Role of Magnetic Field in Self-Oscillation of Nanomagnet Excited by Spin Torque

Tomohiro Taniguchi†, Shingo Tamaru, Hiroko Arai, Sumito Tsunegi, Hitoshi Kubota, and Hiroshi Imamura
National Institute of Advanced Industrial Science and Technology (AIST), Spintronics Research Center,
Tsukuba, Ibaraki 305-8568, Japan

Abstract—The critical current of the self-oscillation of spin torque oscillator (STO) consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer was studied by solving the Landau-Lifshitz-Gilbert (LLG) equation. We found that the critical current diverged at certain field directions, indicating that the self-oscillation does not occur at these directions. It was also found that the sign of the critical current changed depending on the applied field direction.

Index Terms—spintronics, spin torque oscillator, perpendicularly magnetized free layer, critical current

I. INTRODUCTION

DIRECT current applied to a magnetic tunnel junction (MTJ) exerts spin torque on the magnetization of the free layer. When the energy supplied by the spin torque balances with the energy dissipation due to the damping, the self-oscillation of the magnetization is realized. Spin torque oscillator (STO) [1]-[11] utilizing this self-oscillation is an important spintronics device applicable to microwave generators and recording heads of a high density hard disk drive (HDD) due to its small size, high emission power, and frequency tunability. Recent development of the experimental technique to enhance the perpendicular magnetic anisotropy of CoFeB by adding an MgO capping layer [12]-[14] enabled us to fabricate STO consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer [15]-[18]. This type of STO in the presence of a large applied field (2-3 kOe) showed a large emission power (≈ 0.5 µW) with a narrow linewidth (≈ 50 MHz) [15]. On the other hand, the emission power in the absence of the applied field was of the order of 0.01 µW [16]. These results indicate that the applied field plays a key role in the performance of STO. However, the previous works only focused on the self-oscillation with the perpendicular field, while it is experimentally possible to apply the field in an arbitrary direction.

It is important for STO applications to clarify the dependence of the oscillation properties of STO on the applied field direction because of many reasons. For example, it was shown in Ref. [21] for an in-plane magnetized system that the oscillation frequency and the emission power could be controlled by changing the field direction. Also, when STO is used as a read-head sensor of HDD [22], the dipole field from a recording bit acts as an applied field whose direction depends on the bit information. The dependence of a critical current for the self-oscillation on the applied field direction is also interesting because the critical current determines the power consumption.

In this paper, we study the dependence of the critical current of STO with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer on the applied field direction. The critical current diverges when the field points to certain directions, indicating that the self-oscillation cannot be induced. This singularity arises from the energy balance between the work done by spin torque and the energy dissipation due to the damping. The field directions corresponding to the divergence depend on the perpendicular magnetic anisotropy, the applied field magnitude and direction, and the spin torque parameter. The result implies that the emission power of STO increases (decreases) significantly by tilting the magnetic field to the parallel (anti-parallel) direction of the pinned layer magnetization.

This paper is organized as follows. In Sec. II, we linearize the Landau-Lifshitz-Gilbert (LLG) equation around the equilibrium state of the free layer. In Sec. III, the theoretical formula of the critical current is derived. The dependence of the critical current on the applied field direction is also studied. Section IV is devoted to the conclusion.

† Corresponding author. Email address: tomohiro-taniguchi@aist.go.jp
II. **LLG Equation**

The system we consider is schematically shown in Fig. [1]. We denote the unit vectors pointing in the directions of the magnetization of the free and the pinned layers as \( \mathbf{m} \) and \( \mathbf{p} \), respectively. The \( z \)-axis is parallel to \( \mathbf{p} \) while the \( x \)-axis is normal to the film-plane. The current \( I \) flows along the \( x \)-axis, where the positive current corresponds to the electron flow from the free layer to the pinned layer. The following calculations are based on the macrospin model, which works well as the volume of the free layer decreases. For our parameters shown below, where the radius and thickness of the free layer are 60 and 2 nm, respectively, the macrospin model well reproduced the experimental results such as the current dependence of the oscillation frequency [15]. On the other hand, when the radius and thickness of the free layer become larger than a few hundred and a few nanometers, respectively, the vortex state appears in the free layer [23]-[27]. Although the oscillation frequency of the vortex based STO is very low (typically, on the order of 0.1 GHz), a narrow linewidth on the order of sub MHz [27] is fascinating feature for practical applications. The magnetization dynamics of the macrospin is described by the LLG equation:

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma H_s \mathbf{m} \times (\mathbf{p} \times \mathbf{m}) + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}. \tag{1}
\]

The gyromagnetic ratio and Gilbert damping constant are denoted as \( \gamma \) and \( \alpha \), respectively. The magnetic field is defined by \( \mathbf{H} = -\partial E/\partial(M\mathbf{m}) \), where the energy density is

\[
E = -MH_{\text{appl}} \cdot \mathbf{n}_H - \frac{M(H_K - 4\pi M)}{2}(\mathbf{m} \cdot \mathbf{e}_z)^2. \tag{2}
\]

Here, \( M \), \( H_{\text{appl}} \), \( \mathbf{n}_H = (\sin \theta_H \cos \varphi_H, \sin \theta_H \sin \varphi_H, \cos \theta_H) \), and \( H_K \) are the saturation magnetization, applied field magnitude, unit vector pointing in the applied field direction, and crystalline anisotropy field along the \( x \)-axis, respectively. The spin torque strength, \( H_s \) in Eq. (1), is [28]-[31]

\[
H_s = \frac{\eta I}{2e(1 + \lambda \mathbf{m} \cdot \mathbf{p})MV'}, \tag{3}
\]

where \( V \) is the volume of the free layer. Two dimensionless parameters, \( \eta \) and \( \lambda \), determine the magnitude of the spin polarization of the injected current and the dependence of the spin torque strength on the relative angle of the magnetizations, respectively.

Because the parameter \( \lambda \) plays a key role in the following discussions, explanations on its sign and value are mentioned in the following. The form of Eq. (3) is common for spin torque in not only MTJs but also giant magnetoresistive (GMR) systems [28]-[32], and the theoretical relation between \( \lambda \) and the material parameters depend on the model. For example, Ref. [31] calculated the spin torque from the transfer matrix of an MTJ, and showed that \( \lambda = \eta \eta' \), where \( \eta' \) is the spin polarization of the free layer. The sign of \( \lambda \) is positive (negative) when the MTJ shows the positive (negative) TMR. On the other hand, in the case of a GMR system, Ref. [30] calculated the spin torque from the ballistic spin current in a circuit, and showed that \( \lambda = (A^2 - 1)/(A^2 + 1) \), where \( A = \sqrt{R_F/R_N} \) depends on the resistances of the ferromagnetic (F) electrode and the nonmagnetic (N) spacer. Depending on the ratio \( R_F/R_N \), \( \lambda \) can be both positive and negative.

Before applying the current, the magnetization points to the equilibrium direction corresponding to the minimum of the energy density \( E \). In the spherical coordinate, the polar and azimuth angles \((\theta, \varphi)\) of the equilibrium direction satisfy

\[
H_{\text{appl}} \sin \theta_H \cos \theta \cos(\varphi_H - \varphi) - \cos \theta_H \sin \theta = 0, \tag{4}
\]

\[
H_{\text{appl}} \sin \theta_H \sin \theta \sin(\varphi_H - \varphi) - (H_K - 4\pi M) \sin^2 \theta \sin \varphi \cos \varphi = 0. \tag{5}
\]

We introduce a new coordinate \( XYZ \) in which the \( Z \)-axis points to \((\theta, \varphi)\) direction. The transformation matrix from the \( xyz \)-coordinate to the \( XYZ \)-coordinate is given by

\[
R = \begin{pmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{pmatrix},
\]

\[
\cos \varphi & \sin \varphi & 0 \\
-\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\].

(6)

The magnetic field in the \( XYZ \)-coordinate is \( \mathbf{H} = (H_{XX} \mathbf{m}_X + H_{XY} \mathbf{m}_Y + H_{YZ} \mathbf{m}_Z + H_{XX} \mathbf{m}_X + H_{YY} \mathbf{m}_Y + H_{ZZ} \mathbf{m}_Z) \), where \( H_{ij} \) are the \( i \)-components of \( \mathbf{H} \) proportional to \( m_j \) [33],[34]. The explicit forms of \( H_{ij} \) are

\[
H_{XX} = (H_K - 4\pi M) \cos^2 \theta \cos^2 \varphi, \tag{7}
\]

\[
H_{YY} = (H_K - 4\pi M) \sin^2 \varphi, \tag{8}
\]

\[
H_{XY} = -H_{XX} = -H_K \cos \theta \sin \varphi \cos \varphi, \tag{9}
\]

\[
H_{ZZ} = (H_K - 4\pi M) \sin \theta \cos \theta \cos^2 \varphi. \tag{10}
\]
\[ H_{ZY} = -(H_K - 4\pi M) \sin \theta \sin \varphi \cos \varphi, \quad (11) \]
\[ H_{ZZ} = H_{\text{appl}} [\sin \theta \sin \varphi \cos (\varphi - \varphi_H) + \cos \theta \cos \varphi] + (H_K - 4\pi M) \sin^2 \theta \cos^2 \varphi. \quad (12) \]

Up to the first order of \( m_X \) and \( m_Y \), the LLG equation can be linearized as
\[ \frac{1}{\gamma'} \frac{d}{dt} \begin{pmatrix} m_X \\ m_Y \end{pmatrix} + M \begin{pmatrix} m_X \\ m_Y \end{pmatrix} = \begin{pmatrix} H_{s0} \sin \theta \\ 0 \end{pmatrix}, \quad (13) \]
where the components of the \( 2 \times 2 \) matrix, \( M \), are given by
\[ M_{1,1} = -H_{XY} + \alpha (H_{ZZ} - H_{XX}) - H_{s0} (\cos \theta + \tilde{\lambda} \sin \theta), \quad (14) \]
\[ M_{1,2} = (H_{ZZ} - H_{YY}) - \alpha H_{XY}, \quad (15) \]
\[ M_{2,1} = -H_{XY} + \alpha (H_{ZZ} - H_{XX}) - H_{s0} \cos \theta, \quad (16) \]
\[ M_{2,2} = H_{XY} + \alpha (H_{ZZ} - H_{YY}) - H_{s0} \cos \theta, \quad (17) \]
where \( H_{s0} = \hbar \eta I/[2e(1 + \lambda \cos \theta)MV] \) and \( \tilde{\lambda} \) is given by
\[ \tilde{\lambda} = \frac{\lambda \sin \theta}{1 + \lambda \cos \theta}. \quad (18) \]

### III. CRITICAL CURRENT

Equation (13) indicates that the time-evolutions of \( m_X \) and \( m_Y \) are described by \( \exp[\gamma \pm i \sqrt{\det[M] - (1/2)^2 - \text{Tr}[M]/2}] \), where \( \det[M] \) and \( \text{Tr}[M] \) are the determinant and trace of the matrix \( M \), respectively. The imaginary part of the exponent determines the oscillation frequency of \( m_X \) and \( m_Y \), which is identical to the ferromagnetic resonance frequency in the limit of \( \alpha \to 0 \) and \( H_s \to 0 \). On the other hand, when the real part of the exponent \( -\gamma \text{Tr}[M]/2 \) is positive (negative), the amplitude of \( m_X \) and \( m_Y \) increases (decreases) with the time increases. Therefore, the critical current is determined by the condition \( \text{Tr}[M] = 0 \), and is given by
\[ I_c = \frac{2ae(1 + \lambda \cos \theta)MV}{\hbar\eta(2 \cos \theta + \lambda \sin \theta)} (2H_{ZZ} - H_{XX} - H_{YY}). \quad (19) \]

When the applied field points to the perpendicular direction \( (\mathbf{n}_H = e_x) \) and \( H_K > 4\pi M \), the equilibrium direction is \( (\theta, \varphi) = (\pi/2, 0) \). In this case, the critical current is [18]
\[ \lim_{(\theta, \varphi) \to (90^\circ, 0)} I_c = \frac{4aeMV}{\hbar\eta\lambda} (H_{\text{appl}} + H_K - 4\pi M). \quad (20) \]

Figures 2(a) and (b) show the dependences of the critical current \( I_c \) on the applied field angle \( \theta_H \) for (a) \( 95^\circ \leq \theta_H \leq 100^\circ \) and (b) \( 100^\circ \leq \theta_H \leq 105^\circ \), respectively, in which \( \varphi_H = 0 \). Also, in Fig. 3 \( I_c \) in \( (\theta_H, \varphi_H) \) space is shown, in which \( 95^\circ \leq \theta_H \leq 105^\circ \) and \( -90^\circ \leq \varphi_H \leq 90^\circ \). The values of the parameters are \( M = 1448 \text{ emu/c.c.}, H_K = 18.6 \text{ kOe}, H_{\text{appl}} = 2 \text{ kOe}, V = \pi \times 60 \times 60 \times 2 \text{ nm}^3 \), \( \eta = 0.54, \lambda = \eta^2, \gamma = 1.732 \times 10^7 \text{ rad/(Oe-s)}, \) and \( \alpha = 0.005 \), respectively, which are estimated from the experiments [15, 35, 36]. Two important conclusions are obtained from Eq. (19), Figs. 2(a), 2(b), and 3.

The first conclusion is that the critical current diverges at certain field directions \( (\theta_H, \varphi_H) \) as \( \lim_{(\theta_H, \varphi_H) \to (\theta_H, 0)} I_c = +\infty \) and \( \lim_{(\theta_H, \varphi_H) \to (\theta_H, \pi/2)} I_c = -\infty \) for \( \varphi_H = 0 \), as shown in Figs. 2(a) and 2(b). At \((\theta_H, \varphi_H)\), the condition
\[ 2 \cos \theta + \frac{\lambda \sin^2 \theta}{1 + \lambda \cos \theta} = 0, \quad (21) \]
is satisfied, which means that the denominator of Eq. (19) is zero. It should be noted that Eq. (21) depends on only \( \lambda \) but also the perpendicular magnetic anisotropy and applied field magnitude and direction through Eqs. (4) and (5).

The divergence of the critical current means that the self-oscillation cannot be induced at the field direction satisfying Eq. (21). It should be noted that the self-oscillation is realized when the energy supplied by the spin torque balances with the energy dissipation due to the damping. However, for example, when \( \lambda = 0 \) and the direction of the magnetization is perpendicular to the film plane \( (\theta, \varphi) = (90^\circ, 0)) \), the total energy supplied by the spin torque during a precession of the magnetization around the perpendicular axis is zero, as mentioned in Ref. [18]. Then, the critical current, Eq. (21), in the limit of \( \lambda \to 0 \) diverges, and a steady oscillation does not occur by the spin torque. Equation (21) should be regarded as the generalized condition for an arbitrary pointing field, which as a special case corresponds to \( \lambda = 0 \) for the perpendicular field, i.e., the energy supplied by the spin torque is zero when the field, anisotropy, and \( \lambda \) satisfy Eq. (21). A significant reduction of the emission power is expected in experiments when the field points to the direction \( (\theta_H, \varphi_H) \).

The second conclusion is that the sign of the critical current changes from positive to negative when crossing the line of the divergence of \( I_c \) in Fig. 3 from left to right: see also Figs. 2(a) and (b). To understand this point, it is convenient to consider the case in which a large field \( (H_{\text{appl}} \gg H_K - 4\pi M) \) points to the parallel (P) or anti-parallel (AP) direction of \( p \). Then, the system can be regarded as the in-plane magnetized system. It is known that the sign of the critical current for the switching from P to AP state in the in-plane magnetized system is opposite to that for the switching from AP to P state.
[37]. In our definition, the sign of the critical current from P (AP) to AP (P) state is positive (negative), which is consistent with the sign of \( I_c \) shown in Figs. 2 (a) and (b). One of the important conclusions in Ref. [15] is that only the positive current can exert the self-oscillation of the magnetization in this type of STO. However, the present result suggests that the negative current can also exert the self-oscillation when the field tilts to the anti-parallel direction of \( p \).

Finally, let us briefly discuss the relation between the present result and the STO applications. In STO applications, it is desirable to obtain a large emission power by using a low bias current. The emission power depends on the precession direction and the STO applications. In STO applications, it is important to obtain a large emission power by using a low bias current. The emission power depends on the precession direction.

In conclusion, the critical current of STO consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer in the presence of an applied field pointing to an arbitrary direction was calculated. The critical current diverged at certain field directions, which meant that the self-oscillation could not be induced by the spin torque. The field direction corresponding to this singularity depended on the perpendicular magnetic anisotropy, the applied field magnitude and the direction, and the spin torque parameter. The divergence arose from the fact that the spin torque could not supply the energy to the free layer at these directions. It was also found that the sign of the critical current changed depending on the field direction.

ACKNOWLEDGMENT

The authors would like to acknowledge H. Maehara, A. Emura, T. Yorozu, M. Konoto, K. Yakushiji, T. Nozaki, A. Fukushima, K. Ando, and S. Yuasa. This work was supported by JSPS KAKENHI Number 23226001.

REFERENCES

[1] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, Nature, vol.425, pp.380-383, 2003.

[2] W. H. Rippard, M. R. Pufall, S. Kaka, S. E. Russek, and T. J. Silva, Phys. Rev. Lett., vol.92, pp.027201, 2004.

[3] J. C. Sankey, I. N. Krivorotov, S. I. Kiselev, P. M. Baraganca, N. C. Emley, R. A. Buhrman, and D. C. Ralph, Phys. Rev. B, vol.72, pp.224427, 2005.

[4] I. N. Krivorotov, N. C. Emley, R. A. Buhrman, and D. C. Ralph, Phys. Rev. B, vol.77, pp.054440, 2008.

[5] W. H. Rippard, M. R. Pufall, M. L. Schneider, K. Garello, and S. E. Russek J. Appl. Phys., vol.103, pp.053914, 2008.

[6] J. Sinha, M. Hayashi, Y. K. Takahashi, T. Taniguchi, M. Drapeko, S. Mitani, and H. Ohno, Nature Mater., vol.9, pp.621-624, 2010.

[7] S. Tsunegi, H. Kubota, Y. Suzuki, K. Yakushiji, A. Fukushima, S. Yuasa, and K. Ando, J. Appl. Phys., vol.105, pp.07D131, 2009.

[8] H. Kubota, S. Ishibashi, T. Satoya, T. Nozaki, A. Fukushima, K. Yakushiji, K. Ando, Y. Suzuki, and S. Yuasa J. Appl. Phys., vol.111, pp.07C723, 2012.

[9] H. Kubota, K. Yakushiji, A. Fukushima, S. Tamaru, M. Konoto, T. Nozaki, S. Ishibashi, T. Satoya, S. Yuasa, T. Taniguchi, H. Arai, and H. Imamura, Appl. Phys. Express, vol.6, pp.103003, 2013.

[10] Z. Zeng, G. Fineciochio, B. Zhang, P. K. Amri, J. A. Katine, I. N. Krivorotov, Y. Hua, J. Langer, B. Azzerboni, K. L. Wang, and H. Jiang, Sci. Rep., vol.3, pp.1426, 2013.

[11] T. Taniguchi, H. Arai, H. Kubota, and H. Imamura, IEEE Trans. Magn., vol.50, pp.140404, 2014.

[12] T. Taniguchi, H. Arai, S. Tsumegi, S. Tamaru, H. Kubota, and H. Imamura, Appl. Phys. Express, vol.6, pp.123003, 2013.

[13] H. Arai and H. Imamura, Appl. Phys. Express, vol.7, pp.023007, 2014.

[14] Y. Zeng, G. Fineciochio, B. Zhang, P. K. Amri, J. A. Katine, I. N. Krivorotov, Y. Huai, J. Langer, B. Azzerboni, K. L. Wang, and H. Jiang, Sci. Rep., vol.3, pp.1426, 2013.

[15] The effect of the perpendicular field (\( n_H = e_x \)) [15]. On the other hand, the present calculation shown in Fig. 2 (a) implies that the emission power increases significantly by slightly tilting the magnetic field from the perpendicular direction to the parallel direction of \( p \) because \( n_z \) with a large amplitude at the low current is expected due to the decrease of the critical current \( I_c \). However, a further tilting of the magnetic field leads to a decrease of the oscillation amplitude because the oscillation is limited to the region \( n_z > 0 \). Also, a complex magnetization dynamics may happen due to a large difference of the directions between the applied field and the anisotropy field. These considerations imply an existence of an optimum direction of the applied field for a high emission power, which will be an important work to pursue in the future.

IV. CONCLUSIONS

In conclusion, the critical current of STO consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer in the presence of an applied field pointing to an arbitrary direction was calculated. The critical current diverged at certain field directions, which meant that the self-oscillation could not be induced by the spin torque. The field direction corresponding to this singularity depended on the perpendicular magnetic anisotropy, the applied field magnitude and the direction, and the spin torque parameter. The divergence arose from the fact that the spin torque could not supply the energy to the free layer at these directions. It was also found that the sign of the critical current changed depending on the field direction.