Superstring Theories

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Abstract
This is a short review of superstring theories, highlighting the important concepts, developments and open problems of the subject.

1 Introduction

String theory postulates that all elementary particles in nature correspond to different vibration states of an underlying relativistic string. In the quantum theory both the frequencies and the amplitudes of vibration are quantized, so that the quantum states of a string are discrete. They can be characterized by their mass, spin and various gauge charges. One of these states has zero mass and spin equal to 2ℏ, and can be identified with the messenger of gravitational interactions, the graviton. Thus string theory is a candidate for a unified theory of all fundamental interactions, including quantum gravity.

In this short review article we discuss the theory of superstrings as consistent theories of quantum gravity. The aim is to provide a quick (mostly lexicographic and bibliographic) entry to some of the salient features of the subject for a non-specialist audience. Our treatment is thus neither complete nor comprehensive – there exist for this several excellent expert books, in particular by Green, Schwarz and Witten [1] and by Polchinski [2]. An introductory textbook by Zwiebach [3] is also highly recommended for beginners. Several other complementary reviews on various aspects of superstring theories are available on the internet [4]; some more will be given as we proceed.

2 The five superstring theories

Theories of relativistic extended objects are tightly constrained by anomalies, i.e. quantum violations of classical symmetries. These arise because

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the classical trajectory of an extended $p$-dimensional object (or “$p$-brane”) is described by the embedding $X^\mu(\zeta^a)$, where $\zeta^a=0,\ldots,p$ parametrize the brane worldvolume, and $X^\mu=0,\ldots,D-1$ are coordinates of the target space. The quantum mechanics of a single $p$-brane is therefore a $(p+1)$-dimensional quantum field theory, and as such suffers a priori from ultraviolet divergences and anomalies. The case $p=1$ is special in that these problems can be exactly handled. The story for higher values of $p$ is much more complicated, as will become apparent later on.

The theory of ordinary loops in space is called closed \textbf{bosonic string} theory. The classical trajectory of a bosonic string extremizes the Nambu-Goto action (proportional to the invariant area of the worldsheet)

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d^2\zeta \sqrt{-\det(G_{\mu\nu}\partial_{\alpha}X^\mu\partial_{\beta}X^\nu)} ,$$

(1)

where $G_{\mu\nu}(X)$ is the target-space metric, and $\alpha'$ is the Regge slope (which is inversely proportional to the string tension and has dimensions of length squared). In flat spacetime, and for a conformal choice of worldsheet parameters $\zeta^{\pm} = \zeta^0 \pm \zeta^1$, the equations of motion read:

$$\partial_+ \partial_- X^\mu = 0 \quad \text{and} \quad \eta_{\mu\nu} \partial_\pm X^\mu \partial_\pm X^\nu = 0 ,$$

(2)

with $\eta_{\mu\nu}$ the Minkowski metric. The $X^\mu$ are thus free two-dimensional fields, subject to quadratic phase-space constraints known as the \textbf{Virasoro conditions}. These can be solved consistently at the quantum level in the critical dimension $D = 26$. Otherwise the symmetries of eqs. are anomalous: either Lorentz invariance is broken, or there is a conformal anomaly leading to unitarity problems.\footnote{For $D < 26$, unitary non-critical string theories in highly curved rather than in the originally flat background can be constructed.}

Even for $D = 26$, bosonic string theory is, however, sick because its lowest-lying state is a \textbf{tachyon}, i.e. it has negative mass squared. This follows from the zeroth-order Virasoro constraints,

$$m^2 = -p^M p_M = \frac{4}{\alpha'} (N_L - 1) = \frac{4}{\alpha'} (N_R - 1) ,$$

(3)

where $N_L(N_R)$ is the sum of the frequencies of all left(right)-moving excitations on the string worldsheet. The negative contribution to $m^2$ comes from quantum fluctuations, and is analogous to the well-known Casimir energy. The tachyon has $N_L = N_R = 0$. Its presence signals an instability of Minkowski spacetime, which in bosonic string theory is expected to decay, possibly to some lower-dimensional highly-curved geometry. The details of how this happens are not, at present, well understood.

The problem of the tachyon is circumvented by endowing the string with additional, anticommuting coordinates, and requiring space-time \textbf{supersymmetry} \footnote{There exist two standard descriptions of the superstring: the \textbf{Ramond-Neveu-Schwarz} (RNS) formulation, where the anticommuting coordinates $\psi^\mu$ carry a space-time vector index, and the \textbf{Green-Schwarz} (GS) formulation in which they transform as a}. This is a symmetry that relates string states with integer spin, obeying Bose-Einstein statistics, to states with half-integer spin obeying Fermi-Dirac statistics. There exist two standard descriptions of the superstring: the \textbf{Ramond-Neveu-Schwarz} (RNS) formulation, where the anticommuting coordinates $\psi^\mu$ carry a space-time vector index, and the \textbf{Green-Schwarz} (GS) formulation in which they transform as
Anomaly cancellation leads to five consistent superstring theories, all defined in $D = 10$ flat space-time dimensions. They are referred to as type IIA, type IIB, heterotic $SO(32)$, heterotic $E_8 \times E_8$, and type I. The two type II theories are given (in the RNS formulation) by a straightforward extension of eqs. (2):

$$\partial_+ \partial_- X^\mu = \partial_\pm \psi^\mu_{\pm} = 0 \quad \text{and} \quad \eta_{\mu\nu} \psi^\mu_{\pm} \partial_\pm X^\nu = 0 .$$

(4)

The left- and right-moving worldsheet fermions can be separately periodic or antiperiodic – these are known as Ramond (R) and Neveu-Schwarz (NS) boundary conditions. Ramond fermions have zero modes obeying a Dirac $\gamma$-matrix algebra, and which must thus be represented on spinor space. As a result out of the four possible boundary conditions for $\psi^\mu_{\pm}$ and $\psi'^\mu_{\pm}$, namely NS-NS, R-R, NS-R or R-NS, the first two give rise to states that are space-time bosons, while the other two give rise to states that are space-time fermions. Consistency of the theory further requires that one only keep states of definite worldsheet fermion parities – an operation known as the GSO (for Gliozzi-Scherk-Olive) projection. This operation removes the would-be tachyon, and acts as a chirality projection on the spinors. The IIA and IIB theories differ only in the relative chiralities of spinors coming from the left and right Ramond sectors.

The fact that string excitations split naturally into non-interacting left and right movers is crucial for the construction of the heterotic strings. The key idea is to put together the left-moving sector of the $D = 10$ type II superstring and the right-moving sector of the $D = 26$ bosonic string. A subtlety arises because the left-right asymmetry may lead to extra anomalies, under global reparametrizations of the string worldsheet. These are known as modular anomalies, and we will come back to them in the following section. Their cancellation imposes stringent constraints on the zero modes of the unmatched (chiral) bosons in the right-moving sector. The free-field expansion of these bosons can be written as:

$$X(\zeta^-) = x_R + \alpha' p_R \zeta^- + \sqrt{\alpha'} \sum_{n \neq 0} \frac{i}{n} a_n e^{-2in\zeta^-} ,$$

(5)

where bold-face letters denote sixteen-component vectors. Modular invariance then requires that the generalized momentum $p_R$ take its values in a sixteen-dimensional, even self-dual lattice. There exist two such lattices, and they are generated by the roots of the Lie groups $Spin(32)/Z_2$ and $E_8 \times E_8$. They give rise to the two consistent heterotic string theories.

In contrast to the type II and heterotic theories, which are based on oriented closed strings, the type I theory has unoriented closed strings as well as open strings in its perturbative spectrum. The closed strings are the same as in type IIB, except that one only keeps those states
that are invariant under orientation reversal ($\zeta^+ \leftrightarrow \zeta^-$). Open strings must also be invariant under this flip, and can furthermore carry point-like (Chan-Paton) charges at their two endpoints. This is analogous to the flavor carried by quarks at the endpoints of the chromoelectric flux tubes in QCD. Ultraviolet finiteness requires that the Chan-Paton charges span a 32-dimensional vector space, so that open strings transform in bifundamental symmetric or antisymmetric representations of $SO(32)$. For a thorough review of type I string theory see reference [7].

3 Interactions and effective theories

Strings interact by splitting or by joining at a point, as is illustrated in figure 1. This is a local interaction that respects the causality of the theory. To compute scattering amplitudes one sums over all worldsheets with a given set of asymptotic states, and weighs each local interaction with a factor of the string coupling constant $\lambda$. The expansion in powers of $\lambda$ is analogous to the Feynman-diagram expansion of point-particle field theories. These latter are usually defined by a Lagrangian, or more exactly by a functional-integral measure, and they make sense both for off-shell quantities as well as at the non-perturbative level. In contrast, our current formulation of superstring theory is in terms of a perturbatively-defined S-matrix. The advent of dualities has offered glimpses of an underlying non-perturbative structure called M-theory, but defining it precisely is one of the major outstanding problems in the subject.\footnote{One approach consists in trying to define a second-quantized string field theory. This is reviewed in the contribution \cite{8} in the present volume}

Another important expansion of string theory, very useful when it comes to extracting space-time properties, is in terms of the characteristic string length $l_s = \sqrt{\alpha'}$. At energy scales $E l_s \ll 1$ only a handful of massless string states propagate, and their interactions are governed by an effective low-energy Lagrangian. In the type-II theories the massless bosonic states (or rather their corresponding fields) consist of the metric $G_{\mu\nu}$, a scalar field $\Phi$ called the dilaton, and a collection of \textbf{antisymmetric n-form fields} coming from both the NS-NS and the R-R sectors. For type IIA these latter are a NS-NS 2-form $B_2$, a R-R 1-form $C_1$, and a R-R...
The leading-order action for these fields reads:

$$S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \left[ \sqrt{-G} e^{-2\Phi} (R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2) ight. \
- \sqrt{-G} \left( \frac{1}{2}|F_2|^2 + \frac{1}{2}|F_4 - C_1 \wedge H_3|^2 - \frac{1}{2}B_2 \wedge F_4 \wedge F_4 \right) \right], \tag{6}$$

where $F_2 = dC_1$, $H_3 = dB_2$ and $F_4 = dC_3$ are field strengths, the wedge denotes the exterior product of forms, and $|F_n|^2 = \frac{1}{n!} F_{\mu_1 \ldots \mu_n} \wedge F^{\mu_1 \ldots \mu_n}$. The dimensionful coupling $\kappa$ can be expressed in terms of the string-theory parameters, $2\kappa^2 = (2\pi)^7 \lambda^2 \alpha'$. A similar expression can be written for the IIB theory, whose R-R sector contains a 0-form, a 2-form and a 4-form potential, the latter with self-dual field strength.

The action (6), together with its fermionic part, defines the maximally-supersymmetric non-chiral extension of Einstein’s gravity in ten dimensions called type-IIA supergravity [9]. The dilaton and all antisymmetric tensor fields belong to the supermultiplet of the graviton – they provide together the same number of (bosonic) states as a ten-dimensional non-chiral gravitino. Supersymmetry fixes furthermore completely all two-derivative terms of the action, so that the theory defined by (6) is (almost) unique.\(^3\) It is, therefore, not surprising that it should emerge as the low-energy limit of the (non-chiral) superstring theory. This latter provides, however, an ultraviolet completion of an otherwise non-renormalizable theory, a completion which is, at least perturbatively, finite and consistent.

The finiteness of string perturbation theory has been, strictly-speaking, only established up to two loops – for a recent review see [10]. However, even though the technical problem is open and hard, the qualitative case for all-order finiteness is convincing. It can be illustrated with the torus diagram which makes a one-loop contribution to string amplitudes. The thin torus of figure 2 could be traced either by a short, light string propagating (virtually) for a long time, or by a long, heavy string propagating for a short period of time. In conventional field theory these two virtual trajectories would have made distinct contributions to the amplitude, one

\(^3\)There exists in fact a massive extension of IIA supergravity, which is the low-energy limit of string theory with a non-vanishing R-R ten-form field strength.
in the infrared and the second in the ultraviolet region. In string theory, on the other hand, they are related by a modular transformation (that exchanges $\zeta^0$ with $\zeta^1$) and must not, therefore, be counted twice. A similar kind of argument shows that all potential divergences of string theory are infrared—they are therefore kinematical (i.e. occur for special values of the external momenta), or else they signal an instability of the vacuum and should cancel if one expands around a stable ground state.

The low-energy limit of the heterotic and type I string theories is $N=1$ supergravity plus super Yang-Mills. In addition to the $N=1$ graviton multiplet, the massless spectrum now also includes gauge bosons and their associated gauginos. The two-derivative effective action in the heterotic case reads:

$$S_{\text{het}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \ e^{-2\phi} \left[ R + 4\partial^\mu \Phi \partial^\nu \Phi + \frac{\kappa^2}{g_{\text{YM}}} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) \right. \\
\left. - \frac{1}{2} \left| dB_2 - \frac{\kappa^2}{g_{\text{YM}}} \omega_{\text{gauge}}^3 \right|^2 \right] + \text{fermions}, \quad (7)$$

where $\omega_{\text{gauge}}^3 = \text{Tr}(\text{Ad}A + \frac{2}{3}A^3)$ is the Chern-Simons gauge 3-form. Supersymmetry fixes again completely the above action—the only freedom is in the choice of the gauge group and of the Yang-Mills coupling $g_{\text{YM}}$. Thus, up to redefinitions of the fields, the type I theory has necessarily the same low-energy limit.

The $D=10$ supergravity plus super Yang-Mills has a hexagon diagram that gives rise to gauge and gravitational anomalies, similar to the triangle anomaly in $D=4$. It turns out that for the two special groups $E_8 \times E_8$ and $SO(32)$, the structure of these anomalies is such that they can be cancelled by a combination of local counterterms. One of them is of the form $\int B_2 \wedge X_8(F,R)$, where $X_8$ is an 8-form quartic in the curvature and/or Yang-Mills field strength. The other is already present in the lower line of expression (7), with the replacement $\omega_{\text{gauge}}^3 \rightarrow \omega_{\text{gauge}}^3 - \omega_{\text{Lorentz}}^3$, where the second Chern-Simons form is built out of the spin connection. Note that these modifications of the effective action involve terms with more than two derivatives, and are not required by supersymmetry at the classical level. The discovery by Green and Schwarz that string theory produces precisely these terms (from integrating out the massive string modes) was called the “first superstring revolution.”

4 D-branes

A large window into the non-perturbative structure of string theory has been opened by the discovery of D(irichlet)-branes, and of strong/weak-coupling duality symmetries. A $D_p$-brane is a solitonic $p$-dimensional excitation, defined indirectly by the property that its worldvolume can attach open-string endpoints (see figure 3). Stable $D_p$-branes exist in the type-IIA and type-IIB theories for $p$ even, respectively odd, and in the type I theory for $p = 1$ and 5. They are charged under the R-R $(p+1)$-form potential or, for $p > 4$, under its magnetic dual. Strictly-speaking, only for $0 \leq p \leq 6$ do $D$-branes resemble regular solitons (the word stands for ‘solitary waves’). The $D7$-branes are more like cosmic strings,
the D8-branes are domain walls, while the D9-branes are spacetime filling. Indeed, type-I string theory can be thought as arising from type-IIB through the introduction of an orientifold 9-plane (required for tadpole cancellation) and of thirty two D9-branes.

The low-energy dynamics of a Dp-brane is described by a supersymmetric abelian gauge theory, reduced from 10 down to $p+1$ dimensions. The gauge-field multiplet includes $9-p$ real scalars, plus gauginos in the spinor representation of the R-symmetry group $SO(9-p)$. These are precisely the massless states of an open string with endpoints moving freely on a hyperplane. The real scalar fields are Goldstone modes of the broken translation invariance, i.e. they are the transverse-coordinate fields $\vec{Y}(\xi^a)$ of the D-brane. The bosonic part of the low-energy effective action is the sum of a Dirac-Born-Infeld (DBI) and a Chern-Simons like term:

$$I_p = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\det(\hat{G}_{ab} + \mathcal{F}_{ab})} - \rho_p \int \sum n \hat{C}_n \wedge e^\mathcal{F},$$

where $\mathcal{F}_{ab} = \hat{B}_{ab} + 2\pi\alpha' F_{ab}$, hats denote pullbacks on the brane of bulk tensor fields (for example $\hat{G}_{ab} = G_{\mu\nu} \partial_a Y^\mu \partial_b Y^\nu$), $F_{ab}$ is the field strength of the worldvolume gauge field, and in the CS term one is instructed to keep the $(p+1)$-form of the expression under the integration sign. The constants $T_p$ and $\rho_p$ are the tension and charge-density of the D-brane. As was the case for the effective supergravities, the above action receives curvature corrections that are higher order in the $\alpha'$ expansion. Note however that a class of higher-order terms have been already resummed in expression (8). These involve arbitrary powers of $F_{ab}$, and are closely related (more precisely ‘T-dual’, see later) to relativistic effects which can be important even in the weak-acceleration limit. When referring to the D9-branes of the type I superstring, the action includes the Green-Schwarz terms required to cancel the gauge anomaly.
The tension and charge-density of a D\(_p\)-brane can be extracted from its coupling to the (closed-string) graviton and R-R (\(p+1\))-form, with the result:

\[
T^2_p = \rho^2_p = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^{3-p}.
\] (9)

The equality of tension and charge follows from unbroken supersymmetry, and is also known as a Bogomol’nyi-Prasad-Sommerfield (BPS) condition. It implies that two or more identical D-branes exert no net static force on each other, because their R-R repulsion cancels exactly their gravitational attraction. A non-trivial check of the result (9) comes from the Dirac quantization condition (generalized to extended objects by Nepomechie and Teitelboim). Indeed, a D\(_p\)-brane and a D(6-\(p\))-brane are dual excitations, like electric and magnetic charges in four dimensions, so their couplings must obey

\[
2\kappa^2 \rho_p \rho_{6-p} = 2\pi k \quad \text{where} \quad k \in \mathbb{Z}.
\] (10)

This ensures that the Dirac singularity of the long-range R-R fields of the branes does not lead to an observable Bohm-Aharonov phase. The couplings (9) obey this condition with \(k = 1\), so that D-branes carry the smallest allowed R-R charges in the theory.

A simple but important observation is that open strings living on a collection of \(n\) identical D-branes have matrix-valued wavefunctions \(\psi_{ij}\), where \(i, j = 1, \ldots, n\) label the possible endpoints of the string. The low-energy dynamics of the branes is thus described by a non-abelian gauge theory, with group \(U(n)\) if the open strings are oriented, and \(SO(n)\) or \(Sp(n)\) if they are not. We have already encountered such Chan-Paton factors in our discussion of the type I superstring. More generally, this simple property of D-branes has lead to many insights on the geometric interpretation and engineering of gauge theories, which are reviewed in the present volume in references [11]. It has also placed on a firmer footing the idea of a brane world, according to which the fields and interactions of the Standard Model would be confined to a set of D-branes, while gravitons are free to propagate in the bulk (for reviews see references [12],[13]). It has, finally, inspired the gauge/string-theory or AdS/CFT correspondence [14],[15] on which we will comment later on.

5 Dualities and M theory

One other key role of D-branes has been to provide evidence for the various non-perturbative duality conjectures. Dual descriptions of the same physics arise also in conventional field theory. A prime example is the Montonen-Olive duality of four-dimensional, N=4 supersymmetric Yang-Mills, which is the low-energy theory describing the dynamics of a collection of D3-branes. The action for the gauge field and six associated scalars \(\Phi^I\) (all in the adjoint representation of the gauge group \(G\)) is

\[
S_{N=4} = -\frac{1}{4g^2} \int d^4x \text{Tr}\left(F_{\mu\nu}F^{\mu\nu} + 2 \sum_I D_\mu \Phi^I D^\mu \Phi^I + \sum_{I<J} 2[\Phi^I, \Phi^J]^2\right) - \frac{\theta}{32\pi^2} \int d^4x \text{Tr}(F_{\mu\nu}^* F^{\mu\nu}) + \text{fermionic terms}.
\] (11)
Consider for simplicity the case $G = SU(2)$. The scalar potential has flat directions along which the six $\Phi^I$ commute. By a $SO(6)$ R-symmetry rotation we can set all but one of them to zero, and let $< \text{Tr}(\Phi^I \Phi^I) > = v^2$ in the vacuum. In this ‘Coulomb phase’ of the theory a $U(1)$ gauge multiplet stays massless, while the charged states become massive by the Higgs effect. The theory admits furthermore smooth magnetic-monopole and dyon solutions, and there is an elegant formula for their mass:

$$M = v |n_{el} + \tau n_{mg}|, \quad \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

and $n_{el}(n_{mg})$ denotes the quantized electric (magnetic) charge. This is a BPS formula that receives no quantum corrections. It exhibits the $SL(2, \mathbb{Z})$ covariance of the theory,

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad \text{and} \quad (n_{el}, n_{mg}) \rightarrow (n_{el}, n_{mg}) \left( \begin{array}{cc} a & b \\ c & d \end{array} \right)^{-1}. \quad (13)$$

Here $a, b, c, d$ are integers subject to the condition $ad - bc = 1$. Of special importance is the transformation $\tau \rightarrow -1/\tau$, which exchanges electric and magnetic charges and (at least for $\theta = 0$) the strong- with the weak-coupling regimes. For more details see the review \[16\].

The extension of these ideas to string theory can be illustrated with the strong/weak-coupling duality between the type I theory, and the $\text{Spin}(32)/\mathbb{Z}_2$ heterotic string. Both have the same massless spectrum and low-energy action, whose form is dictated entirely by supersymmetry. The only difference lies in the relations between the string and supergravity parameters. Eliminating the latter one finds:

$$\lambda_{\text{het}} = \frac{1}{2\lambda_I} \quad \text{and} \quad \alpha'_{\text{het}} = \sqrt{2} \lambda_I \alpha'_I, \quad (14)$$

It is, thus, very tempting to conjecture that the strongly-coupled type I theory has a dual description as a weakly-coupled heterotic string. These are, indeed, the only known ultraviolet completions of the theory \[7\]. Furthermore, for $\lambda_I \gg 1$ the D1-brane of the type I theory becomes light, and could be plausibly identified with the heterotic string. This conjecture has been tested successfully by comparing various supersymmetry-protected quantities (such as the tensions of BPS excitations and special higher-derivative terms in the effective action), which can be calculated exactly either semiclassically, or at a given order in the perturbative expansion. Testing the duality for non-protected quantities is a hard and important problem, which looks currently out of reach.

The other three string theories have also well-motivated dual descriptions at strong coupling $\lambda$. The type IIB theory is believed to have a $SL(2, \mathbb{Z})$ symmetry, similar to that of the N=4 super Yang Mills.\[4\] The type IIA theory has a more surprising strong-coupling limit: it grows one extra dimension (of radius $R_{11} = 1/\sqrt{\alpha'_{I}}$), and can be approximated at low energy by the maximal eleven-dimensional supergravity of Cremmer, Julia and Scherk. This latter is a very economical theory – its

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\[4\]Note that $\lambda$ is a dynamical parameter, that changes with the vacuum expectation value of the dilaton $< \phi >$. Thus dualities are discrete gauge symmetries of string theory.
massless bosonic fields are only the graviton and a three-form potential $A_3$. The bosonic part of the action reads:

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left( R - \frac{1}{2} |F_4|^2 \right) - \frac{1}{12\kappa_{11}^2} \int A_3 \wedge F_4 \wedge F_4 . \quad (15)$$

The electric and magnetic charges of the three-form are a (fundamental?) membrane and a solitonic fivebrane. Standard Kaluza-Klein reduction on a circle [17] maps $S_{11D}$ to the IIA supergravity action [9], where $G_{\mu \nu}$, $\phi$ and $C_1$ descend from the eleven-dimensional graviton, and $B_2$ and $C_3$ from the three-form $A_3$. Furthermore, all BPS excitations of the type IIA string theory have a counterpart in eleven dimensions, as summarized in the table below. Finally, if one compactifies the eleventh dimension on an interval (rather than a circle), one finds the conjectured strong-coupling limit of the $E_8 \times E_8$ heterotic string.

| tension | type-IIA | $\mathcal{M}$ on $S^1$ | tension |
|---------|----------|----------------------|---------|
| $\frac{\sqrt{\pi}}{\kappa_{10}} (2\pi \sqrt{\alpha'})^3$ | D0-brane | K-K excitation | $\frac{1}{R_{11}}$ |
| $T_p = (2\pi \alpha')^{-1}$ | string | wrapped membrane | $2\pi R_{11} \left( \frac{2\pi^2}{\kappa_{11}} \right)^{1/3}$ |
| $\frac{\sqrt{\pi}}{\kappa_{10}} (2\pi \sqrt{\alpha'})^{-1}$ | D2-brane | membrane | $T^M_2 = \left( \frac{2\pi^2}{\kappa_{11}} \right)^{1/3}$ |
| $\frac{\sqrt{\pi}}{\kappa_{10}} (2\pi \sqrt{\alpha'})^{-1}$ | D4-brane | wrapped five-brane | $R_{11} \left( \frac{2\pi^2}{\kappa_{11}} \right)^{2/3}$ |
| $\frac{\pi}{\kappa_{10}} (2\pi \alpha')$ | NS-five-brane | five-brane | $\frac{1}{2\pi} \left( \frac{2\pi^2}{\kappa_{11}} \right)^{2/3}$ |
| $\frac{\sqrt{\pi}}{\kappa_{10}} (2\pi \sqrt{\alpha'})^{-3}$ | D6-brane | K-K monopole | $\frac{2\pi^2 R_{11}^2}{\kappa_{11}}$ |

Table 1: BPS excitations of type IIA string theory, and their counterparts in $\mathcal{M}$ theory compactified on a circle of radius $R_{11}$.
The web of duality relations can be extended by compactifying further
to $D \leq 9$ dimensions. Readers interested in more details should consult
Polchinski’s book [2] or one of the many existing reviews of the subject
[18][4]. In nine dimensions, in particular, the two type II theories, as
well as the two heterotic superstrings, are pairwise T-dual. T-duality
is a perturbative symmetry (thus firmly established, not only conjectured)
which exchanges momentum and winding modes. Putting together all
the links one arrives at the fully-connected web of figure 4. This makes the
point that all five consistent superstrings, and also eleven-dimensional su-
pergravity, are limits of a unique underlying structure called M theory.5
A background-independent definition of M-theory has remained elusive.
Attempts to define it as a matrix model of D0-branes, or by quantizing
a fundamental membrane, proved interesting but incomplete. The diffi-
culty stems from the fact that in a generic background, or in $D = 11$
Minkowski spacetime, there is only a dimensionful parameter fixing the
scale at which the theory becomes strongly-coupled.

6 Other developments and outlook

We have not discussed in this brief review some important developments
covered in other contributions to the encyclopedia. For the reader’s con-
venience, and for completeness, we enumerate (some of) them giving the
appropriate cross-references:

- **Compactification.** To make contact with the Standard Model of
  particle physics, one has to compactify string theory on a six-dimensional
  manifold. There is an embarassment of riches, but no completely realistic
  vacuum and, more significantly, no guiding dynamical principle to help
  us decide – see [19]. The controlled (and phenomenologically required)
  breaking of spacetime supersymmetry is also a problem.

- **Conformal field theory and quantum geometry.** The algebraic
tools of 2D conformal field theory, both bulk and boundary – see [20],
play an important role in string theory. They allow, in certain cases,

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5For lack of a better definition, ‘M’ is sometimes also used to denote the D=11 supergravity
plus supermembranes, as in figure 4.
a resummation of $\alpha'$ effects, thereby probing the regime where classical geometric notions do not apply.

- **Microscopic models of black holes.** Charged extremal black holes can be modeled in string theory by BPS configurations of D-branes. This has lead to the first microscopic derivation of the Bekenstein-Hawking entropy formula, a result expected from any consistent theory of quantum gravity. As with the tests of duality, the extension of these results to neutral black holes is a difficult open problem – see [21].

- **AdS/CFT and holography.** A new type of (holographic) duality is the one that relates supersymmetric gauge theories in four dimensions to string theory in asymptotically anti-de Sitter spacetimes. The sharpest and best tested version of this duality relates N=4 super Yang Mills to string theory in $AdS_5 \times S_5$. Solving the $\sigma$-model in this latter background is one of the keys to further progress in the subject – see [14].

- **String phenomenology.** Finding an experimental confirmation of string theory is clearly one of the most pressing outstanding questions. There exist several interesting possibilities for this – cosmic strings, large extra dimensions, modifications of gravity, primordial cosmology – see the review [22]. Here we point out the one supporting piece of experimental evidence : the unification of the gauge couplings of the (supersymmetric, minimal) Standard Model at a scale close to, but below the Planck scale, as illustrated in figure 5. This is a generic ‘prediction’ of string theory, especially in its heterotic version.

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