Naturalness upper bounds on gauge mediated soft terms

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Abstract
After a general discussion about the quantitative meaning of the naturalness upper bounds on the masses of supersymmetric particles, we compute these bounds in models with gauge-mediated soft terms. We find interesting upper limits on the right-handed slepton masses that, unless the messenger fields are very light, disfavor minimal models with large messenger content. Deep unphysical minima, that however turn out to be not dangerous, are usually present in such models. The \(\mu\)-problem can be solved by adding a light singlet only at the price of a large amount of fine tuning that gives also rise to heavy sparticles and large \(\tan \beta\).

1 Introduction
In supersymmetric theories the \(Z\)-boson mass is determined as a function of the soft terms in the Higgs potential. For this reason it is unnatural that these soft terms be much larger than \(M_Z\). In specific models that try to understand the origin of the soft terms, specific relations hold between the sfermion masses and the Higgs soft terms, allowing to place specific naturalness bounds on sparticle masses. The case of supergravity mediated soft terms \([1]\) has been extensively studied under the assumptions of universal boundary conditions \([2, 3, 4]\) or with relations suggested by unification theories \([5, 6]\). In this paper we will study the case of gauge-mediated soft terms \([7]\). If soft terms are mediated by ordinary gauge interactions, the combination of Higgs soft mass terms relevant for the determination of \(M_Z\) is typically much larger than the small soft mass terms of the right-handed sleptons, \(\tilde{e}_R\), that must therefore be particularly light for naturalness reasons. Even though some crucial ingredients of the Higgs potential, the \(\mu\) and \(B \cdot \mu\) terms, cannot be mediated by gauge interactions \([8]\), we discuss how, under reasonable assumptions, it is nevertheless possible to compute and quantify the naturalness bounds. Our result is that in minimal models, unless cancellations between the known and the unknown missing contributions of more than one order of magnitude are present, the right-handed sleptons are lighter than 100 GeV. Although possible, such large cancellations have a small probability of about 10%.

In section 2 we reconsider the general problem of quantifying the naturalness bounds in supersymmetric theories. In section 3 we compute the naturalness bounds in a wide class of minimal gauge mediation scenarios with the MSSM as the low energy theory. We also present in appendix A the exact solutions to the one-loop renormalization group equations (RGEs). In section 4 we focus on the models (‘NMSSM’) where the \(\mu\) problem is solved by adding a light singlet to the observable MSSM fields. We find that, with gauge-mediated soft terms, the actual minimum of the potential is unphysical unless some parameters are unnaturally fine-tuned in such a way that also results in heavy sparticles and large \(\tan \beta\).
2 Naturalness bounds

In supersymmetric models the $Z$ boson mass is obtained, by minimizing the potential, in function of the soft terms that enter in the Higgs potential. For example, in the MSSM at tree level, one has

$$M_Z^2 = -2|\mu|^2 + 2\frac{m_{h_u}^2 - m_{h_d}^2 \tan^2 \beta}{\tan^2 \beta - 1}$$  \hspace{1cm} (1)

where $\mu$ is the ‘$\mu$-term’ and $m_{h_u}^2$ ($m_{h_d}^2$) is the soft mass of the Higgs fields $h_u$ ($h_d$) that gives mass to up quarks (down quarks and leptons). Since there are contributions of both signs it is possible, although unnatural, that the sparticle be much heavier than the $Z$ boson due to accidental cancellations between different terms in eq. (1). Naturalness considerations imply upper bounds on the sparticle masses, but it is difficult to give them an unambiguous quantitative meaning.

Naturalness bounds had originally been quantified \cite{2} by constraining the sensitivity of $M_Z^2$ with respect to variations of a set of parameters $\{\varphi\}$ (soft terms, gauge and Yukawa couplings) chosen to be the ‘fundamental’ ones. More in detail, it was required that the fractional variation of $M_Z$ (or $M_Z^2$) with respect to fractional variations of the parameters $\varphi$

$$\Delta[p] \equiv \frac{\varphi \Delta M_Z^2}{M_Z^2} \quad \Delta \equiv \max_p \Delta[\varphi]$$  \hspace{1cm} (2)

be smaller than some maximum allowed value, $\Delta < \Delta_{\text{lim}}$, originally chosen as 10. Following the lines of ref. \cite{3,4}, where this definition has been criticized, let us now show some examples where it turns out to be too restrictive or inadequate.

i. Let us first try to apply it to the case in which the supersymmetry breaking scale, and consequently the $Z$-boson mass, are dynamically determined, for example through gaugino condensation in a ‘hidden’ sector. In such a case

$$M_Z \approx M_{\text{Pl}} \cdot e^{-c/g_H^2},$$

where $g_H$ is the hidden sector gauge coupling constant renormalized at $M_{\text{Pl}}$ and $c$ is a numerical constant, dependent on the one-loop coefficient of the hidden gauge group $\beta$-function and on the mediation scheme. Even though $\Delta[g_H] \sim \ln M_{\text{Pl}}/M_Z$ is quite large, there is nothing unnatural.

ii. As a second more practical example, let us consider the sensitivity of the $Z$-boson mass with respect to the top quark Yukawa coupling, $\lambda_t$. Since the pole top mass and the soft terms of the fields $h_u$, $Q$ and $\tilde{t}_R$ involved in $\lambda_t$ all run towards an infrared fixed point, they are very weak functions of $\lambda_t(M_{\text{GUT}})$. The fine-tuning can thus be significant if we choose $M_t$ as a free parameter, even when the soft terms are of $\mathcal{O}(M_Z^2)$, but becomes very mild if we choose $\lambda_t$ renormalized at a high scale. More generally, definition (2) depends on the parametrization of the parameter space in a possibly significant way.

In ref. \cite{4} these criticisms have been taken into account by substituting the sensitivity $\Delta$ of eq. (2) with a differently normalized one, $\Delta/\langle \Delta \rangle$ obtained dividing $\Delta$ by a suitably defined mean sensitivity $\langle \Delta \rangle$. Somewhat less restrictive bounds have been obtained in this way. Since the numerical difference is not totally irrelevant, we will discuss such issues, choosing a simpler criterion based on solid arguments that, in some relevant case, turns out to be numerically equivalent to the original one \cite{4}. We will now show that there is an unnaturally small probability $p \approx \Delta^{-1}$ that the single ‘contributions’ to $M_Z^2$ in eq. (1) be $\Delta$ times bigger than their sum, $M_Z^2$.

Of course the difference between the ‘interesting’ case of eq. (1) and the one of example i. is the possibility of cancellations between different contributions. As a consequence, while in example i. small variations of the parameters can only render $M_Z^2$ orders of magnitude smaller or larger, in the case of interest small variations of the parameters can make $M_Z^2$ negative (or more exactly, erase the SM-like minimum when the particular point $M_Z^2 = 0$ is reached), leading to an undesired qualitative change in the physics. This simple fact is at the basis of the difference between the two cases: it is only in the actually interesting case, eq. (1), that imposing an upper limit on $M_Z$ forces the parameter space to
shrink to a small region. In example i. instead, an upper limit on the $Z$-boson mass, $M_Z < M_Z^{\text{lim}}$, is satisfied in a large part of the parameter space $g_H < g_H^{\text{lim}}$. Only in the first case a too light $Z$ boson is unnatural. This difference between the two cases cannot be seen if one only looks at the derivatives of $M_Z^2$, that determine the fine-tuning as in eq. (3). In fact, even though in both cases $M_Z^2$ has a strong dependence on $\varphi$, it is only in the ‘actual’ case, where it is possible for $M_Z^2$ to vanish, that a first order Taylor expansion of $M_Z^2(\varphi + \delta \varphi)$ gives a correct approximation of the real behavior. In this case, a $Z$ boson much lighter than the soft terms, as can be obtained for some value $\varphi$ of the parameters, is characteristic of only a small range $\varphi - \delta \varphi \ldots \varphi$, where $\delta \varphi \approx \varphi/\Delta[\varphi]$. We now have to quantify the probability that the parameters $\varphi$ lie in a given small range.

In the case that the value of some particular parameter $\varphi$ (for example $\lambda_t$, or $g_3$, or $g_{\text{GUT}}$) is known with some error $\sigma_\varphi$, the probability that $\varphi$ lies in some small range $\delta \varphi$, compatible with the measured value, is $p \sim \delta \varphi/\sigma_\varphi = \Delta[\varphi]^{-1} \cdot (\varphi/\sigma_\varphi)$. Since $M_t$ and $g_3$ are today known with a ~5% error the corresponding naturalness bounds are not the most interesting ones. Even though, for this reason, we have not presented the obvious precise expression of $p$ in terms of the probability distribution of the measured parameters, it is useful to remark that such definition evades also the criticism discussed in ii. In simpler terms, this happens because the variation of $M_Z^2$ is the same when $M_t$ takes its extremely allowed values, or when $\lambda_t(M_{\text{GUT}})$ takes its corresponding ones.

Let us now consider the opposite case of parameters that have not been measured, such as the soft terms. Since we have no reason for asserting that the particular values $\varphi - \delta \varphi \ldots \varphi$ necessary to get the desired cancellation are more probable than, say, any value less than $\varphi$, we must use $\sigma_\varphi \sim \varphi$. The probability $p$ of getting an accidental cancellation more precise than $\Delta[\varphi]^{-1}$ is thus $p \sim \delta \varphi/\sigma_\varphi = \Delta[\varphi]^{-1}$. In other words the choice of a limiting allowed value of the sensitivity $\Delta[\varphi]$ is nothing more than a choice of a ‘confidence limit’ on unprobable cancellations. To conclude, we choose to normalize the probability so that it is 1 in situations in which we see nothing of unnatural, namely when no accidental cancellation is present.

We have thus justified the criterion that we employ. For definiteness we will now briefly exemplify it, computing the corresponding bounds in the case of the ‘MSSM with universal soft terms at the unification scale’. In this case we obtain, using the RGE-improved tree-level potential, and for $\lambda_t$ values near to its infrared fixed point

$$M_Z^2 = -2|m|^2 + \frac{2 + \mathcal{O}(1) \tan^2 \beta}{\tan^2 \beta - 1} m_0^2 + \frac{\mathcal{O}(1.5) + \mathcal{O}(10) \tan^2 \beta}{\tan^2 \beta - 1} M_Z^2. \tag{3}$$

The dependence on $A_0$ is negligible. Imposing $\Delta < \Delta_{\text{lim}}$ we get\(^1\) for moderate values of $\tan \beta \sim 2$,

$$|\mu| \lesssim M_Z \sqrt{\frac{\Delta_{\text{lim}}}{2}}, \quad m_0^2 \lesssim M_Z^2 \frac{\Delta_{\text{lim}}}{2}, \quad M_Z \lesssim M_Z \sqrt{\frac{\Delta_{\text{lim}}}{10}}. \tag{4}$$

The corresponding bounds on the sparticle masses can be easily derived from their expression in terms of $\mu$, $m_0$ and $M_Z$\(^2\), and from eq. (3), that connects the various soft parameters.

We now want to apply these considerations to the more predictive case of soft terms mediated by MSSM gauge interaction.

### 3 Gauge-mediated soft terms

In a large class of minimal models the soft terms are predicted, at the messenger mass $M_U$, in terms of only one scale, here called $M_0$:

$$M_i(M_U) = \frac{\alpha_i(M_U)}{\alpha_{\text{GUT}}} M_0, \quad m_R^2(M_U) = \eta \cdot c_R M_i^2(M_U), \quad A_i^a(M_U) = 0, \tag{5}$$

\(^1\)A more complicate and realistic example can be found in ref. [6].

\(^2\)We have considered the variations of $M_Z^2$ at fixed $\tan \beta$; other authors prefer to keep fixed the ‘$B$-parameter’ of the Higgs potential, obtaining an higher value of $\Delta$. The difference is however not much significative, provided that $M_Z^2$ is extracted from the minimization of the full one-loop potential, as we do in this article.
where $M_i$ are the three gaugino mass parameters, $m^2_R$ are the soft mass terms for the fields $R = \tilde{Q}, \tilde{u}_R, \tilde{d}_R, \tilde{e}_R, \tilde{L}, h^u, h^d$, $g$ is a generation index and $a = u, d, e$. The various quadratic Casimir coefficients $c_R^g$ are listed in Table 1. Finally, $\eta$ parametrizes the different minimal models. For example $\eta = (n_5 + 3n_{10})^{-1/2}$ in models with only one supersymmetry breaking singlet, with $n_5$ copies of messenger fields in the $5 \oplus \bar{5}$ representation of SU(5), and $n_{10}$ copies in the $10 \oplus \overline{10}$ representation \cite{8,10}. Values of $\eta$ bigger than one are possible if more than one supersymmetry-breaking singlet is present, since an $R$-symmetry can suppress the gaugino masses with respect to the scalar masses.

Before being able to compute the naturalness bounds we must face the problem that gauge interactions alone cannot mediate the (Peccei-Quinn breaking) ‘$\mu$-term’, as well as the corresponding ‘$B \cdot \mu$-term’, so that we control only a part of the contributions to $M^2_Z$ in eq. (3). Even worse, it is also possible that the unknown physics required to solve the problem gives rise to non minimal contributions to the other soft terms, more probably only to the ones in the Higgs sector \cite{8}.

If, in the worst case, also the sfermions received unknown non-minimal contributions, not only would it be impossible to convert the upper bounds on the Higgs soft terms into upper bounds on the sfermion masses but also all the predictive power of gauge-mediation would be lost. For this reason we do not study this possibility.

If instead unknown physics mediates new contributions only to the Higgs soft terms, the sfermion masses would be unaffected apart from a RGE correction due to the hypercharge couplings. This correction, that vanishes in the non-minimal models of ref. \cite{8}, is non negligible only if the masses of the Higgs fields $h_u$ and $h_d$ receive largely different extra contributions. Since such new contributions would enter additively in the expression for $M^2_Z$ (at least until the relevant potential is the RGE-improved MSSM one), they would not affect the naturalness bounds on $M_0$. This bound is, in fact, not evaded if unknown non minimal contributions to $M^2_Z$ partially cancel the minimal ones: we are exactly going to compute how unnatural is this possibility. We could, of course, parametrize the unknown contributions to the Higgs masses but the corresponding naturalness bounds would not give rise to interesting upper bounds on the sfermion masses.

For these reasons we can limit ourselves to compute the bounds on the minimal gauge-mediated
contributions, parametrized by $M_0$. We can now begin our analysis observing that, since the right-handed sleptons, $\tilde{e}_R$, are the lightest sfermions\footnote{For moderate $\tan \beta$ the right-handed sleptons are degenerate. We will discuss in the following the case of large $\tan \beta$, where a stau state becomes even lighter.} the bounds on their masses are more interesting than the bounds on the other heavier sfermions. Secondly, this bound is strong, because $M_2^2$, as given in (4), receives contributions proportional to the large squark masses via the soft parameter $m_{h_u}^2$ of the Higgs field $h_u$ coupled to the top quark.

We can elucidate these points comparing this situation with what happens in the other well motivated case of supergravity mediated soft-terms. Even making the strong and unmotivated assumption of universal soft terms at the unification scale, $m_{h_u}^2(M_{\text{GUT}}) = m_0^2$, the naturalness bounds are not very interesting. In fact, in the limit where the sparticle spectrum is dominated by the contributions from $m_0^2$, all the sparticle masses, and in particular the Higgs mass parameters that enter the MSSM minimization conditions, are of the same order, $m_{h_u}^2(M_Z) = \mathcal{O}(m_0^2)$ so that the naturalness bounds, computed in (4), are not very interesting:

$$ m_{\tilde{e}} \approx m_0 \lesssim 600 \text{ GeV} \sqrt{\frac{\Delta_{\text{lim}}}{100}} \quad (6) $$

On the contrary in minimal gauge-mediation models where soft mass terms are generated by gauge interactions, there is a hierarchy between the lighter sleptons, that interact only weakly, and the heavier squarks and higgses, with strong or $\lambda_t$ interactions (see eq. (5)). This hierarchy enhances the naturalness bounds on the slepton masses. This difference between the two cases of supergravity and gauge-mediated soft terms is exemplified in figure 1, where we plot the naturalness upper bounds on the right-handed $\tilde{e}_R$ mass and on the gaugino mass parameter $M_1$. Note that in the unnatural regions we are exploring $\mu$ is so large that $M_1$ coincides, approximately, with the lightest neutralino mass. We have chosen a small value of $M_U = 10^5$ GeV $\ll M_{\text{GUT}}$: for this reason the bound on the gaugino masses are weaker than in the supergravity case, where the running starts from $M_{\text{GUT}}$. Since the dependence on $\tan \beta$ is similar in the two cases, we have fixed $\tan \beta = 2.$

Figure 2: The naturalness upper bound $\Delta < 100$ (thick lines) and $\Delta < 10$ (thin lines) on the right-handed selectron mass compared with its present experimental bound (horizontal dot-dashed line) and plotted as function of $\tan \beta$ for $M_U = 10^6$ GeV in figure 2a, and as function of $\eta$ for $\tan \beta = 2$ in figure 2b.
Having shown the interest of the bounds on the $\tilde{e}_R$ masses, we can now pass to a more detailed analysis. The $Z$ boson mass can be easily determined as function of the supersymmetric parameters by minimizing the RGE-improved tree-level potential, that takes into account the dominant quantum effects, generated at all the scales from $M_U$ down to $Q \approx M_Z$ and thus enhanced by large logarithms. As is well known [11], in this approximation the result has a strong unphysical dependence on the choice of the low-energy scale $Q$. This dependence can be reduced using the better approximation of minimizing the one-loop effective potential [11, 12], that we have employed in producing the numerical results plotted in the figures. However, since the top/stop sector gives the dominant contribution to the one-loop effective potential, a good approximation is obtained making the stop corrections negligible via an appropriate choice of the scale $Q \sim m_{\tilde{t}}$ and including only the top correction [12] to $M_2^2$ in eq. (1)

$$M_2^2 \rightarrow (1 + \delta_{\text{top}})M_2^2,$$

with

$$\delta_{\text{top}} = \frac{1}{(4\pi)^2} \frac{\sin^2 \beta}{1 - \cot^2 \beta} \frac{g_Y^2}{g_2^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2}$$

where $\lambda_t$ is the top quark Yukawa coupling in the MSSM. Using now the solutions (12) to the RGE given in appendix A we get

$$(1 + \delta_{\text{top}})M_2^2 \approx -2\mu^2 + 1.2 \frac{0.5 + 5\eta}{1 - \cot^2 \beta} \frac{(0.1 + 2\eta) \cot^2 \beta}{M_0^2} + \cdots$$

(8)

where we have fixed $M_U = 10^5$ GeV. We remark that, in our approximation, the possible non minimal additional contributions to the Higgs soft terms, represented by the dots in eq. (8), enter additively even after RGE rescaling. This does not happen in the one-loop quantum potential. For this reason the full one-loop result has also a (negligible) dependence on the remaining free parameters: the sign of $\mu$ and the possible non-minimal contributions to the Higgs soft masses. In our plots we have set them to zero and chosen $\mu > 0$.

For a typical value of $\delta_{\text{top}} \approx 0.6$, the naturalness bound on the right-handed slepton masses is, taking also into account the $D$-term contribution to the them

$$M_{\tilde{e}_R} \lesssim M_Z \left( -\sin^2 \theta_W \cos 2\beta + \Delta_{\text{lim}} \frac{(\eta + 0.025)(1 - \cot^2 \beta)}{14\eta + 1.5 + (5.5\eta + 0.3)\cot^2 \beta} \right)^{1/2}.$$

(9)
We now discuss the dependence of this bound on the various parameters. Expression (4) says that, as usual, the bound becomes stronger for small \( \tan \beta \). In the limit \( \tan \beta \to 1 \) the tree level Higgs potential has no quartic couplings, so that the one loop corrections become important and the full numerical result must be used. We show this dependence in figure 2a, for \( M_U = 10^6 \, \text{GeV} \) and for different minimal models. The horizontal dot-dashed line represents the present lower experimental bound on the \( \tilde{e}_R \) mass.

The dependence on the parameter \( \eta \) is plotted in figure 2b. We see that the bounds are stronger for smaller values of \( \eta \), obtained with a large messenger field content. This is mainly a reflection of the fact that in such cases the selectron is so light that it usually is the Lightest Supersymmetric Particle (apart for the gravitino) \[10\]. Unless the messenger mass is either near to the unification scale or so small that the LSP decays within the detector, and unless there are cancellations of more than one order of magnitude, a large messenger field content is disfavored by the experimental lower bound on the \( \tilde{e}_R \) mass.

In figure 2a we finally show, in the minimal model with \( \eta = 1 \), the dependence of the naturalness bounds \( \Delta_{\text{lim}} < 10 \) and \( \Delta_{\text{lim}} < 100 \) on the messenger mass as a contour-plot in the plane \( (M_U, \tan \beta) \). We see that one-loop effects become more important in the case of a more unnaturally splitted spectrum. Due to the compensation of different effects (higher \( M_U \) means a longer running but also smaller squark masses at \( M_U \)) the bounds are not much dependent on \( M_U \). For the lowest possible values of \( M_U \) the running is so short that the \( \lambda_t \)-induced negative contributions to \( m_{h_a}^2 \) are just sufficient to cancel the gauge-mediated contribution, giving rise to much weaker naturalness bounds on the sfermion masses. At the light of our naturalness considerations, a fully natural acceptable spectrum is possible in this case.

In the same way we could obtain the corresponding upper bounds on the masses of the other supersymmetric particles. Since the typical correlations between different sparticle masses have been carefully studied \[10\] we do not need to report here the less interesting bounds on the heavier sfermions, even if they could be more stable with respect to non minimal corrections. We also avoid discussing the bounds on the Higgs masses because, since they contribute directly to \( M_Z^2 \) in eq. (4), these bounds are not characteristic of the specific model of soft terms we are considering.

Before concluding, we observe that in the case of large \( \tan \beta \sim m_t/m_b \), a stau becomes significantly lighter than the other sleptons. This is due both to the renormalization corrections induced by \( \lambda_t \), and to the left/right mixing term in the stau mass matrix. Since, especially for \( \Delta < 100 \), the stau masses depend in an important way also on the unknown \( \mu \) term, we limit ourselves to observe that the naturalness bound on the lightest \( \tilde{\tau} \) mass is typically two times stronger than the ones we have so far presented on the \( \tilde{e}_R \) mass. As usual \[13\], additional cancellations may be necessary to satisfy the other minimization condition that should give a large value of \( \tan \beta \).

Finally, as an aside remark, we point out that — again because the Higgs soft term \( m_{h_a}^2 \) is large and negative — unless there are large non-minimal corrections to the soft terms, when \( M_U \gtrsim 10 \, \text{TeV} / \lambda_{b, \tau} \) the combination \( m_{h_a}^2 \equiv m_{h_a}^2 + m_{\tilde{L}}^2 \) turns out to be negative. In this case the SM-like minimum is not the unique vacuum state: the MSSM potential develops other deeper unphysical minima with \( \langle h_u \rangle \approx (\tilde{L}) \) \[14\]. The value of the fields at the minimum is given by the scale \( Q_0 \) at which \( m_{h_a}^2(Q_0) = 0 \), and is typically one or two orders of magnitude below the mediation scale \( M_U \). Alternatively, if \( Q_0 \) is sufficiently large, non renormalizable operators (for example those responsible of the neutrino masses) can stabilize the potential along the direction \( \tilde{L}h_u \) at scales smaller than \( Q_0 \).

However this kind of unphysical minimum is not a problem, since, as we will now discuss, the evolution of the universe will likely prefer the SM-like minimum.

First of all, the only minimum present during the high-temperature phase is the symmetric one, \( \phi = 0 \). If the relevant fields \( \phi = \{ h_u, \tilde{L}, \tilde{Q}, \ldots \} \) are not displaced from it when the universe cools down below the Fermi scale, the electroweak phase transition \[13\] results in the usual SM-like vacuum. If, on the contrary, the relevant fields have a large vacuum expectation value \( \phi_R \) at the end of inflation (as would happen if the unphysical minimum were present also during inflation\[4\]), they will rapidly oscillate around the symmetric minimum with amplitude decreasing as \( \phi(T) = \phi_R \times (T/T_R) \) where

\footnote{A non zero \( \phi_R \) after inflation can be pushed to the origin \( \phi = 0 \) during the pre-heating stage in models where the inflation field decays resonantly to a field coupled to the flat direction \( \tilde{L}h_u \) \[14\].}
$T_R$ is the reheating temperature\footnote{We are interested in the case where $\phi \gtrsim T$, so that the fermions coupled to $\phi$ receive large masses that prevent its decay.}. If $T_R > \phi_R$ thermal effects are therefore sufficient to solve the cosmological problem posed by the unphysical minimum.

Fortunately, the quantum corrections that give rise to the unphysical minima are not present during inflation\footnote{This model ('NMSSM') has been extensively studied in the case of supergravity-mediated soft terms\cite{15} finding that there are often inequivalent physically acceptable minima with different values of $\langle S \rangle$, sometimes together with unphysical minima where, for example $\langle S \rangle = 0$. We point out that, in such cases, considering the thermal corrections to the potential and discussing the electroweak phase transition, for example along the lines of ref.\cite{15}, it is possible to compute which one of the various minima — not necessarily the deepest one — is selected by cosmological evolution. For example, if $m_{h^u}^2, m_S^2 > 0$ and $m_{h^d}^2 < 0$ the chosen minimum will be the one that at zero temperature is not separated by a potential barrier from the symmetric state where $S = h_u = h_d = 0$. This property makes simple to find it numerically. If deeper minima are present, it is also necessary to check if the tunneling rate is small enough. If instead $m_S^2$ is negative there are two different such minima and identifying the selected one becomes a very cumbersome numerical problem.}

4 The NMSSM with gauge-mediated soft terms

An elegant attempt to overcome the difficulty of generating the $\mu$-term consists in modifying the field content of the MSSM in such a way that the $\mu$-term arises through the vacuum expectation value of a singlet $S$ with superpotential obtained replacing\footnote{The reason is that the soft parameters of the singlet field, $m_S^2$, $A_\kappa$ and $A_\lambda$, zero at tree level, remain too small with respect to $m_{h^u}$\cite{10,11}. The only possibility of avoiding this unfortunate situation consists in having a light and fine-tuned messenger spectrum such that $m_{h^u}^2(Q) \sim m_S^2$. In this case $M_Z^2 = O(m_S^2)$ so that this fine-tuning also guarantee heavy sfermions and gauginos. Note also that in such a case $\tan \beta$, as obtained from the NMSSM minimization condition,

$$\sin 2\beta = \frac{2\kappa \lambda S^2 - A_\lambda \lambda S}{m_{h^u}^2 + m_{h^d}^2 + \lambda^2 (2S^2 + v^2)} \sim \frac{m_S^2}{m_{h^d}(\tan \beta)},$$

is large, $\tan \beta \sim 50$. When $m_S^2$ is not much smaller than 1/50 of the 'natural' value of the Higgs soft terms, $\sim m_{h^u}(M_U)$, the necessary fine tuning is not much worse than the minimal one, $\Delta \gtrsim 50$, anyway necessary to obtain a large $\tan \beta$\footnote{In this case $M_Z^2 = O(m_S^2)$ so that this fine-tuning also guarantee heavy sfermions and gauginos. Note also that in such a case $\tan \beta$, as obtained from the NMSSM minimization condition,\footnote{We are interested in the case where $\phi \gtrsim T$, so that the fermions coupled to $\phi$ receive large masses that prevent its decay.}}.

\begin{equation}
\mu h^u h^d \rightarrow \lambda S h^u h^d - \frac{\kappa}{3} S^3
\end{equation}

so that $\lambda \langle S \rangle = \mu$. The `$B \cdot \mu$' term is also naturally generated.

5 Renormalization of gauge-mediated soft terms

Neglecting all couplings except the gauge and the top Yukawa ones, the solutions to the one loop RGEs between the messenger mass $M_U$ and $M_Z$ may be written in terms of analytic functions and
only one function, \( \lambda_i^{\text{max}}(E) \), calculable only numerically. We abbreviate the various functions of \( E, \varphi(E) \), as \( \varphi_E \) and define \( t_E \equiv (4\pi)^{-2} \ln M_{\text{GUT}}^2/E^2 \).

For simplicity we assume, consistently with their measured values, that the gauge couplings satisfy unification relations so that we can parametrize \( \alpha_i \) (\( i = 1, 2, 3 \)) and \( \lambda_i \) in terms of \( \alpha_{\text{GUT}} \) and \( \lambda_{\text{GUT}} \) in the usual way

\[
\alpha_i(E) = \frac{\alpha_{\text{GUT}}}{[1 + b_i g_{\text{GUT}}^2(E)]},
\]

\[
\rho_E \equiv \frac{\lambda_i^E(\varphi_E)}{\lambda_i^{\text{max}}(E)} = \frac{1 + \lambda_i^{\text{max}}(E) \varphi_E}{\alpha_{\text{GUT}}^{-1}},
\]

where we have defined

\[
E_{\alpha}(E) = \prod_i \left[ \frac{\alpha_i(E)}{\alpha_{\text{GUT}}} \right]^{c_i^a/b_i}, \quad \lambda_i^{\text{max}}(E) = \left[ 2b_i \int_{\ln(E')} \frac{E_{\alpha}(E')}{E_{\alpha}(E)} d \ln(E') \right]^{-1/2}.
\]

The \( \beta \)-function coefficients \( b_i \), listed in table I, include only the contribution from the MSSM fields. Consequently \( \alpha_{\text{GUT}} \approx 1/24 \) and \( \lambda_{\text{GUT}} \) are then just parameters in terms of which it is convenient to express the solutions of the RGE below \( M_{\text{GUT}}^2 \) for the gaugino masses \( M_i \), for the trilinear terms \( A_g^a \) (where \( a = u, d, e \) and \( g = 1, 2, 3 \) is a generation index) and for the soft masses \( m_R^2 \) of the fields \( R \).

The values of these parameters at the Fermi scale corresponding to the boundary conditions \( \rho_i \) are

\[
M_i = M_0 \cdot \alpha_i/\alpha_{\text{GUT}}
\]

\[
A_g^a = (x_a^a + b_g^{a \hat{f}})M_0
\]

\[
m_R^2 = m_R^2(M_U) + x_R^2 M_0^2
\]

The soft masses of the fields involved in the top quark Yukawa coupling get the additional corrections

\[
m_{h_u}^2 = m_{h_u}^2(M_U) + x_{h_u}^2 M_0^2 - \frac{1}{2} I
\]

\[
m_{Q_3}^2 = m_{Q_3}^2(M_U) + x_{Q_3}^2 M_0^2 - \frac{1}{6} I
\]

\[
m_{t_R}^2 = m_{t_R}^2(M_U) + x_{t_R}^u M_0^2 - \frac{1}{3} I
\]

where we have defined

\[
x_{h_u}^R \equiv \sum_{i=1}^3 c_i^{h_u}/b_i \left[ 1 - \frac{\alpha_i^2(M_U)}{\alpha_{\text{GUT}}^2} \right], \quad \phi_i \equiv \rho_i - 6 \lambda_i^{2} \ln(E), \quad \Phi_i \equiv \phi_i - 6 \lambda_i^{2} \ln(E)(c_i^a g_i^a).\]

and

\[
I \equiv \left[ m_{h_u}^2(M_U) + m_{Q_3}^2(M_U) + m_{t_R}^2(M_U) \right] \cdot \frac{\rho_Z - \rho_U}{1 - \rho_U} + M_0^2 \left[ \frac{(1 - \rho_Z)(\rho_Z - \rho_U)}{(1 - \rho_U)^2} (\Phi_U + x_{1U}^u)^2 + 2 \frac{1 - \rho_Z}{1 - \rho_U}(\phi_Z - \Phi_U)(\phi_U + x_{1U}^u) \right. - (\Phi_U - \phi_Z) \cdot \left. \frac{\rho_Z - \rho_U}{1 - \rho_U}(\Phi_U + x_{2U}^u) \right]
\]

\[
I' \equiv (\phi_Z - \phi_U) + \frac{\rho_Z - \rho_U}{1 - \rho_U}(x_{1U}^u + \phi_U)
\]

Table 1: \textit{Values of the RGE coefficients in the MSSM.} The coefficients \( c_i^h \) and \( c_i^d \) are equal to \( c_i^L \).
All the coefficients are listed in table [1].

References

[1] R. Barbieri, S. Ferrara and C.A. Savoy, *Phys. Lett.* **119B** (1982) 343; P. Nath, R. Arnowitt and A. Chamseddine, *Phys. Rev. Lett.* **49** (1982) 970.

[2] R. Barbieri and G.F. Giudice, *Nucl. Phys.* **B306** (1988) 63.

[3] B. de Carlos and J.A. Casas, *Phys. Lett.* **B309** (1993) 320.

[4] G.W. Anderson and D.J. Castaño, *Phys. Lett.* **B347** (1995) 300 and *Phys. Rev.* **D52** (1995) 1693.

[5] M. Carena and C.E.M. Wagner, *Nucl. Phys.* **B452** (1995) 45; S. Dimopoulos and G.F. Giudice, *Phys. Lett.* **B357** (1995) 573.

[6] A. Strumia, hep-ph/9609286.

[7] L. Alvarez-Gaume, M. Claudson and M.B. Wise, *Nucl. Phys.* **B207** (1982) 96.

[8] G. Dvali, G.F. Giudice and A. Pomarol, *Nucl. Phys.* **B478** (1996) 31.

[9] L. Ibáñez, C. Lopez and C. Muñoz, *Nucl. Phys.* **B256** (1985) 218; A. Boquet, J. Kaplan and C.A. Savoy, *Nucl. Phys.* **B262** (1985) 299; R. Barbieri, L. Hall and A. Strumia, *Nucl. Phys.* **B445** (1995) 219.

[10] S. Dimopoulos, S. Thomas and J.D. Wells, *Phys. Lett.* **B357** (1995) 573; J.A. Bagger et al., *Phys. Rev.* **D55** (1997) 3188.

[11] G. Gamberini, G. Ridolfi and F. Zwirner, *Nucl. Phys.* **B331** (1990) 331.

[12] R. Arnowitt and P. Nath, *Phys. Rev.* **D46** (1992) 3981; V. Barger, M.S. Berger and P. Ohmann, *Phys. Rev.* **D49** (1994) 4908.

[13] R. Rattazzi and U. Sarid, *Phys. Rev.* **D53** (1996) 1553;

[14] H. Komatsu, *Phys. Lett.* **B215** (1988) 323; J.A. Casas, A. Lleyda and C. Muñoz, *Nucl. Phys.* **B471** (1996) 3.

[15] A. Kusenko, P. Langacker and G. Segre, *Phys. Rev.* **D54** (1996) 5824.

[16] A. Riotto, E. Roulet and I. Wilja, *Phys. Lett.* **B390** (1997) 73.

[17] A. Strumia, *Nucl. Phys.* **B482** (1996) 24.

[18] P. Nilles, M. Srednicki and D. Wyler, *Phys. Lett.* **B120** (1983) 346; M. Dine and A.E. Nelson, *Phys. Rev.* **D48** (1993) 1277.

[19] J. Ellis, J.F. Gunion, H.E. Haber, L. Rozkowzki and F. Zwirner, *Phys. Rev.* **D39** (1989) 844; F. Franke and H. Fraas, *Phys. Lett.* **B336** (1994) 415; B. Ananthanarayan and P.N. Pandita, *Phys. Lett.* **B353** (1995) 70; S.F. King and P.L. White, hep-ph/9505320.

[20] M. Dine, A.E. Nelson and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362.