Analysis of Bicycle Headway Distribution, Saturation Flow and Capacity at a Signalized Intersection using Empirical Trajectory Data

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Abstract
Predicting the bicycle flow capacity at signalized intersections of various characteristics is crucial for urban infrastructure design and traffic management. However, it is also a difficult task because of the large heterogeneity in cycling behavior and several limitations of traditional capacity estimation methods. This paper proposes several methodological improvements, illustrates them using high-resolution trajectory data collected at a busy signalized intersection in the Netherlands, and investigates the influence of key variables of capacity estimation. More specifically, it shows that the (virtual) sublane width has a significant effect on the shape of the headway distribution at the stop line. Furthermore, a new method is proposed to calculate the saturation headway (a key variable determining capacity), which excludes the cyclists initially located close to the stop line using a distance-based rule instead of a fixed number (as is usually done in practice). It is also shown that the saturation headway is quite sensitive to the sublane width. Moreover, a new, empirically based method is proposed to identify the number of sublanes that can be accommodated in a given cycle path, which is another key influencing variable. This method yields considerably lower estimates of the number of sublanes than traditional methods, which rely solely on the (available) cycle path width. Finally, the authors show that methodological choices such as the sublane width and the method used to estimate the number of sublanes have a considerable effect on capacity estimates. Therefore, this paper highlights the need to define a sound methodology to estimate bicycle flow capacity at signalized intersections and proposes some steps to move toward that direction.

The ability to predict the bicycle flow capacity of cycle paths at signalized intersections is crucial for urban infrastructure design and traffic management. Capacity (maximum hourly rate at which bicycles can pass the stop line of cycle paths) depends on various key variables, such as the geometric characteristics of cycle paths, the interference with other traffic modes (e.g., car traffic), the traffic control schemes, and the cyclist headway distributions. Estimation of the latter is a difficult task because of the large heterogeneity in cycling behavior. Previous studies on headway distribution models mainly focus on motorized vehicles, and can be split into two categories. The first states that all the headways can be modeled from a single distribution. Examples are the normal distribution (1), the exponential distribution (2), the gamma distribution (3), and the lognormal distribution (4). The second category consists of composite headway models. In this case, the total headway distribution is made up of two different headway distributions, resulting from whether the vehicle/bicycle is following the vehicle/bicycle in front or moving freely. Examples of composite headway models are a semi-Poisson model (5), a generalized queueing model (6), and a distribution-free approach (7). Applications of headway distribution models on cyclist data are rare. A distribution-free (or rather, non-parametric) model estimation approach following the theory of composite headways has been performed on a cyclist intersection in the Netherlands (8).

Another key variable of capacity estimation is the saturation flow, which is often calculated as the inverse of the average headway observed between cyclists and...

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Yuan et al

their leaders once the queue is moving in a stable manner, multiplied by the number of cyclist sublanes that the cycle path can accommodate. There are multiple examples of empirically derived saturation flows, and various values of saturation flows have been reported. A study in the Netherlands reported 3,000–3,500 cyclists per hour for a 0.78 m-wide cyclist lane (9). An empirical analysis in California and Colorado suggested a maximum flow rate of 4,500 cyclists per hour on a 2.43 m-wide cycle path, and it concluded that cyclist saturation headway started at the sixth cyclist in a queue (10). A case study in Beijing, China found 1,836–2,088 cyclists per hour for a 1.25 m-wide path (11). A 2 m-wide cycle-track in Santiago de Chile was reported to have a saturation flow of 4,657 cyclists per hour, whereas for Tavistock Square in London the saturation flow was equal to 4,320 cyclists per hour for a path with the width of 1 m (12). Another point of interest related to saturation flow calculation is the cycle path width. Usually, cyclists can form multiple queues as the “sublane” of a bicycle path is not fixed; this is different from the lane usage of motorized vehicles. In the literature, many different sublane widths have been suggested, for example, 0.78 m in the Netherlands (9), 1.00 m in Germany (13) and even 1.60 m in Norway (14). However, there is a lack of a unified paradigm to compute saturation flow rate and thus the bicycle flow capacity. Besides, there are several limitations in traditional calculation methods. For instance, the calculation of the number of sublanes in Botma and Papendrecht was solely based on the available cycle path width, and ignored leader–follower relations in queue discharge processes (9); the calculation of saturation headway in Raksumon and Khan suffered from the problem of headways close to zero, because leader–follower pairs were not clearly identified (10).

Generally, the capacity of a cycle path at a signalized intersection (C) is calculated as follows (9, 10):

\[ C = q_s \cdot \frac{T_e}{T} \]  
(1)

where:

\[ q_s = \varphi \cdot \frac{1}{h_s} \]  
(2)

and:

\[ T_e = T_G - T_L + T_{YG} \]  
(3)

In Equations 1–3: \( q_s \) denotes the saturation flow; \( h_s \) denotes the saturation headway between leader–follower pairs; \( T \) denotes the total cycle time; \( T_e \) is the effective green time; \( T_G \) is the total green time; \( T_L \) is the start-up lost time; \( T_{YG} \) is the period of the yellow phase in which cyclists still pass the stop line; and \( \varphi \) is the number of virtual sublanes in a given cycle path. This paper proposes a series of methodological improvements on capacity estimation, illustrates them using high-resolution trajectory data collected at a busy intersection in the Netherlands, and investigates the influence of key variables. More specifically, it shows that the (virtual) sublane width has a significant effect on the shape of the headway distribution at the stop line. Furthermore, a new method is proposed to calculate the saturation headway \( h_s \), which excludes the cyclists initially located close to the stop line using a distance-based rule instead of a fixed number (as is usually done in practice), and the authors show that the saturation headway quite sensitive to the sublane width. Moreover, a new, empirically based method is proposed to identify the number of sublanes \( \varphi \) available to cyclists in a given cycle path, which is another key variable influencing capacity. The authors show that this method yields considerably lower estimates of the number of available sublanes than traditional methods, which rely solely on the (available) cycle path width. Finally, it is shown that methodological choices such as the values of the sublane width, as well as the method used to estimate the number of available sublanes, have a considerable impact on capacity estimates.

The rest of this paper is structured as follows. The next section first describes the data set used in this study, and then illustrates the calculations of several key variables of capacity estimation. The results of the analyses are then presented. The final section contains the conclusions of this study and some suggestions for future research.

Data and Methods

This section describes the empirical trajectory data set analyzed in this study, and then it describes the method used to measure cyclist headways at the stop line as well as the methods proposed to estimate the saturation headway, the saturation flow, the start-up lost time and thus the capacity of cycle paths at signalized intersections. For clarification and easy reference purposes, all the variables introduced in this work and their definitions are listed in Table 1.

Data Set Characteristics

The data set analyzed in this study consists of the trajectories of 691 cyclists moving along a cycle path that leads to a signalized intersection in Amsterdam (the Netherlands). The cycle path is 2 m wide, unidirectional, and segregated from car traffic. Traffic demand for this cycle path is relatively high in peak hours, and consists of people riding both bicycles and scooters. In this data set the percentage of scooters is very low; hereafter we
refer to these as cyclists in both cases with an assumption that there is no effect on the analysis results. Access to the intersection is regulated by a traffic light.

The trajectories were derived from top-view video images using the methodology described in Goni-Ros et al. (15). The video images were obtained using two cameras mounted on a 10m pole, which was placed next to the cycle path (more details about this type of installation can be found in Duives [16]). The two camera views can be seen in Figure 1 in (15). The two camera views combined covered 20m of cycle path upstream of the traffic light. The frame rate varied between 5 and 10fps during the video recordings.

Traffic along this cycle path stretch was recorded between 12:45 and 19:00 h on June 6, 2016. By looking at the video footage, the queue discharge periods that

Table 1. Variables and Their Definitions (in Alphabetical Order of the Notation Names)

| Notations | Definitions |
|-----------|-------------|
| C         | capacity of a cycle path at a signalized intersection |
| d         | distance to the stop line, \( d \in D \) |
| \( d_s \) | distance threshold leading to the most accurate estimate of the saturation headway |
| D         | a set containing various consecutive values of \( d \) |
| \( h_c(d) \) | average headway of cyclists closer to the stop line than \( d \) |
| \( h_f(d) \) | average headway of cyclists farther from the stop line than \( d \) |
| \( h_i^P(d) \) | piecewise linear function fitting to the \((d, h_i(d))\) points |
| \( h_i \) | headway of cyclist \( i \) |
| \( h_s \) | saturation headway between leader–follower pairs |
| \( i \)  | cyclist index, \( i = 1, 2, \ldots, N \). It gives the rank of the cyclists regarding the time when they pass cross-section \( x_{ref} \) |
| j         | index of the leader of cyclist \( i \) (\( j < i \)) |
| k         | index of queue discharge period |
| \( M_c(d) \) | a set containing the indices \( i \) of all cyclists whose initial longitudinal position \( x_i \) is higher than \( x_{ref} - d \) for a given value of distance \( d \) |
| \( M_f(d) \) | a set containing the indices \( i \) of all cyclists whose initial longitudinal position \( x_i \) is lower than \( x_{ref} - d \) for a given value of distance \( d \) |
| N         | total number of cyclists in a queue |
| \( N_{c}^P(d) \) | number of cyclists standing closer to the stop lane than \( d \) in a queue discharge period \( k \) |
| \( N_{f}^P(d) \) | number of cyclists farther from the stop lane than \( d \) in a queue discharge period \( k \) |
| \( q_k \)  | linear function fitting to the \((d, N_{c}^P(d))\) data points of all the selected discharge periods saturation flow |
| \( R_k^P \) | number of cyclists who are in each position \( \lambda \) of each queue discharge period \( k \) |
| \( t_c \)  | headway increment because of the start-up reaction and acceleration |
| \( t_i \)  | time instant when cyclist \( i \) passes cross-section \( x_{ref} \) |
| \( t_G \)  | time instant when the green phase starts |
| T         | total cycle time |
| \( T_e \) | effective green time |
| \( T_G \) | total green time |
| \( T_L \) | start-up lost time |
| \( T_{YG} \) | period of the yellow phase where cyclists still pass the stop line |
| w         | width of a virtual sublane for each cyclist |
| W         | physical width of a cycle path |
| \( W_a \) | available cycle path width |
| \( W_d \) | actually used cycle path width |
| \( x_i \) | longitudinal position of cyclist \( i \) |
| \( x_{ref} \) | position of a reference cross-section along the roadway used to calculate headways |
| \( y_i(t) \) | lateral displacement of cyclist \( i \) at time instant \( t \) |
| \( \lambda \) | index of cyclist position in a leader–follower chain, \( \lambda = 1, \ldots, \Lambda \) |
| \( \Lambda_k \) | total number of chain positions in each queue discharge period \( k \) |
| \( \varphi \) | number of virtual sublanes in a given cycle path |
| \( \varphi_E \) | empirical number of sublanes |
| \( \varphi_T \) | theoretical number of sublanes |
| \( K_N \) | kernel distribution function, which is a non-parametric function |
| \( N(\mu_N, \sigma_N) \) | normal distribution function |
| \( \mu_N \) and \( \sigma_N \) | mean and standard deviation of the headways |
| \( X(\mu_X, \sigma_X) \) | lognormal distribution function |
| \( \mu_X \) and \( \sigma_X \) | mean and standard deviation of the natural logarithm of the headways |
met the following criteria were selected (see also Goñi-Ros et al. [15]): (a) the queue is formed by at least seven cyclists; (b) no more than two cyclists within the queue are riding scooters; (c) all cyclists move toward the stop line and pass it without getting out of the path; (d) no pedestrian crosses the cycle path during the queue discharge process; (e) the discharge process is not affected by downstream traffic conditions; and
(f) all cyclists pass the stop line before the end of the green phase.

The total number of selected queue discharge periods was 57, with an average queue size of 12.12 cyclists, 2% of which were scooters (see also Goñi-Ros et al. [15]). For every selected period, the trajectories of all cyclists forming part of the queue were derived using a procedure that comprises six steps: (1) video clip decomposition; (2) manual cyclist tracking; (3) height transformation; (4) orthorectification and scaling; (5) time coding; and (6) trajectory merging. Please refer to Goñi-Ros et al. [15] for a full description of this procedure.

**Definition of Headway of a Cyclist**

In road traffic, the (gross) time headway—or simply *headway*—of a vehicle *n* is defined as the difference between the time when a certain part (e.g., rear bumper) of the preceding vehicle in the same lane (i.e., vehicle *n*−1) passes a reference cross-section along the roadway (*x*<sub>ref</sub>) and the time when the same part of vehicle *n* passes that cross-section (17). Thus defined, the value of the headway is a direct consequence of the longitudinal acceleration behavior of the driver, which is mostly influenced by the movement of the preceding vehicle in the same lane, and can be used to calculate the capacity of a roadway (18). In bicycle traffic, however, the definition of headway is less straightforward, as cyclists can generally not be allocated to well-defined lanes. This makes it difficult to identify the preceding cyclist that constrains the longitudinal acceleration of a given cyclist.

To overcome this issue, headways can be defined based on the concept of *virtual sublane*, as proposed, for example, by Hoogendoorn and Daamen (8) and Botma and Papendrecht (9). For a given queue discharge period, let us assign an index *i* to every cyclist in the queue, where the index gives the rank of the cyclists regarding the time they pass cross-section *x*<sub>ref</sub> (*i*=1, 2, ..., *N*, where *N* is the total number of cyclists in the queue). Furthermore, let *t*_<i> denote the time instant when cyclist *i* passes cross-section *x*<sub>ref</sub>, and let *y*<sub>i</sub>(*t*_<i>) denote the lateral displacement of cyclist *i* at time instant *t*_<i>. Then, the following rule can be used to determine leader *j* (see also Figure 1a). The leader of cyclist *i* is the cyclist with the largest index *j*<*i* for which:

\[
|y_j(t_j) - y_i(t_i)| \leq \frac{w}{2}
\]

where *w* > 0 denotes the width of a virtual sublane for each cyclist.

Parameter *w* corresponds to the minimum width of the space right in front of the cyclist that needs to be empty so that he/she can move forward in a safe manner. The value of *w* is around 75–80 cm in Hoogendoorn and Daamen (8) and Botma and Papendrecht (9), 100 cm in Brilon et al. (13), and 160 cm in Allen et al. (14). This difference between studies is partially as a result of the different bicycle points used to define lateral positions *y*_<i> (e.g., center or end of handlebar) in each study. Another reason is the lack of knowledge on the size of the lateral influencing area of cyclists (swerving and swaying), which leads to different assumptions on the value of *w*.

Here, the leader of cyclist *i* is defined as the cyclist that passed cross-section *x*<sub>ref</sub> earlier than cyclist *i* closer in time, within the virtual sublane used by cyclist *i* (which has a width equal to *w*). Once cyclist *i* has been assigned a leader *j*, his/her headway *h*<sub>i</sub> can be calculated as follows (see also Figure 1a):

\[
h_i = t_i - t_j
\]

(5)

Note that using the method described above, it is not possible to assign a leader to some cyclists, particularly those who are initially located close to the stop line (because no cyclist passes the stop line before them within their virtual lane). For these cyclists, it is assumed that *t*_<i> in Equation 5 is the time instant when the green phase starts (*t*_<G>), as shown in Figure 1a for the calculation of *h*<sub>i</sub>.

**Headway Distribution Modeling**

Using the method described in above, the headways of cyclists at the intersection stop line (i.e., *x*<sub>ref</sub> was defined as the cross-section of the stop line) in the 57 selected queue discharge periods were calculated. As will be discussed later, the headways at the stop line are relevant for the estimation of saturation headway, start-up lost time, and thus cycle path capacity.

A histogram of the headways of all cyclists who have a leader (all following headways) was built. Three different functions were fitted to the headway data to determine the shape of the distribution: (a) normal distribution function, \(N(\mu_N, \sigma_N)\); (b) lognormal distribution function, \(\chi(\mu_X, \sigma_X)\); and (c) kernel distribution function, \(K_N\). The first and second functions are well-known parametric functions, where parameters \(\mu_N\) and \(\sigma_N\) correspond to the mean and standard deviation of the headways, and parameters \(\mu_X\) and \(\sigma_X\) correspond to the mean and standard deviation of the natural logarithm of the headways. Instead, \(K_N\) is a non-parametric function, which has been fit using the MATLAB function fitdist(x,’kernel’) with a normal smoothing function and default parameter values.

Note that a critical influencing factor of the headway calculation method (see “Definition of Headway of a Cyclist”) is the virtual sublane width (*w*). To investigate the sensitivity of the headway distribution to the virtual...
sublane width, the authors calculated the headways, built the histograms and fit the distribution functions for various values of \( w \).

**Calculation of Saturation Headway**

Similarly to road traffic (19), the saturation headway for cyclists can be defined as the average headway between cyclists and their leaders once the queue is moving in a stable manner. Like in road traffic, the first group of cyclists in a queue generally have longer headways than cyclists located more upstream (10). The main reason is that they generally lose time reacting to the traffic light (15) and they pass the stop line when still accelerating to their desired speed, whereas the other cyclists pass the stop line at more or less constant speed. For this reason, in practice, the headways of the first group of cyclists are excluded when calculating the saturation headway (10).

As mentioned above, when calculating the saturation headway, the basic assumption is that once the queue moves in a stable manner, the average headway stays constant, whereas the average headway of the first group of cyclists is longer. Therefore, the main challenge is to determine which headways need to be excluded from the calculation. For example, Raksuntorn and Khan analyzed the discharge process on a 2.43 m-wide cycle path and estimated that the headways of the first five cyclists in the queue need to be excluded (10); note, however, that they did not calculate the headways based on the concept of virtual sublane (see “Definition of Headway of a Cyclist”), so their headways are not necessarily the difference in passing times between follower–leader pairs. Here, the authors propose to exclude not a fixed number of cyclists (like in Raksuntorn and Khan [10]), but those cyclists who are initially closer than a certain distance to the stop line (\( d_s \), distance threshold). The reason is that the authors deem the initial distance to the stop line to influence the speed of cyclists at the stop line more than their ranking regarding their initial proximity to the stop line. In other words, beyond a certain initial distance \( d_s \), cyclists are assumed to have enough space to accelerate to their desired speed before crossing the stop line, and only then they are included in the calculation of the saturation headway.

To determine the distance threshold \( d_s \), leading to the most accurate estimate of the saturation headway, a model-based approach was used (see also Figure 1b). Let \( M_c(d) \) and \( M_f(d) \) denote two disjoint sets containing the indices \( i \) of all cyclists (in all selected discharge periods) whose initial longitudinal position \( x_i \) is higher and lower than \( x_{\text{ref}} - d \), respectively, for a given value of distance \( d \). Here, the headways of cyclists who did not have a leader were excluded. Firstly, a set \( D \) containing various consecutive values of \( d \) was defined, and the average headways contained in sets \( M_c(d) \) and \( M_f(d) \) calculated for all values of \( d \) contained in set \( D \). Thus, the average headway of cyclists closer to the stop line than \( d \) and the average headway of cyclists farther from the stop line than \( d \) (which can be denoted by \( h_c(d) \) and \( h_f(d) \), respectively) were obtained for all the specified values of \( d \). Secondly, the authors visualized the results in a plot where the axes were \( d \) and the average headway in set \( M_f(d) \) (see Figure 1b). A visual inspection of this plot shows that the average headway of cyclists initially located farther from the stop line than \( d \) decreases more or less linearly with \( d \) and becomes quite stable after a certain value of \( d \). Thirdly, the value of \( d \) that leads to the most accurate estimate of the saturation headway (i.e., \( d_s \)) was identified by fitting the following piecewise linear function to the \((d, h(d))\) points (note that the data points for \( d > 12 \) m were not used to fit the piecewise linear function, because for \( d > 12 \) m the number of elements (cyclists) in set \( M_f(d) \) is too low and the average headway \( h_s(d) \) varies too much for different values of \( d \)). (see also Figure 1b):

\[
\tilde{h}_s(d) = \begin{cases} 
    a_1 + b_1 \cdot d & \text{if } d < d_s \\
    A_1 & \text{if } d \geq d_s 
\end{cases}  
\]

which has three parameters: \( a_1 \), \( b_1 \) and \( d_s \), and \( A_1 = a_1 + b_1 \cdot d_s \).

Once the distance threshold \( d_s \) has been derived, the saturation headway \( h_s \) can be defined as the average headway of all cyclists initially located farther from the stop line than \( d_s \) (see also Figure 1c):

\[
h_s = \tilde{h}_s(d_s) \tag{7}
\]

Note that the distance threshold \( d_s \) may be different for different values of \( w \). The authors calculated the saturation headway using the headway data derived using different virtuals sublane widths (\( w \)) to investigate the sensitivity of the saturation headway to the value of \( w \). In all cases, the piecewise linear model (Equation 6) fits the \((d, h_s(d))\) data points reasonably well (\( R^2 \) higher than 0.96).

**Calculation of Start-Up Last Time**

As discussed in “Calculation of Saturation Headway”, the first group of cyclists in the queue (more specifically, those located closer to the stop line than \( d_s \)) generally experience a start-up reaction and acceleration time, which causes them to have longer headways at the stop line on average than the following cyclists. For the latter, the influence of start-up reaction and acceleration on their headways at the stop line is minimal, so their headway is shorter (\( h_s \), on average).
The average headway of the first group of cyclists in the queue (both with and without a leader) can be decomposed into two elements: the saturation headway ($h_s$) and a headway increment ($t_c$) resulting from the start-up reaction and acceleration, which can thus be defined as follows (see also Figure 1c):

$$t_c = \bar{h}_c(d_c) - h_s \quad (8)$$

Similar to the work in Rakstuntorn and Khan (10), the start-up lost time ($T_L$) can then be defined as the headway increment ($t_c$) multiplied by the expected number of cyclists standing closer to the stop line than $d_c$ before the traffic light turns green ($N_E^c(d_c)$):

$$T_L = N_E^c(d_c) \cdot t_c \quad (9)$$

Variable $N_E^c(d_c)$ in Equation 9 is determined using a linear model estimated by fitting the following linear function to the $(d, N_E^c(d))$ data points of all the selected discharge periods $k$ (see Figure 1d):

$$N_E^c(d) = a_2 + b_2 \cdot d \quad (10)$$

where: $a_2$ and $b_2$ are constants; $d$ denotes an element (distance value) of set $D$ (see “Calculation of Saturation Headway”); and $k$ is an index identifying the queue discharge period.

As shown in Figure 1d, variable $N_E^c(d_c)$ is obtained by substituting $d$ in Equation 10 by the value of $d_c$, which is derived as explained in “Calculation of Saturation Headway”. The start-up lost time assuming different virtual sublane widths ($w$) was calculated to investigate the sensitivity of $T_L$ to the value of $w$.

**Calculation of the Number of Sublanes**

The number of virtual sublanes ($\phi$) can be calculated either theoretically (which is the traditional approach, used for example in Botma and Papendrecht (9)) or based on empirical observations as proposed in this work. Theoretically, $\phi$ could be calculated as follows (9):

$$\phi_T = \frac{W_a}{w} \quad (11)$$

where $W_a$ is the available cycle path width, which is the path width that is actually used by cyclists if we look at their positions on the ground ($W_a$) plus $w/2$ on each side (so plus $w$ in total):

$$W_a = W_a + w \quad (12)$$

From this definition of available cycle path width, it follows that in some sites $W_a$ may not be equal to the physical width of the cycle path ($W$). For example, in this study’s site, the cycle path is segregated from both the road and the sidewalk, so cyclists tend to use the full width of the cycle path ($W_a \approx W$); therefore, $W_a > W$. Instead, in the sites analyzed in Rakstuntorn and Khan (10), the cycle paths are not segregated from the road, so cyclists do not use the full width of the cycle path ($W_a < W$), which implies that the difference $W_a - W$ is probably lower and maybe even negative.

This theoretical definition of $\phi$ may overestimate the number of virtual sublanes that can be accommodated in a given cycle path. An alternative method to calculate $\phi$ is to determine the actual number of virtual sublanes observed in empirical data ($\phi_E$). For this purpose, the authors propose the following innovative approach. For a given discharge period $k$, the leader–follower chains (as shown in Figure 1e) are built. Then, the number of cyclists $R^k_\lambda$ who are in each position $\lambda$ of a chain are counted, where $\lambda = 1, \ldots, \Lambda^k$. For example, $\lambda = 1$ means the first position in a chain, and in Figure 1e there are two cyclists who are in the first position of a chain, so $R^1_1 = 2$. The total number of chain positions ($\Lambda^k$) differs among queue discharge periods.

The number of virtual sublanes of a given discharge period can then be defined as follows:

$$\phi_E^k = \frac{\Lambda^k}{\Lambda^k} \sum_{\lambda=1}^{\Lambda^k} R^k_\lambda, \quad (13)$$

and the number of virtual sublanes ($\bar{\phi}_E$) is the average value of $\phi_E^k$ in all queue discharge periods.

The saturation flow ($q_s$ in Equation 2) and the capacity of the cycle path ($C$ in Equation 1) were calculated assuming different sublane width ($w$) and using different estimates of the number of sublanes $\phi$ (theoretical and empirical methods). The traffic control scheme parameters were set to $T_G = 20$ s, and $T_{YG} = 4$ s and $T \in \{60, 120\}$ s.

**Results**

This section presents the results of the previously described analyses, which have been performed using the empirical trajectory data set described in the same section.

**Headway Distribution Functions and Influence of Virtual Sublane Width**

The histograms shown in Figure 2, a–d provide an overview of the empirical headway distributions at the stop line with different virtual sublane widths ($w$). These figures show that the change in virtual sublane width has a significant influence on the shape of the headway distribution. In general, headways decrease as the width of...
virtual sublane increases: the average headway decreases from 2.16 to 1.28s if $w$ increases from 0.7 to 2.0m (see Table 2). This makes sense, because with greater values of $w$ cyclists can be assigned to leaders that are laterally more distant and pass the stop line closer in time (see “Definition of Headway of a Cyclist”). However, the headway varies considerably among cyclists: the range between first (Q1) and third quantiles (Q3) is 0.80–1.36s wide depending on the sublane width (see Table 2). As expected, the kernel distribution function fits the headway histogram best, followed by the lognormal distribution which also fits the data reasonably well (see Figure 2, a–e). Besides, it is observed that the average headway of cyclists who have a leader is generally smaller than the one derived from all cyclists (with and without a leader) (see Table 2).

**Figure 2.** Headway distributions and negative log likelihood with three distribution functions with respect to different sublane widths. Note only the headways of cyclists who have a leader (following headways) have been included in this analysis. (a) Headway distribution with $w = 0.80$ m; (b) headway distribution with $w = 1.00$ m; (c) headway distribution with $w = 1.20$ m; (d) headway distribution with $w = 1.40$ m; (e) negative log likelihood of three distribution functions with different sublane widths.
Table 2. Values of Different Variables with Respect to the Sublane Width $w$

| Sublane Width $w$ (m) | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|-----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Headway of cyclists that have a leader (s) | Mean | 2.16 | 2.05 | 1.93 | 1.84 | 1.79 | 1.70 | 1.66 | 1.61 | 1.54 | 1.49 | 1.44 | 1.37 | 1.32 | 1.28 |
| Median | 1.82 | 1.70 | 1.61 | 1.54 | 1.50 | 1.43 | 1.41 | 1.37 | 1.33 | 1.30 | 1.26 | 1.19 | 1.18 | 1.16 |
| 1st Quantile (Q1) | 1.29 | 1.21 | 1.16 | 1.12 | 1.11 | 1.08 | 1.07 | 1.02 | 0.98 | 0.94 | 0.91 | 0.85 | 0.82 | 0.79 |
| 3rd Quantile (Q3) | 2.65 | 2.49 | 2.35 | 2.20 | 2.17 | 2.03 | 2.00 | 1.96 | 1.83 | 1.79 | 1.74 | 1.67 | 1.59 |
| Headway of all cyclists (s) | Mean | 2.30 | 2.19 | 2.07 | 1.98 | 1.91 | 1.83 | 1.78 | 1.73 | 1.65 | 1.59 | 1.55 | 1.47 | 1.43 | 1.38 |
| Median | 2.00 | 1.85 | 1.76 | 1.68 | 1.65 | 1.58 | 1.54 | 1.50 | 1.43 | 1.40 | 1.33 | 1.29 | 1.26 | 1.22 |
| Q1 | 1.38 | 1.30 | 1.24 | 1.20 | 1.17 | 1.12 | 1.11 | 1.10 | 1.08 | 1.02 | 0.98 | 0.94 | 0.88 | 0.86 | 0.84 |
| Q3 | 2.85 | 2.73 | 2.56 | 2.45 | 2.37 | 2.23 | 2.19 | 2.14 | 2.01 | 1.98 | 1.92 | 1.83 | 1.80 | 1.73 |
| Normal distribution $N$ | Parameter $\mu_N$ | 2.16 | 2.05 | 1.93 | 1.84 | 1.79 | 1.70 | 1.66 | 1.61 | 1.54 | 1.49 | 1.44 | 1.37 | 1.32 | 1.28 |
| Parameter $\sigma_N$ | 1.26 | 1.20 | 1.13 | 1.07 | 1.03 | 0.96 | 0.95 | 0.91 | 0.91 | 0.91 | 0.89 | 0.87 | 0.83 | 0.82 |
| Lognormal distribution $\lambda$ | Parameter $\mu_\lambda$ | 0.63 | 0.57 | 0.52 | 0.47 | 0.45 | 0.40 | 0.37 | 0.34 | 0.28 | 0.24 | 0.20 | 0.13 | 0.09 | 0.05 |
| Parameter $\sigma_\lambda$ | 0.52 | 0.52 | 0.51 | 0.50 | 0.50 | 0.51 | 0.52 | 0.55 | 0.59 | 0.60 | 0.63 | 0.65 | 0.67 |
| Distance threshold $d_e$ (m) | 8.25 | 7.75 | 8.25 | 8.25 | 8.00 | 7.75 | 7.75 | 7.00 | 6.75 | 7.00 | 6.50 | 6.75 | 7.00 | 6.75 |
| Saturation headway $h_s$ (s) | Mean | 2.12 | 1.66 | 1.52 | 1.45 | 1.42 | 1.40 | 1.36 | 1.34 | 1.29 | 1.26 | 1.21 | 1.16 | 1.12 | 1.10 |
| Median | 1.57 | 1.43 | 1.32 | 1.23 | 1.25 | 1.22 | 1.21 | 1.20 | 1.18 | 1.15 | 1.11 | 1.08 | 1.03 | 1.03 |
| Q1 | 1.09 | 1.08 | 0.98 | 0.94 | 0.95 | 0.94 | 0.93 | 0.91 | 0.89 | 0.86 | 0.81 | 0.77 | 0.75 | 0.75 |
| Q3 | 2.20 | 2.15 | 1.97 | 1.77 | 1.72 | 1.66 | 1.61 | 1.60 | 1.54 | 1.52 | 1.46 | 1.43 | 1.38 | 1.36 |
| Start-up lost time $T_s$ (s) | Mean | 3.15 | 3.18 | 3.86 | 4.04 | 4.05 | 3.69 | 3.77 | 3.66 | 3.40 | 3.17 | 3.22 | 3.07 | 2.98 |
| Median | 3.26 | 3.92 | 4.12 | 4.46 | 4.18 | 4.01 | 3.93 | 3.89 | 3.84 | 3.29 | 3.28 | 3.14 | 3.09 | 2.89 |
| Q1 | 3.10 | 2.65 | 3.26 | 3.41 | 3.03 | 2.73 | 2.66 | 2.63 | 2.36 | 1.97 | 1.95 | 1.44 | 1.61 | 1.36 |
| Q3 | 3.97 | 3.76 | 4.76 | 5.49 | 5.71 | 5.60 | 5.63 | 5.15 | 4.79 | 4.41 | 4.47 | 4.35 | 4.38 | 4.12 |
| Theoretical number of sublanes ($\phi_T$) | Mean | 3.86 | 3.50 | 3.22 | 3.00 | 2.82 | 2.67 | 2.54 | 2.43 | 2.33 | 2.25 | 2.18 | 2.11 | 2.05 | 2.00 |
| Median | 2.18 | 2.04 | 1.93 | 1.86 | 1.82 | 1.72 | 1.68 | 1.63 | 1.59 | 1.52 | 1.51 | 1.49 | 1.45 | 1.46 |
| Q1 | 2.17 | 2.00 | 1.83 | 1.82 | 1.80 | 1.67 | 1.63 | 1.63 | 1.50 | 1.40 | 1.38 | 1.33 | 1.30 |
| Q3 | 1.83 | 1.75 | 1.69 | 1.63 | 1.63 | 1.40 | 1.40 | 1.31 | 1.23 | 1.13 | 1.13 | 1.11 | 1.10 |
| Saturation flow $q_s$ (cyc./h) | Mean | 8060 | 7610 | 7645 | 7456 | 7133 | 6866 | 6741 | 6532 | 6495 | 6449 | 6476 | 6579 | 6574 |
| Median | 4561 | 4435 | 4573 | 4626 | 4617 | 4424 | 4452 | 4452 | 4376 | 4437 | 4497 | 4463 | 4563 | 4800 |

Distance Threshold, Saturation Headway and Start-Up Lost Time

The distance threshold $d_e$ used to exclude the first group of cyclists of the queue when calculating the saturation headway (see “Calculation of Saturation Headway”) decreases slightly with the increasing sublane width (see Figure 3a and Table 2). For sublane widths between 0.7 and 2.0m, $d_e$ is estimated to be 6.5–8.25m, which implies the exclusion of the first 6.1–7.6 cyclists, according to the linear model shown in Figure 1d. This is a slightly higher number than in Raksuntorn and Khan (10), in which the authors estimated that the first five cyclists needed to be excluded, although the available width of the cycle path analyzed in their study (10) is slightly lower than in this study’s site.

The saturation headway decreases with the increasing sublane width (see Figure 3b and Table 2), as expected. More specifically, the saturation headway decreases from 1.72 to 1.10s if $w$ increases from 0.7 to 2.0m.
Furthermore, $h_s$ is a highly stochastic variable (the range between Q1 and Q3 of $h_s$ is 0.61–1.11s depending on the value of $w$ (see Table 2)). For values of $w$ between 1.00 and 1.40m (which can be considered as reasonable values of $w$), the saturation headway equals 1.34–1.45s.

The start-up lost time, instead, seems to increase for $w = [0.7, 1.1]$m and decrease for $w = [1.1, 2.0]$m (see Figure 3c). For values of $w$ between 1.00 and 1.40m, $T_L$ is within the range of 3.66–4.04s. This variable is highly stochastic, as the range between Q1 and Q3 of $T_L$ is 0.87–2.87s depending on the value of $w$ (see Table 2). Thus, the value of $T_L$ is again slightly higher than the $T_L$ value reported in Raksuntorn and Khan (10), which was 2.5s, as more cyclists (1.1–2.6 more) are included in the first cyclist group of the queue in the present study.

**Saturation Flow and Capacity Estimation**

Clearly, the proposed empirical method to calculate the average number of virtual sublanes $C_{22}$ (see “Calculation of the Number of Sublanes”) yields considerably lower
estimates than the theoretical method, especially for lower values of \( w \) (see Figure 3d). The estimates obtained using the empirical method are between 1.46 and 2.18 virtual sublanes (depending on the value of \( w \)), 27–44% less than the theoretical method. This may be explained by the fact that cyclists do not always sufficiently utilize/occupy the space of cycle paths when they form leader–follower pairs during queue-formation processes. Note, however, that \( \varphi_E \) is a stochastic variable, as the range between Q1 and Q3 of \( \varphi_E \) is 0.37–0.67 depending on the value of \( w \) (see Table 2).

The fact that the observed number of sublanes is much lower than the theoretically assumed number of sublanes has enormous implications for the calculation of the saturation flow (and thus the capacity). As shown in Figure 3e and Table 2, the saturation flow calculated using the mean empirical estimates of \( \varphi (\varphi_E) \) and saturation headway (\( h_s \)) is around 4350–4800 cyc./h depending on the value of \( w \) (values similar to those observed by Rakusuntorn and Khan (10) and Seriani et al. (12) in cycle paths of similar widths). 27–43% lower than the saturation flow calculated using the theoretical estimates of \( \varphi (\varphi_T) \). Also, the saturation flow changes less with \( w \) if the empirical estimate of \( \varphi \) is used (see Figure 3e). This has crucial implications for the estimation of capacity (as shown in Figure 3f, and Table 2).

Conclusions

The ability to predict the bicycle flow capacity at signalized intersections of various characteristics is crucial for urban infrastructure design and traffic management. However, it is also a difficult task because of the large heterogeneity in cycling behavior as well as several limitations of traditional capacity estimation methods (e.g., the ignorance of leader–follower relations). This study presented an improved methodology to estimate the saturation flow and the capacity of cycle paths at signalized intersections. More specifically, this methodology includes a method to measure headways between leader–follower pairs using the concept of virtual sublane, an improved method to calculate the saturation headway and the start-up lost time (using a distance-based rule), as well as a new empirically based method to estimate the number of virtual sublanes that the cycle path can accommodate. Estimates of these variables were derived from cyclist trajectory data of a busy intersection in Amsterdam (the Netherlands). One of the main findings is that saturation headway, start-up lost time, and number of virtual sublanes are highly stochastic variables. This needs to be taken into account when interpreting the saturation flow and capacity estimates calculated using the means of these variables. For a reasonable range of \( w \) (which is considered to be 1.00–1.40 m), the authors estimated (for the site in Amsterdam) that the saturation headway is 1.34–1.45 s depending on the value of \( w \), the start-up lost time is 3.66–4.04 s, the number of virtual sublanes (measured using the empirical method) is 1.63–1.86 (whereas the theoretical number is 2.43–3.00), and the saturation flow is 4,376–4,626 cyc./h. The saturation flow estimates are significantly lower (27–43% lower) than if the theoretical method is used to calculate the number of virtual sublanes (\( \varphi_T = W_a/w \)), as is usually done in practice.

Further research is necessary to identify the lateral influencing area of cyclists and to define the virtual sublane width based on empirical evidence. Moreover, it is necessary to analyze more sites with various cycle path widths \( (W_a) \) to establish an empirical relation between number of virtual sublanes and \( W_a \), which would allow more accurate prediction of the capacity of cycle paths of various width and other characteristics to be made, and also for optimal widths of cycle paths to be derived. This is very useful for cycle path design and traffic management. Alternative methods to measure the number of sublanes can also be explored. In this work, no distinction was made between scooters and different types of bicycles (e.g., electric bicycles, and human-powered bicycles including regular bicycles, cargo bicycles, racing bicycles). Note, however, that the percentage of scooters in this data set was very low. As future work, the influence of the presence of scooters, and different composition of bicycle types on the calculation of bicycle headway and capacity should be investigated. Also the influence of traffic composition should take into account different traffic conditions. This is not trivial, as traffic operations are different for regular and saturated flow conditions. For the latter, conflicts caused by speed differences among road users would not be so different dependent on the composition, as the flow is fully packed in this situation.

Acknowledgments

This research was supported by the ALLEGRO project (Unravelling slow mode traveling and traffic: with innovative data to create a new transportation and traffic theory for pedestrians and bicycles), which is funded by the European Research Council (Grant Agreement No. 669792), and the Amsterdam Institute for Advanced Metropolitan Solutions.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: BG-R, YY, MP; data collection: BG-R, WD; analysis and interpretation of results: YY, BG-R; manuscript preparation: YY, BG-R, MP; comments to draft manuscript: WD, SPH. All authors reviewed the results and approved the final version of the manuscript.
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The Standing Committee on Bicycle Transportation (ANF20) peer-reviewed this paper (19-04329).