Spin in a Fluctuating Field: The Bose(+Fermi) Kondo models

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I consider models with an impurity spin coupled to a fluctuating gaussian field with or without additional Kondo coupling of the conventional sort. In the case of isotropic fluctuations, the renormalisation group flows for these models have controlled fixed points when the autocorrelation of the gaussian field \( h(t) \), \( \langle Th(t)h(0)\rangle \sim \frac{1}{\tau^2} \) with small positive \( \epsilon \). In absence of any additional Kondo coupling, I get powerlaw decay of spin correlators, \( \langle TS(0) \rangle \sim \frac{1}{\tau^2} \). For negative \( \epsilon \), the spin autocorrelation is constant in long time limit. The results agree with calculations in Schwinger Boson mean field theory. In presence of a Kondo coupling to itinerant electrons, the model shows a phase transition from a Kondo phase to a field fluctuation dominated phase. These models are good starting points for understanding behaviour of impurities in a system near a zero-temperature magnetic transition. They are also useful for understanding the dynamical local mean field theory of Kondo lattice with Heisenberg (spin-glass type) magnetic interactions.

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Theory of metallic spin glasses provides an example where Kondo effect and magnetic fluctuations compete on par at the local mean field theory level [1]. The local mean field theory of this model has an impurity spin coupled to a fluctuating gaussian Weiss field in addition to a Kondo coupling of the conventional sort. The treatment of metallic spin glass, so far, has dealt with Kondo effect in a cavalier fashion [2,3]. We do not have too much insight into the fluctuating gaussian Weiss field in addition to a Kondo coupling of the conventional sort. The treatment of metallic fluctuations at the local mean field theory level [1]. The local mean field theory of this model has an impurity spin coupled to a fluctuating gaussian Weiss field in addition to a Kondo coupling of the conventional sort. In the case of isotropic fluctuations, the system is isotropic, i.e., \( h(t) = h(t) = h(t) \). This is a nontrivial problem. In a path integral formulation using coherent state representation for the spin, one gets the effective action

\[
A_{\text{eff}} = \int \int d\tau_1 d\tau_2 \sum_{ab} \gamma_a \gamma_b D^{ab}(\tau_1 - \tau_2) S^a(\tau_1) S^b(\tau_2) + i\text{Berry Phase}[\vec{S}(\tau)]
\]

after integrating out the Weiss field. If one could ignore the Berry phase term, this would be a simple classical long-range spin model. We know a lot about such model. For example, in the paramagnetic phase, spin correlator have the same power law, as that of the long range coupling. However, when the system is isotropic, i.e., \( \gamma_a \gamma_b D^{ab}(\tau) = \gamma^2 D(\tau) \delta^{ab} \), the Berry phase term changes the behaviour of the system completely. The spin autocorrelation has a power law which is, generically, different from the field-field correlator exponent.

Let us begin by considering the special case where \( \epsilon = 0 \). Renormalisation of the coupling involves integrals of the form \( \int_0^\infty d\tau_2 \int_{-\infty}^0 d\tau_1 D^{ab}(\tau_2 - \tau_1) \) which are logarithmically divergent. The perturbative beta-functions for the couplings turn out to be

\[
\beta(\gamma_\parallel) = -\frac{1}{2} \gamma_\parallel \gamma_\perp^2
\]

\[
\beta(\gamma_\perp) = -\frac{1}{4} \gamma_\perp (\gamma_\parallel^2 + \gamma_\perp^2)
\]
From this point onward I discuss mostly the symmetric case, $\gamma_z = \gamma_\perp = \gamma$.

$$\beta(\gamma) = -\frac{1}{2} \gamma^3 + o(\gamma^5)$$ (6)

Now I consider small nonzero $\epsilon$. Dimension analysis of the coupling $\gamma$ tells us that

$$\beta(\gamma) = \epsilon \gamma - \frac{1}{2} \gamma^3 + \cdots$$ (7)

This equation has a stable nontrivial fixed point at $\gamma_\ast = \epsilon$ when $\epsilon > 0$. Note that $\gamma \to -\gamma$ is a symmetry. Hence, sometimes it is more useful to write the beta function in terms of $g = \gamma^2$.

$$\beta(g) = \epsilon g - g^2 + \cdots$$ (8)

To calculate the spin-spin auto correlation, we evaluate the correction perturbatively in $\gamma_\ast$. Including the second order correction in $\gamma$ to the spin autocorrelation, $< T_{\tau} S(\tau) S(0) > \sim 1 - \gamma_\ast^2 \log \tau$ leading to a power-law

$$< T_{\tau} S(\tau) S(0) > \sim 1/\tau$$ (9)

upto order $\epsilon$.

It is quite remarkable that the Schwinger boson calculation gives the same power-law, for arbitrary $\epsilon$. This calculation is justified for spin belonging to certain representations of $SU(N)$ when $N \to \infty$. The case $\epsilon = 1$ appeared in the self-consistent mean field theory of a gapless spin-fluid phase [4].

The Bose-Kondo model has some interesting applications as an impurity model. One of them is the problem of a non-magnetic site in an antiferromagnet near a quantum critical point. The usual procedure for deriving the long distance field theory for quantum antiferromagnets is to take pairs of neighbouring sites and associate a staggered magnetisation order parameter with the pair. Then one writes down the effective hamiltonian in terms of this order parameter. When there is a non-magnetic site, there will be a pair with one magnetic and one non-magnetic site. The corresponding effective hamiltonian would have a spin coupled at one site to the order parameter of a sigma-model, the long distance theory of the anti-ferromagnet. This model may be of use in studying effects of Zn doping in cuprates.

When the sigma model is in the the quantum critical region, analysing this impurity model is hard. For dimensions near two, the order parameter fluctuations are far from gaussian. However, for spatial dimension $d = 3 - \epsilon$, $\epsilon$ small one can replace the sigma-model by a $\phi^4$ field theory and use $\epsilon$ expansion to treat the interactions. Upto order $\epsilon^2$, one can use the result of the gaussian approximation. I believe this leads to impurity contributions to the susceptibility which diverge like $1/T^{1-\epsilon}$ as a function of temperature $T$ for small $\epsilon$. Since these spins are not very effectively quenched, a small concentration of them can easily lead to a magnetic state.

So much for the Bose-Kondo model with just a Gaussian Weiss field. What happens when it is in competition with the usual Kondo effect? Let there be some additional fermionic degrees of freedom interacting with the spin through antiferromagnetic Kondo coupling $J$. Clearly, for $\epsilon > 0, J$ is irrelevant, for small $J$. For big enough Kondo coupling, there is a Kondo dominated phase where $< T_{\tau} S(\tau) S(0) > \sim 1/\tau^2$.

The physics is captured by the beta functions:

$$\beta(J) = -\frac{1}{2} \gamma^2 J + J^2 + \cdots$$ (10)
$$\beta(\gamma) = \epsilon \gamma - \frac{1}{2} \gamma^3 + \cdots$$ (11)

The modification of the magnetic coupling due to Kondo effect shows up in the term of the order $-\gamma J^2$ in $\beta(\gamma)$. However, to determine the fixed points to the lowest order in $\epsilon$, we don’t need this.

In terms of $g = \gamma^2$

$$\beta(J) = -\frac{1}{2} g J + J^2 + \cdots$$ (12)
$$\beta(g) = \epsilon g - g^2 + \cdots$$ (13)
These equations have an unstable fixed point at

\[
\begin{align*}
g_* &= \epsilon \\
J_* &= \frac{\epsilon}{2}
\end{align*}
\]  

(14)  
(15)

Notice the line in the \( g - J \) plane which flows into the unstable fixed point. Points above that line flow to the strong Kondo fixed point. Points below the line flow to the magnetic fixed point \( J = 0, g = \epsilon \).

At the unstable fixed point, spin autocorrelation ought to have a different power law. However, up to order \( \epsilon \) the exponent remains the same.

This phase diagram can be obtained in a different limit, where \( \epsilon \) is arbitrary but the spin belongs to an antisymmetric representation of \( SU(N) \) with \( N \to \infty \). One can treat the Kondo effect by slave boson mean field theory. In the magnetic fluctuation dominated phase, the slave boson does not condense.

All the magnetic fixed points discussed so far, the magnetic fluctuation dominated one as well as the fixed point at the border of the Kondo phase, depend crucially on isotropy. Anisotropy is a relevant perturbation around these fixed points. This is another aspect of Bose-Kondo models which differs from the usual fermionic Kondo model. In the usual Kondo model, anisotropies in antiferromagnetic couplings renormalise away. For the Bose-Kondo model with anisotropic couplings, the biggest coupling, say \( \gamma_z \), wins. One might as well ignore the other couplings, \( \gamma^x \) and \( \gamma^y \), in that case. Such problems have been discussed before [5] in a slightly different context.

In conclusion, I have studied the problem of a spin coupled to a fluctuating power-law correlated gaussian field, with or without additional Kondo coupling of the conventional sort. These models have interesting phase transitions and critical behaviour, when the magnetic fluctuations are isotropic. Apart from being interesting in their own right, such impurity models provide a good starting point for understanding the local mean field theory of the Kondo lattice with Heisenberg spin-glass type magnetic interactions [6].

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