Deterministic entanglement of superconducting qubits by parity measurement and feedback

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The stochastic evolution of quantum systems during measurement is arguably the most enigmatic feature of quantum mechanics. Measuring a quantum system typically steers it towards a classical state, destroying the coherence of an initial quantum superposition and the entanglement with other quantum systems. Remarkably, the measurement of a shared property between non-interacting quantum systems can generate entanglement, starting from an uncorrelated state. Of special interest in quantum computing is the parity measurement, which projects the state of multiple qubits (quantum bits) to a state with an even or odd number of excited qubits. A parity meter must discern the two qubit-excitation parities with high fidelity while preserving coherence between same-parity states. Despite numerous proposals for atomic, semiconducting and superconducting qubits, realizing a parity meter that creates entanglement for both even and odd measurement results has remained an outstanding challenge. Here we perform a time-resolved, continuous parity measurement of two superconducting qubits using the cavity in a three-dimensional circuit quantum electrodynamics architecture and phase-sensitive parametric amplification. Using postselection, we produce entanglement by parity measurement reaching 88 per cent fidelity to the closest Bell state. Incorporating the parity meter in a feedback-control loop, we transform the entanglement generation from probabilistic to fully deterministic, achieving 66 per cent fidelity to a target Bell state on demand. These realizations of a parity meter and a feedback-enabled deterministic measurement protocol provide key ingredients for active quantum error correction in the solid state.

Recent advances in nearly quantum-limited amplification and improved qubit coherence times in three-dimensional circuit quantum electrodynamics (3D cQED) architectures have allowed investigations of the gradual collapse of single-qubit wavefunctions in the solid state, following previous fundamental studies in atomic systems. The continuous measurement of a joint property extends this study to the multiquubit setting, resolving the projection to states which are inaccessible via individual qubit measurements. In a two-qubit system, the ideal parity measurement transforms an unentangled superposition state $|\psi^+\rangle = ((|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$ into Bell states:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

and

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

for odd and even outcome, respectively. Beyond generating entanglement between non-interacting qubits, parity measurements allow deterministic two-qubit gates and play a key role as syndrome detectors in qubit error correction. A heralded parity measurement has been recently realized for nuclear spins in diamond. By minimizing measurement-induced decoherence at the expense of single-shot fidelity, Pfaff et al. generated highly entangled states with 3% success probability. Here we realize the first solid-state parity meter that produces entanglement with unity probability.

Our parity meter realization exploits the dispersive regime in two-qubit cQED. Qubit-state dependent shifts of a cavity resonance (here, the fundamental of a 3D cavity enclosing transmon qubits $Q_A$ and $Q_B$) allow joint qubit readout by homodyne detection of an applied microwave pulse transmitted through the cavity (Fig. 1a). The temporal average $V_{\text{int}}$ of the homodyne response $V(t)$ over the time interval $[t_1, t_2]$ constitutes the measurement needle, with expectation value

$$V_{\text{int}} = \langle V(t) \rangle = \text{Tr}(O \rho)$$

where $\rho$ is the two-qubit density matrix and the observable $O$ has the general form

$$O = \beta_0 + \beta_A \sigma_A^\dagger + \beta_B \sigma_B^\dagger + \beta_{BA} \sigma_A^\dagger \sigma_B^\dagger$$

where $\sigma^q_j$ is the Pauli operator for qubit $q$. The coefficients $\beta_0, \beta_A, \beta_B$ and $\beta_{BA}$ depend on the strength $G$, frequency $f_5$ and duration $\tau$ of the measurement pulse, and also on the cavity linewidth $\kappa$ and the frequency shifts $2\Delta_A$ and $2\Delta_B$ of the fundamental mode when $Q_A$ and $Q_B$ are individually excited from $|0\rangle$ to $|1\rangle$. The necessary condition for realizing a parity meter is $\Delta_A = \Delta_B = 0$ (i.e., a trivial offset). A simple approach, pursued here, is to set $f_5$ to the average of the resonance frequencies for the four computational basis states $|ij\rangle$ ($i, j \in \{0, 1\}$) and to match $\Delta_A = \Delta_B$. We engineer this matching by targeting specific qubit transition frequencies $f_A$ and $f_B$ below and above the fundamental mode during fabrication and using an external magnetic field to fine-tune $f_5$ in situ (Extended Data Fig. 1). We align $\Delta_A$ to $\Delta_B$ to within $\sim 0.06\kappa = 2\pi \times 90$ kHz (Fig. 1b). The ensemble-average $\langle V(t) \rangle$ confirms nearly identical high response for odd-parity computational states $|01\rangle$ and $|10\rangle$, and nearly identical low response for the even-parity $|00\rangle$ and $|11\rangle$ (Fig. 1c). The transients observed are consistent with the independently measured $\kappa, \Delta_A$ and $\Delta_B$ values, and the 4-MHz bandwidth of the Josephson parametric amplifier (IPA; Fig. 1a) at the front end of the output amplification chain (see Extended Data Fig. 2 for a detailed schematic of the set-up). Single-shot histograms (Fig. 1d) demonstrate the increasing ability of $V_{\text{int}}$ to discern states of different parity as $t_2$ grows (keeping $t_1 = 0$), and its inability to discriminate between states of the same parity. The histogram separations at $t_2 = 400$ ns give $|\beta_{BA}|/|\beta_0| < 0.02$ (Extended Data Fig. 3). Moving beyond the description of the measurement needle, we now investigate the collapse of the two-qubit state during parity measurement. We prepare the qubits in the maximal superposition state $|\psi^+\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$, apply a parity measurement pulse for $t_5$, and perform tomography of the final two-qubit density matrix $\rho$ with and without conditioning on $V_{\text{int}}$ (Fig. 2a). We choose a weak parity measurement pulse exciting $n_s = 2.5$ intra-cavity photons on average in the steady-state, at resonance. A delay of $3.5\kappa = 350$ ns is inserted to deplete the cavity of photons before performing tomography. The tomographic joint readout is also carried out at $f_5$, but with 14 dB higher power, at which the cavity response is weakly nonlinear and sensitive to both single-qubit terms and two-qubit correlations ($\beta_A = \beta_B = \beta_{BA}$, see Extended Data Fig. 3), as required for tomographic reconstruction.

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Figure 1 | Realization of cavity-based two-qubit parity readout in circuit QED. a, Simplified diagram of the experimental set-up. Single- and double-junction transmon qubits (Qa and Qb, respectively) dispersively couple to the fundamental mode E of a 3D copper cavity enclosing them. The transition frequency of Qb is tuned by a static magnetic field B generated by an external coil. Parity measurement is performed by homodyne detection of the qubit state-dependent cavity response using phase-sensitive Josephson parametric amplification (JPA). Following further amplification at 4 K by a low-noise semiconductor amplifier (HEMT) and room temperature, the signal is demodulated and integrated. A field-programmable-gate-array (FPGA) controller closes the feedback loop that achieves deterministic entanglement by parity measurement (Fig. 4). b, Matching of the dispersive cavity shifts realizing a parity measurement. Different colours correspond to qubits prepared in |00⟩ (black), |01⟩ (blue), |10⟩ (red) and |11⟩ (purple). c, Ensemble-averaged homodyne response ⟨Vp⟩ for qubits prepared in the four computational basis states (same colour code). d, Curves, corresponding ensemble averages of the running integral ⟨Vimp⟩ of ⟨Vp⟩ between t = 0 and t = t. Single-shot histograms (5,000 counts each) of Vimp are shown in 200-ns increments.

The ideal continuous parity measurement gradually suppresses the unconditioned density matrix elements ρijkl = ⟨ij|ρ|kl⟩ connecting states with different parity (either i ≠ k or j ≠ l), and leaves all other coherences (off-diagonal terms) and all populations (diagonal terms) unchanged. The experimental tomography reveals the expected suppression of coherence between states of different parity (Fig. 2b, c). The temporal evolution of |01⟩, with near full suppression by τp = 400 ns, is quantitatively matched by a master-equation simulation of the two-qubit system (see Methods). Tomography also unveils a non-ideality: albeit more gradually, our parity measurement partially suppresses the absolute coherence between equal-parity states, albeit more gradually, our parity measurement partially suppresses the absolute coherence under ensemble averaging. We emphasize that this imperfection is technical rather than fundamental. It can be mitigated in the odd subspace by perfecting the a.c. Stark phase shift induced by intra-cavity photons on basis states of the same parity.

Figure 2 | Unconditioned two-qubit evolution under continuous parity measurement. a, Pulse sequence including preparation of the qubits in the maximal superposition state |Ψ⟩ = |ψ⟩ |ψ⟩, parity measurement and tomography of the final two-qubit state |Ψ⟩ using joint readout. b, Absolute coherences |ρ01,10⟩, |ρ01,01⟩, |ρ00,11⟩ following a parity measurement with variable duration τp. Free parameters of the model are the steady-state photon number on resonance nss = 2.5 ± 0.1, the difference (ωm − ωk)/π = 235 ± 4 kHz, and the absolute coherence values at τp = 0 to account for few-percent pulse errors in state preparation and tomography pre-rotations. Note that the frequency mismatch differs from that in Fig. 1b owing to its sensitivity to measurement power. Error bars, standard deviation of 15 repetitions of tomography, with 22,500 averages per set of pre-rotations. c, d, Extracted density matrices for τp = 0 (c) and τp = 400 ns (d), by which time coherence across the parity subspaces (grey columns) is almost fully suppressed, while coherence persists within the odd-parity (orange columns) and even-parity (green columns) subspaces. See Extended Data Fig. 4 for the temporal evolution with parity measurement off and Extended Data Figs 5 and 6 for two-qubit tomography at other values of τp and nss, respectively.
The ability to discern parity subspaces while preserving coherence within each opens the door to generating entanglement by parity measurement on $|\psi\rangle^2$. For every run of the sequence in Fig. 2, we discriminate $V_{\text{int}}$ using the threshold $V_{\text{th}}$ that maximizes the parity measurement fidelity $F_P$ (Fig. 3a). Assigning the parity measurement result $M_P = +1$ (–1) to $V_{\text{int}}$ below (above) $V_{\text{th}}$, we bisect the tomographic measurements into two groups, and obtain the density matrix for each. We quantify the entanglement achieved in each case using concurrence $C$ as the metric$^{21}$, which ranges from 0% for an unentangled state to 100% for a Bell state. As $t_p$ grows (Fig. 3b), the optimal balance between increasing $F_P$ at the cost of measurement-induced dephasing and intrinsic decoherence is reached at approximately 300 ns (Fig. 3c). Postselection on $M_P = \pm 1$ achieves $C_{M_P = +1} = 45 \pm 3\%$ and $C_{M_P = -1} = 17 \pm 3\%$, with each case occurring with probability $p_{\text{success}} = 50\%$. The higher performance for $M_P = -1$ results from lower measurement-induced dephasing in the odd subspace, consistent with Fig. 2.

The entanglement achieved by this probabilistic protocol can be increased with more stringent postselection. Setting a higher threshold $V_{\text{th}}$ achieves $C_{M_P = -1} = 77 \pm 2\%$ but keeps $p_{\text{success}} = 20\%$ of runs. Analogously, using $V_{\text{th}} = 0.5$ achieves $C_{M_P = +1} = 29 \pm 4\%$ with similar $p_{\text{success}}$ (Fig. 3d, e). However, increasing $C$ at the expense of reduced $p_{\text{success}}$ is not evidently beneficial for quantum information processing. For the many tasks calling for maximally-entangled qubit pairs (ebits), one may use an optimized distillation protocol$^{20}$ to prepare one ebit from $N = 1/E_{\text{S}}(\rho)$ pairs in a partially-entangled state $\rho$, where $E_{\text{S}}$ is the logarithmic negativity$^{23}$. The efficiency $\mathcal{E}$ of ebit generation would be $\mathcal{E} = p_{\text{success}}E_{\text{S}}(\rho)$. For postselection on $M_P = -1$, we calculate $\mathcal{E} = 0.31$ ebits per run using $V_{\text{th}}$ and $\mathcal{E} = 0.20$ using $V_{\text{int}}$. Evidently, increasing entanglement at the expense of reducing $p_{\text{success}}$ is counterproductive in this context.

Motivated by the above observation, we finally demonstrate the use of feedback control to transform entanglement by parity measurement from probabilistic to deterministic, that is, $p_{\text{success}} = 100\%$. Whereas initial proposals in cQED focused on analogue feedback schemes$^{24}$, here we adopt a digital strategy$^{25}$. Specifically, we use a custom-built programmable controller to apply a π pulse on QA conditional on measuring $M_P = +1$ (using $V_{\text{th}}$, Fig. 4). In addition to switching the two-qubit parity, this pulse lets us choose which odd-parity Bell state to target by selecting the phase $\varphi$ of the conditional pulse. To optimize deterministic entanglement, we need to maximize overlap to the same odd-parity Bell state for $M_P = -1$ (Fig. 4b) as for $M_P = +1$ (Fig. 4c). For the targeted state $|\Phi^-\rangle$, this requires cancelling the deterministic a.c. Stark phase $\varphi = 0.73\pi$ accrued between [00] and [11] when $M_P = +1$. This is accomplished by choosing $\varphi = (\pi - \varphi)/2$, which clearly maximizes the entanglement obtained when no postselection on $M_P$ is applied (Fig. 4c, d). The highest deterministic $C = 34\%$ achieved is lower than for our best probabilistic scheme, but the boost to $p_{\text{success}} = 100\%$ achieves a higher $\mathcal{E} = 0.41$ ebits per run. Our experiment extends the fundamental study of continuous measurement$^{6,17}$ in superconducting circuits to the multiqubit scenario, providing a test-bed for the investigation of wavefunction projection and induced dephasing. Furthermore, the implemented parity meter generates entanglement for any measurement result, making it suitable for deterministic quantum information processing protocols. Specifically, the combination of parity measurement with digital feedback realizes a multiqubit measurement-based protocol in the solid state made deterministic through feedback, as achieved with photonic$^{26}$, ionic$^{27,28}$ and atomic$^{29}$ systems. Future experiments will target the complementary use of analogue feedback control to combat measurement-induced dephasing$^{30}$. Integration of analogue feedback will refine the control over quantum measurement and feedback$^{15}$ required to extend quantum coherence by active control methods.
METHODS SUMMARY

Device parameters. Lorentzian best fits to cavity transmission (Fig. 1b) yield $\kappa = \kappa_{\text{on}} + \kappa_{\text{off}} = 2\pi \times (1.56 \pm 0.01$ MHz) and \( \{X_{\text{on}}, X_{\text{off}}\}/\pi = \{-4.03 \pm 0.02, -4.21 \pm 0.02\} \) MHz. From room-temperature characterization, we estimate asymmetric output/input couplings \( \kappa_{\text{on}}/\kappa_{\text{off}} = 8 \). The qubits have transition frequencies \( \{f_{\text{on}}, f_{\text{off}}\} = \{5.52, 7.80\} \) GHz, relaxation times \( \{T_{1,\text{on}}, T_{1,\text{off}}\} = (22.7, \mu\text{s}, \) and pure dephasing times \( \{T_{2,\text{on}}, T_{2,\text{off}}\} = (11.8, \mu\text{s} \) (see also Extended Data Fig. 8). Using the method detailed in ref. 31, we estimate a residual excitation of 1% (2%) probability for QA(Qb).

Readout signal processing. In Fig. 1b we probe the cavity with a pulse \( (t_0 = 1.4) \) at variable frequency, after preparing the qubits in one of the four computational states. The cavity transmission is acquired with homodyne detection at 10 MHz intermediate frequency. In Fig. 1c and d the cavity response \( (t_0 = 2.5) \), first amplified by the JPA, is demodulated with 0 intermediate frequency (measurement, local oscillator, and pump tones are provided by the same generator). For each shot, the average homodyne signal over a 2.5 \( \mu\text{s} \) window preceding state preparation is subtracted. This subtraction mitigates the infiltration of low-frequency fluctuations in the JPA bias. In Figs 2–4, \( t = 100 \text{ ns} \) and \( t = t_0 + 150 \text{ ns} \), experimentally found to maximize \( F_s \). Similarly, an offset integrated over 2.5 \( \mu\text{s} \) is subtracted from each \( V_{\text{moc}} \) (Extended Data Fig. 9).

Model. The system is described by the dispersive Hamiltonian:

\[
\hat{H}/\hbar = \left(\omega_{\text{r}} - \sum_{q=A,B} X_q \sigma_3^q\right) \hat{a}^\dagger \hat{a} - \sum_{q=A,B} \frac{1}{2} \omega_{\text{r}} \sigma_3^q + \omega \left[\hat{a}^\dagger e^{-im_0 t} + \hat{a} e^{+im_0 t}\right]
\]

The cavity-mediated qubit-qubit interaction \( \hat{J} (\sigma_3^a \sigma_3^b + \sigma_3^b \sigma_3^a) \) is disregarded, as \( J \) vanishes for \( X_q = X_q \). We model the evolution of \( \rho \) following the method of quantum trajectories in refs 8, 9, 21 and 32. The stochastic master equation, valid for \( t < T_1, T_2 \) and \( t > \), is:

\[
\frac{d\rho}{dt} = \frac{1}{i\hbar} [\hat{H}, \rho]dt + \sum_{q=A,B} \left( \frac{1}{2T_1} D[\sigma_+^q] \rho + \frac{1}{2T_2} D[\sigma_-^q] \rho \right) dt + \sum_{q=A,B} \mathcal{L}_{\text{X}_q} \left[ \frac{1}{2} \text{Im} \langle \hat{X}_q^* \hat{X}_q \rangle + \text{Re} \langle \hat{X}_q^* \hat{X}_q \rangle \right] \Pi_{\text{X}_q} \Pi_{\text{X}_q} dt + \sqrt{\kappa_{\text{eff}}} M[\Pi_{\text{X}} e^{i\phi}] \rho dW(t)
\]

with operators \( \Pi_{\text{X}} = |\langle \langle \rangle \rangle | \) and \( \sum_{q=A,B} \Pi_{\text{X}_q}, \) super-operators \( D(\theta) \rho = \Theta \rho \Theta^\dagger - \frac{1}{2} (\Theta^\dagger \Theta \rho + \Theta \rho \Theta^\dagger - 2 \langle \Theta + \Theta^\dagger \rangle \rho \) and \( \mathcal{L}_{\text{X}}(\theta) \rho = \Theta \rho \Theta^\dagger - \langle \Theta + \Theta^\dagger \rangle \rho \). Here, \( \phi \) is the homodyne-detection phase set by the JPA pump, and \( \mathcal{X}_q = X_q - X_{q,0} \), where \( X_{q,0} = |\langle \langle \rangle \rangle | X_q \). The dynamics of \( \hat{X}_q \) in the frame rotating at \( \omega_{\text{r}} \) is given by:

\[
\dot{X}_q = -i\omega_{\text{r}} t - i(\omega_{\text{r}} - \omega_{\text{off}} + X_q) X_q - \frac{K}{2} X_q
\]

\( dW \) is the noise in the homodyne record:

\[
V_q(t) dt \propto \sqrt{\kappa_{\text{eff}}} (\Pi_{\text{X}} e^{i\phi} + \Pi_{\text{X}}^\dagger e^{i\phi}) dt + dW
\]

Quantum trajectories are unravelled by numerically solving equation (1) with \( dW = 1 \text{ ns} \) and a Wiener white-noise process \( dW \) (zero mean, variance \( dt \) generated pseudo-randomly. For each trajectory, \( V_{\text{moc}} \) is obtained using the same integration and offset-subtraction parameters as in the experiment. The unconditioned \( \rho \) is obtained by solving equation (1) without the last term.
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Author Contributions D.R. fabricated the device. D.R. and C.A.W. performed the measurements. D.R., C.A.W. and G.d.L. analysed the data. M.D., Ya.M.B. and L.D.C. provided theory support. M.U.T. and R.N.S. produced the feedback controller. K.W.L. designed the JPA. D.R., G.d.L. and L.D.C. wrote the manuscript with feedback from all authors. L.D.C. designed and supervised the project.

Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to L.D.C. (l.dicarlo@tudelft.nl).
Extended Data Figure 1 | Spectroscopy of the two-qubit and cavity system. The transition frequency of $Q_B$ is tuned by applying magnetic flux through its SQUID loop with an external coil. $Q_A$ ($f_A = 5.52 \text{ GHz}$) is a single-junction transmon and thus not tunable. $Q_B$ was designed tunable to allow trimming of the dispersive-shift matching condition. However, the maximal frequency of $Q_B$ ($f_B = 7.80 \text{ GHz}$) is still approximately 20 MHz lower than needed for a perfect match of dispersive shifts. Thus, we flux bias $Q_B$ at this maximal frequency, which is also optimal for coherence. Inset, higher resolution spectroscopy of the avoided crossing of $Q_B$ with the cavity fundamental mode ($f_r = 6.55 \text{ GHz}$), revealing a minimum splitting of 167 MHz.
Extended Data Figure 2 | Detailed schematic of the experimental set-up. Complete wiring of electronic components outside and inside the $^3$He/$^4$He dilution refrigerator (Leiden Cryogenics CF-650). Readout and qubit-drive pulses, shaped by a Tektronix AWG5014 and two Tektronix AWG520, enter the cavity via a single transmission line. The cavity output is reflected by the JPA, which is biased by a superconducting coil and a strong pump tone, bending its resonance down to $f_P$ and providing parametric amplification. The signal is further amplified at the 3 K stage (Caltech Cryo1-12, 0.06 dB noise figure) and at room temperature (two Miteq AFS3-04000800-10-ULN amplifiers, 0.8 dB noise figure). Demodulation to baseband is provided by a generator at $f_P$, also used for readout and pump. Two phase shifters allow adjusting the relative phase between the three tones at $f_P$. The demodulated signal is split into three separate arms after amplification by a Stanford Research Systems SR445A. One arm stabilizes the JPA flux bias via an ADwin-GOLD processor programmed as a PID controller. In the second arm, the signal is filtered by a bias tee, amplified with a custom-built amplifier, and integrated and thresholded by the FPGA. The FPGA conditionally triggers a $\pi$ pulse from an AWG520 (Fig. 4). The third arm connects to an AlazarTech ATS9870 digitizer for data storage and processing after a second SR445A amplification stage. Red colour highlights the key components of the feedback loop.
Extended Data Figure 3 | Readout configuration for parity measurement and state tomography. a, Histograms for the computational basis for the parity measurement $M_P$ ($t_p = 300$ ns, $\bar{n}_i = 2.5$), as in Fig. 3a. At this measurement power, states within each parity subspace are largely indistinguishable (see also Fig. 1b and Extended Data Fig. 7). For an ideal parity measurement, $\beta_A = \beta_B = 0$. We extract $\beta_A = 0.0146$ mV, $\beta_B = -0.123$ mV, $\beta_{B/A} = -6.25$ mV and $\beta_0 = 7.46$ mV. b, Histograms for the tomography measurement (integration time 850 ns, $\bar{n}_i = 60$). At this power, the cavity response is nonlinear (critical photon number $n_{crit} < 60$), causing the resonance for $|10\rangle$ to bend towards lower frequency. As the resonance for $|01\rangle$ is instead power-independent, this effect discriminates $|01\rangle$ from the other states. This gives the joint readout the sensitivity to single and two-qubit terms required to perform state tomography. Averaging of raw tomography measurements yields $\beta_A = -8.10$ mV, $\beta_B = 9.10$ mV, $\beta_{B/A} = -12.8$ mV and $\beta_0 = 17.1$ mV. Digitizing the single shots with threshold $V_D = 32$ mV gives $\beta_A = 0.424$, $\beta_B = -0.360$, $\beta_{B/A} = 0.379$ and $\beta_0 = 0.540$.  

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Extended Data Figure 4 | Temporal evolution of two-qubit superposition state with and without continuous parity measurement. Comparison of the unconditioned two-qubit evolution during parity measurement (filled symbols, same data as in Fig. 2b) and during a delay of the same duration $t_P$ (open symbols). In the latter case, the decay of $|\rho_{ij,kl}|$ is solely due to intrinsic qubit decoherence. Evidently, measurement-induced dephasing dominates over intrinsic qubit dephasing. For reference, we estimate that entanglement ($C > 0$) would be achieved in the deterministic scheme provided the net qubit dephasing rate $1/T_2^A + 1/T_2^B < 1/0.4 \mu s^2$, under the experimental conditions of Fig. 4.
Extended Data Figure 5 | Two-qubit evolution under continuous parity measurement. Unconditioned and conditioned state tomography of the final two-qubit state similar to Figs 2 and 3, but at more values of \( \tau_P \) and using the threshold \( V_{th} \) optimizing parity readout fidelity (\( n_{th} = 2.5 \)). Middle row, unconditioned evolution. For \( \tau_P = 0 \), there is only a 10 ns buffer between state preparation and tomography, instead of the 350 ns used in Figs 2–4 and all other \( \tau_P \) values here. The uniformity of \( |\rho_{ijkl}| \) for \( \tau_P = 0 \) (<4% relative difference) attests to the preparation fidelity of the initial maximal superposition state. Top row, evolution conditioned on \( V_{int} > V_{th} \) (\( M_P = -1 \)); bottom row, evolution conditioned on \( V_{int} < V_{th} \) (\( M_P = +1 \)).
Extended Data Figure 6 | Two-qubit unconditioned evolution and conditioned concurrence for different measurement strengths. 

a–f, Experiment as in Figs 2 and 3 with measurement strength corresponding to \( \bar{n}_m = 0.6 \pm 0.1 \) (a, d), \( 1.4 \pm 0.1 \) (b, e) and \( 3.9 \pm 0.1 \) (c, f). The best-fit frequency mismatch \( (\chi_A - \chi_B) / \pi \) (see also Extended Data Fig. 3) is \( 182 \pm 32 \) kHz (a, d), \( 220 \pm 18 \) kHz (b, e) and \( 275 \pm 7 \) kHz (c, f). Concurrence is calculated after postselection on \( V_{int} < V_{th} \) \( (M_p = +1) \) or \( V_{int} > V_{th} \) \( (M_p = -1) \).
Extended Data Figure 7 | Cumulative histograms of parity measurements.
The four computational states are subjected to a parity measurement with $\tau_p = 300 \, \text{ns}$, $R_n = 2.5$, as in Fig. 3a. At the optimal threshold $V_{th}$ (dashed line), the average errors in determining the parity are $e_e = 0.13$, $e_o = 0.11$, yielding a parity measurement fidelity of $F_p = 1 - e_e - e_o = 0.76$ (corrected for residual qubit excitations, see Methods). In a similar manner, we define the distinguishability within each parity subspace as the fidelity of the measurement discriminating between those states, yielding 0.03 for the even subspace and 0.02 for the odd.
Extended Data Figure 8 | Frequency-dependent coherence times of Qb.

Energy relaxation times $T_B^1$ (filled circles) and $T_A^1$ (square) below the fundamental cavity resonance are consistent with the single-mode Purcell effect\textsuperscript{34} and a coupling strength $g/\pi = 167$ MHz at the Qb-cavity avoided crossing, as extracted from spectroscopy (Extended Data Fig. 1). We attribute the lower $T_B^1$ above the fundamental resonance to the effect of higher cavity modes. Pure dephasing times $T_Q^2$ (open circles) are in excellent agreement with the first-order approximation for flux noise\textsuperscript{35} with spectral density $S_f(v) = A^2/|f|$ and best-fit $A = (1.9 \pm 0.1) \times 10^{-7} \phi_0$ (dashed line), with $\phi_0$ the flux quantum.
Extended Data Figure 9 | Pulse timing and measurement integration windows. Extended view (not to scale) of the pulse sequence used in Figs 2–4, showing also the integration windows used for parity measurement ($t_{\text{int}}$ for signal and $t_{\text{off}}$ for offset) and tomographic joint readout. All specified time intervals are expressed in ns. Qubit control is performed with DRAG pulses\textsuperscript{36} with Gaussian envelopes on the main quadrature ($\sigma = 6$ ns, $4\sigma$ total duration) and derivative-of-Gaussian envelopes of optimized amplitude on the other. Single-qubit pulses are applied sequentially (Q\textsubscript{B} first), with 10 ns buffer between them. The tomography measurement pulse is 1 $\mu$s long, and the homodyne response integrated for 850 ns starting after the first 100 ns.
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