GAS-DYNAMIC SHOCK HEATING OF POST-FLARE LOOPS DUE TO RETRACTION FOLLOWING LOCALIZED, IMPULSIVE RECONNECTION

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\textbf{ABSTRACT}

We present a novel model in which field lines shortening after localized, three-dimensional reconnection heat the plasma as they compress it. The shortening progresses away from the reconnection site at the Alfvén speed, releasing magnetic energy and generating parallel, compressive flows. These flows, which are highly supersonic when $\beta \ll 1$, collide in a pair of strong gas-dynamic shocks at which both the mass density and temperature are raised. Reconnecting field lines initially differing by more that 100$^\circ$ can produce a concentrated knot of plasma hotter that 20 MK at the loop’s apex, consistent with observations. In spite of these high temperatures, the shocks convert less than 10% of the liberated magnetic energy into heat—the rest remains as kinetic energy of bulk motion. These gas-dynamic shocks arise only when the reconnection is impulsive and localized in all three dimensions; they are distinct from the slow magnetosonic shocks of the Petschek steady-state reconnection model.

\textit{Key words:} MHD – shock waves – Sun: flares

1. INTRODUCTION

Magnetic reconnection has long been proposed as a mechanism for heating coronal plasma. In one early model (Kopp & Pneuman 1976, see Figure 1), reconnection occurs between open field lines separated by a vertical current sheet (red line) creating new closed field lines (post-flare loops, gray). Closing these field lines stops the solar wind upflow in a gas-dynamic shock (GDS) that Kopp & Pneuman (1976) estimated would raise the temperature by 80%. Cargill & Priest (1982) found the direct magnetic energy conversion by reconnection to be a far more effective source of heating in this same model. It could raise the temperature of post-flare loops by up to a factor of 3 to 6 MK. Even this higher value is, however, insufficient to explain the 15–20 MK temperatures observed at apices of post-flare loops in soft X-ray and Fe xxiv EUV emission (Warren et al. 1999).

Recent theoretical investigations have revealed that electric fields large enough for fast magnetic reconnection can be self-consistently produced by a wide range of small-scale mechanisms, provided only that they are localized within a segment of the current sheet (Birn et al. 2001; Biskamp & Schwarz 2001). All such processes generate reconnection flows resembling the model of Petschek (1964; see inset (a) of Figure 1) with slow magnetosonic shocks (SMSs, blue lines) outside the non-ideal region at which the magnetic field is deflected and weakened, thereby heating the plasma. When reconnection is localized in both space and time the SMSs close back together across a finite layer of horizontal field, as shown in inset (b) of Figure 1 (Semenov et al. 1983; Biernat et al. 1987). This field forms the top of a “hairpin” comprising all the flux closed since the onset of reconnection.

As the hairpin flux sheet retracts its field lines become much shorter and release substantial magnetic energy. This magnetic energy is converted almost entirely into kinetic energy rather than partly into heat as in Petschek’s steady-state model (Semenov et al. 1998). In a strictly two-dimensional version of the model the mass of the shortened tube accumulates in the tip of the the retracting hairpin. This “snowplowing” artifact is absent when reconnection is also localized in the third dimension ($\zeta$) and there is a magnetic field component in that direction. With the horizontal field component (sometimes called a “guide field”) the current sheet separates field with angle $\pm \zeta$ (tan $\zeta = B_y / B_x$; see the “front view” of Figure 1).

Flux reconnect within a patch and over a finite interval forms a $\Lambda$-shaped flux tube (gray) similar to magnetospheric flux transfer events (Russell & Elphic 1978; Lee et al. 1993; Otto 1995). The tube is distinguished from the surrounding field (the flux layers) by the distinct connectivity given it through reconnection. The different field line geometry, such as the bend, produces dynamics in the tube entirely different from those in the surrounding flux layers, with which it has little subsequent interaction.

Recently Linton & Longcope (2006) studied the relaxation of this post-reconnection flux tube using three-dimensional magnetohydrodynamic (MHD) simulation. They found that the perpendicular dynamics of the tube, as it moves between the flux layers, is well approximated by the equations for a thin magnetic flux tube (Spruit 1981), although with $\beta < 1$. They presented an analytic solution in which the post-reconnection tube shortens at the Alfvén speed, converting magnetic energy into kinetic energy. Linton & Longcope (2006) did not investigate the dynamics parallel to the tube or any associated thermal effects of shortening.

This Letter demonstrates that flux tube shortening is a powerful, inevitable mechanism for heating post-flare loops which has not been previously investigated. The magnetic forces responsible for shortening also drive compressive parallel flows at the Alfvén speed. At very low $\beta$, these are high-Mach number flows whose collision naturally generates very strong shocks. The shocks are distinct from the SMSs of Petschek reconnection and are driven by reconnection-initiated, perpendicular dynamics. This is in contrast to previous investigations of flux tube shocks wherein acoustic wave-steepening or pressure differences were considered as drivers (Herbold et al. 1985; Thomas & Montesinos 1991).
2. POST-RECONNECTION FLUX TUBE DYNAMICS

We begin by assuming that a localized, transient, fast magnetic reconnection has occurred, by an unspecified physical mechanism, within an otherwise static current sheet. This will, as just discussed, leave a Λ-shaped flux tube, initially at rest. Due to its sharp bend it is out of equilibrium, and magnetic forces start it sliding downward between the magnetic layers separated by the current sheet.

The tube’s retraction, unhindered by the external flux layers, can be modeled using the thin flux tube equations of Spruit (1981) and subsequent authors (see, e.g., the review by Fisher et al. 2000). While the high-β flux tubes in those previous investigations are confined by the pressure of the unmagnetized convection zone, our post-reconnection tube has very low β and is confined by the magnetic pressure of the flux layers outside the current sheet. We assume sufficient collisionality to justify the use of MHD equations throughout.

The tube is assumed thin enough to be described only by its axis. Internal properties such as the magnetic field strength $B_t$, pressure $p_i$, and mass density $ρ_i$ are functions only of the axial position. The tube is also thin enough for fast magnetosonic waves to establish pressure balance across its diameter virtually instantaneously. This assumption constrains the internal properties to match those outside the flux tube: $B_t^2/8π + p_i = B_e^2/8π + p_e$, assumed to be uniform and constant.

We will also assume the plasma β to be always small. The main force on a section of tube is therefore the magnetic tension due to curvature of the axis $B_t^2(∂i/∂ℓ)/4π$, where $ℓ$ is arc length and $i = ∂x/∂ℓ$ is the unit tangent vector. Since it is ultimately the Lorentz force, it is natural that this force is strictly perpendicular to the axis ($i · ∂i/∂ℓ = 0$).

The pressure gradient, $-i · ∂p_i/∂ℓ$, is formally smaller, by a factor of $β$, than the magnetic tension. It is, however, the only force parallel to the axis, and is essential to arresting internal pile-up of mass. We therefore retain terms involving pressure to first order in $β$.

Velocity evolution is governed by a momentum equation with only these two forces. Conservation properties become apparent when arc length is replaced with $μ$, the integrated mass per unit flux:

\[ \frac{dμ}{dℓ} = \frac{ρ_i}{B_t} \cdot \frac{∂}{∂μ} \left( \frac{B_i}{4π} \right) \]

includes the parallel pressure gradient after use of pressure balance, $B_t = B_e + 4π(p_e - p_i)/B_e$, valid to first order in $β$. The momentum per unit flux of any section of tube,

\[ P = \int \frac{ρ_i v}{B_i} dℓ = \int v dμ, \]

changes only through forces (per unit flux) from the ends of the section, $B_i i/4π$, directed parallel to the axis.

3. SHOCK RELATIONS IN THIN FLUX TUBES

Momentum conservation leads to a set of shock relations for thin flux tubes. Consider two straight sections with uniform properties (designated 1 and 2), separated by an abrupt change at coordinate $μ_0$. The length scale of this change is large compared with the tube radius but otherwise small enough that we hereafter
call it a “discontinuity” and “corner.” This feature moves through space with constant velocity \( \mathbf{u} \) while its Lagrangian coordinate changes at constant rate \( \mu_0 \). We assume that the properties of the straight sections do not change as this happens. Fluid velocities on either side of the discontinuity differ from \( \mathbf{u} \), but components perpendicular to the tangent vector match that of \( \mathbf{u} \): \( (\mathbf{v} - \mathbf{u}) \times \mathbf{i} = 0 \).

The component of relative velocity parallel to the tangent vector, \( \mathbf{v}_\parallel = \mathbf{i} \cdot (\mathbf{v} - \mathbf{u}) \), represents a flow across the discontinuity. The mass flux (per magnetic flux) through the discontinuity, \( \dot{\mu}_0 \), must equal the mass flux across points on either side of it, since the corner moves without changing. For positions \( \mu_1 \) and \( \mu_2 \), separated by fixed distances from \( \mu_0 \), this means

\[
\dot{\mu}_2 = -\rho_i v_i / (\mathbf{B}_1 \cdot \mathbf{i} / 4\pi) = -\rho_i v_i / (\mathbf{B}_2 \cdot \mathbf{i} / 4\pi) = \dot{\mu}_1 = \dot{\mu}_0. \tag{3}
\]

The flux tube between \( \mu_2 \) and \( \mu_1 \) does not change so its momentum (per magnetic flux) is constant,

\[
\mathbf{P} = \|\dot{\mu}\mathbf{v}\| + \|\mathbf{B}_i / 4\pi\| = 0, \tag{4}
\]

denoting \( \|f\| = f_2 - f_1 \). Since \( \mathbf{v} = \mathbf{u} + v_i \mathbf{i} \) away from the discontinuity momentum constancy may be written

\[
\left(\mathbf{B}_i / 4\pi - \rho_i v_i^2 / (\mathbf{B}_i \cdot \mathbf{i} / 4\pi) - \left[\rho_i / B_e + \rho_i v_i^2 / (\mathbf{B}_i \cdot \mathbf{i} / 4\pi)\right] \right) \hat{i} = 0, \tag{5}
\]

where \( \langle f \rangle = (f_2 + f_1)/2 \). The vectors \( \|f\| \) and \( \|\hat{i}\| \) are orthogonal so Equation (5) represents two independent equations. As long as the bend is not a hairpin (\( \|\hat{i}\| \neq 0 \)) the \( \|\hat{i}\| \) component of momentum conservation, to lowest order in \( \beta \), is equivalent to gas-dynamic momentum conservation: \( \|\rho_i \| = 0 \). To the same order in \( \beta \), Equation (3) gives the gas-dynamic mass continuity equation: \( \|\rho_i v_i \| = 0 \). Provided there is no heat flow from the external field or across \( \mu_1 \) or \( \mu_2 \) then the sum of kinetic and thermal energy within the tube section will also be conserved. This provides one additional, independent constraint on \( \rho_i, p_i, \) and \( v_i \) across the discontinuity. (Since \( \mathbf{B}_i \) can be related directly to \( p_i \), it can be eliminated from energy conservation.)

The foregoing describes three relations between six quantities which do not include the magnetic field’s strength or direction. The relations are the traditional gas-dynamic Rankine–Hugoniot conditions (Courant & Friedrichs 1948), but for flows inside a flux tube (Ferriz-Mas & Moreno-Insertis 1987). The other component of Equation (5), to lowest order in \( \beta \),

\[
1 - \left(4\pi \rho_i / B_i^2\right) v_i^2 \|\hat{i}\|^2 = 0 \tag{6}
\]

constitutes one more relation, which does involve the field direction.

From values on one side the Rankine–Hugoniot relations may be satisfied in two different ways by values on the other (see, e.g., Courant & Friedrichs 1948). They may be satisfied nontrivially by a unique set of different values, or they may be satisfied trivially by the same set of values (i.e., \( \|\rho_i\| = \|v_i\| = \|p_i\| = 0 \)). In the nontrivial case, the three different quantities satisfy three independent constraints and cannot be forced, in general, to satisfy a fourth. The leading factor of Equation (6), however, constitutes a fourth independent constraint so it will not in general vanish; it is therefore necessary that \( \|\hat{i}\| = 0 \) (there is no bend). This means that a thin, low-\( \beta \) flux tube can support discontinuities in internal quantities only at a GDS within a straight section of tube.

If, on the other hand, there is a bend in the flux tube (\( \|\hat{i}\| \neq 0 \)), the only way to satisfy Equation (6) is for the pre-factor to vanish. Since the nontrivial solution of all three Rankine–Hugoniot conditions would overdetermine the system, they must be satisfied trivially, without discontinuity. In other words \( \rho_i, p_i, \) and \( v_i \) are continuous at \( \mu_0 \) and satisfy the relation

\[
v_i = \mathbf{B}_e / 4\pi \rho_i = v_{\text{Alfvén}}, \tag{7}
\]

directed along the bisection of the bend (the mean direction of the curvature force).

Internal quantities are continuous across the bends, so \( \rho_2 = \rho_1 = \rho_e \) and \( p_2 = p_1 = p_e \). From a reference frame moving
downward at \(-2v_{A,e} \sin \zeta\), sections 2 and 3 appear to form a classic shock tube, with inflow at
\[
M_2 = \frac{|v_{x,2}|}{c_{s,e}} = \sqrt{\frac{8}{\gamma \beta_e}} \sin^2(\zeta/2),
\]
where \(\gamma = 5/3\) is the ratio of specific heats. Since \(\beta_e \ll 1\), this Alfvénic inflow can have extremely high Mach number when field lines of significantly different orientation reconnect.

This supersonic inflow is brought to rest, \(v_{x,3} = 0\), by a GDS moving outward at \(v_s\). This stopping shock, equivalent to a piston moving into stationary fluid at speed \(|v_{x,2}|\), is a classic problem (see, e.g., Section 69 from Courant & Friedrichs 1948), for which the solution is
\[
v_s/|v_{x,2}| = \sqrt{M_2^{-2} + (\gamma + 1)^2/16 - \frac{1}{4}(3 - \gamma)}.
\]
The density ratio, \(\rho_3/\rho_2 = 1 + |v_{x,2}|/v_s\), following from mass conservation, approaches the well known limit \(\rho_3/\rho_e = (\gamma + 1)/(\gamma - 1) = 4\) at large Mach numbers.

The post-shock pressure, \(\beta_3 = \beta_e [1 + \gamma M_2^2 (1 + v_s/|v_{x,2}|)]\), also follows from the shock relations, and even in the limit of vanishing pre-shock pressure (\(\beta_e \ll 1\)) can be significant: \(\beta_3 \simeq 4(\gamma + 1) \sin^2(\zeta/2)\). The low-\(\beta\) assumption thereby imposes a limit on the reconnection angle; only for \(\zeta < 36^\circ\) is \(\beta_3 < 0.1\) and \(\beta_3 > 1\) when \(\zeta > 67^\circ\).

The post-shock temperature is more conveniently expressed with respect to \(T^A_L = (m_p/k_B) v^2_A e/2\), than to the initial temperature. For example, 15 G field immersed in \(n_e = 3 \times 10^8\ cm^{-3}\) plasma has a characteristic temperature \(T^A_L = 2 \times 10^8\ K\). The plasma beta is \(\beta_e = T_e/T^A_L\), so a 2 MK coronal plasma will have \(\beta_e \approx 10^{-2}\). Figures 3(a)–3(b) show the post-shock temperature, \(T_3 = T^A_L \beta_3 \rho_e/\rho_3\), over a range of \(\rho_{3+}\) parameter space. For \(\beta_e = 0.01\) and \(\beta_3 = 1/3\) (the edge of the light gray area, \(\zeta \simeq 50^\circ\)) the post-shock plasma will be \(T_3 = 0.1T^A_L = 20\ MK\).

The total energy release initiated by reconnection, \(\Delta W = 2v^2_A e \rho_0 \sin^2(\zeta/2)\), is the initial kinetic energy of all mass within the \(L = 2v_A e\) of tube affected by retraction. This is greater than the energy decrease resulting from shortening the tube by \(\Delta L = 2L \sin^2(\zeta/2)\), due to additional work done by the background magnetic field expanding into the vacated volume. In our idealized model this work exactly doubles the energy, but in more realistic scenarios the factor may be somewhat different.

The moving mass is deflected downward by the GDS, converting a portion of its kinetic energy into thermal energy. The kinetic energy (per area) converted to thermal energy, \(\rho_3 v^2_{x,3} v_s\), constitutes a fraction
\[
\frac{\Delta E_t}{\Delta W} = \frac{\rho_3 v^2_{x,3} v_s}{\rho_0 v^2_A e v_{x,2}} = 2 \left(1 + \frac{v_s}{|v_{x,2}|}\right) \sin^4(\zeta/2),
\]
of the total (see Figure 3(c)). Cases of vanishing initial pressure (\(\beta_e \ll 1\)) will have a fraction \((\gamma + 1) \sin^2(\zeta/2)\) of the released energy thermalized. Compression work done on larger initial pressure will raise this fraction slightly, but all cases with \(\beta_3 < 1/3\) thermalize less than 10% of the released energy.

5. SUMMARY

The foregoing illustrates, through a simplified analytic model, a process we believe must be common in the flaring corona.

Figure 3. Post-shock temperature and thermal energy across the \(\zeta–\beta_e\) parameter space. The top panel (a) shows contours of \(T_3\) whose levels can be determined from intersection with the left axis (\(\zeta = 0\)) since \(T_3 = T_e\) there. Solid contours show \(T_3 = 0.01, 0.1, 1\) and \(1\) (all in units of \(T^A_L\)). Light and dark gray regions show \(0.33 < \beta_e < 1\) and \(\beta_3 > 1\) respectively. The middle panel (b) shows \(T_3\) vs. \(\zeta\) for the values \(T_e = \beta_e = 0.01\) (squares) and \(0.1\) (triangles). The bottom dashed curve is the limit for the case \(\beta_e = 0\). The bottom panel (c) shows, for the same values of \(T_e = \beta_e\), the fraction of released energy thermalized by the GDS. Symbols show the same point in each panel.

Localized reconnection does not directly dissipate magnetic energy, but rather initiates its release through subsequent shortening of field lines. This shortening propagates from the reconnection site at the Alfvén speed, converting energy from magnetic to kinetic form. The shortening flux tubes compress plasma within them, raising its temperature as they do so. Due to the supersonic (Alfvénic) flows this compression occurs at strong shocks, which raise the temperature far beyond that from adiabatic compression.

Both the GDS in our model and the SMS in Petschek’s model result, ultimately, from shortening or weakening of magnetic field lines. The two-dimensional Petschek model permits shortening only perpendicular to the symmetry direction, accompanied by weakening. The shock is therefore an SMS whose normal is mostly perpendicular to the sheet. After three-dimensional patchy reconnection, on the other hand, field lines also shorten in the erstdwhile symmetry direction which is the orientation of the shock normal (±\(\zeta\)). The GDSs are disconnected from the diffusion region, and are instead features of the ideal relaxation following reconnection. This new scenario requires two-step energy conversion, from magnetic to kinetic to thermal energy, in contrast to the SMSs which thermalize magnetic energy directly where they weaken the field.

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