Interpretations of Elastic Electron Scattering

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June 28, 2016

Abstract. Elastic scattering of relativistic electrons from the nucleon yields Lorentz invariant form factors that describe the fundamental distribution of charge and magnetism. The spatial dependence of the nucleon’s charge and magnetism is typically interpreted in the Breit reference frame which is related by a Lorentz boost from the laboratory frame, where the nucleon is at rest. We construct a toy model to estimate how the charge and magnetic radii of the nucleon are modified between the Breit and lab. frames. This has implications for the ratio of the proton electric to magnetic elastic form factors as a function of momentum transfer as well as for determinations of the proton charge radius. Predicted corrections based on the model are provided for the rms charge radii of the deuteron, the triton, and the helium isotopes.

Keywords. proton radius – proton form factors – elastic electron scattering – Lamb shift – few body nuclei

PACS. 14.20.Dh – 13.40.Gp – 25.30.Bf

Consider relativistic elastic electron scattering from the nucleon. In single photon exchange approximation, the unpolarized elastic eN scattering cross section in the lab. system (nucleon at rest) can be written

$$\frac{d\sigma}{d\Omega} = \sigma_M f_{rec} \cdot F^2,$$

where the Mott cross section is

$$\sigma_M = \left[ \frac{\alpha \cos \theta/2}{2 E_e \sin^2 \theta/2} \right]^2$$

and the recoil factor is $f_{rec} = E_e/E'_e$. Here the incident electron has energy $E_e$, the scattered electron has energy $E'_e$ and the scattering is through angle $\theta$; also $\alpha$ is the fine-structure constant. In all expressions, the extreme relativistic limit is taken, namely the electron mass is ignored with respect to its energy.

The square of the form factor, $F^2$, can be expressed in terms of the electric and magnetic Sachs form factors $G_{p,n}^E$ and $G_{p,n}^M$ of the proton and neutron, respectively. The normalizations at $Q^2 = 0$ are $G_{E}^p(0) = G_{M}^p(0)/\mu_p = G_{M}^n(0)/\mu_n = 1$ (the neutron charge form factor is irregular, but may be fixed with respect to the proton case). As usual, one has

$$W_1 = \tau G_M^2$$

$$W_2 = \frac{1}{1 + \tau} [G_E^2 + \tau G_M^2],$$

where $\tau = |Q^2|/4m_N^2$ with $m_N$ the nucleon mass. Using these the square of the elastic form factor can be written
in several equivalent forms:

\[ F^2 = W_2 + 2W_1 \tan^2 \theta / 2 \]  
\[ = \frac{1}{(1 + \tau)\epsilon} \left[ \epsilon G_E^2 + \tau G_M^2 \right] \]  
\[ = v_L W_L + v_T W_T, \]

where the relative flux of longitudinally polarized virtual photons \( \epsilon \) is given by

\[ \epsilon^{-1} = 1 + 2(1 + \tau) \tan^2 \theta / 2. \]

Using the fact that for elastic eN scattering one has

\[ \rho \equiv \frac{Q^2}{q^2} = \frac{1}{1 + \tau} \]

we obtain

\[ v_L = \rho^2 = \frac{1}{(1 + \tau)^2}, \]  
\[ v_T = \frac{1}{2} \rho + \tan^2 \theta / 2 = \frac{1}{2(1 + \tau)} \epsilon^{-1}. \]

The longitudinal and transverse responses are related to the Ws or to the Sachs form factors via

\[ W_L = (1 + \tau)G_E^2 \]  
\[ W_T = 2\tau G_M^2. \]

One definition of a “proton charge radius” — the one commonly used in analyses of elastic electron-proton scattering — is obtained through the derivative of \( G_E^p(Q^2) \) with respect to the invariant 4-momentum transfer, \( Q^2 \), namely via

\[ \left[ -\frac{6 dG_E^p(Q^2)}{dQ^2} \right]_{Q^2=0} \equiv \left( r_{E,p}^{\text{nom}} \right)^2. \]

Since both \( G_E^p(Q^2) \) and \( Q^2 \) are invariants, the so-defined quantity with dimensions of length is also. We denote this type of radius, \( r_{E,p}^{\text{nom}} \), as the momentum-space proton charge radius. It is a convenient quantity to employ when inter-comparing data on elastic ep scattering. What it is not is the RMS charge radius of the proton. In principle the latter would be found by taking the charge distribution of the proton in its rest frame, weighting by \( r^2 \), integrating and taking the square root to obtain the coordinate-space charge radius \( r_{E,p}^{\text{coord}} \). In some modeling or approaches such as lattice QCD one works in coordinate space and proceeds in this manner. Non-relativistically the two radii are the same, and were the same logic to be applied to heavy nuclei, even relativistically the differences would be negligible. However, the nucleon (and to a lesser extent few-body nuclei) is exceptional: the difference between the two definitions is larger than the present uncertainties in the data. Purists would say that only the momentum-space quantity should be used when discussing elastic ep scattering, since this is intrinsically a process where different frames are involved. In the lab frame, for instance, the proton starts at rest, but must recoil after absorbing the momentum transferred via the exchanged virtual photon; and choosing the Breit frame does not make the problem go away, for then the proton must be moving in both the initial and final states (see below).

Were only electron scattering to be considered, it is hard to argue with this point of view, and it is largely our non-relativistic bias that motivates us to attempt to define a quantity that we hope is some sort of radius. Unfortunately, typical modeling is not done in a fully covariant way, but rather is done by attempting to provide a picture in a specific frame (for instance, through some approach such as lattice QCD or employing models such as the bag model) and then Fourier transforming to obtain a form factor. For any model that is not boostable this procedure incurs the same difficulty as the above for the two types of radius. Furthermore, as discussed briefly later where electronic and muonic atomic hydrogen are mentioned, there are other measurements involving the concept of a proton charge radius that one wants to use to explore whether or not they yield radius values that are compatible with those found via electron scattering. An important question is: which type of radius do these other measurements probe?

Nevertheless, let us not take the point of view that only the invariant momentum-space radius defined above is relevant and at least attempt to evaluate the potential uncertainty in employing a specific definition of the radius (later generalized to all four radii for the nucleon) with consequence for comparisons of electron scattering determinations with those from the Lamb shifts in electronic and muonic hydrogen. We start by summarizing some of the basics in the Breit frame.

The Breit frame is defined by zero energy transfer of the virtual photon and reversal of the 3-momentum of the target between the initial and final states. Thus, one has \( \omega_B = 0 \) and \( q_B = \sqrt{Q^2} \). Since in that frame (with \( z \)-axis along the momentum transfer vector) one has the nucleon entering with 3-momentum \(-p_B\) and leaving with 3-momentum \(+p_B\), one has \( p_B = q_B / 2 \). Thus, the relativistic \( \gamma \)-factor relative to the lab. frame for the nucleon in that frame is

\[ \gamma = \sqrt{1 + \tau}. \]

We note that, no matter what reference frame is adopted, the nucleon before and after the scattering must have different momenta. In the lab frame the nucleon begins at rest, but must recoil with the full momentum in the final state. In the Breit frame (the “brick-wall” frame as discussed above) it must enter with momentum \(-q_B / 2\) and leave with momentum \(+q_B / 2\). In any case, Lorentz boosts are involved and accordingly it is reasonable to expect some consequences from these such as Lorentz contractions and time dilations. For instance, one might expect a moving nucleon to be flattened (as nuclei are in relativistic heavy-ion collisions) with corresponding modification of its “charge distribution”. In a covariant description in momentum space the spinors and operators are typically handled appropriately; however, if one tries to interpret the Fourier transforms, problems arise and concepts such
as the charge and spin distributions are not simply these Fourier transforms, as to a large extent they are for heavier systems such as nuclei. One might just work in momentum space and never confront these issues, although there is strong sentiment that some interpretation should be attempted in coordinate space and in the following we try to do this with a “toy model” that, while certainly not a fundamental one, is motivated by our intuition. Note that scattering involves these problems of reference frames and boosts, whereas the Lamb shift does not have the same issue for there one has an electron or muon bound to a proton with a single frame to consider. Of course, these atomic systems have their own issues (relativistic corrections, recoil effects, etc.)

We now attempt to formulate a non-relativistic version of eN scattering. Assume that one puts a single nucleon into the lowest level in a very deep harmonic oscillator (HO) potential to avoid any recoil problem. In effect, in this “toy model” one can imagine there being a very strong trap to hold the nucleon essentially at rest; indeed, a nucleon in a heavy nucleus is confined and not allowed to carry the full recoil momentum when electrons are scattered from it, providing an example where this situation actually occurs. Then, using, for instance, the tables of [15] or the review article of [16] or the tables of [15], one can compute the multipole matrix elements of the C0 (Coulomb monopole) and M1 (magnetic dipole) elastic scattering operators. As discussed in the standard literature for electron scattering from nuclei, and those papers in particular, the differential cross section may be written as in eq. [16] where, using the non-relativistic limit for the current operators together with 1s1/2 harmonic oscillator wave functions, one obtains the following:

\[ \langle 1s_{1/2} | M_0^{\text{Coul}} | 1s_{1/2} \rangle = F_1 \langle 1s_{1/2} | M_0 | 1s_{1/2} \rangle \quad (16) \]

\[ \langle 1s_{1/2} | T_1^{\text{mag}} | 1s_{1/2} \rangle = \frac{\alpha}{m_N} \langle 1s_{1/2} | \Delta_1(q \mathbf{x}) | 1s_{1/2} \rangle \]

\[ -\frac{1}{2} (F_1 + F_2) \cdot \langle 1s_{1/2} | \Sigma_1(q \mathbf{x}) | 1s_{1/2} \rangle \] \quad (17)

Here, following [16], we use the Dirac and Pauli single-nucleon form factors, as is conventional for non-relativistic treatments of electron scattering from nuclei. Either working directly with the harmonic oscillator wave functions and the explicit forms for the current multipole operators (see [16]) or (which is easier) using the tables of [15] one finds for the three required reduced matrix elements

\[ \sqrt{4\pi} \langle 1s_{1/2} | M_0(q \mathbf{x}) | 1s_{1/2} \rangle = \sqrt{2} e^{-y} \] \quad (18)

\[ \sqrt{4\pi} \langle 1s_{1/2} | \Delta_1(q \mathbf{x}) | 1s_{1/2} \rangle = 0 \] \quad (19)

\[ \sqrt{4\pi} \langle 1s_{1/2} | \Sigma_1(q \mathbf{x}) | 1s_{1/2} \rangle = 2e^{-y}. \] \quad (20)

Each is proportional to \( \exp(-y) \) where \( y = (bq/2)^2 \) with \( b \) the HO parameter. However, one must multiply by the center-of-mass correction which in the non-relativistic HO shell model can be computed; it is a multiplicative factor of \( f_{cm} = \exp(+y/A) = \exp(+y) \) for \( A = 1 \) and cancels the above factor, leaving only the remaining factors obtained by using the above-cited tables. Performing the detailed developments one can obtain the form factors in the HO shell model and, comparing the covariant expressions in eqs. [12] and [13] with these results, one can establish a correspondence between the non-relativistic so-defined quantities and their relativistic counterparts

\[ F_1^2 = F_1^2 \leftrightarrow (1 + \tau)G_E^2 \] \quad (21)

\[ F_2^2 = \left[ \frac{q}{2m_N} \right]^2 (F_1 + F_2)^2 \leftrightarrow 2\tau G_M^2 \] \quad (22)

where \( F_1 \) and \( F_2 \) are the Dirac and Pauli form factors of the nucleon. Note that for the transverse form factor the left-hand expression involves the square of the 3-momentum transfer, whereas the right-hand side involves the square of the 4-momentum transfer. Equivalently, to “de-relativize” the covariant expressions one should define the following non-relativistic expressions:

\[ G_E^{nr} \equiv \sqrt{1 + \tau} G_E^{rel} \] \quad (23)

\[ G_M^{nr} \equiv \frac{1}{\sqrt{1 + \tau}} G_M^{rel}. \] \quad (24)

\[ F^{nr} = \frac{[R_p]^{rel}}{G^{nr}_{M/p}} \] \quad (25)

and is shown in Fig. 1. See [4] for references to the data and to the so-called G\textit{K}ex vector meson based model [5] shown as a solid line in the figure. The “de-relativized” ratio is then immediately given by

\[ [R_p]^{nr} = (1 + \tau) [R_p]^{rel} \] \quad (26)

and is shown in Fig. 2. Note that this has the boost factor squared (going as \( 1 + \tau \) and shown in the right panel as a red line) rather than just linearly as in the individual form factors. Clearly this introduces large modifications at high momentum transfers. Indeed, the “de-relativized” results are relatively flat as functions of \( Q^2 \) and differ from unity by less than roughly 20%.

Next, one might attempt to “de-relativize” the definition of the RMS radii using the same procedures. The usual definition is obtained by expanding the \( J_0 \) spherical Bessel function for low momentum transfer to obtain

\[ r_E^{rel} \equiv \left[ \frac{3}{2m_N} \left( \frac{d}{dr} G_E \right) \right] \tau \rightarrow 0 \] \quad (27)

This is simply a re-writing of eq. [14]. That is, we identify \( r_E^{rel} \) with what was called \( r_E^{mom} \) at the beginning of
Fig. 1. (Color online) $[R_p]^{re}$ vs. $Q^2$ for the data in [17–21] together with the VMD-based curve [4, 5].

Fig. 2. (Color online) Left panel: $[R_p]^{nr} \equiv (1 + \tau) [R_p]^{re}$ vs. $Q^2$ with data and GKex curve as in Fig. 1 together with $1 + \tau$ curve referenced to the right-hand axis.
the paper. We then proceed to “de-relativize” to obtain a quantity that might be more intuitive, based on our toy model. For this one must include the $\sqrt{1 + \tau}$ factor in eq. (23) obtaining at small momentum transfer

$$G_{nr}^E = \left(1 + \frac{1}{2}\tau + \cdots\right) \left(1 - \frac{2}{3}\tau m_N^2 \left[r_{E,p}^{rel}\right]^2 + \cdots\right)$$

(28)

$$= 1 - \frac{2}{3}m_N^2 \left[r_{E,p}^{rel}\right]^2 \frac{1 - \frac{1}{2}\tau + \cdots}{\tau}$$

(29)

$$\equiv 1 - \frac{2}{3}\tau m_N^2 \left[r_{E,p}^{nr}\right]^2 + \cdots$$

(30)

and leading to the relationship

$$r_{E,p}^{nr} = \sqrt{\left[r_{E,p}^{rel}\right]^2 - \Delta},$$

(31)

where one has

$$\Delta_p \equiv \frac{3}{4m_p^2} = 0.0332 \text{ fm}^2$$

(32)

$$\Delta_n \equiv \frac{3}{4m_n^2} = 0.0331 \text{ fm}^2$$

(33)

for protons and neutrons, respectively. The same arguments for the magnetic form factor where the required boost factor is now $1/\sqrt{1 + \tau}$ (see eq. 24) leads to the expression

$$r_{M,p}^{nr} = \sqrt{\left[r_{M,p}^{rel}\right]^2 + \Delta}.$$

(34)

We note that the same result for the proton charge form factor was obtained in [14], arguing from a very different point of view: see also [11], on which that work is based.

It is important to understand that these differences between relativistic and non-relativistic radii do not go away if electron scattering data are obtained at ever smaller values of the momentum transfer. As the above expressions clearly show, the relativistic boost factor arising from $(1 + \tau)^{\pm 1/2}$ deviates from unity at order $Q^2$; however, that is the order needed to extract the charge or magnetic radii. In other words, being locked together at the same order when expanding in powers of $Q^2$ the effects can never be separated, no matter how small the momentum transfer becomes.

Specifically, using the Bernauer value for the proton rms charge radius [6], and the PDG values [22] for the proton magnetic, neutron charge, and neutron magnetic rms radii, one has the following:

$$r_{E,p}^{nr} = \sqrt{(0.879)^2 - 0.0332} = 0.860 \pm 0.008 \text{ fm}$$

(35)

$$r_{M,p}^{nr} = \sqrt{(0.777)^2 + 0.0332} = 0.7981 \pm 0.013 \pm 0.010 \text{ fm}$$

(36)

$$r_{M,n}^{nr} = \sqrt{(0.862)^2 + 0.0331} = 0.8810^{+0.008}_{-0.006} \text{ fm}$$

(37)

$$\left[r_{E,n}^{nr}\right]^2 = -0.1161 - 0.0331 = -0.1492 \pm 0.0022 \text{ fm}^2 .$$

(38)

Here the uncertainties are taken from the Bernauer work [6] and from the PDG compilation [22], respectively. Note that the electric result for the neutron is traditionally expressed as the square of the radius, which is negative.

What has been obtained above from electron scattering through the simple toy model are radii that start from the quantities measured in electron scattering $(r_{mom}^{mom} \equiv r_{E,p}^{rel})$ to quantities that are more like $r_{coord}^{coord}$. The toy model employed in this work thus leads us to make the identification $r_{coord}^{coord} \leftrightarrow r_{E,p}^{nr}$. Namely, when coordinate-space radii are wanted, rather than the momentum-space ones measured in electron scattering, the toy model motivates taking into account the corrections discussed above. While the model is not “fundamental”, at least the fact that the relativistic and non-relativistic quantities so-defined are different should be cause for some concern that the concept of a charge or magnetic radius is not totally robust.

Let us take this one step further and bring in the values of the “proton charge radius” determined via the Lamb shifts in electronic and muonic hydrogen which are known to have some charge-distribution dependence because the proton is not a point particle, but has a finite extent. Note that these atomic systems are described in coordinate space and not, as for electron scattering, in momentum space. In the course of developing the formalism for studies of atomic hydrogen it is natural to invoke the Fourier transform of the proton charge distribution to inter-relate what is desired in this case with what is measured in electron scattering; however, this is the essence of the issue. Namely, it would appear to be more natural that the Lamb shift problem involves the coordinate-space radius $r_{E,p}^{coord}$ and not its momentum-space analog, i.e., to use $r_{E,p}^{nr}$ not its relativistic partner. One might be tempted to use the fact that the effective value of $QR_{E,p}$ is very small for these atomic systems ($\sim 10^{-5}$); however, the above argument on the “locking” of the boost factor with the radius shows that the smallness of this product is not sufficient to make relativistic and non-relativistic radii effectively the same. It is not the scale of momentum transfer that is critical (as long as it is small enough to allow only terms of quadratic order to be considered), but the fact that a scattering process and measurements of an atomic system are more naturally studied in momentum space and coordinate space, respectively. We assume that analysis of the Lamb shift entails using the coordinate-space version of the proton charge radius which, in our toy model, means $r_{E,p}^{nr}$.

The proton charge radius discrepancy has arisen from the different values resulting from a precise determination using the Lamb shift in muonic hydrogen $(0.84087 \pm 0.00026 \pm 0.00029)$ [8] disagreeing with the CODATA 2010 value $(0.8775 \pm 0.0051)$ [23], largely determined by the most precise value resulting from elastic electron proton scattering [6]. This amounts to more than 4% difference, whereas the stated total uncertainty in the Bernauer electron scattering result is quoted as 0.9%. On the other hand, the correction resulting from the boost between Breit and lab. frames as calculated in our toy model decreases the electron scattering determination of $r_{E,p}$ to-
Fig. 3. Timeline of recent determinations \[6, 9, 23–29\] of the proton charge radius \(r_{E,p}\). For earlier experimental results see \[7\].

The Bernauer value for the proton charge radius, namely
\[
r_{E,p}^{\text{mom}} = r_{E,p}^{\text{rel}}
\]
is indicated with a black cross lying at 0.879, while its corrected value
\[
r_{E,p}^{\text{coord}} = r_{E,p}^{\text{nr}}
\]
is indicated by the white cross at 0.860. The shaded bands show the 1\(\sigma\) and 2\(\sigma\) uncertainties about the latter and may then be compared with the Lamb shift value indicated by a star and lying at about 0.84.

Towards the muonic hydrogen value. The resulting discrepancy in the different determinations of \(r_{E,p}\) is cut in half using the corrected value and now differs from the muonic Lamb shift value by only about 2%. Further, the corrected value here for the proton charge radius is not inconsistent with the value determined using the hydrogen atom, within experimental uncertainty. Fig. 3 shows a selection of determinations of the proton rms charge radius since the year 2000 including all the work cited here. In particular, it shows the electron scattering determination from Bernauer and our corrected value with both 1\(\sigma\) and 2\(\sigma\) uncertainties. If one accepts the ideas we have developed in this paper, then one is left with only a small amount to explain and, the remaining discrepancy being commensurate with the size of \(\alpha\), might lead one to question the other model dependences in the problem (experimental systematic uncertainties, radiative corrections, etc.)

The \(Q^2\)-independent recoil correction to the rms charge radius derived from elastic electron scattering can also be evaluated for the deuteron, the triton, and the helium isotopes. For heavier nuclei, the correction factor above is replaced as follows:

\[
\Delta = \frac{3}{4m_N^2} \rightarrow \Delta \cdot \left( \frac{m_N}{m_A} \right)^2,
\]

where \(m_A\) is the mass of that heavier system. In Table II we summarize the results based on our model. We note that the corrected deuteron rms charge radius from electron scattering is in excellent agreement with the recent high precision value obtained from measurements of the Lamb shift in muonic deuterium. We provide predictions for the rms charge radii of the helium isotopes, which are being determined to high precision in ongoing experiments that employ measurement of the Lamb shift in muonic atoms. New high precision measurements of the Lamb shift in electronic hydrogen are in progress and results are expected soon.

We have also considered how the conclusions here can be validated by experiment. We point out that the correction we derive cannot be separated by going to lower \(Q^2\) in electron scattering experiments nor by comparison of elastic electron and muon scattering on the proton. However, based on the estimates made here, a precision comparison of electron scattering from the proton with electron scattering from the deuteron at low \(Q^2\) should deviate at the level of about 1.5%, since the corrections we derive differ at this level. For highest precision, such measurements should be carried out using the same apparatus and systematics must be minimized. We note that internal radiative corrections, which arise mainly from the incident and
scattered electrons, should be quite similar for proton and deuteron targets.

In summary, we argue that corrections between the Breit and lab. frames are important in interpreting the form factors of the nucleon as determined in relativistic electron scattering. We have constructed a toy model to estimate these corrections. In this toy model, the observed significant decrease of the ratio of the proton elastic form factors as a function of $Q^2$ is understood as a predominantly relativistic effect. Furthermore, in this toy model, the proton charge radius as determined in electron scattering has a correction that reduces its value towards that resulting from the precision determination using the Lamb shift in muonic hydrogen. The toy model provides predictions for the rms charge radii of the deuteron, triton, and helium isotopes.

The ideas presented here should not be viewed as providing a detailed model of how boost effects might enter in electron scattering from the nucleon, but as a warning that they could play a role in electron scattering, but not in determinations of the Lamb shift. Much more satisfying would be to have a model of the nucleon that could be boosted, although it is not obvious that such a model exists at present. For instance, in models where relativistic quarks are confined via a “bag” one must confront the problem of how to boost the latter in going between the frames that inevitably enter in electron scattering. One possible, but different, related case that could be studied to test the ideas is that of relativistic (covariant, boostable) modeling of the deuteron[32]. There, on the one hand, one could directly compute the elastic form factor, while on the other hand one could compute the ground-state charge distribution and then Fourier transform it to momentum space. Upon comparing the two results it is likely that differences will be found that relate to the boost issues raised in the present study.

We thank Jan Bernauer for valuable discussion. The authors’ research is supported by the Office of Nuclear Physics of the U.S. Department of Energy under grant Contract Numbers DE-SC0011090 and DE-FG02-94ER40818.

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Table 1. Comparison of charge radii measured by electron scattering from various light nuclei and the corrected values following the procedure proposed herein. The muonic determinations for the proton and the deuteron (preliminary value - see [31]) are also given. We note that high precision values for the helium isotopes will be forthcoming from muonic atom experiments.
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