Generalized Chaplygin gas as geometrical dark energy

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Abstract

The generalized Chaplygin gas provides an interesting candidate for the present accelerated expansion of the universe. We explore a geometrical explanation for the generalized Chaplygin gas within the context of brane world theories where matter fields are confined to the brane by means of the action of a confining potential. We obtain the modified Friedmann equations, deceleration parameter and age of the universe in this scenario and show that they are consistent with the present observational data.

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1 Introduction

In recent times the generalized Chaplygin gas (gCg) model has been proposed as an alternative to both the cosmological constant and quintessence in explaining the accelerated expansion of our universe. The Chaplygin gas model describes a transition from a universe filled with dust-like matter to an accelerated expanding stage. The generalized Chaplygin gas model, introduced in [1] and elaborated in [2], is described by a perfect fluid obeying an exotic equation of state

\[ p = -\frac{B}{\rho^\beta}, \]

where \( B \) is a positive constant and \( 0 < \beta \leq 1 \). The standard Chaplygin gas corresponds to \( \beta = 1 \). An attractive feature of the model is that it can naturally explain both dark energy and dark matter [3]. The reason is that the Chaplygin gas behaves as dust-like matter at the early stages of the evolution of the universe and as a cosmological constant at late times. The Chaplygin gas appears as an effective fluid associated with \( d \)-branes [4, 5] and can also be derived from the Born-Infeld action [6]. An interesting range of models have been found to be consistent with the SNe Ia data [7], CMB experiments [8] and other observational data [9]. The cosmological implications of the Chaplygin gas model have been intensively investigated in recent literature [10, 11].

Over the past few years, models with extra dimensions have been proposed in which the standard fields are confined to a four-dimensional (4D) world viewed as a hypersurface (the brane) embedded in a higher dimensional space-time (the bulk) through which only gravity can propagate. The most popular model in the context of brane world theory is that proposed by Randall and Sundrum (RS). The so-called RSI model [12] proposes a mechanism to solve the hierarchy problem with two branes, while in the RSII model [13] a single brane with positive tension is considered where 4D Newtonian gravity is recovered at low energies even if the extra dimension is not compact. This mechanism provides us with an alternative to the compactification of extra dimensions.

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The cosmological evolution of such a brane universe has been extensively investigated and effects such as a quadratic density term in the Friedmann equations have been found \[14\]-\[16\]. This term arises from the imposition of the Israel junction conditions which is a relationship between the extrinsic curvature and energy-momentum tensor of the brane and results from the singular behavior in the energy-momentum tensor. There has been concerns expressed over applying such junction conditions in that they may not be unique. Indeed, other forms of junction conditions exist so that different conditions may lead to different physical results \[17\]. Furthermore, these conditions cannot be used when more than one non-compact extra dimension is involved. Against this background, an interesting higher-dimensional model was introduced in \[18\] where particles are trapped on a 4-dimensional hypersurface by the action of a confining potential $V$. In \[19\], the dynamics of test particles confined to a brane by the action of such a potential at the classical and quantum levels were studied and effects of small perturbations along the extra dimensions investigated. Within the classical limits, test particles remain stable under small perturbations and the effects of the extra dimensions are not felt by them, making them undetectable in this way. The quantum fluctuations of the brane cause the mass of a test particle to become quantized and, interestingly, the Yang-Mills fields appear as quantum effects. Also, in \[20\] a confining potential formalism was used to confine particles to an arbitrary manifold in a higher dimensional Euclidean space. In \[21\], a brane world model was studied in which the matter is confined to the brane through the action of such a potential, rendering the use of any junction condition unnecessary and predicting a geometrical explanation for the accelerated expansion of the universe. In a related work \[22\], a brane scenario was studied where the $m$-dimensional bulk is endowed with a Gauss-Bonnet (GB) term and the localization of matter on the brane is again realized by means of a confining potential. It was shown that in the presence of the GB term, the universe accelerates faster than brane models without the GB term. The behavior of an anisotropic brane world with Bianchi type I and V geometry was studied in \[23\] along the same lines.

In this paper, we follow \[21\] and consider an $m$-dimensional bulk space without imposing the $Z_2$ symmetry. As mentioned above, to localize matter on the brane, assumed to be thin, a confining potential is used rather than a delta-function in the energy-momentum tensor. The resulting Friedmann equations on the brane are modified by an extra term that may be interpreted as the generalized Chaplygin gas, providing a possible phenomenological explanation for the accelerated expansion of the universe. It should be emphasized that the model presented in this work is different from that introduced in \[24, 25\] in that the latter provides no account for the confinement of matter to the brane.

2 The model

In this section we present a brief review of the model proposed in \[19, 21\]. Consider the background manifold $\nabla_4$ isometrically embedded in a pseudo-Riemannian manifold $V_m$ by the map $\mathcal{Y}: \nabla_4 \rightarrow V_m$ such that

$$G_{AB}Y^A_{\mu}Y^B_{\nu} = g_{\mu\nu}, \quad G_{AB}Y^A_{\mu}N^B_a = 0, \quad G_{AB}N^A_a N^B_b = \bar{g}_{ab} = \pm 1,$$

(2)

where $G_{AB}$ ($\bar{g}_{\mu\nu}$) is the metric of the bulk (brane) space $V_m(\nabla_4)$ in arbitrary coordinates, $\{Y^A\}$ ($\{x^a\}$) is the basis of the bulk (brane) and $N^A_a$ are $(m - 4)$ normal unit vectors orthogonal to the brane. Perturbation of $\nabla_4$ in a sufficiently small neighborhood of the brane along an arbitrary transverse direction $\xi$ is given by

$$Z^A(x^\mu, \xi^a) = Y^A + (L_\xi Y)^A,$$

(3)

where $L$ represents the Lie derivative and $\xi^a$ ($a = 1, 2, ..., m - 4$) is a small parameter along $N^A_a$ parameterizing the extra noncompact dimensions. By taking $\xi$ orthogonal to the brane, we ensure gauge independency \[19\] and have perturbations of the embedding along a single orthogonal extra direction $N_a$, giving the local coordinates of the perturbed brane as

$$Z^A_{,\mu}(x^\nu, \xi^a) = Y^A_{,\mu} + \xi^a N^A_{a,\mu}(x^\nu).$$

(4)
In a similar manner we see that since the vectors $\mathcal{N}^A$ depend only on the local coordinates $x^\mu$, they do not propagate along the extra dimensions. The above assumptions lead to the embedding equations of the perturbed geometry

$$g_{\mu\nu} = \mathcal{G}_{AB} Z^A_{\mu\nu} Z^B_{\mu\nu}, \quad g_{\mu a} = \mathcal{G}_{AB} Z^A_{\mu\nu} N^B_a, \quad \mathcal{G}_{AB} N^A_a N^B_b = g_{ab}. \quad (5)$$

If we set $N^A_a = \delta^A_a$, the metric of the bulk space can be written in the following matrix form

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\mu\nu} + A_{\mu c} A^c_{\nu} & A_{\mu a} \\ A_{\nu b} & g_{ab} \end{pmatrix}, \quad (6)$$

where

$$g_{\mu\nu} = \bar{g}_{\mu\nu} - 2\xi^a \bar{K}_{\mu a} + \xi^a \xi^b \bar{K}_{\mu c a} \bar{K}_{\nu b}, \quad (7)$$

is the metric of the perturbed brane, so that

$$\bar{K}_{\mu a} = -\mathcal{G}_{AB} Y^A_{\mu a} N^B_{a\nu}, \quad (8)$$

represents the extrinsic curvature of the original brane (second fundamental form). We use the notation $A_{\mu c} = \xi^d A_{\mu d c}$ where

$$A_{\mu c d} = \mathcal{G}_{AB} N^A_{\mu a} N^B_{c d} = \bar{A}_{\mu c d}, \quad (9)$$

represents the twisting vector fields (the normal fundamental form). Any fixed $\xi^a$ signifies a new perturbed geometry, enabling us to define an extrinsic curvature similar to the original one by

$$\bar{K}_{\mu a} = -\mathcal{G}_{AB} Z^A_{\mu a} N^B_{a\nu} = \bar{K}_{\mu a} - \xi^b (\bar{K}_{\mu a} \bar{K}_{\nu b} + \bar{K}_{\gamma a} \bar{K}^\gamma_{\nu b}) \ . \quad (10)$$

Note that definitions (6) and (10) require

$$\bar{K}_{\mu a} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}. \quad (11)$$

In geometric language, the presence of gauge fields $A_{\mu a}$ tilts the embedded family of sub-manifolds with respect to normal vectors $\mathcal{N}^A$. According to our construction, the original brane is orthogonal to normal vectors $\mathcal{N}^A$. However, equation (5) shows that this is not true for the deformed geometry. Let us change the embedding coordinates and set

$$\mathcal{X}^A_{\mu} = Z^A_{\mu} - g^{ab} N^A_a A_{b\mu}. \quad (12)$$

The coordinates $\mathcal{X}^A$ describe a new family of embedded manifolds whose members are always orthogonal to $\mathcal{N}^A$. In this coordinates the embedding equations of the perturbed brane is similar to the original one, described by equation (2) so that $Y^A$ is replaced by $\mathcal{X}^A$. This new embedding of the local coordinates are suitable for obtaining induced Einstein field equations on the brane. The extrinsic curvature of the perturbed brane then becomes

$$K_{\mu a} = -\mathcal{G}_{AB} \mathcal{X}^A_{\mu a} N^B_{a\nu} = \bar{K}_{\mu a} - \xi^b (\bar{K}_{\mu a} \bar{K}_{\nu b} + \bar{K}_{\gamma a} \bar{K}^\gamma_{\nu b}) = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \quad (13)$$

which is the generalized York’s relation and shows how the extrinsic curvature propagates as a result of the propagation of the metric in the direction of extra dimensions. The components of the Riemann tensor of the bulk written in the embedding vielbein $\{\mathcal{X}^A_{\alpha}, N^A_a\}$, lead to the Gauss-Codazzi equations [26]

$$R_{\alpha\beta\gamma\delta} = 2g^{ab} K_{\gamma a} K_{\delta b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{\alpha} \mathcal{X}^B_{\beta} \mathcal{X}^C_{\gamma} \mathcal{X}^D_{\delta}, \quad (14)$$

$$2K_{\alpha\beta\gamma\delta} = 2g^{ab} A_{\gamma a} K_{\delta b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{\alpha} N^B_{c} \mathcal{X}^C_{\gamma} \mathcal{X}^D_{\delta}, \quad (15)$$
where $R_{ABCD}$ and $R_{\alpha\beta\gamma\delta}$ are the Riemann tensors for the bulk and the perturbed brane respectively. By contracting the Gauss equation (14) on $\alpha$ and $\gamma$ and defining

$$Q_{\mu\nu} = -g^{ab} \left( K_{\mu a} K_{\nu b} - K_{\nu a} K_{\mu b} \right) + \frac{1}{2} \left( K_{\alpha\beta a} K^{\alpha\beta a} - K_a K^a \right) g_{\mu\nu},$$  \hspace{1cm} (16)

which is an independently conserved quantity, that is $Q_{\mu\nu,\nu} = 0$ [24]. Using the decomposition of the Riemann tensor into the Weyl curvature, the Ricci tensor and the scalar curvature, we obtain the 4D Einstein equations as

$$G_{\mu\nu} = G_{AB} \chi^A \chi^B + Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{(m - 3)}{(m - 2)} g^{ab} R_{AB} N_a^A N_b^B g_{\mu\nu}$$

$$- \frac{(m - 4)}{(m - 2)} R_{AB} \chi^A \chi^B + \frac{(m - 4)}{(m - 1)(m - 2)} \mathcal{R} g_{\mu\nu},$$ \hspace{1cm} (17)

where $\mathcal{E}_{\mu\nu} = g^{ab} C_{ABCD} N^A_a \chi^B \chi^C$ is the electric part of the Weyl tensor $C_{ABCD}$. Now, let us write the Einstein field equations in the bulk space

$$G_{AB}^{(b)} + \Lambda^{(b)} g_{AB} = \alpha^* S_{AB},$$ \hspace{1cm} (18)

where $\alpha^* = \frac{1}{M_*^2}$. In this equation $\Lambda^{(b)}$ is the cosmological constant of the bulk space with $S_{AB}$ consisting of two parts

$$S_{AB} = T_{AB} + \frac{1}{2} \mathcal{V} g_{AB},$$ \hspace{1cm} (19)

where $T_{AB}$ is the energy-momentum tensor of the matter confined to the brane through the action of the confining potential $\mathcal{V}$. We require $\mathcal{V}$ to satisfy three general conditions: it should have a deep minimum on the non-perturbed brane, depends only on extra coordinates $\xi^a$ and the gauge group representing the subgroup of the isometry group of the bulk space should be preserved by it.

Although the explicit form of such a potential is of no consequence in the present work, for the sake of completeness it would be useful to mention a result presented in [19] where a confining potential whose role, as the name suggests, is to trap test particles on the brane was obtained by invoking the above properties and assuming that the brane is located at $\xi^a = 0$, with the result

$$\mathcal{V}(\xi) = \frac{1}{2} w^2 g_{ab} \xi^a \xi^b,$$ \hspace{1cm} (20)

where $w$ is a constant much larger than the scale of curvature of the brane. This potential is clearly of the harmonic oscillator type, forcing a test particle leaving the brane along the extra dimension back to its initial position on the brane which coincides with the location of minimum of the potential. It is conceivable that other equivalent potentials having different forms but possessing the same properties and playing the same role can be envisaged.

Using Einstein equations (18), we obtain

$$R_{AB} = -\frac{\alpha^*}{(m - 2)} G_{AB} S + \frac{2}{(m - 2)} \Lambda^{(b)} g_{AB} + \alpha^* S_{AB},$$ \hspace{1cm} (21)

and

$$\mathcal{R} = -\frac{2}{m - 2} (\alpha^* S - m \Lambda^{(b)}).$$ \hspace{1cm} (22)

Substituting $R_{AB}$ and $\mathcal{R}$ from the above into equation (17), we obtain

$$G_{\mu\nu} = Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{(m - 3)}{(m - 2)} \alpha^* g^{ab} S_{ab} g_{\mu\nu} + \frac{2\alpha^*}{(m - 2)} S_{\mu\nu} - \frac{(m - 4)(m - 3)}{(m - 1)(m - 2)} \alpha^* S g_{\mu\nu}$$

$$+ \frac{(m - 7)}{(m - 1)} \Lambda^{(b)} g_{\mu\nu}.$$ \hspace{1cm} (23)
As was mentioned in the introduction, localization of matter on the brane is realized in this model by the action of a confining potential. This can simply be done by taking
\[
\alpha \tau_{\mu \nu} = \frac{2 \alpha}{m - 2} T_{\mu \nu}, \quad T_{\mu a} = 0, \quad T_{ab} = 0,
\]
where \(\alpha\) is the scale of energy on the brane. Now, the induced Einstein field equations on the original brane can be written as
\[
G_{\mu \nu} = \alpha \tau_{\mu \nu} - \frac{(m - 4)(m - 3)}{2(m - 1)} \alpha \tau g_{\mu \nu} - \Lambda g_{\mu \nu} + Q_{\mu \nu} - \mathcal{E}_{\mu \nu},
\]
where \(\Lambda = -\frac{(m - 7)}{(m - 1)} \Lambda^{(b)}\) and \(Q_{\mu \nu}\) is an independently conserved quantity which may be considered as an energy-momentum tensor of the generalized Chaplygin gas (gCg). This matter is a candidate for a unified dark matter-energy scenario which is parameterized by an equation of state \(p = -\frac{B}{\rho}\), where \(B\) and \(\beta\) are arbitrary constants. The speed of sound \(v_s^2 = -\frac{dp}{d\rho}\) in the gCg is defined as \(v_s^2 = -\frac{\beta p}{\rho}\). Ultimately, we have three different types of ‘matter’ on the right hand side of equation (25), namely, ordinary confined conserved matter represented by \(\tau_{\mu \nu}\), the matter represented by \(Q_{\mu \nu}\) which will be discussed later and finally, the Weyl matter represented by \(\mathcal{E}_{\mu \nu}\).

The geometrical approach considered here is based on three basic postulates, namely, the confinement of the standard gauge interactions to the brane, the existence of quantum gravity in the bulk and finally, the embedding of the brane world. All other model dependent properties such as warped metric, mirror symmetries, radion or extra scalar fields, fine tuning parameters like the tension of the brane and the choice of a junction condition are left out as much as possible in our calculations [25]. In the next section, we discuss the cosmological implications of our model. As was mentioned above, the results do not depend on the precise shape of the potential.

### 3 Friedmann equations, deceleration parameter and age of the universe

In what follows we will analyze the influence of the extrinsic curvature terms on a FRW universe, regarded as a brane embedded in a 5-dimensional bulk with constant curvature (\(\mathcal{E}_{\mu \nu} = 0\)). The FRW line element is given by
\[
d s^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right].
\]
The confined source is the perfect fluid given in co-moving coordinates by
\[
\tau_{\mu \nu} = (\rho + p) u_\mu u_\nu + pg_{\mu \nu}, \quad u_\mu = -\delta_\mu^0, \quad p = (\gamma - 1)\rho.
\]
The constant curvature bulk is characterized by the Riemann tensor
\[
R_{ABCD} = k_\ast (g_{AC} g_{BD} - g_{AD} g_{BC}),
\]
where \(g_{AB}\) denotes the bulk metric components in arbitrary coordinates and \(k_\ast\) is either zero for the flat bulk, or proportional to a positive or negative bulk cosmological constant respectively, corresponding to two possible signatures \((4, 1)\) for the \(dS_5\) bulk and \((3, 2)\) for the \(AdS_5\) bulk. We take, in the embedding equations, \(g^{55} = \varepsilon = \pm 1\). With this assumption the Gauss-Codazzi equations reduce to
\[
R_{\alpha \beta \gamma \delta} = \frac{1}{\varepsilon} (K_{\alpha \gamma} K_{\beta \delta} - K_{\alpha \delta} K_{\beta \gamma}) + k_\ast (g_{\alpha \gamma} g_{\beta \delta} - g_{\alpha \delta} g_{\beta \gamma}),
\]
Also the effective field equations derived in the previous section with constant curvature bulk can be written as

\[ G_{\mu\nu} = \alpha \tau_{\mu\nu} - \lambda g_{\mu\nu} + Q_{\mu\nu}. \]  

(31)

Here, \( \lambda \) is the effective cosmological constant in four dimensions with \( Q_{\mu\nu} \) being a completely geometrical quantity given by

\[ Q_{\mu\nu} = \frac{1}{\varepsilon} \left[ (KK_{\mu\nu} - K_{\mu\alpha}K_{\nu}^{\alpha}) + \frac{1}{2} \left( K_{\alpha\beta}K_{\alpha\beta} - K^2 \right) g_{\mu\nu} \right], \]

(32)

where \( K = g^{\mu\nu}K_{\mu\nu} \). Using the York's relation

\[ K_{\mu\nu a} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \]

(33)

we realize that in a diagonal metric, \( K_{\mu\nu a} \) is diagonal. After separating the spatial components, the Codazzi equations reduce to (here \( \alpha, \beta, \gamma, \sigma = 1, 2, 3 \))

\[ K_{\alpha\gamma a,\sigma} + \dot{a} K_{\alpha\gamma a} = K_{\alpha\sigma a,\gamma} + \dot{a} K_{\alpha\sigma a}, \]

(34)

\[ K_{\alpha\gamma a,0} + \dot{a} K_{\alpha\gamma a} = -\frac{\dot{a}}{a} K_{\alpha0 a}. \]

(35)

The first equation gives \( K_{1a,\sigma}^{1} = 0 \) for \( \sigma \neq 1 \), since \( K_{1a}^{1} \) does not depend on the spatial coordinates. Repeating the same procedure for \( \alpha, \gamma = i, i = 2, 3 \), we obtain \( K_{2a,\sigma}^{2} = 0 \) for \( \sigma \neq 2 \) and \( K_{3a,\sigma}^{3} = 0 \) for \( \sigma \neq 3 \). Assuming \( K_{1a}^{1} = K_{2a}^{2} = K_{3a}^{3} = b_a(t) \), where \( b_a(t) \) are arbitrary functions of \( t \), the second equation gives

\[ K_{00 a} = -\left( \frac{\dot{b}_a a}{a} + b_a \right). \]

(36)

For \( \mu, \nu = 1, 2, 3 \) we obtain

\[ K_{\mu\nu a} = b_a g_{\mu\nu}. \]

(37)

Assuming that the functions \( b_a \) are equal and denoting \( b_a = b, h = \frac{\dot{b}}{b} \) and \( H = \frac{\dot{a}}{a} \), we find from equation (32) that

\[ K_{\alpha\beta}K_{\alpha\beta} = b^2 \left( \frac{h^2}{H^2} + 2 \frac{h}{H} + 4 \right), \quad K = b \left( \frac{h}{H} + 4 \right), \]

(38)

\[ Q_{\mu\nu} = -\frac{3b^2}{\varepsilon} \left( \frac{2h}{3H} + 1 \right) g_{\mu\nu}, \quad \mu, \nu = 1, 2, 3, \quad Q_{00} = \frac{3b^2}{\varepsilon}. \]

(39)

As we noted before, \( Q_{\mu\nu} \) is an independently conserved quantity, suggesting the same behavior as that of an energy-momentum tensor of an uncoupled non-conventional energy source. With this analogy we take the gCg model as a practical example and define \( Q_{\mu\nu} \) as an exotic fluid and write

\[ Q_{\mu\nu} \equiv \frac{1}{\alpha} \left[ (\rho_{ch} + p_{ch}) u_\mu u_\nu + p_{ch} g_{\mu\nu} \right], \quad p_{ch} = -\frac{B}{\rho_{ch}}, \]

(40)
where $B$ is a positive constant and $0 < \beta \leq 1$ [2]. Comparing $Q_{\mu \nu}, \mu, \nu = 1, 2, 3$ and $Q_{00}$ from equation (40) with the components of $Q_{\mu \nu}$ and $Q_{00}$ given by equation (39), we obtain

\[ p_{ch} = \frac{-3b^2}{\alpha \varepsilon} \left( \frac{2h}{3H} + 1 \right), \quad \rho_{ch} = \frac{3b^2}{\alpha \varepsilon}. \]  

(41)

Equation (40) was chosen in accordance with the weak-energy condition corresponding to a positive energy density and negative pressure with $\varepsilon = +1$. Use of the above equations leads to an equation for $b(t)$

\[ \left( \frac{2ab}{3\dot{ab}} + 1 \right) = B \left( \frac{3b^2}{\alpha \varepsilon} \right)^{-(1+\beta)}, \]  

(42)

for which the solution is

\[ b(t) = \left( \frac{\alpha \varepsilon}{3} \right)^{\frac{1}{2}} \left[ B + \frac{C}{a^{3(1+\beta)}} \right]^{\frac{1}{2(1+\beta)}}, \]

(43)

where $C$ is an integration constant. Using equation (41) and this solution, the energy density of the gCg becomes

\[ \rho_{ch} = \rho_{ch0} \left[ B_{s} + \frac{(1 - B_{s})}{a^{3(1+\beta)}} \right]^{\frac{1}{(1+\beta)}}, \]  

(44)

where $\rho_{ch0}$ is the gCg density at the present time, $B_{s} = B \rho_{ch0}^{-(1+\beta)}$ is a dimensionless quantity related to the speed of sound for the gCg today, $v_{s}^2 = \beta B \rho_{ch0}^{-(1+\beta)}$ and $C = \rho_{ch0}^{(1+\beta)} - B$. From equation (44), we see that for $B_{s} = 0$ the gCg behaves like matter, $\rho_{ch} \sim a^{-3}$, whereas for $B_{s} = 1$ it behaves as a cosmological constant, $\rho_{ch} \sim \text{constant}$ and when $0 < B_{s} < 1$, the model predicts a behavior as that between a matter phase in the past and a negative dark energy regime at late times. This particular behavior of the gCg inspired some authors to propose a unified scheme for the cosmological “dark sector,” an interesting idea which has been considered in many different contexts.

The contracted Bianchi identities in the bulk space, $G^{AB};_{A} = 0$, using equation (18), imply

\[ \left( T^{AB} + \frac{1}{2} V G^{AB} \right);_{A} = 0. \]  

(45)

Since the potential $V$ has a minimum on the brane, the above conservation equation reduces to

\[ \tau^{\mu \nu};_{\mu} = 0. \]

(46)

For a perfect fluid as a confined matter source, the time evolution of the energy density of the matter is given by

\[ \rho = \rho_{0} a^{-3\gamma}. \]  

(47)

Taking equation (31) and using the geometrical energy density for $Q_{\mu \nu}$, the modified Friedmann equations on the brane become

\[ \left( \frac{\ddot{a}}{a} \right)^{2} + \frac{k}{a^{2}} = \frac{\lambda}{3} + \frac{\alpha}{3} \rho_{0} a^{-3\gamma} + \frac{\alpha}{3 \varepsilon} \rho_{ch0} \left[ B_{s} + \frac{(1 - B_{s})}{a^{3(1+\beta)}} \right]^{\frac{1}{(1+\beta)}}, \]  

(48)

\[ \frac{\ddot{a}}{a} = \frac{\lambda}{3} - \frac{\alpha}{6} \rho_{0} a^{-3\gamma} (3\gamma - 2) + \frac{\alpha}{3 \varepsilon} \rho_{ch0} \left[ B_{s} + \frac{(1 - B_{s})}{a^{3(1+\beta)}} \right]^{\frac{1}{(1+\beta)}} \left[ B_{s} - \frac{(1 - B_{s})}{2a^{3(1+\beta)}} \right]. \]  

(49)
Now, we rewrite the Friedmann equation (48) in terms of the redshift \( z = \frac{a_0}{a} - 1 \) and of the observational parameter \( \Omega \), as

\[
H^2 = H_0^2 \left\{ \Omega_k (1 + z)^2 + \Omega_\Lambda + \Omega_m (1 + z)^3 \gamma + \frac{\Omega_{ch}}{\varepsilon} \left[ B_s + (1 - B_s)(1 + z)^3(1 + \beta) \right]^{\frac{1}{1 + \beta}} \right\},
\]

where \( H_0 \) is the present value of the Hubble parameter, \( \Omega_m, \Omega_\Lambda \) and \( \Omega_k \) are respectively, the confined matter, cosmological constant and spatial curvature relative density parameters where \( \Omega_k \) is associated with the geometrical dark energy through \( \Omega_{ch} = \frac{\alpha \rho_{ch0}}{3H_0^2} \). The deceleration parameter \( q \) defined by \( q = -\frac{\ddot{a}}{a^2} \) becomes

\[
q = -\Omega_\Lambda + \frac{\Omega_m}{\varepsilon} (1 + z)^3 - \frac{\Omega_{ch}}{\varepsilon} \left[ B_s + (1 - B_s)(1 + z)^3(1 + \beta) \right]^{\frac{1}{1 + \beta}} \left[ B_s - \frac{1}{2} (1 - B_s)(1 + z)^3(1 + \beta) \right] \Omega_k (1 + z)^2 + \Omega_\Lambda + \Omega_m (1 + z)^3 + \frac{\Omega_{ch}}{\varepsilon} \left[ B_s + (1 - B_s)(1 + z)^3(1 + \beta) \right]^{\frac{1}{1 + \beta}}
\]

We consider \( \Omega_\Lambda = 0 \) and show that, within the context of the present model, the extrinsic curvature can be used to account for the accelerated expansion of the universe. Figure 1 shows the behavior of the deceleration parameter as a function of the redshift for selected values of \( \Omega_m, B_s \) and \( \beta \) for signature \( \varepsilon = +1 \). Note that this behavior is much dependent on the range of the values that \( B_s \) can take and is insensitive to the parameter \( \beta \). For having an accelerating universe with \( \Omega_m = 0.3 \), the value of \( B_s \) should lie in the range \( 0.5 \leq B_s \leq 1 \). Also the value of \( B_s \) determines the acceleration redshift \( z_a \). For \( B_s = 1 \) and \( B_s = 0.6 \) with \( \Omega_m = 0.3 \), the accelerated expansion begins at \( z_a = 0.67 \) and \( z_a = 0.07 \) respectively.

Had we taken the bulk signature to be \((3, 2)\) or \( \varepsilon = -1 \), equation (41) would have represented a fluid with negative energy and positive pressure, causing an unexpected behavior of the expansion of the universe. In order to better visualize this behavior, the deceleration parameter has been plotted as a function of the redshift for selected values of \( \Omega_m \) and \( \Omega_{ch} \) in figure 1.

The age of the universe in FRW models is given by

\[
t_0^E = \frac{1}{H_0} \int_0^1 \frac{dx}{\left[ \Omega_m + (1 - \Omega_m) \right]^{\frac{1}{2}}},
\]

where \( H_0^{-1} = 9.8 \times 10^9 h^{-1} \) years and the dimensionless parameter \( h \), according to modern data, is about 0.7. Hence, in the flat matter dominated universe with \( \Omega_{total} = 1 \) the age of the universe would be only 9.3 Gyr, whereas the oldest globular clusters yield an age of about 12.5 with an uncertainty of 1.5 Gyr [27]. We find the age of the universe for our model by direct integration of the Friedmann equation (48)

\[
t_0^B = \frac{1}{H_0} \int_0^1 \frac{dx}{\left\{ \Omega_k + \Omega_m x^{-3\gamma + 2} + x^2 \Omega_{ch} \left[ B_s + \frac{(1 - B_s)}{x(1 + \beta)} \right]^{\frac{1}{1 + \beta}} \right\}^{\frac{1}{2}}}
\]

where

\[
\Omega_k = -\frac{k}{H_0^2}, \quad \Omega_m = \frac{\alpha \rho_{om}}{3H_0^2}, \quad \Omega_{ch} = \frac{\alpha \rho_{ch0}}{3H_0^2},
\]

For a flat, matter dominated universe with \( \Omega_m = 0.3 \) and \( B_s = 1 \) this leads to a prediction for the age of the universe of about 13.2 Gyr. It shows that the age of the universe in our model is longer than the FRW model. In figure 2 we have plotted the dimensionless age parameter \( H_0 t_0 \) as a function of \( \Omega_m \) for some selected values of \( B_s \) and \( \beta \). This behavior shows that for a fixed value of \( \Omega_m \) the predicted age of the universe is longer for larger values of \( B_s \). Note also that the age parameter \( H_0 t_0 \) is an almost insensitive function of parameter \( \beta \) but that it depends strongly on variations of \( B_s \). This means that age considerations will be much more efficient for imposing constraints on the speed of sound, \( B_s \), than the values of the parameter \( \beta \) [28].
Finally, it would be appropriate to compare the predicted age of the universe in our model to the Randall-Sundrum brane model where the effects of the brane parameters and dark energy on the age of the universe have been studied. It has been shown that the effect of the quadratic term $\rho^2$, resulting from the imposition of the Israel junction conditions on the energy density term is to lower significantly the age of the universe. This term effectively contributes as a positive pressure, making brane models less accelerating. This problem can be avoided if we accept the existence of dark energy with $p = -\frac{4}{3}\rho$ (phantom matter) on the brane since it has a very strong influence on increasing the age of the universe \cite{29}.

4 Conclusions

In this paper, we have studied a brane world model in which the matter is confined to the brane through the action of a confining potential, rendering the use of any junction condition redundant. We have shown that in a FRW universe embedded in a constant curvature $dS_5$ bulk the accelerated expansion of the universe can be explained in a purely geometrical fashion based on the extrinsic curvature. We have established a correspondence with the phenomenological gCg dark energy model and have extended the predictions of geometrical matter in the more general case where the relation between $p$ and $\rho$ is not linear. Finally, we have found that the age of the universe in our model is longer than that predicted by the FRW models and hence more in line with the present observational data.
References

[1] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511 265 (2001).

[2] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 66 043507 (2002).

[3] N. Bilic, G. B. Tupper and R.D. Viollier, Phys. Lett. B 535 17 (2002).

[4] J. C. Fabris, S. V. B. Goncalves and P. E. de Souza, Gen. Rel. Grav. 34 2111 (2002).

[5] M. Bordemann and J. Hoppe, Phys. Lett. B 317 315 (1993),
N. Ogawa, Phys. Rev. D 62 085023 (2000).

[6] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Lett. B 575 172 (2003).

[7] M. Makler, S. Q. de Oliveira and I. Waga, Phys. Lett. B 555 1 (2003),
Y. Gong and C. K. Duan, Class. Quant. Grav. 21 3655 (2004),
Y. Gong, JCAP 0503 007 (2005).

[8] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Lett. B 575 172 (2003),
L. Amendola, F. Finelli, C. Burigana and D. Carturan, JCAP 0307 005 (2003),
M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 67 063003 (2003).

[9] R. Bean, O. Dore, Phys. Rev. D 68 023515 (2003),
Z. H. Zhu, Astron. Astrophys. 423 421 (2004).

[10] L. P. Chimento and R. Lazkoz, Phys. Lett. B 615 146 (2005),
T. Barreiro, A. A. Sen, Phys. Rev. D 70 124013 (2004),
M. K. Mak and T. Harko, Phys. Rev. D 71 104022 (2005),
P. F. Gonzalez-Diaz, Phys. Lett. B 562 1 (2003),
L. P. Chimento, Phys. Rev. D 69 123517 (2004),
G. M. Kremer, Phys. Rev. D 68 123507 (2003),
D. Podolsky, Astro. Lett. 28 434 (2002).

[11] U. Debnath, A. Banerjee and S. Chakraborty, Class. Quant. Grav. 21 5609 (2004),
M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 70 083519 (2004),
R. R. R. Reis, I. Waga, M. O. Calvao and S. E. Joras, Phys. Rev. D 68 061302 (2003),
T. Multamaki, M. Manera and E. Gaztanaga, Phys. Rev. D 69 023004 (2004),
P. P. Avelino, L. M. G. Beca, J. P. M. de Carvalho and C. J. A. P. Martins, JCAP 0309 002 (2003).

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 3370 (1999).

[13] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 4690 (1999).

[14] P. Brax and C. van de Bruck, Class. Quant. Grav. 20 R201 (2003),
D. Langlois, Prog. Theor. Phys. Suppl. 148 181 (2003),
R. Maartens, Reference Frames and Gravitomagnetism, ed. J Pascual-Sanchez et. al., (World Scientific,2001), p.93-119.

[15] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B 565 269 (2000).

[16] P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B 477 285 (2000).

[17] R. A. Battye and B. Carter, Phys. Lett. B 509 331 (2001).
[18] V. A. Rubakov and M. E. Shaposhnikov, *Phys. Lett.* B **125** 136 (1983),
K. Akama, “Pregeometry” in Lecture Notes in Physics, 176, Gauge Theory and Gravitation, Proceedings, Nara, 1982, edited by K. Kikkawa, N. Nakanishi and H. Nariai, 267-271 (Springer-Verlag, 1983).

[19] S. Jalalzadeh and H. R. Sepangi, *Class. Quant. Grav.* **22** 2035 (2005).

[20] P. C. Schuster and R. L. Jaffe, *Ann. Phys.* **307** 132 (2003).

[21] M. Heydari-fard, M. Shirazi, S. Jalalzadeh and H. R. Sepangi, *Phys. Lett.* B **640** 1 (2006).

[22] M. Heydari-fard and H. R. Sepangi, *Phys. Rev.* D **75** 064010 (2007).

[23] M. Heydari-fard and H. R. Sepangi, *Phys. Lett.* B **649** 1 (2007).

[24] M. D. Maia, E. M. Monte, J. M. F. Maia and J. S. Alcaniz, *Class. Quant. Grav.* **22** 1623 (2005).

[25] M. D. Maia, E. M. Monte and J. M. F. Maia, *Phys. Lett.* B **585** 11 (2004).

[26] L. P. Eisenhart 1966 *Riemannian Geometry*, Princeton University Press, Princeton NJ (1966).

[27] L. M. Krauss and B. Chaboyer, *Science* **299** 65 (2003).

[28] A. Dev, J. S. Alcaniz and D. Jain, *Phys. Rev.* D **67** 023515 (2003).

[29] W. Godlowski and M. Szydlowski, *Gen. Rel. Grav.* **36** 767 (2004),
M. P. Dabrowski, W. Godlowski and M. Szydlowski, *Int. J. Mod. Phys.* D **13** 1669 (2004).