Bayesian Maximum Entropy Applied to Fluid Measurement

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Abstract. To alleviate the ill-posedness and non-linearity of electrical capacitance tomography, Bayesian maximum entropy (BME) mapping method is used to mix another source of information. They are some observation points, which are hard data. They can be seen as error free. Three two-phase flow patterns are preset to verify the method. Results show that the BME prediction maps are more accuracy than ECT reconstructed image. BME images have more clear edges and those sizes are more similar to the original maps. Two quality assessments demonstrate that BME maps has a lower relative error(RE) and higher correlation coefficient(CC), which verify the effectiveness of the BME mapping method in fluid measurement.

1. Introduction
Two-phases flow is commonly exist in many industrial processes such as chemical, nuclear and petroleum industries [1]. Process tomography (PT) is widely used for the measurement of two-phase flow characteristics [2]. The PT technique has the advantages of being a non-invasive measurement, having a fast response and being of low cost. However, due to the ill-posedness and non-linearity of the inverse problem, the accuracy of reconstructed images is low [3].

To improve the accuracy of PT images (ECT image is used in this paper), some observation points are mixing by BME. BME theory was presented by Christakos in the 1990s [4]. It initiates the era of modern geostatistics. BME provides a systematic and rigorous approach for integrating various knowledge bases into spatiotemporal analysis. In this paper, one sources of information is the ECT image, which represents soft data. The other one is observation points, which represents hard data. It may be seen as error free.

2. Method

2.1. Bayesian Maximum Entropy
Based on dispersed observations with different levels of uncertainties, statistical spatial data fusion is becoming a popular tool for spatial prediction [5]. Similar to that employed by Douaik et al [6], a Bayesian spatial prediction approach is adopt based on the Bayesian maximum entropy(BME) [4]. As a spatial prediction method, BME can incorporate a wide variety of information with different uncertainties.
Using BME, the target flow pattern map is considered as a random variable. A specific realization of the random variable is $\chi = (\chi_{\text{hard}}, \chi_{\text{soft}}, \chi_k)$, where: $\chi_{\text{hard}}$ is the hard data (accurate); $\chi_{\text{soft}}$ is the soft data (vague); $\chi_k$ is the data will be estimated through BME. $P_{\text{hard}}$, $P_{\text{soft}}$ and $P_k$ are the locations of $\chi_{\text{hard}}$, $\chi_{\text{soft}}$ and $\chi_k$, respectively. To maximum the information about the target random variable over a certain area, three stages are introduced:

1. The first or prior stage. An initial probability distribution will be generated across space on the general knowledge base (G). Physical laws as well as other sources of information may be included as mean trend and the covariance function for the target parameter.

2. The second stage. The site-specific knowledge available is organized into hard and soft data, which is denoted by S.

3. The third of integration (posterior) stage. The site-specific knowledge S is assimilated with G. The complete probability law at each estimation site will be contained in the final prediction.

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The mathematical formulation of the above stages is as follows. Consider a parameter such as electrical capacitance $X(s)$. It is constrained by $G$ as

$$G(s) = \hat{g}_\alpha(x_{\text{map}}), \alpha = 1, 2, ..., N_G$$

$$\hat{g}_\alpha(x_{\text{map}}) = \int g_\alpha(\chi_{\text{map}}) f_G(\chi_{\text{map}}) d\chi_{\text{map}}$$

Where: $x_{\text{map}}$ is a vector which indicates the variables associated with $X(s)$ at a set $s_{\text{map}}$ of mapping sites. $g_\alpha(\chi_{\text{map}})$ are a set of functions of $\chi_{\text{map}}$. Its expectation is known from G [7]. In our case, G consist of the mean

$$mX(s) = E[X(s)]$$

And covariance function

$$C(s, s') = E[(X(s) - mX(s))(X(s') - mX(s'))]$$

Where: $E[.]$ indicates statistical expectation. $f_G(\chi_{\text{map}})$ is the multivariate probability distribution function (pdf) based on G, which represents the prior knowledge. G is used to maximize the expected amount of information, which is represented by Shannon’s entropy $\text{ENT}$:

$$\text{Ent} = -\int f_G(\chi_{\text{map}}) \log[f_G(\chi_{\text{map}})] d\chi_{\text{map}}$$

Equation (5) may be maximized by introducing the Lagrange multiplier $\mu_\alpha$:

$$L[f_G(\chi_{\text{map}})] = -\int f_G(\chi_{\text{map}}) \log[f_G(\chi_{\text{map}})] d\chi_{\text{map}} - \sum \mu_\alpha \left[ \int g_\alpha(\chi_{\text{map}}) f_G(\chi_{\text{map}}) d\chi_{\text{map}} - E[g_\alpha(\chi_{\text{map}})] \right]$$
The values of $\mu_a$ are obtained by solving equation (5) and equation (6). Solving the system of equation (5) leads to

$$f_G(\chi_{map}) = A \exp\left[\sum_{a} \mu_a g_a(\chi_{map})\right]$$

(7)

Where: A is a normalization constant [8]. Knowing $\mu_a$ and hence the G-based prior pdf (equation (6)) means that the first stage has completed.

At the second stage, the site specific knowledge S is organized as hard and soft data.

At the third stage, to obtain the total knowledge $K = G \cup S$, the prior pdf is updated with the site-specific knowledge S. It is represented as the BME pdf given by:

$$f_K(\chi^k) = (BA)^{-1} \int \exp\left[\sum_{a} \mu_a g_a(\chi_{map})\right]d\chi_{soft}$$

(8)

Where: B is a normalization coefficient. The BME pdf is non-Gaussian in general. It contains spatial dependency and accuracy of the data. The soft data used in this paper is in the form of pdfs. Serre provide the following solution for the pdf-type,

$$f_K(\chi_k | \chi_{hard}, f_S(\chi_{soft})) = \frac{\int f_G(\chi_k, \chi_{hard}, \chi_{soft}) f_S(\chi_{soft}) d\chi_{soft}}{\int f_G(\chi_{hard}, \chi_{soft}) f_S(\chi_{soft}) d\chi_{soft}}$$

(9)

2.2. Quality Assessments

In this paper, images are analyzed in two aspects: relative error and correlation coefficient, with:

Relative error (RE),

$$\delta = \frac{\sum_{i=1}^{n} (\tilde{z}_i - z_i)}{\sum_{i=1}^{n} z_i} \times 100\%$$

(10)

And Correlation coefficient (CC),

$$r = \frac{\sum_{i=1}^{n} (\tilde{z}_i - \tilde{z}) (z_i - \bar{z})}{\sqrt{\sum_{i=1}^{n} (\tilde{z}_i - \bar{z})^2 \sum_{i=1}^{n} (z_i - \bar{z})^2}} \times 100\%$$

(11)

Where: $\tilde{z}$ is the predicted image using BDF. z is the original image reconstructed by ECT. A smaller relative error means the image suffers from lower noise effects. The image which is more similar to the preset one has a larger correlation coefficient.

3. Result and Discussion

To verify the effectiveness of BME mapping method, three typical two-phase flow patterns are preset. They are the core flow, the scattered flow and the sedimented flow. They are widely existed in industry process. There are 6400 pixels each map. The hard data is 100 observation points, which distributed evenly in the measurement area. They are error free. The other source of information is the image reconstructed by Landweber method. It is an interval reconstruction algorithm, which can reconstruct a higher accuracy image than Linear Back Projection (LBP). The ECT image is soft data, which expressed in terms of pdf. The BME algorithm is realized in the SEKS-GUI software libraries.
As Fig. 1 shows, the BME estimated map has a more accuracy edges. The image reconstructed by ECT has a smooth edge. As shown in Fig. 2, the shape of object can be recognized after BME prediction. The object in ECT image is rough although the true one is square. In Fig. 3, the two objects stick together. The objects in the BME prediction map are distributed clearly.

The quality assessment result is shown in Table 1. It indicates that the prediction one has a lower RE and a high CC. It shows that the prediction one has a low error and more relative to the original one, which is expected in fluid measurement.

**Figure 1.** Prediction results of Scattered Flow.

**Figure 2.** Prediction results of Core Flow.

**Figure 3.** Prediction results of Sedimented Flow.

**Table 1.** Quality Assessment.

| Flow Pattern       | Core Flow | Scattered Flow | Sedimented Flow |
|--------------------|-----------|----------------|-----------------|
|                    | ECT       | BME            | ECT             | BME            | ECT             | BME             |
| Relative Error (%) | 33.23     | 5.82           | 36.75           | 15.84          | 24.77           | 11.37           |
| Correlation Error  | 82.85     | 96.88          | 80.95           | 92.84          | 83.01           | 92.63           |

**4. Conclusion**

In this paper, to improve the quality of images reconstructed by ECT, the BME mapping method is applied to the fluid measurement. As an important character of flow, flow pattern is predicted with BME. Three typical two-phase flow patterns (i.e. the core flow, the scattered flow and the sedimented flow)
are preset to test the method. As the auxiliary source of information, some observation points are taken. Result shows that the prediction one has a more clearly edge. The size and shape of object in the BME map is more nearly to the true one. The objects which stick together in the ECT image can be distributed successfully after fusion. Two quality assessments also show the effectiveness of the method. The fusion map has a low error and more relative to the original one. The effectiveness of BME in fluid measurement is verified in this paper.

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