Polarized Deep Inelastic Diffractive Scattering near the Light Cone

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Polarized inclusive deep-inelastic scattering is formulated in the light cone expansion. The QCD evolution of the leading twist distribution functions is derived. It is shown that the twist–2 contribution to the structure functions \( g_2^{D(3)} \) is obtained via \( g_1^{D(3)} \) by a Wandzura–Wilczek relation.

1. Introduction

Inclusive unpolarized and polarized deeply inelastic diffractive scattering at high energies and momentum transfer is one of the important processes in lepton–nucleon scattering. As found by experiment, cf. [1], there are interesting relations between the cross sections of these processes and those of inclusive deeply inelastic scattering: \( i \) the scaling violations of both processes are quite similar and \( ii \) the ratio of the differential cross sections in \( x \) and \( Q^2 \) are widely constant in the whole kinematic domain and are of \( O(1/8...1/10) \). Whereas the latter aspect cannot be understood with perturbative methods the former calls for a rigorous analysis in perturbative QCD. In recent analyses [2,3] this aspect has been investigated both for the unpolarized and the polarized case on the basis of the light–cone expansion. By this method the semi-exclusive processes of diffractive scattering could be related to forward scattering processes at short distances, for which similar evolution equations as in the deep inelastic case apply. Moreover a Callan–Gross and Wandzura–Wilczek relation between the twist–2 contributions of the diffractive structure functions were derived. In this note we give a summary of these papers.

2. Scattering Cross Section and Formalism

The process of deep inelastic diffractive scattering is \( l + p \rightarrow l' + N' + \text{hadrons} \), with a significant rapidity gap between \( N' \) and the remaining hadrons. The differential scattering cross section for single–photon exchange is given by

\[
d^5\sigma_{\text{diffr}} = \frac{1}{2(s - M^2)} \frac{1}{4} dP S^{(3)} S^{(3)} e^4 Q^2 L_{\mu\nu} W_{\mu\nu},
\]

with \( L_{\mu\nu} \) and \( W_{\mu\nu} \) the leptonic and hadronic tensors. Using current conservation, P \ and \ T invariance and the hermiticity relation for the hadronic tensor one finds a representation of the hadronic tensor in terms of four unpolarized and eight polarized structure functions [4].
We will henceforth consider the case of small values of \((p_2 - p_1)^2 = t\). In this limit the outgoing and incoming proton momenta are related by \(p_2 = (1 - x_p)p_1\) and the cross section depends on two unpolarized and two polarized structure functions only

\[
W_{\mu\nu} = \left( g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1 + \left( p_{1\mu} - q_\mu \frac{p_{1\cdot q}}{q^2} \right) \left( p_{1\nu} - q_\nu \frac{p_{1\cdot q}}{q^2} \right) \frac{W_2}{M^2} \\
+ i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{p_{1\cdot q}} g_1 + i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda (p_{1\cdot q} S^\sigma - S_{q\cdot p_1}^\sigma)}{(p_{1\cdot q})^2} g_2 ,
\]

(2)

with \(MW_1 \rightarrow F_1\) and \((p_{1\cdot q}/M)W_2 \rightarrow F_2\) for \(Q^2, p_{1\cdot q} \rightarrow \infty\). Eq. (2) is considered in the generalized Bjorken limit: \(Q^2, p_{1\cdot q}, p_{2\cdot q} \rightarrow \infty\) and \(x = Q^2/(2p_{1\cdot q}), \eta = q.(p_2 - p_1)/q.(p_2 + p_1)\) is fixed. The non-forward variable \(\eta\) is related to another variable often used, \(x_p\), by \(x_p = -2\eta/(1 - \eta)\). In the limit \(t \rightarrow 0\) the above structure functions depend on the three variables \(x, x_p\) and \(Q^2\).

Since for diffractive processes the outgoing proton is well separated in rapidity from the diffractively produced hadrons (rapidity gap), one may apply A. Mueller’s generalized optical theorem [4] to calculate the scattering cross section. This is done moving the outgoing proton into an incoming anti-proton and considering the absorptive part of deep inelastic forward scattering off the state \(\langle p_1, -p_2, S_1|\) summing over all final-state spins. Note that under this interchange \(t\) is kept space-like. Due to this operation we may now evaluate the Compton–operator

\[
\hat{T}_{\mu\nu}(x) = RT \left[ J_\mu \left( \frac{x}{2} \right) J_\nu \left( -\frac{x}{2} \right) S \right] \]

(3)

between the above states for forward scattering. We represent this operator in terms of a vector and an axial-vector operator, which are in turn related to the associated scalar and pseudo-scalar operators, through which we introduce the respective operator expectation values, see [3,4] defining non–forward parton densities \(f_{x_5}^q(z_+, z_-), \)

\[
\langle p_1, -p_2|O_5^q(\kappa_+ x, \kappa_- x)|p_1, -p_2\rangle = x_p \int Dz \ e^{-i\kappa_- x_p z} f_5^q(z_+, z_-), \\
\langle p_1, S_1, -p_2|O_5^q(\kappa_+ x, \kappa_- x)|p_1, S_1, -p_2\rangle = x S \int Dz \ e^{-i\kappa_- x_p z} f_5^q(z_+, z_-) ,
\]

(4)

with \(S \equiv S_1\) and \(p_{1\pm} = p_2 \pm p_1\). Here we neglect sub-leading components \(\propto \pi_{-p_{1+}} - p_{-}/\eta\). After passing a series of steps, see [3,4], we may express the hadronic tensor in this approximation by one unpolarized and one polarized distribution function, \(f_D^q\) and \(f_{5D}^q\), respectively. For quarks and anti-quarks these distribution functions, which are the diffractive parton distributions, read

\[
f_{(5)}^{D}(\pm 2\beta, \eta, Q^2) = a \int_{\frac{2-2x_p}{x_p + 2x_p}} \frac{\rho^2}{2-2x_p} \ d\rho f_{(5)}(\rho, \pm 2\beta + \rho(2 - x_p)/x_p; Q^2) .
\]

(5)

The upper sign refers to quarks, the lower to anti-quarks, and \(a = -1\) in the unpolarized case, \(a = 1/x_p\) in the polarized case, where \(\beta = x/x_p\).
3. Relations between Structure Functions

The diffractive structure functions $F_1^D$ and $g_1^D$ obey the representation

$$F_1^D(\beta, \eta, Q^2) = \sum_{q=1}^{N_f} e_q^2 \left[ f_q^D(\beta, x_P, Q^2) + \bar{f}_q^D(\beta, x_P, Q^2) \right]$$

$$g_1^D(\beta, \eta, Q^2) = \sum_{q=1}^{N_f} e_q^2 \left[ f_q^D(\beta, x_P, Q^2) + \bar{f}_q^D(\beta, x_P, Q^2) \right].$$

(6)

After some calculation one finds for the twist–2 contributions to the hadronic tensor the relations

$$F_2^D(\beta, \eta, Q^2) = 2x F_1^D(\beta, \eta, Q^2)$$

$$g_2^D(\beta, \eta, Q^2) = -g_1^D(\beta, \eta, Q^2) + \int_{\beta}^{1} \frac{d\beta'}{\beta'} g_1^D(\beta', \eta, Q^2).$$

(7)

The Callan–Gross relation between the structure functions depending on $\beta$ is modified due to the emergence of $x$, while the Wandzura–Wilczek relation holds in the new variable $\beta \in [0, 1]$. The emergence of the integral term in one of the above relations is due to a basic connection between a vector–valued non–forward distribution function and the associated scalar one [5]. The corresponding term exceptionally cancels in the Callan–Gross relation but is present in most relations of this type, see also [6,7].

4. Evolution Equations

The evolution equations of the diffractive parton densities can be formulated starting with the evolution equations for the scalar quark and gluon operators in the flavor non–singlet and singlet case, see e.g. [5].

$$\mu^2 \frac{d}{d\mu^2} O^A(\kappa_+ \bar{\kappa}, \kappa_- \bar{\kappa}; \mu^2) = \int D\kappa' \gamma^{AB}(\kappa_+, \kappa_-; \mu^2) O_B(\kappa'_+ \bar{\kappa}, \kappa'_- \bar{\kappa}; \mu^2),$$

(8)

with $\mu$ the factorization scale. Forming expectation values as in the foregoing section one notices that the evolution does not depend on the value of the light-cone mark $\kappa_+$, which can be set to 0. Moreover the all-order rescaling relation

$$\gamma^{AB}(\kappa_+, \kappa_-; \mu^2) = \sigma^{d_A} \gamma^{AB}(\sigma \kappa_+, \sigma \kappa_-; \mu^2),$$

(9)

where $d_A = 2 + d_A - d_B, d_q = 1, d_G = 2$, is applied. After some calculation one finds the following evolution equations

$$\mu^2 \frac{d}{d\mu^2} f_A^D(\beta, x_P; \mu^2) = \int_{\beta}^{1} \frac{d\beta'}{\beta'} P_A^B \left( \frac{\beta}{\beta'}; \mu^2 \right) f_B^D(\beta', x_P; \mu^2).$$

(10)

These equations apply both to the unpolarized and polarized diffractive parton densities of twist–2 to all orders in the coupling constant. In the calculation the absorptive part of the distribution was taken, which identifies the original momentum fraction, cf. [29, 33].
those in $\vartheta$. If compared to the case of deep-inelastic scattering the evolution is here not in $x$ but in the new variable $\beta$. Otherwise the same evolution equations are obtained. This holds also for higher twist operators, for which the argument is exactly the same as given in this section, see [2,3].

5. Conclusions

We derived the twist–2 evolution equations for the unpolarized and polarized twist–2 diffractive parton distributions considering these processes in the light–cone expansion at short distances. We also derived relations between the diffractive structure functions in the unpolarized and polarized case. The observed similarity of the scaling violations between deep-inelastic diffractive and deep–inelastic structure functions was shown in the present approach being due to the same structure of evolution equations which act in the former case on the variable $\beta$ and in the latter on $x$. Polarized deep–inelastic scattering was not yet observed nor studied in detail in a larger kinematic domain. It would be interesting to know if the pattern of relations as observed for unpolarized scattering repeats and the predictions of the present paper can be verified. The COMPASS experiment and polarized experiments at future high-energy facilities could clarify this.

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