Influence of Porous Material MR Structure on its Flow Characteristics

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Abstract. Existing presentation of MR (metal rubber) material structure by its hydraulic (average) diameter of inner space for hydrodynamic equations is non-complete for understanding of influence of structure on hydrodynamic characteristics of MR material. Analysis of existing results of MR structure researches for porous structures shows that relative geometrical size of porous structure provides large influence on its average diameter. Therefore, it is proposed in a present research to determine a thick-wall and thin-wall porous details made of MR material. It is shown that placement of elements of solid phase is connected with value of parameter of density of pore distribution by size $\alpha$, which is typical for gamma-distribution. Thus, it is proposed to use the dimensionless parameter $\alpha$ as characteristics of non-uniformity of MR material macrostructure. It is proved that for hydraulic equations the structure of porous detail made of material MR for direction perpendicular to liquid flow is determined univalent by effective hydraulic diameter proposed in the present paper. This diameter depends not only on average pores diameter but on parameter $\alpha$ too. Generalized dependency is obtained, it is possible to use it for calculation of hydraulic losses both in thick-wall and thin-wall details made of MR material by well-known and new technologies of manufacturing..

1. Introduction
Effective application of porous materials for hydro-gas-dynamic and heat-transport units is provided by choice of rational inner structure of materials which can improve its transport properties. It is evident that clear understanding of dependency of hydrodynamic characteristics of porous details of its structure features is necessary for it [1–6]. In the present time, a MR (metal rubber) material is used widely for manufacturing of porous details of engines and aircrafts. This material is made of metal wire spirals pressed as a shape of ready product by different ways proposed, for example, in [7–10]. The part made of MR material is shown in the Figure 1.
In [7, 8] details and units made of MR material for application for damping systems are presented. In [8, 9] ways of porous details manufacturing of MR material and research methods of these detail’s characteristics are given. Details manufactured by these technologies are applied as hydrodynamic elements of engines and power plants pipelines [3].

However, existed methods of obtaining of hydrodynamic characteristics of porous details made of MR material often provide large calculation errors [11].

The aims of the present research are analysis of existed equations for calculation of hydraulic losses in details made of porous MR material, and obtaining of parameters of this material structure, which determine a process of filtration of working liquids.

It is well-known that hydraulic losses for porous details depend on physical properties of working media, kinematic characteristics of flow and inner features of porous structure. For common case a dependency between all these factors is presented as equation

\[ \frac{\Delta p}{L} = f(V_x, D_x, \rho, \mu) \]  \hspace{1cm} (1)

here \( \Delta p = p_1 - p_2 \) is changing of pressure; \( p_1 \) is enter pressure; \( p_2 \) is exit pressure; \( L \) is length of porous detail; \( V_x \) is characteristic velocity of working media flow; \( D_x \) is characteristic size of detail porous structure; \( \rho \) is density of working media, \( \mu \) is coefficient of dynamic viscosity for liquid [3, 6, 11 etc].

There are many researches of influence of different parameters on value \( \Delta p/L \). For example, it is proved in [3] that laminar, turbulent and transitional from laminar to turbulent conditions of flow may be presented in details made of MR material.

In [3, 6 and 11] two dimensionless complexes were obtained by methods of theory of similarity and analysis of dimensionality:

\[ \pi_1 = \frac{\Delta p D_x}{L V_x^2 \rho} \] \hspace{1cm} (2)

\[ \pi_2 = \frac{\mu}{V_x D_x \rho} \] \hspace{1cm} (3)

Analogously to pipeline hydraulic, it is possible to transform these complexes to equations

\[ \xi = \eta = \frac{2 \Delta p D_x}{L V_x^2 \rho}; \hspace{1cm} Re = \frac{V_x D_x \rho}{\mu} \] \hspace{1cm} (4)

here \( \xi \) or \( \eta \) are coefficients of friction resistance, and \( Re \) is Reynolds number [3].

Thus, it is possible to present the equation (1) as criterial: for laminar flow

\[ \xi = \frac{A}{Re} \] \hspace{1cm} (5)
for transitional and turbulent flow (V>Vk) for Re > 28

\[
\xi = \frac{A}{Re} + B
\]  

(6)

Constant coefficients A1, A2, B for dependencies (5), (6) are obtained experimentally [3]. Authors of [3, 11] suggested to choose the hydraulic diameter of porous media as characteristic linear size of MR material structure.

Structure of MR material has no blind alleys, and all inner surface of material is moistened by liquid. Therefore, authors of [3, 11, 12 and 13] suggested choosing in absence of obliteration as characteristic size of porous media the hydraulic diameter of inner space

\[
D_s = d_s = d_c = \frac{\Pi d_{\Pi}}{(1 - \Pi)}
\]  

(7)

here \(d_s\) and \(d_c\) are hydraulic and average diameters of porous space of MR material; \(d_{\Pi}\) is wire diameter, \(\Pi\) is porousness of material.

An important peculiarity, as author of [3] think, is determination of hydraulic diameter in equation (7) mostly by structure characteristics of porous media: porousness \(\Pi\) and wire diameter \(d_{\Pi}\).

Thus, by results of [3], it is possible to characterize the structure of MR material by any constant coefficient \(A\) and hydraulic diameter \(d_s\) (7), which determines the size of pores of porous detail.

However experimental results for MR material presented on Figure 2 contradicts to this statement and shows that coefficient \(A\) for equations (5) and (6) is non-constant value.

Thus, it is possible to suppose that coefficient \(A_1\) of dependency (5) is parameter of structure too. Therefore, it is necessary to solve two important problems.

Problem 1. On a basis of analysis and synthesis of results of existing works on calculation of hydraulic losses in porous details made of MR material to obtain the geometrical parameter for more correct describing of MR structure.

It is possible to divide methods of porous materials structure on direct ones and indirect ones (it is described, for example, in [2]). A development of software and computing allows numerical modelling
of porous structures [14]. It allows obtaining of structures with preliminary required properties. MR material has stochastic structure. For correct description of MR structure, it is possible to obtain the most reliable data with direct methods of structure research. These methods provide immediate data which doesn’t need additional interpretation and treatment. It does not lead to additional errors for data, and obtained results are more reliable.

Let we consider of MR material inner structure research. Cylindric and plane details made of MR material were used as the researched specimens. Porousness Π (measured in %) of specimens was 55 – 85%. Statistic method of micro-section studying was used for research. It allowed obtaining of pores distribution by size for section of porous specimen with different structures. A distance between contours of elements of solid framework of porous space was taken as diameter of pore, as in [15]. The distance between little parts of framework contour was considered as random value equal to length of line segment d between two points on a surface of solid phase.

Obtained data allows calculation of statistic average distance dc and statistic dispersion Dc by equations [15]:

\[
d_{cs} = \sum_{i=1}^{k} d_{ci} f_{i} \Delta d_{i}
\]

\[
D_{c} = \sum_{i=1}^{k} (d_{ci})^{2} f_{i} \Delta d_{i} - d_{c}^{2} = \sigma_{c}^{2}
\]

Here dc is average value of distance for interval i; \( \sigma_{c} \) is mean square deviation for value d. The value of average distance for each interval was obtained as \( d_{ci} = (d_{i-1} + d_{i})/2 \), and \( \Delta d_{i} = d_{i} - d_{i-1} \).

Relative frequency \( f_{i}^{*} \) of appearance of size d in interval [di-1; di] was calculated by equation

\[
f_{i}^{*} = \frac{m_{i}}{N(d_{i} - d_{i-1})} = \frac{\Delta F_{i}^{*}}{\Delta d_{i}}
\]

here \( m_{i} \) is quantity of size value from interval i; di-1; di are boundaries of interval i; \( N = \sum_{i=1}^{k} m_{i} \) is total number of measurement; \( \Delta F_{i}^{*} \) is frequency of appearance of d in interval [di-1; di].

Analysis of results of research shows that average size of pores dc (7) is equal to average distance dc (8) (Figure 3) for detail with relative thickness \( \delta_{o} = \delta_{f} / S_{c} > 1 \), here \( \delta_{f} \) is minimal thickness of detail; Sc is diameter of wire spiral.

It was found that for details made of MR material and for relative thickness of wall \( \delta_{f} / S_{c} < 1 \) the average distances dc are not coincide with dependency (7) (Figure 4).

It is necessary to notice that for obtaining of average diameter by dependency (7) areas of moistening surfaces of wire spirals of MR material were taken into account, but areas of moistening surfaces of case in which the porous detail situated were not taken into account.

Therefore, the average diameter was obtained in [3, 7] for porous structure of MR material only. If to suppose that porous detail in the case is absent (condition \( \Pi = 1 \)), by equation (7) the value \( d_{c} \rightarrow \infty \). It is not right, because in this situation the hydraulic diameter is equal to hydraulic diameter of hole in the case, which contains the porous detail. For example, for ring channel \( d_{c} = d_{w} = 2\delta_{f} \).
Figure 3. Average distance between little parts of framework of details made of MR material for $\delta_f / S_c > 1$: $\nabla$ is average distance between little parts for radial section of specimen, $\bullet$ is average distance between little parts for axial section of specimen; 1 is calculated dependency (5).

Figure 4. Average distance between little parts of framework of details made of MR material for $\delta_f / S_c < 1$: $\nabla$ is average distance between little parts for radial section of specimen, $\bullet$ is average distance between little parts for axial section of specimen; 1 is dependency (5), 2 is dependency (9).

If to take into account the area of moistening surface of case contacted with MR material, the equation for hydraulic diameter of ring-type porous structure will be as

$$d_g = d_{cs} = \Pi d_{l1} / (1 + d_{l1} / 2\delta_f) - \Pi$$

(11)

If porous element is absent ($\Pi = 1$), value of average diameter will be equal to hydraulic diameter of hole in the case contain the porous structure. For example, for ring channel it will be equal $d_c = d_c = 2\delta_f$, it coincides with reality and with a rule of boundary transition.

Results of calculation by dependency (11) and experimental data are presented on Figure 3. As it is seen from Figure 4, dependency (11), in a difference on dependency (7), presents the experimental results for condition $\delta_f / S_c < 1$ more precisely.

Thus, it is possible to distinguish two types of porous details made of MR:
- thick-wall, for condition $\delta_f / S_c > 1$;
- thin-wall, for condition $\delta_f / S_c < 1$.

It is necessary to notice that for condition $\delta_f / S_c > 1$ calculation fulfilled for value of average diameter dc by equations (7) and (11) for $\delta_f / D_c = 1$, have difference between their values not more than 9%. If ratio of parameters increases, difference of values obtained by equations (7) and (11) approaches to 0.

Thus, dependency (11) generalizes both of cases. It is more general equation for middle diameter of pores in MR material both for thick-wall and thin-wall details.

Subsequently in the present paper the value of average diameter of pore dc is calculated by equation (11).

By results of structure researches the histograms were obtained for different types of researched specimens and different relative frequencies $f_i*(d)$, calculated by equation (10).
Figure 5. Density of distribution of distance d for plane specimens (\(\Pi = 0.6; d\Pi = 50\) micrometers; \(d_{c} = 75\) micrometers): a) is radial section; b) is axial section; 1 is density of \(\gamma\)-distribution, \(d_{c}\) is average distance between contours of elements of solid framework

An example of these histogram is presented on Figure 5.

By the shape of histograms, it is possible to conclude that density of pores size distribution is non-symmetric.

If to compare the shape of histogram and a shape of curve 1 on Figure 4, it is possible to propose a gamma-distribution for modelling of pores size distribution in details made of MR material. Function of density of distribution in this case is

\[
f(d) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} d^{\alpha-1} e^{-\beta d} \quad (12)
\]

here \(\Gamma(\alpha)\) is noncomplete gamma-function; \(\alpha\) and \(\beta\) are parameters of distribution obtained from equations

\[
\alpha = \beta d_{c} = d_{c}^{2} / D_{c} = (1 / K_{\beta}) \quad (13)
\]

\[
\beta = d_{c} / D_{c} \quad (14)
\]

Here \(K_{\beta}\) is coefficient of variation obtained by equation \(K_{\beta} = \sigma / d_{c}\), \(\sigma\) is mean square deviation of value to be determined [16].

Function \(f(d)\) of density of pores size distribution (12) is presented on Figure 5 as continuous line 1 and determined by size of pores \(d\) and parameters \(\alpha\) and \(\beta\), obtained by experimental researches of MR material porous structure. However, qualification of obtained histograms on coinciding with gamma-distribution by criterion of Pearson didn’t show 100% certainty of chosen function (12). However, it is possible to notice for this function that its region of definition is \(\alpha \in [0, \infty)\), \(\beta \in [0, \infty)\), \(d \in [0, \infty)\).

The proposed rule for pores size distribution isn’t contradicted to reality of MR material porous structure for boundary values of pores diameter.

If parameter \(\alpha \to 0\), \(d \to 0\). It means that the porous structure has discontinuity.

If parameter \(\alpha \to \infty\), the function \(f(d)\) tends to logarithmic normal distribution. For \(\alpha = \infty\) \(D = 0\). The structure is ideal porous media, and its average diameter is \(d_{c} = d\) [6]. However, it is impossible to obtain \(\alpha \to \infty\) for MR material structure on technological reasons of stochastic porous structure as a result of MR material manufacturing. As it is presented in data of experimental research of porous structure for detail made of MR value of \(\alpha\) is possible in a range from 0.6 to 3.6.

It is known that the shape of curve \(f(d)\) is determined by parameter \(\alpha\), and parameter \(\beta\) is a scale multiplier [16].
As generalization of presented analysis, it is possible to suppose that parameter \( \alpha \) of function (12) determines adequately the pores size distribution for MR material.

This connection of parameter \( \alpha \) and pores size distribution it is possible to consider, for example, as connection of distance between contours of elements of solid framework of porous space. It is possible to see it clearly for one-dimensional linear model presented on Figure 6.

Figure 6. Linear models for analysis of discrete structures; a is porous media for \( \alpha \to \infty, D=0 \); B is porous media for \( \infty > \alpha > 0, D \neq 0 \); Lc is basic length of model; lc is average distance between surfaces of little parts; dч is diameter of little part

Models of structure on Figures a and b have the same basic length \( Lc \) and average distance between surfaces of little parts \( lc \), However, structure parameters \( \alpha \) are different for these models. As it is seen from Figure 6b, distance between surfaces of little parts and placement of solid phase are different from these parameters of Figure 6a.

It is evidently that value of parameter of distribution \( \alpha \) is connected with position of elements of solid phase relatively each other independently on rule of pores size distribution. Therefore, it is possible to use the non-dimensional parameter \( \alpha \) as characteristics of inhomogeneity of macrostructure for MR material and other porous media with any other single-modal rule of pores size distribution.

To compare the porous media and to estimate an influence of their structures on processes of mass transport in it a scale characteristic of pore volume is necessary. It is proposed to use as this characteristic for porous media the average pore diameter \( dc \).

Let for two porous media the average diameters are equal \( dc1 = dc2 \), but \( \alpha1 \neq \alpha2 \). However, the relative position of framework elements is different, and it is possible to suppose that influence of porous media on process of mass transport is not the same.

Therefore, comparison of porous media with different structure inhomogeneity by average diameter of pores are non-correct. Only for equality of parameter of distribution \( \alpha \) and \( dc \) these structures to be compared have equal geometric configuration of porous space.

Therefore, it is proposed to use a geometric model from for estimation of influence of variation of pores by size in a volume of porous detail on hydraulic properties of porous structure. This model is set of capillary with different diameter. A size of capillary in direction perpendicular to working media flow is changed by arbitrary rule. For direction of working media flow the size of pores doesn’t change. All pores are connected with each other hydraulically.

It is proposed to use for this media as characteristic size of equation (1) its hydraulic diameter as

\[
D_h = \frac{4F}{k}
\]

here \( F \) is area of flow passage section in porous media; \( k \) is moistening perimeter.

It is possible to obtain the area of flow passage section for all pores of separate volume of porous media as
\[ F = \sum_{i=1}^{N} \frac{\pi \cdot d_i^2}{4} p(d_i). \]

A moistening perimeter of flow passage for all pores of separated volume of porous media equal to

\[ k = \sum_{i=1}^{N} \pi \cdot d_i p(d_i) \]

By equation (15) and dependencies for \( F \) and \( k \) it is possible to obtain equation for characteristic size of equation (1) as

\[ D_s = d_{fh} = d_{cs} \left( 1 + K^2 \right) \tag{16} \]

here \( d_{cs} \) is average diameter of pores obtained from equation (11); \( d_{fh} \) is effective hydraulic diameter of porous media with arbitrary single-mode rule of pores size distribution.

By equation (13) it is possible to obtain equation for \( d_{fh} \) for MR material as

\[ d_{fh} = d_{cs} \left( 1 + 1/ \alpha \right) \tag{17} \]

It is seen from equation (17) that the characteristic size of MR material porous structure is determined by two parameters \( d_{cs} \) and \( \alpha \).

Therefore, it is possible to assert that effective hydraulic diameter from equation (17) is more correct for determination of structure of porous detail made of MR material than this diameter from equation (7).

Problem 2. Obtaining of equations for calculation of hydraulic losses in MR material.

If to take into account equations (4), it is possible to obtain coefficient of resistance \( \xi_{fh} \) and Reynolds number \( \text{Re}_{fh} \) as

\[ \xi_{fh} = \frac{2 \Delta p L^2 d_{cs} (1 + 1/ \alpha)}{LV \rho}, \tag{18} \]

\[ \text{Re}_{fh} = \frac{V \rho d_{cs} (1 + 1/ \alpha)}{\mu} \tag{19} \]

Dependency of \( \xi_{fh} \) and \( \text{Re}_{fh} \) is obtained by equation

\[ \xi_{fh} = A_2 / \text{Re}_{fh} \tag{20} \]

for laminar flow and

\[ \xi_{fh} = A_3 / \text{Re}_{fh} + B_1 \tag{21} \]

for transient flow.

It is possible to obtain values of constants \( A_2, A_3 \) and \( B_1 \) from experimental researches of hydraulic losses in MR material by method presented in [3]. If to take into account these values and equations (18), it is possible to present equations (20) and (21) as

\[ \frac{\Delta P}{L} = \frac{240}{2 \Pi d_{cs}^2 (1 + 1/ \alpha)^2 } \mu V \tag{22} \]

for laminar flow and
\[
\frac{\Delta P}{L} = \frac{220 \mu V}{2 \pi d_{cs}^2 (1 + 1/\alpha)^2} + \frac{2 \rho V^2}{2 \pi d_{cs}^2 (1 + 1/\alpha)}
\]  

(23)

for transient flow of liquid in MR material.

Critical value of Reynolds number is \( Re_{hk} = 10 \).

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**Figure 7.** Dependency of hydraulic loss coefficient \( \xi_{fh} \) on Reynolds number \( Re_{fh} \) (\( d\Pi = 0.09 \text{ mm} \): \( \square \) - \( \Pi = 0.741, \alpha = 1.56 \); \( \blacksquare \) - \( \Pi = 0.752, \alpha = 1.48 \); \( \circ \) - \( \Pi = 0.608, \alpha = 1.32 \); \( \bullet \) - \( \Pi = 0.614, \alpha = 1.30 \)

Graphic interpretation of equations (20) and (21) is presented on Figure 7. Experimental data in a limit of error (15…20%) coincides with calculation results by equations (19). As characteristic linear velocity \( Vx \) an average velocity of flow in pores is taken. It is obtained by average volume velocity \( V \) and porousness \( \Pi \). The effective hydraulic diameter is taken as characteristic size of porous structure. This diameter takes into account both the average diameter of pores \( d_{cs} \), and relative variation of its size \( 1/\alpha \). As it is follow from equations (22) and (23), \( \alpha \) in the range of its changing has influence on hydraulic losses in porous details made of MR material.

It is possible to use equations (22) and (23) for calculation of hydraulic losses in details made of MR material by well-known and new developed technologies.

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**2. Conclusion**

1. The existed presentation of MR material by its hydraulic (average) diameter for hydrodynamic equations is not completely enough for understanding of influence of material structure on its hydrodynamic characteristics.
2. Researches of MR material structure by direct observation methods show, that for condition \( \delta_f/D_c < 1 \) (differently of \( \delta_f/D_c > 1 \)) the relative minimal geometric size of porous detail has large influence on average diameter of porous media. Therefore, it is useful to separate this type of detail as a type of thin-wall structures made of MR material.
3. The relative placement of solid phase elements connects with value of parameters of density of pores size distribution \( \alpha \). Therefore, it is possible to use the dimensionless parameter \( \alpha \) as characteristic of inhomogeneity of macrostructure of MR material.

The scale characteristics of porous volume as average diameter of pores \( d_{cs} \) and parameter of inhomogeneity \( \alpha \) are necessary for comparison of porous media and estimation of influence of its structure in processes of mass transport.
4. It is possible to determine the structure of porous detail made of MR material for hydraulic equations by effective hydraulic diameter \( dh_{fh} \) obtained by equation (17).
5. More precise dependencies for calculation of hydraulic losses in details of MR material are obtained. These dependencies take into account peculiarities of inner structure of detail determined by average diameter of pores $d_{cs}$ and parameter of structure inhomogeneity $\alpha$.

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