Integers in quantum physics can arise either from symmetry, for example, the quantized angular momentum associated with rotational symmetry, or from topological origins, such as the quantized Hall conductivity in the integer quantum Hall effect (IQHE).\(^1\) The key distinction between these two scenarios is their robustness to perturbations; discreteness in the former breaks down in the presence of perturbations which remove the symmetry, whilst in the latter, it remains preserved even under relatively strong perturbations (such as disorder, system geometry, and so forth). Such integer indices, which are protected by topology, are called topological invariants.

From a practical standpoint, it is of interest to measure and make use of such indices. The most notable experimental measurement of a topological invariant is the quantized Hall conductivity of the IQHE in semiconductor quantum dots.\(^2\) From a practical standpoint, it is of interest to measure and make use of such indices. The most notable experimental measurement of a topological invariant is the quantized Hall conductivity of the IQHE in semiconductor quantum dots.\(^2\)

In this study, we examine a new form of nonvolatile magnetic storage, in which the writing process entails a conventional writing field, but whose electrical readout process is topologically protected and is consequently robust against weak disorder and perturbations. The basis for our proposed device is the Hall effect mediated by the \(k\)-space Berry curvature in the presence of spin-orbit coupling (SOC). In a related paper, Qi \textit{et al.}\(^3\) proposed a general two-dimensional model for the quantum anomalous Hall effect for general SOC systems, and found that the charge Hall conductivity is topologically quantized in analogy with the IQHE. Here, we study a practical realization of this system, which should exist on the metallic surfaces of three-dimensional (3D) topological insulators (TIs). In particular, we devise a TI-based magnetic memory cell, in which a bit is stored via the exchange coupling of the TI surface states (SSs) induced by magnetic doping. The magnetism induces a finite \(k\)-space Berry curvature in the SSs, thereby driving the Hall effect.\(^4\) The readout (Hall) voltage of the cell is related directly to the Hall conductivity, which is highly sensitive to the magnetization of the surface (i.e., the stored bit) but which is insensitive to weak disorder, cell imperfections, and cell geometry.

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We propose a memory device based on magnetically doped surfaces of three-dimensional topological insulators. Magnetic information stored on the surface is read out via the quantized Hall effect, which is characterized by a topological invariant. Consequently, the readout process is insensitive to disorder, variations in device geometry, and imperfections in the writing process.

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The Hall conductivity of the SSs can be quantified via the Kubo formula, which reveals its direct relation to the Berry curvature, i.e.,

\[
\sigma_{xy}(k) = \frac{e^2}{h} \int \frac{d^2 k}{(2\pi)^2} \left( f_T - f_0 \right) \Omega_z(k),
\]

where

\[
f_T(k) = \begin{cases} 
1, & \text{if } E_k < E_F + k_B T \\
\exp \left[ \frac{E_k - E_F}{k_B T} \right] + 1, & \text{otherwise} 
\end{cases}
\]

is the Fermi distribution of band \( \tau = \pm \), \( k_B T \) is thermal energy, and the integration is carried out over all the occupied states. The limits of integration in the expression for \( \sigma_{xy} \) are governed by the position of the Fermi level \( E_F \) with respect to the bands. When the Fermi level \( E_F \) lies inside the energy gap [see Fig. 1(c)] so that the VB is completely full and the CB is completely empty (\( f_- = 1, f_+ = 0 \)), the Hall conductivity in the low-temperature limit takes on the half-quantized value,

\[
\sigma_{xy} = \frac{e^2}{h} \int_0^\infty \frac{d^2 k}{(2\pi)^2} \Omega_z(k) = -\frac{e^2}{2h} \text{sgn}(m),
\]

which is a topological invariant. This situation is reminiscent of the perfectly quantized Hall conductivity in the IQHE. Here, \( \sigma_{xy} \) is finite despite an insulating bulk state due to the presence of conducting edge channels, just as in the IQHE.

**Reading and writing to memory cell.** In our memory cell, a bit is stored by the magnetization \( M \) of the FM-doped TI surface, with, say, a “1” (“0”) being stored by an upward (downward) pointing \( M \). Writing to the cell would require a writing field whose field strength exceeds the magnetic coercivity of the surface. Previous works indicated that magnetically doped Bi₂Te₃ should have a coercivity of \( H_C \sim 0.01 T \), which has been measured by experiment (Mn-doped Bi₂Te₃). The soft magnetic anisotropy may be viable in MRAM and magnetic sensor applications, as it reduces the required switching field. The stored bit is read out from the cell via the Hall voltage \( V_H \), which is inversely proportional to \( \sigma_{xy} \),

\[
V_H = \frac{I}{\sigma_{xy}},
\]

where \( I \) is the current flowing through the device (n.b., this equation applies only when the Fermi level lies in the gap and the longitudinal conductivity \( \sigma_{zz} \) vanishes). A current source provides the current \( I \) across the surface, whilst Hall electrodes are attached to the lateral sides to measure \( V_H \) as depicted in Fig. 1(a). In Fig. 2 (main), we plot \( \sigma_{xy} \) as a function of \( m \) for various temperatures. Let us initially focus on the low-temperature case \( T = 0 K \) (blue, solid line), where we assume the Fermi level to lie at \( E_F = 0 \) meV, in the middle of the VB and CB. The stored bit can be read out simply by measuring \( V_H \) and determining its sign. In this case, \( \sigma_{xy} \) is half-quantized at \( e^2/2h \) for \( m \neq 0 \) and is topologically robust to cell imperfections. For a typical driving current of \( I = 1 \mu A \), this corresponds to a readout voltage of \( V_H = \pm 47 mV \).

At finite temperatures \( T > 0 K \), the Fermi distribution of the carriers must be factored into the calculation of \( \sigma_{xy} \). Moreover, the size of the gap is important as the insulating behavior of the bulk can be destroyed by thermal excitations from the VB to CB (we require \( \Delta > k_B T \)). Increasing the gap size whilst maintaining \( E_F \) to lie within the gap may be achievable, for example, using the doping technique outlined in ref. 10. There, it was found that doping Bi₂Se₃ with Fe resulted in an opening of a gap together with an upward shift of the Fermi level into the CB (making it n-type).
Fermi level could then be reshifted back into the gap by introducing nonmagnetic p-type dopants. Refining this two-step procedure of (i) opening the gap, and (ii) shifting the Fermi level into the induced gap could potentially accommodate very large gaps, whilst maintaining the Fermi level to lie inside the gap. Figure 2 (main) shows the effect of increasing $T$ well beyond 0 K. In our calculations, we assumed that the Fermi level lies at $E_F = 0$ meV, i.e., always within the gap. Figure 2 (main) shows that the thermal effect diminishes $\sigma_{xy}$ from its quantized value at $T = 0$ K, but that the accuracy is improved with increasing $|m|$. A large $|m|$ is also beneficial as it helps to preserve bulk insulating behavior as discussed above. For illustration, we indicate the points $m = k_b T$ for each $T > 0$ K (corresponding to $\Delta = 2k_b T$). From a mean field perspective, the exchange splitting $\Delta = 2|m|$ is given by $\Delta = nJ(S)_{\sigma}^{18}$ where $n$ is the doping concentration, $J$ is the exchange coupling, and $\langle S \rangle$ is the expectation of the local spin at saturation. Using typical values of $J = 50$ meV nm$^2$ (ref. 16) and $\langle S \rangle = 1.5 \mu_B$ for Mn-doped Bi$_2$Te$_3$ (ref. 17), a value of $m = 30$ meV corresponds to a doping concentration of $n = 0.8$ nm$^{-2}$, which is of the order of typical values.\textsuperscript{19} In Fig. 2 (inset), we study the effect of $E_F \neq 0$ for $T > 100$ K [for illustration see Fig. 1(c)], which indicates a general broadening effect. The sloped regions coincide with the condition $|m| < E_F$, where the Fermi level lies inside the CB. In our device, it is desirable to ensure that $|m| > E_F$, such that $\sigma_{xy}$ is quantized (apart from scaling by the Fermi distribution). Greater accuracy of $\sigma_{xy}$ is achieved for $|m| \gg E_F$.

We note that in our analysis of $\sigma_{xy}$, we considered only the contribution from the single Dirac cone residing on the top surface of Fig. 1(a). In practice, the remaining five surfaces must also be considered as required by the Fermi doubling, or Nielsen-Ninomiya theorem.\textsuperscript{20-23} If these surfaces are gapless, the Hall conductivity in eq. (3) is no longer quantized. However, quantization may be restored if the surfaces are gapped, in which case $\sigma_{xy}$ for the whole system takes on integer multiples of $e^2/h$. In our memory device setup in Fig. 1(a), only the magnetization $M$ of the top surface is subjected to a change in orientation due to an applied magnetic field, e.g., the remaining surfaces are doped with hard magnetic dopants and/or shielded from the applied field. Hence, the memory state is detected only by the change in the Hall conductivity contribution from the top surface due to orientation of $M$ on that surface.

A topologically invariant readout process is attainable from the point of view of robustness to impurities and geometrical imperfections, such as edge roughness, in analogy with the IQHE. It also alleviates the use of voltage comparators, which traditionally compare the readout signal with a threshold voltage to determine stored bits; such processes are prone to noise, which may lead to errors in bit detection. Moreover, once $|m|$ is sufficiently large, $\sigma_{xy}$ exhibits only a weak dependence on $m$. In practice, the writing process is not perfect; $m$ is not fully switched to the vertical direction and will exhibit spatial fluctuations. Our proposed memory cell ensures a constant readout voltage even in the presence of such imperfections.

Despite the advantages of a topological readout process, several challenges are anticipated, such as opening up a sufficiently large gap for high-temperature operation (achieving large $|m|$). Furthermore, the Curie temperature $T_C$ of magnetic TI surfaces needs to be improved drastically. Presently, experiments reveal that $T_C \lesssim 20$ K for Mn\textsuperscript{17} and Fe\textsuperscript{32-34}–doped Bi$_2$Te$_3$. However, there are hopes of increasing $T_C$ beyond 100 K\textsuperscript{18} in the same spirit as magnetic III–V semiconductors,\textsuperscript{25,26} for the mechanisms of magnetism in the two systems are analogous.\textsuperscript{17}

In summary, we have proposed a memory cell based on magnetically doped TIs. Writing information to the cell entails switching the cell magnetization. The readout process is facilitated by the Hall effect, which is a function of the stored information. The Hall voltage is a topological quantity that is insensitive to details such as edge roughness, the presence of impurities and defects, and imperfect writing.

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1) D. J. Thouless: Topological Quantum Numbers in Nonrelativistic Physics (World Scientific, Singapore, 1998).

2) K. von Klitzing, G. Dorda, and M. Pepper: Phys. Rev. Lett. 45 (1980) 494.

3) X.-L. Qi, Y.-S. Wu, and S.-C. Zhang: Phys. Rev. B 74 (2006) 085308.

4) J. Zang and N. Nagaosa: Phys. Rev. B 81 (2010) 245125.

5) D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan: Nature 452 (2008) 970.

6) Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan: Nat. Phys. 5 (2009) 398.

7) C. L. Kane and E. J. Mele: Phys. Rev. Lett. 95 (2005) 146802.

8) R. R. Biswas and A. V. Balansky: Phys. Rev. B 81 (2010) 234505.

9) L. A. Wray, S.-Y. Xu, Y. Xia, D. Hsieh, A. V. Fedorov, Y. S. Hor, R. J. Cava, A. Bansil, H. Lin, and M. Z. Hasan: Nat. Phys. 7 (2011) 32.

10) Y. L. Chen, J.-H. Chu, J. G. Analytis, Z. K. Liu, K. Igarashi, H.-H. Kuo, X. L. Qi, S. K. Mo, R. G. Moore, D. H. Lu, M. Hashimoto, T. Sasagawa, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z. X. Shen: Science 329 (2010) 659.

11) K. Nomura and N. Nagaosa: Phys. Rev. B 82 (2010) 161401.

12) T. Yokoyama, J. Zang, and N. Nagaosa: Phys. Rev. B 81 (2010) 241410.

13) T. Santos, J. S. Moodera, K. V. Ramamurthy, E. Negusse, J. Holroyd, J. Dvorak, M. Liberati, Y. U. Izderza, and E. Arenholz: Phys. Rev. Lett. 101 (2008) 147201.

14) R. Karplus and J. M. Luttinger: Phys. Rev. 95 (1954) 1154.

15) H.-Z. Lu, W.-Y. Shan, W. Yao, Q. Niu, and S.-Q. Shen: Phys. Rev. B 81 (2010) 115407.

16) I. Garate and M. Franz: Phys. Rev. Lett. 104 (2010) 146802.

17) Y. S. Hor, P. Roushan, H. Beidenkopf, J. Seo, D. Qu, J. G. Checkelsky, L. A. Wray, D. Hsieh, Y. Xia, S.-Y. Xu, D. Qian, M. Z. Hasan, N. P. Ong, A. Yazdani, and R. J. Cava: Phys. Rev. B 81 (2010) 195203.

18) R. Yu, W. Zhang, H.-Z. Zhang, S.-C. Zhang, X. Dai, and Z. Fang: Science 329 (2010) 61.

19) D. A. Abanin and D. A. Pesin: Phys. Rev. Lett. 106 (2011) 136802.

20) H. B. Nielsen and M. Ninomiya: Nucl. Phys. B 185 (1981) 20.

21) H. B. Nielsen and M. Ninomiya: Nucl. Phys. B 193 (1981) 173.

22) M. Z. Hasan and C. L. Kane: Rev. Mod. Phys. 82 (2010) 3045.

23) J. Goryo: J. Phys. Soc. Jpn. 80 (2011) 043704.

24) V. A. Kulkbachinski, A. Yu. Kaminskii, K. Kindo, Y. Narumi, K. Suga, P. Lostak, and P. Svanda: Physica B 311 (2002) 292.

25) H. Ohno, A. Shen, F. Matsukura, A. Oiwa, A. Endo, S. Katsumoto, and Y. Iye: Appl. Phys. Lett. 69 (1996) 363.

26) M. Wang, R. P. Campion, A. W. Rushforth, K. W. Edmonds, C. T. Foxon, and B. L. Gallagher: Appl. Phys. Lett. 93 (2008) 132103.