To bind or not to bind: $\Lambda\Lambda$ hypernuclei and $\Xi$ hyperons

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Faddeev calculations suggest that: (i) $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$ for the recent KEK emulsion event is compatible with fairly weak $\Lambda\Lambda$ potentials $V_{\Lambda\Lambda}$ such as due to the Nijmegen soft-core NSC97 model; (ii) the isodoublet $^{5}_{\Lambda\Lambda}\text{H} - ^{\Lambda\Lambda}_{\Lambda\Lambda}\text{He}$ hypernuclei are particle-stable even in the limit $V_{\Lambda\Lambda} \to 0$; and (iii) $^{4}\Lambda\Lambda\text{H}$ within a $\Lambda\Lambda\text{d}$ model is particle-stable. However, four-body $\Lambda\Lambda\text{pn}$ Faddeev-Yakubovsky calculations do not produce a bound state for $^{4}_{\Lambda\Lambda}\text{H}$ even for $V_{\Lambda\Lambda}$ considerably stronger than required to reproduce $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$, in contrast to the normal situation (e.g. $^{10}\text{Be}$) where a four-body Faddeev-Yakubovsky calculation yields stronger binding than that due to a suitably defined three-body Faddeev calculation. For stranger systems, Faddeev calculations using $\Lambda\Xi$ interactions which simulate model NSC97 suggest that $^{6}_{\Lambda\Xi}\text{He}$ marks the onset of nuclear stability for $\Xi$ hyperons.

1. INTRODUCTION AND INPUT

Information on hyperon-hyperon ($YY$) interactions is not readily available from experiments in free space. It is almost exclusively limited to the study of strangeness $S=-2$ hypernuclear systems. This information is crucial for extrapolating into multi-strange hadronic matter [1], for both finite systems and in bulk [2], and into neutron stars [3].

The recent unambiguous identification of $^{6}_{\Lambda\Lambda}\text{He}$ in the KEK hybrid-emulsion experiment E373 [4] is consistent with a scattering length $a_{\Lambda\Lambda}\sim-0.8$ fm [5], indicating a considerably weaker $\Lambda\Lambda$ interaction than that specified by $a_{\Lambda N}\sim-2$ fm [6] for the $\Lambda N$ interaction. With such a relatively weak $\Lambda\Lambda$ interaction, and since the three-body system $\Lambda\Lambda N$ is unbound (comparing it with the unbound $\Lambda nn$ system [7]), the question of whether or not the onset of binding in the $S=-2$ hadronic sector occurs at $A=4$ becomes highly topical.

Here I review primarily a recent Faddeev-Yakubovsky calculation for $^{4}_{\Lambda\Lambda}\text{H}$ as a four-body $\Lambda\Lambda\text{pn}$ system [8], taking into account properly all possible rearrangement channels. I also briefly review calculations for other $\Lambda\Lambda$ hypernuclei and $\Lambda\Xi$ hypernuclei [8].

The $YN$ and $YY$ s-wave interaction input potentials consisted of combinations of three Gaussians with different ranges, such as those used by Hiyama et al. [10]. A similar form was used for the $pn$ triplet interaction and the results proved almost identical to those derived using the Malfliet-Tjon potential MT-III [11]. Of the several $\Lambda\Lambda$ potentials due to OBE models which are shown in Fig. 1, NSC97e is the weakest one, of the order of magnitude required to reproduce $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$. The $\Lambda\Lambda$ interaction is fairly weak for all six versions ($a$)-(f) of the Nijmegen soft-core model NSC97 [12], and versions e and f

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Figure 1. ΛΛ potentials in OBE models. Figure 2. NSC97-model YY potentials.

provide a reasonable description of single-Λ hypernuclei [6]. Several YY potentials fitted to reproduce the low-energy parameters of the NSC97 model [12] are shown in Fig. 2. We note that the ΛΞ interaction is rather strong, considerably stronger within the same version of the model (here e) than the ΛΛ interaction.

2. RESULTS

Figure 3 demonstrates a nearly linear correlation between Faddeev-calculated values of $\Delta B_{\Lambda\Lambda}(^6\text{He})$ and $\Delta B_{\Lambda\Lambda}(^5\Lambda\Lambda\text{H}, ^5\text{He})$, using several ΛΛ interactions including (the lowest-left point) $V_{\Lambda\Lambda} = 0$. The ΛΛ incremental binding energy $\Delta B_{\Lambda\Lambda}$ is defined by

$$\Delta B_{\Lambda\Lambda}(^A_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}(^A_{\Lambda\Lambda}Z) - 2\bar{B}_{\Lambda}((A-1)\Lambda Z),$$

where $B_{\Lambda\Lambda}(^A_{\Lambda\Lambda}Z)$ is the ΛΛ binding energy of the hypernucleus $^A_{\Lambda\Lambda}Z$ and $\bar{B}_{\Lambda}((A-1)\Lambda Z)$ is the (2J+1) average of $B_{\Lambda}$ values for the $(A-1)\Lambda Z$ hypernuclear core levels. $\Delta B_{\Lambda\Lambda}$ increases monotonically with the strength of $V_{\Lambda\Lambda}$, starting in approximately zero as $V_{\Lambda\Lambda} \to 0$, which is a general feature of three-body models such as the $\alpha\Lambda\Lambda$, $^3\text{H}\Lambda\Lambda$ and $^3\text{He}\Lambda\Lambda$ models used in these $s$-wave Faddeev calculations [5]. The $I = 1/2$ $^5\Lambda\Lambda\text{H} - ^5\Lambda\Lambda\text{He}$ hypernuclei are then found to be particle stable for all the ΛΛ attractive potentials here used.

2.1. $^4\text{H}$ in a $\Lambda\Lambda d$ model

ΛΛ binding-energy values ($B_{\Lambda\Lambda}$) calculated within a $\Lambda\Lambda d$ s-wave Faddeev calculation [8] are shown in Fig. 4 as function of the ΛΛ scattering length $a_{\Lambda\Lambda}$ for two $d$ potentials, one of an exponential shape and the other one of an isle shape, both of them fitted to the low-energy parameters of an $s$-wave Faddeev calculation for $\Lambda pn$ which uses model
NSC97f for the underlying ΛN interaction, yielding $B_\Lambda(\Lambda^3H(1^+)) = 0.19 \text{ MeV}$.\(^2\)

The roughly linear increase of $B_{\Lambda\Lambda}$ with the strength of $V_{\Lambda\Lambda}$ holds generally \([5]\) in three-body ΛΛC models (C standing for a cluster) as is evident also from Fig. 3. The solid squares in Fig. 3 correspond to a ΛΛ interaction fitted to $B_{\Lambda\Lambda}(\Lambda^6\text{He})$. The onset of particle stability for $\Lambda^4\text{H}(1^+)$ requires a minimum strength for $V_{\Lambda\Lambda}$ which is exceeded by the choice of $B_{\Lambda\Lambda}(\Lambda^6\text{He})$ \([4]\) as a normalizing datum. Disregarding inessential complications due to spin it can be shown that, for essentially attractive ΛΛ interactions and for a static nuclear core $d$, a two-body Λ$d$ bound state implies binding for the three-body ΛΛ$d$ system \([13]\).

2.2. $\Lambda\Lambda^4\text{H}$ in a ΛΛ$pn$ model

For two identical hyperons and two essentially identical nucleons (upon introducing isospin) as appropriate to a ΛΛ$pn$ model calculation of $\Lambda\Lambda^4\text{H}$, the 18 Faddeev-Yakubovsky components reduce to seven independent components satisfying coupled equations. Six rearrangement channels are involved in the s-wave calculation \([8]\) for $\Lambda\Lambda^4\text{H}(1^+)$:

\begin{equation}
(\Lambda NN)_{S=\frac{1}{2}} + \Lambda, \quad (\Lambda NN)_{S=\frac{3}{2}} + \Lambda, \quad (\Lambda NN)_{S=\frac{1}{2}} + N
\end{equation}

for 3+1 breakup clusters, and

\begin{equation}
(\Lambda\Lambda)_{S=0} + (NN)_{S=1}, \quad (\Lambda N)_{S} + (\Lambda N)_{S'}
\end{equation}

with $(S, S') = (0, 1) + (1, 0)$ and $(1, 1)$ for 2+2 breakup clusters.

\(^2\)Using model NSC97e, with $B_\Lambda(\Lambda^3H) = 0.07 \text{ MeV}$, does not alter the conclusions listed below.
Using $V_{\Lambda\Lambda}$ which reproduces $B_{\Lambda\Lambda}(\Lambda\Lambda^6\text{He})$, the four-body calculation converges well as function of the number $N$ of the Faddeev-Yakubovsky basis functions allowed in, yet yielding no bound state for the $\Lambda\Lambda pn$ system, as demonstrated in Fig. 5 by the location of the ‘$\Lambda\Lambda pn$’ curve above the horizontal straight line marking the ‘$\Lambda + \frac{3}{2}H$ threshold’.

In fact these Faddeev-Yakubovsky calculations exhibit little sensitivity to $V_{\Lambda\Lambda}$ over a wide range. Even for considerably stronger $\Lambda\Lambda$ interactions one gets a bound $\Lambda\Lambda^4H$ only if the $\Lambda N$ interaction is made considerably stronger, by as much as 40%. With four $\Lambda N$ pairwise interactions out of a total of six, the strength of the $\Lambda N$ interaction (about half of that for $NN$) plays a major role in the four-body $\Lambda\Lambda pn$ problem. Here it appears impossible to prove that, for essentially attractive $\Lambda\Lambda$ interactions and for a non static nuclear core $d$ (made out of dynamically interacting proton and neutron), a $\Lambda d$ bound state implies binding for the $\Lambda\Lambda d$ system. It is unlikely that incorporating higher partial waves, and $\Lambda\Lambda - \Xi N$ coupling effects, will change this qualitative feature.

2.3. $^{10}_{\Lambda\Lambda}\text{Be}$

For heavier $\Lambda\Lambda$ hypernuclei, the relationship between the three-body and four-body models is opposite to that found here for $^{4}_{\Lambda\Lambda}H$: the $\Lambda\Lambda C_1 C_2$ calculation provides higher binding than a properly defined $\Lambda\Lambda C$ calculation yields (with $C = C_1 + C_2$) due to the attraction induced by the $\Lambda C_1 - \Lambda C_2$, $\Lambda\Lambda C_1 - C_2$, $C_1 - \Lambda\Lambda C_2$ four-body rearrangement channels that include bound states for which there is no room in the three-body $\Lambda\Lambda C$ model. The

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\[\text{This threshold was obtained as the asymptote of the } \Lambda pn \text{ s-wave Faddeev calculation; note that, in agreement with Fig. 4, the asymptote of the } \Lambda\Lambda d \text{ curve is located below this threshold.}\]
binding energy calculated within the four-body model increases 'normally' with the strength of $V_{AA}$ \cite{5}. This is demonstrated in Fig. 1 for $^{10}_{ΛΛ}Be$ using several $ΛΛ$ interactions, including $V_{AA} = 0$ which corresponds to the lowest point on each one of the straight lines. The origin of the dashed axes corresponds to $ΔB_{AA} = 0$. The fairly large value of 1.5 MeV for $ΔB_{AA}(^{10}_{ΛΛ}Be)$ in the limit $V_{AA} → 0$ is due to the special $αα$ cluster structure of the $^8Be$ core. The correlation noted in the figure between $^{10}_{ΛΛ}Be$ and $^{6}_{ΛΛ}He$ calculations, and the consistency between various reports on their $B_{AA}$ values, are discussed in Ref. \cite{5} and by Hiyama in these proceedings.

2.4. The onset of $Ξ$ stability

![Figure 7. s-wave Faddeev calculations for the level schemes of $^{6}_{ΛΞ}H$ and $^{6}_{ΛΞ}He$.](image)

Figure 7. s-wave Faddeev calculations for the level schemes of $^{6}_{ΛΞ}H$ and $^{6}_{ΛΞ}He$.

If model NSC97 indeed provides a valid extrapolation from fits to $NN$ and $YN$ data, and recalling the strongly attractive $^1S_0$ $ΛΞ$ potentials due to this model (Fig. 2), it is only natural to search for stability of $A = 6, S = -3$ systems obtained from $^{6}_{ΛΛ}He$ replacing one of the $Λ$’s by $Ξ$. Faddeev calculations \cite{9} for the isodoublet hypernuclei $^{6}_{ΛΞ}H$ and $^{6}_{Lambda Ξ}He$, considered as $αΛΞ^-$ and $αΛΞ^0$ three-body systems respectively, indicate that $^{6}_{ΛΞ}He$ is particle-stable against $Λ$ emission to $^{5}_{ΛΛ}He$ for potentials simulating model NSC97, particularly versions $e$ and $f$, whereas $^{6}_{ΛΞ}H$ is unstable since $M(Ξ^-) > M(Ξ^0)$ by 6.5 MeV. This is demonstrated in Fig. 7. Nevertheless, predicting particle stability for $^{6}_{ΛΞ}He$ is not independent of the assumptions made on the experimentally unexplored $Ξα$ interaction which was extrapolated from recent data on $^{12}C$ \cite{14}; hence this prediction cannot be considered conclusive.

\footnote{Recall that the $I = \frac{1}{2}^3_{ΛΛ}H - ^5_{ΛΛ}He$ hypernuclei, within a $ΛΛC$ Faddeev calculation, are particle stable even in the limit $V_{AA} → 0$.}
3. CONCLUSION

I have presented a first ever four-body Faddeev-Yakubovsky calculation [8] for $\Lambda\Lambda^4H$ using $NN$ and $\Lambda N$ interaction potentials that fit the available data on the relevant sub-systems, including the binding energy of $^3\Lambda H$. No $\Lambda\Lambda^4H$ bound state was obtained for a wide range of $\Lambda\Lambda$ interactions, including that corresponding to $B_{\Lambda\Lambda}(^6\Lambda\Lambda^4He)$. This non-binding is due to the relatively weak $\Lambda N$ interaction, in stark contrast to the results of a ‘reasonable’ three-body $\Lambda d$ Faddeev calculation. More experimental work is needed to decide whether or not the events reported recently in the AGS experiment E906 [15] correspond to $\Lambda\Lambda^4H$.

Accepting the predictive power of model NSC97, our calculations suggest that $\Lambda\Lambda^6\Xi^0He$ may be the lightest particle-stable $S = -3$ hypernucleus, and the lightest and least strange particle-stable hypernucleus in which a $\Xi$ hyperon is bound (and not $\Lambda\Lambda\Lambda\Xi^0He$ for $S = -4$ as argued in Ref. [11]). Unfortunately, the direct production of $\Lambda\Xi$ hypernuclei is beyond present experimental capabilities, requiring the use of $\Omega^-$ initiated reactions.

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