Coupled-mode theory of field transfer processes in surface plasmon resonance structures

D V Nesterenko1,4,*, S Hayashi2,3 and Z Sekkat3,5,6
1Image Processing Systems Institute RAS - Branch of the FSRC “Crystallography and Photonics” RAS, Samara, Russia
2Kobe University, Kobe, Japan
3Moroccan Foundation for Science, Innovation and Research (MAScIR), Rabat, Morocco
4Samara National Research University, Samara, Russia
5Osaka University, Suita, Japan
6Mohammed V University in Rabat, Rabat, Morocco
*E-mail address: dtnesteren@gmail.com
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Abstract. We study the response of the metal-dielectric planar structures supporting surface plasmon polariton modes to an external electromagnetic field. Analysing the field transfer processes within the coupled-mode theory, we demonstrate the role of non-resonant and resonant responses in the appearance of Fano line shapes in spectra of the metal-dielectric interfaces, Otto and Kretschmann ATR configurations.

1. Introduction

Surface plasmon polariton (SPPs) modes supported by metal-dielectric interfaces can be excited using high-index prisms in the attenuated total reflection (ATR) configurations in planar plasmonic structures [1, 2]. The optical properties of surrounding media determine the resonance characteristics of SPP modes and can be characterized by the analysis of resonances in ATR spectra. Experimental and theoretical angle-scan ATR spectra in the Kretschmann and Otto configurations exhibit strongly asymmetric resonance line shapes. However, the SPP ATR resonances have not been considered as asymmetric ones yet. The asymmetry in the resonance profiles was first explained by Fano as an interference of responses from discrete and continuum states of physical systems [3]. To explain the origins of asymmetric resonance line shapes in the ATR spectra of the planar plasmonic structures, we apply coupled-mode (CM) theory to analyze the field transfer processes and provide a deeper insight into the general Fano-like phenomena of mode coupling. The fundamental mechanisms of field transfer and coupling in the planar plasmonic structures are crucial for analysis and design of general resonance structures that support modes of different nature.

2. Coupled-mode theory at metal-dielectric interfaces

The CM approach in optics was implemented for analysis of Fano resonances in terms of interaction of electromagnetic fields [4, 5]. We consider the generation of evanescent field at the metal-dielectric interface by an external wave. In the case of near-field enhancement by resonant excitation of an SPP
mode, the overall response from the interface includes non-resonant and resonant components [6]. The three field transfer processes are distinguished in the formation of these response components. The outcoupling of the non-resonant component of evanescent field, resonant coupling to and outcoupling from the SPP mode to the resonant component are schematically demonstrated in figure 1.

![Figure 1. Field transfer processes of coupling between the external wave and an SPP mode at the metal-dielectric interface.](image)

For the coupling process of an evanescent-field with a SPP mode \( \psi_{\text{SPP}} \propto \exp(i \gamma_{\text{SPP}} x) \) propagating along the interface in a two-layer metal-dielectric system, the dynamic equation for \( \psi_{\text{SPP}} \) induced by incoupling of an evanescent wave \( H \) can be written as

\[
\frac{1}{i \gamma_{\text{SPP}}} \frac{d \psi_{\text{SPP}}}{dx} = \psi_{\text{SPP}} + \kappa H ,
\]

where \( \kappa \) is a coefficient of coupling between the mode and external waves.

The total outcoupled evanescent wave \( H_t \) is composed of a component generated by the non-resonant outcoupling of \( H \) with the continuum coefficient \( r^\circ \) of near-field enhancement and a component resulted from the resonant outcoupling of the mode \( \psi_{\text{SPP}} \) with coupling coefficient \( -\kappa \). Expression for \( H_t \) can be represented as

\[
H_t = r^\circ H - \kappa \psi_{\text{SPP}} .
\]

Assuming arbitrary propagation constant \( \alpha \) along \( x \) axis for the incoupling wave, \( H \propto \exp(i \alpha x) \), the spectral dependence of the mode amplitude \( \psi_{\text{SPP}} \) is represented by a Lorentzian

\[
\psi_{\text{SPP}} = \frac{\gamma_{\text{SPP}}}{\alpha - \gamma_{\text{SPP}}} \kappa H .
\]

The overall near-field enhancement coefficient \( r \equiv H_t / H \) can be obtained by substituting Eq. (3) into Eq. (2) as the interference of the non-resonant and resonant components

\[
r = r^\circ - \frac{\gamma_{\text{SPP}}}{\alpha - \gamma_{\text{SPP}}} \kappa^2 .
\]

These components are indistinguishable in the overall response registered by a detector. Eq. (4) can be represented in a general form of a Fano formula

\[
r = r^\circ \frac{\alpha - \gamma_{\text{SPP}} \left( 1 + \frac{\kappa^2}{r^\circ} \right)}{\alpha - \gamma_{\text{SPP}}} .
\]

Here, \( \kappa^2 \gamma_{\text{SPP}} \) is defined as the complex amplitude of a single SPP mode response, the relation of the resonance amplitude to the continuum amplitude \( \kappa^2 \gamma_{\text{SPP}} / r^\circ \) determines a shift of the zero parameter.
of the function \( r \) from \( \gamma_{\text{SPP}} \). All the considered parameters \( \kappa \), \( \gamma_{\text{SPP}} \), and \( r' \) are the intrinsic parameters of the two-layer structure that depend on the permittivities of both layers.

Based on Eq. (5), we can conclude that the field enhancement spectra \( |r|^{2} \) in the metal-dielectric interfaces are described by the Fano line shape. The appearance of the non-resonant and resonant components in the near-field enhancement coefficient can be also demonstrated analytically by the rigorous electromagnetic theory.

3. Coupled-mode theory in ATR geometries

A metal-dielectric structure considered in the previous section can be coupled to the prism by the spacer layer in two ways. In the well-known Otto and Kretschmann configurations, the spacer layer is a dielectric and a metal, respectively. Excitation of SPP modes at a metal-dielectric interface is performed by evanescent waves generated by a high-index prism under ATR conditions. These configurations can be represented by a three-layer structure, in which the prism and outer layer are denoted as semi-infinite layers \( L_{0} \) and \( L_{2} \), respectively; the spacer layer of a finite thickness \( h \) separates the layers \( L_{0} \) and \( L_{2} \) and is denoted as \( L_{1} \) as depicted in figure 2. Therefore, the spacer layer \( L_{1} \) is a dielectric in the Otto configuration and a metal in the Kretschmann configuration.

![Figure 2. Field transfer processes of SPP mode excitation in three-layer structures.](image)

We develop CM theory for the field transfer processes in three-layer resonance systems. Neglecting the non-resonant reflection from the metal-dielectric interface, the four field transfer processes can be outlined. As schematically demonstrated in figure 2, an incident propagating plane wave \( H \) is split onto a beam \( \phi \) non-resonantly reflected from the prism/spacer layer interface and an evanescent wave in the spacer layer. The generated evanescent field of the complex amplitude \( \tau \) at the spacer layer/outer layer interface resonantly incouples to a SPP mode \( \psi_{\text{SPP}} \) with coupling coefficient \( \kappa \).

The excited SPP mode \( \psi_{\text{SPP}} \) resonantly outcouples into the evanescent field with coefficient \( -\kappa \). The generated evanescent wave exponentially decays with distance from the metal-dielectric interface. At the prism/spacer layer interface, the decayed evanescent field generates two waves. One is a wave propagating in the prism. The amplitude of the propagating wave is proportional to the decay coefficient \( \rho \). The second wave is an evanescent wave, which reflects back to the spacer layer/outer layer interface and couples again to \( \psi_{\text{SPP}} \). The amplitude of this evanescent wave at the spacer layer/outer layer interface is proportional to the decay coefficient \( \eta \). Due to the resonant outcoupling, the reflected wave \( \rho \) and the evanescent wave \( \eta \), which is coupled back to the SPP mode, have resonant behavior.

Based on the field transfer processes, we can write generalized CM equations to describe the field transfer dynamics for the outgoing reflected wave \( H_{r} \) represented as a sum of the resonant and non-resonant components

\[
\frac{1}{i\gamma_{\text{SPP}}} \frac{d\psi_{\text{SPP}}}{dx} = \psi_{\text{SPP}} - \eta \kappa \psi_{\text{SPP}} + \tau \kappa H ,
\]  

(6)
\[ H_i = \phi H - \rho \kappa \psi_{SPP} . \]  

(7)

The amplitude of \( \psi_{SPP} \) is obtained from Eq. (6) as a Lorentzian function:

\[ \psi_{SPP} = \frac{\gamma_{SPP}}{\alpha - \gamma_{SPP}(1-\eta \kappa^2)} \tau \kappa H . \]  

(8)

The reflection coefficient \( r \equiv H_i / H \) is obtained by substitution of Eq. (8) into Eq. (7)

\[ r = \phi \frac{\alpha - \gamma_{SPP}[1-(\eta - \rho \tau / \phi)\kappa^2]}{\alpha - \gamma_{SPP}(1-\eta \kappa^2)} . \]  

(9)

The position and depth of the SPR dip are determined by the zero parameter \( \gamma_{SPP}[1-(\eta - \rho \tau / \phi)\kappa^2] \).

In the three-layer system, the mode self-coupling \(-\eta \kappa^2 \gamma_{SPP}\) in the spacer layer leads to the shift of \(-\eta \kappa^2 \gamma_{SPP}\) in the pole and zero values in respect to its intrinsic value \( \gamma_{SPP} \). The weaker field decay in the spacer layer and stronger coupling degree result in a larger shift of the propagation constant. In addition to the influence of self-coupling \(-\eta \kappa^2 \), the resonance dip parameter in the three-layer system is perturbed by the coupling of external fields with the mode and non-resonant reflection from the structure.

The reflection coefficient \( r \) can be also expressed as the interference of the non-resonant reflection from the prism interface and a resonant component arising from the SPP excitation that results in a sum of a Lorentzian function and a continuum

\[ r = \phi \frac{\gamma_{SPP}}{\alpha - \gamma_{SPP}(1-\eta \kappa^2)} \rho \tau \kappa^2 . \]  

(10)

Eqs. (9) and (10) represent Fano formulas, and the reflectivity \( |r|^2 \) exhibits an asymmetric Fano line shape. Therefore, the presented model of the field transfer processes can be effectively used to describe the asymmetric resonance responses in the ATR structures.

4. Conclusions

The general expressions obtained by the CM theory for the spectral responses of the two- and three-layer planar plasmonic structures demonstrate that the structures acts like non-tunable and tunable resonators, respectively. In both cases, the appearance of resonant and non-resonant components in the response results in the Fano interference, where the origins of non-resonant component are different from those of the resonant one. The resonance features of the asymmetric Fano line shape in spectra of the considered structures are determined by the coupling strength between the SPP mode and the external field. In the case of the three-layer structure, the spacer layer allows the self-interaction of the SPP mode that modifies the position and line shape of the Fano resonances. The obtained results may open a new avenue in studies on field transfer processes in matter.

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