The electromagnetic wave amplification in thin superconducting film in non-linear mixed state

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Abstract. In this paper the interaction of electromagnetic waves with the moving fluxons in the thin superconducting film on the dielectric substrate is considered. It is shown that in the case of flux-flow described by the Larkin-Ovchinnikov model, the amplification of electromagnetic wave can be observed. In this case, amplification can be achieved when the fluxons velocity is less than the phase velocity of the electromagnetic wave. It is revealed that the thin superconducting film in mixed state demonstrated nonreciprocal properties for backward and forward waves. The results obtained in this work allow to create amplifiers based on cuprate superconductors.

1. Introduction
As is known, when the electromagnetic wave interacts with the moving subsystem, wave amplification can be observed. In this case the phase velocity of the electromagnetic wave should be equal to the velocity of motion of the subsystem. Well-known examples are electromagnetic wave amplification in traveling wave and ultrasonic wave amplification in piezoelectric semiconductors [1]. The electromagnetic wave interaction with the moving fluxons lattice in type II superconductors can lead to amplification of electromagnetic [2] and ultrasonic waves [3]. We need very high velocities of fluxons in order to obtain electromagnetic waves amplification in the thin superconducting film. The vortex velocity ~3 km s\(^{-1}\) is getting interesting not only for amplification of electromagnetic waves, but also for other applications. Currently the ultra-fast vortex motion at velocities of 10–15 km s\(^{-1}\) in niobium carbide is demonstrated [4]. The destruction of the low-dissipative state of fluxons can be explained by flux-flow instability when the normal electrons leave the vortex cores [5]. To achieve the maximum current close to the pair-breaking current and high vortex velocities we need fast cooling of quasiparticles.

The equality of the speed of motion of the subsystem and the phase speed is not necessary for electromagnetic wave amplification. As shown in [3], the effect of amplification of longitudinal ultrasonic waves in type II superconductors can be observed at speeds of vortex structure much lower than the speed of these waves. The interaction between an electromagnetic wave and Abrikosov vortices in a type II superconductor in Larkin-Ovchinnikov state can lead to feasibility of amplifying [6]. Let us show that amplification of an electromagnetic wave in structure containing thin superconducting film in the dynamic mixed state can also be observed at velocities of the vortex structure that are less than the phase velocity of the wave.
2. Methods
Let us consider reflection from the structure that consists of the thin type II superconducting film with the thickness $d \ll \lambda$, where $\lambda$ is the London penetration depth, located on the dielectric substrate (figure 1). The electromagnetic wave falls on the thin superconductor from a semi-infinite dielectric at the angle $\theta_1$ and exits into the dielectric at the angle $\theta_2$. The external magnetic field $B_{y0}$ is directed perpendicular to the boundary of the superconducting film. This field $B_{y0}$ does not exceed the second critical field for superconductor. The transport current flows through the superconducting film along the Oz. Under the action of the Lorentz force the fluxons lattice begins to move with velocity $v$ along the Ox axis. In this case, the current density $j_{z0}$ exceeds the critical one. We consider the case, when the good heat dissipation take place in the superconducting layer and there is a thermodynamic equilibrium between the lattice and thermostat. The energy relaxation time due to electron-phonon collisions is larger than or of the same order as the electron-phonon interaction time. Consider the temperature close to the critical temperature $T_c$ because in this case the effect will be maximum.

![Figure 1. The structure with superconducting film.](image)

The behavior of the fluxon structure in mixed state is described by Larkin–Ovchinnikov model [7]. According to the modified model described in the work [8] the viscosity coefficient of magnetic vortex has the following form:

$$\eta(v) = \eta(0) \frac{1}{1 + (v/v^*)^2},$$  \hspace{1cm} (1)

$$v^* = \left[ (1-t)^{1/2} D[14\zeta(3)]^{1/2} \right]^{1/2} \left( 1 + \frac{a}{\sqrt{D\tau_e}} \right),$$ \hspace{1cm} (2)

$$\eta(0) = 0.45 \frac{\sigma_n T_c}{D} (1-t)^{1/2}$$ \hspace{1cm} (3)

where $v^*$ is the velocity corresponding to vortex instability, $D$ is the diffusion coefficient, $t = T/T_c$, $\tau_e$ is the normal electron energy relaxation time, $a$ is the distance between vortices, $\sigma_n$ is the normal conductivity of a superconductor, $\zeta(3)$ is Riemann zeta function for 3. These expressions are correct near fields $B/B_{c2} < 0.4$, where $B_{c2}$ is the second critical field. Note that the instability of the vortex structure appears most strongly at temperatures close to the critical temperature [9].

If the thickness of the superconducting layer $d$ is much less than the, we can take into account the presence of this thin layer in the form of a special boundary condition. Consider a superconductor layer at the boundary $y=0$. In the inertialess approximation and without taking into account the elastic “rigidity” of the vortex lattice, the boundary condition is written in the following form [2]:

$$\frac{2\Phi_0}{\eta(0)v^2} \frac{\partial B_y}{\partial t} + \frac{1}{j_{z0}} \left[ \frac{1}{j_{z0}^2} \frac{4\Phi_0^2}{\eta(0)^2 v^2} \right] \frac{\partial B_x}{\partial \xi} \frac{B_{y0}}{d} = \frac{1}{j_{z0}} \left[ \frac{1}{j_{z0}^2} \frac{4\Phi_0^2}{\eta(0)^2 v^2} \right] \frac{\partial}{\partial \xi} \left[ H_{x1} - H_{x2} \right].$$  \hspace{1cm} (4)
where \( j_{z0} \) is the current density in superconducting layer, \( H_{x1} = H(y = 0) \), \( H_{x2} = H(y = d) \), \( \Phi_0 \) is the magnetic flux quantum.

We can write the boundary condition (4) in the form of a matrix connecting the electrical and magnetic fields at the boundaries \( y=d \) and \( y=0 \):

\[
\begin{pmatrix}
E_z(y=0) \\
H_x(y=0)
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ A & 1 \end{pmatrix} \begin{pmatrix} E_z(y=d) \\
H_x(y=d)
\end{pmatrix}
\]

where \( A = \frac{d j_{z0}^2}{B y_0} \left[ 1 \pm \frac{4 \Phi_0^2 j_{z0}^2}{\eta(0)^2 v_*^2} \right]^{1/2} \),

\[
\frac{2 \Phi_0}{\eta(0) v_*^2} \left[ \frac{1 \pm \frac{4 \Phi_0^2 j_{z0}^2}{\eta(0)^2 v_*^2}}{\frac{\Phi_0}{\eta(0) v_*^2}} \right]^{1/2} \frac{k_x}{\omega j_{z0}}
\]

where \( k_x \) is the projection of the wave vector onto the Ox axis, \( \omega \) is the cyclic frequency. The upper sign corresponds to the forward wave, the lower one to the backward wave.

3. Results
We calculated the reflection coefficient \( R \) from the thin superconducting film using the matrix method and the matrix from expression (5) [10]. The dependence of the reflection coefficient on the current density \( j_{z0} \) is shown in figures 2 and 3.

![Figure 2](image)

**Figure 2.** The dependence of reflection coefficient on the transport current density for different thicknesses of superconducting film \( d \). Forward wave. Structure: thin superconducting film MgO on substrate SrTiO\(_2\). The thickness of substrate is 8 \( \mu \)m, \( B_{y0}=1 \) T, angle of incidence \( \theta_1 = 0.5 \), \( \omega = 10^{10} \text{ s}^{-1} \), \( v_* = 1000 \text{ m/s} \), \( \eta(0) = 1 \cdot 10^{-6} \text{ N} \cdot \text{s/m}^2 \).

It is seen from figures 2 and 3, the electromagnetic wave can demonstrate amplification due to the energy of the Abrikosov vortex lattice. Furthermore, this amplification is observed at low velocities of the vortex structure about \( v = 1 \pm 2 \text{ km/s} \). Such values can be observed in cuprate superconductors and other widely used superconductors. An important feature is the nonreciprocal character of the investigated structure. When the direction of electromagnetic wave propagation coincides with the direction of vortex motion in superconductor (forward wave), we can observe the amplification at small values of the transport current density about \( 3.8 \cdot 10^5 \text{ A/m}^2 \). When we consider the backward wave, the amplification is observed at higher current density about \( 2.4 \cdot 10^{11} \text{ A/m}^2 \). Such high current values in the instability region of mixed state can lead to the destruction of superconductivity. Therefore, we can talk about amplification only for the forward wave.
Figure 3. The dependence of reflection coefficient on the transport current density for different thicknesses of superconducting film $d$. Backward wave. Structure is the same as in figure 2. $B_{so}=1\ T$, angle of incidence $\theta_1=0.5^\circ$. $\omega=10^{10}\ s^{-1}, v^*=1000\ m/s$, $\eta(0)=1\cdot10^{-6}\ N\cdot s/m^2$.

4. Conclusion
The results of reflection coefficient calculation obtained in this work can be used to create superconducting amplifiers and switches for the terahertz and optical ranges.

References
[1] Grigoriev A D, Ivanov V A and Molokovsky S I 2018 Microwave Electronics (Cham: Springer International Publishing AG) p 154
[2] Glushchenko A G and Golovkina M V 2007 Technical Physics 52(10) 1366-8 doi: 10.1134/S1063784207100192
[3] Gutlyanskii E D 2005 JETP Letters 82 72-6 doi: 10.1134/1.2056630
[4] Dobrovolskiy O V, Vodolazov D Y, Porat F, Sachser R, Bevz V M, Mikhailov M Y, Chumak A V and Huth M 2020 Nat Commun. 11 3291 doi: 10.1038/s41467-020-16987-y
[5] Vodolazov D Y, Ilin K, Merker M and Siegel M 2016 Supercond. Sci. Technol. 29 025002 doi:10.1088/0953-2048/29/2/025002
[6] Golovkina M V 2010 Bulletin of the Russian Academy of Sciences: Physics 74 1669-73 doi: 10.3103/S1062873810120105
[7] Larkin A I and Ovchinnikov Y N 1975 Sov. Phys. JETP 41 960
[8] Doettinger S, Huebener R and Kühle A 1995 Physica C 251 285-9 doi: 10.1016/0921-4534(95)00411-4
[9] Bezuglyi A I, Shklovskij V A, Vovk R V, Bevz V M, Huth M and Dobrovolskiy O V 2019 Phys. Rev. B 99 174518 doi: 10.1103/PhysRevB.99.174518
[10] Golovkina M V 2009 Periodic semiconductor structures with metamaterials Proc. Int. Siberian Conf. on Control and Communications SIBCON-2009 (Tomsk: TUSUR) 133-7 doi:10.1109/SIBCON.2009.5044843