Abstract

We review the result for the BFKL Pomeron intercept at $N = 4$ Supersymmetric Yang-Mills model in the form of the inverse coupling expansion

$$j_0 = 2 - 2\lambda^{-1/2} + \lambda^{-1} + 1/4\lambda^{-3/2} + 2(3\zeta_3 + 1)\lambda^{-2} + (18\zeta_3 + 361/64)\lambda^{-5/2} + (39\zeta_3 + 447/32)\lambda^{-3/2} + O(\lambda^{-7/2}),$$

which has been calculated in [1]-[5] with the use of the AdS/CFT correspondence.

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1. Introduction

Pomeron is the Regge singularity of the $t$-channel partial wave [6] responsible for the approximate equality of total cross-sections for high energy particle-particle and particle-antiparticle interactions valid in accordance with the Pomeronanchuck theorem [7]. In QCD the Pomeron is a colorless object, constructed from reggeized gluons [8].

The investigation of the high energy behavior of scattering amplitudes in the $N = 4$ Supersymmetric Yang-Mills (SYM) model [9, 10, 11] is important for our understanding of the Regge processes in QCD. Indeed, this conformal model can be considered as a simplified version of QCD, in which the next-to-leading order (NLO) corrections [12] to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [8] are comparatively simple and numerically small. In the $N = 4$ SYM the equations for composite states of several reggeized gluons and for anomalous dimensions (AD) of quasi-partonic operators turn out to be integrable at the leading logarithmic approximation [13, 14]. Further, the eigenvalue of the BFKL kernel for this model has the remarkable property of the maximal transcendentality [10]. This property gave a possibility to calculate the AD of the twist-2 Wilson operators in one [15], two [16, 17], three [17], four [18, 19] and five [20] loops using the QCD results [21] and the asymptotic Bethe ansatz [22] improved with wrapping corrections [19] in an agreement with the BFKL predictions [8, 10].

On the other hand, due to the AdS/CFT-correspondence [23, 24, 25] in $N = 4$ SYM some physical quantities can be also computed at large couplings. In particular, for AD of the large spin operators Beisert, Eden and Staudacher constructed the integral equation [26] with the use the asymptotic Bethe-ansatz. This equation reproduced the known results at small coupling constants and is in a full agreement (see [27, 28]) with large coupling predictions [29, 30].

With the use of the BFKL equation in a diffusion approximation [9, 11], strong coupling results for AD [29, 30] and the pomeron-graviton duality [31] the Pomeron intercept was calculated at the leading order in the inverse coupling constant (see the Erratum[1] to the paper [17]). Similar results were obtained also in Ref. [23]. The Pomeron-graviton duality in the $N = 4$ SYM gives a possibility to
construct the Pomeron interaction model as a generally covariant effective theory for the reggeized gravitons [32].

Below we review the strong coupling corrections to the Pomeron intercept \( j_0 = 2 - \Delta \) in next orders. These corrections were obtained in Refs. [1, 2, 3, 4, 5] (see also a short review in [33]) with the use of the recent calculations [34]-[37] of string energies.

Below we review the strong coupling corrections to the Pomeron intercept, following studies in [3, 4, 5]. In particular, in the strong coupling regime \( \lambda \to \infty \)

\[ j_0 = 2 - \Delta, \]

where the leading contribution \( \Delta = 2/\sqrt{\lambda} \) was calculated in Refs. [1, 2]. Below we show the several terms in the strong coupling expansion of the Pomeron intercept, following studies in [3, 4, 5].

2. BFKL equation at small coupling constant

The eigenvalue of the BFKL equation in \( N = 4 \) SYM model has the following perturbative expansion [3, 10] (see also Ref. [11])

\[
\begin{align*}
\omega &= \frac{A}{4\pi^2} \left[ \chi(\gamma_{\text{BFKL}}) + \delta(\gamma_{\text{BFKL}}) \frac{A}{16\pi^2} \right] \\
\lambda &= g^2 N_c,
\end{align*}
\]

where \( \lambda \) is the 't Hooft coupling constant. The quantities \( \chi \) and \( \delta \) are functions of the BFKL anomalous dimension

\[ \gamma_{\text{BFKL}} = \frac{1}{2} + iv \]

and are presented below [3, 10]

\[
\begin{align*}
\chi(\gamma) &= 2\Psi(1) - \Psi(1 - \gamma), \\
\delta(\gamma) &= \Psi'(1) + \Psi'(1 - \gamma) + 6\zeta_3 - 2\zeta_2 \chi(\gamma) \\
&- 2\Phi(1) - 2\Phi(1 - \gamma).
\end{align*}
\]

Here \( \Psi(z) \) and \( \Psi'(z), \Psi''(z) \) are the Euler \( \Psi \) function and its derivatives. The function \( \Phi(\gamma) \) is defined as follows

\[ \Phi(\gamma) = 2 \sum_{k=0}^{\infty} \frac{1}{k + \gamma} \beta'(k + 1), \]

where

\[ \beta'(z) = \frac{1}{4} \left[ \Psi\left( \frac{z + 1}{2} \right) - \Psi\left( \frac{z}{2} \right) \right]. \]

Due to the symmetry of \( \omega \) to the substitution \( \gamma_{\text{BFKL}} \to 1 - \gamma_{\text{BFKL}} \), expression (1) is an even function of \( \nu \)

\[ \omega = \omega_0 + \sum_{n=1}^{\infty} (-1)^n D_n \nu^{2n}, \]

where

\[
\begin{align*}
\omega_0 &= 4 \ln 2 \frac{A}{4\pi^2} \left[ 1 - \frac{\lambda}{16\pi^2} \right] + O(\lambda^3), \\
D_n &= 2 \left( \frac{\gamma}{2\lambda} - 1 \right) \frac{A}{4\pi^2} \left( \frac{\gamma}{2\lambda} \right)^{n} + \frac{\delta(2m)}{(2m)!} \frac{\lambda^4}{64\pi^4} + O(\lambda^5). \end{align*}
\]

According to Ref. [10] we have

\[ \tau_1 = 2\zeta_2 + \frac{1}{2\ln 2} \left( 11\zeta_3 - 32 L_{33}^2 \left( \frac{\pi}{2} \right) - 14\pi \zeta_2 \right) \approx 7.5812, \]

where (see [38])

\[ L_{33}(x) = - \int_0^x \ln^2 2 \sin \left( \frac{\gamma}{2} \right) dy. \]

3. AdS/CFT correspondence

Due to the energy-momentum conservation, the universal AD of the stress tensor \( T_{\mu\nu} \) should be zero, i.e.,

\[ \gamma(j = 2) = 0. \]

It is important, that the AD \( \gamma \) contributing to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations [39] does not coincide with \( \gamma_{\text{BFKL}} \) appearing in the BFKL equation. They are related as follows [12, 40]

\[ \gamma = \gamma_{\text{BFKL}} + \frac{\omega}{2} = \frac{j}{2} + iv, \]

where the additional contribution \( \omega/2 \) is responsible in particular for the cancelation of the singular terms \( \sim 1/\gamma^3 \) obtained from the NLO corrections [1] to the eigenvalue of the BFKL kernel [12]. Using above relations one obtains

\[ \nu(j = 2) = i. \]
As a result, from eq. (7) for the Pomeron trajectory we derive the following representation for the correction $\Delta$ [12] to the graviton spin 2

$$\Delta = \sum_{m=1}^{\infty} D_m. \tag{16}$$

According to [12] and [16], we have the following small-$\nu$ expansion for the eigenvalue of the BFKL kernel

$$j = 2 \sum_{m=1}^{\infty} D_m \left((-1)^m \nu^{2m} - 1\right), \tag{17}$$

where $\nu$ is related to $\gamma$ according to eq. (14)

$$\nu^2 = -\left(\frac{j}{2} - \gamma\right)^2. \tag{18}$$

On the other hand, due to the ADS/CFT correspondence the string energies $E$ in dimensionless units are related to the AD $\gamma$ of the twist-two operators as follows [24, 25]

$$E^2 = (j - 2\gamma)^2 - 4 \tag{19}$$

and therefore we can obtain from (15) the relation between the parameter $\nu$ for the principal series of unitary representations of the Möbius group and the string energy $E$

$$\nu^2 = -\left(\frac{E^2}{4} + 1\right). \tag{20}$$

This expression for $\nu^2$ can be inserted in the r.h.s. of Eq. (17) leading to the following expression for the Regge trajectory of the graviton in the anti-de-Sitter space

$$j = 2 \sum_{m=1}^{\infty} D_m \left[\left(\frac{E^2}{4} + 1\right)^m - 1\right]. \tag{21}$$

4. Pomeron intercept

We assume, that eq. (21) is valid also at large $j$ and large $\lambda$ in the region $1 \ll j \ll \sqrt{\lambda}$, where the strong coupling calculations of energies were performed [34, 37]. These energies can be presented in the form

$$E^2 = \frac{\sqrt{\lambda}}{2} \left[ h_0(\lambda) + \frac{h_1(\lambda)}{\sqrt{\lambda}} \right] \tag{22}$$

where

$$h_0(\lambda) = a_{00} + \sum_{m=1}^{\infty} \frac{a_{im}}{\lambda^{m/2}}. \tag{23}$$

The contribution $\sim S^2$ can be extracted directly from the Basso result [35] taking $J_{im} = 2$ according to [36]:

$$h_0(\lambda) = \frac{I_2(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} + \frac{2}{\sqrt{\lambda}} = \frac{I_1(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} - \frac{2}{\sqrt{\lambda}}, \tag{24}$$

where $I_k(\sqrt{\lambda})$ is the modified Bessel functions. It leads to the following values of coefficients $a_{im}$

$$a_{00} = 1, \quad a_{01} = -\frac{1}{2}, \quad a_{02} = a_{03} = \frac{15}{8}, \quad a_{04} = \frac{135}{128}, \quad a_{05} = -\frac{45}{32}. \tag{25}$$

The contribution $\sim S^2$ has been evaluated recently in [5]. It leads to the following values of coefficients $a_{1m}$ ($\zeta_3$ and $\zeta_5$ are the Euler $\zeta$-functions)

$$a_{10} = \frac{3}{4}, \quad a_{11} = \frac{3}{2} \left(\frac{1}{8} - \zeta_3\right), \quad a_{12} = \frac{9}{4} \left(\zeta_3 + \frac{3}{8}\right), \quad a_{13} = \frac{3}{16} \left(\zeta_3 + 20\zeta_5 - \frac{37}{2}\right). \tag{26}$$

The coefficients $a_{im}$ for $m \geq 2$ are currently not known. However, as it was discussed in [5], from an analysis of classical energy, with semi-classical corrections, it is in principle possible to extract the coefficients $a_{im}$ and $a_{1m}$ for all $m$ values. For $m = 2$, we have [5]

$$a_{20} = \frac{3}{16}, \quad a_{21} = \frac{15}{8} \left(\zeta_3 + \zeta_5 - \frac{17}{60}\right). \tag{27}$$

Comparing the l.h.s. and r.h.s. of (21) at large $j$ values gives us the coefficients $D_m$ (see Appendix [5]).

Note that our expression (19) for the string energy $E$ differs from a definition, in which $E$ is equal to the scaling dimension $\Delta_2$. But eq. (19) is correct, because it can be presented as $E^2 = \left(\Delta_2 - 2\right)^2 - 4$ and coincides with Eqs. (45) and (3.44) from Refs. [28] and [25], respectively.

\[\text{Footnotes go here.}\]
pomeron intercept for finite
method presented in [42] could be used to build the numerical
BFKL results [8] have been reproduced from integrability. The
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of the Pomeron intercept
A in [4]) and \( \Delta \) in eq. (28) below.

To obtain an unified expression for the position of the
Pomeron intercept \( j_0 \) for arbitrary values of \( \lambda \), an interpolation between weak and strong
coupling regimes has been used in [4]. Following the
method of Refs. [16, 17, 41], it is possible to write
a simple algebraic equation for
\[ j_0 = j_0(\lambda) \]
whose solution will interpolate \( j_0 \) for the full \( \lambda \) range (see ref. [4]).

On Fig. 1, we plot the Pomeron intercept \( j_0 \) as a function of
the coupling constant \( \lambda \). The behavior of the
Pomeron intercept \( j_0 \) is similar to that found in QCD with some additional
assumptions (see ref. [43]). Indeed, for the often-used values 0.1 \( \div \) 0.3
of OCD strong coupling constant \( \alpha_s \) (i.e., respectively, for the values \( z = 0.5 \div 1 \) in Fig. 1) we have the predictions
\[ j_0 = 0.15 \div 0.3 \]
for the pomeron intercept \( j_0 \) in QCD. It corresponds to the \( j_0 \) values commonly
used in QCD phenomenology (see, for example, [43]-[48] and the reviews [49] and references therein).

5. Conclusion

We have shown the intercept of the BFKL Pomeron in the \( N = 4 \) SYM at weak coupling
regime and demonstrated an approach to obtain its
values at strong couplings (for details, see refs. [4] and [5]).

At \( \lambda \to \infty \), the correction \( \Delta \) for the Pomeron
intercept \( j_0 = 2 - \Delta \) has the form
\[
\Delta = \frac{2}{\lambda^{1/2}} \left[ 1 + \frac{1}{2\lambda^{1/2}} - \frac{1}{8\lambda} \left( 1 + 3\zeta_3 \right) \frac{1}{\lambda^{1/2}} \right. \\
\left. - \left( \frac{361}{128} + 9\zeta_3 \right) \frac{1}{\lambda^{2}} - \left( \frac{447}{64} + \frac{39}{2}\zeta_3 \right) \frac{1}{\lambda^{3/2}} \right] + \mathcal{O}\left( \frac{1}{\lambda^2} \right). \tag{28}
\]

Note that the corresponding expansion at strong
couplings for the Odderon intercept in the \( N = 4 \)
SYM has been obtained recently in [50].

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References

[1] A. V. Kotikov, L. N. Lipatov, A. I. Onushienco and V. N. Velizhanin, Phys. Lett. B 632 (2006) 754
[arXiv:hep-th/0404092v5].
[2] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, JHEP 0712 (2007) 005 [arXiv:hep-th/0603115]
[3] M. S. Costa, V. Goncalves and J. Penelones, JHEP 2012 (2012) 091 [arXiv:1209.4355 [hep-th]]
[4] A. V. Kotikov and L. N. Lipatov, Nucl. Phys. B 874 (2013) 889 [arXiv:1301.0882 [hep-th]]
[5] N. Gromov, F. Levkovich-Maslyuk, G. Sizov and S. Valatka, JHEP 1407 (2014) 156 [arXiv:1402.0871 [hep-th]]
[6] G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. 7 (1961) 394; V. N. Gribov, Sov. Phys. JETP 14 (1962) 1395 [Zh.
Eksp. Teor. Fiz. 41 (1961) 1962]; Nucl. Phys. 22 (1961) 249.
[7] I. Ya. Pomeranchuk, Sov. Phys. JETP 7 (1958) 499 [Zh.
Eksp. Teor. Fiz. 34 (1958) 725]; L. B. Okun and I. Ya.
Pomeranchuk, Sov. Phys. JETP 3 (1956) 307 [Zh. Eksp.
Teor. Fiz. 30 (1956) 424].
[8] L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338; V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B
60 (1975) 50; E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 44 (1976) 443; E. A. Kuraev, L. N.
Lipatov and V. S. Fadin, Sov. Phys. JETP 45 (1977) 199; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys.
28 (1978) 822; I. I. Balitsky and L. N. Lipatov, JETP Lett. 30 (1979) 355.
[9] A. V. Kotikov and L. N. Lipatov, Nucl. Phys. B582 (2000) 19.
[10] A. V. Kotikov and L. N. Lipatov, Nucl. Phys. B661 (2003) 19; in: Proc. of the XXXV Winter School, Repino,
S’Peterburg, 2001 [hep-ph/0112356],
[11] V. S. Fadin and R. Fiore, Phys. Lett. B 661 (2008) 139 [arXiv:0712.3901 [hep-ph]]; V. S. Fadin, R. Fiore

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\[ \text{Fig. 1. (color-online). The results for } j_0 \text{ as a function of } z (\lambda = 10^3). \]
