Violation of $k_\perp$ factorization in quark production from the Color Glass Condensate

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We examine the violation of the $k_\perp$ factorization approximation for quark production in high energy proton-nucleus collisions. We comment on its implications for the open charm and quarkonium production in collider experiments.

1. Introduction

Semi-hard processes, where $\sqrt{s} \gg m_{q_\perp} \gg \Lambda_{\text{QCD}}$, contribute significantly to particle production in high-energy collider experiments due to the large density of the small-$x$ gluons. The $k_\perp$ factorization formalism [1] systematically resums corrections of $(\alpha_s \ln(s/q_\perp^2))^n$ from gluon branchings in perturbative QCD. In this framework, the particle production cross-section is expressed as a convolution of a hard matrix element and unintegrated distributions of gluons in the hadrons with definite transverse momentum $k_{i\perp}$ and longitudinal fraction $x_i$ in each projectile hadron ($i=1, 2$).

Multiple-scattering (higher twist) effects become important at small $x$ due to the large density of small-$x$ gluons. It is expected to be the origin of the Cronin enhancement and $p_\perp$ broadening of hadrons observed in nuclear experiments. It is also relevant for the nuclear suppression of quarkonium production.

The simplest situation for studying the impact of higher twist effects on $k_\perp$ factorization is in proton-nucleus (pA) collisions, wherein the proton is dilute and the nucleus is dense. The $k_\perp$ factorization formalism was examined in the color glass condensate framework [2]. It is shown that the factorization is recovered when one keeps only the terms that are of the lowest order in the charge sources $\rho_{p,A}$ of the projectiles [3]. The cross-sections at the leading order in $\rho_p$, but at all orders in the dense source $\rho_A$ of the nucleus are obtained analytically. Gluon production by the “2-to-1” processes is shown to be $k_\perp$-factorizable [4, 5, 6] whereas the quark production is generally not [7, 8, 9].

Here we report the numerical estimates for the $k_\perp$ factorization breaking in quark production within the McLerran-Venugopalan (MV) model [10]. We briefly discuss open charm production and quarkonium suppression in pA collisions in this framework.
2. Violation of $k_\perp$ factorization in quark pair production

The quark pair production cross-section is obtained as [7]:

$$\frac{d\sigma}{d^2p_\perp d^2q_\perp dy dy} = \frac{\alpha_s^2 N}{8\pi^4(N^2-1)} \int_{k_{1\perp},k_{2\perp}} \frac{\delta(p_\perp + q_\perp - k_{1\perp} - k_{2\perp})}{k_{1\perp}^2 k_{2\perp}^2} \times \left\{ \int_{k_{1\perp},k_{1}'\perp} \text{tr}_d \left[ (\bar{q} + m)T_{qq}(\bar{y} - m)\gamma^0 T_{qq}^\dagger \gamma^0 \right] \phi_{A}^{\bar{q}q}(k_{2\perp}; k_{1\perp}, k_{1}'\perp) \right. $$

$$+ \int_{k_{1\perp}} \text{tr}_d \left[ (\bar{q} + m)T_{qq}(\bar{y} - m)\gamma^0 T_{g}^\dagger \gamma^0 + \text{h.c.} \right] \phi_{A}^{qg}(k_{2\perp}; k_{1\perp}) $$

$$\left. + \text{tr}_d \left[ (\bar{q} + m)T_{g}(\bar{y} - m)\gamma^0 T_{g}^\dagger \gamma^0 \right] \phi_{A}^{gg}(k_{2\perp}) \right\} \varphi_p(k_{1\perp}), \tag{1}$$

where the explicit forms for the Dirac matrices $T_{qq}(k_{1\perp}, k_{1\perp})$ and $T_{g}(k_{1\perp})$ are given in [7]. Here $\varphi_p(l_{\perp}) \equiv (\pi^2 R_p^2 l_{\perp}^2 / l_{\perp}^2)$ F.T. $\langle \rho_p^a(0) \rho_p^a(x_{\perp}) \rangle$ is the unintegrated gluon distribution for the proton, and F.T. denotes the Fourier transformation. One needs, however, three nuclear distributions defined as (see Eqs. (42), (43) and (45) in [7])

$$\phi_{A}^{\bar{q}q}(l_{\perp}) \equiv \frac{\pi^2 R_A^2 l_{\perp}^2}{g^2 N} \text{F.T. tr } \langle U(0)U^\dagger(x_{\perp}) \rangle,$$

$$\phi_{A}^{qg}(l_{\perp}; k_{1\perp}) \equiv \frac{2\pi^2 R_A^2 l_{\perp}^2}{g^2 N} \text{F.T. tr } \langle \bar{U}(x_{\perp})t^a U^\dagger(y_{\perp})U(0) \rangle,$$

$$\phi_{A}^{gg}(l_{\perp}; k_{1\perp}; k_{1}'\perp) \equiv \frac{2\pi^2 R_A^2 l_{\perp}^2}{g^2 N} \text{F.T. tr } \langle \bar{U}(0)t^a \bar{U}^\dagger(y_{\perp})U(x_{\perp})t^b U^\dagger(y_{\perp}) \rangle, \tag{2}$$

where $U$ and $\bar{U}$ denote the path-ordered exponentials of the gauge fields in the nucleus in the adjoint and fundamental representations, respectively, and describe the multiple scatterings of the gluon and the quarks. The average $\langle \cdots \rangle$ is taken over the Gaussian distribution of the color charge sources characterized by the saturation scale $Q_s^2$.

$k_{\perp}$ factorization is violated by the transverse structure of the quark pair probed by the momentum $k_{\perp}^{(1)}$ from the nucleus since each quark from the pair can resolve and interact with several gluons from the nucleus. If any of the transverse masses $m_{q_{\perp}}$ and $m_{p_{\perp}}$ of the produced quarks is large compared with the typical rescattering scale, $Q_s$, we can neglect $k_{\perp}^{(1)}$ in $T_{qq}(k_{1\perp}, k_{1\perp}^{(1)})$ and recover the $k_{\perp}$ factorized formula thanks to the sum rule for $\phi_A$’s:

$$\int_{k_{1\perp},k_{1}'\perp} \phi_{A}^{\bar{q}q}(l_{\perp}; k_{1\perp}, k_{1}'\perp) = \int_{k_{1\perp}} \phi_{A}^{qg}(l_{\perp}; k_{1\perp}) \phi_{A}^{gg}(l_{\perp}; k_{1\perp}) \quad .$$

In Fig. 1 we compare the exact result with the $k_{\perp}$ factorized approximation for single charm quark production. The breaking is relatively small for the saturation momentum $Q_s^2 = 1 \text{ GeV}^2$, which may be the relevant scale for RHIC at central rapidity. At $Q_s^2 = 15, 25 \text{ GeV}^2$ (corresponding to very forward rapidities in the proton fragmentation region at RHIC and LHC) the correction can be as large as 40% at $q_{\perp} \sim Q_s$. For the bottom quark production the violation is smaller. To assess the model-dependence of our results, we compute them now, shown in Fig. 2, with a non-local Gaussian model known to be the
asymptotic solution of renormalization equations for $x$ evolution\cite{11}; non-linear evolution effects reduce the magnitude of the violation of $k_{\perp}$ factorization.

In Fig. 3 shown is the total $P_{\perp}$ distribution of the charm quark pair with the fixed invariant masses $M=3.1, 4, 8$ GeV. In the $k_{\perp}$ factorized approximation (thin curves), either quark or antiquark exchanges all the momentum from the nucleus and we see the bump structure near $Q_s$, reflecting the gluon distribution of the nucleus. The bump is smeared out due to multiple scatterings of both the quark and antiquark in the full formula. Integrating over $P_{\perp}$, we show in Fig. 4, the magnitude of factorization breaking in the invariant mass spectrum of the pair.

3. Phenomenology

We study the importance of small-$x$ distributions in D meson production by convoluting the single quark spectrum with an appropriate fragmentation function\cite{12}. We find, however, the production spectrum is determined not by the quark distribution with $q_{\perp}\ll Q_s$, but largely by the tail part $\propto 1/q_{\perp}^4$ of the MV model. Moreover, in order to assess
the rapidity dependence of open charm production, the $x$-dependence of the unintegrated gluon distributions should be taken into account, which requires going beyond the MV model. Our results on open charm production will be reported elsewhere[13].

The $Q_s^2$-dependence of the pair spectrum (divided by the charge density $\mu_A^2$) is displayed in Fig. 5. At larger $M$, where the high-density effects are diminished, all curves converge to a single one. The multiple scatterings of the pair quarks suppress the yield in the low $M$ region. (The overall cross-section is of course enhanced with increasing $Q_s^2$.) One can get an idea about the normal suppression of the quarkonium production in the pA collisions, relying on the color evaporation picture. We show the nuclear modification ratio, $R_{pA}$, for the pairs with $M$ less than the open charm threshold $2M_D$, as a function of $Q_s^2$. The suppression pattern fits the form $1/(Q_s^2)\alpha$ with $\alpha \sim 0.42$, and not the frequently assumed exponential form. One should note here that $Q_s^2 \sim A^{1/3}$ in the MV model.

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