I. Constructing gravitational field using photonic waveguide lattice with nonuniform coupling coefficients

Inspired by transformation optics, we use femtosecond laser written photonic waveguide lattice to mimic gravitational field and investigate the dynamic evolution of boson in close proximity of black hole. Firstly, we start by considering the line element of a two-dimensional Schwarzschild spacetime:

\[ ds^2 = (1 - 2M/\rho)dt^2 - (1 - 2M/\rho)^{-1}dr^2 \]  

(S1)

where \( M \) is the mass of black hole, and nature units have been adopted (\( G = c = 1 \)). In our work, we are interested in the short distance behavior of Hawking radiation near the horizon (\( \rho \approx 2M \)), the Schwarzschild metric can be written as:

\[ ds^2 = \alpha^2 r^2 dt^2 - dr^2 \]  

(S2)
where \( \alpha = 1/2 r_s \), \( r_s = 2M \), \( r(\rho) = \sqrt{8M(\rho - 2M)} \). What is more interesting is that the Schwarzschild metric near the horizon of black hole has the same form as the line element of the Rindler [1].

While the quantum evolution of boson for null geodesic satisfies \( ds = 0 \), which means \( |dr/r| = \alpha dt \), then we can obtain the evolution function of boson near to black hole as

\[
  r = r_0 e^{\alpha t},
\]

(S3)

where \( r_0 \) is initial position. The quantum evolution depends on the curvature of black hole \( \alpha \), which means that the quantum evolution is faster near the event horizon of a black hole with small radius.

For the spacetimes based on Eq. (S1), the evolution of boson similar to that in an optical media with an effective refraction index based on transformation optics [2–4] [Fig. S1(c)]:

\[
  n = \sqrt{-g_{11}/g_{00}} = 1/\alpha r,
\]

(S4)

where \( g_{00} = \alpha^2 r^2 \), \( g_{11} = -1 \). In our work, we use evanescently coupled photonic waveguide lattice with designed coupling coefficient to achieve the required inhomogeneous effective refractive index. The dynamics of single-photon wave packet in photonic waveguide lattice can be described by a set of coupled discrete Schrödinger equations, which are derived from Schrödinger-type paraxial wave equation by employing the tight-binding approximation:

\[
  i\partial \varphi_m/\partial z = \beta_0 \varphi_m - \kappa_m \varphi_{m-1} - \kappa_{m+1} \varphi_{m+1},
\]

(S5)

where \( \varphi_m \) is the complex field amplitude of site \( m \), \( z \) is the propagation distance along the waveguides mapping the time variable, \( \beta_0 \) is on site energy of each waveguide, and parameter \( \kappa_m \) represents the coupling strength between the adjacent sites. Taking coupling coefficients as \( \kappa_m = \kappa_{m+1} = \kappa \) and substituting the complex field amplitude with the plane wave solution \( \varphi_m = A \cdot exp(i\beta_r md + i\beta_z z) \), we can obtain the dispersion connecting transverse and
longitudinal dynamics as [Fig. S1(b)]:

\[ \beta_z = \beta_0 - 2\kappa \cos (\beta_r d), \]  
\( (S6) \)

where \( A \) is the amplitude of plane wave, \( \beta_r \) and \( \beta_z \) are transverse and longitudinal wavevector respectively, \( d \) is waveguide spacing \((r = md)\). After photon evolves in the such a waveguide over distance \( \Delta z \), each transverse component gains a phase \( \Phi = \beta_z (\beta_r) \Delta z \), and the corresponding transverse shift of a wave centered around \( \beta_r \) is \( \Delta r = \partial \Phi / \partial \beta_r = (\partial \beta_z / \partial \beta_r) \Delta z \). Due to that the propagation distance \( z \) in the coupled waveguide equation plays the role as the time \( t \) in Schrödinger equation, hence when exciting the coupled waveguide with the transverse wavevector as around \( \beta_r = \pi/2d \), we can define the velocity of wavepacket in such system as [Fig. S1(b)]:

\[ v = \frac{\Delta r}{\Delta z} = \frac{\partial \beta_z}{\partial \beta_x} = 2\kappa d. \]  
\( (S7) \)

And the refractive index of such photonic lattice is

\[ n = 1/v = 1/2\kappa d. \]  
\( (S8) \)

Comparing Eq. (S3) and (S7), to construct gravitational field of 1+1 dimensional Schwarzschild black hole in evanescently coupled waveguides, the coupling coefficients should satisfy

\[ \kappa = \alpha_r/2d = \alpha m/2, \]  
\( (S9) \)

where we take discrete the continuous function with lattice \( r = md \). Therefore, by constructing the coupling coefficient \( \kappa \) increasing linearly with waveguide site \( m \) [Fig. S1(d)], we can achieve effective refractive index required by 1+1 dimensional black hole. In Fig. S1(e) and Fig. S1(f), we show the dynamic evolution of single-photon wave packet in the flat space and near the event horizon of black hole, which clearly show the evolution has exponential form near the event horizon of black hole compared with linear evolution in the flat space.
Figure S1: (a) The schematic of evanescently coupled waveguides. (b) The dispersion relation $\beta_z(\beta_r)$. The green line indicates the transverse wavevector as $\beta_r = \pi/2d$. (c) The effective index corresponds to 1 + 1 dimensional Schwarzschild black hole given by transformation optics. (d) The corresponding coupling coefficient in evanescently coupled waveguides which can mimic the gravitational field of black hole. (e) The evolution of photon in the flat space. (f) The evolution of photon near the event horizon of black hole.

II. The Dirac equation near the event horizon of black hole

Besides the dynamic behavior of photon as boson, we can also investigate the evolution of fermion pair generated by Hawking Radiation in close proximity to black hole. We start with the Dirac equation in the 1 + 1 dimensional spacetime for a massless particle is

$$\gamma^\mu \nabla_\mu \varphi = 0,$$  \hspace{1cm} (S10)

where $\mu$ is 0 for time and 1 for spatial 1 dimensional direction, $\varphi = (a, b)^T$ ($T$ stands for transpose) is a two-component spinor, $\gamma^\mu$ are the Dirac gamma matrices, and the covariant derivative $\Delta_\mu = \partial_\mu + \Omega_\mu$. Considering the metric Schwarzschild black hole according to
Eq. (S1), we obtain the element of metric as
\[ g_{00} = (\alpha r)^2, \]
\[ g_{11} = -1, \]
\[ g^{00} = 1/(\alpha r)^2, \]
\[ g^{11} = -1 \]

Then we can get the calculated Christoffel symbol as
\[ \Gamma^i_{kl} = \frac{1}{2} g^{im} \left( \frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right), \]
the Einstein convention is adopted here where the repeated indices are summed over. We have for nonzero Christoffel symbol as
\[ \Gamma^0_{01} = \Gamma^0_{10} = 1/r, \quad \Gamma^1_{00} = \alpha^2 r. \]

The calculated spin connection \( \omega^a_{bv} \) can be directly obtained as
\[ \omega^a_{bv} = e^a_\mu \partial_v (e^\mu_b) + e^a_\nu e^\sigma_b \Gamma^\mu_{\sigma v}, \]
where \( e^a_\mu \) (we use the convention that latin indices \( a, b \) are used to label local inertial coordinates and greek indices \( \mu, \nu \) for general coordinates) is \textit{vielbein} and satisfy the equation \( e^a_\mu e^b_\nu g_{\mu \nu} = \eta_{ab}, e^a_\mu e^b_\nu \eta_{ab} = g_{\mu \nu} \), where \( \eta_{ab} = \text{diag}(1, -1) \) is the Minkowski metric. Then we obtain \( e_0^0 = 1/\alpha r, e_1^1 = 1 \). Based on Eq. (S13), the nonzero spin connection is
\[ \omega_{010} = -\omega_{100} = \alpha. \]

Using the spin connection, we can obtain the calculated spinor \( \Omega_v = \frac{1}{4} \omega_{abv} \sigma^{ab} \), where \( \sigma^{ab} = [\gamma^a, \gamma^b] / 2 \). Then nonzero spinor is
\[ \Omega_0 = \frac{1}{4} \omega_{010} \sigma^{01} + \frac{1}{4} \omega_{100} \sigma^{10} = \frac{\alpha}{4} \left[ \gamma^0, \gamma^1 \right]. \]
Therefore, the Eq. (S9) can be written as

\[ i\partial_t \varphi = -i\alpha r \gamma_0 \gamma_1 \partial_r \varphi - \frac{i\alpha}{4} [\gamma^0, \gamma^1] \varphi. \]  

(S17)

After choosing \( \gamma^0 = \sigma_z, \gamma^1 = i\sigma_y \), the above equation can be simplified as

\[ i\partial_t \varphi = h\varphi = -i\alpha r \sigma_x \partial_r \varphi - \frac{i\alpha}{2} \sigma_x \varphi, \]  

(S18)

where \( h \) is Hamiltonian density.

III. The quantum evolution of single-photon wave packet in close proximity to an artificial black hole

In order to investigate the quantum evolution of single-photon wave packet near the event horizon of black hole, we assume that the Eq. (S17) has the solution

\[ \varphi = \frac{1}{\sqrt{r}} \begin{pmatrix} a \\ b \end{pmatrix} e^{ikr - iEt}, \]

where \( k_r \) is the wavevector of the plane wave, \( E \) is the frequency of the plane wave, and then the Eq. (S17) can be written as

\[ E \begin{pmatrix} a \\ b \end{pmatrix} = \alpha k_r r \begin{pmatrix} b \\ a \end{pmatrix}. \]  

(S19)

Therefore, the solution has two energy state \( E = \pm \alpha k_r r \). For positive energy state \( E_+ = \alpha k_r r \), the solution is \( \varphi_+ (r, t) = \frac{1}{\sqrt{2r}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{ik_r r - iE_+ t} \); and for negative energy state \( E_- = -\alpha k_r r \), the solution is \( \varphi_- (r, t) = \frac{1}{\sqrt{2r}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{ik_r r - iE_- t} \).

In order to consider the dynamic behavior of single-photon wave packet with positive-negative energy solution close to the black hole, the time-depended position operator in the
Heisenberg pictures can be written as

\[ r_H(t) = e^{iht/\hbar}e^{-iht/\hbar} \]

\[ = r + \frac{it}{\hbar}(-i\hbar \sigma_x) r + \left( \frac{1}{2!} \right) \left( \frac{it}{\hbar} \right)^2 (-i\hbar \sigma_x)^2 r \]

\[ + \left( \frac{1}{3!} \right) \left( \frac{it}{\hbar} \right)^3 (-i\hbar \sigma_x)^3 r \]

\[ + \left( \frac{1}{4!} \right) \left( \frac{it}{\hbar} \right)^4 (-i\hbar \sigma_x)^4 r + \ldots \]

\[ + \left( \frac{1}{n!} \right) \left( \frac{it}{\hbar} \right)^n (-i\hbar \sigma_x)^n r + \ldots \]

\[ = \left[ 1 + \frac{1}{2!} (\alpha t)^2 + \frac{1}{4!} (\alpha t)^4 + \frac{1}{2k!} (\alpha t)^{2k} \right] r \]

\[ + \sigma_x \left[ \alpha t + \frac{1}{3!} (\alpha t)^3 + \frac{1}{(2k + 1)!} (\alpha t)^{2k+1} \right] r \]

\[ = \sigma_x \sinh(\alpha t) r + \cosh(\alpha t) r. \]  

(S20)

We consider a single-photon Gaussian wave packet of positive energy state as

\[ \varphi_+(r, 0) = N \int dk_\nu \frac{1}{\sqrt{2\nu}} e^{-\nu^2 (k_\nu - k_0)^2 / 2} e^{ik_\nu r} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \]

\[ = N \sqrt{\pi} \frac{1}{w} e^{-\frac{r^2}{2w^2}} e^{ik_0 r} \left( \begin{array}{c} 1 \\ 1 \end{array} \right), \]  

(S21)

where \( N \) is normal coefficient, and the packet centered at \( k_0 \) is characterized by a width of \( w \).

A direct calculation gives the evolution of the wave packet as

\[ \bar{r}_+(t) = \langle \varphi_+(r, 0) | r_H(t) | \varphi_+(r, 0) \rangle \]

\[ = \langle \varphi_+(r, 0) | \sigma_x \sinh(\alpha t) r + \cosh(\alpha t) r | \varphi_+(r, 0) \rangle \]

\[ = \frac{N^2 \pi^{3/2}}{2w} e^{\alpha t} \]  

\[ = r_0 e^{\alpha t}, \]  

(S22)

\[ \bar{v}_+(t) = \partial \bar{r}_+(t) / \partial t = \alpha \bar{r}_+(t). \]
The single-photon Gaussian wave packet of negative energy state is
\[
\varphi_-(r, 0) = N \int dk_r \cdot \frac{1}{\sqrt{r}} e^{-\frac{w^2(k_r - k_0)^2}{2}} e^{ik_r r} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{S23}
\]
and the dynamic evolution can be described as
\[
\tau_-(t) = \langle \varphi_-(r, 0) | r_H(t) | \varphi_-(r, 0) \rangle
\]
\[
= \langle \varphi_-(r, 0) | \sigma_x \sinh(\alpha t) r + \cosh(\alpha t) r \varphi_-(r, 0) \rangle
\]
\[
= \frac{N^2 \pi^{3/2}}{2w} e^{-\alpha t} \tag{S24}
\]
\[
= r_0 e^{-\alpha t},
\]
\[
\tau_-(t) = \partial \tau_-(t)/\partial t = -\alpha \tau_-(t).
\]

By comparing the time evolution of wave packet with different energy, we obtain that the positive energy state has positive velocity and exponentially escapes away from black hole, while the negative state propagates toward and eventually stops around the black hole. The evolution process is analogy of Hawking Radiation. Owing to vacuum fluctuations, particle-antiparticle pair is generated close to the event horizon of a black hole. One of the particles with negative energy falls into the black hole while the other escapes. In order to preserve total energy, the particle that fell into the black hole must have had a negative energy (with respect to an observer far away from the black hole). This causes the black hole to lose mass to an outside observer; it would appear that the black hole has just emitted a particle.

**IV. Discrete Dirac equation near the event horizon of black hole in the waveguide lattice**

In order to discrete the continuous Hamiltonian in the photonic lattice, we start single-particle Hamiltonian $h$ with [5]:
\[
H = \frac{1}{2} \int d\varphi (h \varphi)^+ \varphi + \frac{1}{2} \int d\varphi^+ h \varphi. \tag{S25}
\]
Figure S2: The schematic of optical lattice. $a_m$ and $b_m$ respectively corresponds to waveguide in the upper layer and lower layer, $\beta_0$ is on site energy of each waveguide, $\kappa_0$ is the nearest neighbor coupling coefficient in same layer, $\kappa_{1m}$ and $\kappa_{2m}$ are coupling coefficient between the upper and lower layer.

Considering two-component spinor $\varphi = (a, b)^T$, the above equation is written as:

$$H = \frac{i}{2} \int dr ar (\partial_r a^+ b - a^+ \partial_r b) + h.c$$  \quad (S26)

Furthermore, we discrete the continuous function with lattice as shown Fig. S2:

$$r = md, \quad \partial_r a^+ = \frac{a^+_{m+1} - a^+_{m}}{d}, \quad \partial_r b = \frac{b_m - b_{m-1}}{d}. \quad (S27)$$

The discrete Hamiltonian is

$$H = \frac{i}{2} \sum_m \alpha m \left( a^+_{m+1} b_m + a^+_{m} b_{m-1} - 2 a^+_{m} b_m \right). \quad (S28)$$

Then we obtain coupling equation in the optical lattice:

$$i \frac{da_m}{dt} = [a_m, H] = i\alpha (m - 1/2) b_{m-1} - i\alpha m b_m, \quad (S29)$$

$$i \frac{db_m}{dt} = [b_m, H] = -i\alpha (m + 1/2) a_{m+1} + i\alpha a_m.$$  

After taking $\tilde{a}_m = i^{2m} a_m$, $\tilde{b}_m = i^{2m+1} b_m$, the above equation is:

$$i \frac{d\tilde{a}_m}{dt} = -\alpha (m - 1/2) \tilde{b}_{m-1} - \alpha m \tilde{b}_m, \quad (S30)$$

$$i \frac{d\tilde{b}_m}{dt} = -\alpha (m + 1/2) \tilde{a}_{m+1} - \alpha m \tilde{a}_m.$$
Figure S3: (a) The evolution of fermion pair with positive ($E_+$) and negative ($E_-$) energy state. (b)-(h) The intensity of waveguide section at different time

V. The inhomogeneous coupling between bi-layer waveguide

Furthermore, if we construct bi-layer lattice fabricated by femtosecond laser written technology as shown in Fig. S2, considering the nearest neighbor, the coupled wave equation can be described as:

$$
i\frac{d}{dz}a_m = \beta_0 a_m - \kappa_0 (a_{m+1} + a_{m-1}) - \kappa_{1m} b_{m-1} - \kappa_{2m} b_m,$$

$$
i\frac{d}{dz}b_m = \beta_0 b_m - \kappa_0 (b_{m+1} + b_{m-1}) - \kappa_{1m} a_{m+1} - \kappa_{2m} a_m,$$

(S31)

where $a_m$ and $b_m$ respectively correspond to waveguide site in the upper layer and lower layer, $\beta_0$ is on site energy of each waveguide, $\kappa_0$ is the nearest neighbor coupling coefficient in same layer, $\kappa_{1m}$ and $\kappa_{2m}$ are coupling coefficient between the upper and lower layer. When assuming the approximated solution \( \begin{pmatrix} a_m \\ b_m \end{pmatrix} = \begin{pmatrix} A \exp(i\beta_r m d - i\beta_z z) \\ B \exp(i\beta_r m d - i\beta_z z) \end{pmatrix} \) in local lattice, then we obtain $a_{m\pm1} = \exp(\pm i\beta_r d)a_m$, $b_{m\pm1} = \exp(\pm i\beta_r d)b_m$, the above equation can be written as:

$$
i\frac{d}{dz}a_m = (\beta_0 - 2\kappa_0 \cos(\beta_r d)) a_m - \kappa_{1m} b_{m-1} - \kappa_{2m} b_m,$$

$$
i\frac{d}{dz}b_m = (\beta_0 - 2\kappa_0 \cos(\beta_r d)) b_m - \kappa_{1m} a_{m+1} - \kappa_{2m} a_m.$$  
(S32)
If assuming \[
\begin{pmatrix}
\tilde{a}_m \\
\tilde{b}_m
\end{pmatrix}
= \begin{pmatrix}
 a_m \exp \left[ -(\beta_0 - 2\kappa_0 \cos (\beta_r d)) z \right] \\
 b_m \exp \left[ -(\beta_0 - 2\kappa_0 \cos (\beta_r d)) z \right]
\end{pmatrix},
\]
the above equation can be written as:
\[
\begin{align*}
\imath \frac{d\tilde{a}_m}{dz} &= -\kappa_{1m} \tilde{b}_{m-1} - \kappa_{2m} \tilde{b}_m, \\
\imath \frac{d\tilde{b}_m}{dz} &= -\kappa_{1m} \tilde{a}_{m+1} - \kappa_{2m} \tilde{a}_m.
\end{align*}
\tag{S33}
\]

By comparing the Eq. (S29) and Eq. (S32), if we manipulate the coupling coefficient \(\kappa_{1m} = \alpha (m - 1/2), \kappa_{2m} = \alpha m\) depending on waveguide site \(m\), the quantum evolution of massless fermion pair with positive and negative energy generated by black hole can be emulated by the designed bi-layer optical lattice. And in the photonic lattice the propagation distance \(z\) plays the role as the time \(t\), longitudinal wavevector \(\beta_z\) plays the role as the energy \(E\), transverse wavevector \(\beta_r\) plays the role as the wavevector \(k_r\) of the Dirac fermion. Fig. S3 shows the evolution of fermion near the event horizon of black hole according to number calculation based on Eq. (S30). There are two branches which respectively corresponds to fermion with positive and negative energy as shown in Fig. S3(a): The positive energy state exponentially escapes away from black hole, and negative state propagates toward and eventually stops around the black hole. Fig. S3(b)-Fig. S3(h) shows the intensity of waveguide section at different time where the propagation distance plays as role of time in the waveguide.

In order to clearly clarify the positive and negative state in the waveguide, the dispersion relation at the site \(m\) in the momentum space according to Eq. (S30) can be given as:
\[
\beta_z^m = \beta_0 - 2\kappa_0 \cos (\beta_r d) \\
\pm \sqrt{\left( \kappa_{1m} - \kappa_{2m} \right)^2 + 4\kappa_{1m}\kappa_{2m}\cos^2 (\beta_r d/2)}
\tag{S34}
\]
In Fig. S4, we show the two modes maintain separated with the change of parameters \(\kappa_{1m}\) and \(\kappa_{2m}\).

After taking adiabatic approximation and assuming \(\kappa_{2m} - \kappa_{1m} = \alpha/2 \ll \kappa_{1m}(\kappa_{2m})\) and \(\kappa_{1m} \approx \kappa_{2m} = \alpha m\), the Eq. (S33) can be written as \(\beta_z^m = \beta_0 - 2\kappa_0 \cos (\beta_r d) \pm 2\alpha m \cos (\beta_r d/2)\). Due to translational symmetry in the propagation, the \(\beta_z\) is a conserved quantity. In the initial...
Figure S4: The spectrum of the bi-layer waveguide lattice. The two modes maintain separated with the change of parameters $\kappa_{1m}$. The inset (i) and (ii) give the dispersion relation of the fabricated bi-layer lattice, where $\kappa_0$ is adopted as $0.15\ mm^{-1}$, $\beta_0$ is uniform and is adopted as 0 in calculation, $\kappa_1$ is adopted as $0.5\ mm^{-1}$ for inset (i) and $1.0\ mm^{-1}$ for inset (ii) respectively, and $\kappa_2 = \kappa_1 + 0.005\ mm^{-1}$. For the whole lattice, $\kappa_1$ varies from 0.5 to $1.0\ mm^{-1}$ in step of $0.005\ mm^{-1}$. 
time of the exciting waveguide, $\beta_z$ is taken as $\beta_z = \beta_0 - 2\kappa_0$. Then the group velocity of positive and negative wave packet is:

\[
\begin{align*}
v_+ &= \frac{\partial \beta^+_z}{\partial r} = \alpha m, \\
v_- &= \frac{\partial \beta^-_z}{\partial r} = -\alpha m.
\end{align*}
\tag{S35}
\]

The Eq. (S34) is corresponding to Eq. (S21) and Eq. (S23) well. All these equations unveil that the positive energy state escapes away from black hole with increasing positive velocity, while the negative energy state propagates toward to black hole with decreasing negative velocity.

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