Asymptotically anti-de Sitter spacetimes
in topologically massive gravity

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Abstract

We consider asymptotically anti-de Sitter spacetimes in three-dimensional topologically massive gravity with a negative cosmological constant, for all values of the mass parameter $\mu$ ($\mu \neq 0$). We provide consistent boundary conditions that accommodate the recent solutions considered in the literature, which may have a slower fall-off than the one relevant for General Relativity. These conditions are such that the asymptotic symmetry is in all cases the conformal group, in the sense that they are invariant under asymptotic conformal transformations and that the corresponding Virasoro generators are finite. It is found in particular that at the chiral point $|\mu l| = 1$ (where $l$ is the anti-de Sitter radius), one must allow for logarithmic terms (absent for General Relativity) in the asymptotic behavior of the metric in order to accommodate the new solutions present in topologically massive gravity, and that these logarithmic terms make both sets of Virasoro generators nonzero even though one of the central charges vanishes.
I. INTRODUCTION

Following the lead of [1], topologically massive gravity with a negative cosmological constant [2] has received a great deal of renewed interest in the last year. The theory is described by the action

\[ I[e] = 2 \int \left[ e^a \left( d\omega_a + \frac{1}{2} \epsilon_{abc} \omega^b \omega^c \right) + \frac{1}{6} \ell^2 \epsilon_{abc} e^a e^b e^c \right] + \frac{1}{\mu} \int \omega^a \left( d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^b \omega^c \right) \]  

so that the field equations read

\[ G^\mu_\sigma - \frac{1}{l^2} \delta^\mu_\sigma - \frac{1}{\mu} C^\mu_\sigma = 0, \]

where \( \mu \neq 0 \) is the mass parameter, \( l \) is the AdS radius, and \( C^\mu_\sigma := \epsilon^{\mu\nu\rho} \nabla_\nu \left( R_{\rho\sigma} - \frac{1}{4} g_{\rho\sigma} R \right) \), stands for the Cotton tensor. The case \( |\mu l| = 1 \) is known as the chiral point and has been advocated in [1] to enjoy remarkable properties.

In the absence of the topological mass term, the relevant asymptotic behavior of the metric is given by [5]

\[ \Delta g_{rr} = f_{rr} r^{-4} + O(r^{-5}), \]
\[ \Delta g_{rm} = f_{rm} r^{-3} + O(r^{-4}), \]
\[ \Delta g_{mn} = f_{mn} + O(r^{-1}). \]

Here \( f_{\mu\nu} = f_{\mu\nu}(t, \phi) \), and the indices have been split as \( \mu = (r, m) \), where \( m \) includes the time and the angle. We have also decomposed the metric as \( g_{\mu\nu} = \bar{g}_{\mu\nu} + \Delta g_{\mu\nu} \), where \( \Delta g_{\mu\nu} \) is the deviation from the AdS metric,

\[ ds^2 = -(1 + r^2/l^2)dt^2 + (1 + r^2/l^2)^{-1}dr^2 + r^2 d\phi^2. \]

The boundary conditions (3) fulfill the following three crucial consistency requirements explicitly spelled out in [6]:

- They are invariant under the anti-de Sitter group.
- They decay sufficiently slowly to the exact anti-de Sitter metric at infinity so as to contain the “asymptotically anti-de Sitter” solutions of the theory of physical interest (in this case, the BTZ black holes [7]).
- But at the same time, the fall-off is sufficiently fast so as to yield finite charges.
It was actually found in [5] that the asymptotic conditions (3) are invariant not just under $SO(2, 2)$ but under the bigger infinite-dimensional conformal group in two dimensions. The Poisson brackets algebra of the corresponding charges (given by surface integrals at infinity) gives two copies of the Virasoro algebra with a central charge equal to $c = 3l/(2G)$.

If one changes the theory, the asymptotic behavior of the physically interesting solutions might be different and the asymptotic conditions might therefore have to be modified in order to accommodate the new solutions of physical interest. This was investigated at length in [8, 9, 10] for anti-de Sitter gravity with scalar fields in any number of dimensions (see also [11, 12]). It was found that the standard anti-de Sitter boundary conditions indeed had to be relaxed in that case, but that the charges remained finite thanks to a delicate cancellation of divergences between the relaxed terms in the metric and contributions from the scalar fields.

The same phenomenon occurs if one modifies the action of pure Einstein gravity by the topological mass term, as in [13] above. Indeed, as observed in [13], the metric could acquire then a slower decay to the anti-de Sitter metric at infinity for a class of physically interesting linearized solutions. For a generic value of $\mu l > -1$, an exact asymptotically AdS solution describing a chiral pp-wave was found in [14], and further developed in [15], whose metric reads

$$ds^2 = l^2 \frac{dr^2}{r^2} - r^2 dx^+ dx^- + F(x^-) r^{1-\mu l} (dx^-)^2,$$

where $F(x^-)$ is an arbitrary function and $x^\pm = t^\pm \pm \phi$. This solution is to be compared with the AdS metric written in the same coordinates,

$$ds^2 = \left( 1 + \frac{r^2}{l^2} \right)^{-1} \left( dr^2 - \frac{r^2}{4} (dx^+)^2 + dx^2 - \left( \frac{l^2}{2} + r^2 \right) dx^+ dx^-\right),$$

and one sees that the $F(x^-) r^{1-\mu l}$ term spoils the asymptotic behavior (3).

The purpose of this note is to provide a consistent set of new boundary conditions that accommodate these solutions with slower decay at infinity and that are yet compatible with the full conformal symmetry, for all values of the mass parameter. It turns out that the analysis carries many features in common with the scalar case studied previously. (The asymptotic study of topologically massive gravity has been carried out recently in [16] at the chiral point. While we agree with the asymptotic form of the metric and the symmetries given in that paper, we do find however that both set of Virasoro generators are generically
nonzero, a fact that shows that the theory with these boundary conditions cannot be chiral [17].

Because the computations are rather cumbersome, and because the logic follows the scalar case situation, we shall, in this note, only report the results and discuss some of their properties. The full details will be provided elsewhere [18].

II. RANGE $0 < |\mu l| < 1$ OF THE MASS PARAMETER

We first consider the most intricate case, which occurs when the mass parameter $\mu$ fulfills the condition $0 < |\mu l| < 1$.

A. Asymptotic conditions

We have found that there are two consistent sets of boundary conditions fulfilling the three consistency requirements repeated in the introduction. The existing solutions given in the literature fulfill one or the other set of boundary conditions. We shall first give the boundary conditions and we shall then explain how one verifies that they are indeed consistent.

Negative chirality. The boundary conditions are in that case

$$
\Delta g_{rr} = f_{rr} r^{-4} + \cdots \\
\Delta g_{r+} = f_{r+} r^{-3} + \cdots \\
\Delta g_{r-} = h_{r-} r^{-2-\mu l} + f_{r-} r^{-3} + \cdots \\
\Delta g_{++} = f_{++} + \cdots \\
\Delta g_{+-} = f_{+-} + \cdots \\
\Delta g_{--} = h_{--} r^{1-\mu l} + f_{--} + \cdots 
$$

(6)

where $f_{\mu \nu}$ and $h_{\mu \nu}$ depend only on $x^+$ and $x^-$ and not on $r$. We use the convention that the $f$-terms are the standard deviations from AdS already encountered in (3), while the $h$-terms represent the relaxed terms that need to be included in order to accommodate the solutions of the topologically massive theory with slower fall-off. We see that only the negative chirality $h$-terms $h_{r-}$ and $h_{--}$ are present, hence the terminology.
Positive chirality. The boundary conditions are in that case

\[
\begin{align*}
\Delta g_{rr} &= f_{rr}r^{-4} + \cdots \\
\Delta g_{r+} &= h_{r+} r^{-2+\mu l} + f_{r+}r^{-3} + \cdots \\
\Delta g_{r-} &= f_{r-}r^{-3} + \cdots \\
\Delta g_{++} &= h_{++} r^{1+\mu l} + f_{++} + \cdots \\
\Delta g_{+-} &= f_{+-} + \cdots \\
\Delta g_{--} &= f_{--} + \cdots 
\end{align*}
\]

(7)

with only the positive chirality \(h\)-terms \(h_{r+}\) and \(h_{++}\).

Although the known solutions [14, 15] are of a given chirality and hence completely covered by the above boundary conditions, one might try to be more general and include both chiralities simultaneously. This cannot be done, however, in a manner that is compatible with the other consistency requirements as it will be explained below.

B. Asymptotic symmetry

One easily verifies that both sets of asymptotic conditions are invariant under diffeomorphisms that behave at infinity as

\[
\begin{align*}
\eta^+ &= T^+ + \frac{l^2}{2r^2} \partial_- T^- + \cdots \\
\eta^- &= T^- + \frac{l^2}{2r^2} \partial_+ T^+ + \cdots \\
\eta^r &= -\frac{r}{2} (\partial_+ T^+ + \partial_- T^-) + \cdots
\end{align*}
\]

(8)

where \(T^\pm = T^\pm(x^\pm)\). The \(\cdots\) terms are of lowest order and do not contribute to the surface integrals. Hence, the boundary conditions are invariant under the full conformal group in two dimensions, generated by \(T^+(x^+}\) and \(T^-(x^-)\).

C. Surface integrals

We shall compute the conserved (Virasoro) charges within the canonical formalism, “à la Regge-Teitelboim” [19]. The canonical analysis of topologically massive gravity has been performed in [4, 20]. As noticed in [21], there is a useful choice of variables allowing one to write topologically massive gravity with a cosmological constant as a Chern-Simons theory.
In this case, since the action is already written in first order, the Hamiltonian formalism can be readily done once the torsion constraint is incorporated as an additional constraint \[4\]. This enables one to skip the standard and somewhat awkward procedure associated with higher order derivatives.

The charges that generate the diffeomorphisms \(8\) take the form \[19\]

\[
H[\eta] = \text{"Bulk piece"} + Q_+ [T^+] + Q_- [T^-],
\]

(9)

where the bulk piece is a linear combinations of the constraints with coefficients involving \(\eta^+, \eta^-,\) and \(\eta^r\), which has been explicitly worked out in \[4\], and where \(Q_+ [T^+]\) and \(Q_- [T^-]\) are surface integrals at infinity that involve only the asymptotic form of the vector field \(\eta^+, \eta^-,\) and \(\eta^r\). On shell, the bulk piece vanishes and \(H[\eta]\) reduces to \(Q_+ [T^+] + Q_- [T^-]\).

Evaluating the variation of the bulk piece in \(9\) under the asymptotic conditions \(6\) or \(7\), one obtains that the surface terms at infinity should obey:

\[
\delta Q_\pm[T^\pm] = \left(1 \pm \frac{1}{\mu l}\right) \delta Q_0^\pm[T^\pm],
\]

where

\[
\delta Q_0^\pm[T^\pm] := \frac{2}{l} \int T^\pm \delta f^\pm d\phi,
\]

is exactly the same expression as that valid for the standard asymptotic behavior. The Virasoro charges are then easily integrated to yield

\[
Q_\pm[T^\pm] = \frac{2}{l} \left(1 \pm \frac{1}{\mu l}\right) \int T^\pm f^\pm d\phi
\]

(10)

(up to additive constants). The details will be given in \[18\]. What happens is that the diverging pieces associated with the slower fall-off \(h_{--}\) or \(h_{++}\) disappear in \(\delta Q_\pm[T^\pm]\) so that \(Q_\pm\) is given by \[10\], and hence the charges acquire no correction involving the terms associated with the relaxed behavior. One can then view \(h_{--}\) (or \(h_{++}\)), which cannot be gauged away, as defining a kind of “hair.” This situation is analogous to the one found for a scalar field with mass \(m\) in the range \(m_B^2 < m^2 < m_B^2 + 1/l^2\) (where \(m_B\) is the Breitenlohner-Freedman bound \[22\]). There are then two possible admissible behaviors (two “branches”) for the scalar field, and the analysis proceeds as here when only the branch with slower behavior is switched on \[10\].

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1 In the scalar field case, one can switch on simultaneously the two branches in a manner compatible with
Under an asymptotic conformal transformation (8), \( f_{++} \) and \( f_{--} \) are straightforwardly found to transform as

\[
\delta _{\eta }f_{++} = 2f_{++}\partial _{+}T^{+} + T^{-}\partial _{-}f_{++} + T^{+}\partial _{+}f_{++} - l^{2}(\partial _{+}T^{+} + \partial _{+}^{2}T^{+})/2 ,
\]

\[
\delta _{\eta }f_{--} = 2f_{--}\partial _{-}T^{-} + T^{-}\partial _{-}f_{--} + T^{+}\partial _{+}f_{--} - l^{2}(\partial _{-}T^{-} + \partial _{-}^{2}T^{-})/2 .
\]

On shell, one verifies that

\[
\partial _{+}f_{--} = 0 = \partial _{-}f_{++}
\]

and so (11) and (12) reduce to

\[
\delta _{\eta }f_{++} = 2f_{++}\partial _{+}T^{+} + T^{+}\partial _{+}f_{++} - \frac{l^{2}}{2}(\partial _{+}T^{+} + \partial _{+}^{2}T^{+}) ,
\]

\[
\delta _{\eta }f_{--} = 2f_{--}\partial _{-}T^{-} + T^{-}\partial _{-}f_{--} - \frac{l^{2}}{2}(\partial _{-}T^{-} + \partial _{-}^{2}T^{-}) .
\]

As \( \beta _{+}\delta _{\eta }\int f_{++}Y^{+}d\phi \sim [Q_{+}(Y^{+}), Q_{+}(T^{+}) + Q_{-}(T^{-})] \) (with \( \beta _{\pm } = 2l^{-1}(1 \pm (\mu _{l})^{-1}) \)) and \( \beta _{-}\delta _{\eta }\int f_{--}Y^{-}d\phi \sim [Q_{+}(Y^{+}), Q_{+}(T^{+}) + Q_{-}(T^{-})] \) (with \( Y^{+} \) and \( Y^{-} \) the asymptotic conformal transformation associated with a second spacetime diffeomorphism \( \xi ^{+}, \xi ^{-} \) and \( \xi ^{r} \)), one can easily infer from (14) and (15) that \( Q_{+}(Y^{+}) \) and \( Q_{-}(T^{-}) \) commute with each other and each fulfills the Virasoro algebra with central charges

\[
c_{\pm } = \left(1 \pm \frac{1}{\mu _{l}}\right)c
\]

(see [23] for general theorems).

### III. RANGE \(|\mu _{l}| > 1\) OF THE MASS PARAMETER

Take for definiteness \( \mu _{l} \) positive and hence \( > 1 \). Solving the equations starting from infinity shows that again, one should expect both chiralities to be present, taking exactly the same form as (6) and (7) above. However, the positive chirality blows up at infinity...
\( \Delta g_{++} \) dominates the background) and the space is not asymptotically of constant curvature. So, if \( h_{++} \neq 0 \), the space is not asymptotically anti-de Sitter. For this reason, one must set \( h_{++} = 0 \). But the other \( h_{--} \)-term is subdominant with respect to \( f_{--} \), so that the asymptotic negative chirality behavior reproduces (3). The same analysis holds when \( \mu l \) is negative (with an interchange of the roles of the two chiralities). Therefore, the behavior of the metric can be taken to be (3). The asymptotic derivation of the charges and the central charges proceeds then straightforwardly (no divergence to be canceled) and yields

\[
Q_\pm [T^\pm] = \frac{2}{l} \left( 1 \pm \frac{1}{\mu l} \right) \int T^\pm f_{\pm \phi} \quad \text{(17)}
\]

with central charges

\[
c_\pm = \left( 1 \pm \frac{1}{\mu l} \right) c. \quad \text{(18)}
\]

IV. THE CHIRAL POINT

A. Asymptotic behavior

Hereafter we only consider \( \mu l = 1 \), since the case of \( \mu l = -1 \) just corresponds to the interchange \( x^+ \leftrightarrow x^- \).

In the case of \( \mu l = 1 \), the appropriate asymptotic behavior for \( \Delta g_{\mu \nu} \) reads

\[
\begin{align*}
\Delta g_{rr} &= f_{rr} r^{-4} + \cdots \\
\Delta g_{r+} &= f_{r+} r^{-3} + \cdots \\
\Delta g_{r-} &= h_{r-} r^{-3} \ln(r) + f_{r-} r^{-3} + \cdots \\
\Delta g_{++} &= f_{++} + \cdots \\
\Delta g_{+-} &= f_{+-} + \cdots \\
\Delta g_{--} &= h_{--} \ln(r) + f_{--} + \cdots
\end{align*}
\quad \text{(19)}
\]

where \( f_{\mu \nu} \) and \( h_{--} \) depend only on \( x^\pm = \frac{r}{l} + \phi \). This behavior accommodates the known solutions with constant curvature at infinity \cite{14, 15, 24}, whose metric is given by

\[
ds^2 = l^2 \frac{dr^2}{r^2} - r^2 dx^- dx^- + F(x^-) \log(r) \left( dx^- \right)^2.
\quad \text{(20)}
\]

with \( F(x^-) \) being an arbitrary function.
B. Asymptotic symmetry

Just as for $\mu l \neq 1$, the asymptotic conditions are invariant under diffeomorphisms that behave at infinity as in Eq. (8), where the $\cdots$ terms are again of lowest order and do not contribute to the surface integrals. Hence, the boundary conditions are invariant under the conformal group in two dimensions, generated by $T^+(x^+)$ and $T^-(x^-)$.

Under the action of the Virasoro symmetry, one obtains

$$\delta \eta h_{--} = 2h_{--}\partial_- T^- + T^- \partial_- h_{--} + T^+ \partial_+ h_{--}$$

(21)

and

$$\delta \eta f_{++} = 2f_{++}\partial_+ T^+ + T^- \partial_+ f_{++} + T^+ \partial_+ f_{++} - l^2 (\partial_+ T^+ + \partial_+ T^+)/2.$$  

(22)

The field equations are easily verified to imply that

$$\partial_- f_{++} = 0, \text{ and } \partial_+ h_{--} = 0.$$  

Note that this time the equations do not impose $\partial_+ f_{--} = 0$ and furthermore, the transformation rule of $f_{--}$ also differs from the one found off the chiral point.

C. Conserved charges for $\mu l = 1$

Evaluating the variation of the surface charges using the expressions of [4] with the asymptotic conditions (19) one obtains:

$$\delta Q_+ = \frac{4}{l} \int T^+ \delta f_{++} d\phi,$$

and

$$\delta Q_- = \frac{2}{l} \int T^- \delta h_{--} d\phi.$$  

This implies (up to additive constants)

$$Q_+ = \frac{4}{l} \int T^+ f_{++} d\phi,$$

and

$$Q_- = \frac{2}{l} \int T^- h_{--} d\phi.$$  

The crucial new feature found here, apparently overlooked in the previous literature, is that $Q_- [T^-]$ does not vanish identically. Rather, the relaxation term $h_{--}$ does contribute to it. This behavior is somehow similar to what occurs for scalar fields that saturates the BF bound. One may verify explicitly that on definite solutions, $Q_- [T^-]$ is not zero [18]. Indeed, for the metric (20), $h_{--} = F(x^-)$ which in general does not vanish.
From the variations (21) and (22) of \( h_- \) and \( f_{++} \) and the asymptotic field equations, one finds that both \( Q_+[T^+] \) and \( Q_-[T^-] \) fulfill the Virasoro algebra with the central charge

\[
c_+ = 2c, \quad c_- = 0.
\]  

(23)

Even though \( Q_-[T^-] \) does not vanish, the central charge \( c_- \) is zero because the inhomogeneous terms \( -\ell^2 \left( \partial_- T^- + \partial_3 T^- \right) / 2 \) are absent from \( \delta_\eta h_- \).

In this paper we have exhibited the boundary conditions appropriate to accommodate the solutions of topologically massive gravity found in the literature with a slower decay at infinity than the one for pure standard gravity discussed in \( \text{[5]} \). These boundary conditions fulfill the consistency conditions listed in the introduction. The analysis proceeds very much as in the case of anti-de Sitter gravity coupled to a scalar field \( \text{[8, 9, 10]} \) and the results turn out to be comparable.

A question not addressed here is the exact physical relevance of the new solutions which the more liberal boundary conditions enable one to consider. One might question whether they should be included \( \text{[17]} \) and it is not clear what one loses if one does not include them, i.e., if one sticks to the more restrictive boundary conditions of \( \text{[5]} \). Note that for the scalar field, the softening of the boundary conditions leads to interesting developments. In particular, this enlarges the space of admissible solutions to include hairy black holes \( \text{[8, 11, 25]} \), solitons and instantons \( \text{[26]} \).

We have also shown that with the new boundary conditions, the Virasoro generators with both chiralities are actually nonzero at the chiral point (while one chiral set of them does vanish under the boundary conditions of \( \text{[5]} \)). The corresponding central charge vanishes, however. This puzzling fact should be understood from the point of view of conformal field theory.

A longer version of this paper, with detailed proofs and more information on the charges of various solutions is in preparation \( \text{[18]} \).

Note added: After this paper was posted on the arXiv, we received comments by various colleagues (i) confirming the intriguing result established above that at the chiral point, the left-moving generators have a zero central charge even though they are not identically zero; and (ii) investigating the interpretation of this result in terms of a dual logarithmic CFT. We thank A. Strominger and D. Grumiller for kindly providing us this information prior to publication.
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