A Question About
Standard Cosmology and Extremely Dense Stars’ Collapsing

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We ask if the conventional variable separation techniques in the studying of standard cosmology and the collapsing of extremely dense stars introduce Newton’s absolute space-time concepts. If this is the case, then a completely relative cosmology is needed. We build the basic frame-works for such a cosmology and illustrate that, the observed luminosity-distance v.s. red-shift relations of supernovae can be explained naturally even without any conception of dark energies.

I. OUR QUESTIONS

Either in the standard cosmology, or in the a spherical collapsing star, we are told to start with a homogeneous and isotropic fluid ball, write down the dynamic equation describing its evolutions as,

\[ G_{\mu \nu} = -8\pi GT_{\mu \nu}, \] (1)

and the general expressions for energy momentum tensor, \( T_{\mu \nu} = \rho u_{\mu} u_{\nu} + p(g_{\mu \nu} + u_{\mu} u_{\nu}), \) in the co-moving reference frame is written as

\[ T_{\mu \nu} = \text{diag}(\rho, p, p, p) \]

if no radiation and/or no dark energy is involved

\[ = \text{diag}(\rho, 0, 0, 0) \] (2)

In solving eq(1), we are told to start from the metric ansatz

\[ ds^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2\theta d\phi^2), \] (3)

using variable separation techniques, setting

\[ U(t, r) = a^2(t)f(r), \]
\[ V(t, r) = a^2(t) r^2, \]
\[ f(r) = \frac{1}{1 - kr^2}, k = 0, \pm 1 \] (4)

and finally obtain Friedmann Equation

\[ \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \] (5)

Our question is, can the factorized (and independently, and at the putting epoch, all the matter particles are at rest and of no inter-gravitation at all. Since otherwise, the expansion or contraction of basic particles in the system will produce pressures. The \( U(t, r) \) and \( V(t, r) \) factorizable metric eqs(3) + (4) is only a special kind of solutions of Einstein equation. This solution requires us to have a globally defined scale factor \( a(t) \) and a co-moving reference mesh in the universe or in the collapsing star. At the initial time of “universe creation”, such as inflation beginning, Ekpyrotic universe’s Brane-Brane-collision-point, or the collapsing beginning epoch of the extremely dense stars, all the objects in the system were put on the co-moving mesh instantaneously and independently, but homogeneously and isotropically. For this reason, all these objects are of no inter-gravitation, and no pressures will be introduced at the initial time. On the contrary, the \( U(t, r) \) and \( V(t, r) \) non-factorizable solutions also exist and in those solutions, all the objects in the system are all inter-gravitated from the beginning. We have no epochs at which those objects’s inter-gravitation can be set to zero, hence no epochs at which the pressures originated from those objects’ expanding or falling down can be neglected.

Physically speaking, assuming the metric function of the system can be factorized as \( U(t, r) = a^2(t)f(r), \) probably introduces Newton’s absolute space-time in our studies. In this space-time concepts, all the objects in the system were put on the co-moving meshes (absolute space) instantaneously and independently at the initial epochs. In the following evolutions, the distances between any two space points increase or decreases at the same speed, because the scale factor \( a(t) \) (absolute time) is defined globally in the system. So if we were put on a given point in the system, we must see some points are running away from us at infinitely speed, as long as those points are infinitely away from us. Obviously, this contradicts special relativity.

II. MAXIMUMLY SYMMETRIC SPACE?

Some people may tell us the fact that our universe should be described by Friedmann-Robertson-Walker metric is the requirement of symmetry. It has nothing to do with other things such as those we stated in the

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previous section. To this argument, we would like to point out that, the symmetries referred to by these peoples are symmetries of an absolute space, its operating definition depends on the existence of a signal which can be infinitely speedy. So, our question may be asked as, is cosmological principle in the standard cosmology expressed in an anti-relativity way? (The case of extremely dense stars’ collapsing theory has similar problem.) Our meaning is, if special relativity is considered in the definition of homogeneity and isotropy, then we must consider the finiteness of signal transferring speed, i.e., the homogeneity and isotropy must be a statement which can be tested by signals no more speedly than light.

So, special relativity may suggest that, to build a completely relative cosmology, we may have to find a new expression of cosmological principles which is consistent with the finiteness of light’s tranferring speed.

III. CONSTRUCTURE A COMPLETELY RELATIVE THEORY OF COSMOLOGY AND BLACK HOLES’ FORMATION

If we discard the usual definition of cosmological principle or similar expressions in the extremely dense stars collapsing case which is based on the existence of signals which have infinitely large tranferring speeds, we will have no co-moving mesh anymore on which the basic particles in our systems can be put instantaneously and independently, and the physical distances between any two matter particles in our systems can not be factorized as the multiplying of an only-time-dependent scale factor and an absolutely defined co-moving distance. In this case the function $U(t, r)$ and $V(t, r)$ is non-factorizable as in eq(3).

Techniquely analyzing, the Hubble recession of the basic molecules of cosmological gases or the falling down of the matter particles in the extremely dense stars make the energy momentum tensor of the system non-diagonal, hence make the function $U(t, r)$ and $V(t, r)$ non-factorizable. To emphasize this point, let us rewrite the metric of our systems in the following,

$$ds^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2\theta d\phi^2). \quad (6)$$

Denote the four velocity of basic molecules of the cosmological gases or that of the matter particles in the extremely dense stars as

$$u^\mu = (u^0, u^1, 0, 0), \text{ but}$$

$$\frac{u^1}{u^0} = v, \quad -(u^0)^2 + (u^1)^2U(t, r) = -1, \quad (7)$$

where $v$ is appropriate Hubble recession or falling down velocity of the basic particles in our studying system. It is an observable quantity and its value depends on $(t, r)$.

The usual hubble parameter is defined as

$$H = \frac{1}{r} v = \frac{1}{r} \frac{dr}{dt}. \quad (8)$$

From eqs(7) we can solve

$$u^0 = \frac{1}{\sqrt{1 - U v^2}}, u^1 = \frac{v}{\sqrt{1 - U v^2}}. \quad (9)$$

So the energy momentum tensor describing our cosmological gas or the matter particles in the extremely dense stars is

$$T^\mu\nu = \rho u^\mu u^\nu + p(g^\mu\nu + u^\mu u^\nu),$$

$$p = \frac{1}{3} \mu v^2, \text{we are not sure if}$$

$$\rho \text{ should depend on (t, r) or not}$$

$$= \rho \left[ \begin{array}{cccc}
\frac{1+Uv^2}{1-Uv^2} & \frac{v(1+Uv^2)}{1-Uv^2} & 0 & 0 \\
\frac{v(1+Uv^2)}{1-Uv^2} & \frac{(1+3Uv^2)}{3U(1-Uv^2)} & 0 & 0 \\
0 & 0 & 3v^2 & 0 \\
0 & 0 & 0 & \frac{2}{3v\sin \theta} \\
\end{array} \right]. \quad (10)$$

In the standard cosmology, an $r$ dependent matter density violates cosmological principle, which says that the matter density in our universe is homogeneous and isotropic on large scales. While in the conventional black hole theory, a $(t, r)$ dependent matter distribution inside the horizon of the black holes violates “no-hair” theorem.

However, we are intending to believe that the energy density $\rho$ appearing in the above energy momentum tensor should depend on $t$ and $r$. Because the standard cosmology definition of homogeneity and isotropy (or similar expressions in the extremely collapsing stars have the same problem) depends on a globally defined co-moving reference mesh. It is on this co-moving reference mesh that the matter distribution in our universe are homogeneous and isotropic. We have explained this globally defined co-moving reference mesh may just be Newton’s absolute space, so the homogeneity and isotropy definitions on this reference mesh cannot be tested by experiment. The definition which can be tested by experiment should depends on an experiment tool, such as light. So we have to introduce a four velocity

$$c^0 = (c^0, c^1, 0, 0), \text{ but}$$

$$c^1 = 1, \quad -(c^0)^2 + (c^1)^2U(t, r) = 0, \quad (11)$$

Only observed by $c^0$, our universe is homogeneous and isotropic, and the black hole are “no-hair”, i.e.,

$$T^\mu\nu c_\mu c_\nu = \rho_{\text{aver}}, \quad T^{\mu\nu}(g_{\mu\nu} + c_\mu c_\nu) = 0, \quad \rho_{\text{aver}} \text{ only depends}$$

$$\text{on (t, neither t nor r).} \quad (12)$$

Eq(12) is our definition of cosmological principle or similar statement about the matter distributions in the extremely collapsing stars.
It is worth noting that superficially looking eq(11) means that \( U(t, r) = 1 \) (we will use “\( = \)” denoting identical relations and “\( \equiv \)” denoting definition relations). Is this the case? No, this only means that

\[
U(t, r)_{r=t} = 1, \quad (13)
\]
i.e., along the trace of light \( c^t = c^0 \) hence \( dr = dt \) and hence \( r = t \), the function \( U(t, r) \) takes value 1. In some sense, this can be looked as an assumption which has counterpart in standard cosmology as, \( U(t, r) \)’s dependence on \( t \) and \( r \) is factorized globally.

Starting from eq(12), explicit calculations will give us

\[
\rho_{\text{aver}}(t) = \rho(t, r)(1 + \frac{u^2}{3})(u \cdot c)^2, \\
0 = \rho(t, r)[(-1 + \dot{v}^2) + (1 + \frac{u^2}{3})(u \cdot c)^2], \quad (14)
\]

From which we can solve

\[
\rho(t, r) = \frac{\rho_{\text{aver}}(t)}{(1 + \frac{u^2}{3})(u \cdot c)^2}, \\
v(t, r) = 2U + F - (9U^4 - 18U^3 - 19U^2 - 36U)\frac{1}{(U^2 - 3U)}F^{-1}, \\
F = \frac{1}{2}(216U^2 + 1564U^3 - 864U^4 + 108U^5} \\
+ \sqrt{4(-36U - 19U^2 - 18U^3 + 9U^4)^3} \\
+ (216U^2 + 1564U^3 - 864U^4 + 108U^5)^2) \frac{1}{4}. \quad (15)
\]

Let us make a little plume of our logics here. In eq(9), we express the four velocity of the basic particles in our systems (universe or the inside space of extremely dense stars) in terms of \( U(t, r) \) and \( v(t, r) \), in eq(10) we express the energy momentum tensor describing the systems in terms of \( U(t, r) \) and \( v(t, r) \), while in eqs(12)+(14)+(15) according to cosmological principle (or similar statement such as black holes ‘no-hair’ theorem), we derive out the explicit dependence of \( v(t, r) \) on \( U(t, r) \). So if we substitute the results in eq(10) back into eq(11), we will have an energy momentum tensor expressed in terms of \( U(t, r) \) and \( \rho_{\text{aver}}(t) \), which can be solved from Einstein equations and energy momentum conservation law uniquely.

### IV. ENERGY MOMENTUM CONSERVATION AND EINSTEIN EQUATIONS

In standard cosmology or the conventional black hole formation theory, energy momentum conservation has simple forms, for the no-radiation and no-dark-energy cosmology, it is \( d(\rho a \cdot r_{co})^3 = 0 \); for the with-radiation or/and with-dark-energy case, \( d(\rho a \cdot r_{co})^3 + pd a \cdot r_{co})^3 = 0 \), where \( a \) is the only-time-dependent scale factor of the universe, while \( r_{co} \) is the time-independent co-moving distance.

However, if what we analyzed in the previous sections is the fact, then our current standard cosmology must be some kinds of approximation which is applicable only in short time evolution or small space phenomenology of the strict relative cosmology we proposed in the above section. In this case, the energy momentum conservation law \( T_{\mu \nu} = 0 \) can be integrated to give

\[
\int_0^r \frac{\rho_{\text{aver}}(t)(1 - \dot{v}^2)}{(1 + \frac{\dot{v}^2}{3})(1 - U^2)^2}\sqrt{U(t, r)V(t, r)}dr = \text{const.} \quad (16)
\]

We do not consider radiation and dark energy in our new frame-work of cosmology in this paper. Einstein equation

\[
R_{\mu \nu} = -8\pi G(T_{\mu \nu} - \frac{1}{2}g_{\mu \nu}T) \quad (17)
\]
has the following non-trivial component,

\[
\begin{align*}
R_{01} &= \frac{\dot{V}'}{V} - \frac{V'}{2V^2} - \frac{\dot{U}V'}{2UV} \\
&= 8\pi G \rho_{\text{aver}}(t) \frac{Uv}{(1 - Uv)^2}
\end{align*}
\]

\[
\begin{align*}
R_{00} &= \frac{\ddot{U}}{2U} + \frac{\dot{V}^2}{V} - \frac{\dot{U}^2}{4U^2} - \frac{\dot{V}^2}{2V} \\
&= 8\pi G \rho_{\text{aver}}(t) \left[ \frac{(1 - \frac{\dot{v}^2}{3})(1 - Uv^2)}{2(1 + \frac{\dot{v}^2}{3})(1 - Uv)^2} - \frac{1}{(1 - Uv)^2} \right]
\end{align*}
\]

\[
\begin{align*}
R_{11} &= \frac{V''}{V} - \frac{V'U'}{2V^2} - \frac{U''V'}{2UV} - \frac{\ddot{U}}{4U} - \frac{\dot{U}^2}{2V} \\
&= 8\pi G \rho_{\text{aver}}(t) \left[ \frac{U(1 + \dot{v}^2)(1 - Uv^2)}{2(1 + \frac{\dot{v}^2}{3})(1 - Uv)^2} - \frac{U}{(1 - Uv)^2} \right]
\end{align*}
\]

\[
\begin{align*}
R_{22} &= -1 + \frac{V''}{2U} - \frac{V'U'}{4U^2} - \frac{\ddot{V}U}{2U} - \frac{\dot{V}U}{4U} \\
&= -8\pi G \rho_{\text{aver}}(t) \left[ \frac{V(1 - \frac{\dot{v}^2}{3})(1 - Uv^2)}{2(1 + \frac{\dot{v}^2}{3})(1 - Uv)^2} \right]
\end{align*}
\]

So in our frame-work of cosmology, to predict experiments such as super-novaes’ luminosity-distance v.s. red-shift relations, we need substitute the expressions of \( v(t, r) \) in eq(10) into eq(15) and combine with eqs(13)+(14), then solve the resulting equations to get the function \( U(t, r) \). As long as \( U(t, r) \) is obtained, we can get the Hubble recession velocity v.s. \( (t, r) \) relation, from which the super-novaes’ luminosity-distance v.s. red-shift relation should be calculated.

Obviously, the forms of eq(10)+eq(15) are so complicated that almost no attempts of directly solving the
In in the first section of this paper, we ask if the conventional variable separation techniques in the studying of standard cosmology and the collapsing of extremely dense stars introduce Newton’s absolute space-time concepts. In the second section we point out that a maximally symmetric metric may only be used to describe an absolute space-time in which the signal transferring speed can be infinitely large. In our real universe, the speedestly transferring signal is light, so its describing metric cannot have the maximally symmetric properties.

In the third section of the paper, we build the basic frame-work of a completely ralitive cosmology, in which the cosmological principle is expressed consistently with special relativity. In standard cosmology, this is not the case. In the fourth section, we provide the basic equations controlling the evolutions of the quantities involved in our completely relative cosmology. In the fifth section, we prove that in a completely relative cosmology, the current observed luminosity-distance v.s. red-shift relations of supernovaes may be explained naturally without assuming that our universe is accelerately expanding.

If what we criticized here of standard cosmology is the fact, i.e., Friedmann-Robertson-Walker metric cannot be the correct metric describing our universe on large scales and long time evolution processes, then we will have to change it revolutionally. After the changes, we think at least four problems in our current cosmology will appear differently or even will not appear at all: (i)dark energy and cosmological constant problem, (ii)the primordial singularity problem, (iii)horizon problem and flattness problem and (iv) the necessity of inflations. Probably, the singularity problem of black holes will have different apperance either, because our current black hole formation theory is based on the extremely dense stars collapsing.

Although we have not get definite solutions from the basic equations we provide in the fourth section of this paper, we think such solution must exist. While the tasks of searching for such solutions must be challenging and meaningful, both physically and mathematically. It is a

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**FIG. 1:** Inferring that our universe is expanding accelerately: if $S_1$ and $S_2$ are two supernovaes $r$ and $2r$ away from us and if we observe photons from them have red-shift $z_1$ and $z_2$ (upper figure), then if we observe photons from two supernovaes also with red-shift $z_1$ and $z_2$, but are $r$ and $2r+\Delta r$ away from us (down figure, $\Delta r > 0$), in standard cosmology we infer that our universe is accelerately expanding.

System is to be successful. We are now working by a different strategy, that is, guess a solution such as $ds^2 = -dt^2 + e^{2\frac{t}{v}(dr^2 + r^2d\Omega_2^2)}$, note this solution has satisfy eq(13) already, then using(13) to get the function $\rho_{aver}(t)$, and check if the resulting function $\rho_{aver}(t)$ can satisfy eq(16) or not. Of course, if we require the solution come back to the usual Friedmann-Robertson-Walker metric, we can add extra requirements on our guess starting metric functions $U(t, r)$ and $V(t, r)$. We have not obtained definite conclusions on this aspect to provide here. But we think this is a challengeful and meaningful tasks both on physics and mathematics.

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**V. IS OUR UNIVERSE ACCELERATELY EXPANDING?**

Inferring that our universe is accelerately expanding from supernovae’s luminosity-distance v.s. red-shift relation is a well known and widely accepted conclusion, we illustrate the basic principles in FIG.1. In standard cosmology, if $S_1$ and $S_2$ are two supernovaes with distances $r$ and $2r$ away from us, then if $S_1$ are recessing away from us at speed $v$, we infer that $S_2$ is doing so at speed $2v$, please see the upper figure of FIG.1. This logics of reasoning is based on the Friedmann-Robertson-Walker metrics, the physical distance $r = a(t) \cdot r_{co}, a(t)$ is defined globally.

Our view point is, on large scales, we cannot write the metrics of our universe as Friedmann-Robertson-Walker type, we cannot define a globally meaningful scale factor $a(t)$ so that physical distance can be factorized as $r = a(t) \cdot r_{co}$. If $S_1$ is a supernovae with distance $r$ away from us and it is recessing from us at speed $v$, we can only infer that if $S_2$ is $2r$ away from us, $S_2$ is recessing away from $S_1$ at speed $v$, while the relative recessing speed between $S_2$ and us should be calculated according to the velocity addition law of special relativity, $v_{2O} = \frac{2v}{1-v^2}$. Obviously this is smaller than $2v$. To get a relative speed $v_{2O} = 2v$, $S_2$ must be put on $2r + \Delta r$ away from us, please see the down figure of FIG.1.

So if what we say (standard cosmology’s space-time is Newton’s absolute one) is correct, then the luminosity-distance v.s. red-shift relations detected in the current observations may do not mean that our universe is accelerately expanding, it only means that, on large scales, the Friedmann-Robertson-Walker metric is not the correct metric which describes our universe.
completely new area of general relativity and cosmology.

**About references** about observations which indicate that our universe is accelerately expanding and an non-zero dark energies existing, we referred to [2, 3, 4, 5, 6, 7, 8]; about theoretical studying of dark energies, we referred to [8, 9, 10, 11, 12]; about inflation and Ekpyrotic universes we referred to [13, 14, 15]; about the non-factorizable solutions of Einstein equations we referred to [1, 17, 18]; professor A.D.Linde thinks that the solutions we provided in [17] may have relevance to [16]. We thank very much to professor A.D.Linde to inform us this point.

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