Gravitational Smearing of Minimal Supersymmetric Unification Predictions

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Abstract

The prediction for $\alpha_s$ in the minimal supersymmetric SU(5) grand unified theory is studied in the presence of a gravitationally-induced dimension-five operator. Unless the coefficient of this operator is small, the correlation between $\alpha_s$ and the mass scale which governs proton decay to $K\nu$ is destroyed. Furthermore, a reduction of the experimental uncertainty in $\alpha_s$ would not provide a significant test of the theory.

PACS numbers: 04.60.+n,11.30.Pb,12.10.-g.

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1This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by NSF grant PHY-90-21139.

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The standard model of elementary particles and interactions is specified by eighteen fundamental parameters. The only one which has been successfully predicted to a high level of accuracy is the weak mixing angle. In simple supersymmetric versions of grand unified theories (GUTs) it is predicted \[1\] to be 0.233, in full agreement with its experimental value \[2\] of 0.2325 ± 0.0008. Over the past few years several groups \[3, 4, 5, 6\] have studied what information may be extracted from this unification of gauge couplings. It is frequently stated that to further test GUTs, the strong coupling should be measured more precisely \[7, 5\]. Some claim that this would determine the scale of superpartner masses, others that such improved accuracy would help pin down the proton decay rate in the minimal model \[8\].

The renormalisation group equations for the gauge couplings provide three equations relating several quantities. One of these equations determines the GUT gauge coupling to be about 1/25, but beyond this fact it will not concern us further. A second provides a relationship among the various superheavy particle masses. The third predicts one of the low energy gauge couplings; although this is frequently chosen to be the weak mixing angle, we choose instead \[4\] to predict the strong coupling \(\alpha_s\), because we want to address the question of whether more information would be gained by reducing the error bar on its experimental value.

In a general GUT the spectrum of superheavy states is likely to be complicated. Logarithmic threshold corrections to the prediction for \(\alpha_s\) may arise from each non-degenerate (“split”) SU(5) multiplet, and some such corrections will be present in every GUT \[8\]. The minimal number of SU(5) representations whose states are not degenerate is two: one contains the superheavy (mass \(M_X\)) and the light gauge particles, the other contains the superheavy (mass \(M_{tr}\)) and the light members of the multiplet responsible for spontaneous electroweak symmetry breaking. In addition, the remnants \(\Sigma\) of the representation which breaks SU(5) to SU(3) × SU(2) × U(1) have masses \(M_\Sigma\) which generically differ from those of the Goldstone bosons eaten by the superheavy gauge particles. It is well known that (at one-loop order) the prediction for \(\alpha_s\) does not depend on \(M_X\). Recently it was pointed out \[4, 5\] that there is also no dependence on \(M_\Sigma\) — a fact overlooked in the previous analysis of threshold corrections \[8\]. This raises the interesting possibility that, in certain simple GUT models, the only significant dependence of the \(\alpha_s\) prediction on the superheavy sector is through the mass parameter \(M_{tr}\), which in these models controls the rate of proton decay. Since \(\alpha_s\) increases with \(M_{tr}\), an improved experimental upper limit on \(\alpha_s\) could reduce the upper bound on \(M_{tr}\). In that case, super-Kamiokande could definitively test this theory \[5\].

In this letter we study the extent to which these claims are spoiled by the presence
of higher-dimensional operators generated at the Planck scale \[9, 10, 11\]. Although we study the minimal supersymmetric SU(5) theory, for a wide class of GUTs the superheavy corrections are at least as large as those we consider. We assume that the mass scale suppressing such nonrenormalizable operators is the reduced Planck mass \(\hat{M}_P \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \text{GeV}\) since this is the combination which enters quantum-gravity calculations. At first sight the Planck scale corrections appear to change the prediction for \(\alpha_s\) only at the 1\% level, which is the ratio of the naive GUT scale (roughly \(M_X\)) to the reduced Planck mass. But \(M_X\) may easily be higher than the naive expectation if \(M_Z\) is reduced. Furthermore, these corrections are enhanced by several numerical factors. As we show below, the result is a significant modification to the prediction of \(\alpha_s\), and since the sign of this effect is unknown, the prediction is spread out considerably.

To one-loop order, and in the absence of gravitational corrections, gauge coupling unification is embodied in the three renormalization-group equations relating the values of the gauge couplings at the Z mass, \(\hat{\alpha}^{-1} \equiv \hat{\alpha}^{-1}(m_Z) = (\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1})\), and the common gauge coupling \(\alpha_G\) at the GUT scale \(M_G\):

\[
\hat{\alpha}^{-1} = \alpha_G^{-1} \hat{\mathbf{1}} - \sum_a \hat{\mathbf{b}}_a \ln \left( \frac{M_a}{M_G} \right)
\]

Here \(\hat{\mathbf{1}} \equiv (1, 1, 1)\) and \(\hat{\mathbf{b}}_a \equiv \bar{b}_a/(2\pi)\) where \(\bar{b}_a\) are the three beta-function coefficients for the particle labeled by \(a\). The sum extends over all particles in the model, and \(M_a\) denotes the mass threshold at which each is integrated out. (We neglect electroweak-breaking effects in the SUSY mass spectrum, and treat the top quark as being degenerate with the Z. These effects are small relative to the dominant uncertainties in the experimental inputs and in the gravitational corrections.) All of the standard model particles are already present at the initial scale \(m_Z\); we then include the second Higgs doublet at \(m_{H_2}\), the squarks at an average mass \(m_{\tilde{q}}\), the sleptons at their average mass \(m_{\tilde{l}}\), the gluinos at \(m_{\tilde{g}}\), the higgsinos at \(m_{\tilde{H}}\), the color-triplet component of the 5 of Higgs at \(M_{\Sigma}\), and finally the superheavy gauge bosons and their superpartners (‘X’) at \(M_X\). The GUT scale is the highest mass threshold, above which all particles fill complete SU(5) multiplets. The experimental inputs are derived from \[3\] \(s^2 \equiv \sin^2 \theta_W = 0.2325 \pm 0.0008\) and \(1/\alpha = 127.9 \pm 0.1\) using \(\hat{\alpha}^{-1} = (\frac{3}{5}(1-s^2))/\alpha, s^2/\alpha, 1/\alpha_s\). The two-loop contributions to \(\hat{\alpha}^{-1}\) are incorporated using typical values for all the parameters. We also include in these contributions the terms needed to translate the experimental inputs given in the \(\overline{\text{MS}}\) scheme into the \(\overline{\text{DR}}\) scheme appropriate for step-function corrections in supersymmetric theories \[12\]. The result is a term \(\Delta_2 = (0.65, 1.09 + 2/12\pi, 0.55 + 3/12\pi)\) which must be added to
the right-hand side of (1).

We concentrate on the predictions of (1) for \( \alpha_s \) and \( M_{tr} \), and ignore the prediction for \( \alpha_G \). Therefore we consider (well-known) linear combinations of (1) which do not involve \( \alpha_G \). Define for convenience (up to an irrelevant overall normalization) one projection vector \( \vec{P}_1 \) by requiring \( \vec{P}_1 \cdot \vec{T} = 0 \) and \( \vec{P}_1 \cdot \vec{\beta}_X = 0 \), and another projection vector \( \vec{P}_2 \) by requiring \( \vec{P}_2 \cdot \vec{T} = 0 \) and \( \vec{P}_2 \cdot \vec{\beta}_{tr} = 0 \). We choose

\[
\vec{P}_1 = (-1, 3, -2); \quad \vec{P}_2 = (5, -3, -2).
\]

The dot product of \( \vec{P}_1 \) with (1) will be independent of \( \alpha_G \) and of \( M_X \) and \( M_\Sigma \), and therefore will furnish a simple expression for \( \alpha_s \) in terms of the low-energy parameters and the Higgs-triplet mass. The dot product of \( \vec{P}_2 \) with (1) will be independent of \( \alpha_G \) and of \( M_{tr} \) and the light Higgs sector, and will relate the masses of the superheavy gauge multiplet and the superheavy 24. Finally, the unification scale \( M_G \) enters (1) only through the combination \( (\ln M_G) \sum \beta_a \propto \vec{\beta}_1 \) and so it, too, is projected out; thus any other scale may be used in these dot products, and we choose that scale for convenience to be \( m_Z \).

Note that in the dot product with \( \vec{P}_1 \) both the \( X \) and the \( \Sigma \) were projected out. The reason is simple. The \( \vec{\beta} \) for any complete SU(5) multiplet, and in particular \( \vec{\beta}_{24} \) for the 24, contributes equally to the running of all gauge couplings, so it is proportional to \( \vec{T} \) and hence orthogonal to \( \vec{P}_1 \). The \( \vec{\beta}_{GB} \) for the Goldstone mode components of the 24 is proportional to the \( \vec{\beta}_X \) of the superheavy \( X \) since they carry the same quantum numbers, and so it too is orthogonal to \( \vec{P}_1 \). Therefore their difference \( \vec{\beta}_{24} - \vec{\beta}_{GB} = \vec{\beta}_\Sigma \) also satisfies \( \vec{P}_1 \cdot \vec{\beta}_\Sigma = 0 \) and the \( \Sigma \) does not make a threshold contribution to this equation. Similarly, both the Higgs doublet and the triplet are projected out in the dot product with \( \vec{P}_2 \). The \( \Sigma \) could contribute if it were split, which is not the case in the minimal model but would be the case in most extensions. An example of such a contribution in the minimal model is provided by the gauginos: they would also be projected out since they carry quantum numbers complementary to those of the \( X \), but their masses are widely split by renormalization-group running so they make a significant (and calculable) contribution to the predictions for \( \alpha_s \).

To obtain specific predictions, we need the mass spectrum of the model. In the minimal model the weak-scale masses are determined to a good approximation by the four mass parameters \( m_0 \) (the common scalar mass), \( m_{1/2} \) (the common gaugino mass at the GUT scale), \( \mu \) (the coupling of the two Higgs doublets in the superpotential) and \( m_{H_2} \) (the mass of the second Higgs doublet—we take the first to be degenerate with the \( Z \)). For our purposes the following simplified spectrum will suffice: \( m_{\tilde{q}} \simeq \sqrt{m_0^2 + 6m_{1/2}^2} \), \( m_{\tilde{l}} \simeq \sqrt{m_0^2 + 4m_{1/2}^2} \), \( m_{\tilde{g}} \simeq 2.7m_{1/2} \), \( m_{\tilde{\omega}} \simeq 0.8m_{1/2} \) and \( m_{\tilde{H}} \simeq \mu \).
By applying the projections (2) to (4) and including the two-loop term $\Delta_Z$ we find

$$\frac{2}{\alpha_s} + \frac{6}{5\pi} \ln \frac{M_{tr}}{m_Z} = f_1(s^2, m_0, m_{1/2}, \mu, m_{H_2})$$  \hspace{1cm} (3)

and

$$\frac{2}{\alpha_s} + \frac{6}{\pi} \ln \frac{M_{\Sigma}}{m_Z} + \frac{12}{\pi} \ln \frac{M_X}{m_Z} = f_2(s^2, m_{1/2})$$  \hspace{1cm} (4)

where

$$f_1 = \frac{3(6s^2 - 1)}{5\alpha} - \frac{3}{20\pi} \ln \frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 4m_{1/2}^2} - \frac{2}{\pi} \ln \frac{2.7}{0.8} + \frac{4}{5\pi} \ln \frac{\mu}{m_Z} + \frac{1}{5\pi} \ln \frac{m_{H_2}}{m_{1/2}} - 1.52$$

$$\simeq 27.9 + 0.4\sigma + \frac{1}{\pi} \ln \frac{\mu^{4/5}m_{1/2}^{1/5}}{m_Z},$$  \hspace{1cm} (5)

$$f_2 = \frac{3(1 - 2s^2)}{\alpha} - \frac{3}{4\pi} \ln \frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 4m_{1/2}^2} - \frac{2}{\pi} \ln(2.7 \cdot 0.8) - \frac{4}{\pi} \ln \frac{m_{1/2}}{m_Z} + 1.13 + \frac{1}{\pi}$$

$$\simeq 206.2 - 0.6\sigma - \frac{3}{4\pi} \ln \frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 4m_{1/2}^2} - \frac{4}{\pi} \ln \frac{m_{1/2}}{m_Z},$$  \hspace{1cm} (6)

and $\sigma \equiv (s^2 - 0.2325)/0.008$.

Eq. (5) can be viewed as a prediction for $M_{\Sigma}$. It shows that one can raise the X mass by lowering the mass of the $\Sigma$ without affecting the prediction for $\alpha_s$. Since there are no experimental consequences of a light $\Sigma$, we focus on (4). To avoid excessive fine-tuning and retain the motivation for supersymmetric unification, we restrict $m_0$, $m_{1/2}$, $\mu$ and $m_{H_2}$ to lie below one TeV. (Our results are not sensitive to the exact value of this cutoff.) By varying these four parameters between the weak scale and a TeV, and varying $s^2$ within one standard deviation of its central value (namely $|\sigma| \leq 1$), we obtain a predicted range of $\alpha_s$ as a function of $M_{tr}$. This range is the region between the two black curves in Fig. 1.

We now turn to the effects of quantum gravity on these predictions. In the absence of a specific and predictive theory of quantum gravity, we can only estimate these effects by including higher-dimension operators that would arise in an effective theory once the Planck-scale degrees of freedom have been integrated out. Following Hill [9] and Shafi and Wetterich [10], we restrict our attention to the dominant dimension-five operator

$$\delta \mathcal{L} = \frac{c}{2M_P} \text{tr} (GG\Sigma),$$  \hspace{1cm} (7)

where $G \equiv G_a T^a$ is the field-strength tensor of the SU(5) gauge field and the generators are normalized to $\text{tr} \ T^a T^b = \frac{1}{2} \delta^{ab}$. The mass scale suppressing this operator is the reduced Planck mass $M_P \simeq 2.4 \times 10^{18}$ GeV as discussed above. We have
also conservatively included a factor of $1/2$ to account for the two identical operators $GG$. The remaining coefficient $c$ is unknown without further assumptions; we have no reason to think it is less than $O(1)$ in magnitude. SU(5) is broken by the vacuum expectation value $\langle \Sigma \rangle \equiv v T^0$ where $T^0 = \text{diag}(2,2,-3,-3)/2\sqrt{15}$. This breaking induces superheavy masses $M_X = \sqrt{\frac{2}{3}} g_5 v$ from the covariant derivative $D_\mu \Sigma = \partial_\mu \Sigma a T^a + ig_5 X_{\mu a} \Sigma_b [T^a, T^b]$, a triplet mass $M_\nu = \sqrt{\frac{2}{15}} \lambda_5 v$ from the term $\frac{3}{5} M_\nu \Sigma \Sigma \Sigma = \Sigma \Sigma \Sigma$ (after a fine-tuning to make the doublets light), and a $\Sigma$ mass $M_\Sigma = \frac{i}{\sqrt{12}} \lambda_{24} v$ from the term $\frac{i}{3} M_\nu \Sigma^2 + \frac{i}{3} \lambda_{24} \Sigma \Sigma^2$. It also modifies the kinetic terms of the standard-model gauge bosons through $\delta L$:

$$L_{\text{gauge}} = -\frac{1}{4} (F F)_{U(1)} \left[ 1 + \frac{c}{2 M_P} \left( \frac{v}{2 \sqrt{15}} \right) \right] - \frac{1}{2} \text{tr} (GG)_{SU(2)} \left[ 1 + \frac{c}{2 M_P} \left( \frac{v}{2 \sqrt{15}} \right) \right] - \frac{1}{2} \text{tr} (GG)_{SU(3)} \left[ 1 + \frac{c}{2 M_P} \left( \frac{v}{\sqrt{15}} \right) \right]. \quad (8)$$

Consequently the three gauge couplings are not degenerate at the GUT scale. The first term in the equations for unification (9) must be replaced by $\alpha_G^{-1} \overrightarrow{T} \rightarrow \alpha_G^{-1} (\overrightarrow{T} + \overrightarrow{c})$ where $\overrightarrow{c} \equiv (cv/2 M_P) \left( \frac{-1}{2 \sqrt{15}}, \frac{3}{2 \sqrt{15}}, \frac{1}{15} \right)$. We have absorbed the sign of $v$ into $c$, so $v$ and $\lambda_{5,24}$ are by definition positive.

By applying the projection operators to the modified unification equations, we obtain:

$$\frac{2}{\alpha_s} + \frac{6}{5 \pi} \ln \frac{M_\nu}{m_Z} - \sqrt{\frac{12 c}{5 \pi}} \frac{v}{2 M_P} \alpha_G = f_1 (s^2, m_0, m_{1/2}, \mu, m_{H_2}) \quad (9)$$

and

$$\frac{2}{\alpha_s} + \frac{9}{\pi} \ln \frac{5}{12} + \frac{12}{5} \ln g_5 + \frac{6}{\pi} \ln \lambda_{24} + \frac{18}{\pi} \ln \frac{v}{m_Z} = f_2 (s^2, m_0, m_{1/2}) \quad (10)$$

In (9) we use the zeroth-order expression for $\alpha_G = g_5^2/4 \pi \simeq 1/25$ in the coefficient of $\overrightarrow{c}$. Eq. (10) has no direct gravitational contributions, since it turns out that $\overrightarrow{P} \cdot \overrightarrow{c} = 0$; we have merely rewritten it using the above expressions for the superheavy masses. The magnitude of the gravitational smearing may be readily estimated from (9). If $v \sim 2 \times 10^{17} \text{GeV}$ and $|c| \sim 1$ then the prediction for $\alpha_s$ is corrected by $\sim 10\%$.

To be more precise, we study the predictions of the two equations (9) and (10) in the five unknowns $\{\alpha_s, M_\nu, \lambda_5 = \sqrt{\frac{2}{5}} M_\nu/v, \lambda_{24}, c\}$. A nonzero $c$ couples the two equations and makes an exact analytic solution impossible. Instead, they may be solved analytically to a good approximation ($\sim \pm 1.5\%$ in $\alpha_s$ and $\sim \pm 30\%$ in $M_\nu$), or numerically to a high precision. The analytic expressions are:

$$\alpha_s \simeq 0.132 \left( 1 - 0.024 \sigma - 0.02 \ln \frac{M_\nu^{1/5} m_{H_2}^{1/5}}{m_Z} + 0.025 \ln \frac{M_\nu}{3 \times 10^{16} \text{GeV}} \right)$$

\[\dagger\] The coefficient of the last term in (11) should be changed from $-0.025$ to $-0.04$ for large values (0.14-0.15) of $\alpha_s$ in order to achieve the desired accuracy.
\[-0.025 \, c \, (1 - 0.1 \sigma) \left( \frac{m_{1/2}}{m_Z} \right)^{-2/9} \lambda_{24}^{-1/3} \]  \hspace{1cm} (11)

and

\[ M_{\text{tr}} \simeq (3 \times 10^{16} \text{ GeV}) \lambda_5 (1 - 0.1 \sigma) \lambda_{24}^{-1/3} \left( \frac{m_{1/2}}{m_Z} \right)^{-2/9}. \]  \hspace{1cm} (12)

Numerically, one subtracts (10) from (9) and solves the resulting equation for \( M_{\text{tr}} \):

\[ f_1 - f_2 + \frac{6}{\pi} \ln \frac{\lambda_{24}}{\lambda_5^3} + \frac{6}{\pi} \ln 4\pi \alpha_G = -\frac{84}{5\pi} \ln \frac{M_{\text{tr}}}{m_Z} - \frac{6c}{5\alpha_G \lambda_5 M_P}. \]  \hspace{1cm} (13)

The solution(s) can then be used to find the corresponding \( \alpha_s \). Since we have no reason to presume that the scalar couplings \( \lambda_{5,24} \) are particularly small or particularly large, we allow them to vary between 0.1 and 3, and also let \( c \) vary between \(-1\) and \(1\). For each such choice of \( \lambda_5, \lambda_{24} \) and \( c \), we obtain numerically a region of allowed \( (\alpha_s, M_{\text{tr}}) \) values when we scan \( m_0, m_{1/2}, \mu, m_{H_2} \) and \( s^2 \) over the same ranges as before. The overlap of all these regions is shown as the gray area in Fig. 1; it represents the region allowed in the minimal supersymmetric SU(5) model by our present knowledge of \( s^2 \), our suspicions about the ranges of superpartner masses, our assumptions about the scalar couplings in the superpotential [15], and our ignorance of the true theory at the GUT scale. The domain of predictions for \( \alpha_s \) is greatly increased by the possible Planck-scale corrections, and the correlation between \( \alpha_s \) and the parameter \( M_{\text{tr}} \) relevant to proton decay is largely blurred away.

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considered nonrenormalizable operators in SU(5), showing that the unification
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[14] We note in passing that the non-supersymmetric minimal SU(5) model cannot be
resurrected by these gravitational effects. Eqs. (9) and (10) can easily be rewritten
for this simpler model using the appropriate beta-function coefficients and mass
spectrum, yielding expressions for $\alpha_s$ and for $M_{\Sigma}$. The first such expression shows
that to obtain $\alpha_s > 0.10$ one needs large gravitational corrections, which require
$v > 10^{16}\text{GeV}$ for any plausible choice of $\sin^2 \theta_W$. The second expression then
shows that $M_{\Sigma}$ is proportional to $v^{-22}$ (compare with $v^{-3}$ in the supersymmetric
version); if $v$ is increased from its unperturbed value of $\sim 10^{14}\text{GeV}$ to the extent
required by $\alpha_s$ then $M_{\Sigma}$ is reduced to intolerably low values. Of course, if $v$ is
at or above $\tilde{M}_P$ then both (9) and (10) are completely changed, and $M_{\Sigma}$ can be
made reasonably large, but in that case we are no longer considering an effective
SU(5) model.
[15] How large can the gravitational smearing become if we relax the lower bound of 0.1 on $\lambda_{24}$? For $c \geq 0$ there is always a (unique) solution to (13), so one could lower the prediction for $\alpha_s$ as far as desired by lowering $\lambda_{24}$. However, for $c < 0$ there may be 2, 1 or 0 solutions. For large $\lambda_{24}$ two solutions exist, one above $M_{tr}^{cr} \equiv -14\lambda_5\alpha_G\hat{M}_P/\pi c$ and one below. (We have chosen the lower one in the above estimates and in the figure.) These two merge into $M_{tr}^{cr}$ at some critical value of $\lambda_{24}$ which depends on the other parameters, while for smaller $\lambda_{24}$ no solutions exist. Thus the largest corrections occur when $M_{tr} = M_{tr}^{cr}$, in which case the gravitational correction term in (3) becomes exactly $84/5\pi$. Such a correction raises even the lowest possible uncorrected $\alpha_s$ value, namely $\sim 0.118$, to above 0.17. We learn that any conceivable future experimental determination of $\alpha_s$ may be accommodated using some value of $\lambda_{24}$. We find that $\lambda_{24}$ need never be taken less than 0.015 for such large corrections, while the corresponding value of $M_{tr}$ can be adjusted by changing $\lambda_5$, thus allowing the entire experimentally-relevant region in the $(\alpha_s, M_{tr})$ plane.

[16] After this work was completed, we learned of a paper by P. Langacker and N. Polonsky (Pennsylvania preprint UPR-0513T) in which the effects of nonrenormalizable operators arising at the Planck scale have also been studied.

**Figure Captions**

**Fig. 1:** The prediction in the minimal supersymmetric SU(5) model of $\alpha_s$ as a function of the color-triplet mass $M_{tr}$. The region between the two black curves accounts for the possible variation of the light superpartner masses and the second Higgs doublet mass between 100 GeV and 1 TeV, and for the variation of $\sin^2\theta_W$ between 0.2317 and 0.2333, but does not incorporate any Planck-scale corrections. The shaded region adds the gravitational corrections, with the restrictions that $0.1 \leq \lambda_{5,24} \leq 3$ and that $|c| \leq 1$. 