The Higgs Mass as a Signature of Heavy SUSY

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Abstract
We compute the mass of the Higgs particle in a scheme in which SUSY is broken at a large scale $M_{SS}$ well above the electroweak scale $M_{EW}$. Below $M_{SS}$ one assumes one is just left with the SM with a fine-tuned Higgs potential. Under standard unification assumptions one can compute the mass of the Higgs particle as a function of the SUSY breaking scale $M_{SS}$. For $M_{SS} \gtrsim 10^{10}$ GeV one obtains $m_H = 126 \pm 3$ GeV, consistent with CMS and ATLAS results. For lower values of $M_{SS}$ the values of the Higgs mass tend to those of a fine-tuned MSSM with $m_H \lesssim 130$ GeV. These results support the idea that the measured value of the Higgs mass at LHC may be considered as indirect evidence for the existence of SUSY at some (not necessarily low) mass scale.
1 Introduction

The evidence \[1,2\] obtained by the CMS and ATLAS experiments at CERN of a scalar particle with the properties of a Standard Model (SM) Higgs particle with mass \(m_H \simeq 126\) GeV is a crucial piece of information to unravel the origin and characteristics of the Electroweak (EW) symmetry breaking. This mass value is compatible with the region allowed by the MSSM which is \(m_H \lesssim 130\) GeV. Still getting a value of the Higgs mass of order 125 GeV in the MSSM requires a certain amount of fine-tuning. On the other hand within the SM any value from the LEP bound up to almost 1 TeV could have been possible. Thus one might interpret the experimental results as pointing in the direction of some sort of (fine-tuned) SUSY.

Building on ideas discussed in \[3\], in the present paper we try to answer the following question. Imagine the SM is extended to the MSSM above a certain scale \(M_{SS}\) not necessarily tied to the EW scale, but possibly much higher. If that is the case, a fine-tuning of the underlying theory would be required in order for a Higgs doublet to remain massless. Under those circumstances, what would be the mass of the fine-tuned Higgs?

Although the question sounds too generic to have a sharp answer it turns out that under standard unification assumptions a concrete answer may be given. In particular, assuming the unification of Higgs mass parameters \(m_{H_u} = m_{H_d}\) at the GUT/String scale and a minimally fine-tuned Higgs below the SUSY breaking scale \(M_{SS}\), then one obtains a definite prediction for the Higgs mass as a function of the SUSY breaking scale. Although the experimental error from the top quark mass as well as the SUSY spectra introduce some degree of uncertainty, the results, exemplified in fig.\[1\] show that for \(M_{SS} \gtrsim 10^{10}\) GeV the value of the Higgs mass is centered around 126 ± 3 GeV. Below that scale this mass depends more on the details of the SUSY breaking mass parameters but the maximum value is bound by 130 GeV, corresponding to a standard fine-tuned MSSM with \(M_{SS} \simeq 10 – 100\) TeV.

This predictivity is remarkable, since the SM by itself would allow for a large range of consistent values with e.g. much higher values for the Higgs mass of order e.g. 150-300 GeV or higher. The fact that experimentally \(m_H \simeq 126\) GeV then renders strong support to the idea of SUSY being realized at some, possibly large, mass scale. Even if SUSY particles are not found at LHC energies the particular value of the Higgs mass points to an underlying SUSY at some higher scale. This is of course due to the fact that SUSY, even spontaneously broken at an arbitrarily high energy scale, relates de quartic Higgs selfcoupling to the EW gauge couplings.
2 Traces from high energy SUSY and a minimally fine-tuned Higgs

There is at present no experimental evidence at LHC for the existence of SUSY particles. This, combined with earlier experimental limits, severely constraint the idea of low energy SUSY and indicates the necessity of some degree of fine-tuning of parameters of the order of at least 1-0.1 percent \[4, 5\]. If no evidence of SUSY particles is found at the 14 TeV LHC the general idea of low-energy SUSY as a solution to the hierarchy/fine-tuning problem will be strongly questionable. On the other hand, as recently emphasized in \[3\], even if SUSY is not present at the EW scale to solve the hierarchy problem, there are at least three reasons which suggest that supersymmetry could be present at some scale \(M_{SS}\) above the EW scale and below the unification/string scale. The first is the fact that SUSY is a substantial ingredient of string theory which is, as of today, the only serious contender for an ultraviolet completion of the SM. The second reason is that SUSY guarantees the absence of scalar tachyons which are generic in non-SUSY string vacua. Thirdly, and independently from any string theory consideration, a detailed study of the non-SUSY SM Higgs potential consistent with the measured Higgs mass indicates that there is an instability at scales above \(\simeq 10^{10} \text{ GeV} \) \[6, 7\]. Although in principle one can live in a metastable vacuum, supersymmetry would stabilize the vacuum in a natural way if present at an energy scale \(\lesssim 10^{10} \text{ GeV}\).

Let us then consider a situation in which SUSY is broken at some high scale \(M_{SS}\) with \(M_{EW} \ll M_{SS} \ll M_C\), where \(M_C\) is the unification/string scale. For previous work on a fine-tuned Higgs in a setting with broken SUSY at a high scale see e.g.\[8, 9, 10, 11, 12\]. With generic SUSY breaking soft terms one is just left at low-energies with the SM spectrum. In addition the scalar potential should be fine-tuned so that a Higgs doublet remains light so as to trigger EW symmetry breaking. One would say that no trace would be left from the underlying supersymmetry. However this is not the case \[9\]. Since dimension four operators are not affected by spontaneous SUSY breaking, the value \(\lambda(M_{SS})\) of the Higgs self-coupling at the \(M_{SS}\) scale will be given in the MSSM by the (tree level) boundary condition

\[
\lambda_{SUSY}(M_{SS}) = \frac{1}{4}(g_2^2 + g_1^2) \cos^2 2\beta
\]

which is inherited from the D-term scalar potential of the MSSM. Here \(g_{1,2}\) are the EW gauge couplings and \(\beta\) is the mixing angle which defines the linear combination of the two \(SU(2)\) doublets \(H_u, H_d\) of the MSSM which remains massless after SUSY...
breaking, i.e.,

\[ H_{SM} = \sin \beta H_u + \cos \beta H_d^* . \]  

(2.2)

Thanks to this boundary condition, for any given value of \( \tan \beta \) one can compute the Higgs mass as a function of the SUSY breaking scale \( M_{SS} \).

Schematically the idea is to run in energies the values of \( g_1, g_2 \) up to the given \( M_{SS} \) scale. For any value of \( \tan \beta \) one then computes \( \lambda(M_{SS}) \) from eq. (2.1). Starting with this value we then run down in energies and obtain the value for the Higgs mass from \( m^2_H(Q) = 2v^2 \lambda(Q) \). Threshold corrections at both the EW and SUSY scales have to be included. This type of computation for different values of \( \tan \beta \) was done e.g in ref. [13], [14], [15], [16]. We show results for a similar computation in fig. (grey bands). The Higgs mass may have any value in a broad band below a maximum around 140 GeV. One may easily understand the general structure of these curves. The mass is higher for higher \( \tan \beta \) since the tree level contribution to the Higgs mass through eq. (2.1) is higher. On the other hand the Higgs mass slowly grows with larger \( M_{SS} \) as expected.

What we want to emphasize here is that the natural assumption of Higgs soft mass unification at the unification scale \( M_C \), i.e.

\[ m_{H_u}(M_C) = m_{H_d}(M_C) \]  

(2.3)

leads to a much more restricted situation with trajectories in the \( m_{higgs} - M_{SS} \) plane rather than a wide band. Note that this equality is quite generic in most SUSY, unification or string models. In particular it appears in gravity mediation as well as in almost all SUSY breaking schemes, including those arising from compactified string theory, see e.g. [17].

Indeed, to see this let us recall what is the general form for Higgs masses in the MSSM at the scale \( M_{SS} \),

\[
\begin{pmatrix}
H_u & H_d^*
\end{pmatrix}
\begin{pmatrix}
m^2_{H_u} & m^2_3 \
m^2_3 & m^2_{H_d}
\end{pmatrix}
\begin{pmatrix}
H_u^* \\
H_d
\end{pmatrix}.
\]  

(2.4)

where we will take \( m^2_3 \) real. If all these mass terms were zero we would get two Higgs doublets in the massless spectrum. However this would require extra unnecessary fine-tuning. The \textit{minimal Higgs fine-tuning} would only require a single Higgs doublet to remain at low-energies\footnote{This is a particular realization of the \textit{Extended Survival Hypothesis} of ref. [18] (see also [19]).}. This is achieved for a single fine-tuning \( m_4 = m^2_{H_u} m^2_{H_d} \). The massless eigenvector is then

\[ H_{SM} = \sin \beta H_u + \cos \beta H_d^* \]  

(2.5)
with
\[ \tan \beta = \frac{|m_{H_u}|}{|m_{H_d}|}. \]  
(2.6)

If the origin of this fine-tuning is understood in terms of the fundamental SUSY breaking parameters \( \text{s}\)\hspace{1mm}\( \text{c}\)\hspace{1mm}\hspace{1mm}\text{a}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{h}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{g}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{a}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{w}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{i}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{g}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{a}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{w}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{i}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{g}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{a}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{w}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{i}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{g}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{a}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{w}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{i}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}\text{n}\hspace{1mm}\hspace{1mm}\hspace{1mm}g\hspace{1mm}\hspace{1mm}\hspace{1mm}a\hspace{1mm}w\text{n}\hspace{1mm}n\hspace{1mm}a\hspace{1mm}g\hspace{1mm}a\wtext{w}\text{i}n\text{n}g\text{a}\wtext{a}w\text{i}n

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We will allow to vary the top mass with an error $m_t = 173.1 \pm 0.7$ GeV obtained from the average from Tevatron [20] and CMS and ATLAS results as in ref. [21]. We will neglect the error from $\alpha_3$ which is much smaller than that from the top quark mass. To extract the value of the top Yukawa coupling $h_t(m_t)$ we take into account the relationship between the pole top-quark mass $m_t$ and the corresponding Yukawa coupling in the $\overline{MS}$ scheme [22]

$$h_t(m_t) = \frac{m_t}{v}(1 + \delta_t)$$  \hspace{1cm} (3.3)

where the dominant one-loop QCD corrections may be estimated [22, 14, 16]

$$\delta_t^\text{QCD}(m_t) = -\frac{4}{3\pi}\alpha_3(m_t) - 0.93\alpha_3^2(m_t) - 2.59\alpha_3^3(m_t) \approx -0.0605.$$  \hspace{1cm} (3.4)

One then obtains $h_t(m_t) = 0.934$. We run now the couplings $g_1, g_2$ and $h_t$ up to the given scale $M_{SS}$. We do this by solving the RGE at two loops for the SM couplings. Those equations are shown for completeness in appendix A.

### 3.2 Computing $\tan\beta$ and $\lambda(M_{SS})$

With $g_{1,2}(M_{SS})$ at hand we want now to compute the value of $\lambda(M_{SS})$ from eq. (2.1). To do that we need to compute $\tan\beta(M_{SS})$ from eq. (2.6), which in turn requires the computation of the running of the masses $m_{H_u}, m_{H_d}$ from the unification scale at which $m_{H_u} = m_{H_d}$ down to $M_{SS}$.

The value of the unification scale $M_C$ is usually obtained from the unification of gauge coupling constants. In our case, with two regions respectively with the SM (below $M_{SS}$) and the MSSM (in between $M_{SS}$ and $M_C$) the value of $M_C$ is not uniquely determined. In fact it is well known that precise unification is only obtained for $M_{SS} \approx 1$ TeV, as in standard MSSM phenomenology [23]. However, approximate unification around a scale $M_C \approx 10^{14} - 10^{15}$ GeV is anyway obtained for much higher values of $M_{SS}$, even in the limiting case with $M_{SS} \approx M_C$ in which case SUSY is broken at the unification scale, so a simple approach would be to take $M_C \approx 10^{15}$ GeV to compute the running of $\tan\beta$. We find more interesting instead to achieve consistent gauge coupling unification from appropriate threshold corrections. In particular, in a large class of string compactifications like F-theory $SU(5)$ GUT’s there are small threshold corrections respecting the boundary condition at the GUT scale [24, 25, 3]

$$\frac{1}{\alpha_1(M_C)} = \frac{1}{\alpha_2(M_C)} + \frac{2}{3\alpha_3(M_C)}.$$  \hspace{1cm} (3.5)
This boundary condition is consistent (but more general) than the usual GUT boundary conditions $\alpha_3 = \alpha_2 = 5/3\alpha_1$. It arises for example from F-theory $SU(5)$ GUT’s \[^{20}\] once fluxes along the hypercharge direction are added to break the $SU(5)$ symmetry down to the SM \[^{25, 27}\]. Using the RGE for gauge couplings in both SM and MSSM regions (at two loops for the gauge couplings and one loop for the top Yukawa) one finds that unification of couplings is neatly obtained at a scale $M_C$ related with $M_{SS}$ by the approximate relationship

$$\log M_C = -0.23 \log M_{SS} + 16.77 .$$ (3.6)

As one varies $M_{SS}$ in the range $1 \text{ TeV-}M_C$ one obtains $M_C \simeq 10^{16} - 10^{14}$ GeV. This relation changes very little compared to the one obtained just using the RGE at one loop in ref.\[^3\]. To compute $\tan \beta(M_{SS})$ we will use as unification scale the $M_C$ obtained from eq.(3.6) consistent with gauge coupling unification. It is important to remark though that this has very little impact in the numerical results obtained, there is no detail dependence on the value of $M_C$ as long as it remains in the expected $10^{14} - 10^{17}$ GeV region.

To compute $\tan \beta$ at $M_{SS}$ one solves the RGE for the Higgs mass parameters $m_{H_u}, m_{H_d}$. At this point one needs to make some assumptions about the structure of the SUSY-breaking soft terms of the underlying MSSM theory. We will thus assume a standard universal SUSY breaking structure parametrized by universal scalar masses $m$, gaugino masses $M$ and trilinear parameter $A$. The results are independent from the value of the B parameter which is fixed by the fine-tuning condition at $M_{SS}$. Given these uncertainties it is enough to use the one-loop RGE for the soft parameters, which were analytically solved in ref.\[^{28}\]. As described in \[^3\] one has

$$\tan \beta(M_{SS}) = \left| m_{H_d}(M_{SS}) \right| / \left| m_{H_u}(M_{SS}) \right|$$

with

$$m_{H_u}^2(t) = m^2 + \mu^2 q^2(t) + M^2 g(t)$$

$$m_{H_d}^2(t) = m^2 h(t) - k(t) A^2 + \mu^2 q^2(t) + M^2 e(t) + A M f(t)$$ (3.7)

where $m, M, A, \mu$ are the standard universal CMSSM parameters at the unification scale $M_C$, $t = 2 \log(M_C/M_{SS})$ and $q, g, h, k, e, f$ are known functions of the top Yukawa coupling $h_t$ and the three SM gauge coupling constants. Except for regions with large $\tan \beta$, appearing only for low $M_{SS}$, one can safely neglect the bottom and tau Yukawa couplings, $h_b = h_\tau = 0$. For completeness these functions are provided in Appendix B. The value taken for $h_t$ to perform the running of soft terms is a bit subtle since at $M_{SS}$ one has to match the $h_t^{SM}$ value obtained from the SM running up to $M_{SS}$ with
the SUSY value $h_t^{\text{SUSY}}$ which are related by

$$h_t^{\text{SM}} = h_t^{\text{SUSY}} \sin \beta .$$

(3.8)

Since the value of $h_t^{\text{SUSY}}$ depends on $\beta$ through eq. (3.8), the computation of $\tan \beta$ is done in a self-consistent way: a value is given to $\sin \beta (M_{\text{SS}})$, $h_t^{\text{SUSY}}$ is run up in energies and one has a tentative $h_t(M_C)$. One then runs $m_{H_u}/m_{H_d}$ down in energies and computes $\tan \beta$ at $M_{\text{SS}}$. When both values for $\beta$ at $M_{\text{SS}}$ agree the computation of $\tan \beta$ is consistent.

Once computed the value of $\tan \beta$ as described above, one then obtains $\lambda (M_{\text{SS}})$ from eq.(2.1). In addition there are threshold corrections at $M_{\text{SS}}$ induced by loop diagrams involving the SUSY particles. The leading one-loop correction is given by

$$\delta \lambda (M_{\text{SS}}) = \frac{1}{(4\pi)^2} 3 h_t^4 \left( 2 X_t - \frac{X_t^2}{6} \right).$$

(3.9)

where $h_t$ is the SUSY top Yukawa coupling at $M_{\text{SS}}$ and the stop mixing parameter $X_t$ is given by

$$X_t = \frac{(A_t - \mu \cot \beta)^2}{m_Q m_U} .$$

(3.10)

with $m_Q (m_U)$ the left(right)-handed stop mass. This term comes from finite corrections involving one-loop exchange of top squarks. There are further correction terms which are numerically negligible compared to this at least for not too low $M_{\text{SS}}$, in which case the SUSY spectrum becomes more spread and further threshold corrections become relevant, see e.g. [14]. We have computed this parameter $X_t$ using the one loop RGE for the soft parameters that are provided in Appendix B and the value of $\tan \beta$ obtained above.

### 3.3 Computing the Higgs mass

Starting from $(\lambda + \delta \lambda)(M_{\text{SS}})$ one runs back the self-coupling down to the EW scale (using the SM RGE at two loops) and computes the Higgs mass at a scale $Q$ (taken as $Q = m_t$) through

$$m_H^2 = 2 v^2 (\lambda(Q) + \delta^{\text{EW}} \lambda(Q)) ,$$

(3.11)

where $v = 174.1$ GeV is the Higgs vev and $\delta^{\text{EW}} \lambda(Q)$ are additional EW scale threshold corrections. At one-loop these corrections are given by [29]

$$\delta^{\text{EW}} \lambda = -\frac{\lambda G_F M_Z^2}{8\pi^2 \sqrt{2}} (\xi F_1 + F_0 + F_3/\xi) \approx 0.011 \lambda$$

(3.12)
Figure 1: Higgs mass versus SUSY breaking scale $M_{SS}$. The grey bands correspond to the Higgs mass for different values of $\tan\beta$, for $X_t = 0$, without imposing unification of Higgs soft parameters. The other colored bands correspond to imposing $\tan\beta$ values consistent with unification of soft terms, $m_{H_u} = m_{H_d}$. Results are shown for a choice of universal soft terms $M = \sqrt{2}m$, $A = -3/2M$ and four values for the $\mu$-term. The stop mixing parameter $X_t$ is computed from the given soft parameters. The width of the bands correspond to the error from the top quark mass which is taken to be $m_t = 173.1 \pm 0.7$. The horizontal band corresponds to the ATLAS and CMS average Higgs mass result.

where $\xi = m_H^2/M_Z^2$ and the functions $F_1, F_0, F_3$ depend only on EW parameters and are shown in appendix C for completeness. This completes the computation procedure for the Higgs mass as a function of $M_{SS}$.

Figure 1 plots the value of $m_H^2$ as a function of $M_{SS}$. For definiteness we plot the results for universal soft terms with $M = \sqrt{2}m$, $A = -3/2M$. This choice of values is motivated by modulus dominance SUSY breaking in string scenarios, see e.g. [30],[17]. However, as we will explain later, other different choices for soft parameters $m, M, A$ lead to analogous results. The grey bands correspond to the computation of
Figure 2: The black line shows the value of the SM self-coupling \( \lambda \) as a function of \( M_{SS} \), using as input the LHC Higgs data. The remaining curves show values of \( \lambda_{SUSY} \) consistent with \( m_{H_u}(M_C) = m_{H_d}(M_C) \) for different values of \( \mu \). When these \( \lambda_{SUSY} \) lines cross the \( \lambda \) curve the SUSY model is consistent with LHC Higgs data.

The mass for \( \tan \beta = 1, 2, 4, 50 \) and \( X_t = 0 \). The results are similar to those obtained in ref.[13], [14], [15], [16]. The other colored bands correspond to the Higgs mass values obtained under the assumption of Higgs parameter unification as in eq.(2.3). Results are displayed for a mu-term \( \mu = -M/4, -M/2, -3/4M, -M \) with the value for \( X_t \) computed from the obtained running soft terms. The width of the grey and colored bands corresponds to the error from the top quark mass. Finally the horizontal band corresponds to the average CMS and ATLAS results for the Higgs mass (we take \( m_H = 125.5 \pm 0.54 \), see [16]).

The figure shows that above a scale \( \simeq 10^{10} \) GeV the value of the Higgs mass is contained in the range

\[
m_H = 126 \pm 3 \text{ GeV}.
\]

This is remarkably close to the measured value at LHC and supports the idea that SUSY and unification underly the observed Higgs mass. This result is quite independent of

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The results are very weakly dependent on the sign of \( \mu \) through the \( X_t \) appearing in the threshold corrections.
the details of the soft terms. Below 10⁹ GeV the Higgs mass becomes more model
dependent. In particular the Higgs mass is reduced as |µ| increases. This is easy to
understand from eq. (3.7) since for larger µ the ratio m_H/u/m_H/d approaches one, yielding
tanβ ≃ 1. One still gets a Higgs mass consistent with LHC results for not too large
|µ|. As one approaches M_{SS} ≃ 10 − 100 TeV one reaches the region of standard fine-
tuned MSSM with a Higgs mass which may be as large as 130 GeV. As we approach
that region our treatment becomes incomplete since some neglected SUSY threshold
corrections beyond those in (3.12) become important, and the SUSY spectrum spreads
out. However, that region corresponds to the well understood situation of the MSSM
with a heavy SUSY spectrum with masses in the 10-100 TeV region.

Let us finally note that, within uncertainties, the figure also favours values for the
SUSY breaking scale M_{SS} ≲ 10^{13} GeV.

One may also interpret graphically the above results in terms of the unification of
the SM Higgs self-coupling λ_{SM} and the SUSY predicted self-coupling λ_{SUSY} = (g_1^2 +
g_2^2)cos^22β/4. This is depicted in fig. 2, in which we have not included the uncertainty
from the m_t error to avoid clutter. Note that the dependence of λ_{SUSY} on M_{SS} is
qualitatively similar to the running of λ_{SM}. This may be understood as follows. In
the definition of λ_{SUSY}, (g_1^2 + g_2^2) runs very little and remains practically constant. On
the other hand one has cos^22β = (m_{H_u}^2 - m_{H_d}^2)/(m_{H_u}^2 + m_{H_d}^2)^2. The difference on the
numerator goes like h_t^4, which is also the order of the leading correction to the λ_{SM}
coupling.

4 Model dependence

In this section we discuss different model dependent possibilities which arise depending
on the structure of the underlying soft terms. With sufficiently precise information
about the top quark and Higgs masses one may obtain interesting constraints on the
possible structure of the SUSY-breaking terms.

Let us concentrate first in the case with universal soft terms and µ = −M/2 but
still keeping the relationships M = √2m_t, A = −3/2M. As we said these values are
interesting since, as discussed in ref. [3], they may be understood as arising from a
Giudice-Masiero mechanism in a modulus dominance SUSY breaking scheme. The
dependence of the Higgs mass as a function of M_{SS} in this particular case is shown
in fig. 1 with the red band, a zoom is provided in fig. 3. Given the uncertainties, in
this particular case (µ = −M/2) essentially any value for M_{SS} in the 10⁴ − 10^{14} GeV
Figure 3: Higgs mass versus SUSY breaking scale $M_{SS}$ for $\mu = -M/2$ (red band). Its width reflects the uncertainty on $m_t = 173.1 \pm 0.7$. The grey bands, as in fig[1] show the Higgs mass for several values of $\tan \beta = 1, 2, 4, 50$ and are displayed to guide the eye.

region is consistent with the observed Higgs mass, although regions around $10^4 - 10^5$ and $10^8 - 10^{10}$ GeV are slightly favoured. This second possibility with $M_{SS} \simeq 10^{10}$ GeV was explored in [3] (see also [11]) in which it was argued that such intermediate SUSY breaking may be interesting for two additional reasons. On one hand this scale naturally appears in string compactifications in which SUSY breaking is induced by closed string fluxes. Indeed in such a case one has [3]

$$M_{SS} \simeq \sqrt{2g_s/\alpha_G(M_C^2/M_p)},$$  \hfill (4.1)

where $g_s$ is the string coupling, $\alpha_G$ is the unified fine structure constant and $M_p$ is the Planck mass. For $g_s \simeq 1$ and $M_C \simeq 10^{14}$ GeV one indeed gets $M_{SS} \simeq 10^{10}$ GeV. The second reason is that in those constructions an axion with a scale $F_a \simeq M_C/(4\pi)^2 \simeq 10^{12}$ GeV appears, which is consistent with axions providing for the dark matter in the

\footnote{An additional interesting property is that for $M_{SS} \gtrsim 10^{10}$ GeV such models do not require the implementation of doublet-triplet splitting nor R-parity preservation. No Polony problem is present either [3].}
Figure 4: Left: Evolution of the SM Higgs selfcoupling $\lambda(t)$ and the combination $\lambda_{SUSY} = \frac{(g_1^2(t) + g_2^2(t))/4}{\cos^2(2\beta)(M_{SS})}$ in the model with $\mu = -M/2$ and an intermediate scale $M_{SS} \approx 3 \cdot 10^{10}$ GeV. They unify at $M_{SS}$ where SUSY starts to hold. Right: Values of the 3-d generation squark soft masses $m_{Q,U,D}$ as well the Higgs mass parameters $m_{H_u}, m_{H_d}, \mu$ and trilinear $A_t$ at the scale $M_{SS}$ obtained from the running below the unification scale $M_C$.

In this case, using eqs. (3.6), (4.1) one obtains $M_{SS} = 2.49 \times 10^{10}$ GeV and $M_C = 2.43 \times 10^{14}$ GeV. Values this low for the unification scale can still be compatible with proton decay constraints [3]. Computing the Higgs mass following the procedure described in the previous section one obtains in this case

$$m_H = 126.1 \pm 1.2 \text{ GeV}$$

where the error includes only that coming from the top mass uncertainty. This is clearly consistent with the findings at ATLAS and CMS. In this scheme with an intermediate scale $M_{SS}$ the Higgs self-coupling unifies with its SUSY extension as depicted in fig.4 (left). The soft masses evolve logarithmically from $M_C$ down to $M_{SS}$ as depicted in fig.4 (right). The value of $\tan\beta$ increases as the value of $m_{H_u}^2$ decreases and $m_{H_d}^2$ remains almost constant, so that $\tan\beta$ increases as $M_{SS}$ decreases.

It is interesting to explore how relaxing the above mentioned relationships $M = \sqrt{2}m$, $A = -3/2M$ modify the results for the Higgs mass. In fig.5 (up) we show how the prediction for the Higgs mass is changed as one varies the value of $m$ away from $m = M/\sqrt{2}$. The figure remains qualitatively the same but one observes that as $m/M$ increases the Higgs mass tends to be lighter. Above $M_{SS} \approx 10^7$ GeV the Higgs mass remains in the region $m_H \approx 126 \pm 3$ GeV. The effect of varying $A$ away
Figure 5: Higgs mass versus SUSY breaking scale $M_{SS}$ for $\mu = -M/2$. Up: for various values of the scalar mass parameter $m$ in units of the gaugino mass $M$; Down: for various values of the trilinear $A$ parameter.

from $A = -3/2M$ is also shown in fig.5 (down). Although we have not included the error coming from the top quark mass to avoid clutter, one concludes that the overall structure remains the same and the Higgs mass stays around $126 \pm 3$ GeV for $M_{SS} \gtrsim 10^{10}$ GeV. However now values of $M_{SS}$ in between 100 TeV and $10^{10}$ GeV are consistent with the observed Higgs mass for particular choices of soft terms.

Let us finally comment that our results do not directly apply to the case of Split SUSY [8, 12] in which one has $M, \mu, \ll m$, since then the effect of light gauginos and Higgssinos should be included in the running below $M_{SS}$. In that case however it has been shown (see e.g. [13, 14, 15]) that split SUSY is only consistent with a 126 GeV
Higgs for $M_{SS} \lesssim 100$ TeV and no intermediate scale scenario is possible. Essentially Split SUSY becomes a fine-tuned version of the standard MSSM. One relevant issue is also that in Split SUSY, due to the smallness of gaugino masses, in running down from the unification scale the scalar quarks of the third generation may easily become tachyonic, which restricts a lot the structure of the possible underlying SUSY breaking terms [12].

5 Discussion

In this paper we have argued that the evidence found at LHC for a Higgs-like particle around $m_H \simeq 126$ GeV supports the idea of an underlying Supersymmetry being present at some (not necessarily low) mass scale. Even if the SUSY breaking scale is high, SUSY identities for dimension four operators remain true to leading order. In particular, the quartic Higgs self-coupling $\lambda$ is related to the EW gauge couplings at the SUSY breaking scale, yielding constraints on the Higgs mass. The presence of SUSY restores the stability of the SM Higgs potential which otherwise becomes unstable at high scales.

It is remarkable that the simple assumption of Higgs mass parameter unification $m_{H_u} = m_{H_d}$ at the unification scale and minimal fine-tuning directly predict a Higgs mass in the range $m_H = 126 \pm 3$ GeV, consistent with LHC results, for a SUSY scale $\gtrsim 10^{10}$ GeV. For smaller values of $M_{SS}$ the Higgs mass tends to the value of a standard fine-tuned MSSM scenario with $m_H \lesssim 130$ GeV. Both situations with (relatively) low and High scale SUSY are consistent with the Higgs data (see e.g. fig 3). Since in the context of the SM any value from e.g 100 GeV to 1 TeV would have been possible, one may interpret this result as indirect evidence for an underlying SUSY.

It has been argued that the fact that $\lambda \simeq 0$ near the Planck scale and that the SM Higgs potential seems to be close to metastability could have some deep meaning [31]. In our setting with SUSY at a large scale the quartic coupling $\lambda$, is always positive definite and no such instability arises. The smallness of $\lambda$ is due to the fact that the EW couplings $g_1, g_2$ are small and that the boundary condition $m_{H_u} = m_{H_d}$ at the unification scale keeps $\tan \beta$ close to one for $M_{SS} \gtrsim 10^{10}$ GeV. As discussed in the previous section, such a situation with an intermediate scale SUSY breaking and gauge coupling unification may be naturally embedded in string theory compactifications like those resulting from F-theory $SU(5)$ unification. The embedding into string theory is also suggested in order to understand the required fine-tuning in terms of the string
landscape of compactifications.

LHC at 13 TeV will be able to test the low SUSY breaking regime for squark and gluino masses of the order of a few TeV. If no direct trace of SUSY or any other alternative new physics is found at LHC, the case for a fine-tuning/landscape approach to the hierarchy problem will become stronger. Still, as we have argued, heavy SUSY may be required for the stability of the Higgs potential and we have shown that the value $m_H \simeq 126$ GeV is generic for $M_{SS} \gtrsim 10^{10}$ GeV (see e.g. figures 1 and 5), hinting to a heavy SUSY scale.

One apparent shortcoming of High scale SUSY is that we lose the possibility of using the lightest neutralino as a dark matter candidate. In this context the case of an intermediate scale SUSY breaking $M_{SS} \simeq 10^{10}$ GeV is particularly interesting. Indeed, as recently discussed in [3] (see also [32, 33]), such scale may be compatible with an axion with decay constant $F_a \simeq 10^{12}$ GeV, appropriate to provide for the required dark matter in the universe. Furthermore, gauge coupling unification may elegantly be accomodated due to the presence of small threshold corrections as discussed in [3].

Although a large scale for SUSY makes it difficult to test this idea directly at accelerators, indirect evidence could be obtained. Improved precision on the measured values of both the top quark and the Higgs masses (e.g. at a linear collider) can make the constraints on specific High SUSY breaking models and the Higgs mass predictions more precise, along the lines discussed in the previous section. Going beyond the next to leading order in the Higgs mass computation would also be required, see [21]. If those measurements were precise enough, specific choices of soft terms and SUSY breaking scenarios could be ruled out or in. Additional evidence in favour of an intermediate scale SUSY scenario could come from dark matter axion detection in microwave cavity search experiments like ADMX [34]. Furthermore, since in models with large SUSY breaking scale the unification scale typically decreases, proton decay rates could also be at the border of detectability [3]. Finally, if any deviation from the SM expectations is found at low energies (like e.g. an enhanced Higgs rate to $\gamma\gamma$) the idea of a large SUSY scale with a fine-tuned Higgs would be immediately ruled out.

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A Renormalization group equations

Here we first present the renormalization group equations at two loops for the SM couplings (the three gauge couplings, the top Yukawa and the Higgs quartic coupling).

\[
\frac{dg_1}{dt} = \frac{1}{(4\pi)^2} \frac{41}{6} g_3^3 + \frac{g_1^3}{(4\pi)^4} \left( \frac{199}{18} g_1^2 + \frac{27}{6} g_2^2 + \frac{44}{3} g_3^2 - \frac{17}{6} h_t^2 \right) \quad (A.1)
\]

\[
\frac{dg_2}{dt} = -\frac{1}{(4\pi)^2} \frac{19}{6} g_3^2 + \frac{g_2^3}{(4\pi)^4} \left( \frac{9}{6} g_1^2 + \frac{35}{6} g_2^2 + 12 g_3^2 - \frac{3}{2} h_t^2 \right) \quad (A.2)
\]

\[
\frac{dg_3}{dt} = -\frac{1}{(4\pi)^2} \frac{7}{6} g_3^3 + \frac{g_3^3}{(4\pi)^4} \left( \frac{11}{6} g_1^2 + \frac{9}{2} g_2^2 - 26 g_3^2 - 2 h_t^2 \right) \quad (A.3)
\]

\[
\frac{dh_t}{dt} = \frac{1}{(4\pi)^2} \left( \frac{9h_t^2}{2} - \frac{17g_2}{12} - \frac{9g_2^2}{4} - 8g_3^2 \right) + \frac{1}{(4\pi)^4} h_t \left( -12h_t^4 + \frac{6\lambda^2}{4} - \frac{12}{2} \lambda h_t^2 + \frac{131}{16} g_1^2 h_t^2 + \frac{225}{16} g_2^2 h_t^2 + 36 g_3^2 h_t^2 + \frac{1187}{216} g_1^4 - \frac{23g_2^4}{4} - 108g_3^4 \right) \quad (A.4)
\]

\[
\frac{d\lambda}{dt} = \frac{1}{(4\pi)^2} \left( 12\lambda h_t^2 - 9\lambda \left( \frac{g_1^2}{3} + g_2^2 \right) - 43h_t^4 + \frac{3}{4} g_1^4 + \frac{3}{4} g_2^4 - \frac{9}{4} g_2^4 + 12\lambda^2 + \frac{1}{(4\pi)^4} \left( \frac{312}{8} \lambda^3 + \frac{36}{4} \lambda^2 (g_1^2 + 3g_2^2) - \frac{1}{2} \lambda \left( -\frac{629}{24} g_1^4 - \frac{39}{4} g_1^2 g_2^2 + \frac{73g_2^4}{8} \right) + \frac{305g_3^6}{16} - \frac{289}{48} g_1^2 g_2^4 - \frac{559}{48} g_1^4 g_2^2 - \frac{379}{48} g_1^6 - \frac{32g_3^4 h_t^4}{3} - \frac{9}{4} g_1^4 h_t^2 + \frac{1}{2} \lambda h_t^2 \left( \frac{85}{6} g_1^2 + \frac{45g_2^2}{2} + 80g_3^2 \right) + g_1^2 h_t^2 \left( -\frac{19}{4} g_1^2 + \frac{21g_2^2}{2} \right) - \frac{144}{4} \lambda^2 h_t^2 - \frac{3}{2} \lambda h_t^4 + 30h_t^6 \right) \quad (A.5)
\]

And finally the RGE (at 2 loops for gauge couplings, leading order in $h_t$) for the SUSY case:

\[
\frac{dg_1}{dt} = \frac{11g_1^3}{(4\pi)^2} + \frac{g_1^3}{(4\pi)^4} \left( \frac{199}{9} g_1^2 + 9g_2^2 + \frac{88}{3} g_3^2 - \frac{26}{3} h_t^2 \right) \quad (A.6)
\]

\[
\frac{dg_2}{dt} = \frac{g_2^3}{(4\pi)^2} + \frac{g_2^3}{(4\pi)^4} \left( 3g_1^2 + 25g_2^2 + 24g_3^2 - 6h_t^2 \right) \quad (A.7)
\]

\[
\frac{dg_3}{dt} = -\frac{3g_3^3}{(4\pi)^2} + \frac{g_3^3}{(4\pi)^4} \left( \frac{11}{3} g_1^2 + 9g_2^2 + 14g_3^2 - 4h_t^2 \right) \quad (A.8)
\]

\[
\frac{dh_t}{dt} = \frac{h_t}{(4\pi)^2} \left( 6h_t^2 - \frac{13g_1^2}{9} - 3g_2^2 - \frac{16g_3^2}{3} \right) \quad (A.9)
\]
B  RGE solutions for the soft terms

Here we display all the functions that appear in the solution of the RGE for the Higgs mass parameters $m_{H_u}$ and $m_{H_d}$ (see ref. [28]).

First we define the functions

$$E(t) = (1 + \beta_3 t)^{16/(3b_3)} (1 + \beta_2 t)^{3/(b_2)} (1 + \beta_1 t)^{13/(3b_1)} , \quad F(t) = \int_0^t E(t')dt' \quad (B.1)$$

with $\beta_i = \alpha_i(0)b_i/(4\pi)$ and $t = 2 \log(M_c/M_{SS})$. The beta-functions coefficients for the SUSY case are $(b_1, b_2, b_3) = (11, 1, -3)$ and we define $\alpha_0 = \alpha(0) = \alpha_i(0) = g_i^2(0)/(4\pi^2)$ for $i = 2, 3$, $\alpha_1(0) = (3/5)\alpha(0) = g_1^2(0)/(4\pi^2)$ where $\alpha_0$ is the unified coupling at $M_c$. In our case the couplings do not strictly unify, only up to 5% corrections. In the numerical computations we take the average value of the three couplings at $M_c$, which is enough for our purposes.

We then define the functions in eqs. (3.7)

$$q(t)^2 = \frac{1}{(1 + 6Y_0 F(t))^{1/2}} (1 + \beta_2 t)^{3/b_2} (1 + \beta_1 t)^{1/b_1} ; \quad h(t) = \frac{1}{2}(3/D(t) - 1)$$

$$k(t) = \frac{3Y_0 F(t)}{D(t)^2} ; \quad f(t) = -\frac{6Y_0 H_3(t)}{D(t)^2} ; \quad D(t) = (1 + 6Y_0 F(t)) \quad (B.2)$$

$$e(t) = \frac{3}{2} \left( \frac{(G_1(t) + Y_0 G_2(t))}{D(t)} + \frac{(H_2(t) + 6Y_0 H_4(t))^2}{3D(t)^2} + H_8 \right)$$

where $Y_0 = Y_t(0)$ and $Y_t = h_t^2/(4\pi)^2$. The functions $g, H_2, H_3, H_4, G_1, G_2$ and $H_8$ are independent of the top Yukawa coupling, only depend on the gauge coupling constants and are given by

$$g(t) = \frac{3}{2} \frac{\alpha_2(0)}{4\pi} f_2(t) + \frac{1}{2} \frac{\alpha_1(0)}{4\pi} f_1(t)$$

$$H_2(t) = \frac{\alpha_0}{4\pi} \left( \frac{16}{3} h_3(t) + 3h_2(t) + \frac{13}{15} h_1(t) \right)$$

$$H_3(t) = tE(t) - F(t)$$

$$H_4(t) = F(t)H_2(t) - H_3(t)$$

$$H_5(t) = \frac{\alpha_0}{4\pi} \left( -\frac{16}{3} f_3(t) + 6f_2(t) - \frac{22}{15} f_1(t) \right)$$

$$H_6(t) = \int_0^t H_2(t')^2 E(t')dt'$$

$$H_8(t) = \frac{\alpha_0}{4\pi} \left( -\frac{8}{3} f_3(t) + f_2(t) - \frac{1}{3} f_1(t) \right)$$
\[ G_1(t) = F_2(t) - \frac{1}{3}H_2(t)^2 \]
\[ G_2(t) = 6F_3(t) - F_4(t) - 4H_2(t)H_4(t) + 2F(t)H_2(t)^2 - 2H_6(t) \]
\[ F_2(t) = \frac{a_0}{4\pi} \left( \frac{8}{3}f_3(t) + \frac{8}{15}f_1(t) \right) \]
\[ F_3(t) = F(t)F_2(t) - \int_0^t E(t')F_2(t')dt' \]
\[ F_4(t) = \int_0^t E(t')H_5(t')dt' \]

(B.3)

where \( f_i(t) \) and \( h_i(t) \) are defined by
\[
f_i(t) = \frac{1}{\beta_i}(1 - \frac{1}{(1 + \beta_i t)^2}) ;
\]
\[
h_i(t) = \frac{t}{(1 + \beta_i t)}.
\]

(B.4)

The low energy of the top mass may be obtained from the solutions of the one-loop renormalization group equations, divided into two pieces, SUSY and non-SUSY, i.e. (here \( Y_t = h_t^2/(16\pi^2) \))
\[
Y_t(m_t) = \sin^2\beta Y_t(M_{SS}) \frac{E'(t_{EW})}{(1 + (9/2)\sin^2\beta Y_t(M_{SS})F'(t_{EW}))}
\]

(B.5)

where
\[
Y_t(M_{SS}) = Y_t(M_c) \frac{E(t_{SS})}{(1 + 6Y_t(M_c)F(t_{SS}))}
\]

(B.6)

The functions \( E, F \) are as defined above, with \( t_{SS} = 2\log(M_c/M_{SS}) \) and \( t_{EW} = 2\log(M_{SS}/M_{EW}) \), while the functions \( E', F' \) are analogous to \( E, F \) but replacing the \( b_i \) and anomalous dimensions by the non-SUSY ones, i.e.
\[
E'(t) = (1 + \beta'_1 t)^{8/(b_1^{NS})}(1 + \beta'_2 t)^{9/(4b_2^{NS})}(1 + \beta'_3 t)^{17/(12b_3^{NS})},
\]
\[
F'(t) = \int_0^t E'(t')dt'
\]

(B.7)

with \( \beta'_i = \alpha_i(M_{SS})b_i^{NS}/(4\pi) \), \( b_i^{NS} = (41/6, -19/6, -7) \) and \( t = t_{EW} \). For the anomalous dimensions we have made the change in the definition of \( E(t) \) \((13/9, 3, 16/3) \rightarrow (17/12, 9/4, 8) \). And we take the value of \( h_t(m_t) \) computed in eq. (3.3) taking into account the threshold corrections at electroweak scale. For this particular computation we take actually as electroweak scale the top mass, so \( t_{EW} = 2\log(M_{SS}/m_t) \).
following equations for the running of the soft parameters:

\[
A_t(t) = \frac{A}{D(t)} + M(H_2(t) - \frac{6Y_0H_3(t)}{D(t)}) \\
\mu(t) = \mu_0q(t) \\
m^2_t(t) = M^2(-3\frac{\alpha_2(0)}{4\pi} f_2(t) + \frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m^2_\tau(t) = -\frac{1}{3}m^2 + M^2(-\frac{8}{3}\frac{\alpha_3(0)}{4\pi} f_3(t) + \frac{\alpha_2(0)}{4\pi} f_2(t) - \frac{5}{9}\frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m^2_{D}(t) = m^2 + 2M^2(\frac{4}{2}\frac{\alpha_3(0)}{4\pi} f_3(t) + \frac{3}{4}\frac{\alpha_2(0)}{4\pi} f_2(t) + \frac{1}{36}\frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m^2_{U}(t) = \frac{2}{3}m^2_{H_u}(t) - \frac{2}{3}\mu^2(t) - m^2_\epsilon(t) \\
m^2_{Q}(t) = \frac{1}{2}m^2_B(t) - \frac{1}{2}m^2_4(t) + \frac{1}{2}m^2_U(t) \\
\] (B.8)

C  Threshold corrections at the EW scale

The functions appearing in the computation of the threshold corrections to the Higgs self-coupling at the weak scale are given by [29]:

\[
F_1 = 12 \log \left[ Q \frac{1}{m_h} \right] + \frac{3\log[z]}{2} - \frac{1}{2}Z \left[ 1 \xi \right] - Z \left[ c^2 W \xi \right] - \log[c^2 W] + \frac{9}{2} (\frac{25}{9} - \frac{\pi}{\sqrt{3}}) \] (C.1)

\[
F_0 = -12 \log \left[ Q \frac{1}{M_Z} \right] \left( 1 + 2c^2 W - \frac{2m^2}{M_Z^2} \right) + \frac{3c^2 W \xi \log[c^2 W] - 1}{1 - c^2 W} + 2Z \left[ 1 \xi \right] + 4c^2 W Z \left[ c^2 W \xi \right] + \\
+ \frac{3c^2 W \log[c^2 W]}{s^2 W} + 12c^2 W \log[c^2 W] - \frac{15}{2} (1 + 2c^2 W) - \\
- \frac{3m^2_1}{2Z} \left( 2Z \left[ \frac{m^2_1}{M_Z^2} \right] - 5 + 4 \log \left[ \frac{m^2_1}{M_Z^2} \right] \right) \] (C.2)

\[
F_3 = 12 \log \left[ Q \frac{1}{M_Z} \right] \left( 1 + 2c^4 W - \frac{4m^4_1}{M_Z^2} \right) - 6Z \left[ 1 \xi \right] - 12c^4 W Z \left[ c^2 W \xi \right] - 12c^4 W \log[c^2 W] + \\
+ 8 (1 + 2c^4 W) + \frac{24m^4_1}{M_Z^2} \left( Z \left[ \frac{m^2_1}{M_Z^2} \right] - 2 + \log \left[ \frac{m^2_1}{M_Z^2} \right] \right) \] (C.3)

where $\xi = m^2_H/M_Z^2$, $c_W = \cos \theta_W$, $s_W = \sin \theta_W$ and

\[
Z(z) = \begin{cases} 
2\zeta \arctan \left[ \frac{1}{\xi} \right] & \text{for } z > 1/4 \\
\zeta \log[1 + \xi] & \text{for } z < 1/4 
\end{cases} \] (C.4)

where $\zeta = \sqrt{\text{Abs}[1 - 4z]}$. In the computation we have taken the central experimental values for $M_Z$, $m_t$ and $s_W$ given by eqs.(3.1,3.2) and the tree level value for the Higgs mass, i.e. $m^2_h = 2\lambda v^2$ with $v = 174.1$.  

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