Nucleon electromagnetic form factors in a quark-gluon core model

Xian-Qiao Yu

School of Physical Science and Technology, Southwest University, Chongqing 400715, China

We study the nucleon electromagnetic form factors in a quark-gluon core model framework, which can be viewed as an extension of the Isgur-Karl model of baryons. Using this picture we derive nucleon electromagnetic dipole form factors at low $Q^2$ and the deviation from the dipole form at high $Q^2$, that are consistent with the existing experimental data.

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The nucleon elastic electromagnetic form factors (FFs) are very important for understanding the dynamics of the nucleons' constituents. There has been much activity in the measurement of proton and neutron elastic electromagnetic FFs in the last decade. High accuracy experimental data on nucleon electromagnetic FFs obtained in recent years [1, 2, 3, 4] indicate that nucleon electromagnetic FFs can be well fitted by a simple dipole formula at low $Q^2$:

$$G_{E}^{p} = G_{M}^{p}/\mu_{p} = G_{M}^{n}/\mu_{n} = 1/(1 + Q^2/0.71\text{GeV}^2)^2, \quad (1)$$

here $Q^2 = -q^2$, $q$ is momentum transfer in elastic electron-nucleon scattering and $\mu_p$ and $\mu_n$ are magnetic moments of proton and neutron respectively. This has spurred a significant reevaluation of the nucleon and pictures of its structure [3].

Starting from the relation between nucleon electromagnetic FFs and nucleon intrinsic charge (magnetic moment) density distributions, this note will give a possible origin of the dipole FF. Strict analysis of the dynamics of nucleon constituents should start from QCD. Because the complication of QCD non-perturbation, various phenomenological models are developed. Quark potential model [6, 7, 8] treats the nucleon as three constituent quarks bound state, the effect of gluons is buried within constituent quarks, which are considered as quasi-particles. Hadrons bag model [9, 10, 11, 12] assumes the nucleons’ three non-interacting quarks are confined in a bag of finite dimension. These models have absorbed or ignored other degrees of freedom beyond the three quarks. If we take into account the gluon degree of freedom in nucleon and assume gluons in nucleon contract under their own strong self-interactions, we find a different method of describing nucleon from which the dipole FF can be derived easily.

The idea of this picture about nucleon structure comes from a comparison between nucleon and triatomic molecule. We suppose a quark at a place in space, unlike its electric charge located at the definite place where it is, its color charge will diffusely spread out around it due to gluon emission and absorption, this leads to that most of the quark’s color charge is carried by the gluon cloud around it. Comparing nucleon with triatomic molecule, three valence quarks corresponding approximately to three atomic nuclei, the gluon cloud around valence quark is just like electron cloud around atomic nucleus. In the central core of three valence quarks, gluon cloud will overlap and become dense. We may assume that the dense gluon cloud will contract under its own strong interaction to a compact gluon cluster (Boros et al. have suggested virtual gluon clusters exist in nucleon in reference [13], where gluon clusters mean a group of gluons), three light valence quarks, part of their color charge have been transferred to the compact gluon cluster, moving around the nucleus composed of gluons. The spin-independent interaction between one valence quark and the gluon nucleus take the following form:

$$V = -\alpha/r + V_{conf}, \quad (2)$$

where $\alpha$ is a positive constant and $V_{conf}$ is the confining potential. We call this picture quark-gluon core structure model of nucleon, as shown in Fig. 1.

We find that the above hypothesis combined with quantum mechanics appears sufficient for the derivation of nucleon electromagnetic dipole FFs at low $Q^2$. In the following I shall sketch the derivation briefly.

We assume that the mass of the gluon cluster is much larger than that of the quark, in this case, setting the gluon nucleus at the origin of coordinate, we write the non-relativistic Hamiltonian for the system as

$$H = H_0 + H_{pert}, \quad (3)$$
FIG. 1: The configuration for quark-gluon core structure model of nucleon

where

\[ H_0 = -\frac{\hbar^2}{2m} \nabla_1^2 - \frac{\hbar^2}{2m} \nabla_2^2 - \frac{\hbar^2}{2m} \nabla_3^2 - \frac{\alpha}{r_1} - \frac{\alpha}{r_2} - \frac{\alpha}{r_3}, \]  

(4)

and

\[ H_{\text{pert}} = \frac{\beta}{r_{12}} - \frac{\beta}{r_{13}} - \frac{\beta}{r_{23}} + \sum_{i<j}(V_{i,j}^{\text{conf}} + V_{i,j}^{\text{hyp}}), \]  

(5)

\[ V_{i,j}^{\text{hyp}} = \frac{2\alpha_s}{3m_i m_j} \left[ \frac{8\pi}{3} \hat{S}_i \cdot \hat{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} \left( 3\hat{S}_i \cdot \vec{r}_{ij} \hat{S}_j \cdot \vec{r}_{ij} - \hat{S}_i \cdot \hat{S}_j \right) \right], \]  

(6)

but its form between the valence quark and the gluon core is unknown. If we take a harmonic oscillator type confining potential, the picture described here can be viewed as an extension of the Isgur-Karl model\[\text{[14]}\]. We can use it to study the masses of baryons, which I hope to discuss in a separate paper. Here we are interested in nucleon electromagnetic form factors. The eigenstates of the Hamiltonian\[\text{[4]}\] are well known. For nucleon, all the three valence quarks in the \(1s\) state, the ground state wave function is

\[ \psi_0(r_1, r_2, r_3) = \psi_{100}(r_1)\psi_{100}(r_2)\psi_{100}(r_3) = \frac{b_0^{9/2}}{\pi^{3/2}} \exp \left[ -b_0(r_1 + r_2 + r_3) \right], \]  

(7)

where \(b_0 = \alpha m/\hbar^2\). It is impossible to solve accurately the eigen-wave functions of Hamiltonian\[\text{[3]}\]; we assume that the approximate ground state wave function of Hamiltonian\[\text{[3]}\] has the same form as Eq.(7), that is

\[ \psi(r_1, r_2, r_3) = \frac{b^{9/2}}{\pi^{3/2}} \exp \left[ -b(r_1 + r_2 + r_3) \right], \]  

(8)

where \(b = \alpha' m/\hbar^2\) and \(\alpha'\) is a effective coupling constant. A direct result of Eq.(8) is that the electric charge and magnetic moment density distributions in nucleon are a function of exponential type

\[ \rho(r) = \frac{b^3}{\pi} \exp[-2br]. \]  

(9)

After carrying through Fourier transformation

\[ F(q^2) = \frac{4\pi}{q} \int \rho(r) \sin(qr) r dr, \]  

(10)

we get nucleon electromagnetic form factors.
\[ G_E^p(Q^2) = \frac{G_E^p(Q^2)}{\mu_p} = \frac{G_M^p(Q^2)}{\mu_p} = G_D(Q^2), \]
\[ G_E^n(Q^2) = 0, \]
where
\[ G_D(Q^2) = \frac{1}{(1 + Q^2/4b^2)^2}, \]
which is dipole formula supported by many experiments. We notice that a small but definite deviation from zero is observed for the neutron electric form factor \( G_E^n \) at low \( Q^2 \) by recent high-precision data from double-polarization measurements, which can be explained by the small mass and interaction differences between up quark and down quark in our model. Look back to Eq. (4), where we have assumed up quark and down quark to have the same mass and coupling constant. Denoting \( m_u \) and \( m_d \) as the mass of up quark and down quark, respectively, \( \alpha_u \) and \( \alpha_d \) represent respectively the effective coupling constant involving of \( u \) and \( d \) quark, and we have the following expression for the neutron electric form factor
\[ G_E^n(Q^2) = \frac{2}{3} \left[ \frac{1}{(1 + Q^2/4b_u^2)^2} - \frac{1}{(1 + Q^2/4b_d^2)^2} \right], \]
where \( b_u = \alpha_u' m_u / \hbar^2 \) and \( b_d = \alpha_d' m_d / \hbar^2 \). If we assume that \( m_u = m_d = m \) and \( \alpha_u' = \alpha_d' = \alpha' \), we find that constraints from dipole formula and energy spectra of nucleons suggest \( \alpha' = 3.17 \) and \( m = 133 MeV \). Supposing that the mass difference between up quark and down quark is 7 MeV (we set \( m_d = 133 MeV \) and \( m_u = 140 MeV \)) and the effective coupling constant symmetry breaking is about 9% (we set \( \alpha'_d = 3.17 \) and \( \alpha'_u = 3.47 \)), we get the neutron electric form factor \( G_E^n \) as shown in Fig. 2, which is consistent with the existing experimental data.

![Fig. 2: The neutron electric form factor \( G_E^n \) as a function of \( Q^2 \). Data are from references.](image)

In the above discussions, we interpret the Fourier transforms of nucleon charge(magnetization) densities as the electromagnetic FFs. This identification is only appropriate for a non-relativistic(static) system. However, if the wavelength of the probe is much larger than the Compton wavelength of the nucleon with mass \( M_N \), i.e. if \( |Q^2| \gg M_N^2 \), one needs to take the effect of relativity into account and consequently the physical interpretation of the FFs becomes complicated. Recently, Kelly has used a relativistic prescription to relate the electromagnetic FFs to the nucleon charge and magnetization densities, accounting for the Lorentz contraction of the densities in the Breit frame relative to the rest frame. We follow this treatment in reference to give the nucleon electromagnetic FFs at high \( Q^2 \).

Let \( \rho_{ch}(r) \) and \( \rho_m(r) \) represent the spherical charge and magnetization densities in the nucleon rest frame, the related intrinsic FFs can be obtained through a Fourier-Bessel transform as
\[ \tilde{\rho}(k) = \int_0^\infty dr r^2 j_0(kr) \rho(r), \]
with \( k \equiv |q| \) being the wave vector in the nucleon rest frame. At low \( Q^2 \), the nucleon is a non-relativistic system and the intrinsic FFs are just the the electromagnetic FFs \( G_E(Q^2) \) and \( G_M(Q^2) \) (called Sachs FFs in the literature) that have been discussed above. However, at high \( Q^2 \) where the nucleon moves with velocity \( v = \sqrt{\tau/(1 + \tau)} \) relative to the rest frame, here \( \tau = Q^2/4M_N \), a Lorentz boost with \( \gamma^2 = (1 - v^2)^{-1} = 1 + \tau \) is involved\[4\]. This Lorentz boost leads to a contraction of the nucleon densities as seen in the Breit frame and hence the intrinsic FFs defined by Eq.\[15\] needs to replace \( k^2 \) with \( Q^2/(1 + \tau) \).

The relativistic relationships between the intrinsic FFs \( \tilde{\rho}(k) \) and the Sachs FFs \( G(Q^2) \) measured by electron scattering at finite \( Q^2 \) are not unambiguous. There exist different prescriptions in the literature which can be written in the form

\[
\tilde{\rho}_{ch}(k) = G_E(Q^2)(1 + \tau)^{\lambda_E}, \tag{16}
\]

\[
\mu_N\tilde{\rho}_m(k) = G_M(Q^2)(1 + \tau)^{\lambda_M}, \tag{17}
\]

where \( \lambda \) is a model-dependent constant. To account for the asymptotic \( 1/Q^4 \) FFs obtained by the perturbative QCD at very large \( Q^2 \), Mitra and Kumari\[27\] proposed the choice \( \lambda_E = \lambda_M = 2 \). Following this choice we calculate the nucleon electromagnetic FFs at very high \( Q^2 \) as shown in Fig. 3.

![FIG. 3: The proton electric form factor \( G_E^p \) in unit of \( G_D \) and the nucleon magnetic form factor \( G_M \) in units of \( \mu_N G_D \) as a function of \( Q^2 \).](image)

From Fig. 3 we can see that \( G_E^p/G_D = G_M/\mu_N G_D \approx 0.7 \) at high \( Q^2 \), which is consistent with the existing experimental data for \( G_E^p/\mu_p G_D \) in the range of momentum transfer from \( Q^2 = 19.5 \) to \( 31.3 \) (GeV/c)^2\[28\]. The experimental data at higher \( Q^2 \) values is not available nowadays and will become available in the near future that will provide a critical test of our calculations.

If we choose \( \lambda_E = 1.9 \) and \( \lambda_M = 2 \), we obtain \( \mu_p G_E^p(Q^2) \approx 0.4\text{GeV}^4 \) and \( \mu_n G_M^p(Q^2) \approx 0.3\text{GeV}^4 \) at \( Q^2 \approx 10 - 30\text{GeV}^2 \), that are consistent with the perturbative QCD results calculated by Chernyak and Zhitnitsky\[29\]. They also predicted that \( \mu_p G_M^p(Q^2) \) \( \to 0 \) at \( Q^2 \to \infty \)\[29\], which disagrees with the predictions of Eq.\[17\] for the choice \( \lambda_E = 1.9 \) and \( \lambda_M = 2 \). Considering that the unique relativistic relationships between the intrinsic FFs \( \tilde{\rho}(k) \) and the Sachs FFs \( G(Q^2) \) do not exist, there might be large uncertainties in Eq.\[17\] at \( Q^2 \to \infty \). Eventually the uncertainties can be extracted in the future form factor measurements at higher \( Q^2 \) values.

We consider the gluon core inside nucleon as a quasi-particle. Further, studies along this line will show the properties of such quasi-particle, for example its spin. These properties are helpful information for our quantitative understanding of nucleon.

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