Spin Correlations in Top-Quark Pair Production

at $e^+e^-$ Colliders

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Abstract

We show that top-quark pairs are produced in an essentially unique spin configuration in polarized $e^+e^-$ colliders at all energies above the threshold region. Since the directions of the electroweak decay products of polarized top-quarks are strongly correlated to the top-quark spin axis, this unique spin configuration leads to a distinctive topology for top-quark pair events which can be used to constrain anomalous couplings to the top-quark. A significant interference effect between the longitudinal and transverse W-bosons in the decay of polarized top-quarks is also discussed. These results are obtained at leading order in perturbation theory but radiative corrections are expected to be small.
Since the top-quark, with mass in the range of 175 GeV [1,2], decays electroweakly before hadronizing [3], there are significant angular correlations between the decay products of the top-quark and the spin of the top-quark. Therefore, if top-quark pairs are produced with large polarizations, there will be sizable angular correlations between the decay products and the spin orientation of each top-quark, as well as between the decay products of the top-quark and the decay products of the top anti-quark. These angular correlations depend sensitively on the top-quark couplings to the $Z$ and the photon, and to the $W$ and $b$-quark. Many authors have proposed to use the angular information in top-quark events produced at $e^+e^-$ colliders to constrain deviations from Standard-Model couplings [4-8]. In most of these studies, the top-quark spin is decomposed in the helicity basis, i.e. along the direction of motion of the quark. Recently, Mahlon and Parke [9] have shown that this decomposition in the helicity basis is far from the optimal decomposition for top-quarks produced at the Tevatron. In this paper we address the issue of what is the optimal decomposition of the top-quark spins for $e^+e^-$ colliders. Here, we only consider top-quark production at energies that are above the threshold region [10]. We give the differential cross-section for top-quark pair production for a generic spin basis. We have found that there is a decomposition for which the top-quark pairs are produced in an essentially unique spin configuration at polarized $e^+e^-$ colliders. We call this spin basis the “Off-diagonal” basis because the contribution from like-spin pairs vanishes to leading order in perturbation theory. Finally, we discuss the angular correlations between the decay products of the top-quarks, and point out a significant interference effect between the longitudinal and transverse W-bosons in the decays of the polarized top-quarks.

The generic spin basis we will consider in this paper is shown in Figure 1. This choice is motivated by two features of the Standard Model prediction for top-quark pair production at $e^+e^-$ colliders at leading-order in perturbation theory; the transverse top-quark polarization is zero, and CP is conserved. Therefore, we have defined our general basis so that the spins of the top quark and anti-quark are in the production plane, so there is no transverse polarization, and the spin four-vectors are back-to-back in the zero momentum frame, so
that states with opposite spins are CP eigenstates.

The top-quark spin states are defined in the top-quark rest-frame, where we decompose the top spin along the direction \( \hat{s}_t \), which makes an angle \( \xi \) with the anti-top momentum in the clockwise direction. Similarly, the top anti-quark spin states are defined in the anti-top rest-frame, along the direction \( \hat{s}_{\bar{t}} \), which makes the same angle \( \xi \) with the top momentum also in the clockwise direction. Thus, the state \( t_\uparrow \bar{t}_\uparrow \) (\( t_\downarrow \bar{t}_\downarrow \)) refers to a top with spin in the \( +\hat{s}_t \) (\( -\hat{s}_t \)) direction in the top rest-frame, and an anti-top with spin \( +\hat{s}_{\bar{t}} \) (\( -\hat{s}_{\bar{t}} \)) in the anti-top rest-frame.

Using this generic spin basis, the Standard Model leading-order polarized cross-sections for top-quark pair production at center-of-mass energy \( \sqrt{s} \), top-quark speed \( \beta \) and top-quark scattering angle \( \theta^* \), are given by

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \rightarrow t_\uparrow \bar{t}_\uparrow) = \frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \rightarrow t_\downarrow \bar{t}_\downarrow) = \left( \frac{3\pi\alpha^2}{2s\beta} \right) |A_{LR}\cos\xi - B_{LR}\sin\xi|^2,
\]

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \rightarrow t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) = \left( \frac{3\pi\alpha^2}{2s\beta} \right) |A_{LR}\sin\xi + B_{LR}\cos\xi \pm D_{LR}|^2. \quad (1)
\]

Here \( \alpha \) is the QED fine structure constant and the quantities A, B and D depend on the kinematic variables \( \beta \) and \( \cos\theta^* \), and on the fermion couplings to the photon and Z-boson, in the following way

\[
A_{LR} = \left[ (f_{LL} + f_{LR}) \sqrt{1 - \beta^2 \sin\theta^*} \right]/2,
\]

\[
B_{LR} = \left[ f_{LL} (\cos\theta^* + \beta) + f_{LR} (\cos\theta^* - \beta) \right]/2,
\]

\[
D_{LR} = \left[ f_{LL} (1 + \beta \cos\theta^*) + f_{LR} (1 - \beta \cos\theta^*) \right]/2, \quad (2)
\]

with

\[
f_{IJ} = Q_{\gamma}^I(e)Q_{\gamma}^J(t) + Q_{Z}^I(e)Q_{Z}^J(t) \left( \frac{1}{\sin^2\theta_W} \right) \left( \frac{s}{(s - M_Z^2) + iM_Z\Gamma_Z} \right), \quad (3)
\]

where \( M_Z \) is the Z mass, \( \Gamma_Z \) is the Z width, and \( I, J \in (L, R) \). The electron couplings are given by
\[ Q_\gamma(e) = -1, \quad Q^L_Z(e) = \frac{2 \sin^2 \theta_W - 1}{2 \cos \theta_W}, \quad Q^R_Z(e) = \frac{\sin^2 \theta_W}{\cos \theta_W}, \quad (4) \]
and the top-quark couplings are given by
\[ Q_\gamma(t) = \frac{2}{3}, \quad Q^L_Z(t) = \frac{3 - 4 \sin^2 \theta_W}{6 \cos \theta_W}, \quad Q^R_Z(t) = \frac{-2 \sin^2 \theta_W}{3 \cos \theta_W}. \quad (5) \]
The angle \( \theta_W \) is the Weinberg angle. In the limit \( s \gg M_Z^2 \) the products of fermion couplings for top-quark production are
\[ f_{LL} = -1.19, \quad f_{LR} = -0.434, \quad f_{RR} = -0.868, \quad f_{RL} = -0.217, \quad (6) \]
where, as throughout this paper, we use \( \sin^2 \theta_W = 0.232 \).

The cross-sections for \( e^- R e^+ L \) may be obtained from eqns. (1-3) by interchanging \( L \) and \( R \) as well as \( \uparrow \) and \( \downarrow \). These cross-sections can be conveniently derived using the spinor helicity method for massive fermions described in [9].

Our generic basis reproduces the familiar helicity basis for the special case
\[ \cos \xi = \pm 1, \quad (7) \]
for which the top-quark spin is defined along its direction of motion. Substituting eqn. (7) in our general polarized cross-section expressions (1), we recover the well known helicity cross-sections
\[ \frac{d\sigma}{d\cos \theta^*} (e_L^- e_R^+ \to t_L \bar{t}_L) = \frac{d\sigma}{d\cos \theta^*} (e_L^- e_R^+ \to t_R \bar{t}_R) = \left( \frac{3\pi \alpha^2}{8s} \beta \right) |f_{LL} + f_{LR}|^2 (1 - \beta^2) \sin^2 \theta^*, \]
\[ \frac{d\sigma}{d\cos \theta^*} (e_L^- e_R^+ \to t_R \bar{t}_L \text{ or } t_L \bar{t}_R) = \left( \frac{3\pi \alpha^2}{8s} \beta \right) |f_{LL}(1 \pm \beta) + f_{LR}(1 \pm \beta)|^2 (1 \mp \cos \theta^*)^2. \quad (8) \]

Another useful basis is the “Beamline basis” [3], in which the top-quark spin axis is the positron direction in the top rest-frame, and the top anti-quark spin axis is the electron direction in the anti-top rest-frame. In terms of the spin angle \( \xi \), this corresponds to
\[ \cos \xi = \frac{\cos \theta^* + \beta}{1 + \beta \cos \theta^*}. \quad (9) \]
The polarized cross-sections in this basis are

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\uparrow \bar{t}^\uparrow) = \frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\downarrow \bar{t}^\downarrow) = \left( \frac{3\pi\alpha^2}{2s} \beta \right) |f_{LR}|^2 \frac{\beta^2 (1 - \beta^2) \sin^2 \theta^*}{(1 + \beta \cos \theta^*)^2},
\]

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\downarrow \bar{t}^\downarrow) = \left( \frac{3\pi\alpha^2}{2s} \beta \right) |f_{LR}|^2 \frac{\beta^4 \sin^4 \theta^*}{(1 + \beta \cos \theta^*)^2},
\]

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\uparrow \bar{t}^\downarrow) = \left( \frac{3\pi\alpha^2}{2s} \beta \right) \left| f_{LL} (1 + \beta \cos \theta^*) + f_{LR} \frac{(1 - \beta^2)}{(1 + \beta \cos \theta^*)} \right|^2.
\]  

(10)

Note that in this basis, three out of the four polarized cross-sections are proportional to $|f_{LR}|^2$ which is much smaller $|f_{LL}|^2$. These three components are further suppressed by at least two powers of $\beta$. The remaining component, $t^\uparrow \bar{t}^\downarrow$, will therefore account for most of the total cross-section. Hence the top-quark spin is strongly correlated with the positron spin direction determined in the top-quark rest-frame.

The production threshold for top-quarks of mass 175 GeV is far above the $Z$-boson pole. In this region, the $Z$ width is negligible and we can take the factors $f_{IJ}$ as real. With real $f_{IJ}$’s one can choose a basis in which the $t^\uparrow \bar{t}^\uparrow$ and $t^\downarrow \bar{t}^\downarrow$ components vanish identically, see eqn. (1). In this basis, which we refer to as the “Off-diagonal basis”, the spin angle $\xi$ is given by

\[
\tan \xi = \frac{(f_{LL} + f_{LR}) \sqrt{1 - \beta^2 \sin \theta^*}}{f_{LL} (\cos \theta^* + \beta) + f_{LR} (\cos \theta^* - \beta)}
\]  

(11)

and the polarized cross-sections are

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\uparrow \bar{t}^\uparrow) = \frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\downarrow \bar{t}^\downarrow) = 0,
\]

\[
\frac{d\sigma}{d\cos\theta^*} (e^-_L e^+_R \to t^\uparrow \bar{t}^\downarrow or t^\downarrow \bar{t}^\uparrow) = \left( \frac{3\pi\alpha^2}{8s} \beta \right) \left[ f_{LL} (1 + \beta \cos \theta^*) + f_{LR} (1 - \beta \cos \theta^*) \right]
\]

\[
\pm \sqrt{(f_{LL} (1 + \beta \cos \theta^*) + f_{LR} (1 - \beta \cos \theta^*))^2 - 4 f_{LL} f_{LR} \beta^2 \sin^2 \theta^*} \right]^2.
\]  

(12)

For $|f_{LL}| \gg |f_{LR}|$ only the $t^\uparrow \bar{t}^\downarrow$ component is substantially different from zero, and to leading
order in $f_{LL}/f_{LR}$, the $t_+ \bar{t}_+$ component is given by the same expression as for the Beamline basis\[1\]. In general there are two “Off-diagonal” bases for fermion pair production, one for $e^- e^+_R$ scattering and the other for $e^- e^+_L$, since in general $f_{LL}/f_{LR} \neq f_{RL}/f_{RR}$. However for top-quark production the two ratios are approximately equal in both sign and magnitude, so that the two “Off-diagonal” bases are almost identical. In the rest of this paper we will only use the Off-diagonal basis for $e^- e^+_R$ defined by eqn. (11) even when discussing $e^- e^+_L$ scattering.

To illustrate the different spin bases we now consider top-quark pair production, at a 400 GeV $e^+e^-$ collider. We take the top-quark mass to be 175 GeV ($\beta \sim 0.5$). Fig. 2 shows the dependence of the spin direction angle, $\xi$, on the scattering angle, $\theta^*$, for the helicity, Beamline and Off-diagonal bases. The Beamline basis lies close to the Off-diagonal basis for all scattering angles. For $\cos \theta^* \near 0$ there is a marked difference between the helicity basis and the other two bases. Note that as $\beta \to 1$, both the Beamline and Off-diagonal bases approach the helicity basis. Therefore, for lepton colliders with center-of-mass energies greater than 1 TeV, all three bases are equivalent.

The cross-sections for producing $t\bar{t}$ pairs of definite helicities are shown in Fig. 3 for both $e^- e^+_R$ and $e^- e^+_L$ scattering. While the dominant components of the signal are $t_L \bar{t}_R$ for a left-handed electron and $t_R \bar{t}_L$ for a right-handed electron, other spin components make up more than 40% of the total cross-section. In the Off-diagonal basis, in contrast, only one spin component is appreciably non-zero for all values of the scattering angle; $t_+ \bar{t}_+$ for $e^- e^+_R$ and $t_+ \bar{t}_+$ for $e^- e^+_L$, see Fig. 4. All other components are more than two and a half

\[1\] The polarized cross-section formulae (1) are also valid for top-quark pair production in $q\bar{q}$-scattering, with an appropriate change of couplings. The Off-diagonal basis for $q\bar{q}$-scattering is given by eqn. (11) with $f_{LL} = f_{LR} = f_{RL} = f_{RR}$, for which $\tan \xi = \sqrt{1 - \beta^2} \tan \theta^*$ with

$$\frac{d\sigma}{d\cos \theta^*}\left(q^-_L \bar{q}^+_R \to t_+ \bar{t}_+ \text{ or } t_+ \bar{t}_-\right) = 0$$

and

$$\frac{d\sigma}{d\cos \theta^*}\left(q^-_L \bar{q}^+_R \to t_+ \bar{t}_- \text{ or } t_- \bar{t}_+\right) = \left(\frac{\alpha_s^2}{9\beta}\right) \left[1 \pm \sqrt{1 - \beta^2 \sin^2 \theta^*}\right]^2.$$
orders of magnitude smaller than these dominant contributions. Thus, when defined in the Off-diagonal basis, the spins of the top quark and anti-quark produced in $e_L^- e_R^+$ and $e_R^- e_L^+$ scattering are essentially determined. This continues to hold for top-quark pair production at higher energies. In Fig. we show, for $e_L^- e_R^+$ collisions, the fraction of top-quark events in the $t_{\uparrow}\bar{t}_{\downarrow}$ configuration, defined in the Off-diagonal basis, as a function of the top-quark speed $\beta$. Also shown is the fraction of top-quark pairs in the $t_L\bar{t}_R$ helicity configuration. The Off-diagonal basis gives a very clean $t_{\uparrow}\bar{t}_{\downarrow}$ spin state for all values of $\beta$ in these collisions. Similarly, the spin state $t_{\downarrow}\bar{t}_{\uparrow}$ dominates $e_R^- e_L^+$ collisions at all energies.

We do not show here cross-section plots for the Beamline basis, since they are almost identical to those of Fig. except that the non-dominant contributions are now at the 1% level for a 400 GeV collider. For both the Beamline basis and the Off-diagonal basis, the contribution of higher order corrections to the non-dominant components is expected to increase their total contribution to the few percent level.

Since the top-quark pairs are produced in an unique spin configuration, and the electroweak decay products of polarized top-quarks are strongly correlated to the spin axis, the top-quark events at $e^+e^-$ collider have a very distinctive topology. Deviations from this topology would signal anomalous couplings. In the Standard Model, the predominant decay mode of the top-quark is $t \rightarrow bW^+$, with the $W^+$ decaying either hadronically or leptonically. For definiteness we consider here the decay $t \rightarrow bW^+ \rightarrow be^+\nu$. The differential decay width of a polarized top-quark depends non-trivially on three angles. The first is the angle, $\chi_{tw}$, between the top-quark spin and the direction of motion of the $W$-boson in the top-quark rest-frame. Next is the angle between the direction of motion of the $b$-quark and the positron in the W-boson rest-frame. We call this angle $\pi - \chi_{we}$. Finally, in the top-quark rest-frame, we have the azimuthal angle, $\Phi$, between the positron direction of motion and the top-quark spin around the direction of motion of the $W$-boson.

The differential polarized top-quark decay distribution in terms of these three angles is given by
\[ \frac{1}{\Gamma_T} \frac{d^3 \Gamma}{d \cos \chi_w^t d \cos \chi_e^w d \Phi} = \frac{3}{16\pi (m_t^2 + 2m_W^2)} \left[ m_t^2 (1 + \cos \chi_w^t) \sin^2 \chi_e^w \\
+ m_W^2 (1 - \cos \chi_w^t)(1 - \cos \chi_e^w)^2 + 2m_tm_W (1 - \cos \chi_e^w) \sin \chi_e^w \sin \chi_w^t \cos \Phi \right], \] (13)

where \( m_t \) is the top-quark mass, \( m_W \) is the \( W \) mass, and \( \Gamma_T \) is the total decay width (we neglect the \( b \)-quark mass). The first and second terms in (13) give the contributions of longitudinal and transverse \( W \)-bosons respectively. The interference term, given by the third term in (13), does not contribute to the total width, but its effects on the angular distribution of the top-quark decay products are sizable. Fig. 6 shows contour plots of the differential angular decay distribution in the \( \chi_e^w - \chi_w^t \) plane\(^2\), after integrating over the azimuthal angle \( \Phi \). Fig. 7 shows analogous contours integrated over \( \Phi \) for positive (solid lines) and for negative (dashed lines) values of \( \cos \Phi \) separately. The pronounced difference between these is related to the size of the interference term, which can be seen from the \( \Phi \)-distribution

\[ \frac{1}{\Gamma_T} \frac{d \Gamma}{d \Phi} = \frac{1}{2\pi} \left[ 1 + \frac{3\pi^2m_tm_W}{16(m_t^2 + 2m_W^2)} \cos \Phi \right]. \] (14)

For a 175 GeV top-quark the coefficient in front of the cosine term has a value equal to 0.59, therefore the maximum and minimum values of this distribution are approximately 4 to 1.

There are also significant correlations of the angle between the top-quark spin and the momentum of the \( i \)-th decay product, \( \chi_i^t \), measured in the top-quark rest-frame. The differential decay rate of the top-quark is given by

\[ \frac{1}{\Gamma_T} \frac{d \Gamma}{d \cos \chi_i^t} = \frac{1}{2} \left[ 1 + \alpha_i \cos \chi_i^t \right], \] (15)

where \( \alpha_b = -0.41, \alpha_\nu = -0.31 \) and \( \alpha_{e^+} = 1 \), for \( m_t = 175 \) GeV, see ref. [11].

In summary, we have presented simple analytic expressions for the polarized cross-section for top-quark pair production in polarized \( e^+e^- \) colliders. For a particular choice of axes, we take \( M_W = 80 \) GeV.
the Off-diagonal basis, not only do the like-spin contributions vanish, but one spin configuration dominates the total cross-section. In this configuration, the top-quark spin is strongly correlated with the positron spin direction determined in the top-quark rest-frame. The subsequent electroweak decays of the top-quark pair give decay products whose angular distributions are highly correlated with the parent top-quark spin. Top-quark pair events thus have a distinctive topology. This topology is sensitive to the top-quark couplings to the $Z$-boson and to the photon, which determine the orientation and the size of the top-quark and top anti-quark polarizations, as well as to the top-quark couplings to the $W$ and the $b$-quark, which determine its decay distributions. Angular correlations in top-quark events may therefore be used to constrain deviations from the Standard Model. We have also shown that the interference between the longitudinal and transverse $W$-bosons has a significant impact on the angular distribution of the top-quark decay products, and thus will provide additional means for testing the Standard Model predictions for top-quark decays.

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FIG. 1. The scattering process in the center-of-mass frame (a), in the top-quark rest-frame (b) and in the top anti-quark rest-frame (c). $s_t$ ($s_{\bar{t}}$) is the top (anti-top) spin axis.
FIG. 2. The dependence of the spin angle, $\xi$, on the scattering angle, $\theta^*$, for the helicity, Beamline and Off-diagonal (defined for $e^-e^+$ scattering) bases for a 175 GeV top-quark produced by a 400 GeV $e^+e^-$ collider.
FIG. 3. The differential cross-sections for producing top-quark pairs at a 400 GeV $e^+e^-$ collider in the following helicity configurations: $t_L\bar{t}_R$ (LR), $t_R\bar{t}_L$ (RL), and the sum of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ (LL+RR), for left-handed and right-handed electron beams.
FIG. 4. The differential cross-sections for producing top-quark pairs at a 400 GeV $e^+e^-$ collider in the following spin configurations in the Off-diagonal basis (defined for $e_L^- e_R^+$ scattering): $t\bar{t}_\perp$ (UD), $t\bar{t}_\parallel$ (DU), and the sum of $t\bar{t}_\perp$ and $t\bar{t}_\parallel$ (UU+DD), for left-handed and right-handed electron beams.
FIG. 5. The fraction of top-quark pairs in the dominant spin configuration in the Off-diagonal basis and in the helicity basis, as a function of the top-quark speed, $\beta$, in $e^-_Le^+_R \rightarrow t\bar{t}$ scattering. The solid line gives $\sigma(e^-_Le^+_R \rightarrow t\bar{t}_L)/\sigma_T$, defined in the Off-diagonal basis. The dot-dashed line gives $\sigma(e^-_Le^+_R \rightarrow t_L\bar{t}_R)/\sigma_T$ in the helicity basis. Here $\sigma_T$ is the total cross-section for $e^-_Le^+_R \rightarrow t\bar{t}$. 
FIG. 6. Contours of the top-quark decay distribution, eqn. (13), integrated over all Φ, in the $\chi_e^w - \chi_t^w$ plane.
FIG. 7. Contours of the top-quark decay distribution, eqn. (13), integrated over $\Phi$ for $\cos \Phi > 0$ (solid), and for $\cos \Phi < 0$ (dashed) in the $\chi_w^e - \chi_w^t$ plane. The solid and dashed curves join continuously at the edges of the plot, where the interference term is zero.