Penguin Pollution in $B \to J/\psi V$ Decays and Impact on the Extraction of the $B_s - \bar{B}_s$ mixing phase

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We formulate the most general time-dependent decay distributions of $B_s \to J/\psi (\to l^+ l^-) \phi (\to K^+ K^-)$ in which the direct CP violation is explicitly incorporated. We then investigate the $B \to J/\psi V$ decays in the perturbative QCD approach where $V$ is a light vector meson. Apart from the leading-order factorizable contributions, we also take into account various QCD corrections and the hard-spectator diagrams. With the inclusion of these sizable corrections, our theoretical results for CP-averaged branching ratios, polarization fractions, CP-violating asymmetries, and relative phases are in good consistency with the available data. Based on the global agreement, we further explore the penguin contributions and point out that the $\phi_s$ extracted from $B_s \to J/\psi \phi$ can be shifted away by $O(10^{-3})$.

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I. INTRODUCTION

In the standard model of particle physics, the source of CP violation arises from the non-vanishing complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix which governs the quark mixing. Constraints stemming from the unitarity of CKM matrix can be pictorially represented as triangles, the length of whose sides are products of CKM matrix elements and the angles are relative phases between them. Due to the comparable size of the three sides, the so-called (bd) triangle from CKM matrix can be pictorially represented as triangles, the length of whose sides are products of CKM matrix elements and thus is also of special interest.

The observation of significant non-zero value would be the signal for new physics. Since the CKM factors $V_{cb}$ and $V_{cs}$ can evolve in time and get mixed with each other. These states at $t=0$ can evolve in time and get mixed with the each other. These states at $t$ can be denoted as $B_s(t)$ and $\bar{B}_s(t)$. Since both the $B_s^0$ and $\bar{B}_s^0$ can decay into the same final state like $J/\psi \phi$, there is an indirect CP asymmetry (CPA) between the rates of $B_s(t) \to J/\psi \phi$ and $\bar{B}_s(t) \to J/\psi \phi$, quantified by

$$\text{Im} \left[ \frac{q}{p} \frac{\bar{A}_f}{A_f} \right]. \tag{1}$$

Here the $A_f$ and $\bar{A}_f$ are the amplitudes for the $B_s$ and $\bar{B}_s$ meson decays which are dominated by the $b \to c\bar{c}s$ transition at the quark level. Since the CKM factors $V_{cb}$ and $V_{cs}$ in $A_f$ and $\bar{A}_f$ are real in the standard parametrization of CKM, the indirect CPA defined in Eq. (1) measures the phase in $q/p$ defined with the form: $\phi_s = -\arg(q/p)$. This phase $\phi_s$ is tiny in the SM, and in particular $\phi_s = -2\beta_s = -2\arg(-V_{ts} V^*_{tb} / (V_{cs} V^*_{cb})) [1]:$

$$\phi_s = (-0.036 \pm 0.002) \text{ rad}. \tag{2}$$

The observation of significant non-zero value would be the signal for new physics.

The analogue of the $\phi_s$ has benefited a lot from the measurements of time-dependent observables in $B_s/\bar{B}_s \to J/\psi \phi$, the golden-channel $B \to J/\psi K_s$. In the experimental retrospect, many progresses have been made in the past few years. Thanks to the large amount of data sample collected on the Tevatron and LHC experiments, the result for $\phi_s$ is getting more and more precise [2–6]. Recently based on the data of $1.0fb^{-1}$ collected at 7 TeV in 2011, the LHCb collaboration gives [7]

$$\phi_{J/\psi \phi} = (0.07 \pm 0.09 \pm 0.01) \text{ rad}, \tag{3}$$

which is in agreement with the standard model when the errors are taken into account. Moreover new alternative channels are proposed and in particular the $B_s \to J/\psi f_0(980)$ is believed to have the supplementary power in reducing the error in $\phi_s$ [8, 9]. A characteristic feature of this mode is that $f_0(980)$ is a $0^{-+}$ scalar meson, and thus the final state $J/\psi f_0$ is a CP eigenstate. In contrast with the $B_s \to J/\psi \phi$, it does not request to perform the angular decomposition, and therefore the
Some calculation formulas are the counterpart of the direct CP asymmetry is incorporated. Instead of using the flavor SU(3) symmetry to relate the effects in CPA measurement. To do so, we will first use the helicity amplitudes to derive the time-dependent angular distributions, in which amplitudes and penguin amplitudes. In particular, the perturbative QCD factorization approach (pQCD) \( \phi \) as the analysis is greatly simplified. Agreement on branching ratios is found between theoretical calculation [10–13] and experimental measurements [14–16], while the \( \phi \) is reported by the LHCb collaboration [7].

\[
\phi_{s/J/\psi f_0}^{s/J/\psi f_0} = (-0.14^{+0.17}_{-0.16} \pm 0.01) \text{ rad.} \tag{4}
\]

On the theoretical side, although decays of the \( B_s/B_s^0 \) meson into \( J/\psi(\phi/f_0) \) are mainly governed by the \( b \to c \bar{c} s \) transition at the quark level, there are indeed penguin contributions with non-vanishing different weak phases. Thus the indirect CPA can be shifted away from the \( \phi \). Though intuitively penguin contribution is expected to be small in the SM, a complete and reliable estimate of its effects by some QCD-inspired approach is not yet available. Such estimate will become mandatory soon especially when confronted with the gradually-reducing experimental error. As a reference, after the upgrade of LHC the error can be diminished to \( \Delta \phi \sim 0.008 \) [6].

The main purpose of this work is to estimate the penguin contributions in the \( B \to J/\psi V \) decays and explore the impact to the CPA measurement. To do so, we will first use the helicity amplitudes to derive the time-dependent angular distributions, in which the direct CP asymmetry is incorporated. Instead of using the flavor SU(3) symmetry to relate the effects in \( B_s \to J/\psi f \) and the counterpart of \( B \) decay modes [17, 18], we will adopt the QCD-based factorization approach to directly compute both tree amplitudes and penguin amplitudes. In particular, the perturbative QCD factorization approach (pQCD) [19–22] will be used in this work, the same approach that has been applied to study the \( B \to J/\psi P \) [23–25] and estimate the penguin contribution to \( \Delta S \) in \( B \to J/\psi K_S \) [26]. Recent development of this approach can be found in Refs. [27, 28]. In the calculation apart from the leading order contributions, we will also include the next-to-leading order corrections in \( \alpha_s \), which are sizable especially to penguin contributions.

The rest of this paper is organized as follows. We derive the time-dependent angular distributions in Sec. II. Section III is devoted to the ingredients of the basic formalism in the pQCD approach. The analytic expressions for the decay amplitudes of \( B_{u/d}/s \to J/\psi V \) modes in the pQCD approach are also collected in this section. The numerical results and phenomenological analysis for the CP-averaged branching ratios, CP-averaged polarization fractions, relative phases, and CP-violating asymmetries of the considered decays are given in Sec. IV. We summarize this work and conclude in Sec. V. Some calculation formulas are relegated to the appendix.

## II. HELICITY-BASED ANGULAR DISTRIBUTIONS OF \( B_s \to J/\psi(\to l^+l^-)\phi(\to K^+K^-) \)

The decay distributions can be expressed in terms of helicity angles, \( \theta_K, \theta_1, \phi \). The convention on the kinematics in \( B_s/\bar{B}_s \to \phi(\to K^+K^-)J/\psi(\to l^+l^-) \) is illustrated in Fig. 1. The moving direction of the \( K^+K^- \) pair in the \( B_s/\bar{B}_s \) rest frame is chosen as the \( z \) axis. The polar angle \( \theta_K \) is defined as the angle between the flight direction of \( K^+ \) (\( l^+ \)) and the \( z \) axis in the \( \phi (J/\psi) \) rest frame. \( \phi \) is the azimuthal angle between the two decay planes of \( \phi \) and \( J/\psi \).

![Kinematics of B_s/Bar B_s to J/psi(->l^+l^-)phi(->K^+K^-).](image)

FIG. 1. Kinematics of \( B_s/\bar{B}_s \to J/\psi(\to l^+l^-)\phi(\to K^+K^-) \). The moving direction of the \( K^+K^- \) pair in the \( B_s/\bar{B}_s \) rest frame is chosen as the \( z \) axis. The polar angle \( \theta_K \) is defined as the angle between the flight direction of \( K^+ \) (\( l^+ \)) and the \( z \) axis in the \( \phi (J/\psi) \) rest frame. \( \phi \) is the azimuthal angle between the two decay planes of \( \phi \) and \( J/\psi \).
Considering the mixing of \( B_s - \bar{B}_s \), the generic time-dependent decay rate is written as
\[
\Gamma(t) = N e^{-\Gamma t} \left\{ \frac{|A|^2 + |\bar{A}|^2}{2} \cosh \frac{\Delta \Gamma t}{2} + \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta m t) - \Re(A^* \bar{A}) \sinh \frac{\Delta \Gamma t}{2} - \Im(A^* \bar{A}) \sin(\Delta m t) \right\},
\]
\[
\Gamma(t) = \left| \frac{p}{q} \right|^2 N e^{-\Gamma t} \left\{ \frac{|A|^2 + |\bar{A}|^2}{2} \cosh \frac{\Delta \Gamma t}{2} - \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta m t) - \Re(A^* \bar{A}) \sinh \frac{\Delta \Gamma t}{2} + \Im(A^* \bar{A}) \sin(\Delta m t) \right\},
\]
where \( N \) is a normalization constant to compensate the difference of the decay width and amplitudes, \( \Delta m = m_H - m_L \), \( \Delta \Gamma = \Gamma_L - \Gamma_H \), and \( \Gamma = (\Gamma_L + \Gamma_H)/2 \). Here \( A \) and \( \bar{A} \) are functions of angular variables explicitly given below.

**TABLE I.** The angular and time-dependent functions used in Eqs. (7) and (8), as discussed in the text. In the amplitude \( A^i_j \), the superscript \( j \) denotes the spin of the \( K^+ K^- \) state, while the subscript \( i = 0, |, \perp \), corresponds to the three polarization configurations. \( A^0_0 \) is also usually referred to as \( A_S \) in the literature. Some abbreviations have been used for cosine and sine functions: \( c_K = \cos \theta_K \), \( s_K = \sin \theta_K \).

| \( f_k \) | \( N_k \) | \( a_k \) | \( b_k \) | \( c_k \) | \( d_k \) |
|---|---|---|---|---|---|
| \( c_K^2 \) | \( A^0_0 \) | \( \frac{1}{2} \left[ \sin(\delta_0^0 - \delta_0^0) - |\lambda_0^0\lambda_0^0| \right] + \frac{i}{2} \left[ |\lambda_0^0| \sin(\delta_0^0 - \delta_0^0) - \phi_0^0 \right] \) | \( -\frac{2|\lambda_0^0|}{1+|\lambda_0^0|^2} \sin(\phi_0^0) \) | \( \frac{1-|\lambda_0^0|^2}{1+|\lambda_0^0|^2} \) | \( -\frac{2|\lambda_0^0|}{1+|\lambda_0^0|^2} \sin(\phi_0^0) \) |
| \( -\sqrt{2} c_K s_K c_\phi \) | \( A^0_0 \) | \( \frac{1}{2} \left[ \cos(\delta_0^0 - \delta_0^0) + |\lambda_0^0\lambda_0^0| \right] + \frac{i}{2} \left[ |\lambda_0^0| \cos(\delta_0^0 - \delta_0^0) - \phi_0^0 \right] \) | \( -\frac{2|\lambda_0^0|}{1+|\lambda_0^0|^2} \cos(\phi_0^0) \) | \( \frac{1-|\lambda_0^0|^2}{1+|\lambda_0^0|^2} \) | \( -\frac{2|\lambda_0^0|}{1+|\lambda_0^0|^2} \cos(\phi_0^0) \) |
| \( \sqrt{2} c_K c_\phi s_K c_\phi \) | \( A^0_0 \) | \( \frac{1}{2} \left[ \sin(\delta_0^0 - \delta_0^0) - |\lambda_0^0\lambda_0^0| \right] + \frac{i}{2} \left[ |\lambda_0^0| \sin(\delta_0^0 - \delta_0^0) + \phi_0^0 \right] \) | \( \frac{1}{2} |\lambda_0^0| \sin(\delta_0^0 - \delta_0^0) + \frac{1}{2} |\lambda_0^0| \cos(\delta_0^0 - \delta_0^0) \) | \( \frac{1}{2} |\lambda_0^0| \sin(\delta_0^0 - \delta_0^0) + \frac{1}{2} |\lambda_0^0| \cos(\delta_0^0 - \delta_0^0) \) |

The full decay amplitudes can be calculated using the helicity amplitudes and for detailed result we refer the readers to Appendix A. In the presence of the S-wave \( K^+ K^- \) constituent the angular distribution for \( B_s \to J/\psi \to l^+ l^- \) at the time \( t \) of the state that was a pure \( B_s \) at \( t = 0 \) is given as
\[
\frac{d^4 \Gamma(t)}{dm_K^2 \cos \theta_K \cos \theta_l d\phi} = \sum_{k=1}^{10} h_k(t) f_k(\theta_K, \theta_l, \phi),
\]
where the time-dependent functions \( h_k(t) \) are given as
\[
h_k(t) = \frac{3}{4\pi} e^{-\Gamma t} \left\{ a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} + c_k \cos(\Delta m t) + d_k \sin(\Delta m t) \right\}.
\] (8)

For the state that was a \( \bar{B}_s \) at \( t = 0 \), the signs of \( c_k \) and \( d_k \) should be reversed. The explicit results for these coefficients are collected in Table I, in which we have used
\[
\lambda_j' = \eta_j' \frac{q \cdot A_j}{p \cdot A_j} \equiv |\lambda_j'| e^{-iu_j'}.
\] (9)

\( \eta_j' \) is the CP-eigenvalue of the final state, \( j \) is the spin of the \( K^+ K^- \) system and \( i \) is the polarization/helicity configuration of the final state. Assuming that the quantity \( \lambda_j' \) is not sensitive to the helicity, the quantities in table I will be simplified to the form given in table II. Results in this table are in agreement with Ref. [29].

### TABLE II. The angular and time-dependent functions used in Eqs. (7) and (8), under the assumption that the \( \lambda_j' \) is not sensitive to the polarization. Some abbreviations have been used for cosine and sine functions: \( c_K = \cos \theta_K, s_K = \sin \theta_K \).

| \( f_k \) | \( N_k \) | \( a_k \) | \( b_k \) | \( c_k \) | \( d_k \) |
|---|---|---|---|---|---|
| \( \frac{c_K^2 s_i^2}{\lambda_{i/A_i}^2} \) | \( \frac{1}{\lambda_{i/A_i}^2 + |\lambda_{i/A_i}|^2} \) | 1 | \( -\frac{2|\lambda|}{1+|\lambda|^2} \cos(\phi) \) | \( \frac{1-|\lambda|^2}{1+|\lambda|^2} \) | \( \frac{2|\lambda|}{1+|\lambda|^2} \sin(\phi) \) |
| \( \frac{1}{\lambda_{i/A_i}} \) | \( \frac{|\lambda_{i/A_i}|}{|\lambda_{i/A_i}|^2 + \lambda_{i/A_i}^2} \) | 1 | \( -\frac{2|\lambda|}{1+|\lambda|^2} \cos(\phi) \) | \( \frac{1-|\lambda|^2}{1+|\lambda|^2} \) | \( \frac{2|\lambda|}{1+|\lambda|^2} \sin(\phi) \) |
| \( \frac{1}{\lambda_{i/A_i}} \) | \( \frac{|\lambda_{i/A_i}|}{|\lambda_{i/A_i}|^2 + \lambda_{i/A_i}^2} \) | 1 | \( \frac{2|\lambda|}{1+|\lambda|^2} \cos(\phi) \) | \( \frac{1-|\lambda|^2}{1+|\lambda|^2} \) | \( -\frac{2|\lambda|}{1+|\lambda|^2} \sin(\phi) \) |
| \( \sqrt{2} \frac{s_K c_K s_i c_K c_{\phi}}{\lambda_{i/A_i}^2} \) | \( \frac{|\lambda_{i/A_i}|}{|\lambda_{i/A_i}|^2 + \lambda_{i/A_i}^2} \) | \( \frac{1}{|\lambda_{i/A_i}|} \) | \( \frac{2|\lambda|}{1+|\lambda|^2} \cos(\phi) \) | \( \frac{1-|\lambda|^2}{1+|\lambda|^2} \) | \( \frac{2|\lambda|}{1+|\lambda|^2} \sin(\phi) \) |
| \( \sqrt{2} \frac{s_K c_K s_i c_K c_{\phi}}{\lambda_{i/A_i}} \) | \( \frac{|\lambda_{i/A_i}|}{|\lambda_{i/A_i}|^2 + \lambda_{i/A_i}^2} \) | \( \frac{1}{|\lambda_{i/A_i}|} \) | \( \frac{2|\lambda|}{1+|\lambda|^2} \cos(\phi) \) | \( \frac{1-|\lambda|^2}{1+|\lambda|^2} \) | \( -\frac{2|\lambda|}{1+|\lambda|^2} \sin(\phi) \) |

### III. PERTURBATIVE QCD CALCULATION

Because of the large mass of the bottom quark, for convenience, we will work in the rest frame of \( B \) meson, where \( B \) denotes any of the \( B_d, B_u, B_s \) mesons. Throughout this paper, we will use light-cone coordinate \( (P^+, P^−, P_T) \) to describe the meson’s momenta with the definitions
\[
P^\pm = \frac{p_0 \pm p_3}{\sqrt{2}} \quad \text{and} \quad P_T = (p_1, p_2);
\] (10)

Then for \( B \to J/\psi V \) decays, the involved three meson momenta in the light-cone coordinates can be written as
\[
P_1 = \frac{m_B}{\sqrt{2}} (1, 1, 0_T), \quad P_2 = \frac{m_B}{\sqrt{2}} (1 - r_2^2, r_2^2, 0_T), \quad P_3 = \frac{m_B}{\sqrt{2}} (r_3^2, 1 - r_2^2, 0_T),
\] (11)

respectively, where the \( J/\psi (V) \) meson moves in the plus (minus) \( z \) direction carrying the momentum \( P_2 (P_3) \), \( r_2 = m_{J/\psi}/m_B \), and \( r_3 = m_V/m_B \). In the numerical calculations, the higher-order terms of \( r_3 \) can be neglected, as \( r_3^2 \sim 0.036 \) is not large in \( B_s \to J/\psi \phi \) mode. The longitudinal and transverse polarization vectors of vector meson are denoted by \( \epsilon^L \) and \( \epsilon^T \), respectively. The explicit forms of \( \epsilon^L_2 \) and \( \epsilon^T_3 \) can be chosen as
\[
\epsilon^L_2 = \frac{m_B}{\sqrt{2m_{J/\psi}}} (1 - r_2^2, -r_2^2, 0_T) \quad \text{and} \quad \epsilon^L_3 = \frac{m_B}{\sqrt{2m_V}} (-r_3^2, 1 - r_2^2, 0_T).
\] (12)
And the transverse ones are parameterized as \( \epsilon_T^2 = (0, 0, 1_T) \) and \( \epsilon_T^3 = (0, 0, 1_T) \).

Putting the (light) quark momenta in \( B, J/\psi \) and \( V \) mesons as \( k_1, k_2, \) and \( k_3 \), respectively, we have

\[
k_1 = (x_1 P_1^+, 0, k_1T), \quad k_2 = x_2 P_2 + (0, 0, k_2T), \quad k_3 = x_3 P_3 + (0, 0, k_3T).
\]

Then, for \( B \to J/\psi V \) decay, the integration over \( k_1^{-1}, k_2^{-1}, \) and \( k_3^{-1} \) will conceptually lead to the decay amplitude in the pQCD approach,

\[
\mathcal{A}(B \to J/\psi V) \sim \int dx_1 dx_2 dx_3 b_1 b_2 b_3 \times \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{J/\psi}(x_2, b_2) \Phi_V(x_3, b_3) H(x_1, b_1, t) S_i(x_i) e^{-S(t)} \right].
\]

where \( b_i \) is the conjugate space coordinate of \( k_iT \), and \( t \) is the largest energy scale in function \( H(x_i, b_i, t) \). The large logarithms \( \ln(m_W/t) \) are included in the evolution of Wilson coefficients \( C(t) \).

### A. Wave Functions and Distribution Amplitudes

The heavy \( B \) meson is usually treated as a heavy-light system and its light-cone wave function can generally be defined as

\[
\Phi_{B, \alpha \beta, ij} \equiv \langle 0| b_{\beta j}(0) g_{\alpha i}(z)| B(P) \rangle = \frac{i \delta_{ij}}{\sqrt{2N_c}} \int dx d^2 k e^{-i(xP - z^+ - k_T^+)} \{(P + m_B) \gamma_5 \phi_B(x, k_T)\}_\alpha \beta,
\]

where the indices \( i, j \) and \( \alpha, \beta \) are the Lorentz indices and color indices, respectively, \( P(m) \) is the momentum(mass) of the \( B \) meson, \( N_c \) is the color factor, and \( k_T \) is the intrinsic transverse momentum of the light quark in \( B \) meson. Note that, in principle, there are two Lorentz structures of the wave function to be considered in the numerical calculations, however, the contribution induced by the second Lorentz structure is numerically small and approximately negligible [30, 31].

In Eq. (15), \( \phi_B(x, k_T) \) is the \( B \) meson distribution amplitude and obeys to the following normalization condition,

\[
\int_0^1 dx \phi_B(x, b = 0) = \frac{f_B}{2\sqrt{2N_c}}.
\]

where \( b \) is the conjugate space coordinate of transverse momentum \( k_T \) and \( f_B \) is the decay constant of \( B \) meson. For \( B \) meson, the distribution amplitude in the impact b space has been proposed [32, 33]

\[
\phi_B(x, b) = N_B x^2(1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x m_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right].
\]

Based on lots of calculations of form factors and other well-known decay modes of \( B \) meson in the pQCD approach in recent years, the shape parameter \( \omega_B \) has been fixed at about 0.40 GeV [32] by using the rich experimental data on the \( B \) mesons, while \( \omega_B = 0.50 \) GeV for the \( B_s \) meson [34].

The nonlocal matrix elements associated with longitudinally and transversely polarized \( J/\psi \) mesons are decomposed into

\[
\Phi^{L}_{J/\psi, \alpha \beta, ij} \equiv \langle J/\psi(P, \epsilon_L)|\bar{c}(z)_{\beta j} c(0)|\alpha_i|0 \rangle
\]

\[
= \frac{\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx d^2 p e^{-i(xP - z^+)} \left\{ m_{J/\psi} \epsilon_L \phi^{L}_{J/\psi}(x) + \epsilon_L P \phi^{T}_{J/\psi}(x) \right\}_{\alpha \beta},
\]

\[
\Phi^{T}_{J/\psi, \alpha \beta, ij} \equiv \langle J/\psi(P, \epsilon_T)|\bar{c}(z)_{\beta j} c(0)|\alpha_i|0 \rangle
\]

\[
= \frac{\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx d^2 p e^{-i(xP - z^+)} \left\{ m_{J/\psi} \epsilon_T \phi^{T}_{J/\psi}(x) + \epsilon_T P \phi^{\epsilon T}_{J/\psi}(x) \right\}_{\alpha \beta}.
\]

respectively, which define the twist-2 distribution amplitudes \( \phi^{L}_{J/\psi} \) and \( \phi^{T}_{J/\psi} \), and the twist-3 distribution amplitudes \( \phi^{T}_{J/\psi} \) with the \( c \) quark carrying the fractional momentum \( xP \). With the inclusion of the relativistic corrections, the distribution amplitudes for the \( J/\psi \) have been derived as [35]

\[
\phi^{L}_{J/\psi}(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x (1 - x) \left[ \frac{x (1 - x)}{1 - 2.8x(1 - x)} \right]^{0.7},
\]

\[
\phi^{T}_{J/\psi}(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x (1 - x)^2 \left[ \frac{x (1 - x)}{1 - 2.8x(1 - x)} \right]^{0.7},
\]

\[
\phi^{\epsilon T}_{J/\psi}(x) = 1.67 \frac{f_{J/\psi}}{2\sqrt{2N_c}} [1 + (2x - 1)^2] \left[ \frac{x (1 - x)}{1 - 2.8x(1 - x)} \right]^{0.7},
\]

(20)
in which the twist-3 ones $\phi^{\nu,v}$ vanish at the end points due to the additional factor $[x(1- x)]^{0.7}$.
Up to twist-3, the light-cone wave function of a light vector meson is
\[
\Phi^{L}_{V,\alpha\beta,ij} \equiv (V(P, \epsilon_{L}))|\bar{q}(z)_{ij}q(0)_{\alpha i}|0\rangle = \frac{\delta_{ij}}{\sqrt{2N_{c}}} \int_{0}^{1} dx e^{ixP\cdot z} \left\{ m_{V} \gamma_{\mu} \phi_{V}(x) + \gamma_{\nu} \phi_{V}^{\nu}(x) \right\}_{\alpha\beta},
\]
\[
\Phi^{T}_{V,\alpha\beta,ij} \equiv (V(P, \epsilon_{T}))|\bar{q}(z)_{ij}q(0)_{\alpha i}|0\rangle = \frac{\delta_{ij}}{\sqrt{2N_{c}}} \int_{0}^{1} dx e^{ixP\cdot z} \left\{ m_{V} \gamma_{\mu} \phi_{V}(x) + \gamma_{\nu} \phi_{V}^{\nu}(x) \right\}_{\alpha\beta}.
\]
for longitudinal polarization and transverse polarization, respectively, and $x$ denotes the momentum fraction carried by quark in the meson, and $n = (1,0,0_{T})$ and $v = (0,1,0_{T})$ are dimensionless light-like unit vectors. We adopt the convention $\epsilon_{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$.
The twist-2 distribution amplitudes for the longitudinally and tranversely polarized vector meson can be parameterized in terms of Gegenbauer polynomials:
\[
\phi_{V}(x) = \frac{3f_{V}}{\sqrt{2N_{c}}} x(1-x) \left[ 1 + 3a_{1V}^{||} (2x-1) + a_{2V}^{||} \frac{3}{2} (5(2x-1)^2 - 1) \right],
\]
\[
\phi_{V}^{\nu}(x) = \frac{3f_{V}^{\nu}}{\sqrt{2N_{c}}} x(1-x) \left[ 1 + 3a_{1V}^{\nu} (2x-1) + a_{2V}^{\nu} \frac{3}{2} (5(2x-1)^2 - 1) \right],
\]
Here $f_{V}$ and $f_{V}^{\nu}$ are the decay constants of the vector meson with longitudinal and transverse polarization, respectively.

### TABLE III. Input values of the decay constants of the light vector mesons (in MeV)

| $f_{V}$ | $f_{V}^{\rho}$ | $f_{V}^{\omega}$ | $f_{V}^{\rho}$ | $f_{V}^{K^{*}}$ | $f_{V}^{K^{*}}$ | $f_{V}^{\phi}$ | $f_{V}^{\phi}$ |
|--------|---------------|----------------|---------------|----------------|----------------|----------------|----------------|
| 209 ± 2 | 165 ± 9 | 195 ± 3 | 145 ± 10 | 217 ± 5 | 185 ± 10 | 231 ± 4 | 200 ± 10 |

The decay constants $f_{J/\psi}$ and $f_{V}$ can be extracted from the data on $V^{0} \to l^{+}l^{-}$ and $\tau \to V^{-}\bar{\nu}$ [36], while the transverse decay constants $f_{V}^{\nu}$ are taken from Ref. [37]. We collect these quantities in Tab. III. For the Gegenbauer moments of the light-vector mesons, we take the recent updates [38]:
\[
a_{1K^{*}}^{||} = 0.03 \pm 0.02, \quad a_{1K^{*}}^{\rho} = 0.11 \pm 0.09, \quad a_{2\rho}^{||} = a_{2\omega}^{||} = 0.15 \pm 0.07, \quad a_{2\rho}^{\nu} = 0.18 \pm 0.08;
\]
\[
a_{1K^{*}}^{\perp} = 0.04 \pm 0.03, \quad a_{2K^{*}}^{\perp} = 0.10 \pm 0.08, \quad a_{2\rho}^{\perp} = a_{2\omega}^{\perp} = 0.14 \pm 0.06, \quad a_{2\rho}^{\nu} = 0.14 \pm 0.07.
\]
The asymptotic forms of the twist-3 distribution amplitudes $\phi_{V}^{\nu}$ and $\phi_{V}^{\nu,\alpha}$ are:
\[
\phi_{V}(x) = \frac{3f_{V}^{\nu}}{2\sqrt{2N_{c}}} (2x-1)^{-2}, \quad \phi_{V}^{\nu}(x) = -\frac{3f_{V}^{\nu}}{2\sqrt{2N_{c}}} (2x-1),
\]
\[
\phi_{V}^{\nu}(x) = \frac{3f_{V}^{\nu}}{8\sqrt{2N_{c}}} (1 + (2x-1)^2), \quad \phi_{V}^{\nu,\alpha}(x) = -\frac{3f_{V}^{\nu}}{4\sqrt{2N_{c}}} (2x-1).
\]
In the pQCD approach, the above choices of vector meson distribution amplitudes can successfully explain the measured $B \to K^{*}\phi$, $B \to K^{*}\rho$ and $B \to \rho\rho$ polarization fractions together with the right branching ratios [39, 40].

### B. Perturbative Calculations

For the considered $B_{u/d/s} \to J/\psi V (V = \rho, \omega, \phi, K^{*})$ decays, the related weak effective Hamiltonian $H_{\text{eff}}$ can be written as [41]
\[
H_{\text{eff}} = \frac{G_{F}}{\sqrt{2}} \left\{ V_{cb}^{*}V_{cD}[C_{1}(\mu)O_{1}^{\mu}(\mu) + C_{2}(\mu)O_{2}^{\mu}(\mu)] - V_{tb}^{*}V_{tD}\sum_{i=3}^{10} C_{i}(\mu)O_{i}(\mu) \right\} + \text{H.c.},
\]
with the Fermi constant $G_{F} = 1.16639 \times 10^{-5}\text{GeV}^{-2}$, the light $D = d, s$ quark, and Wilson coefficients $C_{i}(\mu)$ at the renormalization scale $\mu$. The local four-quark operators $O_{i}(i = 1, \cdots, 10)$ are written as
where the upper(lower) sign applies, when
the color indices
runs through
is chosen as the largest energy scale in the gluon and/or the quark propagators of a given Feynman diagram, in order to suppress
instance, in Ref. \([B, C]\) electroweak penguin operators
\(O_{10} = (\bar{D}_a b_a)_{V-A} \sum_{q'} (\bar{q}_b' q_{b_a}')_{V-A} ; \)
with the color indices \(\alpha, \beta\) and the notations \((\bar{q} q')_{V,A} = \bar{q}^\gamma (1 \pm \gamma_5) q'\). The index \(q'\) in the summation of the above operators runs through \(u, d, s\), and \(b\). The standard combinations \(a_i\) of Wilson coefficients are defined as follows,
\[
\begin{align*}
a_1 &= C_2 + \frac{C_1}{3}, \\
a_2 &= C_1 + \frac{C_2}{3}, \\
a_i &= C_i + \frac{C_{i\pm1}}{3} (i = 3 - 10) ,
\end{align*}
\]
where the upper(lower) sign applies, when \(i\) is odd(even).
In the perturbative QCD approach, the scale \(t\) in Wilson coefficients \(C_i(t)\), hard-kernel \(H(x_i, b_i, t)\) and Sudakov factor \(e^{-S(t)}\) is chosen as the largest energy scale in the gluon and/or the quark propagators of a given Feynman diagram, in order to suppress the higher order corrections and improve the convergence of the perturbative calculation. In the range of \(t < m_b\) or \(t \geq m_b\), the number of active quarks is \(N_f = 4\) or \(N_f = 5\), respectively. The explicit expressions of the LO and NLO \(C_i\) can be found, for instance, in Ref. \([41]\).
In the leading order calculation, the leading order Wilson coefficients \(C_i(m_W)\), the leading order RG evolution matrix \(U(t, m)^{(0)}\) from the high scale \(m\) down to \(t < m\) and the leading order \(\alpha_s(t)\) are used:
\[
\alpha_s(t) = \frac{4\pi}{\beta_0 \ln \left[ t^2 / \Lambda_{QCD}^2 \right]},
\]
where \(\beta_0 = (33 - 2N_f) / 3\). In the NLO contributions, the NLO Wilson coefficients \(C_i(m_W)\), the NLO RG evolution matrix \(U(t, m, \alpha)\) ( see Eq. (7.22) in Ref. \([41]\)) and the \(\alpha_s(t)\) at two-loop level will be used:
\[
\alpha_s(t) = \frac{4\pi}{\beta_0 \ln \left[ t^2 / \Lambda_{QCD}^2 \right]} \left\{ 1 - \frac{\beta_1}{\beta_0} \ln \left[ \ln \left[ \frac{t^2 / \Lambda_{QCD}^2}{\Lambda_{QCD}^2} \right] \right] \right\},
\]
where $\beta_0 = (33 - 2N_f)/3$, $\beta_1 = (306 - 38N_f)/3$. Using $\Lambda^{(S)}_{QCD} = 0.225$ GeV, we get $\Lambda^{(4)}_{QCD} = 0.287$ GeV (0.326 GeV) for LO (NLO) case. As discussed in Ref. [42, 43], it is reasonable to choose $\mu_0 = 1.0$ GeV as the lower cut-off of the hard scale $t$. In the numerical integrations we will fix the values $C_i(t)$ at $C_i(1.0)$ whenever the scale $t$ runs below the scale $\mu_0 = 1.0$ GeV, unless otherwise stated.

The decay amplitudes $\mathcal{M}^{(\sigma)}$ for $B \to J/\psi (P_2, \epsilon^*_2) (P_3, \epsilon^*_3)$ decays can be decomposed into products of independent Lorentz structures

$$\mathcal{M}^{(\sigma)} = \epsilon^{2\mu}_{\rm h} (\sigma) \epsilon^{\nu*}_{3\nu}(\sigma) \left[ a g^{\mu\nu} + \frac{b}{m_{J/\psi} m_V} P^{\mu}_1 P^{\nu}_1 + i \frac{c}{m_{J/\psi} m_V} \epsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{3\beta} \right],$$

$$\equiv m^2_B \mathcal{M}_L + m^2_B \mathcal{M}_N \epsilon^*_3(\sigma) \cdot \epsilon^*_3(\sigma) + i M_T \epsilon^{\alpha\beta\gamma\rho} \epsilon^*_{2\alpha}(\sigma) \epsilon^*_{3\beta}(\sigma) P_2, P_3 p,$$

(36)

where the superscript $\sigma$ denotes the helicity states of two mesons with $L(T)$ standing for the longitudinal (transverse) component. And the definitions of the amplitudes $\mathcal{M}_{i}(i = L, N, T)$ in terms of the Lorentz-invariant amplitudes $a, b$ and $c$ are

$$m^2_B \mathcal{M}_L = a \epsilon^*_2(L) \cdot \epsilon^*_3(L) + \frac{b}{m_{J/\psi} m_V} \epsilon^*_2(L) \cdot P_3 \epsilon^*_3(L) \cdot P_2,$$

$$m^2_B \mathcal{M}_N = a \epsilon^*_3(T) \cdot \epsilon^*_3(T),$$

$$m^2_B \mathcal{M}_T = \frac{c}{r_2 r_3}.$$

We shall compute these amplitudes $\mathcal{M}_L, \mathcal{M}_N, \mathcal{M}_T$ individually.

By taking various of contributions from the relevant Feynman diagrams into consideration, the total decay amplitudes for $B \to J/\psi V$ channels are given as

$$\mathcal{M}^h(B \to J/\psi V) = F^h f_{J/\psi} \left\{ V^*_{s\bar{s}} V_{c\bar{c}}(s) \frac{a_2 - V^*_{s\bar{s}} V_{s\bar{s}}(s)}{a_3 + a_5 + a_7 + a_9} \right\}$$

$$+ M^h \left\{ V^*_{s\bar{s}} V_{c\bar{c}}(s) C_2 - V^*_{s\bar{s}} V_{s\bar{s}}(s) \left( C_2 - C_6 - C_8 + C_10 \right) \right\},$$

(38)

where the superscript $h$ standing for the three polarizations $L, N, T$, respectively. We adopt $F$ and $M$ to stand for the contributions of factorizable and non-factorizable diagrams from $(V - A)(V - A)$ operators. The leading-order factorization amplitudes derived from Fig. 2 for three polarizations can be read as,

$$F^L = \zeta \frac{8 \pi C_F m^2_B}{F^2} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 b_3 b_4 \phi_B(x_1, b_1)(r_2 - 1)$$

$$\times \left\{ \left[ (r_2^2 - 1)x_3 - 1 \right] \phi_V(3x) + r_3 (2x - 1) \phi_V^* (3x) - r_3 [2(r_2^2 - 1)x + r_3^2 + 1] \phi_V^* (x) \right\} h_{f_s}(x_1, x_3, b_1) E_{f_s}(t_a) - \left[ 2r_3 \phi_V^* (x) \right] h_{f_s}(x_3, x_1, b_3) E_{f_s}(t_b),$$

(39)

$$F^N = \zeta \frac{8 \pi C_F m^2_B}{F^2} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 b_3 b_4 \phi_B(x_1, b_1) r_2$$

$$\times \left\{ - \left[ (r_2^2 - 1)x_3 - 1 \right] x_3 \phi_V(3x) + (r_2 - 1) \phi_V^* (3x) + r_3 (r_2 - 1)x_3 - 2] \phi_V^* (x) \right\} E_{f_s}(t_a)$$

$$\times h_{f_s}(x_1, x_3, b_1, b_3) + r_3 (r_2 - 1) \left[ (r_2^2 - 1) \phi_V^* (x) - \phi_V^* (3x) \right] h_{f_s}(x_3, x_1, b_3, b_1) E_{f_s}(t_b),$$

(40)

$$F^T = \zeta \frac{16 \pi C_F m^2_B}{F^2} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 b_3 b_4 \phi_B(x_1, b_1) r_2$$

$$\times \left\{ - [r_3 x_3 \phi_V^* (x) - \phi_V^* (3x) + r_3 (r_2 - 1)x_3 - 2] \phi_V^* (x) \right\} h_{f_s}(x_1, x_3, b_1, b_3)$$

$$\times E_{f_s}(t_a) - r_3 \left[ (r_2^2 - 1) \phi_V^* (x) - \phi_V^* (3x) \right] h_{f_s}(x_3, x_1, b_3, b_1) E_{f_s}(t_b),$$

(41)

where the factor $\zeta = 1$ except $\zeta = -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ for $\rho^0, \omega$. 
For the nonfactorizable spectator diagrams, Eq. 2(c) and 2(d), all three meson wave functions are involved. The integration of $b_3$ can be performed using the function $\delta(b_3 - b_1)$, leaving only integration of $b_1$ and $b_2$. For the $(V - A)(V - A)$ operators, the corresponding decay amplitude is

\[
M^L = \frac{16\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_2 \phi_B(x_1, b_1) (r_2^2 - 1) \\
\times \left\{ \phi_V(x_3) - 2r_3 \phi_V^2(x_3) \right\} \left[ x_3 \phi_{J/\psi}(x_2) + (2x_2 - x_3) r_2^2 \phi_{J/\psi}(x_2) \\
- 2r_2 r_c \phi_{J/\psi}^2(x_2) \right\} h_{nfs}(x_1, x_2, x_3, b_1, b_2) E_{nfs}(t_{nfs}),
\]

(42)

\[
M^N = \frac{32\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_2 \phi_B(x_1, b_1) \\
\times \left\{ (r_2^2 - 1) \left[ r_c \phi_{J/\psi}(x_2) - r_2 x_2 \phi_{J/\psi}^2(x_2) \right] \phi_T^2(x_3) \right\} + r_3 \left[ r_c (1 + r_2^2) \phi_{J/\psi}^3(x_2) - r_2 (x_2 (1 + r_2^2) \\
+ x_3 (1 - r_2^2) \phi_{J/\psi}^2(x_2) \right] \phi_T^2(x_3) \right\} h_{nfs}(x_1, x_2, x_3, b_1, b_2) E_{nfs}(t_{nfs}),
\]

(43)

\[
M^T = -\frac{64\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_2 \phi_B(x_1, b_1) \\
\times \left\{ r_c \phi_{J/\psi}(x_2) - r_2 x_2 \phi_{J/\psi}^2(x_2) \right\} \phi_T^2(x_3) + r_3 \left[ -r_c (1 + r_2^2) \phi_{J/\psi}^3(x_2) + r_2 (x_2 (1 + r_2^2) \\
+ x_3 (1 - r_2^2) \phi_{J/\psi}^2(x_2) \right] \phi_T^2(x_3) \right\} h_{nfs}(x_1, x_2, x_3, b_1, b_2) E_{nfs}(t_{nfs}).
\]

(44)

where $r_c = m_c / m_B$ with $m_c$, the charm quark mass.

FIG. 3. (Color online) Typical Feynman diagrams contributing to $B \to J/\psi V$ decays known at Next-to-Leading Order level, in which the contributions will be combined into the Wilson coefficients associated with the factorizable contributions.

As was pointed out in Ref. [24], for these considered $B \to J/\psi V$ decays, only the vertex corrections (see Fig. 3a-3d) will contribute at the current NLO level, in which their effects can be combined into the Wilson coefficients associated with the factorizable contributions [44]:

\[
a_2^h = C_1 + \frac{C_2}{N_c} + \frac{\alpha_s C_F}{4 \pi N_c} C_2 \left( -18 + 12 \ln \frac{m_b}{\mu} + f_1^h \right),
\]

(45)

\[
a_3^h = C_3 + \frac{C_4}{N_c} + \frac{\alpha_s C_F}{4 \pi N_c} C_4 \left( -18 + 12 \ln \frac{m_b}{\mu} + f_1^h \right),
\]

(46)

\[
a_4^h = C_5 + \frac{C_6}{N_c} + \frac{\alpha_s C_F}{4 \pi N_c} C_6 \left( 6 - 12 \ln \frac{m_b}{\mu} - f_1^h \right),
\]

(47)

\[
a_5^h = C_7 + \frac{C_8}{N_c} + \frac{\alpha_s C_F}{4 \pi N_c} C_8 \left( 6 - 12 \ln \frac{m_b}{\mu} - f_1^h \right),
\]

(48)

\[
a_6^h = C_9 + \frac{C_{10}}{N_c} + \frac{\alpha_s C_F}{4 \pi N_c} C_{10} \left( -18 + 12 \ln \frac{m_b}{\mu} + f_1^h \right).
\]

(49)
with the function $f_I^h$,
\[ f_I^h = f_I + g_I(1 - r^2) , \quad f_I^\pm = f_I , \] (50)
where the functions $f_I$ and $g_I$ read as [44]
\[ f_I = \frac{2\sqrt{2N_c}}{f_{J/\psi}} \left[ \int_0^1 dx_2 \phi_{J/\psi}^L(x_2) \left\{ \frac{2r^2x_2}{1 - r^2(1 - x_2)} + \frac{3}{1 - r^2(1 - x_2)} \ln x_2 \right\} \right. 
\left. + \left( -\frac{1}{1 - r^2(1 - x_2)} + \frac{1}{1 - r^2(1 - x_2)} - \frac{2r^2x_2}{(1 - r^2(1 - x_2))^2} \right) r^2x_2 \ln(r^2x_2) \right.
\left. + \left( 3(1 - r^2) + 2r^2x_2 + \frac{2r^2x_2^2}{1 - r^2(1 - x_2)} \right) \ln(1 - r^2) - i\pi \right\} 
\int_0^1 dx_2 \phi_{J/\psi}^T(x_2) \left\{ -8x_2^2 \frac{\ln x_2}{1 - r^2(1 - x_2)} + \frac{8r^2x_2^2 \ln(r^2x_2)}{1 - r^2(1 - x_2)} - \frac{8r^2x_2^2}{1 - r^2(1 - x_2)} \right\} , \] (51)
and
\[ g_I = \frac{2\sqrt{2N_c}}{f_{J/\psi}} \left[ \int_0^1 dx_2 \phi_{J/\psi}^L(x_2) \left\{ -\frac{4x_2}{1 - r^2(1 - x_2)} \ln x_2 + \frac{r^2x_2}{1 - r^2(1 - x_2)^2} \ln(1 - r^2) \right\} 
\left. + \left( \frac{1}{1 - r^2(1 - x_2)^2} - \frac{1}{1 - r^2(1 - x_2)^2} + \frac{2(1 + r^2 - 2r^2x_2)}{(1 - r^2(1 - r^2(1 - x_2)^2))} \right) r^2x_2 \ln(r^2x_2) - i\pi \frac{r^2x_2}{1 - r^2(1 - x_2)^2} \right\} 
\left. + \int_0^1 dx_2 \phi_{J/\psi}^T(x_2) \left\{ \frac{8x_2^2}{(1 - r^2(1 - x_2))} \ln x_2 - \frac{8r^2x_2^2}{(1 - r^2(1 - x_2))} \ln(r^2x_2) \right\} \right] , \] (52)
respectively.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will present the theoretical predictions on the CP-averaged BRs, CP-averaged polarization fractions, and CP-violating asymmetries for those considered $B \to J/\psi V$ decay modes. In numerical calculations, central values of the input parameters will be used implicitly unless otherwise stated.

A. Input quantities

The masses (in units of GeV) and $B$ meson lifetime (in ps) are taken from Particle Data Group [36]
\[ m_W = 80.41 , \quad m_B = 5.28 , \quad m_{B_s} = 5.37 , \quad m_b = 4.8 ; \]
\[ \tau_{B_s} = 1.641 , \quad \tau_{B_d} = 1.519 , \quad \tau_{B_s} = 1.497 , \quad m_{J/\psi} = 3.097 . \] (53)
For the CKM matrix elements, we adopt the Wolfenstein parametrization up to corrections of $\mathcal{O}(\lambda^5)$ and the updated parameters $A = 0.811 , \lambda = 0.22535 , \rho = 0.131^{+0.026}_{-0.013}$ and $\eta = 0.345^{+0.013}_{-0.014}$ [36].

B. CP-averaged Branching Ratios, Polarization Fractions, and Relative Phases

In this subsection, we will analyze the CP-averaged BRs of the considered $B \to J/\psi V$ decays in the pQCD approach. For $B \to J/\psi V$ decays, the decay rate can be written explicitly as,
\[ \Gamma = \frac{G_F^2 |P_c|}{16\pi m_B^2} \sum_{\sigma = L,T} \mathcal{M}^{(\sigma)} \mathcal{M}^{(\sigma)} \] (54)
where $|P_c| = |P_{2\omega}| = |P_{3\omega}|$ is the momentum of either of the outgoing vector mesons. Using the decay amplitudes obtained in last section, it is straightforward to calculate the CP-averaged BRs with uncertainties as displayed in Eqs. (58)-(61). The
dominant errors are induced by the shape parameters $\omega_B = 0.40 \pm 0.04 (\omega_B = 0.50 \pm 0.05)$ GeV for $B (B_s)$ meson, the uncertainties of the decay constants $f_M$, the Gegenbauer moments $a_i$ for the light vector mesons, the charm quark mass $m_c = (1.50 \pm 0.15)$ GeV, and CKM matrix elements ($\bar{\rho}, \bar{\eta}$), respectively. It is worthwhile to stress that the variation of the CKM parameters has almost no effects on the CP-averaged BRs and polarization fractions of these considered $B_{(s)} \to J/\psi V$ decays in the pQCD approach and thus will be neglected in the numerical results as shown in Eqs. (58)-(61) and Eqs. (90)-(98).

The theoretical predictions in the pQCD approach for the CP-averaged BRs of the decays under consideration within errors are the following,

- for $b \to \bar{s}$ decay channels,

$$\text{BR}(B_d \to J/\psi K^{*0}) = 1.23^{+0.30}_{-0.23} (\omega_B) + 0.16 (f_M) + 0.10 (a_i) + 0.22 (m_c) \quad [1.23^{+0.42}_{-0.36}] \times 10^{-3}, \quad (55)$$

$$\text{BR}(B_u \to J/\psi K^{*+}) = \frac{\tau_{B_d}}{\tau_{B_d}} \cdot \text{BR}(B_d \to J/\psi K^{*0})$$

$$= 1.33^{+0.32}_{-0.25} (\omega_B) + 0.17 (f_M) + 0.11 (a_i) + 0.24 (m_c) \quad [1.33^{+0.45}_{-0.39}] \times 10^{-3}, \quad (56)$$

$$\text{BR}(B_s \to J/\psi \phi) = 1.02^{+0.29}_{-0.22} (\omega_B) + 0.14 (f_M) + 0.06 (a_i) + 0.16 (m_c) \quad [1.02^{+0.36}_{-0.30}] \times 10^{-3}; \quad (57)$$

- for $b \to \bar{d}$ decay channels,

$$\text{BR}(B_d \to J/\psi \rho^0) = 2.7^{+0.7}_{-0.5} (\omega_B) + 0.3 (f_M) + 0.2 (a_i) + 0.5 (m_c) \quad [2.7^{+1.0}_{-0.7}] \times 10^{-5}, \quad (58)$$

$$\text{BR}(B_u \to J/\psi \rho^+) = 2 \cdot \frac{\tau_{B_d}}{\tau_{B_d}} \cdot \text{BR}(B_d \to J/\psi \rho^0)$$

$$= 5.8^{+1.5}_{-1.0} (\omega_B) + 0.6 (f_M) + 0.2 (a_i) + 1.1 (m_c) \quad [5.8^{+2.2}_{-1.6}] \times 10^{-5}, \quad (59)$$

$$\text{BR}(B_d \to J/\psi \omega) = 2.3^{+0.4}_{-0.3} (\omega_B) + 0.2 (f_M) + 0.1 (a_i) + 0.3 (m_c) \quad [2.3^{+0.6}_{-0.7}] \times 10^{-5}; \quad (60)$$

$$\text{BR}(B_s \to J/\psi K^{*0}) = 4.1^{+1.1}_{-0.8} (\omega_B) + 0.5 (f_M) + 0.3 (a_i) + 0.7 (m_c) \quad [4.1^{+1.4}_{-1.1}] \times 10^{-5}. \quad (61)$$

where the values as given in the square parentheses are obtained by adding the various errors in quadrature.

Experimentally, the available measurements of the branching ratios for the considered decay modes are as follows [36, 45, 46],

$$\text{BR}^\text{ex.}(B_d \to J/\psi K^{*0}) = (1.34 \pm 0.06) \times 10^{-3}, \quad (62)$$

$$\text{BR}^\text{ex.}(B_u \to J/\psi K^{*+}) = (1.43 \pm 0.08) \times 10^{-3}; \quad (63)$$

$$\text{BR}^\text{ex.}(B_s \to J/\psi \phi) = (1.09^{+0.28}_{-0.23}) \times 10^{-3}, \quad (64)$$

$$\text{BR}^\text{ex.}(B_d \to J/\psi \rho^0) = (2.7 \pm 0.4) \times 10^{-5}, \quad (65)$$

$$\text{BR}^\text{ex.}(B_u \to J/\psi \rho^+) = (5.0 \pm 0.8) \times 10^{-5}, \quad (66)$$

$$\text{BR}^\text{ex.}(B_d \to J/\psi \omega) = (2.4 \pm 0.7) \times 10^{-5}; \quad (67)$$

$$\text{BR}^\text{ex.}(B_s \to J/\psi K^{*0}) = (4.4 \pm 0.9) \times 10^{-5}. \quad (68)$$

where various errors from experimental measurements have been added in quadrature for $B_d \to J/\psi \omega$ and $B_s \to J/\psi K^{*0}$ decays, respectively. It is necessary to stress that the LHCb results for $B_s$ decays correspond to the time-integrated quantities, while theory predictions made in the above refer to the branching fractions at $t = 0$ [47], and may differ by 10%. The precision of the measurements for the $b \to d$ modes will be rapidly improved along with the more and more data samples collected in the LHC experiments.

From the above results, one can see that most of our pQCD predictions on CP-averaged branching ratios up to NLO precision agree well with the existing experimental measurements within the uncertainties. Meanwhile, one can observe that the decay rates for the $b \to s$ transition processes, i.e., $B_{u,d} \to J/\psi K^*$ and $B_s \to J/\psi \phi$, are generally much larger than those for the $b \to d$ transition ones, i.e., $B_{u,d} \to J/\psi \rho$, $B_d \to J/\psi \omega$, and $B_s \to J/\psi K^{*0}$. This is due to the CKM hierarchy for two kinds of process: the CKM factors $V_{cb} V_{cs}$ in $b \to s$ are about 4 times larger than the $V_{cb} V_{cd}$ for $b \to d$ process. The remanent but small differences arise from the SU(3) symmetry breaking effects in the hadronic parameters, such as decay constants, mesonic masses, distribution amplitudes, etc. This analysis may provide theoretical ground for the use of SU(3) symmetry in $B$ and $B_s$ decays to hunt for a scalar glueball in Ref. [48].

Here, we will also explore some interesting relations among those considered decay channels,
• The ratio $R_{\omega/\rho}$ between the branching ratios of $B_d \to J/\psi \omega$ and $B_d \to J/\psi \rho^0$ decays can be defined as,

$$R_{\omega/\rho}^{th} \equiv \frac{BR(B_d \to J/\psi \omega)}{BR(B_d \to J/\psi \rho^0)} \approx 0.85 ,$$

which is in good consistency with the LHCb measurement [46] within errors,

$$\frac{BR(B_d \to J/\psi \omega)}{BR(B_d \to J/\psi \rho^0)} = 0.89 \pm 0.19_{(\text{stat})}^{+0.07}_{-0.13_{(\text{syst})}} , \quad (70)$$

Theoretically, both of these two decays embrace the same component at the quark level, which means the involved QCD behavior is similar, and the differences between their CP-averaged branching ratios are just from their decay constants and masses of the relevant mesons.

• The ratio for branching fractions of $B_s \to J/\psi \bar{K}^*0$ and $B_d \to J/\psi K^*0$ decays is predicted as

$$R_{s/d}^{th} \equiv \frac{BR(B_s \to J/\psi \bar{K}^*0)}{BR(B_d \to J/\psi K^*0)} \approx 0.0333 ,$$

which agrees well with that shown in Ref. [45]

$$\frac{BR(B_s \to J/\psi \bar{K}^*0)}{BR(B_d \to J/\psi K^*0)} = (3.43^{+0.34}_{-0.36} \pm 0.50)\% , \quad (72)$$

and also with the CDF results [49]

$$\frac{BR(B_s \to J/\psi \bar{K}^*0)}{BR(B_d \to J/\psi K^*0)} = 0.062 \pm 0.009_{(\text{stat})} \pm 0.025_{(\text{syst})} \pm 0.008_{(\text{frag})} , \quad (73)$$

where the branching ratio of $B_s \to J/\psi K^*0$ measured by CDF Collaboration is $[8.3 \pm 1.2_{(\text{stat})} \pm 3.4_{(\text{syst})} \pm 1.0_{(\text{frag})} \pm 0.4_{(\text{norm})}] \times 10^{-5}$ [49].

• The ratio for the branching ratios of two $B_s$ decay channels can be read theoretically as,

$$R_{K^*/\phi}^{th} \equiv \frac{BR(B_s \to J/\psi K^*0)}{BR(B_s \to J/\psi \phi)} \approx 0.040 ,$$

which is also in good agreement with the entry derived from the available data [36, 45],

$$\frac{BR(B_s \to J/\psi K^*0)}{BR(B_s \to J/\psi \phi)} \approx 0.040 , \quad (75)$$

• In those two $\bar{b} \to s$ transition modes, the theoretical ratio of $BR(B_d \to J/\psi K^*0)$ to $BR(B_s \to J/\psi \phi)$ is

$$R_{d/s}^{th} \equiv \frac{BR(B_d \to J/\psi K^*0)}{BR(B_s \to J/\psi \phi)} \approx 1.21 ,$$

which is consistent well with that provided by the existing data [36],

$$\frac{BR(B_d \to J/\psi K^*0)}{BR(B_s \to J/\psi \phi)} \approx 1.22 , \quad (77)$$

We have also computed the polarization fractions for $B \to J/\psi V$ decay modes. Based on the helicity amplitudes (37), we can define the transversity amplitudes as

$$A_L = -\xi \mu^2 B M_L , \quad A_\parallel = \xi \sqrt{2} \mu^2 B M_N , \quad A_\perp = \xi r_2 r_3 \sqrt{2(r^2 - 1)} \mu^2 B M_T . \quad (78)$$
for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor \( \xi = \sqrt{G_F^2 \rho_\pi/(16\pi m_\pi^2 \Gamma)} \) and the ratio \( r = P_2 P_3/(m_2^2 r_2 r_3) \). These amplitudes satisfy the relation,

\[
|A_L|^2 + |A_0|^2 + |A_\perp|^2 = 1 .
\] (79)

following the summation in Eq. (54). The polarization fractions \( f_L, f_0 \) and \( f_\perp \) can thus be read as,

\[
f_L = \frac{|A_L|^2}{|A_L|^2 + |A_0|^2 + |A_\perp|^2} = |A_L|^2 ,
\] (80)

The CP-averaged polarization fractions for those \( B \to J/\psi K^* \) decays are

- for \( B_{u/d} \to J/\psi K^* \) decays,
  
  \[
  f_L(B_{u/d} \to J/\psi K^*) = 48.7^{+0.8}_{-0.7}(\omega_B)\sqrt{1.6}(f_M)^{+3.8}_{-3.8}(a_i)^{+0.7}_{-1.2}(m_c) \quad [48.7^{+4.3}_{-4.3}] \% ,
  \] (81)
  
  \[
  f_0(B_{u/d} \to J/\psi K^*) = 30.3^{+0.4}_{-0.5}(\omega_B)^{+2.2}_{-2.1}(a_i)^{+1.2}_{-0.9}(m_c) \quad [30.3^{+2.7}_{-2.5}] \% ,
  \] (82)
  
  \[
  f_\perp(B_{u/d} \to J/\psi K^*) = 21.0^{+0.2}_{-0.3}(\omega_B)^{+0.8}_{-0.9}(f_M)^{+1.6}_{-1.8}(a_i)^{+0.2}_{-0.0}(m_c) \quad [21.0^{+1.8}_{-2.1}] \% ;
  \] (83)

which agree with the LHCb measurement [50]:

\[
\begin{align*}
  f_L(B_{u/d} \to J/\psi K^*) &= (22.7 \pm 0.4 \pm 1.1) \% , \\
  f_\perp(B_{u/d} \to J/\psi K^*) &= (20.1 \pm 0.4 \pm 0.8) \% .
\end{align*}
\] (84, 85)

- for \( B_s \to J/\psi \phi \) mode,

\[
\begin{align*}
  f_L(B_s \to J/\psi \phi) &= 50.7^{+0.8}_{-0.8}(\omega_B)^{+1.5}_{-1.3}(f_M)^{+3.1}_{-3.0}(a_i)^{+0.5}_{-1.1}(m_c) \quad [50.7^{+3.6}_{-3.6}] \% , \\
  f_0(B_s \to J/\psi \phi) &= 29.8^{+0.6}_{-0.7}(\omega_B)^{+0.8}_{-0.7}(f_M)^{+1.7}_{-1.7}(a_i)^{+1.2}_{-0.8}(m_c) \quad [29.8^{+2.3}_{-2.0}] \% , \\
  f_\perp(B_s \to J/\psi \phi) &= 19.4^{+0.4}_{-0.3}(\omega_B)^{+0.7}_{-0.6}(f_M)^{+1.4}_{-1.3}(a_i)^{+0.4}_{-0.0}(m_c) \quad [19.4^{+1.7}_{-1.5}] \% ;
\end{align*}
\] (86, 87, 88)

which are also in agreement with the recent measurement by the LHCb Collaboration [2]:

\[
\begin{align*}
  f_L &= 0.497 \pm 0.013(\text{stat}) \pm 0.030(\text{syst}) , \\
  f_\perp &= 0.237 \pm 0.015(\text{stat}) \pm 0.012(\text{syst}) ;
\end{align*}
\] (89)

- for \( B_{u/d} \to J/\psi \rho \) decays,

\[
\begin{align*}
  f_L(B_{u/d} \to J/\psi \rho) &= 51.8^{+0.9}_{-0.7}(\omega_B)^{+1.7}_{-1.6}(f_M)^{+2.8}_{-2.6}(a_i)^{+0.0}_{-0.4}(m_c) \quad [51.8^{+3.4}_{-3.2}] \% , \\
  f_0(B_{u/d} \to J/\psi \rho) &= 27.7^{+0.5}_{-0.4}(\omega_B)^{+0.9}_{-0.9}(f_M)^{+1.4}_{-1.4}(a_i)^{+0.8}_{-0.5}(m_c) \quad [27.7^{+2.1}_{-1.7}] \% , \\
  f_\perp(B_{u/d} \to J/\psi \rho) &= 20.4^{+0.3}_{-0.3}(\omega_B)^{+0.7}_{-0.8}(f_M)^{+1.3}_{-1.2}(a_i)^{+0.5}_{-0.2}(m_c) \quad [20.4^{+1.6}_{-1.9}] \% ;
\end{align*}
\] (90, 91, 92)

- for \( B_d \to J/\psi \omega \) mode,

\[
\begin{align*}
  f_L(B_d \to J/\psi \omega) &= 53.5^{+0.9}_{-0.8}(\omega_B)^{+2.2}_{-2.0}(f_M)^{+2.6}_{-2.5}(a_i)^{+0.0}_{-0.5}(m_c) \quad [53.5^{+3.3}_{-3.3}] \% , \\
  f_0(B_d \to J/\psi \omega) &= 26.9^{+0.6}_{-0.5}(\omega_B)^{+1.1}_{-1.0}(f_M)^{+1.5}_{-1.4}(a_i)^{+0.8}_{-0.4}(m_c) \quad [26.9^{+2.1}_{-1.8}] \% , \\
  f_\perp(B_d \to J/\psi \omega) &= 19.5^{+0.3}_{-0.3}(\omega_B)^{+1.0}_{-1.0}(f_M)^{+1.3}_{-1.1}(a_i)^{+0.5}_{-0.2}(m_c) \quad [19.5^{+1.7}_{-1.5}] \% ;
\end{align*}
\] (93, 94, 95)
\[ f_\perp (B_s \rightarrow J/\psi \bar{K}^*) = 20.1^{+2.0}_{-1.9} \% \quad \text{(98)} \]

while the measurement for the polarization fractions for \( B_s \rightarrow J/\psi \bar{K}^* \) decay LHCb Collaboration [45] are

\[ f_L = 0.50 \pm 0.08 \pm 0.02, \quad f_\parallel = 0.19^{+0.10}_{-0.08} \pm 0.02. \quad \text{(99)} \]

In terms of the transversity amplitudes, we study their relative phases \( \phi_\parallel \) and \( \phi_\perp \)

\[ \phi_\parallel \equiv \arg \frac{A_\parallel}{A_L} + \pi \quad \text{and} \quad \phi_\perp \equiv \arg \frac{A_\perp}{A_L} + \pi. \quad \text{(100)} \]

We then predict the CP-averaged relative phases for the considered \( B \rightarrow J/\psi V \) decays as

- for \( B_{u/d} \rightarrow J/\psi K^* \) decays,
  \[ \phi_\parallel = 2.65^{+0.10}_{-0.08} \quad \text{rad} , \quad \phi_\perp = 2.59^{+0.08}_{-0.11} \quad \text{rad} ; \quad \text{(101)} \]
- for \( B_s \rightarrow J/\psi \phi \) mode,
  \[ \phi_\parallel = 2.74^{+0.07}_{-0.07} \quad \text{rad} , \quad \phi_\perp = 2.65^{+0.08}_{-0.08} \quad \text{rad} ; \quad \text{(102)} \]
- for \( B_{u/d} \rightarrow J/\psi \rho \) decays,
  \[ \phi_\parallel = 2.58^{+0.10}_{-0.08} \quad \text{rad} , \quad \phi_\perp = 2.52^{+0.09}_{-0.11} \quad \text{rad} ; \quad \text{(103)} \]
- for \( B_d \rightarrow J/\psi \omega \) mode,
  \[ \phi_\parallel = 2.58^{+0.09}_{-0.11} \quad \text{rad} , \quad \phi_\perp = 2.54^{+0.10}_{-0.12} \quad \text{rad} ; \quad \text{(104)} \]
- for \( B_s \rightarrow J/\psi \bar{K}^* \) mode
  \[ \phi_\parallel = 2.67^{+0.09}_{-0.08} \quad \text{rad} , \quad \phi_\perp = 2.59^{+0.10}_{-0.11} \quad \text{rad} ; \quad \text{(105)} \]

where various errors from the input parameters have been added in quadrature.

C. CP-violating Asymmetries

As for the direct CP-violating asymmetry in these considered modes, considering the involved three polarizations, whose definitions are as follows,

\[ A_{CP}^{\text{dir}} = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma} = \frac{\left| \mathcal{M}(\bar{B} \rightarrow \bar{f}_{\text{final}}) \right|^2 - \left| \mathcal{M}(B \rightarrow f_{\text{final}}) \right|^2}{\left| \mathcal{M}(B \rightarrow f_{\text{final}}) \right|^2 + \left| \mathcal{M}(\bar{B} \rightarrow \bar{f}_{\text{final}}) \right|^2} \quad \text{(106)} \]

where \( \Gamma \) and \( M \) denote the decay rate and decay amplitude of \( B \rightarrow J/\psi V \) decays, respectively, and \( \bar{\Gamma} \) and \( \overline{M} \) are the charge conjugation one correspondingly. It is conventional to combine the three polarization fractions in Eq. (80) with those of its CP-conjugate \( \bar{B} \) decay, and to quote the six resulting observables corresponding to tranversity amplitudes as direct induced CP asymmetries [51]. The direct CP asymmetries in transversity basis can be defined as

\[ A_{CP}^{\text{dir}, \alpha} = \frac{f_\alpha - f_\alpha}{f_\alpha + f_\alpha} \quad (\alpha = L, \parallel, \perp) \quad \text{(107)} \]
where the definition of \( \tilde{f} \) is same as that in Eq.(80) but for the corresponding \( \tilde{B} \) decay. The direct CP-violating asymmetries for those \( B \to J/\psi V \) decays are

\[
\begin{align*}
A_{CP}^{\text{dir}}(B \to J/\psi K^*) &= 3.19^{+0.42}_{-0.32} \omega_B + 0.12 (f_M) + 0.12 (a_t) + 0.12 (m_c) \\
&\times (\text{CKM}) \times 10^{-4}, \\
A_{CP}^{\text{dir}}(B_s \to J/\psi \phi) &= 2.66^{+0.45}_{-0.37} \omega_B + 0.10 (f_M) + 0.11 (a_t) + 0.10 (m_c) \\
&\times (\text{CKM}) \times 10^{-4}, \\
A_{CP}^{\text{dir}}(B \to J/\psi \rho) &= -5.15^{+0.37}_{-0.35} \omega_B + 0.18 (f_M) + 0.22 (a_t) + 0.21 (m_c) \\
&\times (\text{CKM}) \times 10^{-3}, \\
A_{CP}^{\text{dir}}(B \to J/\psi \omega) &= -5.06^{+0.55}_{-0.68} \omega_B + 0.19 (f_M) + 0.22 (a_t) + 0.20 (m_c) \\
&\times (\text{CKM}) \times 10^{-3}, \\
A_{CP}^{\text{dir}}(B_s \to J/\psi K^{*0}) &= -4.15^{+0.78}_{-0.70} \omega_B + 0.18 (f_M) + 0.27 (a_t) + 0.26 (m_c) \\
&\times (\text{CKM}) \times 10^{-3}.
\end{align*}
\]

We also give the results for direct CP asymmetries corresponding to three polarizations,

- for \( B_{u/d} \to J/\psi K^* \) decays,

\[
\begin{align*}
A_{CP}^{\text{dir},L} &= 3.04^{+0.66}_{-0.54} \times 10^{-4}, \\
A_{CP}^{\text{dir},||} &= 3.39^{+0.47}_{-0.37} \times 10^{-4}, \\
A_{CP}^{\text{dir},\perp} &= 3.26^{+0.64}_{-0.44} \times 10^{-4};
\end{align*}
\]

- for \( B_s \to J/\psi \phi \) mode,

\[
\begin{align*}
A_{CP}^{\text{dir},L} &= 2.69^{+0.62}_{-0.49} \times 10^{-4}, \\
A_{CP}^{\text{dir},||} &= 2.71^{+0.39}_{-0.40} \times 10^{-4}, \\
A_{CP}^{\text{dir},\perp} &= 2.52^{+0.55}_{-0.40} \times 10^{-4};
\end{align*}
\]

- for \( B_{u/d} \to J/\psi \rho \) decays,

\[
\begin{align*}
A_{CP}^{\text{dir},L} &= -4.15^{+0.78}_{-0.96} \times 10^{-3}, \\
A_{CP}^{\text{dir},||} &= -6.32^{+0.72}_{-0.81} \times 10^{-3}, \\
A_{CP}^{\text{dir},\perp} &= -6.09^{+0.84}_{-1.09} \times 10^{-3};
\end{align*}
\]

- for \( B_d \to J/\psi \omega \) mode,

\[
\begin{align*}
A_{CP}^{\text{dir},L} &= -3.84^{+0.81}_{-0.96} \times 10^{-3}, \\
A_{CP}^{\text{dir},||} &= -6.63^{+0.74}_{-0.90} \times 10^{-3}, \\
A_{CP}^{\text{dir},\perp} &= -6.36^{+0.82}_{-1.16} \times 10^{-3};
\end{align*}
\]

- for \( B_s \to J/\psi K^{*0} \) mode

\[
\begin{align*}
A_{CP}^{\text{dir},L} &= -3.82^{+1.14}_{-0.96} \times 10^{-3}, \\
A_{CP}^{\text{dir},||} &= -4.62^{+0.80}_{-0.71} \times 10^{-3}, \\
A_{CP}^{\text{dir},\perp} &= -4.27^{+0.76}_{-1.05} \times 10^{-3};
\end{align*}
\]

in which various errors have been again added in quadrature.

D. Impact of Penguin Contamination

Based on the encouraging agreements of our theoretical calculations with the available data, we study the penguin impacts on the mixing-phase in \( B_s \to J/\psi \phi \) decay:

\[
\phi_s^{\text{eff}} = -\text{arg} \left[ \frac{q}{p} \frac{A_f}{A_f} \right] = \phi_s + \Delta \phi_s,
\]

where \( j \) denotes three polarization configurations. We extract the quantity \( \Delta \phi_s \) from our pQCD numerical evaluations given in three polarizations as follows,

\[
\begin{align*}
\Delta \phi_s(L) &\approx 0.97^{+0.04+0.12}_{-0.05-0.15} \times 10^{-3}; \\
\Delta \phi_s(\|) &\approx 0.84^{+0.02+0.00}_{-0.01-0.03} \times 10^{-3}; \\
\Delta \phi_s(\perp) &\approx 0.80^{+0.03+0.00}_{-0.01-0.02} \times 10^{-3}.
\end{align*}
\]

where the dominant error is from the variation of the shape parameter \( \omega_B \) in the distribution amplitude of \( B_s \) meson, and various uncertainties have been added in quadrature. The deviation of \( \Delta \phi_s \) from the \( \phi_s \) is found to be of \( \mathcal{O}(10^{-3}) \) in the standard model with the pQCD approach by taking into account the known NLO contributions, specifically, vertex corrections. This finding can be stringently examined in the ongoing LHCb experiment and under-designed Super B factory and may provide an important standard model reference for verifying the existing new physics from the \( B_s \to J/\psi \phi \) data.
TABLE IV. The factorization amplitudes (without CKM factors) for the hadronic \( B \to J/\psi \nu \nu \) decays with three polarizations in the pQCD approach, where only the central values are quoted for clarification.

| Decay modes | Tree Operators | Penguin Operators \((\times 10^{-2})\) |
|-------------|----------------|----------------------------------|
| \( B_{u/d} \to J/\psi K^* \) | \( L \) \(-0.278 + i0.959\) | \(-1.653 + i2.571\) |
| \( N \) \(-0.163 - i0.775\) | \(-0.356 - i2.020\) |
| \( T \) \(-0.179 - i0.635\) | \(0.179 - i1.617\) |
| \( B_s \to J/\psi \phi \) | \( L \) \(-0.175 + i0.933\) | \(-1.190 + i2.376\) |
| \( N \) \(-0.155 - i0.713\) | \(0.186 - i1.784\) |
| \( T \) \(-0.177 - i0.563\) | \(0.009 - i1.378\) |
| \( B_{u/d} \to J/\psi \rho \) | \( L \) \(0.220 - i0.620\) | \(1.032 - i1.665\) |
| \( N \) \(0.105 + i0.472\) | \(-0.198 + i1.231\) |
| \( T \) \(0.117 + i0.398\) | \(-0.099 + i1.014\) |
| \( B_d \to J/\psi \omega \) | \( N \) \(-0.093 - i0.426\) | \(0.203 - i1.112\) |
| \( T \) \(-0.104 - i0.356\) | \(0.106 - i0.910\) |
| \( B_s \to J/\psi \bar{K}^* \) | \( N \) \(-0.115 - i0.606\) | \(0.153 - i1.506\) |
| \( T \) \(-0.137 - i0.495\) | \(0.007 - i1.208\) |

Here, we also calculate the modules of amplitudes for \( B_s \to J/\psi \bar{K}^{*0} \) and \( B_s \to J/\psi \phi \) decays,

\[
\left| A'_L \right| \approx 0.814^{+0.140}_{-0.133}, \quad \left| A_L \right| \approx 0.949^{+0.159}_{-0.152}, \tag{120}
\]

\[
\left| A''_L \right| \approx 0.617^{+0.110}_{-0.099}, \quad \left| A_L \right| \approx 0.730^{+0.126}_{-0.111}, \tag{121}
\]

\[
\left| A'_L \right| \approx 0.513^{+0.078}_{-0.090}, \quad \left| A_L \right| \approx 0.590^{+0.109}_{-0.096}. \tag{122}
\]

which result in the ratios between \( |A'(B_s \to J/\psi \bar{K}^{*0})| \) and \( |A(B_s \to J/\psi \phi)| \),

\[
\left| \frac{A'_L}{A_L} \right| = 0.858^{+0.206}_{-0.190}, \quad \left| \frac{A''_L}{A_L} \right| = 0.845^{+0.210}_{-0.188}, \quad \left| \frac{A'_L}{A_L} \right| = 0.869^{+0.234}_{-0.204}. \tag{123}
\]

V. SUMMARY

Up to this date, the CKM mechanism has successfully described almost all available data on flavor physics and CP violation, which has continued to motivate more precise tests of CP violation in the heavy flavor sector. The fact that the \( B_s - \bar{B}_s \) mixing angle \( \phi_s \) is tiny provides an ideal test of the CKM paradise and offers an opportunity to probe the new physics. To achieve this goal renders the precise predictions/measurements important. The reduction of experimental uncertainties seems to have a promising prospect in near future, due to the large amount of data sample (to be) collected at LHC and the forthcoming Super KEKB factory.

On the other side, it is in necessity to learn about theoretical contamination. What has been explored in this work is an attempt to fill this gap. We have computed both tree and penguin amplitudes in the perturbative QCD approach, in which the leading-order contributions and various QCD corrections are taken into account. With the inclusion of these sizable corrections, our theoretical results for CP-averaged branching ratios, polarization fractions, CP-violating asymmetries, and relative phases are in good consistency with the available data. Based on the global agreement, we have explored the penguin contributions and discussed the impact on \( \phi_s \) extracted from \( B_s \to J/\psi \phi \). Adopting the \( k_T \) factorization approach, we found that the results can be shifted by \( 10^{-3} \), and the future experiments can examine this prediction.

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Appendix A: The angular distributions of $B \to J/\psi(\to l^+l^-)V_J(\to K^+K^-)$

In the following, we will give some details to derive the angular distributions for which we use the helicity amplitudes and adopt the convention in Refs [52, 53]. For the hadronic $B_s$ decays into the intermediate $J/\psi V_J$, we generalize the expressions in Eq. (38) to the $K^+K^-$ with arbitrary angular momentum $J$:

$$\mathcal{M}(V_J, i), \quad \overline{\mathcal{M}}(V_J, i)$$

(A1)

with $i = 0, \perp, \parallel$ or 0, ±.

The angular decomposition of the amplitude square and the interference are given as

$$\frac{d^4|A|^2}{dm^2_{KK}d\cos\theta_K d\cos\theta_J d\phi} = \left[|H_0|^2 \sin^2 \theta_J + |H_1|^2(1 - \cos^2 \phi \sin^2 \theta_J) + |H_\perp|^2(1 - \sin^2 \phi \sin^2 \theta_J) + \text{Re}[H_0 H_\perp^* \sin(2\theta_J) \cos \phi + \text{Im}[H_0 H_\perp^* \sin(2\theta_J) \sin \phi + \text{Im}[H_\perp H_\parallel^* \sin^2 \theta_J \sin(2\phi)]ight],$$

$$\frac{d^4\overline{AA}^*}{dm^2_{KK}d\cos\theta_K d\cos\theta_J d\phi} = \left[\overline{\mathcal{M}}_0 H_0^* \sin^2 \theta_J + \overline{\mathcal{M}}_1^* H_1^*(1 - \cos^2 \phi \sin^2 \theta_J) + \overline{\mathcal{M}}_\perp H_\perp^*(1 - \sin^2 \phi \sin^2 \theta_J) + \frac{1}{2}(\overline{\mathcal{M}}_0 H_\perp^* + \overline{\mathcal{M}}_1^* H_\parallel^*) \sin(2\theta_J) \cos \phi + \frac{-i}{2}(\overline{\mathcal{M}}_0 H_\perp^* - \overline{\mathcal{M}}_1^* H_\parallel^*) \sin(2\theta_J) \sin \phi + \frac{-i}{2}(\overline{\mathcal{M}}_1^* H_\parallel^* - \overline{\mathcal{M}}_1^* H_\perp^*) \sin^2 \theta_J \sin(2\phi)\right],$$

(A2)

with the functions $H_i$

$$H_0 = \sum_{J=0,1,2,...} Y^0_J(\theta_K, 0) H_0^J, \quad H_{\parallel/\perp} = \sum_{J=1,2,...} Y^{-1}_J(\theta_K, 0) H_{\parallel/\perp}^J,$$

with $N_J = 3\sqrt[3]{m_{J/\psi}^2/(64\pi m_B^2)}$. The function $\lambda$ is related to the magnitude of the $J/\psi$ momentum in $B_s$ meson rest frame: $\lambda \equiv \lambda(m^2_{B_s}, m^2_{KK}, m^2_{J/\psi}) = (2m_B|\vec{p}_{J/\psi}|)^2$, and $\lambda(a^2, b^2, c^2) = (a^2 - b^2 - c^2)^2 - 4b^2c^2$. $L_{V_J}$ is the line shape of the $V_J$ for decays into $K^+K^-$:

$$L_{V_J} = \frac{i}{m_{KK}^2 - m_{V_J}^2 + i m_{V_J} \Gamma_{V_J}} \sqrt{m_{V_J} \Gamma_{V_J} \to K^+K^-},$$

(A3)

in the narrow width limit which is normalized to

$$\int dm^2_{KK}|L_{V_J}|^2 = B(V_J \to K^+K^-).$$

(A4)

The functions $H_{L/Ri}$ are defined by

$$H_0^J = \sqrt{N_J} \mathcal{M}(V_J, 0) L_{V_J} \equiv |H_0^J| e^{i\delta_0^J},$$

$$H_{\parallel/\perp}^J = \sqrt{N_J} \mathcal{M}(V_J, ||/\perp) L_{\phi_J} \equiv |H_{\parallel/\perp}^J| e^{i\delta_{\parallel/\perp}^J},$$

$$\overline{\mathcal{M}}_0 = \frac{q}{p} \sqrt{N_J} \mathcal{M}(V_J, 0) L_{\phi_J} \equiv |\overline{\mathcal{M}}_0^J| e^{i\delta_0^J},$$

$$\overline{\mathcal{M}}_{\parallel/\perp}^J = \frac{q}{p} \sqrt{N_J} \mathcal{M}(V_J, ||/\perp) L_{V_J} \equiv |\overline{\mathcal{M}}_{\parallel/\perp}^J| e^{i\delta_{\parallel/\perp}^J}. $$

(A5)

Through definition, the $B_s - \bar{B_s}$ mixing phase has been incorporated in $\delta_{\parallel}^J$. These distributions are based on the same convention on helicity angles with Refs. [29, 54], but different with those in Ref. [55, 56].
Appendix B: Related Functions in the perturbative QCD factorization

We show here the function \( h_i \)'s, coming from the Fourier transformations of the function \( H^{(0)} \),

\[
h_{fs}(x_1, x_3, b_1, b_3) = K_0 \left( \frac{\sqrt{x_1 x_3 (1 - r_2^2)}}{m_B b_1} \right) \left[ \theta(b_1 - b_3)K_0 \left( \frac{\sqrt{x_3 (1 - r_2^2)}}{m_B b_1} \right) \right. \\
\left. \cdot I_0 \left( \frac{\sqrt{x_3 (1 - r_2^2)}}{m_B b_3} \right) + \theta(b_3 - b_1)K_0 \left( \frac{\sqrt{x_3 (1 - r_2^2)}}{m_B b_3} \right) \right. \\
\left. \cdot I_0 \left( \frac{\sqrt{x_3 (1 - r_2^2)}}{m_B b_1} \right) \right] S_t(x_3),
\]

(B1)

\[
h_{afs}(x_1, x_2, x_3, b_1, b_2) = \left\{ \theta(b_2 - b_1)I_0(m_B \sqrt{x_1 x_3 (1 - r_2^2) b_1} K_0(m_B \sqrt{x_1 x_3 (1 - r_2^2) b_2}) \right. \\
+ (b_1 \leftrightarrow b_2) \left. \right\} \cdot \left( \begin{array}{cc}
K_0(m_B F_1(b_2)), & \text{for } \frac{F_1^2}{2} > 0 \\
\frac{\pi}{2} I_0(m_B \sqrt{F_1^2 b_2}), & \text{for } \frac{F_1^2}{2} < 0
\end{array} \right),
\]

(B2)

where \( J_0 \) is the Bessel function, \( K_0 \) and \( I_0 \) are the modified Bessel functions with \( K_0(-ix) = -(\pi/2)Y_0(x) + i(\pi/2)J_0(x) \), and \( F_1 \)'s are defined by

\[
F_{1,2}^2 = (x_1 - x_2) |x_3 + (x_2 - x_3) r_2^2| + 2 r_2^4,
\]

(B3)

The threshold resummation form factor \( S_t(x_i) \) is adopted from Ref. [32]

\[
S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1+c)} |x(1-x)|^c,
\]

(B4)

where the parameter \( c = 0.3 \). This function is normalized to unity.

The Sudakov factors used in the text are defined as

\[
S_{ab}(t) = s \left( x_1 P_1^+, b_1 \right) + s \left( x_3 P_3^-, b_3 \right) + s \left( (1 - x_3) P_3^-, b_3 \right) \\
- \frac{1}{\beta_1} \left[ \ln \frac{\ln(t/\Lambda)}{-\ln(b_1 \Lambda)} + \ln \frac{\ln(t/\Lambda)}{-\ln(b_3 \Lambda)} \right],
\]

(B5)

\[
S_{cd}(t) = s \left( x_1 P_1^+, b_1 \right) + s \left( x_2 P_2^+, b_2 \right) + s \left( (1 - x_2) P_2^+, b_2 \right) \\
+ s \left( x_3 P_3^-, b_1 \right) + s \left( (1 - x_3) P_3^-, b_1 \right) \\
- \frac{1}{\beta_1} \left[ 2 \ln \frac{\ln(t/\Lambda)}{-\ln(b_1 \Lambda)} + \ln \frac{\ln(t/\Lambda)}{-\ln(b_2 \Lambda)} \right],
\]

(B6)

The scale \( t_i \)'s in the above equations are chosen as

\[
t_a = \max(\sqrt{x_3 (1 - r_2^2) m_B}, 1/b_1, 1/b_3),
\]

\[
t_b = \max(\sqrt{x_1 (1 - r_2^2) m_B}, 1/b_1, 1/b_3),
\]

\[
t_{nfs} = \max(\sqrt{x_1 x_3 (1 - r_2^2) m_B}, \sqrt{(x_1 - x_2) |x_3 + (x_2 - x_3) r_2^2| + r_2^4 m_B}, 1/b_1, 1/b_2).
\]

(B7)

The scale \( t_i \)'s are chosen as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.
