PWM rectifier impedance modelling and analysis

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Abstract. PWM rectifiers are widely used in the field of new energy power generation. The study of small signal impedance modeling of PWM rectifier is helpful to analyze the stability of new energy grid connected impedance. In this paper, the small-signal impedance model of the PWM rectifier is first established, and then the impedance model of the PWM rectifier is scanned through MATLAB using the impedance sweep method, and compared with the analytical expressions in this paper, the accuracy of the impedance model of PWM rectifier established in this paper is verified.

1. Introduction

PWM rectifier is widely used in new energy grid-connection due to its advantages of unit power operation, sinusoidal current at the grid side, and stable voltage at the DC side, etc. Many power transformation systems, such as wind turbines, photovoltaic grid-connection converters and energy storage converters, have adopted PWM rectifier as grid-connection device [1-4]. However, after the PWM rectifier is connected to the power grid, it will interact with other power electronic devices and affect the stability of the entire power grid [5-7]. Therefore, establishing the impedance model of PWM rectifier for the stability of power grid is the premise of analyzing the stability of electric power electronic power system.

At present, scholars at home and abroad have done a lot of research on impedance modeling and stability analysis of PWM rectifier [8-11] and other aspects. In reference [8], a decoupled linearized small signal model based on three-phase voltage and phase current was established in a synchronous rotating coordinate system (d-q coordinate system), and an impedance model was obtained by combining the voltage and current double closed-loop control strategy. Literature [9] proposed a method to determine the stability of the grid-connected system by using the output impedance of the rectifier and the grid impedance. Literature [10] puts forward a stability evaluation criterion of power system based on impedance. For the cascade system composed of PWM rectifier and backstage grid-connected equipment, the steady-state working point impedance can be measured by current injection method for its dual-port small signal model, and then the stability of the system can be judged. In reference [11], according to the physical characteristics of the output side of PWM rectifier, divides it into impedance and admittance and establishes an equivalent circuit for analysis.

According to the above method, this paper firstly establishes a small signal model of PWM rectifier in the d-q system, and then according to its voltage outer loop and current inner loop control strategy
of impedance model, reuse based on current impedance scanning method to get the static working point of the disturbance of the impedance characteristics, compare the two methods and, finally, the simulation results prove the validity of the method of impedance scanning.

2. Structure and principle of three-phase PWM rectifier

Figure 1 is the circuit topology diagram of three-phase PWM rectifier with pure resistance when it operates independently [12]. In the figure, \( V_{p}, V_{b}, \) and \( V_{c} \) represent the ideal three-phase balanced grid voltage; \( L \) represents the filter inductance on the AC side of the three-phase PWM rectifier; \( i_{a}, i_{b}, \) and \( i_{c} \) represent the grid current; the three-phase full bridge PWM circuit is composed of six switch tube, \( S_{a}, S_{b}, S_{c}, S_{d}, S_{e}, \) and \( S_{f} \); \( V_{a}, V_{b}, \) and \( V_{c} \) represent the voltage of the three-phase PWM rectifier after modulation; \( C_{DC} \) is the DC side capacitor; \( V_{dc} \) and \( I_{dc} \) is the output voltage and current of three-phase PWM rectifier DC side respectively; \( R \) is the DC-side pure resistive load.

![Figure 1. Three-phase PWM rectifier structure diagram.](image1)

![Figure 2. Three-phase PWM rectifier structure diagram.](image2)

The three-phase PWM rectifier could convert the input three-phase AC voltage and current into DC voltage and current; in order to ensure the normal operation of the later stage converter, the voltage outer loop and current inner loop are used in the control strategy to ensure the stability of the output voltage on the DC side; and at the same time, ensure that the AC side power factor is 1, so that the current reactive component of the three-phase PWM rectifier and the cascade system of the latter stage is minimized.

The control strategy of the three-phase PWM rectifier is shown in Figure 2[13]. Since it is difficult to realize the control of voltage and current without steady-state difference in the ABC static coordinate system, and the time-varying AC quantity is not easy to control, so it is converted into a DC quantity in the d-q rotating coordinate system for decoupling control, and establish the corresponding double-loop control strategy of current inner loop and voltage outer loop. Under normal circumstances, the reference value of the output voltage on the DC side is given \( v_{dc} \), and the voltage outer loop controller outputs the reference value \( i_{d} \) of active current through PI control; the reference value \( i_{q} \) of the reactive current is generally set to 0.

3. Small signal model of three-phase PWM rectifier

To obtain a linear, time-invariant small-signal model of a three-phase PWM rectifier, the average mathematical model of the three-phase PWM rectifier needs to be established first, and the specific derivation process is as follows:

\[
\begin{bmatrix}
\frac{dv_{pd}}{dt} \\
\frac{dv_{pg}}{dt} \\
\frac{dv_{ps}}{dt}
\end{bmatrix} = L \begin{bmatrix}
\frac{di_{pd}}{dt} \\
\frac{di_{pg}}{dt} \\
\frac{di_{ps}}{dt}
\end{bmatrix} + \begin{bmatrix}
v_{a} + v_{m0} \\
v_{b} + v_{m0} \\
v_{c} + v_{m0}
\end{bmatrix}
\]

(1)
After averaging the switches:

\[
\frac{d}{dt} \begin{bmatrix} i_s \\ i_d \end{bmatrix} = \frac{1}{L} \begin{bmatrix} v_{sp} \\ v_{sq} \end{bmatrix} - \frac{1}{L} \begin{bmatrix} 2d_d - db - dc \\ 2db - da - dc \end{bmatrix} V_{dc}
\]

(3)

\[
\frac{dV_{dc}}{dt} = \frac{1}{C_{dc}} \left[ d_d \, db \, dc \right] \begin{bmatrix} i_s \\ i_d \end{bmatrix}^T - \frac{1}{RC_{dc}} V_{dc}
\]

(4)

The average model in the ABC static coordinate system obtained by equations (2-3) and (2-4) is still a time-varying model, and its steady-state operating point could not be obtained. In order to derive the output impedance model of the three-phase PWM rectifier, the time-varying average model in the stationary coordinate system is transformed into the time-invariant average model in the d-q rotating coordinate system. The specific method is to transform the ABC static coordinate system into a d-q rotating coordinate system, to synchronize the angular velocity of the d-axis rotation with the space vector of the AC quantity in the static coordinate system; the d-q coordinate transformation is performed on the average model in the static coordinate system to obtain the d-q rotating coordinate system. The average model under the system changes the time-varying variable in the average model into a time-invariant variable, which could be obtained after simplification:

\[
\frac{d}{dt} \begin{bmatrix} i_s \\ i_d \end{bmatrix} = \frac{1}{L} \begin{bmatrix} v_{sp} \\ v_{sq} \end{bmatrix} - \frac{1}{L} \begin{bmatrix} 0 \\ \omega \end{bmatrix} \begin{bmatrix} 0 \\ i_s \end{bmatrix}
\]

(5)

Similarly, the formula could be obtained:

\[
\frac{dV_{dc}}{dt} = \frac{3}{2C_{dc}} \left[ d_d \, db \, dc \right] \begin{bmatrix} i_s \\ i_d \end{bmatrix}^T - \frac{V_{dc}}{RC_{dc}}
\]

(6)

The mathematical model at this time is a time-invariant model, but it is still a nonlinear model and could not be directly used to derive the output impedance. At this time, the average mathematical model of the three-phase PWM rectifier in the d-q coordinate system is DC, and a steady-state operating point could be found, as the average mathematical model is considered to be linear in a small range around it. Add small signal disturbance \( \frac{\Delta x}{x} \) to the steady-state working points of formula (2-5) and (2-6), which is \( x = X + \xi \), \( u = U + \eta \), and ignore the quadratic term of small signal disturbance and eliminate the steady-state quantity. The small signal model of three-phase PWM rectifier could be obtained as follows:

\[
\frac{d}{dt} \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \frac{1}{L} \begin{bmatrix} \eta_{sp} \\ \eta_{sq} \end{bmatrix} - \frac{1}{L} \begin{bmatrix} D_s \xi \\ D_d \eta \end{bmatrix} V_{dc} - \frac{1}{L} \begin{bmatrix} 0 \\ \omega \end{bmatrix} \begin{bmatrix} 0 \\ \xi \end{bmatrix}
\]

(7)

\[
\frac{dV_{dc}}{dt} = \frac{3}{2C_{dc}} \left[ D_s \xi + D_d \eta \right] \begin{bmatrix} i_s \\ i_d \end{bmatrix}^T - \frac{V_{dc}}{RC_{dc}}
\]

(8)

**Figure 3.** Equivalent circuit diagram of three-phase PWM rectifier small signal model.
The corresponding equivalent circuit diagram at this time is shown in Figure 3:

Transform the small signal model in the time domain into a small signal model in the frequency domain:

\[
\begin{bmatrix}
\hat{\phi}(s)
\end{bmatrix}
= \frac{1}{sL} \begin{bmatrix}
\hat{V}_{ph}(s)
\end{bmatrix}
- \frac{1}{L} \begin{bmatrix}
D_d
\end{bmatrix} \begin{bmatrix}
\hat{I}_{ch}(s)
\end{bmatrix}
- \frac{1}{L} \begin{bmatrix}
D_q
\end{bmatrix} \begin{bmatrix}
\hat{I}_{ch}(s)
\end{bmatrix}
V_{dc}
- \begin{bmatrix}
0
-\omega
\end{bmatrix}
\begin{bmatrix}
\hat{V}_{dc}(s)
\end{bmatrix}
\]

\[
\hat{V}_{dc}(s) = \frac{3}{2C_{dc}} D_d \begin{bmatrix}
\hat{I}_{ch}(s)
\end{bmatrix}
- \frac{3}{2C_{dc}} D_q \begin{bmatrix}
\hat{I}_{ch}(s)
\end{bmatrix}
\begin{bmatrix}
I_d
I_f
\end{bmatrix}
\frac{V_{dc}}{RC_{dc}}
\]

(9)

After the above derivation, the small signal model of three-phase PWM rectifier is obtained. Combined with the control strategy of voltage outer loop and current inner loop, the d-axis DC output impedance of the three-phase PWM rectifier is finally obtained:

\[
Z_o(s) = \frac{Ls + r + G_i(s)}{(sC_{dc} + 1/R) [Ls + r + G_i(s)] + [1.5V_{dc} - 1.5LsL + r]G_i(s)G_i(s)/V_{dc}}
\]

(10)

4. Simulation analysis of small signal model of three-phase PWM rectifier

A closed-loop control simulation model of three-phase PWM rectifier is built by using simulation software. The simulation parameters are shown in Table 1.

| Parameter | Value |
|-----------|-------|
| Input AC line voltage | 100V |
| Output DC voltage | 200V |
| AC filter inductance | 1.8mH |
| DC filter capacitance | 2.2mF |
| Pure load | 36Ω |

**Parameters of voltage outer loop controller**

- Scale: 0.75
- Points: 50

**Current inner loop controller parameters**

- Scale: 10
- Points: 400
- Switching frequency: 10kHz

The simulation conditions are as follows: the entire simulation time of the three-phase PMW rectifier is 0.3s. For the three-phase PWM rectifier, the double closed-loop control strategy of current inner loop and voltage outer loop is adopted. The output voltage of DC side is compared with the reference value of DC side. The rear output is the reference value of the current inner loop, and the initial voltage on the DC side is 160V. The current inner loop compares the current sampling of the three phases after inductance filtering with the reference value of the current inner loop, and sets the q-axis current command to 0. When the simulation time is 0.15s, 36Ω resistance is added in parallel on the DC side, and at 0.25s, 36Ω resistance is added in parallel. The simulation output waveform results are shown in Figure 4.
It could be obtained from Figure 4: Figure a) is the three-phase AC side voltage waveform when the DC load resistance is stabilized. It could be seen that the three-phase AC voltage waveform has been stable throughout the simulation time, indicating that the entire system is stable at this time. Figure b) is the three-phase AC current waveform when the DC load resistance is added, and it could be seen that the voltage and current phases are the same, and the AC side implements unit power factor control. Figure c) is the DC output voltage waveform for the entire simulation time. It could be seen from the figure that when the load is added, the DC voltage has a sudden drop of about 10V, but the system could quickly return to the original state, indicating that the control structure and parameter designs are reasonable, and the system is stable at this time.

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**Figure 4.** Three-phase PWM rectifier simulation waveform.

Build a three-phase PWM rectifier through Matlab / Simulink, and get the A-phase voltage and current waveform figure 5 of its AC side. It could be seen from the figure that under the d-q axis double closed-loop control system, the AC voltage and current are in the same phase to achieve unit power Factor operation. Figure 6 is the DC side voltage waveform diagram, which is controlled by the voltage outer loop, and the voltage is stable at the expected value.

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**Figure 5.** Three-phase PWM rectifier ac side voltage and current waveforms.

**Figure 6.** DC side voltage waveform.
In order to verify the correctness of the impedance expression based on the small signal model, it is necessary to measure the output impedance of the DC side of the multi-module power electronic transformer through experiments [14]. The method of measuring DC impedance is the same as the principle of establishing a small signal model, that is, the system works at a selected steady-state operating point, and a disturbance signal is input to measure its response in the system. Select the current disturbance injection method, the measuring principle is shown in Figure 7:

As shown in Figure 7, according to the circuit relationship, the following formula could be obtained. Generally speaking, the injected impedance current is mainly shunted to the power supply side. The impedance could be calculated by the detected disturbance current and disturbance response voltage.

\[
\hat{i}_{dc} = \hat{i}_{dcs} + \hat{i}_{dcl}
\]

(11)

\[
\frac{\hat{i}_{dcs}}{\hat{i}_{dcl}} = \frac{Z_l}{Z_s}
\]

(12)

\[
Z_s = \frac{\hat{u}_{dc}}{\hat{i}_{dcs}}
\]

(13)

According to the above principle, the actual measurements through Matlab / Simulink are as follows. It can be seen from the Figure 8 that the amplitude-frequency characteristics and phase-frequency characteristics obtained by using the scanning impedance at different steady-state operating points are basically consistent with the bode diagram obtained by the mathematical model of small signal impedance, which verifies the correctness of the mathematical model of small signal impedance described above.

![Figure 7](image7.png)

**Figure 7.** Measuring schematic diagram.

![Figure 8](image8.png)

**Figure 8.** Verify the output impedance of the three-phase PWM rectifier on the DC side.
5. Summary
This paper analyzes the structure and control strategy of the PWM rectifier, and simulates the time-domain waveform of the PWM rectifier. On this basis, a small-signal impedance model of the PWM rectifier is established. The simulation model of the PWM rectifier is established through MATLAB and the impedance Bode diagram of the PWM rectifier is obtained through the impedance scanning method. Compared with the Bode diagram of impedance mathematical model, the accuracy of the small signal impedance model established in this paper is proved.

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