Hybrid Block Method Algorithms for Solution of First Order Initial Value Problems in Ordinary Differential Equations

Ajileye G*, Amoo SA and Ogwumu OD

Department of Mathematics and Statistics, Federal University Wukari, Wukari, Taraba State, Nigeria

Abstract

In this paper, we consider the derivation of hybrid block method for the solution of general first order Initial Value Problem (IVP) in Ordinary Differential Equation. We adopted the method of Collocation and Interpolation of power series approximation to generate the continuous formula. The properties and feature of the method are analyzed and some numerical examples are also presented to illustrate the accuracy and effectiveness of the method.

Keywords: Collocation; Interpolation; Linear multistep method; Hybrid and power series polynomial

Introduction

In recent times, the integration of Ordinary Differential Equations (ODEs) is carried out using some kinds of block methods. In this paper, we propose an order six block integrator for the solution of first-order ODEs of the form:

\[ y' = f(x, y), \quad y(a) = y_a, \quad x \in [a, b] \]  

(1)

where \( y \) is continuous within the interval of integration \([a, b]\). We assume that \( f \) satisfies Lipschitz condition which guarantees the existence and uniqueness of solution of eqn. (1). For the discrete solution of (1) by linear multi-step method has been studied by authors like [1] and continuous solution of eqns. (1) and [2-4]. One important advantage of the continuous over discrete approach is the ability to provide discrete schemes for simultaneous integration. These discrete schemes can be reformulated as general linear methods (GLM) [5]. The block methods are self-starting and can be applied to both stiff and non-stiff initial value problem in differential equations. More recently, authors like [6-10] and to mention few, these authors proposed methods ranging from predictor- corrector to hybrid block method for initial value problem in ordinary differential equation.

In this work, hybrid blocks method with two off-grid using Power series expansion [11,12]. This would help in coming up with a more computationally reliable integrator that could solve first order differential equations problems of the form eqn. (1).

Derivation of Hybrid Method

In this section, we intend to construct the proposed two-step LMMs which will be used to generate the method. We consider the power series polynomial of the form:

\[ P(x) = \sum_{j=0}^{n} a_j x^j \]  

(2)

which is used as our basis to produce an approximate solution to (1.0) as

\[ y(x) = \sum_{j=0}^{m+t-1} a_j x^j \]  

(3)

And

\[ y'(x) = \sum_{j=0}^{m+t-1} j a_j x^{j-1} = f(x, y) \]  

(4)

where \( a \) are the parameters to be determined, \( m \) and \( t \) are the points of collocation and interpolation respectively. This process leads to \((m+t-1)\) unknown coefficients, which are to be determined by the use of Maple 17 Mathematical software.

Hybrid Block Method

Using eqns. (3) and (4), \( m=1 \) and \( t=5 \) our choice of degree of polynomial is \((m+t-1)\). Eqn. (3) interpolates at the point \( x=x_n \) and eqn. (4) is collocated at \( x=0, \frac{1}{2}, \frac{3}{2}, 2 \) which lead to system of equation of the form

\[ \sum_{j=0}^{m+t-1} a_j x_n^j = y_{n+1} \quad i = 0 \]  

(5)

\[ \sum_{j=0}^{m+t-1} j a_j x_n^j = f_{n+1} \quad i = 0, \frac{1}{2}, \frac{3}{2}, 2 \]  

(6)

With the mathematical software, we obtain the continuous formulation of eqns. (5) and (6) of the form

\[ y(x) = a_0 y_n + h \left[ \beta_0 f_n + \beta_1 f_{n+1} + \beta_2 f_{n+2} + \beta_3 f_{n+3} + \beta_4 f_{n+4} + \beta_5 f_{n+5} \right] \]  

(7)

After obtaining the values of \( a \) and \( \beta_i = 0 \) and \( i = \left( 0, \frac{1}{2}, \frac{3}{2}, 2 \right) \) in eqn. (7)

We evaluated at the point \( x = x_{n+1} \), \( f = \left( 0, \frac{1}{2}, \frac{3}{2}, 2 \right) \) which gives the following set of discrete schemes to form our hybrid block method.

*Corresponding author: Ajileye G, Department of Mathematics and Statistics, Federal University Wukari, Wukari, Taraba State, Nigeria, Tel: 2348034906427; E-mail: ajisco4live@yahoo.com
Received April 16, 2018; Accepted May 30, 2018; Published June 19, 2018

Citation: Ajileye G, Amoo SA, Ogwumu OD (2018) Hybrid Block Method Algorithms for Solution of First Order Initial Value Problems in Ordinary Differential Equations. J Appl Comput Math 7: 390. doi: 10.4172/2168-9679.1000390

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The hybrid block method is zero stable.

Where

\[ A^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B^{(0)} = \begin{bmatrix} 29 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

and

\[ A^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B^{(1)} = \begin{bmatrix} 180 \\ 1440 \\ 270 \\ 720 \end{bmatrix} \]

The first characteristic polynomial of the scheme is

\[ \rho(\lambda) = \det[\lambda A^{(0)} - A^{(1)}] \]

\[ \rho(\lambda) = \det \begin{bmatrix} \lambda & 0 & 0 & -1 \\ 0 & \lambda & 0 & -1 \\ 0 & 0 & \lambda & -1 \\ 0 & 0 & 0 & \lambda & -1 \end{bmatrix} \]

\[ \lambda = \lambda_1 = \lambda_2 = 0 \text{ or } \lambda = 1 \]

We can see clearly that no root has modulus greater than one (i.e. \( \lambda \leq 1 \)) \( \forall i \). The hybrid block method is zero stable.

**Numerical Examples**

**Problem 1**

\[ y' = y, \quad y(0) = 1, \ h = 0.1 \]

**Exact solution**

\[ y(x) = \exp(x) (\text{Table 1}). \]

**Problem 2**

\[ y' = 0.5(1 - y), \quad y(0) = 0.5, h = 0.1 \]

**Exact solution**

\[ y(x) = 1 - 0.5e^{-0.5x} (\text{Table 2}). \]

**Discussion of Result**

We observed that from the two problems tested with this proposed...
block hybrid method the results converges to exact solutions and also compared favorably with the existing similar methods (see Tables 1 and 2).

Conclusion

In this paper, we have presented Hybrid block method algorithm for the solution of first order ordinary differential equations. The approximate solution adopted in this research produced a block method with stability region. This made it to perform well on problems. The block method proposed was found to be zero-stable, consistent and convergent.

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| x    | Exact Solution | Scheme           | Error in Scheme | Error [2] |
|------|----------------|------------------|-----------------|-----------|
| 0.1  | 1.105170918075648 | 1.105170917860730 | 2.149179E-10    | 1.22622039551945e-05 |
| 0.2  | 1.221402758160170  | 1.221402757765120  | 4.7505E-10      | 1.35518338201958e-05 |
| 0.3  | 1.349858680776603  | 1.349858680768490  | 7.875129E-10    | 1.49770979570133e-05 |
| 0.4  | 1.491824697641270  | 1.491824696480820  | 1.16045E-09     | 1.655225270247307e-05 |
| 0.5  | 1.648721270700128  | 1.648721269097010  | 1.603118E-09    | 1.82930683154669e-05 |
| 0.6  | 1.822118800390509  | 1.822118798264440  | 2.126069E-09    | 2.021696710463598e-05 |
| 0.7  | 2.013752707470747  | 2.013752704742920  | 2.741277E-09    | 2.234320409576696e-05 |
| 0.8  | 2.225409249249248  | 2.225409250303090  | 5.989459E-09    | 2.469305938346267e-05 |
| 0.9  | 2.459603111111111  | 2.459603106852120  | 4.30483-09      | 2.729005110868599e-05 |
| 1.0  | 2.718218218218218  | 2.718218218218218  | 4.337016E-09    | 3.01601708373664e-05 |

| x    | Exact Solution | Scheme           | Error in Scheme | Error [7] |
|------|----------------|------------------|-----------------|-----------|
| 0.1  | 0.524385287749643 | 0.524385287750861 | 1.218026E-13    | 5.574430e-012  |
| 0.2  | 0.547581290982020 | 0.547581290981880 | 1.399991E-12    | 3.946177e-012  |
| 0.3  | 0.569646011787471 | 0.569646011786286 | 1.184941E-12    | 8.13832e-012   |
| 0.4  | 0.590634623461009 | 0.590634623462548 | 1.538991E-12    | 3.436118e-011   |
| 0.5  | 0.610996684642927 | 0.61099668463187  | 1.110000E-12    | 1.929743e-010   |
| 0.6  | 0.629590889659141 | 0.629590889658614 | 5.270229E-12    | 1.879040e-010   |
| 0.7  | 0.647655955140643 | 0.647655955142752 | 2.10989E-12     | 1.776335e-010   |
| 0.8  | 0.664839976982180 | 0.664839976996201 | 1.297895E-11    | 1.724676e-010   |
| 0.9  | 0.681185924189113 | 0.681185924158290 | 3.06229E-11     | 1.847545e-010   |
| 1.0  | 0.696734670143683 | 0.696734670139561 | 4.121925E-11    | 3.005770e-010   |

Table 1: Comparison of approximate solution of problem 1.

Table 2: Comparison of approximate solution of problem 2.