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AIP Advances 9, 125105 (2019); https://doi.org/10.1063/1.5117340
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Cite as: AIP Advances 9, 125008 (2019); doi: 10.1063/1.5124436
Submitted: 15 August 2019 • Accepted: 11 November 2019 • Published Online: 3 December 2019

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ABSTRACT

When a strong magnetic field diffuses into a metal, the metal is ablated by Joule heating accompanying the magnetic diffusion process, and the metal’s resistance changes violently with the fast-growing temperature. This results in the formation of a so-called “nonlinear diffusion wave” characterized by a sharp “wave-front” where the magnetic field abruptly decays. A metal has its own threshold magnetic field value, which is determined by the critical temperature of the metal. If the constant vacuum magnetic field \( B_0 \) is above the threshold value \( B_c \), the magnetic diffusion process can be approximately described by sharp-front diffusion wave theory [B. Xiao et al., Physics of Plasmas 23, 082104 (2016)], which gives a simple formula to describe the velocity of the diffusion process. However, if \( B_0 \) is below \( B_c \), the sharp-front diffusion wave theory is no longer applicable. In this situation, one would need another type of sharp-front diffusion wave theory (type II theory) to describe the magnetic diffusion behaviors. In type II theory, the sharp-front diffusion wave velocity depends on three parameters, i.e., the magnetic boundary condition \( B_0 \), the critical temperature \( T_c \), and the cold metal resistance \( \eta \). The dependence of the velocity on these three parameters is analyzed in detail in this paper.

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I. INTRODUCTION

The diffusion of a strong magnetic field in metals is one of the key factors that affect the results of magnetic-driven experiments (such as magnetic-driven flyer\textsuperscript{1,2} and magnetic flux cumulation\textsuperscript{3}). Most metals’ resistance has a dramatic dependence on temperature, so the penetration of strong magnetic fields into a metal is a highly nonlinear process. When the magnetic diffusion starts, Joule heat is produced simultaneously. The Joule heat raises the metal’s temperature and then the resistivity of metal increases, which in turn accelerates magnetic diffusion and the Joule heat production rate. This forms a positive feedback of magnetic field diffusion. While the massive Joule heating generated in the magnetized region accelerates the magnetic field diffusion, the high conductivity in the relatively unmagnetized region hinders it. This results in the formation of a “nonlinear diffusion wave”\textsuperscript{4,5} characterized by a sharp wave-front where the magnetic field abruptly decays.

Many literature studies have contributed to studying the characteristics of strong magnetic diffusion in metals. Garanin\textsuperscript{6} pointed out that the magnetic field needs to be above a threshold value in order for the surface of the metal to be burned fast into plasmas. This was verified many years later in experiments.\textsuperscript{10} Schnitzer\textsuperscript{7} studied the diffusion process of strong magnetic fields into semi-infinite compressible metallic conductors via a simplified resistivity model \( \sigma = \sigma_0 \left( \frac{T}{T_c} \right)^{-\gamma} \), wherein the magnetic-diffusion equation is coupled with a thermal-energy balance, and analyzed the propagation of the magnetic diffusion wave-front. Xiao\textsuperscript{8} studied the sharp-front wave of strong magnetic field diffusion in solid metals by simplifying the Burgess resistivity model\textsuperscript{9} to step shape and gave an approximate formula to describe the diffusion velocity, which is applicable for the condition that the boundary magnetic field is above a threshold value \( B_c = \sqrt{2\mu_0 J_c} \). In this paper, we show that when \( B_0 \) is below \( B_c \), the diffusion process can still behave in the manner of a “sharp-front” wave. This sharp-front magnetic diffusion wave, although similar in appearance to the magnetic diffusion wave of \( B_0 > B_c \), obeys quite a different rule to it.

The remaining parts of this paper are organized as follows. In Sec. II, we establish the mathematical description of the problem;
The physical process is the penetration of a vacuum magnetic field into a metal slab, as illustrated in Fig. 2. The constant vacuum magnetic field \( B_0 \) is out of the metal slab, and the initial magnetic field in the metal slab is zero. The initial resistivity of the metal slab is \( \eta_\text{i} \), and its initial temperature is room temperature \( T_0 \). After the diffusion starts, the vacuum magnetic field diffuses into the slab and the Joule heat is generated as a result. When the Joule heat raises the temperature of the slab’s magnetized region to the critical point \( T_c \), it leads to an abrupt increase in the resistivity, which in turn accelerates the magnetic diffusion process. As a result, a “wave-front line” will appear between the regions of resistivity \( \eta_\text{i} \) with \( \eta_\text{i} \) and propagate deeper. In this process, kinematic movements are not considered and the heat diffusion rate, which is much smaller than the magnetic diffusion rate, is also neglected.

For this physical model, as pointed out in Xiao’s paper, the metal has a threshold magnetic field value \( B_c = \sqrt{2\rho_0 J_c} \), in which \( J_c \) is the heat energy required to heat the metal from room temperature to the gasification point per volume [e.g., \( J_c = c_p(T_c - T_0) \) when \( c_p \) and \( \rho_0 \) are constant]. As described in Ref. 14, when the magnetic field \( B_0 \) is above \( B_c \), the solution of the magnetic diffusion equation can be approximately described by the curve shown in Fig. 3, and from this solution, the wave velocity can be derived analytically as

\[
V_c = \frac{\eta_\text{i}}{\mu_0} \frac{B_0 - B_c}{B_c} \frac{1}{\eta_\text{i}}
\]

(4)

For the case of \( B_0 < B_c \), however, the situation becomes more complex. In this case, the magnetic diffusion curve still has a sharp wave-front as shown in Fig. 4 (to differentiate from that in Fig. 3, we call this the second type of sharp-front wave) but it is difficult to acquire the approximate analytical solution because the curve of the wave-front can no longer be described in a simple exponential form. What we can do in this paper is obtain the characteristics of the second type of sharp-front wave mainly by numerical simulations.

### III. NUMERICAL SIMULATION

We set the metal’s physical parameters in our simulation as follows: room temperature is set as \( T_0 = 300 \) K, with the critical temperature \( T_c \) varying around 5000 K; the density is \( \rho = 7.86 \) g cm\(^{-3}\); and the heat capacity is set to be a constant value \( c_p = 0.45 \) J g\(^{-1}\) K\(^{-1}\); the initial resistivity varies around \( \eta_\text{i} = 9.7 \times 10^{-8} \) \( \Omega \) m. Since the final description of the characteristics of the second type of sharp-front wave should be independent of the actual values of the physical parameters, a self-consistency test for two different numerical simulation methods is conducted; in Sec. III B, the law of the second type of sharp-front wave is studied by simulation. Conclusions are given in Sec. IV.

### II. PHYSICAL PROBLEM MODELING

The diffusion of the magnetic field in metals can be described by the one-dimensional equations

\[
\frac{\partial}{\partial t} B(x, t) = \frac{\partial}{\partial x} \left( \frac{\eta}{\mu_0} \frac{\partial}{\partial x} B(x, t) \right),
\]

(1)

\[
c_p \frac{\partial}{\partial t} (T(x, t)) = \eta \frac{\partial}{\partial t} B(x, t),
\]

(2)

\[
\eta = \eta(T),
\]

(3)

where \( B(x, t) \) and \( T(x, t) \) are the distributions of the magnetic field and temperature, respectively, \( c_p \) and \( \rho \) are the specific heat capacity and density, \( \mu_0 \) is the space permeability, and \( \eta(T) \) is the electrical resistance of the metal whose dependence on the temperature is given by a resistivity model. In this paper, a simplified step-shaped model is designed according to the general characteristics of the metal’s resistivity,\(^{13,14}\) as shown in Fig. 1.

The physical process of the magnetic field diffusion into a metal slab, as illustrated in Fig. 2. The constant vacuum magnetic field \( B_0 \) is out of the metal slab, and the initial magnetic field in the metal slab is zero. The initial resistivity of the metal slab is \( \eta_\text{i} \), and its initial temperature is room temperature \( T_0 \). After the diffusion starts, the vacuum magnetic field diffuses into the slab and the Joule heat is generated as a result. When the Joule heat raises the temperature of the slab’s magnetized region to the critical point \( T_c \), it leads to an abrupt increase in the resistivity, which in turn accelerates the magnetic diffusion process. As a result, a “wave-front line” will appear between the regions of resistivity \( \eta_\text{i} \) with \( \eta_\text{i} \) and propagate deeper. In this process, kinematic movements are not considered and the heat diffusion rate, which is much smaller than the magnetic diffusion rate, is also neglected.

For this physical model, as pointed out in Xiao’s paper, the metal has a threshold magnetic field value \( B_c = \sqrt{2\rho_0 J_c} \), in which \( J_c \) is the heat energy required to heat the metal from room temperature to the gasification point per volume [e.g., \( J_c = c_p(T_c - T_0) \) when \( c_p \) and \( \rho_0 \) are constant]. As described in Ref. 14, when the magnetic field \( B_0 \) is above \( B_c \), the solution of the magnetic diffusion equation can be approximately described by the curve shown in Fig. 3, and from this solution, the wave velocity can be derived analytically as

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For the case of \( B_0 < B_c \), however, the situation becomes more complex. In this case, the magnetic diffusion curve still has a sharp wave-front as shown in Fig. 4 (to differentiate from that in Fig. 3, we call this the second type of sharp-front wave) but it is difficult to acquire the approximate analytical solution because the curve of the wave-front can no longer be described in a simple exponential form. What we can do in this paper is obtain the characteristics of the second type of sharp-front wave mainly by numerical simulations.
parameters, we choose these parameters to make the simulations more sensible and convenient for analysis.

A. Self-consistency test of two simulation methods

The resistivity \( \eta_L \) is far larger than \( \eta_s \), so we focus on the characteristics of the second type of sharp-front wave in which the resistivity \( \eta_L \) is approximated to infinity. In this issue, two methods can be used. One is to make \( \eta_L = k \eta_s \), where \( k \) is a large number. The other method is to make \( \eta_L = \infty \) directly, and in this method, a moving magnetic field boundary needs to be used. The simulation results by these two methods are shown in Fig. 4. It can be seen that at the same moment, the greater the resistivity \( \eta_L \) used in the first method, the deeper the inflection point of the "wave-front", and the more approximate the corresponding magnetic field distribution curve is to that of \( \eta_L = \infty \). So, we can say that the simulation results obtained by these two methods are mutually verifiable when the value of \( \eta_L \) used in the first method is large enough. However, the time step lengths for magnetic diffusion calculation in these two simulation methods are respectively determined by \( \eta_L \) and \( \eta_s \), so the simulation speed of the first method will be much slower than that of the second one. Due to this limitation, the first method cannot simulate the situation wherein the value of \( \eta_L \) is too large. Therefore, in the following part, we will study the second type of sharp-front wave based on the second simulation method.

B. The law of the second type of sharp-front wave

After the diffusion starts, the magnetic field begins to penetrate into the metal. If the Joule heat raises the temperature of the metal to the critical point \( T_c \), then the resistivity will bump up to infinity and instantaneously make the rest of the diffusion process accomplished. The magnetic field of the totally magnetized region will achieve the same strength of the vacuum boundary magnetic field. The "wave-front" will appear between regions of resistivity \( \eta_L \) and \( \eta_s \) and propagate deeper. The numerical simulation results of the second type of sharp-front wave at different moments are shown in Fig. 5(a). The conventional magnetic diffusion process with a constant resistance \( \eta_s \) is shown in Fig. 5(b) for comparison.

In principle, the relationships between the sharp-front wave velocity \( V_c(t) \) and the parameters \( (B_0, \eta_s/\mu_0, T_c) \) can be obtained by varying the three parameters in the simulation. However, to do it directly in this way will cost too much time, and worse, make the relation \( V_c(B_0, \eta_s/\mu_0, T_c; t) \) a complicated expression that is not convenient for use. It is better to reduce the parameters according to the characteristics of the magnetic diffusion equations as much as possible before deriving their relations numerically.

In the magnetic diffusion Eq. (1), the first order time derivative of the magnetic field is proportional to the coefficient \( \eta_s/\mu_0 \), and since \( \eta_s \) is the only value for the resistance in the model, \( V_c \) must be proportional to \( \eta_s/\mu_0 \) in a fixed magnetic field configuration (which can be described by the wave-front displacement \( X_c \)). That means

\[
V_c(\eta_s/\mu_0; X_c) \propto \eta_s/\mu_0.
\]

In Ref. 8, it is pointed out that the magnetic diffusion must depend only on the self-similar variable \( x/\sqrt{t} \). This indicates a relation \( X_c \propto \sqrt{t} \). In the magnetic diffusion process: (a) 5 \( \mu s \) and (b) 10 \( \mu s \).
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FIG. 5. (a) Results of the second type of sharp-front wave at different moments. (b) The conventional magnetic diffusion process.

of \( V_c \equiv dX_c/dt \), it can lead to the relation

\[
V_c(X_c) \propto \frac{1}{X_c}. \tag{6}
\]

Relations (5) and (6) are confirmed by simulations shown in Figs. 6(a) and 6(b).

By taking relations (5) and (6) into account, the relation

\[
V_c(B_0, \eta_s/\mu_0, T_c; t) = \frac{\eta_s \mu_0 h(B_0, T_c)}{X_c(t)} \tag{7}
\]

or equivalently

\[
V_c(B_0, \eta_s/\mu_0, T_c; t) = \sqrt{\eta_s \mu_0 h(B_0, T_c)} \frac{1}{\sqrt{t}} \tag{8}
\]

In the following part, we are going to clarify that the two parameters \( B_0 \) and \( T_c \) can be further reduced to only one. We analyze as follows (please note that talking about \( T_c \) is equivalent to talking about \( J_c \); in the following analysis, we temporarily use \( J_c \) instead of \( T_c \) for convenience of discussion). Assuming that one couple of parameters \( (B_0, J_c) \) and \( (B_0', J_c') \), where \( B_0' = aB_0 \) (\( a \) = const), could result in the same evolution of the magnetic diffusion curve, then the electric currents calculated in these two cases are related via

\[
\frac{j}{j'} = \frac{1}{\mu_0} \frac{\partial B}{\partial x} = \frac{1}{\mu_0} \frac{\partial aB}{\partial x} = aj \tag{9}
\]

Then, the Joule heat produced at \( X_c \) for the two cases is related by

\[
J_c(J_c') = a^2 J_c(J_c'). \tag{10}
\]

This result means that for a couple of parameters \( (B_0, J_c) \) and \( (B_0', J_c') \), there exists corresponding data \( (aB_0, a^2 J_c) \) which can lead to the same sharp-front wave curve. Thus, we can define a dimensionless parameter as \( \gamma_c = \frac{B_0^2}{(2\mu_0 J_c)} \), and the dependence of \( V_c \) on \( (B_0, T_c) \) is simplified to the dependence on \( \gamma_c \). The simulation result shown in Fig. 7 confirms this analysis: assuming that the resistivity \( \eta_s \) is the same, for simulations with different values of \( B_0 \) and \( T_c \), the same moving track of the wave-front \( X_c(t) \) could be obtained once the values of \( \gamma_c \) are equal.

FIG. 6. (a) The linear relations between \( V_c \) and \( \eta_s/\mu_0 \). (b) The linear relations between \( V_c \) and \( 1/X_c \). [Note that in Fig. 6(b), the appearance of the turning point is due to the numerical error when \( X_c \) is very small.]
Then, the function of $\gamma_c$ is

$$f(\gamma_c) = 2/(4.26 - 4.12\gamma_c).$$  \hspace{1cm} (15)

Taking the fitting results (13) and (15) into consideration, we can rewrite Eq. (10) equivalently in the form

$$f(\gamma_c) = \frac{2\eta_0/\mu_0}{(46.3 - 165.9\gamma_c + 209.5\gamma_c^2 - 91.3\gamma_c^3)^2}, \quad \gamma_c \in [0, 0.8].$$

$$f(\gamma_c) = \frac{2\eta_0/\mu_0}{(4.26 - 4.12\gamma_c)^2}, \quad \gamma_c \in [0.8, 0.95].$$  \hspace{1cm} (16)

**IV. CONCLUSION**

In this paper, a simplified resistivity model is used to study the nonlinear magnetic diffusion wave when the vacuum boundary magnetic field $B_0$ is below the threshold magnetic field value $B_c$. Approximate formulas for the description of the sharp-front diffusion wave velocity are obtained. The velocity $V_c$ depends on three parameters, i.e., the boundary magnetic field $B_0$, the critical temperature $T_c$, and the cold metal resistance $\eta_c$, of which the dependence on $B_0$ and $T_c$ can be simplified to the dependence on the dimensionless parameter $\gamma_c = B_0^2/(2\mu_0J_c)$. A key difference of this second type of magnetic diffusion wave (i.e., for $B_0 < B_c$) from the one for $B_0 > B_c$ is that this "second-type" velocity is proportional to the cold metal resistance $\eta_c$, while the "first-type" one is proportional to the resistance of the burned metal $\eta_c$. This means that the second type of magnetic diffusion wave velocity is usually much smaller than the first one. The final formulas of the second type of sharp-front wave can be used to evaluate the magnetic field diffusion rate expediently when the outer magnetic field is below the threshold magnetic field value.

**ACKNOWLEDGMENTS**

This work was supported by the Foundation of China Academy of Engineering Physics (Grant No. 2015B0201023) and the National...
Natural Science Foundation of China (Grant Nos. 11571293, 11672276, and 11532012).

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