ANALYSIS OF THE VERTEXES $\Xi_Q^*\Xi_Q'V$, $\Sigma_Q^*\Sigma_QV$ AND RADIATIVE DECAYS

$\Xi_Q^* \rightarrow \Xi_Q'\gamma$, $\Sigma_Q^* \rightarrow \Sigma_Q\gamma$

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Abstract

In this article, we study the vertexes $\Xi_Q^*\Xi_Q'V$ and $\Sigma_Q^*\Sigma_QV$ with the light-cone QCD sum rules, then assume the vector meson dominance of the intermediate $\phi(1020)$, $\rho(770)$ and $\omega(782)$, and calculate the radiative decays $\Xi_Q^* \rightarrow \Xi_Q'\gamma$ and $\Sigma_Q^* \rightarrow \Sigma_Q\gamma$.

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1 Introduction

The charm and bottom baryons which contain a heavy quark and two light quarks are particularly interesting for studying dynamics of the light quarks in the presence of a heavy quark. They serve as an excellent ground for testing predictions of the quark models and heavy quark symmetry [1, 2]. The three light quarks form an $SU(3)$ flavor triplet $3$, two light quarks can form diquarks of a symmetric sextet and an antisymmetric antitriplet, i.e. $3 \times 3 = \bar{3} + 6$. For the $S$-wave charm baryons, the $\frac{1}{2}^+$ antitriplet states ($\Lambda_c^+, \Xi_c^+, \Xi'_c^0$), and the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ sextet states ($\Omega_c, \Sigma_c, \Xi_c^*$) and ($\Omega_c^*, \Sigma_c^*, \Xi_c^{*0}$) have been well established; while the corresponding bottom baryons are far from complete, only the $\Lambda_b$, $\Sigma_b$, $\Sigma_b^*$, $\Xi_b$, $\Omega_b$ have been observed [3]. Furthermore, several new excited charm baryon states have been observed by the BaBar, Belle and CLEO Collaborations, such as $\Lambda_c(2765)^+$, $\Lambda_c^*(2880)$, $\Lambda_c^*(2940)$, $\Sigma_c^*(2800)$, $\Xi_c^*(3077)$, $\Xi_c^{*0}(2980)$, $\Xi_c^{*0}(3077)$ [4, 5, 6].

In Ref.[7], we assume the charm mesons $D_{s0}(2317)$ and $D_{s1}(2460)$ with the spin-parity $0^+$ and $1^+$ respectively are the conventional $c\bar{s}$ states, and calculate the strong coupling constants $\langle D_{s0}\phi|D_{s0}\rangle$ and $\langle D_{s1}\phi|D_{s1}\rangle$ with the light-cone QCD sum rules, then take the vector meson dominance of the intermediate $\phi(1020)$, study the radiative decays $D_{s0} \rightarrow D_{s1}\gamma$ and $D_{s1} \rightarrow D_{s}\gamma$. In Refs.[8, 9], we calculate the masses and the pole residues of the $\frac{1}{2}^+$ heavy baryons $\Omega_Q$ and the $\frac{3}{2}^+$ heavy baryons $\Omega_Q^*$ with the QCD sum rules. Moreover, we study the vertexes $\Omega_Q^*\Omega_Q\phi$ with the light-cone QCD sum rules, then assume the vector meson dominance of the intermediate $\phi(1020)$, and calculate the radiative decays $\Omega_Q^* \rightarrow \Omega_Q\gamma$ [10].

In this article, we extend our previous works to study the vertexes $\Xi_Q^*\Xi_Q'V$ and $\Sigma_Q^*\Sigma_QV$ with the light-cone QCD sum rules [5], then assume the vector meson dominance of the intermediate $\phi(1020)$, $\rho(770)$ and $\omega(782)$, and calculate the radiative decays $\Xi_Q^* \rightarrow \Xi_Q'\gamma$ and $\Sigma_Q^* \rightarrow \Sigma_Q\gamma$ to complete our works on radiative decays among the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ sextet states ($\Omega_Q, \Sigma_Q, \Xi_Q$) and ($\Omega_Q^*, \Sigma_Q^*, \Xi_Q^{*0}$). In Ref.[11], Aliev et al. study the radiative decays.

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2The results of the strong coupling constants among the nonet vector mesons, the octet baryons and the decuplet baryons will be presented elsewhere.
\( \Sigma_Q^* \rightarrow \Sigma_Q \gamma, \Xi_Q^* \rightarrow \Xi_Q \gamma \) and \( \Sigma_Q^* \rightarrow \Lambda_Q \gamma \) with the light-cone QCD sum rules, where the light-cone distribution amplitudes of the photon are used.

The light-cone QCD sum rules carry out the operator product expansion near the light-cone \( x^2 \approx 0 \) instead of the short distance \( x \approx 0 \), while the nonperturbative hadronic matrix elements are parameterized by the light-cone distribution amplitudes instead of the vacuum condensates [12, 13, 14]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [14, 15, 16]. The \( \rho \Sigma \), \( \rho \Sigma \Sigma \), \( \rho \Xi \Xi \) and other strong coupling constants of the nonet vector mesons with the octet baryons have been calculated using the light-cone QCD sum rules [17, 18, 19]. In Refs. [20, 21], Aliev et al study the strong coupling constants of the pseudoscalar octet baryons comprehensively. In Refs. [22, 23], we study the strong decays \( \Delta^{++} \rightarrow p \pi, \Sigma^{*} \rightarrow \Sigma \pi \) and \( \Sigma^{*} \rightarrow \Delta \pi \) using the light-cone QCD sum rules. Moreover, the coupling constants of the vector mesons \( \rho \) and \( \omega \) with the baryons are studied with the external field QCD sum rules [24]. Recently, the strong coupling constants among the light vector mesons and the heavy baryons are calculated with the light-cone QCD sum rule in the leading order of heavy quark effective theory [25].

The article is arranged as follows: we derive the strong coupling constants \( g_1, g_2 \) and \( g_3 \) of the vertexes \( B_Q^* B_Q V \) with the light-cone QCD sum rules in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

## 2 The vertexes \( B_Q^* B_Q V \) with light-cone QCD sum rules

We parameterize the vertexes \( B_Q^* B_Q \phi \), \( B_Q^* B_Q \rho \) and \( B_Q^* B_Q \omega \) with three tensor structures due to Lorentz invariance and introduce three strong coupling constants \( g_1, g_2 \) and \( g_3 \) [26],

\[
\langle B_Q(p+q)|B_Q^*(p)|\phi(q)\rangle = \frac{1}{\sqrt{2}} U(p+q) \left[ g_1(q_\mu \epsilon/ - \epsilon_\mu q) \gamma_5 + g_2(P \cdot \epsilon_\mu - P \cdot q \epsilon_\mu) \gamma_5 \right. \\
\left. + g_3(q \cdot \epsilon_\mu - q^2 \epsilon_\mu) \gamma_5 \right] U^\mu(p),
\]

\[
\langle B_Q(p+q)|B_Q^*(p)|\rho_0/\omega(q)\rangle = \frac{1}{\sqrt{2}} U(p+q) \left[ g_1(q_\mu \epsilon/ - \epsilon_\mu q) \gamma_5 + g_2(P \cdot \epsilon_\mu - P \cdot q \epsilon_\mu) \gamma_5 \right. \\
\left. + g_3(q \cdot \epsilon_\mu - q^2 \epsilon_\mu) \gamma_5 \right] U^\mu(p),
\]

where the \( U(p) \) and \( U_\mu(p) \) are the Dirac spinors of the heavy baryons states \( B_Q \) (\( \Xi'_Q, \Sigma_Q \)) and \( B_Q^* \) (\( \Xi_Q^*, \Sigma_Q^* \)) respectively, the \( \epsilon_\mu \) is the polarization vector of the mesons \( \phi(1020) \), \( \rho(770) \) and \( \omega(782) \), and \( P = \frac{2p+q}{2} \).

In the following, we write down the two-point correlation functions \( \Pi_\mu(p,q) \),

\[
\Pi_\mu^{\phi/\rho_0}(p,q) = i \int d^4x e^{-ipx} \langle 0|T \{ J(0) \bar{J}_\mu(x) \} |\phi/\rho_0(q)\rangle, \tag{3}
\]

\[
\begin{align*}
J^\phi(x) &= \epsilon^{ijk} q^T_i(x) C\gamma_{\mu} s_j(x) \gamma_5 \gamma^\mu Q_k(x), \\
J^\Sigma(x) &= \epsilon^{ijk} q^T_i(x) C\gamma_{\mu} q'_j(x) \gamma_5 \gamma^\mu Q_k(x), \\
J^\Sigma_\mu(x) &= \epsilon^{ijk} q^T_i(x) C\gamma_{\mu} s_j(x) Q_k(x), \\
J^\Sigma_\mu(x) &= \epsilon^{ijk} q^T_i(x) C\gamma_{\mu} q'_j(x) Q_k(x), \tag{4}
\end{align*}
\]
where $Q = c, b$ and $q, q' = u, d$, the $i, j, k$ are color indexes, the Ioffe type heavy baryon currents $J(x)$ ($J^z(x)$, $J^\Sigma(x)$) and $J_\mu(x)$ ($J^\Sigma_\mu(x)$, $J^\Phi_\mu(x)$) interpolate the $1^+$ baryon states $\Xi_Q, \Sigma_Q$ and the $3^+$ baryon states $\Xi_Q', \Sigma_Q'$, respectively, the external vector states $\phi(1020)$ and $\rho(770)$ have the four momentum $q_\mu$ with $q^2 = M^2_{\phi/\rho}$. The quark constituents of the vector mesons $\rho_0$ and $\omega$ are $\frac{1}{\sqrt{2}}(|\bar{u}u| - |d\bar{d}|)$ and $\frac{1}{\sqrt{2}}(|\bar{u}u| + |d\bar{d}|)$ respectively, the isospin triplet meson $\rho_0$ and isospin singlet meson $\omega$ have approximately degenerate masses. We assume that the vector mesons $\rho_0$ and $\omega$ have similar light-cone distribution amplitudes, and obtain the corresponding strong coupling constants by symmetry considerations, as the $\omega$-meson light-cone distribution amplitudes have not been explored yet, see Appendix A for detailed discussions.

Basing on the quark-hadron duality [15, 16], we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J(x)$ and $J_\mu(x)$ into the correlation functions $\Pi_\mu(p, q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the heavy baryons $\Xi_Q', \Sigma_Q$ and $\Xi_Q, \Sigma_Q$, we get the following results,

$$
\Pi^{\phi/\rho_0}_\mu(p, q) = \frac{\langle 0|J(0)|B_Q(q + p)\rangle\langle B_Q(q + p)|B_Q^\varphi(p)\phi/\rho(q)\rangle\langle B_Q^\varphi(p)|J_\mu(0)|0\rangle}{M^2_{B_Q} - (q + p)^2} \left\{ \frac{g_1}{M_{B_Q} + M_{B_Q^\varphi}} \not\! p_\gamma q_\mu - \frac{g_2}{2} \not\! p_\gamma q^2_\mu - g_3 \not\! p_\gamma q_\mu \cdot \not\! p_\mu \cdot q_\mu + \cdots \right\} 1/\sqrt{2} + \cdots, \tag{5}
$$

where the following definitions have been used,

$$
\langle 0|J(0)|B_Q(p)\rangle = \lambda_{B_Q} U(p, s),
\langle 0|J_\mu(0)|B_Q^\varphi(p)\rangle = \lambda_{B_Q^\varphi} U_\mu(p, s),
\sum_s U(p, s)\bar{U}(p, s) = \not\! p + M_{B_Q},
\sum_s U_\mu(p, s)\bar{U}_\nu(p, s) = -(\not\! p + M_{B_Q^\varphi}) \left( g_{\mu\nu} - \frac{\gamma_\mu\gamma_\nu}{3} - \frac{2p_\mu p_\nu}{3M_{B_Q^\varphi}^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3M_{B_Q^\varphi}^2} \right), \tag{6}
$$

the factors 1 and $\frac{1}{\sqrt{2}}$ correspond to the correlation functions $\Pi^{\phi}_\mu(p, q)$ and $\Pi^{\rho_0}_\mu(p, q)$ respectively. The current $J_\mu(x)$ couples not only to the spin-parity $J^P = \frac{3}{2}^+$ states, but also to the spin-parity $J^P = \frac{1}{2}^-$ states. For a generic $\frac{1}{2}^-$ resonance $B_Q^\varphi$, $\langle 0|J_\mu(0)|B_Q^\varphi(p)\rangle = \lambda^* (\gamma_\mu - 4p_\mu M^*_{B_Q^\varphi})U^*(p, s)$, where $\lambda^*$ is the pole residue, $M^*$ is the mass, and the spinor $U^*(p, s)$ satisfies the usual Dirac equation $(\not\! p - M^*)U^*(p) = 0$. In this article, we choose the tensor structures $\not\! p_\gamma q_\mu, \not\! p_\gamma q_\mu \cdot \not\! p_\mu \cdot q_\mu$ and $\not\! p_\gamma q_\mu \cdot \not\! p_\mu \cdot q_\mu$, the baryon state $B_Q^\varphi$ has no contamination, for example, we can study the contribution of the $\frac{1}{2}^-$ baryon state $\bar{B}_Q^\varphi$ to the correlation.
functions $\Pi_\mu^\phi(p, q)$,

$$
\Pi_\mu^\phi(p, q) = \frac{\langle 0|J(0)|B_Q(q + p)\rangle\langle B_Q(q + p)|\tilde{B}_Q^\phi(p)|J_\mu(0)|0 \rangle}{[M_{B_Q}^2 - (q + p)^2][M_{B_Q}^2 - p^2]} + \ldots
$$

$$
= \lambda_{B_Q} \frac{g + \not{p} + M_{B_Q}}{M_{B_Q}^2 - (q + p)^2} \left[ g_V \not{q} + ig_T \frac{\epsilon^\alpha \sigma_{\alpha\beta} q^\beta}{M_{B_Q} + M_*} \right] \gamma_5 \not{p} + M_* \left[ \gamma_\mu - 4 \frac{p_\mu}{M_*} \right] + \ldots
$$

$$
f_1(\gamma, p, q, \epsilon) \gamma_\mu + f_2(\gamma, p, q, \epsilon) p_\mu + \ldots,
$$

where we introduce the strong coupling constants $g_V$ and $g_T$ to parameterize the vertexes $\langle B_Q(q + p)|\tilde{B}_Q^\phi(p)\phi(q)\rangle$, the notations $f_1$ and $f_2$ are functions of $\gamma_\alpha, \gamma_5, \epsilon_\alpha, p_\alpha$ and $q_\alpha$, here we order the Dirac matrixes as $\not{q}, \not{p}, \not{\epsilon}, \not{\gamma}_5$.

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_\mu(p, q)$ in perturbative QCD. The calculations are performed at the large space-like momentum regions $(q + p)^2 \ll 0$ and $p^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the "full" propagator of a massive quark in the presence of the quark and gluon condensates firstly \cite{12} \cite{16},

$$
S_{ij}(x) = \frac{i \delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} m_s}{4\pi^2 x^2} - \frac{\delta_{ij} \langle \bar{s}s \rangle}{12} \not{x} + \frac{i \delta_{ij} x^2}{1152} \delta_{ij} \langle \bar{s} g_s \sigma Gs \rangle \not{x} - \frac{i}{16\pi^2 x^2} \int_0^1 dv G_{ij}^\mu (vx) \left[ (1 - v) \not{x} \sigma^{\mu\nu} + v \sigma^{\mu\nu} \not{x} \right] + \ldots,
$$

$$
S_{ij}^Q(x) = \frac{i}{(2\pi)^4} \int d^4 k e^{-ikx} \left\{ \frac{\delta_{ij}}{k - m_Q} - \frac{g_s G_{ij}^{\alpha\beta}}{4} \sigma_{\alpha\beta} (k + m_Q) + \frac{\langle \bar{s} G_{ij}^{\alpha\beta} \rangle}{k^2 - m_Q^2} \frac{k^2 + m_Q k}{(k^2 - m_Q^2)^4} + \ldots \right\},
$$

where $\langle \bar{s} g_s \sigma Gs \rangle = \langle \bar{s} g_s \sigma G_{\alpha\beta} G^{\alpha\beta} \rangle$ and $\langle \bar{s} G_{ij} \rangle = \langle \bar{s} G_{ij}^{\alpha\beta} G^{\alpha\beta} \rangle$ (the corresponding full propagators $U_{ij}(x)$ and $D_{ij}(x)$ of the quarks $u$ and $d$ respectively can be obtained with a simple replacement), then contract the quark fields in the correlation functions $\Pi_\mu(p, q)$ with Wick theorem, and obtain the following results:

$$
\Pi_\mu^{\Xi_\phi}(p, q) = i e^{ik} e^{i\epsilon'} j' k' \int d^4 x e^{-ipx} \gamma_5 \gamma^\alpha S_{k'k}(x) Tr \left[ \gamma_\alpha \langle 0|s_j(0)\bar{s}_j(x)\phi(q)\rangle \gamma_\mu C U / D_{ij'}^T(x) \gamma_5 (-x) C \right],
$$

$$
\Pi_\mu^{\Xi_\phi}(p, q) = i e^{ik} e^{i\epsilon'} j' k' \int d^4 x e^{-ipx} \gamma_5 \gamma^\alpha S_{k'k}(x) Tr \left[ \gamma_\alpha S_{jj'}(x) \gamma_\mu C \langle 0|q_i(0)\bar{q}_{i'}(x)\rho(q)\gamma_5 C \right],
$$

$$
\Pi_\mu^{\Xi_\rho}(p, q) = i e^{ik} e^{i\epsilon'} j' k' \int d^4 x e^{-ipx} \gamma_5 \gamma^\alpha S_{k'k}(x) Tr \left[ \gamma_\alpha S_{jj'}(x) \gamma_\mu C \langle 0|q_i(0)\bar{q}_{i'}(x)\gamma_5 C \right],
$$

4
\[ \Pi_\mu^{\Sigma^p \Sigma^\rho} (p, q) = A \epsilon^{ijk} \epsilon^{ij'}k' \int d^4xe^{-ipx} \]
\[
\left\{ \bar{\psi} \gamma_\alpha S_{Q}^{kk'} (-x) Tr \left[ \gamma_\alpha (0|u_j (0) \bar{u}_{j'} (x)|\rho (q)) \gamma_\mu C U_{ij'}^T (-x) C \right] + \bar{\psi} \gamma_\alpha S_{Q}^{kk'} (-x) Tr \left[ \gamma_\alpha U_{jj'} (-x) \gamma_\mu C (0|u_i (0) \bar{u}_{i'} (x)|\rho (q)) T C \right] \right\}, \quad (11)
\]

Here we take isospin limit for the quarks \( u \) and \( d \), the symmetry factor \( A = 1 \), \( \bar{\omega} = -1 \) for the channels \( \Sigma^* (Qud) \Sigma (Qud) \rho \), and \( A = \pm 2 \) and \( \bar{\omega} = 1 \) for the channels \( \Sigma^* (Quu) \Sigma (Quu) \rho \) and \( \Sigma^* (Qdd) \Sigma (Qdd) \rho \) respectively.

Performing the Fierz re-ordering to extract the contributions from the two-particle and three-particle vector meson light-cone distribution amplitudes respectively, then substituting the full \( q \) and \( Q \) quark propagators into the correlation functions in Eqs. (9-11) and completing the integral in the coordinate space, finally integrating over the variable \( k \), we can obtain the correlation functions \( \Pi_\mu (p, q) \) at the level of quark-gluon degree of freedom.

In calculation, the two-particle and three-particle vector meson light-cone distribution amplitudes have been used [27, 28, 29, 30]. The parameters in the light-cone distribution amplitudes are scale dependent and are estimated with the QCD sum rules [20, 30]. In this article, the energy scale \( \mu \) is chosen to be \( \mu = 1 \) GeV.

Taking double Borel transform with respect to the variables \( Q_1^2 = -p^2 \) and \( Q_2^2 = -(p + q)^2 \) respectively, then subtracting the contributions from the high resonances and continuum states by introducing the threshold parameter \( s_0 \) (i.e. \( M_{2n} \rightarrow \frac{1}{y} \int_0^{s_0} dss^{n-1}e^{-\frac{y}{s}} \)), finally we can obtain 30 sum rules for the strong coupling constants \( g_1, g_2 \) and \( G3 = -\left( M_{BQ} + M_{DQ} \right) g_1 - M_\phi / \rho_0 \left( \frac{m_2}{2} + G3 \right) \) respectively, the explicit expressions are presented in the appendix A.3

3 Numerical result and discussion

The masses of the established hadrons are taken from the Particle Data Group \( M_\phi = 1.019455 \) GeV, \( M_\rho = 0.77549 \) GeV, \( M_\omega = 0.78265 \) GeV, \( M_{\Xi^+} = 2.6459 \) GeV, \( M_{\Sigma^+} = 2.5184 \) GeV, \( M_{\Sigma^0} = 2.5175 \) GeV, \( M_{\Sigma^*} = 2.5180 \) GeV, \( M_{\Sigma^*} = 5.8290 \) GeV, \( M_{\Xi} = 5.8364 \) GeV, \( M_{\Xi} = 2.5756 \) GeV, \( M_{\Xi} = 2.5779 \) GeV, \( M_{\Sigma^*} = 2.4502 \) GeV, \( M_{\Xi^*} = 2.4529 \) GeV, \( M_{\Xi^*} = 2.45376 \) GeV, \( M_{\Xi^*} = 5.8078 \) GeV, and \( M_{\Xi^*} = 5.8152 \) GeV [3]. In calculation, we take the average values of the masses in each isospin multiplet and neglect the small isospin splitting in the heavy baryon multiplet.

The parameters which determine the vector meson light-cone distribution amplitudes are \( f_\phi = (0.215 \pm 0.005) \) GeV, \( f_\phi = (0.186 \pm 0.009) \) GeV, \( a_1^\parallel = 0.0 \), \( a_1^\perp = 0.0 \), \( a_2^\parallel = 0.18 \pm 0.08 \), \( a_2^\perp = 0.14 \pm 0.07 \), \( \zeta_3^\parallel = 0.024 \pm 0.008 \), \( \zeta_3^\perp = 0.0 \), \( \omega_3^\parallel = -0.045 \pm 0.015 \), \( \kappa_3^\parallel = 0.0 \), \( \omega_3^\perp = 0.09 \pm 0.03 \), \( \lambda_3^\parallel = 0.0 \), \( \lambda_3^\perp = 0.0 \), \( \omega_3^\parallel = 0.20 \pm 0.08 \), \( \lambda_3^\parallel = 0.0 \), \( \zeta_4^\parallel = 0.00 \pm 0.02 \), \( \omega_4^\parallel = -0.02 \pm 0.01 \), \( \zeta_4^\perp = -0.01 \pm 0.03 \), \( \zeta_4^\parallel = -0.03 \pm 0.04 \), \( \kappa_4^\parallel = 0.0 \), \( \kappa_4^\perp = 0.0 \) for the \( \phi \)-meson; and \( f_\rho = (0.216 \pm 0.003) \) GeV, \( f_\rho = (0.165 \pm 0.009) \) GeV, \( a_1^\parallel = 0.0 \), \( a_1^\perp = 0.0 \), \( a_2^\parallel = 0.15 \pm 0.07 \), \( a_2^\perp = 0.14 \pm 0.06 \), \( \zeta_3^\parallel = 0.030 \pm 0.010 \), \( \lambda_3^\parallel = 0.0 \), \( \omega_3^\parallel = -0.09 \pm 0.03 \),

3Here we present some technical details in calculating the correlation functions \( \Pi_\mu^{\Sigma^\rho \Sigma^\phi} (p, q) \) to illustrate
we take the simple Ioffe type interpolating currents, which are constructed by considering
studied using the QCD sum rules, the pole residues are not calculated. In this article,
\( \rho \omega (0) \Pi \) the procedure,
\( m \lambda \) and \( \bar{\lambda} \) and pole residues \( \lambda_{BQ} \) and \( \lambda_{BQ}^\prime \) are determined by the following correlation functions,
\[ \Pi_{\mu\nu}(p) = i \int d^4xe^{ipx} \langle 0| T \{ J_\mu(x)\bar{J}_\nu(0) \} | 0 \rangle, \]
\[ \Pi(p) = i \int d^4xe^{ipx} \langle 0| T \{ J(x)\bar{J}(0) \} | 0 \rangle, \] (13)
\[ J^\Xi(x) = e^{ijk}u_i^T(x)C\gamma_\mu s_j(x)\gamma_5\gamma^\mu Q_k(x), \]
\[ J^\Sigma(x) = e^{ijk}u_i^T(x)C\gamma_\mu d_j(x)\gamma_5\gamma^\mu Q_k(x), \]
\[ J^\Xi_\mu(x) = e^{ijk}u_i^T(x)C\gamma_\mu s_j(x)Q_k(x), \]
\[ J^\Sigma_\mu(x) = e^{ijk}u_i^T(x)C\gamma_\mu d_j(x)Q_k(x). \] (14)

In Refs. [32, 33], the masses of the heavy baryon states containing one heavy quark are studied using the QCD sum rules, the pole residues are not calculated. In this article, we take the simple Ioffe type interpolating currents, which are constructed by considering
the procedure,
\[ \Pi_\mu(p, q) = \frac{ie^{ijk}e^{ij\kappa}}{12} \int d^4xe^{-ipx}\langle 0| \bar{s}(x)\gamma_5\gamma_\alpha[s_\mu q_\alpha - \epsilon_\alpha q_\mu]d^4xe^{i(k-p-q)x} \frac{1}{k^2 - m_Q^2 x^2} + \cdots \]
\[ = \frac{3\bar{f}_5M_5}{32\pi^2} \gamma_5\gamma_\alpha \frac{\epsilon_\alpha q_\mu - \epsilon_\alpha q_\mu}{1 - u} \int_0^1 du \int_0^1 d^4x \frac{\Gamma(\varepsilon)}{(p + uq)^2 - m_Q^2} | \varepsilon \rightarrow 0 + \cdots \]
\[ \rightarrow [\gamma \not\! q_\alpha \epsilon_\mu - \gamma \not\! q_\alpha \epsilon_\mu] \frac{\bar{f}_5M_5}{16\pi^2} \int_0^1 du \int_0^1 d^4x \frac{\Gamma(\varepsilon)}{(p + uq)^2 - m_Q^2} | \varepsilon \rightarrow 0 + \cdots \]
\[ + \lambda_{\Xi} \bar{M}_{\Xi} \lambda_{\Xi} \exp \left[ -\frac{\bar{m}^2 + u(1-u)M_\Xi^2}{M^2} \right] \delta(u - u_0) + \cdots \] (take double Borel transform)
\[ + \lambda_{\Sigma} \bar{M}_{\Sigma} \lambda_{\Sigma} \exp \left[ -\frac{\bar{m}^2 + u(1-u)M_\Sigma^2}{M^2} \right] \delta(u - u_0) + \cdots \] (subtract continuum contributions)
\[ = \frac{\lambda_{\Xi} \bar{M}_{\Xi} \lambda_{\Xi} \exp \left[ -\frac{\bar{m}^2 + u(1-u)M_\Xi^2}{M^2} \right]}{M^2} + \cdots \]

For technical details about the Borel transform, one can consult the excellent review [14].
the diquark theory and the heavy quark symmetry [34, 35]. We insert a complete set of intermediate baryon states with the same quantum numbers as the current operators $J(x)$ and $J_{\mu}$ into the correlation functions $\Pi(p)$ and $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [15, 16]. After isolating the pole terms of the lowest states $\Xi^*_Q, \Xi_Q, \Sigma^*_Q$ and $\Sigma_Q$, we obtain the following results:

$$\Pi_{\mu\nu}(p) = \lambda^2_{B_Q} \frac{M_{B_Q} + p}{M^2_{B_Q} - p^2} [-g_{\mu\nu} + \cdots] + \cdots ,$$

$$\Pi(p) = \lambda^2_{B_Q} \frac{M_{B_Q} + p}{M^2_{B_Q} - p^2} + \cdots ,$$

we choose the tensor structures $g_{\mu\nu}, \delta g_{\mu\nu}, 1$ and $\delta$ for analysis. After performing the standard procedure of the QCD sum rules, we obtain sixteen sum rules for the heavy baryons states $B_Q^*$ and $B_Q$.

$$\chi^2_{\mu} e^{-\frac{M_i^2}{M^2}} = \int_{\Delta_i} s^0 \, ds \rho_i^A(s) e^{-\frac{s}{M^2}} ,$$

$$\chi^2_{\mu} M_i e^{-\frac{M_i^2}{M^2}} = \int_{\Delta_i} s^0 \, ds \rho_i^B(s) e^{-\frac{s}{M^2}} ,$$

where the $i$ denote the channels $\Xi^*_Q, \Xi_Q, \Sigma^*_Q$ and $\Sigma_Q$ respectively; the $s^0$ are the corresponding continuum threshold parameters and the $M^2$ is the Borel parameter. The thresholds $\Delta_i$ can be sorted into two sets, we introduce the $q\bar{q}, qs$ to denote the light quark constituents in the heavy baryon states to simplify the notations, $\Delta_{q\bar{q}} = m_Q^2, \Delta_{qs} = (m_Q + m_s)^2$. The explicit expressions of the spectral densities $\rho_i^A(s)$ and $\rho_i^B(s)$ are given in the appendix B.

Differentiate the Eq.(16) with respect to $\frac{1}{M^2}$, then eliminate the pole residues $\lambda_i$, we can obtain the sixteen sum rules for the masses of the heavy baryon states $B_Q^*$ and $B_Q$,

$$M_i^2 = \frac{\int_{\Delta_i} s^0 \, ds \rho_i^{A/B}(s) e^{-\frac{s}{M^2}}}{\int_{\Delta_i} s^0 \, ds \rho_i^{A/B}(s) e^{-\frac{s}{M^2}}} .$$

In the conventional QCD sum rules [15, 16], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. We impose the two criteria on the heavy baryon states to choose the Borel parameter $M^2$ and threshold parameter $s_0$, the values are shown in Table 1. Finally we obtain the values of the masses and pole resides of the heavy baryon states $B_Q^*$ and $B_Q$, which are shown in Table 2.

From Table 2, we can see that the average values of the masses with the tensor structures $\delta g_{\mu\nu}, 1$ and $\delta$ can reproduce the experimental data approximately for the established heavy baryon states. So it is reasonable to take the average values $M_{\Xi_b^*} = 5.98 \text{ GeV}$ and $M_{\Xi_b} = 5.95 \text{ GeV}$ for the un-established bottom baryon states $\Xi_b^*$ and $\Xi_b$ in numerical analysis. The values of the pole residues from different tensor structures differ greatly from each other in some channels, for example, $\Xi_b^*, \Sigma_b, \Xi_b', \Sigma_b'$. In this article, we take the average values and assume uniform uncertainties (about 20%) for the pole residues in all
Table 1: The Borel parameters $M^2$ and threshold parameters $s_0$ for the heavy baryon states.

| State | $M^2$ (GeV$^2$) | $s_0$ (GeV$^2$) |
|-------|-----------------|-----------------|
| $\Xi^*_b$ | 5.0 – 6.0 | 46.0 ± 0.5 |
| $\Xi'_b$ | 4.8 – 5.6 | 44.5 ± 0.5 |
| $\Sigma^*_b$ | 5.0 – 6.0 | 45.0 ± 0.5 |
| $\Sigma_b$ | 4.8 – 5.6 | 43.5 ± 0.5 |
| $\Xi^*_c$ | 2.0 – 3.0 | 11.0 ± 0.5 |
| $\Xi'_c$ | 2.0 – 2.8 | 10.5 ± 0.5 |
| $\Sigma^*_c$ | 2.0 – 3.0 | 10.5 ± 0.5 |
| $\Sigma_c$ | 1.9 – 2.7 | 10.0 ± 0.5 |

Table 2: The masses and pole residues of the heavy baryon states from the sum rules with different tensor structures. The masses $M$ are in unit of GeV and the pole residues $\lambda$ are in unit of $10^{-2}$GeV$^3$.

| State | $\rho g_{\mu\nu} / \rho (M)$ | $g_{\mu\nu} / \rho (M)$ | $\rho g_{\mu\nu} / \rho (\lambda)$ | $g_{\mu\nu} / \rho (\lambda)$ |
|-------|-----------------|-----------------|-----------------|-----------------|
| $\Xi^*_b$ | 6.04 ± 0.14 | 5.92 ± 0.20 | 4.8 ± 1.0 | 4.2 ± 1.3 |
| $\Xi'_b$ | 6.04 ± 0.10 | 5.85 ± 0.15 | 9.0 ± 1.8 | 6.0 ± 1.3 |
| $\Sigma^*_b$ | 5.95 ± 0.14 | 5.72 ± 0.25 | 4.2 ± 1.0 | 3.3 ± 1.3 |
| $\Sigma_b$ | 5.96 ± 0.10 | 5.73 ± 0.16 | 8.0 ± 1.7 | 5.0 ± 1.2 |
| $\Xi^*_c$ | 2.60 ± 0.20 | 2.68 ± 0.18 | 3.1 ± 0.8 | 3.1 ± 0.7 |
| $\Xi'_c$ | 2.65 ± 0.14 | 2.54 ± 0.17 | 6.2 ± 1.5 | 4.4 ± 0.9 |
| $\Sigma^*_c$ | 2.50 ± 0.20 | 2.59 ± 0.19 | 2.7 ± 0.7 | 2.7 ± 0.7 |
| $\Sigma_c$ | 2.54 ± 0.15 | 2.42 ± 0.20 | 5.4 ± 1.4 | 3.6 ± 1.0 |

channels, the uncertainties originate from the parameters other than the Borel parameter $M^2$ are about 20%, we subtract the uncertainties originate from the Borel parameter from the total uncertainties to avoid double counting. The values of the pole residues are $\lambda_{\Xi^*_b} = (4.5±0.8) \times 10^{-2}$ GeV$^3$, $\lambda_{\Xi'_b} = (3.1±0.5) \times 10^{-2}$ GeV$^3$, $\lambda_{\Sigma^*_b} = (7.5±1.5) \times 10^{-2}$ GeV$^3$, $\lambda_{\Xi^*_c} = (5.3±1.0) \times 10^{-2}$ GeV$^3$, $\lambda_{\Sigma^*_c} = (3.8±0.7) \times 10^{-2}$ GeV$^3$, $\lambda_{\Sigma_b} = (2.7±0.5) \times 10^{-2}$ GeV$^3$, $\lambda_{\Xi'_c} = (6.5±1.2) \times 10^{-2}$ GeV$^3$, and $\lambda_{\Sigma_c} = (4.5±0.9) \times 10^{-2}$ GeV$^3$. The threshold parameters are taken as $s_0 = (11.0±1.0)$ GeV$^2$, $(10.5±1.0)$ GeV$^2$, $(45.0±1.0)$ GeV$^2$ and $(46.0±1.0)$ GeV$^2$ in the channels $\Xi^*_c(\Xi'_c)$, $\Sigma^*_c(\Sigma_c)$, $\Xi^*_b(\Sigma_b)$ and $\Sigma^*_b(\Sigma_b)$ respectively; the Borel parameters are taken as $M^2 = (2.0 – 3.0)$ GeV$^2$ and $(5.0 – 6.0)$ GeV$^2$ in the charm and bottom channels respectively. Those parameters are determined by the two-point QCD sum rules to avoid possible contaminations from the high resonances and continuum states. In calculation, we observe that the values of the strong coupling constants $g_1$, $g_2$ and G3 are insensitive to threshold parameters $s_0$.

The main uncertainties originate from the parameters $\lambda_{B_Q}$, $\lambda_{B'_Q}$ (as the strong coupling constants $g_1$, $g_2$ and G3 $\propto \frac{1}{\lambda_{B_Q}\lambda_{B'_Q}}$) and $m_Q$, the variations of those parameters can lead to relatively large changes for the numerical values, and almost saturate the total
uncertainties, i.e. the variations of the two hadronic parameters $\lambda_{Bq}$ and $\lambda_{B\bar{q}}$, lead to an uncertainty about $20\% \times \sqrt{2} = 28\%$, and the variations of the $m_Q$ lead to an uncertainty about $(10-20)\%$, refining those parameters is of great importance. In the case of the sum rules for the strong coupling constants $g_2$, the values are not stable enough with variations of the Borel parameter, additional uncertainties are introduced, the total uncertainties are very large, see Table 3. The contributions from the strong coupling constants $g_2$ to the radiative decay widths are very small comparing with the corresponding ones from the $g_1$, the predictions are insensitive to the Borel parameter. Although there are many parameters in the light-cone distributions amplitudes [29, 30], the uncertainties originate from those parameters are rather small. In calculation, we neglect the contributions from the high dimension vacuum condensates, such as $\langle f_{abc} G^a G^b G^c \rangle$, $\langle \bar{q} q \rangle \langle \alpha_s G G \rangle$, $\langle \bar{s} s \rangle \langle \alpha_s G G \rangle$, etc. They are greatly suppressed by the large numerical denominators and additional inverse powers of the Borel parameter $\frac{1}{M^2}$, and would not play any significant roles. Furthermore, we neglect some terms involving the light-cone distributions amplitudes $\tilde{f}(\bar{u}_0)$ and $\tilde{f}(\bar{u}_0)$ in case of the contributions from the terms $f(\bar{u}_0)$ are small, as

$$\tilde{f}(\bar{u}_0) \approx 40\%, \quad \tilde{f}(\bar{u}_0) \approx 10\%. \quad (18)$$

Taking into account all the uncertainties of the revelent parameters, finally we obtain the numerical results of the strong coupling constants $g_1$, $g_2$ and $G_3$, which are shown in the Table 3. We estimate the uncertainties $\delta$ with the formula $\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 |x_i - \bar{x}_i|^2}$, where the $f$ denote strong coupling constants $g_1$, $g_2$ and $G_3$, the $x_i$ denote the revelent parameters $m_Q$, $\langle \bar{q} q \rangle$, $\langle \bar{s} s \rangle$, $\cdots$. In the numerical calculations, we take the approximation $\left( \frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2 \approx |f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i)|^2$ for simplicity. For the central values of the strong coupling constants, $g_{B_u^b B_\mu^V} = (70 - 80)\%$, $G_{B_u^b B_\mu^V} = (80 - 90)\%$, the heavy quark symmetry works rather well. Those strong coupling constants in the vertexes $B_Q^* V_Q V$ are basic parameters in describing the interactions among the heavy baryon states, once reasonable values are obtained, we can use them to perform phenomenological analysis.

The radiative decays $B_Q^* \to B_Q \gamma$ can be described by the following electromagnetic lagrangian $\mathcal{L}$,

$$\mathcal{L} = -eQ_b \bar{b} \gamma_\mu b A^\mu - eQ_c \bar{c} \gamma_\mu c A^\mu - eQ_s \bar{s} \gamma_\mu s A^\mu - eQ_u \bar{u} \gamma_\mu u A^\mu - eQ_d \bar{d} \gamma_\mu d A^\mu, \quad (19)$$

where the $A_\mu$ is the electromagnetic field. From the lagrangian $\mathcal{L}$, we can obtain the decay amplitudes with the assumption of the vector meson dominance, $eT = \langle B_Q(p) \gamma(q) | \mathcal{L} | B_Q^*(p+\cdots) \rangle$. \hfill \eject
| Vertexes | $-g_1$(GeV$^{-1}$) | $-g_2$(GeV$^{-2}$) | G3 |
|----------|------------------|------------------|----|
| $\Xi_c^+ \Xi_c^+ \phi$ | $2.98^{+1.18}_{-1.81}$ | $0.62^{+0.63}_{-0.31}$ | $9.75^{+3.87}_{-2.68}$ |
| $\Xi_c^+ \Xi_c^+ \rho$ | $3.54^{+1.39}_{-0.97}$ | $0.47^{+0.46}_{-0.24}$ | $13.61^{+5.12}_{-3.76}$ |
| $\Sigma_c^{*+} \Sigma_c^{*+} \rho_0$ | $-3.54^{+1.39}_{-0.97}$ | $-0.47^{+0.46}_{-0.24}$ | $-13.61^{+5.12}_{-3.76}$ |
| $\Sigma_c^{*+} \Sigma_c^{*+} \rho_0$ | $7.07^{+3.09}_{-2.17}$ | $0.97^{+0.87}_{-0.49}$ | $25.00^{+10.94}_{-7.53}$ |
| $\Sigma_c^{*+} \Sigma_c^{*+} \rho_0$ | $0$ | $0$ | $0$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $0$ | $0$ | $0$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.97^{+0.54}_{-0.49})$ | $-25.00^{+10.94}_{-7.53}$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.11^{+0.16}_{-0.11})$ | $20.14^{+8.54}_{-6.12}$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.07^{+0.14}_{-0.05})$ | $26.68^{+11.64}_{-8.25}$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.07^{+0.14}_{-0.05})$ | $26.68^{+11.64}_{-8.25}$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.13^{+0.27}_{-0.10})$ | $47.05^{+19.72}_{-14.71}$ |
| $\Sigma_c^{*0} \Sigma_c^{0} \rho_0$ | $(0.13^{+0.27}_{-0.10})$ | $47.05^{+19.72}_{-14.71}$ |

Table 3: The values of the strong coupling constants $g_1$, $g_2$ and G3.

| Channels | $\Gamma$ (KeV) |
|----------|----------------|
| $\Xi_c^+ \Xi_c^+ \gamma$ | $0.96^{+1.41}_{-0.67}$ |
| $\Xi_c^{0*} \Xi_c^{0*} \gamma$ | $1.26^{+0.80}_{-0.49}$ |
| $\Sigma_c^{++} \Sigma_c^{++} \gamma$ | $6.36^{+9.79}_{-3.31}$ |
| $\Sigma_c^{+} \Sigma_c^{+} \gamma$ | $0.40^{+0.34}_{-0.21}$ |
| $\Sigma_c^{0} \Sigma_c^{0} \gamma$ | $1.58^{+1.98}_{-0.82}$ |
| $\Xi_b^{0} \Xi_b^{0} \gamma$ | $0.047^{+0.103}_{-0.036}$ |
| $\Xi_b^{-} \Xi_b^{-} \gamma$ | $0.066^{+0.045}_{-0.027}$ |
| $\Sigma_b^{++} \Sigma_b^{++} \gamma$ | $0.12^{+0.12}_{-0.06}$ |
| $\Sigma_b^{-} \Sigma_b^{-} \gamma$ | $0.0076^{+0.0019}_{-0.0040}$ |
| $\Sigma_b^{-} \Sigma_b^{-} \gamma$ | $0.030^{+0.082}_{-0.016}$ |

Table 4: The widths of the radiative decays $B_Q \rightarrow B_Q \gamma$. 
\[ e T = -e Q s \eta^*_\mu (B_Q(p) | \bar{s} \gamma^\mu s | B_Q^*(p + q)) - e Q u \eta^*_\mu (B_Q(p) | \bar{u} \gamma^\mu u | B_Q^*(p + q)) \]
\[ -e Q d / \eta^*_\mu (B_Q(p) | \bar{d} \gamma^\mu d | B_Q^*(p + q)) + \cdots \]
\[ = -e Q s \eta^*_\mu f_\phi M_\phi \epsilon_\mu \frac{i}{q^2 - M_\phi^2} \langle \phi(q) B_Q(p) | B_Q^*(p + q) \rangle \]
\[ -e Q u \eta^*_\mu \frac{1}{\sqrt{2}} f_\rho M_\rho \epsilon_\mu \frac{i}{q^2 - M_\rho^2} \langle \rho(q) B_Q(p) | B_Q^*(p + q) \rangle \]
\[ + e Q d \eta^*_\mu \frac{1}{\sqrt{2}} f_\omega M_\omega \epsilon_\mu \frac{i}{q^2 - M_\omega^2} \langle \omega(q) B_Q(p) | B_Q^*(p + q) \rangle \]
\[ -e Q d \eta^*_\mu \frac{1}{\sqrt{2}} f_\omega M_\omega \epsilon_\mu \frac{i}{q^2 - M_\omega^2} \langle \omega(q) B_Q(p) | B_Q^*(p + q) \rangle + \cdots , \quad (20) \]

where the \( \eta_\mu \) is the polarization vector of the photon. In the heavy quark limit, the matrix elements \( \langle B_Q(p) | \bar{Q} \gamma_\mu Q | B_Q^*(p + q) \rangle \propto M_{\bar{Q}Q}^2 \), and can be neglected, so we consider only the contributions from the intermediate vector mesons \( \phi(1020) \), \( \rho(770) \) and \( \omega(782) \). The photon can be viewed as emitted from the light diquark system while the heavy quark is unaffected by the emission process.

From the strong coupling constants \( g_1 \) and \( g_2 \), we can obtain the decay widths \( \Gamma_{B_Q^* \rightarrow B_Q \gamma} \),

\[ \Gamma_{B_Q^* \rightarrow B_Q \gamma} = \frac{\alpha \left( M_{B_Q^*}^2 - M_{B_Q}^2 \right)}{16 M_{B_Q}^3} \sum_{ss'} | T |^2 , \quad (21) \]

the numerical values are shown in Table 4.

There have been many works focusing on the radiative decays of the \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) heavy baryon sextets \( B_s^0 \) and \( B_6 \) to the \( \frac{1}{2}^+ \) heavy baryon antitriplet \( B_3 \), \( B_s^0 \rightarrow B_3 \gamma \) and \( B_6 \rightarrow B_3 \gamma \), such as the light-cone QCD sum rules \([31] \), the heavy hadron chiral perturbation theory \([37, 38] \), the combination of the heavy quark symmetry and the light diquark \( SU(2N_f) \times O(3) \) symmetry \([39] \), the relativistic three-quark model \([40, 41] \), etc. The works on the radiative decays \( B_s^0 \rightarrow B_6 \gamma \) are very few, some decay channels are studied in the constituent quark model \([42] \) and the non-relativistic potential model \([43] \). Combining with our previous work on the radiative decays \( \Omega_Q^* \rightarrow \Omega_Q \gamma \) \([10] \), we perform systematic studies for the radiative decays \( B_s^0 \rightarrow B_6 \gamma \) with the light-cone QCD sum rules. The strong decays \( B_s^0 \rightarrow B_6 \pi \) are forbidden due to the unavailable phase space, while the radiative channels are not phase space suppressed and become relevant, although the electromagnetic strength is weaker than that of the strong interaction. The properties of the charm baryon states would be studied at the BESIII and PANDA \([44, 45] \), where the charm baryon states are copiously produced at the \( e^+e^- \) and \( pp \) collisions. The LHCb is a dedicated \( b \) and \( c \)-physics precision experiment at the LHC (large hadron collider). The LHC will be the world’s most copious source of the \( b \) hadrons, and a complete spectrum of the \( b \) hadrons will be available through gluon fusion. In proton-proton collisions at \( \sqrt{s} = 14 \text{ TeV} \), the \( b \bar{b} \) cross section is expected to be \( \sim 500 \mu b \) producing \( 10^{12} b \bar{b} \) pairs in a
standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ [36]. The present predictions for the radiative decays can be tested at the BESIII, PANDA and LHCb.

4 Conclusion

In this article, we parameterize the vertexes $B^*_Q B Q V$ with three tensor structures due to Lorentz invariance, study the corresponding three strong coupling constants with the light-cone QCD sum rules, then assume the vector meson dominance of the intermediate $\phi(1020)$, $\rho_0(770)$ and $\omega(782)$ as the contributions from the $J/\psi$ and $\Upsilon$ are negligible in the heavy quark limit, and calculate the radiative decay widths $\Gamma_{B^*_Q \to B Q \gamma}$. The predictions can be tested by the experimental data at the BESIII, PANDA and LHCb in the future. Although the values of the strong coupling constants $g_2$ are not stable enough with variations of the Borel parameter, the Borel parameter dependence of the radiative decay widths is very weak, as the main contributions come from the strong coupling constants $g_1$. The heavy quark symmetry works rather well for the strong coupling constants $g_1$ and G3. The strong coupling constants in the vertexes $B^*_Q B Q V$ are basic parameters in describing the interactions among the heavy baryon states, once reasonable values are obtained, we can use them to perform phenomenological analysis.

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Appendix

Appendix A

The 30 sum rules for the strong coupling constants $g_1$, $g_2$ and G3 in different channels,

\[
\begin{align*}
\frac{\Xi_f \Xi_f}{g_1 g_1} &= \frac{1}{\lambda_{\Xi_f} \lambda_{\Xi_f}} \left( M_{\Xi_f} + M_{\Xi_f} \right) \exp \frac{M_{\Xi_f}^2 + M_{\Xi_f}^2 - 2u_0 \tilde{u}_0 M_f^2}{2M^2} \\
&- \frac{u_0 f_{\phi} M_{\phi} g_1^{(v)}(\tilde{u}_0)}{8\pi^2} M^4 E_1(x) \int_0^1 dt t (1 - t) e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{u_0 m_Q^2 f_{\phi} M_{\phi} \bar{g}_1^{(a)}(\bar{u}_0) \rho_{s\Sigma}(\bar{u}_0)}{144M^2} \int_0^1 dt t e^{-\frac{m_f^2}{2M^2}} \\
&- \frac{u_0 f_{\phi} M_{\phi} \bar{g}_1^{(a)}(\bar{u}_0)}{32\pi^2} M^4 E_1(x) \int_0^1 dt t e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{u_0 m_Q^2 f_{\phi} M_{\phi} \bar{g}_1^{(a)}(\bar{u}_0) \rho_{s\Sigma}(\bar{u}_0)}{576M^2} \int_0^1 dt (1 - t) e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{m_Q^2 \tilde{f}_{\phi} \tilde{g}_{\phi} g_1^{(a)}(\bar{u}_0)}{16\pi^2} M^4 E_1(x) \int_0^1 dt e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{m_Q^2 \tilde{f}_{\phi} \tilde{g}_{\phi} g_1^{(a)}(\bar{u}_0) \rho_{s\Sigma}(\bar{u}_0)}{288M^2} \int_0^1 dt e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{u_0 f_{\phi} M_f^3}{16\pi^2} M^2 E_0(x) \int_0^1 dt \int_0^{u_0} \frac{d\alpha_s}{\alpha_s} \int_0^{1 - \alpha_s} \frac{d\alpha_g}{\alpha_g} \int_0^{1 - \alpha_s} \frac{d\alpha_g}{\alpha_g} (1 - 2v) A(\alpha_i) + \mathcal{V}(\alpha_i) e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{f_{\phi} M_f}{16\pi^2} M^4 E_1(x) \int_0^1 dt \frac{d}{du_0} \int_0^{u_0} \frac{d\alpha_s}{\alpha_s} \int_0^{1 - \alpha_s} \frac{d\alpha_g}{\alpha_g} (1 - v) A(\alpha_i) + \mathcal{V}(\alpha_i) e^{-\frac{m_f^2}{2M^2}} \\
&+ \frac{1}{\lambda_{\Xi_f} \lambda_{\Xi_f}} \left( M_{\Xi_f} + M_{\Xi_f} \right) \exp \frac{M_{\Xi_f}^2 + M_{\Xi_f}^2 - 2u_0 \tilde{u}_0 M_f^2}{2M^2} \\
&- \langle \bar{q} \bar{q} \sigma G q \rangle f_{\phi}^{+} \phi_{\perp}(\bar{u}_0) \frac{m_Q^2}{M} \left( 1 + \frac{1}{M^2} \right) + \frac{m_Q^4 f_{\phi}^{+} M_f^2 \langle \bar{q} \bar{q} \sigma G q \rangle A_{\perp}(\bar{u}_0)}{96M^6} \\
&- \langle \bar{q} \bar{q} \sigma G q \rangle f_{\phi}^{+} \phi_{\perp}(\bar{u}_0) \frac{m_Q^2}{M} \left( 1 + \frac{1}{M^2} \right) + \frac{u_0 \langle \bar{q} \bar{q} \sigma G q \rangle f_{\phi}^{+} M_f^2 C_{\perp}(\bar{u}_0)}{24M^2} \left( 1 + \frac{M^2}{M^2} \right) \left( 1 + \frac{1}{M^2} \right),
\end{align*}
\]
\[
\hat{g}_2 \hat{g}^\phi = \frac{1}{\lambda \Xi \lambda \Xi} \exp \left( \frac{M^2_{\Xi \phi} + M^2_{\Xi \phi'} - 2u_0 \bar{u}_0 M^2_{\phi}}{2M^2} \right) \\
\left\{ \frac{u_0 f_\phi M_\phi \left[ \tilde{\phi}_\parallel(\bar{u}_0) - \tilde{g}'_{\perp}(\bar{u}_0) \right]}{4\pi^2} \int_0^1 dt t(1-t)e^{-\frac{\bar{m}_Q^2}{M^2}} \\
\frac{u_0 m^2_Q f_\phi M_\phi \left[ \tilde{\phi}_\parallel(\bar{u}_0) - \tilde{g}'_{\perp}(\bar{u}_0) \right]}{72M^4} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} \right\} \\
\left\{ \frac{u_0 m^2_Q f_\phi M_\phi \left[ \tilde{\phi}_\parallel(\bar{u}_0) - \tilde{g}'_{\perp}(\bar{u}_0) \right]}{288M^4} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} \right\} \\
\frac{f_\phi M_\phi}{8\pi^2} \int_0^1 dt t \int_0^{u_0} d\alpha_x \int_{u_0 - \alpha_x}^{1-\alpha_x} d\alpha_g \frac{A(\alpha_x) + (1 - 2v)\mathcal{V}(\alpha_x)}{\alpha_g} e^{-\frac{\bar{m}_Q^2}{M^2}} \right\}
\}
\frac{1}{\lambda \Xi \lambda \Xi} \exp \left( \frac{M^2_{\Xi \phi} + M^2_{\Xi \phi'} - 2u_0 \bar{u}_0 M^2_{\phi}}{2M^2} \right) \\
\left\{ \frac{2u_0 (q\bar{q}) f_\phi \tilde{B}_\perp(\bar{u}_0)}{3M^2} - \frac{u_0 (q\bar{q} \sigma Gq) f_\phi \tilde{B}_\perp(\bar{u}_0)}{6M^4} \left( 1 + \frac{m^2_Q}{M^2} \right) \right\} , \tag{23}
\]
\[
G3_{\Xi_Q^\prime \Xi_Q^\prime} = \frac{1}{\lambda_{\Xi_Q} \lambda_{\Xi_Q}} \exp \left( \frac{M_{\Xi_Q}^2 + M_{\Xi_Q^\prime}^2 - 2u_0 \bar{u}_0 M_{\phi}^2}{2M^2} \right) \\
\left\{ \begin{array}{l}
\int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} \frac{1}{8\pi^2} \left[ \tilde{\phi}_\| (\bar{u}_0) - \tilde{g}_\| (\bar{u}_0) \right] M^4 E_1(x) \int_0^1 dt (1 - t) e^{-\frac{\bar{m}_Q^2}{M^2}} \\
- \frac{m_Q^2 f_{\phi} M_{\phi} (1 + t) e^{-\frac{\bar{m}_Q^2}{M^2}}}{144 M^2} \\
- \frac{m_Q^2 f_{\phi} M_{\phi} (1 - t) e^{-\frac{\bar{m}_Q^2}{M^2}}}{144 M^2} \\
- \frac{u_0 f_{\phi} M_{\phi}^2}{8\pi^2} \frac{1}{M^2 E_0(x)} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} \int_{u_0 - \alpha}^{u_0} d\alpha \int_0^{1-\alpha} d\alpha' \int_0^{1-\alpha} d\alpha' e^{-\frac{\bar{m}_Q^2}{M^2}} \right. \\
+ \left. \left[ \begin{array}{l}
\frac{1}{\lambda_{\Xi_Q} \lambda_{\Xi_Q}} \exp \left( \frac{M_{\Xi_Q}^2 + M_{\Xi_Q^\prime}^2 - 2m_Q^2 - 2u_0 \bar{u}_0 M_{\phi}^2}{2M^2} \right) \\
\left\{ \begin{array}{l}
\int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} \frac{1}{6} \left[ \tilde{\phi}_\| (\bar{u}_0) - \tilde{g}_\| (\bar{u}_0) \right] M^4 E_0(x) + \frac{\bar{q} g_s G g}{24} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
- \frac{m_Q^2 f_{\phi} M_{\phi}^2}{96 M^6} \frac{1}{24} \left[ \tilde{g}_\| (\bar{u}_0) - \tilde{g}_\| (\bar{u}_0) \right] M^2 A_{\phi} (\bar{u}_0) + \frac{\bar{q} g_s G g}{24} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
+ \frac{\bar{q} g_s G g}{3} \frac{M_{\phi}^2}{12 M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \right) \\
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\[
\begin{align*}
g_{1}^{\prime} & = \pm \frac{1}{\lambda_{\Xi_{Q}}^{\prime}} \exp \left( \frac{M_{\Xi_{Q}}^{2} + M_{\Xi_{Q}}^{2} - 2u_{0}\bar{u}_{0}M_{\rho}^{2}}{2M^{2}} \right) \\
\{ & - u_{0}f_{\rho}M_{\rho}g_{1}^{(v)}(\bar{u}_{0}) \frac{M^{4}E_{1}(x)}{8\pi^{2}} \int_{0}^{1} dt(1-t)e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{u_{0}m_{Q}^{2}f_{\rho}M_{\rho}g_{1}^{(v)}(\bar{u}_{0})}{144M^{2}} \frac{\langle \alpha_{S}GG \rangle_{\pi}}{\pi} \int_{0}^{1} dt \frac{1}{t^{2}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
- & \frac{m_{s}f_{\rho}^{\perp}(\bar{u}_{0})}{8\pi^{2}} M^{4}E_{1}(x) \int_{0}^{1} dt e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{m_{s}^{2}f_{\rho}^{\perp}(\bar{u}_{0})}{144M^{2}} \frac{\langle \alpha_{S}GG \rangle_{\pi}}{\pi} \int_{0}^{1} dt \frac{1}{t^{2}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
- & \frac{u_{0}m_{s}^{2}f_{\rho}^{\perp}M_{\rho}^{2}\tilde{C}_{\perp}(\bar{u}_{0})}{8\pi^{2}} M^{2}E_{0}(x) \int_{0}^{1} dt e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
- & \frac{u_{0}f_{\rho}M_{\rho}}{32\pi^{2}} M^{4}E_{1}(x) \frac{d}{du_{0}} g_{1}^{(a)}(\bar{u}_{0}) \int_{0}^{1} dt(1-t)e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{u_{0}m_{Q}^{2}f_{\rho}^{\perp}M_{\rho}^{2}\tilde{C}_{\perp}(\bar{u}_{0})}{576M^{2}} \frac{\langle \alpha_{S}GG \rangle_{\pi}}{\pi} \int_{0}^{1} dt \frac{1}{t^{2}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
- & \frac{\tilde{f}_{\rho}M_{\rho}g_{1}^{(a)}(\bar{u}_{0})}{16\pi^{2}} M^{4}E_{1}(x) \int_{0}^{1} dt e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{m_{Q}^{2}f_{\rho}M_{\rho}g_{1}^{(a)}(\bar{u}_{0})}{144M^{2}} \frac{\langle \alpha_{S}GG \rangle_{\pi}}{\pi} \int_{0}^{1} dt \frac{1}{t^{2}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{u_{0}f_{\rho}M_{\rho}^{3}}{16\pi^{2}} M^{2}E_{0}(x) \int_{0}^{1} dt \int_{0}^{u_{0}} d\alpha_{g} \int_{\alpha_{0}^{-1}}^{1-\alpha_{0}} d\alpha_{\lambda} \frac{(1-2v)A(\alpha_{\lambda}) + V(\alpha_{\lambda})}{\alpha_{\lambda}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \\
+ & \frac{f_{\rho}M_{\rho}}{16\pi^{2}} M^{4}E_{1}(x) \int_{0}^{1} dt \frac{d}{du_{0}} \int_{0}^{u_{0}} d\alpha_{g} \int_{\alpha_{0}^{-1}}^{1-\alpha_{0}} d\alpha_{\lambda} (1-v) \frac{A(\alpha_{\lambda}) + V(\alpha_{\lambda})}{\alpha_{\lambda}} e^{-\frac{m_{Q}^{2}}{M^{2}}} \}
\end{align*}
\]
\[ g_2 = \pm \frac{1}{\lambda_{\xi_1} \lambda_{\xi_2}} \exp \frac{M_{s_1}^2 + M_{\bar{s}_1}^2 - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \]
\[ \left\{ \begin{array}{l}
\frac{u_0 m_s \langle \bar{s} \rangle f_\rho M_\rho g_\perp^{(v)}(\bar{u}_0)}{12} - \frac{\langle \bar{s} \rangle f_\rho M_\rho \tilde{C}_\perp(\bar{u}_0)}{6} \\
\frac{u_0 m_s \langle \bar{s} \rangle f_\rho M_\rho \tilde{C}_\perp(\bar{u}_0)}{2} \left( 1 + \frac{m_\rho^2}{M^2} \right) \\
\frac{\langle \bar{s} \rangle f_\rho M_\rho \tilde{C}_\perp(\bar{u}_0)}{24} \left( 1 + \frac{m_\rho^2}{M^2} \right) \\
\frac{u_0 m_s \langle \bar{s} \rangle f_\rho M_\rho \tilde{C}_\perp(\bar{u}_0)}{48} \left( 1 + \frac{m_\rho^2}{M^2} \right) - \frac{u_0 m_s \langle \bar{s} \rangle f_\rho M_\rho \tilde{C}_\perp(\bar{u}_0)}{32} \left( 1 + \frac{m_\rho^2}{M^2} \right) \end{array} \right\} , \quad (25) \]
\[
\pm \frac{1}{\lambda \tilde{\xi} Q} \exp \left( \frac{M^2_{\xi Q} + M^2_{\bar{Q}} - 2m^2_{\bar{Q}} - 2u_0 \bar{u}_0 M^2_\rho}{2M^2} \right) \times \left\{ \frac{u_0 m_s \langle \bar{s}s \rangle f_\rho M_\rho \left[ \tilde{\phi}_{\parallel}\left(\bar{u}_0\right) - \tilde{g}_\perp^{(v)}\left(\bar{u}_0\right) \right]}{6M^2} \right. \\
- \frac{u_0 m_s \langle \bar{s}s \rangle f_\rho M^2_\rho \tilde{A}\left(\bar{u}_0\right)}{24M^4} \left(1 + \frac{m^2_Q}{M^2}\right) \\
- \frac{u_0 m_s \langle \bar{s}g_s \sigma Gs \rangle f_\rho M_\rho \left[ \tilde{\phi}_{\parallel}\left(\bar{u}_0\right) - \tilde{g}_\perp^{(v)}\left(\bar{u}_0\right) \right]}{36M^4} \left(1 + \frac{m^2_Q}{M^2}\right) \\
+ \frac{2u_0 \langle \bar{s}s \rangle f_\rho^{\perp} M^2_\rho \tilde{B}_\perp\left(\bar{u}_0\right)}{3M^2} - \frac{u_0 \langle \bar{s}g_s \sigma Gs \rangle f_\rho^{\perp} M^2_\rho \tilde{B}_\perp\left(\bar{u}_0\right)}{6M^4} \left(1 + \frac{m^2_Q}{M^2}\right) \\
- \frac{u_0 m_s \langle \bar{s}s \rangle f_\rho M_\rho g_\parallel^{(a)}\left(\bar{u}_0\right)}{24M^2} \right\} ,
\]
\[
G3_{\Xi_0} = \pm \frac{1}{\lambda_{\Xi_0} \lambda_{\Xi_0}^*} \exp \left( \frac{M_{\Xi_0}^2 + M_{\Xi_0}^2 - 2u_0 \bar{u}_0 M^2}{2M^2} \right) \left\{ \begin{array}{l}
\frac{f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{8\pi^2} M^2 E_1(x) \int_0^1 dt \left[ (1 - t) e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
- \frac{m_{\Xi_0}^2 f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{144M^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
+ \frac{m_{\Xi_0}^2 f_\rho M_\rho A(\bar{u}_0)}{32\pi^2} M^2 E_0(x) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
+ \frac{m_{\Xi_0}^2 f_\rho M_\rho A(\bar{u}_0)}{576M^4} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
- \frac{m_{\Xi_0}^2 f_\rho M_\rho A(\bar{u}_0)}{8\pi^2} M^4 E_1(x) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
- \frac{f_\rho M_\rho M_{\Xi_0}^2 \frac{M^2}{\Xi_0} \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{4\pi^2} M^2 E_0(x) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
+ \frac{m_{\Xi_0}^2 f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{72M^4} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dt \left[ \frac{1}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
- \frac{f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{32\pi^2} M^4 E_1(x) \int_0^1 dt \left[ (1 + t) e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
+ \frac{m_{\Xi_0}^2 f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{576M^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dt \left[ \frac{1 + t}{t^2} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \\
- \frac{u_0 f_\rho M_\rho \left[ \tilde{g}_{\parallel} (\bar{u}_0) - \gamma_\perp (\bar{u}_0) \right]}{8\pi^2} M^2 E_0(x) \int_0^1 dt \left[ \frac{1 - \gamma_\perp}{\alpha_g^3} \int_{\alpha_g - \alpha_g}^{1 - \gamma_\perp} d\alpha_g A(\alpha_g) - \frac{V(\alpha_g)}{\alpha_g} e^{-\frac{\bar{u}_0^2}{M^2}} \right] \right\} \right.
\]
\[
\pm \frac{1}{\lambda_{\Xi}^{\prime} \lambda_{\Xi}} \exp \left( \frac{M_{\Xi}^2 + M_{\Xi}^2 - 2m_0^2 - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \right) \frac{m_s \langle \bar{s}s \rangle f_\rho M_\rho [\bar{\phi}_\parallel (\bar{u}_0) - \bar{g}_\parallel (\bar{u}_0)]}{12} \\
- \frac{m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}(\bar{u}_0) - m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}(\bar{u}_0)}{48M^2} \left( 1 + \frac{m_\phi^2}{M^2} \right) \\
- \frac{m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}(\bar{u}_0) - m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}(\bar{u}_0)}{72M^2} \left( 1 + \frac{m_\phi^2}{M^2} \right) \\
- \frac{\langle \bar{s}s \rangle f_\rho \bar{A}_\perp (\bar{u}_0)}{6} M^2 E_0(x) + \frac{\langle \bar{s}s \rangle f_\rho \bar{A}_\perp (\bar{u}_0)}{24} \left( 1 + \frac{m_\phi^2}{M^2} \right) \\
+ \frac{m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}_\perp (\bar{u}_0)}{96M^6} \left( 1 + \frac{m_\phi^2}{M^2} \right) + m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}_\perp (\bar{u}_0) \left( 1 + \frac{m_\phi^2}{M^2} \right) \\
- \frac{\langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}_\perp (\bar{u}_0)}{12M^2} \left( 1 + \frac{m_\phi^2}{M^2} \right) \\
+ \frac{m_\phi \langle \bar{s}s \rangle f_\rho M_\rho M_{\Xi}^2 \bar{A}_\perp (\bar{u}_0)}{48} \left( 1 + \frac{2m_\phi^2}{M^2} \right) \right),
\]
(27)
\[ g_1^{\Sigma Q_{Q\rho}} = \pm \frac{1}{\lambda_{\Sigma Q} \lambda_{\Sigma Q}^* (M_{\Sigma Q} + M_{\Sigma Q}^*)} \exp \frac{M_{\Sigma Q}^2 + M_{\Sigma Q}^* - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \]

\[ = \frac{1}{\lambda_{\Sigma Q} \lambda_{\Sigma Q}^* (M_{\Sigma Q} + M_{\Sigma Q}^*)} \exp \frac{M_{\Sigma Q}^2 + M_{\Sigma Q}^* - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \]

\[ \left\{ - u_0 f_{\rho} M_{\rho} g_\perp^{(v)} (\bar{u}_0) \frac{M^4 E_1(x)}{2\pi^2} \int_0^1 dt t (1-t) e^{-\frac{\bar{m}_Q^2}{M^2}} \right\} \]

\[ + \frac{u_0 m_{Q}^2 f_{\rho} M_{\rho} g_\perp^{(v)} (\bar{u}_0)}{36M^2} \frac{\alpha s G G}{\pi} \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\bar{m}_Q^2}{M^2}} \]

\[ + \frac{u_0 f_{\rho} M_{\rho} g_\perp^{(a)} (\bar{u}_0)}{8\pi^2} \frac{M^4 E_1(x)}{M^4} \frac{g_\perp^{(a)} (\bar{u}_0)}{du_0} \frac{1}{dt} \frac{1-t}{t^2} e^{-\frac{\bar{m}_Q^2}{M^2}} \]

\[ + \frac{m_{Q}^2 f_{\rho} M_{\rho} g_\perp^{(a)} (\bar{u}_0)}{72M^2} \frac{\alpha s G G}{\pi} \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\bar{m}_Q^2}{M^2}} \]

\[ + \frac{u_0 f_{\rho} M_{\rho}^3}{4\pi^2} M^2 E_0(x) \int_0^1 dt \int_0^{u_0} d\alpha \int_{u_0 - \alpha}^{1-\alpha} d\alpha_g (1 - 2v) A(\alpha_i) + \mathcal{V}(\alpha_i) e^{-\frac{\bar{m}_Q^2}{M^2}} \]

\[ + \frac{f_{\rho} M_{\rho}^3}{4\pi^2} M^4 E_1(x) \int_0^1 dt \frac{d}{du_0} \int_0^{u_0} d\alpha \int_{u_0 - \alpha}^{1-\alpha} d\alpha_g (1 - v) A(\alpha_i) + \mathcal{V}(\alpha_i) e^{-\frac{\bar{m}_Q^2}{M^2}} \]
\[ g_2^{* \Sigma_Q \rho} = \pm \frac{1}{\lambda_{\Sigma Q} \lambda_{\Sigma_Q}^{*}} \exp \left( \frac{M_{\Sigma_Q}^2 + M_{\Sigma_Q}^{2*} - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \right) \]

\[
\left\{ \frac{u_0 f_\rho M_\rho}{\pi^2} \left[ \bar{\phi}_\parallel (\bar{u}_0) - \bar{g}_\perp (\bar{u}_0) \right] \right\} M^2 E_0(x) \int_0^1 dt t (1 - t) e^{-\frac{\bar{m}_Q^2}{M^2}}
\]

\[
- \frac{u_0 m_Q^2 f_\rho M_\rho}{18M^4} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}} + \frac{u_0 m_Q^2 f_\rho M_\rho^3 A(\bar{u}_0)}{72M^6} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}}
\]

\[
+ \frac{u_0 m_Q^2 f_\rho M_\rho^3}{72M^4} \int_0^1 dt t e^{-\frac{\bar{m}_Q^2}{M^2}}
\]

\[
+ \int_0^1 dt \int \bar{u}_0 d\alpha \int_{\alpha_0 - \alpha_a}^{1 - \alpha_a} d\alpha_g A(\alpha_i) + (1 - 2\nu) V(\alpha_i) e^{-\frac{\bar{m}_Q^2}{M^2}} \right\}
\]

\[\pm \frac{1}{\lambda_{\Sigma Q} \lambda_{\Sigma_Q}^{*}} \exp \left( \frac{M_{\Sigma_Q}^2 + M_{\Sigma_Q}^{2*} - 2m_Q^2 - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \right) \]

\[
\left\{ \frac{8u_0 \langle \bar{q} q \rangle f_\rho^3 M_{\rho}^2 B_\perp (\bar{u}_0)}{3M^2} - \frac{2u_0 \langle \bar{q} g_s \sigma G q \rangle f_\rho^3 M_{\rho}^2 B_\perp (\bar{u}_0)}{3M^4} \left( 1 + \frac{m_Q^2}{M^2} \right) \right\}, \quad (29)
\]
\[
G_{3\Sigma_0, \Sigma Q}\rho_0 = \pm \frac{1}{\lambda_{\Sigma_0} \lambda_{\Sigma Q}} \exp \left( \frac{M_{\Sigma_0}^2 + M_{\Sigma Q}^2 - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \right) \\
\left\{ f_{\rho} M_{\rho} \left[ \tilde{\phi}_\parallel (\bar{u}_0) - \tilde{g}_\perp^{(v)} (\bar{u}_0) \right] + \frac{m_{Q}^2 f_{\rho} M_{\rho} \left[ \tilde{\phi}_\parallel (\bar{u}_0) - \tilde{g}_\perp^{(v)} (\bar{u}_0) \right]}{36M^2} \frac{\alpha_s g G_{GQ}}{\pi} \int_0^1 dt \frac{1 - t}{t^2} e^{-\frac{\alpha_s^2}{m^2}}, \\
- \frac{f_{\rho} M_{\rho}^3 \tilde{A}(\bar{u}_0)}{8\pi^2} M^2 E_0(x) \int_0^1 dt t e^{-\frac{\alpha_s^2}{m^2}}, \\
+ \frac{m_{Q}^2 f_{\rho} M_{\rho}^3 \tilde{A}(\bar{u}_0)}{144M^4} \left[ \frac{\alpha_s g G_{GQ}}{\pi} \right] \int_0^1 dt \frac{1}{t^2} e^{-\frac{\alpha_s^2}{m^2}}, \\
- \frac{\tilde{f}_{\rho} M_{\rho} M^4 E_0(x) \gamma_0 (\bar{u}_0)}{8\pi^2} \left[ \frac{\alpha_s g G_{GQ}}{\pi} \right] \int_0^1 dt (1 + t) e^{-\frac{\alpha_s^2}{m^2}}, \\
- \frac{u_0 f_{\rho} M_{\rho}^3}{4\pi^2} M^2 E_0(x) \int_0^1 dt \int_0^{u_0} \sigma \left[ \int_0^{1 - \sigma} \frac{A(\alpha_i) - V(\alpha_i)}{\alpha_i} e^{-\frac{\alpha_s^2}{m^2}} \right] e^{-\frac{\alpha_s^2}{m^2}} \right\} \\
+ \frac{1}{\lambda_{\Sigma_0} \lambda_{\Sigma Q}} \exp \left( \frac{M_{\Sigma_0}^2 + M_{\Sigma Q}^2 - 2m_{Q}^2 - 2u_0 \bar{u}_0 M_{\rho}^2}{2M^2} \right) \\
\left\{ - \frac{2 \langle \bar{q} q \rangle f_{\rho} \gamma_0 (\bar{u}_0)}{3} M^2 E_0(x) + \frac{\langle \bar{q} g \sigma G q \rangle f_{\rho} \gamma_0 (\bar{u}_0)}{6} \left[ 1 + \frac{m_{Q}^2}{M^2} \right], \\
- \frac{m_{Q}^2 f_{\rho} M_{\rho}^3 \langle \bar{q} g \sigma G q \rangle A_\perp (\bar{u}_0)}{24M^6} + \frac{\langle \bar{q} q \rangle f_{\rho} M_{\rho}^2 A_\perp (\bar{u}_0)}{6} \left[ 1 + \frac{m_{Q}^2}{M^2} \right], \\
+ \frac{4 \langle \bar{q} q \rangle f_{\rho} M_{\rho}^2 B_\perp (\bar{u}_0)}{3M^2} \left[ 1 + \frac{m_{Q}^2}{M^2} \right] \right\}, \quad (30)
\]

where \(\bar{u}_0 = 1 - u_0\), \(\bar{f}_\phi = f_{\phi} - f_{\phi} \frac{2m_{\Sigma_0}}{M_{\rho}}\), \(\bar{f}_\phi \perp = f_{\phi} \perp - f_{\phi} \frac{2m_{\Sigma_0}}{M_{\rho}}\), \(\bar{f}_{\rho} = f_{\rho} - f_{\rho} \frac{m_{\Sigma_0} + m_{\rho}}{M_{\rho}}\), \(\bar{f}_{\rho} \perp = f_{\rho} \perp - f_{\rho} \frac{m_{\Sigma_0} + m_{\rho}}{M_{\rho}}\), \(M_{\rho}^2 = M_0^2 = 2M^2\) and \(u_0 = \frac{M_{\Sigma_0}^2}{M_{\Sigma_0}^2 + M_{\rho}^2} \approx \frac{1}{2} = \frac{M_{\Sigma_0}^2}{M_{\Sigma_0}^2 + M_{\rho}^2} \approx \frac{1}{2}, v = \frac{u_0 - \alpha_s}{\alpha_g} \left( \frac{u_0 - \alpha_s}{\alpha_g} \right)\), \(\bar{m}_{Q}^2 = \frac{m_{Q}^2}{M_{\Sigma_0}^2 + M_{\rho}^2}\), \(\bar{E}_0(x) = 1 - (1 + x + x^2 + \cdots + x^n) e^{-x}, x = \frac{M_{\Sigma_0}^2}{M_{\Sigma_0}^2 + M_{\rho}^2}\).
respectively where the couplings correspond to the vertexes $\Xi^+\Xi^0\rho_0$, $\Xi^0\Xi^0\rho_0$, $\Sigma^{++}\Sigma^+\rho_0$, $\Sigma^+\Sigma^0\rho_0$ and $\Xi^0\Xi^0\rho_0$ respectively. The strong coupling constants $g_1$, $g_2$ and $G_3$ in the vertexes $\Sigma^{++}\Sigma^+\rho_0$ and $\Sigma^+\Sigma^0\rho_0$, $\Sigma^0\Sigma^0\rho_0$ vanish in the isospin symmetry limit. For some technical details involving the three particle vector-mesons ($\phi$ and $\rho_0$) light-cone distribution amplitudes, one can consult Ref. [36].

The quark constituents of the vector mesons $\rho_0$ and $\omega$ are respectively. For example, the correlation functions $\Pi_\mu^{\Xi^+\Xi^0\rho/\omega}(p,q)$ and $\Pi_\mu^{\Xi^0\Xi^0\rho/\omega}(p,q)$ can be decomposed as

\[
\Pi_\mu^{\Xi^+\Xi^0\rho/\omega}(p,q) = \frac{i}{\sqrt{2}} \epsilon^{ijk} \epsilon^{j'k'} \int d^4xe^{-ip\cdot x} \gamma_5 \gamma_i S^k_{C} (x) \nonumber \\
\times Tr \left[ \gamma_0 S_{jj'}(-x) \gamma_\mu C (0) | u_i(0) \bar{u}_{j'}(x) | u(q) \right] T_C ,
\]

\[
\Pi_\mu^{\Xi^0\Xi^0\rho/\omega}(p,q) = \frac{i}{\sqrt{2}} \epsilon^{ijk} \epsilon^{j'k'} \int d^4xe^{-ip\cdot x} \gamma_5 \gamma_i S^k_{C} (x) \nonumber \\
\times Tr \left[ \gamma_0 S_{jj'}(-x) \gamma_\mu C (0) | d_i(0) \bar{d}_{j'}(x) | d(q) \right] T_C ,
\]

respectively, where the couplings

\[
\langle 0 | u_i(0) \bar{u}_{j'}(0) | u(q) \rangle |_\rho = \langle 0 | d_i(0) \bar{d}_{j'}(0) | d(q) \rangle |_\rho \propto f_\rho (f_\rho^\perp) M_\rho^n , 
\]

\[
\langle 0 | u_i(0) \bar{u}_{j'}(0) | u(q) \rangle |_\omega = \langle 0 | d_i(0) \bar{d}_{j'}(0) | d(q) \rangle |_\omega \propto f_\omega (f_\omega^\perp) M_\omega^n ,
\]

the $n$ is an integer, and the $\pm$ correspond to the vector mesons $\rho_0$ and $\omega$ respectively. The isospin triplet meson $\rho_0$ and isospin singlet meson $\omega$ have approximately degenerate masses, i.e. $M_\omega/M_\rho \approx 98.5\%$. The $\omega$-meson light-cone distribution amplitudes have not been explored yet, we assume that the vector mesons $\rho_0$ and $\omega$ have similar light-cone distribution amplitudes, and take the approximation $M_\omega = M_\rho$, $f_\omega = f_\rho$, $f_\omega^\perp = f_\rho^\perp$, $\cdots$ for the hadronic parameters and obtain the strong coupling constants involving the vector meson $\omega$ by symmetry considerations, which are shown in Table 3. Such an approximation is not crude, for example, if we study the masses and decay constants of the vector mesons $\rho_0$ and $\omega$ using the interpolating currents $J^\mu_\rho = \frac{1}{\sqrt{2}} (u\gamma_\mu u - d\gamma_\mu d)$ and $J^\mu_\omega = \frac{1}{\sqrt{2}} (u\gamma_\mu u + d\gamma_\mu d)$ respectively with the QCD sum rules, the resulting values are almost degenerate.

Appendix B

The spectral densities of the heavy baryon states $\Xi^+_Q$, $\Xi^0_Q$, $\Sigma^+_Q$ and $\Sigma^0_Q$ at the level of quark-gluon degrees of freedom,

\[
\rho^{\Xi^+_Q}_\perp (s) = \frac{1}{128\pi^3} \int_{t_i}^1 dt(t + 2)(1 - t)^2 (s - m_Q^2)^2 + \frac{m_s \langle \bar{s}s \rangle}{16\pi^2} \int_{t_i}^1 dt t^2 - \frac{m_s \langle \bar{q}q \rangle}{8\pi^2} \int_{t_i}^1 dt t + \frac{m_s \langle \bar{s}g_sG_s \rangle}{96\pi^2} \int_{0}^1 dt t \delta(s - m_Q^2) + m_s \frac{3 \langle \bar{q}g_sG_s \rangle - \langle \bar{s}g_sG_s \rangle}{96\pi^2} \delta(s - m_Q^2)
\]

\[
- \frac{m_Q^2}{1152\pi^2} \left( \frac{\alpha_s G_F}{\pi} \right) \int_{t_i}^1 dt \left( \frac{t + 2}{t^2} \right)^2 \delta(s - m_Q^2) + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{6} \delta(s - m_Q^2)
\]

\[
- \frac{1}{384\pi^2} \left( \frac{\alpha_s G_F}{\pi} \right) \int_{t_i}^1 dt (2 - t) ,
\]

(33)
\[
\rho_{\Xi_Q}^B(s) = \frac{m_Q}{128\pi^4} \int_{t_i}^1 dt (t+2)(1-t)^2(s-\bar{m}_Q^2)^2 + \frac{m_s m_Q \langle \bar{s}s \rangle}{16\pi^2} \int_{t_i}^1 dt \left[ -\frac{m_s m_Q \langle \bar{q}q \rangle}{8\pi^2} \right] \int_{t_i}^1 dt + \\
\frac{m_s m_Q \langle s\bar{g}_s \sigma G \rangle}{96\pi^2} \int_0^1 dt \delta(s-\bar{m}_Q^2) + \frac{m_s m_Q \left[ 3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle \right]}{96\pi^2} \delta(s-\bar{m}_Q^2) \\
- \frac{m_Q}{152\pi^2} \left( \alpha_s GG \right) \frac{\langle \bar{s}g_s \sigma Gs \rangle}{\pi} \int_{t_i}^1 dt \frac{t^4 - 3t^3 + 3t^2 + 9t - 4}{t^2} + \frac{m_Q \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{6} \delta(s-\bar{m}_Q^2) \\
- \frac{m_Q}{152\pi^2} \left( \alpha_s GG \right) \frac{\langle s\bar{g}_s \sigma Gs \rangle}{\pi} \int_0^1 dt \frac{t^3 - 3t + 2}{t} \bar{m}_Q^2 \delta(s-\bar{m}_Q^2),
\]

(34)

\[
\rho_{\Xi_Q}^A(s) = \frac{1}{128\pi^4} \int_{t_i}^1 dt (t+2)(1-t)^2(s-\bar{m}_Q^2)^2 + \frac{\langle \bar{q}q \rangle^2}{6} \delta(s-\bar{m}_Q^2) \\
- \frac{m_Q^2}{152\pi^2} \left( \alpha_s GG \right) \frac{\langle \bar{s}g_s \sigma Gs \rangle}{\pi} \int_{t_i}^1 dt \frac{t^4 - 3t^3 + 3t^2 + 9t - 4}{t^2} \delta(s-\bar{m}_Q^2) \\
- \frac{1}{384\pi^2} \left( \alpha_s GG \right) \frac{\langle s\bar{g}_s \sigma Gs \rangle}{\pi} \int_{t_i}^1 dt t(2-t),
\]

(35)

\[
\rho_{\Xi_Q}^B(s) = \frac{m_Q}{128\pi^4} \int_{t_i}^1 dt (t+2)(1-t)^2(s-\bar{m}_Q^2)^2 + \frac{m_Q \langle \bar{q}q \rangle^2}{6} \delta(s-\bar{m}_Q^2) \\
- \frac{m_Q}{152\pi^2} \left( \alpha_s GG \right) \frac{\langle \bar{s}g_s \sigma Gs \rangle}{\pi} \int_{t_i}^1 dt \frac{t^4 - 3t^3 + 3t^2 + 9t - 4}{t^2} \\
- \frac{m_Q}{152\pi^2} \left( \alpha_s GG \right) \frac{\langle s\bar{g}_s \sigma Gs \rangle}{\pi} \int_0^1 dt \frac{t^3 - 3t + 2}{t} \bar{m}_Q^2 \delta(s-\bar{m}_Q^2),
\]

(36)

\[
\rho_{\Xi_Q}^A(s) = \frac{1}{32\pi^4} \int_{t_i}^1 dt (t-1)^3(s-\bar{m}_Q^2)(5s - 3\bar{m}_Q^2) - \frac{m_s \langle \bar{q}q \rangle}{4\pi^2} \int_{t_i}^1 dt \\
+ \frac{m_s \langle \bar{s}s \rangle}{4\pi^2} \int_{t_i}^1 dt (t-1) \left[ 3 + s \delta(s-\bar{m}_Q^2) \right] \\
- \frac{m_s \langle s\bar{g}_s \sigma Gs \rangle}{24\pi^2} \int_0^1 dt \left[ 2 + \frac{s}{M^2} \right] \delta(s-\bar{m}_Q^2) \\
+ \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{16\pi^2} \delta(s-\bar{m}_Q^2) + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{3} \delta(s-\bar{m}_Q^2) \\
+ \frac{1}{96\pi^2} \left( \alpha_s GG \right) \int_{t_i}^1 dt (4-5t) + \frac{1}{96\pi^2} \left( \alpha_s GG \right) \int_{t_i}^1 dt (1-t) \bar{m}_Q^2 \delta(s-\bar{m}_Q^2) \\
- \frac{m_Q^2}{288\pi^2} \left( \alpha_s GG \right) \frac{\langle s\bar{g}_s \sigma Gs \rangle}{\pi} \int_0^1 dt \frac{(1-t)^3}{t^2} \left[ 2 + \frac{s}{M^2} \right] \delta(s-\bar{m}_Q^2),
\]

(37)
\[ \rho_{\Sigma Q}^B(s) = \frac{3m_Q}{64\pi^4} \int_{t_i}^1 dt (1-t)^2 (s - \tilde{m}_Q^2)^2 - \frac{m_s m_Q \langle \bar{q}q \rangle}{2\pi^2} \int_{t_i}^1 dt + \frac{m_s m_Q \langle \bar{s}s \rangle}{8\pi^2} \int_{t_i}^1 dt \\
+ \frac{m_s m_Q [6 \langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{48\pi^2} \delta(s - m_Q^2) + \frac{2m_Q \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{3} \delta(s - \tilde{m}_Q^2) \\
+ \frac{m_Q}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt \left[ -3 - 2t + \frac{2}{t^2} \right] \\
- \frac{m_Q}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt \left( \frac{1-t}{t} \right) \tilde{m}_Q \delta(s - \tilde{m}_Q^2), \] (38)

\[ \rho_{\Sigma Q}^A(s) = \frac{1}{32\pi^4} \int_{t_i}^1 dt (1-t)^3 (s - \tilde{m}_Q^2)(5s - 3\tilde{m}_Q^2) + \frac{\langle \bar{q}q \rangle^2}{3} \delta(s - m_Q^2) \\
+ \frac{1}{96\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt (4 - 5t) + \frac{1}{96\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt (1-t)\tilde{m}_Q \delta(s - \tilde{m}_Q^2) \\
- \frac{m_Q^2}{288\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt \left( \frac{1-t}{t^2} \right) \left[ 2 + \frac{s}{M^2} \right] \delta(s - \tilde{m}_Q^2), \] (39)

\[ \rho_{\Sigma Q}^B(s) = \frac{3m_Q}{64\pi^4} \int_{t_i}^1 dt (1-t)^2 (s - \tilde{m}_Q^2)^2 + \frac{2m_Q \langle \bar{q}q \rangle^2}{3} \delta(s - m_Q^2) \\
+ \frac{m_Q}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt \left[ -3 - 2t + \frac{2}{t^2} \right] \\
- \frac{m_Q}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{t_i}^1 dt \left( \frac{1-t}{t} \right) \tilde{m}_Q \delta(s - \tilde{m}_Q^2), \] (40)

where \( \tilde{m}_Q^2 = \frac{m_Q^2}{t_i^2} \), \( t_i = \frac{m_Q^2}{s} \).

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