Proposed LRM Early Termination Method: LRM ETM is based on observing sign alterations of a small cluster in \( m_{\rightarrow c} \) messages. Since the sign parts of the LLR values are utilized for hard-decision in the decision part of BP, observing sign alterations of \( m_{\rightarrow c} \) during successive iterations can be used to determine whether estimated data bits change. If the estimated data bits stop changing for a number of consecutive iterations, it can be assumed that decoder successfully converged. To be able to get lowest average iteration amounts, \( \Gamma_{LC} \) value should be as low as possible.

Instead of \( m_{\rightarrow c} \) messages, our proposed method observes sign alterations of \( m_{c\rightarrow \rightarrow} \) messages that specify \( m_c \). Therefore, our method doesn’t require performing “Decision()” at each decoding iteration. On the other hand, proposed LRM method is basically based on the fact that \( m_{c\rightarrow \rightarrow} \) messages with lower absolute LLR values are less reliable among entire \( m_{c\rightarrow \rightarrow} \) messages and they converge later than messages that have higher absolute LLR values. Therefore, we observe only LRM which is a small cluster of LLR values to determine successful convergence. This simplification also reduces the computational complexity of ETM section significantly. Determination of LRM which means finding the smallest absolute LLR values in all \( m_{c\rightarrow \rightarrow} \) messages, can be easily done by using a selection algorithm. We use quickselect algorithm which has low computational complexity [14]. LRM ETM determines LRM to observe sign alterations after running decoder for a few iterations. This is because LT BP decoder typically needs a few iterations to propagate initial channel LLR values. We call these threshold for iteration numbers as deterministic condition of LRM (DC-LRM).

It is easy to see that larger DC-LRM value increases probability of choosing accurate LRM because better propagation occurs when iteration number increases. On the other hand, DC-LRM shouldn’t be larger than minimum iteration number that decoder converged to keep average iteration number as low as possible. DC-LRM values are chosen as 45, 28, 22, 18 and 15 for 0.5, 1.0, 1.5, 2.0 and 2.5dB according to simulations, respectively. DC-LRM values for different systems can be determined by simulations and previously loaded to a look-up table. LT BP decoding process with proposed LRM method is presented in Algorithm 2.
complexities to simplify the comparison. In the table, $N$ is coded packet length, $K$ is uncoded packet length, $\lambda_1$ is the fraction of VN s of degree 1, $d_v$ is CN degree, $\rho_d$ is the fraction of CNs of degree $d_v$ and $\Delta_{max}$ is maximum CN degree. $N_B$ symbolizes number of LRM determined by $N_B = B \times N_{max}$, where $B$ is the percentage to determine the amount of LRM. $N_{max}$ is number of all $m_{max}$ messages and calculated by $N_{max} = NQ(1)$. $Q(1)$ is average degree of degree distribution chosen for LT code [8]. As we mentioned above, LRM method performs quickselect algorithm only one time for whole decoding process to determine less reliable messages. The quickselect uses less than $2N_{max}$ compare operations to find the smallest $N_B$ items of an array with length $N_{max}$. [7] We add the average effect of quickselect to computational complexities for each iteration by $2N_{max}/\log(N)$ comparisons in the table. Here, $\log(N)$ is average iteration number.

It should be also emphasized that all operations required for CSR method are performed in every decoding iteration until decoding is terminated, while the operations for LRM method start after decoder runs DC-LRM iterations which does not emphasised in Table 1.

**Numerical results:** In this section, we evaluate the bit error rate (BER) performances of LT BP decoding algorithm with and without ETMs over binary-input additive white Gaussian noise (BIAGWN) channel by simulation works. Also, computational complexities of ETMs and average iteration amounts of BP algorithm with LRM and CSR ETMs are compared. For all simulation works and complexity analyzes, we consider the following degree distribution, $\Omega(x) = 0.0088x + 0.4942x^2 + 0.1663x^3 + 0.0738x^4 + 0.083x^5 + 0.056x^6 + 0.0372x^7 + 0.056x^{19} + 0.025x^{66} + 0.003x^{66}$ [8], code rate of 1/2, data packet length of 4000 and fixed iteration number of 100.

Average simulation times of ETMs for decoding a code block are compared in Table 2 with considered simulation parameters (CSR with $\Gamma_{LC} = 5$ and LRM with $\Gamma_{LC} = 1, B = %5$). Results show that required computation time of LRM method is significantly lower than CSR. Note that timing results demonstrate only ETM section of decoding process. Furthermore, decoder with proposed LRM method has small average iteration amounts compared to decoder with CSR as shown in Table 4. This provides additional reduction in computation time of whole decoding process.

**Conclusion:** In this paper, we developed a new early termination method for LT BP decoder to avoid redundant processes which cause high computational complexity, decoding latency and energy dissipation. Simulation results and complexity analyzes show that proposed LRM method significantly lower complexity and computation time of early termination section in decoder without BER performance degradation and decreases the average iteration amounts compared to conventional CSR ETM. The method can be easily applied to code families which can be decoded by BP such as low density parity check (LDPC) codes, polar codes and Raptor codes. The best way to compare ETMs can be done by hardware implementation which will be held in future.

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**Table 1:** Complexities of ETMs for single iteration

| Operation          | CSR | LRM |
|--------------------|-----|-----|
| Addition           | $N + K(1 - \lambda_1)$ | $N_B$ |
| abs., sign., XOR   | $N\sum_{\Delta_{max}}{d_v, \rho_d}$ | $N_B$ |
| Compare            | $K$ | $N_B + 2N_{max}/\log(N)$ |

**Table 2:** Average iteration amounts of LT BP decoder with ETMs and LT BP decoder successfully converged

| $E_b/N_0$ (dB) | Decoder Convergence | CSR | LRM |
|---------------|---------------------|-----|-----|
| 0.5           | 90.74               | 91.65 | 91.53 |
| 1.0           | 41.25               | 45.19 | 43.73 |
| 1.5           | 28.65               | 32.45 | 30.15 |
| 2.0           | 22.84               | 26.70 | 24.04 |
| 2.5           | 19.42               | 23.33 | 20.37 |

**Table 3:** Average computation time of ETMs for decoding a code block

| $E_b/N_0$ (dB) | CSR(ms) | LRM(ms) | Reduction (%) |
|---------------|---------|---------|---------------|
| 0.5           | 80.18   | 69.61   | 91.58          |
| 1.0           | 59.99   | 3.07    | 92.13          |
| 1.5           | 26.93   | 2.04    | 92.54          |
| 2.0           | 21.80   | 1.72    | 92.11          |
| 2.5           | 19.28   | 1.58    | 91.80          |