THE EFFECT OF THE PAIRING INTERACTION ON THE ENERGIES OF ISOBAR ANALOG RESONANCES IN $^{112-124}$Sb AND ISOSPIN ADMIXTURE IN $^{100-124}$Sn ISOTOPES

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In the present study, the effect of the pairing interaction and the isovector correlation between nucleons on the properties of the isobar analog resonances (IAR) in $^{112-124}$Sb isotopes and the isospin admixture in $^{100-124}$Sn isotopes is investigated within the framework of the quasiparticle random phase approximation (QRPA). The form of the interaction strength parameter is related to the shell model potential by restoring the isotope invariance of the nuclear part of the total Hamiltonian. In this respect, the isospin admixtures in the $^{100-124}$Sn isotopes are calculated, and the dependence of the differential cross section and the volume integral $J_F$ for the Sn($^3$He,t)Sb reactions at $E(^3$He$)=200$ MeV occurring by the excitation of IAR on mass number $A$ is examined. Our results show that the calculated value for the isospin mixing in the $^{100}$Sn isotope is in good agreement with Colo et al.’s estimates ($4 - 5\%$), and the obtained values for the volume integral change within the error range of the value reported by Fujiiwara et al. ($53\pm5 \text{ MeV fm}^3$). Moreover, it is concluded that although the differential cross section of the isobar analog resonance for the ($^3$He,t) reactions is not sensitive to pairing correlations between nucleons, a considerable effect on the isospin admixtures in $N\approx Z$ isotopes can be seen with the presence of these correlations.

1. Introduction

With the development of the heavy mass ion accelerators technology, many isotopes being proton rich within the range of $A=80-100$ have been established. The studies on such isotopes, far away from the beta stability valley, are very useful in understanding many nuclear aspects. The investigation on the isospin admixture effect in the ground state of these proton
rich nuclei can be considered as one of the possible candidate studies. It is well known that the isospin admixture effects of the nuclear states are very important in the estimates of the effective vector coupling constants based on Fermi transitions, and in the description of the energies and the widths of the analog states and the isospin multiplets [1-4].

The isospin mixing is basically caused by the Coulomb potential, or more accurately by that part of Coulomb potential which changes over the nuclear volume. Since the very small symmetry energy tries to minimize the difference in the proton and the neutron systems, Coulomb forces are more dominant in nuclei for which the proton and the neutron numbers are close to each other. Therefore, the isospin mixing can take large values in the ground state of the proton rich nuclei.

The physics of the exotic nuclei, characterized by the very unusual ratios of the neutrons and the protons, can be considered as a test for the already well-established models of the nuclear structure that are used to describe the stable systems. Since the parameters and the interactions used in the usual shell-model calculations are determined in order to reproduce the properties of the known nuclei, they may not always be appropriate for use in the calculation of the nuclei approaching to the drip lines.

The isospin admixtures in the nuclear ground states have been calculated in various models. The two liquid hydrodynamic model is used to estimate the energies of the collective isovector monopole excitation with an isospin of $T = T_0 + 1$ [5]. The admixture of this excited state to the ground state with an isospin of $T = T_0$, caused by the Coulomb potential, turned out to be small $\approx (0.1 - 0.3)$% for all stable nuclei with $A > 40$. Quantitative estimates performed by using the shell model [6-8] are approximately an order of magnitude larger than the estimate of Bohr and Mottelson. There are several effects which may cause such a difference in the mentioned estimates. First, the shell model calculations were performed in the particle-hole approximation and used a limited number of configurations. Second, the residual isovector interaction was either neglected, or it was included in the Tamm-Dankoff approximation. Third, the residual interaction was not related to the shell-model potential in the self-consistent way. The residual isovector forces in separable form are derived for a given form of the shell-model potential [9]. The derivation is based on the isotopic invariance of the nuclear forces. The form of the interaction strength parameters is related to the potential given by the self-consistency relations. It has been shown that the $T = T_0 + 1$ isospin admixture in the ground state of the parent nucleus can be determined by the sum of the square of the nuclear matrix elements for beta decay $(N + 1, Z - 1)$ [9]. The absolute values of the admixtures, however, are in some cases several times larger than the hydrodynamic estimate. Theoretical calculations for
the isospin mixing in proton rich nuclei between $A = 80 - 100$ have been performed [10-20]. The Hartree-Fock calculations for the very proton-rich nuclei in the region around $A = 80 - 100$ predict the isospin mixing in an order of $3 - 5\%$ [13]. With the inclusion of the random phase approximation in the calculations, the isospin mixing increased by an amount of $15 - 20\%$ [10]. These estimates are still 2-3 times larger than the Bohr and Mottelson ones [5]. The pairing correlations between nucleons were not considered in all studies mentioned above. However, it has been stated in Ref. [10] that these correlations could be of primary importance for medium and heavy nuclei.

In the present study, based on the spherical single-particle wave functions and energies with the pairing and the residual isovector interaction treated in QRPA, the isospin admixtures in the ground states of the $^{100-124}$Sn isotopes and the differential cross sections for the charge-exchange reactions ($^3$He,t) at $E(3\text{He}) = 200$ MeV occurring by the excitation of the IAR states in the $^{112-124}$Sb were investigated. In the calculations, considering the restoration of the isotopic invariance for the nuclear part of the total Hamiltonian [9,21] the effective interaction strength ($\gamma_\rho$) in the quasiparticle space has been obtained in such a way that it is self consistent with the Woods-Saxon form of the shell model potential. Thus, this method makes our model free of any adjustable parameters.

2. Hamiltonian

Let us consider a system of nucleons in a spherical symmetric average field interacting via pairing forces. In this case, the corresponding single quasiparticle Hamiltonian of the system is given by

$$\hat{H}_{\text{sqp}} = \sum_{\tau,j,m} \varepsilon_{\tau}^{(j)} \alpha_{jm}^\dagger(\tau)\alpha_{jm}(\tau), \quad (\tau = n,p), \quad (1)$$

where the $\varepsilon_{\tau}^{(j)}$ is the single quasiparticles energy of the nucleons with angular momentum $j$, and the $\alpha_{jm}^\dagger(\tau)(\alpha_{jm}(\tau))$ is the quasiparticle creation (annihilation) operator. The shell model potential contains the isovector terms in nuclei having different proton and neutron numbers in addition to the Coulomb potential and proton-neutron mass difference which breaks the isotopic invariance (if the proton-neutron mass difference is neglected)

$$\left[\hat{H}_{\text{sqp}} - V_c, \hat{T}^{\rho}\right] \neq 0, \quad (2)$$
where $V_c$ is the Coulomb potential given by the following expression:

$$V_c = \sum_{i=1}^{A} v_c(r_i) \left( \frac{1}{2} - t_z^i \right), \quad t_z^i = \begin{cases} 1/2, & \text{for neutrons;} \\ -1/2, & \text{for protons.} \end{cases}$$

(3)

with the radial part of the Coulomb potential

$$v_c(r) = \frac{e^2(Z - 1)}{Z} \int \frac{\rho_p(r')}{|r - r'|} d^3r'.$$

(4)

Here, $\rho_p(r')$ is to the proton density distribution in ground state. The isospin operators $\hat{T}^\rho$ are defined in the following way:

$$\hat{T}^\rho = \frac{1}{2} \left[ \hat{T}_+ + \rho \hat{T}_- \right] = \begin{cases} \hat{T}_x, & \rho = +1; \\ i\hat{T}_y, & \rho = -1. \end{cases}, \quad \hat{T}_\pm = \sum_{k=1}^{A} \hat{t}_\pm^k,$$

(5)

where $\hat{t}_\pm^k = \hat{t}_x^k \pm i\hat{t}_y^k$ are the raising and the lowering isospin operators. The broken symmetry should be restored by the residual interaction $\hat{h}$ which obey the following equation [9,21]

$$\left[ \hat{H}_{s qp} + \hat{h} - V_c, \hat{T}^\rho \right] = 0,$$

(6)

where $\hat{h}$ is defined as [22]

$$\hat{h} = \sum_{\rho = \pm 1} \frac{1}{A\gamma_\rho} \left[ \hat{H}_{s qp} - V_c, \hat{T}^\rho \right]^\dagger \left[ \hat{H}_{s qp} - V_c, \hat{T}^\rho \right].$$

(7)

$\gamma_\rho$ is an average of double commutator in the ground state,

$$\gamma_\rho = \frac{\rho}{2} \left\langle 0 \left| \left[ \left[ \hat{H}_{s qp} - V_c, \hat{T}^\rho \right], \hat{T}^\rho \right] \right| 0 \right\rangle.$$

(8)

Such a form of the residual interaction allows us to treat the Coulomb mixing effects of the isospin simply and self-consistently. Let us note that the derivation of Eq. (7) uses only one assumption, namely the separability of the residual interactions. Thus, the restoration of the isotopic invariance for the nuclear part of the Hamiltonian is satisfied, and the total Hamiltonian operator can be written in the form of

$$\hat{H} = \hat{H}_{s qp} + \hat{h}.$$ 

(9)
3. Isobar Analog States

In this section, we shall consider the isobaric $0^+$ excitations in odd-odd nuclei generated from the correlated ground state of the parent even-even nucleus by the charge-exchange forces and use the eigenstates of the single quasiparticle Hamiltonian $\hat{H}_{sqp}$ as a basis. The basis set for the neutron-proton quasiparticle pair creation $\hat{A}_{jn,jp}^\dagger$ and annihilation $\hat{A}_{jn,jp}$ operators is defined as:

$$\hat{A}_{jn,jp}^\dagger = \frac{1}{\sqrt{2jn+1}} \sum_m (-1)^{jn-m} \alpha_{jn,m}^\dagger \alpha_{jn,-m}^\dagger,$$  \hspace{1cm} (10)

$$\hat{A}_{jn,jp} = \left(\hat{A}_{jn,jp}^\dagger\right)^\dagger.$$

The bosonic commutation relations of these operators in quasi-boson approximation [10] are given by

$$\left[\hat{A}_{jn,jp}, \hat{A}_{jn,jp}'\right] \approx \delta_{jn,jn} \delta_{jp,jp'}, \quad \left[\hat{A}_{jn,jp}, \hat{A}_{jn,jp}'\right] = 0. \hspace{1cm} (11)$$

In the quasiparticle space, the effective interaction $\hat{h}$ and the average of the double commutator $\gamma_\rho$ can be written in the form of

$$\hat{h} = - \sum_{jn,jp,jn',jp'} \frac{1}{4\chi_\rho} E_{jn,jp}^\rho E_{jn,jp'}^\rho \left( \hat{A}_{jn,jp} - \rho \hat{A}_{jn,jp}^\dagger \right) \left( \hat{A}_{jn,jp'} - \rho \hat{A}_{jn,jp'}^\dagger \right), \hspace{1cm} (12)$$

$$\chi_\rho = -\gamma_\rho = \sum_{jn,jp} b_{jn,jp}^\rho b_{jn,jp}^\rho , \quad b_{jn,jp}^\rho = \frac{1}{2} (\bar{b}_{jn,jp} + \rho \bar{b}_{jn,jp}) ,$$

with

$$E_{jn,jp}^\rho = \left\{ \varepsilon_{jn,jp} b_{jn,jp}^\rho + \frac{1}{2} \left( \varphi_{jn,jp} - \rho \varphi_{jn,jp} \right) \right\} ,$$

$$\varepsilon_{jn,jp} = \varepsilon_{jn} + \varepsilon_{jp} ,$$

$$b_{jn,jp} = \sqrt{2jp+1} \left\langle j_p \parallel \hat{T}_- \parallel j_n \right\rangle u_{jp} v_{jn} ,$$

$$\bar{b}_{jn,jp} = \sqrt{2jp+1} \left\langle j_p \parallel \hat{T}_- \parallel j_n \right\rangle u_{jn} v_{jp} ,$$

$$\varphi_{jn,jp} = \sqrt{2jp+1} \left\langle j_p \parallel \hat{T}_- v_c(r) \parallel j_n \right\rangle u_{jp} v_{jn} ,$$

$$\varphi_{jn,jp} = \sqrt{2jp+1} \left\langle j_p \parallel \hat{T}_- v_c(r) \parallel j_n \right\rangle u_{jn} v_{jp} ,$$

$$\varphi_{jn,jp} = \left[ \varphi_{jn,jp}^2 + (E_{j\tau} - \lambda_{j\tau})^2 \right]^{1/2} , \hspace{1cm} (13)$$
\[
\begin{align*}
    u_{j_\tau} &= \left[ \frac{1}{2} \left( 1 + \frac{E_{j_\tau} - \lambda_\tau}{\varepsilon_{j_\tau}} \right) \right]^{1/2}, \\
    v_{j_\tau} &= \left[ \frac{1}{2} \left( 1 - \frac{E_{j_\tau} - \lambda_\tau}{\varepsilon_{j_\tau}} \right) \right]^{1/2},
\end{align*}
\]
where \( C_\tau, E_{j_\tau} \) and \( \lambda_\tau \) correspond to the correlation function, the single particle energies for nucleons and the chemical potential, respectively. \( v_{j_\tau}, u_{j_\tau} \) is the occupation (unoccupation) amplitude. In QRPA, the collective \( 0^+ \) states, generated by the effective interaction \( \hat{h} \), are accepted as one-phonon excitations and can be described as follows:

\[
|\psi_i\rangle = \hat{Q}_i^\dagger |0\rangle = \sum_{j_n,j_p} \left( r_{j_n,j_p}^i \hat{A}_{j_n,j_p}^\dagger - s_{j_n,j_p}^i \hat{A}_{j_n,j_p} \right) |0\rangle,
\]

where \( r_{j_n,j_p}^i \) and \( s_{j_n,j_p}^i \) are the amplitudes for the neutron-proton quasiparticle pair, \( \hat{Q}_i^\dagger \) is the phonon creation operator, and \( |0\rangle \) is the phonon vacuum corresponding to the ground state of the even-even nucleus, i.e.

\[
\hat{Q}_i |0\rangle = 0.
\]

Assuming that the phonon operators obey the commutation relations given below

\[
\langle 0 \left| \left[ \hat{Q}_i, \hat{Q}_j^\dagger \right] \right| 0 \rangle = \delta_{ij}, \quad \langle 0 \left| \left[ \hat{Q}_i, \hat{Q}_j \right] \right| 0 \rangle = 0,
\]
we obtain the following orthonormalization condition for the amplitudes:

\[
\sum_{j_n,j_p} \left( r_{j_n,j_p}^i r_{j_n,j_p}^{i'} - s_{j_n,j_p}^i s_{j_n,j_p}^{i'} \right) = \delta_{ii'}.
\]

The eigenvalues and the eigenfunctions of the restored Hamiltonian (9) can be obtained by solving the equation of motion in QRPA

\[
\left[ \hat{H}_{sqp} + \hat{h}, \hat{Q}_i^\dagger \right] |0\rangle = \omega_i \hat{Q}_i^\dagger |0\rangle.
\]

Here the \( \omega_i \)'s are the energies of the isobaric \( 0^+ \) states. Employing the conventional procedure of QRPA, we obtain the dispersion equation for the excitation energy of the isobaric \( 0^+ \) states as follows:

\[
\begin{multline}
\lambda + 1 - \sum_{j_n,j_p} \frac{\varepsilon_{j_n,j_p} \left( E_{j_n,j_p}^{+1} \right)^2}{\varepsilon_{j_n,j_p}^2 - \omega_i^2} \left( \chi + 1 - \sum_{j_n,j_p} \frac{\varepsilon_{j_n,j_p} \left( E_{j_n,j_p}^{-1} \right)^2}{\varepsilon_{j_n,j_p}^2 - \omega_i^2} \right) \\
- \omega_i^2 \sum_{j_n,j_p} \left[ \frac{\varepsilon_{j_n,j_p} E_{j_n,j_p}^{+1} E_{j_n,j_p}^{-1}}{\varepsilon_{j_n,j_p}^2 - \omega_i^2} \right]^2 = 0.
\end{multline}
\]
The amplitudes for the neutron-proton quasiparticle pair can then be expressed analytically in the following form:

\[ r_{jn,jp}^i = \frac{1}{\sqrt{Z(\omega_i)}} \left( \frac{E_{jn,jp}^{-1} + L(\omega_i)E_{jn,jp}^1}{\varepsilon_{jn,jp} - \omega_i} \right), \]

\[ s_{jn,jp}^i = \frac{1}{\sqrt{Z(\omega_i)}} \left( \frac{E_{jn,jp}^1 - L(\omega_i)E_{jn,jp}^{-1}}{\varepsilon_{jn,jp} + \omega_i} \right), \]

(20)

with

\[ L(\omega_i) = \chi - \sum_{jn,jp} \frac{\varepsilon_{jn,jp}(E_{jn,jp}^1)^2}{\varepsilon_{jn,jp}^2 - \omega_i^2}, \]

\[ \omega_i \sum_{jn,jp} \frac{\varepsilon_{jn,jp}E_{jn,jp}^1E_{jn,jp}^{-1}}{\varepsilon_{jn,jp}^2 - \omega_i^2}. \]

The quantity \( Z(\omega_i) \) is determined from the normalization condition given in Eq. (17).

An analog state is contained in the solutions of the Eq. (19). This is easily seen in case of the constant Coulomb potential

\[ \langle j_p \| \hat{t}_- v_c(r) \| j_n \rangle = \Delta E_C \langle j_p \| \hat{t}_- \| j_n \rangle, \quad \Delta E_C = \text{constant}. \]

(21)

The equation (19) now contains the solution \( \omega_k = \Delta E_C \), corresponding to the average energy of the single quasi-particle Coulomb shift of the nuclei \((N,Z)\) and \((N-1,Z+1)\). From Eqs. (14), (17) and (20), it follows that

\[ \hat{Q}_k^\dagger \mid 0 \rangle_{\omega_k=\Delta E_C} = \frac{1}{\sqrt{2T_0}} \hat{t}_- \mid 0 \rangle, \]

(22)

i.e., this solution describes the IAR state. Because Eq. (16) is fulfilled, the isospin is exactly conserved in all states. This latter fact is a natural consequence of the isospin invariance of the self-consistent nuclear Hamiltonian.

4. Fermi Beta Transitions

It has been shown in Ref. [9] that if the pairing correlations between nucleons are not considered, two independent isobaric excited \( 0^+ \) states occur from the ground state of parent nucleus: the \( T_z = T_0 - 1 \) states including Isobar Analog Resonance (IAR) in the nucleus with the number \((N-1,Z+1)\) (the isotopic spin of the parent nucleus is assumed as \( T = T_0 \)), and the \( T_z = T_0 + 1 \) excited states in the nucleus with the number \((N+1,Z-1)\). It must also be noted that both branches in question are
dependent on the mother nucleus due to the Fermi transition and the matrix elements for the $\beta^\pm$ transitions obeying Fermi sum rule.

Let us now mention how the above situation changes with the presence of the pairing forces. First of all, there will be no two independent isobaric $0^+$ excited states, and these states will occur in both nuclei with the number $(N-1, Z+1)$ and $(N+1, Z-1)$ due to the violation of particle number conservation. Secondly, the total probability of the $\beta$ transition from both nuclei $(N+1, Z-1)$ and $(N-1, Z+1)$ to the parent nucleus $(N,Z)$ increases in such a way that the Fermi sum rule is fulfilled.

The isobaric $0^+$ states in the neighbor odd-odd nuclei $(N-1, Z+1, N+1, Z-1)$ is characterized by the Fermi transition matrix elements between these states and the ground state of the neighbor even-even nuclei [4]. One could obtain the following Fermi transition matrix elements by using the wave functions in Eq.(14):

$$M_{\beta^-}^i = \langle 0 \left| \hat{Q}_i, \hat{T}_- \right| 0 \rangle = \sum_{j,n,j_p} \left( r_{j_n,j_p}^i b_{j_n,j_p} + s_{j_n,j_p}^i \overline{b}_{j_n,j_p} \right), \quad (23)$$

$$M_{\beta^+}^i = \langle 0 \left| \hat{Q}_i, \hat{T}_+ \right| 0 \rangle = \sum_{j,n,j_p} \left( r_{j_n,j_p}^i \overline{b}_{j_n,j_p} + s_{j_n,j_p}^i b_{j_n,j_p} \right). \quad (24)$$

It is possible to show that the transitions in question obey the Fermi sum rule

$$\sum_i \left\{ |M_{\beta^-}^i|^2 - |M_{\beta^+}^i|^2 \right\} = 2T_0 = N - Z. \quad (25)$$

In case of the constant Coulomb potential (21), only the matrix elements of the analog resonance $\beta$ decay is non-vanishing, and $M_{\beta^-}^i (\omega_i = \Delta E_C) = \sqrt{2T_0}$. This matrix element exhausts the full strength of the Fermi transition. The separation of the IAR state appearing in Eqs. (19-22) from the other states and the collection of all the $\beta$ transition strength on this state are the evidence for the conservation of the isotopic invariance for the nuclear part of the shell model Hamiltonian.

5. Isospin Structure of the Ground State for the Parent Nuclei

When the isospin structure of the ground state for the nuclei considered in the present study is investigated, it is observed that the isospin impurity of the ground states and IAR are related to the matrix elements of the $\beta^\pm$
transitions ($M_{\beta\pm}$) [5]. Expanding the ground state wave function in terms of the pure isospin components $|T, T_z\rangle$, we obtain

$$|0\rangle = a|T_0, T_0\rangle + b|T_0 + 1, T_0\rangle, \quad a^2 + b^2 = 1. \quad (26)$$

This expansion states that the ground state of the parent nucleus will contain only the $T_0 + 1$ isospin admixtures caused by the isovector Coulomb potential. The expectation value of the square of the isospin in the ground state of the parent nucleus can be expressed as

$$\langle 0 | \hat{T}^2 | 0 \rangle = T_0(T_0 + 1) + \sum_i |M^{i+}_\beta|^{2}. \quad (27)$$

On the other hand, the following expression for the same quantity can be obtained from Eq. (26)

$$\langle 0 | \hat{T}^2 | 0 \rangle = T_0(T_0 + 1) + 2b^2(T_0 + 1). \quad (28)$$

From Eqs. (27) and (28), it follows that the $T_0 + 1$ isospin admixture in the ground state of the parent nucleus is determined by the sum of the squares of the beta decay matrix elements from the isobaric states of the nucleus $(N + 1, Z - 1)$:

$$b^2 = [2(T_0 + 1)]^{-1} \sum_i |M^{i+}_\beta|^{2}. \quad (29)$$

This result obtained in Ref [9] is given over the sum of the $\beta^+$ transition matrix elements from the parent nucleus to the neighbor odd-odd ($N + 1, Z - 1$) nuclei. However, in the similar studies, the $T_0 + 1$ isospin admixture is usually determined by the Coulomb mixing of the ground state with the isovector monopole excited states in the same even-even nucleus [13,23].

6. Differential Cross Sections of $^{112–124}$Sn($^3$He,t)$^{112–124}$Sb Reactions

If the nucleon motion of the $^3$He beam is much faster than the relative Fermi motion of the nucleons in $^3$He, the ($^3$He,t) reaction is expected to become simple like the (p,n) reaction at intermediate energies. The experiment in Ref. [24] indicates that the complex contribution from the Fermi motion of the nucleons in $^3$He and in the triton is negligible in the charge-exchange reactions at the bombarding energy around 150 MeV/nucleon.

The differential cross sections of zero degrees for the excitation of the IAR can be written as [25,26]

$$\left(\frac{d\sigma}{d\Omega}\right)_F (q \approx 0, \quad \theta = 0) = \left(\frac{\mu}{\pi \hbar^2}\right)^2 \left(\frac{k_f}{k_i}\right)^2 N_F J^2_F B(F), \quad (30)$$
where $J_F$ is the volume integral of the central part of the effective interaction, $N_F$ is the distortion factor which may be approximated by the function $\exp(-x A^{1/3})$ [26], $\mu$ and $k$ denote the reduced energy divided by $c^2$ and the wave number in the center of mass system, respectively. The value of $x$ is taken from Ref. [24] and $B(F) = \left| M_{\beta^{- IAR}}^{i} \right|^2$ is the reduced matrix elements.

7. Results and Discussions

In this section, the isospin admixtures for the $^{100-124}$Sn isotopes, the IAR energies in the $^{112-124}$Sb isotopes, and the differential cross section for the Sn($^3$He,t)Sb reactions at $E(^3$He)$=200$ MeV occurring by the excitation of the IAR state were numerically calculated by considering the pairing interaction between nucleons. In the calculations, the Woods-Saxon Potential with Chepurnov parametrization [27] was used, and the correlation function ($C_n = 12/\sqrt{A}$) was chosen in accordance with Ref. [23]. The basis used in our calculation contain all neutron-proton transitions which change the radial quantum number $n$ by $\Delta n = 0, 1, 2, 3$. The left-hand side of the sum-rule in Eq.(25) containing the overlap integrals $\langle n||p \rangle$ is fulfilled with the approximately $\sim 1\%$ accuracy.

Our results for the $T_0 + 1$ isospin admixtures in the ground state of the $^{100-124}$Sn isotopes with (solid line) and without (dotted line) pairing correlations between nucleons have been presented in Fig. 1. As seen from Fig. 1, the admixture amplitude ($b^2$) decreases with the increasing N-Z value in both cases since according to Pauli principle the number of the $\beta^+$ transitions with $\Delta n \neq 0$ which makes a considerable contribution to the sum in Eq. (27) will decrease as the neutron number increases. It can also be seen obviously from this figure that the pairing correlations between nucleons lead to shift the isospin admixture values ($b^2$) to the lower ones. This effect is more pronounced in case of the small N-Z values although it vanishes for the large N-Z values. This is an expected result because these pairing correlations are more dominant when the isovector potential is small. The nearness of the isospin admixture values for the $^{100}$Sn isotope are due to the weak effect of the pairing correlations in the closed shell nuclei.

In Fig. 2, our model results (solid line) for the quantity $b^2$ in $^{100-124}$Sn isotopes have been compared with the hydrodynamic model results [23] (dashed line) and the values calculated by using the formula given in Eq. (11) of Ref. [13] (dotted line). Our results are closer to those given in Ref. [13] and two times larger than the hydrodynamic models for the $^{100}$Sn isotope. The difference between the results of the different models diminishes with an increasing mass number $A$. The calculated results shown in Fig. 2 are also numerically given in Table 1.
Fig. 1. Dependence of the contribution of the $T_{\alpha} + 1$ isomultiplet states to the ground of even-even Sn isotopes on mass number $A$. The solid and dashed lines correspond to the cases with and without the pairing forces, respectively.

Table 1. The numerical results for the quantity $b^2$ obtained by the different models and our model in $^{100-124}$Sn isotopes

| $A$  | Bohr and Mottelson [23] | Calculation results of Ref. [13] | This work |
|------|-------------------------|---------------------------------|-----------|
| 100  | 1.885                   | 4.067                           | 4.100     |
| 102  | 0.955                   | 1.908                           | 2.037     |
| 104  | 0.645                   | 1.197                           | 1.421     |
| 106  | 0.490                   | 0.847                           | 0.930     |
| 108  | 0.397                   | 0.641                           | 0.702     |
| 110  | 0.334                   | 0.506                           | 0.691     |
| 112  | 0.290                   | 0.411                           | 0.543     |
| 114  | 0.257                   | 0.343                           | 0.462     |
| 116  | 0.231                   | 0.290                           | 0.398     |
| 118  | 0.211                   | 0.249                           | 0.356     |
| 120  | 0.194                   | 0.217                           | 0.306     |
| 122  | 0.179                   | 0.190                           | 0.276     |
| 124  | 0.167                   | 0.168                           | 0.248     |
The dependence of the IAR energies for the $^{100-124}$Sb isotopes on mass number $A$ has been shown in Fig. 3. The IAR energies were calculated from the ground state of the Sb isotopes. As seen from Fig. 3, the pairing forces try to shift the IAR energies to the lower values in all Sb isotopes studied here. The corresponding energy shift for the $^{104-122}$Sb isotopes changes from 529 keV to 53 keV. The pairing correlations between nucleons play an important role on the IAR energy shift (up to 0.5 MeV) in the light isotopes as it was in the case of the isospin admixture ($b^2$). In Table 2, the calculated values of the IAR energies for $^{112-124}$Sb isotopes based on different models have been compared with the experimental ones. Our results are in agreement to the experimental ones compared with the results in Ref [28].

The differential cross-section for the $^{112-124}$Sn($^3$He,t)$^{112-124}$Sb reactions at $E(^3$He$)= 200$ MeV occurring by the excitation of the IAR states has been calculated by using the formula in Eq. (30). In this calculation, the value for the central volume integral is taken as $J_F = 53$ MeV fm$^3$ [24] and the distortion factor $N_F$ is considered as $N_F = \exp(-0.7A^{1/3})$. Our model results with (solid line) and without (dotted line) pairing correlations have been compared with the experimental values [29] (dashed line) in Fig. 4. From the corresponding curves, it is obvious that the contribution of the pairing
Fig. 3. Dependence of the IAR energies on mass number A in the Sb isotopes. The solid and dotted lines correspond to the cases with and without the pairing forces, and points show the experimental values [29].

The value of $J_F = 53 \pm 5$ MeV fm$^3$ for the volume integral has been obtained in Ref. [24] by considering the $(^3\text{He},t)$ reaction at $E(^3\text{He}) = 450$ MeV. In our study, the volume integral $J_F$ has been calculated by using the experimental value [29] of the differential cross-section for the $(^3\text{He},t)$
reaction at E(\(^3\)He)= 200 MeV. The calculation results have been depicted in Fig. 5. The value of the volume integral \(J_F\) has a tendency to decrease with some fluctuations as the mass number \(A\) increases. Here, the effect of pairing forces are also weak. Our calculated \(J_F\) values for the mass number region, \(A = 112 - 124\), are changing within the range of the value of 53 \(\pm\) 5 MeV fm\(^3\) given in Ref. [24].

\[
\text{Fig. 4. Dependence of the differential cross-section, } \frac{d\sigma}{d\Omega} \text{ (in mb/sr), for the Sn}(^3\text{He},t)\text{Sb reactions at } E(\(^3\)\text{He})= 200 \text{ MeV occurring by the excitation of the IAR state on mass number } A. \text{ The solid (dotted) and dashed lines correspond to the cases with (without) the pairing correlations, and the experimental values [29].}
\]

In summary, the effect of the pairing interaction and the isovector correlation between nucleons on the properties of the IAR state and the \(T_0 + 1\) isospin admixture in even-even isotopes has been investigated. The form and the strength parameters of the interaction has been related to the shell model potential by the self consistency relations. These relations make our model free of any adjustable parameters. As a result of our calculations, it has been observed that the \(T_0 + 1\) isospin admixture in \(N \approx Z\) isotopes is sensitive to the pairing interactions although the differential cross-section of the IAR state for the \(^3\text{He},t\) reactions is not sensitive to these correlations. The value of 4.236\% obtained for the \(T_0 + 1\) isospin admixture \(b^2\) in \(^{100}\text{Sn}\) isotope shows a good agreement with the value of 4 – 5\% [13]. Moreover, it is found that the calculated values for the volume integral based on the experimental differential cross-section values [29] of the \(^3\text{He},t\) reactions at E(\(^3\)He)= 200 MeV were in agreement with the value of 53 \(\pm\) 5 MeV fm\(^3\) [24].

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Fig. 5. Dependence of the volume integral $J_F$ (in MeV fm$^3$) obtained from the Sn($^3$He,t)Sb reaction occurring by the excitation of the IAR state at E($^3$He)= 200 MeV, $\theta = 0^\circ$ on mass number A. The solid and dotted lines are the cases with and without the pairing forces.

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