Equilibration and baryon densities attainable in heavy-ion collisions

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Kinetic equilibration of the matter and baryon densities attained in central region of colliding Au+Au nuclei in the energy range of \sqrt{s_{NN}} = 3.3–39 GeV are examined within the model of the three-fluid dynamics. It is found that the kinetic equilibration is faster at higher collision energies: the equilibration time (in the c.m. frame of colliding nuclei) rises from \sim 5 fm/c at \sqrt{s_{NN}} = 3.3 GeV to \sim 1 fm/c at 39 GeV. The chemical equilibration, and thus thermalization, takes longer. We argue that for informative comparison of predictions of different models it is useful to calculate an invariant 4-volume (\textit{V}_4), where the proper density the equilibrated matter exceeds certain value. The advantage of this 4-volume is that it does not depend on specific choice of the 3-volume in different studies and takes into account the lifetime of the high-density region, which also matters. The 4-volume \textit{V}_4 = 100 fm\textsuperscript{4}/c is chosen to compare the baryon densities attainable at different energies. It is found that the highest proper baryon density increases with the collision energy rise, from \textit{n}_B/\textit{n}_0 \approx 4 at 3.3 GeV to \textit{n}_B/\textit{n}_0 \approx 30 at 39 GeV. These highest densities are achieved in the central region of colliding system.

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I. INTRODUCTION

The main goal of high-energy heavy-ion research is to explore the properties of strongly interacting matter, particularly its phase structure. Initial and final stages of the heavy-ion collisions are non-equilibrium. The interest is mainly focused on properties of the equilibrated matter, which take place in the intermediate stage of the collisions and its evolution is frequently described by hydrodynamical models. One of the main questions is what energy and baryon densities can be accessed by means of heavy-ion collisions? At LHC and top RHIC collision energies systems with a very small net baryon density but rather high temperature are formed, while it is expected that the creation of the high baryon densities occurs at more moderate collision energies, such as those at the Nuclotron-based Ion Collider Facility (NICA) in Dubna \textsuperscript{1} and the Facility for Antiproton and Ion Research (FAIR) in Darmstadt \textsuperscript{2} under construction, at the low-energy end of the Relativistic Heavy Ion Collider (RHIC), i.e. the Beam Energy Scan (BES) program at RHIC, and at planned J-PARC-HI facility \textsuperscript{3}. In the present paper we are interested precisely in these moderate-energy heavy-ion collisions.

The question of highest attainable energy and baryon densities in the NICA-FAIR energy range was first addressed in Ref. \textsuperscript{4}, where predictions of different models where compared. However, the initial equilibration of the matter was not analyzed in Ref. \textsuperscript{4}. Besides, since the time of Ref. \textsuperscript{4} the models themselves were refined based on numerous experimental data from BES RHIC. Therefore, it is reasonable to repeat and extend the analysis of Ref. \textsuperscript{4}. Within the Ultra-relativistic Molecular Dynamics (UrQMD) model \textsuperscript{5, 6} and the Quark-Gluon String Model (QGSM) \textsuperscript{7, 8} this was done in Refs. \textsuperscript{9, 10} at NICA and FAIR energies. The question of highest attainable energy and baryon densities is closely related to degree of the baryon stopping in nuclear collisions, the discussion of which was recently resumed in Ref. \textsuperscript{11}.

In the present paper we present the analysis within the model of the three-fluid dynamics (3FD) \textsuperscript{12} in a wider, i.e. NICA/FAIR/BES-RHIC, energy range. Since the time of Ref. \textsuperscript{4} the 3FD model \textsuperscript{13} was supplemented by two equations of state (EoS) involving the deconfinement transition \textsuperscript{14}, i.e. a first-order phase transition and a smooth crossover one, which turned out to be the most successful in reproduction of various observables. Such kind of analysis has been already started in Refs. \textsuperscript{15, 16}. In the present paper we report a more quantitative results for the central region of the colliding system.

II. THE 3FD MODEL

The main part of the hydro models \textsuperscript{17-23}, that are designed for describing evolution of the baryon-rich matter, takes their initial conditions from third-party kinetic codes. Unlike those hybrid hydro models, the 3FD model \textsuperscript{12} takes into account finite stopping power of nuclear matter right within the 3FD evolution. The finite stopping power results in a counterstreaming regime of leading baryon-rich matter. This nonequilibrium regime is modeled by two interpenetrating baryon-rich fluids initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles are attributed to a fireball (f) fluid. Each of

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these fluids is governed by conventional hydrodynamic equations coupled by friction terms in the right-hand sides of the Euler equations. These friction terms describe energy–momentum loss of the baryon-rich fluids. A part of this loss is transformed into thermal excitation of these fluids, while another part gives rise to the particle production into the fireball fluid. Thus, the 3FD approximation is a minimal way to simulate the early-stage nonequilibrium at high collision energies. Similar concepts were used in recently developed hybrid models [20–23]. Unlike the 3FD, these hybrid models deal with a single equilibrated fluid that however does not involve all the matter of colliding nuclei. Therefore, this kind of hybrid hydrodynamics contains source thermics describing gain of the equilibrated matter in the course of the collision. This is similar to the production of the f-fluid in the 3FD.

The counterstreaming of the p and t fluids takes place only at the initial stage of the nuclear collision. At later stages the baryon-rich (p and t) fluids have already either partially passed through each other or partially stopped and unified in the central region. The f-fluid also is entrained by the unified baryon-rich fluid but is not that well unified with the latter, thus keeping its identity even after the initial unification of the baryon-rich fluids. In particular, the friction between the baryon-rich and net-baryon-free fluids is the only source of dissipation at the expansion stage.

The physical input of the present 3FD calculations is described in Ref. [13]. The 3FD simulations were performed with three different equations of state (EoS’s): a purely hadronic EoS [24] and two versions of the EoS with the deconfinement transition [14], i.e. the first-order phase transition (1PT) and crossover one. In the present paper we demonstrate results only with the 1PT and crossover EoS’s as the most successful in reproduction of various observables in the considered energy range.

III. MATTER EVOLUTION IN CENTRAL REGION OF COLLIDING NUCLEI

Figure 1 presents evolution of the matter in central region of colliding nuclei Au+Au at impact parameter b = 2 fm. Similarly to Ref. [4], the figure displays evolution of various quantities in the central box placed around the origin \( r = (0, 0, 0) \) in the frame of equal velocities of colliding nuclei: \( |x| \leq 2 \text{ fm}, |y| \leq 2 \text{ fm} \) and \( |z| \leq \gamma_{\text{cm}} \text{ 2 fm} \), where \( z \) is the direction of the beam and \( \gamma_{\text{cm}} \) is the Lorentz factor associated with the initial nuclear motion in the c.m. frame. At high collision energies, the Lorentz contraction can be so high that the central box of a fixed (rather than Lorentz contracted) longitudinal size may turn out to be half-empty. That would result in incorrect determination of densities in such a box. The size of the box is chosen to be large enough that the amount of matter in it can be representative to conclude on the medium properties and to be small enough to consider the matter in it as a homogeneous medium. The matter in the box still amounts to a minor part of the total matter of colliding nuclei.

One of the advantages of this central box is that the matter is at rest in it due to symmetry considerations. Therefore, the baryon and energy densities can be expressed as a direct sum of partial densities of partial densities of different fluids

\[
\begin{align*}
n_B &= n_p u_p^0 + n_t u_t^0 + n_f u_f^0, \\
\varepsilon &= \varepsilon_p u_p^0 + \varepsilon_t u_t^0 + \varepsilon_f u_f^0,
\end{align*}
\]

where \( n_\alpha \) and \( \varepsilon_\alpha \) are proper (i.e. in the local rest frame) baryon and energy densities of different fluids (\( \alpha = p, t, f \), respectively, and \( u_\alpha^0 \) stands for the 0-component of the hydrodynamic 4-velocity of the \( \alpha \)-fluid. Notice that \( n_f \equiv 0 \) by construction of the 3FD model, \( u_t^0 = 1 \) and \( u_p^0 = u_f^0 \) by definition of the central box.

Longitudinal (\( P_{\text{long}} \)) and transverse (\( P_{\text{tr}} \)) pressures

\[
\begin{align*}
P_{\text{long}} &= T_{zz}, \quad \text{(in the beam direction),} \\
P_{\text{tr}} &= (T_{xx} + T_{yy})/2
\end{align*}
\]

are defined in terms of the total energy–momentum tensor

\[
T^{\mu\nu} \equiv T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}
\]

being the sum of conventional hydrodynamical energy–momentum tensors of separate fluids

\[
T^{\mu\nu}_\alpha = (\varepsilon_\alpha + P_\alpha)u_\alpha^\mu u_\alpha^\nu + g^{\mu\nu} P_\alpha.
\]

The initial stage of the nuclear collision is nonequilibrium. This is manifested in the fact that the longitudinal (\( P_{\text{long}} \)) and transverse (\( P_{\text{tr}} \)) pressures are different, see upper panels in Fig. 1. Criterion of the equilibration, that we use in the present paper, is equality of longitudinal and transverse pressures with the accuracy no worth than 10%. Time instants, when the equilibration happens, are marked by star symbols on the curves in Fig. 1. As seen, the nonequilibrium stage lasts from \( \sim 5 \text{ fm}/c \) at \( \sqrt{s_{NN}} = 3.3 \text{ GeV} \) to \( \sim 1 \text{ fm}/c \) at collision energy of 39 GeV. This is somewhat shorter that reported in the previous study of Ref. [10]. The reason is that equilibration criterion in [10] was the equality of longitudinal and transverse pressures with the accuracy better than 10%, which differs from “no worth” in the present study. The \( P_{\text{long}} \) and \( P_{\text{tr}} \) values become equal with the accuracy \( \sim 10\% \) and remain at this level for some time. This is why the “no worth” criterion gives shorter thermalization times than the “better” criterion. In fact, the presently used “no worth” criterion is natural for the chosen size of the box. The matter in this box is not perfectly homogeneous, which results in a certain difference in the \( P_{\text{long}} \) and \( P_{\text{tr}} \) values. Note that similar \( \sim 10\% \) difference appear even at later stages of the expansion.

We consider only so-called kinetic equilibration, characterized the equality of longitudinal and transverse components of the pressure. Because of that we does not
FIG. 1: Time evolution of the longitudinal and transverse pressure, net baryon density, energy density, temperature, and fraction of the quark-gluon phase (QGP) in the central region of central (b = 2 fm) Au+Au collision at various collision energies (√s_{NN}). Left column corresponds to the crossover EoS, while the right one – to the 1PT EoS. Star symbols on the curves mark the time instant of the equilibration.
call it thermalization. The chemical equilibration takes longer time, if ever, because the f-fluid retains its identity even after the unification of the baryon-rich fluids. This is in agreement with the analysis within the UrQMD and QGSM models [9, 10], which indicates that the complete thermalization (including the chemical one) requires a longer time. The kinetic equilibration also take longer in the UrQMD and QGSM models.

\[ V_4(\tilde{n}_B) = \int d^4x \, \Theta(n_B(x) - \tilde{n}_B), \tag{7} \]

where \( \Theta(x) \) is the step function being equal 1 for \( x > 0 \) and 0 otherwise. The \( V_4(\tilde{n}_B) \) is an invariant measure of the space–time region, where the \( n_B \) value remains high. Note that the calculation of \( V_4 \) is not restricted by the central box described above. The \( V_4 \) values for different collision energies \( \sqrt{s_{NN}} \) are presented in Fig. 2. As seen, the \( V_4 \) values rapidly decrease with the rise of \( \tilde{n}_B \). We choose the level of \( V_4 = 100 \text{ fm}^4/c \) to compare the baryon densities attainable at different different energies. This 4-volume can be viewed, for example, as \( 5 \times 5 \times 2 \text{ fm}^3 \times 2 \text{ fm}^3/c \), i.e. consisting of quite reasonable 3-volume and time period. The baryon densities reached in the 4-volume not smaller than \( 100 \text{ fm}^4/c \) are displayed Fig. 3. As seen, a very high density is attained at the collision density of 39 GeV, this is not surprising because 70% of the baryon charge gets stopped in the central Au+Au collision at this energy, as it is argued in Refs. [15, 16].

The maximal densities achieved in collisions at NICA-FAIR energies are comparable with those found within the UrQMD and QGSM models [9] in a small central cell of \( 0.5 \times 0.5 \times 0.5 \text{ fm}^3 \) size while considerably higher than those [9] in a large central cell of \( 5 \times 5 \times 5 \text{ fm}^3 \). In contrast to the 3FD results, these highest values are achieved at the nonequilibrium stage in terms of Ref. [9], because the chemical equilibrium is still absent. Lower baryon densities are reported in Ref. Ref. [10] because the analysis was performed in a larger cell \( 2 \times 2 \times 2 \text{ fm}^3 \). Thus, results obtained within the same UrQMD model [9, 10] cannot be directly compared. Moreover, the observation that the
maximal baryon density does not rise in the large box of Ref. [9] and moderately large box of Ref. [10] may signal that a box of a fixed longitudinal size contain the main part of the colliding system (in longitudinal direction) at the stage of the maximal compression. Therefore, it is necessary to use the same central box (and better a Lorentz contracted one) for informative comparison of predictions of different models. It would be even better if the analysis is performed in terms of the invariant 4-volume described above.

For completeness, we also present the evolution of temperature in the fourth row of the columns in Fig. 1. It is displayed only from the moment of equilibration, i.e. when it has a physical sense. One should keep in mind that the temperature is a EoS-dependent quantity. In the regions, where the QGP dominates or vise versa, the hadronic phase prevails in both considered scenarios (see the bottom raw of panels in Fig. 1). The temperatures are also very similar for both EoS’s. If the phases are different, the temperatures are also different even in spite of very similar baryon and energy densities.

The evolution of the pressure and densities is very similar for the 1PT and crossover EoSs, as seen from Fig. 1 in spite of difference of these EoSs. It takes place because the friction forces in the QGP were independently fitted for each EoS [13] in order to reproduce observables. Therefore, the presented evolution predominantly reflects the dynamics of collisions necessary to explain the experimental data. The bottom raw of panels in Fig. 1 displays evolution of the fraction of the quark-gluon phase (QGP). This QGP-fraction evolution demonstrates that the considered 1PT and crossover EoSs are very different. The mixed phase is rapidly passed in the 1PT EoS while the QGP-fraction becomes close to unity only at the highest considered energies $\sqrt{s_{NN}} > 20$ GeV and for a quite short period of time.

IV. SUMMARY

We estimated the baryon densities of the equilibrated matter achievable in the central region of colliding Au+Au nuclei at NICA/FAIR/BES-RHIC collision energies. The analysis was performed within the 3FD model. We analyzed the kinetically equilibrated matter characterized by equality of the longitudinal and transverse components of the pressure. It is found that the kinetic equilibration is faster at high collision energies: the equilibration time (in the c.m. frame of colliding nuclei) decreases from $\sim 5$ fm/c at $\sqrt{s_{NN}} = 3.3$ GeV to $\sim 1$ fm/c at 39 GeV. The chemical equilibration, and thus thermalization, takes longer [9].

We argue that for informative comparison of predictions of different models it is useful to calculate a 4-volume, where the proper density of the equilibrated matter exceeds certain value. The advantage of this 4-volume is that it does not depend on specific choice of the 3-volume in different studies. Different choices of the 3-volume prevent us even from direct comparison or results obtained within the same model [9] [10]. In addition, the period of time during which a high density exists also matters. The 3-volume and the time period are non-invariant quantities, while the 4-volume is an invariant quantity.

This 4-volume is calculated as a function of a threshold density. To compare the baryon densities attainable at different different energies, we choose the level of $V_4 = 100$ fm$^4$/c. This 4-volume can be viewed as consisting of quite reasonable 3-volume and time period. It is found that the highest proper baryon density increases with the collision energy rise, from $n_B/n_0 \approx 4$ at 3.3 GeV to $n_B/n_0 \approx 30$ at 39 GeV. These highest densities are achieved in the central region of colliding system, as it is indicated by analysis of the dynamics of these collisions [15] [16].

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