Theory on the Temperature Dependence
of Giant Magnetoresistance

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Abstract

The temperature dependence of the giant magnetoresistance (GMR) for currents parallel and perpendicular to the multilayer plane, is discussed by taking account of the random exchange potentials, phonon scatterings and spin fluctuations. The effect of spin fluctuations, which plays an important role at finite temperatures, is included by means of the static functional-integral method developed previously by the present author. Our model calculations well explain the observed features of the parallel and perpendicular GMR of Fe/Cr and Co/Cu multilayers recently reported by Gijs et al.

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I. INTRODUCTION

The giant magnetoresistance (GMR)\textsuperscript{1} in magnetic multilayers is one of the most attractive subject in current solid-state physics. In recent years much progress has been made in understanding the GMR and its related phenomena.\textsuperscript{1} One of the important aspects of the GMR is its temperature dependence. A careful study of the temperature dependence of GMR is not only important in understanding its mechanism but also very useful to its realistic applications. Most of the magnetic multilayers are fabricated with transition metals such as Fe, Ni and Co. It would be instructive to briefly discuss the temperature dependence of the resistivity of \textit{bulk} transition metals, before we study the temperature dependence of the GMR or of the resistivity of transition-metal multilayers.

It has been reported that when the temperature is raised from $T = 0$ K, the resistivity of Fe\textsuperscript{2} or Co\textsuperscript{3} gradually increases up to the Curie temperature, where it has a cusp (see Fig.1). This characteristic temperature dependence of the resistivity is interpreted as due to the contributions from impurity, phonon and magnetic terms. The last contribution is classically discussed as spin-disorder scatterings with the use of the s-d model.\textsuperscript{4} Lately, a modern theory on the itinerant-electron magnetism has accounted for it in terms of spin fluctuations.\textsuperscript{5}

It has been well known that d-electrons in transition metals show both the localized and itinerant character: the Curie-Weiss susceptibility and the large specific heat peak near the Curie temperature are easily explained by the localized-spin model whereas the non-integral ground-state moment and the large linear-specific heat coefficient favor the band model. It has been realized that the effect of spin fluctuations plays essential roles to reconcile the duality of d electrons.\textsuperscript{6} The finite-temperature band theory, which has
been proposed by Hasegawa,\textsuperscript{7} includes the effect of spin fluctuations by means of the static functional-integral method combined with the coherent potential approximation (CPA). Spin fluctuations including spin waves are shown to yield the $T^2$ contribution to the resistivity at $T \simeq 0$ by several approaches.\textsuperscript{8} This type of theories\textsuperscript{8} is, however, valid only at very low temperatures. In our finite-temperature theory,\textsuperscript{7} spin fluctuations are regarded as localized, static modes with the adopted approximations. This method has proved useful in understanding the overall finite-temperature properties of transition metals, alloy and multilayers,\textsuperscript{9} covering both below and above the Curie temperature.

By employing the finite-temperature band theory,\textsuperscript{7} we discussed in previous papers\textsuperscript{10} the temperature dependence of the MR ratio for currents parallel to the multilayer plane. The observed temperature dependences of Fe/Cr,\textsuperscript{10d,11} NiCo/Cu, NiFe/Cu and CoFe/Cu\textsuperscript{12} multilayers have been shown to be well explained by our theory. It has been pointed out\textsuperscript{9c} that a multilayer in which the normal and inverse GMR\textsuperscript{13} coexist, may have an interesting temperature dependence beneficial for real applications.

One of the purposes of the present paper is to generalize our theory\textsuperscript{10} to the perpendicular GMR, whose experimental\textsuperscript{14–16} and theoretical study\textsuperscript{17–20} has been currently performed. The other purpose is to include the phonon contribution to the conductivity calculation, which was neglected in our previous study.\textsuperscript{10} The paper is organized as follows: In the Sec.II, we present our formulation applying our finite-temperature band theory to the GMR. Numerical calculations of the parallel and perpendicular GMR of Fe/Cr and Co/Cu multilayers are reported in Sec.III. Supplementary discussions are given in Sec.IV.

\textbf{II. \textsc{c}alcul\textsc{a}tion \textsc{m}ethod}
A. An Adopted Model and the Expression of GMR

We adopt an A/B multilayer consisting of magnetic A and nonmagnetic B atoms with the simple-cubic (001) interface. The layer parallel to the interface is assigned by the index $n (= 1 - N_f)$. The thickness of the A and B layers is assumed to be thinner than the mean free path and sufficiently thin compared with the spin diffusion length. It is assumed that atoms A and B are randomly distributed on layer $n$ with the concentrations of $x_n$ and $y_n$, respectively ($x_n + y_n = 1$). The film is described by the single-band Hubbard model, in which the atomic potential (the on-site interaction) is assumed to be given by $\varepsilon^A$ and $\varepsilon^B$ ($U^A$ and $U^B$) when a given lattice site is occupied by A and B atoms, respectively.

In order to study the finite-temperature properties of the magnetic film, we apply the functional-integral method within the static approximation to the Hubbard Hamiltonian. The partition function is evaluated by calculating the partition function of the effective one-electron Hamiltonian including the random charge and exchange fields with the Gaussian weight. The charge field is include by the saddle-point approximation and the exchange field by the alloy-analogy approximation with the CPA. The energy-dependent coherent potential for an $s$-spin electron ($s = \uparrow, \downarrow$) on the layer $n$, $\Sigma_{ns}(\varepsilon)$, is determined by the CPA condition. The coherent potentials, the average of the magnetic moments on the layer $n$, $\langle M_n \rangle$, and its root-mean-square (RMS) value, $\langle (M_n)^2 \rangle^{1/2}$, are calculated self-consistently, details having been given in Ref. 7.

When we employ the CPA, the conductivity of the film is given by

$$\sigma_{\xi\eta} = \left( \frac{e}{\hbar} \right)^2 \frac{1}{\pi} \int d\varepsilon \left( -\frac{\partial f}{\partial \varepsilon} \right) \text{Tr} (v_{\xi} \text{Im} G v_{\eta} \text{Im} G) \quad (\xi, \eta = x, y, z),$$

provided the vertex correction is neglected. In Eq.(1) $v_{\xi}$ is the velocity operator and $G$ is
the Green function matrix. The conductivities for currents parallel (∥) and perpendicular (⊥) to the film layer are given by\textsuperscript{10,18,20}

\[
\sigma^\parallel = \left( \frac{e}{\hbar} \right)^2 \int d\varepsilon \left( -\frac{\partial f}{\partial \varepsilon} \right) \sum_s \nu_s^\parallel(\varepsilon) \left( \frac{1}{N_f} \right) \sum_n \sum_m \frac{a_{nms} \tau_{nms}}{(\Delta_{ns} + \Delta_{ms})}, \quad (2)
\]

\[
\sigma^\perp = \left( \frac{e}{\hbar} \right)^2 \int d\varepsilon \left( -\frac{\partial f}{\partial \varepsilon} \right) \sum_s \nu_s^\perp(\varepsilon) \left[ \left( \frac{1}{N_f} \right) \sum_n \Delta_{ns} \right]^{-1}, \quad (3)
\]

with

\[
\nu^\lambda(\varepsilon) = \hbar^2 \sum_{k\lambda} v^2_{\lambda} \delta(\varepsilon - \varepsilon_{k\lambda}) \quad (\lambda = ||, \perp), \quad (4)
\]

\[
\tau_{nms} = \delta_{nm} + (1 - \delta_{nm}) \left( \frac{(\Delta_{ns} + \Delta_{ms})^2}{((\Lambda_{ns} - \Lambda_{ms})^2 + (\Delta_{ns} + \Delta_{ms})^2)} \right), \quad (5)
\]

which is valid within the Born approximation. In Eqs. (2)-(5) \( \Lambda_{ns} = \text{Re} \Sigma_{ns}(\varepsilon) \), \( \Delta_{ns} = |\text{Im} \Sigma_{ns}(\varepsilon)| \), \( \Sigma_{ns} \) is the coherent potential of an \( s \)-spin electron on layer \( n \), and \( a_{nls} \) and \( \nu^\lambda \) are specified by the electronic structure of the film (see Eqs. (19) and (20) in Ref. [10a]).

Analytic expressions given by Eqs. (2)-(5) have clear physical meaning. When currents flow parallel to the plane, an \( s \)-spin electron propagating successively from a site on layer \( n \) to a site on layer \( m \), is scattered with the strength proportional to \( \Delta_{ns} \) and \( \Delta_{ms} \), respectively, and its conductivity is given as a sum of such processes with the weight of \( a_{nms} \tau_{nms} \).\textsuperscript{10}

On the contrary, in the case of the perpendicular current, the \( s \)-spin conductivity is given as of a series circuit of resistivities on successive layers, each of which is proportional to \( \Delta_{ns} \).\textsuperscript{18,21} In both cases, the total conductivity is a sum of the up- and down-spin channels.

The so-called spin-flop process is implicitly included through the spin-fluctuation term which is responsible to a decrease in layer magnetization, as will be shown shortly. In the next section, we will employ our formalism in a semi-phenomenological way to discuss the temperature dependence of the MR ratio.
B. A Semi-phenomenological Study of GMR

We adopt a system consisting of magnetic ($M_1, M_2$) and nonmagnetic ($N_1, N_2$) layers, whose thickness are $M$ and $N$, respectively. Bulk scatterings are assumed to be important in these layers, related discussion will be given in Sec.IV. When magnetic moments on $M_1$ and $M_2$ layers are in the antiferromagnetic (AF) configuration, the real and imaginary parts of the coherent potentials are given by\(^{10}\)

\[
\Lambda_{ns}^{AF} - i\Delta_{ns}^{AF} = \Lambda_s - i\Delta_s \quad \text{for } n \in M_1, \tag{6}
\]

\[
= \Lambda_{-s} - i\Delta_{-s} \quad \text{for } n \in M_2, \tag{7}
\]

\[
= \Lambda_0 - i\Delta_0 \quad \text{for } n \in N_1, N_2. \tag{8}
\]

Using Eqs.(2)-(8), we get the parallel and perpendicular conductivities given by\(^{10}\)

\[
\sigma_{AF\parallel} = \sum_s \left\{ \frac{2c_{MM}^{AF}}{(\Delta_s + \Delta_{-s})} + \frac{c_{NN}^{AF}}{\Delta_0} + 4c_{MN}^{AF} \left( \frac{1}{\Delta_s + \Delta_0} + \frac{1}{\Delta_{-s} + \Delta_0} \right) + d_M^{AF} \left( \frac{1}{2\Delta_s} + \frac{1}{2\Delta_{-s}} \right) + \frac{d_N^{AF}}{\Delta_0} \right\}, \tag{9}
\]

\[
\sigma_{AF\perp} = \left( \frac{e}{h} \right)^2 \nu^\perp N_f \sum_s \left\{ \frac{1}{M\Delta_s + M\Delta_{-s} + 2N\Delta_0} \right\}, \tag{10}
\]

with

\[
c_{MM}^{AF} = N_f^{-1}(e/h)^2\nu^\parallel \sum_{n \in M_1} \sum_{m \in M_2} a_{nm}\tau_{nm}, \tag{11}
\]

\[
d_{M}^{AF} = N_f^{-1}(e/h)^2\nu^\parallel \sum_{n \in M_1} \sum_{m \in M_1} a_{nm}\tau_{nm}, \tag{12}
\]

and $c_{NN}^{AF}$, $c_{MN}^{AF}$, and $d_N^{AF}$ are given by similar expressions. We employed the $T = 0$ limit of Eqs. (2) and (3) because the relevant temperature is much less than the Fermi energy, $\varepsilon_F$.

In Eqs.(9)-(12) $\nu^\lambda = \nu^\lambda(\varepsilon_F)$, $N_f = 2(M + N)$, and the spin dependence in $a_{nms}$ and $\tau_{nms}$ is neglected. Subscripts, MM, NN and MN, denote the contributions from the interlayer scatterings between magnetic layers, between nonmagnetic layers, and between magnetic
and nonmagnetic layers, respectively. On the contrary, the single subscript, M (N), expresses the contribution from the \textit{intralayer} scatterings within magnetic (nonmagnetic) layers.

On the contrary, when magnetic moments on the subsequent magnetic layers are in the ferromagnetic (F) configuration, the real and imaginary parts of the coherent potentials are given by\textsuperscript{10}

\[
\Lambda_{ns}^{F} - i\Delta_{ns}^{F} = \Lambda_s - i\Delta_s \quad \text{for } n \in M_1, M_2, \quad (13)
\]

\[
= \Lambda_0 - i\Delta_0 \quad \text{for } n \in N_1, N_2. \quad (14)
\]

We get the parallel and perpendicular conductivities given by\textsuperscript{10}

\[
\sigma^{F\parallel} = \sum_s \left\{ \frac{c_{MM}^{F}}{\Delta_s} + \frac{c_{NN}^{F}}{\Delta_0} + \frac{8c_{MN}^{F}}{(\Delta_s + \Delta_0)} + \frac{d_{M}^{F}}{\Delta_s} + \frac{d_{N}^{F}}{\Delta_0} \right\}, \quad (15)
\]

\[
\sigma^{F\perp} = \left( \frac{e}{h} \right)^2 \nu_{\perp} N_f \sum_s \left\{ \frac{1}{2M\Delta_s + 2N\Delta_0} \right\}. \quad (16)
\]

The MR ratio, \( \Delta R/R \), is given from Eqs. (9), (10), (15) and (16), by

\[
\left( \frac{\Delta R}{R} \right)^{\lambda} \equiv \left( \frac{R^{AF} - R^{F}}{R^{F}} \right) = \frac{(\sigma^{F} - \sigma^{AF})}{\sigma^{AF}} = \frac{(\alpha - \beta)^2}{4\alpha\beta} X^{\lambda} \quad (\lambda = \parallel, \perp), \quad (17)
\]

with

\[
\alpha = \Delta_{\uparrow}/\Delta_0, \quad \beta = \Delta_{\downarrow}/\Delta_0, \quad (18)
\]

\[
X^{\parallel} = \left[ 1 + g_0 \frac{(\alpha + \beta)^2}{\alpha\beta} + g_1 \left( \frac{N}{M} \right) (\alpha + \beta) \left( \frac{1}{\alpha + 1} + \frac{1}{\beta + 1} \right) + g_2 \left( \frac{N}{M} \right)^2 (\alpha + \beta) \right]^{-1}, \quad (19)
\]

\[
X^{\perp} = \left[ 1 + \left( \frac{N}{M} \right) \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) + \left( \frac{N}{M} \right)^2 \left( \frac{1}{\alpha\beta} \right) \right]^{-1}. \quad (20)
\]

In Eq. (19) \( g_0, g_1 \) and \( g_2 \) are defined by\textsuperscript{10}

\[
\frac{d_M}{4c_{MM}} = g_0, \quad \frac{2c_{MN}}{c_{MM}} = g_1 \left( \frac{N}{M} \right), \quad \left( \frac{c_{NN} + d_N}{2c_{MM}} \right) = g_2 \left( \frac{N}{M} \right)^2. \quad (21)
\]
The expression for the GMR given by Eqs. (17) and (20) is just the same as that derived by Edwards et al. using the resistor network model and has been employed for an analysis of \((\Delta R/R)^\perp\)\(^{14}\).

Setting \(N = 0\) in Eqs. (17)-(20), we get

\[
\left( \frac{\Delta R}{R} \right)^\parallel = \frac{(\alpha - \beta)^2}{4\alpha\beta \left[ 1 + g_0(\alpha + \beta)^2 / \alpha\beta \right]},
\]

(22)

\[
\left( \frac{\Delta R}{R} \right)^\perp = \frac{(\alpha - \beta)^2}{4\alpha\beta},
\]

(23)

and the ratio of the parallel GMR to the perpendicular one is given by

\[
\frac{(\Delta R/R)^\parallel}{(\Delta R/R)^\perp} = \frac{1}{1 + g_0(a + 1)^2 / a} \leq 1 \quad (a = \alpha / \beta).
\]

(24)

When electrons flow perpendicular to the layer plane, all electrons pass through the adjacent two magnetic layers. On the contrary, it is not the case for currents parallel to the plane; some electrons go through only the one of magnetic layers without probing the other magnetic layer. The second \(g_0\) term of the denominator of Eq.(22) denotes this contribution. Itoh et al.\(^{20}\) claim that the anisotropy of the velocity operator : \(\nu^\parallel / \nu^\perp \geq 1\) is the main mechanism leading to \((\Delta R/R)^\parallel \leq (\Delta R/R)^\perp\). The factor, \(\nu^\parallel\) or \(\nu^\perp\), is not, however, relevant because it is cancelled out when the MR ratio given by Eq.(17) is calculated.

The temperature dependence of the GMR arises from those of \(\alpha\) and \(\beta\), which is expressed in terms of the coherent potential of the film (Eq.(18)), whose imaginary part in the magnetic (\(M_1\) or \(M_2\)) layer is given within the Born approximation by\(^{10}\)

\[
\Delta_s = \Delta_s^r + \Delta_s^s + \Delta_s^p,
\]

(25)
with
\[
\Delta^r_s = \pi \rho_s x y [\tilde{\varepsilon}^A - \tilde{\varepsilon}^B - s \left( \frac{U^A}{2} \right) \langle M^A \rangle]^2, \tag{26}
\]
\[
\Delta^s_s = \pi \rho_s x \left( \frac{U^A}{2} \right)^2 \left[ \langle (M^A)^2 \rangle - \langle M^A \rangle^2 \right], \tag{27}
\]
\[
\Delta^p_s = P_m \rho_s Z(T/\Theta_m), \tag{28}
\]
where \(\tilde{\varepsilon}^A\) and \(\tilde{\varepsilon}^B\) are the spin-independent Hartree-Fock potentials and \(\rho_s\) is the density of states of an \(s\)-spin electron at the Fermi level. The first term (\(\Delta^r_s\)) in Eq. (25) arises from the scattering due to random Hartree-Fock potentials for an \(s\)-spin electron: the second term (\(\Delta^s_s\)) comes from the effect of spin fluctuations: the third term (\(\Delta^p_s\)) is introduced for phonon scatterings whose explicit form will be given shortly (Eq.(45)).

On the other hand, the imaginary part of the coherent potential in the nonmagnetic (\(N_1\) or \(N_2\)) layer, is given by
\[
\Delta_0 = \Delta_0^r + P_n \rho_0 Z(T/\Theta_n), \tag{29}
\]
where the first and second terms denote the contributions from random potentials and phonons, respectively, and \(\rho_0\) is the density of states at the Fermi level of the nonmagnetic metal. In Eqs.(28) and (29) \(\Theta_m\) and \(\Theta_n\) are Debye temperatures, and \(P_m\) and \(P_n\) are related with the electron-phonon interactions in magnetic and nonmagnetic metals.

Using Eqs. (18), (26)-(29), we get \(\alpha\) and \(\beta\) given by
\[
\alpha = A \left(1 + \gamma(T)\right) \frac{[xy(B + m(T))^2 + x(\mu(T))^2 - m(T)^2] + p_m Z(T/\Theta_m)}{(1 + p_0 Z(T/\Theta_n))}, \tag{30}
\]
\[
\beta = A \left(1 - \gamma(T)\right) \frac{[xy(B - m(T))^2 + x(\mu(T))^2 - m(T)^2] + p_m Z(T/\Theta_m)}{(1 + p_0 Z(T/\Theta_n))}, \tag{31}
\]
with

$$m(T) = \langle M^A \rangle / M_0,$$

$$\mu(T) = \sqrt{\langle (M^A)^2 \rangle / M_0},$$

$$\gamma(T) = (\rho_\uparrow - \rho_\downarrow) / (\rho_\uparrow + \rho_\downarrow),$$

$$A = \pi \rho (U^A M_0 / 2)^2 / \Delta_r^0,$$

$$B = (2/U^A M_0) (\tilde{\epsilon}_B - \tilde{\epsilon}_A),$$

$$p_m = P_m / \pi (U^A M_0 / 2)^2,$$

$$p_0 = P_n \rho_0 / \Delta_r^0 = p_m \ A \ (P_n / P_m) \ (\rho_0 / \rho),$$

where $\rho = (1/2)(\rho_\uparrow + \rho_\downarrow)$ and $M_0$ is the ground-state magnetic moment.

At $T = 0K$ where $m(0) = \mu(0) = 1$ and $\gamma(0) = \gamma_0$, Eqs. (30) and (31) become

$$\alpha_0 = \alpha(T = 0) = xy \ A \ (1 + \gamma_0)(B + 1)^2,$$

$$\beta_0 = \beta(T = 0) = xy \ A \ (1 - \gamma_0)(B - 1)^2,$$

from which the coefficients $A$ and $B$ are expressed in terms of $\alpha_0$, $\beta_0$ and $\gamma_0$ as

$$A = \frac{1}{4xy} \left( \sqrt{\frac{\alpha_0}{1 + \gamma_0}} - \sqrt{\frac{\beta_0}{1 - \gamma_0}} \right)^2,$$

$$B = \left( \sqrt{\frac{\alpha_0}{1 + \gamma_0}} + \sqrt{\frac{\beta_0}{1 - \gamma_0}} \right) / \left( \sqrt{\frac{\alpha_0}{1 + \gamma_0}} - \sqrt{\frac{\beta_0}{1 - \gamma_0}} \right).$$

The normalized magnetic moment, $m(T)$, and its RMS value, $\mu(T)$, are in principle calculated with the use of the finite-temperature band theory.\textsuperscript{9} We here, however, adopt simple, analytic expressions of $m(T)$ and $\mu(T)$ for our model calculation, given by\textsuperscript{10}

$$m(T) = \sqrt{1 - (T/T_C)^2}, \quad \mu(T) = 1.$$
The temperature dependence of the spin asymmetry $\gamma(T)$ defined by Eq.(34) is assumed to be given by

$$\gamma(T) = \gamma_0 \ m(T).$$  \hspace{1cm} (44)

As for the phonon contribution given by $Z(T/\Theta_m)$ in Eqs.(28) and (29), we adopt the simple Grüneisen function:

$$Z(T/\Theta_m) = (T/\Theta_m)^{5} \int_{0}^{\Theta_m/T} dy \frac{y^5}{(e^y-1)(1-e^{-y})},$$  \hspace{1cm} (45)

which is $124.43 (T/\theta_m)^5$ at $T/\Theta \ll 1$ and $T/4\Theta_m$ at $T/\Theta_m \gg 1$.

Now we may calculate the MR ratio, $\Delta R/R$, as a function of temperature with the use of Eqs. (17), (19), (20), (30), (31), (41)-(45), when we treat $\alpha_0$, $\beta_0$, $\gamma_0$, $g_0$, $g_1$, $g_2$, $T_C$, $\Theta_m$, $\Theta_n$, $p_m$, $p_0$, and $y$, as input parameters. Our strategy for calculating the temperature- and layer-thickness-dependent MR ratio is as follows: We first determine the parameters, $\alpha_0$, $\beta_0$ and $\gamma_0$ to be consistent with the band calculation, and also $g_0$, $g_1$ and $g_2$ so as to reproduce the $N$ dependence of the observed, ground-state parallel GMR. Then fixing there six parameters thus determined, we calculate the finite-temperature GMR with the additional parameters, $T_C$, $\Theta_m$, $\Theta_n$, $p_m$, $p_n$ and $y$, which can be properly chosen, as will be discussed in the model calculations of the next section.

III. MODEL CALCULATIONS

A. Fe/Cr Multilayers

Gijs et al.$^{15}$ have observed both the parallel and perpendicular GMR for a sample of (3 nm Fe + 1.0 nm Cr) multilayer, whose results are plotted by circles and squares in Fig.2, respectively.
Firstly we consider the case of $T = 4.2$ K. We determine the value of $\gamma_0 = 0.4$ from the ground-state band calculation of $\rho_\uparrow/\rho_\downarrow = 2.3^{22}$ We adopt $\alpha_0 = 7.9$ and $\beta_0 = 1.0$, leading to $B = 3.38$ (Eq.(42)), which is consistent with the value estimated from Eq.(36) by using the band parameters such as $\varepsilon^{\text{Fe}}$ etc. We choose the parameters of $g_0 = 0.045$, $g_1 = 0.77$, and $g_2 = 3.05$, such that we have a good fit to the envelope of the observed layer-thickness ($t_N$) dependence of parallel GMR in (3 nm Fe + $t_N$ Cr) multilayers.$^{11}$

Next we consider the MR ratio at finite temperatures. We assume the Curie temperature of the multilayer of $T_C = 1000$ K because the thickness of the Fe layers of the adopted Fe/Cr multilayers$^{11,15}$ is sufficiently thick to sustain the Curie temperature of bulk Fe. The Debye temperatures of Fe and Cr are assumed to be $\Theta_m = \Theta_n = 460$ K. The phonon parameters, $p_m$ and $p_0$, can be determined as follows: The total resistivity, $R$, of a *pure, bulk* metal is given from Eqs.(2), (3) and (30), by

$$R(T) \propto \left\{(1 + s \gamma_0 m(T))(\mu(T)^2 - m(T)^2 + p_m Z(T/\Theta_m))\right\}^{-1},$$

from which the ratio of the phonon contribution, $R_p$, to the total resistivity at $T = T_C$ is given by $r_p \equiv R_p(T_C)/R(T_C) = p_m Z(T_C/\Theta_m)/[1 + p_m Z(T_C/\Theta_m)]$. The value of $p_m = 0.69$ is chosen from the experimental data of $r_p = 0.27$ of bulk Fe (Fig.1(a)).$^2$ We calculate $p_0$ by $p_0 = p_m A (\rho_0/\rho)$ derived from Eq.(38) with $P_n = P_m$ and $\rho_0/\rho = 0.7^{22}$ The parameters discussed above are summarized in Table 1. The solid curve in Fig.1(a) expresses the resistivity, $R(T)$, of bulk Fe calculated by using Eqs.(43) and (46) with $\gamma_0 = 0.4$ and $p_m = 0.69$, which well reproduces the observed data.$^2$

The last parameter $y$, which expresses a concentration of nonmagnetic atoms in the magnetic layer and which depends on a sample employed in a experiment, is treated as
an adjustable parameter. The parallel and perpendicular GMR of the Fe/Cr multilayer calculated with \( y = 0.002, 0.005 \) and 0.01 are shown in Fig.2. Our calculation with \( y = 0.005 \) well explains both the \((\Delta R/R)\parallel\) and \((\Delta R/R)\perp\) observed by Gijs et al.\textsuperscript{15}

In order to study the temperature dependence of the GMR in more detail, we show in Fig.3, \( \Delta_s \) (s =\( \uparrow, \downarrow \)) as a function of the temperature. When the temperature is raised, \( \Delta_{\uparrow} \) and \( \Delta_{\downarrow} \) increase because of the contributions from spin fluctuations and phonons. Then the ratio, \( \Delta_{\uparrow}/\Delta_{\downarrow} (= \alpha/\beta) \), changes from 7.9 at \( T = 0 \) to unity at \( T \geq T_C \). Fig.3 also shows the decomposition of \( \Delta_s \) to various contributions from random potentials (\( \Delta^r_s \)), spin fluctuations (\( \Delta^s_s \)) and phonons (\( \Delta^p_s \)). We note that at \( T = T_C \), \( \Delta^s_s/\Delta = 0.70 \), \( \Delta^p_s/\Delta = 0.26 \) and \( \Delta^s_s/\Delta^p = 2.65 \). This shows a significant spin-fluctuation contribution, as suggested from the resistivity data of bulk Fe.\textsuperscript{2}

B. Co/Cu Multilayers

We have performed a similar calculation to explain the temperature dependence of parallel and perpendicular GMR of the (1.2 nm Co + 1.1 nm Cu) multilayer observed by Gijs et al.\textsuperscript{16} We adopt \( \alpha_0 = 0.7, \beta_0 = 8.4 \) (\( \beta_0/\alpha_0 = 14 \)),\textsuperscript{16} and \( \gamma_0 = -0.7 \) which comes from the ground-state band calculation of \( \rho_{\uparrow}/\rho_{\downarrow} \sim 0.15 \) of bulk Co.\textsuperscript{22} We cannot determine the values of \( g_0, g_1 \) and \( g_2 \) because the layer-thickness dependence of the parallel GMR of this series of samples has not been reported. Then we tentatively adopt \( g_0 = 0.13, g_1 = 0.39 \) and \( g_2 = 0.11 \) by scaling the data of similar Co/Cu multilayer\textsuperscript{23} as to reproduce the observed ground-state value of \((\Delta R/R)\parallel = 0.43\).\textsuperscript{16} The Curie and Debye temperatures are taken to be \( T_C = 1400 \) K and \( \Theta_m = \Theta_n = 445 \) K. We adopt \( p_m = 1.62 \) from the observed ratio of \( r_p = 0.56 \) for bulk Co (Fig.1(b)),\textsuperscript{3} and \( \rho_0/\rho = 0.3 \).\textsuperscript{22} Adopted
parameters are shown in Table 1. The solid curve in Fig.1(b) denotes the temperature-dependent resistivity of bulk Co calculated by using Eqs.(43) and (46) with $\gamma_0 = -0.7$ and $p_m = 1.62$.

The calculated $(\Delta R/R)^\parallel$ and $(\Delta R/R)^\perp$ of the Co/Cu multilayer are shown in Fig.4, where $y$ is treated as an adjustable parameter. Both the parallel and perpendicular GMR observed by Gijs et al.\textsuperscript{16} are fairly well explained by our calculation with $y = 0.005$.

Figure 5 expresses the temperature dependence of $\Delta_s$ and its components, $\Delta_s^r$, $\Delta_s^p$ and $\Delta_s^p$, which shows that at $T = T_C$, $\Delta_s^r/\Delta = 0.42$, $\Delta_s^p/\Delta = 0.53$ and $\Delta_s^p/\Delta = 0.79$. Comparing these figures with the corresponding ones of Fe/Cr systems, we note that spin-fluctuation contribution in Co/Cu multilayer is less significant than in Fe/Cr multilayer.

This fact is expected to be the main reason why the observed temperature dependence of the GMR in Co/Cu multilayer is less considerable than that in Fe/Cr multilayers.

IV. CONCLUSION AND DISCUSSION

We have discussed the temperature dependence of the GMR for currents parallel and perpendicular to the multilayer plane. We have included contributions from the random exchange potentials, spin fluctuations and phonons, which are considered to be main scattering mechanisms yielding the resistivity in transition-metal multilayers. Our model calculations have accounted for the following features of the observed GMR:\textsuperscript{11,15,16} (1) both the parallel and perpendicular GMR are significantly temperature dependent than the (average) layer moment, (2) $(\Delta R/R)^\perp$ is larger than $(\Delta R/R)^\parallel$, (3) the temperature dependence of $(\Delta R/R)^\perp$ is more significant than that of $(\Delta R/R)^\parallel$, and (4) the temperature dependence of GMR in Co/Cu multilayers is less considerable than that in Fe/Cr multilayers.
multilayers. The effect of spin fluctuations plays an important role to account for these three items whereas phonons play a secondary role. In fact, the items (1)-(3) can be explained without invoking phonons.\textsuperscript{10}

In our phenomenological analysis, we have assumed that the bulk scattering is predominant. On the contrary, when we take into account only the interface scattering, the expression for the GMR is given again by Eqs.\textsuperscript{(17)-(20)} but with $M$ replaced by $I$, the thickness of the interface, and with $\alpha$ and $\beta$ expressed in terms of the quantities relevant to the interface. Then, they have ostensibly similar $T$ and $N$ dependence to those in which only the bulk scattering is included. It is possible to extend our analysis taking into account both the interface and bulk scatterings, although the calculation becomes laborious because it inevitably needs much number of parameters. Among many parameters, the most important ones are $g_0$, $\alpha_0$, $\beta_0$ and $y$; $(\Delta R/R)_{\parallel}$ generally becomes smaller than $(\Delta R/R)_{\perp}$ by $g_0$, and the essential feature of the temperature dependence of the GMR is determined by the ratio of $\alpha_0/\beta_0$ and $y$.\textsuperscript{10,12}

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| parameter | Fe/Cr | Co/Cu |
|-----------|-------|-------|
| $\alpha_0$ | 7.9 | 0.7 |
| $\beta_0$ | 1.0 | 8.4 |
| $\gamma_0$ | 0.4 | -0.7 |
| $\rho_0/\rho$ | 0.7 | 0.3 |
| $T_C$ (K) | 1000 | 1400 |
| $\Theta_{m,n}$ (K) | 460 | 445 |
| $p_m$ | 0.69 | 1.62 |
| $g_0$ | 0.045 | 0.13 |
| $g_1$ | 0.77 | 0.39 |
| $g_2$ | 3.05 | 0.11 |

**Table 1** Parameters adopted for numerical calculations (see text).
Figure Captions

Fig. 1 The temperature dependence of the observed resistivity (circles) of (a) bulk Fe (Ref.3) and (b) Co (Ref.4); the calculated resistivity, $R(T)$, and its phonon term, $R_p(T)$, above $T_C$ are shown by solid and dotted curves, respectively, results being normalized by $R_C = R(T_C)$.

Fig. 2 The temperature dependence of the parallel ($\parallel$) and perpendicular $\Delta R/R$ ($\perp$) of (3 nm Fe + 1.0 nm Cr) multilayers. Dotted, solid and dashed curves denote the calculated results with $y = 0.002$, 0.005 and 0.01, respectively; circles (squares) expressing the observed parallel (perpendicular) GMR (Ref.15).

Fig. 3 The temperature dependence of $\Delta_s$ of up-spin (solid curves) and down-spin electrons (dashed curves) calculated with $y = 0.005$ for the (3 nm Fe + 1.0 nm Cr) multilayer. Also shown are their decomposition to various contributions from the random exchange potentials ($\Delta^r_s$), spin fluctuations ($\Delta^s_s$) and phonons ($\Delta^p_s$); the calculated results being normalized by $\Delta_C = \Delta_s(T_C)$.

Fig. 4 The temperature dependence of the parallel ($\parallel$) and perpendicular $\Delta R/R$ ($\perp$) of (1.2 nm Co + 1.1 nm Cu) multilayers. Dotted, solid and dashed curves denote the calculated results with $y = 0.002$, 0.005 and 0.01, respectively; circles (squares) expressing the observed parallel (perpendicular) GMR (Ref.16).

Fig. 5 The temperature dependence of $\Delta_s$ of up-spin (solid curves) and down-spin electrons (dashed curves) calculated with $y = 0.005$ for the (1.2 nm Co + 1.1 nm Cu) multilayer. See a caption of Fig.3.