White dwarfs in massive gravity

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Motivated by the importance of observational results by advanced LIGO concerning the existence of massive gravitons, interesting results of massive gravity and the possibility of massive white dwarfs more than the Chandrasekhar limit ($1.44 M_\odot$, in which $M_\odot$ is mass of the sun), we study white dwarfs in massive gravity. Firstly, we consider the modified TOV equation in massive gravity and solve this equation by numerically using the Chandrasekhar’s equation of state. Our results show that the maximum mass of white dwarfs in massive gravity can be more than $3 M_\odot$. Then we investigate the effects of parameters of massive gravity on other interesting properties of white dwarfs such as mass-radius relation, mass-central density relation, Schwarzschild radius, average density and compactness. Finally, we study the dynamical stability condition for massive white dwarfs in this gravity and show that these stars have dynamical stability.

I. INTRODUCTION

General relativity (GR) is a successful theory of gravity in which gravitons in this theory are massless spin 2 degree of freedom. In order to build up a massive theory with a massive spin 2 particle propagation, one can add an interaction term to the Einstein-Hilbert action which are interpreted as a graviton mass. This theory of gravity can describe our Universe which is currently undergoing acceleration expansion without introducing a cosmological constant [1, 2]. In addition, this theory of gravity can illustrate the dark energy problem. Massive gravity modifies gravity by weakening it at the large scale compared with GR which allows the Universe to accelerate, but its predictions at small scales are the same GR. On the other hand, massive gravity will result into graviton having a mass of $m$ which in case of $m \rightarrow 0$, the effect of massive gravity is vanished and this theory reduce to GR. It is notable that, it was shown that mass of graviton is very small in the usual weak gravity environments, but becomes much larger in the strong gravity regime such as; black holes and compact objects [3]. Current experimental data from the observations of gravitational waves by advanced LIGO require the mass of graviton to be smaller than the inverse period of orbital motion of the binary system, that is $m = 1.2 \times 10^{-22} \text{ev/c}^2$ [4]. Accordingly, there were numerous developments in the massive gravity theories in recent years [5–10].

Fierz and Pauli in 1939 introduced a class of massive gravity theory in flat background [5]. In other words, Fierz and Pauli added the interaction terms at the linearized level of GR, this theory called Fierz and Pauli massive (FP massive) gravity. Then van Dam, Veltman and Zakharovit found out that FP massive theory suffer from discontinuity in predictions which so called van Dam-Veltman-Zakharov (vDVZ) discontinuity [6, 7]. In order to remove vDVZ discontinuity, Vainshtein found that the origin of this problem is related to this fact that the prediction made by the linearized theory cannot be trust inside some characteristic Vainshtein’s radius, hence Vainshtein proposed the mechanism for the nonlinear massive gravity which can be use to recover the predictions made by GR [9]. On the other hand, Boulware and Deser in Ref. [10], explored that such nonlinear generalizations usually generate an equation of motion which has a higher derivative term yielding a ghost instability in the theory, so called Boulware-Deser (BD) ghost. However, these problems, arising in the construction of the massive gravity have been resolved in the last decade by first introducing Stückelberg fields [11]. This permits a class of potential energies depending on the gravitational metric and an internal Minkowski metric (reference metric). In Refs. [12, 13], de Rham, Gabadadze and Tolley (dRGT) introduced a new version of massive gravity which was free of vDVZ discontinuity and BD ghost [14] in arbitrarily dimensions. Although the equations of motion have no higher derivative term in the dRGT massive gravity, but finding exact solutions in this theory of gravity is difficult. However, Black hole solutions in dRGT massive gravity have been obtained by some authors in Refs. [15–18]. In the astrophysics context, Katsuragawa et al. evaluated the obtained neutron stars in this theory and showed that the massive gravity leads to small deviation from the GR [19]. From the perspective of cosmological, bounce and cyclic cosmology [20], cosmological behavior [21], and another interesting properties have been studied in Refs. [22–24].

It is notable that, modification in the introduced reference metric in dRGT theory leads to the possibility of introduction of different classes of dRGT like massive theories [22]. One of theories proposed by Vegh which has

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applications in gauge/gravity duality \[26\]. Indeed this theory is similar to dRGT massive gravity with a difference that its reference metric is a singular one. Graviton in this massive gravity may behave like a lattice and exhibits a Drude peak \[26\]. It was shown that for arbitrary singular metric, this theory of massive gravity is ghost-free and stable \[27\]. Black hole solutions in this gravity have been obtained in Refs. \[28\], \[29\]. The existence of van der Waals behavior in extended phase space form the obtained black holes in this massive gravity have been studied in Refs. \[30\], \[33\]. It was pointed out that it is possible to have a heat engine for non-spherical black holes in Ref. \[31\]. In addition, magnetic solutions of such theory have been addressed in Refs. \[35\], \[36\].

On the other hand, in recent years, some peculiar super-luminous type Ia supernovae(SNIa) are of particular interest \[32\]–\[37\], it has been suggested that the progenitor mass to explain such a supernovae lie in the range 2.1 – 2.8\(M_\odot\) \[40\], \[41\], which exceeds significantly the Chandrasekhar mass limit 1.44\(M_\odot\). Initially, the authors explain such overluminous type Ia supernovae by proposing the existence of superstrong uniform magnetic fields to generate a super-Chandrasekhar white dwarf, and get the new mass limit of white dwarfs as 2.58\(M_\odot\) \[43\], however, it suffers several severe stability issues \[44\], such as neutronization induced by inverse beta decay \[45\] and dynamical instability \[46\]. As a result, the models to explain the peculiar type Ia supernovae include rotation white dwarfs \[47\], electrical charge distribution white dwarfs \[48\] and modification to Einstein’s gravity in white dwarfs \[49\], \[50\] appear. In this work, we investigate the influence of the massive gravity on the properties and stability of white dwarfs.

The article is organized as follows: after a short introduction, we present the modified TOV in massive gravity. In Sec. III, we recall the Chandrasekhar’s equation of state and study its properties such as the energy conditions, stability and the Le Chatelier’s principle. Then, we investigate the properties of white dwarfs in massive gravity such as average density, compactness and dynamical stability. Some closing remarks are given in the last section.

II. BASIC EQUATIONS

The action of Einstein-massive gravity is given by \[26\]

\[
I = -\frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R + m^2 \sum_{i} c_i U_i (g, f) \right] + I_{\text{matter}},
\]

where \(R\) and \(m\) are the Ricci scalar and the mass of graviton. \(f\) and \(g\) are a fixed symmetric tensor and metric tensor, respectively. In addition, \(c_i\)'s are constants and \(U_i\)'s are symmetric polynomials of the eigenvalues of the 4 \(\times\) 4 matrix \(K_{\mu\nu} = \sqrt{g^{\alpha\beta} f_{\alpha\beta}}\) (for 4-dimensional spacetime) where they can be written in the following forms

\[
U_1 = [K], \quad U_2 = [K]^2 - [K^2],
\]
\[
U_3 = [K]^3 - 3 [K] [K^2] + 2 [K^3],
\]
\[
U_4 = [K]^4 - 6 [K^2] [K]^2 + 8 [K^3] [K] + 3 [K^2]^2 - 6 [K^4].
\]

By variation of Eq. (1) with respect to the metric tensor \(g_{\mu\nu}\), the equation of motion for EN-massive gravity can be written as

\[
G_{\mu\nu} + m^2 \chi_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},
\]

where \(G\) is the gravitational constant, and also, \(G_{\mu\nu}\) and \(c\) are the Einstein tensor and \(c\) the speed of light in vacuum, respectively. \(T_{\mu\nu}\) denotes the energy-momentum tensor which comes from the variation of \(I_{\text{matter}}\) and \(\chi_{\mu\nu}\) is the massive term with the following explicit form

\[
\chi_{\mu\nu} = \frac{c_1}{2} (U_1 g_{\mu\nu} - K_{\mu\nu}) - \frac{c_2}{2} (U_2 g_{\mu\nu} - 2U_1 K_{\mu\nu} + 2K_{\mu\nu}^2)
\]
\[
- \frac{c_3}{2} (U_3 g_{\mu\nu} - 3U_2 K_{\mu\nu} + 6U_1 K_{\mu\nu}^2 - 6K_{\mu\nu}^3)
\]
\[
- \frac{c_4}{2} (U_4 g_{\mu\nu} - 4U_3 K_{\mu\nu} + 12U_2 K_{\mu\nu}^2 - 24U_1 K_{\mu\nu}^3 + 24K_{\mu\nu}^4).
\]

Considering a spherical symmetric space-time in 4-dimensional as

\[
g_{\mu\nu} = H(r) dt^2 - \frac{dr^2}{S(r)} - r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right),
\]
where $H(r)$ and $S(r)$ are unknown metric functions. The reference metric is given \[28, 29\]

$$f_{\mu\nu} = \text{diag}(0, 0, C^2r^2, C^2r^2 \sin^2 \theta),$$ (5)

in which $C$ is a positive constant. Considering the neutron star as a perfect fluid with the following energy-momentum tensor as

$$T^{\mu\nu} = \left( c^2 \rho + P \right) U^\mu U^\nu - P g^{\mu\nu},$$ (6)

where $P$ and $\rho$ are the pressure and density of the fluid which are measured by the local observer, respectively, and $U^\mu$ is the fluid four-velocity. The metric function $S(r)$ has been extracted in the following form \[51\]

$$S(r) = 1 - \frac{m^2}{2} C \left( \frac{c^4 r^2 + c^2 C^2}{c^2 r} \right),$$ (7)

in which $M(r) = \int 4\pi r^2 \rho(r) dr$. The modified TOV equation in Einstein-massive gravity has been obtained as \[51\]

$$\frac{dP}{dr} = \frac{G(c^2 B(r) + 4\pi r^3 P) - \frac{m^2 c^2 e^C}{4}}{\left( \frac{m^2 c^2 e^{C^2}}{2} + 2GM(r) + c^2 r\left( m^2 c^2 C^2 - 1 \right) \right)} \left( c^2 \rho + P \right).$$ (8)

Considering the obtained modified TOV equation, we want to investigate the properties of white dwarfs in massive gravity in the next section.

### III. EQUATION OF STATE

we use the Chandrasekhar’s equation of state (EoS), which are constituted of electron degenerate matter,

$$k_F = \hbar \left( \frac{3\pi^2 \rho/(m_p \mu_e)}{4} \right)^{1/3}$$ (9)

and

$$P = \frac{8\pi c}{3(2\pi h)^3} \int_0^{k_F} \frac{k^2}{(k^2 + m^2 c^2)^{1/2}} k^2 dk,$$ (10)

where $k$ is the momentum of electrons, $m_p$ is the mass of a proton, $\mu_e$ is the mean molecular weight per electron (we choose $\mu_e = 2$ for our work), $\hbar = h/2\pi$, $h$ is the Planck’s constant. The Chandrasekhar’s EoS of the electron degenerate matter was shown in Fig. 1.

#### A. Energy conditions

In order to more investigation of the introduced Chandrasekhar’s EoS of the electron degenerate matter, we study the energy conditions such as the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) at the center of obtained white dwarfs by using this EoS. These conditions are as

$$\text{NEC} \rightarrow P_c + \rho_c \geq 0,$$ (11)

$$\text{WEC} \rightarrow P_c + \rho_c \geq 0, \quad \& \quad \rho_c \geq 0,$$ (12)

$$\text{SEC} \rightarrow P_c + \rho_c \geq 0, \quad \& \quad 3P_c + \rho_c \geq 0,$$ (13)

$$\text{DEC} \rightarrow \rho_c > |P_c|,$$ (14)

where $P_c$ and $\rho_c$ are the pressure and density at the center of white dwarfs ($r = 0$), respectively. Using Fig. 1 and the mentioned conditions \[11,14\], our results are presented in table 1. According to Fig. 1 and table 1, we observe that all energy conditions are satisfied. So, the Chandrasekhar’s EoS of the electron degenerate matter introduced in Eq. 10 is suitable.
TABLE I: Energy conditions at the center of obtained white dwarfs of Chandrasekhar’s EoS.

| $\rho_c (10^{12} \text{ kg/m}^3)$ | $P_c (10^{15} \text{ kg/m}^3)$ | NEC | WEC | SEC | DEC |
|-------------------------------|-------------------|-----|-----|-----|-----|
| 150.6590                      | 4.0515            | ✓   | ✓   | ✓   | ✓   |

B. Stability

In order to evaluate the Chandrasekhar’s EoS for a physically acceptable model, one expects that the velocity of sound ($v = \sqrt{\frac{dP}{d\rho}}$) be less than the light’s velocity ($c$) [52, 53]. In other words, stability condition is in the form $0 \leq v^2 \leq c^2$. Therefore, by considering this stability condition and Fig. 1 and comparing them with diagrams related to speed of sound-density relationship plot in Fig. 2, it is evident that this EoS satisfies the inequality $0 \leq v^2 \leq c^2$.

FIG. 1: Chandrasekhar’s equation of state.

FIG. 2: Sound speed ($v^2/c^2 \times 10^{-18}$) versus density ($\rho \times 10^{14} \text{(kg/m}^3\text{)}$).
than Chandrasekhar limit ($M_{\text{Max}}$) of white dwarf in massive gravity and by employing the Chandrasekhar’s EoS can be more than this limit ($1.44M_\odot$). Here, we would like to see whether the maximum mass of white dwarfs increase as $m^2c_2$ and $C$ (tables II and III). Our calculations show that the maximum mass of white dwarf in massive gravity can be more than $3M_\odot$. In addition, the variation of maximum mass versus radius ($M - R$) is also shown in right panels of Figs. 3 and 4.

C. Le Chatelier’s principle

There is another important principle which is related to the matter of star so called Le Chatelier’s principle. The matter of star satisfies $dP/d\rho \geq 0$ which is a necessary condition of a stable body both as a whole and also with respect to the non-equilibrium elementary regions with spontaneous contraction or expansion (Le Chatelier’s principle), see Ref. [54], for more details. Our calculation show that, Le Chatelier’s principle is established for the Chandrasekhar’s EoS (see Fig. 2).

Our investigations indicate that the Chandrasekhar’s EoS satisfies both energy and stability conditions, and also this EoS respect to Le Chatelier’s principle.

IV. PROPERTIES OF WHITE DWARFS

Using the famous Chandrasekhar limit, the mass limit of the white dwarf is obtained about $1.44M_\odot$. On the other hand, the explosion of peculiar type Ia supernovae provoke us to rethink the maximum mass of white dwarfs. Hence, the maximum mass of white dwarf is still an open question. Here, we would like to see whether the maximum mass of white dwarf in massive gravity and by employing the Chandrasekhar’s EoS can be more than this limit ($1.44M_\odot$). Then we want to study the effects of massive’s parameter on properties of the obtained massive white dwarfs such as; Schwarzschild radius, average density, compactness and dynamical stability. For this purpose, we consider the mass of graviton about $1.78 \times 10^{-65}g$, which was extracted in Ref. [52]. Our results indicate that by considering the spacial values for the parameters of massive gravity, the maximum mass of white dwarf is an increasing function of $m^2c_2$ and $C$ (tables IV and V). Our calculations show that the maximum mass of white dwarf in massive gravity can be more than Chandrasekhar limit ($M_{\text{Max}} > 1.44M_\odot$). In other words, our results predict that the mass of white dwarfs in this gravity can be in the range upper than $3M_\odot$. Also, by decreasing the values of $m^2c_2$ and $C$ less than $-10^{-3}$ and $10^{-2}$, respectively, the maximum mass and radius of white dwarfs are not affected. In other words, considering the value of $m^2c_2$ and $C$ about $-10^{-3}$ and $10^{-2}$, respectively, the maximum mass and radius of white dwarfs reduce to the obtained results of Einstein gravity. It is notable that the variation of $m^2c_1$ has very interesting effects. In this case, the maximum mass and radius of white dwarfs are constant and by variation of $m^2c_1$ (see the table IV).

In order to more investigation, we plot the mass of white dwarf versus the central mass density ($M - \rho_c$) in left panels of Figs. 3 and 4. This figures show that, the maximum mass of white dwarfs increase as $m^2c_2$ and $C$ increase. In addition, the variation of maximum mass versus radius ($M - R$) is also shown in right panels of Figs. 3 and 4.

| $m^2c_2$ | $M_{\text{Max}}(M_\odot)$ | $R$(km) | $R_{\text{Sch}}$(km) | $\bar{\rho}$(10$^{12}$kg m$^{-3}$) | $\sigma$(10$^{-2}$) |
|---------|-----------------|------|-----------------|-----------------|---------|
| $-1 \times 10^{-4}$ | 1.41 | 871 | 4.17 | 1.01 | 0.48 |
| $-1 \times 10^{-3}$ | 1.41 | 871 | 4.17 | 1.02 | 0.48 |
| $-1 \times 10^{-2}$ | 1.43 | 875 | 4.19 | 1.02 | 0.48 |
| $-1 \times 10^{-1}$ | 1.63 | 913 | 4.37 | 1.02 | 0.48 |
| $-2 \times 10^{-1}$ | 1.86 | 954 | 4.57 | 1.02 | 0.48 |
| $-4 \times 10^{-1}$ | 2.34 | 1030 | 4.93 | 1.02 | 0.48 |
| $-6 \times 10^{-1}$ | 2.86 | 1101 | 5.27 | 1.02 | 0.48 |
| $-8 \times 10^{-1}$ | 3.41 | 1168 | 4.79 | 1.02 | 0.48 |

| $C$ | $M_{\text{Max}}(M_\odot)$ | $R$(km) | $R_{\text{Sch}}$(km) | $\bar{\rho}$(10$^{12}$kg m$^{-3}$) | $\sigma$(10$^{-2}$) |
|-----|-----------------|------|-----------------|-----------------|---------|
| 0.01 | 1.41 | 870 | 4.17 | 1.02 | 0.48 |
| 0.1 | 1.42 | 871 | 4.17 | 1.02 | 0.48 |
| 0.5 | 1.52 | 892 | 4.27 | 1.02 | 0.48 |
| 1.0 | 1.86 | 954 | 4.57 | 1.02 | 0.48 |
| 1.5 | 2.46 | 1048 | 5.02 | 1.02 | 0.48 |
| 2.0 | 3.41 | 1168 | 5.59 | 1.02 | 0.48 |
TABLE IV: Structure properties of white dwarf in massive gravity for $C = 1$ and $m^2 c_2 = -2 \times 10^{-1}$.

| $m^2 c_1$ | $M_{\text{max}} (M_\odot)$ | $R$ (km) | $R_{\text{Sch}}$ (km) | $\rho$ ($10^{12}$ kg m$^{-3}$) | $\sigma$ ($10^{-2}$) |
|-----------|-----------------|-------|-----------------|----------------|----------------|
| $1 \times 10^{-13}$ | 1.86 | 953 | 4.56 | 1.02 | 0.48 |
| $1 \times 10^{-12}$ | 1.86 | 953 | 4.56 | 1.02 | 0.48 |
| $1 \times 10^{-11}$ | 1.86 | 954 | 4.57 | 1.02 | 0.48 |
| $1 \times 10^{-10}$ | 1.86 | 956 | 4.58 | 1.02 | 0.48 |
| $-1 \times 10^{-11}$ | 1.86 | 953 | 4.56 | 1.02 | 0.48 |
| $-1 \times 10^{-12}$ | 1.86 | 953 | 4.56 | 1.02 | 0.48 |
| $-1 \times 10^{-13}$ | 1.86 | 950 | 4.55 | 1.02 | 0.48 |

FIG. 3: Gravitational mass versus central density (radius) for $C = 1$ and $m^2 c_1 = 1 \times 10^{-11}$. Left diagrams: gravitational mass versus central mass density for $m^2 c_2 = -1.0 \times 10^{-1}$ (solid line), $m^2 c_2 = -2.0 \times 10^{-1}$ (dotted line), $m^2 c_2 = -3.0 \times 10^{-1}$ (dashed line), $m^2 c_2 = -5.0 \times 10^{-1}$ (dashed-dotted line) and $m^2 c_2 = -7.0 \times 10^{-1}$ (dashed-dotted-dotted line). Right diagrams: gravitational mass versus radius for $m^2 c_2 = -1.0 \times 10^{-1}$ (solid line), $m^2 c_2 = -2.0 \times 10^{-1}$ (dotted line), $m^2 c_2 = -3.0 \times 10^{-1}$ (dashed line), $m^2 c_2 = -5.0 \times 10^{-1}$ (dashed-dotted line) and $m^2 c_2 = -7.0 \times 10^{-1}$ (dashed-dotted-dotted line).

FIG. 4: Gravitational mass versus central density (radius) for $m^2 c_1 = 1 \times 10^{-11}$ and $m^2 c_2 = -1 \times 10^{-1}$. Left diagrams: gravitational mass versus central mass density for $C = 1.0$ (solid line), $C = 1.5$ (dotted line), $C = 2.0$ (dashed line), $C = 2.3$ (dashed-dotted line) and $C = 2.5$ (dashed-dotted-dotted line). Right diagrams: gravitational mass versus radius for $C = 1.0$ (solid line), $C = 1.5$ (dotted line), $C = 2.0$ (dashed line), $C = 2.3$ (dashed-dotted line) and $C = 2.5$ (dashed-dotted-dotted line).
For completeness, in the following, we investigate other properties of white dwarf in massive gravity such as the Schwarzschild radius, average density compactness and dynamical stability.

A. modified Schwarzschild Radius

The Schwarzschild radius for this gravity is obtained as [51]

\[ R_{Sch} = \frac{c (1 - m^2 c_2 C^2)}{m^2 c_1 C} - \frac{\sqrt{c^2 (m^2 c_2 C^2 - 1)^2 - 4m^2 c_1 CGM}}{m^2 c_1 C}. \] (15)

The Schwarzschild radius of white dwarfs obtain in tables II and III, these results show that by increasing the maximum mass and radius of white dwarfs, the Schwarzschild radius increases and the obtained white dwarfs in massive gravity with mass more than the Chandrasekhar limit are out of the Schwarzschild radius (see tables II and III). In other words, different parameters of massive gravity have different behavior on the Schwarzschild radius. for example, by considering the negative value of \( m^2 c_2 \) and increasing \( m^2 c_2 \), the Schwarzschild radius increases (see table II). Also, by increasing \( C \), the Schwarzschild radius increases (see table III). on the other hand, by considering the positive (negative) values of \( m^2 c_1 \) and increasing (decreasing) \( m^2 c_1 \), the Schwarzschild radius does not change (see table IV).

B. Average Density

We can evaluate the average density by using the obtained maximum mass and radius of white dwarfs in the massive gravity. So the average density of white dwarf is given

\[ \rho = \frac{3M}{4\pi R^3}, \] (16)

where the results for variation of the massive parameters are presented in the tables II, III and IV. There is an interesting results about the average density of white dwarfs in massive gravity. The average density of white dwarfs are the same. In other words, by variations of the different parameters, the average density remains fixed (see tables II, III and IV).

C. Compactness

The compactness of a spherical object may be defined as

\[ \sigma = \frac{R_{Sch}}{R}, \] (17)

where may be indicated as the strength of gravity. Here we want to investigate the effects of parameters of massive gravity. Hence, we obtain the values of \( \sigma \) in the tables II, III and IV. Similar to the obtained results for the average density, by considering different values of parameters of massive gravity, the results show that the strength of gravity is the same.

D. Dynamical Stability

Another important quantity is related to the dynamical stability of white dwarfs in massive gravity. Chandrasekhar introduced the dynamical stability of stellar model against the infinitesimal radial adiabatic perturbation in Ref. [56]. Some authors developed this stability condition and applied it to astrophysical cases in Refs. [57–60]. The adiabatic index (\( \gamma \)) is defined in the following form

\[ \gamma = \frac{\rho c^2 + P dP}{c^2 P \frac{dP}{d\rho}}. \] (18)
We will encounter with the dynamical stability when $\gamma$ is more than $4/3$ ($\gamma > 4/3 = 1.33$) everywhere within the obtained white dwarfs. So, we plot two diagrams related to $\gamma$ versus radius for different values of $m^2 c_2$ and $C$ in Fig. 4. Our results show that, these white dwarfs are stable against the radial adiabatic infinitesimal perturbations.

Here, we want to evaluate another property of white dwarfs, therefore, we plot the density (pressure) versus distance from the center of white dwarfs in Figs. 6 and 7. As one can see, the density and pressure are maximum at the center and they decrease monotonically towards the boundary (see Figs. 6 and 7 for more details).

V. CLOSING REMARKS

In this paper, we studied white dwarfs in massive gravity by employing the Chandrasekhar’s EoS. In order to evaluate the Chandrasekhar’s EoS for a physically acceptable model, we investigated the energy conditions, stability ($0 \leq \nu^2 \leq \epsilon^2$) and Le Chatelier’s principle. Our results indicated that the Chandrasekhar’s EoS satisfied these conditions. Then by considering the modified TOV equation in massive gravity, we studied the white dwarfs in this gravity in order to find massive white dwarfs. Our results showed that the maximum mass of white dwarfs can be more than Chandrasekhar limit ($M_{\text{Max}} > 1.44 M_\odot$). Indeed, we extracted massive white dwarfs with mass more than three times of solar mass ($M_{\text{Max}} > 3 M_\odot$). Then we studied other interesting properties of white dwarfs such as Schwarzschild radius, average density, compactness and dynamical stability in order to have physical white dwarfs.
Briefly, we obtained the quite interesting results from massive gravity for the white dwarfs such as:

I) Prediction of maximum mass for white dwarfs more than $3M_\odot$, due to the existence of massive gravitons. In other words, super-Chandrasekhar white dwarfs ($M_{\text{Max}} > 1.44M_\odot$) in massive gravity were acceptable.

II) Massive white dwarfs in the massive gravity had dynamically stable.

III) The Chandrasekhar’s EoS satisfied the energy, stability conditions and Le Chatelier’s principle, simultaneously.

IV) Considering different values of parameters of massive gravity, the average density and the strength of gravity were constant.

Finally, it is notable that rotating, slowly rotating and magnetized white dwarfs \cite{61-69} in the context of massive gravity are interesting topics. We leave these issues for future work.

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\[\text{FIG. 7: Pressure versus radius for } m^2c_1 = 1 \times 10^{-11}. \text{ Left diagrams: for } C = 1, \quad m^2c_1 = -1 \times 10^{-1} \text{ (continuous line), } m^2c_1 = -5 \times 10^{-1} \text{ (dotted line), } m^2c_1 = -8 \times 10^{-1} \text{ (dashed line). Right diagrams: for } m^2c_2 = -1 \times 10^{-1}, \quad C = 1.0 \text{ (continuous line), } \quad C = 2.0 \text{ (dotted line), } \quad C = 3.0 \text{ (dashed line).}\]
