Realization of Superadiabatic Two-qubit Gates Using Parametric Modulation in Superconducting Circuits

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We propose a protocol to realize parametric control of two-qubit coupling, where the amplitude and phase are tuned by a longitudinal field. Based on the tunable Hamiltonian, we demonstrate the superadiabatic two-qubit quantum gate using superconducting quantum circuits. Our experimental results show that the state of qubits evolves adiabatically during the gate operation even though the processing time is close to the quantum limit. In addition, the quantum state transition is insensitive to the variation of control parameters, and the fidelity of a SWAP gate achieved 98.5%. This robust parametric two-qubit gate can alleviate the tension of frequency crowding for quantum computation with multiple qubits.

High fidelity two-qubit quantum gates are a key component in quantum information processing [11, 13]. In the transmon-like superconducting qubit architecture [4], two types of two-qubit gates are generally employed. One requires local magnetic fields to tune the transition frequency of qubits [5, 8]. The other uses all-microwave control [9, 11]. Neither type is optimal. A tunable design introduces an extra dephasing source to the qubits. In addition, with the qubit number increasing, there may not be enough frequency space to sweep the qubit, a problem known as frequency crowding. As for all-microwave control (such as in the case of cross-resonance gates), gates can be operated in fixed-frequency superconducting qubits, which typically feature reduced sensitivity to flux noise. However, in order to obtain efficient gate operation, one must impose stringent requirements on the spectral landscape of the qubits. Moreover, coupling strength is limited by the driving amplitude. Recently, a promising alternative emerged that combines aspects of these two types of gates by using tunable coupling [12–16] and parametric driving architectures [17–21]. But this new approach needs improved protocols to increase gate fidelity and reduce gate time.

Currently, there are two quantum mechanisms to realize two-qubit gates. One is based on the dynamical process, where the quantum state transition can be achieved relatively quickly. However, in this case, gate fidelity relies heavily on the pulse shape, rendering the gate operation unstable against systematic and random fluctuations of control parameters. The other scheme uses adiabatic evolution. Adiabatic control [22, 23] of quantum states is insensitive to the fluctuations of control parameters. However, the adiabatic theorem requires that the slow evolution, which renders the system more susceptible to decoherence. In order to improve gate fidelity, some have proposed a superadiabatic protocol [24] to realize a fast adiabatic control without inducing unwanted excitations during evolution. For such a two-qubit system, the key is to construct the auxiliary Hamiltonian to restore desired adiabatic characteristics [6, 25]. Recent work has realized a superadiabatic two-qubit gate in fixed coupling transmons [26]. However, superadiabatic two-qubit gates that use tunable coupling or parametric modulated coupling to achieve high fidelity and flexibility have not been reported yet.

In this work, we propose a protocol to implement two-qubit superadiabatic (TQSA) gates with a parametric modulation scheme. In our scheme, a parametric modulating field provides fully tunable coupling between two qubits [19, 27, 31], enabling us to construct a target superadiabatic Hamiltonian. We experimentally demonstrate TQSA gates in superconducting circuits consisting of multiple qubits. Using superadiabatic evolution we implement a SWAP gate. We track the state evolution in the \{01, 10\} subspace and find no nonadiabatic error during the process. Then we investigate the robustness of TQSA gates against the variations of control parameters. The fidelity of TQSA gates reaches 98.5%, which is promising for quantum information processing.

The principle of our protocol is as follows. The Hamiltonian of two coupled qubits with one of them modulated by a longitudinal field $\varepsilon(t)$ can be written as

$$H = \frac{\hbar}{2} \sum_{i=1}^{2} \omega_i \sigma^i_z + g \sigma^i_z \sigma^2_z + \sigma^1_+ \sigma^2_+ + \frac{\hbar}{2} f(\varepsilon(t)) \sigma^1_z,$$

(1)

where $\sigma^i_z$ is the Pauli operator and $\sigma^i_+ (\sigma^i_-)$ is the creation (annihilation) operator in the Hilbert space of $i$th qubit $Q_i$. $g$ is the coupling strength between $Q_1$ and $Q_2$. $f(\varepsilon)$ is the nonlinear frequency response to the modulation pulse, and can be determined experimentally [32]. In an interaction frame defined by the instantaneous qubit frequencies, the Hamiltonian is given by

$$H_1 = \hbar g e^{i(\omega_2 - \omega_1) t + i F(t) \sigma^1_+ \sigma^2_z} + h.c.,$$

(2)

where $F(t) = \int f(\varepsilon(t)) dt$. We intentionally choose $F(t)$ to be an adjustable sinusoidal function

$$F(t) = A(t) \sin[(\omega_2 - \omega_1) t + \Delta_L(t) + \varphi_L(t)],$$

(3)
where $A_L(t)$ and $\varphi_L(t)$ indicate frequency detuning and the phase of the longitudinal field, respectively. Using Jacobi-Anger expansion and applying unitary transformation we can rewrite Eq. (2) as

$$H_I(t) = \frac{\hbar}{2} \begin{bmatrix} 0 & 0 & \Delta_L(t) & 2gJ_1(A(t))e^{\imath \varphi_L(t)} & 0 \\ 0 & 0 & 2gJ_1(A(t))e^{-\imath \varphi_L(t)} & -\Delta_L(t) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$  

(4)

where $J_1$ is the Besel function. From Eq. (4) we find that arbitrary gates in the {$|01\rangle$, $|10\rangle$} subspace (such as $\sqrt{iSWAP}$) can be constructed with a properly modulated pulse $\varepsilon(t) = f^{-1}(F(t))$. It is worth emphasizing that if one considers the higher coupling energy level of transmons, such as [11] and [20], a similar Hamiltonian can be constructed to realize a CZ gate [19].

We can use Eq. (4) to construct a TQSA Hamiltonian. Without loss of generality, we perform our protocol in {$|01\rangle$, $|10\rangle$} subspaces. A two-level system coupled with a microwave field with frequency $\omega_m(t)$ and phase $\varphi(t)$ can be expressed as

$$H_0(t) = \frac{\hbar}{2} \begin{bmatrix} \Delta(t) & \Omega_R(t)e^{\imath \varphi(t)} & -\Delta(t) \\ \Omega_R(t)e^{-\imath \varphi(t)} & \Omega(t) & 0 \\ -\Delta(t) & 0 & -\Delta(t) \end{bmatrix},$$  

(5)

where $\Delta(t) = \omega_0 - \omega_m(t)$ represents the detuning between energy gap of the two-level system and the frequency of the microwave field. $\Omega_R(t)$ is the Rabi frequency. The instantaneous eigenstates are $|\lambda_+(t)\rangle = \left[ \cos \frac{\theta(t)}{2} e^{-\imath \varphi(t)/2}, \sin \frac{\theta(t)}{2} e^{\imath \varphi(t)/2} \right]$,  

$$|\lambda_-(t)\rangle = \left[ -\sin \frac{\theta(t)}{2} e^{\imath \varphi(t)/2}, \cos \frac{\theta(t)}{2} e^{-\imath \varphi(t)/2} \right]$$

with different eigenvalues $E_{\pm} = \pm \frac{\hbar}{2} \sqrt{\Omega_R(t)^2 + \Delta^2(t)^2}$, where $\theta(t) = \arctan(\Omega_R(t)/\Delta(t))$. For simplicity, we choose $\varphi(t)$ to be constant, and the auxiliary Hamiltonian $H_I$ in superadiabatic theory can be derived as

$$H_I(t) = i\hbar \begin{bmatrix} 0 & -\theta(t)e^{\imath \varphi} & 0 \\ \theta(t)e^{-\imath \varphi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. $$  

(6)

Therefore, we obtain the Hamiltonian to implement a superadiabatic gate [33, 34]

$$H_S(t) = H_0(t) + H_I(t)$$

$$= \hbar \left[ \begin{bmatrix} \Delta(t) & \Omega_S(t)e^{\imath \varphi + \phi_S(t)} & -\Delta(t) \\ \Omega_S(t)e^{-\imath \varphi + \phi_S(t)} & \Omega_S(t) & 0 \\ -\Delta(t) & 0 & -\Delta(t) \end{bmatrix} \right],$$

(7)

where $\Omega_S(t) = \sqrt{\Omega_R(t)^2 + \varepsilon(t)^2}$, and $\phi_S(t) = \arctan(\theta(t)/\Omega_S(t))$.

Combining Eq. (4) and Eq. (7), we can calculate the parameters in $F(t)$ and thus the modulated pulse $\varepsilon(t)$ to implement TQSA gates. Similarly, we can construct Hamiltonian $H_0(t)$ using Eq. (4) with specific $F(t)$ to realize two-qubit adiabatic (TQA) gates.

We demonstrate our protocol using superconducting quantum circuits [35, 36]. The chip contains eight transmons arranged in an array with nearest-neighbor coupling. Each qubit can be readout with an individual resonator that is coupled to the transmission line on the chip. For four of the eight transmons we replace the Josephson junction with two junctions in parallel (DC SQUID). Therefore, the frequency of those transmons can be tuned by applying pulse through the Z control lines, represented by red lines in Fig. 1(a) [32].

We demonstrate tunable coupling hence the TQSA quantum gate in two coupled qubits $Q_1$ and $Q_2$. $Q_2$ is a fix frequency qubit with frequency $\omega_1/2\pi = 5.9498$ GHz. The frequency of $Q_1$ can be tuned by the combined static bias and fast flux pulse introduced through individual Z control lines [Fig. 1(a)]. In Fig. 1(b) and (c) we show parametric control of the effective coupling strength between $Q_1$ and $Q_2$. According to the theory of parametric modulation [29], frequency is modulated in the form of $\omega_1 = \omega_{10} + f(\varepsilon(t))$, where $\omega_{10}/2\pi$ is the
excited frequency of the ground state and the first excited state of the tunable qubit. $\omega_{10}/2\pi = 6.1567$ GHz is the static bias frequency. The intrinsic coupling strength $g/2\pi$ between $Q_1$ and $Q_2$ is 6.26 MHz, determined by the capacity between the pads of transmons. We operate the system in the dispersive regime where $\Delta >> g$ [29]. Both $Q_1$ and $Q_2$ are coupled to the readout transmission line which is also used for delivering the microwave signal to transmons for $XY$ control. The relaxation (dephasing) time of $Q_1$ is $T_1 = 4.06$ $\mu$s ($T_\phi = 620$ ns) at operation points. For $Q_2$, $T_1 = 3.98$ $\mu$s and $T_\phi = 6.1$ $\mu$s. From Fig. 1(c) we find that the maximum effective coupling is about 3.6 MHz, leading to a quantum limit $T_{QL} = \pi/2g_{eff}^{\max} = 69$ ns. Therefore, the minimum two-qubit gate time for the dynamical scheme is about 70 ns.

Having realized parametric tunable coupling, we can experimentally implement a specific TQSA gate and verify the acceleration of the superadiabatic scheme compared to the adiabatic process. We set the typical time dependent parameters of the TQSA gate as

$$\Omega_{R}(t) = \Omega_0 \sin(T/2\pi)$$
$$\Omega_{A}(t) = \Omega_0 \cos(T/2\pi),$$

where $T$ is the gate duration time and $\varphi = 0$. In order to maximize efficiency we chose $T = 80$ ns, which is subjected to the limitation of the effective coupling strength $g/2\pi = 6.26$ MHz. Using Eq. (6), we obtained the auxiliary Hamiltonian $H_1 = \frac{\hbar c}{2\pi} \sigma_y$. Thus, the $\sigma_y$ component of $H_S(t)$ is a constant value, as shown in Fig. 2(b). By using Eq. (4) and Eq. (7), we can calculate the specific form of $F(t)$ for our experiment with the parameters of $Q_1$ and $Q_2$ [32].

We track the system trajectory to verify the adiabaticity of evolution. The time profile of the experiments is shown in Fig. 2(a). We apply a microwave pulse to $Q_2$, preparing the system in $\{|0\rangle\} = |01\rangle$. Then we perform the TQSA gate, followed by rotating the system with different pulses choosing from the set $\{X/2, Y/2, I\}$, which are realized by qubit dynamic longitudinal gate (QDLG) [19] in the subspace $\{|01\rangle, |10\rangle\}$. The function of the QDLG set is to project and measure the system state on three axes in the subspace $\{|01\rangle, |10\rangle\}$. Finally we measure the system occupation probability of different states using two-qubit joint readout [8] [19]. By applying $H_S(t)$ to the qubits we realize superadiabatic operation. In Fig. 2(c) we show the state evolution trajectory of the TQSA gate in the Bloch sphere spanned by $|01\rangle$ and $|10\rangle$. The qubit state evolves precisely along the meridian predicted by the adiabatic theorem, proving the validity of the TQSA gate. It is noteworthy that the whole procedure time takes 80 ns, which is close to the quantum limit $T_{QL}$. The 10 ns extra time comes from the requirement of the protocol since we use a longitudinal pulse to control coupling strength [32]. Furthermore, we compare our TQSA approach with the TQA routine within the same time, as shown in Fig. 2(c). The TQA approach requires the fulfilling of adiabatic restriction, which is $T > 10T_{QL} = 690$ ns. Without auxiliary Hamiltonian $H_1$, the evolution trajectory deviates dramatically from the designed adiabatic path, which is caused by unwanted transitions between eigenstates of $H_S(t)$. The experimental results indicate that our TQSA scheme successfully accelerates the adiabatic procedure.

Compared to traditional two-qubit gates based on the dynamical procedure, TQSA gates possess the advantage of robustness against parameter fluctuations. The two important parameters for high fidelity gate operations are evolution time $T_c$ and Rabi frequency $\Omega_{zc}$. Here $\Omega_{zc}$ corresponds to the amplitude of the longitudinal field $F(t)$. In the QDLG scheme, the accuracy of gate operation is determined by both $T_c$ and $\Omega_{zc}$. Therefore, the fluctuation of system parameters will significantly affect gate fidelity. To quantify the robustness of gate operation, we performed a SWAP gate using both the TQA and QDLG protocols. The artificial perturbations $\epsilon_{T}$ and $\epsilon_{zc}$ are intentionally added. We choose

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**FIG. 2:** (Color online) (a) Time profile for measuring the evolution trajectory of the state. Dashed vertical lines delineate the four steps of the experiment. The system is initialized in $|01\rangle$ with a blue pulse on $Q_2$. Then we implement the TQSA (TQA) gate, shown as light gray dashed (black solid) line. Rotations (denoted as Rot) are operated with QDLG sets $\{X/2, Y/2, I\}$ (purple dotted line) right after the TQSA/ TQA gates. Finally we apply the readout pulse. (b) The typical component values of $H_S(t)$ in the parameter space to realize the TQSA protocol in experiments. $H_S(t) = \frac{\hbar c}{2}(\Omega_R(t)\sigma_x + \Omega_A(t)\sigma_y + \Delta(t)\sigma_z)$. (c) Experimental results shown on the Bloch sphere represent the evolution of the quantum state in the subspace of $\{|01\rangle, |10\rangle\}$ . Red dots (blue diamonds) represent experimental data of the superadiabatic (adiabatic) scheme, while black solid lines represent the numerical simulation of the two schemes. Numerical simulation results agree well with experimental data.
The gate fidelity. Fig. 3(a) and 3(b) show fidelity as functions of control parameters in superadiabatic and dynamical schemes. (a) Contour plot of transfer fidelity, represented by the population of $|00\rangle$, as a function of normalized parameters $\Omega_x/\Omega_{xc}$ and $T/T_c$ for the dynamical scheme. (b) Contour plot of transfer fidelity of the dynamical scheme as a function of normalized parameters $\Omega_x/\Omega_{xc}$ and $T/T_c$ for the superadiabatic scheme. The inserts in (a) and (b) are numerical simulation results involving the effect of decoherence. (c) Cross section along the black dashed line in (a) and (b). Blue and red circles represent experimental fidelity of dynamical and superadiabatic SWAP gates respectively. Red solid (blue dashed) line is numerical simulation.

$\epsilon, \Omega_x \in [-0.1\Omega_{xc}, 0.1\Omega_{xc}]$ and $\epsilon_T \in [-0.1T_c, 0.1T_c]$. In experiments, we initialize the system state in $|01\rangle$, and set $\Omega_{xc} = 0.36$ g and $T_c = 110$ ns. With varying $\epsilon, \Omega_x$ and $\epsilon_T$ we measure the populations in state $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. In experiments, we test the error of the TQSA $X$ gate with $N$ random reference gates and append a recovery gate $R_e$ making the final system state in $|00\rangle$. Then we measure the occupation probability of $|00\rangle$ and fit it to $P_{ref} = pX_{ref} + B$. We can obtain $pX_{gate}$ using the same method. The error rate is calculated with $r_{X_{gate}} = (1 - pX_{gate}/p_{ref})(d - 1)/d$. Here $d = 2$, similar to the single-qubit IRB. Eliminating system errors with reference sequences, we obtain a 98.5% superadiabatic gate fidelity for $k = 40$. We find that the subspace state leaks to $|00\rangle$ as time increases and the leakage curves coincide with corresponding relaxation time.

Finally, we quantify the error rate of the superadiabatic gate using a method similar to single-qubit interleaved randomized benchmarking (IRB) [41]. The reference gate sequence consists of random gates chosen from $\{I, X, Y, -X, -Y\}$ TQSA gates. From Eq. (5), we can perform $X, Y, -X, -Y$ gates by choosing $\varphi = 0, \pi/2, \pi, 3\pi/2$. With these gates, the system always stays in the instantaneous eigenstate of $H_0(t)$. A specific gate is interleaved with random reference gate sequences, making the measured fidelity irrelevant with regard to state preparation, decoherence, and readout errors. Both the interleaved gates and reference gates evolve in 80 ns with a 2 ns time slot between each other. In this experiment, we test the error of the TQSA $X$ gate with sequence lengths of 1 to 20. As shown in Fig. 4(a), we initialize the system in $|01\rangle$, implement sequences of reference gates and append a recovery gate $R_e$ making the final system state in $|00\rangle$. Then we measure the occupation probability of $|00\rangle$ and fit it to $P_{ref} = pX_{ref} + B$. We can obtain $pX_{gate}$ using the same method. The error rate is calculated with $r_{X_{gate}} = (1 - pX_{gate}/p_{ref})(d - 1)/d$. Here $d = 2$, similar to the single-qubit IRB. Eliminating system errors with reference sequences, we obtain a 98.5% superadiabatic gate fidelity for $k = 40$. We find that the subspace state leaks to $|00\rangle$ as time increases and the leakage curves coincide with corresponding relaxation time.

In summary, we propose and demonstrate a TQSA gate using a parametric modulation protocol in super-
conducting circuits. Using the longitudinal field we can control coupling strength and phase. Tunable coupling enables us to select operation points, which can help avoid frequency crowding. In addition, the superadiabatic gate follows the expected adiabatic trajectory at a speed close to the quantum limit, exhibiting robustness against system or random fluctuations. The combined uniform high fidelity and fast gate speed makes this TQSA gate promising for quantum information research.

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