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ABSTRACT
Different magnetic navigation systems (MNSs) have been investigated for the wireless manipulation of biomedical magnetic robots. Here we propose a novel MNS simply composed of a triad of electromagnetic coils (TEC) that can effectively manipulate a magnetic robot in two-dimensional (2D) environments. We derived a constraint equation of the TEC’s input currents in order to manipulate the magnetic robot’s 2D aligning and propelling motions in a controlled manner. We also examined the TEC’s ability to manipulate those motions by simulating the TEC’s magnetic field and the corresponding magnetic force in terms of several newly defined dimensionless variables. We then constructed the proposed TEC and demonstrated several controlled 2D motions of a magnetic robot to show the validity of the proposed system and the robot manipulation method.

I. INTRODUCTION
Two-dimensional (2D) magnetic robots manipulated by a magnetic navigation system (MNS) have attracted great attention in various biomedical applications such as cell manipulation, drug delivery, and fluid control.1–4 An MNS is an integrated system composed of multiple electromagnetic coils and programmable power supply units and an implemented control system. With various combinations of different types of electromagnetic coils, an MNS can generate many special kinds of magnetic fields in the central region of the system, such as those that are uniform, rotating, oscillating, and form gradients, which can be effectively utilized to control various robot motions.

Different MNSs have been investigated to manipulate 2D magnetic robots precisely and effectively. H. Choi et al. showed that a stationary MNS with two pairs of Helmholtz and Maxwell coils (eight circular coils) could control the 2D motions of a cylindrical micro-robot.5 M. Salehizadeh et al. showed that three pairs of Helmholtz coils (six circular coils) could manipulate multiple magnetic agents in a 2D environment. However, the effort required to control an MNS generally becomes considerably larger as the number of MNS coils increases because the MNS needs to regulate every coil current simultaneously to generate such complex magnetic fields precisely. A large number of coils can also make the MNS structurally and electrically unsuitable to be applied to relatively large-scale applications. For example, an MNS for the manipulation of intravascular magnetic robots should be large enough to accommodate the human body within the system, which is analogous to magnetic resonance imaging systems.8,9 Considering that the magnitude of the magnetic field is inversely proportional to the size of a coil, such MNSs also suffer from a large amount of electric power consumption and corresponding heat dissipation problems.

For this research, we propose a novel MNS simply composed of a triad of electromagnetic coils (TEC), as shown in Fig. 1, that can efficiently manipulate magnetic robots in 2D environments. The TEC is a combination of three identical circular coils placed at the vertices of an equilateral triangle, and it occupies a geometrically minimal space to provide a proper robot working space inside the MNS. Thus, the corresponding control effort, power consumption, and heat dissipation of the TEC can be effectively minimized compared to conventional MNSs with larger numbers of coils.5,6 In this paper, we derived a simplified expression of the TEC’s magnetic field by introducing local and global coordinates and transformation matrices. We also derived a constraint equation from the expression...
to manipulate a magnetic robot’s 2D motions by means of the TEC’s input currents. We then constructed a prototype TEC and a magnetic robot and demonstrated controlled robot motions to verify the proposed structure and manipulation method.

II. GENERATION OF 2D MOTIONS OF A MAGNETIC ROBOT USING A TEC

Generally, the magnetic torque and force exerted on a magnetic robot in a magnetic field can be idealized as the following respective equations:

\[
T = m \times B \tag{1}
\]

\[
F = \nabla (m \cdot B) = \frac{\partial B}{\partial x} m \tag{2}
\]

where \(m\), \(B\), and \(\frac{\partial B}{\partial x}\) are the magnetic moment of the magnetic robot, the external magnetic field, and the Jacobian matrix of the external magnetic field, respectively. Assuming a global origin, \(O\), the magnetic field of the TEC’s \(k\)th coil shown in Fig. 1b at the position of \(x_k = [x_k, y_k, z_k]^T\) with respect to the local origin \((O_k)\), can be simplified as the following equation:

\[
B^{O_k}_k(x_k) = \frac{\mu_0 N R^2 I_k}{2} \frac{3 y_k}{(R^2 + y_k^2)^{3/2}} \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{3}
\]

where \(\mu_0\) is the permeability of free space and \(I_k, x_k, y_k,\) and \(z_k\) are the current and the \(x\), \(y\), and \(z\)-directional components of \(x_k\), respectively. Here, \(R\) and \(N\) are the coils’ common radius and the number of turns, respectively. Assuming a global origin, \(O\), which matches the TEC’s centroid as shown in Fig. 1b, the magnetic field of the \(k\)th coil near the coil axis at the position of \(x = [x \ y \ z]^T\) with respect to the global origin can be expressed as the following equation:

\[
B^O_k(x) = R^O_k B^{O_k}_k(x_k) \tag{4}
\]

where \(R^O_k\) is the \(z\)-directional rotation matrix with a rotation angle of \(\theta_k\) along the \(z\)-axis. The overall magnetic field of the TEC near the centroid can thus be expressed as the following equation:

\[
B^O_{\text{TEC}}(x) = \sum_{k=1}^3 \left[ R^O_k B^{O_k}_k(x_k) \right] \tag{5}
\]

From the geometry of the equilateral triangle, \(\theta_k\) equals \(2\pi(k - 1)/3\) and \(x_k\) equals \(R^O_k \left( x + \frac{\sqrt{3}}{2} R^O_{\hat{\phi}_k} a_{\phi} \right)\) where \(\hat{a}_{\phi}\) is a \(y\)-directional unit vector. On the other hand, because the Jacobian matrix of the TEC’s \(k\)th coil can be directly calculated with respect to the local coordinate, \(x_k\), from Eq. (3), the overall Jacobian matrix of the TEC can be effectively calculated using the chain rule, as follows:

\[
\frac{\partial B^O_{\text{TEC}}}{\partial x} = \sum_{k=1}^3 \left[ R^O_k \frac{\partial B^O_k}{\partial x_k} \right] \tag{6}
\]

Therefore, the magnetic torque and force exerted on the magnetic robot near the central region of the TEC can be calculated using Eq. (1)–(6).

In this research, we derived a constraint equation to manipulate the magnetic robot’s 2D motions via control of the TEC’s input currents. At the centroid of the TEC \((O)\), Eq. (5) and (6) yield simplified equations of the TEC’s magnetic field and its Jacobian matrix, as in the following respective equations:

\[
B^O_{\text{TEC}}(0) = H_1 \sum_{k=1}^3 I_k \begin{bmatrix} -\sin \theta_k & \cos \theta_k & 0 \end{bmatrix}^T \tag{7}
\]

\[
\frac{\partial B^O_{\text{TEC}}}{\partial x} \bigg|_{x=0} = H_2 \sum_{k=1}^3 I_k \begin{bmatrix} 2 - 6 \sin^2 \theta_k & 3 \sin 2 \theta_k & 0 \\ 0 & 2 - 6 \cos^2 \theta_k & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{8}
\]

where \(H_1 = \frac{y^3 \mu_0 N}{108 k}\) and \(H_2 = \frac{27 \mu_0 N}{256 k}\). Assuming that the magnetic robot is freely rotatable and unimpeded in the \(xy\)-plane, it can be considered that the magnetic robot’s orientation always follows (matches) the applied magnetic field \((m/B^O_{\text{TEC}})\). Thus, from Eq. (1) and (2), if the magnetic robot located near the center of the TEC is to move (be propelled) along the direction of \( \hat{a}\) from the \(x\)-axis while aligning in the direction of \(\phi\), the magnetic field of the TEC should satisfy the following respective conditions:

\[
B^O_{\text{TEC}}(0) \times \hat{a}_\phi = 0 \tag{9}
\]

\[
\left[ \frac{\partial B^O_{\text{TEC}}}{\partial x} \right] \bigg|_{x=0} \ B^O_{\text{TEC}}(0) \times \hat{a}_\phi = 0 \tag{10}
\]

where \(\hat{a}_\phi\) and \(\hat{a}_\alpha\) are \(\phi\)- and \(\alpha\)-directional unit vectors, respectively. From Eq. (7)–(10), the constraint equation of the input currents of the TEC can then be derived as the following equation:
\[
I_{\text{TEC}} = \begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \frac{2F_0}{27m_0H_0\cos\theta} \left[ 
\begin{array}{c}
-2\cos\alpha + 2\cos(\alpha - 2\theta) - 6\cos(\alpha + 2\theta) \\
-2\sin\left(\alpha - \frac{\pi}{6}\right) - 2\sin\left(\alpha - 2\theta + \frac{\pi}{6}\right) - 6\cos(\alpha + 2\theta) \\
2\sin\left(\alpha + \frac{\pi}{6}\right) + 2\sin\left(\alpha - 2\theta - \frac{\pi}{6}\right) - 6\cos(\alpha + 2\theta)
\end{array}
\right]
\]

(11)

where \(m_0\) and \(F_0\) are the magnitudes of the magnetic moment and the desired magnetic force of the magnetic robot, respectively. In Eq. (11), the required input currents of the TEC are affected by both the aligning and propelling directions of the magnetic robot. Considering that there exist impermissible aligning directions of the robot and that the dot product of \(I_{\text{TEC}}\) with itself (\(\|I_{\text{TEC}}\|^2\)) is proportional to the overall electric power consumption of the TEC, the most efficient (optimal) aligning direction of the magnetic robot can be calculated by an extremum condition (\(d\|I_{\text{TEC}}\|^2/d\theta = 0\)), once the propelling direction of the magnetic robot is prespecified.

III. RESULTS AND DISCUSSION

We first examined the TEC’s ability to generate a magnetic field capable of manipulating the magnetic robot’s 2D motions. Fig. 2 shows various simulated magnetic fields (aligning directions of the magnetic robot) of the TEC and the corresponding magnetic forces derived from the field gradients under different conditions of \(\theta\) and \(\alpha\). In this case, we used the Biot-Savart law to calculate the magnetic fields rather than using the simplified equation in Eq. (3), in order to examine the validity of the equations derived in this research. We also converted all the variables used in Fig. 2 into dimensionless variables, as shown in Table I, to show the relative variations of the magnetic field and force compared to those at the centroid of the TEC ([\(x', y'] = [0, 0]\)) while removing the effect of scale. Fig. 2 shows that the TEC can generate a relatively uniform magnetic field and force within a specific central area of the TEC in which a magnetic robot could be manipulated in a linear manner.

In this research, we also conducted various experiments to verify the proposed TEC (Fig. 3). We first constructed three identical circular coils using copper wire with a thickness of 1 mm and mounted them at the vertices of an equilateral triangle to construct the TEC, as shown in Fig. 3a. The radii and numbers of turns of the coils were identically 125 mm and 1300 times, respectively. Each coil was connected to an individual programmable power supply and was integrated into a control panel so that every coil current of the TEC could be regulated simultaneously. We then constructed a prototype magnetic robot using a transversely magnetized disk-like NdFeB magnet. The diameter, thickness, and magnetization of the

| FIG. 2. Simulated results of the TEC’s magnetic field (\(B_{\text{TEC}}\)') and force (\(F'\)) with respect to different conditions. The values of (\(\alpha, \theta\)) in Figs. 2a, 2b, 2c, and 2d correspond to (0°, 60°), (30°, 60°), (60°, 0°), and (90°, 0°). |

| TABLE I. Properties of the dimensionless variables of the TEC’s magnetic field. |
| --- |
| Variables | Positions | Current | Magnetic field | Magnetic force |
| --- | --- | --- | --- | --- |
| Dimensionless variables | \(x'\) | \(y'\) | \(I_k'\) | \(B_{\text{TEC}}^{O', T}\) | \(F'\) |
| Relationships with dimensional variables | \(\sqrt{3}/R\) \(\times\) \(\sqrt{3}/R\) | \(\sqrt{3}\) \(x\) | \(\sqrt{3}\) \(y\) | \(R_{\text{TEC}}^{O', T}/R_{\text{TEC}}^{O, T}\) | \(m_0[R_{\text{TEC}}^{O, T}(0)]\) | \(\sqrt{3}m_0[R_{\text{TEC}}^{O', T}(0)]\) |
magnet was 3 mm, 1 mm, and 955 kA/m, respectively. A petri dish filled with silicone oil with viscosity of 97cP was used as the robot’s 2D working environment. A highly viscous fluid can provide a wet friction condition for the robot and at the same time, can reduce overly abrupt motions during manipulation.

To examine the proposed robot manipulation method (Eq. 11), we first observed the translational motions of the magnetic robot along different straight lines, as shown in Figs. 4a–4d. Under the input currents of the TEC shown in Table II (Fig. 2), of which the resultant magnetic forces of the robot are identically 300 μN, the magnetic robot placed at the central point of the petri dish showed sufficiently steady translational motions along the directions at 0, 30, 60, and 90 degrees from the x-axis, with less than 6.5 degrees of deviated angle. We also observed the robot’s consecutive translational motions along a programmed pathway, as shown in Figs. 4e and 4f. These figures show the overlapped images of the magnetic robot moving along a triangular-shaped pathway. In Fig. 4e, we calculated the optimal aligning directions of the robot at every different propelling direction (Step I, II, and III) in the pathway and applied the corresponding currents to the TEC in series, as shown in Table II. In Fig. 4f, on the other hand, we applied different sets of currents to the TEC as shown in Table II. In this case, the aligning directions of the magnetic robot slightly changed to 22.4 degrees higher values, respectively, whereas the directions and the magnitudes of the propelling forces remained the same. As a result, the magnetic robot in both cases showed almost the same translational motions...
along the programmed pathway with an identical average speed of 0.51 mm/sec and less than 6.5 degrees of the deviated angle. However, the TEC in Fig. 4f consumed more than 5 times more electric power than the one in Fig. 4e. Thus, the TEC can minimize the electric power consumption by aligning the magnetic robot along the optimal direction during manipulation. However, it was seen that the magnetic robot tends to show apparent deviations from the desired pathway as it moved outside of the square with an approximate area of 1,600 mm$^2$ shown in Fig. 3b, which may be derived from the nonlinearly distributed off-axis magnetic field of the TEC. Therefore, the magnetic robot should be manipulated within a specific working area of the TEC to minimize unwanted deviations, unless an appropriate closed-loop control system is added to the MNS.

### IV. CONCLUSIONS

In this paper, we proposed one MNS that can effectively manipulate magnetic robots in 2D environments. The newly developed TEC is geometrically compact and electrically efficient compared to conventional systems. Experimental results verified the proposed TEC and the robot manipulation method. Further investigations, such as for localization, mapping, and closed-loop control of the magnetic robot, remain for future work to improve the magnetic robot’s admissible working area and manipulation accuracy. This research could contribute to the development of structurally and electrically realistic MNSs for various biomedical magnetic robot applications.

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