Research Article

Estimation of Finite Population Mean in Simple and Stratified Random Sampling by Utilizing the Auxiliary, Ranks, and Square of the Auxiliary Information

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In this article, estimating the finite population means under simple random and stratified random sampling schemes. Our proposition is based on the notion of using auxiliary information in a more rigorous fashion. Specifically, we use ranks and squared values of the auxiliary information in addition to observed values of the auxiliary variable. The applicability of the proposed family of estimators is demonstrated by considering real data sets coming from diverse fields of applications. Moreover, the performance comparison is conducted with respect to a recently proposed family of estimators. The findings are encouraging and superior performance of the suggested family of estimators is witnessed and documented throughout the article.

1. Introduction

In this age of aggressive flow of information, the notion of using auxiliary information under the argument of maximum use of available information is well cherished. However, the applicability of supplementary information to enhance the efficiency of estimation procedures estimating the attributes of the population under study has a rich history in the multidisciplinary research literature. The advocacy of the utility of supportive information to assist the more elegant resolve of the estimation problem in hand can be tracked to Pierre–Simon Laplace—an eminent name of the eighteenth century academic circles. While trusted with the sensitive task of estimation of the total population of the eighteenth century France he advised “The register of births, which are kept with care in order to assure the condition of the citizens, can serve to determine the population of great empire without resorting a census of its inhabitants. But for this it is necessary to know the ratio of population to annual the birth.” see [1]. The legitimacy of the aforementioned abstract idea can be witnessed through streams of research, fundamentally aiming to advance the theoretical and methodological frontiers dealing with the incorporation of additional information. For example, the seminal work of [2] instigated the idea of exploiting the underlying correlation structure deriving both the study variable and auxiliary variable. Over the time, many researchers have paid tribute to the notable contribution of [2] by proposing
Section 6, we present the proposed family of estimators in the case of SRS. The performance evaluation is persuaded in Section 7, where general discussions are documented in Section 8.

2. Preliminaries with respect to SRS

2.1. Notation and Symbols. Let \( Z \) be a finite population of \( N \) units, such as \( Z = \{Z_1, Z_2, \ldots, Z_N\} \). We draw a sample of size \( n \) from the population through SRS without replacement (SRSWOR) scheme. Let \( Y_i \) and \( X_i \) are study and auxiliary variables, respectively. Moreover, let us denote ranks and squared values of auxiliary variable as \( R_i \) and \( U_{iy} \), respectively, for the \( ith (i = 1, 2, \ldots, N) \) unit of the population.

Let, \( \bar{Y} = 1/n \sum_{i=1}^{n} y_i \) and \( \bar{X} = 1/n \sum_{i=1}^{n} x_i \) are sample means of the study and auxiliary variable corresponding to the population means \( Y = 1/N \sum_{i=1}^{N} y_i \) and \( X = 1/N \sum_{i=1}^{N} X_i \), respectively. Similarly, let us define \( \bar{R} = 1/n \sum_{i=1}^{n} r_i \) as the sample mean of ranks of auxiliary variable and \( \bar{U} = 1/N \sum_{i=1}^{N} U_i \) as sample mean of squared values of auxiliary variable estimating the corresponding population attributes \( R = 1/N \sum_{i=1}^{N} R_i \) and \( U = 1/N \sum_{i=1}^{N} U_i \), respectively. On these grounds, sample variances of study and auxiliary variables are defined as \( s^2_Y = 1/n - 1 \sum_{i=1}^{n} (y_i - \bar{Y})^2 \) and \( s^2_X = 1/n - 1 \sum_{i=1}^{n} (x_i - \bar{X})^2 \), whereas sample variability of ranks is quantified as \( s^2_r = 1/n - 1 \sum_{i=1}^{n} (r_i - \bar{R})^2 \) and sample variance of squared values of the auxiliary variable is given as \( s^2_u = 1/n - 1 \sum_{i=1}^{n} (u_i - \bar{U})^2 \). Furthermore, let us define error coefficients of variation of \( X, Y, R \), and \( U \) as \( C_x, C_y, C_R, C_U \), respectively, where \( C_x = S_x/\bar{X}, C_y = S_y/\bar{X}, C_R = S_R/\bar{R} \) and \( C_U = S_U/\bar{U} \). We now define error terms as \( e_i = (\bar{Y} - Y)/\bar{Y}, e_1 = (\bar{X} - X)/\bar{X}, e_2 = (\bar{R} - R)/\bar{R}, e_3 = (\bar{U} - U)/\bar{U} \), such that \( E(e_i) = 0, i = 0, 1, 2, 3 \). 

\[
E(e_0) = \lambda C_y C_x \rho_{yx}, \quad E(e_0 e_2) = \lambda C_y C_x \rho_{yx}, \quad E(e_0 e_3) = \lambda C_y C_x \rho_{yx}, 
\]

\[
E(e_1 e_2) = \lambda C_x C_R \rho_{xr}, \quad E(e_1 e_3) = \lambda C_x C_R \rho_{xr}, \quad E(e_2 e_3) = \lambda C_R C_U \rho_{ru}, \quad \lambda = (1/n - 1/N), \quad \text{commonly known as sample fraction}.
\]

The performance evaluation reveals the superior performance of the proposed family in comparison to the \([5]\) family of estimators and thus outperforms the other noted estimators. In addition, our proposition accommodates \([5]\) family as a special case and thus seals the generality of our technique. The rest of the article is arranged in seven major parts. In Section 2, we present preliminaries with reference to Simple Random Sampling (SRS) along with \([5]\) proposed family of estimators. Section 3 is dedicated to the introduction of a proposed family of estimators, whereas the performance investigation is conducted in Section 4. Next, Section 5 documents the preliminaries when the Stratified Random Sampling (SRS) scheme was employed along with the extensions of \([5]\) proposed family to incorporate the stratification existent in the population under study. In Section 6, we present the proposed family of estimators in the case of SRS. The performance evaluation is persuaded in

useful amendments into the original doctrine. For example, \([3]\) proposed the expression for product estimator capitalizing on the exploitation of the negative degree of correlation prevalent between the study variable and the supportive variable. In processional, \([4]\) provided the extensions of the classic ratio estimator and product estimator, namely, ratio-type exponential estimator and product-type exponential estimator, respectively. Yet another domain facilitating the incorporation of additional information in estimation procedure was motivated by the use of more profound functional forms known for producing estimators with minimal standard errors. Under the motivation, \([5]\) proceeded by formulating a generalized family of exponential-based estimators encompassing numerous existing main stream estimators as members of the resultant class. For a based estimators encompassing numerous existing main

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where \( \omega_1, \omega_2, \omega_3 \) are unknown quantities minimizing the MSE of the proposed estimator. The optimal values are simplified as under,

\[
\begin{align*}
\omega_1(\text{opt}) &= \frac{8 - \lambda \theta^2 C_x^2}{8 \left[ 1 + \lambda C_y^2 (1 - \phi_{yx}^2) \right]^2}, \\
\omega_2(\text{opt}) &= \frac{Y \left[ \lambda \delta C_x \left( 1 + \rho_{yx}^2 \right) + (-8C_y + \lambda \theta^2 C_y^2) \left( \rho_{yx} - \rho_{yx} \rho_{yx} \right) \right]}{8XC_x \left( 1 + \rho_{yx}^2 \right) \left[ 1 + \lambda C_y^2 (1 - \phi_{yx}^2) \right]}, \\
\omega_3(\text{opt}) &= \frac{Y (8 - \lambda \theta^2 C_y^2) C_y \left( \rho_{yx} \rho_{yx} - \rho_{yx} \right)}{8RC_x \left( 1 + \rho_{yx}^2 \right) \left[ 1 + \lambda C_y^2 (1 - \phi_{yx}^2) \right]},
\end{align*}
\]

where \( \phi_{yx}^2 = \rho_{yx}^2 + \rho_{yx}^2 + 2 \rho_{yx} \rho_{yx} / 1 - \rho_{yx}^2 \) is the coefficient of multiple determination of \( Y \) on \( X \) and \( R_y \).

Table 1 below comprehends the members of [9] family corresponding to various values of \( a \) and \( b \). Reference [9] provided the expressions of bias and MSE of the family of the estimator as follows:

\[
MSE_{\text{min}} \left( \hat{Y}_{\text{Hag}} \right) = \frac{\lambda \theta^2 \left[ 64C_y^2 (1 - \phi_{yx}^2) - \lambda \theta^2 C_y^2 - 16 \lambda \theta^2 C_y^2 C_y^2 (1 - \phi_{yx}^2) \right]}{64 \left[ 1 + \lambda C_y^2 (1 - \phi_{yx}^2) \right]},
\]

respectively, where \( \theta = aX/aX + b \).

### 3. Proposed Family of Estimators

We now proceed by proposing a new family of estimators based on a more rigorous use of auxiliary information. The general expression of the proposed estimator is as follows:

\[
\hat{Y}_k = \left\{ \kappa_1 \bar{Y} + \kappa_2 (\bar{X} - \bar{X}) + \kappa_3 (\bar{R} - \bar{R}) + \kappa_4 (\bar{U} - \bar{U}) \right\} \\
\exp \left( \frac{-a (\bar{X} - \bar{X})}{a (\bar{X} + \bar{X}) + 2b} \right).
\]

Where \( \kappa_1, \kappa_2, \kappa_3, \) and \( \kappa_4 \) are unknown constants whose values are decided by minimizing the MSE of the proposed family of estimator, given in equation (6). Moreover, similar to that of [9], \( a \) and \( b \) can take varying values and thus provide different members of our proposed family of estimators. Table 2 presents various values of \( a \) and \( b \) and resultant estimators. Under the notion of fair comparison, we consider the same values of \( a \) and \( b \) as those of [9]. Next, we provide the calculations for bias and MSE of our proposition. By using error terms defined in Section 2.1, it is verifiable that the proposed estimator given in equation (6) is re writable as follows:

\[
\hat{Y}_k = \left\{ Y \kappa_1 + Y e_{1} \kappa_1 - X e_{1} \kappa_2 - Re_{2} \kappa_3 - U e_{3} \kappa_4 \right\} \\
\left\{ 1 - \frac{\delta e_{1}}{2} + \frac{3 \delta^2 e_{1}^2}{8} + \cdots \right\}.
\]

On further solving and keeping terms with second degree of \( e_{s} \), we obtain the following equation:

\[
\left( \bar{Y}_k - \bar{Y} \right) = \bar{Y} (\kappa_1 - 1) + \bar{X} e_{1} \kappa_1 - \bar{X} e_{1} \kappa_2 - \bar{R} e_{2} \kappa_3 \\
- \bar{U} e_{3} \kappa_4 - \frac{1}{2} \delta \bar{Y} e_{1} \kappa_1 \\
+ \frac{3}{8} \delta^2 \bar{Y} e_{1} \kappa_1 - \frac{1}{2} \delta \bar{X} e_{1} \kappa_1 \\
+ \frac{1}{2} \delta \bar{U} e_{1} \kappa_4.
\]
Table 1: Members of the [9] family of estimators.

| $a$ | $B$ | $\bar{\Gamma}_{\text{maq}}$ |
|-----|-----|-----------------|
| 1   | $C_x$ | $\bar{\Gamma}_{\text{maq}}^{(1)}$ |
| 1   | $\beta_1(x)$ | $\bar{\Gamma}_{\text{maq}}^{(2)}$ |
| $\beta_2(x)$ | $C_x$ | $\bar{\Gamma}_{\text{maq}}^{(3)}$ |
| $C_x$ | $\beta_2(x)$ | $\bar{\Gamma}_{\text{maq}}^{(4)}$ |
| 1   | $\rho_{yx}$ | $\bar{\Gamma}_{\text{maq}}^{(5)}$ |
| $\beta_2(x)$ | $C_x$ | $\bar{\Gamma}_{\text{maq}}^{(6)}$ |
| $\rho_{yx}$ | $\beta_2(x)$ | $\bar{\Gamma}_{\text{maq}}^{(7)}$ |
| 1   | $N \bar{X}$ | $\bar{\Gamma}_{\text{maq}}^{(10)}$ |

Table 2: Members of the suggested family of estimators.

| $a$ | $b$ | $\bar{F}_K$ |
|-----|-----|---------------|
| 1   | $C_x$ | $\bar{F}_K^{(1)}$ |
| 1   | $\beta_2(x)$ | $\bar{F}_K^{(2)}$ |
| $\beta_2(x)$ | $C_x$ | $\bar{F}_K^{(3)}$ |
| $C_x$ | $\beta_2(x)$ | $\bar{F}_K^{(4)}$ |
| 1   | $\rho_{yx}$ | $\bar{F}_K^{(5)}$ |
| $\beta_2(x)$ | $C_x$ | $\bar{F}_K^{(6)}$ |
| $\rho_{yx}$ | $\beta_2(x)$ | $\bar{F}_K^{(7)}$ |
| 1   | $N \bar{X}$ | $\bar{F}_K^{(10)}$ |

By employing the expectation operator on both sides of equation (8), we attain the expression for bias as follows:

$$\text{Bias}(\bar{Y}_K) = \bar{Y}(k_1 - 1) + \frac{3}{8} \lambda \delta^2 Y_1 C_x^2$$

$$- \frac{1}{2} \lambda \delta Y C_x C_x \rho_{yx} + \frac{1}{2} \lambda \delta \bar{Y} C_x C_x k_2$$

$$+ \frac{1}{2} \lambda \delta Y C_x C_x \rho_{yx} + \frac{1}{2} \lambda \delta Y C_x C_x k_2 \rho_{yx}$$

$$+ \frac{1}{2} \lambda \delta Y C_x C_x \rho_{yx} + \frac{1}{2} \lambda \delta Y C_x C_x k_2 \rho_{yx}$$

The MSE of the proposed family of estimators is obtained by taking the expectation of the square of the equation (8). We obtain MSE as follows:

$$\text{MSE}(\bar{Y}_K) = \bar{Y}^2 - 2 \bar{Y}^2 k_1 + \bar{Y}^2 k_1^2 + \lambda \delta^2 Y^2 k_1^2 C_x^2 - \frac{3}{4} \lambda \delta^2 Y^2 C_x^2 k_1 + \lambda \bar{Y}^2 C_x^2 k_2 + \lambda \bar{Y}^2 C_x^2 k_2$$

$$+ \lambda \bar{X}^2 C_x^2 k_2 + \lambda \bar{U}^2 C_x^2 k_2 + 2 \lambda \bar{X} \bar{R} C_x C_x k_2 \rho_{xy} + 2 \lambda \bar{X} \bar{U} C_x C_x k_2 \rho_{xy}$$

$$+ 2 \lambda \bar{X} \bar{R} C_x C_x k_2 \rho_{xy} + 2 \lambda \bar{Y} \bar{R} C_x C_x k_2 \rho_{xy} + 2 \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy}$$

$$- 2 \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy} - 2 \lambda \bar{Y} \bar{R} C_y C_x k_2 \rho_{xy} - 2 \lambda \bar{Y} \bar{U} C_y C_x k_2 \rho_{xy}$$

$$+ \lambda \bar{Y} \bar{R} C_x C_x k_2 \rho_{xy} - \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy} + 2 \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy}$$

$$+ \lambda \bar{Y} \bar{R} C_x C_x k_2 \rho_{xy} - \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy} + 2 \lambda \bar{Y} \bar{U} C_x C_x k_2 \rho_{xy}.$$
The optimal values of $\kappa_1$, $\kappa_2$, $\kappa_3$ and $\kappa_4$ are found by minimizing equation (10) and are given as follows:

\[
\kappa_{1(\text{opt})} = \frac{(v_1 - 1)(\lambda \theta^2 C_x^2 - 8)}{8\Lambda C_y^2(v_3 + v_4 + v_2 + 1) + 1 + v_1},
\]

\[
\kappa_{2(\text{opt})} = \frac{-4Y(-1/4\theta^2 \Lambda^2 C_x^2(v_1 - 1) - 1/4\theta^2 C_y C_x^2 v_5 + \lambda C_y^2(v_3 + v_4 - v_2 + 1))}{+8C_x + 2C_y v_3},
\]

\[
\kappa_{3(\text{opt})} = \frac{-\gamma C_y v_6(\lambda \theta^2 C_x^2 - 8)}{8RC_y(\Lambda C_y^2(v_3 + v_4 + v_2 + 1) + 1 + v_1)},
\]

\[
\kappa_{4(\text{opt})} = \frac{-\gamma C_y v_5(\lambda \theta^2 C_x^2 - 8)}{8RC_y(\Lambda C_y^2(v_3 + v_4 + v_2 + 1) + 1 + v_1)},
\]

where

\[
v_1 = \rho_{x^2} + \rho_{xu} + \rho_{ru} - 2\rho_{xu}\rho_{ru},
\]

\[
v_2 = \rho_{y_x} + \rho_{y^2} + \rho_{yu} + 2\rho_{yu}\rho_{yru},
\]

\[
v_3 = (\rho_{yx}^2 - 1)\rho_{x^2} + 2\rho_{xr}(\rho_{yx}\rho_{yu} + \rho_{xu}) + 2\rho_{yu}\rho_{ru}(\rho_{yx}\rho_{yu} + \rho_{xu}),
\]

\[
v_4 = (\rho_{yu}^2 - 1)\rho_{x^2} - 2\rho_{yu}\rho_{ru}(\rho_{yu}\rho_{xu} - \rho_{yx}) + (\rho_{yu}^2 - 1)\rho_{xu}^2,
\]

\[
v_5 = -\rho_{yu}\rho_{ru}^2 + (\rho_{yu}\rho_{xr} + \rho_{yru}\rho_{xu})\rho_{ru} - \rho_{yu}\rho_{ru} - \rho_{yu}\rho_{ru} + \rho_{yu},
\]

\[
v_6 = \rho_{yu}\rho_{ru}\rho_{xu} + \rho_{yu}\rho_{xr}\rho_{xu} - \rho_{yu}\rho_{ru}^2 - \rho_{yu}\rho_{ru} - \rho_{yu}\rho_{xu} + \rho_{yu},
\]

\[
v_7 = \rho_{yu}\rho_{ru}\rho_{xu} - \rho_{yu}\rho_{ru}^2 + \rho_{yu}\rho_{xr}\rho_{xu} - \rho_{yu}\rho_{ru} - \rho_{yu}\rho_{ru} + \rho_{yu}.
\]

The minimum MSE of $\hat{Y}_k$ is achieved by substituting optimal values of $\kappa_1$, $\kappa_2$, $\kappa_3$ and $\kappa_4$ is given by the following equation:

\[
\text{MSE}_{\text{min}}(\hat{Y}_k) = \frac{\lambda \gamma^2(\rho_{x^2} + \rho_{yu} - \rho_{ru} + 1)(\lambda \theta^2 C_x^2 - 4)C_y^2 - 1/16\lambda \theta^4 C_x^2 (v_1 - 1)}}{4(v_1 - 1 - \lambda C_y^2(v_3 + v_4 - v_2 + 1))}.
\]

4. Performance Comparison

This section is dedicated to evaluate and compare the performance of the proposed family of estimators relative to [9] family of estimators. To show the superior performance of the proposed family of estimators with respect to [9] family numerically, we need to show that $\text{MSE}_{\text{min}}(\hat{Y}_{\text{proposed}}) - \text{MSE}_{\text{min}}(\hat{Y}_k) > 0$. By comparing MSEs
given in equations (5) and (11), we get a general expression providing the condition for superior performance of the proposed family, as follows:

\[
\left[64\lambda^2 C_{y}^3 (1 - Q_{yx}^2) - 16\lambda^2 C_{x}^2 (1 - Q_{yx}^2)\right] \\
\left[v_1 - 1 - \lambda C_{y}^2 (v_3 + v_4 - v_2 + 1)\right] \\
\left[-\left(v_3 + v_4 - v_2 + 1\right) (\lambda^2 C_{x}^2 - 4) C_{y}^2 - \frac{1}{16} \lambda^2 C_{x}^2 (v_1 - 1)\right]
\]

> 0.

(14)

In the next procession, we empirically quantify the performance of all members of our proposed family (Table 2) by considering one by one comparison with members of [9] family (Table 1).

4.1. Evaluating Empirically. The empirical performance investigation is performed by using three diverse and commonly used following data sets. Reference [9] also considered the same data sets to delineate the applicability of their proposed family.

4.1.1. Dataset 1: [3]. \( y \): Output of the factory and \( x \): Number of workers.

\[\begin{align*}
N &= 80, \quad n = 10, \quad Y = 5182.64, \quad X = 285.125, \quad R = 40.5, \\
U &= 153514.2, \quad \rho_{yx} = 0.914981, \quad \rho_{yr} = 0.983609, \\
\rho_{xu} &= 0.805818, \quad \rho_{ru} = 0.890219, \quad \rho_{ru} = 0.961144, \\
\rho_{rr} &= 0.758839, \quad \rho_{ru} = 0.354194, \quad C_r = 0.948459, \\
C_x &= 0.573765, \quad C_u = 1.673663, \quad \beta_{2(x)} = 3.58078.
\end{align*}\]

4.1.2. Dataset 2: [12]. \( y \): Estimated number of fish caught by marine recreational fishermen in the year 1995 and \( x \): Estimated number of fish caught by marine recreational fishermen in the year 1994.

\[\begin{align*}
N &= 69, \quad n = 10, \quad Y = 4514.9, \quad X = 4954.43, \quad R = 35, \\
U &= 7366, \quad \rho_{yx} = 0.96014, \quad \rho_{yr} = 0.768859, \quad \rho_{ru} = 0.855499, \\
\rho_{ru} &= 0.754346, \quad \rho_{ru} = 0.982814, \\
C_x &= 0.520899, \quad C_y = 1.35089, \quad C_x = 1.42478, \quad C_r = 0.573212, \\
C_u &= 2.86356, \quad \beta_{2(x)} = 9.85055.
\end{align*}\]

4.1.3. Dataset 3: [12]. \( y \): Approximate duration of sleep (in minutes) of persons with age more than 50 years and \( x \): Corresponding age of persons in years.

\[\begin{align*}
N &= 30, \quad n = 5, \quad Y = 384.20, \quad X = 67.2667, \quad R = 15.5, \\
U &= 4607.2, \quad \rho_{yx} = -0.855241, \quad \rho_{yr} = -0.839446, \quad \rho_{ru} = -0.8546303, \\
\rho_{xx} &= 0.988995, \quad \rho_{ru} = 0.9776999, \quad C_x = 0.15579, \quad C_r = 0.13725, \\
C_u &= 0.567456, \quad C_{2(x)} = 0.276536, \quad \beta_{2(x)} = 2.238933.
\end{align*}\]

Table 3 comprehends the performance comparison of ten members of both families presented in Tables 1 and 2. We offer percentage relative efficiencies (PREs) of each member of our family and [9] family with respect to SRS along with PREs with respect to each other. The superior performance of our proposed family is self-evident in Table 3. As log as SRS is concerned, every member of both families outperform the usual estimation strategy. In the case of comparison between both families, the resulting PREs reveal a better performance of our proposed method than [9] family. These findings are consistent for all three populations and all members of respective families.

5. Preliminaries with respect to StRS

Next, we demonstrate the applicability of our proposed method in the estimation of finite population mean when the sample is drawn through the StRS scheme.

5.1. Notation and Symbols. Let us say \( Z \) be a finite population of distinct units of size \( N \), such that \( Z = \{Z_1, Z_2, \ldots, Z_N\} \). Further, let us assume that the population consist of \( L \) homogeneous partitions (starta), each of size \( N_h \), where \( h = \{1, 2, \ldots, L\} \), such that \( \sum_{h=1}^{L} N_h = N \). For the purpose of consistency, we define \( Y, X, R \), and \( U \) as the study variable, auxiliary variable, ranks and squared values of the auxiliary variable taking values \( Y_{ih}, X_{ih}, R_{ih}, \) and \( U_{ih} \), respectively, on the \( i \)th unit belongs to the \( h \)th stratum, where \( i = \{1, 2, \ldots, N_h\} \). Thus, \( W_h = N_h/N \) stays as the weight of \( h \)th stratum. We then draw a sample of size \( nh \) from the \( h \)th stratum using the SRSWOR scheme for the estimation of population mean ensuring that the total sample size \( n = \sum_{h=1}^{L} n_h \).

We now define the population mean of study variable as \( \bar{Y}_{st} = \bar{Y} = \sum_{h=1}^{L} W_{ih} \bar{Y}_{h} \) where population mean of \( Y \) for \( h \)th stratum is \( \bar{Y}_{h} = \sum_{h=1}^{L} Y_{ih}/N_{h} \). Similarly, \( \bar{X}_{st} = \bar{X} = \sum_{h=1}^{L} W_{ih} \bar{X}_{h} \) and \( \bar{X}_{h} = \sum_{h=1}^{L} X_{ih}/N_{h} \) are the population mean of auxiliary variable and population mean of auxiliary in \( h \)th stratum, respectively. Furthermore, \( \bar{R}_{st} = \bar{R} = \sum_{h=1}^{L} W_{ih} \bar{R}_{h} \) and \( \bar{R}_{h} = \sum_{h=1}^{L} R_{ih}/N_{h} \) represent the population mean of ranks and mean of ranks of \( h \)th stratum along with \( \bar{U}_{st} = \bar{U} = \sum_{h=1}^{L} W_{ih} \bar{U}_{h} \) and \( \bar{U}_{h} = \sum_{h=1}^{L} U_{ih}/N_{h} \) define as the population mean of squared values of auxiliary and population mean of squared values in \( h \)th stratum, respectively. Their corresponding sample estimate are given as \( \bar{Y}_{st} = \bar{Y} = \sum_{h=1}^{L} W_{ih} \bar{Y}_{h} \), \( \bar{X}_{st} = \bar{X} = \sum_{h=1}^{L} W_{ih} \bar{X}_{h} \), \( \bar{X}_{h} = \sum_{h=1}^{L} X_{ih}/n_{h} \), \( \bar{R}_{st} = \bar{R} = \sum_{h=1}^{L} W_{ih} \bar{R}_{h} \), \( \bar{R}_{h} = \sum_{h=1}^{L} R_{ih}/n_{h} \), \( \bar{U}_{st} = \bar{U} = \sum_{h=1}^{L} W_{ih} \bar{U}_{h} \) and \( \bar{U}_{h} = \sum_{h=1}^{L} U_{ih}/n_{h} \). Next, we define expression of population variances within stratum such that
Table 3: The PREs of estimators for different choices of \( a \) and \( b \).

| Estimator | \( \hat{Y}_{SRS-Haq} \) | \( \hat{Y}_{SRS-K} \) | \( \hat{Y}_{SRS-Haq} \) | \( \hat{Y}_{SRS-K} \) | \( \hat{Y}_{SRS-Haq} \) | \( \hat{Y}_{SRS-K} \) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \hat{\gamma}_{1} \) | 6307.63 | 6674.07 | 1502.54 | 1608.53 | 375.70 | 377.26 |
| \( \hat{\gamma}_{2} \) | 6182.12 | 6533.81 | 1501.84 | 1607.74 | 375.68 | 377.15 |
| \( \hat{\gamma}_{3} \) | 6342.03 | 6712.51 | 1502.65 | 1608.65 | 375.70 | 377.17 |
| \( \hat{\gamma}_{4} \) | 6173.26 | 6523.92 | 1502.08 | 1608.01 | 375.57 | 377.04 |
| \( \hat{\gamma}_{5} \) | 6309.30 | 6675.89 | 1502.58 | 1608.57 | 375.72 | 377.18 |
| \( \hat{\gamma}_{6} \) | 6306.82 | 6673.13 | 1502.61 | 1608.60 | 375.79 | 377.26 |
| \( \hat{\gamma}_{7} \) | 6303.25 | 6669.13 | 1502.54 | 1608.52 | 375.71 | 377.17 |
| \( \hat{\gamma}_{8} \) | 6342.51 | 6713.06 | 1502.65 | 1608.65 | 375.71 | 377.18 |
| \( \hat{\gamma}_{9} \) | 6167.00 | 6516.94 | 1501.80 | 1607.60 | 375.74 | 377.21 |
| \( \hat{\gamma}_{10} \) | 3993.05 | 4139.73 | 1375.80 | 1486.36 | 375.36 | 377.82 |

\[
\begin{align*}
S_{Y_{\gamma_1}} &= \frac{1}{h-1} \left( \frac{(Y_{\gamma_1} - \bar{Y})^2}{(N_h - 1)} \right), \\
S_{X_{\gamma_1}} &= \frac{1}{h-1} \left( \frac{(X_{\gamma_1} - \bar{X})^2}{(N_h - 1)} \right), \\
S_{U_{\gamma_1}} &= \frac{1}{h-1} \left( \frac{(U_{\gamma_1} - \bar{U})^2}{(N_h - 1)} \right), \\
S_{\rho_{XY}} &= \frac{1}{h} \left( \frac{(X_{\gamma_1} - \bar{X}) (Y_{\gamma_1} - \bar{Y})}{(N_h - 1)} \right).
\end{align*}
\]

where covariances are defined as follows:

\[
\begin{align*}
S_{Y_{\gamma_1}X_{\gamma_1}} &= \frac{1}{h} \left( \frac{(Y_{\gamma_1} - \bar{Y}) (X_{\gamma_1} - \bar{X})}{(N_h - 1)} \right), \\
S_{Y_{\gamma_1}U_{\gamma_1}} &= \frac{1}{h} \left( \frac{(Y_{\gamma_1} - \bar{Y}) (U_{\gamma_1} - \bar{U})}{(N_h - 1)} \right), \\
S_{X_{\gamma_1}U_{\gamma_1}} &= \frac{1}{h} \left( \frac{(X_{\gamma_1} - \bar{X}) (U_{\gamma_1} - \bar{U})}{(N_h - 1)} \right), \\
S_{\rho_{XY}} &= \frac{1}{h} \left( \frac{(X_{\gamma_1} - \bar{X}) (Y_{\gamma_1} - \bar{Y})}{(N_h - 1)} \right).
\end{align*}
\]

Based on above-provided expressions, we now provide correlation coefficients when a stratified sampling scheme is used, such as

\[
\rho_{XY(a)} = \frac{\sum_{h=1}^{L} W_{h}^2 \lambda_{p_{x_{1}}x_{1}} S_{Y_{h}} S_{X_{h}}}{\sqrt{\sum_{h=1}^{L} W_{h}^2 \lambda_{S_{Y_{h}}S_{Y_{h}}}}} 
\]

\[
\rho_{YX(b)} = \frac{\sum_{h=1}^{L} W_{h}^2 \lambda_{p_{x_{1}}x_{1}} S_{X_{h}} S_{Y_{h}}}{\sqrt{\sum_{h=1}^{L} W_{h}^2 \lambda_{S_{X_{h}}S_{X_{h}}}}} 
\]

\[
\rho_{YU(c)} = \frac{\sum_{h=1}^{L} W_{h}^2 \lambda_{p_{x_{1}}x_{1}} S_{Y_{h}} S_{U_{h}}}{\sqrt{\sum_{h=1}^{L} W_{h}^2 \lambda_{S_{Y_{h}}S_{Y_{h}}}}} 
\]

\[
\rho_{UX(d)} = \frac{\sum_{h=1}^{L} W_{h}^2 \lambda_{p_{x_{1}}x_{1}} S_{U_{h}} S_{X_{h}}}{\sqrt{\sum_{h=1}^{L} W_{h}^2 \lambda_{S_{U_{h}}S_{U_{h}}}}} 
\]

where \( \rho_{YX} = S_{Y_{\gamma_1}X_{\gamma_1}} / S_{Y_{\gamma_1}} S_{X_{\gamma_1}}, \rho_{YU} = S_{Y_{\gamma_1}U_{\gamma_1}} / S_{Y_{\gamma_1}} S_{U_{\gamma_1}}, \rho_{UX} = S_{U_{\gamma_1}X_{\gamma_1}} / S_{U_{\gamma_1}} S_{X_{\gamma_1}}, \rho_{XY} = S_{X_{\gamma_1}Y_{\gamma_1}} / S_{X_{\gamma_1}} S_{Y_{\gamma_1}} \) and \( \rho_{RU} = S_{R_{\gamma_1}U_{\gamma_1}} / S_{R_{\gamma_1}} S_{U_{\gamma_1}} \).
\[
E(e_1e_3) = \sum_{t=1}^{L} W_{t}^{2} \lambda_{t} p_{t} x_{t} u_{t} / x_{t} u_{t} / x_{t} u_{t} / x_{t} u_{t} = V_{0101} \quad \text{and} \quad E(e_2e_3) = \sum_{t=1}^{L} W_{t}^{2} \lambda_{t} p_{t} x_{t} u_{t} / x_{t} u_{t} / x_{t} u_{t} = V_{0011}.
\]
The above-mentioned expected values of errors can generally be written as follows:
\[
V_{\text{opt}} = \sum_{t=1}^{L} W_{t}^{2} \lambda_{t} p_{t} x_{t} u_{t} / x_{t} u_{t} / x_{t} u_{t} = V_{0011}.
\]

5.2. Extending the [9] Family under StRS Scheme. We proceed by deriving a general expression of [9] proposition when the StRS method of sampling is under consideration such as,
\[
\bar{y}_{\text{Hog}} = \left\{ \frac{w_{1} (\bar{y}_{st} - \bar{x}) + w_{2} (\bar{X} - \bar{x}) + w_{3} (\bar{R} - \bar{r})}{\exp \left( \frac{a (\bar{X} - \bar{x})}{a (\bar{X} + \bar{x}) + 2b} \right)} \right\},
\]

where \(w_{1}, w_{2}, \) and \(w_{3}\) are unknown constants subject to the constraint of minimizing MSE. We drive the optimal values of \(w_{s}\) as follows:

\[
\begin{align*}
\text{u}_{1(\text{opt})} &= \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0200}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]}, \\
\text{u}_{2(\text{opt})} &= \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0110}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]}, \\
\text{u}_{3(\text{opt})} &= \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0110}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]}. \\
\end{align*}
\]

Furthermore, the bias and MSE of [9] family is derived as follows:
\[
\text{Bias} \left( \bar{y}_{\text{Hog}}^{*} \right) = \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0110}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]},
\]
\[
\text{MSE}_{\text{min}} \left( \bar{y}_{\text{Hog}}^{*} \right) = \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0110}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]},
\]
\[
\begin{align*}
\text{MSE}_{\text{min}} \left( \bar{y}_{\text{Hog}}^{*} \right) &= \frac{8X}{\left[ -V_{0200} V_{1010} + 2V_{0110} V_{1010} V_{1100} - V_{0110}^{2} (1 + V_{2000}) + V_{0020} \left( -V_{1100}^{2} + V_{0200} (1 + V_{2000}) \right) \right]}, \\
\end{align*}
\]

respectively.

Table 4 offers all members of [9] family extended to compensate the StRS scheme.

6. Proposed Family of Estimators for StRS

In this section, we proposed an extended version of our suggested family of estimators (equation (6)) to efficiently accommodate the underlying homogeneous structure prevalent in the population under study. The general estimator is given as follows:
\[
\begin{align*}
\bar{y}_{k}^{*} &= \left\{ \kappa_{1} \bar{y}_{st} + \kappa_{2} (\bar{X} - \bar{x}) + \kappa_{3} (\bar{R} - \bar{r}) + \kappa_{4} (\bar{U} - \bar{u}) \right\}, \\
\exp \left( \frac{a (\bar{X} - \bar{x})}{a (\bar{X} + \bar{x}) + 2b} \right),
\end{align*}
\]

where \(\kappa_{1}, \kappa_{2}, \kappa_{3}, \) and \(\kappa_{4}\) are unknown constants minimizing the MSE of the proposed family. To calculate the bias while
keeping $e'$ s up till the second degree, we obtain the following equation:

$$
\left( \widehat{f}_k - f \right) = \frac{3}{8} \gamma \left( k_1 - 1 \right) + \frac{1}{2} \beta \gamma e_1 k_1
$$

(23)

On further solving, the bias is calculated as follows:

$$
\text{Bias} \left( \widehat{f}_k \right) = \frac{1}{2} \beta \gamma V_{1000} k_1 + \frac{1}{2} \gamma \gamma V_{0020} k_2.
$$

(24)

The MSE is deduced by squaring and taking expectation on both sides of equation (23). We obtain the following equation:

$$
\text{MSE} \left( \widehat{f}_k \right) = \frac{3}{4} \gamma \gamma V_{0020} k_1 + \frac{3}{4} \gamma \gamma V_{0200} k_2 + \frac{3}{4} \gamma \gamma V_{2000} k_3
$$

(25)

The optimal values of $k_1$, $k_2$, $k_3$, and $k_4$ can be determined as follows:
respectively, where

$$J_1 = (-V_{0002}V_{0001} + V_{0002}^2), \quad J_2 = (V_{0002}V_{0010} + V_{0002}V_{0100} - 2V_{0010}V_{0100}V_{0010}),$$

$$J_3 = 16(V_{2000}V_{0001} + V_{0002}V_{0100} - 2V_{0100}V_{0001} - V_{2000}V_{0002}V_{0010}),$$

$$J_4 = (V_{2000}V_{0002} - V_{0010}^2), \quad J_5 = (-V_{2000}V_{0001} + V_{0011}V_{0100}), \quad J_6 = (-V_{0002}V_{0100} + V_{0011}V_{0011}),$$

$$J_7 = (V_{2000}V_{0002} - V_{0100}^2), \quad J_8 = J_6 = (-V_{0002}V_{0100} + V_{1010}V_{0001}), \quad J_9 = (V_{2000} + 1).$$

After performing some simplification we attain the expression of MSE such that,

$$\text{MSE}_{\text{min}}(\hat{Y}_k) = \frac{\bar{Y}_k^2}{64} \left[ \frac{\delta^3J_1V_{0200} + \delta^3V_{0200}(\delta^3J_2 + 16J_3) + G_1 - 64J_4V_{0110} + 128V_{0110}(J_5V_{0101} - J_6V_{1100})}{-J_3V_{0002}V_{0100}^2 - J_4V_{0110}^2 - 2V_{0001}V_{0200}V_{1010}V_{0100} - G_3 - V_{0002}V_{0002}V_{1100}} \right]$$
where

\[
G_1 = \left( -2J_3 V_{0011} + 2V_{1010} V_{1001} V_{0101} + 2J_6 V_{1100} V_{0110} \right),
\]

\[
G_2 = \left( -2J_3 V_{0002} + V_{1001}^2 V_{0020} + V_{0002} V_{1010}^2 \right) V_{0200},
\]

\[
G_3 = \left( 16J_4 V_{0110}^2 + 32V_{0110}(J_3 V_{0101} + V_{1100} J_6) + 16V_{0101}^2 J_7 + 32V_{1100} J_8 - 16J_1 V_{1100}^2 \right) V_{0200}.
\]

Table 5 presents all members of our proposed family while taking into account the underlying stratification.

7. Performance Comparison

In this section, we advance by comparing both families, comprehended in Tables 4 and 5. To establish the efficiency of each member of our family (Table 5), we need to show \( \text{MSE}_{\text{min}}(\overline{Y}_{Ia}) - \text{MSE}_{\text{min}}(\overline{Y}_k) > 0 \), which on simplification provides the general efficiency condition such as,

\[
\left( 64 - 16 \theta^2 V_{0200} \right) \left[ -V_{0200} V_{1010}^2 + 2V_{0110} V_{1010} V_{1100} - V_{0110}^2 (1 + V_{2000}) \right] + \left( 8 \theta^2 V_{0200} - 8 \right) \left( V_{0110}^2 - V_{0020} V_{0200} \right). \]

Moreover, it motivating to witness the superior performance of our estimator, evident through the results of Table 9, for all data sets and for every member of the proposed family.

8. Discussion

This article delineates the developments on a family of estimators inherently capable of more rigorous use of auxiliary information while estimating the finite population mean. We propose a three folded use of auxiliary information where auxiliary information is supplemented through ranks and second raw moments of auxiliary variable. It is then mathematically and numerically demonstrated that the triplet use of extra information enhances the performance of the mean estimating family. The findings are perfectly align with the notion of using auxiliary information to aid the estimation of required attribute; we observe that more rigorous use of relevant information enhances the efficiency of estimating mechanism. The mathematical developments are established along the SRS and SrRS methods of sampling. Furthermore, the proposition is applied to six commonly used data sets to assess the applicability of the introduced family. The performance comparison is conducted with respect to [9] suggested family of estimators. The findings reveal that more efficient use of supportive information
Table 5: Members of the suggested families of estimators.

| a | b | $\tilde{\gamma}_k$ |
|---|---|---|
| 1 | $C_{x(a)}$ | $\tilde{\gamma}_k^{(1)}$ |
| 1 | $\beta_{2(a)(a)}$ | $\tilde{\gamma}_k^{(2)}$ |
| 1 | $C_{x(a)}$ | $\tilde{\gamma}_k^{(3)}$ |
| 1 | $\beta_{2(a)(a)}$ | $\tilde{\gamma}_k^{(4)}$ |
| 1 | $\rho_{YX(a)}$ | $\tilde{\gamma}_k^{(5)}$ |
| 1 | $\beta_{2(a)(a)}$ | $\tilde{\gamma}_k^{(6)}$ |
| 1 | $\rho_{YX(a)}$ | $\tilde{\gamma}_k^{(7)}$ |
| 1 | $\beta_{2(a)(a)}$ | $\tilde{\gamma}_k^{(8)}$ |
| 1 | $\rho_{YX(a)}$ | $\tilde{\gamma}_k^{(9)}$ |
| 1 | $N$ | $\tilde{\gamma}_k^{(10)}$ |

Table 6: Summary statistics for data 1.

| h | $N_h$ | $n_h$ | $W_h$ | $\lambda_h$ | $Y_h$ | $X_h$ | $R_h$ | $U_h$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 127 | 25 | 0.138 | 0.033 | 703.740 | 20804.590 | 64 | 13549 |
| 2 | 117 | 23 | 0.127 | 0.035 | 413.000 | 9211.795 | 59 | 31334 |
| 3 | 103 | 20 | 0.112 | 0.040 | 573.175 | 14309.300 | 52 | 95637 |
| 4 | 170 | 33 | 0.184 | 0.024 | 424.665 | 9478.853 | 86 | 41983 |
| 5 | 205 | 40 | 0.222 | 0.020 | 267.029 | 5569.946 | 103 | 10288 |
| 6 | 201 | 39 | 0.218 | 0.021 | 393.841 | 12997.59 | 101 | 69962 |

Table 7: Summary statistics for data 2.

| h | $N_h$ | $n_h$ | $W_h$ | $\lambda_h$ | $Y_h$ | $X_h$ | $R_h$ | $U_h$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 106 | 9 | 0.124 | 0.102 | 1536.774 | 24375.59 | 54 | 29909 |
| 2 | 106 | 17 | 0.124 | 0.049 | 2212.594 | 27421.7 | 54 | 40225 |
| 3 | 94 | 38 | 0.110 | 0.016 | 9384.309 | 72409.95 | 48 | 30811 |
| 4 | 171 | 67 | 0.200 | 0.009 | 5588.012 | 74364.68 | 86 | 86622 |
| 5 | 204 | 7 | 0.239 | 0.138 | 966.956 | 26441.72 | 103 | 27505 |
| 6 | 173 | 2 | 0.203 | 0.494 | 404.399 | 9843.827 | 84 | 44807 |

Values provided for $S_{Y_h}$, $S_{X_h}$, $S_{R_h}$, $S_{U_h}$, $\rho_{YX_h}$, $\rho_{YR_h}$, $\rho_{YU_h}$, $\rho_{XR_h}$, $\rho_{XU_h}$, $\rho_{UR_h}$.
enables our family of superior performance when compared with the [9]. We anticipate that an alike strategy can be employed for the estimation of population variance but this is left as a future research topic.

### Data Availability

The data sets used to support the study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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