The Quark Distributions in the $\Sigma^+$ Hyperon

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Abstract

We use the meson cloud model and the Sullivan mechanism to estimate the sea flavor asymmetry in the $\Sigma^+$ baryon and calculate the distribution functions of both sea and valence quarks. We find large deviations from SU(3).

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I. INTRODUCTION

By now, there have been a number of experiments which point to a flavor asymmetry in the sea of the proton [1–3]. Although the exclusion principle [4] and charge asymmetry [5,6] may contribute to the observed asymmetries, they are not sufficiently large to explain the data [6]. The two most noteworthy experiments are the deviation of the Gottfried sum rule from 1/3, i.e., $S_G = 0.235 \pm 0.026$ [4] and the recent Drell-Yan measurements of $\bar{d}/\bar{u}$ by E866 [3] and NA51 [2]. The CERN measurement gives $\bar{u}/\bar{d} \simeq 0.51$ at $x = 0.18$ and the Fermilab one obtains the major part of the $x$-distribution of $\bar{d}(x)/\bar{u}(x)$, finding $\bar{d}/\bar{u} \simeq 1.5$ at $x \simeq 0.2$. The most reasonable explanation of these large deviations from unity or 1/3 is that there is a flavor asymmetry in the sea quark distributions; this flavor asymmetry is readily understood if a meson cloud surrounds the quarks, in that a proton can change into a neutron by emitting a $\pi^+ (u\bar{d})$, as first pointed out by Thomas [4] and later by Henley and Miller [8]. The Sullivan process allows one to calculate the $x$-distribution of the asymmetry, as has been done by Speth and collaborators [9] and others (see [9] for references); reasonable agreement with experiment is obtained.

More recently, Alberg et al. [10] have pointed out that a test of the meson cloud model is feasible by carrying out Drell-Yan experiments with $\Sigma$ beams on protons and deuterium. In this model, the flavor asymmetry in the $\Sigma^+$, for instance, is expected to be even larger than in the proton ($p$), so that $\bar{d}/\bar{u}$ should be much larger than 1. This is in contrast to the prediction of SU(3), under which $p \to \Sigma^+$ by $d(\bar{d}) \leftrightarrow s(\bar{s})$, from which it follows that $\bar{d}/\bar{u} < 1$ in the $\Sigma^+$.

In this paper we use the convolution method with the Sullivan process to calculate the valence and sea quark distributions of the $\Sigma^+$ in the meson cloud model.

II. METHODOLOGY

In the convolution method, the physical $\Sigma^+$ wave function is composed of the following Fock states

$$|\Sigma^+\rangle = \sqrt{Z}(|\Sigma^+\rangle_{\text{bare}} + \sum_{MB} \int dyd^2k_{\perp} \phi_{BM}(y, k_{\perp}) |B(y, k_{\perp}); M(1 - y, -k_{\perp})\rangle), \quad (2.1)$$

where $\phi_{BM}(y, k_{\perp})$ is the probability amplitude to find a physical $\Sigma^+$ in a state consisting of a virtual baryon $B$ and a virtual meson $M$ with longitudinal momentum fractions $y$ and $1 - y$, and transverse momenta $k_{\perp} = (k_{\perp} \cos(\varphi), k_{\perp} \sin(\varphi))$ and $-k_{\perp}$, respectively. The wave function renormalization factor $Z$ is a measure of the probability of finding the "bare" $\Sigma^+$, that is the $\Sigma^+$ without a meson cloud in the physical $\Sigma^+$.

We assume that the $\Sigma^+(uuu)$ will have components $\Lambda^0(uus)\pi^+(u\bar{d})$, $\Sigma^0(uds)\pi^+(u\bar{d})$, $\Sigma^+(uuu)\pi^0(\sqrt{2}[d\bar{d} - u\bar{u}])$ and $p(uud)\not{K}^0(\bar{d}s)$ [10]. We neglect higher mass components.
In the infinite momentum frame (IMF, $|\vec{p}| \to \infty$ with $\vec{p}$ the $\Sigma^+$ momentum) the contribution of a certain Fock state, $BM$, to the $\Sigma^+$ quark distribution can be written in terms of its quark components as

$$\delta q_{\Sigma^+}(x) = \sum_{BM} \left( \int_x^1 f_{MB/\Sigma^+}(y) q_M(y) \frac{dy}{y} + \int_x^1 f_{BM/\Sigma^+}(y) q_B(y) \frac{dy}{y} \right),$$

(2.2)

where the splitting functions $f_{MB/\Sigma^+}(y)$ and $f_{BM/\Sigma^+}(y)$ are related to the probability amplitude $\phi_{BM}$ in the IMF via

$$f_{BM/\Sigma^+}(y) = \int_0^\infty dk_{1\perp}^2 |\phi_{BM}(y, k_{1\perp}^2)|^2,$$

(2.3)

$$f_{MB/\Sigma^+}(y) = \int_0^\infty dk_{1\perp}^2 |\phi_{BM}(1 - y, k_{1\perp}^2)|^2.$$

(2.4)

In terms of these splitting functions the wave function renormalization constant $Z$ is given by

$$Z = [1 + \sum_{BM} \langle f_{BM/\Sigma^+} \rangle]^{-1} \equiv [1 + \sum_{BM} \int_0^1 f_{BM/\Sigma^+}(y) dy]^{-1},$$

(2.5)

and the quark distribution functions $q_{\Sigma^+}$ of a $\Sigma^+$ within the Fock state expansion are given as

$$q_{\Sigma^+}(x) = Z(q_{\Sigma^+}^{\text{bare}}(x) + \delta q_{\Sigma^+}(x))$$

(2.6)

where $q_{\Sigma^+}^{\text{bare}}$ is the quark distribution of the bare $\Sigma^+$.

The next step is to calculate the splitting functions $f_{MB/\Sigma^+}$ and $f_{BM/\Sigma^+}$. We do this using time ordered perturbation theory (TOPT) in the IMF, following the steps of reference [9]. In TOPT in the IMF one can write the probability amplitudes $\phi_{BM}(y, k_{1\perp}^2)$ explicitly as

$$\phi_{BM}(y, k_{1\perp}^2) = \sqrt{m_{\Sigma^+} + m_B} V_{\text{IMF}}(y, k_{1\perp}^2) \frac{2\pi \sqrt{y(1 - y)(m_{\Sigma^+}^2 - M_{BM}^2)}}{2\pi \sqrt{y(1 - y)(m_{\Sigma^+}^2 - M_{BM}^2)}}$$

(2.7)

where $M_{BM}^2(y, k_{1\perp}^2)$ is the invariant mass squared of the intermediate $BM$ Fock state

$$M_{BM}^2(y, k_{1\perp}^2) = \frac{m_B^2 + k_{1\perp}^2}{y} + \frac{m_M^2 + k_{1\perp}^2}{1 - y}$$

(2.8)

and $V_{\text{IMF}}$ denotes the vertex function in the IMF-limit. Vertices involving point-like particles automatically fulfill the symmetry relation

$$f_{MB/\Sigma^+}(y) = f_{BM/\Sigma^+}(1 - y)$$

(2.9)

but since hadrons have an extended structure one has to introduce phenomenological vertex form factors which parameterize the unknown microscopic effects. Therefore the vertex function $V(y, k_{1\perp}^2)$ is replaced by $G(y, k_{1\perp}^2) V(y, k_{1\perp}^2)$ and from equation (9) we get the restriction

$$f_{MB/\Sigma^+}(y) = f_{BM/\Sigma^+}(1 - y)$$

(2.9)
\[ G_{BM}(y, k^2_\perp) = G_{MB}(1 - y, k^2_\perp) . \]  

This is satisfied by the exponential form \[ G_{\Sigma^+BM}(y, k^2_\perp) = \exp\left( \frac{m_{\Sigma^+}^2 - M_{BM}^2(y, k^2_\perp)}{2\Lambda^2} \right) , \]  

where \( \Lambda \) is a cut-off parameter which we use in our further calculations.

Equations (3) and (7) allow us to write the splitting functions as

\[ f_{BM/\Sigma^+}(y, k^2_\perp) = \frac{1}{4\pi^2} \frac{m_{\Sigma^+}m_B}{y(1 - y)} \frac{|V_{IMF}|^2}{[m_{\Sigma^+}^2 - M_{BM}^2(y, k^2_\perp)]^2} . \]  

One gets the spin averaged vertex functions in the IMF from the interaction Lagrangian density \( L = ig\phi\gamma_5\pi\phi \) where \( \phi \) denotes a baryon \((\Lambda^0, \Sigma^0, \Sigma^+, p)\) and \( \pi \) a pseudo scalar field \((\pi^+, \pi^0, K^0)\) [11]. For the vertex

\[ \Sigma^+ (helicity = 1/2) \rightarrow baryon (helicity = \lambda) + meson (helicity = \lambda') , \]  

the vertex function \( V^{\lambda\lambda'}_{IMF}(y, k^2_\perp) \) is given by

\[ V^{+0}_{IMF} = \frac{g}{2} \frac{ym_{\Sigma^+} - m_B}{\sqrt{ym_{\Sigma^+}m_B}} , \]  

\[ V^{-0}_{IMF} = \frac{g}{2} e^{-i\phi} \frac{k_\perp}{\sqrt{ym_{\Sigma^+}m_B}} , \]  

and when spin averaged we obtain

\[ |V_{IMF}|^2 = \frac{g^2}{4} \frac{(ym_{\Sigma^+} - m_B)^2 + k_\perp^2}{ym_{\Sigma^+}m_B} . \]  

Hence, in our case the splitting function is given by

\[ f_{BM/\Sigma^+}(y) = \frac{g^2}{16\pi^2} \frac{1}{y^2(1 - y)} \int_0^\infty dk_\perp^2 |G_{\Sigma^+BM}(y, k^2_\perp)|^2 \frac{(ym_{\Sigma^+} - m_B)^2 + k_\perp^2}{[m_{\Sigma^+}^2 - M_{BM}^2(y, k^2_\perp)]^2} . \]  

Using the following coupling constants \( g \) [12]

\[
\begin{align*}
\frac{1}{4\pi} g_{\Lambda^0\pi^+}/\Sigma^+ &= 11.8 \\
\frac{1}{4\pi} g_{\Sigma^0\pi^+}/\Sigma^+ &= 13.0 \\
\frac{1}{4\pi} g_{\Sigma^+\pi^0}/\Sigma^+ &= 13.0 \\
\frac{1}{4\pi} g_{pK^0}/\Sigma^+ &= 2.0
\end{align*}
\]
and the cutoff parameter $\Lambda = 1.08$ GeV for most of our work, we just need the quark distributions in the bare particles as an additional input.

For some of the following, we use Holtmann’s parameterization of the quark distribution function in the bare nucleon in which he assumes a symmetric sea, $\overline{Q}_{bare}$

$$xu_{v,bare}(x) = 0.62x^{0.37}(1-x)^{2.5}(1+11x), \quad (2.18)$$

$$xd_{v,bare}(x) = 0.04x^{0.10}(1-x)^{4.7}(1+102x), \quad (2.19)$$

$$x\overline{Q}_{bare} = 0.11(1-x)^{15.8}, \quad (2.20)$$

$$\overline{Q}_{bare} = u_{sea,bare} = \overline{u}_{sea,bare} = d_{sea,bare} = \overline{d}_{sea,bare} = 2s_{sea,bare} = 2\overline{s}_{sea,bare}. \quad (2.21)$$

In addition to this form for $\overline{Q}_{bare}$, we also take a form which is tied to the recent determination of the gluon distribution

$$x\overline{Q}_{bare} = 0.0124x^{-0.36}(1-x)^{3.8}. \quad (2.22)$$

Since the gluon splits into the ”perturbative” or bare sea quarks, this distribution should be close to that of those quarks. The advantage of this choice will become clear in the next section. For the quark distribution in the pion we take

$$xq_v(x) = 0.99x^{0.61}(1-x)^{1.02}, \quad (2.23)$$

$$xq_{sea}(x) = 0.2(1-x)^{5.0}, \quad (2.24)$$

where 20% of the pion’s momentum is assumed to be carried by the symmetrical sea, which is presumed to be due to gluon splitting.

To begin with, we determined the quark distributions of the bare hyperons by using $SU(3)$ symmetry for the valence quarks; that is we neglected the mass difference between the $s$ and $u$ and $d$ quarks. We do not show the results because we do not believe that this is a realistic choice. Look at the $K^0$. Experiments show that

$$\frac{\overline{u}_{K^-}}{\overline{u}_{\pi^-}} \sim (1-x)^{0.18\pm0.07}. \quad (2.25)$$

Using this we can take the parametrization for the quark distribution in the pion to get the $\overline{u}$ distribution in the $K^-$ and through charge independence the $\overline{d}$ distribution in the $\overline{K}^0$. To get the $s$ distribution in the $K^0$ we assume that the gluon and the light sea quark distributions in the kaon and the pion are the same and hence carry the same momentum fraction in both particles. We also assume the following form of the $s$ quark distribution

$$xs_v(x) \sim x^{0.61}(1-x)^a \quad (2.26)$$
where the parameter \( a \) is determined by matching up the right momentum fraction. The valence quark distributions in the \( \overline{K}^0 \) are then given by

\[
x\overline{d}_v(x) = 1.05x^{0.61}(1 - x)^{1.20},
\]

\[
x\overline{s}_v(x) = 0.94x^{0.61}(1 - x)^{0.86}.
\]

Compared to the pion, where \( d(x) \) and \( u(x) \) look the same, the \( s \) distribution in the \( K^0 \) now peaks at higher \( x \) than the \( \overline{d} \) distribution, which reflects the fact that the \( s \) quark is heavier than the \( u \) and the \( d \).

Our starting point for the bare \( \Sigma^+ \) is Holtmann’s parametrization for the bare nucleon, equations 18-21. Again we assume only a change in the \((1 - x)\)-part of the parametrizations and that the light sea quarks and gluons in the bare \( \Sigma^+ \) and the bare nucleon look the same.

To account for the higher mass of the \( s \) quark we then make the Ansatz

\[
\int_0^1 xu^{\Sigma^+}_{v,\text{bare}}(x)dx = \int_0^1 xu^p_{v,\text{bare}}(x)dx, \quad m_{d,\text{con}}
\]

\[
\int_0^1 xs^{\Sigma^+}_{v,\text{bare}}(x)dx = \int_0^1 xd^p_{v,\text{bare}}(x)dx, \quad m_{s,\text{con}}
\]

where we take model dependent constituent quark masses to get

\[
\frac{m_{d,\text{con}}}{m_{s,\text{con}}} \approx \frac{336\text{MeV}}{540\text{MeV}}
\]

The quark distributions in the bare \( \Sigma^+ \) are then

\[
xu^{\Sigma^+}_{v,\text{bare}}(x) = 0.82x^{0.37}(1 - x)^{3.89}(1 + 11x),
\]

\[
xs^{\Sigma^+}_{v,\text{bare}}(x) = 0.03x^{0.1}(1 - x)^{1.76}(1 + 102x).
\]

We can also determine the distributions in the bare \( \Sigma^0 \) and \( \Lambda^0 \) via charge independence and SU(3):

| \( \Sigma^0 \) | \( \Lambda^0 \) |
|----------------|----------------|
| \( Q^{\Sigma^0}_{v,\text{bare}} \) | \( Q^{\Lambda^0}_{v,\text{bare}} \) |
| \( u^{\Sigma^0}_{v,\text{bare}} \) | \( u^{\Lambda^0}_{v,\text{bare}} \) |
| \( d^{\Sigma^0}_{v,\text{bare}} \) | \( d^{\Lambda^0}_{v,\text{bare}} \) |
| \( s^{\Sigma^0}_{v,\text{bare}} \) | \( s^{\Lambda^0}_{v,\text{bare}} \) |

Since we assume no change in the symmetric sea of the bare particles, the only variance in the antiquark distributions of the physical \( \Sigma^+ \) comes from the different parametrization of the \( K^0 \) which only affects \( d(x) \). There is also a change in \( d(x) \) due to the more realistic \( s \) quark distributions for the \( \Sigma^0 \) and the \( \Lambda^0 \). The splitting \( \Sigma^+ \to p\overline{K}^0 \) is nearly negligible. Hence taking into account the larger \( s \) quark mass does not noticeably change our results for \( \overline{d}/\overline{u} \) and the difference \( \overline{d} - \overline{u} \) from those using SU(3) parameters. However, the valence quark distributions in the physical \( \Sigma^+ \) do change.
III. RESULTS

We first test our model for the measured sea quark distributions in the proton. We show both \((\bar{d} - \bar{u})\) and \(\bar{d}/\bar{u}\) in our model with no \(\Delta\) and no mesons more massive than the pion in Figs. 1 and 2. The comparison with the E866 data shows that \((\bar{d} - \bar{u})\) agrees reasonably with experiment, but that the ratio \(\bar{d}/\bar{u}\) does not turn over towards 1 at higher value of \(x\) with the parameterization of the bare sea used by Holtmann \cite{13}. This problem is also found for other meson cloud and chiral models, as has been recently noted by Peng et al. \cite{17}. However, we find that the ratio does turn over (although not sufficiently fast) for \(\bar{Q}_{bare}^p\) given by Eq. 22. It is true that the inclusion of the \(\Delta\) would help, but we believe that the splitting of the gluon into \(q\bar{q}\) pairs is the dominant cause of the return of the ratio \(\bar{d}/\bar{u}\) towards unity at \(x > 0.3\). We believe that a parameterization for \(\bar{Q}_{bare}^p\) can be found that is consistent with the gluonic data and with the ratio \(\bar{d}/\bar{u}\). For a further discussion, see also \cite{18}.

The four splitting functions for the \(\Sigma^+\) are shown in Fig. 3; it is clear that the contribution from the \(pK^0\) state is very small. In Fig. 4 we show the calculated valence quark momentum distributions. The \(s\) quark distribution peaks at a slightly higher value of \(x\) than that of the \(u\) quark momentum distribution due to its larger mass. The momentum distribution of the sea quarks is shown in Figs. 5-7. We show the distributions for both the Holtmann \cite{13} and the gluonic \cite{14} bare \(\Sigma^+\) distributions. The \(xd\) and \(x\bar{d}\) distributions are slightly different, but the difference is so small that we do not attempt to show it. The difference \((\bar{d} - \bar{u})\) shown in Fig. 8 is, of course, independent of the bare \(\Sigma^+\)'s quark distribution, since it is due to gluon splitting. It is interesting to see in Fig. 9 how much \(\bar{r}_\Sigma \equiv \bar{d}/\bar{u}\) distribution depends on that of the bare sea. In both cases considered the ratio is larger than \(\bar{r}_p\) in the proton. For the Holtmann \cite{13} bare quark distribution \(\bar{r}_\Sigma\) increases with increasing \(x\) and does not turn over. For the gluonic distribution, on the other hand, \(\bar{r}_\Sigma\) does not rise half as high and approaches 1 as \(x \to 0.6\). We show \((\bar{d} - \bar{u})_\Sigma/(\bar{d} - \bar{u})_p\) in Fig. 10 and compare \(\bar{r}_\Sigma\) to \(\bar{r}_p\) in Fig. 11. In the latter figure we note that \(\bar{r}_\Sigma/\bar{r}_p\) falls below 1 for the gluonic-like distribution at high \(x\), but the distribution functions are very small here.

We also tested the influence of the cut-off parameter \(\Lambda\) on our results. Since a higher cut-off primarily makes the amplitudes of the splitting functions larger but does not change their shape very much, the sea quarks will carry more of the \(\Sigma^+\) momentum while the momentum fraction carried by the valence \(s\) quarks stays the same and that of the valence \(u\) quarks gets smaller. The ratio \(\bar{d}/\bar{u}\) gets smaller and the difference \(\bar{d} - \bar{u}\) gets larger with growing \(\Lambda\) [ at all \(x\)].

IV. CONCLUSIONS

In conclusion, we have used the meson cloud model to calculate the valence and sea quark distributions in the \(\Sigma^+\). The same model can of course be used to calculate the valence and sea quark distributions in the \(\Sigma^0, \Lambda^0\) and \(\Sigma^-\). Now that quark distributions in the proton have been well-determined, it is possible to use \(\Sigma^+\) and \(\Sigma^-\) beams and inclusive Drell-Yan reactions on protons, alone, to obtain similar information for the hyperons. Nuclear binding
corrections for deuterium targets are thereby avoided. For instance if \( x(p) \) is large and \( x(\Sigma^\pm) \) is small, the measurement

\[
\sigma(\Sigma^+ p) - \sigma(\Sigma^- p) \propto (\bar{d}^\Sigma - \bar{u}^\Sigma)(\frac{1}{9}d^p - \frac{4}{9}u^p)
\]  

is sensitive to \((\bar{d} - \bar{u})\) in the \( \Sigma^+ \).

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FIG. 1. Comparison of our meson model with data [3] for $(\bar{d} - \bar{u})$.

FIG. 2. Comparison of our model with data for $\bar{d}/\bar{u}$ and data [3]. The light line is for the bare distribution of Eq.(20) and the dark line for Eq.(22).

FIG. 3. The splitting functions for $\bar{K}p$ (short dashes), $\Sigma^+\pi^0$ (long dashes), $\Sigma^0\pi^+$ (heavy line) and $\Lambda^0\pi^+$ (light line).
FIG. 4. Valence quark distributions in the $\Sigma^+$. The solid line is that of the $u$ quarks and the dashed line is that for $s$ quarks.

FIG. 5. The sea $u$ and $\bar{u}$ quark momentum distributions. The light line and long dashes are for Eq.(20) and the heavy line and short dashes for Eq.(22). The dashed lines are the bare distributions and the solid ones for the physical $\Sigma^+$. 

FIG. 6. The sea $s$ and $\bar{s}$ quark momentum distributions. See Fig. 5 for details.
FIG. 7. The sea $d$ and $\bar{d}$ quark momentum distributions. See Fig. 5 for details. Although the distributions differ slightly for $d$ and $\bar{d}$, the differences are small and not shown.

FIG. 8. The difference $(\bar{d} - \bar{u})$ for the $\Sigma^+$. 

FIG. 9. The ratio $\bar{d}/\bar{u}$ for the $\Sigma^+$. 

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FIG. 10. The ratio $\frac{(d-\bar{u})_{\Sigma^+}}{(d-\bar{u})_p}$.

FIG. 11. The ratio $\frac{\bar{r}_{\Sigma^+}}{\bar{r}_p}$. The light line is for Eq.(20) and the heavy one for Eq. (22).