Filamentary Diffusion of Cosmic Rays on Small Scales

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We investigate the diffusion of cosmic rays (CR) close to their sources. Propagating individual CRs in purely isotropic turbulent magnetic fields with maximal scale of spatial variations \(l_{\text{max}}\), we find that CRs diffuse anisotropically at distances \(r \lesssim l_{\text{max}}\). As a result, the CR densities around the sources are strongly irregular and show filamentary structures. We determine the transition time \(t_s\) to standard diffusion as \(t_s \approx 10^5 \text{yr} (l_{\text{max}}/150 \text{pc})^2 (E/\text{PeV})^{-\gamma} (B_{\text{rms}}/4 \mu\text{G})^{-\beta}\), with \(\beta \approx 2\) and \(\gamma \approx 0.25-0.5\) for a turbulent field with Kolmogorov power spectrum. We calculate the photon emission due to CR interactions with gas and the resulting irregular source images.

Introduction.—The suggestion that Galactic cosmic rays (CR) are accelerated using the energy released in supernova (SN) explosions dates back to the 1930’s [1]. This idea was supported initially mainly by the argument that SNe inject sufficient energy into the Galaxy to maintain the observed CR energy density, while later the radio emission observed from SN remnants (SNR) was interpreted as indication for the acceleration of high-energy electrons. Until present, a clear proof for both the acceleration of hadrons and the identity of their sources is still missing [2]. Alternatively, the required power could be provided by acceleration processes which operate at distances and time scales larger than for individual SNRs, e.g. in superbubbles [3].

Main obstacle for the identification of CR sources is the diffusion of CRs in the Galactic magnetic field (GMF), erasing directional information on the position of their sources. The GMF has a turbulent component which varies on scales between \(l_{\text{min}} \lesssim 1 \text{AU}\) and \(l_{\text{max}} \sim\) few to 200 pc. Since CRs scatter on inhomogeneities with variation scales comparable to their Larmor radius, the propagation of Galactic CRs in the GMF resembles a random walk and is well described by the diffusion approximation [4,5].

However, CRs around young sources do not have time to diffuse far away. They should produce an extended gamma-ray halo, which can be detected using Cherenkov telescopes and gamma-ray satellites. If SNR power the Galactic CR population, about ten sources with degree scale large enough to be detected at energies \(E_{\gamma} > 100 \text{GeV}\) in the Galactic plane assuming the sensitivity of Fermi-LAT, while Ref. [6] found 18. Most of these sources were observed also as extended sources up to energies \(E_{\gamma} > 10 \text{TeV}\) by the HESS experiment [7] and have a non-spherical shape. Similarly, the Veritas observations of Tycho show a clear asymmetric extension of TeV photons towards the north of the SNR [8].

The diffusion approximation cannot predict local phenomena which arise below the CR mean free path \(\lambda \ll l_{\text{max}}\). when the local configuration of the turbulent field must lead to observable imprints [3]. Before the present study it has remained unclear to which extent the diffusion approximation is satisfied on intermediate scales \(\lambda \ll l \lesssim l_{\text{max}}\), where large-scale fluctuations of the field lead to local anisotropies, which in turn can explain the irregular images of extended sources found in Refs. [6,8].

We study therefore in this letter the diffusion of CRs on scales comparable to the coherence length of the turbulent GMF, \(\mathcal{O}(100 \text{pc})\). In contrast to earlier studies, we calculate the diffusion tensor propagating individual CRs in specific realizations of the turbulent magnetic field. We find that diffusion is anisotropic even for an isotropic random field and can lead to a filamentary structure of the CR density around young sources. Responsible for these anisotropies are turbulent field modes with variations much larger than the Larmor radius of CRs which mimic a regular field. This effect can be confirmed via the observation of irregular gamma-ray emissions around CR sources.

Cosmic rays in magnetic turbulence.—Since we want to test effects beyond the diffusion approximation, we propagate individual CRs in turbulent magnetic fields \(B(k) \propto \exp(-ikx)\) using the numerical code described in [10,11]. The validity of this code was checked reproducing earlier results from [10,11,13]. We assume that the spectrum \(\mathcal{P}(k)\) of magnetic field fluctuations is static and follows a power-law, \(\mathcal{P}(k) \propto k^{-\alpha}\). The former assumption is justified, because the changes introduced by a finite Alfvén velocity are small for the considered time scales. We fix the mean magnetic field strength \(B_{\text{rms}}^2 \equiv \langle B^2(x) \rangle\) as \(B_{\text{rms}} = 4 \mu\text{G}\), normalising \(B_{\text{rms}}\) for fluctuations bounded by \(l_{\text{min}} = 1 \text{AU}\) and \(l_{\text{max}} = 150 \text{pc}\). In the numerical simulations, we choose \(l_{\text{min}}\) sufficiently small compared to the CR Larmor radius \(R_L\). We also adopt an isotropic spectrum of fluctuations, as we want to demonstrate that even in this case CRs diffuse initially anisotropically.

The spectral index of the turbulent GMF is only weakly constrained, and both Kolmogorov (\(\alpha = 5/3\)) and Kraichnan (\(\alpha = 3/2\)) spectra are consistent with observations [12,14]. As our results do not vary much between these two cases, we present them for a Kolmogorov spectrum. The turbulence is expected to have a Bohm spectrum (\(\alpha = 1\)) only close to shocks, where efficient CR acceleration requires a diffusion coefficient \(D(E) \sim R_L\). As we are interested in time-scales when CRs have al-
to calculate the diffusion tensor in this case is to compute the diffusion coefficients of the turbulent field. Thus the correct procedure is to neglect the backreaction of CRs on the turbulent field, discussed e.g. in [15], which modifies the magnetic field only in a thin layer in front of the shock. We also only consider CRs with $E \gtrsim 1$ TeV, which are those relevant for gamma ray observations above 100 GeV.

**CR diffusion and diffusion tensor**—Investigating the propagation of CRs close to their source requires to inject them localised in space, say at $x = 0$, and to propagate them in a given concrete realization of the turbulent field. We find that diffusion can be strongly anisotropic in a specific field realization, as long as the distance between CRs and the source is $\lesssim l_{\text{max}}$. This anisotropy is washed out averaging over many realisations of the turbulent field. Thus the correct procedure to calculate the diffusion tensor in this case is to compute $D_{ij}^{(b)} = 1/(2M) \sum_{a=1}^{N} \bar{x}_i^{(a)} \bar{x}_j^{(a)}$ for $N$ particles (labeled by the subscript $a$) and injected at $x = 0$ in one single realization $b$. For each of the $M$ realizations one diagonalizes $D_{ij}^{(b)}$ and finds its eigenvalues $d_i^{(b)}$. Then one averages the ordered eigenvalues, $d_1^{(b)} < d_2^{(b)} < d_3^{(b)}$, over the $M$ realizations, $d_i = 1/M \sum_{b=1}^{M} d_i^{(b)}$.

In Fig. 1 we show the three eigenvalues $d_i$ of $D_{ij}$ as function of time for the case of CRs with energy $E = 10^{15}$ eV. We used $M = 10$ realizations, propagating for each $N = 10^4$ particles. At early times $t \lesssim t_*$, the diffusion tensor is strongly anisotropic, and the ratio $d_3/d_1$ between its largest and smallest eigenvalue can reach a factor of a few hundreds in some turbulent field realizations, while it is on average around a factor of a few tens. For $t \gtrsim t_*$, CRs propagate more and more isotropically, approaching the predictions of the diffusion approximation for a purely isotropic turbulent field.

Modes with large scale variations contain most of the power in Kolmogorov turbulence, compared to the smaller scale variation modes which are responsible for the diffusion of TeV–PeV CRs. Particles see modes with large variation scales as local uniform fields and diffuse therefore anisotropically. Once a sufficient fraction of particles moved beyond $|x| \sim l_{\text{max}}$, the anisotropies of different “cells” of size $l_{\text{max}}^3$ are averaged out and the CR densities around sources tend towards the limit predicted by the diffusion approximation. Adding a large scale regular field $B_0$ on top of the turbulence would increase the anisotropies. In this case, CRs are known to diffuse faster in the direction of $B_0$ than in the perpendicular direction.

The dashed line in Fig. 1 represents the average diffusion coefficient $D = 1/M \sum_{b=1}^{M} D^{(b)}$, with $D^{(b)}$ defined as $D^{(b)} = 1/(6Dt) \sum_{a=1}^{N} \bar{x}_i^{(a)} \bar{x}_j^{(a)}$ for the magnetic field realization $b$. For the largest times numerically reachable, $t = 10^5$ yr, the eigenvalues in Fig. 1 and the average $D$ approach a common value, $D \approx 5.5 \times 10^{28}$ cm$^2$/s. Interestingly, $2 \times 10^4$ yr corresponds to $(r^2)^{1/2} = \sqrt{D\tau} \approx l_{\text{max}}$ valid in the isotropic limit. We verified that the limiting value for the average $D$ agrees with the value of the diffusion coefficient computed using CRs with random starting positions. It is consistent, among others, with the computations of [12] for pure random fields. The value of $D(E)$ in our Galaxy is currently only known within a factor $\approx 50$ at $E_0 \sim 10$ GeV, and $5.5 \times 10^{28}$ cm$^2$/s is in the acceptable range extrapolating $D(E)$ to $E = 10^{15}$ eV for a Kolmogorov spectrum [16, 17]. It is a factor $\approx 4$ smaller than the value used in Ref. [3].

Earlier works [4, 12, 13, 18] did not report anisotropic diffusion for several reasons: Diffusion coefficients were computed averaging over several configurations, with random initial positions for particles, or the considered space or time scales were too large. For instance, [4] and [12] find isotropic diffusion in the limit of a vanishing uniform field, $B_0 \ll B_{\text{rms}}$. In both works the diffusion coefficient are calculated averaging over many realisations of the turbulent field. As a result, any anisotropy is averaged out, if the random field is isotropic. We also stress that $t_*$ is much larger than the transition time $\tau_{\text{diff}} \approx 4D/c_2^2$ from the ballistic to the diffusive regime [18].

**Cosmic ray intensity and extrapolation to low $E$.**—In the middle row of Fig. 2 we show the projection of the number density of 1 PeV CRs on an arbitrarily chosen plane of size 600 pc x 400 pc containing the injection point $x = 0$ in the center. We consider here one given realization of the turbulent field, out of the ten used for Fig. 1. The diffusion is confirmed to be strongly anisotropic at early times, 500 yr (left) and 2000 yr (middle panel), and even strongly filamentary. At 7000 yr (right panel), the distribution of CRs slowly tends towards the spherical limit expected for true isotropic diffusion. Most of the nine other configurations display similar filamentary structures at early times. For a few of them, no thin filaments are visible, but the CR distribution around the source is still asymmetric, showing wind-like structures,
FIG. 2: Relative cosmic ray densities around their source projected in the panel planes, for energies $E = 100$ TeV (upper row), 1 PeV (middle row), 10 PeV (lower row) and times $t = 500$ yr (left column), 2 kyr (middle column), 7 kyr (right column). Same field realization in each panel. Each panel corresponds to a 600 pc × 400 pc field-of-view, with the source located in the center.

also visible for some of the observed sources [7].

The upper and lower rows of Fig. 2 present results for $E = 100$ TeV and $E = 10$ PeV, respectively. A comparison of the three rows shows that the period of anisotropic diffusion lasts longer for CRs with lower energy. For instance, the panel with $t = 2$ kyr and $E = 1$ PeV is very similar to the one with $t = 7$ kyr and $E = 100$ TeV, which suggests that the expected scaling $t \propto 1/D(E) \propto E^{-1/3}$ also holds in the case of anisotropic diffusion. To determine this scaling law more quantitatively, we plot the values of $d_3/b_3$, $d_2/b_2$, $d_1/b_1$ and $D(b)$ as function of time for given realizations $b$ in Fig. 3 and look for times with similar $d_3/b_1$: Numerically we find $t_\ast \propto E^{-\gamma}$ with $\gamma = 0.25–0.5$, i.e. a value of $\gamma$ consistent with the theoretical expectation.

Transition time. For the parameters of Fig. 4, one estimates $t_\ast \sim 10^4$ yr. This value is mostly determined by $l_{\text{max}}$, and we find that the naive expectation $t_\ast \propto t_{\text{max}}^2$ holds in a first approximation. Reducing $l_{\text{max}}$ by a factor 6, to 25 pc, the transition happens at 200–300 $\sim 10^4/6^2$ yr in all ten tested configurations. Therefore, our numerical results suggest that the diffusion approximation predictions become valid at

$$t_\ast \sim 10^4 \text{ yr} \left(l_{\text{max}}/150 \text{ pc}\right)^\beta \left(E/\text{PeV}\right)^{-\gamma} \left(B_{\text{rms}}/4 \mu\text{G}\right)^\gamma$$

(1)

with $\beta \simeq 2$ and $\gamma = 0.25–0.5$ for Kolmogorov turbulence. $B_{\text{rms}} \simeq 4 \mu\text{G}$ corresponds to the estimated local value around the Earth from unpolarized synchrotron radiation data [19]. However, the large uncertainties of the turbulent field parameters, in particular of $l_{\text{max}}$, imply that $t_\ast$ may differ significantly from $10^4$ yr for PeV CRs.

The spectral index $\alpha$ does not have a strong impact on $t_\ast$. In contrast, it influences the ratio $d_3/d_1$ at early times. For a Bohm spectrum, we find that the anisotropy at $t \lesssim t_\ast$ is reduced. Indeed, more power is concentrated in small scale modes than in a Kolmogorov spectrum, resulting in a better isotropization of PeV CRs. After averaging over five realizations of the field, we find a factor $d_3/d_1 \approx 3–4$ at early times. Out of these five configurations, only one is Kolmogorov-like with $d_3/b_3 \sim 10$ and visible filaments. The others are just slightly anisotropic.

Gamma-ray emission. High energy protons can scatter on protons of the interstellar gas, producing sec-
FIG. 3: Eigenvalues $d_i^{(b)}$ of the diffusion tensor $D_{ij}$ as function of time $t$ for energies $E = 100$ TeV (left), 1 PeV (middle) and 10 PeV (right). Same field realization as in Fig. 2. Dashed lines for the average diffusion coefficient $D^{(b)} \approx D$.

FIG. 4: Left panel: Relative cosmic ray densities along the lines-of-sight as seen from a specific observer located 500 pc away from the source. $E = 1$ PeV and $t = 1$ kyr, for a given magnetic field realization. Box size is $20^\circ \times 20^\circ$, with the source at (0,0). Right panel: Corresponding relative surface brightness in $\gamma$-rays with energies $E_\gamma \geq 100$ GeV.

ondaries which in turn decay into photons. We simulate cross sections and the final state of proton-proton interactions using QGSJET-II [20], while we use SIBYLL 2.1 [21] for the subsequent decays of unstable particles. The emission of secondaries is strongly forward beamed and CRs in filaments have an anisotropic distribution of momenta. We may expect that the emitted $\gamma$-rays are a good tracker of the underlying CR anisotropies.

For simplicity, we assume a uniform gas density around the source. Then we place an observer at 500 pc distance from the source and integrate the photon flux emitted along the line-of-sight towards the observer. We model the observer as a sphere of 5 pc, which is the smallest size providing reasonable statistics and introducing only a small amount of artificial ‘fuzziness’. The resulting source image is shown in the right panel of Fig. 4. The comparison to the corresponding CR intensity (left panel) shows that the latter can be used to predict the shape of the gamma ray halo. Note that gamma rays emitted via Compton scattering by electrons would also display such anisotropic patterns.

Conclusions.— We studied the diffusion of TeV–PeV CRs on scales $l$ smaller or comparable to the largest scales of magnetic field fluctuations, $l \lesssim l_{\text{max}}$. The propagation of such CRs close to their sources has to be studied in single realizations of the turbulent field. We showed that CRs diffuse anisotropically at early times $t \lesssim t_*$, with $t_*$ from Eq. 4, leading to a filamentary structure of the CR density around their sources. Turbulent field modes with variation scales much larger than the Larmor radius of CRs are responsible for this anisotropic diffusion regime. This effect can explain the observations of irregular gamma-ray halos [6–8] around CR proton and electron sources. If CRs propagate distances $l \gtrsim l_{\text{max}}$, these anisotropies are averaged out. CR densities around sources become isotropic and tend towards those expected from the diffusion approximation.

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[1] W. Baade and F. Zwicky, Proc. Nat. Acad. Sci. 20, 259 (1934).
[2] A. M. Hillas, J. Phys. G 31, R95 (2005).
[3] A. Bykov and I. Toptygin, Astron. Lett. 27, 625 (2001); E. Parizot et al., Astron. Astrophys. 424, 747 (2004) [astro-ph/0405531]. M. Ackermann et al., Science 334, 1103 (2011).
[4] J. Giacalone and J. R. Jokipii, Astrophys. J. 520, 204 (1999).
[5] A. W. Strong, I. V. Moskalenko and V. S. Ptuskin, Ann. Rev. Nucl. Part. Sci. 57, 285 (2007) [arXiv:astro-ph/0701517].
[6] A. Neronov and D. V. Semikoz, Phys. Rev. D 85, 083008 (2012) [arXiv:1201.1690 [astro-ph.HE]].
[7] F. Aharonian et al. Astrophys. J. 636, 777 (2006) [astro-ph/0510397].
[8] V. A. Acciari et al., Astrophys. J. 730, L20 (2011) [arXiv:1102.3871 [astro-ph.HE]].
[9] G. Giacinti and G. Sigl. [arXiv:1111.2526 [astro-ph.HE]].
[10] G. Giacinti et al., Astropart. Phys. 35, 192 (2011) [arXiv:1104.1141 [astro-ph.HE]].
[11] G. Giacinti et al. [arXiv:1112.5599 [astro-ph.HE]]. To appear in JCAP.
[12] F. Casse, M. Lemoine and G. Pelletier, Phys. Rev. D 65, 023002 (2002) [astro-ph/0109223].
[13] D. De Marco, P. Blasi and T. Stanev, JCAP 0706, 027 (2007) [arXiv:0705.1972 [astro-ph]].
[14] R. Beck, E. M. Berkhuijsen and B. Uyaniker, in “Plasma
Turbulence and Energetic Particles in Astrophysics, M. Ostrowski and R. Schlickeiser (Eds.), p. 5, Obser-

[15] B. Reville and A. R. Bell, Mon. Not. R. Astron. Soc. 419, 2433 (2012) [arXiv:1109.5690 [astro-ph.HE]].

[16] D. Maurin et al., astro-ph/0212111

[17] T. Delahaye et al., Astron. Astrophys. 531, A37 (2011) [arXiv:1102.0744 [astro-ph.HE]].

[18] E. Parizot, Nucl. Phys. Proc. Suppl. 136, 169 (2004) [astro-ph/0409191].

[19] R. Beck, AIP Conf. Proc. 1085, 83 (2009) [arXiv:0810.2923 [astro-ph]].

[20] S. Ostapchenko, Nucl. Phys. Proc. Suppl. 151, 143 (2006) [arXiv:hep-ph/0412322]; Phys. Rev. D 74, 014026 (2006) [arXiv:hep-ph/0505259].

[21] R. Engel et al., Proc. 26th ICRC, Salt Lake City, 1, 415 (1999).