Investigation of a viscous fluid drain under the pressure drop with simultaneous filling of a rectangular tank

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Abstract. In this work the study of the hydrodynamic process, realized when a viscous fluid is drain from a triangular tank with simultaneous filling of a rectangular tank in a plane approximation, was carried out. The flow is described system of equations which is composed of Navier-Stokes equations and continuity equation in dimensionless variables. On the solid wall the no-slip conditions are specified. On free surfaces, the continuity conditions for normal and tangential stresses are satisfied. At the initial moment of time the fluid in drain tank is located. At the moment when the free surface reaches the level of the drain orifice, the drain tank is again considered to be filled of fluid. According to this a continuous drain process is carried out until the lower tank is completely filled without gas breakthrough from of the drain tank. The formulated problem is solved numerically using the PLIC VOF method. Parametric studies of this process were conducted. Different regimes of free surfaces formation for both the draining process and the filling process have been identified depending on the values of the determining parameters. The distribution of the kinematic characteristics of the flow is obtained.

1. Introduction

One of the stages of the technological processing of polymeric materials using the method of casting is drain the polymer composition from the mixer with simultaneous filling of the mold. A characteristic feature of the flow, implemented in the process of draining and filling, is the presence of free surfaces in the drain and filled tanks.

At a certain stage of draining, the front of the free surface is deformed with the formation of a funnel and the subsequent gas breakthrough into the drain orifice. The gas breakthrough into the fillable tank leads to the formation of defects of the molded product consolidation. Therefore, there is a need to control time moment when the front of free surface reaches the drain orifice plane. In addition, the proper organization of the forming product process requires a study of the process tank filling draining fluid.

The literature presents studies about the drain of fluid from the tanks. In [1], the funnel formation in the emptying process is investigated both the conical and cylindrical tank in the regime of creeping flow of fluid. In [2], in order to reduce the mass of fluid that remained in the drain tank at the moment of gas breakthrough, the use of a various kinds structural elements was proposed. This leads to the formation of a planer free-surface shape and, as a result, a decrease of the remained fluid in the drain tank. In [3], the results of numerical simulation of a viscous fluid drain process are presented for the creeping flow approximation under the given pressure drop. The evolution of the free surface and the shape of the jet depending on the determining parameters are shown. Regimes with the rapid formation of a funnel are...
identified. In [4], in addition to the drain, the behavior of the free surface in the fillable tank is investigated.

The results of experimental studies on the dip formation on the free surface occurring when the fluid drains from cylindrical and conical tanks through a circular orifice on the bottom are presented in [5-8]. The numerical simulation of the fluid outflow from conical and cylindrical tanks accompanied with the funnel formation is carried out in [9] and [10], respectively.

The purpose of this work is investigation of the process of a viscous fluid drain from a tank under the gravity and pressure drop; identify of the features of the shapes formation of free surfaces; determine of the kinematic characteristics of the flow depending on the value of the determining parameters.

2. Formulation of the Problem
The viscous fluid flow that is realized when a drained from triangular tank and simultaneously filling of a rectangular tank in a plan approximation is considered. In fig. 1 the solution domain is shown schematically.

![Figure 1. Solution domain.](image)

The flow is described equations Navier-Stokes and continuity equations which of in dimensionless variables in Cartesian coordinate system have a form

\[
\text{Ga} \left( \frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} \right) = -\frac{\partial p}{\partial x} + \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]

\[
\text{Ga} \left( \frac{dv}{dt} + u \frac{dv}{dx} + v \frac{dv}{dy} \right) = -\frac{\partial p}{\partial y} + \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + 1,
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.
\]

Here \( u, v \) is the velocity vector projections on the axes of the Cartesian coordinate system \( x, y \); \( t \) is the time; \( p \) is the pressure.

The following values are used as scales of length, speed, time and pressure: \( l \) is the drain orifice width, and the complexes \( pgl^2 / \mu \), \( \mu / pgl \) and \( pgl \), respectively. The formulation of problem includes dimensionless parameters: Galilei number (\( \text{Ga} \)); excess pressure on the free surface in the drain tank (\( P \)) and geometric characteristics. The Galilei number and excess pressure determined by the formulas
PP_

Here \( \mu \) is the viscous of fluid; \( \rho \) is the density of fluid; \( g \) is the gravitational acceleration.

On the solid wall, \( \Gamma_1 \), the no-slip conditions are specified. On free surfaces, \( \Gamma_2 \) and \( \Gamma_3 \), the continuity conditions for normal and tangential stresses are satisfied.

At the initial moment of time the fluid in drain tank is located, wherein free boundaries, \( \Gamma_2 \) and \( \Gamma_3 \), are straight line \( y = H_0 \) and \( y = (H_0 + h + H) \), respectively (fig. 1).

At the moment when the free surface, \( \Gamma_3 \), reaches the level of the drain orifice \( y = (H_0 + h) \), the drain tank is again considered to be filled of fluid to level \( y = (H_0 + h + H) \). According to this a continuous drain process is carried out until the lower tank is completely filled without gas breakthrough from of the drain tank.

3. Method of Solution

The problem formulated is solved numerically using the VOF method [11]. In the internal grid nodes, the finite volume method is used to calculate the components of the velocity vector, and the pressure is calculated using the SIMPLE procedure [12]. The free surface motion is realized by computational technology PLIC-VOF-method [13], which is a modification of the original method VOF.

All the following calculations were performed for the following sizes: \( L = 5 \), \( H_0 = 8 \), \( h = 1 \); \( l = 1 \) and \( H = 4 \) (angular opening of triangle is 90°). The pressure inside the fillable tank was considered to be zero, on the free surface of the fluid in the drain tank it was assumed equal to the dimensionless value of excess pressure \( P \).

![Figure 2](image)

**Figure 2.** The velocity profiles (a) at the section \( y = (H_0 + 0.5h) \), volumetric flow, \( Q \), rate versus time (b) through the section \( y = (H_0 + 0.5h) \) and free surface (c) at time \( t = 34 \) at various grid steps.

For verification of developed algorithm and calculation program, the approximating convergence is tested on the set of grids. The shape of the free surface in the drain tank, the volumetric flow rate and the velocity profile in the drain orifice were chosen as controlled characteristics. Used the following values of defining parameters: \( Ga = 10 \) and \( P = 4 \). Velocity profiles, \( V_y \), at the section \( y = (H_0 + 0.5h) \), volumetric flow rate, \( Q \), versus time through the section \( y = (H_0 + 0.5h) \) and shapes of the free surface on the set of grids at the moment \( t = 34 \) presented in the figures 2a, 2b and 2c, respectively.

| \( \Delta h \) | \( V_y \) (on the axis of symmetry) | \( Q \) (at time \( t = 34 \)) | \( H_y \) (on the axis of symmetry) |
|---|---|---|---|
| 0.1 | -0.366718 | 0.251214 | 11.4958 |
| 0.05 | -0.357353 | 0.240289 | 11.5743 |
| 0.025 | -0.352378 | 0.235777 | 11.6066 |
The maximum value of velocity and maximum value of volumetric flow rate obtained in cross section of the drain orifice, and height of point of free surface which is on the plane of symmetry at time \( t = 34 \) at various grid steps are shown in table 1. According to this data, the approximating convergence of the algorithm is observed.

4. Results and Discussion

In the course of parametric investigations, various modes of formation of free surfaces in a drainage tank were revealed, depending on the values of the governing parameters: the flow with the rapid funnel formation (fig. 3a); drain which maintaining the plane shape of the free surface until the moment of reaching the drain orifice (fig. 3b). Flow patterns are considered until the moment when the free surface front touches the drain orifice.

![Figure 3. The free surface evolution for P = 0: a – Ga = 1, b – Ga = 100.](image)

For a small values of the Galilei number (fig. 3a), an intensive flow of fluid near the plane of symmetry is realized, which leads to the rapid formation of a funnel in the drain tank. For large Ga (fig. 3b), a thin layer of fluid forms on the solid wall during the drain, and a part of the free surface outside this layer moves while maintaining a plane shape.

The rapid formation of a peak in the center of the interface is associated with a gas breakthrough into the fillable tank. A similar behavior of the free surface in the creeping flow regime was studied earlier and described in details in [1].

The volume of remaining fluid in the drain tank at the time of gas breakthrough is one of the parameters which characterize the process of fluid drain. This volume is referred to as \( \Delta V \) and determined by the ratio of the volume of fluid in the drain tank at the initial time to the volume of fluid at the time of formation of the funnel.

| Ga   | 0.1 | 1   | 10  | 100 |
|------|-----|-----|-----|-----|
| \( \Delta V \) | 31  | 30  | 26  | 18  |

Table 2 shows the dependence of the volume of fluid remaining in the drain tank at the time of the gas breakthrough, on the value of the Galilei number.

| P    | 0  | 2   | 4   | 8   | 16  |
|------|----|-----|-----|-----|-----|
| \( \Delta V \) | 26 | 31  | 36  | 42  | 47  |

In the case when in the drainage tank an excess pressure \( P > 0 \) is realized, a flow with the rapid formation of a funnel is observed, similar to the flow at low Galilei numbers values. This trend is reflected in table 3, in which for different values of an excess pressure, \( P \), given the value of the volume of fluid remaining in the drain tank at the time of formation of the funnel.
Figure 4. Volumetric flow rate of fluid depending on the time until the time of gas breakthrough:

\( a \) – vs Ga for \( P = 0 \) (1 – Ga = 100, 2 – Ga = 10, 3 – Ga = 1),

\( b \) – vs P for Ga = 10 (1 – P = 0, 2 – P = 2, 3 – P = 4, 4 – P = 8, 5 – P = 16).

The dependence of the dimensionless volumetric flow rate of fluid through the drain orifice versus time is shown in the fig. 4a for different Galilei number, Ga, in fig. 4b for different values of excess pressure, P.

Figure 5. Topograms of fluid mass distribution for Ga = 10 and P = 0:

\( a \) – \( t = 10 \), \( b \) – \( t = 30 \), \( c \) – \( t = 68 \), \( d \) – \( t = 80 \), \( e \) – \( t = 120 \), \( f \) – \( t = 168 \).

The investigation of the fluid mass distribution entering at different time in the process of the filling of tank makes it possible to estimate the heterogeneity of the spatial distribution of the properties of the molded product material. Figures 5 and 6 demonstrate the evolution of the fluid mass distribution topograms in the filling of tank for different values of the Galilei number. Two adjacent portions of fluid are marked with different colors.

Figure 6. Topograms of fluid mass distribution for Ga = 100 and P = 0:

\( a \) – \( t = 30 \), \( b \) – \( t = 65 \), \( c \) – \( t = 121 \), \( d \) – \( t = 150 \), \( e \) – \( t = 210 \), \( f \) – \( t = 272 \).
The surface separating two adjacent portions of the fluid is traced using the marker particles arranged along the cross section of the drain orifice \( y = H_0 \) when a new drain tank is installed. These particles have no mass and are transported at the fluid velocity. Complex of the marker particles in the flow defines the surface that separates adjacent fluid portions. The motion of the markers is described by the following kinematic condition

\[
\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v. \tag{4}
\]

The coordinates of the marker particles in the flow are calculated by integrating equations (4).

**Conclusion**

Mathematical formulation of the problem for a viscous fluid drain under the pressure drop with simultaneous filling of a rectangular tank was formulated. A numerical solution algorithm based on the PLIC VOF method was developed. Parametric investigations of the process were conducted. The influence of the dimensionless criteria of the problem on the evolution of free surfaces in the drain and fillable tanks was demonstrated. For a small values of the Galilei number, an intensive flow of fluid near the plane of symmetry is realized, which leads to the rapid formation of a funnel in the drain tank. With the Ga increasing in the process of the fluid drain a thin layer of fluid forms on the solid wall during the drain, and a part of the free surface outside this layer moves while maintaining a plane shape. In the case when in the drainage tank an excess pressure \( P > 0 \) is realized, a flow with the rapid formation of a funnel is observed, similar to the flow at low Galilei numbers values.

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