NUCLEAR GEOMETRY OF JET QUENCHING

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Abstract

The most suitable way to study the jet quenching as a function of distance traversed is varying the impact parameter $b$ of ultrarelativistic nucleus-nucleus collision (initial energy density in nuclear overlapping zone is almost independent of $b$ up to $b \sim R_A$). It is shown that $b$-dependences of medium-induced radiative and collisional energy losses of a hard parton jet propagating through dense QCD-matter are very different. The experimental verification of this phenomenon could be performed for a jet with non-zero cone size basing on essential difference between angular distributions of collisional and radiative energy losses.
1 Introduction

The experimental investigation of ultrarelativistic nuclear collisions offers a unique possibility of studying the properties of strongly interacting matter at the high energy density when the hadronic matter is expected to become deconfined and a gas of asymptotically free quarks and gluons is formed. This is called quark-gluon plasma (QGP), in which the colour interactions between partons are screened owing to collective effects (see, for example, reviews [1, 2, 3, 4]).

In recent years, a great deal of attention has been paid to the study of ”hard” probes of QGP – heavy quarkonia and hard partonic jets, which do not appear as constituents of the thermalized system, but can carry information about the earliest stages of its evolution. In particular, the strong suppression of yield of heavy quark vector mesons as $J/\Psi$, $\Psi'$ ($c\bar{c}$ states) and $\Upsilon$, $\Upsilon'$ ($b\bar{b}$ states) is one of the promising signatures of the quark-gluon plasma formation in heavy ion collisions [5]. An intriguing phenomenon is the ”anomalously” small yield of $\Psi$-resonances, observed in Pb-Pb collisions in the NA50 experiment (CERN-SPS) [6] and inconsistent with the conventional model of pre-resonance absorption in cold nuclear matter. Although the interpretation of this phenomenon as a result of the formation of a QGP is quite plausible [7], alternative explanations have also been put forward, such, for example, as $\Psi - h$ rescattering on comoving hadrons [8]. Thus the nature of this ”anomalous” suppression of $\Psi$-resonance production is not yet fully understood, and it should be completely explained in future [9]. For heavier ($b\bar{b}$) systems, a similar suppression effect in super-dense strongly interacting matter is expected at higher temperatures than for $c\bar{c}$, which are expected to be reached in central collisions of heavy ions at the RHIC at BNL and LHC at CERN colliders.

Along with the suppression of heavy quarkonia, one of the processes which may give information about the earliest stages of evolution of the dense matter formed in ultrarelativistic nuclear collisions is the passage through the matter of hard jets of colour-charged partons, pairs of which are created at the very beginning of the collision process (typically, at $\lesssim 0.01$ fm/c) as a result of individual initial hard nucleon-nucleon (parton-parton) scatterings. Such jets pass through the dense parton matter formed due to mini-jet production at larger time scales ($\sim 0.1$ fm/c), and interact strongly with the comoving constituents in the medium, changing its original properties as a result of additional rescatterings. The inclusive cross section for hard jet production processes is still very small for performing a systematic analysis at the SPS energies ($\sqrt{s} \simeq 20$ GeV per nucleon pair), but it increases fast with the energy of collided nuclei. Thus these will play important role in the formation of the initial state at the energies of RHIC ($\sqrt{s} = 200$ GeV per nucleon pair) and LHC ($\sqrt{s} = 5.5$ TeV per nucleon pair) colliders.

The actual problem is to study the energy losses of a hard jet evolving through the dense matter. We know two possible mechanisms of energy losses: (1) radiative losses due to gluon ”bremsstrahlung” induced by multiple scattering [10, 11, 12, 13, 14, 15] and (2) collisional losses
due to the final state interactions (elastic rescatterings) of high $p_T$ partons off the medium constituents \cite{16, 17, 18}. Since the jet rescattering intensity strongly increases with temperature, formation of a super-dense and hot partonic matter in heavy ion collisions (with initial temperature up to $T_0 \sim 1$ GeV at LHC \cite{19}) should result in significantly larger jet energy losses as compared with the case of ”cold” nuclear matter or hadronic gas at $T \lesssim 0.2$ GeV.

Although the radiative energy losses of a high energy parton have been shown to dominate over the collisional losses by up to an order of magnitude \cite{11}, a direct experimental verification of this phenomenon remains an open problem. Indeed, with increasing of hard parton energy the maximum of the angular distribution of bremsstrahlung gluons has shift towards the parent parton direction. This means that measuring the jet energy as a sum of the energies of final hadrons moving inside an angular cone with a given finite size $\theta_0$ will allow the bulk of the gluon radiation to belong to the jet and thus the major fraction of the initial parton energy to be reconstructed. Therefore, the medium-induced radiation will, in the first place, soften particle energy distributions inside the jet, increase the multiplicity of secondary particles, but will not affect the total jet energy. It was recently shown \cite{12, 13} that the radiation of energetic gluons in a QCD medium is essentially different from the Bethe-Heitler independent radiation pattern. Such gluons have formation times exceeding the mean free path for QCD parton scattering in the medium. In these circumstances the coherent effects play a crucial role leading to a strong suppression of the medium-induced gluon radiation. This coherent suppression is a QCD analogue of the Landau-Pomeranchuk-Migdal effect in QED. It is important to notice that the coherent LPM radiation induces a strong dependence of the jet energy on the jet cone size $\theta_0$ \cite{20, 21}.

On the other hand, the collisional energy losses represent an incoherent sum over all rescatterings. It is almost independent of the initial parton energy. Meanwhile, the angular distribution of the collisional energy loss is essentially different from that of the radiative one. The bulk of ”thermal” particles knocked out of the dense matter by elastic scatterings fly away in almost transverse direction relative to the hard jet axis. As a result, the collisional energy loss turns out to be practically independent on $\theta_0$ and emerges outside the narrow jet cone. Thus the relative contribution of collisional losses would likely become significant for jets with finite cone size propagating through the QGP \cite{21}.

In a search for experimental evidences in favour of the medium-induced energy losses a significant dijet quenching (a suppression of high-$p_T$ jet pair yield) \cite{22} and a monojet-to-dijet ratio enhancement \cite{23} were proposed as possible signals of dense matter formation in ultrarelativistic collisions of nuclei. Other possible signatures that could directly measure the energy losses involve tagging the hard jet opposite a particle that does not interact strongly as a $Z$-boson \cite{24} (mostly $q + g \rightarrow q + Z (\rightarrow \mu^+\mu^-)$, but also $q + q \rightarrow q + Z$), or a photon \cite{25}.
(mostly $q + g \rightarrow q + \gamma$, also $q + \bar{q} \rightarrow q + \gamma$). The jet energy losses in dense matter should result in the non-symmetric shape of the distribution of differences in $P_T$ between the Z-boson ($\gamma$) and jet. The above phenomena can be studied in heavy ion collisions [26] with Compact Muon Solenoid (CMS), which is the general purpose detector designed to run at the LHC [27]. Note, that using $\gamma + jet$ channel in this case is complicated due to large background from $jet + jet$ production when one of the jet in an event is misidentified as a photon (the leading $\pi^0$). However the shape of the distribution of differences in $E_T$ between the $\gamma$ and jet is very different for signal and background, and still sensitive to the jet energy losses [26].

The advantage of $\gamma + jet$ and $Z(\rightarrow \mu^+\mu^-) + jet$ channels is that one can determine the average initial transverse momentum of the hard jet, $\langle P_T^{jet} \rangle \approx \langle P_T^{\gamma,Z} \rangle$. It gives the attractive possibility to search for coherent effects in QCD-medium: the dependence of energy losses of the distance traversed can be studied experimentally in different bins of impact parameter distribution of nucleus-nucleus collision, or by varying collided ions and selecting the most central collisions. The intriguing prediction associated with the coherence pattern of the medium induced radiation is that radiative energy losses per unit distance $dE/dx$ depend on the total distance traversed $L$ [12, 13]. The value $dE/dx$ is approaching to being proportional to $L$ for static medium [12], and it has weaker $L$-dependence for the case of expanding medium [13].

The main goal of the present paper is to analyze the possibility of observing the $L$-dependence of jet energy losses $dE/dx$ for realistic nuclear geometry. In particular, we are studying the impact parameter dependence of collisional and radiative jet energy losses in dense QCD-matter, created in ultrarelativistic heavy ion collisions.

2 The geometrical model for jet production in nuclear collisions

Let us to consider the simple geometrical model of jet production and jet passing through a dense matter in high energy symmetric nucleus-nucleus collision. The figure 1 shows the essence of the problem in the plane of impact parameter $b$ of two colliding nuclei $A-A$. The impact parameter $b$ here is the transverse distance between nucleus centers $O_1$ and $O_2$, $OO_2 = -O_1O = b/2$. Let $B(r \cos \psi, r \sin \psi)$ be denoted as a jet (dijet) production vertex, with $r$ being the distance from the nuclear collision axis to the $B$. Then the distance between nucleus centers $(O_1, O_2)$ and vertex $B$ can be found as

$$r_{1,2} = \sqrt{r^2 + \frac{b^2}{4}} \pm r b \cos \psi. \quad (1)$$

The distribution over jet production vertex $B(r, \psi)$ at given impact parameter $b$ is written as
Figure 1: Jet production in high energy symmetric nucleus-nucleus collision in the plane of impact parameter $b$. $O_1$ and $O_2$ are nucleus centers, $OO_2 = -O_1O = b/2$. $B(r \cos \psi, r \sin \psi)$ is the jet (dijet) production vertex; $r$ is the distance from the nuclear collision axis to $B$; $r_1, r_2$ are distances between nucleus centers ($O_1, O_2$) and $B$; $\varphi$ is the jet azimuthal angle; $\varphi_0$ is the azimuthal angle between vectors $r_1$ and $r_2$. 
\[ P_{AA}(r, b) = \frac{T_A(r_1) \cdot T_A(r_2)}{T_{AA}(b)}, \]  

where \[ T_{AA}(b) = \int d^2s T_A(s) T_A(b - s) = \int d\psi \int_0^{r_{max}} r dr T_A(r_1) T_A(r_2) \] is the nuclear overlap function, \[ T_A(r) = A \int_{-\infty}^{+\infty} \rho_A(r, z) dz \] is the nuclear thickness function with nucleon density distribution \( \rho_A(r, z) \). The maximum possible value of \( r \) in nuclear overlapping zone can be estimated from the equation

\[ \max\{r_1(r = r_{max}), r_2(r = r_{max})\} = R_A \]

\( (R_A \) is the radius of the nucleus \( A \) \). This gives

\[ r_{max} = \min\{\sqrt{R_A^2 - \frac{b^2}{4} \sin^2 \psi} + \frac{b}{2} \cos \psi, \sqrt{R_A^2 - \frac{b^2}{4} \sin^2 \psi} - \frac{b}{2} \cos \psi\}. \]

In particular, for the uniform nucleon density distribution, \( \rho_A^{un}(R) = \rho_0 \cdot \Theta(R_A - |R|) \), the nuclear overlap function is equal to \( T_A^{un}(r) = 3A \sqrt{R_A^2 - r^2} / (2\pi R_A^3) \). Then the distribution \( P_{AA}^{un}(r, b) \) is proportional to

\[ P_{AA}^{un}(r, b) \propto \sqrt{R_A^2 - r_1^2(r, \psi, b)} \cdot \sqrt{R_A^2 - r_2^2(r, \psi, b)}. \]

For central \( AA \) collisions \( (b = 0, r_{max} = R_A) \) we get simply \( P_{AA}^{un}(r, b = 0) \propto (R_A^2 - r^2). \)

It is straightforward to evaluate the time \( \tau_L = L \) it takes for jet to traverse the dense zone:

\[ \tau_L = \min\{\sqrt{R_A^2 - r_1^2 \sin^2 \varphi} - r_1\cos \varphi, \sqrt{R_A^2 - r_2^2 \sin^2 (\varphi - \varphi_0)} - r_2\cos(\varphi - \varphi_0)\}, \]

where \( \varphi \) is the azimuthal angle which determines the direction of a jet motion in the transverse plane, and \( \varphi_0 \) is the angle between vectors \( r_1 \) and \( r_2 \). The expression for

\[ \varphi_0 = \arccos \frac{r^2 - b^2/4}{r_1r_2} \]

can be obtained from the condition

\[ r_1r_2 \cos \varphi_0 = r_1 \cdot r_2 = (-b/2 - r \cos \psi) \cdot (b/2 - r \cos \psi) + r^2 \sin^2 \psi = r^2 - b^2/4. \]

Finally, we are going to estimate the dependence of initial energy density in nuclear overlapping zone on impact parameter of the collision. At collider energies the minijet system (the semi-hard gluons, quarks and antiquarks with \( p_T \gtrsim p_0 \sim 1 \div 2 \) GeV/c) in the central rapidity region is typically formed in parton-parton scatterings at very early times, \( \tau_0 \sim 1/p_T \lesssim 1/p_0 \sim 0.1 \)
ferm. It will then serve as initial condition for the further evolution of the system [19]. Strictly speaking, the soft particle production mechanisms (like the decay of the colour field) can also contribute to initial conditions in nuclear interactions. However, the relative strength of soft part decreases strongly with increasing c.m.s. energy of the ion beams. In particular, at LHC energies \( \sqrt{s} = 7 \text{ TeV} \times (2Z/A) \) per nucleon pair the hard and semi-hard processes contribute over 80% to the transverse energy in heavy ion collisions [19]. Moreover, soft processes with small momentum transfer \( Q^2 \sim \Lambda_{QCD}^2 \) \( \ll p_0^2 \) can be partially or fully suppressed, owing to screening of the colour interaction in the dense parton matter produced from the system of minijets in the early stages of the reaction [28]. Therefore, at LHC energies, we will consider only dominant semi-hard contribution to the formation of initial state.

The initial energy density inside the comoving volume of longitudinal size \( \Delta z = \tau_0 \cdot 2\Delta y \) can be estimated using the Bjorken formula [29, 19] as

\[
\varepsilon(\tau = \tau_0) = \frac{\langle E_T^A(|y| < \Delta y) \rangle}{S(b) \cdot \Delta z} = \frac{\langle E_T^A(|y| < \Delta y) \rangle \cdot p_0}{S(b) \cdot 2\Delta y},
\]

where

\[
S_{AA}(b) = \int_0^{2\pi} d\psi \int_0^{r_{max}} rdr = \left( \pi - 2 \arcsin \frac{b}{2R_A} \right) R_A^2 - b \sqrt{R_A^2 - b^2/4}
\]

is the effective transverse area of nuclear overlapping zone at impact parameter \( b \). The total initial transverse energy deposition in mid-rapidity region can be calculated [19] as

\[
\langle E_T^A(b, \sqrt{s}, p_0, |y| < \Delta y) \rangle = T_{AA}(b) \cdot \sigma_N^{\text{jet}}(\sqrt{s}, p_0) \cdot \langle p_T \rangle,
\]

where the first \( p_T \)-moment of inclusive differential minijet cross section \( \sigma_N^{\text{jet}}(\sqrt{s}, p_0) \cdot \langle p_T \rangle \) is determined by the dynamics of nucleon-nucleon interactions at the corresponding c.m.s. energy. Then the dependence of initial energy density \( \varepsilon_0 \) in nuclear overlapping zone on impact parameter \( b \) has the form:

\[
\varepsilon_0(b) \propto T_{AA}(b)/S_{AA}(b),
\]

or

\[
\varepsilon_0(b) = \varepsilon_0(b = 0) \frac{T_{AA}(b)}{T_{AA}(b = 0)} \frac{S_{AA}(b = 0)}{S_{AA}(b)}.
\]

For central \( AA \) collisions we have \( S_{AA}(b = 0) = \pi R_A^2 \) and \( T_{AA}(b = 0) = 9A^2/(8\pi R_A^2) \).

It is worth noting that although this simple geometrical model for jet production in nucleus-nucleus collisions is formally can be applicable up to impact parameter \( b = 2R_A \), the major informative domain of our interest is central and semi-central collisions with \( b \ll R_A \) only. We have the following reasons in favour of this.

1) The contribution of such events to total jet rate is dominant, although these events represent only a few percents of total inelastic \( AA \) cross section [30]. For example, the \( Pb - Pb \)
collisions with impact parameter \( b < 0.9 R_{Pb} = 6 \) fm contribute \( \approx 50\% \) to the total dijet rate at LHC energy, their relative fraction of total cross section being only \( \approx 10\% \) in this case \([20]\).

2) In the most central heavy ion collisions the maximum initial energy density is expected to be achieved in a fairly large (compared with typical hadronic scales) volume, when the effect of super-dense and hot matter formation, like quark-gluon plasma, can be really observable. The result for impact parameter dependence of initial energy density \( \varepsilon_0 \) in nuclear overlapping zone for uniform nucleon density is shown in figure 2: it is very weakly dependent of \( b \) (\( \delta \varepsilon_0 \lesssim 10\% \)) up to \( b \sim R_A \), and decreases rapidly at \( b \gtrsim R_A \). On the other hand, the averaged over all possible jet production vertices proper time \( \langle \tau_L \rangle \) of jet escaping from the dense zone is found to go down almost linearly with increasing impact parameter \( b \) (see the second curve in fig.2). Therefore the variation of impact parameter \( b \) of nucleus-nucleus collision (which can be measured, for example, using the total transverse energy deposition detected in different parts of calorimeters \([31]\)) up to \( b \sim R_A \) gives the possibility to study jet quenching as a function of distance traversed without significant changing initial energy density \( \varepsilon_0 \).

Meanwhile, the weakness of \( b \)-dependence of \( \varepsilon_0 \) gives us the advantage as compared with using of beams of different ions at a fixed bin of impact parameter distribution, when the scaling \( \langle \tau_L \rangle (b = 0) \propto R_A \propto A^{1/3} \) exists. Eq.(13) gives \( \varepsilon_0(b = 0) \propto A^2/R_A^4 \), i.e. \( \varepsilon_0(b = 0) \propto A^{2/3} \).

3) It is well known, that the uniform nucleon density distribution in the nucleus, \( \rho_A^{un}(R) = \rho_0 \cdot \Theta(R_A - |R|) \), can serve as a good approximation for central and semi-central collisions (see figure 4, which shows the nuclear overlap function profile for the uniform \( \Theta \) and the standard Woods-Saxon nucleon densities). The edge effects near the surface of the nucleus, impact parameter dependence of nuclear parton structure functions (“nuclear shadowing”) \([32]\), early transverse expansion of the system and other potentially important phenomena for peripheral \( (b \sim 2R_A) \) collisions are beyond our consideration here.

\[ T_{AA}^{un}(b) = T_{AA}^{un}(b = 0) \left[ 1 - \hat{b} \left[ 1 + \left( 1 - \frac{\hat{b}}{4} \right) \ln \frac{1}{b} + 2 \left( 1 - \frac{\hat{b}}{4} \right) \left( \ln (1 + \sqrt{1 - \hat{b}}) - \frac{\sqrt{1 - \hat{b}}}{1 + \sqrt{1 - \hat{b}}} \right) - \frac{\hat{b}(1 - \hat{b})}{2(1 + \sqrt{1 - \hat{b}})^2} \right] \right], \]

\( \hat{b} = b^2/(4R_A^2) \), the weak \( b \)-dependence of \( \varepsilon_0 \) and approximately linear drop of \( \langle \tau_L \rangle (b) \) being derived for \( b \lesssim R_A \) analytically.
Figure 2: The impact parameter dependence of initial energy density $\varepsilon_0(b)/\varepsilon_0(b = 0)$ in nuclear overlapping zone (solid curve), and the average proper time $\langle \tau_L \rangle / R_A$ of jet escaping from the dense matter (dashed curve) for uniform nucleon density distribution.

Figure 3: The initial energy density $\varepsilon_0(A, b)/\varepsilon_0(A = Pb, b = 0)$ in nuclear overlapping zone versus average proper time $\langle \tau_L \rangle (A, b)/\langle \tau_L \rangle (A = Pb, b = 0)$ of jet escaping from the dense matter for varying atomic weight $A$ at fixed $b = 0$ (dashed curve), and impact parameter $b$ at fixed $A = Pb = 207$ (solid curve) for uniform nucleon density distribution.
The intensity of final state rescattering and collisional and radiative energy losses of hard jet partons in dense QCD-matter, created in nuclear overlapping zone, are sensitive to their initial parameters (energy density, formation time) and space-time evolution [18]. In order to analyze the impact parameter dependence of jet energy losses and jet quenching, we treat the medium as a boost-invariant longitudinally expanding quark-gluon fluid, and partons as being produced on a hyper-surface of equal proper times $\tau = \sqrt{t^2 - z^2}$ [29]. We are expecting this is an adequate approximation for central and semi-central collisions for our semi-qualitative discourse.

The approach relies on an accumulative energy losses, when both initial and final state gluon radiation is associated with each scattering in expanding medium together including the interference effect by the modified radiation spectrum as a function of decreasing temperature $dE/dx(T)$. Note that recently the radiative energy losses of a fast parton propagating through expanding (according to Bjorken’s model) QCD plasma have been explicitly evaluated in [13] as $dE/dx|_{\text{expanding}} = c \cdot dE/dx|_{T_L}$ with numerical factor $c \sim 2$ (6) for a parton created inside (outside) the medium, $T_L$ being the temperature at which the dense matter was left [13].

The total energy losses in transverse direction experienced by a hard parton due to multiple scattering in matter are the result of averaging over the jet production vertex $P_{AA}(r, b)$ (2), the
transfer momentum squared $t$ in a single rescattering and space-time evolution of the medium:

$$\langle \Delta E_T(b) \rangle = \int_0^{2\pi} d\psi \int_0^{r_{\text{max}}} r \cdot dr \frac{T_A(r_1) \cdot T_A(r_2)}{T_{AA}(b)} \int_0^{2\pi} d\varphi \int_0^{\tau_L} d\tau \left( \frac{dE^{\text{rad}}}{dx} (\tau) + \sum_b \sigma_{ab}(\tau) \cdot \rho_b(\tau) \cdot \nu(\tau) \right).$$

(15)

Here $\tau_0$ and $\tau_L$ are the proper time of the plasma formation and the time of jet escaping from the dense zone respectively; $\rho_b \propto T^3$ is the density of plasma constituents of type $b$ at temperature $T$; $\sigma_{ab}$ is the integral cross section of scattering of the jet parton $a$ off the comoving constituent $b$ (with the same longitudinal rapidity $y$); $\nu$ and $dE/dx^{\text{rad}}$ are the thermal-averaged collisional energy loss of a jet parton due to single elastic scattering and radiative energy losses per unit distance respectively.

If the mean free path of a hard parton is larger than the screening radius in the QCD medium, $\lambda \gg \mu^{-1}$, the successive scatterings can be treated as independent [11]. The transverse distance between successive scatterings, $\Delta r_i = (\tau_{i+1} - \tau_i) \cdot v_T = (\tau_{i+1} - \tau_i) \cdot p_T/E$, is determined in linear kinetic theory according to the probability density:

$$\frac{dP}{d(\Delta r_i)} = \lambda^{-1}(\tau_{i+1}) \cdot \exp \left( - \int_0^{\Delta r_i} \lambda^{-1}(\tau_i + s) ds \right),$$

(16)

where the mean inverse free path is given by $\lambda_a^{-1}(\tau) = \sum_b \sigma_{ab}(\tau) \rho_b(\tau)$.

The dominant contribution to the differential cross section $d\sigma/dt$ for scattering of a parton with energy $E$ off the ”thermal” partons with energy (or effective mass) $m_0 \sim 3T \ll E$ at temperature $T$ can be written as [11, 33]

$$\frac{d\sigma_{ab}}{dt} \approx C_{ab} \frac{2\pi \alpha_s^2(t)}{t^2},$$

(17)

where $C_{ab} = 9/4, 1, 4/9$ for $gg$, $gq$ and $qq$ scatterings respectively,

$$\alpha_s = \frac{12\pi}{(33 - 2N_f) \ln (t/\Lambda_{QCD}^2)}$$

(18)

is the QCD running coupling constant for $N_f$ active quark flavours, and $\Lambda_{QCD}$ is the QCD scale parameter which is of the order of the critical temperature, $\Lambda_{QCD} \simeq T_c$. The integrated parton scattering cross section,

$$\sigma_{ab} = \int \frac{dt}{\mu_D^2(\tau)} \int \frac{d\sigma_{ab}}{dt},$$

(19)

is regularized by the Debye screening mass squared $\mu_D^2$.

The collisional energy losses due to elastic scattering with high-momentum transfer have been originally estimated by Bjorken in [16], and recalculated later in [17] taking also into
account the loss with low-momentum transfer dominated by the interactions with plasma collective modes. Since latter process contributes to the total collisional energy losses without the large factor $\sim \ln \left( \frac{E}{\mu_D} \right)$ in comparison with high-momentum scattering and it can be effectively "absorbed" by the redefinition of minimal $t \sim \mu_D^2$ under the numerical estimates, we shall concentrate on collisional energy losses with high-momentum transfer only. The thermal average of such loss can be written as

$$\nu = \left\langle \frac{t}{2m_0} \right\rangle = \frac{1}{2} \left\langle \frac{1}{m_0} \right\rangle \cdot \langle t \rangle \simeq \frac{1}{4T} \int \frac{d\sigma_{ab}}{dt} \frac{3TE/2}{\mu_D^2} ,$$

(20)

The value $\nu$ is independent of total distance traversed and determined by temperature, roughly $\nu \propto T$. Then total collisional energy losses integrated over whole jet path are estimated as $\langle \Delta E_{col} \rangle \propto T^2 \propto \sqrt{\varepsilon_0}$, as it has been pointed out in [10]. The $\tau_L$-dependence of $\Delta E_{col}$ can be weaker than linear for expanding medium ($\Delta E_{col} \propto \tau_L$ for static matter).

The energy spectrum of coherent medium-induced gluon radiation and the corresponding dominated part of radiative energy losses, $dE/dx$, were analyzed in [12, 13] by means of the Schrödinger-like equation whose "potential" is determined by the single-scattering cross section of the hard parton in the medium. For the quark produced in the medium it gives [13, 14]

$$\frac{dE^{rad}}{dx} = \frac{2\alpha_s C_R}{\pi \tau_L} \int d\omega \left[ 1 - y + \frac{y^2}{2} \right] \ln |\cos (\omega_1 \tau_1)| ,$$

(21)

$$\omega_1 = \sqrt{i \left( 1 - y + \frac{C_R}{3} y^2 \right) \bar{k} \ln \frac{16}{\bar{k}} \text{ with } \bar{k} = \frac{\mu_D^2 \lambda_g}{\omega (1 - y)} .$$

(22)

Here $\tau_1 = \tau_L/(2\lambda_g)$, and $y = \omega/E$ is the fraction of the hard parton energy carried by the radiated gluon, and $C_R = 4/3$ is the quark colour factor. A similar expression for the gluon jet can be obtained by substituting $C_R = 3$ and a proper change of the factor in the square bracket in (21), see [13]. The integral (21) is carried out over all energies from $\omega_{\text{min}} = E_{\text{LPM}} = \mu_D^2 \lambda_g$ ($\lambda_g$ is the gluon mean free path), the minimal radiated gluon energy in the coherent LPM regime, up to initial jet energy $E$. The complex form of the expression (21) does not allow us in general case to extract the explicit form of $\tau_L$- and $T$- dependences of $dE/dx^{rad}$. In the limit of "strong" LPM effect, $\omega \gg \mu_D^2 \lambda_g$, we have [12, 13, 21] $dE/dx^{rad} \propto T^3$ and $dE/dx^{rad} \propto \tau_L$.

Anyway, high- and low-momentum parts of collisional energy losses have the same dependence on distance traversed.

The gluons with formation times $\tau_f$ exceeding the time $\tau_L = L$ that are formed outside the medium (the factorization medium-independent component) carry away a fraction of the initial parton energy proportional to $\alpha_s(E)$. This part of gluon radiation produces the standard jet energy profile which is identical to that of a jet produced in a hard process in the vacuum. Hereafter we shall concentrate on the medium-dependent effects and will not include the "vacuum" part of the jet profile.
with logarithmic accuracy. Then total radiative energy losses \( \langle \Delta E_{\text{rad}} \rangle = \int d\tau \cdot dE/dx_{\text{rad}} \) are estimated as \( \langle \Delta E_{\text{rad}} \rangle \propto T_0^3 \propto \varepsilon_0^{3/4} \) and \( \Delta E_{\text{rad}} \propto \tau_L^{\beta} \), where \( \beta \lesssim 2 \) for expanding medium (\( \beta \sim 2 \) in the case of static matter).

In order to simplify numerical calculations (and not to introduce new parameters) we omit the transverse expansion and viscosity of the fluid using the well-known scaling Bjorken’s solution \[29\] for temperature and density of QGP at \( T > T_c \simeq 200 \text{ MeV} \):

\[
\varepsilon(\tau)\tau^{4/3} = \varepsilon_0\tau_0^{4/3}, \quad T(\tau)\tau^{1/3} = T_0\tau_0^{1/3}, \quad \rho(\tau)\tau = \rho_0\tau_0. \quad (23)
\]

Let us remark that the influence of the transverse flow, as well as of the mixed phase at \( T = T_c \), on the intensity of jet rescattering (which is a strongly increasing function of \( T \)) seems to be inessential for high initial temperatures \( T_0 \gg T_c \) \[18\]. On the contrary, the presence of viscosity slows down the cooling rate, which leads to a jet parton spending more time in the hottest regions of the medium. As a result the rescattering intensity goes up, i.e., in fact an effective temperature of the medium gets lifted as compared with the perfect QGP case \[18\]. We also do not take into account here the probability of jet rescattering in nuclear matter, because the intensity of this process and corresponding contribution to total energy losses are not significant due to much smaller energy density in a ”cold” nuclei. For certainty we used the initial conditions for the gluon-dominated plasma formation \( (N_f \approx 0, \rho_q \approx 1.95T^3) \) expected for central \( Pb - Pb \) collisions at LHC \[19\]: \( \tau_0 \simeq 0.1 \text{ fm}/c, T_0 \simeq 1 \text{ GeV} \).

Figure 5 represents the calculated \( \tau_L \)-dependence of coherent medium-induced radiative and collisional energy losses of a quark-initiated jet with initial energy \( E_T^q = 100 \text{ GeV} \). We see that the \( \tau_L \)-dependence of radiative and collisional losses is very different: \( \Delta E_{\text{rad}}(\tau_L) \) grows somewhat stronger than linearly, meanwhile \( \Delta E_{\text{col}}(\tau_L) \) looks rather logarithmic. This results in the corresponding difference in the impact parameter dependence of radiative and collisional losses, the normalized profile of which are presented in figure 6. To make the plot more visual the energy losses \( \langle \Delta E_T(b) \rangle \) are normalized to the corresponding average values at zero impact parameter, \( \langle \Delta E_T^{\text{rad}}(b = 0) \rangle \sim 45 \text{ GeV} \) and \( \langle \Delta E_T^{\text{col}}(b = 0) \rangle \sim 5 \text{ GeV} \) for the parameters used.

For example, decreasing of impact parameter from \( b = 0 \) to \( b = R_{Pb} \) gives \( \sim 30\% \) collisional and \( \sim 50\% \) radiative losses reduction. We have also found that the the form of \( \tau_L \)-dependence of collisional losses is almost independent of scenarios of space-time evolution of QGP (perfect or viscous fluid), \( b \)-dependence of radiative losses being somewhat more sensitive to these effects.

Note that the choice of the scale for a minimal jet energy \( E_T^q \sim 100 \text{ GeV} \) corresponds to the threshold for ”true” QCD-jet recognition against the ”thermal” background jets (statistical fluctuations of the transverse energy flux) with reconstruction efficiency closed to 1 in heavy

\[4\]These estimates are of course rather approximate and model-depending: the discount of higher order \( \alpha_s \) terms, uncertainties of structure functions in the low-\( x \) region, and nuclear shadowing can result in variations of the initial energy density \[19\].
Figure 5: The medium-induced radiative (dashed) and collisional (solid) energy losses of a quark-initiated jet with initial energy $E^q_T = 100$ GeV versus the average proper time $\langle \tau_L \rangle / R_{Pb}$ of jet escaping from the dense matter.

Figure 6: The impact parameter dependence of medium-induced radiative (dashed) and collisional (solid) energy losses of a quark-initiated jet with initial energy $E^q_T = 100$ GeV normalized to the corresponding average values at zero impact parameter.
ion collisions at LHC \cite{26, 27, 34}. We hope that the separation of collisional and radiative contribution to the total energy losses, when doing the experimental data analysis for jets with finite cone size, could be performed basing on essential difference of their angular distribution \cite{20, 21}: the radiative losses are expected to dominate at small jet cone size $\theta_0$, while the relative contribution to collisional losses grows with increasing $\theta_0$.

4 Impact parameter dependence of dijet production rate

In previous section we have analyzed the impact parameter dependence of jet energy losses, which can be directly observed in $\gamma + jet$ and $Z(\rightarrow \mu^+\mu^-) + jet$ production processes. Another observable effect is a suppression of high-$p_T$ jet pair yield (dijet quenching) due to final state rescattering and energy losses. In connection with this, we would like to estimate the impact parameter dependence of jet + jet production rates in heavy ion collisions. The observed number of $\{ij\}$ type dijets with transverse momenta $p_{T1}, p_{T2}$ produced in initial hard scattering processes in minimum bias $AA$ collisions is written as:

$$\frac{dN^{dijet}_{ij}}{dp_{T1}dp_{T2}} = \int_0^\infty d^2b \frac{d^2\sigma^{ij}_{jet}}{d^2b} \cdot \frac{dN^{dijet}_{ij}}{dp_{T1}dp_{T2}}(b) \int_0^\infty d^2b \frac{d^2\sigma^{ij}_{jet}}{d^2b},$$

$$\frac{dN^{dijet}_{ij}}{dp_{T1}dp_{T2}}(b) = \int_0^{2\pi} d\psi \int_0^{r_{max}} rdr T_A(r_1) T_A(r_2) \int_0^{2\pi} d\varphi \int dp_{T1}^2 \sigma^{ij}_{jet}(p_{T1} + \Delta E_T(r, \psi, \varphi, b)) \cdot \delta(p_{T1} - p_T + \Delta E_T(r, \psi, \pi - \varphi, b)), \quad (24)$$

where parton differential cross section $d\sigma_{ij}/dp_T^2$ is calculated in the perturbative QCD:

$$\frac{d\sigma_{ij}}{dp_T^2} = K \int dx_1 \int dx_2 \int d\xi f_i(x_1, p_T^2)f_j(x_2, p_T^2) \frac{d\sigma_{ij}}{d\xi} \delta(p_T^2 - \frac{\hat{t}}{\hat{s}}), \quad (25)$$

d$\sigma_{ij}/d\xi$ expresses the differential cross-section for a parton-parton scattering as a function of the kinematical Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$, $f_{i,j}$ are the structure functions, $x$ is the nucleon-momentum fraction carried by a parton, the correction factor $K$ takes into account higher order contributions. We have tested with the program of S.D. Ellis et al. \cite{35} that next-to-leading order (NLO) corrections are insignificant ($K \sim 1$) for jets with $p_T \geq 50 \div 100$ GeV/c and reasonable cone radius in the $(y, \phi)$-plane $R = 0.3 \div 0.5$ (see also \cite{36}). Note also that the region of sufficiently hard jets, $x_{1,2} \sim \sqrt{\hat{s}/s} \gtrsim 0.2$, almost does not affected by the initial state nuclear interactions like gluon depletion ("nuclear shadowing" of nucleon structure functions) \cite{37}. Anyway, the integrated above the threshold value $p_T^{cut}$ dijet rate,

$$R_{AA}^{dijet}(p_{T1}, p_{T2} > p_T^{cut}) = \int_{p_T^{cut}} dp_{T1} \int_{p_T^{cut}} dp_{T2} \sum_{i,j} \frac{dN^{dijet}_{ij}}{dp_{T1}dp_{T2}}(AA), \quad (27)$$
in $AA$ relative to $pp$ collisions can be studied by introducing a reference process, unaffected by energy losses and with a production cross section proportional to the number of nucleon-nucleon collisions, such as Drell-Yan dimuons or (suitable for LHC) $Z(\rightarrow \mu^+\mu^-)$ production,

$$R_{AA}^{dijet} / R_{pp}^{dijet} = \left( \frac{\sigma_{AA}^{dijet}}{\sigma_{pp}^{dijet}} \right) / \left( \frac{\sigma_{AA}^{DY}(Z)}{\sigma_{pp}^{DY}(Z)} \right).$$

(28)

The cross section $d^2\sigma_{jet}^0/d^2b$ for initially produced jets in $AA$ collisions at given $b$ can be written as [26, 30]:

$$d^2\sigma_{jet}^0 / d^2b (b, \sqrt{s}) = T_{AA}(b)\sigma_{NN}^{jet}(\sqrt{s})\frac{d^2\sigma_{AA}^{in}}{d^2b}(b, \sqrt{s}),$$

(29)

where nucleon-nucleon collision cross section of the hard process $\sigma_{NN}^{jet}$ has been computed with PYTHIA model [38]. The differential inelastic $AA$ cross section is calculated as:

$$d^2\sigma_{in}^{AA} / d^2b (b, \sqrt{s}) = \left[ 1 - \left( 1 - \frac{1}{A^2} T_{AA}(b)\sigma_{NN}^{in}(\sqrt{s}) \right)^{A^2} \right],$$

(30)

with inelastic non-diffractive nucleon-nucleon cross section $\sigma_{NN}^{in} (\simeq 60 \text{ mb for } \sqrt{s} = 5.5 \text{ TeV}).$

Figure 7: The jet + jet rates for $E_T^{jet} > 100 \text{ GeV}$ and $|y^{jet}| < 2.5$ in different impact parameter bins for cases: without energy losses (solid curve), with collisional losses only (dashed curve), with collisional and radiative losses (dotted curve). The rates are normalized to the expected number of events produced in Pb-Pb collisions during two weeks of LHC running.

Figure 7 shows dijet rates $\sigma_{AA}^{in}R_{AA}^{dijet} L (E_T^{jet} > p_T^{cut} = 100 \text{ GeV}, \text{rapidity window } |y^{jet}| < 2.5)$ in different impact parameter bins for three cases: (i) without energy losses, (ii) with collisional losses only, and (iii) with collisional and radiative losses.
losses only, (iii) with collisional and radiative losses. The rates are normalized to the expected number of events produced in Pb-Pb collisions during two weeks \((1.2 \times 10^6 \text{ s})\) of LHC run time, assuming luminosity \(L = 10^{27} \text{ cm}^{-2} \text{s}^{-1}\) \([27]\). The total initial dijet rate with \(E_T^{\text{jett}} > 100 \text{ GeV}\) is estimated as \(1.1 \times 10^7\) events \((gg \rightarrow gg \simeq 60\%, \, qg \rightarrow qg \simeq 30\%, \, qq, gg \rightarrow qq \simeq 10\%)\). Since the dijet quenching is much stronger in central collisions than in peripheral one’s, the maximum and mean values of \(dN/djett/db\) distribution get shifted towards the larger \(b\). The corresponding result for jets with non-zero cone size \(\theta_0\) is expected to be somewhere between (iii) \((\theta_0 \rightarrow 0)\) and (ii) cases. The observation of a dramatic change in the \(b\)-dependence of dijet rates in heavy ion collisions as compared to what is expected from independent nucleon-nucleon interactions pattern, would indicate the existence of medium-induced parton rescattering.

As we have mentioned above, the measurement of the centrality of events can be performed from total transverse energy deposition \(E_T^{\text{tot}}\) in calorimeters, which strongly decreases from central to peripheral collisions \([31]\), roughly as \(E_T^{\text{tot}}(b) \propto T_{AA}(b)\). If jet energy losses \(\langle \Delta E_T^{\text{jett}} \rangle\) \([13]\) or dijet production rates \(R^{\text{dijett}}\) \([27, 28]\) are measured in different bins of \(E_T^{\text{tot}}\), then one can relate \(b\)- and \(E_T^{\text{tot}}\)- dependences of \(F = (\Delta E_T^{\text{jett}}, R^{\text{dijett}})\) using \(E_T^{\text{tot}}-b\) correlation functions \(C_{AA}\):

\[
F(E_T^{\text{tot}}) = \int d^2b F(b)C_{AA}(E_T^{\text{tot}}, b), \quad C_{AA}(E_T^{\text{tot}}, b) = \frac{1}{\sqrt{2\pi}\sigma_{E_T^{\text{tot}}}(b)} \exp \left( -\frac{(E_T^{\text{tot}} - E_T^{\text{tot}}(b))^2}{2\sigma_{E_T^{\text{tot}}}(b)^2} \right), \quad (31)
\]

\[
F(b) = \int dE_T^{\text{tot}} F(E_T^{\text{tot}})C_{AA}(b, E_T^{\text{tot}}), \quad C_{AA}(b, E_T^{\text{tot}}) = \frac{1}{\sqrt{2\pi}\sigma_b(E_T^{\text{tot}})} \exp \left( -\frac{(b - \bar{b}(E_T^{\text{tot}}))^2}{2\sigma_b(E_T^{\text{tot}})^2} \right). \quad (32)
\]

The estimated with the HIJING model \([33]\) accuracy of impact parameter determination \(\sigma_b(E_T^{\text{tot}}) \sim 1 - 2 \text{ fm in AA collisions at LHC}\) \([40]\) seems to be enough to observe the above effects.

5 Conclusions

To summarize, we have considered the impact parameter dependence of medium-induced radiative and collisional jet energy losses in dense QCD-matter, created in ultrarelativistic heavy ion collisions. We have found that this \(b\)-dependence is very different for each mechanism due to coherent effects (the dependence of radiative energy losses per unit distance \(dE/dx\) of total distance traversed). As a consequence, the radiative losses are more sensitive to the impact parameter of nucleus-nucleus collision, which determines the effective volume of nuclear overlapping dense zone, and the space-time evolution of the medium.

A possible way to directly observe the energy losses at different impact parameter (or total detected \(E_T\) deposition) bins, involves tagging the hard jet opposite a particle that does not interact strongly, like in \(\gamma + \text{jet}\) and \(Z(\rightarrow \mu^+\mu^-) + \text{jet}\) production processes. Since initial energy
density $\varepsilon_0$ in dense zone depends on $b$ very slightly ($\delta \varepsilon_0 \lesssim 10\%$) up to $b \sim R_A$, studying $b$-dependence appears to be advantageous than using of different ions at fixed impact parameter $b \sim 0$ (when $\varepsilon_0(b \sim 0) \propto A^{2/3}$). We hope that the separation of collisional and radiative contribution to total energy losses when doing the experimental data analysis for jets with finite cone size could be performed basing on essential difference in their angular distributions.

Another process of interest is high-$p_T$ jet pair production. The expected statistics for dijet rates in heavy ion collisions at LHC will be large enough to study the impact parameter dependence. Since suppression of dijet yield (jet quenching) due to medium-induced energy losses should be much stronger in central collisions than in the peripheral one’s, the maximum and mean values of $dN^{dijet}/db$ distribution predicted to be shifted towards the larger $b$.

Finally, the study of the impact parameter dependences in the hard jet production processes ($jet + jet$, $\gamma + jet$ and $Z + jet$ channels) is important for extracting information about the properties of super-dense QCD-matter to be created in heavy ion collisions at LHC.

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