A simple and tunable switch between slow- and fast-light in two signal modes with an optomechanical system

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Abstract
The control of slow and fast light propagation is a challenging task. Here, we theoretically study the dynamics of a driven optomechanical cavity coupled to a charged nanomechanical resonator (NR) via Coulomb interaction. We find that the tunable switch between slow- and fast-light for two signal modes can be observed from the output field by adjusting the laser-cavity detuning in this system. Moreover, the frequencies of two signal light can be tuned by Coulomb coupling strength. In comparison with previous schemes, the clear advantage of our scheme is that we can simply switch from fast- to slow-light in two signal modes by only adjusting the laser-cavity detuning from \( \Delta = \omega_1 \) to \( \Delta = -\omega_1 \). The proposal may have potential application in optical router and quantum optomechanical memory.

Keywords: slow and fast light, optical resonators, optical switch

1. Introduction

The control of slow and fast light propagation, in the probe transmission in single experiment, is a challenging task. Both theoretical and experimental aspects of the slow- and fast-light have been presented in physics [1, 2]. The first superluminal light propagation was observed in a resonant system [3], where the laser propagates without appreciable shape distortion but experiences very strong resonant absorption. Also, many techniques have been developed to realize slow- and fast-light in atomic vapor systems [4–6] and solid material systems [7, 8]. To reduce absorption, most of those works [4–7] are based on electromagnetically induced transparency (EIT).

On the other hand, opto-mechanical [9–16] systems have advanced rapidly, which are promising candidates for realizing architectures exhibiting quantum behavior in macroscopic structures. Also, a lot of them have been demonstrated experimentally in this systems, for example, quantum information transfer [17], normal mode splitting [13, 18], optomechanically induced transparency (OMIT) [19], frequency transfer [20]. Moreover, slow- and fast-light has also been successfully observed in this system [21], where an optically tunable delay of 50 ns with near-unity optical transparency and superluminal light with a 1.4 \( \mu \)s signal advance. Most recently, many proposals have been proposed in this system, such as slow-light based on an optomechanical cavity with a Bose–Einstein
condensate (BEC) [22] and fast-light in reflection meanwhile slow-light in transmission [23]. Moreover, we have demonstrated an efficient switch between slow- and fast-light work in the microwave regime [24].

However, the previous proposals and experiments all are work in single optical mode. In this letter, we theoretically investigate the slow- and fast-light in two signal modes based on the optomechanical system. Compared to recent proposals [21–23, 25–28], the clear advantage of our scheme is that we can simply switch from slow- to fast-light in two signal modes only by adjusting the effective laser-cavity detuning. Moreover, the two signal frequencies are tunable according to Coulomb coupling strength.

This letter is structured as follows. In section 2 we present the model and the analytical expressions of the optomechanical system and obtain the solutions. Section 3 includes numerical calculations for the two signal modes and tunable switch from slow- to fast-light based on recent experimental parameters. The last section is a brief conclusion.

2. Model and solutions

We begin with the Hamiltonian of the opto-mechanical system. As shown schematically in figure 1, the Hamiltonian is given by [29],

\[ H = \hbar \omega_a a + \left( \frac{p_1^2}{2m_1} + \frac{1}{2} m_2 q_1^2 \right) + \left( \frac{p_2^2}{2m_2} + \frac{1}{2} m_2 q_2^2 \right) - \hbar g a q_1 + \hbar \lambda q_1 q_2 + i \hbar \epsilon (a^d e^{-i \omega_\ell t} - \text{H.c.}) + i \hbar (a^d q_1 e^{-i \omega_\ell t} - \text{H.c.}), \]

(1)

where the first term is for the cavity field with frequency \( \omega_a \) and annihilation (creation) operator \( a (a^d) \). The second (third) term presents the vibration of the charged NR1 (NR2) with frequency \( \omega_1 (\omega_2) \), effective mass \( m_1 (m_2) \), position \( q_1 (q_2) \) and momentum operator \( p_1 (p_2) \) [30]. The fourth term denotes the

Figure 1. Schematic diagram of the system. A high-quality Fabry–Pérot cavity of length \( d \) consists of a fixed mirror and a movable mirror NR1. NR1 is charged by the bias gate voltage \( V_1 \) and subject to the Coulomb force due to another charged NR2 with the bias gate voltage \(-V_2\). The optomechanical cavity is driven by two light fields, one of which is the pump field \( l \) with frequency \( \omega_\ell \) and the other of which is the probe field \( q_1 \) with frequency \( \omega_p \). The output field is represented by \( \epsilon_{out} \). \( q_1 \) and \( q_2 \) represent the small displacements of NR1 and NR2 from their equilibrium positions, with \( l \) is the equilibrium distance between the two NRs.

Figure 2. The absorption \( \text{Re} [\epsilon_{\ell}] \) (red solid line) and dispersion \( \text{Im} [\epsilon_{\ell}] \) (blue dashed line) of the signal light as a function of the frequency \( \Delta = \omega_p - \omega_\ell \). The parameters used as \[ \omega_\ell = 2 \pi \times 947 \times 10^3 \text{ Hz}, m_1 = m_2 = 154 \text{ mg}, \]

\[ \kappa = 2 \pi \times 215 \times 10^3 \text{ Hz}, \varphi_1 = 9 \text{ mW}, \text{ and } \lambda = 8 \times 10^3 \text{ Hz m}^{-2} \]

NR1 couples to the cavity field due to the radiation pressure with the coupling strength \( g = \frac{\kappa}{d} \). The fifth term describes the Coulomb coupling between the charged NR1 and NR2 and \( \lambda = \frac{\hbar^2 C_1}{2 \pi \hbar \omega_\ell^2} \) [29, 31, 32], where the NR1 and NR2 take the charges \( C_1 V_1 \) and \(-C_2 V_2\), with \( C_1 (C_2) \) and \( V_1 (-V_2) \) being the capacitance and the voltage of the bias gate, respectively. \( l \) is the equilibrium distance between the two NRs. \( q_1 \) and \( q_2 \) describes the small displacements of NR1 and NR2 from their equilibrium positions, respectively. Finally, the last two terms correspond to the classical light fields (laser and signal fields) with frequencies \( \omega_\ell \) and \( \omega_\ell \), respectively. Furthermore, \( q_1 \) and \( q_2 \) are related to the laser power \( \varphi \) by \( \varphi_1 = \frac{2 \kappa \varphi}{\hbar \omega_\ell} \) and \( \varphi_2 = \frac{\sqrt{2 \kappa \varphi}}{\hbar \omega_\ell} \).

Hence, under the rotating frequency frame at the frequency \( \omega_p \) of the laser field, the Hamiltonian of the system can be rewritten as,
\[ H = \hbar \Delta a^\dagger a + \left( \frac{p_1^2}{2m_1} + \frac{1}{2} m_1 \omega_1^2 q_1^2 \right) + \left( \frac{p_2^2}{2m_2} + \frac{1}{2} m_2 \omega_2^2 q_2^2 \right) - \hbar g a^\dagger a q_1 + \hbar \lambda q_1 q_2 + i \hbar (a^\dagger e^{-i\delta} - a e^{i\delta}) - \text{H.c.}, \] (2)

where \( \Delta_a = \omega_a - \omega_l \) and \( \delta = \omega_1 - \omega_2 \) are the detunings of the cavity field and signal field from the laser field, respectively.

By taking the corresponding dissipation and fluctuation terms into account, the nonlinear quantum Langevin equations are given by [30]

\[ \begin{align*}
\langle \dot{q}_1 \rangle &= \frac{\langle p_1 \rangle}{m_1}, \\
\langle \dot{p}_1 \rangle &= -m_1 \omega_1^2 \langle q_1 \rangle - \hbar \lambda \langle q_2 \rangle + \hbar g \langle e^\dagger \rangle \langle e \rangle - \gamma_1 \langle p_1 \rangle, \\
\langle \dot{q}_2 \rangle &= \frac{\langle p_2 \rangle}{m_2}, \\
\langle \dot{p}_2 \rangle &= -m_2 \omega_2^2 \langle q_2 \rangle - \hbar \lambda \langle q_1 \rangle - \gamma_2 \langle p_2 \rangle, \\
\langle \dot{a} \rangle &= -[\kappa + i(\Delta_a - g \langle q_1 \rangle)] \langle a \rangle + \epsilon_1 e^{-i\delta},
\end{align*} \] (3)

where \( \gamma_1 \) and \( \gamma_2 \) are the decay rates associated with NR1 and NR2, respectively. As we encounter the expectation values of all the operators in equation (3). Therefore, we drop the quantum Brownian noise and input vacuum noise terms which average to zero [33]. Under the mean field approximation \( \langle Qa \rangle = \langle Q \rangle \langle a \rangle \) [15]. Equation (3) is a set of nonlinear equations and the steady-state response in the frequency domain is composed of many frequency components. In order to acquire the steady-state solutions of the above equations, we make an ansatz [21, 34]

\[ \begin{align*}
\langle q_1 \rangle &= q_{1s} + q_1 e^{-i\delta} + q_1^* e^{i\delta}, \\
\langle p_1 \rangle &= p_{1s} + p_1 e^{-i\delta} + p_1^* e^{i\delta}, \\
\langle q_2 \rangle &= q_{2s} + q_2 e^{-i\delta} + q_2^* e^{i\delta}, \\
\langle p_2 \rangle &= p_{2s} + p_2 e^{-i\delta} + p_2^* e^{i\delta}, \\
\langle a \rangle &= a_s + a e^{-i\delta} + a^* e^{i\delta},
\end{align*} \] (4)

where each quantity contains three items \( O_s, O_+, O_- \) (with \( O \in \{ q_1, p_1, q_2, p_2, a \} \)). In the case of \( O_s \gg O_\pm \), equation (3) can be solved by treating \( O_s \) as perturbations. Substituting the ansatz into equation (3), and ignoring the second-order terms, we can obtain the following steady-state:

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**Figure 3.** The two signal modes of fast group velocity index \( n_g \) as a function of the laser power \( \epsilon_l \) while \( \Delta = \omega_1 \), the parameters used are the same as in figure 2.

**Figure 4.** The absorption \( \text{Re} [\epsilon] \) (red solid line) and dispersion \( \text{Im} [\epsilon] \) (blue dashed line) of the two signal lights as a function of the laser power \( \epsilon_l \) while Coulomb coupling strength \( \lambda = 2 \times 8 \times 10^{35} \text{ Hz m}^{-2} \). Other parameters used are the same as in figure 2.

**Figure 5.** The absorption \( \text{Re} [\epsilon] \) (red solid line) and dispersion \( \text{Im} [\epsilon] \) (blue dashed line) of the two signal lights as a function of the laser power \( \epsilon_l \) while \( \Delta = -\omega_1 \). Other parameters used are the same as in figure 2.
\[ p_{1s} = p_{2s} = 0, \quad q_{1s} = \frac{\hbar |a_s|^2}{m_1 \omega_1^2 - \frac{\hbar^2 \gamma_2}{m_2 \omega_2^2}}, \]

\[ q_{2s} = \frac{\hbar \lambda q_{1s}}{-m_2 \omega_2^2} \quad a_i = \frac{\epsilon_i}{i \Delta + \kappa}, \]

where, \( \Delta = \Delta_0 - g q_{1s} \) is the effective detuning, and the solution of \( a_+ \),

\[ a_+ = \frac{1}{\kappa + i (\Delta - \delta)} - \frac{\hbar^2 |a_0|^2}{\Lambda \times B}, \]

\[ B = 1 + \frac{\hbar^2 |a_s|^2}{\Lambda[\kappa - i (\Delta + \delta)]}. \]

Figure 6. The two signal modes of slow group velocity index \( n_g \) as a function of the laser power \( \epsilon_l \) when \( \Delta = -\omega_l \). Other parameters used are the same as in figure 2.

\[ \epsilon_y \pm 1 + 2\pi \text{Im} \left[ \chi_{\text{eff}}(\omega_i) \right] L_i + 2\pi \omega_1 \text{Im} \left[ \frac{d \chi_{\text{eff}}(\omega_i)}{d \omega_i} \right] L_i, \]

where, \( \chi_{\text{eff}}(\omega_i) \) is the effective susceptibility and is in direct proportion to \( \epsilon_y \). Noticing that the signal light has changed on the reflection and while \( \text{Im} \left[ \chi_{\text{eff}}(\omega_i) \right] L_i = 0 \), the group velocity index should be written as

\[ n_g \pm 1 = 1 + 2\pi \omega_1 \text{Im} \left[ \frac{d \chi_{\text{eff}}(\omega_i)}{d \omega_i} \right] L_i \propto \text{Im} \left[ \frac{\epsilon_y(\omega_i)}{\omega_i} \right] L_i. \]

We can find from this expression when the dispersion is steeply positive or negative, the group velocity can be significantly reduced or increased. In the following section we will present some numerical results.

3. Numerical results and discussion

For illustration of the numerical results, we choose the realistically reasonable parameters to demonstrate the slow and fast light effect based on the optomechanical system. We employ the parameters from the recent experiment [20] in the observation of the normal-mode splitting.

Figure 2 illustrates the behavior of the absorption \( \text{Re} \left[ \epsilon_y(\omega_i) \right] \) and dispersion \( \text{Im} \left[ \epsilon_y(\omega_i) \right] \) of the signal light as a function of \( \delta/\omega_1 = (\delta - \omega_i)/\omega_1 \). We can find obviously that there are two steep negative slopes related to two minima of zero-absorption in the reflective light. The two minima of the zero-absorption in figure 2 can be evaluated by \( \frac{d \text{Re} \left[ \epsilon_y(\omega_i) \right]}{d \omega_i} \bigg|_{\omega_1 = \omega_i, \omega_2 = 0} = 0 \), where \( \omega_i \) and \( \omega_2 \) are the points of the zero-absorption minima. This large dispersive characteristics can lead to the possibility of implementation of fast light effect. As a general rule, OMIT can produce slow-light in the red sideband for transmission. In contrast, here, we focus on the reflected light from the left of the cavity. There is a \( \pi \) phase difference between the transmission light and the reflection light. So it makes reversed behaviour and then produces the fast-light phenomenon. Furthermore, if we turn off the Coulomb coupling between the charged NR1 and NR2, the two steep negative slopes disappear and only one occurs.
Figure 3 shows the two signal modes of group velocity $n_g$ as a function of the laser power with the parameters the same as in figure 2. It is clear that near $\epsilon_l = 0.01$ mW, the obtainable fast-light index can be as large as 6000 times for the two modes’ light frequencies $\omega_1$ and $\omega_2$ respectively. In other words, the output will be 6000 times faster than the input at two different frequencies. Therefore, in our structure one can obtain the fast output light without absorption by only adjusting the effective detuning of laser field from the bare cavity equal to the frequency of the NR1. The physics of the effects can be explained by the radiation pressure coupling an optical mode to a mechanical mode in an optomechanically induced transparency (OMIT) [19, 21]. The OMIT depends on quantum interference, which is sensitive to the phase disturbance. The Coulomb coupling between the NR1 and NR2 breaks down the symmetry of the OMIT interference, and thus the single OMIT window is split into two OMIT [29, 39].

Figure 4 describes the absorption $\text{Re}[\varepsilon_R]$ and dispersion $\text{Im}[\varepsilon_R]$ of the signal light with different Coulomb coupling strength (compared with figure 2). The increase that in the splitting is corresponding to the Coulomb coupling strength between the NR1 and NR2. That is to say, the frequencies of the two signal lights can be tuned by Coulomb coupling strength.

Similarly, the optomechanical system can also implement the two signal modes’ slow light effect without absorption when the effective detuning of laser field from the bare cavity ($\Delta = -\omega_l$). In order to illustrate it more clearly, we plot figures 5 and 6 with the same experimental data as in figure 2. In figure 5, we also describe the theoretical variation of absorption $\text{Re}[\varepsilon_R]$ and dispersion $\text{Im}[\varepsilon_R]$ of the two signal lights as a function of $\delta/\omega_1 = (\delta - \omega_l)/\omega_1$ when the detuning $\Delta = -\omega_l$. From figure 6 we can find that there are two large dispersions related to two very steep positive slopes. It means that there is the two modes’ slow light effect without absorption. Figure 6 shows the group velocity index $n_g$ of two signal modes’ slow light as a function of laser power.

According to the above discussions, it can be found clearly that the optomechanical system provides us a simple and two signal modes’ switch between slow- and fast-light by simply adjusting the effective laser detuning. In experiments, one can fix the two signal fields with frequency $\omega_1 = \omega_1$, $\omega_2$ and scan the effective laser detuning from $\Delta = -\omega_1$ to $\Delta = +\omega_1$, then one can simply switch the two signal fields from slow to fast with two different frequencies.

4. Conclusions

In conclusion, we have investigated tunable fast- and slow-light effects in two signal modes with the optomechanical system. It can provide us a simple and convenient way to switch between slow- and fast-light in two signal modes. The greatest advantage of our system is that we can simply switch from fast- to slow-light by only adjusting the laser-cavity detuning. Moreover, the frequencies of two signal lights can be tuned by Coulomb coupling strength. Different to the scheme proposed in [25], our proposal works in an optomechanical system, and the main advantage of our proposal is the fact that it can be performed at room temperature. It will be great for potential applications, including integrated quantum optomechanical memory, designing novel quantum information processing gates and optical routers.

Finally, we hope that the results of this letter can be tested by experiments in the near future. Recently, Weis et al [19] and Safavi-Naeini et al [21] have reported experimental results on single signal mode slow light in an optomechanical system, maybe one can use a similar experimental setup to test our predicted effects.

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