Gradient-Based Production Optimization in Reservoir Development

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Abstract. One of the core contents of intelligent oilfield is to develop proved oil resources economically and effectively. The way to realize it is to make reservoir development in the optimal state in real time based on the existing production conditions. Water flooding is a commonly used technique after primary oil recovery. However, due to the influence of formation heterogeneity, the injected water breaks through the high permeability layer and traps the crude oil untouched. In order to solve such problems, a new optimal control model is proposed here, which aims to maximize the net present value of reservoir development and production. By solving the mathematical model of development and production, the input and output control parameters of reservoir are optimized in real time to obtain the optimal production scheme. This model is based on the reservoir numerical simulation and optimization theory. By solving the gradient of the control variables, multiple objective strategy schemes (such as the optimal one aiming at oil production, water production and economic benefits) can be optimized according to different needs to assist the oilfield production decision. On the basis of theoretical research, this model is used to optimize the calculation and analysis of reservoir examples, and the results show that the optimal production scheme is in line with the actual oilfield, which provides theoretical and technical support for the intelligent control of oilfield development.

1. Introduction

Nowadays, lots of oil companies have put forward the concept of intelligent control of injection production optimization, which is an emerging technical theory based on the idea of intelligent management of oil fields. This theoretical method can not only timely analyze oil well production data and control reservoir occurrence, but also reduce uncertainty and risk factors in reservoir development and improve reservoir recovery [1-3]. By combining the optimization method with numerical simulation, the injection and production volume of each well in the production cycle can be optimized to achieve the goal of stabilizing oil and controlling water and improve the development effect of the block under the premise of no construction measures [4,5].
In recent years, the algorithms in the field of production optimization can be divided into gradient optimization algorithm and gradient-free algorithm [6-8]. Gradient-free algorithms need to be simulated and estimated according to the time series, which cannot guarantee the monotonically decreasing or increasing of the objective function, but they can find the global optimal solution through large-scale calculation. Relatively speaking, the gradient algorithm is more effective, and also requires simulation estimation according to the time series. For practical problems of reservoir simulation, the number of simulated grids is 100,000, and the time series simulation calculation usually takes several hours, dozens of hours or more. This means that the algorithm based on gradient algorithm is more efficient and feasible for this problem. Moreover, for the objective function, gradient algorithm is more suitable for reservoir production optimization because it performs single-increment calculation, which makes the objective function monotonically increase and can clearly give the objective income under different conditions.

2. Reservoir Production Optimization

Reservoir development optimization is to maximize the efficiency of reservoir development by adjusting the injection and production of oil and water wells, which belongs to the optimal control problem. The purpose of the problem is to maximize the net present value of production by optimizing the production plan of the reservoir to increase the recovery factor.

The object of optimal control theory is the control system. The core problem is how to choose the control strategy for the given control system, and make the system optimal in a certain sense. We can describe it by a mathematical model. Combined with the general model of optimal control and the performance index of reservoir production optimization, the optimal control model for the production optimization is as follows:

\[
\max J = \sum_{m=0}^{N_p-1} \sum_{n=0}^{N_i-1} R^{m,n}(x^{m,n+1},u^n) + \theta(x^n)
\]  (1)

The constraints are described as:

\[
L^{m,n}(x^{m,n+1},x^{m,n},u^n) = 0
\]  (2)

\[
Au^n \leq b \quad \forall n \in (0, \cdots, N_p-1), m \in (0, \cdots, N-1)
\]  (3)

\[
u_{low}^n \leq u^n \leq u_{up}^n
\]  (4)

The specific parameters of the model are as follows:

\(J\) is the objective function, which is mainly composed of two parts: the performance of the optimal control \(R^{m,n}\) (net present value) and the extra cost expense \(\theta\) (such as depreciation or scrapped cost), where \(R^{m,n}\) is the increment of the performance index \(P\) in the \(n\)th sampling period. For the reservoir production optimization problem, it consists of variables such as the flow or saturation function (fluctuation efficiency) of the oil and water well, and its specific expression is:

\[
R^{m,n}(u^n) = \frac{\sum_{j=1}^{N_p} (r_{o,j} Q_{oi,j}^{m,n} - r_{wp,j} Q_{wi,j}^{m,n}) - \sum_{j=1}^{N_p} r_{wi,j} Q_{wi,j}^{m,n}}{(1+b)^{\Delta t^{m,n}}/\Delta t^{m,n}}
\]  (5)

Where, \(Q_{oi,j}\) is the oil production of a single well in the section \(j\) of each time period, \(m^3\); \(Q_{wi,j}\) is the water production of a single well in the section \(j\) of each time period, \(m^3\); \(Q_{wi,j}\) is the water injection volume of a single well in the section \(j\) of each time period, \(m^3\); \(r_{o}\) is the economic factor of oil production; \(r_{wp}\) is the cost factor of water production; \(r_{wi}\) is the cost factor of injection volume; \(\Delta t\) is the time period, \(\text{year}\); \(b\) is the current interest rate; \(N_p\) is the total number of production wells; \(N_i\) is the total number of injection wells; \(m\) is the control time step; \(n\) is the simulated time step in each control time step. \(L^{m,n}\) and
the initial conditions of the reservoir constitute the reservoir dynamic system, which is composed of a fully implicit three-dimensional three-phase black oil model equation. $Au^n \leq b$ is a linear or nonlinear constraint condition; $x^{m,n}$ is a dynamic variable (such as pressure, saturation and composition, etc.); $u^m$ is a control variable (such as oil well production flow pressure and the flow rate of oil and water wells, etc.); $N$ is the total number of control time steps; $N_m$ is the total number of simulation time steps in each control time step.

The problem can be described as follows: while the control variables satisfy the linear constraint condition $u_{low}^m \leq u^m \leq u_{up}^m$ (single well production limit) and the nonlinear constraint condition $Au^m \leq b$ (such as the total injection quantity constraint), the optimal control $u^*(t)$ and corresponding optimal state for maximizing the performance index $J$ are solved.

3. The Solution of Optimal Control Model

3.1. Discrete Maximum Principle

Using discrete maximum principle to solving process is derived from the classical variational theory, the essence of this theory is to combine the objective function equation with the existing equality constraints, convert the equality constraint optimization to unconstrained optimization, and convert the equations (1) to (4). So, the augmented performance index for a production optimization problem is:

$$J_A = J(x^{n+1}, u^n) + \sum_{n=0}^{N-1} \left( \lambda_{n+1}^* \right)^T L^n \left(x^{n+1}, x^n, u^n \right)$$

In equation (6), only the constraints of the black oil model equation are considered. In the calculation, each constraint equation corresponds to a Lagrange multiplier vector, that is, the number of Lagrange multipliers is related to the number of calculation time steps and control variables. If the three-phase black oil model is divided into 2000 grids and 100 control time steps, the number of Lagrange multipliers to be solved should be $3 \times 2000 \times 100 = 6 \times 10^5$.

According to the discrete maximum principle, the first order variation of $J_A$ is:

$$\delta J_A = \sum_{n=0}^{N-1} \left( \frac{\partial J_{A,n}}{\partial x^n} \delta x^n + \frac{\partial J_{A,n}}{\partial x^n} \right) \delta x^n + \sum_{n=0}^{N-1} \left( \frac{\partial J_{A,n}}{\partial u^n} \right) \delta u^n + \sum_{n=0}^{N-1} \left( \frac{\partial J_{A,n}}{\partial \lambda_{n+1}^*} \right) \delta \lambda_{n+1}$$

According to the definition the term $\delta \lambda_{n+1}^{m,n+1}$ is zero, and the necessary condition to obtain the functional extreme value is $\delta J_A = 0$, these terms are independent of each other, so the adjoint equation and the gradient solving equation are obtained, where the adjoint equation is:

$$\begin{align*}
\frac{\partial J_{A,n}}{\partial x^{m,n}} + \frac{\partial J_{A,n}}{\partial x^{m,n}} &= 0, \quad m \leq N \\
\frac{\partial \theta}{\partial x^{N-1,n}} + \frac{\partial J_{A,n}}{\partial x^{N-1,n}} &= 0, \quad m = N
\end{align*}$$

The gradient solving equation is:

$$\frac{\partial J_{A,n}}{\partial u^n} = \sum_{n=0}^{N-1} \left[ \frac{\partial R_{m,n}}{\partial u^n} + \left( \lambda_{n+1}^* \right)^T \frac{\partial L_{m,n}}{\partial u^n} \right] \quad \forall m \in (0, \ldots, N-1)$$

Since the Lagrange multiplier is used in the gradient solution, the equation (8) needs to be solved first. Substitute equation (6) into equation (8), we can get as follows:
\[
\frac{\partial R^{n+1}_m}{\partial x} + \left( \lambda^{n+1}_m \right)^T \frac{\partial L^{n+1}}{\partial x} + \left( \lambda^{n+1}_m \right)^T \frac{\partial L^{n+1}}{\partial x} = 0, \quad m \leq N \quad (a)
\]
\[
\frac{\partial R^{n+1}_m}{\partial x} + \left( \lambda^{n+1}_m \right)^T \frac{\partial L^{n+1}}{\partial x} = 0, \quad m \leq N \quad \text{(Boundary condition for each control step) (b) (10)}
\]
\[
\frac{\partial \theta}{\partial x} + \left( \lambda^{n+1}_m \right)^T \frac{\partial L^{n+1}}{\partial x} = 0, \quad m = N \quad \text{(The final boundary condition) (c)}
\]

In which, \( x^{m,N_\alpha} = x^{n+1,0} \); \( \lambda^{m+1,1} = \lambda^{m,N_\alpha+1} \). In the above equation, there is a final boundary condition for each control time step, and this boundary condition is obtained from the calculation of the previous time step adjoint equation, and is inversely solved in the time series.

3.2. Solution process of partial derivative

For the solving equations derived from the equation (8) and (9), the biggest problem to solving the equation is that each partial derivative calculation, so here are specific to introduce how the partial derivative is calculated.

3.2.1 Calculation of \( L \). The partial derivative of \( L \) includes two main parts: \( \frac{\partial L}{\partial x} \) and \( \frac{\partial L}{\partial u} \).

\[
(1) \frac{\partial L}{\partial x} = \psi'(x) + W(x) - \left[ \eta^{n+1} - \eta^{n} \right] = \left[ \sum_{i=1}^{m} \left( \sum_{p} \left( \lambda_x \rho_p X_p \Delta \Phi_p \right) \right) \right]^n
\]
\[
+ \left[ \sum_{i=1}^{m} \left( \sum_{p} \left( \lambda_x \rho_p X_p (p_p - p_{br}) \right) \right) \right]^n - V \frac{\Delta t}{\Delta t} = 0
\]

Where, \( L \) is the full implicit equation of the black oil model; \( \eta \) is the cumulative term of conservation of matter; \( \psi \) is the internal term of conservation of matter; \( W \) is the injection product of oil and water wells; \( V \) is the volume, \( m^3 \); \( \phi \) is porosity; \( S_p \) is saturation; \( \rho_p \) is density, \( kg/m^3 \); \( X_p \) is the number of moles corresponding to the phase \( p \) component \( c \); \( \Delta t \) is the time interval, \( days \); \( T \) is the conductivity coefficient, \( m^2/d \); \( \lambda_p \) is the fluidity of the phase \( p \), \( 1/cp \); \( \Delta \Phi \) is the potential difference of phase \( p \), \( bar \); \( \eta \) is the time step; \( p_p \) is the grid pressure, \( bar \); \( p_{br} \) is the bottom hole flow pressure.

For the three unknown variables in each grid, the derivatives here are the derivatives of the three phase equation of oil, gas and water with respect to gas phase pressure, oil phase saturation and gas phase saturation respectively. Therefore, the first term in the Jacobian matrix structure diagram is transformed as:

\[
\begin{align*}
\frac{\partial L}{\partial x} &= \begin{bmatrix}
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} \\
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} \\
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x}
\end{bmatrix} \\
\text{where,} \\
L_x &= \begin{bmatrix}
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} \\
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} \\
\frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial x}
\end{bmatrix}
\end{align*}
\]

Here, \( L_g \) is the gas phase flow equation; \( L_o \) is the oil phase flow equation; \( L_w \) is the water phase flow equation; \( P_g \) is the gas phase pressure, \( bar; S_g \) is gas phase saturation; \( S_o \) is oil phase saturation, \( m^3 \).

Here, we get that:

\[
J_{\Delta t \Delta x} = J_{\Delta t}
\]
\[
\frac{\partial L^{n+1}}{\partial x} = \frac{\partial \psi^{n+1}}{\partial x} + \frac{\partial W^{n+1}}{\partial x} - \frac{\partial \eta^{n+1}}{\partial x}
\]
\[ \frac{\partial L^m}{\partial x^{mn}} = \frac{\partial \eta^m(x^{mn})}{\partial x^{mn}} \] (15)

It is found that the partial derivative term in equation (15) is completely consistent with the last term in equation (14), so when calculating this term, it can be retained in the last time step. In the time step \( n \), this term does not need to be calculated, but only needs to directly extract the calculation result of the previous step \( n-1 \), and the calculation of \( \partial L / \partial x \) is completed here. Since the equation calculation process is all completed by matrix form. While the grid dimension of the reservoir is \( N_x \times N_y \times 1 \), that is, for the reservoir with grid number in \( x \) direction and \( y \) direction of single layer \( N_e \), the dimension of \( \partial L^m / \partial x^{mn} \) and \( \partial L^m / \partial x^{mn-1} \) is \( N_e \times N_e \).

The others are only related to the \( W \) in equation (11), because \( Y \) is the function of oil production, water production and water injection, and \( \frac{\partial L}{\partial u} \) is the partial derivative of the flow equation to the control variables of oil-well. So they are only related to oil-well terms. According to equation (5):

\[ \frac{\partial R}{\partial u} = \frac{\Delta t}{1+b} \left[ \sum_{j=1}^{N_e} \left( r_{o,j} \frac{\partial W_{o,j}}{\partial u} - r_{w,j} \frac{\partial W_{w,j}}{\partial u} \right) - \sum_{j=1}^{N_e} \frac{\partial \omega}{\partial u} \right] \] (16)

Where

\[ \frac{\partial W}{\partial u} = W^p \sum_p [\beta, \rho, X_p, \frac{\partial p}{\partial u}] \] (17)

\( W_{o,j} \) is the oil- producing term, \( W_{w,j} \) is the water-producing term, and \( W_{w,j} \) is the water-flooding term.In gradient equation (9), if the number of oil-well control variables here is \( N_o \), then the dimension of \( \partial R / \partial u \) is \( N_o \times 1 \).

Since \( \partial L / \partial u \) is the partial derivative of the control variable, the partial derivative of the other terms in equation (11) is 0 except the oil-well term, that is, \( \frac{\partial L}{\partial u} = \frac{\partial W}{\partial u} \). The dimension is \( N_e \times N_e \); When we take the derivative for \( x \), we ignore the change in density with respect to the state variable, and the derivative is as follow:

\[ \frac{\partial R}{\partial x} = \frac{\Delta t}{1+b} \left[ \sum_{j=1}^{N_e} \left( \frac{\partial W_{o,j}}{\partial x} - r_{o,j} \frac{\partial W_{o,j}}{\partial x} \right) - \sum_{j=1}^{N_e} \frac{\partial \omega}{\partial x} \right] \] (18)

In the formula, the term \( \partial W / \partial x \) is only a component part of the term \( \partial L / \partial x \), so it can also be directly obtained from the Jacobian matrix solved completely implicitly. The dimension of \( \partial R / \partial x \) is \( N_e \times 1 \). As for the two terms \( \partial R / \partial u \) and \( \partial L / \partial u \), they can be calculated in the process of numerical simulation, and the calculation results of each time step can be retained, which can be directly read when the subsequent gradient inverse solution is solved.

### 4. Reservoir Case Studies

The model is a two-dimensional three-phase reservoir model with a grid of \( 11 \times 11 \times 1 \), grid size is \( \Delta x = \Delta y = 30 \, \text{m}, \Delta z = 15 \, \text{m} \). The permeability field is shown in Figure 1. The reservoir pressure is 14.1 MPa, and the bottom hole flow pressure of each oil well is initially set to 12.7 MPa, and the total injection volume is 450 m³/d. The price of crude oil is 2,170 RMB/m³, the cost of processing produced water is 5 RMB/m³, and the interest rate is 0.1. In order to compare the effects before and after optimization, only the production wells are optimized here. The initial bottom hole flow pressure of each well is 13.56 MPa, and the total injection volume of the injection well remains unchanged. The lower boundary of the bottom hole pressure of the production well is set to 8.3 MPa and the upper boundary is 15.8 MPa. The simulated production time has a maximum step size of 30 days, the residual oil saturation is 0.2, and the irreducible water saturation is 0.3. The total production
time is 365 days, and the optimization time step is divided into 4 steps. The distribution diagram of control step saturation at each time before and after optimization is shown as Figure 2.

According to figure 2 the fingering phenomenon is obviously under better control. On the premise of the same injection, the sweep efficiency is greatly improved. Compared with the initial scheme, this scheme has achieved better development results, and it increases the net present value to $4.0 \times 10^7$ RMB.

5. Conclusion
According to the actual situation of oilfield development, this paper establishes a reservoir development control optimization model, and the principle of maximum value is used to solve the model to achieve the purpose of optimizing reservoir development and production. In the gradient solution, based on the principle of maximum value, an improved adjoint model is used, which combines reservoir simulation and gradient solution, and fully implicit digital model calculation is used to replace the calculation of the matrix in the process of maximum value principle. The two calculation processes are effectively combined, the calculation process is simplified, and the calculation efficiency is greatly improved; and it is proved that the method is not only applicable to the oil-water two-phase model, but also applicable to the oil-water-gas three-phase multilayer model. Through case analysis, the correctness and feasibility of the method is proved, which provides theoretical and technical support for the intelligent oilfield system.

Acknowledgments
Supported by the Foundation of State Key Laboratory of Shale Oil and Gas Enrichment Mechanisms Effective Development.

References
[1] Darui Wang. BP Statistical Review of World Energy (2005). Petroleum Exploration and Development. 2006;33(1):98.
[2] ExxonMobil Corporation. Energy Outlook to 2030. technical report from. 2004.
[3] S. Mochizuki, L.A. Saputelli, C.S. Kabir, R. Cramer. Real-Time Optimization: Classification and Assessment. SPE, 90213; 2006.
[4] H.P. Bieker, O. Slupphaug, T.A. Johansen. Real-Time Production Optimization of Oil and Gas Production Systems: A Technology Survey. SPE, 99446; 2007.
[5] D. Zhu, K. Furui. Optimizing Oil and Gas Production by Intelligent Technology. SPE, 102104; 2006.
[6] Goldberg DE. Genetic Algorithms in Search, Optimization, and Machine Learning. New York: Addison-Wesley; 1989.
[7] Horst R, Tuy H. Global Optimization: Deterministic Approaches. Berlin: Springer-Verlag; 1996.
[8] Liu HQ. Special Topic of Reservoir Numerical Simulation[M]. Dongying: Petroleum University Press, 2001, 72-95.