Luminosity-Colour Relations for Red Clump Stars

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Abstract We calibrated the $M_V$, $M_J$, $M_{K_s}$ and $M_g$ absolute magnitudes of red clump stars in terms of colours. $M_V$ and $M_g$ are strongly dependent on colour, while the dependence of $M_J$ and $M_{K_s}$ on colour is rather weak. The calibration of $M_V$ and $M_{K_s}$ absolute magnitudes is tested on 101 RC stars in the field SA 141. The Galactic model parameters estimated with this sample are in good agreement with earlier studies.

Keywords Stars: distances, Stars: late-type, Galaxy: fundamental parameters

1 Introduction

Red Clump (RC) stars are core helium-burning giants. They form a prominent feature in the colour-magnitude diagrams (CMDs) of open clusters. Following the prediction of Cannon (1970), it is known that they are abundant in the solar neighbourhood. In recent years much work has been devoted to studying the suitability of RC stars as a distance indicator. Their absolute magnitudes in the optical range lie from $M_V = +0.7$ mag for those of spectral type G8 III to $M_V = +1$ mag for type K2 III (Keenan & Barnbaum 1999). The absolute magnitude of these stars in the $K_s$ band is $M_{K_s} = -1.61 \pm 0.03$ mag with negligible dispersion. Based on observations of 14 open clusters with $−0.5 < [Fe/H] \leq 0$ dex and $1.58 \leq t \leq 7.94$ Gyr, Grocholski & Sarajedini (2002) found that for RC stars in clusters have $< M_{K_s} >= -1.61 \pm 0.04$ mag.

The dependence of the $I$-band magnitude of RC stars on the metallicity and age was extensively studied in the past from an observational point of view. In the most cases the $I$ band-mean absolute magnitude is insensitive to age and metallicity. Udalski (2000) found that the $M_I$ of RC stars weakly depends on metallicity. Paczynski & Stanek (1998) and Stanek & Garnavich (1998) found little or no variation in $M_I$ with colour and metallicity. Sarajedini (1999) presented observations of eight open clusters, concluding that $M_I$ is less sensitive to metal abundance than $M_V$, but that the dependence on age and metallicity is still not negligible. Zhao et al. (2001) and Kubiak et al. (2002) confirmed the results of Udalski (2000), and theoretical models from Girardi & Salaris (2001) also show a dependence in $I$-band, predicting that an older cluster with higher metallicity has fainter RC stars. Based on the model of Girardi et al. (2000), Salaris & Girardi (2002) stated that $M_K$ is a complicated function of metallicity and age. For age $t > 1.5$ Gyr, it decreases with increasing metallicity, the opposite behaviour with respect to $M_V$ and $M_I$ absolute magnitudes. Pietrzynski et al. (2003) have also investigated the dependence of the mean $K$, $J$, and $I$ absolute magni-
tudes of the RC stars on metallicity and age, as a part of their ongoing Araucaria Project to improve stellar distance indicators. They took deep near-infrared (NIR) J and K images of several fields in LMC, SMC, and the Carina, and Fornax dwarf galaxies and made a comparison between the extinction-corrected K-band RC star magnitudes and some other stellar indicators, particularly the tip of the red giant-branch magnitude, the mean RR Lyrae star V-band magnitude, and the mean K-band magnitude of Cepheid variables at a period of 10 days. This comparison strongly suggests that absolute K-band magnitude of the RC stars have a very weak dependence, (if any) on [Fe/H] over the broad range of metallicities covered by their target galaxies. They conclude that the mean K-band magnitude of the RC stars is an excellent distance indicator with small (if any) population corrections over a wide range in metallicity and age. Pietrzynski et al. (2010) stated that population effects strongly affect both V- and I-band magnitude of RC stars in a complicated way. Therefore, optical V−I photometry of RC stars is not an accurate method for the determination of distances in nearby galaxies, while NIR photometry is a much better way to measure distances with RC stars given its smaller sensitivity to population effects. Laney et al. (2012) determined the mean $M_K$, absolute magnitude for RC stars in the solar neighbourhood to within 2 per cent ($M_K = -1.613 \pm 0.015$ mag) and applied their results to the estimation of the distance of LMC. A mean value for the $M_K$ absolute magnitude with weak dependence on metallicity makes it possible to use this population as a tracer of Galactic structure and interstellar extinction, as several works have fully demonstrated in the last decade (see for example Lopez-Corredoira et al. 2002, 2004; Cabrera-Lavers et al. 2005, 2007, 2008; Bilir et al. 2012, and references therein).

In a recent work, van Helshoecht & Groenewegen (2007) used the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006) infrared data for a sample of 24 open clusters to investigate how the $K_s$-band absolute magnitude of the red clump depends on age and metallicity. They showed that a constant value of $M_{K_s} = -1.57 \pm 0.05$ mag is a reasonable assumption in determination of distances of clusters with metallicity between -0.5 and +0.4 dex and age between 0.31 and 7.94 Gyr. The constant absolute magnitude value of RC stars was also supported with the newly reduced Hipparcos data by Groenewegen (2008) ($M_{K_s} = -1.54 \pm 0.04$ mag).

In this paper, we will contribute to the discussion by a different approach. We aim to calibrate the absolute magnitudes of RC stars in four bands, i.e. V, J, $K_s$ and g, as a function of colour with field stars taken from different photometric surveys. The data are given in Section 2. The location of the RC stars in the Hertzsprung-Russell (H-R) diagram are demonstrated in Section 3. The luminosity-colour relations in optical and near-infrared are given in Sections 4. We tested the absolute magnitudes derived by the procedure in our study in Section 5, and a summary and conclusion is presented in Section 6.

2 The data

We used three sets of data. The BV I data were taken from the Hipparcos catalogue (van Leeuwen 2007) which also provides trigonometric parallaxes. In order to obtain reliable absolute magnitude and distances, only the stars with relative parallax error ($\sigma_\pi/\pi$) is less than or equals to 0.1 are selected. Thus, 32 144 stars are included into the sample with $BVI$ magnitudes. The second set of data consists of $JHK_s$ magnitudes. Within this sample only 32 072 stars were detected in the 2MASS catalogue (Cutri et al. 2003). The Hipparcos stars were not observed in the Sloan Digital Sky Survey (SDSS; York et al. 2000). Hence, the gri magnitudes of the sample stars were evaluated by the transformation equations of Yaz et al. (2010). Thus, the resulting catalogue consisted of three different photometries, i.e. Johnson-Cousins (BVI), 2MASS ($JHK_s$), and SDSS (gri) of all the RC stars within the Solar neighbourhood.

2.1 Lutz-Kelker Correction

The observed trigonometric parallaxes are biased because the volume of space per unit of parallax is not constant. The pioneers of this topic are Trumpler & Weaver (1953). However, Lutz & Kelker (1973) were the first who quantified the bias. Other studies followed the work of Lutz & Kelker (1973). Smith (1987) claimed that Lutz & Kelker (1973)’s methodology seemed to be that of Bayesian statistics. The Lutz-Kelker (LK) bias can be explained as follows. Let $\pi$ and $\sigma_\pi$ be the parallax and its error of a star. Then, one can define a distance $d$ with lower and upper limits, i.e. $d_{\pi+\sigma_\pi}$ and $d_{\pi-\sigma_\pi}$. Stars in a given volume can scatter to the distance $d$. Since the number of stars in the distance interval $(d, d_{\pi-\sigma_\pi})$ are more than the ones in the distance interval $[d_{\pi+\sigma_\pi}, \; d]$, more stars from the interval $(d, d_{\pi+\sigma_\pi})$ will scatter to distance $d$ than the ones in $[d_{\pi+\sigma_\pi}, \; d]$. The result of this effect is that measured parallaxes cause smaller distance estimations. We used
the following equation of [Smith (1987)] to correct the observed Hipparcos parallaxes [van Leeuwen (2007)],

\[ \pi_0 = \pi + \frac{1}{2} + \frac{1}{2} \sqrt{1 - 16(\sigma_\pi / \pi)^2}, \]  

(1)

where \( \pi \) and \( \pi_0 \) are the observed and corrected parallaxes, respectively, and \( \sigma_\pi / \pi \) denotes the error of the observed parallax. According to [Lutz & Kelker (1973)] relative parallax errors, \( \sigma_\pi / \pi \), larger than 0.17 are not reliable.

2.2 Reddening

The \( E(B-V) \) colour excess of stars have been evaluated in two steps. First, we used the maps of [Schlegel et al. (1998)] and evaluated a \( E_\infty(B-V) \) colour excess for each star. Then, we reduced them using the following procedure [Bahcall & Soneira (1980)]:

\[ A_d(b) = A_\infty(b) \left[ 1 - \exp \left( -\frac{1}{H} \right) \right]. \]  

(2)

Here, \( b \) and \( d \) are the Galactic latitude and distance to the star, respectively. \( H \) is the scale height for the interstellar dust which is adopted as 125 pc [Marshall et al. (2006)]. \( A_\infty(b) \) and \( A_d(b) \) are the total absorptions for the model and for the distance to the star, respectively. \( A_\infty(b) \) can be evaluated by means of the following equation:

\[ A_\infty(b) = 3.1 \times E_\infty(B-V). \]  

(3)

\( E_\infty(B-V) \) is the colour excess for the model taken from [Schlegel et al. (1998)]. Then, \( E_d(B-V) \), i.e. the colour excess for the corresponding star at the distance \( d \), can be evaluated via equation,

\[ E_d(B-V) = A_d(b) / 3.1. \]  

(4)

We have omitted the indices \( \infty \) and \( d \) from the colour excess \( E(B-V) \) in the equations. However, we use the terms “model” for the colour excess of [Schlegel et al. (1998)] and “reduced” for the colour excess corresponding to distance \( d \). The total absorption \( A_d \) used in the section and classical total absorption \( A_V \) have the same meaning.

We de-reddened the colours and magnitudes using the \( E(B-V) \) colour excesses of the stars evaluated from the procedures explained above and the following equations from [Fan (1999)] and [Fiorucci & Munari (2003)] for \( V-I \) colour and for the 2MASS photometry.

\[ V_o = V - 3.1 \times E(B-V), \]

\[ (B-V)_o = (B-V) - E(B-V), \]

\[ (V-I)_o = (V-I) - 1.250 \times E(B-V), \]

\[ J_o = J - 0.887 \times E(B-V), \]

\[ (J-H)_o = (J-H) - 0.322 \times E(B-V), \]

\[ (H-K_s)_o = (H-K_s) - 0.183 \times E(B-V). \]

3 Hertzsprung-Russell Diagram of RC Stars

We evaluated the \( V \)-band absolute magnitudes of 144 Hipparcos stars with relative parallax errors less than 0.1 using the following formula and plotted them onto the H-R diagram (Fig. 1) to identify the location of the RC stars:

\[ M_V = V_0 - 5 \log \left( \frac{1000}{\pi(\text{mas})} \right) - 5. \]  

(6)

The sample stars were fitted to Padova isochrones [Marigo et al. (2008)] with metallicities \([M/H] = 0, -0.5, \) and \(-1 \) dex and ages \( t=1, 5, \) and \( 10 \) Gyr. The relatively high condensed region on the evolved segments of the isochrones corresponds to the location of the RC. The borders of the RC stars in the vertical direction have been fixed by using the constraint of [Puzeras et al. (2010)], i.e. \( 2.1 \leq \log g \leq 2.7 \), where \( g \) denotes surface gravity of the RC stars. These borders comprise the region in which RC stars identified in the literature. However, for the borders in the horizontal direction we used their highly concentrated location, i.e. \( 0.7 \leq (B-V)_0 \leq 1.3 \). The number of the RC stars defined in this way are 2576. We separated them into three categories with respect to their Galactic latitudes, i.e. \(|b| \leq 30^\circ, 30^\circ < |b| \leq 60^\circ, \) and \(|b| > 60^\circ.\) Their median \( E(B-V) \) colour excesses are 0.035, 0.020, and 0.012 mag, respectively (Fig. 2). These stars will be used in the following sections to derive colour dependent \( M_V \) absolute magnitude calibrations.

The distances of 2576 RC stars evaluated by the combination of their absolute and de-reddened apparent \( V_0 \) magnitudes show almost a symmetrical distribution (Fig. 3). The median distance is 140 pc. Distribution of stars in \( X-Y \) and \( X-Z \) planes is given in Fig. 4, where \( X, Y, \) and \( Z \) are the heliocentric rectangular coordinates. The distribution of stars in the medians of \( X, Y, \) and \( Z, i.e. +3, -1, \) and -5 pc, respectively show that the distribution of RC stars in the solar neighbourhood is almost symmetrical.

The errors of magnitudes in 2MASS photometry are larger than the optical ones, \( B, V, \) and \( I. \) We selected
Fig. 1  The $M_V - (B - V)_0$ diagram for 32 144 stars, taken from the Hipparcos catalogue, fitted to the Padova isochrones with metallicities $[M/H] = 0$, -0.5, and -1 dex, and ages $t = 1$, 5, and 10 Gyr. The dotted lines show the location of RC stars.

Fig. 2  Distribution of the $E(B - V)$ colour excesses of 2576 RC stars for three Galactic latitudes, $|b| \leq 30^\circ$ (a), $30^\circ < |b| \leq 60^\circ$ (b), and $|b| > 60^\circ$ (c).

Fig. 3  Distance distribution of 2576 RC stars. The median of the distances is $d = 140$ pc.

Fig. 4  Distributions of 2576 RC stars in the $X - Y$ and $X - Z$ planes.
the best quality $J$, $H$, and $K_s$ magnitudes by applying
the constraint “AAA” to avoid the large errors. We
found 25401 stars using this criteria. Then, we eval-
uated the $M_J$ and $M_{K_s}$ absolute magnitudes by means
of the procedure applied to $M_V$ absolute magnitudes,
and plotted them onto the H-R diagram, $M_J - (J - H)_o$
and $M_{K_s} - (J - K_s)_o$, respectively, to identify the lo-
cation of the RC stars (Fig. 5 and Fig. 6) with $J$ and
$K_s$ bands. As in $M_V$, the sample stars were fitted to
the Padova isochrones [Marigo et al. 2008] mentioned
above. The number of RC stars in Fig. 5 and Fig. 6
($N = 499$) are less than the ones in Fig. 1 ($N = 2576$),
due to the constraint “AAA” used to ensure the best
photometric quality.

The errors of the photometric data are given in Fig.
7. One can see that the errors for the optical colour
and magnitudes are smaller than the corresponding
NIR ones. Also, the errors for bright magnitudes are
lower than the faint ones, as expected, and the errors
for $(B - V)_o$ colours are rather smaller than the ones
for $(V - I)_o$ colours.

We do not plot any H-R diagram for the SDSS $(gri)$
data, because they were provided by the transforma-
tion equations, as mentioned above. The equation used
to evaluate the $M_g$ absolute magnitude is given in the
following [Yaz et al. 2010]:

$$M_g - M_J = 2.923(J - H)_o + 3.031(H - K_s)_o + 0.329. \quad (7)$$

4 Luminosity-Colour Relations for RC Stars

We evaluated two different sets of colours for each star
sample mentioned above, i.e. $(B - V)_o$ and $(V - I)_o$;
$(J - H)_o$, and $(H - K_s)_o$; $(g - r)_o$ and $(r - i)_o$ for Johnson-
Cousins, 2MASS and SDSS photometries, respectively,
and combined them with the corresponding absolute
magnitudes ($M_V$, $M_J$, $M_{K_s}$, and $M_g$) evaluated by
means of their apparent magnitudes and Hipparcos par-
allaxes. We separated the $(B - V)_o$ and $(V - I)_o$ colours
into 11 bins, whereas only six bins could be provided for
the $(J - H)_o$, $(H - K_s)_o$, $(g - r)_o$ and $(r - i)_o$ colours
due to smaller number of stars in 2MASS and SDSS
photometries. Table 1 gives the mean colours and ab-
solute magnitudes of the bins in question. Then, we
adapted the colours and absolute magnitudes in Table
1 to the following equations and obtained colour depen-
dent absolute magnitude equations for the RC stars by
regression analysis.
\[ M_V = a_1(B - V)_0 + b_1(V - I)_0 + c_1, \]
\[ M_g = a_2(g - r)_0 + b_2(r - i)_0 + c_2, \]
\[ M_J = a_3(J - H)_0 + b_3(H - K_s)_0 + c_3, \]
\[ M_{K_s} = a_4(J - H)_0 + b_4(H - K_s)_0 + c_4. \] (8)

The numerical values of the coefficients \(a_i\), \(b_i\), and \(c_i\) (\(i = 1, 2, 3, 4\)) estimated by means of regression analysis are given in Table 2.

Table 1 gives the indication of an absolute magnitude gradient with respect to the colours for \(M_V\), \(M_g\) and \(M_J\), whereas it is almost zero for \(M_{K_s}\). We plotted the absolute magnitudes estimated for the bins of different colours in Table 1 versus the corresponding colour with (absolutely) larger coefficient, i.e. \((B - V)_0\), \((J - H)_0\), and \((g - r)_0\) to treat the problem schematically. Fig. 8 confirms the result obtained via Table 1, i.e. there is a linear variation of \(M_V\), \(M_g\) and \(M_J\), whereas the locus of \(M_{K_s}\) is a horizontal line.

We evaluated the differences between the absolute magnitudes estimated using the corresponding equation in (8) as the combination of apparent magnitudes and Hipparcos parallaxes of the stars, and plotted them in Fig. 9. The original values of the absolute magnitudes are those evaluated by the combination of apparent magnitudes and Hipparcos parallaxes of the stars. The standard deviations for the optical and NIR regions are \(\sigma = 0.23\) and \(\sigma = 0.28\) mag, respectively. The dotted lines in the figure correspond to the residuals of \(\pm 1\sigma\).

5 Test of Absolute Magnitudes of RC Stars

We tested the calibrations of \(M_V\) and \(M_{K_s}\) absolute magnitudes on 101 RC stars in the field SA 141. We estimated the Galactic model parameters with CCD \(UBVRI\) photometric data for these RC stars and compared them with the ones appeared in the literature. The procedure is given in the following.

5.1 Identification of the RC stars in SA 141

Siegel et al. (2009) determined the CCD \(UBVRI\) magnitudes of 1299 stars in 1.2 square-degrees in a field in the direction of SA 141 \((l = 246^\circ.33, b = -85^\circ.83)\). We used the \(B\) and \(V\) magnitudes of these stars, identified the RC stars in this field and estimated the Galactic model parameters by using the space densities evaluated for the RC sample as explained in the following. We de-reddened the \(B\) and \(V\) magnitudes by using the interstellar extinction maps of Schlegel et al. (1998).
Table 1 Distribution of colours and absolute magnitudes of the RC stars in 11 bins for the $BV I$ photometry, and six bins for the $JHK_s$ and $gri$ photometries.

|                | Johnson-Cousins | 2MASS | SDSS |
|----------------|-----------------|-------|------|
| $(B-V)_o$      | 0.777           | 0.777 | 0.777|
| $(V-I)_o$      | 0.502           | 0.339 | 0.339|
| $M_V$          | 0.146           | -0.937| -0.337|
| $(J-H)_o$      | 0.142           | -0.910| -1.182|
| $(H-K_s)_o$    | 0.137           | -0.873| -1.149|
| $M_J$          | 0.132           | -0.841| -1.436|
| $M_K_s$        | 0.130           | -0.809| -1.452|
| $(g-r)_o$      | 0.126           | -0.779| -1.467|
| $(r-i)_o$      | 0.751           | 0.777 | 0.782|
| $M_g$          | 0.179           | 0.817 | 0.261|

Fig. 9 Absolute magnitude residuals for $M_V$, $M_J$, and $M_K_s$. Dotted horizontal lines indicate ±1σ residuals.

Table 2 The numerical values of the coefficients $a_i$, $b_i$, and $c_i$ ($i = 1, 2, 3, 4$) in the Eq. (8).

| $i$ | $M_V$         | $M_J$         | $M_K_s$        | $M_g$         |
|-----|---------------|---------------|----------------|---------------|
| 1   | 1.398±0.010   | 3.152±0.075   | 0.706±0.017    | -0.337±0.013  |
| 2   | -0.011±0.005  | -0.213±0.138  | 0.039±0.019    | 0.275±0.143   |
| 3   | -0.577±0.001  | -0.937±0.075  | -1.182±0.034   | -1.312±0.025  |
| 4   | 0.0003        | 0.0009        | 0.0003         | 0.0002        |

and the canonical procedure, as explained Section 2.2. The apparent $V$ magnitude of the stars lie in the interval of $12 < V < 22$ mag and their colour excesses are rather small, i.e. $0.010 < E(B - V) < 0.025$ mag. We added 49 stars with $JHK_s$ magnitudes to this sample which were provided from 2MASS All Sky Catalog of point sources (Cutri et al. 2003) and which could not be observed with $UBVRI$ photometry due to the saturation of the CCDs. Thus, the number of stars in the field increased to 1348. The colour excess for the bright stars adopted as the mean of the colour excesses of 1299 stars, $E(B - V) = 0.016$ mag.

The $V_0 - (B - V)_0$ colour-magnitude diagram (CMD) of 1299 stars observed by Siegel et al. (2009) and the $K_{s0} - (J - K_s)_0$ CMD of 859 stars provided by the 2MASS catalogue (Cutri et al. 2003) are given in Fig. 10. Bright stars which could not be observed with $UBVRI$ photometry are marked with the symbol (+). The colour and magnitude errors for the $JHK_s$ photometry are larger than the ones for $UBVRI$ photometry (Fig. 11). However, 49 stars used in our study are bright ones with best quality labeled as “AAA”. Hence, the large errors in question will not affect our results. The limiting magnitudes of completeness for $V_0$ and $K_{s0}$ are 18.5 and 14.5 mag, respectively (Fig. 12).

The RC stars in Fig. 10 were identified in two steps. First, we applied the colour constraint of Puzeras et al.
Fig. 10  $V_o - (B - V)_o$ and $K_{so} - (J - K_{so})_o$ apparent magnitude colour diagrams for the stars observed in the field SA 141. The symbol (+) denotes the bright stars which could not be observed with $BVI$ photometry due to saturation. The vertical dashed lines indicate the lower and upper limiting colours of the RC stars.

Fig. 11  Optical (a and c) and NIR (b and d) magnitude and colour errors for stars observed in SA 141 star field.

Fig. 12  Limiting magnitude of completeness for $V_o$ (a) and $K_{so}$ (b) magnitudes. The histogram with colours white and black corresponds to all stars (859) observed with $JHK_s$ photometry, whereas the one with black colour is plotted only for the stars with best quality, labeled with AAA (563).
The values in Table 3 are given in a tabular format to facilitate comparison and analysis. Each row represents a different distance range and the corresponding parameters for the density function.

### 5.2.2 Density laws

We adopted the density laws of Basel group (Buser et al. 1998, 1999). Disc structures are usually parameterized in cylindrical coordinates using radial and vertical exponentials:

\[
D_t(x, z) = n_t e^{-z/H} e^{-(x-R_o)/h_i},
\]

where \(z = r \sin b\) is the distance from Galactic plane, \(x\) is the planar distance from the Galactic center, \(R_o\) is the solar distance to the Galactic center (8 kpc; Reid 1993), \(H_i\) and \(h_i\) are the scaleheight and scalelength, respectively, and \(n_t\) is the normalized local density. The suffix \(i\) takes the values 1 and 2, for the thin and thick discs.

The density law for spheroid component is parameterized in different forms. The most common is the de Vaucouleurs (1948) spheroid used to describe the surface brightness profile of elliptical galaxies.

\[
D_s(R) = n_s \exp[10.093(1 - (R/R_o)^{1/4})]/(R/R_o)^{7/8}.
\]

Here, \(n_s\) is the normalized density at the solar radius, \(R\) is the (uncorrected) galactocentric distance in spherical coordinates. \(R\) has to be corrected for the axial ratio \(\kappa = c/a\),

\[
R = [x^2 + (z/\kappa)^2]^{1/2},
\]

where,

\[
x = [R_o^2 + r^2 \cos^2 b - 2R_0 r \cos b \cos l]^{1/2},
\]

with \(l\) and \(b\) being the Galactic longitude and latitude, respectively.

### 5.2.3 Galactic model parameters

We estimated the Galactic model parameters by fitting the density functions in Table 3 derived from the observations (combined from the three population components) to a corresponding combination of the adopted population-specific analytical density laws (Fig. 14). The distance to a star in the line of sight (\(r\)), in our sample, is rather close to its distance from the Galactic plane (\(z\)), due to the position of the field SA 141, i.e., \(b = -85^\circ.83\). Hence, the space densities and the density laws are given as a function of \(z\), instead of \(r\).

We extrapolated the logarithmic density from the nearest point, \(z^* = 2.4\) kpc, to the local space density for giants (\(D^* = 6.64\)) in the literature (Gliese 1969) and used the classical \(\chi^2_{min}\) statistic to estimate Galactic model parameters, which is the most commonly used

### Table 3 Space density function for 101 RC stars in the direction to the field SA 141, calculated by the distances evaluated via the absolute magnitudes based on the procedure in this study.

| \(r_1 - r_2\) (kpc) | \(\Delta V_{1,2}\) (pc\(^3\)) | \(r^*\) (kpc) | \(z^*\) (pc) | \(N\) | \(D^*\) |
|---|---|---|---|---|---|
| 1-3 | 3.11E(6) | 2.41 | 2.40 | 9 | 4.45 |
| 3-5 | 1.19E(7) | 4.24 | 4.22 | 12 | 3.96 |
| 5-8 | 4.72E(7) | 6.83 | 6.81 | 14 | 3.56 |
| 8-12 | 1.46E(8) | 10.38 | 10.36 | 20 | 3.17 |
| 12-16 | 2.89E(8) | 14.28 | 14.24 | 17 | 2.72 |
| 16-20 | 4.76E(8) | 18.22 | 18.17 | 19 | 2.53 |
| >24 | 7.10E(8) | 22.18 | 22.12 | 8 | 2.05 |

### Table 4 Galactic model parameters estimated by the space density function in Table 3.

| Thin Disc | Thick Disc | Halo |
|---|---|---|
| \(n^*\) \((\%)\) | \(H_{pc}\) | \(n^*\) \((\%)\) | \(\kappa\) |
| 6.61 | 5.40 \(\pm\) 0.07 | 5.80 \(\pm\) 0.09 | 975 \(\pm\) 40 | 4.02 \(\pm\) 0.07 | 0.2 \(\pm\) 0.1 | 0.89 \(\pm\) 0.18 |
method in recent studies [Du et al. 2006; Jurić et al. 2008; Bilir et al. 2008]. The results are given in Table 4. There is a good agreement between the Galactic model parameters estimated in this study and the ones appeared in the literature (cf. Karaali et al. 2004; Cabrera-Lavers et al. 2005; Bilir et al. 2006b,c).

6 Discussion

The RC stars are the most numerous population, within the red giants, and they are easy to detect at larger distances from the Sun. Hence, they become a powerful tool to trace the different Galactic components in the Milky Way. A lot of work has been done on this topic in recent years, but even more is still to come with the advent of the recent deeper databases in the NIR as those of UKIRT Infrared Deep Sky Survey (UKIDSS; Lawrence et al. 2007), or VISTA Variables in the Via Lactea (VVV) public survey (Minniti et al. 2010), or even with high quality data at larger wavelengths as the ones that the more recent Wide Field Survey Explorer (WISE; Wright et al. 2010) provides.

In recent years much work has been devoted to studying the suitability of RC stars for application as a distance indicator. The absolute magnitude of these stars in the K band is $M_K = -1.613$ mag with negligible dependence on metallicity (Alves 2000; Laney et al. 2012). Whereas, their absolute magnitudes in the optical range lie from $M_V = +0.7$ mag for those of spectral type G8 III to $M_V = +1$ mag for type K2 III (Keenan & Barnbaum 1999).

In this work, we calibrated the $M_V$, $M_g$, $M_J$, and $M_K$ absolute magnitudes of RC stars in terms of colours. We found that the absolute magnitudes $M_V$ and $M_g$ are depend strongly on colour, whereas the $M_J$ and $M_K$ are weak. The calibrations of $M_V$ and $M_K$ absolute magnitudes are tested on 101 RC stars in the field SA 141. The Galactic model parameters estimated with this sample are in good agreement with the ones appeared in the literature. The data of the RC stars are taken from the Hipparcos catalogue and only stars with relative parallax errors $\sigma_\pi/\pi \leq 0.1$ are considered to obtain reliable absolute magnitudes and distances. The range of the metallicity of stars in the Hipparcos catalogue is narrow. Hence, we did not consider the metallicity effect on the calibration of absolute magnitude in terms of colour.

7 Acknowledgments

This work has been supported by the Scientific and Technological Research Council (TÜBİTAK) 210T114.
We are grateful to Dr. Siegel for providing us the $UBVRI$ photometric data for the field SA 141 and Dr. T. Güver for reading and correcting for grammatical and linguistic aspect of the manuscript.

This research has made use of the NASA/IPAC Infrared Science Archive and Extragalactic Database (NED) which are operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

This research has made use of the SIMBAD, and NASA’s Astrophysics Data System Bibliographic Services.
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This manuscript was prepared with the AAS LATEX macros v5.2.