From the Field Electron Model to the Unified Field Theory

1 Definition of Field Sources.

Maxwell-Lorentz equations for potentials are taken as the basis. Left parts of equations remain unchanged and are considered precise, but field sources in the right part a given a new definition. Spherically symmetrical electron at rest is described in the spherical coordinate system, $S$. There are two sets of currents with different charge signs in each point:

$$\vec{J}_\pm(R, \psi) = \rho_\pm(R, \psi) \cdot \vec{v}(R, \psi), \quad v_\pm(\infty, \psi) = 1, \quad \lambda = \cos \psi = \frac{(\vec{v}, \vec{R})}{vR},$$

and the system of units in which the velocity of light is equal to one is selected.

These currents form stationary current threads of moving Ons — marked elements of continuous charged medium. Continuity equation is satisfied for each charge sign independently. The ends of each current thread rest on infinity and each thread has an opposite one, i.e. coinciding with the direct one to within velocity sign or time.

When coming from the infinity where the field is equal to zero, charged Ons are affected by the field of electron $A_e$ and form pairs of direct and opposite current threads in accordance with equation of On motion in the field

$$J_a = D(A_e).$$

Current thread pairs generate field in correspondence with Maxwell-Lorentz equations

$$A_a = M(J_a),$$

and the sum of all fields generated by all current threads of two charge signs gives us an electron field

$$A_e = \sum_a A_a.$$

An electron is declared as a pair one element of which is the field, $A_e$, and the other element — the whole set of current threads, $J_a$, of two charge signs. Each element of the pair gives birth to itself through its partner.

$$\begin{cases} A_e = \sum_a M(D(A_e)) \\ J_a = D(\sum_a M(J_a)) \end{cases}$$

Operator $M$ affecting $J_a$ gives the field coinciding in form with the integral of sum of retarded and advanced potentials of Lienar-Vichert.
But now, each of two summands is an average of an retarded field of an On of one current thread and an advanced field of an On of an opposite current thread of this pair. Both summands have equal rights and their division into retarded and advanced terms becomes arbitrary.

The natural character and non-eliminability of $M$ operator symmetry relative to time reversal is the strongest reason for accepted definition of field sources, the properties of which are in conformity with the necessity to make vanish the sum of currents in each point of electron at rest and are guaranteed by the invariance of the law of motion with respect to time reversal.

The law of motion is declared a gnosiological derivative of the fundamental fact of existence of identical electrons with all their properties of complete objects in the terms of Maxwell-Lorentz equations with accepted definitions of sources (A) and the system of equations (B).

Formally, the law of motion $D$ is a possible solution of the system of equations, (B), if all values are stationary and spherically symmetrical, currents are uniform and isotropic at the infinity, and $A_x$ asymptotically approaches a Coulomb field for large $R$.

2 The Law of Motion.

An arbitrary motion may be approximated by tangent uniformly accelerated motion when the acceleration is constant in an instant accompanying reference system $\Sigma$, as the simplest one after inertial motion.

Suppose, as a first approximation, that this symmetry is global and all currents in the whole space have the same $W_{\Sigma}$. This $W_{\Sigma}$ is made equal to one by selecting an appropriate system of units.

Three-dimensional equation of such motion

$$(1 - v^2)\ddot{v} + 3(\dot{v}, \ddot{v})\dot{v} = 0$$

is transformed into the following system in the cylindrical coordinate system:

$$\begin{align*}
\ddot{v}(t) &= \{v_x, v_y, 0\} = \{v \cos \alpha, v \sin \alpha, 0\}, \\
\dot{\alpha} &= \chi \frac{(\sqrt{1 - v^2})^3}{v^2}, \chi = \text{const}, \\
\dot{v}^2 &= \frac{(\sqrt{1 - v^2})^3}{v^2}((1 + \chi^2)v^2 - \chi^2).
\end{align*}$$

The special solution is selected such that:

1) for $t \to +\infty$ the velocity is parallel to $OX$;
2) minimum vertex distance $\vec{s}$ and vertex velocity $\vec{\beta}$ are simultaneous at $t = 0$ and orthogonal.

Then the constant is fixed

$$\chi = \mp \beta \gamma, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$
and the solution takes the following form:

\[ \vec{r}(t) = (A \mp \sqrt{1 + \theta^2}) \frac{\vec{s}}{s} + t\vec{\beta}, \quad \vec{v}(t) = \mp \frac{1}{\gamma \sqrt{1 + \theta^2}} \frac{\vec{s}}{s} + \vec{\beta}, \]

\[ \frac{\vec{s}}{s} = \begin{cases} \frac{\pm 1}{\gamma}, \beta, 0 \end{cases}, \quad \frac{\vec{\beta}}{\beta} = \begin{cases} \beta, \pm \frac{1}{\gamma} \end{cases}, \]

\[ A = s \pm 1, \quad \theta = \frac{t}{\gamma}, \quad \Gamma(t) = \frac{1}{\sqrt{1 - v^2}} = \gamma \sqrt{1 + \theta^2}. \]

Suppose that the upper sign in symbols \( \pm \) and \( \mp \) represents positively charged currents, and the lower one — negatively charged. Vertex velocities \( \beta(s) \) increase continuously from \( \beta(1 \mp 1) = 0 \) to \( \beta(\infty) = 1 \). The domain of radius 2 in the origin is unapproachable for negatively charged currents.

Motion in accordance with (G) may be represented in the form:

\[ \vec{W} = \mp \frac{1 - v^2}{A \sqrt{1 + \theta^2} \mp 1} (\vec{r} - t\vec{v}), \quad t = (\vec{r}, \vec{v}) \cdot \left( 1 \pm \frac{A}{\gamma \sqrt{1 + \theta^2} \mp \frac{A}{\gamma}} \right), \]

being similar to the expression:

\[ \vec{W} = \frac{\sqrt{1 - v^2}}{\mu} \left( \frac{d\vec{p}}{dt} - \left( \frac{d\vec{p}}{dt} + (\nabla \mu) \sqrt{1 - v^2} \vec{v} \right) \vec{v} \right), \]

obtained from differentiated with respect to \( t \) inequality

\[ \vec{p} = \frac{\mu}{\sqrt{1 - v^2}} \vec{v}, \]

that coincides with the classic law of motion of a point charge in the field at \( \mu = \text{const} \).

The motion of an On in the field as of an infinitely small element of a current thread cannot depend of the value of its charge \( \Delta q \), and mass and pulse in the following form may be assigned to it:

\[ \Delta \mu = |\Delta q| \cdot \mu(t), \quad \Delta \vec{p} = \frac{\Delta \mu}{\sqrt{1 - v^2}} \vec{v}. \]

Applying the law of conservation of impulsive moment for the motion of an On along current thread in spinless approximation and requiring

\[ \frac{\mu(+\infty)}{\sqrt{1 - v^2}} = 1, \]

independently of \( y(+\infty) \), and selecting appropriate mass unit, we obtain:

\[ \mu(t) = \frac{1}{\gamma \sqrt{1 + \theta^2} \mp \frac{\gamma}{A}}. \]

For vertex velocities satisfying the condition

\[ \gamma = A = s \pm 1, \]

\[ \text{(E)} \]
the whole expression is simplified:

\[ \Gamma = r \pm 1, \quad \Delta \mu = \frac{|\Delta q|}{r}. \]

Expressions (C) and (D) become identical if:

\[ \frac{d\vec{p}}{dt} = \Delta \vec{q} \vec{E}, \quad \vec{E} = -\frac{1}{r(r \pm 1)} \vec{r}, \]

that justifies hyperbolic current field for large \( r \).

Asymptotically similar expression may be obtained from the Lagrangian

\[ L = -|\Delta q||\varphi - \vec{v} \cdot \vec{A}|_{\Sigma} - \Delta q|\varphi - \vec{v} \cdot \vec{A}|_{S}, \]  

which is obtained from classic using substitution \( m_0 \to |\Delta q|\varphi \).

The first terms is taken in instant accompanying reference system \( \Sigma \), and the second — in the system of inertia center \( S \). Then, it is necessary to make a substitution \( \varphi \to \varphi - (\vec{v}, \vec{A}) \)

in each term of (L) and proceed to density.

### 3 Field Generated by Currents.

Application of continuity equation to current field (G) gives us charge density along current thread:

\[ \rho(t, s)\sqrt{1 - v^2} = 1 \pm \frac{\gamma}{\Gamma} \frac{1}{\frac{1}{\gamma} + t}, \]

where normalization is carried out securing independence of this expression of \( y(+\infty) \) at the infinity, and corresponding charge unit is selected. Namely this expression

\[ \rho_S = \rho(t, s)\sqrt{1 - v^2} \]

is the charge density in the system of electron center of mass \( S \), but this question is worth separate consideration.

For vertex velocities (E), differential charge density is equal:

\[ d\rho_S(r, \lambda) = \frac{1}{r} \left( 1 \pm \frac{1}{\Gamma} \frac{1}{1 + v\lambda} \right) \Gamma d(v\lambda). \]

Scalar potential at the distance \( R \) from the center of electron generated by currents from the spherical layer of radius \( r \) with the center in the origin, omitting elementary integration with respect to angle variable, which currents are not dependent of, and a numerical factor, is equal to:

\[ d\varphi = \frac{1}{R} \left( \int \lambda J(\alpha, v, \lambda)d\rho_S(r, \lambda) \right) 4\pi r^2 dr, \]
\[ J(\alpha, v, \lambda) = \frac{1}{4} \int_{-1}^{1} \left( \frac{1}{\sqrt{A_+}} + \frac{1}{\sqrt{A_-}} \right) d\sigma, \]  

\[ A_{\pm} = B_{\pm}^2 - v^2(1 - \lambda^2)(1 - \sigma^2), \quad B_{\pm} = \sqrt{1 + \alpha^2 + 2\alpha\sigma \pm v\lambda(\sigma + \alpha)}, \quad \alpha = \frac{r}{R}. \]

For \( r \leq R \), the expression \((J)\) is degenerates into elementary integral

\[ J(\alpha \leq 1, v, \lambda) = \frac{1}{2v} \ln \left( \frac{1 + v}{1 - v} \right), \quad v = v(r, \lambda), \]

which becomes equal to one at \( v = 0 \), that was the reason why unit charge constant was omitted in \((J)\). For inner points, when \( R < r \), \((J)\) is substantially elliptical and the following integral inequality is satisfied:

\[ \alpha \int_{0}^{1} J(\alpha > 1, v, \lambda) d\lambda = \frac{1}{2v} \ln \left( \frac{1 + v}{1 - v} \right). \]  

Unexpectedly, reverse field has appeared in the inner volume when \( 0 \leq R < r \) with the sign of intensity opposite to intensity of direct field in the outer domain, when \( r \leq R < \infty \).

The intensity of electron field is he sum of intensities of direct and reverse fields of currents from spherical layers covering the whole space.

Vertex velocities \((E)\) do not give needed field.

Approximate calculations of vertex velocities defined by the condition

\[ \gamma = \sqrt{s^2 - 1 \pm 2}, \]  

of field intensity near zero result in the function close to linear:

\[ E(R \ll 1) = \text{const} \cdot R \]

and more realistic field for \( R \gg 1 \).

Consideration of the spin when applying the law of conservation of \( \Omega \) moment for motion along current thread in the hyperbolic approximation for currents changes the mass at rest \( \Delta \mu \) making it less singular and accordingly decreasing the intensity of field necessary to generate \( \Omega \) currents in the vicinity of the center.

The beauty of Lagrangian \((L)\) requires calculations of field and currents on its base.

Interpretation of the hyperbolic solution \((G)\) has been made assuming that the integral of intensities of reversed fields generated by spherical layers from \( R \) to \( \infty \) exceeds the integral from \( 1 \mp 1 \) to \( R \) for all \( R \) by absolute value. This represents the field of electron similar to the field of stationery negative charge distributed near the center as the residue of prevailing reverse field of positively charged currents. Such concept of electron field structure seems to be preferable.

The model obtained by substitution: \((\pm) \to (\mp)\) not affecting signs of electron field and its mass cannot be excluded yet.
Appendix.

Expansion the integral (J) in an infinite series takes the following form:

\[
J(\alpha > 1, v, \lambda) = \frac{1}{\alpha P} \left\{ \left(1 + \frac{1}{3}\right) \delta^2 + \left(\frac{1}{5} + \frac{Q}{3} p_{10}\right) \delta^4 + \left(\frac{1}{7} + \frac{Q}{5} p_{12}\right) \delta^6 + \left(\frac{1}{9} + \frac{Q}{7} p_{14} + \frac{Q^2}{5} p_{20}\right) \delta^8 + \cdots \right\},
\]

\[
P = 1 - v^2 \lambda^2 b, \quad b = \frac{\alpha^2 - 1}{\alpha^2}, \quad Q = 4(\alpha^2 - 1)\lambda^2(1 - \lambda^2), \quad \delta^2 = \frac{v^2}{\alpha^2 P},
\]

\[
p_{10} = 1,
\]
\[
p_{12} = \frac{2 \cdot 3}{1 \cdot 2} \left(1 - \frac{2 \cdot 1}{1 \cdot 3}\right) \lambda^2,
\]
\[
p_{14} = \frac{3 \cdot 4}{1 \cdot 2} \left(1 - \frac{2 \cdot 2}{1 \cdot 3}\lambda^2 + \frac{2 \cdot 4 \cdot 1}{1 \cdot 3 \cdot 5}\lambda^4\right),
\]
\[
p_{16} = \frac{4 \cdot 5}{1 \cdot 2} \left(1 - \frac{2 \cdot 3}{1 \cdot 3}\lambda^2 + \frac{2 \cdot 4 \cdot 3}{1 \cdot 3 \cdot 5}\lambda^4 - \frac{2 \cdot 4 \cdot 6 \cdot 1}{1 \cdot 3 \cdot 5 \cdot 7}\lambda^6\right),
\]
\[
\ldots = \ldots;
\]
\[
p_{20} = 1,
\]
\[
\ldots = \ldots;
\]
\[
\ldots = \ldots.
\]

Differentials of intensities of direct and reverse fields generated by currents from spherical layers take the following forms:

\[
(r \leq R): \quad d\vec{E}_\pm(R, r) = \pm \frac{4\pi}{R^2} \int_\lambda \frac{1}{2v} \ln \left(\frac{1 + v}{1 - v}\right) rd\psi rdr \frac{R}{r},
\]
\[
(R < r_\ast): \quad d\vec{E}_\pm(R, r_\ast) = \pm 8\pi R \int_\lambda R(\alpha, v, \lambda) r_\ast d\rho \frac{d\psi}{r_\ast^2} \frac{R}{R},
\]
\[
R(\alpha, v, \lambda) = A_{20} v^2 + 2(A_{40} - bA_{41}) v^4 + 3(A_{60} - 2bA_{61} + b^2 A_{62}) v^6 + \cdots, \quad (J^0)
\]

\[
A_{20} = \frac{1}{3} - \lambda^2,
\]
\[
A_{40} = \frac{1}{5} - \lambda^2 + \frac{2 \cdot 1}{1 \cdot 3}\lambda^4,
\]
\[
A_{60} = \frac{1}{7} - \lambda^2 + \frac{2 \cdot 2}{1 \cdot 3}\lambda^4 - \frac{2 \cdot 4 \cdot 1}{1 \cdot 3 \cdot 5}\lambda^6,
\]
\[
A_{80} = \frac{1}{9} - \lambda^2 + \frac{2 \cdot 3}{1 \cdot 3}\lambda^4 - \frac{2 \cdot 4 \cdot 3}{1 \cdot 3 \cdot 5}\lambda^6 + \frac{2 \cdot 4 \cdot 6 \cdot 1}{1 \cdot 3 \cdot 5 \cdot 7}\lambda^8,
\]
\[
\ldots = \ldots;
\]
\[
\ldots = \ldots.
\]

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