Monte Carlo simulation of gas free molecular flow in turbo molecular pump’s inlet tube

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ABSTRACT
Due to the beaming effect of inlet tube and reflection effect of turbine cover in the turbo molecular pump, the positional and angular distribution of incident gas molecules (IGM) on the turbine blades would no longer be uniformly distributed. Based on the test particle Monte Carlo method, a new self-defining procedure is proposed and it can be more accurate to describe the status of IGM into turbine blades. The results show that the influence of rotor speed on the transmission probability is indistinctive. Meanwhile, the incident density between the tip and root of the blades is distinct. The positional beaming effect and the angular beaming effect are enhanced with the lengthening of the tube and less molecule diverge towards the tube wall. The positive transmission probability of first-stage turbine blades using our new method is different from those obeying the cosine distribution under the same number of incident molecules. The calculation results of new method verified by another procedure are reasonable and accurate.

1. Introduction
A turbo molecular pump (TMP) is currently one of the most widely used apparatus for obtaining high and ultrahigh vacuum. A number of investigations have been carried out on the pumping principle \cite{1}, key structures \cite{2–6} and manufacturing technologies \cite{7} of the pump. Among the studies on effective pumping speed of TMP, especially the pumping mechanism, at the turbine blades region, either with the 2D model \cite{8–10} or 3D model \cite{11–15}, most of them are based on the hypothesis that the incident gas molecules (IGM) distribute uniformly at the turbine blades surface \cite{8–15}. Between circular inlet surface of the tube and the annular surface of turbine blades, the flow resistance of the inlet tube is usually disregarded or simply regarded as that of a short tube. However, the former studies on the beaming effect for the IGM through a straight circular tube illustrate the following results \cite{16–18}: the IGM distribute uniformly at the inlet surface of the tube, but lose uniformity when they arrive at the turbine blades, and will gather towards the centre of the tube. Most of these dense molecular beams hit, unfortunately, the turbine cover in the centre of turbine rotor, and then are reflected back. That significantly reduces the number of gas molecules which arrive at the turbine blades and thereby decreases the effective pumping speed. At the same time, the gas molecules entering the turbine blades no longer uniformly distributed along the radial direction either. These facts inevitably bring the original error to the former theoretical research based on the assumption of uniform distribution. Meanwhile, these facts also lead to a decrease the actual pumping speed of TMP, and the decline cannot be simply estimated as the loss of tube flow resistance.

According to previous studies \cite{19–22}, Monte Carlo simulation has been applied to many fields. A new self-defining procedure is proposed to describe the IGM density distribution, angular distribution and move trace through inlet tube of TMP by the test particle Monte Carlo (TPMC) simulation method in this paper. It simulates the process of the gas molecules flow from the inlet surface of tube to the turbine blades surface and calculates the transmission probability, density distribution and angular distribution at the turbine blades surface. Thus, this paper gives a more accurate description for the status of molecules entering the first-stage turbine blades and detailed flow field characteristics of the gas molecular flow in the TMP’s inlet tube, and provides reference data for the theoretical study and practical application on TMP’s pumping performance.

2. Hypotheses and models
2.1. Basic hypotheses
According to the kinetic theory of gases in the free molecular flow range, all the simulations are based on the following basic hypotheses \cite{16}:

(1) The IGM distribute uniformly on the entire inlet surface of the tube.

(2) The gas molecules’ flow direction complies with the cosine distribution when they inject the inlet surface and leave the solid reflective surface (including the turbine cover surface and the cylinder inner wall surface of the tube) without surface diffusion.

(3) Molecules move trace along the ray, and the collisions between molecules are ignored.
(4) The particle velocity complies with Maxwellian velocity distribution when they leave the stationary surface.

2.2. Geometric model of the tube

Select a straight cylinder tube as the research object, which is most widely used in inlet of TMP (Figure 1). Thus, the tube inlet diameter (regarded as the vacuum container’s pumping port) is equal to the outer diameter of the turbine blades, which is defined as \( D \); at the central part of the turbine rotor, the cover’s diameter is defined as \( d \) (i.e. the inner diameter of the turbine blades). The annular region between \( d \) and \( D \) is the actual pumping region of the turbine rotor. The tube length is defined as \( L \) (i.e. the distance between the tube inlet surface and the turbine blades surface). For the further analysis, the region of inlet surface of tube, turbine cover, the annular region between \( d \) and \( D \) and the inner wall surface of tube is regarded as \( A_1 \), \( A_2 \), \( A_3 \) and \( A_4 \), respectively.

3 Theory and algorithms

3.1. Theoretical analysis of flow field characteristics

The move paths of the gas molecules are also shown in Figure 1. Moreover, Table 1 elaborates the possible move direction of IGM in TMP’s inlet tube.

When the gas pressure in the vacuum container and the pumping capacity of TMP are constant, the gas molecules form a steady non-equilibrium flow in the inlet tube. Since the molecular incident density at \( A_1 \) is steady, the molecular incident and reflection density at \( A_2 \) and \( A_4 \) are also steady.

Because of the beaming effect [18] and non-equilibrium flow, the gas molecular density at \( A_2 \) and \( A_3 \) is not uniform along the radial direction which causes the non-uniform reflection intensity of gas molecules from those surfaces. In addition, since the entire geometry is axisymmetric, the density and velocity distribution of gas molecules are also axisymmetric in each region.

3.2. Simulation procedure and method

Based on the TPMC method [23], the simulation procedure for inlet tube of TMP is compiled by VB 6.0 software (Figure 2). In order to verify the feasibility and reliability of the self-defining procedure, the transmission probability of tube with different slenderness ratios is calculated by two different methods. The computation results show good quantitative agreement (Section 3.6).

3.3. Parameter symbols definition and selection

In view of the geometric similarity of the gas flow under the condition of free molecular flow, a dimensionless method is adopted in all of the size calculations. The tube’s diameter \( D \) is regarded as the reference quantity; the turbine cover’s diameter and the tube length are represented by the relative sizes \( d/D \) and \( L/D \).

The meanings of symbols and subscripts in the article are listed in Table 2.

In this paper, the structural parameters of the inlet tube model are chosen as followed: the tube diameter \( D \) is the reference quantity, taken as 1; the turbine cover diameter \( d \) is represented by \( d/D \) and selected five values: 0.4, 0.45, 0.5, 0.55, 0.6; the tube length \( L \) is represented by \( L/D \) and selected eight values: 0.1, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.7, 1, 2, 3, 4, 5; and the rotor speeds are selected two values: 24,000 and 60,000 rpm, respectively. These two rotor speeds are selected

![Figure 1](https://via.placeholder.com/150)

**Figure 1.** (Colour online) Schematic of molecular paths in inlet tube and TMP model.

| Table 1. The possible move direction of IGM. |
|---------------------------------------------|
| Gas molecules launching surface | Receive surface |
|---------------------------------|----------------|
| \( A_1 \) | \( A_2 \), \( A_4 \), \( A_3 \) |
| \( A_2 \) | \( A_1 \) |
| \( A_4 \) | \( A_2 \), \( A_4 \), \( A_1 \), \( A_3 \) |

![Figure 2](https://via.placeholder.com/150)

**Figure 2.** The flow chart of the simulation procedure.
to calculate, mainly because high rotor speed is fit for small pumping speed TMP and low rotor speed is fit for large pumping speed TMP.

3.4. Grid division

In order to investigate the density distribution at A3 and A4, the A1 is divided into 20 concentric annular regions and the same is true for the whole A2 and A3. And the A4 is divided by the same annulus width as the one of concentric annular region of A1. For example, \( L/D = 1 \), the annulus number of A4 is 20; \( L/D = 2 \), the number is 40 and so on. Meanwhile, this is also for the convenience of counting the number of gas molecules that reach each region. The schematic diagrams of concentric annular region divided for A1 and A4 are shown in Figure 3.

3.5. Determination of test particles number

The computation data for the different number of test particles \((N = 10^5, 0.3 \times 10^6, 0.5 \times 10^6, 0.2 \times 10^7, 0.5 \times 10^7, 0.8 \times 10^7 \) and \( 10^7 \)) are shown in Figure 4 by employing geometric parameters \( L/D = 1 \), \( d/D = 0.5 \) and \( n = 24,000 \) rpm. A stable value of transmission probability and computer simulation time can be found from Figure 4. Accurate results and the shortest simulation time are what we want. Consequently, considering these two performance parameters, \( 10^6 \) particles is chosen for following simulation, which can provide an accurate result and an optimal simulation time.

3.6. Verification of self-defining procedure

Based on TPMC method by Molflow+ [24] software and the self-defining procedure, some key data were obtained as shown in Table 3. The Molflow+ is a Monte Carlo simulator package developed at CERN. It simulates gas molecule flow in a vacuum chamber under the condition of free molecular flow. But the software cannot simulate the condition that the boundary has a speed. Instead, the self-defining procedure can simulate the condition of considering the rotation of the turbine cover. In the dimensionless calculation model, we converted the actual rotating speed of the turbine cover into a dimensionless one. The conversion process is that the simulating rotating speed is equal to the actual rotating speed multiplied by the actual diameter.

There is a good match by comparing the transmission probability from Table 3. Compared with previously published data [25], the results are also very reliable. Thus, the reliability of this self-defining procedure is verified through comparison with different methods and known numerical results. The error analysis of the two methods will be further studied in the future.

4. Results and discussion

4.1. The calculation of transmission probability

The transmission probability defined as a ratio is given by the following equation:

\[
P = \frac{N_3}{N_1}.
\]  
(1)

The variation of transmission probability versus \( L/D \) at different \( d/D \) and \( n \) values are shown in Figure 5.

From above curves it can be seen that, as \( d/D \) and \( L/D \) become larger, the total transmission probability decreases, which is not only caused by the increase of turbine cover diameter and tube length but also by the positional beaming effect.

For different rotor speeds, the variations in transmission...
probability are indistinctive. Thus, rotor speed of TMP has a weaker effect on the transmission probability of the tube.

For qualitative analysis later, the regression analysis for the calculation data of transmission probability is carried out by Statistical Product and Service Solutions (SPSS) software. According to variation curves of transmission probability in Figure 5, the fitting equations of these curves are as follows.

When \( n = 24,000 \text{ rpm} \), the transmission probability is given by the following equation:

\[
P = \begin{cases} 
(\frac{d}{D})^{-0.033} - 0.96 \\
+10.92 \exp \left(-0.472 \frac{L}{D}\right) \\
[0.85(d/D) - 0.2] \cdot \ln \left(\frac{L}{D}\right) \\
-0.173(d/D) + 0.52 
\end{cases} \quad L/D \leq 1 \quad (2)
\]

When \( n = 60,000 \text{ rpm} \), the transmission probability is given by the following equation:

\[
P = \begin{cases} 
(\frac{d}{D})^{-0.022} - 0.954 \\
+11.905 \exp \left(-0.47 L/D\right) \\
[0.07 \cdot (d/D) - 0.198] \\
-0.14 \cdot (d/D) + 0.514 
\end{cases} \quad L/D \leq 1 \quad (3)
\]

The error analysis of above equations is shown in Table 4, which present both equations meeting the general calculation requirements.

### 4.2. Incident density distribution at \( A_4 \)

Calculating incident density distribution at \( A_4 \) is aimed to obtain the molecular angle distribution of turbine blades surface (Section 4.5). For easy statistics and calculations, the inner wall is divided by the several equal-interval annuluses along the axial direction. The annulus closest to the inlet surface is marked as number 1. The further the inlet surface is, the larger the mark number is. Therefore, the number is gradually increased from inlet surface to turbine blades surface.

The relative incident density \( W_i \) of each annulus region on \( A_4 \) is given by Equation (4). When \( d/D = 0.5 \), the variation of relative incident density along the axial direction for three typical tube lengths are shown in Figure 6:

\[
W_i = \frac{N_i / S_i}{N_1 / S_1} \quad (4)
\]

When the turbine cover diameter and rotor speed is constant, the trends among the curves are similar for different tube lengths (Figure 6). These curves are all linear reduction at first, but fluctuation at last (about \( 0.3 \text{ times} L/D \) to \( A_3 \)). The longer the tube is, the smaller the slope is, and the difference between the maximum and minimum relative incident density is larger. For instance, the difference between maximum and minimum is 0.707 for \( L/D = 3 \), while that is 0.345 for \( L/D = 0.3 \).

In order to quantitatively describe the relationship between relative incident density \( W \) and relative position of tube \( Lx/D \), and calculate the angular distribution at \( A_3 \), the fitting formula

| Table 3. Transmission probability calculated by Molflow+ and self-defining procedure with \( n = 0 \text{ rpm} \). |
|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| \( d/D \) |          | \( L/D = 0.1 \) |          | \( L/D = 0.5 \) |          | \( L/D = 1 \) |          | \( L/D = 2 \) |          | \( L/D = 5 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.4       | 0.75635   | 0.75639   | 0.57857   | 0.57772   | 0.45873   | 0.45855   | 0.32909   | 0.32949   | 0.18257   | 0.18361   |
| 0.5       | 0.67103   | 0.67082   | 0.52512   | 0.52520   | 0.42506   | 0.42531   | 0.31156   | 0.31121   | 0.17708   | 0.17740   |

| Table 4. The error result value of regression analysis. |
|----------------------------------------|----------------------------------------|
|                                       | \( n = 24,000 \text{ rpm} \) | \( n = 60,000 \text{ rpm} \) |
|----------------------------------------|----------------------------------------|----------------------------------------|
| Residual sum of squares                | \( L/D \leq 1 \)                | \( L/D > 1 \)                  | \( L/D \leq 1 \)            | \( L/D > 1 \)                  |
| \( \sum_{x}^{2} \)                    | 0.003                                | 0.000                                | 0.004                                | 0.000                                |
| \( R^2 \)                             | 0.985                                | 0.998                                | 0.988                                | 0.998                                |
The average incident density at each annular facet is calculated, respectively. The relative incident density $E$ of each annulus region is given by the following equation:

$$E(r/R) = \frac{N_3(r/R)S_3(r/R)}{N_3S_3}.$$  \hspace{1cm} (6)

When $n = 60,000$ rpm, the curves of $E(r/R)$ at different $L/D$ and different $d/D$ are shown in Figure 7(a,b), respectively. In Figure 7(a), the relative incident density at $A_3$ decreases along the radius direction, and this is agreement with the actual situation because of the influence of positional beamming effect. In Figure 7(b), a considerable deviation of incident density between the blades tip and root can be found. For instance, when $d/D = 0.4$, $n = 60,000$ rpm and $L/D = 2$, the deviation between the blades tip and root reaches 20%. The result is far away from the former literature results based on the molecular uniform incident at the turbine blades surface, and will have a major impact on the TMP pumping performance evaluation such as its transmission probability.

The fitting formula of $E$ is given in Equation (7) by SPSS software. The residual sum of squares is 0.016, the mean square of residua is 0 and $R^2$ is 0.974, so Equation (7) can meet the calculation requirements generally:

$$E = -1.796(r/R)^3 + 3.137(r/R)^2 - 2.042(r/R) + 37.977 - 0.001(L/D)^{0.741} + 36.375(d/D)^{-0.002}.$$  \hspace{1cm} (7)

### 4.3 Incident density distribution along radius at $A_3$

The molecular incident density along the radial direction in the turbine blades surface needs to be calculated for the further calculation of the TMP pumping capacity. The turbine cover and turbine blades surface ($A_2 + A_3$) are divided into 20 concentric annular region, and the diameter difference between adjacent annular is 0.05 times $D$ (Figure 3(a)). The radial position of each annular facet is marked by its relative central radius $r/R$. For instance, the largest facet ($r/R$ ranges from 0.95 to 1) is marked as $r/R = 0.975$, while its adjacent annular facet ($r/R$ ranges from 0.90 to 0.95) is marked as $r/R = 0.925$ and so on.

The molecular incident density along the radial direction in the turbine blades surface (A2 + A3) are divided into 20 concentric

### 4.4 Calculation formula of molecular incident density at $A_3$

Based on above results, the gas molecular incident density $n_3(r/R)$ of any location at $A_3$ can be directly estimated according to

![Figure 6](image-url) (Colour online) The variation of relative incident density at $A_4$ along axial direction for different tube lengths when $d/D = 0.5$.

![Figure 7](image-url) (Colour online) The variation of relative density $E$ at $A_3$ along the radial direction for different $L/D$, and with (a) $d/D = 0.5$; for different $d/D$, and with (b) $L/D = 2$. 
the gas molecular incident density \( n_1 \) at \( A_1 \). The calculation formula of \( n_3(r/R) \) and \( n_1 \) are given by Equations (8) and (9), respectively:

\[
n_3(r/R) = \frac{N_3(r/R)}{S_3(r/R)}, \tag{8}
\]

\[
n_1 = \frac{N_1}{S_1}. \tag{9}
\]

Combining Equations (1) and (6), Equation (8) can be reduced to Equation (10):

\[
n_3(r/R) = n_1 \cdot P \cdot E(r/R) \cdot \frac{S_1}{S_3}. \tag{10}
\]

**4.5. The angular distribution of IGM at \( A_3 \)**

The molecules reaching \( A_3 \) is composed with the direct incident molecules from \( A_1 \) and reflected molecules from \( A_4 \) as mentioned above. The relative emission probability at each point of the \( A_3 \) is defined in a unit solid sphere to reflect the 3D solid characteristics of molecular angular distribution and its basic formula has been derived [16]. The molecular angular distributions of different tube lengths at different position, \( r/R = 23/33.5, r/R = 26/33.5, r/R = 29/33.5 \) and \( r/R = 32/33.5 \), are calculated by MATLAB program based on structural parameters of F-63/55 TMP (Table 5). This program considers the rotor speed of the turbine cover, which is different from the previous procedure [16]. The calculation results are shown in Figures 8–10. The direct incident molecules still obey the cosine distribution, which forms an area that overlaps with the unit sphere in each graph.

Figures 8–10 describe the angular distribution from a different position on outlet of tube with different tube lengths. In each graph, gas molecules are all emitted at (0,0). They are symmetrical from the view of \( x \)-axis direction in (a) graph. The left half of the (b) graph represents the molecules that are directed towards the inner wall of tube, whereas the right half of the graph represents the molecules directed towards the turbine cover. Most molecules are deviated towards the inner wall from view of \( y \)-axis direction. The positional beaming effect and the angular beaming effect are enhanced with the lengthening of the tube, so less molecules diverge towards the inner wall. This will lead to the result of the transmission probability of first-stage turbine blades which is different from that obeying the cosine distribution under the same number of incident molecules.

Compared with the previous results [16], the influence of the turbine cover on the molecules reflection was taken into account in this calculation and this is closer to the actual condition. The results can provide a more accurate reference for predicting the performance of TMP.

**4.6. Example**

The pumping performance of F-63/55 TMP is calculated to test the accuracy of Equation (10), which is used in the laboratory.

Table 5. The structural parameters of F-63/55 TMP.

| Parameter                | F-63/55 TMP          |
|-------------------------|----------------------|
| Number of blades        | 16                   |
| Inner diameter of blades (mm) | 44             |
| Outer diameter of blades (mm) | 67            |
| Blade angle (°)         | 30                   |
| Blade height (mm)       | 6                    |
| Blade thickness (mm)    | 0.8                  |

Figure 8. (Colour online) The angular distribution from different position on outlet of tube with \( L/D = 0.3 \); (a) view of \( x \)-axis direction and (b) view of \( y \)-axis direction.
Three different methods were used to calculate the pumping performance separately such as geometric-mean method, integral-mean method and integral method [26]. The geometric-mean method and integral-mean method are based on uniform distribution regular of a gas molecule at turbine blades inlet plane. The integral method obeys non-uniform distribution regular and the gas molecular incident density is calculated by Equation (10) for different tube lengths.

The five blade angles (20°, 25°, 30°, 35° and 40°) are selected to calculate the positive transfer probability ($M_{12}$) of first-stage turbine blades. In this paper, the $M_{12}$ is employed for describing the pumping speed characteristic of TMP. The positive transfer probability calculated by three different methods is shown in Table 6.

From Table 6, it is found that the positive transfer probabilities calculated by three different methods are not the same in...
Table 6. The positive transfer probability calculated by three different methods.

| Different calculation method | $20^\circ$ | $25^\circ$ | $30^\circ$ | $35^\circ$ | $40^\circ$ |
|-----------------------------|-----------|-----------|-----------|-----------|-----------|
| Geometric-mean method        | 0.2200    | 0.3323    | 0.4457    | 0.5367    | 0.6131    |
| Integral-mean method         | 0.2264    | 0.3393    | 0.4527    | 0.5400    | 0.6178    |
| Integral method              |           |           | L/D = 0.2 | 0.2240    | 0.3362    |
|                             |           |           | L/D = 0.5 | 0.2241    | 0.3364    |
|                             |           |           | L/D = 1   | 0.2244    | 0.3367    |

the case of the same turbine blades structure and incident molecules number. With the lengthening of the tube, the transfer probability of tube decreases but the angular beaming effect are enhanced. In this sense, the number of molecules that vertically enter the turbine blades increases and these molecules are easier to cross turbine blades. Thus, the first-stage turbine blades positive transfer probability increases.

Compared the calculation results, the positive transfer probability for the geometric-mean method is less than that for an integral method and the positive transfer probability for the integral method is less than that for the integral-mean method. According to the former analysis, it is found that more molecules close to the blades root with the enhancement of positional beaming effect. This effect causes the positive transfer probability less than that for integral-mean method obeying uniform distribution regular by the weak pumping capacity on blades root. The more molecules directly enter turbine blades with the enhancement of angular beaming effect and this will promote the pumping capacity of blades. So the positive transfer probability obeying non-uniform distribution regular is greater than that for the geometric-mean method obeying uniform distribution regular. The above analysis proved that the integral method obeying non-uniform distribution regular is more reasonable than two other methods in the present study.

5. Conclusion

In the free molecular flow range, the gas molecular flow in the inlet tube of TMP is simulated by self-defining procedure and the angular distribution of gas molecules at turbine blades surface is calculated by MATLAB program. Based on above results and analysis, the main conclusions are as followed:

(1) With the lengthening of the tube, the transmission probability almost decreases exponentially. The influence of rotor speed for the transmission probability is indistinctive. The calculation formula of transmission probability is given considering positional beaming effect and reflection effect of turbine cover.

(2) The relative molecular incident density at turbine blades surface decreases along the radius direction and a considerable deviation of incident density is found between the blades tip and root. When $d/D = 0.4$, $n = 60,000$ rpm and $L/D = 2$, the relative deviation can reach 20%. This conclusion will have a major impact on the accurate evaluation for pumping performance of TMP such as its transmission probability. What is more, the calculation formula of molecular incident density at turbine blades is given.

(3) The positional beaming effect and the angular beaming effect are enhanced with the lengthening of the tube. The molecules diverging towards the tube wall become less. This result will lead to the transmission probability of first-stage turbine blades which is different from obeying the cosine distribution under the same number of incident molecules.

The above calculation results distinguish from the hypotheses obeying uniform incidence at position and cosine distribution at direction in previous studies. The new self-defining procedure can more accurately describe the status of molecules entering the first-stage turbine blades. The resulting impact will be contrasted in the future.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

[1] Becker W. The turbomolecular pump, its design, operation and theory; calculation of the pumping speed for various gases and their dependence on the forepump. Vacuum. 1966;16:625–632.
[2] Bottauscio O, Casaro F, Chiampi M, et al. High-speed drag-cup induction motors for turbo-molecular pump applications. IEEE Trans Magn. 2006;42:3449–3451.
[3] Spagnoli M, Cerruti R, Helmer J. Turbo molecular pump design for high pressure operation. J Vac Sci Technol A. 1998;16:1151–1156.
[4] Iqbal M, Wasy A, Batani D, et al. Design modification in rotor blade of turbo molecular pump. Nucl Inst Methods Phys Res A. 2012;678:88–90.
[5] Osterstrom GE. A new type of turbomolecular vacuum pump bearing. J Vac Sci Technol A. 1983;1:224–227.
[6] Wang S, Ninokata H, Merzari E, et al. Numerical study of a single blade row in turbo molecular pump. Vacuum. 2009;83:1106–1117.
[7] Hablanian MH. Engineering aspects of turbomolecular pump design. Vacuum. 2007;82:61–65.
[8] Schneider TN, Katsimichas S, de Oliveira CRE, et al. Empirical and numerical calculations in two dimensions for predicting the performance of a single stage turbomolecular pump. J Vac Sci Technol A. 1998;16:175–180.
[9] Sengil N, Edis FO. Fast cell determination of the DSMC molecules in multi-stage turbo molecular pump design. Comput Fluids. 2011;45:202–206.
[10] Antoniou AG, Valamontes SE, Panos CN, et al. The turbo molecular pump in molecular state. Vacuum. 1995;46:709–715.
[11] Schneider TN, Katsimichas S, de Oliveira CRE, et al. Analysis of three-dimensional single stage and two dimensional multistage models of flows in turbo molecular pumps. Vacuum. 1997;48:449–453.
[12] Chandran M, Krishna Murthy MV. A Markov chain model for a turbo molecular pump – theory and experiments. Vacuum. 1997;48:899–911.
[13] Katsimichas S, Goddard AJH, Lewington R, et al. General geometry calculations of one-stage molecular flow transmission probabilities for turbomolecular pumps. J Vac Sci Technol A. 1995;13:2954–2961.
[14] Wang S, Ninokata H. The pumping performances of the turbo molecular pump simulated by direct simulation Monte Carlo method. Prog Nucl Energ. 2005;47(1):664–671.
[15] Chang YW, Jou RY. Direct simulation of pumping characteristics in a fully 3d model of a single-stage turbomolecular pump. Appl Surf Sci. 2001;169–170:772–776.
[16] Zhong SW, Ji GZ, Han J. The positional and angular distribution of molecules flowing through cylindrical tube in free molecular flow. Phys Proc. 2012;32:513–524.
[17] Krasuski P. Angular distribution of flux at the exit of cylindrical tubes. J Vac Sci Technol A. 1987;5:2488–2492.
[18] Zhang SW, Han J, Zhang ZJ. Computer simulation of positional beaming effect of molecular flow in straight cylindrical pipeline by Monte Carlo method. ICC Sci Informat Technol. 2008;26:486–491.

[19] Shevkunov SV, Vorontsov-Velyaminov PN, Martsinovski AA. A new Monte Carlo method for direct calculation of the critical size and the formation work of a microdrop. Mol Simul. 1990;5:119–132.

[20] Nezbeda I, Kolaf J. A new version of the insertion particle method for determining the chemical potential by Monte Carlo simulation. Mol Simul. 1991;5:391–403.

[21] Moučka F, Nezbeda I. The multi-particle sampling method in Monte Carlo simulations on fluids and its efficient implementations. Mol Simul. 2010;36:526–534.

[22] Sweatman MB, Quirke N. Modelling gas adsorption in slit-pores using Monte Carlo simulation. Mol Simul. 2001;27:295–321.

[23] Tuer TW, Springer GS. A test particle Monte Carlo method. Comput Fluids. 1973;1:399–417.

[24] CERN. A Monte-Carlo Simulator package developed at CERN, Geneva: CERN; [cited 2017 Nov]. Available from: http://test-molflow.web.cern.ch/.

[25] O’Hanlon JF. A user’s guide to vacuum technology. 3rd ed. Hoboken (NJ): Wiley; 2003.

[26] Wang XD, Zhang L, Ba DC. Calculations of pumping behavior of turbo molecular pump and error analysis. Chin J Vac Sci Technol. 2016;36:432–435.