Measurements of the Composite Fermion masses from the spin polarization of 2-D electrons in the region $1 < \nu < 2$

R. Chughtai, V. Zhitomirsky, R.J. Nicholas  
Department of Physics, Oxford University, Clarendon Laboratory, Parks Rd., Oxford, OX1 3PU, U.K.

M. Henini  
School of Physics and Astronomy, Nottingham, NG7 2RD, U.K.

(January 8, 2022)

Measurements of the reflectivity of a 2-D electron gas are used to deduce the polarization of the Composite Fermion hole system formed for Landau level occupancies in the regime $1 < \nu < 2$. The measurements are consistent with the formation of a mixed spin CF system and allow the density of states or ‘polarization’ effective mass of the CF holes to be determined. The mass values at $\nu = 3/2$ are found to be $\sim 1.9m_e$ for electron densities of $4.4 \times 10^{11}$ cm$^{-2}$, which is significantly larger than those found from measurements of the energy gaps at finite values of effective magnetic field.

The fractional quantum Hall effect has been very successfully described in terms of the picture of Composite Fermions [1, 2], however considerable uncertainties exist about both the Composite Fermion masses and the role of spin and the formation of partially polarized states [3, 4]. In particular Park and Jain [3] have pointed out that the CF mass measured from activation measurements of the energy gap may be substantially smaller than the thermodynamic or “polarization” mass which determines the equilibrium spin populations of the carriers, an idea which has received some recent experimental support [5]. In this paper we describe a direct measurement of the polarization of the 2D Composite Fermion (CF) gas and deduce values for the polarization mass.

In the Composite Fermion model 2D electrons at filling factor $\nu = 1/2$ form a new collective state with quasiparticle excitations, known as Composite Fermions, and zero effective magnetic field. If the Zeeman energy is small compared to the equivalent Fermi energy in the Composite Fermion system the spin polarization of the 2D electrons around $\nu = 1/2$ deviates from 100% even at zero temperature. Cyclotron-like quantization of the energy spectra of the Composite Fermions away from $\nu = 1/2$ causes oscillations in the spin polarization of the 2D electrons at integer filling factors for Composite Fermions. Simultaneously, transport properties of the 2DEG become controllable by the ratio of the cyclotron energy for the Composite Fermions to the Zeeman energy [6, 7]. In spite of this evidence for partial polarization of the CF system direct measurements of the CF Fermi wavevector suggest that the system is close to fully polarized both at $\nu = 1/2$ [6] and at $\nu = 3/2$ [8].

In this work we report direct measurements of the properties of Composite Fermion holes in the region of level occupancy $1 < \nu < 2$. For small values of the Zeeman energy a very convenient way to treat the unoccupied states in the zeroth Landau level in the region $1 < \nu < 2$ is as “holes” [4] with an effective filling factor given by $\nu_h = 2 - \nu_e$ relative to completely filled electron sublevels of both spins. The properties of such holes in the region $1 < \nu_e < 2$ are equivalent to the properties of electrons in the region $0 < \nu_e < 1$ if the energy scale for the Coulomb interaction is much less than the cyclotron energy. Thus a new Composite Fermion system is generated around filling factor $\nu_h = 1/2$ which corresponds to the filling factor $\nu_e = 2 - 1/2 = 3/2$. One should expect similar but much more significant oscillations in the spin polarization of the 2D holes away from the filling factor $\nu_e = 3/2$ [9] as the relative energy of the Zeeman splitting is considerably smaller with respect to the Coulomb interaction. Evidence of such oscillations has been observed before [10] in the magnetic field dependence of the spin polarization of electrons at low density on the top of a strong monotonic fall of the electron spin polarization towards $\nu = 2$. In principle there is a simple relation between the spin polarization $P_e$ of 2D electrons and the spin polarization $P_h$ of 2D holes $P_h \nu_h = P_e \nu_e$, but in the region close to $\nu_e = 2$ both $\nu_h$ and $P_e$ become too small to allow reliable extraction of the hole spin polarization from the measured electron value. Here we measure directly the spin polarization of the holes in the zeroth Landau level of 2D electrons by measuring the hole population with interband reflectivity. At temperatures below 1.5K we find that the spin polarization of the holes in the region $1 < \nu < 2$ agrees very well with a picture based on Composite Fermions with a relatively large effective mass.

We have measured magnetoreflectivity from a 2DEG formed in a pair of doped GaAs single quantum wells with widths of 120Å and 150Å and with carrier densities of $4.4 \times 10^{11}$ cm$^{-2}$. The quantum wells were grown using a 300Å thick Al$_{0.33}$Ga$_{0.67}$As spacer followed by a 300Å...
thick layer of doped Al$_{0.33}$Ga$_{0.67}$As:Si with a doping level $10^{18}$ cm$^{-3}$ and a final 250Å thick cap layer of GaAs. The measurements used a standard projection lamp giving unpolarized illumination of the sample in a 20T He$_3$ insert with a base temperature of 0.4K. Reflected light from the sample was analysed using an in-situ circular polariser and detected and dispersed with a CCD camera and monochromator.

Fig.1 shows a false colour plot of the optical reflectivity for both $\sigma^+$ and $\sigma^-$ polarizations for the 150Å SQW. This is made from a series of reflectivity spectra measured with a small magnetic field step at 0.4K. White corresponds to high intensities of the reflected signal and black to a weaker signal. The strength of the magnetoreflectivity signal is determined by the density of unoccupied states and hence the density of holes in the Landau levels. The absorption coefficient is directly related to dispersion in the refractive index through the Kramers-Kronig relation and it has been shown theoretically that the magnitude of the excursion of the reflectivity around the transition energy is proportional to the absorption coefficient $[1]$. The direct reflectivity spectra measured at 14.5T are shown as an example of this analysis in the lower part of Fig.1. At high magnetic fields the reflectivity shows a set of different lines corresponding to the allowed transitions between Landau levels of the valence and conduction bands with the same Landau level index $N$. For a well resolved spin splitting in the 2D electron gas we start to detect $N$-$N$ transitions only after empty states appear in the $N$th electron Landau level at filling factor $\nu = 2N + 2$ for the upper spin sublevel ($\sigma^+$ polarization) and at $\nu = 2N + 1$ for the lower spin sublevel ($\sigma^-$ polarization). This simple single particle behaviour however changes dramatically for absorption into the two spin sublevels of the zeroth Landau level. For $\nu < 2$ at first we see only transitions to the upper spin state, but beyond $\nu = 5/3$ we start to detect transitions to both spin sublevels in contrast to the predictions of a single particle picture.

In Fig.2 we show the filling factor dependence of the relative intensities $(I^+)$ and $(I^-)$ of the two lines from the interband $0 \downarrow - 0 \downarrow$ and $0 \uparrow - 0 \uparrow$ transitions extracted from the magnetoreflectivity spectra for $\sigma^+$ and $\sigma^-$ polarized light. The sum of the intensities $(I^+ + I^-)$ for both lines agrees well with the the theoretically expected proportionality to $(2 - \nu)/\nu$ for the density of empty states thus confirming the validity of our analysis procedure. We estimate from this that the relative values of the intensities are correct to within $\pm 5\%$. The spin polarization of the holes in the zeroth Landau level should be equal to the polarization of the intensities of the interband transitions $P_h = (I^+ - I^-)/(I^+ + I^-)$. In Fig.3 we show $P_h$ for the zeroth Landau level for temperatures in the range from 2.7K to 0.4K. At temperatures below 1.5K we find that the $P_h$ agrees very well with a picture based on Composite Fermions with a $g$-factor close to that of

![FIG. 1. Upper section shows false colour plots of reflectivity for the $\sigma^-$ (a) and $\sigma^+$ (b) polarizations. Some LH transitions are also visible with opposite polarization dependence. The lower sections show typical spectra measured at 14.5 T which illustrate the magnitude of the reflectivity feature as used in the subsequent analysis.](image)

\[ \text{Energy, eV} \]

\[ \text{Intensity} \]

\[ \text{Magnetic Field, T} \]

\[ \text{Extracted intensity} \]

\[ \text{0.33} \]

\[ \text{0.33} \]

\[ \text{0} \]

\[ \text{0} \]

\[ \text{1.53} \]

\[ \text{1.55} \]

\[ \text{1.57} \]

\[ \text{1.59} \]
We assume that the population difference at zero effective field from simple Pauli paramagnetism (or polarization mass \[3\]) of the Composite Fermions are able to deduce the density of states effective mass or partial filling of only a single Composite Fermion hole \(\nu\). From the absolute spin polarization at \(\nu = \nu_{\text{h}} - \nu_{\text{h}}\), where \(\nu_{\text{h}}\) is the number of CF holes. The fact that we are using a parabolic dispersion means that the same equation would hold for the electron polarization \(P_e\) in terms of the electron density \(n_e\) and would lead to the same values of mass for CF electrons and holes.

Using the measured 60% hole spin polarization and remembering that at \(\nu_e = 3/2\), \(n_h = \frac{1}{3} n_e\) we find that \(m^{*} g^{*} / m_e = 0.60\) and the ratio \(n_h / n_e = 4\). The electron g-factor depends on both the quantum well confinement and the magnetic field and can be calculated using k.p theory \[12\] which may be approximated by the following expression:

\[
g = 2 - 20 \left( \frac{1}{1.5 + \Delta E_g} - \frac{1}{1.85 + \Delta E_g} \right) + 0.062 \quad (2)
\]

where \(\Delta E_g\) accounts for the increase in the band gap due to the electron and hole confinement and cyclotron energies. This expression gives a g-factor of -0.32 for the 150Å quantum well which gives a value of \(m^{*}_{e} \sim 1.9 m_e\) for the CF holes. Fig.3 also shows the spin polarization of the holes for a second 120Å SQW, which has a significantly smaller g-factor of -0.23 (because of stronger confinement) but the same carrier density. This sample shows a similar qualitative behaviour around \(\nu = 3/2\) of only 0.40 due to the lower effective g-factor. The polarization value gives a similar value for the effective mass of 1.74\(m_e\) and confirms the general picture.

We can further confirm the large value of the effective mass by examining the temperature dependence of the spin polarization of the CF holes moving in finite effective field. The total \((B)\) and effective \((B^*)\) fields for the CF hole levels are given by

\[
B = \frac{2p + 1}{3p + 2} n_e \Phi_0 \quad \text{and,} \quad B^* = \frac{1}{3p + 2} n_e \Phi_0 \quad (3)
\]

where \(p = \pm 1, 2, \ldots\), \(\Phi_0\) is the flux quantum and for the present samples \(n_e \Phi_0 = 18T\). For the case of \(p = \pm 2\) when the Zeeman and CF cyclotron energies are comparable we expect a high temperature limit for the polarization of 50%, as there will be \(1/2\) filled levels of spin up and a \(1/2\) filled level of spin down. The low temperature
behaviour will now depend on which splitting is largest with $g^*\mu_B B > \hbar e B^*/m^*$ leading to an increase in polarization as $T \to 0$ and $g^*\mu_B B < \hbar e B^*/m^*$ leading to a falling polarization. The low temperature limits for the two cases should be 100% and 0% for totally unbrodened levels. The actual values will be less than these limits due to the disorder present and the fact that the lowest temperature studied is only 0.4K. However, the direction of the temperature dependence will be independent of the disorder unless this is highly asymmetric. For $p = -2$ $(\nu_h = 2/3, \nu_e = 4/3)$ and $p = +2$ $(\nu_h = 2/5, \nu_e = 8/5)$ the high $T$ limit (2.7K) is $P = 50\%$ as expected but for $p = -2$ the polarization falls to $\sim 40\%$ and for $p = +2$ it rises to $\sim 75\%$. These two cases give us the limits $1.2 < m^* < 2.0$, with the fact that the polarization at $p = +2$ is closer to its fully polarized value suggesting that the mass is closer to the upper limit. Around $\nu_e = 3/2$ $(\nu_h = 1/2)$ there is a rapid depolarization of the spin system with temperature due to the increasing thermal population of the upper spin sublevel, which is consistent with the behaviour being dominated by the small energy scale of the bare spin splitting. By contrast at $\nu_e = 1$ $(\nu_h = 1)$ there is a maximum in the spin polarization of the holes which is much more robust than these other features due to the formation of the Quantum Hall ferromagnet and exchange enhancement of the spin gap, although the polarization is not 100% due to spin mixing phenomena which will be the subject of a further publication [13].

The values which we have deduced for the mass are high compared with values based on activation measurements of the energy gap [13–15], exactly as predicted by Park and Jain [3], and it should be born in mind that the value of the carrier density is quite high in the samples studied here and we would expect that the mass would increase due to the $\sqrt{B}$ dependence of the Coulomb energy on magnetic field as observed in activation measurements for $\nu_e < 1$ [3]. The value for $m^* g^*/m_e$ is also substantially higher than found from tilted field [4,16,17] and thermal acivation [17] measurements around $\nu = 3/2$ in lower density samples. Du et al [4] in particular concluded that there was a dependence on the effective field of $m^* g^*/m_e = 0.264 + 0.05 B^*$ which would give a value of $m^* g^*/m_e = 0.49$ for $\nu = 4/3$ in our samples. Similarly the conclusion of Leadley et al [3] was that the mass alone also depended on the effective field as $m^*/m_e = 0.51 + 0.083 B^*$ which would give a mass of 1.91$m_e$ at $\nu_e = 1/3$ in our samples. The present measurements suggest instead that the polarization mass is determined only by the total carrier density and the fact that the polarization approaches $\nu = 3/2$ smoothly shows that there is no evidence for any divergence of the mass as $B^* \to 0$ as has been previously suggested in both theory [2] and experiment [13]. One question is whether the $\nu_h = 1/2$ CF hole masses should be compared with the mass at the equivalent (low) electron densities or with the value appropriate to the total electron density. In the present case the parabolic dispersion assumption implies that the CF hole and electron masses should be the same and therefore makes it likely that the high values for the CF hole mass are due to the high total carrier density. Even higher values for the masses have been deduced in a recent optical measurement of electron polarization deduced indirectly from photoluminescence polarization by Kukushkin et al [6]. Values of 2.27$m_e$ were found both at $\nu_e = 1/2$ with $n_e = 1.26 \times 10^{11} \text{cm}^{-2}$ and at $\nu_e = 3/2$ with $n_e = 6.3 \times 10^{11} \text{cm}^{-2}$. The higher density result seems consistent with our data, but not that at the lower density.

In conclusion, we have demonstrated that a direct measurement of the spin polarization of the 2DEG can be made through measurements of the absorption deduced from the reflectivity. This allows us to observe a partially polarized CF gas and to deduce the polarization mass of the CF holes which is found to be significantly higher than the values deduced from measurements of the activation energy gaps at fractional filling factors.

**Acknowledgements:** We are grateful to the UK- EPSRC for continued support.

---

1. J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
2. B. I. Halperin et al., Phys. Rev. B **47**, 7312 (1993).
3. K. Park and J. K. Jain, Phys. Rev. Lett. **80**, 4237 (1998).
4. R.R. Du et al. Phys. Rev. Lett. **75**, 3926 (1995); R.G. Clark et al. Phys. Rev. Lett. **62**, 1536 (1989); J.P. Eisenstein et al. *ibid.* 1540;
5. D.R. Leadley et al. Phys. Rev. Lett. **79**, 4247 (1997)
6. I.V. Kukushkin, K. von Klitzing and K. Eberl Phys. Rev. Lett. **82**, 3665 (1999)
7. S Kronmuller et al. Phys. Rev. Lett. **82**, 4070 (1999)
8. J.P. Eisenstein et al. Phys. Rev. B **41**, 7910 (1990); L.W. Engel et al. Phys. Rev. B **45**, 3418 (1992)
9. R.L. Willett et al. Phys. Rev. Lett. **71**, 3846 (1993); W. Kang et al. Phys. Rev. Lett. **71**, 3850 (1993); V.J. Goldman et al. Phys. Rev. Lett. **72**, 2065 (1994); J.H. Smet et al. Phys. Rev. Lett. **77**, 2272(1996)
10. R.L. Willett and L.N. Pfeiffer. Surf. Sci. **38**, 361 (1997)
11. E.L. Ivchenko, A.V. Kavokin, V.P. Kochereshko et al., Phys. Rev. B **46**, 7713(1992)
12. R.J. Nicholas et al. Semicond. Sci. Technol. **11**, 1477 (1996)
13. D. R. Leadley, M. van der Burgt, R. J. Nicholas, C. T. Foxon, and J. J. Harris Phys. Rev. B **53**, 2057 (1996)
14. R.R. Du et al. Phys. Rev. Lett. **70**, 2944 (1993)
15. D.R. Leadley et al. Phys. Rev. Lett. **72**, 1906 (1994)
16. P.J. Gee et al. Phys. Rev. B **54**, R14313 (1996)
17. R.R. Du et al. Phys. Rev. B **55**, R7351 (1997)
18. R.R. Du et al. Solid State Commun. **90**, 71 (1993)
19. V. Zhitomirsky et al. (unpublished)