Vortex matter in $U(1) \times U(1) \times \mathbb{Z}_2$ phase-separated superconducting condensates

Julien Garaud$^{1,2}$ and Egor Babaev$^1$

$^1$Department of Theoretical Physics, Royal Institute of Technology, Stockholm, SE-10691 Sweden
$^2$Department of Physics, University of Massachusetts Amherst, MA 01003 USA

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We study the properties of vortex solutions and magnetic response of two-component $U(1) \times U(1) \times \mathbb{Z}_2$ superconductors, with phase separation driven by intercomponent density-density interaction. Such a theory can be viewed arising from the breakdown of SU(2) symmetry by a bi-quadratic interaction between the components of the field. Depending on the symmetry breaking term, there are two ground-state phases. One where both components of the doublet are equal (miscible state) and one where only one component assumes non-zero expectation value (immiscible state). In the latter phase, the spectrum of topological excitations contains both domain walls and vortices. We show the existence of another kind of excitation that has properties of both topological excitations at the same time. They combine vorticity together with a circular domain walls, interpolating between inequivalent broken states, that shows up as ring of localized magnetic flux. Asymptotically, this resembles a vortex carrying multiple flux quanta, but because the magnetic field is localized at a given distance from the center this looks like a pipe. The isolated multi-quantum pipe-like vortices can be either stable or meta-stable, even if the system is not type-I. We also discuss the response of such a system to an externally applied magnetic field.

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In the recent years, there have growing interest in models of superconductivity described by more than one superconducting condensate. This interest follows from the growing number of known material that are described by multiple condensates. One could mention multi-band superconductors such as MgB$_2$ or iron based superconductors. Also multi-component models apply to describe unconventional superconductors such as Sr$_2$RuO$_4$ that is an exotic superconductors with chiral $p_x + ip_y$ pairing symmetry or heavy fermion compounds such as UPt$_3$.

The macroscopic physics of multi-component superconductors is described by Ginzburg-Landau free energy with multiple condensates. That is, a field theory of multiple complex scalars charged under the same U(1) gauge field. There, new physics that have no counterpart in single component systems arises. This comprises vortices carrying fractional amount of flux quantum or non-monotonic intervortex interactions originating in the additional length scales associated with the extra condensates, for a review see.

Multi-component models with bi-quadratic density-density interaction are discussed for example in the context of superconductors with pair density wave order or in the context of interface superconductors such as SrTiO$_3$/LaAlO$_3$. Here we investigate the properties of topological defects in two-component models, in an immiscible phase where there is strong bi-quadratic interaction between condensates that penalizes coexistence of both condensates. This is modeled by a field theory of a doublet of complex fields that have a $U(1) \times U(1) \times \mathbb{Z}_2$ symmetry. In the immiscible case that occurs for strong bi-quadratic interaction, the ground state spontaneously breaks a $U(1) \times \mathbb{Z}_2$ part of the symmetry of the theory.

We show that despite only one condensate exists in the ground state, the topological defects’ physics is dramatically altered because of the existence of the suppressed condensate. Depending on the values of the symmetry breaking term, two ground-state phases with different broken symmetries are found. In the first phase, the ground state spontaneously breaks the $U(1) \times U(1)$ part of the symmetry and both components of the doublet are equal and non-zero. In the second phase, only one component assumes non-zero ground state density and the ground state spontaneously breaks $U(1) \times \mathbb{Z}_2$. There, the $U(1)$ part associated with the vanishing condensate, is unbroken. Here, we are principally interested in the latter phase, where the spectrum of topological excitations features both domain walls and vortices.

We demonstrate that in this phase, the coexistence of both kind of topological defects gives interesting defects that are vortices, but comprising a domain wall as well. That is, it resembles asymptotically to a vortex carrying multiple flux quanta, but the magnetic field is localized along a circular domain wall at a given distance from the center. The overall object looks like a pipe. We thus refer to these configurations as pipe-like vortices, in analogy with the discussion of pipe-like vortices in current-carrying two-component condensates.

Below, we introduce the simple two-component Ginzburg-Landau model that have $U(1) \times U(1) \times \mathbb{Z}_2$ symmetry. We then characterize the different possible ground-state phases of that model. Finally, we numerically investigate the properties of vortices within the phase where both components cannot coexist. There we demonstrate that there exist a regime, where pipe-like vortices form and they are stable. We eventually discuss the response to an external applied magnetic field.

The model considered here is a Ginzburg-Landau model of two charged condensates described by two com-
plex fields $\psi_1$ and $\psi_2$. These can be cast into a single complex vector $\Psi$, as $\Psi^1 = (\psi^1_1, \psi^1_2)$. The theory can be written as a theory with a global SU(2) symmetry that is explicitly broken by an extra inter-component term:

$$\mathcal{F} = \frac{B^2}{2} + \frac{1}{2} |D\Psi|^2 + \frac{\Lambda}{2} (\Psi^1 \Psi - \Psi^2 \bar{\Psi})^2 + \delta |\psi_1|^2 |\psi_2|^2. \quad (1)$$

In addition to the coupling to the vector potential $A$ of the magnetic field, through the kinetic term $D \equiv \nabla + ieA$ (and $|D\Psi|^2 := (D\Psi)^\dagger D\Psi$), the two condensates interact through the inter-component bi-quadratic density interaction ($\Lambda + \delta) |\psi_1|^2 |\psi_2|^2$. The theory is thus invariant under local U(1) transformations $A \rightarrow A - \nabla \chi$ and $\psi_a \rightarrow e^{i\chi} \psi_a$, for arbitrary $\chi(x)$. The potential has an SU(2) symmetry that is explicitly broken by the last term when $\delta \neq 0$. When $\delta = 0$, the theory is sometimes called semilocal SU(2) $\times$ U(1) since it has a both a global SU(2) symmetry and a local U(1) symmetry group. The theory is thus invariant under the discrete operation that permutes both condensates $\psi_1 \leftrightarrow \psi_2$.

The U(1) $\times$ U(1) $\times$ Z2 symmetry of the theory is spontaneously broken by the ground state. Depending on the symmetry breaking parameter $\delta$, there are two different phases of the model (1). When $\delta < 0$, both condensates have the same density $|\psi_1| = |\psi_2| = \Lambda \Psi_0 / (\delta + 2\Lambda)$. We call this regime the A-phase, or miscible regime. In the B-phase, the immiscible regime we are principally interested in, only one condensate has non-zero density $|\psi_1| = |\psi_2| = (\Psi_0, 0)$ or $(0, \Psi_0)$ and it is stable when $\delta > 0$. This is summarized in Fig. 1. In the higher symmetry state where the SU(2) symmetry is not explicitly broken ( $\delta = 0$), only the total density is fixed $|\psi_1|^2 + |\psi_2|^2 = \Psi_0^2$ and there is a continuous degeneracy to choose the relative density.

Symmetrywise both phases are different. In the A-phase, the ground state breaks the U(1) $\times$ U(1) symmetry and since it is invariant under permutation of $\psi_1$ and $\psi_2$, it has an unbroken Z2 symmetry. On the other hand, the B-phase spontaneously breaks the U(1) $\times$ Z2 part. The unbroken U(1) symmetry is associated to the condensate that has zero density. There, the topological defects associated with the broken Z2 symmetry are domain-walls interpolating between the two inequivalent ground states $(\Psi_0, 0)$ and $(0, \Psi_0)$. On the other hand, vortices are the topological defects associated with broken U(1) symmetries. In two spatial dimensions, closed domain walls are topologically trivial and thus collapse for dynamical reasons. We show below that interaction with vortices can change that behaviour.

We consider field configurations varying in the xy plane and assume invariance with respect to translations along the z axis. To investigate the properties of topological defects, we numerically minimize the free energy (1) within a finite element framework. That is, for a given choice of parameters, a starting configuration with the desired winding is created and the energy is minimized with a non-linear conjugate gradient algorithm. The results of these simulations for vortices in the B-phase, for small $\delta$, are shown in Fig. 2. First, consider a configuration carrying a single flux quantum. There, the component that have non-zero ground state density $\psi_1$ forms a vortex. At the core of $\psi_1$, because there is less density, it becomes beneficial to give a non-zero value to $\psi_2$. The suppressed condensate $\psi_2$ condenses in the vortex core. The corresponding interface energy is positive, so it is preferable to minimize it.

For multiple quanta configurations, the is a competition between the type-2-like repulsion originating in vortices of $\psi_1$ and the attraction to minimize the interface energy of $\psi_2$ that condenses in the core. For small $\delta$, it is always preferred to form a bound state of vortices in order to minimize the interface energy. It results in a circular domain at the center of which $\psi_1 = 0$ and the condensate $\psi_2 = \Psi_0$. At a certain distance depending on the number of enclosed flux quanta, $\psi_1$ recovers its ground state density while $\psi_2$ is completely suppressed. Thus there is a circular domain wall while outgoing from the vortex center. Since the ground state is realized both at the center and asymptotically, the magnetic field is screened everywhere except at the domain wall. The overall configuration looks like a pipe, thus we refer to this as a pipe-like vortex. Pipe-like vortices are different from type-1 multi-quanta vortices for which the magnetic field is non zero at the center. The pipe-like vortices we find here somehow remind pipe-like solutions found for vortex configurations carrying persistent longitudinal current, in the $\delta = 0$ case where the model has a global symmetry $\times$ U(1) $\times$ U(1) $\times$ Z2.
SU(2) symmetry.\textsuperscript{11} The reason for screening and the flux localization along the pipe is further discussed later in the paper.

At this point, it is important to recall that the B-phase has domain wall excitations that interpolate between $(\Psi_0,0)$ and $(0,\Psi_0)$. Thus the interface previously mentioned is exactly such a domain wall. For small $\delta$, they have very small energy and their energy increases while going deeper in the B-phase. There, we can expect a change of behaviour because domain walls are more energetic and condensation in the vortex core becomes much smaller. We find that indeed, deeper in the B-phase, isolated vortices becomes preferred from pipe-like vortices. Nevertheless, pipe-like vortices may be constructed here but they are metastable. That is, they still can exist but only as local minima of the energy functional. We find that typically configurations carrying a small number of flux quanta easily decay into vortices during the relaxation process, as shown on the second line. Solutions with larger $N$ form but their energy is larger than the one of $N$ isolated vortices. Note that the energy difference becomes smaller as the number flux quanta increases.

In substantially strong external field, vortex matter usually forms dense lattices. The previous results for isolated vortices inform about the low field physics. There, as discussed above, vortex configuration show very interesting structure comprising between the two kind of topological defects that the theory allows. This may result in quite unusual properties of the solutions in high field. To investigate this, we simulate the response of the system to an external field $\mathbf{H} = H_z \mathbf{e}_z$ perpendicular to the plane. For this, the Gibbs free energy $\mathcal{G} = \mathcal{F} - \mathbf{B} \cdot \mathbf{H}$ is minimized, with requiring that $\nabla \times \mathbf{A} = \mathbf{H}$ on the boundary.\textsuperscript{14} Note that since it is a finite sample with boundary Meissner currents the total flux through the sample does not have to be quantized, even in the standard vortex state.

We start by considering solutions in external field, at the boundary between A- and B- phases. At the point $\delta = 0$ of the phase diagram, where the theory has the SU(2) symmetry, there are no stable type-2 vortices.\textsuperscript{2,15,16} Note that these vortices can be stabilized by having a twist of the phase in the $z$ direction.\textsuperscript{17,18} This is somehow akin to have a symmetry breaking term in the potential. Only short pieces of such twisted vortices are stable as they develop an unstable mode similar to hydrostatic Plateau-Rayleigh instability.\textsuperscript{19} There are no stable isolated vortices in the theory with SU(2) symmetry, does not means that its response in external field is trivial. Indeed, isolated vortices exhibit the spreading

| $N$ | $|\psi_1|^2$ | $|\psi_2|^2$ |
|-----|-------------|-------------|
| 1   | 1.0x10$^3$  | 6.9x10$^3$  |
| 2   | 7.1x10$^3$  | 5.4x10$^3$  |
| 3   | 6.3x10$^3$  | 5.6x10$^3$  |
| 4   | 6.1x10$^3$  | 4.8x10$^3$  |

| $N$ | $|\psi_1|^2$ | $|\psi_2|^2$ |
|-----|-------------|-------------|
| 1   | 3.6x10$^3$  | 2.7x10$^3$  |
| 2   | 3.6x10$^3$  | 2.7x10$^3$  |
| 3   | 2.0x10$^3$  | 1.5x10$^3$  |
| 4   | 1.5x10$^3$  | 1.0x10$^3$  |

Figure 2. (Color online) – Vortex solutions in the B-phase of Fig. 1, for the symmetry breaking parameter $\delta = 0.02$. The first column shows the magnetic field and second and third column respectively show the densities $|\psi_1|^2$ and $|\psi_2|^2$. The different lines show configurations with different vorticity $N = 1, 2, 3, 4$ respectively. In the B-phase, only one condensate has non-zero ground state density. Here this is $\psi_1$, while $\psi_2$ vanishes asymptotically. Despite not being in a type-1 regime, objects carrying multiple flux quanta are formed. The essential difference from type-1 vortices in that here the magnetic field vanishes at the vortex center.

Figure 3. (Color online) – Deep into the B-phase, vortex structures are substantially different from those obtained for small $\delta$. Here, displayed quantities are the same as in Fig. 2 and the symmetry breaking parameter is $\delta = 0.4$. It is no more beneficial to condense $\psi_2$ inside the vortex core. As a result, vortices are simply ordinary vortex solutions embedded in the $U(1) \times U(1) \times \mathbb{Z}_2$ theory.\textsuperscript{(1)} Nevertheless, pipe-like vortices may be constructed here but they are metastable. That is, they still can exist but only as local minima of the energy functional. We find that typically configurations carrying a small number of flux quanta easily decay into vortices during the relaxation process, as shown on the second line. Solutions with larger $N$ form but their energy is larger than the one of $N$ isolated vortices. Note that the energy difference becomes smaller as the number flux quanta increases.
Figure 4. (Color online) – Solution in an external field in the very special case of an SU(2) symmetric potential ($\delta = 0$) and the external field corresponds to 274 flux quanta going through the sample’s area. The other parameters are the same as in Fig. 2. The panels on the first row display the magnetic field and the phase difference $\phi_{12} = \phi_2 - \phi_1$. The second line shows the densities $|\psi_1|^2$ and $|\psi_2|^2$ respectively. Here, although isolated vortices are unstable, they nonetheless form in the external field.

Figure 5. (Color online) – Solution in an external field in the B-phase, for the same parameters as in Fig. 2. There the displayed quantities are the same as in Fig. 4 and the external field corresponds to 313 flux quanta going through the sample’s area.

Figure 6. (Color online) – Solution in an external field, corresponding to 175.6090 flux quanta going through the sample’s area. The parameters are the same as in Fig. 2, but slightly deeper into the B-phase $\delta = 0.2$. Displayed quantities are the same as in Fig. 4. We see that there are alternating regions populated by vortices of different condensates. These alternating regions correspond (approximately) to the two inequivalent ground states in the B-phase and thus they are separated by a domain wall that carries flux as in Fig. 2. The second panel showing the phase difference tells about position of singularity in both condensates.

instability in the SU(2) case. However, there is additional constraint in external field. In Fig. 4, we show that indeed the magnetic response is non-trivial. There are lines of vortices in a given condensate alternating with lines of vortices in the other one. Within a line vortices pair and form some kind of dimer. Note that finite size effects and interaction with Meissner currents here play some role in deforming the lattice structure. That is, a perfect lattice can form only for tuned domains with periodic boundary conditions. The results in Fig. 4 should be understood as a typical state which would form in experiment, in mesoscopic samples.

For small values of $\delta$, the behaviour in external field mixes the behaviour reported for the SU(2) symmetry in Fig. 4 and those deep into the B-phase shown Fig. 6. In this regime, displayed in Fig. 5, the dimers start to merge together. Here again, finite size effects and interaction with Meissner currents makes it very difficult to have a perfect lattice in this kind of simulation. It is interesting to note, that even when the stability properties of the topological excitations are completely different, then the magnetic response can be quite similar to the one at the point with the SU(2) symmetry.

Deeper into the B-phase, the response to an external applied field starts to be quite different from those closer to the $\delta = 0$ point. In Fig. 6, we show that there are alternating regions populated by vortices of different condensates. These alternating regions correspond (approximately) to the two inequivalent ground states in the B-phase. These regions are separated by a domain wall that carries flux as in Fig. 2. The domain walls form some kind of spiral covering the whole area of the sample. Such a pattern indicates that there is an important interplay between the two kind of topological defects that the theory supports.

In Fig. 3, we showed that deep into the B-phase, isolated vortices are preferred to pipe-like vortices. The
The component is as SU(2) potential supplemented by a term that explicitly density interaction. It can be seen as model having an of two-component superconductor that have density-
ties of topological defects in a Ginzburg-Landau model seen from Fig. 8.

The reason why the magnetic flux is localized along the domain wall can be understood as follows. From Ampère’s law $\nabla \times B + J = 0$, the total supercurrent reads as $J = e\text{Im}(\Psi^\dagger D\Psi)$ and the contribution due to each component is $J_d = e\text{Im}(\psi_n^\dagger D\psi_n)$. For pipe-like vortices, the supercurrent of both components flow in opposite directions. $J_2$ due to $\psi_2$ screens the magnetic field inside the domain, while $\psi_1$ is responsible for screening in the exterior. As a result, the only region where the magnetic flux penetrates is the domain wall. The structure of the superconducting currents in the pipe-like vortices can be seen from Fig. 8.

In this article, we investigated the physical properties of topological defects in a Ginzburg-Landau model of two-component superconductor that have density-density interaction. It can be seen as model having an SU(2) potential supplemented by a term that explicitly breaks it down to $U(1) \times U(1) \times \mathbb{Z}_2$. Depending on the symmetry breaking parameter $\delta$, this model has two physically different phases. When $\delta > 0$, the discrete $\mathbb{Z}_2$ part of the symmetry is spontaneously broken, while when $\delta < 0$ it is not. We have been focusing on the former phase where condensates cannot coexist and the

ground state is either ($\Psi_0, 0$) or (0, $\Psi_0$). There, two kind of topological defects are possible: domain wall or vortices.

We have shown that, form small symmetry breaking term, vortices form bound state carrying multiple quanta of flux and exhibit properties of domain walls at the same time. The resulting configuration is a cylindrical inclusion of the component $\psi_2$ inside a whole domain where $\psi_1 = \Psi_0$. The interface between both regions is a (cylindrical) domain wall where the flux is localized and that resembles to a pipe. These pipe-like vortices are stable only for small $\delta > 0$. However, deeper in the phase separated regime, they can still exist, but they are meta-stable.

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* garaud.phys@gmail.com

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