Simple theory for scotogenic dark matter with residual matter-parity

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Dark matter stability can result from a residual matter-parity symmetry surviving spontaneous breaking of an extended gauge symmetry. We propose the simplest scotogenic dark matter completion of the original SVS theory [1], in which the “dark sector” particles as well as matter-parity find a natural theoretical origin in the model. We briefly comment on its main features.

I. INTRODUCTION

The nature of dark matter remains mysterious, though a lot of progress has been made on what dark matter should not be [2]. Many particle dark matter candidates have been proposed in agreement with astrophysical and cosmological observations, in particular the so-called Weakly Interacting Massive Particles, or WIMPs, have attracted a lot of attention. From a theory point of view it would be desirable that the particle dark matter candidate should obey two requirements:

1. fit in a broader scheme accounting for other shortcomings of the standard model,
2. have its stability on cosmological scales naturally protected by a symmetry.

The existence of supersymmetry would provide a WIMP candidate, the Lightest supersymmetric particle, though it fails to obey the above requirements, since its stability is assumed as a result of R-parity conservation, an ad hoc symmetry [3]. Moreover, the LSP does not relate to other problems of the standard model except, possibly, the technical aspects associated to the hierarchy problem.

Neutrino mass generation is one of the basic open challenges in particle physics and it could well be that it may be directly related to the understanding of dark matter. Indeed, WIMP dark matter could mediate neutrino mass generation [4]. This idea, realized within the simplest standard model gauge structure, has been studied in many papers over the past few years [5–11].

When the gauge symmetry is extended, it can happen that there is a “dark symmetry” called matter-parity, that remains conserved after spontaneous symmetry breaking. In this case the lightest odd-particle will be automatically
stable and can play the role of dark matter. Indeed, this has been shown to be the case in the context of the SU(3) ⊗ SU(3)L ⊗ U(1)X ⊗ U(1)N electroweak extension of the standard model [12–15].

Here we construct a non-supersymmetric scenario for scotogenic dark matter in which dark matter stability results naturally from the residual matter-parity symmetry. The construction provides the simplest dark matter completion of the original SVS theory [1] by incorporating “automatically” a stable scotogenic dark matter candidate.

The theory is minimal, as it uses only particles already present in the original SVS theory to make up the “dark” sector, with the residual matter-parity resulting from the extended symmetry breaking dynamics. This way it provides an elegant origin for the scalar dark doublet introduced ad hoc in other dark matter constructions, of the Inert Higgs Doublet type [16–20]. The latter is naturally identified here with the inert electroweak doublet contained in one of the triplet Higgs scalars required to ensure adequate breaking of the extended SU(3)L gauge symmetry. If lightest, its stability becomes automatic because of the residual matter-parity gauge symmetry.

This following material is organized as follows: in Sec. II we sketch the theory setup and quantum numbers, in Sec. III we summarize the scalar sector and in Sec. IV we describe the Yukawa couplings and the neutrino mass generation mechanism. Finally in Sec. V we give a short Discussion and conclude.

II. THE MODEL

Our starting point is a variant of the model introduced in [12] based on the SU(3) ⊗ SU(3)L ⊗ U(1)X ⊗ U(1)N gauge symmetry. The main motivation for the extra U(1)N is to allow for a fully gauged B−L symmetry [21, 22]. In our model, electric charge and B−L are embedded into the gauge symmetry as

\[ Q = T_3 - \frac{T_8}{\sqrt{3}} + X, \]

\[ B - L = -\frac{2}{\sqrt{3}} T_8 + N, \]

with \( T_i \) (\( i = 1, 2, 3, ..., 8 \)), \( X \) and \( N \) as the respective generators of SU(3)_L, U(1)_X and U(1)_N.

In the present model, after spontaneous symmetry breaking (SSB) a residual discrete symmetry arises as a remnant from the B−L symmetry breakdown. Its role is analogous to that of R-parity in supersymmetric theories, we call it matter-parity, \( M_P = (-1)^{3(B-L)+s} \). The stability of the lightest \( M_P \)-odd particle leads to a potentially viable WIMP dark matter candidate.

The particle content of the model is shown in Table II. Here, left-handed leptons \( l_{aL} \); \( a = 1, 2, 3 \) transform as triplets under SU(3)_L,

\[ l_{aL} = \begin{pmatrix} \nu_a \\ e_a \\ N_a \end{pmatrix}_L, \]

and the third component is precisely the \( M_P \)-odd singlet fermion needed in a scotogenic neutrino mass generation mechanism. Anomaly cancellation requires that two generations of quarks \( q_{iL}; i = 1, 2 \) must transform as anti-triplets and one as a triplet [1]²,

\[ q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ D_i \end{pmatrix}_L \]

\[ q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ U_3 \end{pmatrix}_L \]

¹ A singlet scalar is also added to break the degeneracy of the neutral scalars, needed to close the scotogenic neutrino mass loop.

² Here we follow mainly the gauged B-L extension of the original reference. Many other works exist, see also [23–30].
This choice predicts three generations of quarks and leptons (the same as the number of colors), an important feature of this class of models.

| Field | SU(3)$_c$ | SU(3)$_L$ | U(1)$_X$ | U(1)$_N$ | $Q$ | $M_P = (-1)^{3(B-L)+2s}$ |
|-------|-----------|-----------|-----------|-----------|-----|-------------------------|
| $q_{iL}$ | 3 | 3 | 0 | 0 | $(-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3})^T$ | $(- -, +)^T$ |
| $q_{3L}$ | 3 | 3 | $\frac{1}{3}$ | $\frac{2}{3}$ | $(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3})^T$ | $(+, +)^T$ |
| $u_{aR}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $+$ |
| $d_{aR}$ | 3 | 1 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $+$ |
| $U_{3R}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{4}{3}$ | $\frac{2}{3}$ | $-$ |
| $D_{1R}$ | 3 | 1 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $-\frac{1}{3}$ | $-$ |
| $l_{aL}$ | 1 | 3 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $(0, -1, 0)^T$ | $(+ -)^T$ |
| $e_{aR}$ | 1 | 1 | $-1$ | $-1$ | $-1$ | $+$ |
| $\nu_{1R}$ | 1 | 1 | 0 | $-4$ | 0 | $-$ |
| $\nu_{3R}$ | 1 | 1 | 0 | 5 | 0 | $+$ |
| $N_{aR}$ | 1 | 1 | 0 | 0 | 0 | $-$ |
| $\eta$ | 1 | 3 | $-\frac{1}{3}$ | $\frac{1}{3}$ | $(0, -1, 0)^T$ | $(+ -)^T$ |
| $\rho$ | 1 | 3 | $\frac{2}{3}$ | $\frac{1}{3}$ | $(1, 0, 1)^T$ | $(+ -)^T$ |
| $\chi$ | 1 | 3 | $-\frac{2}{3}$ | $-\frac{2}{3}$ | $(0, -1, 0)^T$ | $(- -)^T$ |
| $\phi$ | 1 | 1 | 0 | 2 | 0 | $+$ |
| $\sigma$ | 1 | 1 | 0 | 1 | 0 | $-$ |

TABLE I. 3311 model particle content ($a = 1, 2, 3$ and $i = 1, 2$ represent generation indices). Note the non-standard charges of “right handed neutrinos” $\nu_R$.

Besides the fields contained in [12], the model contains only one scalar singlet $\sigma$. This field will play an important role in the neutrino mass generation mechanism by breaking the degeneracy of the real and imaginary parts of the scalar exchanged in the scotogenic loop. The original scotogenic proposal includes a dark SU(2)$_L$ doublet. A key observation of the present work is that such dark doublet is already present in the original SVS model when promoted to a SU(3)$_c$ ⊗ SU(3)$_L$ ⊗ U(1)$_X$ ⊗ U(1)$_N$ gauge symmetry, it is naturally identified with the first two components of the $\chi$ triplet, that are already $M_P$-odd.

Notice the unconventional U(1)$_N$ charges of the $\nu_R$ fields. Due to this choice the tree level neutrino mass is absent. The two $\nu_{iR}$ neutrinos can acquire a majorana mass after spontaneous symmetry breaking (SSB) by the inclusion of a scalar field transforming as $(1, 1, 0, 8)$ and $\nu_{3R}$ requires a scalar with quantum numbers $(1, 1, 0, -10)$. In the present work we do not include those fields in order to keep the analysis of the scalar sector as simple as possible.

The gauged $B - L$ symmetry is spontaneously broken by two units as the singlet scalar $\phi$ develops a vacuum expectation value (VEV), leaving a discrete remnant symmetry $M_P = (-1)^{3(B-L)+2s}$. The most general VEV alignment for the scalar fields compatible with the preservation of $M_P$ symmetry is

$$
\langle \eta \rangle = \frac{1}{\sqrt{2}} (v_1, 0, 0)^T, \quad \langle \rho \rangle = \frac{1}{\sqrt{2}} (0, v_2, 0)^T, \quad \langle \chi \rangle = (0, 0, w)^T, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} \Lambda, \quad \langle \sigma \rangle = 0. \quad (5)
$$

In this work we will assume the hierarchy $w, \Lambda \gg v_1, v_2$, such that the SSB pattern of the model is

$$
SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_N \\
\downarrow w, \Lambda \\
SU(3)_C \times SU(2)_L \times U(1)_Y \times M_P \\
\downarrow v_1, v_2 \\
SU(3)_C \times U(1)_Q \times M_P.
$$

(6)
Tree level light active neutrino masses are forbidden in this basic setup. Small masses for the light active neutrinos are only generated at one loop level via a radiative seesaw mechanism mediated by the CP-even and CP-odd parts of the first component of the SU(3)_L scalar triplet \( \chi \) as well as by the gauge singlet right handed Majorana neutrinos, as shown in Figure 1.

### III. Scalars

In this section we discuss the scalar sector of our model. The scalar multiplets are decomposed as follows,

\[
\eta = \left( \begin{array}{c} \frac{v_1 + s_1 + i \alpha_1}{\sqrt{2}} \\ \frac{v_2 + s_2 + i \alpha_2}{\sqrt{2}} \\ \frac{v_3 + s_3 + i \alpha_3}{\sqrt{2}} \end{array} \right), \quad \rho = \left( \begin{array}{c} \frac{\rho_1^+}{\sqrt{2}} \\ \frac{\rho_2^+}{\sqrt{2}} \\ \frac{\rho_3^+}{\sqrt{2}} \end{array} \right), \quad \chi = \left( \begin{array}{c} \frac{\chi_1^+}{\sqrt{2}} \\ \frac{\chi_2^+}{\sqrt{2}} \\ \frac{\chi_3^+}{\sqrt{2}} \end{array} \right), \quad \phi = \frac{\Lambda + s_\phi + i a_\phi}{\sqrt{2}}, \quad \sigma = \frac{s_\sigma + i a_\sigma}{\sqrt{2}}. \tag{7}
\]

The scalar potential invariant under the symmetries of the model takes the form:

\[
V = \mu_1 \rho^\dagger \rho + \mu_2 \chi^\dagger \chi + \mu_3 \eta^\dagger \eta + \mu_4 \phi^\dagger \phi + \mu_5 \sigma^\dagger \sigma \\
+ \lambda_1 (\rho^\dagger \rho)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)^2 \\
+ \lambda_4 (\rho^\dagger \rho)(\chi^\dagger \chi) + \lambda_5 (\rho^\dagger \rho)(\eta^\dagger \eta) + \lambda_6 (\chi^\dagger \chi)(\eta^\dagger \eta) \\
+ \lambda_7 (\rho^\dagger \chi)(\chi^\dagger \rho) + \lambda_8 (\rho^\dagger \eta)(\eta^\dagger \rho) + \lambda_9 (\chi^\dagger \eta)(\eta^\dagger \chi) \\
+ \lambda_{10} (\phi^\dagger \phi)(\rho^\dagger \rho) + \lambda_{11} (\phi^\dagger \phi)(\chi^\dagger \chi) + \lambda_{12} (\phi^\dagger \phi)(\eta^\dagger \eta) \\
+ \lambda_{13} (\sigma^\dagger \sigma)(\rho^\dagger \rho) + \lambda_{14} (\sigma^\dagger \sigma)(\chi^\dagger \chi) + \lambda_{15} (\sigma^\dagger \sigma)(\eta^\dagger \eta) \\
+ \lambda_{16} (\phi^\dagger \phi)^2 + \lambda_{17} (\sigma^\dagger \sigma)^2 + \lambda_{18} (\phi^\dagger \phi)(\sigma^\dagger \sigma) + \lambda_{19} [(\sigma^\dagger \phi)(\eta^\dagger \chi) + \text{h.c.}] \\
+ \frac{1}{\sqrt{2}} \left[ -\mu_4 \rho \eta \chi + \mu_5 \phi \sigma \sigma + \mu_6 (\eta^\dagger \chi) \sigma + \text{h.c.} \right],
\tag{8}
\]

where the \( \lambda_k \) (\( k = 1, 2, \cdots, 19 \)) are dimensionless parameters whereas the \( \mu_r \) (\( r = 1, 2, \cdots, 5 \)), \( \mu_t, \mu_s, \mu_u \) are dimensionful parameters. Note that, to ensure the preservation of \( M_D \), we assume \( \mu_5^2 > 0 \). The minimization condition of
the scalar potential yields the following relations:

\[ \mu_2 = \frac{v_1 w \mu_t - v_2}{2v_2} \left( \lambda_\lambda^2 \lambda_3^2 + \lambda_4^2 \right), \]
\[ \mu_2 = \frac{v_1 v_2 \mu_t - w}{2w} \left( \lambda_\lambda \lambda_3^2 + \lambda_4^2 \right), \]
\[ \mu_2 = \frac{v_2 w \mu_t - v_2}{2v_2} \left( \lambda_\lambda \lambda_3^2 + \lambda_4^2 \right), \]
\[ \mu_3 = -\frac{1}{2} \left( \lambda_\lambda^2 + \lambda_\lambda \lambda_3^2 + \lambda_4^2 \right). \]

From the analysis of the scalar potential, we find that the squared mass matrix for charged scalars, in the basis \((\eta_2^+, \rho_1^+, \chi_2^+, \rho_3^+)\) versus \((\eta_2^-, \rho_1^-, \chi_2^-, \rho_3^-)\), takes the form:

\[ M_C = \begin{pmatrix} M_{C}^{(1)} & 0_{2 \times 2} \\ 0_{2 \times 2} & M_{C}^{(2)} \end{pmatrix}, \]
\[ M_{C}^{(1)} = \frac{1}{2} \left( v_1 v_2 \lambda_8 + w \mu_1 \right) \begin{pmatrix} \frac{v_1}{v_2} & 1 \\ \frac{v_2}{v_1} & 1 \end{pmatrix}, \]
\[ M_{C}^{(2)} = \frac{1}{2} \left( w v_2 \lambda_7 + v_1 \mu_1 \right) \begin{pmatrix} \frac{v_1}{v_2} & 1 \\ \frac{v_2}{v_1} & 1 \end{pmatrix}. \]

Note that the squared mass matrix \( M_C \) has two vanishing eigenvalues which correspond to the Goldstone bosons

\[ G^\pm = \pm \frac{v \eta_{1,2} - v \rho_{1,2}}{\sqrt{v_{1,2}^2 + v_{1,2}^2}}, \]
\[ G'^\pm = \pm \frac{w \chi_{1,2} - v \rho_{1,2}}{\sqrt{w_{1,2}^2 + v_{1,2}^2}}. \]

associated to the longitudinal components of the \( W^\pm \) and \( W'^\pm \). The massive eigenstates are the physical charged scalar bosons \( H_1^\pm \) and \( H_2^\pm \)

\[ H_1^\pm = \frac{v_2 \eta_{1,2} + v_1 \rho_{1,2}}{\sqrt{v_{1,2}^2 + v_{1,2}^2}}, \]
\[ H_2^\pm = \frac{w \chi_{1,2} + v_1 \rho_{1,2}}{\sqrt{w_{1,2}^2 + v_{1,2}^2}}, \]
\[ m_{H_1^\pm}^2 = \frac{\left( v_1^2 + v_2^2 \right) \left( w \mu_1 + \lambda_\lambda v_1 v_2 \right)}{2v_1 v_2}, \]
\[ m_{H_2^\pm}^2 = \frac{\left( v_1^2 + v_2^2 \right) \left( v_1 \mu_1 + \lambda_\lambda v_1 v_2 \right)}{2v_1 v_2}. \]

Concerning the neutral scalar sector, we find that the squared mass matrix for CP-even and CP-odd neutral scalars, in the basis \((s_1, s_2, s_3, s_\phi, s'_1, s'_3, s_\sigma)\) and \((a_1, a_2, a_3, a_\phi, a'_1, a'_3, a_\sigma)\), are respectively given by:

\[ M_S = \begin{pmatrix} M_S^{(1)} & 0_{1 \times 4} \\ 0_{1 \times 4} & M_S^{(2)} \end{pmatrix}, \]
\[ M_S^{(1)} = \begin{pmatrix} 2 \lambda_3 v_2^2 + \frac{w v_2 \mu_t}{2v_2} & v_1 v_2 \lambda_5 - \frac{w \mu_t}{2} & w v_1 \lambda_6 - \frac{w \mu_t}{2} & \lambda v_1 \lambda_12 \\ v_1 v_2 \lambda_5 - \frac{w \mu_t}{2} & 2 \lambda_2 v_2^2 + \frac{w v_2 \mu_t}{2v_2} & w v_2 \lambda_4 - \frac{w \mu_t}{2} & \lambda v_2 \lambda_10 \\ \lambda v_1 \lambda_12 & \lambda v_2 \lambda_10 & 2 \lambda_2^2 \mu_{16} & w \lambda_{11} \\ w \lambda_{11} & 2 \lambda_2^2 \mu_{16} & \lambda_{19} & \lambda_{19} \end{pmatrix}, \]
\[ M_S^{(2)} = \frac{1}{2} \begin{pmatrix} v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) \\ v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) \\ v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) & v_1 \left( \lambda v_{19} + \mu_u \right) \end{pmatrix}. \]
and

\[
M_A = \begin{pmatrix} M_A^{(1)} & 0_{4 \times 3} \\ 0_{3 \times 4} & M_A^{(2)} \end{pmatrix},
\]

\[
M_A^{(1)} = \frac{1}{2} \begin{pmatrix} w \frac{\mu_1}{v_1} & w \mu_t & v_2 \mu_t & 0 \\ w \mu_t & w \frac{\mu_1}{v_2} & v_1 \mu_t & 0 \\ v_2 \mu_t & v_1 \mu_t & \frac{1}{2} w \mu_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},
\]

\[
M_A^{(2)} = \frac{1}{2} \begin{pmatrix} v_1 (w_1 (\Lambda_{19} + \mu_u)) & -v_1 \theta_3 - v_2 \mu_t & 0 \\ -v_1 \theta_3 - v_2 \mu_t & w_1 (\Lambda_{19} + \mu_u) & 0 \\ v_1 (\Lambda_{19} - \mu_u) & w (\mu_u - \Lambda_{19}) & 0 \end{pmatrix}.
\]

The block \(M_A^{(1)}\) contains one small eigenvalue associated to the standard model Higgs field. Assuming the hierarchy \(\Lambda, w, \mu_t \gg v_1, v_2\) the latter can be identified with

\[
h \approx \frac{v_1 s_1 + v_2 s_2}{\sqrt{v_1^2 + v_2^2}}, \quad m_h^2 = O(v_{1,2}^2),
\]

and three heavy Higgs bosons, given as,

\[
H_1 \approx \frac{v_2 s_1 - v_1 s_2}{\sqrt{v_1^2 + v_2^2}}, \quad m_{H_1}^2 \approx \frac{(v_1^2 + v_2^2) w \mu_t}{2 v_1 v_2},
\]

\[
H_2 \approx \cos \xi_3 s_3 - \sin \xi_4, \quad m_{H_2}^2 \approx \lambda_{16} A^2 + \lambda_2 w^2 - \sqrt{\lambda_{16}^2 A^4 + \lambda_2^2 w^4 + \lambda_{11}^2 A^2 w^2 - 2 \lambda_2 \lambda_{16} A^2 w^2},
\]

\[
H_3 \approx \sin \xi_3 + \cos \xi_4, \quad m_{H_3}^2 \approx \lambda_{16} A^2 + \lambda_2 w^2 + \sqrt{\lambda_{16}^2 A^4 + \lambda_2^2 w^4 + \lambda_{11}^2 A^2 w^2 - 2 \lambda_2 \lambda_{16} A^2 w^2}.
\]

The matrix \(M_A^{(1)}\) contains three Nambu-Goldstone bosons

\[
G_1 = \frac{v_1 a_1 - v_2 a_2}{\sqrt{v_1^2 + v_2^2}}, \quad G_2 = \frac{v_1 a_1 - w a_2}{\sqrt{v_1^2 + w^2}}, \quad G_3 = a_\phi,
\]

related to the longitudinal components of the \(Z, Z', Z''\) gauge bosons, plus a heavy CP-odd massive state

\[
A_1 = \frac{v_2 w a_1 + v_1 w a_2 + v_1 v_2 a_3}{\sqrt{(v_2 w)^2 + (v_1 w)^2 + (v_1 v_2)^2}}, \quad m_{A_1}^2 = \frac{\mu_t (v_1^2 w^2 + w_1^2 w^2 + v_2^2 v_1^2)}{2 v_1 v_2 w},
\]

The CP-even states \(s_1', s_3'\) and \(s_\sigma\) mix according to the squared mass matrix \(M_S^{(2)}\), that can be diagonalized by the transformation

\[
\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ G_4 \end{pmatrix} = U \begin{pmatrix} s_1' \\ s_3' \\ s_\sigma \end{pmatrix} = \begin{pmatrix} w \cos \theta_3 + v_1 \sin \theta_3 & w \cos \theta_3 - v_1 \sin \theta_3 & 0 \\ v_1 \sin \theta_3 + w \cos \theta_3 & -v_1 \sin \theta_3 + w \cos \theta_3 & 0 \\ v_1 \sin \theta_3 & -v_1 \sin \theta_3 & 0 \end{pmatrix} \begin{pmatrix} s_1' \\ s_3' \\ s_\sigma \end{pmatrix},
\]

with

\[
tan 2\theta_s = \frac{2 v_1 w \sqrt{v_1^2 + w^2} (\lambda_{19} A + \mu_u)}{v_1 w (-2 \mu_5^2 - \Lambda (\lambda_{18} A + \mu_u) - \lambda_{13} v_2^2 - \lambda_{15} v_2^2 + \lambda_9 (v_1^2 + w^2) - \lambda_{14} w^2) + v_2 \mu_4 (v_1^2 + w^2)},
\]
yielding two heavy physical real scalars \( \varphi_1 \) and \( \varphi_2 \) with squared masses

\[
m_{\varphi_{1,2}}^2 = \frac{1}{4v_1w} \left\{ v_1 w \left( \lambda_1 \Delta^2 + 2 \mu_5^2 + 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) + \left\{ v_1 w \left( \lambda_1 \Delta^2 + 2 \mu_5^2 + 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) - 4v_1 w \left( v_1^2 + w^2 \right) \left( v_2 \mu_1 \left( \lambda_1 \Delta^2 + 2 \mu_5^2 + 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) + v_1 w \left( \lambda_9 \left( \lambda_1 \Delta^2 + 2 \mu_5^2 + 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) - (\lambda_9 \Lambda + \mu_u)^2 + \lambda_9 \lambda_3 v_2^3 + \lambda_1 v_2 v_1^2 \mu_t + \lambda_9 \lambda_1 v_1^3 w \right) \right\}^{1/2},
\]

and the Goldstone mode \( G_4 \). The CP-odd scalars \( \alpha'_1, \alpha'_3 \) and \( a_\sigma \) have a similar fate, since the squared mass matrix \( M_A^{(2)} \) can be diagonalized by the transformation

\[
\begin{pmatrix}
\tilde{\varphi}_1 \\
\tilde{\varphi}_2 \\
G_5
\end{pmatrix}
= U^\alpha
\begin{pmatrix}
\alpha'_1 \\
\alpha'_3 \\
a_\sigma
\end{pmatrix},
\]

with mixing angle

\[
\tan 2\theta_a = \frac{2v_1 w \sqrt{v_1^2 + w^2} \left( \mu_u - \lambda_9 \Lambda \right)}{v_1 w \left( -\lambda_1 \Delta^2 - 2 \mu_5^2 + 2 \mu_\Lambda - \lambda_3 v_2^2 - \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) - \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right)}.
\]

The real scalars \( \tilde{\varphi}_1 \) and \( \tilde{\varphi}_2 \) acquire squared masses

\[
m_{\tilde{\varphi}_{1,2}}^2 = \frac{1}{4v_1w} \left\{ v_1 w \left( \lambda_1 \Delta^2 + 2 \mu_5^2 - 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) + \left\{ v_1 w \left( \lambda_1 \Delta^2 + 2 \mu_5^2 - 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) - 4v_1 w \left( v_1^2 + w^2 \right) \left( v_2 \mu_1 \left( \lambda_1 \Delta^2 + 2 \mu_5^2 - 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) + v_1 w \left( \lambda_9 \left( \lambda_1 \Delta^2 + 2 \mu_5^2 - 2 \mu_\Lambda + \lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_9 \left( v_1^2 + w^2 \right) + v_2 \mu_1 \left( v_1^2 + w^2 \right) \right) - (\lambda_9 \Lambda - \mu_u)^2 + \lambda_9 \lambda_3 v_2^3 + \lambda_1 v_2 v_1^2 \mu_t + \lambda_9 \lambda_1 v_1^3 w \right) \right\}^{1/2},
\]

and the Goldstone boson \( G_5 \) combines with \( G_4 \) into a neutral complex Goldstone associated with the non-Hermitian gauge boson \( X^0 \). Notice that in the limit \( \mu_\Lambda, \mu_u \to 0 \), one obtains a degenerate physical scalar spectrum \( m_{\tilde{\varphi}_{1,2}}^2 = m_{\varphi_{1,2}}^2 \). This degeneracy is broken in our model by the inclusion of the scalar singlet \( \sigma \), a feature required to implement the scotogenic neutrino mass generation approach.

IV. YUKAWA SECTOR

The Yukawa interactions and mass terms for fermions are given by

\[
-L_{\text{Yukawa}} = y_{\nu_{\alpha}L} \bar{\nu}_{\alpha} L \rho e_R + y_{\nu_{\alpha}L}^{\nu} \bar{\nu}_{\alpha} L \chi N_R + \frac{M_{ab}}{2} \bar{N}_a \sigma_R N_b + y_{\nu_{\alpha}L} \bar{\nu}_{\alpha} L \rho^* u_R + y_{\nu_{\alpha}L}^{\nu} \bar{\nu}_{\alpha} L \chi \rho^* u_R + \bar{Y}_{L} \chi \rho^* u_R + y_{\nu_{\alpha}L} \bar{\nu}_{\alpha} L \rho^* u_R + y_{\nu_{\alpha}L}^{\nu} \bar{\nu}_{\alpha} L \chi \rho^* u_R + y_{\nu_{\alpha}L} \bar{\nu}_{\alpha} L \rho^* u_R + y_{\nu_{\alpha}L}^{\nu} \bar{\nu}_{\alpha} L \chi \rho^* u_R + \text{h.c.}
\]
After the spontaneous breakdown of the SU(3) ⊗ SU(3)_L ⊗ U(1)_X ⊗ U(1)_N gauge symmetry, the Yukawa interactions generate the following mass matrices for quarks:

\[
M_U = \begin{pmatrix}
-y_u^{11} & -y_u^{12} & -y_u^{13} & 0 \\
-y_u^{21} & -y_u^{22} & -y_u^{23} & 0 \\
y_u^{31} & y_u^{32} & y_u^{33} & 0 \\
0 & 0 & 0 & y_U^w
\end{pmatrix},
\]

(30)

\[
M_D = \begin{pmatrix}
y_d^{11} & y_d^{12} & y_d^{13} & 0 & 0 \\
y_d^{21} & y_d^{22} & y_d^{23} & 0 & 0 \\
y_d^{31} & y_d^{32} & y_d^{33} & 0 & 0 \\
0 & 0 & 0 & y_D^w & y_D^w \\
0 & 0 & 0 & y_D^w & y_D^w
\end{pmatrix}.
\]

(31)

Due to the U(1)_N symmetry assignments, there are no tree level mixing between the exotic and standard model (SM) quarks, and therefore the Cabibbo-Kobayashi-Maskawa (CKM) matrix is unitary. As indicated by Eqs. (30) and (31), both SU(3)_L scalar triplets η and ρ are needed to generate the up- and down-type SM quark masses, whereas the third triplet χ, responsible for the spontaneous breaking of the SU(3)_L ⊗ U(1)_X symmetry, produces the exotic quark masses.

Turning to the charged lepton sector, only the SU(3)_L scalar triplet ρ contributes to the charged lepton mass matrix, given by

\[
M_l = \begin{pmatrix}
y_e^{11} & y_e^{12} & y_e^{13} \\
y_e^{21} & y_e^{22} & y_e^{23} \\
y_e^{31} & y_e^{32} & y_e^{33}
\end{pmatrix} \frac{v_2}{\sqrt{2}}.
\]

(32)

Notice that, after SSB, the “dark” or \(M_P\)-odd fermions \(N_L\) and \(N_R\) mix through the mass matrix

\[
M_N = \begin{pmatrix}
0 & y^N \omega \\
(y^N \omega)^T & M_M
\end{pmatrix},
\]

(33)

in the basis \((N^c_L, N_R)\). Using the general method in Eq.(3.1) of [31] this matrix can be diagonalized perturbatively by a unitary transformation, defining six physical Majorana states denoted by \(S^a_R\) through

\[
\begin{pmatrix}
N^c_L \\
N_R
\end{pmatrix} = U S_R,
\]

(34)

such that \(M' = U^T M_N U = \text{diag}(M_a)\). In the following discussion, only the lower blocks of the unitary matrix \(U\) will be relevant. We adopt the following notation for the relation between \(N_R\) and \(S_R\):

\[
N_a R = U_{a\alpha} S_{\alpha R}.
\]

(35)

For simplicity, we will assume that the entries of the matrix in Eq.(33) are real, and that the matrix \(U\) becomes orthogonal.

A. Neutrino masses

Concerning the neutrino sector, the light active neutrino masses are produced by a radiative one-loop seesaw mechanism, thanks to the remnant \(M_P\) discrete symmetry preserved after the SSB of the U(1)_N gauge symmetry.
Then, according to Fig. 1 the one loop level light active neutrino mass matrix is given by

\[ y_{ab}^N N^L_a aL \mathcal{N}_{bR} = y_{ab}^N \left( \tau_a^\dagger \tau_a \right)_L \left( \frac{s_i^2 + i a^i}{\sqrt{2}} \right) N_{bR} + y_{ab}^N \mathcal{N}_{aL} \left( \frac{w + s_3 + i a_3}{\sqrt{2}} \right) N_{bR}. \] (36)

The first term in the above relation is the necessary interaction between neutrinos and singlet fermions through an inert SU(2) scalar doublet, while the second term gives rise to the Dirac mass blocks in Eq. (33). Thus, in the physical basis, the relevant terms for the generation of neutrino masses are

\[
- \mathcal{L} \supset y_{ab}^N \tau_{aL}^\dagger \tau_{bR} + \frac{M_{ab}}{2} \mathcal{N}_{aR} \mathcal{N}_{bR} \\
= \left( y^N \right)_{aa} U^a_1 \tau_{aL} \tilde{\varphi}_a \mathcal{S}_{aR} + i \left( y^N \right)_{aa} U^a_{11} \tau_{aL} \tilde{\varphi}_a \mathcal{S}_{aR} \\
+ \left( y^N \right)_{aa} U^a_3 \tau_{aL} \mathcal{G}_a \mathcal{S}_{aR} + i \left( y^N \right)_{aa} U^a_{11} \tau_{aL} \mathcal{G}_a \mathcal{S}_{aR} \\
+ \sum_{\alpha=1}^6 \frac{M_{a\alpha}}{2} \mathcal{S}_{aR} \mathcal{S}_{aR} + h.c.
\] (37)

Then, according to Fig. 1 the one loop level light active neutrino mass matrix is given by

\[
(M_{\nu})_{ab} = \sum_{\alpha=1}^6 \sum_{i=1}^2 \left( y^N \right)_{aa} \left( y^N \right)_{ba} M_\alpha \left( U^a_{11} \right)^2 \frac{M^2_{\tilde{\varphi}_i}}{M^2_{\mathcal{S}_{\alpha}}} \ln \left( \frac{M^2_{\mathcal{S}_{\alpha}}}{M^2_{\mathcal{S}_{\alpha}}} \right) - \left( U^a_{11} \right)^2 \frac{m^2_{\tilde{\varphi}_i}}{M^2_{\mathcal{S}_{\alpha}}} \ln \left( \frac{m^2_{\tilde{\varphi}_i}}{M^2_{\mathcal{S}_{\alpha}}} \right). \] (38)

where the mass splitting between the CP even and CP odd scalars running in the internal lines of the loop is generated from the \( \frac{\mu_3}{\sqrt{2}} \tilde{\sigma} \sigma \) and \( \frac{\mu_2}{\sqrt{2}} (n^\dagger \chi) \sigma \) trilinear scalar interactions. Thus, the tiny values of the light active neutrino masses can be attributed to the loop suppression, as well as to the smallness of the trilinear scalar couplings \( \mu_3 \) and \( \mu_2 \), which in turn produce a small mass splitting between the virtual CP even and CP odd scalars.

V. DISCUSSION

In this letter we have explored the idea that dark matter stability results from a residual matter-parity symmetry that survives the spontaneous breaking of an extended gauge symmetry. For the latter we have taken the SU(3) \( \otimes \) SU(3)\textsubscript{L} \( \otimes \) U(1)\textsubscript{X} \( \otimes \) U(1)\textsubscript{N} symmetry, proposing the simplest scotogenic dark matter completion of the original SVS theory [1]. In our new construction the “dark sector” particles are clearly identified with states already present in the original picture. The only new state added is a new singlet scalar in order to break the degeneracy of the neutral scalars. The latter is needed to close the scotogenic Majorana neutrino mass loop. The theory provides a natural origin for the scalar dark doublet introduced \textit{ad hoc} in other dark matter constructions, such as the inert Higgs dark matter scenarios. The latter is simply identified with the electroweak doublet part one of the triplet Higgs scalars required for adequate breaking of the extended SU(3)\textsubscript{L} gauge symmetry. Assuming this scalar to be the lightest “odd particle” it will be dark matter, with its stability naturally ensured by the residual matter-parity gauge symmetry. This gives an elegant scotogenic realization of inert doublet scenarios of dark matter, extensively explored in a number of recent works [32–41].
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