The effect of supernova natal kicks on compact object merger rate

Krzysztof Belczyński and Tomasz Bulik
Nicolaus Copernicus Astronomical Center, Bartycka 18, 00-716 Warszawa, Poland

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Abstract. Mergers of compact objects may lead to different astrophysical phenomena: they may provide sources of observable gravitational radiation, and also may be connected with gamma-ray bursts. Estimate of the rate with which such mergers take place are based on assumptions about various parameters describing the binary evolution. The distribution of one of these parameters - the kick velocity a neutron star receives at birth will strongly influence the number and orbital parameters of compact objects binaries. We calculate these effect using population synthesis of binary stars and show that the expected compact object merger rate changes by a factor of 30 when the kick velocity varies within its current observational bounds.

Key words: general relativity: gravitational waves — stars: binaries, evolution

1. Introduction

The gravitational wave detectors LIGO and VIRGO will soon be operational. Their completion brings up the question of the possible sources of gravitational waves and also their brightness and observable rate. The most often considered sources of gravitational waves are mergers of compact objects; neutron stars or black holes. A number of authors have already considered these phenomena, and several estimates have been published [Narayan et al., 1991; Phinney, 1991; Tutukov and Yungelson, 1993; Portegies Zwart and Spreeuw, 1996].

There are two approaches leading to the calculation of the merger rate. The first method, let us call it the experimental approach, is based on the observational fact that we do see three binary systems of neutron stars: PSR1913+16 (Taylor and Weisberg, 1989), PSR1534+12 (Wolszczan, 1991), and PSR2303+46 (Taylor and Dewey, 1988). Based on this number, and considering various observational selection effects, like e.g. the pulsar beam width, one can estimate the number of such systems in our Galaxy. Given the observed orbital parameters for these systems one can then calculate the lifetime of each one and thus obtain the expected merger rate in the Galaxy. This approach suffers from several weaknesses: it is based on observations of only three objects, so small number statistics strongly affects the results. Moreover, the estimate of the rate is based on theoretical assumptions regarding the selection effects. These assumptions may lead to systematic errors of uncertain value.

The other approach that we can use, (let us call it a theoretical one), is based on population synthesis of binary systems. All compact objects that will finally merge, must have their origin as ordinary stars in binary systems. First studies in that field were performed for Monte Carlo simulations of radio pulsars by Dewey and Cordes (1987). Thus it seems that by modeling the evolution of stellar binaries and the statistical properties of large ensembles of binaries one can calculate the expected population of compact object binaries. Analyzing this population and then considering the systems that will have merged within the Hubble time one can also estimate the present compact object merger rate in the Galaxy. This calculation requires the supernova rate in the Galaxy, and also the fraction of all stars that are in binaries. This "theoretical approach" also suffers from the fact that it is based on several assumptions. In order to create a population of binaries one requires distributions of initial parameters, like the mass of the primary star or the initial mass ratio in the binary. Some stages in the stellar evolution require parameterization as well, e.g. mass loss and angular momentum loss through stellar wind, mass exchange in the common envelope phase etc. Another parameter that may strongly influence the number of compact object binaries as well as their lifetime is the value of the velocity kick that a neutron star receives as a result of supernova explosion. The importance of this parameter has been shown by Portegies Zwart and Spreeuw (1996).

The measurement of the distribution of kicks in supernova explosions is not easy. One approach that can be taken is to measure velocities of pulsars and use this distribution as the distribution of supernova kicks. Here one has to take into account selection effects, like for example the fact the fast neutron stars may leave the Galaxy and not be visible as radio pulsars. A comprehensive study of
Beyond 10M⊙ the distribution of the masses of the primary stars that we use follows Beležni and Brown (1998),
\[ \Psi(M) \propto M^{-1.5} \] (1)

Other authors have used similar distributions, e.g. Portegies Zwart and Verbunt (1996) use the exponent of −1.7. We restrict the range of the masses to the interval between 10M⊙ to 50M⊙ as we want at least the more massive star to undergo a supernova explosion.

We adopt the following distribution of the mass ratio \( q \)
\[ \Phi(q) \propto \text{const} \] (2)

There is some uncertainty regarding \( \Phi \). Portegies Zwart and Verbunt (1996) use \( \Phi \propto (1 + q)^{-2} \), while Tutukov and Yungelson (1994) use \( \Phi \propto q \). Yet another distribution is used in the simulation by Pols and Marinus (1994), who use a distribution slightly peaked for small values of \( q \) and falling down when \( q \) goes to unity.

Initial binary eccentricity \( e \) is assumed to have value between 0 and 1 and its initial distribution is taken from Duquennoy and Mayor (1991):
\[ \Xi(e) = 2e. \] (3)

The distribution of the initial semi-major axis \( a \) used in population synthesis codes is flat in the logarithm, i.e.
\[ \Gamma(a) \propto \frac{1}{a}. \] (4)

Both stars are initially massive main sequence stars, with the radii of at least 4–5 R⊙, so we take minimum value of \( a \) to be 10 R⊙. However if the sum of radii of two zero age main sequence stars exceeds 10 R⊙, we chose the minimal separation to be twice the radius of the primary component. This should give the stars the space to evolve without merging before they go through their main sequence lifetime. Nevertheless some systems might be born with high eccentricity, and thus be in contact and merge forming a single very massive star. At this point we stop our calculation as we are not interested in this type of mergers in our study.

Although very wide binaries with periods up to 10^{10} days are observed (Duquennoy and Mayor, 1991) and models with initial separation up to 10^6 R⊙ are used (Portegies Zwart and Verbunt, 1996), we have set the maximal separation to 10^5 R⊙ as wider binaries are very unlikely to survive two supernova explosions and form a compact object system. Our models showed that setting the upper limit to 10^6 R⊙ does not change the results noticeably.

We assume that the distribution of the kick velocity a newly born neutron star receives in a supernova explosion is a three dimensional Gaussian and parameterize it by its width \( \sigma_v \). We vary \( \sigma_v \) to determine the dependence of the merger rate and properties of the compact object binaries population on this parameter. Alternatively, we draw the natal kicks from a weighted sum two Gaussians (Dewey and Cordes, 1987): 80 percent with the width 175 km s^{-1} and 20 percent with the width 700 km s^{-1}.

This should give the stars the space to evolve without merging before they go through their main sequence lifetime.
2.2. Evolution of binaries

In our calculations we neglect the effects of stellar wind on evolution of binaries.

Tidal circularization takes place in binaries for which the size of any component (or both components) is large in relation to their separation \( r \) (Zahn, 1975). This happens when the stars evolve to the giant stage, and/or if the initial binary separation is small. We follow the prescription proposed by Portegies Zwart and Verbunt (1996); the circularization takes place if the stellar radius of one component is larger than 0.2 of the periastron binary separation. The orbital elements \((a, e)\) change with conservation of angular momentum until the new binary separation is 5 stellar radii of the component in which tidal effects take place, or the binary is totally circularized \((e = 0)\).

Binary stars may undergo mass transfer and common envelope phases in different evolutionary stages. Depending on the physical conditions this will result in the change of mass of each star, mass loss from the system, and also in change of the size and shape of the orbit. We describe below three schematic evolutionary paths and accompanying mass transfer types used in the code.

2.2.1. Evolution for small mass ratios, \((q < 0.8897)\)

The more massive star (primary) evolves faster in the binary. We use the following approximate formula for the main sequence lifetime of a star with mass \( M_i \): \( \text{TSyS} = 20 \times 10^6 \left( M_i/10M_\odot \right)^{-2} \text{ yrs.} \) After this time the primary evolves to the giant stage which lasts about 20 per cent of its main sequence lifetime, and increases its size. For \( q < 0.8897 \) the secondary is still on main sequence at the end of primary giant stage. If the giant primary overfills its Roche lobe mass transfer to the main sequence companion (star 2, with mass \( M_2^f \)) and mass loss takes place. We denote this regime by type I mass transfer. The mass transfer continues until the giant is stripped of its envelope, and thus the final mass of the giant will be just that of the its helium core. We use the approximation \( M_1^f = 0.3 M_1^i \) (Bethe and Brown, 1998). A part of the envelope mass \( 0.7 M_1^i \) is lost from the system while another part is transferred to the companion. The fraction of mass transferred to the companion is proportional to the square of the mass ratio \((\text{Vrancken et al., 1991; Bethe and Brown, 1998})\), so the mass of the companion after the mass transfer is

\[
M_2^f = M_1^i (q + 0.7q^2),
\]

where \( q \) is the initial mass ratio.

The orbit was already circularized by tidal forces, when the giant was filling its Roche lobe, and now its size changes due to mass transfer and mass loss. We describe the change in semi-major axis by (Pols and Marinus, 1994)

\[
a_f/a_i = \left( \frac{M_1^f M_2^f}{M_1^i M_2^i} \right)^{-2} \left( \frac{M_1^i + M_2^i}{M_1^f + M_2^f} \right)^{2\beta+1},
\]

where \( a \) is binary separation, \( M_1 \) is the mass of the giant losing material, \( M_2 \) is the mass of its main sequence companion and indices \( i, f \) correspond respectively to the values before and after the Roche lobe overflow. The parameter \( \beta \) describes the specific angular momentum of material leaving binary (Pols and Marinus, 1994). The above equation assumes that \( \beta \) is constant with time. The value of \( \beta \) is uncertain; typically the values such as \( \beta = 6 \) or \( \beta = 3 \) are used, for discussion see Pols and Marinus (1994). Here all calculations are performed for \( \beta = 6 \). The initial primary, now either a helium star (after the mass transfer) or a giant goes supernova, leaving behind a neutron star. The initial secondary evolves, becomes a giant and may begin to transfer material to the newly formed neutron star increasing its mass, and possibly turning it to a black hole. We assume that the maximum mass of a neutron star is \( 2.2M_\odot \).

In describing accretion on a neutron star we follow Bethe and Brown (1998), and we will call this regime type II mass transfer. A neutron star accretes matter while moving in orbit through the extended envelope of the giant. We use the Bondi-Hoyle accretion rate

\[
\dot{M} = \rho \pi v R^2 \ac
\]

where \( \rho \) is the density in the vicinity of the compact object of mass \( M_2 \), \( v \) is its velocity, and \( R_\ac = 2GM_2/v^2 \) is the accretion radius. Consideration of the energy loss equations lead to the final mass of the compact object (Bethe and Brown, 1998)

\[
M_1^f = 2.4 \times \left( \frac{M_2^i}{M_1^i} \right)^{1/6},
\]

We assume that the giant loses its envelope so its final mass is just that of the helium core \( M_2^f = 0.3 M_2^i \). The orbital separation follows from the energy integration i.e. equations 5.18 through 5.23 in (Bethe and Brown, 1998):

\[
a_f = 0.145 \times a_i \left( \frac{M_1^i}{M_2^f} \right).
\]

Finally the initial secondary undergoes a supernova explosion, and provided that the system survives, we obtain a compact object binary, consisting of either two neutron stars or of a black hole and a neutron star.

2.2.2. Evolution of two stars for intermediate mass ratio, \((0.8897 < q < 0.95)\).

When \( q > 0.8897 \) the secondary is already a giant when the primary explodes as a supernova. The upper limit is explained in the next subsection.

For the case of small orbital separations the evolution begins in a similar way as for small \( q \) described above. The evolution begins in a similar way as in the case of small mass ratio, and the primary may transfer mass to the secondary as described by type I above. When the
secondary becomes a giant the primary is already a helium star, stripped of its envelope. The secondary may transfer mass to the primary and we approximate this process also by type I. The primary explodes as a supernova and forms a neutron star, which may accrete from the secondary, provided that it is still a giant. The mass transfer in this phase is described by type II. The secondary undergoes a supernova explosion and if the system survives we obtain a compact object binary.

In the case of large orbital separations the first stage of mass transfer from the giant to the main sequence star (type I) does not occur. The stars might begin to interact only when both of them are already in the giant stage. In this case they lose the common envelope. To describe this stage of evolution we assume that the giants lose the entire envelopes and become helium stars of 30 per cent of the initial giant masses. We then approximate the change in the size of the orbit by:

\[ \frac{a_f}{a_i} \approx \left[ 1 + \frac{(M_i^1 + M_i^2)M_i^2(1-q^2)}{\alpha_{CE} M_i^1 M_i^2} \right]^{-1} \]  

(10)

As above \( a \) is binary separation, \( M_1, M_2 \) are the masses of primary and secondary. The indices \( i, f \) correspond to the values before and after common envelope phase, respectively, and \( \alpha_{CE} \) describes the efficiency of the orbital energy expenditure in the dispersal of common envelope. Equation (10) was adopted from Yungelson and Tutukov (1993) and describes the change of the orbital separation \( a \) following decrease of the orbital energy which was used to eject a common envelope. They have also showed that \( 0.6 \leq \alpha_{CE} \leq 1 \), and we have performed our calculations for different values: \( \alpha_{CE} = 0.4, 0.6, 0.8, 1.0 \). Our calculations show that the results (the production rate of the compact object binaries, and their properties) are almost indistinguishable for these three values, so for simplicity we present our results for one value of \( \alpha_{CE} = 0.8 \).

Similarly to the case of small orbital separations if the secondary is still a giant it may transfer mass to the neutron star formed in the supernova explosion of the primary. This mass transfer is described as type II.

2.2.3. Evolution of two stars of almost equal mass, \( (q > 0.95) \)

Two stars almost simultaneously (within 10% of their main sequence lifetime) leave main sequence and become giants. If their radii are large enough in relation to the periastron orbital separation, the orbit is tidally circularized. If the binary separation is small enough the giants may finally overfill their Roche lobe and enter a common envelope phase, which we describe by equation (10). The two stars go supernova one after another, and if the system survives, we obtain a compact object binary consisting of two neutron stars.

![Fig. 1. An example evolutionary path leading to formation of a black hole neutron star binary. Units of mass are solar masses, and units of distance are astronomical units: (a) initial binary; (b) after tidal circularization when star 1 expanded to giant size; (c) system after first mass transfer, star 1 explodes as supernova; (d) system after first supernova; (e) after tidal circularization when star 2 expanded to giant size; (f) system after second mass transfer, star 2 explodes as supernova; (g) system after second supernova: black hole neutron star binary.](image-url)

After each process which changes the orbital parameters and sizes of the components we check if the components sizes are not larger than the periastron binary separation.

2.3. Supernova explosion

The supernova explosions are very likely to have an impact on the binary systems. We assume that in each supernova explosion a neutron star with mass of 1.4 \( M_\odot \) is formed and we neglect the interaction of the companion star with the envelope ejected in the supernova explosion.

We draw a random time for a supernova explosion in the orbital motion of the binary. We find the relative position and velocity of the two stars for this moment. The exploding star is then replaced by a neutron star and we add the kick velocity to its orbital velocity, while the ex-
Fig. 3. Possible results of the evolution of a binary with the initial orbital separation $a = 20R_\odot$ and eccentricity $e = 0.5$ for different values of the initial primary mass $M$ and the initial mass ratio $q$. The top left panel shows the systems that were disrupted in the first supernova explosion. The solid line corresponds to the initial secondary mass $10M_\odot$. The top right panel shows the systems in which the neutron star, born in the first supernova explosion, merged with the companion. The bottom left panel shows the systems disrupted in the second supernova explosion. The crosses in the bottom right panel show the neutron star binaries and the filled circles are the black hole neutron star binaries. We present one thousand of each type of binaries. In this calculation we used the Cordes & Chernoff (1997) kick velocity distribution.

The expanding supernova shell carries away a part of the total momentum of the binary. If the energy of such a system is less than zero the system is bound and we calculate the parameters of a new orbit of the binary and its new spatial velocity. For a detailed discussion of the effect of sudden mass loss and a random kick velocity on binary parameters see Hills (1983).

After the first supernova explosion we verify if the neutron star does not collide and merge with the companion,
Fig. 4. Possible results of the evolution of a binary with the initial primary mass $20 M_\odot$ and the mass ratio $q = 0.5$. The top left panel shows the systems disrupted in the first supernova explosion, the top right panel shows the systems in which the neutron star, born in the first supernova explosion, merged with the companion. The bottom left panel shows the systems disrupted in the second supernova explosion. The bottom right panel shows the black hole neutron star binaries, and there are no neutron star binaries formed for this mass ratio ($q = 0.5$). We present one thousand of each type of binaries. In this calculation we used the Cordes & Chernoff (1997) kick velocity distribution.

i.e. if the radius of the companion is larger than the peri-astron binary separation.

2.4. Energy loss through gravitational radiation

Once a compact object binary is formed its orbit will evolve because of the gravitational wave energy loss. Orbital energy loss through radiation of gravitational waves becomes important once a compact object binary is in a tight and/or highly eccentric orbit. The evolution of the semi-major axis $a$ and eccentricity of the orbit $e$, in a bi-
Fig. 2. An example evolutionary path leading to formation of a neutron star neutron star binary formation: (a) initial binary; (b) after tidal circularization when both stars expanded to giant sizes, system ejects common envelope; (c) system after common envelope phase, star 1 explodes as supernova; (d) system after first supernova, now star 2 explodes as supernova; (e) system after second supernova: neutron star neutron star binary.

The lifetime of a compact object binary is given by

\[ t_{\text{merge}} = \frac{5c^5a^4(1 - e^2)^{7/2}}{256G^3M_1M_2(M_1 + M_2)} \left( 1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right)^{-1} \]  

where \( a \) is the semi-major axis of the orbit, \( e \) is its eccentricity, and \( M_1, M_2 \) are the masses of the compact objects.

3. Results

The code described above allows to generate populations of compact object binaries and trace their statistical properties. In this paper we mainly concentrate on the dependence of these properties as a function of the kick velocity distribution. In Figures 1 and 2 we present two example evolutionary paths leading to formation of compact object binaries: black hole neutron star binary and double neutron star systems.

Let us first consider different evolutionary paths of the binaries depending on their initial parameters. We first consider a model with the Cordes and Chernoff (1997) kick velocity distribution. Tracing the evolution now depends on four parameters: \( M \) (primary initial mass), \( q \) (initial mass ratio), \( a \) (initial semi-major axis), and \( e \) (initial eccentricity). In order to visualize the evolutionary effects we fix two of them and present the types of systems obtained in the course of the binary evolution. In Figure 3 we fix the initial orbital separation \( a = 20R_\odot \) and eccentricity \( e = 0.5 \). The graphs are empty in the lower left part for which \( M\times q < 10M_\odot \). This is the region for which the secondary is not massive enough to undergo a supernova explosion. However, in some systems for which the primary mass is \( M > 20M_\odot \), the mass of the secondary is increased by accretion when the primary goes through the giant phase. Most of the systems end up either by disruption in the first supernova explosion or by a spiral in of the neutron star to the secondary (top panels) The remaining systems may be disrupted in the second supernova explosion. The compact object binary population (bottom right panel) is bimodal. The neutron star neutron star binaries are formed from the systems with \( q \) very close to unity, while the black hole neutron star systems are primarily formed when \( q \) is intermediate. Thus the initial distribution of the mass ratio \( q \) has a strong influence on the production rates of these two types of compact object binaries. The number of systems shown in each panel does not correspond to the actual production rates. We present one thousand systems in each panel. The actual calculation produces 51% systems with the secondary not massive enough to undergo a supernova explosion, 40% systems torn after the first explosion, 6.2% of systems merged after the first explosion, 2.7% systems...
torn after the second explosion, and 0.35% of compact object binaries.

In Figure 6 we present the results of the evolution of systems for which we fix the initial primary mass and the initial mass ratio, while varying the the initial orbital parameters $a$ and $e$. The systems with small orbital separations and high eccentricity merge in the early phase of the evolution and are not considered in this paper. Disruption after the first supernova explosion may occur to all systems. The population of systems that end up merging after the first explosion, are disrupted in the second explosion, or form a compact object binary originate from the same region of the parameter space. One should note that because of the circularization of orbits already in the initial stages of the binary evolution the final population is not very sensitive to the initial eccentricity. On the other hand the distribution of the initial orbital separation is important, as the compact object binaries originate in systems.

**Fig. 6.** Distribution of the initial orbital compact binary systems parameters in the space spanned by the period $P$ and eccentricity $e$, for four different Gaussian distributions of the kick velocity $\sigma_v = 0 \text{ km s}^{-1}$ - top left panel, $200 \text{ km s}^{-1}$ - top right panel, $400 \text{ km s}^{-1}$ - bottom left panel, and $800 \text{ km s}^{-1}$ - bottom right panel. Each panel contains 1000 objects.
with relatively small orbital separations (see bottom right panel in Figure 3). One should note that all the compact object binaries shown in Figure 4 are black hole neutron star binaries. Double neutron star systems are formed only when $q$ is nearly unity in our simulations. As before the number of systems shown in each panel does not correspond to the actual production rates, and show one thousand systems in each panel. The actual calculation produces 12% systems born in contact, 43% systems with the secondary not massive enough to undergo a supernova explosion, 34% systems torn after the 1st explosion, 0.7% systems merged after the first explosion, 0.50% systems torn in the second explosion and only 0.15% mergers.

We present the dependence of the production rates of different types of compact object binaries on the width of the kick velocity distribution $\sigma_v$ in Figure 3. The number of double neutron star systems that merge within the Hubble time increases with the kick velocity, while the production rate of black hole neutron star systems becomes smaller. These two rates are nearly equal when the kick velocity is roughly that given by Cordes and Chernoff (1997).

In Figure 3 we present the distributions of the orbital parameters of compact object binaries for a few representative values of the width of the kick velocity distribution $\sigma_v$. Each panel contains one thousand compact object binaries. To understand these plots let us first consider the properties of the population of objects just before the second supernova explosion in the case $\sigma_v = 0$ km s$^{-1}$. Some of the systems are wide, with the eccentricity varying from zero to unity, however, most of the systems populate a region with eccentricities above $\approx 0.45$. These systems originate in binaries with small initial mass ratios. A characteristic property of this group is a correlation between eccentricity and period. Objects in this group originate in systems with small initial value of $q$. The masses of the primary compact objects in these systems have a narrow distribution, the mass of the secondary after the accretion phase weakly depends on the initial mass before accretion - see equation (1) and is typically $M'_2 \approx 2.3 M_\odot$. The mass of the secondary just before the second supernova explosion, is the mass of the helium core of secondary star and is in the range $3.0 M_\odot < M'_2 < 5.5 M_\odot$. The orbital parameters of such system after an explosive mass loss (second supernova explosion) depend only on the total mass loss, see e.g. Pols and Maruris (1994). In our calculation the newly formed neutron star has a mass of 1.4 $M_\odot$, thus the relative mass loss $x = (1.4 + M'_2)/(M'_2 + M'_2)$ is in the range $0.47 < x < 0.69$. Systems that loose more than half of the mass ($x < 0.5$) are unbound. Other systems become eccentric and the eccentricity is given by $e' = (1 - x)/x$. Hence the lowest eccentricity a system can have after the second supernova explosion in our simulations is $e' = 0.44$.

![](image.png)

**Fig. 7.** Cumulative distributions of the $t_{\text{merg}}$ - the lifetimes of compact object binaries for four different kick velocities: the solid line for $\sigma_v = 0$ km s$^{-1}$, the short dashed line for $\sigma_v = 200$ km s$^{-1}$, the long dashed line for $\sigma_v = 400$ km s$^{-1}$, and the dash dotted line for $\sigma_v = 800$ km s$^{-1}$. Each line was plotted from a distribution of 1000 objects.

The relation between the new orbital period $P'$ and the new eccentricity $e'$ for different relative mass loss $x$ is

$$P = P_0 \frac{(1 + e')^{1/2}}{(1 - e')^{3/2}},$$

(12)

where $P_0$ is the orbital period before the explosion. This relation defines the curved shape of the distribution in the $P, e$ diagram. The objects populating the right hand side of the $P, e$ plot, originate from systems with intermediate or large $q$. The intermediate $q$ objects went through accretion onto the neutron star (regime II of a mass transfer described above) and therefore similar reasoning as above applies to them. However systems with high initial value of $q$ results in wide binaries (periods larger than 100 days) with different eccentricities. The compact object binaries in Figure 3 (top left panel) with eccentricities below 0.45 and orbital periods smaller than $10^3$ days originate in binaries of intermediate and large initial mass ratio.

In the case of non zero kick velocity the systems with high $q$ are very unlikely to survive. In fact there are only two such systems on the plot for $\sigma_v = 200$ km s$^{-1}$, and none for higher velocities. The escape velocity in long period systems is low, and if such systems existed after the first supernova event, they have are very likely disrupted in the second supernova explosion.

As the kick velocity is increased the shape of the distribution in the $P, e$ diagram also changes. In the case $\sigma_v = 200$ km s$^{-1}$ there is a small fraction of high ec-
centrivity, low period systems. This region of the parameter space fills up as the kick velocity increases, see lower panel in Figure 4 where the case $\sigma_v = 400 \text{ km s}^{-1}$ and $\sigma_v = 800 \text{ km s}^{-1}$ are shown. This is due to the fact that with the increasing kick velocity the long period systems are easier to disrupt and the fraction of surviving short period systems becomes larger. We note that the distributions shown in Figure 4 are similar to those obtained previously (Portegies Zwart and Spreeuw, 1996; Tutukov and Yungelson, 1993).

In Figure 5 we present the cumulative distributions of the lifetimes of compact object binaries for the same set of kick velocities as in Figure 4. In the case of no kick velocity $\sigma_v = 0 \text{ km s}^{-1}$ the distribution is bimodal: the systems with small $q$ merge typically within the Hubble time (which we take to be 15 Myr). However the systems with higher $q$ remain in wide orbits and their merger times exceed the Hubble time. With the increasing kick velocity only the tightly bound systems survive. The lifetime of a system scales with the fourth power of the semi-major axis, and therefore the median decrease with increasing kick velocity. In the case of $\sigma_v = 200 \text{ km s}^{-1}$ the median lifetime is $\approx 4 \times 10^8$ years, and it decreases by a factor of four when the kick velocity is doubled. For the highest kick velocity velocity $\sigma_v = 800 \text{ km s}^{-1}$ the median lifetime is only $\approx 3 \times 10^7$ years. One should note however, that the distribution of $t_{\text{merge}}$ is very skewed, and we present it on a logarithmic axis. Thus there is a long tail extending to about the Hubble time. Most of the mergers take place in the Hubble time and only a small fraction of the total population lasts longer.

Finally in Figure 6 we present $f_{\text{merge}}$ – the fraction of all binaries with the primary star more massive than $10 M_\odot$ that produce a pair of compact objects in a binary system. We calculate $f_{\text{merge}}$ for a large range of the kick velocity distribution widths. To calculate each point in Figure 6 we calculated one thousand of compact binaries. This fraction falls down very approximately exponentially with the increasing kick velocity. We have fitted a modified exponential to this dependence

$$f_{\text{merge}} \propto \exp(-4.21 - 8.51 \times 10^{-3} \sigma_v + 2.6 \times 10^{-6} \sigma_v^2). \quad (13)$$

The fit is shown by the dashed line in Figure 6 and is accurate to about 6%. Note that equation (13) can be used to determine the merger fraction for any kick velocity distribution that can be expressed as a linear combination of three dimensional Gaussians. Equation (13) can be used together with Figure 5 to obtain the production rates of any type of compact object binaries as a function of the width of the kick velocity distribution.

The actual compact object merger rate in the Galaxy can be calculated given the observed supernova rate, the fraction of stars that exist in binaries, and assuming some form of the star formation history. Assuming that there is one supernova explosion every fifty years in the Galaxy and that binary fraction is 50%, we denote the supernova rate as $0.02 f_{\text{SN}} \text{ year}^{-1}$, and the binary fraction as $0.5 f_{bin}$, where $f_{\text{SN}}$ and $f_{bin}$ are factors of the order of unity.

In the simplest case when we assume that the star forming process has been going at the same rate throughout the history of the Galaxy we obtain the compact object merger rate

$$r = 0.0001 \times f_{\text{SN}} \times f_{bin} \times f_{\text{merge}} \text{ year}^{-1}, \quad (14)$$

where $f_{\text{merge}}$ is expressed in percents. Taking values of $f_{\text{merge}}$ from our Figure 6, we may calculate number of expected compact objects gravitational merging events, which for, let say, $\sigma_v = 100 \text{ km s}^{-1}$ will be 0.0002 per year which yields approximately 1 event per 5000 years per Galaxy.

Since the compact object merger rate is directly proportional to $f_{\text{merge}}$ it also depends exponentially on the kick velocity distribution width! The assumption about the constant star formation throughout the history of the Galaxy is in fact not really crucial. As we have seen most of the mergers take place within a few times $10^8$ years even for rather small velocity kicks, therefore in reality we are only concerned about the star forming history in the last $10^9$ years. The star forming rate in the Galaxy has most probably been constant over the last $5 \times 10^9$ years (Miller and Scalo, 1979).

The calculation of the detection rate in gravitational wave detectors (Abramovici et al., 1992) requires a number of additional assumptions, like for example the galaxy density in our local and far neighborhood etc., for a discussion see e.g. Phinney (1991); Chernoff and Finn...
Regardless of the assumptions the detection rate is proportional to the compact object merger rate in the Galaxy, provided that stellar populations are similar in other galaxies. When calculating the expected rates one has to take into account the masses of the systems that merge. The volume of the space in a flux limited sample of events scales as \( \propto M^{5/2} \) (Tutukov and Yungelson, 1993). Thus mergers of heavy objects like a neutron star and a black hole, or a pair of black holes are visible in a larger volume and may yield a similar observational rate despite the fact that they are not as frequent as the double neutron star mergers.

4. Conclusions

We have modeled the evolution of binary systems using a Monte Carlo code. The results of the simulations are consistent with the results of other codes (Portegies Zwart and Yungelson, 1998; Lipunov et al., 1997; Portegies Zwart and Verbunt, 1996). Using this code we find that the merger rate of compact object binaries and consequently the detection rate in gravitational wave detectors falls approximately exponentially with the width of kick velocity distribution. While the code that we use is far from describing all the details of binary stellar evolution we must emphasize that it produces similar results to the ones obtained elsewhere and in this work we only concentrate on the relative scaling of the resultant merger rate with the kick velocity in a supernova explosion.

The exact shape of the kick velocity distribution is very difficult to measure. Cordes and Chernoff (1997) and Bethe and Brown (1998) use a distribution which is a weighted sum of two Gaussian distributions: 80 percent with the width 175 km s\(^{-1}\), 20 percent with 700 km s\(^{-1}\); Portegies Zwart and Spreeuw (1996) use a Gaussian with the width 450 km s\(^{-1}\). On the other hand Iben and Tutukov (1996) argue that no velocity kicks are required at all, however the lack of pulsars in wide binaries suggests that at least a small kick of a few tens of km s\(^{-1}\) must be present (Portegies Zwart et al., 1997). Thus, the velocity kick determination remains uncertain. Consequently the detection rate estimates in gravitational wave detectors may be uncertain by this amount. Approximating the kick velocity distribution by a single Gaussian profile and changing its dispersion, we calculated the merger rate for a wide range of velocity kicks. Changing the width of the assumed profile within the values proposed by other authors, namely from \( \sigma_{\text{min}} = 0.0 \) to \( \sigma_{\text{max}} = 500 \) km s\(^{-1}\) results in decrease of the merger rate by a factor of 30. The expected number of compact object mergers varies by more than an order of magnitude when the kick velocity goes from 200 km s\(^{-1}\) (the value preferred in population studies) to 500 km s\(^{-1}\) (the measured in the observed population of pulsars).

The measurements of gravitational wave signals may thus allow some determination of the kick velocities. The rates and also perhaps measurements of the "chirp" masses, \( M = \mu M_\odot \) where \( \mu \) and \( M \) are the reduced and total mass of binary system (Chernoff and Finn, 1993), and their distribution will pose yet another constraint on the stellar evolution.

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