The Generation and Exchange of Entanglement via Zeeman Effect

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Abstract

In this paper we show a new way to generate entanglement via two identical three-level atoms splitting in the magnetic field interacting with the cavity field. By the system we investigate, We can acquire the EPR state, multi-dimensional entangled states etc. which are more stable than usual realization by high-energy-level Rydberg atoms and we can realize the local exchange operator too. We also achieve the goal of maintaining long- time entanglement between atoms. At last, by using the procedure of local exchange, we put forward an experimental scheme for quantum feedback.
I. INTRODUCTION

It is well known that entanglement has been variously studied and how to get entanglement fast and stably plays key roles in the quantum information processing. Multipartite-entanglement is a great resource which is not only of importance for test of quantum mechanics against local hidden theory[1], but also useful in quantum teleportation, dense coding and quantum cryptography[2]. Most of research in quantum non-locality and quantum information is based on entanglement of two-level particles. Entangled states for two-level particles have been observed for photons, atoms in cavity QED, and ions in trap[3,4]. Zou et al.[5] have present a scheme to generate a maximally entangled state of two three-level atoms with a non-resonant cavity by cavity collisions. A scheme for generating entangled states for multilevel atoms in a thermal cavity has been proposed[6]. Teleporting entanglement of cavity-field states has also been presented[7]. Recently, the system of three-level atom in two-mode field has been exactly calculated[8]. Most of the work concentrate on two-level atoms’ manipulation and evolution with the cavity field. Although a scheme for generating entangled states for multilevel atoms has been proposed[6], its content still bases on the two-level atoms.

In this paper, we give a new way to realize the entanglement of three-level atoms, we investigate two “L” type three-level atoms whose spin are both zero interacting with the cavity field in the magnetic field. First, we consider two identical three-level atoms Zeeman-splitting in the magnetic field plus the cavity field, As it will be shown, we can easily acquire EPR states, multi-dimensional entangled states, etc. The atoms we use here are normal atoms which are different from three-level Rydberg atoms with high energy levels. It will be more convenient to realize the entanglement between atoms and the entanglement is more stable. And at the end of our paper, we present an experimental scheme for quantum feedback.

II. GENERATION OF ENTANGLEMENT

We exploit two zero-spin atoms interacting with electromagnetic field in a stable-constant magnetic field. In each atom one electron has been excited to $p$ state. For convenience we denote $|ij\rangle$ as the state with $l = i, m = j$. Due to electric dipole transition rule, the transition in the subspace of $l = 1$ will not be allowed. The transition can be allowed only
between the subspaces of \( l = 1 \) and \( l = 0 \). The Lamb shift of the atoms can be ignored because it is far smaller than the system’s resonant energy. Under this approximation, only subsequent transitions \(|1, \pm 1\rangle \leftrightarrow |00\rangle\) will appear. Then the interaction between the atoms and the electromagnetic field can be written as

\[
g \left[ (l_1^+ + l_2^+) a + (l_1^- + l_2^-) a^\dagger \right]
\]  

(1)

where \( g \) is the interaction coefficient between the atoms and the electromagnetic field, and \( l_i^\pm = |1\rangle_i\langle 0| + |0\rangle_i\langle -1|, l_i^- = (l_i^+)\dagger \)

The interaction between the atoms and the magnetic field and the interaction between the atoms are \(- (\vec{\mu}_1 + \vec{\mu}_2) \cdot \vec{B}, \quad \lambda \vec{\mu}_1 \cdot \vec{\mu}_2\) respectively. Suppose the direction of the magnetic field is along \( z \) axis, we can simplify (3) as below

\[- (\mu_{1z} + \mu_{2z}) B_z + \lambda \mu_{1z} \mu_{2z} \]  

(2)

where \( \lambda \) is the interaction coefficient between the magnetic moments. The equation above can be rewritten as

\[\beta (l_{1z} + l_{2z}) + \alpha l_{1z} \cdot l_{2z} \]

(3)

where \( \beta = e/2\mu \) and \( \alpha = \lambda e^2/4\mu^2 \). And finally we obtain the Hamiltonian of the system

\[H = \omega \ a^\dagger a + \beta (l_{1z} + l_{2z}) + \alpha l_{1z} \cdot l_{2z} \]

(4)

\(\omega \) is the frequency of the electromagnetic field.

The Hamiltonian in the interaction picture is

\[H^{(I)}_i = \alpha \ l_{1z} l_{2z} + \]

\[g \left[ e^{i(\beta - \omega)t} (l_1^+ + l_2^+) a + e^{-i(\beta - \omega)t} (l_1^- + l_2^-) a^\dagger \right] \]

(5)

Then consider the exact resonance, we can neglect the phase terms \( e^{i(\beta - \omega)t} \) and \( e^{-i(\beta - \omega)t} \).

The interaction Hamiltonian in the interaction picture can be simplified to

\[H^{(I)}_i = g \left[ (l_1^+ + l_2^+) a + (l_1^- + l_2^-) a^\dagger \right] + \alpha l_{1z} l_{2z} \]

(6)

It is easy to find that \( a^\dagger a + l_{1z} + l_{2z} \) is the conversation quantity of the system, thus we can choose states \(|N⟩_p|l_{1z} l_{2z}⟩_a\) as the complete basis of the system with \( a \) and \( p \) indicating
atoms and photon respectively and $N$ is the number of photons. Suppose $a\dagger a + l_{1z} + l_{2z} = n$ then we can obtain the invariant space of the system. The complete basis of invariant space are: $|n+2\rangle_p|-1,-1\rangle_a$, $|n+1\rangle_p|0,-1\rangle_a$, $|n+1\rangle_p|-1,0\rangle_a$, $|n\rangle_p|1,-1\rangle_a$, $|n\rangle_p|0,0\rangle_a$, $|n\rangle_p|-1,1\rangle_a$, $|n-1\rangle_p|1,0\rangle_a$, $|n-1\rangle_p|0,1\rangle_a$, $|n-2\rangle_p|1,1\rangle_a$. Thus, in this invariant subspace, the interaction Hamiltonian in the interaction picture can be expressed as $9 \times 9$ matrix below

$$
\begin{pmatrix}
\alpha & g\sqrt{n+2} & g\sqrt{n+2} & 0 & 0 & 0 & 0 & 0 & 0 \\
g\sqrt{n+2} & 0 & 0 & g\sqrt{n+1} & g\sqrt{n+1} & 0 & 0 & 0 & 0 \\
g\sqrt{n+2} & 0 & 0 & 0 & g\sqrt{n+1} & g\sqrt{n+1} & 0 & 0 & 0 \\
0 & g\sqrt{n+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & g\sqrt{n+1} & g\sqrt{n+1} & 0 & 0 & 0 & g\sqrt{n} & g\sqrt{n} & 0 \\
0 & 0 & g\sqrt{n+1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & g\sqrt{n} & g\sqrt{n} & 0 & 0 & 0 & g\sqrt{n-1} \\
0 & 0 & 0 & 0 & g\sqrt{n} & g\sqrt{n} & 0 & 0 & g\sqrt{n-1} \\
0 & 0 & 0 & 0 & 0 & 0 & g\sqrt{n-1} & g\sqrt{n-1} & \alpha
\end{pmatrix}
$$

Because $\alpha \ll \beta, g$, under the fist order approximation, we can simplify the above matrix with $\alpha = 0$. With this simplification the matrix is still too large, we can choose appropriate $n$ to simplify the above matrix furthermore, for convenience, we can set $n$ to be zero, then the matrix can be simplified below

$$
H_i^{(I)} = \begin{pmatrix}
0 & g\sqrt{2} & g\sqrt{2} & 0 & 0 & 0 \\
g\sqrt{2} & 0 & 0 & g & 0 \\
g\sqrt{2} & 0 & 0 & 0 & g & g \\
0 & g & 0 & 0 & 0 \\
0 & g & g & 0 & 0 \\
0 & 0 & g & 0 & 0
\end{pmatrix}
$$

(7)

Then we can obtain the evolving state in Schrodinger picture:

$$
|\psi(t)\rangle = \exp(-iH_0t)\exp(-iH_i^{(I)}t)|\psi(0)\rangle|_{n=0,\omega=\beta} = \exp(-iH_i^{(I)}t)|\psi(0)\rangle|_{n=0,\omega=\beta}
$$
\[
\psi(0) = |\psi(0)\rangle = |0\rangle_p |1, -1\rangle_a = \\
\begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}
\]

then we can obtain the evolving state below

\[
|\psi(t)\rangle = \frac{\sqrt{2}}{2} \left[ -1 + \cos (\sqrt{7}gt) \right] |2\rangle_p |1, -1\rangle_a - \frac{i\sqrt{7}}{14} \left[ \sqrt{7} \sin (gt) + \sqrt{7} \sin (\sqrt{7}gt) \right] |1\rangle_p |1, 0\rangle_a + \frac{i\sqrt{7}}{14} \left[ \sqrt{7} \sin (gt) - \sqrt{7} \sin (\sqrt{7}gt) \right] |1\rangle_p |-1, 0\rangle_a + \frac{1}{7} \left[ -1 + \cos (\sqrt{7}gt) \right] |0\rangle_p |0, 0\rangle_a + \frac{1}{14} \left[ 6 + 7 \cos (gt) + \cos (\sqrt{7}gt) \right] |0\rangle_p |1, -1\rangle_a + \frac{1}{7} \left[ -1 + \cos (\sqrt{7}gt) \right] |0\rangle_p |0, 0\rangle_a + \frac{1}{14} \left[ 6 - 7 \cos (gt) + \cos (\sqrt{7}gt) \right] |0\rangle_p |1, -1\rangle_a
\]
When $t = \frac{2n\pi}{\sqrt{2}g}$, the state evolves to
\[
|\psi(t)\rangle = -\frac{i}{2} \left[ \sin \left( \frac{2n\pi}{\sqrt{2}g} \right) \right] |1\rangle_p |0, -1\rangle_a + \frac{i}{2} \left[ \sin \left( \frac{2n\pi}{\sqrt{2}g} \right) \right] |1\rangle_p |-1, 0\rangle_a \\
+ \frac{1}{2} \left[ 1 + \cos \left( \frac{2n\pi}{\sqrt{2}g} \right) \right] |0\rangle_p |1, -1\rangle_a + \frac{1}{2} \left[ 1 - \cos \left( \frac{2n\pi}{\sqrt{2}g} \right) \right] |0\rangle_p |-1, 1\rangle_a
\]  
(11)

by detecting the photons, we can obtain the two-atom entangled state immediately
\[
\frac{1}{\sqrt{2}} \left( |0, -1\rangle_{12} - |-1, 0\rangle_{12} \right)
\]  
(12)

Now we change the direction of the electromagnetic field and let it along x axis. After setting appropriate time we can obtain the entangled state
\[
\frac{1}{\sqrt{2}} \left( |1, -1\rangle_{12} + |-1, 1\rangle_{12} \right)
\]  
(13)

With the same process we can obtain the entangled state $\frac{1}{\sqrt{2}} \left( |1, 1\rangle_{12} + |-1, 1\rangle_{12} \right)$ with the electromagnetic field along y axis.

### III. REALIZATION OF LOCAL EXCHANGE OPERATOR

Furthermore, using the system we study, we can easily realize long-time entanglement between atoms and realize a local exchange. By the step below we can keep the information in the entangled atoms. Suppose the initial entangled state is $c_1 |0, -1\rangle_{12} + c_2 |-1, 0\rangle_{12}$ and these two atoms are in two cavity, respectively. Before the spontaneous radiation, we send two atoms whose states are both in $|{-1}\rangle$ into the two cavity respectively. In each cavity the conversation number $n = -1$, as it is shown above, we can easily get the evolving state as below
\[
|\psi(t)\rangle = e^{i\omega t} \begin{pmatrix}
\cos \left( \sqrt{2}gt \right) & -\frac{i\sin \left( \sqrt{2}gt \right)}{\sqrt{2}} & -\frac{i\sin \left( \sqrt{2}gt \right)}{\sqrt{2}} \\
\frac{i\sin \left( \sqrt{2}gt \right)}{\sqrt{2}} & \frac{1}{2} \left[ 1 + \cos \left( \sqrt{2}gt \right) \right] & \frac{1}{2} \left[ 1 - \cos \left( \sqrt{2}gt \right) \right] \\
\frac{i\sin \left( \sqrt{2}gt \right)}{\sqrt{2}} & \frac{1}{2} \left[ -1 + \cos \left( \sqrt{2}gt \right) \right] & \frac{1}{2} \left[ 1 + \cos \left( \sqrt{2}gt \right) \right]
\end{pmatrix} |\psi(0)\rangle |n = -1, \omega = \beta\rangle
\]  
(14)

when the evolving time of the system pass through $t = \frac{(2n+1)\pi}{\sqrt{2}g}$, we can obtain the exchange of states as below
\[
|\psi(0)\rangle = |0\rangle_p |0, -1\rangle_a = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow |\psi(t)\rangle |t = \frac{(2n+1)\pi}{\sqrt{2}g}\rangle = |0\rangle_p |-1, 0\rangle_a = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\]
As is shown above, we can realize local exchange process. Using this property we can keep long-time entanglement as below

\[ \langle 0 \rangle_p | -1, 0 \rangle_a = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow |\psi(t)\rangle_{t=(2n+1)p \sqrt{2g}} = |0\rangle_p |0, -1\rangle_a = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

where the first and the third atoms are in a cavity, and the second and the forth atoms are in the other. Both cavities are in vacuum states. We can see that after a period of evolution time the entanglement between the first two atoms is transferred to the last two atoms by proper arrangement of the atoms’ time of flight. After we choose a proper evolution time which is much smaller than the average spontaneous radiation time, we can persist the entanglement and store the information in the entangled state for a long time.

**IV. AN EXPERIMENTAL SCHEME OF QUANTUM FEEDBACK**

Furthermore, with the local exchange procedure above we can realize some kind of quantum feedback shown in the figure below. However, the conception of quantum feedback here is a little different from the traditional opinions about quantum feedback presented in paper [10,11]. The problem is that when we have a system which is designed for consistently producing the maximally entangled states, how we can obtain the information about the quality of the entangled states by our experimental system without changing them such as moving the instrument or adjusting the magnetic field or changing other necessary parts. In the experimental scheme shown below, with the local exchange procedure we can not only check the quality of our experimental system but also give positive feedback for our system.
The generation procedure of the first and the second atoms entangled state has been shown in sec II. And the procedure that the assisted third and fourth flight atoms finish the exchange process in the cavity one and cavity two has been shown in Eq. (15). The third and fourth atoms finally bring the entangled information of the first and second atoms out from the experimental system. After we measure the entangled state of the third and the fourth atoms, we can acquire the information such as the quality, the stability and the efficiency of our experimental system without changing them. From the result of the measurement we can adjust our experimental system and ameliorate the quality of the productivity of the experimental system. Such feedback procedure may need some classical information from the measurement of output states, but it has advantage that we have no need to "change" or "move" our experimental system. As we all know that quantum system is so sensitive and subtle that if we "change" or "move" any part of our experimental system, the results would be completely different. Accordingly, following the cyclic feedback procedure above and accumulating useful improvement, we will finally get a sound system to produce entangled states.
V. CONCLUSIONS

In summary, we have proposed a way to realize two-atom maximally entangled states and generate multi-atoms entangled states. By the model we use, we realize long-time entanglement and can keep the information in the entangled state for a long time. Finally, we also realize local exchange operator. Furthermore, by using the local exchange procedure, we also present an experimental scheme of quantum feedback.

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