RELATIVISTIC DEUTERON STRUCTURE FUNCTION AT LARGE $Q^2$

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Abstract

The deuteron deep inelastic unpolarized structure function $F_2^D$ is calculated using the Wilson operator product expansion method. The long distance behaviour, related to the deuteron bound state properties, is evaluated using the Bethe–Salpeter equation with one particle on–mass–shell. The calculation of the ratio $F_2^D/2F_2^N$ is compared with other convolution models showing important deviations in the region of large $x$. The implications in the evaluation of the neutron structure function from combined data on deuterons and protons are discussed.

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1 Introduction

Since the discovery of the EMC \cite{1} effect it became clear that the nucleon structure functions inside nuclei are different from the ones determined for free nucleons. In the quark–parton model point of view, this means that the quark and the anti–quark sea distributions, in the nucleon, are changed by the surrounding nucleons. Traditional methods \cite{2,3} that considered only Fermi motion of the nucleons are not sufficient to describe the experimental data. Several efforts have been done to explain this deviation, mainly along two different approaches. The first is QCD motivated \cite{4} and assumes a $Q^2$ rescaling, i.e., $F_A^N(x, Q^2)/A = F_N^N(x, \xi Q^2)$, where $\xi$ is a $A$–dependent free parameter determined by the best fit to the data. The other typical approach \cite{5}−\cite{12} is based on the nuclear degrees of freedom namely, the nucleon–nucleon interaction involving mesonic exchange contributions and other nuclear structure effects. In this work special attention is given to the method described in Ref. \cite{10,11}, which relies on the Wilson operator product expansion (OPE). This is a systematic and self–consistent approach inspired in nuclear physics concepts which will be used here in the calculation of the deuteron structure function $F_D^D$.

A comparison \cite{13} between the $Q^2$ rescaling model and the OPE model through the moments $M_n(F_A^1)$ of the structure functions, shows that both methods agree in the intermediate region of $x$ ($0.2 < x < 0.7$) corresponding to small values of $n$. For large $n$, it seems that the OPE is more appropriate since it describes the experimental data in the region of large $x$. The phenomenological nature of the $Q^2$ rescaling method results from the fact that it is hard to predict the value of $\xi$ within QCD. On the other hand, the OPE method has no free parameters and, in particular, the deuteron structure function can be obtained in a consistent way.

Ref.\cite{12} provides a general covariant method to describe deep–inelastic scattering (DIS) that does not rely on the Bjorken limit. In this model the deuteron structure function depends upon the off–shell nucleon structure function and the Bethe–Salpeter deuteron wave function. However, there is no factorization, which implies that, in general, the convolution formula is not valid. The usual ambiguities related to the off–shellness problems in DIS are not present, but the nucleon distribution function, which is an important tool for the interpretation of other processes involving the deuteron, is not consistently obtained within this formalism.

Some important considerations about parton distribution functions and QCD analysis of DIS rely on the experimental structure functions of the nucleons. In general, the neutron structure function is extracted from combined experimental data on the proton and deuteron \cite{14}, taking into account only the Fermi motion of the nucleons in the deuteron. The question of whether the EMC effect is a feature only of the heavy nuclei or should also be considered as leading to an important correction in light nuclei must be discussed. In principle one should take into account the binding effects and the meson exchange currents, required to describe the EMC effect in heavy nuclei, also in light nuclei. It has been shown \cite{15} that the additional corrections are important in the deuteron and that the extracted neutron structure function is model dependent due to the nuclear contributions. This is particularly important at large $x$ where the discrepancy with the data is of the order $\simeq 25\%$ and exceeds the experimental error bars \cite{14}. It is thus clear that, from a nuclear physics point of view, one should consider corrections to the Fermi motion of the nucleons even for light nuclei.

In this paper the deuteron structure function is calculated using a relativistic formalism, based on the Wilson operator product expansion method, following Ref. \cite{10,11}.
The deuteron Bethe–Salpeter amplitudes are obtained from a quasi–potential equation with one particle on–shell \[\text{[13]}\]. Section 2 is an overview of the formalism and presents the relevant expressions used in the calculation. The main features of the relativistic deuteron wave function used in our calculations are described in section 3 and in section 4 we discuss the results of the ratio $F_D^2 / 2F_N^2$ through a comparison with other approaches.

## 2 The Formalism

The basic feature of the Wilson operator product expansion is the possibility of factorizing the DIS amplitude into two pieces in the limit $Q^2 \to \infty$, one associated with the long distance behaviour and the other with the short distance behaviour. The pertubative QCD formalism determines the large $Q^2$ behaviour of the amplitude. On the other hand, a nuclear physics approach can be used to deal with the target nuclei, where the long distance behaviour can in general be predicted from the bound state properties.

According to the OPE method \[\text{[10, 11]}\] the deuteron structure function $F_D^2$ satisfies the relation

$$
\int_0^1 dx \, x^{n-2} F_D^2(x, Q^2) = \sum_a C^{(2)}_{a, n-1}(Q^2) \mu_{n/D}^{a/n+1},
$$

where $x = Q^2 / 2P_D.q$ is the usual Bjorken scaling variable. The sum in the second member of Eq.(1) runs over different fields of the theory, according to the twist–2 approximation. The Wilson coefficients $C^{(2)}_{a, n}$ are target independent, giving the short distance behaviour of $F_D^2$. In the impulse approximation, it is found that these coefficients are identical to the moments of the structure function $F_a^2$ of nucleons ($a = N$) or mesons ($a = B$),

$$
C^{(2)}_{N,n}(Q^2) = M_n(F_N^2) \quad \quad C^{(2)}_{B,n}(Q^2) = M_n(F_B^2).
$$

Noting that the moments are defined as

$$
M_n(F) = \int_0^1 dx \, x^{n-1} F(x, Q^2),
$$

we obtain from Eqs.(1),(2)

$$
M_n(F_D^2) = M_n(F_N^2) \mu_{n+1/D}^{N/D} + M_n(F_B^2) \mu_{n+1/D}^{B/D},
$$

where $\mu_{n+1/D}$ is interpreted as the moment of an effective distribution function of nucleons or mesons in the deuteron. In the leading twist approximation, these functions are given by

$$
< P_D \mid O_{\mu_1, \ldots, \mu_n}^{a_1, \ldots, a_n} \mid P_D > = P_D^{\mu_1} \ldots P_D^{\mu_n} \mu_{n/D}^{a_{n/D}}
$$

where $O_{\mu_1, \ldots, \mu_n}$ is the set of local operators that provide the basis for the operator product expansion. The matrix elements from Eq.(4) can be explicitly evaluated using the Mandelstam method \[\text{[17]}\] and in particular, the following result holds in the impulse approximation

$$
\mu_{n/D}^{N/D} = \frac{1}{2(P_D.e)^n} \int \frac{d^4k}{(2\pi)^3} \tilde{\chi}(p_1, p_2)S(p_1)\xi(S(p_1))\chi(p_1, p_2)S^{-1}(p_2)
\times \delta \left( p_2^2 - m_N^2 \right) (p_1.e)^{n-1} + (1 \leftrightarrow 2)
$$

3
where \( p_1, p_2 \) are the nucleon momenta in the deuteron and \( k = (p_1 - p_2)/2 \). The vector \( \epsilon \) is chosen in such a way that \( \epsilon^2 = 0 \) and \( \epsilon \propto \vec{q} \) where \( \vec{q} \) is the photon momentum with \( q^2 = -Q^2 \). In particular, if the transferred 3–momentum \( \vec{q} \) is along the z–axis one can choose \( \epsilon = (1,0,0,-1) \). This prescription is used in extracting \( \mu_{n/D}^{N/D} \) from the twist–2 operator expansion.

Expression (6) is similar to the one presented in Ref. \[11\], but includes a \( \delta \)–function in the integrand which means that the distribution function moment \( \mu_{n/D}^{N/D} \) is calculated with one particle on–mass–shell. The difference is related to the fact that the Bethe–Salpeter amplitudes \( \tilde{\chi}, \chi \) used in this work, are determined by a quasi–potential equation with one particle on–mass–shell \[16\]. This approximation includes some off–shell effects (the other particle is completely off–shell) and it reproduces successfully the bound state properties including the deuteron form factors \[18\].

Applying the inverse Mellin transform to Eq.(4), defined as

\[
F(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} dn x^{-n} M_n(F),
\]

and taking into account the explicit expression for \( \mu_{n/D}^{N/D} \) one recovers the convolution formula

\[
F_D^2(x) = \int_x^1 dy f^{N/D}(y) F^2_N(x/y) + \text{MEC}
\]

where

\[
f^{N/D}(y) = \frac{i}{2P_{D,\epsilon}} \int \frac{d^4k}{(2\pi)^3} \tilde{\chi}(p_1,p_2)S(p_1)\chi(p_1,p_2)S^{-1}(p_2) \times \delta \left( p_2^2 - m_N^2 \right) \left\{ \theta(p_1,\epsilon) \delta \left( y - \frac{p_1,\epsilon}{P_{D,\epsilon}} \right) + \frac{p_1,\epsilon}{P_{D,\epsilon}} \theta(-p_1,\epsilon) \right\} + (1 \leftrightarrow 2)
\]

is the distribution function of nucleons in the deuteron. The meson exchange currents (MEC) also contributes to the deuteron structure function and the corresponding distribution function can be found in an analogous way \[11\].

Notice that \( f^{N/D} \) given by Eq.(8) takes into account the binding and off–shell effects in the deuteron, including the features needed to describe the EMC effect. It can be shown, using the Bethe–Salpeter normalization condition, that the distribution function satisfies the baryonic number relation,

\[
\int_0^1 dy f^{N/D}(y) = 2.
\]

The average value of the momentum carried by the nucleons is,

\[
<y> = \int_0^1 dy y f^{N/D}(y),
\]

which can be written in the form

\[
<y> = 1 - \delta_N,
\]

where \( \delta_N \) is that part of the deuteron momentum carried by the mesons. The value of \( \delta_N \) controls the magnitude of the binding effects. Since the function \( f^{N/D}(y) \) is strongly peaked at \( y = 1/2 \), in a first approximation one has \( f^{N/D}(y) \simeq 2\delta(y - 1/2) \), resulting
in $\delta_N = 0$. In this case the deuteron is assumed to be made of two non-interacting quasi-free nucleons, where binding effects are neglected. This approximation leads to $F_2^D(x) \simeq 2 F_2^N(2x)$, and corresponds to a non-smearing motion of the nucleons. A better estimative of $F_2^D$ can be obtained if we expand $F_2^N$ around $y = <y>/2$ in the integrand of Eq.(8). The result is given by

$$F_2^D(x) = 2 F_2^N(2x/y) + \frac{1}{2} (y^2) - \frac{y^2}{2} \frac{\partial^2 F_2^N(x/y)}{\partial y^2} |_{y = <y>/2} + \ldots$$  \hspace{1cm} (13)$$

3 Deuteron vertex

In relativistic field theory we can describe the deuteron by the Bethe–Salpeter (BS) vertex $\chi_s$, where $s = -1, 0, 1$ is the deuteron helicity. This function is the Fourier transform of matrix elements involving the nucleon fermionic fields,

$$\chi_s = \int d^4 x e^{ik.x} <0 | T\Psi(x/2)\Psi(-x/2) | P_D, s > \hspace{1cm} (14)$$

and satisfies the homogeneous BS equation [19]. It depends on the deuteron total momentum $P_D$ and on the nucleons relative momentum $k$. According to the Gross on-mass-shell prescription [16] this vertex can be parameterized by,

$$\chi_s = F \xi + \frac{G}{m_N} (k.\xi) + \frac{(P_D/2 - \bar{k} - m_N)}{m_N} (H \xi + \frac{I}{m_N} (k.\xi)),$$  \hspace{1cm} (15)

where $\xi_{\mu}(P_D, s)$ are the deuteron polarization functions, a set of spin dependent space–like orthogonal vectors satisfying the following identities

$$\xi_{\mu}(P_D, s) P_D^\mu = 0 \hspace{1cm} \xi^*(P_D, s') \xi_{\mu}(P_D, s) = -\delta_{s's} \hspace{1cm} (16)$$

The scalar amplitudes $F, G, H$ and $Y$ depend only on the variable $a = [(P_D.k)/m_D]^2 - k^2$ and are determined covariantly by solving the BS equation with one particle on-mass-shell. The solutions can be found in Ref.[20].

The NN interaction used involves the meson exchange ($\sigma, \omega, \pi, \rho$) with the parameterization taken from the first article in Ref.[16]. We also consider two different couplings for the pion namely, a pseudovector (PV) and a mixed pseudoscalar–pseudovector (PS–PV) coupling, which is characterized by the parameter $\lambda \in [0,1]$ defined in the pion–nucleon vertex as

$$g_\pi \left( \lambda \gamma^5 + (1 - \lambda) \frac{ \not{q}}{2m_N} \gamma^5 \right)$$

where $g_\pi$ is the pion coupling constant. The PV coupling leads to a deuteron wave function consisting only of S and D states. The percentage of P states is negligible for small $\lambda$ but increases with $\lambda$ and for the PS–PV coupling corresponding to $\lambda = 0.4$ is 0.45%. In this case the interference of the P and S states can lead to a significant effect. In our calculations we consider two solutions, one obtained with $\lambda = 0$ (BS0) and the other with $\lambda = 0.4$ (BS04). Both solutions are consistent with the deuteron static properties.
4 Results and Conclusions

The extraction of the neutron structure function from combined experimental data on the proton and deuteron can now be discussed. The formalism presented here indicates that one should use Eq. (8)–(9) to analyse this problem consistently in the large $Q^2$ region where the Wilson operator product expansion is reliable. The on–shell condition given in Eq. (6) corresponds to a well established and successful procedure in nuclear physics. In order to get some physical insight into the corrections introduced by Eq. (9) one has to compare the results obtained by different approaches. In Fig. (1) we present the results of the ratio $F_D^2/2F_N^2$ as a function of $x_N = (m_D/m_N)x$, for the Bethe–Salpeter formalism with a simpler deuteron model (BSsm) [11], a calculation using a non–relativistic deuteron wave function (NR) [10], a light–cone calculation (LC) [21] and finally the Bethe–Salpeter (BS) formalism with one particle on–mass–shell. In the latter calculations we used the convolution formula (8) with the corresponding distribution function given by Eq. (9), while in the other approaches the results were obtained using the expansion (13). The nucleon structure function $F_N^2$ is taken from Ref. [13].

It is clear that for small values of $x_N$ all different approaches agree. However, for $x_N > 0.3$ the deviation between the different models increases. The main reason for these differences can be attributed to binding effects. In fact, the value of $\delta_N$ differs substantially in all approaches. It goes from $\delta_N = 0$ in LC calculations to $\delta_N \simeq 5.0 \times 10^{-3}$ in NR calculations with the intermediate value $\delta_N \simeq 3.9 \times 10^{-3}$ for the BSsm. In our calculations we obtain $\delta_N \simeq 7.2 \times 10^{-3}$ and $\delta_N \simeq 6.3 \times 10^{-3}$ for the BS04 and BS0 models, respectively.

The deviation between the two curves BS0 and BS04 is an effect of the deuteron P states, which are present only on the last one. In particular, the BS04 result deviates from all the other calculations even for small values of $x$. This is not surprising since none of those models consider the presence of deuteron P states. The present calculations indicate that there is an ambiguity in the extraction of the neutron structure function from the combined deuteron and proton data. The procedure is model dependent and particularly sensitive to the features of the deuteron wave function.

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Figure 1: Theoretical predictions for the ratio $F_{2D}^D/2F_{2N}^N$ using four different approaches. The curves tagged by BS0 and BS04 are the results of the present calculation. In the first one, we consider a pseudo–vectorial coupling for the pion–nucleon interaction and in the second one a mixed coupling (pseudo–vectorial plus pseudo–scalar) as described in the text. The remaining curves result from calculations using the simplified deuteron BS wave function of Ref.[11] (BSsm), the non–relativistic deuteron wave function of Ref.[10] and the light–cone approach of Ref.[21].
