**Finite Element Analysis for Time Varying Mesh Stiffness behavior of different shapes of spalling**

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**Abstract:** Time Varying gear mesh stiffness is the variation of stiffness for the one contact period of a gear pair. Spalling on gear tooth is one of the most common defects in gear transmission. The loss of surface materials due to tooth spall reduces the Time-Varying Mesh Stiffness (TVMS) of the gear pair. The time varying mesh stiffness is the change in mesh stiffness for a gear pair in one contact period. The evaluation of the TVMS of the pair of gear tooth under gear tooth spalling conditions may be useful in finding its dynamic behavior or instant change in TVMS behavior may be the signal of fault developed. The potential energy method is used for analytical calculation of time varying mesh stiffness of the gear. The tooth spall may be of any shape and in this paper we have assumed the spall to be circular, triangular and elliptical and conducted analytical calculation to find TVMS and compare with healthy gear pair. The same gear pair with three shape of spall and modelled and verified through finite element analysis. The data may be used for further analysis. Also a number of shape may be modelled in future.

**Keywords:** TVMS, Potential Energy, Spalling, FEA.

**I. Introduction**  
Gear is an important mechanical element which is used to transmit power from one shaft to another shaft. Gear is used in many industrial applications such as automobile, marine, aviation etc. Spalling occurs on the gear face of the tooth profile, due to the insufficient lubrications, high service load, bad operating conditions, high contact speed and hence transmission of power is not uniform. The loss of surface materials due to tooth spall reduces the Time-Varying Mesh Stiffness (TVMS) of the gear pair, and thus modifies the vibration response of the gear transmission. The evaluation of the TVMS of the gear tooth pair under gear tooth spalling conditions plays an important role in gear dynamic Simulation and the corresponding fault feature analysis [1]. Based on the probability distribution a new analytical model for tooth pitting was developed and mesh stiffness was investigated [1]. Xihui Liang, et al. study a circular shape spall having specific location, radius and depth was considered. With the help of potential energy equations mesh stiffness of gears with single and multiple pits are evaluated [2]. Yang Luo, et al. Gear tooth spall was studied by curved bottom shape method that better represents the geometries of spall. In this study different shape of spall like circular and ellipsoid are taken and mesh stiffness result were compared with FEA [3]. Ankur Saxena et. al. study the effect of spalling defect on the gear an analytical formulation was developed for the calculations of the TVMS. The TVMS was calculated for rectangular, circular, ellipsoid shape of spall and observed TVMS reduction when spall in in mesh. H Ma, et al. calculated error of TVMS under double tooth engagement by improved analytical method and finite element method l. [5] Yan Ding, F. Reiger In this study spalling is differentiated spalling from pitting. Experimental study in a test rig was done at the contact surfaces about the thickness of 10 microns and spalling appears as a deeper cavity at contact surface about the thickness of 20 to 100 microns. [6] Fakher Chari, Wallid Baccar l given modified the potential energy method for finding the spall and tooth breakage effect on vibration response [7] Z. Chen, Y. Shao [8] In this study six degree of freedom dynamic spur gear pinion model is taken to examine the gear defects such as tooth root crack along the width and depth. Time varying mesh stiffness is calculated analytically and validated by finite element method. So many research was done to find the spalling defect analytically and its comparison either by FEA or dynamic response. In this paper along with circular and elliptical spall a new triangular error is used for TVMS change response analytically and its FEA was done further verification.
Methodology

For the study of time varying mesh stiffness a tooth is considered cantilever and spalling is assumed symmetrical about pitch line. The spalling of triangular, circular and elliptical are taken for study. The shapes are shown in figure 1.

![Fig. 1 Different shapes of spalling for Analysis [4, 3]](image)

The gear tooth geometry is as shown in fig. 2 spalling is located near the pitch circle.

![Fig. 2 The cross section of gear tooth [6]](image)

In this case the gear fault is assumed symmetrical about pitch point so for the analysis the gear may be divided as figure no.2.
The length of DC will depend on the one of the dimension of the fault. The Values of \( x_A, x_B, x_C, x_D \) are the values of distance of points A, B, C and D which has been calculated from reference point A. and y values are useful for finding area and moment that is necessary in further calculations.

For the analysis purpose the teeth may be divided into a number of strips, the area of strip without spall is given by

\[
A = 2yl \quad \text{-------------(1)}
\]

While for the spalled region it is calculated by

\[
A_s = (y + y_s) l \quad \text{-------------(2)}
\]

Where, \( y, y_s \) represent the vertical values of the gear tooth contour line with respect to axis of gear teeth in healthy and spalled region.

During the meshing, when teeth contact force \( F \) is transmitted from one teeth to another teeth along the line of action as shown in fig4. This force \( F \) is having two one is in radial direction and another in tangential direction given by

\[
F_r = F \sin \alpha_1 \quad \text{----------(3)}
\]

\[
F_t = F \cos \alpha_1 \quad \text{-------------(4)}
\]

The bending moment of the teeth is also an important data that is necessary for Bending energy calculations. The value of bending moment for the healthy part of the tooth is given by

\[
M = F_t(x_B - x_A) - F_r y \quad \text{-------------(5)}
\]

For the spalled region it will be

\[
M_s = F_t(x_B - x_A) - F_r y_s \quad \text{-----------------(6)}
\]

In the similar way the moment of inertial for healthy and spalled region of the teeth are given by equation 7 and 8.

\[
I = \frac{1}{2} y^3 l \quad \text{-----------------(7)}
\]

\[
I_s = \frac{1}{2} y_s^3 l \quad \text{-----------------(8)}
\]

**Analytical Calculation:**

The potential energy method is used for all the calculations. The Bending stiffness, Shear Stiffness, Axial Stiffness and Contact stiffness were calculates by using the outcome of the following equations. [4,9].

\[
\frac{1}{K_h} = \int_{0}^{a_2} \frac{R_b \sin \alpha_1 (a_2 - a) \cos \alpha_1}{E A_s} \, \text{d} \alpha \quad \text{-------------(9)}
\]

\[
\frac{1}{K_s} = \frac{2}{E A_s} \left[ 2 R_b (1+ \mu) \sin^2 \alpha_1 (a_2 - a) \cos \alpha_1 \right] \, \text{d} \alpha \quad \text{--------------(10)}
\]

\[
\frac{1}{K_b} = \int_{0}^{a_2} \frac{R_b^2 \left[ 1+ \cos \alpha_1 \sin \alpha_1 \cos \alpha_1 \right] (a_2 - a) \alpha_1}{E I_s} \, \text{d} \alpha \quad \text{-----------(11)}
\]

Contact stiffness- The Hertzian contact stiffness is a constant value along the line of action it is independent of contact position and expressed for healthy gear as-

\[
K_h = \frac{\pi E l}{4(1-\mu^2)} \quad \text{-----------------(12)}
\]

The Hertzian Contact stiffness for faulty gear can be expressed as –

\[
K_h = \frac{\pi E (l-l_s)}{4(1-\mu^2)} \quad \text{--------------(13)}
\]

Where \( E, L, \mu \) represents young’s modulus, tooth contact width and Poisson’s ratio respectively.

The total equivalent mesh stiffness of single tooth pair in meshing can be expressed as-

\[
K_{\text{total}} = \frac{1}{K_h + K_b + K_s + K_a} \quad \text{-----------------(14)}
\]

Where subscripts 1 and 2 denotes the driving gear and driven gear. The total equivalent mesh stiffness of double teeth pair in meshing can be expressed as

\[
K_{\text{total}} = \sum_{i=1}^{2} \frac{1}{K_{hi} + K_{bi} + K_{si} + K_{ai}} \quad \text{-----------(15)}
\]

Modelling for different shape of spall: In this paper the analytical calculation are done by assuming the spall to be symmetrical.
about pitch line. Three types of spall were modelled and the time varying mesh stiffness calculated.

1. Circular spall: A circular spall of radius \( r = 1.5 \text{ mm} \) and depth 1 mm is created at about pitch line. Using the equation of circle
\[
x^2 + y^2 = r^2
\]
where, \( r \) = radius of the circular spall and \( y \) varies from \(-r\) to \(+r\). The length of spall \( l_s = 2x \)

2. Triangular Spall: A triangular spall of size 3mm x 3mm x 3mm and depth 1 mm is created on the gear surface at pitch line. The height of the triangle represents the length of spall and one of the sides that is parallel with the width represents the maximum width of spall. For other regions of the triangle, the width of spall may be calculated manually marking a number of strips.

3. Elliptical Spall: The parametric function of an ellipsoid is given by
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

If \( a = b \) then it will be a circle.

After modelling of the gear, the minor axis will represent the length of spall and the major axis with depth will cause the change in width as width of contact is reduced by width of spall. In this case, the major axis were taken as 5mm and the minor axis was 3 mm.

Gear Modeling: For the analytical as well as the finite element analysis, the gear-pinion arrangement was modeled in software. Gear parameters used in this study are given below in Table 1 [9].

| Parameters                  | Gear | Pinion |
|-----------------------------|------|--------|
| Teeth numbers               | 30   | 25     |
| Mass (Kg)                   | 0.588| 0.46   |
| Tooth width (mm)            | 20   | 20     |
| Modulus of elasticity (Gpa) | 210  | 210    |
| Poisson’s ratio             | 0.3  | 0.3    |
| Shear modulus (Gpa)         | 78   | 78     |
| Shear factor                | 2    | 2      |
| Module                      | 2    | 2      |
| Pressure angle              | 20°  | 20°    |

Table 1: Parameters for gear modeling
**Finite element analysis:**

After analytical calculation model was tested in the ansys software for finding its stiffness. The model is imported in preprocessing, meshed and applied by fixing one and moment applied by other. A force of 1000 N is supposed to apply on the gear teeth of pinion by teeth of gear. The deflection of gear is used for its stiffness value. The number of load position used and data were compared with the analytical data. The Figure 4 is showing the deflection in the teeth when the load is applied.

Figure 3: Finite Element Analysis on gear pair.
Results

The analytical result shown in the form of time varying mesh stiffness for different shape of spall like circular, triangular and elliptical are given in figure below.

![Time varying mesh stiffness with different shapes](image)

Comparison of mesh stiffness of healthy gear in analytical calculations with healthy gear in FEA:

| Gear Condition | Analytical Stiffness (N/m) | FEA Stiffness (N/m) | Variation (%) |
|----------------|----------------------------|---------------------|---------------|
| Healthy        | 312920                     | 317918              | 1.57          |

Time varying mesh stiffness is calculated through analytical calculations and FEA. And maximum percentage variation at a point where double tooth contact occurs is evaluated. Maximum Variation in TVMS is 1.57%

Comparison of mesh stiffness when spall shape is circular with Analytical Result –

| Gear Condition | Analytical Stiffness (N/m) | FEA Stiffness (N/m) | Variation (%) |
|----------------|----------------------------|---------------------|---------------|
| Circular       | 258591                     | 277937              | 6.96          |

Time varying mesh stiffness is calculated through analytical calculations and FEA. And percentage variation in faulty gear having circular spall is evaluated. Maximum Variation in TVMS is 7.98%

Comparison of mesh stiffness when spall shape is elliptical with Analytical Result –

| Gear Condition | Analytical Stiffness (N/m) | FEA Stiffness (N/m) | Variation (%) |
|----------------|----------------------------|---------------------|---------------|
| Elliptical     | 255693                     | 271846              | 6.31          |

Time varying mesh stiffness is calculated through analytical calculations and FEA. And percentage variation in faulty gear having elliptical spall is evaluated. Maximum Variation in TVMS is 6.31%

Comparison of mesh stiffness when spall shape is Triangular with Analytical Result –

| Gear Condition | Analytical Stiffness (N/m) | FEA Stiffness (N/m) | Variation (%) |
|----------------|----------------------------|---------------------|---------------|
| Triangular     | 305642                     | 297235              | 2.69          |

Time varying mesh stiffness is calculated through analytical calculations and FEA. And percentage variation in faulty gear having Triangular spall is evaluated. Maximum Variation in TVMS is 2.69%

Conclusion

Finite element conducted on the analytical results shows a good agreement. So the method is used for different shape of spall. The data may also be used for dynamic analysis and vibration behavior of the system.

References

[1]. Lei, Y., Liu, Z., Wang, D., Yang, X., Liu, H., & Lin, J. (2018). A probability distribution model of tooth pits for evaluating
time-varying mesh stiffness of pitting gears. Mechanical Systems and Signal Processing, 106 (2018) 355-366 Elsevier Publication.

[2] Liang, X., Zhang, H., Liu, L., & Zuo, M. J. (2016). The influence of tooth pitting on the mesh stiffness of a pair of external spur gears. Mechanism and Machine Theory, 106 (2016) 1-15. Elsevier Publication.

[3] Luo, Y., Baddour, N., Han, G., Jiang, F., & Liang, M. (2018). Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features. Engineering Failure Analysis, Elsevier Publication 92 (2018) 430-442.

[4] Saxena, A., Parey, A., & Chouksey, M. (2016). Time varying mesh stiffness calculation of spur gear pair considering sliding friction and spalling defects. Engineering Failure Analysis, 70 (2016) 200-211, Elsevier Publication.

[5] Ma, H., Pang, X., Feng, R., Zeng, J., & Wen, B. (2015). Improved time-varying mesh stiffness model of cracked spur gears. Engineering Failure Analysis, 55, 271-287.

[6] Ding, Y., & Rieger, N. F. (2003). Spalling formation mechanism for gears. Wear, 254 (2003), 1307-1317, Elsevier Publication.

[7] Chaari, F., Baccar, W., Abbes, M. S., & Haddar, M. (2008). Effect of spalling or tooth breakage on gear mesh stiffness and dynamic response of a one-stage spur gear transmission. European Journal of Mechanics - A/Solids, 27 (2008), 691-705, Elsevier Publication.

[8] Chen, Z., & Shao, Y. (2011). Dynamic simulation of spur gear with tooth root crack propagating along tooth width and crack depth. Engineering Failure Analysis, 18 (8), 2149-2164, Elsevier Publication.

[9] Ankur Saxena, Manoj Chouksey, Anand Parey (2017) Effect of Mesh Stiffness of healthy and cracked gear tooth on modal and frequency response characteristics of geared rotor system. Mechanism and Machine Theory, 107 (2017), 261-273, Elsevier Publication.