Nonsingular chattering-free barrier function finite time tracker for perturbed nth-order nonlinear systems and its application to chaotic color image scrambling

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ABSTRACT This study proposes a nonsingular barrier-function-based terminal sliding mode control technique for nth-order nonlinear dynamic systems. Its main objective is to guarantee the finite-time tracking performance in the presence of unmodeled dynamics, external disturbances and parameter variations. The proposed approach is synthesized using a novel barrier function-based terminal sliding surface to ensure the effective estimation of the system perturbations using the barrier adaptation laws, and thereby achieve the desired tracking performance. The dynamics of chaotic models are strongly dependent upon the initial conditions, parameters of the system, parametric uncertainty and external disturbances which are required to be controlled/synchronized by a robust nonlinear control technique. By designing the terminal sliding mode control approach combined with the adaptive control law, the tracking problem of the nth-order nonlinear dynamical system with unmodeled dynamics, parametric variations and external disturbances is investigated. Moreover, the application of the proposed method is studied using the color-image scrambling system. The scrambling keys are created by transmitter chaotic systems, where using the chaotic keys and scrambling techniques, the original color image is encrypted. The performance and applicability of the proposed design is assessed using two practical applications: a chaotic hyper-jerk system and a color image encryption system. The simulation and analytical results obtained with both systems confirmed the ability of the proposed control method to guarantee the finite time convergence of sliding surface, ensure chattering-free dynamics and avoid the singularity problems.

INDEX TERMS Nonlinear system; sliding mode control; adaptive-tuned function; nonsingular; finite time convergence.

I. INTRODUCTION

Nonlinear dynamics are inherently present in most physical systems [1]. Additionally, most practical systems are subject to uncertainties and bound to operate under various constraints and limitations. Designing control approaches for such systems has long been a challenging problem [2, 3]. Numerous nonlinear design approaches were proposed in the literature for uncertain nonlinear systems [4]. Classical approaches, such as nonlinear feedback control, input-output feedback linearization, backstepping control, suffer from some limitations. For instance, input-output linearization assumes the availability of the outputs as well as their relative degree order derivatives. The stability justification of such designs relies on the cancellation of nonlinearities. Hence, model uncertainties can degrade the performance of the controlled system and the robustness of such designs is not guaranteed. Synthesizing control approaches that are capable of tracking a set of reference signals whilst ensuring robustness to both parametric and non-parametric...
uncertainties and external disturbances is still an active research area.

Sliding mode control (SMC) remains one of the popular methods for the stabilization/tracking of uncertain nonlinear systems [5-9]. It owes its popularity to its inherent robustness and insensitivity to matched uncertainties, fast dynamic response, good transient performance and stable dynamics [10-12]. SMC suppresses uncertainties by exactly keeping the sliding variables in the origin using an infinite-frequency control action. Standard SMC designs are developed using linear switching manifolds that specify the expected control performances and are synthesized based on the Lyapunov theory. A major limitation of SMC design, however, is the chattering phenomena emanating from the high-frequency switching of the control input [13-15]. This problem limits its applicability as it can lead to actuator saturation, accelerate the wear and tear of mechanical systems, and/or excite unmodeled dynamics, thus degrading system performance and potentially leading to system instability. Another restriction of the standard SMC approach is the necessity that the relative degree of sliding variable is one. Various procedures have been proposed in the literature to deal with the chattering phenomena. For instance, boundary layer techniques were adopted to reduce chattering. A boundary layer-based SMC design that switches between a discontinuous control and a disturbance estimator (UDE)-based control was proposed in [16] for chattering reduction. High order sliding mode controls (HOSMC) were successful in reducing chattering and accurately keeping the sliding variable at zero [17, 18]. A super-twisting SMC control was designed in [19] to minimize chattering. Disturbance attenuation-based sliding mode control approaches have recently been proposed for systems with mismatched uncertainties [20]. Terminal Sliding Mode Control approaches (TSMC) were successful in achieving finite time convergence and alleviating the chattering phenomena. TSMC designs replace the linear sliding surfaces by nonlinear switching hyperplanes to improve the system’s transient performance and increase its convergence rate [21-24]. However, despite their ability to provide improvements and overcome the drawbacks associated with standard SMC, the above-mentioned techniques still need further improvements in terms of chattering mitigation and convergence speed, especially when implemented to highly nonlinear systems with unmodeled dynamics, parametric variations and external disturbances.

Additionally, most of existing approaches require the availability of information about the upper bounds of the perturbations and even their derivatives (in the case of super-twisting second-order SMC). However, uncertainty bounds are often not exactly known in most practical cases and such requirement often results in SMC designs with largely oversized discontinuous control gains, thus further aggregating the chattering associate with the unmodeled dynamics. This shortcoming has recently motivated research efforts in adaptive SMC approaches with algorithms that dynamically adapt the control gains to prevent their overestimation whilst guaranteeing sliding is maintained [25]. A genetic algorithm (GA) particle swarm optimization (PSO)-based nonsingular terminal sliding mode control (GP-NSTSMC) and an extended Lyapunov-based control design approach were developed in [26] to cope with the limitations of bounded parameter uncertainties and external disturbances. An adaptive second-order SMC approach which eliminates the system uncertainties and adjusts the control gain online was proposed in [27]. An adaptive sliding hypersurface with an adjustable dead-zone scheme was proposed in [28] for the attitude control of a spacecraft with large flexible sun-oriented solar panels to overcome the difficulties arising from the measurement of flexible dynamics coordinates. An adaptation methodology capable of searching for the minimum possible control values due to the equivalent control concept was proposed in [29]. A design that considers an adaptation mechanism that monotonically increases the gains was proposed in [30]. An SMC design in which the controller amplitude is being continuously tuned to counteract the effects of uncertainty was proposed in [31]. An SMC design with adaptive gains that have magnitudes that can increase/decrease as small as possible to not over-bound uncertainties, but sufficiently large to sustain the sliding motion was proposed in [32]. A main advantage of the above-mentioned approaches is their ability to guarantee the finite-time convergence of sliding variables to the neighborhood of the origin without over-estimating gains. However, it is worth noting that the neighborhood size and convergence time depend on the upper bounds of the external disturbances, which are unknown a priori [33]. Hence, a major drawback of these approaches is the fact that they do not guarantee the finite time tracking performance in the presence of unmodeled dynamics, parameter variations, input saturation and external disturbances.

Barrier Lyapunov function (BLF)-based techniques have recently proven their effectiveness in the control and stabilization designs for nonlinear systems with constraints, such as input saturation, full-state constraints, and output constraints [34-37]. A major feature of barrier functions is the fact that their values go to the infinity, when approaching the boundary. Hence, barrier functions boundedness implies that not only the system is stable, but the barriers are not transgressed [38]. A barrier function based approach was shown to maintain the finite-time convergent property whilst properly mitigating the effects of saturation in [39]. A barrier function-based adaptive SMC approach was synthesized in [40] for first-order disturbed systems with unknown bounds. Currently, barrier Lyapunov functions are widely utilized in constraint control designs. For instance, a new adaptive full-state constraint controller is designed in [41] for a class of uncertain multi-input-multi-output (MIMO) underactuated systems. A novel adaptive output-feedback controller was proposed in [42], for a class of uncertain underactuated systems. An important merit of these approaches are their ability to guarantee the convergence and maintenance of
output variables in the pre-defined neighborhood of the origin independent of the disturbance bound, and without over-estimating the gains. The above mentioned approaches, however, implemented linear sliding manifolds, thus only achieved asymptotic convergence, whereby trajectories convergence to the equilibrium will occur in infinite time [43]. Terminal SMC (TSMC) controls are well known for their finite-time convergence property [44-46]. A major drawback of TSMC designs, however, is the singularity problem or unboundedness of the control signal.

In [47], an adaptive non-singular fuzzy finite time decentralized event trigger-based control method was designed for the output-feedback control of large-scale nonlinear systems with external disturbances, unmeasured states, and error constraints. A nonsingular finite time TSMC-based partial stabilizer was proposed in [48] for nonlinear systems with perturbations. The design divided the nonlinear system into two subsystems according to the stability features of the system states. In [49], an adaptive robust non-singular integral TSMC was suggested for the finite time tracking control of disturbed nonlinear systems, where the time-derivative of the fractional power terms are not required in the control design. In [50], two adaptive chattering-free finite-time SMC techniques were recommended for single-input multiple-output (SIMO) nonlinear systems with unknown unmatched uncertainties. An adaptive neuro-fuzzy backstepping hybrid integral finite-time SMC approach was designed in [51] for uncertain fractional nonlinear systems subject to parametric uncertainties and external disturbances. A finite-time adaptive fuzzy nonsingular continuous TSMC technique was suggested in [52] for the control of nonlinear systems with model uncertainties and external disturbances. A fuzzy functional observer adaptive finite-time SMC approach is advised in [53] for the control of nonlinear dynamical systems with matched uncertainties upper with unknown bounds and partially unmeasured states. A robust finite-time adaptive super-twisting SMC stabilizer was designed in [54], for multiple-input multiple-output (MIMO) nonlinear systems with parameter uncertainties and external disturbances. The method was shown to alleviates the chattering problem. To the best of the authors’ knowledge, none of the existing works have considered nonsingular chattering-free finite time tracking control designs for nonlinear systems with external disturbances and parametric uncertainties using the barrier adaptation laws.

Data transmission has been widely used in our daily life in the recent years. However, image encryption or scrambling methods have been extensively applied in commercial and military usages. The motivations for considering chaotic encryption methods over the traditional encryption schemes are originated from the fact that the traditional approaches often exhibit high security issues, high time consumption, key distribution problems and low-efficiency levels. Chaos is employed in various communication applications such as radar [55], spread-spectrum systems [56], secure communication [57], ultra-wide-band communication [58] and image, speech or video encryption. Because of the randomness and rich dynamics of chaotic systems, various chaos theory-based encryption methods have been introduced to the field of multimedia encryption. Chaos-based encryption tactics are fast and the advanced security algorithms provide high sensitivity to the initial conditions, pseudo-randomness property, no periodicity behavior and no parameter dependency. These properties allow for supporting the permutation-diffusion requirements in cryptosystem establishment [59]. The above mentioned advantages along with the fast expansion of engineering applications of chaos theory have motivated recent research efforts in chaotic image cryptography techniques [60, 61].

This paper designs and implements a nonsingular barrier-function-based TSMC approach for nth-order nonlinear dynamical systems and its application to chaotic color image scrambling. Its main contributions are as follows:

- A control approach synthesized using a novel barrier-function based terminal sliding surface for the finite-time robust tracking and stabilization of the nth-order nonlinear systems with unmodeled dynamics, parametric variations and external disturbances.
- A design which not only achieves the desired tracking performance, but it also offers chattering and singularity-free dynamics.
- Successful implementation to a chaotic hyper-jerk and a color image encryption system.

The remainder of this paper is organized as follows. The control problem along with the nth order nonlinear system under consideration are described in section 2. The proposed Nonsingular barrier function-based sliding mode approach is derived in section 3. The applicability and performance of the proposed approach is assessed using two benchmark problems in Sections 4 through 6. Some concluding remarks are finally given in section 6.

II. PROBLEM FORMULATION

Consider the perturbed nth-order nonlinear system described as [62]

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = x_3 \\
\vdots \\
\dot{x}_n = f(x, t) + g(x, t)u
\]

where \( x = [x_1, \ldots, x_n]^T \in R^n \) represent the state vector; \( u \in R \) denotes control input; \( f(x, t) \in R \) and \( g(x, t) \in R \), \( g(x, t) \neq 0 \) specify the perturbed nonlinear continuous functions, which may consist of unmodeled dynamics, parameter variation and external disturbances. The main control objective is to design a finite time tracking control input that enables the states to follow the reference trajectories in the presence of perturbations.
Assumption 1: The functions \( f(x, t) \) and \( g(x, t) \) can be written as nominal parts \( f_0(x, t) \) and \( g_0(x, t) \), and uncertain bounded parts \( \Delta f(x, t) \) and \( \Delta g(x, t) \) as [63]:

\[
\begin{align*}
  f(x, t) &= f_0(x, t) + \Delta f(x, t) \\
  g(x, t) &= g_0(x, t) + \Delta g(x, t).
\end{align*}
\]

(2)

The functions \( f(x, t) \) and \( g(x, t) \) are supposed to be differentiable. Moreover, it is assumed that there are two known functions \( M(x, t) \) and \( N(x, t) \) so that the uncertain functions \( \Delta f(x, t) \) and \( \Delta g(x, t) \) fulfill the subsequent conditions [63]:

\[
\begin{align*}
  |\Delta f(x, t)| &\leq M(x, t) \\
  |\Delta g(x, t)| &\leq N(x, t)
\end{align*}
\]

(3)

for \( x \in \lambda \subseteq \mathbb{R} \), where \( \lambda \) is an open subset of \( \mathbb{R}^n \), which ensures the system states boundedness.

III. MAIN RESULTS

Defining the tracking error signals as \( e = x_1 - x_{1d}, \ldots, e^{(n-1)} = x_n - x_{nd} \), the control purpose is to track the reference trajectories \( x_{1d}, \ldots, x_{nd} \). For this aim, the sliding variable is proposed as:

\[
s = e^{(n-1)} + \sum_{i=1}^{n-1} c_i t^{i-1} e^{(i-1)},
\]

(4)

where \( c_0, c_1, \ldots, c_{n-2} \in \mathbb{R} \) denote the constant scalars. If the system states are on the sliding surface, i.e., \( s = 0 \), we have:

\[
e^{(n-1)} = -\sum_{i=1}^{n-1} c_i t^{i-1} e^{(i-1)}.
\]

(5)

Moreover, the time-derivative of the sliding variable (4) is obtained by:

\[
\dot{s} = e^{(n)} + \sum_{i=1}^{n-1} c_i t^{i-1} e^{(i)} = f(x, t) + g(x, t)u - \dot{x}_{nd} + c_0 \dot{e} + c_1 \dot{e} + \cdots + c_{n-2} e^{(n-1)}.
\]

(6)

To guarantee the finite time convergence of the sliding variable to the origin and remove the chattering phenomenon, the following nonsingular TSMC surface is introduced:

\[
\sigma = k_p s + k_i \int_0^t s(t) \dot{s} dt,
\]

(7)

where \( c \) and \( d \) are odd integers with \( 1 < c < 2 \), and \( k_p, k_i > 0 \) are the proportional and integral scalars.

In the following theorem, the finite time convergence of the TSMC surface to the origin is satisfied and the tracking objective of the desired states using this nonsingular surface is guaranteed.

Theorem 1: Consider the perturbed \( n \)-order nonlinear system (1), sliding variable (4) and the nonsingular TSMC surface (7). The control law is proposed as:

\[
u = -\left[k_p g_0(x, t)\right]^{-1} \left\{ k_p \left(f_0(x, t) - \dot{x}_{nd} + c_0 \dot{e} + c_1 \dot{e} + \cdots + c_{n-2} e^{(n-1)}\right) + k_i \mathcal{E}\right\}
\]

(8)

where \( \eta \) is a positive scalar and \( \frac{\delta}{k_p} \geq M(x, t) + N(x, t) |u|_{\text{max}} \), then the state trajectories of the perturbed \( n \)-th order nonlinear system (1) converge to the TSMC surface in finite time and remain on it afterwards.

Proof: From (1), (6) and (7), one can obtain the time-derivative of the TSMC surface as:

\[
\dot{\sigma} = k_p \dot{s} + k_i \dot{s} = k_p \left(f(x, t) + g(x, t)u - \dot{x}_{nd} + c_0 \dot{e} + c_1 \dot{e} + \cdots + c_{n-2} e^{(n-1)}\right) + k_i \mathcal{E}
\]

(9)

Construct the Lyapunov candidate function as \( V_1 = 0.5 \sigma^2 \), where its time-derivative is obtained as

\[
\dot{V}_1 = \sigma \left[k_p (\Delta f(x, t) + \Delta g(x, t)u) - (\delta + \eta) sgn(\sigma) \right]
\]

(10)

Substituting the control law (8) into the above equation, yields:

\[
\dot{V}_1 = \sigma \left[k_p (\Delta f(x, t) + \Delta g(x, t)u) - (\delta + \eta) sgn(\sigma) \right]
\]

(11)

Thus, based on the Lyapunov stability theorem [64], the state responses of the perturbed \( n \)-order nonlinear system converge from their initial states to the TSMC surface (7) in finite time and remain on it thereafter. \( \square \)

On the other side, it is expected that the functions \( |\Delta f(x, t)| \) and \( |\Delta g(x, t)| \) are unknown but bonded, where \( M(x, t) \) and \( N(x, t) \) are their upper bounds, respectively. In practical usage, these upper bounds are unknown and challenging to determine. In what follows, a novel barrier function adaptive TSMC law is proposed, such that the system perturbations are effectively estimated using the barrier adaptation laws and the tracking objective is attained. The adaptive TSMC law is designed as:

\[
u = -\left[k_p g_0(x, t)\right]^{-1} \left\{ k_p \left(f_0(x, t) - \dot{x}_{nd} + c_0 \dot{e} + c_1 \dot{e} + \cdots + c_{n-2} e^{(n-1)}\right) + k_i \mathcal{E}\right\}
\]

(12)

where \( \eta \) is a positive constant and
\[ \hat{\delta} = \begin{cases} \delta_a, & \text{if } 0 < t \leq \ell \\ \delta_{psb}, & \text{if } t > \ell \end{cases} \]  

(13)

where $\ell$ is the convergence time at which the error trajectories reach the neighborhood $\epsilon$ of nonsingular TSMC surface. The adaptation gains $\delta_a$ and $\delta_{psb}$ are obtained by:

\[ \dot{\delta}_a = \varphi |\sigma| \]

(14)

\[ \delta_{psb} = \frac{|\sigma|}{\varepsilon - |\sigma|}, \]

(15)

where $\varphi$ and $\epsilon$ denote two positive scalars. Using the adaptive tuning law (14), the controller gain $\delta_a$ is increased until the error trajectories reach the neighborhood $\epsilon$ of $\sigma$ at the barrier function time $\ell$. For times that are larger than $\ell$, the adaptive controller gain is converted to the positive-semidefinite barrier function $\delta_{psb}$, which reduces the convergence region and retains the tracking errors there. The following subsections detail the stability procedure of the system:

**Condition (1):** $0 < t \leq \ell$

**Theorem 2:** For the perturbed nth-order nonlinear system (1), sliding variable (4) and the TSMC surface (7), by using the adaptive TSMC input (12) with $\dot{\delta} = \delta_a$ and adaptive-tuning law (14), then the tracking error signals reach the neighborhood $\epsilon$ of the nonsingular TSMC surface in the finite time.

**Proof:** Construct the following Lyapunov function:

\[ V_2 = 0.5 \left\{ \sigma^2 + \frac{1}{\zeta} (\delta_a - \delta)^2 \right\}. \]

(16)

where $\zeta$ is a positive constant. Differentiating the above Lyapunov function, yields:

\[ \dot{V}_2 = \sigma \dot{\sigma} + \frac{1}{\zeta} (\delta_a - \delta) \dot{\delta}_a. \]

(17)

Substituting equations (9) and (14) in (17), yields:

\[ \dot{V}_2 = \sigma \left\{ k_p (f_0(x,t) + g_0(x,t)u - \dot{x}_{nd} + c_0 \dot{e} + c_1 e + \cdots + c_{n-2} e^{(n-1)}) 
+ k_p (\Delta f(x,t) + \Delta g(x,t)u) 
+ k_\iota \delta a \right\} \frac{\varphi}{\zeta} (\delta_a - \delta) |\sigma| \]

(18)

Using the adaptive controller (12) in the above equation, one has:

\[ \dot{V}_2 = -\sigma \left\{ (\dot{\delta} + \eta) \operatorname{sgn}(\sigma) \right\} \]

(19)

\[ \leq -\eta |\sigma| - \dot{\delta} |\sigma| + k_p (|\Delta f(x,t)| + |\Delta g(x,t)u_{max}|) |\sigma| + \frac{\varphi}{\zeta} (\delta_a - \delta) |\sigma| \]

where by adding and subtracting $\delta |\sigma|$ in (19), we have:

\[ \dot{V}_2 \leq -\eta |\sigma| - \dot{\delta} |\sigma| + k_p (|\Delta f(x,t)| + |\Delta g(x,t)u_{max}|) |\sigma| + \frac{\varphi}{\zeta} (\delta_a - \delta) |\sigma| \]

(20)

where because $\eta > 0$, $\frac{\delta}{k_p} \geq M(x,t) + N(x,t) |u_{max}$ and $\frac{\varphi}{\zeta} < 1$, we attain

\[ V_2 \leq -\sqrt{2} \left[ \delta - k_p (M(x,t) + N(x,t) u_{max}) \right] \frac{|\sigma|}{\sqrt{2}} - \sqrt{2} \zeta \left( 1 - \frac{\varphi}{\zeta} \right) |\sigma| \frac{|\delta_a - \delta|}{\sqrt{2} \zeta} \]

(21)

\[ \leq - \Psi |\sigma|^2 \leq -\Psi |\sigma|^2 \]

with

\[ \Psi = \min \left\{ \sqrt{2} \left[ \delta - k_p (M(x,t) + N(x,t) u_{max}) \right] \frac{|\sigma|}{\sqrt{2}} \right\} \]

The error states reach the region $|\sigma| \leq \epsilon$ in finite time.

**Condition (2):** $t > \ell$

**Theorem 3:** Consider the perturbed nth-order nonlinear system (1), sliding variable (4) and the nonsingular TSMC manifold (7). The adaptive TSMC control law (12) with $\dot{\delta} = \delta_{psb}$ is updated as

\[ u = -\left[ k_p f_0(x,t) \right]^{-1} \left\{ k_p (f_0(x,t) - \dot{x}_{nd} + c_0 \dot{e} + c_1 e + \cdots + c_{n-2} e^{(n-1)}) + k_\iota \delta a \right\} \frac{\varphi}{\zeta} (\delta_a - \delta) |\sigma| \]

(22)

then the error states reach the region $|\sigma| \leq \epsilon$ in finite time.

**Proof:** Construct the subsequent Lyapunov function:

\[ V_3 = 0.5 (\sigma^2 + \delta_{psb}^2) \]

(23)

where the time-derivative of the above functional is

\[ \dot{V}_3 = \sigma \dot{\sigma} + \delta_{psb} \dot{\delta}_{psb} \]

(24)
Substituting (9) into the above equation, yields:

\[
\dot{V}_3 = \sigma \left( k_p \left( f_0(x,t) + g_0(x,t)u - \dot{x}_{ad} + c_0e + c_1\dot{e} + \cdots + c_{n-2}e^{(n-1)} + \Delta f(x,t) \right) + \Delta g(x,t)u + k_i\ddot{a} \right) + \delta_{psb}\dot{\delta}_{psb} \tag{25}
\]

From (22) and (25), one has:

\[
\dot{V}_3 = \sigma \left[ k_p \Delta f(x,t) + k_p \Delta g(x,t)u \right] - \left( |\sigma|(\epsilon - |\sigma|) + \eta \right) \text{sgn}(\sigma) + \delta_{psb}\dot{\delta}_{psb} \tag{26}
\]

where by calculating the time-derivative of the barrier function, one obtains:

\[
\dot{V}_3 \leq -\eta|\sigma| - (\epsilon - |\sigma|)^{-1}|\sigma|^2 + |\sigma|[k_p|\Delta f(x,t)| + k_p|\Delta g(x,t)u_{\text{max}}] + \delta_{psb}\epsilon(\epsilon - |\sigma|)^{-2} \text{sgn}(\sigma)\dot{\sigma} \leq -\eta(\epsilon - |\sigma|)^{-1}|\sigma|^2 + k_p(M(x,t) + N(x,t)u_{\text{max}})\eta + \delta_{psb}(\epsilon - |\sigma|)^{-2}|\text{sgn}(\sigma)|\text{sgn}(\sigma) \tag{27}
\]

Eq. (27) can be rewritten as

\[
\dot{V}_3 \leq -\left( \delta_{psb} - k_p(M(x,t) + N(x,t)u_{\text{max}}) \right)|\sigma| - \eta\delta_{psb}\epsilon(\epsilon - |\sigma|)^{-2} - \epsilon(\epsilon - |\sigma|)^{-2}\delta_{psb}\dot{\delta}_{psb} - k_p(M(x,t) + N(x,t)u_{\text{max}}) \tag{28}
\]

where because \(\eta\) and \(\delta_{psb}\) are positive scalars, and \(\delta_{psb} > k_p(M(x,t) + N(x,t)u_{\text{max}})\), we have

\[
\dot{V}_3 \leq -\sqrt{2}\left( \delta_{psb} - k_p(M(x,t) + N(x,t)u_{\text{max}}) \right)\frac{|\sigma|}{\sqrt{2}} - \sqrt{2}\epsilon(\epsilon - |\sigma|)^{-2}\delta_{psb}\dot{\delta}_{psb} - \sqrt{2}\epsilon(\epsilon - |\sigma|)^{-2}\delta_{psb}\dot{\delta}_{psb} - \sqrt{2}(\epsilon - |\sigma|)^{-2}\min\{1, \frac{|\sigma|}{\sqrt{2}}, \frac{\delta_{psb}}{\sqrt{2}}\} \tag{29}
\]

\[
\leq -\sqrt{2}(\delta_{psb} - k_p(M(x,t) + N(x,t)u_{\text{max}}))\frac{\min\{1, \epsilon(\epsilon - |\sigma|)^{-2}\}|\sigma|}{\sqrt{2}} - \sqrt{2}\epsilon(\epsilon - |\sigma|)^{-2}\frac{\delta_{psb}}{\sqrt{2}} \leq -\Lambda \frac{|\sigma|}{\sqrt{2}} + \frac{\delta_{psb}}{\sqrt{2}} \leq -\Lambda V_3^{0.5}
\]

with

\[
\Lambda = \sqrt{2}(\delta_{psb} - k_p(M(x,t) + N(x,t)u_{\text{max}}))\min\{1, \epsilon(\epsilon - |\sigma|)^{-2}\}.
\]

**Remark 1:** In order to eliminate the chattering phenomenon resulting from the discontinuous function \(\text{sgn}(\sigma)\), the designed controller \(u\) can be modified by using the continuous hyperbolic tangent function \(\tanh(\sigma)\). Then, the updated control input is written as:

\[
u = -\left[ k_p g_0(x,t) \right]^{-1} \left\{ k_p(f_0(x,t) - \dot{x}_{ad} + c_0e + c_1\dot{e} + \cdots + c_{n-2}e^{(n-1)} + \Delta f(x,t)) + \frac{k_i}{k_p}c \right\}
\]

IV. APPLICATION IN COLOR IMAGE ENCRYPTION

SMC and BLF control methods are used in various engineering applications. For example, for the purpose of shortening the response time and improving the anti-disturbance performance of the permanent magnet synchronous motor (PMSM) drives, a compound control method that uses an improved non-singular fast terminal sliding mode controller (NFTSMC) and disturbance observer compensation techniques was developed in [65]. Also, by embedding the asymmetric BLF using the power integrator technique, tan SOSC control method was developed in [66] to solve the stabilization problem of the pendulum system via a backstepping-like method. Moreover, the synchronization of the chaotic systems using the SMC and BLF control techniques has various applications in implementation of secure communication systems. In scrambling or secure communication, there are usually child keys, and the keys used in next rounds are made from these child keys. One of the methods for secure communications implementation is chaotic scrambling. Chaotic secure communication systems contain a master nonlinear chaotic system in the transmitter and a slave nonlinear chaotic system in the receiver. In fact, implementing a secure chaotic communication system requires the generation of cryptographic keys by two synchronized chaotic systems. In this regard, the tracking problem can be considered as a synchronization goal. Now, consider two 5-dimensions synchronized chaotic systems in the transmitter and receiver as master and slave systems, respectively, and consider a color image \(P\) measuring \(M \times N\) to be used for scrambling. The procedure of the proposed chaotic scrambling is as follows:

**Step 1.** Consider 5 chaotic sequences \(x_i[n], i = 1,2,\ldots,5\) generated by a transmitter chaotic system then,

\[
D_1 = \text{sort}(x_1[n]) \quad D_2 = \text{sort}(x_2[n]) \quad K_1 = \text{mod}(x_1[n], \text{floor}(x_3[n] - 1)) \quad K_2 = \text{mod}(x_4[n], \text{floor}(x_3[n] - 1)) \quad K_3 = \text{mod}(x_5[n], \text{floor}(x_3[n] - 1)) \tag{30}
\]

where the function \(\text{sort}(x_i[n])\) refers to the generation of an index vector \(D_i\), in accordance with the ascending order of the values in the chaotic sequence \(x_i[n]\). \text{mod}(f,g)\) yields the residue of \(f\) divided by \(g\), whereas \(\text{floor}(\omega)\) enables
rounding the elements of \( \omega \) to the closest integers. Note that the values of the decimal portions after the comma (fraction portion) exist in the chaotic float values \( K_1, K_2 \) and \( K_3 \) obtained from Eq. (30). The mentioned values are converted to 64 bits binary digits and 32 LSBs with the low-valued and high-precision \((Kb_1, Kb_2, Kb_3)\). This procedure is performed to create one million bits for each phase. The new random bit sequences are then generated by:

\[
\begin{align*}
rb_1 &= \text{bitxor}(Kb_1, Kb_2) \\
rb_2 &= \text{bitxor}(Kb_1, Kb_3) \\
rb_3 &= \text{bitxor}(Kb_2, Kb_3) \\
rb_4 &= \text{bitxor}(Kb_1, Kb_2, Kb_3)
\end{align*}
\]

Finally, the random bits \((Rb)\) are found as:

\[
Rb = [rb_1, rb_2, rb_3, rb_4]
\]  

**Step 2.** The two permutation vectors \(D_1\) and \(D_2\), are used to permute the row and column indexes of the original image matrix, respectively. Assuming the original image matrix \(P, M \times N\) is

\[
P^{(0)}_{M \times N} = [r^T_1, r^T_2, \ldots, r^T_M]
\]

where \(r_1, r_2, \ldots, r_M\) are the row vectors. A row permutation using vector \(D_1\) yields:

\[
P^{(1)}_{M \times N} = [r^T_{D_1(1)}, r^T_{D_1(2)}, \ldots, r^T_{D_1(M)}] = [C_1, C_2, \ldots, C_M]
\]

Similarly, the columns are permuted via vector \(D_2\), and the corresponding chaotic image matrix after both row and column permutations becomes

\[
P^{(2)}_{M \times N} = [C_{D_2(1)}, C_{D_2(2)}, \ldots, C_{D_2(M)}]
\]

\[
= [w^T_1, w^T_2, \ldots, w^T_M]
\]

**Step 3.** Convert the scrambled image \(P^{(2)}\) to the vector \(E^{(2)}\) with length \(M \times N\). For more security, the scrambled image vector \(E^{(2)}\) is encrypted with the random bit sequences \((Rb)\) obtained from Eq. (32) by XOR operation as follows:

\[
E = P^{(2)} \oplus Rb
\]

Finally, the encrypted image can be obtained by reshaping the encrypted vector \(E\) to the matrix \(E\) of size \(M \times N\). This encryption procedure is applied to the components of the color image \((R, G, B)\) and the resultant encrypted image is sent to receiver through a public noisy wireless channel using a TX/RX module.

**Step 4.** Once the synchronization procedure defined in section 2 is complete and the chaotic signals at the receiver and transmitter ends are synchronized, then it is possible to recover the original image by reversing the encryption process. The block diagram of the proposed color image encryption system is illustrated in Fig. 1.

**Remark 2.** In the proposed method, a novel barrier-function-based terminal sliding mode controller is suggested for the finite-time robust tracking and stabilization of nth-order nonlinear systems with unmodeled dynamics, parametric variations and external disturbances. This method, not only achieves the desired tracking performance, but also offers a nonsingular chattering-free response. The proposed chaos synchronization and encryption technique is used for chaotic color image scrambling; however, it can be employed for any digital data encryption and scrambling in wireless/wired communication data networks. Because our proposed control approach is dimension-free, then this...
method can be used for synchronization of higher-order chaotic systems, such as hyper-chaotic systems. This approach is very important when the synchronized chaotic systems are used in the implementation of secure communications. Since the suggested method is applied for data encryption purpose, the higher-order chaotic systems create more complex encryption keys. Moreover, the transmitter can change the dimension of the chaotic system at specified intervals (of which the authorized receiver is aware) to barricade the revelation of information by an eavesdropper.

V. SIMULATION RESULTS

In the following section, we assess the applicability and performance of the proposed nonsingular barrier function-based TSMC approach using the following benchmark problems:

Synchronization of nonlinear chaotic systems

In recent decades, control, synchronization, circuit realization and applications of hyper-jerk chaotic systems have been studied well in the literature, because of their simple structure and complex qualitative properties [67-69]. For example, a mathematical model with an electronic circuit for nth-order hyper-jerk system has been proposed in [62]. In this study, various dynamic behavior and typical time series chaotic responses of the proposed design are validated by incorporating bifurcation sequence, numerical simulation, and practical implementation via op-amp devices. Also, an application perspective of the proposed hyper-jerk system is extended to a random pulse generator (RPG).

Consider the following chaotic hyper-jerk system for \( n = 5 \), with nonlinearity \( \arctan(x) \) as the reference trajectories in the transmitter system [70]:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= x_5(t) \\
\dot{x}_5(t) &= -7.9x_1 + 2.06 \tan^{-1}(200x_1) - 9.19x_2 - 4x_3 - 7.278x_4 - x_5
\end{align*}
\]  

(37)

Moreover, the perturbed nth-order nonlinear system (1) with uncertainties, external disturbances and control inputs can be consider as the receiver chaotic system as follows

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= x_5(t) \\
\dot{x}_5(t) &= f_0(x, t) + \Delta f(x, t) + \Delta g(x, t) u(t)
\end{align*}
\]  

(38)

where

\[
\begin{align*}
f_0(x, t) &= -7.9x_1 + 2.06 \tan^{-1}(200x_1) - 9.19x_2 - 4x_3 - 7.278x_4 - x_5
\end{align*}
\]

\[
\begin{align*}
\Delta f(x, t) &= 1.5t + 0.5 \cos(x_1(t)) - 0.3 \cos(x_2(t)) + 1.2 \sin(x_3(t)) - 0.7 \sin(x_4(t)) + 0.25 \cos(x_5(t)) \\
\Delta g(x, t) &= 2t + 0.5 \sin(t) + 0.6 \cos(0.5t) + 0.25 \sin(2\sqrt{t})
\end{align*}
\]  

(39)

In this simulation, the initial states of the transmitter and receiver chaotic systems are selected as \( x_{1d}(0) = -0.05, x_{2d}(0) = 0.2, x_{3d}(0) = 0, x_{4d}(0) = -1.8, x_{5d}(0) = -1.2 \) and \( x_1(0) = 0.55, x_2(0) = -0.1, x_3(0) = -0.25, x_4(0) = 0.05, x_5(0) = 0 \), respectively. The control parameters are determined by trial and error as \( k_p = 19, k_i = 1750, C_0 = 2.32, C_1 = 0.35, C_2 = 9.52, C_3 = 14.21, \ c = 32, d = 28 \) and \( \eta = 0.002 \). The dynamics of the states of the chaotic system are depicted in Fig.2 through Fig.6. It is understood that the states trajectories of the nonlinear chaotic hyper-jerk system (37) are synchronized with the perturbed 5-order nonlinear chaotic system (38) in about 6 seconds. Fig.7 depicts the time histories of the error signals. Note that the error dynamics converge to the equilibrium in about 6 seconds. Therefore, it is determined that the proposed procedure is robust to parametric uncertainties and exhibits an appropriate synchronization performance. The dynamics of the proposed switching surface \( s(t) \), the controller signal \( u(t) \) and nonsingular terminal sliding surface \( \sigma(t) \) are displayed in Fig.8. Note the satisfactory magnitudes of the sliding surface as well as the nonsingular terminal sliding surface.

![Fig.2. Dynamics of the system states x1, x1d.](image)

![Fig.3. Dynamics of the system states x2, x2d.](image)
Simulation results for the color image encryption system

The practicality and application of the proposed approach is assessed in this section using a color image scrambling/encryption system. To this end, we chose a standard color plain image of an adult woman (hereafter, "Lena") measuring 512×512×3 uint8, in JPG format (see Fig. 9-a). The scrambling/encryption keys are created by the transmitter chaotic system. The original image is encrypted using the chaotic keys, scrambling and encryption techniques detailed in subsection 4. Fig. 9-b depicts the encrypted images generated by the proposed process. At the receiver end, the decryption keys are created using the chaotic system. The decrypted image is obtained following the synchronization method and decryption demonstrated in Fig. 9-c. Note that the encrypted images contain uniform distributions, whereas the encrypted images are very similar to noise. It shows that the proposed method exhibits good encryption performance in terms of visual impression.

Remark 3. As shown in Figure 1, two five-order hyper-jerk chaotic systems are employed to implement the cryptographic system. The physical realization of an n-order hyper-jerk chaotic systems was proposed in [62] by using n number of op-amps, some passive components (n capacitors, 2(n−1) resistors) and a nonlinear circuit element. Moreover, in various practical nonlinear systems, the unmodeled dynamics are caused by several factors, such as measuring errors, modeling errors and uncertain perturbations. The parametric variations can be caused by the tolerance of the electrical and electronic components.
and the external disturbances can be created using communication channel effects.

VI. PERFORMANCE ANALYSIS OF THE SUGGESTED CRYPTOSYSTEM

The robustness and security of the suggested chaotic cryptosystem is analyzed in this part. To this end, the correlation test, histogram analysis, pixels change rate number, information entropy, and unified average change intensity are performed.

A. Histogram analysis

A histogram uses a bar graph to display a profile of the intensity-level values in a given image. The levels of intensity are represented in the horizontal axis and typically start at zero and covers the various intensity levels. The image’s intensity levels are represented using vertical bars. The following properties should be fulfilled by image encryption algorithms:

1. The encrypted image’s histogram should be entirely different from the original image’s histogram.

2. The encrypted image’s histogram should have uniform distribution; thus, indicating that probability of existence of any intensity value is the same, and totally random.

The histograms of components R, G, B of the original and encrypted images are depicted in Fig.10. Note that the encrypted image’s histograms are considerably different, more uniform and bare no statistical similarities to the original image. This confirms the success of encrypted images art hiding the information of the original image.

![Histograms of Original and Encrypted Images](image-url)
B. Correlation test

A useful measure to evaluate the encryption quality of image cryptosystem is the correlation between two adjacent pixels in plain and cipher images. The correlation coefficient can be determined using the subsequent formula [72]:

$$\text{Corr}(\alpha, \beta) = \frac{\text{cov}(\alpha, \beta)}{\sqrt{D(\alpha)D(\beta)}}$$  \hspace{1cm} (40)

$$\text{cov}(\alpha, \beta) = \frac{1}{N} \sum_{i=1}^{N} (\alpha_i - E(\alpha))(\beta_i - E(\beta))$$  \hspace{1cm} (41)

where $\alpha$ and $\beta$ represent the adjacent samples values in the decrypted image and $E(\alpha) = \frac{1}{N} \sum_{i=1}^{N} (\alpha_i)$, $D(\alpha) = \frac{1}{N} \sum_{i=1}^{N} (\alpha_i - E(\alpha))^2$, where $N$ refers to the number of samples considered in calculating the correlation. The horizontal correlation of the adjacent samples in the original and encrypted images are depicted in Fig. 11. Note that the correlation factors of encrypted images are very small, thus implying that no detectable correlation exists between the original image and its encrypted counterpart. This confirms that the proposed chaotic encryption method has great security with respect to statistical attack.

C. Image encryption quality assessment via IE, UACI, and NPCR

The following metrics are considered to assess the image encryption’s quality: Information Entropy (IE), Number of Pixel Change Rate (NPCR), Unified Average Change Intensity (UACI).

The information entropy for a given image provides a measure of the complexity of encrypted data, and is found as [73]

$$IE(\Phi) = \sum_{i=1}^{255} \Phi(\Phi_i) \log \left( \frac{1}{\Phi(\Phi_i)} \right)$$  \hspace{1cm} (42)

where $\Phi(\Phi_i)$ represents the probability of the variable $\Phi_i$ and entropy is calculated in the form of bits. The value of information entropy for the truly random source is equal to eight [74]. The closer the information entropy to eight, the better the encryption quality. Note that the value of IE generated by the proposed encryption method is 7.9925, which is very close to 8, thus confirming the quality of the encryption.

In what follows, we will examine whether the proposed encryption algorithm can resist differential attacks. The following evaluation factors are employed to measure strength of the encryption procedure: NPCR and UACI. That is, the variations in the encryption, when difference between the original images is small, are measured using UACI and NPCR. Consider $E\text{I}_1$ and $E\text{I}_2$ as the encrypted images after/before changes in one pixel of original image at the position $m, n$ and $B(m, n)$ as a bipolar array defined by:

$$B(m, n) = \begin{cases} 1 & \text{if } E\text{I}_1(m, n) \neq E\text{I}_2(m, n) \\ 0 & \text{if } E\text{I}_1(m, n) = E\text{I}_2(m, n) \end{cases}$$  \hspace{1cm} (43)

The NPCR and UACI can be calculated from the following formulas [75]:

![Fig.11. (a), (b), (c) Horizontal correlation of the components R, G, B of original image; (d), (e), (f) Horizontal correlation of components R, G, B of encrypted image.](image-url)
where $S$ represents the number pixels in the original image; $F$ represents the largest allowable value in the encrypted image. The optimal values of NPCR and UACI are 99.61% and 33.46%, correspondingly [75]. The obtained values for the proposed encryption method are $\text{NPCR}=99.5921$ and $\text{UACI}=33.4102$, which are very close to optimal values. According to the above outcomes, one concludes that the proposed cryptosystem is able to perfectly conceal the image information.

D. Classical attacks

Common cryptography attacks range from plaintext-only attack, selected plaintext attack, selected ciphertext attack, to ciphertext-only, with the selected plaintext as the most powerful [76]. An encryption algorithm that resist against such attack is considered resistant to other attacks. The security of cryptosystems is assessed based on the Kerckhoff’s principle. This latter case states that the security of system should only depend on confidentiality of cryptographic keys. Note the sensitivity of the proposed algorithm to parameters of system and initial conditions of chaotic system. A change in any of these variables results in the generation of a new set of chaotic keys. This implies that each ciphered image will result in the unique former plain and former ciphered values. This confirms the ability of the planned algorithm to resist plaintext/ciphertext attacks.

VII. CONCLUSIONS

This paper proposed a nonsingular barrier-function-based TSMC approach with to solve for the tracking problem of nth-order nonlinear dynamic systems with unmodeled dynamics, parameter variations and external disturbances. The proposed technique was derived using a novel barrier-function-based terminal sliding surface to ensure a nonsingular chattering-free performance. Implementation of the proposed approach to a chaotic hyper-jerk system showed that the error signals converge to the origin in finite time, thus confirming the ability of the proposed method to mitigate the unmodeled dynamics, parametric variations and external disturbances which display the suitable synchronization/ tracking performance of the suggested control approach. Additionally, the amplitude of the proposed control signal was found to be proper and the control input did not exhibit any chattering. Its implementation to the design of a color image encryption system further confirmed its robustness and security. Overall, the proposed approach guarantees the finite-time convergence of the sliding surface, ensures chattering-free dynamics and circumvents the singularity problem. Finally, it is worth noting that the proposed chaos synchronizer and encryption technique can be used for any digital data encryption and scrambling in wireless/wired communication networks.

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