Impedance-Based Whole-System Modeling for a Composite Grid via Frame-Dynamics Embedding

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Abstract—The paper establishes a methodology to overcome the difficulty of dynamic-preserving frame alignment in ac impedance models, and thereby enables impedance-based whole-system modeling of converter-generator composite power systems. The methodology is based on an intermediary steady frame between local and global frames, which separates the global (constant) and local (dynamic) parts of the frame alignment. The local frame dynamics can be fully embedded into local impedance models via a unified transformation law. Compared to start-of-the-art impedance-based models, the proposed method retains both frame dynamics and scalability, and is generally applicable to almost all generators and converters. The methodology is used to analyze the dynamic interaction between rotors and phase-locked loops (PLLs) in a composite grid, which yields very interesting findings on the rotor-PLL stability in a low-inertia grid.

Index Terms—Impedance/Admittance, Power Systems, Power Electronics, Dynamic Modeling

I. INTRODUCTION

The major power systems of the world are transforming from ones dominated by sources employing synchronous generators (e.g., hydro, gas, and nuclear) to composite systems in which synchronous generators and power-electronic converters interfaced sources (e.g. wind, solar, battery) co-exist. Such a composite grid gives rise to new dynamics behaviors and stability threats, and calls for new approaches for system modeling and analysis.

The most generic approach to model a composite grid is the state-space method [1]–[4], which covers all internal dynamics of every machines (including generators, converters, transmission and distribution facilities, and loads) in the grid, and offers insights into the root cause of each oscillation mode via participation analysis [5]–[10]. However, state-space models rely on detailed knowledge of hardware design and control algorithms of each machine, which may not be available for power electronic converters since they are not usually disclosed by manufacturers nor standardized across different suppliers.

Impedance (or equivalently, admittance) models are taken as a useful alternative to the state-space method [11]–[15]. It describes the dynamic behavior of a machine by the transfer function from the port (input and output) without looking into internal details, and can be measured or validated online where high-fidelity analytical models are not available. Impedance models prove to preserve all information on dynamics (in the sense of small-signal analysis) as long as every state is controllable and observable from the port, and therefore can get equivalent results to those of the state-space method in system stability analysis [16].

The impedance-based method have very successful application in low-voltage dc power systems [17]–[21], but meet a fundamental difficulty in ac power systems. Machines (converters and generators) in ac systems are usually modeled in local rotating frames aligned to each rotor or phase-locked loop (PLL) angle, which need to be aligned to a global reference frame in the whole-system model. This frame alignment is trivial in state-space models since all angles are represented in the states which enables straightforward transformations between the local and global frames. The impedance model, on the other hand, does not explicitly carry angle information since the input and output only contain voltage and current. It has been proposed that the angles derived from power flow analysis can be used to enable impedance transformation to the global frame [2], but this implies discarding the dynamics of the frames themselves since the frame angles are assumed constant [22]. Nonetheless, frame dynamics reflect important information in stability (rotor angle stability in synchronous generators, and PLL stability in power electronic converters) and cannot be neglected. To avoid this problem, every machine has to be modeled in the global frame from the beginning, which makes the modeling procedure highly complicated and not scalable [23]. That is, the impedance of each machine cannot be defined locally but are affected by external dynamics in the global frame. As a result, impedance models are limited to simple systems such as a single-converter-infinite-bus system where the global frame is aligned to the infinite bus [24]–[25], or a stand-alone single-converter-load system where the global frame is aligned to the internal frame of the converter [27].

In this paper, we establish a methodology to overcome this difficulty. In particular, we define an impedance transformation to embed frame dynamics into impedances locally, so that they can be aligned to the global frame without losing dynamic information. Compared to impedance modeling directly in the global frame [23], the proposed methodology retains scalability, that is, the impedance of each machine can be modeled or measured locally without referring to the external global frame, and the local impedances can be interconnected modularly in whole-system analysis. The proposed frame-dynamics-embedding impedance transformation is generally applicable to almost all three-phase ac electric systems, includ-
Fig. 1. Frame alignment in state-space modeling: frame angle $\delta_n$ contained in states enables straightforward frame transformation.

Fig. 2. Frame alignment in impedance modeling: embedding frame dynamics into local impedance model.

Fig. 3. Illustration of a swing frame $dq$ (such as the rotor frame of a synchronous generator) which swings around a steady frame $d'q'$ by a dynamic angle $\epsilon$.

II. FRAME ALIGNMENT IN WHOLE-SYSTEM MODELING

We start from reviewing how frame alignment is performed in state-space models to demonstrate why frame alignment is difficult in impedance-based models. As illustrated in Fig. 1, each machine in the system is modeled in its local frame $dq_n$ ($n = 1, 2, \cdots, N$) first. The angle difference $\delta_n$ between a local frame and the global frame is determined by the state equation below

$$\dot{\delta}_n = \omega_n - \omega_1$$

in which $\omega_n$ is the local frame speed of $dq_n$ governed by each rotor or PLL. Since $\delta_n$ is preserved in the states, the local-global frame transformations in a state-space model is straightforward. In an impedance model, however, only current and voltage are visible, so there is no straightforward ways to identify the angles of transformations between local and global frames. This is the fundamental difficulty of impedance-based whole-system modeling of ac grids.

To solve this problem, we propose a method to embed frame dynamics (that is, the dynamics of $\delta_n$) into impedances. We first split a local frame into two sub-frames, namely the steady frame and swing frame. The steady frame is defined as the steady-state operating point of a local frame: it rotates at a constant speed $\omega_0$ and contains no dynamics. The swing frame, on the other hand, is aligned to the dynamic rotor or PLL angle, and swings around the steady frame by an angle $\epsilon$, as illustrated in Fig. 3.

The steady frame serves as an intermediary between a local frame and the global frame. Based on this intermediary frame, the impedance transformation from local to global can be performed in two steps: (i) from a local swing frame to a local steady frame, and (ii) from a local steady frame to the global steady frame, as shown in Fig. 2. This two-step transformation separates the global (constant) and local (dynamic) part of the frame alignment, and thereby retains both frame dynamics and scalability in the impedance model.

The second-step transformation (ii) is global but constant. It does need external information, that is, the angle $\xi_n$ between the local steady frame and the global steady frame, but $\xi_n$ is hold still in dynamic modeling and can be obtained from power flow analysis before hand. The impedance transformation law for (ii) is well understood and we put the result below without proof:

$$Z_{d_nq_n} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

$$Z_{d'q'} = \begin{pmatrix} Z_{11} & Z_{12}e^{j2\xi_n} \\ Z_{21}e^{-j2\xi_n} & Z_{22} \end{pmatrix}$$

in which $Z_{d_nq_n}$ is the impedance of the $n$th machine in
its local steady frame, and $Z_{d'd'q'++}$ is the corresponding impedance in the global steady frame. The complex signal method is used here to represent the impedance for concise expression \[\text{[28]-[30]}. \text{It is clear from [2] that the diagonal entries } Z_{11} \text{ and } Z_{22} \text{ are unchanged after the transformation, and the anti-diagonal entries are simply modified by a phase angle } 2\epsilon n. \text{No extra dynamics are introduced by this transformation.}

The first-step transformation (i), on the other hand, is local but dynamic. It carries all frame dynamics and can be defined locally without referring to external information. The impedance transformation law for this step is non-trivial and is discussed in detail in the next section.

III. FRAME-DYNAMICS-EMBEDDING IMPEDANCE TRANSFORMATION

The impedance transformation from a local swing frame to a local steady frame is the essential step to embed frame dynamics in local impedances, and is therefore named frame-dynamics-embedding impedance transformation in this paper. The transformation law for a generic machine (generators and converters) is discussed below.

Following Fig. 3 let the angle swing between the swing frame $dq$ and the steady frame $d'q'$ be $\epsilon$. Now we find how the dynamics of $\epsilon$ can be represented in the impedance. We use complex signals $u_{dq\pm} = u_d \pm ju_q$ and $u_{d'q'\pm} = u_{d'} \pm ju_{q'}$ to represent the current or voltage in $dq$ and $d'q'$ frames, and the transformation in between is

\[
\begin{pmatrix}
u_{d'q'} + \\
u_{d'q'} - \\
u_{dq} + \\
u_{dq} -
\end{pmatrix} = \begin{pmatrix}
1 & \epsilon e^j\epsilon & 0 & 0 \\
0 & 1 & \epsilon e^{-j\epsilon} & 0 \\
0 & 0 & 1 & \epsilon e^{-j\epsilon} \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
u_{dq} + \\
u_{dq} - \\
u_{dq} + \\
u_{dq} -
\end{pmatrix}.
\]

We can linearize (3) to see the effect of the angle perturbation

\[
\begin{pmatrix}
\Delta u_{d'q'} + \\
\Delta u_{d'q'} - \\
\Delta u_{dq} + \\
\Delta u_{dq} -
\end{pmatrix} = \begin{pmatrix}
\Delta u_{dq} + \\
\Delta u_{dq} - \\
\Delta u_{dq} + \\
\Delta u_{dq} -
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \cdot \epsilon
\]

in which $\Delta u$ is the deviation from the the operating point $u_{0\pm}$ which is the same for both $u_{dq\pm}$ and $u_{d'q'\pm}$ (because the swing frame and steady frame are aligned in the steady state). Written in a compact form, (4) becomes

\[
\Delta u_{d'q'++} = \Delta u_{dq+-} + U_0 \cdot \epsilon
\]

in which $\Delta u_{dq+-} = (\Delta u_{dq} , \Delta u_{dq})^T$, $\Delta u_{d'q'++} = (\Delta u_{d'q'} , \Delta u_{d'q'})^T$, and $U_0 = (j u_{0+} , -j u_{0-})^T$.

The frame perturbation $\epsilon$ can be governed by either current (in the case of a synchronous generator) or voltage (in the case of a PLL-controlled converter). Taking the former as an example, we have

\[
\epsilon(s) = K_i(s) \Delta i_{dq+-}(s)
\]

in which $K_i(s)$ is the transfer function representing the current-governed frame dynamics.

Let the impedance in the swing frame be represented as

\[
\Delta u_{dq+-} = Z_{dq+-} \Delta i_{dq+-}
\]

in which $v$, $i$ and $Z$ denote voltage, current, and impedance respectively. Equation (7) can be transformed to the steady frame using (6) and (5), from which we get the impedance transformation law below:

\[
Z_{d'q'++} = (Z_{dq+-} + V_0 K_i)(I + I_0 K_i)^{-1}
\]

in which $Z_{dq+-}$ and $Z_{d'q'++}$ are the impedance in the swing and steady frame respectively, $I$ is the unit matrix, $V_0 = (j u_{0+} , -j u_{0-})^T$ represents the steady-state voltage, and $I_0 = (j i_{0+} , -j i_{0-})^T$ represents the steady-state current.

The admittance transformation law is similar:

\[
Y_{d'q'++} = (Y_{dq+-} + I_0 K_v)(I + V_0 K_v)^{-1}
\]

in which $Y_{dq+-}$ and $Y_{d'q'++}$ are the impedance in the swing and steady frame respectively and $K_v$ represents the voltage-governed frame dynamics.

The impedance/admittance transformations above can be visualized by the signal flow diagrams in Fig. 4 and Fig. 5. Taking (Fig. 4) as an example, the current disturbance induces the frame swing $\epsilon$ which in turn affects the representation of the current and the voltage in the steady frame. As a result, the frame dynamics is embedded into the impedance in the steady frame.

After the impedance of all machines are transformed to the global steady frame, they can be interconnected to build the impedance model of the whole system. In the conventional impedance models, a system is separated into the source and load and the stability are evaluated by the impedance ratio in between. In a composite power system, however, there are multiple sources and loads interconnected in a meshed network, which makes the conventional source-load separation very difficult if not impossible to be applied. To solve this problem, we developed the formulation in Fig. 6.

Fig. 4. Signal flow in an impedance model with current-governed frame dynamics.

Fig. 5. Signal flow in an admittance model with voltage-governed frame dynamics.
\( Z_m(s) \) represents the impedances of all machines (including generators, converters and loads) written as a diagonal matrix:

\[
Z_m(s) = \begin{pmatrix} Z_{m1}(s) & & \\ & Z_{m2}(s) & \\ & & \ddots \end{pmatrix}
\]

\( Y_b(s) \) is the nodal admittance matrix of the network and represents the dynamics of the transmission lines (note that each element of \( Y_b(s) \) is in \((R + jX + sL)^{-1}\) format to preserve the dynamic part \( sL \), in contrast to the conventional nodal admittance matrix in \((R + jX)^{-1}\) format with the dynamics neglected). Under this formulation, the stability of the whole system is determined by the interaction between \( Z_m(s) \) and \( Y_b(s) \), which form a closed loop. To evaluate stability, we perturb the closed-loop system by injecting small disturbances \( \hat{v}(s) \) or \( \hat{i}(s) \) at the loop joining points

\[
\hat{v}(s) = (\hat{v}_1(s), \hat{v}_2(s), \ldots, \hat{v}_N(s))^\top
\]

\[
\hat{i}(s) = (\hat{i}_1(s), \hat{i}_2(s), \ldots, \hat{i}_N(s))^\top
\]

The resulted system responses under the disturbances are determined by the transfer functions below:

\[
\Delta i_b(s) = (I + Y_b Z_m)^{-1} Y_b \cdot \hat{v}(s) = \hat{Y}(s) \cdot \hat{v}(s)
\]

\[
\Delta v_m(s) = Z_m (I + Y_b Z_m)^{-1} \cdot \hat{i}(s) = \hat{Z}(s) \cdot \hat{i}(s)
\]

in which \( \Delta i_b(s) \) and \( \Delta v_m(s) \) are the system responding voltages and currents under the disturbances of \( \hat{v}(s) \) or \( \hat{i}(s) \)

\[
\Delta i_b(s) = (i_{b1}(s), i_{b2}(s), \ldots, i_{bN}(s))^\top
\]

\[
\Delta v_m(s) = (v_{m1}(s), v_{m2}(s), \ldots, v_{mN}(s))^\top
\]

and \( \hat{Z}(s) \) and \( \hat{Y}(s) \) are the closed-loop impedance and admittance of the whole system.

The stability of the system can then be evaluated from the poles of \( \hat{Z}(s) \) or \( \hat{Y}(s) \). \( \hat{Z}(s) \) and \( \hat{Y}(s) \) are \( N \times N \) matrices and each element shall share the same unstable poles, since the network is connected and any unstable oscillation shall be propagated throughout all nodes in the end. However, this is only right in theory because the poles seen by different elements have different observability (represented by residues) \([\cdot]\), and particular modes can only be effectively detected at the most relevant nodes. This raise another question of the optimal location for impedance evaluation and measurement, which is beyond the scope of this paper and will be resolve in future works.

The whole-system model in Fig. 6 and be visualized as the circuit in Fig. 7. The disturbance injection \( \hat{v}(s) \) is equivalent to small voltage sources in series with each machines, and \( \hat{i}(s) \) is equivalent to small current sources in parallel with each machines. The elements of \( \hat{Z}(s) \) and \( \hat{Y}(s) \) describes the resulted voltage and current disturbances throughout the system, from which the stability of system can be evaluated.

![Fig. 6. Whole-system dynamic modeling by the closed-loop formulation of nodal admittance matrix and machine impedance matrix.](image)

![Fig. 7. Equivalent circuit to visualize the effect of perturbation injection \( \hat{v}(s) \) and \( \hat{i}(s) \).](image)

### IV. Composite Converter-Generator Grid

This section applied the proposed methodology to the whole-system modeling of a composite converter-generator grid. Using the impedance transformation in the preceding section, we first find the frame-dynamics-embedded impedance for both synchronous generators and PLL-controlled converters in local steady frames, and then interconnect them for whole-system analysis. This leads to very interesting results on the interaction between rotor and PLL in a low-inertia grid.

#### A. Synchronous Generator

The state equations of a synchronous generator in the local swing frame \( dq \) is

\[
\dot{\psi}_d = v_d - R_i_d + \omega \psi_q
\]

\[
\dot{\psi}_q = v_q - R_i_q - \omega \psi_d
\]

\[
\dot{\omega} = (T_e - T_m - D \omega) / J
\]

\[
\dot{\theta} = \omega
\]

in which

\[
\psi_d = L_i_d, \quad \psi_q = L_i_q - \psi_f, \quad T_e = \psi_f i_d.
\]

The symbols are defined as \( \psi, v \) and \( i \) being the flux-linkage, voltage, and current respectively, \( \psi_f \) is the field flux-linkage, \( \omega \) and \( \theta \) are the rotor speed and angle, \( T_e \) and \( T_m \) are the electrical and mechanical torque, \( R \) and \( L \) are the stator resistance and inductance respectively, \( D \) is the damping torque coefficient, and \( J \) is the rotor inertia. We use the constant flux-linkage model (\( \psi_f \) constant) which combines the total armature reaction in \( L_i \) and we assume \( T_m \) to be constant on the basis that the prime-mover’s speed governor is slow compared to the fast transients under consideration. A single pole-pair with no saliency is considered to simplify the model without losing the essential properties that we wish to illustrate. All variables are in motor convention and the \( q \)
axis is aligned to the field flux so that $i_d$ and $i_q$ represent active and reactive current seen by the generator respectively.

Linearizing the state equation and using Laplace transformation, we get the impedance in the local swing frame

$$Z_{dq} = \left( \begin{array}{cc} Z_L(s) - \omega_0 L & 2 s H(s) \vspace{1mm} \\ \omega_0 L & Z_L(s) \end{array} \right) + \left( \begin{array}{c} -s \psi_0 \vspace{1mm} \\ s \psi_0 \end{array} \right) \right)$$

or equivalently in the complex-signal form

$$Z_{dq^+} = \left( \begin{array}{cc} Z_L(s_1) & 1 \vspace{1mm} \\ Z_L(s_{-1}) \end{array} \right) + \frac{1}{s H(s)} \left( \begin{array}{cc} j \psi_0^+ & j \psi_0^- \vspace{1mm} \\ -j \psi_0^- & -j \psi_0^+ \end{array} \right)$$

in which $Z_L(s) = sL + R$, $H(s) = 2(sD + sF)/\psi_f$, $s_{\pm} = s \pm j\omega_0$, and $\psi_{0\pm}$ is the operating point of $\psi_{dq}$ in complex form (that is, $\psi_{0\pm} = \psi_{d0} \pm j\psi_{q0}$).

Now we transform $Z_{dq^+}$ into the steady frame. The swing $\epsilon$ is determined by the perturbation of the rotor angle $\Delta \theta$, which in turn is governed by electric torque $\Delta T_e$ proportional to current $\Delta i_d$. That is,

$$\epsilon(s) = \Delta \theta(s) = \Delta \omega(s)/s = \frac{2}{s H(s)} \Delta i_d(s)$$

From which we get the voltage-governed frame dynamics

$$K_i = \frac{1}{s H(s)} \left( \begin{array}{cc} 1 & 1 \vspace{1mm} \end{array} \right).$$

The steady-frame impedance can then be obtained using the transformation law \ref{eq:transformation}:

$$Z_{dq^+} = \left( \begin{array}{cc} Z_L(s_1) & 1 \vspace{1mm} \\ Z_L(s_{-1}) \end{array} \right) + \frac{1}{s H(s)} \left( \begin{array}{cc} s_1 & s_1 \vspace{1mm} \\ s_{-1} & s_{-1} \end{array} \right)$$

in which $M(s) = 2(sD + sF - i_{dq}\psi_f)/\psi_f^2$ where $i_{dq}$ denotes the steady-state $i_d$.

It is worth noting that the steady-frame impedance $Z_{dq^+}$ may not always be stable and its stability is related to the reactive current $i_{dq}$ of the generator. The impedance is stable only when $i_{dq}$ is negative, that is, when the generator is absorbing reactive power. This leads to very interesting implications on the stability of rotor-PLL interaction, as explained in the subsection after the next.

### B. PLL-Controlled Converter

The steady-frame impedance for a PLL-controlled grid-connected converter can be found in a similar way to that of a synchronous generator, except that the frame dynamics of a PLL is governed by voltage rather than current. Therefore, we need to invoke the admittance transformation in \ref{eq:admittance_trans} to embed the frame dynamics, and invert the admittance matrix for the corresponding impedance. We simply list the result here without going to details.

State equations:

$$\dot{i}_{dc} = (e_d i_d + e_q i_q - P_{dc})/(v_{dc} C_{dc})$$

$$\dot{\gamma}_{vc} = (v_{vc} - \gamma_{vc}) k_{vc}$$

$$\dot{i}_{id} = (i_d - i_{id}) k_{id}$$

$$\dot{i}_{iq} = (i_q - i_{iq}) k_{iq}$$

$$\dot{\gamma}_{pl} = \delta_{pl} k_{pl}$$

$$\dot{\omega} = (\gamma_{pl} + \delta_{pl} k_{pl} - \omega)/\tau_{pl}$$

$$\dot{\theta} = \omega$$

in which

$$i_{dc} = k_{p,vc} (v_{dc} - v_{dc}) + \gamma_{vc}$$

$$e_d = (i_d - i_{id}) k_{p, id} + \gamma_{id}$$

$$e_q = (i_q - i_{iq}) k_{p, id} + \gamma_{iq}$$

$$\delta_{pl} = \arctan(v_{dc}/v_{dc})$$

The in the state equations, $\gamma$ denotes the integrator for each proportional-integral (PI) control loop, $k_p$ and $k_i$ represent the corresponding PI gain, and the superscript * indicates the PI control references. $v_{dc}$ is the dc-link voltage, $P_{dc}$ is the power injected into the dc-link from the prime-mover (e.g. a PV array or a wind turbine), $e_d$ and $e_q$ are the converter internal voltage, $v_{dc}$ and $v_{dc}$ are the external voltage applied at the converter terminal, $L$ is the filtering inductance, $\omega$ and $\theta$ are the PLL frequency and angle, and $\delta_{pl}$ is the PLL angle error.

Again, $\epsilon = \Delta \theta$ from which we get the voltage-governed frame dynamics:

$$K_v = \frac{1}{2 s H_{pl}} \left( \begin{array}{cc} 1 & -1 \vspace{1mm} \end{array} \right),$$

where

$$H_{pl} = v_{dc} + \frac{v_{dc}^2}{k_{p, pl} s + k_{i, pl}}$$

Linearizing the state equations yields the admittance in the local swing frame, which can then be transformed to the local steady frame via the admittance transformation in \ref{eq:admittance_trans}. This process is tedious so we turn to Matlab symbolic calculation to find the expression, and the Matlab scripts can be found at https://spiral.imperial.ac.uk/.

### C. Rotor-PLL Interaction

Now we connect the frame-dynamics-embedded impedance models for different machines in the global steady frame for whole-system stability analysis. We focus on the stability of rotor-PLL interaction in a low-inertia system, which is less studied in literature and shows the advantage of the proposed methodology. As a specific example, we investigate a wind farm connected to a low-inertia transmission system, as shown in Fig. \ref{fig:windfarm}. The typical four-generator configuration in \ref{fig:windfarm} is used and one of the synchronous generators is replaced by
a off-shore Type-IV wind farm. The turbine-side converters of the Type-IV wind turbines are modeled as constant power sources injected into the dc link capacitors, and the grid-side converters use the standard proportional-integral (PI) vector control with constant power factor.

We model the impedance of all wind converters and synchronous generators in their local swing frame, transform them to the local steady frame to embed the frame dynamics, and then align to the global frame for whole system analysis. We put current disturbance $\hat{i}(s)$ at the point of common coupling (PCC) and evaluate the stability of the whole system by the impedance $\hat{Z}(s)$ seen by $\hat{i}(s)$. The poles of $\hat{Z}(s)$ are plotted for different parameters and power flow condition, and the results are summarized in Fig. 9 and Fig. 10. It is interesting to see that the system is not stable when the following three conditions are met together (i) the total inertia of the system is low, (ii) the dc-link control loop and PLL of the wind converters are slow (1Hz), and (iii) the synchronous generators are injecting reactive power to the grid.

This phenomenon can be explained by the impedance model in Subsection (a). When the dc-link control and the PLL of wind converters are slower than the inertia response, the converters are seen as a constant current source injected into synchronous generators, and the stability of the whole system $\hat{Z}(s)$ is determined by the impedance of the synchronous generators. According to (21), the impedance is not stable when the generator is generating reactive power. On the other hand, if either the dc-link control or the PLL is faster than the inertia response, the wind converters are seen as a constant power source injected into the grid, which is equivalent to constant electric torque on synchronous generators and therefore will not interact with the rotors. The leads to the conclusion that the dc-link control and PLL should be fast enough to make the converters appear as constant power sources in the timescale of inertia, which is the new insights generated by the impedance-based whole-system model.

V. SIMULATION RESULTS

A series of simulation results are presented to illustrate and validate the key findings of the paper. The same configuration of Fig. 8 is used in the simulation.

We first demonstrate the effect of frame-dynamics-embedding impedance transformation between the local swing frame and local steady frame. A time-domain impedance measurement [31] is conducted on a synchronous generator in both the local swing frame and local steady frame, and the results are compared with the theoretical models in Fig. 11. Since the steady-frame impedance may not be stable, admittances instead of impedances are presented here. In both the steady-frame and swing-frame models in Fig. 11 (a-b), there is a resonant peak at the fundamental frequency (60Hz) which represents the flux dynamics $(s + j\omega_0)L$ of the winding inductance. In the steady-frame model, however, there is an extra resonant peak in the admittance spectrum at 1Hz, which represents the frame dynamics of the rotor swing. This resonant peak is missing in the swing-frame model, validating that the transformation from the swing frame to the steady
frame embeds frame dynamics into the model. If the rotor speed $\omega$ is held constant by setting the rotor inertia $J$ to $\infty$, the frame dynamics disappears and the steady- and swing-frame admittances become the same, as shown in Fig. 11(c). This further confirms that the extra resonant peak in Fig. 11(c) does come from the embedded frame dynamics. The results from measurement of admittance by signal injection in the time-domain simulation agree with the theoretical models in all three cases except for minor phase errors which are believed to be caused by the time delays in the discrete-time sampling, showing the very high accuracy of the models.

Now we verify the key results of whole-system stability analysis based on impedance models. We simulated the system in Fig. 8 via electromagnetic transient (EMT) simulation in Matlab/Simulink, and the results (in Fig. 12) are compared with the theoretic prediction in the previous section, which agree with each other. At 1s, the bandwidths of dc-link control and PLL are reduced from 10Hz to 1Hz, which results in the instability of the low-inertia system. The oscillation is eliminated by increasing the bandwidth of dc-link control [Fig. 12(a)] or PLL [Fig. 12(b)] or both [Fig. 12(c)] to 10Hz at 2s, which confirms the major conclusion of our model. Notably, $v_{dc}$ (i.e., the dc link voltage of wind converter) is re-stabilized faster in (a) because of the increased bandwidth of dc link control; and $\delta$ (i.e., the angle difference between voltages of the wind converter and the synchronous generator) is re-stabilized faster in (b) due to the increased bandwidth of PLL. (c) shows the best system stability performance due to the recovery of both two bandwidths.

VI. CONCLUSIONS

The methodology described in this paper established a generic procedure to represent the dynamics of a composite converter-generator power system via pure impedance-based models. This methodology overcomes the fundamental difficulty of dynamic frame alignment in impedance modeling by
embedding the frame dynamics in impedance itself via the transformation between the steady frame and the swing frame. The yielded whole-system model leads to new insights into the interaction mechanism between PLLs and rotors in a low inertia grid with recommended solution given.

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