Tightly Bound Composite Higgs$^{*\dagger}$

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Abstract

We explain general features of the tightly bound composite Higgs models proposed in recent years; walking technicolor, strong ETC technicolor, and a top quark condensate, etc.. These models are all characterized by the large anomalous dimension due to nontrivial short distance dynamics in the gauged Nambu-Jona-Lasinio models (gauge theories plus four-fermion interactions).

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1 Introduction

It is well known that the Higgs sector in the standard model is precisely the same as the sigma model except that \( \langle \sigma \rangle = f_\pi = 93 \text{MeV} \) is simply replaced by the Higgs vacuum expectation value (VEV) \( F_\pi = 250 \text{GeV} \), roughly 2600 times scale-up. Since the sigma model is a low energy effective theory of QCD reproducing the spontaneous chiral symmetry breaking (\( S\chi SB \)) due to the quark-antiquark pair condensate, one is naturally led to speculate that there might exist a microscopic theory for the Higgs sector, with the Higgs VEV being replaced by the fermion-antifermion pair condensate due to yet another strong interaction called Technicolor (TC)\(^1\).

Unfortunately, the original version of TC was too naive to survive the FCNC (flavor-changing neutral current) syndrome\(^1\). It was not the end of the story, however. QCD-like theories (simple scale-up’s of QCD) turned out not to be the unique candidate for the underlying dynamics of the Higgs sector. Actually, there have been proposed varieties of composite Higgs models which, though equally behaving as the sigma model in the low energy region, still have different high energy behaviors than QCD; Walking Technicolor\(^2,3\), Strong ETC Technicolor\(^4\) and the Top Quark Condensate (Top Mode Standard Model)\(^5\), etc.. In this talk I would like to give a general description of such new possibilities in terms of the large anomalous dimension, based on the explicit dynamical model, the gauged Nambu-Jona-Lasinio (NJL) model (gauge theory plus four-fermion interactions).

2 Large Anomalous Dimension and Tightly Bound States

Such a different high energy behavior than QCD simply reflects the relatively short distance dynamics relevant to the composite Higgs. Using the operator product expansion (OPE) and the renormalization-group equation (RGE) for the condensed fermion propagator \( iS^{-1}(p) = \frac{\not{p} - \Sigma(-p^2)}{p^2} \) (in Landau gauge), we may write the dynamical mass \( \Sigma(-p^2) \) (“nonlocal order parameter”) in the high energy region (we switch over to Euclidean momentum hereafter; \(-p^2 \rightarrow p^2\)):

\[
\Sigma(p^2) \overset{p \gg F_\pi}{\approx} \frac{\bar{\psi}\psi}{-p^2} \mu \cdot \exp \left[ \int_0^t \gamma_m(t') dt' \right],
\]

(2.1)

where \( t \equiv \ln(p/\mu) \) and \( \mu = O(F_\pi) \) is the renormalization point. The canonical scaling \( 1/p^2 \) is modified due to the anomalous dimension \( \gamma_m(t) \). Similarly, the
fermion-antifermion condensate ("local order parameter") at a certain high energy scale $\Lambda$ takes the form
\[
\langle \bar{\psi}\psi \rangle_{\Lambda} = Z_m^{-1}(\bar{\psi}\psi)_{\mu} = \exp \left[ \int_{t_0}^{t^\Lambda} \gamma_m(t')dt' \right] \cdot \langle \bar{\psi}\psi \rangle_{\mu} \simeq -\Lambda^2\Sigma(\Lambda^2),
\] (2.2)

where $t_\Lambda \equiv \ln(\Lambda/\mu)$ and the last equation follows from comparison with Eq.(2.1) at $p^2 = \Lambda^2$.

Now, the ($N_f$-flavored) QCD has a logarithmically vanishing anomalous dimension $\gamma_m \simeq A/2\bar{t}$, with $A = 24/(33 - 2N_f)$ and $\bar{t} \equiv \ln(p/\Lambda_{QCD}) = t + \ln(\mu/\Lambda_{QCD})$, which yields only a logarithmic correction (enhancement) to the canonical one:
\[
\exp \left[ \int_{t_0}^{t^\Lambda} \gamma_m(t')dt' \right] \simeq \exp \left[ \frac{A}{2} \ln(\bar{t}/\bar{t}_{\mu}) \right] = \left( \frac{\ln \frac{p}{\Lambda_{QCD}}}{\ln \frac{\mu}{\Lambda_{QCD}}} \right)^\frac{A}{2}.
\] (2.3)

On the other hand, if the theory has a non-vanishing anomalous dimension $\gamma_m(t) \simeq \gamma_m \neq 0$ due to non-vanishing coupling constant (behaving as a non-trivial ultraviolet (UV) fixed point/peudo fixed point) at high energies, then we have a power enhancement instead of the above logarithmic one:
\[
\exp \left[ \int_{t_0}^{t^\Lambda} \gamma_m(t')dt' \right] \simeq e^{\gamma_m t} = \left( \frac{p}{\mu} \right)^{\gamma_m}.
\] (2.4)

Accordingly, we have power-enhanced order parameters:
\[
\Sigma(p^2) \simeq \frac{\langle \bar{\psi}\psi \rangle_{\mu}}{-\mu^2} \left( \frac{p}{\mu} \right)^{-2+\gamma_m},
\]
\[
\langle \bar{\psi}\psi \rangle_{\Lambda} \simeq \left( \frac{\Lambda}{\mu} \right)^{\gamma_m} \cdot \langle \bar{\psi}\psi \rangle_{\mu}.
\] (2.5)

This is actually the mechanism that Holdom\cite{Holdom} proposed without explicit dynamics to resolve the FCNC and the light pseudo NG bosons problems of TC, by simply assuming $1 < \gamma_m$. In fact, in the extended technicolor (ETC) scenario\cite{ETC} with $\mu = \Lambda_{TC}$ and $\Lambda = \Lambda_{ETC} (\gg \Lambda_{TC})$, quarks/leptons mass is given by $m \simeq \langle \bar{\psi}\psi \rangle_{\Lambda_{ETC}}/\Lambda_{ETC}^2$ and $m_{\text{pseudoNG}} \sim \langle \bar{\psi}\psi \rangle_{\Lambda_{ETC}}/\Lambda_{TC}\Lambda_{ETC}$, both of which are thus enhanced by the factor $(\Lambda_{ETC}/\Lambda_{TC})^{\gamma_m}$.

Such a large enhancement also amplifies the small symmetry violation of high energy parameters. This fact was first utilized by Miransky, Tanabashi and Yamawaki\cite{Miransky, Tanabashi} in the proposal of a top quark condensate ($m_t \gg m_b$) and was re-emphasized in a slightly different context\cite{Yamawaki2}. 

A large anomalous dimension actually implies a tightly bound Nambu-Goldstone (NG) bosons due to relatively short distance dynamics. In fact, it is well known that the amputated Bethe-Salpeter amplitude of the NG bosons at zero NG-boson-momentum is related to $\Sigma(p^2)$ through the Ward-Takahashi identity for the axialvector vertex:

$$\chi^a(p, p + q)|_{q^2=0} = \frac{1}{F_\pi} r^a \gamma_5 \Sigma(p^2) \sim \left(\frac{p}{\mu}\right)^{2+\gamma_m}$$

(2.6)

(in the $SU(2)_L \times SU(2)_R/SU(2)_V$ case). Thus in QCD with $\gamma_m \simeq 0$ we find $\chi_\pi \sim (p/\mu)^{-2}$ and hence the radius of the composite $\langle r \rangle \simeq \mu^{-1} \simeq F_\pi^{-1}$. In this case the sigma model description breaks down at the order of $O(F_\pi)$. On the other hand, in the extreme case of $\gamma_m \simeq 2$ we have $\chi_\pi \sim \text{const.}$ and hence $\langle r \rangle \simeq \Lambda^{-1}$ (almost point-like, or very tightly bound), in which case the sigma model description persists up to the high energy scale $\Lambda$.

3 Tightly Bound Composite Higgs Models

Walking Technicolor\[2, 3\]

Do such explicit dynamical models as have a large anomalous dimension really exist? The answer is "yes". It was first pointed out by Yamawaki, Bando and Matumoto\[3\] that the technicolor within the ladder Schwinger-Dyson (SD) equation (with the gauge coupling constant fixed - non-running) possesses an $S\chi_{SB}$ solution with a large anomalous dimension:

$$\gamma_m \simeq 1,$$

$$\Sigma(p^2) \sim \frac{1}{p} \quad (p \gg \Lambda_{TC}),$$

$$\langle \bar{\psi}\psi \rangle_{\Lambda_{ETC}} \simeq \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right) \cdot \langle \bar{\psi}\psi \rangle_{\Lambda_{TC}},$$

(3.1) (3.2) (3.3)

and hence resolves the long standing problems of the old TC mentioned above. (Essentially the same observation was also made by Akiba and Yanagida\[3\] without notion of the anomalous dimension.)

The above feature is actually the essence of the "walking TC", a generic name currently used (see Appelquist et al.\[3\]) for a wider class of TC’s with slowly running ($A \gg 1$) gauge coupling including the non-running ($A \to \infty$, "standing") case as an extreme case. In order for the walking TC be a realistic solution of the FCNC problem, however, it must be very close to the
standing limit anyway (see Bando et al. [3]). In the standing limit the \( S\chi SB \) solution exists only when the gauge coupling \( \alpha \equiv e^2/4\pi \) exceeds a critical value \( \alpha_c = \pi/(3C_F) \) \((C_F:\) quadratic Casimir of the fermion representation). Hence the critical coupling plays a role of a nontrivial UV fixed point. Thus the walking TC may be viewed as the TC with a nontrivial UV fixed point/pseudo fixed point, with the coupling being kept close to the critical coupling all the way up to \( \Lambda_{ETC} \) scale. It should be mentioned that Holdom [2] earlier recognized essentially the same dynamics through a purely numerical analysis of the ladder-type SD equation.

**Strong ETC Technicolor**

Next we come to the TC with even larger anomalous dimension, \( 1 < \gamma_m < 2 \), which have been proposed [4], based on the explicit dynamics of the gauged NJL model in the framework of the ladder SD equation [7], with the ETC/preonic interactions being simulated by the four-fermion interactions (see Yamawaki et al. [3]). Actually, based on the \( S\chi SB \) solution [8], we have a big enhancement of order parameters due to a large anomalous dimension:

\[
1 < \gamma_m = 1 + \sqrt{1 - \frac{\alpha}{\alpha_c}} < 2, \tag{3.4}
\]

\[
\Sigma(p^2) \sim p^{-1+\sqrt{1-\frac{\alpha}{\alpha_c}}}, \tag{3.5}
\]

\[
\langle \bar{\psi}\psi \rangle_{\Lambda_{ETC}} \simeq \left( \frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{1+\sqrt{1-\frac{\alpha}{\alpha_c}}} \cdot \langle \bar{\psi}\psi \rangle_{\Lambda_{TC}} . \tag{3.6}
\]

This *in principle* can yield more enhancement of the quark mass, say, up to \( O(\Lambda_{TC}) \) (if one takes \( \gamma_m \approx 2 \)).

**Top Quark Condensate**

Once we have taken a TC with such an extremely tight bound composite Higgs with \( \gamma_m \approx 2 \), we may consider a much simpler alternative. As such a top quark condensate (top mode standard model) was proposed by Miransky, Tanabashi and Yamawaki (MTY) [5], based on the \( S\chi SB \) *solution of the SD equation* for the gauged NJL model (this time QCD plus four-fermion interactions) with \( \alpha_{QCD} \ll 1 \). Actually, we find

\[
\gamma_m \simeq 1 + \sqrt{1 - \frac{\alpha_{QCD}}{\alpha_c}} \simeq 2 - \frac{\alpha_{QCD}}{2\alpha_c} \simeq 2 - \frac{\Lambda}{2t}, \tag{3.7}
\]
\[ \Sigma(p^2) \sim \left( \ln \frac{p}{\Lambda_{QCD}} \right)^{-\frac{4}{7}}, \tag{3.8} \]

\[ \langle \bar{t} t \rangle_\Lambda \simeq \left( \frac{\Lambda}{m_t} \right)^2 \left( \ln \frac{\Lambda}{m_t} \right)^{-\frac{4}{7}} \cdot \langle \bar{t} t \rangle_{m_t}, \tag{3.9} \]

where we have taken \( \mu = m_t \). Using such a very slowly damping solution of the SD equation, MTY predicted a rather large top mass \( m_t \simeq 250 \text{GeV} \) (for \( \Lambda \simeq 10^{19} \text{GeV} \)) and a Higgs boson mass \( m_H \simeq 2m_t \). The idea of the Higgs boson being a \( \bar{t} t \) composite was also proposed independently by Nambu\[5\] in a different context. An elegant reformulation of the MTY model was further made by Bardeen, Hill and Lindner (BHL)\[5\] who newly included loop effects of the composite Higgs (and \( SU(2) \times U(1) \) gauge interactions) and thereby modified the above MTY prediction into somewhat smaller value \( m_t \simeq 220 \text{GeV} \). (If we switch off such newly included effects in the BHL formulation, we can actually recover the original MTY result.)\[9\]

Although current LEP data seem to indicate somewhat smaller value for \( m_t \), there have been suggested many possible variations and modifications reducing the above predicted value.\[9\]

4 Renormalization of the Gauged NJL Model

All the previous analysis of the gauged NJL model was based on the SD equation which has an explicit ultraviolet cutoff \( \Lambda \). The continuum limit \( \Lambda \to \infty \) can be taken à la Miransky\[12\] by fine-tuning the bare coupling(s) to the critical point (line) so as to keep the fermion dynamical mass finite. This defines the \( \beta \) function and the anomalous dimension with respect to the bare couplings only in the \( S\chi SB \) phase where the dynamical mass exists. On the other hand, the OPE and the RGE analysis we have been talking about should be formulated in terms of the renormalized couplings in the continuum theory in the symmetric phase as well as the \( S\chi SB \) phase. Actually, it was not until recently that such a procedure was accomplished by Kondo, Tanabashi and Yamawaki\[11\]. Of course, this renormalization breaks down at \( \alpha \to 0 \) (pure NJL limit) which is obviously non-renormalizable.

Let us consider QED plus chiral-invariant four-fermion interaction \((G_0/2) [\bar{\psi}\psi]^2 + (\bar{\psi}\gamma_5\psi)^2\) in the ladder approximation as the simplest gauged-NJL model with a standing gauge coupling, \( A \to \infty \). By using auxiliary fields \( \sigma \)
and $\pi$, we can rewrite the original Lagrangian into an equivalent one;

$$\mathcal{L} = \bar{\psi} i \gamma_5 D^\mu \psi - \frac{1}{G_0} \left( \frac{\sigma^2 + \pi^2}{2} - m_0 \sigma \right) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}, \quad (4.1)$$

with $m_0$ being the bare mass of the fermion.

Now, the multiplicative renormalization can be done phase-independently through an effective potential written solely in terms of the auxiliary fields $\sigma$ and $\pi$. Here we use an effective potential obtained by Bardeen and Love\cite{13} (we set $\pi = 0$):

$$-4 \pi^2 V(\sigma) = \Lambda^2 g^{-1} m_0 \sigma + \Lambda^2 \left( g^{* -1} - g^{-1} \right) \frac{\sigma^2}{2} + \Lambda^4 C \frac{2 - \omega}{\alpha/\alpha_c} \left( \frac{\sigma}{\Lambda} \right)^\frac{4}{2 - \omega} + \Lambda^4 O\left( \left( \frac{\sigma}{\Lambda} \right)^\frac{4 + \omega}{2 - \omega}, \left( \frac{\sigma}{\Lambda} \right)^4 \right), \quad (4.2)$$

where

$$g \equiv \frac{G_0 \Lambda^2}{4 \pi^2} = \frac{1}{4} (1 + \omega)^2 \equiv g^*, \quad \omega \equiv \sqrt{1 - \frac{\alpha}{\alpha_c}} \quad (0 < \alpha < \alpha_c), \quad (4.3)$$

and $C$ is a certain constant depending on $\omega$. It is evident that the phase is determined by the sign of the coefficient of $\sigma^2$ term, i.e., $g = g^*$ is the critical line discovered by Kondo, Mino and Yamawaki\cite{8} and by Appelquist, Soldate, Takeuchi and Wijewardhanai\cite{8} through the ladder SD equation. It should be emphasized that this potential holds both in the symmetric and the $S\chi SB$ phases.

Now, we renormalize the above effective potential as follows:\cite{11}

$$\Lambda^{1 - \omega} \sigma = \mu^{1 - \omega} \sigma_R, \quad (4.4)$$
$$\Lambda^{2 \omega} (g^{* -1} - g^{-1}) = \mu^{2 \omega} (g_R^{* -1} - g_R^{-1}), \quad (4.5)$$
$$\Lambda^{1 + \omega} g^{-1} m_0 = \mu^{1 + \omega} g_R^{-1} m_R, \quad (4.6)$$

where $\sigma_R, m_R$ (current mass) and $g_R$ are renormalized at the renormalization point $\mu$. (Note that $g_R^*$ ($0 < g_R^* < \infty$) is left arbitrary and the simplest choice would be $g_R^* = g^*$.) Through this renormalization we in fact have a renormalized effective potential $V(\sigma_R)$ ($\Lambda \to \infty$). Thus we have obtained a sensible (interacting) continuum theory defined on the critical line, $g \to g^*$ as $\Lambda \to \infty$. The theory has been renormalized at zero momentum ($q_\mu = 0$) of the auxiliary field.
As to \( q_\mu \neq 0 \), we need to know the propagator of \( \sigma \) which was calculated by Appelquist, Terning and Wijewardhana\[14\]. Remarkably enough, their \( \sigma \) propagator is also renormalized via the above renormalization condition:\[11\]

\[
i(4\pi^2)\Delta_R^{-1}(-q^2) = -\alpha_c\mu^2(q^2/\mu^2)\omega/(g^*\omega\alpha) + \mu^2(g_R^{-1} - g_R^{-1}) \text{ at } \Lambda \to \infty.
\]

From (4.5) and (4.6) we immediately obtain \( \beta(g_R) \) and \( \gamma_m(g_R) \) by using the \( \mu \) independence of the bare parameters:\[11\]

\[
\beta(g_R) = 2\omega g_R \left(1 - \frac{g_R}{g_R^*}\right), \\
\gamma_m(g_R) = 1 - \omega + 2\omega\frac{g_R}{g_R^*}.
\]

(4.7) \hspace{1cm} (4.8)

Thus the continuum theory does have a nontrivial UV fixed line \( g_R = g_R^* \). The anomalous dimension at the UV fixed line becomes very large;

\[
\gamma_m = 1 + \sqrt{1 - \frac{\alpha}{\alpha_c}} \quad (0 < \alpha < \alpha_c),
\]

(4.9)

which thus encompasses a variety of tightly bound composite Higgs models; walking TC \( (\gamma_m \simeq 1) \), strong ETC technicolor \( (1 < \gamma_m < 2) \) and the top quark condensate \( (\gamma_m \simeq 2) \). It is explicitly checked that the OPE is consistent with this renormalization and the large anomalous dimension.\[11\]

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