Prediction of crack growth direction by Strain Energy Sih’s Theory on specimens SEN under tension-compression biaxial loading employing Genetic Algorithms

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Abstract. Crack growth direction has been studied in many ways. Particularly Sih’s strain energy theory predicts that a fracture under a three-dimensional state of stress spreads in direction of the minimum strain energy density [1]. In this work a study for angle of fracture growth was made, considering a biaxial stress state at the crack tip on SEN specimens. The stress state applied on a tension-compression SEN specimen is biaxial one on crack tip, as it can observed in figure 1.

A solution method proposed to obtain a mathematical model considering genetic algorithms, which have demonstrated great capacity for the solution of many engineering problems. From the model given by Sih one can deduce the density of strain energy stored for unit of volume at the crack tip as

\[ dW = \frac{1}{2E} \left( \sigma_x^2 + \sigma_y^2 \right) - \frac{\nu}{E} \left( \sigma_x \sigma_y \right) dV \]  

(1)

From equation (1) a mathematical deduction to solve in terms of \( \theta \) of this case was developed employing Genetic Algorithms, where \( \theta \) is a crack propagation direction in plane x-y. Steel and aluminum mechanical properties to modelled specimens were employed, because they are two of materials but used in engineering design. Obtained results show stable zones of fracture propagation but only in a range of applied loading.

1. Introduction

In many works developed by Sih [2-4] he has proposed a theory of fracture based on the field strength of the local strain energy density which marks a fundamental departure from the classical and current
concepts. The theory requires no calculation on the energy release rate and thus possesses the inherent advantage of being able to treat all mixed mode crack extension problems for the first time. Unlike the conventional theory of $G$ and $k$ which measures only the amplitude of the local stresses, the main parameter in Sih’s Energy Theory, “strain energy density” factor $S$, is also direction sensitive. The difference between $k$ (or $G$) and $S$ is analogous to the difference between an scalar and a vector.

In accordance with the Griffith-Irwin theory if can be viewed as a scalar theory in that it specifies only the critical value of a scalar $G_{1c}$ (or $K_{1c}$) at incipient fracture. The direction of crack propagation is always preassumed to be normal to the load. Moreover, the crack front must be straight in such a way that $G$ or $k$ does not vary along the leading edge of the crack. In addition, a scalar theory cannot yield the correct material parameter when two or more stress intensity factors are present along the crack border. The $S$-factor in the Sih theory behaves like a director: It senses the direction of least resistance by attaining a stationary value with respect to the angle $\theta$. As it will be shown, the stationary value of $S_{\text{min}}$ can be used as an intrinsic material parameter whose value at the point of crack instability $S_{cr}$ is independent of the crack geometry and loading state.

Practically all the engineering structures contain imperfections or defects that can become fractures. In this context, it is very important to know the integrity of a structure the way it affects the propagation of a crack and determine the magnitude of the damage when this condition takes place. In an investigation, Rodriguez and collaborators [5] showed the relationship that exists among the dimensions of a crack SEN specimen and the biaxial loading condition applied. In that work results were obtained that relate the scalar factor $k$ as being dependent of the geometric and loading conditions. On the other hand, in the development of this work very interesting results are shown in relation to the already mentioned paper.

2. Study cases
In the development of this investigation, specimens $SEN$ were considered (Figure 1), which have geometry and loading conditions indicated in Table I. In all cases only opening load was changed keeping constant all the other parameters.

Figure 1 Model of cracked plate ($SEN$ specimen). a) Geometry and dimensions, b) Stress state at the crack tip.
As it was mentioned previously, the mechanical properties of materials employed, correspond to steel and aluminum (Table 2)

### Table 1. Cases of Study

| \( P_x (N) \) | \( P_y (N) \) | \( L (m) \) | \( w (m) \) | \( a (m) \) |
|---|---|---|---|---|
| 1000 | 1000 | 0.1016 | 0.1016 | 0.005 |
| 2000 | | | | |
| 3000 | | | | |
| 4000 | | | | |
| 5000 | | | | |
| 6000 | | | | |
| 7000 | | | | |
| 8000 | | | | |
| 9000 | | | | |
| 10000 | | | | |
| 11000 | | | | |

### Table 2. Mechanical Properties of Specimens

| Mechanical Properties | Material |
|-----------------------|----------|
| Steel \( E \) (GPa)   | Aluminum |
| 250                   | 50       |
| Poisson’s ratio \( \nu \) | 0.29 | 0.33 |

3. Deduction of local energy density due to a tension-compression biaxial stress state

According to the Sih’s theory [6] for an elastic material, the strain energy stored in the element \( dV = dx \ dy \ dz \) under a biaxial stress state is given by equation (1). In this case, the stresses \( \sigma_x \) and \( \sigma_y \) are given by

\[
\sigma_x = -\frac{k_1}{2r^{1/2}} \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)]
\]  

(2)

\[
\sigma_y = -\frac{k_1}{2r^{1/2}} \sin(\theta/2)[\sin(\theta/2)\sin(3\theta/2) + 1]
\]  

(3)

also, strain energy density function is obtained by means of

\[
\frac{dW}{dV} = \frac{1}{r} \left[ a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 \right] + ...
\]  

(4)

and \( S \) factor is given by

\[
S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2
\]  

(5)
In this case \( k_2 = k_3 = 0 \) and \( k_1 = \sigma a^{1/2} \).

Substituting (2) and (3) into (1) and (5) and finished differentiating with respect to \( \theta \) and setting \( \frac{\partial S}{\partial \theta} = 0 \), mathematical model is obtained to prediction of crack growth direction, who is

\[
\frac{\partial S}{\partial \theta} = \frac{\sigma^2 a}{4E} \left[ 2 \cos(\theta/2) \sin^2(\theta/2) \sin^3(\theta/2) + 3 \sin(3\theta/2) \cos(\theta/2) \sin(\theta/2) + 3 \sin(\theta/2) \cos(\theta/2) \cos(\theta/2) + \sin(\theta/2) \cos(\theta/2) \sin(\theta/2) \right] - \frac{v \sigma^2 a}{2E} \left[ - \frac{1}{2} \sin^2(\theta/2) \sin^4(\theta/2) + 3 \sin(\theta/2) \cos(\theta/2) \cos(\theta/2) \sin(\theta/2) \right] + \frac{3}{4} \sin^2(\theta/2) \sin^2(3\theta/2) \cos^2(\theta/2) + \frac{1}{2} \sin^2(\theta/2) - \frac{1}{2} \cos^2(\theta/2) \right]
\]

Equation (6) is expressed in terms of crack growth angle \( \theta \), but to resolve it is necessary a computational method. In this case a procedure based on Genetic Algorithms was employed.

4. Genetic Algorithm description [7]
A typical genetic algorithm starts with a randomly generated population composed by genes, locus, allele, chromosome, genotype, variables and phenotype [8, 9, 10] (Figure 2). Individuals are probabilistically selected by evaluating an objective function. This gene has converged when at least 95% of individuals in the population share the same value for that gene. It is said then that the population converges when all the genes have converged. The steps to make a genetic algorithm, as defined in [8], are shown in the diagram of Figure 3.

![Figure 2 Chromosome binary representation](image)

*Initial Population* is created randomly and it is encoded within the chromosome of an array with variable length. The coding can be done in a binary representation [8, 11], based on the domain of each variable (Figure 4).
5. Solution of study cases

By applying the genetic algorithm developed to solve the equation 6, results shown in the figure 5 were obtained. It can be observed that a wide area of stability of fracture propagation exists.
On the other hand tests carried out with aluminum specimens throw the results are shown in figure 7.

Figure 5 Fracture propagation angle under variable loading on steel specimen

Figure 6 Fracture propagation angle under variable loading on aluminum specimen

6. Discussion
In the proposed study cases it is clearly shown that the test specimens have a square geometry with a constant and very small crack length (5 mm), which was made in a way to observe the crack behavior trajectory when a small fracture is initiated. The application of big loads was made to observe firstly the effect due to of strength of the employed material, their mechanical properties and mainly to study the behavior of the crack when they initialize and the loads suddenly vary. Also, the Energy Sih’s Theory seemingly coincides only in a limited range of loads, with results reported in other works [5, 12, 13, 14] and mainly with the hypothesis that the compressive axial load stabilizes the direction of fracture propagation in a considerable way when variable opening loads are applied. Also the genetic algorithm developed, sample the capacity of the solution in engineering problems of this type, in this case applied to fracture mechanics.

7. Conclusions
When observing the results obtained by means of application of Genetic Algorithms to resolve fracture propagation problems, very interesting conclusions were obtained, which can be enunciated in the following way:
1. For steel specimens crack initiation occurs at negative angle it initially takes place at 1000 N opening load which suggest crack direction instability.
2. It can be seen in Figure 5 that between 2000 to 10,000 N of opening load, the crack propagation angle is stable.
3. In Figure 5 as well, it can be observed a drastic change for the opening load between 10,000 to 11,000 N. From this it is apparent that this crack becomes unstable.
4. The behaviour graphic for aluminum specimens and those for steel are similar, even though those for steel are more unstable. However these have a zone between 3000 to 10,000 N, in which crack propagation is stable.
5. For steel specimens in the interval 10,000 to 11,000 N a drastic change is introduced on the direction of the crack propagation angle, these suggest that this is a zone of crack instability.

8.- Acknowledgement
The authors gratefully acknowledge the financial support from the Mexican government by de Consejo Nacional de Ciencia y Tecnología and the Instituto Politécnico Nacional, Escuela Superior de Ingeniería Mecánica y Eléctrica.

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