Future Probes of the Primordial Scalar and Tensor Perturbation Spectra: Prospects from the CMB, Cosmic Shear and High-Volume Redshift Surveys

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Detailed study of the scalar and tensor perturbation spectra can provide much information about the primordial fluctuation-generator, be it inflation or something else. The tensor perturbation spectrum may be observable through its influence on CMB polarization, but only if the tensor-to-scalar ratio, \( r \equiv T/S \), is greater than about \( 10^{-5} \). The tensor tilt can be measured with an error of \( \sigma(n_T) \) that decreases with \( r \) from 0.1 at \( r = 0.001 \) to 0.02 at \( r = 1 \). Current CMB constraints on the scalar perturbation spectrum can be improved by higher-resolution CMB observations and/or by tomographic cosmic shear observations. These can both shrink errors on the tilt \( (n_S) \) and running \( (n_S' \equiv dn_S/d\ln k) \) to the \( 10^{-3} \) level. Stunning as these results would be, it may become very desirable to improve upon them an order of magnitude further in order to study the expected departures from \( n_S' = 0 \). Such improvements are likely to require observation of three-dimensional clustering over very large volumes. Unfortunately, to get down to the \( 10^{-4} \) level will require a sparse spectroscopic redshift survey with about \( 10^7 \) galaxies spread over a volume less than but comparable to that of the observable Universe.

I. INTRODUCTION

Inflation is doing remarkably well with respect to observations. The evidence that structure in the Universe formed from adiabatic, nearly scale-invariant fluctuations is very strong. The mean spatial curvature has been determined with high accuracy and is consistent with zero. The perturbations are highly Gaussian. These are all predictions of inflation, of varying degrees of robustness. Topological defect models, formerly inflation’s chief competition, have been strongly ruled out (e.g., [4]).

Despite this great success we still have little understanding of the physics that led to these initial perturbation spectra. There are many different models of inflation. As Michael Turner said in his talk at this meeting, we have a Landau-Ginzburg theory, but we are missing the underlying BCS theory.

To discriminate among inflationary models, to assist in the quest for a BCS theory of inflation, and perhaps to discriminate between inflation and alternatives (such as we heard from Steinhardt and from Durrer), we want to take the determination of the primordial tensor and scalar perturbation spectra to a qualitatively new level of precision. This point was also emphasized in Albrecht’s talk.

I will first briefly review the current constraints on the scalar perturbation spectra before turning to the future. The discussion of future experiments will start with the constraints on the tensor perturbation spectra from CMB experiments. We will then look at how current CMB data can be complimented either with higher-resolution CMB data and/or tomographic cosmic shear. Finally, I consider results that can in principle be achieved by very high-volume spectroscopic redshift surveys.

II. NON-ZERO RUNNING?

A question of great relevance to inflationary models is the significance of evidence for running of the spectral index. The evidence from CMB data alone is very weak: \( n_S' \equiv dn_S/d\ln k = -0.055 \pm 0.038 \). Combining CMB data with the Croft et al. matter power spectrum inferred from high-resolution observations of the Ly\textsubscript{α} forest results in \( n_S' = -0.031 \pm 0.017 \). Other authors working with the same datasets have since found much looser constraints on \( n_S' \). The looser bound on \( n_S' \) is due to their marginalization over the mean ionizing flux as a function of redshift. The importance of marginalizing over this parameter, which leads to a large degeneracy between spectral index and amplitude, was pointed out in

We had much discussion at this meeting about constraints from combining CMB data with low-resolution spectra from the Sloan Digital Sky Survey (SDSS), as presented by Hui and by Seljak. Although lower resolution, there are thousands of these spectra allowing for very small statistical errors. Possible systematic errors have not yet been understood and controlled well enough to allow for any firm detections of \( n_S' \neq 0 \).

To summarize the current situation: there is no strong evidence for \( n_S' \neq 0 \), but very interesting results may be coming soon from SDSS quasar spectra combined with WMAP.

III. CMB AND TENSOR PERTURBATIONS

One can decompose a polarization pattern on the sky into curl-free modes (E modes) and divergence-free modes (B modes). Each scalar three-dimensional Fourier mode only has one direction, that given by the wavevector \( k \), and therefore only leads to E modes. Ten-
sor Fourier modes (gravitational waves) have the direction given by $k$ and also an orientation given by the polarization of the gravitational wave. They can therefore generate B modes in addition to E modes. Since, at least in linear perturbation theory, scalar perturbations do not generate B modes, whereas tensor perturbations do, the B mode has been proposed as a means to detect the tensor perturbations.

At second order in perturbation theory we cannot solve for just one Fourier mode at a time and then sum up the resulting solutions; the evolution of one Fourier mode is affected by the presence of others. This mode-mode coupling leads to the generation of B modes even from scalar perturbations \[ \mathbf{6} \]. The dominant second order effect is gravitational lensing. The lensing-induced B modes can obscure the gravitational wave contribution to the B modes. If the lensing-induced B modes are not cleaned from the B-mode map, one can only detect the tensor signal (at 3$\sigma$) if $r > r_{\text{th}} = 1.7 \times 10^{-4}, 7 \times 10^{-5}$ or, $2.4 \times 10^{-5}$ for $\tau = 0.05, 0.1$ or 0.2 respectively.

Gravitational lensing leads to off-diagonal correlations in Fourier space \[ \mathbf{10, 11, 12, 13} \]. These can be used to reconstruct the lensing potential \[ \mathbf{14, 15} \]. With the lensing potential thus reconstructed, the maps can be unlensed. A perfectly unlensed map would have no scalar B modes in it. Unavoidable imperfections in the lensing potential reconstruction mean the unlensed map will have some residual B mode, even in the absence of tensor perturbations. These residual B modes prevent detection of gravity waves unless $r > r_{\text{th}} = 1.4 \times 10^{-5}, 6 \times 10^{-6}$ or $2 \times 10^{-6}$ for $\tau = 0.05, 0.1$ or 0.2. These limit calculations were first done by \[ \mathbf{16, 17} \].

As shown in \[ \mathbf{35} \], if $r \geq 0.01$ then we can learn something about the shape of the tensor B mode power spectrum. The possible constraints on $n_T$ as a function of $r$ are shown in Fig. 1.

Tensor spectra from inflationary models with a single field in slow roll obey the ‘consistency equation’, $r = -5n_T \mathbf{18}$. Note that, more generally, $r = f(\Omega) n_T$ and that $f(0) \approx -7$ and $f(0.7) \approx -5 \mathbf{19, 20}$. From the figure we can see that for $r \geq 0.03$ we can make a significant test of the consistency relation. The consistency equation applies for all single-field models to first order in the slow roll parameters.

IV. CMB AND SCALAR PERTURBATIONS

Constraints to the scalar perturbation spectra can be improved by pushing to higher angular resolution than the $\sim 13'$ of the WMAP’s highest frequency channel. It can not be improved arbitrarily though because of the exponential suppression of power that sets in at the Silk damping scale, due to photon diffusion during recombination. High sensitivity can to some degree fight against this exponential cutoff, but not to indefinitely high $\ell$.

Here we show results expected for three different experiments (with parameters specified in Table I for the amplitude of the primordial gravitational potential power spectrum, $P_\Phi(k_f)$, $n_S$, and $n'_S$ where

$$\ln |P_\Phi(k)| = \ln |P_\Phi(k_f)| + (n_S(k_f) - 3) \ln \frac{k}{k_f} + n'_S \ln \frac{k}{k_f}$$

We also include the expected constraints on $w$ and $m_\nu$. The errors in Table II are what would result from a simultaneous fit to these parameters plus $\Omega_b h^2$, $\Omega_m h^2$, $\theta_s$ (the angular size of the sound horizon), the Helium mass fraction and $\tau$. We assume the mean spatial curvature is zero. See \[ \mathbf{21} \] for more details.

Are these highly precise measurements of $n_S$ and $n'_S$ valuable? The difference $n_S - 1$ is first order in the slow-roll parameters and $n'_S$ is second order. Thus we expect

\begin{table}[h]
\centering
\begin{tabular}{llllccc}
\hline
Experiment & $r_T^f$ & $r_B^f$ & $f$ (GHz) & $\theta_b$ & $\Delta r$ & $\Delta \rho$ \\
\hline
Planck & 2000 & 3000 & 100 & 9.2' & 5.5 & $\infty$ \\
& & & 143 & 7.1' & 6 & 11 \\
& & & 217 & 5.0' & 13 & 27 \\
SPTpol ($f_{sky} = 0.1$) & 2000 & 3000 & 217 & 0.9' & 12 & 17 \\
& & & 217 & 3.0' & 1 & 1.4 \\
\hline
\end{tabular}
\caption{Experimental specifications.}
\end{table}
The signal covariance has the structure

\[ S_{lmz,lm'z'} = C_{l}^{zz'} \delta_l \delta_{mm'} \]

and the noise covariance is entirely diagonal:

\[ N_{lmz,lm'z'} = N_{l}^{2} \delta_l \delta_{mm'} \delta_{zz'} \]

The average ellipticity of galaxies, \( \gamma_{rnm} \), leads to an error in the estimated real-space convergence map with variance \( \gamma_{rnm}^{2}/N_{l}^{2} \) where \( N_{l}^{2} \) is the mean number of galaxies in each pixel in redshift bin \( z \). This error is uncorrelated from pixel to pixel and from redshift bin to redshift bin. It leads to an error in the Fourier-transformed convergence with variance

\[ N_{l}^{2} = \gamma_{rnm}^{2}/N_{l}^{2} \]  

For specificity we take \( \Omega_{l}^{pix} = 11 \) sq. arcmin and \( \gamma_{rnm} = 0.2 \). The mean number of galaxies in each pixel we take to be

\[ N_{l}^{pix} = 1.42 z^{1.1} \exp[-(z/1.2)^{1.2}] \]

for \( \Delta z = 0.4 \). This is a fit to the extrapolation from Nagashima et al. 2002 assuming a limiting magnitude in R of 26 (Tony Tyson, private communication). We also assume that half of these galaxies in the 1.2 < z < 2.5 range will not be able to use because we will not be able to get sufficiently accurate photometric redshifts.

As the pixel size increases, the number of galaxies increases and the statistical error drops (equivalently, in Fourier space, \( l^{2}N_{l} \) increases with \( l \) since \( N_{l} \) is constant). We do not expect any observations to keep the systematic errors smaller than \( 10^{-4} \) on scales above 10'. With our modeling of the statistical errors, the rms statistical error on the shear maps drops below \( 10^{-4} \) on scales of 5 degrees. For this reason, we quote our results without \( l < l_{c} \) where \( l_{c} = 180/5 = 36 \).

The auto- and cross-angular power spectra for the convergence, \( C_{l}^{zz'} \), are given by (e.g., 30)

\[ C_{l}^{zz'} = \frac{\pi^{2}l}{2} \int drrW_{z}^{*}(r)W_{z'}^{*}(r)\Delta_{l}^{2}(k, r) \]

with \( l = kr \) and

\[ W_{z}^{*}(r) = \frac{1}{2} \frac{\sqrt{2}}{\bar{n}_{i} r} \int_{r(z_{i1})}^{r(z_{i2})} dr' \frac{r'}{r'(r' - r)} N_{i}(z')dz'/dr' \Theta(r' - r) \]

where \( z_{i1} = z_{i} - \Delta z/2, z_{i2} = z_{i} + \Delta z/2 \) and \( n_{i} \) is

\[ \bar{n}_{i} = \int_{r(z_{i1})}^{r(z_{i2})} dr' N_{i}(z')dz'/dr' \]

and we have assumed that the r.m.s. are evenly distributed through the redshift bin.

In addition to the convergence, \( \kappa \), there are also the lensing potential, \( \phi \) and the deflection angle \( \Delta \Phi \). They are related in real space by \( d = \nabla \phi \) and \( \kappa = \nabla^{2} \phi/2 \). Their power spectra are related by

\[ 2C_{l}^{-2}(z) \Delta \phi/(2\pi) = l(l+1)c_{l}^{d}(\phi)d(\phi)/(2\pi) = l^{2}(l+1)^{2}c_{l}^{\phi}(z)\Delta \phi/(2\pi) \]

TABLE II: Standard deviations expected from Planck, SPTpol and CMBpol. From 21.

| Experiment    | \( m_{n} \) (eV) | \( w_{z} \) | \( \ln P_{s}^{0} \) | \( n_{S} \) | \( n_{S}' \) |
|---------------|-----------------|-------------|-----------------|-----------|-----------|
| Planck        | 0.14            | 0.28        | 0.016           | 0.0074    | 0.0032    |
| SPTpol        | 0.11            | 0.34        | 0.018           | 0.01      | 0.0057    |
| SPTpol + Planck| 0.082          | 0.22        | 0.016           | 0.0057    | 0.0027    |
| CMBpol        | 0.031           | 0.088       | 0.011           | 0.0024    | 0.0014    |

The shapes of galaxies are distorted by gravitational lensing. If we knew the shapes of the galaxies in the absence of lensing we could then infer the lensing convergence, \( \kappa \), from the lens-induced alteration of the shape. Although we have no way of knowing the unlensed shape of an individual galaxy, we expect that an average over a large number of galaxy images would be perfectly cylindrical. The average departure from a completely cylindrical shape can thus be used as a measure of the amount of lensing. From proper analysis of the average galaxy shape in some pixel, the lensing convergence, \( \kappa \), can be recovered.

We model the data \( \kappa \) (the convergence vector with index running over \( l, m \) and redshift bin \( z \)) as a Gaussian random field with zero mean and contributions to the total variance from both signal and noise:

\[ \langle \kappa \kappa \rangle = S + N. \]

The signal covariance has the structure

\[ S_{lmz,lm'z'} = C_{l}^{zz'} \delta_l \delta_{mm'} \]

V. CMB+COSMIC SHEAR

Adding cosmic shear to CMB data can improve constraints of cosmological parameters, as was studied in the first CMB+cosmic shear forecasting paper 21. A number of other studies have followed, extending to use of photometric redshifts to allow tomography 25 and to examine constraints possible on \( m_{n} \) and \( w_{z} \).

Other studies have followed, extending to use of photometric redshifts to allow tomography 25 and to examine constraints possible on \( m_{n} \) and \( w_{z} \).
In Fig. 2 we see the convergence power spectra for four redshift slices: \( z = 0.6, z = 1, z = 2.2 \) and \( z = 1100 \). Errors on \( z = 1100 \) are those expected from Planck. Errors on the others are those expected from the 30,000 sq. degree LSST survey described above with bins in redshift of width \( \Delta z = 0.4 \). The convergence rises monotonically with redshift since the lensing contribution from each redshift interval adds incoherently to that of the previous redshift interval.

**FIG. 2:** Convergence power spectra for four redshift slices: \( z = 0.6, z = 1, z = 2.2 \) and \( z = 1100 \). Errors on \( z = 1100 \) are those expected from Planck. Errors on the others are those expected from a 30,000 sq. degree survey to \( m_R = 26 \) with bins in redshift of width \( \Delta z = 0.4 \).

In Fig. 3 we plot the correlation, \( r_{l,l'}^{zz'} \equiv C_{l,z}^{zz'} / \sqrt{C_{l,z}^{zz} C_{l,z'}^{zz'}} \), for \( z = 0.6, 1 \) and 2.2 with \( z' \) fixed to 1100. The correlation increases with increasing redshift as the window functions become better matched. In other words, the correlation rises with increasing redshift since as \( z \) goes towards 1100 a greater fraction of the lensing structures is common to both.

By differencing the auto power spectra with appropriate weightings one can isolate how much lensing power is coming from each redshift range, and therefore the power spectrum of the matter in each redshift range. To forecast errors on the parameters that govern the matter power spectrum, we need not do this differencing. Our statistical model of the data is completely specified by the auto and cross–power spectra and we need only understand our errors on these and how they vary as we vary the model parameters.

To forecast errors we calculate the Fisher matrix

\[
F_{pp'} = \frac{1}{2} \text{Tr} [S_p W S_{p'} W]
\]

where the subscript \( , p \) means differentiation with respect to parameter \( a_p \) and

\[
W \equiv (S + N)^{-1}.
\]

The inversion of \( W \) is numerically tractable because of its block–diagonal structure:

\[
W_{l,mz,z'm'} = W_l^{zz'} \delta_{mm'} \delta_{zz'}. \tag{14}
\]

Since photometric redshifts are completely unproven for \( z > 3 \) we make no use of galaxies at \( z > 3 \). To reduce sensitivity to the non-linear evolution which we may not have modeled well enough we restrict ourselves to \( l < 1000 \). Results are shown in Table III.

There is not much to be gained by going deeper, at least with our \( z \)-independent \( l \) cutoff at \( l = 1000 \). The reason is that the number density of galaxies is high enough in each redshift bin that the \( C_l \) measurements are sample-variance limited for \( l < 1000 \). For the lower redshift bins there is not much to be gained by going to higher \( l \) because that puts us into the fully non-linear regime which has little to no shape information. Also in the non-linear regime our formalism for forecasting the uncertainty breaks down. But for higher redshift bins one can in principle push to higher \( l \) and gain by it. The \( l \) that divides linear from non-linear is higher at high redshift for two reasons: 1) a given \( k \) projects into a higher
TABLE III: Standard deviations expected from WMAP (after 2 years), Planck (without reconstruction of \(C_l^{dd}\)), WMAP + LSST and Planck (with reconstruction of \(C_l^{dd}\)) + LSST.

| Experiment                  | \(m_\nu\) (eV) | \(w_x\) | \(\ln P_k^b\) | \(n_S\) | \(n'_S\) |
|-----------------------------|----------------|--------|----------------|--------|--------|
| WMAP                        | 1.2            | 2.4    | 0.061          | 0.055  | 0.018  |
| Planck (no lensing)         | 0.32           | 0.45   | 0.016          | 0.0077 | 0.0035 |
| WMAP+LSST                   | 0.085          | 0.023  | 0.023          | 0.0004 | 0.0027 |
| Planck+LSST                 | 0.041          | 0.019  | 0.012          | 0.0042 | 0.0021 |

Although we need full sky to get these exquisitely small errors, we only need full sky to beat down sample variance, not to accurately determine fluctuation power on large angular scales.

VI. CMB + HIGH-VOLUME REDSHIFT SURVEYS

The errors on \(n'_S\) from WMAP+LSST or from higher-resolution CMB observations, are just at the level of \((n_S - 1)^2\) if \(n_S \approx 0.95\). These experiments may very well leave us only with tantalizing one or two \(\sigma\) determinations of \(n'_S\). We may want to do better.

Better CMB experiments will not do it. Better tomographic cosmic shear experiments probably will not do it. We can understand why with a simple expression for the error on a power-spectral index, \(n\), given errors on the power \(P_1\) and \(P_2\) at two different scales, \(k_1\) and \(k_2\):

\[
\sigma^2(n) = \frac{\sigma^2(\ln P_1) + \sigma^2(\ln P_2)}{(\ln(k_2/k_1))^2}. \tag{15}
\]

First note that gaining dynamic range (increasing \(k_2/k_1\)) only reduces the error on \(n\) logarithmically. Extending dynamic range is not a way to dramatically improve the constraints on \(n\). The only way to dramatically reduce \(\sigma(n)\) is to make better measurements of the power over around a decade (or more) in wavenumber. 2-dimensional surveys can not do this. Even with 9 uncorrelated measures of fluctuation power in redshift bins between zero and last-scattering we can reduce the CMB alone errors by at most a factor of 3.

Thus we explore the constraining power of three-dimensional measurements of clustering. In particular, we consider a spectroscopic redshift survey. See [21] for more discussion of future redshift surveys. For simplicity, we ignore effects of redshift distortions and scale-dependent galaxy bias, which should be included in a more detailed study. Note that very deep cosmic shear surveys may be invaluable for calibrating the galaxy-density relationship for these spectroscopic surveys (in order to constrain the bias), and for providing the targets for spectroscopic follow-up.

For a survey of volume \(V\), the error on the power in a band of width \(\Delta k\) is [22, 33]:

\[
\Delta P(k) = \sqrt{\frac{2}{N_k}} \left( P(k) + \frac{1}{nb^2} \right). \tag{16}
\]

where \(N_k\) is the number of independent samples of \(\delta(k)\) with \(-\Delta k/2 < |k| < \Delta k/2\) and is given by

\[
N_k = 4\pi k^2\Delta k \frac{V}{(2\pi)^3} = k^2\Delta k/(2\pi^2)V. \tag{17}
\]

The mean number density of galaxies, \(\bar{n}\), determines the shot-noise contribution to the variance, \(1/(\bar{n}b^2)\) where \(b\) is the galaxy bias such that \(P_b(k) = b^2P(k)\).
An optimal survey with fixed number of galaxies $N_g = \bar{n} V$ (optimal from a solely statistical error point of view) will have a volume such that $1/\bar{n} = b^2 P(k)$ on the scale of interest where $b$ is the bias of the galaxies. In this case

$$\frac{\Delta P(k)}{P(k)} = \frac{\sqrt{8/N_g}}{2} \left[ N_g b^2 \Delta^2 (k) \left( \frac{\Delta k}{k} \right) \right]^{-1/2}$$

(18)

where $N_g = \bar{n} V$ is the number of galaxies in the survey and $\Delta^2 (k) \equiv k^3 P(k)/(2\pi^2)$.

Solving for $N_g$ we find

$$N_g = 1.5 \times 10^9 \left[ \frac{10^{-4}}{\Delta P/P} \right]^2 \frac{2}{b^2 \Delta^2 (k)} \left[ \frac{k}{\Delta k} \right].$$

(19)

This is a lot of galaxies. And unfortunately, most of them are at high redshift (if we want $\Delta P/P = 10^{-4}$). If the volume is spherical with coordinate distance radius of $R$ then

$$R = 7 h^{-1} Gpc \left( \frac{0.2 h Mpc^{-1}}{k} \right) \left( k/\Delta k \right)^{1/3} \left[ \frac{10^{-4}}{\Delta P/P} \right]^{2/3}.$$

(20)

For reference, the distance to the horizon is about 14 Gpc.

One might conceivably do better by extending to higher $k$ than the fiducial $0.2 h$ Mpc$^{-1}$. The main benefit is the reduced volume one has to sample ($V \propto k^{-3}$, see above equation) to achieve the same value of $\Delta P/P$. The higher this is pushed, however, the more one must worry about scale-dependent bias. And when one wants $\Delta P/P$ as small as $10^{-4}$ even a tiny amount of unknown scale-dependence can lead to highly significant systematic error.

VII. CONCLUSIONS

If inflation happened at sufficiently high energy scales ($V^{1/4} \gtrsim 2 \times 10^{15}$ GeV) and nature is kind to us with respect to astrophysical foregrounds (dust, synchrotron radiation, etc.) then we can detect the influence of tensor perturbations in the CMB polarization. If the energy scale is slightly higher, we will be able to verify (or rule out) the inflation consistency equation.

For measurement of the scalar perturbation spectrum, constraining $n_S$ and $n'_S$ to better than $10^{-3}$ may be impossible. This level can be reached with a post-Planck CMB polarization mission or by combining WMAP or Planck with all-sky tomographic cosmic shear observations. Spectroscopic redshift surveys, even with benign assumptions about galaxy bias, are unlikely to improve our ultimate constraints on $n_S$ and $n'_S$.

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