Dynamical baryogenesis through Complex Hybrid Inflation

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Abstract. We propose a hybrid inflation model with a complex waterfall field which contains an interaction term that breaks the $U(1)$ global symmetry associated to the waterfall field charge. We show that the asymmetric evolution of the real and imaginary parts of the complex field during the phase transition at the end of inflation translates into a charge asymmetry [1].

We know that only 4% of the total material content of the universe is of baryonic nature, the type of matter we seem to understand thanks to the Standard Model of Particle Physics and nucleosynthesis process of the early universe.

However, even in this well understood case one fundamental question remains open: why this baryonic component is almost completely made of matter and not of antimatter? This is usually quoted as the baryonic asymmetry problem. Sakharov in Ref.[2] has shown that any quantum field theory could generate a baryonic asymmetry if three conditions are satisfied: no-conservation of the baryonic charge, CP and C violation, and out-of-equilibrium condition for the Universe. The electroweak standard model cannot produce the baryonic asymmetry of the Universe as the condition to be out of equilibrium cannot be fulfilled at the electroweak phase transition.

In this paper, we shall present a particular model of hybrid inflation[3, 4] in which a complex scalar field is the responsible for both the symmetry breaking that puts an end to inflation and for the production of a baryonic asymmetry in the early universe.

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The model
We consider that during the early inflationary universe there were two scalar fields, the inflaton \( \phi \) and another complex one \( a \) wearing a charge which we call baryonic charge. These fields are both minimally coupled to gravity, and endowed with a scalar potential of the form

\[
V(\phi, a) = \frac{1}{4\lambda^2} \left( M^2 - \lambda^2 |a|^2 \right)^2 + \left( \frac{m^2}{2} + \frac{g^2}{2} |a|^2 \right) \phi^2 + \frac{\delta}{4} a^2 \phi^2 + \text{c.c.} \quad (1)
\]

This potential resembles the hybrid potential for inflation, and for \( \delta = 0 \), the complex field could be identified with the waterfall field of standard hybrid inflation[5]. There are six free parameters and we shall impose to the potential to be \( CP \) invariant.

The Lagrangian of our model is \( CP \) conserving, but we shall show that the dynamics of the waterfall field will generate an effective \( CP \) asymmetry at the very end of inflation. Potential (1) is one of simplest choices we can do, as it is invariant under the discrete symmetry \( a \to -a \), whereas the waterfall field \( a \) is coupled to the inflaton.

Our primary quantity of interest, the charge density associated to the waterfall field \( a \) is given by \( n_a \equiv \text{Im}(a^* \dot{a}) \), and obeys a Boltzmann equation in the form

\[
\dot{n}_a + 3H n_a = -|\delta| |a|^2 \phi^2 \sin(2\theta) , \quad (2)
\]

where \( \theta \) is the complex phase of \( a \). Eq. (2) clearly illustrates that any source for the charge density comes from the potential term with the \( \delta \) coupling. But such source needs non-zero field values in order to modify the charge density \( n_a \).

Inflationary dynamics
As for inflation, we will assume the conditions for the so-called vacuum inflation (VI)[3, 4]. First, the inflaton field accomplishes the constraint \( \phi \gg M/g \) before the beginning of inflation. As we will show below, that constraint is modified in our extended model, but the value \( M/g \) still gives the correct order of magnitude to start an inflationary stage. In this case, the effective mass of the waterfall field is positive definite, and this makes \( |a| \to 0 \).

Second, the constant term in potential (1) is initially the dominant one. From the Friedmann equation of cosmology, this means that the Hubble parameter during inflation is almost constant, its value given by \( H_0 \simeq (\sqrt{2\pi/3 \lambda^2}) (M^2/m_{Pl}) \), where \( m_{Pl} \) is the Planck mass. Hence, the second condition for VI can also be written as \( m \ll H_0 \).

Once inflation starts, the waterfall field is located near the local minimum \( |a| = 0 \), whereas the inflaton field obeys the slow-roll equation \( 3H_0 \dot{\phi} \simeq -m^2 \phi \). The universe expands almost exponentially with time, and the scale factor of the universe evolves in the form \( R = R_{END} e^{-N} \), where we have defined the number of e-foldings to the end of inflation as \( N \equiv H_0 (t_{END} - t) \). The subscript ’end’ in all quantities means their values at the end of inflation. In this approximation, the solution for the inflaton field is

\[
\phi(t) = \phi_{END} \exp \left( (m^2/3H_0^2)N \right) . \quad (3)
\]

The only possibility to put an end to VI is by the destabilization of the waterfall field. In the case \( \delta = 0 \), that proceeds as in standard hybrid inflation with a real waterfall field[5]. However, our model presents some additional features we are to study below.
Rainfall dynamics

In order to grasp the dynamics of the fields, we need to know the critical points of the scalar field potential. As in the standard case, there are two critical points of physical interest. The first one is a local maximum, and is located at \( \phi = |a| = 0 \), and the corresponding value of the scalar potential is \( V(0, 0) = M^4/(4\lambda^2) \); this is considered a false vacuum because it is unstable. The critical value of \( \theta \) at this point cannot be resolved, but it will depend on the initial conditions and on the particular evolutionary path that may have carried the waterfall field into \( |a| = 0 \). The second critical point, which corresponds to the true vacuum of the system, is a global minimum and is located at \( \phi = 0 \) and \( \lambda|a|/M = 1 \). Again, the complex phase \( \theta \) cannot be resolved but this time because the global minimum of the system is degenerate. We describe the motion of the complex waterfall field around the local minimum \( |a| = 0 \) as that of small classical oscillations. In this regime, the waterfall field is not coupled to the inflaton, so its real and imaginary parts can be considered a separate pair of coupled and damped harmonic oscillators of the form

\[
\ddot{Q}_\pm = -3H_0\dot{Q}_\pm - m_\pm^2(\phi)Q_\pm ,
\]

\[
m_\pm^2(\phi) = (g^2 \pm |\delta|) \phi^2 - M^2 ,
\]

where the \( \pm \) signs are in correspondence, and the fields \( Q_\pm \) are the perturbations of the real and imaginary parts of \( a \), respectively.

Critical points for the rainfall fields

As seen in Eqs. (4), there are two special points in the slow-roll down of the inflaton field for which the masses of the oscillators vanish, \( \phi_\pm = M/\sqrt{g^2 \pm |\delta|} \), and then \( \phi_+ < \phi_- \). Next, we can use the slow-roll inflaton’s solution, Eq. (3), and the assumption \( m \ll H_0 \), to give the explicit time-dependence of the mass terms,

\[
m_\pm^2(t) \simeq \frac{M^2m^2}{3H_0}(t_\pm - t) ,
\]

where \( t_\pm \) are the corresponding times of \( \phi_\pm \) such that

\[
\frac{m^2}{3H_0}(t_+ - t_-) = \ln \left( \frac{\phi_-}{\phi_+} \right) = \frac{1}{2} \ln \left( \frac{1 + |\delta|/g^2}{1 - |\delta|/g^2} \right) \simeq \frac{|\delta|}{g^2} .
\]

Therefore, the exact solution of Eqs. (4) can be given in terms of Airy functions as follows

\[
Q_+(t) = C_+ R^{-3/2}f_+[u_+(t)] ,
\]

\[
Q_-(t) = C_- R^{-3/2}f_-[u_-(t)] ,
\]

where

\[
f_\pm(u) = \text{AiryAi}(u_\pm) + C^*_\pm \text{AiryBi}(u_\pm) ,
\]

\[
u_\pm(t) = \frac{1}{4} \left( \frac{3H_0}{M^2m^2} \right)^{2/3} \left[ 9H_0^2 - 4m_\pm^2(t) \right] ,
\]

being \( C_\pm \) and \( C^*_\pm \) arbitrary constants.

The normal modes \( Q_\pm \) are destabilized at different times as the inflaton field rolls down to the origin of coordinates. We notice that \( Q_- \) is destabilized first from the false vacuum once
\(\phi \sim \phi_-\), and \(Q_+\) is destabilized next at \(\phi \sim \phi_+\). It is at this point that we can considered the system as completely destabilized, as both oscillation modes now grow and roll down to the true vacuum.

However, all the results in Eqs. (7,8) and (9,10) are valid under the assumption that inflation proceeds up to \(t = t_+\), and then if \(t_+ \lesssim t_{\text{end}}\). We can estimate the value of \(t_{\text{end}}\) if we assume that for \(t > t_-\) the mode \(Q_-\) follows the effective minimum at fixed \(\phi\) once the latter falls below \(\phi_-\)[4]. The end of inflation would happen at the time the slow-roll parameter \(\epsilon(\phi_{\text{end}}) \approx 1\); from this and after some involved algebra, we obtain the strong constraint

\[
\frac{\phi_-}{\phi_{\text{end}}} \approx 1 + \frac{\sqrt{\pi}}{2} \left(\frac{\phi_-}{m_{\text{Pl}}}\right) .
\]

This result confirms that inflation ends almost instantaneously soon after the critical point (\(\phi_-\) in our case) is reached, as described in more detail in[4, 11, 7].

We shall take for granted that condition (11) is satisfied so that \(\phi_+ \gtrsim \phi_{\text{end}}\), and then that we fulfill the aforementioned restriction \(t_+ \lesssim t_{\text{end}}\). Hence, Eqs. (6) and (11) can be combined together to impose an \textit{upper limit} on \(|\delta|/g^2\). The charge asymmetry we calculate below depends on the latter ratio, which is \textit{maximum} for \(\phi_+ = \phi_{\text{end}}\). Therefore, we assume hereafter that the latter condition applies.

On the other hand, Eqs. (4) are also the equations of motion for the quantum fluctuations of the waterfall field. It has been shown in[11], that these quantum fluctuations are negligible if they are very massive, i.e. \(m_+ > H_0\). But once their masses are smaller than \(H_0\), which happens around the times at which the modes become massless, \(m_+ = 0\), the fluctuations acquire an amplitude of the order \(Q_+ \sim H_0/2\pi\). Such an assumption is not exact, but it is a reasonable one because the contributions to the full amplitude made by different wavelengths are comparable; for further details see[11]. Therefore, in order to fix the arbitrary constants in Eqs. (7,8), we impose the conditions[12, 6]

\[
Q_\pm(t) \approx \pm \frac{H_0}{2\pi}, \quad \dot{Q}_\pm(t) \approx -\frac{1}{3H_0} \frac{\partial V}{\partial Q_\pm} .
\]

These conditions represent the slow-roll motion of the \(Q\)-fields close to their instability points.

Notice that the normal modes \(Q_\pm\) are evaluated at the points they are effectively massless, and that the slow-roll condition suggests \(\dot{Q}_\pm(t_\pm) \approx 0\). The latter also implies that \(C^*_- = C^*_-^2\), and then both normal modes \(Q_\pm\) can be described by the same function, \(f_- = f_+ = f\). Therefore, Eqs. (7,8) now read

\[
Q_+(t) = \pm \frac{H_0}{2\pi} \left(\frac{R_+}{R}\right)^{3/2} \frac{f[u_+(t)]}{f[u_+(t_+)]} , \quad (13a)
\]
\[
Q_-(t) = \pm \frac{H_0}{2\pi} \left(\frac{R_-}{R}\right)^{3/2} \frac{f[u_-(t)]}{f[u_-(t_-)]} . \quad (13b)
\]

in which \(R_\pm\) indicates the values of the scale factor at which the respective mode becomes massless.

\textbf{Generation of the Charge asymmetry}

Any charge asymmetry in the waterfall field should happen in between the times \(t_\pm\), where both scalar fields \(\phi\) and \(a\) have enough amplitude to source the Boltzmann equation (2). That the

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2 The condition \(C^*_+ = C^*_+\) also suggests that the initial conditions imposed on \(Q_\pm\) at \(t \ll t_\pm\) were \textit{symmetrical}. As we shall show, our model can generate a baryonic asymmetry even if there is none induced by initial conditions.
produced charge is not null critically depends on the asymmetric evolution of the oscillation modes $Q_{\pm}$ which allows to generate a CP asymmetry during the time interval $\Delta t = t_1 - t_2$; for $\delta = 0$ the modes are destabilized at the same time and no charge would be generated. The final result after Eqs. (12) and (13) is then

$$|n_a| \simeq \frac{M^2 m_Q^2}{5 \pi^2 H_0} \left( \frac{9 H_0^2 \delta}{m^2 g^2} \right) \exp \left( - \frac{9 H_0^2 \delta}{2 m^2 g^2} \frac{f[u_+(t_+)]}{f[u_-(t_-)]} \right).$$

(14)

Notice that because the proximity of $u_-$ and $u_+$, the last ratio on the r.h.s. of Eq. (13b) is of order of unity, and we will take it as such in our calculations below.

The argument in the exponential term in Eq. (14) can be related to the COBE normalization condition[4, 8, 7], so that

$$\frac{9 H_0^2 \delta}{2 m^2 g^2} = \frac{5 \sqrt{3} \pi}{4 \sqrt{2}} \frac{\lambda}{\delta H} \approx 2.93 \times 10^{-4} \frac{\lambda}{g^2},$$

(15a)

$$\delta H \approx \frac{2 \sqrt{6 \pi g} (M/m_P)^5}{3 \delta} \left( \frac{M}{m_P} \right)^2 \approx 10^{-5}.$$  

(15b)

where the very last equality is the known CMB constraint on primordial perturbations. Eq. (15a) is a fundamental result, as it relates the asymmetric evolution of modes $Q_{\pm}$ with the amplitude of inflationary density perturbations. However, we should stress out that such a relation is only valid if $|\delta| \neq 0$.

To make an order of magnitude estimation, we will assume that reheating occurs promptly at the end of inflation so that the reheating temperature is $T_{reh} = (30/(4 \pi^2 g_* \lambda^2))^{1/4} M$. If the total entropy is produced in the reheating stage, the baryonic asymmetry produced in the model presented here is estimated to be

$$\left| \frac{n_a}{s} \right| \approx 1.77 \frac{g_*^{3/4}}{q_+} \frac{M^2}{m_P^2} x e^{-x^2},$$

(16)

where variable $x^2$ in the exponential is the term given in Eq. (15a), and $g_*$ ($q_*$) represents the density (entropy) degrees of freedom, respectively.

Let us take that $q_* = q_+ \approx 10^2$, and estimate the maximum charge asymmetry carried by the waterfall field; this is accomplished for $x = 1/\sqrt{2}$. If the expected asymmetry is of order $\sim 10^{-10}$, then we get the lower bound $(M/m_P) > 10^{-5}$. On the other hand, the combined constraints of the amplitude and of the spectrum of primordial perturbations gives the upper bound $(M/m_P) < 5 \times 10^{-5} \lambda/g[7]$.

It is not easy to satisfy both constraints in the general case. We should note that $\lambda < g$ would not be allowed, and so the constraints seem to prefer $\lambda > g$. Actually, all parameters can be resolved in the case $g \sim \lambda$, for which we get $(M/m_P) \sim 10^{-5}$ and $\lambda \sim 10^{-4}$. This scenario may not be the most realistic, as the VEV of the waterfall field is just below the Planck scale, and the inflaton field should be of the same order before the beginning of inflation. In all other cases, we can summarize the results by normalizing $\lambda = 1$, and get the following range for the remaining free parameter $0.05 < g < 0.5$; correspondingly we get $10^{-5} < (M/m_P) < 10^{-2}$.

Once destabilized, the inflaton and waterfall fields start to roll down to one of the minima of the potential, around which it starts oscillating. We can describe these oscillations as small perturbations around the (degenerate) global minimum. In consequence, the source on the r.h.s. of the Boltzmann equation (2) can be considered a second order effect in those perturbations, so that we do not expect the production of substantial charge asymmetry after the end of inflation.
The charge of the waterfall field can be easily translated in a baryonic asymmetry once the \( a \) field decays into fermions,

\[
\left| \frac{n_B}{s} \right| \simeq \kappa \Delta B \frac{\Gamma_{\Delta B}}{\Gamma_a} \left| \frac{n_a}{s} \right|
\]

(17)

where \( \Gamma_a \) is the \( a \)-total width, \( \Gamma_{\Delta B} \) is the rate of \( a \)-decays into fermions for a given \( \Delta B \), and \( \kappa \) is the suppression factor due to interactions of the waterfall fields with the inflaton. For instance, the channels violating the charge of the waterfall field without producing fermions are \( \Gamma(a \to 2\phi) \) or \( \Gamma(a + a \to \phi + \phi) \), which are proportional respectively to \( g^4/\lambda^2 \) and \( |\delta|^2 \). These channels are suppressed for some of the values allowed by the constraints discussed above.

**Conclusion**

The mechanism presented throughout this paper to generate a baryonic asymmetry is generic and illustrates how such an asymmetry can be produced at the phase transition of hybrid inflation. The model appears to be the simplest realization, as there is no need of extra fields and the term added to the standard potential is not complicated.

For simplicity too, we chose to call baryonic charge to the one associated to the complex waterfall field. However, in realistic models, it would be better to choose another charge (for instance the \( B - L \)-charge in place of the \( B \)-charge) for the \( a \) field to avoid any suppression of the baryonic asymmetry at further stages in the evolution of the Universe.

Nevertheless, how this asymmetry could be transferred to the observed matter-antimatter asymmetry, and how to keep it until our present time are still open questions. Any answer crucially depends on the coupling of the waterfall field to ordinary matter and the reheating mechanism after inflation; this is work under research and will be reported elsewhere[13].

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