Robust parameter design of inventory policy considering the risk preference of decision makers

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Abstract. Aiming at reducing the risk in inventory management of supply chain system suffering from the influence of uncertainties, we propose a strategy to analyse the optimal inventory policy considering different risk preference of decision makers. The proposed method is a combination of the mean-conditional value-at-risk and the response surface model. From the perspective of robust optimization, the proposed method is adopted to solve the inventory control problems in a multi-echelon supply chain system, the effectiveness of the proposed method is verified in the simulation model. Furthermore, the comparison analysis between the proposed method and the dual-response surface method is conducted in this paper, and the comparison results shows the superiority of the proposed method in improving the robustness of supply chain system.

1. Introduction

With the integration and standardization of enterprises’ product or service, the competition among enterprises has increasingly evolved into the competition among supply chains. In supply chain management, inventory policy plays an important role. Inventory management aims at improving customer service level, reducing operating costs, and gaining competitive advantages for the enterprises. Inventory policy is usually determined with economic order quantities and order points based on customer demand, storage cost, and order cost. Even though, inventory policy based on economic order quantities is an optimal solution for a firm, it may not contribute to a robust supply chain system in the big picture. Because as a member of the supply chain system, the enterprise’s inventory management is not only affected by its own factors, but also restricted by other factors from the supply chain system. Therefore, it is necessary to coordinate and optimize the inventory policy of the enterprises on the chain from the perspective of the whole supply chain system [1].

Scarf and Clark [2] put forward mathematical model to study the multi-echelon inventory management problem of supply chain system. To describe the uncertainties and dynamic relations in the supply chain system, it is much difficult to study the supply chain with mathematical model. In this case, computer simulation plays an important role. In order to study the optimization problem of inventory policy in the supply chain system, scholars have developed a variety of supply chain simulation models, summarized as production-inventory system, inventory-distribution system, and production-inventory-distribution system, of which two-echelon supply chain inventory simulation models are the most widely used [3,4], and multi-echelon supply chain inventory simulation models are getting more attention gradually [5,6]. In particular, the classic four-echelon
supply chain simulation model called "beer game" [7] is used to study various behaviors of the supply chain as soon as it is proposed [8-12]. Uncertainties will have an impact on the performance of each participant of supply chain system. In order to solve such problems, some scholars pointed out that robust parameter design can be introduced into the field of simulation and optimization, which aims to make the responses insensitive to the interference of uncertainties by adjusting the level of control factors [13]. Taguchi method is a generally adopted for robust parameter design, where orthogonal array is used to arrange experiment trials and the location and dispersion modelling method is adopted to analyze the experiment results. Taguchi method helps to find out the optimal setting of variables in their discrete levels. When it comes to searching for the optimal solution in continuous levels of variables, response surface method based on sequential experiment design and analysis needs to be considered [14]. Box and Draper [15] introduced the idea of robust parameter design into the response surface method and proposed the dual-response surface method. The dual-response surface method takes variance as an indicator of risk measurement, and solves the simulation optimization problem through a mean-variance response surface model. When variance is used as an indicator of risk measurement, both the upper deviation and the lower deviation are taken into account. Although the upper deviation also deviates from expectation, it does not constitute an actual loss, which reflects the deficiency of taking variance as the risk measurement [16]. Value-at-risk (VAR) is a popular unilateral risk measurement which considers the minimum loss that a decision maker would have to take under normal market condition and a given confidence level $\beta \in [0,1]$ The Conditional value-at-risk (CVAR), is also a way to measure the unilateral risk, which takes the value-at-risk as the target loss of the decision makers, and gives the average loss when the loss of decision makers exceeds the value-at-risk. Compared with the value-at-risk, the conditional value-at-risk has the advantages of consistency, monotony, sub-additivity, positive secondary and translation immutability [17].

In practice, profit maximization or cost minimization are usually considered as the objective in the decision of inventory policy, which implies the assumption that the decision makers are risk neutral. However, the risk preference of decision makers may be risk seeking or risk averse. Therefore, in practice, the supply chain strategy may deviate from the optimal solution obtained based on the hypothesis of risk neutral [18]. Based on the above points, this paper focuses on the decision of inventory policy in a multi-echelon supply chain system, proposes a method combining the conditional value-at-risk and response surface model to solve the robust optimization problem considering different risk preference of decision makers. The impact of different risk preference on the optimal solution is discussed.

2. Four-echelon supply chain model

In order to study the impact of inventory policy on the performance of the supply chain, a four-echelon supply chain simulation model is adopted to model the dynamic of orders and stocks in the supply chain system. The adopted simulation model is a variant of the classic “beer game”, shown in Figure 1. The model consists of eight retailers, four wholesalers supplying products to retailers, two distributors supplying products to wholesalers, and one factory that produces and supplies products to distributors. The simulation model is operated in Arena simulation software [19]. The model involves the following main assumptions: (1) Each participant of the supply chain has unlimited storage capacity, and products and orders are only transferred between adjacent levels; (2) An order arrives every day, and each level of the supply chain must satisfy orders from their downstream participants as much as possible. In the case of a stock-out, unfilled orders will be recorded as backorders, and will generate corresponding penalty cost; (3) Lost sale is only allowed at the retail level, that is, unfilled orders at the retail level may either be backlogged for later delivery or immediately lost, which incurs either penalty cost or lost sale cost respectively; (4) In each period, each level of the supply chain must pay the penalty cost for backorders, and pay the
inventory holding cost for the products in its own warehouse; (5) Both ordering products from upstream and shipping products to downstream are subject to delays, which generates order lead time and delivery lead time accordingly; (6) The goal of retailers, wholesalers, distributors and factory is to maximize the total profit of the supply chain system.

2.1. Demand structure and lead time
In the model of “beer game” given above, the material and information flows of a single product across the four-echelon supply chain system is shown in Figure 2. In the simulation model, there are two sources of uncertainties, namely, customer demand and lead time. Customer demand only appears at the retail level, which is recorded as a random variation denoted as \( D \). Considering no demand variation \( D = 0 \), the customer demand is assumed to be a constant 25. When demand variation is considered \( D = 1 \), the customer demand follows a Poisson distribution \( P(\eta) \) with \( \eta = 25 \), shown in Figure 3. The demand of factory, distributors, and wholesalers is determined by the orders from neighbouring downstream customers and their own inventory position.
Lead time is also a random variation which is composed of order time and delivery time. The customer’s ordering information directly reaches the retailers, so the order time of the retailers is 0 day, while the distributors, wholesalers and factory each have a fixed order time of 2 days. The delivery time is the sum of the fixed delivery time and the fluctuated delivery time, the fixed delivery time is assumed to be 2 days for each level except the factory, and 1 day for the factory. The fluctuated delivery time is a random variable. Assuming the fluctuated delivery time follows a Poisson distribution with \( P(0.1) \), while when its value is equal to 0.5, the floating delivery time follows the \( P(0.5) \) distribution. The values of the order time and delivery time for each level as shown in Table 1.

### Table 1. The setting of lead time.

| Level    | Order lead time (Days) | Delivery lead time (Days) | High variation \( P(0.1) \) | Low variation \( P(0.5) \) |
|----------|------------------------|---------------------------|-----------------------------|-----------------------------|
| Retailer | 0                      |                           | 2\( \cdot \)Poisson (0.1)   | 2\( \cdot \)Poisson (0.5)   |
| Distributor | 2                    |                           | 2\( \cdot \)Poisson (0.1)   | 2\( \cdot \)Poisson (0.5)   |
| Wholesaler | 2                    |                           | 2\( \cdot \)Poisson (0.1)   | 2\( \cdot \)Poisson (0.5)   |
| Factory  | 2                      |                           | 2\( \cdot \)Poisson (0.1)   | 2\( \cdot \)Poisson (0.5)   |

#### 2.2 Inventory structure

The Arena “beer game” simulation model mainly tracks inventory indicators: on-hand inventory, on-order inventory, and backorders. On-hand inventory refers to the actual amount of inventory physically in stock, on-order inventory refers to the amount of product that have been placed but have not been received (either in transit or in production), and backorders refer to the amount of order yet to be satisfied. The above three indicators can be combined to calculate the net inventory position using the following formulation. \( L \) is the index for supply chain level, where \( L = \{ \text{retailer}, \text{wholesaler}, \text{distributor}, \text{factory} \} \). The net inventory position is a sign of whether replenishment is need. This paper adopts continuous review policy, when the net inventory position of each level is lower than the reorder point \( r \) at this level, the order command will be triggered, and the order quantity is equal to the reorder quantity \( Q \) at this level.

\[
\text{Inventory position}(L) = \text{Inventory onhand}(L) + \text{Inventory onorder}(L) - \text{Backorder}(L)
\]  

(1)
2.3. Price, cost and profit structure
The Arena “beer game” simulation model tracks the cost and profit day by day across the supply chain. The cost and profit parameters are shown in Table 2. The cost mainly involves order cost, holding cost, penalty cost and lost sale cost. Profit at each level is calculated by subtracting cost from sales revenue at that level. \( D \) is the index for day of the month. The daily operating cost and profit is calculated for each level by the following formulation, respectively:

\[
\text{Cost}(D,L) = \begin{cases} 
\text{Order cost}(L) \times \{0, \text{if no order placed}, \text{otherwise } 1\} + \text{holding cost}(L) \times \text{on-hand inventory}(L) \\
+ \text{penalty cost}(L) \times \text{Backorder}(L) + \{\text{lost sale cost} \times \text{lost sales, if } L = 1, \text{otherwise } 0\}
\end{cases}
\]

(2)

\[
\text{Profit}(D,L) = \text{unit profit}(L) \times \text{units sold}(L) - \text{Cost}(D,L)
\]

(3)

Table 2. Parameter settings of related cost and profit.

|                      | Retailer | Wholesaler | Distributor | Manufacturer |
|----------------------|----------|------------|-------------|--------------|
| Order cost:$/unit    | 25       | 50         | 100         | 200          |
| Holding cost:$/day * unit | 0.125   | 0.25       | 0.223       | 0.16         |
| Penalty cost:$/day * unit | 5       | 2.5        | 1.25        | 1            |
| Profit:$/unit        | 4        | 3.75       | 3.5         | 3.25         |
| Lost sale cost:$/unit| 10       | -          | -           | -            |

2.4. Simulation logic of “beer game” model
The simulation logic of “beer game” model is mainly composed of three parts: (1) Demand and order processing process; (2) Order generation; (3) Order delivery and receiving. Figure 4 shows the order processing logic of the retailer. The order processing logic of retailers, wholesalers, distributors and factory are similar, except for the following details: When a retailer’s on hand inventory level is lower than the amount of customer demand, unfilled demand may either be backlogged for later delivery or immediately lost. To be specific, the retailer needs to determine whether the customer allows for later delivery. If yes, then a backorder is created, otherwise this coming order becomes sale loss. Other than retailer, unfilled demand for wholesalers, distributors and factory only become backorders that require later delivery. Unlike the distributing parts (retailer, wholesalers, and distributors), when the factory reaches its reorder point, they will not place an order from upstream, instead, they will set up to organize production for replenishment.

![Figure 4. Simulation logic at the retail level.](image-url)
2.5. Operation environment of “beer game” model

In order to study the impact of system behaviour on the supply chain performance during the process of the product’s production and distribution, we set the simulation running length as 360 days, and the profit unit is $1000. In order to get a stable running results, a 30-day warm-up time is set before each simulation starts to run. Table 5 shows the analysis of the changes in various inventory levels during the 30-day warm-up time, in which the initial on-hand inventory of each retailer, distributor, wholesaler, and factory is 150, 300, 500, and 300, respectively. The reorder point and the reorder quantity are \{100, 200, 400, 200\} and \{100, 150, 300, 700\}, respectively. Through the 30-day warm-up time, it can be found that the levels of on-hand inventory and on-order inventory at each level tend to be stable obviously, which proves that the warm-up time of 30 days is effective.

![Figure 5. Analysis of various inventory levels during the 30-day warm-up time.](image)

In order to make running results of simulation experiment more realistic and reduce the error, 30 replications are performed for each simulation run (as shown in Figure 6).

![Figure 6. Result of time persistent statistics under 30 replications.](image)
Arena tracks every change in the reported inventory changes over 30 replications, the half-width means the half width of 95% confidence interval. The expression of half width as follow:

$$\text{half-width} = t_{(1-\alpha/2)} \times \frac{s}{\sqrt{n}}$$  \hspace{1cm} (4)$$

Where \( n \) represents the number of repeated experiments and \( s \) represents the sample standard deviation of the repeated experiments. The smaller the half-width, the more accurate the estimation accuracy of the time persistent statistics. Taking on-hand inventory factory for example, the half-width of retailer, wholesaler, distributor and factory are 1.58, 3.25, 5.22 and 5.19, respectively. The half-width of on-hand inventory at each level is small, which meets the requirement of estimation accuracy. And the amount of on-hand inventory shows an increasing trend from downstream to upstream of the supply chain. In another aspect, which is a vivid demonstration of the phenomenon of bullwhip effect in multi-echelon supply chain, indicating that the simulation model can accurately reflect the real system.

3. Multi-response robust optimization design of supply chain with risk preference

3.1. Multi-response optimization strategy

The continuous deepening of economic globalization and the increasing diversification and personalization of customer demand, the uncertainties in supply chain have become prominent. Supply chain risk arises from the uncertainties of supply chain [20]. The risk caused by uncertainties is far more harmful to the entire supply chain than the sum of the impact on individual enterprises. The variations of customer demand and lead time are the most common uncertainties that cause supply chain risk. Different decision makers have different risk preference and make different decisions on the uncertainties, which will lead to the divergence of supply chain performance. In order to incorporate the decision makers’ risk preference into the robust optimization design of the supply chain, both mean and variance of the response must be considered in the objective function of robust optimization. Variance, as a bilateral risk measurement, is not suitable for measuring some responses such as profit that generates risk only when its value is lower than the expectation. Therefore, this paper proposes to replace the variance with the conditional value-at-risk and adopts the weighting method to construct an optimization model, in which the size of the weighting coefficient reflects the degree of risk averse. This paper selects two responses to measure the supply chain performance: total profit and customer service level. The optimization goal of this paper is to solve the optimal supply chain inventory policy under the condition of profit maximization and customer service level not less than 95%. Finally, the robust optimization objective functions of supply chain multi-response inventory policy considering different risk preference are expressed as follows [21]:

\[
\begin{align*}
\max_{d \in \mathbb{R}^n} & \{E[f(d,e)] + \lambda V[f(d,e)]\} \\
\text{s.t.} & \quad E[g(d,e)] \geq \alpha \\
\max_{d \in \mathbb{R}^n} & \{E[f(d,e)] + \lambda CVAR_{(\alpha)}[f(d,e)]\} \\
\text{s.t.} & \quad E[g(d,e)] \geq \alpha
\end{align*}
\]

(5)  \hspace{1cm} (6)

Equation (1) constructs a robust optimization objective function based on the dual-response surface methodology, which uses variance as a risk measurement. While Equation (2) adopts the conditional value-at-risk instead of variance as a risk measurement to construct a robust optimization objective function of the supply chain inventory policy. \( \lambda \in [0,1] \) reflects the degree of risk averse of the decision makers, the greater the \( \lambda \), the higher the degree of risk averse.
\[ \alpha \in [0.100\%] \] represents the threshold of customer service level, \( d \) represents the vector composed of controllable factors, \( e \) represents the vector composed of noise factors, \( f(d,e) \) and \( g(d,e) \) represent analytical expressions of total profit and customer service level, respectively, which are unknown and highly nonlinear. of the mean of profit response \( E[f(d,e)] \) and the variance of profit response \( V[f(d,e)] \). In addition, mathematical models of the conditional value-at-risk of profit response \( CVAR_{1-\beta}[f(d,e)] \) and the mean of service level (CSL) \( E[g(d,e)] \). Using the response surface modelling method to model the result of simulation experiments, which is possible to build a high-precision approximate model that reflects the input and output relationships of complex systems, thereby optimizing the system. Assume that the fitted response surface models are respectively:

\[
\hat{\theta}_f = f_{\text{mean}}(d,e), \quad \delta_f = f_{\alpha}(d,e), \quad \hat{\gamma}_{1-\beta} = f_{\text{CVAR}_{1-\beta}}(d,e), \quad \hat{\theta}_g = g_{\text{mean}}(d,e),
\]

the approximate models of Equation (1) and Equation (2) are Equation (3) and Equation (4), respectively:

\[
\min \hat{\theta}_f + \lambda \delta_f \quad \text{s.t.} \quad \hat{\theta}_g \geq \alpha \quad (7)
\]

\[
\min \hat{\theta}_f + \lambda \hat{\gamma}_{1-\beta} \quad \text{s.t.} \quad \hat{\theta}_g \geq \alpha \quad (8)
\]

3.2. Implementation process of robust optimization

3.2.1. Taguchi method.

Taguchi method has been widely used in robust optimization design, which is proposed by Professor Taguchi of Japan in the 1980s, and advocates the robust design into the products’ production process, by controlling the source of products’ production to resist the loss caused by noise or uncontrollable factors. Therefore, Professor Taguchi puts forward three design methods, in which parameter design is regarded as the core. Parameter design aims to introduce noise factors such as operating conditions and environmental conditions into the design process, and gradually seeks the "robust controllable factor levels" based on the simulation results. In the supply chain model proposed in this paper, the reorder points and the reorder quantities on each level are set as controllable factors, which are expressed as \( \{F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8\} \), (as shown in Table 3). Noise factors are difficult to control artificially, which are usually represented as variations in the simulation model. In the model proposed in this paper, the variations of customer demand and lead time on each level are adopted as the noise factors, which can be expressed as \( \{D, S_1, S_2, S_3, S_4\} \) (as shown in Table 3). In order to obtain a robust combination of controllable factors under the influence of noise factors, a cross array, including an inner array and an outer array, is used to arrange experiments. The inner array is used to arrange the controllable factors in trials through a control array of factorial design, while the outer array is used to study the difference among the same trial of the inner array with different settings of noise factors [22]. The purpose of arranging outer array is to conduct stability analysis on the experiment points determined by the inner array. According to the number of controllable factors and noise factors and the number of levels corresponding to each factor, the inner array adopts the orthogonal \( L_{27}(3^8) \), while the outer array adopts the orthogonal \( L_9(2^5) \). The purpose of adopting the orthogonal experiments is to greatly reduce the number of experiments. Due to the orthogonality of orthogonal experiments, a few
representative experiments can be selected from a large number of the experimental design, and the optimal parameter combination can be obtained through fewer experiments.

This paper adopts the location and dispersion modelling method to analyze the experiment results. The mean of response samples of repeated experiments with noise factors are used as the location measurement, and the sample standard deviation of repeated experiments are taken as the dispersion measurement. Factors that significantly affect the location measurement are defined as the location factors, while factors that significantly affect the dispersion measurement are defined as the dispersion factors. When using the location and dispersion modelling method, different strategies are adopted according to different types of quality characteristics [23]. Quality characteristics can generally be divided into three categories which can be obtained from the simulation results: “the larger the better”, “the nominal the best” and “the smaller the better”. This paper selects the total profit and customer service level as responses which both belong to “the larger the better”. For “the larger the better” problem, referring to the larger the location and the smaller the dispersion, the better. To be specific, the levels of location factors are selected firstly to maximize the location measurement, then the levels of dispersion factors are selected to minimize the dispersion measurement. The experiment design adopts the cross array, there are 27 trials in the inner array and 8 trials in the outer array. A total of at least 216 trials should be conducted, and replications should be conducted for each experiment to reduce the experiment error.

**Table 3.** Parameter settings of controllable factors and noise factors.

| Control factors                        | Low level | Medium level | High level |
|----------------------------------------|-----------|--------------|------------|
| F1 (Reorder point of retailer)         | 75        | 100          | 150        |
| F2 (Reorder point of distributor)      | 150       | 200          | 300        |
| F3 (Reorder point of wholesale)        | 300       | 400          | 600        |
| F4 (Reorder point of factory)          | 150       | 200          | 300        |
| F5 (Reorder quantity of Retailer)      | 75        | 100          | 150        |
| F6 (Reorder quantity of distributor)   | 150       | 175          | 200        |
| F7 (Reorder quantity of wholesale)     | 200       | 300          | 500        |
| F8 (Reorder quantity of factory)       | 600       | 700          | 900        |

| Noise factors                          | Low level | High level |
|----------------------------------------|-----------|------------|
| D (Customer demand variation)          | 0.1       | 0.5        |
| S1 (Delivery time variation of retailer)| 0.1       | 0.5        |
| S2 (Delivery time variation of distributor) | 0.1       | 0.5        |
| S3 (Delivery time variation of wholesaler)| 0.1       | 0.5        |
| S4 (Delivery time variation of factory) | 0.1       | 0.5        |

3.2.2. Fractional factorial experiment.
There are eight controllable factors in the simulation model, considering that not all controllable factors have a significant impact on the responses, we need to conduct variable selection to search the significant factors that influence the output responses. There are two classic experiment design methods, namely, full factorial design and fractional factorial design. Fractional factorial design not allow us to estimate all the main effects and interactions, which can conduct variable selection with fewer experiments when the number of factors is large [24].

3.2.3. Response surface methodology (RSM).
Response surface methodology is a popular parameter optimization method to model and optimize stochastic, dynamic, and complex systems like supply chain, which uses an approximation of the real objective function. RSM allows the identification of the relations between independent factors and response. RSM follows a sequential process that aims to achieve optimal response value by searching for the optimal combination of controllable factors. Central composite design is the most
common design method in response surface experiment design, which consists of a full factorial design or a fractional factorial design that includes center points and a new set of axial points on the surface of the cube, so that the nonlinear relationship between the factors and the response can be evaluated [25].

The response surface modelling method is the most convenient and effective choice to obtain the optimal combination of controllable factors at the continuous factor level. In order to obtain the optimal response value, first-order regression models are adopted to fit the relation between controllable factors and each response. If there is a curvature in the experiment region, we need to add the type of experiment design points and construct the second-order regression models to further estimate the nonlinear function relation between factors and responses. Finally, according to ANOVA to determine whether the second-order regression models are valid.

3.2.4. Solve the optimization models.
According to the response surface models established in step 3, the optimization models of Equation (3) and (4) are constructed base on chapter 3.1, and the different values of \( \lambda \) are adopted, which are equal to 0, 0.25, 0.5, 0.75 and 1, respectively. The weight of optimization model is increasing, indicating that decision makers are increasingly trend to risk averse.

4. Evaluation of supply chain inventory policy under different risk preference
Carrying out the simulation experiment according to the across array described in step 1 of Chapter 3.2, the mean values and the standard deviation values of the total profit response and customer service level response are obtained, then the main effects plots for means and standard deviation are drawn. According to the main effect plots, it is vivid to know how each factor influence the responses in location and dispersion perspective. The main effects plots of total profit as shown in Figure 7. According to the size of the main effect of each factor on the mean and dispersion of the response, we can obtain the classification of the factors. Figure 7 shows the optimal combination of the location factors that maximizes the mean values of total profit are \( \{F_1,F_2,F_4,F_6\} \), while the optimal combination of the dispersion factors that minimizes the sample standard deviation of total profit are \( \{F_1,F_2,F_5,F_6\} \). As a result, the optimal combination of controllable factors can be obtained as \( \{F_1,F_2,F_3,F_4,F_5,F_6,F_7\} = \{100,150,400,150,100,150,300,700\} \), similarly we can get the optimal combination of controllable factors for maximizing the response of customer service level is \( \{F_1,F_2,F_3,F_4,F_5,F_6,F_7\} = \{150,300,300,200,150,200,500,700\} \).

When multi-responses are considered simultaneously in the RSM design, it is necessary to find the factors that significantly influence both responses through simulation experiments. In step 2 of Chapter 3.2, fractional factorial experiment has the function of variable selection. \( 2^{k-2} \) fractional factorial experiment is conducted to select significant factors that influence the total profit and the customer service level under the extremely unstable combination of noise factors \( (D=1,S_1 = S_2 = S_3 = S_4 = 0.5) \). As shown in Figure 8, the first Pareto chart of this experiment results shows that \( \{F_2,F_3,F_1,F_5\} \) are the significant factors for the total profit, and the second Pareto chart of this experiment results shows that \( \{F_1,F_5,F_3,F_6,F_7\} \) are the significant factors for the customer service level. Finding the same factors from their respective significant factors, recorded as \( \{F_1,F_2,F_3,F_5\} \), which is four factors that must be considered in the multi-response response surface design for the bi-objective problems of total profit and customer service level maximization.
The factorial second-order responses \( F \) with design.

Through points or face-centered as recorded second-order response the factor \( F \) performance and obtain points, response \( \tilde{F} \) total of inventory experiment analysis independent is full for the factor

The significant chain as CSL the result Equation models composite these surface experiment the response the be variance, significant experiment between 3

Figure 8. Pareto charts of total profit and customer service level.

Then, the response surface modelling method is used to obtain the optimal inventory parameter combination and performance of supply chain under the continuous factor level. According to step 3 of Chapter 3.2, the next step is to design a \( (2^4 + 7) \) full factorial experiment with center points using the significant factors \( \{ F_1, F_2, F_3, F_4 \} \) to estimate the first-order fitting function relationship between the responses and independent variables. Through analysis of variance, the full factorial experiment with center points either for total profit maximization or for CSL maximization shows a significant evidence of curvature. According the sequential principle of the RSM, we need to add \( (2 * 4) \) face-center points into the former full factorial experiment, then the former full factorial experiment is transformed into Central composite Face-centered (CCF) design. The levels of significant factors are set according to the optimal experiment points obtained by the Taguchi method, as shown in Table 4. According to the CCF experiment for profit maximization, the ANOVA result shows the derived second-order response model is significant. Similarly, the second-order response model of CSL can be obtained. According to the CCF experiment, the second-order response surface models for the mean, the variance \( (\sigma^2) \) and the conditional value-at-risk of total profit are marked as \( \hat{\delta}_1 = f_{\text{mean}}(d,e), \hat{\delta}_2 = f_{\sigma^2}(d,e), \hat{\gamma}_{1-(1-\delta)} = f_{\text{VaR}_{1-(1-\delta)}}(d,e) \), respectively, and the second-order response surface model for the mean of CSL is recorded as \( \hat{\gamma}_\theta = g_{\text{mean}}(d,e) \), as shown in Equation (9-12), and the analysis of variance confirmed that these second-order models are effective.
Table 4. Experiment data fitting the second-order response surface models.

| Number | \{F_1,F_2,F_3,F_4\} | Mean of profit | Variance of profit | CSL | CVAR |
|--------|---------------------|---------------|---------------------|-----|------|
| 1      | \{100,220,400,100\} | 803.459       | 76.763              | 97.1% | -785.387 |
| 2      | \{80,220,520,75\}  | 790.859       | 258.473             | 89.6% | -757.697 |
| 3      | \{120,220,520,75\} | 805.600       | 98.020              | 97.6% | -785.178 |
| 4      | \{80,180,280,125\} | 744.504       | 274.068             | 90.0% | -710.356 |
| .....  | .....               | .....         | .....               | ..... | ..... |
| 28     | \{100,200,400,100\} | 803.459       | 76.763              | 97.4% | -785.387 |
| 29     | \{120,180,520,125\} | 782.835       | 77.842              | 98.6% | -764.636 |
| 30     | \{100,200,400,100\} | 803.459       | 76.763              | 97.1% | -785.387 |
| 31     | \{120,180,280,75\}  | 767.816       | 623.868             | 96.6% | -716.295 |

\[ f_{\text{mean}}(F_1,F_2,F_3,F_4) = 338 + 1.086 F_1^2 + 3.86 F_2^2 + 0.725 F_3^2 - 2.822 F_4 - 0.01169 F_1^2 - 0.01131 F_2^2 - 0.001322 F_3^2 + 0.000029 F_4 + 0.002986 F_1^2 - 0.0087 F_2 F_3 + 0.001605 F_2 F_4 \] (9)

\[ f_{\alpha}(F_1,F_2,F_3,F_4) = 9449 + 26.23 F_1 - 103.3 F_2 - 0.54 F_3 + 0.2543 F_2^2 - 0.00641 F_3^2 - 0.05759 F_2^2 - 0.01322 F_3 F_4 \] (10)

\[ f_{\beta}(F_1,F_2,F_3,F_4) = -15590 - 44.3 F_1 + 176.7 F_2 + 1.89 F_3 + 0.0285 F_4 - 0.434 F_3^2 - 0.01201 F_2^2 + 0.0977 F_3 F_4 \] (11)

\[ g_{\text{mean}}(F_1,F_2,F_3,F_4) = -0.247 + 0.01434 F_1 + 0.00291 F_2 + 0.000038 F_3 + 0.000038 F_4 + 0.000148 F_1 F_2 - 0.0006 F_2^2 - 0.000008 F_3^2 - 0.000007 F_3 F_4 - 0.000004 F_2 F_3 + 0.000003 F_3 F_4 \] (12)

After obtaining the above response surface models, constructing the optimization models (3) and (4) previously proposed, and then the Matlab software is used to solve the optimal inventory parameter combination and optimal performance of supply chain, where \( \alpha = \beta = 95\% \). The optimization model (3) uses the dual-response surface method that adopts the variance of total profit as risk measurement to estimate the risk that uncertainties bring to supply chain performance. However, Variance as a bilateral risk measurement, both the upper deviation and the lower deviation are taken into account. This paper proposes to replace the bilateral risk measurement (variance) with the unilateral risk measurement (conditional value-at-risk), constructing the mean-variance value-at-risk optimization model based on the Equation (4) to solve the multi-response optimization problem with risk preference. The effectiveness of the proposed method is verified by simulation analysis. The results of simulation experiment are shown in Table 5.

Table 5. Optimal parameter levels and total profit under different risk preference.

| Optimization | This paper’s method s | Mean-variance method |
|--------------|-----------------------|----------------------|
| \( \lambda = 0 \) | \{102,199.5, 434.9, 75\} | \{102,199.5, 434.9, 75\} | 815.6737 | 815.6737 |
| \( \lambda = 0.25 \) | \{110,1280.7, 529.3, 207.8\} | 793.1796 | \{154,2203.9, 635.9, 100.9\} | 760.7952 |
| \( \lambda = 0.5 \) | \{141,1203.8, 6321,1083\} | 768.075 | \{154,6203.7, 676,9107.3\} | 744.9251 |
| \( \lambda = 0.75 \) | \{141,22303.7, 6416,1088\} | 764.5581 | \{154,72203.5, 6958,1074\} | 736.2088 |
| \( \lambda = 1 \) | \{141,32637.7, 6468,1089\} | 762.5326 | \{154,82304.3, 706,9107.1\} | 730.6272 |

As shown in Table 5, when \( \lambda = 0 \), that is to say, without considering the risk of the supply chain, the results of the two methods are the same, and the performance of the supply chain is optimal at this time. In other cases, the total profit obtained using the method proposed in this paper is slightly higher than the total profit obtained by the mean-variance method. In particular, when \( \lambda = 0.25 \), the gap between the optimization results of the two methods is the largest, the total profit of the method
proposed in this paper exceeds the mean-variance method of $330,000. This also confirms that the conditional value-at-risk is more suitable to measure the risk of supply chain inventory policy, which can help solve more accurate optimal combination of control parameters and optimize the performance of the supply chain.

In addition, as shown in Table 5, the total profit obtained by both methods decreases with the risk parameter (\(\lambda\)) increased. Compared with the mean-variance model, this paper’s method has a slight decrease in total profit. The change of risk parameter has a significant influence on the total profit, while there is no obvious influence on the combination level of decision variables. Table 5 provides the decision reference for supply chain decision makers with different risk preference, and provides technical support for management decisions.

5. Conclusion
This paper introduces the unilateral risk measurement-conditional value-at-risk into the multi-response inventory policy optimization problem with risk preference. The proposed method is a combination of the mean-conditional value-at-risk optimization model and response surface methodology. In order to verify the effectiveness of the proposed method, the comparison analysis between the proposed method and the dual-response surface method is conducted in this paper. From the perspective of total profit, the method proposed in this paper is superior to the mean-variance method. Considering the same risk preference, the total profit of the proposed method is always higher than that of the mean-variance method based on the dual-response surface method. The reason is that the latter takes into account the unnecessary risk, which has a negative impact on total profit.

It should be pointed out that this paper only studies the problem of robust optimization design of inventory policy. This method can be extended to solve other supply chain problems, such as the cooperation problem of supply chain’s participants. In addition, only the risk of profit is considered in the optimization objective function, how to include multiple risks of responses is worthy of further study.

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