Statistical Gauge Theory for Relativistic Finite Density Problems

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A relativistic quantum field theory is presented for finite density problems based on the principle of locality. It is found that, in addition to the conventional ones, a local approach to the relativistic quantum field theories at both zero and finite density consistent with the violation of Bell like inequalities should contain, and provide solutions to at least three additional problems, namely, 1) the statistical gauge invariance 2) the dark components of the local observables and 3) the fermion statistical blocking effects, base upon an asymptotic non-thermo ensemble. An application to models are presented to show the importance of the discussions.

I. INTRODUCTION

Besides its relevance to domestic processes like in heavy ion collisions, in nuclear matter, a consistent theory for relativistic finite density systems is neither less interesting fundamentally due to its relevance to profound cosmological questions like the mechanism for baryogenesis, the nature of dark matter, etc., nor can it be expected to be trivially derivable from the theoretical framework at zero density.

Although such kind of theory, which adopts the basic framework of the field theoretical representation of non-relativistic many body systems exists and is widely used in literature, at least two important qualities that a correct theory for high energy physics should possess is lacking. The first one is that the essential ingredient of the relativistic spacetime, namely the principle of locality, has not been properly addressed. This is because in the grand canonical ensemble base on which such a theory is built, the chemical potential is a global quantity in spacetime. As it is well known in the studies of few and many body relativistic systems that it is not enough to simply adopt the relativistic kinematics for each particles in the system when the interaction is present. The less well understood relative time between particles inside the system exists due to the lack of a frame independent definition of simultaneity in the relativistic spacetime which constitutes another fundamental difference between non-relativistic systems based upon Newtonian spacetime and the relativistic ones. The second one is related to the fact that unlike in the non-relativistic condensed matter systems, the relativistic extended theory has not been systematically tested since nuclear matter at equilibrium to which this theory should be applied is hard to prepare domestically to allow such tests. In addition, question posed for hadronic system are most likely different from the ones posed for a non-relativistic condensed matter system.

This situation leaves room for new theoretical possibilities to propose a theoretical framework that can cover not only the non-relativistic situations, which is a specially limit of the theory when resolution of observation or energy is low, but also the relativistic ones based on the principle of locality. While the first above mentioned quality can be implemented theoretically, as it is done in the following, the full implementation of the second one is beyond the scope of a theoretical exploration. But at least a theoretical exploration can improve the situation by try to answer questions mostly likely to be experimentally studied in a hadronic system from domestic to astronomical scale and from zero to finite density. Here again, as it is shown in the sequel, the principle of locality plays a key role.

The global and local measurements in quantum systems is compared in section II. The limitation of a global measurement in study relativistic quantum systems is discussed. A 4-vector field, called the primary statistical gauge field, is introduced. An asymptotic grand canonical ensemble is introduced in section III. The consequence of the statistical gauge invariance is studied and a semi-quantitative non-perturbative method for study some qualitative aspects of the problem is introduced. The instability of the asymptotic grand canonical ensemble is revealed in section IV where the asymptotic non-thermo ensemble is shown to be necessary. Section V presents the low energy effective action for the primary statistical gauge field and its dynamical characteristics. It is shown that the statistical blocking effects result in the absence of a long range force that couples to baryon or nucleon number in the normal phase of the hadronic matter. A brief summary is given in section VI.

II. GLOBAL AND LOCAL MEASUREMENTS
A. Global Measurements

The non-relativistic many body theory for a system with variable particle number is based upon the Grand Canonical Ensemble (GCE) [1]. The partition function for the system in the GCE is related to its Hamiltonian $\hat{H}$ and its particle number $\hat{N}$ in the following way

$$Z = \lim_{\Omega \to \infty} \text{Tr} e^{-\beta (\hat{H} - \mu \hat{N})},$$

where $\Omega$ is the volume of the system, $\beta = 1/T$ with $T$ the temperature and $\mu$ is the chemical potential. The limit $\Omega \to \infty$ is the thermodynamical limit in which the equilibrium statistical mechanics describes physical many body system at equilibrium. A measurement of the ground state fermion number density in GCE which is given by

$$n = \lim_{T \to 0} \frac{\partial}{\partial \mu} \left( \frac{T}{\Omega} \ln Z \right)$$

corresponds to a global measurement. Global measurements are most frequently carried out in condensed matter systems with a size smaller than or comparable to our human body (of order 1 meter) which permit us to study the system “as a whole”. The GCE is highly successful in describing these situations. Questions posed by high-energy physics, astrophysics and cosmology are somewhat different. The size of the systems compared to that of the resolution of the measuring apparatus are normally much larger, therefore one actually is capable of study either more details of the probed systems due to the reduction of the wave length of the probing system or only small fraction of the probed system due to the impossibility of covering the whole probed system by the probing system at the same time. One is in effect doing local measurements in these later situations.

B. Local Measurements

Global measurements are in principle not directly definable since simultaneity of two events separated from each other by a space-like distance depends on reference frame in special relativity. Measurements in relativistic space-time are in principle local ones in space-time. The value of a global quantity is obtained from an integration over the results of a complete set of local measurements carried out at the same time. In addition, the property of asymptotic freedom in strong interaction makes the local measurements relevant to the study of the properties of fundamental current quarks at short distances where they are point like and almost free particles.

Local measurements in quantum field theory are realized by exerting an external local field to the system at the space-time point interested, and, the measured quantity are deduced from the response of the system to the external field. For finite density systems, a Lorentz 4-vector local field $\mu^\alpha(x)$, called the primary statistical gauge field, is introduced. The ground state expectation value of fermion number density can be expressed as the functional derivative

$$\rho(x) = \frac{\delta \ln Z}{\delta \mu^\alpha(x)}$$

with $Z$ a functional of $\mu^\alpha$. It corresponds to a local measurement in the ground state at the space-time point $x$.

C. The dark component of local observables

That $\rho \equiv \overline{\rho}$ is not mathematically warranted. The GCE expression for $\overline{\rho}$ in Eq. 3 suffers from an ambiguity related to the thermodynamical limit of $\Omega \to \infty$ which is taken ahead of the partial derivative over the chemical potential $\mu$ in Eq. 2. Since the contributing eigenvalues of the number operator $\hat{N}$ to Eq. 3 are proportional to $\Omega$, the partial derivation over $\mu$ is not well defined in Eq. 3 due to the fact that $\mu$ is multiplied by an infinite number in Eq. 1. Eq. 3 does not contain such an ambiguity since the modification of the system due to a local change in the primary statistical gauge field $\mu^\alpha(x)$ does not modify the system by an infinite amount. So Eq. 3 is an exact expression for the particle number density for an uniform system.

The difference

$$\rho_D = \rho - \overline{\rho}$$

(4)
is called the dark component of the local observable in the ground state. It is known that \( \rho \) equals to \( n \) for a system made of non-interacting fermions and bosons which has a Hamiltonian (or Lagrangian) quadratic in the fields corresponding to the particles. This equality is also expected to hold for weakly interacting systems. As the strength of the interaction or the non-linearity of the system increases, the dark component \( \rho_D \) is expected to develop non-vanishing value. For a system whose behavior can be approximately described by an ensemble of quasi-particles, as it is demonstrated in the following, \( \rho \) contains contributions from spatially extensive quasi-particles of the system that can propagate indefinitely in time. So \( \rho_D \) contains all other contributions to \( \rho \) due to localized and transient quantum fluctuations of the fields.

III. THE ASYMPTOTIC GRAND CANONICAL ENSEMBLE

A. The evaluation of conserved local observables

Unlike in a weakly interacting theory in which the spectra of the system are not modified in some essential way from the corresponding free system, the number (density) of the particle can be found by a counting of the particles inside the system. The particle number density in a strongly interacting theory is obscured by the fact that in certain phases of a strongly interacting system the original particles are either confined or altered in a fundamental way due to phase transitions in the system. It makes a straightforward counting of these particles difficult since some type of them do not appear in the set of initial and final physical states at all. In these more general case, one should start from the evaluation of the original field theoretic definition of the particle number density operator. For any conserved particle numbers, like the baryon number, the evaluation can be simplified by counting the particle number of the system when the strength of the interaction is adiabatically switched off in the remote past. These conserved numbers are not going to be changed as the interaction strength is fully switched on long before the measurement time in the near past and near future. These conserved numbers are selected using the asymptotic grand canonical ensemble (AGCE). It is defined as the grand canonical ensemble of the corresponding free system in the remote past (or future) when the interaction effects are adiabatically switched off.

So \( \rho \) follows from the asymptotic grand canonical ensemble (AGCE), which is defined above, with the Lagrangian density that underlies \( Z \) given by

\[
\mathcal{L}' = \frac{1}{2} \bar{\Psi}(i\partial + \mu \sigma_3 - m)\Psi + \mathcal{L}_B + \mathcal{L}_{\text{int}}
\]

(5)

and \( m \) the fermion mass, \( \mathcal{L}_B \) the Lagrangian density of boson fields of the system and \( \mathcal{L}_{\text{int}} \) the interaction between the boson and fermion fields. An 8 component “real” fermion field \( \Psi \) is used for the discussion with \( \sigma_3 \) the third Pauli matrices acting on the upper and lower 4-component of \( \Psi \). In most of the cases, \( \mathcal{L}_{\text{int}} = -\bar{\Psi}(\Sigma(f) - m)\Psi \) with \( \Sigma \) a function(al) of the boson fields represented by \( f \) so that the fermion degrees of freedom can be integrated out leading to an effective Euclidean action for the boson fields

\[
S_{\text{eff}}[f, \mu] = \frac{1}{2} \text{Sp} \ln \left[ \frac{\bar{\Psi}(i\partial + \mu \sigma_3 - \Sigma(f))\Psi}{\bar{\Psi}(i\partial + \mu \sigma_3 - m)\Psi} \right] + \int d^4x [\mathcal{L}_B(f) + \mu \bar{\rho} - \bar{\epsilon}]
\]

(6)

with “Sp” the functional trace, \( \bar{\epsilon} \) the average energy density of the corresponding free system of fermions (\( \Sigma = 0 \)) and \( \bar{\rho} \) is the ground state expectation value of fermion number density to be discussed in the following at the “chemical potential” \( \mu = \sqrt{\mu_{\alpha}^2} \), where \( \mu_{\alpha} \) is the ground state value of \( \mu^\alpha \) normally provided by the external conditions. The partition functional

\[
Z = \int D[f] \exp(S_{\text{eff}})
\]

(7)

is then a functional of \( \mu^\alpha \) and \( \ln Z \) its effective action.

B. The super-selection sector

The first term in Eq. (6) is invariant under the \( U(1) \) statistical gauge transformation \( \mu^\alpha(x) \rightarrow \mu^\alpha(x) - \partial^\alpha \Lambda(x) \), which corresponds to the conservation of the fermion number. Since \( \mu^\alpha \) is a local field, its excitation represents certain collective excitations of the system. Such a view introduces infinite many extra degrees of freedom since there is
no such a field in the original theory. These extra degrees of freedom are eliminated by restricting the representing Hilbert space.

Superselection-sector in the representing Hilbert space (SIRHS) containing physical states correspond to the primary statistical gauge field exists and can be selected using a “Gauss Law” constraint. For the statistical gauge invariant system, it can be imposed differently on the physical eigenstates of the Hamiltonian without causing contradictions. States in a SIRHS that is identified by a coordinate dependent complex function \( \varsigma \) satisfy

\[
\langle \psi^i_\varsigma | (\hat{\rho} + \nabla \cdot \vec{\pi}_u) | \psi^j_\varsigma \rangle = \delta_{E_E} N_{ij\varsigma}
\]

(8)

with \( \vec{\pi}_u \) the “statistical electric field”, \( | \psi^k_\varsigma \rangle \) a physical state that has energy \( E_k \), \( \delta_{E_E} \) taking zero or unity value if \( E \neq E' \) or \( E = E' \) (assuming that \( E \) is discrete before length of the time interval in which the system is confined is let to go to infinity) and \( N_{ij} \) independent of space-time. \( \hat{\rho} + \nabla \cdot \vec{\pi}_u \) is the generator of local statistical gauge transformations, so the physical states in a SIRHS change a common (coordinate dependent) phase under a specific gauge transformation rather than remains invariant. The special condition \( \varsigma = 0 \) is hitherto been used for any gauge field in literature. This is true for dynamical gauge fields but not necessarily for statistical gauge fields. Clearly, the later condition is a special case of the former.

Due to the conservation of the fermion number, the SIRHS of the system with fixed fermion number selected \( [4] \) by the AGCE is invariant during the time evolution. In the AGCE, the dependence of \( \overline{\rho} \) upon the space-time independent \( \mu \) remains the same whether there is interaction in the system or not. It is given by

\[
\overline{\rho} = N_\rho \mu^3/3\pi^2
\]

(9)

for a massless system, with \( N_\rho \) the total internal degrees of freedom besides the spin of the fermion. \( \overline{\rho} \) does not has a simple dependence on \( \mu \), which can be identified with the chemical potential in uniform case, especially when there is certain kind of phase transition that modifies the excitation spectra of the system.

In some sense, the AGCE is a GCE for the initial state of the system under the time evolution, it is a canonical ensemble for the interaction effects since the invariant SIRHS (under the time evolution) is fixed in the remote past (or future). The time component of the local primary statistical gauge field \( \mu^A \) does not necessarily correspond to the chemical potential, which is a global parameter. For example, in a system of baryons in thermo equilibrium, the primary statistical gauge field \( \mu^A \) is mainly non-vanishing around nucleons and nuclei; the chemical potential, on the other hand, is a constant throughout the system.

C. Relevant results from lattice simulations

There are interesting lattice Monte-Carlo studies of the finite density problems \( [4,5] \) based upon GCE for a finite lattice using Glasgow method. It is shown using the chiral Gross-Neveu model that for a given chemical potential, the fermion number density of the system has two stable values, which are reached by generating the Monte-Carlo ensemble at either finite or zero density. The first stable one correspond to the one given by the AGCE \( \rho - \mu \) relation and the second one correspond to the GCE prediction for a massive system of quasi-particles. The exact results for the global treatment of the problem crossover from the later \( \rho - \mu \) curve to the former one near the chiral symmetry restoration \( \mu = \mu_c \). This behavior can be interpreted as that before the chiral symmetry restoration, the first \( \rho - \mu \) curve for the AGCE is a local minimum of the (Euclidean) action of the various configurations in the Monte-Carlo ensemble with the GCE curve for the massive system of quasiparticles the absolute one. Near the chiral symmetry restoration point, their role exchanges. The existence of a stable local minimum for the AGCE \( \rho - \mu \) curve is a direct consequence of the conservation of the fermion number in the model used. It is expected that some of the puzzles encountered in the lattice gauge theory studies at finite density, like the early on set of the baryon number before the chiral symmetry restoration, can be understood in the light of the AGCE since the finite size effects of the lattice gauge theory study based upon GCE corresponds to, to some degree, a low resolution local measurement of an infinite system \( [2,3] \).

D. Semi-quantitative discussions

This above interpretation for the origin of the dark component of \( \overline{\rho} \) can be substantiated by using a cluster decomposition approximation of the partition functional \( Z[\mu] \) \( [3] \) which is expected to provide a non-perturbative picture for the \( \mu \) dependence of the fermion number density at sufficiently low space-time resolutions. In the crudest approximation, the ground state fermion number density can be written as \( [2] \).
\[ \rho_{\Delta\Omega}(\sigma, \mu) = \sqrt{\frac{\alpha}{\pi}} \int_{-\mu}^{\mu} d\sigma' e^{-\alpha \sigma'^2} \rho_{\infty}(\sigma + \sigma', \mu), \]  

\(10\)

where \(\bar{\sigma}\) is the ground state expectation value of \(\sigma\),

\[ \rho_{\infty}(\sigma, \mu) = \frac{N_g}{3\pi^2} (\mu^2 - \sigma^2)^3/2 \]  

\(11\)

is the ground state fermion number density in the GCE with fermion mass \(\sigma\). Here \(\Delta\Omega\) is the space-time volume that can be resolved in the observation. \(\alpha\) is proportional to \(\Delta\Omega\) for large enough \(\Delta\Omega\); it approaches a constant value for small \(\Delta\Omega\) which scales with the inverse mass gap of the \(\sigma\)' excitation of the system. Clearly,

\[ \rho_{\Delta\Omega}(\sigma, \mu) \rightarrow \rho_{\infty}(\sigma, \mu) = \pi \]  

\(12\)

as \(\Delta\Omega \rightarrow \infty\). This qualitative picture shows that the result of a local measurement of the fermion number density approaches that of the global one as the resolution gets lowered; in the limit of the zero resolution, the result approaches that of the global measurement as expected.

The properties of the fermion propagator can also be discussed using the cluster decomposition of the partition functional. Following the same line of reasoning, it can be shown that the fermion propagator \(S_F(x_1, x_2)\) has the following qualitative properties for a space-like separation between \(x_1\) and \(x_2\): 1) when the distance between \(x_1\) and \(x_2\) is large, \(S_F(x_1, x_2)\) behaves much like that of a genuine particle with mass \(\bar{\sigma}\) 2) when the distance between \(x_1\) and \(x_2\) is comparable to the dimension of \(\Delta\Omega\) for which \(\alpha\) turns into a constant, \(S_F(x_1, x_2)\) is a superposition of the free fermion propagator with a mass \(\bar{\sigma} + \bar{\sigma}'\) weighted by a Gaussian weight peaked at \(\bar{\sigma}\). In the first case, the propagation of the fermions inside the system can be well approximated by that of a quasi-particle. In the second case, the quasi-particle picture for a fermion is not sufficient due to the fluctuation of the “mass term” can not be suppressed. For a time-like separation, the qualitative picture discussed above is expected to be still true.

The results of non-relativistic statistical mechanics and its relativistic extension based on GCE are approached by the ones derived from the local theory under two extreme conditions: 1) vanishing energy or zero resolution limit and 2) spatial uniformity limit. While the first limit can be achieved in hadronic processes, the second one can only be possibly realized in quark-gluon plasma or weak coupling limit since the confining phase of QCD is not a uniform situation for the quark degrees of freedom in the view of the local theory even at equilibrium. This is discussed above.

In principle, a constant chemical potential can only be coupled to the hadronic degrees of freedom in the confining phase of QCD but not the quark degrees of freedom. In practice, when the quasi-particle provides a satisfying set of degrees of freedom in describing the physical problem, an effective chemical potential for the quasi-particle or constituent quarks can be used only in an approximate sense in the local theory.

**E. The implications**

The existence of the dark component for local fermion number density has interesting implications. Take the vacuum state of a massless strongly interacting system in which the chiral symmetry is spontaneously broken down for example. The originally gapless vacuum state acquires an excitation gap. If only the massive quasi-particles are taken into account, the baryon number density are expected to be non-vanishing only when \(\mu\) is larger than the mass of the quasi-particles. The local fermion number density is however finite as long as \(\mu\) is not zero. A natural question arises as to what this dark component of fermion number corresponding to? It is reasonable to conjecture that this dark component of fermion number corresponds to those fermion states that are localized and non-propagating \(1\) similar to the Anderson localization in a condensed matter system. The random potential here is self-generated within the system by the transient and localized random quantum fluctuations of the \(\sigma\) and other boson fields. Such a conjecture is amenable to future studies.

At the fundamental level, the dark component of local observables is of pure quantum in origin related to the space-like correlations between local measurements in the relativistic quantum world that are shown to exist in experimental observations \(2\), which are what reality manifests itself \(10\). This is because the size of the correlated cluster used in studying the vacuum state (in the Euclidean space-time) tends to zero in the classical limit (\(\hbar \rightarrow 0\)) as a result of the classical relativistic causality, which suppresses the dark component for any finite resolution observation \(4\).

**IV. THE INSTABILITY OF AGCE AND THE ASYMPTOTIC NON-THERMO ENSEMBLE**
A. The instability of AGCE

The Euclidean effective action for the boson field given by Eq. 6 can be evaluated in the usual way \[3\]. Since the effective action given by Eq. 6 is a canonical functional of \( \mu^\alpha \), we can make a Legendre transformation of it, namely

\[
\tilde{S}_{\text{eff}} = S_{\text{eff}} - \int d^4x \bar{\rho}_\alpha \tilde{J}^\alpha \tag{13}
\]

with \( \tilde{J}^\alpha \) the ground state current of fermion number density, to make it a canonical functional of the fermion density in order to study the the stability of the vacuum state against fluctuations in fermion number.

The vacuum state is defined as the special ground state that has the lowest possible “energy density” amongst all other ground states, each one of which has the lowest “energy density” under a set of corresponding external constraints.

The effective potential characterizing the above mentioned energy density can be defined using \( \tilde{S}_{\text{eff}} \) for space-time independent background field:

\[
V_{\text{eff}} = -\frac{\tilde{S}_{\text{eff}}}{V_4} \tag{14}
\]

with \( V_4 \) the volume of the space-time box that contains the system. The stability of the vacuum state against the fluctuations in fermion number density around \( \rho = 0 \) can be studied using \( V_{\text{eff}} \), which is a canonical function of \( \rho \).

Minimization of \( V_{\text{eff}} \) with respect to \( \mu \) gives the vacuum value \( \rho_{\text{vac}} \) of the system since the vacuum \( \rho \) is a known function of the vacuum \( \mu \) in AGCE.

The local finite density theory based upon the AGCE developed after considering the above ingredients is examined by applying it to the chiral symmetry breaking phase of the half bosonized Nambu Jona-Lasinio model for the 3+1 dimension and the chiral Gross-Neveu model for 2+1 dimensions with its effective potential given by

\[
V_{\text{eff}} = -N_g \int \frac{d^Dp}{(2\pi)^D} \left[ \ln \left( 1 + \frac{\sigma^2}{p_+^2} \right) + \ln \left( 1 + \frac{\sigma^2}{p_-^2} \right) \right] + \frac{1}{4G_0} \sigma^2 + \overline{\tau}, \tag{15}
\]

where “\( D \)” is the space-time dimension, \( G_0 \) is the coupling constant and \( p_\pm^2 = (p_0 \pm i\mu)^2 + p^2 \). It can be shown that \( \partial^2 V_{\text{eff}} / \partial \mu^2 \big|_{\mu=0} < 0 \) as long as \( \sigma \neq 0 \) and \( D \geq 2 \), which implies that the \( \sigma \neq 0 \) state with \( \overline{\tau} = 0 \) is not stable against fermion number fluctuations. Such a conclusion is both theoretically and physically unacceptable.

B. The asymptotic non-thermo ensemble

It appears that the AGCE is not sufficient for the local finite density theory, a more general ensemble, called the asymptotic non-thermo ensemble (ANTE) is introduced to cure this pathology. The ANTE in the remote past of the system is not necessarily a strict thermal one. This is because the \( \sigma \neq 0 \) phase, which is called the \( \alpha \)-phase of the massless quark system, is known to be condensed with macroscopic number of bare fermion-antifermion pairs. These pairs occupy the low energy states of the system; they block other fermions from further filling of these states. This kind of statistical blocking effect is not encoded into Eq. 15, which starts the time evolution of the system from a set of initial states having zero number of fermion-antifermion pairs. The later initial states do not overlap with the states having a finite density of such pairs in the thermodynamic limit. Its effects can however be included in the boundary condition for the system like what has been done for the finite density cases. Therefore, it is proposed that the initial ensemble of quantum states in the ANTE for the system at zero temperature and density are those ones with negative energy states filled up to \( E = -\epsilon \) rather than \( E = 0 \) and with positive energy states filled up to \( E = \epsilon \) rather than empty. It is expected that such a state can has sufficient overlap with the true vacuum state of the system for properly determined \( \epsilon \). In the ANTE, the effective potential corresponding to Eq. 15 is expressed as

\[
V_{\text{eff}} = -N_g \int \frac{d^Dp}{(2\pi)^D} \left[ \ln \left( 1 + \frac{\sigma^2}{p_+^2} \right) + \ln \left( 1 + \frac{\sigma^2}{p_-^2} \right) \right] + \frac{1}{4G_0} \sigma^2 + \overline{\tau}_(+) + \overline{\tau}_(-) - \overline{\tau}, \tag{16}
\]

where
\[ \mathcal{C} = 2N_g \int_{\mathcal{C}} p^{D-1} \frac{d^{D-1}p}{(2\pi)^{D-1}}. \]  

Here the upper boundary of the radial \( p \) integration in the D-1 dimensional momentum space is denoted as 

\[ \mu_\pm \equiv \mu \pm \epsilon. \]  

The contour \( \mathcal{C} \) for the (complex) \( p_0 \) integration is shown in Fig. 1 in which both the original Minkowski contour and its Euclidean distorted contour are drawn. The effective potential for the \( \sigma \neq 0 \) case in ANTE given by the above equation has two sets of minima, the first set contains \( \mu = \pm \mu_{\text{vac}} \) and \( \epsilon = 0 \) points, the second one includes \( \mu = 0 \) and \( \epsilon = \pm \epsilon_{\text{vac}} \) ones with finite \( \mu_{\text{vac}} \) and \( \epsilon_{\text{vac}} \). The second solutions correspond to the absolute minima, namely, the vacuum state. Thus there is no instability against quantum fluctuation over \( \rho \).

**C. Model studies**

The vacuum and ground states of the strong interaction could have different phases from the \( \alpha \)-phase \([12,13]\). They are characterized by a condensation of diquarks. Such a possibility is interesting because it may be realized in the early universe, in astronomical objects and events, in heavy ion collisions, inside nucleons \([15,17]\) and nuclei, etc.

For the possible scalar diquark condensation in the vacuum, a half bosonized model Lagrangian is introduced \([18,3]\), which reads

\[ L_I = \frac{1}{2} \bar{\Psi} \sigma \Psi + \bar{\Psi} \chi \gamma^5 \chi O_{(-)} - \frac{1}{4G_0} (\sigma^2 + \chi^2) + \frac{1}{2G_0} \chi^2, \]

where \( \sigma, \chi \) are auxiliary fields with \( (\chi^c)^\dagger = -\chi^c \) and \( G_0, G_\chi \) are coupling constants of the model. \( \chi^c \) are raising and lowering operators respectively in the upper and lower 4 components of \( \Psi \).

This model has two non-trivial phases. The vacuum expectation of \( \sigma \) is non-vanishing with vanishing \( \chi^2 = -\chi^c \chi^c \) in the \( \alpha \)-phase. The vacuum state in the \( \alpha \)-phase is condensed with quark-antiquark pairs. The vacuum expectation of \( \sigma \) is zero with finite \( \chi^2 \) that spontaneously breaks the \( U(1) \) statistical gauge symmetry in the second phase, which is called the \( \omega \)-phase. There is a condensation of correlated scalar diquarks and antidiquarks in the color \( \mathbf{3} \) and \( \mathbf{3} \) states in the \( \omega \)-phase of the vacuum state.

Since diquarks and antidiquarks are condensed in the \( \omega \)-phase, it is expected that an exchange of the role of \( \epsilon \) and \( \mu \) occurs. It is found to be indeed true: there are also two sets of minima for the effective potential, the first set is the one with finite \( \mu = \pm \mu_{\text{vac}} \) and \( \epsilon = 0 \) and the second set corresponds to \( \mu = 0 \) and finite \( \epsilon = \pm \epsilon_{\text{vac}} \) in the \( \omega \)-phase of the model. But here the absolute minima of the system in the \( \omega \)-phase correspond to the first set of solutions, in which the \( \text{CP/T} \) invariance and baryon number conservation are spontaneously violated due to the presence of a finite vacuum \( \mu_{\text{vac}} \). This conclusion is also applicable to the \( \beta \)-phase of models \([12,13]\) with vector fermion pair and antifermion pair condensation, in which the chiral symmetry \( SU(2)_L \times SU(2)_R \) is also spontaneously broken down.

**D. The implications**

Although it is found that \( \epsilon_{\text{vac}} \leq \sigma_{\text{vac}} \) in all cases that were studied, the effects of a finite vacuum \( \epsilon \) can still have dynamical consequences \([12,13]\) since at short distances (between \( x_1 \) and \( x_2 \)), the propagation of the fermion is not described by the quasi-particle propagator which has a definite mass (\( \sigma_{\text{vac}} \) in the half-bosonized four fermion interaction model). The fluctuation in the effective quasi-particle mass are finite at short distances so that there will be a finite tail of the mass term that goes below \( \epsilon_{\text{vac}} \) which causes real dynamical effects some of which are discussed in Refs. \([13]\).

Quasi-particles in an interacting system can not propagate indefinitely so that the fluctuation in their effective mass term can be ignored. The quasi-particles in the system are scattered constantly when they propagate inside the system with a finite time to go as a free particle before being scattered. Therefore the presence of the statistical blocking effects characterized by a finite \( \epsilon \) has observable effects in principle. One of them are discussed in the following related
to the absence of additional long range force to the gravity in the α-phase, which is believed to be the phase in which the strong interaction vacuum state is at at the present day conditions.

V. THE DYNAMICS OF THE PRIMARY STATISTICAL GAUGE FIELD

The equilibrium configuration of the primary statistical gauge field $\mu^\alpha$ can has non-trivial topology which will be studied in other works. The vibration of the primary statistical gauge field $\mu^\alpha(x)$ around its equilibrium configuration represents certain collective excitations of the system.

The dynamics of the primary statistical gauge field are generated by $S_{\text{eff}}^{(\mu)} = \ln Z$. Due to the statistical gauge invariance, the leading order in the derivative expansion of $S_{\text{eff}}^{(\mu)}$ in terms of $\mu^\alpha$ can be expressed as the following:

$$S_{\text{eff}}^{(\mu)} = \int d^4x \left[ -\frac{Z^{(\mu)}}{4} f_{\mu\nu} f^{\mu\nu} + \frac{N g}{\pi^2} \vec{\tau} \cdot \mu' \cdot \mu' + \ldots \right]$$

with $f^{\alpha\beta} = \partial^\alpha \mu^\beta - \partial^\beta \mu^\alpha$ and $\vec{\tau}$ the ground state value of $\tau$; in the phase where local statistical $U(1)$ gauge symmetry is spontaneously broken down and before considering electromagnetic interaction,

$$S_{\text{eff}}^{(\mu)} = \frac{1}{2} \int d^4x \left[ \tau \tau - g^{\alpha\beta} \Pi^{(\mu)} \right] \mu'_\alpha \mu'_\beta + \ldots$$

In both of the situations with slow varying $\mu'_\alpha$, $Z^{(\mu)}$ and $\Pi^{(\mu)}$, which can be extracted from the time-ordered current–current correlator $< T j^\alpha(x) j^\beta(x') >$, are approximately constants.

The excitation corresponds to $\mu^\alpha$ is massive for the vacuum state in the α-phase due to the statistical blocking effects. There is no long range force associated with $\mu^\alpha$ in the α-phase. This conclusion is important for the current theory to be consistent with empirical facts related to the absence of long range force between charge neutral objects besides the gravity. This is because the primary statistical gauge field $\mu^\alpha$ is massless due to the statistical gauge invariance before considering the effects of the statistical blocking. The exchange of massless objects generates long range force between the interacting objects.

It can be shown that after including the electromagnetic interaction, the spatial component of $\mu^\alpha$ excitation is massless for the vacuum state in the β- and ω- phases in the rest frame of matter.

VI. SUMMARY

In summary, it is found that general local relativistic quantum field theory (in the sense of including both the zero and the finite density situations) contains dark components for local observables, statistical gauge invariance and fermionic blocking effects. The proper boundary condition for a consistent theory is that of an optimal ANTE. The rich implications of the finding presented here remain to be explored.

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[1] See, e.g., K. Huang, “Statistical Mechanics,” Willy, New York, 1963.

1So far, the search for the “fifth force” has been turning out null results.
FIG. 1. The $p^0$ integration contour $C$ for the effective potential. The thick lines extending to positive and negative infinity represent the branch cuts of the logarithmic function. Curve “I” is the original $p_0$ integration contour for the theory in the Minkowski space-time. Curve “II” corresponds to the $p_0$ integration contour for the theory in the Euclidean space-time.