Mapping of geo-location influence on the uncertainty level of GNSS observations

Ahmed H H Alboabidallah¹, Husham H Rashid ², Mahdi M Ali ³ and Firas N Jaafer⁴

¹ TECB, Middle Technical University, Iraq, Baghdad, ©https://orcid.org/0000-0003-1753-8440, ahmed.alboabidallah@mtu.edu.iq, ahmed.alboabidallah@students.plymouth.ac.uk
² ITB, Middle Technical University, Iraq, Baghdad, Husham.alameri@mtu.edu.iq, Alamrihusham.1976@gmail
³ TECB, Middle Technical University, Iraq, Baghdad, Dr.mahdimustafa55@mtu.edu.iq, Mahdimustafago@gmail.com
⁴ Karbala Refinery, Karbala, Iraq, Firas.n.jaafer@gmail.com

Abstract. The random errors of differential Global Navigation Satellite Systems (GNSS) are statistically identifiable by an uncertainty component for each coordinate axis at each observed station. Literature reflects a noticeable correlation between stations’ geo-location and the uncertainty components. In this study, the multi-temporal correlation between uncertainty components in easting and northing was confirmed with moderate correlation coefficients of $R^2=0.68$ and 0.59 respectively. However, a low $R^2$ of 0.38 was obtained for the elevation component. Quantified uncertainties were mapped using first-order polynomial, quadratic polynomial, and kriging. The first-order polynomial revealed a slightly higher residual level than the quadratic polynomial. However, they both performed correspondingly for the validation points, whereas Kriging showed a clear case of an over-fitting. Therefore, the first-order polynomial was considered as a suitable scheme. Geo-statistical analysis of Easting and Northing components showed that the uncertainty is not uniform over the study area. It also showed that although the uncertainty is not purely continuous, it has a significant continuity. A geo-location based uncertainty map layers were produced based on the geo-statistical analysis result. The map concluded two layers represent the resultant, and the resultant orientation. Results represent an example of the possibilities to produce meaningful maps of uncertainties.

1. Introduction
The issue of GNSS uncertainty analysis has received considerable critical attention. This focus has been mainly used for error reduction. Therefore, most studies traced adjustable, explicable, systematic errors. Literature deals with errors, such as absolute phase-centre errors [1], tropospheric zenith delay [2], Ionospheric errors [3], and Tidal displacements [4].

Nevertheless, numerous studies have attempted to identify non-systematic errors (uncertainty) of the differential GNSS. The temporal variability of GNSS systems was argued to be the main reason for the variability of the uncertainty level of GNSS stations [4, 5]. However, the spatial variability has been only analysed as a source of systematic errors of the geoid undulation [6]. Although uncertainties are of unknown sources, two types of factors that influence the accuracy were explored in literature. On one hand, some factors were found to be uniformly influencing the uncertainty such as group delay...
differential, Ionospheric delay [7, 8], and non-systematic clock parameters’ uncertainty [5]. On the other hand, a number of factors are reported to be functions of the spatial location such as the error of double differencing scheme between the observed point and the nearby IGS stations [3, 5], and geographical features of the observed region [9]. Considering this evidence, it seems that a part of the consequential variation can be predictable and varies based on the geospatial location [10].

This indicates a need to understand the nature of the spatially varies factors and how they can affect the optimum network design. Throughout this paper, the term ‘uncertainty’ refers to the stochastic non-systematic error represented with standard deviation. The central idea is that uncertainty can be split into two components. One component is uniform all over the study area, while the other one is geospatially variable. These components are distinctive and hence, they can be both mapped. Error mapping has been used in literature [9-12]. However, it has not been used to map non-systematic errors (Uncertainty).

The main aim is to explore the spatial dependant component of the uncertainty. Three objectives were set to achieve this aim. The first was to study the temporal stability of the uncertainty over a period of one week. The second was to explore the share of the geo-location uncertainty and investigate the continuity. The third was to map the uncertainty. Understanding the share of each component will help to determine the importance of each component.

2. Materials and Methods

2.1. GNSS data collection and processing

Twenty points were observed inside a small study area of about 9km2 located in an arid area near Khan El-Atshan village between Karbala and Alnajaf governorates (Figure 1). There is a future plan to construct an airport [13] and to set a new Continuously Operating Reference Stations (CORS) points at this are. The area was selected as a sample semi-flat area. The high level of sky visibility minimizes the effect of multipath all over the study area [14]. The geo-location of this area provides another advantage due to the fact that the geoid undulation, which is the geoid-ellipsoid separation, between GRS80 ellipsoid and the EMG08 geoid, is relatively small; According to the map shown in Figure 1, the geoid undulation in the study area is about -1m, compared to a national range of between -17m and 22m.

The rapid static observations were made using Topcon’s HiPer II 72 channel L1/L2 GPS-GLONASS receiver. However, it was used in GPS only mode in order to reduce the expected model complexity. A fix antenna height was applied for all observations to avoid variations in pseudo-range multipath errors at different stations [15].

The procedure uses the International GNSS Service (IGS) stations as a Continuously Operating Reference Stations (CORS) existing control station. Each observation includes occupying the objected points for a period of one hour. The time period was decided based on the recommendation of Berber et.al, (2012) [16]. All observations were carried out over a short time period of two days. The aim was to minimize the temporally varied uncertainties [4]. Eight points were observed a second time four days later to provide statistics to analyse temporal variations.

The collected data were processed by Geoscience Australia Online GPS Processing Service (AUSPOS) version 2.3. The service uses Bernese GNSS 5.2 software in “a baseline by baseline mode using triple-differences” mode that uses the closest baseline (closest two IGS points). The processed observation AUSPOS report, for each point, lists the reduced geodetic and projected coordinates and the coordinates’ uncertainties based on the ITRF2014 reference frame. The uncertainty is attributed to the 95% confidence level as a function of observation quality, inter-relationship with IGS baseline, and geo-location. Three uncertainty indications are provided for each point, namely uncertainty in easting coordinate, uncertainty in northing coordinate, and uncertainty in elevation.
The points were filtered based on the reported ambiguity resolution per baseline [17]; four points showed an ambiguity resolution of less than 85% and therefore were excluded in the next analysis steps. The filtration aimed to eliminate outliers that may affect the error-mapping model.

2.2. Statistical analysis
The GNSS is dynamic. Hence, many errors affecting parameters vary every part of second. Still, geo-location-based parameters are more temporally consistent. Therefore, the variation of uncertainty over time was explored. The analysis regressed uncertainties of two multi-temporal sets of observations over specific geo-locations. The regression results are shown in Figure 2. An Analysis of variance (ANOVA) was also applied between the two sets. The Coefficient of Determination ($R^2$) and the ANOVA's $p$-values, listed in Table 1, reflect moderate correlations for easting and northing uncertainties and a mild correlation for elevation uncertainty. Accordingly, only easting and northing uncertainties were geospatially analysed.
2.3. Geo-statistical analysis

To analyse the dependence of the uncertainty level on the geo-location, the uncertainty level was separated into two components. The first component is geo-location dependent and can be mapped as a continuum surface. The other one is spatially discreet and therefore can only be defined at the observation points. Therefore, the share of this component can be quantified by the residuals over the observed stations. Kriging [18], and polynomial surface fittings [3] have been used widely for GNSS error analysis. First-order polynomial, quadratic polynomial and kriging schemes were implemented. Uncertainties of easting and northing were analysed individually. Prior to analysing the data, observations were separated into training and validation sets. The training set, of twelve points, to produce the model. The validation set consisted of four points, and was used to test the bias. The root mean square errors of training and validation tests are listed in Table 2. First-order fitting, Figure 3, shows an increase in the uncertainty level toward the southeast for the easting coordinates, and toward the south-west for the northing coordinates. The quadratic polynomial fitting, Figure 4, has a similar behaviour with a dome-shaped surface of uncertainty level. The kriging scheme, Figure 5, yields in a compound, contouring. However, it has similar trending to the other schemes.

Table 2: Root mean square errors of training and validation sets for the three schemes

|                  | 1st-order Polynomial | Quadratic Polynomial | Kriging |
|------------------|----------------------|----------------------|---------|
|                  | RMS$_E$ | RMS$_N$ | RMS$_E$ | RMS$_N$ | RMS$_E$ | RMS$_N$ |
| Training Set     | 0.023   | 0.013   | 0.020   | 0.011   | 0       | 0       |
| Validation Set   | 0.041   | 0.038   | 0.039   | 0.033   | 0.034   | 0.037   |

Figure 3. Results of 1st-order polynomial scheme uncertainty map (on the top) and residual mapping (on the bottom) for A- Easting uncertainty, B-Northing uncertainty.
Figure 4. Results of quadratic polynomial scheme uncertainty map (on the top) and residual mapping (on the bottom) for A- Easting uncertainty, B-Northing uncertainty.

Figure 5. Results of kriging scheme uncertainty map (on the top) and residual mapping (on the bottom) for A- Easting uncertainty, B-Northing uncertainty.
2.4. Uncertainty Mapping
With the limited number of observations, a major advantage of the first order polynomial is that it has only three constants and hence, can provide more redundancy compared to the other two models. In addition, the use of the second order polynomial and kriging does not significantly improve the fitting accuracy. Therefore, the uncertainty was mapped based on the general trends of first order polynomial result shown in Figure 3. The produced map represents the geo-location based component of the uncertainty. Uncertainties on both easting and northing directions were quantified jointly, by calculating their resultant and anisotropy orientation. Simple triangle calculation was used; resultant (R) and orientation (θ) were computed by using:

\[ R = \sqrt{U_E^2 + U_N^2} \]  
\[ \theta = \tan^{-1} \frac{U_N}{U_E} \]

The resulted uncertainty map is shown in Figure 6; the contour line map represents the uncertainty quantity while the lines represent the orientation.

![Figure 6](imageURL)

**Figure 6.** Resultant geo-location based uncertainty, contour lines represent the uncertainty level and the yellow arrows represent anisotropy uncertainty orientation.

3. Discussion
When two GNSS points are observed, two variables differ between these stations, namely time and geo-location. Time variability has been verified, in literature, as the dominating variable. However, previous studies evaluating GNSS uncertainty have not explored the spatial variability of non-
systematic errors. An initial objective of the project was to identify the spatial variation of the uncertainty. Results of the statistical analyses of multi-temporal observations showed that the geo-location-based uncertainty can be significant for easting and northing coordinates. On the other hand, the moderate correlation reviled an underlying variability. This can be interpreted as a result of the dominating temporal variability of the GNSS.

An obvious finding to emerge from the geospatial analysis is that higher-order fitting, quadratic polynomial and kriging, does not improve the RMSE of the validation set compared to the first-order polynomial. This means that the spatial variability of uncertainty inside the study area is linear. However, the trend of the variation is relatively steep. This suggests the possibility of higher-order behaviour for larger areas. It can also be interpreted to be a case of over-fitting due to the limited number of observations.

The RMSEs of the validation set, listed in Table 2, is high (RMSEE=0.04, RMSEN=0.04) compared to the average uncertainty level of around 0.07m. This finding is partially consistent with that of previous studies that the geo-location effect is small [4, 5]. However, it also showed that the mapped variation is statistically significant; the final map of the uncertainty, Figure 5, shows that the geospatial uncertainty variation cannot be neglected. This supports conclusions of Li et al., 2015 [12] that errors can vary based on the stations’ geo-locations due to the variation of geo-location based parameters. It is important to bear in mind the possible bias in these results due to the small scale and limited number of observations. The archived positive result, while preliminary, can open the door for a further investigation with wider coverage and more GNSS stations.

4. Conclusions
This study set out to investigate the GNSS uncertainty associated with the spatial variation of observations. Simple statistical and geo-statistical analyses were used to achieve this aim. ANOVA analysis approved partial Multi-temporal synchrony of uncertainties between geo-location and uncertainty. This supports the hypothesis that there is an uncertainty component that is temporally-independent. The geo-statistical analysis showed that this component can be mapped geospatially based on the observed station coordinates. It also showed that, although being small, the share of this geospatial based component can be significant. Taken together, the current results highlight the importance of GNSS non-systematic analysis. It approved the possibility to map the geo-location-based component as well.

Due to practical constraints, this paper cannot provide a comprehensive review of global behaviour of GNSS uncertainty, but it laid the groundwork for future research into geospatial dependence of GNSS uncertainties.

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