The Thinking Process of Students in Representing Images to Symbols in Fractions

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Abstract. Thinking is the development of ideas and concepts within a person caused by the process of establishing relationships between parts of the information stored in a person. In the early stages, the object of thought is accepted by the senses, processed in the brain into concepts / ideas, and expressed into a representation. Elementary students who are generally in a concrete operational stage desperately need this representation to facilitate their understanding. This research is a qualitative research with the subject of 12 students grouped into 3 groups, namely: (1) group representation in the usual way; (2) representation group with percent; (3) representation group by degrees. This research is intended to describe the thinking process of students in representing images to symbols on fractional materials. The results show that the representations of these three groups are essentially similar. All of them follow the APOS process (action, process, object and schema). The difference only occurs in group 1, where group 1 tends to think procedurally, while groups 2 and 3 think analytically.

Keywords: thinking process, representations and fractions.

1. Introduction

Thinking is the development of ideas and concepts within a person [2]. The development of these ideas and concepts takes place through the process of establishing relationships between parts of the information stored in a person which is the meanings [18]. The process by which an object is captured by the student's senses. Then go into the brain for processing (with the concept / idea) which is translated through a language called representation.

Three types of mental model represented by Bruner, namely: (1) Enactive representation (enactive) is a motor sensory representation formed by action or movement; (2) Iconic Representation (iconic) relates to image or perception, which is a learning stage where knowledge is represented in the form of visual imagery, drawings, or diagrams depicting concrete or concrete activities contained in enactive stage; (3) Symbolic Representation (symbolic) relates to the language of mathematics and symbols.

Bruner also stated that representation plays a role in the process of solving mathematical problems. Manipulation of symbols, equations, tables, graphics, words and problem solving [14] depends on the student's representational skills. Students must experience many representations of different mathematical concepts [16].

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Representation has a meaning describing the relationship between objects with symbols [10]. The relationship between these objects and symbols is more concrete to represent more abstract concepts that serve as aids in solving mathematical problems [17]. Representations by [11] are everything students make to internalize and demonstrate their work is called representation.

Four representations according to [6], namely verbal repository, informational images, decorative images, and number lines that have a significant effect on students' mathematical problem solving abilities.

Identify five different types of representational systems in mathematical concepts that occur in learning mathematics and problem solving [13], namely: (1) experience-based "scripts"; (2) manipulative models; (3) drawings or diagrams; (4) oral symbols; (5) written symbols. The five different types of representational systems are described as follows:

![Figure 1. Five different types of representation systems [13]](image)

Not only the five different types of representational systems are important, but the (translation) relationship between representational modes, as shown by the double arrows crossing in the middle of the image is also important. A concept that has been understood when in a representational system should be able to be displayed in another form either in a system of representation (transformation) or in other representational systems. between different representations has the value of whether a student understands the conceptual problem. Some ways to show between representations are to ask students to restate the problem in their own words, draw diagrams to illustrate the problem, or act out.

Various representations need to be done by primary school students in understanding a concept that describes something else in some way [7]. Why? because students at the age of 7 to 11 years are in a concrete period. This concrete period of students is no longer dominated by perception, because it is able to think logically in solving problems through representation. Mental acts by students can go both ways interrelated with real and concrete objects.

Four stages of model emergence from real situation to formal situation (figure 2) according to [9].

![Figure 2. Level of model appearance from real situation to formal situation](image)

Example students convert integers into fractional forms, students use fractional multiplication procedures with fractions. The model used is a line of numbers or rectangles. Where the model is a bridge for students to move from formal to informal situation. The rectangular model can be in a referential position (model from real situation) or general (general model that can be used for various mathematical issues).
Fractional concept is not a simple concept. The uniqueness of fractions, different from the original and integer numbers, sometimes makes it difficult for students to understand ([15]; [8]) and makes it difficult to introduce to students [3]. The complexity of the characteristics and the understanding of the concept of fractions cannot be understood in a relatively short time[19]. Understanding the concept of fractions, starting with the real objects to obtain something abstract that is manipulating representatives (representations) fragments of real / concrete (enactive). Then the activity is expressed with images (iconic). Finally, the concept is represented as a symbol or mathematical notation (symbolic).

The process of describing the concept of this fraction has not been done by other researchers, usually done only to see the construction of students on fractions both based on apos theory [1], and research representations that have been described above. In this study, a description of students' thought processes in repressing images to symbols is seen from APOS Theory.

APOS Theory is divided into 4 stages; action-process-object-scheme [5]. This theory believes to be able to describe how the process of mathematical knowledge formed in students, and then can be used to see whether students have reached a certain stage or not.

APOS theory embraces the principle that there is a close relationship between the nature of mathematical concepts and their development in one's mind [5]. In constructing a concept [5], students have a tendency to use actions (actions), processes (processes), and objects (objects). The next three are organized in a scemas to solve math problems. This theory is known as APOS Theory (actions, Processes, Objects and Scemas).

The APOS theory for children in the concrete operation phase can be seen in Figure 3 depicting how mental constructions in APOS Theory is the formation of actions, which are reflected (interiorized) into processes, then encapsulated into objects (object), then the object can be decomposed (de-encapsulated) into a process. Action, process and object can be organized into a scheme (scemas), hereinafter abbreviated as APOS.

![Schema](image)

**Figure 3. Stages in Constructing Mathematics Knowledge for Elementary School Students**

Figure 3 represents the implementation of APOS Theory for children at the concrete operation stage. The illustration of the image illustrates how Action applied to Objects physically raises abstract Mathematical Objects in the child's mind. At the concrete operation stage, the Object on which the student's actions must be concrete. Objects that arise from encapsulation in an interiorized action are abstract, as for postsecondary students.

2. **Research methods**

This research includes explorative research type with qualitative approach [4], which is based on task and subject interview result. Subjects selected from the results of communication with teachers and pre-test results conducted by researchers to the subject. Then the subject is getting as many as 12 students from 30 students who have finished pre-test. The 12 students consisted of 7 students working in the usual way, 4 students with percent, and 1 student using degree. Problems are given in the form of routine problems and students are asked to solve the problem with think aloud. The given problem consists of a circular pitch problem consisting of a pink color with a half (consisting of 5 parts) and one part of 10 parts on a circle, orange color with 1 medium section (2 parts) of 10 parts on a circle, a
blue color and green one piece each of the 10 sections on the circle that the subject will finish by
writing the fractions in pink and orange and explaining how to get the answer.

From the results of 12 interviewed students, in grouped with 3 groups, namely group 1 thinking
process by representing fractions in the usual way, group 2 thinking process by representing fractions
with percent and group 3 thinking process by representing fractions by the way of degree.

3. Results and Discussion
Referring to the identification of five different types of representation systems in problem solving
[13], the students' thinking process in representing fractional problems can be seen in three subjects
who solve the fractional problem by using the usual means, percent, and level. Then the form of
subject thinking process is done by APOS approach [1].

This thinking process can be seen in the flow presented in each of the following ways:

3.1. The flow of thinking process S1 represents fractions in the usual way
The thinking process of S1 in representing the problem on the usual fractions only describes the
surface properties of the problem and tends to use standard procedures.

![Figure 4. The work of S1](image)

The thinking process of the work of S1 can be seen in Chart 1 below.

![Chart 1. Process of Thinking S1](image)

The initial scheme that S1 has on integrated problem solving is greater than the scheme on the
problem. Action performed S1 in the form of observing, cutting, counting, moving the hand to get a
fraction and write down how to find answers (shown yellow in the chart). While the process is done
S1 looking for ways (determine the number of ways) to complete the fraction on the issue of the circle.
The way it is done is the usual way (brown color). S1 knows there are other ways to complete the
fraction, but the way it is done is the usual way. Because S1 feels easier and more understanding in its
workmanship. The object set S1 is in the parts of the circle that are cut as large (blue). Next S1 do the
scheme of pink and orange (green) part.

3.2. Groove thinking process S2 with percent fraction representation
The thought process of S2 in representing the problem on fractions using a percent marked perception cuts the circle with the fingers into equal parts. Then on the piece of the circle is written fractions in percent.

**Figure 5.** The work of S2

The thinking process of the work of S2 can be seen in Chart 2 below.

**Chart 2.** Process of Thinking S2

The existing scheme (percent material is the result of construction) on the S2 in an integrated problem solving is greater than the scheme on the problem. Action done S2 in the form of observing, cutting, calculating and writing how to get the fractional value (shown yellow in the chart). While the process in doing S2 looking for ways (determine the number of ways) to complete the fraction on the issue of the circle (brown color). S2 does problem solving by using percent, because it is easier to understand in the process. The object set S2 is in the parts of the circle that are cut as large as moving the finger, then for the asked to write the fraction (blue). Next S2 menskemakan section of pink and orange (green) by explaining to the researchers at the time of the interview.

3.3. The flow of thought processes S3 represents fractions with degrees

The process of thinking S3 in representing the fractional problem with the degree of connecting the scheme to the scheme of construction results in degree in solving the problem.

**Figure 6.** The work of S3

The results of S3 work above are still not quite right on the 360°, 10 part division, then when S3 is interviewed and recalculated (without limited intervention, [12]), then S3 improves by moving hands on paper. The thought process of the improved S3 work on this work can be seen in Exhibit 3 below.
Chart 3. Process of Thinking S3

The scheme is connected to the degree that S3 has on integrated problem solving is greater than the scheme on the problem. S3 actions are observing, cutting, counting, writing how to get fractional values and writing how to find answers (shown yellow in the chart). While the process is done S3 determine the number of ways to solve the fraction on the issue of the circle (brown color). How to do is how to use the degree, because it is easier to understand and do it. The object specified S3 is the parts of the circle that are cut as large as moving the finger, then for the asked to write the fraction (blue). Next S3 menskemakan section of pink and orange (green) by explaining to the researchers at the time of the interview.

4. Conclusion
Students 'thinking processes in representing images to symbols in fractions are grouped into three groups: students' thinking processes in representing fractions in the usual way, students' thinking processes in representing fractions by percent, and students' thinking processes in representing fractions with degrees. The first group tends to think procedurally, while the second and third groups think analytically.

This process of thinking explores previously owned schemes (either a scheme already owned or a construction scheme of fractional learning) to solve almost the same fractional problems, from actions, processes, objects and schemes to fractions either with the fractional settlement, percent or degree. Students can perform image representations to symbols on fractions properly and correctly. It is characterized by the subject of developing ideas and concepts in solving fractional problems.

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