Research on dynamics of hybrid pouring robot and attitude stability control of ladle

Long Li$^{1,2,3}$, Chengjun Wang$^{2,3}$, Yongcun Guo$^{2,3}$ and Hongtao Wu$^1$

Abstract
In this paper, the vibration characteristics of the designed hybrid truss-type movable heavy-load pouring robot (pouring robot) and the pose control of the parallel working arm are studied. The dynamic-static method is used to simplify the truss and parallel working arm of the pouring robot into a forced vibration system composed of spring, damping, and mass body, based on this the 9-degree-of-freedom vibration equation of the pouring robot is established, the analysis of the amplitude–frequency characteristics shows that the four input vibrations of the pavement have basically the same influence on the ladle, which will cause the ladle to resonate. Kane's method is used to establish the lower mobility dynamic equation of the parallel working arm, the Jacobian matrix of force-driven and torque-driven coupling are given. Based on the vibration characteristics of the pouring robot and the dynamic model of the parallel working arm, an adaptive sliding mode control method with radial basis function neural network compensator and Newton–Euler iterative estimator are proposed to realize the pose control of the ladle, the Lyapunov theorem proves the stability of the control method, the simulation results show that adaptive sliding mode control has better control performance, faster response speed, higher convergence accuracy, and better robustness than traditional sliding mode control algorithm. The paper provides a reference and research basis for the suppression of ladle vibration when the pouring robot is transferred, which is affected by the road roughness.

Keywords
Parallel mechanism, dynamic model, sliding mode control, posture tracking

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Introduction
Pouring is the process of injecting molten metal from the ladle into the mold cavity. The common pouring methods are bottom injection type, pouring type, pneumatic type, and so on. In this paper, a hybrid truss-type movable heavy-loading pouring robot is designed using the characteristics of strong load capacity of parallel mechanism, which can realize the adaptability and flexibility requirements of casting equipment in the production of multi-variety small batch products.

The road roughness causes the vibration of the pouring robot, and the severe vibration will cause the spilling of the molten metal. The pouring robot is approximated as the vehicle suspension system. Litak et al., Ackay and Türkay, and Shaohua and Jianying used the Melnikov, Mean, and Galerkin method to study the chaotic vibration of the single-degree-of-freedom vehicle suspension model, the dynamic response of the random road surface, and the nonlinear dynamics of the vehicle. Yang et al. and Cai et al. studied the road excitation of a 2-degree-of-freedom (2-DOF) vehicle model and the nonlinear dynamics of the suspension. Zhu and Ishitobi and Gao et al. studied the spectrum and nonlinear dynamics of a 4-DOF half-vehicle suspension model excited by rough road surface. Zhang et al. and Wang et al. established a 7-DOF vehicle vibration model. This research is based on the standard vehicle model, discussing its modeling method, as well as the evaluation index of the road body excitation on the car body and the carrier; for the
A non-standard vehicle, usually containing complex additional components, the vehicle will have higher DOF and complex vibration state, and there are few modeling and simplification methods in this aspect.

The classical methods for dynamics modeling of parallel mechanisms include Newton–Euler equation, Lagrangian equation, Gaussian minimum restraint principle method, and Kane equation. For the control of the trajectory and pose of the parallel mechanism, the proportional derivative/proportional integral derivative (PD/PID) control method is generally adopted, but the PD/PID cannot track the expected trajectory and pose of high dynamic motion of nonlinear dynamic systems. Sliding mode control (SMC) is a robust control scheme with good ability to overcome nonlinear, time-varying parameters, disturbances, and uncertainties. Ouyang et al. proposed a PD-SMC controller for robot trajectory tracking control, and discussed some adjustment rules of the controller through simulation. Soltanpour et al. and Navabi et al. combine SMC with fuzzy systems to improve controller performance with uncertain structural and non-structural parameters. Fei and colleagues proposed a radial basis function neural network (RBFNN) and double closed-loop recurrent neural network based on adaptive sliding mode controller for a class of nonlinear dynamic systems, that is, all states are measurable and the input gain matrix is unchanged. The improved SMC is used in the above literature and combined with other excellent algorithms to control the pose and trajectory of the parallel mechanism, and achieve good control effects. The real-time control depends on the end attitude sensor, but in the vibration suppression, the real-time acquisition of the end pose is very demanding, and an excellent sensor is required.

In this paper, the dynamic–static method is used to simplify the vibration of the ladle into a forced vibration, and then establish the 9-DOF vibration equation of the pouring robot; Kane’s method is used to establish the dynamic equation of the parallel working arm, the form of the Jacobian matrix of the rotational drive and linear drive coupling, and the dynamic equation of the force and torque are given; the ASMC with RBFNN compensator and Newton–Euler estimator is proposed, the method estimates the state variables through the changes of the driving, thereby realizing the real-time control of the pose, and suppresses the influence of road roughness during the transition of the ladle.

Vibration model of parallel pouring robot

Vibration model of the whole machine

Figure 1 is a schematic diagram of the structure of a hybrid truss-type movable pouring robot. The design dimensions of the whole machine are 2150 mm × 1200 mm × 2000 mm, the maximum load is 60 kg, and the design working space of the parallel working arm is 100 mm × 50 mm × 400 mm.

To study the dynamic characteristics of the pouring robot under the excitation of rough road surface, the pouring-vehicle–ground interaction can be effectively and reasonably predicted, and the movement of the parallel working arm can be realized to avoid the splashing of molten metal in the ladle due to severe vibration. This paper assumes that the vibration behavior of the casting robot conforms to linear rules and is simplified to the body, unsprung mass, spring, suspension, and tire composition, and a 9-DOF vibration model as shown in Figure 2 is established. Figure 3 is a simplified model of the truss of the pouring robot (composed of 4–8 in Figure 1). According to the dynamic–static method in the material mechanics, the truss is simplified into the forced vibration of the 2-DOF elastic system.
In Figure 3, the counterweight and ladle are simplified to the mass bodies \(m_{jb}\) and \(m_{pc}\) applied to the ends of the truss. The truss is approximately a homogeneous rod with a unit mass of \(q\). The lengths from the support beam \((s)\) to the ends the truss are \(l_{jb}\) and \(l_{pc}\), respectively, and the gravitational acceleration is \(g\). Then, based on the superposition principle of material deformation, the static displacement of the ladle is

\[
w_{jb} = -\frac{m_{jb}gl_{jb}^3}{3EI} qg_{jb}^2 + \frac{8m_{jb}gl_{jb}^3 + 3m_{qjb}g_{jb}^3}{24EI}
\]

(1)

In equation (1), \(E\) and \(I\) are the elastic modulus and the moment of inertia, respectively. Obviously, the quality and position of the ladle play a major role in the vibration of the ladle. So, rewrite equation (1)

\[
w_{jb} = \frac{m_{jb}}{k_{jb}} q_{jb}^2 + \frac{-24EI}{1 + (3m_{qjb}/8m_{jb})} k_{jb} w_{jb} - F_d = 0
\]

(2)

where \(k_{jb}\) is the spring constant, then the vibration equation of the ladle is

\[
(m_{jb} + m_{qjb})w_{jb} + c_{jb} w_{jb} + k_{jb} w_{jb} - F_d = 0
\]

(3)

In equation (3), \(m_{qjb} = m_{qjb}\), \(c_{jb}\) is damping, which can be obtained by experimental or theoretical formula, and \(F_d\) is external force. The same method can be used to obtain the vibration equation of the counterweight.

In Figure 2, \(z_{jb}\) and \(z_{pc}\) are the vertical displacements of the ladle and the counterweight, respectively; \(z_c\) is the vertical displacement of the centroid of the moving platform of the pouring robot; \(\varphi\) is the pitch angle of the moving platform; \(\theta\) is the roll angle of the moving platform; \(l_1\) is half of the front axle track; \(l_2\) is half of the rear axle track; \(l_3\) is the distance from the center of mass to the front axle track; \(l_4\) is the distance from the center of mass to the rear axle track; \(m_i\) is the mass of the mobile platform located at the origin of the coordinates \(c_i\) and \(J_{xc}\) and \(J_{xt}\) are the pitching moments of inertia of the mobile platform. \(z_{li}\) \((i = 1-4)\) is the vertical displacement of the support point; \(c_i\) and \(k_i\) are the damping and stiffness of the suspension, respectively; \(m_{li}\) is the total mass of the suspension and the tire; \(c_{li}\) and \(k_{li}\) are the damping and stiffness of the tire, respectively; \(q_i\) is the vertical displacement from the ground. \(m_d\) is the mass of the lifting device, the center of mass is located in the middle of the lifting device, the length is \(l_{ds}\). \(l_3\) is the distance from the lifting device to the center of mass of the mobile platform, and the lifting device is fixedly connected with the mobile platform.

Assuming that \(\varphi\) and \(\theta\) are small, the vertical displacement relationship between the four support points of the vehicle body is

\[
z_{l1} = z - l_3\varphi - l_1\theta \\
z_{l2} = z + l_3\varphi - l_2\theta \\
z_{l3} = z - l_3\varphi + l_1\theta \\
z_{l4} = z + l_3\varphi + l_2\theta
\]

(4)

The 9-DOF differential vibration equation of the pouring robot is established by the isolation method. The matrix form can be written as

\[
M\ddot{X} + C\dot{X} + KX = C_iQ + K_cQ
\]

(5)

In equation (5)

\[
X = \begin{bmatrix} z_{jb} & z_{pc} & \varphi & z_{l1} & z_{l2} & z_{l3} & z_{l4} \end{bmatrix}^T
\]

(6)

\[
Q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T
\]

(7)

In equation (8), \(J_{yj}\) and \(J_{xz}\) are the moments of inertia of the lifting device in the \(y_j\) and \(x_z\) directions. \(M\) is the mass matrix, \(C\) is the damping matrix, \(K\) is the stiffness matrix, \(K_c\) and \(C_c\) are input matrix, see Appendix 1 for the coefficient matrix \(M, C, K, K_c\), and \(C_c\). \(X\) is the displacement vector, and \(Q\) is the vertical displacement vector of the ground.

### Numerical analysis of vibration of parallel pouring robot

To analyze the influence of the road roughness on the vibration characteristics of the ladle, we can obtain the state space equation of the pouring robot from equation (5)

\[
\begin{bmatrix} \dot{X} \\ \ddot{X} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K_c & -M^{-1}C_c \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \end{bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}K_c \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}
\]

(8)

In equation (13), \(\begin{bmatrix} X & \dot{X} \end{bmatrix}^T\) is the state quantity of the pouring robot, \(Y\) is the output, and equations (4)–(8) are compiled into MATLAB and brought into the model parameters in Table 1. Figure 4 shows the amplitude–frequency characteristics of \(q_1, q_2, q_3\), and \(q_4\) on the ladle vibration. The amplitude and frequency characteristics of \(q_1\) and \(q_3\) are the same, and the
amplitude and frequency characteristics of \( q_2 \) and \( q_4 \) are the same, indicating that \( q_1 \), \( q_3 \), and \( q_2 \), \( q_4 \) have the same effect on the vibration of the ladle. Further analysis shows the trend of the amplitude-frequency characteristics in Figure 4 is similar, the amplitude is close and the change is stable; when the frequency \( f = 9.21 \), 24.66, 131.56, and 142.85 Hz, the amplitude of the ladle reaches the peak value and the peak frequency is the same, at this time, the ladle resonates with the road surface input. But compared with the vibration frequency of the general ground,\(^{22} f = 131.56 \) and 142.85 Hz are high-frequency resonances, and the risk of resonance is low. In the design of the pouring robot, the natural frequency of the ladle \( (f = 9.21 \) and 24.66 Hz) should be moved to a lower or higher frequency by changing the relevant structural parameters, so as to avoid the ladle resonating when the pouring robot is working.

In Figure 5, taking a type of aperiodic deceleration band input model as an example (equation (9)), it can be seen that the vibration of the ladle is compound vibration, and the acceleration and displacement changes are irregular, the maximum displacement and acceleration are 0.15 m and 10 m/s\(^2\), respectively, and the amplitude fluctuates sharply. When the pouring robot is working on this kind of road surface, the vibration of the ladle will inevitably cause the vibration of the molten metal, in severe cases, it will cause a spill accident. Therefore, the active movement of the parallel working arm is required to offset or reduce the vibration of the ladle.

\[
\begin{align*}
q_1 &= q_3 = 0.04 \sin 3t + 0.02 \sin 6t \\
q_2 &= q_4 = 0.04 \sin(3t + 0.66) + 0.02 \sin(6t + 0.66)
\end{align*}
\]  

\( q_1 \) and \( q_3 \) have the same effect on the vibration of the ladle, \( q_2 \) and \( q_4 \) have the same effect on the vibration of the ladle, \( f = 9.21 \), 24.66, 131.56, and 142.85 Hz, the amplitude reaches the peak value and the peak frequency is the same, at this time, the ladle resonates with the road surface input. But compared with the vibration frequency of the general ground,\(^{22} f = 131.56 \) and 142.85 Hz are high-frequency resonances, and the risk of resonance is low. In the design of the pouring robot, the natural frequency of the ladle \( (f = 9.21 \) and 24.66 Hz) should be moved to a lower or higher frequency by changing the relevant structural parameters, so as to avoid the ladle resonating when the pouring robot is working.

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### Dynamic modeling of parallel working arm

To realize the active motion of the pouring robot to reduce the vibration of the ladle, the dynamic equation of the parallel working arm must be established. In this paper, the Kane method is used to establish the dynamic equation of the parallel working arm.

Figure 6 is a simplified diagram of the parallel working arm simplified to a 4-UPU (U stands for universal joint and P stands for moving pair) parallel mechanism.\(^{19} ABCD \) stands for fixed platform, \( abcd \) stands for moving platform, \( \gamma \) is the inclination of the ladle, and \( C(c) \) is located on the extension of the midpoint of \( AB(ab) \). \( m, n, k, M, N, \) and \( K \) represent the relevant structural dimensions of the fixed platform and the moving platform, \( d_1, d_2, d_3, \) and \( d_4 \) represent the length of the four branches. The UPU can be decomposed into an RRPRR structure. \( S_j(i = 1–4; \ j = 1–5) \) represent the vector for the motion pair. \( S_{11} || S_{22}, \ S_{13} || S_{15}, \ S_{12} || S_{23}, \) and \( S_{32} || S_{42} \).

**The velocity, acceleration, and partial velocity of the centroid of each component of the branch**

Taking branch 1 as an example, the change of \( S_{11} \) from the moving coordinate system to the fixed coordinate system is

\[
\begin{align*}
Ou &= RS_{11} + Oo \\
Ou &= S_{15} + S_{13}  \\
S_{13} &= S_{13}/L_{13}
\end{align*}
\]  

In equation (10), \( s_{13} \) is the unit vector of \( S_{13}, L_{13} = ||S_{13}|| \) is the rod length, \( R \) is the rotation matrix,
and RS_{11} = q_{11}. By deriving equation (10), the velocity \( \dot{S}_{13} \) and acceleration \( \ddot{S}_{13} \) of the branch, the angular velocity \( \omega_{13} \) and the angular acceleration \( \alpha_{13} \) can be obtained

\[
\begin{align*}
\dot{S}_{13} &= w \times q_{11} + \dot{\omega}_0 + L_{13} \dot{S}_{13} + w_{13} \times S_{13} \\
L_{13} &= s_{13} \cdot \dot{S}_{13} \\
w_{13} &= \frac{s_{13} \times \dot{S}_{13}}{L_{13}} = \frac{s_{13} \times (w \times q_{11} + \dot{\omega}_0)}{L_{13}} \\
\ddot{S}_{13} &= a \times q_{11} + w \times (w \times q_{11} + \dot{\omega}_0) + \dot{\omega}_0 + L_{13} \ddot{S}_{13} + 2w_{13} \times L_{13} \dot{S}_{13} + a_{13} \times S_{13} \\
L_{13} &= s_{13} \times \ddot{S}_{13} + L_{13} \dot{w}_{13} \times w_{13} \\
\alpha_{13} &= \frac{s_{13} \times \ddot{S}_{13} + 2L_{13} \dot{w}_{13}}{L_{13}}
\end{align*}
\]

where \( w \) is the angular velocity of the moving platform, and the moving pair of the branch 1 is composed of upper and lower parts. Therefore, it is necessary to consider the coordinate transformation of the centroid to the fixed coordinate system as

\[
\begin{align*}
\mathbf{r}_{\text{up}13} &= T_{13} \mathbf{r}_{\text{up}13} \\
\mathbf{r}_{\text{down}13} &= T_{13} \mathbf{r}_{\text{down}13} + \mathbf{c}_{13} \\
I_{\text{up}13} &= T_{13} I_{\text{up}13} T_{13}^T \\
I_{\text{down}13} &= T_{13} (I_{\text{down}13} + \mathbf{m}_{\text{down}13} L_{13} \text{diag}(1,1,1)) T_{13} \tag{13}
\end{align*}
\]

In equation (13), \( \mathbf{r}_{\text{up}13}, I_{\text{up}13}, \mathbf{r}_{\text{down}13}, \) and \( I_{\text{down}13} \) represent the position vector and moment of inertia of the upper and lower centroids of the branch 1 in the moving coordinate system, respectively. \( \mathbf{r}_{\text{down}13}, \mathbf{r}_{\text{up}13}, I_{\text{up}13}, \) and \( I_{\text{down}13} \) represent the position vector and moment of inertia of the upper and lower centroids of the branch 1 in the fixed coordinate system, respectively, \( \mathbf{c}_{13} = [00 L_{13}]^T \). The transformation matrix \( T_{13} \) of the centroid of the branch 1 to the fixed coordinate system is

\[
T_{13} = \begin{bmatrix} Y \times s_{13} & Y \times s_{13} & Y \times s_{13} \end{bmatrix} \left[ \begin{array}{ccc} Y \times s_{13} & Y \times s_{13} & Y \times s_{13} \\ Y \times s_{13} & Y \times s_{13} & Y \times s_{13} \\ Y \times s_{13} & Y \times s_{13} & Y \times s_{13} \end{array} \right] \tag{14}
\]

According to the simultaneous equations (11)–(13), the velocity and acceleration of the centroid of the upper and lower parts of the chain 1 can be obtained

\[
\begin{align*}
\mathbf{r}_{\text{up}13} &= \frac{s_{13} \times (w \times q_{11} + \dot{\omega}_0)}{L_{13}} + \mathbf{s}_{13} \times (w \times q_{11} + \dot{\omega}_0) \times \mathbf{s}_{13} \\
\mathbf{a}_{\text{up}13} &= \alpha_{13} \times \mathbf{r}_{\text{up}13} + w_{13} \times (w_{13} \times \mathbf{r}_{\text{up}13}) \\
\mathbf{r}_{\text{down}13} &= \frac{s_{13} \times (w \times q_{11} + \dot{\omega}_0)}{L_{13}} + \mathbf{s}_{13} \times (w \times q_{11} + \dot{\omega}_0) \times \mathbf{s}_{13} + w_{13} \times (w_{13} \times \mathbf{r}_{\text{down}13}) + 2L_{13} \mathbf{w}_{13} \times \mathbf{s}_{13} \\
\mathbf{a}_{\text{down}13} &= \alpha_{13} \times \mathbf{r}_{\text{down}13} + w_{13} \times (w_{13} \times \mathbf{r}_{\text{down}13}) + 2L_{13} \mathbf{w}_{13} \times \mathbf{s}_{13} \tag{14}
\end{align*}
\]
The 4-UPU mechanism designed in this paper is a 3-translational and 1-rotation parallel mechanism, and the generalized angular velocity \( \omega = (w_x, w_y, w_z) \) and velocity \( \dot{\omega} = (\dot{w}_x, \dot{w}_y, \dot{w}_z) \) are selected.

\[
\begin{align*}
\dot{\varphi}_{313} &= (\varphi_{313}, \varphi_{313}, \varphi_{313})^T \\
S_{13} &= (s_{13}, s_{13}, s_{13})^T \\
\dot{\varphi}_{313} &= \dot{\varphi} = (\dot{w}_x, \dot{w}_y, \dot{w}_z)
\end{align*}
\]

(15)

Equation (15) is brought into 14 for partial expansion and is organized into equations for \( \dot{w}_x, \dot{w}_y, \dot{w}_z \). Thus, the derivation rule can be used to obtain the partial velocity of the upper and lower parts of the branch 1 relative to the generalized velocity \( \dot{w}, \dot{v} \)

\[
\begin{align*}
\dot{w}_{i3} &= \frac{s_{i3} \dot{v}_{i3} + i_{i3} \cdot n_{i3}}{L_{i3}} \\
\dot{w}_{i3} &= \frac{\dot{s}_{i3} + s_{i3} \cdot j_{i3}}{L_{i3}} \\
\dot{w}_{i3} &= \frac{s_{i3} \dot{v}_{i3} + i_{i3} \cdot k_{i3}}{L_{i3}}
\end{align*}
\]

(16)

**The velocity, acceleration, and partial velocity of the centroid of each component of the moving platform**

The moving coordinate of the moving platform is at the center of mass, and the velocity and acceleration of the moving platform are: \( \dot{v}_o = \dot{\varphi}_o, a_o = \alpha_o = \dot{\varphi}_o \). Then, the generalized partial velocity \( \dot{v}_{\alpha, \alpha} = i, \dot{v}_{\alpha, \beta} = j \), \( \dot{v}_{\alpha, \gamma} = k, \dot{v}_{\alpha, \gamma} = 0, \dot{v}_{\alpha, \gamma} = 0, \dot{v}_{\alpha, \gamma} = 0 \), and \( \dot{w}_{\alpha, \gamma} = j \).

**Dynamic equations of parallel working arm**

In Figure 6, the rotating pair \( S_{13} \) of the branch 4 is given as \( (w_4) \)

\[
\begin{align*}
w_4 &= w_{43} \cdot j = \frac{s_{43} \cdot (w \cdot q_{41} + \dot{\varphi}_o) j}{L_{43}} \\
&= w[j(s_{43} \cdot q_{41} - s_{31} \cdot q_{41} \cdot j)] - \dot{\varphi}_o(s_{43} \cdot j)
\end{align*}
\]

Then, the Jacobian of the parallel working arm is

\[
J = \begin{bmatrix}
\ddot{s}_{313}^T & \dot{s}_{13} \cdot s_{13}^T \\
\ddot{s}_{232}^T & \dot{s}_{23} \cdot s_{23}^T \\
\ddot{s}_{333}^T & \dot{s}_{33} \cdot s_{33}^T \\
\ddot{\varphi}_{43}^T & \dot{\varphi}_{43} \cdot \varphi_{43}^T
\end{bmatrix}
\]

The forces on the parallel working arm include the driving force of the branch \( f = (f_1, f_2, f_3, T_d) \), and \( f_1, f_2, f_3 \) and \( T_d \) are in the same direction as \( S_{13}, S_{23}, \) and \( S_{33}, \) and \( T_d \).

In equations (17) and (18), \( F_1 \) is the generalized active force of the branch, \( F_1 \) is the generalized active force of the external force and the external moment, \( F_1 \) is the generalized active force of gravity; \( F_1^* \) is the inertial force of the moving platform, and \( F_1^* \) is the generalized inertial force of the upper and lower parts of the branch. \( L_1, L_1, \) and \( L_1 \) represent the moment of inertia of the moving platform and the moment of inertia of the upper and lower parts of the branch, respectively. \( m_1, m_1, \) and \( m_1 \) are the masses of the branches and the upper and lower parts of the moving platform, \( w_o = w, \) \( \alpha_o \) is the angle acceleration of the moving platform.

In summary, the dynamic equation of parallel parallel working arm is as follows

\[
f = \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
T_d
\end{bmatrix} = -J^T (F_2 + F_3 + F_1^* + F_2^*) \quad (19)
\]

Further, the driving force and driving moment of each branch chain can be obtained by equation (19) as follows

\[
f = \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
T_d
\end{bmatrix} = -J^T (F_2 + F_3 + F_1^* + F_2^*) \quad (20)
\]

**Numerical analysis of dynamics of parallel working arm**

Equations (10)–(20) are compiled into MATLAB and the structural parameters are shown in Table 2. The arc track in the XZ plane is taken as an example. The center point \( \omega_{center} = (0.09, -0.825) \) m, the arc radius \( d_0 = 0.1 \) m, rotate at a constant speed \( \omega_{center} = 0.3 \times (0, 0, 1.0) \) rad/s, and move for one cycle without load. The driving force \( f \) of each branch changes with time, as shown in Figure 7.

Figure 7 shows that due to the interaction between different components, the driving force/torque is increased
or decreased. The force and moment at the start and end points after one cycle of motion are equal, reflecting the dynamic characteristics of the mechanism and proving the correctness of the Kane algorithm.

Control scheme design

When discussing ladle control, there is a problem of rigid-liquid coupling, but combined with the previous research work\textsuperscript{23,24,25} and the actual working environment, the liquid is not sensitive to the vertical displacement, and the road input does not not change sharply, the ladle also has a sloshing margin. Therefore, to facilitate the control study, the pouring liquid is considered as the quality of the ladle.

For equation (19), it can be organized into a dynamic standard form

\[
\mathbf{J}^T \mathbf{f} = \mathbf{M}_p \mathbf{q}_{\dot{p}} + \mathbf{C}_p \mathbf{q}_{\dot{p}} + \mathbf{G}_p
\]  

(21)

In equation (21), \( \mathbf{M}_p \) represents the mass matrix of the parallel working arm, \( \mathbf{C}_p \) represents the Coriolis force and centripetal force of the parallel working arm, \( \mathbf{G}_p \) represents gravity, and \( \mathbf{q}_{\dot{p}} = [x, y, z, \gamma]^T \). Obviously, the actual dynamic equation can be written as

\[
\mathbf{J}^T (\mathbf{f} + \mathbf{\delta}_f) = (\mathbf{M}_p + \delta_{M_1}) \mathbf{\dot{q}}_{\dot{p}} + (\mathbf{C}_p + \delta_{C_1}) \mathbf{\dot{q}}_{\dot{p}} + (\mathbf{G}_p + \delta_{G_1}) + \mathbf{J}^T (\mathbf{f} + \mathbf{\delta}_f) + \mathbf{\delta}_fd
\]  

(22)

In equation (22), \( \delta_{M_1}, \delta_{C_1}, \delta_{G_1} \) represent uncertain dynamic models, \( \delta_{f_1}, \delta_{f_2}, \delta_{f_3} \) represent uncertain friction, and \( \delta_{f_4} \) represent uncertain drive fault, friction between various components and environmental disturbances, respectively. Rewrite equation (22)

\[
\mathbf{J}^T \mathbf{f} = \mathbf{M}_p \mathbf{q}_{\dot{p}} + \mathbf{C}_p \mathbf{q}_{\dot{p}} + \mathbf{G}_p + \zeta
\]

\[
\zeta = \delta_{M_1} \mathbf{\dot{q}}_{\dot{p}} + \delta_{C_1} \mathbf{\dot{q}}_{\dot{p}} + \delta_{G_1} + \mathbf{J}^T \mathbf{f} + \delta_{f_4} - \mathbf{J}^T \mathbf{f}
\]  

(23)

Standard SMC

The author found that the PD control has a certain effect on the fixed-point control of the parallel working arm, so the standard sliding surface is selected as

\[
s_{s_{n_m}} = \mathbf{e}_{da} + K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt
\]  

(24)

where \( \mathbf{e}_{da} = \mathbf{Q}_{pa} - \mathbf{Q}_{pa} \), \( \mathbf{Q}_{pa} \) and \( \mathbf{Q}_{pa} \) define the desired and actual positions, respectively. The values of \( K_D \) and \( K_P \) correspond to the natural frequencies and damping of each branch.

**Proof 1.** The Lyapunov function of the standard sliding surface can be designed as

\[
V_{s_{n_m}} = \frac{1}{2} s_{s_{n_m}}^T s_{s_{n_m}}
\]  

(25)

\[
\dot{V}_{s_{n_m}} = s_{s_{n_m}}^T \dot{s}_{s_{n_m}} = s_{s_{n_m}}^T (\mathbf{e}_{da}
\]

\[
= s_{s_{n_m}}^T (\mathbf{e}_{da} + K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt)
\]

\[
= s_{s_{n_m}}^T \left( \mathbf{Q}_{pa} - \mathbf{Q}_{pa} + K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt \right)
\]

\[
= s_{s_{n_m}}^T \left( \mathbf{Q}_{pa} - \mathbf{M}_p^{-1} \left( \mathbf{J}^T \mathbf{f} + \mathbf{\delta}_f - \mathbf{\delta}_fd \right) - \mathbf{Q}_{pa} + \zeta \right)
\]

\[
+ K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt
\]  

(26)

\[
f_a = f_a^T \left[ \mathbf{M}_p \mathbf{\dot{q}}_{\dot{p}} + \mathbf{C}_p \mathbf{\dot{q}}_{\dot{p}} + \mathbf{G}_p + \ldots \right]
\]

\[
\mathbf{M}_p (K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt) + \lambda \text{sign}(s_{s_{n_m}})
\]  

(27)

Bringing equation (27) into equation (26), we get

\[
\dot{V}_{s_{n_m}} = s_{s_{n_m}}^T s_{s_{n_m}} = \mathbf{f}_{sa}^T \left[ \mathbf{M}_p \mathbf{\dot{q}}_{\dot{p}} + \mathbf{C}_p \mathbf{\dot{q}}_{\dot{p}} + \mathbf{G}_p + \ldots \right]
\]

\[
\mathbf{M}_p (K_D \mathbf{e}_{da} + K_P \int \mathbf{e}_{da} dt) + \lambda \text{sign}(s_{s_{n_m}})
\]  

(28)

\( (*)*_{s_{n_m}} \) and \( (*)*_{da} \) represent the parameters of the desired pose and the actual pose. In equation (28), since \( \mathbf{M}_p^{-1} \) is the inverse of the mass matrix of the parallel working arm, it must be a principal diagonal matrix.

**Assumption 1.** The total uncertainty parameter \( \zeta \) has a boundary that satisfies \( ||\zeta|| < \lambda \) and \( \lambda > 0 \) is a constant.

**Proof by equation (32) and Assumption 1**

\[
\dot{V}_{s_{n_m}} = s_{s_{n_m}}^T s_{s_{n_m}} < 0
\]  

(29)

Therefore, the control law given by equations (24)–(29) can achieve the control target \( \mathbf{e}_{da} \rightarrow 0 \).

But, there are still several problems with the standard SMC. First, the real \( \mathbf{Q}_{pa}, \mathbf{Q}_{na} \), and \( \mathbf{Q}_{pa} \) are difficult to measure at the same time. Second, the upper boundary is considered too conservative, and then the chattering of the sliding surface is not considered.

**ASMC**

Since the displacement and rotation of the driving in the parallel working arm are independent variables, the change of the driving \( \mathbf{s} \) can be measured, and the pose of the moving platform can be estimated by Newton iteration method.

\[
\begin{bmatrix}
\end{bmatrix}
\]
\[ Q_{pe(a+1)} = Q_{pe(a)} + J_u^{-1}(s_a + 1 - s_u) \]  
(30)

where \( Q_{pe} \) is the estimated pose, \( u \) represents the number of iterations, and equations (24) and (27) are rewritten to obtain the \( s_{asmc} \) of the adaptive sliding surface and the \( f_e \) of estimated control force

\[ s_{asmc} = \hat{e}_e + K_D e_e + K_p \int e_e \, dt \]  
(31)

\[ f_e = J_e^T \left[ M_p \hat{Q}_{pe} + C_p \hat{Q}_{pe} + G_pe + \cdots M_p (K_D \hat{e}_e + K_p e_e) + \text{diag}(\text{sign}(s_{asmc}))\xi \right] \]  
(32)

\( r_e \) indicates the estimated pose parameter, \( \hat{e}_e = Q_{pe} - Q_{pc} \). From equations (27) and (32), we obtain the error of the actual output force \( f_a \) and the estimated control force \( f_e = f_a - \Gamma \). The boundary of the pose disturbed by the outside is uncertain, so this paper calculates the uncertainty boundary based on the dynamic characteristics of the parallel working arm and the estimated force

\[ \xi_{u+1} = \left[ \eta_1 f_{ed, \text{max}} + \eta_2 (f_{ed, \text{max}} - f_{ed, \text{min}}) \right] \]  
(33)

\( f_{ed, \text{max}} \) is the global maximum output force to maintain the ladle pose. Similarly, \( f_{ed, \text{min}} \) is the minimum global output force to maintain the ladle pose; \( \eta_1 \) and \( \eta_2 \) represent the weight factors, and both are diagonal matrix.

Because the RBFNN has a simple structure and good global convergence, it is often used in approximation problems, and the response speed is fast. Therefore, this paper intends to use RBFNN as a compensator for uncertain parameters. The RBFNN architecture uses the simplest input layer \( Q' = [Q_r, Q_p, Q_h] \), an output layer \( \xi = [f_{p1}, f_{p2}, f_{p3}, T_p] \), a hidden layer. The activation function \( H = [h_1, h_2, \ldots, h_m]^T \) selects the Gaussian function, and \( C_{ij} \) and \( \sigma_j \) are the center and width of the \( j \)th node of the hidden layer

\[ h_l = \exp \left( \frac{\|Q_l - C_l\|}{2\sigma_l} \right) \quad j = 1, 2, \ldots, m \]

Then, the output of the RBFNN is written as

\[ W^T H = \sum_{\rho=1}^m W_{\rho j} h_j \quad \rho = 1, 2, 3, 4 \]  
(34)

where \( W_{\rho j} \) is the connection weight, and equation (22) is rewritten as

\[ J^T (f + W^T H) = M_p \hat{Q}_p + C_p \hat{Q}_p + G_p + \xi \]  
(35)

**Proof 2.** The Lyapunov function of the adaptive sliding surface can be designed as

\[ V_{asmc} = \frac{1}{2} s_{asmc}^2 s_{asmc} + \frac{1}{2} \hat{W}^T \hat{W} \]  
(36)

\[ \dot{V}_{asmc} = s_{asmc}^T \dot{s}_{asmc} + \frac{1}{2} \hat{W}^T \hat{W} \]

where \( \hat{W} = W_e - W_e f_e - f_e - \Gamma \) and equation (35) are brought into equation (37)

\[ \dot{V}_{asmc} = s_{asmc}^T \dot{s}_{asmc} + \frac{1}{2} \hat{W}^T \hat{W} \]

Assumption 2. Under the condition of friction, uncertain dynamics model, environmental disturbance and driving fault, the total uncertainty parameter \( \Gamma_{\text{total}} \) can be obtained by designing appropriate weight coefficients \( \eta_1 \) and \( \eta_1 \)

\[ |\Gamma_{\text{total}}| < \xi \]

Assumption 3. Adaptive law can choose

\[ W_e = \frac{1}{\xi} s_{asmc}^T M_p^{-1} H_a \quad \xi = (e_1, e_2, e_3, e_4)^T \]

From Assumptions 1 and 2, equation (37) can be obtained

\[ \dot{V}_{asmc} = s_{asmc}^T \dot{s}_{asmc} + \frac{1}{2} \hat{W}^T \hat{W} < 0 \]

Proof 2 shows that the ASMC of the design can achieve \( e_d \rightarrow 0 \). To alleviate the chattering of the sliding surface, the continuous function \( \Phi(s) \) is used instead of \( \text{sign}(s) \)

\[ \Phi(s_i) = \frac{s_i}{s_i + \phi_i} \]  
(39)

where \( \phi_i \) is a small positive constant.

**Numerical analysis of control schemes**

To verify the proposed ASMC algorithm, the algorithm is compiled into MATLAB, and the flow chart is shown in Figure 8. Because of the numerical analysis,
the driving vector is obtained by the actual pose parameter through the Jacobian change; the pose of the ladle is mainly disturbed by the ground, so plays a leading role in the $\xi$, and other factors are ignored in the numerical analysis. Import the vibration data of the ladle in section “Numerical analysis of vibration of parallel pouring robot” into MATLAB, and since the amplitude–frequency characteristic of Figure 5 is composite vibration, the molten metal is not sensitive to high-frequency, low-amplitude vibration, so a low-pass filter is introduced to filter out high-frequency vibration. The RBFNN uses the difference between the ideal dynamic equation and the actual dynamic equation as the training sample. The parallel working arms are listed in Table 2, and the adaptive sliding mode parameters are listed in Table 3.

In Figures 9 and 10 (intercepting the first 8 s), it can be seen that the ASMC control law based on the RBFNN and the pose estimator is able to follow the desired pose, and relative to the general SMC control rate has a higher (about 2 times) convergence speed, a higher convergence accuracy, and a smaller error fluctuation in the desired pose.

**MATLAB/Simulink—ADAMS co-simulation**

To further verify the reliability of the ASMC control law, the MATLAB/Simulink—ADAMS co-simulation was established. The three-dimensional (3D) model was built in Creo according to the parameters of Table 2 and imported into ADAMS, as shown in Figure 11.

---

**Table 2.** Structural parameters of parallel working arm.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $M$ (m)   | 0.374 | $I_a$ (Nm) | diag(0.2, 0.2, 0.04) |
| $m$ (m)   | 0.330 | $I_a$ (Nm) | diag(0.2, 0.2, 0.04) |
| $N$ (m)   | 0.680 | $r_{upa}$ (m) | (0.0, 0.0, 0.1)$^T$ |
| $n$ (m)   | 0.365 | $r_{down}$ (m) | (0.0, 0.0, 0.1)$^T$ |
| $K$ (m)   | 0.300 | $I_a$ (Nm) | diag(20, 20, 17) |
| $k$ (m)   | 0.150 | $m_a$ (kg) | 5 |
| $m_0$ (kg) | 100 | $m_a$ (kg) | 5 |

**Table 3.** Parameters of adaptive sliding mode.

| Control parameter | Value |
|-------------------|-------|
| $K_D$             | $[4, 4, 4, 4]^T$ |
| $K_p$             | $(150, 150, 150, 150)^* \eta$ |
| $\eta_1$          | $[0.5, 0.5, 0.5, 0.5]^T$ |
| $\eta_2$          | $[0.2, 0.2, 0.2, 0.2]^T$ |
| $\xi$             | $[15, 15, 15, 15]^T$ |
| $C_{i,j}$         | $\begin{bmatrix}
-0.0481 & -0.0472 & -0.0465 & -0.0463 \\
-0.0459 & -0.0454 & -0.0451 & -0.0449 \\
-0.0448 & -0.0447 & -0.0447 & -0.0447 \\
0.0447 & 0.0447 & 0.0447 & 0.0448 \\
0.0449 & 0.0451 & 0.0454 & 0.0459 \\
0.0463 & 0.0465 & 0.0472 & 0.0481 \\
0.0068 & & & 
\end{bmatrix}$ |

**Figure 8.** Flowchart of the ASMC of the parallel working arm.

**Figure 9.** Position and angle results in $x$, $y$, $z$, and $\gamma$ directions.
Add the necessary fixed pairs, moving pairs and rotating pairs, and the quality of each part is set according to Table 2. The driving force/driving torque in MATLAB is used as the ADAMS input, and the motion parameters of the ladle in the ADAMS are input as MATLAB. Apply the acceleration data in section “Numerical analysis of control schemes” to the ladle, and the simulation time is still set to 8 s. The result is shown in Figure 12.

Comparing Figures 10 and 12, the numerical analysis and simulation results are basically consistent. This indicates that the ASMC control law is correct and effective, the ASMC control law is better than the SMC control law, and the proposed pose control method has excellent performances such as fast response speed, small overshoot, and high tracking precision. It also verifies the correctness of the dynamic modeling of the parallel working arm established in this paper.

**Conclusion**

In this paper, the designed pouring robot is taken as the research object, and the pose control method is taken as the research goal. The vibration dynamics equation and the parallel working dynamics equation of the pouring robot are studied. Based on RBFNN and Newton iteration, a method suitable for pose
control of pouring robot is proposed. The main conclusions are as follows:

1. Using the static-dynamic method to simplify the cantilever structure of the pouring robot into a 2-DOF forced vibration system, and based on the isolation method, the 9-DOF vibration dynamics equation of the whole machine is established. The vibration analysis in the frequency domain and the time domain shows: there are multiple resonance peaks in the whole vibration frequency range, and the low-frequency resonance interval includes the road frequency. So, the design should change the structure to avoid resonance as much as possible.

2. The Jacobian with rotation and translation coupling is established. The 4-DOF dynamic equation of the parallel working arm driven by torque and force is established by Kane method. The numerical analysis verifies the correctness of the model.

3. Using the sliding mode controller of RBFNN and Newton iterative estimator to track the pose of the ladle, Lyapunov theory proves the stability of the control. The simulation results show that the proposed method has good control performance, fast response, high convergence precision, and excellent robustness.

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ORCID iD
Long Li https://orcid.org/0000-0002-4079-0241

References
1. Cervellero PB and Mckelvie JJ. Induction melt shop technologies: part 2. *Found Manage Technol* 1998; 5: 46–50.
2. Wang CJ, Li L, Guo YC, et al. Hybrid trussed mobile heavy duty casting robot. China Patent 201710-682226.8, 2017.
3. Litak G, Borowiec M, Friswell MI, et al. Chaotic response of a quarter car model forced by a road profile with a stochastic component. *Chaos Soliton Fract* 2009; 39(5): 2448–2456.
4. Akcay H and Türkay S. Influence of tire damping on actively controlled quarter-car suspensions. *J Vib Acoust* 2011; 133(5): 054501.
5. Shaohua L and Jianying R. Dynamical analysis on a two dimensional nonlinear vehicle-pavement system. In: *World automation congress 2012*, Puerto Vallarta, Mexico, 24–28 June 2012, pp. 1–4. New York: IEEE.
6. Yang Z, Liang S, Zhu Q, et al. Chaotic vibration and comfort analysis of nonlinear full-vehicle model excited by consecutive speed control humps. *Math Prob Eng* 2014; 2014: 370634.
7. Cai H, Du P, Liu H, et al. Dynamic response characteristic analysis of vehicle frame based on virtual simulation technology. In: *2011 international conference on electric information and control engineering*, Wuhan, China, 15–17 April 2011, pp. 2133–2136. New York: IEEE.
8. Zhu Q and Ishitobi M. Chaotic vibration of a nonlinear full-vehicle model. *Int J Solids Struct* 2006; 43(3–4): 747–759.
9. Gao W, Zhang N and Du HP. A half-car model for dynamic analysis of vehicles with random parameters. In: *Proceedings of the 5th Australasian congress on applied mechanics*, Brisbane, QLD, Australia, 10–12 December 2007, pp. 595–600. Canberra, ACT, Australia: Engineers Australia.
10. Zhang HJ, Guo ZP, Si JP, et al. Parametric identification of a vehicle suspension dynamic model. *J Vib Shock* 2013; 32(23): 145–150.
11. Wang J, Li TJ and Meng LQ. Study on dynamic characteristics of vehicle based on the whole vehicle model with 7 freedom. *J Anhui Sci Technol Univ* 2013; 27(1): 72–76.
12. Schiehlen W. Multibody system dynamics: roots and perspectives. *Multibody Syst Dyn* 1997; 1(2): 149–188.
13. Kumar V, Nakra BC and Mittal AP. A review on classical and fuzzy PID controllers. *Int J Intell Con Syst* 2011; 16(3): 170–181.
14. Ouyang PR, Acob J and Pano V. PD with sliding mode control for trajectory tracking of robotic system. *Robotics Chim-Int Manuf* 2014; 30(2): 189–200.
15. Soltanpour MR, Khooban MH and Soltani M. Robust fuzzy sliding mode control for tracking the robot manipulator in joint space and in presence of uncertainties. *Robotica* 2014; 32(3): 433–446.
16. Navabi H, Sadeghnejad S, Ramezani S, et al. Position control of the single spherical wheel mobile robot by using the fuzzy sliding mode controller. *Adv Fuzz Syst* 2017; 2017: 2651976.
17. Fei J and Ding H. Adaptive sliding mode control of dynamic system using RBF neural network. *Nonlin Dyn* 2012; 70(2): 1563–1573.
18. Fei J and Lu C. Adaptive sliding mode control of dynamic systems using double loop recurrent neural network structure. *IEEE T Neur Net Lear* 2018; 29(4): 1275–1286.
19. Li L, Wang C and Wu H. Research on kinematics and pouring law of a mobile heavy load pouring robot. *Math Probl Eng* 2018; 2018: 8790575.
20. Liu HW. *Material mechanics*. Beijing, China: Higher Education Press, 2017.
21. Liu ZQ. Research on the key technology of heavy forklift truck automatic shift. Hefei, China: Hefei University of Technology, 2013.
22. Chen JP, Chen WW, Zhu H, et al. Modeling and simulation on stochastic road surface irregularity based on Matlab/Simulink. *Trans Chin Soc Agr Mach* 2010; 41(3): 11–15.
23. Li L, Wang CJ and Wu HT. Trajectory planning of parallel mechanism for pouring robot. *Curr Sci* 2019; 116(11): 1829–1839.
24. Enriquez-Zarate J, Valencia-Palomo G, Lopez-Estrada FR, et al. Efficient predictive vibration control of a building-like structure. *Asian J Contr*. Epub ahead of print 30 January 2019. DOI: 10.1002/asjc.2015.
25. Enriquez-Zarate J and Silva-Navarro G. Ground-borne vibration control in a building-like structure using multiposition feedback combined with sliding mode control. *Noise Control Eng J* 2016; 64(5): 668–676.
Appendix I

Mass matrix, damping matrix, and stiffness matrix of the pouring robot

\[
M = \begin{bmatrix}
    m_{pb} + m_{qib} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & m_{pc} + m_{qib} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & m_c + m_{qib} & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & J_{yc} + J_{ydl} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & J_{xc} + J_{xdl} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & m_{q1} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & m_{q2} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{q3} \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{q4}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
    -c_{pb} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{pc} & -c_{pc} & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{pc} & c_{pc} & c_{pc} + c_{pc} & c_{pc} - c_{pc} & -c_{pc} - c_{pc} & c_{pc} + c_{pc} & c_{pc} - c_{pc} & -c_{pc} - c_{pc} \\
    c_{pc} & c_{pc} & c_{pc} + c_{pc} & c_{pc} - c_{pc} & -c_{pc} - c_{pc} & c_{pc} + c_{pc} & c_{pc} - c_{pc} & -c_{pc} - c_{pc} \\
    0 & 0 & -c_{pc} & 0 & 0 & c_{pc} & c_{pc} & -c_{pc} \\
    0 & 0 & -c_{pc} & 0 & 0 & c_{pc} & c_{pc} & -c_{pc} \\
    0 & 0 & -c_{pc} & 0 & 0 & c_{pc} & c_{pc} & -c_{pc} \\
    0 & 0 & -c_{pc} & 0 & 0 & c_{pc} & c_{pc} & -c_{pc}
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
    -k_{pb} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    k_{pc} & -k_{pc} & 0 & 0 & 0 & 0 & 0 & 0 \\
    k_{pc} & k_{pc} & k_{pc} + k_{pc} & k_{pc} - k_{pc} & -k_{pc} - k_{pc} & k_{pc} + k_{pc} & k_{pc} - k_{pc} & -k_{pc} - k_{pc} \\
    k_{pc} & k_{pc} & k_{pc} + k_{pc} & k_{pc} - k_{pc} & -k_{pc} - k_{pc} & k_{pc} + k_{pc} & k_{pc} - k_{pc} & -k_{pc} - k_{pc} \\
    0 & 0 & -k_{pc} & 0 & 0 & k_{pc} & k_{pc} & -k_{pc} \\
    0 & 0 & -k_{pc} & 0 & 0 & k_{pc} & k_{pc} & -k_{pc} \\
    0 & 0 & -k_{pc} & 0 & 0 & k_{pc} & k_{pc} & -k_{pc} \\
    0 & 0 & -k_{pc} & 0 & 0 & k_{pc} & k_{pc} & -k_{pc}
\end{bmatrix}
\]

\[
C_t = \begin{bmatrix}
    -c_{t1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -c_{t1} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -c_{t1} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -c_{t1} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & -c_{t1} & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & -c_{t1} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & -c_{t1} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & -c_{t1} & 0 & 0
\end{bmatrix}
\]