Introduction to superconductivity in metals without inversion center

V. P. Mineev¹, M. Sigrist²

¹ Commissariat à l’Energie Atomique, INAC/SPSMS, 38054 Grenoble, France
² Theoretische Physik ETH-Hongerberg, CH-8093 Zurich, Switzerland

(Dated: May 28, 2009)

This Chapter gives a brief introduction to some basic aspects metals and superconductors in crystal without inversion symmetry. In a first part we analyze some normal state properties which arise through antisymmetric spin-orbit coupling existing in non-centrosymmetric materials and show its influence on the de Haas-van Alphen effect. For the superconducting phase we introduce a multi-band formulation which naturally arises due the spin splitting of the bands by spin-orbit coupling. It will then be shown how the states can be symmetry classified and their relation to the original classification in even-parity spin-singlet and odd-parity spin-triplet pairing states. The general Ginzburg-Landau functional will be derived and applied to the nucleation of superconductivity in a magnetic field. It will be shown that magneto-electric effects can modify the standard paramagnetic limiting behavior drastically.

PACS numbers:

I. INTRODUCTION

Motivated by the discovery of the non-centrosymmetric heavy Fermion superconductor CePt₃Si, the physics of unconventional superconductivity in materials without inversion symmetry has recently become a subject of growing interest. The lack of inversion symmetry, a key symmetry for Cooper pairing, combined with unconventional pairing symmetry is responsible for a number of intriguing novel properties. In a short time the list of new superconductors in this class has been enlarged by compounds such as U₁₂, CeRhSi₃, CeIrSi₃, Y₃C₁₆, and Li₂(Pd₁₋ₓPtₓ)₃B₆. In all listed heavy fermion compounds superconductivity appears in combination with a magnetic quantum phase transition suggesting the presence of strong electron correlation effects. Thus, it is widely believed that magnetic fluctuations are likely responsible for inducing here unconventional Cooper pairing. For other materials correlation effects seem to be less relevant. Nevertheless, some of them show unexpectedly features of unconventional pairing. While in some cases experimental results give rather clear suggestions on the gap symmetry, the definite identification of pairing states is far from concluded.

The microscopic theory of superconductivity in metals without inversion has a long history predating these recent experimental developments²⁻⁹,¹⁰,¹¹. Specific aspects such as the possibilities of inhomogeneous superconducting states¹²⁻¹⁵ and the magneto-electric effect¹⁴,¹⁵ in this type of materials have been discussed already in the nineties of the last century. Moreover, general symmetry aspects of non-centrosymmetric superconductors have been addressed rather early in Refs.¹⁶⁻¹⁸. A wide variety of physical phenomena connected with non-centrosymmetricity have since been studied by many groups:

- paramagnetic limitations of superconductivity and the helical vortex state¹⁹⁻²⁹
- paramagnetic susceptibility³⁰⁻³² and the magnetic field induced superconducting gap structure³³
- Josephson and quasiparticle tunneling³⁴,³⁵, surface bound states³⁶,³⁷, and vortex bound states³⁸
- London penetration depth³⁹ and the magnetic field distribution⁴⁰
- effects of impurities⁴¹⁻⁴³
- upper critical field⁴⁴⁻⁴⁵
- nuclear magnetic relaxation rate⁴⁶⁻⁴⁷
- general forms of pairing interaction⁴⁸
- inhomogeneous superconducting states in the absence of external field⁴⁹

In this Chapter we give a brief introduction to several topics in this context, leaving most of the special aspects of non-centrosymmetric superconductivity to other Chapters.
II. NORMAL STATE

The absence of inversion symmetry is imprinted into the electronic structure through spin-orbit coupling effects. Already the normal state of non-centrosymmetric metals bears intriguing features which result from the specific form of spin-orbit coupling. In this section we discuss the electronic spectrum.

A. Electronic states in non-centrosymmetric metals

Our starting point is the following Hamiltonian of non-interacting electrons in a crystal without inversion center:

$$H_0 = \sum_{\mathbf{k}} \sum_{\alpha\beta=\uparrow,\downarrow} [\xi(\mathbf{k}) \delta_{\alpha\beta} + \gamma(\mathbf{k}) \cdot \sigma_{\alpha\beta}] a_{\mathbf{k}\alpha}^\dagger a_{\mathbf{k}\beta}$$

where $a_{\mathbf{k}\alpha}^\dagger$ $(a_{\mathbf{k}\alpha})$ creates (annihilates) an electronic state $|\mathbf{k}\alpha\rangle$. Furthermore, $\xi(\mathbf{k}) = \varepsilon(\mathbf{k}) - \mu$ denotes the spin-independent part of the spectrum measured relative to the chemical potential $\mu$, $\alpha, \beta = \uparrow, \downarrow$ are spin indices and $\sigma$ are the Pauli matrices. The sum over $\mathbf{k}$ is restricted to the first Brillouin zone. The second term in Eq. (1) describes the antisymmetric spin-orbit (SO) coupling whose form depends on the specific non-centrosymmetric crystal structure.

The pseudovector $\gamma(\mathbf{k})$ satisfies $\gamma(-\mathbf{k}) = -\gamma(\mathbf{k})$ and $g \gamma(g^{-1}\mathbf{k}) = \gamma(\mathbf{k})$, where $g$ is any symmetry operation in the generating point group $G$ of the crystal (see below). The usual symmetric spin-orbit coupling which is present also in centrosymmetric crystals yields a new spinor basis (pseudospinor) $\alpha, \beta$ in Eq. (1), which retains the ordinary spin-1/2 structure with complete SU(2)-symmetry. This is different for the antisymmetric spin-orbit coupling. The effect of the antisymmetric spin-orbit coupling is a spin splitting of the band energy with $\mathbf{k}$-dependent spin quantization axis which removes the SU(2)-symmetry.

Depending on the purpose it is more convenient to express the Hamiltonian (1) in the initial 2x2 matrix form (spinor representation) or in its diagonal form (band representation). The energy bands are given by

$$\xi_{\pm}(\mathbf{k}) = \xi(\mathbf{k}) \pm |\gamma(\mathbf{k})|$$

with the Hamiltonian

$$H_0 = \sum_{\mathbf{k}} \sum_{\lambda=\pm} \xi_{\lambda}(\mathbf{k}) c_{\mathbf{k}\lambda}^\dagger c_{\mathbf{k}\lambda},$$

where the two sets of electronic operators are connected by a unitary transformation,

$$a_{\mathbf{k}\alpha} = \sum_{\lambda} u_{\alpha\lambda}(\mathbf{k}) c_{\mathbf{k}\lambda},$$

with

$$(u_{1\lambda}(\mathbf{k}), u_{1\lambda}(\mathbf{k})) = \left( \begin{array}{c} |\gamma| + \lambda \gamma_3, \\ \lambda (\gamma_2 + i \gamma_3) \end{array} \right) \frac{1}{\sqrt{2} |\gamma| (|\gamma| + \lambda \gamma_3)}.$$

The normal-state electron Green’s functions in the spinor representation can be written as

$$\hat{G}(\mathbf{k}, \omega_n) = \sum_{\lambda=\pm} \hat{\Pi}_{\lambda}(\mathbf{k}) G_{\lambda}(\mathbf{k}, \omega_n),$$

where

$$\hat{\Pi}_{\lambda}(\mathbf{k}) = \frac{1 + \lambda \hat{\gamma}(\mathbf{k}) \sigma}{2}$$

are the band projection operators and $\hat{\gamma} = \gamma/|\gamma|$. The Green’s functions in the band representation have then the simple form

$$G_{\lambda}(\mathbf{k}, \omega_n) = \frac{1}{i\omega_n - \xi_{\lambda}(\mathbf{k})},$$

where $\omega_n = \pi T (2n + 1)$ is the Matsubara frequency.
The Fermi surfaces defined by the equations $\xi_{\pm}(\mathbf{k}) = 0$ are split, except at specific points or lines where $|\gamma(\mathbf{k})| = 0$ is satisfied. The band dispersion functions $\xi_{\pm}(\mathbf{k})$ are invariant with respect to all operations of $G$ and the time reversal operations $K = i\sigma_z K_0$ ($K_0$ is the complex conjugation). The states $[\mathbf{k}, \lambda]$ and $K[\mathbf{k}, \lambda]$ belonging to the band energies $\xi_{\pm}(\mathbf{k})$ and $\xi_{\pm}(-\mathbf{k})$, respectively, are degenerate, since the time reversal operation yields $K[\mathbf{k}, \lambda] = t_\lambda(\mathbf{k}) - \mathbf{k}, \lambda$, where $t_\lambda(\mathbf{k}) = -t_\lambda(-\mathbf{k})$ is a nontrivial phase factor. For the eigenstates of $H_0$, defined by (9), this phase factor takes the form,

$$t_\lambda(\mathbf{k}) = -\lambda \frac{\gamma_\alpha(\mathbf{k}) + i\gamma_\beta(\mathbf{k})}{\sqrt{\gamma_\alpha^2(\mathbf{k}) + \gamma_\beta^2(\mathbf{k})}}. \quad (9)$$

Finally we turn to the basic form of the antisymmetric spin-orbit coupling as it results from the non-centrosymmetric crystal structures. Here we ignore the Brillouin zone structure and use only the expansion for small momenta $\mathbf{k}$ leading to basis functions satisfying the basic symmetry requirements of $\gamma(\mathbf{k})$. For the cubic group $G = O$, the point group of $\text{Li}_2(\text{Pd}_{1-x}\text{Pt}_x)\text{B}$, the simplest form compatible with symmetry requirements is

$$\gamma(\mathbf{k}) = \gamma_0 \mathbf{k}, \quad (10)$$

where $\gamma_0$ is a constant. For point groups containing improper elements, i.e., reflections and rotation-reflections, expressions become more complicated. The full tetrahedral group $G = T_d$, which is relevant for $Y_2C_3$ and possibly $\text{KOs}_2\text{O}_6$, the expansion of $\gamma(\mathbf{k})$ starts with third order in the momentum,

$$\gamma(\mathbf{k}) = \gamma_0 [k_x(k_y^2 - k_z^2)\hat{x} + k_y(k_z^2 - k_x^2)\hat{y} + k_z(k_x^2 - k_y^2)\hat{z}]. \quad (11)$$

This is sometimes called Dresselhaus spin-orbit coupling, and was originally discussed for bulk semiconductors of zinc-blend structure. The tetragonal point group $G = \mathbf{C}_{4v}$, relevant for $\text{CePt}_3\text{Si}$, $\text{CeRhSi}_3$ and $\text{CeIrSi}_3$, yields the antisymmetric spin-orbit coupling

$$\gamma(\mathbf{k}) = \gamma_\perp (k_y\hat{x} - k_x\hat{y}) + \gamma_\parallel k_x k_y k_z (k_x^2 - k_y^2)\hat{z}. \quad (12)$$

In the purely two-dimensional case, setting $\gamma_\parallel = 0$ one recovers the Rashba interaction, which is often used to describe the effects of the absence of mirror symmetry in semiconductor quantum wells.

### B. de Haas - van Alphen effect

An experimental way of observing the spin-splitting of the Fermi surface is the de Haas - van Alphen effect which can help to estimate the magnitude of the antisymmetric spin-orbit coupling. The single-electron Hamiltonian can be extended to include the magnetic field as follows:

$$H_0 = \sum_\mathbf{k} \sum_{\alpha\beta = \uparrow, \downarrow} [\xi(\mathbf{k})\delta_{\alpha\beta} + \gamma(\mathbf{k})\sigma_{\alpha\beta} - \mu_B \mathbf{H}\sigma_{\alpha\beta}]a^\dagger_{\mathbf{k}\alpha} a_{\mathbf{k}\beta}. \quad (13)$$

The last term describes the Zeeman interaction for an external magnetic field $\mathbf{H}$, with $\mu_B$ being the Bohr magneton. The orbital effect of the field can be included by replacing $\mathbf{k} \rightarrow \mathbf{k} + (e/\hbar c)\mathbf{A}(\mathbf{r})$, where $\mathbf{r} = i\nabla \mathbf{k}$ is the position operator in the $\mathbf{k}$-representation.

The eigenvalues of the Hamiltonian (13) are

$$\xi_\pm(\mathbf{k}, \mathbf{H}) = \xi(\mathbf{k}) + \lambda|\gamma(\mathbf{k})| - \mu_B |\mathbf{H}|. \quad (14)$$

There are two Fermi surfaces determined by the equations

$$\xi(\mathbf{k}, \mathbf{H}) = 0. \quad (15)$$

For certain directions and magnitudes of $\mathbf{H}$ there may be accidental degeneracies of the Fermi surfaces, determined by the equation $\gamma(\mathbf{k}) = \mu_B |\mathbf{H}|$. However, there are no symmetry reasons for such intersections.

An important property of the Fermi surfaces is the fact that their shapes depend on the magnetic field in a characteristic way, which can be directly probed by dHvA experiments. Note that while at $H = 0$ time reversal symmetry guarantees $\xi_\pm(-\mathbf{k}) = \xi_\pm(\mathbf{k})$, the loss of time reversal symmetry for $H \neq 0$ yields, in general, $\xi_\pm(-\mathbf{k}, \mathbf{H}) \neq \xi_\pm(\mathbf{k}, \mathbf{H})$, i.e. the Fermi surfaces do not have inversion symmetry.
Including now the coupling of the magnetic field to the orbital motion of the electrons we derive in quasi-classical approximation the Lifshitz-Onsager quantization rules:\footnote{56}

\[ S_\lambda(\epsilon, k_H) = \frac{2\pi e H}{\hbar c} \left[ n + \alpha_\lambda(\Gamma) \right]. \] (16)

Here \( S_\lambda \) is the area of the quasi-classical orbit \( \Gamma \), in the \( k \)-space defined by the intersection of the constant-energy surface \( \epsilon_\lambda(k) = \epsilon \) with the plane \( k \cdot \mathbf{h} = k_H (\mathbf{h} = \mathbf{H}/H) \). Moreover, \( n \) is an integer number \( (n \gg 1) \), and \( 0 \leq \alpha_\lambda(\Gamma) < 1 \) is connected with the Berry phase of the electron as it moves along \( \Gamma \). The value of \( \alpha_\lambda(\Gamma) \) does not affect the dHvA frequency discussed below.

The dHvA signal contains contributions from both bands and can be approximately decomposed into the form,

\[ M_{\text{osc}} = \sum_\lambda A_\lambda \cos \left( \frac{2\pi F_\lambda}{H} + \phi_\lambda \right), \] (17)

where \( A_\lambda \) and \( \phi_\lambda \) are the amplitudes and the phases of the oscillations. The amplitudes are given by the standard Lifshitz-Kosevich formula and the dHvA frequencies \( F_\lambda \) are related to the extremal (with respect to \( k_H \)) cross-section areas of the two Fermi surfaces,

\[ F_\lambda = \frac{\hbar c}{2\pi e} S_\lambda^{\text{ext}}. \] (18)

In addition to the fundamental harmonics \( \lambda = 1 \), the observed dHvA signal also contains higher harmonics with frequencies given by multiple integers of \( F_\lambda \).

It is interesting to consider the field dependence of the band energies \( \lambda = 1 \), which yield

\[ S_\lambda^{\text{ext}}(H) = S_\lambda^{\text{ext}}(0) + A_\lambda(\mathbf{h}) H + B_\lambda(\mathbf{h}) H^2 + \ldots. \] (19)

Inserting this in Eq. (17) the term linear in \( H \) contributes to the phase shift, similar to the paramagnetic splitting of Fermi surfaces in centrosymmetric metals. The quadratic term is responsible of the magnetic field dependence of the dHvA frequencies. This is a specific feature of non-centrosymmetric metals which could be observable, if the Zeeman energy is at most of comparable magnitude as the spin-orbit coupling.

For illustration, let us look at the example of a three-dimensional elliptic Fermi surface with \( \xi(k) = \frac{k^2}{2m_-} + \frac{k^2}{2m_+} - \epsilon_F \), where \( m_- \) and \( m_+ \) are the effective masses. The extremal (maximum) cross-sections of the Fermi surfaces \( \Gamma \) correspond to \( k_z = 0 \). Introducing the Fermi wave vector \( k_F \) via \( \epsilon_F = \hbar^2 k_F^2/2m_\perp \), we obtain

\[ S_\lambda^{\text{ext}}(H) = \pi k_F^2 \left[ 1 - \lambda \frac{\chi_\perp k_F}{\epsilon_F} \left( 1 + \frac{\mu_B^2 H^2}{2\chi_\parallel k_F^2} \right) \right]. \] (20)

In this approximation we assumed that the Zeeman energy is small compared to the spin-orbit band splitting, which in turn is much smaller than the Fermi energy: \( \mu_B H \ll |\chi_\perp k_F| \ll \epsilon_F \). Based on this result it is also possible to obtain an estimate of the strength of the spin-orbit coupling.

We use the expressions (18) and (20) to calculate the difference of the dHvA frequencies for the split bands:

\[ F_- - F_+ = \frac{2e}{\hbar c} \gamma_\perp k_F m_\perp \left( 1 + \frac{\mu_B^2 H^2}{2\gamma_\parallel k_F^2} \right) \] (21)

For example, from the frequencies of the "a" and "β" dHvA frequency branches in LaPt\(_3\)Si\(_6\), \( F_\alpha = 1.10 \times 10^5 \text{Oe} \) and \( F_\beta = 8.41 \times 10^5 \text{Oe} \), and \( m_\perp \simeq 1.5 m_e \), we obtain for the spin-orbit splitting of the Fermi surfaces: \( |\gamma_\perp k_F| \approx \mu_\alpha^2 H \approx 10^3 \text{K,} \) which is in reasonable agreement with the results of band structure calculations.\footnote{56,58} According to Eq. (21), the magnetic field effect on \( F_- - F_+ \) in the range of fields used in Ref.\footnote{58} (up to 1T) should be of the order of a few percent. In this way the interplay of the Zeeman splitting and the spin-orbit coupling results in a deformation of the Fermi surface is responsible for a field dependence of the dHvA frequencies, an effect absent in centrosymmetric metals.

### III. SUPERCONDUCTING STATE

In this section we turn to the discussion of some novel aspects of the superconducting state in non-centrosymmetric materials. Here we can consider only a few examples, while a wider range of other phenomena will be discussed in other Chapters of this book.
A. Basic equations

After our introductory discussion of single-electron properties we now include electron-electron interactions to examine the implications of non-centrosymmetry on Cooper pairing. Therefore we retain among all interactions only those terms corresponding to the Cooper channel and formulate it in the band representation. The general form is given by

\[
H_{\text{int}} = \frac{1}{2V} \sum_{k,k'} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} V_{\lambda_1\lambda_2,\lambda_3,\lambda_4}(k,k') c_{\lambda_1,k}^\dagger c_{\lambda_2,-k,\lambda_4}^\dagger c_{-k',\lambda_3} c_{k',\lambda_1}. \tag{22}
\]

It is reasonable to assume \(\lambda_1 = \lambda_2\) and \(\lambda_3 = \lambda_4\), such that only intra-band pairing is considered. Inter-band pairing is usually suppressed, since the spin-orbit coupling induced band splitting would require that electrons far from the Fermi surfaces would have to pair which is unlikely, if the energy scale of the band splitting strongly exceeds the superconducting energy scale\(^{57}\). Introducing the notation \(\lambda_1 = \lambda_2 = \lambda\) and \(\lambda_3 = \lambda_4 = \lambda'\), we obtain:

\[
H_{\text{int}} = \frac{1}{2V} \sum_{kk'q} \sum_{\lambda\lambda'} V_{\lambda\lambda'}(k,k') c_{\lambda,k}^\dagger c_{\lambda',-k'}^\dagger c_{-k',\lambda'} c_{k',\lambda}, \tag{23}
\]

where

\[
V_{\lambda\lambda'}(k,k') = t_\lambda(k)t^*_\lambda(k') V_{\lambda\lambda'}(k,k'). \tag{24}
\]

Since under time reversal the creation and annihilation operators behave as

\[
Kc_{\lambda,\lambda}^\dagger = t_\lambda(k)c_{\lambda,\lambda}^\dagger, \quad Kc_{\lambda',\lambda} = t^*_\lambda(k)c_{\lambda,\lambda'}, \tag{25}
\]

\(V_{\lambda\lambda'}(k,k')\) represents the pairing interaction between time-reversed states. The amplitude \(V_{\lambda\lambda'}(k,k')\) is even in both \(k\) and \(k'\) due to the anticommutation of fermionic operators and is invariant under the point group operations: \(V_{\lambda\lambda'}(gk, gk') = V_{\lambda\lambda'}(k,k')\). The gap functions of the superconducting state can be expressed as, \(\Delta_\lambda(k) = t_\lambda(k)\tilde{\Delta}_\lambda(k)\), in each band, where \(\tilde{\Delta}_\lambda\) transforms according to one of the irreducible representations of the crystal point group\(^{59,60}\).

The Gor'kov equations in each band read

\[
(i\omega_n - \xi_\lambda(k)) G_\lambda(k,\omega_n) + \tilde{\Delta}_{k\lambda} F_\lambda^\dagger(k,\omega_n) = 1
\]

\[
(i\omega_n + \xi_\lambda(-k)) F_\lambda^\dagger(k,\omega_n) + \tilde{\Delta}_{k\lambda}^\dagger G_\lambda(k,\omega_n) = 0. \tag{27}
\]

The gap functions obey the self-consistency equations

\[
\tilde{\Delta}_{k\lambda} = -T \sum_n \sum_{k'} \sum_{\lambda'} \tilde{V}_{\lambda\lambda'}(k,k') F_{\lambda'}(k',\omega_n). \tag{28}
\]

The resulting Green’s functions are then,

\[
G_\lambda(k,\omega_n) = \frac{i\omega_n + \xi_\lambda(-k)}{(i\omega_n - \xi_\lambda(k))(i\omega_n + \xi_\lambda(-k)) - \Delta_{k\lambda}\tilde{\Delta}_{k\lambda}^\dagger} \tag{29}
\]

\[
F_\lambda(k,\omega_n) = \frac{-\Delta_{k\lambda}}{(i\omega_n - \xi_\lambda(k))(i\omega_n + \xi_\lambda(-k)) - \Delta_{k\lambda}\tilde{\Delta}_{k\lambda}^\dagger}. \tag{30}
\]

and the quasiparticle excitation energies for each band have the form

\[
E_{k\lambda} = \frac{\xi_\lambda(k) - \xi_\lambda(-k)}{2} \pm \sqrt{\left(\frac{\xi_\lambda(k) + \xi_\lambda(-k)}{2}\right)^2 + \tilde{\Delta}_{k\lambda}\tilde{\Delta}_{k\lambda}^\dagger}. \tag{31}
\]

which becomes in case of time reversal symmetry,

\[
E_{k\lambda} = \pm \sqrt{\xi_\lambda(k)^2 + \tilde{\Delta}_{k\lambda}\tilde{\Delta}_{k\lambda}^\dagger}. \tag{32}
\]

These forms are analogous to that of a multi-band superconductor\(^58\), apart from the fact that in the non-centrosymmetric case the two bands to not possess spin degeneracy but rather correspond to a type of spinless
fermions, since their spinors on each band are subject to a momentum dependent projection. This distinction becomes more apparent, if we write the Gor’kov equations in the initial spinor basis (spin up and down),

\[ (i\omega_n - \xi(k) - \gamma_k \sigma) \tilde{G}(k, \omega_n) + \Delta_k \tilde{F}^\dagger(k, \omega_n) = 1 \]
\[ (i\omega_n + \xi(-k) + \gamma_{-k} \sigma) \tilde{F}^\dagger(k, \omega_n) + \Delta_k \tilde{G}(k, \omega_n) = 0, \]

where \( \xi(k) = \varepsilon_k - \mu, \)

\[ \tilde{G}(k, \omega_n) = \tilde{\Pi}_+ G_+(k, \omega_n) + \tilde{\Pi}_- G_-(k, \omega_n), \]
\[ \tilde{F}^\dagger(k, \omega_n) = \tilde{g}^\dagger \{ \tilde{\Pi}_+ F^\dagger_+(k, \omega_n) + \tilde{\Pi}_- F^\dagger_-(k, \omega_n) \}, \]
\[ \tilde{\Delta}_k = \{ \tilde{\Pi}_+ \tilde{\Delta}_{k,+} + \tilde{\Pi}_- \tilde{\Delta}_{k,-} \} \tilde{g}, \]

with \( \tilde{g} = i\hat{a}_y. \) Examining the form of the gap function reveals that in the non-centrosymmetric superconductor both even-parity spin-singlet and odd-parity spin-triplet pairing are mixed, since no symmetry is available to distinguish the two. Therefore, we may write

\[ \tilde{\Delta}_k = \frac{\tilde{\Delta}_{k,+} + \tilde{\Delta}_{k,-}}{2} \tilde{g} + \frac{\tilde{\Delta}_{k,+} - \tilde{\Delta}_{k,-}}{2} \tilde{\gamma}_k \sigma \tilde{g}. \]

The odd-parity component is represented by a vector which is oriented along the vector \( \gamma_k \) of the spin-orbit coupling. In this discussion the only approximation entering so far is the absence of inter-band pairing.

### B. Critical temperature

For the discussion of the instability condition at the critical temperature and the topology of the quasiparticle gap we can use the symmetry properties of the pairing interaction matrix \( V_{\lambda \lambda'}(k, k') \) mentioned earlier. The momentum dependence of the matrix elements can be represented in a spectral form decomposed in products of the basis functions of irreducible representations of \( G. \) It is generally sufficient to consider only the part of the pairing potential based on one irreducible representation \( \Gamma \) corresponding to the superconducting state with maximal critical temperature. In a simplified formulation, this could be represented by even basis functions on the two bands, \( \phi_{+,i}(k) \) and \( \phi_{-,i}(k), \)

\[ V_{\lambda \lambda'}(k, k') = -V_{\lambda \lambda'} \sum_{i=1}^d \phi_{+,i}(k) \phi_{+,i}^*(k'), \]

While \( \phi_{+,i}(k) \) and \( \phi_{-,i}(k) \) both belong to the same symmetry representation, their momentum dependence does not have to be exactly the same. The basis functions are assumed to satisfy the following orthogonality conditions:

\[ \langle \phi_{+,i}(k) \phi_{+,j}(k) \rangle = \delta_{ij}, \]

where the angular brackets denote the averaging over the \( \lambda \)-th Fermi surface. The coupling constants \( V_{\lambda \lambda'} \) form a Hermitian matrix, which becomes real symmetric, if the basis functions are real. The gap functions take the form

\[ \tilde{\Delta}_\lambda(k) = \sum_{i=1}^{d_{\lambda}} \eta_{\lambda,i} \phi_{\lambda,i}(k), \]

and \( \eta_{\lambda,i} \) are the superconducting order parameter components in the \( \lambda \)-th band.

As an example, consider a superconducting state with the order parameter transforming according to a one-dimensional representation \( \tilde{\Delta}_\lambda(k) = \eta_\lambda \phi_\lambda(k). \) The linearized gap equations Eq. (25) acquire simple algebraic form

\[ \eta_+ = (g_{++} \eta_+ + g_{+-} \eta_-) S_1(T), \]
\[ \eta_- = (g_{+-} \eta_+ + g_{--} \eta_-) S_1(T), \]

where

\[ g_{\lambda \lambda'} = V_{\lambda \lambda'} N_{0\lambda'}, \]

(42)
and $N_{0\lambda} = \langle \phi_\lambda(k) \rangle^2 N_{0\lambda}(k) \rangle$ is the weighted average angular dependent density of states over the $\lambda$-th Fermi surface. Note that for multidimensional representations (dimensional $d_T$) due to the crystal point symmetry the values of $N_{0\lambda} = \langle |\phi_{\lambda,i}(k)|^2 N_{0\lambda}(k) \rangle$ are equal for all components $i = 1,..d_T$ and all components $\eta_{\lambda,1},..\eta_{\lambda,d_T}$ separately satisfy the same system of equations (41).

The function $S_1(T)$ is

$$S_1(T) = 2\pi T \sum_{n \geq 0} \frac{1}{\nu_n} \ln \frac{2\gamma \epsilon_c}{\pi T},$$

where $\ln \gamma = 0.577\ldots$ is the Euler constant, $\epsilon_c$ is an energy cutoff for the pairing interaction, which we assume to be the same for both bands. From Eq. (41) we obtain then the following expression for the critical temperature:

$$T_{c0} = \frac{2\gamma \epsilon_c}{\pi} \exp \left(-\frac{1}{g}\right),$$

where

$$g = \frac{g_{++} + g_{--}}{2} + \sqrt{\left(\frac{g_{++} - g_{--}}{2}\right)^2 + g_{+-}g_{-+}}$$

is the effective coupling constant. For multidimensional representations the critical temperature is the same for all $d_T$ components of $\eta_{\lambda,i}$ of the order parameter. The particular combination of amplitudes $\eta_{\lambda,i}$ in the superconducting state below $T_c$ is determined by the nonlinear terms in the free energy or self-consistent equation, which depend on the symmetry of the dominant pairing channel.

The solution of Eq. (41) ($\eta_{+}, \eta_{-}$) corresponding to the eigenvalue $S_1(T_c)$ determines two unequal order parameter components $\Delta_\lambda(k) = \eta_{+}\phi_\lambda(k)$. In the spinor representation (38) both singlet and triplet parts of the order parameter are present. Pure singlet or pure triplet pairing occurs only under rather restrictive conditions. First, the momentum dependence of the gap function in both bands is the same $\Delta_\lambda(k) = \eta_{+}\phi_\lambda(k)$. Second, $g_{++} = g_{--}$ and $g_{+-} = g_{-+}$ is realized. Then we obtain two solutions of equations Eq. (41) with

$$\eta_{+} = \eta_{-},$$

$$\eta_{+} = -\eta_{-}.$$  

The critical temperature of the state (46) corresponding to the singlet part is $T_{c0}^s = (2\epsilon_c/\pi)\exp^{-1/g_s}$, where $g_s = g_{++} + g_{-+}$. The critical temperature of the spin triplet state (47) is $T_{c0}^t = (2\epsilon_c/\pi)\exp^{-1/g_t}$, where $g_t = g_{++} - g_{-+}$. If $g_{+-} > 0$ then $T_{c0}^t > T_{c0}^s$ and the phase transition occurs to the state (46). While at $g_{+-} < 0$ we see that $T_{c0}^t > T_{c0}^s$ and the phase transition occurs to the state (47).

C. Zeros in the quasiparticle gap

On the one hand, the zeros in the gap for elementary excitations are dictated by the symmetry of superconducting state or its superconducting class which is a subgroup $\mathcal{H}$ of the group of symmetry of the normal state $G \times K \times U(1)$, where $G$ is the point group, $K$ is the group of time reversal, $U(1)$ is the gauge group. The procedure to find symmetry dictated nodes is described in Ref.59. Let us consider the possible superconducting states (40) and their nodes for $CePt_3Si$ with point group symmetry $C_{4v}$. This group has four one-dimensional irreducible representations, $A_1, A_2, B_1, B_2$, and one two-dimensional representation, $E$. Examples of even basis functions of these irreducible representations are

| $\Gamma$ | $\phi_T(k)$ | nodes |
| --- | --- | --- |
| $A_1$ | $k_x^2 + k_y^2 + ck_z^2$ | $k_x = 0, k_y = 0, k_z = \pm k_y$ |
| $A_2$ | $k_x k_y (k_x^2 - k_y^2)$ | $k_x = \pm k_y$ |
| $B_1$ | $k_x^2 - k_y^2$ | $k_x = 0, k_y = 0$ |
| $B_2$ | $k_x k_y$ | $k_x = 0, k_y = 0$ |
For $E$ state the basis functions are $\phi_{E1}(k) = k_x k_z$ and $\phi_{E2}(k) = k_y k_z$ and it leads to the order parameter for the Fermi surfaces $\lambda$,

$$\tilde{\Delta}_\lambda(k) = \eta_{\lambda,1}\phi_{\lambda,E1}(k) + \eta_{\lambda,2}\phi_{\lambda,E2}(k).$$

The symmetry of superconducting state and the corresponding node positions depend on the particular choice of amplitudes $\eta_{\lambda,i}$. Under weak coupling conditions the combination generating least nodes is most stable, corresponding here to $(\eta_{\lambda,1}, \eta_{\lambda,2}) = \eta_{\lambda}(1, \pm i)$ and is a time reversal symmetry violating phase with a line node on the plane $k_z = 0$ and point nodes at $k_x = k_y = 0$.

On the other hand, the fact that their gaps on the two Fermi surfaces are composed of an even and an odd parity part, can also lead to nodes which are not symmetry protected, as discussed in Ref. 42.

### D. The amplitude of singlet and triplet pairing states

The coupling constants $V_{\lambda\lambda'}$ we have used in previous considerations can be expressed through the real physical interactions between the electrons naturally introduced in the initial spinor basis where BCS type Hamiltonian has the following form:

$$H_{int} = \frac{1}{4V} \sum_{kq'q}\sum_{\alpha\beta\gamma\delta} [V^g(k, k')^\dagger(i\sigma_2)^{\dagger}(i\sigma_2)_{\alpha\beta}\gamma_{\delta}^\dagger + V_{ij}^u(k, k')(i\sigma_2)^{\dagger}\gamma_{\delta}^\dagger] a_{k+q}\beta a_{-k, \alpha}^\dagger a_{-k, \alpha} a_{-k', \gamma}^\dagger a_{-k', \gamma} a_{q, \delta}^\dagger, \tag{49}$$

where the amplitudes $V^g(k, k')$ and $V_{ij}^u(k, k')$ are even and odd with respect to their arguments correspondingly. The unitary transformation (41) transforms the pairing Hamiltonian (49) to the band representation (22). If we neglect inter-band pairing, it is reduced to (23) and (24) with the amplitudes given by the following expression

$$\tilde{V}_{\lambda\lambda'}(k, k') = \frac{1}{2} V^g(k, k')(\sigma_0 + \sigma_x)_{\lambda\lambda'} + \frac{1}{2} V_{ij}^u(k, k')\gamma_i^\dagger(k)\gamma_j^\dagger(k')(\sigma_0 - \sigma_x)_{\lambda\lambda'}. \tag{50}$$

The explicit derivation is given in Ref. 48, where the similar procedure was also made for more general interactions mediated by phonons or spin fluctuations. It can be shown that the pairing given by the amplitude $V^g(k, k')$ in the initial spinor basis including the simple $s$-wave pairing $V^g(k, k') = const$ does not induce any inter-band pairing channel.

To illustrate the origination of the singlet and triplet pairing channels let us consider a superconductor with tetragonal symmetry $C_{4v}$, and Rashba spin-orbital coupling $\gamma(k) = \gamma_\perp(\hat{z} \times k)$, for a spherical Fermi surface. We describe the pairing by the following model, which is compatible with all symmetry requirements:

$$V^g(k, k') = -V_g, \quad V_{ij}^u(k, k') = -V_u(\gamma_i(k)\gamma_j(k')), \tag{51}$$

where $V_g$ and $V_u$ are constants. This type of pairing interaction yields the superconducting state with full symmetry of the tetragonal group $C_{4v}$, transforming according to unit representation $A_1$ both in singlet and in triplet channels.

With Eq. (50) we arrive at the band representation:

$$\tilde{V}_{\lambda\lambda'}(k, k') = -\frac{1}{2} V_g(\sigma_0 + \sigma_x)_{\lambda\lambda'} - \frac{1}{2} V_u(\sigma_0 - \sigma_x)_{\lambda\lambda'}, \tag{52}$$

Thus, this pairing interaction is even simpler than that (eqn. (59)) considered in the previous subsection. So, in our model the gap functions in the two bands Eqs. (40) are: $\Delta_\lambda(k) = \eta_\lambda \varphi_{A_1}(k)$ with $k$ independent functions $\varphi_{A_1}(k) = 1$.

The amplitudes $\eta_\lambda$ satisfy the equations

$$\eta_\lambda = \sum_{\lambda'} g_{\lambda\lambda'} \pi T \sum_n \frac{\eta_{\lambda'}}{\sqrt{\omega_n^2 + \eta_{\lambda'}^2}}, \tag{53}$$

where

$$g_{\pm \pm} = \frac{V_g + V_u}{2} N_{0\pm}, \quad g_{\pm \mp} = \frac{V_g - V_u}{2} N_{0\mp}. \tag{54}$$

The critical temperature is given by Eq. (44).
According to the Eq. (33) the singlet and triplet parts of the order parameter are determined by the order parameter amplitudes in different bands. For the ratio of triplet to singlet amplitude in vicinity of \( T_c \) we find:

\[
    r = \frac{\eta_+ - \eta_-}{\eta_+ + \eta_-} = \frac{2g_{+-} + g_{++} - g_{--} - \sqrt{D}}{2g_{+-} + g_{--} - g_{++} + \sqrt{D}},
\]

where \( D = (g_{++} - g_{--})^2 + 4g_{+-}g_{--} \). It is easy to see that for \( V_g = 0 \) the triplet component of the order parameter vanishes identically: \( r = 0 \). On the other hand, for \( V_g = 0 \) the singlet component of the order parameter disappears: \( r^{-1} = 0 \). Generally the relative weight of singlet and triplet component in the order parameter depends on the ratio of pairing interactions decomposed into even and odd parity channel.

A simple BCS type of model with

\[
    V^q(k, k') = -V_g \quad \text{and} \quad V_{ij}^u(k, k') = 0
\]

yields in the band representation

\[
    \tilde{V}_{\lambda \lambda'}^{BCS}(k, k') = -\frac{1}{2} V_g (\sigma_0 + \sigma_x)_{\lambda \lambda'}.
\]

and gives rise to purely spin singlet pairing within our notion.

### E. Ginzburg-Landau formulation

The Ginzburg-Landau theory is a very efficient tool to discuss a wide variety of phenomena of the superconducting state, in particular, the instability conditions at the critical temperature. We will derive the Ginzburg-Landau functional from a microscopic starting point with the aim to address in the following chapter the influence of the magneto-electric effect on the nucleation of superconductivity in a magnetic field, i.e. the modification of paramagnetic limiting in a non-centrosymmetric metal.

For this purpose we extend the self-consistent equation (28) to the case where magnetic fields are present and the superconducting order parameter has a weak spatial dependence,

\[
    \tilde{\Delta}_\lambda(k, q) = T \sum_n \sum_{k'} \tilde{V}_{\lambda \nu}(k, k') G_\nu(k', \omega_n) G_\nu(-k' + q, -\omega_n) \tilde{\Delta}_\nu(k', q).
\]

Near the critical temperature where one can use the normal metal Green functions \( G^0_{\lambda \nu}(k, \omega_n) \) which yield then the linearized gap equation to examine the instability condition. This equation can be derived from the free energy functional of the form

\[
    F = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left\{ \sum_{\lambda \nu} \eta_{\lambda \lambda'}^+(q) \tilde{V}_{\lambda \lambda'}^{-1}(q) \eta_{\nu \nu'}(q) \right\} - \sum_{\nu} T \sum_n \int \frac{d^3k}{(2\pi)^3} \tilde{\Delta}_\nu^+(k, q) G_\nu(k, \omega_n) G_\nu(-k + q, -\omega_n) \tilde{\Delta}_\nu(k, q).
\]

The corresponding normal metal electron Green function \( G_{\lambda}(k, \omega_n) = (i\omega - \xi_{\lambda}(k, H))^{-1} \) in a magnetic field is determined by the electron energies \( \xi_{\lambda}(k, H) \),

\[
    \xi_{\lambda}(k, H) = \xi(k) + \lambda|\gamma(k)| - \mu_B H \approx \xi_{\lambda}(k) - m_{\lambda}(k) H,
\]

where \( \xi_{\lambda}(k) = \xi(k) + \lambda|\gamma(k)| \) and the second term on the right-hand side is the analog of the Zeeman interaction for non-degenerate bands with the form:

\[
    m_{\lambda}(k) = \lambda \mu_B \gamma(k),
\]

which is valid everywhere except for the vicinity of the band crossing points, where the approximation of independent non-degenerate bands fails. In the standard centrosymmetric metal the magnetic field splits the Fermi surfaces into majority and minority spin surfaces. Here the spin-splitting is imposed at the outset by the spin-orbit coupling. The effect of the magnetic field is a non-centric deformation of the band and shape of the Fermi surfaces. As we will see this will influence the superconducting condensate nucleated in a magnetic field.
The normal electron Green function is then approximated as

\[ G_\lambda(k, \omega_n) = \frac{1}{i\omega_n - \xi_\lambda(k) + m_\lambda(k)H}. \]  

(62)

Since the gap function weakly depends on energy in the vicinity of the Fermi surface, one can integrate the products of two Green's functions with respect to \( \xi_\lambda = \xi_\lambda(k) \):

\[ N_{0\lambda} \int d\xi_\lambda G_\lambda(k, \omega_n)G_\lambda(-k + q, -\omega_n) = \pi N_{0\lambda} L_\lambda(k, q, \omega_n), \]  

(63)

where

\[ L_\lambda(k, q, \omega_n) = \frac{1}{|\omega_n| + i\Omega_\lambda(k, q) \text{sign} \omega_n}, \]  

(64)

depends only on \( \hat{k} \), the direction of \( k \),

\[ \Omega_\lambda(k, q) = \frac{v_\lambda(k)q}{2} - m_\lambda(k)H, \]  

(65)

with \( v_\lambda(k) = \partial \xi_\lambda(k)/\partial k \) being the Fermi velocity in the \( \lambda \)-th band.

The Ginzburg-Landau free energy in usual coordinate representation (as well as the Ginzburg-Landau equations) can be obtained from the Taylor expansion of Eqs. (58) and (59) in powers of \( \Omega_\lambda(k, q) \), by the replacement

\[ q \rightarrow D = -i\nabla - eA(r) \]  

(66)

in the final expressions. In the following we will put \( \hbar = c = 1. \) The special form of \( \Omega_\lambda(k, q) \) introduces a novel gradient terms in the free energy of non-centrosymmetric superconductors. Instead of powers of \( q \) it contains powers of \( \Omega_\lambda(k, q) \). This can lead to the formation of nonuniform superconducting state known as \textit{helical phases} and \textit{magneto-electric effect} in a magnetic field.

IV. MAGNETO-ELECTRIC EFFECT AND THE UPPER CRITICAL FIELD

The term "magneto-electric effect" in non-centrosymmetric superconductors encompasses several intriguing features. It has been discussed on a phenomenological level by introducing additional linear gradients terms to the Ginzburg-Landau free energy, so-called Lifshitz invariants, like

\[ \eta^*(r) \tilde{K}_{ij} H_i D_j \eta(r) \]  

(67)

Here \( \eta(r) \) denotes the order parameter of superconductor, \( H \) is magnetic field and \( D = -i\nabla - 2eA(r) \) is the gauge-invariant gradient. First predicted by Levitov, Nazarov and Eliashberg, the magneto-electric effect was studied microscopically by several authors \cite{10,14,15,19,23}. In this context several observable effects have been predicted: (i) the existence of a helically twisted superconducting order parameter in a magnetic field in two and three dimensional cases and spontaneous supercurrents in a 2D geometry \cite{15,19,21,23,26,29} and near the superconductor surface \cite{28} as well as along junctions of two superconductors with opposite directions of polarization \cite{22}, (ii) the augmentation of the upper critical field oriented perpendicular to the direction of the space parity breaking \cite{23,26}, (iii) magnetic interference patterns of the Josephson critical current for a magnetic field applied perpendicular to the junction \cite{26}.

The presence of Lifshitz invariants \cite{67}, however, can mislead to invalid conclusions and a careful analysis of different contributions to an effect is mandatory. This is indeed true for the influence of the magneto-electric effect on paramagnetic limiting. Moreover, the notion of helical phase has to be considered carefully as it may imply wrong pictures. In this section we would like to give insight into this subtleties by discussing the magnetic field dependence of the effective critical temperature in the Ginzburg-Landau framework.

A. One band case

Before considering the intrinsic multi-band situation due to the spin splitting of the electron band, for simplicity we restrict ourselves to a one-band situation, i.e. we ignore one of the two bands. This band shall be characterized by an isotropic density of states at the Fermi energy, \( N_{0+}(\hat{k}) = N_+ \).
The Ginzburg-Landau free energy for this one-band case with a one-component order parameter can be derived from Eqn. [59]

\[ F = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{2}{V_{++}} - N_{0+}S_1(T) + N_{0+}S_3(T)\langle (\phi^2(k)\Omega(k, q))^2 \rangle \right\} |\eta(q)|^2, \]  

(68)

where we restrict to the second order terms. This is sufficient to analyze the instability conditions. Here, \( \phi(k) \) describes the superconducting state and is an even function belonging to one of one-dimensional representations of \( \Omega \) is given by (65), \( \langle ... \rangle \) means the averaging over the Fermi surface, the function \( S_1(T) \) is given by eqn. [63] and

\[ S_3(T) = \pi T \sum_{n} \frac{1}{|\omega_n|^3} = \frac{7\zeta(3)}{4\pi^2T^2}. \]  

(69)

The Ginzburg-Landau free energy functional in real space can be obtained through a Fourier transformation and leads to

\[ F = \int d^3r \left\{ \alpha(T - T_{c0})|\eta|^2 + \eta^* \left[ K_1(D_x^2 + D_y^2) + K_2D_z^2 + K_{ij}H_iH_j + Q_{ij}H_iH_j \right] \eta \right\}, \]  

(70)

where \( \alpha = N_{0+}/2T_{c0}, T_{c0} = (2\gamma\epsilon_0/\pi) \exp(-2/V_{++}N_{0+}), \)

\[ K_1 = \frac{N_{0+}S_3}{8}\langle \phi^2(k)v_z^2(k) \rangle, \quad K_2 = \frac{N_{0+}S_3}{8}\langle \phi^2(k)v_z^2(k) \rangle, \]

(71)

\[ K_{ij} = -\frac{\mu_B N_{0+}S_3}{2}\langle \phi^2(k)\gamma_i(k)v_j(k) \rangle, \quad Q_{ij} = \frac{\mu_B^2N_{0+}S_3}{2}\langle \phi^2(k)\gamma_i(k)\gamma_j(k) \rangle. \]  

(72)

The term linear in \( H \) incorporates the magneto-electric effects while the term quadratic in \( H \) describes the paramagnetic effect. Therefore \( Q_{ij}|\eta|^2 \) is connected with the change of the paramagnetic susceptibility in the superconducting phase compared with the normal state (Pauli) susceptibility. In particular, \( Q_{ij} \) vanishes when there is no change of the paramagnetic susceptibility. These coefficients have to be compared with those of a spin singlet state of a centrosymmetric superconductor, \( Q_{ij}^{(0)} = \mu_B^2N_0S_3/2 \) which, assuming \( N_{0+} = N_0 \), is larger than above \( Q_{ij} \) due to the fact that \( 1 = \langle \phi^2(k) \rangle \geq \langle \phi^2(k)\gamma_i(k)\gamma_j(k) \rangle \).

We consider now the two illustrative cases, the point group \( C_{4v}, D_4 \) which are characterized by the pseudovectors

\[ \gamma(k) = \gamma_\perp(k_\perp x - k_\perp y) + \gamma_\| k_xk_y(k_x^2 - k_y^2)k^\|z \] for \( C_{4v}, \)
\[ \gamma(k) = \gamma_\perp(k_\perp x + k_\perp y) + \gamma_\| k_xk^\|z \] for \( D_4. \)

(73)

For symmetry arguments and using above expressions we find the following relations for the coefficients,

\[ K_{xy} = -K_{yx} \neq 0 \quad \text{and} \quad K_{ij} = 0 \quad \text{otherwise} \]
\[ Q_{xx} = Q_{yy} \neq Q_{zz} > 0 \quad \text{and} \quad Q_{ij} = 0 \quad \text{otherwise} \]

(74)

for \( C_{4v}, \) where presumably \( |K_{zz}| \ll |K_{xy}| \) and \( Q_{zz} \ll Q_{xx} \) due to large number of nodes in the \( k \)-dependence of the \( \gamma_\| \)-part of \( \gamma(k) \), and

\[ K_{xx} = K_{yy} \neq 0, K_{zz} \neq 0 \quad \text{and} \quad K_{ij} = 0 \quad \text{otherwise} \]
\[ Q_{xx} = Q_{yy} \neq Q_{zz} > 0 \quad \text{and} \quad Q_{ij} = 0 \quad \text{otherwise} \]

(75)

for \( D_4. \)

1. **Symmetry \( C_{4v}, H \parallel \hat{z} \)**

In the case of \( C_{4v} \) for the field directed parallel to \( z \)-axis \( H = H(0,0,1) \) the terms linear in gradients and \( H \) is absent. The standard solution \( \eta = e^{i\psi}f(x) \) of the GL equation

\[ \left\{ \alpha(T - T_{c0}) + K_1 \left[ -\frac{\partial^2}{\partial x^2} + \left( -i \frac{\partial}{\partial y} + 2\gamma Hx \right)^2 \right] + Q_{zz}H^2 \right\} \eta = 0, \]  

(76)
is degenerate in respect to $q_y$. The magnetic field dependence of critical temperature is

$$T_c = T_{c0} - \frac{2eK_1}{\alpha} H - \frac{Q_{zz}}{\alpha} H^2$$  \hspace{1cm} (77)

Both the orbital (linear in $H$) and paramagnetic (quadratic in $H$) depairing effect are present. Compared to the ordinary spin-singlet case, however, the effect of the paramagnetic limiting is weaker here due to $Q_{zz} < Q_{zz}^{(0)}$. It is important to note here that no magneto-electric effect comes into play here.

2. Symmetry $D_4$, $\mathbf{H} \parallel \hat{z}$

The situation is quite different for the uniaxial crystals with point symmetry group $D_4$ (or $D_6$). The GL equation includes gradient terms in the field direction and acquires the form

$$\left\{ \alpha(T - T_{c0}) + K_1 \left[ \frac{\partial^2}{\partial x^2} + \left( -i \frac{\partial}{\partial y} + 2eH_x \right)^2 \right] + iK_{zz} H \frac{\partial}{\partial z} - K_2 \frac{\partial^2}{\partial z^2} + Q_{zz} H^2 \right\} \eta = 0.$$  \hspace{1cm} (78)

The solution can be written as

$$\eta = e^{i q_y y} e^{i q_z z} f(x),$$  \hspace{1cm} (79)

which remains degenerate with respect to the wavevector $q_y$, but not with respect to $q_z$ which is used to maximize the critical temperature to

$$T_c = T_{c0} - \frac{2eK_1}{\alpha} H + \left( \frac{K_{zz}^2}{4K_2} - Q_{zz} \right) \frac{H^2}{\alpha}.$$  \hspace{1cm} (80)

This corresponds to the finite wavevector

$$q_z = \frac{K_{zz} H}{2K_2}.$$  \hspace{1cm} (81)

Note that this wave vector could also be absorbed into the vector potential without changing the physically relevant results: $\mathbf{A} \rightarrow \mathbf{A} + \nabla \chi$ with $\chi = -q_z z / 2e$.

The simple paramagnetic depairing effect is weakened due to magneto-electric response of the system. Adjusting the nucleation of the superconducting phase to the shifted Fermi surface, as incorporated in the wavevector $q_z$, recovers some of the strength of the nucleating condensate. This is a specific effect of the non-centrosymmetric superconductor and has its conceptional analogue in the FFLO phase for centrosymmetric spin singlet superconductors, where the condensate also nucleates with finite momentum Cooper pairs in order to optimize the pairing of degenerate quasiparticles on the split Fermi surface.

3. Symmetry $C_{4v}$, $\mathbf{H} \perp \hat{z}$

Now we turn the magnetic field into the basal plane $\mathbf{H} = H (\cos \varphi, \sin \varphi, 0)$, and impose a gauge to have the vector potential $\mathbf{A} = H z (\sin \varphi, - \cos \varphi, 0)$. The corresponding GL equation take the form

$$\left\{ \alpha(T - T_{c0}) + K_1 (D_x^2 + D_y^2) - K_2 \frac{\partial^2}{\partial z^2} + K_{xy} (H_x D_y - H_y D_x) + Q_{xx} H^2 \right\} \eta = 0,$$  \hspace{1cm} (82)

where

$$D_x = -i \frac{\partial}{\partial x} + 2eH_y z, \quad D_y = -i \frac{\partial}{\partial y} - 2eH_x z.$$  \hspace{1cm} (83)

Like in ordinary superconductors the solution of this equation have the Abrikosov form

$$\eta(\mathbf{r}) = \exp \left[ i (\mathbf{p} \times \mathbf{r})_z \right] f(z),$$  \hspace{1cm} (84)
where we write $\mathbf{p} = p\mathbf{H}/H$ as a vector parallel to the magnetic field ($\left((\mathbf{p} \times \mathbf{r})_z\right)$ denoting the $z$-component of the vector $\mathbf{p} \times \mathbf{r}$), and $f(z)$ satisfies the resulting renormalized harmonic oscillator equation

$$
\left\{ \alpha(T - T_{c0}) + K_1(2eH)^2(z - z_0)^2 - K_2 \frac{\partial^2}{\partial z^2} + \left( Q_{xx} - \frac{K_{xy}^2}{4K_1} \right) H^2 \right\} f(z) = 0, 
$$

(85)

with the shifted equilibrium position

$$
z_0 = (2eH)^{-1} \left( p + \frac{K_{xy}}{2K_1} H \right). 
$$

(86)

Thus, the vector $\mathbf{p}$ is absorbed into the shift $z_0$ and does not appear anywhere else in the equation. Then the corresponding eigenvalue determines the magnetic field dependence of optimized critical temperature:

$$
T_c = T_{c0} - \frac{2e\sqrt{K_1K_2}}{\alpha} H + \left( \frac{K_{xy}^2}{4K_1} - Q_{xx} \right) \frac{H^2}{\alpha}. 
$$

(87)

In the used gauge the eigenstates are degenerate with respect to $p$ and acquire the same structure as the usual Landau degeneracy. Nevertheless, the characteristics of the non-centrosymmetry incorporated in the $K_{ij}$-terms appears in the expression of $T_c$. Similar to the previous case of $D_4$ with $\mathbf{H} \parallel z$ the magneto-electric effect yields a reduction of the paramagnetic limiting term. This renormalization is surprisingly strong in general, as we can see when we return to the expressions which we had derived for the different coefficients. We obtain for the last term in Eq. (87),

$$
\left[ \frac{K_{xy}^2}{4K_1} - Q_{xx} \right] \frac{H^2}{\alpha} = \left[ \frac{\langle \phi^2(k) \phi_x(k) \rangle^2}{\langle \phi^2(k) \phi_z(k) \rangle} - \langle \phi^2(k) \phi_z^2(k) \rangle \right] \frac{\mu_B^2 H^2 N_a S_3}{2\alpha}. 
$$

(88)

Considering the simplified picture of a parabolic band with $\mathbf{v}(k) = k/m^*$ and a Rashba spin-orbit coupling $\hat{\gamma} = k \times \hat{z}$ (setting $\gamma_z(k) = 0$) we find the amazing result that the two terms exactly cancel and the paramagnetic effect is completely suppressed. This effect can be immediately obtained, if we perform the gauge transformation

$$
\mathbf{q} \rightarrow \mathbf{q} + \frac{2\mu_B m^*(\hat{z} \times \mathbf{H})}{k_F}. 
$$

(89)

already in Eq. (85) and so eliminating the paramagnetic term at the outset. However, it is important to notice that this exact cancellation is a consequence of the simplified forms of the band structure and the spin-orbit coupling term. Taking more realistic band structure effects into account it is obvious that this identity does not hold anymore in general. Nevertheless, our results suggests that the magneto-electric effect can, in principle, yield a substantial contribution to eliminate the paramagnetic limiting also for fields in the basal plane.

4. Symmetry $D_4$, $\mathbf{H} \perp \hat{z}$

It is easy to see that this case is analogue to the situation for the field along the $z$-axis and has only quantitative differences. Thus also here we encounter a reduction of the paramagnetic limit due to the magneto-electric effect yielding

$$
T_c = T_{c0} - \frac{2e\sqrt{K_1K_2}}{\alpha} H + \left( \frac{K_{xy}^2}{3K_1} - Q_{xx} \right) \frac{H^2}{\alpha}, 
$$

(90)

where also the same considerations concerning the gauge freedom apply as in the case of $\mathbf{H} \parallel \hat{z}$ apply.

5. Two-dimensional case, symmetry $C_{4v}$, $\mathbf{H} \perp \hat{z}$

The simplest way to pass from 3D to 2D situation it is to introduce $\delta(z)$ function potential well into 3D GL equation \[^3\]. It is equivalent to the theory used by Tinkham\[^4\] for the calculation of the upper critical field in a thin film with thickness $d << \xi$ for a field parallel to the film. Thus, we consider the instability equation

$$
\left\{ \alpha(T - T_{c0}) - K_1(D_z^2 + D_y^2) - K_2 \frac{\partial^2}{\partial z^2} + K_{xy}(H_x D_y - H_y D_x) + Q_{xx} H^2 \right. 
$$

$$
\left. - \frac{2K_2}{d} \delta(z) \right\} \eta = 0, 
$$

(91)
where $d$ is a length of the order of the film thickness that is in pure 2D case it is an atomic scale length. This eigenvalue equation has the solution
\[
\eta(r) = A \exp\left[i(p \times r)_z\right] \exp\left(-\frac{|z|}{d}\right),
\]
(92)
where $p = p\mathbf{H}/H$ is a vector with arbitrary length directed along magnetic field. This then determines the critical temperature as a function of the applied magnetic field.
\[
\alpha(T - \tilde{T}_{c0}) + K_1(2eH)^2\langle(z - z_0)^2\rangle + \left(Q_{xx} - \frac{K_{xy}^2}{4K_1}\right)H^2 = 0.
\]
(93)
Here $\tilde{T}_{c0}$ is the critical temperature in the absence of a magnetic field, corresponding to $d^2 = K_2/\alpha(\tilde{T}_{c0} - T_{c0})$. Moreover, $\langle\ldots\rangle$ denotes the expectation value using the wave function $\exp\left(-|z|/d\right)$ and $z_0$ is determined by the same expression as in the 3D case
\[
z_0 = (2eH)^{-1} \left(p + \frac{K_{xy}}{2K_1}H\right).
\]
(94)
Hence, we obtain for the critical temperature
\[
T_c = \tilde{T}_{c0} + \left[\frac{K_{xy}^2}{4K_1} - Q_{xx}\right] \frac{H^2}{\alpha} - \frac{K_1}{\alpha}(z_0^2 + d^2/2)(2eH)^2.
\]
(95)
The critical temperature reaches obviously a maximal value at $z_0 = 0$, i.e. for
\[
p = -\frac{K_{xy}}{2K_1}H.
\]
(96)
The upper critical field shows also here the square root temperature dependence usual for thin films in a parallel magnetic field\cite{41}. Under special conditions (e.g. rotation symmetry around the normal vector of the film) the expression in the square parenthesis in Eq. (95) may vanish, as described above. Then, unlike in usual superconductors, non-centrosymmetric superconductors follow the standard Tinkham behavior unchanged by paramagnetic contributions.

In view of strong inequality $d \ll 1/\sqrt{2eH}$ the complete suppression of 2D superconducting state ($T_c(H) = 0$) is reached in the field which exceeds the orbital critical field in the 3D case \cite{87}.

B. Two band case

While the one-band picture discussed so far gives useful insights into the influence of the magneto-electric effect on the upper critical field, in particular, in the context of paramagnetic limiting, in reality there are at least two split bands whose Fermi surface allows for the nucleation of a condensate in a finite magnetic field. In the two-band picture the situation is somewhat more complex, so that we restrict here to a few aspects only which, we believe, are relevant in this context without attempting to give a complete overview. We base our analysis on the formalism introduced for the homogeneous superconducting phase in section 3.2. We use also an order parameter belonging to a one-dimensional representation on the two Fermi surfaces, labeled by $\lambda = \pm$, $\hat{\Delta}_\chi(\mathbf{k}, r) = \eta_\lambda(r)\phi_\lambda(\mathbf{k})$. Moreover we restrict our discussion of the case of the point group $C_{4v}$ with an in-plane magnetic field. Then the linearized Ginzburg-Landau equation is given by
\[
\eta_+ = g_+ [S_1(T) - \hat{L}_+] \eta_+ + g_- [S_1(T) - \hat{L}_-] \eta_-, \\
\eta_- = g_- [S_1(T) - \hat{L}_+] \eta_+ + g_+ [S_1(T) - \hat{L}_-] \eta_-,
\]
(97)
with the operators $\hat{L}_\lambda$,
\[
\hat{L}_\lambda = N_{0\lambda}^{-1} \left[K_{1\lambda}(D_x^2 + D_y^2) - K_{2\lambda} \frac{\partial^2}{\partial z^2} + \lambda K_{xy\lambda}(H_x D_y - H_y D_x) + Q_{xx\lambda} H^2\right],
\]
(98)
where the coefficients are defined through the straightforward generalization of Eqs. (71) and (72) to the two-band case with the gap functions $\phi_\lambda(\mathbf{k})$, the Fermi velocity components $v_{\lambda,i}(\mathbf{k})$ and the densities of states $N_\lambda$ taken in the corresponding band.
Similar to the one-band case the solutions of this equation system can be cast into the Abrikosov form

$$\begin{align*}
\begin{pmatrix}
\eta^+(r) \\
\eta^-(r)
\end{pmatrix} &= \begin{pmatrix}
f^+(z) \\
f^-(z)
\end{pmatrix} \exp[i(p \times r)z],
\end{align*}$$

(99)

where again $p = pH/H$ and the functions $f^+(z)$, $f^-(z)$ satisfy to the system of equations

$$\begin{align*}
f^+ &= g_{++}[S_1(T) - \hat{M}_+]f^+ + g_{+-}[S_1(T) - \hat{M}_-]f^- , \\
f^- &= g_{-+}[S_1(T) - \hat{M}_+]f^+ + g_{--}[S_1(T) - \hat{M}_-]f^-.
\end{align*}$$

(100)

Using the same gauge as in the one-band example, the new operator $\hat{M}_\lambda$ is then

$$\hat{M}_\lambda = N_{0\lambda}^{-1}\left[K_{1\lambda}(2eH)^2(z - z_{0\lambda})^2 - K_{2\lambda}\frac{\partial^2}{\partial z^2} + \left(Q_{xx\lambda} - \frac{K_{xy\lambda}^2}{4K_{1\lambda}}\right)H^2\right],$$

(101)

$$z_{0\lambda} = (2eH)^{-1}\left(p + \lambda \frac{K_{xy\lambda}}{2K_{1\lambda}}\right).$$

(102)

As in the one-band case the eigen states of this system possesses the Landau degeneracy represented through the equilibrium positions of the coupled harmonic oscillators, $z_{0+}$ and $z_{0-}$, which both depend on $p$. Through the substitution

$$z = Z + \frac{p}{2eH}$$

we can formulate the equation system so that $p$ is eliminated and $z_{0\lambda} \rightarrow Z_{0\lambda},$

$$Z_{0\lambda} = \lambda \frac{K_{xy\lambda}}{4eK_{1\lambda}}.$$

(103)

The general solution of Eq.(100) can be found only numerically. Here we limit ourselves to a variational solution of the form,

$$\begin{align*}
f^+(Z) &= C_+ \exp\left\{-\frac{eH\sqrt{K_{1+}(Z - Z_{0+})^2}}{\sqrt{K_{2+}}}\right\}, \\
f^-(Z) &= C_- \exp\left\{-\frac{eH\sqrt{K_{1-}(Z - Z_{0-})^2}}{\sqrt{K_{2-}}}\right\}.
\end{align*}$$

(104)

In the following calculations, taking into account that the band splitting is much less than the Fermi energy $|\gamma_\perp|k_F \ll \varepsilon_F$, we neglect the difference of the Fermi velocities $v_\lambda(k)$ and the densities of states $N_{0\lambda}$ of the two bands. This case the values of $Z_{0\lambda} = \lambda Z_0$ for different bands differ each other only by sign. Returning to the free energy functional and integrating over $Z$ we obtain new variational equations for the coefficients $C_+$ and $C_-: 

$$\begin{align*}
C_+ &= g_{++}[S_1(T) - M(H)]C_+ + Ig_{+-}[S_1(T) - M(H)]C_- , \\
C_- &= Ig_{-+}[S_1(T) - M(H)]C_+ + g_{--}[S_1(T) - M(H)]C_-.
\end{align*}$$

(105)

with

$$M(H) = N_0^{-1}\left[2eH\sqrt{K_1K_2} + \left(Q_{xx} - \frac{K_{xy}^2}{4K_1}\right)H^2\right],$$

(106)

and

$$I = \exp(-2eHZ_0^2).$$

(107)

The system (105) has the same form as the system (11) in the absence of magnetic field. Hence for the critical temperature we obtain

$$T_c = \tilde{T}_c(1 - M),$$

(108)

where he temperature $\tilde{T}_c$ is given by the same formula (11) as $T_c$ taking in mind the substitutions $g_{+-} \rightarrow \tilde{g}_{+-} = Ig_{+-}, \ g_{-+} \rightarrow \tilde{g}_{-+} = Ig_{-+}$. The product $2eHZ_0^2 \approx eHm^*^2/(k_Fm)^2$ is much less than unity generally, except for heavy fermion or layered superconductors.
Thus, in the two-band situation the paramagnetic suppression of superconducting state $\propto -Q_{xx} H^2$ is substantially weakened by the magneto-electric effect $\propto K_{xy} H^2/4K_1$. The latter has a finite value so long we work in the limit $\mu_B H \ll |\gamma_\perp| k_F$.

This conclusion is qualitatively valid in general, although our discussion was done by a simple variational approach only. Moreover, assuming $g_{++} = g_{--}$ and $g_{+-} = g_{-+}$, as it was done in the absence of magnetic field (see eqns. (46) and (47)), we come to the solution of (105) with either pure singlet or with pure triplet pairing. The effect of paramagnetic limiting takes place identically in both cases. This underlines directly that the weakening of the paramagnetic limiting in the non-centrosymmetric superconductors is not connected to the formation of a mixed singlet-triplet state. The important point is the spin-orbital splitting of the bands. The Pauli spin susceptibility in the superconducting state is especially sensitive to the spin-orbital splitting of the quasi-particle states.

Several approximations to the two-band model have been used in literature, some of which can obscure the subtleties of non-centrosymmetric superconductors. In particular, the discussion of the spin susceptibility in the superconducting phase as discussed in\cite{30,31,32} have to be considered with caution in view of the magneto-electric effects which are neglected there. The adjustment of the superconducting state to the field-induced shifts of the Fermi surface yield a correction to the spin susceptibility which is not negligible as our discussion in the Ginzburg-Landau regime show. The subtle two-band effects, however, often make quantitative predictions difficult\cite{32}.

\section{V. Conclusion}

In this chapter we have given an overview on some theoretical aspects of non-centrosymmetric superconductors. Unlike symmetric spin-orbit coupling found in centrosymmetric metals, the antisymmetric spin-orbit coupling has a spectacular influence on the electronic bands through a specific spin splitting of the quasi-particle states. Superconductivity as a Fermi liquid instability is naturally influenced by such a modification of the electronic states. Superconductivity then has always a multi-band character. Moreover, parity does no longer provide a good quantum number to classify the superconducting phases.

One of the physically most remarkable aspects of non-centrosymmetric superconductivity is connected with magneto-electricity, the peculiar connection between supercurrents and spin polarization. We have considered one aspect in this context, namely its influence on paramagnetic limiting. This effect is of interest in strongly correlated electron systems where the coherence length is generally small due to the enhanced masses like in heavy Fermion compounds. Here ordinary orbital depairing in a magnetic field is weak, such that the upper critical field reaches magnitudes where paramagnetic limiting through spin polarization becomes visible. Interestingly already on the basis of symmetry considerations it is possible to arrive at very interesting predictions which are borne out in some of the non-centrosymmetric heavy Fermion superconductors.

\begin{thebibliography}{99}
\itemsep=0pt

1. E. Bauer, G. Hilscher, H. Michor, Ch. Paul, E. W. Scheidt, A. Grivanov, Yu. Seroegin, H. Noël, M. Sigrist, and P. Rogl, Phys. Rev. Lett. \textbf{92}, 027003 (2004).
2. T. Akazawa, H. Hidaka, T. Fujiwara, T. C. Kobayashi, E. Yamamoto, Y. Haga, R. Settai, and Y. Onuki, J. Phys.: Condens. Matter \textbf{16}, L29 (2004).
3. N. Kimura, K. Ito, K. Saitoh, Y. Umeda, H. Aoki, T. Terashima, Phys. Rev. Lett. \textbf{95}, 247004 (2005).
4. I. Sugita, Y. Okuda, H. Shishido, T. Yamada, A. Thanizhavel, E. Yamamoto, T. D. Matsuda, Y. Haga, T. Takeuchi, R. Settai, and Y. Onuki, J. Phys. Soc. Jpn. \textbf{75}, 043703 (2006).
5. G. Amano, S. Akutagawa, T. Muranaka, Y. Zenitani, and J. Akimitsu, J. Phys. Soc. Jpn. \textbf{73}, 530 (2004).
6. K. Togano, P. Badica, Y. Nakamori, S. Orimo, H. Takeya, and K. Hirata, Phys. Rev. Lett. \textbf{93}, 247004 (2004); P. Badica, T. Kondo, and K. Togano, J. Phys. Soc. Jpn. \textbf{74}, 1014 (2005).
7. G. Schuck, S. M. Kazakov, K. Rogacki, N. D. Zhigadlo, and J. Karpiński, Phys. Rev. 73, 144506 (2006).
8. L. N. Bulaevskii, A. A. Guseinov, and A. I. Rusanov, Zh. Eksp. Teor. Fiz. \textbf{71}, 2356 (1976) [Sov. Phys. JETP \textbf{44}, 1243 (1976)].
9. L. S. Levitov, Yu. V. Nazarov, and G. M. Eliashberg, Pis’ma Zh. Eksp. Teor. Fiz. \textbf{41}, 365 (1985) [JETP Letters \textbf{41}, 445 (1985)].
10. V. M. Edel’stein, Zh. Eksp. Teor. Fiz. \textbf{95}, 2151 (1989) [Sov. Phys. JETP \textbf{68}, 1244 (1989)].
11. L. P. Gor’kov and E. I. Rashba, Phys. Rev. Lett. \textbf{87}, 037004 (2001).
12. V. P. Mineev, Pis’ma Zh. Eksp. Teor. Fiz. \textbf{57}, 659 (1993) [JETP Letters \textbf{57}, 680 (1993)]; V. P. Mineev, Physica B \textbf{199-200}, 215 (1994).
13. V. P. Mineev and K. V. Samokhin, Zh. Eksp. Teor. Fiz. \textbf{105}, 747 (1994) [Sov. Phys. JETP \textbf{78}, 401 (1994)].
\end{thebibliography}
The interband pairing leads to the interesting possibility of existence of nonuniform superconducting states even in the absence of external magnetic fields. These states, however, could realized in the noncentrosymmetric compounds with the SO band splitting smaller than the superconducting critical temperature. To the best of our knowledge, in all noncentrosymmetric compounds discovered to date the relation between the two energy scales is exactly the opposite: the SO band splitting exceeds all superconducting energy scales by order of magnitude, completely suppressing the interband pairing. Both uniform and nonuniform states, however, could realized in the noncentrosymmetric compounds with

14 V. M. Edel'stein, Phys. Rev. Lett. 75, 2004 (1995).
15 V. M. Edel'stein, J. Phys.: Condens. Matter 8, 339 (1996).
16 K. V. Samokhin, E. S. Zijlstra, and S. K. Bose, Phys. Rev. B 69, 094514 (2004). See also Erratum 70, 069902(E) (2004).
17 I. A. Sergienko and S. H. Curnoe, Phys. Rev. B 70, 214510 (2004).
18 V. P. Mineev, Int. J. Mod. Phys. B 18, 2963 (2004).
19 S. K. Yip, Phys. Rev. B 65, 144508 (2002).
20 V. Barzykin and L. P. Gor'kov, Phys. Rev. Lett. 89, 227002 (2002).
21 O. V. Dimitrova, M. V. Feigel'man, Pis'ma v ZhETF 78, 1132 (2003); Phys. Rev. B 76, 014522 (2007).
22 V. M. Edel'stein, Phys. Rev. B 67, 020505 (2003).
23 K. V. Samokhin, Phys. Rev. B 70, 104521 (2004).
24 P. A. Frigeri, D. F. Agterberg, A. Koga, and M. Sigrist, Phys. Rev. Lett. 92, 097001 (2004). See also Erratum 93, 099903(E) (2004).
25 V. P. Mineev, Phys. Rev. B 71, 012500 (2005).
26 R. P. Kaur, D. F. Agterberg, and M. Sigrist, Phys. Rev. Lett. 94, 137002 (2005).
27 S. Fujimoto, Phys. Rev B, 72, 024515 (2005).
28 M. Oka, M. Ishioka, and K. Machida, Phys. Rev B, 73, 214509 (2006).
29 D. F. Agterberg and R. P. Kaur, Phys. Rev B, 75, 064511 (2007).
30 P. A. Frigeri, D. F. Agterberg and M. Sigrist, New J. Phys. 6, 115 (2004).
31 K. V. Samokhin, Phys. Rev. Lett. 94, 027004 (2005).
32 K. V. Samokhin, Phys. Rev. B 76, 094516 (2007).
33 S. Fujimoto, Phys. Rev. B 76, 184504 (2007).
34 T. Yokoyama, Y. Tanaka, and J. Inoue, Phys. Rev. B 72, 220504(R) (2005).
35 K. Borkje and A. Sudbo, Phys. Rev. B 78, 224520 (2008).
36 K. V. Samokhin, Phys. Rev. B 78, 144511 (2008).
37 K. V. Samokhin, Phys. Rev. B 72, 054514 (2005).
38 N. Hayashi, K. Wakabayashi, P. A. Frigeri and M. Sigrist, Phys. Rev. B 73, 024504 (2006).
39 Chi-Ken Lu and Sundkip Yip, Phys. Rev. B 77, 054515 (2008).
40 V. M. Edel'stein, Phys. Rev. B 72, 172501 (2005).
41 P. A. Frigeri, D. F. Agterberg, I. Milat, and M. Sigrist, Eur. Phys. J. B 54, 435 (2006).
42 V. P. Mineev and K. V. Samokhin, Phys. Rev. B 75, 184529 (2007).
43 K. V. Samokhin, Phys. Rev. B 78, 224520 (2008).
44 K. V. Samokhin, Phys. Rev. B 78, 144511 (2008).
45 K. V. Samokhin, Phys. Rev. B 72, 054514 (2005).
46 N. Hayashi, K. Wakabayashi, P. A. Frigeri and M. Sigrist, Phys. Rev. B 73, 092508 (2006).
47 K. V. Samokhin and V. P. Mineev, Phys. Rev. B 77, 104520 (2008).
48 V. P. Mineev and K. V. Samokhin, Phys. Rev. B 78, 144503 (2008).
49 G. Dresselhaus, Phys. Rev. 100, 580 (1955); L. M. Roth, Phys. Rev. 173, 755 (1968).
50 E. I. Rashba, Fiz. Tverd. Tela (Leningrad) 2, 1224 (1960) [Sov. Phys. Solid State 2, 1109 (1960)].
51 V. P. Mineev and K. V. Samokhin, Phys. Rev. B 72, 212504 (2005).
52 E. M. Lifshitz and L. P. Pitaevskii, Statistical Physics, Part 2 (Butterworth-Heinemann, Oxford, 1995).
53 G. P. Mikitik and Yu. V. Sharlai, Phys. Rev. Lett. 82, 2147 (1999).
54 F. D. M. Haldane, Phys. Rev. Lett. 93, 206602 (2004).
55 S. Hashimoto, T. Yasuda, T. Kubo, H. Shishido, T. Ueda, R. Settai, T. D. Matsuda, Y. Haga, H. Harima, and Y. Onuki, J. Phys.: Condens. Matter 16, L287 (2004).
56 The interband pairing leads to the interesting possibility of existence of nonuniform superconducting states even in the absence of external magnetic fields. These states, however, could realized in the noncentrosymmetric compounds with the SO band splitting smaller than the superconducting critical temperature. To the best of our knowledge, in all noncentrosymmetric compounds discovered to date the relation between the two energy scales is exactly the opposite: the SO band splitting exceeds all superconducting energy scales by order of magnitude, completely suppressing the interband pairing, both uniform and nonuniform.
57 H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Lett. 3, 552 (1959).
58 V. P. Mineev and K. V. Samokhin, Introduction to Unconventional Superconductivity (Gordon and Breach, London, 1999).
59 M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
60 M. Tinkham, Introduction to Superconductivity 2nd ed. (McGraw-Hill, New York, 1996).
61 Y. Yanase and M. Sigrist, J. Phys. Soc. Jpn. 76, 124709 (2007).