Generation of isolated intense vortex laser with transverse angular momentum

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Abstract
A scheme is proposed to explore the generation of isolated intense vortex laser pulse with transverse angular momentum (AM), which implies that the total AM is non-collinear with the propagation direction. When two non-collinear vortex beams impinge on a solid thin target symmetrically on the same side, the generated harmonics containing the contributions of the two input pulses are emitted from the target at a predicted angle. The longitudinal AM of the harmonics can be predicted from the AM conservation regarding the photons involved in the high-harmonic generation process. The asymmetry of the energy flux in the vertical direction is confirmed as the transverse AM generation source. As an example, the related phenomenon of the fourth order harmonic has been well confirmed by theoretical analysis and three-dimensional particle-in-cell simulations.

1. Introduction

Since Allen et al \cite{1,2} demonstrated that a vortex beam possesses an orbital angular momentum (OAM), which makes it a beneficial and powerful probing tool, the research on its generation and application have been extensively. A vortex beam exhibits a helical wavefront that is described by the phase component of \( \exp(i\ell\alpha) \), where \( \alpha \) is the azimuthal coordinate and the integer number \( \ell \) is the topological charge. For a general vortex beam, OAM of each photon it carries is \( \ell\hbar \) in the propagation direction. While the application of the vortex beam with an intensity \( <10^{17} \text{ W cm}^{-2} \) has been realized in various fields \cite{3-9}, the generation and application of relativistic vortex beam applying plasma as the nonlinear medium has been a subject of interest in recent years \cite{10-16}, such as the positive particle acceleration \cite{17,18}, high mode vortex beam generation \cite{19-21}, and specific scientific law correction \cite{22}, etc.

Among the aforementioned research on the generation and application of vortex beams, only the longitudinal angular momentum (AM) along the propagation direction is included. Unlike the longitudinal AM, the transverse AM is orthogonal to the linear momentum of light (longitudinal direction) and includes the transverse spin AM (SAM) and transverse OAM. There is growing interest in these optical fields because of their highly different physical properties and wide promising application fields. For the general intense optical fields, most of the observed transverse SAM is transient and localized \cite{23,24}, usually occurring in tightly focused beams and evanescent waves of tightly confined waveguides. More recently, it should be pointed out that the beam having only transverse OAM which is called spatiotemporal optical vortices is achievable. Completely different from the twist structure of the vortex beam only having longitudinal OAM, the electric field of such kind of beam spins around the axis perpendicular to its propagation plane, just like a photonic wheel \cite{25}. It has been generated in the experiments and the transverse OAM can be controllable by applying a spiral phase in the spatial frequency–frequency domain \cite{26-30}, which promises broader
applications [31, 32] because it provides a new freedom degree to control. For example, it is promising to be applied in optical spanners with arbitrary three-dimensional (3D) orientation by combining the transverse OAM and longitudinal OAM [33–35], although the intensity of the spatiotemporal optical vortex is limited in the nonrelativistic regime. For the optical fields in the relativistic region, a vortex beam of tilted OAM can be generated from grating using the relativistic vortex laser pulse [36], the transverse OAM of which scales with the diffraction order.

In this work, we considered a quite different scheme to generate the isolated vortex with transverse AM. In this scheme, two non-collinear vortex beams impinge on a solid thin target symmetrically on the same side with a large crossing angle, such as 90°, harmonics that contained the contributions of the two input pulses emitted from the target. Besides the variable \(y(z)\), which induced the well-known longitudinal OAM, the phase term of the harmonic also changed with the variable \(x(z)\), which might have induced the helical structure of the phase in the \(xz\) plane, and resulted in the transverse OAM, where \(x\) is defined as the longitudinal direction (also is the laser propagation direction) and \(y(z)\) is the transverse direction. Further, the 3D particle in cell (PIC) simulations confirmed the generation of harmonics with transverse AM due to the asymmetry of the energy flux in the vertical direction (\(z\) direction). The transverse AM observed in our scheme can propagate with the optical fields in space. Meanwhile, the longitudinal AM of the harmonics can be predicted from the AM conservation regarding the photons involved in the high-harmonic generation (HHG) process, as demonstrated by the theoretical and 3D PIC simulation results.

HHG is one of the most applied methods of getting vortex beams with a high topological charge for various applications. Regarding the present laser level, using a gas exposed to light (>10\(^{14–16}\) W cm\(^{-2}\)) [37–42] or plasma target exposed to intense light (>\(10^{18}\) W cm\(^{-2}\)) [19–21, 43–47] is the most promising approaches toward HHG. It has been confirmed that the topological charge of the harmonics is proportional to its order [19, 38, 39, 47] when only one driving pulse is used. When more driving pulses are applied, the generated harmonics will contain the information of all the input pulses, thus availing increased possibilities for applications. For example, an isolated harmonic with low topological charge can be obtained when one major Gaussian pulse and another interrupted vortex beam non-collinearly focused on the gas [42, 48–50]. Regarding the relativistic driving pulses, an attempt to interact the two head-on circularly polarized (CP) vortex pulses with a thin target has been reported [20]. It was observed that the topological charge and frequency of the generated harmonics depend on the tunable information contained in both driving pulses. Recently, based on the non-collinear plasma HHG process, angularly isolated harmonics carrying the information of the two input lasers at large emission angles have been reported [51]. In this work, by overlapping two non-collinear vortex driving lasers on the plasma target, the spatial structure of the driving field is imprinted on the electron dynamics and hence on the vortex harmonic generation. For the same reason, the generated harmonics containing the information of the two incident laser pulses has been isolated spatially, thus one can adjust the incident lasers for the different harmonic emission to satisfy different requirements. More importantly, the harmonics contain both the longitudinal and transverse OAMs which depends on the incident angles and the topological charges of the input lasers, which is the key point we care more about in this work.

2. Theoretical model

Figure 1 exhibits the proposed scheme. Two linearly polarized Laguerre–Gaussian (LG) laser pulses, \(a_1\) and \(a_2\), with frequencies and topological charges of \((\omega, l_1)\) and \((2\omega, l_2)\), irradiate on a thin target at the same time. The normalized vector potential of the laser amplitude acting on the target was \(a = a_1 + a_2 (a = eA/m_e\gamma c^2\), where \(A\) is the vector potential, \(c\) is the speed of light in vacuum, \(m_e\) is the electron mass, and \(e\) is the electron charge). \(a_1\) and \(a_2\) are used as

\[
\begin{align*}
    a_1 &= a_0 \cos(\omega t - kx \cos(\theta) - ky \sin(\theta) + l_1\alpha_1) \hat{z}, \\
    a_2 &= a_0 \cos(2\omega t - 2kx \cos(\theta) + 2ky \sin(\theta) + l_2\alpha_2) \hat{z},
\end{align*}
\]

where \(\alpha_1 = \arctan\left(\frac{\gamma \cos(\theta)}{k} - \frac{\gamma \sin(\theta)}{k}\right)\), \(\alpha_2 = \arctan\left(\frac{\gamma \cos(\theta) + \gamma \sin(\theta)}{-k}\right)\) are the azimuthal angles of the two driving pulses, and \(k = \omega/c\) is the wavenumber. After the Fourier expansion of the source term \(a/\gamma\) (where \(\gamma = \sqrt{1 + a^2}\)), which is the nonlinear part that contributed to HHG, the following terms could be obtained:

\[
\frac{a}{\gamma} \approx \left[ \cos\left(\frac{(3m + 1)\omega t - (3m + 1)kx \cos(\theta)}{\gamma} + (m - 1)ky \sin(\theta) + ((m + 1)l_1\alpha_1 + ml_2\alpha_2)\right) \right. \\
+ \left. \cos\left(\frac{(3m + 2)\omega t - (3m + 2)kx \cos(\theta)}{\gamma} + (m - 1)ky \sin(\theta) + ((m + 1)l_1\alpha_1 + (m + 1)l_2\alpha_2)\right) \right] \hat{z}, \tag{1}
\]
where \( m = 0, 1, 2, \ldots \). It can be observed that when \( m = 1 \), the fourth harmonic, 
\[
\cos(4\omega t - 4k_x \cos(\theta)) + (2l_1 \alpha_1 + l_2 \alpha_2) z,
\]
containing the information of the two driving pulses is emitted in the \( x \) direction and isolated from the driving pulses. Here we should note the emission direction can be obtained from the expression or the phase matching equation, however, the harmonic propagates with the light speed in vacuum, so the corresponding linear momentum regarding the vector length should be \( nk \), where \( n \) is the harmonic order. The whole interaction system, including the plasma, should be considered to accomplish the phase matching for the non-collinear case, that is, to ensure the conservation of the linear momentum regarding the vector length. Thus, the linear momentum regarding the vector length of the fourth harmonic should be 4\( k \) instead of 4\( k \cos(\theta) \). According to the energy and linear momentum conservation equations, each photon of such fourth harmonic with the phase \( \theta_{4,\alpha_{4k}} = (2l_1 \alpha_1 + l_2 \alpha_2) \) can be identified to originate from two photons of \( \alpha_1 \) and one of \( \alpha_2 \). For an extreme case where the normal incidence was \( \theta = 0 \), the corresponding phase expression was \( \theta_{4,\alpha_{4k}} = (2l_1 + l_2) \arctan(y/z) \), where \( \alpha_{4k} = \alpha \arctan(y/z) \), which determines \( l_{4k} = (2l_1 + l_2) \) and agrees well with the results in the previous report [20]. For the case where \( \theta = \pi/4, \theta_{4,\alpha_{4k}} = 2l_1 \arctan(y-x)/\sqrt{2z} + l_2 \arctan(y-x)/\sqrt{2z} \), besides the variable \( y/z \) which contributes to OAM in the \( x \) direction (i.e. the propagating direction), the phase term of the fourth harmonic also changed with the variable \( x/z \) and resulted in a transverse OAM in the \( y \) direction. For simplicity, \( \tan(\alpha) \sim \alpha \) is adopted, which is reasonable for small \( \alpha \), then \( \alpha_1 = (y-x)/\sqrt{2z}, \alpha_2 = (y+x)/\sqrt{2z} \), and the phase of the fourth harmonic can be written as \( \theta_{4,\alpha_{4k}} = -2(1.2y/z - 0.71x/z) \) for \( l_1 = -1, l_2 = -1 \) and \( l_{4k} = -2.12 \) for the former and \( l_{4k} = -0.71 \) for the latter because the fourth harmonic propagates in the \( x \) direction.

Considering from the OAM conservation, we know that in the case of oblique incidence, the OAM of the reflected LG laser pulse is along its propagation direction. The change of OAM in the horizontal direction of the target has been transferred to the target, whereas the OAM component in the normal direction is conserved before and after the reflection since OAM is a pseudovector. Similarly, in the present case of the two crossing LG laser pulses, the OAM component in the \( x \) direction (the normal direction of the target) should be conserved for the photons before and after the HHG process. The OAM of the fourth harmonic in the \( x \) direction is expected to be \( l_{4k} = -2.12 \) and -0.71 for the \( l_1 = -1, l_2 = -1 \) and \( l_1 = -1, l_2 = 1 \) cases according to the OAM vector addition of photons. This expectation agrees well with the aforementioned theoretical results. As for the transverse OAM of the fourth harmonic, although the \( (x/z) \) term contributes to the generation of transverse OAM, the specific values would not be 0.71 and 2.12 as in the \( l_{4k}\alpha_{4k} \) expression because the fourth harmonic propagates in the \( x \) direction instead of the \( y \) direction.

### 3. 3D PIC simulation and discussion

To confirm the above analysis, 3D PIC simulations have been performed with the EPOCH code [52]. The normalized vector potential of the driving LG beam is defined by the following:

\[
a(\mathbf{LG}_p) = a_0 \left( \frac{\sqrt{2} \pi}{r_0} \right) \exp \left( -\frac{r^2}{r_0^2} \right) \exp (i \omega t) \exp (i \omega x) \exp (i \omega t) (-1)^p L_p^1 \left( \frac{2r^2}{r_0^2} \right) \sin \left( \frac{\pi t}{2r_0} \right).
\]

![Figure 1. Schematic of the proposed scheme. (a) Two intense laser pulses, \( a_1(\omega, l_1) \) and \( a_2(2\omega, l_2) \), irradiated symmetrically on a thin target at a large crossing angle \( \theta \), and the generated fourth harmonic containing the information of the two driving pulses was emitted in the target normal direction according to the conservation of linear momentum. (b) Configuration of the 3D PIC simulation box. The driving laser field distribution when \( \theta = 45^\circ \) and the coordinate axis \( x'(y') \) is 45\(^\circ\) anticlockwise to the axis \( x(y) \), indicating the \( x-y \) plane becomes the \( x'y' \) plane after rotating 45\(^\circ\) around \( z \) axis in the anticlockwise direction.](image)
A linearly polarized LG pulse $a_1$ with frequency $\omega$ corresponding to the wavelength $\lambda = 1 \, \mu\text{m}$ propagates along the $+x'$ direction, and $r = \sqrt{y'^2 + z'^2}$. Another pulse $a_2$ with frequency $2\omega$ propagates along the $-y'$ direction and $r = \sqrt{x'^2 + z'^2}$. $a_0 = 5$, $r_0 = 4 \, \mu\text{m}$, and $t_0 = 7T$, where $T$ is the period of the pulse $a_1$. The topological charges of the $a_1$ and $a_2$ pulses are $l_1 = -1$ and $l_2 = \pm 1$, respectively. They focus on the target at the same angle $\theta = \pi/4$ symmetrically, as shown in figure 1(b). In the PIC simulation, the coordinate system $x'y'z'$ is used, which is $\pi/4$ anticlockwise to the $xyz$ rotation around the $z$ axis. That is, the $x$-$y$ plane becomes the $x'y'$ plane after rotating $45^\circ$ around $z$ axis in the anticlockwise direction. The dimension of the simulation box is $30 \, \mu\text{m}(x') \times 30 \, \mu\text{m}(y') \times 30 \, \mu\text{m}(z')$, which corresponds to a window of $800 \times 800 \times 800$ cells with 15 particles per cell. The thin target, which has a $0.5 \, \mu\text{m}$ thick, has a density $n_0 = 20n_c$, where $n_c = 1.1 \times 10^{21} \, \text{cm}^{-3}$ is the critical density for the pulse $a_1$. At $t = 0$, the laser pulses enter the simulation box.

As expected, the fourth harmonic is emitted in the normal direction (the $x$ direction) of the target, as shown in figures 2(a) and (c) in the cases of $l_1 = 1$ and $l_2 = -1$. Here we note that when the target is thin enough and of the above-critical density to render it non-transparent, harmonics will be emitted from both the front and rear of the target. For the parameters we used, harmonics in the front of the target are more intense and only higher order harmonics in the rear of the target can be observed due to its higher critical density. Therefore, we only focus on the reflected direction for the analysis since the harmonic in both sides exhibits the same characteristic, except the intensity. Besides of the fourth harmonic, there are also other order harmonics. (The complete spectrum in $k$-space and its corresponding electric field including all the harmonics except the fundamental components are shown in supplementary material). In the reflected directions of the two input lasers, there are harmonics with the odd order $(2n-1)\omega_{1,2}$, which are only related to one input pulse ($\omega_1$ or $\omega_2$). For the others originating from both the input lasers, they are separated (isolated) spatially because of the phasing matching [51]. Here we choose the fourth harmonic as an example to analyze the OAM due to its relatively higher intensity. The fourth harmonic propagates in the $x$ direction in both cases, as determined by the same linear momentum conservation. However, as shown in figures 2(b) and (d), the distribution in the section plane (perpendicular to the propagation direction) are quite different. For the $l_2 = 1$ case, the phase change is close to $2\pi$ according to the donut-shaped distribution of its electric field, indicating that the topological charge should be $\sim 1$. Conversely, in the $l_2 = -1$ case, the phase jumps three times in one loop, indicating that the topological charge is $\sim 3$. Figure 3 shows the electric field isosurfaces of the fourth harmonic of the full length and within a $1 \, \mu\text{m}$ distance in the propagation direction, thereby exhibiting a clarified structure (one helix and three helixes for the two cases, respectively). Notably, the field is split into two main parts along the $z = 0$ axes in the section planes in the both cases. In figures 2(b) and (d) and figure 3, the field is weaker near the $z = 0$ axis. The formation of such a structure is due to the superposed field of both driving pulses. With the head-on propagation of the input
lasers in the y direction, the superposed field exhibits a fork-like grating pattern (as shown in supplementary material). Two main parts of the superposed field (also is the interference field of the two input lasers) are separated along z = 0 axis, especially when the peaks of the two pulse reach the target, and harmonics with similar structures are emitted.

The corresponding phase as shown in figure 4 demonstrates that the main phase helical structure is in yz plane, which is reasonable because the longitudinal OAM dominates overwhelmingly and in good agreement with the results of figures 2 and 3. Figures 5(a) and (b) show the longitudinal and transverse OAM carried by each photon of the fourth harmonic. OAM of the fourth harmonic is calculated by $\vec{J} = \vec{r} \times \vec{P}$, where $\vec{P} = \varepsilon_0 (\vec{E} \times \vec{B})$ is the linear momentum and $\varepsilon_0$ is the dielectric constant. OAMs carried by each photon in the propagation direction ($oam_x$) are approximately $-2.4 \hbar$ and $-0.8 \hbar$ for the $l_2 = -1$ and 1 cases, which is in good agreement with the expected values ($-2.12 \hbar$ and $-0.71 \hbar$) obtained from the theoretical analysis. Regarding oam_y, in the perpendicular direction, we focus more on the value, which is $\sim 0.085 \hbar$ for $l_2 = 1$, whereas oam_y, carried by each photon increases to $0.285 \hbar$ for $l_2 = -1$. Thus, the tilted angles of the total OAM deviating from the propagating direction are $\sim 6^\circ$ and $\sim 6.8^\circ$ in both cases. This is quite different from the findings of previous research in which there is no generation of transverse OAM. For example, regarding the fourth harmonics in the reflected direction of $a_1 (a_2)$, the AM carried by each photon tends to be a constant ($\pm 4 \hbar$) and there is no net transverse OAM.

Next, we focus on the origination of the transverse OAM. The total transverse OAM of the fourth harmonics can be calculated by $J_y = z_0 \cdot p_x - x_0 \cdot p_z = J_{y1} - J_{y2}$, where $J_{y1} = z_0 \cdot p_x$, $J_{y2} = x_0 \cdot p_z$, $x_0 = x - x_c$, $y_0 = y - y_c$, $z_0 = z - z_c$, and $(x_c, y_c, z_c)$ is the coordinate of the fourth harmonic center. For the $l_2 = -1$ and 1 cases, in figure 5(c), it is observed that the transverse OAM originates mainly from $J_{y2}$. Since the coordinates of the harmonic center are the same in the two cases, as confirmed by our calculation, the main source of the transverse OAM is $p_z$, and it is expected to be asymmetric based on the coordinate $x_0$. $p_z$ also implies the fourth harmonic energy flux in z direction. It is well-known that when an LG laser obliquely irradiates a solid target, the asymmetrical shear stress in the vertical direction (z direction) of the incident
Figure 4. Phase isosurface of the fourth harmonic (a), (c) within a distance of 1 μm in the propagation direction (the phase larger than 0 is denoted by red color and the phase smaller than 0 is denoted by blue color) and (b), (d) the phase distribution in yz plane at x = −10 μm in the case of (a), (b) l_1 = −1, l_2 = 1 and (c), (d) l_1 = −1, l_2 = −1.

Figure 5. Evolution of the (a) longitudinal OAM (oam_x) in the propagation direction and (b) transverse OAM (oam_y) in the perpendicular direction carried by each photon of the fourth harmonic in both the l_2 = −1 and l_2 = 1 cases. (c) Evolution of the total transverse OAM and its different origination components in both the l_2 = −1 and l_2 = 1 cases. The black and red dotted lines denote the total transverse OAM calculated by (j_y = z_0 · P_x − x_0 · P_z = j_y^1 − j_y^2) of the fourth harmonics for l_2 = −1 and l_2 = 1. The blue and burgundy dotted lines denote the transverse OAM component calculated by (j_y^1 = z_0 · P_x) for l_2 = −1 and l_2 = 1. The green and navy dotted lines denote the transverse OAM component calculated by (j_y^2 = x_0 · P_z) for l_2 = −1 and l_2 = 1. (d) Evolution of the linear momentum in the z direction P_z = \varepsilon_0 (E_x · B_y − E_y · B_x), of the fourth harmonic in both the l_2 = −1 and l_2 = 1 cases.

plane would induce the generation of p_z and the deflection of the laser to the vertical direction [22]. In this scheme, the net p_z of the fourth harmonic is also observed after the interaction (figure 5(d)), but still much smaller than p_x. Figures 6(a)–(d) show the 3D distributions of p_z in the relative coordinate system (x_0, y_0, z_0) for the l_2 = −1 and l_2 = 1 cases. Evidently, p_z has an asymmetrical structure regarding the coordinate x_0. From the comparison of figures 6(a) and (b), the asymmetrical phenomenon in l_2 = −1 is more obvious than that in l_2 = 1, which induces a much higher transverse OAM. We notice that p_z has a similar vortex
structure as $p_z$ (figures 6(c) and (d)). This is reasonable because they are the two components of the energy flux of the same harmonic.

It is also well-known that a CP laser pulse exhibits an intrinsic AM, i.e. the SAM. The SAM carried by each photon depends on the state of the laser rotation [1, 2]. Here, s linearly polarized lasers is applied instead of the CP lasers to check the OAM to avoid the uncertainty that arose from SAM. However, similar harmonic structures have also been observed with the CP lasers (see Part I of the supplementary material). To further verify the theoretical and simulation results of the generation of longitudinal and transverse AM, we have conducted a set of 3D simulations for the $l_2 = 2$ and $l_2 = -2$ cases. The simulation results agree well with the theoretical expectation (Part II of the supplementary material).

It should be noted that the angle between the pulses is chosen to $\pi/2$ to make sure that the two input laser pulses are both emitted perpendicularly to the boundaries to avoid the unphysical influence, such as the uneven front profile of the pulse. Actually, the OAM, especially the transverse OAM which results from the obliquely incident angle will change when the angle is changed. And it is expected that the transverse OAM tends to be smaller with the decrease of the angle. For an extreme example, we have checked that the harmonics do not carry any transverse OAM when the two collinear input lasers head-on collide on the target. In addition, as we analyzed, both the longitudinal and transverse OAMs of the fourth harmonic depend proportionally on the topological charge of the driving pulse. It is in accordance with the simulation results in figure 7, which shows both the oam$_x$ and oam$_y$ increase with the topological charge of the input vortex lasers. Therefore, one can adjust the topological charge of the incident lasers for the harmonic emission with different OAM.
4. Conclusions

In conclusion, we propose a scheme to obtain intense vortex beams carrying transverse AM from laser plasma interaction when two non-collinear vortex beams impinge on a solid thin target. This inevitably present new opportunities for applications in intense vortex required fields. The two driving beams contribute to HHG, indicating that AM and the frequency can be tunable. The theoretical analysis and 3D PIC simulations have confirmed that the intense vortex beam carrying transverse AM results from the crossing angle of the involved photons from the two incident beams, while the longitudinal AM is conserved for the photons involved in the HHG process and can be predicted. Further, the present scheme also avails an approach to produce angularly isolated high harmonic vortex beams since the emission direction follows the linear momentum conservation [51].

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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