The Polarizability of the Deuteron

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Abstract

The scalar and tensor polarizabilities of the deuteron are calculated using the recently developed effective field theory that describes nucleon-nucleon interactions. Leading and next-to-leading order contributions in the perturbative expansion predict a scalar electric polarizability of $\alpha_{E0} = 0.595 \text{ fm}^3$. The tensor electric polarizability receives contributions starting at next-to-leading order from the exchange of a single potential pion and is found to be $\alpha_{E2} = -0.062 \text{ fm}^3$. We compute the leading contributions to the scalar and tensor magnetic polarizabilities, finding $\beta_{M0} = 0.067 \text{ fm}^3$ and $\beta_{M2} = 0.195 \text{ fm}^3$.

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I. INTRODUCTION

Efforts to develop a systematic treatment of nucleon-nucleon interactions [1–19] have culminated in an effective field theory with consistent power counting [19]. The leading contribution to two nucleons scattering in an S-wave comes from local four-nucleon operators. Contributions from pion exchanges and from higher derivative operators are suppressed by additional powers of the external nucleon momentum and by powers of the light quark masses. The technique successfully describes the $NN$ scattering phase shifts up to center-of-mass momenta of $p \sim 300$ MeV per nucleon [19] in all partial waves.

To accommodate the unnaturally large scattering lengths in S-wave nucleon-nucleon scattering, fine-tuning is required, which in turn can complicate power counting in the effective field theory. Dimensional regularization with power divergence subtraction (PDS), described in [19], provides a consistent power counting scheme. Since the deuteron is the lightest nucleus and does not have irreducible forces between three or more nucleons, it provides a unique laboratory for studying the strong interactions. Being bound by only $2.2$ MeV, the characteristic momentum of the nucleons in the deuteron is $\sim 40$ MeV and should be well described by the effective field theory which is valid below the scale $\Lambda_{NN} \sim 300$ MeV [19]. The electromagnetic moments and form factors of the deuteron have been explored with this new effective field theory [20]. The charge radius and form factor receive contributions from leading and next-to-leading (NLO) orders in the expansion with the theoretical result reproducing the measured values within the uncertainty coming from the omission of higher order terms. The magnetic moment and form factor receive contributions at leading order from only the nucleon magnetic moments. At next-to-leading order the deuteron magnetic moment determines a combination of counterterms that appear in the Lagrange density at this order. The quadrupole moment first appears from the exchange of a single potential pion. Pion-pole contributions, multiple potential pion exchange, and higher dimension operators contribute only at higher orders in the expansion.

Unlike the electromagnetic form factors, the electric and magnetic polarizabilities of an object are a direct measure of its “deformation” due to the presence of external electric and magnetic fields. Extensive experimental and theoretical progress has been made in understanding the electric and magnetic polarizabilities of the nucleon (for an overview see [21]). Chiral perturbation theory provides a systematic theoretical analysis in the single nucleon sector, with one-loop pion graphs dominating the electric polarizability, e.g. [22–26]. The magnetic susceptibility, on the other hand, is dominated by the $\Delta$-pole and has a significant uncertainty associated with it, e.g. [27]. Recently, the discussion has been extended to include “generalized polarizabilities,” the amplitudes appropriate for interactions with electrons [28–30].

Theoretical understanding of the polarizability of the deuteron has been expressed in terms of meson exchange potential models [31–40] (for an excellent discussion see [37]). In this work we present a model independent, analytic computation of the electric polarizabilities of the deuteron to NLO and the magnetic polarizabilities to leading order in the effective field theory describing nucleon-nucleon interactions.
II. EFFECTIVE FIELD THEORY FOR NUCLEON-NUCLEON INTERACTIONS

The terms in the effective Lagrange density describing the interactions between nucleons, pions, and photons can be classified by the number of nucleon fields that appear. It is convenient to write

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \ldots, \]  

(2.1)

where \( \mathcal{L}_n \) contains \( n \)-body nucleon operators.

\( \mathcal{L}_0 \) is constructed from the photon field \( A^\mu = (A^0, \mathbf{A}) \) and the pion fields which are incorporated into an SU(2) matrix,

\[ \Sigma = \exp \left( \frac{2i\Pi}{f} \right), \quad \Pi = \left( \begin{array}{cc} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{array} \right), \]  

(2.2)

where \( f = 132 \) MeV is the pion decay constant. \( \Sigma \) transforms under the global SU(2)_L \times SU(2)_R chiral and U(1)_{em} gauge symmetries as

\[ \Sigma \rightarrow L\Sigma R^\dagger, \quad \Sigma \rightarrow e^{i\alpha Q_{em}}\Sigma e^{-i\alpha Q_{em}}, \]  

(2.3)

where \( L \in SU(2)_L, R \in SU(2)_R \) and \( Q_{em} \) is the charge matrix,

\[ Q_{em} = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right). \]  

(2.4)

The part of the Lagrange density without nucleon fields is

\[ \mathcal{L}_0 = \frac{1}{2}(E^2 - B^2) + \frac{f^2}{8} \text{Tr} D_\mu \Sigma D^\mu \Sigma^\dagger + \frac{f^2}{4} \lambda \text{Tr} m_q(\Sigma + \Sigma^\dagger) + \ldots. \]  

(2.5)

The ellipsis denote operators with more covariant derivatives \( D_\mu \), insertions of the quark mass matrix \( m_q = \text{diag}(m_u, m_d) \), or factors of the electric and magnetic fields. The parameter \( \lambda \) has dimensions of mass and \( m^2_\pi = \lambda(m_u + m_d) \). Acting on \( \Sigma \), the covariant derivative is

\[ D_\mu \Sigma = \partial_\mu \Sigma + i e [Q_{em}, \Sigma] A_\mu. \]  

(2.6)

When describing pion-nucleon interactions, it is convenient to introduce the field \( \xi = \exp(i\Pi/f) = \sqrt{\Sigma} \). Under SU(2)_L \times SU(2)_R this transformations as

\[ \xi \rightarrow L\xi U^\dagger, \]  

(2.7)

where \( U \) is a complicated nonlinear function of \( L, R, \) and the pion fields. Since \( U \) depends on the pion fields it has spacetime dependence. The nucleon fields are introduced in a doublet of spin 1/2 fields

\[ N = \left( \begin{array}{c} p \\ n \end{array} \right). \]  

(2.8)

that transforms under the chiral SU(2)_L \times SU(2)_R symmetry as \( N \rightarrow UN \) and under the U(1)_{em} gauge transformation as \( N \rightarrow e^{i\alpha Q_{em}}N \). Acting on nucleon fields, the covariant derivative is


\[ D_\mu N = (\partial_\mu + V_\mu + ieQ_{\text{em}}A_\mu)N, \]  

(2.9)

where

\[ V_\mu = \frac{1}{2}(\xi D_\mu \xi^\dagger + \xi^\dagger D_\mu \xi) \]
\[ = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi + ieA_\mu (\xi^\dagger Q\xi - \xi Q\xi^\dagger)). \]  

(2.10)

The covariant derivative of \( N \) transforms in the same way as \( N \) under \( SU(2)_L \times SU(2)_R \) transformations (i.e. \( D_\mu N \rightarrow UD_\mu N \)) and under \( U(1) \) gauge transformations (i.e. \( D_\mu N \rightarrow e^{iQ_{\text{em}}A_\mu}D_\mu N \)).

The one-body terms in the Lagrange density are

\[
\mathcal{L}_1 = N^\dagger \left(iD_0 + \frac{D^2}{2M_N}\right)N - \frac{ig_A}{2}N^\dagger \sigma \cdot (\xi D\xi^\dagger - \xi^\dagger D\xi)N \\
+ \frac{e}{2M_N}N^\dagger \left(\kappa_0 + \frac{\kappa_1}{2}[\xi^\dagger \tau^3 \xi + \xi \tau^3 \xi^\dagger]\right)\sigma \cdot BN \\
+ 2\pi\alpha_E^{(N^0)}N^\dagger NE^2 + 2\pi\alpha_E^{(N^1)}N^\dagger \tau^3 NE^2 + 2\pi\beta_M^{(N^0)}N^\dagger NB^2 + 2\pi\beta_M^{(N^1)}N^\dagger \tau^3 NB^2 + \ldots, \quad (2.11)
\]

where \( \kappa_0 = \frac{1}{2}(\kappa_p + \kappa_n) \) and \( \kappa_1 = \frac{1}{2}(\kappa_p - \kappa_n) \) are isoscalar and isovector nucleon magnetic moments in nuclear magnetons, with

\[ \kappa_p = 2.79285, \quad \kappa_n = -1.91304. \]

The isoscalar and isovector electric polarizabilities of the nucleon are \( \alpha_E^{(N^0)} \) and \( \alpha_E^{(N^1)} \) while the corresponding magnetic quantities are \( \beta_M^{(N^0)} \) and \( \beta_M^{(N^1)} \). Experimentally, it is found that

\[ \alpha_p = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4} \text{ fm}^3, \quad \beta_p = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4} \text{ fm}^3, \quad (2.13) \]

for the proton \([11,12]\) while the two measurements of the neutron electric polarizability \( \alpha_n = 12.0 \pm 1.5 \pm 2.0 \) \([13]\) and \( \alpha_n = 0 \pm 5 \) \([14]\) indicate a sizable uncertainty.

The two-body Lagrange density is

\[
\mathcal{L}_2 = -\left(C_0^{(S_1)} + D_2^{(S_1)}\lambda\text{Tr}m_q\right)(N^T P_i N)^\dagger(N^T P_i N) \\
+ \frac{C_2^{(S_1)}}{8}\left[(N^T P_i N)^\dagger\left(N^T P_i \mathbf{D}^2 + \mathbf{D}^2 P_i - 2\mathbf{D}P_i \mathbf{D}\right)N\right] + \text{h.c.} \\
+ eL_1(N^\dagger \sigma \cdot BN)(N^\dagger N) + eL_2(N^\dagger \sigma \cdot B\tau^a N)(N^\dagger \tau^a N) \\
+ 2\pi\alpha_d(N^T P_i N)^\dagger(N^T P_i N)E^2 + 2\pi\beta_d(N^T P_i N)^\dagger(N^T P_i N)B^2 + \ldots, \quad (2.14)
\]

where \( P_i \) is the spin-isospin projector for the spin-triplet channel appropriate for the deuteron

\[ P_i \equiv \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2, \quad \text{Tr}P_i^\dagger P_j = \frac{1}{2}\delta_{ij}. \]

(2.15)

The \( \sigma \) matrices act on the nucleon spin indices, while the \( \tau \) matrices act on isospin indices. The local operators responsible for \( S - D \) mixing do not contribute at either leading or NLO. The \( C_0^{(S_1)} , C_2^{(S_1)} \) and \( D_2^{(S_1)} \) coefficients are determined from \( NN \) scattering to be

\[ \text{...} \]
\[ C_0^{(3S_1)}(m_\pi) = -5.51 \text{ fm}^2, \quad D_2^{(3S_1)}(m_\pi) = 1.32 \text{ fm}^4, \quad C_2^{(3S_1)}(m_\pi) = 9.91 \text{ fm}^4, \]  

where we have chosen to renormalize the theory at a scale \( \mu = m_\pi \) in the PDS scheme [19]. A linear combination of the coefficients \( L_{1,2} \) contribute to the magnetic moment of the deuteron, but neither they nor the coefficients \( \alpha_4 \) and \( \beta_4 \) contribute to the deuteron polarizability at the order to which we are working. The four individual nucleon polarizability counterterms also do not contribute at the order to which we are working. In eq. (2.14) we have only shown the leading terms of the expansion in meson fields, namely the terms we need for our leading plus NLO calculation.

Since we are working with a field theory, no ambiguities arise from how we choose to define the interpolating fields. We can consistently neglect all operators that vanish by the equations of motion [45,46]. An operator that vanishes by the equations of motion makes a contribution to an observable that has exactly the same form as higher dimension operators that are present in the theory. Further, such operators can be removed by field redefinitions, and therefore it is consistent to work with a Lagrange density that does not contain operators that vanish by the equations of motion.

Another point of interest is that we do not need to include the \( \Delta \) as an explicit degree of freedom in the theory. In order to have a theory that is well defined for processes involving momenta up to \( \sim 1 \text{ GeV} \), the \( \Delta \) must be included as a dynamical object [47]. However, the power counting for the effective field theory describing the nucleon-nucleon interaction outlined in this section is limited to momenta less than \( p \sim \Lambda_{NN} \sim 300 \text{ MeV} \). The momentum scale making the dominant contribution to graphs involving the \( \Delta \) is approximately \( p \sim \sqrt{M_N(M_\Delta - M_N)} \sim 500 \text{ MeV} \), higher than \( \Lambda_{NN} \). Therefore, the \( \Delta \) is not included as a dynamical object, but its effects are included in the coefficients of the local operators (as are the effects of all particles not included as dynamical degrees of freedom).

III. COMPUTING THE POLARIZABILITIES OF THE DEUTERON

The Lagrange density described in the previous section in terms of nucleon field operators matches onto an effective theory describing the dynamics of the deuteron field

\[
\mathcal{L}_D = \mathcal{D}^\dagger_a \left( iD_0 + \frac{D^2}{2M_D} \right) \mathcal{D}^a - i\mu_D \epsilon^{abc} \mathcal{D}^\dagger_a \mathcal{D}_b \mathcal{B}_c \\
+ 2\pi\alpha_{E0} \mathcal{D}^\dagger_a \mathcal{D}^a \mathbf{E}^2 + 2\pi\beta_{M0} \mathcal{D}^\dagger_a \mathcal{D}^a \mathbf{B}^2 \\
+ 2\pi\alpha_{E2} \left[ \mathcal{D}^\dagger_a \mathcal{D}_b + \mathcal{D}^\dagger_b \mathcal{D}_a - \frac{2}{3}\delta_{ab} \mathcal{D}^\dagger_c \mathcal{D}_c \right] \mathbf{E}^a \mathbf{E}^b \\
+ 2\pi\beta_{M2} \left[ \mathcal{D}^\dagger_a \mathcal{D}_b + \mathcal{D}^\dagger_b \mathcal{D}_a - \frac{2}{3}\delta_{ab} \mathcal{D}^\dagger_c \mathcal{D}_c \right] \mathbf{B}^a \mathbf{B}^b .
\]  

\( \mathcal{D}^a \) is an operator that annihilates a deuteron, and its spin index takes values \( a = 1, 2, 3 \). The covariant derivative acting on the deuteron field is \( D_\mu = \partial_\mu + ieQA_\mu \). The coefficient \( \mu_D \) is the deuteron magnetic moment and has been determined from eqs.(2.11) and (2.14) to NLO in [20]. The scalar electric polarizability of the deuteron is \( \alpha_{E0} \), and the scalar magnetic polarizability is \( \beta_{M0} \). These operators give rise to interactions that are independent of the alignment of the deuteron with respect to an applied electromagnetic field. The tensor
electric polarizability of the deuteron is $\alpha_{E2}$, and the tensor magnetic polarizability is $\beta_{M2}$. These operators give rise to interactions that depend upon the alignment of the deuteron with respect to an applied electromagnetic field. In order to compute $\alpha_{E0,E2}$ and $\beta_{M0,M2}$ we will use the formalism developed in [20]. The polarizabilities $\alpha$ and $\beta$ each have perturbative expansions in powers of $Q = m_\pi/\Lambda_{NN} \sim \sqrt{M_N}\mathcal{B}/\Lambda_{NN}$, where $\mathcal{B}$ is the deuteron binding energy. We denote the contribution from a given order by a superscript, e.g. the scalar electric polarizability

$$\alpha_{E0} = \alpha_{E0}^{(-4)} + \alpha_{E0}^{(-3)} + \ldots,$$

and similarly for the other polarizabilities.

In matching onto the deuteron effective Lagrange density in eq. (3.1) from the Lagrange density describing nucleon dynamics, eqs. (2.11) and (2.14), we will recover the coefficient of each operator order by order in the $Q$ expansion, including the coefficient of the operator $D^aD^bD^a$. At leading order in the loop expansion, we find this operator to have a coefficient $\frac{1}{4M_N}$, while at next-to-next-to-leading order (NNLO) it will have a coefficient $\frac{1}{4M_N} + \frac{\mathcal{B}}{8M_N^2}$. When determined to all orders it will sum to $\frac{1}{2(2M_N-\mathcal{B})} = \frac{1}{2M_D}$, which we have written in eq. (3.1). However, it is important to realize that we will only recover interactions coming from this operator order by order in the expansion.

It is instructive to begin by power counting the contributions from operators that appear in the Lagrange density eqs. (2.11) and (2.14). Consider the single loop graph of Fig. 1 between a source that creates a spin triplet $NN$ pair and one that annihilates it, minimally coupling to two photons. In terms of the expansion parameter $Q$, a non-relativistic loop integral scales as $Q^{-5}$, a non-relativistic nucleon propagator as $Q^{-2}$, and a gradient operator as $Q^1$. In order to match onto the $E^2$ or $B^2$ operator in the deuteron Lagrange density (3.1), we must find the coefficient of $\omega^2$ or $k^2$ by expanding the graph in powers of $\omega$ and $k$, respectively, where $(\omega, k)$ is the photon four-momentum. Therefore, this diagram will scale like $Q^{-1}\sum_n \left(\frac{M_N\omega|k|^2}{Q^2}\right)^n$. Wave function renormalization introduces a factor of $Q$, and therefore we find that the leading contribution to the electric polarizabilities arising from this graph is of order $Q^{-4}$. This power counting also shows that the $C_2(\mu)$ operator and the exchange of a single potential pion contribute at order $Q^{-3}$ after wave function renormalization. The magnetic polarizabilities receive contributions from all the graphs shown in Fig. 1 at order $Q^{-2}$. The four-nucleon polarizability counterterms, $\alpha_4$ and $\beta_4$, appearing in (2.14) contribute at order $Q^1$, and can be safely neglected. Counterterms for the individual nucleon polarizabilities appearing in (2.11) contribute at order $Q^0$. Meson loop corrections that are the dominant contribution to the electric polarizability of the nucleon, for instance Fig. 3, appear at order $Q^{-1}$, three orders higher in the expansion than the leading order contributions. Therefore, it is probable that the polarizabilities of the individual nucleons will not be extracted from the polarizabilities of the deuteron.

We will compute the leading and NLO contributions to the scalar and tensor electric polarizability of the deuteron, and the leading contributions to the magnetic scalar and tensor polarizabilities. The loop graphs in Fig. 1 give the leading contribution to the deuteron polarizabilities, which after wave function renormalization are

$$\alpha_{E0}^{(-4)} = \frac{\alpha M_N}{32\gamma^4}.$$
FIG. 1. Leading order contributions to the deuteron polarizabilities. The crossed circles denote operators that create or annihilate two nucleons with the quantum numbers of the deuteron. The dark solid circles correspond to the photon coupling via the nucleon kinetic energy operator (minimal coupling), while the light solid circles denote the nucleon magnetic moment operator. The solid lines are nucleons. Only the graph with the nucleons minimally coupled to the electromagnetic field contributes to the electric polarizability. The photon crossed graphs are not shown. The bubble chain arises from insertions of the four nucleon operator with coefficient $C_{0}^{(1S_{0})}$.

\[\begin{align*}
\alpha_{E2}^{(-4)} &= 0 \\
\beta_{M0}^{(-2)} &= \frac{\alpha}{2M_{N}} \left[ -\frac{1}{16\gamma^2} + \frac{2(k^{(0)})^2 + (k^{(1)})^2}{3\gamma^2} + \frac{(k^{(1)})^2 M_{N}}{6\pi \gamma} A_{-1}^{(1S_{0})}(-B) \right] \\
\beta_{M2}^{(-2)} &= -\frac{\alpha}{2M_{N}} \left[ \frac{(k^{(0)})^2 - (k^{(1)})^2}{\gamma^2} - \frac{(k^{(1)})^2 M_{N}}{2\pi \gamma} A_{-1}^{(1S_{0})}(-B) \right],
\end{align*}\]

where $\gamma = \sqrt{M_{N}B}$ and $\alpha = 1/137$ the fine structure constant. The scattering amplitude in the $1S_{0}$ channel for a centre of mass energy $E$ is

\[A_{-1}^{(1S_{0})}(E) = \frac{-C_{0}^{(1S_{0})}}{1 + C_{0}^{(1S_{0})} M_{N} \gamma / (\mu - \sqrt{-M_{N}E - i\varepsilon})},\]

which is of order $Q^{-1}$. The coefficient $C_{0}^{(1S_{0})}$ has been determined from nucleon-nucleon scattering in the $1S_{0}$ channel to be $C_{0}^{(1S_{0})} = -3.34 \text{ fm}^2$.

At NLO there are contributions from the exchange of a single potential pion, Fig. [3], and from the operator with coefficient $C_{2}^{(\mu)}$, Fig. [4]. The operator with coefficient $D_{2}^{(\mu)}$ does not contribute to the polarizability of the deuteron. We find that at order $Q^{-3}$
FIG. 2. A contribution to the deuteron electric polarizability from a graph that also contributes to the polarizability of the nucleon. The crossed circles denote operators that create or annihilate two nucleons with the quantum numbers of the deuteron. The solid circles correspond to the photon coupling via the nucleon kinetic energy operator (minimal coupling). Dashed lines are pions and solid lines are nucleons.

\[ \alpha_{E_0}^{(-3)} = \frac{\alpha M_N^2}{64 \pi \gamma^3} C_2(\mu) (\mu - \gamma)^2 + \frac{\alpha g_A^2 M_N^2}{384 \pi f^2} \frac{m_\pi^2 (3m_\pi^2 + 16m_\pi \gamma + 24\gamma^2)}{\gamma^3 (m_\pi + 2\gamma)^4} \]

\[ \alpha_{E_2}^{(-3)} = -\frac{\alpha g_A^2 M_N^2}{80 \pi f^2} \frac{2m_\pi^3 + 11m_\pi^2 \gamma + 16m_\pi \gamma^2 + 8\gamma^3}{\gamma^2 (m_\pi + 2\gamma)^4} \]  

(3.5)

The leading contribution to the tensor polarizability of the deuteron \( \alpha_{E_2} \) comes from the exchange of a potential pion. The explicit renormalization scale dependence that appears in eq. (3.5) is compensated by the \( \mu \) dependence of the coefficient \( C_2(\mu) \), which scales \( \sim 1/\mu^2 \) [19].

Numerically, we find that the electric polarizabilities are

\[ \alpha_{E_0} = \alpha_{E_0}^{(-4)} + \alpha_{E_0}^{(-3)} + \ldots = 0.386 \text{ fm}^3 + (0.153 + 0.057) \text{ fm}^3 + \ldots = 0.595 \text{ fm}^3 + \ldots \]  

(3.6)

where the dots denote contributions higher order in the expansion. The first term in parenthesis on the second line of eq. (3.6) comes from the \( C_2(\mu) \) operator while the second term arises from the exchange of a single potential pion. The convergence of the expansion for \( \alpha_{E_0} \) appears to be approximately the same as it is for the static electromagnetic moments [20], with each order being suppressed by between 1/3 and 1/2, consistent with the \( \Lambda_{NN} \) expansion [19]. We expect that the uncertainty in this numerical value is roughly \( \Delta \alpha_{E_0} \approx 0.1 \text{ fm}^3 \) from an estimate of the next order contribution. For the tensor polarizability we find

\[ \alpha_{E_2} = \alpha_{E_2}^{(-4)} + \alpha_{E_2}^{(-3)} + \ldots = -0.062 \text{ fm}^3 + \ldots \]  

(3.7)

where we recall \( \alpha_{E_2}^{(-4)} = 0 \). The fractional uncertainty in \( \alpha_{E_2} \) is much greater than that for \( \alpha_{E_0} \) as it has a vanishing leading order contribution, and we naively estimate an uncertainty of \( \Delta \alpha_{E_2} \approx 0.03 \text{ fm}^3 \). The leading contribution to the magnetic polarizabilities are
FIG. 3. Graphs from potential pion exchange that contribute to the deuteron polarizabilities at NLO. The crossed circles denote operators that create or annihilate two nucleons with the quantum numbers of the deuteron. The solid circles correspond to the photon coupling via the nucleon or meson kinetic energy operator (minimal coupling) or from the gauged axial coupling to the meson field. Dashed lines are mesons and solid lines are nucleons. Photon crossed graphs are not shown.

\[ \beta_{M0}^{(-2)} = 0.067 \text{ fm}^3, \]
\[ \beta_{M2}^{(-2)} = 0.195 \text{ fm}^3. \]

The large values for the magnetic polarizabilities come from the isovector magnetic moment of the nucleon, \( \kappa_1 \).

It is informative to decompose these interactions into the polarizabilities of the individual magnetic substates of the deuteron. The electric polarizability of the \( m = \pm 1 \) states of the deuteron is \( \alpha_{E0} - \frac{2}{3} \alpha_{E2} \) while the polarizability of the \( m = 0 \) state is \( \alpha_{E0} + \frac{4}{3} \alpha_{E2} \), and similarly for the magnetic polarizabilities, we find

\[ \alpha_{E}^{m=\pm 1} = 0.637 \text{ fm}^3 + \ldots, \quad \beta_{M}^{m=\pm 1} = -0.063 \text{ fm}^3 + \ldots \]
\[ \alpha_{E}^{m=0} = 0.511 \text{ fm}^3 + \ldots, \quad \beta_{M}^{m=0} = 0.327 \text{ fm}^3 + \ldots, \]

where the ellipsis denotes higher order contributions. Numerical evaluation of our analytic results agree within uncertainties with values for the electric polarizabilities obtained from potential models. To a very high degree of precision potential models predict a scalar electric polarizability of \( \alpha_{E0} = 0.6328 \pm 0.0017 \text{ fm}^3 \) \cite{37}, which is remarkably consistent with the “zero-range” limiting value of \( \alpha_{E0} = 0.632 \pm 0.003 \text{ fm}^3 \) \cite{31}. Further, the calculations of \cite{40} find \( \alpha_{E}^{m=\pm 1} = 0.669 \text{ fm}^3 \) and \( \alpha_{E}^{m=0} = 0.555 \text{ fm}^3 \). We expect to deviate from these values at higher orders in the effective field theory expansion as generally potential models do not properly describe chiral dynamics, pion propagation, and relativistic effects. The power of the effective field theory formalism is that such effects can be included systematically and the natural size of higher order terms is known. Effective field theories will never match the
precision of potential or other models because their reliance on only the symmetries of the underlying theory and experimental results necessarily makes them less restrictive. However, the fact that effective field theory descriptions provide a model independent treatment makes them an important tool in the study of low energy processes.

**IV. CONCLUSIONS**

We have performed a model independent computation of the scalar and tensor polarizabilities of the deuteron in the effective theory describing nucleon-nucleon interactions. The scalar electric polarizability receives leading order contributions while the tensor electric polarizability begins at NLO in the power counting, and so is sizably smaller than the scalar polarizability. The tensor electric polarizability arises only from the exchange of potential pions at NLO. The leading contribution to the magnetic polarizability of the deuteron comes from the magnetic moments of the individual nucleons in addition to their minimal coupling to the electromagnetic field. From this analysis we conclude that it is unlikely that the neutron polarizabilities will be extracted from the deuteron polarizabilities.

We are encouraged that this effective field theory calculation reproduces the results of potential model calculations. The benefits of using an effective theory are that closed form expressions have been obtained for the polarizabilities, and through the universal couplings in the Lagrange density we can see how QCD relates a variety of two nucleon processes in a
systematic way. Further, higher order terms not considered in this work are parametrically smaller that those we have presented. In subsequent work we will present the photon-deuteron scattering cross section over a range of photon energies.

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