Possible unitarity of black hole evaporation

Boguslaw Broda

Department of Theoretical Physics, Faculty of Physics and Applied Informatics, University of Łódź, 90-236 Łódź, Pomorska 149/153, Poland

Abstract

In the framework of finite-dimensional Fock space models, for a fixed given mean number of particles \( \bar{n}_k \), blackbody-like or another, it is shown that there are, in the space \( S \) of all pure states, a multi-dimensional subspace \( s_{\bar{n}_k} \) of initial pure states and a corresponding multi-dimensional subspace \( S_{\bar{n}_k} \) of final pure states yielding \( \bar{n}_k \), which are mutually related by a unitary transformation. In consequence, in particular, it follows that the blackbody form of the Hawking spectrum does not contradict unitarity of black hole evaporation.

Keywords: black hole information loss, black hole information paradox, black hole information problem, Hawking radiation, unitarity of black hole evaporation

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1. Introduction

The black hole information (loss) paradox/problem appears to be one of the most interesting and important intellectual challenges for theoretical physicists for more than last 40 years. In short, the problem consists in difficulty in explaining the status of unitarity of the process of evaporation of black holes. According to standard picture of semiclassical gravity (quantum field theory of "matter" field on a classical black hole background) a black hole (quantumly) evaporates, due to the Hawking effect, and finally transmutes into blackbody radiation. In consequence, we are confronted with the annoying situation where various distinct initial pure states can possibly be transformed into the same final "structureless" blackbody radiation. Due to this "many to one" process, initial information could be irreversibly lost. Alternatively, more formally (in the language of quantum mechanics), an initial pure state can possibly be transformed into a final mixed state. Thus, complete evaporation of a black hole could mean loss of unitarity, in contradiction with fundamentals of (standard)
quantum mechanics. A wider discussion of the problem, some strategies towards its possible solution, as well as extensive literature can be found in recent review articles (see e.g. [1, 2, 3]).

In the present paper we propose a novel, specific approach to the issue of unitality of the process of complete evaporation of black holes. Our conclusions are in accordance with unitarity and conservation of information (in contradiction with, e.g., [4]). The novelty of our approach consists in generality and purely “geometric”/“kinematic” analysis not related to any possible particular dynamics of black hole evolution. (It is interesting to note that according to [5] Hawking radiation is a purely kinematic effect.) We present our arguments in three steps, in the form of the following three models in finite-dimensional Hilbert spaces:

1) Toy Model (“the Universe on the Bloch sphere” or “the qubit Universe”),
2) more realistic (2) Fermion Fock space model, and (3) Boson Fock space model.

The models are fairly elementary, and the idea is quite straightforward. In short, the idea is to show that there is a “large” (in the sense of dimension) subspace $\mathcal{S}_{\bar{n}_k}$ of different pure states in the whole space $\mathcal{S}$ of final pure states (or in the corresponding Hilbert space $\mathcal{H}$) yielding the same (almost arbitrary) given mean number of particles $\bar{n}_k$. Therefore, the process which can seem, at first glance, to be “many to one” can actually be “one to one” (and unitary), because there is “enough room” in the space $\mathcal{S}$ to accommodate it. One should admit that we do not prove that the actual process of black hole evaporation is of such a type (“one to one” and unitary), nor do we present any physical implementation (e.g. Hamiltonian), but we only show that the observed blackbody shape of the Hawking spectrum does not imply non-unitarity of black hole evaporation. In fact, our approach is more general, namely it is not restricted to the context of evaporation of black holes, because no explicit particular form (blackbody or another) of the mean number of particles $\bar{n}_k$ enters our analysis.

Besides, we would like to emphatically stress that we exclusively operate pure states — nowhere do mixed states or density matrices appear, explicitly or implicitly, in our considerations. In particular, $\bar{n}_k$, blackbody or another, is a mean of the particle number operator $\hat{n}_k$ in a pure state (see [3]). In our approach, in a sense, thermality of the blackbody spectrum is simulated by an appropriately chosen pure state $|\bar{n}_k\rangle$ (see [10, 17, 28]).

2. General idea and the Toy Model

In his famous work [6] Hawking derived a formula for a mean number of particles $\bar{n}_k$ ($k$ — mode number), generally defined as a quantum average

$$\bar{n}_k \equiv \langle \hat{n}_k \rangle,$$

where $\hat{n}_k$ is the particle number operator for the mode $k$. Assuming an appropriate state for averaging in [1] we get

$$\langle \hat{n}_k \rangle = \sum_{k'} |\beta_{kk'}|^2,$$
2.1 General idea

where $\beta_{kk'}$ are the Bogoljubov coefficients, which are determined by geometry. In the simplest treatment the mean number of particles $\bar{n}_k$ appears to be black-body. In fact, blackbody spectrum of radiation from a black hole is modified in a number of ways which are extensively discussed in [7]. Actually, what counts from our point of view is the total mean number of particles $\bar{n}$, rather than usually discussed temporary quantities, but as is mentioned in Section 1 our analysis is not sensitive to any particular form of $\bar{n}_k$.

2.1. General idea

Our first important observation is that we have a “huge multitude” of pure states yielding an (almost) arbitrary fixed mean number of particles $\bar{n}_k$. More precisely, we have a “large”, in the sense of low codimension, subspace $S_{\bar{n}_k}$ in the space $S$ of all pure states (or in the Hilbert space $\mathcal{H}$) for almost any arbitrarily chosen $\bar{n}_k$ (the only restriction on $\bar{n}_k$ is given, depending on the case, by the mild condition [11] or [24], or [33]). Therefore, having first given an explicit form of $\bar{n}_k$ and provided we are able to determine the corresponding subspace $S_{\bar{n}_k}$ of pure states, we can choose any point/state (out of infinitely many, in the sense of multi-dimensional continuum) $|\bar{n}_k\rangle$ yielding, by virtue of the definition of $S_{\bar{n}_k}$, the average with expected predefined values, i.e.,

$$\langle \bar{n}_k | \hat{n}_k | \bar{n}_k \rangle = \bar{n}_k.$$  \hfill (3)

Our second important observation is that we can next perform a unitary transformation $U(-t)$ on $S_{\bar{n}_k}$ which is now interpreted as a subspace of final “thermality imitating” states (in our paper we exclusively deal with pure states), obtaining another subspace $s_{\bar{n}_k}$ (actually, because of unitarity of $U(-t)$, $s_{\bar{n}_k}$ is isometric to $S_{\bar{n}_k}$ in the sense of the complex metric on $\mathcal{H}$) interpreted as a subspace of possible initial states. The unitary transformation $U(-t)$ corresponds to evolution backward (the minus sign) in time. Thus, we can conclude that the “huge multitude” of distinct initial pure states belonging to $s_{\bar{n}_k}$ unarily (according to $U(t)$) evolves towards “huge multitude” of distinct pure states belonging to the subspace $S_{\bar{n}_k}$ with all the states that yield, by virtue of the construction, the fixed predefined mean number of particles $\bar{n}_k$.

2.2. Toy Model

Now, let us consider the Toy Model (which we can call “the Universe on the Bloch sphere” or “the qubit Universe”). Its only role is to explicitly elucidate and visualize (because of low dimension) our main idea. As a chief postulate of the model we assume that the whole Universe consists of only one fermion mode (2-level system). Its Hilbert space $\mathcal{H} = \mathbb{C}^2$ is 4-dimensional in real sense (in this paper we only operate real dimensions), and in the Fock space base $\{|0\rangle, |1\rangle\}$ any state $|\psi\rangle \in \mathcal{H}$ can be expressed as

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle,$$

$\alpha_0, \alpha_1 \in \mathbb{C}.$  \hfill (4)

Because of normalization ($\langle \psi | \psi \rangle = 1$) and of arbitrariness of phase, pure states for this system are parameterized by points on the 2-dimensional Bloch sphere.
Then, the general linear combination (4) can be specified as

\[ |\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \varphi < 2\pi, \]

where \( \theta \) and \( \varphi \) are the polar and azimuthal angle on \( S^2 \), respectively. From the point of view of quantum mechanics any state of the Universe is uniquely given by a point on \( S^2 \), and arbitrary unitary (e.g. time) evolution \( U(t) \) of the Universe corresponds to rotation of \( S^2 \), i.e., \( U(t) \in SO(3) \).

Now, we would like to determine the entire (sub)space \( S\bar{n} \) of states \( |\bar{n}\rangle \) yielding the predefined mean number of particles \( \bar{n} \), where obviously

\[ 0 \leq \bar{n} \leq 1. \]  

(6)

Since \( k = 1 \), the mode number \( k \) has been skipped in this subsection, and the mean number of particles \( \bar{n} \) is now a single number belonging to the interval (6). In general, the state we are looking for, expressed as

\[ |\bar{n}\rangle = \bar{\alpha}_0 |0\rangle + \bar{\alpha}_1 |1\rangle \]  

(7)

(in our paper the “bar” over coefficients denotes their particular values — complex conjugation is denoted by the “asterisk”), should satisfy the two conditions:

\[ \langle \bar{n} | \bar{n} \rangle = 1, \quad \langle \bar{n} | \hat{n} | \bar{n} \rangle = \bar{n}. \]  

(8)

Utilizing the angular parametrization on the Bloch sphere (5) we easily find

\[ \bar{n} = |\bar{\alpha}_1|^2 = \sin^2 \frac{\theta}{2}. \]  

(9)

Thus, finally the solution of the problem (full set of solutions of Eqs.(8) modulo phase) assumes the following explicit form

\[ |\bar{n}\rangle = \sqrt{1 - \bar{n}} |0\rangle + e^{i\varphi} \sqrt{\bar{n}} |1\rangle. \]  

(10)

Eq.(10) says that all the points (interpreted by us as final pure states) on the circle (“parallel”) \( S\bar{n} \) parameterized by the azimuthal angle \( \varphi \) and determined by the “latitude” (on \( S^2 \)) given by the polar angle (see (2)) \( \theta = 2 \arcsin \sqrt{\bar{n}} \), yield the same predefined \( \bar{n} \). We can (thought) rotate the circle (parallel) \( S\bar{n} \) (“evolution backward in time \( U(-t) \)”) obtaining another circle (not a parallel, in general) \( S\bar{n} \) parametrizing all the states (interpreted as initial states) which are transformed (in the course of the “proper time evolution \( U(t) \)”) into states on \( S\bar{n} \) with the predefined \( \bar{n} \). Thus, in general, we have a “one to one” unitary relation between points on isometric circles on \( S^2 \), where parallels play a distinguished role of “thermality imitating” states. Since for \( \bar{n} = 0, 1 \) the circle \( S\bar{n} \) degenerates to a point (a pole), we can impose a mild restriction on admissible values of \( \bar{n} \), removing the boundary values of the interval (6), and putting

\[ 0 < \bar{n} < 1. \]  

(11)

An example situation is explicitly illustrated in Fig.1.
Geometric statement: Rotation $U(\alpha)$ through the angle $\alpha = \pi/2$ in the plane of Figure transforms the circles $s_1$ and $s_2$ on the sphere $S^2$ (and also other circles) into the circles $S_1$ and $S_2$, respectively. Polar angle coordinates of the circles $S_1$ and $S_2$ are $\theta_1$ and $\theta_2$, respectively, and the points on the circles are parametrized by the azimuthal angle $\varphi$. In particular, the points $x_1, x_2 \in s_2$ are transformed into the points $X_1, X_2 \in S_2$, respectively. Quantum-mechanical statement: 2 distinct initial pure states $x_1$ and $x_2$ unitarily (after “time $\pi/2$”) evolve into 2 distinct pure states $X_1, X_2$, respectively, with the same mean number of particles $\bar{n} = \sin^2 \frac{\theta_2}{2}$.

3. Fock space models

In this section we introduce two more realistic models, based on fermion Fock space and boson Fock space, respectively. To technically simplify our discussion (algebraization of the problem), as well as to make it more quantitative and rigorous, we impose some cutoffs on the Fock spaces, which implies finite dimensional Hilbert spaces.

3.1. Fermion Fock space model

First, we consider a fermion model defined on the antisymmetric Fock space $F^n_A$, where $m$ is a number of fermion modes. Here, the cutoff simply means that the number of modes $m$ is finite. Generalizing the linear expansion (4) we can express any state $|\psi\rangle \in F^n_A$ as a linear combination

$$|\psi\rangle = \sum_n \alpha_n |n\rangle, \quad \alpha_n \in \mathbb{C},$$

(12)

where, for convenience, we define a multi-index $n \equiv n_1 \ldots n_m$ with $n_k = 0, 1$ ($k = 1, \ldots, m$).

Analogously to the presentation (7) the points/states $|\bar{n}\rangle \in \tilde{S}_n$, i.e., those normalized and satisfying the condition (3), can be expressed by the sum

$$|\bar{n}\rangle = \sum_n \bar{\alpha}_n |n\rangle,$$

(13)
where for the multi-parameter \( \vec{n} \) we assume \( 0 \leq \tilde{n}_k \leq 1 \), and the “tilde” means the space of states before identification (denoted by “/ \( \sim \)” ) of states differing by phase, e.g. \( S_{\vec{n}} = \tilde{S}_{\vec{n}} / \sim \). Normalization condition for (13) reads

\[
\langle \vec{n} | \vec{n} \rangle = \sum_{n} |\tilde{\alpha}_n|^2 = 1,
\]

whereas the condition (3) yields the following system of \( m \) quadratic equations

\[
\langle \vec{n} | \hat{n}_k | \vec{n} \rangle = \sum_{n_k} |\tilde{\alpha}_{n_k}|^2 = \tilde{n}_k,
\]

where, for further convenience, we define another multi-index \( n_k \equiv n_1 \ldots 1_k \ldots n_m \), i.e., the \( k \)th index assumes a constant value \( (n_k =) 1 \), and consequently there is no summation with respect to \( n_k \) in (15). Furthermore, a bit extending the domain of the index \( k \), and introducing a new auxiliary index \( p = 0, 1, \ldots, m \) instead of \( k ( = 1, \ldots, m ) \), and next denoting \( n_0 \equiv \vec{n} \) and \( \tilde{n}_0 \equiv 1 \), we can write down the quadratic equation (14) and the system (15) in the following compact elegant unified form

\[
\sum_{n_p} |\tilde{\alpha}_{n_p}|^2 = \tilde{n}_p, \quad p = 0, 1, \ldots, m.
\]

Thus, the (sub)space \( \tilde{S}_{\vec{n}} \) is defined as a solution of the system of \( m + 1 \) quadratic equations (16). Fortunately, to proceed further we do not need an explicit form of \( S_{\vec{n}} \), as in the case of the Toy Model of Section 2, where global analysis has been performed. Since we only aim to determine the (co)dimension of \( \tilde{S}_{\vec{n}} \) (and of \( S_{\vec{n}} \)), we can confine ourselves to a purely local discussion.

Our strategy is first to find only a single non-degenerate (in the sense explained latter) solution of the quadratic system (16), and next to show that it can be infinitesimally extended in sufficiently many directions. It is straightforward to check that the following “(tensorial-product) Bloch-type” state

\[
|\vec{n}; \phi \rangle \equiv \sum_{n} \tilde{\alpha}_n(\phi) |n\rangle, \quad \phi = \varphi_1, \ldots, \varphi_m, \quad 0 \leq \varphi_k < 2\pi, \quad k = 1, \ldots, m,
\]

where (cf. (5))

\[
\tilde{\alpha}_n(\phi) \equiv \tilde{\alpha}_{n_1 \ldots n_m}(\varphi_1, \ldots, \varphi_m) = \prod_{k=1}^{m} \left( \delta_{n_k}^0 \cos \frac{\theta_k}{2} + \delta_{n_k}^1 e^{i\varphi_k} \sin \frac{\theta_k}{2} \right),
\]

with (cf. (9))

\[
\sin^2 \frac{\theta_k}{2} = \tilde{n}_k,
\]

solves the system (16). Actually, the formulas (17, 19) define the whole \( m \)-dimensional torus \( T^m \equiv S^1 \times \cdots \times S^1 \) of solutions of the system (16). Since
our analysis is supposed to be local, we only need a single solution of the system (16), and therefore, to simplify our further considerations we put \( \phi = 0 \).

To find a solution of the system (16) in an infinitesimal vicinity of the Bloch-type solution (17–19) at the point \( \phi = 0 \) on the torus \( T^m \), we insert to (16) the expansion
\[
\tilde{\alpha}_n = \tilde{\alpha}_n (0) + z_n, \tag{20}
\]
where \( z_n \) is a (complex) infinitesimal variation around the solution \( \tilde{\alpha}_n (0) \). Thus, we get a system of \( m + 1 \) linear equations
\[
\text{Re} \sum_{n_p} \tilde{\alpha}_{n_p} (0) z_{n_p}^* = 0, \quad p = 0, 1, \ldots, m, \tag{21}
\]
which define \( m + 1 \) hyperplanes tangent to the respective \( m + 1 \) quadrics determined by the system (16). The maximal possible rank of the matrix of the coefficients entering the system (21) is obviously \( m + 1 \), and such a situation (the most desirable one) geometrically corresponds to a non-degenerate intersection of the hyperplanes tangent to the quadrics. Since to determine the rank of a matrix, one usually invokes determinants, let us calculate the determinant of the matrix constructed from the columns containing the following coefficients: \( \tilde{\alpha}_{00} (0), \tilde{\alpha}_{10} (0), \tilde{\alpha}_{01} (0), \ldots, \tilde{\alpha}_{00} (0), \ldots, \tilde{\alpha}_{00} (0) \). The matrix reads
\[
M_A = \begin{bmatrix}
\tilde{\alpha}_{00} (0) & * & \cdots & * \\
\tilde{\alpha}_{10} (0) & * & \ddots & * \\
\vdots & \ddots & \ddots & * \\
\tilde{\alpha}_{00} (0) & & & * \\
\end{bmatrix}, \tag{22}
\]
where \( M_A \) is an upper triangular matrix (more precisely, all entries of \( M_A \), except possibly the 1st row and the main diagonal, are zero). Then, by virtue of (18)
\[
det M_A = \tilde{\alpha}_{00} (0) \prod_{j=1}^{m} \tilde{\alpha}_{00-j,0} (0) = \prod_{j=1}^{m} \sin \frac{\theta_j}{2} \cos^m \frac{\theta_j}{2}. \tag{23}
\]
From (23) it follows that the rank of the system (21) is in fact maximal (= \( m + 1 \)), and there is no degeneracy, provided we impose the condition \( 0 < \theta_k < \pi \), which corresponds (by virtue of the relationship (19)) to a very mild restriction on \( \tilde{n}_k \) (cf. (11)),
\[
0 < \tilde{n}_k < 1, \quad k = 1, \ldots, m, \tag{24}
\]
in comparison with all theoretically admissible values: \( 0 \leq \tilde{n}_k \leq 1 \).

### 3.2 Boson Fock space model

Let us now switch to a boson model defined on the symmetric Fock space \( \mathcal{F}_{m,N}^s \) with cutoffs \( m \) and \( N \), where \( m \) is a finite number of boson modes, and a fixed finite number of levels, the same for each boson mode, is equal to \( N + 1 \).
3.2 Boson Fock space model

In principle, \( N(\geq 1) \) can be arbitrary, but for our needs it should be sufficiently large, i.e.,

\[
\tilde{n}_k < N < +\infty, \quad k = 1, \ldots, m.
\] (25)

As a single-mode Bloch-type state (with \( \varphi = 0 \)) in the boson case we assume

\[
|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |N\rangle,
\] (26)

where \( 0 \leq \theta \leq \pi \). Executing calculations similar to those in Section 2 we obtain, as analog of (9), the relationship

\[
\frac{\tilde{n}_k}{N} = \sin^2 \frac{\theta}{2}.
\] (27)

In turn, the tensorial-product Bloch-type state (with \( \phi = 0 \)) is now (cf.(17))

\[
|\bar{n}; 0\rangle = \sum_n \bar{\alpha}_n (0) |n\rangle,
\] (28)

where (cf.(18))

\[
\bar{\alpha}_n (0) = \prod_{k=1}^{m} \left( \delta^0_{n_k} \cos \frac{\theta_k}{2} + \delta^N_{n_k} \sin \frac{\theta_k}{2} \right),
\] (29)

with (cf.(19))

\[
\sin^2 \frac{\theta_k}{2} = \frac{n_k}{N}.
\] (30)

For the multi-index \( n \) in the boson case we assume \( n_k = 0, 1, \ldots, N \), and for the multi-parameter \( \bar{n} \), \( 0 \leq \bar{n}_k < +\infty \).

The matrix analogous to (22) is now the (upper triangular) matrix

\[
M_S = \begin{bmatrix}
\bar{\alpha}_{00\ldots0} (0) & * & \cdots & * \\
& \bar{\alpha}_{N0\ldots0} (0) & \ddots & \\
& & \ddots & \\
& & & \bar{\alpha}_{00\ldots N} (0)
\end{bmatrix},
\] (31)

and its determinant,

\[
\det M_S = \prod_{j=1}^{m} \sin \frac{\theta_j}{2} \cos \frac{\theta_j}{2},
\] (32)

is exactly of the same form as \( \det M_A \) (see (23)). The relations (27) and (32) impose a very mild restriction on \( \bar{n}_k \) (cf. (24)),

\[
0 < \bar{n}_k < +\infty, \quad k = 1, \ldots, m,
\] (33)

in comparison with all theoretically admissible values: \( 0 \leq \bar{n}_k < +\infty \).
3.3 Summary of the Fock space models

In the case of the fermion Fock space model as well as in the case of the boson one, for fairly general admissible mean number of particles $\bar{n}_k$, respectively, we showed that the tensorial-product Bloch-type state (17–19) and (28–30), respectively, determines a non-degenerate intersection in the corresponding (finite dimensional) Hilbert space $H = F_{A}^m$ and $H = F_{S}^m$, respectively. More precisely, the intersecting hyperplanes tangent to the intersecting quadrics defined by the system (16) (with indices $n_k = 0, 1$ and $n_k = 0, 1, \ldots, N$, respectively) are "in general position". Therefore, the intersection of the quadrics is non-degenerate (a true intersection rather than a contact point) and general analysis applies: Each quadric (equation in the system of the $m + 1$ equations (16)) imposes one condition and reduces dimension of the subspace in $H$ by one.

In particular, codimension of the intersection of the whole set of the quadrics (16) equals the codimension of the intersection of the set of the hyperplanes tangent to these quadrics, and is equal to $m + 1$. Then, $\dim \tilde{S_n} = \dim H - (m + 1)$ and $\dim S_n = \dim \tilde{S_n} - 1 = \dim H - m - 2$. The last subtraction is justified provided the action of the group $U(1)$, corresponding to identification of states differing by phase, proceeds tangentially to $\tilde{S_n}$ at the point $\bar{\alpha}_n(0)$. Glancing at the system (21) we can observe that imaginary parts, $y_n \equiv \text{Im} z_n$, of the infinitesimal variations $z_n$ are arbitrary (unrestricted by the system (21)), and therefore (infinitesimally) $\tilde{S_n}$ can extend freely in imaginary directions, and this is exactly the direction of (infinitesimal) action of the $U(1)$.

To quantify the term "large" or "huge multitude" used in Section 2 in the context of (co)dimension of $S_n$ we should compare dimensions of all relevant spaces. To begin with, for the Hilbert space $H$ we have $\dim H = 2(N + 1)^m$, where $N = 1$ or $N$ is a cutoff (see (25)), in the fermion case ($F_{A}^m$) or in the boson case ($F_{S}^m$), respectively. The normalization condition lowers dimension of the space of states by one, likewise identification of states differing by phase ($\tilde{S} \rightarrow S$) [9, 10]. Hence, dimension of the space $S$ of all states $\dim S = \dim \tilde{S} - 1 = \dim H - 2 = 2(N + 1)^m - 2$. Therefore, $S_n$ is a $2(N + 1)^m - m - 2$-dimensional subspace in the $[2(N + 1)^m - 2]$-dimensional space $S$ of all states (incidentally, codimension of $S_n$ in $S$ is equal to the number of the modes $m$).

Interpreting $S_n$ as a subspace of final (pure) states, and performing a unitary transformation determined by $U(t)$ (understood as evolution backward in time), we obtain an isometric to $S_n$ subspace $s_n$ interpreted as a subspace of initial states. By virtue of the construction, unitary (e.g. time) evolution given by $U(t)$ transforms all pure states belonging to $s_n$ into pure states belonging to $S_n$ which yield the same mean number of particles $\bar{n}_k$.

4. Final remarks

In our approach we have assumed the most standard course of events concerning the fate of an evaporating black hole: at the very beginning of the evolution we deal with a collapsing star determined by a pure state, at the very end, we only deal with blackbody(-like) radiation from a black hole which
completely evaporated. In a sense, our philosophy is in the spirit of S-matrix approach, where we exclusively refer to (and compare) initial and final states without any resort to possible dynamics (for a recent dynamical proposal see [13]). Importantly, no mixed states, nor entropy considerations appear in our analysis, explicitly or implicitly. Moreover, comparing, in the framework of our finite-dimensional Hilbert space models, a subspace $S_{\bar{n}}$ of final pure states corresponding to a predefined mean number of particles $\bar{n}_k$, e.g. blackbody-like, to the whole space $S$ of final pure states, we can observe that the order of growth of codimension of $S_{\bar{n}}$ is only logarithmic in dimension of $S$. Therefore, since $S_{\bar{n}}$ is a (sub)space parametrizing a really “huge multitude” of all potential final states with given $\bar{n}_k$, and $s_{\bar{n}}$ (isometric to $S_{\bar{n}}$) is a (sub)space of potential initial states, there is “enough room” in the space $S$ to accommodate unitarily realized time evolution.

Recapitulating, we would like to stress that we do not prove that the actual process of black hole evaporation is unitary, nor do we address any difficulties possibly occurring in the course of the black hole evolution, but we only demonstrate that the blackbody form of the Hawking spectrum does not contradict unitarity of black hole evaporation. For other proposals concerning unitary evaporation of black holes see e.g. [14] [15] [16] [17].

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