Charge qubit driven via the Josephson nonlinearity

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Abstract

We study the novel nonlinear phenomena that emerge in a charge qubit due to the interplay between a strong microwave flux drive and a periodic Josephson potential. We first analyse the system in terms of the linear Landau–Zener–Stückelberg model, and show its inadequacy in a periodic system with several Landau–Zener crossings within a drive period. Experimentally, we probe the quasienergy levels of the driven qubit with an LC cavity, which requires the use of linear response theory. We show also that our numerical calculations are in good agreement with the experimental data.

1. Introduction

Superconducting circuits have made it possible to study fundamental quantum physics with parameter values inaccessible to conventional atomic, molecular, or optical systems [1]. The coupling between a superconducting two-level system, a qubit, with either zero-point [2–4] or driven [5–8] microwave radiation has become a paradigm within the field. Physical understanding of such systems is of fundamental interest due to the possible application in quantum information processing.

One example of such a novel system is a superconducting circuit that consists of a single-Cooper-pair transistor (SCPT), the qubit, inductively coupled to an LC cavity, as depicted in figure 1(a). It has been used in several experiments to demonstrate, e.g., the Josephson inductance [9], the coupling between the vibrational and electric states of an artificial molecule [10], and exceptionally large Stark and generalized Bloch–Siegert shifts [11] that emerge from strong external driving. In this paper, we will concentrate on the effects that are caused by the nonlinear coupling between the qubit and the magnetic flux, originating in the sinusoidal Josephson potential.

We examine the nonlinear flux dependence by driving the qubit strongly with a periodic flux modulation \( \varphi(t) = \varphi_b + \varphi_{ac} \sin \omega t \) through the superconducting loop. Here \( \varphi_b, \varphi_{ac}, \) and \( \omega \) are referred to as the dc bias, the ac amplitude, and the drive frequency of the flux, respectively. The strong drive can be seen to cause either transitions between adiabatic qubit states, or formation of microwave dressed, i.e. quasienergy, states. Previously, we have used the cavity to probe the quasienergy spectrum of the driven qubit [11, 12], and also studied the influence of the chosen qubit basis on the apparent shifts in the locations of the multi-photon resonances of the qubit under strong drive [13]. Here, we first neglect the coupling with the cavity and discuss the effects of the strong drive in terms of interference occurring between multiple Landau–Zener (LZ) transitions [14], i.e. the Landau–Zener–Stückelberg (LZS) model [15], and in terms of the quasienergies [12, 16]. We emphasize that due to the periodicity of the qubit energy levels with respect to the flux, the LZS model should be revised under ultra-strong driving.
impedance of the cavity

Nonlinearly driven charge qubit. (a) Schematics of the Supercond. Sci. Technol. spectrum of the system as the drive amplitude explanation for the novel nonlinear features that appear in the probing the quasienergies of the driven system. We give an systems [17]. Next, we treat the cavity as a small perturbation of a superconducting qubit and two microscopic two-level transitions during one period of oscillation.

avoided crossing, the system can make an LZ transition, which are ε\(_{\text{LC}}\) collected during the free adiabatic evolution in between the LZ transitions during one period of oscillation.

i.e. when two or more avoided crossings are traversed within a drive period, similarly as in a tripartite system consisting of a superconducting qubit and two microscopic two-level systems [17]. Next, we treat the cavity as a small perturbation probing the quasienergies of the driven system. We give an explanation for the novel nonlinear features that appear in the spectrum of the system as the drive amplitude \(\phi_{\text{ac}}\) is increased. Finally, we make a comparison with the measurement data obtained with the experimental realization of the setup. We omit the detailed explanations of the experimental aspects that can be found in our earlier publications [9–11].

2. Driven qubit in nonlinear regime

We write the SCPT in the charge qubit limit resulting in the complete Hamiltonian of the coupled system:

\[
\hat{H} = \hbar \omega_c \hat{a}^\dagger \hat{a} + \frac{\varepsilon_0}{2} \hat{\sigma}_z - \frac{E_1}{2} \{\cos[\eta(\hat{a}^\dagger + \hat{a}) + \varphi(t)]\hat{\sigma}_x - d \sin[\eta(\hat{a}^\dagger + \hat{a}) + \varphi(t)]\hat{\sigma}_y\}.
\]

We see that the scaled magnetic flux of the qubit comprises the cavity and external components, \(\phi = \eta(\hat{a}^\dagger + \hat{a})\) and \(\varphi(t) = \pi \Phi(t)/\Phi_0\), respectively. In the above, we have defined \(\omega_c\) as the cavity frequency, \(\varepsilon_0 = 4E_c(1 - 2n)\) as the energy difference between the charge states, \(E_c = e^2/2C\) as the charging energy of a single electron, \(E_1 = E_{11} + E_{12}\) as the total Josephson energy, and \(d = (E_{11} - E_{12})/E_1\) as the asymmetry of the two junctions. Due to the sinusoidal dependence on flux, the coupling between the cavity and the qubit can be highly nonlinear even at the quantum limit, depending on the value of the parameter \(\eta = \sqrt{\pi Z/R_K}\), where the characteristic impedance of the cavity \(Z = \sqrt{L/C}\) and \(R_K = h/e^2\) is the resistance quantum. However, in our experimental setup we have \(L = 410\) pH, \(C = 10\) pF, and thus \(\eta \sim 0.03 \ll 1\). This means that the nonlinearities can be revealed only by driving the qubit strongly with an external signal. Also, in the limit of a small number of cavity quanta, the cavity coupling can be treated as a small perturbation leading to the qubit (system) and probe Hamiltonians

\[
\hat{H}_S = -\frac{E_1}{2} \left[ \cos \varphi(t)\hat{\sigma}_x - d \sin \varphi(t)\hat{\sigma}_y \right],
\]

\[
\hat{H}_P = \hbar \omega_c \hat{a}^\dagger \hat{a} + \hbar g(\hat{a}^\dagger + \hat{a})[\sin(\varphi(t)\hat{\sigma}_x + d \cos(\varphi(t)\hat{\sigma}_x)]
\]

respectively. In the above, the linear coupling between the qubit and the cavity is given by \(h g = \eta E_3/2\). Additionally, we bias the system with the charge degeneracy, \(n_K = 0.5\), in order to minimize the charge noise and to highlight the nonlinearities in the system.

We first neglect the influence of the cavity probe (3) and consider the strongly driven qubit Hamiltonian (2) by rotating into the \(\hat{\sigma}_y\) eigenbasis:

\[
\hat{H}_S = -\frac{E_1}{2} \left[ \cos \varphi(t)\hat{\sigma}_z - d \sin \varphi(t)\hat{\sigma}_x \right].
\]

The energy separation of the qubit can be controlled with the static flux bias \(\phi_0\), however, the dependence is periodic due to the sinusoidal coupling. In the following, we choose the bias fluxes from the interval \(\phi_0 \in [\pi/2, \pi]\), and the other choices are obtained by employing the periodicity and the symmetry of the energies with respect to the bias flux. In the absence of driving, the uncoupled (\(d = 0\)) qubit energies \(\varepsilon_{\pm} = \pm \frac{1}{2}E_1\cos \phi_0\) are degenerate when \(\phi_0 = \pi/2\). We refer to these as diabatic energies, and depict them in figure 1 with dashed lines. The introduction of coupling lifts the degeneracy, resulting in adiabatic energies \(E_{\pm} = \pm \frac{1}{8}E_1/\cos^2 \phi_0 + d^2 \sin^2 \phi_0\) with an avoided crossing at the diabatic degeneracy (figure 1, solid lines). The minimum gap is given by \(\Delta = dE_1\) at the diabatic degeneracy. Periodic modulation of the flux bias leads to oscillations in the adiabatic energy splitting \(E(t) = E_{+}(t) - E_{-}(t)\) of the qubit, shown schematically in figure 1(c).

We have first solved the steady state population \(P_+\) of the qubit’s excited state as a function of the flux bias \(\phi_0\) and the drive amplitude \(\phi_{\text{ac}}\) by using the Bloch equations with relaxation times \(T_1 = 15\) ns and \(T_2 = 30\) ns. The parameter values \(E_3/\hbar = 27\) GHz, \(d = 0.19\) [11], and \(\omega/2\pi = 2.1\) GHz are obtained from the experimental realization of the setup. We have neglected dephasing, and chosen values for \(T_1\) and \(T_2\) that are larger than in the experiments in order to resolve the individual resonances. The combined effect of driving and nonlinear coupling leads to rather elaborate interference pattern of \(P_+\), as shown in figure 2. In the following, we will discuss the locations of resonances, and address the differences from the conventional LZS model that arise from the periodic dependence on the control parameter.

2.1. LZS model

Many previous experiments on strongly driven qubits [6–8, 18, 19] and other systems [20, 21] have been discussed.
in terms of the LZS model [15]. The heart of the model is the
discretization of the dynamics into free adiabatic phase
evolution, interrupted with instantaneous LZ transitions at
the locations of the avoided crossings. In our system, when
$\varphi_{ac} > \varphi_{b} - \pi/2$, the flux bias reaches the minimum gap
at least twice in the driving period. These values of the
driving amplitude are located above the dashed line, starting
at least twice in the driving period. These values of the
locations of the avoided crossings. In our system, when
we have denoted the location of the resonance with $m = 4$ photons with the solid vertical line. Clearly, as one increases the drive amplitude
$\varphi_{ac}$, one also has to change the bias $\varphi_{b}$ in order to stay on resonance. In certain regions of the amplitude, the average energy is below 4$\hbar \omega$
with all values of $\varphi_{b}$, and we say that the resonance is collapsed. After the first Bessel zero, the $m = 4$ resonance is revived and then again
collapsed. The extreme of the resonance is located at the first non-zero Bessel maximum. The locations of collapses and the revival of the
$m = 4$ resonance are indicated with horizontal arrows and are in accordance with a steady state maximum, which can be identified as the
$m = 4$ diabatic resonance (13) from figure 3(a).

Figure 2. Steady state population $P_{+}$ of the qubit’s excited state in the $\varphi_{b}$–$\varphi_{ac}$ plane (left), together with a sketch of the geometric collapse
(right). The blue curves indicate average energies $\bar{\epsilon}$ (16) with flux bias values ranging from $\pi/2$ to $\pi$, with the step $\pi/28$. As an example,
we have denoted the location of the resonance with $m = 4$ photons with the solid vertical line. Clearly, as one increases the drive amplitude
$\varphi_{ac}$, one also has to change the bias $\varphi_{b}$ in order to stay on resonance.
In our system, the energies are periodic due to the Josephson
coupling (see figure 1). When $\varphi_{ac} > 3\pi/2 - \varphi_{b}$, the system reaches the next avoided crossing and the corresponding LZ
processes should be included in the discretized dynamics in the
LZS model. Thus, the linear LZS model should hold only
within the dashed diamond indicated in figure 2. Elsewhere
the linear LZS model cannot be expected to reproduce the $P_{+}$
calculated for the periodic qubit energy.

For now, we study amplitudes $\varphi_{ac}$ in the validity range
$[0, 3\pi/2 - \varphi_{b}]$ of the linear LZS model. First, we assume
that the system is in the adiabatic ground state and study
the excited state population after a single driving period. We
call this the St"uckelberg probability $P_{St}$ to separate it from
the population $P_{+}$ that is obtained by averaging over many
periods. The cycle consists of two LZ passages. During
the cycle, the system collects two phases during the free adiabatic
evolution (see figure 1(c)):

$\zeta_{1} = \frac{1}{2\hbar} \int_{t_{1}}^{t_{2}} E(t) \, dt,$ \hspace{1cm} $\zeta_{2} = \frac{1}{2\hbar} \int_{t_{2}}^{t_{1} + 2\pi/\omega} E(t) \, dt. \hspace{1cm} (6)$

The system can evolve along two different paths which
gives rise to interference, similar to a Mach–Zehnder
interferometer. Starting the cycle at a time between $t_{1}$ and $t_{2}$,
the important quantity is the adiabatic phase $\zeta_{2}$, accumulated
in between the LZ passages, that induces St"uckelberg
oscillations [22] of the excited state population:

$P_{St} = 4P_{LZ}(1 - P_{LZ}) \sin^{2}(\zeta_{2} + \zeta_{S}). \hspace{1cm} (7)$

Here, $\zeta_{S} = \pi/4 + \delta (\ln \delta - 1) + \arg[\Gamma(1 - i\delta)]$ is the so-called
Stokes phase [23], collected during non-adiabatic transitions.

Successive driving periods can also interfere, and by
averaging over many periods we end up with the modulated
Stückenberg, i.e. the steady state, population [15]

$P_{+} = \frac{P_{St}}{2 \sin^{2} \xi}, \hspace{1cm} (8)$

similar to a multi-pass Mach–Zehnder interferometer, or to a
multi-slit diffraction grating. In the above,

$\sin^{2} \xi = P_{St} + \left[ (1 - P_{LZ}) \sin \zeta_{+} - P_{LZ} \sin \zeta_{-} \right]^{2}, \hspace{1cm} (9)$

$\zeta_{+} = \zeta_{1} + \zeta_{2} + 2\zeta_{S} - \pi, \hspace{1cm} (10)$

$\zeta_{-} = \zeta_{1} - \zeta_{2}. \hspace{1cm} (11)$

We see that in order to observe maximum $P_{+}$ we need to
minimize $\sin^{2} \xi$. This gives us the conditions for the phases
\( \zeta_+ \) and \( \zeta_- \):

\[
\zeta_+ = \ell \pi, \quad \zeta_- = m \pi, \tag{12} \]

with integers \( \ell \) and \( m \). In addition, we should maximize \( P_{\text{St}} \) by choosing \( \zeta_+ - \zeta_- = 2k\pi \) with integer \( k \). However, the clearest features in the Bloch equation based results in figure 2 are the points around which \( P_+ \) changes rapidly. These correspond to the \( P_{\text{St}} = 0 \) which is obtained by

\[
\zeta_+ - \zeta_- = (2k + 1)\pi. \tag{14} \]

Equations (12) and (13) have a simple physical interpretation. When the driving is weak, \( P_{\text{LZ}} \ll 1 \), and the system follows the adiabatic state, in accordance with the adiabatic theorem [24]. In this adiabatic limit, the locations of the resonances are determined by \( \zeta_+ \), which we will in the following refer to as the adiabatic phase. Similarly, when the driving is strong, \( P_{\text{LZ}} \sim 1 \), and the bending of the energy levels at the avoided crossing is not important due to the strong tendency to tunnel between the adiabatic states. In other words the system follows the diabatic states and the diabatic phase \( \zeta_- \) gives the relevant locations of resonances. We call this the diabatic limit. Moreover, by neglecting altogether the effects due to the asymmetry \( d \), we end up with an analytic result

\[
\zeta_- \approx -\pi E_1 \frac{\hbar \omega}{\hbar \omega} J_0(\varphi_{ac}) \cos \varphi_b = m \hbar \omega. \tag{15} \]

By combining with equation (13), we obtain the multi-photon resonance condition 

\[ -E_1 J_0(\varphi_{ac}) \cos \varphi_b = m \hbar \omega. \]

We have plotted the adiabatic and diabatic resonance conditions in figure 3(a). By properly choosing the parameters, we can see a shift from adiabatic to diabatic following simply by increasing the LZ probability, e.g. by increasing the amplitude of the drive. A rough estimate for the location of the transition is given when the LZ parameter 

\[ 2\pi \delta = 1, \]

denoted in figure 3(a) with a dashed black line. In figure 3(a), we have also plotted the locations (14) of the zeros of the Stueckelberg population. Those indicate destructive interference after each period, a phenomenon often referred to as coherent destruction of tunnelling [25] (CDT).

Overall, we see that within the validity region of the linear LZS model, the locations of resonances in the steady state population \( P_+ \) follow nicely the prediction given by the 'stroboscopic' time evolution (8). Also, the magnitudes of the multi-photon resonances are modulated, due to the single-period Stueckelberg factor (7). The insufficiency of the linear LZS model when \( \varphi_{ac} > 3\pi/2 - \varphi_0 \) is most clearly seen in the mismatch between the locations of CDT in the numerical excited state population \( P_+ \) and the crossing points of the diabatic resonances and the Stueckelberg zeros. Finally, we stress that the choice of the initial time of the first driving period is ambiguous. By starting the cycle at a time between \( t_2 \) and \( t_1 + 2\pi/\omega \), one obtains relations (7)–(11) where the roles of the phases \( \xi_1 \) and \( \xi_2 \) are exchanged. The experimentally relevant steady state population is obtained by properly averaging over these two possibilities. Here, we put emphasis only on the resonance conditions (12) and (13) together with the locations of CDT, which are independent on the choice of initial time of the cycle.

The locations and magnitudes of the resonances are best given in the quasienergy formalism [16], which works also outside the validity region of the linear LZS model and provides an intuitive physical interpretation in terms of microwave dressed states of the qubit. To demonstrate this, we plot in figure 3(b) the difference between the two quasienergies in a Brillouin zone of the driven qubit [12]. The coupling between the qubit and the driving field is the strongest at the locations of multi-photon resonances, which
is clearly seen as the overlap between the LZS resonance conditions (12) and (13), and the valleys and ridges of the quasienergy landscape.

Quasienergies of the driven qubit have been thoroughly investigated before [11, 12, 26] and, thus, are not studied further in detail here. Instead, we concentrate on the novel effects due to the periodic coupling of the control parameter, e.g., the characteristic bending of the resonances away from the avoided crossing as the drive amplitude is increased. This will be discussed in terms of geometric collapse in section 2.2. Finally, we stress that with large drive amplitudes, the LZS model should be revised to take into account the periodicity of the energy levels with respect to the control parameter. In addition, even though the LZS model predicts well the locations of the population extrema of the qubit states, the inclusion of the probe Hamiltonian (3) is difficult within the model. For this reason, the experimental results need to be analysed within the quasienergy formalism (see section 3).

2.2. Geometric collapse

The characteristic bending of the multi-photon resonances at the diabatic limit can be understood by studying the mean energy of the qubit within a drive period. Consider an energy surface with a nonlinear dependence over a control parameter \( \phi \). When the parameter is varied periodically, the mean of the energy over an oscillation period is generally not given by the mean of the parameter, i.e. \( \bar{\epsilon} \neq \epsilon(\bar{\phi}) \). This geometric shift in the effective energy separation can be seen as the rectification of the drive, allowing the control over the energy with the oscillation amplitude, in addition to the possible dc controls.

In general, the mean of the energy has to be calculated numerically. Nevertheless, we obtain a qualitative understanding by considering the diabatic limit where we can approximate \( d = 0 \). Thus, the mean energy splitting of the qubit is analytic and given by

\[
\bar{\epsilon} = -E_J J_0(\varphi_{ac}) \cos \varphi_b,
\]

which is exactly the same term as in the diabatic multi-photon resonance condition (15). We see that on average, the energy difference between the two qubit levels vanishes at the zeros of the Bessel function \( J_0 \), sketched in figure 2(b). We refer to this as geometric collapse, as it has its origin in the special topology of our circuit. To give an example, we have also indicated the location of the multi-photon resonance with \( m = 4 \). Clearly, as the amplitude \( \varphi_{ac} \) is increased, one needs to increase \( \varphi_b \) in order to stay on resonance with \( 4h \omega_0 \). We note that in the vicinity of the zeros of \( J_0 \), the average energy \( \bar{\epsilon} < 4h \omega_0 \), irrespective of the value of the bias flux \( \varphi_b \), indicating the disappearance of the resonances. The chosen resonance can be revived by further increasing the amplitude. Eventually, even the \( m = 1 \) resonance will vanish due to the attenuation of \( J_0 \) with increasing \( \varphi_{ac} \). This discussion is in good qualitative agreement with steady state population \( P_m \) in figure 2(a), and also with the experimental results presented in figure 4.

An effect similar to geometric collapse is observed in any periodic lattice under periodic temporal drive, e.g., with the quasienergy minibands in periodic superlattices that interact with intense far-infrared laser radiation [27]. There the phenomenon is referred to as dynamical collapse. However, in superlattices the drive couples linearly to the position operator, whereas in our system the coupling is explicitly nonlinear both in the parameter \( \varphi_b \) and in the operator. Consequently, the dynamical collapse in our system does not depend on the frequency of the ac drive, contrary to the case with superlattices.

3. Absorptive reflection measurement and probe susceptibility

We use the reflection protocol [28] to measure the quantum circuit consisting of the strongly driven qubit perturbed weakly by the cavity. The measured quantity is the reflection coefficient \( \Gamma(o_p) = V_{out}(o_p)/V_{in}(o_p) \), where \( V_{in} \) is the incoming probe signal sent at frequency \( o_p \approx \omega_c \) via a transmission line with characteristic impedance \( Z_0 \), and \( V_{out} \) is the signal reflected at the boundary of the transmission line and the studied circuit with impedance \( Z \). By using these notions, the reflection coefficient can be expressed as

\[
\Gamma = \frac{Z - Z_0}{Z + Z_0}.
\]
Without the strong driving, cavity-induced qubit excitations would not be possible due to the large detuning $dE_{R0}/\hbar - \omega_c \gg 1/T_2$, which places the system in the dispersive regime [9].

However, when the qubit is driven strongly, its energy bands deform substantially into quasienergies, which energetically allow the cavity-induced excitations. By changing the flux bias $\phi_b$, we observe a decrease in the probe reflection $|\Gamma|$ whenever the cavity is on resonance with the driven qubit. We interpret this as the energy flow [29] from the cavity into the qubit, which has to be compensated by an increased absorption (decreased reflection) from the transmission line. At resonance, the energy difference between the two quasienergy levels must be equal to the energy quanta of the probe. With the experimental values of $\omega_f/2\pi = 2.1$ GHz and $\omega_{op}/2\pi = 3.47$ GHz, the resonance condition can be written either as $\Delta_q = \hbar (\omega_{op} - \omega)$, or as $\Delta_q = \hbar (2\omega - \omega_{op})$. Thus with a fixed probe frequency, one can measure two distinct quasienergy contours, as shown in figure 4. We denote this type of qubit measurement as the absorptive approach to distinguish it from the more widely applied dispersive one (see, e.g., [30]).

To accurately reproduce the measured reflection coefficient $\Gamma$ (17), the probe Hamiltonian (3) is considered in the semiclassical limit where quantum correlations between the cavity and the qubit are neglected. In this limit, the probe Hamiltonian (3) is written as

$$\hat{H}_p(t) = \psi_c(t) \left( \frac{\hbar}{2} \left[ \sin \psi(t) \sigma_+ + d \cos \psi(t) \sigma_\times \right] \right),$$

where $\psi_c(t) = \psi_c (e^{-i\omega_{op}t} + e^{i\omega_{op}t})$ and the constant energy of the cavity is neglected. The probe Hamiltonian can be expressed also as

$$\hat{H}_p(t) = \psi_c (t) \left( \frac{\hbar}{2} \left[ \sigma_+ + d \cos \psi(t) \right] \right),$$

where $\hat{I}_q$ is the current operator for the qubit. In the limit of $\eta \ll 1$, we have that $\psi_c \ll \psi_{ac}$ and the probe Hamiltonian can be considered as a weak harmonic perturbation on the qubit. Then, the impedance of the qubit $Z_q$ can be calculated by applying the linear response theory [31].

We first introduce the generalized probe susceptibility [31] $\alpha(\omega_{op}) = \alpha' (\omega_{op}) + i\alpha'' (\omega_{op})$ which determines the response $\langle \hat{I}_q \rangle (\omega_{op}) = \alpha (\omega_{op}) \psi_c$ to the probe Hamiltonian (19). The response implies the probe impedance $Z_q (\omega_{op}) = i\omega_{op} \alpha^{-1} (\omega_{op})$. To calculate the generalized probe susceptibility $\alpha$, we note that the absorption rate $P(\omega_{op})$ corresponding to $\hat{H}_p$ (19) is proportional to the imaginary part of the susceptibility $\alpha'' (\omega_{op})$ as $P(\omega_{op}) = 2\gamma_{ac}^2 \alpha'' (\omega_{op}) / \hbar$. On the other hand, the absorption rate $P(\omega_{op})$ can be evaluated in the quasienergy basis by applying Fermi’s golden rule for quasienergy transitions [12]:

$$P(\omega_{op}) = \frac{\gamma_{ac}^2}{\hbar} \sum_{i_f} \frac{\gamma_f |\langle u_f | \hat{I}_q | u_i \rangle|^2}{(\omega_{op} - \omega_i)^2 + 1/4 \gamma_f^2}.$$

The matrix element between the final and initial quasienergy states $\langle u_f | \hat{I}_q | u_i \rangle$ is evaluated in the quasienery basis, $\omega_q = (\epsilon_f - \epsilon_i) / \hbar$ (for absorptive transitions $\omega_{q} > 0$) denotes the energy difference of the quasienergy levels, and $\gamma_q$ is the width (inverse lifetime) of the transition. The Kramers–Kronig relations relate the real and the imaginary parts of the probe susceptibility.

Finally, the probe susceptibility $\alpha = \alpha' + i\alpha''$ and the probe impedance $Z_q = i\omega_{ac} \alpha^{-1}$ can be calculated by exploiting the quasienergy structure and the probe transitions. To relate to the reflection coefficient $\Gamma$, we model the studied circuit as an LCR-oscillator (cavity) with impedance $Z_{LCR}$ in parallel with the impedance $Z_q$ of the driven qubit. This as a whole is in series with the coupling capacitance $C_c = 1.3$ pF, producing the total impedance $Z$. The resistance $R = 1.0$ kΩ models the losses in the oscillator. Within this approach, the magnitude and the phase of the reflection coefficient are calculated from equation (17) and plotted in the $\varphi_b - \varphi_{ac}$ plane in figure 4 where we show also a comparison with the experimental data with the same parameters.

The calculated reflection coefficient shows a good agreement with the experiments, both in the magnitude and the phase, as seen in figures 4(a) and (b), respectively. The measured quasienergy contours display geometric collapse as they bend towards larger $\varphi_b$ when the drive amplitude $\varphi_{ac}$ is increased. This is the clearest indicator of the nonlinear coupling of the drive, as explained by the LZS model. Also, the transition from the adiabatic to diabatic behaviour is visible, similar to figure 3(a).

4. Conclusions

We have presented a study of a charge qubit driven strongly via the nonlinear and periodic Josephson potential. The LZS model shows good agreement with the full numerical Bloch solution of the excited state population. Outside the validity range of the linear LZS model, we see deviations from the multi-photon resonance conditions, which we relate to additional evolutionary paths that lead to more complicated interference within a driving period. Thus, future work includes the revision of the LZS model including the possibility of multiple LZ crossings within a driving period. The absorptive response of the driven qubit to a weak probe cannot be calculated within the LZS model. Instead, we have calculated the probe absorption spectrum of the quasienergies by employing the Fermi’s golden rule. Absorption was shown to occur at the locations where the probe is on resonance with a quasienergy separation, and the resulting spectrum was in good accordance with the experimental data.

We have revealed the inherent nonlinearities within the system by driving with strong microwave flux, resulting in, e.g., the characteristic bending of the resonances and ultimately to the geometric collapse. Moreover, by enhancing the coupling between the cavity and the qubit in our circuit, we see a possibility of a novel study of nonlinear interactions at the quantum limit.

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