Rapid Information Transfer in Networks with Delayed Self Reinforcement

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The cohesiveness of response to external stimuli depends on rapid distortion-free information transfer across the network. Aligning with the information from the network has been used to model such information transfer. Nevertheless, the rate of such diffusion-type, neighbor-based information transfer is limited by the update rate at which each individual can sense and process information. Moreover, models of the diffusion-type information transfer do not predict the superfluid-like information transfer observed in nature. The contribution of this article is to show that self reinforcement, where each individual augments its neighbor-averaged information update using its previous update, can (i) increase the information-transfer rate without requiring an increased, individual update-rate; and (ii) capture the observed superfluid-like information transfer. This improvement in the information-transfer rate without modification of the network structure or of the bandwidth of each agent can lead to better understanding and design of networks with fast response.

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INTRODUCTION

The speed of information transfer across the network can impact the cohesiveness and effectiveness of the network’s response to external stimuli. Aligning with the information from a network has been used to model a range of information transfer in nature such as information diffusion in social networks [1,2], complex networks [3,5], and flocking dynamics, e.g., [6,12]. In general, a faster information-transfer rate can be achieved by increasing the alignment strength, i.e., by scaling up the individual update that is based on information from neighbors. Nevertheless, such an increase in the alignment strength will require an increase in the information-update rate, which is limited by each individual’s ability to sense and process external stimuli. Hence there is a limit to the maximum rate of information transfer possible with a fixed update rate.

The main contribution of this paper is to develop a self-reinforcement approach that can increase the information transfer without the need to change the network structure or the bandwidth (information-update rate) of the individual agents. Rather, the proposed approach uses delayed versions of the previous updates from the network to self reinforce the current update and improve the overall network response. Such faster response rate with limited individual performance (update rate) [13] can influence current studies in group decision making, e.g., [14,15], models of cohesiveness in groups, e.g., [9,12], and interactions between layers of networks [3], improve communication of engineered swarms such as robots [16,17], and potentially lead to better understanding of response to external stimuli in biological systems [18].

Another challenge with current models of the neighbor-averaged diffusive information transfer is that they do not predict the superfluid-like information transfer observed in biological flocking [12,19]. Superfluid-like information transfer leads to undamped propagation of the radial acceleration across the flock, which is important to achieve equal-radius (parallel) trajectories for cohesive maneuvers [12]. Nevertheless, superfluid-like models also require an increase in update rate for fast response. This article shows that current diffusive models can be modified to capture the superfluid-like information transfer observed in nature without the need to increase the bandwidth of the individual agents. Since delays are available in neural circuits, the delayed self-reinforcement (DSR) method might be potential mechanism to explain the superfluid-like observations.

MODELS WITH AND WITHOUT DSR

The alignment of each individual $i$ based on the information available to its neighbors $N_i$ is modeled below. Let the new information $i_{k+1}$ for the $i^{th}$ individual be found from the information update given by

$$i_{k+1} = i_k + \beta [I_k - I_{k-1}]$$

where different integers $k$ represent the update time instants $t_k = k\delta_t$, the time interval between updates $\delta_t$, depends on the reaction time of the individual, $\beta$ is the DSR gain on the previous update, $K_s$ is the alignment strength, and $\Delta_s(k)$ is the average difference in the information between the individual and its $|N_i|$ neighbors in the network

$$\Delta_s(k) = \frac{1}{|N_i|} \sum_{j \in N_i} [I_j(k) - I_j(k)]$$

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In flocking, the network connections depend on metric distance or topological distance [20]. In this model, the set of neighbors $N_i$ also includes the information source $I_i$ when the individual $i$ is a leader with direct access to the source. This model corresponds to the standard diffusion-based information update if the DSR gain $\beta$ in Eq. (1) is set to zero, e.g., [7 8].

**Information transfer improvement with DSR** For a given system update time $\delta$, DSR can lead to substantial performance improvement when compared to the standard diffusive information update without the DSR as illustrated in Fig. 1. The system comprised of 225 individuals placed in a $25 \times 25$ regular array, where the spacing in the $x$ and $y$ direction was 1 m. The neighborhood $N_i$ of each individual was considered to be a disc of radius $r = 1.2$ m from the individual $i$. Thus, the average distance of individuals in the neighborhood was $a = 1$m. The leader is the individual shown as a solid black dot in Fig. 1(a). The initial value of the source information and of all the individuals were zero. The source information is switched to one at the start of the simulations. Without DSR, the information transfer becomes unstable as the alignment strength is increased to $K_s = 101$ from $K_s = 100$. Therefore, the alignment strength was selected to be high ($K_s = 100$) to enable fast response, but smaller than the value that causes instability. Then, the DSR gain was selected to yield a fast response without substantial overshoot. With the DSR gain selected to avoid oscillations, $\beta = 0.96$, the DSR leads to a substantial (more than an order) reduction in the settling time (i.e., the time needed for all the individual responses to become close (within 2%) and stay close to the maximum value of the source information) — from 69s to 1.72s. Without DSR, similar substantial improvements are not possible since increases in the alignment strength leads to instability. Thus, for a given update rate, the proposed DSR leads to better transfer of rapidly changing information when compared to the standard case without the DSR.

**IMPACT OF DSR ON FLOCKING**

DSR can improve the cohesiveness of flocking maneuvers, when the orientation of each individual is considered to be the information $I$ being transferred using local alignment to neighbors. To illustrate, the position components $x_i$, $y_i$ of each individual is updated as

\[
\begin{align*}
    x_i(k+1) &= x_i(k) + v \delta t \cos I_i, \\
    y_i(k+1) &= y_i(k) + v \delta t \sin I_i,
\end{align*}
\]

where $v$ is the fixed speed of each individual. To focus on the impact of orientation-information transfer on the maneuver, other effects such as speed changes or strategy changes to maintain spacing between individuals or density are not included in the simulations, e.g., as studied in [1 3 9 21 23]. Note that the set of neighbors can change during these simulations, however, the initial spacing is selected to ensure that each individual has at least two neighbors at the start of the simulations.

The maneuver with DSR is more cohesive, for both uniform and random initial distribution, as seen in the similarity of the initial and final formations when compared to the case without the DSR, and also seen in the Videos V1-V4. Even with the addition of noise in the information update, the overall motion remains cohesive, see Video V3. In Fig. 2, the turn movement (blue solid line) of the leader is similar to that of an individual which is farther away, which is an important feature in biological flocks which exhibit equal-radius (parallel) trajectories [12]. In contrast, without DSR, the final direction of the leader (slope of the solid red line) is different from that of individuals farther away. Moreover, the slower transfer of turn-information leads to a larger turn radius without the DSR when compared to the case with the DSR. The time shift $\Delta_{t,c}$ needed for the individual radial acceleration to best correlate with the radial acceleration of the leader varies linearly with distance $d$ from the leader (for individuals close to the leader), with the DSR approach, as seen in Fig. 2(d). The overall speed of information transfer across the network is 47 m/s, where the correlation time delay $\Delta_{t,c}$ is 0.389 s for a distance of 18.38 m. Moreover, the magnitude of the radial acceleration does not reduce substantially with distance from the leader, as seen in Fig. 2(c). Both these features, linearity of the information transfer with time and low distortion, are indicative of superfluid-like flow of information observed in nature that cannot be explained by standard diffusion models [12]. Thus, the proposed DSR captures the superfluid-like turning maneuvers observed in nature.
SUPERFLUID-LIKE BEHAVIOR WITH DSR

To understand the impact of the DSR gain $\beta$ selection on capturing the superfluid-like behavior in the results in Fig. 1, the information update in Eq. 1 is first rewritten as

$$\frac{\beta}{\delta_t} \left\{ [I_i(k + 1) - I_i(k)] - [I_i(k) - I_i(k - 1)] \right\} + \frac{1}{\alpha} \beta \left[ I_i(k + 1) - I_i(k) \right] = -K_s \Delta_i(k),$$

and then approximated, when the update interval $\delta_t$ is small compared to the information-transfer response, as

$$\beta \delta_t \frac{d^2}{dt^2} I(t) + (1 - \beta) \frac{d}{dt} I(t) = \frac{a^2}{4} K_s \nabla^2 I(t)$$

where $a$ is the average distance to the neighbors and $\nabla^2$ represents the Laplacian. This approximate model captures a broad set of behaviors. As the DSR gain tends to one, the damping term $(1 - \beta)$ tends to zero and the overall behavior changes from overdamped (e.g., $\beta = 0$) to being critically damped (e.g., $\beta = 0.96$) to oscillatory undamped (e.g., $\beta = 0.98$), as seen in Fig. 1. Large oscillations can lead to distortions in the information loss, and ideally the DSR gain $\beta$ is tuned to be close to critical damping. For small DSR gain the DSR dynamics approximates the overdamped standard diffusion-type information transfer

$$\frac{d}{dt} I(t) = \frac{a^2}{4} K_s \nabla^2 I(t).$$

With a larger DSR gain, the DSR dynamics approximates the superfluid-type information transfer, i.e.,

$$\frac{d^2}{dt^2} I(t) = \frac{a^2 K_s}{4 \delta_t} \nabla^2 I(t) = c^2 \nabla^2 I(t)$$

where a smaller update time $\delta_t$ (which is possible if the individuals can respond faster) leads to a larger speed of information propagation $c$.

Both the standard diffusive model and the second-order superfluid-type model in Eq. 5 can achieve faster information transfer, similar to the case with the use of DSR, as seen in Fig. 3. The superfluid-like simulations, were computed based on Eq. 5 as

$$I(k + 1) = I(k) + \hat{I}(k) \delta_t$$

$$\hat{I}(k + 1) = \hat{I}(k) - \frac{(1-\beta)}{\delta_t} \hat{I}(k) \delta_t + \frac{K_s}{\beta \delta_t} \Delta_i(k) \delta_t$$

where the update rate was $\delta_t = 1.246 \times 10^{-4}$ s. The settling time with the standard diffusive model is 1.72
transfer with DSR is close to the expected value for the standard diffusion model. The speed of information distance with the DSR. In contrast, the information transfer distance is linear in time $t$ with the DSR. In contrast, the information transfer distance $d$ is proportional to the square root of time $\Delta t$ with the standard diffusion model.

The speed of information transfer with DSR is close to the expected value for the superfluid case from the expression of $c$ in Eq. 7. In particular, with an average distance of $a = 1$, the predicted speed $c$ in Eq. 7 is $c = 50$ m/s. This is close to the speed of information transfer seen in the results in Fig. 3, where information is transferred over a distance of $18.38$ m in $0.39$ s, i.e., at a speed of $47$ m/s.

In summary, the use of the DSR neighbor-based alignment achieves the superfluid-type information transfer, and increases the overall information transfer rate in the network without requiring a corresponding increase in individual information-update rate. In contrast, current superfluid-like model and standard diffusion models can only achieve the faster information transfer by increasing the individual, information-update rate.

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FIG. 3. Information transfer similar to the case with DSR (Fig. 1(b)) is achieved with: (a) standard diffusion model by increasing the alignment strength $K_s$ from 100 to 4011 and decreasing the update time from 0.01 s to $2.49 \times 10^{-4}$ s; and (b) with the superfluid-type model in Eq. 5. The time delay $\Delta t$ between the leader and other individuals to reach 0.1 as a function of the distance $d$ from the leader: (c) the information transfer distance $d$ is proportional to the square-root of time $\Delta t$ for the standard diffusive model without DSR for individuals close to the leader and (d) linear for the DSR case and for the superfluid-like model.