Further considerations concerning claims for deeply bound kaon atoms and reply to criticisms

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Abstract. We briefly review the experiments of KEK and FINUDA, that claim evidence for deeply bound kaon states, from the perspective of recent theoretical papers and experiments that provide an alternative explanation of the peaks seen. At the same time we show that recent criticisms raised by Akaishi and Yamazaki, and exposed by Akaishi in this Conference, have no base.

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1 Introduction

A brief story of the recent events around deeply bound kaon atoms could be made as follows: Chiral potentials [12345] provide potentials of depth around 50 MeV attraction at a width around 100 MeV at $\rho = \rho_0$. With these potentials, deeply bound states of binding energy around 30-40 MeV are obtained, but with a width of the order of 100 MeV which would preclude the observation of peaks [6]. A next step of the development appeared with the claims of a very large attractive potential in light nuclei [7,8] (AY), around 650 MeV at the center of the nucleus in [8] and with the matter compressed to 10 times nuclear matter density. An experiment was made at KEK with the $K^-$ absorption at rest in $^4He$ [8] and a peak was seen and attributed first to a strange triaryon, since its interpretation as a deeply kaon bound state would contradict the predictions of [7], but afterwards it was reinterpreted as a deeply bound kaon atom since it would match with the corrected version of the potential in [8]. The FINUDA collaboration made the same experiment in different nuclei and found a broad peak in the $\Lambda p$ back to back invariant mass spectrum which was attributed to the existence of the $K^-pp$ bound state [10]. With the interpretation of these peaks in clear contradiction with the predictions of the chiral potentials, theoreticians come into the scene: Oset and Toki (OT) [11] write a paper indicating that the peak seen at KEK is no proof of a kaon bound state since it can be interpreted in terms of absorption of the $K^-$ by a pair of nucleons going to $\Sigma p$, with the daughter nucleus left as spectator. Parallely, Oset, Magas, Ramos and Toki (MORT) write a paper [12] and provide an alternative explanation of the peak seen at FINUDA as coming from $K^-$ absorption in the nucleus going to $\Lambda N$, followed by the rescattering of the $\Lambda$ or the nucleon with the daughter nucleus. After that, a KEK like experiment is made at FINUDA looking at proton spectra following $K^-$ absorption at rest and a peak is indeed found in $^6Li$ [13] which, thanks to the measurement of pions in coincidence, allows the authors to interpret it as coming from $K^-$ absorption by a pair of nucleons going to $\Sigma p$, with the daughter nucleus left as spectator, just the explanation offered by OT [11] for the KEK peak. Incidentally, a second peak seen in the KEK experiment when making a cut of slow pions, and attributed to $K^-$ absorption by a pair of nucleons going to $\Lambda p$ in [11] is also seen in [13] as a feeble signal and associated to the $\Lambda p$ mechanisms as suggested in [11]. This peak is, however, much better seen, as a very narrow peak, in the the $\Lambda p$ back to back invariant mass spectrum of the FINUDA experiment [10].

In between, two novelties have appeared from the japanese side, the experiment has been redone, with an inclusive measurement, without the cuts and acceptance of [9], and the peaks seem to disappear as reported by Iwasaki in this Conference [14]. It was, however, indicated in the discussion that the useful measurement is the one with cuts that reduces background and stresses peaks, which has not been redone [15], and that the the FINUDA data on the proton spectrum showing the KEK like peaks is there to be also seriously considered. The other novelty is the paper by Akaishi and Yamazaki [16] criticizing both the approaches of [11] and [12], and the extra criticism of Akaishi in this Conference criticizing the chiral approach, because of the "unrealistic range" of the interaction used. Actually, no range is used for the interaction because, as we shall see below, all recent versions of the chiral approach rely upon...
the N/D method and dispersion relations, which only requires the knowledge of the interaction on shell, and the range of the interaction never appears in the formalism. In what follows we show that the recent criticism of Akaishi and Yamazaki, in [16] and of Akaishi in his talk have no base.

2 The chiral approach and the N/D method

The chiral approach of [17], with the on-shell factorization of the potential and the $t$-matrix, is based on the N/D method. One can find a systematic and easily comprehensible derivation of the ideas of the N/D method applied for the first time to the meson baryon system in [18], which we reproduce here below and which follows closely the similar developments used before in the meson-meson interaction [19]. One defines the transition $T$–matrix as $T_{ij}$ between the coupled channels which couple to certain quantum numbers. For instance in the case of $K\pi$ scattering studied in [18] the channels with zero charge are $K^-\pi^+$, $K^0\pi^0$, $\pi^0\Sigma^0$, $\pi^+\Sigma^−$, $\pi^-\Sigma^+$, $\pi^0\Lambda$, $\pi^-\Sigma^+$, $K^0\Xi^0$, $K^0\Xi^0$. Unitarity in coupled channels is written as

$$ImT_{ij} = T_{i\rho}T_{\rho j}^*$$

where $\rho_i \equiv 2Mq_i/(8\pi W)$, with $q_i$ the modulus of the c.m. three–momentum, and the subscripts $i$ and $j$ refer to the physical channels. This equation is most efficiently written in terms of the inverse amplitude as

$$Im \ T^{-1}(W)_{ij} = -\rho(W)\delta_{ij}$$

The unitarity relation in Eq. (2) gives rise to a cut in the $T$–matrix of partial wave amplitudes, which is usually called the unitarity or right–hand cut. Hence one can write down a dispersion relation for $T^{-1}(W)$

$$T^{-1}(W)_{ij} = -\delta_{ij} \ g(s_i) + V^{-1}(W)_{ij} ,$$

with

$$g(s_i) = \frac{a_i(s_0)}{2} + \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \ \rho(s')_{ij} / (s' - s)(s' - s_0),$$

where $s_i$ is the value of the s variable at the threshold of channel $i$ and $V^{-1}(W)_{ij}$ indicates other contributions coming from local and pole terms, as well as crossed channel dynamics but without right–hand cut. These extra terms are taken directly from $\chi PT$ after requiring the matching of the general result to the $\chi PT$ expressions. Notice also that $g(s_i)$ is the familiar scalar loop integral of a meson and a baryon propagators.

One can simplify the notation by employing a matrix formalism. Introducing the matrices $g(s) = diag \ (g(s_i))$, $T$ and $V$, the latter defined in terms of the matrix elements $T_{ij}$ and $V_{ij}$, the $T$-matrix can be written as:

$$T(W) = [I - V(W) \cdot g(s)]^{-1} \cdot V(W),$$

which can be recast in a more familiar form as

$$T(W) = V(W) + V(W)g(s)T(W)$$

Now imagine one is taking the lowest order chiral amplitude for the kernel $V$ as done in [18]. Then the former equation is nothing but the Bethe Salpeter equation with the kernel taken from the lowest order Lagrangian and factorized on shell, the same approach followed in [17], where different arguments were used to justify the on shell factorization of the kernel. The kernel $V$ plays the role of a potential in ordinary Quantum Mechanics.

The on shell factorization of the kernel, justified here with the N/D method, renders the set of coupled Bethe Salpeter integral equations a simple set of algebraic equations.

The important thing to note is that both the kernel and the $T$ matrix only appear on shell, for a value of $\sqrt{s}$. The range of the interaction is never used. The loop function is made convergent via a subtraction in the dispersion relation, or equivalently a cut off in the three momentum as used in [17], which is proved to be equivalent to the subtraction method in [18]. Akaishi in his talk confuses this cut off in the loop of propagators with the range of the interaction, when they have nothing to do with each other. Even more, the theory must be cut off independent, which means, one can change arbitrarily the cut off by introducing the appropriate higher order counterterms. As a consequence of this, all pathologies of the interaction pointed out by Akaishi in his talk are a pure invention, which has nothing to do with the physics of the problem.

3 Interpretation of the narrow FINUDA peaks and KEK peaks

In [10], for absorption in a sample of $^6Li$, $^7Li$, $^{12}C$, a narrow peak is seen at $M_f = 2340 MeV$ of the back to back $Ap$ system and a wider one at $M_f = 2275 MeV$, see Fig. 1. Let us assume $^7Li$ for simplicity of the discussion. The first thing to recall is the experience of pion absorption that concluded that at low pion energies the absorption was dominated by a direct two body process (even if later on there would be rescattering of the nucleons in the nucleus, giving rise to what was called indirect three body absorption in contrast with the possible direct three body absorption which had a small rate at low energies [20].) We consider the $K^-$ two nucleon absorption mechanism, disregarding the one body mechanisms which do not produce $Ap$ back to back, see Fig. 1.

**Origin of the narrow peak at $M_f = 2340 MeV$**

We have the reaction,

$$K^-pp + (^5H\text{spectator}) \rightarrow Ap + (^5H\text{spectator}) \quad (7)$$

The kinematics of the reaction is as follows: Let $P$ be the total momentum of the $K$-nucleus system, and $p_1$, $p_2$ and $p_3$ the momenta of the $\Lambda$, $p$ and $^5H$ spectator respectively. We have

$$(P - p_3)^2 = (p_1 + p_2)^2 = M_{12}^2$$  

(8)
from where we deduce that

\[ \Delta(M_{12}) = M(K + \Lambda p) \Delta(E_p)/M_{12} \quad (9) \]

This would lead to \( \Delta(M_{12}) \sim 10\text{MeV} \) for absorption in \(^4\text{He}\) and \( \Delta(M_{12}) \sim 1\text{MeV} \) for \(^7\text{Li}\) if one takes as representative of the Fermi momentum of the quasidetron or \( pp \) pairs \( 150\text{ MeV} \) for \(^4\text{He}\) and \( 50\text{ MeV} \) for \(^7\text{Li}\) as suggested in \[10\]. This produces a dispersion of the \( p \) momentum in the CM of the same order of magnitude. This quantity is smaller than the main source for \( p \) momentum dispersion which is the boost of the proton from the CM of \( \Lambda p \) to the frame where the \( \Lambda p \) has the Fermi momentum of the initial NN pair, \( p_{NN} \).

The boost is easily implemented requiring only nonrelativistic kinematics. We have

\[ p_p = p_{CM} + m_p V; \quad V = p_{NN}/M_{12} \quad (10) \]

\[ \Delta(p_p)^2 = m_p^2 V^2/3 \quad (11) \]

\[ \Delta(p_p) = \pm 35 \text{ MeV}/c (11 \text{ MeV}/c) \quad (12) \]

for \( p_{NN} = 150 \text{ MeV}/c (50 \text{ MeV}/c) \) \quad (13)

Hence we would have a dispersion of proton momentum of \( \pm 35 \text{ MeV} \) for \( K^- \) absorption in \(^4\text{He}\) and \( \pm 11 \text{ MeV} \) in the \(^7\text{Li}\) case. The exercise has been done for \( K^- \) absorption going to \( \Lambda p \) but the results are the same if one has \( K^- \) absorption going to \( \Sigma p \). This latter reaction was the one suggested by OT in \[11\] to explain the KEK peak seen in Fig. 2 lower left figure of the panel, around \( 475 \text{ MeV} \). The dispersion of Eq. (12) would roughly agree with the peak.

We should note that the peaks can be made more narrow, as we have checked numerically by: 1) assuming absorption from a \( 2p \) orbit of the \( K^- \), 2) forcing the \( \Lambda p \) pair to go back to back, 3) putting restrictions on the pion momenta.

It is interesting to observe in this respect that in the figure of the KEK experiment in the case when the slow pions are selected (lower right figure in the panel) one can see also a peak in the momentum distribution at \( p \sim 545\text{MeV} \), which was identified in \[11\] as coming from \( K^- \) absorption going to \( \Lambda p \) with the daughter nucleus as a spectator. It is interesting to see that such a signal, "a feeble signal around \( 580 \text{MeV}/c \)" is seen even in the inclusive spectrum of \[13\], see Fig. 3, and correctly identified there as coming from \( K^- \) absorption in \(^6\text{Li}\) going to \( \Lambda p \) (note one has smaller binding of the nucleons here than one has in \(^4\text{He}\) and there is no loss of energy as in the case of a thick target of \[9\]).

**Fig. 1.** \( \Lambda p \) invariant mass distribution of back to back pairs following \( K^- \) absorption in a mixture of nuclei, \(^6\text{Li}, ^7\text{Li}\) and \(^{12}\text{C}\). The inset of the figure shows data corrected for the detector acceptance. From \[10\].

**Fig. 2.** Proton spectra following \( K^- \) absorption in \(^4\text{He}\). Lower two figures: left with high pion momentum cut, right with lower pion momentum cut. From \[9\].

**Fig. 3.** Proton momentum distribution following the absorption of \( K^- \) in \(^6\text{Li}\) from \[13\].

Coming back to the absorption of \( K^- \) in \(^4\text{He}\) it should be noted that the candidate reaction for the peak at \( 475 \text{ MeV}/c \) is the reaction with the rate

\[ \Sigma^- p d \quad 1.6 \% \quad (14) \]

which has been measured by \[22\]. A fraction of this reaction can go with the \( d \) as a spectator, and then it is
worth mentioning that the fraction of the cross section of this peak is estimated in [9] at less than 1%, of the order of 0.34 % according to [23].

4 Interpretation of the wide FINUDA peak

Next we turn to the wide FINUDA peak in the experiment [10]. This peak was interpreted as naturally coming from the absorption of a $K^-$ from the nucleus going to $\Lambda N$ followed by a rescattering of the nucleon or the Lambda with the remnant nucleus [12]. This is the equivalent of the quasielastic peak which appears in all inclusive nuclear reactions with a similar width which is due to the Fermi motion of the nucleons. In [12] a calculation was done for the mixture of the different nuclei, as in the experiment and the results are seen in Fig. 4.

![Graph](image)

**Fig. 4.** Theoretical calculation of [12] versus experiment of the $\Lambda p$ invariant mass distribution of back to back pairs following $K^-$ absorption in a mixture of nuclei, $^6$Li, $^7$Li and $^{12}$C. Histogram theory, bars data from [10].

5 Claims of no peaks in Akaishi and Yamazaki

Another of the points in the work of [16] is that the peaks predicted by OT in [11] and MORT in [12] are unrealistic and that a proper calculation does not produce any peaks. The curious results are a consequence of a calculation in [10] that:

1) Considers absorption by all four particles at once in the $^4He$ case, disregarding the dynamics found from pion absorption. The spectra essentially reflect phase space with four particles in the final state.

2) Does not consider absorption by two N with spectator remnant nucleus.

3) Does not consider angular cuts or particles in coincidence.

4) Does not consider rescattering of particles.

And with all this an obvious broad spectrum is obtained.

Consequently with their finding the authors of [16] write textually:

"OT further insist that the same $K^-NN$ absorption mechanism at rest persists in the case of heavier targets as well ($^7$Li and $^{12}$C). However, this proposal contradicts the FINUDA experiment [10], in which they reconstructed an invariant mass spectrum of $M_{inv}(\Lambda p)$. Contrary to the naive expectation of OT, the spectrum shows no such peak at 2340 MeV/$c^2$."

This assertion could not be more illuminating of the criticism raised. The peak that Akaishi and Yamazaki claim that we predict and does not exist is the one seen exactly at 2340 MeV/$c^2$ in Fig. 3 of [10], which we have reproduced in Fig. 4.

In case there could be some doubts about this peak let us quote textually what the authors of [10] say regarding this peak:

"On the other hand, the detector system is very sensitive to the existence of the two nucleon absorption mode $K^- + np \rightarrow \Lambda p$ since its invariant mass resolution is 10 MeV/$c^2$ FWHM. The effect of the nuclear binding of two protons is only to move the peak position to the lower mass side of the order of separation energies of two protons ($\sim$ 30 MeV), and not to broaden the peak. A sharp spike around 2.34 GeV/$c^2$ may be attributed to this process."

Incidentally this mechanism is the one proposed by [11] to explain the peak at 545 MeV/$c$ in the proton spectrum when the slow pion cut is made in [9]. This peak is also the one that shows in the inclusive momentum spectrum of [13] mentioned there as a "feeble signal " and with the same interpretation.

With their claims than no peaks should be seen from these processes AY obviously also contradict the clear peak seen in [13] around 500 MeV/$c$, see Fig. 3 which, with the detection in coincidence of the pions coming from $\Sigma \rightarrow \pi N$ decay, the authors of [13] unmistakeably relate to the $K^-NN \rightarrow \Sigma N$ process, the mechanism proposed by OT to explain the lower peak of the spectrum in [9].

Another of the "proofs" presented in [16] is a spectrum of $K^-^4He \rightarrow \Lambda d n$ in which no peak around 560 MeV/$c$ is seen, as one could guess from our interpretation of the process. The comparison is, however, inappropriate. First, the number of counts is of the order of three counts per bin, as average. Second, the rate of $K^-^4He \rightarrow \Sigma^-p d$ of 1.6 % according to [22], and the rate of the peak of the KEK experiment that we attribute to this process, with a value of the order of 0.3 %, indicate that only a fraction of this process will go with the $d$ as a spectator, leading to a peak that can only be seen with far better statistics and resolution than the one in the spectrum of the $K^-^4He \rightarrow \Lambda d n$ experiment [22].

Finally, AY present another calculation to prove that the broad FINUDA invariant mass peak requires an explanation based on the $K^-pp$ bound system by 115 MeV. Their results are presented in Fig. 7 of their paper. The
calculation made is, however, simply unacceptable. They make the following assumptions:

1) Calculation in $^4He$ and compare to experiment which is a mixture of $^6Li$, $^7Li$, $^{12}C$.
2) Direct absorption by four nucleons.
3) No dynamics, just phase space.
4) Has no rescattering, shown by Magas to be essential to account for the peak.

And with this calculation they claim that the $K^-pp$ cluster is bound by 115 MeV !!!!.

We should also mention here another example of inappropriate comparison. In Fig. 7 of [10], which aims at describing the wide FINUDA peak, a vertical line is plotted with the label OT, presenting this as the position predicted by OT for this FINUDA peak. This comparison is out of place because OT in [11] never attempted to predict this broad FINUDA peak. This is done by MORT in [12], requiring a different mechanism, the rescattering of the proton or the $\Lambda$ after $K^-$ absorption by two nucleons [10], which automatically produces a peak at lower invariant masses.

6 conclusions

- Akaishi and Yamazaki criticisms of Oset Toki and Magas et al. are unfounded.
- AY potential with 10 $\rho_0$ compressed matter should not be considered serious.
- The claims of KEK and FINUDA for deeply bound kaons were unfounded.
- The new FINUDA data on $p$ spectrum following $K^-$ absorption in $^6Li$ has been very clarifying, showing KEK like peaks and interpreting them with the suggestion of Oset and Toki.
- The new calculations of Dote and Weise [24], and Schevchenko, Gal, Mares [25] predicting a bound $K^-pp$ state with 50-70 MeV binding, but more that 100 MeV width, have brought new light to this issue. They do not support the deeply bound narrow $K^-pp$ systems claimed by FINUDA.
- The new measurements of $^4He$ X rays by Hayano, Iwasaki et al. [26] are very important to clarify the issue. They clearly contradict predictions of Akaishi based on his potential.
- Interesting results from COSY, Buescher et al from $p \rightarrow K^+K^- \ 3He$ in the same direction [27], clearly rejecting such large $K^- \ 3He$ potentials.

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