The Low-Mass Limit for Total Mass of W UMa-type Binaries

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ABSTRACT

The observations of W UMa type stars show a well-defined short-period limit of 0.22 d, which is equivalent to a lower mass limit of approximately 1 M⊙ for the total binary mass. It is currently believed that cool contact binaries are formed from detached binaries losing angular momentum (AM) via a magnetized wind. Orbital evolution of detached binaries with various component masses was followed until the primary component reached the critical Roche surface and the Roche lobe overflow (RLOF) began. It was assumed that the minimum initial, i.e., ZAMS, orbital period of such binaries is equal to 2 d and that the components lose AM just as single stars. According to the mass-dependent formula for AM loss rate of single stars, derived in this paper, the AM loss time scale increases substantially with decreasing stellar mass. The formula was applied to binaries with the initial primary component masses between 1.0 M⊙ and 0.6 M⊙ and two values of mass ratio q = 1 and 0.5.

Detailed calculations show that the time needed to reach RLOF by a 1 M⊙ primary is of the order of 7.5 Gyr, but it increases to more than 13 Gyr for a binary with an initial primary mass of 0.7 M⊙. Binaries with less massive primaries have not yet had time to reach RLOF even within the age of the Universe. This sets a lower mass limit for the presently existing contact binaries at about 1.0 M⊙ – 1.2 M⊙, in a good agreement with observations.

1. Introduction

W UMa-type stars are binaries of spectral type F0–K5 whose components are in a physical contact. The observed orbital period distribution of the binaries is asymmetric, with a narrow maximum at 0.35 d–0.40 d, a long-period tail extending to about 1 d (possibly up to 1.5 d) and with a sharp short-period cut-off at 0.22 d (Rucinski 1992, 1998a, Szymański, Kubiak and Udalski 2001, Paczyński et al. 2006). A field W UMa star with the shortest known period is CC Com (Porb = 0.22 d), discovered more than 40 years ago (Hoffmeister 1964). Several thousand new variables have been detected since then (mostly during the massive stellar photometry projects like OGLE, MACHO or ASAS), but the record period remains unbeaten. This shows that the existence of the cut-off period does not result from observational selection but is real. Primary components of W UMa stars are main sequence (MS) objects, hence the lower limit for orbital period of 0.22 d translates into a lower limit for the primary mass of about 0.6 M⊙ and for the total binary mass of about 1.0 M⊙ – 1.2 M⊙, with some dependence on the mass ratio. Why do we not observe less massive contact binaries?

*Recently a W UMa star with a slightly shorter period of 0.215 d has been detected by Weldrake et al. (2004) in the globular cluster 47 Tuc.
According to the current view, contact binaries of W UMa-type are formed from short period, detached binaries with components cool enough to possess subphotospheric convection zones. Such stars exhibit chromospheric-coronal activity and lose angular momentum (AM) via a magnetized wind. The activity level varies with rotation rate. Young, single stars rotate rapidly and are very active as the observations of members of young clusters show (Barnes 2003 and references therein) but they spin-down with age and their activity level decreases correspondingly (Wilson 1963, Kraft 1967, Skumanich 1972). For binaries with synchronously rotating components AM loss (AML) results in the actual spin-up of both components. At the same time the orbit tightens until the Roche lobe overflow (RLOF) by the more massive component occurs. Mass transfer to the less massive component leads to the formation of a contact binary. The precise formation mechanism is still a matter of controversy (Yakut and Eggleton 2005, Stepień 2006 and references therein) but irrespective of which one is correct, the phase of RLOF must occur prior to the contact phase. The characteristic time scale of reaching RLOF is of the order of several Gyr (Mochnacki 1981, Vilhu 1982, Stepień 1995, 2006) but its dependence on binary mass is poorly known. With the apparently simplest assumption of mass independent AML rate the shorter time scale is obtained for lower mass binaries (Stepień 1995). This should result in existence of a number of low mass contact binaries with periods beyond 0.22 d, which is in a clear contradiction to observations. Lack of such systems invalidates the assumption.

Rucinski (1992) tried to explain the existence of the cut-off period on physical grounds but his “full convection limit” applies only to stars significantly cooler than the observed limit. Stepień, Schmitt and Voges (2001) conjectured that the AML rate of ultra-fast rotators (URF) i.e., stars with rotation periods shorter than about 0.4 d, decreases compared to stars with slightly longer periods. This conjecture is based on the observations of X-ray activity of active stars. The X-ray flux of UFR decreases with increasing rotation rate – a feature called supersaturation (Randich et al. 1996). Assuming that X-ray flux scales with AML rate Stepień et al. (2001) argued that the time scale for reaching contact by very low mass binary may increase beyond the age of Universe (note that a binary with two 0.5 M⊙ components will not reach contact until \( P_{\text{orb}} \approx 0.2 \) d whereas for two 0.2 M⊙ components this happens when \( P_{\text{orb}} \approx 0.1 \) d). Supersaturation may, indeed, increase the time scale for reaching contact but, as more detailed calculations show below, this increase alone is not sufficient to prevent low mass binaries from forming contact binaries in the Hubble time (see Section 3). On the other hand, observations of young stellar clusters indicate that AML rate of single stars decreases with stellar mass (Sills, Pinsonneault and Terndrup 2000, Barnes 2003).

These facts substantiate a re-discussion of the expression for AML rate derived earlier by Stepień (1995) to allow for its mass dependence. This is done in Section 2. It is shown that the formulas given by Stepień (1995) can be reformulated under specific assumptions to include mass dependence. The resulting AML rate decreases indeed with stellar mass decreasing. As a result, the time scale for reaching RLOF increases rapidly for low mass binaries. For
initial masses of a primary component lower than about 0.7 \( M_\odot \) (a more precise value depends on mass ratio) and initial orbital periods equal to 2 d or more, it exceeds the Hubble time. Detailed calculations are presented in Section 3. The supersaturation effect increases further this time scale but is of secondary importance. Section 4 contains the discussion and summary of the main conclusions.

2. Angular Momentum Loss Rate of Single Stars, Revisited

Based on observations of chromospheric-coronal activity levels and rotation rates of single stars with known age and mass it is possible to find an empirical activity-rotation-age relation for a star of a given mass. The relation can be used to verify possible mechanisms of AML and to determine the activity-related AML rate as a function of stellar age. Unfortunately, no observational data of a comparable quality exist for close binaries. Cool close binaries are very active but their present orbital AM depends not only on the amount of AM lost in the past but, primarily, on initial conditions. Evolutionary advanced systems, with inverted mass ratios, could have lost an unknown amount of AM during a common envelope phase. Because we will be interested only in the approach to contact, we adopt a simplified assumption that the total, activity-related AML and mass loss (ML) of a detached binary is equal to the sum of individual component losses, treated as single stars, i.e., neglecting any influence of their proximity on these rates. In addition, an assumption of synchronous rotation is made for orbital periods shorter than a few days. We need then expressions describing AML rate and ML rate of single, cool dwarfs applicable to stars of different masses.

The pioneer observations of rotation rate of single active stars of different age were obtained by Kraft (1967). Later, Skumanich (1972) obtained his famous activity-age and rotation-age relations. These data indicated that chromospheric activity and rotation rate were closely related to each other and that they both decreased with age. Weber and Davis (1967) developed a theory of the solar wind and calculated the present-day AML rate of the Sun. Noyes et al. (1984) showed in their seminal paper that the CaII H and K core emission flux of MS stars of different spectral types correlates tightly with the Rossby number \( Ro = P_{\text{rot}}/\tau_c \), where \( \tau_c \) is a mass dependent parameter called turnover time. Its values can be obtained from theoretical models of convection zones (e.g., Kim and Demarque 1996) or found empirically (e.g., Stępień 1994). Noyes et al. (1984) derived a polynomial fit to the data but they noted that an exponential fit equally well describes the relation between the chromospheric flux and \( Ro \) (see also Stępień 1994).

Based on theoretical considerations of Mestel (1984) on AML via a magnetized wind Stępień (1995) obtained two different formulas for AML rate of the
form (see his Eqs. (8) and (9))

\[
\frac{dH_{\text{spin}}}{dt} \propto \omega \dot{M}^\alpha R^\beta M^\gamma B^\delta.
\]

(1)

Here \(\dot{M}, R, M\) and \(B\) denote the mass loss rate by the wind, stellar radius, mass and an intensity of the surface magnetic field. The exponents \(\alpha, \beta, \gamma\) and \(\delta\) depend on geometry of the magnetic field, parametrized by the exponent \(n\) in the relation \(B(r) \propto r^{-n}\), with \(r\) being a distance from the star. Depending on the adopted value of gas velocity at the Alfvén surface (equal to sound speed, as assumed by Mestel 1984 or to escape velocity, as assumed by Kawaler 1988) a slightly different functional dependence of the four exponents on \(n\) is obtained (see Stępień 1995). For the adopted value of \(n\) (see below) their numerical values do not differ much between both models so, without giving a preference to any of them, an average value for each exponent will be used. The field \(B\) in Eq. (1) can be replaced with a surface-averaged magnetic field

\[
B \equiv \overline{B}_{\text{surf}} = B_{\text{obs}} f_{\text{mag}}.
\]

(2)

Here both quantities: \(B_{\text{obs}}\) and \(f_{\text{mag}}\) – the filling factor characterizing a fraction of the stellar surface covered by \(B_{\text{obs}}\), are obtained directly from observations. The empirical data indicate that \(B_{\text{obs}}\) scales approximately as \(g^{1/2}\) where \(g\) is the gravitational acceleration (Saar 1996), hence \(B_{\text{obs}} \propto M^{1/2} R^{-1}\).

The magnetic filling factor \(f_{\text{mag}}\) correlates well with the Rossby number (Saar 1990, Montesinos and Jordan 1993). The dependence is equally well described by a power fit or an exponential fit. Here the relation derived by Stępień (1991) and essentially identical to the one obtained by Montesinos and Jordan (1993) will be used

\[
f_{\text{mag}} = Fe^{-Ro/Rf}
\]

(3)

where \(F = 0.87\) and \(R_f = 0.57\). Note that \(F\) describes the maximum fraction of the stellar surface covered by magnetic fields in the limit of \(\omega \rightarrow \infty\). In his later discussion of the “most reliable” magnetic measurements Saar (1996) recommended \(F = 0.58\), but more recent observations suggest again a value around 0.8–0.9 (Valenti and Johns-Krull 2001), so the original value is retained.

The final expression for the surface intensity of the magnetic field (apart from a numerical coefficient) is

\[
B \propto FM^{1/2} R^{-1} e^{-Ro/Rf}.
\]

(4)

Based on observations of early G-type active stars in several stellar clusters obtained by Barry, Cromwell and Hege (1987), supplemented with the solar observations, Stępień (1988) found that the exponent \(\delta\) in Eq. (1) is equal to \(1.7 \pm 0.5\). From this value the geometrical factor \(n\) can be calculated and used to obtain values of the other three exponents \(\alpha, \beta\) and \(\gamma\). Their resulting values are: \(\alpha = 0.15, \beta = 2.2\) and \(\gamma = 0.7\) with a crude uncertainty of 50\%. Because \(\alpha\) is very close to zero, we can neglect the dependence of the AML rate on \(\dot{M}\). Within the
estimated uncertainties two other exponents do not differ significantly from the nearest integers, so we put $\beta = 2$ and $\gamma = 1$. This produces a simple, parametric formula easy to handle and discuss. Note that with a parametric approximation $R \approx M$ (in solar units, see below) we replace in fact $\beta + \gamma = 2.9$ with the integer 3. With these replacements the formula for AML rate becomes

$$-\frac{dH_{\text{spin}}}{dt} = C \omega R^2 M e^{-1.7 \frac{R_0}{R_f}}$$

(5)

where $C$ is a coefficient of proportionality determined by Stepień (1988) together with $\delta$.

After substituting numerical values for $C$ and $R_f$, and recalculating the units, the final formula for AML rate of a single star is obtained

$$-\frac{dH_{\text{spin}}}{dt} = (7 \pm 2) \times 10^{-10} \omega R^2 M e^{-\frac{R_0}{0.335}}$$

(6)

where time is now in years, $\omega$ in d$^{-1}$, and stellar radius and mass are in solar units.

The above formula can be compared with the relations suggested by Kawaler (1988) and modified later by Chaboyer et al. (1995) to allow for a saturation effect

$$-\frac{dH_{\text{spin}}}{dt} = K \omega^3 R^{0.5} M^{-0.5}, \quad \omega < \omega_{\text{crit}},$$

(7)

$$-\frac{dH_{\text{spin}}}{dt} = K \omega^{\omega_{\text{crit}}^2 R^{0.5} M^{-0.5}}, \quad \omega \geq \omega_{\text{crit}}.$$ (8)

Here $K$ is a constant of proportionality and $\omega_{\text{crit}}$ is a limiting angular velocity beyond which the saturation regime occurs. In general, $\omega_{\text{crit}}$ is expected to be a function of stellar mass. Krishnamurthi et al. (1997) conjectured that $\omega_{\text{crit}} \propto \tau_c^{-1}$, where $\tau_c$ is the convective turnover time taken from Kim and Demarque (1996).

Although Kawaler (1988) started also from equations given by Mestel (1984), his final formula for AML rate differs from the one obtained by Stepień (1995) and generalized here for different stellar masses. The main difference between Eq. (6) and Eqs. (7)–(8) comes from a different scaling of the surface magnetic field. Based on early magnetic observations Kawaler assumed that the total stellar magnetic flux is proportional to angular velocity, i.e., $B \propto R^{-2} \omega$. This leads to Eq. (7). The scaling becomes $B \propto R^{-2}$ for the saturated state. Stepień (1995), on the other hand, adopted the scaling $B \propto B_{\text{obs}} \exp(-R_0/R_f)$ with the exponential term describing the period dependence of the filling factor $f_{\text{mag}}$. This term describes in Eq. (6) the $\omega$ – dependence of the AML rate over the whole considered range of angular velocities hence only one equation suffices for both, saturated and unsaturated regime. For short rotation periods, in a saturated regime, the exponential term is nearly constant, and Eq. (6) gives: $-dH_{\text{spin}}/dt \propto \omega$, i.e., the same as Eq. (8) suggested by Chaboyer et al. (1995), whereas for moderate and long rotation periods the formula reproduces the Skumanich law (see Fig. 2 in Stepień, 1988). Note that Eq. (6) describes correctly not only a saturated state but it also gives a quantitative scaling of $\omega_{\text{crit}}$ identical
to the one suggested later by Krishnamurthi et al. (1997). Assuming that the saturation regime is separated from unsaturated by a specified value of the exponential term (the same for all stars), we obtain from Eq. (6) \( R_{\text{crit}} \) and \( R_f \) with the critical value of the Rossby number \( R_{\text{crit}} = 2\pi/\omega_{\text{crit}} \tau_c \). This leads to \( \omega_{\text{crit}} \propto \tau_c^{-1} \). As we see, the proposition of Krishnamurthi et al. (1997), which correctly predicts time evolution of AM of low mass stars in young clusters, finds an independent support from the purely empirical \( B_{\text{surf}} - \omega \) relation (Stepień 1991).

To sum up, the \( \omega \)–dependence of \( -dH_{\text{spin}}/dt \) given by Eq. (6) is approximately the same as given by Eqs. (7)–(8) and the predictions about time-dependence of the rotation rate of solar mass stars, obtained with either set of equations, give essentially the same results. This is not surprising if one remembers that these formulas were calibrated using the present solar rotation period and present solar AML rate. That does not have to be so, however, when we apply the formulas to low mass stars. Due to the apparently different dependence of Eq. (6) and Eqs. (7)–(8) on \( M \) and \( R \), predictions about AML of stars with masses substantially lower than the Sun may diverge. This needs a closer look.

The spin AM of a star is

\[
H_{\text{spin}} = I \omega = k^2 R^2 M \omega \tag{9}
\]

where \( I \) is the stellar moment of inertia and \( kR \) is a gyration radius. As observations show, radii of low mass stars are numerically close to their masses (both expressed in solar units, Lopez-Morales and Ribas 2005 and references therein), i.e., \( R \approx M \). With such a scaling the factor \( (R/M)^{1/2} \) appearing in Eqs. (7)–(8) is constant down the MS. As a result, Eq. (7) gives \( d\omega/dt \propto 1/I \propto M^{-3} \), for the unsaturated regime, i.e., for spun-down stars (see also Barnes 2003). Eq. (8) gives seemingly the same result but here an additional, mass-dependent factor \( \tau_c^{-2} \) appears, so the complete mass-dependent term is \( 1/M^3 \tau_c^2 \). For \( \tau_c \) increasing at least as fast as \( M^{-3/2} \) the spin-down rate decreases with decreasing mass. Otherwise, the spin-down rate increases with mass decreasing. Unfortunately, the correct \( \tau_c(M) \) relation is not known; there exist a number of relations in the literature, both theoretical and empirical. They do not differ much for late F and G-type stars but they diverge for K and M-type substantially. The most recent theoretical values are given by Kim and Demarque (1996) and the recent empirical values are given by Stepień (2003). Spin-down rate resulting from Eq. (6) does not depend explicitly on \( M \), but the exponential term depends on mass via \( \tau_c \). Fig. 1 compares the spin-down rates obtained from Eq. (6) and Eqs. (7)–(8). Its top part shows the spin-down rate for the Sun, assuming the initial rotation period of 1 d and the normalization of both rates to the same initial value. As we see, the Kawaler-Chaboyer and Stepień formulas predict very similar spin-down rates, except for rotation periods longer than about 35 d–40 d for which Eq. (6) predicts a significantly lower rate. The middle part of Fig. 1 shows the same relations for a 0.5 \( M_\odot \) and the turnover time taken from the 200 Myr isochrone of Kim and Demarque (1996). Both predictions agree well except, again, for
very slowly rotating stars. If, however, the turnover time is taken from Stepien (2003) the spin-down rate, predicted by the Kawaler-Chaboyer formulas, turns out to be higher by a factor of \( \approx 3 \) than that, predicted by Eq. (6) already for fast rotating stars and the difference increases with the increasing rotation period (the bottom part of the Fig. 1). It is also higher than that for solar mass stars (compare with Fig. 1, top), contrary to predictions by Eq.(6) which give the same spin-down rate in the limit of fast rotation for all considered masses. Eq. (6) will be used for the rest of the present paper.

Adopting \( k^2 \approx 0.1 \) for lower MS stars, we obtain from Eq. (6) the expression for spin down rate

\[ -\frac{d\omega}{dt} = 7 \times 10^{-9} \omega e^{-Ro/0.335}. \] (10)

As we see, the spin down rate of single stars depends on mass only via \( Ro \).

Applicability of Eqs. (6) and (10) to low mass stars requires the assumption that the constant \( C \) in Eq. (5), which is determined from the solar type stars, can also be applied to lower mass stars, i.e., that the time scale for spin down of fast rotating stars (we will call it initial time scale) is the same for all considered masses. It is seen from Eq. (10) that the initial time scale is equal to \( \approx 1.6 \times 10^8 \) yr. It increases with age, e.g., it is equal to \( \approx 2.2 \times 10^{10} \) yr for the present Sun. This value agrees well with the empirical determination of the solar spin-down rate by Pizzo et al. (1983). Time dependence of the spin down rate for stars of different masses can be calculated with the use of Eq. (10) after substituting the value of \( \tau_c \) corresponding to the considered mass.

For slow enough rotation, i.e., when \( Ro/0.335 \gg 1 \), which implies \( 3P_{\text{rot}} \gg \tau_c \), we can develop the exponential expression in Eq. (10) into power series, and with the first order term retained, we obtain

\[ -\frac{d\omega}{dt} \propto \frac{1}{\tau_c}. \] (11)

which gives, after integration,

\[ \omega \propto \frac{t}{\tau_c} \quad \text{or} \quad P_{\text{rot}} \propto t\tau_c. \] (12)

For coeval stars (i.e., members of an intermediate age or old stellar cluster) \( t = \text{const} \) and \( P_{\text{rot}} \sim \tau_c \), i.e., the \( P_{\text{rot}}(B-V) \) relation follows the \( \tau_c(B-V) \) relation. Observations confirm this prediction (Soderblom 1985, Stepien 1989). The formula cannot be applied to young clusters whose members are still in the phase of contraction towards ZAMS because it does not allow for the change of the stellar moment of inertia. Long contraction time of very low mass stars may be a likely reason that M type stars rotate anomalously rapidly in Hyades, in a marked difference to more massive stars (Radick et al. 1987).

\(^{1}\)Yakut and Eggleton (2005) used Eq. (6) in their models of binary star evolution but with an additional term flattening the slope of the relation plotted in Fig. 1 for long periods.

\(^{2}\)Note a slight difference in a value of the numerical coefficient, compared to Eq. (14) in Stepien (1995).
3. Low-Mass Limit for W UMa Stars

3.1 Angular Momentum Loss of a Close Binary

Assuming that the total AML of a close binary results only from the spin AML due to magnetized winds of both components, we have

\[ \frac{dH_{\text{tot}}}{dt} = \frac{dH_{\text{spin,1}}}{dt} + \frac{dH_{\text{spin,2}}}{dt} \]  

where the total AM of a binary consists of orbital AM and spin AM of the components. Unless the mass ratio of the components is extreme, the spin AM can safely be neglected because it is always 1.5–2 orders of magnitude smaller than orbital AM. Adopting this approximation we have

\[ H_{\text{tot}} \approx H_{\text{orb}} = \frac{G^2 M_1^3}{3 M_{\text{tot}}^2 a^{1/2}} q(1+q)^{-2} \]  

where \( G \) is the gravity constant, \( M_{\text{tot}} = M_1 + M_2 \), \( a \) is a semi-major axis and \( q = M_1/M_2 \).

Using Eq. (10) in the limit of short orbital period (i.e., when the Rossby numbers of both components are small and the rotation is synchronous), Eq. (13) becomes

\[ \frac{dH_{\text{orb}}}{dt} = -4.9 \times 10^{41} \left( R_1^2 M_1 + R_2^2 M_2 \right)/P_{\text{orb}} \]  

where masses and radii of the components are in solar units, period is in days, time in years and the orbital AM is in cgs units.

Eq. (15) is the basic equation describing the evolution of the orbital AM of a close binary star. Because AML rate is inversely proportional to the orbital period it increases correspondingly for very short periods.

The semi-axis \( a \) is connected with the orbital period by the third Kepler law

\[ a = 4.21 M_{\text{tot}}^{1/3} P_{\text{orb}}^{2/3} . \]  

Here, again, \( M_{\text{tot}} \) and \( a \) are in solar units and \( P_{\text{orb}} \) in days. Two more equations, describing the sizes of the Roche lobes of both components, will also be needed. Eggleton (1983) derived the formulas approximating the effective radii of the Roche lobes \( r_1 \) and \( r_2 \) to better than 1%

\[ \frac{r_1}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} , \]  

\[ \frac{r_2}{a} = \frac{0.49 q^{-2/3}}{0.6 q^{-2/3} + \ln(1 + q^{-1/3})} . \]  

3.2 Mass Loss

Magnetized winds carry away not only AM but also mass. Our knowledge of the ML rate by stellar winds is still very poor. The solar ML is equal to \( 2-3 \times 10^{-14} \)
Wood et al. (2002) determined recently ML rates of several single active stars from observations of their astrospheres. The results show that the expected ML rate of the most active, solar type stars is of the order of $1 - 2 \times 10^{-11} \, M_\odot/yr$. Other estimates, based on radio observations of several active M-type stars, indicate that their present ML rates do not exceed a few times $10^{-12} \, M_\odot/yr$ (van den Oord and Doyle 1997). A similar result was reached by Lim and White (1996). The upper limit of 4× solar ML rate (at a level of 1σ) was found for Proxima (Wargelin and Drake 2002). A maximum, i.e., saturated value of $10^{-11} \, M_\odot/yr$ is adopted in the present paper as a reasonable compromise for solar type stars. Wood et al. (2002) also found that ML rate per unit area is best correlated with X-ray flux per unit area (usually denoted by $F_X$). It means that rapidly rotating stars emitting the saturated X-ray flux at the maximum, mass independent level (Vilhu and Walter 1987), will lose mass per unit area also at the maximum, mass independent level. The total ML rate of a given star is obtained by multiplying this ML rate by the stellar surface area. With the saturated value of $10^{-11} \, M_\odot/yr$ for the solar mass star we obtain

$$\dot{M} = -10^{-11} M^2$$

(19)

where the scaling $R \approx M$ (in solar units) is used again. Eq. (19) is included into the model calculations.

Spherically symmetric ML from a component of a binary results in an increase of the orbital period whereas the AML (at constant mass) results in its decrease. If both processes take place, the period can increase or decrease, depending on their relative importance. As discussed by Mochnacki (1981), orbital period decreases when the relative AML rate (i.e., $dH_{\text{orb}}/H_{\text{orb}}$) is at least 5/3 times larger than the relative ML rate (i.e., $dM_{\text{tot}}/M_{\text{tot}}$). When varying the ML and AML rates within uncertainties one should be aware of this limitation, particularly at the initial state when the binary has maximum orbital AM. The relative AML rate will then be minimum and adopting too high initial ML rate results in widening of the binary orbit instead of tightening it (see also Yakut and Eggleton 2005).

As it was mentioned in Section 1, UFRs with rotation periods shorter than about 0.4 d show the effect of supersaturation – their X-ray flux levels off and then decreases with decreasing period, contrary to what is observed for longer periods (Randich et al. 1996). If AML rate scales with the X-ray flux (which is not proved but seems reasonable) it should also level off and decrease (somewhat) for short periods. The effect of supersaturation was introduced to Eq. (15) by assuming $P_{\text{orb}} = 0.4$ for periods shorter than 0.4 d. This influenced only results for least massive binaries which reach periods significantly shorter than 0.4 d at the approach to RLOF. The resulting increase of the time scale of approach was less than 1 Gyr (less than 10% of the total time of approach). We conclude that a decrease of efficiency of AML due to supersaturation alone is not sufficient to explain deficiency of low mass contact binaries.
3.3 Binaries with Equal Mass Components

In this Section we discuss the process of AML in binaries with identical components, \( q = 1 \). Five different binaries with initial component masses equal to 1.0 \( M_\odot \), 0.9 \( M_\odot \), 0.8 \( M_\odot \), 0.7 \( M_\odot \) and 0.6 \( M_\odot \) will be discussed. Evolutionary increase of stellar radii is significant only in case of the most massive stars considered here i.e., with initial masses equal to 1.0 \( M_\odot \) and 0.9 \( M_\odot \). For less massive stars this increase is negligible even in the Hubble time. Eq. (15) was integrated for the initial value of \( P_{\text{orb}} = 2 \) d and, simultaneously, Eq. (19) was applied to each component.

Table 1 lists values of the initial parameters of the considered binaries and of the same parameters when RLOF begins. Initial radii of the stars with masses 1.0 \( M_\odot \), 0.9 \( M_\odot \), and 0.8 \( M_\odot \) were taken from models of VandenBerg (1985). They are somewhat larger than those from more recent models but the newest observational data indicate that the observed radii of low mass stars are, in fact, systematically larger by about 10–15% compared to the recent models and agree better with the older models (Lopez-Morales and Ribas 2005). Values of the initial radii of stars with masses \( \leq 0.7 \ M_\odot \) were assumed to be numerically equal to their masses. With the adopted mass-radius scaling, the exact values of radii have no influence on AML rate. The first row gives initial values and the second row gives the values of the binary parameters when RLOF begins. For two most massive binaries the component radii at this age are assumed to be close to the TAMS radii of stars with masses appropriately decreased by stellar winds. For all other, less massive stars, the values of stellar radii equal numerically to mass (both in solar units) are assumed. Time evolution of the orbital period of the binaries from Table 1 is shown in Fig. 2. As we see,
only two binaries with the most massive initial components lose enough AM to form a contact binary within the age of the Galactic disk (≈ 10 Gyr). Their total mass at the time of RLOF is equal to 1.86 M_☉ and 1.66 M_☉, respectively. Binaries in globular clusters can reach contact within the cluster age for the initial component masses as low as 0.77 M_☉, with their present values close to 0.7 M_☉. Binaries with initial component masses lower than 0.7 M_☉ have not lost enough AM within the age of the Universe to form contact systems and they remain in a detached state.

3.4 Binaries with \( q = 0.5 \)

The most recent observational data suggest that binaries “like to be twins” (Halbwachs et al. 2004, Pinsonneault and Stanek 2006), i.e., their substantial fraction has \( q \) close to one. There exists, however, also a population of binaries with \( q \) having a broad maximum around 0.5. We consider now the AML of four such binaries. Table 2 gives details on the discussed binaries in the same way as Table 1. Fig. 3 shows the results.

As we see, the more massive component can fill its critical Roche lobe within the age of the Galactic disk if the initial total mass of a binary is equal to or higher than ≈1.2 M_☉. The total mass of such binaries at the time of reaching contact is about 1.1 M_☉. Primaries in globular cluster binaries can reach their critical Roche lobe within the cluster age if their initial mass is not lower than about 0.7 M_☉. The total mass of such a binary at the time of reaching contact is about 1 M_☉.

| age [Gyr] | masses [M_☉] | radii [R_☉] | \( P_{\text{orb}} \) [days] | \( a \) [R_☉] | \( H_{\text{orb}} \) \( \times 10^{21} \) |
|---------|-------------|-------------|----------------|-----------|----------------|
| 0       | 1.0±0.5     | 0.9±0.5     | 2.0            | 7.64      | 6.82          |
| 7.3     | 0.93±0.48   | 1.0±0.48    | 0.34           | 2.30      | 3.47          |
| 0       | 0.9±0.45    | 0.82±0.45   | 2.0            | 7.38      | 5.72          |
| 8.6     | 0.84±0.43   | 0.91±0.43   | 0.31           | 2.07      | 2.80          |
| 0       | 0.8±0.4     | 0.76±0.4    | 2.0            | 7.09      | 4.70          |
| 10.6    | 0.74±0.38   | 0.74±0.38   | 0.23           | 1.66      | 2.08          |
| 0       | 0.7±0.35    | 0.74±0.35   | 2.0            | 6.79      | 3.77          |
| 13.0    | 0.64±0.34   | 0.64±0.34   | 0.20           | 1.44      | 1.58          |
4. Discussion and Conclusions

The parameter free formula for AML rate of near solar mass stars, derived by Stepien (1988, 1995), was extended to lower mass stars. The results show that the spin down rate of a single star depends on its mass solely via turnover time entering an exponential term. This term is of the order of unity for rapidly rotating stars (i.e., rotating in the saturation regime) but it decreases with decreasing angular velocity i.e., for moderately and slowly rotating stars. The formula shows that the spin down rate is mass independent in the saturation limit, hence the AML rate for such stars is directly proportional to the stellar moment of inertia.

To follow the formation of contact binaries, the derived formula for AML rate was applied to close binaries with initial orbital periods short enough for synchronously rotating components to be in a saturation regime. It was assumed that the total AM of a binary can be approximated by orbital AM (which rules out systems with extreme mass ratio) and that the total AML is a sum of AM losses of both components treated individually, i.e., neglecting any possible influence of the proximity effects on AML rate of each star. In addition to AML, mass is also lost by the wind. Following Wood et al. (2002) the constant saturated value of ML per unit area of the stellar surface was adopted for all considered stars. This value was multiplied by stellar surface area to obtain the total ML of a star. Similarly as in case of AML, ML of a binary was assumed to be a sum of losses of both stars, neglecting the proximity effects. This assumption is in contrast to the approach of e.g., Eggleton and Kiseleva-Eggleton (2002) and Yakut and Eggleton (2005) who assumed a strong tidal enhancement of the mass loss, following the suggestion by Tout and Eggleton (1988).

The set of parameters needed to follow the orbit evolution consists of initial (i.e., ZAMS) component masses, initial orbital period, AML rate and ML rate. To reduce the parameter space a fixed value of 2 d for the initial orbital period was adopted. This value is close to the expected minimum orbital period of a binary formed in the fragmentation process. Observations of binary T Tau stars and of the youngest clusters are in agreement with this value (for a discussion see Stepien 1995). With a scaling $R\approx M$ (in solar units) the AML rate and ML rate depend only on the component masses and orbital period (see Eqs. (15) and (19)). With orbital period fixed, component masses become the only free parameters of the model. Evolution of the orbital parameters of binaries with a range of masses and two initial mass ratios, $q=1$ and 0.5 was computed until RLOF by a primary occurred.

The results show that the approach to RLOF takes at least several Gyr for all considered cases and the duration of this time depends mainly on the initial mass of the primary. The initial mass ratio plays a secondary role. For primaries massive enough that their MS life time is close to the time of approach to RLOF their radii increase significantly during the process of approach which speeds up RLOF. As a result, mass transfer begins within the age of the Galactic disk for binaries with minimum initial masses of primary components equal
to 0.9 M⊙–1.0 M⊙. For slightly less massive primaries, with masses around 0.7 M⊙–0.8 M⊙, RLOF occurs within the age of globular clusters. Binaries with primaries less massive than 0.7 M⊙ do not reach RLOF within the age of the Universe. Assuming that RLOF by a primary is a necessary condition for formation of a contact binary, the results of the present investigation predict a lower mass limit for the total mass of an immediate progenitor of a contact binary at the level of about 1.1 M⊙–1.2 M⊙ in the Galactic disk and at the level of about 1.0 M⊙–1.1 M⊙ in globular clusters (see Tables 1 and 2). This limit will further be decreased by a possible (additional to the stellar wind) mass loss during the mass exchange process and by the wind operating in the contact phase. The least massive known W UMa type stars in the solar vicinity have total masses of 1 M⊙–1.1 M⊙ (Pribulla et al. 2003), with the uncertainty of at least 10%. This limit agrees very well with the results of the present investigation.

There is a number of uncertainties which may influence the final results. A value of 2 d for the minimum initial orbital period may look too restrictive. While this limit finds a support from theoretical as well as observational results we cannot rule out a possibility that under special conditions some binaries may lose an excessively large fraction of their orbital AM early in evolution and form a ZAMS binary with a period significantly shorter than 2 d. This may involve e.g., interactions with the third body, collisions within a dense environment or excessive AML in the pre-MS phase of evolution. The time to reach RLOF will then be correspondingly shorter for such binaries, as can be seen from Figs. 1 and 2. This is difficult to estimate how often such situations can occur, but observations suggest that they are quite rare. Only one pre-MS binary star is known with a period shorter than 2 d. This is HD 155555 with P_{orb} = 1.7 d (Mathieu 1994, Strassmeier and Rice 2000) and two next shortest periods are P_{orb} = 2.4 d. Some authors treat the initial orbital period as a free parameter which can assume values starting from a fraction of a day (e.g., Webbink 1977a,b, Yakut and Eggleton 2005). Such binaries can reach RLOF within 1–2 Gyr. If many progenitors of contact binaries had very short periods on ZAMS, we should observe several W UMa type stars in young and intermediate age clusters. This is not the case. Observations show that W UMa type binaries are extremely rare in stellar clusters younger than about 4–4.5 yr (Kaluzny and Rucinski 1993, Rucinski 1998b), which suggests that the typical time interval needed to reach RLOF must be of the order of several Gyr. In fact, only one W UMa type binary is a certain member of an intermediate age cluster; this is TX Cnc in Praesepe.

Both formulas, describing AML rate and ML rate, were calibrated by observations hence they are burdened with uncertainties. The numerical coefficient in Eq. (15) is determined within an accuracy of about 30%. Its value was determined from observations of solar type stars in clusters of different age (Barry et al. 1987) which indicated a smooth decrease of rotation velocity with age. Recent observations, obtained by Pace and Pasquini (2004), suggest however that the average stellar rotational velocity shows a sharp decrease at the age of 1 Gyr and then remains nearly constant. New, more accurate and numerous
observations are needed to resolve this controversy. Lowering the AML rate by 30% increases the time to RLOF by 3–4 Gyr, depending on the binary mass, which, in turn, raises the lower limit for the total mass of W UMa stars to more than 1.5 M⊙. This seems to be ruled out by observations. Increasing the AML rate by 30% shortens the time to RLOF by about 3–4 Gyr. Binaries with primary masses as low as 0.6 M⊙ can reach RLOF within the age of globular clusters. Such stars are still very close to ZAMS and their orbital periods must reach a value shorter than 0.2 d when RLOF occurs (see Tables 1 and 2). Such low mass stars certainly cannot form contact W UMa type stars by a mechanism proposed by Stępień (2006) in which the primary is hydrogen depleted at RLOF. The problem is, however, whether such binaries can form a long living contact binary by any mechanism (see below). Regarding ML, the coefficient in Eq. (19) is known only within a factor of 2. Its increase by this factor lengthens enormously the time to RLOF. This is due to the fact that the spherically symmetric ML acts always towards the lengthening of the orbital period (opposite to AML). There exists a critical value of the ML rate for each AML rate such that when ML rate exceeds this value, the period will lengthen (Mochnacki 1981). ML rate two times higher than the value given by Eq. (19) is still lower than the critical value but it is close to it. In consequence, orbital periods decrease but the time scale of this decrease becomes very long – longer than the age of the Galactic disk even for primaries with masses equal to 1 M⊙. Such a high lower limit seems again to be ruled out by the observations. With a coefficient in Eq. (19) decreased by a factor of two, the time to RLOF shortens by about 1–2 Gyr, depending on the binary mass and the lower mass limit for a primary to reach RLOF within the age of the Galactic disk is reduced to 0.8 M⊙ and to 0.7 M⊙ for globular clusters. Such values are not in disagreement with the observations, which indicates that the ML rate adopted in the present paper may be somewhat overestimated rather than underestimated.

A few simple scaling rules were used when deriving the formulas for AML and ML rates. They could be replaced with more accurate relations. It is felt, however, that the best way to decrease significantly the uncertainties connected with the modeling of AML would be to obtain high quality observations of rotation rates of stars of different age and mass. With \( P_{\text{rot}}(t, M) \) known accurately one can infer time and mass derivatives of this function and, with stellar moment of inertia calculated, AML rate can be obtained. Similarly, new, more accurate and numerous observations of ML in different stars will constrain ML rate as a function of age and mass. A substantial uncertainty is connected with possible proximity effects in close binary stars. Can they be neglected, as assumed in this paper or are they of primary importance as assumed e.g., by Eggleton and his group?

The results of the present investigation confirm and extend the conclusions reached by Stępień (2006). For binaries with primary masses \( \geq 1 \, M_\odot \) (\( \geq 0.9 \, M_\odot \) in globular clusters) the time scale for AML is of the same order as evolutionary time scale. As a result, such stars expand due to depletion of hydrogen in their cores and reach their Roche lobe (which shrink at the same time due to AML) within several Gyr when the orbital period is of the order of 0.4 d. Mass transfer
begins and after mass ratio reversal a contact binary can be formed with large enough orbital AM for stability of the system. Less massive stars stay essentially close to ZAMS. Decreased AML rate of low mass stars, together with lack of evolutionary expansion lengths the time to RLOF up to, and beyond the Hubble time. This is why we do not observe low mass contact binaries. One may wonder, however, what is the fate of a low mass binary if it happens to lose a large fraction of its AM due to other mechanisms mentioned above. The orbital period of such a binary at RLOF must be as short as 0.15 d–0.2 d. In fact, we know one detached system with such a short period, which is close to RLOF. This is OGLE BW3 V38 with component masses equal to 0.44 \( M_\odot \) and 0.41 \( M_\odot \) and \( P_{\text{orb}} = 0.198 \) d (Maceroni and Montalbán 2004). Mass transfer in a low mass binary with the component masses equal to 0.8 \( M_\odot \) and 0.4 \( M_\odot \) was modeled by Webbink (1977b). His results indicate that a large fraction of mass and AM must be lost from the system during the mass transfer process and the remaining binary will very likely be unstable leading to merger of both components. It seems that such binaries have simply too few AM to survive mass transfer. In binaries with \( q \) close to 1 mass transfer may not be so dramatic but further AML due to stellar wind and (not considered here) gravitational radiation should quickly lead to mass shedding through the outer Lagrangian points and merging of both components (Rasio and Shapiro 1995). This may explain why not a single low mass contact binary is observed.

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Fig. 1. The spin down rate of a single star as a function of its angular velocity. Solid lines describe the spin down rates resulting from Eq. (6) and derived by the present author, and dotted lines are based on Eqs. (7)–(8), derived by Kawaler (1988) and modified by Chaboyer et al. (1995). The rates for the 1 $M_\odot$ star were normalized at $P_{\text{rot}} = 1$ d (top). The middle part gives the predicted spin down rates assuming that the turnover time of a 0.5 $M_\odot$ star is 3.11 times longer than that for the Sun (Kim and Demarque 1996) and the bottom part gives the same rates assuming that the turnover time of 0.5 $M_\odot$ star is 1.65 times longer than that of the Sun (Stępień 2003).
Fig. 2. Orbital period of binaries with $q = 1$ as a function of age. The consecutive curves describe period evolution of binaries with different total masses, as indicated. Lower end of each curve corresponds to the instant when the primary component reaches the Roche lobe.

Fig. 3. Orbital period of binaries with $q = 0.5$ as a function of age. The consecutive curves describe period evolution of binaries with different total masses, as indicated. Lower end of each curve corresponds to the instant when the primary component reaches the Roche lobe.