Neutrino masses and oscillations

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Abstract.
I summarize the status of three–neutrino oscillations that follow from combining the relevant world’s data. The discussion includes the small parameters $\alpha \equiv \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}}$ and $\sin^2 \theta_{13}$, which characterize the strength of CP violation in neutrino oscillations, the impact of oscillation data on the prospects for probing the absolute scale of neutrino mass in $\beta \beta_0$ and the robustness of the neutrino oscillation interpretation itself in the presence of non-standard physics. I also comment on the theoretical origin of neutrino mass, mentioning recent attemps to explain current oscillation data.

Keywords: neutrino masses, neutrino oscillations
PACS: 26.65.+t, 13.15.+g, 14.60.Pq, 95.55.Vj, 98.80.C, 12.60

INTRODUCTION

The discovery of neutrino oscillations has marked a turning point in our understanding of nature and has brought neutrino physics to the center of attention of the particle, nuclear and astrophysics communities. Here I summarize the determination of neutrino mass and mixing parameters in neutrino oscillation studies following Ref. [1] to which the reader is referred for details on data analysis and experimental references. For future neutrino oscillation projects see Ref. [2]. The structure of the three-flavour lepton mixing matrix in various gauge theories of neutrino mass was given in [3]. Current neutrino oscillation data are well described by its simplest unitary form, with is no sensitivity to CP violation. The effect of Dirac CP phases in oscillations and Majorana phases in $\beta \beta_0$ constitute the main challenge for the future. The interpretation of the data requires good calculations of solar and atmospheric neutrino fluxes [4, 5], neutrino cross sections and experimental response functions, as well the inclusion of matter effects [6, 7] in the Sun and the Earth.

SOLAR AND KAMLAND DATA

The solar neutrino data includes the rates of the chlorine experiment ($2.56 \pm 0.16 \pm 0.16 \text{ SNU}$), the results of the gallium experiments SAGE ($66.9^{+3.9}_{-3.8}^{+3.6}_{-3.2} \text{ SNU}$) and GALLEX/GNO ($69.3 \pm 4.1 \pm 3.6 \text{ SNU}$), as well as the 1496–day Super-K data (44 bins: 8 energy bins, 6 of which are further divided into 7 zenith angle bins). The SNO data include the data from the salt phase in the form of the neutral current (NC), charged current (CC) and elastic scattering (ES) fluxes, the 2002 spectral day/night data (17 energy bins for each day and night period) and the 391–day data. The analysis includes not only the statistical errors, but also systematic uncertainties such as those of the eight solar neutrino fluxes.

KamLAND detects reactor anti-neutrinos at the Kamiokande site by the process $\bar{\nu}_e + p \rightarrow e^+ + n$, where the delayed coincidence of the prompt energy from the positron and a characteristic gamma from the neutron capture allows an efficient reduction of backgrounds. Most of the incident $\bar{\nu}_e$’s come from nuclear plants at distances of $80 – 350 \text{ km}$ from the detector, far enough to probe large mixing angle (LMA) oscillations. To avoid large uncertainties associated with the geo-neutrino flux an energy cut at 2.6 MeV prompt energy is applied for the oscillation analysis.

The first KamLAND data correspond to a 162 ton-year exposure gave 54 anti-neutrino events in the final sample, after all cuts, while $86.8 \pm 5.6$ events are predicted for no oscillations with $0.95 \pm 0.99$ background events, consistent with the no–disappearance hypothesis at less than $0.05\%$ probability. This gave the first evi-
idence for the disappearance of reactor neutrinos before reaching the detector, and thus the first terrestrial confirmation of oscillations with \( \Delta m^2_{\text{sol}} \). Additional KamLAND data with a somewhat larger fiducial volume of the detector were presented at Neutrino 2004, corresponding to a 766.3 ton-year exposure. In total 258 events have been observed, versus 356.2 ± 23.7 reactor neutrino events expected in the case of no disappearance and 7.5 ± 1.3 background events. This leads to a confidence level of 99.995% for \( \bar{\nu}_e \) disappearance. Moreover evidence for spectral distortion consistent with oscillations is obtained.

A very convenient way to bin the latest KamLAND data is in terms of \( 1/E_{\text{pr}} \), rather than the traditional bins of equal size in \( E_{\text{pr}} \). Various systematic errors associated to the neutrino fluxes, backgrounds, reactor fuel composition and individual reactor powers, small matter effects, and improved \( \bar{\nu}_e \) flux parameterization are included [1]. This singles out the LMA solution from the previous “zoo” of alternatives [8]. The stronger evidence for spectral distortion in these data also leads to improved \( \Delta m^2_{\text{sol}} \) determination, substantially reducing the allowed region of oscillation parameters. From this point of view KamLAND has played a key role in the resolution of the solar neutrino problem. Assuming CPT invariance one can directly compare the information obtained from solar neutrino experiments with the KamLAND reactor results.

**ATMOSPHERIC AND K2K DATA**

The first evidence for neutrino oscillations was the zenith angle dependence of the \( \mu \)-like atmospheric neutrino data from the Super-K experiment in 1998, an effect also seen in other atmospheric neutrino experiments. However, though appealing, the original oscillation interpretation was certainly not unique [9]. Today, thanks to the accumulation of upgoing muon data, and the observation of the dip in the \( L/E \) distribution of the atmospheric \( \nu_\mu \) survival probability, the signature for atmospheric neutrino oscillations has become clear. The data include Super-K charged-current atmospheric neutrino events, with the \( e \)-like and \( \mu \)-like data samples of sub- and multi-GeV contained events grouped into 10 zenith-angle bins, with 5 angular bins of stopping muons and 10 through-going bins of up-going muons. We do not use \( \nu_\tau \) appearance, multi-ring \( \mu \) and neutral-current events, since an efficient Monte-Carlo simulation of these data would require further details of the Super-K experiment, in particular of the way the neutral-current signal is extracted from the data. We employ the latest three-dimensional atmospheric neutrino fluxes given in [5].

\( \nu_\mu \) disappearance over a long-baseline probing the same \( \Delta m^2 \) region relevant for atmospheric neutrinos is now available from the KEK to Kamioka (K2K) neutrino oscillation experiment. Neutrinos produced by a 12 GeV proton beam from the KEK proton synchrotron consist of 98% muon neutrinos with a mean energy of 1.3 GeV. The beam is controlled by a near detector 300 m away from the proton target. Comparing these near detector data with the \( \nu_\mu \) content of the beam observed by the Super-K detector at a distance of 250 km gives information on neutrino oscillations.

The data K2K-I sample \( (4.8 \times 10^{19} \text{ protons on target}) \) gave 56 events in Super-K, whereas 80.1\(^{+6.2}_{-5.4} \) were expected for no oscillations. The K2K-II data correspond to \( 4.1 \times 10^{19} \text{ protons on target} \), comparable to the K2K-I sample. Altogether they give 108 events in Super-K, to be compared with 150.9\(^{+11.6}_{-10.0} \) expected for no oscillations. Out of the 108 events 56 are so-called single-ring muon events. This data sample contains mainly muon events from the quasi-elastic scattering \( \nu_\mu + p \rightarrow \mu + n \), and the reconstructed energy is closely related to the true neutrino energy. The K2K collaboration finds that the observed spectrum is consistent with the one expected for no oscillation only at a probability of 0.11%, whereas the best fit oscillation hypothesis spectrum has a probability of 52%.

One finds that the neutrino mass-squared difference inferred from the \( \nu_\mu \) disappearance in K2K agrees with atmospheric neutrino results, providing the first confir-
mation of oscillations with $\Delta m^2_{\text{ATM}}$ with accelerator neutrinos. Unfortunately in the current data sample K2K gives a rather weak constraint on the mixing angle, due to low statistics. However, although the determination of $\sin^2 \theta_{\text{ATM}}$ is completely dominated by atmospheric data, K2K data already start constraining the allowed $\Delta m^2_{\text{ATM}}$ region [1]. In particular, there is a constraint on $\Delta m^2_{\text{ATM}}$ from below, which is important for future long-baseline experiments, since these are drastically affected if $\Delta m^2_{\text{ATM}}$ lies in the lower part of the $3\sigma$ range indicated by current atmospheric data alone.

### THREE-NEUTRINO OSCILLATIONS

The first systematic study of the effective lepton mixing matrix in gauge theories of massive neutrinos was given in [3]. For some models this matrix can be taken as approximately unitary. For three neutrinos, this gives

$$K = \omega_{23} \omega_{13} \omega_{12}$$

where each factor is effectively $2 \times 2$ and contains an angle and a CP phase. Two of the three angles are involved in solar and atmospheric oscillations, so we set $\theta_{12} \equiv \theta_{\text{sol}}$ and $\theta_{23} \equiv \theta_{\text{ATM}}$. The last angle in the three-neutrino leptonic mixing matrix is $\theta_{13}$,

$$\omega_{13} = \begin{pmatrix} c_{13} & 0 & e^{-i \phi_{13}} s_{13} \\ 0 & 1 & 0 \\ -e^{-i \phi_{13}} s_{13} & 0 & c_{13} \end{pmatrix}.$$ for which only an upper bound currently exists. All three phases are physical [10], one corresponds to the one present in the quark sector (Dirac-phase) and affects neutrino oscillations, while the other two are associated to the Majorana nature of neutrinos and show up in neutrinoless double beta decay and other lepton-number violating processes, but not in conventional neutrino oscillations [10, 11].

Current neutrino oscillation experiments are insensitive to CP violation, thus we neglect all phases. In this approximation three-neutrino oscillations depend on the three mixing parameters $\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}$ and on the two mass-squared differences $\Delta m^2_{\text{SOL}} \equiv \Delta m^2_{12} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{\text{ATM}} \equiv \Delta m^2_{23} \equiv m^2_3 - m^2_2$ characterizing solar and atmospheric neutrinos. The hierarchy $\Delta m^2_{\text{SOL}} \ll \Delta m^2_{\text{ATM}}$ implies that one can set, to a good approximation, $\Delta m^2_{\text{SOL}} = 0$ in the analysis of atmospheric and K2K data, and $\Delta m^2_{\text{ATM}}$ to infinity in the analysis of solar and KamLAND data. Apart from the data already mentioned, the global oscillation analysis also includes the constraints from the CHOOZ and Palo Verde reactor experiments.

The results of the global three-neutrino analysis are summarized in Fig. 1 and in Tab. 1, taken from Ref. [1]. In the upper panels of the figure the $\Delta \chi^2$ is shown as a function of the parameters $\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}, \Delta m^2_{21}, \Delta m^2_{13}$, minimized with respect to the undisplayed parameters. The lower panels show two-dimensional projections of the allowed regions in the five-dimensional parameter space. The best fit values and the allowed $3\sigma$ ranges of the oscillation parameters from the global data are summarized in Tab. 1. This table gives the current status of neutrino oscillation parameters.

As it has long been noted, in a three-neutrino scheme CP violation disappears when two neutrinos become degenerate [3] or when one angle vanishes, such as $\theta_{13}$ [12]. All genuine three–flavour effects involve the mass hierarchy parameter $\alpha \equiv \frac{\Delta m^2_{\text{SOL}}}{\Delta m^2_{\text{ATM}}}$ and the mixing angle $\theta_{13}$.
The left panel in Fig. 2 gives the parameter \( \alpha \), namely the ratio of solar over atmospheric splittings, as determined from the global \( \chi^2 \) analysis of [1].

The right panel in Fig. 2 gives \( \Delta \chi^2 \) as a function of \( \sin^2 \theta_{13} \) for different data samples. One finds that the KamLAND-2004 data have a surprisingly strong impact on this bound. Before KamLAND-2004 the bound on \( \sin^2 \theta_{13} \) from global data was dominated by the CHOOZ reactor experiment, together with the determination of \( \Delta m^2_{13} \) from atmospheric data. However, including KamLAND-2004 the bound becomes comparable to the reactor bound. Note also that, since the reactor bound on \( \sin^2 \theta_{13} \) deteriorates quickly as \( \Delta m^2_{13} \) decreases (see Fig. 3), the improvement is especially important for lower \( \Delta m^2_{13} \) values. In Fig. 3 we show the upper bound on \( \sin^2 \theta_{13} \) as a function of \( \Delta m^2_{13} \) from CHOOZ data alone compared to the bound from an analysis including solar and reactor neutrino data. One sees that, although for larger \( \Delta m^2_{13} \) values the bound on \( \sin^2 \theta_{13} \) is dominated by CHOOZ, for \( \Delta m^2_{13} \lesssim 2 \times 10^{-3} \text{eV}^2 \) the solar and KamLAND data become relevant.

Altogether, the bound on \( \sin^2 \theta_{13} \) contributes significantly to the overall global bound 0.047 at 3\( \sigma \) for 1 d.o.f. As shown in [1] such an improved \( \sin^2 \theta_{13} \) bound follows mainly from the strong spectral distortion found in the 2004 sample.

Future long baseline reactor and accelerator neutrino oscillation searches [13], as well as studies of the day/night effect in large water Cerenkov solar neutrino experiments such as UNO or Hyper-K [14] could bring more information on \( \sin^2 \theta_{13} \) [15]. With neutrino physics entering the precision age it is necessary to scrutinize also the validity of the unitary approximation of the lepton mixing matrix in future experiments, given its theoretical fragility [3].

### ABSOLUTE NEUTRINO MASS SCALE

On general grounds neutrino masses are expected to be Majorana [3], a fact that may explain their relative smallness with respect to other fermion masses. Neutrino oscillation data are insensitive to the absolute scale of neutrino masses and also to the fundamental issue of whether neutrinos are Dirac or Majorana particles [10, 11]. Hence the importance of neutrinoless double beta decay [16]. The significance of the \( \beta \beta_{0v} \) decay is given...
by the fact that, in a gauge theory, irrespective of the mechanism that induces $\beta \beta_{0\nu}$, it is bound to also yield a Majorana neutrino mass [17], as illustrated in Fig. 4. Quantitative implications of the “black-box” argument are model-dependent, but the theorem itself holds in any “natural” gauge theory.

Now that oscillations are experimentally confirmed we know that $\beta \beta_{0\nu}$ must be induced by the exchange of light Majorana neutrinos. The corresponding amplitude is sensitive both to the absolute scale of neutrino mass as well as the two Majorana CP phases that characterize the minimal 3-neutrino mixing matrix [3]. Fig. 5 shows the estimated average mass parameter characterizing the neutrino exchange contribution to $\beta \beta_{0\nu}$ versus the lightest neutrino mass. The calculation takes into account the current neutrino oscillation parameters from [1] and the nuclear matrix elements of [18] and compares with experimental sensitivities. The upper (lower) panel corresponds to the cases of normal (inverted) neutrino mass spectra. In these plots the “diagonals” correspond to the case of quasi-degenerate neutrinos [19, 20], which give the largest $\beta \beta_{0\nu}$ amplitude. In contrast to the normal hierarchy, where a destructive interference of neutrino amplitudes is possible, the inverted neutrino mass hierarchy implies a “lower” bound for the $\beta \beta_{0\nu}$ amplitude. An exception to the rule that there is no lower bound on $\beta \beta_{0\nu}$ in normal hierarchy models is provided by the model in [21].

Future experiments [22] will extend the sensitivity and provide an independent check of the Heidelberg-Moscow claim [23]. More information on the absolute scale of neutrino mass will also come from future beta decays searches [24] and cosmology [25].

**FIGURE 4.** $\beta \beta_{0\nu}$ and Majorana mass are equivalent [17].

**FIGURE 5.** $\beta \beta_{0\nu}$ amplitude and current oscillation data.

**FIGURE 6.** The seesaw mechanism.

**THE ORIGIN OF NEUTRINO MASS**

Neutrino mass arise from the dimension-five operator $\ell \ell \phi \phi$ where $\phi$ the $SU(2) \otimes U(1)$ Higgs doublet and $\ell$ is a lepton doublet [26]. Nothing is known from first principles about the mechanism that induces this operator, its associated mass scale or flavour structure. The neutrino masses that result from it once the electroweak symmetry breaks down are expected to be Majorana. This may explain why neutrino masses are much smaller than those of the other fermions. This may happen either because the operator is suppressed by a large scale in the denominator, or else suppressed by a small scale in the numerator. Both ways are viable and can be made natural.

The most popular case is the seesaw mechanism which induces small neutrino masses from the exchange of heavy states that may come from unification. Small neutrino masses are induced either by heavy $SU(2) \otimes U(1)$ singlet “right-handed” neutrino exchange (type I) or heavy scalar bosons exchange (type II), in a nomenclature opposite from the original one in [3]. The effective triplet seesaw term has a flavor structure different from
the type-I term, contributing to the lack of predictivity of seesaw schemes. An attempt to recover predictivity within the seesaw approach by appealing to extra symmetries [27] is given in [20]. The model predicts maximal $\theta_{23}$, $\theta_{13} = 0$, and naturally large $\theta_{12}$, though unpredicted. Moreover, if CP is violated $\theta_{13}$ becomes arbitrary but the Dirac CP violation phase is maximal [28]. The model gives a lower bound on the absolute neutrino mass $m_\nu \gtrsim 0.3$ eV, requires a light slepton below 200 GeV, and gives large rates for flavour violating processes.

Amongst “bottom-up” models we mention those where neutrino masses are given as radiative corrections [29, 30] and models where low energy supersymmetry is the origin of neutrino mass [31]. The latter are based on the idea that R parity spontaneously break [32], leading to a very simple effective bilinear R parity violation model [33]. In this case the neutrino spectrum is typically hierarchical, with the atmospheric scale generated at the tree level and the solar scale radiatively “calculable” [34]. For the parameters that reproduce the masses indicated by current data, typically the lightest supersymmetric particle decays in the detector, and its decay properties correlate with neutrino mixing angles, a test that can be made, e. g. at the LHC.

**ROBUSTNESS OF OSCILLATIONS**

The general effective model-independent description of the seesaw at low-energies was given in [3]. It is characterized by $(n,m)$, $n$ the number of $SU(2) \otimes U(1)$ isodoublet and $m$ the number of $SU(2) \otimes U(1)$ isosinglet leptons. In the mass basis the $(3,3)$ seesaw model has 12 mixing angles and 12 CP phases (both Dirac and Majorana-type) characterizing its full $3 \times 6$ charged current seesaw lepton mixing matrix and non–diagonal neutral current [3]. The nontrivial structure of charged and neutral current weak interactions with non-unitary lepton mixing matrix is a general feature of seesaw models [3] and lead to dimension-6 terms non-standard neutrino interactions (NSI), as illustrated in Fig. 7. Such sub-weak strength $\varepsilon G_F$ operators can be of two types: flavour-changing (FC) and non-universal (NU). In inverse seesaw-type models [35] the non-unitary piece of the lepton mixing matrix can be sizable and hence the induced NSI may be phenomenologically important [36]. Sizable NSI strengths may also arise in radiative neutrino mass models and in supersymmetric unified models [37].

Non-standard physics may in principle affect neutrino propagation properties and detection cross sections [38]. In their presence, the Hamiltonian describing atmospheric neutrino propagation has, in addition to the standard oscillation part, another term $H_{SSI}$

$$H_{SSI} = \pm \sqrt{2} G_F N_f \begin{pmatrix} 0 & \varepsilon \\ \varepsilon & \varepsilon' \end{pmatrix}.$$  

(2)

Here $+(-)$ holds for neutrinos (anti-neutrinos) and $\varepsilon$ and $\varepsilon'$ parameterize the NSI: $\sqrt{2} G_F N_f \varepsilon$ is the forward scattering amplitude for the FC process $\nu_\mu + f \rightarrow \nu_\tau + f$ and $\sqrt{2} G_F N_f \varepsilon'$ represents the difference between $\nu_\mu + f$ and $\nu_\tau + f$ elastic forward scattering. Here $N_f$ is the number density of the fermion $f$ along the neutrino path.

In the 2–neutrino approximation, the determination of atmospheric neutrino parameters $\Delta m^2_{\text{ATM}}$ and $\sin^2 \theta_{\text{ATM}}$ was shown to be practically unaffected by the presence of NSI on down-type quarks ($f = d$) [39]. Future neutrino factories will substantially improve this bound [40].

In contrast, the oscillation interpretation of solar neutrino data is “fragile” in the presence of non-standard interactions [41], with a new “dark side” solution (with $\sin^2 \theta_{\odot} \simeq 0.7$ [41]), essentially degenerate with the conventional one, present even after combining with data from reactors. On the other hand, it has been shown [42] that, even a small residual non-standard interaction of neutrinos in the $e - \tau$ channel leads to a drastic loss in sensitivity in the $\theta_{13}$ determination at a neutrino factory.

![Flavour-changing effective operator](image-url)
It is therefore important to improve the sensitivities on NSI, another window of opportunity for neutrino physics in the precision age.

Work supported by Spanish grant BFM2002-00345 and by the EC RTN network MRTN-CT-2004-503369.

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