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1. Editor’s notes

1. At least two fundamental problems in the field of SPM are finally solved. These and other exciting results are announced in the present issue.

2. The Mathematics Subject Classification list is currently being revised. Visit http://www.msc2010.org/ and make your suggestions. The web page promises that “Mathematical Reviews and Zentralblatt für Mathematik will carefully consider all feedback and use it in preparing their joint MSC revision to be used starting in 2010.”

3. Recently, the online versions of papers published by AMS journals do not contain the tex source. Thus, henceforth I will often not give abstracts of their papers which I announce here (unless the abstract is plain text). You will only get the title and link to the full paper. This is not very informative, so I urge authors to submit relevant abstracts directly to me.

Contributions to the next issue are, as always, welcome.

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2. Research announcements

2.1. Selection Principles and special sets of reals: Open problems. This is the chapter on SPM and special sets of reals, to appear in: Open Problems in Topology II (E. Pearl, ed.), Elsevier Science, 2007.

We give a selection of major open problems involving selection principles, diagonalizations, and covering properties for sets of real numbers.

http://arxiv.org/math/math.GN/0609601
Boaz Tsaban

2.2. Winning the pressing down game but not Banach Mazur. Let $S$ be the set of those $\alpha \in \omega_2$ that have cofinality $\omega_1$. It is consistent relative to a measurable that player II (the nonempty player) wins the pressing down game of length $\omega_1$, but not the Banach Mazur game of length $\omega + 1$ (both starting with $S$).

http://arxiv.org/math/math.LO/0609655
Jakob Kellner, Matti Pauna, and Saharon Shelah
2.3. **Ramsey classes of topological and metric spaces.** This paper is a follow up of the authors programme of characterizing Ramsey classes of structures by a combination of model theory and combinatorics. This relates the classification programme for countable homogeneous structures (of Lachlan and Cherlin) to the proof techniques of the structural Ramsey theory. Here we consider the classes of topological and metric spaces which recently were studied in the context of extremally amenable groups and of the Urysohn space. We show that Ramsey classes are essentially classes of finite objects only. While for Ramsey classes of topological spaces we achieve a full characterization, for metric spaces this seems to be at present an intractable problem.

2.4. **More on partitioning triples of countable ordinals.**

2.5. **Countable compact Hausdorff spaces need not be metrizable in ZF.** We show that the existence of a countable, first countable, zero-dimensional, compact Hausdorff space which is not second countable, hence not metrizable, is consistent with ZF.

2.6. **Every topological group is a group retract of a minimal group.** We show that every Hausdorff topological group is a group retract of a minimal topological group. This first was conjectured by Pestov in 1983. Our main result leads to a solution of some problems of Arhangelskii. One of them is the problem about representability of a group as a quotient of a minimal group (Problem 519 in the first edition of *Open Problems in Topology*). Our approach is based on generalized Heisenberg groups and on groups arising from group representations on Banach spaces and in bilinear mappings.

2.7. **The complexity of classifying separable Banach spaces up to isomorphism.** It is proved that the relation of isomorphism between separable Banach spaces is a complete analytic equivalence relation, i.e., that any analytic equivalence relation Borel reduces to it. Thus, separable Banach spaces up to isomorphism provide complete invariants for a great number of mathematical structures up to their corresponding notion of isomorphism. The same is shown to hold for

1. complete separable metric spaces up to uniform homeomorphism,
2. separable Banach spaces up to Lipschitz isomorphism, and
3. up to (complemented) biembeddability,
4. Polish groups up to topological isomorphism, and
5. Schauder bases up to permutative equivalence.

Some of the constructions rely on methods recently developed by S. Argyros and P. Dodos.

http://arxiv.org/math/math.FA/0610289
Valentin Ferenczi, Alain Louveau, and Christian Rosendal

2.8. Reals \( n \)-generic relative to some perfect tree. We say that a real \( X \) is \( n \)-generic relative to a perfect tree \( T \) if \( X \) is a path through \( T \) and for all \( \Sigma^0_n(T) \) sets \( S \), there exists a number \( k \) such that either \( X|k \in S \) or for all \( \sigma \in T \) extending \( X|k \) we have \( \sigma \notin S \). A real \( X \) is \( n \)-generic relative to some perfect tree if there exists such a \( T \). We first show that for every number \( n \) all but countably many reals are \( n \)-generic relative to some perfect tree. Second, we show that proving this statement requires ZFC\(^-\) + "\( \exists \) infinitely many iterates of the power set of \( \omega \)". Third, we prove that every finite iterate of the hyperjump, \( O^{(n)} \), is not 2-generic relative to any perfect tree and for every ordinal \( \alpha \) below the least \( \lambda \) such that \( \sup_{\beta<\lambda}(\beta\text{'th admissible}) = \lambda \), the iterated hyperjump \( O^{(\alpha)} \) is not 5-generic relative to any perfect tree. Finally, we demonstrate some necessary conditions for reals to be 1-generic relative to some perfect tree.

http://arxiv.org/math/math.LO/0610306
Bernard A. Anderson

2.9. Nagata’s conjecture and countably compactifications in generic extensions. Nagata conjectured that every \( M \)-space is homeomorphic to a closed subspace of the product of a countably compact space and a metric space. This conjecture was refuted by Burke and van Douwen, and A. Kato, independently.

However, we can show that there is a c.c.c. poset \( P \) of size \( 2^\omega \) such that in \( V^P \) Nagata’s conjecture holds for each first countable regular space from the ground model (i.e. if a first countable regular space \( X \in V \) is an \( M \)-space in \( V^P \) then it is homeomorphic to a closed subspace of the product of a countably compact space and a metric space in \( V^P \)). In fact, we show that every first countable regular space from the ground model has a first countable countably compact extension in \( V^P \), and then apply some results of Morita. As a corollary, we obtain that every first countable regular space from the ground model has a maximal first countable extension in model \( V^P \).

http://arxiv.org/math/math.GN/0610432
Lajos Soukup

2.10. A Class of Groups in Which All Unconditionally Closed Sets are Algebraic. It is proved that, in any subgroup of a direct product of countable groups, the property of being an unconditionally closed set in the sense of Markov coincides with that of being an algebraic set.

http://arxiv.org/math/math.GR/0610430
Ol’ga V. Sipacheva
2.11. A $c_0$-saturated Banach space with no long unconditional basic sequences. We present a Banach space $\mathcal{X}$ with a Schauder basis of length $\omega_1$ which is saturated by copies of $c_0$ and such that for every closed decomposition of a closed subspace $X = X_0 \oplus X_1$, either $X_0$ or $X_1$ has to be separable. This can be considered as the non-separable counterpart of the notion of hereditarily indecomposable space. Indeed, the subspaces of $\mathcal{X}$ have “few operators” in the sense that every bounded operator $T : X \to \mathcal{X}$ from a subspace $X$ of $\mathcal{X}$ into $\mathcal{X}$ is the sum of a multiple of the inclusion and a $\omega_1$-singular operator, i.e., an operator $S$ which is not an isomorphism on any non-separable subspace of $X$. We also show that while $\mathcal{X}$ is not distortable (being $c_0$-saturated), it is arbitrarily $\omega_1$-distortable in the sense that for every $\lambda > 1$ there is an equivalent norm $\| \cdot \|$ on $\mathcal{X}$ such that for every non-separable subspace $X$ of $\mathcal{X}$ there are $x, y \in S_X$ such that $\| \cdot \| / \| \cdot \| \geq \lambda$.

http://arxiv.org/math/math.FA/0610562
Jordi Lopez Abad and Stevo Todorcevic

2.12. Spaces of continuous functions over Dugundji compacta. We show that for every Dugundji compact $K$ the Banach space $C(K)$ is 1-Plichko and the space $P(K)$ of probability measures on $K$ is Valdivia compact. Combining this result with the existence of a non-Valdivia compact group, we answer a question of Kalenda.

http://arxiv.org/math/math.FA/0610795
Taras Banakh and Wieslaw Kubis

2.13. Varia: Ideals and Equivalence Relations, beta-version. We present a selection of basic results on Borel reducibility of Borel ideals and equivalence relations, especially those with comparably short proofs. The focal point are reducibility/irreducibility results related to some special equivalences like $E_0, E_1, E_2, E_3, E_\infty, Z_0$, and Banach-induced equivalences $l_p$, in particular several dichotomy theorems. The bulk of results included in the book were obtained in the 1990s, but some rather recent theorems are presented as well, like Rosendal’s proof that Borel ideals are cofinal within Borel equivalences of general form.

http://arxiv.org/math/math.LO/0610988
Vladimir Kanovei

2.14. Equivariant embedding of metrizable $G$-spaces in linear $G$-spaces. Given a Lie group $G$ we study the class $\mathcal{M}$ of proper metrizable $G$-spaces with metrizable orbit spaces, and show that any $G$-space $X \in \mathcal{M}$ admits a closed $G$-embedding into a convex $G$-subset $C$ of some locally convex linear $G$-space, such that $X$ has some $G$-neighborhood in $C$ which belongs to the class $\mathcal{M}$. As corollaries we see that any $G$-ANE for $\mathcal{M}$ has the $G$-homotopy type of some $G$-CW complex and that any $G$-ANR for $\mathcal{M}$ is a $G$-ANE for $\mathcal{M}$.

http://arxiv.org/math/math.GN/0611239
Aasa Feragen
2.15. **Squares of Menger-bounded groups.** Using a portion of the Continuum Hypothesis, we prove that there is a Menger-bounded (also called $o$-bounded) subgroup of the Baire group $\mathbb{Z}^\mathbb{N}$, whose square is not Menger-bounded. This settles a major open problem concerning boundedness notions for groups, and implies that Menger-bounded groups need not be Scheepers-bounded.

http://arxiv.org/math/math.GN/0611353
Michal Machura, Saharon Shelah, and Boaz Tsaban

2.16. **$\kappa$-Fréchet-Urysohn property of $C_k(X)$.** For a Tychonoff space $X$, we denote by $C_k(X)$ the space of all real-valued continuous functions on $X$ with the compact open topology. A space $X$ is said to be $\kappa$-Fréchet-Urysohn if for every open subset $U$ of $X$ and every $x \in U$, there exists a sequence $\{x_n\}_{n \in \mathbb{N}}$ in $U$ converging to $x$. In this paper, we show that $C_k(X)$ is $\kappa$-Fréchet-Urysohn iff every moving off family of compact subsets of $X$ has a countable subfamily which is strongly compact-finite. In particular, we obtain that every stratifiable Baire space $C_k(X)$ is an $M_1$-space.

http://dx.doi.org/10.1016/j.topol.2005.10.014
Masami Sakai

2.17. **How to drive our families mad.** Given a family $F$ of pairwise almost disjoint sets on a countable set $S$, we study families $F^+$ of maximal almost disjoint (mad) sets extending $F$.

We define $a^+(F)$ to be the minimal possible cardinality of $F^+ \setminus F$ for such $F^+$, and $a^+(\kappa) = \sup\{a^+(F) : |F| \leq \kappa\}$. We show that all infinite cardinal less than or equal to the continuum continuum can be represented as $a^+(F)$ for some almost disjoint $F$ and that the inequalities $\aleph_1 = a < a^+(\aleph_1) = \mathfrak{c}$ and $a = a^+(\aleph_1) < \mathfrak{c}$ are both consistent.

We also give a several constructions of mad families with some additional properties.

http://arxiv.org/math/math.LO/0611744
Sakaé Fuchino, Stefan Geschke, and Lajos Soukup

2.18. **Hurewicz sets of reals without perfect subsets.** We show that even for subsets $X$ of the real line which do not contain perfect sets, the Hurewicz property does not imply the property $S_1(\Gamma, \Gamma)$, asserting that for each countable family of open $\gamma$-covers of $X$, there is a choice function whose image is a $\gamma$-cover of $X$. This settles a problem of Just, Miller, Scheepers, and Szeptycki. Our main result also answers a question of Bartoszyński and the second author, and implies that for $C_p(X)$, the conjunction of Sakai’s strong countable fan tightness and the Reznichenko property does not imply Arhangel’skii’s property $\alpha_2$.

http://arxiv.org/math/math.GN/0612148
Dusan Repovs, Boaz Tsaban, and Lyubomyr Zdomskyy

2.19. **The spectrum of characters of ultrafilters on $\omega$.** We show the consistency of the set of regular cardinals which are the character of some ultrafilter on omega is not convex. We also deal with the set of $\pi \chi$-characters of ultrafilters on $\omega$. 
2.20. **Spaces of functions with countably many discontinuities.** Let $\Gamma$ be a Polish space and let $K$ be a separable and pointwise compact set of real-valued functions on $\Gamma$. It is shown that if each function in $K$ has only countably many discontinuities then $C(K)$ may be equipped with a $T_p$-lower semicontinuous and locally uniformly convex norm, equivalent to the supremum norm.

2.21. **Can groupwise density be much bigger than the non-dominating number?** We prove that $g$ (the groupwise density number) is smaller or equal to $b^+$ (the successor of the minimal cardinality of a non-dominated subset of $\mathbb{N}^\mathbb{N}$).

2.22. **Productive local properties of function spaces.** We characterize the spaces $X$ for which the space $C_p(X)$ of real valued continuous functions with the topology of pointwise convergence has local properties related to the preservation of countable tightness or the Fréchet property in products. In particular, we use the methods developed to construct an uncountable subset $W$ of the real line such that the product of $C_p(W)$ with any strongly Fréchet space is Fréchet. The example resolves an open question.

2.23. **Pinning quasi orders with their endomorphisms.** Some general properties of abstract relations are closely examined. These include generalizations of linearity, and properties based on ‘pinning’ an inequality by a pair of families of endomorphisms. To each property we try to associate a canonical definition of an augmentation (or diminishment) that augments (or diminishes) any given relation to one satisfying the desired property. The motivation behind this study was to identify properties distinguishing between the product ordering and the eventual dominance ordering of the irrationals (the family of functions from $\mathbb{N}$ into $\mathbb{N}$), and furthermore to identify their relationship as a member of a natural class of augmentations.

2.24. **A game on the universe of sets.** In set theory without the axiom of regularity, we consider a game in which two players choose in turn an element of a given set, an element of this element, etc.; a player wins if its adversary cannot make any next move. Sets that are winning, i.e. have a winning strategy for a player, form a
natural hierarchy with levels indexed by ordinals. We show that the class of hereditarily winning sets is an inner model containing all well-founded sets, and that all four possible relationships between the universe, the class of hereditarily winning sets, and the class of well-founded sets are consistent. We describe classes of ordinals for which it is consistent that winning sets without minimal elements are exactly in the levels indexed by ordinals of this class. For consistency results, we propose a new method for getting non-well-founded models. Finally, we establish a probability result by showing that on hereditarily finite well-founded sets the first player wins almost always.

http://arxiv.org/math/math.LO/0612636
Denis I. Saveliev

2.25. Algebraic characterizations of measure algebras. We present necessary and sufficient conditions for the existence of a countably additive measure on a complete Boolean algebra.

http://arxiv.org/math/math.FA/0612598
Thomas Jech

3. Problem of the Issue

We recall from [3] that a topological space $X$ has the Pytkeev property, if for every $x \in X$ and a subset $A \subset X$ such that $x \in A \setminus A$, there exists a countable family $\mathcal{N}$ of infinite subsets of $A$ which forms a $\pi$-network at $x$.

Tychonoff spaces $X$ such that $C_p(X)$ has the Pytkeev property were characterized in [4] and [5] as follows: $C_p(X)$ has the Pytkeev property iff for every $\omega$-shrinkable (clopen) $\omega$-cover $\mathcal{U}$ of $X$, there exists a sequence $(\mathcal{U}_n)_{n \in \mathbb{N}}$ of infinite subsets of $\mathcal{U}$ such that $\{\bigcap \mathcal{U}_n : n \in \mathbb{N}\}$ is an $\omega$-cover of $X$.

According to a recent result of A. Miller [5, Theorem 18], the mentioned covering property of $X$ implies that $X$ has a strong measure zero with respect to any totally-bounded metric on $X$. So it is natural to ask whether this covering property implies that $X$ has a strong measure zero with respect to any $\omega$ metric on it, which in the realm of zero-dimensional spaces$^1$ is known to be equivalent to the Rothberger covering property $S_1(O, O)$ [1]. But we do not even know the answer to the following basic question.

**Problem 3.1.** Does the Pytkeev property of $C_p(X)$ imply the Menger property $S_{\text{fin}} (O, O)$ of $X$?

This question is also motivated by the subsequent theorem.

**Theorem 3.2** (Tsaban-Zdomskyy [6]). Let $X$ be a Tychonoff space. If $C_p(X)$ has the Pytkeev property and $X$ satisfies $\bigcup_{\text{fin}} (O, \Omega)$, then $X$ satisfies $\bigcup_{\text{fin}} (O, \Gamma)$ as well as $S_1(O, O)$.

$^1$The Pytkeev property of $C_p(X)$ implies that $X$ is zero-dimensional [4].
Recall from [2] that a zero-dimensional space $X$ has the Gerlits-Nagy property $(\ast)$ if and only if it has the properties $U_{fin}(\mathcal{O},\Gamma)$ and $S_1(\mathcal{O},\mathcal{O})$.

Lyubomyr Zdomskyy

References

[1] D. H. Fremlin and A. W. Miller, *On some properties of Hurewicz, Menger and Rothberger*, Fundamenta Mathematica 129 (1988), 17–33.

[2] A. Nowik, M. Scheepers, and T. Weiss, *The algebraic sum of sets of real numbers with strong measure zero sets*, J. Symbolic Logic 63 (1998), 301–324.

[3] E. G. Pytkeev, *On maximally resolvable spaces*, Proceedings of the Steklov Institute of Mathematics 154 (1984), 225–230.

[4] M. Sakai, *The Pytkeev property and the Reznichenko property in function spaces*, Note di Matematica 22 (2003), 43–52.

[5] P. Simon and B. Tsaban, *On the Pytkeev property in spaces of continuous functions*, Proceedings of the American Mathematical Society, to appear. [http://arxiv.org/math/math.GN/0606286](http://arxiv.org/math/math.GN/0606286)

[6] B. Tsaban, L. Zdomskyy, work on the Pytkeev property, in progress.
4. Unsolved problems from earlier issues

Issue 1. Is $\binom{\Omega}{\Gamma} = \binom{\Omega}{T}$?

Issue 2. Is $\cup_{\text{fin}}(\mathcal{O}, \Omega) = S_{\text{fin}}(\Gamma, \Omega)$? And if not, does $\cup_{\text{fin}}(\mathcal{O}, \Gamma)$ imply $S_{\text{fin}}(\Gamma, \Omega)$?

Issue 4. Does $S_1(\Omega, T)$ imply $\cup_{\text{fin}}(\Gamma, \Gamma)$?

Issue 5. Is $p = p^*$? (See the definition of $p^*$ in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying $S_1(\mathcal{B}_\Gamma, \mathcal{B})$?

Issue 8. Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \mathcal{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?

Issue 9 (CH). Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?

Issue 10. Is $\text{cov}(\mathcal{M}) = \mathfrak{d}$? (See the definition of $\mathfrak{d}$ in that issue.)

Issue 11. Does $S_1(\Gamma, \Gamma)$ always contain an element of cardinality $\mathfrak{b}$?

Issue 12. Could there be a Baire metric space $M$ of weight $\aleph_1$ and a partition $\mathcal{U}$ of $M$ into $\aleph_1$ meager sets where for each $\mathcal{U}^\prime \subset \mathcal{U}$, $\bigcup \mathcal{U}^\prime$ has the Baire property in $M$?

Issue 14. Does there exist (in ZFC) a set of reals $X$ of cardinality $\mathfrak{d}$ such that all finite powers of $X$ have Menger’s property $S_{\text{fin}}(\mathcal{O}, \mathcal{O})$?

Issue 15. Can a Borel non-$\sigma$-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there an uncountable $X \subseteq \mathbb{R}$ satisfying $S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?

Issue 17 (CH). Is there a totally imperfect $X$ satisfying $\cup_{\text{fin}}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^\mathbb{N}$?

Issue 18 (CH). Is there a Hurewicz $X$ such that $X^2$ is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of $C_p(X)$ imply the Menger property of $X$?

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Previous issues. The previous issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, at http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in \LaTeX. The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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