Research progress on calculation method of tooth surface friction coefficient

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Abstract. Firstly, the research significance of tooth surface friction is introduced. The difficulties in calculating the friction factor of tooth surface are analysed. Secondly, the current calculation method of the friction factor of the tooth surface is introduced systematically. the method of calculating the friction factor of the tooth surface based on elastohydrodynamic lubrication theory and the method of calculating the friction factor of the tooth surface based on the friction characteristics test of the tooth surface.

1. Research significance of tooth surface friction

Gear mechanism is the most widely used transmission mechanism. Scientists have never stopped studying the gear mechanism. Because of its complicated mechanism, tooth surface friction is always one of the hot and difficult points in the study. The main effects of gear surface friction on gear transmission include: reduction of gear transmission efficiency, aggravation of gear tooth failure (tooth surface wear, tooth surface pitting, tooth surface cementation, etc.), system vibration and noise [1, 2]. As the two-stage gear reducer, its transmission efficiency is about 97.6%~98.5%, and its power loss mainly comes from gear, bearing and oil seal. Among them, bearing loss accounts for about 25%~35% of the total power loss, oil seal loss for about 0.5%~1.5% of the total power loss, and gear loss accounts for about 65%~70% of the total power loss. Tooth surface friction is the main reason of gear loss. Therefore, the study of tooth surface friction has great economic benefits and is of great significance to improve the performance of gear transmission system.

The second paragraph is indented. According to the theory of gear transmission, the two gears are pure rolling at the joints, and the gears are rigid, so there is no friction in gear transmission. In the actual generation, the gear is not rigid, and there is inevitable elastic deformation. The elastic deformation makes the gear meshing no longer pure rolling in theory, and the actual machining accuracy and installation precision cannot achieve the theoretical accuracy, which also makes the actual meshing different from the theory. No longer pure rolling, there will be tooth surface friction. Compared with the general contact surface friction, the particularity of the friction mainly lies in four aspects: 1) in the process of gear transmission, the size and direction of tooth surface friction change with the change of speed, load and coincidence degree[3];2) factors such as material property, surface roughness, temperature distribution of tooth surface, density and viscosity of lubricating oil, etc. can directly or indirectly affect the friction of tooth surface[4];3) during transmission, there may be both line contact and surface contact between tooth surfaces[5];4) there are many types of gear transmission and complex working conditions. Current theories and experimental calculation methods cannot fully
reflect the actual situation. Therefore, it is more difficult to calculate the friction factor of tooth surface than that of contact surface. According to different research methods, the current calculation methods of tooth surface friction factor are divided into two categories: 1) calculation method of tooth surface friction factor based on elastohydrodynamic lubrication theory; 2) calculating the friction factor of tooth surface based on the friction characteristic test of tooth surface.

2. Calculation method of friction factor of tooth surface based on elastohydrodynamic lubrication theory

The use of lubricants has become quite common in modern industry. The friction pairs of the mechanism are lubricated to different degrees to form various lubricating states. According to the formation principle and characteristics of lubricating oil film, lubrication state can be divided into: 1) hydrodynamic lubrication; 2) hydrostatic lubrication; 3) elastohydrodynamic lubrication (elastohydrodynamic lubrication for short); 4) film lubrication; 5) boundary lubrication; 6) dry friction state. In the actual mechanical structure, two or more lubrication states often exist in the same friction pair, which is called mixed lubrication. The following describes the calculation methods of friction factors of tooth surfaces in three states of elastohydrodynamic lubrication, boundary lubrication and mixed lubrication [6].

2.1. Elastic hydrodynamic lubrication

The current elastohydrodynamic lubrication theory and the calculation formula of friction factor are established for the stable state. The typical calculation method of Dowson-Higginson theory is the numerical method of linear contact isothermal total membrane elastic flow [7].

According to the elastohydrodynamic lubrication theory, the formula for calculating the friction factor of linear contact isothermal total membrane elastohydrodynamic numerical solution was obtained:

\[
\int \frac{u_2 - u_1}{F_n h_{min}} \int_{-a}^{+a} \eta dx
\]

where \( u_1, u_2 \) are the meshing speed of the two gears, \( F_n \) is the normal force of the gear; \( h_{min} \) is the minimum oil film thickness at the mesh point; \( \eta \) is viscosity of lubricating oil. \( h_{min} \) is a very important evaluation indicator. And its empirical formula is given as follow:

\[
h_{min} = 2.65G^{0.54}U^{0.7}W^{-0.13}
\]

where \( G \) is the material parameter; \( U \) is the speed parameter; \( W \) is the load parameters.

The Dowson formula has been proved by many experiments. As an important result of the ideal elastohydrodynamic stage, it is widely accepted and used in the calculation of high side transmission. It should be pointed out that the above calculation formula can be more practical to solve the friction problem of involute tooth profile. However, for other types of tooth profile types, the direct application of this formula will cause large errors. In addition, under the condition of light load, high speed and low elastic modulus, the calculation error will also increase.

2.2. Boundary lubrication

Boundary lubrication was first proposed by Hardy in 1919 to describe a lubrication state between liquid lubrication and dry friction. Later, the contribution of F.P Browdon, D Tabor et al. made the development of boundary lubrication theory more and more perfect, and it was called as an important theoretical basis to improve the lubrication performance of gear transmission.

In gear drive, boundary lubrication is objective. For example, when entering the area near meshing, driven tooth tip is scraped along the active tooth profile, and the dynamic oil film is damaged, which mainly exists in the form of boundary lubrication. The boundary lubrication mechanism is complex and difficult to be tested and analyzed. Therefore, there is still no unified calculation formula and the
application is still in the empirical stage. The adhesion effect and furrow effect in friction wear of tooth surface are significantly influenced by boundary lubrication.

The friction factor of boundary lubrication measured by the test is generally: \( f = 0.1 \sim 0.2 \). For the calculation of friction factor of tooth surface under boundary lubrication, Buckingham [8] semi-empirical formula is more used:

\[
B_0 = 0.05e^{-0.125V_s} + 0.002\sqrt{V_s}
\]  

\[ (3) \]

2.3. Mixed lubrication
As mentioned above, the friction factor of tooth surface in gear transmission changes significantly with the change of speed, load size, coincidence degree, tooth shape and other factors. Affected by the above factors, the lubrication state of gear teeth oscillates between elastohydrodynamic lubrication and boundary lubrication. As a matter of fact, mixed lubrication is the contact state widely existed in gear transmission, and it is a common combination of elastohydrodynamic lubrication, boundary lubrication and film lubrication.

Based on the theory of elastohydrodynamic lubrication and combined with experimental research, Kelley, Lemanski, Martin and others successively proposed different formulas for calculating the friction factor. However, due to the randomness of tooth surface roughness and the time-varying surface contact state of gear teeth in the process of rolling and relative sliding, the friction characteristics of gear teeth in the mixed lubrication state are very complicated, so far no perfect physical model and relevant theories have been established.

Another method for calculating the friction factor of the tooth surface in the mixed lubrication state is that the comprehensive friction factor \( f \) is composed of the friction factor \( f_a \) in the boundary lubrication state and part of the liquid friction factor \( f_{EHD} \). The formula is as follows:

\[
f = f_a q_a + f_{EHD} q_{EHD}
\]  

\[ (4) \]

where \( q_a \) and \( q_{EHD} \) are respectively the load factor of peak contact and the load factor of elasto-flow lubricating oil film, which are all measured by corresponding experiments. Both of them satisfy \( q_a + q_{EHD} = 1 \). \( f_a \) is determined by the contact properties of the surface and can be measured by experiments. \( f_{EHD} \) is not a constant, but a function of the sliding roll ratio of the meshing gear teeth.

3. The method of calculating the friction factor of tooth surface based on the test of friction characteristics of tooth surface
As mentioned above, the friction factor of tooth surface is affected by loading, tooth surface material, tooth surface roughness, tooth shape, temperature, lubrication state, lubricating oil viscosity and other factors. In addition, the friction factors of tooth surfaces are time-varying and strongly nonlinear. Therefore, the physical model based on elastohydrodynamic lubrication theory is very difficult to analyze and solve the tooth surface friction. Moreover, the calculation formula is too simplified, which affects the reliability of the result. Therefore, the method of calculating the friction factor of the tooth surface is developed by using the friction characteristics of the tooth surface.

3.1. The tooth surface factor is calculated based on the weight pendulum method
Hori[9] et al. proposed a new gravity pendulum method which can accurately measure the friction coefficient of a pair of gears under quasi-static conditions. And the gravity pendulum method effectively removes the influence of bearing loss on the friction factor of tooth surface. The gravity pendulum method equates the energy loss caused by friction of tooth surface to the energy loss caused by gravity during the oscillation process. The friction factor of tooth surface is calculated by measuring the Angle of gravity pendulum. The gravity pendulum test bed and its test device are shown in the following Figure 1 and Figure 2.
As shown in figure 2, the internal gear and pinion are connected by flexible hinge, and the center of the two gears, $O_1, O_2$, coincide with fulcrum of flexible hinge. The pinion is fixed, the internal gear can rotate, and the meshing point of the two gears is point A. At the same time, the center of gravity pendulum rotation is installed on the horizontal position of the base circle of the inner gear, so that the cycloid of gravity pendulum and the occurrence line of involute tooth profile coincide. In other words, the gravity pendulum's suspension point and meshing point A are always in a straight line. When gravity swings to and fro, the gears also engage in small areas. Since the two gear centers are on the fulcrum of the flexible hinge, the center distance between the two gears remains unchanged during the meshing process. Therefore, the actual motion at mesh point A can be accurately simulated with no bearing loss. In the experiment, it is assumed that the energy loss of gravity pendulum is all due to tooth surface friction. The pendulum angle at any moment is measured by the plane mirror and the laser transmitter, and the energy loss of tooth surface friction is calculated, and then the friction factor of tooth surface is calculated.

The energy loss in the process of gravity pendulum oscillation $E$ computation formula is as follows:

$$E = Mgh(\cos\theta_i - \cos\theta_{i+2N})$$  \hspace{1cm} (5)

where $M$ is the mass of the gravity pendulum, $g$ is the acceleration of gravity, $h$ is the distance from the center of gravity pendulum rotation to the center of gravity of the pendulum. The meshing process of internal gear and pinion includes the meshing interval of one tooth and the meshing interval of two teeth. As the calculation of two-tooth meshing includes single-tooth meshing, the calculation method of two-tooth meshing is only introduced below.
\[
\begin{align*}
W_1 &= 2\mu_1\lambda_1 Mg (1 - \frac{r_{g1}}{r_{g2}}) e_1 \sum_{j=1}^{i+2N-1} \theta_j \\
W_2 &= 2\mu_2\lambda_2 Mg (1 - \frac{r_{g1}}{r_{g2}}) e_2 \sum_{j=1}^{i+2N-1} \theta_j
\end{align*}
\]

where \(W_1\) and \(W_2\) are the friction energy loss caused by two meshing points respectively; \(\mu_1\) and \(\mu_2\) are the friction coefficients at the mesh point; \(\lambda_1\) and \(\lambda_2\) are the load sharing ratios; \(r_{g1}\) and \(r_{g2}\) are the radius of the base circle; \(e_1\) and \(e_2\) are the distance between mesh point and node; \(\theta_j\) is the angle of gravity pendulum.

It is assumed that the load distribution ratio of the two meshing points is equal to the friction factor. Therefore, the total friction loss is:

\[
W = W_1 + W_2 = \mu Mg (1 - \frac{r_{g1}}{r_{g2}}) (e_1 + e_2) \sum_{j=1}^{i+2N-1} \theta_j
\]

According to E=W, the formula of tooth surface friction factor at meshing point is deduced as follows:

\[
\mu = \frac{h (\cos \theta_i - \cos \theta_i + 2N)}{(1 - \frac{r_{g1}}{r_{g2}}) (e_1 + e_2) \sum_{j=1}^{i+2N-1} \theta_j}
\]

The gravity pendulum method not only simulates the meshing process of the gear accurately, but also eliminates the influence of bearing loss. Therefore, the measurement precision is higher. In addition, the distribution of the friction coefficient of the entire tooth surface can be measured by changing the initial meshing point of the gear. However, the gravity pendulum method also has its shortcomings. Firstly, the gravity pendulum method measures the friction factor of the tooth surface under quasi-static conditions, so it cannot reflect the influence of rotating speed on the friction factor. Secondly, the effect of the temperature rise on the friction of the tooth surface is neglected.

3.2. Calculation method of tooth surface friction factor based on the equivalence principle of power loss and friction loss

At present, the method of measuring gear efficiency based on power flow is used to calculate the friction factor of tooth surface. There are two main methods for measuring gear efficiency: open power flow (Figure 3) and closed power flow (Figure 4) [10]. The former uses a loading device (mechanical brake, electromagnetic dynamometer or magnetic powder brake) to consume the energy transmitted by the gear drive. Its advantages are consistent with the actual working situation, simple and easy to operate, and convenient to install the experimental device. The disadvantage is that the power consumption is high, especially in the case of long time test (such as fatigue test). The latter adopts the method of output power feedback to input. The power supply only supplies the power consumed by the friction resistance in the gear transmission, which can reduce the power consumption. Therefore, this experimental scheme is widely adopted.
The total power loss of the gear box is calculated by measuring the speed and torque of input and output with the rotational speed and torque sensor. The transmission efficiency of gear box is calculated. The friction loss in the bearing is separated from the loss in the gear box, and the friction factor of the tooth surface is calculated as follows [11]:

\[
\mu = \frac{\eta}{\pi \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \left( \varepsilon_1^2 - \varepsilon - 1 - \varepsilon_1 \varepsilon_2 \right)}
\]  

(9)

In fact, the factors influencing the efficiency of gear box include bearing loss, oil loss, wind loss, etc. Whether the friction loss of the tooth surface can be effectively separated directly affects the reliability of the method. In addition, the calculation method based on the transmission efficiency of power flow gear is used to calculate the "effective" or "equivalent" friction factor of gear box under this working condition, that is, both sliding friction and rolling friction are included. Therefore, it cannot reflect the real friction of different contact points in the actual meshing period of gear teeth.

4. Conclusions
The importance of the friction of the tooth surface and the attention to this issue has promoted its development. In this paper, the method of calculating the friction factor of tooth surface is introduced theoretically and experimentally. It can be seen that there are many deficiencies in the calculation of the friction factor of tooth surface both theoretically and experimentally. There are mainly the following aspects:

1) The basic theory needs to be perfected. Based on the current lubrication theory, there are several basic lubrication states during gear meshing. Therefore, the in-depth study of basic lubrication state is of great significance for the friction of tooth surface.

2) During the test, the actual working condition of the gear cannot be simulated, or the influence of other factors in the system cannot be eliminated. Therefore, the accuracy and reliability of the test results are reduced.

3) Gear mechanism and condition type is complex. A test is usually effective only for specific gear types or conditions. Therefore, the conclusion also has considerable limitations.

To sum up, tooth surface friction is the result of the comprehensive action of various factors. The complexity is not only in the variety of factors, but also in the mutual influence of various factors. For example, tooth surface friction causes the tooth surface temperature to rise. The increase of temperature affects the viscosity of lubricant and the material properties of tooth surfaces. Therefore, the combined action of the friction generating machine and various parameters is a problem that needs further consideration.

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