Born Effective Charges and Infrared Response of LiBC

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(Dated: October 29, 2018)

Abstract. Calculations of the zone center optical mode frequencies (including LO-TO splitting), Born effective charges \( Z^{\alpha}_{\nu} \) for each atom, dielectric constants \( \varepsilon_0 \) and \( \varepsilon_\infty \), and the dielectric response in the infrared, using density functional linear response theory, are reported. Calculated Raman modes are in excellent agreement with experimental values (170 cm\(^{-1}\) and 1170 cm\(^{-1}\)), while it will require better experimental data to clarify the infrared active mode frequencies. The Born effective charges \( Z^{\alpha}_{\nu} \) have surprisingly different values for B and C, and (ii) show considerable anisotropy. Relationships between the effective charges and LO-TO splitting are discussed, and the predicted reflectivity in the range 0 – 1400 cm\(^{-1}\) is presented. These results hold possible implications for Li removal in LiBC, and C substitution for B in MgB\(_2\).

PACS numbers: 77.22.-d, 78.30.-j, 63.20.-e

I. INTRODUCTION

Discovery of superconductivity near 40 K in MgB\(_2\)\(^1\) and the rapid development in the understanding of the microscopic mechanism (a specific type of electron-phonon coupling) has spurred extensive work on its properties and the discovery of closely related materials. Interest in LiBC and its Li-deficient derivatives has been spurred by calculations\(^2\) that the electron-phonon coupling will be stronger, and consequently the superconducting transition temperature is expected to be higher, than MgB\(_2\). LiBC is isostructural to, and isovalent with, MgB\(_2\) to the extent possible in a ternary compound, as we discuss more below.

LiBC was first reported by Wörle et al.\(^3\) who provided the crystal structure, and also reported hole doping by removal of Li that changed it from reddish in color to black, and increased the conductivity. Recently there has been renewed interest in synthesizing this compound and studying its properties. Bharathi et al.\(^4\) reported synthesis for varying starting concentrations of Li, but did not report any samples becoming metallic. Hlinka and coworkers reported Raman scattering measurements on few-micron size crystallites,\(^5\) finding modes at 170 cm\(^{-1}\) and 1176 cm\(^{-1}\), which were recently confirmed by Renker et al.\(^6\) (171 and 1167 cm\(^{-1}\)). Souptel et al.\(^7\) reported synthesis of samples for several flux concentrations, and like Bharathi et al.\(^8\) found conductivities characteristic of lightly doped semiconductors.\(^9\) Beginning optical studies on LiBC have been reported by Pronin et al.\(^10\) using an infrared microscope on small crystals, providing infrared (IR) reflectivity in the 400-1900 cm\(^{-1}\) range and measurements of the dielectric behavior at lower frequencies. We will return below to discussion of the implications of the Raman and IR data for the vibrational properties of LiBC.

As for possible doping (de-intercalation of Li), all of these reports are probably consistent with very low doping, too low to drive the material conducting (in spite of the broad bands that promote doped-hole conduction). The susceptibility data indicate small concentrations of local moments, typical of semiconductors with point defects. Zhao, Klavins, and Liu, in their efforts at Li de-intercalation, observed evaporation of Li upon vacuum annealing, with no sign of superconductivity in the annealed samples.\(^11\) The amount of Li remaining in the sample has not yet been characterized, however (x-rays are not sensitive enough), so definitive results remain to be obtained.

The purpose of this paper is to obtain in more detail the character of the intrinsic (insulating) phase of LiBC. Phonon frequencies calculated by linear response techniques using the full-potential linear muffin-tin orbital (FP-LMTO) method were reported by An et al.\(^10\) In this paper we use related by extended methods to calculate not only the phonon frequencies (again, with different codes) but the dynamic effective charge tensor \( Z^{\alpha}_{\nu} \), the longitudinal – transverse (LO-TO) mode splittings, oscillator strengths of the IR active phonons, and related dynamic dielectric behavior. Since Li is known, from calculation, to be ionic in this compound, one could anticipate \( Z^{Li}_{Li} \approx +1 \), and because B and C lie side-by-side in the periodic table, that \( Z^{B}_{C} \approx Z^{C}_{B} \) and of course \( Z^{*}_{B} + Z^{*}_{C} = -Z^{*}_{Li} \) from the acoustic sum rule. We find much more interesting behavior, however; \( Z^{*}_{B} \) and \( Z^{*}_{C} \) are of opposite sign, and large – they behave like strongly charged cation and anion, respectively. We analyze this behavior, and also comment on the possible relevance this behavior has for the difficulty in de-intercalating Li from LiBC, and also for C doping in MgB\(_{2-x}\)C\(_x\).
II. CALCULATIONAL METHODS

The calculations were carried using the ABINIT code with Troullier-Martins pseudopotentials\[11\], Teter parametrization\[12\] of the Ceperley-Alder exchange-correlation potential, and 216 k-points in the irreducible wedge of the Brillouin zone. The kinetic energy cutoff for the plane waves was 60 Hartree. The f-sum rule (relative to unity)\[13\] was satisfied with to 1.0005, which is one measure that the basis set is effectively complete. The experiment lattice constants (a = 2.752 Å, c = 7.058 Å)\[8\] of LiBC (space group P6\(_3\)/mmc, No. 194) were used. The calculation of effective charges follows the formalism of Gonze and Lee\[14\] as implemented in the ABINIT code.

The crystal structure of LiBC, pictured in Fig. 1 is a direct generalization of that of MgB\(_2\). The B and C atoms form a flat graphene sheet, with B and C atoms alternating around each hexagonal unit. The stacking of B-C layers is alternating, such that each B has C neighbors along the \(\hat{c}\) direction, and similarly for C. This stacking, which doubles the unit cell volume over that of MgB\(_2\), indicates that B-C bonding along \(\hat{c}\) (to the extent that it occurs) is preferable to B-B and C-C bonding. Possibly this stacking results in a favorable Madelung energy, although real static charges associated with B and C separately are almost impossible to define or calculate. The Li ions lie in the interstitial site (center of inversion) between the centers of the hexagonal units above and below, coordinated at equal distances with six B atoms and six C atoms.

III. RESULTS

A. Vibrational Frequencies

The character of the various zone center modes in LiBC were presented earlier by An et al.\[10\] For completeness, we repeat them here, with our calculated frequencies (in cm\(^{-1}\)), with macroscopic electric field contributions neglected; those corrections and longitudinal optical (LO) - transverse optic (TO) splittings are discussed below. Factor group analysis\[10\] at the \(\Gamma\) point yields 15 optical modes: 

\[2A_2u + 2B_1g + B_2u\] with motions along the \(\hat{c}\) direction, and 
\[2E_1u + 2E_2g + E_3u\] with motions in the \(a-b\) plane (the latter type are all two-fold degenerate \(E\) modes). The \(2A_2u\) and \(2E_1u\) are infrared active and the \(2E_2g\) modes are Raman active.\[5, 6, 8, 10\]

- \(\omega = 169\): \(E_2g\), B-C layers sliding against each other
- \(\omega = 299\): \(B_2g\), B-C layers beating against each other along \(\hat{c}\)
- \(\omega = 292\): \(E_2u\), Li layers sliding against each other
- \(\omega = 346\): \(E_1u\), Li layers sliding against the B-C layers
- \(\omega = 407\): \(A_2u\), Li layers beating against the B-C layers, along \(\hat{c}\)
- \(\omega = 510\): \(B_1u\), Li layers beating against each other along \(\hat{c}\)
- \(\omega = 803\): \(A_2u\), B-C puckering mode, all B atoms move oppositely to all C atoms, Li sites become inequivalent
- \(\omega = 829\): \(B_2g\), B-C puckering mode, B moves with C atoms above/below it, Li sites remain equivalent
- \(\omega = 1143\): \(E_1u\), B-C bond stretching mode, the two layers are out-of-phase
- \(\omega = 1153\): \(E_2g\), B-C bond stretching mode, layers in phase

Our calculated values are also provided in Table I, where LO-TO splittings, where they arise, are also reported. Compared with the observed Raman active modes at 170 and 1176 cm\(^{-1}\)\[8\] and 171 and 1167 cm\(^{-1}\)\[6\], the calculated values, 169 and 1153 cm\(^{-1}\), are in excellent agreement.

The LO-TO splitting for displacement parallel to the layers occurs for the “layer sliding” mode at 346 cm\(^{-1}\) (\(\Delta\omega = 25\) cm\(^{-1}\)), and for the B-C “bond
TABLE I: Zone center optical modes of LiBC. The first column (E = 0) is for no electric field, the second is for the field lying in the plane (E \perp \hat{c}) and the third is for E \parallel \hat{c}. The 2A_{2u} and 2E_{1u} are infrared active and the 2E_{2g} modes are Raman active. \Delta \omega^2 = \omega^2_{\text{TO}} - \omega^2_{\text{LO}}.

| modes  | Phonon frequency (cm\(^{-1}\)) | \sqrt{\Delta \omega^2} (cm\(^{-1}\)) |
|--------|----------------------------------|-------------------------------------|
| \(E_{2g}\) | 169 169 169  \(\vec{E} = 0\) | \(\vec{E} \parallel a - b \vec{E} \parallel \hat{c}\) |
| \(E_{2u}\) | 292 292 292 | 346 346 346 |
| \(B_{2g}\) | 299 299 299 | 346 346 346 |
| \(E_{1u}\) | 346 346 346 | 346 346 346 |
| \(A_{2u}\) | 407 407 507 | 510 510 510 |
| \(B_{1g}\) | 803 803 828 | 200 200 200 |
| \(B_{1u}\) | 829 829 829 | 469 469 469 |
| \(B_{2g}\) | 1143 1143 1143 | 1143 1143 1143 |
| \(E_{2g}\) | 1153 1153 1153 | 1153 1153 1153 |

stretching” mode at 1143 cm\(^{-1}\) (\(\Delta \omega = 92\) cm\(^{-1}\)). For \(\hat{c}\) polarization, the “layer beating” mode is at 407 cm\(^{-1}\) (\(\Delta \omega = 100\) cm\(^{-1}\)), and the B-C “puckering” mode at 803 cm\(^{-1}\) is split by only \(\Delta \omega = 25\) cm\(^{-1}\). The large LO-TO splittings of the “layer beating” and “bond stretching” modes suggest they involve large effective charges, as we confirm below.

Data on IR active modes have been published by Pronin et al.\[^{13}\] who fit reflectivity data with modes at 540, 620, 700 (less clear) and 1180 cm\(^{-1}\). The highest (and strong in the data) mode is close to our calculated value (TO 1143 cm\(^{-1}\), LO 1235 cm\(^{-1}\)), but the others are difficult to assign. The data, with three or four positions of structure, apparently include contributions from \(\hat{c}\) axis polarization, \(a - b\) plane polarization, and all angles between. Bharathi et al.\[^{8}\] have presented IR absorption spectra with main peaks at \(\sim 380, 950,\) and 1200 cm\(^{-1}\). The lower and upper of these are not far from our calculated values for \(E_{1g}\) modes (Table I), but the nearest IR mode to the 950 cm\(^{-1}\) peak is our \(A_{2u}\) mode at 803-828 cm\(^{-1}\).

B. Effective Charges and Oscillator Strengths

In a non-cubic lattice the Born (dynamical) effective charge of an ion becomes a tensor. In hexagonal symmetry such as in LiBC, the tensor is diagonal and reduces to two values \(Z_{xx}^* = Z_{yy}^* \equiv Z_{11}^*\) and \(Z_{zz}^* = Z_{11}^*\). These charges, which show considerable anisotropy in LiBC, are given in Table II. The acoustic sum rule \(\sum_k Z_k^* = 0\) is fulfilled to within 0.01, suggesting well converged calculations. In spite of the fact that they are neighbors in the periodic table and that both B and C are comfortable forming the graphene structure layer, when they alternate in this graphene layer their effective charges are vastly different: substantial in size but different in sign. Given their similarities, the effective charge sum rule, and that Li is ionized, one might perhaps expect values close to +1, 0, and -1 (or possibly +1, \(\frac{1}{2}\), and \(-\frac{1}{2}\)) for Li, B, and C, respectively. They are, on (angular) average, much closer to the values +1, +2, and -3, which are not so far from the bizarre ionic configuration (only one with closed shells) of +1, +3, and -4 for Li, B, and C respectively that would reflect closed shells on all ions. It must be recognized, of course, that the \(Z^*\) values are dynamic only, and reflect the effects of covalency with respect to some reference ionic value; however, the reference values here are unclear (except for Li, which clearly should be +1).

Due to the expected strong covalency in the B-C bonds, it perhaps is not surprising that the B and C effective charges are strongly altered from (smaller) reference ionic values. For \(\hat{c}\) axis polarization, the Li value itself is unusual (almost 50% larger than its nominal value), indicating Li is definitely involved in the interlayer coupling. We return to this important feature below. This partial Li covalency is probably connected with the difficulty in deintercalating it from the LiBC lattice.\[^{13}\]\[^{14}\]

Another view of the charge response can be obtained from the “mode effective charges” defined by Gonze and Lee\[^{15}\], which for the symmetry of LiBC can be written for each polarization as

\[
\vec{Z}^* = \sum_k \frac{Z_k^* \vec{U}_k}{\sum_{\kappa'} Z_{\kappa'}^* \vec{U}_{\kappa'} \cdot \vec{U}_{\kappa'}}.
\]

Here each ion effective charge is weighted by the mode eigenvector that is normalized without the ion mass factor that occurs in the usual normalization of \(U_{\kappa\alpha}\) (\(\kappa\) is the atom index, \(\alpha\) is the Cartesian coordinate). The mode effective charge vectors indicate how strongly a mode will couple to an electric field, and are listed in Table III for the IR active modes in LiBC. For the \(\hat{c}\) axis polarized modes, \(|\vec{Z}^*| = 3.00\) for the (softer) layer beating mode and 2.10 for the (harder) B-C puckering mode. For planar displacements, the (soft) layer sliding mode has the value 1.52, and the (hard) B-C bond stretching mode value is 5.50. These trends follow those found by Zhong et al.\[^{16}\] in perovskites,\[^{17}\] where displacements that modulated the “covalent” bonding produced the largest mode effective charge. The value of
TABLE II: The Born effective charges of LiBC. The acoustic sum rule \( \sum_s Z_s^a = 0 \) is fulfilled to within 0.01. \( Z^v = (1/3) \text{Tr}Z \pm 0.05 \) 

| \( Z^v \) | \( Z^t \) | \( Z^* \) |
|-------|-------|-------|
| 0.81  | 2.37  | -3.17 |
| 1.46  | 0.61  | -2.07 |
| 1.03  | 1.78  | -2.80 |

3.00 for the layer beating mode reflects the substantial Li “covalent” character for \( \hat{c} \) axis displacements.

C. Infrared Response

The static electronic dielectric constants are calculated to be \( \varepsilon_\infty = 12.95 \), \( \varepsilon_\infty = 11.24 \). These values tend to be overestimated in LDA calculations such as those used here, presumably due to the underestimation of the bandgap. Dielectric constants are also somewhat dependent (at the \( \sim 5\% \) level) on the choice of pseudopotential. According to the generalized Lyddane-Sachs-Teller (LST) relation, 

\[ \varepsilon_0 = \varepsilon_\infty \prod_m \frac{\omega_{LO,m}^2}{\omega_{TO,m}^2} \]  

which is used separately for each polarization, the static dielectric constants \( \varepsilon_0 \) of both directions are 17.4 and 18.5, respectively. In the GHz range (\( \sim 0.03 \) cm\(^{-1}\)), \( \varepsilon \approx 35 \) was reported by Pronin et al.\(^8\), but this value is not necessarily expected to be intrinsic.

The relationship between the LO-TO splitting and the Born effective charges is given by the relation\(^9\),\(^10\) (which holds for parallel and perpendicular vibrations separately),

\[ \sum_m [\omega_{LO,m}^2 - \omega_{TO,m}^2] = \frac{4\pi}{\varepsilon_\infty V_0} \sum_\kappa \frac{(Z^a_{\kappa,\alpha\alpha} e)^2}{M_\kappa} \]  

\[ = \sum_\kappa \Omega_{\kappa,\kappa}^2 \]  

In this relation, \( m \) goes over the IR active modes of the given polarization, \( M_\kappa \) the ionic mass of aton \( \kappa \), \( V_0 \) the volume of the primitive unit cell, and the right hand side has been expressed in terms of screened ionic plasma frequencies \( \Omega_{\kappa,\kappa} \) that indicate how much each ion contributes to the combined strength of the resonances. For in-plane polarization, Li, B, and C contribute 6%, 26%, and 58%; the Li ion has minor effect and the response is dominated by B-C bond stretching. For \( \hat{c} \) axis polarization, the contributions are 43%, 5% and 51% respectively; here the B is almost irrelevant, and the Li contribution is comparable to that of C.

TABLE III: Mode effective charges of the IR active optical modes

| modes        | Directions | a     | b     | c     |
|--------------|------------|-------|-------|-------|
| layer sliding \( E_{1u} \) |            | 0.65  | -1.37 | 0.00  |
|              |            | 1.37  | 0.65  | 0.00  |
| layer beating \( A_{2u} \) |            | 0.00  | 0.00  | 3.00  |
| B-C puckering \( A_{2g} \) |            | 0.00  | 0.00  | -2.10 |
| bond stretching \( E_{2g} \) |            | -5.50 | -0.10 | 0.00  |
|              |            | -1.0  | 5.50  | 0.00  |

The contributions can be examined mode by mode. The dielectric function can be expressed in terms of contributions from the IR active modes \( (m) \) as

\[ \frac{\epsilon(\omega)}{\epsilon_\infty} = 1 + \frac{4\pi}{\varepsilon_\infty V_0} \sum_m \frac{S_m}{\omega_{TO,m}^2 - \omega^2} \]  

\[ = 1 + \sum_m \frac{\omega_{LO,m}^2 - \omega_{TO,m}^2}{\omega_{TO,m}^2 - \omega^2}. \]  

The second line follows from the relation between the mode oscillator strength \( S_m \) and the corresponding LO-TO splitting, provided in Table II in terms of \( \Delta\omega \). For in-plane polarization, large LO-TO splitting of the bond stretching \( E_{1u} \) mode at 1143 (1235) cm\(^{-1}\) is the result of the large B and C effective charges; the splitting of the layer sliding mode at 346 (371) cm\(^{-1}\) is an order of magnitude less. For \( \hat{c} \) axis polarization, the effective charges are “derived” 70% from the 407 (507) cm\(^{-1}\) layer beating mode with large LO-TO splitting involving the larger \( Z^* \) for Li, and only 30% derives from the 803 (828) cm\(^{-1}\) B-C puckering mode.

From Eq. 4 the zero frequency limit is given by

\[ \frac{\epsilon(\omega)}{\epsilon_\infty} - 1 = \sum_m \left( \frac{\omega_{LO,m}^2}{\omega_{TO,m}^2} - 1 \right) \]  

which is different from the approximate “generalized Lyddane-Sachs-Teller” relation of Eq. 2 that is often quoted. It is not as different as it looks. The “textbook” form of the dielectric function in terms of its zeroes and poles is

\[ \frac{\epsilon(\omega)}{\epsilon_\infty} = \prod_m \frac{\omega_{LO,m}^2 - \omega^2}{\omega_{TO,m}^2 - \omega^2}. \]  

While this expression has the same poles (at \( \omega_{TO,m} \)) as Eq. 4 the positions of its zeroes, at \( \omega_{LO,m} \) in this expression, are different from the zeroes in the general expression Eq. 4. However, zeroes have to occur between each of the poles, and the zeroes of
FIG. 2: Calculated reflectivity spectrum $R(\omega)$ for LiBC for polarization in the $a-b$ layer (top panel) and along the $c$ axis (bottom panel), calculated from Eq. 6. The dashed lines mark the positions of the TO and the LO modes for each IR active mode (two of them for each polarization). The damping was chosen to be 3% of the frequency for each mode.

Eq. 6 are determined not only by $\omega_{LO,m}$ but also by the polarization arising from other LO modes with $m' \neq m$. The differences are small, however. For in-plane polarization, the zeroes of $\epsilon(\omega)$ from Eq. 4 are 367 cm$^{-1}$ and 1236 cm$^{-1}$ compared to the calculate “LO” frequencies of 371 and 1235 cm$^{-1}$, respectively. (Note that the lower is pushed down, while the higher is increased, for the case of two LO modes as we have here.) For $c$ axis polarization, Eq. 4 gives 499 and 833 cm$^{-1}$ compared to the calculated values of 507 and 828 cm$^{-1}$. These differences will be difficult to resolve experimentally.

IV. DISCUSSION AND SUMMARY

The calculated Raman-active frequencies are in excellent agreement with the data of two groups. The IR modes remain to be verified and understood. The spectra are polarization dependent, and the spectra presented by Pronin et al. and by Bharathi et al. are not representative of pure polarization data.

Since it will be useful for comparison with single crystal data, in Fig. 2 we present our predicted reflectivity $R(\omega)$ spectrum for both in-plane and perpendicular polarization. The damping (which for a clean undoped sample would be due to anharmonicity) has been chosen for each mode to be 3% of the frequency (zero damping curves are not very representative). If the damping is not too large, and there is no reason to expect large anharmonicity in LiBC, then both TO and LO frequencies can be obtained with small uncertainty.

It was mentioned in the Introduction that calculations have indicated that, if partial Li removal can be achieved, Li$_{1-x}$BC should be a very good superconductor. Such Li extraction is common in many materials, being the process that forms the basis for Li batteries. What our studies here have shown is that (primarily for $c$ axis displacements) Li shows considerable covalency with the B-C layer, with Li-C coupling being the prominent feature. This partial Li covalency is probably connected with the difficulty that several groups have found in de-intercalating it from the LiBC lattice. In addition, Cava, Zandbergen, and Inumaru reported briefly that LiBC seems to be highly resistant to chemical doping.

The finding here that C is very different chemically from B in this system also carries some implications for C replacement of B in MgB$_2$. Recent studies confirm that about 10% of B can be replaced with C while retaining the structure, and the superconducting transition temperature remains high (T$_c$=22 K). If C alloying in the B sublattice were rigid band like, the additional 0.1 electron per cell would nearly fill the important $\sigma$ bands that are responsible for superconductivity, and T$_c$ would be expected to vanish (or nearly so). The great difference between C and B behavior in LiBC reflects strong differences in bonding, and suggests something very different from rigid band “doping.” Preliminary calculations for MgB$_{2-x}$C$_x$ (ordered supercells and virtual crystal) indeed indicate substantial non-rigid-band behavior: C is really different from “B with an extra electron.”

Although calculations such as those described here have been found in several systems to be accurate, it is highly desirable to obtain polarized single crystal IR data to confirm our predictions.
V. ACKNOWLEDGMENTS

W.E.P. acknowledges informative communications with A. Loidl, K. Liu, and J. Hlinka. We have benefitted greatly from the ABINIT project, and from personal communication with X. Gonze. This work was supported by NSF Grant No. DMR-0114818.

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