Participation factors and sub-Gramians in the selective modal analysis of electric power systems

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Abstract: The solution of Lyapunov equations can be represented as a sum of Hermitian matrices corresponding either to particular eigenvalues of the system matrix, or to pairwise combinations of these eigenvalues. These eigen-parts or sub-Gramians proved to be useful for the stability analysis of electric power systems. In this paper we compare and contrast the sub-Gramians and participation factors as applied to the power system state estimation. Using the sub-Gramian approach we introduce the energy participation factor as a new indicator for selective modal analysis. For a stable system it characterizes the participation of i-th mode and initial k-th state in the integrated energy produced in k-th state. We explain the conceptual meaning and practical usefulness of energy participation factors and contrast them with the conventional participation factors in a selective modal analysis of the IEEE 57-bus test model.

Keywords: Selective modal analysis, Participation Factors, sub-Gramians, Energy Participation Factors, electric power systems, small-signal stability analysis

1. INTRODUCTION

One of the most popular methods for studying the small signal stability of electric power systems is a modal analysis based on the calculation of spectrum of the dynamics matrix of the linearized system model. The real part of the eigenvalue closest to the imaginary axis is used as a measure of the system proximity to its margin of stability. This criterion is based on the direct Lyapunov method, which asserts that the non-linear dynamic system is stable at small deviations, if its linearized model is asymptotically stable (Lyapunov, 1893; Polyak and Scherbakov, 2002). This approach can be applied to control systems of different nature, including multi-mode and multivariable-control systems (Vassilyev and Kosov, 2011). In fact, to calculate the degree of stability, it suffices to know only a few eigenvalues closest to the imaginary axis. This problem can be solved by the selective modal analysis, which became a focal area of modal analysis for power systems in recent decades (Arnold, 1951; Martins, 1997; Pavella et al., 2000). Particular attention was paid to the calculation of the participation factors that allow for analysing the relationship between the state variables and the eigenvalues of the linearized dynamic model (Pagola et al., 1989; Pérez-Arriaga et al., 1990; Abed et al., 2000; Garofalo et al., 2002).

An alternative approach to the standard eigen-analysis was proposed in (Yadykin, 2010; Yadykin et al., 2014). In these papers the solution of Lyapunov equation was represented as a sum of Hermitian matrices corresponding either (i) to particular eigenvalues of the system matrix, or (ii) to pairwise combinations of these eigenvalues. Each eigen-part was named sub-Gramian. Sub-Gramians characterize the contribution of eigenmodes or their pairs in the variation of system energy defined by an appropriate Gramian. It was argued in (Yadykin, Kataev et al., 2016; Yadykin, Grobovoy et al., 2016) that the sub-Gramians method allows a more accurate assessment of the resonant interaction between low-frequency inter-area oscillations in electric power systems than the standard analysis of eigenvalues. Such oscillations may occur within the power facility, regional power grid or global power systems (Pai, 1989). The development of renewable energy sources and transmission lines in recent years makes the monitoring of dangerous weakly stable oscillations an urgent problem (Weber and Ali, 2016; Neuman, 2016). It is well known that such oscillations may lead to the occurrence of the voltage avalanche and cascading failures. The loss of stability is accompanied by accumulation of energy in the low-frequency oscillations, which cause the resonant reaction in the system. This observation is confirmed by theoretical analysis of the processes in the
linearized model of the system using $H_2$ norm of its transfer function (Antoulas, 2005). When the system is nearing its stability boundary a square $H_2$ norm of the transfer function tends to plus infinity. This value can serve as a measure of the system proximity to its margin of stability, in much the same way as the distance of eigenvalues from the imaginary axis is used in the standard eigen-analysis. The calculation of energy of weakly stable eigenmodes is also used in the measurement methods to identify the parameters of dangerous oscillations in real time (CIEE report, 2010).

Some examples of using sub-Gramians for the small-signal stability analysis and theoretical comparison of this method with other methods of modal analysis have been already presented in (Yadykin, Kataev et al., 2016; Yadykin, Grobovoy et al., 2016). In this paper we specifically investigate the relation between the participation factors and sub-Gramians. Using the sub-Gramian approach we introduce a concept of energy participation factors as new indicators for selective modal analysis. For a stable system, it characterizes the participation of $i$-th mode and initial perturbation in $k$-th state in the integrated perturbation energy produced in $k$-th state. We explain the conceptual meaning and practical usefulness of energy participation factors and contrast them with the conventional participation factors in a selective modal analysis of the IEEE 57-bus test model. In the following Section we provide some theoretical background. In Section 3 we introduce and explain the concept of energy participation factors. In Section 4 we describe a simulation experiment and discuss the obtained results.

2. THEORETICAL BACKGROUND

2.1. Participation Factors

In this subsection we remind the definition and some properties of the participation factors from (Pagola et al., 1989; Pérez-Arriaga et al., 1990; Garofalo et al., 2002). Consider an autonomous linear time-invariant system

$$\dot{x}(t) = A x(t),$$

where $x \in \mathbb{R}^n$ is a system state vector and $A \in \mathbb{R}^{n \times n}$ is a dynamic matrix with a simple spectrum, which can be represented as

$$A = U \Lambda U^\dagger = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix} \begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_n^T \end{pmatrix},$$

where $U V = V U = I$ and $(\cdot)^T$ is an operation of transpose. Then

$$p_{ki} \equiv u_k^i v_i^k$$

and

$$p_{kij} \equiv u_k^i v_j^i$$

are called participation factors (PF) and generalized participations respectively (Pagola et al., 1989). The PF $p_{ki}$ “weights” the participation of the $i$-th mode in the $k$-th state variable and vice versa. We remind two important properties of generalized participations from (Garofalo et al., 2002).

**Property 1.** The generalized participation $p_{kij}$ is the sensitivity of the $i$-th eigenvalue $\lambda_i$ with respect to the element $a_{jk}$ of $A$, i.e.

$$p_{kij} = \frac{\partial \lambda_i}{\partial a_{jk}} \quad p_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}}$$

(4)

Define the residue matrices $R_i$ as the coefficients in the expansion of the resolvent of the matrix $A$:

$$(I s - A)^{-1} = \frac{R_1}{s - \lambda_1} + \frac{R_2}{s - \lambda_2} + \cdots + \frac{R_n}{s - \lambda_n}$$

(5)

**Property 2.** The generalized participations $p_{kij}$ are the coefficients of the corresponding residue matrix $R_i$, i.e.

$$p_{kij} = e_k^T R_i e_j, \quad R_i = \sum_k p_{kij} e_k \cdot e_j^T,$$

(6)

where $e_j$ and $e_k$ are the $j$-th and $k$-th columns of the identity matrix.

2.2. Gramians and sub-Gramians

In this subsection we remind the definition and some properties of the Gramians and sub-Gramians from (Yadykin, 2010; Yadykin et al., 2014; Yadykin and Iskakov, 2017). An infinite Gramian is a positive definite solution $P = P^* > 0$ of the matrix Lyapunov equation:

$$AP + PA^* = -Q, \quad Q = Q^* > 0$$

(7)

For simplicity, we assume further that the matrix $A$ has a simple spectrum. In this case the solution of (7) can be written as (Yadykin and Iskakov, 2017)

$$P = -\sum_{i,j=1}^n \frac{R_i R_j^*}{\lambda_i + \lambda_j},$$

(8)

where $R_i$ and $R_j$ are the residue matrices defined by (5) corresponding to $\lambda = \lambda_i$ and $\lambda = \lambda_j$ respectively.

In particular, it follows from (5) that

$$\sum_{j=1}^n \frac{R_j^*}{\lambda_i - \lambda_j} = [(\lambda_i I - A)^{-1}]^* = - (\lambda_i I + A^*)^{-1}$$

Substituting this in (8), we obtain another form of spectral decomposition

$$P = -\sum_i R_i Q (\lambda_i I + A^*)^{-1} = -\sum_i (\lambda_i I + A)^{-1} Q R_i^*,$$

(9)

Hermitian parts of matrices in the spectral decompositions (8) and (9)

$$\bar{P}_i = -\{R_i Q (\lambda_i I + A^*)^{-1}\}_{H}, \quad P_{ij} = -\{R_i Q R_j^*\}_{H}$$

(10)

have been named sub-Gramians. Here by $\{\cdot\}_{H}$ we denote the Hermitian part of a matrix. Using sub-Gramians (SG) the decompositions (8) and (9) can be written as

$$P = \sum_{i=1}^n \bar{P}_i = \sum_{i,j=1}^n P_{ij}, \quad \bar{P}_i = \sum_{j=1}^n P_{ij}$$

(11)

The SG $\bar{P}_i$ and $P_{ij}$ characterize the contribution of eigenmodes or their pairs into the asymptotic variation of the
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