Mid-rapidity dependence of pion production in \( p - p \) and \( A - A \) collisions.

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Abstract The calculation of inclusive spectra of pions produced in \( pp \) and \( AA \) collisions as functions of the rapidity \( y \) is presented within the self-similarity approach. It is shown that at not too large rapidities one can obtain an analytical form of the self-similarity function \( \Pi(y, \, p_t) \) dependent on \( y \) and on the hadron transverse momentum \( p_t \). A satisfactory description of the data on rapidity spectra at \( |y| \leq 0.3 \) is presented within the approach applied. The energy dependence of these spectra is also shown to be universal.

1 Introduction

The approach based on similarity of the inclusive spectra of particles produced in hadron–hadron collisions suggested in pioneering papers [1–4] was developed in [5–8]. In [6,8] the similarity of these spectra as a function of the similarity parameter \( \Pi \) dependent of the initial energy \( \sqrt{s} \) in the c.m.s. of colliding particles and transverse masses \( m_{ht} \) of produced hadrons at zero rapidity \( y = 0 \) was demonstrated. A simple form of inclusive spectra was used in [6–8] to describe satisfactorily the spectra at low values of \( m_{ht} \). Further development of this approach was presented in our papers [9–11], where the description of \( m_{ht} \)-spectra was extended to larger values of transverse momenta and initial energies up to a few TeV including contributions of both quarks and gluons to these spectra. The relationship between \( \Pi \) and the Mandelstam variables \( s, \, t \) was obtained in [9]. Moreover, it has been shown that \( \Pi \) cannot be presented in the factorization form as a common function of \( \sqrt{s} \) and \( m_{ht} \). The breakdown of this factorization occurs at not too large initial energies \( \sqrt{s} < 10 \) GeV. It is restored at larger \( \sqrt{s} \) [9]. In fact, this is an advantage of the approach based on the kinematics of four-momentum velocities considered in [5–8], where the parameter \( \Pi \) was obtained using the conservation law of four-momenta and quantum numbers of initial and produced particles, and the minimization principle. At zero rapidity, \( y = 0 \), the form for \( \Pi \) was obtained analytically [8].

Let us note that at non-zero rapidity there are many theoretical models describing the inclusive spectra of hadrons produced in \( A - A \) collisions as functions of \( y \) and \( m_{ht} \); see for example [12–16], and the references therein. There also exist experimental data on these distributions and their fits [17–19]. However, a simultaneous dependence on the rapidities and transverse momenta of the hadrons produced and on the energy of the incident particle was not demonstrated in these papers. Such a dependence is revealed in the present work.

In this paper we extend the approach offered in [9] to the non-zero rapidity region and calculate analytically the similarity parameter \( \Pi \) as a function of \( \sqrt{s}, \, y \) and \( m_{ht} \). Then we calculate the \( y \)-dependence of inclusive spectra of pions produced in \( pp \) and \( AA \) collisions and describe satisfactorily the data at \( |y| \leq 0.3 \) in a wide region of initial energies. We have also confirmed that the distributions over \( y \) and \( m_{ht} \) exhibit a universal energy dependence at low values of these variables, as it has already been shown in [9].

2 The parameter or function of self-similarity \( \Pi \)

The inclusive production of hadron 1 in the interaction of nucleus A with nucleus B,

\[ A + B \rightarrow 1 + \ldots, \quad (1) \]

is satisfied by the conservation law of four-momentum in the following form:

\[ (N_A P_A + N_B P_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2, \quad (2) \]

where \( N_A \) and \( N_B \) are the fractions of four-momentum transmitted by nucleus A and nucleus B; \( P_A, \, P_B, \, p_1 \) are the
four-momenta of the nuclei A and B and particle 1, respectively; \( m_0 \) is the mass of the nucleon; M is the mass of the particle providing for the conservation of the baryon number, strangeness, and other quantum numbers. For \( \pi^- \) mesons \( m_1 = m_\pi \) and \( M = 0 \). For antinuclei and \( K^- \) mesons \( M = m_1 \). For nuclear fragments \( M = -m_1 \). For \( K^+ \) mesons \( m_1 = m_K \) and \( M = m_A - m_K \), \( m_A \) is the mass of the \( \Lambda \) baryon. Let us note that the isospin effects of the produced hadrons and other nuclear effects are out of scope of this approach. Therefore, it is assumed within the self-similarity approach that there is no big difference between the inclusive spectra of \( \pi^+ \) and \( \pi^- \) mesons produced in \( pp \) and \( AA \) collisions. However, there is a difference between similar spectra of \( K^+ \) and \( K^- \) mesons, because the values of \( M \) are different. This is due to the conservation law of strangeness.

In [7] the parameter of self-similarity is introduced in the following form:

\[
\Pi = \min \left[ \frac{1}{2} \left( (u_A N_A + u_B N_B)^2 \right)^{1/2}, \right. 
\]

where \( u_A \) and \( u_B \) are the four-velocities of nuclei A and B.

Then the inclusive spectrum of particle 1 produced in the \( AA \) collision can be presented as a general universal function dependent of the self-similarity parameter:

\[
E d^3 \sigma/dp^3 = A_A^{a(N_A)} A_B^{a(N_B)} F(\Pi) 
\]

where \( a(N_A) = 1/3 + N_A/3 \), \( a(N_B) = 1/3 + N_B/3 \), and \( F \) is the function, its form is presented in [9]:

\[
F(\Pi) = A_q \exp(-\Pi/C_q) + A_g \sqrt{m_{1t}} \exp(-\Pi/C_g) 
(1 - \sigma_{nd}/g(s/s_0)^\Delta) \cdot g(s/s_0)^\Delta. \]

Here \( \Delta = \alpha_P(0) - 1 \) is the excess of the sub-critical Pomeron intercept over 1; \( g = 21 \) mb a constant, which was calculated within the “quasi-eikonal” approximation [20]. We assumed the value \( \Delta = 0.12 \), which is the same value as in our previous paper [9], like in [20]. However, we note that the sensitivity of our results to \( \Delta \) at \( m_{1t} < 1 \) GeV/c^2 is too low because the main contribution to \( F(\Pi) \) comes from the first term in Eq. (5), as shown in [9].

The constants \( A_q = 3.68 \) (GeV/c)^2, \( C_q = 0.147 \); \( A_g = 1.7249 \) (GeV/c)^{-1}, \( C_g = 0.289 \) were obtained in [21,22].

### 3 Analytical solution for self-similarity parameter

An analytical solution for the self-similarity parameter was found in [8]. Here we give a more detailed derivation of the parameter and consider its behavior at small values of \( y \ll 1 \). Equation (2) can be written as follows:

\[
N_A \cdot N_B - \Phi_A \cdot N_A - \Phi_B \cdot N_B = \Phi_M. 
\]

where relativistic invariant dimensionless values have been introduced:

\[
\Phi_A = [(m_1/m_0) \cdot (u_A u_1) + M/m_0]/[(u_A u_B) - 1], \\
\Phi_B = [(m_1/m_0) \cdot (u_B u_1) + M/m_0]/[(u_A u_B) - 1], \\
\Phi_M = (M^2 - m_1^2)/(2m_0^2((u_A u_B) - 1)).
\]

It was shown in [8,9] that for \( AA \) collisions, when \( \Phi_A = \Phi_B = \Phi \), we have the following forms for \( N_A = N_B = N \) and \( \Pi \):

\[
N = N_B = N = 1 + (\Phi M/\Phi^2)^{1/2} \Phi, \\
\Pi = N \cdot c h(Y).
\]

The scalar product of four-dimensional velocities is related to the rapidity of initial particles \( Y \) and the rapidity \( y \) of the produced hadron 1:

\[
(u_A u_B) = c h(2Y); \\
(u_A \cdot u_1) = (m_{1t}/m_1) \cdot c h(-Y - y) = (m_{1t}/m_1) \cdot (c h(Y)c h(y) + s h(Y) s h(y)); \\
(u_B \cdot u_1) = (m_{1t}/m_1) \cdot c h(Y - y) = (m_{1t}/m_1) \cdot (c h(Y)c h(y) - s h(Y) s h(y)).
\]

Here, \( m_{1t} = (m_1^2 + p_{1t}^2)^{1/2} \) is the transverse mass of particle 1. If \( y \ll 1 \), then one can neglect \( s h(y) \) in Eqs. (9, 10) as compared to \( c h(y) \) and we approximately get the following:

\[
(u_A \cdot u_1) \simeq (u_B u_1) \simeq (m_{1t}/m_1) \cdot c h(Y) c h(y).
\]

And in this case

\[
\Phi = \Phi_A = \Phi_B = [m_1/m_0(u_A u_1) + M/m_0]/[(u_A u_B) - 1],
\]

and, approximately, we have the following form for \( \Phi \):

\[
\Phi \simeq [(m_1/m_0) \cdot (m_{1t}/m_1) \cdot c h(Y) \cdot c h(Y) + M/m_0]/[c h(2Y) - 1].
\]

It can be presented as follows:

\[
\Phi \simeq (1/m_0)[m_{1t} \cdot c h(Y) \cdot c h(Y) + M][1/(2s h^2(Y))].
\]

For \( \Phi_M \) we also have the following:

\[
\Phi_M = (M^2 - m_1^2)/(4m_0^2 s h^2(Y)).
\]

Thus, at \( y \ll 1 \)

\[
N = 1 + (\Phi M/\Phi^2)^{1/2} \Phi,
\]

where \( \Phi \) and \( \Phi_M \) are given by Eqs. (14) and (15), respectively.

Since these expressions for \( (u_A \cdot u_1) \) and \( (u_B \cdot u_1) \) do not depend on the projectile mass, we conclude that the above approach is also valid for the projectile hadrons, in particular,
\[ \Pi(s, m_{1t}, y) = \frac{m_1 c h(y)}{2m_0(1 - 4m_0^2/s)} \times \left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{m_1^2 c h^2(y)(1 - 4m_0^2/s)}} \right\}. \]

(17)

For \( pp \to h + X \) inclusive processes the relativistic invariant differential cross-section at small but non-zero rapidity \( y \) has the following form:

\[ E_h \frac{d^3 \sigma_{NN}}{d^3 p_h} \equiv \frac{1}{\pi} \frac{d\sigma}{dm_{1t} dy} = F(\Pi(s, m_{1t}, y)), \]

(18)

where \( F(\Pi(s, m_{1t}, y)) \) is given by Eq. (5) but with \( \Pi(s, m_{1t}, y) \) determined by Eq. (17). For \( AA \to h + X \) processes the differential cross-section is presented by Eq. (17).

The integral of Eq. (18) or the integral of Eq. (4) over the transverse mass of the produced hadron \( m_{1t} \) result in a rapidity \( y \) dependence of the cross-section of hadrons produced in \( pp \) or \( AA \) collision, respectively. Finally, the rapidity distribution can be presented in the following form:

\[ \frac{d\sigma_{NN}}{dy} = 2\pi \int F(\Pi(s, m_{1t}, y))m_{1t} dm_{1t}. \]

(19)

In Figs. 1, 2 and 3 the rapidity distributions of pions produced in AuAu, PbPb and \( pp \) collisions are presented in a wide region of initial energies. A satisfactory description of data with a precision superior to 10% is presented in the rapidity range of the produced particles \(|y| < 0.3\).

As our calculations have shown, the main contribution to \( d\sigma/dy \) given by Eq. (19) is due to the first term of Eq. (5) at low \( y \) values. Therefore, the rapidity distribution can be presented in the following approximate form:

\[ \frac{d\sigma_{NN}}{dy} \simeq A^a(N_A) A^a(N_A) A_q g(s/s_0)^3 m_1 \left[ \frac{m_1 c h(y)}{c h(y) + 2m_0 \delta} \right] C_q \exp(-m_1 c h(y)/(2m_0 \delta)), \]

(20)

where \( \delta = 1 - 4m_0^2/s \). Equation (20) is similar to \( d\sigma_{NN}/dy \) obtained in [12] within the thermal model involving the longitudinal and transverse flows. The difference between our rapidity distribution and the one considered in [12] is the following. We do not include the nuclear thermal effects, which may change the \( y \)-dependence, mainly at \(|y| > 0.3\). Our approach can be applied at \(|y| \leq 0.3\) rather satisfactorily.
The pion rapidity distributions in \( pp \) collisions at the initial momentum \( p_m = 158 \text{ GeV/c} \) (solid line, \( s^{1/2} = 17.28 \text{ GeV} \), 80 GeV/c (long dashed line, \( s^{1/2} = 12.34 \text{ GeV} \)), 40 GeV/c (short dashed line, \( s^{1/2} = 8.77 \text{ GeV} \)), 31 GeV/c (dash-dotted line, \( s^{1/2} = 7.75 \text{ GeV} \)), 20 GeV/c (dash-double dotted line, \( s^{1/2} = 6.27 \text{ GeV} \)) compared to the NA61/SHINE data [19] on the process \( pp \rightarrow \pi^- X \) rily, as seen from Figs. 1, 2 and 3. Equation (20) results in a universal energy dependence of \( d\sigma/dy \), as \((s/s_0)^A\). A more complicated energy dependence of \( d\sigma/dm_{11} \) was obtained in our previous paper [9]. Moreover, it was shown that the data on the \( p_t \)-distributions of pions produced in \( AuAu \) and \( PbPb \) collisions at the STAR and LHC energies as well as in \( pp \) collisions at \(|y| < 0.5\) and a wide range of initial energies are described satisfactorily within the self-similarity approach. This is illustrated in Fig. 4, where our previous results [9] are presented using Eqs. (4, 5). Let us note that the exact form of the self-similarity parameter \( \Pi \) can be obtained from the precise solution of Eq. (3). In this paper we solve this equation approximately neglecting \( sh(y) \) compared to \( ch(y) \) in Eqs. (9, 10). This assumption allows us to get the analytical form of \( \Pi \) at small values of \( y \) and describe satisfactorily the rapidity distributions of pions produced in \( pp \) and \( AA \) collisions at \( y \leq 0.3 \) within a wide range of initial energies.

5 Conclusion

In this paper we have extended the self-similarity approach of analysis of hadron production in \( pp, pA \) and \( AA \) collisions, which was first suggested in [5, 7, 8] and developed in [9] strictly at zero rapidity \( y = 0 \) of the produced hadrons, to the non-zero rapidity region. We achieved it analytically using the conservation laws of four-momenta and quantum numbers of initial and final particles. As the first step, we obtained the analytical form of the self-similarity parameter \( \Pi \) at low rapidities. The validity of our results concerns the rapidity interval \(|y| < 0.3\), as it was shown by the satisfactory description of data concerning pion production in \( pp \) and \( AA \) collisions within a wide range of initial energies. Moreover, we have obtained the universal energy dependence of rapidity distributions using the excess of the sub critical Pomeron intercept over unity, which is well-known from the satisfactory description of numerous data on hadron production in \( pp \) collisions.

The extension of our approach to a broader range of rapidity is the next step of our investigation.

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