Extended paraconductivity regime in underdoped cuprates

S. Caprara\(^1\), M. Grilli\(^1\), B. Leridon\(^2\), and J. Lesueur\(^2\)

\(^1\)Istituto Nazionale per la Fisica della Materia, Unità Roma 1 and SMC Center, and Dipartimento di Fisica Università di Roma “La Sapienza”, piazza Aldo Moro 5, I-00185 Roma, Italy
\(^2\)Laboratoire de Physique Quantique - ESPCI/UPR5-CNRS, 10, Rue Vauquelin - 75005 Paris - France

(Dated: November 23, 2018)

We reconsider transport experiments in strongly anisotropic superconducting cuprates and we find that universal Aslamazov-Larkin (AL) paraconductivity in two dimensions is surprisingly robust even in the underdoped regime below the pseudogap crossover temperature \(T^*\). We also establish that the underlying normal state resistivity in the pseudogap phase is (almost) linear in temperature, with all the deviations being quantitatively accounted by AL paraconductivity. The disappearance of paraconductivity is governed by the disappearance of gaussian pair fluctuations at an energy scale related to \(T^*\).

PACS numbers: 74.72.-h, 74.25.Fy, 74.40.+k, 74.20.De

Recent transport experiments\(^1\) in superconducting cuprates have shown that the paraconductivity effects in the normal state close to the critical temperature \(T_c\) are well described by the following expressions valid in two and three dimensions, respectively,

\[
\Delta \sigma^{\exp}_{D=2} = \frac{\epsilon^2}{16 \hbar d \xi_0 \sinh(\epsilon/\xi_0)},
\]

\[
\Delta \sigma^{\exp}_{D=3} = \frac{\epsilon^2}{16 \hbar \xi_0 \sqrt{2 \epsilon_0} \sinh(2\epsilon/\xi_0)},
\]

where \(d\) is the distance between CuO\(_2\) layers, \(\xi_0\) is the coherence length along the direction perpendicular to the layers, \(\epsilon \equiv \log(T/T_c)\), and \(\epsilon_0 \equiv \log(T^*/T_c)\). Here \(T^*\) is a temperature scale which increases with decreasing doping and appears to follow the characteristic crossover temperature \(T^*\) below which many different experiments in the cuprates detect a pseudogap opening.\(^2\) The above expressions (and the experimental data that they fit well) display two remarkable features. First of all, close to \(T_c\), for small values of \(\epsilon\), they reproduce the Aslamazov-Larkin (AL) form of paraconductivity,\(^3\)

\[
\Delta \sigma^{AL}_{D=2} = \frac{\epsilon^2}{16 \hbar d \xi_0},
\]

\[
\Delta \sigma^{AL}_{D=3} = \frac{\epsilon^2}{32 \hbar \xi_0 \sqrt{\epsilon}}.
\]

These expressions account well for the fluctuating regime near \(T_c\) both in optimally and underdoped cuprates, with YBaCuO\(_{1+x}\) (YBCO) displaying three-dimensional (3D) fluctuations, whereas the other more anisotropic compounds (LSCO and BSCCO) have a two-dimensional (2D) behavior. The fact that the paraconductivity in strongly anisotropic (quasi-2D) underdoped cuprates is described by “traditional” AL fluctuations is at odds with the widespread idea that below the pseudogap formation temperature \(T^*\) particle-particle pairs are formed, which only become phase-coherent at the lower superconducting transition temperature \(T_c\). According to this picture, below the temperature of pair formation the fluctuations would be vortex-driven and should display a Kosterlitz-Thouless behavior, with exponential temperature dependence. On the contrary, it seems a well-established experimental fact that the superconducting fluctuations in the more 2D-like systems (essentially all, but the YBCO) display AL power-law behaviors in \(\epsilon_0 > 1\).\(^4\) Remarkably, in \(D = 2\) the AL theory of paraconductivity does not allow for any fitting parameter besides the experimentally well accessible distance between the 2D layers, which translates the 2D conductivity, with dimensions \(\Omega^{-1}\), in a 3D conductivity with dimensions \(\Omega^{-1} m^{-1}\). Therefore the AL paraconductive behavior observed near \(T_c\) strikingly shows that the establishment of superconducting phase coherence in these materials is not due to a simple condensation of preformed pairs. This by no means implies that preformed pairs are not present below \(T^*\), but simply means that the superconducting coherence is driven by the formation of more loosely bound traditional BCS pairs. Various proposals have already been put forward based on the coexistence of fermionic quasiparticles (eventually forming BCS pairs at \(T_c\)) and more or less bosonic preformed pairs.\(^5\)

The second remarkable feature of the experiments described by Eqs. (1) and (2) regards the exponential suppression of the paraconductivity when \(\epsilon > \epsilon_0\). While it is quite natural that superconducting fluctuations decay when moving away from \(T_c\), no longer contributing to the conductivity, the fact that AL fluctuations survive up to \(T^*\) is surprising. In underdoped cuprates this rapid drop in the AL fluctuations occurs at the temperature scale \(T^* \sim T^*\), which is substantially higher than the superconducting temperature \(T_c\).

In principle one could argue that \(T^*\) is indeed the temperature below which superconducting Cooper-pair fluctuations arise, and therefore it is not surprising that they contribute à la AL to the paraconductivity. However, upon underdoping, \(T^*\) increases, while \(T_c\) decreases. If this is interpreted within a standard scheme of strong-coupling pairing, the phase fluctuations would be (the only) responsible for paraconductivity, in \(D = 2\), and one should rather observe the Kosterlitz-Thouless-like conden-
sation of preformed pairs.

In this paper we focus on these two main features of the paraconductivity experiments. Firstly we critically reexamine the AL theory and the possible occurrence of momentum and/or energy cutoffs in the critical pair fluctuations. This will provide a different perspective on the rapid drop of the paraconductivity above $T^*$, with respect to previous works,$^{16,17}$ but will leave open the question of the mechanism allowing for the long survival of AL fluctuations in the pseudogap phase of underdoped cuprates. Then we will focus on the 2D materials and examine the robustness of the AL paraconductivity at various doping upon varying the assumed normal-state resistivity. Again, our scope is neither to provide a microscopic theory for the normal-state phase nor for its microscopic theory for the normal-state phase nor for its

reexamine the AL theory and the possible occurrence of paraconductivity. The 2D case is the only one of our concern here is to extract the most likely form of the normal-state resistivity in connection to the distinct presence of paraconductivity. This will provide a different perspective on the interplay with pair fluctuations below $T^*$. Our main concern here is to extract the most likely form of the normal-state resistivity in connection to the distinct presence of paraconductivity. This effect adds on top of the reduction associated to the appearance of higher-order momentum. This includes as particular cases, e.g., the effect for an arbitrary expression of $\Omega_q$. We are therefore led to discuss the role of an intrinsic cutoff for the momentum integral in Eq. (5).

The analytical development within a BCS derivation of the effective GL theory leads to a natural momentum cutoff $\sim \xi_D^{-1}$ for higher momenta. This cutoff can alternatively be described as the appearance of higher-order terms in the $q$ dependence of $\Omega_q$, beyond the lower-order term $\sim q^2$. However, neither a strict momentum cutoff $q \leq q_C \sim \xi_D^{-1}$, nor the introduction, e.g., of a $q^4$ term in $\Omega_q$ account for the observed behavior of the paraconductivity.

Based on physical arguments, it was proposed$^{16,17}$ that, rather than a strict cutoff on $q$, a cutoff should be imposed on the energy (namely, the inverse relaxation time) of the collective pair fluctuations. This cutoff, within the standard BCS-GL theory, takes the form $m + \nu q^2 \leq \xi_0^{-2} T_c$ and leads to a sharper reduction with respect to a strict momentum cutoff. This is easily understood by considering that, away from $T_c$, $m$ increases, so that a strict cutoff $\Omega_C$ on $\Omega_q \approx m + \nu q^2$ amounts to a strict momentum cutoff $q^2 \leq q_C^2 \equiv (\Omega_C - m)/\nu$, which decreases with increasing temperature, thus shrinking the region of momenta which contribute to the paraconductivity. This effect adds on top of the reduction associated with an increasing mass $m$, and determines a more rapid decrease at higher temperatures. Nevertheless the experimental suppression of the paraconductivity, fitted by Eqs. (1) and (2), is even stronger. Therefore in the remaining part of this section, we analyze the experimental data within a framework which, although related to the presence of an energy cutoff, rather relies on a model for an effective “density of states” of the collective pair fluctuations. Indeed, we transform the momentum integral into an energy integral, by introducing the effective density of states

$$
\mathcal{N}_D(\Omega) = \int \frac{d^D\mathbf{q}}{(2\pi)^D} q^2 \delta(\Omega - \Omega_q)
$$

for an arbitrary expression of $\Omega_q$ as a function of the momentum. This includes as particular cases, e.g., the effect of a higher-order momentum dependence of $\Omega_q$ with respect to the hydrodynamic $q^2$ dependence, and/or the cutoff condition $\Omega_q \leq \Omega_C$. We observe that the minimum value for $\Omega_q$ is $m$, and therefore

$$
\Delta \sigma_D = \alpha_D \int_m^{+\infty} d\Omega \mathcal{N}_D(\Omega) \mathcal{I}(\Omega; T).
$$

### Footnotes

1. Footnote.

2. Footnote.
This equation is our starting point. For the sake of simplicity we discuss the case in which \( I(\Omega; T) \) has the leading AL expression \( I(\Omega; T) = T/(\Omega^\alpha) \), but the analysis can be easily extended to the case in which \( I(\Omega; T) \) assumes a more complicated dependence on \( \Omega \), which interpolates between the low-\( T \) (\( \Omega \gg T \)) and the high-\( T \) (\( \Omega \ll T \)) regimes.

A sharp energy cutoff \( \Omega \leq \Omega_c \) translates into a vanishing DOS, \( N_D(\Omega) \equiv 0 \) for \( \Omega > \Omega_c \). We relax this condition, and only assume that the DOS vanishes at infinity. More precisely, we write the function to be integrated in Eq. (7) as the derivative of an auxiliary function, \( \alpha_D N_D(\Omega) Z(\Omega; T) = -F_D(\Omega) \), with \( T \) taken as a parameter, and assume that \( F_D(\Omega) \) vanishes as \( \Omega \to +\infty \). Then, evidently \( \Delta \sigma_D = F_D(m) \). Recalling that \( m = \gamma^{-1} \varepsilon_0 \), with \( \varepsilon \equiv \log(T/T_c) \), we can extract \( N_D(\Omega) \) from the interpolating formula for the paraconductivity proposed in Refs. 23, Eq. (1) for \( D = 2 \) and Eq. (2) for \( D = 3 \). Thus we find

\[
N_D(\Omega) = -\frac{d}{\alpha_D Z(\Omega; T)} \frac{d}{d\Omega} \Delta \sigma_D^{\text{exp}}(\varepsilon = \gamma \Omega).
\]

Therefore, we are led to the conclusion that the spectrum of the inverse relaxation time for the collective pair fluctuations is cut off exponentially at higher \( \Omega \) and the characteristic scale for this suppression, \( \Omega_0 \equiv \gamma^{-1} \varepsilon_0 \), increases with decreasing doping, following \( T^* \). The presence of this scale is highly significant and rises the issue of the relation between Cooper pair fluctuations and pseudogap. Since the microscopic interpretation of this finding is beyond the scope of the present work, here we only illustrate two possible interpretations. Coming from high temperatures \( T > T^* \), one can identify \( T^* \) as the mean-field-like temperature for superconductivity, below which the fluctuations bring the critical temperature down to \( T_c \). The bifurcation between \( T^* \) and \( T_c \) around optimal doping can be interpreted in a gaussian GL scheme within a two-gap model.

An alternative interpretation can be proposed starting from \( T_c \) as the temperature above which pair fluctuations set in. The disappearance of pair fluctuations above \( T^* \) can here be interpreted as due to some additional mechanism of strong mixing between the particle-particle and the particle-hole channels. In particular, within a scenario with a quantum critical point around optimal doping, the region above \( T^* \) is characterized by the presence of quantum-critical fluctuations, which can couple to the superconducting fluctuations, and suppress them. These two possibilities are presently under investigation.

— Aslamazov-Larkin paraconductivity in the pseudogap phase — The occurrence of the AL paraconductivity is particularly stringent in 2D systems, where the AL paraconductivity does not contain fitting parameters and takes a universal form with a power-law dependence in \( \varepsilon \) and a definite prefactor [see Eq. (6)]. For this reason, here we concentrate on 2D BSCCO compounds. The choice of a normal-state conductivity (or resistivity) becomes rather natural around optimal doping, where \( \rho_n(T) \) is linear over a wide temperature range. It is in this case that the presence of a AL paraconductivity becomes particularly clear both in \( D = 2,3,8,9,10,11 \) and \( D = 3,22,23,24 \). Remarkably, since the paraconductivity diverges at \( T_c \), the choice of a specific (finite) normal state conductivity \( \sigma_n(T) \) affects little the total conductivity \( \sigma(T) = \sigma_n(T) + \Delta \sigma^{\text{AL}}(T) \), and the divergence of \( \Delta \sigma^{\text{AL}} \) can not be missed by a wrong choice of the normal state. However, the choice of the correct \( \sigma_n \) becomes crucial for the correct description of the paraconductivity away from \( T_c \). Therefore we here systematically investigate how different normal-state resistivities affect the determination of \( \Delta \sigma^{\text{AL}} \) in the resistivity data of Ref. 25. First of all, we notice (see Fig. 2 in Ref. 25) that above a temperature \( T^* \) the resistivity is linear in temperature, while it acquires a downward curvature at lower temperatures. Therefore we assume the normal state resistivity \( \rho_n \) to be described by a straight line above \( T^* \), while a quadratic curve is taken below it. To explore the effects of assuming different \( \rho_n \) we take below \( T^* \) the set of parabolareported in Fig. 1(a). For each choice of the normal-state resistivity we determine the paraconductivity

\[
\Delta \sigma_{D=2}(T) = \frac{1}{\rho(T)} - \frac{1}{\rho_n(T)}
\]

obtaining the data of Fig. 1. The black solid straight line represents the pure AL paraconductivity. Rather naturally, if one chooses the normal state to follow closely the resistivity data [the red circles in the insets], there is little space for the paraconductivity contribution, which rapidly dies above \( T_c \). Nevertheless, one can see that approaching \( T_c \) the paraconductivity [the red circles in the main panels] merges with the (diverging) AL contribution. On the other hand one can choose the normal-state with an upward curvature [the orange triangles in the insets], which emphasizes the difference between the resistivity data and the (supposed) normal-state resistivity. In this case the paraconductivity must be large to bring the large normal-state resistivity down to the observed values. The orange triangles of the main panels represent this large contribution to the paraconductivity. In this case one sees that \( \Delta \sigma_{D=2}(T) \) has the same slope as the pure AL paraconductivity, but has a nearly constant positive offset and is rapidly suppressed around \( \varepsilon \sim 0.5 \), corresponding to \( T \sim T^* \). This last effect simply arises from the “perfect” matching of the linear resistivity data with the assumed linear normal state resistivity for \( T > T^* \). In between the two limiting cases described above, there is the choice of normal state resistivities with small (or vanishing) curvature represented by the blue diamonds of the insets. Quite interestingly, one finds that the related paraconductivity closely follows the pure AL behavior, both for the slope and for the absolute (universal) value. This shows that the resistivity data, not only are compatible with a 2D AL behavior near \( T_c \), but also this behavior extends up to \( T^* \) provided a (nearly) linear normal-state resistivity is assumed. Also in this case, as soon as the temperature reaches \( T^* \), the paraconductivity
This behavior is suggestive of the fact that below $T^*$ the resistivity would be linear were it not for the presence of gaussian Cooper-pair fluctuations giving an AL contributions to the conductivity. These suppress the resistivity below its linear behavior all over the $T < T^*$ region.

— Conclusions — In this paper we critically revisited the paraconductivity data in the cuprates addressing the two main issues: The existence and robustness of the AL paraconductivity, which in underdoped systems survives well above $T_c$, and the rapid suppression of paraconductivity above $T^*$. Regarding the second issue, we recast the problem of the cutoff in the pairing collective-mode fluctuations, showing that the rapid suppression of the pairing fluctuations away from $T_c$ can arise from a rapid suppression of the spectral weight of the pair fluctuations above a characteristic energy scale, which directly involves $T^*$, $\Omega_0 \equiv \gamma^{-1} \log(T^*/T_c)$. As far as the second issue is concerned, we find the surprising result that, assuming a (nearly) linear normal-state resistivity, the measured 2D paraconductivity in BSCCO closely follows the pure AL behavior. It seems to us that the coincidence (revealed at all dopings up to the optimal one) both for the power-law and the universal prefactors between the extracted paraconductivity and the AL behavior can hardly be casual. This suggests that the temperature dependence of the resistivity in BSCCO is given by a normal-state linear contribution, which is decreased below $T^*$ by the 2D AL paraconductivity. If, as it seems natural, this paraconductivity arises from gaussian pair fluctuations, our analysis entails that preformed pairs, if any, do not provide a separate additional conductivity channel.

We acknowledge interesting discussions with C. Castelani, C. Di Castro and M. Aprili. S. C. and M. G. acknowledge financial support from MIUR Cofin 2003, Prot. 2003020239,006, and from ESPCI/CNRS, which they also thank for the warm hospitality.

1 C. W. Luo, et al., Phys. Rev. Lett. 90, 179703 (2003); B. Leridon, et al., Phys. Rev. Lett. 90, 179704 (2003).
2 B. Leridon, et al., Phys. Rev. Lett. 87, 197007 (2001).
3 B. Leridon, et al., J. Superc. 15, 409 (2002).
4 For a review see, e.g., J. L. Tallon and J. W. Loram, Physica C 349, 53 (2001).
5 L. G. Aslamazov and A. I. Larkin, Phys. Lett. A 26, 238 (1968); Sov. Phys. Solid State 10, 875 (1968).
6 G. Balestrino, A. Nigro, R. Vaglio, and M. Marinelli, Phys. Rev. B 39, 12264 (1989).
7 G. Balestrino, et al., Phys. Rev. B 46, 14919 (1992).
8 W. Lang, et al, Phys. Rev. B 51, 9180 (1995).
9 M. R. Cimberle, et al., Phys. Rev. B 55, R14745 (1997).
10 A. A. Varlamov, G. Balestrino, E. Milani, and D.V. Li-vanov, Adv. Phys. 48, 655 (1999).
11 S. R. Curras, et al., Phys. Rev. B 68, 094501 (2003).
12 V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B 65, 3173 (1997).
13 A. Perali, et al., Phys. Rev. B 62, R9295 (2000).
14 J. Ranninger, J. M. Robin, and M. Eschrig, Phys. Rev. Lett. 74, 4027 (1995); J. Ranninger and J. M. Robin, Phys. Rev. B 53, R11961 (1996), and references therein.
15 E. Piegarì and S. Caprara, Phys. Rev. B 67, 214503 (2003).
16 F. Vidal, et al., Europhys. Lett. 59, (2002) 754, and references therein.
17 T. Mishonov and E. Penev, Int. J. Mod. Phys. B, 14, 3831 (2000); T. Mishonov, et al., Phys. Rev. B 68, 054525 (2003).
This relevance of $T^\ast$ as a cutoff scale is at variance with the proposal of Ref. 16, where the energy cutoff of pair fluctuations was related to $\xi_0$. 

18. A. Larkin and A. Varlamov, *Theory of fluctuations in superconductors*, (Clarendon Press, Oxford, 2005).
19. L. Reggiani, R. Vaglio, and A. A. Varlamov, Phys. Rev. B 44, 9541 (1991).
20. S. Caprara, M. Grilli, B. Leridon, and J. Lesueur, unpublished.
21. E. Silva, et al., Phys. Rev. B 64, 144508 (2001).
22. P. P. Freitas, C. C. Tsuei, and T. S. Plaskett, Phys. Rev. B 36, 833 (1987).
23. C. Carballeira, et al, Phys. Rev. B 63 144515 (2001).
24. T. Watanabe, T. Fujii, and A Matsuda, Phys. Rev. Lett. 79, 2113 (1997).