ACCRETION DISKS IN ACTIVE GALACTIC NUCLEI: GAS SUPPLY DRIVEN BY STAR FORMATION

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ABSTRACT

Self-gravitating accretion disks collapse to star-forming (SF) regions extending to the inner edge of the dusty torus in active galactic nuclei (AGNs). A full set of equations including feedback of star formation is given to describe the dynamics of the regions. We explore the role of supernova explosions (SNexp), which act to excite turbulent viscosity, in the transportation of angular momentum in the regions within 1 pc scale. We find that accretion disks with typical rates in AGNs can be driven by SNexp in the regions and metals are produced spontaneously. The present model predicts a metallicity-luminosity relationship consistent with that observed in AGNs. As relics of SF regions, a ring (or belt) consisting of old stars remains for every episode of supermassive black hole activity. We suggest that multiple stellar rings with random directions interact and form a nuclear star cluster after episodes driven by star formation.

Key words: accretion, accretion disks – quasars: general – stars: formation

1. INTRODUCTION

Accretion onto supermassive black holes (SMBHs) is the energy house of active galactic nuclei (AGNs). Applying models of accretion disks to fit the observed big blue bumps in the continuum of AGNs and quasars, one finds that initial mass accretion rates typically accrete with sub-Eddington rates (Laor & Netzer 1989; Sun & Malkan 1989; Brunner et al. 1997; Lü 2008). Observational examination of the Soltan argument for the cosmological growth of SMBHs prefers that they grow episodically through baryon accretion (Yu & Tremaine 2002; Wang et al. 2009a), in agreement with the paradigm of coevolution with their host galaxies. However, what is governing accretion rates of active SMBHs?

We have several relevant clues to understanding the formation of the accretion flows approaching SMBHs at ~1 pc scale. First, the well-known metallicity and luminosity relationship in AGNs and quasars (see a review of Hamann & Ferland 1999) strongly implies that the flows may undergo fast metal production through stellar evolution in active episodes in light of cosmic evolution of metallicity (Dietrich et al. 2003; Shemmer et al. 2004). This gets supports from evidence that there is intense star formation with the top-heavy initial mass function (IMF) within ~0.1 pc in the Galactic center (Paumard et al. 2006; Nayakshin & Sunyaev 2005), which is confirmed by numerical simulations (Nayakshin et al. 2007; Bonnell & Rice 2008; Hobbs & Nayakshin 2009). We may naturally postulate that accretion onto SMBHs is the consequence of metal production. Second, it has long been known that accretion disks in quasars are massive and self-gravitating in outer regions (Paczynski 1978; Kolykhalov & Sunyaev 1980; Shlosman & Begelman 1987; Collin & Zahn 1999). Obviously, the self-gravity is driving intensive star formation that is ongoing there (Goodman 2003), and a star-forming (SF) disk (or regions) appears. It seems that there is no way to avoid the production of metals and the triggering of accretion flows in the SF disk, yielding the metallicity-luminosity relationship. We argue that SNexp plays a leading role in the establishment of the relationship in light of SNexp-excited turbulent viscosity as shown in the ~1 kpc regions by numerical simulations (Wada & Norman 2002; Hobbs et al. 2010) or physical motivation (Wang et al. 2009b; see also Kumar & Johnson 2010), and in ~10^8 Sloan type 2 AGNs (Chen et al. 2009). Furthermore, we are inspired by the recent evidence that suggests that the “visible” inflows displayed by the Hβ line are likely developed from the inner edge of the dusty torus in ~5000 Sloan quasars (Hu et al. 2008a, 2008b). All clues merge into a question: are accretion inflows driven by star formation in the massive self-gravitating disk?

In this Letter, we show the feasibility that an SF disk is able to drive a Shakura–Sunyaev disk (hereafter SS disk) in AGNs and quasars through SNexp-excited turbulent viscosity. We find that such a scenario agrees with observations of metallicity and potentially results in the formation of a nuclear star cluster.

2. BASIC EQUATIONS

Figure 1 shows an illustration of the present model. The outer boundary of the SF disk (R_{out}) is chosen at the inner edge of the torus as suggested by Hu et al. (2008a, 2008b), where dust particles are sublimated at R_{sub} = 1.3L_{UV,46}^{1/2} pc, where L_{UV,46} = L_{UV}/10^{46} erg s^{-1} (Barvainis 1987). The choice of R_{out} = R_{sub} is motivated by a fact that emission from R > R_{sub} is invisible in optics. Collisions of molecular clouds transfer angular momentum outward and supply gas to the SS disk with a rate of M_{out} = 1–20 M_{⊙} yr^{-1} at R_{sub} from the torus (Krolik & Begelman 1988) with a typical mass range of 10^3–10^7 M_{⊙} (Mor et al. 2009). The typical mass of the torus is just at the level required by a single episode of SMBH activity.

2.1. SNexp-driven Gaseous Disk

With the inclusion of the mass dropout from star formation and the injection of gas recycled from SNexp, the mass conservation equation for the SF disk reads

\[ \frac{\partial \Sigma_{\text{gas}}}{\partial t} = \frac{1}{2\pi R} \frac{\partial}{\partial R} \left[ \frac{d}{dR} \left( \frac{R^2 \Omega}{R} \right)^{-1} \frac{\partial G}{\partial R} \right] - \Sigma_{\text{a}} + \Sigma_{\text{SN}}. \tag{1} \]

1 Equation (1) is the modified version from Lin & Pringle (1987) and Wang et al. (2009b). We point out that \( \Sigma_{\text{SN}} \) is at \( t = t_{\text{SN}} \), where \( t_{\text{SN}} \) is the hydrogen main-sequence lifetime of stars depending on the stellar mass. Since stars form in the same gaseous disk, the stellar disk always follows the latter in the current model. Unless the stellar disk was formed in the previous episode, it could be larger than the gaseous disk, leading to reduce the feedback efficiency significantly (Nayakshin et al. 2007).
where $\Sigma_{\text{gas}}$ is the surface density, $\mathcal{G} = -2\pi R^3 \Sigma_{\text{gas}} (d\Omega/dR)$ is the viscous torque, $\nu$ is the kinematic viscosity, $\Omega$ is the angular velocity, $\Sigma_{\text{in}}$ is the surface density of star formation rates, and $\Sigma_{\text{SN}}$ is the injection rate of gas recycled from SNexp. We use the relation $\Sigma_{\text{SN}} = f_c \Sigma_*$, where $f_c$ is the fraction of the SNexp-ejected mass to the total formed stars. For an episodic activity of AGN with $\Delta t_G$ (e.g., Wang et al. 2006), the minimum mass of the stars producing an SNexp should be $M_c / M_\odot \gtrsim 7.0 \Delta t_G^{-0.4} \mathrm{yr}^{-1}$, where the lifetime of hydrogen main-sequence stars is $t_{\text{MS}} = 13 (M_*/M_\odot)^{-2.5} \mathrm{Gyr}$ and $\Delta t_G = \Delta t_G/0.1 \mathrm{Gyr}$. Assuming an IMF as $N(M)dM = N_0 M_*^{-\beta}dM_*$, we have $f_c = \int_{M_{\text{tot}}}^{100M_\odot} M_* N(M_*)dM_*/M_{\text{tot}}$, where $M_{\text{tot}} = \int_{M_{\text{SN}}}^{100M_\odot} M_* N(M_*)dM_*$ is the total mass of the formed stars. It follows that $f_c \sim 0.38$ for the Salpeter IMF ($\beta = 2.35$), $M_{\text{min}} = 1.0 M_\odot$, and $M_c = 7 M_\odot$, meaning that a significant mass fraction of the formed stars will be injected into the SF disk from the SNexp (here we neglect the remnant compact stars after the SNexp). The scale influenced by an SNexp is of the order of $D_{\text{SN}} \sim (E_{\text{SN}}/m_p n_0 c_s^2)^{1/3} \sim 0.1 E_{\text{SN}}^{1/3} (n_{11} T_{13})^{-1/3} \mathrm{pc}$, where $E_{\text{SN}} = E_{\text{SN}}/10^{51}$ erg is the kinetic energy of the SNexp, $n_{11} = n_{\text{gas}}/10^{11} \mathrm{cm}^{-3}$ is the number density of the medium in the SF disk, $c_s \approx 3.0 \times 10^5 T_{13}^{1/2} \mathrm{cm} \mathrm{s}^{-1}$ is the isothermal sound speed, and $T_{13} = T/10^3$ K is the temperature. We find from this simple estimation that the length of the SNexp influence is comparable with the height of the SF disk ($\sim 0.1 \mathrm{pc}$) and that the SNexp-excited turbulence velocity $V_{\text{tur}} \gg c_s$, and hence SNexp plays a key role in the transportation of angular momentum.

In this Letter, we focus on the stationary case, namely, $d\Sigma_{\text{gas}}/dt = 0$. Introducing the parameter $X = R^2 \Omega$, Equation (1) is rewritten as

$$
\frac{1}{2\pi R} \frac{dX}{dR} \frac{d^2 \mathcal{G}}{dX^2} - A (1 - f_c) \Sigma_{\text{gas}}' = 0,
$$

where the Kennicutt–Schmidt (KS) law $\Sigma_* = A \Sigma_{\text{gas}}'$ is used for simplicity in the entire SF disk, $A$ is a constant, and $\gamma$ is the index (Kennicutt 1998). Lynden-Bell & Pringle (1974) show that $d\mathcal{G}/dX = M$ at any radius, where $M$ is the mass rate of the inflows, provides the inner and outer boundary conditions. We employ the $\alpha$-prescription viscosity as $\nu_{\text{SN}} = \alpha_{\text{SN}} V_{\text{tur}} H$, where $\alpha_{\text{SN}}$ is a constant, $V_{\text{tur}}$ is the turbulence velocity driven by SNexp, and $H$ is the thickness of the disk (Wada & Norman 2002; Wang et al. 2009b). The energy equation of the SNexp excited turbulence is given by

$$
\frac{\rho_{\text{gas}} V_{\text{tur}}^2}{t_{\text{dis}}} = \epsilon_{\text{SN}} \rho_{\text{gas}} V_{\text{tur}} = \epsilon f_s \rho_{\text{gas}} E_{\text{SN}},
$$

where $t_{\text{dis}} = H/V_{\text{tur}}$ is the timescale of the turbulence, $\epsilon$ is the efficiency of converting kinetic energy of SNexp into turbulence, and $f_s \rho_{\text{gas}}$ is the density of SNexp rates. Here $f_s$ is a number fraction of stars to the total mass of the formed stars that are able to produce an SNexp during one AGN episode. For a given IMF, we have $f_s = 2.9 N(M_\odot) M_\odot/M_{\text{tot}}$ and $f_s \sim 2.3 \times 10^{-2} M_\odot^{-1}$ for $\beta = 2.35$. Considering $\Sigma_* = \rho_{\text{gas}} H$ and $\Sigma_{\text{gas}} = \rho_{\text{gas}} H$, we have

$$
\frac{V_{\text{tur}}^3}{H} = \epsilon f_s E_{\text{SN}} \Sigma_{\text{gas}}^{-1}.
$$

Following Paczynski (1978), we assume that gas rotates with a Keplerian velocity $\Omega_{K} = (G M_\bullet / R)^{1/2}$, where $G$ is the gravity constant and $M_\bullet$ is the SMBH mass. A vertical equilibrium holds the disk self-gravity, SMBH gravity, and turbulent pressure. We usually have $dP_{\text{tur}}/dZ = (\Omega_{K}^2 + 2\pi G \Sigma_{\text{gas}}) \rho_{\text{gas}}$, where $P_{\text{tur}} = \rho_{\text{gas}} V_{\text{tur}}^2$ is the turbulence pressure and the second term is the self-gravity. Since the star formation supports the thickness of the SF disk, the self-gravity can be neglected (Thompson et al. 2005), allowing us to have an approximate form of vertical-averaged structure,

$$
\frac{P_{\text{tur}}}{H} = \left( \frac{G M_\bullet}{R^3} + 2\pi G \Sigma_{\text{gas}} \right) \rho_{\text{gas}} \approx \Omega_{K}^2 H \rho_{\text{gas}}.
$$

We point out that the above equations are actually averaged in the vertical direction, and the parameters obtained in these equations are the values at the mid-plane of the SF disk. The $\mathcal{G} - \Sigma_{\text{gas}}$ relation is specified for further simplification of Equation (2) through the viscosity $\nu$. With the help of Equations (3)–(5), we have

$$
\frac{d^2 \mathcal{G}}{dX^2} - \left( B / X^2 \right)^2 \mathcal{G} = 0,
$$

yielding an analytical solution in the form of

$$
\mathcal{G}(X) = c_1 X \exp \left( \frac{B}{X} \right) + c_2 X \exp \left( -\frac{B}{X} \right),
$$

where $B = 2GM_\bullet [(1 - f_c)/3\alpha_{\text{SN}}] \epsilon f_s E_{\text{SN}}$ for the Keplerian rotation, and $c_1$ and $c_2$ are two constants determined by the boundary conditions. At the outer radius of the SF disk, only the mass injection rate is known and given by the mechanism of molecular cloud collisions (Krolik & Begelman 1988). We have $d\mathcal{G}/dX|_{X_{\text{out}}} = M_{\text{out}}$, namely,

$$
c_1 (1 - B X_{\text{out}}^{-1}) e^{X_{\text{out}}} + c_2 (1 + B X_{\text{out}}^{-1}) e^{-X_{\text{out}}} = M_{\text{out}},
$$

where $X_{\text{out}} = (GM_\bullet R_{\text{gas}})^{1/2}$. We assume that the inner radius of the SF disk is set to be the self-gravity radius ($R_{\text{sg}}$) of the SS disk, which depends on the SMBH accretion rates ($M_\bullet$). This yields $d\mathcal{G}/dX|_{X_{\text{in}}} = M_\bullet$, and subsequently

$$
c_1 (1 - B X_{\text{sg}}^{-1}) e^{X_{\text{sg}}} + c_2 (1 + B X_{\text{sg}}^{-1}) e^{-X_{\text{sg}}} = M_\bullet,
$$

where $X_{\text{in}} = X_{\text{sg}} = (GM_\bullet R_{\text{sg}})^{1/2}$. 

\footnote{We note that star formation rates may deviate from the KS law along the radii in the SF disk. It is not sufficiently understood (Krumholz et al. 2009), especially for very dense disks.}
2.2. Switching on the Shakura–Sunyaev Disk

The self-gravity instability develops when the Toomre’s parameter $Q \leq 1$. For the SS disk, the instability happens in the middle region of the SS disk at the radius

$$R_{sg} = 1006 \alpha_{0.1}^{14/27} M_8^{-2/27} \dot{M}^{-8/27} R_{Sch},$$  

(10)

where $R_{Sch} = 2GM_8/c^2$ is the Schwartzchild radius, $c$ is the light speed, $M_8 = M_\bullet/10^8 M_\odot$, $\dot{M} = \dot{M}_c^2/L_{Edd} = \dot{M}_\bullet/0.2M_\odot$ yr$^{-3}$, $L_{Edd}$ is the Eddington luminosity, and $\alpha_{0.1} = \Sigma_{gas}/0.1$ is the viscosity parameter in the SS disk (see Kato et al. 1998, Chapter 3). We assume that the SF process quenches within $R_{sg}$ and the SF disks smoothly switch on the SS disks at $R_{sg}$ in terms of surface density. The surface density of the SS disks is

$$\Sigma_{SS} = 8.2 \times 10^8 \alpha^{-4/5} M_8^{1/5} \dot{M}^{-3/5} r_3^{-3/5} M_\odot pc^{-2},$$  

(11)

where $r_3 = R/10^3 R_{Sch}$ giving rise to the expression of the condition ($\Sigma_{gas} = \Sigma_{SS}$) of switching from the SF regions to the SS disk

$$c_1 e^{-\frac{\gamma}{\gamma-1}} + c_2 e^{-\frac{\beta}{\beta-1}} = 3\pi \alpha \dot{M} A E_{SNe} \Sigma_{SS}^\gamma \Omega^{-2}.$$

(12)

It is trivial to get the prolix expression of $c_1$ and $c_2$ from Equations (9) and (12), but we omit them here. We insert $c_1$ and $c_2$ into Equation (8) for the structure of the SF disk. Given $R_{out}$ and $M_{out}$, we can justify if the SF disk is able to drive the SS disk through solving Equations (8), (9), and (12). The accretion rates of the SS disk as well as the structure of the SF disk are obtained self-consistently.

3. ACCRETION FLOWS APPROACHING SMBHS FROM SF DISKS

The empirical KS law $\dot{\Sigma}_* = A\Sigma_{gas}^\gamma$ is used, where $\gamma = 1.4$, $A = 2.5 \times 10^{-4}$, $\dot{\Sigma}_*$ and $\Sigma_{gas}$ are in units of $M_\odot$ yr$^{-1}$ kpc$^{-2}$ and $M_\odot$ pc$^{-2}$, respectively. We employ $E_{SNe} = 10^{51}$ erg, a top-heavy IMF index $\beta = 1.9$, $f_2 = 0.63$, $f_3 = 0.03 M_\odot$, $\alpha_{SS} = 0.2$, and $\alpha_{gas} = 1$ (in light of $D_{SNe} \sim H$) throughout the paper. We take the outer radius $R_{out} = 1.0$ pc and $M_\bullet = 10^8 M_\odot$, but adjust two parameters $\epsilon$ and $M_{out}$ to demonstrate the final accretion rates of SMBHs.

3.1. Structure of the SF Disk

Figure 2(a1) shows that the inflow driven by an SNexp holds at a level of $M_\bullet = 0.5 \sim 2 M_\odot$ at the self-gravity radius ($R_{SG} \sim 0.02$ pc) for $\epsilon = 0.5$. This clearly demonstrates that an SF disk is able to drive an SS disk. Figure 2(b1) plots $\Sigma_*$ along the radial direction. We find that there is a peak of $\Sigma_*$ in the SF disks before switching on an SS disk. This is caused by the star formation and the radial transportation of gas that compete along radii. From the second term of Equation (7), there is a cutoff of $\Sigma_*$ and then an SF disk switches on an SS disk. Figure 2(c1) displays a smooth transition of the surface density from SF disks to SS disks. The bumps in Figure 2(c1) are artificially made by the assumption for simplicity that there is no star formation in the SS disk. This assumption roughly holds since the temperature within $R_{sg}$ could be high enough to efficiently suppress star formation.

The right column of Figure 2 shows a dependence of the structure on $\epsilon$ for fixed $M_{out}$. We find that $M_\bullet$ increases with $\epsilon$, which appears in the exponential as shown in Equation (7). For $M_{out} = 3 M_\odot$ yr$^{-1}$, the accretion rates are about 5%–10% $M_{out}$ from $\epsilon = 0.2$–0.6. The fraction $(1 - \epsilon)$ of the SNexp kinetic energy will be used to heat the gas of the SF disk, and then star formation is quenched close to the middle regions of SS disks. The output of this energy appears in the infrared bands in spectral energy distributions. Figure 3 shows the dependence of accretion rates on the global star formation rates, defined by $\dot{R}_* = \int 2\pi \dot{\Sigma}_* R dR$. For a fixed $\epsilon$, we find $\dot{R}_* \propto M_\bullet^q$, where $q = 1.43$–1.56 insensitive to $M_{out}$.

It is necessary for AGNs and quasars to switch the SF disks from the middle region of the SS disks. We find that for lower
The Shemmer et al. (2004) sample. We employ the relation \( \varepsilon = 1.33 \log(N / \text{C IV}) \) (Hamann et al. 2002) to convert \( N / \text{C IV} \) into \( Z \) for the Shemmer et al. (2004) sample. We employ the relation \( L_{1450} = 2L_{5100} \approx (2 / 9)L_{Bol}, \) \( L_{5100} \approx 9L_{Bol} = 9\eta M_{\odot}c^2, \) where \( \eta \) is the radiative efficiency and \( L_{Bol}, L_{1450}, \) and \( L_{5100} \) are bolometric and specific luminosities at 1450 Å and 5100 Å, respectively. We use \( Z_0 = 0, \) \( Z_1 = 0.1, \) \( Z_2 = 0.1, 0.05, \) \( 0.01 \) for red, green, and blue solid lines, respectively, whereas the dotted lines are for \( \varepsilon_1 = 10. \) The theoretical metallicity–luminosity relation is consistent with the observed.

\( M_{\text{out}} \), the SF disks only support the outer region of the SS disk. In such a case, the SF disk may drive an advection-dominated accretion flow (ADAF) to SMBHs. Excellent examples of this case could be the Galactic center, some elliptical galaxies (e.g., M87), and LINERS, where star formation exhausts most of gas forming an ADAF as discussed by Tan & Blackman (2005). We would like to point out that \( \beta \) used here is only an intermediate top-heavy IMF. It has been suggested that \( \beta = 0.85 \) (Maness et al. 2007) and \( \beta = 0.45 \) (Bartko et al. 2010) in the Galactic center.

3.2. Relation Between Metallicity and Luminosity

Metals are ejected through the SNexp enhancing the metallicity in the SF disk. The mass of the metal element \( -i \) produced by the SNexp is given by

\[
M_{Z} = \int_{m_{Z}}^{m_{Z_out}} \text{d} M_{\odot} N(M_{\odot}) \rho_{\odot} m_{\odot}^{\beta}(M_{\odot}, Z_{\odot})
\]

where \( m_{Z}^{\beta}(M_{\odot}, Z_{\odot}) \) is the ejected mass of the element \( -i \) dependent on the initial mass and metallicity of progenitor stars (Gavilán et al. 2005). With the \( M_{\text{Z}} \), the total ejected metal is then calculated by \( M_{Z} = \sum M_{Z} \). For simplicity, we take an approximation of the metal fraction \( m_{Z} = M_{Z} / M_{\odot} \approx 0.1-0.2 \) (Woosley & Weaver 1986).

Detailed calculations of metallicity involve evolution of stellar populations in the SF disk, advection of metal-enriched gas, and star formation, which make a strong radial dependence of metallicity complicated. However, we study the mean metallicity \( Z \), as the observed, in the SF disk by introducing \( \varepsilon_1 \) and \( \varepsilon_2 \). Considering the radial dependence of metallicity, \( \varepsilon_1 Z \) will be swallowed by SMBHs while \( \varepsilon_2 Z \) is consumed by star formation. The lower limit of metallicity at radius \( R \) is roughly given by \( Z \approx f_{M} m_{Z} / M_{\odot} \), and we have \( \varepsilon_1 \gg Z_{\odot} / Z \sim M_{\odot}^{\beta} / M_{\odot} \sim m_{\odot}^{\beta} / M_{\odot} \approx 5 \) from Figure 2(a), where the (sub/superscript) “in” means the values of parameters at the inner edge of the SF disk, and \( M_{\odot} \) is the local SF rates. On the other hand, its upper limit is \( Z_{\odot} / Z \sim f_{M} m_{Z} / M_{\odot} \sim 30 / (Z / Z_{\odot}) \), where we use \( f_{c} = 0.6 \) and \( m_{\odot} = 0.1 \). So we have \( 5 \leq \varepsilon_1 \leq 30 \) if the mean \( Z \sim Z_{\odot} \). The parameter \( \varepsilon_2 \sim Z_{\odot} / Z \sim 0.1 \) for \( Z_{\odot} = 0.1 Z_{\odot} \).

The total mass of the metal by time \( t \) through integrating the net increase of metals is given by

\[
M_{Z}(t) = \int_{0}^{t} (Z_{0} M_{\text{out}} + \dot{f}_{c} \dot{R}_{c} m_{Z} - \varepsilon_1 Z_{\odot} M_{\odot} - \varepsilon_2 Z_{\odot} \dot{R}_{c}) \text{d}t,
\]

where \( Z_{0} \) is the initial metallicity of the gas supplied from the torus. The first term in the integration is contributed by the initial metal of supplied gas, the second is contributed by the SNexp, the third is advected into the SMBH, and the last one is the consumption of metals during star formation. The total gas of the SF disk is given by

\[
M_{gas}(t) = \int_{0}^{t} 2 \pi \Sigma_{gas}(t, R) R dR.
\]

We obtain the metallicity at time \( t \) by its definition of \( Z(t) = M_{Z}(t) / M_{gas}(t) \).

\[
Z(t) M_{gas}(t) = \int_{0}^{t} (Z_{0} M_{\text{out}} + \dot{f}_{c} \dot{R}_{c} m_{Z} - \varepsilon_1 Z_{\odot} M_{\odot} - \varepsilon_2 Z_{\odot} \dot{R}_{c}) \text{d}t.
\]

This is an integral-differential equation, which is easy to understand in light of \( Z \) as an integral parameter with time. For a steady SF disk, its total mass is a constant, but metals are increasing. After some algebraic manipulations, we have its solution

\[
Z(t) = Z_{\text{max}} + (Z_{0} - Z_{\text{max}}) \exp\left(-\frac{t}{t_{Z}}\right),
\]

where

\[
Z_{\text{max}} = (Z_{0} M_{\text{out}} + f_{c} \dot{R}_{c} m_{Z} / (\varepsilon_1 M_{\odot} + \varepsilon_2 \dot{R}_{c})), \quad t_{Z} = M_{gas}/(\varepsilon_1 M_{\odot} + \varepsilon_2 \dot{R}_{c})
\]

and \( Z(0) = Z_{0} \) is assumed initially. For a sufficiently long time, the last term will tend to zero, and we have

\[
Z_{\text{max}} = Z_{0} (1 - \Lambda) + f_{c} (m_{Z} - Z_{0}) \Lambda \approx \frac{c_{0} f_{c} m_{Z}}{\varepsilon_1 + \varepsilon_2 c_{0} M_{\odot}^{1/4}} M_{\odot}^{1/4}
\]

where \( \Lambda = \dot{R}_{c} / (\varepsilon_1 M_{\odot} + \varepsilon_2 \dot{R}_{c}) \), \( M_{\text{out}} = (1 - f_{c}) \dot{R}_{c} + M_{\odot} \) is used, \( M_{\odot} \) is in units of \( M_{\odot} \ yr^{-1} \), \( \dot{R}_{c} \approx c_{0} M_{\odot}^{1/4} \), and \( c_{0} = 43.03 \) is insensitive to \( M_{\odot} \) for \( e = 0.5 \). The approximation is valid only for \( Z_{0} \ll m_{j} \). We assume for a comparison with observational data that most of quasars reach the state with maximum metallicity, but the episodic age of quasars can be estimated from Equation (14) if the metallicity is measured accurately enough.

Figure 4 shows a comparison of the model with data. Not only is the metal-rich phenomenon in quasars explained, but also the \( Z-L \) relation is reproduced by the present model. The large scatter of the \( Z-L \) relation imply different \( \varepsilon_1, \varepsilon_2, \) and \( c_{0} \) individually.

3.3. Compact Nuclear Star Clusters

The present model may predict the formation of nuclear star clusters commonly found in local galaxies (Kormendy et al. 2009). They are 1–2 orders of magnitude brighter than globular clusters (Côté et al. 2006), have extended star formation histories (Rossa et al. 2006), complex morphologies (Seth et al. 2006), and follow the relation similar to the Mageron relation (Ferrarese et al. 2006). Though the SF disk discussed here is only on a scale of 1 pc, SF is ongoing inside the torus at a few 10 pc scale (Collin & Zahn 1999), which is invisible due to dust extinction. Since AGN types are irrelevant to orientations of their host galaxies (e.g., Munoz-Marín et al. 2007), SMBHs are randomly fed by the dusty torus. This is evidence for random accretion onto the SMBHs, which is derived from the \( \eta \)-equation (Wang et al. 2009a) and is confirmed by calculations based on the semi-analytical theory of mergers (Li et al. 2010). One stellar belt with random direction composed of old stars is accordingly left after one episodic activity. The appearance of the nuclear star cluster could be a natural consequence of the multiple episodes of SMBH activity (Wang et al. 2009a).
et al. 2008; Wang et al. 2009a). The clusters are thus tightly related to the central SMBHs in the present model. Figure 3 suggests that a mass ratio of SMBH and a nuclear star cluster is roughly 0.01–0.1 depending on \( \epsilon \) and \( \beta \) (i.e., star formation history), consistent with the observations (Seth et al. 2008). Detailed comparison with observed properties of the clusters, such as radial-dependent metallicity, SF history, will provide further clues to understanding the physics in galactic centers.

All of the emission from the SF disk will be thermalized with a typical temperature of \( T_{\text{SF}} \sim (L_{\text{SN}}/2\pi R_{\text{SF}}^2 \sigma_{\text{SB}})^{1/4} \sim 2000 L_{143}^{1/4} \) K in the near-infrared band, where \( \sigma_{\text{SB}} \) is the Stefan–Boltzman constant, \( L_{\text{SN}} \), \( 143 \) = \( L_{\text{SN}}/10^{43} \) erg s\(^{-1}\) is the SNexp luminosity. It then contributes a fraction to the observed infrared emissions in quasars, which is worth investigating further.

4. CONCLUSIONS AND DISCUSSIONS

We show that an SF disk supplied by the dusty torus is able to support an SS disk around SMBHs in AGNs and quasars. Such a model naturally produces the observed relationship between metallicity and luminosity. As a natural consequence of the multiple episodes of activity through random accretion onto the holes, a nuclear star cluster will be formed from the remnants of stellar disks. The present model would provide a unified explanation of feeding SMBHs, production of metallicity, and formation of a nuclear star cluster.

We stress that the present paper deals with the stationary structure of SF disks switching to SS disks. A time-dependent model will give the evolution of AGNs driven by the SF disks for the complicated AGN–starburst connection.

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