Numerical analysis for AC losses in single-layer cables composed of rectangular superconducting strips with various lateral $J_c$ distributions

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Abstract. The critical current density ($J_c$) in the superconducting layer in the typical coated conductor is known to be not uniform along a lateral direction of its cross section. Non-uniformity of $J_c$ values in a superconducting layer is the dominant factors of the AC loss characteristics together with its geometrical shape and arrangement in an actual power device. In this paper, we investigated the AC loss characteristics on the single-layer cable composed of thin rectangular superconducting strips with various lateral $J_c$ distributions through the numerical analysis. In our analysis, several rectangular strips were arranged on a cylindrical former, in a parallel way to the conductor length. Numerical calculation of magnetic field distributions and loss values under AC current transmission were performed, by taking into account the locally-varying $J_c$ values in a tape strand and geometrical factors of cables. The influence of non-uniformity of $J_c$ values along a tape width on loss under AC current transmission in the cable conductors was discussed.

1. Introduction
Recently, the fabrication technologies of coated conductors are growing rapidly, as long sample length of several 100 meter class and high critical current density ($J_c$) of MA/cm$^2$ class have become available [1–3]. However, it is experimentally confirmed that the $J_c$ values in the superconducting layers in coated conductors are not uniform along a lateral direction of its cross section [4–6]. Since the magnetic field penetration inside the superconductor with large aspect ratio (AR) strongly depends on the $J_c$ distribution along a width direction, AC losses of coated conductors are strongly influenced by the $J_c$ distributions of them [6–9].

For the applications of coated conductors to AC power cables, they will be assembled into cylindrical shapes to obtain desired current capacity [1, 10]. The magnetic field penetration into each coated conductor inside the cable might be influenced not only by the geometrical factor of conductors [11, 12] but also by the non-uniformity of $J_c$ values in strands. Therefore, it is important to examine the AC loss characteristics in cable conductors using thin strip superconductors with non-uniform $J_c$ distributions, for developing cables with low loss values satisfying the practical demand. In this paper, we numerically studied the AC loss characteristics in single-layer cables using rectangular superconductor strips with various lateral $J_c$ distributions along a lateral direction of a tape. The rectangular strips with AR $> 10^3$ and various lateral $J_c$ distributions were used as the strand for assembling the single-layer cable models. Magnetic field distributions and loss values under AC
current transmission were numerically calculated, by taking into account the locally-varying \( J_c \) values in each tape strand and the arrangements of tape strands and geometrical factors of conductors. Based on the calculated results, the influence of non-uniformity of \( J_c \) values of each tape strand on the losses in single-layer cables was discussed.

2. Calculation procedure

The rectangular superconducting strip with its width \((w_{SC})\) of 5.0 mm and thickness \((t_{SC})\) of 2.0 \(\mu\)m was used as the strand for assembling the single-layer cables. The aspect ratio \(AR (= w_{SC}/t_{SC})\) of the tape section is 2500. The critical current \((I_{c})\) of the individual tape was fixed on 125 A, which corresponds to the average critical current density \(J_{c0} (= I_{c}/w_{SC}t_{SC})\) of 1.25 MA/cm\(^2\). As shown in figure 1, four kinds of lateral \( J_c \) distributions (uniform, peaking, trapezoidal and tilting \( J_c \) profiles) were assumed for the calculation. In addition, it was assumed that all tape strands in the cables have same \( J_c \) distributions. 16 pieces of strand were assembled into cylindrical shape and arranged in a parallel way to the conductor axis with the finite gap \(d_{gap}\) (0–1.0 mm) among the adjacent strands. The total \( I_c \) value of the cable is 2 kA. The specifications of single-layer cables are summarized in Table 1.

Magnetic field distributions and AC losses in the single-layer cable were numerically calculated, with taking into account the actual arrangements and the lateral \( J_c \) distributions of strands [7,12–14]. For the calculation, each rectangular strip inside the cable was regarded as the bundle of straight thin 80000 fibers with their cross sectional area \(dS\). The numbers of meshes along a width and a thickness direction of the strand cross section were set to 1600 and 50, respectively. Under the current amplitude \(I_0\) below \(I_c\), a fiber transports a current fragment \(dl = J_c(r)dS\) outside the current free region, while carrying no current inside the current free region. Here, \(J_c(r)\) represents the critical current density at the position \(r\) in the cross section of a rectangular strip. The origin is taken as an arbitrary

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**Figure 1.** Assumed lateral \( J_c \) distributions of a rectangular strip used as a strand in a single-layer cable. The critical current \((I_c)\) of a strip is fixed to 125 A.

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**Table 1.** Specifications of single-layer cable model used for analysis.

| Strand               | Width of a rectangular strip \(w_s\) | 5.0 mm      |
|----------------------|-------------------------------------|-------------|
|                      | Thickness of a rectangular strip \(t_s\) | 2.0 \(\mu\)m |
|                      | Aspect ratio (AR) of a strip cross section | 2500         |
|                      | Critical current \(I_c\) | 125 A        |
|                      | Average critical current density \(J_{c0} = I_c/w_{SC}t_{SC}\) | \(1.25 \times 10^{10}\) A/m\(^2\) |

| Single-layer cable | Number of strands \(N\) | 16          |
|--------------------|--------------------------|-------------|
|                    | Critical current \(I_c\) | 2000 A      |
|                    | Gap between adjacent strands \(d_{gap}\) | 0–1.0 mm  |
|                    | Cable radius \(R\) | 11.8–14.1 mm |
position in the current free region in a specific strand in the cable. By regarding the cables as the bundle of straight superconductor tapes, the loss density \( Q_d(r) \) per-cycle in the observation point \( r \) in a specific strand in the cable can be expressed as \([7, 12\text{−}14]\)

\[
Q_d(r) = 4J_c(r)\Phi_{\text{total}}(r) = 4J_c(r)\Phi_{\text{self}}(r) + \Phi_{\text{other}}(r)
\]  

(1)

Here, \( \Phi_{\text{total}}(r) \) is the total magnetic flux at peak current \( I_0 \) passing through between the current-free region and observation point \( r \), \( \Phi_{\text{self}}(r) \) and \( \Phi_{\text{other}}(r) \) are the magnetic flux at peak current \( I_0 \) generated by the specific strand own and by the other strands, respectively. The shape of current free region in each strand under \( I_0 < I_c \) is determined based on the fact that \( \Phi_{\text{total}}(r) \) shows the minimum at current free region in a tape strand \([7, 12, 13]\). The AC loss values per-unit length of the cable \( P_t \) (W/m) are obtained by integrating \( Q_d(r) \) over the whole part of superconductor region in the cable, so that it is expressed as follows:

\[
P_t = f \int_{\text{Cable}} Q_d(r) dS = 4f \int_{\text{Cable}} J_c(r)\Phi_{\text{self}}(r) + \Phi_{\text{other}}(r) dS
\]  

(2)

where \( f \) is the frequency of sinusoidal transport current \( I(t) = I_0\sin(2\pi ft) \). By using equation (2), the \( P_t \) values in the single-layer cable were calculated as a function of the \( I_0 \) values below \( I_c \). In this paper, the \( f \) value was fixed to 50 Hz.

3. Results and discussion

Figure 2 shows the current amplitude dependence of loss values \( (P_t) \) in single layer cables \( (d_{\text{gap}} = 0.50 \text{ mm}) \) using rectangular superconducting strips with different lateral \( J_c \) distributions. Also shown are the analytical loss values of monoblock model for a solid superconducting cylinder with an outer diameter of 25.5 mm \( (= w_S \times N) \) and a wall thickness of 2 \( \mu \text{m} \) \([11, 15]\). As can be seen, the \( P_t \) values for all cables are considerably larger than the analytical values for monoblock model. The deviation of the calculated results from the analytical values becomes remarkable with increasing the current amplitude \( I_0 \). This is mainly attributed to the polygonal cross sectional shapes for actual cables and the finite gaps between adjacent strands \([11, 12]\). In addition, it is also evident that the \( P_t \) values for cables are significantly influenced by the lateral \( J_c \) distributions in each strand. In case that each strand has peaking \( J_c \) distribution, the \( P_t \) values are almost 3 times higher than the case with uniform \( J_c \) distributions at \( I_0 < 1500 \text{ A} \). Next, the \( P_t \) values for the cable using strands with trapezoidal \( J_c \) distribution are close to the values for the case with uniform \( J_c \) distributions at \( I_0 > 1700 \text{ A} \), but they deviate from the results with uniform \( J_c \) at \( I_0 < 1500 \text{ A} \). This deviation becomes more remarkable with decreasing \( I_0 \). These results indicate that the decrease of \( J_c \) values near the edge part of strand causes significant increase of AC losses in single-layer cables. On the other hand, for the case with tilting...
$J_c$ distribution, the $P_t$ values are 50–60% larger than those with uniform $J_c$ distributions in whole current range.

Figure 3 shows the loss values $P_t$ in single-layer cables at various fixed current amplitude $I_0$, as a function of the gap $d_{gap}$ of adjacent strands. As can be seen, the $P_t$ values for all cables are substantially reduced as the $d_{gap}$ values decrease. The dependence of the $P_t$ values on the $d_{gap}$ values becomes more remarkable at lower current amplitude. Our previous study indicates that the reduction of loss values with decreasing the $d_{gap}$ values is caused by the suppression of magnetic flux passing through at the edge part of each strand [12]. It is also evident that the lateral $J_c$ distributions in strands have strong influence on the losses in cables regardless the $d_{gap}$ values. In higher current range $I_0 > 0.7I_c$, the difference in the $P_t$ values among the cables using strands with different lateral $J_c$ distributions becomes more remarkable with decreasing $d_{gap}$ values below 0.3 mm.

To examine the influence of non-uniformity of $J_c$ values in strands on the losses in single-layer cables, the lateral distributions of loss density per-cycle $Q_d$ on the outer surface of a strand are shown in figures 4 for $I_0 = 0.9I_c$ (= 1800 A) and 5 for $I_0 = 0.48I_c$ (= 960 A), respectively. As can be seen, the most of losses in single-layer cables generates near the edge part of each strand. This is very similar to the case for an isolated rectangular strip [7, 13]. It is also evident that the absolute values of $Q_d$ near the edge part are reduced as the $d_{gap}$ values decrease, which is mainly attributed to the magnetic flux generated by adjacent tapes [12]. The influence of $d_{gap}$ values on the $Q_d$ distributions becomes more

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**Figure 3.** AC losses $P_t$ for single-layer cables plotted against the gap between adjacent strands $d_{gap}$: (a) $I_0 = 0.9I_c$ (= 1800 A), (b) $I_0 = 0.73I_c$ (= 1460 A) and (c) $I_0 = 0.48I_c$ (= 960 A). In (b) and (c), the analytical values of monoblock model for a superconducting cylinder with an outer diameter of 25.5 mm (= $w_S \times N$) and a wall thickness of 2 μm are shown.
remarkable in lower current amplitude. Furthermore, it is clearly confirmed that the distributions of $Q_d$ of a strand inside the single-layer cable are strongly influenced by the non-uniformity of $J_c$ values in each strand. In the case that each strand has peaking $J_c$ distribution, flux penetration regions with $Q_d > 0$ are significantly extended toward the centre part regardless the $I_0$ values. This leads to the largest loss values among all the cables shown in figures 2 and 3. For the case with trapezoidal $J_c$ distribution, the losses mainly generate at $|x| > 1.5\text{mm}$ at $I_0 = 0.9I_c$, which is almost same as the case with uniform $J_c$. At $I_0 = 0.5I_c$, however, the flux penetration regions are extended toward the centre part of strand, as well as the case with peaking $J_c$ distribution. According to this fact, the loss values for the case with trapezoidal $J_c$ distributions become considerably larger than the case with uniform $J_c$ at lower current amplitude $I_0 = 0.5I_c$. Finally, the $Q_d$ distributions for the case with tilting $J_c$ distribution are asymmetrical on the centre of a strand. Comparing with the results for uniform $J_c$, flux penetration from the right edge part ($x > 0$) with lower $J_c$ becomes deeper, while the penetration from the left edge part ($x < 0$) with higher $J_c$ does shallower. The extension of flux penetration region from right edge part is more significant compared with the suppression of flux penetration from left edge part. Therefore, the loss values for the cable using strand with tilting $J_c$ is larger than the case with uniform $J_c$, as shown in figures 2 and 3.

![Figure 4](Image1.png)  
**Figure 4.** Distributions of loss density $Q_d$ per-cycle on the broad face of a rectangular strip in single-layer cables at $I_0 = 0.9I_c (= 1800 \text{A})$: (a) $d_{gap} = 0.75 \text{mm}$ and (b) $d_{gap} = 0.30 \text{mm}$.

![Figure 5](Image2.png)  
**Figure 5.** Distributions of loss density $Q_d$ per-cycle on the broad face of a rectangular strip in single-layer cables at $I_0 = 0.48I_c (= 960 \text{A})$: (a) $d_{gap} = 0.75 \text{mm}$ and (b) $d_{gap} = 0.30 \text{mm}$.
From these results, it is suggested that the AC losses in cables composed of coated conductors as strands may be increased by the non-uniform lateral $J_c$ distribution inside each strand. The magnetic flux preferentially penetrates from the lower $J_c$ part inside the strand and large loss generation occurs there. Particularly, the lower $J_c$ values near the edge part of strand than in center part lead to the significant increase of losses. The improvement of uniformity of $J_c$ values along a lateral direction of coated conductor has crucial importance to suppress the AC loss generation inside the cables, together with the optimization of cable structures.

4. Conclusion
We investigated the AC loss characteristics on single-layer cables using rectangular superconductor strips with various lateral $J_c$ distributions through the numerical calculations. The calculations of losses under AC current transmission were performed with taking into account the actual arrangements and the lateral $J_c$ distributions of tape strands in cables. The calculated results show that the loss values in single-layer cables are strongly affected not only by the arrangement of strands inside a cable but also by the non-uniformity of $J_c$ values along a lateral direction of each strand. The lower $J_c$ values near the edge part of a strand lead to the significant increase of losses. This is mainly attributed to the deeper flux penetration in going from the edge part to the center part of each strand. These results indicate that the improvement of uniformity of $J_c$ values along a lateral direction in coated conductors is quite important for reducing the AC loss generated inside the cables, together with the optimization of cable structures.

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