Event-triggered Group Consensus for First-order Multi-agent Systems Based on Directed Topology

To cite this article: Shaolei Zhou et al 2019 J. Phys.: Conf. Ser. 1267 012063

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Event-triggered Group Consensus for First-order Multi-agent Systems Based on Directed Topology

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Abstract. Event-triggered group consensus problem is addressed in this paper. Based on the assumption named the in-degree balance, a directed first-order multi-agent system is constructed, which consists of two subgroups. To analyze the problem, the Laplacian matrix of the initial system is decomposed to be two matrices with less row and column dimension. Then the initial state vector can be substituted by a new one, each element in which is the error of two adjacent elements of the initial state vector. Then a new reduced-order system is constructed and the consensus problem is converted to a stability problem. With the designed Lyapunov function, the validity of the proposed algorithm is proved. The lower-bound of the inter-event time interval is also presented. Simulation results are presented to demonstrate the effectiveness of the protocol.

1. Introduction

Multi-agent systems have been applied broadly to areas such as robots[1, 2], unmanned vehicles[3] and computer science[4], which in return attracts more interests from researchers. In recent years, scholars have put much attention to consensus problem in multi-agent systems, and have done research in more particular aspects like group consensus[5-8] for example.

Consensus means that all the agents in the system converge to a certain state. And talking about group consensus, there exist at least two groups in the system, and agents in the same group converge to a same state, while those belonging to different groups may converge to a different stable one. In early research, Yu and Wang[5] proposed first-order group consensus problem based on undirected topology, and converted the fixed topology into switching topology by double-tree-form transformation[6], which can construct a reduced-order system, and communication delays are also taken into consideration. Feng[7] studied a system consisting of agents with double-integrator dynamics. With a special matrix whose zero eigenvalue’s algebraic multiplicity is four, the system can reach time-varying consensus. Sufficiency and necessity analysis is presented. Miao and Ma[8] proposed an algorithm with nonlinear input constraints for continuous-time/discrete-time systems, the validity of which was verified by Lyapunov function and LaSalle’s invariance principle.

Since a large number of studies are based on the assumption that the agents can detect and communicate with neighbours all the time, while in reality making the electronic equipment work keep communicating costs much, some scholars put forward an event-triggered strategy[9-11]. With this algorithm, agents can update their control inputs at discrete time instants, which can dramatically decrease the cost. Studies utilizing this strategy are still few. Tu et al[12] did research on leader-following multi-agent systems and put their attention on fixed and switching topologies with event-triggering mechanism. Ma et al[13] designed even-triggered functions for centralized and
decentralized conditions, and analysed the distributed event-triggered problem with a triggering function depending on the number of each agent’s neighbours[14]. Yan et al[15] studied the switching topology with disturbance. These above studies are all based on undirected topology.

Inspired by the existing studies, this paper further investigate the event-triggered group consensus for first-order multi-agent systems based on directed topology. The system consists two subgroups, and a centralized event-triggered function is designed. The directed Laplacian matrix of the system is decomposed for the convenience of analysis. A Lyapunov function is constructed and a sufficient condition is presented. Simulation results verify the validity of the proposed protocol.

The rest of the paper is as follows. Section 2 presents some preliminaries and introduced some lemmas. Section 3 gives the theoretical result and section 4 gives the simulation counterpart.

2. Problem Formulation
A multi-agent system can be described by a directed or undirected graph \( G = (\mathcal{V}, \mathcal{E}, A) \). Considering \( n \) agents in the system, there is a set of nodes \( \mathcal{V} = \{v_1, v_2, \cdots, v_n\} \) consisting of \( n \) elements, and a corresponding set of edges \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \). An edge of the graph \( G \) is described as \( e_{ij} = (v_i, v_j) \). An undirected graph is connected if there is a path between any two distinct nodes, and if any two nodes can be linked by a directed path, the directed graph can also be called connected. In a directed graph, if there is only one node that has no parent and all the other nodes has exactly one parent, and the special node can connect any other nodes by the paths, then it is called the root. If the topology of the system is undirected, a symmetrical adjacency matrix \( A = [a_{ij}] \) is an \( n \) order square matrix. And if the topology is directed, normally the adjacency matrix \( A \) is asymmetrical. The Laplacian matrix \( L = [l_{ij}] \) can be derived from \( A \) by the following way: \( l_{ij} = -a_{ij} \) if \( i \neq j \), and \( l_{ii} = \sum_{j \neq i} a_{ij} \). It is obvious that \( L \) has a right eigenvector \( [1, 1, \cdots, 1]^T \) and 0 is the corresponding eigenvalue. The set of neighbours of agent \( i \) is written as \( \mathcal{N}_i = \{v_j \in \mathcal{V} | (v_i, v_j) \in \mathcal{E}\} \).

To analyse the group consensus problem, suppose that there are \( n + m \) agents in the system, firstly the agents may be listed in a rank by their index \( i \in \{1, 2, \cdots, n + m\} \). The former \( n \) agents belong to the first group called \( \mathcal{I}_1 \), and the latter \( m \) agents belong to the second group called \( \mathcal{I}_2 \). Then the adjacency matrix \( A \) is a block matrix \( A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \), where \( A_{11} (A_{22}) \) is the inner adjacency matrix in \( \mathcal{I}_1 (\mathcal{I}_2) \), and \( A_{12} (A_{21}) \) stands for the links from \( \mathcal{I}_2 (\mathcal{I}_1) \) to \( \mathcal{I}_1 (\mathcal{I}_2) \).

Suppose that all the agents in the system follow the following dynamics

\[
\dot{x}_i = u_i, \forall i \in \mathcal{I}
\]

in which \( x_i, u_i \in \mathbb{R} \).

Lemma 1. The multi-agent system is said to reach group consensus[6] if the states of agents satisfy

\[
\lim_{t \to \infty} \|x_i - x_j\| = 0, \forall i, j \in \mathcal{I}_1
\]

\[
\lim_{t \to \infty} \|x_i - x_j\| = 0, \forall i, j \in \mathcal{I}_2
\]

Lemma 2. For a graph Laplacian matrix \( L \in \mathbb{R}^{N \times N} \) and a full row rank matrix \( E \in \mathbb{R}^{(N-1) \times N} \) defined as follows
There exists a matrix $M \in \mathbb{R}^{N(N-1)}$ such that $L = ME$ [16].

Lemma 3 (Cauchy-Schwarz Inequality).

$$\sum_{k=1}^{n} a_k b_k \leq \left( \sum_{k=1}^{n} a_k^2 \right)^{\frac{1}{2}} \left( \sum_{k=1}^{n} b_k^2 \right)^{\frac{1}{2}}$$

Lemma 4. The Frobenius-norm of a matrix is compatible to the 2-norm of a vector, that is

$$\|Ax\|_2 \leq \|A\|_F \|x\|_2$$

Where $\|\cdot\|_2$ is the 2-norm and $\|\cdot\|_F$ is the Frobenius-norm [17].

Assumption 1. The system is in-degree-balanced [5] if

$$\sum_{j=n+1}^{n+m} a_{ij} = 0, i \in \mathcal{I}_1$$

$$\sum_{j=1}^{n} a_{ij} = 0, i \in \mathcal{I}_2$$

To solve the event-triggered group consensus problem, a new protocol is proposed

$$u_i = \sum_{j=1}^{n} a_{ij} \left[ x_j(t_k) - x_i(t_k) \right] + \sum_{j=n+1}^{n+m} a_{ij} x_j(t_k), i \in \mathcal{I}_1$$

$$u_i = \sum_{j=1}^{n+m} a_{ij} \left[ x_j(t_k) - x_i(t_k) \right] + \sum_{j=1}^{n} a_{ij} x_j(t_k), i \in \mathcal{I}_2$$

Where $a_{ij} \geq 0$ is the element in $A$, and $t_k$ is the last event time instant when the control input $u_i$ updates, and $x_i(t_k)$ is the reference state of agent $i$.

For each agent, the state measure error vector [11] is defined as

$$e_i = x_i(t_k) - x_i(t), t \in [t_k, t_{k+1})$$

List all the variables in vector $x = [x_1, x_2, \ldots, x_{n+m}]^T$, $x(t_k) = [x_1(t_k), x_2(t_k), \ldots, x_{n+m}(t_k)]^T$, and $e = [e_1, e_2, \ldots, e_{n+m}]^T$, then $e = x(t_k) - x$.

Based on Lemma 2, the Laplacian matrix $L$ can be decomposed

$$L = \hat{M} \hat{E}$$

in which $\hat{E} = \begin{bmatrix} E_{(\cdot\cdot\cdot \cdot)} & 0 \\ 0 & E_{(\cdot\cdot\cdot \cdot)} \end{bmatrix}$. And use another variable $\bar{x}_i$

$$\begin{cases} \bar{x}_i = x_i - x_{i+1}, i = 1, 2, \ldots, n-1 \\ \bar{x}_i = x_i - x_{i+n}, i = n+1, n+2, \ldots, n+m-1 \end{cases}$$

to substitute $x_i$, then an $n + m - 2$ dimensional vector $\bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{n-1}, \bar{x}_{n+1}, \bar{x}_{n+2}, \ldots, \bar{x}_{n+m-1}]^T$ is obtained. In fact, by Lemma 2 and equation (6), it is obvious that

$$\bar{x} = \hat{E} x$$

(11)
Similarly, we get $\dot{\tilde{e}} = \hat{E} \tilde{x}$ and $\dot{x}(t_k) = \hat{E} x(t_k)$.

Based on measurement error, a triggering function $f(x, e)$ would be defined in the next section.

3. Theoretical results

In this section, on the foundation of graph theories and matrix theories, and referring to the analysis in [11], group consensus of multi-agent system based on directed topology will be considered.

Given the system dynamics (1) and control protocol (7), the system will reach group consensus if it is triggered when

$$\|\tilde{e}\|_2 = \alpha \frac{\|\hat{E} \tilde{M} \tilde{x}\|_r}{\|\hat{E} \tilde{M}\|_r}$$

(12)

where $0 < \alpha < 1$.

Proof: Under assumption 1, it is obvious that

$$\sum_{j=1}^{n+m} a_{ij} x_i = 0$$

(13)

Then equation (4) can be rewritten as

$$u_i = \sum_{j=1}^{n+m} a_{ij} \left[ x_j(t_k) - x_i(t_k) \right], i \in I_1$$

(14)

$$u_i = \sum_{j=1}^{n+m} a_{ij} \left[ x_j(t_k) - x_i(t_k) \right], i \in I_2$$

which means

$$\dot{x}_i = u_i = -L x(t_k)$$

(15)

And by equation(8), it can obtain that

$$\dot{\tilde{x}} = -\hat{E} \tilde{M} \tilde{x}(t_k)$$

(16)

A candidate Lyapunov function can be written as

$$V = \frac{1}{2} \tilde{x}^T M^T E^T \tilde{x}$$

(17)

Then the derivative of $V$ is

$$\dot{V} = \dot{\tilde{x}}^T M^T E^T \dot{\tilde{x}}$$

$$= -\tilde{x}^T M^T E^T \tilde{M} \tilde{x}(t_k)$$

$$= -\tilde{x}^T M^T E^T \tilde{E} M \tilde{x}$$

$$= -\tilde{x}^T M^T E^T \tilde{E} \tilde{M} - \tilde{x}^T M^T E^T \dot{\tilde{M}}$$

(18)

Since $\tilde{x}^T M^T E^T \tilde{M} \tilde{x} = \|\tilde{M} \tilde{x}\|^2_2$, and based on Lemma 3, $\tilde{x}^T M^T E^T \dot{\tilde{M}} \tilde{x} \leq \|\tilde{M} \tilde{x}\|^2_2 \|\dot{\tilde{M}}\|^2_2$, so that
\[ V \leq -\left\| \hat{E}Mx \right\|_2^2 + \left\| \hat{E}Mx \right\|_F \left\| e \right\|_2 \]  

(19)

So that if condition (12) is satisfied and \( \left\| \hat{E}Mx \right\|_2 \neq 0 \), it can obtain that

\[ V \leq (\alpha - 1)\left\| \hat{E}Mx \right\|_2^2 \]

(20)

Because \( 0 < \alpha < 1 \), it is obvious that \( V \) is negative.

Given the system dynamics (1), the control protocol (7) and the error function (12), the inter-event time interval \( t_{k+1} - t_k \) has a lower bound \( \tau \) which is strictly positive

\[ \tau = \frac{\alpha}{(1 + \alpha)\left\| \hat{E}M \right\|_F} \]

(21)

Proof: Based on equation (12), it can obtain that the time derivate of \( \left\| \hat{E}Mx \right\|_2 \) is

\[
\frac{d}{dt} \left( \frac{||e||_2}{||\hat{E}Mx||_2} \right) = \frac{\hat{e}^T \hat{e}}{||\hat{E}Mx||_2^2} - \frac{||\hat{E}Mx||_2^2}{||\hat{E}Mx||_2^2} \left( \hat{E}M \right)^T \left( \hat{E}Mx \right) \\
\leq \frac{||\hat{e}||_2^2}{||\hat{E}Mx||_2^2} + \frac{||\hat{e}||_2^2}{||\hat{E}Mx||_2^2} \left( \hat{E}M \right)^T \left( \hat{E}M \right) \\
= \frac{||\hat{e}||_2^2}{||\hat{E}Mx||_2^2} + \frac{||\hat{e}||_2^2}{||\hat{E}Mx||_2^2} \left( \hat{E}M \right)^T \left( \hat{E}M \right) \\
\leq \left( 1 + \frac{||\hat{e}||_2^2}{||\hat{E}M||_F} \right)^2
\]

(22)

Let

\[ y(t, y_0) = \frac{||e||_2}{||\hat{E}Mx||_2} \]

(23)

It is obvious that \( \dot{y} > 0 \), \( \dot{y} \leq \left( 1 + \left\| \hat{E}M \right\|_F \right)^2 \), and \( y(t, y_0) \leq \bar{y}(t, y_0) \) where \( \ddot{y} = \left( 1 + \left\| \hat{E}M \right\|_F \right)^2 \). Since
\[
\bar{y} = \frac{\| \hat{E} \bar{M} \|_F t + 1 - c}{\| \hat{E} \bar{M} \|_F c - \| \hat{E} \bar{M} \|_F^2 t}
\]

(24)
in which \( c \) is a constant. Because \( y_0 = 0 \), it can obtain that \( c = 1 \), and \( \bar{y} = t \left( 1 - \| \hat{E} \bar{M} \|_F \right)^{-1} \).

Based on

\[
\bar{y}(\tau, 0) = \frac{\tau}{1 - \| \hat{E} \bar{M} \|_F \tau} = \frac{\alpha}{\| \hat{E} \bar{M} \|_F}
\]

(25)
we know that

\[
\tau = \frac{\alpha}{(1 + \alpha) \| \hat{E} \bar{M} \|_F}
\]

(26)
and the proof is complete.

4. Simulation results
The effectiveness of the previous theoretical results will be justified by computer simulation results.

Consider a multi-agent system of 3+4 agents, meaning that the first group contains 3 agents and the second one contains 4. And the Laplacian matrix of the system is as follows.

\[
L = \begin{bmatrix}
0 & 0 & 0 & -1 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 & 0 & -1 & 1 \\
0 & -1 & 1 & -1 & 1 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 \\
-1 & 1 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 \\
\end{bmatrix}
\]

(27)
And the corresponding \( \hat{M} \) and \( \hat{E} \) are

\[
\hat{M} = \begin{bmatrix}
0 & 0 & 0 & -1 & -1 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 \\
0 & -1 & 0 & -1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 & -1 & 0 \\
-1 & -1 & 0 & 0 & 0 & -1 \\
\end{bmatrix}, \quad \hat{E} = \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
\end{bmatrix}
\]

(28)
The initial state vector of the agents is \( x(0) = [17.6, 4.6, 82.4, 4.35, 60.56, 32.415, 49.6]^T \), and the parameter \( \alpha = 0.1 \). Figure 1 shows the changing process of the agents’ state. It is obvious that the former 3 agents converge to one same state and the latter 4 agents converge to another different one.
Figure 1. The state converging process.

Figure 2. The control inputs of agents.

Figure 2 shows the control inputs $-Lx(t_k)$ of the agents. Since it is assumed that the agents have great ability so that they can change their state at any rate, the control inputs at the beginning can be very large. But in practical applications the ability of agents is limited, so the future studies should concentrate on event-triggered consensus of limited-ability multi-agent systems.

The event time series is presented in figure 3. At the beginning, since there exists a great difference among the states of agents, the system triggers quite frequently, which results into the fact that in figure 3, the former time intervals between triggering time instants is very short. After a converging process the states of agents don’t vary that much, so the intervals gradually increase to a fairly large value.

Figure 3. Figure with short caption (caption centred).
Figure 4. Norm of error vector and the reference value during the former 2 seconds.

Figure 5. Norm of error vector and the reference state value during the latter 8 seconds.

Figure 4 and 5 show the norm of the error vector $\hat{e}$ and the reference value $\alpha \left\| \hat{E}M \hat{x} \right\|_2 / \left\| \hat{E} \hat{M} \right\|_F$.

When $\left\| \hat{e} \right\|_2 > \alpha \left\| \hat{E}M \hat{x} \right\|_2 / \left\| \hat{E} \hat{M} \right\|_F$, the triggering condition is satisfied, then the agents update their reference state $\hat{x}(t_k)$. In figure 4, during the former 2 seconds, the system triggers more frequently than in figure 5 during the latter 8 seconds, which is consistent with the conclusion we obtain from figure 3.

5. Conclusions
The event-triggered group consensus of multi-agent system is considered in this article, which is based on a first-order directed topology. The agents are separated into 2 groups, and all the agents have a same triggering algorithm. Theoretical analysis is supported by simulation results, and the validity of the proposed protocol is verified. Studies in the future will concentrate on the limited-ability multi-agent system and the application of the proposed protocol in it.
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