Little-Parks effect and multiquanta vortices in a hybrid superconductor–ferromagnet system

A Yu Aladyshkin, A S Mel’nikov, D A Ryzhov
Institute for Physics of Microstructures, Russian Academy of Sciences, 603950, Nizhny Novgorod, GSP-105, Russia
E-mail: alay@ipm.sci-nnov.ru

Abstract. Within the phenomenological Ginzburg-Landau theory we investigate the phase diagram of a thin superconducting film with ferromagnetic nanoparticles. We study the oscillatory dependence of the critical temperature on an external magnetic field similar to the Little-Parks effect and formation of multiquantum vortex structures. The structure of a superconducting state is studied both analytically and numerically.

PACS numbers: 74.25.Dw, 74.25.Op, 74.78.Fk

Submitted to: J. Phys.: Condens. Matter
Little-Parks effect [1], i.e. oscillations of the critical temperature $T_c$ of multiply-connected superconducting samples in an applied magnetic field $H$, is one of the striking phenomenon demonstrating coherent nature of superconducting state. Such oscillatory behaviour of $T_c(H)$ is not a distinctive feature of a superconducting thin-wall cylinder and can be observed also in superconductors with columnar defects and holes (see references [2, 3, 4]) and mesoscopic simply-connected samples of the size of several coherence lengths [5, 6]. Generally, the oscillations of $T_c$ with a change in the external magnetic flux are caused by the transitions between the states with different vorticities (winding numbers) characterizing the circulation of the phase of the order parameter. For a system with cylindrical symmetry the vorticity parameter just coincides with the angular momentum of the Cooper pair wave function. The states with a certain angular momentum $m$ can be considered as $m$-quanta vortices. Experimental and theoretical investigations of these exotic vortex structures (multiquanta vortices and vortex molecules) in mesoscopic superconductors have attracted a great deal of attention. As we change an external homogeneous magnetic field multiquanta vortices and vortex molecules can transform one into another via first or second order phase transitions.

In this paper we focus on another possibility to create multiquantum vortex states: nucleation of superconducting order parameter in a hybrid system consisting of a thin superconducting film and an array of magnetic nanoparticles. The interest to such structures is stimulated by their large potential for applications (e.g., as switches or systems with a controlled artificial pinning). The enhancement of the depinning critical current density $j_c$ has been observed experimentally for superconducting films with arrays of submicron magnetic dots [7, 8, 9], antidots [10], and for superconductor–ferromagnet (S/F) bilayers with domain structure in ferromagnetic films [11]. The matching effects observed for magnetic and transport characteristics were explained in terms of commensurability between the flux lattice and the lattice of magnetic particles. Vortex structures and pinning in the S/F systems at rather low magnetic fields (in the London approximation) have been analysed in papers [12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

Provided the thickness of a superconducting film is rather small as compared with the coherence length, the critical temperature of the superconducting transition as well as the structure of superconducting nuclei should be determined by a two-dimensional distribution of a magnetic field component $B_z(x, y)$ (perpendicular to the superconducting film plane) induced by the ferromagnetic particles. Obviously, the highest critical temperature corresponds to the nuclei which appear near the lines of zeros of $B_z$ due to the mechanism analogous to the one responsible for the surface superconductivity (see, e.g., reference [22]) and domain wall superconductivity [23, 24, 25]. Provided these lines of zeros have the shape of closed loops, the winding number of a superconducting nucleus will be determined by the magnetic flux through the loop. Thus, changing this flux (e.g., increasing an external $H$ field applied along the $z$ axis) we can control the winding number. The resulting phase transitions between the multiquantum states with different $m$ can cause the oscillations of $T_c$. Such oscillatory behaviour has been, in fact, observed in reference [26] for a Nb film with an array of GdCo particles. Note that a change in a slope of the phase transition curve $T_c(H)$ (which is probably a signature of the transition discussed above) has been also found in reference [27] for a Pb film with CoPd particles. Provided the dimensions of the sample in the $(xy)$ plane are compared with the coherence length, we can expect a rather complicated picture which is influenced both by the sample edges and by the
distribution of an inhomogeneous magnetic field. For a model step-like profile of the magnetic field the resulting phase transitions between different types of exotic vortex states in a mesoscopic disc have been studied numerically in reference 28. The interplay between the boundary effects and magnetic field inhomogeneity influences also the formation of multiquantum vortex states around a finite size magnetic dot embedded in a large area superconducting film 29, 30. The transitions between different multiquantum vortex states with a change in magnetic field and magnetic dot parameters were studied in references 28, 29, 30 for certain temperature values. These effects are closely related to the ones observed in mesoscopic and multiply-connected samples, and consequently, we can expect that the oscillations of \( T_c(H) \) (analysed below) should also be a common feature of multiply-connected superconductors and thin-film systems with magnetic dots.

In this work we do not consider the magnetic phase transitions in the mixed state for \( T < T_c \) and focus on the oscillatory behaviour of \( T_c(H) \) in a large area superconducting film caused only by the quantization associated with the characteristics of the inhomogeneous magnetic field produced by ferromagnetic particles. We neglect the influence of the edge and proximity effects in the S/F system and consider a nanoparticle only as a source of a small-scale magnetic field. Our further consideration is based on the linearized Ginzburg-Landau model:

\[
- \left( \nabla + \frac{2\pi i}{\Phi_0} \mathbf{A} \right)^2 \Psi = \frac{1}{\xi^2(T)} \Psi.
\]

Here \( \Psi(r) \) is the order parameter, \( \mathbf{A}(r) \) is the vector potential, \( \mathbf{B}(r) = \nabla \times \mathbf{A}(r) \), \( \Phi_0 \) is the magnetic flux quantum, \( \xi(T) = \xi_0/\sqrt{1-T/T_c}\) is the coherence length, and \( T_c\) is the critical temperature of the bulk superconductor at \( B = 0 \). For the sake of simplicity we neglect the interference effects between the superconducting nuclei appearing near different nanoparticles (i.e. assume the interparticle distance to be rather large as compared with the superconducting nucleus size) and consider a single magnetic particle with a fixed magnetic moment chosen perpendicular to the film plane \( xy \). For a rather thin film (of the thickness less than the coherence length) we can neglect the influence of the field components \( B_x, B_y \) in the film plane and consider an axially symmetrical two-dimensional problem 11 in the field \( B_z(r) = H + b(r) \), where \( b(r) \) is the \( z \)-component of the field induced by the ferromagnetic particle and \( r, \theta, z \) is a cylindrical coordinate system. Choosing the gauge \( A_\theta(r) = Hr/2 + a(r) \) one can find the solution of the equation (1) in the form \( \Psi(r) = g_m(r) \exp(i m \theta)/\sqrt{T} \), where \( m \) is the vorticity, and \( g_m(r) \propto r^{[|m|+1/2}] \) for \( r \to 0 \). The function \( g_m(r) \) should be determined from the equation:

\[
- \frac{d^2 g_m}{dr^2} + \left( \frac{(\Phi(r)/\Phi_0 + m)^2}{r^2} - \frac{1}{4r^2} \right) g_m = \frac{1}{\xi^2(T)} g_m.
\]

Here \( \Phi(r) = 2\pi r A_\theta(r) \) is the total flux through the circle of radius \( r \). The lowest eigenvalue \( 1/\xi^2(T) \) of the Schrödinger-like equation (2) defines the critical temperature \( T_c \) of the phase transition into a superconducting state.

Obviously, for rather small fields \( H \) the superconducting order parameter can nucleate either far from the magnetic particle \( (r \to \infty) \) where the critical temperature \( T_c^H \) is defined by the homogeneous field \( B_h = H \) or in the region close to the circle of the radius \( r_0 \) where \( B_h(r_0) = 0 \) and \( T_c \) is controlled by the slope of \( B_h \) at \( r = r_0 \) and by the flux through the area of the radius \( r_0 \). In the first case we obtain \( 1 - T_c^H/T_c0 = 2\pi |H|\xi_0^2/\Phi_0 \). For the second case we can analyse the behaviour of \( T_c(H) \)
assuming that the characteristic length scale $\ell$ of the order parameter nucleus is much less than the characteristic scale of the magnetic field distribution. Within such local approximation (similar to the one used in reference [25] for the description of domain wall superconductivity) we can expand the flux in powers of the distance from $r_0$:

$$\frac{\Phi(r)}{\Phi_0} + m \simeq \left( \frac{\Phi(r_0)}{\Phi_0} + m \right) + \frac{\pi r_0 B'_z(r_0)}{\Phi_0} (r - r_0)^2.$$ 

This local approximation is valid under the following conditions:

$$\left| \frac{B''_z(r_0)}{B'_z(r_0)} \right| \ell \ll 1 \quad \text{and} \quad \frac{\ell}{r_0} \ll 1.$$

Introducing a new coordinate $t = (r - r_0)/\ell$ we obtain the dimensionless equation

$$- \frac{d^2 g}{dt^2} + (t^2 - Q)^2 g = E g,$$

where the parameters $E$ and $Q$ are given by the expressions:

$$E = \ell^2 \left( 1 - \frac{T}{T_{c0}} \right), \quad \ell = \sqrt{\frac{\Phi_0}{\pi |B'_z(r_0)|}}, \quad Q = - \left( \frac{\Phi(r_0)}{\Phi_0} + m \right)^2 \sqrt{\frac{\Phi_0}{\pi r_0^2 B'_z(r_0)}}. \quad \text{(4)}$$

We obtain $E(Q) \simeq Q^2 + \sqrt{-2Q}$ when $Q \ll 1$, and $E(Q) \simeq 2\sqrt{Q}$ when $Q \gg 1$. The minimal value of $E(Q)$ is $E = E_{\text{min}} \simeq 0.904$ at $Q \simeq 0.437$. The final expression for the critical temperature reads:

$$1 - \frac{T_c}{T_{c0}} = \frac{\ell^2}{\ell^2} \left[ \min_m E \left( - \left( \frac{\Phi(r_0)}{\Phi_0} + m \right)^2 \sqrt{\frac{\Phi_0}{\pi r_0^2 B'_z(r_0)}} \right) + O \left( \frac{\ell^2}{r_0^2} \right) \right]. \quad \text{(5)}$$

The superconducting nuclei are localized near the ferromagnetic particle at a distance $r_0$. The states with different energetically favorable winding numbers $m$ correspond to the multiquantum vortex structures very similar to the ones observed in a mesoscopic disc. As we change an external field $H$, we change the flux $\Phi(r_0)$ and, thus, change the energetically favorable vorticity number and position of the nucleus.

To investigate the details of the oscillatory behaviour discussed above we consider a particular case of a small ferromagnetic particle which can be described as a point magnetic dipole with a magnetic moment $M = Mz_0$ placed at a height $h$ over the superconducting film. The corresponding expressions for the field and the vector potential are:

$$b(r) = \frac{M(2h^2 - r^2)}{(r^2 + h^2)^{3/2}}, \quad a(r) = \frac{Mr}{(r^2 + h^2)^{3/2}}. \quad \text{(6)}$$

Introducing $f_m(r) = g_m(r)/\sqrt{r}$ and a dimensionless coordinate $\rho = r/h$ we obtain the equation (2) in the form:

$$- \frac{1}{\rho} \frac{d}{d \rho} \left( \rho \frac{df_m}{d \rho} \right) + \left( \frac{3\sqrt{3}}{2} N_t \left[ \frac{H}{b_0} \rho + \frac{\rho}{(1 + \rho^2)^{3/2}} \right] + \frac{m}{\rho} \right)^2 f_m = \frac{h^2}{\ell^2} \left( 1 - \frac{T_c}{T_{c0}} \right) f_m, \quad \text{(7)}$$

where $N_t = 4\pi M/(3\sqrt{3}h\Phi_0)$ is the dimensionless flux through the area with the positive field $b(r)$ and $b_0 = b(0) = 2M/h^3$. In the limit of small fields $H \to 0$ the nucleation of superconductivity occurs at large distances $\rho$ and critical temperatures for different winding numbers $m$ are very close. Thus, in this limit the critical
temperature is equal to $T_c^H$ and is not sensitive to the presence of the dipole. Below this temperature we obtain a lattice of singly quantized vortices (with the concentration determined by $H$) which is surely disturbed under the dipole. Note, that the behaviour of $T_c$ in this low field regime should modify provided we take account of a finite distance between the magnetic particles. For large absolute values $|H|$ (much larger than the maximum field induced by the dipole) we obtain the following asymptotical behaviour of $T_c$: $1 - T_c/T_{c0} = 2\pi \xi_0^2 (-H - b_0)/\Phi_0$ for negative $H$ and $1 - T_c/T_{c0} = 2\pi \xi_0^2 (H - b_0/(25\sqrt{5}))/\Phi_0$ for positive $H$ values (here $-b_0/(25\sqrt{5})$ is the minimum of the dipole field). The superconductivity nucleates near the minima of the total field $|B_z|$ and, thus, is localized near the dipole. In the intermediate field region ($-1 < H/b_0 < 1/(25\sqrt{5})$) we should expect the oscillatory behaviour of $T_c$ discussed above. The number of oscillations is controlled by the parameter $N_f$. We have carried out the numerical calculations of equation (7) for the various $N_f$ values. For the numerical analysis of the localized states of equation (7) we approximated it on an equidistant grid and obtained the eigenfunctions $f_m(\rho)$ and eigenvalues by the diagonalization method of the tridiagonal difference scheme. The results of these calculations as well as the analytical dependence of $T_c$ given by the expression (5) are shown in figure 1.

**Figure 1.** Critical temperature as a function of an external magnetic field for $N_f = 4$ (a) and $N_f = 10$ (b). Solid line is a result of direct numerical simulations of equation (7). Dash line is obtained from the analytical formula (5). Certain winding numbers $m$ for different parts of the phase transition line are shown.

We observed a remarkable asymmetry of the phase transition curve ($T_c(H) \neq T_c(-H)$) which is caused by the difference in the distributions of positive and negative parts of the dipole field $b(r)$: the maximum positive field ($b_0$) is much larger than the absolute value of the minimum negative field ($b_0/(25\sqrt{5})$). As a result, the $T_c$ oscillations appear to be most pronounced for negative $H$ which compensate the positive part of the dipole field. Taking $M \sim 3 \times 10^{-11} G \cdot cm^3$ (for a ferromagnetic particle with dimensions $300 \, nm \times 300 \, nm \times 300 \, nm$ and magnetization $\sim 10^3 G$), $h \sim 300 \, nm$ we obtain $N_f \sim 10$, $b_0 \sim 10^3 G$ and the characteristic scales of $T_c$ oscillations $\Delta H \sim 100 \, Oe$, $\Delta T_c \sim 10^{-2} T_{c0} \sim 0.1 \, K$ for a Nb film with $\xi_0 \sim 40 \, nm$.
Little-Parks effect in hybrid superconductor–ferromagnet system

We expect that the oscillatory behaviour of \( T_c \) can be observable, e.g., in magnetoresistance measurements of thin superconducting films with arrays of ferromagnetic particles. The superconducting nuclei localized near the particles should result in the partial decrease in the resistance below the oscillating \( T_c(H) \). As we decrease the temperature below \( T_c(H) \) the superconducting order parameter around a single particle becomes a mixture of angular harmonics with different \( m \) values and we can expect the appearance the phase transitions similar to the ones discussed in papers [28, 30]. With the further decrease in temperature the whole film becomes superconducting and resistivity turns to zero.

Acknowledgments

We would like to thank A. I. Buzdin, A. A. Fraerman, Yu. N. Nozdrin, and I. A. Shereshevskii for stimulating discussions. This work was supported, in part, by the Russian Foundation for Basic Research, Grant No. 03-02-16774, Russian Academy of Sciences under the Program 'Quantum Macrophysics', Russian State Fellowship for young doctors of sciences (MD-141.2003.02), University of Nizhny Novgorod under the program BRHE and 'Physics of Solid State Nanostructures'.

References

[1] Little W A and Parks R D 1962 \textit{Phys. Rev. Lett.} \textbf{9} 9; Parks R D and Little W A \textit{Phys. Rev.} 1964 \textbf{133} A97
[2] Buzdin A I 1993 \textit{Phys. Rev. B} \textbf{47} 11416
[3] Bezryadin A, Buzdin A I and Pannetier B 1994 \textit{Phys. Lett. A} \textbf{195} 373
[4] Bezryadin A and Pannetier B 1995 \textit{Journ. Low. Temp. Phys.} \textbf{98} 251
[5] Buisson O, Gandit P, Rammal R, Wang Y Y and Pannetier B 1999 \textit{Phys. Rev. B} \textbf{57} 13817; Jaddalah H T, Rubinstein J and Sternberg P 1999 \textit{Phys. Rev. Lett.} \textbf{82} 2935; Yampolskii S V and Peeters F M 2000 \textit{Phys. Rev. B} \textbf{62} 9663
[6] Chibotaru L F, Ceulemans A, Bruyndoncx V and Moshchalkov V V 2000 \textit{Nature} \textbf{408} 833; Chibotaru L F, Ceulemans A, Bruyndoncx V and Moshchalkov V V 2001 \textit{Phys. Rev. Lett.} \textbf{86} 1323
[7] Martin J I, Vélez M, Nogués J and Schuller I K 1997 \textit{Phys. Rev. Lett.} \textbf{79} 1929; Martin J I, Vélez M, Hoffmann A, Schuller I K and Vicent J L 1999 \textit{Phys. Rev. Lett.} \textbf{83} 1022
[8] Morgan D J and Ketterson J B 1998 \textit{Phys. Rev. Lett.} \textbf{80} 3614
[9] Van Bael M J, Temst K, Moshchalkov V V and Bruynseraede Y 1999 \textit{Phys. Rev. B} \textbf{59} 14674; Van Bael M J, Van Look L, Lange M, Bekaert J, Bending S J, Grigorenko A N, Moshchalkov V V and Bruynseraede Y 2002 \textit{Physica C} \textbf{369} 97
[10] Van Bael M J, Raedts S, Temst K, Swerts J, Moshchalkov V V and Bruynseraede Y 2002 \textit{J. Appl. Phys.} \textbf{92} 4531
[11] Garsia-Santiago A, Sánchez F, Varela V, Tejada J 2000 \textit{J. Appl. Phys.} \textbf{87} 2990
[12] Sonin E B, 1988 \textit{Pis’ma v Zh. Tekh. Phys.} \textbf{14} 1640 (1988) [1988 \textit{Sov. Tech. Phys. Lett.} \textbf{14} 714]
[13] Tokman I D 1992 \textit{Phys. Lett. A} \textbf{166} 412
[14] Lyuksyutov I F and Pokrovsky V L 1998 \textit{Phys. Rev. Lett.} \textbf{81} 2344
[15] Sášek R and Hwa T 2000 \textit{Preprint} cond-mat/0003462
[16] Bulaevsky L N, Chudnovsky E M and Maley M P 2000 \textit{Appl. Phys. Lett.} \textbf{76} 2594
[17] Bespyatykh Yu I and Wasilevski W 2001 \textit{Fiz. Tverd. Tela (Leningrad)} \textbf{43} 215 [2001 \textit{Sov. Phys.–Solid State} \textbf{43} 1224]; Bespyatykh Yu I, Wasilevski W, Gajdek M, Nikitin I P and Nikitov S A 2001 \textit{Fiz. Tverd. Tela (Leningrad)} \textbf{43} 1754 [2001 \textit{Sov. Phys.–Solid State} \textbf{43} 1827]
[18] Erdin S, Lyuksyutov I F, Pokrovsky V L and Vinokur V M 2002 \textit{Phys. Rev. Lett.} \textbf{88} 017001; Erdin S, Kayali A F, Lyuksyutov I F and Pokrovsky V L 2002 \textit{Phys. Rev. B} \textbf{66} 014414
[19] Helseth L E 2002 \textit{Phys. Rev. B} \textbf{66} 104508
[20] Milošević M V, Yampolskii S V and Peeters F M 2002 \textit{Phys. Rev. B} \textbf{66} 174519; Milošević M V, Yampolskii S V and Peeters F M 2003 \textit{Journ. Low Temp. Phys.} \textbf{130} 321
[21] Laiho R, Lähderanta E, Sonin E B and Traito K B 2003 Phys. Rev. B 67 144522
[22] Saint-James D, Sarma G and Thomas E J 1969 Type-II Superconductivity (Pergamon Press, New York) chapter 4
[23] Buzdin A I, Bulaevskii L N and Panyukov S V 1984 Zh. Eksp. Teor. Phys. 87 299 [1984 Sov. Phys. - JETP 60 174]
[24] Buzdin A I and Mel’nikov A S 2003 Phys. Rev. B 67 020503
[25] Aladyshkin A Yu, Buzdin A I, Fraerman A A, Mel’nikov A S, Ryzhov D A and Sokolov A V, submitted to Phys. Rev. B
[26] Otani Y, Pannetier B, Nozières J P and Givord D 1993 J. Magn. Magn. Mater. 126 622
[27] Lange M, Van Bael M J, Bruynseraede Y and Moshchalkov V V 2002 Preprint cond-mat/0209101
[28] Milošević M V, Yampolskii S V and Peeters F M 2002 Phys. Rev. B 66 024515
[29] Cheng S L and Fertig H A 1999 Phys. Rev. B 60 13107
[30] Marmorkos I K, Matulis A and Peeters F M 1996 Phys. Rev. B 53 2677