Determination of the $\pi NN$ Form Factor from the Threshold Pion Production of $pp$ Scattering

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Abstract

It is shown that the threshold productions of $\pi^0 pp$, $\pi^+ np$ and $\pi^+ d$ from $pp$ collisions can be consistently described by a model consisting of a pion s-wave rescattering and $NN$ pair-terms of heavy-meson exchanges. The large difference between $\sigma^{tot}(pp \rightarrow \pi^+ d)$ and $\sigma^{tot}(pp \rightarrow \pi^+ np)$ is understood from the orthogonality of the deuteron and the $np$ scattering wave functions. In a calculation using the Paris potential to account for the initial and final $NN$ interactions, it is found that the data can be best reproduced by using a soft $\pi NN$ form factor with $\Lambda_{\pi} = 650$ MeV for a monopole form. This is consistent with an earlier study of pion production in the $\Delta$ excitation region.

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With the development of the Cooler facility at the Indiana University Cyclotron Facility (IUCF), accurate data of threshold pion production from proton-proton (pp) scattering have become available. The data of $pp \to pp\pi^0$ [1] had led to the identification of the short-range pion production mechanisms [2–4]. In this paper we demonstrate that by also including the new data of the $pp \to \pi^+ np$ [5] and $pp \to \pi^+ d$ [6] reactions in a theoretical analysis, we can extract information concerning the range of the $\pi NN$ form factor. Furthermore, we will show that the large difference between the cross sections of $pp \to \pi^+ d$ and $pp \to \pi^+ np$ can be understood from the behavior of the final np wave functions.

Following the previous investigations [2–4, 7, 8], the s-matrix for the production of a pion with a charge $\alpha$ from a $pp$ collision can be written as (we use the normalization of [9])

$$S_{fi} = -2\pi i\delta^{(4)}(p_1 + p_2 - p_f - k) \langle f|A_\alpha|\chi^{(+)}_{p_1 p_2}\rangle$$

where $|\chi^{(+)}_{p_1 p_2}\rangle$ is the initial $pp$ scattering wave function, and $\langle f|$ is either the deuteron wave function in $pp \to \pi^+ d$ reaction or the np scattering wave function $\langle \chi^{(-)}_{p_1 p_2}\rangle$ in $pp \to \pi^+ np$ reaction. Following the previous approach [2], the plane-wave matrix element of the production operator $A_\alpha$ is defined by taking a nonrelativistic limit of the lowest-order Feynman amplitudes calculated from a model interaction Hamiltonian. For our present purpose, it is sufficient to consider the following interaction Hamiltonians for $\pi$, $\omega$, $\sigma$, and $N$ fields

$$H(x) = H_\pi(x) + H_s + H_\sigma(x) + H_\omega(x)$$

where

$$H_\pi(x) = \frac{f}{m_\pi} \bar{\psi}(x)\gamma_5\gamma^\mu \vec{\sigma}\psi(x) \cdot \partial_\mu \vec{\phi}_\pi(x)$$

$$H_s(x) = \frac{4\pi\lambda_1}{m_\pi} \bar{\psi}(x)\vec{\phi}_\pi(x) \cdot \vec{\phi}_\pi(x)\psi(x) + \frac{4\pi\lambda_2}{m_\pi^2} \bar{\psi}(x)\vec{\sigma}\psi(x) \cdot \vec{\phi}(x) \times \left( \frac{\partial}{\partial t} \vec{\phi}(x) \right)$$

$$H_\sigma(x) = g_\sigma \bar{\psi}(x)\psi(x)\phi_\sigma(x)$$

$$H_\omega(x) = g_\omega \bar{\psi}(x)\gamma^\mu\psi(x)\partial_\mu \phi_\omega(x)$$

Here $H_\pi$ describes the familiar pseudovector $\pi NN$ coupling. The $\pi N$ s-wave interaction is described by $H_s$, as introduced in Ref. [7]. In this work, we use the vaules $\lambda_1 \simeq 0.005$ and
\( \lambda_2 \simeq 0.05 \) calculated \[ \textnormal{from the } \pi N \textnormal{ s-wave scattering lengths of } [10]. \] The heavy meson-exchanges are induced by \( H_\sigma \) for a isoscalar-scalar \( \sigma \) meson and \( H_\omega \) for a isoscalar-vector \( \omega \) meson. The isovector mesons \( \delta \) and \( \rho \) can be included \[ \textnormal{but will be neglected in this work for simplicity.} \]

The pion s-wave rescattering term of the production operator \( A_\alpha \) is calculated from \( H_\pi \) and \( H_\delta \). The \( N\bar{N} \) pair-terms of \( \sigma \) and \( \omega \) exchange amplitudes are calculated from \( H_\pi \) and second orders in \( H_\sigma \) and \( H_\omega \). To ensure that there is no double counting in the calculation of Eq. (1), only the negative energy component of the nucleon propagator in the heavy-meson Feynman amplitudes is kept. The resulting terms are called the \( N\bar{N} \) pair-terms. To be consistent with the employed nonrelativistic \( NN \) potentials for calculating the initial and final \( NN \) wave functions, we make the usual nonrelativistic expansion of the Feynman amplitudes. In terms of momentum variables and pion isospin \( (N(p_1) + N(p_2) \rightarrow N(p'_1) + N(p'_2) + \pi(k, \alpha)) \), we obtain the following form

\[
A_\alpha = \left[ T_{\alpha}^{(1)} + T_{\alpha}^{(\pi)} + T_{\alpha}^{(\sigma)} + T_{\alpha}^{(\omega)} \right] + (1 \leftrightarrow 2) \quad (4)
\]

where

\[
T_{\alpha}^{(1)} = G_0 \left\{ \delta(\vec{p}_2 - \vec{p}'_2) (2\pi)^3 \left[ \vec{\sigma}(1) \cdot \vec{k} - \frac{\omega_k}{2m_N} \vec{\sigma}(1) \cdot (\vec{p}_1 + \vec{p}'_1) \right] (-1)^\alpha \tau_{-\alpha}(1) \right\} 
\]

\[
T_{\alpha}^{(\pi)} = G^{(\pi)} \left\{ \frac{\vec{\sigma}(1) \cdot (\vec{p}_2 - \vec{p}'_2)}{m_\pi^2 - m_\rho^2} \left[ \frac{8\pi \lambda_1}{m_\pi} (-1)^\alpha \tau_{-\alpha}(2) + i \frac{8\pi \lambda_2}{m_\pi} \omega_k \omega_q \frac{2m_\pi}{m_N} (-1)^\alpha \tau_{-\alpha}(1) \right] \right\} 
\]

\[
T_{\alpha}^{(\sigma)} = G^{(\sigma)} \left\{ \frac{\omega_k}{m_N^2} \frac{g_\rho^2}{m_N^2} \left( \vec{\sigma}(1) \cdot (\vec{p}_1 + \vec{p}'_1 + \vec{k}) (-1)^\alpha \tau_{-\alpha}(1) \right) \right\} 
\]

\[
T_{\alpha}^{(\omega)} = G^{(\omega)} \left\{ \frac{\omega_k}{m_N^2} \frac{g_\omega^2}{m_N^2} \left[ \vec{\sigma}(1) \cdot (\vec{p}_2 + \vec{p}'_2) + \frac{i}{2} (\vec{p}_2 - \vec{p}'_2) \cdot \left( \vec{\sigma}(1) \times \vec{\sigma}(2) \right) \right] (-1)^\alpha \tau_{-\alpha}(1) \right\} \quad (5)
\]

with

\[
\left\{ \begin{array}{l}
G_0 = \frac{i}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \frac{L}{m_\pi}, \\
G^{(i)} = G_0 \left[ F^{(i)}(\Lambda_i, q^2) \right]^2.
\end{array} \right.
\]

Here we set \( \vec{q} = \vec{p}_2 - \vec{p}'_2 \), \( \omega_k = \sqrt{m_\pi^2 + k^2} \), \( \omega_q = \sqrt{m_\pi^2 + q^2} \), \( \tau_{\pm} = \frac{\tau_1 \pm i \tau_2}{\sqrt{2}} \), \( \tau_0 = \tau_z \), and \( \alpha \) is the z-component of pion isospin. The form factors \( F^{(i)}(\Lambda_i, q^2) \) are introduced to regularize meson-NN vertices.
Let us first consider the pion s-wave rescattering term $T^\pi$. Its main feature is that
\[ \lambda_1 \simeq 0.005 \ll \lambda_2 \simeq 0.05. \]
Since the matrix element of the isospin operator $(\vec{\tau}_1 \times \vec{\tau}_2)$ between two $T = 1$ states vanishes, only the very weak $\lambda_1$ term of $T^\pi$ can have a contribution to the $pp \to pp\pi^0$ cross section. That is why the calculations \[2-4,8\] including only $T^{(1)} + T^{(\pi)}$ failed to account for the total cross section of $pp \to pp\pi^0$. It was found \[2-4\] that the discrepancies with the data can be removed by introducing the $N\bar{N}$ pair terms of heavy-meson exchange. A more general approach was introduced in Ref. \[2\] to calculate the $N\bar{N}$ pair-terms from realistic $NN$ potentials. For the present purpose, it is sufficient to use the simple model defined in Eqs. (5). We choose the monopole form factor $F^{(i)}(\Lambda_i, \vec{q}^2) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + \vec{q}^2}$. All of the two nucleon wave functions in Eq. (1) are calculated from the Paris potential \[11\]. We found that the threshold $pp \to pp\pi^0$ total cross section data can be best reproduced if we choose $g_\sigma^2/4\pi = 6$ and $\Lambda_\sigma = 1000$ MeV/c, for the $\sigma$-exchange, and $g_\omega^2/4\pi = 10$ and $\Lambda_\omega = 1400$ MeV for the $\omega$-exchange. In considering only the total cross sections, one can also fit the data by keeping only the $\sigma$-exchange term with $g_\sigma^2/4\pi = 12$ and $\Lambda_\sigma = 2000$ MeV/c. This is similar to the findings of Refs. \[3\] and \[4\]. All results presented below are from the $\sigma$-exchange model. The results for $\sigma + \omega$-exchange model are very similar and are therefore omitted.

Once the heavy meson-exchange mechanism has been determined in the study of the $pp \to pp\pi^0$ reaction, the only freedom of the model is the form factor $F^{(\pi)}(\Lambda_\pi, \vec{q}^2)$ of $T^{(\pi)}$. Since the $T = 0$ final $np$ states are involved in the charged pion production, the large $\lambda_2$ term of $T^{(\pi)}$ will contribute to the cross section. Its contribution depends on the range $\Lambda_\pi$ of the $\pi NN$ form factor. We find that all of the threshold pion data can be best reproduced with $\Lambda_\pi = 650$ MeV/c. Our results are the solid curves displayed in Fig. 1. In the same figure, we also show the results (dotted curves) without including the $N\bar{N}$ pair-terms of heavy meson-exchanges. The heavy meson-exchange mechanisms clearly dominate the $pp \to pp\pi^0$ cross section; but also plays a significant role in charged pion productions.

An interesting feature of Fig. 1 is that at energies near the $\pi^+np$ production threshold (292.30 MeV), the cross sections of the $\pi^+d$ production are almost two orders of magnitude larger than the $\pi^+np$ production. This can be understood as follows. In this near threshold
energy region, the final $np$ pair is mainly in the $^1S_0(T = 1)$ and $^3S - ^3D_1(T = 0)$ states. The production of the $^1S_0(T = 1)np$ state is very weak mainly because the large $\lambda_2$ term of the pion rescattering $T^{(\pi)}$ does not contribute (as in the case of $pp \rightarrow pp\pi^0$). The difference between the $\pi^+d$ and $\pi^+np$ productions is therefore only in their final radial wave functions of the $np$ system. With the same $^3S_1 - ^3D_1$ quantum number, the $np$ scattering wave function should be orthogonal to the deuteron wave function. As illustrated in Fig. 2, this comes about from any realistic $NN$ potential. The orthogonality is due to the change of the sign of the $np$ scattering wave function in the region where the deuteron wave function is large. Consequently, the $\pi^+np$ production amplitude involves a cancellation in the integration over the production operator, and hence is very much suppressed. The results shown in Fig. 1 reflect the correctness of the employed Paris potential in describing this wave function effect.

We have found that our predictions are very sensitive to the range $\Lambda_\pi$ of the form factor $F^{(\pi)}$. This is not surprising since the threshold pion production involves a large momentum transfer at the $\pi NN$ vertex. In Fig. 3, we show that the calculations with $\Lambda_\pi = 1000$ MeV/c overestimate significantly the charged pion production cross sections. This is consistent with the results from the earlier work \[12\] in the $\Delta$-excitation region.

The results presented above are from the calculations using the $\pi NN$ coupling constant $f^2/4\pi = 0.079$ of Ref. \[10\]. In recent $\pi N$ \[13\] and $NN$ \[14,15\] phase shift analyses, a smaller value of $f^2/4\pi = 0.075$ was obtained. We have also carried out calculations using this smaller $\pi NN$ coupling constant. A slightly larger $\pi NN$ cutoff $\Lambda = 680$ MeV is needed to obtain an equally good fit to all of the data shown in Figs.1 and 3.

In conclusion, we have shown that all of the threshold pion production data from $pp$ collisions can be described by a model consisting of the pion s-wave rescattering term and the $N\bar{N}$ pair terms of heavy meson-exchanges. The large differences between the $\pi^+d$ and $\pi^+np$ production are shown to be due to the orthogonality between the $np$ scattering wave function and the deuteron bound state. Because of the isospin character of the pion rescattering term, the $pp \rightarrow pp\pi^0$ data can be used to pin down the $N\bar{N}$ pair terms of the heavy meson-exchanges. The charged pion production can therefore be used to determine...
the $\pi NN$ form factor. A soft $\pi NN$ form factor with $\Lambda_\pi = 650$ MeV for a monopole form is found to be consistent with the data. To end, we mention that the present investigation is based on the distorted-wave formulation, Eq.(1), which was used in all of the previous studies. To make further progress, it is necessary to consider the coupling between the final $\pi^0 pp$, $\pi^+ np$ and $\pi^+ d$ channels. This is a highly nontrivial task. Our effort in this direction will be reported elsewhere.

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FIGURES

FIG. 1. The total cross sections (solid curves) calculated from using the Paris potential and $g_\sigma^2/4\pi = 16$, $\Lambda_\sigma = 2000$ MeV/c for the $\sigma$-exchange model of $A_{\alpha}$ are compared with the data. The dotted curves do not include the $N\bar{N}$ pair-terms of heavy meson exchanges.

FIG. 2. The solid curve is the $^3S_1$ deuteron bound state wave function. The dotted and dashed curves are the $^3S_1$ scattering wave function for $E_L = 1$ and 10 MeV. The wave functions are generated from the Paris potential.

FIG. 3. The solid (dotted) curves are the calculated total cross section using the cutoff $\Lambda_\pi = 650$ (1000) MeV for the $\pi NN$ form factor. The results are from calculations using the Paris potential and $g_\sigma^2/4\pi = 16$, $\Lambda_\sigma = 2000$ MeV/c for the $\sigma$-exchange model of $A_{\alpha}$. 
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