The Action with Manifest E7 Type Symmetry

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Abstract: We generalize Cremmer-Julia 1st order action of $\mathcal{N} = 8$ supergravity with manifest $E_7(7)$ symmetry to cases of $\mathcal{N} = 6$ with manifest $SO^*(12)$ and $\mathcal{N} = 5$ with manifest $SU(1,5)$ duality symmetries. These U dualities belong to groups of type E7 which do not admit a symmetric symplectic bilinear invariant. Therefore the 2d order classical action derived from the one with manifest E7 type duality has a ghost vector field which, under appropriate boundary conditions, decouples. We show that when classical $\mathcal{N} \geq 5$ supergravities are deformed by a candidate UV divergence the ghost field does not decouple. We argue that this means that U duality and supersymmetry explain the mysterious cancellation of UV infinities at $L = 4$, $\mathcal{N} = 5$ in d=4. The same argument suggests that the absence of duality and supersymmetry anomalies would imply higher-loop UV finiteness of $\mathcal{N} \geq 5$ supergravities.
1 Introduction

The $E_7(7)$ duality symmetry in $\mathcal{N} = 8$ supergravity [1–3] was discovered by Cremmer and Julia. In general case of extended supergravities the scalars are coordinates of the $G/H$ coset space, where $G$ is a U duality group and $H$ is an isotropy. Duality symmetries in nonlinear electrodynamics and extended supergravity were studied in [4, 5].

In $\mathcal{N} = 5, 6, 8$ supergravities the relevant duality groups $G$ are: $SU(1, 5), SO^*(12), E_7(7)$. They are known as groups of type $E_7$, [6], [7]. The isotropy groups $H$ are: $U(5), U(6), SU(8)$.
for $\mathcal{N} = 5, 6, 8$ supergravities, respectively. $\mathcal{N} \geq 5$ supergravities, and their deformation due to candidate UV divergences, were studied recently in [8], developing the deformation proposal [9]. A significant progress in studies of extended supergravities was achieved there, by making the analysis of duality symmetry universal for all $\mathcal{N} \geq 5$ supergravities, based on symplectic section formalism [10]. These theories are anomaly free as shown in [11, 12], and therefore one expects that symmetries control quantum corrections. However, an attempt to extract an information about UV infinities in $\mathcal{N} \geq 5$ in [8], based on a deformation proposal [9], was inconclusive.

A part of duality symmetry, based on soft limit on scalars, have been shown to explain some of the properties of UV finite amplitudes in extended supergravities. For example, in [13] it was argued that $\mathcal{N} = 8$ supergravity is protected from UV divergences in $d=4$ up to 6 loop order, based on supersymmetry and $E_{7(7)}$ symmetry. This explains the computations in [14, 15] which have shown UV finiteness at 3 and 4 loops. But the prediction of [13] is still to be confirmed at 5 and 6 loops in $\mathcal{N} = 8$ $d=4$. The analysis in [13], based on soft limit on scalars due to $E_{7(7)}$ symmetry, is inconclusive starting from 7 loops.

Meanwhile for $\mathcal{N} = 8$ other arguments were given about all-loop finiteness based on the light-cone formalism [16] or on $E_{7(7)}$ symmetry in the vector sector of the theory [17]. The $E_{7(7)}$ symmetry argument in [17] was disputed in [9], where a proposal was made that the $E_{7(7)}$ symmetry can be restored even in presence of the candidate counterterms. The proposal is based on a construction of the source of deformation with a manifest $E_{7(7)}$ symmetry, where instead of a physical vector in representation $28, 28$ of $SU(8)$ one has to use a symplectic doublet with twice as many vector fields. An improved version of this proposal was developed in [18] and it was applied to the Born-Infeld theory, as well as Born-Infeld theory with higher derivatives [19]. The proof of consistency of the deformation proposal in bosonic theory was given in [9] only at a base point of the moduli space, where all scalar fields vanish.

The compatibility of the deformation proposal with supersymmetry was questioned in [20] and in [21] and obstructions to this deformation were pointed out. The actual computations in $d=4$ $\mathcal{N} = 8$ at 7-loop level, which would resolve the issues above, are far too difficult, and results are not expected anytime soon.

However, for $\mathcal{N} = 5$ it became known about four years ago that UV divergences are absent at 3 and 4 loop level [22]. Until recently there was no explanation of these computations. The current situation is the following. The soft scalar limit analysis in [13] for $\mathcal{N} = 8$ was generalized for the case of $\mathcal{N} = 5, 6$ in [23]. The result is that consistency of the soft limit on scalars of amplitudes with duality and supersymmetry for $\mathcal{N} \geq 5$ requires that at the loop order $L = \mathcal{N} - 2$ the theory is protected from UV divergences. Thus, $\mathcal{N} = 5$ has to be UV finite at 3 loops, which explains the computation in [22]. $\mathcal{N} = 6$ has to be UV finite at 4
loops, \( \mathcal{N} = 8 \) has to be UV finite at 6 loops, which are predictions still to be validated. The case of a critical loop order

\[
L_{cr} = \mathcal{N} - 1
\]  

(1.1)

for all these theories, \( \mathcal{N} = 5 \) at 4 loops, \( \mathcal{N} = 6 \) at 5 loops, \( \mathcal{N} = 8 \) at 7 loops, remains elusive when only the soft limit analysis of amplitudes following from duality is combined with supersymmetry. A harmonic superspace analysis of available supersymmetric and duality invariant counterterms was performed in [24], where UV divergence was predicted at \( L_{cr} = \mathcal{N} - 1 \) for \( \mathcal{N} \geq 5 \).

Thus, not a single explanation of the \( \mathcal{N} = 5, L_{cr} = \mathcal{N} - 1 = 4 \) UV finiteness discovered four years ago in [22] is available at present. Why UV infinities in 82 diagrams cancel? We show the corresponding set of diagrams in the Appendix A. The first hint for such an explanation is the fact that the proof of consistency of the deformation [9] in supergravity with scalars is still missing, see more on this in our Appendix B. The second hint is in [20, 21], where the supersymmetry obstructions to the deformation proposal in [9] were exposed.

Here we start a new direction of investigation using the 1st order formalism [2] with manifest \( E_7(7) \) symmetry and generalizing it to \( \mathcal{N} \geq 5 \) supergravities with their relevant dualities. The most important for our purpose property of theories with \( E_7 \) type symmetries is the absence of a bilinear symplectic symmetric invariant. This is a fundamental reason why we encounter bad ghosts.\(^1\) In classical theory these bad ghosts decouple, as shown in [2]. However, when deformations due to candidate UV divergences are included, bad ghosts do not decouple, as we will show below. In what follows we will use the word ‘ghost’ in the paper meaning the ‘bad ghosts’, with exception of Appendix C where both ‘good’ and ‘bad’ will be defined.

We will conclude that the 2d order deformed theory, which follows from the 1st order one, with guaranteed \( E_7 \) symmetry, is inconsistent since it has ghosts. Meanwhile, when one starts with the consistent 2d order theory without ghosts, the proof of \( E_7 \) symmetry of the deformed 2d order theory in [9] is not valid in presence of scalars, as we argue in our Appendix B.

In view of the new observations that \( E_7 \) symmetry is inconsistent with the 4-vector UV divergence in \( \mathcal{N} = 5 \) supergravities, and assuming unbroken supersymmetry, which requires all other 4-point UV divergences to show up in computations with the same factor as a 4-

\(^1\)We remind in Appendix C that there are good ghosts and bad ones. The good ones, introduced originally by Faddeev and Popov and called by them ‘fictitious particles’, are present in Lorentz covariant gauges in gauge theories. Their role is to cancel the bad ghosts so that the physical amplitudes are equivalent to the ones in the unitary canonical gauges, where all ghosts are absent and only physical fields propagate.
vector one, we suggest that these observations explain the UV finiteness of $\mathcal{N} = 5$ at four loop order, which was not explained so far.

2 The manifestly E7 invariant vector action for $\mathcal{N} \geq 5$ supergravity

We explain here the important technical features of the symplectic section formalism, developed in [10] and used in the studies of UV infinities of perturbative supergravity in [20] and, more recently, in [8].

In $\mathcal{N} = 8, 6, 5$ the number of physical vectors is $n_v = (28, 16, 10)$ respectively. However, the manifest E7 type symmetry in these models requires that the action depends on duality doublets, which have twice an amount of vectors: $n_{2v} = (56, 32, 20)$. Therefore we will need also another vector duality doublet in the action, as well as a Lagrange multiplier to a duality invariant constraint and a doublet depending on scalars, to be able to construct an action, classical or deformed, with a manifest E7 type symmetry.

2.1 Bilinear symplectic invariants and graviphotons

Consider a $n_{2v}$-dimensional real symplectic vector of field strengths

$$\mathcal{F} \equiv \begin{pmatrix} F^A \\ G_A \end{pmatrix}. \quad (2.1)$$

that transforms in the $\mathbf{56}, \mathbf{32}, \mathbf{20}$ of the corresponding duality groups. The scalars of the theory are described by the symplectic section

$$\mathcal{V}_{AB} \equiv \begin{pmatrix} f^A_{AB} \\ h^A_{AB} \end{pmatrix}, \quad (2.2)$$

where $A, B = 1, \cdots, \mathcal{N}$ are an antisymmetric pair of indices that are raised and lowered by complex conjugation. The period matrix is defined as follows $h^A_{AB} = F^A_{\Lambda} f^{\Sigma} \Sigma_{AB}$. For the symplectic product $\langle \ | \rangle$, we use the convention

$$\langle A \ | B \rangle \equiv B^\Lambda A_\Lambda - B_\Lambda A^\Lambda. \quad (2.3)$$

The graviphoton field strength is defined by

$$T_{AB} \equiv \langle \mathcal{V}_{AB} \ | \mathcal{F} \rangle, \quad (2.4)$$

and its self- and anti-selfdual parts are

$$T_{AB}^{\pm} \equiv \langle \mathcal{V}_{AB} \ | \mathcal{F}^\pm \rangle, \quad T_{*AB}^{\pm} \equiv \langle \mathcal{V}^{AB} \ | \mathcal{F}^{\pm} \rangle. \quad (2.5)$$

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Gravitiphotons are duality invariant, they transform under compensating $SU(N)$ transformations only. In classical $\mathcal{N} \geq 5$ supergravity in absence of fermions there is a linear twisted self-duality constraint

$$
T_{AB}^+ = h_{\Lambda AB} F^\Lambda_{\mu\nu} - f_{\Lambda A}^B G^\Lambda_{\mu\nu B} = 0, \quad T^{*-AB} = \bar{h}^\Lambda_{AB} F_{\mu\nu}^\Lambda - \bar{f}^{\Lambda AB} G_{\mu\nu}^\Lambda = 0. \quad (2.6)
$$

It results in the relation between $G$ and $F$, so that only one of them is independent

$$
G^+ = \mathcal{N} F^+, \quad G^- = \mathcal{N} F^-.
$$

This gives a correct amount of the physical degrees of freedom for vectors, $28, 16, 10$ and is one-half of the symplectic representation of the E7 type symmetry for $E_7(7)$, $SO^*(12)$ and $SU(1,5)$ duality, respectively.

### 2.2 Vectors and fermions in classical theory

In classical supergravity adding fermions in the context of duality requires to add to the bosonic part of the action which is quadratic in $F$, also a term linear in $F$, which is scalar dependent and quadratic in fermions as well as a scalar dependent term quartic in fermions. The $\mathcal{H}$-covariant combination of fermions $\mathcal{O}_{\mu\nu AB}^+, \mathcal{O}_{\mu\nu AB}^-$ has terms with products of two spin 1/2 fields, two gravitino’s and a spin 1/2 and a gravitino, shown for example in eq. (7) in [1] and in eq. (2.22) in [3] in $\mathcal{N} = 8$ theory.

We present the relevant part of the classical $\mathcal{N} \geq 5$ supergravity action in terms of a symplectic section formalism [10] which was used recently in [8] in the bosonic theory without fermions, here we also include fermions. Our discussion is universal for $\mathcal{N} = 5, 6, 8$.

$$
S = F \tilde{G} - i T_{AB}^+ \mathcal{O}^{-AB} + i T^{*-AB} \mathcal{O}^+_{AB}. \quad (2.8)
$$

The modified constraint in presence of fermions relating $G$ to $F$ and scalars and fermions is

$$
T_{\mu\nu AB}^+ \equiv h_{\Lambda AB} F_{\mu\nu}^{+\Lambda} - f_{\Lambda A}^B G_{\mu\nu B}^+ = i \mathcal{O}^+_{AB}, \quad (2.9)
$$

which means that

$$
G_{\mu\nu}^+ = -i \int_{\Lambda}^{1AB} \mathcal{O}_{AB}^+ + \mathcal{N}_{\Lambda\Sigma} F^{+\Sigma}. \quad (2.10)
$$

We can also present the action (2.8) as

$$
S = i F^- G^- - i F^+ G^+ - i T_{AB}^- \mathcal{O}^{-AB} + i T^{*-AB} \mathcal{O}^+_{AB}, \quad (2.11)
$$

since $\tilde{G} = i (G^- - G^+)$. We define the constraint in presence of fermions as

$$
T_{\mu\nu AB}^+ \equiv h_{\Lambda AB} F_{\mu\nu}^{+\Lambda} - f_{\Lambda A}^B G_{\mu\nu B}^+ - i \mathcal{O}_{AB}^+ = 0. \quad (2.12)
$$

Now we prepared the tools we need to present manifestly E7 type symmetric actions for classical $\mathcal{N} \geq 5$ supergravities.
2.3 The 1st order action

The action in the 1st order formalism depending on vectors is manifestly invariant under $G$-duality, i.e. $E_7$ type symmetry, as well as under $H$ isotropy group, $SU(8), U(6), U(5)$ for $N = 8, 6, 5$ respectively.

$$\mathcal{L}^{1st} = -\langle \mathcal{F}_1 | \tilde{\mathcal{F}}_2 \rangle + iT^*_{AB} + i L^{AB+} T_{AB2}^+ O + h.c. \quad (2.13)$$

This action is the same, in different notations, as the 1st order formalism action presented for $N = 8$ in [1, 2]. Here

$$-\langle \mathcal{F}_1 | \tilde{\mathcal{F}}_2 \rangle \equiv -\tilde{F}_2^\Lambda G_{1\Lambda} + \tilde{G}_{2\Lambda} F_1^\Lambda = iF_2^+ G_1^+ - iG_2^+ F_1^+ + h.c. \quad (2.14)$$

The manifestly $E_7$ type and $H$-invariant action (2.13) depends on 2 independent vector symplectic doublets. The first doublet is a field strength

$$\mathcal{F}_1 = dA_1 \quad \text{off - shell} \quad (2.15)$$

with the doublet 1-form vector potential $A_1$:

$$\mathcal{F}_{1\mu\nu} = \partial_{[\mu} A_{1\nu]} = \begin{pmatrix} F^\Lambda_{1\mu} \\ G_{1\Lambda} \end{pmatrix}_{\mu\nu} = \begin{pmatrix} \partial_{[\mu} B_{1\nu]} \\ \partial_{[\mu} C_{1\nu]} \end{pmatrix}. \quad (2.16)$$

The second doublet is an antisymmetric tensor (off shell it is not a field strength)

$$\mathcal{F}_{2\mu\nu} = \begin{pmatrix} F^\Lambda_{2\mu} \\ G_{2\Lambda} \end{pmatrix}_{\mu\nu}. \quad (2.17)$$

The Lagrange multiplier $L^{AB+}$ and a gravi photon constraint $T_{AB2}^+ O$ in eq. (2.12), and their conjugates, are $E_7$-duality invariants, they transform under $H$. The symbol $T_{AB2}^+ O$ in the action means that it depends on the vector doublet $\mathcal{F}_2$ and on fermions

$$T_{AB2}^+ O \equiv \langle \mathcal{V}_{AB} | \mathcal{F}_2^+ \rangle - iO_{AB}^+. \quad (2.18)$$

Off shell we treat $\mathcal{F}_1$ and $\mathcal{F}_2$ in an asymmetric way. This has an advantage that the equation of motion over $A_1$ will produce the requirement that on shell $d\mathcal{F}_2 = 0$. For this to happen, it is necessary that $\mathcal{F}_1$ appears in the action only once, in a bilinear invariant formed with two doublets

$$-\int d^4x \langle \mathcal{F}_1 | \tilde{\mathcal{F}}_2 \rangle = -\int d^4x \langle dA_{1\mu} | \tilde{\mathcal{F}}_2 \rangle. \quad (2.19)$$

Meanwhile, $\mathcal{F}_2$ appears in the first term, in the second term in the interaction with fermions, and also in the third term with a Lagrange multiplier in our action (2.13).
3 E7 type covariant equations of motion

After partial integration one finds that the vector potential $A_1^{\mu}$ is a Lagrange multiplier to a doublet field equation

$$\frac{\delta S}{\delta A_1^{\mu}} = 0 \quad \Rightarrow \quad \partial_\nu \tilde{F}_2^{\mu\nu} = 0. \quad (3.1)$$

Therefore on shell

$$F_2 = dA_2 \quad \text{on} - \text{shell} \quad (3.2)$$
i.e. on shell there is a second doublet vector potential $A_2^{\mu}$ and $F_{2\mu\nu} = \partial_{[\mu}A_{2\nu]}$.

$$F_{2\mu\nu} = \partial_{[\mu}A_{2\nu]} = \begin{pmatrix} F_2^B \\ G_2^C \end{pmatrix}_{\mu\nu} = \begin{pmatrix} \partial_{[\mu}B_{2\nu]} \\ \partial_{[\mu}C_{2\nu]} \end{pmatrix}. \quad (3.3)$$

We now differentiate the action (2.13) over the Lagrange multiplier and we find that

$$\frac{\delta S}{\delta L^{AB+}} = 0 \quad \Rightarrow \quad T_{AB2}^{+} + \partial_{[\mu}B_{2\nu]} = 0. \quad (3.4)$$

Next equation of motion is over $F_2$. The relevant terms in the action are

$$iF_2^{+}G_1^{+} - iG_2^{+}F_1^{+} + iO^{+}(\hat{h}F_2^{+} - \hat{f}G_2^{+}) + iL^{+}(hF_2^{+} - fG_2^{+} - iO^{+}). \quad (3.5)$$

We differentiate the action over $F_2^{+\Sigma}$ and $G_2^{+\Sigma}$ and find

$$\frac{\partial \mathcal{L}}{\partial F_2^{+\Sigma}} = 0 \quad \Rightarrow \quad G_1^{+\Sigma} + \partial_{[\mu}B_{2\nu]} = 0, \quad (3.6)$$

$$\frac{\partial \mathcal{L}}{\partial G_2^{+\Sigma}} = 0 \quad \Rightarrow \quad F_1^{+\Sigma} + \partial_{[\mu}C_{2\nu]} = 0. \quad (3.7)$$

We solve these equations eliminating the Lagrange multiplier and find that

$$G_1^{+\Sigma} = N_{\Lambda\Sigma} F_1^{+\Sigma} - i\hat{f}_{\Lambda}^{+} O_{AB}^{+} \quad \Rightarrow \quad T_{AB1}^{+} = 0. \quad (3.8)$$

Compare this with the eq. (2.10) which solves the constraint (2.12) applied to $F_2$, i.e. when $T_{AB2}^{+} = 0$ so that

$$G_2^{+\Sigma} = N_{\Lambda\Sigma} F_2^{+\Sigma} - i\hat{f}_{\Lambda}^{+} O_{AB}^{+} \quad \Rightarrow \quad T_{AB2}^{+} = 0. \quad (3.9)$$

Thus we see that on shell there is a complete symmetry between the first and the second E7 doublet

$$F_{1\mu\nu} = \begin{pmatrix} \partial_{[\mu}B_{1\nu]} \\ \partial_{[\mu}C_{1\nu]} \end{pmatrix}, \quad F_{2\mu\nu} = \begin{pmatrix} \partial_{[\mu}B_{2\nu]} \\ \partial_{[\mu}C_{2\nu]} \end{pmatrix}, \quad (3.10)$$

and

$$T_{AB1}^{+} = 0, \quad T_{AB2}^{+} = 0. \quad (3.11)$$
All equations of motion are manifestly E7 type covariant! The symmetry between $F_{1\mu\nu}$ and $F_{2\mu\nu}$ is restored on shell.

Note that with $G_2$ and $G_1$ depending on $F_2$ and $F_1$ we still have twice the number of fields versus physical vectors. Namely, with independent $28, 16, 10$ for $B_{1\nu}$ and $28, 16, 10$ for $B_{2\nu}$ we still have a double set of fields, in $\mathcal{N} = 8, 6, 5$ respectively. We will see how in the 2d order formalism the field $B_{1\nu} + B_{2\nu}$ will become a physical field, whereas $B_{1\nu} - B_{2\nu}$ will become a ghost field with a wrong sign of the kinetic term.

4 From the 1st to the 2d order action: ghosts decouple

At the classical 2d order theory half of the E7 type symplectic doublets corresponds to physical vectors, $28, 16, 10$ in $\mathcal{N} = 8, 6, 5$ respectively. There are $28, 16, 10$ equations of motions as well as $28, 16, 10$ Bianchi identities. The E7 type symmetry flips one into another. The corresponding E7 type symmetry is not manifest.

From the manifestly E7 type invariant 1st order action in (2.13) we proceed with derivation of the 2d order action, as follows.

- We integrate over Lagrange multiplier, i. e. we use the condition $T_{AB}^2 + O = 0$ which means that
  \[ G_{2A}^+ = \mathcal{N}_{AB} F_{2}^{\Sigma +} - i f_{AB}^{-1} O_{AB}^+ . \]  

The remaining action
  \[ - \langle \mathcal{F}_1 \mid \tilde{\mathcal{F}}_2 \rangle + iT_{2}^{+ AB} O_{AB}^+ + h.c. \]  

depends on $F_{1\mu\nu} = \partial_{[\mu} B_{1\nu]}$, $G_{1\mu\nu} = \partial_{[\mu} C_{1\nu]}$, on $F_2$, as well as on scalars and fermions.

- We integrate the action over $C_{\mu 1}$ now. The only term depending on it is $C_{\mu 1} \partial_{\nu} F^{\mu \nu}_2$. This equation is solved if $F_{2\mu\nu} = \partial_{[\mu} B_{2\nu]}$

- Our 1st order action becomes
  \[ -i G_{2}^{+} F_{1}^{+} + iT_{2}^{+ AB} O_{AB}^+ + h.c. \]  

With account of the constraint on $G_2$ in (4.1) it becomes
  \[ -i F_{1}^{+ A} \mathcal{N}_{\Sigma} F_{2}^{\Sigma +} - (F_{1}^{+ A} + F_{2}^{+ A}) f_{AB}^{-1} O_{AB}^+ - f_{AB}^{1CD} \tilde{f}^{AB} O_{CD}^+ O_{AB}^+ + h.c. \]  

4.1 Why there are ghosts?

The fundamental reason why we are facing ghosts in the 2d order formalism originating from the 1st one, is that the manifest E7 type symmetry can only operate with the number of
fields twice as big as the number of physical degrees of freedom. Technically, we see this as follows. Let us look carefully at the kinetic term

\[-iF_1^{+\Lambda}N_{\Lambda\Sigma}F_2^{\Sigma+} + h.c.\]  \hspace{1cm} (4.5)

and use notation

\[F_1 + F_2 \equiv 2F, \quad F_1 - F_2 \equiv 2F,\]  \hspace{1cm} (4.6)

so that \(F + F = F_1, \ F - F = F_2,\) and

\[-i(F + F)^{+\Lambda}N_{\Lambda\Sigma}(F - F)^{\Sigma+} + h.c. = -iF^{+\Lambda}N_{\Lambda\Sigma}F^{\Sigma+} + iF^{+\Lambda}N_{\Lambda\Sigma}F^{\Sigma+} + h.c.\]  \hspace{1cm} (4.7)

The same can be presented as

\[F^{\Lambda}\tilde{G}_\Lambda - F^{\Lambda}\tilde{\xi}_\Lambda = -\frac{1}{4}F^2_{\mu\nu} + \frac{1}{4}\tilde{F}^2_{\mu\nu} + \cdots\]  \hspace{1cm} (4.8)

Thus we see that the combination of the fields \(F_1 + F_2 = \partial_{[\mu}B_{1\nu]} + \partial_{[\mu}B_{2\nu]}\) behaves as a normal vector field with the correct sign of the kinetic term, whereas the combination \(F_1 - F_2 = \partial_{[\mu}B_{1\nu]} - \partial_{[\mu}B_{2\nu]}\) has a wrong sign of the kinetic term and is therefore qualified as a ghost vector field. In classical theory we will see that the ghost is decoupled, however, in a theory deformed by candidate counterterms, they do not decouple.

If we would not be restricted by E7 type symmetry property that there are no symmetric bilinear invariants, we would be able to add to the action terms like

\[\frac{1}{2}(-iF_1^{+\Lambda}N_{\Lambda\Sigma}F_1^{\Sigma+} + h.c.), \quad \frac{1}{2}(-iF_2^{+\Lambda}N_{\Lambda\Sigma}F_2^{\Sigma+} + h.c.).\]  \hspace{1cm} (4.9)

In such case we would have only physical kinetic terms for \(F_1 + F_2 = \partial_{[\mu}B_{1\nu]} + \partial_{[\mu}B_{2\nu]}\)

\[-i\frac{1}{2}(F_1 + F_2)^{+\Lambda}N_{\Lambda\Sigma}(F_1 + F_2)^{\Sigma+} + h.c.),\]  \hspace{1cm} (4.10)

and the ghosts \(F_1 - F_2 = \partial_{[\mu}B_{1\nu]} - \partial_{[\mu}B_{2\nu]}\) would not appear in the kinetic terms. The issue of inconsistency would not have appeared.

But for groups of type E7, such terms as shown in (4.9) cannot originate from the 1st order action, and kinetic terms for ghosts might cause a problem, depending on the interaction Lagrangian.

4.2 Why ghosts decouple in classical supergravity action?

Our action (4.4) has the following dependence on all vector fields

\[-iF^{+\Lambda}N_{\Lambda\Sigma}F^{\Sigma+} + iF^{+\Lambda}N_{\Lambda\Sigma}F^{\Sigma+} - 2F^{+\Lambda}f^{-1AB}_{\Lambda}O^{+}_{AB} + h.c.\]  \hspace{1cm} (4.11)
Only normal vectors interact with fermions, ghost vector fields do not interact with fermions. They do interact with scalars and gravity as one can see from the second term in eq. (4.11). In [2] the argument was given as to why the vector ghosts decouple. It was suggested to use classical equations of motion for the vector fields. For the normal vector one finds that

\[ \partial_{\mu} \tilde{G}^{\mu \nu} = \frac{\partial L_{\text{int}}^F}{\partial A_{\nu}}, \]

(4.12)

where \( L_{\text{int}}^F \) is a term where the normal vector interacts with fermions. One can check that the corresponding action does not vanish on shell with account of field equations.

For the ghost vector field one finds, looking at eq. (4.4), that they are decoupled from fermions so that

\[ \partial_{\mu} \tilde{G}^{\mu \nu} = 0. \]

(4.13)

Here a boundary condition on \( B_{1\nu} - B_{2\nu} = 2 A_{\nu} = 0 \) at infinity is imposed consistently, as explained in [2], and the relevant part of the action, upon integration by part, vanishes on shell

\[ \int \bar{F}^{\Lambda} \tilde{g}_{\Lambda} = \int \delta_{\nu} \partial_{\mu} \tilde{G}^{\mu \nu} = 0. \]

(4.14)

We are therefore left with the normal vector field and the 2d order action, including fermions, is

\[ -i F^{+A} N_{\Lambda \Sigma} F_{\Sigma}^{\Sigma} - 2 F^{+A} f^{1AB} \mathcal{O}_{AB}^+ - f^{-1CD} \mathcal{F}^{AB} \mathcal{O}_{CD}^+ \mathcal{O}_{AB}^+ + h.c. \]

(4.15)

This is the standard 2d order \( \mathcal{N} \geq 5 \) supergravity action for vectors and fermions coupled to scalars. We have generalized the \( \mathcal{N} = 8 \) setting in [2] to the case of \( \mathcal{N} = 6, 5 \).

5 Quartic deformation due to candidate UV divergence

When the candidate UV divergences are added to the classical action, bosonic linear twisted self-duality constraint is deformed following the proposal in [9]. Technically, the deformed constraint is taken in the \( \mathcal{H} \) covariant, \( \mathcal{G} \) invariant form

\[ T_{AB}^{+ \text{def}} \equiv T_{AB}^+ - \lambda \frac{\delta I(T^{-}, T^{++})}{\delta T^{++AB}} = 0, \]

(5.1)

as suggested in [18] and developed in details for \( \mathcal{N} \geq 5 \) supergravities in [8]. The solution of the constraint (5.1) is known either in closed form or as a series in \( \lambda \). Since the deformation terms are complicated, we will first study bosonic actions without fermions.
5.1 A solution of the deformed constraint for $G_2$

A quartic in vectors deformation in the form given in [20] is

$$ I(T^-, T^{++}) = 2 \partial_\mu \partial_\nu T^-_{AB} \alpha \beta \partial^\mu T^-_{CD} \alpha \beta \partial^\nu T^{++} + CD \alpha \beta + \cdots $$

(5.2)

The required expression in (5.1) $\frac{\delta I(T^-, T^{++})}{\delta T^{++}}$ is cubic in graviphotons $T$ and has 8 contributions, starting with

$$ \frac{\delta I(T^-, T^{++})}{\delta T^{++}} = 2 \partial_\mu \partial_\nu T^-_{AB} \alpha \beta \partial^\mu T^-_{CD} \alpha \beta \partial^\nu T^{++} + CD \alpha \beta + \cdots $$

(5.3)

In the approximation that we use the classical constraint $T_{AB}^+ = 0$ ignoring the higher $\lambda$ terms one can replace graviphotons as follows: $T^- \Rightarrow \tilde{f}^{-1} F^-, T^{++} \Rightarrow f^{-1} F^+$. The constraint becomes

$$ T_{AB}^{+ def} \equiv h F^+ - f G^+ - \lambda 2 \partial_\mu \partial_\nu \tilde{f}^{-1} F^- AB \partial^\mu \tilde{f}^{-1} F^- CD \partial^\nu f^{-1} F^+ + \cdots = 0. $$

(5.4)

We can solve it for $G$ in terms of $F$ and scalars and we find

$$ G^+ = N F^+ - 2 \lambda \tilde{f}^{-1} \partial_\mu \partial_\nu \tilde{f}^{-1} F^- AB \partial^\mu \tilde{f}^{-1} F^- CD \partial^\nu f^{-1} F^+ + \cdots $$

(5.5)

According to our strategy, we will look at the constraint on $F_2$ which will come with the Lagrange multiplier in the 1st order action

$$ L^{1st} = -\langle F_1 | \tilde{F}_2 \rangle + i L^{AB+} T_{AB2}^{+ def} + h.c. $$

(5.6)

Thus the solution of the constraint $T_{AB2}^{+ de f} = 0$ is

$$ G_2^+ = N F_2^+ - 2 \lambda \tilde{f}^{-1} \partial_\mu \partial_\nu \tilde{f}^{-1} F_2^- AB \partial^\mu \tilde{f}^{-1} F_2^- CD \partial^\nu f^{-1} F_2^+ + \cdots $$

(5.7)

The first term in $G_2$ is linear in $F_2$ but the deformation term is cubic in $F_2$. Higher order in $\lambda$ will contain higher powers in $F_2$ in the solution for $G_2$.

5.2 A solution of the deformed constraint for $G_1$

For the purpose of deriving the 2d order action from the 1st order one, we could have just observed that all equations of motion are duality covariant since the action is manifestly duality invariant. However, it is interesting to derive the constraint on $F_1$ which in the classical case was derived in Sec. 4 and was shown to be the same as the one for $F_2$. 

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Deformed equations of motion take into account that the deformed constraint is complicated and, for example, $T_{AB}^{+\text{def}}$ depends not only on $F^+$ and $G^+$ but also on $F^-$ and $G^−$

\[
\frac{\delta L^{\text{def}}}{\delta F^+_2} = 0 \Rightarrow G^+_1 - L^{+AB} \frac{\delta T_{AB2}^{+\text{def}}}{\delta F^+_2} + L^{AB} \frac{\delta T_{2}^{+\text{def}AB}}{\delta F^+_2} = 0 ,
\]

(5.8)

\[
\frac{\delta L^{\text{def}}}{\delta G^{+2}_2} = 0 \Rightarrow F^{1\Sigma}_1 + L^{+AB} \frac{\delta T_{AB2}^{+\text{def}}}{\delta G^{+2}_2} - L^{+AB} \frac{\delta T_{2}^{+\text{def}AB}}{\delta G^{+2}_2} = 0 .
\]

(5.9)

We can add the conjugate equations and eliminate the Lagrange multipliers, find the solution for them in terms of vectors. It take the following form

\[
L^{+AB} = (YF^+_1 + ZF^-_1)^{AB} .
\]

Here $Y$ and $Z$ depend on scalars and on $F_2$ and $G_2$. Once the solution for the Lagrange multipliers is plugged into an expression defining $G_1$, we find that $G^+_1 = (VF^+_1 + WF^-_1)^{\Sigma} = G^+_1(F_1, G_2, f, h)$ where $V, W$ now depend on $F_2$ and $G_2$. Note that the constraint imposed by the equation for Lagrange multipliers on $F_2$ leads to a solution for $G_2$ in terms of $F_2$ only, see eq. (5.7)

\[
T_{AB2}^+ - \lambda \frac{\delta I(T^-, T^2_1)}{\delta T_{2\Sigma}^-} = 0 \Rightarrow G_{2\Sigma} = G_{2\Sigma}(F_2, f, h) .
\]

(5.10)

When this equation is solved, one finds that $G_{2\Sigma}$ depends on scalars and $F_2$, it does not depend on $F_1$ by construction since it solves the equation $T_{2\Sigma}^{+\text{def}} = 0$ which depends only on $F_1$. In general, one finds that due to a quartic deformation $G_2$ is given by an infinite series in powers of $F_2$. The asymmetry between $F_1$ and $F_2$, as opposite to the classical supergravity, is not restored on shell:

\[
G_2^{\text{def}} = G_2^{\text{def}}(F_2, f, h) , \quad G_1^{\text{def}} = G_1^{\text{def}}(F_1, F_2, f, h) ,
\]

(5.11)

and

\[
\frac{\delta G_2^{\text{def}}}{\delta F_1} = 0 , \quad \frac{\delta G_2^{\text{def}}}{\delta F_2} \neq 0 .
\]

(5.12)

This is opposite to the classical case where we have found that

\[
G_2 = G_2(F_2, f, h) , \quad G_1 = G_1(F_1, f, h) , \quad \frac{\delta G_2}{\delta F_1} = \frac{\delta G_1}{\delta F_2} = 0 .
\]

(5.13)

Quartic in vectors deformation breaks the on shell symmetry between $F_1$ and $F_2$. This already suggests that we may encounter a related problem in deriving a 2d order action in a deformed theory.

6 From the 1st to the 2d order with deformation: ghosts do not decouple

We start with the action in (5.6). As in classical case we integrate the Lagrange multiplier and find $G_2$ as a functional of $F_2$ and scalars. We integrate over $C_1$ and identify $F_{2\mu\nu}$ with
\[ \partial_{[\mu}B_{2\nu]} \]. The terms remaining in the action are

\[ -iG^+_2 F^+_1 + h.c. \quad (6.1) \]

We use the expression for \( G^2 \) in (5.7) which we have found by resolving the constraint when integrating over the Lagrange multiplier and find

\[ -iF^+_1 N F^+_2 - 2\lambda F^+_1 f^{-1} \partial_\mu \partial_\nu \bar{f}^{-1} F^{-1}_2 \partial^\mu \bar{F}^{-1}_2 \partial^\nu F^+_2 + \cdots + h.c. \quad (6.2) \]

We replace as before \( F^+_1 = F + \Phi \), \( F^+_2 = F - \Phi = F_2 \),

\[ -iF^+ N F^+ + i\Phi^+ N\Phi^+ - 2\lambda (F + \Phi)^+ f^{-1} \partial_\mu \partial_\nu \bar{f}^{-1} (F - \Phi)^- \partial^\mu \bar{f}^{-1} (F - \Phi)^- \partial^\nu F^{-1}_2 + h.c. \quad (6.3) \]

The kinetic terms for normal vectors \( F \) and for ghosts vectors \( \Phi \) are as before in a classical case, the normal have a correct sign, the ghosts have a wrong sign. But now, when the deformation terms is are present and \( \lambda \neq 0 \), we find interaction terms between normal vectors \( F \) and ghosts vectors \( \Phi \). Besides the single term shown here, there are many other terms at the level \( \lambda \), as well as an infinite series of terms with higher and higher powers of \( \lambda \) and increasing powers of vectors. All these terms have normal vectors coupled to ghosts vectors. Thus, the ghosts do not decouple, and therefore the 2d order action following from the 1st order one with manifest E7 type symmetry is inconsistent if local candidate counterterms deform the action and the constraint.

One may ask the question: Is it possible that in the presence of \( \lambda \) the ghost field is not anymore \( F_1 - F_2 \) but it picks up some \( \lambda \)-dependent term\(^2\) and this more complicated expression for the deformed ghost field \( \Phi^{def} \) decouples?

Note that we have an action where \( F_{1\mu \nu} \) from the beginning was \( \partial_{[\mu}B_{1\nu]} \), and after integrating the action over \( C_{\mu 1} \) using \( C_{\mu 1} \partial_\nu \bar{F}^2_{2\nu} \) we found that \( F_{2\mu \nu} = \partial_{[\mu}B_{2\nu]} \). Thus we look for a deformation of the ghost vector field like

\[ \Phi^{def}_{\mu \nu} = F_{1\mu \nu} - F_{2\mu \nu} = \partial_{[\mu}B_{1\nu]} - \partial_{[\mu}B_{2\nu]} + \lambda \partial_{[\mu}B_{3\nu]} \quad (6.4) \]

so that it can be implemented via the change of variables in the functional integral, where we integrate over \( B_{1\nu} \) and \( B_{2\nu} \).

\[ B_{1\nu} - B_{2\nu} \Rightarrow B_{1\nu} - B_{2\nu} + \lambda B_{3\nu} . \quad (6.5) \]

To decouple the vector ghost we need to find out if from the four vector coupling term in the action depending on \( \Phi \) we can extract the expression in the form of \( \partial_{[\mu}B_{3\nu]} \), so that in the first approximation in \( \lambda \) we can absorb the four-vector term into a deformation of the kinetic

\(^2\)We are grateful to R. Roiban for this question.
term. The four vector can be given in the form $\mathbb{F}_{\mu
u}^+ X^{\mu
u} + cc$. The expression for $X^{\mu\nu}$ is cubic in $F$ and $\mathbb{F}$, and function of scalars. To be able to absorb this terms into a redefinition of the kinetic term we have to show that $X^{\mu\nu} = \mathcal{N} Y^{\mu\nu}$ where $Y_{\mu\nu} = \partial_{[\mu} B_{3\nu]}$. However, $Y_{\mu\nu}$ is a complicated expression cubic in vectors and it is not of the form $\partial_{[\mu} B_{3\nu]}$ off shell. Therefore the decoupling of a vector ghosts is not possible, once $\lambda \neq 0$.

7 Discussion

Our main result here is the derivation of the 1st order action with manifest duality symmetry of the E7 type for classical $\mathcal{N} \geq 5$ supergravities given in eq. (2.13). The action is based on a universal symplectic approach used recently in [8] to study the deformation of supergravities due to candidate counterterms in perturbative theory. It was developed earlier in the context of supersymmetric black hole attractors in [10]. The defining feature of this action is that the first term in it, a bilinear invariant
\[
\langle \mathcal{F}_1 \mid \tilde{\mathcal{F}}_2 \rangle \equiv \tilde{\mathcal{F}}_2^\Lambda \mathcal{F}_1_{\Lambda} - \tilde{\mathcal{F}}_2_{\Lambda} \mathcal{F}_1^\Lambda,
\]
vanishes for a single duality doublet; only an antisymmetric in two doublets invariant is possible for E7 type groups.

Our 1st order action (2.13) is valid for the most interesting maximal supersymmetry case of $\mathcal{N} = 8$, as well as for $\mathcal{N} = 5$, where there is an information about 4 loop UV finiteness in d=4, which was not explained until now. In $\mathcal{N} = 8$ case we have reproduced the main result of Cremmer and Julia [2]: when deriving 2d order action from the manifestly $E_{7(7)}$ invariant 1st order action, one encounters ghosts, but they decouple. This renders the classical theory without additional local terms in the action, associated with UV divergences, ghosts-free and preserving E7 type symmetry.

We also found here that when the theory is deformed via a local four-vector candidate counterterm, the ghosts of the 2d order action, following from the E7 invariant 1st order action, do not decouple. But other terms, like any vector independent 4-point deformation due to a candidate UV divergence, are not forbidden by the E7 type argument here. Thus our duality symmetry argument forbids only one of the possible various 4-point candidate counterterms.

Other 4-point terms, there are about 50 of them, supersymmetric partners of the 4-vector candidate UV divergences, are not forbidden by the manifest duality argument that we have developed in this paper. Supersymmetry anomaly would invalidate our choice of the four-vector deformation in eq. (5.2) as a single representative of all possible terms in a supersymmetric candidate counterterm. Only unbroken supersymmetry requires all four-point terms in a given candidate UV divergence to come up in perturbative loop computations.
with the same factor as the one in the four-vector case. Duality anomaly would clearly invalidate our argument, which is based on unbroken duality symmetry.

It is important to stress that our E7 type duality-supersymmetry argument for the absence of UV divergences is valid for $\mathcal{N} \geq 5$ perturbative supergravity to all loop orders. Its validity depends on the validity of our assumption that both duality and supersymmetry have no anomalies when loop computations are performed, and predictions of these symmetries are respected.

In conclusion, we argued here that, in absence of duality and supersymmetry anomalies, $\mathcal{N} \geq 5$ supergravities are predicted to be perturbatively UV finite. This argument explains the mysterious cancellation of the 82 diagrams in $\mathcal{N} = 5$, 4 loop theory in four dimensions, discovered in [22], see Appendix A where we bring up these diagrams. As far as we know, no other explanation of this cancellation was proposed during the last four years since the result in [22] was published. Our result obtained here implies that at the level of four loop theory in four dimensions, $\mathcal{N} = 5$ theory has no duality and no supersymmetry anomalies, and therefore has to be UV finite at this level.

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A Cancelation of UV divergences in 82 diagrams at $L = 4$ in $\mathcal{N} = 5$

In Fig. 1 we show the set of diagrams from [22] where the UV divergences cancel. The reason why the UV infinities in this set of diagrams cancel is explained above using the action with a manifest E7 symmetry.

\textsuperscript{3}An investigation in [25] shows an inconsistency between the deformation proposal [9] and linearized supersymmetry. It suggests an independent explanation of 4 loop $\mathcal{N} = 5$ UV finiteness.

\textsuperscript{4}In the recent talk by Z. Bern at ‘QCD meets gravity”, this situation was described as follows: No standard symmetry explanation is known for $\mathcal{N} = 5$ at 4 loops. We would like to clarify the definition of ‘standard symmetry’. Duality symmetry requires a well studied vanishing soft limit on amplitudes with scalars, but this does not explain UV finiteness of $\mathcal{N} = 5$ at 4 loops [23]. However, duality symmetry in the vector sector, which has to be satisfied in addition to soft limits, as we have argued in this paper, does explain it.
Figure 1. 82 diagrams in $\mathcal{N} = 5$, 4 loops. The individual diagrams are UV divergent in $d=4$, but the sum of all diagrams has no UV divergences [22].

B Deformation proposal in presence of scalars

Duality symmetry in supergravity with the second order action rotates vector field equations into Bianchi identities. In the second order formalism these are treated in an asymmetric way: the action depends on $n_v = (28, 16, 10)$ vector potentials $B_\mu$ via $F = dB$. The Bianchi identity $dF = 0$ for $F = dB$ are valid off-shell, whereas equations of motion $dG = 0$ with $G = \frac{1}{2} \delta L}{\delta F}$ are only valid on shell. Therefore $G = d\mathcal{C}$ only in virtue of field equations. The dual vector $C_\mu$ is not present in the action, $G$ is the function of $F$ and scalar and vector fields, and the analysis of duality symmetry in the second order formalism relies on the fact that $\delta S = \int GB\tilde{G}$. Meanwhile, in supergravity there is also a Noether-Gaillard-Zumino identity [4], [5],[17]:

$$\int d^4x \ G b \tilde{G} = \int d^4x \left[ \frac{\delta}{\delta \varphi} \left( \frac{\delta \mathcal{L}_v}{\delta \varphi} \right) + h.c. \right]. \quad (B.1)$$

Here $\mathcal{L}_v$ is a vector dependent part of the action. In absence of scalars the right hand side of this identity vanishes. This supports the proof of the consistency of the deformed theory in
the Appendix\footnote{It was explained there that the dependence on scalars was suppressed for simplicity. However, looking at eq. (B.1) we see now that in absence of scalars the expansion in $\lambda$ was guaranteed to provide $\int d^4x Gb\tilde{G} = 0$ order by order in $\lambda$, but only in absence of scalars.} in [9], where the action is expanded in a deformation parameter $\lambda$

$$S = S_d + \lambda S^{(1)} + \lambda^2 S^{(2)} + \cdots$$

(B.2)

Here $S^{(1)}$ is the candidate counterterm which is known [17] to violate the condition $\int d^4x Gb\tilde{G}$ at the $\lambda^2$ level. The terms of the order $\lambda^n$ in the proposal [9], indeed, are capable of restoring the condition $\int d^4x Gb\tilde{G} = 0$ order by order in absence of scalars. But for extended supergravities where scalars are present in the coset space $\frac{G}{H}$ and duality groups are of the type $E_7$, the right hand side of eq. (B.1) is not vanishing, in general. In any case, in supergravity with scalars duality symmetry was never proven to be valid in deformed theory on shell.

In classical theory it is also not very clear how the NGZ identity (B.1) is consistent with $\int d^4x Gb\tilde{G} = 0$ condition when scalars are present. Note that $E_{7(7)}$ symmetry in [2] was not associated with the duality current conservation requiring that $\int d^4x Gb\tilde{G} = 0$, as it was suggested later in [4]. In [1, 2] the emphasis was on the existence of the manifest $E_{7(7)}$ invariant 1st order action underlying the 2d order action, where ghosts where shown to decouple. We have followed here the setting of [1, 2], we have confirmed it for classical theory and found that the deformation of the action with manifest duality by the candidate counterterms makes the corresponding 2d order theory inconsistent.

C Ghosts: the good ones (FP fictitious particles), and the bad ones

In gauge theories like non-Abelian Yang-Mills theories and gravity and supergravity the Hamiltonian quantization is not as simple as in theories without gauge symmetry. Besides, the quadratic part of the classical action is degenerate, due to gauge symmetry. This complicates the derivation of the Feynman rules, as it is known for a very long time [26, 27].

The solution to these problems is known both in the Lagrangian Lorentz covariant formulation as well as in the canonical, Hamiltonian non-Lorentz covariant one, [28–35]. The conditions for the equivalence of the physical S-matrix in the covariant and the canonical formulation are also known.

To find the perturbative Feynman rules one has to perform a gauge-fixing in these theories. In Lorentz covariant gauges the gauge-fixing procedure enforces to add to the classical fields, constrained by the gauge-fixing condition, some additional fields, which are often called ‘ghosts’. In the beginning, when they were first introduced in [29, 30] these new fields in the effective action producing the Feynman rules were called by Faddev-Popov (FP) in [30] ‘fictitious particles’, since they do propagate but have unconventional properties which makes
them different from normal particles. These are now called FP ghosts in Yang-Mills theories and gravity, and also NK ghosts [34, 35] in supergravity.

The reason why these FP and NK ghosts are the good ghosts is the following. In Lorentz covariant gauges the number of propagating degrees of freedom exceeds the number of physical degrees of freedom. These extra degrees of freedom would spoil the equivalence theorem for gauge theories which states that on shell physical amplitudes are gauge-independent. These is why good ghosts, or ‘ficticious particles’ are added to the Feynman rules, so that they cancel the contribution of the classical fields which in covariant gauges include fields with wrong sign kinetic terms, i. e. bad ghosts. Meanwhile, in the canonical gauge there are only physical degrees of freedom propagating, both good ghosts and bad ghosts are absent. The equivalence of physical amplitudes in covariant and canonical gauges is proven at all orders in perturbation theory.

A interesting example of a somewhat related situation is a recent computation in [36] of the contribution to the physical amplitude of the trace of the metric in general relativity. In the Lorentz covariant gauge $\partial_\mu t_{\mu\nu} = 0$, where the metric is $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $t_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{4} \eta_{\mu\nu} h$, the quadratic part of the classical action is

$$
- \frac{1}{4} \partial_\mu t_{\nu\lambda} \partial^\mu t^{\nu\lambda} + 3 \frac{3}{32} \partial_\mu h \partial^\mu h
$$

We can see that the physical transverse, traceless field $t_{\nu\lambda}$ has a correct sign kinetic term, whereas trace of the metric field $h$ has a wrong sign and therefore can be qualified as a bad ghost. Does it mean that this bad ghost shows up in a pole in a physical 4-graviton amplitude and makes the theory inconsistent? In this case of general relativity the answer is ‘no’. The field $h$ couples to $t_{\mu\nu}$ at the level of a cubic 3-graviton vertex. This leads to an effective 4-point non-local amplitude with the pole, generated by an $h$-exchange, called $Y_4$ in [36]. In addition, in this covariant gauge there is also a particular contact term with 4 fields $t_{\mu\nu}$ in the action called $\bar{X}_4$ in [36]. Both terms seem to signal a possibility that the presence of a bad ghost field $h$, with a wrong sign kinetic term, might affect the value of the physical 4-graviton amplitude.

However, it was observed in [36], using a special choice of the polarization tensors of asymptotic fields, that the non-local term $Y_4$ as well as a contact term $\bar{X}_4$ both vanish. This is a direct confirmation of the fact that physical amplitude is gauge-independent and the result in the Lorentz covariant gauge is the same as in the canonical gauge. At the tree level studied in [36] the contribution from bad ghost field $h$ disappeared by the convenient choice of the on shell gauge condition. Note that in the example in [36] the loop contribution due to $h$ was not studied. The FP good ghosts contribute only in loop diagrams where the contribution of a bad ghost field $h$ together with the contribution of the FP ‘good ghosts’ must cancel,
according to equivalence theorem. In this way, the amplitude in a covariant gauge is the same as in a canonical gauge, where only physical degrees of freedom are propagating.

In our case above, with the deformation, the bad ghost vector, with a wrong sign kinetic term, couples to a normal vector. There is no choice of the gauge-fixing condition which removes the bad ghost and its coupling, as opposite to the case of the trace of the metric in general relativity. There are no FP type good ghosts which would cancel the $F$ field contribution. Therefore the bad ghost case described in Sec. 6 makes the theory inconsistent in presence of deformation induced by UV infinities.

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