Passive control for a class of T-S fuzzy systems with memory controller

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Abstract. Taking the advantage of fuzzy systems, a class of T-S fuzzy models used to describe nonlinear systems. Firstly, by designing a memory state feedback controller, we draw the conclude of robust stability and the conditions of passive control. Secondly, using Lyapunov stability and linear matrix inequality technology in MATLAB, we can utilize the gain matrix about the controller of the memory state feedback. Finally, a simulation example is given to illustrate the effectiveness of the proposed method.

1. Introduction

In some industrial systems, there exist some uncertainties and time-delay phenomena of controlled variables. As we know, norm boundaries can be used to describe the uncertainties in the systems. For the time-delay phenomena of controlled variables, we can use memory control to achieve the related goals of the systems. Some scholars have done relevant research on this problem [1] [2] [3] [4]. In recent years, passive control of uncertain systems has drawn the attention of scholars. Passivity provides a new research direction for the study of non-linear systems, and corresponding results emerge endlessly [5] [6]. Based on the advantages of passivity, passive control has been applied to a variety of dynamic systems.

For example, in reference [5], the discrete hybrid power system is given in the form of piecewise affine and piecewise polynomial. There for, sufficient conditions for passivity analysis and synthesis are given by piecewise quadratic function and piecewise polynomial function. Reference [6] studies passivity performance analysis based on networked control systems, and considers network-induced delay, packet loss and quantization. By using Lyapunov-Krasovskii function method and free weight matrix technology, a passive controller is designed to satisfy the specified passivity performance. In reference [7], a class of time-delay neural networks is studied. Based on Lyapunov-Krasovskii function method, neuron activation function and time-varying delay are introduced, and a new delay-dependent passivity condition is given in the form of LMI. In reference [8], passive sliding mode control is studied for a class of uncertain nonlinear singular time-delay systems. By introducing relaxation matrix, sufficient conditions for the system to satisfy the dynamic quadratic stability and robust passivity of sliding mode are given in the form of LMI. In this paper, a memory state output feedback controller is designed for a class of T-S fuzzy systems with uncertainties. The robust passive control of T-S fuzzy systems is studied by using Lyapunov stability and linear matrix inequality (LMI) technique.
2. Preliminaries and problem formulation

Lemma 1 [9]: Matrices of $U, V$ and $F$ are achieved, for any scalar $\varepsilon > 0$, on the condition of $F^TF \leq I$, we have

$$UFV + V^T F^T U^T \leq \varepsilon UU^T + \varepsilon^{-1} V^T V$$

The total fuzzy model is described as follows [2].

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(\theta) [(A_i + \Delta A_i)x(t) + B_iu(t) + B_{i2}\omega(t)]$$

$$y(t) = \sum_{i=1}^{n} C_i x(t)$$

where $y(t) \in \mathbb{R}^q$ is the output of system, $\omega(t)$ is external perturbation input, $C_i (i = 1, \ldots, m)$ is digital matrices.

Provided the following fuzzy controller with the function of memory state feedback is used

$$u(t) = \sum_{i=1}^{n} \mu_i(\theta) [K_i x(t) + K_{i2} x(t - \tau(i))]$$

where $K_i, K_{i2} \in \mathbb{R}^{n \times n}$ are the gain matrix, from (2).

By (2) and (1), the closed-loop system can be composed by the following equation:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(\theta) \sum_{j=1}^{n} \mu_j(\theta) [\bar{A}_i x(t) + B_iu(t) + B_{i2}\omega(t)]$$

$$y(t) = \sum_{i=1}^{n} C_i x(t)$$

let $\bar{A}_i = A_i + \Delta A_i$

Definition: For a fuzzy system (3), if there is a constant $\lambda > 0$, the following inequalities are always established

$$2\int_{0}^{T} y(t)^T w(t) dt \geq -\lambda \int_{0}^{T} w(t)^T w(t) dt$$

Then the fuzzy system (3) is strictly passive.

3. Main results

Theorem 1: For a given gain matrix $K_i, K_{i2} (i = 1, \ldots, m)$ with memory state feedback, if there are positive definite matrices $P, Q$ and constants $\lambda \geq 0$, all admissible uncertainties about the closed-loop system (3) make the following linear matrix inequality (5) hold, then the system (3) is passive.

$$\begin{bmatrix}
P\bar{A}_i + \bar{A}_i^T P + PB_i K_{i2} + K_{i2}^T B_i^T P + Q & PB_i K_{i2} & PB_{i2} + \bar{A}_i^T C_i^T + K_{i2}^T B_{i2}^T C_i^T \\
K_{i2}^T B_{i2}^T P & -Q & K_{i2}^T B_{i2}^T C_i^T \\
C_i A_i + B_{i2} K_{i2} + B_{i2}^T P & C_i B_{i2} K_{i2} & -\lambda I
\end{bmatrix} < 0$$

Proof: we construct Lyapunov function candidate:

$$V(x(t)) = V_1(x(t)) + V_2(x(t))$$

where $V_1(x(t)) = x(t)^T P x(t), V_2(x(t)) = \int_{t-\tau}^{t} x(s)^T Q x(s) ds$

The time derivative of $V(x(t))$ is given by

$$\dot{V}(x(t)) = \dot{V}_1(x(t)) + \dot{V}_2(x(t))$$

$$= \sum_{i=1}^{n} \mu_i(\theta) \sum_{j=1}^{n} \mu_j(\theta) [2x(t)^T P(\bar{A}_i + B_i K_{i2}) x(t) + 2x(t)^T P B_i K_{i2} x(t - \tau) + 2x(t)^T P B_{i2} \omega(t) + x(t)^T Q x(t - \tau) - x(t - \tau)^T Q x(t - \tau)]$$

$$= \sum_{i=1}^{n} \mu_i(\theta) \sum_{j=1}^{n} \mu_j(\theta) \begin{bmatrix} x(t) \\ x(t - \tau) \\ \omega(t) \end{bmatrix} \text{sym} \begin{bmatrix} x(t) \\ x(t - \tau) \\ \omega(t) \end{bmatrix}$$
where $\Pi = \begin{bmatrix} P\bar{A} + \bar{A}^T P + PB_iK_{ij} + K_{ij}^T B_i^T P + Q & PB_{i,2j} & PB_i \\ K_{ij}^T B_i^T P & -Q & 0 \\ B_{i,2j}^T P & 0 & 0 \end{bmatrix}$

Hence, for $\forall t \geq 0$, we have

$$\dot{V}(x(t)) - 2\gamma(t)^T w(t) - \lambda o(t)^T o(t) = \sum_{i=1}^{m} \mu_i(\theta) \sum_{j=1}^{m} \mu_j(\theta) \begin{bmatrix} x(t) \\ o(t) \end{bmatrix}^T \Lambda \begin{bmatrix} x(t) \\ o(t) \end{bmatrix}$$

where $\Lambda = \begin{bmatrix} P\bar{A} + \bar{A}^T P + PB_iK_{ij} + K_{ij}^T B_i^T P + Q & PB_{i,2j} + \bar{A}_i^T C_i + K_{ij}^T B_i^T C_i^T \\ K_{ij}^T B_i^T P & -Q & K_{ij}^T B_i^T C_i^T \\ C_i A_i + C_i B_i K_{ij} + B_{i,2j}^T P & C_i B_{i,2j} & -\lambda I \end{bmatrix}$

From formula (5) of the theorem

$$\dot{V}(x(t)) - 2\gamma(t)^T w(t) - \lambda o(t)^T o(t) < 0$$

From zero initial condition, we can get the integral from zero to $T(T > 0)$, on both sides of the equation.

$$2\int_0^T \gamma(t)^T w(t) dt + \lambda \int_0^T o(t)^T o(t) dt > V(x(T)) \geq 0$$

so, we have

$$2\int_0^T \gamma(t)^T w(t) dt + \lambda \int_0^T o(t)^T o(t) dt > \dot{V}(x(T)) \geq 0$$

According to the definition of passivity, closed-loop system (3) is strictly passive.

**Theorem 2**: Consider uncertain system (3), if there are some asymmetric positive definite matrices $\bar{P}$ and $\bar{Q}$, matrix $\bar{R}_{i}$ and $\bar{K}_{2j}$, some positive constants $\varepsilon_i$ ($i = 1, 2, \cdots, m$) and $\bar{\lambda}$, satisfying the following LMIs:

$$
\begin{bmatrix}
A\bar{P} + \bar{A}^T \bar{P} + B_i\bar{K}_{ij} + K_{ij}^T B_i^T \bar{P} + \bar{Q} & B_{i,2j} & B_2 + \bar{A}_i^T C_i + K_{ij}^T B_i^T C_i^T \\
K_{ij}^T B_i^T \bar{P} & -\bar{Q} & K_{ij}^T B_i^T C_i^T \\
C_i A_i + C_i B_i \bar{K}_{ij} + B_{i,2j}^T \bar{P} & C_i B_{i,2j} & -\lambda I
\end{bmatrix} < 0
$$

Then (2) is a memory state feedback fuzzy controller of the fuzzy system (1) and a passive controller of the system.

**Proof**: firstly, it is derived from Formula (5) of Theorem 1.

$$
\begin{bmatrix}
P A_i + \bar{A}_i^T P + PB_iK_{ij} + K_{ij}^T B_i^T P + Q & PB_{i,2j} & PB_i + \bar{A}_i^T C_i + K_{ij}^T B_i^T C_i^T \\
K_{ij}^T B_i^T P & -Q & K_{ij}^T B_i^T C_i^T \\
C_i A_i + C_i B_i K_{ij} + B_{i,2j}^T P & C_i B_{i,2j} & -\lambda I
\end{bmatrix} + \begin{bmatrix} P\Delta A_i + \Delta \bar{A}_i^T P & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} < 0
$$

by $\Delta A_i = H_i F_i(t) E_i$ [2], according to schur lemma[10] and lemma 1, (8) satisfies

$$
\begin{bmatrix}
P A_i + \bar{A}_i^T P + PB_iK_{ij} + K_{ij}^T B_i^T P + Q & PB_{i,2j} & PB_i + \bar{A}_i^T C_i + K_{ij}^T B_i^T C_i^T \\
K_{ij}^T B_i^T P & -Q & K_{ij}^T B_i^T C_i^T \\
C_i A_i + C_i B_i K_{ij} + B_{i,2j}^T P & C_i B_{i,2j} & -\lambda I
\end{bmatrix} \leq \begin{bmatrix} P & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

By diagonal matrix $\text{diag}\{P^{-1}, P^{-1}, I, \varepsilon_i, I\}$, let $\bar{P} = P^{-1}$, $\bar{Q} = P^{-1} Q P^{-1}$, $\bar{K}_{ij} = K_{ij} P^{-1}$, $\bar{K}_{2j} = K_{2j} P^{-1}$, according to the right of (9).
We let
\[
\overline{\lambda} = \begin{bmatrix}
A_{i} P + \overline{P} A_{i}^{T} + B_{i} K_{i,j} + \overline{K}_{i,j}^{T} B_{i}^{T} + \overline{Q} & B_{i} K_{i,j} + B_{i}^{T} C_{i}^{T} + K_{i,j}^{T} B_{i}^{T} C_{i}^{T} & e_{i} E_{i} & e_{i} E_{i}^{T} & -\epsilon I \\
C_{i} A_{i} P + C_{i} B_{i} K_{i,j} + B_{i}^{T} C_{i}^{T} & -\overline{Q} & \overline{K}_{i,j}^{T} B_{i}^{T} C_{i}^{T} & 0 & 0 \\
\epsilon_{i} E_{i}^{T} & 0 & 0 & -\epsilon I \\
H_{i}^{T} \overline{P} & 0 & 0 & 0 & -\epsilon I \\
\end{bmatrix},
\]
by (7). \(\overline{\lambda} < 0\), so the fuzzy system (1) is robust stable under the controller (2).

4. Numerical example

Considering the system (3) is composed by the two rules, where
\[
A_{1} = \begin{bmatrix}
5 & 0 \\
1 & 8 \\
\end{bmatrix}, \quad A_{2} = \begin{bmatrix}
2 & 1 \\
0 & 5 \\
\end{bmatrix}, \quad B_{1} = \begin{bmatrix}
3 & 0 \\
0 & 2 \\
\end{bmatrix}, \quad B_{2} = \begin{bmatrix}
3 & 0 \\
0 & 3 \\
\end{bmatrix}, \quad B_{21} = \begin{bmatrix}
4 & 0 \\
0 & 4 \\
\end{bmatrix}, \quad B_{22} = \begin{bmatrix}
6 & 0 \\
0 & 3 \\
\end{bmatrix},
\]
\[
H_{1} = H_{2} = \begin{bmatrix}
3 & 0 \\
0 & 3 \\
\end{bmatrix}, \quad E_{1} = E_{2} = \begin{bmatrix}
2 & 0 \\
0 & 2 \\
\end{bmatrix}, \quad C_{1} = C_{2} = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}, \quad \tau = 0.2
\]

We have the following feasible solution
\[
\overline{P} = \begin{bmatrix}
10.3918 & -0.5094 \\
-0.5094 & 4.7184 \\
\end{bmatrix}, \quad \overline{Q} = \begin{bmatrix}
268.8647 & 0.0737 \\
0.0737 & 227.5302 \\
\end{bmatrix}, \quad K_{21} = 1.0e-008 \begin{bmatrix}
-0.8569 & 0 \\
0 & -0.2436 \\
\end{bmatrix},
\]
\[
K_{22} = \begin{bmatrix}
-0.188032 & -2.2003 \\
-2.3479 & -48.0299 \\
\end{bmatrix}, \quad K_{23} = 1.0e-008 \begin{bmatrix}
-0.9674 & 0 \\
0 & -0.1311 \\
\end{bmatrix}, K_{24} = \begin{bmatrix}
-1.188032 & -2.2003 \\
-2.3479 & -48.0299 \\
\end{bmatrix},
\]
\[
\epsilon_{1} = 75.4001, \epsilon_{2} = 98.7694, \lambda = 900.2704.
\]

We give the initial conditions as
\[
x(0) = \begin{bmatrix}
-1 \\
-1 \\
\end{bmatrix}, \quad F_{1}(t) = \sin(\pi t) \begin{bmatrix}
0.5 \\
0 \\
\end{bmatrix}, \quad F_{2}(t) = \sin(\pi t) \begin{bmatrix}
0.5 \\
0 \\
\end{bmatrix}, \quad \alpha(t) = 0.4 \sin(10 \pi t) \begin{bmatrix}
1 \\
1 \end{bmatrix}, \quad \alpha_{1}(t) = 1 - \alpha(t) \begin{bmatrix}
1 \\
1 \\
\end{bmatrix}.
\]

The following membership functions are chosen \(\alpha_{2}(t) = 1 - \alpha_{1}(t)\), \(\alpha_{1}(t) = 1/(1 + e^{-\lambda x})\).

The state output is shown below.

![Figure 1](image)

Figure 1. where “- -” and “—“are the state \(x_{1}\) and \(x_{2}\) output.

5. Conclusion

As uncertainty and time-delay are common phenomena in practical systems, a T-S fuzzy system with state uncertainties is studied, and memory state feedback controller is used to solve the time-delay problem. By using the norm boundaries of uncertainties and designing a fuzzy memory state.
feedback controller, not only the robust stability of the system but also the passive control of the system is achieved.

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