New insights on non-perturbative Yang-Mills

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Abstract.

In this talk we review some recent results on the infrared properties of the gluon and ghost propagators in pure Yang-Mills theories. These results are obtained from the corresponding Schwinger-Dyson equation formulated in a special truncation scheme, which preserves gauge invariance. The presence of massless poles in the three gluon vertex triggers the generation of a dynamical gluon mass (Schwinger mechanism in $d = 4$), which gives rise to an infrared finite gluon propagator and ghost dressing function. As a byproduct of this analysis we calculate the Kugo-Ojima function, required for the definition of the non-perturbative QCD effective charge within the pinch technique framework. We show that the numerical solutions of these non-perturbative equations are in very good agreement with the results of $SU(3)$ lattice simulations.

Keywords: Non-perturbative QCD, Schwinger-Dyson, Infrared propagators

In the last three decades we have accumulated ample experimental evidence corroborating our conviction that QCD is indeed the theory of the strong interactions. Most of these experimental data come from short distance processes, whose theoretical description can be performed by means of perturbation theory. The reason why perturbation theory describes so successfully the experimental results in the ultraviolet (UV) region is due to one of the most intrinsic characteristics of QCD, namely asymptotic freedom [1]. On the other hand, at large distances, another extraordinary phenomenon of the QCD manifests itself: the color confinement, which prevents the fundamental excitations of the theory (quarks and gluons) from appearing as free particles; only color singlets are observed as asymptotic states.

One of the major challenges of the strong interactions is to explain confinement from first principles. One possible way to gain some insights on the nature of the confinement, and how the infrared (IR) dynamics of the theory works, is to study the Green’s functions of the fundamental degrees of freedom, gluons, quarks and ghosts. Even though it is well-known that these quantities are not physical, since they depend on the gauge-fixing scheme and the parameters used to renormalize them, they do capture crucial aspects of the underlying perturbative and non-perturbative dynamics. In addition, when appropriately combined, they give rise to physical observables.

The most widely employed tools to explore the IR dynamics of the Yang-Mills theories are: (i) the lattice simulation, where Monte Carlo techniques are used, after discretizing space-time and imposing periodic boundary conditions on the finite volume, and (ii) the Schwinger-Dyson equations (SDE), which form an infinite set of integral equations governing the dynamics of the off-shell QCD Green’s functions.

In this talk we will review some of the recent advances on the IR behavior of the gluon and ghost propagator in a pure Yang-Mills theory, obtained within the truncation
scheme of the SDE based on the pinch technique (PT) [2] and its connection with the background field method (BFM) [3]. We compare our SDE results with the available lattice data [4, 5, 6], and we discuss how these results are best interpreted by assuming the generation of an effective gluon mass [7, 8, 9].

In the Landau gauge the full gluon propagator, $\Delta_{\mu\nu}(q)$, is transverse, and its general form is given by

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2), \quad \text{with} \quad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2},$$

where the scalar function, $\Delta(q^2)$, is related to the all order self-energy $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q^2)$ through $\Delta^{-1}(q^2) = q^2 + i\Pi(q^2)$. The full self-energy $\Pi_{\mu\nu}(q)$ is expressed in terms of an infinite tower of coupled integral equation governing the dynamics of all n-point functions of the theory. For practical purposes, this tower of equations, known as gluon SDE, can be only employed after devising a self-consistent truncation scheme which selects a tractable subset of these equations, without compromising the crucial characteristics of the theory.

Devising such a scheme, however, is very challenging, especially in the context of non-Abelian gauge theories, such as QCD. One of the main difficult resides in the fact that one cannot truncate the conventional gluon SDE, in any obvious way, without violating the basic transversality relation $q^\mu\Pi_{\mu\nu}(q) = 0$, imposed by the BRST symmetry.

Recently a new truncation scheme, that respects gauge invariance at every level of the “dressed-loop” expansion, has been proposed for the gluon and ghost SDEs [8]. This particular truncation scheme is based on the pinch technique (PT) [2] and its correspondence with the background field method (BFM)[3], and implements a drastic modification at the level of the building blocks of the SD series. The PT enables the construction of new, effective Green’s functions endowed with very special properties; most importantly, they are independent of the gauge-fixing parameter, and satisfy QED-like Ward identities (WI) instead of the usual Slavnov-Taylor identities [2]. Within this formalism one trades the conventional SD series for another one, written in terms of the new Green’s functions, and then truncates this new series, by keeping only a few terms in...
a “dressed-loop” expansion, maintaining exact gauge-invariance. In Fig. 1, we show the new diagrams composing the the PT-BFM self-energy, to be denoted as \( \Pi_{\mu\nu}(q) \). Notice that the vertices appearing in the diagrams of Fig. 1 are not the conventional ones, but rather the vertices constructed using the Feynman rules of the BFM [3].

One of the most interesting properties of \( \Pi_{\mu\nu}(q) \) is the way its transversality is enforced. More specifically, the SDE is composed of “one-loop” and “two-loop” dressed blocks that are individually transverse. In fact, the resulting pattern is even more restrictive: the gluon and ghost diagrams form separate transverse blocks, represented by the boxes in Fig. 1. In this way, we have that the gluonic sector of the “one-loop” dressed blocks satisfies \( q^\mu[(a_1) + (a_2)]_{\mu\nu} = 0 \), while for the ghost \( q^\mu[(a_3) + (a_4)]_{\mu\nu} = 0 \); similarly for the “two-loop” blocks, we have \( q^\mu[(a_5) + (a_6)]_{\mu\nu} = 0 \) for the gluonic part, and \( q^\mu[(a_7) + (a_8) + (a_9) + (a_{10})]_{\mu\nu} = 0 \) for the ghost sector. The fact that the transversality is enforced “block-wise” allows for a self-consistent truncation of the full gluon SDE [8].

In addition, the connection between the conventional \( \Delta(q^2) \) and the PT-BFM \( \hat{\Delta}(q^2) \) are done via a powerful formal identity [10] stating that

\[
\Delta(q^2) = \left[ 1 + G(q^2) \right]^2 \hat{\Delta}(q^2),
\]

with \( G(q^2) \) defined from \( \Lambda_{\mu\nu}(q) \), Fig. 1,

\[
\Lambda_{\mu\nu}(q) = g_{\mu\nu}G(q^2) + \frac{q_{\mu}q_{\nu}}{q^2}L(q^2) = -iC_A g^2 \int_k H_{\mu\rho}^{(0)} D(k + q) \Delta^{\rho\sigma}(k) H_{\sigma\nu}(k, q),
\]

where \( C_A \) is the Casimir eigenvalue of the adjoint representation \([C_A = N \text{ for } SU(N)]\), and \( \int_k \equiv \mu^{2d} \frac{(2\pi)^d}{d!} \int dq^d k \), with \( d = 4 - \epsilon \) the dimension of space-time. The vertex \( H_{\mu\nu} \), appearing in Eq. (3), is related to the full gluon-ghost vertex, \( \Gamma_{\nu}(k, q) \), by the STI

\[
q^\mu H_{\mu\nu}(k, q) = -i\Gamma_{\nu}(k, q).
\]

It is important to keep in mind that the auxiliary function \( G(q^2) \), appearing in the definition of \( \Lambda_{\mu\nu}(q) \), plays an instrumental role in the PT-BFM framework, since only with it we are able to connect the \( \Delta(q^2) \) and \( \hat{\Delta}(q^2) \). Interestingly enough, and in the Landau gauge only, \( G(q^2) \) coincides with the so-called Kugo-Ojima (KO) function; this latter function, and in particular its value in the deep IR, is intimately connected with the corresponding well-known confinement criterion [11, 12].

In one loop dressed approximation, the PT-BFM self-energy is given by \( \hat{\Pi}_{\mu\nu}(q) = [(a_1) + (a_2) + (a_3) + (a_4)] P_{\mu\nu}(q) \), and using the identity (2), we can express the gluon SDE of Fig. 1 as an integral equation involving only \( \Delta(q^2) \), in the following way [9]

\[
\Delta^{-1}(q) = \frac{q^2 + i[(a_1) + (a_2) + (a_3) + (a_4)]}{[1 + G(q^2)]^2}.
\]

Moreover, as shown in Fig. 1, the ghost SDE is the same as in the conventional formulation, namely

\[
iD^{-1}(q) = q^2 + iC_A g^2 \int_k \Gamma_{\mu} \Delta_{\mu\nu}(k) \Gamma^\nu(k, q) D(q + k),
\]
where $\Gamma_\mu$ is the standard (asymmetric) gluon-ghost vertex at tree-level, and $\Gamma_\mu(k, q)$ its fully-dressed counterpart, with $k$ representing the momentum of the gluon and $q$ the one of the outgoing ghost.

Next, we use for the two vertices appearing in Eq.(3) and (5) their tree-level values, $H_{\mu\nu}(k, q) = ig_{\mu\nu}$, and $\Gamma_\mu(k, q) = -q_\mu$ respectively. Then, setting $f(k, q) \equiv (k \cdot q)^2/k^2q^2$, one may show that [13]

$$F^{-1}(q^2) = Z_c + g^2C_A \int [1 - f(k, q)]\Delta(k)D(k + q),$$

$$1 + G(q^2) = Z_c + \frac{g^2C_A}{d-1} \int [(d-2) + f(k, q)]\Delta(k)D(k + q),$$

$$L(q^2) = \frac{g^2C_A}{d-1} \int [1 - d f(k, q)]\Delta(k)D(k + q),$$  \hspace{1cm} (6)

where $F(q^2)$ is the dressing function of the ghost propagator defined as $D(q^2) = iF(q^2)/q^2$.

It is important to mention that there exists a powerful formal identity relating $F(q^2)$, $G(q^2)$, and $L(q^2)$, namely $F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$ [14].

In addition to its formal derivation [14], the above relation has been recently obtained at the level of the SDEs defining these three quantities [13]. Adding the three equations of (6) we can verify that above identity is indeed satisfied under the approximations employed.

One of the most crucial ingredients of this truncation scheme is the Ansatz employed for the PT-BFM vertex $\bar{\Gamma}_{\mu\alpha\beta}(q, k_1, k_2)$, appearing in the diagram $(a_1)$ of Fig.1,

$$\bar{\Gamma}_{\mu\alpha\beta} = \Gamma_{\mu\alpha\beta}^{(0)} + \frac{q_\mu}{q^2} [\Pi_{\alpha\beta}(k_2) - \Pi_{\alpha\beta}(k_1)],$$  \hspace{1cm} (7)

whose essential feature is the presence of massless pole terms, required for triggering the Schwinger mechanism. A very detailed discussion about the properties of the above Ansatz can be found in [9]. After using the vertex of Eq.(7) in the gluon SDE of (4), we obtain a lengthy equation that we do not report here.

The solutions obtained for the Green’s function of the Eqs.(4) and (6) are shown in Fig. 2, and are compared with the corresponding lattice data [5, 6]. Unfortunately, as far we know, no lattice results exists for $L(q^2)$, and therefore in the last panel of Fig. 2 we present only our SDE prediction for it.

Note that, we obtain a good qualitative agreement with the lattice result for $\Delta(q^2)$, $F(q^2)$, and $G(q^2)$. More specifically, in the case of the gluon propagator, we clearly see that both SDE and lattice results are infrared finite, since $\Delta(0) > 0$. Such feature can be associated to a purely non-perturbative effect that gives rise to a dynamical gluon mass, which saturates the gluon propagator in the IR. The appearance of the gluon mass is also responsible for the infrared finiteness of the ghost dressing function, $F(q^2)$, which is clearly shown on the right upper panel of Fig. 2 [9, 15]. Therefore both, lattice and SDE results, are clearly at odds with the KO confinement scenario, which requires an enhanced ghost dressing function. In addition, SDE and lattice [6] find no evidence of $G(0) = -1$, which is required for the realization of the KO confinement scenario. Specifically, the large-volume lattice simulations of [6] find that $G(q^2)$ saturates in the
deep IR around approximately $G(0) = -0.6$, which is excellent agreement with the value obtained from a recent SDE analysis [12]. It is important to mention that, very recently, the PT-BFM truncation scheme has been also applied successfully in the case of Yang-Mill in 3d [16].

Using the all ingredients presented so far, and remembering the fact that the new PT-BFM Green’s functions satisfy Abelian-like WIs, we may construct the renormalization-group-invariant quantity $\hat{\Delta}(q^2)$, defined as [13, 17]

$$\hat{\Delta}(q^2) = g^2(\mu^2) \frac{\Delta(q^2, \mu^2)}{[1 + G(q^2, \mu^2)]^2}.$$  \hspace{1cm} (8)

From $\hat{\Delta}(q^2)$ we can extract a dimensionless quantity that corresponds to the non-perturbative generalization of the QCD effective charge, given by

$$4\pi\alpha(q^2) = [q^2 + m^2(q^2)]\hat{\Delta}(q^2),$$  \hspace{1cm} (9)

where $m^2(q^2)$ is a momentum-dependent gluon mass, and $\alpha(q^2) = g^2(q^2)/4\pi$.

Then, assuming a power-law running mass of the type $m^2(q^2) = m^2/(q^2 + m^2)$ [18](shown in the left panel of Fig. 3), and using the results obtained from the SDE solutions, we obtain the effective charge shown in the right panel of Fig. 3. As we can see, $\alpha(q^2)$ saturates at an IR finite value, and displays the correct UV behavior [13, 17]. The presence of the IR fixed point in the behavior of $\alpha(q^2)$ is another manifestation of the appearance of the gluon mass [19] that tames the Landau pole, allowing for a smooth connection between the IR and UV regions of $\alpha(q^2)$.

**FIGURE 2.** The SDE results for $\Delta(q^2)$, $F(q^2)$, $G(q^2)$, and $L(q^2)$ compared with the corresponding lattice data.
FIGURE 3. The power-law running mass $m(q^2)$ with $m = 700$ MeV (left panel). The non-perturbative QCD effective charge, $\alpha(q^2)$, of Eq.(9) obtained from the solution of the SDE (right panel).

We have presented the basic characteristics of the SDEs formulated within the PT-BFM framework. We have seen that the infrared finiteness of the gluon propagator and ghost dressing function are associated to the generation of a dynamical gluon mass, which is also responsible for the appearance appearance of an IR fixed point in the QCD effective charge. In addition, we have shown that the SDE results for the Green’s functions are in nice agreement with the data of large-volume lattice simulations.

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