Graph-based Detection of Multiuser Impulse Radio Systems

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Abstract - Impulse-Radio (IR) is a wideband modulation technique that can support multiple users by employing random Time-Hopping (TH) combined with repeated transmissions. The latter is aimed at alleviating the impact of collisions. This work employs a graphical model for describing the multiuser system which, in turn, facilitates the inclusion of general coding schemes. Based on factor graph representation of the system, several iterative multiuser detectors are presented. These detectors are applicable for any binary linear coding scheme. The performance of the proposed multiuser detectors is evaluated via simulations revealing large gains with low complexity.

Keywords -

I. Introduction

Impulse-radio systems use short baseband pulses. In Time-Hopping Impulse Radio (TH-IR) systems the information is encoded in the polarity of the pulses (Pulse amplitude modulation - PAM), or in the position of the pulses (Pulse position modulation - PPM). To support multiple access, additional delay is introduced per pulse. The added delay, which is unique for each user, is (pseudo) random, and is assumed to be known to the receiver. Additionally, in order to improve performance, each information bit is typically transmitted several times. As a result of the above measures, the probability of catastrophic collision (multiple-user interference) is minimized. In-depth treatment of Impulse Radio communications is given by Win and Scholtz [1]. While impulse radio is typically associated with Radio-Frequency (RF) communications, Visible Light Communications (VLC) is yet another data communication technique that can employ short (light) pulses for signalling, and can hence benefit from the algorithms and results presented herein.

TH-IR was analyzed in the past for several channel models and interference types, see e.g. [2], [3], [4]. Scholtz [5] suggested employing this communication scheme for supporting multiple access. The Multiuser interference (MUI) was modeled as Gaussian noise and the receiver employed a matched filter to detect a specific user [5], [1]. Two different receiver architectures were defined: AIRM A (Analog impulse radio multiple access) and DIRMA (Digital IRMA) receivers [6]. In another work [7] the choice of a different time hopping sequence was discussed, a pseudo-chaotic sequence, aimed at alleviating the MUI problem. Deterministic sequences designed to mitigate MUI altogether were also proposed [8], [9]. To improve system performance and better deal with MUI in the framework of single-user detection, several authors proposed more general coding and modulation schemes, see e.g. [10], [11], [12], [13].

Rather than treating the signals received from the many users as interference, one can benefit by performing MultiUser Detection (MUD), thus extracting the data associated with all users. In general, MUD is a computationally intensive task. To alleviate this problem, Fishler and Poor [14] suggested using an iterative multiuser detector for the simple repetition-based code in order to achieve good performance with low-complexity. Wang et al. [15] suggested using the same iterative detector as [14] with different message passing aimed at reducing computational complexity.

In this paper we present several iterative multiuser receivers. Rather than treating the signals received from the many users as interference, one can benefit by performing MultiUser Detection (MUD), thus extracting the data associated with all users. In general, MUD is a computationally intensive task. To alleviate this problem, Fishler and Poor [14] suggested using an iterative multiuser detector for the simple repetition-based code in order to achieve good performance with low-complexity. Wang et al. [15] suggested using the same iterative detector as [14] with different message passing aimed at reducing computational complexity.

LDPC codes were originally introduced by Gallager [19], [20] in 1962, and "reintroduced" in recent years. It was the introduction of Turbo convolutional Codes with efficient iterative decoding (which exhibit notably low error probabilities at low SNR) that triggered the search for such codes - including LDPC codes on this family of codes (see e.g. [21] [22] [23]).

In this work we present several iterative multiuser receivers for the original TH-IR system. Then, we study a modified system where all users employ arbitrary linear coding, and generalize the above multiuser receiver for this setting. The proposed receivers are based on factor graph representation of the complete system. Comparison with the classical repetition-based systems reveal the significant improvements possible with the proposed practical approach. The rest of this manuscript is organized as follows. Section II reviews the system model. In Section III we present iterative multiuser detectors. Finally, Section IV provides simulation results for...
the detectors introduced in the previous section using several
codes including LDPC codes. Some of the results reported
herein appeared in [24].

II. SYSTEM MODEL

A typical UWB TH-IR signal can be described by the
following general model:

\[ s^{(k)}_{tr}(t) = \sum_{j=-\infty}^{\infty} b_{j/N_f}^{k} w_{tr}(t - jT_f - c^{k}\tau_c), \quad (1) \]

where \( S^{(k)}_{tr} \) is the transmitted signal of the \( k \)th user; \( T_f \) is
the nominal pulse repetition time; \( w_{tr} \) is the transmitted pulse
shape; \( b_i^{k} \) is the \( i \)th symbol transmitted by user \( k \); \( c^{k}\tau_c \) is
the time hopping sequence used by user \( k \); \( N_f \) is the number of
frames in which a symbol is transmitted; \( \tau_c \) is the chip size.

A user repeats every information symbol in \( N_f \) different
frames, where each frame consists of \( N_c \) chips, also referred
to as slots. The time hopping sequence employed by each user is
a set of values chosen randomly from \{0, 1, \ldots, N_c - 1\}. Usually \( N_c < T_f/\tau_c \) for avoiding inter-symbol-interference
(ISI). In this work the information symbols are assumed to be
binary digits (bits) and the signaling is binary-phase shift
keying (BPSK).

The system consists of \( K \) transmitting users and one re-
ceiver, where the different users are centrally coordinated and
synchronized by the receiver (base station) [25], [26], [27].

The channel over which the signal is transmitted can be
frequency selective; it is assumed that the combined response of
the channel and pulse shape is such that the inter-symbol
interference is negligible, and the channel characteristics are
slowly varying in time. The received signal is perturbed by
additive white Gaussian noise (AWGN).

The received signal is given by

\[ r(t) = \sum_{k=1}^{K} A_k \sum_{j=-\infty}^{\infty} b_{j/N_f}^{k} w_{rx}(t - jT_f - c^{k}\tau_c) + n(t), \quad (2) \]

where \( K \) denotes the number of users in the system; \( w_{rx} \) is
the received waveform associated with one transmitted pulse;
\( A_k \) is the amplitude associated with user \( k \); \( n(t) \) denotes an
additive white Gaussian noise process.

In particular, this channel model can be encountered in
scenarios where the dominant propagation path is the line-
of-sight, and \( \tau_c \) is chosen to satisfy \( \tau_c > Sup\{w_{rx}\} \), where
\( Sup\{\cdot\} \) is the support of the received pulse \( w_{rx} \). See also [14]
and the references therein.

A. Discrete Time model

The receiver tracks the gains associated with the different
users. The received signal (2) goes through a matched filter
whose output is sampled every \( \tau_c \) seconds. Denote by \( r[i] \)
the vector of samples at the output of the matched filter
corresponding to the \( i \)th information symbol. The size of this
vector is \( N_f N_c \), and it can be described by the following equation

\[ r[i] = S[i] A b[i] + n[i], \quad (3) \]

where \( S[i] \) is a \( [N_f N_c \times K] \) matrix describing the slots used
by the different users for transmitting their \( i \)th information
symbol. The matrix is defined by

\[ S[i]_{lk} = \begin{cases} 1 & \text{if } c_l^{k}((i-1)N_f + \lfloor \frac{j}{N_c} \rfloor N_c) = l - \lfloor \frac{j}{N_c} \rfloor N_c \\ 0 & \text{otherwise.} \end{cases} \quad (4) \]

\( A \) is the gain between the \( k \)th transmitter and the receiver,
\( A = diag(A_1, A_2, \ldots, A_k) \); \( b[i] \) is the information vector
transmitted by the \( K \) users at the \( i \)th interval, \( b[i] = [b_1^k, b_2^k, \ldots, b_K^k]^T \); \( n \) is a zero-mean Gaussian random vector
with correlation matrix \( \sigma^2_n I \), where \( \sigma^2_n = \frac{N_c}{2} \).

Without loss of generality we can examine the first information
symbol and therefore omit the time index \( i \), consequently
(3) can be rewritten as

\[ r = S A b + n. \quad (5) \]

Figure 1 depicts a toy example of transmitting one informa-
tion symbol in a 2-user system with the following parameters:
\( N_f = 3 \) frames, \( N_c = 5 \) chips per frame. The rectangles
represent the slots used by the users.

\[ \text{Fig. 1. System illustration} \]

III. ITERATIVE MULTIUSER DETECTION FOR TH-IR
SYSTEMS

A major drawback of optimum Multi User Detection (MUD) is implementation complexity. In this section we try to
alleviate this problem by presenting several iterative detectors
of reduced complexity.

We first introduce a simple iterative detector for the original
TH-IR system settings presented in Section II. Then, we
propose a graph-based description of the same system, and
develop the corresponding MAP-based iterative detector. Fi-
lly, we introduce coding into the original system and develop
the appropriate graph-based iterative multi-user detector.

A. ID detector

The first detector we present is a (very) low complexity
iterative detector based on intuition, rather than mathematical
arguments. It is therefore referred to as ID (Intuition-driven)
detector. It is tailored for detecting repetition-based transmis-
sions, and may be characterized as a Gallager-type decoder.

Returning to the toy example of Figure 1, the corresponding
S matrix (4) is given by

\[ S^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (6) \]

and the graph we associate with the ID detector is given in
Figure 2.

The detector consists of two stages performed in an iterative
manner. In the first stage, the estimated value of a specific
The output message passed to the check node is defined as

\[ m_0 = r_i - \sum_{l=1}^{k-1} A_{i,l} * m_l, \]  

where \( A_{i,l} \) is the amplitude of user \( i \) as seen by input node \( i \).

With the aid of Figure 3(b), we proceed to describe the check nodes. Any check node is connected to exactly \( N_f \) input nodes (due to the repetition nature of the transmission). The message computed by a check node (to be passed back to an input node) is defined as

\[ m_0 = \text{sgn} \left( \sum_{l=0}^{N_f-1} m_l \right). \]  

Note that for the first iteration, the output from all check nodes is initialized to zero. Finally, after the last iteration, the output of the detector is taken from the check nodes, yet it includes all the inputs as follows

\[ \hat{b}_k = \text{sgn} \left( \sum_{l=0}^{N_f-1} m_l \right). \]  

\section*{B. 3-stage Factor-Graph (FG3) detector}

In this section we present an iterative (soft) detector based on a 3-stage factor-graph description of the system. While the proposed detector is aimed at decoding the repetition-based transmission, it is of great interest as it lays the ground for introducing arbitrary linear coding into the system.

Henceforth, we let \( y \) represent the received vector \( r \), i.e. \( y = r \). The output of the MAP decoder (for the \( k \)th bit) is given by

\[ \hat{b}_k = \arg \max_{b_k = \pm 1} \{ p(b_k | y) \} \]

\[ = \arg \max_{b_k = \pm 1} \left\{ \sum_{b \setminus b_k} p(y | b) * \frac{p(b)}{p(y)} \right\}, \]  

where \( \sum_{-b_k} \) denotes summing over all values of the vector \( b \) excluding those containing \( -b_k \). Since all input vectors, \( \{b\} \) are equiprobable,

\[ \hat{b}_k = \arg \max_{b_k = \pm 1} \left\{ \sum_{-b_k} p(y | b) \right\} \]

\[ = \arg \max_{b_k = \pm 1} \left\{ \sum_{b_0, \ldots, b_{K-1}} p(y | b_0^1, \ldots, b_{K-1}^1) \right\}. \]  

Using the following definition

\[ \mathbb{E}(x_1, \ldots, x_i) = \begin{cases} 1 & \text{if } x_1 = \ldots = x_i, \\ 0 & \text{otherwise} \end{cases}, \]  

one can write

\[ \hat{b}_k = \arg \max_{b_k = \pm 1} \left\{ \sum_{b_0^1, \ldots, b_{K-1}^1} \mathbb{E}(b_0^1, \ldots, b_{K-1}^1) \right\}. \]  

\[ \prod_{k=1}^{K} \mathbb{E}(b_0^k, \ldots, b_{N_f-1}^k). \]  

(14)
Note that the subscript \( j \), denoting the frame index, has been added though one expects all \( N_f \) estimations \( \hat{b}_j^k, j \in \{0, 1, \ldots, N_f - 1\} \) to produce the same value.

Since the channel is memoryless

\[
p(y|b_0^1, \ldots, b_{N_f-1}^1, \ldots, b_K^1, \ldots, b_{N_f-1}^K) = \prod_{i=1}^{N_f N_c-1} p(y_i|b_{yi}),
\]

where \( b_{yi} \) represents the transmitted bits from all users employing chip slot \( i \). Finally, we have

\[
\hat{b}_j^k = \arg\max_{b_j^k=\pm 1} \left\{ \sum_{-b_j^k}^{N_f N_c-1} \prod_{i=1}^{K} p(y_i|b_{yi}) \prod_{k=1}^{N_f} \mathbb{E}(b_0^k, \ldots, b_{N_f-1}^k) \right\}.
\]

This, so-called, "sum-of-products" function can be calculated using a message passing algorithm operating iteratively on a bipartite graph, termed factor graph [28], [29]. A factor graph typically consists of two types of nodes - function nodes and variable nodes. In Figure 4(a) we depict a graph corresponding to Equation (16).

The graph consists of three types of nodes. The squares represent function nodes, which are divided into two types. The first type of function nodes are associated with the function \( p(y_i|b_{yi}) \); there are exactly \( N_f N_c \) such nodes. We shall denote these as \( P \) function nodes. The second type of function nodes are associated with \( \mathbb{E}(b_0^k, \ldots, b_{N_f-1}^k) \); there are exactly \( KN_f \) such nodes. Denote this type of nodes as \( E \) function nodes. Nodes indicated by circles represent the variable nodes for \( b_j^k \); there are exactly \( KN_f \) such nodes. Denote these nodes as \( b \) variable nodes. Note that the edges connecting the \( P \) function nodes with the \( b \) variable nodes are completely defined by the matrix \( S \). Finally, as will be shown later on, by omitting the \( b \) variable nodes, the graph reduces to the one shown for the ID detector.

The messages passed by the algorithm shall be denoted by \( \mu(x), x = \pm 1 \). \( \mu_{p-v} \) represents a message passed from a \( P \) function node to a \( b \) variable node, \( \mu_{v-E} \) represents a message from a \( b \) variable node to an \( E \) function node, while \( \mu_{E-v} \) is a message from an \( E \) function node to a \( b \) variable node. Next, we provide explicit description of the messages associated with the different types of nodes.

1) \( P \) function nodes: The \( \mu_{p-v} \) messages, calculated at the \( P \) function nodes, are

\[
\mu_{p-v}(+1) = \sum_{x_1, \ldots, x_j} p(y_j + 1, x_1, \ldots, x_j) \prod_{j=1}^{J} \mu_j(x_j);
\]

\[
\mu_{p-v}(-1) = \sum_{x_1, \ldots, x_j} p(y_j - 1, x_1, \ldots, x_j) \prod_{j=1}^{J} \mu_j(x_j),
\]

where \( x_i \) is a binary value conveyed by an edge connected to an adjacent variable node. W.l.o.g. \( \mu(\pm 1) \) denotes the output message passed to the variable node of index \( 0 \), while \( \mu_j(x_j) \) denotes an input message originating at an adjacent \( b \) variable node. (There are \( J \) input edges, with \( j \) being the edge index.) We shall henceforth omit the cumbersome directive pointers, \( p \to v \) and \( v \to p \), as it will always be easy to realize the correct flow.

Rather than passing two messages, one may use a single message in the form of the log likelihood ratio (LLR). Let \( r \) be defined as follows

\[
r \equiv \frac{\mu(+1)}{\mu(-1)} = \frac{x_j + 1}{x_j - 1}.
\]

Finally, using (19) the LLR, \( l = \log(r) \), is given by

\[
l = \log \frac{\sum_{x_1, \ldots, x_j} p(y_j + 1, x_1, \ldots, x_j) \prod_{j=1}^{J} e^{x_j}}{\sum_{x_1, \ldots, x_j} p(y_j - 1, x_1, \ldots, x_j) \prod_{j=1}^{J} e^{x_j}}.
\]

2) \( b \) variable nodes: It is easy to see that each of the \( b \) variable nodes is connected to exactly one \( P \) function node on the left and one \( E \) function node on the right. In this case the output messages are the same as the input messages.

3) \( E \) function nodes: For this type of nodes we have

\[
r = \frac{\sum_{x_1, \ldots, x_{N_f-1}} \mathcal{E}(1, x_1, \ldots, x_{N_f-1}) \prod_{j=1}^{N_f-1} \mu_j(x_j)}{\sum_{x_1, \ldots, x_{N_f-1}} \mathcal{E}(-1, x_1, \ldots, x_{N_f-1}) \prod_{j=1}^{N_f-1} \mu_j(x_j)}
\]

\[
= \frac{\sum_{x_1, \ldots, x_{N_f-1}} \mathcal{E}(1, x_1, \ldots, x_{N_f-1}) \prod_{j=1}^{N_f-1} r_j^{x_j + 1}}{\sum_{x_1, \ldots, x_{N_f-1}} \mathcal{E}(-1, x_1, \ldots, x_{N_f-1}) \prod_{j=1}^{N_f-1} r_j^{-x_j + 1}}.
\]
Recalling the definition of $\Xi$, (13), it is easily verified that
\[
 r = \frac{\prod_{j=1}^{N_f-1} r_j^{1+\delta_j}}{\prod_{j=1}^{N_j} r_j^{\gamma_j}} = \prod_{j=1}^{N_j} r_j,
\] (22)
and the LLR is simply
\[
l = \log(r) = \sum_{j=1}^{N_j-1} l_j.
\] (23)

After the last iteration is performed, the output of the detector is taken from the $E$ function node:
\[
\hat{b}^k = \text{sgn}(\sum_{j=1}^{N_j} l_{j}).
\] (24)

We described a multiuser detector based on factor graph representation of the system. Recall that Fishler and Poor (FP) [14] presented an iterative multiuser detector for the same system that follows the turbo principle. Interestingly, the FG3 detector turns out to be the same as the FP detector although a different model is employed for describing the system. This assertion may not be obvious by simply comparing at the mathematical manipulations one can move from the set of equations presented herein to those used by FP. It can be shown that the FP and FG3 detectors are not MAP-achieving since the associated graphs are not cyclic-free.

C. CFG3 detector

The FG3 proposed model and detection technique shall now serve as the basis for the introduction of a coded system. The receiver to be developed in this section is aimed at providing a solution for a system employing arbitrary linear coding. It shall hence be denoted as coded-FG3 (CFG3).

A linear code is typically defined by its parameters $[n, k, d]$, where $n$ is the code length, $k$ is the dimension of the code (not to be confused with the number of users), and $d$ denotes the minimum Hamming distance of the code. Hence, in a coded TH-IR system, the number of frames $N_f$ will satisfy $N_f = n$, and the system rate is $\frac{n}{n}$.

Referring to Equation (16), the function $\Xi(\cdot)$ represents the fact that the bits associated with a specific user must all be the same. When a code $[n, k, d]$ is used, we shall replace $\Xi(\cdot)$ with a new function, $PC$, representing a Parity Check equation.

The parity check matrix $H$ is an $(n-k) \times n$ binary matrix
\[
PC = \prod_{i=1}^{n-k} PC_i,
\] (25)
where $PC_i$ is defined as follows
\[
PC_i = \begin{cases} 
1 & \text{if } \left(\sum_{j,h_{i,j}=1} b_j = 1 \right) \text{ where the sum is over GF}2 \\
0 & \text{otherwise}
\end{cases}
\] (26)

As an example, Figure 4(b) depicts a factor graph of a linear code when the latter is a repetition code whose parity check matrix $H$ is given by
\[
H = \begin{pmatrix} 
1 & 0 & 0 & \ldots & 1 \\
0 & 1 & 0 & \ldots & 1 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 1 & 1 \\
\end{pmatrix}.
\]

Although this graph is different from Figure 4(a), which means that two different detectors are to be used, both graphs target the same system and code.

The $b$ variable nodes have one input (form the left), but typically several outputs (connected to PC function nodes). The messages to be calculated and passed to the output are
\[
\mu(1) = \prod_i \mu_i(1),
\] (27)
and hence
\[
r = \frac{\mu(1)}{\mu(-1)} = \prod_i r_i \\
l = \log(r) = \sum_i l_i.
\] (28)

The messages to be calculated at the PC function nodes are associated with single parity check equations. These are the same messages used in LDPC decoding:
\[
l = 2\tanh^{-1}(\prod_i \tanh \frac{l_i}{2}).
\] (29)

After iterating all messages through the graph, the final marginalization is performed at the $b$ variable nodes
\[
\hat{b}^k = \text{sgn}(\sum_{j=1}^{N_j} l_j). 
\] (30)

It can be shown that by appropriately choosing the code, the associated graph can be made cyclic-free, and hence CFG3 is MAP-achieving.

IV. Simulation Results

A. Conventional repetition-based system

We first study the original system employing a repeated transmission scheme. We shall consider two different detectors: the FG3 detector and the CFG3 detector. In all presented simulations the number of slots per frame is $N_c = 20$, while the number of users varies, $K = \{3, 10, 30\}$.

Figures 5 hold the simulation results. The performance of the FG3 and the CFG3 detectors are practically the same. Therefore, whenever using repetition-based coding, we shall only consider the FG3 detectors.
B. Repetition vs. LDPC-based systems

We next compare the performance of a repeated transmission system (using ID and FG3 detectors) with an LDPC-based coded system (using CFG3 detector). The parameters of the LDPC code chosen are \((n = 120, k = 56, R = 0.4667)\) - Mackay code 120.64.3.109 [30]. It has been chosen because of its relatively short length\(^1\); no attempt has been made to optimize it for our system. Repetition code of rate \(\frac{1}{3}\) have been chosen for comparison.

Figures 6 and 7 pertain to the repetition code with ID and FG3 detectors, respectively, while Figure 8 pertains to the LDPC code-based system employing the CFG3 detector. The ID and FG3 detectors exhibit an error floor which is irreducible due to the fact that system performance is limited by Multi User Interference (MUI). To support this assertion note how the error floor increases with the number of users. The FG3 detector exhibits better error floor performance than the ID detector, as expected.

Clearly, the LDPC-based system with the CFG3 detector behaves differently. First, the BER curve is much sharper as might be expected of a coded system. Second, as demonstrated in Figure 8, when the number of users increases, the \(E_b/N_0\) threshold-point also increases. Still, once the threshold is passed, the slopes associated with all cases are similar. Increasing the number of users amounts to adding more noise, which leads us to the next observation: another threshold that can be clearly identified corresponds to the number of users - beyond a certain number of users, the system collapses.

In general, we argue that the LDPC-based system handles MUI much better than the alternative approaches mainly because it employs a large number of frames \((N_f)\). Consequently, "catastrophic" collisions among different users are much less common.

Complexity-wise we argue as follows. ID performs only simple addition operations. Calculation of the message at the input nodes \((8)\) requires \(O(K^1)\) additions, where \(K^1\) is the number of colliding users. Check node calculation \((9)\) requires \(O(N_f)\) additions. Recall that the graphs associated with the ID and FG3 systems are the same. The two detectors differ only in the calculations carried out at the input nodes. Calculating \((20)\), instead of \((8)\), increases complexity by \(O(2^{K^1})\) [14]. Moving from FG3 to CFG3, as the input nodes remain the same, the added complexity is that of decoding. In our example, LDPC coding has been employed, and therefore the additional complexity is given by \(C_{LDPC} \cdot K\), where \(C_{LDPC}\) is the complexity required for decoding the LDPC code used.

Compared to ID, FG3 better treats MUI for the price of increased complexity at the input side (left-hand side of the graph). CFG3 add coding gain for the price of increased complexity at the coding side (right-hand side of the graph). In conclusion, among the three described schemes (for the same \(E_b/N_0\)), the LDPC-based system provides the best performance both in terms of BER and overall system throughput.

C. Performance dependency on the number of users

In a multi-user environment users often join and leave the system. Analyzing the performance of the system for different numbers of users is therefore of interest.

A system with \(N_c = 20\) slots per frame is considered: ID and FG3 detectors are employed for a repetition-based transmission scheme with \(N_f = 3\) \((rate = 1/3)\); CFG3 detector with LDPC code \((n = 120, k = 56, R = 0.4667)\) is also considered.

Figures 9 and 10 present the simulation results for \(E_b/N_0 = 20dB\) and \(E_b/N_0 = 5dB\), respectively. For \(E_b/N_0 = 20dB\),

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\(^1\)Recall that the number of frames, \(N_f\), in our system model is actually the code length. Since the total number of time slots in the system is \(N_f N_c\), reasonable simulation times are when using shorter codes.
MUI is the dominant impairment. The performance of the ID and FG3 detectors gradually deteriorate as the number of users is increased from 3 to 40. FG3 exhibits better behavior throughout. The LDPC coded system with CFG3 detector performs quite differently. No errors are observed until the number of users reaches a certain threshold, 33 in this simulation. Once the threshold is passed, performance degradation is steep. Exact position of the threshold and its behavior depend on system parameters.

In the second simulation, shown in Figure 10, the additive noise is not negligible. The CFG3 detector behaves much like before: as long as the number of users is below a certain threshold, performance degradation is quite moderate. Also notice that the threshold moved to the left due to the existence of noticeable noise. The ID and FG3 detectors perform much worse than the CFG3 detector with FG3 being only slightly superior to the ID detector.

D. Performance dependency on iterations

The influence of the number of iterations on performance has so far been overlooked; a fixed number of 8 iterations was used throughout. Notably, increasing the number of iterations benefits the LDPC-based system more than the alternatives. As an illustrative example Figure 11 presents the BER performance of the three detectors, as a function of $E_b/N_0$, for $K = 20$ users and different number of iterations. The performance of the ID and FG3 detectors converge in only 4 iterations. More iterations are required for the LDPC-based system to converge. In the single-user case (simulation results not shown), the number of iterations required for the LDPC-based system to converge is smaller than in the multi-user case. One may argue that this follows from the fact that the graph associated with the multi-user case is much more involved.

E. Some after simulation comments

In a single-user system, the user can clearly employ all the time slots for its own usage. In the case of high SNR, with two-level modulation, the channel capacity is upper bounded by $1\text{[bit/chip]}$. Since each frame provides $N_c$ transmit opportunities, the capacity of the single-user system is upper bounded by $C = N_c\text{[bits/frame]}$. Let us now consider the CFG3-based system with $K$ users. The code rate used was 0.46 and $N_c = 20$. The CFG3 detector hardly produced any bit errors as long as the number of users satisfied $K < 33$. The overall system throughput in this case is $C = 14.72\text{[bits/frame]}$ as compared to the above upper bound of $C = N_c = 20\text{[bits/frame]}$. With an optimized LDPC code of greater length, the obtained throughput is expected to grow closer to this bound.

V. CONCLUSIONS

This work is concerned with iterative receivers for a multiuser TH-IR system. In particular, we study the gain achieved
by introducing coding into the originally uncoded system. Several iterative multi-user detectors have been presented based on factor graph representation of the complete system. These detectors are general and can support any binary linear coding scheme. Yet another strength of the proposed approach is that the graph used may be extended to account for multipath components (owing to the nature of the UWB channel). This is an interesting topic for future work.

Simulation results for the above mentioned detectors are presented for several codes including LDPC codes. It is demonstrated that for 20 users, BER of $10^{-3}$ can be achieved with higher system rate and more than 4dB gain in $E_b/N_0$ when using an LDPC-coded system. Furthermore, it is shown that the achievable performance of the original system quickly saturates at relatively poor BER.

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