Digital tracking control of the boost converter: prediction-based digital redesign approach

Cheng Shion Shieh
Department of Electrical Engineering, Far-East University, Tainan, Taiwan

E-mail: csshieh@mail.feu.edu.tw

Abstract. Several continuous-time tracking controllers of the DC-to-DC boost converters had been addressed, but few literature touched the digital tracking control because that was a challenging theoretical problem which consists of nonlinear, non-minimum phase and saturated control. Given a continuous-time nonlinear boost converter with continuous tracking control, this paper proposes a new sampled-data feedback laws from the available analog counterpart via digital redesign such that the state trajectories of the sampled-data closed-loop systems close to the continuous-time one in the sense of state-matching. An existing boost converter circuit has been demonstrated to visualize the feasibility of the proposed methodology.

1. Introduction
DC-to-DC converter and DC-to-AC inverter are very important topics in power conversion and are usually accomplished by some classical continuous circuits. The power transfer efficiency of power conversion often is influenced by used control methodology. Hence, the control issue of power conversion is still a major concern. The control of an inverter is closely related to the tracking control problem of the basic converter [1]. Recently the converter or inverter device is often nonlinear, non-minimum phase system with saturated control. Therefore, the control problem is much more complicated and is a challenging problem even in the regulator case (DC-to-DC conversion) due to its dynamical characteristics.

Analysis of the dynamical properties of the boost converter with control strategies had been addressed [2-3]. Most of the previous work dealing with the continuous control of converter/inverter. However the sampled-data systems are widespread more and more because most controlled plants encountered in engineering applications are continuous while controls are digitally implemented using computers. In order quickly to transfer continuous system to sampled-data one, a digital redesign is an efficient design tool of digital control [4-6]. That is, a continuous controller is designed first, and discretized afterward to obtain an equivalent digital controller. The advantages are that the existing continuous-time methods are more intuitive and better established than their discrete-time counterparts and, furthermore, the sampling period can be selected based on the known continuous-time closed-loop bandwidth [7]. That is just what this paper wants to construct a digital tracking controller for DC-to-DC converter by digital redesign.

This paper is organized as follows: Section 2 formulates the digital redesign problem of the tracking control of the booster converter. The proposed digital control is developed in Section 3, which control performance the DC-DC conversion state matching is satisfied between continuous
system and sampled-data one. An illustrative example is shown in Section 4. Finally, in Section 5, conclusions are presented.

2. Problem formulation

The solved diagram of the boost converter with digital control is shown in Fig. 1. It is well known that when the switching frequency is high, it is possible to model the average behavior of this power converter. The given DC-to-DC converter expression is shown as [1]:

\[ L \dot{x}_{1c} = -u_c x_{2c} + v_{in} \]  
(1a)

\[ C \dot{x}_{2c} = u_c x_{1c} - \frac{x_{2c}}{R} \]  
(1b)

\[ y_c = x_{2c} \]  
(1c)

where the \( v_{in} \) is supposed to be known constant voltage, the \( x_{1c} \) is the current across the inductor, \( x_{2c} \) is the voltage in the capacitor, and the control variable \( u_c \in [0, 1] \), known as the duty cycle, is the average value within a communication cycle. Throughout this paper, the state symbols represent average values.

We rewrite system (1) in general form

\[ \dot{x}_c = f(x_c) + g(x_c)u_c \]  
(2a)

\[ y_c = Cx_c \]  
(2b)

where

\[ x_c = \begin{bmatrix} x_{1c} \\ x_{2c} \end{bmatrix}; f(x_c) = \begin{bmatrix} f_1(x_c) \\ f_2(x_c) \end{bmatrix} = \begin{bmatrix} \frac{v_{in}}{L} \\ -\frac{x_{2c}}{RC} \end{bmatrix}; g(x_c) = \begin{bmatrix} \frac{-x_{2c}}{L} \\ \frac{x_{1c}}{C} \end{bmatrix}; C = [0, 1] \]

The Cortes et al. [1] had proposed an easy-to-implement continuous control law shown as

\[ u_c(t) = \frac{v_{in} + K(x_{1c}(t) - x_{1r})}{x_{2c}(t)} \]  
(3)

where \( K \) is a positive constant and \( x_{1r} \) is given by [1]

\[ x_{1r} = \frac{x_{2r}^2}{Rv_{in}} \]  
(4)

where \( x_{1r} \) and \( x_{2r} \) represent the steady-state value of the inductor current and the voltage in the capacitor. The Cortes et al. also proposed the following proposition to define the range of parameters \( R, L \) and \( C \) in (1).

The proposition [1]: If \( x_{2r} = a + b\sin(\omega t) \) then

\[ \frac{v_{in}(a+b\Delta x_{1c})}{(a+b)\Delta x_{1c}} T_s < \frac{Rv_{in}^2}{2bw(a+b/2)} \]  
(5)
\[
\frac{a + b \cdot v_{in}}{R \Delta x_{2c}} T_s < C \frac{E_x}{R_{bw} (a + b/2)}
\]  
(6)

where \(\Delta x_{1c}, \Delta x_{2c}\) and \(T_s\) denote the maximum allowed ripple in the inductor, the capacitor voltage and the switching period, respectively. \(E_x\) satisfies the inequality \(\|x_2^2 - x_2^d\| < E_x\). In short, a given continuous DC-to-DC boost converter system with continuous controller (3), we should construct \(u_d(t)\) piecewise-constant input to replace the original continuous control \(u_c(t)\), which is expressed as

\[
u_d(t) = u_d(kT) \text{ for } kT \leq t < (k + 1)T
\]  
(7)

where \(T\) is the sampling period. A zero-order hold is utilized in (7).

Fig. 2 shows the traditional digital design schema but it is hard to do digital modeling because the input function \(g(x)\) is non-constant. This paper offers a different issue of digital tracking control methodology based on the proposed modified digital redesign diagram, described in Section 3, to solve the digital tracking control of the DC-to-DC boost converter.

**Figure 2.** Traditional digital redesign block for the boost converter [1]

### 3. Digital redesign of boost converter

Because the digital controller needs feedback digital state of the controlled plant, the digital modeling of the boost converter (2) must be solved. However, it is hard to achieve a digital state from the model (2a) for \(g(x)\) non-constant. Here we represent the modified digital redesign block diagram Fig. 3 to replace the traditional one in Fig. 2 for solving the digital states. The proposed design diagram is described as bellows.

Rewrite the continuous model (2a) in the following:

\[
\begin{bmatrix}
\dot{x}_{1c} \\
\dot{x}_{2c}
\end{bmatrix}
= 
\begin{bmatrix}
0 & \frac{-u_c}{L} \\
\frac{u_c}{C} & -\frac{1}{RC}
\end{bmatrix}
\begin{bmatrix}
x_{1c} \\
x_{2c}
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix}
\begin{bmatrix}
v_{in}
\end{bmatrix}
\]

\(\triangleq \dot{x}_c = Ax_c + Bv_{in}\)  
(8)
where $A \triangleq \begin{bmatrix} 0 & -u_c/L \\ u_c/C & -1/RC \end{bmatrix}$; $B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$. The system matrix $A$ is floating according to the given continuous control $u_c$ and input matrix $B$ is constant. The solution of the continuous-time controlled system (8), $x_c(t)$, at a time $t = t_v = kT + vT$ for a design parameter $0 \leq v \leq 1$ is found to be

$$x_c(t_v) = e^{A(t_v-kT)}x_c(kT) + \int_{kT}^{kT+vT} e^{A(kT+vT-\tau)}Bv_in d\tau$$

which, since $B$ and $v_in$ are constant, reduces to

$$x_c(t_v) = e^{A(t_v-kT)}x_c(kT) + \int_{kT}^{kT+vT} e^{A(kT+vT-\tau)}Bv_in d\tau = G^y x_c(kT) + H^v v_in$$

(9a)

where $G^v = (e^{AT})^v = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^v$ and $H^v = (G^v - I)A^{-1}B = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}^v$. While the solution to the sampled-data controlled system, $x_d(t_v)$ at $t = t_v = kT + vT$, is obtained as

$$x_d(t_v) = G^y x_d(kT) + H^v v_in$$

(9b)

$$\triangleq \begin{bmatrix} x_{1d}(t_v) \\ x_{2d}(t_v) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^v \begin{bmatrix} x_{1d}(kT) \\ x_{2d}(kT) \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}^v v_in$$

(10)

**Existing continuous controller**

$$u_c(t) = \frac{V_{in} + K(x_{1c} - x_{1r})}{x_{2c}}$$

**Digital Control $u_d(kT)$**

$$\dot{x}_d = A(u_c) + BV_m$$

**Sampling**

**Proposed digital one**

$$\dot{x}_d = f(x) + g(x)u_d(t)$$

**Figure 3. Modified digital redesign block for the boost converter [1]**

The continuous control (3) at time $t_v$ is shown as

$$u_c(t_v) = \frac{V_{in} + K(x_{1c}(t_v) - x_{1r})}{x_{2c}(t_v)}$$

(11)
From the viewpoint of digital redesign principle, state matching requirement $x_c(t_v) = x_d(t_v)$ under the assumption that $x_c(kt) = x_d(kt)$, it is necessary to have $u_c(t_v) = u_d(kt)$. This makes the following prediction-based digital controller:

$$u_d(t_v) = u_c(t_v) = \frac{v_{in} + k(x_{1c}(t_v) - x_{1r})}{x_{2c}(t_v)} = \frac{v_{in} + k(x_{1d}(t_v) - x_{1r})}{x_{2d}(t_v)}$$ (12)

After substitution (10) into (12), we obtain the digital form of (12) based on $v=1$ shown as

$$u_d(kt) = \frac{v_{in} + k(G_{11}x_{1d}(kt) + G_{12}x_{2d}(kt) + h_1v_{in} - x_{1r})}{G_{21}x_{1d}(kt) + G_{22}x_{2d}(kt) + h_2v_{in}}$$ (13)

Fig. 3 shows the proposed digital control algorithm schematically for the nonlinear boost converter (2) and (3). It is important to note that the matrix $A$ in (8) is obtained via the existing continuous controller $u_c$. Then calculate the gains $G_{11}, G_{12}, G_{21}$ and $G_{22}$ in (9) to obtain the predicted discrete-time controller $u_d(kt)$.

4. Illustrative example

We use the proposed digital controller for the boost converter to generate a reference sinusoidal signal $x_{2r}(t) = 100 + 30\sqrt{2}\sin(20\pi t)$ over a load of $220\Omega$. A DC-power supply of $v_{in}=48V$ and a switch commutation frequency $f_c = 1/T_c$ of 30kHz are used. The maximum admissible error is $E_c = 0.1\|x_{2r}\|$, and the maximum ripples are $\Delta x_1 = 0.05\|x_{2r}\|^2/(Rv_{in})$ and $\Delta x_2 = 0.02\|x_{2d}\|$. (5) and (6) are taken the form

$$11.04mH < L < 16.34mH, 5.02\mu F < C < 28.53\mu F$$ (14)

The continuous control gain $K=20$, capacitor $L=12mH$ and inductor $C=15\mu F$ in (8) are set same as reference [1].

Numerical simulations are carried out on Simulink of Matlab using the proposed method with a sampled-hold period 0.005 seconds. The 0.005 second period is based on the sampling theorem that the sampling frequency is greater than 2 times the controlled frequency. The frequency controlled in this paper is $f=10$ for $2\pi f=20\pi$. Its corresponding period is 0.1 seconds. Based on the sampling theorem, the sampling frequency of this computer simulation is at least 20 or higher, that is, the best digital simulation can be obtained by a cycle greater than 0.05 seconds. Fig. 4 shows the state trajectories for applying both continuous-time and digitally redesigned controller to the boost converter circuit. Furthermore, the control laws for both the continuous-time and the digitally redesigned controllers are presented in Fig. 5. The simulation result illustrates $x_d(t)$ is very close to $x_c(t)$.

**Figure 4.** Trajectories of the boost converter under the digitally redesigned controller and original continuous controller for the boost converter circuit [1]
5. Conclusion
A prediction-based digital redesign technique has been proposed for finding a dynamic digital control law in (13) from the given analog counterpart in (11) for the boost converter system, and it can be implemented using microcomputers, which satisfies a state-matching requirement. The simulation result, state-matching, demonstrated the effectiveness of the developed method.

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