Non-Thermal Radio and Gamma-Ray Emissions from a Supernova Remnant by Blast Wave Breaking Out of the Circumstellar Matter

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Abstract

We calculated synchrotron radio emission and $\gamma$-ray emission due to bremsstrahlung, inverse-Compton scattering, and $\pi^0$-decay from the remnant of supernova that exploded in the circumstellar matter (CSM) formed by the progenitor’s stellar wind. This sort of situation is a possible origin of mixed-morphology supernova remnants (SNRs), like W 49B, which exhibit recombination-radiation spectra in X-ray emission. Fermi $\gamma$-rays from SNRs: while the luminosities in $\gamma$-rays come from the dense matter around MM SNRs. Because of low density, the $\gamma$-ray luminosity is dominated by inverse-Compton scattering, which is higher than the $\pi^0$-decay luminosity by an order of magnitude. The total $\gamma$-ray luminosity, including bremsstrahlung, is on the order of $10^{33}$ erg s$^{-1}$ lower than the typical value of $10^{35}–10^{36}$ erg s$^{-1}$ observed from mixed-morphology SNRs. However, if, e.g., $\sim 10\%$ of accelerated protons interact with some matter of density of $\sim 100$ cm$^{-3}$, the $\pi^0$-decay $\gamma$-ray luminosity would be enhanced to be comparable with the observed value.

Key words: ISM: supernova remnants — stars: circumstellar matter

1. Introduction

About 70% of galactic supernova remnants (SNRs) exhibit a shell-like morphology in radio wavelength (Green 2009). The radio shells are due to synchrotron emission by non-thermal electrons on the order of GeV. Many of these SNRs also exhibit a shell-like morphology in X-rays, due to bremsstrahlung and line emission by thermal electrons on the order of keV. The electrons are heated and/or accelerated by interstellar shocks of SNRs. On the other hand, $\sim 10\%$ of the SNRs that have radio shells exhibit center-filled thermal X-rays, and are called mixed-morphology (MM) SNRs (e.g., Rho & Petre 1998). Recently, Suzaku revealed recombination radiation, which is hardly expected from typical shell-like SNRs, in X-ray spectra of six MM SNRs so far observed: IC 443 (Yamaguchi et al. 2009), Westernhout 49B (Ozawa et al. 2009), G359.1−0.5 (Ohashi et al. 2011), W 28 (Sawada & Koyama 2012), W 44 (Uchida et al. 2012), and G346.6−0.2 (Yamauchi et al. 2013).

MM SNRs are also characteristic in $\gamma$-ray emission. Fermi detected GeV $\gamma$-rays from SNRs: while the luminosities in the 1–100 GeV band of shell-like SNRs are $10^{33}–10^{35}$ erg s$^{-1}$ (Abdo et al. 2010b; Castro & Slane 2010; Katagiri et al. 2011; Abdo et al. 2011; Giordano et al. 2012; Tanaka et al. 2011), the luminosities of MM SNRs are distinctively higher, $10^{35}–10^{36}$ erg s$^{-1}$ (Abdo et al. 2009, 2010a, 2010c, 2010d, 2010e; Castro & Slane 2010). This may imply that such intense $\gamma$-rays come from the dense matter around MM SNRs. Actually, for these MM SNRs, interactions with molecular clouds are suggested by 1720 MHz OH maser (Frail et al. 1994; Yusef-Zadeh et al. 1995; Frail et al. 1996; Green et al. 1997; Claussen et al. 1997; Hewitt & Yusef-Zadeh 2009) and/or near-infrared observations (Keohane et al. 2007). The $\gamma$-ray spectra of observed SNRs are not always single power-laws, but exhibit a break at an energy of $\sim 1–5$ GeV, above which their slopes are steepened.

X-ray characteristics different from that of shell-like SNRs and possible interactions with molecular clouds may suggest that MM SNRs are remnants of core-collapse explosions of massive stars surrounded by H II regions, stellar-wind matter, and molecular clouds, as in star-forming regions. Recombination radiation in X-rays has been predicted by Itoh and Masai (1989) for the remnant of a supernova surrounded by its progenitor’s stellar wind matter. They show the X-ray spectra due to rarefaction (adiabatic cooling) caused when the blast wave breaks out of the wind matter to expand rapidly into the ambient interstellar medium. Recently, Shimizu, Masai, and Koyama (2012, hereafter Paper I), extended this work to non-spherically-symmetric stellar wind matter, and found that recombination X-rays exhibit a center-filled morphology like MM SNRs with various shapes, depending on the viewing...
direction. They also suggest that a synchrotron radio shell is located outside, surrounding the X-ray emitting region.

If rarefaction caused by the shock break-out is the origin of the recombination X-rays found in MM SNRs, it is naturally of our interest whether the SNR model can explain observed radio and γ-ray emissions as well. Hence, in the present paper, we consider non-thermal particles, which can be accelerated by the shock of the SNR model discussed in Paper I, and emission thereby from the blast-shocked ISM shell. In the following section, we describe the SNR model presented in Paper I and the particle acceleration. In section 3 we provide calculations of non-thermal radio flux and γ-ray luminosity, and discuss the results in section 4.

2. Model

2.1. Supernova Remnant

We consider model B2 of Paper I for a model of MM SNR. This model describes evolution during which the initially spherically symmetric ejecta interact with anisotropic circumstellar matter (CSM). Outside the CSM, a uniform interstellar medium (ISM) of density $0.016 \, \text{cm}^{-3}$ is assumed. Such a low density can be possible for an H II region formed by the progenitor. The ejecta have an initial kinetic energy of $2 \times 10^{51} \, \text{erg}$ and a mass of $10 \, \text{M}_\odot$, and therefore the initial velocity of the blast wave is $8.5 \times 10^3 \, \text{km} \, \text{s}^{-1}$. For a comparison, we also calculated the evolution of SN without CSM, which expands directly into the uniform ISM of the same density.

The CSM is composed of stellar wind matter blown by the progenitor in its pre-supernova phase. If the progenitor rotated rapidly, the stellar wind may have an anisotropic density distribution. We assume that the CSM is concentrated in the equatorial plane. The density on the equatorial plane is 4-times higher than that in the polar direction at the same radius. The mass-loss rate is $5 \times 10^{-5} \, \text{M}_\odot \, \text{yr}^{-1}$ at a wind velocity of $100 \, \text{km} \, \text{s}^{-1}$. The inner and outer radii of the CSM are $0.07 \, \text{pc}$ and $3 \, \text{pc}$ in the equatorial direction. These radii imply that the wind activity lasts $3 \times 10^4 \, \text{years}$, and then ceases $6 \times 10^7 \, \text{years}$ before the explosion. The mass of the CSM is $1.5 \, \text{M}_\odot$, which was obtained from the period of the wind activity and the mass-loss rate.

The wind parameters are possible for B[e] supergiants or luminous blue variables (LBVs). For example, radio observations suggest that W9, which is a B[e] supergiant in Westerlund 1, has a wind velocity of $\sim 10^3 \, \text{km} \, \text{s}^{-1}$ and a mass-loss rate of $\sim 10^{-4} \, \text{M}_\odot \, \text{yr}^{-1}$ (Dougherty et al. 2010). LBVs typically have a wind velocity of $\sim 10^2 \, \text{km} \, \text{s}^{-1}$ and a mass-loss rate of $\sim 10^{-5} - 10^{-4} \, \text{M}_\odot \, \text{yr}^{-1}$ (Humphreys & Davidson 1994). Although LBVs were considered not to explode as supernova, recent observations show that a progenitor of SN 2005gl is an LBV (Gal-Yam et al. 2007). Also, observations of type IIb supernovae suggest interactions between ejecta and dense CSM, which have a wind velocity of $\sim 10^2 - 10^3 \, \text{km} \, \text{s}^{-1}$, and a mass-loss rate of $\sim 10^{-5} - 1 \, \text{M}_\odot \, \text{yr}^{-1}$ (e.g., Kiewo et al. 2012).

The blast wave breaks out of the CSM at $\sim 450 \, \text{yr}$ in the equatorial direction, and then is rapidly accelerated. A rarefaction wave propagates inward from the CSM–ISM contact interface. This causes a rapid adiabatic expansion, and thus cooling of the once-shocked CSM and ejecta, which result in recombination-radiation X-rays. Since the ISM is rarefied enough to make a density difference of factor $\sim 10$ at the CSM–ISM interface, rarefaction occurs for a mass-loss rate of $5 \times 10^{-5} \, \text{M}_\odot \, \text{yr}^{-1}$, assumed here (cf. Moriya 2012). After that, the blast wave propagates to form a shocked shell into the ISM, while the second reverse-shock propagates inward.

2.2. Particle Acceleration

We consider diffusive shock acceleration by the blast wave: part of the thermal particles of the shocked ISM are scattered across the shock by magnetic inhomogeneities, and gain momentum. The energy spectrum of accelerated particles is expressed in the form (Bell 1978)

$$N(E) = K(E + mc^2)(E^2 + 2mc^2E)^{-(\mu+1)/2}$$

with

$$K = \xi(n)(\mu - 1)(E_{\text{inj}}^2 + 2mc^2E_{\text{inj}})^{(\mu - 1)/2}$$

for $E_{\text{inj}} \leq E \leq E_{\text{max}}$. Here, $E = (\gamma - 1)mc^2$ is the kinetic energy of a particle of mass $m$ with the Lorentz factor, $\gamma$. 

![Fig. 1. Contours of the number density of shocked matter averaged over the line of sight in the equatorial plane of model B2 at 700 yr. Black lines represent those for reverse-shocked ejecta. Gray lines represent those for the shocked ISM. "RR X-rays" means recombination-radiation X-rays.](https://academic.oup.com/pasj/article-abstract/65/3/69/1527187)
E_{\text{inj}} is the injection kinetic energy, E_{\text{max}} is the maximum kinetic energy of accelerated particles, and \( \xi \) is the injection efficiency. The injection efficiency is defined as the ratio of the number density of accelerated to thermal particles.

The diffusive shock acceleration results in a single power-law energy spectrum, as described above. On the other hand, \( \gamma \)-ray observations of SNRs suggest that the energy spectrum of particles is not simply a single power-law, but, for instance, Fermi-observed SNRs show a break at an energy of \( \sim 1\text{–}5\text{ GeV} \), as mentioned in section 1. Since the cooling time at the break energy is much longer than the age, this break may reflect acceleration processes. Therefore, for the energy spectrum of accelerated particles we assume a broken power-law,

\[
N(E) = \begin{cases} 
K E^{-\mu}, & \text{for } E < E_b, \\
K E_b^{\mu - \mu_2} E^{-\mu_2}, & \text{for } E \geq E_b,
\end{cases}
\]  

(3)

where \( E_b \) is a break energy of 10 GeV, taken so as to make a GeV break in the \( \gamma \)-ray spectrum. The spectral indexes are assumed to be \( \mu = 2 \) and \( \mu_2 = 2.3 \) which is a medium value of the spectral index of cosmic-ray sources (e.g., Putze et al. 2011).

For the injection energy, \( E_{\text{inj}} \), and efficiency, \( \xi \), we consider that particles in the high-energy tail of the thermal distribution are injected into the acceleration process, as

\[
E_{\text{inj}}-p,e = \lambda_{p,e} kT_{p,e},
\]  

(4)

with a constant \( \lambda \), where the characters “p” and “e” mean proton and electron, respectively. Then, the relation between \( \xi \) and \( \lambda \) is given by

\[
\xi_{p,e} = \frac{\int_{E_{\text{inj}}-p,e}^{\infty} f_{p,e}(E) dE}{\int_0^{\infty} f_{p,e}(E) dE} = 1 - \text{erf}(\lambda_{p,e}^{1/2}) + \frac{2}{\pi^{1/2}} \lambda_{p,e}^{1/2} e^{-\lambda_{p,e}},
\]  

(5)

where \( f(E) \) is the Maxwell distribution function, and \( \text{erf} \) is the error function.

We determined \( \xi_p \) for the pressure of accelerated protons to be equal to 10% of the ram pressure of ISM enters in the blast wave. The pressure of accelerated particles is given by

\[
P_{\text{CR}} = \frac{1}{3} \int_{p_{\text{inj}}}^{p_{\text{max}}} N'(p) p v dp
\]  

\[
\simeq \frac{1}{3} \xi (n_e) c p_{\text{inj}} \left[ \ln \left( \frac{2 p_b}{m c} \right) + \frac{1}{\mu_2 - 2} \left( \frac{p_b c}{E_b} \right)^{-\mu_2 + 2} \right].
\]  

(6)

where we use \( \mu = 2 \) in the last expression. Here, \( v \) is the particle velocity, \( p \) is the momentum of a particle, \( p_{\text{inj}} = (2m E_{\text{inj}})^{1/2} \) is the injection momentum, \( p_{\text{max}} = E_{\text{max}} / c \) is the maximum momentum, \( p_b \) is the break momentum, and \( N'(p) dp = N(E) dE \). In the last expression in equation (6), \( p_{\text{inj}} \ll mc \) and \( p_{\text{max}} \gg p_b > mc \) are considered. The injection efficiency of protons is roughly proportional to the blast-wave velocity, \( V_s \), because \( \xi_p \propto V_s^2 / p_{\text{inj}} \propto V_s^2 / T_s^{1/2} \propto V_s \).

The injection efficiency of protons reaches a maximum value of \( \sim 2 \times 10^{-4} \) at \( \sim 530 \text{ yr} \), and then decreases to \( 5 \times 10^{-5} \) at \( \sim 10000 \text{ yr} \). We determine \( \xi_e \) for the pressure \( P_{\text{CR}-e} \) of accelerated electrons not to exceed the pressure \( P_{\text{CR}-p} \) of accelerated protons. The ratio of the pressure of accelerated electrons to protons is

\[
\frac{P_{\text{CR}-e}}{P_{\text{CR}-p}} \simeq 0.05 \frac{\xi_e}{\xi_p} \left( \frac{E_{\text{inj}-e}}{E_{\text{inj}-p}} \right)^{1/2}.
\]  

(7)

If the injection energy of electrons is the same as that of protons, \( \xi_e \lesssim 20 \xi_p \) follows. In the following, we express \( \xi_e \) in units of \( \xi_p \).

The maximum energy, \( E_{\text{max}} \), is determined by the time-scales of the energy gain and loss. Adiabatic loss due to SNR expansion is negligible through the age concerned here. The dominant loss process is synchrotron radiation and inverse-Compton scattering for electrons. Assuming that (1) the mean free path of a particle is its gyration radius (Bohm limit), (2) the shock compression ratio is 4, and (3) accelerated particles are relativistic (\( \gamma \gg 1 \)), we estimate the time-scales of acceleration and radiation loss as

\[
\tau_{\text{acc}} \simeq \frac{32 y m e c^3}{3 e B V_s^2},
\]  

(8)

and

\[
\tau_{\text{loss(electron)}} \simeq \frac{6 \pi m_e c}{\gamma \sigma_T (B_s^2 + 8\pi U_{\text{CMB}})},
\]  

(9)
respective, where $\sigma_T$ is the Thomson scattering cross section, $e$ is the elementary electric charge, $B$ is the strength of the magnetic field in the shock downstream (lower), as functions of the elapsed time after the explosion. Gray line represents the maximum energy of the protons in the SNR evolution without CSM.

In figure 3 we show the time evolution of $E_{\text{max}}$ and $B$. For protons, $E_{\text{max}}$ is determined by $t_{\text{acc}} \sim t_{\text{age}}$, and reaches $\sim 1300$ TeV at $\sim 700$ yr, while $\sim 800$ TeV at $\sim 430$ yr in the case without CSM. For electrons, $E_{\text{max}}$ is determined by $t_{\text{acc}} \sim t_{\text{loss}}(\text{electron})$, and its maximum is about $10$ TeV at the moment of the break-out. At $\sim 700$ yr, just after the break-out, $E_{\text{max}}$ takes its maximum/minimum for protons/electrons because of a rapid increase of the shock velocity and the magnetic field. For the explosion energy of $2 \times 10^{51}$ erg assumed, the total energy of accelerated protons is $1 \times 10^{49}$ erg at $\sim 700$ yr and $\sim 2 \times 10^{50}$ erg at $\sim 10000$ yr.

3. Non-Thermal Radiation

We calculated the non-thermal emission from the blast-shocked ISM shell, in which accelerated particles are confined for the shorter time of $t_{\text{age}}$ or $R_b/D$. Here, $D$ is the diffusion coefficient, and is taken to be in the form of $D_{10}(E/10^9\text{ GeV})(B/10^8\mu \text{G})^{-3}$ cm$^2$ s$^{-1}$ with a numerical coefficient, $D_{10}$. Observations of cosmic-rays suggest that $D \sim 10^{28}$ cm$^2$ s$^{-1}$ at 10 GeV (Berezinskii et al. 1990). On the other hand, near SNRs, GeV and TeV observations suggest $D \sim 10^{29}$ cm$^2$ s$^{-1}$ at 10 GeV (e.g., Li & Chen 2012). We adopt $D_{10} = 3 \times 10^{27}$ so that $D \sim 10^{28}$ cm$^2$ s$^{-1}$ for $B = 3 \mu$G, a typical field in interstellar space, and $D \sim 10^{29}$ cm$^2$ s$^{-1}$ for $B \sim 100 \mu$G, which could be attained for SNRs.

3.1. Synchrotron Radio

Synchrotron radiation at a frequency of $v = 1$ GHz is mainly emitted by electrons with energies of $\sim 2(\nu/1 \text{ GHz})^{1/2}(B/100 \mu \text{G})^{-1/2}$ GeV. According to Ginzburg and Syrovatskii (1965), a flux $F_{\text{syn}}(v)$ of synchrotron radiation emitted from electrons with the broken power-law spectrum is

$$F_{\text{syn}}(v) = \frac{1}{4\pi d^2} \int \frac{\sqrt{3eB}}{2m_e c^2} K_e \left( \frac{16m_e^3 e^5 v}{3e} \right)^{-(\mu - 1)/2}$$

$$\times \left[ \int_{x_i}^{x_f} F(x)x^{(\mu - 3)/2} dx \right]^{1/2}$$

$$+ E_{b}^{\mu - \mu_2} \left( \frac{16m_e^3 e^5}{3e B} \right)^{(\mu - \mu_2)/2} \int_{x_i}^{x_f} F(x)x^{(\mu_2 - 3)/2} dx \right]$$

$$\times 4\pi r^2 dr,$$

where

$$x = \frac{16m_e^3 e^5 v}{3e B E^2}.$$  

Here, $K_e$ is $K$ of electrons [see equation (3)], $d$ is the distance to the SNR, $r$ is the radius from the center of the SNR, and $F(x)$ is the synchrotron function. The characters “$\text{inj}$,” “$b$,” and “$\text{max}$” correspond to the injection, break, and maximum energy, respectively. The integration interval of $r$ is given by the shocked ISM shell defined in the beginning of section 3. Figure 4 shows the time evolution of the 1 GHz flux for the magnetic field in figure 3. Since the blast wave is only slightly decelerated, the radio flux continues to increase through $\sim 10000$ yr (see section 4).

3.2. Bremsstrahlung $\gamma$-Rays

Relativistic electrons emit bremsstrahlung $\gamma$-rays by interacting with target protons. The number of the bremsstrahlung photons emitted from electrons with the broken power-law spectrum per unit time per unit energy per unit volume is (Blumenthal & Gould 1970)

$$\frac{dN_\gamma}{dtdhvdV}$$

$$= \frac{4e^2 c^2 n_T}{\hbar v} \left[ \ln \left( \frac{4hv}{m_e c^2} \right) - \frac{1}{2} \right]$$

$$\times \int_{hv}^{E_{\text{max}}} dE N_e E^{-2} \left( \frac{4}{3} E^2 - \frac{4}{3} E hv + (hv)^2 \right)$$
per unit volume is (Blumenthal & Gould 1970)

\[
\frac{dN_y}{dt d\Omega dE_y} = 8\pi^2 \frac{E_{\text{CM}}^{\mu+1}}{h^2 c^2 (m_e c^2)^{\mu-1}} \left( \frac{m_e c^2}{E_{\text{CM}}} \right)^{\mu+1} \left( kT_{\text{CMB}} \right)^{\mu+1/2} \Gamma \left( \frac{\mu+1}{2} \right) \left( \frac{\mu+5}{2} \right) \zeta \left( \frac{\mu+5}{2} \right) \\
\times 3 \frac{\left( \frac{E_{\text{CM}}^{\mu+1}}{E_{\text{CM}}} \right)^{\mu/2}}{\left( \frac{E_{\text{CM}}^{\mu+1}}{E_{\text{CM}}} \right)^{\mu/2}} \left( \frac{m_e c^2}{E_{\text{CM}}} \right)^{\mu+1/2},
\]

where \( \zeta \) is the zeta function and \( E_{\text{CM}} \) is the maximum energy of accelerated electrons. Using equation (14), we obtain an inverse-Compton \( \gamma \)-ray luminosity by

\[
L_{\text{IC}} = \int \int \frac{dN_y}{dt d\Omega dE_y} 4\pi r^2 dr dhv.
\]

The integration interval of \( r \) is given by the shocked ISM shell defined in the beginning of section 3. Figure 4 shows the time evolution of the \( \gamma \)-ray luminosity due to inverse-Compton scattering in the 1–100 GeV band.

### 3.4. \( \pi^0 \)-Decay \( \gamma \)-Rays

Relativistic protons emit neutral \( \pi^0 \)s through inelastic collisions with protons, and then the \( \pi^0 \)s decay into two \( \gamma \)-ray photons. We now calculate the \( \pi^0 \)-decay \( \gamma \)-ray luminosity, using the parameterized cross section of an inelastic proton–proton collision,

\[
\sigma_{\text{inel}}(E_p) \simeq 3 \left[ 0.95 + 0.06 \ln \left( \frac{E_p}{1 \text{ GeV}} \right) \right] \times 10^{-26} \text{ cm}^2,
\]

and \( \delta \)-function approximation of number of \( \pi^0 \)s emitted per unit time per unit energy per unit volume.
$$\frac{dN_\pi}{dt d\bar{E}_\pi dV} = \frac{cnT}{f_{30}} \sigma_{\text{inel}} \left( \frac{m_pc^2 + \bar{E}_\pi}{f_{30}} \right) N_p \left( \frac{m_pc^2 + \bar{E}_\pi}{f_{30}} \right),$$

(17)

which are used in Aharonian and Atoyan (2000). Here, $f_{30} \simeq 0.17$ is the mean fraction of the kinetic energy of a proton transferred to a $\pi^0$ per collision, $\bar{E}_p = \gamma m_p c^2$ is the total energy of a proton, and $\bar{E}_\pi = \gamma m_\pi c^2$ is the total energy of $\pi^0$. The number of $\pi^0$-decay photons emitted from protons with the broken power-law spectrum per unit time per unit energy per unit volume is

$$\frac{dN_\pi}{dt dhvdV} \approx 2 \int_{E_{\text{min}}}^{\infty} \frac{1}{\bar{E}_\pi} \frac{dN_\pi}{dt d\bar{E}_\pi dV} d\bar{E}_\pi \approx 2 \int_{h_0}^{\infty} \frac{1}{\bar{E}_\pi} \frac{dN_\pi}{dt d\bar{E}_\pi dV} d\bar{E}_\pi \approx 3 \times 10^{-26} \frac{2cnT K_p}{f_{30}} \times \left\{ \begin{array}{l} \frac{1}{\mu} \left( \frac{h}{f_{30}} \right)^{-\mu} \left( 0.95 + 0.06 \ln \left( \frac{h}{f_{30}} \right) + \frac{1}{\mu} \right) \\
+ E_b^{-\mu} \left( \frac{1}{\mu_2} - \frac{1}{\mu} \right) + 0.06 \left( \frac{1}{\mu_2} - \frac{1}{\mu} \right)^2 \\
+ 0.06 \left( \frac{1}{\mu_2} - \frac{1}{\mu} \right) \ln \left( \frac{E_b}{1\text{ GeV}} \right) \right\}, \\
\text{for } h \leq f_{30} E_b, \\
E_b^{-\mu+\mu_2} \left( \frac{h}{f_{30}} \right)^{-\mu_1} \left( 0.95 + 0.06 \ln \left( \frac{h}{f_{30}} \right) + \frac{1}{\mu_2} \right), \\
\text{for } h > f_{30} E_b, \end{array} \right.$$

(18)

where $E_{\text{min}} = h + m_\pi^2 c^4/(4h)$ is the minimum pion energy needed to produce a photon of energy $h$, and $K_p$ is the $K$ of protons [see equation (3)]. In the second expression of equation (18), $h > m_\pi^2 c^2$ and $\bar{E}_\pi > m_\pi c^2$ are considered because we consider only photons above 1 GeV. In the last expression of equation (18), we approximate the proton energy spectrum as the relativistic form and the variable of the spectrum as $(m_pc^2 + \bar{E}_\pi/f_{30}) \sim \bar{E}_\pi/f_{30}$, because $\bar{E}_\pi/f_{30} \gtrsim (1\text{ GeV}/17) \sim 6\text{ GeV} > m_\pi c^2$. Using equation (18), we obtain the $\pi^0$-decay $\gamma$-ray luminosity by

$$L_\pi = \int \int h \frac{dN_\pi}{dt dhvdV} 4\pi r^2 dr dh.$$

(19)

The integration interval of $r$ is given by the shocked ISM shell, defined in the beginning of section 3. As in the calculation of the bremsstrahlung luminosity, we used $\langle n^2 \rangle$ instead of $\langle n \rangle^2$. Figure 4 shows the time evolution of the $\gamma$-ray luminosity due to $\pi^0$-decay in the 1–100 GeV band.

4. Discussion

For a low-density ISM of density 0.016 cm$^{-3}$ in the present model, supposed for an H II region (e.g., formed by the progenitor and extended to a few tens parsec), the blast wave is only slightly decelerated over a period of $\sim 10000$ yr. As a result, in the context of diffusive shock acceleration described in section 2, the radio flux continues to increase, because the increase of the emission measure overcomes the decrease of the magnetic field strength. Consequently, for about ten thousand years, combination-readiation X-rays are observed from the irregular-shape inner part of the SNR (see figure 1 and Paper I), while the radio emission of tens Jy is observed from the blast-shocked ISM shell.

In the beginning of the Sedov/Taylor phase, where the blast wave is being decelerated significantly as $V_c \propto r^{-3/5}$, the radio flux turns to decrease slowly as $r^{-3/10}$, and then approaches nearly constant values as the magnetic field approaches its interstellar value ($\sim 3 \mu G$) and $T_e$ approaches $T_p$. Also, the inverse-Compton $\gamma$-rays turns to decrease as $\propto r^{-1/5}$ in the Sedov/Taylor phase, while $\pi^0$-decay $\gamma$-rays are nearly constant. This sort of analysis done is also for the phase $\lesssim 10000$ yr with the relation $V_c \propto r^{-\alpha}$, where $\alpha$ is given by the hydrodynamical calculation, and gives a good agreement with the computed time evolution of the radio and $\gamma$-ray emission shown in figure 4. It should be noted that $s \sim 0.4$ at 10000 yr, yet smaller than the Sedov value of $s = 3/5$; also, the SNR is in the transient phase to the Sedov/Taylor regime.
Again, because of the low density, the \(\gamma\)-ray luminosity of the shocked ISM shell is dominated by inverse-Compton scattering through out the SNR evolution of concern. However, \(\pi^0\)-decay \(\gamma\)-rays could be enhanced by interactions with dense external matter, e.g., dense H I gas, molecular clouds, or a cavity wall formed by the stellar wind of the progenitor. If 10\% of accelerated protons interact with such matter of density \(n \sim 100\) cm\(^{-3}\), the luminosity, \(L_{\gamma}\), would exceed \(10^{35}\) erg s\(^{-1}\) at a few thousand year, comparable to the typical \(\gamma\)-ray luminosity of MM SNRs. Interactions with molecular clouds are suggested in many MM SNRs based on OH maser and/or near-infrared observations. The interaction with H I gas is suggested in RX J1713.7–3946 by observations (Fukui et al. 2012), and may also be expected in MM SNRs.

Interaction with some dense external matter may be realized on the \(\gamma\)-ray to radio flux ratio. We show a ratio of 1–100 GeV to 1 GHz flux in figure 5. One can see that this ratio is systematically high for MM SNRs compared to 1–100 GeV to 1 GHz flux in figure 5. One can see that the ratio can be a measure of the density, \(n_{\gamma}/\gamma\), of the matter with which the particles interact. For \(B \sim 100\) \(\mu\)G, the high ratios observed from MM SNRs may be explained by \(\pi^0\)-decay if the density of the target matter, \(n_T > 10\) cm\(^{-3}\), is higher than the typical ISM density (\(< 1\) cm\(^{-3}\)). The high ratios could also be explained by inverse-Compton if \(B \lesssim 10\) \(\mu\)G. Such a field may be possible for shell-like SNRs, but unlikely for MM SNRs, which exhibit a rather high radio flux.

Finally, we mention the effect of the CSM, stellar wind matter. An important effect of the CSM is that the shock break-out raises the maximum energy, \(E_{\text{max}}\), to \(\sim 1300\) TeV for protons (see figure 3). Since \(E_{\text{max}} \propto BV_{\gamma}^2t \propto V_{\gamma}^3\), \((E_{\gamma}/M_{\gamma})^{3/2}\), where \(E_{\gamma}\) and \(M_{\gamma}\) are the initial kinetic energy of ejecta and the ejecta mass, respectively, \(E_{\text{max}}\) would exceed \(\sim 3000\) TeV, the cosmic-ray knee energy, for a 2-times larger value of \(E_{\gamma}/M_{\gamma}\) than that in the present model.

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