Retrial queueing model with two heterogeneous server using matrix geometric method

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Abstract: We consider a retrial queuing system with heterogeneous servers. The two servers have service rate $\mu_1$ for server 1 and $\mu_2$ for server 2 where $\mu_1 \neq \mu_2$. Server 1 is always obtainable and server 2 is intermittently obtainable. Server 2 goes to execute some important dissimilar jobs when the queue distance is increasing. By introducing the Markov Process $\{R(t), I(t); t \geq 0\}$ the stationary analysis has been carried out using Matrix Geometric Technique (MGT). Some numerical outcomes are also obtained.

Keywords: Heterogeneous Server, Retrial queue, Idle, Markovian Process, Exponential Distribution, intermittently obtainable Server

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1. Introduction

Retrial queueing models are queueing models in which an arriving customer who find all servers engaged may retry for service another time later a random total of time. In the present study, an increasing interest in m/m/2 retrial queueing model with Poisson arrival and two heterogeneous server. If there is a free server when a patron comes from outside the system, this patron commences to be served instantly and depart the system afterwards the service is consummated. Upon arrival, if the server 1 and server 2 is working or intermittently obtainable, the arriving patron will join the orbit with probability $\theta$ and then retry for service according to an FCFS discipline. We assume that retrial times are exponentially distributed with parameter $\gamma_1$ for working or $\gamma_0$ for intermittently obtainable.

Interference in service occur for a random length of time but occur only after the service in hand is completed. This service is termed as intermittently obtainable service. Retrial queues have been extensively used to replica many complications/situations in telephone system, call centers, telecommunication networks, computer networks and in routine life. In the important concept of Steady state Probabilities have been found using Matrix geometric technique.

Accessible bibliography on retrial queues has been investigated by Artalejo [1]. On the busy period of the M/G/1 retrial queue was studied by Artalejo and Lopez-Herrero [2]. Retrial queueing systems were proposed by Artalejo and Gomez-Corral [3]. Analysis of a multi-server re-trial queue with search of customers from the orbit has been introduced by Chakravarty etal[4]. Threshold policies for controlled retrial queues with heterogeneous servers were analyzed by Efrosinin and Breuer [5]. Utilization of idle time in an M/G/1 queueing system have been studied by Levy and Yechiali [6]. Matrix-Geometric solutions in stochastic models have been investigated by Neuts[7]. Ponomarov and
Lebedev [8] have studied in finite Source retrial queues with state-dependent service rate. Numerical analysis of optimal control polices for queueing systems with heterogeneous servers have been investigated by Rykov and Efrosinin [9]. On the busy period distribution of the M/G/2 queueing system has been proposed by Wiens [10]. The queue length in an M/G/1 batch arrival retrial queue was studied by Yamamuro [11].

Generally, Interference occur in working period. Interference occur in the form of server breakdown or due to the unobtainability of the server. Queueing system with service interruptions have been introduced by Federgrune and Green [12]. Mitran and Avi-Itzhak [13] proposed a many-server queue with service interruptions. Nain [14] introduced inqueueing systems with service interruptions. A single server queue with service interruptions was analyzed by Takine and Sengupta [15]. A retrial queueing model with unreliable server in k policy was studied by Seenivasan et al. [16]. A multi serverretial queue with break down and geometric loss was analyzed by Kalyanaraman and Seenivasan [17].

The rest of the paper is organized as follows: In section 2, the mathematical model has been introduced and the analysis is carried out. In section 3, the model analyzed using a numerical example. For a detailed review of main results and the literature of multi server queues one may refer the monograph by Falin and Templeton [18].

2. Model Description

We determine the two heterogeneous server retrial queue in which patron arrive according to a Poisson process at rate $\lambda_0$ for intermittently obtainable and $\lambda_1$ for working. The two servers have heterogeneous service rate $\mu_1$ for server 1 and $\mu_2$ for server 2 where $\mu_1 \neq \mu_2$. The service time at both servers follow exponential disbursed. Server 1 is always obtainable and server 2 is intermittently obtainable. Server 2 goes to swift some crucial dissimilar/odd jobs when the queue disparate is larger than or equal to zero. But before going to perform these odd jobs, the server 2 must first complete his in hand service. The obtain ability time of server 2 follows exponential distribution with rate $\nu$.

Patron that find the server dormant on arrival will occupy the server instantly and commence being served. On consummation of the service, they depart the system. On arrival, if the server 1 and server 2 is working or intermittently obtainable, the arriving patron will join the orbit with probability $\theta$ and then retry for service according to an FCFS discipline. We presume that retrial times are exponentially disbursed with limitation $\gamma_1$ for working or $\gamma_0$ for intermittently obtainable. The structure of retrial queuing model is shown in figure 1.

![Figure 1. The structure of retrial queuing model](image-url)
Let $P(t) = (R(t), I(t))$ be the state of the process at time $t$, where $R(t)$ is the number of patrons in the orbit and $I(t)$ is defined as:

$$I(t) = \begin{cases} 
0 & \text{if server 2 is \textit{Intermittently obtainable}} \\
1 & \text{if server 2 is \textit{Working}} \\
2 & \text{if server 2 is \textit{Idle}}
\end{cases}$$

Then $\{R(t), I(t); t \geq 0\}$, a Markov Process with the state space. $R(t)$ be the number of patrons in the orbit and $R(t) \in \{0, 1, 2, 3, \ldots\}$. The state space of the process is $\{0, 1, 2, 3, \ldots\} \times \{0, 1, 2\}$. In equilibrium the infinitesimal matrix is $Q = (q_{ij})$ and is defined as

$$Q = \begin{pmatrix}
A_0 & B_0 & 0 & 0 & \cdots \\
B_2 & B_1 & B_0 & 0 & \cdots \\
0 & B_2 & B_1 & B_0 & \cdots \\
0 & 0 & B_2 & B_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$

Where

$$A_0 = \begin{pmatrix} -(\nu + \mu_1 + \theta) & \nu & \mu_1 \\ 0 & -(\theta + \mu_1 + \mu_2) & \mu_1 + \mu_2 \\ \lambda_0 & \lambda_1 & -(\lambda_0 + \lambda_1) \end{pmatrix}; \quad B_0 = \begin{pmatrix} \theta & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} -(\nu + \mu_1 + \theta) & \nu & \mu_1 \\ 0 & -(\theta + \mu_1 + 2\mu_2) & \mu_1 + \mu_2 \\ \lambda_0 & \lambda_1 & -(\lambda_0 + \lambda_1 + \gamma_0 + \gamma_1) \end{pmatrix}; \quad B_2 = \begin{pmatrix} 0 & 0 & 0 \\ \mu_2 & 0 & 0 \\ \gamma_0 & \gamma_1 & 0 \end{pmatrix}$$

Let $R$ and $I$ be the stationary random variable for the number of patrons in the orbit and the status of the server 2.

We define $P_{ij} = \{R=i, I=j\}$ where $(i, j) \in$ state space set. The stationary probability matrix $P$ is given by $P = (P_0, P_1, P_2, \ldots)$ where $P_i = (P_{0i}, P_{1i}, P_{2i})$ for $i \geq 0$.

The stationary probability matrix $P$ is solved by using $PQ = 0$.

$$P_0A_0 + P_1B_2 + \ldots = 0 \quad (1)$$
$$P_0B_0 + P_1B_1 + P_2B_3 + \ldots = 0$$
$$P_1B_0 + P_2B_1 + P_3B_3 + \ldots = 0$$
$$\vdots$$
$$P_iB_0 + P_{i+1}B_1 + P_{i+2}B_2 = 0 \quad (2)$$

And

$$P_i = P_0R^i, \quad i \geq 1 \quad (3)$$

From equation $(1)$

$$P_0A_0 + P_1B_2 = 0$$
\[ P_0(A_0 + R B_2) = 0 \]  
and  
\[ P_i B_2 + P_{i+1} B_1 + P_{i+2} B_2 = 0 , \]  
using Eq.(3), we have

\[ P_0 R (B_0 + R B_1 + R^2 B_2) = 0, \quad i \geq 1. \]  
(5)

The normalizing equation is given by

\[ P_0 [I - R]^{-1} e = 1 \]  
(6)

Here ‘e’ is column vector of appropriate length of 1’s.

The matrix form R is the least solution to the matrix form non-linear equalization

\[ B_0 + R B_1 + R^2 B_2 = 0, \]  
(7)

Where R larger than or equal zero and it is an irreducible natural number matrix form of spectral radius smaller one. An iterative method can be used to calculate R as follows.

\[ R_0 = 0 \]  
(8),

\[ R_{n+1} = -B_0 B_1^{-1} - R_2^2 B_2 B_1^{-1}, \quad n \geq 0. \]  
(9)

For a Markov process with such generators, Neuts [7] has obtained the stable condition as

\[ PB_0 e < PB_2 e \]  
(10)

Where the row vector \( P = (P_0, P_1, P_2) \) is obtained from the infinitesimal generator \( B = B_0 + B_1 + B_2 \).

B is given by

\[
  B = \begin{pmatrix}
    -(\nu + \mu_1) & \nu & \mu_1 \\
    \mu_2 & -(\mu_1 + 2\mu_2) & \mu_1 + \mu_2 \\
    \lambda_0 + \gamma_0 & \lambda_1 + \gamma_1 & -(\lambda_0 + \lambda_1 + \gamma_0 + \gamma_1)
  \end{pmatrix}
\]  
(11)

It can be shown that B is irreducible and that the row vector P is unique such that

\[ PB = 0 \quad \text{and} \quad Pe = 1 \]  
(12)

From Eq.(12), we have

\[ P_1 = \xi_0 P_0 \]
\[ P_2 = \xi_1 P_0 \]
and

\[ P_0 = [1 + \xi_0 + \xi_1]^{-1} \]

Where

\[ \xi_0 = \frac{(\lambda_0 + \gamma_0)[\nu(\mu_1 + \mu_2)] + (\lambda_1 + \gamma_1)[\nu(\mu_1 + \mu_2)] + \mu_1 \mu_2 (\lambda_1 + \gamma_1)}{(\mu_1 + \mu_2)(\mu_1 + 2\mu_2)(\lambda_0 + \gamma_0) + \mu_1 \mu_2 (\lambda_0 + \gamma_1)} \]

\[ \xi_1 = \frac{[(\nu + \mu_1)(\mu_1 + 2\mu_2) - \mu_1 \nu]}{[(\mu_1 + 2\mu_2)(\lambda_0 + \gamma_0) + \mu_1 (\lambda_1 + \gamma_1)]} \]

The stability condition takes the form

\[ \theta(P_0 + P_1) < \mu_2 P_1 + (\gamma_0 + \gamma_1) P_2 \]

Once if we obtain the R matrix then the probability vectors \( P_i \)'s (i \geq 1) are obtained from Equations (6) and (3).

3. Numerical Example

Now we provide the numerical results for the model debated in the section aloft. Our aim is to display the consequence of a parameter on system properties. By changing \( \lambda_0, \mu_1 \) and \( \mu_2 \). Totally six examples are presented in the section.
The changes in the value of one parameter $\lambda_0$ at time keeping the other parameter values constant Example 3.1. Example 3.2 are presented in below.

3.1. Example.
For $\lambda_0 = 0.7$, $\lambda_1=0.7$, $\mu_1=1.5$, $\mu_2=2.5$, $v = 0.6$, $\gamma_0 = 0.4$, $\gamma_1 = 0.9$, $\theta=0.6$, and the R matrix is given by

$$ R = \begin{pmatrix} 0.3567 & 0.0774 & 0.3128 \\ 0.0872 & 0.1214 & 0.2282 \\ 0 & 0 & 0 \end{pmatrix} $$

Table 1. Probability vectors

|       | P_0         | P_1         | P_2         | P_3         | P_4         | P_5         | P_6         | Total     |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|
| P_0   | 0.1568      | 0.0652      | 0.0255      | 0.0098      | 0.0037      | 0.0014      | 0.0005      | 0.1568    |
| P_1   | 0.1068      | 0.0251      | 0.0081      | 0.0030      | 0.0011      | 0.0004      | 0.0002      | 0.1068    |
| P_2   | 0.4733      | 0.0734      | 0.0261      | 0.0098      | 0.0037      | 0.0014      | 0.0005      | 0.4733    |
| P_3   | 0.7369      | 0.1634      | 0.0597      | 0.0226      | 0.0085      | 0.0032      | 0.0012      | 0.7369    |

By utilizing the overhead R model, the prospect vectors $P_i = P_0 R^i$, $i = 1,2,3,...$, Where $P_0$ is calculated from the relation $P_0[R_0+A_0]= 0$ and normalization condition $P_0[I-R]^i e = 1$ for the numerical parameter designated aloft, the row vector $P_0$ is provided by $P_0 = (0.1568, 0.1068, 0.4733)$. Further the remaining vectors $P_i$’s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3,...$ and are bestowed in Table 1. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, ...$ the last column represent the sum of the three components. It is substantiated that the total probability is 0.9963 $\approx 1$.

3.2. Example.
For $\lambda_0 = 0.8$, $\lambda_1=0.7$, $\mu_1=1.5$, $\mu_2=2.5$, $v = 0.6$, $\gamma_0 = 0.4$, $\gamma_1 = 0.9$, $\theta=0.6$, and the R matrix is given by

$$ R = \begin{pmatrix} 0.3697 & 0.0786 & 0.3104 \\ 0.0965 & 0.1223 & 0.2264 \\ 0 & 0 & 0 \end{pmatrix} $$

Table 2. Probability vectors

|       | P_0         | P_1         | P_2         | P_3         | P_4         | P_5         | P_6         | Total     |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|
| P_0   | 0.1673      | 0.0719      | 0.0291      | 0.0116      | 0.0046      | 0.0018      | 0.0007      | 0.1673    |
| P_1   | 0.1044      | 0.0259      | 0.0088      | 0.0034      | 0.0013      | 0.0005      | 0.0002      | 0.1044    |
| P_2   | 0.4457      | 0.0756      | 0.0282      | 0.0110      | 0.0044      | 0.0017      | 0.0007      | 0.4457    |
| P_3   | 0.7174      | 0.1734      | 0.0661      | 0.0260      | 0.0103      | 0.0040      | 0.0016      | 0.7174    |

Table 2. Probability vectors

|       | Total     |
|-------|-----------|
| Total | 0.9988    |
By utilizing the overhead R model, the prospect vectors \( P_i = P_0R^i \), \( i = 1, 2, 3, \ldots \), where \( P_0 \) is calculated from the relation \( P_0[A_0 + RB_2] = 0 \) and normalization condition \( P_0^T[I - R]^{-1}e = 1 \) for the numerical parameter designated aloft, the row vector \( P_0 \) is provided by \( P_0 = (0.1673, 0.1044, 0.4457) \). Further the remaining vectors \( P_i \)'s are obtained from \( P_i = P_0R^i \), \( i = 1, 2, 3, \ldots \) and are bestowed in Table 2. In the table column 2, 3 and 4 represent the three components of \( P_i \), \( i = 0, 1, 2, \ldots \) the last column represent the sum of the three components. It is substantiated that the total probability is 0.9988 \( \approx 1 \).

The changes in the value of one parameter \( \mu_1 \) at time keeping the other parameter values constant Example 3.3, Example 3.4 are presented in below.

### 3.3. Example.

For \( \lambda_0 = 0.7, \lambda_1=0.7, \mu_1=1.6, \mu_2=2.5, \nu = 0.6, \gamma_0 = 0.4, \gamma_1 = 0.9, \theta = 0.6 \), and the R matrix is given by

\[
R = \begin{pmatrix}
0.3426 & 0.0751 & 0.3170 \\
0.0840 & 0.1196 & 0.2314 \\
0 & 0 & 0
\end{pmatrix}
\]

| \( P_i \) | \( P_0 \) | \( P_1 \) | \( P_2 \) | \( P_3 \) | \( P_4 \) | \( P_5 \) | Total |
|----------|--------|--------|--------|--------|--------|--------|-------|
|          | 0.1548 | 0.1068 | 0.4898 | 0.0620 | 0.0233 | 0.0086 | 0.0032 | 0.1068 |
|          | 0.1548 | 0.1068 | 0.4898 | 0.0620 | 0.0233 | 0.0086 | 0.0032 | 0.1068 |
|          | 0.0233 | 0.0076 | 0.2314 | 0.0032 | 0.0012 |       |       |       |
|          | 0.0086 | 0.0027 | 0.2314 | 0.0032 | 0.0012 |       |       |       |
|          | 0.0032 | 0.0010 | 0.2314 | 0.0032 | 0.0012 |       |       |       |
|          | 0.0012 | 0.0004 | 0.2314 | 0.0032 | 0.0012 |       |       |       |
| Total    |        |        |        | 0.7514 | 0.1602 | 0.0562 | 0.0075 | 0.0028 |
|          |        |        |        | 0.7514 | 0.1602 | 0.0562 | 0.0075 | 0.0028 |

By utilizing the overhead R model, the prospect vectors \( P_i = P_0R^i \), \( i = 1, 2, 3, \ldots \), where \( P_0 \) is calculated from the relation \( P_0[A_0 + RB_2] = 0 \) and normalization condition \( P_0^T[I - R]^{-1}e = 1 \) for the numerical parameter designated aloft, the row vector \( P_0 \) is provided by \( P_0 = (0.1548, 0.1068, 0.4457) \). Further the remaining vectors \( P_i \)'s are obtained from \( P_i = P_0R^i \), \( i = 1, 2, 3, \ldots \) and are bestowed in Table 3. In the table column 2, 3 and 4 represent the three components of \( P_i \), \( i = 0, 1, 2, \ldots \) the last column represent the sum of the three components. It is substantiated that the total probability is 0.9985 \( \approx 1 \).

### 3.4. Example.

For \( \lambda_0 = 0.7, \lambda_1=0.7, \mu_1=1.7, \mu_2=2.5, \nu = 0.6, \gamma_0 = 0.4, \gamma_1 = 0.9, \theta = 0.6 \), and the R matrix is given by

\[
R = \begin{pmatrix}
0.3295 & 0.0731 & 0.3211 \\
0.0811 & 0.1179 & 0.2346 \\
0 & 0 & 0
\end{pmatrix}
\]
Table 4. Probability vectors

|     | Total  |
|-----|--------|
| $P_0$ | 0.1522 0.1063 0.5038 | 0.7623 |
| $P_1$ | 0.0588 0.0237 0.0738 | 0.1563 |
| $P_2$ | 0.0213 0.0071 0.0244 | 0.0528 |
| $P_3$ | 0.0076 0.0024 0.0085 | 0.0185 |
| $P_4$ | 0.0027 0.0008 0.0030 | 0.0065 |
| $P_5$ | 0.0010 0.0003 0.0011 | 0.0024 |
| Total | 0.9964 |

By utilizing the overhead R model, the prospect vectors $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$, where $P_0$ is calculated from the relation $P_0[A_0 + RB_2] = 0$ and normalization condition $P_0[I - R]^{-1}e = 1$ for the numerical parameter designated aloft, the row vector $P_0$ is provided by $P_0 = (0.1522, 0.1063, 0.5038)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$ and are bestowed in Table 4. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is substantiated that the total probability is $0.9964 \approx 1$.

3.5. Example.
For $\lambda_0 = 0.7$, $\lambda_1=0.7$, $\mu_1=1.5$, $\mu_2=2.6$, $\nu = 0.6$, $\gamma_0 = 0.4$, $\gamma_1 = 0.9$, $\theta=0.6$, and the R matrix is given by

$$R = \begin{pmatrix} 0.3561 & 0.0749 & 0.3116 \\ 0.0861 & 0.1176 & 0.2264 \\ 0 & 0 & 0 \end{pmatrix}$$

Table 5. Probability vectors

|     | Total  |
|-----|--------|
| $P_0$ | 0.1583 0.1055 0.4785 | 0.7423 |
| $P_1$ | 0.0655 0.0243 0.0732 | 0.1630 |
| $P_2$ | 0.0254 0.0078 0.0259 | 0.0591 |
| $P_3$ | 0.0094 0.0028 0.0097 | 0.0222 |
| $P_4$ | 0.0037 0.0011 0.0037 | 0.0085 |
| $P_5$ | 0.0014 0.0004 0.0014 | 0.0032 |
| Total | 0.9983 |

By utilizing the overhead R model, the prospect vectors $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$, where $P_0$ is calculated from the relation $P_0[A_0 + RB_2] = 0$ and normalization condition $P_0[I - R]^{-1}e = 1$ for the numerical parameter designated aloft, the row vector $P_0$ is provided by $P_0 = (0.1583, 0.1055, 0.4785)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$ and are bestowed in Table 5. In the
table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. The total probability is substantiated to be $0.9983 \approx 1$.

3.6. Example.
For $\lambda_0 = 0.7$, $\lambda_1=0.7$, $\mu_1=1.5$, $\mu_2=2.7$, $\nu = 0.6$, $\gamma_0 = 0.4$, $\gamma_1= 0.9$, $\theta=0.6$, and the R matrix is given by

$$R = \begin{pmatrix}
0.3557 & 0.0727 & 0.3106 \\
0.0851 & 0.1140 & 0.2247 \\
0 & 0 & 0
\end{pmatrix}$$

Table 6. Probability vectors

|       | $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | Total |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $P_0$ | 0.1592| 0.1038| 0.4820| 0.0096| 0.0037| 0.0014| 0.7450|
| $P_1$ | 0.0655| 0.00234| 0.0728| 0.0095| 0.0036| 0.0004| 0.1617|
| $P_2$ | 0.0253| 0.0074| 0.0256| 0.0095| 0.0036| 0.0004| 0.0586|
| $P_3$ | 0.0096| 0.0027| 0.0095| 0.0095| 0.0036| 0.0004| 0.0218|
| $P_4$ | 0.0037| 0.0010| 0.0036| 0.0036| 0.0036| 0.0004| 0.0083|
| $P_5$ | 0.0014| 0.0004| 0.0014| 0.0014| 0.0014| 0.0004| 0.0032|
| Total |       |       |       |       |       |       | 0.9980|

By utilizing the overhead R model, the prospect vectors $P_i = P_0 R^i$, $i = 1,2,3,\ldots$, where $P_0$ is calculated from the relation $P_0[A_0+RB_2] = 0$ and normalization condition $P_0 [I - R]^{-1} e = 1$ for the numerical parameter designated aloft, the row vector $P_0$ is provided by $P_0 = (0.1592, 0.1038, 0.4820)$ . Further the remaining vectors $P_i$’s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3,\ldots$ and are bestowed in Table 6. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is substantiated that the total probability is $0.9980 \approx 1$.

4. Conclusion
In this paper, we observed a retrial queuing system with intermittently obtainable server. We studied static probability vector by appealing MGM. Moreover, we conducted numerical analysis by presuming especially values to the parameters.

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