Preparing the State of a Black Hole

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Abstract
Measurements of the mass or angular momentum of a black hole are onerous, particularly if they have to be frequently repeated, as when one is required to transform a black hole to prescribed parameters. Irradiating a black hole of the Kerr-Newman family with scalar or electromagnetic waves provides a way to drive it to prescribed values of its mass, charge and angular momentum without the need to repeatedly measure mass or angular momentum throughout the process. I describe the mechanism, which is based on Zel’dovich-Misner superradiance and its analog for charged black holes. It represents a possible step in the development of preparation procedures for quantum black holes.

1 Introduction

In general relativity a classical black hole in equilibrium is singled out by just a few parameters. The more mundane are mass $M$, electric charge $Q$ and angular momentum vector $J$ in which case we have a Kerr-Newman black hole [1]. One could add parameters: magnetic monopole, skyrmion number [2] and the like, but I shall confine myself to the short list in order to make the story clear. Much of classical black hole physics consists of considering processes and phenomena in connection with a particular Kerr-Newman black hole or sequence of Kerr-Newman black holes. Little thought is given as to how to produce a black hole with sharp values of $M$, $Q$ and $J$. This is no trivial task. For example, total gravitational collapse, accompanied as it probably always is by matter ejection, radiation, etc., is apt to yield a black hole whose parameters are not well known. It would seem that to prepare a black hole with sharp values of $M$, $Q$ and $J$ one would have to follow the collapse by various transformations of the hole while continuously keeping track of the values of these parameters.

Whereas measuring $Q$ of a black hole is relatively straightforward, measuring $M$ or $J$ is no small matter! Mass is measurable only gravitationally, basically by keeping track of a neutral test particle moving under the influence of the hole. $J$ can also be deduced from the hole’s influence on a neutral test particle. However, the metric component through which $J$ primarily expresses itself, the same that gives rise to the Lense-Thirring effect [1], is relatively weak and rapidly decreasing with distance

\[^1\text{Dedicated to Mario Novello on the occasion of his sixtieth birthday.}\]
from the hole (the effect is just becoming accessible to modern space technology). So the determination of $J$ is even harder than that of $M$. Of course, $J$ can also be determined by first measuring the black hole’s magnetic dipole moment from its effects on a charged test particle. However, such a moment exists only for a charged black hole, and to infer $J$ from the measurement, both $M$ and $Q$ (or at least their ratio [1]) must also be known. Since a series of measurements would seem to be required to prepare a black hole state with prescribed parameters, one would seem to be faced with the need for a sequence of difficult multiple measurements.

The problem of preparing a black hole state is compounded in the quantum domain, Quantum Kerr black holes should have available states with definite mass, charge and squared angular momentum, as well as definite $z$ component of angular momentum [3]; this is so because all these quantities are represented in usual physics by commuting hermitian operators. In quantum theory of ordinary systems one measures an observable $\hat{O}$, and if the measurement gives the eigenvalue $o_i$ of $\hat{O}$, then one knows the system has been prepared in the eigenstate of $\hat{O}$ whose eigenvalue is $o_i$ (if $o_i$ is a degenerate eigenvalue we need more observables to specify the prepared state). How do we prepare a quantum black hole state in this sense? Since, as discussed, the black hole observables are not so easily measured even classically, we may enquire whether there is a better way to prepare a quantum black hole state?

As a preparation for this challenge I discuss in this paper a method for preparing a classical Kerr-Newman black hole in a prescribed state, a method which requires only one—initial—set of measurements, as opposed to a sequence of measurements. In pedagogical spirit I first set out the method as applied to spherical black holes (Sec. 2), then describe the necessary modifications for treating rotating black holes (Sec. 3), and only then join all parts for the treatment of generic Kerr-Newman black holes (Sec. 4). Although my treatment is classical, I also discuss (Sec. 5) how to overcome difficulties for the method arising from Hawking radiation.

Unless otherwise stated, I work in units with $G = c = 1$.

## 2 Preparing a Reissner-Nordström state

I first explain the basic ideas in the simple context of spherical black holes. Consider a Reissner-Nordström black hole ($J = 0$) of mass $M$ and charge $Q$ with $|Q| \leq M$. The event horizon area of the hole is

\[
A = A(M, Q) \equiv 4\pi(M + \sqrt{M^2 - Q^2})^2
\]

Taking the total differential we have

\[
\Theta_{RN} \ dA = dM - \Phi dQ
\]

\[
\Theta_{RN} \equiv \frac{1}{2} \sqrt{M^2 - Q^2} A^{-1}
\]

\[
\Phi \equiv (4\pi/A)^{1/2} Q.
\]
Here $\Phi$ is the electric potential of the black hole ($-1 < \Phi < +1$).

Our tool for transforming the black hole will be a charged Klein-Gordon field coupled to electromagnetism with charge $\varepsilon$. The question of the field’s rest mass $m_s$ becomes relevant because, as will become clear, we would like the energy $\hbar \omega$ of the corresponding quanta—which must exceed $m_s$—to be able to take up all values in the range $[0, |\varepsilon \Phi|]$. Although the known scalar fields of these kind, e.g. the charged $\pi$ field, are popularly construed as being quite massive, in our units $m_\pi \sim 10^{-16} |\varepsilon_\pi|$. Thus I foresee no problems in covering the desired range of $\hbar \omega$ except for Reissner-Nordström black holes extremely close to the Schwarzschild limit (say $|\Phi| \ll 10^{-13}$).

We shall assume that surrounding the black hole at large distance is a device—the *radiator* for short—capable of irradiating the hole with coherent waves of this (bosonic) scalar field. Coherent waves are the same solutions of the scalar field equation that we would use as modes in the quantum treatment of the field. In addition we shall need a *monitor* that keeps track of the intensity of the incident and scattered waves.

Let us suppose a train of *spherical* wave modes of this sort with (positive) frequency $\omega$ and $\varepsilon$ of the *same sign* as $Q$ converge concentrically onto the hole, and are partially absorbed and partially reflected off it. Now if the hole absorbs a total of $N$ quanta from the waves, the remainder of which is scattered back, then obviously the hole’s parameters undergo changes $\delta M = N \hbar \omega$ and $\delta Q = N \varepsilon$. According to Eq. (2),

$$\delta A = \hbar N \Theta_{RN}^{-1} (\omega - \varepsilon \Phi / \hbar) \quad (5)$$

Naively we would assume $N > 0$. Indeed, if $\omega > \varepsilon \Phi / \hbar$ we then have from this equation that $\delta A > 0$ in harmony with Hawking’s area theorem [4]. However, nobody can stop us from taking $\omega < \varepsilon \Phi / \hbar$. In fact, our tacit assumption that $m_s \ll |\varepsilon|$ is tantamount to allowing $\hbar \omega \ll |\varepsilon|$. Since for the Reissner-Nordström black hole $|\Phi| \leq 1$, we can in fact arrange for $\omega < \varepsilon \Phi / \hbar$, unless, of course, the hole is very close to Schwarzschild. Since we can make $N$ large (by taking the incident wave strong) the process is classical, yet Eq. (5) seems to suggest that $\delta A < 0$ in contradiction with Hawking’s theorem. One is thus forced to accept that when $\omega < \varepsilon \Phi / \hbar$ then $N < 0$, i.e., the scattering *amplifies* the wave [5]. This is the electromagnetic counterpart of Zel’dovich-Misner superradiance from a Kerr black hole [6, 7]. All this means that the reflection coefficient $R_{RN}$ for the scalar waves off the black hole in the vicinity of the *transition point* $\omega = \varepsilon \Phi / \hbar$ may be expanded in a Taylor series of the form [8]

$$R_{RN} = 1 - \alpha_{RN} \cdot (\omega - \varepsilon \Phi / \hbar) + O \left( (\omega - \varepsilon \Phi / \hbar)^3 \right) \quad (6)$$

with some $\alpha_{RN}(\varepsilon, M, Q) > 0$.

If $\omega$ is tuned exactly to $\varepsilon \Phi / \hbar$, our result tells us that the wave’s intensity will not be affected upon scattering—although the wave may thereby acquire a phase shift. Thus neither $M$ nor $Q$ will change. This equilibrium between hole and waves at the transition point $\omega = \varepsilon \Phi / \hbar$ is a stable one. For suppose the black hole parameters are sightly shifted by an external agency from the transition point so that $\omega$ slightly
exceeds $\varepsilon \Phi / h$. This is the normal regime and the black hole will absorb radiation: 

$$\delta Q = N \varepsilon$$

and 

$$\delta M = N h \omega$$

with $N > 0$. Then from Eq. (4) we find that this results in the change 

$$\delta \Phi = N \varepsilon (M + (M^2 - Q^2)^{1/2})^{-1},$$

so that $\varepsilon \delta \Phi > 0$. Thus the black hole evolves back in the direction of the transition point $\omega = \varepsilon \Phi / h$, and cannot stop until it gets there. Similarly, if the black hole is perturbed into the superradiant regime $(N < 0)$, $\varepsilon \delta \Phi < 0$ and again the hole evolves back to the transition point. Thus the Reissner-Nordström transition point is an attractor.

We can exploit this stability to prepare a Reissner-Nordström hole with any prescribed parameters, $\bar{M}$ and $\bar{Q}$. Our starting point can be any Reissner-Nordström black hole whose horizon area $A$ lies below $\bar{A} \equiv A(\bar{M}, \bar{Q})$. For if $A$ were bigger, we could never reach the desired state classically—by Hawking’s area theorem—but would require for this the assistance of some quantum effect, like Hawking radiation, which acts very slowly for large black holes. So how do we determine $A$ without having to measure $M$?

This can be done by exposing the hole to a (very weak) test charged scalar wave of the type mentioned earlier. The idea is to use this as a probe of the black hole state which has negligible back effect. By sweeping the $\omega$ of the wave back and forth and having the monitor check the wave’s intensity gain upon scattering, we can zero in on the transition point $\omega = \varepsilon \Phi / h$ without significantly affecting any of the hole’s parameters. Thus we can determine $\Phi$ directly. (In case the sign of $\varepsilon$ was chosen infelicitously, zeroing-in is impossible, and the opposite sign must be selected.) Now according to Eq. (4), if we can also find $Q$, we know $A$. Measuring $Q$ involves no complications. We may, for example, measure the electric field at a couple of points far away from the hole along the radial direction, and fit a Coulomb law to the measurements to isolate the value of the initial charge, $Q_0$. This is much easier than measuring the gravitational field because actual charged particles, which we would use as probes in both types of measurements, have a large charge-to-mass ratio in our units ($10^{18}$ for the electron). Thus we have a way to check if the initial horizon area, $A_0$, complies with the requirements from Hawking’s theorem. And from Eq. (1), we can determine the initial mass:

$$M_0 = (A_0 / 16\pi)^{1/2}(1 + 4\pi Q_0^2 / A_0)$$  \hspace{1cm} (7)

We next cause $A$ to increase at constant charge $Q_0$ until it reaches $\bar{A}$. This is easily done, for example, by gradually dropping neutral matter with zero angular momentum into the hole; according to Eq. (2), at constant charge we indeed need to increment the hole’s mass in order to increase its horizon area. From Eq. (1) we know that the final mass in this process must be (recall that $Q = Q_0$ throughout the process)

$$M_1 = (A / 16\pi)^{1/2}(1 + 4\pi Q_0^2 / A).$$  \hspace{1cm} (8)

Comparison of this with Eq. (7) will tell us how much energy to add to the hole to reach $\bar{A}$.

The process just described will in general change $\Phi$ (but no its sign) and shift the transition point. But we can again lock the hole to the new transition point by
repeating the original $\omega$-sweeping procedure. Once the hole is at the transition point, we increase the intensity of the waves and slightly shift the said $\omega$ in the direction of $\epsilon \Phi / \bar{h}$, where $\Phi$ is computed from $\bar{Q}$ and $\bar{M}$ by means of Eq. (4). By the stability condition we know that the black hole will follow suit and change its parameters so that its instantaneous $\Phi$ evolves in the direction of the desired $\Phi$. Thus by slowly shifting $\omega$ in the same sense, we drag the actual $\Phi$ in the desired direction. (If the initial $Q$ is opposite in sign to $\bar{Q}$, we shall have to switch the sign of $\epsilon$ as $\Phi$ passes through zero.) The question is, are we changing $A$ by this procedure?

A look at Eq. (5) informs us that the change in $A$ in the process amounts to

$$\delta A = \Theta_{RN}^{-1} \cdot (1 - \epsilon \Phi / \bar{h} \omega) \cdot \delta M.$$  

Thus if the sweep of $\omega$ is carried out sufficiently slowly so that the hole stays very close to the transition point ($|1 - \epsilon \Phi / \bar{h} \omega| \ll 1$), the concomitant change in $A$ will be entirely negligible on the scale of the changes of $M$ and $Q$. This is just another example of an adiabatic black hole process which leaves the horizon area unchanged [9, 10, 11]. When $\omega$ reaches the value $\epsilon \Phi / \bar{h}$, we have managed to bring the hole to the potential $\Phi$ and area $\bar{A}$. Because Eqs. (1) and (4) can be solved uniquely for $M$ and $Q$, we have obtained the black hole with the prescribed mass $\bar{M}$ and charge $\bar{Q}$.

3 Preparing a Kerr state

For a Kerr black hole ($Q = 0$) with mass $M$ and angular momentum $J$ about a specified axis ($J \leq M^2$), the analogs of Eqs. (1)-(4) are

$$A = A(M, J) = 4\pi [r_+^2 + (J/M)^2]$$

$$\Theta_K \, dA = dM - \Omega \, dJ$$

$$\Theta_K \equiv \frac{1}{2} \sqrt{M^2 - (J/M)^2} \, A^{-1}$$

$$\Omega \equiv 4\pi (J/MA)$$

Here $r_+ \equiv M + \sqrt{M^2 - J^2/M^2}$ and $\Omega$ is the hole’s rotational angular frequency ($-(2M)^{-1} < \Omega < +(2M)^{-1}$). Henceforth I will identify the hole’s rotation axis with the $z$-axis. More appropriate here than the charged scalar field of Sec. 2 is the electromagnetic—hence also bosonic—field. It is more suitable because its masslessness guarantees the existence of photons with energies $\hbar \omega$ as low as desired. In the sequel we shall have need of quanta frequencies which fall below $|\Omega|$ times a small integer, and for macroscopic black holes this means a very low $\omega$.

Now focus on an electromagnetic mode of frequency $\omega$ with the asymptotic angular form of a vector spherical harmonic $\mathcal{Y}_{\mu}(\theta, \phi)$ (with the azimuthal angle $\phi$ and the integer $\mu$—the $z$ component of angular momentum—both referred to the $z$ axis). Suppose that when the radiator irradiates the black hole with a train of such wave-modes, the hole absorbs $\mathcal{N}$ quanta. Then $\delta M = \mathcal{N} \hbar \omega$ while $\delta J = \mathcal{N} \hbar \mu$, and it can be seen from Eq. (10) that

$$\delta A = \hbar \mathcal{N} \Theta_{K}^{-1} (\omega - \mu \Omega).$$
In the regime $\omega > \mu \Omega$ the assumption that $\mathcal{N} > 0$ implies that the horizon area increases, in harmony with Hawking’s theorem. However, for $\omega < \mu \Omega$ there will be a conflict with the theorem unless we assume that $\mathcal{N} < 0$, i.e., that the black hole reinforces the wave. This is the original Zel’’dovich-Misner superradiance. In analogy with Eq. (6) we may infer that the reflection coefficient in the vicinity of the transition point $\omega = \mu \Omega$ looks like

$$R_K = 1 - \alpha_K \cdot (\omega - \mu \Omega) + O((\omega - \mu \Omega)^3)$$

(14)

with $\alpha_K(\mu, M, J) > 0$.

Again, if we tune $\omega$ to $\mu \Omega$, the wave is neither amplified nor depleted: equilibrium of the hole with the waves ensues at the transition point. This, again, is a stable equilibrium. For if we push the hole slightly off the transition point so that $\omega$ exceeds $\mu \Omega$, the hole will absorb: $\delta M = \mathcal{N} \hbar \omega$ and $\delta J = \mathcal{N} \hbar \mu$ with $\mathcal{N} > 0$. From Eq. (12) we find that the consequent $\mu \delta \Omega$ is a sum of two terms, both positive by virtue of the constraint $J \leq M^2$. The absorption thus acts to bring the hole back to the transition point $\omega = \mu \Omega$. Similarly, if we perturb the hole into the superradiant regime, the corresponding $\mu \delta \Omega$ will be negative, which again tends to bring the hole back to the transition point. Therefore, the Kerr transition point is also an attractor.

Of course, we have only verified stability at the transition point against perturbations which preserve the $z$ axis. What if we tilt this axis slightly from the axis set by the radiator (with respect to which the mode $Y_{j\mu}(\theta, \phi)$ is determined)? Such a slight tilt would mean that the waves falling on the hole would be mostly in the primary mode $Y_{j\mu}(\theta, \phi)$ but with a slight admixture of various other values of $\mu$. Some of these secondary modes might actually superradiate while the primary mode acts normally, or vice versa. There is no question, however, of this new feature overturning our mentioned conclusion. The very weakness of the secondary modes would cause their effects on the hole to be swamped by that of the primary; the convergence $\mu \Omega \to \omega$ would still take place. And because the hole is gaining or loosing angular momentum about the direction imposed by the radiator, it is intuitively clear that the black hole’s rotation axis $z$ will gradually line up with the primary mode’s axis. The Kerr black hole at the transition point is stable overall.

So suppose we are given a Schwarzschild or a Kerr black hole and required to prepare it in a Kerr state with prescribed parameters $\bar{M}$ and $\bar{J}$ about a prescribed axis. As in the previous example, this can be done classically only if the initial horizon’s area is below $\bar{A} = \mathcal{A}(\bar{M}, \bar{J})$. I have not hit upon an appropriate analog of the method of Sec. 2 to determine the area in the pure Kerr (or pure Schwarzschild) case. However, it stands to reason that the initial $\bar{A}$ can be determined from the scattering cross section of the hole to very weak incident plane waves. I will not go into the details of such a method, but just assume they can be worked out. By using this procedure once only we can determine the initial horizon area, $A_0$, of the black hole. If the area condition is satisfied, we proceed as follows.

First we cause the radiator to irradiate the hole with a (very weak) test wave which is asymptotically of $Y_{j\mu}(\theta, \phi)$ form (as defined by the arbitrarily selected radiator
axis), and let its frequency sweep across the spectrum while the said axis is allowed to range over the sphere. All this while the monitor checks the intensity of the scattered wave. We can thus home in on the transition point $\omega = \mu \Omega$ and on the axis of the hole—the $z$ axis. This is the combination for which the amplification factor $aK$ for the wavemode in question is unity. (If the sign of $\mu$ has been chosen inappropriately, homing-in is impossible, and the opposite sign must be selected.) The process just described does not increase $A$ significantly due to the test wave’s weakness.

Thus we have determined the initial horizon area, $A_0$, axis direction and the angular frequency, $\Omega_0$, of the hole. And from Eqs. (12) and (9) we can determine the initial mass

$$M_0 = (A_0/16\pi)^{1/2}(1 - A_0\Omega_0^2/4\pi)^{-1/2}$$

from which follows the initial angular momentum $J_0 = (M_0A_0\Omega_0/4\pi)$. Next we cause $A$ to grow to $\bar{A}$ by gradually adding neutral matter or waves with zero angular momentum to the hole; according to Eq. (10), to raise $A$ under this condition we indeed need to make $M$ larger. And from Eq. (9) we see that the final mass of the process (recall that $J$ is fixed at $J_0$) is given by the Christodoulou-style formula [12]

$$M_1 = (\bar{A}/16\pi)^{1/2}(1 + 64\pi^2J_0^2/\bar{A}^2)^{1/2}$$

The difference $M_1 - M_0$ is the energy that must be added to raise the horizon area to $\bar{A}$.

Suppose now that the determined $z$ axis (which was unaffected by the step just carried out) is collinear with the prescribed rotation axis. Then as in the Reissner-Nordström case, we may now “drag” the parameter $\Omega$ from $\Omega_0$ to the $\bar{\Omega}$ corresponding to $\bar{M}$ and $\bar{J}$ by the simple expedient of consistently shifting $\omega$ slightly away from the instantaneous transition point in the direction of $\mu\Omega$. (Of course, in case $\Omega_0$ and $\bar{\Omega}$ are opposite in sign, it will be necessary to switch the sign of $\mu$ as $\Omega$ passes through zero.) If, on the contrary, the determined $z$ axis proves not collinear with the prescribed rotation axis, we first drag $\Omega$ to zero converting the hole into a Schwarzschild one. Whether the original black hole was Schwarzschild or Kerr, we may now, by irradiating the hole with a wave of angular form $Y_{j\mu}(\theta, \phi)$ with $\mu$ defined with respect to the prescribed axis, drag $\Omega$ from zero to $\bar{\Omega}$. Of course the final black hole’s rotation will be about the prescribed axis. According to Eq. (13), the increment in area $A$ during the two processes in this paragraph is $\Theta_K^{-1} \cdot (1 - \mu\Omega/\omega) \cdot \delta M$. Thus by changing $\omega$ sufficiently slowly so that $\mu\Omega$ keeps pace with it, we can accomplish the transformation with as small a change of $A$ as desired. Since $A$ had already reached the desired value $\bar{A}$, and since Eqs. (9) and (12) can be solved uniquely for $M$ and $J$ in terms of $A$ and $\Omega$, we have reached the prescribed values $\bar{M}$ and $\bar{J}$.
4 Preparing a generic Kerr-Newman state

The generic stationary—Kerr-Newman—black hole has parameters $M$, $Q$ and $J$ subject to the condition $M \geq (Q^2/2 + \sqrt{Q^4/4 + J^2})^{1/2}$. The analogs of Eqs. (4), (9) and (12) are here [1]

\[ A = A(M, J, Q) = 4\pi[r_+^2 + (J/M)^2] \]

\[ \Theta_{KN} dA = dM - \Omega dJ - \Phi dQ \]

\[ \Theta_{KN} \equiv \frac{1}{2} \sqrt{M^2 - Q^2 - (J/M)^2} A^{-1} \]

\[ \Omega \equiv 4\pi(J/M) \]

\[ \Phi \equiv 4\pi Qr_+/A \]

with $r_+ \equiv M + \sqrt{M^2 - Q^2 - (J/M)^2}$.

Let us unify the discussions of Secs. 2 and 3 by considering charged wavemodes with definite angular momentum. These could refer to either a scalar or a vector field since both are bosonic. We suppose that at the outset the axis with respect to which $\mu$ is defined coincides with the hole’s rotation axis—the $z$-axis. The analysis is then more complicated than before, but it should come as no surprise, in view of the form of Eq. (18), that the transition point between normal and superradiant regimes resides at $\omega = \varepsilon\Phi/\hbar + \mu\Omega$. In analogy with Eq. (6) and (14) we find the reflection coefficient in the vicinity of the transition point to be

\[ R_{KN} = 1 - \alpha_{KN} \cdot (\omega - \varepsilon\Phi/\hbar - \mu\Omega) + O((\omega - \varepsilon\Phi/\hbar - \mu\Omega)^3) \]

with some $\alpha_{KN}(\varepsilon, \mu, M, Q, J) > 0$.

Suppose we perturb the black hole so that it shifts into the superradiant regime, $\omega < \varepsilon\Phi/\hbar + \mu\Omega$. This, of course, leads to reinforcement of the wave at expense of the hole: $\delta M = N\hbar\omega$, $\delta J = N\hbar\mu$ and $\delta Q = N\varepsilon$ with $N < 0$ this time. It takes a laborious calculation (I used Mathematica to do the algebra) to show that, whatever the ratio $\varepsilon/\mu$, these increments give $\varepsilon\delta\Phi/\hbar + \mu\delta\Omega < 0$. Thus the black hole changes in such a sense as to approach the transition point $\omega = \mu\Omega + \Phi\varepsilon/\hbar$. Likewise, if we push the hole into the normal regime, it will absorb, $\varepsilon\delta\Phi/\hbar + \mu\delta\Omega > 0$, and as a consequence it will again be driven towards the transition point. The Kerr-Newman transition point is thus also an attractor. Just as in Sec. 3 we can argue that this property is maintained in the face of perturbations of the hole’s rotation axis.

How to prepare a state with prescribed parameters $\bar{M}$, $\bar{Q}$ and $\bar{J}$ starting from a given Kerr-Newman hole? As previously, we first have to check that $A \leq \bar{A} \equiv A(\bar{M}, \bar{Q}, \bar{J})$. Following the example of Sec. 2, we cause the radiator to irradiate the black hole with two separate wavemode trains, one of asymptotically spherical ($\mu = 0$) scalar modes with charge $\varepsilon$, and the second of electromagnetic modes ($\varepsilon = 0$) all with the same $j$ and azimuthal index $\mu \neq 0$. The radiator is supposed to sweep the frequency of the first mode, through a broad range and, with help of the monitor, tune it to the value $\varepsilon\Phi/\hbar$ (as before, this requires the appropriate choice for the sign
of \( \varepsilon \). Likewise, the radiator is charged with aligning its reference axis with the hole’s and tuning the frequency \( \omega_2 \), of the second mode to \( \mu \Omega \) (again after a felicitous choice of sign of \( \mu \)). Thus are the initial values \( \Phi_0 \) and \( \Omega_0 \) determined. We also determine \( Q_0 \) this once only from its Coulomb field as in Sec. 2. Now by substituting \( r_+ \) from Eq. (21) and \( J/M \) from Eq. (20) in the expression for \( A(M, Q, J) \), Eq. (17), we have

\[
A = \frac{4\pi}{(\Phi/Q)^2 + \Omega^2}.
\]

(23)

Thus we can determine the initial area, \( A_0 \), and obtain \( Q_0 \) and \( (J/M)_0 \) into the bargain.

If \( A_0 < \bar{A} = A(\bar{M}, \bar{Q}, \bar{J}) \), we proceed to determine the initial mass, \( M_0 \), by eliminating \( r_+ \) between its definition and Eq. (21), replacing \( J_0/M_0 \) from Eqs. (20), and simplifying with help of Eq. (23):

\[
M_0 = \frac{1}{2}(Q_0/\Phi_0) + 2\pi(Q_0^3/\Phi_0 A_0)
\]

(24)

We follow this by gradual addition of \textit{neutral} matter or waves with \textit{zero} angular momentum to the hole in order to raise its area to \( \bar{A} \). According to Eq. (18) this indeed requires an increase in \( M \). Solving Eq. (17) for \( M \) we get for the mass at the end of the process (recall that \( J_0 \) and \( Q_0 \) are unchanged) the Christodoulou-Ruffini style formula [13]

\[
M_1 = \left[ \frac{\bar{A}}{16\pi} \left( 1 + \frac{4\pi Q_0^2}{A} \right)^2 + \frac{4\pi J_0^2}{A} \right]^{1/2}
\]

(25)

The quantity \( M_1 - M_0 \) is the energy we are required to add to the hole.

Of course the envisaged process has caused \( \Phi \) and \( \Omega \) to drift away from the transition points. We, therefore, again sweep the frequencies of the two wavemodes to recover the two corresponding transition points. We then shift the two mode frequencies away from the two transition points in the directions of \( \varepsilon \bar{\Phi}/\hbar \) and \( \mu \bar{\Omega} \), respectively. This has the effect of dragging the black hole’s rotational angular frequency and electric potential to the prescribed values \( \bar{\Omega} \) and \( \bar{\Phi} \). (As before, if initial and prescribed values differ in sign, we have to switch the sign of \( \varepsilon \) or \( \mu \) when \( \Phi \) or \( \Omega \), respectively, pass through zero.) Again, slowness of the dragging guarantees that the horizon’s area does not increase significantly on the scale of the overall changes in the black hole’s parameters. At the end of the envisaged process the desired \( \bar{M} \), \( \bar{Q} \) and \( \bar{J} \) will have been reached because, as easily checked from the expressions (17), (20) and (21), particular values of \( \bar{A}, \bar{\Omega} \) and \( \bar{\Phi} \) correspond to a unique set of values for \( \bar{M}, \bar{Q} \) and \( \bar{J} \).

5 Beating the Hawking radiance

All the above discussions ignored Hawking radiance which poses some problems for our method. For instance, because it emerges in a range of modes, it tends to cause
the hole parameters to drift away from any prescribed values, e.g. away from the transition point if set there initially. This destabilization be counteracted if the radiation incident from the radiator is sufficiently strong. We now determine how strong it must be.

It is well known that Hawking radiation from a Kerr-Newman black hole is thermal—apart from a distortion in the mean occupation numbers of the various modes. Thus we can estimate the rate, as measured with respect to global time, at which the hole loses mass to the radiation by using the Stefan-Boltzmann law appropriate to Hawking’s temperature $T_H = \frac{4\hbar}{\Theta_{\text{KN}}}$ and radiating area $A$:

$$|\dot{M}_H| \approx \frac{\pi^2 A T_H^4}{60\hbar^3} = \frac{4\pi^2 \hbar [M^2 - Q^2 - (J/M)^2]^2}{15A^3} \leq \frac{\hbar}{240\pi M^2}$$  \hspace{1cm} (26)

This must be compared with $\hbar \omega$ times the rate at which quanta are absorbed from the radiator wavemodes (if $\omega - \varepsilon \Phi - \mu \Omega > 0$) or added to them (if $\omega - \varepsilon \Phi - \mu \Omega < 0$), namely

$$|\dot{M}|_{\text{rad}} = |1 - R_{\text{KN}}| \cdot N \hbar \omega \cdot (\Delta \omega/2\pi) \hspace{1cm} (27)$$

in both cases. Here $N$ is the occupation number of the incoming modes while the factor $(\Delta \omega/2\pi)$ is the usual rate of flow of modes in one dimension—since we keep $\mu$ and $j$ fixed—assuming that those modes span a bandwidth $\Delta \omega$. Since the radiator operates near the transition point we can use Eq. (22) to approximate $R_{\text{RN}}$. Now $\alpha_{\text{KN}}$ has dimensions of length and it measures absorption by the hole, so it is intuitively clear that it must be of order $\sqrt{A}$; for orientation I take $\alpha_{\text{KN}} = \sqrt{A} \geq 2\sqrt{\pi M}$. Thus provided we take

$$N \gg \frac{\omega/\Delta \omega}{240\sqrt{\pi} |1 - \varepsilon \Phi/(\hbar \omega) - \mu \Omega/\omega| (M \omega)^3},$$  \hspace{1cm} (28)

the rate of change of $M$ attributable to the radiator will strongly dominate $|\dot{M}_H|$. The hole will then be stabilized at the transition point against Hawking’s radianc since the last causes relatively slow changes in $M$, as well as in $Q$ and $J$.

As an illustration let us consider a plain Kerr black hole locked into its transition point by a train of electromagnetic wavemodes with definite $\mu \geq 1$. If the quality parameter $|1 - \mu \Omega/\omega|$, which measures the hole’s departure from the transition point, is $10^{-4}$, the r.h.s. of condition (28) is actually smaller than $24(M\mu\Omega)^{-3}(\omega/\Delta \omega)$. Since the quanta are bosons, the condition is thus easy to satisfy if we do not insist on almost monochromatic incoming radiation $(\Delta \omega/\omega$ not very small compared to unity), and if the black hole is not almost a Schwarzschild one $(M \Omega$ not very small compared to unity). Stabilizing a generic Kerr hole at its transition point against Hawking radiation drift is thus relatively easy.

The Unruh radiation [15], which emerges spontaneously in a range of superradiant modes, is quite similar to Hawking’s in this same respect. Thus its destabilizing effects can be suppressed as well. Neither radiation need cause problems during the stage
in which the frequency \( \omega \) is swept to locate the black hole’s transition point because the sweep can be made quickly so that little spontaneous radiation is emitted in the interim.

However, as mentioned, the stage in which \( \Omega \) or \( \Phi \) are “dragged” has to be protracted, for otherwise the process would fail to be adiabatic and the horizon area \( A \) would increase substantially. We do not want this to happen because the first step of our process has already set \( A \) to its final value. But—so it would seem—if the dragging is performed too slowly, a lot of Hawking radiation will get emitted in the interim with consequent quantum decrease of \( A \). This last eventuality would complicate the procedure we have set forth for preparing the black hole state.

Fortunately the same condition (28), which guarantees that Hawking’s radiance does not destabilize the hole from its transition point, insures that the radiance generates negligible changes of the horizon area in the course of dragging. Let us calculate the change in the logarithm of \( A \) which comes from Hawking’s radiance (subscript “H”) in terms of mass changes attributable to the radiator (subscript “rad”):

\[
(\Delta \ln A)_H = \int \frac{\dot{A}_H}{A} dt = \int \frac{\dot{A}_H}{A M_{\text{rad}}} dM_{\text{rad}}
\]  

(29)

Now according to Eq. (18), the fact that Hawking’s radiance carries away angular momentum (charge) of the same sign as the hole’s angular momentum (charge) implies (remember that \( \dot{M}_H \) and \( \dot{A}_H \) are both negative) that \( |\dot{A}|_H < \Theta_{KN}^{-1} |\dot{M}|_H \). Thus it follows from Eq. (29) that

\[
|\Delta \ln A|_H < \int \frac{|\dot{M}|_H}{\Theta_{KN} A|\dot{M}_{\text{rad}}|} |dM_{\text{rad}}|
\]  

(30)

where we have exploited the fact that \( \dot{M}_{\text{rad}} \) and \( dM_{\text{rad}} \) are of like sign. We now substitute here from Eq. (26) the second form for \( |\dot{M}|_H \) as well as \( |\dot{M}_{\text{rad}}| \) from (27) with the previously discussed value \( \alpha_{KN} \geq 2\sqrt{\pi}M \). Now because by condition (28) most of \( M \)'s change is attributable to the radiator, \( dM_{\text{rad}} \approx dM \) and \( M_{\text{rad}} \approx \dot{M} \). In view of Eqs. (17) and (19) we now have

\[
|\Delta \ln A|_H < \frac{\omega/\Delta \omega}{120\sqrt{\pi} |1 - \bar{\epsilon}\Phi/\hbar\omega - \mu\Omega/\omega| N(M\omega)^3} |d\ln M|
\]  

(31)

It then follows from condition (28) that \( |\Delta \ln A|_H \ll |\Delta \ln M| \), i.e., the fractional change in \( A \) which is attributable to the Hawking radiance is negligible compared to the overall fractional change in \( M \) (or in the other hole parameters for that matter) during “dragging”. As already mentioned a the end of Secs. 2 and 3, the change in \( A \) directly attributable to the radiator is also entirely negligible on this scale. Thus one can indeed carry out slow dragging of \( \Omega \) and \( \Phi \) without having the horizon area increase significantly.

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