Optimization of helicopter rotor blade performance by spline-based taper distribution using neural networks based on CFD solutions

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ABSTRACT

The taper distribution along the span of a helicopter blade is defined using a novel method applied for the first time and considered as the main contribution of this work. This method uses cubic splines to generate modified blade shapes. The thrust and the torque values, computed by a 3-D Reynolds Average Navier Stokes solver, are used to train a Neural Networks based model. After that a constrained optimization is conducted based on this model for two different rotor speeds under hover condition. The optimization variables are the chord lengths at three different span locations: root, mid-span and tip. The optimization constraints are the torque or thrust values of the original blade and the practical limits for the chord lengths. Two optimum cases are investigated: maximum Figure of Merit with greater thrust and maximum Figure of Merit with less torque than the baseline. The major challenge of this work is to use the taper distribution as the only design parameter to obtain comparable results to other studies in literature in which more than one parameter is used. The results show that the Figure of Merit can improve by around 5% and the torque can be reduced by around 20%.

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Introduction

One of the main challenging missions for the designers is to increase the helicopter rotor blade performance in hover condition. The helicopter rotor flow is complicated due to vortical motions and instabilities. This means that more accurate analyses with high fidelity tools are required. Computational Fluid Dynamics (CFD) is a strong tool that is able to capture the complicated flow features of the helicopter flows. However, the CFD analysis of rotor blades is harder compared to the analysis of fixed-wing.

Renzoni et al. (2000) have developed a three dimensional time-accurate Euler equations solver for overlapping structured grids to analyze the aerodynamics of rotor flow. Their method was successfully applied to different helicopter test cases.

A framework based on unsteady compressible 3-D Navier Stokes equations was developed by Gecgel (2009). The framework has been applied to solve the flowfields around single and coaxial helicopter blades. The rotor flowfields were successfully captured and compared using different turbulent models.

Pomin and Wagner (2002) have carried out compressible Reynolds Averaged Navier Stokes (RANS) equations for 7A helicopter rotor blade in hover condition. They have coupled the RANS solver with a finite element model which is based on Timoshenko beam theory to account for the aeroelastic effects. The results were in good agreement with the experimental data.

Servera, Beaumier, and Costes (2001) have coupled the HOST dynamics code and the WAVES Euler aerodynamic solver to calculate a flexible rotor trim for steady forward flight. Their weak coupling method was applied to 7A and 7AD rotors and the results obtained have shown a pronounced improvement for the predictions of torsion and pitching moment.

Park and Kwon (2004) have developed a 3-D Euler solver to simulate the unsteady helicopter rotor aerodynamics on unstructured grid in hover and forward flights. They have concluded that the solver was a grid efficient and robust for the predictions of complicated unsteady rotor flowfields.

Another 3-D Euler solver based on a finite volume upwind scheme was developed by Chen, McCroskey, and Obayashi (1991) to compute the flowfields around helicopter rotor blade during forward flight condition. Their results were successfully compared with the experimental data.

Srinivasan and Baeder (1993) have addressed the capabilities of a free wake Euler and Navier Stokes
CFD methodology (TURNS) in calculating and capturing the helicopter rotor blade flowfields and acoustic details under hover and forward flight conditions. Their results were in good agreement with the experimental ones.

Allen (2004) has developed an unsteady multiblock multigrid Euler solver and a structured multiblock grid generator to simulate the rotor in forward flight. His solver and grid generator were validated against the experimental results.

Conlisk (1997) explains in details the aerodynamic challenges corresponding to the analysis of helicopter rotors. He concluded that, enhancing the performance of helicopter rotor blades is usually done by changing the distribution of the blade chord, blade twist and airfoil section along the span of the blade.

Different optimization techniques were conducted in the literature to find the optimum rotor blade shape for higher performance.

Le Pape and Beaumier (2005) present a procedure to optimize the rotor geometry using CFD tools for a case study. Their procedure is a local gradient-based optimization method. The objective is to maximize the figure of merit and the design variables are the chord, sweep, anhedral, twist distribution and the airfoils locations. They have shown that it is possible to improve the figure of merit in hover condition.

Dumont, Le Pape, Peter, and Huberson (2011) have used a gradient-based discrete adjoint method coupled with a RANS solver to optimize a blade shape for maximum figure of merit using large number of shape parameters as design variables including, twist, chord and tip modifications. Their method succeeded in providing interesting blade shapes which results in a higher figure of merit using low computer resources.

Allen and Rendall (2013) have stated that they were the first to implement a free form design optimization using compressible CFD for a rotor blade. Their optimization method is based on an efficient domain element shape parameterization. The objective was to reduce the torque. The thrust, root moments and internal volume were considered as the constraints of optimization. The results they obtained lead to large geometric deformations which exhibited a significant reduction in torque values.

The researchers Choi, Lee, Potsdam, and Alonso (2014) have optimized the UH-60A rotor blade using an adjoint-based optimization technique and a Navier-Stokes solver in a time-spectral form. They have used the leading edge location, twist distribution and chord length at different locations as the design variables to reduce the torque. They achieved a 5% decrease in torque with almost no loss of thrust.

According to the wind tunnel tests conducted by McVeigh and McHugh (1984), a significant advance in conventional rotor capability has been demonstrated. They have concluded that more enhancements can be attained from planform-structural tailoring.

An optimization approach for the design of helicopter rotor blades was described by Walsh, Bingham, and Riley (1987). The aim of the optimization was to minimize the horsepower during hover flight condition while keeping a satisfying performance during forward flight.

Vua and Leeb (2015) have presented the results of their optimization study to minimize the power required during hover and forward flight. They have combined CFD tools with a geometry generator which uses taper ratio, twist, blade root chord, taper initiation point, and airfoil distribution function coefficients as the design variables. The power was reduced by about 4%. The characteristics of the airfoil have also been improved.

Elfarra and Kaya (2018) have investigated the effect of parabolic taper distribution along the span of Caradonna–Tung helicopter rotor blade on the rotor performance. They have studied different blade shapes by setting the maximum chord length at different spanwise locations while keeping the planform area the same as the baseline blade area. The authors have concluded that, the helicopter rotor performance can be significantly improved when the maximum chord length is 1.3 times the baseline blade chord length.

Some researchers have implemented different learning methods and soft computing techniques for real-life applications. Chuntian and Chau (2002) have presented a 3-person multi-objective conflict decision model based on reservoir flood control. Their simple model can be generalized to any multi-person multi-objective conflict decision problems and can be implemented for real-life problems.

Wu and Chau (2011) attempted to accurately model the rainfall-runoff (R-R) transform using two different techniques coupled to an artificial neural network (ANN): the modular artificial neural network (MANN) and the singular spectrum analysis (SSA). They found that there is no pronounced improvement of using MANN over ANN. However, coupling ANN with SSA method leads to a better R-R prediction model. It also eliminates the lag effect of the ANN method.

Moazenzadeh, Mohammadi, Shamshirband, and Chau (2018) have simulated the evaporation component at two meteorological stations in Iran using support vector regression (SVR) and a firefly algorithm – coupled SVR (SVR-FA).

Samadianfard et al. (2019) have implemented both of empirical equations and learning techniques including support vector regression, gene expression programing,
model trees and adaptive neuro-fuzzy inference system to evaluate the relations between the global solar radiation (GSR) and several meteorological variables for real site in Iran. The results have shown that the SVR gave good GSR modeling trends.

Baghban, Jalali, Shafiee, Ahmadi, and Chau (2019) have expanded an adaptive network-based fuzzy interference system (ANFIS) code to create an accurate general model for the nanofluid viscosity prediction. The code has been expanded based on many independent variables.

Yaseen, Sulaiman, Deo, and Chau (2019) have used the enhanced version of extreme learning machine (EELM) in modeling the river flow forecasting. The results have shown that the EELM model is a robust and expert model than can be used in real-life water and river applications.

There are many researches about the Caradonna–Tung helicopter rotor blade, some of which can be seen in references (Doerffer & Szulc, 2008; Elfarra, Kaya, & Kadioglu, 2018; Mohd & Barakos, 2012; Steijl, Barakos, & Badcock, 2006).

So far in the studies shown in literature, most of the optimization methods, if not all, use the chord distribution as a design variable (among other variables) to attain a certain objective function. However, the chord distribution is shown in the investigated papers above, linearly changes from one point to another. Also, the researchers usually combine the optimum chord distribution with an optimum twist and other optimum shape parameters to enhance the performance of the rotor blade. However, increasing the number of design parameters increases the number of samples and hence results in more computations and more CPU time.

The main contribution of the current study is to change the chord distribution by passing a cubic spline between the points which define the leading edge and trailing edge of the blade. So the chord is not linearly changing, but it changes with curvature. It might be difficult to obtain high-performance results for a helicopter blade by optimizing only one design variable. However, in this study, it is shown that using the spline-based taper distribution as the only design variable leads to interesting results which are comparable to the optimization results that use more than one design parameter. This means that good improvement in the rotor performance is attained for less number of computations.

The Caradonna–Tung helicopter rotor blade (Caradonna & Tung, 1981) is the one selected as the baseline for the optimization. The objective is to maximize the Figure of Merit (FM) and minimize the torque and hence minimize the power. The shape of the blade is changed by altering the taper distribution while keeping the area of the blade fixed. The taper distribution is changed by using cubic splines which modify the corresponding chord lengths at root, mid-span and tip locations, hence producing various samples. A three-dimensional RANS CFD software is used to compute the flow around the samples and report their performance parameters such as thrust and torque. A Neural Networks (NN) model is obtained by using the CFD results of thrust and torque values which correspond to the different generated samples. The samples are generated by changing the taper distribution using the cubic spline method. Once the NN model is determined, the optimization procedure starts. In this paper, two constrained optimization processes are performed; maximum FM with thrust constrained and maximum FM with minimum torque. The flowfields of all the samples and cases are computed assuming that the blades are rigid. The optimization is carried out separately for two rotor speeds; 1250 and 1750 RPM.

**Methodology**

**Flow solver**

The three-dimensional flowfield are computed using the commercial FINE/Turbo CFD package of NUMECA International. The software is a density-based, 3-D, and multi-block finite volume solver for structural grids. For external flow over rotating blades, the RANS equations are solved in a rotating frame of reference for absolute velocities to assure a uniform flow at the external boundaries and a rotating flow around the blade. This guarantees that the rotation of the blades does not affect the far field flow. Hence, the velocities there can be considered as absolute velocities. The RANS equations in the rotating frame are given below:

\[
\frac{\partial}{\partial t} \int_V Q dV + \oint_S (F \cdot n) dS - \oint_S (F_\nu \cdot n) dS = \int_S s_T dV
\]

where the vector of conservative variables, Q, the inviscid flux, F, the viscous flux, F_\nu, and the source term s_T are given as

\[
Q = \begin{bmatrix}
\tilde{\rho} \\
\tilde{\rho}u_1 \\
\tilde{\rho}u_2 \\
\tilde{\rho}u_3 \\
\tilde{\rho}v_0 + k
\end{bmatrix}, \quad F = \begin{bmatrix}
\tilde{\rho} \tilde{w}_j \\
\tilde{\rho} \tilde{w}_1 \tilde{w}_j + \tilde{\rho} \delta_{1j} \\
\tilde{\rho} \tilde{w}_2 \tilde{w}_j + \tilde{\rho} \delta_{2j} \\
\tilde{\rho} \tilde{w}_3 \tilde{w}_j + \tilde{\rho} \delta_{3j} \\
\tilde{\rho} \tilde{h}_0 \tilde{w}_j + k \tilde{w}_j
\end{bmatrix}, \quad s_T = \begin{bmatrix}
\rho \tilde{\nu}_j \\
\rho \tilde{\nu}_1 \tilde{\nu}_j + \delta_{1j} \\
\rho \tilde{\nu}_2 \tilde{\nu}_j + \delta_{2j} \\
\rho \tilde{\nu}_3 \tilde{\nu}_j + \delta_{3j} \\
\rho \tilde{h}_0 \tilde{\nu}_j + k \tilde{\nu}_j
\end{bmatrix}
\]
\[
F_v = \begin{cases} 
0 \\
\tilde{r}_{1j} - \tau^T_{1j} \\
\tilde{r}_{2j} - \tau^T_{2j} \\
\tilde{r}_{3j} - \tau^T_{3j} \\
\tilde{u}_{ij} - \tilde{q}_j + \Theta^T_{ij} 
\end{cases}
\]

\[
s_T = \begin{cases} 
0 \\
-\bar{\rho}(\omega \times \mathbf{u}) \\
0 
\end{cases}
\]

where \( w_i \) is the relative velocity component in \( x_i \), \( u_i \) is the absolute velocity component in \( x_i \), \( \rho \) is the density, \( e_0 \) is total energy, \( p \) is the pressure, \( h_0 \) is total enthalpy, \( k \) is the turbulent kinetic energy, \( \tau^T_{ij} \) is the Reynolds stress tensor, \( \Theta^T_{ij} \) includes the turbulent heat flux tensor \( q^T_{ij} \) and other turbulent terms, \( \delta_{ij} \) is the Kronecker delta and \( \omega \) is the angular velocity.

The mesh is generated using AutoGrid5 software of NUMECA. The O4H grid topology is implemented in the mesh generation around the blade. A schematic of the 5-block grid is shown in Figure 1.

The block structure of the mesh in Figure 1 is explained below:

- Block 1 is an O block surrounding the blade
- Blocks 2 and 3 are H blocks upstream and downstream the blade respectively. They are part of the farfield boundaries.
- Blocks 4 and 5 are other H blocks located at the top and bottom of the blade.

**Boundary conditions**

The computation of the flowfield around the helicopter rotor blades requires the implementation of three types of boundary conditions; solid (at the blade and the hub), farfield and periodic boundary conditions. The solid boundaries have been treated as adiabatic walls with non-slip condition. Where, the flow variables at the external boundaries are calculated using Riemann invariants which are based on the freestream values of pressure and temperature. Please notice that the freestream velocity in hover condition is zero.

**Caradonna–Tung helicopter rotor blade**

Caradonna and Tung (1981) have conducted their experimental study of an untapered and untwisted helicopter rotor blade in the Army Aeromechanics Laboratory’s hover test facility. The rotor is two-bladed with a precone angle of 0.5 degree and aspect ratio of 6. The symmetric NACA 0012 airfoil section is used from root to tip. The experimental study was applied for different collective pitch angles and at different rotor speeds. Details about the blade can be found in (Caradonna & Tung, 1981).

The Caradonna–Tung blade has been used as the baseline blade in the current research. The geometry of the rotor was generated such that the blades are mounted to the hub via a circular cross-section. The transition from the circular section to the airfoil section starts at 10% from the rotor center of the blade span.

**Validation**

The validation study is performed according to the experimental helicopter blade of Caradonna–Tung (Caradonna & Tung, 1981). The 3-D grid was generated for a single rotor blade while the other blade was accounted for by applying the periodic boundary condition. The mesh is a structured mesh and it contains around 7 million points (about 1 million cells). The mesh is shown in Figure 2.

The relation between the first layer thickness and the desired \( y^+ \) value might be expressed in terms of a truncated solution of Balsius equation as follow:

\[
y_{wall} = 6 \left( \frac{V_{ref}}{v} \right)^{-7/8} \left( \frac{L_{ref}}{2} \right)^{1/8} y^+ \tag{3}
\]

where \( y_{wall} \) is the first layer thickness, \( V_{ref} \) is the reference velocity (in the current study it is taken as the blade tip speed), \( v \) is the kinematic viscosity of the fluid (air) and \( L_{ref} \) is a reference length (the chord length of the baseline blade).

For the rotational speed of 1750 RPM, and for a desired value for \( y^+ \) of around 1.0–2.0, the first layer thickness is calculated as nearly \( 3 \times 10^{-6} \) m. Similarly, for a 1250 RPM rotational speed, the \( y_{wall} \) is computed as about \( 4 \times 10^{-6} \) m. Since, the Spalart–Allmaras (SA) turbulence model requires a \( y^+ \) value of less than 10.0, such a \( y_{wall} \) value of \( 3 \times 10^{-6} \) m is suitable for the used turbulence model at the tested RPM speeds. A Turbulence model study was conducted (not shown in this paper).
and four models were tested; $k$-$\epsilon$ (Launder Sharma), $k$-$\epsilon$ with Extended Wall Function, Shear Stress Transport (SST) and Spalart-Allmaras. It was found that SA model gave fare enough close results to the experimental data at less computational time compared to the other tested models. That is why the turbulence model used in all the computations of this study is the SA model.

To speed up the computations, parallel computing with 16 processors has been used. A typical converging computation takes about 70–80 min. It should be noticed that all the meshes generated in this research have the same topology and settings as the mesh of the baseline blade.

The validation case presented in this section corresponds to the experimental data for a collective pitch angle of 8 degrees and at rotational speeds of 1250 and 1750 RPM. The results for the pressure coefficients calculated at different spanwise sections at speed of 1250 RPM are validated against the experimental data in Figure 3. While the computed sectional lift (thrust) coefficients at speed of 1750 RPM are compared to the experimental data in Figure 4. The computational results for the pressure coefficients match very well against the experimental data for both 1250 RPM (Figure 3) and 1750 RPM (obtained in a previous study by Elfarra & Kaya, 2018). However, the sectional lift coefficient is not in a very well agreement to the experimental results especially at the blade tip. The sectional lift coefficient in both the experiment and CFD is calculated from the integration of the chordwise pressure coefficients at the required section. The pressure was measured in the experiment at a fewer number of points (around 26 points) compared to the CFD results in which the pressure was calculated at around 200 points. So it is believed that the error in matching is mainly due to the integration process. Similar results were also obtained by another work (Mohd & Barakos, 2012).

**Cubic Splines**

The chord length distribution along the span is determined using the cubic splines as explained below:

1. The chord lengths at the root, mid and tip locations along the span are given as input
2. Fit two cubic polynomials between the root and mid and mid and tip chord length values. Fitting is such that the chord length value at the mid is continuous together with the first and the second derivatives with respect to the span direction.
3. The missing 1st derivative values at the root and at the tip are determined using an optimization process in which the generated chord length distribution is as close as possible to the original chord length distribution.
4. The constraint of the optimization process is that the planform area of the blade is equal to the baseline area.

The mathematical formulations for the cubic spline are given in Equation (4).

\[
\begin{align*}
  f_1(y) &= a_1(y - y_{\text{root}})^3 + b_1(y - y_{\text{root}})^2 \\
                   &\quad + c_1(y - y_{\text{root}}) + d_1 \\
  f_2(y) &= a_2(y - y_{\text{mid}})^3 + b_2(y - y_{\text{mid}})^2 \\
                   &\quad + c_2(y - y_{\text{mid}}) + d_2
\end{align*}
\]

where $y$, is the coordinate in the spanwise direction.

The conditions to determine the unknown parameters, $a_1, b_1, c_1, \ldots, d_2$ are given in Equation (5).

\[
\begin{align*}
  f_1(y_{\text{root}}) &= c_{\text{root}} \\
  f_1(y_{\text{mid}}) &= c_{\text{mid}} \\
  f_2(y_{\text{mid}}) &= c_{\text{mid}} \\
  f_2(y_{\text{tip}}) &= c_{\text{tip}} \\
  \frac{df_1}{dy}{|y = y_{\text{mid}}} &= \frac{df_2}{dy}{|y = y_{\text{mid}}} \\
  \frac{d^2f_1}{dy^2}{|y = y_{\text{mid}}} &= \frac{d^2f_2}{dy^2}{|y = y_{\text{mid}}}
\end{align*}
\]

(5)
There are 6 conditions against 8 unknowns. Therefore two more conditions are needed. One condition is the area constraint:

\[ \int_{y_{\text{root}}}^{y_{\text{mid}}} f_1(y) dy + \int_{y_{\text{mid}}}^{y_{\text{tip}}} f_2(y) dy = A \]  

(6)

where \( A \) is the baseline area. The last condition comes from the solution of the following optimization problem:

\[
\min_{a_1, b_1, \ldots, a_2} \int_{y_{\text{root}}}^{y_{\text{mid}}} (f_1(y) - c)^2 dy + \int_{y_{\text{mid}}}^{y_{\text{tip}}} (f_2(y) - c)^2 dy
\]  

(7)

where \( c \) is the fixed chord length of the baseline blade. The reason for the last condition (Equation (7)) is to obtain shapes which are as close as possible to the baseline shape to avoid unpractical planforms.

**Neural networks model**

A feedforward backpropagation system is used in the learning process. This system is sort of an artificial neural network (ANN) model which consists of a series of layers starting with the input layer and ending with the output layer. The layers in between are called hidden layers. Each one of those layers is attached to the neighboring layer through a transfer function. The output layer might also use the transfer function before generating the output. The data which correspond to the transfer function are the independent parameters which are transported by neurons in a similar way to the neurons transported in a biological system. The data
enter a certain layer is linearly weighted and gathered in accordance to the total number of neurons in this particular layer. The main aim of the NN model is to train the network by finding the weight matrices and bias vectors.

The feedforward networks of the NN model are used to map the inputs to the outputs. A one-hidden layer feedforward network is constructed to fit the computed values of torque and/or thrust as functions of chord lengths at root, mid-span and tip locations. In this study, the hidden layer contains 5 neurons. The reason behind selecting 5 neurons is explained as follow. There are three inputs (root, mid and tip chord lengths) to the NN for a two-parameter outputs (thrust and torque). Based on the previous experiences, 5 neurons with the activation function of \( \tanh \) are known to be sufficient to train such a system without overfitting (Kaya & Elfarra, 2019). The hyperbolic tangent function is selected as the transfer function of the hidden layer. Figure 5 shows a schematic plot of the used neural network model. In the figure, \( w^h \) and \( w^o \) are the linear combination weights of the data which enter the hidden layer and the output layers, respectively. \( b^h \) and \( b^o \) are the bias of the layers. Notice that Input 1, in Figure 5, is the chord length at blade root, Input 2 is chord length at the mid-span of the blade and Input 3 is the chord length at the blade tip. On the other hand, Output 1 and Output 2 correspond to the computed values of thrust and torque.

The fitted function and the parameters which will be evaluated by mapping from the inputs to the outputs are shown in Equation (8). 17 unknowns (elements of \( w^h \), \( w^o \), \( b^h \), \( b^o \)) exist in total which are calculated by the NN training. The \( \tanh \) function in Equation (4) is the selected hyperbolic tangent function. This function is implemented to its array variables in an element by element fashion. In this study, the training of the applied Neural Network model is conducted by using the Levenberg-Marquardt backpropagation method (Hagan & Menhaj, 1994). Any converged backpropagation should yield a vanishing gradient value.

\[
\begin{bmatrix}
\text{Output 1} \\
\text{Output 2}
\end{bmatrix}_{2 \times 1} = [w^o]_{2 \times 3} \cdot \tanh \left( [w^h]_{3 \times 2} \cdot \begin{bmatrix}
\text{Input 1} \\
\text{Input 2}
\end{bmatrix}_{2 \times 1} + [b^h]_{3 \times 1} \right) + [b^o]_{2 \times 1}
\]

(8)

**Optimization**

The training of the neural network is carried out with a specific performance criterion to determine the values of the bias and weights in the transfer function (Equation (8)). The weights are shown by \( w \) and the bias by \( b \) in this equation. After that, an optimization process is conducted to find the extrema of this analytical function which fits a relation between the input and the output.

In the current research, two optimization problems are solved as shown below:

1. **Thrust constrained – figure of merit maximization:**
   The optimization problem given by Equation (9) is solved for a maximum figure of merit while keeping the thrust more than or equal to the baseline...
thrust. In this problem, there are no constraints on the torque values.

\[
\text{maximize} \quad \text{Figure of Merit} \\
\text{Input 1, Input 2,} \quad \text{Thrust} = \text{Output 1(Input 1, Input 3)} \\
\text{Input 2, Input 3} \quad \geq \text{Thrust}_{\text{baseline}} \\
\text{subject to} \quad l_1_{\text{min}} \leq \text{Input 1} \leq l_1_{\text{max}} \\
\quad l_2_{\text{min}} \leq \text{Input 2} \leq l_2_{\text{max}} \\
\quad l_3_{\text{min}} \leq \text{Input 3} \leq l_3_{\text{max}} \\
\] (9)

(2) Torque constrained – figure of merit maximization: The optimization problem given by Equation 10 is solved for a maximum figure of merit while keeping the torque value less than or equal to the baseline torque. In this problem, there are no constraints on the thrust values. This case corresponds to the minimum torque and hence minimum energy requirement.

\[
\text{maximize} \quad \text{Figure of Merit} \\
\text{Input 1, Input 2,} \quad \text{Torque} = \text{Output 2(Input 1, Input 3)} \\
\text{Input 2, Input 3} \quad \geq \text{Torque}_{\text{baseline}} \\
\text{subject to} \quad l_1_{\text{min}} \leq \text{Input 1} \leq l_1_{\text{max}} \\
\quad l_2_{\text{min}} \leq \text{Input 2} \leq l_2_{\text{max}} \\
\quad l_3_{\text{min}} \leq \text{Input 3} \leq l_3_{\text{max}} \\
\] (10)

Where the Figure of Merit is defined in Equation (11) as

\[
\text{FM} = \frac{C_T^{3/2}}{\sqrt{2C_Q}} \\
\] (11)

where \(C_T\) and \(C_Q\) are the thrust and torque coefficients respectively.

The problems in Equations (9) and (10) can be solved by transforming the constrained optimization into an unconstrained optimization. This is accomplished by adding the constraints to the objective function as exterior penalty functions (Garret, 2001). The final formulations of the problems are shown in Equations (12) and (13). Those problems are solved by using the conjugate gradient method (Douglass, 1964) which is a

Table 1. The generated samples.

| Cases | Root | Mid-Span | Tip |
|-------|------|----------|-----|
| 1 (baseline) | 1.0  | 1.0      | 1.0 |
| 2     | 0.6  | 0.6      | 0.6 |
| 3     | 0.6  | 0.6      | 1.0 |
| 4     | 0.6  | 0.6      | 1.4 |
| 5     | 0.6  | 1.0      | 0.6 |
| 6     | 0.6  | 1.0      | 1.4 |
| 7     | 0.6  | 1.4      | 0.6 |
| 8     | 0.6  | 1.4      | 1.0 |
| 9     | 0.6  | 1.4      | 1.4 |
| 10    | 0.6  | 1.4      | 1.4 |
| 11    | 1.0  | 0.6      | 0.6 |
| 12    | 1.0  | 0.6      | 1.0 |
| 13    | 1.0  | 0.6      | 1.4 |
| 14    | 1.0  | 1.0      | 0.6 |
| 15    | 1.0  | 1.0      | 1.4 |
| 16    | 1.0  | 1.4      | 0.6 |
| 17    | 1.0  | 1.4      | 1.0 |
| 18    | 1.0  | 1.4      | 1.4 |
| 19    | 1.4  | 0.6      | 0.6 |
| 20    | 0.3  | 1.0      | 0.3 |
| 21    | 1.0  | 1.4      | 0.3 |
| 22    | 0.3  | 1.0      | 1.0 |

Figure 7. Blade shapes: (a) baseline blade, (b) Case 8, (c) Case 12, (d) Case 16, (e) Case 19, (f) Case 21.
gradient-based optimization algorithm.

maximize \( \text{Figure of Merit} + r_p \{ \max \{0, -\text{Output 1} + \text{Thrust}_{\text{baseline}}\}^2 \) 
\( + \max \{0, \text{Input 1} + l_1_{\max}\}^2 \)
\( \text{Input 1, Input 2, Input 3} + \max \{0, -\text{Input 1} + l_1_{\min}\}^2 \)
\( \max \{0, \text{Input 2} + l_2_{\max}\}^2 \)
\( + \max \{0, -\text{Input 2} + l_2_{\min}\}^2 \)
\( \max \{0, \text{Input 3} + l_3_{\max}\}^2 \)
\( + \max \{0, -\text{Input 3} + l_3_{\min}\}^2 \) \}

(12)

maximize \( \text{Figure of Merit} + r_p \{ \max \{0, \text{Output 1} - \text{Torque}_{\text{baseline}}\}^2 \) 
\( \text{Input 1, Input 2, Input 3} + \max \{0, -\text{Input 1} + l_1_{\min}\}^2 \)
\( \max \{0, \text{Input 2} - l_2_{\max}\}^2 \)
\( + \max \{0, -\text{Input 2} + l_2_{\min}\}^2 \)
\( \max \{0, \text{Input 3} - l_3_{\max}\}^2 \)
\( + \max \{0, -\text{Input 3} + l_3_{\min}\}^2 \) \}

(13)

**Figure 8.** Convergence history of NN training performance at 1250 RPM.

**Figure 9.** Response surfaces given by NN at 1250 RPM. Dots show the CFD computed values. (a) FM, (b) Thrust, (c) Torque.
At the start of the optimization, a small value has been assigned to the multiplier, \( r_p \), in Equations (12) and (13). Then, \( r_p \), is increased by a factor during the optimization iterations. The initial multiplier in this work has been selected as 1, while the factor is set at 3.

The optimization process is summarized in the flow chart given in Figure 6.

The process starts with the generation of the samples based on the cubic spline method. The CFD solutions give the flowfields around the blades (the generated samples). The torque and thrust values for each sample are calculated based on the CFD solutions. The NN is trained to fit a function between input parameters (which define the blade shape) and the output parameters (which are the torque and thrust values). After that, the figure of merit based on the torque and thrust values given by the NN model is calculated using Equation (11). The figure of merit is maximized in two different ways which means that two single-objective optimization problems are solved. In the first way, the FM is maximized such that the torque is less than or equal to the torque of the baseline blade. In the other optimization way, the FM is maximized such that the thrust force is greater than or equal to the thrust value of the baseline blade.

The reason why two separate single objective optimizations are implemented instead of using multi-objective optimization for both the torque and thrust at the same time is that, in the multi-objective optimization of this problem, it was not possible to obtain a textbook-style Pareto front. The Pareto front that was obtained was flat either horizontally or vertically. Therefore, a single objective optimization for maximizing FM directly by constraining the thrust or torque was applied.

**Design of experiment**

The taper distribution is obtained by fitting a cubic spline which satisfies the chord length values at the blade root, mid-span and tip locations provided that the blade planform area is kept the same as the baseline blade area. The taper stacking point location is kept fixed at 25% chord as in the original blade.

The cases in the design of experiment are shown in Table 1. As seen in this table, the flowfields of 22 samples

![Figure 10](image-url). Response surfaces given by NN at 1750 RPM. Dots show the CFD computed values. (a) FM, (b) Thrust, (c) Torque.
need to be computed by the CFD solver. The 22 solved samples are then fed into the NN model. Some of the generated cases are sketched in Figure 7. Please notice that Out of the 22 samples from the design of experiment, 19 samples were used for training. The remaining 3 samples were randomly selected for testing. It should be clear that the thickness of the sample blades is variable and different from the baseline blade thickness. The thickness will increase at some locations (according to the chord length) and decrease at some other locations along the blade span. However, since the area is kept fixed, the volume of the blade is expected to be very similar. All the cases analyzed in the current study are investigated at speeds of 1250 and 1750 RPM under pitch angle of 8 degrees.

**Results and discussion**

The optimization process is conducted separately for two different rotor speeds; 1250 and 1750 RPM. The trained Neural Network converged to the required performance (below $1.0 \times 10^{-6}$) nearly after 500 iterations. The gradient of the backpropagation method has decreased to about $10^{-9}$. This value is very close to zero. The performance of training is defined as the mean square error of the residual of Equation (4) and its convergence is shown in Figure 8 for the 1250 RPM rotor speed.

The response surfaces of the figure of merit, thrust and torque values determined by using the trained NN model at 1250 and 1750 RPM speeds are given in Figures 9 and 10 respectively. Figure 9(a) represents the response surface of the FM at different root, mid and tip chord ratios. Similarly, Figure 9(b,c) show the response surface for the thrust and torque respectively at different chord ratios in the root, mid and tip. It is noticed in the figures that the maximum FM and minimum torque are obtained when both of the root and mid chord ratios are large while the tip root ratio is small. On the other hand, the maximum thrust is obtained when the mid and tip ratios are high while the root chord ratio is small.

The response surfaces for the rotor speed of 1750 RPM are plotted in Figure 10. Similar conclusion can be drawn from Figure 10(a–c). Again, for higher FM and lower torque, the root and mid chord ratios should be kept large while the tip chord ratio should be small.

As mentioned before, two optimization problems (cases) are solved. The first case corresponds to constrained thrust for maximum FM and the second case belongs to constrained torque for maximum FM. The results of optimization are shown in Tables 2 and 3.

**Table 2.** Enhancement in FM, thrust and torque values for the optimum blades.

| Optimization Case                  | RPM  | Increase in FM (%) | Increase in Thrust (%) | Decrease in Torque (%) |
|------------------------------------|------|---------------------|------------------------|------------------------|
| Thrust constrained FM maximization | 1250 | 1.79                | 0.00                   | 1.77                   |
| Torque constrained FM maximization |      | 4.94                | -11.25                 | 20.35                  |
| Thrust constrained FM maximization | 1750 | 1.40                | 0.00                   | 1.40                   |
| Torque constrained FM maximization |      | 4.47                | -11.09                 | 19.82                  |

**Table 3.** Optimum taper distribution.

| Optimization Case                  | RPM  | Normalized chord length |
|------------------------------------|------|-------------------------|
|                                    |      | Root | Mid-span | Tip    |
| Thrust constrained FM maximization | 1250 | 0.438 | 1.367    | 0.842  |
| Torque constrained FM maximization |      | 1.400 | 1.400    | 0.300  |
| Thrust constrained FM maximization | 1750 | 0.548 | 1.326    | 0.881  |
| Torque constrained FM maximization |      | 1.400 | 1.400    | 0.300  |

**Table 4.** Comparison for the optimum cases between the NN and the CFD results.

| Optimization Case                  | RPM  | NN Results | CFD Results |
|------------------------------------|------|------------|-------------|
|                                    |      | Thrust (N) | Torque (N.m) | Thrust (N) | Torque (N.m) |
| Thrust constrained FM maximization | 1250 | 566.0      | 66.6        | 566.5      | 66.7        |
| Torque constrained FM maximization |      | 502.7      | 54.0        | 503.8      | 54.2        |
| Thrust constrained FM maximization | 1750 | 1140.0     | 132.8       | 1141.0     | 132.9       |
| Torque constrained FM maximization |      | 1013.6     | 108.0       | 1014.7     | 108.1       |

**Figure 11.** Optimum blade shapes. (a) Constrained torque – FM maximization for 1250 and 1750 RPM, (b) Constrained thrust – FM maximization for 1250 RPM, (c) Constrained thrust – FM maximization for 1750 RPM.
It should be noticed that the optimum shapes are obtained within the design of experiment limits such that the chord length does not exceed 1.4 times the baseline chord at any location and also it can’t be less than 0.3 times the baseline chord at any position along the span.

Table 2 shows the optimum values of FM, thrust and torque. It is clear in this table that conducting a thrust constrained – FM maximization yields an increase of around 2% in the FM for the 1250 RPM speed and 1.4% for the 1750 RPM speed. The torque has also decreased by a similar percentage as the FM. However, there was no enhancement on the thrust values for both of the rotor speed cases.

More enhancements in the FM and torque were attained for the torque constrained – FM maximization case. Here, the figure of merit has increased by around 5% and 4.5% for the 1250 and 1750 RPM speeds respectively. The torque has significantly decreased by around 20% in both of the speed cases which means a pronounced decrease in the power requirement. However, the drawback of this optimization case is that the thrust has also decreased.

![Figure 12. Chordwise skin friction coefficient distribution for the torque constrained FM maximization case at various spanwise sections at 1250 RPM.](image)
The optimum taper distribution which corresponds to each optimization problem and rotor speed is shown in Table 3. In this table, it is noticed that the optimum taper distribution of the constrained torque – FM maximization for both rotor speeds converge to the same taper distribution of 1.4 times the baseline chord at the root and at the mid and 0.3 times the baseline chord at the tip.

For the constrained thrust – FM maximization, it is noticed that, for both of the speeds, the chord length is minimum at the root and maximum at the mid. Those results are in agreement with the results of the generated samples which are represented by Figures 9 and 10.

The flowfields of the optimum cases obtained from the Neural Networks model are computed using CFD. The CFD results are compared to the NN optimization results in Table 4. It is seen that the estimated values of thrust and torque using NN optimization for the optimum cases are very close to the computed values using CFD. The average deviation from the CFD results is less than 0.36%.

The optimum blade shapes are shown in Figure 11. One may observe that the optimum blade shapes for the constrained thrust – FM maximization of both of the rotor speeds are very similar.

The skin friction coefficient distributions around the chord at various spanwise sections of the optimum blade shape corresponding to the constrained torque – FM maximization optimum blade (right) and baseline blade (left) at 1750 RPM.

Figure 15. Comparison of vorticity contours (upper) and vorticity contour isolines (lower) between the torque constrained FM maximization optimum blade (right) and baseline blade (left) at 1750 RPM.

Figure 16. Comparison of pressure contours between the torque constrained FM maximization optimum blade and baseline blade at 1250 RPM. Left: pressure side (lower surface), right: suction side (upper surface).

Figure 17. Comparison of pressure contours between the torque constrained FM maximization optimum blade and baseline blade at 1750 RPM. Left: pressure side, right: suction side.
maximization case are plotted and compared to the baseline blade results at 1250 RPM in Figure 12. The results in Figure 12 show the deviation in the friction coefficient of the baseline blade from the optimum blade in the spanwise sections from root to tip. The sectional skin friction coefficient is calculated by integrating the chordwise skin friction distribution at different sections for both of the baseline blade and the optimum blade. The optimum blade is the blade which gives a maximum figure of merit value at lowest torque. The results are compared in Figure 13. The figure shows a reduction in the skin friction coefficient for the optimum case at most of the spanwise sections. The average reduction in skin friction drag is computed as around 20%.

The vorticity contours are plotted for the torque constrained FM maximization case and compared with the baseline blade for both 1250 and 1750 RPM rotor speeds in Figures 14 and 15 respectively. A significant reduction in vorticity in the vicinity of the trailing edge region is noticed in the case of the optimum blade for both of

![Comparison of pressure coefficient distributions at various span sections contours between the torque constrained FM maximization optimum blade and baseline blade at 1750 RPM.](image-url)
rotational speeds. This observation explains the reduction in torque caused by the optimum shape.

The static pressure contours at the suction and pressure sides are also plotted for the optimum blade (torque constrained FM maximization case) and the baseline blade in Figures 16 and 17. The difference in the static pressure between the optimum blade and the baseline blade is not very clear in those figures except at the blade tips where the suction side of the optimum blade has less pressure compared to the original blade. It is known that the difference in the static pressure causes the difference in the thrust force. To clarify this point, the pressure coefficient distributions at different span locations are plotted in Figure 18 for the speed of 1750 RPM. It is clear in this figure that, the pressure difference between the pressure side (PS) and the suction side (SS) of the baseline blade is in general more than the pressure difference in the optimum blade except towards the blade tips. This notice explains the reduction in thrust for the optimum case where the pressure difference between the PS and SS is relatively small compared to the baseline blade case.

Conclusion

In the current study, the blade shape of the Caradonna–Tung helicopter rotor blade is changed to maximize the figure of merit and to minimize the torque. Minimizing the torque for a helicopter rotor blade, results in power requirement reduction. The shape modification is conducted by optimizing the chord lengths at three different spanwise locations; root, mid and tip without changing the area of the blade. The variation in the chord lengths is attained by using cubic splines. A model based on Neural Networks and CFD computations is constructed. Two constrained optimization problems are solved based on this model; maximum FM with thrust constrained and maximum FM with torque constrained.

The optimization process is conducted for two rotor speeds. It is clear from the optimization results that a reduction of about 1.6% in torque and increase of about 1.6% in FM is possible while keeping the same thrust value as baseline. The torque can be minimized to a significant less value than the baseline torque value by conducting a torque constrained – FM optimization without any restrictions on the thrust. In this case, the torque can reduce up to around 20% with around 5% increase in the figure of merit. This case can be considered as a large enhancement for the helicopter rotor power reduction. In literature, a reduction of 18% in torque was possible but with the usage of more number of design parameters including the chord distribution.

Analyzing the optimum shapes obtained as outputs of the optimization problems, one observes that, for maximum FM and minimum torque, the chord length should be large at both the root and mid sections along the span while it should be small at the root. The disadvantage of this optimum blade case is that it yields less thrust compared to the baseline value.

The optimum shape which keeps the thrust force same as the baseline value with slight increase in FM and slight decrease in the torque has the least chord length at the root, maximum chord length at the mid and a moderate chord length at the tip.

The common result of both of the optimization problems is that, the chord length at the mid of the blade span should be maximum to increase FM and decrease the torque.

The main limitation of this work is that not all the generated samples have feasible shape to be studied. Such shapes were eliminated from the list of samples. As a future work, it is intended to increase the number of the points through which the spline is passing from 3 to 5. In this case many new shapes can be generated and an improvement in the performance is expected. In addition to that, similar work will be conducted for helicopter blades in the forward flight condition.

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