Experiments with Conflict Analysis in Mixed Integer Programming
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Abstract

The analysis of infeasible subproblems plays an important role in solving mixed integer programs (MIPs) and is implemented in most major MIP solvers. There are two fundamentally different concepts to generate valid global constraints from infeasible subproblems. The first is to analyze the sequence of implications obtained by domain propagation that led to infeasibility. The result of the analysis is one or more sets of contradicting variable bounds from which so-called conflict constraints can be generated. This concept has its origin in solving satisfiability problems and is similarly used in constraint programming. The second concept is to analyze infeasible linear programming (LP) relaxations. The dual LP solution provides a set of multipliers that can be used to generate a single new globally valid linear constraint. The main contribution of this short paper is an empirical evaluation of two ways to combine both approaches. Experiments are carried out on general MIP instances from standard public test sets such as MIPLIB2010; the presented algorithms have been implemented within the non-commercial MIP solver SCIP. Moreover, we present a pool-based approach to manage conflicts which addresses the way a MIP solver traverses the search tree better than aging strategies known from SAT solving.

1 Introduction: MIP and Conflict Analysis

In this paper we consider mixed integer programs (MIPs) of the form

$$c^* = \min \{ c^t x \mid Ax \geq b, \ell \leq x \leq u, x \in \mathbb{Z}^k \times \mathbb{R}^{n-k} \}, \quad (1)$$

with objective function $c \in \mathbb{R}^n$, constraint matrix $A \in \mathbb{R}^{m \times n}$, constraint left-hand side $b \in \mathbb{R}^m$, and variable bounds $\ell, u \in \mathbb{R}^n$, where $\mathbb{R} := \mathbb{R} \cup \{ \pm \infty \}$. 

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Furthermore, let $\mathcal{N} = \{1, \ldots, n\}$ be the index set of all variables. Let $\mathcal{I} \subseteq \mathcal{N}$ such that $x_i \in \mathbb{Z}$ for all $i \in \mathcal{I}$, i.e., the set of variables that need to be integral in every feasible solution.

When omitting the integrality requirements, we obtain the linear program (LP)

$$c^*_{LP} = \min \{c^t x | Ax \geq b, \ell \leq x \leq u, x \in \mathbb{R}^n\}. \quad (2)$$

The linear program (2) is called LP relaxation of (1). The LP relaxation provides a lower bound on the optimal solution value of the MIP (1), i.e., $c^*_{LP} \leq c^*$. In LP-based branch-and-bound [11, 18], the most commonly used method to solve MIPs, the LP relaxation is used for bounding. Branch-and-bound is a divide-and-conquer method which splits the search space sequentially into smaller subproblems that are (hopefully) easier to solve. During this procedure we may encounter infeasible subproblems. Infeasibility can be detected by contradicting implications, e.g., derived by domain propagation, or by an infeasible LP relaxation. Modern MIP solvers try to learn from infeasible subproblems, e.g., by conflict analysis. Conflict analysis for MIP has its origin in solving satisfiability problems (SAT) and goes back to [21]. Similar ideas are used in constraint programming, e.g., see [14, 15, 25]. First integrations of these techniques into MIP were independently suggested by [12], [24] and [2]. Further publications suggested to use conflict information for variable selection in branching, to tentatively generate conflicts before branching [3, 16], and to analyze infeasibility detected in primal heuristics [6, 7].

Today, conflict analysis is widely established in solving MIPs. The principal idea of conflict analysis, in MIP terminology, can be sketched as follows.

Given an infeasible node of the branch-and-bound tree defined by the subproblem

$$\min \{c^t x | Ax \geq b, \ell' \leq x \leq u', x \in \mathbb{Z}^k \times \mathbb{R}^{n-k}\} \quad (3)$$

with local bounds $\ell \leq \ell' \leq u' \leq u$. In LP-based branch-and-bound, the infeasibility of a subproblem is typically detected by an infeasible LP relaxation (see next section) or by contradicting implications.

In the latter case, a conflict graph gets constructed which represents the logic of how the set of branching decisions led to the detection of infeasibility. More precisely, the conflict graph is a directed acyclic graph in which the vertices represent bound changes of variables and the arcs $(v, w)$ correspond to bound changes implied by propagation, i.e., the bound change corresponding to $w$ is based (besides others) on the bound change represented by $v$. In addition to these inner vertices which represent the bound changes from domain propagation, the graph features source vertices for the bound changes that correspond to branching decisions and an artificial sink vertex representing the infeasibility. Then, each cut that separates the branching decisions from the artificial infeasibility vertex gives rise to a valid conflict constraint. A conflict constraint consists of a set of variables with associated bounds, requiring that in each feasible solution at least one of the variables has to take a value outside
these bounds. Note that in general, this is not a linear constraint and that by using different cuts in the graph, several different conflict constraints might be derived from a single infeasibility. A variant of conflict analysis close to the one described above is implemented in SCIP, the solver in which we will conduct our computational experiments. Also, a similar implementation is available in the FICO Xpress-Optimizer.

This short paper consists of two parts which are independent but complement each other in practice. The first part of this paper (Section 2) focuses on a MIP technique to analyze infeasibility based on LP theory. We discuss the interaction, differences, and commonalities between conflict analysis and the so-called dual ray analysis. Although both techniques have been known before, e.g., [2, 22], this will be, to the best of our knowledge, the first published direct comparison of the two. In the second part (Section 3), we present a new approach to drop conflicts that do not lead to variable bound reductions frequently. This new concept is an alternative to the aging scheme known from SAT. Finally, we present computational experiments comparing the techniques described in Section 2 and 3.

2 Analyzing Dual Unbounded Solutions

The idea of conflict analysis is tightly linked to domain propagation: conflict analysis studies a sequence of variable bound implications made by domain propagation routines. Besides domain propagation, there is another important subroutine in MIP solving which might prove infeasibility of a subproblem: the LP relaxation. The proof of LP infeasibility comes in form of a so-called “dual ray”, that is a list of multipliers on the model constraints and the variable bounds. Those give rise to a globally valid constraint that can be used similarly to a conflict constraint. In this section, we discuss the analysis of the LP infeasibility proof in more detail.

2.1 Analysis of Infeasible LPs: Theoretical Background

Consider a node of the branch-and-bound tree and the corresponding subproblem

\[
\begin{align*}
\min & \quad c^t x \\
\text{subject to} & \quad Ax \geq b, \quad \ell' \leq x \leq u', \quad x \in \mathbb{Z}^k \times \mathbb{R}^{n-k}
\end{align*}
\]

(4)

defined by local bounds \( \ell \leq \ell' \leq u' \leq u \). The dual LP of the corresponding LP relaxation of (4) is given by

\[
\begin{align*}
\max & \quad y^t b + \ell^t \ell' + u'^t u' \\
\text{subject to} & \quad A^t y + \ell + u \leq c, \quad y, \ell, u \in \mathbb{R}^n_{\geq 0}, \quad u \in \mathbb{R}^n_{\leq 0},
\end{align*}
\]

(5)

where \( A_i \) is the \( i \)-th column of \( A \), \( \ell_i = \max\{0, c_i - y^t A_i\} \), and \( u_i = \min\{0, c_i - y^t A_i\} \). By LP theory each unbounded ray \((\gamma, \ell, u)\) of (5) proves infeasibility
A ray is called unbounded if multiplying the ray with an arbitrary scalar \( \alpha > 0 \) will not change the feasibility. Note, in this case it holds
\[
r_i = \max\{0, -y^t A_i\} \quad \text{and} \quad r_i = \min\{0, -y^t A_i\}.
\]
Moreover, the Lemma of Farkas states that exactly one of the following two systems is satisfiable
\[
(F_1) \quad \begin{cases} 
Ax & \geq b \\
\ell' & \leq x & \leq u'
\end{cases} \quad \forall \begin{cases} 
\gamma^t A + \bar{r} + \bar{\tau} & \leq 0 \\
\gamma^t b + \bar{r} \ell' + \bar{\tau} u' & > 0
\end{cases} \quad (F_2)
\]
It follows immediately, that if \( F_1 \) is infeasible, there exists an unbounded ray \((\gamma, \bar{r}, \bar{\tau})\) of \((5)\) satisfying \(F_2\). An infeasibility proof of \((4)\) is given by a single constraint
\[
\gamma^t Ax \geq \gamma^t b, \quad (6)
\]
which is an aggregation of all rows \(A_j\) for \(j = 1, \ldots, m\) with weight \(\gamma_j > 0\). Constraint \((6)\) is globally valid but violated in the local bounds \([\ell', u']\) of subproblem \((4)\). In the following, this constraint will be called \textit{proof-constraint}.

2.2 Conflict Analysis of Infeasible LPs

The analysis of an infeasible LP relaxation, as it is implemented in SCIP, is a hybrid of the theoretical considerations made in Section 2.1 and the analysis of the conflict graph known from SAT. To use the concept of a conflict graph, all variables with a non-zero coefficient in the proof-constraint are converted to vertices of the conflict graph representing bound changes; global bound changes are omitted. Those vertices, called the \textit{initial reason}, are then connected to the artificial sink representing the infeasibility. This neat idea was introduced in [1]. From thereon, conflict analysis can be applied as described in Section 1.

In practice, the proof-constraint is often quite dense, and therefore, it might be worthwhile to search for a sparser infeasibility proof. This can be done by a heuristic that relaxes some of the local bounds \([\ell'', u'']\) that appear in the proof-constraint. Of course, the relaxed local bounds \([\ell'', u'']\) with \(\ell < \ell'' \leq \ell' \leq u' \leq u'' < u\) still need to fulfill
\[
\gamma^t b + \bar{r} \ell'' + \bar{\tau} u'' > 0.
\]
The more bounds can be relaxed that way, the smaller gets the initial reason and consequently the stronger are the derived conflict constraints. Note again that these constraints do not need to be linear, if general integer or continuous variables are present.

2.3 Dual Ray Analysis of Infeasible LPs

The proof-constraint is globally valid but infeasible within the local bounds. It follows immediately by the Lemma of Farkas that the \textit{maximal activity}
\[
\Delta_{\text{max}}(\gamma^t A, \ell', u') := \sum_{i \in N: \gamma^t A_i > 0} (\gamma^t A_i) u'_i + \sum_{i \in N: \gamma^t A_i < 0} (\gamma^t A_i) \ell'_i
\]
of $\gamma^t Ax$ w.r.t. variable bounds $[\ell', u']$ is strictly less than the corresponding left-hand side $\gamma^t b$.

Instead of creating an “artificial” initial reason, the proof-constraint might also be used directly for domain propagation in the remainder of the search. It is a conic combination of global constraints, i.e., it is itself a valid (but redundant) global constraint. In contrast to the method described in Section 2.2, using a dual unbounded ray as a set of weights to aggregate model constraints yields exactly one linear constraint.

The proof-constraint along with an activity argument can be used to deduce local lower and upper variable bounds $[\ell', u']$. For any $i \in \mathcal{N}$ with a non-zero coefficient in the proof-constraint the maximal activity residual is given by

$$
\Delta_{\max}^i(\gamma^t A, \ell', u') := \sum_{j \in \mathcal{N} \setminus i : \gamma^t A_j > 0} (\gamma^t A_j)u'_j + \sum_{j \in \mathcal{N} \setminus i : \gamma^t A_j < 0} (\gamma^t A_j)\ell'_j,
$$
i.e., the maximal activity over all variables but $x_i$. Hence, valid local bounds are given by

$$
\frac{\gamma^t b - \Delta_{\max}^i(\gamma^t A, \ell', u')}{a_i} \left\{ \begin{array}{ll}
\leq & \text{if } a_i > 0 \\
\geq & \text{if } a_i < 0
\end{array} \right\} x_i.
$$

This is the so-called bound tightening procedure [9] which is widely used in all major MIP solvers, for all kinds of linear constraints.

Just like the dual ray might be heuristically shrunk to get a short initial reason for conflict analysis, it might be worthwhile to alter the proof-constraint itself before using it for propagation. This can include the application of presolving steps such as coefficient tightening to the constraint, projecting out continuous variables or applying mixed-integer rounding to get an alternative globally valid constraint which might be more powerful to propagate.

Finally, instead of generating a valid constraint from the dual ray, one could equivalently use the ray itself to simply check for infeasibility [22, 23] or to estimate the objective change during branch-and-bound and to derive branching decisions therefrom. While in Section 2.2 we described a way to reduce LP infeasibility analysis to conflict analysis based on domain propagation, one could as well try to generate a dual ray by solving the LP relaxation after having detected infeasibility by propagation.

3 Managing of Conflicts in a MIP Solver

Maintaining and propagating large numbers of conflict constraints might slow down a solver and create a big burden memory-wise. For instances with a high throughput of branch-and-bound nodes, a solver like SCIP might easily create hundreds of thousands of conflicts within an hour of running time. In order to avoid a slowdown or memory short-coming, an aging mechanism is used within SCIP. Once again, aging is a concept inspired by SAT and CP solving. Every
time a conflict constraint is considered for domain propagation an age counter (individually for each constraint) is increased if no deduction was found. If a deduction is found, the age will be reset to 0. If the age reaches a predefined threshold the conflict constraint is permanently deleted.

In SAT and CP, this mechanism is a well-established method to drop conflict constraints that do not frequently propagate. In the case of MIP solving, there are two main differences concerning the branch-and-bound search. First, domain propagation is most often not the most expensive part of node processing. Second, SAT and CP solvers often use a pure depth-first-search (DFS) node selection, while state-of-the-art MIP solvers use some hybrid between DFS and best-estimate-search or best-first-search (see, e.g., [2, 5, 24]). Therefore, it frequently happens that the node processed next is picked from a different part of the tree.

In the following, we describe a pool-based approach to manage conflict constraints. Here, a pool refers to a fixed-size array that allows direct access to a particular element and which is independent of the model itself. The conflict pool is used to manage all conflict constraints, independently whether they were derived from domain propagation or an infeasible LP relaxation. The number of constraints that can be stored within the conflict pool at the same is limited. In our implementation the maximal size of the conflict pool depends on the number of variables and constraints of the presolved problem. However, the pool provides space for at least 1 000 and at most 50 000 conflict constraints at the same time. The conflict pool allows a central management of conflict constraints independently from the model constraints, i.e., they can be propagated, checked or deleted separately, without the need to traverse through all constraints.

To drop conflict constraints that don’t lead to deductions frequently we implemented an update-routine that checks the conflict pool regularly, e.g., any time we create the first new conflict at a node. Moreover, we still use the concept of aging to determine the conflict constraints that are rarely used in propagation. Within this update procedure the oldest conflict constraints are removed.

Beside of the regular checks, the conflict pool is updated every time a new improving incumbent solution is found. Conflict constraints might depend on a (previous) best known solution, e.g., when the conflict was created from an LP whose infeasibility proof contained the objective cutoff. Such conflicts become weaker whenever a new incumbent is found and the chance that they lead to deductions becomes smaller the more the incumbent improves. Due to this, for each conflict constraint involving an incumbent solution we store the corresponding objective value. If this value is sufficiently worse than the new objective value, the conflict constraint will be permanently deleted. In our computational experiments (cf. Section 4) we use a threshold of 5%.
4 Computational Experiments

In our computational experiments, we compare combinations of the techniques presented in this paper: conflict analysis and dual ray analysis. To the best of our knowledge, most major MIP solvers either use conflict analysis of infeasible LPs and domain propagation (e.g., SCIP, FICO Xpress-Optimizer) or they employ dual ray analysis (e.g., Gurobi, SAS). We will refer to the former as the conflict setting and to the latter as the dualray setting. We compare those to a setting that uses conflict analysis and dual ray analysis simultaneously, the combined setting. Finally, we consider an extension of the combined setting that uses a pool for conflict management, the setting combined+pool.

All experiments were performed with the non-commercial MIP solver SCIP \cite{2013} (git hash 60f49ab, based on SCIP 3.2.1.2), using SoPlex 2.2.1.3 as LP solver. The experiments were run on a cluster of identical machines, each with an Intel Xeon Quad-Core with 3.2GHz and 48GB of RAM; a time limit of 3600 seconds was set.

We used two test sets: the Miplib2010 \cite{2010} benchmark test set and a selection of instances taken from the Miplib 3.1.8, Miplib2003 \cite{2003}, Miplib2010, the Cor@l \cite{2009} collection, the Alu\cite{2007} and the Markshare \cite{2010} test sets. From these we selected all instances for which (i) all of the above settings need at least 100 nodes, (ii) at least one setting finishes within the time limit of 3600 seconds, and (iii) at least one setting analyzes more than 100 infeasible subproblems successfully. We refer to this test set as the Conflict set, since it was designed to contain instances for which conflict or dual ray analysis is frequently used.

Aggregated results on the number of generated nodes and needed solving time can be found in Table 1. Detailed results can be found in Table 2 and 5 in the appendix.

We use the conflict setting as a base line (since it used to be the SCIP default), for which we give actual means of branch-and-bound nodes and the solving time. For all other settings, we instead give factors w.r.t. the base line. A number greater then one implies that the setting is inferior and a number less than one implies that the setting is superior to the conflict setting.

First of all, we observe that solely using dual ray analysis is inferior to using conflict analysis on both test sets and w.r.t. both performance measures. Note that we used a basic implementation of dual ray analysis; a solver that solely relies on it might implement further extensions that decrease this difference in performance, see also Section \cite{2013}. However, the combination of conflict and dual ray analysis showed some significant performance improvements. We observed a speed-up of 3% and 18% on Miplib2010 and Conflict, respectively. Moreover, the number of generated nodes could be reduced by 5% and 25%, respectively.

Finally, on the Conflict test set, the combined setting solved one instance more than the conflict setting and five more than the dualray setting. We take those results as an indicator that the two techniques complement each other.

\footnote{The instances are part of the contributed section of Miplib2003}
Table 1: Aggregated computational results. Columns marked with # show the number of solved instances. Columns 3 and 4 show the shifted geometric mean of absolute numbers of generated nodes ($n$, shift = 100) and needed solving time in seconds ($t$, shift = 10), respectively. All remaining columns show the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Column 3 and 4, respectively.

other nicely. In an additional experiment, we also tested to apply conflict analysis solely from domain propagation or solely from infeasible LPs. Both variants were inferior to the conflict setting and are therefore not discussed in detail (cf. Table 4 and 5 in the appendix).

To partially explain the different extent of the improvements on both tests set, we would like to point out that in the Miplib2010 benchmark set, there are only 31 instances which fulfill the filtering criteria mentioned above for the Conflict set. On those, the combined setting is 7.2% faster and needs 15.6% less nodes than the conflict setting.

Looking at individual instances, there are a few cases for which the combined setting is the clear winner, e.g., neos-849702 or bnatt350. For neos-849702 and bnatt350, the dualray setting has a timeout, while the conflict setting is a factor of 6.2 and 1.83 slower, respectively, than the combined setting. At the same time, ns1766074 shows the largest deterioration from using a combined setting, being a factor of 1.63 slower than conflict and a factor of 1.06 slower than dualray.

As can be seen in Table 1, using a conflict pool in addition to an aging system makes hardly any difference w.r.t. the overall performance.

5 Conclusion and Outlook

In this short paper we discussed the similarities and differences of conflict analysis and dual ray analysis in solving MIPs. Our computational study indicates that a combination of both approaches can enhance the performance of a state-of-the-art MIP solver significantly. On instances where the analysis of infeasible subproblems succeeds frequently, the solving time improved by 17.3% and the number of branch-and-bound nodes by 24.5%. In contrast to that, using a pool-based approach in addition to an aging mechanism to manage conflict constraints showed hardly any impact.

There are several instances for which using either dual ray analysis or conflict analysis exclusively outperformed the combination of both. Thus, we will
plan to investigate a dynamic mechanism to switch between both techniques. Furthermore, applying dual ray analysis for infeasibility deduced by domain propagation as well as using more preprocessing (e.g., mixed integer rounding, projecting out continuous variables, etc.) techniques to modify constraints derived from dual ray analysis appear as promising directions for future research.

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A Appendix

A detailed overview of all computational results on MIPLIB2010 and CONFLICT test set can be found in Table 2-5. For each table we use the conflict setting as a base line, for which we give actual means of branch-and-bound nodes and the solving time. For all other settings, we instead give factors w.r.t. the base line. A number greater than one implies that the setting is inferior and a number less than one implies that the setting is superior to the conflict setting.

A comparison between the conflict, dualray, combined, and combined+pool setting can be found in Table 2 and 3.

In addition, results for applying conflict analysis solely from domain propagation or solely from infeasible LPs can be found in Table 4 and 5.
Table 2: Detailed computational results on MiPLib2010 test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict        | dualray         | combined         | combined+pool    |
|-------------------|-----------------|-----------------|------------------|------------------|
|                   | $n$  | $t$  | $n$  | $n_Q$ | $t$  | $t_Q$ | $n$  | $n_Q$ | $t$  | $t_Q$ |
| 30n20b8           | 51   | 268  | 933  | 18.294 | 1591.27 | 5.933 | 51   | 1.000 | 269.69 | 1.006 |
| acc-tight5        | 920  | 140  | 357  | 0.388  | 76.88   | 0.551 | 1107 | 1.203 | 153.85 | 1.102 |
| aflow40b          | 16398 | 333  | 160367| 0.978  | 888.21  | 0.666 | 135529| 0.826  | 634.29 | 0.476 |
| air04             | 218  | 74   | 180  | 0.826  | 73.71   | 0.997 | 218  | 1.000 | 69.62  | 0.942 |
| app1-2            | 2    | 3600 | 71   | 35.500 | 2100.69 | 0.584 | 2    | 1.000 | 3600.00 | 1.000 |
| ash608gpia-3col   | 7    | 13   | 297  | 42.429 | 36.69   | 2.720 | 7    | 1.000 | 13.44  | 0.996 |
| bab5              | 21993 | 3600 | 29410| 0.978  | 888.21  | 0.666 | 135529| 0.826  | 634.29 | 0.476 |
| beasleyC3         | 83224 | 3600 | 1278442| 1.536 | 3600.00 | 1.189 | 989653| 1.189  | 3600.00 | 1.006 |
| biella1           | 10755 | 1711 | 9961  | 0.926  | 1683.80 | 0.984 | 9134 | 0.849  | 156.83 | 0.546 |
| bienst2           | 242186| 486  | 16973 | 0.701  | 326.13  | 0.671 | 148427| 0.613  | 333.62 | 0.686 |
| binkar10_1        | 245091| 380  | 24554 | 1.002  | 355.62  | 0.936 | 274286| 1.119  | 426.07 | 1.122 |
| bley_xll          | 1    | 238  | 1    | 1.000  | 236.30  | 0.991 | 1    | 1.000 | 238.00 | 0.998 |
| bnatt350          | 4433  | 287  | 167031| 37.679 | 3600.00 | 12.540 | 1913 | 0.432  | 156.83 | 0.546 |
| core2536-691      | 2335  | 1295 | 2051  | 0.878  | 1731.98 | 1.337 | 2335 | 1.000 | 1297.94 | 1.002 |
| cov1075           | 1195289| 3600 | 1193248| 0.998  | 3600.00 | 1.000 | 1202635| 1.000 | 1297.94 | 1.002 |
| csched10          | 398144 | 3600 | 386528 | 0.971  | 3600.00 | 1.000 | 366770| 0.921  | 3600.00 | 1.000 |
| danoint           | 1043564| 3600 | 950242 | 0.911  | 3600.00 | 1.000 | 1015127| 0.973  | 3600.00 | 1.000 |
| dfn-gwn-UUM       | 46722  | 96   | 46670 | 0.999  | 95.51   | 0.989 | 46456 | 0.994  | 95.95  | 0.999 |
| eil33-2           | 865   | 59   | 865   | 1.000  | 57.51   | 0.976 | 865  | 1.000 | 59.31  | 1.000 |
| eilB101           | 12775 | 406  | 12775 | 1.000  | 408.54  | 1.066 | 12775| 1.000 | 405.95 | 1.000 |
| enlight13        | 1    | 1    | 1    | 1.000  | 0.50    | 1.000 | 1    | 1.000 | 0.50   | 1.000 |
| enlight14        | 1    | 1    | 1    | 1.000  | 0.50    | 1.000 | 1    | 1.000 | 0.50   | 1.000 |
| ex9               | 1    | 36   | 1    | 1.000  | 36.33   | 1.004 | 1    | 1.000 | 35.89  | 0.992 |
| glass4            | 5039212| 3074 | 6252477| 1.241  | 3600.00 | 1.171 | 4069131| 0.807  | 3600.00 | 1.171 |
| gmu-35-40         | 5403540| 3600 | 6607070| 1.223  | 3600.00 | 1.000 | 5371633| 0.994  | 3600.00 | 1.000 |

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| Instance          | conflict       | dualray       | combined      | combined+pool |
|-------------------|----------------|---------------|---------------|---------------|
|                   | n   | t   | n   | t   | n   | t   | n   | t   |
| iis-100-0-cov     | 88800| 631 | 88800| 1.00 | 630.10| 0.999| 88800| 1.00 | 630.63| 1.000| 88800| 1.000| 630.81| 1.000|
| iis-bupa-cov      | 180506| 2425 | 180506| 1.00 | 2324.71| 1.001| 180506| 1.00 | 2635.21| 1.086| 180506| 1.000| 2425.22| 1.000|
| iis-pima-cov      | 8090 | 241 | 8090 | 1.00 | 241.09| 0.999| 8090 | 1.00 | 242.11| 1.003| 8090 | 1.000 | 240.73| 0.998|
| lecstrong4-4-obj  | 2309 | 209 | 29448| 12.275| 3600.00| 19.399| 9925 | 4.137| 688.75| 2.563| 9925 | 4.137 | 685.39| 2.551|
| m100n500k41       | 3292898| 3600 | 3565574| 1.083| 3600.00| 1.000| 3592592| 1.091| 3600.00| 1.000| 356478| 1.083| 3600.00| 1.000|
| macrophage        | 878022| 3600 | 865342| 0.986| 3600.00| 1.000| 903545| 1.029| 3600.00| 1.000| 903961| 1.030| 3600.00| 1.000|
| map18             | 307  | 269 | 307  | 1.00 | 265.56| 0.986| 307  | 1.00 | 271.94| 1.010| 307  | 1.000 | 268.08| 0.996|
| map20             | 385  | 256 | 385  | 1.00 | 255.38| 0.998| 385  | 1.00 | 255.75| 1.000| 385  | 1.000 | 256.51| 1.003|
| mcsched           | 20358| 206 | 18218| 0.895| 169.02| 0.821| 20358| 1.000| 193.54| 0.941| 20358| 1.000 | 193.74| 0.942|
| msk-250-1-100-1   | 633761| 317 | 578416| 1.000| 316.41| 0.999| 633761| 1.000| 317.71| 1.003| 633761| 1.000 | 317.71| 1.003|
| mine-166-5        | 6651 | 37  | 4551 | 0.684| 28.03 | 0.763| 6651 | 1.000| 36.60 | 0.997| 6651 | 1.000 | 36.39 | 0.991|
| mine-90-10        | 45418| 210 | 137594| 3.030| 254.67| 1.213| 37025| 0.815| 158.11| 0.973| 57388| 1.264 | 219.77| 1.047|
| mcs98-ip          | 648  | 3600| 1658 | 2.559| 3600.00| 1.000| 602  | 0.929| 3600.00| 1.000| 599  | 0.924 | 3600.00| 1.000|
| mspip16           | 59   | 2142| 121  | 2.051| 2720.47| 1.270| 59   | 1.000| 2158.66| 1.008| 59   | 1.000 | 2136.90| 0.997|
| mzvz11            | 7274 | 1307| 8588 | 1.181| 1930.21| 1.412| 7259 | 0.998| 1278.05| 0.935| 7271 | 1.000 | 1271.60| 0.930|
| n3div36           | 123273| 3600| 130853| 1.061| 3600.00| 1.000| 122843| 0.997| 3600.00| 1.000| 122601| 0.995 | 3600.00| 1.000|
| n3ses24           | 6   | 3600| 6    | 1.000| 3600.00| 1.000| 6    | 1.000| 3600.00| 1.000| 6    | 1.000 | 3600.00| 1.000|
| n4-3              | 73370| 926 | 88693| 1.209| 1172.36| 1.266| 90878| 1.239| 1210.23| 1.307| 90878| 1.239 | 1200.98| 1.297|
| neos-1109824      | 31218| 102 | 42132| 1.350| 156.36| 0.962| 35139| 1.196| 183.91| 1.192| 35139| 1.196 | 182.62| 1.191|
| neos-1337407      | 224112| 3600| 24452| 1.077| 3600.00| 1.000| 226899| 1.012| 3600.00| 1.000| 225408| 1.006 | 3600.00| 1.000|
| neos-1396125      | 32266| 707| 159271| 4.936| 2391.59| 3.813| 44694| 1.385| 1037.98| 1.467| 44694| 1.385 | 1029.89| 1.456|
| neos-1601936      | 324  | 3600| 2465 | 7.608| 3600.00| 1.000| 335  | 1.034| 3600.00| 1.000| 298  | 0.920 | 3600.00| 1.000|
| neos-476283       | 3201 | 438 | 3201 | 1.000| 437.63| 1.000| 3201 | 1.000| 438.42| 1.002| 3201 | 1.000 | 435.71| 0.996|
| neos-866190       | 149683| 1077| 61808| 0.413| 461.05| 0.428| 149683| 1.000| 1072.85| 0.996| 149683| 1.000 | 1077.56| 1.001|
| neos-849702       | 34345| 648 | 287514| 8.371| 3600.00| 5.559| 1207 | 0.835| 98.09 | 0.151| 1207 | 0.835 | 98.27 | 0.152|

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Table 2: Detailed computational results on MiPLib2010 test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict | dualray | combined | combined+pool |
|-------------------|----------|---------|----------|---------------|
|                   | $n$      | $t$     | $n$      | $n_Q$ | $t$       | $t_Q$ | $n$      | $n_Q$ | $t$     |
| neos-916792       | 75359    | 248     | 75359    | 1.000 | 241.38    | 0.975 | 75359    | 1.000 | 238.03  | 0.962 |
| neos-934278       | 52       | 3600    | 52       | 1.000 | 3600.00   | 1.000 | 52       | 1.000 | 3600.00 | 1.000 |
| neos13            | 162158   | 3600    | 164002   | 1.011 | 3600.00   | 1.000 | 162805   | 1.004 | 3600.00 | 1.000 |
| neos18            | 4919     | 27      | 76980    | 15.650 | 241.72    | 0.975 | 4919     | 1.000 | 26.93   | 0.860 |
| net12             | 2922     | 3600    | 5301     | 1.814 | 3600.00   | 1.000 | 2805     | 0.960 | 3114.80 | 0.865 |
| netdiversion      | 60       | 3600    | 61       | 1.000 | 3600.00   | 1.000 | 84       | 1.400 | 3600.00 | 1.000 |
| newdano           | 3337954  | 3600    | 3301689  | 0.989 | 3600.00   | 1.000 | 3344550  | 1.002 | 3600.00 | 1.000 |
| noswot            | 584962   | 140     | 1810982  | 3.096 | 353.15    | 2.517 | 586950   | 1.003 | 145.09  | 1.034 |
| ni1208400         | 2373     | 3600    | 2296     | 0.968 | 2432.96   | 0.676 | 2885     | 0.960 | 3114.80 | 0.865 |
| ni1688347         | 4308     | 231     | 32931    | 7.644 | 870.60    | 3.777 | 2781     | 0.646 | 259.46  | 0.919 |
| ni1758913         | 8        | 3600    | 2        | 0.250 | 3600.00   | 1.000 | 1        | 0.125 | 3600.00 | 1.000 |
| ni1766074         | 915997   | 971     | 1029467  | 1.124 | 1583.71   | 1.631 | 1005021  | 1.097 | 1317.95 | 1.357 |
| ni1830653         | 39491    | 351     | 73960    | 1.873 | 615.22    | 1.752 | 29502    | 0.646 | 259.46  | 0.919 |
| opm2-z7-s2        | 8798     | 2208    | 5350     | 0.608 | 2211.72   | 1.002 | 8798     | 1.000 | 2209.04 | 1.000 |
| pg5                | 300472   | 1383    | 298798   | 0.994 | 1318.28   | 0.953 | 300472   | 1.000 | 1333.86 | 0.965 |
| pigeon-10         | 1288262  | 3600    | 12348998 | 0.959 | 3600.00   | 1.000 | 12571973 | 0.976 | 3600.00 | 1.000 |
| pw-myciel4        | 567066   | 3600    | 627543   | 1.107 | 2682.55   | 0.745 | 298632   | 0.527 | 3600.00 | 1.000 |
| qiu                | 11256    | 49      | 11256    | 1.000 | 48.61     | 0.990 | 11256    | 1.000 | 44.65   | 0.910 |
| rail507           | 993      | 287     | 1183     | 1.191 | 284.98    | 0.993 | 993      | 1.000 | 285.31  | 0.994 |
| ran16x16          | 34928    | 270     | 313667   | 0.920 | 241.53    | 0.907 | 303203   | 0.889 | 239.69  | 0.887 |
| reblock67         | 128157   | 235     | 74055    | 0.578 | 138.03    | 0.587 | 127595   | 0.996 | 215.33  | 0.916 |
| rmatr100-p10      | 768      | 162     | 768      | 1.000 | 163.23    | 1.005 | 768      | 1.000 | 162.70  | 1.002 |
| rmatr100-p5       | 463      | 503     | 463      | 1.000 | 502.14    | 0.999 | 463      | 1.000 | 502.11  | 0.998 |
| rmine6            | 562634   | 1146    | 807204   | 1.435 | 1549.12   | 1.351 | 562634   | 1.000 | 1141.78 | 0.996 |
| rocII-4-11        | 15293    | 2126    | 35576    | 2.326 | 3600.00   | 1.693 | 11683    | 0.764 | 1608.66 | 0.757 |

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| Instance     | conflict | dualray | combined | combined+pool |
|--------------|----------|---------|----------|--------------|
|              | $n$      | $t$     | $n_Q$    | $t_Q$        | $n$      | $n_Q$    | $t$     | $t_Q$        |
| rococoC10-001000 | 298459  | 2527    | 353552   | 2180.44      | 0.863   | 289711   | 0.971   | 2359.33      | 0.934   | 289711   | 0.971   | 2356.48      | 0.933   |
| roll3000     | 1111977  | 3600    | 1177662  | 3600.00      | 1.000   | 1150992  | 1.035   | 3600.00      | 1.000   | 1151494  | 1.036   | 3600.00      | 1.000   |
| satellites1-25 | 471     | 1787    | 1982     | 3600.00      | 2.014   | 721      | 1.531   | 1286.73      | 0.720   | 721      | 1.531   | 1291.66      | 0.723   |
| sp98ic       | 15804   | 3600    | 17206    | 3600.00      | 1.000   | 17270    | 1.093   | 3600.00      | 1.000   | 17307    | 1.095   | 3600.00      | 1.000   |
| sp98ir       | 5623    | 75      | 7980     | 3600.00      | 1.187   | 5623     | 1.000   | 76.06        | 1.016   | 5623     | 1.000   | 75.24        | 1.005   |
| tanglegram1  | 61      | 743     | 61       | 3600.00      | 1.034   | 61       | 1.000   | 740.19       | 0.997   | 61       | 1.000   | 740.07       | 0.997   |
| tanglegram2  | 3       | 6       | 3        | 3600.00      | 1.034   | 3       | 1.000   | 6.43         | 0.995   | 3       | 1.000   | 6.80        | 1.056   |
| timtab1      | 843361  | 435     | 872675   | 3600.00      | 1.23      | 905328  | 1.073   | 544.35       | 1.253   | 905328  | 1.073   | 515.82       | 1.187   |
| triptim1     | 1      | 499     | 1       | 1000.00      | 1.005   | 1       | 1.000   | 501.62       | 1.002   | 1       | 1.000   | 509.57       | 1.021   |
| unitcal_7    | 35212   | 1499    | 38141    | 3600.00      | 1.106   | 43582   | 1.238   | 2018.70      | 1.347   | 43582   | 1.238   | 1830.40      | 1.221   |
| vpphard      | 1322    | 3600    | 3858     | 3600.00      | 1.000   | 2388    | 1.806   | 3600.00      | 1.000   | 2388    | 1.806   | 3600.00      | 1.000   |
| zib54-UUE    | 487119  | 3600    | 505411   | 3600.00      | 1.000   | 481143  | 0.988   | 3600.00      | 1.000   | 480906  | 0.987   | 3600.00      | 1.000   |
| geom.        | 8491.43 | 617.78  | 12356.545 | 1.455   | 724.631  | 1.173   | 8065.819  | 0.943   | 603.184  | 0.976   | 7958.263  | 0.937   | 692.033      | 0.975   |
| sh. geom. [100, 10] | 14381.73 | 685.80  | 19636.864 | 1.365   | 800.395  | 1.167   | 13737.172 | 0.955   | 670.337  | 0.977   | 13769.376 | 0.957   | 668.945      | 0.975   |
Table 3: Detailed computational results on CONFLICT test set. The table shows the absolute number of generated nodes \((n)\) and needed solving time in seconds \((t)\), as well as the relative number of generated nodes \((n_Q)\) and needed solving time \((t_Q)\) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict | dualray | combined | combined+pool |
|-------------------|----------|---------|----------|---------------|
| acc-tight5        | 920      | 357     | 0.388    | 1107          | 1.203          |
| allow40b          | 16398    | 16036   | 0.978    | 135529        | 0.826          |
| alu4_7            | 279      | 1643    | 5.889    | 250           | 0.896          |
| alu4_8            | 649      | 28296   | 43.599   | 710           | 1.094          |
| alu5_7            | 1704     | 6467    | 3.795    | 1368          | 0.803          |
| alu5_8            | 7492     | 212946  | 8.423    | 4473          | 0.597          |
| alu6_7            | 14365    | 277059  | 19.287   | 6024          | 0.896          |
| alu6_8            | 47135    | 739660  | 15.692   | 40659         | 0.863          |
| alu7_7            | 71481    | 739480  | 10.345   | 59906         | 0.838          |
| bell3a            | 23108    | 21050   | 0.911    | 21050         | 0.911          |
| bell5             | 1132     | 1346    | 1.189    | 1132          | 1.000          |
| biella1           | 10755    | 9961    | 0.926    | 9134          | 0.849          |
| bienst1           | 12591    | 15883   | 1.261    | 14827         | 0.613          |
| bienst2           | 242186   | 169723  | 0.701    | 14827         | 0.613          |
| binkar10_1        | 245091   | 245654  | 37.578   | 274286        | 1.119          |
| bnatt350          | 4433     | 166584  | 37.578   | 1913          | 0.432          |
| enigma            | 1172     | 760     | 0.648    | 1172          | 0.648          |
| Lectsched-4-obj   | 2399     | 29505   | 12.299   | 9925          | 1.119          |
| leu               | 601      | 530     | 0.882    | 576           | 0.958          |
| markshare_3_0     | 4841     | 1843    | 0.381    | 1475          | 0.305          |
| markshare_3_2     | 5018     | 2861    | 0.570    | 3049          | 0.608          |
| markshare_3_3     | 4826     | 2645    | 0.548    | 2343          | 0.485          |
| markshare_3_4     | 4352     | 2349    | 0.701    | 1375          | 0.410          |
| markshare_3_5     | 2481     | 1421    | 0.573    | 1609          | 0.649          |
| markshare_4_0     | 643068   | 168883  | 0.263    | 129999        | 0.202          |

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| Instance          | conflict | dualray | combined | combined+pool |
|-------------------|----------|---------|----------|---------------|
|                   | n        | t       | $n_Q$    | $t_Q$         | $n$  | $n_Q$ | $t$     | $t_Q$         |
| markshare _4_1    | 531216   | 137     | 214185   | 0.403         | 219.32 | 0.160 | 108056   | 0.203         | 14.81 | 0.108 |
| markshare _4_2    | 369816   | 104     | 136123   | 0.368         | 13.52  | 0.130 | 10105    | 0.273         | 15.20  | 0.146 |
| markshare _4_4    | 420963   | 82      | 204375   | 0.485         | 20.46  | 0.251 | 89807    | 0.213         | 13.36  | 0.164 |
| markshare _4_5    | 504816   | 118     | 274293   | 0.543         | 27.82  | 0.236 | 145753   | 0.289         | 19.14  | 0.162 |
| mas74             | 3420026  | 685     | 3577586  | 1.046         | 717.66 | 1.047 | 3420026  | 1.000         | 689.28 | 1.006 |
| mas76             | 516981   | 96      | 516356   | 0.994         | 86.56  | 0.901 | 516981   | 1.000         | 93.60  | 0.974 |
| miki-250-1-100-1  | 633761   | 315     | 578416   | 0.913         | 285.29 | 0.905 | 633761   | 1.000         | 316.66 | 1.005 |
| mine-166-5        | 6651     | 37      | 4551     | 0.684         | 27.87  | 0.755 | 6651     | 1.000         | 36.23  | 0.982 |
| mine-90-10        | 45418    | 214     | 137594   | 3.030         | 254.94 | 1.191 | 37025    | 0.815         | 156.94 | 0.733 |
| misc03            | 134      | 1       | 134      | 1.000         | 1.06   | 0.955 | 135      | 1.007         | 1.09   | 0.982 |
| misc07            | 29168    | 21      | 38819    | 1.331         | 23.36  | 1.125 | 31784    | 1.090         | 21.97  | 1.058 |
| mzzv11            | 7274     | 1393    | 8588     | 1.181         | 1996.33| 1.433 | 7259     | 0.998         | 1277.69| 0.917 |
| mzzv42z           | 3050     | 731     | 2320     | 0.761         | 529.74 | 0.724 | 3392     | 1.112         | 901.81 | 1.233 |
| neos-1061020      | 4040     | 1070    | 2704     | 0.669         | 701.54 | 0.656 | 10200    | 2.525         | 1600.51| 1.496 |
| neos-1109824      | 31218    | 163     | 42132    | 1.350         | 155.53 | 0.956 | 35139    | 1.126         | 182.47 | 1.122 |
| neos-1126860      | 5055     | 597     | 7015     | 1.388         | 589.86 | 0.989 | 5055     | 1.000         | 569.51 | 0.954 |
| neos-1173026      | 383075   | 1193    | 1305899  | 3.409         | 3291.14| 2.758 | 103261   | 0.270         | 389.05 | 0.326 |
| neos-1208069      | 6041     | 457     | 161143   | 26.675        | 2583.86| 5.656 | 6782     | 1.123         | 701.05 | 1.535 |
| neos-1208135      | 8457     | 1139    | 60146    | 7.112         | 2193.83| 1.926 | 3651     | 0.432         | 492.98 | 0.433 |
| neos-1215259      | 2880     | 101     | 1645     | 0.571         | 56.00  | 0.554 | 7630     | 2.649         | 185.91 | 1.840 |
| neos-1215891      | 3890     | 1409    | 80000    | 20.567        | 3600.00| 2.554 | 11676    | 3.002         | 2221.22| 1.576 |
| neos-1223462      | 1472     | 819     | 1878     | 1.276         | 1020.01| 1.245 | 561      | 0.381         | 442.74 | 0.541 |
| neos-1281048      | 1384     | 12      | 1221     | 0.882         | 10.75  | 0.930 | 1243     | 0.898         | 11.04  | 0.955 |
| neos-1396125      | 32266    | 708     | 159271   | 4.936         | 2393.14| 3.381 | 44694    | 1.385         | 1029.00| 1.454 |
| neos-1402020      | 55079    | 18      | 26789    | 0.486         | 9.57   | 0.545 | 55079    | 1.000         | 17.83  | 1.016 |

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| Instance       | conflict | dualray | combined | combined+pool |
|----------------|----------|---------|----------|---------------|
|                | $n$      | $t$     | $n$      | $t$           | $n$      | $t$     | $n$   | $t$   |
| neos-1440225   | 1461     | 47      | 5584     | 3.822         | 12608    | 8.630  | 315.83 | 6.663 |
| neos-1460265   | 4367     | 66      | 26519    | 0.607         | 30942    | 0.708  | 59.61  | 0.906 |
| neos-1461051   | 3401     | 13      | 301005   | 88.505        | 3991     | 1.173  | 13.56  | 1.609 |
| neos-1480121   | 5608250  | 3000    | 2359174  | 0.421         | 565      | 0.000  | 0.94   | 0.000 |
| neos-1582420   | 59182    | 621     | 31105    | 0.526         | 56913    | 0.962  | 596.55 | 0.960 |
| neos-1620807   | 1812678  | 1543    | 6291083  | 3.471         | 1722467  | 0.950  | 1520.50| 0.985 |
| neos-430149    | 25985    | 24      | 59691    | 2.297         | 27247    | 1.049  | 24.35  | 1.007 |
| neos-503737    | 9330     | 354     | 1743     | 0.187         | 2257     | 0.242  | 133.17 | 0.376 |
| neos-504674    | 36880    | 78      | 27888    | 0.756         | 32657    | 0.885  | 69.83  | 0.898 |
| neos-538867    | 46052    | 53      | 119585   | 2.597         | 42703    | 0.927  | 48.08  | 0.904 |
| neos-538916    | 29500    | 42      | 50111    | 1.699         | 29547    | 1.002  | 42.35  | 1.005 |
| neos-551991    | 5448     | 319     | 10487    | 1.925         | 5448     | 1.000  | 318.02 | 0.997 |
| neos-555298    | 27669    | 200     | 37347    | 1.350         | 11145    | 0.403  | 178.05 | 0.888 |
| neos-584851    | 351      | 27      | 424      | 1.208         | 351      | 1.000  | 26.37  | 0.994 |
| neos-585192    | 1577     | 35      | 1967     | 1.247         | 1478     | 0.937  | 32.89  | 0.952 |
| neos-595925    | 27416    | 59      | 14296    | 0.521         | 22257    | 0.812  | 50.22  | 0.847 |
| neos-686190    | 149638   | 1079    | 61807    | 0.413         | 149683   | 1.000  | 1085.06| 1.006 |
| neos-717614    | 265333   | 414     | 530703   | 2.000         | 15048    | 0.057  | 30.81  | 0.075 |
| neos-785912    | 267      | 107     | 797      | 2.985         | 319      | 1.195  | 115.74 | 1.081 |
| neos-791021    | 636      | 1787    | 188      | 0.296         | 271      | 0.426  | 1034.35| 0.579 |
| neos-803219    | 22155    | 35      | 24978    | 1.127         | 22478    | 1.015  | 37.22  | 1.066 |
| neos-803220    | 55512    | 89      | 48674    | 0.877         | 51137    | 0.921  | 103.62 | 1.164 |
| neos-806323    | 10041    | 29      | 14620    | 1.456         | 7717     | 0.769  | 22.50  | 0.776 |
| neos-807639    | 6123     | 16      | 3509     | 0.573         | 3575     | 0.584  | 13.34  | 0.820 |
| neos-807705    | 9210     | 24      | 5174     | 0.562         | 5240     | 0.569  | 15.41  | 0.636 |

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Table 3: Detailed computational results on CONFLICT test set. The table shows the absolute number of generated nodes \((n)\) and needed solving time in seconds \((t)\), as well as the relative number of generated nodes \((n_Q)\) and needed solving time \((t_Q)\) w.r.t. Columns 2 and 3 \((\text{conflict})\). All changes in the number of nodes or solving time of at least 5\% are highlighted in bold and blue \((\text{improvement})\) and italic and red \((\text{deterioration})\).

| Instance          | conflict |   | dualray |   | combined |   | combined+pool |   |
|-------------------|----------|---|---------|---|----------|---|--------------|---|
|                   | \(n\)  | \(t\) | \(n_Q\) | \(t\) | \(n_Q\) | \(t\) | \(n_Q\) | \(t\) |
| neos-810286       | 945     | 502 | 381     | 0.403 | 122.82  | 0.245 | 1015     | 1.074 |
| neos-810326       | 2441    | 104 | 1188    | 0.487 | 61.46   | 0.589 | 3144     | 1.288 |
| neos-827015       | 187     | 1376| 569     | 3.043 | 3268.56 | 2.376 | 187      | 1.000 |
| neos-831188       | 4962    | 483 | 3861    | 0.778 | 372.08  | 0.770 | 5236     | 1.055 |
| neos-839859       | 18823   | 1130| 20996   | 1.115 | 1214.60 | 1.075 | 13830    | 0.735 |
| neos-848845       | 36361   | 923 | 31112   | 8.556 | 3600.00 | 3.900 | 3316     | 0.091 |
| neos-849702       | 34345   | 647 | 299757  | 8.728 | 3600.00 | 5.562 | 1207     | 0.035 |
| neos-862348       | 1516    | 30  | 5222    | 3.445 | 38.85   | 1.315 | 2007     | 1.324 |
| neos-863472       | 262229  | 220 | 714473  | 2.725 | 453.29  | 2.056 | 328043   | 1.351 |
| neos-868622       | 66990   | 244 | 35286   | 0.527 | 218.88  | 0.898 | 66990    | 1.000 |
| neos-892255       | 1612    | 448 | 930     | 0.577 | 288.40  | 0.643 | 1612     | 1.000 |
| neos-905856       | 4334    | 55  | 33444   | 7.717 | 220.39  | 3.979 | 11715    | 2.703 |
| neos-906865       | 48822   | 122 | 49529   | 1.014 | 123.62  | 1.010 | 49279    | 1.009 |
| neos-912023       | 1827    | 21  | 53733   | 29.411| 239.03  | 11.291| 6968     | 3.814 |
| neos-914441       | 354     | 227 | 180     | 0.508 | 155.90  | 0.686 | 1350     | 3.814 |
| neos-942323       | 2567    | 16  | 4610    | 1.796 | 18.95   | 1.220 | 2382     | 0.928 |
| neos18            | 4919    | 27  | 76980   | 15.650| 145.81  | 5.498 | 4919     | 1.000 |
| net12             | 1514    | 1690| 3903    | 2.578 | 3600.00 | 2.130 | 1372     | 0.906 |
| ns1208400         | 58492   | 141 | 1816882 | 3.096 | 354.55  | 2.507 | 586950   | 1.003 |
| ns1688347         | 2495    | 3600| 2296    | 0.920 | 2454.32 | 0.682 | 1031     | 0.413 |
| ns1766074         | 4308    | 228 | 32931   | 7.644 | 874.84  | 3.831 | 2781     | 0.646 |
| ns1830653         | 39491   | 351 | 73960   | 1.873 | 619.02  | 1.762 | 28852    | 0.731 |
| pg5_34            | 300472  | 1332| 298798  | 0.994 | 1317.25 | 0.989 | 300472   | 1.000 |
| prod1             | 29058   | 18  | 29822   | 1.026 | 15.83   | 0.885 | 29058    | 1.000 |

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Table 3: Detailed computational results on Conflict test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance | conflict | dualray | combined | combined+pool |
|----------|----------|---------|----------|---------------|
| n        | t        | n       | t        | t_Q           |
| n_Q      | t_Q      | n       | t        | t_Q           |
| n_Q      | t_Q      | n       | t        | t_Q           |
| prod2    | 78750    | 63      | 150765   | 1.914         |
| pw-myciel4 | 569026   | 3600    | 627543   | 1.103         |
| reblock67 | 128157   | 235     | 74055    | 0.578         |
| rmn6     | 562634   | 1144    | 807204   | 1.435         |
| rococoC10-00000 | 298459   | 2527    | 353552   | 1.185         |
| rout     | 49976    | 58      | 137696   | 2.755         |
| satellites1-25 | 471     | 1859    | 1982     | 4.208         |
| timtab1  | 843361   | 434     | 872675   | 1.035         |
| geom.    | 16183.61 | 101.28  | 26196.478| 1.619         |
| sh. geom. [100, 10] | 16769.05| 143.22  | 27097.915| 1.616         |
Table 4: Detailed computational results on Miplib2010 test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict dualray | inf. LP | dom. prop. |
|-------------------|------------------|---------|------------|
|                   | $n$   | $t$   | $n$   | $n_Q$ | $t$   | $t_Q$ | $n$   | $n_Q$ | $t$   | $t_Q$ |
| 30n20b8           | 51    | 268   | 933   | 18.294| 1591.27 | 5.933 | 1207  | 23.667| 1477.92 | 5.511 | 6754  | 132.431| 3600.00 | 13.123 |
| acc-tight5        | 920   | 140   | 357   | 0.388 | 76.88  | 0.551 | 207   | 0.225 | 69.40  | 0.497 | 1615  | 1.755 | 280.53 | 2.009 |
| afollow40b        | 163981| 1333  | 160367| 0.978 | 388.21 | 0.666 | 158904| 0.969 | 1259.54 | 0.945 | 199108| 1.214 | 1486.33 | 1.115 |
| air04             | 218   | 74    | 180   | 0.826 | 73.71  | 0.997 | 180   | 0.826 | 73.85  | 0.999 | 218   | 1.000 | 69.87  | 0.945 |
| app1-2            | 7     | 13    | 7     | 42.129| 36.69  | 5.933 | 297   | 42.429| 36.65  | 5.511 | 297   | 1.000 | 13.423 | 0.996 |
| ash608gpia-3col   | 1     | 7     | 1     | 42.129| 36.69  | 5.933 | 297   | 42.429| 36.65  | 5.511 | 297   | 1.000 | 13.423 | 0.996 |
| bab5              | 21993 | 3600  | 29410 | 1.337 | 3600.00| 1.073 | 23609 | 1.000 | 28573  | 1.115 | 542564| 1.861 | 3600.00 | 1.000 |
| beasleyC3         | 832249| 3600  | 127842| 1.536 | 3600.00| 1.000 | 542564| 0.652 | 3600.00| 1.000 | 1548537| 1.861 | 3600.00 | 1.000 |
| biella1           | 10755 | 1711  | 9961  | 0.926 | 1683.80| 0.984 | 9356  | 0.870 | 1460.30| 0.853 | 7155  | 0.665 | 950.74 | 0.556 |
| bienst2           | 242186| 486   | 169723| 0.701 | 326.13 | 0.671 | 184318| 0.761 | 333.64 | 0.686 | 136397| 0.563 | 295.24 | 0.607 |
| binkar10_1        | 245091| 380   | 245654| 1.002 | 355.63 | 0.936 | 247948| 1.012 | 360.19 | 0.948 | 242089| 0.988 | 376.38 | 0.991 |
| bley_xll          | 1     | 238   | 1     | 236.30 | 1.000 | 0.991 | 1     | 1.000 | 238.66 | 1.001 | 1     | 1.000 | 233.57 | 0.980 |
| bnatt350          | 4433  | 287   | 167031| 37.679| 3600.00| 12.999| 148078| 33.404| 3600.00| 12.540| 4949  | 1.116 | 281.28 | 0.980 |
| core2536-691      | 2335  | 1295  | 2051  | 0.878 | 1731.98| 1.337 | 2051  | 0.878 | 1733.00| 1.338 | 2335  | 1.000 | 1295.13| 1.000 |
| cov1075           | 1195289| 3600  | 1193248| 0.998| 3600.00| 1.000 | 120446| 1.008 | 3600.00| 1.000 | 1195478| 1.000 | 3600.00| 1.000 |
| cshed010          | 398144| 3600  | 386528| 0.971 | 3600.00| 1.000 | 386405| 0.971 | 3600.00| 1.000 | 410822| 1.032 | 3600.00| 1.000 |
| danoint           | 1043564| 3600  | 950242| 0.911| 3600.00| 1.000 | 1030940| 0.988| 3600.00| 1.000 | 982068| 0.941 | 3256.07| 0.904 |
| dfn-gwin-UUM      | 46722 | 96    | 46670  | 0.999| 95.01 | 0.989 | 46300  | 0.985| 95.92 | 0.999 | 46722  | 1.000 | 96.39  | 1.003 |
| eil33-2           | 865   | 59    | 865   | 1.000| 57.51 | 0.976 | 865   | 1.000| 58.41 | 0.991 | 865   | 1.000 | 57.28  | 0.972 |
| enlight13         | 12775 | 406   | 12775  | 1.000| 408.54| 1.006 | 12775  | 1.000| 409.03| 1.008 | 12775 | 1.000 | 405.62 | 0.999 |
| enlight14         | 1     | 1     | 1     | 1.000| 0.50  | 1.000 | 1     | 1.000| 0.50  | 1.000 | 1     | 1.000 | 0.50  | 1.000 |
| ex9               | 1     | 36    | 1     | 36.33| 1.004 | 1     | 1.000| 35.84 | 0.991 | 1     | 1.000 | 37.02 | 1.024 |
| glass4            | 5039212| 3074  | 6252477| 1.241| 3600.00| 1.171 | 4871908| 0.967| 3600.00| 1.171 | 2397374| 0.476 | 1430.13| 0.465 |
| gmu-35-40         | 5403540| 3600  | 6607070| 1.223| 3600.00| 1.000 | 6882538| 1.274| 3600.00| 1.000 | 5433976| 1.005 | 3600.00| 1.000 |

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Table 4: Detailed computational results on MiPLIB2010 test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance            | conflict n | dualray n | inf. LP n | LP dom. n | prop. n | conflict t | dualray t | inf. LP t | LP dom. t | prop. t |
|---------------------|------------|-----------|-----------|-----------|---------|------------|-----------|-----------|-----------|---------|
| iis-100-0-cov       | 88800      | 88800     | 1.000     | 630.10    | 0.999   | 88800      | 1.000     | 630.28    | 0.999     | 88800   |
| iis-bupa-cov        | 180506     | 180506    | 1.000     | 2426.71   | 1.001   | 180506     | 1.000     | 2425.33   | 1.000     | 180506  |
| iis-pima-cov        | 8090       | 8090      | 1.000     | 241.09    | 0.999   | 8090       | 1.000     | 240.66    | 0.997     | 8090    |
| lecshced-4-obj      | 2399       | 2399      | 1.000     | 29448.12  | 13.399  | 20488      | 8.540     | 1649.51   | 6.139     | 5744    |
| m100n500k4r1        | 329289     | 329289    | 1.000     | 56585.36  | 27.605  | 56585      | 1.000     | 56585     | 1.000     | 56585   |
| macrophage           | 878022     | 878022    | 1.000     | 865342    | 0.983   | 863221     | 0.984     | 863221    | 0.983     | 863221  |
| map18                | 307        | 307       | 1.000     | 265.56    | 0.986   | 307        | 1.000     | 266.44    | 0.989     | 307     |
| map20                | 385        | 385       | 1.000     | 255.38    | 0.998   | 385        | 1.000     | 254.63    | 0.995     | 385     |
| mcsched              | 20358      | 20358     | 1.000     | 169.02    | 0.824   | 169.02     | 1.000     | 169.02    | 1.000     | 169.02  |
| mik-250-1-100-1      | 633761     | 633761    | 1.000     | 287.90    | 0.909   | 287.90     | 1.000     | 287.90    | 1.000     | 287.90  |
| mine-166-5           | 6651       | 6651      | 1.000     | 254.67    | 1.213   | 254.67     | 1.000     | 254.67    | 1.000     | 254.67  |
| mine-90-10           | 45418      | 45418     | 1.000     | 2645.91   | 1.235   | 2645.91    | 1.000     | 2645.91   | 1.000     | 2645.91 |
| mcsched              | 7274       | 7274      | 1.000     | 1930.21   | 1.432   | 1930.21    | 1.000     | 1930.21   | 1.000     | 1930.21 |
| mcsched              | 123273     | 123273    | 1.000     | 130853    | 1.061   | 130853     | 1.000     | 130853    | 1.000     | 130853  |
| n3div36              | 31218      | 31218     | 1.000     | 52693     | 1.697   | 52693      | 1.000     | 52693     | 1.000     | 52693   |
| n3seq24              | 73370      | 73370     | 1.000     | 934.17    | 1.092   | 934.17     | 1.000     | 934.17    | 1.000     | 934.17  |
| neos-1109824         | 3201       | 3201      | 1.000     | 3201      | 1.000   | 3201       | 1.000     | 3201      | 1.000     | 3201    |
| neos-1337307         | 224112     | 224112    | 1.000     | 248820    | 1.110   | 248820     | 1.000     | 248820    | 1.000     | 248820 |
| neos-1396125         | 32266      | 32266     | 1.000     | 36926     | 1.144   | 36926      | 1.000     | 36926     | 1.000     | 36926   |
| neos-1601936         | 3201       | 3201      | 1.000     | 333       | 1.028   | 333        | 1.000     | 333       | 1.000     | 333     |
| neos-476283          | 149683     | 149683    | 1.000     | 63808     | 0.413   | 63808      | 1.000     | 63808     | 1.000     | 63808   |
| neos-686190          | 4345       | 4345      | 1.000     | 287514    | 8.371   | 287514     | 1.000     | 287514    | 1.000     | 287514 |
| neos-849702          | 149683     | 149683    | 1.000     | 63808     | 0.413   | 63808      | 1.000     | 63808     | 1.000     | 63808   |

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Table 4: Detailed computational results on MiPLIB2010 test set. The table shows the absolute number of generated nodes ($n$) and needed solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict | dualray | inf. LP | dom. prop. |
|-------------------|----------|---------|---------|------------|
|                   | $n$      | $t$     | $n$     | $n_Q$      |
|                   |          |         |         | $t$        |
|                   |          |         |         | $t_Q$      |
| neos-916792       | 75359    | 248     | 75359   | 1.000      |
|                   |          |         |         | 241.38     |
|                   |          |         |         | 0.975      |
| neos-934278       | 52       | 3600    | 52      | 1.000      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| neos13            | 162158   | 3600    | 164002  | 1.011      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| neos18            | 4919     | 27      | 76980   | 15.650     |
|                   |          |         |         | 146.41     |
|                   |          |         |         | 5.488      |
| net12             | 2922     | 3600    | 5301    | 1.814      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| netdiversion      | 60       | 3600    | 41      | 0.683      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| newdano           | 3337954  | 3600    | 3301689 | 0.989      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| newswot           | 584962   | 140     | 1810982 | 3.096      |
|                   |          |         |         | 353.15     |
|                   |          |         |         | 2.517      |
| ml208400          | 2373     | 3600    | 2296    | 0.968      |
|                   |          |         |         | 2432.96    |
|                   |          |         |         | 0.676      |
| ml688347          | 4308     | 231     | 32931   | 7.644      |
|                   |          |         |         | 870.60     |
|                   |          |         |         | 3.777      |
| ml7587913         | 8        | 3600    | 2       | 0.250      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| ml767674          | 915997   | 971     | 1029467 | 1.124      |
|                   |          |         |         | 1499.56    |
|                   |          |         |         | 1.544      |
| ml1830653         | 39491    | 351     | 73960   | 1.879      |
|                   |          |         |         | 615.22     |
|                   |          |         |         | 1.752      |
| opl2-z7-x2        | 8798     | 2208    | 5350    | 0.608      |
|                   |          |         |         | 1327.74    |
|                   |          |         |         | 0.601      |
| pg5_34            | 300472   | 1383    | 298798  | 0.994      |
|                   |          |         |         | 1318.28    |
|                   |          |         |         | 0.953      |
| pigeon-10         | 1288262  | 3600    | 12348998| 0.959      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.000      |
| pw-myciel4        | 567066   | 3600    | 627543  | 1.107      |
|                   |          |         |         | 2682.55    |
|                   |          |         |         | 0.745      |
| qiu               | 11256    | 49      | 11256   | 1.000      |
|                   |          |         |         | 48.61      |
|                   |          |         |         | 0.990      |
| rail507           | 993      | 287     | 1183    | 1.191      |
|                   |          |         |         | 368.52     |
|                   |          |         |         | 1.288      |
| ran16x16          | 340928   | 270     | 313667  | 0.920      |
|                   |          |         |         | 245.13     |
|                   |          |         |         | 0.907      |
| reblock67         | 128157   | 235     | 74055   | 0.578      |
|                   |          |         |         | 138.03     |
|                   |          |         |         | 0.587      |
| rmatr100-p10      | 768      | 162     | 768     | 1.000      |
|                   |          |         |         | 163.23     |
|                   |          |         |         | 1.005      |
| rmatr100-p5       | 463      | 503     | 463     | 1.000      |
|                   |          |         |         | 502.14     |
|                   |          |         |         | 0.999      |
| rmine6            | 562634   | 1146    | 807204  | 1.435      |
|                   |          |         |         | 1549.12    |
|                   |          |         |         | 1.351      |
| rocII-4-11        | 15293    | 2126    | 35576   | 2.326      |
|                   |          |         |         | 3600.00    |
|                   |          |         |         | 1.693      |

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| Instance       | conflict | dualray | inf. LP | dom. prop. |
|----------------|----------|---------|---------|------------|
|                | $n$      | $t$     | $n_{Q}$ | $t_{Q}$    | $n$ | $n_{Q}$ | $t$ | $t_{Q}$ |
| rococoC10-001000 | 298459  | 2527    | 353552  | 1.185  | 0.863 | 217769 | 0.730 | 1595.62 | 0.631 | 361057 | 1.210 | 2740.30 | 1.084 |
| roll3000       | 1111977 | 3600    | 1177662 | 1.059  | 3600.00 | 1.000 | 932197 | 0.838 | 3600.00 | 1.000 | 1055375 | 0.949 | 3600.00 | 1.000 |
| satellites1-25  | 471     | 1787    | 1982    | 4.298  | 3600.00 | 2.014 | 6212   | 13.189 | 1930.49 | 1.080 | 1088   | 2.297 | 3600.00 | 2.014 |
| sp98ic         | 15804   | 3600    | 17206   | 1.089  | 3600.00 | 1.000 | 17299  | 1.095 | 3600.00 | 1.000 | 17294  | 1.094 | 3600.00 | 1.000 |
| sp98ir         | 5623    | 75      | 7980    | 1.419  | 88.89   | 1.187 | 8102   | 1.441 | 89.17   | 1.194 | 5623   | 1.000 | 75.03   | 1.002 |
| tanglegram1    | 61      | 743     | 61      | 1.000  | 767.68  | 1.034 | 61     | 1.000 | 744.40  | 1.003 | 61     | 1.000 | 747.47  | 1.007 |
| tanglegram2    | 3       | 6       | 3       | 1.000  | 6.43    | 0.998 | 3      | 1.000 | 6.60    | 1.025 | 3      | 1.000 | 6.41    | 0.995 |
| timtab1        | 843361  | 435     | 872675  | 1.035  | 488.01  | 1.123 | 863707 | 1.024 | 441.25  | 1.015 | 3612719 | 4.284 | 1665.76 | 3.833 |
| triptim1       | 1       | 499     | 1       | 1.000  | 501.62  | 1.005 | 1      | 1.000 | 507.33  | 1.016 | 1      | 1.000 | 500.72  | 1.003 |
| unitcal_7      | 35212   | 1499    | 38141   | 1.083  | 1657.13 | 1.106 | 423050 | 1.208 | 1851.53 | 1.235 | 35212  | 1.000 | 1481.98 | 0.989 |
| vpphard        | 1322    | 3600    | 38558   | 2.918  | 3600.00 | 1.000 | 2824   | 2.156 | 3600.00 | 1.000 | 924    | 0.609 | 3600.00 | 1.000 |
| zib54-UUE      | 487119  | 3600    | 505411  | 1.038  | 3600.00 | 1.000 | 475796 | 0.977 | 3600.00 | 1.000 | 502361 | 1.031 | 3600.00 | 1.000 |
| geom.          | 8491.43 | 617.78  | 12356.545 | 1.455 | 724.631 | 1.173 | 11550.090 | 1.360 | 708.962 | 1.148 | 9768.339 | 1.150 | 663.745 | 1.074 |
| sh. geom. [100, 10] | 14381.73 | 685.80 | 19636.864 | 1.365 | 800.395 | 1.167 | 18728.848 | 1.302 | 783.014 | 1.142 | 16243.066 | 1.129 | 736.524 | 1.074 |
Table 5: Detailed computational results on Conflict test set. The table shows the absolute number of generated nodes \((n)\) and needed solving time in seconds \((t)\), as well as the relative number of generated nodes \((n_Q)\) and needed solving time \((t_Q)\) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5\% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance          | conflict | dualray | inf. LP | dom. prop. |
|-------------------|----------|---------|---------|------------|
| acc-tight5        | n \(920\) | t \(139\) | n \(357\) | t \(0.388\) | t \(0.577\) |
| aflow40b          | n \(163981\) | t \(1338\) | n \(160367\) | t \(0.978\) | t \(945.95\) |
| alu4_7            | n \(649\) | t \(1\)  | n \(28296\) | t \(43.599\) | t \(17.09\) |
| alu5_7            | n \(1704\) | t \(2\)  | n \(6467\) | t \(3.795\) | t \(3.97\) |
| alu5_8            | n \(7492\) | t \(4\)  | n \(212946\) | t \(28.423\) | t \(71.31\) |
| alu6_7            | n \(14365\) | t \(11\) | n \(739480\) | t \(10.345\) | t \(680.82\) |
| alu6_8            | n \(47135\) | t \(33\) | n \(739660\) | t \(15.692\) | t \(380.08\) |
| alu7_7            | n \(71481\) | t \(76\) | n \(1132\) | t \(1.189\) | t \(0.50\) |
| bell3a            | n \(23108\) | t \(6\)  | n \(23108\) | t \(0.911\) | t \(4.12\) |
| bell5             | n \(1132\) | t \(1\)  | n \(1346\) | t \(1.189\) | t \(0.61\) |
| biella1           | n \(10755\) | t \(1710\) | n \(10755\) | t \(0.926\) | t \(1685.98\) |
| bierst1           | n \(12591\) | t \(44\) | n \(15883\) | t \(1.961\) | t \(53.32\) |
| bierst2           | n \(242186\) | t \(487\) | n \(169723\) | t \(0.701\) | t \(1654.63\) |
| binkar10_1        | n \(245091\) | t \(379\) | n \(245654\) | t \(0.938\) | t \(1457.89\) |
| bnatt350          | n \(4433\) | t \(288\) | n \(166584\) | t \(37.578\) | t \(12.299\) |
| enigma            | n \(1172\) | t \(1\)  | n \(760\) | t \(0.648\) | t \(0.50\) |
| lectsched-4-obj   | n \(2399\) | t \(268\) | n \(29505\) | t \(12.299\) | t \(85.40\) |
| lseu              | n \(601\) | t \(1\)  | n \(530\) | t \(0.882\) | t \(0.50\) |
| markshare_3_0     | n \(4841\) | t \(1\)  | n \(1843\) | t \(0.381\) | t \(0.50\) |
| markshare_3_2     | n \(5018\) | t \(1\)  | n \(2861\) | t \(0.570\) | t \(0.50\) |
| markshare_3_3     | n \(4826\) | t \(1\)  | n \(2645\) | t \(0.548\) | t \(0.50\) |
| markshare_3_4     | n \(3352\) | t \(1\)  | n \(2349\) | t \(0.701\) | t \(0.50\) |
| markshare_3_5     | n \(2481\) | t \(1\)  | n \(1421\) | t \(0.573\) | t \(0.50\) |
| markshare_4_0     | n \(643068\) | t \(121\) | n \(168883\) | t \(0.263\) | t \(16.77\) |

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| Instance          | n     | t     | n_Q    | t_Q    | n     | t     | n_Q    | t_Q    |
|-------------------|-------|-------|--------|--------|-------|-------|--------|--------|
| markshare_4_1     | 531216| 137   | 21485  | 0.403  | 21.93 | 0.160 | 491671 | 0.926  |
| markshare_4_2     | 369816| 104   | 136123 | 0.368  | 13.52 | 0.130 | 359908 | 0.973  |
| markshare_4_4     | 420963| 82    | 204375 | 0.485  | 20.46 | 0.251 | 398662 | 0.947  |
| markshare_4_5     | 504816| 118   | 274295 | 0.543  | 27.82 | 0.236 | 1106341| 2.192  |
| mas74             | 3420026| 685   | 3577586| 1.046  | 717.66| 1.047 | 3431529| 1.003  |
| mas76             | 519681| 96    | 516536 | 0.994  | 86.56 | 0.901 | 519458 | 1.000  |
| mzk-250-1-100-1   | 633761| 315   | 578416 | 0.913  | 285.29| 0.905 | 578416 | 0.913  |
| mine-166-5        | 6651  | 37    | 4551   | 0.684  | 27.87 | 0.755 | 5176   | 0.778  |
| mine-90-10        | 45418 | 214   | 137594 | 3.030  | 254.94| 1.191 | 53696  | 1.182  |
| misc03            | 134   | 1     | 134    | 1.000  | 1.06  | 0.955 | 128    | 0.955  |
| misc07            | 29168 | 21    | 38819  | 1.531  | 23.36 | 1.125 | 33828  | 1.160  |
| mzzv11            | 7274  | 1393  | 8588   | 1.181  | 1996.33| 1.433 | 10482  | 1.441  |
| mzzv42z           | 3050  | 731   | 2320   | 0.761  | 529.74| 0.724 | 1470   | 0.482  |
| neos-1061020      | 4040  | 1070  | 2704   | 0.669  | 701.54| 0.656 | 4530   | 1.121  |
| neos-1109824      | 31218 | 163   | 42132  | 1.550  | 155.53| 0.956 | 52693  | 1.688  |
| neos-1126860      | 5055  | 597   | 7015   | 1.588  | 589.86| 0.989 | 8011   | 1.585  |
| neos-1173026      | 383075| 1193  | 1305899| 3.409  | 3291.14| 2.578 | 1091276| 2.849  |
| neos-1208669      | 6041  | 457   | 16143  | 26.675 | 2983.86| 5.656 | 12846  | 2.126  |
| neos-1208335      | 8457  | 1139  | 60146  | 7.119  | 2193.83| 1.926 | 33349  | 3.913  |
| neos-1215259      | 2880  | 101   | 1645   | 0.571  | 56.00  | 0.554 | 4310   | 1.497  |
| neos-1215891      | 3890  | 1409  | 80007  | 20.567 | 3600.00| 2.554 | 5344   | 1.374  |
| neos-1223462      | 1472  | 819   | 1887   | 1.876  | 1020.01| 1.245 | 586    | 0.398  |
| neos-1281048      | 1384  | 12    | 1221   | 0.882  | 10.75 | 0.930 | 1120   | 0.809  |
| neos-1396125      | 32666 | 708   | 159271 | 4.936  | 2393.14| 3.381 | 151173 | 4.685  |
| neos-1420205      | 55079 | 18    | 26789  | 0.486  | 9.57  | 0.545 | 20946  | 0.380  |

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Table 5: Detailed computational results on Conflict solving time in seconds ($t$), as well as the relative number of generated nodes ($n_Q$) and needed solving time ($t_Q$) w.r.t. Columns 2 and 3 (conflict). All changes in the number of nodes or solving time of at least 5% are highlighted in bold and blue (improvement) and italic and red (deterioration).

| Instance       | conflict | dualray | inf. LP | dom. prop. |
|----------------|----------|---------|---------|------------|
|                | $n$      | $t$     | $n_Q$   | $t_Q$      | $n$      | $n_Q$   | $t$     | $t_Q$     |
| neos-1440225   | 1461     | 47      | 5584    | 3.822      | 127980   | 87.598  | 2620.81 | 55.291    | 1756     | 1.202    | 56.22   | 1.186   |
| neos-1460265   | 4367     | 66      | 26519   | 0.607      | 23940    | 0.548   | 52.86   | 0.803     | 10826    | 0.248    | 40.66   | 0.618   |
| neos-1461051   | 3401     | 13      | 301005  | 88.505     | 546261   | 160.618 | 408.46  | 32.213     | 3608     | 1.061    | 13.10   | 1.033   |
| neos-1480121   | 5608250  | 3600    | 2359174 | 0.421      | 17799189 | 3.174   | 3600.00 | 1.000      | 32071    | 0.058    | 58.65   | 0.016   |
| neos-1582420   | 59182    | 621     | 31105   | 0.526      | 17976    | 0.304   | 323.46  | 0.520     | 27241    | 0.460    | 396.75  | 0.638   |
| neos-1620807   | 1812678  | 1543    | 6291083 | 3.471      | 4881156  | 2.693   | 1483.78 | 0.961      | 1654801  | 0.913    | 1400.46 | 0.907   |
| neos-430149    | 25985    | 24      | 59691   | 2.297      | 7813     | 0.837   | 260.84  | 0.736      | 5797     | 0.621    | 232.06  | 0.655   |
| neos-503737    | 9330     | 354     | 1743    | 0.187      | 7813     | 0.837   | 260.84  | 0.736      | 36880    | 1.000    | 77.56   | 0.997   |
| neos-504674    | 36880    | 78      | 27888   | 0.756      | 32446    | 0.880   | 66.97   | 0.861      | 36880    | 1.000    | 77.56   | 0.997   |
| neos-538867    | 46052    | 53      | 119585  | 2.597      | 105752   | 2.296   | 68.48   | 1.287      | 40285    | 0.875    | 47.81   | 0.899   |
| neos-538916    | 29500    | 42      | 50111   | 1.699      | 49107    | 1.665   | 41.35   | 0.981      | 30631    | 1.038    | 43.95   | 1.043   |
| neos-551991    | 5448     | 319     | 10487   | 1.925      | 10487    | 1.925   | 1019.42 | 1.915      | 5448     | 1.000    | 321.11  | 1.006   |
| neos-555298    | 27669    | 200     | 37347   | 1.550      | 39681    | 1.434   | 333.98  | 1.666      | 7968     | 0.288    | 98.32   | 0.490   |
| neos-584851    | 351      | 27      | 424     | 1.208      | 424      | 1.208   | 26.37   | 0.994      | 351      | 1.000    | 26.50   | 0.998   |
| neos-585192    | 1577     | 35      | 1967    | 1.547      | 2300     | 1.458   | 39.45   | 1.141      | 1683     | 1.067    | 34.59   | 1.001   |
| neos-595925    | 27416    | 59      | 14296   | 0.521      | 15337    | 0.559   | 37.96   | 0.640      | 9774     | 0.357    | 27.71   | 0.467   |
| neos-686190    | 149683   | 1079    | 61805   | 0.413      | 74326    | 0.497   | 629.68  | 0.584      | 30924    | 0.207    | 245.01  | 0.227   |
| neos-717614    | 265333   | 414     | 530763  | 2.600      | 238841   | 9.009   | 3600.00 | 8.705      | 575150   | 2.168    | 989.23  | 2.392   |
| neos-785912    | 267      | 107     | 797     | 2.985      | 436      | 1.296   | 99.28   | 0.928      | 6577     | 24.639   | 585.12  | 5.467   |
| neos-791021    | 636      | 1787    | 188     | 0.296      | 2738     | 4.305   | 2466.50 | 1.380      | 423      | 0.665    | 792.06  | 0.443   |
| neos-803219    | 22155    | 35      | 24978   | 1.127      | 25400    | 1.146   | 38.17   | 1.094      | 32205    | 1.545    | 45.11   | 1.293   |
| neos-803220    | 55512    | 89      | 48674   | 0.877      | 55071    | 0.992   | 89.53   | 1.006      | 51782    | 0.933    | 80.60   | 0.905   |
| neos-806323    | 10041    | 29      | 14620   | 1.436      | 10165    | 1.012   | 29.62   | 1.021      | 10117    | 1.008    | 28.52   | 0.983   |
| neos-807639    | 6123     | 16      | 3509    | 0.573      | 6265     | 1.023   | 16.42   | 1.009      | 6245     | 1.020    | 16.46   | 1.012   |
| neos-807705    | 9210     | 24      | 5174    | 0.562      | 4870     | 0.529   | 14.70   | 0.607      | 5270     | 0.572    | 15.47   | 0.639   |

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| Instance                  | conflict | dualray | inf. LP | dom. prop. |
|---------------------------|---------|---------|---------|------------|
| n                         | t       | n       | t       | n          |
| n_{Q}                     | t_{Q}   | n_{Q}   | t_{Q}   | n_{Q}      |
| neos-810286               | 945     | 502     | 381     | 100        |
| neos-810326               | 2441    | 104     | 1188    | 1188       |
| neos-827015               | 187     | 1376    | 569     | 517        |
| neos-831188               | 4962    | 483     | 3861    | 4011       |
| neos-839859               | 18823   | 1130    | 20996   | 23734      |
| neos-848845               | 36361   | 923     | 31122   | 141118     |
| neos-849702               | 34345   | 647     | 299757  | 258988     |
| neos-862348               | 1516    | 30      | 5222    | 3581       |
| neos-863472               | 26229   | 220     | 714473  | 432913     |
| neos-886822               | 66990   | 244     | 35286   | 84519      |
| neos-922255               | 1612    | 448     | 930     | 1516       |
| neos-905856               | 4334    | 55      | 33444   | 8228       |
| neos-906865               | 48822   | 122     | 49529   | 49183      |
| neos-912023               | 1827    | 21      | 53733   | 221277     |
| neos-914441               | 354     | 227     | 180     | 166        |
| neos-942323               | 2567    | 16      | 4610    | 22291      |
| neos18                    | 4919    | 27      | 76980   | 352971     |
| net12                     | 1514    | 1690    | 3903    | 5342       |
| noswot                    | 580602  | 141     | 1810982 | 1130205    |
| noswot                    | 2495    | 3600    | 2296    | 519        |
| noswot                    | 4308    | 228     | 32931   | 19314      |
| noswot                    | 915997  | 978     | 102947  | 936139     |
| noswot                    | 39491   | 351     | 73960   | 64211      |
| noswot                    | 300472  | 1332    | 298798  | 300472     |
| noswot                    | 29058   | 18      | 29822   | 29558      |

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| Instance       | conflict | dualray | inf. LP | dom. prop. |
|----------------|----------|---------|---------|------------|
|                | $n$      | $t$     | $n$     | $t$        | $n$     | $t$     | $n$     | $t$        |
| prod2          | 78750    | 63      | 150765  | 104.46     | 113606  | 1.443   | 91.29   | 1.441      |
|                |          |         |         |            |         |         |         |            |
| pw-myciel4     | 569026   | 3600    | 627543  | 2708.26    | 619421  | 1.089   | 3600.00 | 1.000      |
|                |          |         |         |            |         |         |         |            |
| reblock67      | 128157   | 235     | 74055   | 1.103      | 101852  | 0.795   | 175.47  | 0.746      |
|                |          |         |         |            |         |         |         |            |
| mine6          | 562634   | 1144    | 807204  | 1.435      | 1010123 | 1.795   | 2175.24 | 1.902      |
|                |          |         |         |            |         |         |         |            |
| rococoC10-001000 | 298459 | 2527    | 353552  | 1.185      | 217769  | 0.730   | 1594.96 | 0.631      |
| rout           | 49976    | 58      | 137696  | 2.755      | 168051  | 3.563   | 135.10  | 2.347      |
| satellites1-25 | 471      | 1859    | 1982    | 4.208      | 6212    | 13.189  | 1930.08 | 1.038      |
| timtab1        | 843361   | 434     | 872675  | 1.035      | 863707  | 1.024   | 441.72  | 1.017      |
| geom.          | 16183.61 | 101.28  | 26196.47 | 1.619    | 26064.206 | 1.611  | 135.274 | 1.336      |
|                |          |         |         |            |         |         |         |            |
| sh. geom. [100, 10] | 16769.05 | 143.22  | 27097.915 | 1.616  | 27119.469 | 1.617  | 189.288 | 1.322      |

|                |         |         |         |            |         |         |         |            |
|                |         |         |         |            |         |         |         |            |
|                |         |         |         |            |         |         |         |            |
|                |         |         |         |            |         |         |         |            |