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SVD for Enhanced Explosives Detection Using NQR

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Abstract

Nuclear Quadrupole Resonance (NQR) is a solid-state radio frequency (RF) spectroscopic technique, allowing the detection of many high explosives and narcotics. Unlike metal detectors and ground penetrating radar (GPR), NQR detectors measure signals that are unique to the explosives, thus can provide a lower false alarm rate. The practical use of NQR is restricted by the inherently low signal-to-noise ratio (SNR) of the observed signals, a problem that is further exacerbated by the presence of strong RF interference (RFI), so a robust detection method is required. Singular value decomposition (SVD) based on Hankel matrix is proposed to detect the NQR signal in this paper. The original signal can be decomposed into the linear superposition of a series of component signals by SVD using Hankel matrix, and the process of constructing the Hankel matrix shows that a component signal can be produced by averaging the elements on the inverse diagonal of the $A_i = \sigma_i u_i v_i^T$. Then an eigendecomposition based spectrum estimator called “MUltiple Sign al Characterization” (MUSIC) is applied to estimate power spectrum of each component signal, so the RFI and the noise are canceled by using the priori knowledge of the NQR frequency. Finally, an average power detector is applied to detect NQR signal. The result using measured data shows that the proposed method can provide robust NQR detection performance.

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1. Introduction

In the 21st century, a week, or even a day without a report on bombing in the news has become rare. As a consequence, detecting a threat before it causes harm is a prominent public interest. However, there is a frightening variety of threats of which one has to be aware: those that have already been encountered and further ones of which nobody except the initiator(s) may have the slightest idea. Consequently, a systematic approach is assumed to be helpful to identify carefully all explosives.

Nuclear Quadrupole Resonance (NQR) is a solid-state radio frequency (RF) technique that can be used to detect the presence of quadrupolar nuclei [1-5], such as the nucleus prevalent in many explosives and narcotics.

Two types of signal are often measured when NQR is used to detect explosive components [6, 7]. The first, called the free induction decay (FID) is the signal monitored immediately following a pulse. The second, is an echo
train, which is produced by subjecting the sample to a series of pulses, in this case, the higher SNR can be obtained in a short time.

Unfortunately, NQR signals are inherently weak and vulnerable both to the thermal noise of the coil and any external radio frequency interference (RFI). In many NQR applications, RF interference (RFI) can be a major concern. Often, extra RFI mitigation needs to be employed, including passive methods which use specially designed antennas to cancel far-field RFI or active methods which require extra antennas to measure the background RFI.

A frequency domain least-mean-square (LMS) algorithm has achieved reasonable performance for RFI mitigation in [3]. Reference [4] exploited the signal-of-interest (SOI) free samples, deriving the Subspace-based EvaluAtion of QUadrupole resonance signals Exploiting Robust methods (SEAQUER) and Robust Correlation Domain Approximate Maximum Likelihood (RCDAML) detectors, both of which are able to efficiently reduce the influence of RFI.

In reference [8], it’s proved that original signal can be decomposed into the linear superposition of a series of component signals by SVD using Hankel matrix. Simultaneously, it’s pointed out that SVD has the very similar signal processing effect as wavelet transform when Hankel matrix is used.

Singular value decomposition (SVD) based on Hankel matrix is adopted to detect the NQR signal in this paper. Then an eigendecomposition based spectrum estimator called “MUltiple Signal Characterization” (MUSIC) [9] is applied to estimate power spectrum of each component signal, so the RFI and the noise are canceled by using the priori knowledge of the NQR frequency. Finally, an average power detector is applied to detect NQR signal. Fig.1 shows the processing of NQR signal.

![Fig.1 Block diagram of the processing of NQR signal](image)

### 2. SVD using Hankel matrix

The observed sequence is expressed as $X = \left[ x(1), x(2), \ldots, x(N) \right]$, and the Hankel matrix is $p \times q$ orders constructed by using the aforementioned data.

$$
A = 
\begin{bmatrix}
  x(1) & x(2) & \cdots & x(q) \\
  x(2) & x(3) & \cdots & x(q+1) \\
  \vdots & \vdots & \ddots & \vdots \\
  x(p) & x(p+1) & \cdots & x(N)
\end{bmatrix}_{pq}
$$

where, $p + q - 1 = N$, $p \geq q$ and $A \in \mathbb{C}^{pq}$. Singular value decomposition (SVD) is applied to Hankel matrix $A$. Let

$$
A = U \Sigma V^T
$$

where $U$ and $V$ are orthogonal matrices representing a sine and cosine transform, and $\Sigma$ is the diagonal matrix of singular values.
where both \( \mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p] \in \mathbb{C}^{p \times p} \) and \( \mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q] \in \mathbb{C}^{q \times q} \) are orthogonal matrixes, \((\cdot)^T\) denotes the transpose, and \( \sum = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_q) \) is a singular value matrix.

In order to separate the NQR signal from the measured data using SVD, equation (2) can be rewritten by row vector \( \mathbf{u}_i \) and \( \mathbf{v}_j \) as

\[
\mathbf{A} = \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T + \sigma_2 \mathbf{u}_2 \mathbf{v}_2^T + \ldots + \sigma_q \mathbf{u}_q \mathbf{v}_q^T
\]  

(3)

where \( \mathbf{u}_i \in \mathbb{C}^{p \times 1} , \mathbf{v}_j \in \mathbb{C}^{q \times 1} , i = 1, 2, \ldots, q \). According to the theory of SVD, \( \mathbf{u}_i \) are orthogonal with each other, standard orthogonal bases of \( p \) dimension space constituted by \( \mathbf{u}_i \), \( \mathbf{v}_j \) are also orthogonal with each other, standard orthogonal bases of \( q \) dimension space constituted by \( \mathbf{v}_j \). Let \( \mathbf{A}_i = \sigma_i \mathbf{u}_i \mathbf{v}_i^T \), then \( \mathbf{A}_i \in \mathbb{C}^{pq \times pq} \), the process of constructing the Hankel matrix shows that a component signal \( \mathbf{P}_i \) can be produced by averaging the elements on the inverse diagonal of the \( \mathbf{A}_i \), it can be modelled as,

\[
\mathbf{A}_i = \begin{bmatrix}
    x_1(1) & x_1(2) & \cdots & x_1(q) \\
    x_2(2) & x_2(3) & \cdots & x_2(q+1) \\
    \vdots & \vdots & \ddots & \vdots \\
    x_p(p) & x_p(p+1) & \cdots & x_p(N)
\end{bmatrix}
\]  

(4)

It is essentially that the original signal can be decompounded into the \( p \) and \( q \) dimension spaces by using SVD with Hankel matrix. And the obtained component signal \( \mathbf{P}_i \) naturally reflects that the average of the projections which the original signal projects onto the \( ith \) base-vector \( \mathbf{v}_i \) and \( \mathbf{u}_i \), respectively, i.e. the average of the similarity extent of original signal to these standard orthogonal bases. A decomposition of the original signal is completed, according to this manner of these \( \mathbf{A}_i \) to form components.

And then, (3) and (4) show that the original signal is consisted of the obtained component signal \( \mathbf{P}_i \) by a simple linear accumulation,

\[
\mathbf{P}_1 + \mathbf{P}_2 + \ldots + \mathbf{P}_q = \mathbf{X}
\]  

(5)

The advantage of the simple linear accumulation is that the separation of a sub-signal from the original signal is just subtracting it from the original signal. The phase of each sub-signal separated is kept the same as the original signal by subtraction, i.e. the phase of zero offset. (5) is the signal reconstruction formula using SVD method based on Hankel matrix, the significance of which is that the expected sub-signals are selected to accumulation, and hence the character extraction to the original signal can be fulfilled.

The information theory is usually used to determine the number of original signals. Here we adopt the Akaike information criterion (AIC) [10]. The AIC is defined as,

\[
\hat{k}_{AIC} = \text{arg min}_k AIC(k)
\]  

\[
= \text{arg min}_k \left( -\log L_k + \frac{k(2p-k)}{N} \right)
\]  

(6)

where

\[
L_k = \frac{\prod_{j=1}^{p} l_j}{\left( \sum_{j=1}^{p} l_j \right)^{p-k}}
\]  

(7)
where \( l_j \) in (6) and (7) are the eigenvalues of covariance matrix \( \mathbf{R} = E\left( \mathbf{A} \mathbf{A}^H \right) \), and \( l_1 \geq l_2 \geq \cdots \geq l_p \).

Because of the construction characteristic of the Hankel matrix, i.e. each row of the matrix is formed by shifting original signal to the right for a bit, and there is strong correlation between each row. The characteristic of singularity values of the row-strong-correlated matrix is that the first \( l_1 \) singularity values are much larger than others, and the last \( p - l \) singularity values have very minor differences. So most of energy is focused on the first \( l_1 \) singularity vectors \( \mathbf{P}(i = 1, 2, \ldots, l) \), which are corresponding to the first \( l_1 \) singularity values. Thus, the main part of original signal can be described by \( \sum \mathbf{P} \).

Additionally, under the condition of \( p \geq q \), the closer between the values of \( p \) and \( q \), the better the signal is separated. Because when signal energy is constant, \( q \) singularity values are computed from the SVD method based on Hankel matrix, and the corresponding energy is divided into \( q \) parts. Hence, when the demanded conditions are satisfied, the larger the value of \( q \), the better the original signal is separated.

3. Detection Algorithms based on Power Spectrum Estimation

3.1. Data Model

The FID signal is obtained by the narrowband receiver can be well modeled as a damped sinusoid [1, 3, 4].

\[
s(n) = \alpha_0 e^{-\beta_0 n} + w(n), \quad n = 0, 1, \ldots, N - 1
\]

where \( \alpha_0 \), \( \beta_0 \) and \( \omega_0 \) represent the (complex) amplitude, damping constant and frequency, respectively, and \( w(n) \) is an additive colored noise. In practice, the frequency of the NQR signal is often a function of the environmental parameters, such as temperature.

3.2. Power Spectrum Estimation of the NQR signal

Clearly, one could estimate the power spectrum, to find the distribution of power over frequency, and use this as the basis for a detector. Unfortunately, NQR signals are inherently damping characteristic, which causes the effective sampling time is very short, the frequency resolution of the general spectrum estimation is quite lower. The algorithm of spectrum estimation with high frequency resolution is compelled to seek.

MUSIC is a subspace technique that assumes the NQR signal can be modelled as a complex exponential in white noise, and is based upon eigen decomposition of the autocorrelation matrix into two subspaces, the noise subspace and the signal subspace [9]. Let

\[
\mathbf{R}_i = \mathbf{U} \sum \mathbf{U}^H
\]

where \( \mathbf{R}_i \) is the autocorrelation matrix of the \( i \)th component signal. \( \sum = diag(\sigma_1^2, \ldots, \sigma_M^2) \), and \( \sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_M^2 \) denote the eigenvalues of \( \mathbf{R}_i \), \( M \) is the dimension of \( \mathbf{R}_i \).

Denote the unit-norm eigenvectors associated with \( \sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2 \) by \( U_1, U_2, \ldots, U_M \), and those corresponding to \( \sigma_{m+1}^2, \sigma_{m+2}^2, \ldots, \sigma_M^2 \) by \( U_{m+1}, U_{m+2}, \ldots, U_M \). Also define

\[
\mathbf{S} = [U_1, U_2, \ldots, U_m]
\]

\[
\mathbf{G} = [U_{m+1}, U_{m+2}, \ldots, U_M]
\]

\( \mathbf{S} \) and \( \mathbf{G} \) represent the signal subspace and the noise subspace, respectively. The MUSIC spectra estimation is obtained by projecting the \( \mathbf{a}(\omega) \) corresponding to different frequencies onto the noise subspace.

\[
\hat{\mathbf{P}}_{\text{MUSIC}}(\omega) = \frac{1}{\mathbf{a}(\omega) \mathbf{G}^H \mathbf{a}(\omega)}
\]

where \( \mathbf{a}(\omega) = [1, \exp(j\omega), \exp(j2\omega), \ldots, \exp(j(M - 1)\omega)]^T \).
3.3. Average Power Detector

Once these component signals have been determined, according to the prior of the NQR frequency, \( f_0 \), the window of the effective frequency is set, and the components whose frequency out of the window are cancelled. The powers of the residual components are averaged within window.

\[
\bar{S}_a = \frac{1}{KL} \sum_{k=1}^{K} \sum_{i=p_{\max}-\frac{f_0(L/2)}{2}}^{p_{\max}+\frac{f_0(L/2)}{2}} \hat{P}(k,i)
\]

(14)

where \( K \) is the number of the residual components. \( L \) is the length of window, and the subscript \( p_{\max} \) denotes the NQR frequency index.

Two hypotheses are made in explosives detection,

\[
\begin{cases}
H_1: \text{present} \\
H_0: \text{absent}
\end{cases}
\]

And the optimal test statistic is

\[
\Lambda(\bar{S}_a) = \frac{p(\bar{S}_a | H_1)}{p(\bar{S}_a | H_0)}
\]

(15)

where \( p(\cdot) \) is the pdf of the average power \( \bar{S}_a \). If \( \Lambda \) is greater than a set threshold, \( \gamma_2 \), \( H_1 \) is assumed to be true.

4. Numerical examples

The algorithms were evaluated using real data. Two sets of data, one containing signals from Ammonium Nitrate (AN) and another not, were collected in a shielded environment. The mass of the AN sample used was 250g. The sample was placed within a solenoid within a Faraday shield. There are 200 data files, and each data file consists of 2000 echo trains.

Fig.2 shows the estimation of frequency. The figure shows that the frequency of the signal from AN is around 2.5 kHz, and the background is around 1.5 kHz. From the figure we would distinguish between the two sets of data. The central frequency is chosen as 2.5 kHz, and the window width can be 7, 11, 15 or 19.

As we are primarily concerned with detection, we now investigate receiver operator characteristic (ROC) curves for the detectors based on real data, illustrated in Fig.3. The figure shows the performances of the MUSIC-based detector and the Capon-based detector, the former is better than the later. And after the processing of SVD, the performances of both detectors are significantly improved; the SVD-Capon-based detector is the best. Window width is 11.

![Fig.2 The estimation of frequency](image1)

![Fig.3 ROC curves using real data for MUSIC, Capon detectors](image2)
5. Conclusions

In this paper, we evaluated the performance of average power detectors based on MUSIC and Capon after the processing of SVD. The results show that the SVD-Capon based detector performed best out of those compared, and the SVD using Hankel matrix can improve the performance of the detectors. It is shown that the SVD has two functions, canceling the most of the RFI and filtering the noise. Further work includes incorporating techniques to cope with RFI.

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