Mathematical modelling of winter concreting

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Abstract. The aim of this study is to obtain the simple, convenient for the practical application of calculation formulas using the nonlinear submodels obtained by a group analysis of the differential equations for calculating the temperature fields during of the winter concreting of the building structures hardening in the cold. The paper presents the results of the experimental concreting of a fragment of a reinforced concrete column at the different heat treatment modes and their comparison with the calculated indices.

1. Introduction
The transition of the construction at the beginning of the twentieth century to year-round work required guaranteed quality of the concrete structures in the winter conditions. The main problem of winter concreting is to provide a concrete hardening with the necessary strength. The strength directly depends on the aging temperature, the task of its prediction has become central [1–10]. This required from the builders to predict the changes in a temperature and an increase in the strength of hardened concrete. Many scientists and practitioners of building production have proposed the methods of the calculation [3–10], but they all have certain disadvantages. Winter concreting theorists have been looking for the ways to equip the practical engineers with simple but reliable calculation methods, convenient for an implementation in the building practice using the elementary calculation tools, deliberately assuming the inaccurate results.

Today, engineers-practitioners of the construction companies for daily calculations during the concreting in winter use not very accurate, but easy-to-use methods for the predicting temperature conditions based on the very rough approximations of the solutions of the linear Fourier differential equation [3–10]. This leads to a significant inaccuracy of the calculations. Another important reason for this inaccuracy is the fact that the nonlinear process is described by a linear differential equation.

In this paper, to study the winter concreting process, we use a nonlinear model of the heat distribution in an inhomogeneous rod in the presence of a non-stationary heat source. This work is an attempt to use the simple models obtained in [11] by a group analysis of the differential equations, in the winter concreting, for practical use in the solving problems of the modeling of the temperature fields under the conditions of a hardening of the building structures in the cold. The results of a experimental concreting of the reinforced concrete column fragments at various heat treatment modes
and their comparison with the calculated indicators are presented.

2. Non-Linear Model of Temperature Distribution in a Column during Winter Concreting

The model describing the nonlinear process of a heat propagation in an inhomogeneous rod in the presence of a non-stationary heat source is given by the equation

$$\frac{\partial T}{\partial t} = \frac{1}{\alpha} \left( x T^{\beta} \frac{\partial T}{\partial x} \right) + \gamma(t) T,$$

where $T = T(t, x)$ is a temperature of the rod at a point $x \in (\infty, \infty)$ at a time $t$; $\alpha$ is a parameter characterizing the inhomogeneity of the rod; $\beta$ is a parameter characterizing the nonlinearity of the process; $\gamma(t) > 0$ is a non-stationary coefficient of a heat input (this coefficient also takes into account the thermal diffusivity of the rod material, that is essential for the non-stationary thermal processes).

The parameters $\alpha$ and $\beta$ are arbitrary real constants. They are determined empirically. We assume that a condition

$$\alpha \beta \gamma'(t) \neq 0,$$

is satisfied. This condition means that the process is nonlinear, the rod is inhomogeneous and there is a non-stationary heat source.

For the convenience of writing down the formulas below, we introduce a function that is related to the function $\gamma(t)$ by the relation

$$\gamma(t) = \frac{1}{\beta} \left( \ln(\varepsilon'(i)) \right), \quad \varepsilon'(i) > 0, \quad \left( \ln(\varepsilon'(i)) \right)^* \neq 0.$$

The function $\varepsilon(t)$ is expressed through the function $\gamma(t)$ according to the formula

$$\varepsilon(t) = k_1 \exp(\beta \int_{t_0}^{t} \gamma(t) \, dt) + k_2,$$

where $k_1$ and $k_2$ are arbitrary real constants.

Many mathematical models of physics and continuum mechanics are formulated in the form of linear and quasi-linear differential equations. Mathematical model is a description of the real scheme by mathematical language. The symmetry analysis of the equations of the models of physics and mechanics of continuous media is one of the most effective ways to obtain quantitative and qualitative characteristics of the physical processes [12–14]. The main task of the symmetry analysis of differential equations is to study the set of the solutions of these equations. All algorithms of the symmetry analysis are the preparation for achieving this purpose. The modern concept of the symmetry analysis is understood as the fullest using of the group of transformations admitted by the equations of model primarily to obtain and research the exact solutions. Exact solutions allow us to describe the specific physical processes. Exact solutions can be used as test solutions in numerical calculations, which perform in the studies of the real processes. Exact solutions allow us to assess the degree of adequacy of a given mathematical model to the real physical processes, after carrying out experiments corresponding to these decisions, and estimating the deviations that arise.

In [11], 13 significantly different invariant submodels defined by the exact solutions of the nonlinear differential equation (1) under conditions (2) and (3) were obtained by the methods of group analysis of differential equations [12–14]. They contain arbitrary parameters $\alpha$, $\beta$ and a
function $\varepsilon(t)$. These submodels can be used to describe the distribution of the heat in the column during a winter concreting in the presence of an external non-stationary heat source. In the present work, we took $\varepsilon(t) = t^2 + t + 1$, that corresponds to an economical heating mode with a time-dependent non-stationary heat transfer coefficient $\gamma(t) = \frac{1}{\beta(t+0.5)}$, inversely proportional to a time.

For the calculated description of the performed experiments, we used the following two submodels of the temperature distribution in the column: 1) submodel for which the exact temperature value is determined by the formula

$$T_{t,exact}(t, x) = \alpha^{1-\alpha} x^{\beta+1} \left( \varepsilon'(t) \right)^{1-\beta}.$$  (5)

2) submodel for which the exact temperature value is determined by the formula

$$T_{t0,exact}(t, x) = \left( \varepsilon'(t) \right)^{1-\beta} x^{\beta+1}.$$  (6)

The purpose of further research is to determine the values of the parameters at which these submodels adequately describe the experiments, performed by the authors of the paper.

3. Experimental Studies

Experimental studies of the temperature fields in a concrete structure were carried out by measuring of the temperature in a model column body of a square section of $200 \times 200$ mm and a height of 1000 mm (Figure 1). Formwork was made from laminated plywood. (GOST 3916.1-96) 18 mm thick. As a heater, a heating wire, widely used in production conditions, was used. Heating wire PNSV, diameter 1.6 mm with a heat-resistant electrical insulation from polyvinyl chloride 0.8 mm thick (GOST TU 16.K71-013-88) a loop length of 50 m was fixed on the inner surface of the formwork with a pitch of 20 mm along all side faces (Figure 2). The heating wire was connected to the electrical network, voltage 220 V. The change in an electrical power was carried out using a laboratory autotransformer AOSN-20-220-75UHL4.

In the center of the column was a thermal solution of chromel-kopel thermocouples (GOST 6616-74) with a pitch of 150 mm. The model was placed in the AK-2FR refrigerator, the control unit of which provides for the adjustment of the operating temperature in the range from 0 to -25 °C. At the top and bottom of the column was a heat insulation (polystyrene).

For the convenience of the conducting studies of the various modes of a heat treatment of the structure and eliminating the effect of a cement exothermy, a model body was used as a concrete mixture. It was a mixture consisting of the same materials and accepted in the same proportions as the concrete mixture, with the difference that instead of cement, ground sand was used with a specific surface $S_{sp} = 2900$ cm² / g equal to the specific surface of the cement. Such material was obtained in a laboratory ball mill. To equalize the electrophysical properties, NaC12 was added to the composition. The water-holding ability of the mixture was provided by the addition of a bentonite clay. The model body prepared in the laboratory according to its physical parameters — mass, stiffness, thermophysical characteristics, and resistivity — practically did not differ from the real concrete mixtures. The initial temperature of the mixture was 22.2 °C.

| Table 1. The composition of the model body accepted for research. |
|---------------------------------------------------------------|
| Materials | Consumption of materials per 1 m³, kg |
|---------------------------------------------------------------|
|---------------------------------------------------------------|
Standard Concrete
B 22.5

| Crushed stone diabase FR 5-20. GOST 10268-70 | 1250 | 1250 |
| Sand quartz river Krivodanovsky quarry MKR = 1.8. GOST 10268-70 | 530 | 530 |
| Portland cement M 400. GOST 10178-68 | 450 | - |
| Ground sand (SSp = 2900 cm²/g) | - | 450 |
| Water technical GOST 2874-54 | 180 | 180 |
| Volumetric mass of concrete, kg/m³ | 2410 | 2410 |

The following experiments were carried out, which differ in the heating capacity of the column:
1 - low power: voltage 11 V, current 3 A,
2 - medium power: voltage 27.3 V; current 6.21 A,
3 - high power: voltage 37.8 V; Current 9.19 A.
A heating occurred within 25 hours. All thermocouples during the heating showed almost the same temperature, which we will designate further on \(T_{n, \text{exp}}\), where \(n = 1\) for the low heating power, \(n = 2\) for the medium heating power, \(n = 3\) for the high heating power.

4. Comparison of the Results of Experimental Studies with Calculated Indicators
In all of the graphs below, the experimental curves describe the temperature increment \(T = T_{n, \text{exp}} - 22, 2^\circ\). These graphs are assigned a number \(n\) depending on the heating power. Due to the nonlinearity of the equation (1), its exact solutions (5), (6) obviously cannot approximate these temperature increments. Therefore, to obtain the calculation formulas approximating these increments, correction additive terms and coefficients are introduced into formulas (5), (6) by the formula

\[
T_n = \lambda T_{n, \text{exact}} (t, 0.5) - \mu \left( n = 1, 10 \right),
\]

where the numbers \(\lambda\) and \(\mu\) take specific values for each calculation formula (7). Generally speaking, this is the usual practice of applying the obtained theoretical formulas to describe the real processes. The best approximation for the case of three different heating powers is given by the following calculation formulas given in Table 2.

| Figure No. | Coefficient \(\alpha\) | Coefficient \(\beta\) | Calculation formula (7) | Calculation curve |
|------------|-----------------|-----------------|------------------|------------------|
| Low Power Case, 33VA | | | | |
| 4. | 2.0 | 4.0 | \(T_1 = 7T_{1, \text{exact}} (t, 0.5) - 8\) | \(T = T_1\) |
| 5. | 1.44 | 3.0 | \(T_1 = 5T_{1, \text{exact}} (t, 0.5) - 7\) | \(T = T_1\) |
| 6. | 1.45 | 4.0 | \(T_{10} = 8T_{10, \text{exact}} (t, 0.5) - 10\) | \(T = T_{10}\) |
| 7. | 2.0 | 2.0 | \(T_{10} = 2T_{10, \text{exact}} (t, 0.5) - 3\) | \(T = T_{10}\) |

| Medium Power Case, 169.5 VA | | | | |
| 8. | 1.43 | 2.0 | \(T_1 = 7T_{1, \text{exact}} (t, 0.5) - 10\) | \(T = T_1\) |
| 9. | 1.45 | 4.0 | \(T_1 = 26T_{1, \text{exact}} (t, 0.5) - 30\) | \(T = T_1\) |
| 10. | 2.0 | 4.0 | \(T_{10} = 25T_{10, \text{exact}} (t, 0.5) - 29\) | \(T = T_{10}\) |
| 11. | 1.44 | 3.0 | \(T_{10} = 15T_{10, \text{exact}} (t, 0.5) - 19\) | \(T = T_{10}\) |

| High Power Case, 347.4 V | | | | |
| 12. | | | | |

4
| Figure № | Coefficient $\alpha$ | Coefficient $\beta$ | Calculation formula (7) | Calculation curve |
|---------|----------------------|----------------------|--------------------------|------------------|
| 12.     | 2.0                  | 4.0                  | $T_1 = 42T_{1,\text{exact}}(t, 0.5) - 51$ | $T = T_1$        |
| 13.     | 1.44                 | 3.0                  | $T_1 = 27T_{1,\text{exact}}(t, 0.5) - 31$ | $T = T_1$        |
| 14.     | 1.45                 | 4.0                  | $T_{10} = 45T_{10,\text{exact}}(t, 0.5) - 50$ | $T = T_{10}$     |

4. 1. Graphs of the calculated curve and experimental curves for different powers

![Figure 1.](image1)

![Figure 2.](image2)

![Figure 3.](image3)

![Figure 4.](image4)
4. Conclusion
The aim of this study was to obtain the simple, convenient for the practical application of calculation formulas using the nonlinear submodels, obtained by a group analysis of the differential equations for calculating temperature fields during of the winter concreting of the building structures hardening in the cold.

The object of study was a square column with a side of 200 mm and a height of 1000 mm. A heating wire 50 m long was fixed on the formwork along the column with a pitch of 20 mm on all side faces. The thermocouples were installed in the center with a step of 150 mm. At the top and bottom of the column was heat insulation (polystyrene). The mixture was a model body. Experiments were carried out, differing in the heating capacity of the column: 1) low power, 2) average power, 3) high power.

So, we took two submodels, described by the exact solutions of the nonlinear differential equation that describes the nonlinear process of heat propagation in an inhomogeneous rod in the presence of a non-stationary heat source in order to obtain the calculation formulas. The calculated formulas, obtained using these submodels adequately describe the experimental results. A particularly accurate description was obtained with low and medium heating power. We received 11 graphs, which show the results of experiments and calculated results.

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