Semi-empirical bound on the $^{37}$Cl solar neutrino experiment

Waikwok Kwong and S. P. Rosen

Department of Physics, University of Texas at Arlington
Arlington, Texas 76019-0059

ABSTRACT

The Kamiokande measurement of energetic $^8$B neutrinos from the sun is used to set a lower bound on the contribution of the same neutrinos to the signal in the $^{37}$Cl experiment. Implications for $^7$Be neutrinos are discussed.

Energetic $^8$B neutrinos from the sun have been detected in the Kamiokande experiment [1] at about one half the rate predicted by the Standard Solar Model (SSM) [2]. These same neutrinos must also interact with the $^{37}$Cl detector [3] and so it is important to understand their contribution to the measured $^{37}$Cl signal. By comparing this contribution to the total signal, we can extract information about other parts of the solar neutrino spectrum, especially $^7$Be.

We find that, even allowing for neutrino flavor oscillations, the Kamiokande experiment imposes a bound on the $^{37}$Cl signal that does not leave much room for a significant contribution from $^7$Be neutrinos. This finding is not inconsistent with the latest results from the $^{71}$Ga experiments [4,5], and so we may refine the statement of the solar neutrino problem to read: Where have all the $^7$Be neutrinos gone?

Since the basic physical process in the Kamiokande and $^{37}$Cl experiments are different, the former being neutrino–electron scattering and the latter neutrino capture on $^{37}$Cl, we must follow a semi-empirical method to relate them to one another. In Kamiokande, the calculated signal involves a convolution over $\phi(E_{\nu})$, the SSM spectrum of $^8$B neutrinos with energy $E_{\nu}$, the differential cross section for scattered electrons with kinetic energy $T$, and the electron resolution function $\theta(T, T')$ which represents the probability that $T$ will appears as $T'$ in an actual measurement. We
call this function \( \phi \sigma(\nu_e; E_\nu) \) and plot in Fig. 1 its normalized shapes as a function of \( E_\nu \) for two choices of \( \theta(T, T') \): The first is a Gaussian shape that closely approximates the actual experimental resolution [6], the second is a \( \delta \)-function representing perfect resolution, and both assume \( 7.5 \leq T' \leq 15 \) MeV. Notice that because of the experimental resolution, the first case has developed a significant tail below the 7.5 MeV threshold. Only the first case with the experimental resolution will be used for calculations below.

In the \( ^{37}\text{Cl} \) experiment, the relevant quantity is the product of \( \phi(E_\nu) \) with the total capture cross section [7] for neutrinos of energy \( E_\nu \) on \( ^{37}\text{Cl} \). We call this function \( \phi \sigma(^{37}\text{Cl}; E_\nu) \) and plot its normalized shape also in Fig. 1. The integral of \( \phi \sigma(^{37}\text{Cl}; E_\nu) \) gives the \( ^{8}\text{B} \) contribution to the SSM signal in \( ^{37}\text{Cl} \), \( R_{\text{SSM}}(^{7}\text{Be}; ^{37}\text{Cl}) \).

Comparing the normalized functions for the two experiments, we see that they are remarkably similar to one another, especially at the high energy end. We therefore write

\[
\frac{\phi \sigma(^{37}\text{Cl}; E_\nu)}{\int \phi \sigma(^{37}\text{Cl}; E_\nu) dE_\nu} = \alpha \frac{\phi \sigma(\nu_e; E_\nu)}{\int \phi \sigma(\nu_e; E_\nu) dE_\nu} + r(E_\nu) , \tag{1}
\]

where \( \alpha \) is a constant whose value is maximized subject to the condition that the remainder function \( r(E_\nu) \) be everywhere positive. It turns out that the largest value of \( \alpha \) is 0.93, and so we obtain an inequality

\[
\phi \sigma(^{37}\text{Cl}; E_\nu) \geq 0.93 \frac{R_{\text{SSM}}(^{8}\text{B}; ^{37}\text{Cl})}{R_{\text{SSM}}(\text{Kam})} \phi \sigma(\nu_e; E_\nu) . \tag{2}
\]

The next step of the argument is to note that the actual quantity measured in these experiments involves the product of \( \phi \sigma \) with an electron-neutrino “survival probability” \( P(E_\nu) \) which, in general, may be a function of the neutrino energy \( E_\nu \). If \( P(E_\nu) \) represents some, possibly energy-dependent, reduction of the \( ^{8}\text{B} \) spectrum, or an oscillation into a sterile neutrino, then we find from Eq. (2) that

\[
\int \phi \sigma(^{37}\text{Cl}; E_\nu) P(E_\nu) dE_\nu \geq 0.93 \frac{\int \phi \sigma(\nu_e; E_\nu) P(E_\nu) dE_\nu}{R_{\text{SSM}}(\text{Kam})} R_{\text{SSM}}(^{8}\text{B}; ^{37}\text{Cl})
\]

or

\[
R(^{8}\text{B}; ^{37}\text{Cl}) \geq 0.93 (0.50 \pm 0.08) (6.1 \text{ SNU}) = (2.84 \pm 0.45) \text{ SNU} , \tag{3}
\]

where we have used the most recent result from the Kamiokande experiment [1]. This falls within the errors of the twenty-year average of the Davis value [3]

\[
\langle R_{\text{Davis}} \rangle = 2.32 \pm 0.23 \text{ SNU} , \tag{4}
\]
but is somewhat on the high side. Note that the bound in Eq. (3) also holds in the simple case of a reduction of the total $^8$B flux with no change in the spectral shape.

Next, consider the case of oscillations of solar electron-neutrinos into $\nu_\mu$ or $\nu_\tau$, or some combination thereof. The signal observed in Kamiokande is then given by

$$R(\text{Kam}) = \int \left( \phi \sigma(\nu_e; E_\nu) P(E_\nu) + [1 - P(E_\nu)] \phi \sigma(\nu_\mu e; E_\nu) \right) dE_\nu ,$$

where we must now distinguish between the cross sections for electron-neutrinos and muon- or tau-neutrinos. As is well known [7] the latter cross section lies somewhere between 1/6 and 1/7 of the former in magnitude and is very similar in shape for energetic neutrinos. For our case it is an extremely good approximation to set

$$\sigma(\nu_\mu e; E_\nu) = 0.148 \sigma(\nu_e e; E_\nu).$$

We can then rewrite Eq. (5) in the form

$$\int \phi \left( \sigma(\nu_e e; E_\nu) - \sigma(\nu_\mu e; E_\nu) \right) P(E_\nu) dE_\nu = R(\text{Kam}) - \int \phi \sigma(\nu_\mu e; E_\nu) dE_\nu ,$$

or

$$0.852 \int \phi \sigma(\nu_e e; E_\nu) P(E_\nu) dE_\nu = R(\text{Kam}) - 0.148 R_{SSM}(\text{Kam}) .$$

From Eqs. (2) and (7) and the Kamiokande data [1], we see that the contribution of the $^8$B neutrinos must be bounded in the case of flavor oscillations by

$$R(\text{Kam}) \geq 0.93 \frac{\int \phi \sigma(\nu_e e; E_\nu) P(E_\nu) dE_\nu}{R_{SSM}(\text{Kam})} R_{SSM}(^8\text{B}; ^{37}\text{Cl})$$

$$= 0.93 \frac{(0.50 \pm 0.08) - 0.148}{0.852} (6.1 \text{ SNU})$$

$$= (2.34 \pm 0.53) \text{ SNU} .$$

To show that the above argument really does provide lower bounds on the $^7$Be neutrino contribution to the $^{37}$Cl experiment, we consider the special case in which, inspired by the non-adiabatic MSW solution [8], we take the electron-neutrino survival probability to be [9]

$$P(E_\nu) = e^{-C/E_\nu} ,$$

where $C$ is a constant to be determined by fitting the Kamiokande data. When there is either no oscillation, or oscillation into a sterile neutrino, we find

$$C = 6.9^{+1.8}_{-1.5} \text{ MeV} \quad \text{and} \quad R(\text{Kam}) = 3.0 \pm 0.5 \text{ SNU} .$$
Allowing for neutrino oscillations, we find instead

\begin{equation}
C = 8.8^{+2.6}_{-2.0} \text{ MeV} \quad \text{and} \quad R(^8\text{B},^{37}\text{Cl}) = 2.5 \pm 0.5 \text{ SNU}.
\end{equation} 

(11)

Both rates are larger than the corresponding lower bounds in Eqs. (8) and (8) respectively.

When compared with the Davis result of Eq. (4), our bounds on the energetic \(^8\text{B}\) neutrino contribution in Eq. (3) and (8) do not leave much room for the 1.8 SNU coming from all other sources, or the 1.1 SNU from \(^7\text{Be}\) neutrinos alone. Indeed, the contribution from all other sources, call them \(X\), is given in the two cases we have considered by

\[
R(X,^{37}\text{Cl}) \leq \begin{cases} 
-0.52 \pm 0.51 \text{ SNU} & \text{(no oscillations)}, \\
-0.02 \pm 0.58 \text{ SNU} & \text{(with oscillations)}.
\end{cases}
\]

(12)

At the 95\% confidence limit, this means

\[
R(X,^{37}\text{Cl}) \leq \begin{cases} 
0.32 \text{ SNU} & \text{(no oscillations)}, \\
0.93 \text{ SNU} & \text{(with oscillations)}.
\end{cases}
\]

(13)

Assuming that the \(^7\text{Be}\) contribution is approximately 1.1/1.8, or 60\% of this, we find it to be:

\[
R(^7\text{Be},^{37}\text{Cl}) < \begin{cases} 
0.20 \text{ SNU} & \text{(no oscillations)}, \\
0.57 \text{ SNU} & \text{(with oscillations)}.
\end{cases}
\]

(14)

To pursue this line of argument further, we can set lower bounds on the contribution of the \(^8\text{B}\) neutrinos to the \(^{71}\text{Ga}\) experiments. Replacing the absorption cross section of \(^{37}\text{Cl}\) by that of \(^{71}\text{Ga}\) everywhere [10], we obtain an inequality similar to Eq. (3) but with \(\alpha = 0.81\). The bounds on the \(^8\text{B}\) contribution to the \(^{71}\text{Ga}\) experiments are

\[
R(^8\text{B},^{71}\text{Ga}) \geq \begin{cases} 
5.7 \pm 0.9 \text{ SNU}, & \text{(no oscillations)}, \\
4.7 \pm 1.1 \text{ SNU}, & \text{(with oscillations)}.
\end{cases}
\]

(15)

The corresponding values in the \(e^{-C/E}\) model,

\[
R(^8\text{B},^{71}\text{Ga}) = \begin{cases} 
6.6 \pm 1.1 \text{ SNU}, & \text{(no oscillations)}, \\
5.5 \pm 1.3 \text{ SNU}, & \text{(with oscillations)},
\end{cases}
\]

(16)

are again larger than their counterparts in Eq. (13).
Combining the bounds of Eq. (15) with the latest $^{71}\text{Ga}$ results [4,5],

$$R(^{71}\text{Ga}) = \begin{cases} 79 \pm 12 \text{ SNU}, & \text{GALLEX} \\ 73 \pm 19 \text{ SNU}, & \text{SAGE} \\ 77 \pm 10 \text{ SNU}, & \text{(combined)} \end{cases}$$

we find an interesting situation, namely that the sum of the signals from pp neutrinos, $^7\text{Be}$ neutrinos, and other non-$^8\text{B}$ sources is very close to the SSM prediction of $^{71}\text{SNU}$ for pp neutrinos alone:

$$R(^{71}\text{Ga}) - R(^8\text{B}, ^{71}\text{Ga}) \leq \begin{cases} 72 \pm 12 \text{ SNU}, & \text{(no oscillations)} \\ 73 \pm 12 \text{ SNU}, & \text{(with oscillations)} \end{cases}$$

Scaling up the $^7\text{Be}$ neutrino bounds in Eq. (14) by the ratio of the capture cross sections on $^{71}\text{Ga}$ and $^{37}\text{Cl}$, we find that the bounds on the $^7\text{Be}$ neutrino contribution to the $^{71}\text{Ga}$ signals are:

$$R(^{7}\text{Be}, ^{71}\text{Ga}) < \begin{cases} 6.0 \text{ SNU}, & \text{(no oscillations)} \\ 17.4 \text{ SNU}, & \text{(with oscillations)} \end{cases}$$

at the 95% confidence level. It will be interesting to test these bounds by direct observation of the $^7\text{Be}$, or pp neutrinos themselves.

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Note added: After this work was completed, the authors learned from Prof. David Schramm that he had obtained a bound in the non-oscillation case similar to that in Eq. (3).

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**Figure caption**

**Fig. 1.** Normalized shapes of $\phi \sigma$ for various experiments.
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