Short Communication

The CODATA 2017 values of $h$, $e$, $k$, and $N_A$ for the revision of the SI

D B Newell, F Cabiati, J Fischer, K Fujii, S G Karshenboim, H S Margolis, E de Mirandés, P J Mohr, F Nez, K Pachucki, T J Quinn, B N Taylor, M Wang, B M Wood and Z Zhang

Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants

E-mail: dnewell@nist.gov

Received 2 August 2017, revised 19 October 2017
Accepted for publication 20 October 2017
Published 29 January 2018

Abstract

Sufficient progress towards redefining the International System of Units (SI) in terms of exact values of fundamental constants has been achieved. Exact values of the Planck constant $h$, elementary charge $e$, Boltzmann constant $k$, and Avogadro constant $N_A$ from the CODATA 2017 Special Adjustment of the Fundamental Constants are presented here. These values are recommended to the 26th General Conference on Weights and Measures to form the foundation of the revised SI.

Keywords: international system of units, fundamental constants, SI redefinition

(Some figures may appear in colour only in the online journal)

1. Introduction

The international system of units (SI) has been slowly evolving from an artifact based system to one based on values of fundamental constants and invariant properties of atoms. The quantitative limitations of the last remaining base unit of the SI defined by an artifact, the kilogram, have been known since at least the third verification of national kilogram prototypes (Quinn 1991, Girard 1994). As a consequence the possible role of the fundamental constants in replacing the kilogram has been discussed in earnest for nearly three decades. International consensus on the foundation of a new system of units based on exactly defined values of the Planck constant $h$, elementary charge $e$, Boltzmann constant $k$, and Avogadro constant $N_A$ was reached during the 24th meeting of the General Conference on Weights and Measures (CGPM 2011). Progress in the accuracy and consistency of the research results has enabled the 106th International Committee for Weights and Measures (CIPM) to recommend proceeding with the adoption of the revised SI (CIPM 2017).

The Committee on Data for Science and Technology (CODATA), through its Task Group on Fundamental Constants (TGFC), periodically provides the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry. Because of this role, the CGPM invited the CODATA TGFC to carry out a special least-squares adjustment (LSA) of the values of the fundamental physical constants to provide values for defining constants to form the foundation of the revised SI (CGPM 2011). The results of that adjustment are given here, namely, the numerical values of $h$, $e$, $k$, and $N_A$, each with a sufficient number of digits to maintain consistency between the present and revised SI as proposed by the Consultative Committee for Units (CCU) and agreed to by the CIPM (CIPM 2016). These numbers are recommended to the 26th CGPM to establish the revised SI when it convenes in November 2018.

Chair

Original content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
2. The CODATA 2017 special adjustment

The input data for the CODATA 2017 Special Adjustment includes the input data used in the final CODATA 2014 regular adjustment on which the 2014 recommended values are based. Of these data, which are given in tables XV–XIX of Mohr et al. (2017a), the following were omitted: the four cyclotron frequency ratios of hydrogenic carbon to the proton, items B8, B9, B11, and B12 that have been superseded by the 2016 atomic mass evaluation (Huang et al. 2017, Wang et al. 2017), and all measurements of the Newtonian constant of gravitation G. Key data that were published or accepted for publication before the 1 July 2017 closing date of the CODATA 2017 Special Adjustment and have a significant impact on the determination of \( h \), \( e \), \( k \), and \( N_A \) are listed in table 1. The full list of data considered for the CODATA 2017 Special Adjustment is given in tables 2–5 in Mohr et al. (2018). Of note are data that are not included for the same reasons they were omitted from the 2014 adjustment. In particular, the measurements in muonic hydrogen and deuterium that have led to the proton radius ‘puzzle’ were not included. These data would have no effect on the 2017 values of \( h \), \( e \), \( k \), and \( N_A \), but will be reconsidered for the next CODATA periodic adjustment.

The CODATA 2017 Special Adjustment follows the same procedures as the previous periodic CODATA adjustments of the fundamental constants (Mohr and Taylor 2000, 2005, Mohr et al. 2008a, 2008b, Mohr et al. 2012a, 2012b, Mohr et al. 2016a, 2016b). Details of the Special Adjustment analysis are given in Mohr et al. (2018). In general, the measure the CODATA TGFC uses for consistency of an input datum is the normalized (or reduced) residual of that datum given by the LSA, that is, the difference between an input datum and its adjusted value divided by the input datum uncertainty. If a residual for an input datum is larger than two, the TGFC identifies the fundamental constant primarily influenced by that datum as well as other input data that influence the same constant. The uncertainties of this subset of input data are multiplied by a factor that is large enough that the relevant residuals are two or less. To achieve consistency, multiplicative expansion factors were applied to the uncertainties of two subsets of input data corresponding to two adjusted constants for the 2017 Special Adjustment.

The first subset consists of the eight input data for the Planck and Avogadro constants listed in table 1, relevant to the adjusted value of the Planck constant. The uncertainties of these input data are multiplied by a factor of 1.7. With this expansion of the uncertainties of the eight data, five have relative standard uncertainties \( u_t \) at or below 50 \( \times \) 10\(^{-9}\), with two at or below 20 \( \times \) 10\(^{-9}\), where the latter includes results from both the Kibble balance and the x-ray crystal density (XRCD) methods.

The second subset of expanded data consists of the input data that determine the relative atomic mass of the proton: the 2016 atomic mass evaluation value of \(^1\text{H}\) and the cyclotron frequency ratio of hydrogenic carbon to the proton, items B2 and B12, respectively, of table 4 in Mohr et al. (2018). Coincidentally, an expansion factor of 1.7 was also appropriate.

### Table 1.

| Source | Identification* | Quantityb | Value | Rel. stand. uncert \( u_t \) |
|---|---|---|---|---|
| Schlamminger et al. (2015) | NIST-15 | \( h \) | 6.626 069 36(38) \( \times \) 10\(^{-34}\) J s | 5.7 \( \times \) 10\(^{-8}\) |
| Wood et al. (2017) | NRC-17 | \( h \) | 6.626 070 133(60) \( \times \) 10\(^{-34}\) J s | 9.1 \( \times \) 10\(^{-9}\) |
| Haddad et al. (2017) | NIST-17 | \( h \) | 6.626 069 934(88) \( \times \) 10\(^{-34}\) J s | 1.3 \( \times \) 10\(^{-8}\) |
| Thomas et al. (2017) | LNE-17 | \( h \) | 6.626 070 40(38) \( \times \) 10\(^{-34}\) J s | 5.7 \( \times \) 10\(^{-8}\) |
| Azuma et al. (2015) | IAC-11 | \( N_A \) | 6.022 140 95(18) \( \times \) 10\(^{23}\) mol\(^{-1}\) | 3.0 \( \times \) 10\(^{-8}\) |
| Azuma et al. (2015) | IAC-15 | \( N_A \) | 6.022 140 70(12) \( \times \) 10\(^{23}\) mol\(^{-1}\) | 2.0 \( \times \) 10\(^{-8}\) |
| Bartl et al. (2017) | IAC-17 | \( N_A \) | 6.022 140 52(76) \( \times \) 10\(^{23}\) mol\(^{-1}\) | 1.2 \( \times \) 10\(^{-8}\) |
| Kuramoto et al. (2017) | NMIJ-17 | \( N_A \) | 6.022 140 78(15) \( \times \) 10\(^{23}\) mol\(^{-1}\) | 2.4 \( \times \) 10\(^{-8}\) |
| Moldover et al. (1988) | NIST-88 | \( R \) | 8.314 470(15) J mol\(^{-1}\) K\(^{-1}\) | 1.8 \( \times \) 10\(^{-6}\) |
| Pitre et al. (2009) | LNE-09 | \( R \) | 8.314 467(23) J mol\(^{-1}\) K\(^{-1}\) | 2.7 \( \times \) 10\(^{-6}\) |
| Sutton et al. (2010) | NPL-10 | \( R \) | 8.314 468(26) J mol\(^{-1}\) K\(^{-1}\) | 3.2 \( \times \) 10\(^{-6}\) |
| Pitre et al. (2011) | NLE-11 | \( R \) | 8.314 455(12) J mol\(^{-1}\) K\(^{-1}\) | 1.4 \( \times \) 10\(^{-6}\) |
| Pitre et al. (2015) | NLE-15 | \( R \) | 8.314 4615(84) J mol\(^{-1}\) K\(^{-1}\) | 1.0 \( \times \) 10\(^{-6}\) |
| Gavioso et al. (2015) | INRIM-15 | \( R \) | 8.314 4743(88) J mol\(^{-1}\) K\(^{-1}\) | 1.1 \( \times \) 10\(^{-6}\) |
| Pitre et al. (2017) | LNE-17 | \( R \) | 8.314 4614(50) J mol\(^{-1}\) K\(^{-1}\) | 6.0 \( \times \) 10\(^{-7}\) |
| Podesta et al. (2017) | NPL-17 | \( R \) | 8.314 4603(58) J mol\(^{-1}\) K\(^{-1}\) | 7.0 \( \times \) 10\(^{-7}\) |
| Feng et al. (2017) | NIM-17 | \( R \) | 8.314 459(17) J mol\(^{-1}\) K\(^{-1}\) | 2.0 \( \times \) 10\(^{-6}\) |
| Gaiser et al. (2017) | PTB-17 | \( A_e(\text{He})/R \) | 6.221 140(12) \( \times \) 10\(^{-8}\) m\(^3\) K J\(^{-1}\) | 1.9 \( \times \) 10\(^{-6}\) |
| Qu et al. (2017) | NIM/NIST-17 | \( kR/h \) | 2.083 6630(56) \( \times \) 10\(^{10}\) Hz K\(^{-1}\) | 2.7 \( \times \) 10\(^{-6}\) |

* IAC: International Avogadro Coordination; INRIM: Istituto Nazionale di Ricerca Metrologica, Torino, Italy; LNE: Laboratoire national de métrologie et d’essais, Trappes and La Plaine-Saint-Denis, France; NIM: National Institute of Metrology, Beijing, PRC; NIST: National Institute of Standards and Technology, Gaithersburg, MD, and Boulder, CO, USA; NMIJ: National Metrology Institute of Japan, Tsukuba, Japan; NPL: National Physical Laboratory, Ted- dington, UK; NRC: National Research Council Canada, Ottawa, Canada; PTB: Physikalisch-Technische Bundesanstalt, Braunschweig and Berlin, Germany.

b \( h \): Planck constant; \( N_A \): Avogadro constant; \( R \): molar gas constant; \( A_e(\text{He})/R \): molar polarizability of \(^4\text{He}\) gas to the molar gas constant quotient; \( kR/h \): Boltzmann constant to Planck constant quotient.
in this case, although its application has no effect on the 2017 values of \(h\), \(e\), \(k\), and \(N_A\).

### 3. Results

Figure 1 shows values of \(h\) inferred from the key input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is \(\pm 20\) parts in \(10^9\) and the outer grey band is \(\pm 50\) parts in \(10^9\). KB: Kibble balance; XRCRD: x-ray-crystal-density.

**Table 2.** The CODATA 2017 adjusted values of \(h\), \(e\), \(k\), and \(N_A\).

| Quantity | Value | Rel. stand. uncert |
|----------|-------|-------------------|
| \(h\) | \(6.626070150(69) \times 10^{-34}\) Js | 1.0 \(\times 10^{-8}\) |
| \(e\) | \(1.602176634(83) \times 10^{-19}\) C | 5.2 \(\times 10^{-9}\) |
| \(k\) | \(1.38064903(51) \times 10^{-23}\) J K\(^{-1}\) | 3.7 \(\times 10^{-7}\) |
| \(N_A\) | \(6.022140758(62) \times 10^{23}\) mol\(^{-1}\) | 1.0 \(\times 10^{-8}\) |

A requirement by the CGPM (2011) is that the revised SI be consistent with the present SI. In the SI prior to redefinition, the following quantities have exactly defined values: the international prototype of the kilogram \(m(K) = 1\) kg, the vacuum magnetic permeability \(\mu_0 = 4\pi \times 10^{-7}\) H m\(^{-1}\), the triple point of water \(T_{\text{TPW}} = 273.16\) K, and the molar mass of carbon-12, \(M^{(12}\text{C}) = 0.012\) kg mol\(^{-1}\). In the revised SI, these quantities are determined experimentally with associated uncertainties. As stated in the agreed upon CCU recommendation (CIPM 2016), the number of digits for the exact numerical values of \(h\), \(e\), and \(N_A\) to define the revised SI are determined by requiring that the numerical values of \(m(K)\), \(\mu_0\), and \(M^{(12}\text{C})\) remain consistent with their previous exact values within their relative standard uncertainties given by the CODATA 2017 Special Adjustment. The number of digits for \(k\) is chosen such that \(T_{\text{TPW}}\) is equal to 273.16 K within a relative standard uncertainty at the level which \(T_{\text{TPW}}\) can be realized (CCT 2017). The recommended exact numerical values of \(h\), \(e\), \(k\), and \(N_A\) to establish the revised SI are given in table 3.

**Table 3.** The CODATA 2017 values of \(h\), \(e\), \(k\), and \(N_A\) for the revision of the SI.

| Quantity | Value |
|----------|-------|
| \(h\) | \(6.62607015 \times 10^{-34}\) Js |
| \(e\) | \(1.602176634 \times 10^{-19}\) C |
| \(k\) | \(1.380649 \times 10^{-23}\) J K\(^{-1}\) |
| \(N_A\) | \(6.02214076 \times 10^{23}\) mol\(^{-1}\) |

### 4. Summary

Sufficient progress has been achieved towards meeting the recommendations for redefining the SI in terms of exact values of fundamental constants. The recommended exact numerical values of \(h\), \(e\), \(k\), and \(N_A\) to establish the revised SI based on fundamental constants are given. A detailed description of the unique 2017 CODATA special adjustment is given by Mohr et al (2017). The next regular CODATA periodic adjustment of the fundamental constants, CODATA 2018, will also be unique as it will be the first one based on the exact fundamental constants of the revised SI.
Acknowledgment

The CODATA Task Group on Fundamental Constants thanks the CGPM for inviting it to play a significant role in the international effort to establish a revised SI for the 21st century, arguably the most important change to the International System of Units since its formal adoption in 1960.

ORCID iDs

H S Margolis  https://orcid.org/0000-0002-8991-3855

References

Azuma Y et al 2015 Metrologia 52 360
Bartl G et al 2017 Metrologia 54 693
CCT 2017 Recommendation T1 of the 28th CCT Meeting (June 2017) www.bipm.org/cc/CCT/Allowed/Summary_reports/Recommendation-CCT-T1-2017-EN.pdf
CGPM 2011 Resolution 1 of the 24th CGPM www.bipm.org/en/CGPM/db/24/1/
CIPM 2016 Decision CIPM/105-15 of the 105th CIPM www.bipm.org/en/committees/cipm/meeting/105.html
CIPM 2017 Decision CIPM/2017-xx of the 106th CIPM www.bipm.org/en/committees/cipm/meeting/106.html
Feng X J, Zhang J T, Gillis K A, Mehl J B, Moldover M R, Zhang K and Duan Y N 2017 Metrologia 54 748
Fujii K, Massa E, Bettin H, Kuramoto N and Mana G 2018 Metrologia 55 L1
Gaiser C, Fellmuth B, Haft N, Kuhn A, Thiele-Krivoi B, Zandt T, Fischer J, Jusko O and Sabuga W 2017 Metrologia 54 280
Gavioso R M, Madonna Ripa D, Steur P P M, Gaiser C, Truong D, Guianvarc’h C, Tarizzo P, Stuart F M and Dematteis R 2015 Metrologia 52 S274
Girard G 1994 Metrologia 31 317
Haddad D, Seifert F, Chao L S, Possolo A, Newell D B, Pratt J R, Williams C J and Schlamminger S 2017 Metrologia 54 633
Huang W J, Audi G, Wang M, Kondev F G, Naimi S and Xu X 2017 Chin. Phys. C 41 030002
Kuramoto N, Mizushima S, Zhang L, Fujita K, Azuma Y, Kurokawa A, Okubo S, Inaba H and Fujii K 2017 Metrologia 54 716
Mohr P J, Newell D B and Taylor B N 2016a Rev. Mod. Phys. 88 035009
Mohr P J, Newell D B and Taylor B N 2016b J. Phys. Chem. Ref. Data 45 043102
Mohr P J, Newell D B, Taylor B N and Tiesinga E 2018 Metrologia 155 125
Mohr P J and Taylor B N 2000 Rev. Mod. Phys. 72 351
Mohr P J and Taylor B N 2005 Rev. Mod. Phys. 77 1
Mohr P J, Taylor B N and Newell D B 2008a Rev. Mod. Phys. 80 633
Mohr P J, Taylor B N and Newell D B 2008b J. Phys. Chem. Ref. Data 37 1187
Mohr P J, Taylor B N and Newell D B 2012a Rev. Mod. Phys. 84 1527
Mohr P J, Taylor B N and Newell D B 2012b J. Phys. Chem. Ref. Data 41 043109
Moldover M R, Trusler J P M, Edwards T J, Mehl J B and Davis R S 1988 Phys. Rev. Lett. 60 249
Pitre L, Guianvarc’h C, Sparasci F, Guillou A, Truong D, Hermier Y and Himbert M E 2009 C. R. Phys. 10 835
Pitre L, Risegari L, Sparasci F, Plimmer M D, Himbert M E and Giuliano Albo P A 2015 Metrologia 52 S263
Pitre L et al 2017 Metrologia 54 856
Pitre L, Sparasci F, Truong D, Guillou A, Risegari L and Himbert M E 2011 Int. J. Thermophys. 32 1825
Podesta M D, Mark D F, Dymock R C, Underwood R, Bacquart T, Sutton G, Davidson S and Machin G 2017 Metrologia 54 683
Qu J, Benz S P, Coakley K, Rogalla H, Tew W L, White R, Zhou K and Zhou Z 2017 Metrologia 54 549
Quinn T J 1991 IEEE Trans. Instrum. Meas. 40 81
Schlamminger S, Steiner R L, Haddad D, Newell D B, Seifert F, Chao L S, Liu R, Williams E R and Pratt J R 2015 Metrologia 52 L5
Sutton G, Underwood R, Pitre L, de Podesta M and Valkiers S 2010 Int. J. Thermophys. 31 1310
Thomas M, Ziane D, Pinot P, Karcher R, Imanalieva A, Pereira Dos Santos F, Merlet S, Piquemal F and Espel P 2017 Metrologia 54 468
Wang M, Audi G, Kondev F G, Huang W J, Naimi S and Xu X 2017 Chin. Phys. C 41 030003
Wood B M, Sanchez C A, Green R G and Liard J O 2017 Metrologia 54 399