Cosmic–Ray Neutrino Annihilation on Relic Neutrinos Revisited:
A Mechanism for Generating Air Showers above the
Greisen–Zatsepin–Kuzmin Cutoff

Thomas J. Weiler
Department of Physics & Astronomy, Vanderbilt University, Nashville, TN 37235
and
Department of Physics, University of Wisconsin, Madison, WI 53706
email: weilertj@ctral1.vanderbilt.edu

Abstract
If neutrinos are a significant contributor to the matter density of the universe, then
they should have $\sim eV$ mass and cluster in galactic (super) cluster halos, and possibly
in galactic halos as well. It was noted in the early 1980’s that cosmic ray neutrinos with
energy within $\delta E/E_R = \Gamma Z / M_Z \sim 3\%$ of the peak energy $E_R = 4 (eV/m_\nu) \times 10^{21}$ eV
will annihilate on the nonrelativistic relic antineutrinos (and vice versa) to produce the
$Z$–boson with an enhanced, resonant cross section of $O(G_F) \sim 10^{-32}$ cm$^2$. The result
of the resonant neutrino annihilation is a hadronic $Z$–burst 70% of the time, which
contains, on average, thirty photons and 2.7 nucleons with energies near or above the
GZK cutoff energy of $5 \times 10^{19}$ eV. These photons and nucleons produced within our
Supergalactic halo may easily propagate to earth and initiate super–GZK air showers.
Here we show that the probability for each neutrino flavor at its resonant energy to
annihilate within the halo of our Supergalactic cluster is likely within an order of
magnitude of 1%, with the exact value depending on unknown aspects of neutrino
mixing and relic neutrino clustering. The absolute lower bound in a hot Big Bang
universe for the probability to annihilate within a 50 Mpc radius (roughly a nucleon
propagation distance) of earth is 0.036%. From fragmentation data for $Z$–decay, we
estimate that the nucleons are more energetic than the photons by a factor $\sim 10$.
Several tests of the hypothesis are indicated.
1 The Cosmic Ray Puzzle Above \(10^{20}\) eV

The recent discoveries by the AGASA\cite{1}, Fly’s Eye\cite{2}, Haverah Park\cite{3}, and Yakutsk\cite{4} collaborations of air shower events with energies above the Greisen–Zatsepin–Kuzmin (GZK) cutoff of \(\sim 5 \times 10^{19}\) eV presents an outstanding puzzle in ultrahigh-energy cosmic-ray physics. It was anticipated that the highest-energy cosmic primaries would be protons from outside the galaxy, perhaps produced in active galactic nuclei (AGNs). It was also anticipated that the highest energies for protons arriving at earth would be \(\sim 5 \times 10^{19}\) eV\cite{5}. The origin of this GZK cutoff is degradation of the proton energy by the resonant scattering process \(p + \gamma_{2.7K} \rightarrow \Delta^* \rightarrow N + \pi\) when the proton is above the resonant threshold for \(\Delta^*\) production; \(\gamma_{2.7K}\) denotes a photon in the 2.7K cosmic background radiation. For every mean free path \(\sim 6\) Mpc of travel, the proton loses 20\% of its energy on average\cite{6}. A proton produced at its cosmic source with an initial energy \(E_p\) will on average arrive at earth with only a fraction \(\sim (0.8)^{D/6\text{Mpc}}\) of its original energy. Since AGNs are hundreds of megaparsecs away, the energy requirement at an AGN source for a proton which arrives at earth with a super–GZK energy is unrealistically high\cite{7}. Of course, proton energy is not lost significantly if the highest energy protons come from a rather nearby source, \(\sim 50\) to 100 Mpc\cite{8}. However, no AGN sources are known to exist within 100 Mpc of earth. Hence, the observation of air shower events above \(5 \times 10^{19}\) eV challenges standard theory\cite{9}. In principle, there exists a check of the cutoff dynamics. The expected energy losses for protons above the cutoff should lead to an event pile–up at energies just below the cutoff\cite{10}. At present, the data is too sparse to rule in or out an event pile–up at \(\sim 5 \times 10^{19}\) eV.

A primary nucleus mitigates the cutoff problem (energy per nucleon is reduced by \(1/A\)), but has additional problems: above \(\sim 10^{19}\) eV nuclei should be photo–dissociated by the 2.7K background\cite{11}, and possibly disintegrated by the particle density ambient at the astrophysical source. Gamma–rays and neutrinos are other possible primary candidates for the highest energy events. The gamma–ray hypothesis appears inconsistent with the time–development of the Fly’s Eye event, but is not ruled out for this event\cite{6}. However, the

\[\text{\footnotesize{1The suggestion has been made that hot spots of radio galaxies in the Supergalactic plane at distances of tens of megaparsecs may be the sources of the super–GZK primaries.}}\]
mean free path for a $\sim 10^{20}$ eV photon to annihilate on the radio background to $e^+e^-$ is believed to be only 10 to 40 Mpc,\textsuperscript{[3]} and the density profile of the Yakutsk event\textsuperscript{[4]} showed a large number of muons which may argue against gamma–ray initiation. Concerning the neutrino hypothesis, the Fly’s Eye event occurred high in the atmosphere, whereas the expected event rate for early development of a neutrino–induced air shower is down from that of an electromagnetic or hadronic interaction by six orders of magnitude\textsuperscript{[3]}. On the other hand, there is good evidence that some of the highest–energy primaries have common arrival directions, with arrival times displaced by $\sim 10^8$ s. Such event–pairing argues for a common source of some duration emitting stable neutral primaries\textsuperscript{[12]}. Neutrino primaries satisfy this criterion. Charged–particle primaries should display bending in cosmic magnetic fields, leading to an energy–dependent divergence in arrival directions and arrival times.

Some exotic particles\textsuperscript{[13, 14]} and exotic dynamics\textsuperscript{[15]} have been introduced in an attempt to circumvent the problems of conventional primaries and conventional interactions, and thereby accommodate the highest energy events. However, most of the new proposals come with their own peculiar difficulties\textsuperscript{[16, 17]}, so new ideas are still welcomed.

1.1 A Possible Resolution to the Puzzle

The purpose of this article is to revisit a neutrino–annihilation mechanism which may produce photons and baryons with with energies above the GZK cutoff, and to calculate the number of photons and nucleons which would arrive at earth with super–GZK energies and interact in our atmosphere. In 1982 it was shown\textsuperscript{[18]} that the annihilation channel of cosmic–ray neutrinos on relic neutrinos should produce an absorption dip in the high–energy cosmic neutrino flux if the target neutrinos, the relics predicted by hot Big Bang cosmology, have a mass large compared to their relic temperature $T_\nu \sim 10^{-4}$ eV. Scattering of an ultrahigh energy neutrino on the nonrelativistic relic neutrino background resonantly produces a $Z$–boson at the energy $E_{\nu_j}^R = M_Z^2/2m_{\nu_j} = 4 (eV/m_{\nu_j}) \times 10^{21}$ eV ($j$ labels the three light neutrino types). The width of the resonant energy for annihilation is $\delta E_{\nu_j}^R/E_{\nu_j}^R \sim 2\Gamma_Z/M_Z \sim 0.06$. The annihilation process converts neutrinos with energy within 0.03 of the peak resonant energy $E_{\nu_j}^R$ into highly energetic $Z$–bosons. The $Z$ produced in each neutrino annihilation immediately decays (its lifetime is $3 \times 10^{-25}$ s in its rest frame). 70% of the
$Z$ decays are hadronic, with a final state known to include on average fifteen neutral pions and 1.35 baryon–antibaryon pairs\textsuperscript{[19]}. The fifteen $\pi^0$'s decay to produce thirty high–energy photons. The $Z$ is highly boosted, with $\gamma_Z = E_\nu/M_Z \sim 10^{10}$ for super–GZK energies. Consequently, its decay products are relativistically beamed into a cone of half–angle $\theta \sim 1/\gamma_Z \sim 10^{-10}$. We will refer to the collimated high–energy end product of this $Z$ production and hadronic decay as a “$Z$–burst.” If the $Z$–burst points in the direction of earth and occurs close enough to earth so that the nucleons may propagate to earth without too much energy attenuation, or the photons without too much absorption, then one or more of the photons and nucleons in the burst may initiate a super–GZK air shower at earth, as illustrated in Fig. 1.

Let us call the distance over which a stable particle can propagate without losing more than an order of magnitude of its energy the GZK distance. For a photon it is 10 to 40 Mpc, with the exact number depending on the strengths of the diffuse radio and infrared background. For a proton it is $D_{\text{GZK}} \sim -[\ln(1/10)/\ln(0.8)] \times 6$ Mpc $\sim$ 50 to 100 Mpc. We seek the probability for resonant annihilation of a cosmic ray $\nu$ and a relic $\bar{\nu}$ (or vice versa) into the $Z$–boson, within the GZK distance of earth (the “GZK zone”). The rate of $Z$–burst production within the GZK zone will be greatly enhanced if the relic neutrinos cluster in the potential wells within the GZK zone\textsuperscript{[18, 20]}, which includes the wells of the Local Supercluster, the Local Group of galaxies, and of course, our own Galaxy. Some clustering is expected if the relic neutrinos are nonrelativistic, \textit{i.e.} with mass considerably above their temperature, $T_\nu \sim 2K \sim 10^{-4}$ eV.

The mean energy per hadron from a $Z$–burst will be about 40 times less than $E_{\nu_j}^R$, since the mean multiplicity in $Z$ decay is about 40 \textsuperscript{[19]}. Although the energy per particle is diluted, the thirty photons from $\pi^0$–decays and the 2.7 baryons in the final state serve to amplify the propagating particle flux above the GZK cutoff by a factor of $\sim 30$. We will soon argue that the mean energy per baryon is about ten times larger than the mean energy per photon.

Two crucial elements are required for this mechanism to produce super–GZK air showers. They are:

(i) the existence of a neutrino flux at $\gtrsim 10^{21}$ eV, and

(ii) the existence of a neutrino mass in the 0.1 to 10 eV range.

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A third ingredient greatly enhances the rate for observed $Z$–bursts. It is:

(iii) a significant clustering of the relic neutrino density within the GZK zone, e.g. in the potential wells of our Supergalactic cluster, Galactic cluster, or possibly even our Galactic halo.

We discuss the plausibility of each of these ingredients in turn.

2 Neutrino Flux, Mass, and Relic Density

The observations of the highest–energy cosmic rays tell us that some primaries are produced at $10^{20}$ eV or more. The actual energies of particles at their sources may well be higher than $10^{20}$ eV, since particles may lose energy while emerging from the high–density environment of the source, and, as noted earlier, nucleons lose energy by scattering on the 2.7K photon background. It is likely that whatever mechanism produces these most energetic particles also produces charged pions of comparable energy. Thus, one may expect neutrino production at ultrahigh energy, coming from pion decay and subsequent muon decay\cite{21}. The much smaller opacity for neutrinos as compared to protons in dense sources such as AGNs makes credible the possibility of a neutrino flux considerably above the proton flux at highest energies.\footnote{There is also the possibility that the highest energy neutrinos originate in quark jets, which themselves result from decay of some supermassive relic particles, in which case the neutrino flux greatly exceeds the proton flux\cite{14}.}

The mean relic neutrino density of the universe is predicted by hot Big Bang cosmology. The density of relic neutrinos with mass below an MeV (the neutrino decoupling temperature) is given by a relativistic Fermi-Dirac distribution characterized by a single temperature parameter. The distribution is that of relativisitic neutrinos, even though the neutrinos are nonrelativistic today, because the distribution is determined at the decoupling epoch. As a result of photon reheating from the era of $e^+e^− \rightarrow \gamma \gamma$ annihilation, the neutrino temperature $T_\nu \sim 1.95$K is predicted to be a factor of $(4/11)^{1/3}$ less than that of the photon temperature, $T_\gamma = 2.73$K. The resulting mean neutrino number density is $<n_\nu> = (3\zeta(3)/4\pi^2)T_\nu^3 = 54$ cm$^{-3}$ for each light flavor $j$ with an equal number density for each antineutrino flavor.\footnote{If neutrinos are Dirac particles and if there are some lepton number violating processes in the early

\[ \]
the temperature relation to the relic photon density which is measured. Consequently, the predicted mean density of \( <n_{\nu_j}> = 54 \text{ cm}^{-3} \) must be considered firm.

The mean energy density in neutrinos is obtained by multiplying together mass and mean density and summing over light flavors. The result, as a fraction of the closure density, is

\[ \Omega_{\nu} = \sum_j m_{\nu_j} / (92 h_{100}^2 \text{ eV}) \]

with \( h_{100} \) being the present value of the Hubble parameter \( H_0 \) in units of 100 km/s/Mpc. The observational constraint on the Hubble parameter gives

\[ 0.5 \leq h_{100} \leq 1 \]

with lower values in the range favored. One sees that a neutrino mass, or a sum of neutrino masses, up to tens of eV is consistent with hot Big Bang cosmology. Perhaps more importantly, one sees that a neutrino mass around or above an eV is required if neutrino hot dark matter is to contribute in any significant way to the evolution of large-scale structures. Some have argued that the existence of the largest observed structures necessitates the existence of hot dark matter\cite{22}. Furthermore, the simplest explanation for the anomalous atmospheric–neutrino flavor–ratio\cite{23} is neutrino oscillations driven by a mass–squared difference of \( \sim 0.01 \text{ eV}^2 \) \cite{24}, which implies a mass of at least 0.1 eV. The recent LSND measurement is claimed\cite{25} to also indicate a neutrino mass, of order 1 eV.

\[ \text{2.1 Gravitational Clustering of Neutrinos} \]

Some nonrelativistic matter, whether charged or neutral, is expected to have fallen into gravitational potential wells and “cooled”. For charged matter the energy loss mechanism is dominantly bremsstrahlung during two–body scattering. For neutral (“dissipationless”) matter, the mechanism (termed “violent relaxation”) is many–body gravitational scattering wherein some particles gain energy at the expense of other particles as a consequence of the time–varying gravitational potential\cite{26}. Examples of collapse and capture of dissipationless matter are seen in models of gravitational infall\cite{27}. A consequence of clustering for massive neutrinos is that the mean free path for annihilation is much shorter in large–scale potential wells, and the efficiency for neutrino to hadron conversion is accordingly much higher. Let universe, then the numbers of neutrinos and antineutrinos need not be equal; a nonzero chemical potential is effectively introduced into the neutrino Fermi–Dirac distribution. We note that any lepton asymmetry necessarily increases the number of neutrinos plus antineutrinos, but we do not develop this possibility further here. For unpolarized Majorana neutrinos, there can be no chemical potential.
us determine the scaling law that governs the increase in annihilation rate when neutrinos are clustered rather than distributed uniformly throughout the universe. Density scales with clustering length \( L \) and the number of cluster sites \( N \) within the volume of the visible universe \( V_U \) as \( V_U/N L^3 \). The path length across the neutrino cluster scales as \( L/V_U^{1/3} \). Putting the factors together, and estimating \( V_U \sim D_H^3 \) with \( D_H \equiv c H_0^{-1} \), the annihilation rate for a neutrino cosmic ray traversing the cluster therefore scales as \( D_H^2/NL^2 \). It is useful to write \( V_U = D_H^3 = N L_{ss}^3 \) to define the mean distance \( L_{ss} \) between neighboring cluster sites. Then, \( N \) may be eliminated to yield the final form \( L_{ss}^3/L^2 D_H \) of the geometrical scaling factor for annihilation within a cluster of size \( L \) and mean separation \( L_{ss} \). If the neutrino clustering scale is less than the GZK distance of 50 to 100 Mpc, then the \( Z \)–burst production rate within the GZK zone also scales with the same \( L_{ss}^3/L^2 D_H \) law.\footnote{An average over all cosmic ray trajectories would show no enhanced annihilation rate. The fraction of rays that intersect any relic halo varies as \( NL^2 \) with clustering, to compensate the \( 1/NL^2 \) increase in annihilations within the halos. The fact that an earthbound observer sees only the \( 1/NL^2 \) enhancement factor is due to our “preferred” vantage point within or nearby a neutrino cluster.} We will implement this scaling law shortly.

3 Neutrino Annihilation on the Relic Background

For neutrinos with the standard weak interaction, there is little hindrance to their propagation over cosmic distances except for nonzero mass neutrinos with energy near the resonant energy on the \( Z \)–pole\cite{18,20,28}. The annihilation cross section \( \sigma_{\text{ann}}(\nu_j + \bar{\nu}_j \rightarrow Z) \) is first order in the Fermi constant \( G_F \), whereas neutrino scattering cross sections are order \( G_F^2 \). The relatively large cross section of the resonant process is somewhat mitigated by the narrowness of the \( Z \) pole (2.5 GeV is the width) since it is the energy–integrated cross sections which determine the annihilation rate of a flux distributed in energy. The resonant contribution should dominate the energy–integrated order \( G_F^2 \) scattering cross sections in the energy region of interest, particularly if the neutrino flux has a falling spectrum above \( E_R \). In any case, non–resonant scattering contributions can only add to the resonantly produced super–GZK flux we calculate here.
Define the invariant, energy–averaged annihilation cross section by the following integral over the Z pole: 
\[ <\sigma_{\text{ann}}> \equiv \int \frac{ds}{M_Z^2} \sigma_{\text{ann}}(s), \]
with \( s \) the square of the energy in the center of momentum frame. The standard model value for this cross section is 
\[ <\sigma_{\text{ann}}> = 4\pi G_F/\sqrt{2} = 4.2 \times 10^{-32}\text{cm}^2 \]
for each neutrino type (flavor or mass basis), independent of any neutrino mixing–angles since the annihilation mechanism is a neutral current process. The energy of the neutrino annihilating at the peak of the Z–pole is 
\[ E_{\nu_j}^R = \frac{M_Z^2}{2m_{\nu_j}} = 4(eV/m_{\nu_j}) \times 10^{21}\text{eV}. \]
For a given resonant energy \( E_{\nu_j}^R \), only neutrinos with the \( j^{th} \) mass \( m_j \) may annihilate.

The energy–averaged annihilation cross section \( <\sigma_{\text{ann}}> \) is the effective cross section for all neutrinos within 
\[ \frac{1}{2}\delta E_R/E_R = \Gamma_Z/M_Z = 3\% \]
of their peak annihilation energy. (We will sometimes use \( E_R \) generically for resonant energy, as we do here, with the understanding that there are really three different resonant energies, one for each neutrino mass.) We will refer to neutrinos with resonant flavor \( j \) and with energy in the resonant range \( 0.97 E_{\nu_j}^R \) to \( 1.03 E_{\nu_j}^R \) as “resonant neutrinos.”

### 3.1 Rate of Annihilation to Z–Bursts

In a universe where the neutrinos are nonrelativistic but unclustered the mean annihilation length for neutrinos at their resonant energy would be 
\[ \lambda = \left( <\sigma_{\text{ann}}> <n_{\nu_j}> \right)^{-1} = 4.4 \times 10^{29}\text{cm}. \]
A cosmic ray neutrino arriving at earth from a cosmically distant source will have traversed approximately a Hubble distance of space, 
\[ D_H \equiv cH_0^{-1} = 0.9 h_1^{-1}_100 \times 10^{28}\text{cm}. \]
In an unclustered relic neutrino sea the annihilation probability for such a neutrino at its resonant energy is 
\[ D_H/\lambda_{\text{ann}} = 2.0 h_1^{-1}_100 \%. \]
For each 50 Mpc of travel through the mean neutrino density, the probability for a neutrino with resonant energy to annihilate to a Z–boson is \( 3.6 \times 10^{-4} \). The branching fraction for a Z to decay to hadrons is 70% . Consequently, one part in 4000 of the resonant neutrino flux will be converted into a Z–burst containing ultrahigh energy photons and nucleons within the 50 Mpc GZK–zone of earth in this unclustered universe. This value of 0.025% for the probability of a resonant neutrino

\[ ^5 \text{This annihilation length is not too much larger than the Hubble size of the universe. It is this rough equality of lengths which led to the first observation of the possible absorption dip in the neutrino spectrum for neutrinos originating from cosmologically–distant sources. The absorption dip is significantly enhanced by the much higher relic densities present in the more–compact early universe.} \]
creating a \( Z \)-burst within the GZK zone is the absolute minimum in a Big Bang universe.

### 3.2 Neutrino Clustering Included

With “local” clustering of the relic neutrinos, the probability for annihilation within the GZK zone is significantly enhanced. The probability is given by that found for a Hubble distance of travel in a non–clustered universe, scaled by the geometric factor of Sec. 2.1. The resulting annihilation probability is \((L_{ss}^3/D_H L^2)(2.0 h_{100}^{-1} \%)\), for a cluster of size \( L \) and mean cluster–cluster separation distance \( L_{ss} \). Let us define some fiducial values for cluster sizes and mean cluster–to–cluster distances.\(^6\) We take \( D_S = 20 \) Mpc for the diameter of the Virgo Supercluster, the only supercluster well within our GZK zone, and \( 100 \) Mpc, typical of the distance across a cosmic void, for the supercluster mean separation distance \( D_{SS} \). We take \( D_C = 5 \) Mpc for the typical diameter of galactic clusters and \( D_{CC} = 50 \) Mpc as the typical distance between neighboring galactic clusters. For our Local Group of \( \sim 20 \) galaxies, we take \( D_{Gp} = 2 \) Mpc, and \( D_{Gp–Gp} = 20 \) Mpc for the mean separation distance between groups. \( D_G = 50 \) kpc is a typical diameter of galactic halos (including our own) and \( D_{GG} = 1 \) Mpc is a typical distance between neighboring galaxies. With these fiducial values for sizes and separation distances, the respective density enhancements \((L_{ss}/L)^3\) are of order \( 100, 10^3, 10^3, \) and \( 10^4 \), for the Supercluster, Cluster, Local Group, and Galactic halo, assuming that neutrino clustering is more or less as efficient as baryonic clustering for these scales. The annihilation–probability geometric factors are \((L_{ss}^3/D_H L^2) \sim 0.9, 1.7, 0.7, \) and \( 0.14 h_{100} \), respectively, for neutrino clustering on the scales of the Supercluster, Cluster, Local Group, and Galactic halo, again assuming efficient neutrino clustering. Including the 70% hadronic branching ratio of the \( Z \), one then gets the probabilities \( 1.3\%, 2.4\%, 1.0\%, \) and \( 0.2\% \) (independent of \( h_{100} \)), for \( Z \)-burst production by a neutrino of relevant flavor at resonant energy traversing the Supercluster, Cluster, Local Group, and Galactic halo, respectively.

\(^6\)We realize that a cluster or halo size is not well defined, but it is useful to pretend that they are for purposes of illustrating the annihilation enhancement expected from neutrino clustering. We will see shortly that in fact it is the integrated column density of relic neutrinos that determines the annihilation rate. Our use of constant densities within well–defined cluster radii may be easily translated into column densities.

\(^7\)Distances from earth to the centers of the nearest rich clusters are estimated in \(^2\) to be 11, 40, 50, and \( 60 h_{100}^{-1} \) Mpc for Virgo, Pisces, Perseus, and Coma, respectively.
Clustering on the small scale of the Galactic halo gives the smallest probability. The larger clusters all give a robust probability, within a factor of two of each other, and of order of a per cent. This is our main result, which we repeat for emphasis: the probability for cosmic ray neutrinos at their resonant energy to annihilate within the \( \sim 50 \) Mpc zone of earth is likely within an order of magnitude of 1\%, with the exact value depending on unknown aspects of neutrino mixing and relic neutrino clustering.

We have assumed here that the neutrino cluster is local, in that it contains our earth. In this case the scaling law and the enhanced probability applies for the full sky of cosmic rays. A “non–local” cluster will also yield this enhanced probability, but only for cosmic rays with arrival directions within the solid angle of the distant cluster. A detailed calculation of the \( Z \)–burst rate, including the possibility of anisotropic neutrino clustering, will be presented in the next two subsections.

The (in)efficiency of neutrino clustering warrants our attention. One expects the relic neutrinos to be less clustered than the baryons, especially on scales as small as the Galactic halo, for several reasons. First of all, neutrinos do not dissipate energy as easily as electrically charged protons do. Secondly, neutrinos have a much larger Jeans (“free–streaming”) length than do baryons at the crucial time when galaxies start to grow nonlinearly. And thirdly, Pauli blocking presents a significant barrier to clustering of light–mass fermions\[^{30}\]. As a crude estimate of Pauli blocking, one may use the zero temperature Fermi gas as a model of the gravitationally bound halo neutrinos. Requiring that the Fermi momentum of the neutrinos not exceed the virial velocity \( \sigma \sim \sqrt{MG/L} \) within the cluster, one gets \( \xi = n_{\nu}/54 \text{cm}^{-3} \sim 10^3(m_{\nu}/\text{eV})^3(\sigma/200\text{km/s})^{-3} \). The virial velocity within our Galaxy is a couple hundred km/sec, whereas virial velocities for rich galactic clusters are a thousand km/s or more. Thus it appears that Pauli blocking allows significant clustering on the Galactic scale only if \( m_{\nu} \gtrsim 1 \) eV, but allows clustering on the larger scales for \( m_{\nu} \gtrsim 0.1 \) eV.

The free–streaming argument also favors clustering on the larger scales\[^{9}\]. This is just as well,

\[^{8}\text{An anonymous referee mentioned yet another potential obstacle to the clustering of neutrinos on the Galactic scale. It is that the thermal velocity of relic neutrinos is comparable to the virial or escape velocity if } m_{\nu} \gtrsim 0.1 \text{ eV. This result is obtained by noting that it is the neutrino’s momentum which has red–shifted since decoupling, leading to } m_{\nu}v_{\nu}^{\text{Th}} = p_D/z_D \approx E_D/z_D \sim 3T_D/z_D = 3T, \text{ which implies } v_{\nu}^{\text{Th}} \sim 3T/m_{\nu} \sim 5 \times 10^{-4}/(m_{\nu}/\text{eV}). \]
for we have shown that it is the larger scales of clustering that give the $O(1\%)$ probability for annihilation to a Z-burst.

### 3.3 A More Careful Rate Calculation

The rate calculation may be carried out more carefully. Let $F_{\nu_j}(E_\nu, x)$ denote the flux of the $j^{th}$ neutrino flavor, as would be measured at a distance $x$ from the source, with energy within $dE$ of $E_\nu$. The units of $F_{\nu_j}(E_\nu, x)$ are neutrinos/energy/area/time/solid angle. This flux may be quasi-isotropic (“diffuse”), as might arise from a sum over cosmically-distant sources such as AGNs; or it may be highly directional, perhaps pointing back to sources within our Supergalactic plane. The distinction between diffuse and discrete fluxes affects the arrival direction of the super-GZK events, but not the rate calculation. The change in the neutrino flux due to resonant annihilation is $dF_{\nu_j}(E_\nu, x) = -F_{\nu_j}(E_\nu, x) \frac{dx}{\lambda_j(E_\nu, x)}$, where the mean annihilation length is $\lambda_j(E_\nu, x) = [\sigma_{ann}(E_\nu) n_{\nu_j}(x)]^{-1}$ as before. $dF_{\nu_j}(E_\nu, x)/dx$ is therefore the production rate per unit length of Z’s with energy within $dE$ of $E_\nu$, per unit area and unit solid angle. Integrating the absorption equation over the distance $D$ from the emission site to earth gives the depletion in the neutrino flux at energy $E_\nu$:

$$
\delta F_{\nu_j}(E_\nu, D) = F_{\nu_j}(E_\nu, 0) \left[ 1 - \exp\{-\sigma_{ann}(E_\nu) S_j(D)\} \right],
$$

(1)

with $\delta F_{\nu}(E_\nu, D) \equiv F_{\nu}(E_\nu, 0) - F_{\nu}(E_\nu, D)$. The relic neutrino column density is defined as

$$
S(D) \equiv \int_0^D dx \, n_{\nu_j}(x).
$$

(2)

Integrating this equation over neutrino energy then gives the total rate (per unit area and unit solid angle) of resonant annihilation, i.e. Z-burst production, over the distance $D$:

$$
\delta F_{\nu_j}(D) \equiv \int dE_\nu \, \delta F_{\nu_j}(E_\nu, D) = \int dE_\nu \, F_{\nu_j}(E_\nu, 0) \left[ 1 - \exp\{-\sigma_{ann}(E_\nu) S_j(D)\} \right].
$$

(3)

For a narrow resonance the flux may be evaluated at the resonant energy and removed from the integral. The remaining integral may be made an invariant by using $E_\nu = sE_R/M_Z^2$ to write

$$
\delta F_{\nu_j}(D) = E_R F_{\nu_j}(E_R, 0) \int \frac{ds}{M_Z^2} \left[ 1 - \exp\{-\sigma_{ann}(s) S_j(D)\} \right].
$$

(4)
Contact is made with our earlier estimate if $\sigma_{\text{ann}}(s)S(D)$ is small compared to one. Then it is sufficient to keep the first nonzero term in the Taylor series of the bracketed factor. This leads to

$$\delta F_{\nu_j}(D) \approx E_R <\sigma_{\text{ann}}> S_j(D) F_{\nu_j}(E_R,0). \quad (5)$$

where the averaged annihilation cross section is as defined before, $<\sigma_{\text{ann}}> \equiv \int \frac{d\sigma}{M_Z^2}\sigma_{\text{ann}}(s)$, which is equal to $4.2 \times 10^{-32}\text{cm}^2$. Eqn. $(5)$ shows that for $S(D) \ll 1/\sigma_{\text{ann}}(s)$, the rate for Z–bursts depends linearly on the relic neutrino column density, which of course scales as $1/NL^2$, satisfying our scaling law. For $S(D) \ll 1/\sigma_{\text{ann}}(s)$, the attenuation of the neutrino flux over the distance from the source to the GZK zone will be small, and we may set $D$ in Eqn. $(5)$ equal to the GZK distance to get the Z–burst rate within the GZK zone.

### 3.4 Anisotropic Clustering

If the neutrino cluster is not isotropic with respect to our preferred position at earth, the Z–burst rate will not be isotropic either. The anisotropic rate is easily accounted for by generalizing the distance $D$ to be vector $\hat{D}$. The column density integral in Eqn. $(2)$ then becomes an integral of $n_{\nu_j}(\hat{x})$ along the vector $\hat{D}$. There is weak evidence that the super–GZK events may correlate with the Supergalactic plane. Such a correlation would arise naturally in the model presented here if the SG plane provided either the super–GZK neutrino flux or the potential well in which neutrinos are clustered (or both).

### 3.5 Possible Flavor–Mixing of Neutrinos

It is possible, in fact it is probable, that massive neutrinos exhibit mixing in analogy to the quark sector of the standard model of particle physics. In the mixed case, the flavor states are unitary mixtures of the mass states. Letting $\alpha = e, \mu, \tau$ label flavor and $j = 1, 2, 3$ label mass, one writes $|\nu_\alpha> = \sum_j U_{\alpha j}|\nu_j>$. Then each neutrino flavor at the resonant energy of a given mass state has a nonzero probability to annihilate, but with an extra probability factor of $|U_{\alpha j}|^2$. For example, the $\nu_\mu$’s and $\nu_e$’s from pion and mu decay will annihilate at the resonant energy of $m_2$ with the probability factors $|U_{\mu 2}|^2$ and $|U_{e 2}|^2$, respectively,
times what we have calculated above. We remark that if the lepton mixing mirrored the known mixing among the quarks, then the heaviest of the three neutrino mass states should be mainly mixed with the third generation $\nu_{\tau}$. If this were the case, then $|U_{\mu 3}|^2$ and $|U_{e 3}|^2$ would be small, and annihilation with a significant probability at the lowest resonant–energy $E_{R3}$ would arise only if there were a substantial $\nu_{\tau}$ flux (of unspecified origin). However, recent data from the SuperKamiokande Collaboration validates earlier evidence that the $\nu_{\tau}$ and $\nu_{\mu}$ may be near–maximally mixed (see “Note added” at the paper’s end), in which case significant $\nu_{\mu}$ annihilation on $\nu_3$ is expected.

We have calculated annihilation probabilities assuming no mixing. Without mixing, each flavor–type has a unique mass, and the resonant energy of each neutrino flavor is unique. If there is mixing, the factors $|U_{\alpha j}|^2$ can be easily multiplied in.

### 3.6 Dirac Versus Majorana Neutrinos

In addition to the flavor–mixing issue, another subtlety arises if the massive neutrinos are of Dirac–type as opposed to Majorana–type. As the universe expanded and the momenta of the relic neutrinos red–shifted, they evolved to the unpolarized nonrelativistic state which they occupy today. As a result, if the neutrino is a Dirac particle, then the sterile right–handed neutrino and the sterile left–handed antineutrino fields are populated equally with the two active fields. Therefore, for Dirac neutrinos the active densities available for annihilation with the incident high energy neutrino are half of the total densities, and the $Z$–burst production probability is half of what we quote in this article. In contrast, for Majorana neutrinos there are no sterile fields and the total densities are active in annihilation. Majorana neutrinos are favored over Dirac neutrinos in currently popular theoretical models with nonzero neutrino mass.

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9 The phenomenon of neutrino oscillations will occur when neutrinos are mixed, and it will affect the flavor populations of the cosmic–ray neutrinos. However, it will not affect the calculation of annihilation, because the $\nu - \bar{\nu}$ annihilation process requires just a single transformation from flavor to mass basis, so the phase differences induced between mass states are not observed.

10 If $U_{\mu 3}$ and $U_{e 3}$ are small, then the vacuum oscillation probabilities for $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ are also small.
4 Z–bursts and the p, n, γ Flux Above $10^{20}$ eV

Having argued that cosmic neutrino/relic neutrino annihilation in our galactic halo is a possible candidate for Z–burst events, let us study more closely the particle spectrum contained in the burst. The decay products of the Z are well–known from the millions of Z’s produced at LEP and at the SLC.[19]. The respective branching fractions for Z–decay into hadrons, neutrino–antineutrino pairs, and charged lepton pairs are 70%, 20%, and 10%. The mean multiplicity $<N>$ in hadronic Z decay is about 40 particles, of which, on average, 17 are charged pions, 9 are neutral pions, 4.4 are kaons, one is the eta meson, 3.3 are light–quark vector mesons (mainly the $\rho^0$ and $K^*$’s) which decay to 1.4 kaons, 0.6 $\pi^0$’s, and 3.2 $\pi^\pm$’s, on average; 0.6 are light–quark tensor mesons, 0.8 are $D$ mesons, 0.5 are $B$ mesons, and importantly, 1.35 are baryon–antibaryon pairs which become 2.7 nucleons and antinucleons. The one eta meson decays on average to 0.8 hard photons (two–photon decay), and through three–body decays to 1.2 $\pi^0$’s, and 0.56 $\pi^\pm$’s. We count the 0.8 hard photons of the eta as 0.4 $\pi^0$ for simplicity. The 1.3 heavy–quark $D$ and $B$ mesons have many modes available for their decays. We estimate their average collective final state as 1.3 kaons and 3 pions. The total of seven kaons decay to two– and three–body final states including on average another 3 neutral pions and 5 charged pions. The total pion and nucleon count for the Z–burst is then, 15 $\pi^0$’s, 28 $\pi^\pm$’s, and 2.7 nucleons (we now mean “nucleons” to include the antinucleons as well).

Among the 15 $\pi^0$’s, we have seen that 9 are produced directly in Z–decay, while the other 6 arise from decays of various hadronic resonances. A comparison of the data[19] for the momentum spectra for direct pions and for protons produced in Z–decay reveals that the proton momentum is about three times that of the direct pion, on average. Taking into account the differing masses as well, we estimate that the boosted mean energy per proton is larger than that of a direct pion by a factor $\sim 3.5$. The energy of the six $\pi^0$’s produced through resonance decays may be softer yet. Weighting the direct and secondary pions appropriately then (we take the latter to be softer by $\sim 3$), we arrive at a factor of about six for the softness of the mean pion energy compared to the mean nucleon energy. Since the photon on average carries half of the parent pion energy, the mean energy of the
\[\pi^0\text{-decay photons in a } Z\text{-burst is expected to be less than that of the nucleons by an order of magnitude. The energetics arguments we have presented here are qualitative. The next step would be to quantitatively obtain the energy spectrum of the super–GZK nucleons and photons by boosting the output of a simulation program such as ISAJET}^{[32]}, \text{which is fitted to the final state data in } Z\text{-decay.}

The fate of the charged–pions' decay products is somewhat complicated. Each charged pion goes through a decay chain which results in an \(e^\pm\) and three neutrinos.\(^{[4]}\) At the very least, this decay chain provides a mechanism for injecting copious \(e^\pm\)’s into any gravitational well rich in relic neutrinos. Each of the four leptons from the decay chain carries on average a quarter of the charged pion’s energy. The produced neutrinos are below the original resonant energy of their parent cosmic ray neutrino, so they are not relevant for our purposes of creating and identifying super–GZK candidates (we do not consider the possibility of a second resonant energy nearby the first). The \(e^\pm\)'s are, however, another potential source of super–GZK photons, via the inverse Compton (IC) process. The boost factors for the \(e^\pm\)'s are \(\gamma_e \sim \frac{E_R}{160}/m_e \sim 5 (\text{eV}/m_\nu) \times 10^{13}\). When a photon with incident energy \(\omega\) and incident angle \(\theta\) with respect to the \(e^\pm\) velocity is scattered through an angle \(\alpha\), the final photon energy as a fraction of the incident \(e^\pm\) energy is \((1 - \cos \theta)(1 - \cos \alpha + \frac{m_e}{2\omega\gamma_e})^{-1}\). Thus, photons with initial energy satisfying \(\omega \gg E_{\text{crit}}\), with \(E_{\text{crit}} \equiv \frac{m_e}{\gamma_e} \sim 10^{-8} (m_\nu/\text{eV}) \text{ eV}\), will scatter “catastrophically” off the \(e^\pm\)'s to acquire an energy of order \(E_R/160 \sim 2 (\text{eV}/m_\nu) \times 10^{19} \text{ eV}\). \(E_{\text{crit}}\) is in the radio range, \(\sim 10 \text{ MHz or } \sim 10 \text{ m}\). Photons in the 2.73K microwave background have energies greatly exceeding \(E_{\text{crit}}\), and may acquire super–GZK energies from IC–scattering, depending on the value of \(m_\nu\). However, the \(e^\pm\)–photon scattering cross–section for photons with energy exceeding \(E_{\text{crit}}\) is in the Klein–Nishina regime, down considerably from the Thomson cross-section, by roughly the factor \((E_{\text{crit}}/2\omega) \ln(2\omega/E_{\text{crit}})\).

\(^{[11]}\)The boost factor for the charged and neutral pions is typically \(\gamma_\pi = \frac{E_R}{40}/m_\pi = 7 (\text{eV}/m_\nu) \times 10^{11}\), which leads to mean decay lengths \(c\gamma_\pi\tau_\pi\) of \(5 \times (\text{eV}/m_\nu) \times 10^{14} \text{ cm} \sim 30 (\text{eV}/m_\nu) \text{ AU}\) and \(20 (\text{eV}/m_\nu) \text{ km}\), respectively. The boost factor for the muon in the charged pion decay chain is typically \(0.75 E_R/40/m_\mu = 7 (\text{eV}/m_\nu) \times 10^{11}\), very similar to the pion boost, leading to a mean decay length for the muon of \(5 \times 10^{16} \text{ cm} \sim 0.02 \text{ pc}\). These decay lengths are very short compared to the mean pion and muon interaction lengths in the photons and hydrogen of the intergalactic or even interstellar medium, so the decays are not impeded by interactions or absorption.
In contrast to the CMB photons, any ambient radio photons with energies far below $E_{\text{crit}}$ will scatter with the full strength of the Thomson cross section ($\frac{8\pi\alpha^2}{m_e^2} = 6.7 \times 10^{-25}\text{cm}^2$); however, they will not gain a significant fraction of the $e^\pm$’s energy. Magnetic fields in the Supergalactic cluster, Local Group, or the Galactic halo may also degrade the energy of the $e^\pm$’s via synchrotron emission\textsuperscript{3}. A numerical simulation including modeling of the radio, IR and starlight backgrounds, and the magnetic fields, may be needed to determine the outcome of the energy loss of the $e^\pm$’s. In what follows, we will conservatively ignore the possible contribution to the super–GZK photon flux from catastrophic IC–scattering of the 28 $e^\pm$’s, and focus on the 30 $\pi^0$–decay photons and the 2.7 nucleons in the $Z$–burst.

In the $Z$–burst, the 30 photons from $\pi^0$–decay and the 2.7 nucleons are the candidate primary particles for inducing super–GZK air showers in the earth’s atmosphere. The 2.7 nucleons may be protons or neutrons. With its ten–minute rest–frame lifetime enhanced by the boost factor $\gamma_N \sim E_R/ < N > m_N \sim (\text{eV}/m_\nu) \times 10^{11}$, the neutron will typically travel $ct_n\gamma_N \sim (\text{eV}/m_\nu) \text{ Mpc}$, and then transfer virtually all of its energy to a proton when it decays. The a priori photon to nucleon ratio is about 10 on average.\textsuperscript{13} However, the hardness of the nucleon spectrum compared to the photon spectrum mitigates this ratio if a selection is made for the very highest energy particles. Moreover, the differing attenuation reactions and attenuation lengths of nucleons and photons will affect the observable event ratio. Also, the development of photon–initiated air–showers at $E \sim 10^{20}$ eV is skewed by

\textsuperscript{12}Integration of the energy loss formula for relativistic $e^\pm$’s due to synchrotron emission and IC–scattering in the Thomson regime gives\textsuperscript{33} $E_e(x) = E_e(0)(1 + \beta E_{20} x)^{-1}$, with the distance $x$ measured in parsecs, $E_{20} \equiv E_e(0)/10^{20}$ eV, and $\beta = 80 (\rho_\gamma + 0.6 B^2/8\pi)$. Here, $\rho_\gamma$ is the energy density of the photon background with energy $\ll E_{\text{crit}}$ in units of eV/cm$^3$, and $B$ is the ambient magnetic field in units of microgauss. The electron or positron loses 90% of its energy in a distance $x_{90} = (10/\beta E_{20}) \text{ pc}$. Across cosmological distances, there is only an upper limit on $B$ of $\sim 10^{-3}$; models typically give $B \sim 10^{-6}$ or less. However, in clusters of galaxies $B \sim 1$ has been measured, probably generated by jets from radio galaxies\textsuperscript{34}. With $B \gtrsim 1$, $x_{90}$ is less than a parsec for $E_e \gtrsim 5 \times 10^{20}$ eV. In our Galaxy, $B$ is typically 3, increasing to about 30 when averaged over the inner AU of our solar system, and to about $10^6$ within 1000 km of the earth’s surface. The fraction of $e^\pm$ energy transferred to an individual photon through the synchrotron process is typically small, $\sim 10^{-20} \gamma_e B_\perp$, where $B_\perp$ is the magnetic field perpendicular to the $e^\pm$ trajectory.

\textsuperscript{13}As just noted above, there is the possibility of even more super–GZK photons resulting from IC–scattering of the CMB by the $\sim 28 e^\pm$’s liberated in the decay of the $\pi^\pm$’s.
the LPM effect\cite{35} and by high–altitude photon–absorption on the earth’s magnetic field\cite{17}, which may affect their identification and measurement. Finally, the average values for the multiplicities and energies presented here must be used with some caution, since fluctuations in multiplicity and particle-types per event, and in energy per individual particle, are large\cite{17}.

5 Further Signatures from \(Z\)–bursts

The particle spectrum in \(Z\)–decay and the Lorentz factor of the \(Z\), \(\gamma_Z = E_R/M_Z = M_Z/2m_\nu = 0.9 \times 10^{11}\), determine the possible signatures of \(Z\)–bursts. Going beyond simple shower–counting above the GZK cutoff, we comment on some of the possible \(Z\)–burst characteristics:

(i) The \(Z\)–decay products which in the \(Z\) rest frame lie within the forward hemisphere are boosted into a highly–collimated lab–frame cone of half–angle \(1/\gamma_Z = 2 (m_\nu_j/eV) \times 10^{-11}\) radians. \(Z\)–bursts originating within \(20 (eV/m_\nu_j)\) parsecs of earth, if directed toward the earth, arrive with a transverse spatial spread of less than one earth diameter. It is possible for the decay products of a single not–to–far distant \(Z\)–burst to initiate multiple air showers. A large area surface array (\textit{e.g.} the Auger project\cite{36}) or an orbiting all–earth observing satellite (\textit{e.g.} the OWL proposal\cite{37}) could search for these nearly coincident showers.

(ii) The mean number of baryon–antibaryon pairs per hadronic \(Z\)–decay is 1.35. Baryon number conservation requires each hadronic \(Z\) decay to contain an integer number of baryon–antibaryon pairs. If the baryon and antibaryon both strike the earth’s atmosphere, but are sufficiently separated in arrival time, repeater events may be observed. If the number of baryon pairs per hadronic shower is governed by Poisson statistics, then the probabilities for 0, 1, 2, 3, 4, and 5 pairs are 26\%, 35\%, 24\%, 10\%, 4\%, and 1\%, respectively. In estimating the difference in arrival time of two nucleons, it may be useful to work with the rapidities of the baryon and antibaryon, since rapidity is an additive variable under boosts. The rapidity difference \(\Delta y = y - \bar{y}\) is invariant under boosts. The difference in arrival times of two

\textsuperscript{14}Simulations on an event–by–event basis are possible with Monte Carlo programs such as ISAJET; an additional layer of reality may be added by also simulating the development of the electromagnetic cascade in the cosmic medium.
baryons is $\Delta t = D_Z (\tanh y - \tanh \bar{y})$, where $D_Z$ is the distance of the $Z$–burst from earth; the difference in arrival times of a photon and baryon is $\Delta t = D_Z (1 - \tanh y)$. Correlated baryon–antibaryon rapidity distributions may be sought in terrestrial $Z$–decay data. If QCD implements baryon number conservation locally, then $\Delta y$ will generally be small.

(iii) The energy of the $Z$–bursts are fixed at $4 \left( \text{eV}/m_{\nu} \right) \times 10^{21} \text{ eV}$ by the neutrino mass(es). The energy of individual particles produced in the burst can approach this value but cannot exceed it. This may serve to distinguish the $Z$–burst hypothesis from some recent speculations for super–GZK events based on SUSY or GUT–scale physics\[14\], in which cutoff energies are expected to be much higher.

(iv) From the highest super–GZK event energy $E_{\text{max}}^\text{max}$, one can deduce an upper bound on the neutrino mass of $m_{\nu} < M_Z^2/2E_{\text{max}}^\text{max} = 4 \left( 10^{21} \text{eV}/E_{\text{max}}^\text{max} \right) \text{ eV}$. Similarly, from the mean energy $<E>$ of super–GZK events one can estimate the mass of the participating neutrino flavor via $m_{\nu} = M_Z^2/2E_R \sim M_Z^2/(2<N><E>) \sim 0.5 \left( 10^{20} \text{eV}/<E> \right) \text{ eV}$; if there is a selection bias toward events at higher energy, then this formula gives a lower bound on the neutrino mass.

(v) There could be a “neutrino pile–up” at two to three decades of energy below $E_R$. These pile–up neutrinos are the result of the hadronic decay chain which includes $Z \rightarrow \sim 28 (\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow \nu_\mu + \bar{\nu}_\mu + \nu_e/\bar{\nu}_e + e^\pm)$; i.e. each of the 70% of the resonant neutrino interactions which yield hadrons produces about 85 neutrinos with mean energy $\sim E_\pi/4 \sim E_R/160$. These neutrinos are in addition to the neutrinos piling up from the decay of pions photo–produced by any super–GZK nucleons scattering on the 2.7K background.

(vi) A very interesting issue is to what extent the boosted $Z$–decay products will contain a copious amount of observable brehmsstrahlung gamma–rays. Even radio and infrared brehmsstrahlung becomes observable after boosting with $\gamma_Z \sim 10^{11}$. For example, a $10^{-5}$ radio photon becomes an MeV gamma–ray, and a $10^{-2}$ eV infrared photon becomes a hard GeV gamma–ray. The glib statement that “the 1/E brehmsstrahlung singularity produces photons with such low energy that they cannot be observed” may hold for terrestrial accelerators, but does not hold for $Z$–bursts. A careful field theory analysis of the extreme infrared in $Z$–decay is required to address this issue\[38\]. It seems possible that $Z$–bursts may generate short duration gamma–ray bursts observable at earth. The hadronization time

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when boosted to the lab frame, $\gamma_Z \times \text{fm}/c \sim 10^{-12}$ s, sets the basic time-scale for particle emission and brehmsstrahlung.

There are a few more remarks that should be made concerning the hypothesis under discussion:

(i) A common problem that arises when trying to explain the super–GZK air showers with new physics concerns the smoothness of the event rate as a function of energy. Both the event rate and the event energy above the GZK cutoff follow extrapolations from below the cutoff. Smooth extrapolations are not necessarily expected when new physics is invoked to explain the super–GZK events. The proposal in this article makes use of standard model particles and interactions only, and so is not automatically subject to this criticism. If the product of the super–GZK neutrino flux per flavor, times the annihilation probability within the GZK zone (which we have estimated to be $10^{-2\pm1}$), times the photon and nucleon multiplicity per burst ($\sim 30$), is comparable to the highest–energy proton flux, then a smooth extrapolation in event rates results. Such an occurrence does not seem unnatural.

(ii) To be more quantitative, the observed rate of super–GZK events can be used to predict the cosmic neutrino flux at the resonant energy, within the context of the neutrino–annihilation hypothesis. Let $\mathcal{L}(> E_{\text{GZK}})$ be the luminosity of super–GZK events (in units of events/area/time/solid angle). The relation between this event luminosity and the neutrino flux is $\mathcal{L}(> E_{\text{GZK}}) = E_R F_{\nu_j}(E_R) \times 10^{-2\pm1} \times (\sim 30)$, which implies $F_{\nu_j}(E_R) = 10^{0.5\pm1} \times E_R^{-1} \mathcal{L}(> E_{\text{GZK}})$. Because the high energy cross-section is several orders of magnitude below the hadronic cross-section at these energies, this neutrino flux is not directly measureable in any neutrino detectors presently under development.

(iii) The highest–energy neutrino cosmic–ray flux should point back to its sources of origin, and the super–GZK event arrival directions should point back to these same sources. However, if the highest–energy neutrino flux is diffuse, then the super–GZK event directions should correlate with the spatial distribution of the relic neutrino density. The solid angles subtended by any nearby halos may offer preferred directions for super–GZK events. As

$^{15}$There is statistically weak evidence for a possible event gap at energies just above the GZK cutoff. There is also weak statistical evidence for the pile–up of cosmic rays just below the GZK cutoff. If either phenomenon is confirmed in the future, then the case for new physics is strengthened.
discussed in the earlier section on neutrino clustering, the Supergalactic plane may be the most probable cluster domain. Perhaps the angular distribution of super–GZK events can be used to perform neutrino–cluster tomography. (We remark that significant flattening of dark halos is suggested by observation[40] and by numerical simulations[41], and that a flattened halo implies a larger number density by a factor of the axes ratio.)

(iv) If the super–GZK events are due to neutrino annihilation on relics as hypothesized here, and if the high–energy neutrino flux is eventually measured, then an estimate of the relic neutrino column density out to $D_{\text{GZK}} \sim 50 \text{ Mpc}$ may be made: the column density of the annihilating neutrino flavor out to $D_{\text{GZK}}$ is $S_{\nuj}^{\text{GZK}} \sim \mathcal{L}(> E_{\text{GZK}})/[< \sigma_{\text{ann}}> F_{\nuj}(E_R) \delta E_R] = 4.5 \times 10^{32} \mathcal{L}(> E_{\text{GZK}})/[E_R F_{\nuj}(E_R)] \text{ cm}^{-2}$. If $F_{\nuj}(E)$ is measured below the resonant energy, an estimate of the neutrino column density can still be made by extrapolating the flux to $E_R$. For example, if a power law is assumed with a spectral index $\alpha$, then $F_{\nuj}(E_R) = (E/E_R)^\alpha F_{\nuj}(E)$. (v) If the cosmic sources of the highest–energy neutrino fluxes are themselves located in potential wells containing bound relic neutrinos, then the neutrinos produced at the resonant energy may well annihilate on the relic neutrinos in the source halo. In such a case, a depleted flux of resonant energy neutrinos would emerge from the source. However, the recessional velocity due to the Hubble flow will red–shift neutrinos with energies just above the resonant value at the source to the resonant energy in our vicinity. Any source with cosmological red–shift $z_s$ above $z = \delta E/E_R = 2\Gamma Z/M_Z \sim 0.06 (0.03$) will provide our Galaxy with a full (half) flux of neutrinos at the resonant energy. If the spectrum of neutrinos from a single source can be measured, one might see the depletion at $E_R$ due to absorption in our halo, and a depletion at $(1 - z_s) E_R$ due to absorption in the source halo.

6 Conclusions

In summary, if one or more neutrino mass is of order $0.1 \text{ to } 1 \text{ eV}$, and if there is a sufficient flux of cosmic ray neutrinos at $\gtrsim 10^{21} \text{ eV}$, then $\nu_{\text{ex}} + \bar{\nu}_{\text{relic}}$ (or vice versa) $\to Z \to \text{hadrons} \to \text{nucleons and photons}$ in our Supergalactic cluster, Local Group, or Galactic halo may be the origin of air shower events above the GZK cutoff. An abundant number of possible signatures to validate or invalidate this hypothesis were presented in this work. The weakest
link in the hypothesis is probably the large neutrino flux required at the resonant energy, well above the GZK cutoff. Such a flux severely challenges conventional source models in two ways: first of all, source dynamics must be able to produce the neutrino flux above $10^{20}$ eV, and secondly, any concomitant photon flux must not violate existing upper limits. The super–GZK photon flux produced in non–local $Z$–bursts is also a potential liability, for it will cascade down to produce an abundance of gamma–rays which must not violate existing upper bounds.

A logical next step toward validating or invalidating the hypothesis is a numerical simulation of the details of the model[42]. The more pressing details amenable to simulation are the energy spectrum of the super–GZK nucleons and photons, the cluster densities of the relic neutrinos, and the flux of gamma–rays resulting from cascadading of the highest–energy photons produced in the $Z$–bursts and at the neutrino source. If future simulations and the inevitable increase in event statistics validates the hypothesis, then highest–energy air–showers are our window to the relic neutrino gas liberated from the primordial early–universe plasma when the universe was only one second old!

**Acknowledgements:** This work is supported in part by the U.S. Department of Energy grant no. DE-FG05-85ER40226. Some of this work was performed at the Aspen Center for Physics. This paper has benefitted from discussions with Francis Halzen and Venya Berezinskii. I thank Stuart Wick for a careful reading of the manuscript.

**Notes added:**

1. As this manuscript was being completed, a related work appeared on the net, astro-ph/9710029, by D. Fargion, B. Mele and A. Salis.

2. Two recent experiments have updated the neutrino “anomalies” which are most simply interpreted in terms of oscillations of neutrinos having nonzero mass. The neutrino mass values which emerge from these data analyses are within the range of interest for producing $Z$-bursts at energies just above the GZK cutoff. The SuperKamiokande experiment finds that the neutrino-oscillation explanation of the anomalous flavor-ratio of atmospheric neu-
trinos requires $\nu_\mu \rightarrow \nu_\tau$ oscillations (or $\nu_\mu \rightarrow \nu_s$ oscillations to a sterile neutrino $\nu_s$) with a mass–squared difference of $0.5 \times 10^{-3} \lesssim \delta m^2_{\text{atm}} \lesssim 6 \times 10^{-3}$ eV$^2$ and mixing angle $\sin^2 2\theta_{\text{atm}} \approx 0.8$. This implies a roughly 50% overlap in probability for the $\nu_\mu$ with a mass eigenstate having a mass of at least 0.02 eV. The LSND experiment has additional evidence for $\nu_\mu$–$\nu_e$ flavor conversion, which continues to support the inference of a mass–squared difference in the range $0.3$ eV$^2 < \delta m^2_{\text{LSND}} < 10$ eV$^2$, and a small mixing probability (although the KARMEN experiment rules out part of the LSND allowed region). If this LSND result is validated with future experiments, it implies some overlap of both $\nu_\mu$ and $\nu_e$ with a mass eigenstate having a mass of at least 0.5 eV.

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Figure Caption:

Fig. 1. Schematic diagram showing the production of a $Z$–burst resulting from the resonant annihilation of a cosmic ray neutrino on a relic neutrino. If the $Z$–burst occurs within the GZK zone ($\sim 50$ Mpc) and is directed towards the earth, then photons and nucleons with energy above the GZK cutoff may arrive at earth and initiate air showers.
$D_{GZK} \sim 50 \text{ Mpc}$

$\nu_{\text{RElic}} \rightarrow \nu_{\text{cosmic ray}}$

$15\pi^0 \rightarrow 30\gamma$

$2.7 \text{ nucleons}$

$28\pi \rightarrow e^\pm, \nu\bar{\nu}$