Study on Design Speeds of Metro Inter-stations for Traction Energy Saving and Travel Time Reduction

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Abstract: The increment of design speed of metro system will increase train energy cost while reduce passengers’ travel time. To determine reasonable design speeds of different inter-stations in a line, the factors including passenger demand, operation cost and interval distance need to be taken into account. However, the common practice adopts uniform design speed for the whole line, which is unsuitable for the metro line with large variations on distances of different inter-stations. Therefore, this paper proposed a design speed optimization model with given line conditions and train control strategies to minimize the comprehensive cost, including train traction energy and passenger travel time. Case studies on a Beijing metro line show that the overall cost is reduced by 4.57% through allowing varied design speeds across different inter-stations in comparison with a uniform design speed for all inter-stations. Sensitivity analysis shows that when the interval distances are 900 m and 1700 m, the recommended design speeds are 80km/h and 100km/h, respectively.

1. Introduction
As the metro has high-speed, convenient and large-capacity characteristics, it has become an important means of transportation to alleviate urban traffic congestion. Under the background of the booming domestic urban rail transit projects, the reasonable design speed of train has an important impact on operating cost of the enterprise (e.g. train energy consumption) and passenger service levels (e.g. travel time).

Many scholars have conducted in-depth research on the design speed. In terms of design speed selection, Su Mei et al. analyzed the impact of passenger comfort, interval distance, train weight and marshalling on design speed of railways in the city. Nong Xingzhong comprehensively analyzed and demonstrated the selection of train design speed based on taking civil engineering, electromechanical equipment and vehicle purchase cost into consideration. The selection of the design speed should consider the vehicle purchase cost. Xu Dexin analyzed the factors affecting the design speed of urban rail transit vehicles, and then concluded that the primary basis for determining the design speed is generally the interval distance. However, the above studies did not quantify factors such as passenger service level and enterprise operating costs.

In the research on the quantitative relationship between design speed and passenger service level, Guo Huan proposed the concept of generalized travel cost and calculation formulas to analyze the impacts of different design speeds on passenger demand. Feng Xujie et al. established a multi-objective nonlinear optimization model for the intercity train speed, and analyzed the influence of passenger time value and minimum attendance rate on the optimal operating speed of intercity trains. Lu Yongchen studied the design speed of suburban railways and established the relationship
between average interval distance, passenger travel time, curve radius and maximum design speed. The above researches mainly study the relationship between design speed and passenger time value regarding to the perspective of passengers. But features related to enterprises have rarely been incorporated into the determination of design speed.

In terms of enterprise operating cost, some scholars analyze the relationship between design speed and energy consumption through simulation. Based on computer-aided simulation, Feng Xuesong\cite{7} found that the increasing speed of traction energy consumption for high-speed trains is more than twice the increase in target speed. Subsequently, Feng Xuesong\cite{8} simulated the operation of two Chinese EMU on a smooth track, and analyzed the relationship between design speed and operational energy consumption and transportation efficiency. Zhang Xiong\cite{9} took the Metro B-type vehicle as an example to simulate and study the impact of design speed on travel time, traction energy consumption and vehicle purchase cost. These researches are based on quantitative analysis, but does not comprehensively consider energy consumption and travel time to establish a mathematical optimization model that can assist in the calculation of the optimal design speed.

Therefore, this paper comprehensively considers traction energy consumption and travel time, and establishes a design speed optimization model for metro interval. The Brute Force is used to solve the optimization design speed of each interval on the line. Finally, the effectiveness of the model and algorithm is verified by the Beijing metro.

2. Problem statement
The design speed is the maximum speed allowed for the train. According to technical standards of metro system design, the design speed directly affects the various operational indicators of the metro. As a result, it is the main basis for the design work of the metro system\cite{10}.

In practice, the design speed of a line is generally uniform without the consideration of the length of the interval, resulting the following disadvantages. On the one hand, when the interval distance is small, there will be cases where the train starts decelerating into the station when the design speed has not been attained in the early acceleration process. On the other hand, when it is large, the train has traction characteristics that make the train accelerate to a higher design speed, but it is limited by the current design speed and cannot further increase the speed. Therefore, it is difficult to weigh the different interval distance for the uniform design speed of the whole line. It is necessary to consider the influence of the interval distance when designing the speed.

The design speed should consider the train energy consumption and passenger time value. Figure 1 shows the variation of traction energy consumption and travel time when the train operating in the section with different design speeds. It can be seen that increasing the design speed can reduce passenger travel time, but will increase traction energy consumption. The former is due to the positive correlation between the average travel speed and the interval design speed. When the length of the interval keeps the same, the travel time becomes small with the increase of design speed. The latter is because there is a quadratic function revealing the relationship between the unit basic resistance of railway and the operating speed. The effect of the letter, as the speed increases, the traction energy consumption will increase with the square of the speed. In summary, it is necessary to weigh the profits of both operators and passengers when determining the design speed of the interval.
To this end, this paper takes the interval design speed as the decision variable. Then, a comprehensive design speed optimization model is proposed from the perspective of operators and passengers. The model aims at minimizing the comprehensive cost of traction energy consumption and passenger travel time, and finds the optimal interval design speed, so that the corresponding operators and passengers' benefits are satisfied and unified.

The hypothesis of this paper is as follows. Firstly, the train is regarded as a single mass point in the train kinematics model. Secondly, since the line timetable does not exist in the interval design speed optimization stage, it is assumed that the train runs in the time-saving mode. Finally, the train full load rate is fixed. The population of passengers out of the behavior of rigid demand, do not increase with the growth of speed.

3. Model

3.1. Objective function

In this paper, considering the traction energy consumption and travel time, the two are multiplied by the corresponding coefficients and converted into money as the objective function. The formula is as follows:

\[ C = \alpha E + \beta T \]

Where:
\( C \) —— The combined cost of traction energy consumption and travel time, RMB;
\( \alpha \) —— electricity price per kWh;
\( \beta \) —— The time cost per passenger per hour determined by GDP is calculated as follows.

\[ \beta = \frac{GDP \times P_{ij} \times R_y \times t_{day}}{T_{year} \times t_{day}} \]

Where:
\( GDP \) —— Annual per capita GDP, yuan;
\( P_{ij} \) —— The maximum number of passengers on the train, person;
\( R_y \) —— Full load rate of the train;
\( T_{year} \) —— One-year working day, day;
\( t_{day} \) —— Average working time of one day, h;

In this paper, the design speed \( v_{max} \) of each interval on the line is used as the decision variable to study the traction energy consumption \( E(v_{max}) \) and travel time \( E(v_{max}) \) of the train running in the
interval. The calculation formula for traction energy consumption is:

\[ E(v^{\text{max}}) = \int_0^X F(v, x, v^{\text{max}})dx / 3600 \] (3)

Where:
- \( E \) — Traction energy consumption, kWh;
- \( F(v, x, v^{\text{max}}) \) — The value of the traction force corresponding to the position of the train at position \( x \), kN;
- \( v \) — The running speed of the train at position \( x \), km/h;
- \( v^{\text{max}} \) — The design speed of the train at position \( x \), km/h;
- \( 3600 \) — The unit conversion factor valuing 3600.

The calculation formula for travel time is:

\[ T(v^{\text{max}}) = \int_0^1 \frac{1}{v} dx \] (4)

According to the basic theory of train traction calculation, the train is affected by the traction force \( F \), the resistance \( W \) and the braking force \( B \) during the running process. The acceleration per step of the train is the quotient of the current combined force and the total mass of the train. The formula is as follows:

\[ a_i = \frac{F - W - B}{M * 10^3} = f - w - b \] (5)

The unit of the total mass of the train is ton, and the unit of the combined force of the train is N; \( f \), \( w \), \( b \) are the unit traction force, the unit basic resistance, and the unit braking force; the unit basic resistance is composed of air resistance, additional slope resistance, additional resistance of the curve and additional resistance of the tunnel.

\[
\begin{align*}
w &= w_0(v) + w_i(s) + w_r(s) + w_c(s) \\
w_0(v) &= a_1v^2 + b_1v + c_1 \\
w_i(s) &= i \\
w_r(s) &= \frac{600}{R} \\
w_c(s) &= 0.00013L_i * d
\end{align*}
\] (6)

In the formula, \( w_0(v) \), \( w_i(s) \), \( w_r(s) \) and \( w_c(s) \) are respectively the air resistance, the additional slope resistance, the additional resistance of the curve and the additional resistance of the tunnel for the following speed and position; \( v \) is the current speed of the train (in km/h), \( s \) is the current position of the train (in m), \( a_1, b_1, c_1 \) are constant; \( i \) is the slope of the current position; and \( R \) is the curve radius of the current position (in m).

3.2. Constraints

(1) Parking accuracy constraints

In order to ensure that the position of the train door matches the position of the screen door of the platform after the train stops, it is necessary to put forward higher requirements for the parking accuracy of the train.

\[ \int \Delta x \leq X \] (7)

(2) Speed limit constraint

In order to ensure safe operation, the train needs to meet the operating speed boundary constraint, and the speed at the starting and ending point is 0 km/h. And the speed of the train on the way can not exceed the speed limit, namely:
\[
\begin{align*}
\begin{cases}
v(0) = 0, v(X) = 0 \\
0 \leq v(x) \leq v^{\text{max}}
\end{cases}
\end{align*}
\] (8)

(3) Acceleration constraints
In order to meet the passenger's comfort requirements, the acceleration of the train during accelerating and braking time is \( a_{\text{accelerate}} \) and \( a_{\text{brake}} \), which cannot be higher than a maximum value, namely:
\[
\begin{align*}
\begin{cases}
a_{\text{accelerate}} \leq a^{\text{max}} \\
a_{\text{brake}} \leq a^{\text{max}}
\end{cases}
\end{align*}
\] (9)

(4) Design speed constraints
According to the design standard of the metro, the design speed of the interval generally takes 10 times of the effect, that is:
\[
v^{\text{max}} = 10 \times m \quad (m = 6, 7, 8, 9 \ldots)
\] (10)

4. Algorithm
In order to obtain the global optimal design speed of the interval, the Bruce Force algorithm is used to solve the model to verify the validity of the model.

Brute Force is a kind of force algorithm. There is no pre-processing process. You can compare and calculate the target value in any order to get the optimal value. This algorithm is often used for string matching. The flow is based on a main string and a substring, the characters in the main string are sequentially compared with the characters in the string to find the position index of the substring that appears for the first time in the main string. When the matching is successful, the next character is performed with the same process, until all feasible solutions are searched to obtain exact solutions.

In order to find the lowest comprehensive cost of the interval, this paper first enumerates all possible design speeds in a certain order, and then calculates the traction energy consumption and travel time based on the interval distance, line conditions and total train mass. Finally, the lowest comprehensive cost will be selected from all comprehensive costs corresponding to various design speeds. The algorithm design flowchart is shown in Figure 2.
Initialize design speed $v_{\text{max}}$ and variable $i=0$

Traction calculation of saving time driving strategy

Overall cost $C_j$

$i<M$

$v_{\text{max}}=v_{\text{max}}+i*10$

Traction calculation of Time-saving strategy

Calculate overall cost $C_2$

$C_j > C_2$  \[ S \]

$C_j = C_2$

$i=i+1$

Minimum overall cost $C_j$

Fig.2 Flowchart of Algorithm

5. Case studies
This paper selects the actual metro line data and vehicle parameters of Beijing metro for case studies. Firstly, the influence of interval distance on interval design speed is studied with a single station interval, and then a complete line is simulated to verify the effectiveness of the model and algorithm.

By enquiry, the average unit price of electricity consumption in China's metro transportation is about 1 yuan / kWh. In 2017, the per capita GDP was 59,261 yuan, and the working day of the year was 250 days.

5.1. Comparison of different interval distance
By simulating the interval between the two stations, the influence of interval distance on the design speed is analyzed. Table 1 shows the optimal design speeds for different intervals; Figures 3 and 4 show the overall cost trends of the 925m and 1739m intervals, respectively.

| Interval                          | Interval distance(m) | Design speed(km/h) | Overall cost (yuan) |
|----------------------------------|----------------------|--------------------|---------------------|
| Tian an men East - Tian an men West | 925                  | 80                 | 96.56               |
| Liang xiang University City West - Liang xiang University City | 1739                  | 100                | 144.94              |

Table. 1 Optimal design speeds for different intervals
Figure 3 Trends in the comprehensive cost of the 925-meter interval as a function of design speed

Figure 4 Trends in the comprehensive cost of the 1739-meter interval as a function of design speed

It can be seen from Table 2 that the optimization design speeds of different interval distance are different. When the interval distance is 925m, the optimization design speed is 80km/h; when the interval distance is 1739m, the optimization design speed is 100km/h. It can be seen from Fig. 3 that when the interval distance is short, the overall cost tends to decrease first and then increase as the design speed increases. The main reason is that the train runs in the interval with the time-saving strategy. When the design speed of interval is less than 80km/h, the reduction rate of the travel time is greater than the increase rate of the traction energy consumption as the design speed increases, so the overall cost curve decreases; When it is greater than 80km/h, the rate of increase of traction energy consumption is greater than the rate of reduction of travel time, so the overall cost curve rises. It can be seen from Fig. 4 that when the interval distance is long, the overall cost decreases as the design speed increases. The reason for the decrease in the overall cost curve is the same as in Figure 3.

5.2. Metro Yanfang Line Simulation

In order to verify the validity of the model and algorithm, a complete line is simulated. Taking the Yanfang Line as an example, the optimization design speed of each interval is searched one by one by this method. Table 2 shows the optimal design speed of all sections of the line; Figure 5 shows the speed-displacement curve of the simulated line; Table 3 shows the comprehensive cost comparison of the method and the whole line using the unified design.

| number | interval               | Interval distance(m) | Design speed (km/h) |
|--------|------------------------|----------------------|---------------------|
| 1      | Yanfang-Fangshan Cheng | 2076                 | 100                 |
| 2      | Fangshan City - Rao Lefu | 1777                 | 100                 |
| 3      | Rao Lefu - Magezhuang  | 968                  | 80                  |
| 4      | Magezhuang - Dashihe East | 1928                | 100                 |
| 5      | Dashihe East-Star City | 2104                 | 100                 |
| 6      | Star City-Yancun       | 1649                 | 100                 |
| 7      | Yancun-Zicaowu         | 1210                 | 90                  |

It can be seen from Table 3 that the optimization design speed of the third interval and the seventh interval are 80 km/h and 90 km/h, and the optimization design speeds of other intervals are 100 km/h. The interval design speed increases as the interval distance length increases. When the interval distance is less than 1500m, the optimal design speed is 80kmh or 90km/h; when the interval distance is greater than 1500m, the optimized design speed is 100km/h.
Figure 5 Full line velocity-displacement curve

| Table 3 Comparison of comprehensive cost between optimized method and uniform design speed |
|---------------------------------|-----------------|-----------------|
| Travel time (s) | Traction energy (kWh) | Overall cost (yuan) |
| Method of this paper | 634 | 222.2 | 970.32 |
| line design speed is 80km/h | 681 | 211.1 | 1014.68 |
| line design speed is 90km/h | 653 | 214.1 | 984.64 |
| line design speed is 100km/h | 631 | 229.5 | 974.08 |

It can be seen from Table 3 that the overall cost of the method is the smallest compared with that of the uniform design speed of the whole line. Compared with the line design speeds of 80km/h, 90km/h and 100km/h, the comprehensive cost of the proposed method reduce by 4.57%, 1.48% and 0.39% respectively. Taking the design speed of 80km/h as an example, the line has five sections exceeding 1500m. According to the method, a high design speed (100 km/h) with a long interval of station spacing has a small overall cost, so the overall cost is lower than the line design speed of 80 km/h.

6. Conclusion

In this paper, the optimization problem of interval design speed for a metro line is studied. From the benefits of both operators and passengers, the optimization model of interval design is established with the goal of minimum cost, including a and b. The solution was acquired by the Brute Force algorithm, and a case study was conducted on a line of the Beijing metro. The results show:

The interval optimal design speed will increase as the interval distance increases. When the interval distance is about 900 meters, the design speed should be 80km/h; when the interval distance is about 1700m, the design speed should be 100km/h.

Compared with the line to adopt a uniform design speed, optimizing the design speed of each interval one by one can obtain a lower overall cost. The more intervals greater than 1500 m, the better optimizing effect that this method achieves compared with the case that the design speed of the whole line is unified to 80 km/h.

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