Spinor condensates with a laser-induced quadratic Zeeman effect

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We show that an effective quadratic Zeeman effect can be generated in $^{52}\text{Cr}$ by proper laser configurations, and in particular by the dipole trap itself. The induced quadratic Zeeman effect leads to a rich ground-state phase diagram, can be used to induce topological defects by controllably quenching across transitions between phases of different symmetries, allows for the observability of the Einstein-de Haas effect for relatively large magnetic fields, and may be employed to create $S = 1/2$ systems with spinor dynamics. Similar ideas could be explored in other atomic species opening an exciting new control tool in spinor systems.

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Spinor Bose-Einstein condensates (BEC) have recently attracted a growing interest. A spinor gas is formed by atoms in two or more internal states, which can be simultaneously confined by optical dipole traps (1). Spinor BECs present a rich variety of possible ground states, including ferromagnetic and polar phases for spin-1 BECs (2,3), and an additional cyclic phase for the spin-2 case (4,5). Elegant topological classifications of the possible spinor ground-states have been recently proposed (6,7). The spinor dynamics has been also actively studied, in particular the coherent oscillations between the different spinor components (8). In addition, a spinor gas has been recently quenched across a transition between phases of different symmetries, inducing topological defects (9,10).

The recent creation of a Chromium BEC (11) opens new interesting possibilities for the spinor physics. The ground state of $^{52}\text{Cr}$ is $^7\text{S}_3$, constituting the first accessible example of a spin-3 BEC. The spin-3 BEC presents a novel rich ground-state phase diagram at low magnetic fields (12,13,14). In particular, the existence of biaxial spin-nematic phases (12) opens fascinating links between the spin-3 BECs and the physics of liquid crystals. In addition, $^{52}\text{Cr}$ has a large magnetic moment $\mu = 6\mu_B$, where $\mu_B$ is the Bohr magneton, i.e. six times larger than that of alkali atoms. The corresponding large dipole-dipole interaction (DDI) can lead to novel effects in the BEC physics (15). In particular, dipolar effects were observed for the first time ever in quantum gases in the expansion of a Chromium BEC (16). The DDI plays also a significant role in the spinor dynamics, since it violates spin conservation, allowing for the transfer of spin into center-of-mass angular momentum, i.e. the equivalent to the Einstein-de Haas effect (EdH) (13,17). Interestingly, the EdH and other dipolar effects may be also observed in $^{87}\text{Rb}$ spinor BECs since, in spite of its low $\mu$, the DDI may be significant when compared to the low energy scales associated with the spinor physics (18,19).

In the presence of an external magnetic field, $B$, the different Zeeman sublevels (with quantum number $m$) of a spinor BEC acquire different shifts due to the linear Zeeman effect (LZE), $\Delta E_{LZE}(m) = g\mu_B B m$, with $g$ the Landé factor. The LZE plays no role in the spinor dynamics of short-range interacting BECs, because spin is never violated, and hence the LZE may be gauged out. On the contrary, the dipole-induced EdH is largely prevented by the LZE, even for rather low magnetic fields (12,13,17,19). In addition to the LZE, and due to the underlying hyperfine structure, a quadratic Zeeman effect (QZE), $\Delta E_{QZE}(m) \propto B^2m^2$ cannot be neglected, since it indeed becomes very important for the understanding of typical spinor BECs. The QZE is not present in $^{52}\text{Cr}$ due to the absence of hyperfine structure but can be induced by a quasi-resonant light field (20). The big advantage of a light induced QZE consists mainly in its tunability. In particular Gerbier et al. have recently employed an off-resonant microwave field to induce a QZE of the appropriate sign and resonantly control the spinor dynamics in Rb atoms in optical lattices (21).

In this Letter, we show that a light-induced QZE, tuned independently from the magnetic field, opens promising ways of control for $^{52}\text{Cr}$, and in general for other spinor gases. In the first part of the Letter we analyze the rich ground-state phase diagram introduced by the QZE. In particular modifications of the induced QZE may quench across transitions between phases with different symmetries, and could lead to the observation of topological defects (9,10). In the second part of the Letter, we discuss how the effective QZE may be manipulated such that a large EdH effect may be observed even in the presence of a relatively large, and even fluctuating magnetic field. Moreover, we show that the induced QZE can be used to engineer $S = 1/2$ systems with spinor dynamics, differing significantly from standard binary BECs (22), where spinor dynamics is absent.

The ground-state of $^{52}\text{Cr}$ ($^7\text{S}_3$) can be coupled by

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optical dipole transitions to different excited P-states ($^7P_{2,3,4}$). For sufficiently detuned lasers from these transitions, an m-dependent Stark shift can be induced in the $^7S_3$ manifold. We show in the following that such a shift mimics a QZE in $^{52}$Cr, despite the absence of nuclear spin. To illustrate this fact, we shall initially simplify the actual experimental situation (which is discussed in detail below), and consider a π-polarized laser (in the z-direction) on the particular optical transition $^7S_3 \to ^7P_3$. For a sufficiently large laser detuning $\Delta$ from the transition frequency, the induced Stark shift for a given state $m$ of the $^7S_3$ manifold is provided by $\Delta E(m) \approx \hbar \Omega_0(m)^2/\Delta$, where the Rabi frequency is given by $\Omega_0 = (eE/\hbar)^{7P_3, m|z|^7S_3, m}$ (due to selection rules only transitions to the same $m$ are possible in the example considered), where $e$ is the electron charge, and $E$ is the electric field of the laser. The optical dipole moment of the transition is easily calculate from the corresponding Clebsch-Gordan coefficients, $|J = 3, m; j = 1, 0J = 3, m\rangle = m/2\sqrt{3}$. As a consequence a QZE $\Delta E(m) = \alpha m^2$ is induced, where $\alpha$ is a function of the applied intensity and the detuning, but independent of the magnetic field. Similarly, for transitions to other $^7P$ states, with other polarizations and detunings, general shifts of the form $\Delta E(m) = \gamma m + \alpha m^2$ can be generated. In the following we absorb $\gamma$ in the LZE.

![FIG. 1: Phases as a function of $\tilde{p}$ and $\tilde{a}$. See text for details.](image)

Exact calculations taking into account the excited states $^7P_{2,3,4}$ and $^7P_{3,4,3}$ show that $\alpha/2\mu_B \approx \pm 2 \text{ mG}$ can be obtained with few mW of π polarized light with a wavelength of 430 nm (red detuning respect the $^7P$ states) and 424 nm (blue detuning), respectively. However, a heating rate of ≈ 2 $\mu$K/s due to off-resonance light scattering would destroy the condensate in few tens of ms. To increase the lifetime we can increase the detuning and the applied light power. Interestingly, the dipole trap laser at 1064 nm normally used to condense $^{52}$Cr atoms [11] for 20 W focused to 30 μm induces a QZE $\alpha/2\mu_B \approx 7 \text{ mG}$ for π polarization and $\alpha/2\mu_B \approx -3 \text{ mG}$ for a combination of $\sigma^+$ and $\sigma^-$ polarizations. This is due to a discrepancy of 10% between the values of the linewidths of the excited states $^7P_{2,3,4}$ [23]. For such dipole trap the lifetime of the condensate can be a few seconds. Using combinations of different polarizations to change $\alpha$ can cause two photon Raman coupling between different sublevels severely affecting the spinor ground state. The latter can be prevented by combining different lasers with orthogonal polarizations and with randomized relative phases.

In the following we consider an optically trapped Chromium BEC with $N$ particles under the influence of the above mentioned QZE. The corresponding Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{V}_{sr} + \hat{V}_{dd}$. The single-particle part, $\hat{H}_0$, includes the trapping energy and the LZE and QZE:

$$\hat{H}_0 = \int d\mathbf{r} \sum_m \hat{\psi}^\dagger_m \left[ -\hbar^2 \nabla^2 / 2M + U_{\text{trap}} + pm + \alpha m^2 \right] \hat{\psi}_m,$$

(1)

where $\hat{\psi}_m$ ($\hat{\psi}_m$) is the creation (annihilation) operator in the $m$ state, $M$ is the atomic mass, $U_{\text{trap}}(\mathbf{r})$ is the trapping potential, and $p = g_{\mu_B}B + \gamma$, with $g = 2$ for $^{52}$Cr. The short-range interactions are given by [2]

$$\hat{V}_{sr} = \frac{1}{2} \int d\mathbf{r} \sum_{S=0}^6 g_S \hat{P}_S(\mathbf{r}),$$

(2)

where $\hat{P}_S$ is the projector on the total spin $S = (0, 2, 4, 6)$, $g_S = 4\pi\hbar^2a_S/M$, and $a_S$ is the s-wave scattering length for a total spin $S$. The DDI $\hat{V}_{dd}$ is given by

$$\hat{V}_{dd} = \frac{c_d}{2} \int d\mathbf{r} \int d\mathbf{r}’ \left[ \frac{1}{|\mathbf{r} - \mathbf{r}’|^3} \hat{\psi}^\dagger_m(\mathbf{r}) \hat{\psi}_m(\mathbf{r}’) \right]$$

$$\left[ S_{mn} \cdot S_{m’n’} - 3(S_{mn} \cdot \mathbf{e})(S_{m’n’} \cdot \mathbf{e}) \right] \hat{\psi}_n(\mathbf{r}) \hat{\psi}_n(\mathbf{r}’),$$

(3)

where $S = (S_x, S_y, S_z)$, $S_x, y, z$ are the spin-3 matrices, $c_d = \mu_0 \mu_B^2 g^2 / 4\pi (= 0.004 g_6)$ for $^{52}$Cr, with $\mu_0$ the magnetic permeability of vacuum, and $e = (\mathbf{r} - \mathbf{r}’)/|\mathbf{r} - \mathbf{r}’|$. We first discuss the ground-state of the spin-3 BEC. We consider mean-field (MF) approximation $\psi_m(\mathbf{r}) \approx \sqrt{N} \psi_m(\mathbf{r})$. In order to simplify the analysis of the possible ground-state solutions we employ the single-mode approximation (SMA): $\psi_m(\mathbf{r}) = \Phi(\mathbf{r})\psi_m$, with $|d\mathbf{r}^2|\Phi(\mathbf{r})|^2 = 1$, $\beta = \int d\mathbf{r}^2|\Phi(\mathbf{r})|^4$. Considering a magnetic field in the z-direction, and following the procedure discussed in Ref. [13], we obtain the expression for the energy per particle $E = N\beta\epsilon/2$

$$\epsilon = \tilde{p}(S_z) + \tilde{\alpha}(S_z^2) + \tilde{c}_1(S_x^2) + \frac{4c_2}{t} |\Theta|^2$$

$$+ c_3 \left( \frac{3(S_z^2)^2}{2} - 12(S_x^2)^2 + 12|S_x^2|^2 + 2|S_x^2|^2 \right)^2$$

(4)

where $S_z = S_x + iS_y$, $\Theta = \sum_m (-1)^m \psi_m \psi_{-m}/2$, $\tilde{p} = 2p/N\beta$, $\tilde{\alpha} = 2a/\Omega_0$, $\tilde{c}_1 \approx c_1 - c_3/2$, $c_1 = (g_6 - g_3)/18$, $c_2 = g_0 + (-55g_2 + 27g_4 - 5g_6)/33$, $c_3 = g_2/126 - g_4/77 + g_6/198$. For the case of $^{52}$Cr [24] $c_0 \approx 0.659$, $c_1 \approx 0.059g_6$, $c_2 \approx g_0 + 0.374g_6$, and $c_3 \approx -0.002g_6$. The
value of $a_0$ is unknown. In the following we assume $g_0 = g_0$. Compared to the expression of Ref. [13], (1) contains an extra term $\tilde{\alpha} (S_2^2)$ corresponding to the induced QZE. This new term leads to a rich physics of new phases.

We have minimized by means of simulated annealing Eq. (1) with respect to $\psi_m$, under the constraints $\sum_m |\psi_m|^2 = 1$ and $\langle S_+ \rangle = 0$. Fig. 1 shows the corresponding phase diagram as a function of the LZE ($\tilde{p}$) and the QZE ($\tilde{\alpha}$). For sufficiently large values of $\tilde{\alpha}$ and $\tilde{p}$, different ferromagnetic phases ($F_m$) are possible: $F_{-3}$ ($\tilde{p} > 5\tilde{c}_1 + 5\tilde{\alpha} + 75\tilde{c}_3/2$, $F_{-2}$ ($3\tilde{c}_1 + 3\tilde{\alpha} - 27\tilde{c}_3/2 < \tilde{p} < 5\tilde{c}_1 + 5\tilde{\alpha} + 75\tilde{c}_3/2$, $F_{-1}$ ($\tilde{\alpha} + \tilde{c}_1 - 21\tilde{c}_3/2 - c_2/7 < \tilde{p} < 3\tilde{c}_1 + 3\tilde{\alpha} - 27\tilde{c}_3/2$), and $F_0$ ($\tilde{p} < \tilde{\alpha} + \tilde{c}_1 - 21\tilde{c}_3/2 - c_2/7$). For sufficiently negative $\tilde{\alpha}$, a polar phase ($P$) transforms continuously into a ferromagnetic phase ($F_{-3}$) (the transformation is complete at $\tilde{p} = 6\tilde{c}_1 - 2c_2/21 \simeq 0.22g_0$). In addition to these phases, as shown in Fig. 1, uniaxial ($U$) and biaxial ($B$) spin-nematic phases are possible [12], depending on the eigenvalues $\lambda_{1,2,3}$ of the nematic tensor $\langle S_i S_j - S_j S_i \rangle/2$ ($i, j = x, y, z$). The phase $U_1$ (CY$_{-3,-2}$ in Ref. [13]) fulfills $\lambda_1 > \lambda_2 > \lambda_3$, and transforms continuously into $F_{-3}$. The phase $U_3$ (CY$_{1,-2}$ in Ref. [13]) is a discotic phase satisfying $\lambda_1 = \lambda_2 > \lambda_3$, and transforms continuously into $F_{-1}$. Note that $U_1$ and $U_3$ become degenerated at $\tilde{\alpha} = 0$ as pointed out in Ref. [13]. In addition, different biaxial phases (with $\lambda_1 \neq \lambda_2 \neq \lambda_3$) occur, characterized by $\langle S_\pm^2 \rangle \neq 0$. $B_{1(3)}^{(1)}$ is basically a modification of $P$, $B_{1(3)}^{(3)}$ is the CY$_{-3,-1,1,3}$ phase in Ref. [13]. $B_3^{(1)}$ is a modification of $F_0$. At the boundaries between $F_{-3}$, $F_{-2}$ and $F_{-1}$ other biaxial phases appear, $B_{1(2)}^{(2)}$, $B_{2(2)}^{(2)}$, and $B_{3(2)}^{(2)}$, which are respectively modifications of $F_{-3}$, $F_{-2}$ and $F_{-1}$. Although a detailed analysis of the symmetries of the different phases is beyond the scope of this Letter, we would like to note, that following the classification scheme proposed in Ref. [7], the phases $P$, $U_1$ and $U_3$ have very different symmetry properties, in particular $P$ transforms as an hexagon, $U_1$ as a pyramid with pentagonal base, and $U_3$ as a tetrahedron, and hence a controlled change in the induced QZE at low magnetic fields may lead to very interesting quantum phase-transition dynamics [25], which is left for further investigations.

The induced QZE can play an important role in the observation of the EdH. As discussed in Refs. [12, 17], the DDI violates spin conservation. If the Hamiltonian preserves a cylindrical symmetry around the dipole direction, the conservation of the total angular momentum, leads to the transfer of spin into center-of-mass angular momentum, resembling the EdH. However, Larmor precession prevents the EdH even for relatively small $B$ (although other interesting dipolar effects may occur even for a fixed magnetization [19]). We show in the following that the QZE may be employed to induce under realistic conditions a degeneracy between two neighboring states of the ground-state manifold, allowing for a large EdH even for relatively large magnetic fields.

Following Ref. [13] we analyze the spinor dynamics within the MF approximation, but abandoning the SMA. The dynamics of the different components is provided by seven coupled non local non linear Schrödinger equations:

$$i\hbar \frac{\partial}{\partial t} \psi_m(r) = \left[ -\frac{\hbar^2 \nabla^2}{2M} + U_{\text{trap}} + pm + \alpha m^2 \right] \psi_m + N \left[ c_0 m + m(c_1 f_z + c_4 A_0) \right] \psi_m + N \left[ c_1 f_- + 2c_4 A_- \right] S_{m,m-1}^+ \psi_{m-1} + N \left[ c_1 f_+ + 2c_4 A_+ \right] S_{m,m+1}^- \psi_{m+1} + \left( -1 \right)^m 2N c_2 \psi_m^* \psi_m + N c_3 \sum_{n \neq m} \nabla_{ij} \left( S_i^I S_j^I \right)_{mn} \psi_n,$$

where $n(r)$ is the total density, $f_i(r) = \langle S_i(r) \rangle$, $A_0 = \sqrt{6\pi/5} \sqrt{S^3 / 3 \pi r_a} + \Gamma_{1,-} + \Gamma_{-1,+}$, $A_{\pm} = \sqrt{6\pi/5} \Gamma_{0,\pm} / \sqrt{6} \mp \Gamma_{\pm,\pm} + \Gamma_{\pm,2,\pm}$, $\Gamma_{m,n} = \int dr' f_i(r') Y_{2m}(r - r') / |r - r'|^3$, $S_{m,m+1}^\pm = (1/2)(m + 1)$, with $Y_{2m}$ spherical harmonics.

We consider at $t = 0$ all atoms in $m = -3$. Without DDI, spin conservation restricts the system to $m = -3$ (scalar BEC). The DDI allows for a transfer into $m = -2$, although Larmor precession limits this transfer to very small magnetic fields ($B \simeq 0.1$ mG). In the following, we show that the QZE optimizes such transfer for much larger $B$. For simplicity we consider a 2D BEC [24], i.e. we assume in the z-direction a strong harmonic potential of frequency $\omega_z$. Hence $\psi_m(r) = \phi_0(r) \psi_m(\rho)$, where $\phi_0(\rho) = \exp[-z^2/2l_z^2]|l_z \sqrt{\pi}/\sqrt{\omega_z}$, with $l_z = \sqrt{\hbar}/m \omega_z$. We consider an additional harmonic confinement of frequency $\omega_{xy}$ on the xy-plane. Fig. 2 shows the population in $m = -3$, for the particular case of $\omega_z = 2kZ$, $\omega_{xy} = 50Z$, and $N = 2 \times 10^4$ atoms. We consider the dipole direction in the y-direction. If $p = 0$ and $\alpha = 0$ a clear transfer from $m = -3$ to other modes is observed due to the coherent spin relaxation induced by the DDI. If a magnetic field of 20mG is applied in the absence of QZE the transfer to other modes is completely suppressed, and a scalar BEC in $m = -3$ is recovered. The presence of the induced QZE alters the situation significantly, since for $\alpha = p/5$, the $m = -3$ and $-2$ states become degenerated. In that case a significant population is again transferred between $m = -3$ and $m = -2$. Note, however, that due to technical reasons the magnetic field typically fluctuates around a given value, and hence the degeneration is not fulfilled at any time. We have taken this into account in our simulations by randomly varying $B = B_0 + \delta B$, for $\delta B = 1$ mG, such that $B_0$ and $\alpha$ satisfy the degeneration condition. As shown in Fig. 2 as long as the degeneration is fulfilled in average, the induced QZE allows for a large spin relaxation even for large and even fluctuating magnetic fields. It is crucial, however, to control the exter-
nal magnetic fields to avoid spurious polarization components in every laser, since e.g. a residual magnetic field of 100 μG transversal to the quantization axis would induce a coupling of 100s−1 between the m sublevels. This (single-particle) effect could obscure the EdH. However, the time-scale for the EdH, τEdH, is inversely proportional to the atomic density, contrary to the spurious single-particle transfer time-scale, τSP, which is independent of it. Hence a sufficiently large density can allow for τEdH < τSP, and hence a clear observation of the EdH.

As mentioned above, α = p/5 induces a degeneracy between the doublet {−3, −2}. The next Zeeman state, m = −1, is separated by a gap 2p/5 from the doublet, and hence (even for low B) the combination of LZE and QZE shields the doublet from the other Zeeman states, transforming the S = 3 problem into an effective S = 1/2 one. Note that contrary to typical S = 1/2 systems, in which there is no spinor dynamics, spin relaxation couples both levels. In this sense, the induced QZE allows for a fundamentally new physical situation, indeed the simplest spinor BEC system with spinor dynamics. Moreover, an effective spin-1/2 system may be generated for any pair {m, m + 1} if α = −p/(2m + 1), i.e. six S = 1/2-systems with different collisional properties are possible. They will be studied in detail elsewhere.

Summarizing, a QZE can be induced in 52Cr by proper laser configurations, in particular by the dipole trap itself. The QZE can be controlled independently of the magnetic field, leads to a rich variety of ground-state phases, can be used to rapidly quench through quantum phase transitions, allows for an observable EdH effect for relatively large magnetic fields, and permits S = 1/2 systems with spinor dynamics. Similar ideas could be explored in other atomic species opening an exciting new control tool in spinor systems.

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26. Our 2D results on population transfer can be extrapolated to 3D. However, the 2D case leads to interesting physics in itself. If the dipole direction, d, is on the xy-plane, the cylindrical symmetry around d is broken, and the transfer −3 → −2 does not lead to a rotation of the −2 cloud (although non-rotating patterns are observed).
If d is along z, the cylindrical symmetry is preserved, and the system rotates, but the transfer is slowed down, due to the averaging of the DDI [13].