Transport properties of Rashba conducting strips coupled to magnetic moments with spiral order

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Magnetic and transport properties of a conducting layer with Rashba spin-orbit coupling (RSOC) magnetically coupled to a layer of localized magnetic moments are studied on strips of varying width. The localized moments are free to rotate and they acquire an order that results from the competition between the magnetic exchange energy and the kinetic energy of the conduction electrons. By minimizing the total Hamiltonian within the manifold of variational spiral orders of the magnetic moments, the phase diagram in the space of the interlayer exchange \( J_{sd} \), and the ratio of the Rashba coupling to the hopping integral, \( \lambda/t \) was determined. Two main phases with longitudinal spiral order were found, one at large interlayer coupling \( J_{sd} \) with uniform order in the transversal direction, and the other at small \( J_{sd} \) showing a transversal staggered order. This staggered spiral order is unstable against an antiferromagnetic (AFM) for large values of \( \lambda/t \). In both spiral phases, the longitudinal spiral momentum that departs from the expected linear dependence with the RSOC for large values of \( \lambda/t \). Then, various transport properties, including the longitudinal Drude weight and the spin Hall conductivity, inside these two phases are computed in linear response, and their behavior is compared with the ones for the more well-studied cases of a fixed ferromagnetic (FM) and AFM localized magnetic orders.

I. INTRODUCTION

There is currently an increasing interest in studying and developing new systems and devices that could process information using the electron spin, which is the essence of the field of spintronics [1–3]. In particular, a considerable number of possibilities stem from the implementation of effective couplings derived from microscopic spin-orbit (SO) interactions, chief among them the Rashba spin-orbit coupling (RSOC) which appears in systems with structural inversion asymmetry and leads to the appearance of transversal spin currents and the spin Hall effect [4–7].

It has been recently noticed [8, 9] that a strong spin torque can be induced on a two-dimensional (2D) conducting layer with Rashba SOC coupled to a ferromagnetic (FM) layer. This process was observed when an electrical current flows in the plane of a Co layer with asymmetric Pt and AlO\(_x\) interfaces [10–12]. Even more recently, it has been discussed the possibility of an analogous relativistic SO torque in conducting layers containing RSOC in the presence of antiferromagnetic (AFM) layers [13]. This possibility was suggested that could be realized in bulk Mn\(_2\)Au, which although is centrosymmetric, it can be divided into two sublattices that separately have broken inversion symmetry and form inversion partners. Various other heterostructures could be considered as containing a subsystem of localized magnetic moments with FM or AFM order [14]. It has also been noticed that in AFM coupled systems, the spin-orbit torque could drive magnetic walls with velocities a magnitude greater than in FM ones [15].

In addition to these FM and AFM orders that are fixed by large exchange interactions between the localized magnetic moments, or by the structure of the materials, the case in which the magnetic moments are allowed to move in order to minimize the total energy, has also been studied [16, 17]. In particular it has been shown that the spiral order of the localized magnetic moments along the longitudinal \( x \)-axis is driven by the SO interaction in the conduction strip [17], which induces a spin rotation or chiral precession, around the transversal \( y \)-axis, with \( k_{B,x} \sim \lambda/t \). The electron spin spiral in the uncoupled Rashba conducting strip had been first noticed by Ref. [18]. Using numerically exact Monte Carlo calculations within the spin-fermion decoupling, it has been also shown that in two-dimensional systems, the FM and AFM orders are unstable against various other types of magnetic orders, mainly spiral orders [19].

Hence, the main motivation for this work is to examine transport properties of Rashba conducting strips coupled to a layer of magnetic moments with the spiral order that minimizes the total energy for each set of parameters. Spirals, as well as other magnetic orders that may be present at oxide heterostructures, such as skyrmions, have been studied but within effective, spin-only, models [20]. In most of this previous work, infinite two-dimensional systems were considered and a parabolic band was assumed [16, 17, 21].

It should be noticed that although in conventional semiconductor heterostructures [22] or at the interface LaAlO\(_3\)/SrTiO\(_3\) [23], the spin-orbit parameter \( \alpha_R \), related to the coupling \( \lambda \) as \( \alpha_R = \lambda a \), where \( a \) is the lattice constant, is up to \( 10^2 \) eV Å, a number of compounds where \( \alpha_R \) is more than 2 orders of magnitude higher have been found. This is the case of BiTeI [24, 25], in the BaIrO\(_3\)/BaTiO\(_3\) heterostructure [26], and in the CH\(_3\)NH\(_3\)PbBr\(_3\) organic-inorganic perovskite [27]. In these cases, \( \lambda/t \approx 0.3 \), which justifies the range of \( \lambda/t \) between 0 and 1 adopted in the present study. It is also important to emphasize that RSOC can be varied by an electric field perpendicular to the strip plane, and hence

\[ \lambda/t \]
the spiral state could be accordingly modified. This electric control of magnetic order is at the heart of magnetic ferroelectrics or multiferroics [28].

Finally, an array of recently proposed materials and devices presents strong Rashba SO couplings and involve sizable electron fillings [29, 30]. Then, in the present work, the coupled system with quarter-filled band for the conduction electrons will be examined for varying Rashba SO interaction and exchange coupling \( J_{sd} \), on strips of various widths between the minimal value \( W = 2 \) and large enough values to represent the infinite width limit.

The outline of the paper is the following. In Section II the model here studied is defined and some methodological details are provided. Then, in Section III the phase diagram in the \( \lambda/t-J_{sd} \) plane for strips of various widths at quarter filling is presented. In Section IV the behavior of the Rashba helical currents introduced in Ref. [31], is discussed, and in Section V results for transport properties, the longitudinal optical conductivity and the spin Hall conductivity are presented. Finally, in Section VI a summary is provided together with a suggestion of possible application of the present results to spintronic devices.

II. MODEL AND METHODS

The system to be studied in the present work is schematically shown in Fig. (a)-(b). A slab of conduction electrons that are to undergo conventional or spin conserving hopping and Rashba-type or spin flipping hopping (bottom slab in both pictures) is coupled by the exchange integral \( J_{sd} \) to a slab of localized magnetic moments (upper slab). Both slabs are modelled by a single layer. This heterostructure is similar to the one studied for the ferro- or antiferromagnetic orders of the magnetic layer [8, 10, 21]. The crystal structure of the whole system is assumed cubic, with the layers belonging to the \( x, y \)-plane as shown in Fig. (a)-(b). The Hamiltonian for the resulting model on the \( x, y \)-plane is [13, 19]:

\[
H_{1\sigma} = H_0 + H_{\text{int}}
\]

\[
H_0 = -t \sum_{<l,m>,\sigma} (c_{l\sigma}^\dagger c_{m\sigma} + H.c.) + \lambda \sum_l [c_{l+\hat{x}\downarrow}^\dagger c_{l\uparrow}^\dagger - c_{l+\hat{x}\uparrow}^\dagger c_{l\downarrow} + i(c_{l+\hat{y}\uparrow}^\dagger c_{l\downarrow}^\dagger + c_{l+\hat{y}\downarrow}^\dagger c_{l\uparrow}) + H.c.]
\]

\[
H_{\text{int}} = -J_{sd} \sum_l S_l \cdot s_l + J \sum_{<l,m>} S_l \cdot S_m
\]

where \( H_0 \) corresponds to the conducting layer, and it includes the hopping and the RSOC terms with coupling constants \( t \) and \( \lambda \), respectively, which connects nearest neighbor sites on the square lattice. Since both terms contribute to the total kinetic energy, the normalization \( t^2 + \lambda^2 = 1 \) was imposed, and naturally its square root is adopted as the unit of energy. With this normalization, the kinetic energy turns out to be approximately constant as \( \lambda/t \) is varied [19] with all the remainder parameters held fixed. The interacting part of the Hamiltonian contains a ferromagnetic exchange coupling between conduction electron spins \( s_l \) and localized magnetic moments \( S_l \), with strength \( J_{sd} \), and an exchange magnetic interaction between localized magnetic moments with coupling \( J \). This exchange term favors a FM (\( J < 0 \)) or an AFM (\( J > 0 \)) order of the localized moments.

This system is schematically shown in Fig. (a)-(b). The plane of localized magnetic moments (upper slab in both pictures) is coupled by the exchange integral \( J_{sd} \) to the layer where conduction electrons are able to undergo conventional hopping and Rashba type hopping (bottom slab).

The localized magnetic moments are assumed classical variables with modulus equal to one, and hence they are described in spherical coordinates by the angles \((\theta, \varphi)\). Notice that by taking \(|S| = 1\), the magnitude of the physical magnetic moments has been absorbed in \( J_{sd} \).

Model (1) will be studied by exact diagonalization on strips \( L \times W \), where periodic boundary conditions are assumed in the longitudinal (\( x \)-axis) direction, and open boundary conditions on the transversal (\( y \)-axis) direction. The electron filling, as in all lattice models, is defined as the total number of electrons divided by the total number of orbitals of the conducting layer, which for the present single-orbital model is equal to the number of sites \( N = LW \). For all the sets of parameters considered, the condition of closed shell, that is, that all the degenerate single-electron eigenvalues up to the Fermi level were included, was verified in order to avoid spurious values of the physical properties computed. In some

- **FIG. 1.** (Color online) (a), (b) Schematic depiction of the system considered in the present work consisting of a layer of localized magnetic moments coupled by an exchange \( J_{sd} \) to a conducting layer with hopping \( t \) and Rashba SO coupling \( \lambda \). The magnetic moments present a spiral ordering in the longitudinal direction that is uniform (panel (a)) or staggered (panel (b)) in the transversal or \( y \)-direction. A charge current along the strip or \( x \)-direction that could be injected after a voltage bias is applied to the strip’s ends, is also shown inside the conducting layer. (c) Schematic phase diagram of model Eq. (1) on strips at \( n = 0.5 \). The main phases are the uniform spiral (SP) shown in (a), the staggered spiral (s-SP) shown in (b), and the antiferromagnetic (AFM) phase.
cases, this condition was enforced by adding a small twist $\Phi_x = 10^{-7}$ in the boundary conditions along $x$.

The non coplanar spiral order of the magnetic moments is defined for the angle $\theta_l$ and a uniform azimuthal angle, $\varphi_l$, as:

$$\theta_{x,y} = k_\theta \cdot (x, y)$$  \(2\)

with spiral momentum $k_\theta = (k_{\theta,x}, k_{\theta,y}) = 2\pi (m/L, m'/W)$, $m, m'$ integers. In the following, the azimuthal angle will be considered uniform, that is, $\varphi_l = \varphi$. Special cases are the FM order, with $m = m' = 0$, and the AFM order, with $m = L/2$, $m' = W/2$.

The system defined by the Hamiltonian given by Eq. (1) will be studied at zero temperature and in linear response. For each set of parameters, $\lambda/t, J_{sd}, J, W, L$, and $n$, and for each pair of integers $(m, m')$, $m \in [0, L]$ and $m' \in [0, W]$, the total energy is computed. The optimal spiral state for that set of parameters is the one corresponding to the pair $(m, m')$ for which the minimum value of the total energy is obtained. All physical properties for each set of parameters will be computed for the corresponding optimal spiral momentum. Most of the calculations were performed on clusters containing up to 8192 sites, although most of the results reported below were obtained for 512 × 512 clusters.

The parameter $\lambda/t$ was varied in the interval $[0, 1]$, and $J_{sd}$ was varied between 0 and 15. Since the effect of $J$ is somewhat trivial, in the following it will be set equal to zero. For $J = 0$, the FM order exists only for $\lambda = 0$, and the AFM order appears in a finite region of the $\lambda/t$-$J_{sd}$ plane at $n = 0.5$, as discussed in the following section. For this density, the FM or AFM orders exist for all the range of $\lambda/t-J_{sd}$ for $|J| \gtrless 1$. However, to compute physical properties for these two orders is technically much simpler to fix the corresponding values of $m, m'$, as mentioned above, while setting $J = 0$.

All the transport properties studied below involve the charge current, which is the sum of the spin-conserving current, $J_{\sigma,\mu}, \sigma = \uparrow, \downarrow, \mu = x, y$, which is the expectation value of the operator:

$$\hat{j}_{\sigma,\mu} = \sum c_{\uparrow,\mu,\sigma} c_{\downarrow,\sigma} - H.c.$$, \(3\)

in units where the electron charge $e = 1$, and of the spin-flipping current, $J_{SO,\mu}$ which is the expectation value of the operator:

$$\hat{j}_{SO,\mu} = -i\lambda (c_{\uparrow,\mu, \uparrow} - c_{\downarrow,\mu, \downarrow} - H.c.)$$ \(4\)

Other physical quantities involving also the transversal spin currents, will be defined below.

III. PHASE DIAGRAM AT QUARTER FILLING

Let us start by examining the phase diagram in the $\lambda/t$-$J_{sd}$ plane at $n = 0.5, J = 0$. This phase diagram, schematically shown in Fig. (1c), is approximately valid for all strip widths, from the narrowest strip that could contain Rashba helical currents and spin polarization across its section, which corresponds to $W = 2$ [32], up to $W = 32$, for which results are virtually indistinguishable from those of $W = 64$.

This diagram contains the main phases to be examined in this study. At large $J_{sd}$, for all values of $\lambda/t$ in the interval $(0, 1]$, a spiral (SP) order of the localized magnetic moments, with a spiral momentum $(k_{\theta,x}, 0)$, and $\varphi = 0$ or $\pi$, shown in Fig. (1a), is present. As $J_{sd}$ is reduced below $J_{sd} \approx 6$, another interesting order appears, the “staggered” spiral (s-SP) phase, with $k_{\theta} = (k_{\theta,x}, \pi)$, $\varphi = 0$ or $\pi$, shown in Fig. (1b), which exists for $\lambda/t > 0$ up to a value $(\lambda/t)^*$ where the magnetic slab enters into an AFM, $(\pi, \pi)$, phase that in turn extends up to $\lambda/t = 1$. The boundary between the s-SP and AFM regions is located at $\lambda/t \sim 0.65$ for $J_{sd} = 2.5$, and at $\lambda/t \sim 0.8$ for $J_{sd} = 5$. This crossover between s-SP and AFM phases, with a jump in the longitudinal spiral momentum from $\approx \pi/2$ to $\pi$, is of first order since there are two minima in the energy as a function of $k_{\theta,x}$. For fixed $\lambda/t$, the crossover between the s-SP and SP phases as $J_{sd}$ is varied, is also first order. For $J_{sd} \lesssim 1$ there are many competing phases depending strongly on the parameters of the model.

An important feature in these spiral phases is the following. As it can be observed in Fig. (2a), the spiral momentum along the strip axis, $k_{\theta,x}$, decreases almost linearly from zero to $\approx -\pi/2$ as $\lambda/t$ increases from zero.
to one, for \( J_{sd} = 10 \). This linear behavior is apparent from the interpolation of the results for \( W = 32 \) up to \( \lambda/t \leq 0.6 \). However, for larger values of \( \lambda/t \), \( k_{0,x} \) clearly starts to deviate from that linear behavior. A similar behavior is shown in Fig. 2(b) for \( J_{sd} = 7.5 \), also within the SP region. Within the staggered spiral phase, for \( J_{sd} = 5 \), it can be also observed an almost linear decrease of the longitudinal spiral momentum \( k_{0,x} \), from zero to \( \approx \pi/2 \) as \( \lambda/t \) increases from zero to its maximum value before entering in the AFM phase (Fig. 2(c)). As said above, this transition is of first order, and within the AFM phase, the SP order corresponds to the first excited state within the subset of states considered. The same behavior is observed for \( J_{sd} = 2.5 \) except that in this case the AFM phase starts at a lower value of \( \lambda/t \) (Fig. 2(d)). These results correspond to \( \varphi = 0 \). The same chirality of the reported spiral states is recovered for \( \varphi = \pi \) and reversing the sign of \( k_{0,x} \).

Since, as discussed in the Introduction, the spiral order of the localized magnetic moments is driven by the conduction electrons, it is expected that \( J_{sd} \) will make this conducting-induced spiral order on the localized magnetic slab to survive for larger values of \( \lambda/t \). This corresponds to the behavior shown in Fig. 2 where it can also be noticed that the spiral momentum is roughly independent of \( J_{sd} \) for a given value of \( \lambda/t \), as long as the spiral order exists. It should also be emphasized that the relationship \( k_{0,x} \sim \lambda/t \) remains valid except when approaching the value of \( k_{0,x} = \pi/2 \), for the strip geometry here considered. This departure of the linear behavior could be due to higher order effects in \( \lambda/t \), involving virtual processes through \( J_{sd} \), which were neglected in the first order calculation in Ref. [17].

Notice also that in the absence of Rashba SO coupling, \( \lambda = 0 \), and for large \( J_{sd} \), as in the conventional double-exchange model, and as exemplified by manganites [33], localized spins acquire a FM, \((0,0)\) state, in order to favour the kinetic energy of conduction electrons. For small values of \( J_{sd} \), on the other hand, for \( \lambda = 0 \), the localized moments present a \((0,\pi)\) order. Hence, although the variation of \( k_{0,x} \) with \( \lambda/t \) seems a result of the precession of single conducting electrons, the presence of the SP, s-SP or AFM phases depends on the values of \( J_{sd} \), \( \lambda/t \), and as it will be mentioned below, also on the electron filling, through the full many-body nature of the system.

The location of the boundaries between different phases is mildly dependent on the strip width \( W \) in the proximity of quarter-filling. For \( n = 0.25 \) the phase diagram contains essentially the SP phase for all values of \( \lambda/t \) and \( J_{sd} \gtrsim 3 \), with a similar linear dependence of \( k_{0,x} \) with \( \lambda/t \) above discussed for the \( n = 0.5 \). The s-SP phase has disappeared. At \( J_{sd} = 2.5 \), \( W = 4 \), the localized moments have a \((\approx \pi/4,0)\) order for \( \lambda = 0 \), and \( k_{0,x} \) decreases by increasing \( \lambda/t \) but with a smaller slope than for the cases shown in Fig. 2. Although this electron filling was not exhaustively explored, the comparison with the results for \( n = 0.5 \), suggests that many-body effects determine the possible magnetic phases of the system. This conclusion stems from the well-known behavior of Kondo lattice models, where an effective interaction between the localized magnetic moments mediated by the conduction electrons appears at an effective level. This effective interaction makes the order of the magnetic layer to depend on the filling of the conduction layer.

Generalized Kondo lattice models like the one here studied, present a phase separated state close to half-filling in two dimensions in the absence of RSOC, as it is well-known from studies in the context of manganites [33]. The presence of phase separation has also been discussed for the Rashba system at the LaAlO\(_3\)/SrTiO\(_3\) interface [34, 35]. Previous calculations for model [11] on the square lattice, with \( J_{sd} = 10 \), in the whole range of \( \lambda/t \) in \([0,1]\), have shown that phase separation occurs close to half-filling, \( n \gtrsim 0.75 \), and moreover, that actually it is suppressed by increasing \( \lambda/t \) [19]. The situation discussed in [34, 35] could be present for smaller \( J_{sd} \), particularly in the AFM region, but its precise determination is out of the scope of the present work.

Finally, it should be noticed that the reported spiral states are the true ground state states as it results from Monte Carlo simulations up to \( \lambda/t \approx 0.5 \). For larger SO couplings, this phase is unstable towards various other orderings, one of them is the already mentioned AFM for small \( J_{sd} \), which is also the true ground state. Moreover, Monte Carlo calculations show that for \( \lambda/t \gtrsim 0.7 \) and \( J_{sd} \) above the AFM region, the \((\approx \pi/2,0)\) spiral becomes unstable against other states with a maximum of the magnetic structure factor near \((0,\pi/2)\). These other states have an energy much lower than the one for the spiral state with the spiral momentum equal to that maximum of the magnetic structure factor, indicating that they are not spiral but a distinct, so far unknown, order, probably a lattice of skyrmions [20, 32].

IV. RASHBA HELICAL CURRENTS

The Rashba helical currents (RHC), with counter-propagating spin-up and spin-down electron currents at each link at the lattice [31], appear due to the presence of RSOC in the longitudinal directions on a closed strip in equilibrium, that is in the absence of any external electromagnetic field. Their presence can be inferred at an effective level [31] or by the mathematical structure of the RSOC Hamiltonian [34, 35]. A breaking of translation invariance by adopting boundaries [31], or due to the presence of impurities [34] is necessary for the existence of the RHC. The RHC are qualitative different to the spin currents (studied in the next section) involved in the spin Hall effect that appear in the transversal direction as a response to an injected charge current in the longitudinal direction. These helical currents have been observed in multiterminal devices (see references in [31]).

In the following, results for the RHC correspond to the current of spin-up electrons at each chain of the strip.
FIG. 3. (Color online) Current of spin-up electrons on each chain as a function of the depth of the chain \( \nu \) (\( \nu = 0 \), edge, \( \nu = 1 \), center chain), for \( \lambda/t = 0.4 \) and, (a) \( J_{sd} = 10 \) (SP), and (b) \( J_{sd} = 5 \) (s-SP). Current of spin-up electrons on the edge chain, \( \nu = 0 \), as a function of \( \lambda/t \) for (c) \( J_{sd} = 10 \) and (d) \( J_{sd} = 5 \). Strip widths \( W \) are indicated in the plot. In (c) and (d) the edge currents for the fixed FM and AFM localized magnetic slab are also included with dashed lines. In (d) the fixed AFM results have been divided by 4 in order to fit in the scale of the plot. Electron filling \( n = 0.5 \).

In Figs. 3(a) and (b), the current of spin-up electrons at each chain on the planar strip, along the \( x \)-axis, is shown for the SP (\( J_{sd} = 10 \)) and s-SP (\( J_{sd} = 5 \)) regions respectively, as a function of the chain depth \( \nu \) (\( \nu = 0 \), edge, \( \nu = 1 \), center chain), for \( \lambda/t = 0.4 \). As expected, in both cases, the RHCs become concentrated at the strip edges as \( W \) is increased, and the decay of \( J_{\nu}(\nu) \) with \( \nu \) is faster for the staggered SP phase than for the SP one, where oscillations can be still observed for \( W = 32 \).

To study the dependence of the RHC with \( \lambda/t \), only the currents on the strip edge chain (\( \nu = 0 \)) will be considered. Results for \( J_{\nu}(\nu) \) as a function of \( \lambda/t \) are shown in Figs. 3(c) and (d) for the SP (\( J_{sd} = 10 \)) and s-SP (\( J_{sd} = 5 \)) regions respectively, for various strip widths. Notice that in Fig. 3(c), SP region, the sign of the currents has been changed in order to have a better comparison with the s-SP case.

It can be observed that the dependence of \( |J_{\nu}(\nu)| \) with \( \lambda/t \) is similar for both the SP and s-SP states, particularly for larger strip width where the results are also smoother, but the RHC are larger for the SP case. There is an approximately quadratic dependence for small \( \lambda/t \), as predicted in Ref. [31], saturating as \( \lambda/t \) approaches one. Notice an irregular behavior in the case of the s-SP (Fig. 3(d)) for large \( \lambda/t \), when the spiral order becomes an excited state and the system enters in the AFM region.

It is also interesting to compare the present results, where as said above, the spiral order is driven by the SO coupling in the conducting strip, to the two cases more considered in the previous literature [37] where a FM or an AFM order is fixed by a large exchange coupling \( |J| \) for all values of \( \lambda/t \) and \( J_{sd} \). By comparing the SP state with the fixed FM system, one should notice that the RHC for the later (dashed line in Fig. 3(c)) is approximately three times smaller than for the SP one for the same \( W = 32 \) strip, that is, the SP order favours the tendency to induce the RHC. On the other hand, by comparing the s-SP state with the fixed AFM order, it is remarkable that the RHC for the later (dashed line in Fig. 3(d)) are much larger than those for the staggered SP state for \( \lambda/t \lesssim 0.5 \), and of course much larger than the ones for the fixed FM case as originally noticed in Ref. [37].

It will be examined in the following section if any of these behaviors translate into transport properties that are more conventionally experimentally measured and more relevant for spintronics applications.

V. TRANSPORT PROPERTIES

The zero temperature optical conductivity is defined as the real part of the linear response to the electric field and can be written as [38]:

\[
\sigma(\omega) = D \delta(\omega) + \sigma^{reg}(\omega)
\]

\[
= D \delta(\omega) + \frac{\pi}{L} \sum_{n \neq 0} \frac{|\langle \Psi_n | j_x | \Psi_0 \rangle|^2}{E_n - E_0} \delta(\omega - (E_n - E_0))
\]

where \( |\Psi_n\rangle \) are the eigenstates of the total Hamiltonian with energy \( E_n \), \( |\Psi_0\rangle \) is the ground state, and the paramagnetic current along the \( x \)-direction is defined in terms of the currents defined in Eqs. 3(4) as:

\[
\hat{j}_x = \hat{j}_{hop,x} + \hat{j}_{SO,x}
\]

\[
\hat{j}_{hop,x} = \hat{j}_{\uparrow,x} + \hat{j}_{\downarrow,x}
\]

\[
\hat{j}_{\sigma,x} = \sum_l \hat{j}_{\sigma,l,x}, \quad \sigma = \uparrow, \downarrow
\]

\[
\hat{j}_{SO,x} = \sum_l \hat{j}_{SO,l,x}
\]

The Drude weight \( D \) is calculated from the f-sum rule as:

\[
\frac{D}{2\pi} = -\frac{\langle H_{0,x} \rangle}{2L} - I_{reg}
\]

where \( K_x \equiv -\langle H_{0,x} \rangle \) is the total kinetic energy of electrons along the \( x \)-direction, and

\[
I_{reg} = \frac{1}{L} \sum_{n \neq 0} \frac{|\langle \Psi_n | j_x | \Psi_0 \rangle|^2}{E_n - E_0}
\]

is the integral over frequency of \( \sigma^{reg} \) defined in Eq. 5. Clearly, \( I_{reg} \), and hence \( D \), will have contributions from matrix elements \( i_{\mu} = |\langle \Psi_n | j_{\mu,x} | \Psi_0 \rangle|^2, (\mu = hop, SO) \), and \( i_{crossed} = 2Re[\langle \Psi_n | j_{hop,x} | \Psi_0 \rangle \langle \Psi_0 | j_{SO,x} | \Psi_n \rangle] \).

The spin Hall conductivity is the main quantity describing the spin Hall effect because it involves spin
currents appearing in the transversal direction when a charge current is applied to a conductor in the longitudinal direction. The spin Hall conductivity \( \sigma_{xy} \) is defined as the \( \omega = 0 \) limit of the spin-charge transversal response function given by the Kubo formula, at zero temperature \[39, 40\]:

\[
\sigma_{xy}^{\text{sc}}(\omega) = -\frac{e}{\pi \hbar} \sum_{n} \sum_{m} \frac{\langle \Psi_n | \hat{j}_{y}^s | \Psi_m \rangle \langle \Psi_m | \hat{j}_{x}^s | \Psi_n \rangle}{(E_n - E_m)^2 - \omega^2} \quad (9)
\]

where \( \hat{j}_{y}^s \) is the spin current along the \( y \)-direction.

In the first sum, the summation is performed only over states with energies \( E_n \) larger than the Fermi energy \( E_F \), and in the second sum only over states with energies \( E_m < E_F \). The spin current operator at each bond connecting sites \( l \) and \( l + \hat{v} \), where \( \hat{v} \) is the unit vector along the \( x \) or \( y \) directions, follows from the local spin conservation operator equation in the absence of external torques: \( \sum_{\hat{v}} \hat{j}_{l,\hat{v}} + \partial S^I_l / \partial \tau = 0 \) (\( \tau \) is the time).

For the total \( \hat{j}_{y}^s \) the following expression is obtained \[37\]:

\[
\hat{j}_{y}^s = \hat{j}_{\text{hop},y}^s + \hat{j}_{\text{SO},y}^s,
\]

\[
\hat{j}_{\text{hop},y}^s = \frac{1}{2} (\hat{J}_{\uparrow,y} - \hat{J}_{\downarrow,y}),
\]

\[
\hat{j}_{\text{SO},y}^s = -\frac{\lambda}{2} \sum_{l} (c_{l+y}^\dagger c_{l+y}^\dagger - c_{l+y}^\dagger c_{l+y}^\dagger + H.c.) \quad (10)
\]

This expression of \( \hat{j}_{y}^s \) is the second quantized equivalent to the one formulated in first quantization and using a parabolic kinetic energy that was considered in previous calculations of the spin Hall conductivity \[39, 40\].

By replacing the longitudinal charge currents and the transversal spin currents into Eq. (10), in the same way as it was done for the optical conductivity, the contributions to the double sum can be classified according to the conserving or non-conserving currents involved. For example, there is a contribution from terms \( \langle \Psi_n | \hat{j}_{\text{hop},y}^s | \Psi_m \rangle \langle \Psi_m | \hat{j}_{\text{hop},x}^s | \Psi_n \rangle \), for short, and another contribution from \( \langle \Psi_n | \hat{j}_{\text{SO},y}^s | \Psi_m \rangle \langle \Psi_m | \hat{j}_{\text{SO},x}^s | \Psi_n \rangle \).

In Fig. 4(a), the integral of the regular part of the longitudinal optical conductivity, \( I_{\text{reg}} \), defined in Eq. (8), is shown for various strip widths as a function of \( \lambda/t \) for the spiral state with \( J_{sd} = 10 \). A strong increase of \( I_{\text{reg}} \) can be observed as \( W \) is increased and for large \( \lambda/t \).

Both matrix elements \( i_{\text{hop}} \) and \( i_{\text{SO}} \) contribute to \( I_{\text{reg}} \), with a slightly larger contribution from \( i_{\text{hop}} \), and the contribution from the crossed matrix elements changes sign from negative to positive at \( W = 8 \). The Drude peak, shown in Fig. 4(b), has two noticeable behaviors. In the first place, \( D \) decreases as the strip width increases in the whole range of \( \lambda/t \). This is clearly due to a reduction of the kinetic energy with increasing \( W \), since \( I_{\text{reg}} \) is negligible for small \( \lambda/t \). For large \( W \) and \( \lambda/t \gtrsim 0.4 \), a further decrease in \( D \) can be observed, in this case due to the increase in \( I_{\text{reg}} \) shown in Fig. 4(a). The Drude peak for the fixed FM state \[37\], for \( W = 32 \), is close to the one for the SP state for small \( \lambda/t \), but it is clearly larger for large \( \lambda/t \), indicating the expected favouring of transport by the FM state.

Fig. 5(c) shows the two nonzero contributions to the spin Hall conductivity \( \sigma_{xy} \) for various strip widths as a
function of $\lambda/t$ for the SP state. It can be seen that the main contribution, $\langle h_x h_y \rangle$, increases with both $W$ and $\lambda/t$, while the $(SO_x h_y)$ contribution decreases with $W$ and it even changes sign for large $\lambda/t$. The total $\sigma_{sH}$ is shown in Fig. 4(d), showing a clear enhance with increasing $W$, saturating at $W = 32$. The spin Hall conductivity in the Rashba strip with $W = 32$ coupled to a FM layer was also added for comparison. For the FM state, $\sigma_{sH}$, is about one order of magnitude smaller than the one of the SP order, as it can be seen in Fig. 4(d). It should also be noticed that $\sigma_{sH}$ for the fixed FM state is entirely due to the transitions $(SO_x h_y)^{\lambda/t}$.

Let us now examine transport properties for the staggered spiral state. In the following, in the whole interval $0 < \lambda/t \leq 1$ only results for the s-SP state will be reported even when it is an excited state and the true ground state is the AFM state, as discussed in Section III.

In Fig. 5(a), $I_{reg}$ is shown for various strip widths as a function of $\lambda/t$ for $J_{sd} = 5$. By comparing these results with those for the SP phase shown in Fig. 4(a) it is clear that $I_{reg}$ is smaller for the s-SP state, for the same values of $\lambda/t$ and $W$. In the second place, while in the SP case, for a given $W$, $I_{reg}$ has a smooth behavior with $\lambda/t$, particularly for wider strips, in the staggered case there is a clear jump at $\lambda/t = (\lambda/t)^*$ at which the s-SP state becomes an excited state. As in the SP case, the contribution from the matrix elements $t_{hopp}$ is slightly larger than the one from $iSO$, but for the staggered case the contribution from the crossed matrix elements are negative for all $\lambda/t$ and $W$. This behavior is translated to the Drude weight as shown in Fig. 5(b), where, for each $W$, a clear jump is observed at the same values of $\lambda/t$ where a jump appears in $I_{reg}$. Overall, the Drude weight is larger for the staggered case, where it is also absent the suppression of the kinetic energy term pointed out regarding Fig. 4(b). This is partially due to a smaller value of the conducting-magnetic slabs, $J_{sd}$ adopted for the staggered case. As expected, the Drude peak of the fixed AFM state, also included in Fig. 5(b) for $W = 32$, is strongly suppressed as $\lambda/t$ increases, which is opposed to the behavior above discussed for the FM state.

With respect to the spin Hall conductivity, it can be observed in Fig. 5(c) that, similarly to what was noticed for the SP case, in the s-SP state the main contribution comes from the terms $\langle h_x h_y \rangle$. Both contributions suffer a jump again at $(\lambda/t)^*$. The $\langle h_x h_y \rangle$ becomes larger with both $\lambda/t$ and $W$, while the $(SO_x h_y)$ contribution becomes negative and increases in absolute value with both $\lambda/t$ and $W$. As a result, the total $\sigma_{sH}$ shown in Fig. 5(d), increases with $W$ converging at $W \approx 32$, and reaching a maximum for each $W$ at $(\lambda/t)^*$. Another important conclusion is that $\sigma_{sH}$ for the staggered spiral phase is about a factor of 3 larger than the one for the spiral phase shown in Fig. 4(d). The values of $\sigma_{sH}$ for the fixed AFM state, $W = 32$, were also included in Fig. 5(d) for the sake of comparison. Notice that these values were divided by 2, and their sign was changed in order to fit into the plot scale. As for the fixed FM coupled magnetic slab, $\sigma_{sH}$ for the AFM case is entirely due to the $(SO_x h_y)$ contribution. Notice that $\sigma_{sH}$ for the fixed AFM coupled slab is still much larger than the one for the staggered spiral phase.

Finally, it is also interesting to discuss the intra- and inter-band contributions to $\sigma_{sH}$, which correspond to transitions between single-particle states with the same (opposite) chirality. The chirality of each single-particle state is defined by the sign of the $y$-component of the electron spin averaged on that single-particle state. As discussed in Ref. 32, this definition appears as a natural extension to strips of the corresponding concepts used for infinite 2D systems 39–41. Notice also that in the literature alternative definitions have been used 42.

Results for both types of contributions to the spin Hall conductivity, $\langle h_x h_y \rangle$ and $(SO_x h_y)$, for the spiral and staggered spiral states, are shown in Figs. 6(a,c) and Figs. 6(b,d) respectively. For both types of spirals, and for both types of contributions to $\sigma_{sH}$, a first conclusion is that there is approximately the same contribution from both inter and intra band transitions. For the spiral state, this behavior is different than for the fixed FM order where only interband transitions contribute. On the other hand, for the staggered spiral state, the behavior is similar to the one for the fixed AFM order where both types of processes are present although the interband ones are dominant. A second conclusion is that, consistently with the results presented so far, all the contributions have a rather smooth behavior except when the system starts to deviate from the longitudinal
spiral order, which occurs for large $\lambda/t$. This change of behavior is more clear for the $(SO_x h_y)$ contribution, and even more notorious for the staggered spiral state when it becomes an excited state (Figs. 6(d)).

To end this Section, let us briefly discuss the possibility of a spin current along the longitudinal direction. This current has also two components analogous to the expressions for $j_{sp}$ given by Eq. (10). The hopping part, $j_{hop,x}^{sp}$, is conventionally termed the spin polarized longitudinal current. By replacing $j_{sp}$ for $j_{sp}^{h}$ in the expression of the spin Hall conductivity Eq. (9), an analogous Kubo formula at zero frequency for the spin polarized conductivity, $\sigma_{sp}$, would be obtained. Let us call $\sigma_{sp,hh}$ the contribution to $\sigma_{sp}$ from the hopping charge current and $\sigma_{sp,hh}^{spin}$.

VI. CONCLUSIONS

The first conclusion of this work is that when the magnetic exchange between the magnetic moments is negligible, at quarter filling, a spiral (SP) order of the localized magnetic moments with transversal momentum equal to zero, that is, uniform across the strip section, exists for interlayer exchange coupling $J_{sd} \gtrsim 5$, while another spiral order with transversal momentum equal to $\pi$, that is, staggered across the strip section, dominates for $J_{sd} \lesssim 5$. In addition, this s-SP order is unstable towards an antiferromagnetic phase for $\lambda/t$ greater than a $J_{sd}$-dependent value. Both SP/$\pi$-SP and s-SP/AFM crossovers are of first order.

The second conclusion is that both spiral phases have an almost linear dependence of the spiral longitudinal momentum with $\lambda/t$ as long as this momentum is smaller than $\pi/2$. Since this linear behavior is essentially driven by the precession of independent electrons moving on the conducting slab, one could speculate that for large $\lambda/t$, effective interactions mediated by $J_{sd}$ lead to a loss of coherence of the electron gas, particularly for the small wavelengths corresponding to the longitudinal spiral momentum approaching $\pi/2$. In the SP order, there is then a noticeable departure from the linear behavior, and in the s-SP region, the whole spiral is finally replaced by an AFM order. In principle, these spiral states could be detected by neutron scattering techniques. However, since the magnetic layer should be a thin film, and in addition, be part of an heterostructure, there are other techniques that may be more appropriate. For example, there is a recently devised technique to detect spiral states on thin films using quantum sensors that can achieve resolution of a few nanometers [48].

As a summary of transport properties, including the Rashba helical currents, it is clear than in general the SP and s-SP states interpolate between the behaviors previously observed for the fixed FM and AFM orders. Still, there are considerable differences in the amplitudes that these properties have in the SP and s-SP regions, particularly a near factor of two in the spin Hall conductivities. In addition, the momentum or the period of the spiral state can be controlled by external electric fields through the RSOC leading to a multiferroic behavior, and at the same time, the different responses could also be exploited for spintronic applications. These results for transport properties can be experimentally verified by the usual techniques employed for studying the spin Hall effect, as reviewed in [7].

A key ingredient of a device that could take advantage of the present results, and that indicates a departure from the devices containing FM or AFM layers studied so far, is to have a very low exchange coupling $J$ between the localized magnetic moments. These virtually noninteracting magnetic moments could be created in the first place by depositing magnetic Fe or Co atoms or nanoparticles on top of a conducting Rashba layer. A second possibility is to employ a ferromagnetic semiconductor for the magnetic coupled layer, at a temperature above its Curie temperature, which for this class of materials is very low, within a setup similar to that of [10]. For this effect to be robust, the magnetic slab should be a thin film.

To some extent, a variation of $J_{sd}$ could also be achieved for a single device by the application of an external magnetic field perpendicular to the planes, although certainly this would not be practical. On the other hand, it is possible to set an electric field modulation of $J_{sd}$, which was intensively studied in the context of magnetic storage devices [19]. In particular, the mechanism of electric field modulation of the magnetic anisotropy consists in changing the electron occupancy at d-orbitals in coupled 3d-transition metal slabs by shifting the Fermi level and/or modifying the electronic structure close to the Fermi level. Since $J_{sd}$ contains the amplitude of the magnetic moments, it could acquire large values. On the other hand, since $J_{sd}$ is an effective coupling between the conducting and the magnetic layers, it can be made arbitrarily small by interposing layers of nonmagnetic insulating material between the magnetic moments and the Rashba conducting layer as in [8].

It is also worth to notice that the spiral state implies a breaking of translational invariance of the system along the longitudinal direction and hence a modulation at an effective level of the RSOC and hopping couplings. In this sense, it would be interesting to investigate if the presently studied system shows a spin Hall conductivity that remains robust against disorder as in a recently proposed model with modulated RSOC [50].

Further study at finite temperature and in out-of-equilibrium regimes would be necessary to assess the use-
fulness of the various features here reported.

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