Corrections to the hadron resonance gas from lattice QCD and their effect on fluctuation-ratios at finite density

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The ideal Hadron Resonance Gas (HRG) model simply sums over contributions from all known hadronic resonances:

\[ P(T, \hat{\mu}) = \sum_{i \in \text{hadrons}} P_i(T, \hat{\mu}) = -\frac{T}{V} \sum_{i \in \text{hadrons}} \ln Z_i(T, \hat{\mu}) \]

While overall successful in describing QCD thermodynamics, increased precision in lattice results suggest small deviations.

Due to charge conjugation symmetry, an expansion in the fugacity \( e^{\mu} = e^{(B\mu_B + S\mu_S)} \) yields:

\[ P(T, \hat{\mu}_B, \hat{\mu}_S) \simeq \sum_{i \in \text{hadrons}} P_i(T) e^{(B_i \hat{\mu}_B + S_i \hat{\mu}_S)} = \sum_{j,k} P_{BS}^{jk}(T) \cosh(j \hat{\mu}_B - k \hat{\mu}_S) \]

In an ideal hadron gas, only nonzero contributions come from:

- \( P_{00}^{BS} \): \( \pi, f, \omega, \rho, \ldots \)
- \( P_{01}^{BS} \): \( K, \ldots \)
- \( P_{10}^{BS} \): \( p, n, \Delta, \ldots \)
- \( P_{11}^{BS} \): \( \Lambda, \Sigma, \ldots \)
- \( P_{12}^{BS} \): \( \Xi, \ldots \)
- \( P_{13}^{BS} \): \( \Omega, \ldots \)

In general, additional terms exist, due to composite or exotic objects, or interactions

- \( P_{20}^{BS} \): \( p-p, \ldots \)
- \( P_{21}^{BS} \): \( p-\Lambda, \ldots \)
- \( P_{02}^{BS} \): \( K-K, \ldots \)
- \( P_{22}^{BS} \): \( \Lambda-\Lambda, p-\Xi \ldots \)
- \( P_{1, -1}^{BS} \): pentaquark (e.g. \( uu\bar{d}\bar{d} \)), \ldots
- \( P_{12}^{BS} \): \( \Xi-\Xi, \ldots \)
- \( P_{13}^{BS} \): \( \Omega-\Omega, \ldots \)
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From fugacity expansion to Fourier expansion

Lattice simulations, not currently possible at finite (real) chemical potential, can be done at imaginary chemical potential \( (\hat{\mu}_a = i \hat{\mu}_a^I) \). In this case, one gets a Fourier expansion:

\[
P(T, \hat{\mu}_B, \hat{\mu}_S) = \sum_{j,k} P^{BS}_{jk}(T) \cos(j \hat{\mu}_B^I - k \hat{\mu}_S^I)
\]

We performed a 2D scan in the chemical potentials plane at \( (\mu_B, \mu_S) = \frac{\pi}{8}(i, j) \), with \( i = 0, \ldots, 15, j = 0, \ldots, 8 \) (144 points) for \( T = 145, 150, 155, 160 \) MeV.
The fugacity/Fourier expansion coefficients

We calculated different observables containing the $P_{ij}^{BS}(T)$, temperature-by-temperature, at all simulation points:

\[
\begin{align*}
\text{Im } \chi_{10}^{BS} & = \sum_{j,k} j P_{jk}^{BS}(T) \sin(j \hat{\mu}^I_B - k \hat{\mu}^I_S) \\
\text{Im } \chi_{01}^{BS} & = \sum_{j,k} (-k) P_{jk}^{BS}(T) \sin(j \hat{\mu}^I_B - k \hat{\mu}^I_S) \\
\chi_{20}^{BS} & = \sum_{j,k} j^2 P_{jk}^{BS}(T) \cos(j \hat{\mu}^I_B - k \hat{\mu}^I_S) \\
\chi_{11}^{BS} & = \sum_{j,k} (-jk) P_{jk}^{BS}(T) \cos(j \hat{\mu}^I_B - k \hat{\mu}^I_S) \\
\chi_{02}^{BS} & = \sum_{j,k} k^2 P_{jk}^{BS}(T) \cos(j \hat{\mu}^I_B - k \hat{\mu}^I_S)
\end{align*}
\]

At each $T$, we perform a global fit to determine the $P_{ij}^{BS}(T)$.

- We see in general a mild $N_\tau$-dependence, meaning discretization errors are under control
- At most temperatures, most contributions are non-zero. \textbf{Interactions and exotic states do play a role.}
- We can then reconstruct the pressure and its derivatives also at finite (real) $\hat{\mu}_B$
Fugacity expansion: fluctuation ratios at real $\hat{\mu}_B$

i. **A proxy for baryon-strangeness correlation:** In PRD 101 034506 we proposed $\sigma_A^2 / (\sigma_K^2 + \sigma_A^2)$ to be a good proxy for $\chi_{11}^{BS}/\chi_2^S$, even after including some experimental effects. The inclusion of multi-kaon interaction terms $P_{02}^{BS}$, $P_{03}^{BS}$ has a small effect, reinforcing predictions from ideal hadronic model.

ii. **Comparison with STAR data:** We are able to compare to $\chi_3^B/\chi_1^B$ and $\chi_4^B/\chi_2^B$ at different energies. Results are consistent with Taylor expansions (PRD 101 074502 from HotQCD).

Notably, fluctuations in the experiment seem not to deviate largely from QCD matter in equilibrium.