Novel optical solitons to the perturbed Gerdjikov–Ivanov equation via collective variables

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Received: 7 June 2021 / Accepted: 15 July 2021 / Published online: 7 August 2021
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Abstract
The objective of this manuscript is to study the collective variable (CV) technique to explore an important form of Schrödinger equation known as the Gerdjikov–Ivanov (GI) equation which expresses the dynamics of solitons for optical fibers in terms of pulse parameters. These parameters are temporal position, amplitude, width, chirp, phase, and frequency known as CVs. This is an effective and dynamic mathematical gadget to obtain soliton solutions of non-dimensional as well as perturbed GI equations. Moreover, an established numerical scheme that is the fourth-order Runge–Kutta method is exerted for the numerical simulation of the revealing coupled system of six ordinary differential equations which represent all the CVs included in the pulse ansatz. The CV approach is used to determine the evolution of pulse parameters with the propagation distance and illustrated them graphically. Furthermore, figures show the compelling periodic oscillations of pulse chirp, width, frequency and amplitude of soliton. For various values of super-Gaussian pulse parameters, the numerical behavior of solitons to illustrate variations in CVs is provided. Other significant aspects with regards to the current investigation are also inferred.

Keywords Runge–Kutta method · Super-Gaussian solitons · Gerdjikov–Ivanov equation · Collective variables

1 Introduction
In the field of information technology, the transmission mechanism is a crucial component. The significant adoption of optical fiber is in telecommunications (Hasegawa and Frederick 1973; Doran et al. 1955; Zakharov and Stefan 1998; Osman 2016; Khater et al. 2020). This structure has been employed for high-performance and long-distance data transport due to its enhanced bandwidth capabilities. As a result, it is employed to relay signals from communication devices such as the telephone, television and internet between distant areas. Recent research has shown that using dispersion-managed solitons for data transfer will
significantly increase the ability of fiber-optic links (Doran et al. 1955; Zakharov and Stefan 1998). Because of its extensive applications in nonlinear optics, solitons theory is gaining a lot of interest. To investigate communication phenomena, a variety of analytical and numerical methods have been presented (Osman 2016; Khater et al. 2020; Ali et al. 2020).

Many physical processes such as nonlinear optics, electromagnetism, plasma, shallow water wave propagation, fluid dynamics and many more can be represented using nonlinear partial differential equations (Drazin 1992; Kudryashov 2004; Goufoa et al. 2020; Shehata et al. 2019). There are many efficient and compelling methods that have been made to extract the solution of these equations in recent years (Kudryashov 1988; Abdou 2008; Ali et al. 2020; Kumar 2014; Kumar et al. 2020; Inc 2017; Tchier et al. 2017). The nonlinear Schrödinger equation (NSE) is a well-known soliton problem with numerous applications in mathematics and physics (Raza and Arshed 2020; Raza and Zubair 2019; Raza and Javid 2019; Hosseini et al. 2020; Aslan and Inc 2017; Inc et al. 2017; Inc et al. 2016). Many scientists have proposed and investigated extended versions of NSE with higher-order nonlinearity (Raza et al. 2019a, b; Rezazadeh et al. 2021). In these equations, there are three Chen-Lee-Liu equation, Kaup-Newell equation and GI equation with derivative-type non-linearities, which are known as derivative NSEs (Kadkhoda and Jafari 2017; Yépez-Martínez et al. 2021; Hosseini et al. 2020). Several efficient methods exact as well as numerical address the soliton solutions for GI and Perturbed GI equation in the existing literature (Arshed 2018; Arshed et al. 2018; Zulfiqar and Ahmad 2021; Ding et al. 2019; Liu et al. 2019; Almusawa et al. 2021).

In this paper, we have investigated the optical solitons for both dimensionless as well as generalized perturbed GI equations by applying a CV approach. It is a compelling and latest technique to extract exact solutions of different NSEs Al-Qarni et al. (2020); Veljkovic et al. (2015). This strategy deals with conservative and non-conservative models by showing equations of motion, regardless of dissipative terms and nonlinearities. For mathematical approximation, the traditional fourth-order Runge-Kutta method for integration of the corresponding simultaneous equations is further followed. The assumed solution of the governing equations is divided into two parts using this method, one of them is the soliton part and the other is the residual part. The number of CVs that can be applied is determined by the physical system being considered. A transformation is derived to convert the original field equation in terms of CVs. Furthermore, the parameters such as width, amplitude, chirp, temporal position, phase and frequency have an impact on soliton solutions.

The arrangement of this article is as follows: Sect. 2 provides the governing model, the CV technique is illustrated in Sect. 3. The dynamics of the soliton parameter are introduced in sect. 4 and graphical interpretation is given in Sects. 4.1.1, 4.2.1 and 4.3.1. Section 5 shows its discussion and in Sect. 6 conclusion is given.

### 2 Governing equation

Consider the non-dimensional GI equation as:

\[
iz_t + \sigma_1 z_{xx} + \sigma_2 |z|^4 z + i\sigma_3 z^2 z_x^* = 0,
\]  

(1)

where \(z^*(x, t)\) denotes the complex conjugation of the complex values wave structure \(z(x, t)\) with \(x\) as spatial and \(t\) is for the temporal variables correspondingly. The first term of Eq. (1) denotes the temporal evolution of solitons linearly and the last term of Eq. (1)
representing the nonlinear dispersion whereas $\sigma_1$ is the group velocity dispersion and $\sigma_2$ is the coefficient of the quintic form of nonlinearity.

The famous full nonlinearity structure of the perturbed GI-equation is,
\[ \imath \frac{\partial z}{\partial t} + \sigma_1 z_{xx} + \sigma_2 |z|^2 z_x = i[\sigma_4 z_t + \sigma_5 (|z|^{2q})_x + \sigma_6 (|z|^{2q})_x]. \] (2)

where $\sigma_4$ stands for the inter-modal dispersion, $\sigma_5$ depicts the self steepening for the short pulses, $\sigma_6$ is the higher order dispersion effect and $q$ represents effects of full nonlinearities.

### 3 Mathematical methodology

For the description of proposed technique, the solution of GI is assumed to cleave into two parts composing the residual radiation and soliton part. The concept is that the solitons depend on CVs that may represent the pulse, frequency, amplitude, chirp, temporal position and width, etc. Moreover, the initiation of CVs continues in a rise in the phase space of the dynamical arrangement of the soliton boundaries. The residual part of the solution is approximated to zero. The restrictions result to a nonlinear dynamical system of the collective variables which are investigated mathematically.

Next, the soliton field $z(x, t)$ is pursued into $m(x, t)$ denoting the soliton component of the solution, and $n(x, t)$ representing the residuary part, here $t$ stands for the temporal and $x$ represents the spatial component. Thus,
\[ z(x, t) = m(x, t) + n(x, t). \] (3)

$x$ is considered as $x = x(v_1, v_2, ..., v_p)$, for the CVs. This is very significant that the presence of the variables in $m$ contributes an enlargement in the degree of freedom of the model. In this way we customize Eq. (3) relatively to these parameters as.

By employing the accompanying wave conversion as
\[ z(x, t) = m(v_1(x), v_2(x), ..., v_p(x), t) + n(x, t), \] (4)

the residual free energy is provided as in below equation
\[ E = \int_{-\infty}^{\infty} |n|^2 dt = \int_{-\infty}^{\infty} |z - m(v_1(x), v_2(x), ..., v_p(x), t)|^2 dt. \] (5)

Partially differentiate Eq. (5) with respect to $v_k$, we assume $R_k$ to be as the accompanying
\[ R_k = \frac{\partial E}{\partial v_k} = \frac{\partial}{\partial v_k} \int_{-\infty}^{\infty} |n|^2 dt, \]
\[ = \frac{\partial}{\partial v_k} \int_{-\infty}^{\infty} nn^* dt, \]
\[ = \int_{-\infty}^{\infty} \frac{\partial}{\partial v_k} nn^* dt, \]
\[ = \int_{-\infty}^{\infty} (\frac{\partial n}{\partial v_k} n^* + \frac{\partial n^*}{\partial v_k} n) dt. \] (6)

We define inner product in the following way:
\[ \langle \Gamma_1, \Gamma_2 \rangle = \int_{-\infty}^{\infty} \Gamma_1(t) \Gamma_2(t) dt. \] (7)

Hence, by utilizing above relation, we rewrite Eq. (6) in the following way,

\[ R_k = \langle \frac{\partial n}{\partial v_k}, n^* \rangle + \langle n, \frac{\partial n^*}{\partial v_k} \rangle = \langle \frac{\partial n}{\partial v_k}, n^* \rangle + \langle \frac{\partial n}{\partial v_k}, n^* \rangle, \]

\[ = \langle \frac{\partial n^*}{\partial v_k}, n \rangle + \langle \frac{\partial n^*}{\partial v_k}, n \rangle = 2 \text{Re} \left( \langle \frac{\partial n^*}{\partial v_k}, n \rangle \right), \]

\[ = 2 \text{Re} \left( \langle \frac{\partial (z(x, t) - m(v_1, v_2, ..., v_p, t))^*}{\partial v_k}, n \rangle \right), \]

\[ = 2 \text{Re} \left( \langle \frac{\partial z(x, t)}{\partial v_k} - \frac{\partial m^*(v_1, v_2, ..., v_p, t)}{\partial v_k}, n \rangle \right), \]

\[ = -2 \text{Re} \left( \langle \frac{\partial m^*}{\partial v_k}, n \rangle \right) = -2 \text{Re} \left( \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} dt \right). \] (8)

Here, Re is for the real part and \( \frac{\partial z(x, t)}{\partial v_k} = 0 \). Now, From Eq. (4) we write \( n(x, t) \) as

\[ n(x, t) = z(x, t) - m(v_1, v_2, ..., v_p, t), \] (9)

which gives the following relations.

\[ \frac{\partial n}{\partial v_k} = -\frac{\partial m}{\partial v_k}, \]

\[ \frac{\partial n^*}{\partial v_k} = -\frac{\partial m^*}{\partial v_k}. \]

Using the above relations into Eq. (6) gives

\[ R_k = \frac{\partial E}{\partial v_k} = \int_{-\infty}^{\infty} \left( \frac{\partial m^*}{\partial v_k} n^* - \frac{\partial m}{\partial v_k} n \right) dt, \]

\[ = -\int_{-\infty}^{\infty} \left( \frac{\partial m^*}{\partial v_k} n^* + \frac{\partial m}{\partial v_k} n \right) dt, \] (10)

or rather

\[ R_k = -2 \text{Re} \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} n dt. \] (11)

Again, we also get

\[ \dot{R_k} = \frac{dR_k}{dx} = -2 \text{Re} \int_{-\infty}^{\infty} \frac{d}{dx} \left( \frac{\partial m^*}{\partial v_k} n \right) dt, \]

\[ = -2 \text{Re} \int_{-\infty}^{\infty} \left( \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial v_k} + n \frac{\partial m^*}{\partial v_k} \right) dt, \] (12)

and
\[
\frac{\partial m^*}{\partial x} = \sum_{j=1}^{p} \frac{\partial m^*}{\partial v_j} \frac{\partial v_j}{\partial x}.
\]  

(13)

Then by putting Eq. (13) into Eq. (12), we obtain

\[
\dot{R}_k = -2Re \int_{-\infty}^{\infty} \left( \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} + n \frac{\partial}{\partial v_k} \left( \sum_{j=1}^{p} \frac{\partial m^*}{\partial v_j} \frac{\partial v_j}{\partial x} \right) \right) dt,
\]

(14)

Moreover, by Dirac’s Postulate, it is known that once all the changes in variables cannot be equal to zero, then the function itself is approximately zero.

So, \( R_k \) is certainly minimum if

\[
R_k \approx 0,
\]

(15)

and also

\[
\dot{R}_k \approx 0.
\]

(16)

Therefore, from Eqs. (1) and (2) we obtain

\[
i \sigma_1(m + n)_t + i \sigma_2[(m + n)^4](m + n) - \sigma_3(m + n)^2(m + n)^* = 0,
\]

(17)

\[
i \sigma_1(m + n)_t + i \sigma_2[(m + n)^4](m + n) - \sigma_3(m + n)^2(m + n)^* + [\sigma_4(m + n)_t + \sigma_5((m + n)[(m + n)]^{2l})_t + \sigma_6([(m + n)[(m + n)]^{2l}])_t(m + n))] = 0.
\]

(18)

More, from the Eq. (3) we have

\[
z_x = \frac{\partial m(v_1(x), v_2(x), ..., v_p(x))}{\partial x} + \frac{\partial n(x, t)}{\partial x},
\]

(19)

also by combining Eqs. (17) and (18) with Eq. (19), we get

\[
\frac{\partial n}{\partial x} = -\sum_{k=1}^{p} \frac{\partial m}{\partial v_k} \frac{dv_k}{dx} + \xi_l, \quad l = 1, 2
\]

(20)

with

\[
\xi_1 = i \sigma_1(m + n)_t + i \sigma_2[(m + n)^4](m + n) - \sigma_3(m + n)^2(m + n)^*,
\]

(21)

\[
\xi_2 = i \sigma_1(m + n)_t + i \sigma_2[(m + n)^4](m + n) - \sigma_3(m + n)^2(m + n)^* + [\sigma_4(m + n)_t + \sigma_5((m + n)[(m + n)]^{2l})_t + \sigma_6([(m + n)[(m + n)]^{2l}])_t(m + n))] = 0.
\]

(22)

Similarly, we know that

\[
\dot{R}_k = -2Re \left( \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} dt + \left( \sum_{j=1}^{p} \int_{-\infty}^{\infty} \frac{\partial^2 m^*}{\partial v_k \partial v_j} \frac{dv_j}{dx} n \right) dt \right),
\]

(23)
\[
\dot{R}_k = 2Re \sum_{j=1}^{p} \int_{-\infty}^{\infty} \left( \frac{\partial m^* \partial m}{\partial v_k \partial v_j} - \frac{\partial^2 m^*}{\partial v_k \partial v_j} n \right) dt \frac{dv_j}{dx} 2Re \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \xi dt.
\] (24)

Which in compact form can be written as

\[
\dot{R}_k = \frac{\partial R}{\partial v} \dot{V} + U, \quad \text{with} \quad \dot{V} = \left[ \frac{\partial R}{\partial V} \right]^{-1} [U],
\] (25) (26)

where

\[
\frac{\partial R}{\partial v} = \begin{bmatrix}
\frac{\partial R_1}{\partial v_1} & \frac{\partial R_1}{\partial v_2} & \frac{\partial R_1}{\partial v_3} & \frac{\partial R_1}{\partial v_4} & \frac{\partial R_1}{\partial v_5} & \frac{\partial R_1}{\partial v_6} \\
\frac{\partial R_2}{\partial v_1} & \frac{\partial R_2}{\partial v_2} & \frac{\partial R_2}{\partial v_3} & \frac{\partial R_2}{\partial v_4} & \frac{\partial R_2}{\partial v_5} & \frac{\partial R_2}{\partial v_6} \\
\frac{\partial R_3}{\partial v_1} & \frac{\partial R_3}{\partial v_2} & \frac{\partial R_3}{\partial v_3} & \frac{\partial R_3}{\partial v_4} & \frac{\partial R_3}{\partial v_5} & \frac{\partial R_3}{\partial v_6} \\
\frac{\partial R_4}{\partial v_1} & \frac{\partial R_4}{\partial v_2} & \frac{\partial R_4}{\partial v_3} & \frac{\partial R_4}{\partial v_4} & \frac{\partial R_4}{\partial v_5} & \frac{\partial R_4}{\partial v_6} \\
\frac{\partial R_5}{\partial v_1} & \frac{\partial R_5}{\partial v_2} & \frac{\partial R_5}{\partial v_3} & \frac{\partial R_5}{\partial v_4} & \frac{\partial R_5}{\partial v_5} & \frac{\partial R_5}{\partial v_6} \\
\frac{\partial R_6}{\partial v_1} & \frac{\partial R_6}{\partial v_2} & \frac{\partial R_6}{\partial v_3} & \frac{\partial R_6}{\partial v_4} & \frac{\partial R_6}{\partial v_5} & \frac{\partial R_6}{\partial v_6}
\end{bmatrix}.
\] (27)

while

\[
\dot{V} = \begin{bmatrix}
\dot{v}_1 \\
\dot{v}_2 \\
\dot{v}_3 \\
\dot{v}_4 \\
\dot{v}_5 \\
\dot{v}_6
\end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4 \\
U_5 \\
U_6
\end{bmatrix}.
\] (28)

Additionally, we finally have \( U_k \) as follows
\[ U_k = -2 \text{Re} \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \xi dt, \]
\[ = -2 \text{Re} \int_{-\infty}^{\infty} i \sigma_1 \frac{\partial m^*}{\partial v_k} m_n dt - 2 \text{Re} \int_{-\infty}^{\infty} i \sigma_1 \frac{\partial m^*}{\partial v_k} n_n dt \]
\[ - 2 \text{Re} \int_{-\infty}^{\infty} i \sigma_2 |m + n|^4 \frac{\partial m^*}{\partial v_k} m dt - 2 \text{Re} \int_{-\infty}^{\infty} i \sigma_2 |m + n|^4 \frac{\partial m^*}{\partial v_k} n dt \]
\[ - 2 \text{Re} \int_{-\infty}^{\infty} -\sigma_3 (m + n)^2 \frac{\partial m^*}{\partial v_k} m dt - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_4 \frac{\partial m^*}{\partial v_k} \frac{\partial m}{\partial v_k} dt \]
\[ - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_4 \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial v_k} dt - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_5 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} m dt \]
\[ - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_5 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} n dt - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_6 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} m dt \]
\[ - 2 \text{Re} \int_{-\infty}^{\infty} \sigma_6 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} n dt. \] (29)

For which \( k \in 1, 2, \ldots p. \)

**4 Soliton parameter dynamics**

To begin, we obtain the system of equations of motion for the CVs by using the constraint equations previously acknowledged by defining the function \( m(v_1, v_2, \ldots, v_p, t). \) However, we suppose the soliton part of solution \( m \) incorporates six CVs by implementing Gaussian ansatz as:

\[ m = v_1 \exp \left( \frac{i}{2} v_4 (t - v_2)^2 - \frac{(t - v_2)^2}{v_3^2} + iv_5 (t - v_2) + iv_6 \right), \] (30)

where \( v_i, (i = 1, 2, \ldots 6) \) denoting the centre position, width, amplitude, phase, frequency and chirp. The differential equations for CVs are made by utilizing the basic approximation, alternatively mentioned as the lowest order CV hypothesis. This concept sets the residuary part equals to zero, \( (n(x, t) = 0). \) We find \( \frac{\partial R}{\partial v} \) and \( U \) as follows:
For Eq. (1), \( U_i \) \( i = 1, 2, ... 6 \) are calculated as:

\[
U_1 = 0,
\]

\[
U_2 = \sqrt{2\pi v_1^2} \left[ -\frac{3\sigma_1 v_3^3 v_4^4 v_5}{4} - \sigma_1 v_3 v_5^3 - \frac{3\sigma_1 v_5}{v_3} + \frac{\sigma_2 v_4 v_5 v_5^2}{\sqrt{3}} + \frac{\sigma_3 v_3^5 v_5^3 v_5^2}{8\sqrt{2}} - \sigma_3 v_3 v_5 \right].
\]

\[
U_3 = \sqrt{2\pi v_1^2} \left[ - \sigma_1 v_4 \right],
\]

\[
U_4 = \sqrt{2\pi v_1^2} \left[ \frac{3\sigma_2 v_3^3 v_4^4 v_5}{32} + \frac{\sigma_1 v_3^5 v_5^2}{8} - \sigma_1 v_3 - \frac{3\sigma_2 v_3 v_5^3}{72} - \frac{\sqrt{2}\sigma_3 v_3^2 v_5^3 v_5}{32} \right].
\]

\[
U_5 = \sqrt{2\pi v_1^2} \left[ \frac{\sigma_1 v_3 v_5^4 v_4}{2} - \frac{\sigma_3 v_3^2 v_4^3 v_4}{8\sqrt{2}} \right],
\]

\[
U_6 = \sqrt{2\pi v_1^2} \left[ \frac{\sigma_1 v_3^2 v_5^2}{4} + \sigma_1 v_3 v_5^2 + \frac{\sigma_1}{v_3} - \frac{\sigma_2 v_4 v_3}{\sqrt{3}} - \frac{\sigma_3 v_3^2 v_4^2 v_5}{\sqrt{2}} \right].
\]

Similarly, from Eq. (26) and substituting Eqs. (31) and (32–37), we get

\[
v_1 = -\sigma_1 v_1 v_4,
\]

\[
v_2 = \left[ 2\sigma_1 v_5 + \frac{\sqrt{2}\sigma_3 v_5^2}{4} \right],
\]

\[
v_3 = 2\sigma_1 v_3 v_4,
\]

\[
v_4 = -\frac{1}{9\sqrt{3}} \left[ 18\sigma_1 v_4^2 v_5^2 - 72\sigma_1 + 8\sqrt{3}\sigma_2 v_4^3 v_5^2 + 9\sqrt{2}\sigma_3 v_3^2 v_5^3 v_5 \right],
\]

\[
v_5 = \frac{1}{\sqrt{2}} \left[ \sigma_3 v_3^7 v_4 \right],
\]

\[
v_6 = \frac{1}{72v_3^2} \left[ 72\sigma_1 v_3^3 + 144\sigma_1 + 32\sqrt{3}\sigma_2 v_4^3 v_5^2 + 63\sqrt{2}\sigma_3 v_3^2 v_5^3 v_5 \right].
\]
4.1 Geometrical illustration

With Chirp (See Fig. 1)

Fig. 1 Plots of collective variables $v_i, (i = 1, 2, \ldots, 6)$ with the propagation distance $x$ having initial value $v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0$
Without Chirp (See Fig. 2)

4.2 For \( q=1 \)

Equation (2) becomes

\[
\sigma_1(m+n)x_i + i\sigma_2[(m+n)^{3}(m+n) - \sigma_3(m+n)^{2}(m+n) - [\sigma_4(m+n) + \sigma_5(m+n)|m+n|^{2}] + \sigma_6(|(m+n)|^2, (m+n))],
\]

while

\[
(38)
\]
\[ U_1 = -\sqrt{2\pi} \sigma_1 v_3, \]
\[ U_2 = \sqrt{2\pi} v_1^2 \left[ -\frac{3\sigma_1 v_3^3 v_5^3}{4} - \sigma_1 v_3^3 v_5^3 + \frac{3\sigma_1 v_5}{v_3} + \frac{\sigma_2 v_1^4 v_3 v_5}{\sqrt{3}} + \frac{\sigma_3 v_1^2 (v_3^4 v_4^2 + 8v_3^2 v_5^2 - 4)}{8\sqrt{2}v_3} \right. \]
\[ \left. - \frac{\sigma_4 (v_3^4 v_4^2 + 4v_3^2 v_5^2 + 4)}{4v_3} + \frac{\sigma_5 v_1^2 (v_3^4 v_4^2 + 8v_3^2 v_5^2 + 4)}{8\sqrt{2}v_3} + \frac{(\sigma_3 + \sigma_6) v_1^2}{\sqrt{2}v_3} \right] \]
\[ U_3 = -\sqrt{2\pi} v_1^2 \left[ -\sigma_1 v_4 + \frac{3\sigma_4}{4v_3} \right], \]
\[ U_4 = \sqrt{2\pi} v_1^2 \left[ 3\sigma_1 v_3^4 v_5^2 + \frac{\sigma_1 v_3^2 v_5^3}{8} - \sigma_1 v_3 - \frac{\sqrt{3}\sigma_2 v_3^4 v_5^2}{72} - \frac{\sqrt{2}\sigma_3 v_3^2 v_5^3 v_5}{32} - \frac{3\sigma_4 v_3^5}{64} + \frac{3\sigma_3 v_5 v_3^3 v_5}{16\sqrt{2}} \right], \]
\[ U_5 = \sqrt{2\pi} v_1^2 \left[ \frac{\sigma_1 v_3^4 v_5^2}{2} + \frac{\sigma_1 v_3^2 v_5^3}{8\sqrt{2}} - \frac{\sigma_3 v_3^4 v_5^3}{4} - \frac{\sigma_5 v_3^4 v_5^3}{8\sqrt{2}} \right], \]
\[ U_6 = \sqrt{2\pi} v_1^2 \left[ \frac{\sigma_1 v_3^4 v_5^2}{4} + \frac{\sigma_1 v_3^2 v_5^3}{8\sqrt{2}} + \frac{\sigma_1}{v_3} - \frac{\sigma_2 v_3^4 v_5^3}{\sqrt{3}} - \frac{\sigma_3 v_3^4 v_5^3}{\sqrt{2}} - \frac{\sigma_4 v_3^5}{\sqrt{2}} - \frac{\sigma_5 v_3^4 v_5^3}{\sqrt{2}} \right]. \]

Similarly, from Eq. (26) and substituting Eqs. (31) and (39–44), we get

\[ \nu_1 = -\frac{1}{4v_3} \left[ 4\sigma_1 v_1 v_3 v_4 + 3\sigma_4 v_1 - 6\sigma_4 v_3 \right], \]
\[ \nu_2 = 2\sigma_1 v_3 + \sqrt{2}\sigma_2 v_3^3 v_4 + \frac{\sigma_4 v_3^3 v_4^2}{4} + \sigma_4 v_3 v_5 + \frac{\sigma_4 v_3^4}{4} + \sigma_4 - \frac{3\sigma_3 v_5}{4} - \sqrt{2}\sigma_6 v_4^2, \]
\[ \nu_3 = \frac{1}{2v_1} \left[ 4\sigma_1 v_1 v_3 v_4 + 3\sigma_4 v_1 - 2\sigma_4 v_3 \right], \]
\[ \nu_4 = -\frac{1}{18v_3^3} \left[ 144\sigma_1 - 36\sigma_1 v_3^2 v_5^2 - 16\sqrt{3}\sigma_2 v_3^3 v_4 - 18\sqrt{2}\sigma_3 v_3^2 v_5 v_5 + 27\sigma_4 v_3^4 - 72\sigma_4 v_3^5 - 18\sqrt{2}\sigma_5 v_3^2 v_5^3 v_5 \right], \]
\[ \nu_5 = \frac{\sqrt{2}\sigma_3 v_3^2 v_4}{2} + \sigma_4 + \frac{\sigma_4 v_3^4 v_5^3}{4} + \sigma_4 v_4 + \sigma_3 v_3^2 v_4 v_5^2 + \sigma_3 v_3^3 v_4 v_5 + \frac{\sqrt{2}\sigma_5 v_5^2 v_4}{2} - \frac{\sqrt{2}\sigma_6 v_4^2}{2}, \]
\[ \nu_6 = \frac{1}{144v_3^3} \left[ 144\sigma_1 v_3^3 v_5^2 - 288\sigma_1 + 64\sqrt{3}\sigma_2 v_3^3 v_4^3 + 126\sqrt{2}\sigma_3 v_3^2 v_5^3 v_5 + 36\sigma_4 v_3^4 v_5^2 v_5 + 36\sigma_4 v_3^4 v_4 v_5 \right. \]
\[ + 144\sigma_4 v_3^4 v_5^2 + 144\sigma_4 v_3^4 v_5^3 - 27\sigma_4 v_4^3 + 144\sigma_4 v_3^2 v_5 v_5 + 216\sigma_4 v_3^3 - 18\sqrt{2}\sigma_5 v_3^2 v_5^3 v_5 - 72\sqrt{2}\sigma_6 v_3^2 v_5^3 v_5 \right]. \]
4.2.1 Geometrical illustration

With Chirp (See Fig. 3)

Fig. 3 Plots of collective variables \(v_i\), \((i = 1, 2, \ldots, 6)\) with propagation distance \(x\) having initial value \(v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0\)
Without Chirp (See Fig. 4)

Fig. 4 Plots of collective variables $v_i, (i = 1, 2, \ldots, 6)$ with propagation distance $x$ having initial value $v_1 = v_2 = v_3 = 1, v_4 = v_5 = v_6 = 0$

### 4.3 For $q = 2$

Equation (2) becomes

$$\begin{align*}
&i\sigma_1(m+n)m + i\sigma_2[(m+n)^4](m+n) - \sigma_3(m+n)^2(m+n)^2 \left[ \sigma_4(m+n)^3 + \sigma_5((m+n)(m+n))^2 \right] \\
&+ \sigma_6(\{(m+n)^4\}(m+n)))
\end{align*}$$

(45)
while

\[ U_1 = -\sqrt{2\pi} \sigma_4 v_3, \]

\[ U_2 = \sqrt{2\pi} v_1 \left[ -\frac{3\sigma_1 v_3^2 v_5^2}{4} - \sigma_1 v_3 v_5^3 + \frac{3\sigma_1 v_5^3}{v_3} + \frac{\sigma_2 v_1 v_3 v_5}{\sqrt{3}} + \frac{\sigma_3 v_1^2 (v_3^4 v_2^2 + 8 v_3^2 v_5^2 - 4)}{8\sqrt{2} v_3} - \frac{\sigma_4 (v_3^4 v_4^2 + 4 v_3^2 v_5^2 + 4)}{4 v_3^3} + \frac{\sqrt{3} \sigma_5 v_1^4 (v_3^4 v_2^2 + 12 v_3^2 v_5^2 + 4)}{36 v_3} + \frac{4\sqrt{3} (\sigma_5 + \sigma_6) v_1^4}{9 v_3} \right], \]

\[ U_3 = \sqrt{2\pi} v_1^2 \left[ -\sigma_1 v_3 - \frac{3\sigma_4}{4v_3} \right], \]

\[ U_4 = \sqrt{2\pi} v_1^2 \left[ \frac{3\sigma_1 v_3^2 v_4^2}{32} + \sigma_1 v_3^2 v_5^2 - \sigma_1 v_3^3 + \frac{\sqrt{3} \sigma_2 v_1^2 v_3^3}{72} - \frac{\sqrt{2} \sigma_3 v_3^4 v_1^3}{32} - \frac{3\sigma_4^2}{64} + \frac{3\sigma_5 v_1^4 v_3^2}{72} \right], \]

\[ U_5 = \sqrt{2\pi} v_1^2 \left[ \frac{\sigma_1 v_3^2 v_4 v_5^2}{2} - \frac{\sigma_1 v_3^2 v_3^2 v_4}{8} - \frac{\sigma_2 v_1^2 v_3 v_5}{4} - \frac{\sigma_4 v_1^3}{4} - \frac{\sqrt{3} \sigma_5 v_1^4 v_3^3}{36} \right], \]

\[ U_6 = \sqrt{2\pi} v_1^2 \left[ \frac{\sigma_1 v_3^2 v_2^2}{4} + \sigma_1 v_3^2 v_5^2 + \frac{\sigma_2 v_1^2 v_3^3}{\sqrt{3}} + \frac{\sigma_3 v_3^4 v_1^3}{\sqrt{2}} - \frac{\sigma_4 v_3}{4} \right]. \]

Similarly, from Eq. (26) and substituting Eqs. (31) and (46–51), we get

\[ v_1 = -\frac{1}{4v_3} \left[ 4\sigma_1 v_1 v_2 v_3 + 3\sigma_4 v_4 - 6\sigma_4 v_1 \right], \]

\[ v_2 = \frac{2\sigma_1 v_3}{4} + \frac{\sqrt{2} \sigma_1 v_4}{2} - \frac{\sigma_4 v_1^2}{4} - \frac{\sigma_4 v_1^2}{4} + \sigma_4 v_1^2 + \sigma_4 v_1^2 - \frac{5\sqrt{3} \sigma_4 v_1^2}{9} + \frac{4\sqrt{3} \sigma_4 v_1^2}{9} \],

\[ v_3 = \frac{2\sigma_1 v_3}{4} + \frac{\sqrt{2} \sigma_1 v_4}{2} + \frac{\sigma_4 v_1^2}{4} + \frac{\sigma_4 v_1^2}{4} + \sigma_4 v_1^2 + \sigma_4 v_1^2 + \sigma_4 v_1^2 - \frac{5\sqrt{3} \sigma_4 v_1^2}{9} + \frac{4\sqrt{3} \sigma_4 v_1^2}{9} \],

\[ v_4 = \frac{18 \sigma_4}{144 \sigma_3^2 v_3^2} - 288 \sigma_4 + 64 \sqrt{3} \sigma_2 v_1^2 v_3^2 - 16 \sqrt{3} \sigma_2 v_1^2 v_5^2 + 12 \sqrt{2} \sigma_1 v_1^2 v_3^2 + 27 \sigma_2 v_1^2 - 72 \sigma_4 v_5^2 - 16 \sqrt{3} \sigma_3 v_1^2 v_5^2 \],

\[ v_5 = \frac{144 \sigma_1 v_3^2 v_4^2}{2} + \sigma_4 + \frac{\sigma_4 v_1^2}{4} + \frac{\sigma_4 v_1^2}{4} + \sigma_4 v_1^2 + \sigma_4 v_1^2 + \sigma_4 v_1^2 - \frac{5\sqrt{3} \sigma_4 v_1^2}{9} + \frac{4\sqrt{3} \sigma_4 v_1^2}{9} \],

\[ v_6 = \frac{144 \sigma_1 v_3^2 v_4^2}{2} - 288 \sigma_4 + 64 \sqrt{3} \sigma_2 v_1^2 v_3^2 + 12 \sqrt{2} \sigma_1 v_1^2 v_3^2 + 27 \sigma_2 v_1^2 + 27 \sigma_1 v_1^2 - 16 \sqrt{3} \sigma_3 v_1^2 v_5^2 - 64 \sqrt{3} \sigma_3 v_1^2 v_5^2 \].
4.3.1 Geometrical illustration

With Chirp (See Fig. 5)

Fig. 5 Plots of collective variables $v_i$ ($i = 1, 2, \ldots, 6$) with propagation distance $x$ having initial value $v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0$
Without Chirp (See Fig. 6)

**Fig. 6** Plots of collective variables $v_i, (i = 1, 2, \ldots, 6)$ with propagation distance $x$ having initial value $v_1 = v_2 = v_3 = v_4 = v_5 = v_6 = 0$

### 5 Results and discussion

In this part of the manuscript, we addressed numerical calculation of the dynamics of pulse propagation variables, which are frequently shown geometrically to interpret the accomplished achievement of CV method. In addition, the forth order Runge-Kutta strategy is employed to solve the ODEs that result from our method. For the following parameter
values: \( \sigma_1 = \frac{1}{100}, \sigma_2 = \frac{1}{2} \) and \( \sigma_3 = 1 \) for Eq. (1) the dynamics of the attained system is displayed in Figs. 1 and 2. Then for the Eq. (2) dynamics of the equation is described for these parametric values: \( \sigma_1 = \frac{1}{100}, \sigma_2 = \frac{1}{16}, \sigma_3 = 1, \sigma_4 = 0, \sigma_5 = -\frac{1}{2} \) and \( \sigma_6 = \frac{1}{50} \) in Figs. 3, 4, 5, and 6. We have presented a graphical illustration of the suggested model initially in the presence of chirping and without chirping for \( q = 1, 2 \). Whereas chirp is defined as a signal in which the frequency increases or decreases with respect to time.

It is worth mentioning that the pulse inseminates, \( v_1, v_3, v_4 \) and \( v_5 \) which is the soliton’s amplitude, width, chirp, and frequency of the pulse fluctuates periodically with uniform wavelengths but temporal position \( v_2 \) is increasing with propagation distance and phase \( v_6 \) is periodic with small amplitude and behaving decreasingly in Figs. 1 and 2. However, Figs. 3, 4, 5 and 6 illustrate that \( v_1, v_3, v_4 \) and \( v_5 \) are the amplitude of pulse, width, pulse chirp and frequency of the wave oscillate periodically with uniform wavelengths but temporal position \( v_2 \) is increasing and the phase of pulse \( v_6 \) is also periodic having a small amplitude that is almost linear and showing increasing behavior in Figs. 5 and 6. Consequently, it is recognized that the energy depends upon the amplitude of the wave \( v_1 \), pulse width \( v_3 \), chirp \( v_4 \) and frequency of the pulse \( v_5 \). The assumed pulse parameters are influential factors of this study, therefore the dynamics of these parameters possess a significant role in the pulse propagation. Therefore, the CV approach allows for a straightforward study of the obtained equations and the determination of the impact of various parametric values in the GI equation.

6 Conclusion

This piece of research introduced the CV approach for the proposed model and the soliton solutions of this equation are successfully explored in this work. For pulse propagation in optical fibers, the CV technique regulated a system of ordinary differential equations with higher-order nonlinearities. This analysis exposed solitons in the terms of computational effects resulting in a large and distinct class of pulse behavior that arises for a variety of parameter values. The theme of the CV approach is based on the implementation of constraint, \( R_k \approx 0 \), on the CVs \( v_k(k = 1, \ldots, p) \), which gives them sufficient physical importance by setting the Gaussian ansatz function. The major advantage of this technique is that it allows for the derivation of the equations of motion for CVs comparatively. The graphs of acquired solutions provide us admirable thoughts about the dynamics of the GI equation. These consequences may be useful in optical fibers and have a prominent impact on optical communications. To the best of our knowledge, the obtained results are novel and have not been reported before in the literature. This scheme gradually becomes a consistent procedure that can be applied for the solution of numerous NSEs. These outcomes may be proven to be helpful to improve the reliability of optical fibers in telecommunication that are utilized for long-distance and high-performance networking. Mathematicians, engineers, and physicists may find the investigated soliton solutions useful in recognizing the physical significance of this model.

Acknowledgements José Francisco Gómez Aguilar acknowledges the support provided by CONACyT: cátedras CONACyT para jóvenes investigadores 2014 and SNI-CONACyT.

Author contributions ZH: Conceptualization, methodology, writing original draft preparation; NR: methodology, data curation, writing—original draft preparation; JFG: conceptualization, methodology, writing—original draft preparation, software, supervision.
Declarations

Conflict of interest The authors declare no conflict of interest.

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