Effective Couplings of Dynamical Nambu-Goldstone Bosons with Elementary Fermions

Takayuki Matsuki *
Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, JAPAN

Masashi Shiotani †
Faculty of Science and Technology, Kobe University, 1-1 Rokkohdai, Nada, Kobe 657, JAPAN

Assuming dynamical spontaneous breakdown of chiral symmetry for massless gauge theory without scalar fields, we find a method how to construct an effective action of the dynamical Nambu-Goldstone bosons and elementary fermions by using auxiliary fields. Here dynamical particles are assumed to be composed of elementary fermions. Various quantities including decay constants are calculated from this effective action.

11.30.Rd, 11.15.Tk, 11.10.St

I. INTRODUCTION

According to the paper by Nambu and Jona-Lasinio [1] or more precisely the Goldstone theorem, [2] if the Lagrangian without scalar fields has a continuum global symmetry and if this symmetry is spontaneously broken, then there must appear dynamically generated massless spin-0 particles, the so-called Nambu-Goldstone (NG) bosons, the number of which is determined how the symmetry is broken. Then it is necessary to have effective interactions among the NG bosons and elementary particles to study the effects of this symmetry breakdown, either they appear as physical particles with tiny masses or are absorbed by gauge fields to make them massive.

Quite often these NG bosons acquire tiny masses through quantum corrections and/or soft symmetry breaking to give actual physical effects. In these cases we really need an effective Lagrangian to calculate those physical effects. There are a couple of examples of effective Lagrangians known among the NG bosons and elementary particles. For instance, there are the original Nambu-Jona-Lasinio model [1] of four-fermi interactions with spontaneous chiral symmetry breakdown in which the NG boson is completely massless, and the axion model [3] due to breakdown of the Peccei-Quinn symmetry [4] in which the axion (pseudo-NG boson) acquires a little mass through quantum corrections, etc. The former case, which is the first paper to stimulate the study of spontaneous symmetry breakdown, we just need to calculate the ordinary Feynman rules for elementary fermions as well as would-be NG bosons which are expressed as a chain of fermion loops. In the latter case we must assume and/or guess effective interactions with some unknown coupling constants although some constraints are enforced.

There is another interesting example in which people have calculated physical quantities without an effective action. When one calculates the pion decay constant, one uses the effective coupling between the pion (NG boson) and quarks (or nucleons), which was, utilizing the Jackiw-Johnson [5] sum rule, originally derived by Pagels and Stokar [6] applying the Ward-Takahashi (WT) identity to the axial vector vertex. This way of determining the effective coupling, i.e., direct use of the WT identity to the vertex, includes ambiguities. The Kyoto group has used the Bethe-Salpeter (BS) equation to uniquely determine this effective coupling and to calculate it perturbatively. [7] They calculate the expectation value of an axial vector current sandwiched between vacuum and pion state and relates it with the BS amplitude to obtain the pion decay constant. In these approaches there do not appear the dynamical NG bosons. Instead Pagels and Stoker use only the WT identity, and the Kyoto group uses a classical equation for the BS amplitude. Hence it does not seem to be clear how actually the NG boson couples to other elementary and/or dynamical fields as an operator form.

Here we propose one approach to construct an effective Lagrangian among the NG bosons and elementary fields, i.e., the auxiliary field method, which was once in fashion to construct various kinds of models out of four-fermi interactions and also to study chiral symmetry breakdown. [8,9] With this method one can determine uniquely the coupling of the NG boson with fermions as an operator form. In this paper to show how powerful this approach is, we start from the simplest example, $U(1)$ massless gauge theory without scalar fields to derive the coupling among

*E-mail: matsuki@tokyo-kasei.ac.jp
†E-mail: shiotani@tanashi.kek.jp
the dynamically generated NG bosons and fermions, to prove masslessness of the NG boson, and to calculate the decay constant of this NG boson in Section III. Then in Section III we proceed into SU(3) color gauge theory with iso-doublet fermions (up and down quarks) without scalar fields to do the same thing in Section II and to give the pion decay constant. The final Section IV is devoted to the summary and discussions of our method compared with others.

### II. U(1) MASSLESS GAUGE THEORY

The Lagrangian for massless U(1) gauge theory is given by

$$\mathcal{L}_0 = -\frac{1}{4} (F_{\mu \nu})^2 - \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 + \bar{\psi} i D \psi,$$

where $D_\mu = \partial_\mu - igA_\mu$ and $\alpha$ is a gauge parameter. In this paper we use Minkowski metric except for a few equations. In order to introduce auxiliary fields as dynamical ones, we first functionally integrate the partition function over gauge fields $A_\mu$. The resultant four-fermi interaction terms are rewritten by using the Fierz transformation as

$$S = \int d^4x \bar{\psi}(x) i \partial \psi(x) + \frac{1}{2} g^2 \int d^4x \, d^4y \, \bar{\psi}(x) \gamma_\mu \psi(x) D^{\mu \nu}(x-y) \bar{\psi}(y) \gamma_\nu \psi(y)$$

$$= \int d^4x \bar{\psi}(x) i \partial \psi(x) - \frac{1}{2} \int d^4x \, d^4y \left[ \bar{\psi}(x) \psi(y) D(x-y) \bar{\psi}(y) \psi(x) + \bar{\psi}(x) i \gamma_5 \psi(y) D(x-y) \bar{\psi}(x) i \gamma_5 \psi(y) \right] + \ldots,$$

where $\ldots$ expresses other modes and

$$D_{\mu \nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \left[ p^2 g_{\mu \nu} - (1 - \alpha) p_\mu p_\nu \right],$$

$$D(x-y) = \frac{g^2}{4} g^{\mu \nu} D_{\mu \nu}(x-y) = \frac{3 + \alpha}{4} g^2 \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} = -\frac{\lambda}{(x-y)^2},$$

$$\lambda = \frac{(3 + \alpha)g^2}{16\pi^2}.$$

Then by introducing the gaussian term which is quadratic in auxiliary fields and cancels four-fermi terms in Eq. (2) as

$$S_{AF} = \frac{1}{2} \int d^4x \, d^4y \left[ (\phi(x,y) - \bar{\psi}(x)\psi(y)D(x-y)) \right. \left. D^{-1}(x-y) \right. $$

$$\times \left[ (\phi(y,x) - D(y-x)\bar{\psi}(y)\psi(x) \right]$$

$$+ \frac{1}{2} \int d^4x \, d^4y \left[ (\pi(x,y) - \bar{\psi}(x) i \gamma_5 \psi(y)D(x-y)) \right. \left. D^{-1}(x-y) \right. $$

$$\times \left[ (\pi(y,x) - D(y-x)\bar{\psi}(y) i \gamma_5 \psi(x) \right] + \ldots,$$

we obtain [9],

$$S + S_{AF} = \int \, d^4x \bar{\psi}(x) i \partial \psi(x) - \int \, d^4x \, d^4y \, \psi(x) \left[ (\phi(x,y) + i \gamma_5 \pi(x,y)) \psi(y) \right.$$

$$+ \frac{1}{2} \int \, d^4x \, d^4y \left[ (\phi(y,x)D^{-1}(x-y)\phi(y,x) + (\pi(x,y)D^{-1}(x-y)\pi(y,x)) \right. $$

$$\ldots,$$

where $D^{-1}(x-y)$ is nothing but $1/D(x-y)$ and assumed are $\phi(x,y) = \phi(y,x)$ and $\pi(x,y) = \pi(y,x)$. Since main purpose of this paper is to present our idea as simple as possible, we have neglected other modes.

Next we assume that bilocal fields can be decomposed into local fields with continuum indices as follows, which is essential to the following whole arguments in this paper.

2
\[ \phi(x, y) = \phi_0(x - y) + \frac{\sigma_0(x - y)}{f} \sigma \left( \frac{x + y}{2} \right), \quad (6a) \]
\[ \pi(x, y) = \frac{\pi_0(x - y)}{f} \varphi \left( \frac{x + y}{2} \right), \quad (6b) \]

where fields with arguments \( x - y \) are considered to be classical fields and those with \( (x + y)/2 \) to be quantum ones like in the operator product expansion. Also assumed is that \( \phi_0(x - y) \) is a vacuum expectation value (VEV) of \( \phi(x, y) \). The VEV should depend only on \( x - y \) because of translational invariance. The coordinate \( (x + y)/2 \) is the center of mass system. Here a constant parameter \( f \) has massive dimension one, \( \phi_0(x - y) \), \( \sigma_0(x - y) \), and \( \pi_0(x - y) \) dimension five, and \( \sigma((x + y)/2) \) and \( \varphi((x + y)/2) \) dimension one. The difference between quantum and classical fields can be seen by Fourier-transforming fields with arguments \( x - y \) into momentum space, in which case, e.g., \( \pi(x, y) \) can be interpreted as local fields with continuum label of internal momentum. That is,

\[ \pi(x, y) = \frac{\pi_0(x - y)}{f} \varphi \left( \frac{x + y}{2} \right) = \frac{1}{f} \int \frac{d^4q}{(2\pi)^4} e^{-iqr} \tilde{\pi}_0(q) \varphi(X), \]

with \( r = x - y \) and \( X = (x + y)/2 \), which shows \( \pi(x, y) \) is equivalent to \( \tilde{\pi}_0(q) \varphi(X) \) with a continuum label \( q \).

The legitimacy of this decomposition Eqs. (6) may be supported by the following observation. The composite field in question is tightly bound due to the strong coupling which causes chiral symmetry breakdown and hence it can be decomposed into two wave functions as the first approximation, one for internal motion and another for total motion. An internal motion can be described in \( x - y \), difference of coordinates of two particles, and the whole motion in \( (x + y)/2 \), the center of mass system coordinate of two particles. Anyway this is the simplest decomposition of \( \phi(x, y) \) and \( \pi(x, y) \). This could also be justified by checking whether mass for the would-be NG boson, \( \varphi((x + y)/2) \) becomes massless or not which we will see soon.

From symmetrical point of view (rotational invariance), it holds

\[ \sigma_0(x - y) = \pi_0(x - y). \quad (7) \]

Since we are interested in describing a coupling of the NG boson, i.e., \( \varphi((x + y)/2) \), with fermions and in obtaining the equations for \( \phi_0(x - y) \) and \( \pi_0(x - y) \), functionally integrating over fermions and keeping only \( \phi_0(x - y) \), \( \pi_0(x - y) \), and \( \varphi((x + y)/2) \), we obtain

\[ S_{\text{eff}} = \frac{1}{2} \int d^4X d^4r \left[ \phi_0(r)D^{-1}(r)\phi_0(r) + \frac{1}{f^2} \left( \varphi(X) \right)^2 \pi_0(r)D^{-1}(r)\pi_0(r) \right] \]
\[ - i \text{Tr} \ln \left[ i \partial_\tau - \phi_0(r) - i \frac{1}{f} \pi_0(r)\gamma_5 \varphi(X) \right], \]
\[ = \frac{1}{2\lambda} \int d^4X \frac{d^4q}{(2\pi)^4} \left\{ \Sigma(q)\partial_q^2\Sigma(q) + \frac{1}{f^2} \left( \varphi(X) \right)^2 \tilde{\pi}_0(q)\partial_q^2\tilde{\pi}_0(q) \right. \]
\[ - \left. 4i\lambda \ln \left[ \frac{g^2 - \Sigma^2(q)}{f^2 \tilde{\pi}_0^2(q)} \right] \right\}, \quad (8) \]

where \( \lambda \) is given by Eq. (5), \( \text{Tr} \) is to take trace over coordinates \( (x \text{ and } y \text{ or } r = x - y \text{ and } X = (x + y)/2 \text{ or } X \text{ and } q \) where \( q \) is a conjugate momentum to \( r \) and gamma-matrix indices, \( \partial_q^2 \) is a dalaiberman in terms of \( q_\mu \) in Minkowski space and

\[ \Sigma(q) = \int \frac{d^4r}{(2\pi)^4} \phi_0(r) e^{-iqr}, \quad \tilde{\pi}_0(q) = \int \frac{d^4r}{(2\pi)^4} \pi_0(r) e^{-iqr}. \quad (9) \]

In calculating \( \text{Tr} \ln \), we have neglected the term \( \theta_X \) inside since we are not interested in the derivative terms in \( X \).

Now setting \( \varphi(X) = 0 \) in Eq. (8) since the VEV of this field is zero by definition \( \varphi(X) \) is the NG boson and varying \( S_{\text{eff}} \) in terms of \( \Sigma(q) \), we can obtain an equation for \( \Sigma(q) \). This is nothing but the gap equation, i.e., this gives the mass, \( \Sigma(q) \), for a fermion.

\[ \partial_q^2\Sigma(q) + \frac{4i\lambda \Sigma(q)}{q^2 - \Sigma^2(q)} = 0, \quad (10) \]

or
\[ \Sigma(q) = 4i\lambda \int \frac{d^4p}{(2\pi)^2} \frac{1}{(p-q)^2} \frac{\Sigma(p)}{p^2 - \Sigma'^2(p)}, \]  

which becomes a familiar form when converted into one in Euclidean space and in the Landau gauge \( \alpha = 0 \) or \( \lambda = 3g^2/(16\pi^2) \). Here we have used

\[ (\partial_q^2)^{-1} = -\int \frac{d^4p}{(2\pi)^2} \frac{1}{(p-q)^2}. \]

Next extracting only the bilinear terms in \( \varphi(X) \) from Eq. (8), we obtain the mass term for \( \varphi(X) \) as

\[ \frac{1}{2\lambda f^2} \int d^4X \frac{d^4q}{(2\pi)^4} (\varphi(X))^2 \bar{\pi}_0(q) \left[ \partial_q^2 + \frac{4i\lambda}{q^2 - \Sigma^2(q)} \right] \bar{\pi}_0(q), \]

which is identically zero when one identifies \( \bar{\pi}_0(q) \) with \( \Sigma(q) \) and uses Eq. (10). In other words, when one requires masslessness of the NG boson \( \varphi(X) \), then \( \bar{\pi}_0(q) = \Sigma(q) \) is automatically derived up to a constant \( f \). This proves masslessness of the NG boson, \( \varphi(X) \). Back to Eq. (5) this result means that the Yukawa coupling of the NG boson \( \varphi(X) \) with fermions is given by

\[ -\int d^4x d^4y \bar{\psi}(x) \gamma_5 \psi(x) \gamma_5 \psi(y) = -\int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \bar{\psi}(p+q) \gamma_5 \frac{i}{f} \gamma_5 \Sigma \left( p + \frac{q}{2} \right) \varphi(q) \psi(p), \]

in momentum space. Here fields with tilde are Fourier-transformed ones. The Feynman rule for the vertex of \( \bar{\psi}\varphi\psi \), i.e., fermion-anti-fermion-NG boson vertex, is finally given by

\[ \frac{1}{f} \Sigma \left( p + \frac{q}{2} \right) \gamma_5. \]

This corresponds to the Feynman diagram depicted in Fig.1 with a vertex factor given by Eq. (15).

\[ \text{FIG. 1. Bare vertex of } \bar{\psi}\varphi\psi. \]

This result coincides with that derived by using the BS equation [7] and slightly differs from that originally derived by Pagels and Stokar [6] using the WT identity. Eqs. (10) are rewritten as

\[ \phi(x, y) = \frac{\phi_0(x-y)}{f} \left[ f + \sigma \left( \frac{x+y}{2} \right) \right], \]

\[ \pi(x, y) = \frac{\phi_0(x-y)}{f} \varphi \left( \frac{x+y}{2} \right), \]

These equations mean that the fields \( \phi(x, y) \) and \( \pi(x, y) \) should be rescaled by a factor \( \phi_0(x-y)/f \) to obtain ordinary dynamical fields and a scalar field \( \sigma'(X) \) \( (= f \phi(x,y)/\phi_0(x)) \) has a vacuum expectation value \( f \) to become a massive scalar field \( \sigma(X) \).

Now we calculate the decay constant from this vertex. The decay constant in the \( U(1) \) case is given by \( 2f \) and is defined by a matrix element of the axial vector current as

\[ \langle 0 \mid \bar{\psi}(0) \gamma_\mu \gamma_5 \psi(0) \mid \varphi(q) \rangle = -2i f q_\mu. \]
In the $q_\mu \to 0$ limit, our vertex, the Beth-Salpeter vertex, and the Pagels-Stokar vertex, all give the same expression for the decay constant $f$. The factor 2 of Eqs. (17) is explained in the final Section IV, which is peculiar to $U(1)$ theory. The Feynman diagram which corresponds to the left hand side (lhs) of Eq. (17) is depicted in Fig. 2 which gives, keeping only a linear term in momentum $q_\mu$ and with the help of Eq. (15), the final answer as,

$$f^2 = \frac{1}{2(2\pi)^2} \int xdx \frac{\Sigma(x)\left(\Sigma(x) - x\Sigma'(x)/2\right)}{\left[x + \Sigma^2(x)\right]^2},$$

(18)

where momentum space is converted into Eucledean space, i.e., $x = -p^2$. Be careful when one converts the results of Fig. 2 to the r.h.s. of Eq. (17) because $q_\mu$ is an outgoing momentum.

![Fig. 2. Fermion one loop correction to the axial vector current.](image)

III. $SU(3)$ COLOR GAUGE THEORY WITH ISO-DOUBLET

Now that we have described our idea in the former section, we would like to proceed into a more realistic case, $SU(3)$ color gauge theory with massless iso-doublet fermions, $u$ and $d$ quarks.

The difference between $U(1)$ and $SU(3)$ cases is first the color factor, and secondly the flavor number. Although the arguments to obtain the pion decay constant $f_\pi$ in $SU(3)$ are almost parallel to the $U(1)$ case, there are a couple of numerical factor differences. Hence only the equations different from $U(1)$ case are described below. First we have the following coupling between the dynamical scalar and fermions,

$$- \int d^4x d^4y \psi(x) \left[ \phi(x, y) + i\gamma_5\pi^a(x, y)\tau^a \right] \psi(y),$$

(19)

where $\psi(x) = (u(x), d(x))^T$ and $\tau^a$ are the $2 \times 2$ Pauli matrices. Likewise we can easily derive masslessness of the NG boson, $\varphi^a(X)$, and the auxiliary fields given by Eqs. (13) should be replaced with

$$\phi(x, y) = \frac{\phi_0(x - y)}{f_\pi} \left[ f_\pi + \sigma \left( \frac{x + y}{2} \right) \right],$$

(20a)

$$\pi^a(x, y) = \frac{\phi_0(x - y)}{f_\pi} \varphi^a \left( \frac{x + y}{2} \right),$$

(20b)

with

$$\int \frac{d^4r}{(2\pi)^4} \phi_0(r) e^{-irr} = \Sigma(q).$$

(21)

Instead of Eq. (17) for the $U(1)$ axial vector current, the $SU(3)$ axial vector iso-doublet current satisfies

$$\langle 0 | \bar{\psi}(0) \gamma_\mu \gamma_5 \frac{\tau^a}{2} \psi(0) | \varphi^a(q) \rangle = -if_\pi q_\mu.$$

(22)

Note that the coefficient of the rhs of Eq. (22) is 1 instead of 2. The Feynman rule for the vertex $\bar{\psi}\varphi^a\psi$ in momentum space is given by
\[ \frac{1}{f_\pi} \Sigma \left( p^2 + \frac{q^2}{2} \right) \gamma^\tau, \]

and we only give the final expression for \( f_\pi \) as

\[ f_\pi^2 = \frac{N_c}{\left(2\pi \right)^2} \int x dx \frac{\Sigma (x) \left[ \Sigma (x) - x \Sigma' (x)/2 \right]}{\left[ x + \Sigma^2 (x) \right]^2}, \]

where \( N_c = 3 \) is a number of colors. Only be careful that the Fierz transformation of internal degrees of freedom, i.e., \( SU(2) \) flavor as well as \( SU(3) \) color must be taken into account, in which case use has been made of

\[ \delta_{\alpha \beta} \delta_{\delta \gamma} = \frac{1}{N} \delta_{\alpha \gamma} \delta_{\delta \beta} + \frac{4}{N} (T^a)_{\alpha \gamma} (T^a)_{\delta \beta}, \]

\[ (T^a)_{\alpha \beta} (T^a)_{\delta \gamma} = \frac{1}{2} \left( 1 - \frac{1}{N^2} \right) - \frac{4}{N} (T^a)_{\alpha \gamma} (T^a)_{\delta \beta}, \]

\[ \text{Tr} T^a = 0, \quad \text{Tr} (T^a T^b) = \frac{1}{2} \delta_{ab}, \]

where \( T^a \) are generators and \( N \) is a number of dimensions of the group.

**IV. SUMMARY AND DISCUSSION**

In this paper we have shown how useful and easy our approach is and derived that by introducing the auxiliary fields, composite fields have a coupling with fermions and can be decomposed into local fields multiplied by a factor which is given by the solution to the gap equation, that the NG bosons are actually massless, and that the decay constant \( f_\pi \) is a vacuum expectation value of a composite scalar field and calculated by a one-loop fermion diagram given by Fig.2 after taking a limit of \( q^\mu \rightarrow 0 \). These have been done both for \( U(1) \) and \( SU(3) \) massless gauge theories without scalar fields.

Now in this section, we show the relation in the simplest \( U(1) \) case between our method without the WT identity and others using the WT identity. If the chiral symmetry is not broken, the axial vector Ward-Takahashi identity is given by

\[ q_\mu \Gamma_\mu (p + q, p) = S_F^{-1} (p + q) \gamma_5 + \gamma_5 S_F^{-1} (p), \]

where \( S_F (p) \) is a fermion full propagator given by

\[ S_F (p) = \frac{i}{\not{p} - \Sigma (p)}. \]

The usual way to obtain Eq. (26) is to calculate the divergence of the T product,

\[ \int d^4 x e^{-i q x - i p y} \partial^\mu \langle 0 | T (J^5_\mu (x) \psi (y) \bar{\psi} (0)) | 0 \rangle, \]

\[ J^5_\mu (x) = \bar{\psi} (x) \gamma_5 \psi (x) = - \frac{\delta L}{\delta \partial^\mu \psi} \delta \psi. \]

Here \( J^5_\mu (x) \) is the Noether current for the chiral symmetry. Eq. (28a) can be estimated by using the following anti-commutation relation, transformation laws, and conservation equation for the chiral symmetry,

\[ \{ \psi (\vec{x}, x'), \psi (\bar{\vec{y}}, y') \} \big | \ x' = y' = 0 = i \delta^3 (\vec{x} - \vec{y}), \]

\[ [Q^5, \psi (x)] = i \gamma_5 \psi (x), \quad [Q^5, \bar{\psi} (x)] = i \bar{\psi} (x) \gamma_5, \quad Q^5 = \int d^3 x J^5_\mu (x), \]

\[ \partial^\mu J^5_\mu (x) = 0, \]

where \( Q^5 \) is an operator which generates chiral transformation. Negative sign of Eq. (28b) comes from Eqs. (29a). Now let us graphically calculate \( \Gamma_{\mu}^5 (p + q, p) \) when the chiral symmetry is spontaneously broken. In this case, since
only the fermions as well as gauge fields are elementary, a massless pole contribution to $\Gamma^5_{\mu}(p + q, p)$ can be depicted in Fig.3 and the whole $\Gamma^5_{\mu}(p + q, p)$ with a tree vertex is given by,

$$
\Gamma^5_{\mu}(p + q, p) = i \gamma_{\mu} \gamma_5 - 2i \int G(p + q, p) \frac{q_{\mu}}{q^2} + \text{regular part},
$$

(30a)

$$
G(p + q, p) = \frac{1}{f} \Sigma \left( p + \frac{q}{2} \right) \gamma_5,
$$

(30b)

where "regular part" means the remaining terms which are not singular at $q^2 = 0$.

These are the arguments of References [5–7]. Pagels and Stokar [6] have suggested the form of $G(p + q, p)$ so that the WT identity Eqs. (26, 30a) is satisfied and gives a form slightly different from Eq. (30b). The Kyoto group [7] uses Eq. (30b) as a normalization of the BS amplitude. All these arguments have meaning only in the limit of $q^2 \rightarrow 0$. We would like to put stress upon again the reason why the second term in the rhs of Eq. (30a) is added to $\Gamma^5_{\mu}(p + q, p)$.

This is because people so far have treated the NG boson as a composite particle of fermions, not an elementary field, and hence the Feynman diagram for the vertex shown in Fig.3 consisting only of fermions and gauge fields should be included.

On the other hand, in our case we treat the NG boson as an elementary field although dynamical so that we can directly derive the coupling between fermions and the NG boson. Let us see how this affects the above arguments on derivation of the WT identity. Since the NG boson as well as a scalar field are introduced in the effective Lagrangian, the Noether current must be modified so that it includes the contributions from the scalars, which is given by

$$
\tilde{J}_5^\mu(x) = J_5^\mu(x) - \frac{\delta L}{\delta \partial^\mu \varphi} \delta \varphi - \frac{\delta L}{\delta \partial^\mu \sigma'} \delta \sigma',
$$

(31)

where $J_5^\mu(x)$ is given by Eq. (28b). In our case the correct Noether current is given by Eq. (31) and hence replacing $J_5^\mu(x)$ with $\tilde{J}_5^\mu(x)$ in Eqs. (29), the following equations hold,

$$
\begin{align}
\tilde{Q}^5, \psi(x) &= i \gamma_5 \psi(x),
\tilde{Q}^5, \bar{\psi}(x) &= i \bar{\psi}(x) \gamma_5,
\tilde{Q}^5, \sigma'(x) &= 2 \varphi(x),
\tilde{Q}^5, \varphi(x) &= -2 \sigma'(x),
\tilde{Q}^5 &= \int d^3x \tilde{J}_5^\mu(x), \partial^\mu \tilde{J}_5^\mu(x) = 0.
\end{align}
$$

(32a, 32b, 32c)

Using these equations, one can easily show that the same WT identity as Eq. (26) holds since Eqs. (32b) do not affect derivation of this equation.

The second and third terms of the rhs of Eq. (31) are expected to be obtained from the kinetic terms of the scalars, which are, however, not present in the effective Lagrangian obtained after a couple of manipulations. Therefore either that kinetic terms must be generated by quantum corrections or the correction terms to the Noether current should directly be calculated. What we have done in Sect. [3] is a calculation of the correction term directly concerned with the chiral symmetry breakdown. There we have calculated Fig.2 and obtained the term proportional to $\partial^\mu \varphi$ that we need now. That is, we have required that the diagram should satify

$$
- \frac{\delta L}{\delta \partial^\mu \varphi} \delta \varphi = 2f \partial^\mu \varphi(x) + \ldots,
$$

(33)
which determines $f$ in terms of $\Sigma(x)$ after symmetry breaks down. Namely the Feynman diagram corresponding to Fig.3 is now given by Fig.4, which gives massless particle (NG boson) contribution to the vertex. This is a multiplication of two diagrams, Fig.2 and Fig.1 and in between there is a massless NG boson propagator which gives a $1/q^2$ pole term.

Fig.2 gives a factor $-2f q_\mu$, Fig.1 does $\Sigma(p + q/2) \gamma_5/f$, and the $\varphi$ propagator does $i/q^2$. After multiplying all these factors, we obtain $-2i\Sigma(p + q/2) \gamma_5 q_\mu/q^2$, which is exactly equal to the second term of the rhs of the WT identity Eq. (30a).

Finally we would like to describe another simple method to calculate the decay constant or $f$. In order to estimate Eq. (17) we lift global chiral symmetry to local one. That is, we introduce gauge fields associated with local chiral symmetry, calculate some diagram to obtain $f$, and then turn off local symmetry to have the original global chiral symmetry. The advantage to have local symmetry is to have a gauge coupling term with the Noether current, $\bar{\psi}(x) \gamma^\mu \gamma_5 \psi(x)$, as

$$A_5^\mu (x) \bar{\psi}(x) \gamma^\mu \gamma_5 \psi(x), \quad (34)$$

which helps calculate $f$. We here neglect the chiral anomaly as well as its effects due to introduction of local chiral symmetry or $A_5^\mu (x)$. This does not affect the final results. The fermion-anti-fermion-NG boson vertex is given by

$$\tilde{\Gamma}_\mu(p + q, p) = \int d^4x d^4y e^{-iqx - ipy} \langle 0| T (A_5^\mu (x) \psi(y) \bar{\psi}(0)) |0\rangle_{\text{amp}}, \quad (35)$$

which corresponds to a bare vertex $i\gamma^\mu \gamma_5$. Here the subscript "amp" means vertex functions are amputated. Taking an analogy to the ordinary elementary Higgs field, what we need is the following two-point function,

$$\tilde{\Gamma}_\mu(q) = \int d^4x e^{-iqx} \langle 0| T (A_5^\mu(0) \varphi(x)) |0\rangle_{\text{amp}}, \quad (36)$$

which is nothing but $2f q_\mu$ in the limit of $q_\mu \to 0$ which will be shown below. With the help of Eq. (34) Eq. (36) is given by a fermion one-loop diagram depicted in Fig.5 and equating this to $2f q_\mu$ gives the expression for $f$ in terms of $\Sigma(x)$. This is the simplest way to obtain the Pagels-Stokar formula compared with other methods.

To see that the above two-point function Eq. (36) gives $2f q_\mu$, we expand the ordinary "gauge-invariant" Higgs kinetic term,
\[
\frac{1}{2} |D_\mu (\sigma'(x) + i\varphi(x))|^2 \\
= \frac{1}{2} \left[ (\partial^\mu - 2iA^5_\mu) \(\sigma'(x) - i\varphi(x)\) \(\partial_\mu + 2iA^5_\mu\) \(\sigma'(x) + i\varphi(x)\) \right] = 2fA^5_\mu \partial^\mu \varphi + \ldots, \tag{37}
\]
where the charge of the scalar term \(\sigma'(x) + i\varphi(x)\) is -2 as shown by Eq. (32b) and use has been made of \(<0 | \sigma'(x) | 0 > = f\) as given by Eq. (16a). Notice that a relative sign between \(A^5_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi\) and \(2fA^5_\mu \partial^\mu \varphi\) in Eq. (37) is plus which is the same as in Eq. (31).

There are some models in which there are still gauge fields remained after functionally integrating a couple of strongly coupled gauge fields. In this case there remains local symmetry and we have terms similar to Eq. (34) which give, as we have seen, masses to remained gauge fields, which will be a next problem to be solved.

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