Calculation of concentration fields of high-inertia aerosol particles in the flow past a cylindrical fibre

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Abstract. The behaviour of high-inertia aerosol particles’ concentration fields in stationary gas suspension flows around a cylinder is investigated using a numerical solution to the Navier-Stokes equations and the fully Lagrangian approach for four Stokes numbers (Stk = 0.1, 1, 4, 10) and three Reynolds numbers (Re = 1, 10, 100). It has been shown that the points of maximum particle concentration along each trajectory shift downstream both when Stk and/or Re increase.

1. Introduction

The calculation of concentrations of high-inertia aerosol particles in complex gas-suspension flows is an integral part of the modelling of various engineering processes and natural phenomena, including the processes in internal combustion engines and the transfer of anthropogenic air suspensions (aerosols). The spatial distribution of the concentration of particles is required to determine the intensity of heat and mass transfer between the carrier and dispersed phases and the rates of chemical reactions.

There are two groups of methods for calculating the concentration of suspended aerosol particles: Eulerian-Eulerian and Eulerian-Lagrangian. The motion of the carrier phase in both groups is described by the continuity and Navier-Stokes equations written and solved in Eulerian coordinates. Within the first group of methods, the equations of continuity and motion of a dispersed phase are also solved in Eulerian coordinates [1, 2]. Particular interest has been focused on the method of moments [3], based on finding the moments of the distribution function of particles in several phase variables (dimensions, velocities, temperature, composition). Among the disadvantages of the Eulerian-Eulerian methods are the high computational cost and the need to specify the boundary conditions for the dispersed phase.

Within the framework of the second group of methods, the equations of motion of the dispersed phase are solved for individual particles, and trajectories of particle motion are found. The concentration of the dispersed phase in the Eulerian cell is found by counting the number of particles in it at every time step. It was shown in [4] that in order to obtain reliable results each cell should contain about 10,000 particles even in the two-dimensional case. If the mesh is very fine, then the computational cost of the problem becomes very high.

In [5, 6] alternative methods were suggested, based on the solution to the continuity equation of a dispersed phase along fixed trajectories. These are called fully Lagrangian approaches, and are much cheaper computationally, as the particle concentration is calculated based on the mass conservation...
equation. In [5], the continuity equation of a dispersed phase along the trajectory is solved using the Eulerian formulation. It was shown in [4] that this approach could not be used in the points/regions with infinitely high particle concentration, which are formed in flows with crossing particle trajectories.

In [6], the equation of continuity for a dispersed phase along a trajectory is written in Lagrangian coordinates, and the problem of finding the concentration is reduced to that of finding the components of the Jacobi matrix of the Eulerian-Lagrangian coordinate transformation. An important advantage of this method is that it can be applied to cases with singularities in particle concentration fields, where the number density becomes infinitely high.

In our previous paper [7] we presented the results of implementation of the fully Lagrangian approach (FLA), proposed in [6], into the ANSYS Fluent CFD package. The results of the investigation of the distribution of the low inertia particle concentrations in a stationary gas-suspension flow around a cylindrical fibre were presented and compared with those of a variant of the Eulerian-Eulerian approach. In this paper our numerical study is focused on the distribution of concentration of high inertia particles.

2. Mathematical model
Let us consider an isothermal, dilute gas-suspension flow around an infinitely long circular cylinder. As a carrier phase, we take the laminar flow of a viscous incompressible fluid (gas), described by the continuity and Navier-Stokes equations. We assume that the volume and mass fractions of the particles are small, which allows us to ignore the interaction between the particles and their influence on the carrier phase. The particles are assumed to be spherical, and the equation of particle motion is written in the framework of the Stokes drag law. The forces of buoyancy and the added masses are ignored remembering that the density of the particle material is much higher than the density of the carrier phase. Also, we assume that the particles are sufficiently large to allow us to ignore the contributions of the Brownian forces. The equations of motion of inertial particles in this case are written as

$$\frac{dx}{dt} = v,$$

$$\frac{dv}{dt} = \frac{1}{\text{Stk}}(u - v),$$

where $x$ and $v$ are the position and velocity vectors of a particle in the Eulerian coordinates, $u$ is the velocity vector of the carrier phase. The Stokes number Stk is determined as

$$\text{Stk} = \frac{\rho_p d_p^2 U_0}{18 \mu L},$$

where $d_p$ are the diameter and density of the particle material, respectively, $\mu$ is the viscosity of the carrier phase, $U_0$ and $L$ are the characteristic velocity and length.

The equation of continuity for a dispersed phase along a chosen trajectory is written in the form

$$n_p = \frac{n_0 |J|}{\mu},$$

where $n_0$ is the concentration of particles at the starting point of the trajectory. To find the components of the Jacobi matrix $J$, it is necessary to solve the following system of ordinary differential equations along the chosen trajectory [6]

$$\frac{\partial J_{ij}}{\partial t} = \frac{\partial v_i}{\partial x_{i0}} = \omega_{ij},$$

(2)
\[
\frac{\partial \omega_{ij}}{\partial t} = \frac{1}{\text{Stk}} \left( \sum_k \left( J_{kj} \frac{\partial u_i}{\partial x_k} \right) - \omega_{ij} \right).
\]

The continuity and Navier-Stokes equations for the carrier phase are solved by ANSYS Fluent using the finite volume method. The particle trajectories are calculated by solving Equations (1) using the Runge-Kutta method. Equations (2) for the components of the Jacobi matrix of the Eulerian-Lagrangian coordinate transformation are solved along the trajectories using the Runge-Kutta method.

The geometry of the computational domain for a cylinder of radius \( d_c \) is shown in figure 1. The flow of the carrier phase far from the cylinder was assumed to be uniform. The velocity of the carrier phase \( u = U_0 \) was set at the left, upper and lower boundaries of the domain. On the right boundary, gauge pressure drop was set to zero.

![Figure 1. Geometry of the computational domain.](image)

3. Results

Particle concentrations were calculated along 100 trajectories starting at the left boundary of the domain for four Stokes numbers (Stk = 0.1, 1, 4, 10) and three Reynolds numbers of the flow (Re = 1, 10, 100). In figure 2, individual points of the trajectories are shown for four values of Stk at a fixed Re = 10. Points are coloured in shades of grey depending on the concentration of particles: lighter shades correspond to lower concentrations, darker shades correspond to higher concentrations. It can be seen that the region of high concentrations shifts downstream as the Stk increases.

In Figure 3, the curves connecting the points of greatest concentration along each trajectory are shown. It can be seen that these curves shift to the right both with increasing Stk and with increasing Re. The maximum concentration along the trajectories of the deposited particles is always achieved at the point of deposition. Points of the global maximum of the concentration for each Stk are marked by circles. For high-inertia particles (Stk > 1) these points move downstream when Stk and/or Re increase. The global maximum of the concentration of low-inertia particles (Stk < 0.2) is located at the deceleration (stagnation) point in front of the cylinder for all three Re.
Figure 2. Trajectories and the distribution of particle concentrations at Re = 10 for four Stokes numbers (Stk = 0.1 (a), Stk = 1 (b), Stk = 4 (c), Stk = 10 (d)). The points of the trajectories are coloured depending on the concentration of particles.

Figure 3. Curves of the greatest concentration along each trajectory for Re = 1 (a), Re = 10 (b), and Re = 100 (c).
4. Conclusions
The behaviour of the particle concentration in steady-state gas-suspension flows around a cylinder has been investigated for four Stokes numbers (Stk = 0.1, 1, 4, 10) and three Reynolds numbers (Re = 1, 10, 100). It has been shown that the points of the maximum concentration along each trajectory shift downstream when Stk and/or Re increase.

Acknowledgements
This work was supported by RFBR grants (16-51-10024 and 15-01-06135) and the Royal Society (UK) (grant IE 160014).

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