The top quark mass in the minimal top condensation model with extra dimensions

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The minimal dynamical electroweak symmetry breaking through top condensation in the presence of compact large extra dimensions is studied. It is shown that thanks to the power-low evolution of gauge and Yukawa couplings the original BHL predictions for the top quark mass are significantly lowered and even for small cut-off scale $\Lambda \sim \text{few TeV}$ one can obtain experimentally allowed values.

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Despite the success of the Standard Model (SM) in describing the experimental data with impressive accuracy, the physical mechanism behind the electroweak symmetry breaking (EWSB) and somewhat related explanation of the masses and mass hierarchies of elementary particles remains as the most outstanding problem. The top condensation is an interesting mechanism for the dynamical EWSB (see [1] for review and references therein). The beauty and strength of this mechanism lies in the fact that usually the set assumptions are rather limited and that it can explain simultaneously the dynamical generation of the heavy top quark mass as well as all or part of the EWSB. A detailed investigation of the minimal scenario, where the EWSB follows from the top quark condensation alone, was done by Bardeen, Hill and Lindner (BHL) [2]. Eliminating the SM Higgs sector in favor of the following local, attractive, Nambu–Jona-Lasinio–type interaction

$$\mathcal{L}_{NJL}^{4-dim} = G(Q^T_L t_R)(Q^L t_R)$$

(1)

(here $Q^T_L = (t_L, b_L)$ is the third generation left–handed quark doublet and $t_R$ the corresponding right–handed singlet and color and weak isospin indeces are suppressed), in the fermion bubble approximation for sufficiently attractive $G$ ($G > \frac{8\pi^2}{3}$), they have obtained a non-trivial solution to the gap equation which relates the top quark mass $m_{\text{top}}$ with an ultraviolet cut-off $\Lambda$ and four-fermion coupling $G$

$$m_{\text{top}}^2 = \frac{\Lambda^2 - \frac{8\pi^2}{3} G^{-1}}{\ln(\Lambda^2/m_{\text{top}}^2)}$$

(2)

and a W-boson mass

$$m_W^2 = \frac{3g_5^2}{32\pi^2}m_{\text{top}}^2 \left( \ln(\Lambda^2/m_{\text{top}}^2) + \frac{1}{2} \right)$$

(3)

For a large cut-off, $\Lambda \approx 10^{15}$ GeV, the bubble approximation predicts the value for the top quark mass, $m_{\text{top}} \approx 163$ GeV, while $m_{\text{top}} \approx 1$ TeV for smaller cut-off, $\Lambda \approx 10$ TeV. This requires $G$ to be fine tuned in order to cancel the quadratic cut-off dependence in (2).

An improvement for the top quark mass can be achieved by considering a low-energy effective field theory which is just the full SM treating the composite Higgs field as an elementary degree of freedom. The only difference is the compositeness condition that the Higgs field becomes static at high ($\sim \Lambda$) energies, i.e. $Z_H(\Lambda) = 0$, thus recovering from the SM lagrangian together with equation of motion the structure of the basic NJL lagrangian (1). This compositeness condition means that the top quark mass is governed by an infrared fixed-point solution of the renormalization group equations (RGEs) [3]. Unfortunately, the full RGE analyse gives unacceptable predictions for the top quark mass, $m_{\text{top}} \approx 220 \div 430$ GeV for $\Lambda \approx 10^{10} \div 10^4$ GeV [2]. Thus the minimal BHL model as well as its various modifications (two Higgs and supersymmetric versions), while being in many aspects phenomenologically viable [4], has a common drawback predicting the value of the top quark mass too high to fit the experimental data. So, one is led to consider more complex symmetry breaking scenarios with more condensates and more parameters [1,4].

In this paper I consider the minimal BHL model assuming the existence of extra dimensions with relatively large compactification radii which tend to produce lower values for the top quark mass in agreement with experimental data even for lower cut-off scale $\Lambda$, thus avoiding the fine tuning required for large $\Lambda$. Extra space-time dimensions appear naturally in the string theory, and therefore, such an idea is highly motivated from the fundamental point of view. Recently, the possibility of large extra dimensions has received considerable attention and their role has been explored for gauge coupling unification [5,6], for neutrino mass generation [7], for suppersymmetry breaking [8], to provide an alternative solution to the gauge hierarchy problem [9]. Cosmological consequences [10], various phenomenological issues [11] as well as possible collider signatures [12] of large radii extra dimension have also been investigated.

The way compact extra dimensions is implemented in practice is to introduce towers of Kaluza-Klein (KK) excitations associated with gauge and matter fields. The exact details of the spectrum of KK states are, to some extent,
model dependent. Here I closely follow the models described in [5], where the extra dimensions are compactified on $S/Z_2$ orbifold (a circles subjected to further identification $y_\alpha \rightarrow -y_\alpha$, $\alpha = 1, \ldots, \delta$; $\delta$ denotes a number of compact dimensions). The KK exiitations in this case can be decomposed into even $\Phi_+(x,y)$ and odd $\Phi_-(x,y)$ fields

$$
\Phi_+(x,y) = \sum_{n_1=0}^\infty \cdots \sum_{n_4=0}^\infty \Phi^{(n_\alpha)}(x) \cos(n_\alpha y_\alpha / R)
$$

$$
\Phi_-(x,y) = \sum_{n_1=1}^\infty \cdots \sum_{n_4=1}^\infty \Phi^{(n_\alpha)}(x) \sin(n_\alpha y_\alpha / R)
$$

(4)

Here $R$ is the radius of compact dimensions (for simplicity I assume that all extra dimensions have the same radius). Since the appropriate transformation of the fields under the $Z_2$ parity is determined by interactions, half of the original KK states may be projected out according to the $Z_2$ parity of the fields. If only the odd tower is left, the zero mode is missing.

Non-supersymmetric theories can be more straightforwardly embedded into higher dimensions than those of supersymmetric, because KK states no longer need to form $N = 2$ multiplets as it is usually assumed in the supersymmetric case. Thus, as a minimal scenario I assume that the gauge bosons ($Z_2$–even) have KK exiitations, while the chiral SM fermions ($Z_2$–odd) transforming as adjoint representations of each SM gauge symmetry group in order to make corresponding gauge bosons massive [5]. Following to this framework, I assume that in four dimensions besides the ordinary NJL interaction effectively appear an infinite number of four-fermion interactions

$$
\mathcal{L}_{NJL}^{(4+\delta)-\text{dim}} = \sum_{n_1=0}^\infty \cdots \sum_{n_4=0}^\infty G^{n_\alpha}(\overline{Q}_L t_R)(\overline{t}_R Q_L)
$$

(5)

with $G^{n_\alpha} = \frac{1}{M_0^2 + (n_\delta^2/R^2)}$. The set of four-fermion interactions in (5) can be viewed as a result of integration over some heavy ($M_0^2$) state and its KK exiitations ($M_0^2 + n_\delta^2/R^2$) [13]. Introducing an auxiliary fields $H^{n_\alpha} = G^{n_\alpha}(\overline{t}_R Q_L)$ one can rewrite lagrangian (5) in the equivalent form:

$$
\mathcal{L}_{NJL}^{(4+\delta)-\text{dim}} = \sum_{n_1=0}^\infty \cdots \sum_{n_4=0}^\infty \left[ -(M_0^2 + \frac{n_\delta^2}{R^2})|H^{n_\alpha}|^2 + (\overline{Q}_L t_R)H^{n_\alpha} + h.c. \right]
$$

(6)

The set of static fields $H^{n_\alpha}$ acquire gauge invariant kinetic and self-interacting terms after radiative corrections are taken into account. So below the cut-off scale $\Lambda$ one obtains the SM lagrangian describing the interactions of gauge and Higgs bosons and their KK exiitations with each other and with chiral fermions living on the orbifold fixed points:

$$
\mathcal{L}_{NJL}^{(4+\delta)-\text{dim}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \sum_{n_1=0}^\infty \cdots \sum_{n_4=0}^\infty |Z_H| |D_\mu H^{n_\alpha}|^2 - M^2_H H^{n_\alpha} |H^{n_\alpha}|^2
$$

$$
-\frac{1}{2} \lambda (H^{n_\alpha} + H^{n_\alpha})^2 + ((\overline{Q}_L t_R)H^{n_\alpha} + h.c.)
$$

(7)

where $\mathcal{L}_{\text{gauge}}$ and $\mathcal{L}_{\text{ fermion}}$ are the gauge and fermionic kinetic terms, respectively, and $D_\mu = \partial_\mu - \sum_{n_1=0}^\infty \cdots \sum_{n_4=0}^\infty (ig_2 r^{A^{n_\alpha}} + ig_1 B^{n_\alpha})$. In a full analogy with [2] one must demand $Z_H \rightarrow 0$ or, equivalently, rescaling Higgs field and its KK exiitations $H^{n_\alpha} = \frac{h^{n_\alpha}}{\sqrt{Z_H}}$ in order to normalize canonically kinetic terms in (6), top-Yukawa coupling

$$
gh = \frac{1}{Z_H} \rightarrow \infty
$$

(8)

as energy approaches to $\Lambda$.

Now let’s examine how the BHL predictions for the top quark mass are changed in the presence of extra dimensions. First, note that the since the fermions, and particularly top quark, in our minimal approach have no KK exiitations, it is obvious, that in the fermion bubble approximation the results for the top quark mass are exactly the same as in the BHL model (see (2) with $G \equiv G^{n_\delta=0}$). The RGE improvement, however, is expected to be drastically different because of contributions of KK states to $\beta$– and $\gamma$– functions, leading to power–low runing of gauge and Yukawa couplings.
(for the proper treatment of RGEs in extra dimensions see [5]) and corresponding changes to the fixed-point solutions of RGEs (see [14] for the fixed-point solutions in supersymmetric theories with large extra dimensions).

The one-loop diagrams contributing to the anomalous dimensions of the top quark consists of top-Higgs and top-gauge internal states. Since in our minimal approach top quark has no KK excitations each time one pass Higgs or gauge KK threshold the diagrams contribute as the equivalent SM diagrams. This is because the KK number is not conserved since translational invariance is broken in the extra dimensions. The Higgs–gauge loops contributing to the anomalous dimension of the Higgs field also give the same contribution as an equivalent SM diagrams since now KK number must be conserved at vortices. The only contribution of KK modes to gauge coupling β–functions are came from diagrams with gauge loops and loops of Z₂–odd adjoint scalars. Thus, above the energy μ₀ ≈ 1/Λ the RGEs for the top-Yukawa Y₁ ≡ δ₁² and gauge couplings αᵢ (i = 1, 2, 3) are

\[
\frac{dY_i}{d\ln(\mu)} = \frac{3}{2\pi} Y_i^2 + \frac{Y_i}{4\pi} \left[ \frac{3}{2} Y_i - c_i \alpha_i \right] \frac{d\mathcal{J}}{d\ln(\mu)}
\]

\[
\frac{d\alpha_i}{d\ln(\mu)} = \frac{b_i - b_i'}{2\pi} \alpha_i^2 + \frac{b_i'}{2\pi} \alpha_i^2 \frac{d\mathcal{J}}{d\ln(\mu)}
\]

where

\[
b_i = \left( \frac{41}{10}, \frac{19}{6} - \gamma \right)
\]

\[
b'_i = \left( \frac{1}{10}, \frac{41}{6} - \frac{21}{2} \right)
\]

\[
c_i = \left( \frac{17}{12}, \frac{9}{4} - \delta \right)
\]

\[
\mathcal{J} \text{ in (9) is the integral of elliptic Jacobi theta function [5]}
\]

\[
\mathcal{J}(\mu/\mu_0, \delta) = \int_{r/(\mu/\mu_0)^{\delta}}^{r} \frac{dx}{x} [\vartheta_3(0, e^{-x})]^{\delta},
\]

\[
\vartheta_3(u, q) = \sum_{n=-\infty}^{\infty} q^n e^{i2\pi u n}
\]

where \( r = [\Gamma(1 + \delta/2)]^{\delta/2} \) (Γ is the Euler gamma function). Below the compactification scale \( \mu_0 \) top–Yukawa and gauge couplings evaluate according to the usual four–dimensional SM RGEs, which easily recovered from (9) taking \( \delta = 0 \) or, alternatively, \( \frac{d\mathcal{J}}{d\ln(\mu)} = 1 \) while \( \frac{d\mathcal{J}}{d\ln(\mu)} = \mathcal{J}(1, 1, 1) = 2 \).

In FIG. 1 I have plotted the dependence of the top quark mass \( m_{\text{top}} \) on \( \Lambda/\mu_0 \) solving the full set of Eqns. (9) numerically with compositness condition (8) for \( \Lambda = 10^4, 10^7, 10^{13}, 10^{19} \) GeV and for \( \delta = 1, 2, 4, 7 \). The values \( m_{\text{top}}(0) \) (\( \Lambda = \mu_0 \)) in Figure 1 are clearly the BHL predictions \( m_{\text{top}}(0) = m_{\text{BHL}} \). When the radius of extra dimensions is close to \( \Lambda (\Lambda/\mu_0 \lesssim 2.5 \times 10^4) \) for \( \delta = 7 \) or \( 1 \) \( m_{\text{top}} \) is further increased since the gauge contributions to the evolution of top–Yukawa coupling \( Y_i \) are more significant than in the case of four–dimensional SM. This is the direct consequence of our minimal approach when the chiral fermions are assumed to live on the orbifold fixed points and thus do not feel the extra dimensions. However, for larger \( \Lambda/\mu_0 \) \( m_{\text{top}} \) is quickly decreased and one can get the values of top quark mass in the experimentally allowed range even for \( \Lambda = 10^4 \) GeV, \( \delta \geq 5 \) (demanding \( \mu_0 \geq 1 \) TeV). This happens because of power–low \( \left( \frac{1}{\Lambda} \right)^{\delta} \) running of \( Y_i \) in contrast to the logarithmic running \( \ln(\Lambda_i) \) in the SM [5, 13] leading to extremely small values of \( Y_i \) when \( \Lambda/\mu_0 \) and/or \( \delta \) increase even for the small cut–off \( \Lambda \sim \text{few} \) TeV. Thus, the problem of quadratic divergences can be potentially solved within our framework.

To conclude, I have studied the minimal BHL model for the dynamical electroweak symmetry breaking assuming the existence of compact extra dimensions with relatively large radii. It was shown that owing to the power–low evolution of gauge and Yukawa couplings the original BHL predictions for the top quark mass are significantly lowered and, even for small cut–off scale \( \Lambda = \text{few} \) TeV, one can obtain experimentally allowed values \( m_{\text{top}} \), provided the number of extra dimensions with \( R \approx 1 \) TeV⁻¹ to be at least 5. This offers an exciting possibility for future colliders to probe not only the composite nature of the Higgs boson but also the structure of space–time.

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Figure caption

FIG. 1. The top quark mass as a function of $\Lambda/\mu_0$, for $\Lambda = 10^4, 10^7, 10^{13}, 10^{19}$ GeV and for $\delta = 1, 2, 4, 7$. Intersection of the curves at $\Lambda = \mu_0$ corresponds to BHL predictions, while leftgoing arrow indicates the central experimental value for the top quark mass.
