Mathematical Modelling of Risk in Portfolio Optimization with Mean-Gini Approach

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Abstract. Risk and return are the important parameters in the investment. The investors wish to find the trade-off between the risk and return. Investors will be exposed to the risk of loss in their investment. Yitzhaki [1] has introduced the Mean-Gini model in portfolio optimization to minimize the portfolio risk at the target rate of return by using Gini as risk measure. The objective function of the Mean-Gini model is to minimize the portfolio risk at the expected rate of return. The expected rate of return is measured by the mean return of the portfolio. The objective of this paper is to construct an optimal portfolio in the investment by employing the Mean-Gini model. The data of this study consists of stocks that listed in Malaysian stock market. The results of this study show that the investors can minimize the portfolio risk and able to achieve the target rate of return with the Mean-Gini model. Besides that, the composition of the stocks is different in the optimal portfolio. This study is significant because it will help investors in the investment decision analysis in order to minimize the risk.

1. Introduction
Investors wish to find the trade-off between the risk and return in their investment. Investors will be exposed to the risk of loss in their investment. Yitzhaki [1] has introduced the Mean-Gini model in portfolio optimization to minimize the portfolio risk and can achieve the target rate of return. Gini is employed as risk measure in the Mean-Gini model. The expected return is represented by the mean return. The Mean-Gini model has been studied by the past researchers [2-9]. The objective of this paper is to construct an optimal portfolio in the investment by employing the Mean-Gini model with cardinal constraint. The cardinal constraint implies that the investors can determine the number of stocks in the optimal portfolio to make it more realistic and practical for investors [10-16]. The rest of the paper is organized as follows. The next section describes the materials and methods. Section 3 discusses about the empirical results of this study. Section 4 concludes the paper.

2. Materials and Methods

2.1. Data
In this study, the data comprises weekly returns of 20 stocks that listed in Malaysia stock market construction sector. The period of this study covers from July 2011 until June 2016. The Mean-Gini
model is employed in this study by incorporating the cardinal constraint to make it more practical and realistic for investors. The Mean-Gini model is formulated as follows by incorporating the cardinal constraint:

\begin{equation}
\text{Minimize } 2\text{cov}[R_p, F(R_p)]
\end{equation}

Subject to

\begin{equation}
\sum_{i=1}^{n} x_i \mu_i = E(R_p)
\end{equation}

\begin{equation}
\sum_{i=1}^{n} x_i = 1
\end{equation}

\begin{equation}
x_i \geq 0, i = 1, ..., n
\end{equation}

\begin{equation}
\sum_{i=1}^{n} z_i = K
\end{equation}

\begin{equation}
L_i z_i \leq x_i \leq U_i z_i, i = 1, ..., n
\end{equation}

\begin{equation}
z_i \in [0,1], i = 1, ..., n
\end{equation}

where $R_p$ is the portfolio return, $F(R_p)$ is the cumulative probability distribution of the portfolio return, $x_i$ is the weight invested in asset $i$, $\mu_i$ is the expected return of asset $i$, $E(R_p)$ is the portfolio expected return, $z_i$ is the variable that equals to 1 if the asset $i$ is selected in the portfolio and equals to 0 otherwise, $K$ is the number of assets to be selected in the portfolio, $L_i$ is the lower bound of the investment proportion on asset $i$ and $U_i$ is the upper bound of the investment proportion on asset $i$. Objective function (1) defines the portfolio Gini which is twice the covariance between the portfolio return and the cumulative probability distribution of the portfolio return. Constraint (2) ensures that investors can achieve the expected rate of return. Constraint (3) ensures that the sum of weights of the assets equals to one. Constraint (4) ensures that the weights of all the assets are positive. Constraint (5) is the incorporated cardinal constraint to ensure that the number of assets to be selected in the portfolio equals to $K$. The variable $z_i$ is introduced to indicate the asset selection problem with $z_i = 1$ indicates the $i$th asset is selected in the portfolio or otherwise $z_i = 0$ for the constraint (6) and (7).

The optimal portfolio is constructed with the Mean-Gini model with cardinal constraint. Furthermore, the return distribution of the optimal portfolio is displayed in this study. The number of stocks to be invested in the optimal portfolio is set as 10 in this study [11-13].

3. Results
Table 1 shows the optimal portfolio composition of the Mean-Gini model with cardinal constraint.

| Stocks  | Percentage (%) |
|---------|----------------|
| ASUPREM | 0.00           |
| AZRB    | 0.00           |
| BENALEC | 0.23           |
As shown in Table 1, the optimal portfolio of the Mean-Gini model with cardinal constraint consists of 10 stocks which are BENALEC (0.23%), BPURI (2.35%), CRESBLD (7.87%), EKOVEST (8.59%), GADANG (6.34%), GAMUDA (29.91%), HSL (14.80%), IJM (10.49%), KEURO (5.91%) and PRTASCO (13.52%). GAMUDA is the largest component stock in the optimal portfolio which comprises 29.91%. ASUPREM, AZRB, FAJAR, JAKS, KIMLUN, MITRA, MUDAJYA, MUHIBAH, PUNCAK and WCT are not invested in the optimal portfolio because these stocks give the value 0.00%. It indicates that the weight of each stock invested in the optimal portfolio of the Mean-Gini model with cardinal constraint is different.

Table 2 presents the summary statistics of the optimal portfolio of the Mean-Gini model with cardinal constraint.

| Mean-Gini Model | Summary Statistics |
|-----------------|--------------------|
| Portfolio Mean Return | 0.0010 |
| Portfolio Risk | 0.0109 |
| Portfolio Skewness | 1.0882 |
| Portfolio Kurtosis | 9.4257 |

As reported in Table 2, the optimal portfolio of the Mean-Gini model gives the mean return at the rate of 0.0010. The optimal portfolio gives the portfolio risk at 0.0109. It implies that the investors can achieve target rate of return at minimum risk using the Mean-Gini model with cardinal constraint. Besides that, the skewness and kurtosis value of the optimal portfolio is 1.0882 and 9.4257 respectively.
Figure 1 presents the return distribution of the optimal portfolio of the Mean-Gini model.

As displayed in Figure 1, the return distribution of the Mean-Gini optimal portfolio shows positive and negative returns. Moreover, the optimal portfolio return distribution also exhibits skewness.

4. Conclusion
In conclusion, the Mean-Gini model is employed in this study by taking cardinal constraint into consideration in portfolio optimization. It is because the cardinal constraint will make it more realistic and practical for investors. The Mean-Gini model is studied in Malaysia stock market. The results of this study show that the investors can minimize the portfolio risk at the expected rate of return. Furthermore, the investors also can determine the number of stocks in the optimal portfolio with the Mean-Gini model. The future research of this study should be extended to the assets in other countries.

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