Superheavy Dark Matter and Thermal Inflation

T. Asaka, M. Kawasaki, and T. Yanagida

1Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
2Research Center for the Early Universe, University of Tokyo, Tokyo 113-0033, Japan

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Abstract

The thermal inflation is the most plausible mechanism that solves the cosmological moduli problem naturally. We discuss relic abundance of superheavy particle $X$ in the presence of the thermal inflation assuming that its lifetime is longer than the age of the universe, and show that the long-lived particle $X$ of mass $10^{12} - 10^{14}$ GeV may form a part of the dark matter in the present universe in a wide region of parameter space of the thermal inflation model. The superheavy dark matter of mass $\sim 10^{13}$ GeV may be interesting in particular, since its decay may account for the observed ultra high-energy cosmic rays if the lifetime of the $X$ particle is sufficiently long.
A large class of string theories predicts a number of flat directions, called moduli fields $\phi$. They are expected to acquire their masses of order of the gravitino mass $m_{3/2}$ from some nonperturbative effects of supersymmetry (SUSY) breaking. The gravitino mass lies in a range of $10^{-2}$ keV–1 GeV for gauge-mediated SUSY breaking models and in a range of 100 GeV–1 TeV for hidden-sector SUSY breaking models. It is well known that such moduli fields are produced too much as coherent oscillations in the early universe and conflict with various cosmological observations. Therefore, we must invoke a mechanism such as late-time entropy production to dilute the moduli density substantially.

The thermal inflation is the most plausible mechanism to produce an enormous amount of entropy at the late time of the universe’s evolution. In recent articles we have shown that the thermal inflation is very successful in solving the above cosmological moduli problem. Since it produces a tremendous amount of entropy to dilute the moduli density, abundances of any relic particles are substantially diluted simultaneously, which may provide a new possibility for a superheavy $X$ particle to be a part of the dark matter in the universe. In this paper we show that it is indeed the case if the mass of $X$ particle is of order of $10^{12}$–$10^{14}$ GeV and its lifetime is longer than the age of the universe. Such a long-lived $X$ particle is particularly interesting, since its decay may naturally explain the observed ultra high-energy cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff when the lifetime is sufficiently long.

In this paper we consider that the particle $X$ was primordially in the thermal equilibrium and froze out at the cosmic temperature $T = T_f$ when it was nonrelativistic $x_f = m_X/T_f > 1$ (i.e., the $X$ is left as a cold relic.). Then, the present relic abundance of $X$ (the ratio of the present energy density of $X$ to the critical density $\rho_{cr}$) is estimated by using the thermally-averaged annihilation cross section of the $X$, $\langle \sigma_X | v | \rangle$, as

$$\Omega_{X} h^2 = \frac{0.76(n_f + 1)x_f}{g_* (T_f)^2 M_G h^{-2} \rho_{cr} / s_0 \langle \sigma_X | v | \rangle}$$

where $g_* (T_f) \approx 200$ counts the effective degrees of freedom at $T = T_f$, $M_G \approx 2.4 \times 10^{18}$ GeV denotes the reduced Planck scale, and $s_0$ is the present entropy density $\rho_{cr} / s_0 \approx 3.6 \times 10^{-9} h^2$ GeV with the present Hubble parameter $h$ in units of 100 km/sec·Mpc$^{-1}$. Here $n_f$ parametrizes the dependence on $T$ of the annihilation cross section and $n_f = 0$ for s-wave annihilation, $n_f = 1$ for $p$-wave annihilation, etc. Assuming the s-wave annihilation with $\langle \sigma_X | v | \rangle \approx m_X^2 (x_f \approx 7$ for $m_X \sim 10^{13}$ GeV) we obtain

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1 Other candidates for the dark matter in the presence of the thermal inflation are the moduli themselves whose masses are less than about 100 keV, and the axion with a relatively high decay constant $f_a \sim 10^{15}$–$10^{16}$ GeV.

2 For the superheavy particle $X$ to be in the thermal equilibrium, the reheating temperature after the primordial inflation should be higher than $m_X \sim 10^{13}$ GeV. Such a high reheating temperature is realized in e.g. hybrid inflation models. Although a large number of gravitinos are also produced in this case, they are sufficiently diluted by the thermal inflation and become harmless.

3 Nonthermal productions of superheavy particles were discussed in Refs. [13,21].
\[ \Omega_X h^2 \simeq \frac{0.4m_X^2}{M_G (h^2 \rho_{\text{cr}}/s_0)} = 4.3 \times 10^{15} \left( \frac{m_X}{10^{13} \text{ GeV}} \right)^2. \]  

(2)

Therefore, the superheavy particle \( X \) of \( m_X \gtrsim 10^5 \text{ GeV} \) leads to overclosure of the universe if its lifetime is longer than the age of the universe. In order to realize \( \Omega_X h^2 \lesssim 1 \) the dilution factor more than about \( 10^{16} \) is required for \( m_X \sim 10^{13} \text{ GeV} \) for example. However, such a huge dilution may be naturally provided by the thermal inflation. In this paper, we examine whether the thermal inflation could sufficiently reduce the energy density \( \Omega_X \) of the superheavy particle even if it was in the thermal equilibrium, as well as that of the string moduli.

Let us start with reviewing briefly the thermal inflation model \([6,7]\). The potential of the inflaton field \( S \) is given by

\[ V = V_0 - m_S^2 |S|^2 + \frac{|S|^{2n+4}}{M_*^{2n}}, \]  

(3)

where \(-m_0^2\) denotes a soft SUSY breaking negative mass squared which is expected to be of order of the electroweak scale. \( M_* \) denotes the cutoff scale and \( V_0 \) is the vacuum energy. Then the vacuum expectation value of \( S \) is estimated as

\[ \langle S \rangle \equiv M = (n + 2)^{-\frac{1}{n+1}} (m_0 M_*^n)^{\frac{1}{n+1}}, \]

(4)

and \( V_0 \) is fixed as

\[ V_0 = \frac{n+1}{n+2} m_0^2 M_*^2, \]

(5)

so that the cosmological constant vanishes at the true vacuum. The inflaton \( \sigma \) (\( \equiv \text{Re} S \)), which we call it a flaton, obtains a mass \( m_\sigma^2 = 2(n+1)m_0^2 \).

After the thermal inflation ends, the vacuum energy is transferred into the thermal bath through flaton decay and increases the entropy of the universe by a factor \([6,7]\)

\[ \Delta \simeq \frac{V_0}{66 m_\sigma^3 T_R}. \]  

(6)

Here \( T_R \) is the reheating temperature after the thermal inflation and determined by the flaton decay width \( \Gamma_\sigma \) which can be written as

\[ \Gamma_\sigma = C_\sigma \frac{m_\sigma^3}{M_*^2}, \]  

(7)

where \( C_\sigma \) is a dimension-less parameter depending on the decay modes. We consider that the flaton \( \sigma \) dominantly decays into two photons or two gluons, and \( C_\sigma \) is given by

\[ 4 \]  

Here we have assumed that the flaton can not decay into two \( R \)-axions which is an imaginary part of \( S \) to obtain a successful dilution. See Ref. \([6]\).

\[ 5 \]  

We assume that the decay amplitudes are obtained from one-loop diagrams of some heavy particles. See also Ref. \([6]\).
\[ C_\sigma \simeq \begin{cases} 
\frac{1}{4\pi} \left( \frac{\alpha}{4\pi} \right)^2 & \text{for } \sigma \to \gamma\gamma \\
\frac{1}{4\pi} \left( \frac{\alpha_s}{4\pi} \right)^2 & \text{for } \sigma \to gg 
\end{cases} \]  

(8)

Then the reheating temperature is estimated as

\[ T_R = \left( \frac{90}{\pi^2 g_*(T_R)} \right)^{\frac{1}{4}} \sqrt{\Gamma_\sigma M_G} \simeq 0.96 C_\sigma^{\frac{1}{2}} m_\sigma^{\frac{3}{2}} M_\sigma^{\frac{3}{2}} M. \]  

(9)

The entropy-production factor \( \Delta \), therefore, can be written as

\[ \Delta = \frac{M^3}{130(n+2)C_\sigma^{\frac{1}{2}} m_\sigma^{\frac{3}{2}} M_\sigma^{\frac{3}{2}}}, \]  

(10)

and for \( n = 1 \) we obtain

\[ \Delta \simeq \begin{cases} 
1.0 \times 10^{17} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-\frac{3}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to \gamma\gamma \\
6.4 \times 10^{15} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-\frac{3}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to gg 
\end{cases} \]  

(11)

Here the values \( m_\sigma = 100 \text{ GeV} \) and \( M = 10^{10} \text{ GeV} \) correspond to \( M_\sigma = 3.5 \times 10^{18} \text{ GeV} \) for \( n = 1 \). In the following analysis, we take \( n = 1 \) for simplicity. From Eq. (11) we see that the thermal inflation can dilute relic particle density extensively by producing an enormous entropy. Here it should be noted that the reheating temperature should be \( T_R \gtrsim 1 \text{ MeV} \) to keep the big bang nucleosynthesis successful \cite{23}, which leads to the upper bounds on \( M \) from Eq. (9) as

\[ M = 0.96 \, C_\sigma^{\frac{1}{2}} m_\sigma^{\frac{3}{2}} M_\sigma^{\frac{3}{2}} T_R \lesssim \begin{cases} 
2.4 \times 10^{11} \text{ GeV} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to \gamma\gamma \\
3.9 \times 10^{12} \text{ GeV} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to gg 
\end{cases} \]  

(12)

These are translated into the upper bounds on \( M_\sigma \) as

\[ M_\sigma = 3.2 \, C_\sigma^{\frac{1}{2}} m_\sigma^{\frac{3}{2}} M_\sigma^{\frac{3}{2}} T_R \lesssim \begin{cases} 
2.1 \times 10^{21} \text{ GeV} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{2} & \text{for } \sigma \to \gamma\gamma \\
5.4 \times 10^{23} \text{ GeV} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{2} & \text{for } \sigma \to gg 
\end{cases} \]  

(13)

In the presence of the thermal inflation the relic abundance of the superheavy particle \( X \) [Eq. (2)] is reduced by the factor \( \Delta \) in Eq. (11) as

\[ \Delta \simeq \begin{cases} 
1.0 \times 10^{17} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-\frac{3}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to \gamma\gamma \\
6.4 \times 10^{15} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-\frac{3}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} & \text{for } \sigma \to gg 
\end{cases} \]  

(11)

In Ref. \cite{23} the lower bound on the reheating temperature is determined as about 0.5 MeV. Since our definition of the reheating temperature is different by a factor \( \sqrt{3} \), it leads to \( T_R \gtrsim 1 \text{ MeV} \) in our case.
\[ \Omega_X h^2 = \Omega_X^0 h^2 \times \frac{1}{\Delta} = 140 C^\frac{1}{2} \frac{m_X^2 m_\sigma^2}{M^3 M_G^2 (h^{-2} \rho_{cr}/s_0)} \]

\[ \simeq \begin{cases} 
0.04 \left( \frac{m_X}{10^{13} \text{ GeV}} \right)^2 \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^2 \left( \frac{M}{10^{10} \text{ GeV}} \right)^{-3} & \text{for } \sigma \to \gamma \gamma \\
0.68 \left( \frac{m_X}{10^{13} \text{ GeV}} \right)^2 \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^2 \left( \frac{M}{10^{10} \text{ GeV}} \right)^{-3} & \text{for } \sigma \to gg
\end{cases} \]  

(14)

In Fig. 1 we show the contour of \( \Omega_X \) in the parameter space of the thermal inflation model (in \( m_\sigma-M_* \) plane). We find that the thermal inflation can naturally realize \( \Omega_X h^2 \lesssim 1 \) keeping the constraint \( T_R \gtrsim 1 \text{ MeV} \) in a large region of the parameter space.

As mentioned in the introduction, the thermal inflation was originally proposed as a solution to the cosmological moduli problem. Therefore, we turn to examine whether the thermal inflation could sufficiently dilute not only the density of the superheavy particle \( X \) but also that of the string moduli, simultaneously.

When the Hubble parameter becomes comparable to the moduli masses, the moduli \( \phi \) start coherent oscillations and the corresponding cosmic temperature is estimated as

\[ T_\phi \simeq 7.2 \times 10^6 \text{ GeV} \left( \frac{m_\phi}{100 \text{ keV}} \right)^{\frac{1}{2}}. \]

Here notice that the moduli oscillations always begin after the \( X \) freezes out since \( T_f \sim m_X \gg T_\phi \) even for the heavy moduli \( m_\phi \sim m_{3/2} \simeq 100 \text{ GeV} - 1 \text{ TeV} \) predicted in hidden sector SUSY breaking models. Because the initial amplitudes of the oscillations, \( \phi_0 \), are expected to be \( \phi_0 \sim M_G \), the present abundances \( \Omega_\phi h^2 \) of the moduli oscillations are given by

\[ \Omega_\phi h^2 = 2.5 \times 10^{14} \left( \frac{m_\phi}{100 \text{ keV}} \right)^{\frac{1}{2}} \left( \frac{\phi_0}{M_G} \right)^2. \]

(16)

Such a huge energy density of the moduli leads to cosmological difficulties for the typical moduli mass regions predicted in both gauge-mediated SUSY breaking and hidden-sector SUSY breaking scenarios.

However, if the universe experienced the thermal inflation, the moduli abundances are reduced by the factor \( \Delta \) [Eq. (11)] as \[ (\Omega_\phi)_{BB} h^2 = 22C^\frac{1}{2} \frac{m_\phi^2 m_\sigma^2 M_G}{M^3 (h^{-2} \rho_{cr}/s_0)} \left( \frac{\phi_0}{M_G} \right)^2 \]

\[ \simeq \begin{cases} 
2.4 \times 10^{-3} \left( \frac{m_\phi}{100 \text{ keV}} \right)^{\frac{1}{2}} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{\frac{1}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{-3} \left( \frac{\phi_0}{M_G} \right)^2 & \text{for } \sigma \to \gamma \gamma \\
3.9 \times 10^{-2} \left( \frac{m_\phi}{100 \text{ keV}} \right)^{\frac{1}{2}} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{\frac{1}{2}} \left( \frac{M}{10^{10} \text{ GeV}} \right)^{-3} \left( \frac{\phi_0}{M_G} \right)^2 & \text{for } \sigma \to gg
\end{cases}. \]  

(17)

\[ \text{If the moduli masses are less than about 100 MeV, the moduli become stable until the present. On the other hand, for the heavier moduli mass region, } \Omega_\phi \text{ is regarded as the ratio, } (\rho_\phi/s)_{D}/(\rho_{cr}/s_0), \text{ where } (\rho_\phi/s)_{D} \text{ denotes the ratio of the energy density of the moduli oscillations to the entropy density when the moduli decay.} \]
We call these moduli produced at $T = T_\phi$ as “big-bang” moduli. In deriving Eqs. (16) and (17) we have assumed that the energy of the universe is radiation-dominated when the big-bang modulus starts to oscillate at $H \simeq m_\phi$. This assumption is justified for $(m_\phi/100 \text{ keV})^{1/2} (m_X/10^{13} \text{ GeV})^{-2} \gtrsim 2.9$. On the other hand, when the energy at $H \simeq m_\phi$ is dominated by the superheavy particle $X$, the present abundance of the big-bang modulus is related to the abundance of $X$ (14) as

$$\Omega_\phi^{BB} h^2 = \frac{1}{6} \Omega_X h^2 \left( \frac{\phi_0}{M_G} \right)^2.$$ (18)

Therefore, both abundances are comparable for $\phi_0 \sim M_G$.

Furthermore, it should be noticed that the secondary oscillations of the moduli start just after the thermal inflation ends [6]. We call the moduli produced by these secondary oscillations as “thermal inflation” moduli. The present abundances of the thermal inflation moduli are estimated as [6,7]

$$\Omega_\phi^{TI} h^2 = 6.0 \times 10^{-2} \frac{C_{\frac{3}{2}}^2}{m^2_{\sigma} M^2_G (h^{-2} \rho_{cr}/s_0)} \left( \frac{\phi_0}{M_G} \right)^2 \begin{cases} 7.4 \left( \frac{m_\phi}{100 \text{ keV}} \right)^{-2} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-2} \left( \frac{M}{10^{10} \text{ GeV}} \right) \left( \frac{\phi_0}{M_G} \right)^2 & \text{for } \sigma \rightarrow \gamma \gamma \\ 1.2 \times 10^2 \left( \frac{m_\phi}{100 \text{ keV}} \right)^{-2} \left( \frac{m_\sigma}{100 \text{ GeV}} \right)^{-2} \left( \frac{M}{10^{10} \text{ GeV}} \right) \left( \frac{\phi_0}{M_G} \right)^2 & \text{for } \sigma \rightarrow gg \end{cases}.$$ (19)

We see from Eqs. (17) and (19) that the thermal inflation dilutes the moduli density substantially. In fact, it has been shown in Refs. [6,7,24] that two moduli mass regions: (i) $m_\phi \lesssim 1 \text{ MeV}$ and (ii) $m_\phi \gtrsim 10 \text{ GeV}$ survive various cosmological constraints in the presence of the thermal inflation.

We are now at the point to examine whether the thermal inflation can solve the moduli problem and also dilute sufficiently the superheavy particle $X$ at the same time. First, we consider the lighter allowed region of the moduli masses predicted in gauge-mediated SUSY breaking models. In Fig. 2 we show the contour plot of the abundance of $X$ of mass $m_X = 10^{13} \text{ GeV}$ as well as the various constraints in the $m_\sigma$-$M_*$ plane for $m_\phi = 100 \text{ keV}$. Such light moduli are constrained from the overclosure limit. [8] Then the requirements $(\Omega_\phi)_{BB} h^2 \lesssim 1$ and $(\Omega_\phi)_{TI} h^2 \lesssim 1$ put the lower and upper bounds on $M_*$ respectively. Furthermore, the condition $T_R \gtrsim 1 \text{ MeV}$ leads to the upper bound on $M_*$ as represented in Eq. (13). We see from the figures that in the parameter space which survives above constraints one can obtain $\Omega_X h^2 \sim 10^{-6}$ for $m_X = 10^{13} \text{ GeV}$; i.e., the thermal inflation can dilute the abundance of the superheavy particle $X$ sufficiently. Similar results are obtained in the allowed moduli mass region $m_\phi \sim 10^{-2} \text{ keV}$–$1 \text{ MeV}$. For example, when $m_\phi = 10^{-2} \text{ keV}$, the thermal abundance

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8 Notice that for the modulus of mass $m_\phi = 100 \text{ keV}$ the constraint from the diffuse x(\gamma)-ray backgrounds is not so severe as the overclosure limit. It gives a more stringent upper bound on the modulus abundance for $100 \text{ MeV} \sim g_\phi \sim 200 \text{ keV}$ [25].
inflation solves the moduli problem in the parameter regions $m_\phi \simeq 10^{-2}$ GeV–1 GeV and $M_* \simeq 10^{13}$ GeV–10$^{17}$ GeV and in this region we obtain $\Omega_X h^2 \sim 10^{-1}$–1 for $m_X = 10^{13}$ GeV. We find that the thermal inflation is very successful, in a whole range of $m_\phi \simeq 10^{-2}$ keV–1 MeV, to solve the moduli problem as well as to reduce the density of the superheavy particle whose mass is $m_X \sim 10^{13}$ GeV even if it was in the thermal bath in the early universe.

Next, we turn to discuss the case of the heavier moduli of masses $m_\phi \simeq 100$ GeV–1 TeV predicted in hidden-sector SUSY breaking models. In Fig. 3 we show the contour plot of $\Omega_X$ with various constraints for $m_X = 10^{13}$ GeV and $m_\phi = 100$ GeV. Such heavy moduli are severely constrained not to destroy or overproduce light elements synthesized by the big bang nucleosynthesis. It has been found that $(\Omega_\phi)_{BB} h^2$ and $(\Omega_\phi)_{TI} h^2$ should be less than about $10^{-5}$ [26]. Fig. 3 shows that in order to solve the moduli problem, the thermal inflation requires $M_*$ higher than in the previous case, which results in more dilution of $X$, and hence we obtain $\Omega_X h^2 \sim 10^{-9}$–$10^{-7}$ for $m_X = 10^{13}$ GeV.

Although we have only considered the case $n = 1$, similar discussions also hold in the higher $n$ case, except that the scale of $M_*$ becomes higher.

In the present analysis we have taken the cutoff scale of the thermal inflation model $M_*$ as a free parameter. However, it is natural to choose it as the gravitational scale, i.e., $M_* \sim M_G$. If it is the case, the thermal inflation dilutes the string moduli sufficiently only if their masses are $m_\phi \simeq 10^{-1}$ keV–1 MeV. Moreover, as shown in Fig. 1, the dark matter density of the superheavy particle of mass $m_X \simeq 10^{12}$–$10^{14}$ GeV becomes $\Omega_X h^2 \sim 10^{-4}$–1, which is just the mass region required to explain the observed ultra high-energy cosmic rays. The required long lifetime of the $X$ particle may be explained by discrete gauge symmetries [27] or by compositeness of the $X$ particle [28].

In this paper we have shown that the thermal inflation provides indeed a new possibility that the superheavy $X$ particle may form a part of the dark matter in the present universe. However, it gives rise to a new problem, since it also dilutes the primordial baryon asymmetry significantly. Because the reheating temperature of the thermal inflation should be quite low $T_R \sim 1$–10 MeV to dilute $\phi$ and $X$ sufficiently, the electroweak baryogenesis does not work. However, as shown in Ref. [29], the Affleck-Dine mechanism [30] may produce enough baryon asymmetry even with tremendous entropy production due to the thermal inflation, if the moduli are light ($m_\phi \lesssim 1$ MeV) [25]. Therefore, in the present scenario, the light moduli predicted in gauge-mediated SUSY breaking models are favored.

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FIG. 1. The contour lines of the relic abundance ($\Omega_X h^2$) of the superheavy particle $X$ of mass $m_X = 10^{13}$ GeV in the presence of the thermal inflation with $n = 1$ for the cases that a flaton $\sigma$ decays mainly into two gluons (A) and that a flaton $\sigma$ decays mainly into two photons (B). The contour lines are represented by the dotted lines. (The contour line of $\Omega_X h^2 = 1$ is represented by the thick dotted line.) We show the upper bound on the cutoff scale $M_*$ obtained from $T_R > 1$ MeV by the thick solid line.
FIG. 2. The contour lines of the relic abundance \((\Omega_X h^2)\) of the superheavy particle \(X\) of mass \(m_X = 10^{13}\) GeV in the presence of the thermal inflation with \(n = 1\) for the cases that a flaton \(\sigma\) decays mainly into two gluons (A) and that a flaton \(\sigma\) decays mainly into two photons (B). The contour lines are represented by the dotted lines. (The contour line of \(\Omega_X h^2 = 1\) is represented by the thick dotted line.) We show the upper bound on the cutoff scale \(M_*\) obtained from \(T_R > 1\) MeV by the thick solid line. The lower bound on \(M_*\) from \((\Omega_\phi)_{BB} h^2 < 1\) and the upper bound on \(M_*\) from \((\Omega_\phi)_{TI} h^2 < 1\) for the case \(m_\phi = 100\) keV are also shown by the thick solid lines.
FIG. 3. The contour lines of the relic abundance ($\Omega_X h^2$) of the superheavy particle $X$ of mass $m_X = 10^{13}$ GeV in the presence of the thermal inflation with $n = 1$ for the cases that a flaton $\sigma$ decays mainly into two gluons (A) and that a flaton $\sigma$ decays mainly into two photons (B). The contour lines are represented by the dotted lines. We show the upper bound on the cutoff scale $M_*$ obtained from $T_R > 1$ MeV by the thick solid line. The lower bound on $M_*$ from $(\Omega_\phi)_{BB} h^2 < 10^{-5}$ and the upper bound on $M_*$ from $(\Omega_\phi)_{TI} h^2 < 10^{-5}$ for the case $m_\phi = 100$ GeV are also shown by the thick solid lines.
FIG. 4. The contour lines of the relic abundance ($\Omega_X h^2$) of the superheavy particle $X$ in the presence of the thermal inflation with $n = 1$ for the cases that a flaton $\sigma$ decays mainly into two gluons (A) and that a flaton $\sigma$ decays mainly into two photons (B). We take the cutoff scale as $M_\phi = M_G \simeq 2.4 \times 10^{18}$ GeV and $m_\phi = 100$ keV. The contour lines are represented by the dotted lines. (The contour line of $\Omega_X h^2 = 1$ is represented by the thick dotted line.) We show the lower bound on the flaton mass $m_\sigma > 1$ GeV to open the gluon decay channel (A) and $m_\sigma > 3.4$ GeV from $T_R > 1$ MeV (B), and also the upper bound on $m_\sigma$ from $(\Omega_\phi)_{TI} h^2 < 1$ by the thick solid lines.