Subcritical regime of hybrid inflation with modular $A_4$ symmetry

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Abstract

We consider a supergravity model that has the modular $A_4$ symmetry and discuss the interplay between the neutrino mixing and inflation. The model contains right-handed neutrinos that have the Majorana masses and additional Yukawa couplings to the waterfall field. In the model an active neutrino is massless and we find that only the inverted hierarchy is allowed and the Majorana phase is predicted to be around $\pm(120–180)^\circ$ from the observed neutrino mixing data. In the early universe, one of right-handed sneutrinos plays the role of the inflaton field. Focusing on the subcritical regime of the hybrid inflation that is consistent with the cosmic microwave background data, we analyze the dynamics of the scalar sector and derive an upper bound $O(10^{10})$ GeV on the scale of the Majorana mass.
1 Introduction

Inflation, a hypothesis of accelerated expansion of the Universe, is supported by the observation of the cosmic microwave background (CMB) radiation. It is driven by the vacuum energy at the early state of the Universe and it is realized by a slowly-rolling scalar field, called inflaton. According to decades of the observations and analysis of the CMB, some properties of inflation have been revealed. The latest results by Planck collaboration [1] report that the amplitude $A_s$ and spectral index $n_s$ of the scalar perturbation and the ratio $r$ of the tensor mode to the scalar mode are given by $A_s = (2.089 \pm 0.029) \times 10^{-9}$ (68% CL), $n_s = 0.9649 \pm 0.0042$ (68% CL), and $r < 0.056$ (95% CL). The results already exclude some of inflation models and future experiments are expected to measure the observables with more precision.

Besides the accelerated expansion of the Universe, the inflaton field plays another important role, i.e., reheating to create the radiation dominated Universe. Then the production of dark matter and baryons follows. Thus, it is tempting to consider an inflation model that provide the sequence of the thermal history after inflation. Moreover, the model would be motivated if it is controlled by an underlying symmetry. The modular symmetry and supersymmetry are the promising candidates.

The modular symmetry is the discrete symmetry associated with the compactification of the extra dimensions inspired by superstring theory. It has recently caught attention from the phenomenological point of view since Ref. [2] pointed out that it gives a consistent pattern of the neutrino mixing data [3–8]. Variety types of symmetries have been studied so far; for instance, the modular $S_3$ [9], $A_4$ [2,10–52], $A_5$ [53–55], and other modular groups [56–59]. Quark masses and mixings are investigated in Refs. [60–63]. In the modular symmetric framework, soft supersymmetry breaking terms [64] and modular stabilization [65–69] are also discussed. For other phenomenological applications, for example, the application of modular symmetry to grand-unified theories is discussed in Refs. [70–78]. Moreover, in cosmology, models with dark matter candidates [79–86] have been considered and Refs. [87–95] have applied it to leptogenesis.

In the present study we apply the modular $A_4$ symmetry to inflation. Introducing three right-handed (s)neutrinos, we consider one of right-handed sneutrinos plays the role of the inflaton field. Since the typical model of the right-handed sneutrino inflation tends to be chaotic inflation due to the quadratic term, which is already excluded by the CMB observations, we extend the model to the $D$-term hybrid inflation by introducing a new Yukawa term. These days variety of new $D$-term hybrid inflation models have been proposed, depending on the symmetry of the Kähler potential; $R^2$ Starobinsky model [96,97] appears from the superconformal symmetry [98], the chaotic regime emerges from the shift symmetry [99,100], and $\alpha$-attractor [101] comes from their combination, which is called superconformal subcritical hybrid inflation [102]. A generalized version of the inflation model proposed in Ref. [102] is intensitively analyzed in Ref. [103]. In our paper we study the neutrino mixing and the dynamics of inflation based on the model in Ref. [103]. In the framework, an unconventional neutrino mixing pattern and the CP phases are obtained. We find that only the inverted hierarchy is consistent with the data and in
the valid parameters the upper limit for the scale of the Majorana mass is obtained for successful inflation.

This paper is organized as follows. In Sec. 2, we introduce the supergravity model to study. Then neutrino masses and mixing pattern are discussed in Sec. 3. Here we show the predicted CP phases and the effective neutrino mass for the neutrinoless double beta decay. In Sec. 4, the dynamics of inflation is studied both analytically and numerically, especially focusing on the impact of the Majorana mass terms. Sec. 5 contains our conclusion.

2 The model

2.1 Brief review of modular symmetry

The modular symmetry is the geometric symmetry of the two dimensional torus. The two dimensional torus $T^2$ is defined by $\mathbb{C}/\Lambda$, i.e., the complex plane $\mathbb{C}$ divided by the two dimensional lattice $\Lambda = \{ \sum_{i=1}^2 n_i \omega_i | n_i \in \mathbb{Z} \}$ with basis vectors $\omega_i \in \mathbb{C}$. Here, the basis vectors are given by $\omega_1 = 2\pi R$ and $\omega_2 = 2\pi R \tau$ with $R \in \mathbb{R}$, and $\tau = \omega_2/\omega_1$ is the modulus defined in the upper half plane $\mathcal{H} = \{ \tau \in \mathbb{C} | \text{Im}\tau > 0 \}$. The same lattice is constructed using the basis vectors transformed as

$$\left( \begin{array}{c} \omega'_2 \\ \omega'_1 \end{array} \right) = \gamma \left( \begin{array}{c} \omega_2 \\ \omega_1 \end{array} \right), \quad \gamma \in SL(2, \mathbb{Z}), \quad \text{(2.1)}$$



where

$$\Gamma \equiv SL(2, \mathbb{Z}) \equiv \left\{ \gamma = \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) \ | \ a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}. \quad \text{(2.2)}$$

Then, the modulus is transformed as

$$\tau \rightarrow \gamma \tau = \frac{a \tau + b}{c \tau + d}. \quad \text{(2.3)}$$

Eq. (2.3) is called modular transformation. It is seen that the transformation law of $\tau$ is the same for $\gamma$ and $-\gamma$. Then, the group of modular transformation is isomorphic to $PSL(2, \mathbb{Z}) = SL(2, \mathbb{Z})/\pm \mathbb{I} \equiv \bar{\Gamma}$, which is called modular group. The modular group is generated by two generators,

$$S = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right), \quad T = \left( \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right), \quad \text{(2.4)}$$

satisfying

$$(ST)^3 = S^2 = \mathbb{I}. \quad \text{(2.5)}$$

The modulus $\tau$ is transformed by $S$ and $T$ as

$$S : \tau \rightarrow -1/\tau, \quad T : \tau \rightarrow \tau + 1. \quad \text{(2.6)}$$
For a positive integer $N$, the principal congruence subgroup of level $N$ is defined by

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}, \quad (2.7)$$

Note that $\Gamma(1) = \Gamma$ and $-I \not\in \Gamma(N)$ for $N > 2$. Then, we introduce the groups $\Gamma(N)$ as $\Gamma(2) = \Gamma(2)/\pm I$ and $\Gamma(N) = \Gamma(N)/(N > 2)$. The quotient groups $\Gamma_N \equiv \Gamma/\Gamma(N)$ are called finite modular groups and the generators of $\Gamma_N$ have the additional relation,

$$T^N = I. \quad (2.8)$$

It is known that the finite modular groups for $N = 2, 3, 4, 5$ are isomorphic to the non-Abelian discrete groups, $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$, $\Gamma_5 \simeq A_5$, respectively [104]. Under the modular transformation, the holomorphic functions of $\tau$, called the modular forms are transformed as

$$f(\tau) \rightarrow f(\gamma \tau) = (c\tau + d)^k \rho(\gamma) f(\tau). \quad (2.9)$$

Here $\rho(\gamma)$ is a unitary transformation of $\Gamma_N (N > 2)$ and $k$ is non-negative even integer, called modular weight.

In this study, we focus on $\Gamma_3$, i.e., $A_4$. Then, the modular forms $Y = (Y_1, Y_2, Y_3)^T$ with representation of $A_4$ triplet and modular weight 2 are given by [2]

$$Y_1 = \frac{i}{2\pi} \left[ \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau + 1)/3)}{\eta((\tau + 1)/3)} + \frac{\eta'((\tau + 2)/3)}{\eta((\tau + 2)/3)} - 27\eta'(3\tau) \right],$$

$$Y_2 = \frac{-i}{\pi} \left[ \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2 \frac{\eta'((\tau + 1)/3)}{\eta((\tau + 1)/3)} + \frac{\eta'((\tau + 2)/3)}{\eta((\tau + 2)/3)} \right],$$

$$Y_3 = \frac{-i}{\pi} \left[ \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega \frac{\eta'((\tau + 1)/3)}{\eta((\tau + 1)/3)} + \omega^2 \frac{\eta'((\tau + 2)/3)}{\eta((\tau + 2)/3)} \right], \quad (2.10)$$

where $\omega = e^{2\pi i/3}$ and $\eta(\tau)$ is the Dedekind eta-function defined by

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad q = e^{2\pi i \tau}. \quad (2.11)$$

The modular forms $Y_i (i = 1–3)$ satisfy the relation,

$$Y_2^2 + 2Y_1Y_3 = 0. \quad (2.12)$$

They are transformed under $T$ and $S$ transformations as

$$S : Y(-1/\tau) = \tau^2 \rho(S)Y, \quad T : Y(\tau + 1) = \rho(T)Y, \quad (2.13)$$

where

$$\rho(S) = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad \rho(T) = \begin{pmatrix} 1 & \omega & \omega^2 \end{pmatrix}. \quad (2.14)$$

It is assumed that the superfields, denoted as $Z^I$, are also transformed under $A_4$ as

$$Z^I \rightarrow (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) Z^I, \quad (2.15)$$

where the modular weight $k_I$ will be determined later.
\begin{align*}
| & L & E^c = (e^c, \mu^c, \tau^c) & N^c & H_u & H_d & S_{\pm} \\
U(1) & 0 & 0 & 0 & 0 & 0 & \pm q \\
A_4 & 3 & (1, 1''', 1') & 3 & 1 & 1 & 1 \\
\text{weight} & -k_L & (-k_{e^c}, -k_{\mu^c}, -k_{\tau^c}) & -k_{N^c} & -k_{H_u} & -k_{H_d} & -k_{S_{\pm}}
\end{align*}

Table 1: Field contents and their representations and modular weights.

\section{The superpotential and \(\text{Kähler potential}\)}

We consider a supergravity model of inflation inspired by modular \(A_4\) symmetry. In addition to the fields in the minimal supersymmetric standard model (MSSM), we introduce three right-handed neutrinos \(N^c_i\) \((i = 1\ldots3)\) and two new fields \(S_{\pm}\). Here \(S_{\pm}\) are gauge singlets under the MSSM gauge but are charged under a local \(U(1)\) with charge \(\pm q\) \((q > 0)\). They play important roles during inflation.

The superpotential allowed under \(A_4\) symmetry is given by

\begin{equation}
W \supset W_E + W_N + W_{\text{hyb}},
\end{equation}

where

\begin{align*}
W_E &= \alpha_1 e^c H_d (LY)_1 + \alpha_2 \mu^c H_d (LY)_1' + \alpha_3 \tau^c H_d (LY)_1'', \\
W_N &= g_1 (N^c H_u (LY)_{a_1})_1 + g_2 (N^c H_u (LY)_{a_2})_1, \\
W_{\text{hyb}} &= \lambda S_{\pm} S_{\pm} (N^c Y)_1 + \Lambda (N^c N^c Y)_1.
\end{align*}

The superpotential is similar to one considered in Ref. [105], but it is extended to accommodate the modular \(A_4\) symmetry. The contents of the fields are listed in Table 1. The representation and the modular weights of the fields are also shown in the table, which is based on the model considered in Ref. [2, 11]; the right-handed neutrinos and the left-handed lepton doublets \(L_i = (\nu_{Li}, l_{Li})^T\) \((i = e, \mu, \tau)\) are the \(A_4\) triplets and the others are the singlets. Here the three different singlet representations \((1, 1''', 1')\) are assigned to the right-handed charged leptons \(E^c = (e^c, \mu^c, \tau^c)\), respectively. We will discuss the assignment of the modular weights soon later. In the superpotential, \(\alpha_i\) \((i = 1\ldots3)\), \(g_i\) \((i = 1, 2)\), and \(\lambda\) are Yukawa coupling constants, and \(\Lambda\) determines the mass scale of the right-handed neutrinos. By redefinition of the fields, the parameters \(\alpha_i\), \(\lambda\), \(\Lambda\), and \(g_1\) can be taken to be real without the loss of generality.

For \(\text{Kähler potential}\), we adopt a class of the canonical superconformal supergravity model [106]. This type of model can be extended to the model with a parameter \(\alpha\), which corresponds to the parameter of the superconformal \(\alpha\) attractor model [101]. Recently the dynamics of inflation has been analyzed in a generalized version of the model [103]. In order to consider the similar inflation model, we consider the \(\text{Kähler potential}\) based on Ref. [103].

\begin{equation}
K = -3 \log \left( \frac{-N}{3} \right),
\end{equation}

\#1 We adopt the Planck unit, where \(M_{\text{pl}} = 1\) for the Planck mass \(M_{\text{pl}}\).
where
\[ \mathcal{N} = -|Z^0|^n \left( -\frac{\Phi}{3} \right)^{\alpha}, \quad \frac{-\Phi}{3} = 1 - \sum_{I} \frac{|Z_I|^2}{|Z^0|^k}. \] (2.21)

Here we have introduced a positive parameter \( n \) in order to discuss the modular weights of the field generically. We take \( Z^0 = Z^0 = 2 \text{Im} \tau \). In general, \( \Phi \) includes additional terms, such as \((Y N^c N^c)_1\) or \((\tilde{Y} L \tilde{Y} L)_1\) [2, 19]. In the current study we ignore them to focus on the simple setting.\(^2\)

The Kähler potential is further divided into the modular and matter parts as
\[ K = K^\tau + K^m, \]
where
\[ K^\tau = -3n \log(2 \text{Im} \tau), \quad K^m = -3\alpha \log \left( -\frac{\Phi}{3} \right). \] (2.22)
The matter part is constructed to be modular invariant. Then, under the modular transformation (2.3), the Kähler potential is transformed as
\[ K \to K + 3n \left[ \log(c \tau + d) + \log(c \bar{\tau} + d) \right]. \] (2.23)
In the supergravity the combination of the Kähler potential and superpotential, which is defined by
\[ G = K + \log W + \log \bar{W}, \] (2.24)
should be modular invariant. Due to the invariance, the transformation of the superpotential is determined as,
\[ W \to e^{i \alpha(\gamma)(c \tau + d)^{-3n}} W, \]
\[ \bar{W} \to e^{-i \alpha(\gamma)(c \bar{\tau} + d)^{-3n}} \bar{W}. \] (2.25)
Namely, its modular weight is \(-3n\). Consequently, the weight of fields should satisfy
\[ 3n = k_{H_u} + k_{H_d} = k_{S_+} + k_{S_-} + k_{N^c} - 2 = 2k_{N^c} - 2 \]
\[ = k_{N^c} + k_L + k_{H_u} - 2 = k_{E^c} + k_L + k_{H_d} - 2, \] (2.26)
where \( k_{e,e} = k_{\mu,e} = k_{\tau,e} = k_{E^c} \). Those are the generic conditions for the modular weights in the present model. For instance, if we further impose the following conditions:
\[ k_{H_u} = k_{H_d} \equiv k_H, \quad k_{S_+} = k_{S_-} \equiv k_S, \] (2.27)
the modular weights are uniquely determined for a given \( n \) as
\[ k_L = 1, \quad k_{N^c} = k_{E^c} = 2k_S = k_H + 1 = (3n + 2)/2. \] (2.28)
\(^2\)This is also motivated by the results given by Ref. [103]. In the literature it is shown such a simple Kähler potential gives a consistent result with the CMB observations.
The value of $n$ is a free parameter and it may restrict possible terms in the Kähler potential. However, we do not consider this direction seriously in the current study.

After the modulus parameter $\tau$ is fixed, we redefine the chiral superfields as

$$Z^I \rightarrow \hat{Z}^I = Z^I \sqrt{3/|Z^0|^2}.$$  \hspace{1cm} (2.29)

Accordingly it is convenient to reparametrize the Yukawa couplings $(\alpha_i, g_i, \lambda)$ and $\Lambda$ as $(\hat{\alpha}_i, \hat{g}_i, \hat{\lambda})$ and $\hat{\Lambda}$ to give rise to the same form of $W_E + W_N + W_{hyb}$. Hereafter, we will write the model in this field basis and omit the ‘check’ symbol for a simple notation unless otherwise noticed. Namely, we use the superpotential given in Eqs. (2.17) – (2.19) and the Kähler potential $K^{\text{m}}$ of the matter part with the function $\Phi$

$$-\frac{\Phi}{3} = 1 - \frac{1}{3} \sum_I |Z^I|^2.$$ \hspace{1cm} (2.30)

For later calculation, we rewrite $W_{hyb}$ as

$$W_{hyb} = \lambda_i S_i S_i N_i^c + \frac{1}{2} M_{ij} N_i^c N_j^c,$$ \hspace{1cm} (2.31)

where $(\lambda_1, \lambda_2, \lambda_3) = \lambda(Y_1, Y_3, Y_2)$ and

$$M = \Lambda \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}.$$ \hspace{1cm} (2.32)

### 3 Neutrino masses and mixing pattern

In this section, we discuss the lepton sector, especially focusing on the neutrino mixing pattern. From the superpotential, the light neutrinos acquire masses in the seesaw mechanism [107–112]. In our model, however, $S_+$ obtains a vacuum expectation value (VEV), denoted as $\langle S_+ \rangle$, at the global minimum, which leads to an unconventional mass matrix for the light neutrinos [105]. In addition, the components of the mass matrices are given in a limited number of parameters due to $A_4$ symmetry, which results in the characteristic pattern of the neutrino mixing and CP phases in the lepton sector.

#### 3.1 Neutrino masses and PMNS matrices

To give the mass matrices of the leptons, it is convenient to adopt canonically normalized field basis. The canonically normalized field $\hat{Z}^I$ is obtained by

$$\hat{Z}^I = \sqrt{\alpha} Z^I.$$ \hspace{1cm} (3.1)

Here we have used $-\Phi/3 \simeq 1$ at the global minimum. The validity of the approximation is guaranteed by $\langle S_+ \rangle \ll 1$, which will be shown in Sec. 4. Accordingly we absorb the factor $\sqrt{\alpha}$ by introducing

$$(\hat{\alpha}_i, \hat{g}_i, \hat{\lambda}) = \alpha^{-3/2}(\alpha_i, g_i, \lambda), \quad \hat{\Lambda} = \alpha^{-1}\Lambda,$$ \hspace{1cm} (3.2)
to give the same form of the superpotential.

Let us see the neutrino mass matrix. We rewrite Eq. (2.19) as

\[ W_{\text{hyb}} = \hat{\lambda} \hat{S}_+ \hat{\tilde{S}}_- \hat{\tilde{N}}_i^c + \frac{1}{2} \hat{M}_{ij} \hat{\tilde{N}}_i^c \hat{\tilde{N}}_j^c. \]  

(3.3)

where \((\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3) = \hat{\lambda}(Y_1, Y_3, Y_2)\) and

\[
\hat{M} = \hat{\lambda} \begin{pmatrix}
2Y_1 & -Y_3 & -Y_2 \\
-Y_3 & 2Y_2 & -Y_1 \\
-Y_2 & -Y_1 & 2Y_3
\end{pmatrix}.
\]

(3.4)

Then, the mass matrix of the neutrinos in the basis of \((N_1^e, N_2^e, N_3^e, \tilde{S}_-, \nu_{Le}, \nu_{L\mu}, \nu_{L\tau})\)^3 is given by

\[
\mathcal{M} = \begin{pmatrix}
\hat{M} & \hat{M}_D \\
\hat{M}_D^T & O
\end{pmatrix}.
\]

(3.5)

Here \(\hat{M}\) and \(\hat{M}_D\) are \(4 \times 4\) and \(4 \times 3\) matrices, respectively, written by

\[
\hat{M} = \hat{\lambda} \langle \hat{S}_+ \rangle \begin{pmatrix}
\hat{M} / \hat{\lambda} \langle \hat{S}_+ \rangle & Y_1 \\
Y_2 & Y_3
\end{pmatrix},
\]

(3.6)

\[
\hat{M}_D = \langle \hat{H}_u^0 \rangle \begin{pmatrix}
2\hat{g}_1 Y_1 & (\hat{g}_1 + \hat{g}_2)Y_3 & (\hat{g}_1 - \hat{g}_2)Y_2 \\
(\hat{g}_1 - \hat{g}_2)Y_3 & 2\hat{g}_1 Y_2 & (\hat{g}_1 + \hat{g}_2)Y_1 \\
0 & 0 & 0
\end{pmatrix},
\]

(3.7)

where \(\langle \hat{H}_u^0 \rangle\) is the VEV of up-type neutral Higgs. We note that \(\hat{\lambda} \langle \hat{S}_+ \rangle\) corresponds to the scale of the inflaton mass, which will be shown later. Consequently, the light neutrinos mass matrix is obtained by the seesaw mechanism as

\[
M_\nu = -\hat{M}_D^T \hat{M}^{-1} \hat{M}_D.
\]

(3.8)

Using Eq. (2.12), it is straightforward to obtain the mass matrix \(M_\nu\) as

\[
\frac{M_{\nu 11}}{m_{\nu 0}} = 12 \hat{Y}_2^2 \left[ -2(3 + 5\hat{g}) + (-3 + \hat{g})\hat{Y}_2^3 \right],
\]

\[
\frac{M_{\nu 22}}{m_{\nu 0}} = \frac{4(2 + \hat{Y}_2^3)[(1 + \hat{g})\hat{Y}_2^6 + 16\hat{Y}_2^3 + 8(-1 + \hat{g})]}{\hat{Y}_2},
\]

\[
\frac{M_{\nu 12}}{m_{\nu 0}} = \frac{M_{\nu 21}}{m_{\nu 0}} = \frac{-M_{\nu 33}}{2m_{\nu 0}} = (-4 + \hat{Y}_2^3)[4(3 + \hat{g}) + (-3 + 5\hat{g})\hat{Y}_2^3],
\]

(3.9)

\[
\frac{M_{\nu 13}}{m_{\nu 0}} = \frac{M_{\nu 31}}{m_{\nu 0}} = \hat{Y}_2 \left[ (-3 + \hat{g})(16 + \hat{Y}_2^6) + 4(6 - 11\hat{g})\hat{Y}_2^3 \right],
\]

\[
\frac{M_{\nu 23}}{m_{\nu 0}} = \frac{M_{\nu 32}}{m_{\nu 0}} = \frac{-64(1 + \hat{g}) - 48(2 + \hat{g})\hat{Y}_2^3 - 24(1 + \hat{g})\hat{Y}_2^6 + (1 + \hat{g})\hat{Y}_2^9}{2\hat{Y}_2},
\]

\#^3We use the same notation for the fermionic part as the chiral superfield for \(N_i^c\) and the leptons in the MSSM while \(\tilde{S}_-\) is the fermionic part of \(S_-\).
| Normal Hierarchy (3\textsigma range) | Inverted Hierarchy (3\textsigma range) |
|-------------------------------------|---------------------------------------|
| $\sin^2 \theta_{12}$               | 0.269−0.343                           | 0.269−0.343                           |
| $\sin^2 \theta_{23}$               | 0.415−0.616                           | 0.419−0.617                           |
| $\sin^2 \theta_{13}$               | 0.02032−0.02410                       | 0.02052−0.02428                       |
| $\Delta m^2_{21}/10^{-3}$ eV$^2$    | 6.82−8.04                             | 6.82−8.04                             |
| $\Delta m^2_{3l}/10^{-3}$ eV$^2$    | 2.435−2.598                           | −2.581−−2.414                         |

Table 2: Observed data of the neutrino mass squared differences and the neutrino mixing angles from NuFIT 5.0 [115]. Here, $\Delta m^2_{3l} = \Delta m^2_{31} > 0$ (NH) and $\Delta m^2_{3l} = \Delta m^2_{32} < 0$ (IH).

where

$$m_{\nu 0} \equiv \frac{(-1 + \hat{g}) \, \hat{g}_3^2 Y_3 \langle \hat{H}_u \rangle^2}{(8 + \hat{Y}_2^3)^2 \hat{\Lambda}},$$ \hspace{1cm} (3.10)

$$\hat{Y}_1 \equiv \frac{Y_1}{Y_3}, \quad \hat{Y}_2 \equiv \frac{Y_2}{Y_3}, \quad \hat{g} \equiv \frac{\hat{g}_2}{\hat{g}_1}. \hspace{1cm}$$

It is worth notifying that the mass matrix is independent of $\hat{\Lambda} \langle \hat{S}_+ \rangle$. Therefore, the scale of the inflaton mass is not affected by the observations of the neutrino sector. Finally $M_\nu$ is diagonalized by a unitary matrix $U_\nu$ as

$$\text{diag}(m_1, m_2, m_3) = U^T_\nu M_\nu U_\nu.$$ \hspace{1cm} (3.11)

The obtained neutrino masses are then compared with the observed values shown in Table 2. We notice that the lightest neutrino mass becomes zero since rank of $M_\nu$ is two [105]. Therefore, by imposing the condition in which the neutrino mass squared differences are within 3\textsigma range of the observed values, the cosmological upper bound on the sum of light neutrino masses \cite{113,114}

$$\sum_i m_i \leq 120 \text{ meV},$$ \hspace{1cm} (3.12)

is automatically satisfied in our model for both the normal hierarchy (NH) and inverted hierarchy (IH).

Another observable is the PMNS matrix, which is defined as $U \equiv U_l^T U_\nu$. Here $U_l$ is a unitary matrix that diagonalize the charged lepton mass matrix $M_E$. $M_E$ is the same as one studied in Ref. [11]. The result is parametrized in terms of three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$, a Dirac phase $\delta_{\text{CP}}$, and a Majorana phase $\alpha_{21}$;

$$U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13}
\end{pmatrix} \begin{pmatrix}
1 & e^{i\alpha_{21}} \\
e^{-i\alpha_{21}} & 1
\end{pmatrix},$$ \hspace{1cm} (3.13)
where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). The mixing angles are determined by the observations and they are summarized in Table 2. We note that there is only one Majorana phase since one of the light neutrinos is massless. Then, the invariant quantities regarding the CP phases are given by \([116–120]\):

\[
J_{CP} = \text{Im} \left[ U_{e1}U_{\mu2}U_{e2}^*U_{\mu1}^* \right] = s_{23}c_{23}s_{12}c_{12}s_{13}^2 c_{13}^2 \sin \delta_{CP},
\]

\[
I_1 = \text{Im} \left[ U_{e1}^*U_{e2} \right] = s_{12}c_{12}c_{13}^2 s_{13} \sin(\alpha_{21}/2),
\]

\[
I_2 = \text{Im} \left[ U_{e1}^*U_{e3} \right] = c_{12}s_{13} c_{13} \sin(-\delta_{CP}).
\]

In addition, the following relations are useful to determine the CP phases \([91]\):

\[
\cos \delta_{CP} = \frac{|U_{\tau1}|^2 - s_{12}^2 s_{23}^2 - c_{12}^2 c_{23}^2 s_{13}^2}{2c_{12} s_{12} c_{23} s_{23} s_{13}},
\]

\[
\text{Re} \left[ U_{e1}^*U_{e2} \right] = c_{12}s_{12} c_{13} \cos(\alpha_{21}/2),
\]

\[
\text{Re} \left[ U_{e1}^*U_{e3} \right] = c_{12}s_{13} c_{13} \cos(-\delta_{CP}).
\]

### 3.2 Observational consequences

Based on the arguments in the previous subsection, we compute the mixing angles and CP phases for a given set of parameters. The relevant parameters are \( \tau \) and \( \hat{g} \) and we parametrize them as

\[
\tau = \text{Re} \tau + i \text{Im} \tau, \quad \hat{g} = |\hat{g}| e^{i\phi_g},
\]

where \( g = |\hat{g}| \) and \( \phi_g \) is argument of \( \hat{g} \). Considering the fundamental domain \( \mathcal{F} \) of \( \bar{\Gamma} \) for \( \tau \),

\[
\mathcal{F} = \left\{ \tau \in \mathcal{H} \mid |\text{Re} \tau| \leq \frac{1}{2}, \ |\tau| \geq 1 \right\},
\]

we scan the following parameter ranges,

\[
|\tau| \geq 1, \quad |\text{Re} \tau| \leq \frac{1}{2}, \quad \text{Im} \tau \leq 1.65, \quad 0 \leq g \leq 6.5, \quad |\phi_g| \leq 180^\circ.
\]

We use the 3\( \sigma \) data of the mixing angles and mass squared differences from NuFIT 5.0 \([115]\), which are listed in Table 2. In the analysis, we take the VEVs of the up- and down-type Higgs bosons as \( v/2 \) (\( v \approx 246.7 \text{ GeV} \)) by considering so-called high-scale SUSY to give 125 GeV Higgs mass \([121,122]\).

First of all, we have found no allowed region for the NH case. Fig. 1 shows the allowed regions for the \( \tau \) and \( \hat{g} \) for the IH case. We found a specific pattern of the allowed values of \( \tau \) and \( \hat{g} \) is seen. The allowed value of \( \tau \) is limited in \( 1.0 \lesssim |\text{Im} \tau| \lesssim 1.1 \) and \( |\text{Re} \tau| \lesssim 0.1 \), or \( \text{Im} \tau \sim 1.2 \) and \( |\text{Re} \tau| \sim 0.5 \). This is a different feature compared to the previous work, e.g., Ref. \([11]\), where the allowed values of \( \tau \) is distributed more widely. This difference comes from the unconventional pattern of the active neutrino mass matrix (3.8) given by Eqs. (3.6) and (3.7), which comes from additional Yukawa
Figure 1: Allowed region of $\tau$ (left) and $\hat{g} = ge^{i\phi_g}$ (right) for inverted hierarchy. Each dot indicates the value that is consistent with the 3$\sigma$ data of the neutrino oscillation experiments [115] summarized in Table 2.

couplings between the right-handed neutrinos and $S_+$ in Eq. (2.31) and the VEV of $S_+$. Regarding $\hat{g}$, $g$ is distributed as $0.4 \lesssim g \lesssim 5$. The phase $\phi_g$ can take any value in $[-180^\circ, 180^\circ]$, but it is given by a smooth function of $g$. Such behavior is in contrast to the results in Ref. [11]; in the literature, the allowed region for both $g$ and $\phi_g$ are more restricted.

The discovered sets of the parameters lead to a new prediction for the CP phases, which is shown in Fig. 2. In the plot we show the correlation between $\sin^2 \theta_{23}$ and the CP phases. For $\delta_{\text{CP}}$, we found $|\delta_{\text{CP}}| \lesssim 120^\circ$ for any value of $\sin^2 \theta_{23}$ in the 3$\sigma$ range. If $\sin^2 \theta_{23} \simeq 0.55$, on the other hand, then $\delta_{\text{CP}}$ can take any values. Regarding $\alpha_{21}$, we found $|\alpha_{21}| \sim 120$–180$^\circ$ for any value of $\sin^2 \theta_{23}$ in 3$\sigma$ range. An exception is $|\alpha_{21}| \sim 35^\circ$ for $\sin^2 \theta_{23} \simeq 0.42$–0.43. Finally, we found no specific correlation between $\delta_{\text{CP}}$ and $\alpha_{21}$, i.e., $|\alpha_{21}| \sim 120$–180$^\circ$ is predicted while various value of $\delta_{\text{CP}}$ is possible. To summarize, only the IH is allowed and Fig. 2 is the prediction for the CP phases in our model.

Another observable consequence of this model is the neutrinoless double beta decay $(A, Z) \rightarrow (A, Z + 2) + 2e^-$, which is a lepton number violating process in low-energy phenomena. See, e.g., Ref. [123] for review. The decay rate of the process is proportional to the effective neutrino mass-squared, which is given by

$$m_{\text{eff}} = \left| \sum_i m_i U_{ei}^2 \right|. \hspace{1cm} (3.23)$$

The current upper limit on the effective mass of neutrinoless double beta decay from KamLAND-Zen is 36–156 meV [124]. Note that the experimental upper bound on the effective mass is affected by the uncertainty of the nuclear matrix elements in
Figure 2: Predicted CP phases $\delta_{CP}$ and $\alpha_{21}$ from the allowed parameter sets given in Fig. 1. The correlations between $\sin^2 \theta_{23}$ and the CP phases (top-left and top-right) and the correlation between CP phases (bottom) are shown. The gray solid lines show the boundaries of the $3\sigma$ region of the experimental data.
the decay process. In the present model, one of the light neutrinos is massless and we have seen that the IH is allowed. Therefore, the effective neutrino mass is given by \[ m_{\text{eff}}^2 = c_{413}^4 (m_{1}^2 c_{12}^4 + m_{2}^2 s_{12}^4 + 2m_{1}m_{2}c_{12}^2 s_{12}^2 \cos \alpha_{21}). \] (3.24)

The result is shown in Fig. 3. We found $14 \text{ meV} \lesssim m_{\text{eff}} \lesssim 28 \text{ meV}$ for the most parameter sets, where $\alpha_{21} \sim \pm 180^\circ$. One can see that it becomes as large as around $45-47 \text{ meV}$ for $\alpha_{21} \sim \pm 35^\circ$, which can be understood from Eq. (3.24). Namely the term proportional to $\cos \alpha_{21}$ is constructive in this case. The resultant value $\mathcal{O}(10) \text{ meV}$ of $m_{\text{eff}}$ is the same as one shown in Ref. [125]. Since the model predicts a relatively large effective mass, neutrinoless double beta decay events may be observed in future experiments. If the CP phases are constrained, for instance, by considering the leptogenesis [126] in the present framework, then $m_{\text{eff}}$ may be more restrictive. That would be another future work to pursue (see also the discussion in Sec. 4.5).

4 Inflation

$\mathcal{W}_{\text{hyb}}$ induces the $D$-term hybrid inflation. Motivated by the previous studies [102, 103], we consider the subcritical regime of hybrid inflation, where one of right-handed sneutrinos plays the role of the inflaton field. In the present case, however, the additional Majorana mass terms may disturb the dynamics during inflation. We will derive the inflaton potential and see how the additional terms affect the dynamics. In this section, we examine the impact of the heavy Majorana masses on the subcritical
hybrid inflation scenario. To be concrete, we assume

\[ |M_{ij}| \ll |\lambda_i \langle S_+ \rangle| \, . \tag{4.1} \]

Under the assumption, we expect that the same inflation model is obtained as Ref. [103]. We will derive the upper bound for \( M_{ij} \) quantitatively.

### 4.1 New field basis

We start with the basis given in the last paragraph of Sec. 2.2. At the global minimum, the mass matrix squared \( M_{sc}^2 \) of the scalar sector in the basis \((\tilde{N}_c^1, \tilde{N}_c^2, \tilde{N}_c^3, S_-)\)#4 is obtained as

\[ M_{sc}^2 = \tilde{M}^\dagger \tilde{M} \, , \tag{4.2} \]

where \( \tilde{M} \) is equal to Eq. (3.6) without ‘hat’ symbol. Under the condition (4.1), the mass matrix is approximated as

\[ M_{sc}^2 = \langle S_+ \rangle^2 \left( \begin{array}{cccc}
|\lambda_1|^2 & \lambda_2 \lambda_1^* & \lambda_3 \lambda_1^* & 0 \\
\lambda_1^* \lambda_2 & |\lambda_2|^2 & \lambda_3 \lambda_2^* & 0 \\
\lambda_1^* \lambda_3 & \lambda_2 \lambda_3^* & |\lambda_3|^2 & 0 \\
0 & 0 & 0 & \tilde{\lambda}^2
\end{array} \right) + \mathcal{O}(M_{ij} \lambda_k \langle S_+ \rangle) \, , \tag{4.3} \]

where

\[ \tilde{\lambda} \equiv \sqrt{\sum_i |\lambda_i|^2} \, , \tag{4.4} \]

and it can be approximately diagonalized by a unitary matrix \( V \) as

\[ V^\dagger M_{sc}^2 V = \langle S_+ \rangle^2 \left( \begin{array}{c}
0 \\
0 \\
0 \\
\tilde{\lambda}^2
\end{array} \right) + \mathcal{O}(M_{ij} \lambda_k \langle S_+ \rangle) \, , \tag{4.5} \]

\[ V \equiv \left( \begin{array}{cccc}
W & 0 & 0 & 0 \\
0 & W & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_1}{|\lambda_1|}
\end{array} \right) \, , \quad W \equiv \left( \begin{array}{cccc}
\frac{-\lambda_1^* \lambda_3}{\sqrt{\sum_j |\lambda_j|^2}} & \frac{-\lambda_2 \lambda_3^*}{\sqrt{\sum_j |\lambda_j|^2}} & \frac{|\lambda_3 \lambda_2^*|}{\lambda_3^2} & 0 \\
\frac{-\lambda_1^* \lambda_3}{\sqrt{\sum_j |\lambda_j|^2}} & \frac{-\lambda_2 \lambda_3^*}{\sqrt{\sum_j |\lambda_j|^2}} & 0 & \lambda_3 \lambda_2^* \\
\frac{|\lambda_1|}{\lambda_3} & \frac{\lambda_2^*}{\lambda_3} & \frac{|\lambda_3|^2}{\lambda_3^2} & 0 \\
\frac{\lambda_1}{\sqrt{\sum_j |\lambda_j|^2}} & \frac{\lambda_2}{\sqrt{\sum_j |\lambda_j|^2}} & \frac{-\lambda_2 \lambda_3^*}{\lambda_3^2} & \frac{|\lambda_3|^2}{\lambda_3^2}
\end{array} \right) \, . \tag{4.6} \]

Therefore, it is legitimate to use a new basis of \((N_c^1, N_c^2, N_c^3, S_-^T) \equiv V^\dagger (N_c^1, N_c^2, N_c^3, S_-^T) \). In this basis, it is straightforward to find that \( S_\pm^T \) only couple to \( N_c \). Then, the superpotential (2.31) is written as

\[ W_{hyb} = \tilde{\lambda} S_+^T S_-^T N_c^3 + \frac{1}{2} M_{ij} N_i^c N_j^c \, , \tag{4.7} \]

#4 \( \tilde{N}_i^c \) and \( S_\pm \) are scalar components of the chiral superfield \( N_c^i \) and \( S_\pm \), respectively.
where \( M' \equiv W^T M W \) is a \( 3 \times 3 \) matrix of order \( |M_{ij}| \). Therefore, \( \phi \equiv \sqrt{2} \text{Re} \tilde{N}_j^c \) becomes the inflaton field \([103,105]\).

The parameter \( \alpha \) can be taken to various values and different predictions for \( n_s \) and \( r \) are obtained accordingly, which is intensively analyzed in Ref. \([103]\). In the current study, we take \( \alpha = 2/3, \tilde{\lambda} = 0, 959 \times 10^{-3}, \xi = (0.604 \times 10^{16} \text{GeV})^2 \)

and \( q = y = 1 \). Then the best-fit value of observed spectral index \( n_s \), along with the tensor-to-scalar ratio \( r \approx 6 \times 10^{-4} \), is obtained at the last 60 e-folds of the subcritical regime of the hybrid inflation \([103]\). With the parameters, the condition (4.1) gives

\[ |M'_{ij}| \ll 10^{13} \text{GeV}, \tag{4.9} \]

where \( \langle S_+ \rangle = (\xi/2q)(1 + \tilde{\xi})^{1/2} \) and \( \tilde{\xi} \equiv \xi/2q \) has been used, which will be shown in the next subsection. In the following subsections, we will quantitatively examine the condition (4.9) to keep the inflaton dynamics unchanged. To keep the readable analytic expressions, we leave \( \tilde{\lambda}, \xi, q \) and \( y \) as they are.

### 4.2 The scalar potential

The scalar potential is given by \([103]\),

\[
V = V_F + V_D, \tag{4.10}
\]

\[
V_F = \left( -\frac{\Phi}{3} \right)^{-1} 3 \left[ \delta^{ij} W_i \bar{W}_j + \frac{1}{\Delta} |\delta^{ij} W_i \Phi_j - 2 W_j|^2 + \frac{2}{\Phi} |W|^2 \right], \tag{4.11}
\]

\[
V_D = \frac{y^2}{2} \left[ \left( -\frac{\Phi}{3} \right)^{-1} 2q \left( |S_+|^2 - |S'_+|^2 \right) - \xi \right]^2, \tag{4.12}
\]

where \( V_F \) and \( V_D \) are \( F \) and \( D \)-term potentials, respectively. Here \( W_i \equiv \partial W/\partial z^i, \Phi_i \equiv \partial \Phi/\partial z^i, \) and \( \Delta \equiv \Phi - \delta^{ij} \Phi_i \Phi_j, \) and \( z^i \) shows the scalar component of \( Z^I \). In the \( D \)-term potential, \( y \) and \( \xi (> 0) \) are the gauge coupling and the Fayet-Iliopoulos (FI) term related to the local \( U(1) \). Due to the FI-term, \( S'_+ \) acquires a VEV as \( \langle S'_+ \rangle = (\xi/(2q)(1 + \tilde{\xi}))^{1/2} \) at the global minimum.\(^5\) Then, \( s \equiv \sqrt{2}|S'_+| \) is identified with the waterfall field.

During inflation, we expect that the fields except for the inflaton and waterfall fields are stabilized at the origin. Thus the scalar potential during inflation is given as

\[
V(\phi, s) \equiv V|_{\sqrt{2} \text{Re} \tilde{N}_j^c = \phi, \sqrt{2}|S'_+| = s, \text{the others}=0} = \left( -\frac{\Phi(\phi, s)}{3} \right)^{-1} 3 \phi^2 \left[ \frac{\tilde{\lambda}^2}{2} s^2 + \Delta M^2(\phi, s) \right] + \frac{y^2}{8} \left[ \left( -\frac{\Phi(\phi, s)}{3} \right)^{-1} 2q s^2 - 2\tilde{\xi} \right]^2, \tag{4.13}
\]

\(^5\)From Eq. (4.8), it is clear that \( \langle S_+ \rangle \ll 1. \)
where $\Phi(\phi, s) \equiv -3 + (s^2 + \phi^2)/2$. $\Delta M^2(\phi, s)$ is the term that originates in the Majorana mass term, which is given by

$$\Delta M^2(\phi, s) \equiv \sum_{i=1}^{3} |M'_{i3}|^2 - |M'_{33}|^2 \frac{\phi^2}{12} \left( -\frac{\Phi(\phi, s)}{3} \right)^{-1}. \quad (4.14)$$

### 4.3 Critical point

As in the literature [99,100,102,103,105], we focus on the dynamics of the subcritical regime of the hybrid inflation. First of all, the Majorana mass term shifts the critical point value of the hybrid inflation. The critical point value is determined by the mass squared of the waterfall field, which is given by

$$m^2_+ = m^2_{+,0} + \Delta m^2_+,$$

where

$$m^2_{+,0} \equiv qy^2 \xi (\Psi(\phi) - 1),$$

$$\Delta m^2_+ \equiv qy^2 \xi \Psi(\phi) \frac{1}{3\lambda^2} \left( -\frac{\Phi_0}{3} \right)^{-1}$$

$$\times \left[ \Delta M^2(\phi, 0) + |M'_{33}|^2 \frac{\phi^2}{12} \left( -\frac{\Phi_0}{3} \right)^{-1} \right],$$

$$\Psi(\phi) \equiv \frac{\bar{\lambda}^2}{2(2/3)^2 qy^2 \xi} \phi^2, \quad \Phi_0 \equiv \Phi(\phi, 0). \quad (4.17)$$

Here we have given the mass in a canonically normalized basis in accordance with Ref. [103]. Consequently, the critical point value $\phi_c$ is obtained by

$$\phi_c^2 = \phi_{c,0}^2(1 - \delta) + \mathcal{O}(M'_{i3})^4,$$

where $\phi_{c,0}^2 = 2(2/3)^2 qy^2 \xi / \bar{\lambda}^2$ [103] and

$$\delta \equiv \frac{2(2/3)^2 \Delta m^2_+(\phi_{c,0})}{\lambda^2 \phi_{c,0}^2}. \quad (4.20)$$

Imposing $|\delta| \ll 1$, the upper bound on $|M'_{i3}|$ is obtained as

$$\sum_{i=1}^{2} |M'_{i3}|^2 \ll 3\bar{\lambda}^2 - 4qy^2 \xi / 9 \simeq 3 \times 10^{-7} \sim (1 \times 10^{15} \text{ GeV})^2,$$

$$|M'_{33}|^2 \ll \frac{(3\bar{\lambda}^2 - 4qy^2 \xi / 9)^2}{3\lambda^2(3\bar{\lambda}^2 - 8qy^2 \xi / 9)} \simeq 4 \times 10^{-8} \sim (5 \times 10^{14} \text{ GeV})^2. \quad (4.21)$$

Those are weaker bounds compared to Eq. (4.9). Therefore, the critical point value is merely affected by the Majorana mass term.
4.4 Subcritical regime

Below the critical point, the waterfall field grows due to the tachyonic instability \cite{99, 127} and soon relaxes to the local minimum $s_{\text{min}}$ of the classical path.\footnote{We have confirmed that the one-loop potential dominates over the Majorana mass terms near the critical point under the condition (4.9). Thus, the dynamics around the critical point is the same as one studied in Ref. \cite{103}.} $s_{\text{min}}$ is obtained by $\partial V(\phi, s)/\partial s^2 = 0$ as

$$s_{\text{min}}^2 = \frac{3\xi}{q} \left[ \phi \left( 1 + \xi(1 - \Psi(\phi)) \right) \right] \times \left\{ 1 - \Psi(\phi) \left[ 1 + \frac{\xi}{1 + \xi(1 - \Psi(\phi))} \right] \right\},$$

(4.23)

where

$$\Delta_\pm \equiv \left( -\Phi_0^2 / 3 \right)^{-1} \frac{1}{3\lambda^2} \left[ M^2(\phi, 0) \pm |M_{33}'|^2 \phi^2 / 12 \left( -\Phi_0^2 / 3 \right)^{-1} - \xi(1 - \Psi(\phi)) \right].$$

(4.24)

Consequently the potential in the subcritical regime can be expressed by a single field effective potential

$$V_{\text{sub}}(\phi) \equiv V(\phi, s_{\text{min}}(\phi))$$

$$= y^2 \xi^2 \Psi(\phi) \left( 1 - \Psi(\phi) \right) + 1 + \frac{1 + \xi}{3} \Psi(\phi) \left( -\Phi_0^2 / 3 \right)^{-1}$$

$$\times \left\{ \Delta M^2(\phi, 0) - |M_{33}'|^2 \phi^2 / 12 \left( -\Phi_0^2 / 3 \right)^{-1} - \xi(1 - \Psi(\phi)) \right\} + O(M_{i3}^4)$$

$$= y^2 \xi^2 \Psi(\phi) \left( 1 - \Psi(\phi) \right) + 1 + \frac{1}{3 \xi^2} \left( -\Phi_0^2 / 3 \right)^{-1}$$

$$\Delta M^2(\phi, 0) + O(M_{i3}^4, \xi^3).$$

(4.25)

In the last step, we have used $\xi \ll 1$. Therefore, $V_{\text{sub}}$ reduces to the inflaton potential in Ref. \cite{103} when

$$|\Delta M^2(\phi, 0)| \ll 3 \lambda^2 \xi \left( -\Phi_0^2 / 3 \right)^{1 - \Psi(\phi)} \left( 1 - \Psi(\phi) \right),$$

(4.26)

is satisfied. This condition gives rise to upper bounds on $|M_{i3}'|$ as

$$\sum_{i=1}^2 |M_{i3}'|^2 \ll 3 \lambda^2 \xi (1 + \phi_{i,0}^2 / 6) / 2 \sim (2 \times 10^{12} \text{GeV})^2,$$

(4.27)

$$|M_{33}'|^2 \ll 2 \lambda^2 (1 - \phi_{c,0}^2 / 6)^2 / |4 - \phi_{c,0}^2| \sim (9 \times 10^{11} \text{GeV})^2.$$

(4.28)

Those are a bit tighter than the condition (4.9). When the above conditions are satisfied, $s_{\text{min}}$ is also in approximate agreement with the local minimum value of the waterfall field in Ref. \cite{103},

$$s_{\text{min}}^2 = \frac{3\xi}{q} \left[ \phi \left( -\Phi_0^2 / 3 \right) \right] \left[ 1 - \Psi(\phi) \right] + O(\xi \Delta_-).$$

(4.29)
To summarize the inflaton dynamics is not affected if
\[ |M'_{ij}| \ll 10^{12} \text{GeV}, \quad (4.30) \]
is satisfied.\(^7\) In the next subsection, we will confirm the condition numerically.

### 4.5 Dynamics of scalar fields: numerical study

The upper bound (4.30) is obtained under the assumption that the fields except for the inflaton and waterfall fields are stabilized at the origin during inflation. However, we need to investigate the validity of this assumption since the values of \( \tilde{N}'_{1,2} \) and \( \text{Im} \tilde{N}'_3 \) may grow to affect the dynamics of inflation depending on the value of the Majorana masses. We will check the stability of \( \tilde{N}'_{1,2} \) and \( \text{Im} \tilde{N}'_3 \) by solving the equations of motion of \( \tilde{N}'_i \) and \( S' \) numerically and examine the condition (4.30) more quantitatively.

To this end, we define the relevant scalar fields as \( \{ \phi_i, \tau_i, s \} \equiv \varphi^A \) where\(^8\)
\[ \phi_i \equiv \sqrt{2} \text{Re} \tilde{N}'_i, \quad \tau_i \equiv \sqrt{2} \text{Im} \tilde{N}'_i. \quad (4.31) \]
Then, the metric in terms of the field space of \( \varphi^A \) is given by
\[ G_{AB} = \delta_{AB} \left\{ \begin{array}{ll} K^m_{N_i \tilde{N}_i} & \text{for} \quad \varphi^A = \phi, \tau_i \\ K^m_{S_i \bar{S}_i} & \text{for} \quad \varphi^A = s \end{array} \right. \quad (4.32) \]
where \( K_{Z^I Z^J} = \partial^2 K / \partial z^I \partial \bar{z}^J \). The equations of motion of \( \varphi^A \) are
\[ \ddot{\varphi}^A + 3H \dot{\varphi}^A + G^{AB} \frac{\partial V}{\partial \varphi^B} + \Gamma^A_{BC} \dot{\varphi}^B \dot{\varphi}^C = 0. \quad (4.33) \]
Here, dot denotes the time derivative and \( H \) is the Hubble parameter that depends on \( \varphi^A \) and \( \dot{\varphi}^A \). \( G^{AB} \) is the inverse of \( G_{AB} \) and \( \Gamma^A_{BC} \) is the connection defined by
\[ \Gamma^A_{BC} \equiv \frac{1}{2} G^{AD} (G_{DB,C} + G_{DC,B} - G_{BC,D}). \quad (4.34) \]
For the numerical analysis, we take a benchmark point from the allowed region of \( \tau \) and \( \tilde{g} \), which is given by
\[ \text{Re} \tau = 8.88 \times 10^{-4}, \quad \text{Im} \tau = 1.08, \quad g = 4.56, \quad \phi_g = -170^\circ \quad (4.35) \]
With the parameters the corresponding neutrino mixing parameters are
\[ \sin^2 \theta_{12} = 0.305, \quad \sin^2 \theta_{13} = 0.0224, \quad \sin^2 \theta_{23} = 0.572, \quad \delta_{\text{CP}} = -160^\circ, \quad \alpha_{21} = -163^\circ \quad (4.36) \]
\(^7\)We have checked that the stability of the \( \tilde{L}_i H_u \) direction, pointed out by Ref. \[128\], is guaranteed if \( |M'_{ij}| \lesssim 10^{15} \text{GeV} \).
\(^8\)Namely, \( \phi_3 = \phi \) is the inflaton field in the notation of this subsection.
Figure 4: The time evolution of $s$, $\phi_i$, and $\tau_i$ ($i = 1$–$3$) as function of $e$-folds before the end of inflation. The parameters are given in Eq. (4.35) and $\hat{\Lambda} = 10^{10}$ GeV. (Top-left) The waterfall field value $s$ (solid) given by the numerical solution and that of the analytical approximation $s_{\text{min}}$ (dotted) are shown. (Top-right) The inflaton $\phi_3$ (solid) and $\tau_3$ (dashed) are given. (Bottom) $\phi_i$ (solid) and $\tau_i$ (dashed) for $i = 1, 2$ are shown.
Figure 5: The maximum values of $\phi_i$ ($i = 1, 2$) (left) and $\tau_i$ ($i = 1–3$) (right) during inflation as functions of $\Lambda$. Red, blue, and green points correspond to $i = 1$, 2, and 3, respectively.

and $|M'_{ij}|$ is obtained as

$$
|M'_{ij}| = \begin{pmatrix} 5.59 & 3.56 & 9.75 \\ 3.56 & 6.52 & 6.57 \\ 9.75 & 6.57 & 3.53 \end{pmatrix} \times 10^9 \text{GeV} \left( \frac{\hat{\Lambda}}{10^{10} \text{GeV}} \right).
$$

(4.37)

Since we are considering the subcritical regime, the initial values of $\phi$ and $s$ are set to $\phi_{\text{init}}^2 \equiv 0.98\phi_{c0}^2$ and $s_{\text{init}} \equiv s_{\text{min}}(\phi_{\text{init}})$, respectively, at the time $t = 0$, while the other initial field values are set to zero. We have confirmed that similar trajectories of the scalar fields are obtained for slightly different the initial values of the inflaton and waterfall fields.

Fig. 4 shows the time evolution of $\varphi^A$ before the end of inflation as a function of the $e$-folds $N_e$ defined by

$$
N_e(t) = \int_t^{t_{\text{end}}} dt' H,
$$

(4.38)

where $t_{\text{end}}$ is the time at the end of inflation. In the plot, $\hat{\Lambda} = 10^{10} \text{GeV}$ is taken to satisfy the condition (4.30) at a percent level. We confirmed that the trajectory of the waterfall field well agrees with the local minimum $s_{\text{min}}$, given in Eq. (4.29). In addition, we found that the $\tau_i$ ($i = 1–3$) merely move during the inflation. On the other hand, $\phi_1$ and $\phi_2$ grow as large as $\mathcal{O}(<s>)$ as seen in the figure. They are, however, still subdominant components in the scalar potential and they do not have

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#9 The waterfall field value is much smaller than the inflaton one during inflation and the trajectory is almost straight along the inflaton field direction. Therefore, the trajectory of $\phi_1$-$s$ system has no additional impact on the curvature perturbation, which is already studied in Ref. [103].

#10 $\phi_i$ ($i = 1, 2$) are kicked by the term proportional to $\sum_j \text{Re}(M'_{ij}M_{j3})\phi_i\phi_3$. The direction that $\phi_i$ is heading for depends on the sign of $\sum_j \text{Re}(M'_{ij}M_{j3})$. 
any impact on the waterfall-inflaton dynamics. For smaller value of $\hat{\Lambda}$ we found that the field values of $\phi_1$ and $\phi_2$ reduce almost linearly, which is summarized in Fig. 5. Having confirmed the condition (4.30), we derived more quantitative bound,

\[ \hat{\Lambda} \lesssim 10^{10} \text{GeV}, \]  

(4.39)

or equivalently

\[ |M'_{i3}| \lesssim 10^{10} \text{GeV}. \]  

(4.40)

In the bottom panels of Fig. 4, the field growths of $\phi_1$ and $\phi_2$ get accelerated. This is because the Hubble parameter begins to decrease after the end of inflation since the inflaton field oscillates around the minimum to behave as a matter component. After that, the $\phi_1$ and $\phi_2$ start to oscillate with an angular frequency $O(|M_{ij}|)$. We checked this behavior numerically, which is shown in Fig. 6. Here we have assumed that the inflaton field keeps the coherent oscillation. Realistically the inflaton field decays to leptons and Higgsinos or sleptons and Higgses and it reheats the universe. Then the radiation domination follows. Although $\phi_1$ and $\phi_2$ continue the coherent oscillation, we expect that they do not become the dominant component of the universe because their energy density is highly suppressed. In that case, non-thermal leptogenesis by the inflaton field works and it is expected to provide a sufficient number of lepton asymmetry [105, 126, 128–138]. The details depend on the model parameters and they are beyond the scope of our current study. We leave it for our future study.

\section{Conclusion}

We consider a supergravity model that has the modular $A_4$ symmetry. This model accommodates the MSSM augmented by three right-handed neutrino fields that have
the Majorana masses. Additionally, two fields that are charged under a gauged U(1) are introduced. They couple to the right-handed neutrinos via the Yukawa interaction and consequently one of the scalar components plays a role of the waterfall field during inflation and acquires a VEV at the global minimum. With the extension, the pattern of the light neutrino mass, generated by the seesaw mechanism, changes and one of the light neutrinos becomes massless. On top of that, the modular $A_4$ symmetry restricts the mixing pattern. Comparing with the current observations regarding the neutrino mixings, we have found that only the IH case is allowed. The predicted Majorana phase is around $\pm 35^\circ$, the Dirac phase is $[\pm 110^\circ, \pm 90^\circ]$, $[\pm 90^\circ, 110^\circ]$ and $0.42 \lesssim \sin^2 \theta_{23} \lesssim 0.43$, or the Majorana phase is $[-180^\circ, -120^\circ]$, $[120^\circ, 180^\circ]$, the Dirac phase takes values in $[-180^\circ, 180^\circ]$ and $0.42 \lesssim \sin^2 \theta_{23} \lesssim 0.62$. The effective neutrino mass, which determines the decay rate of the neutrinoless double beta decay, is found to be around $14$–$28$ meV and $45$–$47$ meV. Such a relatively large effective mass of $O(10)$ meV will be explored in future experiments.

The supergravity model we consider is based on a hybrid inflation model that has the subcritical inflation regime. Namely, inflation continues below the critical point. In the present model, the three right-handed sneutrinos are the candidates for the inflaton field. Compared to the inflation model studied in Ref. [103], the existence of the Majorana mass terms is a crucial difference, which may affect the inflation dynamics. Imposing the Majorana mass not to affect the inflation dynamics, we have revealed that only one of the right-handed sneutrinos turns out to couple to the waterfall field, then we have derived the upper bound for the Majorana mass scale analytically. We have confirmed the results numerically by solving the equations of motion for scalar fields. It is found the other right-handed sneutrinos grow as large as the VEV of the waterfall field. They are, however, negligible in the total energy density during the inflation if the Majorana mass scale is smaller than $O(10^{10})$ GeV. Though their field values are much suppressed during inflation, the scalar fields continue to oscillate after inflation, which may affect the subsequent thermal history. For instance, they may contribute to the generation of the lepton asymmetry. The details depend on the parameter and we leave it for the future work.

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