Material with Quasi-Zero Stiffness for Vibration Isolation in Civil and Industrial Structures and Buildings

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Abstract. The Paper is devoted to the problem of reduction of vibration level in civil and industrial structures and buildings. It is offered to use two-component material for vibration isolation. A special internal structure, that provides quasi-zero stiffness effect, is designed for this purpose. Use of two components makes the material more stable. Analytical and computer calculations have been made in order to get optimum geometrical parameters of the internal structure.

1. Introduction

Vibration isolation and impact damping are common problems in industrial buildings and structures. High vibration causes cracks and reduces reliability of structures. It is also known that prolonged exposure of vibration and noise harmfully affects on human and causes different chronic diseases [1, 2, 3]. Increasing human comfort requires new ways of vibration isolation. Modern materials and new achievements in 3D manufacturing create opportunities in creating new types of metamaterials. Metamaterials - material with special internal structure that provides new artificial properties. For example, materials with negative Poisson's ratio absent in nature, but due to 3D-printing it's possible to get it [4]. Generally, metamaterials also obtaining special properties connected with acoustics, electrics, optics, electromagnetics, etc.

Modern achievements in 3d printing and 3d manufacturing technology create possibilities of creation of materials with complex internal structure. This way may be used for obtaining special structure in elastic materials, i.e. for improved vibration isolation [5, 6]. Providing special shape of the internal structure of the material provides a non-linear force characteristic. If this material has a flat area on its force characteristics then it can be name as isolator with quasi-zero stiffness [7]. It allows to get much higher degree of reduction of the dynamic forces and enables to isolate a wide range of vibration with high efficiency [9].

But there is a problem: vibration isolators with quasi-zero stiffness are characterized by design complexity. Practical implementation of systems with quasi-zero stiffness is rather difficult.
Convenient design of vibration isolator with quasi-zero stiffness may provide worldwide application of such a material in many fields.

2. Formatting the title, authors and affiliations
In this paper general idea of obtaining quasi-zero stiffness effect is providing pre-buckling effect in two-component material with specified structure. Scheme of internal structure is presented on the Fig. 1

![Figure 1. Internal structure of two-component material with quasi-zero stiffness.](image)

On the Fig. 1 two-component material contains hard and soft elastic materials. Hard material is proposed to be a fluorine rubber, hard rubber, polyurethane. Soft material is proposed to be a soft rubber, silicone, soft damping material.

Two-component material contains hard structure that has pre-buckling condition and may have negative stiffness. But soft material due to positive stiffness makes whole material stable.

3. Mathematical modeling of two-component material with quasi-zero stiffness
On the Fig. 2 a sketch of single cell of the metamaterial is presented.

![Figure 2. Single cell of the metamaterial.](image)

$E_1$ - Young’s modulus of the soft elastic materials; $E_2$ - Young’s modulus of the hard elastic materials; $L, S, t_s, h_s, t, t_0, x_1, x_2$ - geometrical parameters

For mathematical analysis of the metamaterial and calculation of its force characteristic an energy method is used. Deformation of the cell is assessed as a deformation of the following elements: compression of the zone I, compression of the zone II, longitudinal compression of the inclined wall, bending of the inclined wall.

Energy of compression of the zone I equals:
\[ W_{a1} = \frac{E_1 b (L + t_0)}{2 (h_2 + h_s)^2} \left( \Delta x_1 + \Delta x_2 \frac{h_s}{h_s + t_0} \right)^2; \]  

(1)

where \( E_1 \) = Young's modulus of the soft elastic materials; \( E_2 \) = Young's modulus of the hard elastic materials; \( \Delta x_1 \) = decrease of the value \( x_1 \) under load; \( \Delta x_2 \) = decrease of the value \( x_2 \) under load; \( b \) = thickness of the material.

Energy of compression of the zone 2 equals:

\[ W_{a2} = \frac{E_2 b t_s}{h_s^2} \Delta x_2^2; \]  

(2)

Energy of compression of the inclined wall equals:

\[ W_c = \frac{E_2 b t_s}{\sqrt{L^2 + S^2}} \frac{\Delta L^2}{2}; \]  

(3)

where \( \Delta L \) = decrease of the value \( L \) under load.

Energy of bending of the inclined wall equals:

\[ W_b = \frac{1}{8} \frac{E_2 b t_s}{\sqrt{L^2 + S^2}} \Delta y^2; \]  

(4)

where \( \Delta y \) = decrease of the value \( S \) under load.

Consider \( x \) - compression of the material.

Note, that there are the following relationships between parameters:

\[ \Delta x_1 = \frac{E_2}{E_1 + E_2} \cdot x; \]

\[ \Delta x_2 = \frac{E_1}{E_1 + E_2} \cdot x; \]

\[ \Delta L = \sqrt{S^2 + L^2} - \sqrt{(S - \Delta y)^2 + L^2}; \]

\[ \Delta y = S - \left[ \left( h_{s1} - \Delta x_1 \right) - \left( h_y - \frac{h_y}{h_y + t_0} \Delta x_2 \right) \right]; \]

\[ h_{s1} = h_y + S; \]

\[ h_{s2} = h_y + t_0; \]

\[ t_0 = t \sqrt{1 + \frac{S^2}{L^2}}. \]

Total energy equals:

\[ W = 2W_{a1} + 2W_{a2} + W_c + W_b. \]

Force relative to compression \( x \) equals:

\[ F(x) = \frac{\partial W}{\partial x}; \]

Stiffness relative to compression \( x \) equals:

\[ k(x) = \frac{\partial F}{\partial x}. \]
For obtaining quasi-zero effect stiffness at a certain compression equals zero. The lowest stiffness reaches at $x=S$, so
\[ \frac{\partial^2 W}{\partial x^2} (S) = 0. \]

Mathematical analysis this have been done. Particularly, it is obtain that the following parameters provides quasi-zero effect:
\[ \frac{E_1}{E_2} = 0.01; \quad \frac{b}{L} = 1; \quad \frac{h_s}{L} = 0.1; \quad \frac{t_s}{L} = 0.1; \quad \frac{t}{L} = 0.4; \quad \frac{S}{L} = 0.61; \]

For these parameters optimal load equals
\[ \frac{F}{bLE_2} = 0.042. \]

So, if we define parameters $E_2=10$ MPa; $E_1=0.1$ MPa; $L=10$ mm; $b=10$ mm; $h_s=5$ mm; $t_s=5$ mm; $t=4$ mm; $S=6.1$ mm optimal load equals 8 N or 8154 kg per 1 m$^2$.

4. **Designing two-component material with quasi-zero stiffness**

For checking the results of analytical study a computer analysis via Ansys is made. For this purpose Mechanical Static Structural Module with option “large deflections” have been applied. General view of the computer model is presented on the Fig. 3. Static force characteristic obtained by Ansys is presented on the Fig. 4.

**Figure 3.** Computer model of single cell of the metamaterial (soft material is hidden on this Figure).

**Figure 4.** Static force characteristic of the single cell.
Due to computer analysis optimal load equals 8.5 N; compression of the material at this load equals 5.5 mm. As we can see, analytical analysis coincides with computer one.

5. Summary
Two-component metamaterial for vibration isolation is presented in this Paper. High vibration isolation parameters are obtained due to quasi-zero stiffness effect. Application of two components in this material provides more stable parameters relative to one-component material. Such a material can greatly reduces vibration level in civil and industrial buildings and structures.

6. References
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