Physical essence of the “nagel effect” for main reinforcement in an inclined crack of reinforced concrete structures

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Abstract. The physical essence of the “nagel effect” for main reinforcement in an inclined crack of reinforced concrete structures for the model of deformation of reinforced concrete structures includes formation of the first block and the second block of an inclined crack. This effect is considered in constructions under the action of vertical load, with linear displacements (normal to the axis of the beam) and rotations of the fixed end. We can find the unknown $\tau_{ad}$ from the canonical equations for structural mechanics. When we multiply the single diagrams of bending moments caused by displacements $\Delta_1$ and $\Delta_2$ and displacement caused by longitudinal force $N_x(x)$ and increment $\Delta N_x$. In this case, for the “nagel effect”, we take into account the presence of several forces and displacements: the force $P_1$ for the surface for the second block; adhesion to the surface of the reinforcement $\tau_{ad}$ and a special force for the effect of discontinuity ($\Delta T$); displacements for mutual shifts of the crack edges ($\Delta_1$); movement from the width of the crack opening ($\Delta_2 = a_{cr}$); angle of rotation for the fixed ends; variable longitudinal force in the fixed ends.

1. Introduction

For more than 45 years, researchers have been interested in studying the "nagel" effect, a phenomenon seen in experiments related to the resistance of reinforced concrete elements to force and deformation effects. However, the physical nature of this phenomenon remains unclear to date and is determined from empirical considerations.

The physical nature of the “nagel” effect uses the transverse force parameter of the rebar $Q_s$, and other basic parameters [1] such as, in order of taking into account the presence of several forces and displacements: for the force $P_1$ for the surface for the second block; adhesion to the surface of the reinforcement $\tau_{ad}$ and a special force for the effect of discontinuity is the opposite $\Delta T$; displacement for mutual shift of the crack banks, $-\Delta_1$; displacement from the width of the crack opening $\Delta_2 = a_{cr}$; variable longitudinal force in the fixed end $N_x$ and others for the calculation models.

Under these conditions, the disclosure of the essence of the phenomena, which makes it possible to construct a calculation model for determining the “nagel” effect, becomes an extraordinary and urgent task [2-12].
The “nagel effect” takes place in reinforced concrete structures from the action of bending, while in the local areas near the inclined crack there are areas of crushing, punching, and separation [13-23].

For the model of deformation of reinforced concrete structures, some schemes are proposed for the first block and the second block of an inclined crack. For making a scheme for connecting the first and second blocks, are also considered the cross-section I-I and III-III. Here, the crushed area of the second block is situated on the upper surface of the longitudinal reinforcement (or transverse reinforcement). The separation area of the first block is special and forms a new longitudinal crack, – a new scheme for structural mechanics.

2. Methods

In this case, scheme (Fig. 2a) for the “nagel” effect, we take into account the presence of several forces and displacements: the force $P_1$ on the surface of the second block (Fig. 2a); adhesion to the surface of the reinforcement (cubic parabola) and a special force for the effect of discontinuity $\Delta T$ (Fig. 2a, b); displacement for mutual shear of the crack edges, - $\Delta$, (Fig. 2a); displacement from the crack opening width $\Delta_2 = a_{cr}$ (Fig. 2a); the angle of rotation of the fixed end, - $\varphi_1 = a \tan \left( \frac{\Delta a_{cr, max} - \Delta a_{cr, t}}{2d} \right)$ (Fig. 2a); longitudinal force in the slipping attachment end $N_s$ (Fig. 2a). Scheme has a beam with two fixed ends at $l = 3d$, with distances of 0.25d – for $\Delta T$; 0.5d – for $P_1$; 1.25d – for adhesion $\tau_{ad}$ (Fig. 2a).

Let’s consider the first scheme – a beam with two fixed ends: to calculate it, we take the main system (Fig. 2b), obtained as a result of a cut in the middle of the beam for which we give single diagrams of bending moments, cased by the action of vertical load, linear displacements and with rotations of the fixed ends.

In the section of the beam raise internal forces: $X_1 = Q_s$ - transverse force; $X_2$ - bending moment; $X_3$ - distributed alternating bending moment caused by adhesion of the reinforcement. Here $\Delta a_{cr, max}$ is the crack opening at a distance of two diameters from the axis of the reinforcement; $\Delta a_{cr, t}$ - crack opening on the axis of the reinforcement. The increment $\Delta N_s$ for the surface (Fig. 1c) parameters:

$\Delta N_s = 2 \pi r \cdot 1d \cdot \tau_{ad} \cdot \omega$; $(\Delta N_{s, \Delta T} = 2 \pi r \cdot 1d \cdot \Delta T \cdot \omega)$. Here $\omega$ - for the form of adhesion stress $\tau_{ad}$. (Fig. 1d).

The surface of normal stresses $q_1 = 2 \pi r \cdot 1d \cdot \sigma_{cr} \cdot \omega_1$. Here $\omega_1$ - for the normal stress form $q_1$ for the surface (Fig. 1c) and the resultant force $P_1$.

Then the angle of rotation of the fixed end can be obtained:

$$tg(\varphi_1) = \frac{\Delta a_{cr, max} - \Delta a_{cr, t}}{2d};$$

$$\varphi_1 = a \tan \left( \frac{\Delta a_{cr, max} - \Delta a_{cr, t}}{2d} \right). \quad (2)$$

For all internal efforts, single diagrams of bending moments are shown at Fig. 2c-e. In this case, the diagram instead of a cubic parabola can be approximated by a rectangle and a trapezoid for simplicity (Fig. 2e).

$$N_{s, 1} = \sigma_{s, k} \cdot A_s + \frac{\Delta_2}{l_{cr}} \cdot E_s \cdot A_s \rightarrow N_{s, max} = N_{s, 1} + \Delta T \cdot 0.25d + \frac{\Delta_2}{l_{cr}} \cdot E_s \cdot A_s =$$

$$= \sigma_{s, k} \cdot A_s + \Delta T \cdot 0.25d + \frac{\Delta_2}{l_{cr}} \cdot E_s \cdot A_s; \quad (3)$$

$$N_s(x) = f(\tau_{ad}) + f(\Delta_2) = N_{s, max} - 2 \pi r \cdot 1.25d \cdot \tau_{ad} \cdot \omega + \frac{\Delta_2}{l_{cr}} \cdot E_s \cdot A_s. \quad (4)$$
Schemes for taking into account the effort \( N_s = N_s(x) \) in the main system and the corresponding diagram of moments from displacements \( \frac{\Delta}{2} \) are in the Fig. 3c, d (a sinusoid). In this case, the variable force \( N_s(x) \) is a function of the adhesion force \( (\tau_{ad}) \) and \( \frac{\Delta}{l_{cr}} \cdot E_j \cdot A_j \) and the corresponding moment diagram \( (M_{s,x3} = X_3 = N_s(x) \cdot e) \). Then we multiply the single moment diagram from the displacement \( \frac{\Delta}{2} \) (Fig. 3c, d) and the displacement \( \Delta \) i.e. from \( N_s(x) \).

\[
N_{s,2} = N_{s,max} - 2\pi r \cdot 1.25d \cdot \frac{X_3}{e} \cdot \omega + \frac{\Delta}{l_{cr}} \cdot E_j \cdot A_j = \\
= \sigma_{sk} \cdot A_j + \Delta T \cdot 0.25d - 2\pi r \cdot 1.25d \cdot \frac{X_3}{e} \cdot \omega + \frac{\Delta}{l_{cr}} \cdot E_j \cdot A_j. 
\]

(5)

Hence the adhesion to the reinforcement can be found as \( \tau_{ad} = \frac{X_3}{e} \) (Fig. 1c).

Here \( e \) is the eccentricity, from the surface of the reinforcement to center of its the axis.

Then \( M_{s,x3} = X_3 = \tau_{ad} \cdot e \), where \( e \) is the adhesion to the reinforcement. Here the distributed variable bending moment \( X_3 \) can be found through the calculation of statically indeterminate systems by the force method.

Then we obtain the internal forces \( X_1, X_2, X_3 \) for systems (6) and (7).

3. Results and Discussion

Unknowns \( X_1, X_3 \) can be found by the calculation of statically indeterminate schemes for the force method, with the related systems of equations:

\[
\begin{align*}
X_1 \delta_{11} + X_3 \delta_{13} + \Delta_{1,p} &= 0 \\
X_1 \delta_{31} + X_3 \delta_{33} + \Delta_{3,p} &= 0
\end{align*}
\]

(6)

Unknowns \( X_2, X_3 \) we find from another system of equations:

\[
\begin{align*}
X_2 \delta_{22} + X_3 \delta_{23} + \Delta_{2,p} &= 0 \\
X_2 \delta_{32} + X_3 \delta_{33} + \Delta_{3,p} &= 0
\end{align*}
\]

(7)

The coefficients \( \delta_{11}, \delta_{13}, \delta_{31}, \delta_{33}, \delta_{22}, \delta_{23}, \delta_{32} \) and load members \( \Delta_{1,p}, \Delta_{2,p}, \Delta_{3,p} \) of these equations can be found:

\[
\delta_{11} = 2 \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot \frac{1}{3 \cdot EI} = \frac{l^3}{12EI};
\]

(8)

\[
\delta_{13} = -\frac{1}{EI} \left[ 0 + \left( \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot N_{s,1} \cdot e \right) \right] = -\frac{N_{s,1} \cdot e \cdot l^2}{8 \cdot EI};
\]

(9)

\[
\delta_{22} = \frac{1}{EI} \cdot l \cdot 1 \cdot l = \frac{l}{EI};
\]

(10)

\[
\delta_{23} = -\frac{1}{EI} \left[ 1 \cdot l \cdot N_{s,1} \cdot e \right] = -\frac{N_{s,1} \cdot e \cdot l}{EI};
\]

(11)
Figure 1. The upper surface of the main reinforcement for the second block is in concrete of normal stresses $q_i$ (parabola) and the resultant force $P_1$ (crushing of concrete) and the adhesion $\tau_{ad}$ to the surface of the reinforcement: a - diagrams of adhesion (in a form of cubic parabola) and a special force for the effect of breaking the continuity, $-\Delta T$; b, c - a cell of nodes with normal tension $q_i$ and adhesion $\tau_{ad}$; d - surface area of longitudinal reinforcement and local concrete with normal stress $q_i$ and tangential $\tau_{ad}$ for eccentricity $(e)$ to the center $O$ of the reinforcement; e – bending moment in the main reinforcement $M_s = X_2$ and Bernoulli hypothesis; f – the dependence “$\sigma_s - \varepsilon_s$”.

\[
\delta_{31} = \frac{1}{EI} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot 0.125d + N_{s,\max} \cdot e \cdot 0.625d \cdot 0.875d + \right.
\]
\[
+ N_{s,2} \cdot e \cdot 0.625d \cdot 1.19d + \frac{1}{2} \cdot \left( \frac{N_{s,\max}}{2} \cdot e - \frac{N_{s,2}}{2} \cdot e \right) \cdot 0.625d \cdot 1.29d \]
\[
= -\frac{2 \cdot e \cdot d^2}{EI} \cdot [0.31 \cdot N_{s,1} + 0.75 \cdot N_{s,\max} + 0.54 \cdot N_{s,2}];
\]

(12)

\[
\delta_{32} = -\frac{1}{EI} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot 1 + N_{s,\max} \cdot e \cdot 0.625d \cdot 1 + \frac{N_{s,\max} \cdot e}{2} \right.
\]
\[
+ \frac{N_{s,2} \cdot e}{2} \cdot 0.625d \cdot 1 = -\frac{e \cdot d}{EI} \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\max} + 0.31 \cdot N_{s,2}];
\]

(13)

\[
\delta_{33} = \frac{1}{EI} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot N_{s,1} \cdot e + N_{s,\max} \cdot e \cdot 0.625d \cdot N_{s,\max} \cdot e +
\]

\[
\frac{N_{s,2} \cdot e}{2} \cdot 0.625d \cdot 1 \right];
\]

(14)
\[ + \frac{0.625d}{6} \cdot (2 \cdot \left( \frac{N_{s,\text{max}} \cdot e}{2} + \frac{N_{s,2} \cdot e}{2} \right)^2 + 2 \cdot N_{s,2} \cdot e \cdot N_{s,\text{max}} \cdot e + 2 \cdot N_{s,2} \cdot e \cdot N_{s,\text{max}} \cdot e) \] \]

\[ = \frac{e^2 \cdot d}{EI} \cdot (0.25 \cdot N_{s,1}^2 + 0.675 \cdot N_{s,\text{max}}^2 + 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,1} \cdot N_{s,\text{max}}^2) \]  \hspace{1cm} (14)

Figure 2. Calculation scheme of the “nagel” effect: loading scheme (a); equivalent system (forces $X_1$, $X_2$, $X_3$ and external loads $P_1$, $\Delta T$, $\tau_{\text{ad}}$, $\Delta N_s$), displacements ($\Delta_1$, $\Delta_2$) and rotation of the fixed end by an angle $\phi_1$) (b); single diagrams from forces, $X_1$, $X_2$, $X_3$ (c-e)
Figure 3. The calculation scheme of the beam: equivalent system (a), the scheme of application with external loads, displacements and rotation in the fixed end (a, c, e, g), diagrams of bending moments from the applied loads, displacements and rotation, - b - $M_p$; d - $N_M$, f - $M_\Delta$, h - $M_\varphi$. 
One of the cases of possible effect on the beam can be a displacement of the fixed end by an amount \( \Delta_1 \) in the direction perpendicular to the axis of the rod AB. When the fixed end A is rotated by an angle \( \phi_1 \) in the main system, we obtain displacements in the directions of the unknown forces \( X_1, X_2, X_3 \) (when the fixed end is rotated, a crack occurs).

Displacements in the main system in the directions of the unknown forces are equal to:

\[
\Delta_{r,p} = \left[ \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot 0 + \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot d \cdot P_1 \cdot 1d \right] + \left[ \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot N_{s,1} \cdot \Delta_1 \right] + \left[ \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{2}{3} \cdot N_{s,2} \cdot \Delta_1 \right] + 2 \left( \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{2}{3} \cdot 2EI \cdot \frac{\Delta_1}{9d^2} \right) + \left[ \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{3}{4} \cdot 34EI \cdot \phi_1 \right] + \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \cdot \frac{2EI}{3d} \cdot \phi_1
\]

\[
\Delta_{s,p} = 1 \cdot 1d \cdot \frac{1}{2} \cdot P_1 \cdot 1d + 0 + l \cdot \frac{1}{4} \cdot \frac{4EI}{3d} \cdot \phi_1 = \frac{d^2}{2} \cdot P_1 + El \cdot \phi_1;
\]

\[
\Delta_{s,p} = \left[ 0 - 0.625d \cdot N_{s,\max} \cdot e^{\left( \text{N}_{s,\max} + \frac{N_{s,2}}{2} \right)} \cdot e^{-0.625d \cdot P_1 \cdot 1d} \cdot 0.68 \right] + \right[ \frac{-N_{s,1} \cdot e^{-0.25d \cdot 0.08 \cdot N_{s,2} \cdot \Delta_1}}{2} - N_{s,\max} \cdot e^{-0.625d \cdot 0.37 \cdot N_{s,2} \cdot \Delta_1} \right] + \left[ N_{s,1} \cdot e^{-0.25d \cdot 0.08 \cdot 6EI \cdot \frac{\Delta_1}{9d^2}} \right] + \left[ \frac{N_{s,\max} + N_{s,2}}{4} \right] + \left[ \frac{N_{s,\max} \cdot e^{-0.625d \cdot 0.37 \cdot 6EI \cdot \frac{\Delta_1}{9d^2}}}{\text{N}_{s,\max} + \frac{N_{s,2}}{2}} \right] + \left[ \frac{N_{s,\max} \cdot e^{-0.625d \cdot 0.0625 \cdot 2EI \cdot \frac{\phi_1}{3d}}}{\text{N}_{s,\max} + \frac{N_{s,2}}{2}} \right] + \left[ \frac{-N_{s,1} \cdot e^{-0.25d \cdot 0.18 \cdot 4EI \cdot \frac{\phi_1}{3d}}}{2} \right]
\]

\[
\Delta_{s,p} = \left[ N_{s,1} \cdot e^{\left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot El \cdot \phi_1 \right)} \right] + \right[ N_{s,\max} \cdot e^{-0.625d \cdot 0.11d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot El \cdot \phi_1} \right] + \left[ \frac{N_{s,\max} + 3 \cdot N_{s,2}}{4} \cdot e^{-0.425d^2 \cdot P_1 \cdot 0.24d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot El \cdot \phi_1} \right]
\]

Then we solve the system of equations (6):

\[
\begin{cases}
X_1 \delta_{31} + X_3 \delta_{33} + \Delta_{1,p} = 0 \\
X_1 \delta_{31} + X_3 \delta_{33} + \Delta_{3,p} = 0 \\
X_1 = \frac{-X_3 \delta_{33} - \Delta_{1,p}}{\delta_{31}};
\end{cases}
\]

\[
\frac{X_1 \delta_{33} - \Delta_{1,p}}{\delta_{31}} + X_3 \delta_{33} + \Delta_{3,p} = 0;
\]
$$X_3 \left( -\frac{\delta_{13}}{\delta_{11}} \delta_{31} + \delta_{33} \right) = -\Delta_{3,p} + \frac{\Delta_{3,p} \cdot \delta_{33}}{\delta_{11}};$$

$$X_3 = \frac{-\Delta_{3,p} + \frac{\Delta_{3,p} \cdot \delta_{33}}{\delta_{11}}}{-\frac{\delta_{13}}{\delta_{11}} \delta_{31} + \delta_{33}};$$

After substitution we get:

$$X_3 \left( \frac{N_{l,1} \cdot e \cdot l^2}{8 \cdot E_l} \cdot \frac{12EI}{l^3} \cdot \left( -\frac{e \cdot d^2}{EI} \cdot \left[ 0.031 \cdot N_{s,l} + 0.75 \cdot N_{s,max} + 0.54 \cdot N_{s,2} \right] + \right) + \frac{e^2 \cdot d}{EI} \cdot \left[ 0.25 \cdot N_{s,1}^2 + 0.675 \cdot N_{s,max}^2 + 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,2} \cdot N_{s,max} \right] \right) =$$

$$= -N_{s,l} \cdot e \cdot \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \phi_1 \right) - \frac{N_{s,\max}}{4} \cdot e \cdot \left( -0.625d - 0.11d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \phi_1 \right) -$$

$$\quad - N_{s,\max} \cdot e \cdot \left( -0.425d^2 \cdot P_l - 0.24d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \phi_1 \right) +$$

$$\quad + \left[ 0.25 \cdot d \cdot P_l \cdot EI + 0.17 \cdot \frac{EI \cdot \Delta_1}{d} \cdot \left( N_{s,1} + N_{s,2} \right) - 0.44 \cdot \frac{\Delta_1}{d^3} \cdot \left( EI \right)^2 + 0.59 \cdot \left( EI \right)^2 \cdot \frac{\phi_1}{d^2} \right] \times$$

$$\times \left( -\frac{e \cdot d^2}{EI} \right) \cdot \left[ 0.31 \cdot N_{s,1} + 0.75 \cdot N_{s,max} + 0.54 \cdot N_{s,2} \right];$$

$$X_3 = \frac{e \cdot d}{EI} \cdot \left[ 0.5 \cdot N_{s,1}^2 + 0.94 \cdot N_{s,\max} \cdot N_{s,1} + 0.31 \cdot N_{s,2} \cdot N_{s,1} + 0.675 \cdot N_{s,\max} \right] +$$

$$+ 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,2} \cdot N_{s,\max} \right] \cdot \left[ N_{s,1} \cdot \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \phi_1 \right) + N_{s,\max} \left( -0.625d - 0.11d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \phi_1 \right) \right] +$$

$$+ \frac{N_{s,\max} + 3 \cdot N_{s,2}}{4} \left[ -0.425d^2 \cdot P_l - 0.24d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \phi_1 \right] +$$

$$+ \left[ 0.25 \cdot d^3 \cdot P_l + 0.17 \cdot \frac{EI \cdot \Delta_1}{d} \cdot \left( N_{s,1} + N_{s,2} \right) - 0.44 \cdot \frac{\Delta_1}{d^3} \cdot \left( EI \right)^2 + 0.59 \cdot \left( EI \right)^2 \cdot \frac{\phi_1}{d^2} \right] \cdot \left[ 0.31 \cdot N_{s,1} + 0.75 \cdot N_{s,\max} + 0.54 \cdot N_{s,2} \right]; \quad (20)$$

$$X_1 = \frac{-X_3 \delta_{13}}{\delta_{11}} \cdot \frac{\Delta_{1,p}}{\delta_{11}} = X_3 \cdot \frac{N_{s,2} \cdot e \cdot l^2}{8 \cdot E_l} \cdot \frac{12EI}{l^3} - \frac{\Delta_{1,p}}{\delta_{11}} =$$

$$= \frac{0.5 \cdot N_{s,2} \cdot l^2}{d^3} \cdot \left[ 0.5 \cdot N_{s,1}^2 + 0.94 \cdot N_{s,\max} \cdot N_{s,1} + 0.31 \cdot N_{s,2} \cdot N_{s,1} + 0.675 \cdot N_{s,\max} \right] +$$

$$+ 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,2} \cdot N_{s,\max} \right] \cdot \left[ N_{s,1} \cdot \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \phi_1 \right) + N_{s,\max} \left( -0.625d - 0.11d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \phi_1 \right) \right];$$
\[
\begin{align*}
&\frac{N_{s,\text{max}} + 3 \cdot N_{s,2}}{4} \left( -0.425 d^2 \cdot P_1 - 0.24 d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \varphi_1 \right) + \\
&+ \left[ 0.25 \cdot d^2 \cdot P_1 + 0.17 \cdot \Delta_1 \cdot d \cdot (N_{s,1} + N_{s,2}) - 0.44 \cdot \frac{\Delta_1}{d} \cdot EI + 0.59 \cdot EI \cdot \varphi_1 \right] \times \\
&\times [0.31 \cdot N_{s,1} + 0.75 \cdot N_{s,\text{max}} + 0.54 \cdot N_{s,2}] - 0.25 \cdot EI \cdot d \cdot P_1 - 0.16 \cdot \frac{EI}{d} \cdot \Delta_1 \times \\
&\times (N_{s,1} + N_{s,2}) + 0.44 \cdot \frac{\Delta_1 \cdot (EI)^2}{d^3} - 0.59 \cdot (EI)^2 \cdot \varphi_1 \end{align*}
\]

Then we solve the system of equations (7):

\[
\begin{align*}
X_2 \delta_{22} + X_3 \delta_{23} + \Delta_{2,p} &= 0 \\
X_2 \delta_{32} + X_3 \delta_{33} + \Delta_{3,p} &= 0 \\
X_2 &= \frac{-X_3 \delta_{23} - \Delta_{2,p}}{\delta_{22}}; \\
-\frac{X_3 \delta_{23} - \Delta_{2,p}}{\delta_{22}} \delta_{32} + X_3 \delta_{33} + \Delta_{3,p} &= 0; \\
X_3 &= \frac{-\Delta_{3,p} + \frac{\Delta_{2,p} \cdot \delta_{32}}{\delta_{22}}}{\delta_{22}}; \\
-\frac{\Delta_{3,p} + \frac{\Delta_{2,p} \cdot \delta_{32}}{\delta_{22}}}{\delta_{22}} \delta_{22} + \frac{\delta_{22}}{\delta_{22}} \delta_{32} + \delta_{33} &= 0.
\end{align*}
\]

After substitution we get:

\[
\begin{align*}
X_3 \cdot \frac{N_{s,1} \cdot e \cdot l}{EI} \cdot \frac{EI}{l} \cdot \left( -\frac{e \cdot d}{EI} \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} + 0.31 \cdot N_{s,2}] + \\
+ \frac{e^2 \cdot d}{EI} \cdot [0.25 \cdot N_{s,1}^2 + 0.675 \cdot N_{s,\text{max}}^2 + 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,1} \cdot N_{s,\text{max}}] \right) &= \\
= -N_{s,1} \cdot e \cdot \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \varphi_1 \right) - \\
-N_{s,\text{max}} \cdot e \cdot \left( -0.625 d - 0.11 d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \varphi_1 \right) - \\
-\frac{N_{s,\text{max}} + 3 \cdot N_{s,2}}{4} \cdot e \left( -0.425 d^2 \cdot P_1 - 0.24 d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \varphi_1 \right) + \\
+ \left( \frac{d \cdot P_1 \cdot EI}{6} \right) + \left( \frac{(EI)^2 \cdot \varphi_1}{3d} \right) \cdot \left( \frac{e \cdot d}{EI} \right) \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} + 0.31 \cdot N_{s,2}] &; \\
X_3 &= \frac{e \cdot d}{EI} \cdot \left( -0.5 \cdot N_{s,1}^2 + 0.94 \cdot N_{s,\text{max}} \cdot N_{s,1} + 0.31 \cdot N_{s,2} \cdot N_{s,1} + 0.675 \cdot N_{s,\text{max}}^2 + \\
+ 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,2} \cdot N_{s,\text{max}} \right) \left( -N_{s,1} \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} \right) - \\
-\frac{N_{s,\text{max}} + 3 \cdot N_{s,2}}{4} \cdot \left( -0.425 d^2 \cdot P_1 - 0.24 d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \varphi_1 \right) + \\
+ \left( \frac{d \cdot P_1 \cdot EI}{6} \right) + \left( \frac{(EI)^2 \cdot \varphi_1}{3d} \right) \cdot \left( \frac{e \cdot d}{EI} \right) \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} + 0.31 \cdot N_{s,2}] \right)
\end{align*}
\]
-0.06 \cdot EI \cdot \varphi_1 - N_{s,\text{max}} \cdot \left( -0.625 d - 0.11 d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \varphi_1 \right) - \\
\frac{N_{s,\text{max}} + 3 \cdot N_{s,2}}{4} \cdot \left( -0.425 d^2 \cdot P_1 - 0.24 d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \varphi_1 \right) - \\
- \left( \frac{d^2 \cdot P_1}{6} + \frac{EI \cdot \varphi_1}{3} \right) \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} + 0.31 \cdot N_{s,2}] ;

(24)

X_2 = -X_3 \frac{\Delta_{2,p}}{\delta_2} = X_3 \frac{N_{s,1} \cdot e \cdot l}{EI} - \frac{\Delta_{2,p}}{\delta_2} = X_3 \cdot N_{s,1} \cdot e - \frac{\Delta_{2,p}}{\delta_2} = \\
\frac{d}{EI} \cdot [0.5 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} \cdot N_{s,1} + 0.31 \cdot N_{s,2} \cdot N_{s,1} + 0.675 \cdot N_{s,2}^2 + \\
+ 0.26 \cdot N_{s,2}^2 + 0.31 \cdot N_{s,2} \cdot N_{s,\text{max}}] \cdot \left[ -N_{s,1} \cdot \left( -0.01 \cdot N_{s,2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} \right) - \\
-0.06 \cdot EI \cdot \varphi_1 - N_{s,\text{max}} \cdot \left( -0.625 d - 0.11 d \cdot N_{s,2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \varphi_1 \right) - \\
\frac{N_{s,\text{max}} + 3 \cdot N_{s,2}}{4} \cdot \left( -0.425 d^2 \cdot P_1 - 0.24 d \cdot N_{s,2} \cdot \Delta_1 + 0.325 \cdot \frac{EI \cdot \Delta_1}{d} + 0.28 \cdot EI \cdot \varphi_1 \right) - \\
- \left( \frac{d^2 \cdot P_1}{6} + \frac{EI \cdot \varphi_1}{3} \right) \cdot [0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,\text{max}} + 0.31 \cdot N_{s,2}] - \\
\frac{d \cdot EI \cdot P_1}{6} - \frac{(EI)^2 \cdot \varphi_1}{3d} 

(25)

Support reactions and reference moments will be equal to:

\begin{align*}
R_A &= -X_1; \\
R_B &= X_1; \\
M_A &= N_{s,1} \cdot e; \\
M_B &= N_{s,2} \cdot e
\end{align*}

(26)

(27)

The bending moment in main reinforcement \( M_s = X_2 \), taking into account the dependence \( \sigma_s - \varepsilon_s \) (Fig. 1c) and the Bernoulli hypothesis (Fig. 1b). Then:

\begin{align*}
M_s &= X_2 = \sigma_s \varphi_s \varphi_s A_j
\end{align*}

(28)

In this case, longitudinal and transverse forces occur in the reinforcement; from deformations \( \Delta_1 = \Delta_{cre} \) and \( \Delta_2 = a_{cre} \) (\( a_{cre} \) is the crack opening width, \( \Delta_{cre} \) is the shift of the crack benches); normal and shear stress from crushing concrete on the upper surface of longitudinal reinforcement (or transverse reinforcement).

4. Conclusions

1. The physical essence of the “nagel effect” for main reinforcement in an inclined crack of reinforced concrete structures is proposed by the first block and the second block of an inclined crack.

Here the crushing concrete is under the action of normal stresses \( q_i \) (and the resultant force \( P_1 \)) in the second block on the upper surface of the longitudinal (or transversal) reinforcement. The adhesion to the surface of the reinforcement \( \tau_{ad} \) (cubic parabola) and the special force for the effect of breaking the continuity – \( \Delta T \). Hence the adhesion to the surface of the reinforcement is can be found – \( \tau_{ad} = \frac{X_3}{e} \).

Support reactions and reference moments will be equal to:

\begin{align*}
R_A &= -X_1; \\
R_B &= X_1; \\
M_A &= N_{s,1} \cdot e; \\
M_B &= N_{s,2} \cdot e
\end{align*}

(26)

(27)

The bending moment in main reinforcement \( M_s = X_2 \), taking into account the dependence \( \sigma_s - \varepsilon_s \) (Fig. 1c) and the Bernoulli hypothesis (Fig. 1b). Then:

\begin{align*}
M_s &= X_2 = \sigma_s \varphi_s \varphi_s A_j
\end{align*}

(28)

In this case, longitudinal and transverse forces occur in the reinforcement; from deformations \( \Delta_1 = \Delta_{cre} \) and \( \Delta_2 = a_{cre} \) (\( a_{cre} \) is the crack opening width, \( \Delta_{cre} \) is the shift of the crack benches); normal and shear stress from crushing concrete on the upper surface of longitudinal reinforcement (or transverse reinforcement).
Important is the special left side region for the longitudinal reinforcement in the first block with a new longitudinal crack arising, which removes the normal and tangential adhesion stresses without the surface and without the eccentricities of the distributed variable bending moments \( M_{s,3} = X_3 \) for \( \tau_{ad} = \frac{X_3}{e} \).

2. Consider the scheme – a beam with fixed ends: to calculate it, we accept the main system as a result of a cut in the middle of the beam for a single diagrams of bending moments. Under the action of vertical load, with linear displacements (normal to the axis of the beam) and rotations of the fixed ends.

3. In this case, for the “nagel” effect, we take into account the parameters of several forces and displacements: for the force \( P_1 \) for on surface of the second block; adhesion to the surface of the reinforcement \( \tau_{ad} \) (cubic parabola) and a special force for the effect of breaking the continuity – \( \Delta T \); displacement for mutual shift of the crack banks, - \( \Delta \gamma \); moving from the width of the crack opening \( \Delta_2 = a_{rc} \); angle of rotation of the fixed end, - \( \phi = a \tan \left( \frac{\Delta_{irc,max} - \Delta_{irc,\alpha}}{2d} \right) \); variable longitudinal force in the slipping attachment end \( N_e \).

The scheme has a beam with two fixed ends at \( l = 3d \), with a distance of 0.25d for \( \Delta T \); 0.5d to \( P_1 \); 1.25d for \( \tau_{ad} \).

4. In the place of cutting the beam, the following internal forces arise: \( X_1 \) - transverse force; \( X_2 \) - bending moment; \( X_3 = M_{s,3} \), hence the adhesion to the surface of the axis of the reinforcement \( \tau_{ad} = \frac{X_3}{e} \) - distributed alternating bending moment, \( \tau_{ad} \) - adhesion to the surface of the reinforcement.

We find the unknown members of equations from the canonical equations for structural mechanics.

Increment \( \Delta N_e \) for surface \( \Delta N = 2\pi r \cdot 1d \cdot \tau_{ad} \cdot \omega \) and \( \Delta N_{s,AT} = 2\pi r \cdot 1d \cdot \Delta T \).

Support reactions and moments will be equal to: \( R_A = -X_1; \quad R_B = X_1; \quad M_A = N_{s,1} \cdot e; \quad M_B = N_{s,2} \cdot e \).

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