Information filtering by similarity-preferential diffusion processes

An Zeng, Alexandre Vidmer, Matúš Medo and Yi-Cheng Zhang

Department of Physics, University of Fribourg - Chemin du Musée 3, CH-1700 Fribourg, Switzerland

received 3 October 2013; accepted in final form 24 February 2014
published online 13 March 2014

PACS 89.75.-k – Complex systems
PACS 89.65.-s – Social and economic systems
PACS 89.20.Ff – Computer science and technology

Abstract – Recommender systems provide a promising way to address the information overload problem which is common in online systems. Based on past user preferences, a recommender system can find items that are likely to be relevant to a given user. Two classical physical processes, mass diffusion and heat conduction, have been used to design recommendation algorithms and a hybrid process based on them has been shown to provide accurate and diverse recommendation results. We modify both processes as well as their hybrid by introducing a parameter which can be used to enhance or suppress the weight of users who are most similar to the target user for whom the recommendation is done. Evaluation on two benchmark data sets demonstrates that both recommendation accuracy and diversity are improved for a wide range of parameter values. Threefold validation indicates that the achieved results are robust and the new recommendation methods are thus applicable in practice.

Copyright © EPLA, 2014

Introduction. – The amounts of data made available by modern World Wide Web sites far exceed the information capability of any individual. As the user base of these sites expands and increasing part of our lives is monitored and stored online, the problem of information overload becomes more and more acute. Information filtering tools thus have become essential to online users. Recommender systems represent an effective approach to information filtering where patterns in past user actions are analyzed to choose a small number of items that are likely to be appreciated by a given target user [1,2]. Approaches to recommendation include collaborative filtering [3], content-based analysis [4], dimensionality reduction techniques [5], tag-aware algorithms [6], trust-aware algorithms [7] and algorithms based on social impact [8]. An extensive review of various methods and their performance is presented in [9].

Information filtering has recently attracted the attention of physicists who have used network representations of the data, bipartite user-item networks in particular, to devise new recommendation algorithms motivated by heat conduction [10] and mass diffusion [11]. The hybrid of these two algorithms was shown to simultaneously improve recommendation accuracy and diversity [12]. This hybrid has been extended in multiple directions by modifying the initial configuration [13], introducing a ground node [14], and personalizing the hybridization parameter [15]. Network manipulation has been shown to solve the cold-start problem in recommendation [16]. A method for the removal of redundant links has been proposed to increase the efficiency of the recommendation process [17]. Preferential diffusion towards low-degree items has been shown to further enhance diversity and novelty of recommendations [18]. The issue of long-term influence of recommendation on the evolution of information systems has been recently studied [19]. See [20] for a review of recommender systems from a physicists’ perspective.

Unlike the works above which alter the original hybrid method [12] by either manipulating the underlying bipartite network or by changing the hybridization procedure, we propose here to modify the diffusion process itself. In particular, we introduce an additional model parameter which makes it possible to increase the influence of users who are most similar to a target user for whom the recommendation is computed. We demonstrate that the resulting algorithm can improve recommendation accuracy and diversity, while increasing novelty of the recommended items (i.e., decreasing their average degree). The optimal value of the newly introduced parameter strongly varies with data sparsity with sparse data requiring that lesser preference is given to similar users or, even, that their weight is made more equal to the weight of less similar
users. The parameter range over which the recommendation performance improves is broad which simplifies the new method’s application in practice.

**Recommendation algorithms.** — Data produced by many e-commerce systems can be modeled by a bipartite network where users and objects are modeled by two distinct kinds of nodes and links represent the interactions of users with contents. We assume that there are \( I \) item nodes and \( U \) user nodes in total. A particular bipartite network can be represented by an adjacency matrix \( \mathbf{A} \), where the element \( a_{i\alpha} \) equals 1 if user \( i \) has collected (or otherwise interacted with) item \( \alpha \), and 0 otherwise. For the sake of clarity, we label users and items with Latin and Greek letters, respectively. \( k_i \) and \( k_\alpha \) thus denote the degree of user node \( i \) and item node \( \alpha \), respectively.

We first describe the similarity-preferential mass diffusion (SPMD) method. For a target user \( i \) for whom recommendation is done, each of the originally collected objects is assigned with one unit of resource. The resource of each object is then evenly distributed to all the neighboring users who have collected this object who receive \( 1/k_\alpha \) each. User \( j \) receives contribution from item \( \alpha \) only if both \( i \) and \( j \) are connected to this item which is equivalent to \( a_{i\alpha}a_{j\alpha} = 1 \). The final resource received by user \( j \) can be thus written as

\[
 f_{ij}^{(SPMD)} = \sum_{\alpha=1}^{I} \frac{a_{i\alpha}a_{j\alpha}}{k_\alpha}. \tag{1}
\]

The obtained \( f_{ij} \) value can be used to measure the similarity between users \( i \) and \( j \) (see, for example, [21]). In the original mass diffusion method [11], recommendation scores of distinct items are obtained by evenly distributing \( f_{ij} \) to all items connected with user \( j \). As a result, the weight of user \( i \) is directly proportional to the similarity value \( f_{ij} \).

To further boost the weight of the most similar users, we modify the similarity value from \( f_{ij} \) to \( f_{ij}^\theta \) which is then evenly distributed to neighboring items as before. Here \( \theta \) is a tunable parameter which results in the original mass diffusion (MD) method when \( \theta = 1 \), enhances the weight of similar users when \( \theta > 1 \), and suppresses the weight of similar users when \( \theta < 1 \) (note that \( \theta < 0 \) would lead to user weight decreasing with similarity which is not reasonable). The final amount of resource on item \( \beta \) is obtained by summing over all users who may send resource to it and therefore

\[
 f_{i\beta}^{(SPMD)} = \sum_{j=1}^{U} \frac{a_{j\beta}f_{ij}^\theta}{k_j}. \tag{2}
\]

The whole SPMD process is illustrated in fig. 1(a). To prevent recommendation of items that have been already collected by the target user \( i \), we set \( f_{i\beta} \) to zero \( \forall \beta : a_{i\beta} = 1 \). The recommendation list for this user is then obtained by sorting all items according to \( f_{i\beta} \) in a descending order. From the community structure point of view, using \( \theta > 1 \) means that the target user is more likely to be recommended items collected by the users from \( i \)'s community. This is in line with [22] which shows that clustering can emerge through common interests of users in online bipartite networks. A study of improving the link prediction performance by community structure information can be found in [23]. In addition to addressing a different problem (recommendation instead of link prediction), our approach is simpler because it does not require prior community analysis which is computationally expensive and often unreliable [24].

The similarity-preferential heat conduction (SPHC) method is similar to SPMD but follows a heat-conduction motivated formula for user similarity

\[
 f_{ij}^{(SPHC)} = \sum_{\alpha=1}^{I} \frac{a_{i\alpha}a_{j\alpha}}{k_j}. \tag{3}
\]

In the second step, normalization is again with respect to the target node \( \beta \) instead of the initial node \( j \), and thus

\[
 f_{i\beta}^{(SPHC)} = \sum_{j=1}^{U} \frac{a_{j\beta}f_{ij}^\theta}{k_j}. \tag{4}
\]

The whole SPHC process is illustrated in fig. 1(b). When \( \theta = 1 \), this new method simplifies to the original heat conduction (HC) method [12].

Finally, we consider the same nonlinear hybridization of the two algorithms as in [12]. The user similarity becomes

\[
 f_{ij}^{(SPHY)} = \sum_{\alpha=1}^{I} \frac{a_{i\alpha}a_{j\alpha}}{k_\alpha k_j}. \tag{5}
\]
and the final resource values are

\[ f_{i\beta}(\text{SPHY}) = \sum_{j=0}^{U} \frac{a_{j\beta} f_{i}^{\beta}}{k_{\lambda}^j k_{1-\lambda}^j}, \quad (6) \]

The parameter \( \lambda \in [0, 1] \) adjusts the relative weight of the two algorithms with \( \lambda = 0 \) and \( \lambda = 1 \) corresponding to SPHC and SPMD, respectively. When \( \theta = 1 \), the hybrid method of [12] is recovered (we label it as HY).

**Data.** – To evaluate the performance of the new methods, we use two benchmark data sets. The MovieLens data [25] is available at [http://www.grouplens.org/](http://www.grouplens.org/). It contains 1682 movies and 943 users who rated the movies using the integer scale from 1 (worst) to 5 (best). The original data contains \( 10^5 \) ratings. To obtain an unweighted bipartite network, we represent all ratings of 3 or more as a link between a respective user and item. The resulting number of links is 82520. The Netflix data is a subset of the original data set released for the purpose of the Netflix Prize [26] and available at [http://www.netflixprize.com/](http://www.netflixprize.com/). The subset contains 3000 users and 3000 movies chosen from the original data at random and all links among them. Since the rating scale is the same as in MovieLens, we applied the same coarse-graining procedure and obtained 197248 links.

**Evaluation.** – We apply the standard evaluation procedure based on randomly dividing the data into two parts: a training set \( E^T \) containing 90% of all links (unless stated otherwise) and a probe set \( E^P \) containing the rest. Performance of a given algorithm is then assessed based on the position of hidden links from \( E^P \) in recommendation lists based only on the data in \( E^T \). Results presented here are obtained by averaging over 10 independent \( E^P/E^T \) divisions. A more cautious evaluation based on threefold validation is presented at the end of the following section.

**Ranking score** (RS) measures the average rank of \( E^P \) entries in recommendation lists. If rank of item \( \alpha \) in the recommendation list of user \( i \) is \( r_{i\alpha} \), we say that its ranking score is \( RS_{i\alpha} = r_{i\alpha}/L \). By averaging this quantity over all probe entries \( (i, \alpha) \), we obtain the ranking score

\[ RS = \frac{1}{|E^P|} \sum_{(i,\alpha) \in E^P} RS_{i\alpha}. \quad (7) \]

The lower the value, the better the recommendation method.

In real online systems, only the top part of a recommendation list is shown and even when users can usually inspect the rest, they rarely go far down the list [27]. We thus accomplish the ranking score with another accuracy measure, **precision**, which considers only top-\( L \) items in each user’s recommendation list. For user \( i \), precision is computed as

\[ P_i(L) = \frac{d_i(L)}{L} \quad (8) \]

where \( d_i(L) \) is the number of \( i \)’s probe set entries that are present in the top-\( L \) recommendation list (they correspond to successful recommendations). The precision \( P(L) \) for the whole system can be obtained by averaging \( P_i(L) \) over all users with at least one entry in the probe set. We use \( L = 20 \) in all \( L \)-dependent metrics presented here. While absolute results change with \( L \), the relative comparison of methods is largely independent of \( L \).

Along with accuracy, diversity of recommendations is also important for a recommender system [28]. We use two different diversity metrics here. **Personalization** measures the average difference between recommendation lists of distinct users. Denoting the number of items that occur in top \( L \) places of recommendation lists for both user \( i \) and user \( j \) as \( C_{ij}(L) \), the Hamming distance of the lists is

\[ H_{ij}(L) = 1 - \frac{C_{ij}(L)}{L} \quad (9) \]

The extreme values of 0 and 1 correspond to identical and entirely dissimilar lists, respectively. By averaging \( H_{ij}(L) \) over all user pairs to obtain \( H(L) \), one obtains the personalization value of a recommendation method. The higher the value of \( H(L) \), the more personalized the recommendation.

A good recommendation method should not place only high-degree items at the top of the resulting recommendation lists because those popular items are likely to be already known to the users and thus of little practical value. To this end, we measure recommendation **novelty** which focuses on the average degree of recommended items. Denoting the set of items in top \( L \) places of user \( i \)’s list as \( O_L \), novelty can be computed as

\[ N(L) = \frac{1}{UL} \sum_{i=1}^{U} \sum_{\alpha \in O_L} k_{i\alpha}. \quad (10) \]

The lower the value, the more novel the recommendation.
Table 1: Results achieved by different recommendation methods. Parameter values (if any) were always set by maximizing the ranking score for each individual method. Standard deviation values obtained by independent training set-probe set divisions are negligible with respect to differences among the methods.

| Data  | Method | RS | Prec. (20) | HD. (20) | N. (20) |
|-------|--------|----|------------|----------|---------|
| MovieLens | MD     | 0.096 | 0.115 | 0.703 | 278 |
|        | SPMD   | 0.084 | 0.129 | 0.837 | 238 |
|        | HC     | 0.135 | 0.006 | 0.862 | 7 |
|        | SPHC   | 0.122 | 0.013 | 0.904 | 14 |
|        | HY     | 0.075 | 0.129 | 0.903 | 179 |
|        | SPHY   | 0.072 | 0.141 | 0.917 | 174 |
| Netflix | MD     | 0.056 | 0.099 | 0.551 | 1169 |
|        | SPMD   | 0.053 | 0.106 | 0.632 | 1116 |
|        | HC     | 0.113 | 0.000 | 0.763 | 1 |
|        | SPHC   | 0.108 | 0.001 | 0.830 | 2 |
|        | HY     | 0.052 | 0.112 | 0.667 | 1058 |
|        | SPHY   | 0.045 | 0.123 | 0.759 | 955 |

Results. — We first assess the one-parametric SPMD method. As shown in fig. 2(a), (b), the best ranking score is obtained with $\theta > 1$ for both data sets. The optimal value and relative improvement with respect to the original MD method is $\theta^* = 2.6$ and 12.4% (for Movielens) and $\theta^* = 1.9$ and 6.4% (for Netflix), respectively. SPHC can be studied in the same way but as mentioned in [12], recommendations obtained purely with heat diffusion are too peculiar to be actually useful (their accuracy is very low). The resulting values are nevertheless reported in table 1.

As can be seen in fig. 2(c), (d), the ranking score of the two-parametric SPHY method outperforms the best result achievable under $\theta = 1$ in a broad region. The optimal parameter values and the relative improvement with respect to the $\theta = 1$ case is $\theta^* = 2.0$, $\lambda^* = 0.32$ and 5.0% (for Movielens) and $\theta^* = 3.0$, $\lambda^* = 0.40$ and 12.0% (for Netflix). Precision values achieved with SPHY for different parameter settings are shown in fig. 3 (left column). Using the optimal parameter values determined from the ranking score, relative precision improvement is 9.0% and 10.5% for Movielens and Netflix, respectively. These results confirm that introducing the similarity preference in the diffusion process can indeed improve the accuracy of recommendations.

Figure 3 (middle and right column) shows that the impact of $\theta$ on recommendation diversity is strictly monotonous: as $\theta$ grows, personalization increases and novelty improves. The increase of personalization with $\theta$ is a direct consequence of effectively relying on fewer users in producing recommendation for a target user. Since this small group can easily differ for one target user to another, the resulting recommendations are also different and thus personalization is high. The connection between $\theta$ and novelty is less direct but equally apparent. By using the optimal parameters determined from the ranking score, personalization and novelty are improved by 1.6% and 2.4% (for Movielens) and 13.8% and 9.7% (for Netflix), respectively.

Table 1 reports detailed results for all four metrics and all recommendation methods. One can see that for both data sets and all metrics, the previously known methods...
(MD, HC, and HY) are outperformed by their variants employing similarity-preferential diffusion. Even in the case of HC, which is a widely known diversity-favoring method, SPHC achieves significantly higher personalization for both MovieLens and Netflix data. We also examine the performance of our methods in the delicious.com data which is very sparse and features a hierarchically organized social network. Compared to the results in the Movielens and Netflix data, the advantage of SPHY over HY is smaller in the Delicious data.

We further investigate the effect of data sparsity on the HY and SPHY method. To this end, we move fraction $p$ of all links to the probe set and use the remaining $1 - p$ links as the training set (the previously reported results are recovered by setting $p = 0.1$). As $p$ increases (corresponding to sparser input data), the optimal ranking score naturally increases and the difference between HY and SPHY decreases (see fig. 4). The latter observation is no surprise as with very sparse data, information on user similarity is scarce and noisy and enhancing user weight based on this similarity thus yields smaller benefits. Interestingly, the optimal parameter values (again obtained by maximization of the ranking score) depend strongly on the data sparsity. In particular, as data gets very sparse ($p \gtrsim 0.8$ for both data sets), $\lambda^*$ = 1 (corresponding to using only mass diffusion) and $\theta^*$ becomes smaller than one. Both observations are understandable: when the data is sparse, it is better to rely on popular items (hence SPMD instead of SPHY) and make the weight of all users more equal (with $\theta < 1$) as opposed to making it more unequal when the data is dense. In summary, there is a crucial difference in how recommendation methods behave on dense and sparse data sets.

As we have seen above, the parameter values under which SPHY yields the best ranking score strongly depend on the input data. This means that no universally applicable values exist and to employ the recommendation method in practice, one has to learn the optimal parameter values from the data. To mimic this process in evaluation of recommender system, one typically uses a so-called threefold validation where a small part (usually 10% of all data) is moved from the previously introduced testing set to a learning set $E^L$. By comparing the recommendations based on the remaining training set with the learning set, one can determine the optimal parameter values that optimize a given aspect of recommendation (here we stick with the previously used $RS$ minimization as the only criterion). The learned optimal values are then used to produce recommendations based on $E^T \cup E^L$ which are then compared with entries in $E^P$ to finally measure the recommendation performance. In this way, recommendation methods with different number of free parameters can be evaluated and compared. Note that while introducing an additional parameter results in improved or at least unchanged performance in the usual train-probe approach, this is not necessarily the case in the threefold validation approach. From the machine learning point of view, threefold validation helps to avoid model over-fitting [29]. Table 2 demonstrates that the superiority of SPHY over HY is preserved also under threefold validation.

We finally analyze computational complexity of the studied recommendation methods. Among them, MD and HC are parameter free and thus need no learning step. Since they are based on three steps of diffusion on a bipartite network, the computational complexity per user is $O(k^2_u k_o)$ where $k_u$ and $k_o$ are, respectively, the mean degree of users and objects. SPMD, SPHC and HY have all one parameter each. Their computational complexity is $O((n+1)k^2_u k_o)$, where $n$ is the number of evaluation points of their respective parameters in the learning process. SPHY has two parameters, which results in computational complexity $O((n^2+1)k^2_u k_o)$. The number of evaluations points $n$, however, is essentially independent of the data size and thus does not impairs scalability of methods that require the learning step (or steps).

**Conclusion.** – Spreading processes on networks have been extensively used in recommendation [12]. To increase the diversity of recommendations, we proposed two new processes: similarity-preferential mass diffusion (SPMD) and similarity-preferential heat conduction (SPHC). These processes contain a tunable parameter

![Fig. 4: (Color online) The optimal ranking score $RS^*$ of HY and SPHY vs. $p$ in MovieLens (a) and Netflix (b). SPHY’s optimal parameter values vs. $p$ in MovieLens (c) and Netflix (d).](image-url)

| Data    | Method | $RS^*$ | $P(20)$ | $H(20)$ | $N(20)$ |
|---------|--------|--------|---------|---------|---------|
| MovieLens | HY     | 0.081  | 0.113   | 0.88    | 179     |
|          | SPHY   | 0.078  | 0.120   | 0.90    | 168     |
| Netflix  | HY     | 0.054  | 0.103   | 0.61    | 996     |
|          | SPHY   | 0.048  | 0.113   | 0.75    | 862     |
which makes it possible to gradually vary the preference
given to similar users. Our results obtained on two stan-
dard data sets (MovieLens and Netflix) show that the hy-
brid of these two processes improves both diversity and
accuracy of the produced recommendations. Our study of
the optimal parameter values showed that they strongly
depend on sparsity of the input data. When the data is
relatively dense, a substantial increase of the weight of
similar users produces the best results. With extremely
sparse data, user weights are best to be made more equal
by $\theta < 1$. We tested the new hybrid method also by
threefold data division and showed that the observed im-
provement is not only an artifact caused by introducing an
additional parameter but a real effect which can be used
in practice.

The presented modification in essence resembles $K$-NN
collaborative filtering methods which are based on con-
sidering only $K$ most similar users ($K$ nearest neighbors
—hence the abbreviation) in the user-based collaborative
filtering [30]. For suitably chosen $K$, these methods tend
to perform better than unconstrained collaborative filtering
where all users contribute proportionally to their sim-
ilarity value. Note that by introducing the $\theta$-parameter in
the previously known MD, HC, and HY methods, we in-
creased the weight of highly similar users at the expense of
less similar users, thus effectively decreasing the number of
users who contribute to the resulting recommendation
scores. One can make the similarity closer by considering
only $K$ most similar users in MD, HC, and HY but our results
show that the thus-achieved improvements are smaller than those of the presented similarity-preferential
methods. Moreover, the generalization based on $\theta$ is very
easy to implement and does not substantially increase the
methods’ computational complexity.

Our work can lead to many applications. For exam-
ple, the link prediction in directed networks depends on
a so-called “Bi-fan” structure [31]. The similarity prefer-
ence can be introduced as a weighting strategy in this
structure and improve the prediction precision. More gen-
erally, similarity-preferential diffusion can be applied to
any multi-step diffusion process on networks. There are
actually many different global and local diffusion based
methods to estimate the similarity between nodes [20,32].
We believe that the similarity preference mechanism can
improve them.

***

This work was partially supported by the Future and
Emerging Technologies program of the European Commissi-
on FP7-COSI-ICT (project QLectives, grant no. 231200)
and by the Swiss National Science Foundation (grant
no. 200020-143272).

REFERENCES

[1] Resnick P. and Varian H. R., Commun. ACM, 40
(1997) 56.

[2] Adomavicius G. and Tuzhilin A., IEEE Trans. Knowl.
Data Eng., 17 (2005) 734.

[3] Herlocker J. L., Konstan J. A., Terveen L. G., and
Riedl J. T., ACM Trans. Inf. Syst. Secur., 22 (2004) 5.

[4] Balabanovic M. and Shoham Y., Commun. ACM, 40
(1997) 66.

[5] Koren Y., Bell R. and Volinsky C., Computer, 42
(2009) 30.

[6] Zhang Z.-K., Zhou T. and Zhang Y.-C., Physica A,
389 (2010) 179.

[7] Burke R., Lect. Notes Comput. Sci., 4321 (2007) 377.

[8] Zeng W., Zeng A., Shang M.-S. and Zhang Y.-C.,
Eur. Phys. J. B, 86 (2013) 375.

[9] Cacheda F., Carneiro V., Fernández D. and For-
moso V., ACM Trans. Web, 5 (2011) 1.

[10] Zhang Y.-C., Blattner M. and Yu Y.-K., Phys. Rev.
Lett., 99 (2007) 154301.

[11] Zhou T., Ren J., Medo M. and Zhang Y.-C., Phys.
Rev. E, 76 (2007) 046115.

[12] Zhou T., Kuscsik Z., Liu J., Medo M., Wakeling
J. R. and Zhang Y.-C., Proc. Natl. Acad. Sci. U.S.A.,
107 (2010) 4511.

[13] Liu C. and Zhou W.-X., Physica A, 391 (2012) 5704.

[14] Zhou Y., Liu L., Liu W. and Zhang J., PloS ONE, 8
(2013) e70094.

[15] Guan Y., Zhao D.-D., Zeng A. and Shang M.-S.,
Physica A, 392 (2013) 3417.

[16] Zhang F. G. and Zeng A., EPL, 100 (2012) 58005.

[17] Zhang Q.-M., Zeng A. and Shang M.-S., PLoS ONE,
8 (2013) e62624.

[18] Liu L. and Liu W., Phys. Rev. E, 83 (2011) 066119.

[19] Zeng A., Yeung C. H., Shang M.-S. and Zhang Y.-C.,
EPL, 97 (2012) 18005.

[20] Liu L., Medo M., Yeung C. H., Zhang Y.-C., Zhang
Z.-K. and Zhou T., Phys. Rep., 519 (2012) 1.

[21] Zhou T., Liu L. and Zhang Y. C., Eur. Phys. J. B, 71
(2009) 623.

[22] Grujić J., Mitrović M. and Tadić B., Proceedings of
the 16th International Conference on Digital Signal
Processing (DSP) (IEEE) 2009, p. 259.

[23] Yan B. and Gregory S., Phys. Rev. E, 85 (2012) 056112.

[24] Lancichinetti A. and Fortunato S., Phys. Rev. E, 84
(2011) 066122.

[25] Herlocker J. L., Konstan J. A., Borchers A. and
Riedl J., Proceedings of the 22nd ACM SIGIR Confer-
ence on Research and Development in Information Re-
trieval (ACM) 1999, p. 230.

[26] Bennett J. and Lanning S., Proceedings of the KDD
Cup Workshop (ACM) 2007, p. 35.

[27] Lempel R. and Moran S., Proceedings of the 12th Inter-
national World Wide Web Conference (ACM) 2003, p. 19.

[28] Ziegler C. N., McNeen S. M., Konstan J. A. and
Lausen G., Proceedings of the 14th International World
Wide Web Conference (ACM) 2005, p. 22.

[29] Abu-Mostafa Y. S. and Magdon-Ismail M., Learning
From Data (AMLBook) 2012.

[30] Sarwar B., Karypis G., Konstan J. and Riedl J.,
Proceedings of the 10th International World Wide Web
Conference (ACM) 2001, p. 285.

[31] Zhang Q.-M., Liu L., Wang W.-Q., Zhu Y. X. and
Zhou T., PloS ONE, 8 (2013) e65437.

[32] Zeng A. and Liu L., Phys. Rev. E, 83 (2011) 056123.