A Method for Calculating Dynamic Parameters of Inter-satellite Link Signals

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Abstract: Aiming at the complex dynamic changes of inter-satellite link signals, this paper proposes a low-complexity method to calculate dynamic parameters of inter-satellite link signals so as to simulate inter-satellite link signals with complex dynamic characteristics. Based on the precise ephemeris, the algorithm is used to calculate the transmission delay and Doppler frequency of the signals in an inertial frame of reference by using iteration and interpolation. The calculation result is compared with the result obtained by using the simulation software of the global navigation system. It is found that the error of the transmission delay is at the nanosecond level and the error of Doppler frequency is at the Hertzian level. Therefore, the dynamic signal simulation accuracy can meet the requirements of load testing and verification of inter-satellite links. The algorithm is simple to implement.

1. Introduction
As the inter-satellite link technique develops, there is an increasing demand for establishing links between satellites for information transmission. The loads of inter-satellite links can help build the links between satellites and links between a satellite and an earth station, used for satellite measurement and control and data transmission. To test and verify the loads of inter-satellite links during the development process, it is necessary to simulate and generate inter-satellite link signals with real dynamic characteristics.

As satellites orbit around the Earth, the motion pattern is variable and the relative motion between satellites is complex. In addition, inter-satellite link signals are highly dynamic, resulting in a large uncertainty in the time of arrival of signals and a large variation in Doppler frequency [¹]. Satellites need to have a certain capability of receiving signals to effectively receive inter-satellite link signals. Currently, the 54th Institute of Electronic Science and Technology of China (ISTC) has proposed the design and implementation method of Ka-band inter-satellite link signal simulator [²]. However, this paper does not specifically point out the method of modeling and calculating dynamic characteristics of satellite signals. In this paper, based on the orbital characteristics of satellites, a method is proposed to...
calculate dynamic parameters of inter-satellite links based on the precise ephemeris.

2. Dynamic Parameters Calculation of Inter-satellite Links

Satellites orbit the Earth, the distance and relative motion between satellites lead to delays and Doppler frequencies for the signals transmitted between satellites, making the inter-satellite link signals dynamic [3]. This paper describes the signal propagation model in an inertial frame of reference and uses Chebyshev polynomials to fit the positions and velocities of satellites so that the relative distance and relative velocity between satellites can be calculated, to obtain the dynamic parameters (transmission delay and Doppler frequency) of inter-satellite link signals.

2.1. Signal Propagation Model

A link is established between Satellite A and Satellite B. Satellite A, as a receiver, receives signals from Satellite B. Figure 1 shows the propagation of inter-satellite link signals.

An inter-satellite link signal is emitted at the clock time $t_B$ of Satellite B, radiated from the antenna phase center of Satellite B to free space after a delay ($\tau t_B$) in the transmission channel of Satellite B. The link signal propagates in space to the antenna phase center of Satellite A, and is captured by Satellite A at $t_A$ after a delay ($\tau r_A$) in the receiving channel. The parameters in the figure are described as follows [1].

- $\delta t_B$: Clock error of Satellite B when the signal is being emitted
- $\delta t_A$: Clock error of Satellite A when the signal is being received
- $t_B$: Clock time of Satellite B when the signal is being emitted
- $t_A$: Clock time of Satellite A when the signal is being received
- $\tau t_B$: Delay of the emitting channel of Satellite B
- $\tau r_A$: Delay of the receiving channel of Satellite A
- $t_0$: BeiDou time when the signal leaves the antenna phase center of BeiDou
- $t$: BeiDou time when the signal arrives at the antenna phase center of Satellite A
- $\tau$: The time when the signal is propagated in the space

Due to the relative motion of satellites, the transmission delay and Doppler frequency of the signal vary over a wide range of time. To simulate the satellite signal more realistically, the transmission delay and Doppler frequency of the satellite signal need to be calculated every 50 ms according to the above model, and the calculation method is described below. The signal processing module can use the dynamic parameters every 50 ms to simulate inter-satellite link signals with dynamic characteristics.

2.2. Chebyshev Fitting Algorithm

The precise ephemeris describes the precise position and velocity of a satellite in the form of coordinate points, and satellite coordinates are usually given at a 5-min interval. To accurately calculate the transmission delay and Doppler frequency between inter-satellite link signals, the positions and velocities of satellites during the establishment of the inter-satellite link need to be calculated by interpolation based on the precise ephemeris, and the Chebyshev polynomial performs well in fitting the positions and velocities of the satellites.
A Chebyshev polynomial is generated based on the temporal information of the fitted nodes. Assuming that the satellite coordinates or velocities given at an interval of \([t_0, t_0 + \Delta t]\) are fitted with a Chebyshev polynomial of the nth-order, where \(t_0\) and \(\Delta t\) are the Epoch time and the length of the fitted interval, respectively, let \(t \in [t_0, t_0 + \Delta t]\), do the following: 

\[
\tau = \frac{2}{\Delta t} (t - t_0) - 1 \quad \tau \in [-1, 1].
\]

The recurrence formula of the Chebyshev polynomial is obtained as follows.

\[
\begin{aligned}
T_0(\tau) &= 1 \\
T_1(\tau) &= \tau \\
T_n(\tau) &= 2\tau T_{n-1}(\tau) - T_{n-2}(\tau)
\end{aligned}
\]

The satellite coordinates can be represented by using the following polynomial:

\[
[x(t) \ y(t) \ z(t)]^T = \left[ \sum_{i=0}^{n} C_{xi} T_i(\tau) \right] \left[ \sum_{i=0}^{n} C_{yi} T_i(\tau) \right] \left[ \sum_{i=0}^{n} C_{zi} T_i(\tau) \right]
\]

Where, \(n\) is the order of the polynomial to be fitted and \(C_{xi}, C_{yi}, C_{zi}\) are coefficients of the Chebyshev polynomial.

Taking the X coordinate as an example (the same for velocity), the error equation is as follows:

\[
X^T = TC - X
\]

The matrix form is:

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} =
\begin{bmatrix}
T_0(\tau_1) & T_1(\tau_1) & \cdots & T_n(\tau_1) \\
T_0(\tau_2) & T_1(\tau_2) & \cdots & T_n(\tau_2) \\
\vdots & \vdots & \ddots & \vdots \\
T_0(\tau_m) & T_1(\tau_m) & \cdots & T_n(\tau_m)
\end{bmatrix}
\begin{bmatrix}
C_{x0} \\
C_{x1} \\
\vdots \\
C_{xm}
\end{bmatrix} =
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_m
\end{bmatrix}
\]

Where, \(m\) is the number of known coordinate points.

Let \(C = \begin{bmatrix} C_{x0} & C_{x1} & \cdots & C_{xm} \end{bmatrix}^T\), \(X = \begin{bmatrix} X_1 \end{bmatrix}^T\), \(T = \begin{bmatrix} T_0(\tau_1) & T_1(\tau_1) & \cdots & T_n(\tau_1) \\
T_0(\tau_2) & T_1(\tau_2) & \cdots & T_n(\tau_2) \\
\vdots & \vdots & \ddots & \vdots \\
T_0(\tau_m) & T_1(\tau_m) & \cdots & T_n(\tau_m)
\end{bmatrix}\), here the relation \(C = (T^T T)^{-1} T^T X\) can be obtained by using the least squares principle, and the polynomial is as follows:

\[
f(\tau) = C_{x0} + C_{x1} \tau + C_{x2} T_2(\tau) + \cdots + C_{xn} T_n(\tau)
\]

The calculation of the positions and velocities of satellites: Firstly, make the following changes to the calculation time \(Xh\):

\[
\tau = \frac{2}{\Delta t} (Xh - t_0) - 1 \quad \text{where} \quad t_0 \quad \text{and} \quad \Delta t \quad \text{are the Epoch time and the length of the fitted time interval, respectively.}
\]

Then, calculate the Chebyshev polynomial according to the recurrence formula, and finally calculate the orbital information (position or velocity) of satellites at the time of \(Xh\) according to the fitted Chebyshev polynomial.

### 2.3. Calculation of the Signal Transmission Delay

The transmission delay of inter-satellite link signals can be solved by iterating precise ephemeris parameters in the J2000 inertial frame of reference. The specific steps are described as follows.

**Step 1:** The clock time when the signal is being emitted in Satellite B and the delay of the emitting channel of Satellite B are known. According to the clock error parameter of Satellite B, the clock error of Satellite B can be obtained when the signal is being emitted. The formula for calculating the BeiDou
time $t_0$ when the signal leaves the antenna phase center of Satellite B is as follows.

$$t_0 = t_B - \delta t_B + \tau_{t,B} \quad (1)$$

**Step 2:** The transmission delay $\tau$ can be iteratively calculated when the signal reaches the antenna phase center of Satellite A.

a. Set the initial value of the iteration $\tau_i = 0$;

b. According to the method in Section 2.2, the position $\vec{r}_B(t_0)$ of Satellite B in the J2000 inertial frame of reference at the time of $t_0$ is calculated using the precise ephemeris parameters of Satellite B. The position $\vec{r}_A(t_0 + \tau_i)$ of Satellite A in the J2000 inertial frame of reference at the time when the signal reaches the antenna phase center of Satellite A is calculated based on the precise ephemeris parameters of Satellite A. The iterative formula is as follows.

$$\tau_{i+1} = \left(\vec{r}_A(t_0 + \tau_i) - \vec{r}_A(t_0)\right)/c \quad (2)$$

Where, $c$ indicates the speed of light.

c. Set the convergence threshold as $\tau_p$. If $|\tau_{i+1} - \tau_i| \leq \tau_p$, then exit the iteration. Usually, the iteration converges after 2 or 3 times.

2.4. Calculation of the Doppler Frequency

The position $\vec{r}_B(t_0)$ and velocity $\vec{v}_B(t_0)$ at the time of $t_0$ are calculated using the precise ephemeris parameters of Satellite B. The position and velocity of Satellite A in the J2000 inertial frame of reference at the time when the signal reaches the antenna phase center of satellite A are calculated based on the precise ephemeris parameters of Satellite A. In turn, the radial and relative velocity of satellite A with respect to satellite B can be calculated as follows.

$$\Delta v = \left[\left(\vec{v}_A(t_0 + \tau) - \vec{v}_B(t_0)\right) \cdot \left(\vec{r}_A(t_0 + \tau) - \vec{r}_B(t_0)\right)\right] / \left|\vec{r}_A(t_0 + \tau) - \vec{r}_B(t_0)\right| \quad (3)$$

The Doppler frequency $f_d$ of the signal is as follows:

$$f_d = \frac{\Delta v}{c} \times f_c \quad (4)$$

Where, $f_c$ indicates the carrier frequency of the inter-satellite link signal.

3. Test Verification and Result

As one of the three major test systems of the major project of China’s second-generation satellite navigation system, the BeiDou ground test verification system provides a multi-layer, flexible, and controllable system-level simulation test platform that can represent the near real operation state of the BeiDou global system. It consists of the software and hardware simulators of each system, environment segment simulation, control and supporting facilities, global system simulation software, and performance evaluation software that represent major system technical status of project. It can integrate all elements and all systems to conduct the design and verification, project coordination, equivalent operating and simulation tests. The simulation software of the global system can establish accurate orbit models and provide accurate input information for subsystem simulation of environmental segments, and generate the information such as the accurate distance between satellites, relative velocity, and antenna beam, pointing direction for performance evaluation of the equipment to be measured.

To evaluate the accuracy of dynamic parameters obtained by using the algorithm in this paper, this paper establishes the same inter-satellite link test scenario. Through intercepting the calculation results during the test, the results obtained by using the method proposed in this paper are compared with the
inter-satellite signal transmission delay and Doppler frequency generated by using the simulation software of the global system. The comparison results are shown in Table 1 and Table 2.

Table 1 Transmission delay comparison results

| Serial Number | SOW of the time when the link is established/s | Calculation results of the simulation software of the global system/s | Calculation results obtained by using the method in this paper/s | Error/s |
|---------------|-----------------------------------------------|-------------------------------------------------|---------------------------------------------------|--------|
| 1             | 86145                                         | 0.1068560212                                    | 0.1068560219                                      | 0.7E-09 |
| 2             | 86165                                         | 0.1699953905                                    | 0.1699953916                                      | 1.1E-09 |
| 3             | 86184                                         | 0.1305449836                                    | 0.1305449825                                      | -1.1E-09|
| 4             | 86210                                         | 0.1845623141                                    | 0.1845623157                                      | 1.6E-09 |
| 5             | 86243                                         | 0.1704734426                                    | 0.1704734439                                      | 1.3E-09 |
| 6             | 86262                                         | 0.1312213422                                    | 0.1312213414                                      | -0.8E-09|
| 7             | 86313                                         | 0.1316644457                                    | 0.1316644469                                      | 1.2E-09 |
| 8             | 86346                                         | 0.1319510019                                    | 0.1319510006                                      | -1.3E-09|

Table 2 Comparison of Doppler frequency calculation results

| Serial Number | SOW of the time when the link is established/s | Calculation result of the simulation software of the global system /Hz | Calculation result obtained by using the method in this paper /Hz | Error /Hz |
|---------------|-----------------------------------------------|-------------------------------------------------|---------------------------------------------------|----------|
| 1             | 86145                                         | 138392.554                                      | 138392.482                                         | -0.072   |
| 2             | 86165                                         | 196050.6578                                     | 196049.4991                                        | -1.159   |
| 3             | 86184                                         | 40165.21764                                     | 40165.58727                                        | 0.370    |
| 4             | 86210                                         | 136425.336                                      | 136424.472                                        | -0.864   |
| 5             | 86243                                         | 196198.4437                                     | 196200.7595                                        | 2.316    |
| 6             | 86262                                         | 196234.1123                                     | 196237.0154                                        | 2.903    |
| 7             | 86313                                         | 196278.769                                     | 196276.801                                        | -1.968   |
| 8             | 86346                                         | 138400.123                                      | 138402.482                                        | 2.359    |

The calculation results generated by using the simulation software of the global system are standard values. Compared with results obtained by using the algorithm in this paper, the error of transmission delay is less than 2 ns and the error of Doppler frequency is less than 3 Hz, which can meet the accuracy requirements for testing and verification of loads of inter-satellite links, and the calculation accuracy is similar to the satellite simulator of BeiDou ground test and verification system. However, the algorithm of this paper is simple, takes less resources and is easy to implement.

4. Conclusion
This paper proposes a method to calculate dynamic parameters based on precise ephemeris in the inertial frame of reference for the inter-satellite link signal simulation equipment, and verifies the algorithm by comparing the results of this paper with that obtained by using the global simulation software. The algorithm is simple in design, high in accuracy, and easy to implement. It solves the problem of simulating and generating the inter-satellite link dynamic signals.

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