Dual-particle-filtering for Recursive Estimation of Agricultural-machinery Dynamics

Matthew Cooper * Tristan Perez **

* Electrical Engineering and Computer Sc., Queensland University of Technology, Gardens Point, QLD 4000 Australia (e-mail: m13.cooper@connect.qut.edu.au)
** Institute for Future Environments, Queensland University of Technology, Gardens Point, QLD 4000 Australia (e-mail: tristan.perez@qut.edu.au)

Abstract: This paper explores the parameter estimation problem for a differential-drive agricultural vehicle with unknown parameters. The differential-drive configuration is commonly used in robotic applications as well as in large agricultural machinery such as harvesters. We use simulation scenarios to compare the performance of two dual filters for parameter and state estimation: the Dual Liu and West filter (D-L&WF) and a novel Dual Merging particle filter (D-MPF). Our initial results indicate a slightly better performance of the D-MPF and we discuss the limitations and advantages of each filter. Dual-particle filtering techniques offer a great opportunity for applications to agricultural machinery, and to the best of the author’s knowledge, these techniques have not been previously investigated in this domain of application.

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1. INTRODUCTION

The use of automated machinery in agriculture is rapidly developing as an enabling factor of precision agriculture and the potential use of field robotics (Whelan and Taylor, 2013; Zhang and Pierce, 2013). This is driven by the needs of data collection and also the applications related to crop protection and crop management such as weed and crop nutrition control as well as detection and management of pests and diseases. Motion control of agricultural machinery requires high precision; and therefore, the need for mathematical models of vehicle dynamics, namely, the combination of both kinematic models which describe the geometric aspects of motion, and kinetic models, which describe the causal relations between force and motion.

The structure of the mathematical models associated with the dynamics of agricultural machinery can be derived from classical mechanics through the application of Euler-Lagrange procedures for nonholonomic systems (Bloch, 2015). In this paper, we consider a particular agricultural machinery configuration and use classical mechanics to derive a model structure. Then, we investigate the use of different formulations of particle filters (PF) for the dual state-and-parameter estimation problem. The motivation for investigating recursive estimation techniques stems from the changes in the dynamics of the machines due to different implement configurations as well as changes in the rolling resistance due to soil structure.

We adopt a dual-filter approach (one filter for state estimation and one filter for parameter estimation) instead of a joint approach where the state is augmented with the unknown parameter vector through a random-walk model. Rekow (2001) highlights the computational reduction of the dual approach by comparing a joint extended Kalman filter (EKF) setup with to a dual EKF/least mean squares (LMS) filter setup for estimating the states and unknown parameters of a John Deer Tractor. However, both methods show significant bias due to the linearisations used by the EKF which, depending on the nonlinearities in the system, can lead to filter divergence (Gyrgy et al., 2014; St-Pierre and Gingras, 2004; Arulampalam et al., 2002).

Recently, more advanced dual-filtering approaches have been developed such as the Dual unscented Kalman filter (Hong et al., 2015), the dual ensemble Kalman Filter/particle filter (Santitissadeekorn and Jones, 2015), and the dual particle filter (Cui and Kavasseri, 2017). Although particle-filter based approaches are often suitable for highly nonlinear non-Gaussian systems (Gyrgy et al., 2014; Arulampalam et al., 2002), static parameter estimation using a generic particle filter is difficult due to the fact that the parameter space is only explored during the initial stage of the filter, thus quickly making the filter degenerate to only a few particles with significant weights. The filter by Liu and West (2001) (L&WF) mitigates the particle impoverishment problem by adding artificial noise to the parameters with an additional shrinkage step to prevent the particles becoming overly diffused.

The L&WF has become the standard for online Bayesian parameter estimation (Kantas et al., 2015). We hereafter refer to a dual particle filter that uses a L&WF for parameter estimation (Cui and Kavasseri, 2017) as a dual
potential energy), the Lagrangian of the system simply equals the kinetic co-energy, and this can be expressed as,

\[ L(q, \dot{q}) = \frac{1}{2} m \left( \frac{d}{dt} (q_1 - \ell \cos(q_3)) \right)^2 + \frac{1}{2} m \left( \frac{d}{dt} (q_2 - \ell \sin(q_3)) \right)^2 + \frac{1}{2} I \dot{q}_3^2, \]  

(5)

\[ = \frac{1}{2} m q_1^2 + \frac{1}{2} m q_2^2 + \frac{1}{2} (I + m \ell^2) \dot{q}_3^2 + m \ell \dot{q}_1 \dot{q}_3 \sin(q_3) - m \ell \dot{q}_2 \dot{q}_3 \cos(q_3). \]  

(6)

The Lagrange-D'Alembert equation for nonholonomic systems takes the form

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = A(q) \lambda, \]  

(8)

where \( \lambda \) is the vector of Lagrange multipliers (in our case this is scalar since there is only one constraint), and the term \( A(q) \lambda \) provides the forces that ensure the constraint (2) is satisfied. By making partial derivatives and eliminating the Lagrange multiplier, we can obtain the following state-space model:

\[ \dot{v} = -\ell \omega^2 + \frac{1}{m} R(v, \omega) + \frac{1}{m} F_D, \]  

(9)

\[ \dot{\omega} = \frac{m\ell}{(I + m\ell^2)} \omega v + \frac{1}{(I + m\ell^2)} T_r(v, \omega) + \frac{1}{(I + m\ell^2)} T_D, \]  

(10)

where in (9) we have added the thrust force \( F_D \) produced by the drive train of the machine, and the rolling resistance force \( R(v, \omega) \). Similarly, in (10), we have added the torque \( T_D \) produced by the differential steering of the drive train, and the resistance friction torque \( T_r(v, \omega) \). These force and torque models are specified in the following sections. The total driving force on the machine is the sum of thrust of the left and right wheel, namely,

\[ F_D = F_L + F_R = \frac{N}{r} (T_{Lm} + T_{Rm}), \]  

(14)

where \( N \) is the gear-box ratio, \( r \) is the radius of the wheel, and \( T_{Lm} \) and \( T_{Rm} \) are the left and right motor torques respectively. The torque due to the differential driving is given by,

\[ T_D = -d F_L + d F_R = \frac{dN}{r} (-T_{Lm} + T_{Rm}), \]  

(15)

where \( d \) is the offset of the wheel from the vehicle centre line.

The last component of the model structure accounts for the dissipative terms due to friction. The friction is assumed to be dominated by rolling resistance and modelled as Coulomb friction. Each of the driving wheels contribute to resistance. The velocities of the points of contact of the wheels are, \( v_L = v - \ell \omega \), and \( v_R = v + \ell \omega \).

Henceforth, the resistance force on the left and right can be approximated by,

\[ R_L(v, \omega) = -\frac{2}{\pi} m g \mu_d \arctan(100 v_L), \]  

(16)

\[ R_R(v, \omega) = -\frac{2}{\pi} m g \mu_d \arctan(100 v_R), \]  

(17)

consider two coordinate systems. The system \( \{a\} \) is fixed to the Earth reference frame. The latter reference frame is considered inertial. The coordinate system \( \{e\} \) is fixed to the body (vehicle) reference frame. The heading angle of the vehicle is denoted \( \psi \). The parameter \( \ell \) denotes the offset of the centre of mass relative to the point of interest in body coordinates.

To describe the configuration of the machine, we consider the following vector of generalised coordinates:

\[ q = [x_{P/O}, y_{P/O}, \psi]^T, \]  

(1)

where \( x_{P/O} \) and \( y_{P/O} \) are the components of the relative position of the point \( P \) with respect to \( O \) in inertial coordinates.

The machine velocity is orientated along \( e_1 \), and there is no motion along \( e_2 \) due to the nonholonomic constraint associated with the driving wheels:

\[ A^T(q) \dot{q} = 0. \]  

(2)

with \( A^T(q) = [\sin(q_3) - \cos(q_3) 0] \). The nonholonomic constraint (2) does not restrict the pose of the vehicle, but restricts the trajectories that take the vehicle from one pose to another.

The kinetic co-energy of the system is given by,

\[ K^* = \frac{1}{2} m \dot{x}_C^2 + \frac{1}{2} m \dot{y}_C^2 + \frac{1}{2} I \dot{\psi}^2, \]  

(3)

where \( x_C \) and \( y_C \) denote the inertial coordinates of the centre of mass, and \( I \) the moment of inertia of the machine about its centre of mass. Assuming a flat ground (no
where $g$ is the gravitational acceleration, $\mu_d$ is the dynamic coefficient of friction which depends on the soil type and structure, and the total resistance and the resistance torque are,

$$R(v, \omega) = R_L(v, \omega) + R_R(v, \omega),$$  \hspace{1cm} (18)

$$T_R(v, \omega) = d (-R_L(v, \omega) + R_R(v, \omega)).$$ \hspace{1cm} (19)

### 2.2 Model for Estimation

The model describing the dynamics of the vehicle derived in the previous section can be discretised, and this leads to a general discrete time state space model,

$$x_k = f(x_{k-1}, \theta, u_{k-1}) + n_{k-1},$$ \hspace{1cm} (20)

$$y_k = h(x_k, \theta, u_k) + w_k,$$ \hspace{1cm} (21)

where $x_k$ is the $d$-dimensional state vector of the system, $y_k$ is the $n$-dimensional measurement vector, $u_k$ is the $m$-dimensional input vector, $\theta$ is the $p$-dimensional vector of static model parameters and $k \in \{1, \ldots, N\}$ is the discrete time index. The process noise $n_{k-1} \sim \mathcal{N}(0, Q)$ represents the uncertainty in the state-transition model and $w_k \sim \mathcal{N}(0, R)$ represents the uncertainty in the measurement model.

For our agricultural vehicle, the process noise and measurement noise are both known values. The state of the vehicle is given by $x = [v \ \omega \ \psi \ x_{P/O} \ y_{P/O}]^\top$. We assume that all states are part of the measurement, that is, $y = [v \ \omega \ \psi \ x_{P/O} \ y_{P/O}]^\top$. The unknown parameters of the vehicle model are given by $\theta = [m \ \mu_d \ I \ \delta]^\top$. Lastly, the input to the system is given by the left and right wheel thrust forces, namely, $u = [F_L \ F_R]^\top$.

### 2.3 Dual and Joint Filtering

In a dual filter setup, two filters are used: one to estimate the parameters and one to estimate the states. The parameter filter uses the state estimation from the previous time step to update the parameters, and the state filter uses the current parameters to update the current state. This is illustrated in Fig. 2. Another commonly used approach, which will not be adopted in this paper, is the joint filtering method in which the state is augmented with the unknown parameters. The joint filter simultaneously updates states and parameters. For systems with many unknown parameters, augmenting the state vector can cause a significant increase to the state dimension. This may be problematic for particle filters as their required sample size grows exponentially with the dimension of the state vector (Bengtsson et al., 2008; Snyder et al., 2008). Therefore, we adopt the dual filtering approach due to its lower computational cost.

### 3. ADAPTED PARTICLE FILTERS

For the type of applications considered in this paper, we adopt two particle filters. The first one is based on the work of Liu and West (2001), and it is a standard type of dual particle filter used for online parameter estimation in nonlinear non-Gaussian problems. The second algorithm stems from the work of Nakano et al. (2007), which was developed to reduce the particle impoverishment problem by merging ensembles of particles that preserve the first two moments of the posterior. The key features of these algorithms are reviewed in the following.

We use the D-L&WF filter because it is the standard, however, we investigate the use of a novel dual merging particle filter (D-MPF) which aims at addressing some of the problems that the D-L&WF filter has at the expense of an increase in computations. Therefore, in our study, we compare these two filters.

#### 3.1 The Liu and West Particle Filter

The Liu and West filter uses a kernel smoothing procedure to avoid the particle impoverishment problem common in particle-based parameter estimation. At each time step, the parameter particles are sampled from the distribution,

$$\theta_k^{(i)} \sim \mathcal{N}(m_k^{(i)}, h^2 V_{k-1}), \quad i = 1, \ldots, N_p$$ \hspace{1cm} (22)

$$m_k^{(i)} = a \theta_k^{(i)} + (1 - a) \hat{\theta}_k, \quad i = 1, \ldots, N_p$$ \hspace{1cm} (23)

$$V_{k-1} = \sum_{i=1}^N w_k^{(i)} (\theta_k^{(i)} - \hat{\theta}_{k-1})(\theta_k^{(i)} - \hat{\theta}_{k-1})^\top,$$ \hspace{1cm} (24)

where $h$ is the smoothing parameter which determines the size of the kernel over each particle, $a = \sqrt{1 - h^2}$ is the shrinkage parameter and $\hat{\theta}_k$ is the mean of the parameter particle set. In (23), the parameter particles are shrunk towards the mean of the particle set to ensure that the particles do not become overly diffused when perturbing the particles.

The D-L&WF algorithm (see Algorithm 1) uses the standard L&WF for parameter estimation and a generic particle filter for state inference. The set of weights for the parameter particles is denoted by $\{w_{k,\theta}^{(i)}\}_{i=1}^{N_p}$ and the set of weights for the state particles is denoted by $\{w_{k,x}^{(i)}\}_{i=1}^{N_p}$, where $N_p$ is the total number of particles.

Algorithm 2, is used to estimate the state. This is adapted from Arulampalam et al. (2002), which summarises the generic particle filter for nonlinear state estimation, where $\{w_{k,x}^{(i)}\}$ denotes the set of particle weights and $N_T$ denotes the effective sampling size threshold.

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**Fig. 2. Dual particle filtering approach to state and parameter estimation, where $\{x_{k-1}\}_{i=1}^{N_p}$ denotes the set of $N_p$ particles for the state and $\{\theta_{k-1}\}_{i=1}^{N_p}$ denotes the set of parameters for the set.**


Algorithm 1 Dual Liu and West Filter

Sample $x_0^{(i)} \sim p_0(x), \ i = 1, \ldots, N_p$ \hspace{1em} $\triangleright$ Initialisation
Sample $\theta_0^{(i)} \sim p_0(\theta), \ i = 1, \ldots, N_p$
Initialise $w_{0,x}^{(i)} = \frac{1}{N_p}, \ i = 1, \ldots, N_p$
Initialise $w_{0,\theta}^{(i)} = \frac{1}{N_p}, \ i = 1, \ldots, N_p$

for $k = 1, \ldots, N$ do

Parameter Estimation

Assign weights:

$w_{k,x}^{(i)} \propto w_{k-1,x}^{(i)} p(y_k|x_{k-1}, \theta_{k-1}), \ i = 1, \ldots, N_p$

where $\mu_{k} = \mathbb{E}[x_k|x_{k-1}, \theta_{k-1}]$ and $m_{k-1}$ is calculated from (23).

Resample particle set $\{\theta_{k-1}^{(i)}\}_{i=1}^{N_p}$ using a multinomial resampling scheme with weights $\{w_{k,\theta}^{(i)}\}_{i=1}^{N_p}$.

Sample particles from the distribution

$\theta_{k}^{(i)} \sim \mathcal{N}(m_{k}^{(i)}, h^2V_{k-1}), \ i = 1, \ldots, N_p$

where $m_{k}^{(i)}$ and $V_{k-1}$ are given in (23) and (24) respectively.

Assign weights:

$u_{k,\theta}^{(i)} \propto \frac{p(y_k|x_{k-1}^{(i)}, \theta_{k})}{p(y_k|\mu_{k}, m_{k-1}^{(i)})}, \ i = 1, \ldots, N_p$

Normalise weights:

$u_{k,\theta}^{(i)} = \frac{w_{k,\theta}^{(i)}}{\sum_{i=1}^{N_p} w_{k,\theta}^{(i)}}, \ i = 1, \ldots, N_p$

Estimate the mean of the parameters:

$\hat{\theta}_k = \sum_{i=1}^{N_p} \theta_{k}^{(i)} u_{k,\theta}^{(i)}$

State Estimation

$\{x_{k}^{(i)}\}_{i=1}^{N_p}, \{w_{k,x}^{(i)}\}_{i=1}^{N_p} = \text{PF}\{\{x_{k-1}^{(i)}\}_{i=1}^{N_p}, \{w_{k-1,x}^{(i)}\}_{i=1}^{N_p}, y_k, x, u_{k-1}\}$ \hspace{1em} $\triangleright$ Algorithm 2

Estimate the mean of the state:

$\hat{x}_k = \sum_{i=1}^{N} x_{k}^{(i)} w_{k}^{(i)}$

end for

3.2 The Merging Particle Filter

Nakano et al. (2007) introduced the merging particle filter to increase particle diversity and protect particle filters from degenerating due to impoverishment. The merging particle filter uses a set of $M$ ensembles of particles representing $p(x_k|y_1:k, \theta)$ which are merged during the re-sampling phase of the filter. The ensembles of particles are generated by performing $M$ resamples of the current particle set $\{x_{k}^{(i)}\}_{i=1}^{N_p}$ with weights $\{w_{k,x}^{(i)}\}_{i=1}^{N_p}$. This results in the set of particles $\{x_{k}^{(1,1)}, \ldots, x_{k}^{(1,N_p)}, \ldots, x_{k}^{(M,1)}, \ldots, x_{k}^{(M,N_p)}\}$. The merging ensembles $\{x_{j}^{(j,1)}, \ldots, x_{j}^{(j,N_p)}\}$ are subsets of this large particle set which each approximate the posterior distribution $p(x_k|y_1:k, \theta)$.

The set of ensembles are provided with a respective set of merging weights $\{\alpha_j\}_{j=1}^{M}$. These weights must be selected so that the following constraints are satisfied:

$$\sum_{j=1}^{M} \alpha_j = 1, \quad \sum_{j=1}^{M} \alpha_j^2 = 1 \quad (25)$$

By selecting merging weights that satisfy (25), the mean and co-variance of the particles will be approximately retained during the merging procedure. The merged particle set is generated by combining the $M$ ensembles of particles using their respective merging weights,

$$x_{k}^{(i)} = \sum_{j=1}^{M} \alpha_j x_{k}^{(i,j)}, \ i = 1, \ldots, N_p. \quad (26)$$

The MPF was originally developed for state estimation, but the merging procedure can be easily adapted to the parameter inference problem. We have developed a dual MPF (D-MPF) algorithm (see Algorithm 3) where the parameter estimation filter is based of the MPF and the generic particle filter is used for state estimation. The main difference between parameter estimation using the MPF compared to the L&WF, is that the MPF does not add artificial noise to static parameters. Instead, the MPF produces merged particles that approximate the target distribution with the same mean and co-variance, but not necessarily the same shape.

4. CASE STUDY - AGBOT II

For the case study, our aim is to compare the performance of the D-L&WF with that of the D-MPF for estimating the unknown parameters in the model described in Section 2.1. The four unknown parameter are the mass $m$, coefficient
Algorithm 3 Merging Particle Filter

Sample \( x_0^{(i)} \sim p_0(x) \), \( i = 1, \ldots, N_p \) \( \triangleright \) Initialisation
Sample \( \theta_0^{(i)} \sim p_0(\theta) \), \( i = 1, \ldots, N_p \)
Initialise \( w_{0,x}^{(i)} = \frac{1}{N_p} \), \( i = 1, \ldots, N_p \)
Initialise \( w_{0,\theta}^{(i)} = \frac{1}{N_p} \), \( i = 1, \ldots, N_p \)

for \( k = 1, \ldots, N \) do

Parameter Estimation
Sample parameter particles:
\[
\left\{ \theta_k^{(i)} \right\}_{i=1}^{N_p} = \left\{ \theta_{k-1}^{(i)} \right\}_{i=1}^{N_p} \quad \triangleright \text{Static transition model}
\]
Sample state transition particles:
\[
x_{k,0} \sim p(x_k | \hat{x}_{k-1}, \theta_{k}^{(i)}, u_{k-1}) \quad i = 1, \ldots, N_p
\]
Assign weights:
\[
w_{k,\theta}^{(i)} \propto p(y_k | x_k^{(i)}, \theta_{k}^{(i)}) \quad i = 1, \ldots, N_p
\]
Normalise weights:
\[
w_{k,\theta}^{(i)} = \frac{w_{k,\theta}^{(i)}}{\sum_{j=1}^{N_p} w_{k,\theta}^{(j)}} \quad i = 1, \ldots, N_p
\]
Calculate effective sampling size:
\[
N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N_p} w_{k,\theta}^{(i)}^2}
\]
if \( N_{\text{eff}} < N_T \) then

for \( j = 1, \ldots, M \) do

Create \( M \) ensembles
Generate particle ensemble \( \left\{ \theta_k^{(j)} \right\}_{i=1}^{N_p} \) from the particle set \( \left\{ \theta_k^{(i)} \right\}_{i=1}^{N_p} \) with weights 
\[
\left\{ w_{k,\theta}^{(i)} \right\}_{i=1}^{N_p}
\]
end for

Merge particle ensembles:
\[
\theta_k^{(i)} = \sum_{j=1}^{M} \omega_j \theta_k^{(j)} \quad i = 1, \ldots, N_p
\]
Reset weights
\[
w_{k,\theta}^{(i)} = \frac{1}{N_p}
\]
end if

Estimate the mean:
\[
\hat{\theta}_k = \sum_{i=1}^{N} \theta_k^{(i)} w_{k,\theta}^{(i)}
\]

State Estimation

\[
\left[ \phi_{k}^{(i)} \right]_{i=1}^{N_p} = \text{PF} \left( \left[ \phi_{k-1}^{(i)} \right]_{i=1}^{N_p}, \left\{ w_{k-1,x}^{(i)} \right\}_{i=1}^{N_p}, y_k, x, u_{k-1} \right) \quad \triangleright \text{Algorithm 2}
\]

Estimate the mean of the state:
\[
\hat{x}_k = \sum_{i=1}^{N} x_k^{(i)} w_{k}^{(i)}
\]
end for

of friction \( \mu_d \), moment of inertia \( I \) and the distance from the point of interest to the centre of mass \( \ell \). We assume all states of the system are part of the noisy measurement, with a measurement noise covariance of
\[
R = \text{diag}(0.01 \ 0.01 \ 0.0025 \ 0.01 \ 0.0011).
\]
The process noise covariance matrix is set to
\[
Q = \text{diag}(0.001 \ 0.001 \ 2.5 \times 10^{-4} \ 9 \times 10^{-4} \ 1 \times 10^{-4}).
\]

At \( k = 0 \) the state of the system is initiated to \( x_0 = [0 \ 0 \ \pi/2 \ 10 \ 0] \). The simulation of the vehicle model is run for 240 seconds with a sampling time of 0.01 seconds so that the filters have adequate temporal space to converge. We apply sinusoidal force signals to the left and right wheels of the vehicle to excite the system, namely, \( u_k = [F_{L,k} \ F_{R,k}]^T \), where
\[
F_{L,k} = 100 + 800 \sin \left( \frac{2\pi t_k}{15} \right)
\]
\[
F_{R,k} = 900 - 800 \sin \left( \frac{\pi t_k}{15} \right)
\]

We use a set of 1000 particles for the state filters and a set of 10 000 particles for the parameter filters. The initial particles for the parameter filters are sampled uniformly from a \( p \) dimensional hypercube defined in terms of the following constraints: 100 kg \( \leq m \leq 800 \) kg, 0.05 \( \leq \mu_d \leq 0.5 \), 100 kg, \( m^2 \leq I \leq 800 \) kg m\(^2\), 0.5 m \( \leq \ell \leq 3 \) m, with true value of 500 kg, 0.2, 500 kg m\(^2\) and 1.5 m respectively. The initial state particles are sampled from a normal distribution modelled off the process noise, namely, \( \{ x_0^{(i)} \}_{i=1}^{N_p} \sim N(x_0, Q) \). We use an effective sample size threshold \( N_T \) of 0.3\( N_p \) for both dual filters. For the D-L&WF, we use a kernel smoothing value of 0.99. For the D-MPF, we use four merging ensembles with merging weights of \( \alpha = [0.2 \ 0.3 \ (\sqrt{149}/20 + 1/4) \ (-\sqrt{149}/20 + 1/4)] \).

We run the simulation of the parameter estimation procedure of the two filters 100 times and record the errors between the true parameter values and the final filter parameter estimates. The root mean squares of these errors (RMSE) are summarised in Table 1. A more detailed view of the estimation results for the D-L&WF and the D-MPF which tracks the parameter estimates of the filters throughout a single simulation is displayed in Fig. 3.

Table 1. RMSE of the Parameter Estimations from 100 Simulations Using the D-L&WF and D-MPF.

| Parameter | D-L&WF | D-MPF |
|-----------|--------|-------|
| \( m \)   | 14.3726 | 12.2476 |
| \( \mu_d \) | 0.0083  | 0.0076 |
| \( I \)   | 46.6769 | 34.1945 |
| \( \ell \) | 0.0275  | 0.0239 |

5. DISCUSSION

Table 1 shows slight improvements in all the parameter estimates of the D-MPF algorithm over the L&WF. The most significant difference is in the \( I \) parameter, with the D-MPF having a 26.74% lower RMSE when compared to the RMSE of the D-L&WF.

Fig. 3 clearly illustrates the superiority of the D-MPF when compared to the D-L&WF for this application. At the end of the D-L&WF simulation, all true values of the parameters lie outside of the 95% credible interval of the parameter estimates. In contrast, at the end of the D-MPF simulations, most of the true parameter values lie within the 95% credible interval of parameter estimates.

The artificial noise introduced in the D-L&WF has a larger negative impact to the filter performance than the
merging procedure had to the D-MPF. By shrinking and jittering the set of particles in the D-L&WF, the target distribution is perturbed which causes an accumulation of errors over time. The merging procedure in the D-MPF only occurs in the re-sampling step of the algorithm and attempts to preserve the mean and covariance of the target distribution. However, the shape of the distribution may be changed, causing a statistical linearisation error during the resampling phase.

6. CONCLUSION

The paper considers the problem of parameter estimation in the dynamics of agricultural machinery. Due to constraints, such dynamical models are inherently nonlinear. Hence, the paper investigates the application of two dual-filter algorithms. The standard dual Lui and West particle filter is compared with the dual merging particle filter. The former uses kernel smoothing while the latter uses ensembles from the target distribution which are merged. The comparison is based on the results from a numerical simulation study based on a commonly used agricultural machine configuration with a differential drive. The numerical results show good performance of both filters with slightly better parameter estimation results using the dual merging particle filter.

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