Abstract

The magnitude of $m_u$, $m_d$ and $m_s$ is discussed on the basis of Chiral Perturbation Theory. In particular, the claim that $m_u = 0$ leads to a coherent picture for the low energy structure of QCD is examined in detail. It is pointed out that this picture leads to violent flavour asymmetries in the matrix elements of the scalar and pseudoscalar operators, which are in conflict with the hypothesis that the light quark masses may be treated as perturbations.

[Talk delivered at the 2nd IFT Workshop, ”Yukawa Couplings and the Origins of Mass”, Gainesville, Florida, Feb. 1994, to be published by International Press.]
1 Effective low energy theory of QCD

At low energies, the behaviour of scattering amplitudes or current matrix elements can be described in terms of a Taylor series expansion in powers of the momenta. The electromagnetic form factor of the pion, e.g., may be expanded in powers of the momentum transfer $t$. In this case, the first two Taylor coefficients are related to the total charge of the particle and to the mean square radius of the charge distribution, respectively,

$$f_{\pi^+}(t) = 1 + \frac{1}{6}(r^2)_{\pi^+} + O(t^2) .$$

(1)

Scattering lengths and effective ranges are analogous low energy constants occurring in the Taylor series expansion of scattering amplitudes.

For the straightforward expansion in powers of the momenta to hold it is essential that the theory does not contain massless particles. The exchange of photons, e.g., gives rise to Coulomb scattering, described by an amplitude of the form $e^2/(p' - p)^2$ which does not admit a Taylor series expansion. Now, QCD does not contain massless particles, but it does contain very light ones: pions. The occurrence of light particles gives rise to singularities in the low energy domain which limit the range of validity of the Taylor series representation. The form factor $f_{\pi^+}(t)$, e.g., contains a cut starting at $t = 4M_{\pi}^2$, such that the formula (1) provides an adequate representation only for $t \ll 4M_{\pi}^2$. To extend this representation to larger momenta, one needs to account for the singularities generated by the pions. This can be done, because the reason why $M_{\pi}$ is so small is understood: the pions are the Goldstone bosons of a hidden, approximate symmetry. The low energy singularities generated by the remaining members of the pseudoscalar octet ($K^\pm, K^0, \bar{K}^0, \eta$) can be dealt with in the same manner, exploiting the fact that the Hamiltonian of QCD is approximately invariant under $SU(3)_R \times SU(3)_L$. If the three light quark flavours $u, d, s$, were massless, this symmetry would be an exact one. In reality, chiral symmetry is broken by the quark mass term occurring in the QCD Hamiltonian

$$H_{\text{QCD}} = H_0 + H_1 , \quad H_1 = \int d^3x\{m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s\} .$$

For yet unknown reasons, the masses $m_u, m_d, m_s$ however happen to be small, such that $H_1$ can be treated as a perturbation. First order perturbation theory shows that the expansion of the square of the pion mass in powers of $m_u, m_d, m_s$ starts with

$$M_{\pi^+}^2 = (m_u + m_d)B\{1 + O(m_u, m_d, m_s)\} ,$$

(2)

while, for the kaon, the leading term contains the mass of the strange quark,

$$M_{K^+}^2 = (m_u + m_s)B + \ldots \quad M_{K^0}^2 = (m_d + m_s)B + \ldots$$

(3)

This explains why the pseudoscalar octet contains the eight lightest hadrons and why the mass pattern of this multiplet very strongly breaks eightfold way symmetry: $M_{\pi}^2, M_{K}^2$ and $M_{\eta}^2$ are proportional to combinations of quark masses, which are small
but very different from one another, \( m_s \gg m_d > m_u \). For all other multiplets of SU(3), the main contribution to the mass is given by the eigenvalue of \( H_0 \) and is of order \( \Lambda_{\text{QCD}} \), while \( H_1 \) merely generates a correction which splits the multiplet, the state with the largest matrix element of \( \bar{s}s \) ending up at the top.

The effective field theory combines the expansion in powers of momenta with the expansion in powers of \( m_u, m_d, m_s \). The resulting new improved Taylor series, which explicitly accounts for the singularities generated by the Goldstone bosons, is referred to as chiral perturbation theory (\( \chi \)PT). It provides a solid mathematical basis for what used to be called the "PCAC hypothesis" [1, 2, 3, 4].

It does not appear to be possible to account for the singularities generated by the next heavier bound states, the vector mesons, in an equally satisfactory manner. The mass of the \( \rho \)-meson is of the order of the scale of QCD and cannot consistently be treated as a small quantity. Although the vector meson dominance hypothesis does lead to valid estimates (an example is given below), a coherent framework which treats these estimates as leading terms of a systematic approximation scheme is not in sight.

The effective low energy theory replaces the quark and gluon fields of QCD by a set of pseudoscalar fields describing the degrees of freedom of the Goldstone bosons \( \pi, K, \eta \). It is convenient to collect these fields in a \( 3 \times 3 \) matrix \( U(x) \in \text{SU}(3) \). Accordingly, the Lagrangian of QCD is replaced by an effective Lagrangian, which only involves the field \( U(x) \) and its derivatives. The most remarkable point here is that this procedure does not mutilate the theory: if the effective Lagrangian is chosen properly, the effective theory is mathematically equivalent to QCD [1, 4].

On the level of the effective Lagrangian, the combined expansion introduced above amounts to an expansion in powers of derivatives and powers of the quark mass matrix

\[
m = \begin{pmatrix}
m_u & m_d \\
m_d & m_s
\end{pmatrix}
\]

Lorentz invariance and chiral symmetry very strongly constrain the form of the terms occurring in this expansion. Counting \( m \) like two powers of momenta, the expansion starts at \( O(p^2) \) and only contains even terms

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^2 + \mathcal{L}_{\text{eff}}^4 + \mathcal{L}_{\text{eff}}^6 + \ldots
\]

The leading contribution is of the form

\[
\mathcal{L}_{\text{eff}}^2 = \frac{1}{4} F^2 \text{tr}\{\partial_\mu U^\dagger \partial^\mu U\} + \frac{1}{2} F^2 \text{tr}\{m(U + U^\dagger)\}
\]

and involves two independent coupling constants: the pion decay constant \( F \) and the constant \( B \) occurring in the mass formulae [2] and [3]. The expression [4] represents a compact summary of the soft pion theorems established in the 1960’s: the leading terms in the chiral expansion of the scattering amplitudes and current matrix elements are given by the tree graphs of this Lagrangian.
At order $p^4$, the effective Lagrangian contains terms with four derivatives such as
\[
\mathcal{L}^4_{\text{eff}} = L_1 [\text{tr} \{ \partial_\mu U^+ \partial^\mu U \}]^2 + \ldots
\]
as well as terms with one or two powers of $m$. Altogether, ten coupling constants occur \cite{3}, denoted $L_1, \ldots, L_{10}$. Four of these are needed to specify the scattering matrix to first nonleading order. The terms of order $m^2$ in the meson mass formulae \cite{2} and \cite{3} involve another three of these constants. The remaining three couplings concern current matrix elements.

As an illustration, consider again the e.m. form factor $f_{+}(t)$. To order $p^2$, the chiral representation reads \cite{3}
\[
f_{+}(t) = 1 + \frac{t}{F^2} \{ 2L_9 + 2\phi_{\pi}(t) + \phi_{K}(t) \} + O(t^2, tm)
\]
In this example, the leading term (tree graph of $\mathcal{L}^2_{\text{eff}}$) is trivial, because $f_{+}(0)$ represents the charge of the particle. At order $p^2$, there are two contributions: the term linear in $t$ arises from a tree graph of $\mathcal{L}^4_{\text{eff}}$ and involves the coupling constant $L_9$, while the functions $\phi_{\pi}(t)$ and $\phi_{K}(t)$ originate in one loop graphs generated by $\mathcal{L}^2_{\text{eff}}$. The loop integrals contain a logarithmic divergence which is absorbed in a renormalization of $L_9$ – the net result for $f_{+}(t)$ is independent of the regularization used. The representation \cite{3} shows how the straightforward Taylor series \cite{1} is modified by the singularities due to $\pi\pi$ and $K\bar{K}$ intermediate states. At the order of the chiral expansion we are considering here, these singularities are described by the one loop integrals $\phi_{\pi}(t), \phi_{K}(t)$ which contain cuts starting at $t = 4M_\pi^2$ and $t = 4M_K^2$, respectively. The result \cite{3} also shows that chiral symmetry does not determine the pion charge radius: its magnitude depends on the value of the coupling constant $L_9$ – the effective Lagrangian is consistent with chiral symmetry for any value of the coupling constants. The symmetry, however, relates different observables. The slope of the $K_\ell$ form factor $f_{+}(t)$, e.g., is also fixed by $L_9$. The experimental value of this slope \cite{4}, $\lambda_+ = 0.030$, can therefore be used to first determine the magnitude of $L_9$ and then to calculate the pion charge radius. This gives $(r^2)_{\pi+} = 0.42 \text{ fm}^2$, to be compared with the experimental result, $0.44 \text{ fm}^2$ \cite{6}.

2 Scale of the loop expansion

One of the main problems encountered in the effective Lagrangian approach is the occurrence of an entire fauna of effective coupling constants. If these constants are treated as totally arbitrary parameters, the predictive power of the method is equal to zero – as a bare minimum, an estimate of their order of magnitude is needed.

Let me first drop the masses of the light quarks and send the heavy ones to infinity. In this limit, QCD is a theoretician’s paradise: a theory without adjustable dimensionless parameters. In particular, the effective coupling constants $F, B, L_1, L_2, \ldots$ are calculable — the available, admittedly crude evaluations of $F$
and $B$ on the lattice demonstrate that the calculation is even feasible. As discussed above, the coupling constants $L_1, L_2, \ldots$ are renormalized by the logarithmic divergences occurring in the one loop graphs. This property sheds considerable light on the structure of the chiral expansion and provides a rough estimate for the order of magnitude of the effective coupling constants \[7\]. The point is that the contributions generated by the loop graphs are smaller than the leading (tree graph) contribution only for momenta in the range $|p| \lesssim \Lambda_{\chi}$, where \[8\]

$$\Lambda_{\chi} \equiv 4\pi F/\sqrt{N_f}$$

is the scale occurring in the coefficient of the logarithmic divergence ($N_f$ is the number of light quark flavours). This indicates that the derivative expansion is an expansion in powers of $(p/\Lambda_{\chi})^2$ with coefficients of order one. The stability argument also applies to the expansion in powers of $m_u, m_d$ and $m_s$, indicating that the relevant expansion parameter is given by $(M_\pi/\Lambda_{\chi})^2$ and $(M_K/\Lambda_{\chi})^2$, respectively.

3 Low lying excited states

A more quantitative picture can be obtained along the following lines. Consider again the e.m. form factor of the pion and compare the chiral representation \[5\] with the dispersion relation

$$f_{\pi^+}(t) = \frac{1}{\pi} \int_{4M_\rho^2}^{\infty} \frac{dt'}{t' - t} \text{Im} f_{\pi^+}(t') .$$

In this relation, the contributions $\phi_{\pi}, \phi_K$ from the one loop graphs of $\chi$PT correspond to $\pi\pi$ and $K\bar{K}$ intermediate states. To leading order in the chiral expansion, the corresponding imaginary parts are slowly rising functions of $t$. The most prominent contribution on the r.h.s., however, stems from the region of the $\rho$-resonance which nearly saturates the integral: the vector meson dominance formula, $f_{\pi^+}(t) = (1 - t/M_\rho^2)^{-1}$, which results if all other contributions are dropped, provides a perfectly decent representation of the form factor for small values of $t$. In particular, this formula predicts $\langle r^2 \rangle_{\pi^+} = 0.39$ fm$^2$, in satisfactory agreement with observation (0.44 fm$^2$). This implies that the effective coupling constant $L_9$ is approximately given by \[3\]

$$L_9 = \frac{F^2}{2M_\rho^2} .$$

In the channel under consideration, the pole due to $\rho$ exchange thus represents the dominating low energy singularity – the $\pi\pi$ and $K\bar{K}$ cuts merely generate a small correction. More generally, the validity of the vector meson dominance formula shows that, for the e.m. form factor, the scale of the derivative expansion is set by $M_\rho = 770$ MeV.
Analogous estimates can be given for all effective coupling constants at order $p^4$, saturating suitable dispersion relations with contributions from resonances \cite{9,10}, e.g.,

$$L_5 = \frac{F^2}{4M_S^2}, \quad L_7 = -\frac{F^2}{48M_{\eta'}^2},$$

where $M_S \simeq 980$ MeV and $M_{\eta'} = 958$ MeV are the masses of the scalar octet and pseudoscalar singlet, respectively. In all those cases where direct phenomenological information is available, these estimates do remarkably well. I conclude that the observed low energy structure is dominated by the poles and cuts generated by the lightest particles – hardly a surprise.

\section{Magnitude of the effective coupling constants}

The effective theory is constructed on the asymptotic states of QCD. In the sector with zero baryon number, charm, beauty, . . . , the Goldstone bosons form a complete set of such states, all other mesons being unstable against decay into these (strictly speaking, the $\eta$ occurs among the asymptotic states only for $m_d = m_u$; it must be included among the degrees of freedom of the effective theory, nevertheless, because the masses of the light quarks are treated as a perturbation — in massless QCD, the poles generated by the exchange of this particle occur at $p = 0$). The Goldstone degrees of freedom are explicitly accounted for in the effective theory — they represent the dynamical variables. All other levels manifest themselves only indirectly, through the values of the effective coupling constants. In particular, low lying levels such as the $\rho$ generate relatively small energy denominators, giving rise to relatively large contributions to some of these coupling constants.

In some channels, the scale of the chiral expansion is set by $M_\rho$, in others by the masses of the scalar or pseudoscalar resonances occurring around 1 GeV. This confirms the rough estimate (6). The cuts generated by Goldstone pairs are significant in some cases and are negligible in others, depending on the numerical value of the relevant Clebsch-Gordan coefficient. If this coefficient turns out to be large, the coupling constant in question is sensitive to the renormalization scale used in the loop graphs. The corresponding pole dominance formula is then somewhat fuzzy, because the prediction depends on how the resonance is split from the continuum underneath it.

The quantitative estimates of the effective couplings given above explain why it is justified to treat $m_s$ as a perturbation. At order $p^4$, the symmetry breaking part of the effective Lagrangian is determined by the constants $L_4, \ldots, L_8$. These constants are immune to the low energy singularities generated by spin 1 resonances, but are affected by the exchange of scalar or pseudoscalar particles. Their magnitude is therefore determined by the scale $M_S \simeq M_{\eta'} \simeq 1$ GeV. Accordingly, the expansion in powers of $m_s$ is controlled by the parameter $(M_K/M_S)^2 \simeq \frac{1}{4}$. The asymmetry in
the decay constants, e.g., is given by \[ F_K/F_\pi = 1 + \frac{M_K^2 - M_\pi^2}{M_3^2} + \chi \logs + O(m^2) , \] where the term \( \chi \logs \) stands for the chiral logarithms generated by intermediate states with two Goldstone bosons. This shows that the breaking of the chiral and eightfold way symmetries is controlled by the mass ratio of the Goldstone bosons to the non-Goldstone states of spin zero. In \( \chi \)PT, the observation that the Goldstones are the lightest hadrons thus acquires quantitative significance.

5 Light quark masses

A crude estimate for the order of magnitude of the light quark masses was given shortly after the discovery of QCD \[ m_u \approx 4 \text{ MeV} \ , \quad m_d \approx 6 \text{ MeV} \ , \quad m_s \approx 125 - 150 \text{ MeV} \ . \]

Many papers dealing with the issue have appeared since then. Weinberg \[ m_u \] pointed out that \( \chi \)PT leads to an improved estimate for the ratios \( m_u : m_d : m_s \). Using the Dashen theorem \[ m_u \] to account for e.m. self energies, the lowest order mass formulae given in eqs. \[ m_u \] and \[ m_s \] imply \[ m_s \]

\[
\frac{m_u}{m_d} = \frac{M_{K^0}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} ,
\]

\[
\frac{m_s}{m_d} = \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^0}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} .
\]

Numerically, this gives

\[
\frac{m_u}{m_d} = 0.55 \ , \quad \frac{m_s}{m_d} = 20 .
\]

The higher order terms in the chiral expansion generate corrections to the mass formulae \[ m_u \], controlled by the parameter \( (M_K/\Lambda_\chi)^2 \). Roughly, the numerical result \[ m_u \] should therefore hold to within 20 or 30%.

The corrections of \( O(p^4) \) to the above mass formulae were worked out some time ago \[ m_u \]. In particular, it was shown that these corrections drop out when taking the double ratio

\[
Q^2 \equiv \frac{M_{K^0}^2}{M_{\pi}^2} \cdot \frac{M_{K^+}^2 - M_{\pi^0}^2}{M_{K^0}^2 - M_{K^+}^2} .
\]

The observed values of the meson masses thus provide a tight constraint on one particular ratio of quark masses,

\[
Q^2 = \frac{m_s^2 - \hat{m}_s^2}{m_d^2 - m_u^2} \left(1 + O(m^2)\right) ,
\]
with \( \hat{m} = \frac{1}{2} (m_u + m_d) \). The constraint may be visualized by plotting the ratio \( m_s/m_d \) versus \( m_u/m_d \). Dropping the corrections of \( O(m^2) = O(M_K^2/\Lambda^4) \), the resulting curve takes the form of an ellipse,

\[
\left( \frac{m_u}{m_d} \right)^2 + \frac{1}{Q^2} \left( \frac{m_s}{m_d} \right)^2 = 1, \tag{14}
\]

with \( Q \) as major semi-axis (the term \( \hat{m}^2/m_s^2 \) has been discarded, as it is numerically very small). The meson masses occurring in the double ratio (12) refer to pure QCD. Using the Dashen theorem to correct for the e.m. self energies, one obtains \( Q = 24.1 \). For this value of the semi-axis, the ellipse passes through the point specified by Weinberg’s mass ratios, eq. (11).

\section{Corrections to the Dashen Theorem}

The Dashen theorem is subject to corrections from higher order terms in the chiral expansion. As usual, there are two categories of contributions: loop graphs of order \( e^2 m \) and terms of the same order from the derivative expansion of the effective e.m. Lagrangian.

The Clebsch-Gordan coefficients occurring in the loop graphs are known to be large, indicating that two-particle intermediate states generate sizeable corrections; the corresponding chiral logarithms tend to increase the e.m. contribution to the kaon mass difference \([13]\). The numerical result depends on the scale used when evaluating the logarithms. In fact, taken by themselves, chiral logs are unsafe at any scale.

The magnitude of the contributions from the terms of order \( e^2 m \) occurring in the effective Lagrangian is estimated in two recent papers by Donoghue et al. \([14]\) and Bijnens \([17]\). Although the framework used is rather different, they come up with the same conclusion: the corrections are large, increasing the value \( (M_{K^+} - M_{K^0})_{e.m.} = 1.3 \) MeV predicted by Dashen to 2.3 MeV (Donoghue et al.) and 2.6 MeV (Bijnens), respectively.

In the present context, the main point is that even large corrections of this size only lead to a small change in the value of \( Q \), because the mass difference between \( K^+ \) and \( K^0 \) is predominantly due to \( m_d > m_u \) : the value \( Q = 24.1 \) (Dashen) is lowered to \( Q = 22.1 \) (Donoghue et al.) and \( Q = 21.6 \) (Bijnens), respectively. Expressed in terms of the error bars attached to the value of \( Q \) in \([11]\), a doubling of the electromagnetic self-energy modifies the value of \( Q \) by \( 1 \frac{1}{2} \sigma \).

The decay \( \eta \to 3\pi \) provides an independent measurement of \( Q \): writing the decay rate in the form \( \Gamma_{\eta \to \pi^+ \pi^- \pi^0} = \Gamma_0/Q^4 \), \( \chiPT \) predicts the value of \( \Gamma_0 \) in a parameter free manner \([3]\). Although the calculation accounts for all corrections of \( O(p^4) \), the numerical accuracy is rather modest because, due to strong final state interaction effects, the calculated corrections are quite large. Since the quantity \( Q \) enters in the fourth power, the value \( \Gamma_{\eta \to \pi^+ \pi^- \pi^0} = 283 \pm 28 \) eV given by the particle data group \([5]\) still yields a decent measurement: \( Q = 20.6 \pm 1.7 \). The fact that this result is
significantly smaller than the value predicted with the Dashen theorem represents an old puzzle. The problem disappears if the e.m. contribution to the kaon mass difference is significantly larger than indicated by the Dashen theorem. In particular, the values of \( Q \) obtained in [16, 17] are consistent with the one from \( \eta \) decay.

The theoretical uncertainties in the \( \eta \) decay amplitude could be reduced. The calculation referred to above only accounts for the corrections of order \( p^4 \) and includes final state interaction effects and \( \eta \eta' \) mixing only to that order. A dispersive analysis along the lines indicated by Khuri and Treiman [21], which uses the \( \chi \)PT predictions only for the subtraction constants would likely lead to a more accurate estimate of the major semi-axis [23].

7 The ratio \( m_u/m_d \)

Chiral perturbation theory thus fixes one of the two quark mass ratios in terms of the other, to within small uncertainties. The ratios themselves, i.e., the position on the ellipse, are a more subtle issue. Kaplan and Manohar [14] pointed out that the corrections to the lowest order result (11) for \( m_u/m_d \) cannot be determined on purely phenomenological grounds. They argued that these corrections might be large and that the \( u \)-quark might actually be massless. This possibility is of particular interest, because the strong CP problem would then disappear. Several authors [23] have given arguments in favour of \( m_u = 0 \).

Let me show the picture this reasoning leads to. The lowest order mass formulae (2), (3) imply that the ratio \( m_u/m_d \) determines the \( K^0/K^+ \) mass difference, the scale being set by \( M_\pi \):

\[
M_{K^0}^2 - M_{K^+}^2 = \frac{m_d - m_u}{m_u + m_d} \cdot M_\pi^2 + \ldots
\]

The formula holds up to corrections from higher order terms in the chiral expansion and up to e.m. contributions. It is illustrated in figure 1, taken from [24]. The upper curve corresponds to the value \( (M_{K^+} - M_{K^0})_{c.m.} = 1.3 \) MeV, which follows from the Dashen theorem. The correction of Donoghue et al. [19] shifts the result by 1 MeV (lower curve). The horizontal line is the experimental value. Hence the corrections from the higher order terms must generate the contributions shown by the arrows. In particular, if \( m_u \) is assumed to vanish, the lowest order mass formula predicts a mass difference which exceeds the observed value by a factor of four. The disaster can only be blamed on the "corrections" from the higher order terms. It is evident that, under such circumstances, it does not make sense to truncate the expansion at first nonleading order and to fool around with the numerics of the effective coupling constants occurring therein. The conclusion to draw from the assumption \( m_u = 0 \) is that \( \chi \)PT is unable to account for the masses of the Goldstone bosons. The fact that it happens to work remarkably well in other cases must then be accidental. I prefer to conclude that the assumption \( m_u = 0 \) is not tenable.
Figure 1: Sensitivity of the first order prediction for the kaon mass difference to $m_u/m_d$. The two curves differ in the estimate used for the e.m. self energies. E is the experimental value. The dot corresponds to Weinberg’s leading order result.

8 Phenomenological ambiguities

The preceding presentation is an extract of a rapporteur talk given in 1992 [24]. I thought that this coffee was rather cold by now and dealt with it only briefly in the manner described above, concentrating the remainder of the talk on issues which I expected to be more interesting to the audience, such as the role of winding number and the mass of the $\eta'$ [25, 26]. The vigorous discussion triggered by the claim that $m_u = 0$ is firmly ruled out by now and the remarkable statements made by some of the participants indicate, however, that not everyone fully shares this view of the matter. I conclude that the arguments given previously do not suffice and elaborate a little further.

To set up notation, I first briefly review the observation of Kaplan and Manohar [14], who pointed out that the matrix

$$m' = \alpha_1 m + \alpha_2 (m^+)^{-1} \det m$$

transforms in the same manner as $m$. For a real, diagonal mass matrix, the transformation amounts to

$$m_u' = \alpha_1 m_u + \alpha_2 m_d m_s \quad \text{(cycl. } u \rightarrow d \rightarrow s \rightarrow u) \quad .$$

Symmetry alone does therefore not distinguish $m'$ from $m$. If $\mathcal{L}_{\text{eff}}(U, \partial U, \ldots, m)$ is an effective Lagrangian consistent with chiral symmetry, so is $\mathcal{L}_{\text{eff}}(U, \partial U, \ldots, m')$.

Since only the product $B m$ enters the Lagrangian, $\alpha_1$ merely changes the value of the constant $B$. The term proportional to $\alpha_2$ is a correction of order $m^2$ which, upon insertion in $\mathcal{L}^2_{\text{eff}}$ generates a contribution to $\mathcal{L}^4_{\text{eff}}$. The contribution can again be removed by changing some of the coupling constants:

$$B' = B / \alpha_1 \quad , \quad L'_6 = L_6 - \alpha \quad , \quad L'_7 = L_7 - \alpha \quad , \quad L'_8 = L_8 + 2\alpha \quad ,$$

(17)
with \( \alpha = \alpha_2 F^2 / 32 \alpha_1 B \). The effective Lagrangian is therefore invariant under a simultaneous change of the quark mass matrix and of the coupling constants. Accordingly, the meson masses and scattering amplitudes which one calculates with this Lagrangian remain the same. The elliptic constraint considered above, e.g., is invariant under the operation (up to terms of order \((m_u - m_d)^2 / m_s^2\); which were neglected). The chiral representation for the Green functions of the vector and axial currents are also invariant. Since there is experimental information only about masses, scattering amplitudes and matrix elements of the electromagnetic or weak currents and since the chiral representation for these does not distinguish \( m, B, L_i \) from \( m', B', L'_i \), phenomenology does not allow one to determine the magnitude of the constants \( B, L_6, L_7, L_8 \).

We are not dealing with a hidden symmetry of QCD here — this theory is not invariant under the change (16) of the quark masses. In particular, the matrix elements of the scalar and pseudoscalar operators are modified. Consider, e.g., the vacuum-to-pion matrix element of the pseudoscalar density. The Ward identity for the axial current implies that this matrix element is given by

\[
\langle 0 | \bar{s} i \gamma_5 u | \pi^+ \rangle = \sqrt{2} \frac{F_{\pi^+} M_{\pi^+}}{(m_u + m_d)}
\]

The relation is exact, except for electroweak corrections. It involves the physical quark masses and is not invariant under the above transformation. For the ratio of the \( K \) and \( \pi \) matrix elements, the effective Lagrangian yields the following low energy representation:

\[
\frac{\langle 0 | \bar{s} i \gamma_5 u | K^+ \rangle}{\langle 0 | \bar{s} i \gamma_5 u | \pi^+ \rangle} = 1 + \frac{4(M_{K^+}^2 - M_{\pi^+}^2)}{F^2} (4L_8 - L_5) + \chi \log s + O(m^2) .
\]

The relation is analogous to the formula for the ratio of the corresponding matrix elements of the axial currents,

\[
\frac{F_{K^+}}{F_{\pi^+}} = 1 + \frac{4(M_{K^+}^2 - M_{\pi^+}^2)}{F^2} L_5 + \chi \log s + O(m^2) .
\]

While the ratio of decay constants is invariant under the KM transformation, the one of the pseudoscalar densities is not. The main difference between the two cases is that nature is kind enough to provide us with a weak interaction probe, testing the matrix elements of the vector and axial currents at low energies, while a probe which would test those of the scalar and pseudoscalar currents is not available — the Higgs particle is too heavy for this purpose. The observed rates of the decays \( K \to \mu \nu \) and \( \pi \to \mu \nu \) imply \( F_{K^+} / F_{\pi^+} = 1.22 \) and thereby permit a phenomenological determination of the effective coupling constant \( L_5 \), while \( L_8 \) cannot be determined on purely phenomenological grounds.

If the electroweak interactions were not available to probe the low energy structure of QCD, the coupling constant \( L_5 \) would also count among those quantities
for which direct phenomenological information is absent, for the following reason. The field

\[ U' = U\{1 + \alpha_3(U^\dagger m - m^\dagger U) - \frac{1}{3}\alpha_3 \text{tr}(U^\dagger m - m^\dagger U) + O(m^2)\} \]  

(19)

transforms in the same manner as \( U \). For a constant quark mass matrix, the change of variables \( U \to U' \) is equivalent to a change of the effective coupling constants:

\[ L'_5 = L_5 - 6\bar{\alpha}, \quad L'_7 = L_7 + \bar{\alpha}, \quad L'_8 - 3\bar{\alpha}, \quad \bar{\alpha} = F^2\alpha_3/24B \]  

(20)

Hence the \( \chi PT \) results for the masses of the Goldstone bosons and for their scattering amplitudes only involve those combinations of coupling constants which are invariant under this operation. The only difference to the transformation considered above is that (17) leaves the matrix elements of the vector and axial currents invariant, while (20) does not.

The ambiguity pointed out by Kaplan and Manohar does not indicate that the effective Lagrangian possesses any symmetries beyond those of the QCD Lagrangian. It does not concern QCD as such, but originates in the fact that the electromagnetic and weak interactions happen to probe the low energy structure of the system exclusively through vector and axial currents.

### 9 Additive renormalization of \( m \) and all that

One of the claims repeatedly made in the discussion is that the quark masses occurring in the effective Lagrangian must be distinguished from those entering the Lagrangian of QCD. Indeed, this problem does arise within early formulations of the effective Lagrangian technique, which exclusively dealt with the properties of the Goldstone bosons on the mass shell. There, the structure of the effective Lagrangian was inferred from *global* symmetry arguments and the only place where the quark mass matrix entered was through the transformation law of the term which explicitly breaks the symmetries of the QCD Hamiltonian.

The framework used in current work on \( \chi PT \), however, does not rely on global symmetry arguments, but identifies the quark masses through their contributions to the Ward identities, which express the symmetries of the underlying theory in *local* form. The method was introduced ten years ago [2, 4]. It is not limited to on-shell matrix elements, but also specifies the low energy expansion of the Green functions formed with the vector, axial, scalar and pseudoscalar currents. The Ward identities involve the physical quark masses, not some effective low energy version thereof. *For the effective theory to reproduce the low energy structure of the Green functions, the mass matrix occurring in the effective Lagrangian must be identified with the physical one.* Likewise, the magnitude of the effective coupling constants \( L_6, L_7 \) and \( L_8 \) is not an inherently ambiguous issue; the effective theory reproduces the low energy expansion of QCD for precisely one value of these constants.

11
There is a distinction between the currents associated with chiral symmetry and the scalar or pseudoscalar densities in that the latter pick up multiplicative renormalization, contragredient to the renormalization of the quark mass matrix. It is crucial that the renormalization used is mass independent, such that the Green functions of the scalar and pseudoscalar operators are given by the response of the effective action to a local change in the quark mass matrix. In the ratio of matrix elements considered above, the renormalization scale then drops out — this ratio is a perfectly well-defined pure number, which may be estimated, e.g., by means of QCD sum rule techniques. In fact, the sum rule for the two-point functions of the pseudoscalar currents is one of the main sources of information available today for the absolute magnitude of the light quark masses. Once numerical evaluations on a lattice reach sufficiently small quark masses, this method will allow a more precise determination of low energy matrix elements involving the scalar and pseudoscalar currents.

Some authors [27] have pointed out that instantons offer a physical interpretation for the KM-ambiguity. In the field of an instanton, the Dirac operator develops discrete zero modes. As pointed out by 't Hooft [28], these modes undergo an effective interaction which indeed generates a self-energy contribution for the up-quark proportional to $m_u m_s$. The problem with this picture is that the spectrum of the Dirac operator on which it is based is fictitious. Banks and Casher [29] have shown that the spontaneous breakdown of chiral symmetry requires the small eigenvalues of the Dirac operator to be distributed uniformly: the level density $\rho(\lambda)$ approaches the value $\rho(0) = |\langle 0 | \bar{q} q | 0 \rangle| / \pi$ when $\lambda \to 0$. For the model described in [27], the spectrum instead contains an isolated zero mode, such that the level density develops a peak there, $\rho(\lambda) \propto \delta(\lambda)$. The peak may generate large flavour symmetry breaking effects, but at the same time, it shows that the model is in conflict with spontaneous chiral symmetry breakdown. The calculations carried out in [27] are based on the assumption that a framework which is unable to account for the leading term in the effective Lagrangian (the one related to the constant $B \leftrightarrow \langle 0 | \bar{q} q | 0 \rangle$) can be trusted when analyzing the higher order terms, i.e., the corrections.

There is some progress in understanding the level structure in a picture which describes the vacuum as a dense collection of instantons [27]. In that framework, the zero modes of the individual instantons merge into a continuum, such that the fictitious peak mentioned above disappears. For models with a decent spectrum, the symmetry breaking effects should be of reasonable size.

10 The $K^0 - K^+$ mass difference

As pointed out in [24], the lowest order $\chi$PT formula for the $K^0 - K^+$ mass difference disagrees with observation by a factor of four — if $m_u$ is set equal to zero. Since I did not find this puzzle discussed in the literature [23], I do it myself and first try to guess the reasoning put forward by the defence.

Any lawyer worth a fraction of his fee would immediately point out that the
problem disappears if the quark masses occurring in the lowest order formula are replaced by suitable KM-transforms $m'_u, m'_d, m'_s$. In particular, one may take $\alpha_1 = 1$ and fix $\alpha_2$ in such a manner that the ratio of the primed masses agrees with the Weinberg ratios. This amounts to the statement that the chiral perturbation series should be reordered, replacing the expansion in powers of the physical quark masses by an expansion in the primed ones. With the above specification of these, the leading term of the reordered series agrees with the experimental value of the $K^0 - K^+$ mass difference, by construction. So, the first conclusion to draw from $m_u = 0$ is that the expansion of the meson masses in powers of the physical quark masses does not make sense — $\chi$PT must be replaced by a reordered series. Why does this not take care of the problem?

The point is that the operation very strongly distorts the matrix elements of the scalar operators, which are obtained from the effective action by expanding in powers of the physical mass matrix, not by some hand made variant thereof. Consider, e.g., the scalar form factors

$$
\langle K^+|\bar{u}u - \bar{d}d|K^+\rangle = S_{K^+}(t) \quad \text{and} \quad \langle \pi^+|\bar{u}u - \bar{s}s|\pi^+\rangle = S_{\pi^+}(t)
$$

Since the operation $s \leftrightarrow d$ takes a $K^+$ into a $\pi^+$, the two form factors coincide in the SU(3) limit. The difference $S_{K^+}(t) - S_{\pi^+}(t)$ is a symmetry breaking effect of order $m_s - m_d$. According to the Feynman-Hellman theorem \cite{30}, the values of these form factors at $t = 0$ are related to the derivatives of the meson masses with respect to the quark masses. The kaon matrix element of the operator $\bar{u}u$, e.g., represents the first derivative of $M_{K^+}^2$ with respect to $m_u$, such that

$$
S_{K^+}(0) = \left( \frac{\partial}{\partial m_u} - \frac{\partial}{\partial m_d} \right) M_{K^+}^2, \quad S_{\pi^+}(0) = \left( \frac{\partial}{\partial m_u} - \frac{\partial}{\partial m_s} \right) M_{\pi^+}^2.
$$

The expansion of the meson masses in powers of $m_u, m_d, m_s$ starts with \cite{3}

$$
M_{K^+}^2 = (m_u + m_s)B\{1 + (m_u + m_s)K_3 + (m_u + m_d + m_s)K_4 + \ldots \}
$$

$$
M_{\pi^+}^2 = (m_u + m_d)B\{1 + (m_u + m_d)K_3 + (m_u + m_d + m_s)K_4 + \ldots \},
$$

where chiral logarithms and terms of $O(m^3)$ are dropped. Evaluating the derivatives, one readily establishes the low energy theorem

$$
r \equiv \frac{S_{K^+}(0)}{S_{\pi^+}(0)} = \left( \frac{m_s - m_u}{m_d - m_u} \frac{M_{K^0}^2 - M_{K^+}^2}{M_{K^0}^2 - M_{\pi^+}^2} \right)^2 \left\{ 1 + O(m^2) \right\}, \quad (21)
$$

valid up to chiral logarithms (in the present case, these are proportional to $M^2_{\pi}$ and therefore tiny, of order 1%). The theorem is of the same character as the one which leads to the elliptic constraint: the corrections are of second order in the quark masses. The relevant combination of quark masses which occurs here, however, fails to be invariant under the KM-transformation; the relation only holds if the masses entering the ratio $(m_s - m_u)/(m_d - m_u)$ are identified with the physical ones.
Suppose now that $m_u$ vanishes. The formula then predicts that the kaon matrix element is about three times smaller than the pion matrix element (the precise value of $r$ depends on the electromagnetic corrections: using the value $(M_{K^0} - M_{K^+})_{\text{QCD}} = 5.3 \text{ MeV}$, which follows from the Dashen theorem, I obtain $r = 0.30$, while the values $(M_{K^0} - M_{K^+})_{\text{QCD}} = 6.3 \text{ MeV}$ and $6.6 \text{ MeV}$ found by Donoghue et al. \cite{16} and Bijnens \cite{17} lead to $r = 0.36$ and $r = 0.38$, respectively). So, $m_u = 0$ leads to the prediction that the evaluation of the above matrix elements with sum rule or lattice techniques will reveal extraordinarily strong flavour symmetry breaking effects.

Massless up-quarks usually come in a luxurious wrapping with plausible well-known statements of general nature. Indeed, there is no logical contradiction with the proposal that (i) the expansion of the meson masses in powers of the physical quark masses fails and (ii) a suitable reordering of the series leads to meaningful results. Also, the prediction that the matrix elements of the scalar and pseudoscalar operators exhibit dramatic flavour symmetry breaking effects is not in conflict with phenomenology. Although theoretical tools such as sum rules lead to the conclusion that the picture does not make sense, these are more fragile than solid experimental facts. It is conceivable that the present example represents an exception to Einstein’s statement: ”Raffiniert ist der Herrgott, aber boshaft ist er nicht.” Even then, the literature on massless up-quarks leaves much to be desired, as it does not address any of the consequences for the low energy structure of QCD, which are rather bizarre.

11 Does the quark condensate vanish?

Let me finally turn to an entirely different picture \cite{31}, which I do not believe either, but find conceptually more interesting. The picture may be motivated by an analogy with spontaneous magnetization. There, spontaneous symmetry breakdown occurs in two quite different modes: ferromagnets and antiferromagnets. For the former, the magnetization develops a nonzero expectation value, while for the latter, this does not happen. In either case, the symmetry is spontaneously broken (for a discussion of the phenomenon within the effective Lagrangian framework, see \cite{32}). The example illustrates that operators which the symmetry allows to pick up an expectation value may, but need not do so.

The standard low energy analysis assumes that the quark condensate is the leading order parameter of the spontaneously broken symmetry, such that the Goldstone boson masses are proportional to the square root of the quark mass. The proportionality coefficient follows from the relation of Gell-Mann, Oakes and Renner \cite{33}

$$F_\pi^2 M_\pi^2 = (m_u + m_d) |\langle 0 | \bar{u} u | 0 \rangle| + \ldots$$

The simplest version of the question addressed in the papers quoted above is whether the quark condensate indeed tends to a nonzero limit when the quark masses are turned off, like the magnetization of a ferromagnet, or whether it tends to zero, as it is the case for the magnetization of an antiferromagnet. If the second option were
realized in nature, the pion mass would not be determined by the quark condensate, but by the terms of order $m^2$, which the Gell-Mann-Oakes-Renner formula neglects. More generally, one may envisage a situation, where the quark condensate is different from zero, but small, such that, at the physical value of the quark masses, the terms of order $m$ and $m^2$ both yield a significant contribution. In the literature, this framework is referred to as "Generalized $\chi$PT" [31]. The "generalized" and "ordinary" chiral perturbation theories only differ in the manner in which the symmetry breaking effects are treated.

The main problem with $G\chi$PT is that much of the predictive power of the standard framework is then lost. The most prominent example is the Gell-Mann-Okubo formula for the octet of Goldstone bosons, which does not follow within $G\chi$PT. As mentioned above, this prediction of the standard framework is satisfied remarkably well, thus supporting that picture, but it is not unfair to say that this may be a coincidence, like the $\Delta I = \frac{1}{2}$-rule, whose explanation within the Standard Model is also considerably more complex than the rule itself. The original motivation for the study of the generalized setting was the discrepancy between theory and experiment with regard to the $\pi N \sigma$-term, which indeed represented quite a fountain, continuously spitting out new ideas, one more strange than the other. In the meantime, this fountain has run dry, because the discrepancy was shown to be due to an inadequate treatment of a form factor, which enters the relation between the scattering amplitude and the $\sigma$-term matrix element [34]. From my point of view, the generalized scenario is still of some interest, mainly as a kinematical framework, which parametrizes the vicinity of the $\chi$PT predictions and allows one to judge the significance of the experimental information concerning these: the standard framework represents a special case of the generalized one. Although this is not very practical in general, one may even put the cart before the horse and claim that the predictions of $\chi$PT are based on extra assumptions, which the generalized setting does not make and which should be tested experimentally. Indeed, $\chi$PT leads to very strong predictions concerning, e.g., the scattering lengths of $\pi\pi$ scattering [35], which are sensitive to the explicit breaking of chiral symmetry generated by the quark masses and which it is important to test experimentally.

12 Summary and Conclusion

It is difficult for me to summarize the present knowledge concerning the masses of the light quarks in an unbiased manner. The following conclusions unavoidably reflect my own views and mainly rely on work done together with Gasser. These views are dyed with the prejudice that a straightforward expansion of the matrix elements in powers of the light quark masses makes sense without reordering the series and that the Gell-Mann-Oakes-Renner relation is not spoiled by higher order effects.

1. Concerning the absolute magnitude of the quark masses, the most reliable estimates are based on evaluations of QCD sum rules [36]. With the experience
Figure 2: The elliptic band shows the range permitted by the low energy theorem (14). The cross-hatched area is the intersection of this band with the sector selected by the phenomenology of $\eta\eta'$-mixing. The shaded wedge shows the constraint imposed on the ratio $R$ by the mass splittings in the baryon octet.

...gained from the various applications of these, the evaluation leads to a remarkably stable result, the visible noise generated by uncertainties of the input being quite small [37]. The main uncertainty stems from the systematic error of the method, which it is hard to narrow down. The results do not indicate a significant change as compared to the estimates given in [38]. In the long run, lattice evaluations should allow a more accurate determination, but further progress with light dynamical fermions is required before the numbers obtained can be taken at face value.

2. Chiral perturbation theory constrains the ratios $m_s/m_d$ and $m_u/m_d$ to the ellipse specified in equation (14) and displayed in figure 2 (taken from [10]). The magnitude of the semiaxis $Q$ is known to an accuracy of 10%. The value $Q = 24$ shown in the figure relies on the Dashen Theorem. The estimates given in [16, 17] indicate that this theorem receives large corrections from higher order terms, reducing the value of $Q$ by about 10%.

3. The position on the ellipse is best discussed in terms of the ratio

$$R = \frac{m_s - \hat{m}}{m_d - m_u},$$

which represents the relative magnitude of SU(3) breaking compared to isospin breaking. If $m_u$ were to vanish, this ratio would be related to the semiaxis by $R \approx Q \approx 24$ and the ratio of the strange quark mass to $\hat{m} = \frac{1}{2}(m_u + m_d)$ would be given by $m_s/\hat{m} \approx 2Q \approx 48$. The quark mass estimates obtained from QCD sum rules and from numerical evaluations on a lattice are in flat disagreement with this pattern, but confirm the standard picture, where $m_s/\hat{m} \approx Q \approx 24$.

4. The value of $R$ was investigated much before QCD had been discovered. For a long time, it was taken for granted that isospin breaking is an electromagnetic effect. The early literature on the subject is reviewed in [38]. In 1981, Gasser [39] analyzed the nonanalytic terms occurring in the chiral expansion of the octets of meson and baryon masses and demonstrated that these remove the discrepancies...
which had affected previous determinations of the ratio $R$. A thorough discussion of the available information concerning this ratio [8] showed that the data on $\Sigma^+ - \Sigma^-$, $\Xi^0 - \Xi^-$, $K^0 - K^+$ and $\rho\omega$-mixing allow four independent determinations of $R$ (in the case of the proton-neutron mass difference, the higher order corrections turn out to be large, so that this source of information does not significantly affect the overall result). Within the noise visible in the calculation, estimated at 10-15%, the results turned out to be mutually consistent. Adding the uncertainties of the individual values quadratically, the analysis implies $R = 43.5 \pm 2.2$. Discarding the information from $K^0 - K^+$ and from $\rho\omega$-mixing, one instead obtains $R = 43.5 \pm 3.2$, which corresponds to the range depicted as a shaded wedge in figure 2.

5. The above error bars rely on the prejudice that the matrix elements of the operators $\bar{u}u$, $\bar{d}d$, $\bar{s}s$ do not exhibit strong SU(3) breaking effects. These were estimated on the basis of the same rough argument, which identifies the quark mass difference $m_s - \hat{m}$ with the mass splitting within the SU(3) multiplets [4]. When analyzing the meson mass spectrum to second order in the chiral expansion [3], we realized, of course, that the mass spectrum of the Goldstone bosons does not suffice to independently determine the ratios $Q$ and $R$, but the algebraic nature of the ambiguity was pointed out only later, by Kaplan and Manohar [14]. In the opinion of some authors, this ambiguity appears to represent the most interesting aspect of the matter, indicating that the notion of quark mass is fuzzy at low energies, due to an additive renormalization problem, generated by nonperturbative effects, such as instantons, etc. The detailed discussion given above is another attempt at fighting these dangerous ogres, which the innocent observer might take for windmills (earlier attempts are described in [41, 10, 24]). Gradually, Rosinante is getting sick and tired.

6. One of the consequences we did infer from the second order analysis is that the $\eta\eta'$ mixing angle cannot be determined with the Gell-Mann-Okubo formula ($\theta_{\eta\eta'} = -10^\circ$), because there are other effects of the same algebraic order of magnitude, originating in $\mathcal{L}_{\text{eff}}^4$. Using the information about the value of $R$ discussed above, we obtained $\theta_{\eta\eta'} = -20^\circ \pm 4^\circ$. The phenomenological analysis of the decays $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$, performed independently a year later [12], neatly confirmed this prediction.

7. As noted in [4], the result for the coupling constant $L_7$, which follows from the above value of $R$, is consistent with the $\eta'$ dominance formula (8). This indicates that the sum rule used in the derivation of that formula is nearly saturated by the $\eta'$ intermediate state also in the real world, not only in the large $N_c$ limit, where the formula is exact.

8. Turning the argument around, the phenomenology on the mixing angle available today [13] may be used to determine the quark mass ratios. This is indicated by the dashed straight lines in figure 2. Since the data agree with the prediction, it is clear that the information about the mass ratios obtained in this manner is consistent with the one extracted from isospin breaking in the baryon octet. Note, however, that the phenomenological analysis of the $\eta$ and $\eta'$ decays relies on large $N_c$ arguments. Although the resulting picture yields a coherent understanding of
quite a few processes [44, 45], the validity of these arguments needs to be examined more carefully to arrive at firm conclusions [16].

9. The branching ratio $\Gamma(\psi' \rightarrow \psi \pi^0)/\Gamma(\psi' \rightarrow \psi \eta)$ provides further information about $R$. Using the lowest order formula, the data available at the time gave $R = 28^{+7}_{-4}$ [38]. The formula is valid up to SU(3) breaking effects; according to the general rule of thumb, these are expected to be of order 20 or 30%. Since we saw no way of evaluating these, the information derived from this branching ratio did not significantly affect our analysis and was discarded. Meanwhile, the data are slightly more precise [4]; the lowest order formula now yields $R = 31 \pm 4$. Furthermore, Donoghue and Wyler [47] have investigated the second order corrections, using the multipole expansion, which relates the relevant matrix elements to $\langle 0| G_{\mu\nu} \tilde{G}^{\mu\nu} |\pi^0 \rangle$, $\langle 0| G_{\mu\nu} \tilde{G}^{\mu\nu} |\eta \rangle$. As it is the case with the sum rule used to estimate $L_7$, these matrix elements involve pseudoscalar operators and do thus not suffer from the KM-ambiguity. The calculation yields remarkably small SU(3)-breaking effects, such that the value obtained for $R$ remains close to $31 \pm 4$.

The problems faced by this attempt at estimating $R$ are listed in [24]. In particular, as discussed in detail in [48], it is not clear that the relevant matrix elements may be replaced by those of the operator $G_{\mu\nu} \tilde{G}^{\mu\nu}$. The calculation is of interest, because it is independent of other determinations, but at the present level of theoretical understanding, it is subject to considerable uncertainties. In view of these, it appears to me that the numerical result is perfectly consistent with the value $R = 43.4$, which follows from the Weinberg ratios.

As evidenced by the example of $\eta \eta'$ mixing and quite a few others, the estimates of the effective coupling constants which follow from the hypothesis that the low energy structure is dominated by the singularities due to the lowest lying levels lead to a rather predictive framework, which until now has passed all experimental tests. It would be of interest to estimate the symmetry breaking effects in the branching ratio $\Gamma(\psi' \rightarrow \psi \pi^0)/\Gamma(\psi' \rightarrow \psi \eta)$ on the basis of this hypothesis and to see whether they indeed increase the value of $R$ obtained with the lowest order formula. If not, there would be a problem with the simple picture I am advocating here.

10. There are examples, where the higher order corrections turn out to be large, such as $\eta \rightarrow 3\pi$ or scalar form factors. In all cases I know of, one can put the finger on the culprit responsible for the enhancement. Invariably, the problem arises, because the perturbations generated by the quark mass term are enhanced by small energy denominators. In particular, the enhancement due to strong final state interactions in the S-waves often generates sizeable corrections, which are perfectly well understood — $\chi$PT itself predicts how large the S-wave phase shifts are near threshold and that they rapidly grow with energy. Once the origin of the phenomenon is understood, one may take it into account and arrive at a reliable low energy representation, even if the straightforward chiral expansion thereof contains relatively large contributions from higher order terms.

11. In the case of the mass formulae, the low energy singularities do not generate a significant enhancement of the higher order contributions. Accordingly, the esti-
mates of the effective coupling constants mentioned above imply that the Weinberg ratios only receive small corrections. There is some evidence for a 10% decrease in $Q$, related to the e.m. self energies: The Dashen theorem is suspected to receive large corrections, because some of the low energy singularities do generate large symmetry breaking effects in that case.

12. Although the above hypothesis is the most natural setting for an effective theory I can think of, it evidently involves assumptions which go beyond pure symmetry and phenomenology. One may dismiss this hypothesis and the estimates for $R$ obtained with it, requiring only that the mass term of the light quarks, which breaks the chiral symmetry of the QCD Hamiltonian, may be treated as a perturbation. With $m_u = 0$, the lowest order formula for $M_{K^0} - M_{K^+}$ is off by a factor of four, in flat contradiction with this requirement. The disaster can be avoided only if the factor $(m_d - m_u)/(m_d + m_u)$ accounts for at least half of the discrepancy, $(m_d - m_u)/(m_d + m_u) < \frac{1}{2}$. This alone yields $m_u/m_d > \frac{1}{3}$.

13. Accordingly, if $m_u$ is assumed to be equal to zero, $\chi$PT must be thrown overboard. The expansion in powers of the quark masses does then not make sense and the matrix elements of the scalar and pseudoscalar operators must exhibit extraordinarily strong SU(3) breaking effects. These cannot be explained with the low energy singularities listed in the particle data tables. As far as I know, the instanton model of Choi et al. [27] is the only theoretical scenario proposed to substantiate the claim that QCD may give rise to strong flavour symmetry breakings of this type. There, the occurrence of discrete zero modes does indeed produce such effects. The same feature, however, is the reason why the model is inconsistent with spontaneously broken chiral symmetry. Dilute instanton coffee is undrinkable. Bold discussion remarks may warm it up a little, but that merely intensifies the unacceptable flavour characteristics of the brewing.

I am indebted to Hans Bijnens, Jürg Gasser, Marina Nielsen and Daniel Wyler for valuable comments.

References

[1] S. Weinberg, *Physica* A96 (1979) 327.

[2] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.*) 158 (1984) 142.

[3] J. Gasser and H. Leutwyler, *Nucl. Phys.* B250 (1985) 465, 517, 539.

[4] The foundations of the method are discussed in detail in H. Leutwyler, *Ann. Phys. (N.Y.*) 235 (1994) 165.

[5] Review of Particle Properties, *Phys. Rev.* D45 (1992).

[6] S.R. Amendolia et al., *Nucl. Phys.* B277 (1986) 168.
[7] H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, Menlo Park, 1984); H. Georgi and A. Manohar, *Nucl. Phys.* B234 (1984) 189.

[8] M. Soldate and R. Sundrum, *Nucl. Phys.* B340 (1990) 1; R.S. Chivukula, M.J. Dugan and M. Golden, *Phys. Rev.* D47 (1993) 2930.

[9] G. Ecker et al., *Nucl. Phys.* B321 (1989) 311; *Phys. Lett.* B223 (1989) 425.

[10] H. Leutwyler, *Nucl. Phys.* B337 (1990) 108.

[11] H. Leutwyler, *Phys. Lett.* B48 (1974) 431; *Nucl. Phys.* B76 (1974) 413; J. Gasser and H. Leutwyler, *Nucl. Phys.* B94 (1975) 269.

[12] S. Weinberg, in *A Festschrift for I.I. Rabi*, ed. L. Motz (New York Acad. Sci., 1977) p. 185.

[13] R. Dashen, *Phys. Rev.* 183 (1969) 1245.

[14] D. B. Kaplan and A. V. Manohar, *Phys. Rev. Lett.* 56 (1986) 2004.

[15] P. Langacker and H. Pagels, *Phys. Rev.* D8 (1973) 4620; K. Maltman and D. Kotchan, *Mod. Phys. Lett.* A5 (1990) 2457; G. Stephenson, K. Maltman and T. Goldman, *Phys. Rev.* D43 (1991) 860.

[16] J. Donoghue, B. Holstein and D. Wyler, *Phys. Rev.* D47 (1993) 2089.

[17] J. Bijnens, *Phys. Lett.* B306 (1993) 343.

[18] J. Donoghue, B. Holsten and D. Wyler, *Phys. Rev. Lett.* 69 (1992) 3444.

[19] J. Donoghue, Lectures given at the Theoretical Advanced Study Institute (TASI), Boulder, Colorado (1993).

[20] D. Wyler, Proc. XVI Kazimierz Meeting on Elementary Particle Physics, eds. Z. Ajduk et al., World Scientific (1994).

[21] N. Khuri and S. Treiman, *Phys. Rev.* 119 (1960) 1115; C. Roiesnel and T.N. Truong, *Nucl. Phys.* B187 (1981) 293.

[22] A. V. Anisovich, "Dispersion relation technique for three-pion system and the P-wave interaction in $\eta \rightarrow 3\pi$ decay", preprint Petersburg Nuclear Physics Institute, Gatchina TH-62-1993/1931; J. Kambor, C. Wiesendanger and D. Wyler, in preparation; A. V. Anisovich and H. Leutwyler, in preparation.

[23] For a review, see T. Banks, Y. Nir and N. Seiberg, in these Proceedings.
[24] H. Leutwyler, Proc. XXVI Int. Conf. on High Energy Physics, Dallas, Aug. 1992, ed. J. R. Sanford, AIP Conf. Proc. No. 272, (1993).

[25] H. Leutwyler and A. Smilga, *Phys. Rev.* D 46 (1992) 5607;
A. Smilga and J. Stern, *Phys. Lett.* B318 (1993) 531.

[26] E. Shuryak and J. Verbaarschot, *Nucl. Phys.* B 410 (1993) 37;
E. Shuryak and J. Verbaarschot, *Nucl. Phys.* A 560 (1993) 306;
J. Verbaarschot and I. Zahed, *Phys. Rev. Lett.* 70 (1993) 3852;
J. Verbaarschot, *Nucl. Phys.* B 426 (1994) 559; B 427 (1994) 534;
*Phys. Rev. Lett.* 72 (1994) 2531; *Phys. Lett.* B 329 (1994) 351;
A. Smilga and J. Verbaarschot, "Spectral sum rules and finite volume partition function in gauge theories with real and pseudoreal fermions", preprint Univ. Minnesota, TPI-MINN-94/10-T (1994).

[27] K. Choi, C.W. Kim and W.K. Sze, *Phys. Rev. Lett.* 61 (1988) 794;
K. Choi and C.W. Kim, *Phys. Rev.* D40 (1989) 890;
K. Choi, *Nucl. Phys.* B383 (1992) 58; *Phys. Lett.* B292 (1992) 159.

[28] G. t’Hooft, *Phys. Rev.* D14 (1976) 3432.

[29] T. Banks and A. Casher, *Nucl. Phys.* B169 (1980) 103.

[30] H. Hellman, "Einführung in die Quantenchemie", Deuticke, Leipzig (1937);
R. P. Feynman, *Phys. Rev.* 66 (1939) 340.

[31] For a recent exposition of $\chi$PT, see
J. Stern, H. Sazdjan and N. H. Fuchs, *Phys. Rev.* D47 (1993) 3814;
M. Knecht et al., *Phys. Lett.* B313 (1993) 229.
The early literature is reviewed in
M. D. Scadron, Rep. Prog. Phys. 44 (1981) 213.

[32] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Phys. Rev.* B48 (1993) 3190;
H. Leutwyler, *Phys. Rev.* D49 (1994) 3033.

[33] M. Gell-Mann, R. J. Oakes and B. Renner, *Phys. Rev.* 175 (1968) 2195.

[34] J. Gasser, H. Leutwyler and M. Sainio, *Phys. Lett.* B253 (1991) 260.

[35] J. Gasser and H. Leutwyler, *Phys. Lett.* B125 (1983) 321, 325.

[36] A. I. Vainshtein et al., *Sov. J. Nucl. Phys.* 27 (1978) 274;
B. L. Ioffe, *Nucl. Phys.* B188 (1981) 317; B191 (1981) 591(E).

[37] The literature may be traced with the following papers:
C. Adami, E. G. Drukarev and B. L. Ioffe *Phys. Rev.* D48 (1993) 2304;
V. L. Eletsky and B. L. Ioffe, *Phys. Rev.* D48 (1993) 1441;
C. A. Dominguez, C. Van Gend and N. Paver, *Phys. Lett.* B253 (1991) 241;
S. Narison, *Phys. Lett.* B216 (1989) 191;
C. A. Dominguez and E. de Rafael, *Ann. Phys.* 174 (1986) 372;
X. Jin, M. Nielsen and J. Pasupathy, “Calculation of $\langle p\bar{u}u-\bar{d}d\rangle$ from QCD sum rule and the neutron-proton mass difference”, [hep-ph/9405202](https://arxiv.org/abs/hep-ph/9405202).

[38] J. Gasser and H. Leutwyler, *Phys. Reports* 87 (1982) 77.

[39] J. Gasser, *Ann. Phys. (N.Y.)* 136 (1981) 62.

[40] The chiral logarithms occurring in the expansion are worked out in R. F. Lebed and M. A. Luty, *Phys. Lett.* B 329 (1994) 479.

[41] M. de Cervantes, ”Don Quixote”, part I, chapter 8, Madrid (1605).

[42] J. F. Donoghue, B. R. Holstein and Y. C. R. Lin, *Phys. Rev. Lett.* 55 (1985) 2766.

[43] F. Gilman and R. Kauffmann, *Phys. Rev.* D36 (1987) 2761;
Riazuddin and Fayyazuddin, *Phys. Rev.* D37 (1988) 149;
ASP Collaboration, N. A. Roe et al., *Phys. Rev.* D41 (1990) 17.

[44] J. Bijnens, A. Bramon and F. Cornet, *Z. Phys.* C46 (1990) 599.

[45] The DAFNE Physics Handbook, eds. L. Maiani, G. Pancheri and N. Paver, INFN–Frascati (1992).

[46] G. M. Shore and G. Veneziano, *Nucl. Phys.* B381 (1992) 3.

[47] J. Donoghue and D. Wyler, *Phys. Rev.* D45 (1992) 892.

[48] K. Gottfried, *Phys. Rev. Lett.* 40 (1978) 598;
Y. P. Tung and T. N. Yan, *Phys. Rev.* D41 (1990) 155;
M. Luty and R. Sundrum, *Phys. Lett.* B312 (1993) 205.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405330v2