On the geometrical interpretation of locality in anomaly cancellation

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Abstract
A notion of local section of the determinant line bundle is defined giving necessary and sufficient conditions for anomaly cancellation compatible with locality. This definition gives an intrinsic geometrical interpretation of the local counterterms allowed in the renormalization program of quantum field theory. For global anomalies the conditions for anomaly cancellation are expressed in terms of the equivariant holonomy of the Bismut-Freed connection.

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1 Introduction
Anomalies in quantum field theory admit a geometrical interpretation in terms of determinant (or Pfaffian) line bundles (e.g. see [1], [2], [15]). In order to have a well defined quantum field theory the determinant line bundle should be trivial. However, it is well known that this condition is not sufficient for anomaly cancellation due to the locality problem. Hence, to cancel the anomaly the determinant line bundle should admit a special kind of section (a local section) corresponding to the local counterterms allowed in the renormalization program of quantum field theory. In this paper we study the geometrical interpretation of these local sections in terms of the Bismut-Freed connection and we obtain necessary and sufficient conditions for perturbative and global anomaly cancellation.

Let us explain in more detail the locality problem. We consider the action of a group $\mathcal{G}$ on a bundle $E \rightarrow M$ over a compact $n$-manifold $M$. Let $\{D_s : s \in \Gamma(E)\}$ be a $\mathcal{G}$-equivariant family of elliptic operators acting on fermionic fields $\psi \in \Gamma(V)$ and parametrized by $\Gamma(E)$. Then the Lagrangian density $\lambda_D(\psi, s) =$
\(\bar{\psi} i D_s \psi\) is \(\mathcal{G}\)-invariant, and hence the classical action \(A_L(\psi, s) = \int_M \bar{\psi} i D_s \psi\), is a \(\mathcal{G}\)-invariant function on \(\Gamma(V) \times \Gamma(E)\). However, at the quantum level, the corresponding partition function defined by a formal fermionic path integral by \(Z(s) = \int \mathcal{D}\psi \exp(-\int_M \bar{\psi} i D_s \psi)\) could fail to be \(\mathcal{G}\)-invariant. \(Z(s)\) can be defined in terms of regularized determinants of elliptic operators but not in a unique way. There is an ambiguity in the definition of \(Z(s)\) modulo the addition of local counterterms (e.g. see [17]). Due to this ambiguity \(Z\) corresponding partition function defined by a formal fermionic path integral by \(\tilde{Z}\) unique way. There is an ambiguity in the definition of \(\tilde{Z}\) could fail to be \(\mathcal{G}\)-invariant. It can be seen that the modulus of \(Z(s)\) is \(\mathcal{G}\)-invariant. Hence we have \(Z(\phi \cdot s) = \tilde{Z}(s) \cdot \exp(2\pi i \cdot \alpha_\phi(s))\) where \(\alpha : \mathcal{G} \times \Gamma(E) \to \mathbb{R}/\mathbb{Z}\) satisfies the cocycle condition \(\alpha_{\phi_2 \phi_1}(s) = \alpha_{\phi_1}(s) + \alpha_{\phi_2}(\phi_1 s)\). Different definitions of \(Z(s)\) determine different cocycles, but they are cohomologous, in the sense that they satisfy the condition \(\alpha'_\phi = \alpha_\phi + \phi^* \theta - \theta\) for some \(\theta \in \Omega^0(\Gamma(E))\). If \(\alpha\) is an exact cocycle, i.e., if there exists \(\lambda \in \Omega^0(\Gamma(E))\) satisfying \(\alpha_\phi(s) = \lambda(\phi \cdot s) - \lambda(s)\) (*) we can define \(\tilde{Z}' = \tilde{Z} \cdot \exp(-2\pi i \lambda)\) and we have \(Z'(\phi \cdot s) = \tilde{Z}'(s)\). Hence the anomaly can be represented by a cohomology class in \(H^1(\mathcal{G}, \Omega^0(\Gamma(E)), \mathbb{R}/\mathbb{Z})\) \(\approx H^1(\mathcal{G}, \Omega^0(\Gamma(E)))/\mathbb{Z}\) (e.g. see [5], [7], [8]). For perturbative anomalies the group cohomology can be replaced by Lie algebra cohomology. For \(\phi \in \text{Lie} \mathcal{G}\) we define \(a(X) = \frac{\delta a_x}{\delta t}\bigg|_{t=0}\) with \(\phi_t = \exp(tX)\). If \(\mathcal{G}\) is connected condition (*) is equivalent to \(a(X) = L_X \lambda\). Hence the condition for perturbative anomaly cancellation is equivalent to \([a] = 0\) on \(H^1(\text{Lie} \mathcal{G}, \Omega^0(\Gamma(E)))\) (\(a\) is closed by the Wess-Zumino consistency condition).

However, from the physical point of view that is not the end of the story. Physics require that \(Z'\) should be the fermionic path integral of a Lagrangian density, and hence \(\Lambda(s)\) should be a local functional, i.e., it should be of the form \(\Lambda(s) = \int_M \lambda(s)\), where \(\lambda(s)(x)\) is a function of \(s(x)\) and the derivatives of \(s\) at \(x\). If that is the case, we can modify the Lagrangian density to the effective Lagrangian \(\mathcal{L}'(s) = \bar{\psi} i D_s \psi - \lambda(s)\) and the partition function of \(\mathcal{L}'\) is \(Z'\). We say that the topological anomaly cancels if condition (*) is satisfied for a functional \(\lambda \in \Omega^0(\Gamma(E))\), and that the physical anomaly cancels if condition (*) is satisfied for a local functional \(\lambda \in \Omega^0_{\text{loc}}(\Gamma(E))\). Obviously the second condition implies the first, but the converse is not true. Furthermore, if condition (*) is satisfied only for the connected component of the identity \(\mathcal{G}\) on \(\mathcal{G}\) we say that the perturbative (or local) anomaly cancels. If it is satisfied for all the elements of \(\mathcal{G}\) we say that the global anomaly cancels. Hence the perturbative physical anomaly is represented by a cohomology class in the local BRST cohomology \(H^1(\text{Lie} \mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E)))/\mathbb{Z}\), and the global physical anomaly by a class in \(H^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E)))/\mathbb{Z}\).

The condition (*) admits the following geometrical interpretation. The cocycle \(\alpha\) determines an action on the trivial bundle \(\mathcal{L} = \Gamma(E) \times \mathbb{C} \to \Gamma(E)\) by setting \(\phi_\mu(s, u) = (\phi(s), u \cdot \exp(2\pi i \alpha_\phi(s)))\) for \(s \in \Gamma(E)\) and \(u \in \mathbb{C}\). If the action of \(\mathcal{G}\) on \(\Gamma(E)\) is free we can consider the quotient bundle \(\mathcal{L} = (\Gamma(E) \times \mathbb{C})/\mathcal{G} \to \Gamma(E)/\mathcal{G}\), and \(Z\) determines a section of \(\mathcal{L}\). Furthermore, if \(\Lambda\) satisfies equation (*), then \(\exp(2\pi i \Lambda)\) determines a section of unitary norm of \(\mathcal{L}\), i.e., a section of the prin-
principal $U(1)$-bundle $\mathcal{U} = (\Gamma(E) \times U(1))/\mathcal{G} \to \Gamma(E)/\mathcal{G}$. Hence topological anomaly cancellation is equivalent to the existence of a section of $\mathcal{U} \to \Gamma(E)/\mathcal{G}$, and hence to the triviality of $\mathcal{U}$.

In [2] the bundle $\mathcal{U}$ is identified with the determinant line bundle of the family of operators. If $\mathcal{G}$ is connected (i.e. for perturbative anomalies) a necessary and sufficient condition for topological anomaly cancellation is that $c_1(\mathcal{U}) = 0$ on $H^2(\Gamma(E)/\mathcal{G})$. The advantage of this approach to anomalies is that the Atiyah-Singer Index Theorem gives an explicit expression for $c_1(\mathcal{U})$ in terms of characteristic classes. Furthermore, it gives the curvature $\text{curv}(\Xi)$ of the Bismut-Freed connection $\Xi$ on $\mathcal{U}$ (e.g. see [4]). Note the difference with the approach based on group and Lie algebra cohomology, where $\alpha$ and $\mathfrak{a}$ are defined only modulo exact terms and given by complicated expressions on secondary characteristic classes. This approach also gives a geometrical interpretation of the anomaly as a cohomology class in $\Gamma(E)/\mathcal{G}$, and allows the use of topological tools in the study of anomaly cancellation. However, due to the locality problem, the cancellation of topological anomalies only gives necessary conditions for physical anomaly cancellation, but they are not sufficient, i.e., anomalies in field theory can exist even if the corresponding topological anomaly is trivial. In order to take into account locality, it is proposed in [19] (see also [1]) the problem of defining a notion of “local cohomology” giving necessary and sufficient condition for physical anomaly cancellation. This problem was solved in [11] for perturbative anomalies by the introduction of local equivariant cohomology.

In place of working with the cohomology of the quotient $H^2(\Gamma(E)/\mathcal{G})$ we can also consider the $\mathcal{G}$-equivariant cohomology $H^2_{\mathcal{G}}(\Gamma(E))$. For free actions we have $H^2(\Gamma(E)/\mathcal{G}) \simeq H^2_{\mathcal{G}}(\Gamma(E))$, and we can consider $\mathcal{L} = \mathcal{L}(E) \times \mathbb{C} \to \Gamma(E)$ as a $\mathcal{G}$-equivariant line bundle and $\mathcal{U} = \Gamma(E) \times U(1) \to \Gamma(E)$ as a $\mathcal{G}$-equivariant $U(1)$-bundle. Furthermore, the $\mathcal{G}$-equivariant curvature $\text{curv}_\mathcal{G}(\Xi)$ of the Bismut-Freed connection $\Xi$ on $\mathcal{L}$ is given by the equivariant Atiyah-Singer Index Theorem (see [16]). One of the advantages of equivariant cohomology is that it is also well defined for non-free actions. But the most important advantage of $H^2_{\mathcal{G}}(\Gamma(E))$ with respect to $H^2(\Gamma(E)/\mathcal{G})$ is that $\text{curv}_\mathcal{G}(\Xi)$ is a local form, whereas $\text{curv}(\Xi)$ is non-local. In [11] the notions of local forms $\Omega^\bullet_{\text{loc}}(\Gamma(E))$ and local equivariant forms $\Omega^\bullet_{\text{loc,}\mathcal{G}}(\Gamma(E))$ are defined in terms of the jet bundle of $E$. For Gauge and gravitational anomalies we have $\text{curv}_\mathcal{G}(\Xi) \in \Omega^2_{\text{loc,}\mathcal{G}}(\Gamma(E))$. Furthermore, the cancellation of the class of $\text{curv}_\mathcal{G}(\Xi)$ on $H^2_{\text{loc,}\mathcal{G}}(\Gamma(E))$ is equivalent to the cancellation of the perturbative physical anomaly. This approach provides new techniques for the study of anomaly cancellation as the local cohomology $H^2_{\mathcal{G},\text{loc}}(\Gamma(E))$ is very different to the cohomology $H^2(\Gamma(E)/\mathcal{G})$ of the quotient space. It is shown in [10] and [11] that $H^2_{\mathcal{G},\text{loc}}(\Gamma(E))$ is related to the equivariant cohomology of jet bundles and Gelfand-Fuks cohomology of formal vector fields.

The objective of this paper is to give a geometrical interpretation of the preceding results and to generalize the results of [11] to global anomalies. Our starting point for the study of anomaly cancellation is the unitary determinant bundle $\mathcal{U} \to \Gamma(E)$ corresponding to a $\mathcal{G}$-equivariant family of elliptic operators [4]. We consider $\mathcal{U} \to \Gamma(E)$ as a $\mathcal{G}$-equivariant $U(1)$-bundle and the Bismut-
Freed connection $\Xi$ is $G$-invariant. We assume that $\mathcal{U} \to \Gamma(E)$ is a topologically trivial bundle and hence admits global sections. To any section $S$ of $\mathcal{U}$ we associate a group cocycle $\alpha^S$ and a Lie algebra cocycle $a^S$. In this way the different expressions of the cocyle $\alpha$ and the integrated anomaly $a$ obtained from perturbation theory correspond to different sections of $\mathcal{U} \to \Gamma(E)$. Furthermore, $S$ determines a trivialization of $\mathcal{U} \to \Gamma(E)$, and in this trivialization any other section is determined by a function of the form $\exp(2\pi i \Lambda)$. The condition (1) for topological anomaly cancellation is equivalent to the existence of a $G$-equivariant section of the unitary determinant bundle. We obtain necessary and sufficient conditions for the existence of a $G$-equivariant section in terms of the Bismut-Freed connection $\Xi$. Perturbative anomalies are related to the $G$-equivariant curvature of the connection. To study global anomalies we introduce the concept of $G$-equivariant holonomy of a connection $\Xi$. We obtain necessary and sufficient conditions for global anomaly cancellation in terms of the $G$-equivariant holonomy.

To deal with the locality problem, we introduce the notion of local section of the unitary determinant bundle. We say that a section $S: \Gamma(E) \to \mathcal{U}$ is $\Xi$-local if $\rho^S = \frac{1}{2\pi} S^* \Xi \in \Omega^1_{\text{loc}}(\Gamma(E))$. We prove that local sections exist and that for local sections the cocycles $\alpha^S$ and $a^S$ are local. Furthermore, in the trivialization determined by $S$ any other $\Xi$-local section is given by a function of the form $\exp(2\pi i \Lambda)$ for a local functional $\Lambda \in \Omega^0_{\text{loc}}(\Gamma(E))$. The condition (1) is shown to be satisfied for a local functional if and only if there exists a $\Xi$-local $G$-equivariant section of $\mathcal{U} \to \Gamma(E)$. In this way an intrinsic characterization of the cancellation of the physical anomaly is obtained. Finally we obtain necessary and sufficient conditions for physical anomaly cancellation in terms of the $G$-equivariant curvature and holonomy of the Bismut-Freed connection $\Xi$.

In [13] we combine our geometric characterization of global anomaly cancellation with the results of [10] and [14] to analyze gravitational anomaly cancellation (see Section 5 for more details).

## 2 Topological anomalies

In this Section we study conditions for the existence of $G$-equivariant sections of a $G$-equivariant $U(1)$-bundle $\mathcal{U} \to N$. As commented in the Introduction, when they are applied to the determinant line bundle, they give necessary and sufficient conditions for topological anomaly cancellation.

### 2.1 Topological anomalies and group cohomology

We say that a $U(1)$-bundle $\mathcal{U} \to N$ is topologically trivial if there exists an isomorphism of principal $U(1)$-bundles $\mathcal{U} \cong N \times U(1)$. We recall it is equivalent to give a section or a trivialization of $\mathcal{U}$. If $S: N \to \mathcal{U}$ is a section, then $\Psi_S: N \times U(1) \to \mathcal{U}$, $\Psi_S(x, u) = S(x) \cdot u$ is a trivialization of $\mathcal{U}$ that we call the trivialization associated to $S$. Conversely, if $\Psi: N \times U(1) \to \mathcal{U}$ is a trivialization
of \( U \), then \( \Psi \circ S_1 \circ \Psi_N^{-1} \) is a section of \( U \), where \( S_1 \) is the section \( S_1(x) = (x, 1) \) of the trivial bundle \( N \times U(1) \rightarrow N \) and \( \Psi_N : N \rightarrow N \) is the projection of \( \Psi \).

Let \( G \) be a group acting (on the left on \( N \)). A \( G \)-equivariant \( U(1) \)-bundle is a \( U(1) \)-bundle in which \( G \) acts by \((U(1))\)-automorphisms. We say that \( U \rightarrow N \) is a trivial \( G \)-equivariant \( U(1) \)-bundle if there exists a \( G \)-equivariant isomorphism \( U \cong N \times U(1) \) (where \( G \) acts trivially in \( U(1) \)). This condition is equivalent to the existence of a \( G \)-equivariant section \( S : N \rightarrow U \), i.e. such that \( \phi_U \circ S = S \circ \phi_N \) for any \( \phi \in G \).

A necessary condition for a bundle to be a trivial \( G \)-equivariant \( U(1) \)-bundle is that it should be topologically trivial. We want to study conditions for a topologically trivial bundle to be a trivial \( G \)-equivariant bundle. Hence from now on we made the following assumption

\[ (a1) \quad U \rightarrow N \text{ is a } G \text{-equivariant } U(1) \text{-bundle that is topologically trivial, } N \text{ is connected and } H^1(N) = 0. \]

If \( S : N \rightarrow U \) is a section, then \( \phi^{-1}_U \circ S \circ \phi_N \) is also a section of \( U \) and we have \( \phi^{-1}_U \circ S \circ \phi_N = S \cdot \exp(-2\pi i \alpha_S^S) \) for a function \( \alpha_S^S : N \rightarrow \mathbb{R}/\mathbb{Z} \). We note that \( S \) is a \( G \)-equivariant section if and only if \( \alpha_S^S = 0 \) for any \( \phi \in G \). The function \( \alpha_S^S : G \times N \rightarrow \mathbb{R}/\mathbb{Z} \) satisfies the following properties

**Lemma 1**  
(a) (cocycle condition) We have \( \alpha_{S,'}^S(x) = \alpha_S^S(x) + \alpha_S^S(\phi x) \) for any \( \phi, \phi' \in G \).

(b) If \( S'(x) = S(x) \cdot \exp(2\pi i \Lambda(x)) \) is another section of \( U \rightarrow N \) we have \( \alpha_{S'}^{S'} = \alpha_S^S + \Lambda - \phi_N^* \Lambda \).

**Proof.** a) We have \( \phi^{-1}_U \circ S \circ (\phi')_N = \phi^{-1}_U \circ \left( (\phi')^{-1}_U \circ S \circ (\phi')_N \right) \circ \phi_N \)

\[ = \phi^{-1}_U \circ \left[ S \cdot \exp(2\pi i \alpha_S^S) \right] \circ \phi_N = \left[ \phi^{-1}_U \circ S \circ \phi_N \right] \cdot \exp(2\pi i \alpha_S^S \circ \phi_N) \]

\[ = S \cdot \exp(2\pi i \alpha_S^S) \exp(2\pi i \alpha_S^S \circ \phi_N) = S \cdot \exp(2\pi i (\alpha_S^S + \alpha_S^S \circ \phi_N)) \]

b) We have \( \phi^{-1}_U \circ S' \circ \phi_N = \phi^{-1}_U \circ \left[ S \cdot \exp(2\pi i \Lambda) \right] \circ \phi_N = \phi^{-1}_U \circ S \circ \phi_N \cdot \exp(2\pi i (\Lambda \circ \phi_N)) \]

\[ = S \cdot \exp(-2\pi i \alpha_S) \cdot \exp(2\pi i \phi_N^* \Lambda) = S \cdot \exp(-2\pi i (\alpha_S - \phi_N^* \Lambda)) \]

\[ = S' \cdot \exp(-2\pi i (\alpha_S - \phi_N^* \Lambda + \Lambda)). \]

**Remark** 2 In the particular case in which \( \alpha_S \) is constant for any \( \phi \in G \) the cocycle condition is equivalent to \( \alpha_{\phi'} = \alpha_S + \alpha_{\phi'} \), i.e. \( \alpha : G \rightarrow \mathbb{R}/\mathbb{Z} \) is a group homomorphism.

In the trivialization determined by the section \( S \) the action of \( \phi \in G \) on \( N \times U(1) \) is given by \( \phi_U(x, u) = (\phi_N(x), u \cdot \exp(2\pi i \alpha_S^S(x))) \). Conversely, we have the following result

**Proposition 3** If \( \alpha : G \times N \rightarrow \mathbb{R}/\mathbb{Z} \) satisfies the cocycle condition, then \( \phi \cdot (x, u) = (\phi_N(x), \exp(2\pi i \alpha_S^S(x)) \cdot u) \) defines a group action of \( G \) on \( U = N \times U(1) \) and \( U \rightarrow N \) is a \( G \)-equivariant \( U(1) \)-bundle. For the section \( S(x) = (x, 1) \) we have \( \alpha^S = \alpha \).
Proof. It is a group action as we have
\[ \phi' \cdot (\phi \cdot (x, u)) = \phi' \cdot ((\phi_N(x), \exp(2\pi i\alpha_N(x))) \cdot u) \]
\[ = (\phi'_{\phi_N}(x), \exp(2\pi i(\alpha_{\phi}(x) + \alpha'_{\phi_N}(\phi_N(x)))) \cdot u) \]
\[ = ((\phi' \cdot \phi)_N(x), \exp(2\pi i\alpha'_{\phi \cdot \phi}(x)) \cdot u) = (\phi' \cdot \phi) \cdot (x, u). \]

If \( \alpha: N \to \mathbb{R}/\mathbb{Z} \) is a function, we define its differential \( \delta \alpha \in \Omega^1(N) \) by
\[ \delta \alpha = -\frac{\partial}{\partial t} \exp(-2\pi i\alpha) d(\exp(2\pi i\alpha)). \]
For \( \alpha: \mathbb{R} \to \mathbb{R}/\mathbb{Z} \) we define \( \delta \alpha \in \Omega^1(\mathbb{R}) \) by
\[ \delta \alpha = -\frac{\partial}{\partial t} \exp(-2\pi i\alpha(t)) d(\exp(2\pi i\alpha(t))). \]
If \( \alpha = \text{A mod } \mathbb{Z} \) for a real function \( A \in \Omega^0(N) \) then \( \delta \alpha = dA \).

Lemma 4 If \( N \) is connected and \( H^1(N) = 0 \), then for any \( \alpha: N \to \mathbb{R}/\mathbb{Z} \) there exists \( A \in \Omega^0(N) \) such that \( \alpha = A \text{ mod } \mathbb{Z} \). Any other function satisfying this condition is of the form \( A + n \) with \( n \in \mathbb{Z} \). Hence we have \( \Omega^0(N, \mathbb{R}/\mathbb{Z}) \simeq \Omega^0(N)/\mathbb{Z} \).

Proof. As \( d(\delta \alpha) = 0 \) and \( H^1(N) = 0 \), there exist \( A' \in \Omega^0(N) \) such that \( dA' = \delta \alpha \). The function \( \exp(2\pi i\alpha) \exp(-2\pi iA') \) is constant and has modulus one, and hence there exists \( r \in \mathbb{R} \) such that \( \exp(2\pi i\alpha) = \exp(2\pi i(A' + r)) \). We can take \( A = A' + r \). The constant \( r \) is unique modulo \( \mathbb{Z} \), and the result follows.

Hence the cocycle \( \alpha \) can be considered as a map \( \alpha: \mathcal{G} \to \Omega^0(N)/\mathbb{Z} \). We denote by \( Z^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) the space of maps \( \alpha: \mathcal{G} \to \Omega^0(N)/\mathbb{Z} \) satisfying the cocycle condition \( \alpha_{\phi \cdot \phi}(x) = \alpha_\phi(x) + \alpha_{\phi'}(\phi x) \) and by \( B^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) the exact cocycles of the form \( \alpha_{\phi} = \phi_*^\alpha \theta - \theta \) for a function \( \theta \in \Omega^0(N) \). The group cohomology is defined by \( H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) = Z^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z})/B^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \).

As \( H^1(N) = 0 \), from Lemmas 4 and 5 it follows that \( \alpha^S \) determines a cohomology class in \( H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) that does not depend on the section \( S \) chosen. We denote this class by \( [\alpha^S] \in H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) and we have the following

Proposition 5 If \( \mathcal{U} \to N \) is a topologically trivial \( \mathcal{G} \)-equivariant \( U(1) \)-bundle, and \( H^1(N) = 0 \) then the following conditions are equivalent

a) \( \mathcal{U} \to N \) is a trivial \( \mathcal{G} \)-equivariant \( U(1) \)-bundle.

b) There exists a \( \mathcal{G} \)-equivariant section \( S: N \to \mathcal{U} \).

c) \([\alpha^U] = 0 \) on \( H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \).

Proof. That a) and b) are equivalent follows from the equivalence between trivializations and sections.

b)\( \Rightarrow \)c) As \( S: N \to \mathcal{U} \) is \( \mathcal{G} \)-equivariant we have \( \alpha^S = 0 \), and hence \( [\alpha^U] = 0 \).

c)\( \Rightarrow \)b) If \( \alpha^U = 0 \) on \( H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) we chose a section \( S: N \to \mathcal{U} \) and we have \( \alpha^S = \phi_\ast^S \theta - \theta \) for \( \theta \in \Omega^0(N) \). We define the section \( S' = S \cdot \exp(2\pi i\theta) \) and by Proposition 4 b) we have \( \alpha^S_{\phi'} = \alpha^S_\phi - \phi_\ast^S \theta + \theta = 0 \), and hence \( S' \) is \( \mathcal{G} \)-equivariant.

Remark 6 If \( H^1(N) \neq 0 \) then the result is also true, but replacing the cohomology \( H^1(\mathcal{G}, \Omega^0(N)/\mathbb{Z}) \) with \( H^1(\mathcal{G}, \Omega^0(N, \mathbb{R}/\mathbb{Z})) \). We prefer to work with \( \Omega^0(N)/\mathbb{Z} \) as it can be easily generalized to local cohomology.
2.2 Local topological anomalies and Lie algebra cohomology

Let $G_0$ be the connected component with the identity on $G$. Invariance under $G_0$ can be determined in terms of the Lie algebra Lie$G$. The action of $G$ on $U$ induces a homomorphism Lie$G \rightarrow \mathfrak{X}(U)$. If $S: N \rightarrow U$ is a section, for any $X \in$ Lie$G$ the vector $X_U(S(x)) - S_*(X_N(x))$ is vertical and hence we have

$$X_U(S(x)) - S_*(X_N(x)) = 2\pi a^S(X)(x)\xi_U(S(x))$$

(1)

for a function $a^S(X) \in \Omega^0(N)$. The term $a^S$ is the infinitesimal variation of $\alpha^S$. Precisely we have the following

**Proposition 7** If $X \in$ Lie$G$, and $\phi_t = \exp(tX)$ then we have

$$a^S(X) = \frac{\Delta a^S}{\Delta t} \bigg|_{t=0}$$

**Proof.** It follows by taking the derivative with respect to $t$ at $t = 0$ on the equation $(\phi_t)^{-1} \circ S \circ (\phi_t)_N = S \cdot \exp(-2\pi i\alpha^S)$. ■

We conclude that the section $S$ is $G_0$-equivariant if and only if $a^S(X) = 0$ for any $X \in$ Lie$G$.

Let us recall the definition of Lie algebra cohomology. If $b \in$ Hom(Lie$G$, $\Omega^0(N)$) we define $\partial b(X,Y) = X_N(b(Y)) - Y_N(b(X)) - b([X,Y])$ for $X,Y \in$ Lie$G$. The closed elements $Z^1$(Lie$G$, $\Omega^0(N)$) $\subset$ Hom(Lie$G$, $\Omega^0(N)$) are those satisfying $\partial b = 0$. The exact elements $B^1$(Lie$G$, $\Omega^0(N)$) are those of the form $b(X) = L_X\Lambda$ for $\Lambda \in \Omega^0(N)$. We define the Lie algebra cohomology by $H^1$(Lie$G$, $\Omega^0(N)) = Z^1$(Lie$G$, $\Omega^0(N))/B^1$(Lie$G$, $\Omega^0(N)$).

**Proposition 8** We have $\partial a^S = 0$.

**Proof.** By the definition of $a^S$, for any $f \in \Omega^0(U)$ we have

$$X_U(f) \circ S - X_N(f \circ S) = 2\pi a^S(X)(\xi_U(f)) \circ S$$

(2)

By using equation [2] for the function $X_U(f)$ we obtain

$$Y_U(X_U(f)) \circ S - Y_N(X_U(f) \circ S) = 2\pi a^S(Y)(\xi_U(X_U(f))) \circ S$$

$$= 2\pi a^S(Y)(X_U(\xi_U(f))) \circ S$$

(3)

(in the last equation we use that $[X_U, \xi_U] = 0$ as $G$ acts on $U$ by $U(1)$-automorphisms). Equation [2] applied to the function $\xi_U(f)$ gives

$$(X_U(\xi_U(f)) \circ S) - X_N(\xi_U(f) \circ S) = 2\pi a^S(X)(\xi_U(\xi_U(f))) \circ S$$

(4)

And finally, by applying $Y_N$ to equation [2] we obtain

$$Y_N(X_U(f) \circ S) - Y_N(X_N(f \circ S)) = 2\pi Y_N(a^S(X))(\xi_U(f) \circ S) + 2\pi a^S(X)(Y_N(\xi_U(f)) \circ S)$$

(5)
In this Section we study the conditions of topological anomaly cancellation in terms of connections and anomaly cancellation of the cohomology of this complex.

First, we recall the definition of equivariant cohomology in the Cartan model. Let \( G \) be an action of a connected Lie group \( G \) on a manifold \( M \). We denote by \( H^k(N)^G \) the cohomology of the space of \( G \)-invariant forms \( \Omega^k(N)^G \) on \( N \). Let \( \Omega^*_G(N) = (\ast^{\ast} (\operatorname{Lie} G)^*) \otimes \Omega^*(N)^G \) be the space of \( G \)-invariant polynomials on \( \operatorname{Lie} G \) with values in \( \Omega^*(N)^G \) with the graduation \( \operatorname{deg}(\alpha) = 2k + r \) if \( \alpha \in \mathcal{P}^k(\operatorname{Lie} G, \Omega^r(N)) \).

Let \( D : \Omega^0_G(N) \to \Omega^{0+1}_G(N) \) be the Cartan differential, \( (D\alpha)(X) = d(\alpha(X)) - \iota_{X_N} \alpha(X), X \in \operatorname{Lie} G \). On \( \Omega^*_G(N) \) we have \( D^2 = 0 \), and the equivariant cohomology (in the Cartan model) of \( N \) with respect of the action of \( G \) is defined as the cohomology of this complex.

A \( G \)-equivariant 1-form \( \alpha \in \Omega^1_G(N) \) is just a \( G \)-invariant 1-form \( \alpha \in \Omega^1(N)^G \). It is \( D \)-closed if and only if it is \( G \)-basic, i.e., if \( d\alpha = 0 \) and \( \iota_{X_N} \alpha = 0 \) for any \( X \in \operatorname{Lie} G \).

Let \( \varpi \in \Omega^2_G(N) \) be a \( G \)-equivariant 2-form. Then we have \( \varpi = \omega + \mu \) where \( \omega \in \Omega^2(N) \) is \( G \)-invariant and \( \mu \in \operatorname{Hom}(\operatorname{Lie} G, \Omega^0(N))^G \). We have \( D\varpi = 0 \) if and only if \( d\omega = 0 \) and \( \iota_{X_N} \omega = d(\mu(X)) \) for every \( X \in \operatorname{Lie} G \).

If \( \pi : \mathcal{U} \to N \) is a principal \( U(1) \) bundle and \( \Xi \in \Omega^1(\mathcal{U}, i\mathbb{R}) \) is a connection then the curvature form \( \operatorname{curv}(\Xi) \in \Omega^2(N) \) is defined by the property \( \pi^*(\operatorname{curv}(\Xi)) = \frac{i}{2\pi} d\Xi \). The (real) first Chern class of \( \mathcal{U} \) is the cohomology class of \( \operatorname{curv}(\Xi) \). A connection \( \Xi \in \Omega^1(\mathcal{U}, i\mathbb{R}) \) on \( \mathcal{U} \) is \( G \)-invariant if \( \phi^*_U \Xi = \Xi \).
Lemma 10 Let $\Xi$ be a $G$-invariant connection on $U \to N$. Then for any $\rho \in \Omega^1(N)$ such that $d\rho = \text{curv}(\Xi)$ there exists a section $S: N \to U$ such that $\rho = \frac{1}{2\pi} S^*(\Xi)$. Any other section satisfying this condition is of the form $S' \cdot \exp(2\pi ir)$ for $r \in \mathbb{R}$.

Proof. As $U \to N$ is trivial, there exist a section $S_0: N \to U$. As $d\rho^{S_0} = \text{curv}(\Xi) = d\rho$ and $H^1(N) = 0$ we have $\rho = \rho^{S_0} - d\Lambda$ for some $\Lambda \in \Omega^1(N)$. We define the section $S = S_0 \cdot \exp(2\pi i\Lambda)$ and we have $\frac{1}{2\pi} S^*(\Xi) = \rho^{S_0} - d\Lambda = \rho$. If $S'$ is another section satisfying this condition we have $S' = S' \cdot \exp(2\pi ir)$ and $dr = \rho^S - \rho^{S'} = 0$. $lacksquare$

3.2 Local topological anomalies and equivariant curvature

Let $\pi: U \to N$ be a $G$-equivariant principal $U(1)$-bundle and $\Xi$ a $G$-invariant connection. The Maurer-Cartan form on $U(1)$ is denoted by $\vartheta = z^{-1}dz$, and $\xi \in \mathfrak{X}(U(1))$ is the $U(1)$-invariant vector field $\xi(z) = iz$ such that $\vartheta(\xi) = i$. We denote by $\xi_U \in \mathfrak{X}(U)$ the vector field on $U$ corresponding to $\xi$. Given a section $S: N \to U$, we define $\rho^S = \frac{1}{2\pi} S^*\Xi \in \Omega^1(N)$. On the trivialization $\Psi_S: N \times U(1) \to U$ determined by $S$ we have

$$\Psi_S^*\Xi = \vartheta - 2\pi i\rho^S$$

(6)

Conversely, if $\rho \in \Omega^1(N)$ and $S$ is a section of $U \to N$ then the form $\Xi = (\Psi_S^{-1})^*(\vartheta - 2\pi i\rho)$ is a connection form on $U \to N$ with $\rho^S = \rho$.

The following result follows from the definitions of $a^S$, $\rho^S$, $\mu^S$ and equation (6).

Lemma 10 We have
a) $\text{curv}(\Xi) = d\rho^S$.

b) $\mu^S(X) = -\rho^S(X_N) + a^S(X)$ for any $X \in \text{Lie}G$.

c) If $S' = S \exp(2\pi i\Lambda)$ for $\Lambda \in \Omega^1(N)$, then $\rho^{S'} = \rho^S - d\Lambda$.

Note that $a^S$ and $\rho^S$ satisfy the following set of equations, that are similar to the Stora-Zumino descent equations

$$\text{curv}(\Xi) = d\rho^S$$

$$L_X\rho^S + d(a^S(X)) = 0.$$
Proposition 12 If $U \to N$ admits $G_0$-invariant connections then the following conditions are equivalent

1. There exists a $G_0$-equivariant section of $U \to N$.
2. $[a^U] = 0$ on the cohomology $H^1(\text{Lie}G, \Omega^0(N))$.
3. The first $G_0$-equivariant Chern class $c_{1,G_0}(U) \in H^2_{G_0}(N)$ vanishes.

Proof. $p_1 \Rightarrow p_3$ If $S$ is a $G_0$-equivariant section then $a^S = 0$, $\rho^S$ is $G_0$-invariant and by Proposition 10 we have $D\rho^S = 0$.

$p_3 \Rightarrow p_2$ If $D\beta = \text{curv}_{G}(\Xi)$ for a $G_0$-invariant $\beta \in \Omega^1(N)$ then $d\beta = \text{curv}(\Xi)$ and $\iota_{X_N} \beta = -\mu^X(\Xi)$. By Proposition 11 there exists a section $S$ such that $\rho^S = \beta$, and by Proposition 10 we have $a^S(X) = \mu^X(X) + \beta(X_N) = 0$.

$p_2 \Rightarrow p_1$ If for a section we have $a^S(X) = L_X \Lambda$ for $\Lambda \in \Omega^0(N)$, we define $S' = S \exp(2\pi i \Lambda)$ and we have $a^{S'} = a^S(X) - L_X \Lambda = 0$ and $S'$ is $G_0$-equivariant.

As commented in the Introduction anomalies are usually studied in terms of the topology of the quotient space, but we prefer to work with equivariant cohomology because it can be extended to local cohomology. However, the topology of the quotient can be used to obtain necessary conditions for anomaly cancellation.

Suppose that $H \subset G$ is a subgroup that acts freely on $N$ and we have a well defined quotient bundle $U/H \to N/H$. If $U \to N$ admits a $G$-equivariant section $S$, then $S$ is also $H$ invariant and the first Chern class $c_1(U/H) \in H^2(N/H)$ vanishes. Hence we have the following

Proposition 13 If $c_1(U/H) \neq 0$ for some $H \subset G$, then $U \to N$ is a non trivial $G$-equivariant $U(1)$-bundle.

This condition is frequently used to show that an anomaly does not cancel as $c_1(U/H)$ can be computed by using topological techniques. However, we recall that, due to the locality problem, in this way we obtain necessary conditions for physical anomaly cancellation, but they are not sufficient.

3.3 Global topological anomalies and equivariant holonomy

In order to obtain conditions for a $G$-equivariant $U(1)$-bundle to be trivial in terms of a connection we need to obtain a condition analogous to condition $p_3$ and valid for non connected groups. One possibility could be to consider the integer equivariant Chern class. However, we need a condition that should be able to be generalized to local cohomology, and that is not the case for the integer cohomology. We show that this problem can be solved by introducing the equivariant holonomy of a invariant connection. Although it seems to be a natural concept, we have been unable to find a detailed study of it in the literature. In this paper we give only the basic facts needed for our characterization of anomaly cancellation. We left a more detailed study of the equivariant holonomy for a separate paper.
3.4 Equivariant Holonomy

Let $\Sigma$ be a $G$-invariant connection on a $G$-equivariant $U(1)$-bundle $U \to N$ and let $I$ denote the interval $[0,1]$. If $\phi \in G$, we define $C^\phi = \{ \gamma : I \to N : \gamma(1) = \phi(\gamma(0)) \}$, and $C^\phi_\infty = \{ \gamma : I \to N : \gamma(0) = x \text{ and } \gamma(1) = \phi(x) \}$. If $\gamma \in C^\phi_\infty$ and $y \in U$, with $\pi(y) = x$, we denote by $\bar{\gamma} : I \to U$ the $\Xi$-horizontal lift of $\gamma$ with $\bar{\gamma}(0) = y$. We have $\pi(\bar{\gamma}(t)) = \pi(\phi_U(y)) = \phi(x)$, and hence there exists $h \in \mathbb{R}/\mathbb{Z}$ such that $\bar{\gamma}(1) = (\phi_U(y)) \exp(2\pi i h)$. It can be easily seen that $h$ does not depend on the $y$ chosen and we denote it by $\text{hol}^\Xi_\phi(\gamma)$ and call it the $\phi$-equivariant holonomy of $\Xi$ on $\gamma$. Note that for $\phi = 1_G$, the $1_G$-equivariant holonomy coincides with the ordinary holonomy. By using equation (6) we obtain the following

Lemma 14 On the trivialization determined by a section $\phi$ we define $(\phi_U(y)) \exp(2\pi i h)$. Note that for $\phi = 1_G$, the $1_G$-equivariant holonomy coincides with the ordinary holonomy. By using equation (6) we obtain the following

Proposition 15 If $S : N \to U$ is a section of $U$, then for any $\gamma \in C^\phi_\infty$:

$$\text{hol}^\Xi_\phi(\gamma) = \int_0^1 \rho^S - \alpha^S(x).$$

Remark 16 The preceding Proposition can be used to give an alternative definition of the cocycle $\alpha_\phi$, by showing that $\int_x \rho^S - \text{hol}^\Xi_\phi(\gamma)$ does not depend on $\gamma \in C^\phi_\infty$. This is the approach used in [12] to define the Chern-Simons line bundles.

If $\gamma, \gamma' : I \to N$ are curves on $N$, we define the inverse curve $\bar{\gamma}'(t) = \gamma(1-t)$, and if $\gamma(1) = \gamma'(0)$ we define $\gamma \ast \gamma' : I \to \mathbb{R}$ by $\gamma \ast \gamma'(t) = \gamma(2t) \text{ for } t \in [0,1/2]$ and $\gamma \ast \gamma'(t) = \gamma'(2t-1) \text{ for } t \in [1/2,1]$. If $\phi$ is a diffeomorphism of $N$ then we define $(\phi \cdot \gamma)(t) = \phi(\gamma(t))$.

Proposition 17 Let $\Sigma$ be a $G$-invariant connection, $\phi, \phi' \in G$, $x, y \in N$, $\gamma \in C^\phi_\infty$ and let $\zeta$ be a curve joining $y$ and $x$. We have

a) $\phi' \cdot \gamma \in C^\phi_\infty$ and $\text{hol}^\Xi_\phi(\phi' \gamma) = \text{hol}^\Xi_\phi(\gamma)$.

b) $\gamma' = \zeta \ast \gamma \ast (\phi \cdot \bar{\zeta}) \in C^\phi_\infty$ and $\text{hol}^\Xi_\phi(\gamma') = \text{hol}^\Xi_\phi(\gamma)$.

c) $\alpha^S_\phi(x) = \alpha^S_\phi(y) + \int_\gamma (\phi \cdot \rho^S - \rho^S)$.

d) $\delta \alpha^S_\phi = \phi^\ast \rho^S - \rho^S$.

Proof. a) and b) easily follows from the definition of $\text{hol}^\Xi_\phi(\gamma)$ and the invariance of $\Sigma$ and d) follows from c). We prove c). If $\gamma \in C^\phi_\infty$ and $\gamma' = \zeta \ast \gamma \ast (\phi \cdot \bar{\zeta})$ then by using b) we obtain

$$\alpha^S_\phi(x) = \int_\gamma \rho^S - \text{hol}^\Xi_\phi(\gamma) = \int_{\gamma'} \rho^S - \int_{\gamma} \rho^S - \int_{\phi \cdot \bar{\zeta}} \rho^S - \text{hol}^\Xi_\phi(\gamma') = \alpha^S_\phi(y) + \int_{\zeta} (\phi \cdot \rho^S - \rho^S).$$

The following results determines necessary and sufficient conditions for global topological anomaly cancellation.
Theorem 18 If $\Xi$ is a $G$-invariant connection then the following conditions are equivalent

$g_1$) There exists a $G$-equivariant section of $U \to N$.
$g_2$) $[\alpha^H] = 0$ on $H^1(G, \Omega^0(N)/\mathbb{Z})$.
$g_3$) There exists $\beta \in \Omega^1(N)^G$ such that $\text{hol}^\Xi_\phi(\gamma) = \int_\gamma \beta$ for any $\phi \in G$, and $\gamma \in \mathcal{C}^\phi$. Furthermore in that case we have $D\beta = \text{curv}_\phi(\Xi)$.

Proof. We have seen in Proposition [3] that $g_1$ and $g_2$ are equivalent.

$g_3 \Rightarrow g_1)$ First we prove that if there exists $\beta \in \Omega^1_\rho(N)$ such that $\text{hol}^\Xi_\phi(\gamma) = \int_\gamma \beta$, then we have $d\beta = \text{curv}(\Xi)$. We define $\chi = \text{curv}(\Xi) - d\beta \in \Omega^2(N)$ and we choose a section $S$ of $U \to N$. For any loop $\gamma \in \mathcal{C}^\rho_\phi$ with $\gamma = \partial D$ we have $\int_D \chi = \int_D d(\rho^S - \beta) = \int_\gamma (\rho^S - \beta) = \text{hol}^\Xi_\phi(\gamma) = - \alpha^S_\phi(x) = 0$, and hence $\chi = 0$, i.e., $\text{curv}(\Xi) = d\beta$.

As $\text{curv}(\Xi) = d\beta$, by Proposition [14] there exists a section $S'$ such that $\rho^{S'} = \beta$, and for any $\phi \in G$ and $\gamma \in \mathcal{C}^\rho_\phi$ we have $\alpha^S_\phi(x) = \int_\gamma \rho^{S'} - \text{hol}_\phi(\gamma) = 0$, and $S'$ is $G$-invariant. Finally by Proposition [10] we have $\iota_X \beta = \iota_X \rho^{S'} = - \mu^\Xi(X)$ and hence $D\beta = \text{curv}_\phi(\Xi)$.

3.5 Global anomalies and equivariant flat connections

In the study of anomaly cancellation, we start with local anomalies because they are easier to analyze. If the local anomaly cancels, then we study the corresponding global anomaly. By Theorem [12] if the local topological anomaly cancels then there exists $\beta_0 \in \Omega^1(N)^G_0$ such that $d\beta_0 = \text{curv}(\Xi)$ and $\iota_X \beta_0 = - \mu^\Xi(X)$. The first problem to cancel the global anomaly is that the form $\beta_0$ does not need to be $G$-invariant. As $\text{curv}(\Xi)$ is $G$-invariant we have $d(\phi^* \beta_0 - \beta_0) = 0$ and as $H^1(N) = 0$ there exist $\sigma_\phi \in \Omega^0(N)$ such that $d\sigma_\phi = \phi^* \beta_0 - \beta_0$. The function $\sigma_\phi$ is determined modulo a constant. If $\phi \in G_0$ we have $\phi^* \beta_0 - \beta_0 = 0$ and we can take $\sigma_\phi$ constant. Hence $\beta_0$ determines a map $\sigma^{\beta_0} : G/G_0 \to \Omega^0(N)/\mathbb{R}$ and it is easily to see that it satisfies the cocycle condition and hence defines an element $[\sigma^{\beta_0}] \in H^1(G/G_0, \Omega^0(N)/\mathbb{R})$.

Proposition 19 Let $\beta_0 \in \Omega^1(N)^G_0$ be a form such that $d\beta_0 \in \Omega^1(N)^G$. If $H^1(N) = 0$ then there exists $\beta \in \Omega^1(N)^G$ such that $d\beta = d\beta_0$ and $\iota_X \beta = \iota_X \beta_0$ if and only if $[\sigma^{\beta_0}] = 0$ on $H^1(G/G_0, \Omega^0(N)/\mathbb{R})$.

Proof. If $\beta_0$ is $G$-invariant then $\sigma^{\beta_0} = 0$. Conversely, if $[\sigma^{\beta_0}] = 0$ then there exists $\rho \in \Omega^0(N)/\mathbb{R}$ such that $\sigma^{\beta_0}_\phi = \phi^* \rho - \rho$. As $\sigma^{\beta_0} = 0$ for any $\phi \in G_0$ we have $L_X \rho = 0$.

If we define $\beta = \beta_0 - d\rho$ then we have $d\beta = d\beta_0$. Also we have $\phi^* (\beta) = \phi^*(\beta_0) - d\phi^*(\rho) = \phi^*(\beta_0) - d\sigma^{\beta_0}_\phi - d\rho = \phi^*(\beta_0) - (\phi^* \beta_0 - \beta_0) - d\rho = \beta$ and $\iota_X \beta = \iota_X \beta_0 - \iota_X d\rho = \iota_X \beta_0$.}

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Hence \([\sigma^{\beta_0}] \in H^1(\mathcal{U}/\mathcal{G}_0, \Omega^0(N)/\mathbb{R})\) is an obstruction to find a \(\mathcal{G}\)-equivariant section of \(\mathcal{U} \to N\). If this obstruction cancels, there exists \(\beta \in \Omega^1(N)^{\mathcal{G}}\) satisfying \(\text{curv}_G(\Xi) = D\beta\). Then we define a new connection \(\Xi' = \Xi + 2\pi i(\pi^*\beta)\), which is \(\mathcal{G}\)-invariant and we have \(\text{curv}_G(\Xi') = 0\), i.e. \(\Xi'\) is \(\mathcal{G}\)-flat. Hence it is enough to study the case of \(\mathcal{G}\)-flat connections. For \(\mathcal{G}\)-flat connections we have the following

**Proposition 20** If \(\Xi\) is a \(\mathcal{G}\)-flat connection then \(\text{hol}^{\Xi}_{\phi}(\gamma)\) does not depend on \(\gamma \in \mathcal{C}^\phi\), and \(\text{hol}^{\Xi}_{\phi}(\gamma) = 0\) for \(\phi \in \mathcal{G}_0\).

**Proof.** By Proposition 14 we can choose a section \(S\) such that \(\rho^S = 0\). By Proposition 15 we have \(\text{hol}^{\Xi}_{\phi}(\gamma) = -\alpha^S_\phi(x)\) that does not depend on \(\gamma \in \mathcal{C}^\phi\). Furthermore, by Proposition 17(c) the holonomy does not depend on \(x\).

By Proposition 10 we have \(\alpha^S = 0\). Hence \(S\) is \(\mathcal{G}_0\)-equivariant and \(\text{hol}^{\Xi}_{\phi}(\gamma) = \alpha^S_\phi = 0\) for \(\phi \in \mathcal{G}_0\). ■

We conclude that \(\text{hol}^{\Xi}_{\phi}\) determines an element \(\kappa^\Xi \in \text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z})\), by setting \(\kappa^\Xi_\phi = \text{hol}^{\Xi}_{\phi}(\gamma)\) for any \(\gamma \in \mathcal{C}^\phi\). The following result shows that any element of \(\text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z})\) can be represented as the holonomy of a flat connection:

**Proposition 21** For any \(h \in \text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z})\) there exists a \(\mathcal{G}\)-flat connection \(\Xi\) on \(\mathcal{U} \to N\) such that \(\kappa^\Xi = h\).

**Proof.** Using the trivialization associated to a section the problem can be reduced to a trivial bundle. On \(\mathcal{U} = N \times U(1)\) we define \(\alpha_\phi(x) = h(\phi)\) (it satisfies the cocycle condition by Remark 2). By Proposition 8 it defines a \(\mathcal{G}\)-equivariant \(U(1)\)-bundle and \(\Xi = \partial\) is a \(\mathcal{G}\)-flat connection with \(\kappa^\Xi = h\). ■

If \(\kappa^\Xi = 0\) then the bundle is a trivial \(\mathcal{G}\)-equivariant bundle, but if \(\kappa^\Xi \neq 0\) we cannot assert that the bundle is nontrivial. The reason is that a trivial \(\mathcal{G}\)-equivariant bundle can admit flat connections with nontrivial holonomy. An easy example is the following:

**Example 22** We consider the trivial bundle \(\mathcal{U} = \mathbb{R} \times U(1)\) and \(\mathcal{G} = \mathbb{Z}\) acting trivially on \(U(1)\). We define \(\alpha_n(x) = \frac{n}{2} \mod \mathbb{Z}\) for \(n \in \mathbb{Z}\) and the connection \(\Xi = \partial\). For the curve \(\gamma_1(s) = s\) we have \(\text{hol}^{\Xi}_{\phi}(\gamma_1) = \alpha_1(x) = \frac{1}{2} \mod \mathbb{Z} \neq 0\) but \(\mathcal{U}\) is a trivial \(\mathbb{Z}\)-equivariant bundle as for any \(\gamma \in \mathcal{C}^n\) we have \(\kappa^\Xi_n = \int_\gamma \beta\) with \(\beta = \frac{1}{2} dt \in \Omega^1(\mathbb{R})^\mathbb{Z}\).

If \(\Xi\) is \(\mathcal{G}\)-flat, then by Theorem 18 the bundle \(\mathcal{U} \to N\) is trivial if and only if there exists \(\beta \in \Omega^1(N)^{\mathcal{G}}\) such that \(D\beta = 0\) and \(\kappa^\Xi_n = \int_\gamma \beta\) for any \(\gamma \in \mathcal{C}^\phi\). We study this condition in more detail

**Proposition 23** If \(\beta \in \Omega^1(N)^{\mathcal{G}}\) satisfies \(D\beta = 0\) then \(k^\beta_\phi = \int_\gamma \beta\) does not depend on \(\gamma \in \mathcal{C}^\phi\) and \(k^\beta_\phi = 0\) if \(\phi \in \mathcal{G}_0\). Furthermore, if \(\beta = dp\) for a form \(p \in \Omega^0(N)\) then \(k^\beta_\phi = \phi^*p - p\) for any \(\phi \in \mathcal{G}\). If there exists \(p\) satisfying \(\beta = dp\) and \(\mathcal{G}\)-invariant then \(k^\beta_\phi = 0\) for any \(\phi \in \mathcal{G}\).
Lemma 25 The constant functions on \( \Gamma(E) \) are local functions.

Proof. As \( d\beta = 0 \) we have \( \beta = d\rho \) for a function \( \rho \in \Omega^1(N) \) (determined up to a constant). Moreover, we have \( k_\gamma^0(x) = \int_\gamma \beta = \int_\gamma d\rho = \rho(\phi x) - \rho(x) \) that does not depend on \( \gamma \in C_\phi^0 \). Furthermore, \( k_\phi^0 \) does not depend on \( x \) because \( dk_\phi^0 = d(\phi^*\rho - \rho) = \phi^*d\rho - d\rho = \phi^*\beta - \beta = 0 \). We have \( L_X \rho = \iota_X \beta = 0 \) and hence \( \rho \in \Omega^1(N)\phi_0 \) and \( k_\phi^0 = 0 \) for \( \phi \in \mathcal{G}_0 \). If \( \rho \in \Omega^1(N)\phi \) satisfies \( \beta = d\rho \) then \( k_\phi^0 = \phi^*\rho - \rho = 0 \) for any \( \phi \in \mathcal{G} \). 

Hence we have a well defined map \( k : H^1_G(N) \rightarrow \text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z}) \). We define \( K^G(N) = k(H^1_G(N)) \subseteq \text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z}) \) and we have the following

Proposition 24 If \( \Xi \) is a \( \mathcal{G} \)-flat connection then \( \mathcal{U} \rightarrow N \) is a trivial \( \mathcal{G} \)-equivariant bundle if and only if \( \kappa^{\Xi} \in K^G(N) \).

4 Locality and physical anomalies

In this section we introduce the concepts of local forms and local cohomology needed for the study the locality problem in anomaly cancellation. The local cohomology is defined when \( N \) is a submanifold of the space of sections \( \Gamma(E) \) of a bundle \( E \rightarrow M \). It generalizes the concept of a local functional to higher order forms. Furthermore, it is easily extended to local \( \mathcal{G} \)-equivariant cohomology. We introduce the notions of local connection and local section, and we show that the existence of local \( \mathcal{G} \)-equivariant section is equivalent to physical anomaly cancellation.

4.1 Local forms and Jet bundles

Let \( p : E \rightarrow M \) be a bundle over a compact, oriented \( n \)-manifold \( M \) without boundary. We denote by \( J^r E \) its \( r \)-jet bundle, and by \( J^\infty E \) the infinite jet bundle (see e.g. [15] for the details on the geometry of \( J^\infty E \)). We recall that the points on \( J^\infty E \) can be identified with the Taylor series of sections of \( E \). Let \( \Gamma(E) \) be the manifold of global sections of \( E \), that we assume to be not empty. If \( s \in \Gamma(E) \) then we denote by \( j^r_x s \) (resp. \( j^\infty_x s \)) the \( k \)-jet (resp. the \( \infty \)-jet) of \( s \) at \( x \).

We consider \( \Gamma(E) \) as an infinite dimensional Frechet manifold, that we assume to be not empty. Let \( j^\infty : M \times \Gamma(E) \rightarrow J^\infty E \), \( j^\infty(x, s) = j^\infty_x s \) be the evaluation map. In [1] it is defined a map \( \Im : \Omega^{n+k}(J^\infty E) \rightarrow \Omega^k(\Gamma(E)) \), by \( \Im(\alpha) = \int_M (j^\infty)^* \alpha \) for \( \alpha \in \Omega^{n+k}(J^\infty E) \). If \( \alpha \in \Omega^k(J^\infty E) \) with \( k < n \), we set \( \Im(\alpha) = 0 \). We define the space of local \( k \)-forms on \( \Gamma(E) \) by \( \Omega^k_{\text{loc}}(\Gamma(E)) = \Im(\Omega^{n+k}(J^\infty E)) \subseteq \Omega^k(\Gamma(E)) \). We have \( \Im(d\alpha) = d\Im(\alpha) \), and hence if \( \theta \in \Omega^k_{\text{loc}}(\Gamma(E)) \) then \( d\theta \in \Omega^{k+1}_{\text{loc}}(\Gamma(E)) \). The local cohomology of \( \Gamma(E) \), \( H^k_{\text{loc}}(\Gamma(E)) \), is the cohomology of \( (\Omega_{\text{loc}}^k(\Gamma(E)), d) \) and we have \( H^k_{\text{loc}}(\Gamma(E)) \approx H^{n+k}_{\text{loc}}(\Gamma(E)) \) for \( k > 0 \) (see [10]). If \( \theta \in \Omega^k_{\text{loc}}(\Gamma(E)) \) is closed, we denote by \( [\theta] \) its cohomology class in \( H^k(\Gamma(E)) \) and by \( \{\theta\} \) its cohomology class in \( H^k_{\text{loc}}(\Gamma(E)) \).

Lemma 25 The constant functions on \( \Gamma(E) \) are local functions.
Proof. If $c \in \mathbb{R}$ and $\omega \in \Omega^p(M)$ is a volume form on $M$ with $v = \int_M \omega > 0$, then $c = \exists |\xi| q_{\infty}^* \omega|$, where $q_{\infty} : J^\infty E \to M$ is the projection. 

A diffeomorphism $\phi \in \text{Diff} E$ is said to be projectable if there exists $\tilde{\phi} \in \text{Diff} M$ satisfying $\phi \circ p = p \circ \tilde{\phi}$. We denote by $\text{Proj} E$ the space of projectable diffeomorphism of $E$, and by $\text{Proj}^+ E$ the subgroup of elements such that $\phi$ is orientation preserving. Let $\mathcal{G}$ be a Lie group acting on $E$ by elements $\text{Proj}^+ E$. The integration operator extends to a map on equivariant differential forms (see [9]) $\exists : \Omega^n_{g+k}(J^\infty E) \to \Omega^n_{g}(\Gamma(E))$, by setting $(\exists(\alpha))(X) = \exists(\alpha(X))$ for every $\alpha \in \Omega^n_{g+k}(J^\infty E), X \in \text{Lie} \mathcal{G}$. The map $\exists$ induces a homomorphism in equivariant cohomology $\exists : H^n_{g}(J^\infty E) \to H^n_{g}(\Gamma(E))$. We define the space of local $\mathcal{G}$-equivariant $q$-forms on $\Gamma(E)$ by $\Omega^n_{g,\text{loc}}(\Gamma(E)) = \bigoplus_{2k+r=q} (P^k(\text{Lie} \mathcal{G}, \Omega^n_{g,\text{loc}}(\Gamma(E))))) \subset \Omega^n_{g}(\Gamma(E))$, and the local $\mathcal{G}$-equivariant cohomology of $\Gamma(E)$, $H^n_{g,\text{loc}}(\Gamma(E))$, as the cohomology of $(\Omega^n_{g,\text{loc}}(\Gamma(E)), D)$. The map $\exists$ induces a cochain map $\exists : \Omega^n_{g+k}(J^\infty E) \to \Omega^n_{g,\text{loc}}(\Gamma(E))$.

4.2 Local connections and sections

Let $\mathcal{U} \to \Gamma(E)$ be a $\mathcal{G}$-equivariant $U(1)$-bundle. On $\Gamma(E)$ we have the space of local forms, but we do not have an analogous notion on $\mathcal{U}$. As a connection is a form on $\mathcal{U}$, we define the notion of local connection in terms of its equivariant curvature

**Definition 26** We say that a $\mathcal{G}$-invariant connection $\Xi$ on $\mathcal{U} \to \Gamma(E)$ is local if $\text{curv}_\mathcal{U}(\Xi) = \text{curv}(\Xi) + \mu^2 \in \Omega^2_{g,\text{loc}}(\Gamma(E))$.

The motivation for this definition is the equivariant Atiyah-Singer index theorem for families (see [10]) that express the equivariant curvature of the Bismut-Freed connection in terms of characteristic forms. It is shown in [11] that for gravitational and gauge anomalies, the Bismut-Freed connection on the determinant line bundle is a local connection. We make the following assumptions:

(A1) We assume that $\mathcal{U} \to \Gamma(E)$ is topologically trivial, $\Gamma(E)$ is connected and that $H^1(\Gamma(E)) = 0$.

As $\mathcal{U} \to \Gamma(E)$ is topologically trivial, we know that $\text{curv}(\Xi) \in \Omega^2_{\text{loc}}(\Gamma(E))$ is exact. But we cannot assert that it is the differential of a local form. Hence we need to make the following assumption

(A2) $\Xi$ is a $\mathcal{G}$-invariant local connection and $\{\text{curv}(\Xi)\} = 0$ on $H^2_{\text{loc}}(\Gamma(E))$, i.e. there exists $\rho \in \Omega^1_{\text{loc}}(\Gamma(E))$ such that $d\rho = \text{curv}(\Xi)$.

A sufficient condition for (A2) is $H^2_{\text{loc}}(\Gamma(E)) \simeq H^{n+2}(E) = 0$. We also make the following assumption

(A3) $H^1_{\text{loc}}(\Gamma(E)) \simeq H^{n+1}(E) = 0$.

**Definition 27** We say that a section $S : \Gamma(E) \to \mathcal{U}$ is $\Xi$-local if $\rho^S = \frac{1}{2\pi} S^* (\Xi) \in \Omega^1_{\text{loc}}(\Gamma(E))$. 

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We show below that the existence of local $\mathcal{G}$-invariant sections characterizes physical anomaly cancellation and gives an intrinsic characterization of the condition in the Introduction. By Proposition local sections exists if assumption (A2) is satisfied.

**Proposition 28** If $S$ is a $\Xi$-local section, then any other section $S'$ is $\Xi$-local if and only if is of the form $S' = S \cdot \exp(2\pi i \cdot \Lambda)$ for $\Lambda \in \Omega^0_{\text{loc}}(\Gamma(E))$.

**Proof.** If $S' = S \cdot \exp(2\pi i \cdot \Lambda)$ for $\Lambda \in \Omega^0_{\text{loc}}(\Gamma(E))$ then by Proposition we have $\rho^{S'} = \frac{1}{2\pi}(S')^*\rho(\Xi) = \rho^S - d\Lambda \in \Omega^1_{\text{loc}}(\Gamma(E))$.

Conversely, if $S'$ is another local section then $\rho^{S'} \in \Omega^1_{\text{loc}}(\Gamma(E))$ and $d\rho^{S'} = d\rho^S = \text{curv}(\Xi)$. By assumption (A3) $H^1_{\text{loc}}(\Gamma(E)) = 0$ and hence there exists $\Lambda' \in \Omega^0_{\text{loc}}(\Gamma(E))$ such that $\rho^{S'} = \rho^S - d\Lambda'$. By Propositions there exists $r \in \mathbb{R}$ such that $S' = S \cdot \exp(2\pi i \cdot \Lambda') \exp(2\pi i \cdot r) = S \cdot \exp(2\pi i \cdot (\Lambda' + r))$ and we can take $\Lambda = \Lambda' + r \in \Omega^0_{\text{loc}}(\Gamma(E))$ (it is local by Lemma).

### 4.3 Local physical anomalies

If $\Xi$ is a local connection and $S$ is $\Xi$-local section, then we have $\alpha^S(X) = \rho^S(X_N) + \mu^S(X)$ and hence $\alpha^S \in \Omega^1(\text{Lie}_{\mathcal{G}}, \Omega^0_{\text{loc}}(N))$.

By Proposition we have $\partial \alpha^S = 0$. Furthermore, by Assumption (A3) and Propositions we have $\{\alpha^S\} \in H^1(\text{Lie}_{\mathcal{G}}, \Omega^0_{\text{loc}}(\Gamma(E)))$ does not depend on the $\Xi$-local section chosen. We denote this class by $\{\alpha^S\} \in H^1(\text{Lie}_{\mathcal{G}}, \Omega^0_{\text{loc}}(\Gamma(E)))$.

The following Theorem is an extension of the results that shows that our definition of local section determines physical anomaly cancellation. This is important in order to generalize this result to global anomalies.

**Theorem 29** If $\Xi$ is a $\mathcal{G}$-invariant local connection, then the following conditions are equivalent

1. There exists a $\Xi$-local $\mathcal{G}_0$-invariant section of $\mathcal{U} \to \Gamma(E)$.
2. $\{\alpha^S\} = 0$ on the local BRST cohomology $H^1(\text{Lie}_{\mathcal{G}}, \Omega^0_{\text{loc}}(\Gamma(E)))$.
3. $\{\text{curv}_{\mathcal{G}_0}(\Xi)\} = 0$ on $H^2_{\overline{\mathcal{G}_0},\text{loc}}(\Gamma(E))$.

**Proof.** $P_1 \Rightarrow P_3$ If $S$ is a $\Xi$-local $\mathcal{G}_0$-invariant section $\rho^S \in \Omega^1_{\text{loc}}(\Gamma(E))^{\mathcal{G}_0}$ and $d\rho^S = \text{curv}(\Xi)$ and $\mu^S(X) = \alpha^S - \rho(X_N) = -\rho(X_N)$, as we have $\alpha^S = 0$ because $S$ is $\mathcal{G}_0$-invariant.

$P_3 \Rightarrow P_2$ Assume that there exists $\rho \in \Omega^1(\Gamma(E))^{\mathcal{G}_0}$ such that $d\rho = \text{curv}(\Xi)$ and $\mu^S(X) = -\mu^S(X)$. By Proposition there exists a $\Xi$-local section $S$ such that $\rho^S = \rho$ and we have $\alpha^S(X) = \rho(X_N) + \mu^S(X) = 0$.

$P_2 \Rightarrow P_1$ If $S$ is a $\Xi$-local and $\alpha^S(X) = L_X \Lambda$ for $\Lambda \in \Omega^0_{\text{loc}}(\Gamma(E))$ then $S' = S \cdot \exp(2\pi i \Lambda)$ is also local and by Proposition we have $\alpha^{S'}(X) = \alpha^S(X) - L_X \Lambda = 0$ and $S'$ is $\mathcal{G}_0$-invariant.
4.4 Global physical anomalies

In this section we generalize the results of the previous section to obtain necessary and sufficient conditions for global anomaly cancellation.

Proposition 30 If $S$ is a $\Xi$-local section then $\alpha^S \in \Omega^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E))/\mathbb{Z})$.

Proof. Let $\phi \in \mathcal{G}$ and fix $s_0 \in \Gamma(E)$. We have $d(\phi^* \rho^S - \rho^S) = \phi^* \text{curv}(\Xi) - \text{curv}(\Xi) = 0$, and as $H^1_{\text{loc}}(\Gamma(E)) = 0$ there exists $\sigma_\phi \in \Omega^0_{\text{loc}}(\Gamma(E))$ such that $\phi^* \rho^S - \rho^S = \sigma_\phi$. By Proposition 17 if $\gamma$ is a curve joining $s_0$ and $s$ we have $\alpha^S_\phi(s) = \alpha^S_\phi(s_0) + \int_\gamma (\phi^* \rho^S - \rho^S) = \alpha^S_\phi(s_0) + \int_\gamma \sigma_\phi = \alpha^S_\phi(s) - \sigma_\phi(s_0)$.

The result follows because the first and third terms are local by Lemma 25.

We know that $\alpha^S$ satisfies the cocycle condition. And by Assumption (A3) and Propositions 28 and 10 the cohomology class of $\alpha^S$ on $H^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E))/\mathbb{Z})$ does not depend on the $\Xi$-local section chosen. We denote this class by $\{\alpha^S\} \in H^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E))/\mathbb{Z})$, and we have the following.

Theorem 31 If $\Xi$ is a $\mathcal{G}$-invariant local connection, then the following conditions are equivalent

$G_1$) There exists a $\Xi$-local $\mathcal{G}$-equivariant section in $\mathcal{U} \to N$.

$G_2$) $\{\alpha^S\} = 0$ on $H^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E))/\mathbb{Z})$.

$G_3$) There exists $\beta \in \Omega^1_{\mathcal{G},\text{loc}}(\Gamma(E))$ such that $\text{hol}^\Xi_\beta(\gamma) = \int_{\gamma} \beta$ for any $\phi \in \mathcal{G}$, and $\gamma \in C^\phi$.

Proof. $G_1 \Rightarrow G_2$. If $S: \Gamma(E) \to \mathcal{U}$ is a $\Xi$-local $\mathcal{G}$-equivariant section then $\alpha^S = 0$ and hence $[\alpha^S] = 0$ on $H^1(\mathcal{G}, \Omega^0_{\text{loc}}(\Gamma(E))/\mathbb{Z})$.

$G_2 \Rightarrow G_3$. If $\{\alpha^S\} = 0$ on $H^1_{\text{loc}}(\mathcal{G}, \Omega^0_{\text{loc}}(\mathcal{M}, \mathbb{R}))/\mathbb{Z}$ we chose a $\Xi$-local section $S: N \to \mathcal{U}$ and we have $\alpha^S = \phi_N^* \theta - \theta$ for $\theta \in \Omega^0_{\text{loc}}(\mathcal{M}, \mathbb{R})$. Hence $\text{hol}^\Xi_\beta(\gamma) = \int_\gamma \rho^S + \phi_N^* \theta - \theta = \int_\gamma (\rho^S + d\theta)$. We define $\beta = \rho^S + d\theta$ and using Proposition 17 we have $\phi^* \beta = \rho^S + d\alpha^S$ and hence $\beta$ is $\mathcal{G}$-invariant.

$G_3 \Rightarrow G_1$. If $\beta \in \Omega^1_{\mathcal{G},\text{loc}}(\Gamma(E))$ satisfies $\text{hol}^\Xi_\beta(\gamma) = \int_{\gamma} \beta$ we have $\text{curv}(\Xi) = d\beta \in \Omega^2_{\text{loc}}(\Gamma(E))$, and by Proposition 11 there exists a section $S$ such that $\rho^S = \beta \in \Omega^1_{\mathcal{G},\text{loc}}(\Gamma(E))$. By Proposition 15 we have $\alpha^S = 0$, and hence $S$ is $\mathcal{G}$-equivariant. Finally, $S$ is $\Xi$-local as $\text{curv}(\Xi) = d\beta \in \Omega^2_{\text{loc}}(\Gamma(E))$ and we have $\mu^\Xi(X) = \rho^S(X_N) + \alpha^S(X) = \rho^S(X_N) \in \Omega^0_{\text{loc}}(\Gamma(E))$, where we have used that $\alpha^S = 0$ as $\alpha^S = 0$. We conclude that $\text{curv}_\phi(\Xi) \in \Omega^2_{\mathcal{G},\text{loc}}(\Gamma(E))$. $\blacksquare$

By definition any $\mathcal{G}$-flat connection $\Xi$ is local, and any section $S$ such that $\rho^S = 0$ is $\Xi$-local. If we define $K^\mathcal{G}_{\text{loc}}(\Gamma(E)) = k(H^1_{\mathcal{G},\text{loc}}(\Gamma(E))) \subset \text{Hom}(\mathcal{G}/\mathcal{G}_0, \mathbb{R}/\mathbb{Z})$ then we have the following

Proposition 32 If $\Xi$ is a $\mathcal{G}$-flat connection on $\mathcal{U} \to \Gamma(E)$, then there exists a $\Xi$-local and $\mathcal{G}$-equivariant section $S$ if and only if $\kappa^\Xi \in K^\mathcal{G}_{\text{loc}}(\Gamma(E))$. 

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5 Concluding remarks

We have obtained necessary and sufficient conditions for physical anomaly cancellation in terms of the existence of a local $\mathcal{G}$-invariant 1-form $\beta$ satisfying certain conditions. The advantage of this approach with respect to the BRST cohomology is that $\Omega^1_{\text{loc}}(\Gamma(E))^\mathcal{G}$ can be completely determined in terms of the variational bicomplex of $J^\infty E$ and its cohomology can be related to Gelfand-Fuks cohomology of formal vector fields (see [13]). This technique is applied in [13] to study physical gravitational anomalies. It is proved that if the equivariant holonomy does not vanish, it is possible to cancel the physical gravitational anomaly only in dimensions $n = 3 \mod 4$ and by means of a Chern-Simons counterterm.

The problem of global anomaly cancellation in dimension 3 has been recently reanalyzed by Witten in [20] and [21]. In these papers it is conjectured that to cancel the anomaly it is not sufficient with the existence of an invariant partition function. To have an anomaly free theory consistent with the principles of unitarity, locality and cutting and pasting, the phases of the partition functions for different 3-manifolds should also be fixed. We hope that our geometric characterization of anomaly cancellation can be used to study this problem.

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