A new approach for generation of generalized basic probability assignment in the evidence theory

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Abstract
The process of information fusion needs to deal with a large number of uncertain information with multi-source, heterogeneity, inaccuracy, unreliability, and incompleteness. In practical engineering applications, Dempster–Shafer evidence theory is widely used in multi-source information fusion owing to its effectiveness in data fusion. Information sources have an important impact on multi-source information fusion in an environment with the characteristics of complex, unstable, uncertain, and incomplete. To address multi-source information fusion problem, this paper considers the situation of uncertain information modeling from the closed-world to the open-world assumption and studies the generation of basic probability assignment with incomplete information. A new method is proposed to generate the generalized basic probability assignment (GBPA) based on the triangular fuzzy number model under the open-world assumption. First, the maximum, minimum, and mean values for the triangular membership function of each attribute in classification problem can be obtained to construct a triangular fuzzy number representation model. Then, by calculating the length of the intersection points between the sample and the triangular fuzzy number model, a GBPA set with an assignment for the empty set can be determined. The proposed method can not only be used in different complex environments simply and flexibly, but also have less information loss in information processing. Finally, a series of comprehensive experiments basing on the UCI data sets is used to verify the rationality and superiority of the proposed method.

Keywords Generalized evidence theory · Modified generalized combination rule · Generalized basic probability assignment · Multi-source information fusion · Dempster–Shafer evidence theory · Triangular fuzzy number

1 Introduction

In artificial intelligence, information fusion technology plays a significant role in pattern recognition [1, 2]. In recent years, the multi-source information fusion technology that can fuse the collected information to improve the security and stability of the system, is widely used in the practical application [3, 4], such as decision making [5, 6], fault diagnosis [7–9], risk analysis and reliability assessment [10–14], and pattern recognition [15–17]. However, in complex environments, the data received by sensors based on physical hardware technology are often inaccurate and uncertain. If the data are directly fused, the results may be incorrect or counterintuitive. The reason for such a result is that the interference information generated by the complex environment or the malfunctioning sensor is not properly modeled and fused. Therefore, how to effectively solve the modeling and fusion of uncertain information in complex environments is still an open issue. To address this issue, many mathematical theories are adopted, such as fuzzy set theory [18], belief function theory [19, 20], grey theory [14], and probability theory [21, 22]. There are also some methods basing on Z-numbers [23, 24], D-numbers [25, 26], and so on [27, 28].

As a generalization of Bayesian theory [29], Dempster–Shafer evidence theory was put forward by Dempster in 1967 [30] and developed by Shafer in 1976 [31]. Since Dempster–Shafer evidence theory can flexibly handle information with incompleteness and uncertainty, it is widely applied in practical application system [32], like classification [33], clustering [34, 35], reliability analysis [36, 37], correlation analysis [38, 39], multi-attribute decision analysis [40], fault diagnosis [41], and so on [42]. Nevertheless,
Dempster–Shafer evidence theory has many limitations that need to be improved. For example, the key step of how to automatically generate basic probability assignment (BPA), which is the prerequisites for the use of Dempster’s combination rule and the application of Dempster–Shafer evidence theory. However, the method for BPA generation is still a difficult and an open issue. The quality of generated BPA will directly affect subsequent practical applications. If the method of generating BPA is unreasonable, it may cause failure to generate an effective BPA, and the fusion result is likely to be counterintuitive. The other issue is a counterintuitive result will be obtained while fusing some pieces of conflicting evidence [43]. Yager et al. [44] believed the unreasonable fusion results are mainly caused by the normalization of the combination rule, so the modification of Dempster’s combination rule is suggested. Smets and Kennes [45] pointed out that modifying the combination rule usually breaks the good properties of the original evidence fusion rule, like associativity and commutativity. Additionally, if the result is caused by a failure of the sensor, it is unreasonable to modify the combination rule. Therefore, Smets and Kennes [45] proposed a transferable belief model (TBM) where the concepts of the closed world and the open world were introduced. Deng [46] pointed out that the traditional Dempster–Shafer theory cannot resolve those information fusion problems if the framework of discernment was incomplete. Consequently, based on the assumption of the closed world and the open world, the basic framework of generalized evidence theory (GET) was proposed. Su et al. [47] considered fusion results based on the discounting coefficient and the independent evidence was associated with the degree of dependence, and the results based on dependent evidence are suggested to be given a smaller weight in the information fusion step. Zhang et al. [48] put forward a decision-making trial and evaluation laboratory (DEMATEL) method in preprocessing different pieces of evidence by considering the weight of each piece of evidence. In Xiao’s method [49], by considering the effect of the evidences and the similarity among each piece of evidence, a new combination method based on belief function entropy and similarity measure was proposed to deal with conflicting evidences. Other solutions for this issue can be found in [50, 51]. In this paper, we will focus on the process of using multi-source information to generate BPAs in the incomplete framework of discernment, which is the base of data fusion in the evidence theory.

Many BPA generation methods based on the complete frame of discernment have been proposed by researchers. For example, Xu et al. [52] utilized the probability density function to generate BPA for a nested structure. Zhang et al. [53] proposed an approach to generate BPA by utilizing fuzzy numbers. These BPAs are determined according to the normalized dissimilarities between the test samples and training samples, which are obtained by calculating the distance between fuzzy numbers. Yin et al. [54] put forward an approach to calculate the negation of BPA, which is also an important way for uncertain information expression. With the upgrading of detection instruments and other equipments, the ability to obtain information has been improved, which may cause some unknown categories and objects that are out of the framework of discernment. Hence, in reality, most of the frames of discernment can be in the open-world assumption. The open world is also denoted as the incomplete frame of discernment [46]. For incomplete frame of discernment, to model uncertain information, Deng [46] defined the concept of generalized basic probability assignment (GBPA) and proposed the generalized combination rule (GCR) to combine two GBPAs. Following the work, Zhang et al. [55] proposed a method to determine GBPA for the incomplete frame of discernment by using triangular fuzzy numbers. The method can generate GBPA based on the sample data and decrease the conflict among different propositions. Jiang et al. [56] also proposed an improved approach based on the triangular fuzzy number to determine GBPA in the incomplete frame of discernment. Most of the methods have the disadvantages of computational complexity, which may not meet the needs of real-time systems. Also, the recognition accuracy of those methods needs to be improved. Consequently, a new approach for generation of GBPA for multi-source information fusion in the evidence theory is proposed in this work.

In [57], Deng pointed out that the strongly constrained GBPA generation method can generate a GBPA with a null set that is not equal to zero, and its size reflects the possibility that the system is in an incomplete framework of discernment. After that, Zhang et al. [55] put forward a new method using triangular fuzzy numbers to determine GBPA in the incomplete framework of discernment. The extreme value, mean value, standard deviation, and each attribute’s triangular membership function are utilized to generate the fuzzy triangle curves and finally construct the GBPA function with a nested structure which has an assignment for GBPA of the empty set. Then, Jiang et al. [58] found that the GCR still has its limitation. Therefore, a modified generalized combination rule (mGCR) in the framework of GET was proposed. In GET, the mGCR fulfills all GCR’s properties. Inspired by the aforementioned works, a new approach for generation of GBPA is proposed. In comparison with the method in [55], the contributions of the proposed method are distinguished as follows:

1. The proposed method simplifies the generation criterion of \(m(\emptyset)\). In [55], the sum of the generated GBPA values may be more than 1, and the generation criterion of \(m(\emptyset)\) depends on not only whether the sample is intersects with the triangular fuzzy numbers of the
propositions, but also whether the sum of GBPA values of all the other single-subset and multi-subset propositions is greater than 1. In the proposed method, the sum of GBPA values for all propositions is no more than 1. In addition, the generation criterion of \( m(\{\emptyset\}) \) is only that: 1 minus the sum of GBPA values of all single-subset and multi-subset propositions. In this way, the generation criterion of \( m(\{\emptyset\}) \) in this method is simpler.

2. The proposed method improves the strategy for the generation of GBPA. When the sample intersects the triangular fuzzy number representation model of multiple propositions, the original GBPA generation strategy is: The high point of the ordinate is the GBPA value that the sample supports the proposition and the low point of the ordinate is the GBPA value that the sample supports the multi-subset proposition. The GBPA generation strategy in this paper is: The interval values of a higher point of the ordinate minus a lower point of the ordinate is the GBPA value that the sample supports a single-subset or multiple-subset proposition, the lowest point of the ordinate is the GBPA value that the sample supports the multi-subset proposition.

3. The proposed method generalized the ability to model uncertain information in the open-world assumption by generating more nonzero values of \( m(\{\emptyset\}) \). In \cite{55}, \( m(\{\emptyset\}) \) is often given 0 because the sum of non-empty sets’ GBPA values often exceeds 1. In the proposed method, the value 0 will be assigned to \( m(\{\emptyset\}) \) only when the sample and the triangular fuzzy number model of the proposition intersect the vertex. Therefore, the proposed method will have a better chance to generate \( m(\{\emptyset\}) \) with nonzero value for incomplete information, which reduces the loss of external environmental information.

The rest of the work is arranged as follows. The preliminaries are introduced in Sect. 2. The new approach of generating GBPA is proposed in Sect. 3. Experiments with UCI data sets are used to prove the superiority and effectiveness of the proposed method in Sect. 4. The application of the proposed method in a closed world as well as the experiment basing on the improved Dempster combination rule is discussed in Sect. 5. The conclusion is drawn in Sect. 6.

## 2 Preliminaries

### 2.1 Dempster–Shafer evidence theory

Some basic definitions in Dempster–Shafer evidence theory \cite{30, 31} are introduced as follows.

**Definition 1** Assume that \( \Omega = \{\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_N\} \) is a set of \( N \) exhaustive and mutually exclusive elements. \( \Omega \) is the frame of discernment (FOD). The power set of \( \Omega \) consists of \( 2^N \) elements is denoted as follows:

\[
\mathcal{G} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_N\}, \{\theta_1, \theta_2\}, \ldots, \{\theta_1, \theta_2, \ldots, \theta_n\}, \ldots, \Omega\}.
\]

(1)

**Definition 2** A mass function \( m \), also known as basic probability assignment (BPA) or basic belief assignment (BBA), is a mapping from the power set \( \mathcal{G} \) to the interval \([0,1]\). \( m \) satisfies:

\[
m(\emptyset) = 0, \quad \sum_{A \in \Omega} m(A) = 1.
\]

(2)

If \( m(A) > 0 \), then \( A \) is called a focal element. \( m(A) \) reflects the degree of evidence supports for the proposition \( A \).

**Definition 3** In Dempster–Shafer evidence theory, two independent mass functions \( m_1 \) and \( m_2 \) can be fused with Dempster’s rule of combination:

\[
m(A) = (m_1 \oplus m_2)(A) = \frac{1}{1 - k} \sum_{B \cap C = A} m_1(B)m_2(C),
\]

(3)

where \( k \) is a normalization factor defined as follows:

\[
k = \sum_{B \cap C = \emptyset} m_1(B)m_2(C).
\]

(4)

### 2.2 Generalized evidence theory

In generalized evidence theory (GET) \cite{46}, the generalized basic probability assignment (GBPA) corresponds to BPA in Dempster–Shafer evidence theory, which can be used to model uncertain information, and the generalized combination rule (GCR) is used to combine two pieces of evidence.

**Definition 4** Suppose that \( U \) is a frame of discernment in an open world. Its power set, \( 2^U \), is composed of \( 2^U \) propositions, \( \forall A \subset U \). A mass function is a mapping \( m_G : 2^U \rightarrow [0,1] \), that satisfies

\[
\sum_{A \in 2^U} m_G(A) = 1.
\]

(5)

Then, \( m_G \) is the GBPA of the framework of discernment \( U \). In GET, it is not necessary \( m_G(\emptyset) = 0 \).

**Definition 5** In GET, \( \emptyset_1 \cap \emptyset_2 = \emptyset \) represents that there is a null set in the intersection between tow null sets. Given two sets of GBPA \((m_1, m_2)\), the GCR is denoted as follows.
A modified generalized combination rule (mGCR) is proposed in [58].

**Definition 6** Given two sets of GBPAs \((m_1, m_2)\), the modified generalized combination rule is defined as follows:

\[
m(A) = \frac{(1 - m(\emptyset)) \sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - K},
\]

\[
K = \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C),
\]

\[
m(\emptyset) = m_1(\emptyset) \cdot m_2(\emptyset),
\]

\[
m(\emptyset) = 1 \quad \text{if and only if} \quad K = 1.
\]

A modified generalized combination rule (mGCR) is proposed in [58].

2.3 Triangular fuzzy number

Based on the fuzzy set, the theory of fuzzy number is proposed by Zadeh in 1965 [59]. A triangular fuzzy number \((a_0, \mu_a, a_1)\) is expressed as a piecewise liner membership function \(f(x)\). And it can be defined by:

\[
f(x) = \begin{cases} 
0 & x < a_0 \\
\frac{x - a_0}{\mu_a - a_0} & a_0 \leq x \leq \mu_a \\
\frac{a_1 - x}{a_1 - \mu_a} & \mu_a \leq x \leq a_1 \\
0 & x > a_1 
\end{cases}
\]

A triangular fuzzy number is limited to an interval from \(a_0\) to \(a_1\). \(\mu_a\) is the value that it is most likely to be.

3 A new approach for generation of GBPA

Based on the multi-subset propositional representation model of fuzzy mathematics, in the incomplete framework of discernment, a new approach of generating GBPA is proposed. The prerequisite is to have prior knowledge of the maximum, minimum, and mean values in the sample data of all classes. Hence, the proposed method is suitable for practical projects that contain an amount of sample data. The idea of the proposed method is that if a sample is between the maximum and minimum values of multiple classes, it is considered that the sample has a belief of GBPA that supports multiple propositions and a GBPA that supports the single proposition. The detailed algorithmic description is shown in Algorithm 1. The steps of the proposed method in detail are presented as follows.
Algorithm 1 The algorithm for generating GBPA

Input: The attribute value \( a \) of test sample and the triangular fuzzy number \( T \) of \( M \) propositions.

Output: GBPAs donated as \( G \) of the test sample that supports different propositions.

1: Set the minimum of the triangular membership function to 0.01.
2: for each \( T_m \) in \( T \) do
3: \hspace{1em} Set \( k_m^1 = (1-0.01)/(T_m[1]-T_m[0]), b_m^1 = 0.01-T_m[0]*k_m^1 \)
4: \hspace{1em} Set \( k_m^2 = (0.01-1)/(T_m[2]-T_m[1]), b_m^2 = 0.01-T_m[2]*k_m^2 \)
5: \hspace{1em} Set \( y_m^1 = k_m^1 x + b_m^1, y_m^2 = k_m^2 x + b_m^2 \)
6: end for
7: Set \( \text{intersections} = [ ] \)
8: for each \( m \) in \( M \) do
9: \hspace{1em} if \( 0 < y_m^1(a) \leq 1 \) then
10: \hspace{2em} \( \text{intersections}+ = (m, y_m^1(a)) \)
11: \hspace{1em} else if \( 0 < y_m^2(a) \leq 1 \) then
12: \hspace{2em} \( \text{intersections}+ = (m, y_m^2(a)) \)
13: \hspace{1em} else
14: \hspace{2em} \( \text{intersections}+ = (m, 0) \)
15: \hspace{1em} end if
16: end for
17: \( \text{intersections}+ = (\emptyset, 1) \)
18: \( \text{intersections} = \text{sorted}(\text{intersections}, \text{key} = \text{lambda, intersection : intersection[1]}) \)
19: Set \( \Omega \) as a set consisting of all single propositions.
20: Set \( S \) as a set consisting of empty, single propositions and multiple propositions.
21: Set \( G \) as a set of the corresponding proposition's GBPA in \( S \), and initialize it to a zero set.
22: Set \( \text{prev} \) as 0.
23: for each \( \text{intersection} \) in \( \text{intersections} \) do
24: \hspace{1em} if \( \text{intersection}[1] = 0 \) then
25: \hspace{2em} \( G[\Omega] = \text{intersection}[1] - \text{prev} \)
26: \hspace{1em} end if
27: \hspace{1em} \( \Omega = \text{intersection}[0] \)
28: \hspace{1em} \( \text{prev} = \text{intersection}[1] \)
29: end for

Step 1 Set the minimum of the triangular membership function as 0.01. When the attribute value of the test sample is equal to the maximum or minimum of the triangular fuzzy number model, it is unreasonable to assign the membership degree of the attribute to 0. Therefore, the minimum of the triangular membership function is set to 0.01, so that it can avoid unreasonable situations and not change the boundary value of the triangular fuzzy number model too much.

Step 2 Calculate the triangular fuzzy number of intersection points. When the test sample intersects the triangular fuzzy number models, \( m \) intersection points will be produced. Symbolize the ordinate values in an ascending order as \( w_1, w_2, \ldots, w_m \).

Step 2.1 Determine the triangular fuzzy number model. The interval from \( w_m \) to \( w_{m-1} \) belongs to the triangular fuzzy number model that contains the largest number of subsets.

Step 2.2 Calculate the GBPA value. The ordinate of \( w_m \) minus \( w_{m-1} \) is the GBPA that the sample supports the multi-subset proposition. Let \( m(A) \) represent the GBPA value of the focal element \( A \). Figure 1 is an illustration of how to generate a GBPA for test data based on a triangular fuzzy number model. The calculation function of GBPA in this rule is as follows:
Step 3 Calculate the GBPA of \( m(\{C_1, C_2, ..., C_m\}) = w_1 \) 
m\( (\{C_2, ..., C_m\}) = w_2 - w_1 \) 
...
\( m(\{C_{m-1}, C_m\}) = w_{m-1} - w_{m-2} \) 
m\( (\{C_m\}) = w_m - w_{m-1} \).  

(15)

Of the interval is equal to \( m_2(\{A, B\}) \) and so on. Through the aforementioned steps, the generation of GBPAs for the first sensor can be completed.

4 Experiment

In this section, a series of experiments for verification of the proposed method is designed. These experiments not only use the Haberman’s Survival data set to illustrate the generation of GBPA, but also use the Iris data set for checking the rationality and performance of the new approach, as well as analyze the robustness of the proposed method in different data sets.

4.1 Generation of GBPA for Haberman’s Survival data set

In this part, Haberman’s Survival data set is utilized to demonstrate the effectiveness of GBPA generation method in the open-world assumption. The data set contains many cases from a study that was conducted between 1958 and 1970 in the University of Chicago’s Billings Hospital. The data sets include the age of patients at the time of operation, the year of patients’ operation, the detected number of positive axillary nodes, and the patients’ final survival status. Among these cases, 81 patients died within 5 years, and 225 patients survived 5 years or longer.

In the Haberman’s Survival data set, the FOD is set as \( \Theta = \{a, b\} \), event \( a \) represents a patient’s survival for 5 years.

Frame of discernment: \{A,B,C\}

Fig. 1 An illustration of how to generate a GBPA for test data based on a triangular fuzzy number model
or more, and event $b$ represents a patient’s death within 5 years. For events $a$ and $b$, 200 and 72 cases are randomly chosen as the training set, respectively, and the rest are used as the test set. In the proposed method, each attribute is regarded as an independent information source, and the test samples’ GBPAs are generated with a step length of 1. All GBPAs of the test samples are labeled on the triangular fuzzy number representation models independently. The distribution of GBPA for the patients’ age at the time of operation on the framework of discernment is shown in Fig. 3. The distribution of GBPA on the frame of discernment of the patients’ year of operation is shown in Fig. 4.

The distribution of GBPA on the frame of discernment for the detected number of positive axillary nodes is shown in Fig. 5.

Figures 3, 4, and 5 show the following features of the generated GBPAs in the data set.

1) When the test sample is not in the interval of the triangular fuzzy number model of Class a and Class b, as shown in Fig. 4 where the sample’s value is less than 58, the test sample has no intersection with any triangular fuzzy number models. Therefore, it is easy to get: $m(\emptyset) = 1$, and $m(\{a\}) = 0, m(\{b\}) = 0$, which means that the system cannot determine which target the test sample belongs to, or...
which target is more inclined to be belonged to. In practical engineering applications, when Class a and Class b are not within the frame of discernment, it is a reasonable behavior to consider the test sample as an unknown class.

2) When the test sample is in the triangular fuzzy number model of Class a but not in the triangular fuzzy number of Class b, as shown in Fig. 3. The test sample’s value is equal to 32. Only $m(\emptyset)$ and $m(\{a\})$ are available for the GBPA value that is greater than 0. The GBPA of $m(\emptyset)$ decreases along with $m(\{a\})$ increases in the FOD. If the system supports Class a increasingly, it indicates that the system is increasingly convinced that the discriminated class belongs to Class a, rather than unknown class.

3) When the test sample is in the mixed triangular fuzzy number model of Class a and Class b, a nested set is generated. The value of $m(\{a\})$ and $m(\{b\})$ decreases or does not change as the value of $m(\{a,b\})$ increases. The value...
of \( m((a, b)) \) reaches the maximum when the test sample is at the intersection point of the two triangular fuzzy number models of Class \( a \) and Class \( b \). As shown in Fig. 5, the detected number of positive axillary nodes is equal to 7. The system considers that the recognized object is most likely to be belonged to \( \{a, b\} \). In the common interval of triangular fuzzy number models for Class \( a \) and Class \( b \), the value of \( m(\emptyset) \) is related to \( m(a) \) and \( m(b) \), and \( m((a, b)) \). And the relationship can be expressed as \( m(\emptyset) = 1 - m(a) - m(b) - m((a, b)) \).

### 4.2 Application in classification of Iris data set in the open-world assumption

In this part, the UCI data set of Iris \((D)\) is adopted to test the rationality and performance of the proposed method in classification problem with potential unknown class in the open-world assumption. In the Iris data set, there are three classes of Iris named Iris Setosa \((a)\), Iris Versicolor \((b)\), and Iris Virginica \((c)\). Each class has 50 samples and four attributes for each sample. The four attributes are named as sepal length (SL), sepal width (SW), petal length (PL), and petal width (PW). In this experiment, 40 samples are randomly chosen as the training set of each class. The maximum, minimum, and mean values of each attribute in the training set are chosen to construct the triangular fuzzy number model for each class. The rest of the samples are set as the test set to verify the rationality of the triangular fuzzy number model-based GBPA generation method.

The following part shows the process of (1) selecting a test sample from Class \( a \) to generate GBPAs. The sample's values on all four attributes (SL, SW, PL, and PW) are listed in Table 2.

For classification of the test sample, each attribute in the selected sample is considered as an independent information source. In this way, based on each attribute value of the test sample, GBPAs can be calculated according to the intersection of the attribute value in the triangular fuzzy number models. The intersection on the triangular fuzzy number models of the four attributes' values for the randomly selected test sample is shown in Fig. 6.

According to the new GBPA generation method, the GBPA values of each attribute (SL, SW, PL, and PW) for the test samples are calculated based on the intersection point with the triangular fuzzy number models. The GBPAs are calculated and shown in Table 3. It should be noted that, as shown in Fig. 6, if the attribute value is at the boundary of a triangular fuzzy number model, e.g., the value of SW is at the maximum of the triangular fuzzy number model of Class \( b \), the GBPA is 0.01, not 0.

After using the proposed method to generate GBPAs for each attribute of the sample, mGCR was used to fuse the GBPAs generated by the four groups of information sources in Table 3. The information fusion results are shown in Table 4.

It can be seen from the fusion results that the proposed method has a good performance. In the open-world assumption where there may be environmental interference factors such as unknown classes, it still assigns a very high belief on the correct Class \( a \), which means a good recognition effect. The experimental results in this section reflect the rationality and availability of the proposed method.

### 4.3 Classification of Iris data set with incomplete FOD

In this part, the FOD of the data set of Iris is divided into three cases for data modeling shown as follows:

\[
\Theta_1 = \{a, b\}, \Theta_2 = \{a, c\}, \Theta_3 = \{b, c\},
\]

(16)

where \( a \) is on behalf of Iris Setosa, \( b \) is on behalf of Iris Versicolor, and \( c \) is on behalf of Iris Virginica. For example, in the FOD \( \Theta_1 \), for each test sample, four sets of GBPAs will be determined according to their four attributes. If the actual class of the test sample is \( a \), then the maximum belief value should be assigned to \( m((a)) \). Since the Class \( b \) is not

### Table 1 Triangular fuzzy numbers model calculated from the training data set

| Class | SL     | SW         | PL         | PW         |
|-------|--------|------------|------------|------------|
| a     | (4.3000, 4.9975, 5.8000) | (2.3000, 3.4125, 4.2000) | (1.0000, 1.4650, 1.9000) | (0.1000, 0.2525, 0.6000) |
| b     | (4.9000, 5.9775, 7.0000) | (2.0000, 2.7750, 3.4000) | (3.0000, 4.2425, 5.0000) | (1.0000, 1.3275, 1.8000) |
| c     | (4.9000, 6.5050, 7.7000) | (2.2000, 2.9600, 3.8000) | (4.5000, 5.4850, 6.9000) | (1.5000, 2.0150, 2.5000) |

### Table 2 A randomly selected sample from Class \( a \)

| Attribute | SL     | SW         | PL         | PW         |
|-----------|--------|------------|------------|------------|
| Value     | 5.1    | 3.8        | 1.5        | 0.3        |
in the FOD $\Theta_1$, for a test sample whose actual class is $b$, a high belief value is assigned to $m(\emptyset)$, which is reasonable.

The results of the proposed method with three different FODs ($\Theta_1$, $\Theta_2$, and $\Theta_3$) are compared with each other. For each FOD, 40 samples are randomly selected from each class as the training set, and the remaining ten samples are used as the test set. In addition, ten samples are randomly selected from the “unknown class” and added to the test set to serve as the unknown factors in the open-world assumption. Each attribute of the Iris data set is still considered as an independent information source. After using mGCR to fuse all GBPAs of the test sample, a new piece of evidence is generated. The final decision rule in the new piece of evidence is to select the class with the maximum GBPAs generated from the test sample in the open world

| Attribute | $m(a)$ | $m(b)$ | $m(c)$ | $m(a, b)$ | $m(a, c)$ | $m(b, c)$ | $m(a, b, c)$ | $m(\emptyset)$ |
|-----------|--------|--------|--------|-----------|-----------|-----------|-------------|-----------|
| SL        | 0.680  | 0      | 0      | 0.061     | 0         | 0         | 0.133       | 0.126     |
| SW        | 0.503  | 0      | 0      | 0.010     | 0         | 0         | 0           | 0.487     |
| PL        | 0.920  | 0      | 0      | 0         | 0         | 0         | 0           | 0.080     |
| PW        | 0.865  | 0      | 0      | 0         | 0         | 0         | 0           | 0.135     |

Table 4 Data fusion result of four-attribute GBPAs for the test sample in the open-world assumption

| BPA       | $m(a)$ | $m(b)$ | $m(c)$ | $m(a, b)$ | $m(a, c)$ | $m(b, c)$ | $m(a, b, c)$ | $m(\emptyset)$ |
|-----------|--------|--------|--------|-----------|-----------|-----------|-------------|-----------|
| Value     | 0.9981 | 0      | 0      | 0         | 0         | 0         | 0           | 0.0019    |

Table 5 Classification results with the incomplete FOD $\Theta_1 = \{a, b\}$

| Sample    | Actual class | Ideal class | Sample number | Correct number | Accuracy  |
|-----------|--------------|-------------|---------------|----------------|-----------|
| sample$_1$| a            | a           | 20            | 16             | 80.00%    |
| sample$_2$| b            | b           | 20            | 20             | 100.00%   |
| sample$_3$| c            | $\emptyset$| 20            | 20             | 100.00%   |
| Total     | –            | –           | 60            | 56             | 93.30%    |

Fig. 6  The intersection of the randomly selected sample on the triangular fuzzy number model for each attribute

Table 5 Classification results with the incomplete FOD $\Theta_1 = \{a, b\}$

| Sample    | Actual class | Ideal class | Sample number | Correct number | Accuracy  |
|-----------|--------------|-------------|---------------|----------------|-----------|
| sample$_1$| a            | a           | 20            | 16             | 80.00%    |
| sample$_2$| b            | b           | 20            | 20             | 100.00%   |
| sample$_3$| c            | $\emptyset$| 20            | 20             | 100.00%   |
| Total     | –            | –           | 60            | 56             | 93.30%    |
4.4 Classification experiment on different data sets

In this part, the data sets of Seeds, Ecoli, User Knowledge Modeling, and Wholesale Customers are utilized to verify the new approach’s generalization performance in classification with different objects. Before data training, to avoid the impact of abnormal category, some categories of abnormal data are manually filtered out. For example, in the Ecoli data set, the class of inner membrane lipoprotein (imL) and outer membrane lipoprotein (omL) have only four samples, which is not enough to train a data model. In each data set, 70% of the data are used for training, and the rest are the test set. The confusion matrices of the classification are shown in Tables 8, 9, 10, and 11, respectively. With the proposed method, the accuracy of the experimental result for each data set is shown in Table 12. The recognition performance of the proposed method is 82% at least, indicating that the proposed method has a good generalization ability on different data sets.

5 Discussion

The generalization ability and downward compatibility of the proposed method are discussed in this section. In the FOD, if all the GBPAs are assigned on singleton subset propositions, the value of GBPA should be equal to the probability function in probability theory. Another example is that the GCR assigns \( m(\{\emptyset\}) = 0 \) in a closed-world assumption where GCR degenerates to the classical Dempster combination rule [46]. Similarly, the proposed method can also be applied to a system with a complete FOD. In the closed world, only the final step of the proposed method needs to be adjusted. For a complete FOD,

| Sample | Actual class | Ideal class | Sample number | Correct number | Accuracy |
|--------|--------------|-------------|---------------|----------------|----------|
| sample1 | a            | a           | 20            | 20             | 100%     |
| sample2 | b            | \emptyset   | 20            | 20             | 100%     |
| sample3 | c            | c           | 20            | 20             | 100%     |
| Total   | –            | –           | 60            | 60             | 100%     |

| Sample | Actual class | Ideal class | Sample number | Correct number | Accuracy |
|--------|--------------|-------------|---------------|----------------|----------|
| sample1 | a            | \emptyset   | 20            | 20             | 100.00%  |
| sample2 | b            | b           | 20            | 20             | 100.00%  |
| sample3 | c            | c           | 20            | 15             | 75.00%   |
| Total   | –            | –           | 60            | 55             | 91.60%   |

### Table 8 Classification results with the incomplete FOD \( \Theta_2 = \{a, c\} \)

| Class | Class 1 | Class 2 | Class 3 | Class \( \emptyset \) |
|-------|---------|---------|---------|------------------------|
| Class 1 | 17      | 0       | 1       | 3                     |
| Class 2 | 2       | 18      | 0       | 1                     |
| Class 3 | 2       | 0       | 18      | 1                     |

### Table 9 Classification results with the incomplete FOD \( \Theta_3 = \{b, c\} \)

| Class | Class \( cp \) | Class \( im \) | Class \( pp \) | Class \( imU \) | Class \( pp \) | Class \( \emptyset \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------------|
| Class \( cp \) | 37            | 2              | 1              | 1              | 0              | 2                    |
| Class \( im \) | 2              | 20             | 1              | 0              | 0              | 1                    |
| Class \( pp \) | 2              | 0              | 11             | 1              | 1              | 1                    |
| Class \( imU \) | 1              | 0              | 1              | 7              | 0              | 1                    |
| Class \( om \) | 1              | 1              | 0              | 0              | 4              | 0                    |

### Table 10 Classification results with the User Knowledge Modeling data set

| Class | Class \( high \) | Class \( medium \) | Class \( low \) | Class \( very low \) | Class \( \emptyset \) |
|-------|------------------|-------------------|----------------|----------------------|----------------------|
| Class \( high \) | 26               | 2                 | 1              | 0                    | 1                    |
| Class \( medium \) | 2                | 33                | 2              | 0                    | 2                    |
| Class \( low \) | 0                 | 2                 | 31             | 1                    | 1                    |
| Class \( very low \) | 2                | 1                 | 0              | 11                   | 0                    |

### Table 11 Classification results with the Wholesale Customers data set

| Class | Class 1 | Class 2 | Class \( \emptyset \) |
|-------|---------|---------|------------------------|
| Class 1 | 79      | 4       | 6                      |
| Class 2 | 4       | 36      | 2                      |
the GBPA value of \( m(\emptyset) \) should be averagely assigned to the \( 2^N - 1 \) (\( N \) is the cardinality of FOD) propositions on the power set space of FOD, which means that each event may happen with an equal probability in unknown situation.

### 5.1 Compatibility in classification of Iris data set in the closed world

The GBPA value of \( m(\{\emptyset\}) \) for each attribute in Table 3 is evenly distributed to other non-empty elements, so that the BPA is constructed in the closed world. The distribution results of BPAs in the closed world are shown in Table 13, where the fraction represents the belief value obtained from the GBPA value of \( m(\{\emptyset\}) \) in Table 3.

The evidence for different attributes in Table 13 is then fused by Dempster’s combination rule. And the evidence fusion results are shown in Table 14.

From the fusion results in Table 14, it can be seen that as there is no interference from external environmental factors, the proposed method assigns a higher belief value than that in the open world (\( m(\{a\}) = 1 \)) to the correct Class \( a \), which reflects the effectiveness of the new method in a complete FOD situation in the closed-world assumption. It is proved that the proposed method has a good downward compatibility. For practical applications in unstable environments, e.g., the switching between an incomplete FOD in the open-world assumption and a complete FOD in the closed world, the proposed method can always be available, which shows the availability, simplicity, and flexibility of this method.

### 5.2 Robust experiments with different proportions of training set in the closed world

In this subsection, the classification performance of the proposed method is tested by changing the proportion of the training data set in the closed world with a complete FOD. The performance of the proposed method with a changing proportion of training data set will be explored. The training data set will be changed from 10% to 98% among all the samples in the Iris data set to show the robustness of the proposed method. For different proportions of the training data set, the randomized hold-out method is used for ten times to calculate the average value of recognition accuracy. The experimental results are presented in Table 15 and Fig. 7.

Table 15 shows that the training proportion in the data set will be changed from 10 to 98%. The proposed method in a complete FOD can have a recognition accuracy between 62.50% and 91.67% even with the lowest training proportion 10% for the ten times experiments of hold-out method. The highest recognition accuracy is 100.00% in some cases with the training proportion of 74% and 94%. In Fig. 7, the abscissa is the training proportion, the ordinate is the No. \( n \) hold-out method for each training proportion, and the height means the recognition accuracy. In general, the experimental results of the ten times hold-out method for each training proportion show that a higher training proportion leads to a higher recognition accuracy. The average recognition accuracy for different training proportions is listed in Table 16.

The training proportion for the test data set has a positive effect on the precision of classification methods. There is a drawback in the hold-out method. If most of the samples in the data set is included in the training set, then the problem of overfitting may happen and the experimental result may be unstable or inaccurate. To address this shortcoming in the aforementioned experiment, the cross-validation strategy is adopted to supplement the experimental results in Table 16. The cross-validation strategy can improve the stability of the experimental results in comparison with the hold-out

---

**Table 12** Classification accuracy on different data sets

| Data set | Seeds | Ecoli | User Knowledge Modeling | Wholesale Customers |
|----------|-------|-------|-------------------------|---------------------|
| Accuracy | 84.13% | 82.30% | 85.60% | 87.79% |

**Table 13** BPA generated from the test sample in the closed-world assumption

| Attribute | \( m(\{a\}) \) | \( m(\{b\}) \) | \( m(\{c\}) \) | \( m(\{a,b\}) \) | \( m(\{a,c\}) \) | \( m(\{b,c\}) \) | \( m(\{a,b,c\}) \) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| SL        | 0.680 + 0.126  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  |
| SW        | 0.503 + 0.487  | 0.487 + 0.487  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  |
| PL        | 0.920 + 0.080  | 0.080 + 0.080  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  | 0.126 + 0.061  |
| PW        | 0.865 + 0.135  | 0.125 + 0.080  | 0.125 + 0.080  | 0.125 + 0.080  | 0.125 + 0.080  | 0.125 + 0.080  | 0.125 + 0.080  |

**Table 14** The fusion result of the test sample in the closed world

| BPA | \( m(\{a\}) \) | \( m(\{b\}) \) | \( m(\{c\}) \) | \( m(\{a,b\}) \) | \( m(\{a,c\}) \) | \( m(\{b,c\}) \) | \( m(\{a,b,c\}) \) |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Value | 0.9995 | 0.0002 | 0.0001 | 0.0001 | 0 | 0 | 0 |
strategy. As an example, the training proportion of 80% and 90% are chosen for verification of the cross-validation strategy. For these proportions, the Iris data set $D$ can be divided into $k$ subset where $k$ can be a positive integer in these proportions. The cross-validation experiments with ten times tenfold and ten times fivefold are finished independently. The calculation result also represents the classification precision of an interval $[-5, 5]$ nearby. The steps of

| Proportion (%) | No.1 (%) | No.2 (%) | No.3 (%) | No.4 (%) | No.5 (%) | No.6 (%) | No.7 (%) | No.8 (%) | No.9 (%) | No.10 (%) |
|---------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|
| 10            | 70.83    | 91.67    | 75.00    | 79.17    | 66.67    | 91.67    | 62.50    | 75.00    | 87.50    | 87.50     |
| 20            | 87.50    | 95.83    | 79.17    | 91.67    | 75.00    | 87.50    | 87.50    | 87.50    | 91.67    |           |
| 30            | 95.83    | 75.00    | 87.50    | 83.33    | 87.50    | 95.83    | 87.50    | 87.50    | 79.17    |           |
| 40            | 91.67    | 83.33    | 83.33    | 83.33    | 87.50    | 87.50    | 91.67    | 91.67    | 79.17    | 95.83     |
| 50            | 86.67    | 84.44    | 82.22    | 88.89    | 91.11    | 86.67    | 93.33    | 80.00    | 93.33    | 86.67     |
| 60            | 88.33    | 93.33    | 88.33    | 90.00    | 91.67    | 95.00    | 90.00    | 91.67    | 93.33    | 90.00     |
| 70            | 88.89    | 75.00    | 91.67    | 94.44    | 91.67    | 91.67    | 88.89    | 94.44    | 97.22    | 91.67     |
| 72            | 88.89    | 88.83    | 86.11    | 91.67    | 94.44    | 97.22    | 88.89    | 91.67    | 94.44    |           |
| 74            | 100.00   | 83.33    | 94.44    | 86.11    | 94.44    | 91.67    | 80.56    | 94.44    | 88.89    | 88.89     |
| 76            | 88.89    | 91.67    | 94.44    | 91.67    | 88.89    | 88.89    | 80.56    | 94.44    | 91.67    | 86.11     |
| 78            | 88.89    | 88.89    | 91.67    | 94.44    | 94.44    | 91.67    | 94.44    | 91.67    | 86.11    |           |
| 80            | 93.33    | 93.33    | 93.33    | 93.33    | 86.67    | 93.33    | 86.67    | 93.33    | 86.67    | 86.67     |
| 82            | 93.33    | 93.33    | 93.33    | 93.33    | 86.67    | 93.33    | 86.67    | 93.33    | 86.67    |           |
| 84            | 86.11    | 91.67    | 88.89    | 88.89    | 91.67    | 94.44    | 94.44    | 94.44    | 94.44    | 94.44     |
| 86            | 91.67    | 94.44    | 91.67    | 88.89    | 88.89    | 80.56    | 94.44    | 91.67    | 86.11    | 91.67     |
| 88            | 91.67    | 88.89    | 90.56    | 91.67    | 97.22    | 80.56    | 97.22    | 94.44    | 83.33    | 86.11     |
| 90            | 94.44    | 94.44    | 91.67    | 88.89    | 91.67    | 94.44    | 91.67    | 94.44    | 91.67    |           |
| 92            | 94.44    | 94.44    | 88.89    | 91.67    | 94.44    | 91.67    | 86.11    | 88.89    | 91.67    | 91.67     |
| 94            | 94.44    | 100.00   | 97.22    | 91.67    | 91.67    | 91.67    | 91.67    | 91.67    | 91.67    | 91.67     |
| 96            | 97.22    | 86.11    | 91.67    | 88.89    | 97.22    | 91.67    | 91.67    | 94.44    | 94.44    | 91.67     |
| 98            | 91.67    | 97.22    | 88.89    | 91.67    | 88.89    | 94.44    | 97.22    | 88.89    | 88.89    |           |

Fig. 7 The recognition accuracy of different proportions of the training set in the closed world
cross-validation experiment for ten times fivefold are listed as follows:

1. The Iris data set \( D \) is divided into five subsets denoted as \( D = D_1 \cup D_2 \cup D_3 \cup D_4 \cup D_5 \). Each subset is mutually exclusive and has the same size. The stratified sampling is used to keep the reasonable of the data for each subset.
2. Four subsets are used as the training data set, and the other subset is the test data set. After \( k \) times training and testing, the average value for all the experiment results can be calculated.
3. To get the reliable experiment results, repeat the previous step ten times. Finally, the average value can be calculated.
4. Improving the precision data of classification based on the hold-out experiment result and the cross-validation strategy. The new accuracy can be calculated as follows:

\[
Accuracy = \frac{\text{accuracy}_{\text{hold-out}} + \text{accuracy}_{\text{cross-validation}}}{2}.
\]

(17)

Table 17 lists the experiment results of the training proportion 80% and 90%. Similarly, the new accuracy calculation results for the training proportion from 76 to 94% are shown in Table 18.

Figure 8 concludes the recognition accuracy of the classification problem in the Iris data set with the improved method. The abscissa and ordinate are the training proportion of the data set and the recognition accuracy, respectively. In general, a higher training proportion means a higher recognition accuracy. When the number of training samples is sufficient, the proposed method can correctly recognize the unknown classes with a high accuracy. The proposed method also has a good effect for a low training proportion. Figure 8 shows that a training proportion of 10% can contribute to a high recognition accuracy of 78.75%. And a training proportion of 20% can lead to a recognition accuracy of 87.08%. The proposed method may have a good effect in practical applications with little training data. The efficiency and robustness of the proposed method are verified with the experimental results.

5.3 A discussion on Dempster combination rule and the negation of BPA

In this part, the possible drawback of Dempster combination rule in the proposed approach is discussed. In the end, a feasible resolution was put forward to address the potential drawback in classical evidence combination rule. Although the recognition accuracy of the proposed method is satisfactory in the closed-world assumption, the potential shortcoming of using the Dempster combination rule needs a cautious strategy. The reason for the flaw is that the Dempster combination rule is sensitive to \( m(\{X\}) = 0 (X \in 2^\Theta) \) in evidence fusion. Here is a typical case to explain the impact of the shortcoming. Assume there are two sensors for recognizing three classes named A, B, and C. The data obtained from the sensors are as follows:

### Table 16 The average accuracy of hold-out method with different training proportions in the closed world

| Training proportion | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 72% | 74% | 76% | 78% |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Average accuracy    | 78.75% | 87.08% | 86.67% | 87.50% | 87.33% | 91.17% | 90.57% | 90.28% | 90.28% | 89.72% | 91.39% |
| Training proportion | 80% | 82% | 84% | 86% | 88% | 90% | 92% | 94% | 96% | 98% |
| Average accuracy    | 90.67% | 89.72% | 91.11% | 90.00% | 89.17% | 90.83% | 90.56% | 93.33% | 92.50% | 91.67% |

### Table 17 The classification result based on the new accuracy calculation method

| Proportion (%) | No.1 (%) | No.2 (%) | No.3 (%) | No.4 (%) | No.5 (%) | No.6 (%) | No.7 (%) | No.8 (%) | No.9 (%) | No.10 (%) | Average accuracy (%) |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|----------------------|
| 80             | 92.00    | 92.00    | 89.33    | 91.33    | 89.33    | 91.33    | 90.67    | 90.00    | 91.33    | 90.67     | 90.80                |
| 90             | 91.33    | 92.00    | 92.00    | 91.33    | 91.33    | 90.67    | 92.00    | 91.33    | 91.33    | 92.00     | 91.53                |

### Table 18 The classification accuracy for different training proportion based on the hold-out and cross-validation method

| Training part (%) | 76 | 78 | 80 | 82 | 84 | 86 | 88 | 90 | 92 | 94 |
|-------------------|----|----|----|----|----|----|----|----|----|----|
| Modified accuracy (%) | 90.26 | 91.09 | 90.73 | 90.26 | 90.95 | 90.76 | 90.35 | 91.18 | 91.05 | 92.43 |
The fusion results obtained with Dempster combination rule are as follows:

Obviously, Dempster combination rule produces a result that is counterintuitive, which is unreasonable and unreliable. It can be seen that the classical Dempster combination rule cannot work when the evidence has a high conflict. To obtain more available information for conflict evidence fusion, the negation of BPA method [54, 60–62] can be a choice. The negation evidence provides information from the opposite perspective. Using negation of BPA from the original BPA, the following information can be obtained:

\[
\begin{align*}
m_1(\{A\}) &= 0.8, m_1(\{B\}) = 0, m_1(\{C\}) = 0.2, \\
m_2(\{A\}) &= 0, m_2(\{B\}) = 0.8, m_2(\{C\}) = 0.2.
\end{align*}
\] (18)

The fusion results obtained with Dempster combination rule are as follows:

\[
\begin{align*}
m(\{A\}) &= 0, m(\{B\}) = 0, m(\{C\}) = 1.
\end{align*}
\] (19)

5.4 The connection with the Bayesian methods

The evidence theory-based methods [63–65] and the Bayesian fusion methods [66–69] are both theoretical systems constructed based on the set theory. However, from a macro perspective, the Bayesian fusion method focuses on objects with discrete information, while the proposed method using the evidence theory focuses on objects with continuous information. This distinction contributes to the differences in certain application scenarios [63, 66–70]. From a micro point of view, due to the different attributes, they can be used in different classifiers for different cases. Some differences can be summarized as follows.

1. The Bayesian fusion method mainly supports the single proposition for the input information. So, the processing speed can be fast. While the proposed method can support multiple propositions, the processing time may be slow.

2. The Bayesian fusion method is easy to produce a situation where the posterior probabilities of the classifiers are equal, and it is difficult to explain which classifier should be trusted. The proposed method is not easy to cause this problem.

3. The Bayesian fusion method cannot point out the unknown element. The proposed method can point out the potential unknown elements.

4. The Bayesian fusion method uses the posterior probability to judge the results of a certain classifier with a belief of one hundred percent. The proposed method combines...
the results of each classifier with distinguishing the prior information.

6 Conclusions

In the framework of Dempster–Shafer evidence theory, this paper proposes a new approach that can generate both GBPA in the open world and BPA in the closed world. The proposed method is based on a multi-subset propositional representation model using the triangular fuzzy number. The proposed method enhances the ability to model incomplete information in the open-world assumption. Thus, it can detect more unknown information in the external environment and reduce the loss of effective information. Based on the experiments of the Iris data set and many other UCI data sets in the open-world assumption, the experimental result shows that the fusion result of the proposed method can be very distinguishable, and its recognition accuracy in practical applications has a promising performance. We discussed some characteristics and superiorities of the proposed method. The proposed method has a downward compatibility in the closed-world assumption as well as a good robustness performance in different experiments. In addition, even if the system is in an unfamiliar environment with a small number of training sample in the closed-world assumption, a training proportion of 20% can contribute to a recognition accuracy of 87.08% with the proposed method. The availability, rationality, and superiority of the proposed method are verified discussed according to the complex experiments.

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