Search for $D^0$-$\bar{D}^0$ mixing in the decays $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$

The BABAR Collaboration

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Abstract

We present a search for $D^0$-$\bar{D}^0$ mixing in the decays $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ using 230.4 fb$^{-1}$ of data collected with the BABAR detector at the PEP-II $e^+e^-$ collider at SLAC. Assuming $CP$ conservation, we measure the time-integrated mixing rate $R_M = (0.019^{+0.016}_{-0.015} \text{ (stat.)} \pm 0.002 \text{ (syst.)})\%$, and $R_M < 0.048\%$ at the 95\% confidence level. Using a frequentist method, we estimate that the data are consistent with no mixing at the 4.3\% confidence level. We present results both with and without the assumption of $CP$ conservation. By combining the value of $R_M$ from this analysis with that obtained from an analysis of the decays $D^0 \rightarrow K^+\pi^0\pi^0$, we find $R_M = (0.020^{+0.011}_{-0.010})\%$, where the uncertainty is statistical only. We determine the upper limit $R_M < 0.042\%$ at the 95\% confidence level, and we find the combined data are consistent with the no-mixing hypothesis at the 2.1\% confidence level.

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Transitions between the flavor eigenstates $|D^0\rangle$ and $|\bar{D}^0\rangle$ are called $D$ mixing, which is expected to have a very small rate in the Standard Model. Due to significant contributions from long-distance effects, an accurate estimate is difficult to obtain, but typical theoretical estimates of the time-integrated mixing rate are $R_M \sim O(10^{-6} - 10^{-4})$. The BABAR collaboration has previously reported searches for $D$ mixing in the decays to CP-even eigenstates [2], in the decay $D^0 \to K^+\pi^-$ [3], and in semileptonic decays [4]. A recent analysis of the decay $D^0 \to K^+\pi^-\pi^0$ set an upper limit $R_M < 0.054\%$ at the 95% confidence level with a data sample consistent with no mixing at the 4.5% confidence level [5]. The most stringent constraints on $D$-mixing parameters to date have been obtained by analyzing the decay $D^0 \to K^+\pi^-$ [6]; the rate is determined to be $R_M < 0.040\%$ at the 95% confidence level.

We search for the process $|D^0\rangle \to |\bar{D}^0\rangle$ by analyzing the decay of a particle known to be created as a $|D^0\rangle$ [7]. We distinguish doubly Cabibbo-suppressed (DCS) contributions from Cabibbo-favored (CF) mixed contributions by the decay-time distribution in the reconstructed wrong-sign (WS) decay $D^0 \to K^+\pi^-\pi^+\pi^-$. The right-sign (RS) decay $D^0 \to K^-\pi^+\pi^+\pi^-$ is a normalization mode in this analysis.

The two mass eigenstates

$$|D_{A,B}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

generated by mixing dynamics have different masses ($m_{A,B}$) and widths ($\Gamma_{A,B}$), with $|p/q| = 1$ if CP is conserved in mixing. We parameterize the mixing process with the quantities

$$x \equiv 2\frac{m_B - m_A}{\Gamma_B + \Gamma_A}, \quad y \equiv \frac{\Gamma_B - \Gamma_A}{\Gamma_B + \Gamma_A}.$$

For a multibody WS decay, the time-dependent decay rate, relative to a corresponding RS rate, is approximated by [8]

$$\frac{\Gamma_{\text{WS}}(t)}{\Gamma_{\text{RS}}(t)} = \tilde{R}_D + \alpha \tilde{y}' \sqrt{\tilde{R}_D} (\Gamma t) + \frac{\tilde{x}'^2 + \tilde{y}'^2}{4} (\Gamma t)^2$$

where the tilde indicates quantities that have been integrated over the selected phase-space regions. Here, $\tilde{R}_D$ is the integrated DCS branching ratio; $\tilde{y}' = y \cos \delta - x \sin \delta$ and $\tilde{x}' = x \cos \delta + y \sin \delta$, where $\delta$ is an unknown integrated strong-phase difference; $\alpha$ is a suppression factor that accounts for strong-phase variation over the region; and $\Gamma$ is the mean width. The time-integrated mixing rate $R_M = (\tilde{x}'^2 + \tilde{y}'^2)/2 = (x^2 + y^2)/2$ is independent of decay mode and should be consistent among mixing measurements. Additionally, while the branching ratio of DCS to CF decays depends on position in the Dalitz plot, the mixing rate does not.

We also search for CP violation in a mixing signal by fitting to the $D^0 \to K^+\pi^-\pi^+\pi^-$ and $\bar{D}^0 \to K^-\pi^+\pi^-\pi^+$ samples separately. We consider CP violation in the interference between the DCS and mixed contributions, parameterized by an integrated CP-violating–phase $\tilde{\phi}$, as well as CP violation in mixing, parameterized by $|p/q|$. We assume CP invariance in both the DCS and CF decay rates. The substitutions

$$(\alpha \tilde{y}' \cos \tilde{\phi} \pm \beta \tilde{x}' \sin \tilde{\phi}) \quad \text{and} \quad (x^2 + y^2) \rightarrow |p/q|^{\pm 1} (x^2 + y^2)$$

1 INTRODUCTION
are applied to Equation 3 using (+) for $\Gamma(D^0 \to K^-\pi^+\pi^-)/\Gamma(D^0 \to K^-\pi^+\pi^-)$ and (−) for the charge-conjugate ratio. The parameter $\beta$ is analogous to $\alpha$ and accounts for net $\phi$ variation.

2 THE BaBar DETECTOR AND DATASET

We use 230.4 $fb^{-1}$ of data collected with the BaBar detector [9] at the PEP-II $e^+e^-$ collider at SLAC. Charged particles are detected and their momenta measured by a combination of a cylindrical drift chamber (DCH) and a silicon vertex tracker (SVT), both operating within a 1.5 T solenoidal magnetic field. A ring-imaging Cherenkov detector (DIRC) is used for charged-particle identification. Photon energies are measured with a CsI electromagnetic calorimeter (EMC). We use information from the DIRC and energy-loss measurements in the SVT and DCH to identify charged-kaon and -pion candidates. The data set includes $e^+e^-$ collisions at and 40 MeV below the $\Upsilon(4S)$ resonance. All selection criteria were finalized before searching for evidence of mixing in the data.

3 ANALYSIS METHOD

We reconstruct the decays $D^{*+} \to D^0\pi^+_s$ and the charge of the soft pion, $\pi^+_s$, is used to determine the flavor of the $D^0$ candidate. In order to obtain a pure data sample, selection of $D^0$ candidates includes a requirement of center-of-mass momentum greater than 2.4 GeV and the application of strict particle-identification (PID) requirements to the daughters of the $D^0$. We accept decays with an invariant mass $1.815 < m_{K\pi\pi\pi} < 1.915$ GeV/$c^2$ and an invariant mass difference $0.1396 < \Delta m < 0.1516$ GeV/$c^2$, where $\Delta m \equiv m_{K\pi\pi\pi} - m_{K\pi\pi\pi}$. We also require that neither $\pi^+\pi^-$ combination of candidate $D^0$ daughters have an invariant mass within 20 MeV/$c^2$ of the $K^0_S$ value given in the Review of Particle Physics (RPP) [10]. This cut suppresses background from the singly Cabibbo-suppressed decay $D^0 \to K^+K^0\pi^-\pi^-$ followed by $K^0\to\pi^+\pi^-$. The candidate masses and decay times are derived from a vertex fit. First, the $D^0$ and $D^{*+}$ decay vertices are determined in separate geometric fits, and the $\chi^2$ probability of each fit is required to be greater than 0.005. The candidate $D^{*+}$-decay tree is then fit for simultaneously optimal $D^{*+}$ and $D^0$ decay vertices [11] with the $D^{*+}$ decay vertex constrained to the beamspot region. We select events for which the $\chi^2$ probability of this fit is greater than 0.01. From this fit, a $D^0$ decay time, $t_{K\pi\pi\pi}$, and uncertainty, $\sigma_t$, are calculated using the three-dimensional flight path. The full covariance matrix, including correlations between the two vertices, is used in the $\sigma_t$ estimate. For signal events, the mean $\sigma_t$ is near 0.29 ps; we accept decays with $\sigma_t < 0.5$ ps. The world-average $D^0$ lifetime is 0.41 ps [10].

To separate correctly reconstructed decays from background, and to distinguish mixing contributions from DCS contributions, unbinned extended maximum likelihood fits to the data sample are performed. Probability density functions (PDFs) are fit in two stages to the distributions $(m_{K\pi\pi\pi}, \Delta m, t_{K\pi\pi\pi})$. First, the $(m_{K\pi\pi\pi}, \Delta m)$ plane is considered to discriminate between signal and background; optimal PDF parameters are established in these dimensions. Second, a fit to $t_{K\pi\pi\pi}$ is performed, retaining the PDF-shape parameters of the previous fit to construct a three-dimensional likelihood $\mathcal{L}$. The signal yields from the fit to the $(m_{K\pi\pi\pi}, \Delta m)$ plane are listed in Table 1. A simultaneous fit is performed to both the large sample of RS decays and the relatively small sample of WS decays; thus, signal shape parameters associated with the WS sample are precisely determined by the RS.
sample, and all associated systematic uncertainties are suppressed. The fit to these distributions is shown for the WS sample in Figure \(\text{a,b}.\)

Table 1: Signal-candidate yields determined by the two-dimensional fit to the \((m_{K^{0}π^{+}π^{-}π^{+}}, Δm)\) distributions for the WS and RS samples. Uncertainties are those calculated from the fit.

|       | \(D^{0}\) Cand. | \(D^{0}\) Cand. |
|-------|------------------|------------------|
| WS    | (1.162 ± 0.053) \(\times 10^{3}\) | (1.040 ± 0.051) \(\times 10^{4}\) |
| RS    | (3.511 ± 0.006) \(\times 10^{5}\) | (3.492 ± 0.006) \(\times 10^{5}\) |

The sources of background remaining in the sample may be characterized by three categories in the likelihood fits to data. The background that peaks in the \(m_{K^{0}π^{+}π^{-}π^{+}}\) distribution is due to correctly reconstructed \(D^{0}\) decays with a misassociated \(π^{+}\); this category has the decay-time distribution of the RS signal. Second, remaining combinatorial background is present as a nonpeaking component of both distributions. This distribution is empirically described by a Gaussian with a power-law tail. The third category is due to correctly reconstructed \(D^{*+}\) decays with a misreconstructed \(D^{0}\), for which the kaon and a pion have been mistaken for each other. This category has the signal lifetime distribution. A three-dimensional likelihood is maximized in a fit to \(t_{K^{0}π^{+}π^{-}}\), after the shape parameters are determined in the two mass distributions.

The RS PDF is fit to the \(t_{K^{0}π^{+}π^{-}}\) distribution to determine the \(D^{0}\) lifetime and the detector-resolution parameters. The signal shape of \(t_{K^{0}π^{+}π^{-}}\) is an exponential function convolved with a double-Gaussian resolution function. The Gaussians have different widths and means; the width of each Gaussian is a scale factor multiplied by \(σ_t\), which is determined for each event. The two different scale factors are determined by the fit to the data. We find a \(D^{0}\) lifetime consistent with the nominal value.

The WS signal PDF in \(t_{K^{0}π^{+}π^{-}}\) is a function based on Equation 3 convolved with the double Gaussians described above. The \(D^{0}\) lifetime and resolution scale factors and means, determined by the fit to the RS \(t_{K^{0}π^{+}π^{-}}\) distribution, are fixed. We fit the WS PDF to the \(t_{K^{0}π^{+}π^{-}}\) distribution allowing yields and background shape parameters to vary. The fit to the \(t_{K^{0}π^{+}π^{-}}\) distribution is shown for the WS sample in Figure \(\text{c,d}.\)

### 4 SYSTEMATIC STUDIES

We quantify systematic uncertainties by performing the analysis with the following changes, in order of decreasing significance: the selection of events based on \(σ_t\), the decay-time resolution function, the background PDF shape in the \(m_{K^{0}π^{+}π^{-}π^{+}}\) distribution, and the measured \(D^{0}\) lifetime value. The selection requirement of \(σ_t\) may skew the decay-time distribution if there is a correlation between the two distributions; it is investigated by moving the selection criterion from 0.5 ps to 0.6 ps. The double-Gaussian resolution function is investigated by fixing one of the two scale factors to unity, determining the other factor from a fit to the RS data, and performing the decay-time fit to the WS data. The PDF used to describe the background contribution in the \(m_{K^{0}π^{+}π^{-}π^{+}}\) distribution is changed from an exponential to a second-order polynomial. This change allows some fraction of events to be weighted toward background, and so affects the number of events contributing to the mixing signal. Finally, the fitted lifetime from the decay-time fit to the RS data is not as accurate as the value listed in the RPP \[10\]; this systematic uncertainty is estimated by setting the lifetime to the
Figure 1: Distributions of WS data with fitted PDFs (described in Sec. 3) overlaid. The $m_{K\pi\pi}$ distribution (a) requires $0.14487 < \Delta m < 0.14587$ GeV/$c^2$; the $\Delta m$ distribution (b) requires $1.859 < m_{K\pi\pi} < 1.869$ GeV/$c^2$; and the $t_{K\pi\pi}$ distribution (c) requires $\Delta m$ cuts as the same as (a). The $t_{K\pi\pi}$ distribution (d) requires both cuts from (a) and (b). In each of the above histograms, the white area beneath the dotted line represents signal events, the light gray represents swapped $K^\pm\pi^\mp$, the medium gray represents misassociated $\pi^\pm_s$ events, and the dark gray represents remaining combinatorial background.
value given in the RPP. The combined systematic uncertainties for most quantities in Table 2 are smaller than statistical uncertainties by a factor of 5; the systematic uncertainties on \((\alpha\tilde{y}' \cos \tilde{\phi})\) and \((\beta\tilde{x}' \sin \tilde{\phi})\) do not account for correlated uncertainties between the \(D^0\) and \(\bar{D}^0\) samples, and thus are conservatively estimated.

## 5 RESULTS

Table 2: Mixing results assuming \(CP\) conservation (\(D^0\) and \(\bar{D}^0\) samples are not separated) and manifestly permitting \(CP\) violation (\(D^0\) and \(\bar{D}^0\) samples are fit separately). The first listed uncertainty is statistical, and the second is systematic.

|                      | \(CP\) conserved               | \(CP\) violation allowed         |
|----------------------|--------------------------------|----------------------------------|
| \(R_M\)              | \((0.019 \pm 0.016 \pm 0.002)\)% | \((0.017 \pm 0.016 \pm 0.002)\)% |
| \(\alpha\tilde{y}'\) | \(-0.006 \pm 0.005 \pm 0.001\) | \(-0.006 \pm 0.008 \pm 0.006\) |
| \(\alpha\tilde{y}' \cos \tilde{\phi}\) |                                   | \(\beta\tilde{x}' \sin \tilde{\phi}\) |
|                      | \(0.002 \pm 0.005 \pm 0.006\)   |                                  |
| \(|p/q|\)            | 1.1 \(\pm 0.4\) \(\pm 0.1\)    |                                  |

The results of the decay-time fit, both with and without the assumption of \(CP\) conservation in a mixing signal, are listed in Table 2. The statistical uncertainty of a particular parameter is obtained by finding its extrema for \(\Delta \ln L = 0.5\); in finding the extrema, the likelihood is kept maximal by refitting the remaining parameters. Contours of constant \(\Delta \ln L = 1.15, 3.0\), enclosing two-dimensional coverage probabilities of 68.3% and 95.0%, respectively, are shown in Figure 2.

We note that \(\Delta \ln L\) as a function of the quantity \(\text{sign}(\alpha\tilde{y}') \times R_M\) is approximately parabolic. The two-sided interval \(-0.048\% < \text{sign}(\alpha\tilde{y}') \times R_M < 0.048\%\) contains 95% coverage probability; thus, we quote \(R_M < 0.048\%\) as our upper limit on the integrated mixing rate under the assumption of \(CP\) conservation.

A feature of \(\Delta \ln L\) in one dimension is that it changes behavior near \(R_M = 0\) because the interference term (linear in \(t\) in Equation 3) becomes unconstrained. Therefore, we estimate the consistency of the data with no mixing using a frequentist method. Generating 1000 simulated data sets with no mixing, each with 76,300 events representing signal and background in the quantities \(\{m_{K\pi\pi}, \Delta m, t_{K\pi\pi}\}\), we find 4.3% of simulated data sets have a fitted value of \(R_M\) greater than that in the observed data set. We conclude that the observed data are consistent with no mixing at the 4.3% confidence level.

We combine the value of \(R_M\) from this analysis with that obtained from an analysis of the decays \(D^0 \rightarrow K^+\pi^-\pi^0\) [5] by adding the \(\Delta \ln L(R_M)\) curves from the two separate analyses. The \(\Delta \ln L(R_M)\) curves are shown in Fig 3. We extract a central value and an uncertainty from the combined curve using the same procedure as for each individual result. With this method, we find \(R_M = (0.020 \pm 0.011)\%\), where the uncertainty is statistical only. We determine the upper limit \(R_M < 0.042\%\) at the 95% confidence level, and we find the combined data are consistent with the
Figure 2: Left: Contours of constant $\Delta \ln \mathcal{L} = 1.15, 3.0$ in terms of the doubly Cabibbo-suppressed branching ratio and the time-integrated mixing rate. The upward slope of the contour indicates negative interference between DCS and mixed contributions. Right: Contours of constant $\Delta \ln \mathcal{L} = 1.15, 3.0$ in terms of the normalized interference term and the integrated mixing rate, for the $D^0$ and $\bar{D}^0$ samples separately. The hatched regions are physically forbidden.

Figure 3: $\Delta \ln \mathcal{L}$ as a function of $R_M$ for separate and combined results of $K^{+}\pi^{-}\pi^{+}\pi^{-}$ and $K^{+}\pi^{-}\pi^{0}$. (On the axis, the natural logarithm is denoted log.)
no-mixing hypothesis at the 2.1% confidence level, as determined from the $\Delta \ln L(R_M)$ curve.

6 CONCLUSION

We find that the data used in an analysis of $D^0 \to K^+\pi^-\pi^+\pi^-$ are consistent with the no-mixing hypothesis at the 4.3% confidence level. Assuming CP conservation, we measure the time-integrated mixing rate $R_M = (0.019^{+0.016}_{-0.015} \text{ (stat.)} \pm 0.002 \text{ (syst.)})\%$, and $R_M < 0.048\%$ at the 95% confidence level. Furthermore, we combine these results with those of a similar analysis of the decays $D^0 \to K^+\pi^-\pi^0$ [5]. From this combination, we find $R_M = (0.020^{+0.011}_{-0.010} \text{ (stat.)})\%$ and $R_M < 0.042\%$ at the 95% confidence level. The combined data sets are consistent with the no-mixing hypothesis with 2.1% confidence.

7 ACKNOWLEDGMENTS

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References

[1] L. Wolfenstein, Phys. Lett. B 164, 170 (1985); J. F. Donoghue, E. Golowich, B. R. Holstein and J. Trampetic, Phys. Rev. D 33, 179 (1986); A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D 65, 054034 (2002); G. Burdman and I. Shipsey, Ann. Rev. Nucl. Part. Sci. 53, 431 (2003); A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D 69, 114021 (2004).

[2] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 91, 121801 (2003).

[3] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 91, 171801 (2003).

[4] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 70, 091102 (2004).

[5] M. G. Wilson [BABAR Collaboration], in Proceedings of the 4th International Conference on Flavor Physics and CP Violation (FPCP 06), Vancouver, B.C., Canada, 2006, \texttt{arXiv:hep-ex/0605046}
[6] L. M. Zhang et al. [Belle Collaboration], Phys. Rev. Lett. 96, 151801 (2006).
[7] Unless otherwise stated, particle types and decay processes imply also their charge conjugates.
[8] G. Blaylock, A. Seiden and Y. Nir, Phys. Lett. B 355, 555 (1995).
[9] B. Aubert et al. [BABAR Collaboration], Nucl. Instrum. Meth. A 479, 1 (2002).
[10] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592, 1 (2004).
[11] W. D. Hulsbergen, Nucl. Instrum. Meth. A 552, 566 (2005).