Probing Quintessence using BAO imprint on the cross-correlation of weak lensing and post-reionization HI 21 cm signal

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ABSTRACT
In this work we investigate the possibility of constraining a thawing Quintessence scalar field model for dark energy. We propose using the imprint of baryon acoustic oscillation (BAO) on the cross-correlation of post-reionization 21-cm signal and galaxy weak lensing convergence field to tomographically measure the angular diameter distance \( D_A(z) \) and the Hubble parameter \( H(z) \). The projected errors in these quantities are then used to constrain the Quintessence model parameters. We find that independent 600hrs radio interferometric observation at four observing frequencies 916MHz, 650 MHz, 520 MHz and 430MHz with a SKA-1-Mid like radio telescope in cross-correlation with a deep weak lensing survey covering half the sky may measure the binned \( D_A \) and \( H \) at a few percent level of sensitivity. The Monte Carlo analysis for a power law thawing Quinetessence model gives the 1−\( \sigma \) marginalized bounds on the initial slope \( \lambda_i \), dark energy density parameter \( \Omega_\phi \) and the shape of the potential \( \Gamma \) at 86.63%, 10.08% and 9.75% respectively. The constraints improve to 7.66%, 4.39% and 5.86% respectively when a joint analysis with SN and other probes is performed.

Key words: Dark energy, 21-cm cosmology, Weak lensing, Cross-correlation

1 INTRODUCTION
Several decades of independent observations [Perlmutter et al. 1997, Riess et al. 1998, Bamba et al. 2012] confirm that our Universe is currently in an accelerated expansion phase. The cause of such cosmic acceleration is attributed to the so called “Dark energy”, [ Sahni & Starobinsky 2000, Peebles & Ratra 2003, Copeland et al. 2006, Amendola & Tsujikawa 2010] a fluid that violates the strong energy condition. Einstein’s cosmological constant (\( \Lambda \)) with an effective fluid equation of state (EoS) \( P/\rho = w(z) = -1 \) provides the simplest explanation for the cosmic acceleration. While, several cosmological observations are consistent with the concordance LCDM model, there are several inconsistencies from both theoretical considerations (like smallness of \( \Lambda \), the ‘fine tuning problem’), and observations (like the low redshift measurements of \( H_0 \) [Riess et al. 2016]). This has led to many significant efforts in developing alternate scenarios to model dark energy and thereby explaining the cosmic acceleration without requiring a cosmological constant.

Generally speaking there are two ways to tackle the problem. One approach involves modifying the gravity theory itself on large scales [Amendola & Tsujikawa 2010], \( f(R) \) modification to the Einstein action [Khoury & Weltman 2004, Starobinsky 2007, Hu & Sawicki 2007, Nojiri & Odintsov 2007] belongs to this approach of modeling cosmic acceleration. In a second approach the matter sector of Einstein’s field equation is modified by considering a dark energy fluid with some nontrivial dynamics. In both the approaches one may find an effective dark energy EoS which dynamically varies as a function of redshift and in principle can be distinguished from the cosmological constant (\( \Lambda \)). There are many models for dark energy that predict a dynamical equation of state. For example, in the quintessence models, dark energy arises from a time dependent scalar field, \( \phi \) [Ratra & Peebles 1988, Caldwell et al. 1998, Steinhardt et al. 1999, Zlatev et al. 1999, Scherrer & Sen 2008]. However these models still require fine tuning for consistency with observations. A wide variety of phenomenological potentials have been

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explored for quintessence field to achieve $w \approx -1$. In all these models, the minimally coupled scalar field is expected to slowly roll in the present epoch. However, other than a few restricted class of potentials, it is difficult to prevent corrections from various symmetry breaking mechanisms which tends to spoil the slow roll condition \cite{Panda:2011}. Weaker gravitational lensing by intervening large scale structure distorts the images of distant background galaxies. This is attributed to the deflection of light by the fluctuating gravitational field created by the intervening mass distribution and is quantified using shear and convergence of photon geodesics. The statistical properties of these distortion fields are quantified using the shear/convergence power spectrum. These imprint the power spectrum of the intervening matter field, as well as cosmological evolution and thereby carries the signatures of structure formation. Dark energy affects the growth of cosmic structures and geometric distances, which crucially affects the power spectrum of the lensing distortion fields. Thus, weak lensing has become one of the important cosmological probes. Several weak lensing experiments are either on-going or are upcoming, such as the Dark Energy Survey \cite{Abbott:2016}, the Hyper Suprime-Cam survey \cite{Aihara:2018}, the Large Synoptic Survey Telescope \cite{Ivezic:2008}, the WideField Infrared Survey Telescope \cite{Wright:2010, Spergel:2015}, and the Euclid \cite{Laureijs:2011}. The 3D tomographic imaging of the neutral hydrogen (HI) distribution is one of the promising tools to understand large scale structure formation and nature of dark energy \cite{Bharadwaj:2001, Wyithe:2009}. The dominant part of the low density hydrogen gets completely ionized by the end of reionization around $z \approx 6$ \cite{Gallerani:2006}. However, a small fraction of HI survives the complex processes of reionization and is believed to remain housed in the over-dense regions of IGM. These clumpy HI clouds remain neutral amidst the radiation field of background ionizing sources as they are self shielded and are the dominant source of the 21-cm radiation in post-reionization epoch. Intensity mapping of such redshifted 21-cm radiation aims to map out the large scale HI distribution without resolving the individual DLA sources and promises to be a powerful probe of large scale structure and background cosmological evolution \cite{Wyithe:2007, Chang:2008, Bharadwaj:2009, Mao:2008}. Several radio telescopes like the GMRT \cite{Gallo:2006}, MEERKAT \cite{MWA:2015}, CHIME \cite{Chime:2019}, and SKA \cite{SKA:2021} are in the pursuit of detecting the cosmological 21-cm signal for a tomographic imaging \cite{Mao:2008}.

We consider the cross-correlation of HI 21-cm signal with the galaxy weak lensing convergence field. It is known that cross-correlations of individual tracers of IGM often offer crucial advantages over auto-correlations. The systematic noise that arises in the individual surveys is pose less threat in the cross-correlation signal as they appear in the variance. Further, the foregrounds and contaminants of individual surveys are, in most cases, uncorrelated and hence do not bias the cross-correlation signal \cite{Sarkar:2009, Vallinotto:2009}. The cross-correlation of the post-reionization HI 21 cm signal has been extensively studied \cite{Sarkar:2009, GuhaSarkar:2010, Sarkar:2010, Sarkar:2019, Dash:2021}.

The acoustic waves in the primordial baryon-photon plasma are frozen once recombination takes place at $z \approx 1000$. The sound horizon at the epoch of recombination provides a standard ruler which can be then used to calibrate cosmological distances. Baryons imprint the cosmological power spectrum through a distinctive oscillatory signature \cite{White:2005, Eisenstein:1998}. The BAO imprint on the 21-cm signal has been studied \cite{Sarkar:2013, Sarkar:2011}. The baryon acoustic oscillation (BAO) is an important probe of cosmology \cite{Eisenstein:2005, Percival:2007, Anderson:2012, Shoji:2009, Sarkar:2013}, as it allows us to measure the angular diameter distance $D_A(z)$ and the Hubble parameter $H(z)$ using the the transverse and the longitudinal oscillatory features respectively thereby allowing us to put stringent constraints on dark energy models. We propose the BAO imprint on the cross-correlation of 21-cm signal and weak lensing convergence as a probe of Quintessence dark energy. The paper is organized as follows. In Section-2 we discuss the cross-correlation of weak lensing shear/convergence and HI excess brightness temperature. We also discuss the BAO imprint and estimation of errors on the BAO parameters namely the expansion rate $H(z)$, angular diameter distance $D_A(z)$ and the dilation factor $D_L(z)$ from the tomographic measurement of cross-correlation power spectrum using Fisher formalism. In Section-3 we discuss the background and structure formation in quintessence dark energy models and constrain the model parameters using Markov Chain Monte Carlo (MCMC) simulation. We discuss our results and other pertinent observational issues in the concluding section.

2 THE CROSS-CORRELATION SIGNAL

Weak gravitational lensing \cite{Bartelmann:2001} by intervening large scale structure distorts the images of distant background galaxies. This is caused by the deflection of light by the fluctuating gravitational field created by the intervening

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1 http://gmrt.ncra.tifr.res.in/
2 https://arxiv.org/abs/1703.00621
3 http://www.ska.ac.za/meerkat/
4 https://www.mwatelescope.org/
5 http://chime.phas.ubc.ca/
6 https://www.skatelescope.org/
Weak lensing is a powerful cosmological probe as galaxy shear is sensitive to both spacetime geometry and growth of structures. The Weak-lensing convergence field on the sky is given by a weighted line of sight integral of the matter overdensity field $\delta$ as

$$\kappa(\theta) = \int_0^{\chi_s} A_\kappa(\chi) \delta(\chi \theta, \chi) d\chi$$

where $\chi_s$ is the maximum distance to which the sources are distributed and the cosmology-dependent function $A_\kappa(\chi)$ is given by

$$A_\kappa(\chi) = \frac{3}{2} \Omega_m H_0^2 \chi \int_0^{\chi_s} n_s(z) \frac{dz}{d\chi'} \chi' d\chi'$$

where $\chi$ denotes the comoving distance and $a(\chi)$, the cosmological scale factor. The redshift selection function of source galaxies, $n_s(z)$, tends to zero at both low and high redshifts. It is typically modeled as a peaked function, parametrized by $(\alpha, \beta, z_0)$ of the form

$$n_s(z) = N_0 z^\alpha e^{-\left(\frac{z}{\pi}\right)^\beta}$$

and satisfies the normalization condition

$$\int_0^\infty dz \ n_s(z) dz = \bar{n}_g$$

where $\bar{n}_g$ is the average number density of galaxies per unit steradian.

On large scales the redshifted HI 21-cm signal from post-reionization epoch ($z < 6$) known to be biased tracers of the underlying dark matter distribution. We use $\delta_T$ to denote the redshifted 21-cm brightness temperature fluctuations. The post-reionization HI signal has been studied extensively and satisfied the normalization condition. Usually for the investigations involving the 21-cm signal the radial information is retained for tomographic study. The weak-lensing signal, on the contrary consists of a line of sight integral whereby the redshift information is lost. We consider an average over the 21-cm signals from redshift slices and thus lose the individual redshift information but improve the signal to noise ratio when cross-correlating with the weak-lensing field.

We define a brightness temperature field on the sky by integrating $\delta_T(\chi \hat{n}, \chi)$ along the radial direction as

$$T(\hat{n}) = \frac{1}{\chi_2 - \chi_1} \sum_{\chi_1}^{\chi_2} \delta_T(\chi \hat{n}, \chi) \Delta \chi$$

where $\chi_1$ and $\chi_2$ are the comoving distances corresponding to the redshift slices of the 21-cm observation over which the signal is averaged.

Radio interferometric observations of the redshifted 21-cm signal directly measures the complex Visibilities which are the Fourier components of the intensity distribution on the sky. The radio telescope typically has a finite beam which allows us to use the ‘flat-sky’ approximation. Ideally the fields $\kappa$ and $\delta_T$ are expanded in the basis of spherical harmonics. For convenience, we use a simplified expression for the angular power spectrum by considering the flat sky approximation whereby we can use the Fourier basis. Using this simplifying assumption, we may approximately write the cross-correlation angular power spectrum as

$$C_T^{\kappa} = \frac{1}{\pi(\chi_2 - \chi_1)} \sum_{\chi_1}^{\chi_2} \Delta \chi A_T(\chi) A_\kappa(\chi) D_1(\chi) \int_0^\infty dk_1 \left[ 1 + \beta_T(\chi) \frac{k_1^2}{k^2} \right] P(k)$$

where $k = \sqrt{k_1^2 + \left(\frac{\chi}{\pi}\right)^2}$, $D_1$ is the growing mode of density fluctuations, and $\beta_T = f/b_T$ is the redshift distortion factor - the ratio of the logarithmic growth rate $f$ and the bias function and $b_T(k, z)$. The redshift dependent function $A_T$ is given by

$$A_T = 4.0 \text{ mK} b_T \bar{x}_H (1 + z)^2 \left( \frac{\Omega_m h^2}{0.02} \right) \left( \frac{0.7}{H(z)} \right) \left( \frac{H_0}{H(z)} \right)$$

The quantity $b_T(k, z)$ is the bias function defined as ratio of HI-21cm power spectrum to dark matter power spectrum $b_T^2 = P_{HI}(z)/P(z)$. In the post-reionization epoch $z < 6$, the neutral hydrogen fraction remains with a value $\bar{x}_H = 2.45 \times 10^{-2}$ (adopted from Noterdaeme et al. 2009; Zafar et al. 2013). The clustering of the post-reionization HI is quantified using $b_T$. On sub-Jean’s length, the bias is scale dependent (Fang et al. 1993). However, on large scales the bias is known to be scale-independent. The scales above which the bias is linear, is however sensitive to the redshift. Post-reionization HI bias is studied extensively using N-body simulations (Bagla et al. 2010; Guha Sarkar et al. 2012; Sarkar et al. 2016; Carucci et al. 2017). These simulations demonstrate that the large scale linear bias increases with redshift for $1 < z < 4$ (Marin et al. 2010).
We have adopted the fitting formula for the bias $b_T(k, z)$ as a function of both redshift $z$ and scale $k$ \cite{GuhaSarkar2012, Sarkar2016} of the post-reionization signal as

$$b_T(k, z) = \sum_{m=0}^{4} \sum_{n=0}^{2} c(m, n) k^m z^n$$

(7)

The coefficients $c(m, n)$ in the fit function are adopted from \cite{Sarkar2016}. The angular power spectrum for two redshifts is known to decorrelate very fast in the radial direction \cite{Bharadwaj2003}. We consider the summation in Eq. (5) to extend over redshift slices whose separation is more than the typical decorrelation length. This ensures that in the computation of noise for each term in the summation may be thought of as an independent measurement and the mutual covariances between the slices may be ignored.

2.1 The Baryon acoustic oscillation in the angular power spectrum

The sound horizon at the epoch of recombination is given by

$$s(z_d) = \frac{\int_{0}^{a_r} c_i d a}{a^2 H(a)}$$

(8)

where $a_r$ is the scale factor at the epoch of recombination (redshift $z_d$) and $c_i$ is the sound speed given by $c_i(a) = c/\sqrt{3(1 + 3 \rho_b/4 \rho_c)}$ where $\rho_b$ and $\rho_c$ denotes the baryonic and photon densities respectively. The WMAP 5-year data constrains the value of $z_d$ and $s(z_d)$ to be $z_d = 1020.5 \pm 1.6$ and $s(z_d) = 153.3 \pm 2.0$ Mpc \cite{Komatsu2009}. We shall use these as the fiducial values in our subsequent analysis. The standard ruler ‘$s$’ defines a transverse angular scale and a redshift interval in the radial direction as

$$\theta_s(z) = \frac{s(z_d)}{(1 + z)D_A(z)} \quad \delta z_s = \frac{s(z_d)H(z)}{c}$$

(9)

Measurement of $\theta_s$ and $\delta z_s$, allows the independent determination of $D_A(z)$ and $H(z)$. The BAO feature comes from the baryonic part of $P(k)$. Hence we isolate the BAO power spectrum from cold dark matter power spectrum through $P_b(k) = P(k) - P_c(k)$. The baryonic power spectrum can be written as \cite{Hu1996, Seo2007}

$$P_b(k) = A \frac{\sin x}{x} e^{-(k \Sigma_s)^{1.4}} e^{-k^2 \Sigma_n^2 / 2}$$

(10)

where $A$ is a normalization, $\Sigma_s = 1/k_{silk}$ and $\Sigma_n = 1/k_{nal}$ denotes the inverse scale of ‘Silk-damping’ and ‘non-linearity’ respectively. In our analysis we have used $k_{silk} = (3.07 h^{-1} \text{Mpc})^{-1}$ and $k_{nal} = (8.38 h^{-1} \text{Mpc})^{-1}$ from \cite{Seo2007} and $x = \sqrt{k_0^2 s_0^4 + k_0^2 s_0^2}$. We also use the combined effective distance $D_V(z)$ defined as \cite{Eisenstein2005}

$$D_V(z) \equiv \left[ (1 + z)^2 D_A(z)^2 \right]^{1/3}$$

(11)

The changes in $D_A$ and $H(z)$ are reflected as changes in the values of $s_\perp$ and $s_\parallel$ respectively, and the errors in $s_\perp$ and $s_\parallel$ corresponds to fractional errors in $D_A$ and $H(z)$ respectively. We use $p_1 = \ln(s_\perp)$ and $p_2 = \ln(s_\parallel)$ as parameters in our analysis. The Fisher matrix is given by

$$F_{ij} = \sum_k \frac{1}{\sigma_{r_k}^2} \frac{1}{\pi(\chi_2 - \chi_1)^2} \sum_{x_1} \Delta x_1 \chi_1^2 A_T(\chi_1) A_r(\chi_2) D_T^2(\chi_2) \int_0^\infty dk_k \frac{1 + \beta_T(\chi_2) k^2}{k^2} \frac{\partial P_b(k)}{\partial p_i} \frac{\partial P_b(k)}{\partial p_j}$$

(12)

$$= \sum_k \frac{1}{\sigma_{r_k}^2} \frac{1}{\pi(\chi_2 - \chi_1)^2} \sum_{x_1} \Delta x_1 \chi_1^2 A_T(\chi_1) A_r(\chi_2) D_T^2(\chi_2) \int_0^\infty dk_k \frac{1 + \beta_T(\chi_2) k^2}{k^2} \left( \cos x - \frac{\sin x}{x} \right) f_{1j} f_{1i} A e^{-(k_\Sigma_s)^{1.4}} e^{-k^2 \Sigma_n^2 / 2}$$

(13)

where $f_1 = k_0^2 / k^2 - 1$, $f_2 = k_0^2 / k^2$ and $k^2 = k_\parallel^2 + \ell^2 / \chi^2$. The variance $\sigma_{r_k}$ is given by

$$\sigma_{r_k} = \sqrt{\frac{(C_{TT}^t + N_{TT}^e)(C_{TT}^t + N_{TT}^p)}{(2\ell + 1) f_{sky}}}$$

(14)

where $C_{TT}^t$ and $C_{TT}^p$ are the convergence and 21-cm auto-correlation angular power spectra respectively and $N_{TT}^e$ and $N_{TT}^p$ are the corresponding noise power spectra.

The auto-correlation power spectra are given by \cite{Dash2021}

$$C_{TT}^t = \frac{1}{\pi(\chi_2 - \chi_1)^2} \sum_{x_1} \Delta x_1 \chi_1^2 A_T(\chi_1) D_T^2(\chi_1) \int_0^\infty dk_k \frac{1 + \beta_T(\chi_2) k^2}{k^2} P(k)$$

(15)

$$C_{TT}^p = \frac{1}{\pi} \int_0^\infty \frac{d x}{x^2} A_r(\chi_2^2) D_T^2(\chi_2) \int_0^\infty dk_k P(k)$$

(16)

The noise is the convergence power spectrum is dominated by Poisson noise. Thus $N_{TT}^e = \sigma_n^2 / n_g$ where $\sigma_n$ is the galaxy-intrinsic
rms shear [Hu 1999]. The source galaxy distribution is modeled using \((\alpha, \beta, z_0) = (1.28, 0.97, 0.41)\) which we have adopted from Chang et al. (2013). For the survey under consideration, we have taken \(\sigma_e = 0.4\) (Takada & Jain 2004). We use a visibility correlation approach to estimate the noise power spectrum \(\mathcal{N}_e^2\) for the 21-cm signal (Geil et al. 2011, Villaescusa-Navarro et al. 2014, Sarkar & Datta 2015).

\[
\mathcal{N}_e^2 = \left( \frac{T_{\text{sys}}^2 \lambda^2}{A_e} \right)^2 \frac{B}{T_o N_b(U, \nu)}
\]

(17)

where \(T_{\text{sys}}\) is the system temperature, \(B\) is the total frequency bandwidth, \(U = \ell/2\pi\), \(T_o\) is the total observation time, and \(\lambda\) is the observed wavelength corresponding to the observed frequency \(\nu\) of the 21 cm signal. The quantity \(A_e\) is the effective collecting area of an individual antenna which can be written \(A_e = \epsilon \pi (D_d/2)^2\), where \(\epsilon\) is the antenna efficiency and \(D_d\) is the diameter of the dish. The \(N_b(U, \nu)\) is the number density of baseline \(U\) and can be expressed as

\[
N_b(U, \nu) = \frac{N_{\text{ant}}(N_{\text{ant}} - 1)}{2} \rho_{2D}(U, \nu) \Delta U
\]

(18)

where \(N_{\text{ant}}\) is the total number of antenna in the radio array and \(\rho_{2D}(U, \nu)\) is the normalized baseline distribution function which follows the normalization condition \(\int d^2U \rho_{2D}(U, \nu) = 1\). The system temperature \(T_{\text{sys}}\) can be written as a sum of contributions from sky and the instrument as

\[
T_{\text{sys}} = T_{\text{inst}} + T_{\text{sky}}
\]

(19)

where

\[
T_{\text{sky}} = 60 K \left( \frac{\nu}{300 \text{MHz}} \right)^{-2.5}
\]

(20)

We consider a radio telescope with an operational frequency range of 400\(–\)950 MHz. We consider 200 dish antennae in a radio interferometer roughly mimicking SKA1-Mid. The telescope parameters are summarized in Table 1. The full frequency range is divided into 4 bins centered on 916 MHz, 650 MHz, 520 MHz and 430 MHz and 32 MHz bandwidth each. To calculate the normalized baseline distribution function we have assumed that baselines are distributed such that the antenna distribution falls off as \(1/\ell^2\). We also assume that there is no baseline coverage below 30m. We have also assumed \(\Delta U = A_e/\ell^2\).

The BAO feature manifests itself as oscillations in the linear matter power spectrum \(P(k)\) at \(k \approx 0.045 \text{Mpc}^{-1}\). Figure

**Figure 1.** This shows the BAO imprint on the transverse cross correlation angular power spectrum \(C_T^{\ell e}\). To highlight the BAO we have divided by the no-wiggles power spectrum \(C_{T,\text{no-wiggles}}^{\ell e}\) which corresponds to the power spectrum without the baryonic feature. This is shown for three redshifts \(z = 1.0, 1.5, 2.0\).

**Figure 2.** The figure shows the projected 1\(–\)\(\sigma\) error bars on \(H(z)\), \(D_A(z)\) and \(D_v(z)\) at 4 redshift bins where the galaxy lensing and HI-21cm cross correlation signal is being observed. The fiducial cosmology is chosen to be LCDM.
The dynamics of background cosmological evolution is obtained by solving a autonomous system of first order equations

\[ \gamma' = 3(\gamma - 2) + \sqrt{3}\Omega_\phi(2 - \gamma)\lambda, \]

\[ \Omega_\phi' = 3(1 - \gamma)\Omega_\phi(1 - \Omega_\phi), \]

\[ \lambda' = \sqrt{3}\Omega_\phi\gamma^2(1 - \Gamma), \]

\[ b' = -\frac{3}{2}b\Omega_\phi(1 - \gamma) \]
In order to solve the above set of 1st order ODEs numerically, we fix the initial conditions for $\gamma, \Omega_\phi, \lambda$ at the decoupling epoch. For thawing models, the scalar field is initially frozen due to large Hubble damping, and this fixes the initial condition $\gamma_i \approx 0$. The quantity $\Gamma$ which quantifies the shape of the potential is a constant for power law potentials. The parameter $\lambda_i$ is the initial slope of scalar field and measures the deviation of LCDM model. For smaller $\lambda_i$ the EoS ($w_\phi$) of scalar field remain close to cosmological constant, whereas larger values of $\lambda_i$ lead to a significant deviation from LCDM. Assuming the contribution of scalar field to the total energy density is negligibly small in the early universe, we fix the present value of $\Omega_{\phi}$. Similarly, we fix the initial value of $b$ (related to the density parameter for baryons) so that one gets right value of the $\Omega_m = 0.049$ (Aghanim et al. 2020) at the present epoch. Figure 3 shows the dynamical evolution of the EoS of quintessence field for three models. We note that there is no departure from the LCDM at large redshifts but a prominent model sensitive departure for small redshifts. At $z\sim 0.5$ there is almost a $\sim 5\%$ departure of the EoS parameter $w_\phi$ from that of the non-dynamical cosmological constant. The departure of $w_\phi$ from its LCDM value of $-1$, imprints on the growing mode of density perturbations by virtue of the changes that it brings to the Hubble parameter $H(z)$.

Growth of matter fluctuations in the linear regime provides a powerful complementary observation to put tighter constrains on cosmological parameters, and also break the possible degeneracy in diverse dark energy models. We have assumed spatially flat cosmology in our entire analysis and not constrained radiation density, as only dark matter and dark energy are dominant in the late universe. The full relativistic treatment of perturbations for Quintessence dark energy has been studied [Hussain et al. (2016)]. Ignoring super-horizon effects, we note that on sub-horizon scales, ignoring the clustering of Quintessence field, the linearized equations governing the growth of matter fluctuations is given by the ODE (Amendola 2000, 2004)

$$D'_{+} + \left(1 + \frac{\mathcal{H}'(a)}{\mathcal{H}(a)}\right) D''_{+} - \frac{3}{2} \Omega_m(a) D_{+} = 0.$$  

(27)

Here, the prime denotes differentiatation w.r.t to ‘log $a$', $\mathcal{H}$ is the conformal Hubble parameter defined as $\mathcal{H} = a\mathcal{H}$ and $\delta_m$ is the linear density contrast for the dark matter. In order to solve the above ODE, we fix the initial conditions $D_{+}$ grows linearly with $a$ and the first derivative of $\frac{d\delta_m}{da} = 1$ at early matter dominated epoch ($a = 0.001$). We now consider the BAO imprint on the cross-correlation angular power spectrum to make error predictions on Quintessence dark energy parameters which affects both background evolution and structure formation.

### 3.1 Statistical analysis and constraints on model parameters

We choose the following parameters $(h, \Gamma, \lambda_i, \Omega_{\phi})$ to quantify the Quintessence dark energy. We have use uniform priors for these parameters in the Quintessence model. The Hubble parameter at present ($z = 0$) in our subsequent calculations is assumed to be $H_0 = 100h$ $Km/s/Mpc$, thus define the dimensionless parameter $h$. We perform a Markov Chain Monte Carlo (MCMC) analysis using the observational data to constrain the model parameters and evolution of cosmological quantities. The analysis is carried out using the Python implementation of MCMC sampler introduced by [Foreman-Mackey et al. (2013)]. We take flat priors for these parameters with ranges of $h \in [0.5, 0.9]$, $\Gamma \in [-1.5, 1.5]$, $\lambda_i \in [0.5, 0.8]$, $\Omega_{\phi} \in [0.5, 0.8]$.  

We first perform the MCMC analysis for the using the error bars obtained on the binned $H(z)$ and $D_{A}$ from the proposed 21-cm weak lensing cross-correlation. The figure 4 shows the marginalized posterior distribution of the set of parameters and $(h, \Gamma, \lambda_i, \Omega_{\phi})$ the corresponding 2D confidence contours are obtained for the model $V(\phi) \sim \phi$. The results are summarized in table 3.

For a joint analysis, we employ three mainstream cosmological probes, namely cosmic chronometers (CC), Supernovae Ia (SN) and $f_\sigma$. We have used the observational measurements of Hubble expansion rate as a function of redshift using cosmic chronometers (CC) as compiled by Gomez-Valent & Amendola (2018). The distance modulus measurement of type Ia
In this paper, we have explored the cross-correlation signal of weak galaxy lensing and HI 21-cm. From the tomographic constraints are also competitive with other probes (Gupta et al. 2012; Sangwan et al. 2018; Yang et al. 2019).

The BAO estimates of \( H(z) \), \( D_A(z) \) and \( D_V(z) \) over a redshift range \( z \sim 0 - 3 \). The quantities of interest namely \( H(z) \) and \( D_A(z) \) explicitly appears in the lensing kernel and also in the BAO feature of the power spectrum. The cross-angular spectrum involve a radial integral and hence loses the redshift information. We have obtained tomographic information by locating the 21-cm slice at different redshift bins before cross-correlating.

Several observational challenges come in the way of measuring the cosmological 21-cm signal. The 21-cm signal is buried deep under galactic and extra-galactic foregrounds (Ghosh et al. 2011). We have assumed that this key challenge is addressed. Even after significant foreground removal, the cosmological origin of the 21 cm signal can only be ascertained only through a cross-correlation (Guha Sarkar et al. 2010; Carucci et al. 2017; Sarkar et al. 2019). The foregrounds for the two individual probes are expected to be significantly uncorrelated and hence leads to negligible effects in the observing cross-correlation power spectrum. We have not considered systematic error which arises from photometric redshift (or so called photo-z) errors which may significantly degrade the cosmological information in the context of lensing auto-correlation (Takada & Jain 2009).

The BAO estimates of \( H(z) \), \( D_A(z) \) allows us to probe dark energy models. We have considered the quintessence scalar field as a potential dark energy candidate and studied the background dynamics as well as the growth perturbation in linear regime in such a paradigm. A Baysean parameter estimation using our BAO estimates indicate the possibility of good

| Parameters | \( \Omega_{\mathrm{bh}} \) | \( \Gamma \) | \( \lambda_i \) | \( h \) |
|------------|----------------|---------|---------|---------|
| Constraints (BAO only) | 0.660^{+0.064}_{-0.049} | 0.091^{+0.734}_{-1.080} | 0.575^{+0.067}_{-0.050} | 0.723^{+0.038}_{-0.036} |
| Constraints (BAO+CC+f8+SN) | 0.616^{+0.034}_{-0.020} | 0.157^{+0.805}_{-0.950} | 0.548^{+0.040}_{-0.036} | 0.701^{+0.016}_{-0.015} |

Table 3. The parameter values, obtained in the MCMC analysis combining all the data sets are tabulated along the 1 – \( \sigma \) uncertainty.
BAO probe of Quintessence with lensing-21 cross-correlation

constraints on scalar field models. The constraints also improve when joint analysis with other probes is undertaken and reaches precision levels competitive with the existing literature.

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