Non-Minimal Cosmological Model in Modified Yang–Mills Theory

V. K. Shchigolev, G. N. Orekhova

Ulyanovsk State University, 42 L. Tolstoy Str., Ulyanovsk 432000, Russia

Abstract – In the present paper, we consider a model of non-minimal modified Yang-Mills (Y-M) theory in the Friedmann-Robertson-Walker (FRW) cosmology, in which the Y-M field couples to the scalar curvature through a function of its first invariant. We show that cosmic acceleration can be realized due to non-minimal gravitational coupling of the modified Y-M theory. Besides general study, we consider in detail the case of power-law coupling function. We derive the basic equations for the cosmic scale factor in our model, and provide several examples of their solutions.

PACS numbers: 98.80.-k, 98.80.Es, 04.30.-w, 04.62.+v

Key words: Cosmological Model, Non-Minimal Coupling, Yang-Mills Fields, Accelerated Expansion.

1 Introduction

Present accelerated expansion of the universe is well proved in many papers [1]-[8]. In order to explain such unexpected behavior of our universe, one can modify the gravitational theory [9]-[14], or construct various field models of so-called dark energy which equation of state satisfies $\gamma = p/\rho < -1/3$. The most studied models consider a canonical scalar field (quintessence) [15]-[17], a phantom field, that is a scalar field with a negative sign of the kinetic term [18]-[21], or the combination of quintessence and phantom in a unified model named quintom [22]-[25]. In such field dark energy scenarios, the potential choice plays a central role in the determination of the cosmological evolution.

The alternative approach to the problem of accelerated expansion is the consideration of various modifications of the gravity theory. Among such modifications, the theories with non-minimal coupling of a field to gravity are especially attractive [26]-[32]. For instance, scalar tensor theories are generalization of the minimally coupled scalar field theories in a sense that here the scalar field is non-minimally coupled with the gravity sector of the action i.e with the Ricci scalar $R$.

In non-minimal theories, the matter field participates in the gravitational interaction, unlike its counterpart in the minimally coupled case where it behaves as a non gravitational source [33]-[36].

In particular, the discovery of the accelerated expansion of the Universe encourages further development of the Y-M theory, including its non-minimal coupling to gravity. Numerous attempts have been made to consider modified Y-M theories in cosmology as alternatives to the dark energy (see, e.g., [37] and references therein). One of the directions in such a generalization of the Y-M theory is connected with a non-minimal extension of the Y-M field theory. There are several motivations to study non-minimal Y-M theory (see, e.g., [38, 39] and references therein).

Basically, the present study is a sequel to the paper [40], where the modified Y-M theory is investigated under the condition of minimal coupling to gravity. Following the considerations provided in [40], we study a model of non-minimal modified Y-M theory, in which the Y-M field couples to a function of the scalar curvature. We show that the accelerated expansion can be realized due to the non-minimal gravitational coupling of Y-M field. Besides general study, we give some examples of exact solution for the model under consideration, which surely can not cover all possible applications of this research.

*E-mail: vkshch@yahoo.com
†E-mail: gnorehova@mail.ru
2 The model equations

Let the action of our model be presented by a generalization of the modified Y-M action [40] on the case of non-minimal coupling:

\[
S = - \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa} \Psi(F_{ik}^{\alpha\beta}) + \Phi(F_{ik}^{\alpha\beta}) \right),
\]

(1)

where the Yang-Mills tensor is \( F_{ik}^{\alpha\beta} = \partial_i W_k^{\alpha\beta} - \partial_k W_i^{\alpha\beta} + f^{abc} W_i^b W_k^c \), and \( \Psi \) and \( \Phi \) are the arbitrary differentiable functions. In the case of \( \Psi \equiv 1 \), \( \Phi = \frac{1}{16\pi G} F_{ik}^{\alpha\beta} \), this action describes the Einstein-Yang-Mills theory. We assume that the universe space-time is described by a Friedmann-Robertson-Walker (FRW) geometry:

\[
ds^2 = N(t)^2 dt^2 - a^2(t)(dr^2 + \xi^2(r)d\Omega^2),
\]

(2)

where \( \xi(r) = \sin r, \sinh r \) in accordance with a sign of the curvature \( k = +1, 0, -1 \). To study the model based on action (1) in metrics (2), we substitute this metrics into action (1) and take into account that

\[
R = -\frac{6a\ddot{a} - a\dot{a}^2 + a^2N}{a^2N^3}
\]

for this metrics. As a result, we can obtain the following effective Lagrangian per unit solid angle:

\[
L_{\text{eff}} = \frac{3}{8\pi G} \left( \frac{a^2\ddot{a}}{N} + \frac{a\dot{a}^2}{N^2} - \frac{a^2\dot{N}}{N^2} + kaN \right) \Psi(I)\xi^2 - \Phi(I)a^3N\xi^2,
\]

(3)

where \( I = F_{ik}^{\alpha\beta} \). At the same time, the generalized Wu-Yang ansatz for \( SO_3 \) Yang-Mills field can be written down as [41]:

\[
W_0^\alpha = x^\alpha W(r,t), W_\mu^\alpha = \varepsilon_{\mu\rho\sigma} x^\rho \frac{K(r,t) - 1}{er^2} + \left( \delta_\mu^\alpha - \frac{x^\alpha x_\mu}{r^2} \right) S(r,t) \frac{1}{er}.
\]

Substituting

\[
K(r,t) = P(r) \cos \alpha(t), \quad S(r,t) = P(r) \sin \alpha(t), \quad W(r,t) = \dot{a}(t)
\]

into this ansatz, we get the following components of Y-M tensor [41]:

\[
F_{01} = F_{02} = F_{03} = 0, \quad F_{12} = e^{-1}P'(r) \left( m \cos \alpha + l \sin \alpha \right),
\]

\[
F_{13} = e^{-1}P'(r) \sin \theta \left( m \sin \alpha - l \cos \alpha \right), \quad F_{23} = e^{-1} \sin \theta \left( P^2(r) - 1 \right) n,
\]

(4)

presented in the orthonormalized isoframe \( n = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \quad l = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta) \) and \( m = (-\sin \phi, \cos \phi, 0) \). The prime in (4) stands for the derivative with respect to \( r \). As it has been noted in [41], Y-M field [41] possesses only magnetic components. From formulas (2) and (4), it is easy to find that the Y-M invariant \( I = F_{ik}^{\alpha\beta} \) has the following expression:

\[
I = \frac{2}{a^2\xi^2} \left[ 2P^2 + \frac{(P-1)^2}{\xi^2} \right].
\]

(5)

Varying the Lagrangian density (3) over \( P(r) \) and taking into account (5), we obtain the following Euler–Lagrange equation for the Y-M field:

\[
\left\{ P'' - \frac{(P-1)^2}{\xi^2} \right\} \left[ Q(t) \Psi' + a^3\Phi' \right] + P' \frac{\partial I}{\partial r} \left[ Q(t) \Psi'' + a^3\Phi'' \right] = 0,
\]

(6)

where \( \Phi' \equiv d\Phi(I)/dI, \quad \Psi' \equiv d\Psi(I)/dI \) and the following notation is temporarily introduced:

\[
Q(t) = \frac{3}{8\pi G} \left( \frac{a^2\ddot{a}}{N} + \frac{a\dot{a}^2}{N^2} - \frac{a^2\dot{N}}{N^2} + kaN \right).
\]

The particular solution of the Y-M equation in the FRW metrics has been obtained in [41]. It has the form \( P(r) = \xi'(r) = \cos r, \cos h r \) for \( k = +1, -1 \) consequently. It satisfies equation (6) as
well. Indeed, it turns the expression in braces into zero and satisfies, in view of (10), the equality \( \frac{\partial I}{\partial r} = 0 \). It is easy to prove the latter, as for this solution the Y-M invariant depends only on time:

\[
I = I(t) = \frac{6}{e^{2a^4(t)}}.
\]  

(7)

Varying Lagrangian over \( a(t) \) and \( N(t) \) with the subsequent choice of gauge \( N = 1 \), one can obtain the following equations for our model:

\[
\left[ \frac{2\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] \Psi(I) - 4I \Phi'(I) \left[ \frac{2\ddot{a}}{a} - 2\left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] + 16\left( \frac{\dot{a}}{a} \right)^2 I^2 \Phi''(I) = 0,
\]

\[
= 8\pi G \left[ \Phi(I) - \frac{4}{3} I \Phi'(I) \right],
\]

(8)

\[
\left[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] \Psi(I) - 4\left( \frac{\dot{a}}{a} \right)^2 I \Psi'(I) = \frac{8\pi G}{3} \Phi(I),
\]

(9)

3 Several examples of exact solution

(10) The arbitrariness of differentiable functions \( \Psi(I) \) and \( \Phi(I) \) essentially complicates the general analysis of the model equations (8), (9). Therefore, we will consider some special cases. Let us begin with the power-law dependencies:

\[
\Phi(I) = A I^n, \quad \Psi(I) = B I^m,
\]

(10)

where \( A \) and \( B \) are some dimensional constants, free parameters of the model. The substitution of expressions (10) into equations (8), (9) leads to the following set of equations:

\[
(1 - 4m) \left[ \frac{2\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 (1 - 4m) + \frac{k}{a^2} \right] = \frac{8\pi G}{3} \frac{A}{B} (3 - 4n) I^{n-m},
\]

(11)

\[
\left( \frac{\dot{a}}{a} \right)^2 (1 - 4m) + \frac{k}{a^2} = \frac{8\pi G}{3} \frac{A}{B} I^{n-m}.
\]

(12)

As one can see from these equations, the case \( m = 1/4 \) is a specific one. Indeed, from equation (11) with \( m = 1/4 \) it immediately follows that \( n = 3/4 \), i.e. the power of \( I \) in equation (12) is equal to \( n - m = 1/2 \), and the first term in the left-hand-side of this equation is equal to zero. Taking into account (7), we can reduce equation (12) to a ratio between constants \( A \) and \( B \) of the form:

\[
k = \frac{8\sqrt{6\pi G} A}{3e B}.
\]

Thus, the scale factor remains uncertain, or arbitrary. It is easy to prove that the same unclearly interpreted result will turn out, if functions (10) are directly substituted into Lagrangian (3) together with \( N = 3/4 \), \( m = 1/4 \) (or \( \Phi = A(6/e^2)^{3/4}a^{-3}(t) \), \( \Psi = A(6/e^2)^{1/4}a^{-1}(t) \)). Varying then it over \( a(t) \) and \( N(t) \), we again arrive at the same relationship between \( A \) and \( B \). As for the equation for the second derivative of the scale factor, it will be satisfied identically for any \( a(t) \). Putting this case aside, we consider our model with \( m \neq 1/4 \).

Combining (11) and (12), we have the following equation:

\[
\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \frac{A}{B} \frac{[2(n-m)-1]}{(4m-1)} I^{n-m}
\]

(13)

for the second derivative of the scale factor.

It follows from (7) that \( I > 0 \). Therefore, the necessary condition of accelerated expansion of the universe \( (\ddot{a} > 0) \), following from equation (13), is reduced to

\[
\frac{A}{B} \frac{[2(n-m)-1]}{(4m-1)} > 0.
\]

The latter inequality must be solved for \( m > 1/4 \), or \( m < 1/4 \). The result of solving is presented in Table 1.
can be written down as follows: *\[ \text{Table 1. Conditions for the free parameters } A/B, m \text{ and } n - m, \text{ corresponding to accelerated expansion according to equation (13)}. \]

\begin{tabular}{|c|c|c|c|c|}
\hline
Ia & \( A/B > 0 \) & \( m > 1/4 \) & \( n - m > 1/2 \) \\
Ib & \( A/B > 0 \) & \( m < 1/4 \) & \( n - m < 1/2 \) \\
Ic & \( A/B < 0 \) & \( m > 1/4 \) & \( n - m < 1/2 \) \\
Id & \( A/B < 0 \) & \( m < 1/4 \) & \( n - m > 1/2 \) \\
\hline
\end{tabular}

It is necessary to emphasize that equation (13) is employed for the analysis of the accelerated mode of evolution, but is a differential consequence of equation (12). Therefore, the model dynamics is defined by the only independent equation (12). As one aims to solve this equation at the condition of accelerated expansion, it is necessary to take into account Table 1.

From Table 1, it follows that in the case of standard Y-M Lagrangian \((n = 1)\) the accelerated expansion is possible, if (Ia) \( A/B > 0 \) and \( m \in (1/4, 1/2) \), or (Id) \( A/B < 0 \) and \( m < 1/4 \) are realized. The assumption of minimal coupling (that is \( m = 0 \)) corresponds to (Id). However, equation (12) at \( A/B < 0, m = 0, n = 1 \) and \( (k = +1) \) has no the real solution, and the case (Ia) does not correspond to \( m = 0 \). The latter simply means the absence of the accelerated mode in Einstein-Yang-Mills minimal theory \([37]\). The trivial solution with accelerated expansion can be obtained in the case \( A/B < 0, m = 0, n = 1 \), and for the negative sign of curvature. Taking into account (10), and supposing \( A = 1/16 \pi, B = -1 \), we can express equation (12) and its solution as follows:

\[ a^2 = 1 - \frac{G}{c^2} \left( \frac{1}{a^2} \right)^2, \quad a(t) = \sqrt{\frac{G}{c^2} t^2}, \quad (14) \]

where the constant of integration is equal to zero for the sake of simplicity. For this solution, the acceleration equals \( \ddot{a} = (G/c^2)/(|G/c^2| + t^2)^{3/2} > 0 \), and it decreases to zero with time. It is interesting that this model of non-massive Y-M field with linear dependence on \( I = F_{ik} F^{ik} \) leads to the accelerated expansion. However, due to equation (12), this model violates the weak energy condition. Therefore this example of solution (14) is exclusively illustrative.

Let us consider now two examples of exact solution for this model which are not so trivial but rather simple.

\textbf{a)} Let \( n = 1, m = 3/4 \), that is we will consider the case (Ic) from Table 1. Then equation (12) can be written down as follows:

\[ 2 \dot{a}^2 - k = C a, \quad \text{where} \quad C = \frac{8 \pi G}{3} \left( \frac{6}{c^2} \right)^{1/4} \left| \frac{A}{B} \right| \]

The obvious solution for this equation is:

\[ a(t) = \frac{C}{8} (t - C_0)^2 - \frac{k}{C}, \]

where \( C_0^2 \geq 8k/C^2 \). This model experiences constant acceleration: \( \ddot{a} = C/4 \), and the Hubble parameter is equal \( H = \frac{2(t - C_0)}{(t - C_0)^2 - 8k/C^2} \).

\textbf{b)} We now consider an example of exact solution with \( n = m \) that corresponds to (Ib) and (Ic) in Table 1. In these cases, equation (12) can be written down as follows:

\[ M \dot{a}^2 - \delta k = C a^2, \quad (15) \]

where

\[ C = \frac{8 \pi G}{3} \left| \frac{A}{B} \right|, \quad M = |4m - 1|, \quad \delta = \begin{cases} +1 & \text{for } m > 1/4, \\ -1 & \text{for } m < 1/4. \end{cases} \]

For the positive sign of curvature \((k = +1)\) and \( m > 1/4 \), as well as for the negative sign of curvature \((k = -1)\) and \( m < 1/4 \), the exact solution for equation (15) is equal

\[ a(t) = \frac{1}{\sqrt{C}} \sinh \left( \sqrt{\frac{C}{M}} t + C_0 \right), \]

\[ \frac{a(t)}{a_0} = \frac{1}{\sqrt{C}} \sinh \left( \sqrt{\frac{C}{M}} t + C_0 \right), \]

\[ \frac{a(t)}{a_0} = \frac{1}{\sqrt{C}} \sinh \left( \sqrt{\frac{C}{M}} t + C_0 \right), \]

\[ \frac{a(t)}{a_0} = \frac{1}{\sqrt{C}} \sinh \left( \sqrt{\frac{C}{M}} t + C_0 \right), \]
where $C_0$ is an integration constant. For the case of negative curvature ($k = -1$) and $m > 1/4$, or positive curvature ($k = +1$) and $m < 1/4$, we can obtain the following solution for equation (15):

$$a(t) = \frac{1}{\sqrt{C}} \cosh\left(\sqrt{\frac{C}{M}} (t + C_0)\right).$$

where $C_0$ is an arbitrary constant. It is interesting that in these cases, as it can be observed from the special feature of solutions and equation (15), this model behaves similarly to the FRW model in which the only source of gravity is the effective cosmological constant $\Lambda = 3C/M$.

(II) To involve in our consideration the widely discussed modifications of Y-M theory (see, for example, [36]-[39], [42]), we assume that function $\Psi(I) = B I^m$, and $\Phi(I)$ arbitrarily depends on $I$. Then the main equations of our model can be written down as follows:

$$(1 - 4m)\left[\frac{2}{a} \left(1 - 4m + \frac{k}{a^2}\right) - \frac{8\pi G}{3B} \frac{[3\Phi(I) - 4I\Phi'(I)]}{I^m}\right] = \frac{8\pi G}{3B} \frac{\Phi(I)}{I^m}. \quad (16)$$

Combining these equations, it is possible to obtain, instead of (16), the following equation for the second derivative of the scale factor:

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3B} \frac{[1 + 2m(\Phi(I) - 2I\Phi'(I))]}{1 - 4m}. \quad (17)$$

It is easy to verify that (18) is a differential consequence of equation (17) which remains the only independent equation of our model. Nevertheless, from equation (18) it is possible to obtain a necessary condition for the accelerated regime (see Table 2).

| IIa | $B > 0$ | $m > 1/4$ | $\Theta(I) > 0$ |
|-----|---------|-----------|----------------|
| IIb | $B > 0$ | $m < 1/4$ | $\Theta(I) < 0$ |
| IIC | $B < 0$ | $m > 1/4$ | $\Theta(I) < 0$ |
| IIId| $B < 0$ | $m < 1/4$ | $\Theta(I) > 0$ |

Table 2. Conditions for $m$ and function $\Theta(I) = 2I\Phi'(I) - (1 + 2m)\Phi(I)$, corresponding to accelerated expansion according to equation (18).

Let us then consider the case of non-Abelian Lagrangian of the Born-Infeld type (BI):

$$L_{NBI} = \frac{\beta^2}{4\pi} \left[1 + \frac{F_{ik}^a F^{aik}}{\beta^2} - \frac{(\tilde{F}_{ik}^a F^{aik})^2}{16\beta^4} - 1\right],$$

where $\beta$ is the critical intensity of BI-field, $\tilde{F}_{ik}^a$ is a dual Y-M tensor. From formulas (1) and metrics (2), it follows that the second invariant of Y-M field for our solution $\tilde{F}_{ik}^a F^{aik} = 0$. So, we can write down $\Phi(I)$ as

$$\Phi(I) = \frac{1}{16\pi\alpha} \left(\sqrt{1 + 2\alpha I} - 1\right), \quad (19)$$

where $\alpha = 1/2\beta^2$. From the latter, it is easy to find that

$$2I\Phi'(I) - (1 + 2m)\Phi(I) = \frac{1}{16\pi\alpha} \left[[2m + 1](\sqrt{1 + 2\alpha I} - 1) - 4m\alpha I]\right]. \quad (20)$$

In view of inequalities $2\alpha I > 0$ and $\sqrt{1 + 2\alpha I} > 1$, one can find that expression (20) will be positive, only if $m \in (0, 1/4)$. Thus, as it follows from the explicit form of invariant (7), the positivity of expression (20) will occur only after crossing the critical value of scale factor:

$$a(t) > a_{cr} = \left[\frac{48\alpha m^2}{\alpha^2(1 - 4m^2)}\right]^{1/4}.$$
One can see that this case corresponds to (II(d)) in Table 2, if $B < 0$. When the value of (20) is negative, $m$ remains arbitrary. However, the acceleration conditions are different for $m^2 < 1/4$ or $m^2 > 1/4$. The first case corresponds to (II(b)) with $m \in (-1/4, 1/4)$ and $B > 0$. The cosmic acceleration is possible while $0 < a(t) < a_{cr}$. In the case (II(c)) (i.e. for $m > 1/4$, $B < 0$), the accelerated expansion is always possible.

So non-trivial dependence of the model behavior on free parameter $m$ requires a thorough research. Therefore, we are going to give more details and consequences of the model considered here in our further investigation.

4 Conclusion

In summary, the modified non-minimal Y-M theory in FRW non-flat cosmology are studied in this paper. First of all, we have derived the set of main equations which determines the model dynamics: (6), (8), (9). Throughout the last section of the paper, non-minimal coupling to gravity is described by the factor $\Phi(I)$ in the Einstein-Hilbert sector of action (1). The non-trivial solution of the modified Y-M equation (8) proposed by one of the authors (V.K.S) earlier allows us to build several modifications of accelerated cosmic expansion in the frame of non-minimal coupling. Besides general study, we have considered in detail the power-law dependence of $\Phi(I)$ on its argument. We have derived the basic equations for the cosmic scale factor in our model, and have provided several examples of their solutions. This work implies that the cosmological applications of modified Y-M theory with non-minimal coupling to gravity may have more fruitful phenomena, which is worth studying further.

References

[1] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[2] C. B. Netterfield et al., Astrophys. J. 571, 604 (2002).
[3] N. W. Halverson et al., Astrophys. J. 568, 38 (2002).
[4] S. Bridle, O. Lahab, J. P. Ostriker and P. J. Steinhardt, Science 299, 1532 (2003).
[5] D. N. Spergel et al., Astrophys. J. Suppl. Ser. 148, 175 (2003).
[6] C. L. Bennett, et al., Astrophys. J. Suppl. 148, 1 (2003).
[7] M. Tegmark, et al.[SDSS Collaboration], Phys. Rev. D 69, 103501 (2004).
[8] S. W. Allen, et al., Mon. Not. Roy. Astron. Soc. 353, 457 (2004).
[9] G. R. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485, 208 (2000).
[10] S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003).
[11] S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002).
[12] P. S. Apostolopoulos, et al., Phys. Rev. D 72, 044013 (2005).
[13] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007).
[14] F. K. Diakonos and E. N. Saridakis, JCAP 0902, 030 (2009).
[15] M. Sahlen, A. R. Liddle and D. Parkinson, Phys. Rev. D 72, 083511 (2005).
[16] M. Sahlen, A. R. Liddle and D. Parkinson, Phys. Rev. D 75, 023502 (2007).
[17] Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A 22, 883 (2007).
[18] R. R. Caldwell, Phys. Lett. B 545, 23 (2002).
[19] S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005).
[20] X. M. Chen, Y. G. Gong and E. N. Saridakis, JCAP 0904, 001 (2009).
[21] E. N. Saridakis, Nucl. Phys. B 819, 116 (2009).
[22] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005).
[23] Z. K. Guo, et al., Phys. Lett. B 608, 177 (2005).
[24] M.-Z Li, B. Feng, X.-M Zhang, JCAP, 0512, 002 (2005).
[25] M. R. Setare and E. N. Saridakis, JCAP 0809, 026 (2008).
[26] V. Sahni and S. Habib, Phys. Rev. Lett. 81, 1766 (1998).
[27] A. A. Sen and S. Sen, Mod. Phys. Lett. A 16, 1303 (2001).
[28] V. Faraoni, Phys. Rev. D 68, 063508 (2003).
[29] E. Elizalde, S. Nojiri and S. Odintsov, Phys. Rev. D 70, 043539 (2004).
[30] S. Nojiri, S. D. Odintsov and M. Sami, Phys. Rev. D 74, 046004 (2006).
[31] M. R. Setare and E. N. Saridakis, Phys. Lett. B 671, 331 (2009).
[32] M. R. Setare and E. N. Saridakis, JCAP 0903, 002 (2009).
[33] O. Bertolami and P. J. Martins, Phys. Rev. D 61, 064007 (2000).
[34] T.D. Saini, S. Raychaudhury, V. Sahni, A. A. Starobinsky, Phys. Rev. Lett. 85, 1162 (2000).
[35] S. Sen and T. R. Seshadri, Int. J. Mod. Phys. D 12, 445 (2003).
[36] K. Bamba, S. Nojiri and S.D. Odintsov, arXiv: 0803.3384 [hep-th] (2008).
[37] D.V. Gal’tsov, arXiv: 0901.0115 [gr-qc] (2009).
[38] A.B. Balakin, A.E. Zayats, Phys. Let. B 644 294 (2007).
[39] A.B. Balakin, H. Dehnen, A.E. Zayats, Phys. Rev. D 76 124011 (2007).
[40] V.K. Shchigolev, (to appear in Grav.Cosmol.): arXiv:1102.2513 [gr-qc].
[41] V.K. Shchigolev, K. Samaroo, Gen.Relat.Grav., 36, 1661 (2004).
[42] Wen Zhao, Int. J. Mod. Phys. D, 18, 1331 (2009).