Asymptotic Analysis on LDPC-BICM Scheme for Compute-and-Forward Relaying

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Abstract—The compute-and-forward (CAF) scheme has attracted great interests due to its high bandwidth efficiency on two-way relay channels. In the CAF scheme, a relay attempts to decode a linear combination of transmitted messages from other terminals or relays. It is a crucial issue to study practical error-correcting codes in order to realize the CAF scheme with low computational complexity. In this paper, we present an efficient bit-interleaved coded modulation (BICM) scheme for the CAF scheme with phase shift keying (PSK) modulations. In particular, we examine the asymptotic decoding performance of the BICM scheme with low-density parity-check (LDPC) codes by using the density evolution (DE) method. Based on the asymmetric nature of the channel model, we utilize the population dynamics method for the DE equations without the all-zero codeword assumption. The results show that, for two-way relay channels with QPSK and 8PSK modulations, the LDPC-BICM scheme provides higher achievable rate compared with an alternative separation decoding scheme.

I. INTRODUCTION

Reliability and bandwidth efficiency of relays have been crucial issues in relay-based wireless communications including satellite communications and mobile communications. Recently, the compute-and-forward (CAF) scheme [1] attracts great interest because of its high bandwidth efficiency. In the CAF scheme, a relay tries to decode a linear combination of the transmitted signals from other relays and terminals. In the next time slot, the decoded message is transmitted to other relays and terminals. It helps the wireless communication system to increase its bandwidth efficiency. This concept is also crucial in the physical layer network coding, which has been studied extensively [2], [3]. In addition, the CAF scheme is effective for secure wireless communications [4].

In order to investigate the performance limits of the CAF scheme, we discuss the performance under the framework of the two-way relay channel. In the two-way relay channel (Fig. 1), two terminals named A and B try to exchange their messages \(X_A\) and \(X_B\), respectively, through bi-directional connections to the relay \(R\), instead of any direct connection to each other. It has two phases. First, the two terminals simultaneously send their messages to \(R\), which is called the multiple-access (MAC) phase. Second, the relay \(R\) sends the received information to both terminals A and B. In the second phase, it is sufficient that the relay \(R\) broadcasts the modulo sum \(X_A \oplus X_B\) to both terminals A and B because they have their own original messages. Therefore, the relay \(R\) may decode only the modulo sum \(X_A \oplus X_B\) in the MAC phase, which is called the CAF scheme. In contrast, Yedla et al. [5] proposed an efficient decoding strategy for decoding both messages \(X_A\) and \(X_B\) in the MAC phase, which is called the MAC separation decoding (SD) scheme (see Fig. 1). The SD scheme has an advantage in directly decoding the messages without losing information as in the CAF scheme while a decoder for the CAF scheme is much simpler than that for the SD scheme. It is thus crucial to analyze their decoding performance in the MAC phase to realize an efficient and practical relaying technique with high reliability and low complexity.

The key to realize a practical CAF scheme is a proper choice of error-correcting codes. The low-density parity-check (LDPC) code is a leading candidate due to its high reliability against various channel models including an additive white Gaussian noise (AWGN) channel [6]. Sula et al. proposed the LDPC coding technique for the CAF scheme with the binary phase shift keying (BPSK) modulation and evaluated its decoding performance by numerical simulations [7]. The authors theoretically analyzed the LDPC codes and its spatial coupling coding by the density evolution (DE) method and found that they exhibit higher reliability than the SD scheme [8], [9].

For realizing a CAF scheme with non-binary modulations, lattice-based CAF methods based on PAM or QAM modulation have been extensively studied [1], [10]. This is because the additive group property of lattices is well matched to the CAF scheme. Although the lattice-based CAF methods provide near optimal performance when SNR is high enough, there remain two issues regarding its practical implementation. The first one is the decoding complexity. In general, lattice decoding is computationally expensive and harder to implement compared with binary channel coding. The second problem is that the lattice-based method cannot apply to a PSK or QPSK modulation format, which is often employed in satellite communications.
In order to find a possible solution for these problems, we here discuss a bit-interleaved coded modulation (BICM) with \(2^K\)-PSK modulations as the basis of a CAF scheme. In the BICM scheme, a bit intercaler is set between an encoder and a mapper to separate the coding and modulation [11]. It remarkably reduces the computational cost of a decoder in spite of a small loss in decoding performance. Although the LDPC-BICM scheme for the CAF scheme is studied in [12], the analysis is only based on the numerical simulations. This means that theoretical analysis unveiling the achievable rate with belief propagation (BP) decoding remains open.

In this paper, we will present the asymptotic performance analysis of the LDPC-BICM scheme in the MAC phase of the CAF scheme on the two-way relay channel. First, we will propose a DE analysis of the LDPC-BICM scheme without any conventional approximations. Then, the achievable rate of the LDPC-BICM scheme for the CAF scheme is obtained by the DE analysis, which offers theoretical comparison with the alternative SD scheme.

II. LDPC-BICM SCHEME AND DENSITY EVOLUTION

Before describing the main results, we first propose a novel DE analysis of the LDPC-BICM scheme. Although the LDPC-BICM scheme has been extensively studied by a DE analysis [11], [13], the analysis is usually based on some approximations. A well-known approximation is the all-zero codeword assumption [14] in which a transmitter send an all-zero codeword as an instance of random codewords. Another approximation is the assumption on the distribution of extrinsic output values in the extrinsic information transfer (EXIT) chart analysis [15]. Unfortunately, these assumptions do not hold in general cases including the LDPC-BICM scheme on higher order modulations. Here, we will propose an alternative DE analysis based on that for asymmetric channels [16].

For simplicity, we consider the LDPC-BICM scheme for a single-access channel model such as a complex AWGN (CAWGN) channel. Let us consider a binary LDPC code \(C \subseteq \mathbb{F}_2^K\) (\(\mathbb{F}_2 \triangleq \{0, 1\}\)). At a transmitter, a message is encoded to \(x = (x^i_s)_{s=1,\ldots,K} \in C\). In the BICM scheme, the transmitter uses a bit interleaver \(\pi\) to remove correlations in the interleaved signal \(\pi(x)\). The signal \(\pi(x)\) is then modulated by a mapper and transmitted through a channel. A receiver attempts to demap and decode \(\pi(x)\) from the received signal \(y\) and detects the transmitted signal \(x\) using the interleaver \(\pi^{-1}\). Assuming that the code length \(n\) is sufficiently large and an interleaver for \(\pi\) is randomly generated, each element of \(\pi(x)\) is sufficiently uncorrelated and the receiver can decode \(x^i_s\) one by one. It makes the structure of the decoder considerably simple because a standard BP decoder for binary LDPC codes is available only by replacing its log-likelihood ratio (LLR) calculation unit. In fact, if \(p_{Y|\tilde{X}}(y|x)\) is the conditional PDF corresponding to the constellation map and channel model for each \(K\) bits \(\tilde{x} \in \mathbb{F}_2^K\), the LLR for the \(s\)th bit in \(\tilde{x}\) is given by

\[
\hat{\lambda}_s(y) \triangleq \ln \frac{\hat{L}_s[y|1]}{\hat{L}_s[y|0]} \quad (s = 1, \ldots, K),
\]

where \(\hat{L}_s[y|u] (u \in \mathbb{F}_2)\) is the likelihood function of \(\hat{x}\) whose \(s\)th bit is \(u\), which is defined by

\[
\hat{L}_s[y|u] \triangleq \frac{1}{2^{K-1}} \sum_{\tilde{x}:\tilde{x}_s = u} P_{Y|X}(y|\tilde{x}).
\]

The DE method is useful to analyze an asymptotic decoding threshold called BP threshold. Now we introduce the DE method for the LDPC-BICM scheme without conventional assumptions. Let us consider the \((d_v, d_c)\)-regular LDPC-BICM scheme, where \(d_v\) and \(d_c\) represent the variable and check node degrees, respectively. The conditional PDF \(Q^{(l)}(m|u)\) (resp. \(Q^{(l)}(\hat{m}|u)\)) denotes the PDF of a message \(m\) from a variable node to a check node (resp. \(\hat{m}\) from a check node to a variable node) with a transmitted bit \(u\) at the \(l\)th step. Following the DE equations for binary asymmetric memoryless channels [16], we have

\[
P^{(l)}(m|u) = \frac{1}{K} \sum_{s=1}^K \int dy \hat{L}_s[y|u] \prod_{d=1}^{d_v-1} d\hat{m}^d Q^{(l-1)}(\hat{m}^d|u) \times \delta \left(m - \hat{\lambda}_s(y) - \sum_{d=1}^{d_v-1} \hat{m}^d\right),
\]

\[
Q^{(l)}(\hat{m}|u) = \frac{1}{2^{d_c-2}} \sum_{(u^d) \in U^d_2} \int d\hat{m}^d \prod_{d=1}^{d_c-1} d\hat{m}^d P^{(l)}(m^d|u^d) \times \delta (\hat{m} - 2 \tanh^{-1} \left(\prod_{d=1}^{d_c-1} \tanh \left(\frac{m^d}{2}\right)\right)),
\]

where \(U^d_2 \triangleq \{(u^d) \in \mathbb{F}_2^d : \bigoplus_{d=1}^d u^d = 0, u^D = u\}\). To derive these equations, we use a condition that \(\pi\) is uniformly chosen from all the possible permutations. We assume that the bit index \(s\) in (1) becomes an independent random variable in the large-\(n\) limit due to the random permutation. This is a crucial assumption for the above DE analysis.

We then employ the population dynamics (PD) method to solve the DE equations efficiently. The method is popular in statistical physics [20] and have been applied to the DE analysis [8], [9]. Note that a conventional numerical analysis...
based on the fast Fourier transformation is also available but the PD method is more straightforward for evaluating \(3, 4\).

In the PD method, the PDFs \(P(\cdot | u)\) and \(Q(\cdot | u)\) \((u \in \mathbb{F}_2)\) are approximated by histograms of \(N\) samples. The parameter \(N\) is called the population size and the DE equations are exactly solved in the large-\(N\) limit. Each sample is recursively updated by an update rule written in a delta function \(\delta(\cdot)\) in \(3\) or \(4\) up to the \(T\)th iteration step. More detailed description is found in \(8\). In the BICM scheme, it is necessary to add a sampling step of \(s\) in \(3\) to the PD method. The LLR function \(L_s[y/u]\) is chosen by a sampled value of \(s\).

We now demonstrate the DE analysis of the LDPC-BICM scheme on a CAWGN channel. We use the QPSK modulation \((K = 2)\) with the Gray mapping. Figure \(2\) shows the BP thresholds as a function of the code rate \(R = K(1 - d_v/d_c)\). In the PD method, we set \(N = 10^5\) and \(T = 2000\), which is sufficiently large to evaluate a BP threshold for an AWGN channel with the BPSK modulation. For comparison, we show the symmetric information rate (SIR) defined as the mutual information between the transmitted signal and received signal assuming that the transmitted word is chosen uniformly. We find that the DE analysis reasonably obtains the BP thresholds of the LDPC-BICM scheme. It is emphasized that the result is asymptotically exact as shown in \(16\) because the analysis directly treats the asymmetric channel. This is the first case of evaluating BP thresholds by DE equations without any conventional approximations as far as the authors aware of.

### III. CAF and SD Schemes on MAC Phase

In this section, we define the CAF and SD schemes on the MAC phase of the two-way relay channel. Let us define a \(2^K\)-PSK modulation by a constellation mapper \(\mathcal{M} : \mathbb{F}_2^K \to \mathbb{C}\). We assume that two terminals A and B use the QPSK and 8PSK modulations in Fig. 3. For each time slot \(t (= 1, \ldots, n)\), A and B respectively transmit their messages \(X_A^{(t)}, X_B^{(t)} \in \mathbb{F}_2^K\). The two-way relay channel \(18, 19\) is then defined by

\[
Y^{(t)} = \mathcal{M}(X_A^{(t)}) + \mathcal{M}(X_B^{(t)})e^{i\theta} + W^{(t)},
\]

where \(Y^{(t)}(\in \mathbb{C})\) is a received signal at the relay \(R\) at time slot \(t\) and \(\theta\) is the phase difference between two terminals. Here, we assume that the perfect phase synchronization and perfect power control are available at \(R\) and \(\theta\) can be tuned to an arbitrary value. Each element of \(W^{(t)}\) is an i.i.d. complex Gaussian random variable with zero mean and variance \(\sigma^2\). Its PDF is given by

\[
F_c(w; \mu, \sigma^2) \triangleq \frac{1}{\pi \sigma^2} \exp \left(-\frac{|w - \mu|^2}{\sigma^2}\right) \quad (w, \mu \in \mathbb{C}) \tag{6}
\]

We use peak signal-to-noise ratio (PSNR) defined by \(\text{PSNR} \triangleq 10 \log_{10}(\max_{x \in \mathbb{F}_2^K} |r|^2/\sigma^2) \) [dB].

In the SD scheme, both terminals transmit their message with their encoding and the relay \(R\) decodes them simultaneously. In the CAF scheme, the relay \(R\) decodes only their modulo sum. In particular, in the CAF scheme with a degraded channel, to detect the transmitted signals from A and B, the relay \(R\) adapts the decoding scheme with the degraded channel, which is defined as follows. When the input signal is \(Z \in \mathbb{F}_2^K\), the output \(Y\) of the degraded channel is defined by

\[
Y \triangleq \mathcal{M}(X_A) + \mathcal{M}(Z \oplus X_A)e^{i\theta} + W, \tag{7}
\]

where \(X_A\) is the random variable uniformly chosen from \(\mathbb{F}_2^K\), and \(W\) is the complex Gaussian random variable with zero mean and variance \(\sigma^2\). The PDF \(p_{Y|Z}\) is given by

\[
p_{Y|Z}(y|z) \triangleq \frac{1}{2^K} \sum_{x_A, x_B \in \mathbb{F}_2^K, x_A \oplus x_B = z} F_c(y; \mathcal{M}(x_A) + \mathcal{M}(x_B)e^{i\theta}, \sigma^2). \tag{8}
\]

In other words, for each \(z \in \mathbb{F}_2^K\), we have the received constellation at \(R \[17, 18\]\), which is defined by \(M_{z, \theta} \triangleq \{\mathcal{M}(x_A) + \mathcal{M}(x_B)e^{i\theta} : x_A \oplus x_B = z\}\). The total received constellation \(M_{z, \theta}\) is represented by open points in Fig. 3.
IV. ASYMPTOTIC ANALYSIS OF LDPC-BICM SCHEME FOR CAF SCHEME

Now we turn to the main results on the LDPC-BICM scheme for the CAF scheme with the degraded channel. In the BICM scheme, we assume that two terminals use the same binary LDPC code \( C \) and both terminals and the relay use the same random interleaver \( \pi \). Then, the LDPC-BICM scheme is defined as described in Section II because of the linearity of LDPC codes. By using the PDF \( p_{Y|Z} \) in \((8)\), the LLR and the likelihood function read

\[
\lambda_s(y) \triangleq \ln \frac{L_s[y|1]}{L_s[y|0]}, \tag{9}
\]

\[
L_s[y|u] \triangleq \frac{1}{2K-1} \sum_{z:z=u} p_{Y|Z}(y|z), \tag{10}
\]

The received signal is decoded as explained in Section II. The only difference from Section II is to replace the LLR \((4)\) by \((9)\) and the likelihood function \((2)\) by \((10)\), respectively. Consequently, the LDPC-BICM scheme for the CAF scheme has an advantage in a simple decoding structure. Although the BICM scheme is also applicable to the SD scheme, its BP decoder is rather complicated because it decodes a pair of transmitted codewords \((21)\). In addition, the DE analysis in Section II is easily extended to the present LDPC-BICM scheme. In fact, we have the same DE equations with \((3)\) and \((4)\) with the above replacement. We can apply the PD method to the DE equations to estimate BP thresholds.

To compare the CAF and SD schemes, we utilize their SIRs, which express the asymptotic transmission rates of random linear codes. By using the uniform distribution \( P_Z \) on \( F_2^K \) and the PDF \( p_{Y|Z} \) given in \((5)\), the SIR of the CAF scheme with the degraded channel is given by

\[
I(Y; Z) = \sum_{z \in F_2^K} P_Z(z) \int_{\mathbb{C}} dy p_{Y|Z}(y|z) \log_2 p_{Y|Z}(y|z)
- \int_{\mathbb{C}} dy p_Y(y) \log_2 p_Y(y), \tag{11}
\]

while that of the SD scheme reads

\[
\frac{1}{2} I(Y; X_A, X_B) = \frac{1}{2} \int_{\mathbb{C}} dw F_c(w; 0, \sigma^2) \log_2 F_c(w; 0, \sigma^2)
- \frac{1}{2} \int_{\mathbb{C}} dy p_Y(y) \log_2 p_Y(y), \tag{12}
\]

where

\[
p_Y(y) \triangleq \frac{1}{2K} \sum_{x_A, x_B \in F_2^K} F_c(y; \mathcal{M}(x_A) + \mathcal{M}(x_B)e^{i\theta}, \sigma^2).
\]

Note that the SIR of the CAF scheme depends on a labeling of a constellation map while that of the SD scheme does not. It implies that an optimal labeling for the CAF scheme exists as studied in \((17)\). In this paper, we use the constellations in Fig. 2 which show reasonably high SIRs because no labels are overlapped at signal points in their received constellations. As shown in \((19)\), SIRs depend on the phase difference \( \theta \). Figures 4 and 5 respectively show the SIRs of both schemes for the QPSK and 8PSK modulation in Fig. 3 as a function of \( \theta \).

We find that the SIRs of the CAF scheme take maximum values at \( \theta = 0 \). In contrast, the SIR of the SD scheme is maximized at a nontrivial value of \( \theta \) because received constellation points of different transmitted signal pairs, e.g., \((X_A, X_B) = (10, 01), (01, 10)\), are overlapped when \( \theta = 0 \). It shows that the value of \( \theta \) is set properly when we compare both schemes. In the following experiments, we set \( \theta = \pi/4 \) for the QPSK modulation and \( \theta = \pi/8 \) for the 8PSK modulation.

Figure 6 shows the BP thresholds of some LDPC-BICM schemes with the QPSK modulation. We used \( N = 10^5 \) and \( T = 2000 \) in the PD method. The figure also shows the SIRs of the CAF and SD schemes with appropriate \( \theta \) to maximize them. We confirm that, in terms of the SIR, the CAF scheme is superior to the SD scheme in the high rate regime as well as the BPSK modulation \((9)\). On the other hand, when the rate \( R \) is below 1.0, the SD scheme becomes effective because the CAF scheme is based on the degraded channel. In addition, the BP thresholds of the LDPC-BICM scheme for the CAF scheme is superior to the SIR of the SD scheme. For example, the BICM scheme with \((3, 18)\)-regular LDPC codes has about 2.0 dB gain against the SIR of the SD scheme.

We also show the BP thresholds of LDPC-BICM schemes.
Fig. 6. The BP thresholds of the LDPC-BICM scheme (symbols) and SIRs of the CAF and SD schemes with the QPSK modulation (lines) as a function of the PSNR. Each label represents $(d_c, d_v)$ of the regular LDPC code ensemble.

Fig. 7. The BP thresholds of the LDPC-BICM scheme (symbols) and SIRs of the CAF and SD schemes with the 8PSK modulation (lines) as a function of the PSNR. Each label represents $(d_c, d_v)$ of the regular LDPC code ensemble.

The CAF scheme is superior to the SD scheme in terms of the SIR when the rate $R$ is larger than 1.8. For practical LDPC-BICM schemes, the BICM scheme with (3, 18)-regular LDPC codes has 1.0 dB gain compared with the SIR of the SD scheme. It is emphasized that the gain becomes larger if a practical error-correcting code is applied to the SD scheme.

V. SUMMARY

In this paper, we have studied the LDPC-BICM scheme for the CAF scheme with the degraded channel on the two-way relay channel. We have investigated its asymptotic decoding performance by using a novel DE method. The results show that the LDPC-BICM scheme exhibits higher reliability than the alternative SD scheme in the high rate regime. It indicates that the LDPC-BICM scheme for the CAF scheme is an effective and practical coded modulation scheme to realize efficient and reliable relaying because of its simple decoding structure, low computational cost, and high achievable rate.

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