A New Solution Algorithm of Magnetic Azimuth

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Abstract. Comparing with traditional compasses, the digital compass is smaller, lighter, and more reliable. The key of the algorithm which used in the digital compass is sensor calibration and angle solution algorithm. The content researched in this paper is angle solution algorithm, which is based on the measurement data of three-axis accelerometers and three-axis magnetic sensors, solving the angle between the north direction (rolling axis) of the body frame and the north direction of the navigation frame, which aligned with the directions of magnetic north, east, and the local vertical (down). This kind of algorithm is also called magnetic azimuth solution algorithm. A new algorithm called vector cross product method is presented and compared with traditional attitude angle method for principle and accuracy. Advantages and disadvantages of this method are analyzed.

1. Introduction
At present, the common magnetic azimuth solution algorithm used in the digital compass is the traditional attitude angle method. It assumes that the body frame rotates with respect to the navigation frame. The three attitudes (yaw, roll and pitch) are solved by establishing the relationship between the navigation frame and the body frame. The vector in the navigation frame has a projection in the body frame and the relationship between them is the direction cosine matrix. The magnetic azimuth is the yaw [1–3].

Similarly, it may assume that the navigation frame rotates with respect to the body frame and forms a new navigation frame. That means the geomagnetism vector and the gravity vector rotate with respect to the body frame. The relationship between the north direction of the new navigation frame and the north direction of the old navigation frame is established. The angle between them is the magnetic azimuth. That is the basic principle of the vector cross product method [4].

2. Traditional attitude angle method
The navigation frame aligned with the directions of magnetic north, east, and the local vertical (down) is established and expressed as $N\rightarrow E\rightarrow D$. The body frame is aligned with the body and expressed as $X\rightarrow Y\rightarrow Z$. The X-axis is in the symmetry plane of the body pointing at the direction that the body moves along. The Y-axis is perpendicular with the symmetrical plane of the body and pointing right. The Z-axis is also in the symmetrical plane of the body, perpendicular with the direction X and pointing down. As the plane for example, the picture shows below (Figure 1).

The magnetic azimuth $\phi$ is the angle between the X-axis of the body frame and the N-axis of the navigation frame in the horizontal plane.
The method of establishing the frame

According to the principle of the attitude angle method introduced in the first section, the process is described as follows:

1. The gravity acceleration vector and the geomagnetism vector of the navigation frame project to the body frame.

   The projection of the gravity acceleration vector: \( \mathbf{g}_x = C^b_n g_{sl} \)  \( \text{(1)} \)

   The projection of the geomagnetism vector: \( \mathbf{m}_x = C^b_n m_{sl} \)  \( \text{(2)} \)

   where \( \mathbf{g}_x \) is the gravity acceleration vector projected in the body frame, \( \mathbf{m}_x \) is the geomagnetism vector projected in the body frame, \( g_{sl} \) is the gravity acceleration vector in the navigation frame, \( m_{sl} \) is the geomagnetism vector in the navigation frame and \( C^b_n \) is the direction cosine matrix described as follows:

   \[
   C^b_n = \begin{bmatrix}
   \cos \varphi \cos \theta & \sin \varphi \sin \theta & -\sin \theta \\
   \cos \varphi \sin \theta \sin \gamma - \sin \varphi \cos \gamma & \sin \varphi \sin \theta \sin \gamma + \cos \varphi \cos \gamma & \cos \theta \sin \gamma \\
   \cos \varphi \sin \theta \cos \gamma + \sin \varphi \sin \gamma & \sin \varphi \sin \theta \cos \gamma - \cos \varphi \sin \gamma & \cos \theta \cos \gamma
   \end{bmatrix} \tag{3}
   \]

   The gravity acceleration vector is \((0 \ 0 \ g)^T\) and the geomagnetism vector is \((m \cos \beta \ 0 - m \sin \beta)^T\) in the navigation frame, where \( g \) is the gravity acceleration value, \( m \) is the geomagnetism intensity and \( \beta \) is the magnetic inclination.

   Substituting for \( C^b_n \), from eq.3 in eq.1 and eq.2 give the measurements of three-axis accelerometers as \( X_g, Y_g, Z_g \) and three-axis magnetic sensors as \( X_m, Y_m, Z_m \).

2. Establishing the magnetic azimuth \( \varphi \)

   \[ \varphi = \arctan(Y_H / X_H) \]  \( \text{(4)} \)

   where \( Y_H = Z_m \sin \gamma - Y_m \cos \gamma \), \( X_H = X_m \cos \theta + (Y_m \sin \gamma + Z_m \cos \gamma) \sin \theta \),

   \[ \theta = -\arctan(X_g / \sqrt{Y_g^2 + Z_g^2}) \], \( \gamma = \arctan(Y_g / Z_g) \), \( \theta \) is the pitch and \( \gamma \) is the roll.

   It will be seen that the process of solving \( \theta \) and \( \gamma \) is the precondition of the solution of \( \varphi, X_H \) and \( Y_H \) are unnecessary variables. Those unnecessary variables increase the extra errors of the system. In order to avoid the disadvantage of the attitude angle method, a new method called the vector cross product method is introduced.

3. The vector cross product method

   The method of establishing the reference frame is the same as the traditional attitude angle method. It assumes that the navigation frame rotates with respect to the body frame and forms a new navigation frame. That means the geomagnetism vector and the gravity vector rotate with respect to the body frame. The relationship between the north direction of the new navigation frame and the north direction of the old navigation frame is established. The angle between them is the magnetic azimuth.

   The process of the vector cross product method is described as follows:

   1. Establishing the three directions of the new navigation frame:
Accelerometers and magnetic sensors of the system give the new gravity acceleration vector and the new geomagnetism vector. The direction of the gravity vector is the direction of the local vertical. The perpendicular plane of the new gravity acceleration vector is the new horizontal plane. The projection of the new geomagnetism vector on the new horizontal plane is the north direction of the new navigation frame. The east direction \( \vec{N}_{sj} \) and the north direction \( \vec{N}_{xi} \) of the new navigation frame are provided by the following equations:

The east direction: \( \vec{N}_{sj} = \vec{g}_x \times \vec{m}_x \)  \( (5) \)

The north direction: \( \vec{N}_{xi} = \vec{g}_x \times \vec{m}_x \times \vec{g}_x \) \( (6) \)

Normalize: \( \vec{N}_{sj} = \frac{\vec{N}_{sj}}{\|\vec{N}_{sj}\|}, \vec{N}_{xi} = \frac{\vec{N}_{xi}}{\|\vec{N}_{xi}\|} \) \( (7) \)

where \( \vec{g}_x \) is the new gravity acceleration vector and \( \vec{m}_x \) is the new geomagnetism vector.

In order to simplify the calculation, we normalize \( \vec{N}_{sj} \) and \( \vec{N}_{xi} \) to \( \vec{N}_{sj} \) and \( \vec{N}_{xi} \).

(2) Establishing the north direction of the old navigation frame:

It is one of the steps calibrating the digital compass. The old navigation frame is determined by the gravity and magnetic vector known at the position of calibration. Be similar to the method above, the north direction of the old navigation frame \( \vec{N}_0 \) is expressed as follows:

\( \vec{N}_0 = \vec{g}_0 \times \vec{m}_0 \times \vec{g}_0 \) \( (8) \)

where \( \vec{g}_0 \) is the calibration gravity acceleration vector and \( \vec{m}_0 \) is the calibration geomagnetism vector.

(3) Establishing the magnetic azimuth \( \varphi \):

\[ \varphi = \arctan \left( \frac{\vec{N}_0 \cdot \vec{N}_{sj}}{\vec{N}_0 \cdot \vec{N}_{xi}} \right) \] \( (9) \)

According to the equation(9), some aspects need to be pay attention to. First, the denominator of the equation(9) can’t be zero. Second, the range of \( \varphi \) solved by equation(9) is \( \pm 90^\circ \).

Because of the rotation of the navigation frame with respect to the body frame, the north direction of the new navigation frame may be perpendicular with the old navigation frame in some condition. In other words, some attitudes or noises could make the denominator of the equation(9) to be zero in the process of the body rotating and it makes the algorithm can’t work. On the other hand, the range equalling to \( \pm 90^\circ \) can’t satisfy the using, so the algorithm needs to be improved. The process of the improvement and noise-added analysis of the algorithm may be introduced by simulation.

4. The improvement and noise-added analysis of the algorithm

4.1. The improvement of the algorithm

The theoretical measurements of the accelerometers and magnetic sensors are calculated by using the three setting attitude angles of the body and the parameters of the gravitational field and geomagnetic field. The value of the theoretical azimuth equals to the value of the setting yaw and the simulation azimuth is calculated by applying the theoretical measurements in the vector cross product method. Changing the other two setting angles separately, differences between the simulation value and the theoretical value are compared [5].

It is shown through simulation results that the changing of the roll doesn’t influence the solution of the azimuth. At the condition of the setting pitch changed and the setting roll unchanged, angle error is the maximum when the pitch equals to \( \pm 90^\circ \) and a difference between the simulation value and the theoretical value is found to be 180° when the pitch is larger than \( \pm 90^\circ \). Base on the reasons mentioned above, the algorithm needs improving. The method of improvement is to determine the sign of the numerator and the denominator in the equation(9), because the vector cross product method can’t calculate the pitch. The equation(10) is the expression that after improvement. Table1 shows...
comparing results of the algorithm before and after improvement. The theoretical azimuth is chosen to be 30° [6].

\[
(1) \overrightarrow{N_0} \cdot \overrightarrow{N_{sj}} > 0, \overrightarrow{N_0} \cdot \overrightarrow{N_{xi}} < 0 \quad \varphi = \arctan \frac{\overrightarrow{N_0} \cdot \overrightarrow{N_{sj}}}{\overrightarrow{N_0} \cdot \overrightarrow{N_{xi}}} + 180^\circ \\
(2) \overrightarrow{N_0} \cdot \overrightarrow{N_{sj}} < 0, \overrightarrow{N_0} \cdot \overrightarrow{N_{xi}} < 0 \quad \varphi = \arctan \frac{\overrightarrow{N_0} \cdot \overrightarrow{N_{sj}}}{\overrightarrow{N_0} \cdot \overrightarrow{N_{xi}}} - 180^\circ \\
(3) \text{else} \quad \varphi = \arctan \frac{\overrightarrow{N_0} \cdot \overrightarrow{N_{sj}}}{\overrightarrow{N_0} \cdot \overrightarrow{N_{xi}}}
\]

Table 1. Simulation results before and after improvement.

| Pitch θ (degree) | Azimuth before Improvement (degree) | Azimuth after Improvement (degree) |
|------------------|-------------------------------------|-----------------------------------|
| 180              | -150                                | 30                                |
| 150              | -150                                | 30                                |
| -90              | -24.1                               | 155.9                             |
| 0                | 30                                  | 30                                |
| 90               | 12.6                                | 12.6                              |
| 150              | -150                                | 30                                |
| 180              | -150                                | 30                                |

Because the projection of the geomagnetism vector is a point when the pitch equals to ±90°, the denominator in the equation(9) equals to 0. It is the inherent limitations of the geomagnetism navigation and can’t be avoided.

After the improvement of the algorithm, the range of the magnetic azimuth calculated by the vector cross product is ±180° unless the pitch equals to ±90°.

4.2. The noise-added analysis of the algorithm

Adding a certain intensity gauss white noise into the output of the sensors, the true azimuth is solved by the vector cross product method after a period of sampling times. The mean error and the standard error between the theoretical value and the true value are solved.

The errors are calculated again when the true azimuth unchanged but the intensity of the noise changed. Results are compared before and after the changing. At last, simulation is repeated by choosing some angles at the range of ±180° and the conclusion is given.

The theoretical azimuth is given to be 100°. First, the simulation condition is that the coefficient of the gauss white noise is 0.01 and the sampling time is 1000. Second, the coefficient is changed to be 0.02 and the sampling time is unchanged. Results of the simulation are observed and the simulation picture is drawn as follows (Figure 2)

![Figure 2. The simulation results when the coefficient of the noise is 0.01.](image-url)
Results show that if the theoretical azimuth is given to be 100°, the mean error and the standard error of the magnetic azimuth solved when the coefficient of the noise equals to 0.01 are -0.0024° and 0.0793°, solved when the coefficient of the noise equals to 0.02 are 0.0016° and 0.1497°.

The theoretical azimuth is chosen at the range of ±180° at random. Repeat the two steps above. The picture drawn below (Figure 3) is the simulation result when the azimuth equals to +180° and the coefficient of the gauss white noise is 0.01.

![Figure 3. Noise-added simulation when the azimuth equals to 180°.](image)

It will be seen from the picture that when the simulation azimuth equals to +180°, three results may be work out by the vector cross product angles, which are 0°, +180° and -180°. The angle equalling to 0° is a wrong value. It is discovered by simulation that except from a little range around ±180° where this situation occurs, other azimuths are not too influenced by the noise at the range of ±180°. The reason is that the sign of the numerator in the equation(9) is not ensured at that range and the equation (9) have singularities as a result. The singularities can be solved by determining the direction of the geomagnetism simply and the range of the magnetic azimuth can be solved is still ±180°.

5. Comparison by experiment
The vector cross product method avoids the unnecessary variables in the attitude angle method. It may be helpful in improving the precision of the system. So the precisions of the two methods are compared by experiment.

The solution algorithm is influenced much by initial calibration, so the sensors must be calibrated before the experiment. Calibration is to compensate the biases and the assembling errors. The north direction of the geomagnetism is set to be the start angle of the experiment. Sensors work every 15 degrees and the azimuth is solved by the two methods separately. Repeat the experiment and compare the errors.

By initial experiment solution we obtain that the magnetic azimuth range measured by either the attitude angle method or the vector cross product method is ±180°. The mean error and the standard error of the magnetic azimuth solved by the attitude angle method are 0.2999° and 0.5583°, solved by the vector cross product method are -0.0084° and 0.4601°. The azimuth error curve is drawn as follows (Figure 4):

Results show that given the high condition of initial calibration and the same position, the vector cross product method has a higher accuracy than the attitude angle method.
6. Conclusion
In the traditional attitude angle method, except from the yaw that is the magnetic azimuth we need, the other two angles have no significance in the digital compass. Further more, the accumulated error of the system increases because of the other two angles.

Vector cross product method calculates in a different aspect. By establishing the relationship of the three directions of the navigation frame, the gravity acceleration vector and the geomagnetism vector, vector cross product method solves the accumulated error caused by the two other angles in the attitude angle method. The advantages of this method are that it uses the gravity acceleration vector and magnetic vector sensed actual time to establish the new horizontal plane and the magnetic north, and uses the data measured by sensors fully. The disadvantage is that this method is much influenced by the condition of initial calibration.

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