CP asymmetry in $B^+ \rightarrow K^+\pi^0$ and New Physics

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The CP asymmetry in $B^0 \rightarrow K^+\pi^-$ is expected to be similar to that in $B^+ \rightarrow K^+\pi^0$. The experimental data however show $\sim 5\sigma$ difference between the two, leading to the so called $\Delta A_{K\pi}$ puzzle. Employing sum rule(s) following from (approximate) flavour symmetry, we show that it is possible to accommodate the observed experimental values within the standard model (SM) for a narrow range of parameters. Sub-leading terms can bring the theoretical predictions in better agreement with the data. Resolution via modified electroweak penguin contributions is possible for a large CP violating phase generated by the new physics. However, the data on polarization in $B \rightarrow VV(T)$, $B_s-B_s$ mixing (and large CP phase) and $B^+ \rightarrow \tau^+\nu_\tau$ rate can not be simultaneously accommodated within SM or new physics with only enhanced electroweak penguins. A plausible resolution to these, and not spoiling the $B \rightarrow K\pi$ rates and asymmetries, could be a general two Higgs doublet model.

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| Observable | HFAG average |
|------------|--------------|
| $BR(B^0 \rightarrow K^+\pi^-)$ | $(19.4\pm0.6) \times 10^{-6}$ |
| $BR(B^0 \rightarrow K^0\pi^0)$ | $(9.8\pm0.6) \times 10^{-6}$ |
| $BR(B^+ \rightarrow K^+\pi^-)$ | $(12.9\pm0.6) \times 10^{-6}$ |
| $BR(B^+ \rightarrow K^0\pi^0)$ | $(23.1\pm1.0) \times 10^{-6}$ |
| $A_{CP}(K^+\pi^-)$ | $(-9.5\pm1.2)\%$ |
| $A_{CP}(K^0\pi^0)$ | $(5.2\pm2.5)\%$ |
| $A_{CP}(K^0\pi^0)$ | $(9.0\pm2.5)\%$ |
| $C_{KS\pi^0}$ | $0.01\pm0.10$ |
| $S_{KS\pi^0}$ | $0.57\pm0.17$ |

TABLE I: HFAG values \textsuperscript{[1]} for observables in $B \rightarrow K\pi$ system

$\Delta A_{CP}(K^+\pi^0) - A_{CP}(K^+\pi^-)$, has attracted lot of attention. Recently the Belle Collaboration published \textsuperscript{2} an updated and precise measurement of this quantity, which is at variance with SM expectation. From the table one can read off that $\Delta A_{K\pi} = 14.8\pm2.8 \neq 0$ at $\sim 5\sigma$. A non-zero value of $\Delta A_{K\pi}$ has been argued to be a signature of new physics \textsuperscript{3}, since the two amplitudes differ by terms which are small in magnitude. In order to infer that this is in fact a clear signal of NP or that its resolution necessarily calls for NP, it is mandatory to carefully examine all the assumptions generally made in estimating various contributions, and to estimate the impact of the neglected small terms in the analysis. Needless to say that parametric uncertainties need to be kept in mind while making all these estimates.

There are two approaches to non-leptonic B-decays. One is based on the effective theory language and employs operators and short distance coefficients in the low energy effective theory to describe the quark level decays (see for example \textsuperscript{4}). The second is the diagrammatic approach combined with flavour symmetries \textsuperscript{3}. In the present note, combining the available experimental information and theoretical constraints with the flavour symmetries, we revisit the sum rule for CP asymmetry in $B^+ \rightarrow K^+\pi^0$ proposed in \textsuperscript{6}. We invert the sum rule and examine to what extent the inverted relation, ex-

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pressing a certain combination of theoretical parameters, called \( \delta_{EW} \) below, in terms of measurable quantities is satisfied within SM. We find that within the present errors, the observed value for \( \delta_{EW} \) does agree with the SM prediction for the same quantity. We hasten to mention that the biggest uncertainty stems from \( |V_{ub}| \), and therefore requires a better precision on the same to conclude anything about the presence of NP.

The effective Hamiltonian responsible for the non-leptonic \( b \to s \) transitions is given by \ref{eq:heff} \((b \to d \) transitions are described by appropriate changes)

\[
H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i (\lambda_u Q_4^u + \lambda_c Q_5^c) - \lambda_4 \sum_{i=3}^{10} C_i Q_i \bigg]+H.C
\]

where \( \lambda_q = V_{ub}^* V_{qs} \), \( C_i \) are the relevant Wilson coefficients while \( Q_i \) are four fermion operators. Here, \( Q_{1,2,5} \) are the current-current operators, while \( Q_{3-6} \) and \( Q_{7-10} \) are the QCD penguin and electroweak (EW) penguin operators. Operators \( Q_5, Q_6, Q_7 \) and \( Q_8 \) have \((V-A) \otimes (V+A)\) structure while all others have \((V-A) \otimes (V-A)\) structure. The NLO SM Wilson coefficients at scale \( \mu = m_b \) (approximately) read:

\[
C_1 \sim -0.3, \quad C_2 \sim 1.14, \quad C_3 - 6 \sim O(10^{-2}),
\]

\[
C_{7,8} \sim O(10^{-4}), \quad C_9 \sim -1.28 \alpha, \quad C_{10} \sim 0.33 \alpha
\]

Due to the smallness of \( C_{7,8} \), the corresponding contributions can be safely neglected. This also holds for extensions of SM where the \((V-A) \otimes (V+A)\) operators are not tremendously enhanced. Further, \( O_9 \) and \( O_{10} \) can be Fierz transformed into \( O_1 \) and \( O_2 \).

Various \( B \to K \pi \) decay amplitudes are expressed as \ref{eq:A+-}

\[
A^{+0} \equiv \sqrt{3} A(B^+ \to K^+ \pi^0) = -(p + t + c + a)
\]

\[
A^{0+} \equiv A(B^+ \to K^0 \pi^+) = (p + a)
\]

\[
A^{+-} \equiv A(B^0 \to K^+ \pi^-) = -(p + t)
\]

\[
A^{00} \equiv \sqrt{2} A(B^0 \to K^0 \pi^0) = (p - c)
\]

The amplitudes \( p, t, c \) and \( a \) are linear combinations of graphical amplitudes denoting Tree (T), Colour suppressed Tree (C), QCD-Penguin (P), colour allowed and suppressed EW-Penguin (\( P_{EW} \) and \( P_{EW}^C \)), Annihilation (A), W-Exchange (E) and Penguin Annihilation (P/A) diagrams, expected to follow the hierarchy \ref{eq:heff}:

\[
|P| >> |T| \sim |P_{EW}^C| >> |C| \sim |P_{EW}^C| > |A, E, P/A|
\]

Diagrams A, E and P/A involve the spectator quark and are therefore expected to be suppressed \((\propto f^2/m^2)\) and are often neglected. This expectation may not hold true in the presence of significant rescattering. Within the \( B \to K \pi \) system, a large CP asymmetry in \( B^+ \to K^0 \pi^+ \), \( A_{CP}(K^0 \pi^+) \), would indicate rescattering. From the table, it is clear that this is not the case here and therefore amplitude \( a \) can be safely neglected. In such a case, we have \( p = P - P_{EW}^C/3, \quad t = T + P_{EW}^C \) and \( c = C + P_{EW} \), where the u-quark penguin contribution has been absorbed in the definition of \( T \) and \( C \). Further, it is implied that the penguin amplitudes \( P, P_{EW}, P_{EW}^C \) contains the CKM factor \( V_{ub}^* V_{us} \) while all others contain \( V_{ub}^* V_{us} \). Using the unitarity of the CKM matrix, one can eliminate \( V_{ub}^* V_{us} \) in favour of \( V_{ub}^* V_{cb} \) and \( V_{ub}^* V_{ub} \). This has the advantage of employing the experimentally measured elements of the CKM matrix. Another advantage is that in this form, generalization to include NP effects due to modified penguin operators is straightforward - the modified coefficient can be simply made complex to take care of the extra phases, if needed. From the above expressions, one expects \( A_{CP}(K^+ \pi^-) \sim A_{CP}(K^+ \pi^0) \) since the two amplitudes differ by small contributions. The data however point to the contrary \((\Delta A_{K^\pm} \neq 0 \text{ at } \sim 5\sigma)\). Neglecting small contributions, following ratios of CP averaged rates are expected to be (almost) unity within SM:

\[
R = \frac{\Gamma_{av}(B^0 \to K^+ \pi^-)}{\Gamma_{av}(B^+ \to K^0 \pi^+)}
\]

\[
R_c = \frac{2\Gamma_{av}(B^+ \to K^+ \pi^0)}{\Gamma_{av}(B^+ \to K^0 \pi^+)}
\]

\[
R_n = \frac{\Gamma_{av}(B^0 \to K^+ \pi^-)}{2\Gamma_{av}(B^0 \to K^0 \pi^0)}
\]

The data in the table do follow this expectation. Using flavour SU(3) symmetry, it is possible to relate the EW penguin contributions to the tree contributions. Neglecting \( C_{7,8} \), and Fierz transforming \( O_{9,10} \) immediately enables one to express the relevant terms in the effective Hamiltonian such that \ref{eq:heff}:

\[
t + c = T + C + PEW + P_{EW}^C = (T + C)[\delta_{EW} - e^{-i\gamma}] \quad (4)
\]

where \( \gamma \) is the CKM angle \((V_{ub}^* = |V_{ub}| e^{-i\gamma})\)

\[
\delta_{EW} = -3 \frac{|V_{ub}^* V_{ts}|}{2 |V_{ub}^* V_{us}|} C_9 + C_{10} = -3 C_9 + C_{10} \cot \theta_C
\]

\[
\theta_C = \frac{\Gamma_{av}^{s}}{\Gamma_{av}^{b}} = \frac{C_9 + C_{10}}{C_9 + C_{10}} \frac{\Gamma_{av}^{b}}{\Gamma_{av}^{s}} \quad \text{for } V_{ub}^{\text{incl}} \text{ or } V_{ub}^{\text{exp}}
\]

In the above equation, \( \theta_C \) is the Cabibbo angle. It has been argued \ref{eq:heff} that the above combination of Wilson coefficients in \( \delta_{EW} \) is renormalization group invariant to a good accuracy. From the above expression it is clear that the numerical value of \( \delta_{EW} \) sensitively depends on \( |V_{ub}| \) (and to some extent on \( V_{cb} \) also). At present, \( |V_{ub}^{\text{exp}}| \) is very different from \( |V_{ub}^{\text{incl}}| \). To incorporate both the ranges, we take the upper limit as extracted from the inclusive value and the lower limit as suggested by exclusive measurements. Therefore, within SM we have (typical central value employed in literature is \( \delta_{EW} \sim 0.64 \))

\[
0.35 < \delta_{SM}^{\text{EW}} < 0.79
\]

Neglecting the amplitude \( a \) and using the above expressions leads to

\[
R_c = 1 - 2r_c \cos \delta_c (\cos \gamma - \delta_{EW}) + r_c^2 (1 - 2 \delta_{EW} \cos \gamma + \delta_{EW}^2)
\]
where \(r_c = \frac{T_{20}}{m_0} \) and \(\delta_c\) is the strong phase difference between the amplitudes \((T + C)\) and \(p\). Making further use of flavour SU(3) symmetry, and assuming factorization, it is possible to relate the magnitude of \((T + C)\) to the tree dominated decay \(B^+ \to \pi^+ \pi^0\) [12]. Since amplitude \(p\) can be extracted from \(B^+ \to K^0\pi^+\) rate, one arrives at

\[
\begin{equation}
\label{eq:8}
r_c = \zeta_{SU(3)} \sqrt{2} \sqrt{\frac{BR(B^+ \to \pi^+ \pi^0)}{BR(B^+ \to K^0\pi^+)}}
\end{equation}
\]

where \(\zeta_{SU(3)} = \frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_{\pi}} \frac{F^{B \to \pi}(m_K^2)}{\lambda^{1/2}(m_B, m_{\pi}, m_K^2)}\)

encodes the SU(3) breaking corrections within the approximations used. \(F^{B \to \pi}\) is the \(B \to \pi\) form-factor and the function \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\) is the phase space factor. Using HFAG values for the branching ratios, we arrive at \(r_c = 0.16 \pm 0.04\), where we have used the form-factors as in [13] and errors have been added in quadrature (we have slightly inflated the total error to account for departure from factorization assumption). The authors in [6] arrive at the following expression after eliminating \(\delta_c\), and retaining terms to linear order in \(r_c\),

\[
\begin{equation}
\label{eq:9}
\left( \frac{R_c - 1}{\cos \gamma - \delta_{EW}} \right)^2 + \left( \frac{A_{CP}(K^+\pi^0)}{\sin \gamma} \right)^2 = 4r_c^2 + O(r_c^3)
\end{equation}
\]

and conclude that due to the EW penguin contribution, the first term itself can saturate the above sum-rule. Note that the authors have used a different value for \(r_c\) compared to what is quoted above. This difference is essentially due to the form factors and phase space contributions not included in [6].

We follow a slightly different route here. Instead of trying to check whether the sum-rule is satisfied within SM, we invert the above relation and express \(\delta_{EW}\) in terms of quantities which are either directly measured, like rates and asymmetries, or are expressible in terms of measured quantities like \(r_c\).

In Fig.1 we plot \(\delta_{EW}\) as a function of CKM angle \(\gamma\) (the shaded region represents 2σ allowed range for \(\gamma\) [14, 15]) for different values of \(R_c\), \(A_{CP}(K^+\pi^0)\) and \(r_c\). The horizontal dashed lines show the SM range of \(\delta_{EW}\) quoted above. Curves \(a, b\) and \(c\) refer to central, minimum and maximum values (1σ) for the above parameters while \(d\) shows two curves almost indistinguishable where a maximum-minimum combination of the parameters is used. From the plot it is very clear that the curve corresponding to central values of various quantities, curve \(a\), always stays outside of the large SM range of \(\delta_{EW}\). Curves \(c\) and \(d\) on the other hand fall within the SM range, albeit for smaller values of \(\gamma\).

Since the default curve, curve \(a\), stays outside SM range of \(\delta_{EW}\), it is instructive to examine the impact of NP. This will be the case, if for example \(|V_{ub}|\) eventually turns out to be close to the current inclusive value. It is straightforward to incorporate the effects of NP to EW penguins by replacing \(\delta_{EW}\) by \(\delta_{EW} \Delta e^{i\phi}\), where \(\Delta\) denotes deviation from SM scaled by SM value and \(\phi\) is the CP violating phase carried by the NP operators. With this replacement, we arrive at

\[
\begin{equation}
\label{eq:10}
R_{ew}^{\new} = 1 - 2r_c \cos \delta_c (\cos \gamma - \delta_{EW} \Delta \cos \phi) + r_c^2 (1 + \delta_{EW}^2 \Delta^2 + 2\delta_{EW} \Delta [\cos \gamma \cos \phi + \sin \gamma \sin \phi])
\end{equation}
\]

and

\[
\begin{equation}
\label{eq:11}
A_{CP}^{\new}(K^+\pi^0) = -\frac{2r_c}{R_{ew}^{\new}} \sin \delta_c (\sin \gamma + \delta_{EW} \Delta \sin \phi)
\end{equation}
\]

In Fig.2 and Fig.3 we show the allowed range of \(\Delta\) and \(\phi\) for \(\delta_c = -20^\circ\) and \(-10^\circ\) respectively (no solution is found for the range of parameters employed for positive \(\delta_c = 10(20)^\circ\)). We have chosen 1σ experimental range for the observables \(R_c\) and \(A_{CP}(K^+\pi^0)\). All other parameters are held to their central values and \(\delta_{EW} = 0.64\). We have varied \(\Delta\) between \(-2\) to \(2\) and \(\phi\) between \(-\pi\) and \(\pi\). We find that the new CP violating phase \(\phi\) and the magnitude \(\Delta\) should be very large and negative (discrete ambiguities have been ignored at this point). This is consistent with the observation made in [20] in the context of CP asymmetry in \(B \to K\pi\pi^0\). If this kind of a scenario turns out to be true, it will be a clear indication beyond the minimal flavour violation (MFV) hypothesis where the only source of CP (and flavour) violation is the CKM phase.

Given the present errors on various quantities, currently it is not very conclusive to infer physics beyond SM from \(B \to K\pi\) rates and asymmetries. As is clear from the above discussion, \(|V_{ub}|\) plays the most crucial role in reaching the conclusion whether there is a sign of new physics or not. We have merged the two measurements (inclusive and exclusive determinations of \(|V_{ub}|\)) and shown a broader band, within which one can accommodate the SM prediction. In this context, it is important to keep in mind that the amplitude relations have
been obtained after neglecting small contributions. One can ask if inclusion of those neglected pieces can improve the situation. Very recently, the authors in [16], following the operator language, have included (some of the) $1/m_B$ contributions. Employing QCDF [17], the authors have checked for the consistency of the fits to all observables in the $B \to K\pi$ system, as well as by removing the one under consideration. Their results show that including the $1/m_B$ corrections and varying them within very plausible ranges, the rates and CP asymmetries obtained are well in agreement with the HFAG values. They conclude therefore that the inclusion of these formally $1/m_B$ suppressed terms (having a very marginal impact on branching ratios) naturally leads to opposite signs for $A_{CP}(K^+\pi^-)$ and $A_{CP}(K^+\pi^0)$, thereby making the SM prediction of $\Delta A_{K\pi}$ consistent with experiments. Similar conclusions have been reached within PQCD [18] and global fits based on (approximate) flavour $SU(3)$ symmetry [19] where a large colour suppressed tree contribution is needed. Inclusion of some of the neglected small contributions can be effectively seen as modifying the value of $\delta_{EW}$, and can thus bring the data and theory in better agreement. This will then be consistent with the findings of [16].

From this discussion, it is clear that no significant new physics contribution may be needed to address the $B \to K\pi$ puzzle(s). However, it is not just $B \to K\pi$ rates and asymmetries that show a possible tension with SM expectations. A somewhat lower value of $\Delta m_s$, large CP violating phase in the $B_s$ mixing, polarization puzzles in $B \to \tau\nu\tau(V(T))$, sin2$\beta$ from penguin dominated modes (see [1] for the present experimental status of all these), all call for a closer look and have been advocated as hints of physics beyond SM. The latest Belle measurement [21] of $BR(B \to \tau\nu\tau)$ confirms the larger than SM value reported earlier [22]. The largest uncertainty comes from $f_B$ and $V_{ub}$. One can instead look at the ratio $BR(B \to \tau\nu\tau)/\Delta m_d$, which within SM takes the form

$$\frac{BR(B \to \tau\nu\tau)}{\Delta m_d} = \frac{3\pi}{4} \frac{m_s^2 \tau_{B+}}{M_W^2 S(x_t) \eta_{B_d} B_{B_d} |V_{ud}|^2} \times \left(1 - \frac{m_s^2}{m_{B+}^2}\right)^2 \left(\frac{\sin \beta}{\sin \gamma}\right)^2$$

(12)

where $\eta_{B_d} = 0.56$ is the QCD correction factor entering $\Delta m_d$ while $S(x_t)$ encodes the dominant short distance top-quark contribution to the box diagram. This ratio brings out $\sim 2\sigma$ tension between the Lattice values of $B_{B_d}$ and that required by global fits to data [23]. Independent of $BR(B \to \tau\nu\tau)$, employing the same lattice value of $B_{B_d}$ yields a consistent value for $\Delta m_d$ for typical $f_B$ quoted in literature. A large central value (currently the errors are also large) of $BR(B \to \tau\nu\tau)$ as observed can not be easily accommodated within SM, and definitely calls for new physics. Various new physics scenarios have been advocated in the literature in order to resolve $B \to K\pi$ and other $b \to s$ penguin dominated issues. A simple example is the sequential four generation standard model, SM4, which can simultaneously explain $B \to K\pi$ puzzles and $B_s$ mixing while at the same time being consistent with other measurements [24]. However, without invoking extra operator structures like scalar, tensor or right handed operators, it is not feasible to explain polarization puzzle in $B \to \tau\nu\tau$ modes [25]. Therefore, models with only enhanced penguins can not simultaneously explain all these puzzles. Looking at all these hints, we find it very plausible that an extended Higgs sector is in fact needed. A general two Higgs doublet model (g2HDM), allowing for CP violation can in fact resolve most of the above mentioned discrepancies. The CP violating phase in the Higgs sector will be common to Higgs (di- penguin) diagrams and therefore will have no effect on $B_d$ mixing while making a non-negligible contribution to $B_s$ mixing, due to non-zero strange quark mass as opposed to negligible down quark mass. For the same reason, $b \to s\bar{s}s\bar{s}$ penguin processes will receive an additional contribution compared to $b \to s\bar{u}d(\bar{u}u)$. Therefore, there is a possibility of resolving polarization puzzle (via the scalar-pseudoscalar operators in such a model [26]) that shows up in $b \to s\bar{s}$

![FIG. 2: Parameter space in $\Delta$-$\phi$ plane for $\delta_c = -20^\circ$ keeping other parameters fixed at their central values.](image1)

![FIG. 3: Same as in Fig.(2) but with $\delta_c = -10^\circ$.](image2)
penguin dominated modes ($B \to VT$ is an exception and may have to do with a very different hadronic structure of the tensor meson involved). In such a model, the strength of EW penguin operators will also get modified, possibly bridging any gap between theoretical predictions and experimental measurements. One does not expect these modifications to be numerically very large once the latest $b \to s\gamma$ constraints are taken into account. Very roughly speaking, $b \to s\gamma$ rate is practically independent of $\tan\beta$ for $\tan\beta > 2$ \cite{21}, and only places tight constraints on $m_{H^+}$ which can be combined with $B \to \tau\nu\tau$ measurements to eliminate a large region of parameter space. Further immediate constraints come from limits on $B_s \to \mu^+\mu^-$ branching fraction. A detailed phenomenological study in the context of a specific 2HDM will be presented elsewhere.

In this note, using flavour SU(3) symmetry, we have shown that given the present errors on various quantities, mainly $V_{ub}$, rates and asymmetries in $B \to K\pi$ modes do not show any significant deviation from SM expectations. Inclusion of formally suppressed contributions will not significantly affect the rates but can have a large impact on CP asymmetries and can bring theory and data in better agreement. We have also found that if EW penguins are to resolve the discrepancies in the $K\pi$ modes, a large new CP violating phase is needed. Looking at various other observables, a general two Higgs doublet model may offer the simplest resolution to most of the puzzling issues in flavour physics. However, we strongly feel that a precise measurement of $V_{ub}$ and lattice estimation of $f_{B_{d,s}}$ is immediately needed, without which possible NP may remain hidden under the parametric uncertainties.

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