\[ \mathcal{L}_{H}^{\text{light}} = - \frac{g}{2 M_W} \left[ \tilde{m}_l^m (m_i \delta_{ik} - M U^*_{Lai} U_{LRk}) l_R^m + \text{h.c.} \right] H^0, \]

where \( i, k = 1, 2, 3 \) and \( m_{1,2,3} \equiv m_{e,\mu,\tau} \). The interaction with the pseudoscalar neutral field \( \chi \) is given by

\[ \mathcal{L}_{\chi} = - \frac{ig}{2 M_W} \left[ \bar{l}_L m_l l_R + \bar{L}_L J l_R - \text{h.c.} \right] \chi. \]

Proceeding in the same way as for the neutral Higgs scalar, the interaction with the pseudoscalar Higgs field \( \chi \) is given by

\[ \mathcal{L}_{\chi}^{\text{light}} = - \frac{ig}{2 M_W} \left[ \tilde{m}_l^m (m_i \delta_{ik} - M U^*_{Lai} U_{Rak}) l_R^m + \text{h.c.} \right] \chi. \]
D. Flavour changing charged current interactions.

The interaction with the $W^\pm$ is given as usual by

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} (W^- \mu^+ + h.c.),$$

(18)

with

$$J^{\mu^+} = \bar{L}_L \gamma^\mu \nu_L.$$  

(19)

In the mass eigenstate basis, the charged current interaction reads

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} (W^- \tilde{T}_L^m U_{L\alpha k}^\dagger \gamma^\mu \nu_L^m + h.c.),$$

(20)

with $\alpha = 1 \ldots 4$ and $k = 1 \ldots 3$. The $V_{MNS}$ mixing matrix is therefore $4 \times 3$ and given by

$$(V_{MNS})_{\alpha k} \equiv (U_L^\dagger)_{ak}.$$  

(21)

E. Limits from rare lepton flavour changing decays and lepton universality.

Lepton-flavour violating processes are strongly constrained by the experimental limits on rare $\mu$ and $\tau$ decays (see for instance Table I). In the present model, $Z$-mediated FCNC induce tree level corrections to processes like $\mu \rightarrow 3e$ or $\mu \rightarrow e$ in $^{48}\text{Ti}$ while $\mu \rightarrow e\gamma$ receives corrections at one-loop level. We shall consider first the limits coming from tree-level rare tau and muon flavour changing decays and from gauge coupling lepton universality. Limits coming from loop-induced processes like $\mu \rightarrow e\gamma$ will be discussed afterwards.

| Process | Limit |
|---------|-------|
| $Br(\mu \rightarrow e\gamma)$ | $< 1.2 \times 10^{-11}$ [15] |
| $Br(\mu \rightarrow 3e)$ | $< 1.0 \times 10^{-12}$ [16] |
| $Br(\mu \rightarrow e$ in $^{48}\text{Ti}$) | $< 6.1 \times 10^{-13}$ [17] |
| $Br(\tau \rightarrow 3e)$ | $< 7.8 \times 10^{-7}$ [18] |
| $Br(\tau \rightarrow 3\mu)$ | $< 8.7 \times 10^{-7}$ [18] |
1. \( \tau \) and \( \mu \) rare flavour changing decays and lepton universality

The restrictions on rare lepton decays can be translated to indirect bounds on lepton flavour violating branching ratios for \( Z \) decays [19], which are directly connected to the parameters of the Lagrangian. For instance the branching ratio for \( Z \to l_i l_j \) is approximately given, for \( i \neq j \), by

\[
Br(Z \to l_i l_j) \simeq \frac{1}{8} |\beta_{ij}|^2.
\]  

(22)

Using the current experimental limits, one obtains

\[
|\beta_{\mu e}| \leq 1.1 \times 10^{-6} \text{ from } (\mu \to 3e),
\]
\[
|\beta_{\mu e}| \leq 4.0 \times 10^{-7} \text{ from } (\mu \to e \text{ in } ^{48}\text{Ti}),
\]
\[
|\beta_{\tau e}| \leq 2.3 \times 10^{-3} \text{ from } (\tau \to 3e),
\]
\[
|\beta_{\tau \mu}| \leq 2.4 \times 10^{-3} \text{ from } (\tau \to 3\mu).
\]  

(23)

Lepton universality sets limits on the diagonal elements \( \beta_{ii} \). Indirect bounds from unitarity loss in charged currents have been computed for the case of vector-like neutrinos [9] but give much less stringent constraints than direct violations of universality appearing in the model with extra isosinglet charged leptons. Using the one sigma deviations for the effective charged lepton couplings, \( g_{V,A}^{e,\mu,\tau} \), to the \( Z \) [20] as an estimation of the allowed new physics contribution we get

\[
|\beta_{ee}| \leq 0.0007,
\]
\[
|\beta_{\mu\mu}| \leq 0.0011,
\]
\[
|\beta_{\tau\tau}| \leq 0.0013.
\]  

(24)

The last two limits are restrictive enough to impose a bound on the \( \beta_{\mu\tau} \) coefficient stronger than the one obtained from rare tau decays

\[
|\beta_{\mu\tau}| = \sqrt{|\beta_{\mu\mu}| |\beta_{\tau\tau}|} \leq 1.2 \times 10^{-3}.
\]  

(25)

These limits can be translated to constraints on the elements of the charged lepton mass matrix making further assumptions. As examples, we shall consider two theoretically motivated ansätze for \( J_i \).

Case A
Given the fact that experimental constraints are much more stringent for the first families than for the heavier ones and that both the entries in the mass matrix $J$ and the standard lepton masses arise from Yukawa couplings, a reasonable ansatz for the values of the $J_i$ is \[ J_i \sim \frac{m_i}{m_l}. \] Using eq.(26) and assuming that the mixing induced by $K$ is negligible, it is possible to extract limits on $J_i/M$ from the experimental bounds given in eqs.(23,24), the strongest bounds being

\[ S_1 \equiv \frac{J_1}{M} \leq 4.4 \times 10^{-5}, \]
\[ S_2 \equiv \frac{J_2}{M} \leq 0.91 \times 10^{-2}, \] (27)
\[ S_3 \equiv \frac{J_3}{M} \leq 0.036. \]

These constraints come from $\mu \to e$ conversion in nuclei for the first two families and from lepton universality for the third one.

**Case B**

It is however possible to use a different ansatz, motivated by models with extra dimensions [7, 22] reviewed in the Appendix, which leads to a natural suppression for the corrections to the couplings of the first family. Let us assume that in the weak basis, the vectors $J_i$ and $(m_l)_{i3}$ are parallel,

\[ J_i = \lambda (m_l)_{i3}, \] (28)

with $\lambda$ a flavour independent constant. In this situation we can use the following property to compute the value for the $\beta_{ij}$ parameters,

\[ (J^\dagger K)^*_i = K^\dagger_{ij} J_j = \lambda K^\dagger_{ij} (m_l)_{i3} = \lambda m_i (K^R)_{i3} + O \left( \frac{m_l^2}{M} \right)^2, \] (29)

with no summation on the index $i$ and where $m_i$ are the mass eigenvalues of the charged leptons and $K^R$ is defined by

\[ K^\dagger m_l K^R \equiv diag(m_e, m_\mu, m_\tau) + O \left( \frac{m_l^2}{M} \right)^2. \] (30)

Then, we can write

\[ \beta_{ij} = \lambda^2 \frac{m_i (K^R)_{i3} K^R_{3j} m_j}{M^2}. \] (31)
The ratio of FCNC processes is then not only suppressed by masses but also by mixing angles. For instance,
\[
\frac{|\beta_{\mu e}|}{|\beta_{\mu \tau}|} = \frac{m_e |K_{3e}^R|}{m_\tau |K_{3\tau}^R|},
\] (32)
and, apart from the mass suppression, there is an extra suppression by mixing angles if \( |K_{3e}^R| \ll |K_{3\tau}^R| \).

2. \( \mu \rightarrow e\gamma \)

This decay is induced at one-loop level, the constraints coming from this process are thus expected to be less stringent than the ones arising from the processes studied in the previous section. Nevertheless, the experimental progress on \( \mu \rightarrow e\gamma \) expected in the next few years [23], makes it worth studying it in detail. The branching ratio for the transition \( \mu \rightarrow e\gamma \) has the form
\[
Br(\mu \rightarrow e\gamma) = 384\pi^3 \frac{\alpha v^4}{m_\mu^2} |A(m_\mu^2)|^2,
\] (33)
where \( v = (8G_F^2)^{-1/4} \simeq 174 \text{ GeV} \) is the Higgs vacuum expectation value, \( \alpha \) is the fine structure constant and \( A(q^2) \) the form factor coming from the one-loop computation. At one-loop order, the transition amplitude is induced by 3 different classes of Feynmann diagrams respectively due to \( W, Z \) and Higgs exchange. For diagrams with \( W \) exchange, the photon line is attached to the \( W \)'s. This diagram is suppressed in the SM by neutrino masses whereas in our model, due to the non-unitarity of the \( V_{MNS} \), it gives an independent contribution. Let us consider the form factors arising from these three kinds of diagrams\(^2\)
\[
A = A_W + A_Z + A_H.
\] (34)

We evaluate these form factors in the limit \( m_{l,\nu}^2/m_{W,Z,H}^2 \rightarrow 0 \) for the light families and we assume that in our model \( M_D \gg m_{W,Z,H} \) for the isosinglet charged lepton contribution \(^3\). They read, in this limit,

\(^2\) See ref.[24] for details on the form factor computation.

\(^3\) Indeed, experimental search on heavy charged lepton imposes \( M_D > 100.8 \text{ GeV} \) [25].
The branching ratio for $\mu \to e\gamma$ is then given by

$$Br(\mu \to e\gamma) \simeq \frac{6\alpha}{\pi} |U_{L4\mu} U_{L4e}^*|^2 \left( \frac{1}{24} + \frac{13}{48} \frac{m_Z^2}{M_D^2} + \frac{1}{6} \frac{m_H^2}{M_D^2} \right)^2,$$

(38)

Using the strongest limit on $|U_{L4\mu} U_{L4e}^*|$ implied by eqs.(23) and the fact that $M_D^2/m_Z^2,m_H^2,m_Z^2,M_D^2 \gtrsim 1$, this branching ratio is bounded by

$$Br(\mu \to e\gamma) \leq 5.5 \times 10^{-16},$$

(39)

far below the sensitivity of current and planned experiments. As expected, $\mu \to e\gamma$ does not introduce any significant restriction, in a model with isosinglet charged leptons.

F. Extra-dimensional framework for isosinglet charged leptons

As we have emphasized in the introduction, theories with extra dimensions provide an exciting arena for the study of physics beyond the SM. It has been recently shown that models with multi-brane backgrounds \cite{7} naturally present light isosinglet fermions which correspond to the first KK excitations of the heavier SM weak singlets ($\tau_R$ in the leptonic sector). We review in the appendix the details of the KK description of the theory, the relevant features at this point being the low energy spectrum and the form of the Yukawa couplings. In the case of interest, the spectrum consists on the fermion zero modes, which are the SM fermions, plus one relatively light (as compared to the compactification scale) isosinglet charged lepton, mixing mainly with the tau. The rest of the spectrum have masses of the order or the compactification scale which decouples from the low energy physics. The other feature relevant for our discussion is the form of the Yukawa couplings. We take the scalar field responsible for the electroweak symmetry breaking to live in a four-dimensional
boundary. The effective Yukawa coupling for the $n$–th and $m$–th modes satisfy the following factorisation property

$$
\lambda_{ij}^{(nm)} = \frac{\lambda_{ij}^{(5)e}}{\pi R} f_L^{(n)}(0) f_R^{e_j(m)}(0),
$$

(40)

where $\lambda_{ij}^{(5)e}$ is the five-dimensional Yukawa coupling and, for definiteness, we have taken the Higgs field to live in the $y = 0$ boundary. Notice that this factorisation property implies the relation in eq.(28), with

$$
\lambda = \frac{f_R^{e_3(1)}(0)}{f_R^{e_3(0)}(0)}.
$$

(41)

As we have seen above, this property has a very relevant feature from the phenomenological point of view, which is the fact that corrections on the fermion zero mode gauge couplings due to the mixing with the heavy vector-like excitations scale with the masses of the fermion zero modes and are thus only relevant for the heavy fermions [22].

It is now clear how the hierarchical pattern of charged lepton masses is generated. Starting with five-dimensional (thus dimensionful) Yukawa couplings of natural order, $\lambda_{ij}^{(5)e} \sim \pi R$, we can generate effective Yukawa couplings of the appropriate size modifying the localization properties of the different fields and thus their overlapping with the Higgs boundary. Given the relatively large mass of the tau lepton it is natural to have its RH component strongly localized near the Higgs boundary and thus multi-localized (see the Appendix) in the set-up we are considering.

If we ask for the RH tau lepton to be strongly multi-localized, so that it will have a light KK excitation, then it is natural to have all mixing angles in the charged leptonic sector small but the one in the $(2, 3)$ LH sector, which can vary, depending on the specific values of the parameters, from moderately small to maximal. In the first case all bi-maximal mixing has to be generated from the neutrino sector (what can be easily accomplished in the model we have [26]) whereas in the second one only the $(1, 2)$ large mixing comes from the neutrino sector.

$^4$ Notice that from the tiny neutrino masses we could expect the third family doublet not to be too strongly localized at the Higgs boundary, thus the mass of the tau being essentially generated by the order one overlapping of its RH component.
III. EFFECTS OF VECTOR-LIKE CHARGED LEPTONS ON \( \nu \) OSCILLATIONS

A. General parametrisation

The new interactions generated by the extra vector-like charged lepton have important effects on neutrino oscillations. Similar effects occur in the charged current sector in models with extra isosinglet neutrinos \cite{9, 27} while neutral current processes could allow to discriminate between them. In this section we compute the effects of the vector-like charged leptons on neutrino oscillations.

As we have seen in the previous sections, in the case of one vector-like charged lepton, the leptonic mixing in charged currents is described by a \( 4 \times 3 \) matrix \( V_{MNS} \). The mixing among the light families is given by a non-unitary \( 3 \times 3 \) submatrix of \( V_{MNS} \) which, using that deviations from unitarity are necessarily small, we will separate in two terms

\[
(V_{MNS})_{ik} = (U_{MNS})_{ik} + (\delta U)_{ik}, \quad i, k = 1, 2, 3,
\]

where \( U_{MNS} \) is a \( 3 \times 3 \) unitary matrix which accounts, at leading order, for the mixing among the light families \cite{28} and \( \delta U \) describes the small deviations from unitarity. In the following we consider the mass eigenstate basis for the charged leptons and the flavour basis for neutrinos, defined as

\[
\nu_f^L \equiv U_{MNS} \nu_m^L.
\]

The charged current interaction reads in this basis, for the light leptons,

\[
\mathcal{L}_W^{\text{light}} = \frac{g}{\sqrt{2}} \left[ W^\mu \bar{l}^m_L \left( \delta_{ik} + A_{ik}^{NP} \right) \gamma^\mu \nu_f^L + h.c. \right] \equiv \mathcal{L}_W^{SM} + \mathcal{L}_W^{NP},
\]

where \( \mathcal{L}_W^{SM} \), the term proportional to the identity, is equivalent to the SM charge current interaction and \( \mathcal{L}_W^{NP} \) describes the New Physics effects due to the isosinglet charged lepton with coefficient \( A_{ik}^{NP} = \delta U_{ij} (U_{MNS}^\dagger)_{jk} \).

In order to compute the effect of New Physics, we need to know \( U_{MNS} \) and \( \delta U \). For that, we have to diagonalise the charged lepton mass matrix given in eq.(5). To define the SM families, we shall proceed in 2 steps: first, \( M_l \) is diagonalised in blocks so that the heavy charged lepton mass eigenstate is defined. In the second step we make a unitary transformation involving only the light charged leptons, which completes the
diagonalization. This stepwise diagonalisation corresponds to making the following
sequence of rotations, for the LH sector,

\[ U_L \equiv R_{34} \tilde{R}_{24} \tilde{R}_{14}^\dagger R_{12}^\dagger \tilde{R}_{13}^\dagger R_{23}^\dagger, \quad (45) \]

where

\[ \tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R_{ij} \equiv \tilde{R}_{ij}(\delta_{ij} = 0). \quad (46) \]

An explicit form for \( U_L \) can be found in ref.[29]. It is easy to check that making the
rotation in the order defined in eq.(45), we have

\[ U_L^\dagger M_i M_j^\dagger U_L = R_{23} \tilde{R}_{13} R_{12}^\dagger \tilde{R}_{14}^\dagger \tilde{R}_{24}^\dagger R_{34} M_i M_j^\dagger R_{34} \tilde{R}_{24} \tilde{R}_{14}^\dagger R_{12}^\dagger \tilde{R}_{13}^\dagger R_{23}^\dagger 
= R_{23} \tilde{R}_{13} R_{12} \left( \begin{array}{cc} m_i^2 & 0 \\ 0 & M_D^2 \end{array} \right) R_{12}^\dagger \tilde{R}_{13}^\dagger R_{23}^\dagger. \quad (47) \]

Thus \( R_{23} \tilde{R}_{13} R_{12} \) is the mixing matrix connecting the 3 light families. It is important
to recall that since \( M_D \gg J \gg m_{\tau,\mu,e} \), the mixing angles \( \theta_{i4} \) \( (i = 1, 2, 3) \) are well
approximated by

\[ \theta_{i4} e^{i\delta_{i4}} \approx \frac{(U_L^\dagger J)_i}{M}. \quad (48) \]

With this prescription, the \( U_{MNS} \) matrix as defined by eq.(42) is given by

\[ (U_{MNS})_{ik} = (R_{23} \tilde{R}_{13} R_{12})_{ik}, \quad (49) \]

and the New Physics contribution by

\[ A_{ik}^{NP} = \delta U_{ij}(U_{MNS}^\dagger)_{jk} = \left( (U_{MNS} \tilde{R}_{14}^\dagger \tilde{R}_{24}^\dagger R_{34}^\dagger U_{MNS}^\dagger) \right)_{ik} - \delta_{ik}. \quad (50) \]

Using the above parametrisation, \( U_{MNS} \) can be expressed as

\[ U_{MNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}e^{-i\delta_{13}} & s_{13}e^{-i\delta_{13}} \\ -c_{13}e^{i\delta_{13}} & c_{12}s_{13} & c_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}e^{i\delta_{13}} + s_{12}s_{23} & -c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} P, \quad (51) \]

where \( P = \text{diag}(1, e^{i\beta_1}, e^{i\beta_2}) \). The phases \( \beta_{1,2} \) are Majorana-type phases which will not play any rôle in our discussion. The New Physics contribution can be also explicitly
written as

\[ A^{NP} = U_{MNS} \begin{pmatrix} c_{14} & -s_{14}s_{24}e^{-i(\delta_{14} - \delta_{24})} & -s_{14}s_{34}c_{24}e^{-i\delta_{14}} \\ 0 & c_{24} & -s_{24}s_{34}e^{-i\delta_{24}} \\ 0 & 0 & c_{34} \end{pmatrix} U_{MNS}^\dagger - I, \quad (52) \]

with \( I \) denoting the \( 3 \times 3 \) identity matrix.

Note that the full \( 4 \times 3 \) \( V_{MNS} \) matrix contains, apart from \( \delta_{13} \), two extra Dirac-type \( CP \) violating phases. These phases appear always in combination with the small mixing angles \( \theta_{i4} \). The appearance of these two extra Dirac-type phases has to do with the fact that the \( 3 \times 1 \) column matrix \( J \) is complex, with one of the phases eliminated through the rephasing of the isosinglet charged lepton field [13].

Once we have separated the New Physics contributions from the SM Lagrangian, we can use the formalism described in [10, 11] to compute the effect of this new interaction on neutrino asymmetries.

In neutrino factories, neutrinos are usually produced through muon decay. Therefore, assuming that we have a \( \mu^+ \) beam, typically the production process of a neutrino state, \( \nu_\alpha \), in conjunction with a \( e^+ \) is given by

\[ \mu^+ \rightarrow e^+ \nu_\alpha \bar{\nu}_\mu. \quad (53) \]

The production of a neutrino state \( \nu_e \) in conjunction with a \( e^+ \) is also possible through the process

\[ \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\alpha. \quad (54) \]

The detector process, for a wrong sign event, is \( \nu_\rho d \rightarrow \mu^- u \). Between production and detection, the oscillations of \( \nu_\alpha \) from (53) or of \( \nu_e \) from (54) into a \( \nu_\rho \) can take place. The important point to note is that only the process given in (53) is able to interfere with the SM process, \( \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \), due to the fact that they have the same initial and final states \( \mu^+, e^+ \) and \( \bar{\nu}_\mu \). So in the sequel, we shall neglect the contribution coming from process (54) as it does not interfere with SM amplitude, \( i.e. \) its effects are higher order in New Physics compared to process (53).

This approach can be generalised to any flavour and to any production and detection processes [11]. Consider the Lagrangian responsible for the production of a \( \nu_\beta \) in conjunction with a \( l_\alpha^+ \),

\[ \mathcal{L}^s = 2\sqrt{2}G_F (\delta_{\alpha\beta} + \epsilon_{\alpha\beta}^s) (\bar{\nu}_\mu P_L \nu_\mu) (\bar{\nu}_\beta \gamma_\mu P_L l_\alpha), \quad (55) \]
where $P_L$ is defined as usual by $(1 - \gamma_5)/2$. According our notation,

$$\epsilon_{\alpha \beta}^s = (A^{NP*})_{\alpha \beta}. \quad (56)$$

For the detection of a $\nu_\beta$ signalled in the detector by a $l_\alpha^-$, the similar 4 Fermi interaction enters

$$\mathcal{L}^d = 2\sqrt{2}G_F(\delta_{\alpha \beta} + \epsilon_{\alpha \beta}^d) (\overline{\tau}_\alpha \gamma^\mu P_L d) (\overline{\nu}_\mu P_L \nu_\beta). \quad (57)$$

with

$$\epsilon_{\alpha \beta}^d = A_{\alpha \beta}^{NP} \Rightarrow \epsilon^d = \epsilon^{s*}. \quad (58)$$

As done in ref.[10], we define $\nu_e^s$ as the neutrino state that is produced in the source in conjunction with an $e^+$, the production process being $\mu^+ \to e^+ \nu_\alpha \overline{\nu}_\mu$, and $\nu_\mu^d$ the neutrino state that is signalled by $\mu^-$ production in the detector, the detector process being $\nu_\alpha d \to \mu^- u$. The detector process for a wrong sign event is interpreted as due to the oscillation of $\nu_e$ into $\nu_\mu$.

$$|\nu_e^s\rangle = \sum_i \left( (1 + \epsilon_{ee}^s) (U_{MNS})_{ei}^* + \epsilon_{e\mu}^s (U_{MNS})_{\mu i}^* + \epsilon_{e\tau}^s (U_{MNS})_{\tau i}^* \right) |\nu_i^m\rangle$$

$$\equiv [(I + \epsilon^s) U_{MNS}^*]_{ei} |\nu_i^m\rangle, \quad (59)$$

$$|\nu_\mu^d\rangle = \sum_i \left( (1 + \epsilon_{\mu\mu}^d) (U_{MNS})_{\mu i}^* + \epsilon_{\mu e}^d (U_{MNS})_{ei}^* + \epsilon_{\mu\tau}^d (U_{MNS})_{\tau i}^* \right) |\nu_i^m\rangle$$

$$\equiv [(I + \epsilon^d) U_{MNS}^*]_{ei} |\nu_i^m\rangle. \quad (60)$$

We recall that the complex conjugation comes from the fact that once $U_{MNS}$ is defined in the Lagrangian as given by eq.(44), one has [30]

$$|\nu_f^d\rangle = (U_{MNS})_{ef}^* |\nu_i^m\rangle. \quad (61)$$

The observation of new physics in the detection requires not only knowledge on neutrino beams but also on quarks properties within nuclear matter [11]. In particular, present uncertainties on parton distribution and hadronisation for the range of neutrino energy will make very difficult to distinguish the effects of new lepton flavour violating physics from other sources of uncertainties or new physics. So, in the following part of the paper, we shall only compute the effects of lepton flavour changing interactions coming from the production processes which is purely leptonic.
and neglect their effects in detection processes. Of course, our analysis can be trivially extended to include flavour changing effects in detection processes.

The $CP$ asymmetries in vacuum and in matter can be computed using the standard procedure.

**B. Oscillations in vacuum**

It is well known that in the framework of the SM, there is no $CP$ violation in neutrino oscillations in the limit $s_{13} \to 0$. Of course, in the presence of physics beyond the SM, this is not true anymore. Therefore, it is interesting to study what happens in our model when $s_{13} \to 0$, answering the question whether even in this limit, one can expect to observe $CP$ violation in neutrino oscillations in models with extra isosinglet charged leptons.

It should be recalled that FCNC impose strict constraints on the $(U_L)^4_i \equiv S_i$ elements of the $U_L$ lepton mixing matrix. Using the unitarity of $U_L$ and eq.(48), we can relate the angles $\theta_{ij}$ to the $S_i$ coefficients

$$\theta_{ij} e^{i\delta_{ij}} \simeq - (U_{\mu}^\dagger K)_{ij} S_j^\dagger \simeq - (U_{MNS}^\dagger)_{ij} S_j^\dagger.$$  \hfill (62)

The stringent bounds on new contributions to the first family couplings require $S_1 \ll S_{2,3}$. We therefore consider the following texture for $S$

$$S = (0, S_2, S_3).$$  \hfill (63)

In such a case, $\theta_{ij}$ read

$$\theta_{e4} e^{i\delta_{e4}} = s_{12} (c_{23} S_2^* - s_{23} S_3^*) + c_{12} s_{13} e^{-i\delta_{13}} (s_{23} S_2^* + c_{23} S_3^*),$$  \hfill (64)

$$\theta_{\mu4} e^{i\delta_{\mu4}} = -c_{12} (c_{23} S_2^* - s_{23} S_3^*) + s_{12} s_{13} e^{-i\delta_{13}} (s_{23} S_2^* + c_{23} S_3^*),$$  \hfill (65)

$$\theta_{\tau4} = -c_{13} (s_{23} S_2^* + c_{23} S_3^*).$$  \hfill (66)

It is interesting to note from the above equations that as expected when $s_{12} \to 0$, $\theta_{e4} \to S_1 \equiv (U_L)_{4e}$ which is strongly constrained by lepton flavour violating muon decays. But if $s_{12} \approx c_{12} \approx O(1)$, $\theta_{e4}$ is dominated by $S_2$ and $S_3$ contributions which are much less constrained than $S_1$ (see eqs.(27)).
As discussed before, the model with one extra isosinglet charged lepton has two additional phases. For simplicity, we shall choose the phases of $S_2$ and $S_3$ such that $\theta_{\tau 4}$ is real ($\delta_{34} = 0$). From these equations, the following limits can be considered

(A) $s_{13} = 0$

\[
\theta_{e4} = -\tan \theta_{12} \times \theta_{\mu 4},
\]

\[
\delta_{14} = \delta_{24}.
\]

(B) $\delta_{13} = 0$ and $s_{12} = c_{12} = 1/\sqrt{2}$. We can make a perturbative expansion in $s_{13}$

\[
|\delta_{14} - \delta_{24}| \simeq \frac{2\theta_{e4}}{\theta_{e4}} s_{13} \sin \phi + O(s_{13}^2),
\]

\[
\theta_{e4,\mu 4} \simeq \pm \frac{a}{\sqrt{2}} \left( 1 \mp s_{13} \frac{\theta_{\tau 4}}{a} \cos \phi + O(s_{13}^2) \right),
\]

\[
\sin \delta_{14,24} \simeq \sin \phi \left( 1 \mp s_{13} \frac{\theta_{\tau 4}}{a} \cos \phi + O(s_{13}^2) \right),
\]

with $\phi = \arg(c_{23}S_2^* - s_{23}S_3^*)$ and $a = |c_{23}S_2^* - s_{23}S_3^*|$. The upper and lower signs in the equation for $\theta_{e4,\mu 4}$ correspond respectively to $\theta_{e4}$ and $\theta_{\mu 4}$.

Using the relation between $\epsilon^s$ and $A^{NP}$ and expanding $A^{NP}$ in terms of $\theta_{14}$, it is easy to get the expression for $\epsilon^s$ (from now on we denote $U_{MNS}$ simply as $U$)

\[
\epsilon^s_{ij} = -U_{j1}^* U_{i1}^\tau \frac{\theta_{e4}^2}{2} - U_{j2}^* U_{i2}^\tau \frac{\theta_{\mu 4}^2}{2} - U_{j3}^* U_{i3}^\tau \frac{\theta_{\tau 4}^2}{2}
\]

\[
- U_{j2}^* U_{i1}^\tau \theta_{e4} \theta_{\mu 4} e^{i(\delta_{14} - \delta_{24})} - U_{j3}^* U_{i1}^\tau \theta_{e4} \theta_{\tau 4} e^{i\delta_{14}} - U_{j3}^* U_{i2}^\tau \theta_{\mu 4} \theta_{\tau 4} e^{i\delta_{24}}.
\]

Following the formalism described in [10, 11], the dominant contribution to $CP$ asymmetries in the different channels can be easily computed. We can use directly their results just replacing $\epsilon^s_{ij}$ by the expressions given in eq.(70) and $\epsilon^d_{ij} \approx 0$.

Before studying the different channels of oscillations, let us define some notation that is used later,

\[
\Delta m_{ij}^2 \equiv m_i^2 - m_j^2,
\]

\[
\Delta_{ij} \equiv \Delta m_{ij}^2 / (2E),
\]

\[
x_{ij} \equiv \Delta_{ij} L / 2 = \frac{\Delta m_{ij}^2 L}{4E}
\]

\[
= 1.27 \frac{\Delta m_{ij}^2}{eV^2} \times \frac{L}{km} \times \frac{GeV}{E}.
\]
To get an idea of the order of magnitude of $x_{ij}$, let us evaluate them using the data on neutrino atmospheric [1, 2] ($\Delta m_{13}^2 = 3 \times 10^{-3} \text{ eV}^2$, $\tan \theta_{23} = 1$), and for $L_{GS} = 732$ km, which is the distance corresponding to the experiment CERN-Gran Sasso[31], $E = 50$ GeV and in the case of LMA solution of the solar neutrino problems (LMA parameters: $\Delta m_{12}^2 = 10^{-4} \text{ eV}^2$, $\tan \theta_{12} = 1$). For SMA, we used the parameters $\Delta m_{12}^2 = 10^{-6} \text{ eV}^2$, $\tan \theta_{12} = 7.5 \times 10^{-4}$[32, 33].

\[
x_{13} \simeq 0.056, \\
x_{12}^{\text{LMA}} \simeq 0.0019, \\
x_{12}^{\text{SMA}} \simeq 1.9 \times 10^{-5}.
\]

1. $\nu_e - \nu_\mu$ channel

In order to distinguish the characteristic signature of this kind of models, we first discuss the behaviour in different extreme cases of the $CP$ asymmetries, $A_{CP}$ defined as follows

\[
A_{CP} \equiv \frac{P_{\nu_e\to\nu_\mu} - P_{\bar{\nu}_e\to\bar{\nu}_\mu}}{P_{\nu_e\to\nu_\mu} + P_{\bar{\nu}_e\to\bar{\nu}_\mu}} \equiv \frac{P_{\nu\mu} - P_{\bar{\nu}\bar{\mu}}}{P_{\nu\mu} + P_{\bar{\nu}\bar{\mu}}}. \tag{74}
\]

To get the analytical results, we should remember that we expand $P_{e\mu}^{\text{SM}}$ to second order in $s_{13}$ and $P_{e\mu}^{\text{NP}}$ to first order in $s_{13}$ and we assume that $x_{21} \ll x_{31} \ll 1 \Rightarrow L \ll E_\mu (\text{GeV}) \times 262 \text{ km}$. $P_{e\mu}^{\text{SM}}$ is the probability of oscillation between $\nu_e - \nu_\mu$ due to SM and $P_{e\mu}^{\text{NP}}$ is the probability of oscillation $\nu_e - \nu_\mu$ due to New Physics.

Let us consider the case when $\delta_{13} = 0$. In this case all $CP$ violating phases are due to New Physics. As in ref.[10], we consider 2 different limits: one for large value of $s_{13}$ (close to the experimental bound: $|U_{e3}| \leq 0.16$ [34]) and the other for small $s_{13}$ (typically small means for instance, taking the LMA solution for solar neutrinos, $s_{13} \leq 0.01$).

In the “large” $s_{13}$ limit ($x_{21}/x_{31} \ll |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|$), the SM probability is given by

\[
P_{e\mu}^{\text{SM}} = 4x_{31}^2 |U_{e3}U_{\mu3}^*|^2. \tag{75}
\]

The New Physics CP asymmetry $A_{CP}^{NP}$ then reads \footnote{As in the limit $\delta_{13} = 0$, all elements of $U_{MNS}$ are real, we simplify the notation omitting the * in the following.}
\[ A_{CP}^{NP} \simeq -\frac{1}{x_{31}} \Im \left( \frac{\epsilon_{\mu\tau} + \epsilon_{\tau\mu}^*}{U_{e3}^* U_{\mu3}} \right) \]
\[ \simeq \frac{1}{x_{31}} \left( \left( \frac{U_{\mu2} U_{e3}}{U_{e3} U_{\mu3}} \right) \theta_{e4} \theta_{\mu4} \sin(\delta_{14} - \delta_{24}) + \theta_{\mu4} \theta_{\tau4} \sin\delta_{24} \left( \frac{U_{e2}}{U_{e3}} \right) \right) \]
\[ + \theta_{e4} \theta_{\tau4} \sin\delta_{14} \left( \frac{U_{e1}}{U_{e2} U_{\mu2}} \right) . \]  

(76)

It is important to notice that \( A_{CP}^{NP} \) seems to be enhanced by \( U_{e3}^{-1} \) factor. In fact, using eqs(68-69), it is easy to check that the \( U_{e3} \) in denominator is cancelled once we replace the \( U_{MNS} \) elements in terms of the \( \theta_{ij} \) angles.

In the “small” \( s_{13} \) limit \( (x_{21}/x_{31} \gg |(U_{e3} U_{\mu3})/(U_{e2} U_{\mu2})|) \), the SM probability of oscillation is given by

\[ P_{e\mu}^{SM} = 4x_{21}^2 |U_{e2} U_{\mu2}|^2 . \]  

(77)

For the \( CP \) asymmetry, we find

\[ A_{CP}^{NP} \simeq -\frac{1}{x_{21}} \Im \left( \frac{\epsilon_{\mu\tau} + \epsilon_{\tau\mu}^*}{U_{e3}^* U_{\mu3}} \right) \]
\[ \simeq \frac{1}{x_{21}} \left( \left( \frac{U_{e1}}{U_{e2}} \right) \theta_{e4} \theta_{\mu4} \sin(\delta_{14} - \delta_{24}) + \theta_{\mu4} \theta_{\tau4} \sin\delta_{24} \left( \frac{U_{\mu3}}{U_{\mu2}} \right) \right) \]
\[ + \theta_{e4} \theta_{\tau4} \sin\delta_{14} \left( \frac{U_{\mu3} U_{e1}}{U_{e2} U_{\mu2}} \right) . \]  

(78)

In case of bimaximal mixing, eqs.(76-78) can be computed making an expansion in \( s_{13} \) and using eqs(69):

\[ A_{CP}^{NP} \simeq -\frac{1}{\sqrt{2P_{e\mu}^{SM}}} \theta_{\tau4}s_{13} \sin \phi + O(s_{13}^2) . \]  

(79)

Both limits (“large” and “small” \( s_{13} \)) are recovered using the appropriate \( P_{e\mu}^{SM} \).

From this equation, it is easy to get the limit of \( s_{13} = 0 \).

\[ A_{CP}^{NP}(s_{13} = 0) \sim \frac{O(S_1 S_3)}{x_{21}} \approx 0 \]  

(80)

with \( i = 2, 3 \). Thus we can conclude that \( CP \) asymmetries in the \( \nu_\mu - \nu_e \) channel are unobservable for \( s_{13} = 0 \) due to the constraint on the non-observation of rare muons flavour changing decays.
2. $\nu_\tau - \nu_\mu$ channel

Let us now consider $CP$ violation in the $\nu_\tau - \nu_\mu$ channel. In this case, the interesting limit is $s_{13} \to 0 \ (\Rightarrow A_{CP}^{SM} = 0)$. The SM probability is

$$P_{\nu_\tau - \nu_\mu}^{SM} \simeq 4 \sin^2 \frac{x_{31}}{2} |U_{\mu 3}|^2 |U_{\tau 3}|^2,$$

leading to an induced asymmetry

$$A_{NP}^{CP} \simeq \frac{1}{x_{31}} \left( \left( \frac{U_{\tau 2} U_{\mu 1}}{U_{\mu 3} U_{\tau 3}} \right) \theta_{e4} \theta_{\mu 4} \sin(\delta_{14} - \delta_{24}) + \theta_{\mu 4} \theta_{\tau 4} \sin\delta_{24} \left( \frac{U_{\mu 2}}{U_{\mu 3}} \right) \right).$$

We see that the last two terms are non-vanishing in the limit $\delta_{12} \sim \delta_{24}$ and could then give observable effects at neutrino factories. Indeed, even for $s_{13} = 0$, $A_{NP}^{CP}$ can be expressed using eqs(68) as

$$A_{NP}^{CP}(s_{13} = 0) \simeq \frac{c_{23}}{s_{23}} a \theta_{\tau 4} \sin\delta_{24} \tag{83}$$

C. Oscillations in matter

In the general case, matter effects [35] have to be taken into account in neutrino oscillations. They are due to forward $\nu - e$ scattering, the Lagrangian describing them coming from the following 4-fermion interactions

$$\mathcal{L}_{matter} = 2\sqrt{2} G_F (\delta_{ae} \delta_{\beta e} + \epsilon_{\alpha \beta}^e) (\overline{\nu}_\alpha \gamma^\mu P_L e) (\overline{\nu}_\beta P_L \nu_\beta). \tag{84}$$

Using the notation given above, the different coefficients read

$$\epsilon_{e \beta}^m = A_{NP}^{e \beta} = \epsilon_{e \beta}^d,$$

$$\epsilon_{e \mu, \tau}^m = |A_{NP}^{e \mu, \tau}|^2,$$

$$\epsilon_{\mu \tau}^m = A_{NP}^{e \mu} A_{NP}^{e \tau} = \epsilon_{e \tau}^{ss} \epsilon_{\mu e}^{ss}. \tag{87}$$

Thus we see that the first order corrections due to New Physics appear in $\epsilon_{e \mu, \tau}^m$ and $\epsilon_{e \tau}^m$ while $\epsilon_{\mu \tau}^m$, $\epsilon_{\tau \tau}^m$ and $\epsilon_{\mu \tau}^m$ appear at higher order in New Physics interactions.

The general expression for the probability to have $\nu_\alpha^s \to \nu_\beta^d$ is given by

$$P_{\nu_\alpha^s \to \nu_\beta^d} = |\langle \nu_\delta | U^{sd}_{\beta \delta} e^{-iH T U^{rs}_{\alpha \gamma} | \nu_\gamma} \rangle|^2. \tag{88}$$
Matter effects can be described using the following Hamiltonian

\[
H_{\alpha\beta} = \frac{1}{2E_\nu} \left\{ (U_{\text{MNS}})_{\alpha i} \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2_{21} \end{pmatrix} (U_{\text{MNS}})_{i\beta}^\dagger + b \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix} \right\},
\]

with \( b = 2\sqrt{2}G_F n_e E_\nu \) and \( n_e \) the electron density of the matter.

IV. RESULTS

In this section, we shall numerically estimate the values of the \( CP \) asymmetries for the different cases studied in previous section, neglecting matter effects. We shall focus our analysis on the signal-to-noise ratio to emphasize the possibility to measure such \( CP \) asymmetries in neutrino oscillations, in particular for the \( \nu_e - \nu_\mu \) channel, at future neutrino factories. For that, one proceeds as in ref.[36] defining the observable

\[
\overline{A}_{\text{CP}} \equiv \frac{N[\mu^-]/N_0[\mu^-] - N[\mu^+]/N_0[\mu^+]}{N[\mu^-]/N_0[\mu^-] + N[\mu^+]/N_0[\mu^+]},
\]

where

\[
N[\mu^{\pm}] = \frac{N_\mu N_T E_\mu}{\pi L^2 m^2_\mu} \int_{E_{\text{th}}}^{E_\mu} 12 \left( \frac{E_\nu}{E_\mu} \right)^2 \left( 1 - \frac{E_\nu}{E_\mu} \right) \sigma_{\text{CC,CC}}(E_\nu) P_{\mu\nu}(E_\nu) dE_\nu,
\]

\[
N_0[e^{\pm}] = \frac{N_\mu N_T E_\mu}{\pi L^2 m^2_\mu} \int_{E_{\text{th}}}^{E_\mu} 12 \left( \frac{E_\nu}{E_\mu} \right)^2 \left( 1 - \frac{E_\nu}{E_\mu} \right) \sigma_{\text{CC,CC}}(E_\nu) dE_\nu,
\]

with \( \sigma_{\text{CC,CC}}(E_\nu) = \sigma_{\text{CC,\overline{CC}}} E_\nu \) and \( \sigma_{\text{CC,\overline{CC}}} \) given, respectively, by \( 0.67 \times 10^{-38}\text{cm}^2/\text{GeV}, 0.34 \times 10^{-38}\text{cm}^2/\text{GeV} \). \( N_\mu \) is the number of useful muon decays, \( N_T \) is the number of protons in the target detector and \( E_{\text{th}} \) the threshold energy of the detector.

The statistical error \( \Delta A_{\text{CP}} \) is given by

\[
\Delta A_{\text{CP}} \simeq \frac{1}{\sqrt{N[\mu^-] + N[\mu^+]}}.
\]

The signal-to-noise ratio is given by \( \overline{A}_{\text{CP}}/\Delta A_{\text{CP}} \). In order to illustrate the possibility to detect \( A_{\text{CP}} \) at future neutrino factories, it is useful to compute the signal-to-noise ratio using the analytical results we have for \( A_{\text{CP}} \) for the different cases studied in
previous sections. These results were obtained making an expansion in $s_{13}$ (expanding $P_{e\mu}^{SM}$ to second order in $s_{13}$ and $P_{e\mu}^{NP}$ to first order in $s_{13}$) and were valid for short distances, $x_{31} \ll 1 \Rightarrow L \ll E_\mu (\text{GeV}) \times 262 \text{ km}$, (typically at neutrino factories, $E_\mu = 50\text{GeV}$ [4]). To get this number, we use $\Delta m^2_{13} = 3 \times 10^{-3} \text{eV}^2$. Within these assumptions, $A_{CP}$ and $P_{e\mu}^{SM}$ have a very simple energy dependence

$$A_{CP} \sim E_\nu,$$

$$P_{e\mu}^{SM} \sim \frac{1}{E_\nu^2}.$$  \hspace{1cm} (94)

(95)

Using this energy dependence, it is easy to integrate eqs.(91-92). The final signal-to-noise ratio reads

$$\frac{\overline{A}_{CP}}{\overline{\Delta A}_{CP}} \approx A_{CP}(E_\mu) \sqrt{P_{e\mu}^{SM}(E_\mu) \left( \frac{N_\mu N_\tau T E_\mu^3}{\pi L^2 m_\mu^2} \right)^{1/2}} \sqrt{\frac{\sigma_{CC} + \sigma_{CC}^2}{2}}.$$  \hspace{1cm} (96)

To get an estimation, let us evaluate this expression for a total of $10^{21}$ useful muons ($N_\mu$) with an energy $E_\mu = 50$ GeV and a 40kt detector ($N_T \approx 1.1 \times 10^{33}$) and for $L_{GS} = 732$ km.

$$\frac{\overline{A}_{CP}}{\overline{\Delta A}_{CP}} \approx 2 \times 10^3 A_{CP}(E_\mu) \sqrt{P_{e\mu}^{SM}(E_\mu) \times \frac{L_{GS}}{L}}.$$  \hspace{1cm} (97)

Usually, it is said that the signal could be distinguished from the background noise at 99% C.L. if the signal-to-noise ratio is bigger than three. Using eq.(79) and imposing the signal-to-noise ratio to be bigger than three, an lower bound on $a$ and $\theta_{\tau 4}$ can be found,

$$\left( \frac{\overline{A}_{CP}}{\overline{\Delta A}_{CP}} \right) > 3 \Rightarrow a \theta_{\tau 4} \sin \phi > \frac{1}{s_{13}} \frac{1}{2 \times 10^{-3}} \frac{L}{L_{GS}}.$$  \hspace{1cm} (98)

As $a \theta_{\tau 4} \sim S_3^2 \approx \beta_{\tau \tau} < 0.0013$ due to lepton universality (see eqs(24)), the above lower bound is never satisfied for future neutrino factories as they are planned [4].

Thus, the constraints on FCNC in the leptonic sector and violation of lepton universality make the New Physics contribution to $CP$-asymmetries in the $\nu_e - \nu_\mu$ channel unobservable, in models with extra isosinglet charged leptons, with the present design of neutrino factories.

We should stress at this point the importance of a measurement of $CP$ violation in the $\nu_\mu - \nu_\tau$ channel in neutrino oscillations. In this case $CP$ violation mediated by the new charged lepton could be noticeable even in the limit $s_{13} = 0$ what would be
a clear signature of physics beyond the SM. Indeed, proceeding in the same way than before, one gets

\[
\left( \frac{\Delta A_{CP}}{\Delta A_{CP}} \right) > 3 \Rightarrow a \theta_{24} \sin \delta_{24} > \frac{3}{2} \times 10^{-3} \frac{L}{L_{GS}}. \tag{99}
\]

This lower bound can be translated as a lower bound on \(S_3,\)

\[
S_3^2 \equiv \beta_{\tau\tau} \gtrsim \frac{3}{2} \times 10^{-3} \frac{L}{L_{GS}}. \tag{100}
\]

This lower bound is still marginally compatible with the upper bound on \(\beta_{\tau\tau}\) coming from lepton universality. So, in principle, there is still a small window for observation of vector-like charged leptons effects on \(CP\) asymmetries in \(\nu_\mu - \nu_\tau\) channel at neutrino factories. We should emphasise that as \(B\)-factories produce as many tau pairs events as \(B - \bar{B}\) events, the expected improvements on rare lepton flavour changing tau decays at BELLE experiment for instance should allow us to close this window in a very near future if no lepton flavour changing tau decays are observed. And inversely, the observation of a such events at a \(B\)-factory will be a strong motivation to adapt the design of future neutrino factories to be able to probe \(CP\) asymmetries in \(\nu_\mu - \nu_\tau\) channel for the kind of model studied in this paper.

V. CONCLUSION

In this paper, we have studied models inspired by extra-dimensions where new particles naturally arise as isosinglet charged leptons. We have described a general perturbative approach to compute the effects of a vector-like charged lepton on neutrino oscillations and their potential signature at future neutrino factories taking into account all constraints coming from FCNC and lepton universality. This approach is based on model-independent formalism introduced in ref. [10, 11] and can be applied to any extension of the SM with vector-like particles (neutrinos or charged-leptons).

We have shown that in case of leptonic maximal mixings, new physics effects in \(\nu_e - \nu_\mu\) \(CP\) asymmetries are significantly enhanced. But due to FCNC and lepton universality constraints, such effects are out of reach of current neutrino factory design. In this class of models, we can expect to observe some signal from new physics at \(1 - 3\sigma\) in \(\nu_\mu - \nu_\tau\) channel if rare flavour changing tau decays are seen in a very
near future at $B$-factories as BELLE experiment. If such events are seen, there will be a strong motivation to slightly adapt the neutrino factory design to improve their sensitivity to $CP$ asymmetries in $\nu_\mu - \nu_\tau$ channel in order to test this class of models.

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APPENDIX A: KALUZA-KLEIN EXPANSION OF BULK FERMIONS

As we have emphasized in the Introduction, models with extra dimensions represent a well motivated framework for physics beyond the SM with extra vector-like fermions. We review in this appendix the KK expansion of bulk fermions in the class of models introduced in [7]. Let us consider a five-dimensional model with the fifth dimension compactified on the orbifold $S^1/Z_2$, which is a circle of radius $R$ with the $Z_2$ identification $y \leftrightarrow -y$ or, equivalently, an interval $0 \leq y \leq \pi R$ with two boundaries, the orbifold fixed points. A fermion in five dimensions is vector-like, the Dirac representation of $SO(1,4)$ being irreducible, thus it admits a bare Dirac mass which, in order to be non-trivial, has to depend on the extra dimension.

The integral in the fifth dimension of the five-dimensional Lagrangian results in the following four-dimensional Lagrangian (we use the “mostly minus” convention for the metric and $\gamma^4 = i\gamma^5$)

$$\mathcal{L} = \int_0^{\pi R} dy \, \bar{\Psi} \left[ i \gamma^\mu \partial_\mu - M(y) \right] \Psi$$

$$= \int_0^{\pi R} dy \, \left[ \bar{\Psi}_L i \gamma^\mu \partial_\mu \Psi_L + \bar{\Psi}_R i \gamma^\mu \partial_\mu \Psi_R 
+ \bar{\Psi}_R (\partial_y - M(y)) \Psi_L + \bar{\Psi}_L (-\partial_y - M(y)) \Psi_R \right],$$  \hspace{1cm} (A1)

where we have split the vector-like fermion into its two chiral components $\Psi = \Psi_L + \Psi_R$ defined by $\gamma^5 \Psi_{L,R} = \mp \Psi_{L,R}$. Note that in order to have a dynamical field in
the extra dimension, that is the term $\bar{\Psi}_L \partial_y \Psi_R + h.c. \neq 0$, the two chiralities of a fermion necessarily have opposite $Z_2$ parities to cancel the change of sign of $\partial_y$. Thus without loss of generality we can choose the mass term to be odd $M(-y) = -M(y)$. In particular we take it to have a multi-kink structure

$$M(y) = \begin{cases} M, & 0 \leq y \leq \pi a, \\ -M, & \pi a \leq y \leq \pi R, \end{cases} \quad (A2)$$

with $0 \leq a \leq R$. Hermiticity of the Lagrangian requires $M$ to be real but it can be of either sign. To complete a four-dimensional description we expand the five-dimensional fields in a real complete orthonormal basis in the fifth dimension with the coefficients of the expansion being four-dimensional fields, the KK modes,

$$\Psi_{L,R}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} f_{n}^{L,R}(y) \Psi_{L,R}^{(n)}(x). \quad (A3)$$

Inserting this KK expansion in the Lagrangian we obtain the action of an infinite tower of four-dimensional vector-like fermions, except for the zero mode which will be chiral,

$$\mathcal{L} = \sum_{n} \left\{ \bar{\Psi}_{L}^{(n)} \partial_{y} \Psi_{L}^{(n)} + \bar{\Psi}_{R}^{(n)} \partial_{y} \Psi_{R}^{(n)} - m_{n} \left[ \bar{\Psi}_{L}^{(n)} \Psi_{R}^{(n)} + \bar{\Psi}_{R}^{(n)} \Psi_{L}^{(n)} \right] \right\}, \quad (A4)$$

provided the following orthonormality and eigenvalue conditions are satisfied

$$\int_{0}^{\pi R} dy \frac{f_{n}^{L} f_{m}^{L}}{\pi R} = \int_{0}^{\pi R} dy \frac{f_{n}^{R} f_{m}^{R}}{\pi R} = \delta_{n,m}, \quad (A5)$$

$$\left[ \pm \partial_{y} - M(y) \right] f_{n}^{L,R} = -m_{n} f_{n}^{R,L}. \quad (A6)$$

The KK spectrum for this problem has been computed in [7] and can be summarised as follows. (In the following the upper (lower) sign stands for LH (RH) fields.)

There is a massless zero mode for the even chirality with exponential localization

$$f_{0}^{L,R}(y) = A_{L,R} \exp[\mp M|y - \pi a|], \quad (A7)$$

where the normalization constant is given by:

$$A_{L,R} = \sqrt{\frac{\pm 2M \pi R}{2 - \exp[\mp 2M \pi a] - \exp[\mp 2M \pi (R - a)]}}.$$
in the intermediate brane (thus the designation we use hereafter “multi-localized”) for \( M < 0 \), the opposite happens for a RH zero mode. The rest of the spectrum is vector-like with the first massive mode having distinct properties in the case that

\[ \mp 2M \pi a (R - a) > R. \]  \hspace{1cm} (A8)

The condition (A8) coincides with the multi-localization of the zero mode (indeed we use this condition as a quantitative definition of multi-localization in this particular problem). In fact the multi-localization of the zero mode is intimately related to the special properties of the first massive mode. When the condition (A8) is satisfied, the even component of the first KK mode is also exponentially multi-localized

\[ f^{L,R_{\text{even}}}_1 (y) = \begin{cases} A \left( e^{\beta_1 y} + \frac{\beta_1 \mp M}{\beta_1 \pm M} e^{-\beta_1 y} \right), & 0 \leq y \leq \pi a, \\ B \left( e^{-\beta_1 y} + \frac{\beta_1 \mp M}{\beta_1 \pm M} e^{\beta_1 (y - 2\pi R)} \right), & \pi a \leq y \leq \pi R, \end{cases} \]  \hspace{1cm} (A9)

where \( A \) and \( B \) are related by continuity of \( f^{(\text{even})}_1 \) at \( y = \pi a \) provided the wave function does not vanish at this point and by continuity of the derivatives if the wave function vanishes at the intermediate brane. The parameter \( \beta_1 \) is the positive solution of the eigenvalue equation (with the upper sign valid for LH components and the lower one for RH fields)

\[ \beta \left[ 1 - e^{-2\beta \pi R} \right] = \mp M \left[ 1 + e^{-2\beta \pi R} - e^{-2\beta \pi a} - e^{-2\beta \pi (R - a)} \right], \]  \hspace{1cm} (A10)

for even fields and the corresponding with the change \( M \rightarrow -M \) for odd fields so that the two chiralities of a fermion KK mode have the same mass as they should. The mass of the first KK mode is \( m^2_1 = M^2 - \beta^2_1 \) and is, provided condition (A8) is satisfied, always positive and smaller than \( M^2 \). In particular it can be seen that in the case of strong multi-localization the mass of the first KK modes goes exponentially to zero

\[ m^2_1 \approx 2M^2 \left[ e^{\pm 2\pi M a} + e^{\pm 2\pi M (R - a)} - 2e^{\pm 2\pi MR} \right] \rightarrow 0 \]  \hspace{1cm} [\mp M \rightarrow \infty]. \]  \hspace{1cm} (A11)

It is therefore effectively decoupled from the compactification scale (thus from the rest of the KK spectrum) in this limit. In the case that the multi-localization condition is not satisfied this first state has the same properties as the rest of the spectrum which consists on oscillating (thus not localized) states with masses greater than \( M \).
Let us finish this short review of the KK description of extra-dimensional theories writing the Yukawa couplings. The Lagrangian involving the Yukawa couplings, taking the Higgs to live at the \( y = 0 \) boundary, reads

\[
-L_{\text{Yuk}} = \int_0^{\pi R} dy \delta(y) \left\{ \lambda^{(5)} \bar{\Psi} \chi \Phi + h.c. \right\} \\
= \frac{\lambda^{(5)}}{\pi R} \sum_{nm} \Phi \left\{ f_{nL}(0) f_{mR}(0) + h.c. \right\}, \tag{A12}
\]

where we have chosen \( \Psi_L \) and \( \chi_R \) to be even fields and in the second equality we have expanded in KK modes.

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