BPS BLACK HOLES IN N=2 FIVE DIMENSIONAL ADS SUPERGRAVITY

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Abstract

BPS black hole solutions of $U(1)$ gauged five-dimensional supergravity are obtained by solving the Killing spinor equations. These extremal static black holes live in an asymptotic $AdS_5$ space time. Unlike black holes in asymptotic flat space time none of them possess a regular horizon. We also calculate the influence, of a particular class of these solutions, on the Wilson loops calculation.
In the past years a considerable amount of work has been devoted to establish a duality between supergravity and super Yang Mills theories. For example the conformal field theory (CFT) living on the boundary of the five-dimensional anti-de Sitter space (AdS) is expected to be dual (in certain limits) to the four-dimensional super Yang Mills theory. Since this conjecture has been made [1] and further developed in [2], five-dimensional anti-de Sitter spaces have received a great deal of interest.

The aim of this letter is to describe BPS black holes living in an asymptotic AdS vacuum (for the AdS case, Reissner-Nordström solutions have been discussed in [3] and non-abelian monopoles in [4]). To keep these solutions as general as possible we formulate them in terms of D=5, N=2 supergravity with an arbitrary prepotential, i.e. the N=4,8 black holes appear as a special subclass for special choices of the prepotential of the N=2 theory. Because the asymptotic vacuum should be AdS instead of flat Minkowski space, we gauge a U(1) subgroup of the SU(2) automorphism group, which results in a scalar potential that becomes constant at infinity.

In the first part we will describe these black holes as solutions of the Killing spinor equations of gauged D=5, N=2 supergravity [5] and in the second part we ask for the modification of the Wilson-loop calculations as done in [6], [7], [8].

First, we briefly describe the theory of N = 2 supergravity coupled to an arbitrary number n of abelian supermultiplets. N = 2 supergravity theories in five-dimensions can be obtained, for example, by compactifying eleven-dimensional supergravity on a Calabi-Yau 3-folds [9]. The massless spectrum of the compactified theory contains (h(1,1) − 1) vector multiplets with real scalar components. Including the graviphoton, the theory has h(1,1) vector bosons. The theory also contains h(2,1) + 1 hypermultiplets, where h(1,1) and h(2,1), are the Calabi-Yau Hodge numbers. In what follows and for our purposes the hypermultiplets are switched off. The anti-de Sitter supergravity can be obtained by gauging the U(1) subgroup of the SU(2) automorphism group of the N = 2 supersymmetry algebra. This gauging, which breaks SU(2) down to U(1) can be achieved by introducing a linear combination of the abelian vector fields present in the ungauged theory, i.e. A_μ = V_I A^I_μ, with a coupling constant g. To restore supersymmetry, g-dependent and gauge-invariant terms have to be added. In a bosonic background, this amounts to the addition of a scalar potential, (for more details see [10, 11]).

The bosonic part of the effective gauged supersymmetric N = 2 Lagrangian which describes the coupling of vector multiplets to supergravity is given by

\[ e^{-1} \mathcal{L} = -\frac{1}{2} R - \frac{1}{4} G_{I J} F^I_{\mu \nu} F^{J \mu \nu} - \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial_\mu \phi^j + \frac{e^{-1}}{48} \varepsilon^{\mu \nu \rho \sigma \lambda} C_{I J K} F^I_{\mu \nu} F^J_{\rho \sigma} A^K_\lambda \]

\[ + \ g^2 V_I V_J \left( 6 X^I X^J - \frac{9}{2} g^{ij} \partial_i X^I \partial_j X^J \right) \]

where R is the scalar curvature, \( F_{\mu \nu} = 2 \partial_\mu A_\nu \) is the Maxwell field-strength tensor and \( e = \sqrt{-g} \) is the determinant of the Fünfbein \( e^a_m \).

5 The signature (− + + + +) is used. Antisymmetrized indices are defined by: \( [ab] = \frac{1}{2} (ab - ba) \).
The physical quantities in (1) can all be expressed in terms of a homogeneous cubic polynomial $V$ which defines “very special geometry” \[12\].

$$G_{IJ} = -\frac{1}{2} \frac{\partial}{\partial X^I} \frac{\partial}{\partial X^J} (\ln V)|_{V=1}, \quad g_{ij} = G_{IJ} \partial_i X^I \partial_j X^J|_{V=1}, \quad (\partial_i \equiv \frac{\partial}{\partial \phi^i}). \quad (2)$$

For Calabi-Yau compactification

$$V = \frac{1}{6} C_{IJK} X^I X^J X^K = X^I X_I = 1. \quad (3)$$

$V$ is the intersection form, $X^I$ and $X_I$ correspond to the size of the 2 and 4-cycles and $C_{IJK}$ are the intersection numbers of the Calabi-Yau threefold.

Since we are interested in finding BPS solutions in the gauged theory, we display the supersymmetry transformation of the Fermi fields in a bosonic background

$$\delta \psi_\mu = \left( D_\mu + \frac{i}{8} X_I \Gamma_{\mu}^{\nu \rho} - 4 \delta_{\mu}^{\nu} \Gamma_{\rho} \right) F_{\nu \rho}^I + \frac{1}{2} g \Gamma_{\mu} X^I V_I - \frac{3}{2} ig V_I A_I^I \epsilon, \quad \delta \lambda_i = \left( 3 \frac{i}{8} \partial_i X_I \Gamma^{\mu \nu} F_{\mu \nu}^I - \frac{i}{2} g_{ij} \Gamma^{\mu} \partial_\mu \phi^j + \frac{3}{2} ig V_I \partial_i X^I \right) \epsilon \quad (4)$$

where $\epsilon$ is the supersymmetry parameter and $D_\mu$ is the covariant derivative.

The spherically symmetric BPS electric solutions can be obtained by solving for the vanishing of the gravitino and gaugino supersymmetry variation for a particular choice for the supersymmetry parameter. We impose the projection operator condition on the spinor $\epsilon$

$$\epsilon = (ia \Gamma_0 + b \Gamma_1) \epsilon, \quad (5)$$

where $a^2 + b^2 = 1$ and this breaks $N = 2$ supersymmetry to $N = 1$.

We briefly\[11\] describe the procedure of obtaining solutions preserving $N = 1$ supersymmetry. First we start with an ansatz for the metric and gauge field

$$ds^2 = -e^{2V} dt^2 + e^{2W} \left( dr^2 + f^2 r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2 \right) \right), \quad A_I^I = e^{-2U} X^I$$

where the functions $U, V, W$ and $f$ are functions of $r$, and $(\theta, \phi, \psi)$ are the polar coordinates of the 3-sphere. As solution of the gauge field equations we find

$$e^{2U} X_I = \frac{1}{3} H_I$$

where $H_I$ is a harmonic functions which depends on the electric charge $q_I$. The supersymmetry variation of the gaugino and the time component of the gravitino imply the following relations

$$e^{2V} = e^{-4U} f^2, \quad e^{2W} = e^{2U} \frac{1}{f^2}$$

\[6\]More detailed analysis will be given in \[11\].
where we used some relations of special geometry analog to the derivation in [13].

The time and spatial components of the gravitino transformation imply differential constraints on the Killing spinor. These are

\[
(\partial_t - ig) \epsilon = 0 ,
\]

\[
\left( \partial_r - \frac{i}{2f} \left( \frac{1}{r} + 3U' \right) \Gamma_0 - \frac{1}{2} \left( \frac{1}{r} + U' \right) \right) \epsilon = 0 ,
\]

\[
\left( \partial_\phi + \frac{i}{2} \Gamma_{012} \right) \epsilon = 0 ,
\]

\[
\left( \partial_\phi + \frac{i}{2} \sin \theta \Gamma_{013} - \frac{1}{2} \cos \theta \Gamma_{23} \right) \epsilon = 0 ,
\]

\[
\left( \partial_\psi + \frac{i}{2} \cos \theta \Gamma_{014} + \frac{1}{2} \sin \theta \Gamma_{24} \right) \epsilon = 0 .
\]

Going to the rescaled coordinates

\[
X^I = V^{-\frac{1}{3}} Y^I
\]

where \( V = e^{3U} \), one obtains the following solution\(^7\)

\[
\epsilon = e^{igt} e^{-\frac{i}{2} \Gamma_{012} \theta} e^{\frac{i}{2} \Gamma_{23} \phi} e^{-\frac{i}{2} \Gamma_{014} \psi} \phi(r)
\]

\[
\phi(r) = \frac{1}{2} \sqrt{\frac{f + 1}{g^2 r^2 V^2}} \left( 1 - i \Gamma_0 \right) \epsilon_0
\]

(6)

where \( f = \sqrt{1 + g^2 r^2 V^2} \) and \( \epsilon_0 \) is an arbitrary constant spinor. Thus, inserting all terms in our ansatz one obtains

\[
d s^2 = -V^{-4/3} (1 + g^2 r^2 V^2) d t^2 + V^{2/3} \left[ \frac{d r^2}{1 + g^2 r^2 V^2} + r^2 (d \theta^2 + \sin^2 \theta d \phi^2 + \cos^2 \theta d \psi^2) \right]
\]

\[
F_{lm}^I = -\partial_m (V^{-1} Y^I) , \quad V = 1 - C_I J K Y^J Y^K , \quad \frac{1}{2} C_I J K Y^J Y^K = H_I = 3 V_I + \frac{q_I}{r^2}
\]

(7)

Note that the constant parts in the harmonic functions are given by \( V_I \), which fixes the \( U(1) \) that has been gauged. The only deviation from the ungauged case \([13]\) comes via the function \( f^2 = 1 + g^2 r^2 V^2 \). This term however changes completely the singularity structure of the black hole solution. To investigate this in more detail we may consider simple cases were \( V \) can be written as

\[
V = H^n = \left( 1 + \frac{q}{r^2} \right)^n , \quad n = 0, 1, 2, 3 .
\]

\(^7\)The Killing spinors for a general \( AdS_p \times S^q \) geometry are also discussed in \([14]\).
Obviously, the first case \((n = 0)\) defines the \(AdS_5\) vacuum with no black hole. The cases of \(n = 1, 2\) correspond to black holes with a singular horizon and they appear naturally as BPS solutions of \(N = 4, 8\) supergravity. In both cases the scalars are either zero or blow up near the horizon. The last case \((n = 3)\) is an example of a BPS black hole of \(N = 2\) supergravity, which seems to have a regular horizon at \(r \simeq 0\). However this coordinate system is misleading. Defining

\[
\rho^2 = r^2 + q
\]

one finds

\[
ds^2 = -e^{2V}dt^2 + e^{-2V}\Delta^{-1}d\rho^2 + \Delta \rho^2d\Omega_3
\]

\[
e^{2V} = H^\frac{2n}{3} + g^2\rho^2\Delta \quad , \quad \Delta = H^\frac{3n}{2} \quad , \quad \tilde{H} = 1 - \frac{q}{\rho^2}
\]

In the ungauged case \((g=0)\) the horizon is at \(\tilde{H} = 0\) (or \(\rho^2 = q\)), which is regular in the case \(n=3\) or \(\Delta = 1\). But taking into account the gauging the horizon disappeared \((e^{\pm 2V}\) is finite at \(\rho^2 = q\) for \(n = 3)\) and the singularity at \(\rho = 0\) becomes naked. For the other cases \((n = 1, 2)\) the horizon becomes singular. For \(n = 1\) the singular horizon is infinitely far away, i.e. a light signal \((ds^2 = 0)\) would need infinite time to reach any finite distance (null singularity). But for \(n = 2\) the distance to the singular horizon is finite. This is different to the ungauged case \((g = 0)\), where all singular cases have null horizons. Note also, the naked singularity at \(\rho = 0\) for \(n = 3\) (i.e. \(\Delta = 1)\) is only a finite distance away! Certainly, this makes this solution rather suspicious and to overcome this situation one should consider the non-extremal case.

Let us nevertheless ask, what is the influence of this black hole on the Wilson loops as calculated in [6], [7], [8]. For this we calculate the Nambo-Goto action for open strings that are attached to the asymptotic boundary

\[
S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{|\det g_{\alpha\beta}|} \quad , \quad g_{\alpha\beta} = \partial_\alpha X^{\alpha} X^{\beta} G_{MN}
\]

where \(G_{MN}\) is the 5d metric. For the worldsheet coordinates we choose the gauge

\[
\tau = t \quad , \quad \sigma = \theta
\]

where \(\theta\) is the polar angle in \(\Omega_3\) (see figure and the eq. [7]). Obviously, the string will stretch inside the AdS space and thus its position is given by a function \(f(\theta, \rho) = 0\), where \(\rho\) is the radial coordinate. This defining equation can also be expressed as \(\rho = \rho(\sigma = \theta)\). For the induced metric we find therefore

\[
g_{\tau\tau} = \partial_\tau X^M \partial_\tau X^N G_{MN} = G_{00} = -e^{2V}
\]

\[
g_{\sigma\sigma} = \partial_\sigma \rho \partial_\sigma \rho G_{\rho\rho} + G_{\theta\theta} = (\rho')^2 e^{-2V} \Delta^{-1} + \rho^2 \Delta
\]

and thus,

\[
S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{(\rho')^2 \Delta^{-1} + \rho^2 e^{2V} \Delta}.
\]
Following arguments given by Maldacena [7], we use the fact that the Lagrangian does not depend explicitly on $\theta$ and therefore

$$c = \frac{\rho^2 e^{2V} \Delta}{\sqrt{(\rho')^2 \Delta^{-1} + \rho^2 e^{2V} \Delta}}. \quad (15)$$

The constant $c$ can be determined by going at the extremum $\rho_0$ where $\rho' = 0$ i.e.

$$c^2 = \left( \rho^2 e^{2V} \Delta \right)_{\rho = \rho_0}. \quad (16)$$

In addition it follows from (15) that

$$d\sigma = \frac{d\rho}{\rho \Delta e^V \sqrt{\frac{1}{c^2} \rho^2 e^{2V} \Delta - 1}} = \frac{dy}{2g \rho_0 \sqrt{\frac{n}{2} \frac{1}{\rho_0^2} (y - \lambda)^\frac{2}{3} \sqrt{\frac{2n}{3} (y - \lambda)^\delta} - 1}}. \quad (17)$$

where $y = (\rho/\rho_0)^2$, $\lambda = q/\rho_0^2$ and $\delta = \frac{2(3-n)}{3}$ (see also the figure). Furthermore we consider here only the simplest case where $e^{2V} = g^2 \rho^2 \Delta$ (i.e. we neglect the first term), which is a good approximation for the region $0 < q < \rho^2$. Integrating this equation yields the function $\rho = \rho(\sigma = \theta)$ that determines the position of the string in the AdS space. We can also calculate the distance between both endpoints on the boundary

$$L = 2 \int_0^{\theta_L} \sqrt{g_{\sigma\sigma}} d\sigma^2 = \frac{(1 - \lambda)^\frac{\delta}{2}}{g} \int_1^\infty \frac{dy}{y^\frac{n}{2} (y - \lambda)^\frac{\delta}{2} \sqrt{y^\frac{2n}{3} (y - \lambda)^\delta} - (1 - \lambda)^\delta}. \quad (18)$$

Note, we are dealing here with a different asymptotic geometry of $\mathbf{R} \times S_3$ (where $\mathbf{R}$ is the time).
There are some interesting things to notice. First, for \( n = 3 \) \((\delta = 0)\) all \( \lambda \) dependence drops out and \( L \sim 1/g \) becomes independent of \( \rho_0 \) and the charge \( q \), it scales only with cosmological constant. Thus it coincides with the case without black hole. Secondly, for \( \delta \neq 0 \) the integral is finite if \( \lambda < 1 \), i.e. the string is away from the horizon. However, if the horizon comes close to string (\( \lambda \to 1 \)) the integral becomes divergent for \( n = 1 \). However taking the pre-factor into account one finds that in this limit \( L \) behaves like \( L \sim \frac{1}{g}(1-\lambda)^{1-\frac{\delta}{2}} \). Therefore, the string endpoints approach each other \( L \to 0 \) for \( q \to \rho_0^2 \) (see figure) if the horizon becomes large enough. This is different from the (neutral) Schwarzschild black hole, where \( L \to \infty \) if the horizon comes close to the string \( \text{[8]} \).

Finally, one may insert the solution (17) in the action and calculate the energy

\[
E = \frac{T}{2\pi \rho_0^2} \int_1^{\infty} dy \left[ \frac{y^{\gamma/6} (y - \lambda)^{\gamma/4}}{\sqrt{y^{2n/3} (y - \lambda) - (1-\lambda)^{\delta} - 1}} \right]
\]  

(19)

where the last term is the subtraction of the infinite self-energy (see [7], [8]). Obviously, for \( \lambda \to 1 \) or \( q \to \rho_0^2 \) the energy remains finite although the string comes close to the singular horizon and it scales with the charge or the BPS mass of the black hole \( E \sim q \).

It is interesting to note that the energy is independent of \( g \).

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