The Search for Matter with Gravitational Lensing

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Abstract. Gravitational lensing is a powerful tool to detect compact matter on very different mass scales. Of particular importance is the fact that lensing is sensitive to both luminous and dark matter alike. Depending on the mass scale, all lensing effects are used in the search for matter: offset in position, image distortion, magnification, and multiple images. Gravitational lens detections cover three main mass ranges: roughly stellar mass, galaxy mass and galaxy cluster mass scales, i.e. well known classes of objects. Various searches based on different techniques explored the frequency of compact objects over more than 15 orders of magnitude, so far mostly providing null results in mass ranges different from the ones just mentioned. In particular, no population of “compact dark objects” could be detected so far. Combined, the lensing results offer some interesting limits on the cosmological frequency of compact objects in the mass interval $10^{-3} \leq M/M_\odot \leq 10^{15}$, unfortunately still with some gaps in between. In the near future, further studies along these lines promise to fill the gaps and to push the limits further down, or they might even detect new object classes.

1 (Relevant) Basics of Lensing

The basic setup for a gravitational lens scenario is displayed in Figure 1. The three ingredients in such a lensing situation are the source S, the lens L, and the observer O. Light rays emitted from the source are deflected by the lens. For a point-like lens, there will always be (at least) two images $S_1$ and $S_2$ of the source. With external shear – due to the tidal field of objects outside but near the light bundles – there can be more images. The observer sees the images in directions corresponding to the tangents of the incoming light paths.

In Figure 1 the corresponding angles and angular diameter distances $D_L$, $D_S$, $D_{LS}$ are indicated. In the thin-lens approximation, the hyperbolic paths are approximated by their asymptotes. In the circular-symmetric case, the deflection angle is given as

$$\tilde{\alpha}(\xi) = \frac{4GM(\xi)}{c^2} \frac{1}{\xi}.$$  

where $M(\xi)$ is the mass inside a radius $\xi$. In this depiction the origin is chosen at the observer. From the diagram it can be seen that the following relation holds:

$$\beta D_S = \beta D_S + \tilde{\alpha} D_{LS}$$  

(for $\theta, \beta, \tilde{\alpha} \ll 1$; this condition is fulfilled in practically all astrophysically relevant situations). With the definition of the reduced deflection angle as $\alpha(\theta) = (D_{LS}/D_S)\tilde{\alpha}(\theta)$, this can be expressed as:

$$\beta = \theta - \alpha(\theta).$$  

This relation between the positions of images and source can easily be derived for a non-symmetric mass distribution as well. In that case all angles are vector-valued. The two-dimensional lens equation then reads:

$$\vec{\beta} = \vec{\alpha}(\vec{\theta}).$$  

For a point lens of mass $M$, the deflection angle is given by equation (1). Plugging this into equation (3) and using the relation $\xi = D_L \theta$ (cf. Figure 1) one obtains:

$$\beta(\theta) = \theta - \frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 \theta}.$$
Figure 1: a) Setup of a gravitational lens situation: The lens $L$ located between source $S$ and observer $O$ produces two images $S_1$ and $S_2$ of the background source. Relations between the various angles and distances involved in the lensing setup can be derived for the case $\tilde{\alpha} \ll 1$, as formulated in the lens equation (3).

For the special case in which the source lies exactly behind the lens ($\beta = 0$), due to the symmetry a ring-like image occurs whose angular radius is called the Einstein radius $\theta_E$:

$$\theta_E = \sqrt{\frac{4GM}{c^2 D_L D_S}}$$  \hspace{1cm} (6)

The Einstein radius defines the angular scale for a lens situation. For a massive galaxy with a mass of $M = 10^{12} M_\odot$ at a redshift of $z_L = 0.5$ and a source at redshift $z_S = 2.0$ (we used here a Hubble constant of $H = 50 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ and an Einstein-deSitter universe), the Einstein radius is

$$\theta_{E, \text{ galaxy}} \approx 1.8 \sqrt{\frac{M}{10^{12} M_\odot}} \text{ arcsec}$$  \hspace{1cm} (7)

(note that for cosmological distances in general $D_{LS} \neq D_S - D_L$). For a galactic microlensing scenario in which stars in the disk of the Milky Way act as lenses for bulge stars close to the center of the Milky Way, the scale defined by the Einstein radius is

$$\theta_{E, \text{ galactic star}} \approx 0.5 \sqrt{\frac{M}{M_\odot}} \text{ milliarcsec.}$$  \hspace{1cm} (8)

Time scales for galactic microlensing events – i.e. the duration for crossing the Einstein radius – typically range from weeks to months. For cosmological/quasar microlensing, this time scale extends to years; however, caustic crossing events can be as short as a few weeks.

More detailed introductions to gravitational lensing including some historic aspects can be found in [42], or in the textbook [37] by Schneider et al. (1992) and in the more mathematically oriented monograph [32] by Petters et al. (2001).
2 Lensing Effects/Phenomena

Light deflection/gravitational lensing has various effects on background sources. Depending on the mass of the lensing object (from comet-like objects to clusters of galaxies), on the nature of the lensed source (point-like/unresolved or extended), and on the detection (imaging and photometry in the optical and radio regime, timing for gamma rays), the actual observations can cover quite a variety of techniques. Here we consider only “strong” lensing, where the effect can be seen for each case individually (for weak lensing, see Yannick Mellier’s contribution).

2.1 How does matter affect the light of background sources?

The consequences of strong lensing are:

- **Change of position:** This is normally not observable, since we do not have any information on the “unlensed” position of a source; only in “dynamical” situations, in which the image/lens configuration changes with time, this can be observed (e.g., the sun passing in front of stars). Exactly this was, after all, the first detection of light deflection during the famous solar eclipse in 1919 [10].

- **Distortion:** The shape of resolved sources is changed by lensing. The best visualization of this effect are the giant luminous arcs.

- **(De)Magnification:** A few sources are (highly) magnified, most sources are slightly demagnified. This means that the luminosity function of a hypothetical population of cosmological “standard candles” will unavoidably be broadened (see, e.g., [40]).

- **Multiple images:** The most dramatic lensing phenomenon: multiple quasars and multiple galaxy images are observed directly, and via microlensing we have evidence of unresolved multiple images as well.

These effects often occur in combination. A slightly exaggerated visualization is provided in Figure 2 displaced, distorted, (de-)magnified and multiple images of a source shape with a particular brightness profile. Quite a variety of spectacular lensing phenomena have been observed in recent years. In Figure 3 four of the most spectacular examples are presented: Multiply-imaged quasars, Giant luminous arcs, Einstein rings and quasar microlensing.

![Figure 2: a) Magnification distribution in source plane due to a number of point lenses (light grey means high magnification), with specific source profile superimposed; b) Corresponding image configuration plus lens positions](image-url)
2.2 Lensing phenomena:

**Multiple quasars** Multiply-imaged quasars were the first category of lensed systems to be discovered [38]. By now more than 60 multiply-imaged quasar systems have been found, most of them doubles or quadruplets, recently even a six image configuration was discovered [35]. The angular separations range from a few tenth of an arcsecond to about 8 arcseconds. The quasars are typically at redshifts between $1.0 \leq z_Q \leq 4.5$. In almost all cases, the lens is identified to be an intermediate galaxy, in some cases “assisted” by a nearby group of galaxies. Up-to-date tables of multiply-imaged quasars and gravitational lens candidates are provided, e.g., by the CASTLE group [11].

Figure 3: Four examples of strong lensing: a) Double quasar HE1104-1805 (top left, [8]): deconvolved infrared (J-band) image of the two quasar images ($z_Q = 2.316$, $\Delta \theta = 3.2$ arcsec) and the lensing galaxy (at $z_G = 1.66$); b) Giant luminous arcs in cluster CL0024 (top right, [7]): five spectacular images of a high redshift galaxy seen lensed by a galaxy cluster (redshift $z_L = 0.39$) with radius of curvature of about 20 arcseconds. c) Einstein ring B1938 (bottom left, see [17]): circular image with diameter 0.95 arcseconds; d) Microlensing in Q2237+0305 (bottom right, see [45, 46]): the lightcurves of the four images vary independently of each other, intrinsic variability can be excluded.
**Einstein rings** A particular class of lenses are the Einstein rings, circular images of extended background sources. This scenario happens when there is perfect alignment between source, lens and observer. Since the radius of the Einstein ring is proportional to the square root of the mass of the lens, these systems are very good laboratories for weighing galaxies. The most remarkable example so far is the Einstein ring B1938+666 [17]. An infrared HST image shows an almost perfectly circular ring with two bright parts plus the bright central galaxy. By now about a half dozen cases have been found that qualify as Einstein rings [11]. Their diameters vary between 0.33 and about 2 arcseconds. All of them are found in the radio regime, some have optical or infrared counterparts as well.

**Giant luminous arcs and arclets** Rich clusters of galaxies at redshifts beyond $z \approx 0.2$ with masses of order $10^{14} M_\odot$ are very effective lenses if they are centrally concentrated. Since most clusters are not really spherical mass distributions and since the alignment between lens and source is usually not perfect, no complete Einstein rings have been found around clusters. But there are many examples known of spectacular giant luminous arcs which are curved around the cluster center, with lengths up to about 20 arcseconds. Their Einstein radii are of the order of 20 arcseconds, but cases with radii up to 35 arcseconds are known [29]. One of the best known cases is the galaxy cluster CL0024+1654 (redshift $z = 0.39$), with red cluster galaxies and nicely elongated bluish arcs [7]. Images further out are less distorted, but still clearly visibly tangentially elongated: the arclets. General results from the analysis of giant arcs and arclets in galaxy clusters are: clusters of galaxies are dominated by dark matter, and typical “mass-to-light ratios” for clusters obtained from strong (and weak, see below) lensing analyses are $M/L \geq 100 M_\odot/L_\odot$.

**Stellar and quasar microlensing** The lensing action of stellar mass objects is usually called “microlensing”. It comes in two varieties: star-star lensing, or “local” microlensing, where stars in the Galactic disk or halo deflect the light of background stars in the Galactic bulge or in nearby galaxies (LMC, SMC, M31). The second variant is star-quasar lensing, where stars in a distant (lensing) galaxy act as microlenses on a quasar at cosmological distances. In both cases, the action is measured as a characteristic light curve.

**Further examples of lensing** “Millilensing” has been proposed as well, for lensing objects of roughly $10^6 M_\odot$ objects, but has not been observationally confirmed (see below). Weak lensing, the tiny effects of galaxy clusters on background galaxies cannot be detected individually any more, but due to the coherent tangential distortion a signal can be measured when the shapes of thousands of background galaxies are analysed. “Very weak lensing” is used to measure the “cosmic shear”, the effect of the large scale structure of the universe on background galaxies.

### 3 Tracing compact dark/bright matter with Lensing

Gravitational lensing is a good means to detect compact matter along the line of sight. “Compact” in this respect means: the size of the potential lens has to be of order its Einstein radius or smaller. In practice, this means

$$r < 0.02(M/M_\odot)^{0.5} \text{pc},$$

which is easily fulfilled for stellar objects or galaxies. Lensing can also detect gradients of the surface mass density, i.e. smoothly varying surface mass density, (cf. Yannick Mellier’s contribution on (very) weak lensing). However, even a large amount of matter distributed with a constant surface mass density over the sky would not be detectable with point objects (it would affect the sizes of extended background objects, but in order to evaluate it, this would require prior knowledge of unlensed sizes).

Lensing effects can be detected in two ways: static measurements: (a) positions (separations) and shapes of objects are determined which do not change over centuries or longer time scales; and (b) dynamic measurements: changes in brightness or positions are measured, usually on timescales of years or shorter. In the following, we will discuss various mass regimes with respect to what lensing can tell us about a possible cosmic population. What is particularly important: No-shows matter!
3.1 There are few (if any) Machos in galactic halos (or elsewhere)

Paczynski showed in 1986 that microlensing can be used to test whether the halo of the Milky Way is made of compact objects of stellar or substellar mass. Occasionally, one of the hypothetical objects should pass in front of a background star in the Large Magellanic Cloud, magnifying it in a very characteristic way. A few years later, three teams set out to measure this effect: MACHO, EROS, and OGLE. They did detect a number of events, but not as many as one would expect, if the halo was made entirely of such Machos. The latest results of MACHO and EROS are consistent with each other: after 5.7 years of MACHO monitoring of 12 million LMC stars, 13-17 events had been recorded (depending on the exact definition of a microlensing event). The conclusions are: 20% of the Galactic halo could be made of objects in the mass range \(0.15 \leq M/M_\odot \leq 0.9\). The EROS team arrived at similar conclusions: objects smaller than a few solar masses are ruled out as important component of the Galactic halo. Due to the lack of long events (order years or longer), the MACHO team could also put limits on black holes/dark matter objects in the mass range \(1.0 \leq M/M_\odot \leq 30\).

Microlensing can be used to test the compact population of halos of other galaxies as well. In the double quasar Q0957+561, image B is seen through the bright part of the lensing galaxy, 1 arcsec off the center, whereas image A is visible through the halo of this galaxy, about 5 arcsec away from its center. If the halo of this galaxy were made of Machos, then the lightcurve of image A should be affected by occasional microlensing and differ from the image B lightcurve. Analysis of the two lightcurves (that were originally measured in order to determine the time delay in this system, see [21]) shows that they are very similar. They are in fact not more different than at most 0.05 mag, which is also the order of the observational uncertainty. Comparison with numerical simulations shows that any population of halo objects in subsolar down to planetary mass range should have produced larger differences (for moderately small quasar sizes). The conclusion is (see [30, 46]): it can be excluded that the halo of this galaxy is made entirely of objects in the mass range \(10^{-3} \leq M/M_\odot \leq 10^{-1}\).

Summarized: LMC microlensing results AND quasar microlensing results confirm that halos of galaxies cannot be made predominantly of Machos.

However, there is no doubt that lensing objects of stellar mass exist: towards the bulge of the Galaxy, a total of more than 500 microlensing events has been detected, presumably due to low mass main sequence stars [2, 21, 36]. This is a much higher number of events than what was predicted from our knowledge of the Milky Way structure, and it still challenges some theoretical models. The microlensing effect of stellar objects on a quasar has been detected as well, most impressively in the quadruple quasar Q2237+0305 [15, 16]. Again, this can be explained entirely by an ordinary old population of stars in the central parts of the lensing galaxy.

So far we only considered compact stellar mass objects bound in halos of galaxies. However, it could be that there is a cosmological distribution of such objects. Press & Gunn [28] showed in the early 1970s that gravitational lensing is an effective method to detect such a population of condensed objects. Dalcanton et al. (1994) did a study to search for such a cosmological population of objects in the mass range of 0.001 \(\leq M/M_\odot \leq 120\). They investigated the equivalent width distribution of 200 quasars, based on the assumption that microlensing of such objects would magnify the continuum emission of the quasars, because it emerges from a much smaller spatial region than the broad and narrow line regions. If microlensed, hence, this would reduce the equivalent width of such affected quasars. In particular, one would expect this to occur for quasars with higher redshift, since the optical depth of such a distribution of compact objects would increase with increasing source redshift. No such effect was found [4]. Their conclusion was that \(\Omega_{10^{-3}} \leq 10^{1.3} < 0.1\). More recently, it was claimed that the variability of (single) quasars could be caused by just such a population of compact objects [4, 13]; however, this is still under debate and the observational evidence is not yet conclusive.

3.2 Few (if any) million solar mass black holes: kinky VLBI jets in Q0957+561, gamma-ray bursts, double radio sources

There are various arguments that halos of galaxies could also be made by black holes with masses around one million solar masses. This is an interesting mass range, because the Einstein radius of such objects at cosmological distances is of order a milliarcsecond, hence accessible to observations with VLBI. And there are also objects out there to measure the effect: the radio jets of lengths 50 to 100 milliarcseconds are perfect targets for such a test. If there is a significant population of lenses in this mass range between such a radio jet and the observer, they would produce bends and kinks
and holes in such a jets. The problem is that some/most jets have naturally bends and kinks, hence the lensing signature is not unique. However, nature provides us with a good test lab anyway: for multiply-imaged quasars, we have two or more images of such a radio jet. And since this lensing effect acts differently on either of these jets, we are able to see whether such millilensing objects exist from comparing the two radio jets [13]. In the case of the double quasar Q0957+561, this test was done [13]. The close similarity of the two jets excludes scenarios in which more than 10% of the halo is multiply-imaged quasars, we have two or more images of such a radio jet. And since this lensing effect acts differently on either of these jets, we are able to see whether such millilensing objects exist from comparing the two radio jets [39].

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A similar mass range for compact lenses can be explored by searching for millilensed gamma-ray bursts. Nemiroff et al. (2001) recently investigated 774 BATSE-triggered GRBs for evidence of millilensing, i.e. repeated peaks with similar light-curves and spectra [27]. Their null detection allowed them to put limits on the universal matter density in compact objects in the mass range $10^5 \leq M/M_{\odot} \leq 10^9$, excluding a significant population in this interval: $\Omega_{10^5-10^9} < 0.1$.

A study by Wilkinson et al. (2001) investigated 300 compact radio sources with VLBI for possible double sources [14]. They did not find any multiple images with angular separations between $1.5 \leq \Delta \theta$/milliarcsec $\leq 100$, corresponding to a mass range of $10^6 \leq M/M_{\odot} \leq 10^8$. They used this null result to put limits on the matter content in this form of supermassive objects: $\Omega_{10^6-10^8} < 0.01$ (2$\sigma$).

### 3.3 Few (if any) $10^9 - 10^{11}$ M$_{\odot}$ objects: no radio doubles

Compact objects in the mass range $10^9 M_{\odot} - 10^{11} M_{\odot}$ have Einstein radii of a few hundredths to a few tenth of an arcsecond, an angular range difficult to access in the optical regime. However, the interferometric techniques in the radio make it possible to probe it. Augusto & Wilkinson [4] investigated 1665 sources with a mean redshift of $< z > \approx 1.3$ (out of the more than 10000 objects in the JVAS/CLASS catalogue). They searched with MERLIN for double images with angular separations between 90 and 300 milliarcseconds, corresponding to a mass range of $3 \times 10^9 \leq M/M_{\odot} \leq 8 \times 10^{10}$. They did not find a single lens, and their conclusion is that the total matter in the universe in this mass interval cannot exceed $\Omega_{10^9.5-10^{10.9}} < 0.1$ (2$\sigma$).

### 3.4 Many bright $10^{11} - 10^{13}$ M$_{\odot}$ objects: multiple quasars cannot err!

So far, 64 multiple quasars are known (see CASTLE web page, [11]). The separation distribution ranges from 0.33 arcsec to 6.93 arcsec. The lens redshifts (31 of them measured) cover $0.04 \leq z_{\text{Lens}} \leq 1.01$, whereas the source redshifts (44 determined) span the interval from $0.96 \leq z_{\text{Lens}} \leq 4.5$. The mass range of these angular separations corresponds to about $10^{11} - 10^{13}$ solar masses, typical galaxy scales. Are the lenses galaxies? Jackson et al. (1998) investigated this question: in 12 out of 12 lens systems which had originally been discovered in the radio regime, they found a galaxy in the optical or near-infrared [10]. So the question “are there any dark galaxy-mass lenses?” was solved three years ago: the answer was “NO!”.

However, in the meantime the CLASS lens B0827+525 refuses to reveal a visible lens galaxy, at least up to now. Koopmans (2000) called this system the best candidate for a “dark lens” [10]. So the answer now is: “MAYBE?”.

But is is clear that if such a population exists at all, it can make up only a (very) small fraction of all the objects in this mass range.

### 3.5 Giant arcs: lots of dark matter in galaxy clusters

There are roughly 100 giant luminous arc systems known, highly distorted background galaxies around clusters of galaxies. The most well known being cluster 0024+1654 [6] and Abell 2218 [13]. The radii of curvature of the arcs range from 10 arcsec to 35 arcsec [29]. In addition to these most dramatic arcs with occasional length-to-width ratios of 10 or larger, there are numerous arclets and weakly distorted galaxies in these clusters [28]. Techniques for cluster mass reconstruction provide excellent tools to study the (total) mass distribution in clusters and compare with the light distribution (see also contributions by Clowe and Lombardi). The results show: galaxy clusters are dominated by dark matter, consistent with studies based on velocity galaxy dispersions or X-ray analyses.

The frequency of giant arcs can be used in a statistical sense to constrain the underlying cosmological model, because the various versions (flat matter-dominated; open; flat with large cosmological constant) predict largely different arc abundances [13] (see also contribution by Meneghetti).

Searches for arcs are usually “biased”: one looks for them around known (massive) clusters of galaxies. So it is no surprise that all known arcs systems are related to visible/bright galaxy clusters.
Are there ways to test whether “dark clusters” exist with masses in the range of galaxy clusters? There are indeed, since they should produce large separation multiple quasars as well. In a recent study, Phillips et al. (2000, 2001) performed a careful study to search for radio multiples with separations between 6 and 15 arcsec [33] as well as between 15 and 60 arcsec [34]. In the former study, there remain ≤ 1 candidates from 15000 flat-spectrum sources. In the latter investigation, they found no radio multiples, and could provide upper limits on any population of (dark) objects with $10^{13} \leq M/M_\odot \leq 10^{14} M_\odot$. (The once so-called “dark lens” MG2016+112 turned out to be a lensing cluster of galaxies at a redshift of one, [31]).

The bottom line with respect to image splittings of order 30 to 70 arcseconds is: there are lenses galore, i.e. galaxy clusters, but there is no evidence for dark matter concentrations on these mass scales. (Already a decade ago, Nemiroff [26] excluded an $\Omega$-value of more than 25% in compact masses between $10^{10} M_\odot$ and $10^{15} M_\odot$ from lensing studies.)

3.6 Cosmological constant cannot be large

Various authors showed in the late 1980s/early 1990s that the frequency of multiple quasar systems depends on the cosmological model (for a review see [3]): the larger the contribution of the cosmological constant the more multiple-imaged system one expects. There were a few studies recently, which explored this quantitatively. Based on a well-defined sample of optical and radio lenses, Falco et al. (1998) concluded that the cosmological constant has to be smaller than $\Lambda < 0.62$ (2σ), in order to be consistent with the known frequency of lensed systems [12]. Depending on the view, this is just about (in)consistent with values for the cosmological constant as determined from recent supernovae searches at high redshift. More work along these lines is clearly required.

4 Dark matter – bright prospects from Lensing?

Strong lensing is a strong tool for the detection of compact matter in the universe. Studies searching for lensing effects of compact objects cover more than 15 orders of magnitude in mass: from substellar objects ($\approx 10^{-2} M_\odot$) to galaxy clusters ($\approx 10^{14} M_\odot$), unfortunately still with a few gaps in between. A graphical summary of the results to date can be found in Figure 4.

![Figure 4: Limits on Omega from various lensing studies](image-url)
Current gravitational lensing studies have good samples from lensing corresponding to mass scales of (roughly) stellar mass, galaxy mass, and galaxy cluster mass objects. There is evidence for dark matter in galaxies, and even more so in galaxy clusters. Few if any really dark objects have been detected. Although there are only upper limits on these dark lenses, it is obvious that they cannot dominate the universe.

The big optical surveys underway (2dF, SDSS) will find many more lens systems, based on well defined selection criteria which will provide much tighter limits on mass scales of galaxies or larger. Radio surveys will provide data for smaller separation/low mass lenses. And new or more “exotic” aspects of lensing (astrometric microlensing measuring centroid shifts, gamma-ray burst lensing measuring time delays) will bridge the gap between mass scales of $\approx 10^2 M_\odot$ and $\approx 10^6 M_\odot$. So as a matter of fact, prospects are very bright for more facts on (dark) matter.

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References

[1] Alcock C., Allsman R. A., Alves D., et al. (The MACHO collaboration), 2000, ApJ 541, 281
[2] Alcock C., Allsman R. A., Alves D., et al. (The MACHO collaboration), 2000, ApJ 541, 734
[3] Alcock C., Allsman R. A., Alves D., et al. (The MACHO collaboration), 2000, ApJ 550, L69
[4] Augusto, P., Wilkinson, P.N., 2001, MNRAS 320, L40
[5] Bartelmann, M., Huss, A., Colberg, J.M., Jenkins, A., Pearce, F.R., 1998, A&A 330, 1
[6] Carroll, S.M., Turner, E.L., Press, W.H., 1992, ARA&A 30, 499
[7] Colley, W.N. Tyson, J. A.; Turner, E. L., 1996, ApJ 461, L83
[8] Courbin, F., Lidman, C., Magain, P., 1998, A&A 330, 57
[9] Dalcanton, J.J., Canizares, C.R., Granados, A., Steidel, C.C., Stocke, J.T., 1994, ApJ 424, 550
[10] Dyson, F.W., Eddington, A.S., Davidson, C.R., 1920, Mem. Roy. Astron. Soc. 62, 291
[11] Falco, E.E., Impey, C., Kochanek, C.S., Lehar, J., McLeod, B., Rix, H.-W.: WWW: [http://cfa-www.harvard.edu/glensdata/](http://cfa-www.harvard.edu/glensdata/)
[12] Falco, E.E., Kochanek, C.S., Munoz, J.A., 1998, ApJ 494, 47
[13] Garrett, M.A., Clader, R.J., Porcas, R.W., King, L., Walsh, D., Wilkinson, P.N., 1994, MNRAS270, 457
[14] Hawkins, M.R.S., 1996, MNRAS 278, L787
[15] Hawkins, M.R.S., Taylor, A.N., 1997, ApJ 482, L5,
[16] Jackson, N., Helbig, P., Browne, I, Fassnacht, C.D., Koopmans, L. et al., 1998, A&A 334, L33
[17] King, L.J., Jackson, N.J., Blandford, R.D., Bremer, M.N., Browne, I. W. A., de Bruyn, A.G., Fassnacht, C., Koopmans, L., Marlow, D., Nair, S., Wilkinson, P.N. 1997, MNRAS 289, 450
[18] Kneib, J.-P., Ellis, R.S., Smail, I., Couch, W.J., Sharples, R.M., 1996, ApJ471, 643
[19] Koopmans, L.V.E., de Bruyn, A.G., Fassnacht, C.D., Marlow, D.R., Rusin, D. et al., 2000, A&A 361, 815,

\[1\] In fact, even regimes covering mass ranges $10^{-16} \leq M/M_\odot \leq 10^{-6}$ are explored, so-called femto- and pico-lensing, see e.g. [24]
[20] Kundic, T., Turner, E.L., Colley, W.N., Gott, III, R., Rhoads, J.E. et al., 1997, ApJ 482, 75
[21] Lasserre, T., Afonso, C., Albert, J.N. et al. (The EROS collaboration), 2000, A&A 355, L39
[22] Paczyński, B., 1986, ApJ 304, 1
[23] Mellier, Y., 1999, ARA&A 37, 127
[24] Marani, G. F., Nemiroff, R. J., Norris, J. P., Hurley, K., Bonnell, J. T., 1999, ApJ 512, L13
[25] Mortlock, D.J., Webster, R.L., 2000, MNRAS 319, 872
[26] Nemiroff, R. J., 1991, Phys. Rev. Lett., 86, 580
[27] Nemiroff, R. J., Marani, G. F., Norris, J. P., Bonnell, J. T., 2001, Phys. Rev. Lett., 86, 580
[28] Press, W.H., Gunn, J.E., 1973 ApJ 185, 397
[29] Schindler, S., Hattori, M., Neumann, D.M., Böhringer, H., 1997, A&A 317, 646
[30] Schmidt R., Wambsganss, J. 1998, A&A 335, 379
[31] Soucail, G., Kneib, J.P., Jansen, A.O., Hjorth, J., Hattori, M., Yamada, T., 2001, A&A 367, 741
[32] Petters, A.O., Levine, H, Wambsganss, J., 2001 “Singularity Theory and Gravitational Lensing” (Birkhäuser, Basel)
[33] Phillips, P.M., Browne, I.W.A., Wilkinson, P.N., Jackson, N.J. 2000, preprint astro-ph/0011032
[34] Phillips, P.M., Browne, I.W.A., Wilkinson, P.N., 2001, MNRAS 321, 187
[35] Rusin, D., Kochanek, C. S., Norbury, M., Falco, E. E., Impey, C. D., et al., 2001, ApJ 557, 594
[36] Udalski, A., Zebrun, K., Szymański, M., Kubiak, M., Pietrzyński, G., Soszyński, I., Woźniak, P., 2000, Acta Astron. 50, 1
[37] Schneider, P., Ehlers, J., Falco E.E. 1992, “Gravitational Lenses” (Springer, Berlin)
[38] Walsh, D., Carswell, R.F., Weymann, R.J, 1979, Nature 279, 381
[39] Wambsganss, J., Paczyński, B., 1992, ApJ 397, L1
[40] Wambsganss, J., Cen, R., Xu, G., and Ostriker, J. P., 1997, ApJ 475, L81
[41] Wambsganss, J., 1997, MNRAS 284, 172
[42] Wambsganss J., 1998, Living Reviews in Relativity 1/1998-12, pp. 1-80; online: http://www.livingreviews.org/Articles/Volume1/1998-12wamb/
[43] Wambsganss J., Schmidt, R. W., Colley, W.N., Kundic, T., Turner, E., 2000, A&A 362, L37
[44] Wilkinson, P.N., Henstock, D.R., Browne, I.W.A., Polatidis, A.G., Augusto, P. et al. 2001, Phys. Rev. Lett. 86, 584
[45] Woźniak, P.R., Alard, C., Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., Zebrun, K., 2000, ApJ 529, 88
[46] Woźniak, P.R., Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., Soszyński, I., Zebrun, K., 2000, ApJ 540, L65