Dark matter and dark energy in galaxies and astrophysical objects

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Abstract. I describe static configurations of dark matter coupled to the scalar field responsible for the dark energy of the Universe. The mass of the dark matter particles is a function of the scalar field. I discuss first the profile of the dark matter halos in galaxies. In the presence of a scalar field, the velocity of a massive object orbiting the galaxy is not of the order of the typical velocity of the dark matter particles, as in the conventional picture. Instead, it is reduced by a factor encoding the dependence of the dark matter mass on the scalar field. This has implications for dark matter searches. I also describe new solutions of the Einstein equations for compact objects composed of dark matter. The size of these objects can vary between microscopic scales and cosmological distances. I discuss the mass to radius relation and the similarities with conventional neutron stars and exotic astrophysical objects.

1. Introduction
There are various contributions to the energy content of our Universe. The most accessible energy component is baryonic matter, which accounts for $\sim 5\%$ of the total energy density. A component that has not been directly observed is dark matter: a pressureless fluid that is responsible for the growth of cosmological perturbations through gravitational instability. Its contribution to the total energy density is estimated at $\sim 25\%$. The dark matter is expected to become more numerous in extensive halos, that stretch up to 100–200 kpc from the center of galaxies. The component with the biggest contribution to the energy density has an equation of state similar to that of a cosmological constant. The ratio $w = p/\rho$ is negative and close to $-1$. This component is responsible for $\sim 70\%$ of the total energy density and induces the observed acceleration of the Universe [1, 2, 3]. The total energy density of our Universe is believed to take the critical value consistent with spatial flatness.

The difficulty with explaining the very small value of the cosmological constant that could induce the present acceleration has motivated the suggestion that this energy component is time dependent [4, 5]. In the simplest realization, it is connected to a scalar field $\phi$ with a very flat potential. The vacuum energy associated with this field is characterized as dark energy and drives the acceleration. If such a field affects the cosmological evolution today, its effective mass must be of the order of the Hubble scale, or smaller.

It is conceivable that there is a coupling between dark matter and the field responsible for the dark energy [6]. In such a scenario it may be possible to resolve the coincidence problem, i.e. the reason behind the comparable present contributions from the dark matter and the dark energy to the total energy density. The presence of an interaction between dark matter and
the scalar field responsible for the dark energy has consequences that are potentially observable. The cosmological implications depend on the form of the coupling, as well as on the potential of the field [7]. If the scale for the field mass is set by the present value of the Hubble parameter, then the field is effectively massless at distances of the order of the galactic scale. Its coupling to the dark matter particles results in a long range force that can affect the details of structure formation [8, 9, 10, 11].

The attraction between dark matter particles mediated by the scalar field may lead to the formation of compact objects composed primarily of dark matter. I discuss static solutions of the Einstein equations that describe such objects. A class of solutions gives an approximate description of galaxy halos in the presence of the scalar field. I examine the distribution and velocity of dark matter particles in halos and discuss the implications for dark matter searches. I also present solutions that describe exotic objects composed of dark matter and held together by the scalar interaction.

2. The formalism [17]

The dark matter is assumed to consist of a gas of weakly interacting particles. The mass $m$ of the particles depends on the value of a slowly varying classical scalar field $\phi$ [6]. For classical particles, the action of the system can be written as (see ref. [12] and references therein)

$$ S = \int d^4x \sqrt{-g} \left( M^2 R - \frac{1}{2} \frac{\partial \phi}{\partial x^\mu} \frac{\partial \phi}{\partial x^\nu} g^{\mu \nu} - U(\phi) \right) - \sum_i \int m(\phi(x_i))d\tau_i, $$

(1)

with $d\tau_i = \sqrt{-g_{\mu \nu}(x_i)} dx_i^\mu dx_i^\nu$ and the second integral taken over particle trajectories. Variation of the action with respect to $\phi$ results in the equation of motion [12, 13]

$$ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu \nu} \frac{\partial \phi}{\partial x^\nu} \right) = \frac{dU}{d\phi} - \frac{d \ln m(\phi(x))}{d\phi} T_{\mu}^\nu, $$

(2)

where the energy-momentum tensor associated with the gas of particles is

$$ T^{\mu \nu} = \frac{1}{\sqrt{-g}} \sum_i \int d\tau_i \left( \frac{dx_i^\mu}{d\tau_i} \frac{dx_i^\nu}{d\tau_i} \delta^{(4)}(x - x_i) \right). $$

(3)

In the following I present solutions of eq. (2) employing an approximation for the form of the energy-momentum tensor $T^{\mu \nu}$: I assume that it takes the diagonal form $T_{\nu}^\nu = \text{diag}(-\rho, p, p, p)$.

The appropriate metric for stationary, spherically symmetric configurations has the form

$$ ds^2 = -B(r)dt^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + A(r)dr^2. $$

(4)

The Einstein equations are

$$ \frac{1}{r^2} \frac{1}{A} - \frac{1}{r^2} - \frac{1}{r^2} A' = \frac{1}{2M^2} \left( -\frac{1}{2A} \phi'^2 - U(\phi) - \rho \right) $$

(5)

$$ -\frac{1}{2r} A' + \frac{1}{2r} B' - \frac{A' B'}{4B^2} - \frac{B'^2}{4AB^2} + \frac{B''}{2AB} = \frac{1}{2M^2} \left( -\frac{1}{2A} \phi'^2 - U(\phi) + p \right) $$

(6)

$$ \frac{1}{r^2} \frac{1}{A} - \frac{1}{r^2} + \frac{1}{r} \frac{B'}{BA} = \frac{1}{2M^2} \left( \frac{1}{2A} \phi'^2 - U(\phi) + p \right), $$

(7)

where a prime denotes a derivative with respect to $r$. The scale $M$ is defined as $M = (16\pi G)^{-1/2}$, where $G$ is Newton’s constant. The equation of motion (2) for the field $\phi$ becomes

$$ \phi'' + \left( \frac{2}{r} - \frac{A'}{2A} + \frac{B'}{2B} \right) \phi' = A \left[ \frac{dU}{d\phi} + \frac{d \ln m(\phi)}{d\phi} \left( \rho - 3p \right) \right]. $$

(8)
It is more convenient to replace one of the equations of motion by the conservation of the energy-momentum tensor. Combining the only non-trivial conservation equation with the equation of motion (8) gives

\[
\frac{dp}{dr} = (3p - \rho) \frac{d\ln m(\phi)}{d\phi} \frac{d\phi}{dr} - (\rho + p) \frac{1}{2B} \frac{dB}{dr}.
\]  

(9)

The Newtonian limit is obtained by writing \( B = 1 + 2\Phi \) and assuming \( |\Phi| \ll 1 \). One finds

\[
\Phi'' + \frac{2}{r} \Phi' = \frac{1}{4M^2} (\rho + 3p - 2U)
\]  

(10)

for the Newtonian potential. In the same limit the equation of motion of the scalar field becomes independent of the gravitational field and reads

\[
\phi'' + \frac{2}{r} \phi' = \frac{dU}{d\phi} + \frac{d\ln m(\phi)}{d\phi} (\rho - 3p).
\]  

(11)

The most popular models of dark matter assume that it consists of a gas of weakly interacting fermions. At non-zero temperature and density the most important physical implications result from the exclusion principle. These can be taken into account by neglecting particle scattering and assuming that the gas is described by a Fermi-Dirac distribution. This is the essence of the Thomas-Fermi approximation. In must be pointed out, however, that the derivation of the equation of motion (8), on which this discussion is based, neglected the quantum nature of the particles. However, within the Thomas-Fermi approximation the only modification relative to a gas of classical particles is the change of the distribution. When this is taken into account in the energy density and pressure, eq. (8) remains valid. For \( T = 0 \), this has been shown explicitly in ref. [14], starting from the equation of motion of a fermionic field (the Dirac equation) in a curved background.

In a local frame at every point in space the particles are described through the distribution

\[
f(p) = \left[ \exp \left( \frac{\sqrt{p^2 + m^2(\phi(r))} - \mu(r)}{T(r)} \right) + 1 \right]^{-1}.
\]  

(12)

The chemical potential \( \mu \) and temperature \( T \), as measured by a local observer, are functions of the radial coordinate \( r \), while the mass of the particles depends on the local value of the field \( \phi \). The pressure, number density and energy density are given by

\[
p = T(r) \frac{1}{4\pi^3} \int d^3p \ln \left[ \exp \left( -\frac{\sqrt{p^2 + m^2(\phi(r))} - \mu(r)}{T(r)} \right) + 1 \right]
\]  

\[= \frac{1}{4\pi^3} \int d^3p \, f(p) \frac{p^2}{3\sqrt{p^2 + m^2(\phi(r))}}
\]  

(13)

\[
n = \frac{1}{4\pi^3} \int d^3p \, f(p)
\]  

(14)

\[
\rho = \frac{1}{4\pi^3} \int d^3p \, f(p) \sqrt{p^2 + m^2(\phi(r))}.
\]  

(15)

The above quantities are related through

\[
p = \mu n - \rho + Ts,
\]  

(16)

where \( s \) is the entropy density. An important identity is

\[
\frac{\partial p}{\partial \phi} = \frac{d\ln m(\phi)}{d\phi} (3p - \rho) = \frac{d\ln m(\phi)}{d\phi} T^\phi_{\mu}.
\]  

(17)
Combining this identity and the conservation equation (9) results in

\[ \frac{\partial p}{\partial r} = -\left(\rho + p\right) \frac{1}{2B} \frac{dB}{dr}. \]  

(Eq. 18)

Evaluating the \( r \)-derivative of \( p \), given by eq. (13), and employing the relation (16) allows for the rewriting of eq. (18) as

\[ T_s \frac{d \ln (T \sqrt{B})}{dr} + \mu n \frac{d \ln (\mu \sqrt{B})}{dr} = 0. \]  

(Eq. 19)

This equation is satisfied if

\[ T(r) = T_0 / \sqrt{B(r)}, \quad \mu(r) = \mu_0 / \sqrt{B(r)}. \]  

(Eq. 20)

These are the standard expressions that describe the behaviour of a Thomas-Fermi fluid in a gravitational field.

A consistent solution can be obtained by integrating eqs. (5), (7), (8), (9). Eq. (6) is then automatically satisfied. Eq. (9) can be replaced by eqs. (20). In this way, the system of equations to be solved is reduced to eqs. (5), (7), (8).

3. Dark matter halos [17, 18]

The dark matter distribution results in approximately flat rotation curves for objects orbiting the galaxies at distances \( r > \sim 10 \) kpc [15, 16]. An analytical understanding of this behaviour is possible within simple models of the dark matter gas, such as the isothermal sphere. If these simple models are extended through the addition of a scalar field to the theory, an analytical treatment is still feasible. The dark matter in galaxies forms a non-relativistic and non-degenerate gas, with \( p \ll \rho \). In the absence of a scalar field \( (\phi = 0, U = 0, m = m_0) \) one can define the non-relativistic chemical potential as \( \bar{\mu}_0 = \mu_0 - m_0 \). The number density is

\[ n \simeq 2 \left( \frac{m_0 T}{2\pi} \right)^{3/2} \exp \left( -\frac{m_0}{T} + \frac{\mu}{T} \right) \simeq n_0 \exp \left( -\alpha \Phi \right), \]  

(Eq. 21)

with

\[ n_0 = 2 \left( \frac{m_0 T_0}{2\pi} \right)^{3/2} \exp \left( \frac{\mu_0}{T_0} \right), \quad \alpha = \frac{m_0}{T_0}. \]  

(Eq. 22)

The validity of the above expressions requires \( \bar{\mu}_0 < 0 \) and \( T_0 \ll m_0, T_0 \ll |\bar{\mu}_0| \). The gravitational field can be studied in the Newtonian limit. Then, eq. (10) can be written as

\[ \frac{d^2 u}{dz^2} + \frac{2}{z} \frac{du}{dz} + \exp u = 0, \]  

(Eq. 23)

with \( u = -\alpha \Phi, z = \beta r, \beta^2 = \alpha \rho_0 / 4M^2 \), and \( \rho_0 \simeq m_0 n_0 \). The solutions of this equation that are regular at \( z = 0 \) behave as \( u = 2/z^2 \) for large \( z \). For large \( r \) the Newtonian potential varies only logarithmically with \( r \), while the integrated mass of the dark matter scales linearly with \( r \). This leads to flat rotation curves for objects orbiting the galaxy [15].

If the mass of the dark matter particles depends on the field \( \phi \), the number density of the dark matter particles can be approximated by the expression (21) with \( m_0 \) replaced by \( m(\phi) \) [18]. In the weak-field limit, \( B(r) \simeq 1 + 2\Phi(r) \), with \( |\Phi(r)| \ll 1 \). The field \( \phi \) is displaced from its asymptotic \( (r \to \infty) \) value only by a small amount, so that the approximation \( m(\phi) = m(\phi_0) + [dm(\phi_0)/d\phi] \delta \phi \equiv m_0 + m_0' \delta \phi \) can be employed. The value of the field at the center of the galaxy \( (r = 0) \) is denoted by \( \phi_0 \). In the leading order in \( \delta \phi \), one can take
$m'/m \simeq m'_0/m_0$ for all $r$. This treatment is relevant up to a distance $r_1 \sim 100-200$ kpc beyond which the dark matter becomes very dilute. For $r \gtrsim r_1$, $\phi$ quickly becomes constant with a value close to $\phi(r_1) \equiv \phi_1$. This is the value that drives the present cosmological expansion [4, 5, 6]. The cosmic evolution of $\phi_1$ is assumed to be negligible for the time-scales of interest, so that the asymptotic configuration is static to a good approximation.

Within the leading order in $\delta \phi$, $dU/d\phi$ can be approximated by a constant between $r=0$ and $r=\infty$. For the scalar field to provide a resolution of the coincidence problem, the two terms in the r.h.s. of eq. (2) must be of similar magnitude in the cosmological solution. This means that $dU/d\phi$ must be comparable to $(m'_0/m_0)\rho_\infty$. The density $\rho_\infty$ is expected to be a fraction of the critical density, i.e. $\rho_\infty \sim 3$ keV/cm$^3$. On the other hand, for the spherically symmetric solution the energy density in the r.h.s. of eq. (2) is that of the galaxy halo ($\sim 0.4$ GeV/cm$^3$ for our neighborhood of the Milky Way). This makes $dU/d\phi$ negligible in the r.h.s. of eq. (2) for a static configuration. The potential is expected to become important only for $r \to \infty$, where the static solution must be replaced by the cosmological one. Similar arguments indicate that $U$ can be neglected relative to $\rho$. The scale for the field mass is expected to be set by the present value of the Hubble parameter. Then the field is effectively massless at distances of the order of the galactic scale. The above indicate that, if the deviation of the scalar field from its cosmological value is small, the form of the potential plays a negligible role at the galactic level, so that $U = 0$ can be used.

The scalar field generates a new long-range scalar interaction, whose strength relative to the gravitational interaction is determined by the parameter

$$\kappa^2 = 4M^2 \left(\frac{m'_0}{m_0}\right)^2.$$ (24)

If the new interaction is universal for ordinary and dark matter, the experimental constraints impose $\kappa \ll 1$. In this case, it is reasonable to expect a negligible effect in the distribution of matter in galaxy halos. However, if $\phi$ interacts only with dark matter this constraint can be relaxed. This is the assumption made in this talk.

The allowed range of $\kappa$ is limited by the observable implications of the model that describes the dark sector. The dependence of the mass of dark matter particles on an evolving scalar field during the cosmological evolution since the decoupling is reflected in the microwave background. The magnitude of the effect is strongly model dependent. In the models of ref. [7, 19, 20] the observations result in the constraint $\kappa^2 \lesssim 0.01$. In the model of refs. [11, 12] the scalar interaction among dark matter particles is screened by an additional relativistic dark matter species. As a result, the model is viable even for couplings $\kappa^2 \simeq 1$. A similar mechanism is employed in ref. [21]. In this model the interaction between dark matter and dark energy becomes important only during the recent evolution of the Universe. In general, an interaction that is effective for redshifts $z \lesssim 1 - 2$ is not strongly constrained by the observations.

For $p \ll \rho$ one can employ the nonrelativistic chemical potential $\tilde{\mu}_0 = \mu_0 - m_0$. The number density of dark matter is written as

$$n \simeq 2 \left(\frac{m(\phi) T}{2\pi}\right)^{3/2} \exp \left(-\frac{m(\phi)}{T} + \frac{\mu}{T}\right) \simeq n_0 \exp \left(-\alpha \Phi - \tilde{\alpha} \delta \phi\right),$$ (25)

with $n_0$, $\alpha$ given by eqs. (22) and

$$\delta \phi = \phi - \phi_0 = \phi - \phi(r = 0), \quad \tilde{\alpha} = \frac{m'_0}{T_0}.$$ (26)

The energy density of dark matter at the center of the galaxy is $\rho_0 = m_0 n_0$. 

It is important to emphasize that the assumption that the number density is given by eq. (21) does not require the presence of thermal equilibrium. In an alternative approach, followed in ref. [18], the dark matter can be considered as a dilute, weakly interacting gas with an energy-momentum tensor $T_{\mu \nu} = \text{diag}(-\rho, p, p, p)$. In analogy to the model of the isothermal sphere [15], one can assume that $p(r) = \rho(r) \langle v_d^2 \rangle = m(\phi(r)) \rho(r) \langle v_d^2 \rangle$, with a constant velocity dispersion $\langle v_d^2 \rangle \ll 1$. In the weak field limit and for $p \ll \rho$, the conservation of the energy-momentum tensor (9) gives

$$p' = -\rho \Phi' - \frac{m_0'}{m_0}(\delta \phi').$$

Integration of this equation results in

$$n \simeq n_0 \exp \left( -\frac{\Phi}{\langle v_d^2 \rangle} - \frac{m_0'}{m_0} \frac{\delta \phi}{\langle v_d^2 \rangle} \right).$$

By defining an effective temperature $T_0$ through the relation $\langle v_d^2 \rangle = T_0/m_0$, eq. (25) is reproduced. This indicates that the assumption of thermal equilibrium is not required for the emergence of eq. (25). The parameter $T_0$ that appears in the various expressions of section 2 does not correspond necessarily to the physical temperature. In many cases of physical interest it is simply a measure of the typical velocity of the Fermi gas.

In the non-relativistic, weak-field limit, with $p = U = 0$, eq. (10) becomes

$$\Phi'' + \frac{2}{r} \Phi' = \frac{1}{4M^2} \rho_0 \exp \left( -\alpha \Phi - \tilde{\alpha} \delta \phi \right).$$

Similarly, eq. (11) becomes

$$(\delta \phi)'' + \frac{2}{r} (\delta \phi)' = \frac{m_0'}{m_0} \rho_0 \exp \left( -\alpha \Phi - \tilde{\alpha} \delta \phi \right).$$

A linear combination of eqs. (29), (30) results in eq. (23) where now

$$u = -\alpha \Phi - \tilde{\alpha} \delta \phi, \quad z = \beta r,$$

with

$$\beta^2 = \left( \frac{\alpha}{4M^2} + \frac{m_0'}{m_0} \right) \rho_0 = \left( 1 + \kappa^2 \right) \frac{m_0}{T_0} \frac{\rho_0}{4M^2}.$$  

The solutions that are regular at $z = 0$ approach the form

$$u = \ln \left( \frac{2}{z^2} \right) + \frac{1}{\sqrt{z^2}} \left[ d_1 \cos \left( \frac{\sqrt{7}}{2} \ln z \right) + d_2 \sin \left( \frac{\sqrt{7}}{2} \ln z \right) \right] + \ldots$$

for large $z$. Another linear combination of eqs. (29), (30) gives

$$\frac{d^2 v}{dz^2} + \frac{2}{z} \frac{dv}{dz} = 0,$$

with

$$v = \tilde{\alpha} \delta \phi - 4M^2 \frac{m_0'}{m_0} \alpha \Phi = -\kappa^2 \alpha \Phi + \tilde{\alpha} \delta \phi.$$  

The solution of this equation is $v = c_0 + c_1/z$. 

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The velocity \( v \) of a massive baryonic object in orbit around the galaxy, at a distance \( r \) from its center, can be expressed as

\[
\left( \frac{v}{v_c} \right)^2 = \frac{r \Phi'}{v_c^2} = -\frac{z}{2} \left( \frac{du}{dz} + \frac{dv}{dz} \right),
\]

where

\[
v_c^2 = \frac{2}{1 + \kappa^2} \frac{T_0}{m_0} = \frac{2}{1 + \kappa^2} \langle v_d^2 \rangle.
\]

The asymptotic form of \( u(z), v(z) \) indicates that \( v \approx v_c \) for large \( z \). The dominant correction to the leading behaviour arises from the term \( 1/\sqrt{z} \) in eq. (33). The function \( v(z) \) gives a higher order correction. This means that the presence of the field \( \phi \) is not expected to cause significant modifications to the shape of the rotation curves relative to the \( \phi = 0 \) case. For a vanishing field the rotation curves are again governed by the solution of eq. (23), given by eq. (33). This simple analysis indicates that the approximately flat rotation curves outside the galaxy cores are a persistent feature even if the dark matter is coupled to a scalar field through its mass. However, numerical simulations are probably necessary in order to reproduce the detailed form of the curves.

Eq. (37) can be used in order to fix \( T_0/m_0 = \langle v_d^2 \rangle \) for a given value of \( \kappa \). The effect of the scalar field is encoded in the factor \( \kappa^2 = 4M_\phi^2 (m'_0/m_0)^2 \). When this is small, the velocity of an object orbiting the galaxy is of the order of the square root of the dispersion of the velocity of the dark matter particles. If \( \kappa^2 \) is large the orbital velocity can become much smaller than the typical dark matter velocity. This behaviour persists even if baryonic matter is added near the center of the galaxy [18].

The scenario considered here may be more interesting for dark matter searches than the conventional one. For large values of \( \kappa \) the velocity of dark matter particles exceeds significantly the observed rotation velocity (\( \sim 220 \text{ km/s} \) for the Milky Way). The estimated local energy density of dark matter remains the same as in the case with \( \kappa^2 = 0 \). It is \( \sim 0.4 \text{ GeV/cm}^3 \) for our neighborhood of the Milky Way. As a result the flux of dark matter particles towards a terrestrial detector is larger roughly by a factor \((1 + \kappa^2)^{1/2} \) relative to the \( \kappa^2 = 0 \) case.

A detailed calculation of the counting rates in detectors must take into account the motion of the Earth around the Sun and the motion of the Sun through our galaxy. The velocity distribution of dark matter is

\[
f(\vec{v}) \sim \exp \left( -\frac{\vert\vec{v} + \vec{v}_E + \vec{v}_S\vert^2}{2\langle v_d^2 \rangle} \right),
\]

where the magnitude of the velocity of the Earth relative to the Sun is \( \vert\vec{v}_E\vert \approx 30 \text{ km/s} \), and that of the Sun relative to the galactic rest frame \( \vert\vec{v}_S\vert \approx v_c \approx 220 \text{ km/s} \). For \( \kappa \gg 1 \), one has \( \sqrt{\langle v_d^2 \rangle} \gg \vert\vec{v}_E\vert, \vert\vec{v}_S\vert \). The motion of the Earth and the Sun are expected to give only a small modification of the dark matter flux towards the Earth. As a result, the seasonal variation of a possible dark matter signal decreases for increasing \( \kappa^2 \). Typically, the cross section for the elastic scattering of halo particles by target nuclei is independent of the particle velocity for very low velocities [16, 22]. The leading effect of a non-zero value of \( \kappa \) is that the counting rates, that are proportional to the dark matter velocity, are increased by the factor \((1 + \kappa^2)^{1/2} \). This makes the dark matter easier to detect. Existing bounds on dark matter properties from direct searches can be extended to include the case of non-zero \( \kappa \). The bound on the cross section for the interaction of dark matter with the material of the detector must be strengthened by the factor \((1 + \kappa^2)^{1/2} \).
4. New astrophysical objects [17]

In the previous section I described the effect of the scalar field on the distribution of dark matter in galaxy halos. The basic assumption was that the field is not displaced significantly from its value at \( r \to \infty \). In this section I discuss solutions with larger deviations of the field from its asymptotic value. This requires a treatment that goes beyond linearized gravity.

In most models in which \( \phi \) is the dark matter field, the potential \( U(\phi) \) does not have a minimum. The typical case, which I take as a working example, is an exponential potential \( U(\phi) = U_0 \exp(-c\phi/M) \) [4, 6, 7]. For simplicity, I shall also assume that the dark matter mass is linear in \( \phi \). The essence of this assumption is that the mass vanishes for some finite value of \( \phi \), which can be set to zero by an appropriate field shift. The potential \( U(\phi) \) retains its form, as the field shift can be absorbed in the pre-exponential factor. The strong dependence of the mass on \( \phi \) causes the field near dense concentrations of dark matter to deviate strongly from its asymptotic value \( \phi_1 \). The field tends to approach zero in the center of such concentrations, so that the dark matter particles become massless there. The solutions of this section are very different from the solutions of the previous one, as the dark matter gas is now assume to be degenerate \((T = 0)\). It is possible to find solutions with \( T \neq 0 \) following an analogous procedure, even though the technical difficulty may be greater.

The effective temperature of the fermionic gas can be related either to the velocity dispersion of a weakly interacting collection of particles (as for galaxy halos), or to the real temperature of a thermalized gas (as for the interior of stars). The formalism of section 2 can describe static solutions in both cases. The value of the temperature is determined by the ability of a collapsing gas to lose momentum, and the possible interactions between the particles that can establish thermalization. This dynamical issue will not be addressed here. Instead I shall present static solutions that describe a system very different from the galaxy halos discussed in the previous section: a completely degenerate fermionic gas, interacting with a scalar field.

The pure scalar field configuration that solves the Einstein equations in the absence of a potential and dark matter has been discussed in ref. [23]. This solution is expected to be realized for distances \( r > R \), for which the dark matter density becomes negligible (of the order of the critical density of the Universe). The dark matter gas is concentrated in the region \( r \leq R \). The radius of the compact object is \( R \). For \( U = 0 \) and large \( r \), the leading terms of the pure field solution read

\[
\phi = \phi_1 - \gamma M R_s/r \\
B = A^{-1} = 1 - \frac{R_s}{r}.
\] (39)

The free parameter \( R_s \) determines the mass of the object as seen from an observer at infinity

\[
M_{tot} = 8\pi M^2 R_s.
\] (40)

The asymptotic value of the field for \( r \to \infty \) is \( \phi_1 \), while the free parameter \( \gamma \) determines the derivative energy density

\[
K = \frac{1}{2A} |\phi|^2 = \frac{\gamma^2 R_s^2 M^2}{2r^4}.
\] (41)

As long as \( K \gg U \), the potential can be neglected. For distances \( r \) of the order of the Hubble radius this condition is not expected to be satisfied. At such scales the static solution must be replaced by the time-dependent cosmological solution.

The solution of ref. [23] can be modified by adding dark matter in the region \( r \leq R \). This addition can eliminate naturally the singularity that appears in the pure field configuration. The simplest form of the mass of the dark matter particles is

\[
m(\phi) = m_0 + g\phi,
\] (42)

where a Yukawa interaction has been assumed between the dark matter fermions and the scalar field. The mass \( m_0 \) is expected to be associated with a new scale \( \sigma \), different from the Planck
scale $M$. For example, $\sigma$ could be set by the expectation value of a heavy Higgs field in a Grant Unified Theory. In order for the Yukawa interaction not to generate a contribution to the mass much larger than $m_0$, the Yukawa coupling must be taken $g \lesssim \sigma/M$. Then the constant $m_0$ can be absorbed in a shift of $\phi$ of order $M$. In the following I assume that the mass is

$$m(\phi) = \sigma \frac{\phi}{M},$$

(43)

where I have taken $g = \sigma/M$ for simplicity. It must be pointed out that this assumption does not provide an explanation for the smallness of the Yukawa coupling. The appearance of a small parameter is inevitable in a system with two physical scales: the expectation value of $\phi$, taken of order $M$, and the physical mass of the dark matter particles, often assumed to be as low as the TeV range. The situation is similar to the flavour problem in the Standard Model, for which a deeper understanding is lacking.

I also assume that the scalar field has a potential of the form

$$U(\phi) = C \sigma^4 \exp \left( -c \frac{\phi}{M} \right).$$

(44)

with $c = \mathcal{O}(1)$. The value of $\phi$ that is relevant for the present cosmological evolution is given by the requirement that $U(\phi)$ be of the order of the critical energy density $U(\phi) \sim 10^{-47}$ GeV$^4$.

This value

$$\phi_1 = \tilde{\phi}_1 \approx \frac{1}{c} \left[ 108 + \ln C + 4 \ln \left( \frac{\sigma}{\text{GeV}} \right) \right]$$

(45)

must be approached by the static solution for $r \to \infty$. A possible constant contribution $m_*$ to the mass defined in eq. (43) has been absorbed in $\phi$. Through this redefinition $\phi/M \to \phi/M + m_*/\sigma$. I assume implicitly that $m_* = \mathcal{O}(\sigma)$ in order to avoid unnaturally large values of $\phi$.

The equations of motion become more transparent if one defines the dimensionless variables

$$\tilde{\phi} = \frac{\phi}{M}, \quad \tilde{r} = \frac{\sigma^2 r}{M}.$$ 

(46)

All other dimensionful quantities are multiplied with appropriate powers of $\sigma$ only, in order to form dimensionless quantities denoted as tilded. I also define the quantity

$$\tilde{B} = \frac{B}{\mu_0^2} = \frac{B \sigma^2}{\mu_0^2}.$$ 

(47)

One has the relations

$$\tilde{m}(\tilde{\phi}) = \tilde{\phi}, \quad \tilde{\mu}(\tilde{r}) = \frac{1}{\sqrt{\tilde{B}(\tilde{r})}}.$$ 

(48)

The surface of the compact object is defined as the point at which the fermionic density and pressure vanish. If the surface corresponds to a value $\tilde{r} = \tilde{R} = \mathcal{O}(1)$, the physical radius is given by the relation

$$\frac{R}{\text{km}} \simeq 0.34 \left( \frac{\text{GeV}}{\sigma} \right)^2 \tilde{R}. $$

(49)

Similarly, from eq. (40), one finds

$$\frac{M_{\text{tot}}}{M_\odot} \simeq 0.115 \left( \frac{\text{GeV}}{\sigma} \right)^2 \tilde{R}_\odot.$$ 

(50)
Another important characteristic of the solution is the total fermionic number, which is assumed to be conserved. The fermionic number density \( j^0 \) is the time component of a covariant 4-vector. Its value can be deduced from the local density \( n \), through the appropriate tetrad factor \( V^a_{\mu} \), where \( g_{\mu\nu} = V^a_{\mu} V^b_{\nu} \eta_{ab} \). The total fermionic number is

\[
N = \int \sqrt{-g} \, d^4 x \, j^0 = \int_0^\infty 4\pi r^2 \, dr \sqrt{A} \, n = \left( \frac{M}{\sigma} \right)^3 \int_0^\infty 4\pi r^2 \, d\tilde{r} \sqrt{A} \, \tilde{n} = \left( \frac{M}{\sigma} \right)^3 \tilde{N}. \tag{51}
\]

The equations of motion become

\[
\ddot{\phi} + \left[ \frac{1}{\tilde{r}} + \frac{1}{2} \tilde{A} \left( \ddot{\tilde{U}}(\tilde{\phi}) + \frac{1}{2} (\ddot{\tilde{\rho}} - \tilde{\rho}) \right) \right] \tilde{\phi} = A \left[ \frac{d\tilde{U}}{d\tilde{\phi}} + \frac{1}{\phi} (\tilde{\rho} - 3\tilde{\rho}) \right], \tag{52}
\]

\[
\frac{1}{\tilde{r}^2} \tilde{A} - \frac{1}{\tilde{r}^2} - \frac{1}{2} \tilde{A}' = \frac{1}{2} \left( -\frac{1}{2A} \tilde{\phi}'^2 - \tilde{U}(\tilde{\phi}) - \tilde{\rho} \right), \tag{53}
\]

\[
\frac{1}{\tilde{r}^2} \tilde{A} - \frac{1}{\tilde{r}^2} + \frac{1}{\tilde{r} \tilde{B} A} = \frac{1}{2} \left( \frac{1}{2A} \tilde{\phi}'^2 - \tilde{U}(\tilde{\phi}) + \tilde{\rho} \right), \tag{54}
\]

where a prime denotes a derivative with respect to \( \tilde{r} \). One also has

\[
\tilde{n} = \frac{1}{3\pi^2} \left( \tilde{\mu}^2 - \tilde{m}^2 \right)^{3/2}, \tag{55}
\]

\[
\tilde{\rho} = \frac{1}{24\pi^2} \left[ \tilde{\mu} \sqrt{\tilde{\mu}^2 - \tilde{m}^2} \left( 2\tilde{\mu}^2 - 5\tilde{m}^2 \right) + 3\tilde{m}^4 \ln \left( \frac{\tilde{\mu} + \sqrt{\tilde{\mu}^2 - \tilde{m}^2}}{\tilde{m}} \right) \right], \tag{56}
\]

\[
\tilde{\rho} = \frac{1}{8\pi^2} \left[ \tilde{\mu} \sqrt{\tilde{\mu}^2 - \tilde{m}^2} \left( 2\tilde{\mu}^2 - \tilde{m}^2 \right) - \tilde{m}^4 \ln \left( \frac{\tilde{\mu} + \sqrt{\tilde{\mu}^2 - \tilde{m}^2}}{\tilde{m}} \right) \right], \tag{57}
\]

for \( \tilde{\mu} \geq \tilde{m} \), and \( \tilde{n} = \tilde{\rho} = \tilde{\rho} = 0 \) for \( \tilde{\mu} < \tilde{m} \). Finally,

\[
\tilde{U}(\tilde{\phi}) = C \exp \left( -c\tilde{\phi} \right). \tag{58}
\]

One needs four initial conditions for the system of equations (52)–(54). One of them is imposed by the regularity of the spherically symmetric solution at \( \tilde{r} = 0 \); \( \tilde{\phi}'(0) = 0 \). Another one is \( A(0) = 1 \). The value of \( \tilde{B}(0) \) can be chosen arbitrarily. However, the normalization of eqs. (39) implies that \( AB(r \to \infty) = 1 \). This means that \( AB(\tilde{r} \to \infty) = (\mu_0/\sigma)^{-2} \), where the definition (47) was employed. As a result, the choice of \( \tilde{B}(0) \) determines the chemical potential. Finally, \( \phi(0) \) must be chosen so that \( \phi(\tilde{r} \to \infty) \) reproduces correctly the present value \( \tilde{\phi}_1 \) of the scalar field in the cosmological solution, as given by eq. (45).

In fig. 1 the form of the solution for a model with \( \sigma/\text{GeV} = 1 \) is displayed. The chemical potential takes the value \( \mu_0/\sigma \simeq 0.33 \), while the scalar field approaches the value \( \tilde{\phi}_1/M = 1 \) for large \( \tilde{r} \). The potential \( U(\phi) \) has \( c = 1 \), \( C \sim 3 \times 10^{-47} \) and is negligible in the range of distances of interest. The quantities \( \tilde{\phi}, \tilde{\rho}, A, B \) are plotted as functions of \( \tilde{r} \). The scalar field approaches zero near the center of the solution, so that the fermions become almost massless there. The pressure and density of the fermionic gas vanish for \( \tilde{r} \geq \tilde{R} \simeq 46 \). The value of \( \tilde{R} \) determines the radius of the astrophysical object through eq. (49): \( \tilde{R} \simeq 16 \text{ km} \). The metric components \( A \) and \( B \) deviate significantly from 1 in the interior of the solution. For this reason a fully non-linear treatment of the Einstein equations has been necessary. The mass of the object can be deduced from the asymptotic form of \( A \) or \( B \) for \( \tilde{r} \to \infty \) through the second of eqs. (39). One finds \( \tilde{R}_g \simeq 17 \), which corresponds through eq. (50) to \( M_{\text{tot}} \simeq 2.0 M_\odot \). The total fermionic number is \( \tilde{N} \sim 920 \), which gives \( N \simeq 4.7 \times 10^{57} \) through eq. (51). The potential \( U(\phi) \) does not play any...
role in this solution. The reason is that its value is much smaller than the pressure or the field derivative energy.

In fig. 2 the form of the solution is presented if the chemical potential is $\mu_0/\sigma \simeq 0.95$ while the remaining parameters remain the same. The deviations of $\tilde{\phi}$, $\tilde{p}$, $A$, $B$ from 1 are much smaller than in the previous case. The fermionic gas is much more dilute, as its pressure is approximately 20 times smaller. The resulting astrophysical object has $\tilde{R} \simeq 22$, $\tilde{R}_s \simeq 0.41$, $\tilde{N} \simeq 10.5$, which correspond to $R \simeq 7.4$ km, $M_{\text{tot}} \simeq 0.047 M_\odot$. The total fermionic number is $\tilde{N} \simeq 10.5$, which gives $N \simeq 5.4 \times 10^{55}$.

The variation of the chemical potential results in a whole class of solutions depicted by the dashed line in fig. 3. The mass to radius relation $M_{\text{tot}}(R)$ is displayed. This function has a maximum at $R \simeq 25$ km, which separates two branches of the curve. The left branch is denoted by a thinner dotted line in fig. (3) because it corresponds to unstable configurations. For a given value of the total fermionic number $N$ there are two solutions with different values of $M_{\text{tot}}$ [14, 24, 25]. The dotted line corresponds to the branch with larger mass. The solutions of figs. 1 and 2 are denoted by a circle and a square, respectively, on this plot.

If the value of the scale $\sigma$ changes while $\phi_1$ is kept constant, the line $M_{\text{tot}}(R)$ retains its shape, but is rescaled by an overall factor. This is obvious from the form of eqs. (52)–(57), in which $\sigma$ does not appear explicitly after the rescaling of eq. (46), as it has been incorporated in the various dimensionless parameters. If a solution with given values of $\sigma$, $\phi_1$ and $\mu_0$ is known, another solution can be generated through the replacements $\sigma \rightarrow \alpha \sigma$, $\phi \rightarrow \phi$, $r \rightarrow r/\alpha^2$, $\rho \rightarrow \alpha^4 \rho$, $\tilde{p} \rightarrow \alpha^4 \tilde{p}$ and $\phi_1 \rightarrow \phi_1$, $\mu_0 \rightarrow \alpha \mu_0$, $R \rightarrow R/\alpha^2$, $M_{\text{tot}} \rightarrow M_{\text{tot}}/\alpha^2$. In fig. 3 a class of solutions is presented with $\phi_1/M=1$, $\sigma/\text{GeV}=3/2$ (solid curve) that demonstrates this point. Each point of this curve can be obtained from a point of the curve with $\phi_1/M=1$, $\sigma/\text{GeV}=1$ (dashed curve) by dividing the coordinates by a factor $9/4$. Lowering $\sigma$ results in compact objects whose maximum mass can be significantly larger than the solar mass, while their average density falls $\sim \sigma^4$.

Changing the asymptotic value $\phi_1$ of the field, while keeping the scale $\sigma$ constant, modifies the shape of the curve $M_{\text{tot}}(R)$. In fig. 3 $M_{\text{tot}}(R)$ is plotted for $\phi_1/M=3/2$, $\sigma/\text{GeV}=1$ (short-dashed
curve). Comparison with the case $\phi_1/M=1, \sigma/\text{GeV}=1$ (dashed curve) shows that the maximum mass is significantly reduced, while the mass is a decreasing function of the radius for the whole stable branch. The form of the curve is very reminiscent of that of conventional neutron stars. The reason is that for large $\phi_1$ the fractional deviation in the interior of the solution from the asymptotic field value is not very large. The fermionic mass is reduced but does not approach zero. This is similar to the case of nuclear matter, for which the effective reduction of the mass of a free fermion ($\sim 1 \text{ GeV}$) is of the order of the nuclear binding energy ($\sim 15 \text{ MeV}$). In fig. 3 the case with $\phi_1/M=3/2, \sigma/\text{GeV}=2/3$ (dot-dashed curve) is also depicted. This has the same asymptotic fermionic mass as the case $\phi_1/M=1, \sigma/\text{GeV}=1$ (dashed curve).

It is interesting that the upper part of the stable branch of the cases with $\phi_1/M = 1$ has the same form as for fermion soliton stars [14], or fermion Q-stars [24, 25], or strange stars [26]. For these cases the fermionic mass approaches zero in the interior and the binding energy is large, similarly to the fermion stars. The mass is an increasing function of the radius. However, the lower part of the stable branch differs from that of the fermion stars. The mass becomes a decreasing function of the radius, similarly to the conventional neutron stars.

5. Summary

In this talk I discussed some of the implications for astrophysical configurations composed of dark matter of the possible interaction with a scalar field that is responsible for the dark energy of the Universe. I assumed that at cosmological scales the scalar field has an almost constant expectation value. The time dependence associated with the evolution of the dark energy density was neglected, as I assumed that the relevant time scale is very long. The interaction between dark matter and dark energy was modelled by assuming that the mass of the dark matter particles depends on the expectation value of the scalar field. The formalism is based on the Thomas-Fermi approximation, which assumes that in a local frame at every point in space the particles are described by a Fermi-Dirac distribution with position-dependent temperature and chemical potential. The Einstein equations and the equation of motion of the scalar field were given.

In regions of high number density of dark matter particles, the scalar field is shifted from its cosmological value. The shift is in the direction that reduces the dark matter mass, so that the total energy of the configuration is minimized. This deviation from the asymptotic value is the manifestation of the attractive interaction between the dark matter particles mediated by the scalar field.

I discussed the implications for the dark matter halos that comprise the bulk of the matter outside the core of galaxies, up to distances of 100–200 kpc. The approach I described is based
on the assumption that the dark matter can be treated as a gas of thermalized particles. It may seem unlikely that such a formalism could apply to the dark matter halos, as the particles in them interact very weakly. However, the formalism is equivalent to the model of the isothermal sphere, which gives a simple explanation for the approximately flat rotation curves in the absence of the scalar interaction. The effective temperature in this approach is proportional to the dispersion $\langle v_d^2 \rangle$ of the velocity of the dark matter particles.

The presence of the new attractive force does not modify the distribution of dark matter so as to destroy the approximately flat rotation curves. The main new effect is that the velocity of a massive object orbiting the galaxy outside its core is not of the order of the typical velocity of the dark matter particles, as in the conventional picture. Instead, it is reduced by a factor $(1 + \kappa^2)^{1/2}$, where $\kappa^2 = 4M^2 \left[(dm_0/d\phi)/m_0\right]^2$ quantifies the dependence of the dark matter mass $m_0$ on the scalar field. If $\kappa^2$ is large, the typical velocity of the dark matter particles can be significantly larger than the rotation velocity. The latter quantity is directly measurable, and its value is used in order to deduce the velocity of dark matter particles for dark matter searches.

For $\kappa^2 \gtrsim 1$ the typical velocity of dark matter in our neighborhood of the Milky Way exceeds the rotation velocity significantly. The flux of dark matter particles towards a terrestrial detector is strengthened by the factor $(1 + \kappa^2)^{1/2}$ relative to the $\kappa^2 = 0$ case. As a result, the counting rates are increased by the same factor. Existing bounds on dark matter properties from direct searches can be extended to include the case of non-zero $\kappa$. The bound on the cross section for the interaction of dark matter with the material of the detector is strengthened by the factor $(1 + \kappa^2)^{1/2}$. Corrections to this simple picture are also possible through the velocity dependence of various contributions to the counting rates, such as nuclear form factors.

Using the same formalism I also discussed solutions that describe denser compact objects composed of dark matter. In the examples I considered, the dark matter gas has zero temperature and the stability of the configurations is provided by the degeneracy pressure. The particle mass is determined by the scalar field through a Yukawa interaction. The compact objects resemble neutron stars, but are composed of dark matter, while their density can deviate significantly from nuclear density. Similarly to the case of dark matter halos, the potential of the scalar field plays a minor role in the structure of these objects.

Their mass to radius curve has a stable branch whose shape depends on the asymptotic value of the scalar field in Planck units $\phi_1/M$. For $\phi_1/M \gtrsim 1$ the mass of the compact object is a decreasing function of the radius, and the curve resembles strongly the one for neutron stars. The reason is that for large $\phi_1$ the fractional deviation in the interior of the solution from the asymptotic field value is not very large. The fermionic mass is reduced but does not approach zero. This is similar to the case of nuclear matter. For $\phi_1/M \lesssim 1$ the upper part of the stable branch has the same form as for fermion soliton stars [14], or fermion Q-stars [24, 25], or strange stars [26]. For these cases the fermionic mass approaches zero in the interior and the binding energy is large. The mass is an increasing function of the radius. However, the lower part of the stable branch differs from that of fermion stars. The mass becomes a decreasing function of the radius, similarly to neutron stars.

The mass to radius curve also depends on the scale $\sigma$ that determines the mass of the dark matter particles. I assumed that the particle mass is generated by the scalar field through a Yukawa term, with a Planck suppressed Yukawa coupling $\sigma/M$. For $\phi_1/M = \mathcal{O}(1)$ and $\sigma = \mathcal{O}(1)$ GeV, the most massive astrophysical object has $M_{\text{tot}} = \mathcal{O}(1) M_\odot$ and $R = \mathcal{O}(10) \text{ km}$. Changing $\sigma \rightarrow \alpha \sigma$ leads to the rescaling $R \rightarrow R/\alpha^2$, $M_{\text{tot}} \rightarrow M_{\text{tot}}/\alpha^2$. Reducing the scale $\sigma$ results in very massive, but dilute astrophysical objects. For $\sigma = \mathcal{O}(10) \text{ keV}$, one has $M_{\text{tot}} = \mathcal{O}(10^{10}) M_\odot$ and $R = \mathcal{O}(10^{10}) \text{ km}$. These objects are similar to the supermassive neutrino stars hypothesized in ref. [27]. For $\sigma = \mathcal{O}(10^{-9}) \text{ eV}$ the radius approaches the horizon size. Conversely, increasing $\sigma$ gives rise to very dense configurations of smaller size. For $\sigma = \mathcal{O}(1) \text{ TeV}$ and $\phi_1 = \mathcal{O}(1) M$ the mass of an unbound dark matter particle is $\mathcal{O}(1) \text{ TeV}$. The most massive compact object
composed of dark matter particles has $M_{\text{tot}} = O(10^{-6}) M_\odot$ and $R = O(1) \text{ cm}$.

A crucial question is whether the gas of dark matter particles can lose momentum fast enough for dense objects, such as the ones I described, to develop. The dark matter is expected to have only weak interactions, while its coupling to the scalar field is suppressed by the Planck scale. Despite this, it has been demonstrated that massive astrophysical objects can form under gravitational collapse [27, 28], even in the absence of the attraction mediated by the scalar field. For light neutrinos it has been shown in ref. [27] that matter can be expelled in a series of bounces during the collapse, so that a condensed object is left behind. In ref. [28] the case of neutralinos with a mass of 100 GeV has been considered. Through numerical simulations it has been shown that compact objects as large as the solar system and with a mass of the order of the Earth mass start forming at redshifts $z \simeq 60$ and survive until today. The presence of an additional scalar attractive force increases the likelihood of formation of compact objects.

The most promising possibility for the detection of the astrophysical objects I discussed is through gravitational microlensing. The sensitivity depends crucially on the ratio of the object radius to the Einstein radius

$$R_E \approx 0.37 \times 10^{12} \sqrt{\frac{M_{\text{tot}}}{M_\odot}}$$

(59)

for gravitational lensing. The zero-temperature solutions that I described have a mass to radius curve approximately given by eqs. (49), (50). For $\sigma > 10^{-3} \text{ eV}$ they satisfy $R < R_E$, so that the corresponding astrophysical objects are expected to produce a detectable lensing signal. This must be contrasted with the much more dilute dark matter subhalos in the absence of an attractive force mediated by a scalar field [28]. These have size larger than the Einstein radius and are more difficult to detect.

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