Entropy of Anisotropic Universe
and Fractional Branes

S. Kalyana Rama

Institute of Mathematical Sciences, C. I. T. Campus,
Tharamani, CHENNAI 600 113, India.
email: krama@imsc.res.in

ABSTRACT

We obtain the entropy of a homogeneous anisotropic universe applicable, by assumption, to the fractional branes in the universe in the model of Chowdhury and Mathur. The entropy for the 3 or 4 charge fractional branes thus obtained is not of the expected form $E^{3/2}$ or $E^2$. One way the expected form is realised is if $p \to \rho$ for the transverse directions and if the compact directions remain constant in size. These conditions are likely to be enforced by brane decay and annihilation, and by the S, T, U dualities. T duality is also likely to exclude high entropic cases, found in the examples, which arise due to the compact space contracting to zero size. Then the 4 charge fractional branes may indeed provide a detailed realisation of the maximum entropic principle we proposed recently to determine the number $(3+1)$ of large spacetime dimensions.
1. Introduction

We assume that our observed universe is described by string/M theory. At low temperatures, the evolution of the universe is described in the standard way by a low energy effective action. At early times, when the temperature is of the order of string scale, higher modes of the strings are excited and the evolution must be described by stringy variables [1, 2]. Assuming that our universe originated from highly excited and highly interacting strings, we have recently proposed in [3] a maximum entropic principle to determine the number $(3 + 1)$ of large spacetime dimensions. (For earlier ideas on this number, see [4].)

The evolution of the universe at early times, when its temperature is high, is not well understood. Within the context of perturbative string theory, a natural idea has been to assume that the universe consists of gas of strings, their winding modes, and/or gases of various branes, and then obtain the evolution of the universe using a low energy effective action [4, 5, 6].

Recently, Chowdhury and Mathur proposed in [7] a novel model where the early universe consists of mutually BPS intersecting brane and antibrane configurations. The branes (and similarly antibranes) in these configurations form bound states and become fractional, supporting very low energy excitations and creating thereby large entropy for a given energy $E$. The entropy $S$ of an $N$ charge fractional brane configuration is expected to be $1 \simeq E^{2 \frac{N}{2}}$. Hence, the early universe is likely to contain, and be dominated by, such fractional brane configurations because of their high entropy. Chowdhury and Mathur also obtain the energy momentum tensor, $T^{\mu \nu} = \text{diag} (-\rho, p_i)$ where $p_i = w_i \rho$, for these configurations, and solve in complete generality the Einstein’s equations of motion with arbitrary $w_i$’s. See [6] also.

The 4 charge fractional branes, with entropy $S$ expected to be $\simeq E^2$, may provide a detailed realisation of the maximum entropic principle proposed in [3] and may help in understanding the evolution of the universe at early times. In this paper we, therefore, study the relation between the entropy $S$ and the energy $E$. We assume that the evolution of the fractional branes in the universe is given by the evolution of the anisotropic universe with the corresponding $w_i$’s; and that the physical quantities $S$ and $E$ are similarly

\[This \text{ relation is derived with the volume of the system kept fixed. However, for } N = 3, 4\text{ configurations considered here, it turns out to be valid in an expanding universe also as explained later.}\]
related.

Following the standard method, we obtain the entropy $S$ of the anisotropic universe in general, as well as in the asymptotic limit of large $t$. We consider two examples, particular cases of which correspond to the fractional branes in the universe with their $w_i$s obtained as in [7]. Using the results of [7], we then compare the entropy $S$ with its expected form $\simeq E^{\frac{N}{2}}$ for the cases $N = 3, 4$. We find that they do not agree.

Given the importance of 4 charge fractional branes in understanding the early universe, it is important to study how to obtain the expected form for $S$. Hence, we consider a few possible ways of obtaining the expected form for $S$. One of them is that it is the holographic entropy which should obey the expected relation. Another is the following. Our examples show that the expected form can be realised if $w_i = \omega \to 1$ for the transverse directions and if the compact directions remain constant in size. We discuss the physical interpretation of these conditions and argue, in the light of our two examples, that they are likely to be enforced by including brane decay and annihilation processes and by the S, T, U duality symmetries of the string and M theories.

Also, there are high entropic cases in our examples, for example $S \simeq E^X$ with $X > 4$, which arise due to the compact space contracting to zero size. The T duality symmetry of string theory is likely to prevent such contractions. We show that the entropy then remains in accord with one’s expectation that the most entropic object is a Schwarzschild black hole or an isotropic universe containing matter with $\omega = 1$.

This paper is organised as follows. In sections 2 and 3, we present briefly the relevant equations of motion for an anisotropic universe and their general and asymptotic solutions. In section 4, we present the expressions for entropy and, in section 5, two examples. In section 6, we discuss in detail the fractional branes. In section 7, we conclude by mentioning a few issues for further study.

2. Equations of motion

Consider a $D$ – dimensional homogeneous anisotropic universe containing matter with $T^{\mu \nu} = \text{diag} \ (-\rho, p_i)$ where $\rho > 0$, $p_i = w_i \rho$ with $w_i$’s in the range $-1 \leq w_i \leq 1$, and $i = 1, 2, \cdots, D - 1$. The line element is given by

$$ds^2 = -dt^2 + \sum e^{2\lambda_i(t)} dx_i^2$$  \hspace{1cm} (1)
where, here and in the following, the sum is over \( i = 1, 2, \ldots, D - 1 \) unless mentioned otherwise. Defining \( \Lambda = \sum \lambda_i \) and \( b = \sum w_i \lambda_i \) and using natural units with \( 8\pi G = 1 \), the conservation equation \( \nabla_\mu T^\mu_\nu = 0 \) and the Einstein’s equations of motion \( R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu} \) can be written as

\[
\dot{\rho} + (\dot{b} + \dot{\Lambda})\rho = 0 \quad , \quad \dot{\Lambda}^2 - \sum \dot{\lambda}_i^2 = 2\rho \quad (2)
\]

\[
\ddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda} - \dot{\Lambda} - \dot{\lambda}_i^2 = -\rho + p_i = -(1 - w_i)\rho \quad (3)
\]

where overdots denote time derivatives. It follows from the above equations that

\[
\dddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda} = C_i \rho \quad , \quad \dddot{\Lambda} + \dot{\Lambda}^2 = C_\Lambda \rho \quad , \quad \dddot{b} + \dot{b} \dot{\Lambda} = C_b \rho \quad (4)
\]

where the constants \( C_i, C_\Lambda = \sum C_i \), and \( C_b = \sum w_i C_i \) are given by

\[
C_i = w_i + \frac{1 - W}{D - 2} \quad , \quad C_\Lambda = \frac{D - 1 - W}{D - 2} \quad , \quad C_b = U + \frac{W - W^2}{D - 2} \quad (5)
\]

with \( W = \sum w_i \) and \( U = \sum w_i^2 \). The above equations \( (2) - (4) \) have been solved recently by Chowdhury and Mathur in \([7]\). See also \([6]\) for anisotropic solutions in string/brane cosmology. We follow the methods of \([7]\) closely and define

\[
H_i = \dot{\lambda}_i e^\Lambda \quad , \quad H = \dot{\Lambda} e^\Lambda \quad , \quad B = \dot{b} e^\Lambda \quad , \quad T = \rho e^\Lambda \quad . \quad (6)
\]

Note that \( H = \sum H_i, B = \sum w_i H_i \), and that \( T > 0 \) since \( \rho > 0 \). It is straightforward to see that \( (H_i, H, B, T) \) satisfy the equations

\[
\dot{H}_i = C_i T \quad , \quad \dot{H} = C_\Lambda T \quad , \quad \dot{B} = C_b T \quad , \quad \dot{T} = -\dot{b} T \quad . \quad (7)
\]

Define a parameter \( \tau \) by \( \dot{\tau} = T \), equivalently by \( t - t_0 = \int_0^\tau \frac{dt}{T} \). Then \( \dot{f} = T \dot{f}_\tau \) for any function \( f \) where a subscript \( \tau \) denotes \( \tau \)-derivative. Using the above definitions and equations, one now gets

\[
(\lambda_i)_\tau = \frac{H_i}{T e^\Lambda} \quad , \quad \Lambda_\tau = \frac{H}{T e^\Lambda} \quad , \quad b_\tau = -\frac{T_\tau}{T} = \frac{B}{T e^\Lambda} \quad , \quad (8)
\]

\[
H_i = C_i \tau + K_i \quad , \quad H = C_\Lambda \tau + K_\Lambda \quad , \quad B = C_b \tau + K_b \quad . \quad (9)
\]

It follows from equations \( (8) \) that \( (T e^\Lambda)_\tau = H - B \) and, hence,

\[
T e^\Lambda = \frac{C_\Lambda - C_b}{2} \tau^2 + (K_\Lambda - K_b) \tau + K_0 \quad . \quad (10)
\]
The constants \((t_0, K_i, K_Λ, K_b, K_0)\) in the above equations are initial values of \((t, H_i, H, B, T, e^Λ)\) respectively at \(τ = 0\). Equations (8), and then equation \(\dot{τ} = T\), can now be solved to obtain \((λ_i, Λ, b, T)\) and \(t\) in terms of \(τ\). Thus \((λ_i, Λ, b, T)\) are obtained implicitly in terms of \(t\).

### 3. Asymptotic solutions and their properties

The general solutions to the above equations and a thorough discussion of their properties are presented in detail in [7]. Among other things it is shown that, for physical ranges of parameters, the equation \(Te^λ = 0\) has two real roots \(τ_1\) and \(τ_2 > τ_1\) and that the universe expands into future with no singularities.

We are interested here in the asymptotic solutions in the limit \(t \gg 1\), which are also given in [7]. In this limit, we have \(^3\)

\[
e^λ \simeq t^{α_i} , \quad e^Λ \simeq t^{α} , \quad e^b \simeq t^{α_b} , \quad ρ \propto e^{-b-Λ} \simeq t^{-α-α_b} \tag{11}
\]

where \(α_i\)'s are constants, \(α = \sum α_i\), and \(α_b = \sum w_iα_i\). As shown in [7], the limit \(t \gg 1\) corresponds to the following cases:

**(i) \(C_Λ > C_b\):** In this case, \(τ \gg 1\) in the limit \(t \gg 1\) and

\[
α_i = \frac{2C_i}{C_Λ + C_b} , \quad α = \frac{2C_Λ}{C_Λ + C_b} , \quad α_b = \frac{2C_b}{C_Λ + C_b} . \tag{12}
\]

Note that \(α + α_b = 2\), and \(α > 1\) since \(C_Λ > C_b\).

**(ii) \(C_Λ < C_b\):** In this case, \(τ \rightarrow τ_2\) from below in the limit \(t \gg 1\) and

\[
α_i = \frac{H_{i2}}{H_2} , \quad α = 1 , \quad α_b = \frac{B_2}{H_2} \tag{13}
\]

where \((H_{i2}, H_2, B_2)\) are the values of \((H_i, H, B)\) respectively at \(τ = τ_2\). From the definitions of \(H\) and \(B\) it follows that \(H_2 = \sum H_{i2}\) and \(B_2 = \sum w_iH_{i2}\).

\(^2\)Asymptotic solutions are valid when \(τ \gg 1\) or \(τ_2 - τ \ll τ_2\) as the case may be, which translates into \(t \gg 1\) in natural units.

\(^3\)Here and in the following \(\simeq\) denotes that constant factors, numerical as well as dimensionfull ones, which are not needed here are omitted.
(iii) $C_\lambda = C_b$: It is straightforward to obtain detailed solutions for this case also, not given explicitly in [7]. However, it suffices here to note the following.

For generic initial conditions $K_\Lambda \neq K_b$ and the solutions depend on whether $K_\Lambda > K_b$ or $K_\Lambda < K_b$. If $K_\Lambda > K_b$ then $\tau \gg 1$ in the limit $t \gg 1$ since $Te^\lambda > 0$. Hence, this case can be thought of as case (i) with $C_\Lambda = C_b + \epsilon$ in the limit $\epsilon \to 0_+\equiv 0$, equivalently with $\alpha \to 1$ from above. The $\alpha$'s are then given by

$$
\alpha_i = \frac{2C_i}{2C_\Lambda - \epsilon}, \quad \alpha = \frac{2C_\Lambda}{2C_\Lambda - \epsilon}, \quad \alpha_b = \frac{2(C_\Lambda - \epsilon)}{2C_\Lambda - \epsilon}.
$$

Similarly, if $K_\Lambda < K_b$ then $\tau \to \tau_3 \equiv \frac{K_b}{K_b - K_\Lambda}$ from below in the limit $t \gg 1$. Hence, this case can be thought of as case (ii) with $C_\Lambda = C_b - \epsilon$ in the limit $\epsilon \to 0_+\equiv 0$. The $\alpha$'s are then given by

$$
\alpha_i = \frac{H_{i3}}{H_3}, \quad \alpha = 1, \quad \alpha_b = \frac{B_3}{H_3} \text{ (15)}
$$

where $(H_{i3}, H_3, B_3)$ are the values of $(H_i, H, B)$ respectively at $\tau = \tau_3$. From the definitions of $H$ and $B$ it follows that $H_3 = \sum H_{i3}$ and $B_3 = \sum w_i H_{i3}$.

Note that $\alpha \geq 1$ for all the solutions.

4. Entropy, energy, and area

In the following, we assume that the spatial directions denoted by $i = 1, 2, \cdots, n$ are compact and toroidal with coordinate size $l_i$; the transverse spatial directions denoted by $j = 1, 2, \cdots, m = D - 1 - n$ may be non compact, or compact with coordinate sizes $l_j$ assumed to be $\gg l_i$.

In a given direction with scale factor $e^\lambda$, the coordinate and the physical sizes of the horizon within which causal contact is possible are given respectively by

$$
r_H = \int_0^t dt \ e^{-\lambda}, \quad L_H = e^{\lambda} r_H \text{ (16)}
$$

Consider a compact direction, with coordinate size $l$ and scale factor $e^\lambda$. Its physical size is then given by $L \simeq e^{\lambda} l$. If its horizon size $r_H \geq l$, equivalently $L_H \geq L$, then all of that direction is in causal contact and, hence, $L_H$ is to be replaced by $L$. In the following, we take this to be the case for $i = 1, 2, \cdots, n$.
that is, we consider the asymptotic limit \( t > t_c \gg 1 \) where \( t_c \) is the earliest time when \( L_{Hi}(t_c) > L_i, \ i = 1, 2, \cdots, n \). Then \( L_{Hi} \) is to be replaced by \( L_i \) and the coordinate and the physical volume of the \( n \) – dimensional compact space, within which causal contact is possible, are given respectively by

\[
v_c \simeq \prod_{i=1}^{n} l_i, \quad V_c \simeq \prod_{i=1}^{n} L_i. \tag{17}
\]

The coordinate and the physical volume of the horizon in the transverse space are, respectively,

\[
v_{H\perp} \simeq \prod_{j=1}^{m} r_{Hj}, \quad V_{H\perp} \simeq \prod_{j=1}^{m} L_{Hj}. \tag{18}
\]

The total physical volume \( V \) and the physical area \( A \), within which causal contact is possible, are thus given by

\[
V \simeq V_c V_{H\perp}, \quad A \simeq V_c (V_{H\perp})^{m-1}. \tag{19}
\]

When the transverse space is compact, let \( t_\perp \gg t_c \) be the earliest time when \( L_{Hj}(t_\perp) > L_j, \ j = 1, 2, \cdots, m \). Then, equation (19) is still applicable for \( t \ll t_\perp \). For \( t > t_\perp \), however, \( L_{Hj} \) is to be replaced by \( L_j \) and the coordinate and the physical volume of the \( m \) – dimensional transverse space, within which causal contact is possible, are given respectively by

\[
v_{\perp} \simeq \prod_{j=1}^{m} l_j, \quad V_{\perp} \simeq \prod_{j=1}^{m} L_j. \tag{20}
\]

The total physical volume \( V_\perp \), within which causal contact is possible, is now given by \( V \simeq V_c V_{\perp} \simeq v_c v_{\perp} e^\Lambda \).

It can be shown that the conservation equation \( \dot{\rho} + (\dot{b} + \dot{\Lambda})\rho = 0 \) implies that the comoving entropy density \( S_{co} \) is a constant, see the paper by Tseytlin and Vafa in [4]. Following the standard method, the physical entropy density \( s \) is given by

\[
s = e^{-\Lambda} S_{co}. \tag{21}
\]

The physical entropy \( S \) and the physical energy \( E \), defined naturally as those contained in the volume \( V \) within which causal contact is possible, and assumed to be large as is generically the case in an expanding universe, are then given by

\[
S = s V, \quad E = \rho V. \tag{22}
\]
using which $S$ may also be expressed in terms of $E$.

We now evaluate the above quantities for the solutions in the asymptotic limit $t \gg 1$, given in section 3. $e^{\lambda_i}$, $e^\Lambda$, and $\rho$ are then given by equation (11) from which it follows that $L_i \simeq l_i t^{\alpha_i}$, $r_{Hi} \simeq t^{1-\alpha_i}$, $L_{Hi} \simeq t$, and

$$s \simeq t^{-\alpha}, \quad \rho \simeq t^{-\alpha - \alpha_b}.$$  \hspace{1cm} (23)

We take $t > t_c \gg 1$ so that $L_{Hi} > L_i$ for $i = 1, 2, \cdots, n$ and, hence, are to be replaced by $L_i$ – this is possible if $\alpha_i < 1$ which is the case in the examples below. Then, $V_c \simeq v_c t^{\alpha_c}$ where we define

$$\alpha_c = \sum_{i=1}^{n} \alpha_i, \quad \alpha_\perp = \alpha - \alpha_c = \sum_{j=1}^{m} \alpha_j.$$  \hspace{1cm} (24)

Note that, for increasing $t$, the compact space on the whole expands if $\alpha_c > 0$ and contracts if $\alpha_c < 0$ since $V_c$ increases or decreases respectively.

We now consider two cases where the transverse space is (i) non compact, and (ii) compact.

(i) **Transverse space is non compact**

We have $V_{Hnc} \simeq t^m$. The total physical volume $V$ and the physical area $A$, within which causal contact is possible, are then given by

$$V \simeq t^{m+\alpha_c}, \quad A \simeq t^{m-1+\alpha_c}. \hspace{1cm} (25)$$

The physical entropy $S$ and the physical energy $E$ contained in the volume within which causal contact is possible are given by

$$S \simeq t^{m+\alpha_c-\alpha}, \quad E \simeq t^{m+\alpha_c-\alpha-\alpha_b}. \hspace{1cm} (26)$$

Since $\alpha \geq 1$ for all the solutions, the entropy $S$ satisfies the Fischler – Susskind holographic bound $S \leq A$, up to constant factors which are omitted here [8]. The entropy $S$ is given in terms of the energy $E$ as

$$S \simeq E^X, \quad X = \frac{m + \alpha_c - \alpha}{m + \alpha_c - \alpha - \alpha_b}. \hspace{1cm} (27)$$

The exponent $X$ then characterises the amount of entropy for given energy: higher the value of $X$ higher is the entropy.
Note that for the \( \alpha \)'s obtained from equations (12), \( \alpha + \alpha_b = 2 \) and therefore \( X = \frac{m + \alpha_c - \alpha}{m + \alpha_c - 2} \). Also note that, generically, \( \alpha \) and/or \( \alpha_c \) and, hence, the exponent \( X \) depend on \( n \), the dimension of the compact space.

(ii) **Transverse space is compact**

For \( t_c < t \ll t_{\perp} \), the horizon size is less than the size of the compact transverse space and, hence, the transverse space is effectively non compact. The results in 4 (i) are then applicable here also.

For \( t > t_{\perp} \), however, entire transverse space is in causal contact and, hence, \( L_{Hj} \) are to be replaced by \( L_j, j = 1, 2, \cdots, m \), and \( V_{H\perp} \) is to be replaced by \( V_{\perp} \approx t^{\alpha_{\perp}} \). Then, the total physical volume \( V \approx t^\alpha \). The physical entropy \( S \) and the physical energy \( E \) contained in the volume within which causal contact is possible are then given by

\[
S \approx v_c v_{\perp} S_{\text{co}} \approx \text{constant} \ , \quad E \approx t^{-\alpha_b} .
\]  

(28)

The entropy \( S \) is constant because the comoving entropy density \( S_{\text{co}} \) is constant [4] and, since \( t > t_{\perp} \), the comoving coordinate volume of the region within which causal contact is possible is now that of the entire space, namely \( v_c v_{\perp} \), which is also constant; the entropy \( S \) which is a product of these two quantities is, therefore, constant.

The energy varies – typically decreases – because the physical size of the space varies – typically expands – and work is done against/by the pressure. Thus if in all directions either there is no expansion/contraction or there is no pressure then no work is done and, hence, the energy \( E \) must be constant. This is indeed the case since either no expansion/contraction or no pressure in \( i^{\text{th}} \) direction means either \( \alpha_i = 0 \) or \( w_i = 0 \) and thus \( w_i \alpha_i = 0 \); when this is the case for all directions it follows that \( \alpha_b = \sum w_i \alpha_i = 0 \). The energy \( E \approx t^{-\alpha_b} \) is then constant.

Let \( \alpha_b = 0 \). Then, for \( t > t_{\perp} \), \( S \) and \( E \) are both constants and, a priori, the relation between them can be anything. However, for \( t_c < t \ll t_{\perp} \), equation (27) is applicable from which we see that \( X = 1 \) since \( \alpha_b = 0 \) and, hence, \( S \approx E \). If one assumes that this relation holds even for \( t > t_{\perp} \), as

\(^{4}\text{The area is now zero since a compact space has no boundary. Therefore, more rigorous Bousso's formulation [8] has to be used to verify the holographic principle, which will not be pursued here.}\)
seems physically reasonable, then it follows that $S \simeq E \simeq \text{constant}$ for $t > t_\perp$ also.

In the following, unless mentioned otherwise, we assume that the transverse space is non compact; or that if the transverse space is compact then we consider only $t \ll t_\perp$ so that the transverse space is effectively non compact. The results for $t > t_\perp$ are straightforward to obtain.

5. Examples

(i) Consider an example where $\lambda_i = 0$ for the compact directions $i = 1, 2, \cdots, n$. Then $\dot{\lambda}_i = 0$ and, as follows from equations (3) and (4), $w_i = \sigma$ and $C_i = 0$ for $i = 1, 2, \cdots, n$ where $\sigma$ is yet to be determined. Thus, the compact space needs to be supported by an isotropic pressure $p_c = \sigma \rho$ in order for its scale factors to remain constant.

Define $W_\perp = \sum w_j$ and $U_\perp = \sum w_j^2$ where the sum in $\sum_j$ is over the transverse directions $j = 1, \cdots, m$ only. It then follows, after some algebra, that $\sigma = \frac{W_\perp - 1}{m - 1}$ and

$$C_j = w_j + \frac{1 - W_\perp}{m - 1}, \quad j = 1, \cdots, m$$

$$C_\Lambda = \frac{m - W_\perp}{m - 1}, \quad C_b = U_\perp + \frac{W_\perp - W_\perp^2}{m - 1}.$$  \hspace{1cm} (29)

These expressions for the $C$'s and, hence, the subsequent analysis and the results are identical to those of an $(m + 1)$-dimensional universe. In particular, they are independent of $n$, the dimension of the compact space which is now kept at a constant size by a pressure $p_c = \sigma \rho$.

Furthermore let $w_j = \omega$ for $j = 1, \cdots, m$. Thus, the transverse space is isotropic and contains a perfect fluid with pressure $p_\perp = \omega \rho$. Then $W_\perp = m \omega$, $\sigma = \frac{m - \omega}{m - 1}$, $C_i = 0$ for $i = 1, 2, \cdots, n$ by construction, and

$$C_j = \frac{1 - \omega}{m - 1}, \quad j = 1, \cdots, m$$

$$C_\Lambda = \frac{m (1 - \omega)}{m - 1}, \quad C_b = \frac{m \omega (1 - \omega)}{m - 1}.$$  \hspace{1cm} (30)

Note that $C_\Lambda \geq C_b$ since $\omega \leq 1$. Also, it is physically reasonable to think of $\omega = 1$ as $\omega \to 1$ from below, equivalently as $C_\Lambda \to C_b$ from above. Hence,
(α_1, α, α_b) are given by equations (12), and (α_c, α_⊥) by equations (24). Thus, 
\[ \alpha_i = \alpha = 0 \] for \( i = 1, 2, \ldots, n \), \( \alpha_\perp = \alpha \), and \( (\alpha_j, \alpha, \alpha_b) \) for \( j = 1, \ldots, m \) are

given by
\[ \alpha_j = \frac{2}{m(1 + \omega)} \, , \quad \alpha = \frac{2}{1 + \omega} \, , \quad \alpha_b = \frac{2\omega}{1 + \omega} \, . \tag{33} \]

The entropy \( S \) is given in terms of the energy \( E \) as \( S \approx E X_\omega(m) \) where, since \( \alpha_c = 0 \),

\[ X_\omega(m) = \frac{m - \alpha}{m - 2} \, , \quad \alpha = \frac{2}{1 + \omega} \, . \tag{34} \]

Note that \( X_\omega \) and, hence, the entropy \( S \) are independent of \( n \), the dimension of the compact space and are identical to those of an \( (m + 1) \) – dimensional universe. Also, note that the exponent \( X_\omega \) is given by

\[ X_0(m) = 1 \] for \( \omega = 0 \), and by \[ X_1(m) = \frac{m - 1}{m - 2} \] for \( \omega = 1 \).

\( \textbf{(ii)} \) Consider a second example where \( w_i = \sigma \) for \( i = 1, 2, \ldots, n \) with \( n \geq 1 \) and \( w_i = \omega \) for \( i = n + 1, \ldots, D - 1 \). Hence, \( \alpha_1 = \alpha_2 = \ldots = \alpha_n \), \( \alpha_{n+1} = \ldots = \alpha_D = 1 \), \( \alpha_c = n\alpha_1 \), \( \alpha_\perp = m\alpha_{n+1} \), and \( \alpha = \alpha_c + \alpha_\perp \). We assume that \( C_\Lambda > C_b \) or \( \rightarrow C_b \) from above. Then, \( (\alpha_i, \alpha, \alpha_b) \) are given by equations (12) and \( \alpha + \alpha_b = 2 \). Using equations (5), it follows that \( C_b - C_\Lambda = \frac{(1 - \omega^2)}{D - 2} \int f(x) \)

where \( x = \frac{1 - \sigma}{1 - \omega} \) and

\[ f(x) = n(m - 1)x^2 - 2nmx + m(n - 1) \, . \tag{35} \]

Hence, \( C_\Lambda \geq C_b \) if \( x_- \leq x \leq x_+ \) where

\[ x_\pm = \frac{nm \pm \sqrt{\Delta}}{n(m - 1)} \, , \quad \Delta = nm(n + m - 1) \tag{36} \]

are the zeroes of \( f \). Note that, for the example \( \textbf{(i)} \), \( x = \frac{m}{m-1} \) which is where \( f(x) \) is minimum. It is easy to show that \( \alpha_i = \frac{\alpha}{n} \geq 0 \) if \( x \leq \frac{m}{m-1} \) and < 0 if \( x > \frac{m}{m-1} \) for the compact directions \( i = 1, 2, \ldots, n \).

Consider the limit \( \alpha \rightarrow 1 \) from above. Then, \( C_\Lambda \rightarrow C_b \) from above and \( x \rightarrow x_\pm \) from within its allowed range. At \( x = x_\pm \), we have \( \alpha = \alpha_b = 1 \) and, after some algebra,

\[ \alpha_{c\pm} = \frac{n \pm \sqrt{\Delta}}{n + m} \, . \]
\( \alpha_i \)'s then follow from \( \alpha_c = n \alpha_1 \) and \( \alpha_\perp = \alpha - \alpha_c = m \alpha_{n+1} \). The entropy \( S \) is given in terms of the energy \( E \) as \( S \simeq E^{X_\pm} \) where, since \( \alpha = 1 \),

\[
X_\pm = \frac{m + \alpha_c \pm 1}{m + \alpha_c - 2}.
\]

Note that \( \alpha_{c \pm} \) and, hence, \( X_\pm \) and the entropy \( S \) now depend also on \( n \), the dimension of the compact space. Also note that \( X_- < X_1(m) \) since \( \alpha_{c-} > 0 \), and \( X_+ > X_1(m) \) since \( \alpha_{c+} < 0 \) where \( X_1(m) = \frac{m+1}{m-2} \) is defined in equation (34). Some examples of \( (n, m, X_+) \) are:

\[
(2, 8, \frac{6}{3}), \quad (5, 5, \frac{3}{2}), \quad (3, 6, \frac{4}{3}), \quad (7, 3, \sim 4.07).
\]

One would expect the most entropic object in an \((m + 1)\)–dimensional spacetime to be a Schwarzschild black hole or an isotropic universe containing matter with \( \omega = 1 \). For both of them, the exponent \( X \) in the relation \( S \simeq E^X \) is given by \( X = X_1(m) \). Therefore, the fact that \( X \) can be \( > X_1(m) \) is perhaps surprising. Here, we have an \( m \)–dimensional transverse space and an \( n \)–dimensional compact space. As pointed out before, the entropy satisfies the holographic bound, so its violation can not be the reason for \( X \) being \( > X_1(m) \). The origin of this behaviour of \( X \) can, however, be traced back to the fact that \( \alpha_c < 0 \); hence, in the limit \( t \gg 1 \), the compact space is contracting to zero size; equation (26) then leads to a value of \( X_+ \) which is \( > X_1(m) \). See near the end of next section for more discussion.

### 6. Fractional branes in the universe

We assume that our observed universe is described by string/M theory. At low temperatures, the evolution of the universe is described in the standard way by a low energy effective action. At early times, when the temperature is of the order of string scale, higher modes of the strings are excited and the evolution must be described by stringy variables [1, 2].

Assuming that our universe originated from highly excited and highly interacting strings, we have proposed in [3] a maximum entropic principle to determine the number \((3 + 1)\) of large spacetime dimensions. According to this principle, the spacetime configuration that eventually emerges from the highly excited and interacting strings is the one that has maximum entropy.
for a given amount of energy, both assumed to be large. In [3], we considered an isotropic universe with a compact space of constant size which is assumed to be $\gtrsim l_s$, the string length. Hence, the entropy $S \simeq E^{\omega (m)}$ as given in equation (34). The maximum entropic principle then leads to a $(3 + 1)$-dimensional spacetime. As can be easily checked, this result remains valid also for the anisotropic examples considered here although, contrary to an assumption in [3], the compact space contracts to zero size in some cases.

The evolution of the universe at early times, when its temperature is high, is not well understood. Within the context of perturbative string theory, a natural idea has been to assume that the universe consists of gas of strings, their winding modes, and/or gases of various branes, and then obtain the evolution of the universe using a low energy effective action [4, 5]. In some of these models, the resulting cosmology is anisotropic and the corresponding anisotropic solutions have also been obtained [6].

Recently, Chowdhury and Mathur proposed in [7] a novel model where the early universe consists of mutually BPS intersecting brane and antibrane configurations. In such BPS configurations, the intersecting branes (and similarly antibranes) form bound states and become fractional, supporting very low energy excitations and creating thereby large entropy for a given energy. Hence, the early universe is likely to contain, and be dominated by, such fractional brane configurations because of their high entropy. These configurations, as Chowdhury and Mathur explain clearly, are different from string/brane gas in [4, 5, 6].

Assuming all the spatial directions to be toroidal, and the brane antibrane decay or annihilation to be negligible, Chowdhury and Mathur also obtain the energy momentum tensor, $T^{\mu \nu} = diag (-\rho, p_i)$ where $p_i = w_i \rho$, for these configurations with net brane charges vanishing. They also solve in complete generality the Einstein’s equations of motion with arbitrary $w_i$’s, and discuss the properties of the solutions. See [7] for details; see [6] also.

An $N$ charge configuration of the fractional branes consists of $N$ or $N - 1$ stacks, each containing a large number $n_I$ of coincident branes and an equal number of antibranes, intersecting in a mutually BPS way; in the later case, the $N - 1$ stacks also have a common boost with $n_I$ units of Kaluza–Klein momentum. See [7] for intersection rules. For an $N$ charge configuration,
the energy $E$ and the entropy $S$ are given by

$$E = 2 \sum_{I=1}^{N} n_I m_I, \quad S \simeq \prod_{I=1}^{N} \sqrt{n_I}$$

where $m_I$ is the mass of the $I^{th}$ type of brane/boost and is constant; $m_I = \tau_I V_I$ for branes and $= \frac{1}{R}$ for boosts where $\tau_I$ and $V_I$ are the tension and volume of the brane and $R$ is the size of the compact boost direction. The numbers $n_I$, in thermal equilibrium, are obtained by maximising the entropy $S$ with respect to $n_I$ keeping the energy $E$ fixed. It then follows that $n_I m_I = \frac{E}{2N}$ and, hence, that $S \simeq E^{\frac{N}{2}}$.

This relation is derived with the volume of the system kept fixed, see [7] and the references therein for details. As pointed out by a referee, the entropy $S$ and the energy $E$ defined in section 4 in an expanding universe will, in general, have a different relation since now the volume changes as dictated by the relevant equations of motion. However, note that the $N = 3, 4$ configurations considered here, when in a finite volume, describe black holes. The universe containing such configurations may therefore be thought of as containing black hole fluid in a $m+1$ – dimensional non compact spacetime. Such a fluid is known to obey the equation of state $p = \rho$ and the corresponding $S$ and $E$ defined in section 4 then obey the relation $S \simeq E^{\frac{m}{m-1}}$ [9]. Thus, the entropy $S$ is expected to be $\simeq E^3$ for the 3 charge case and $\simeq E^2$ for the 4 charge case. Hence, for these cases, the expected form turns out to be $S \simeq E^2$, the same as that obtained with volume kept fixed which, we reemphasise, is not the case in general. The 4 charge case may thus provide a detailed realisation of the maximum entropic principle proposed in [3].

Let the $N$ charge fractional branes be wrapped along the $n$ – compact directions and smeared uniformly in the transverse space. We assume that the transverse space is non compact; or that if it is compact then its size is sufficiently large so as to correspond to our observed universe. The $w_i$’s in the equation of state $p_i = w_i \rho$ for fractional branes may be obtained as in [7]. We assume that the evolution of the fractional branes in the universe is given

\[^{5}\text{For example, for radiation in a } m + 1 \text{ - dimensional spacetime, } p = \frac{\rho}{m} \text{ and, with volume kept fixed, } S \simeq E^{\frac{m}{m+1}}. \text{ However, in an isotropic expanding universe the volume changes and } S \text{ and } E \text{ defined in section 4 obey a different relation } S \simeq E^{\frac{m}{m(m-1)}}.\]
by the evolution of the anisotropic universe with the corresponding $w_i$’s; and that the physical quantities are similarly related. Hence, for example, the entropy of the fractional branes in the universe is assumed to be given by the entropy of the anisotropic universe with the corresponding $w_i$’s, which can be calculated using the results presented here. We now compare the entropy $S$ of the fractional branes in the universe, thus obtained, with its expected value $\sim E^{n/2}$.

We consider 3 and 4 charge configurations here and denote by, for example, 25B a 3 charge configuration consisting of two intersecting stacks of M2 and M5 branes and with a boost along the common direction; similarly, by 2255 a 4 charge configuration consisting of two stacks of M2 branes and two stacks of M5 branes intersecting. Note that a given $N$ charge configuration can be described equivalently in several ways, all related by a series of S, T, and U dualities. Thus, the 3 charge configurations 222 and 25B are equivalent; and, similarly, the 4 charge configurations 2255 and 555B.

The $w_i$’s for these configurations, calculated as in [7], turn out to be of the form $w_i = \sigma$ for the compact directions $i = 1, 2, \ldots, n$ and $w_i = 0$ for the transverse directions $i = n + 1, \ldots, D - 1$. Hence, $\alpha_1 = \alpha_2 = \cdots = \alpha_n$, $\alpha_{n+1} = \cdots = \alpha_{D-1}$, $\alpha_c = n\alpha_1$, $\alpha_\perp = m\alpha_{n+1}$, $\alpha = \alpha_c + \alpha_\perp$, and $\alpha_b = n\sigma\alpha_1$ where $n + m = D - 1$ with $D = 11$.

Using the formulas in [7], it follows that $(m, \sigma) = (4, -\frac{1}{3})$ for 3 charge configurations, $(m, \sigma) = (3, -\frac{1}{2})$ for 4 charge configurations, and $n = D - 1 - m$. It can be seen that $C_\Lambda > C_b$ for these configurations and, hence, $(\alpha_i, \alpha, \alpha_b)$ are given by equations (12). Using these equations one gets, for all these configurations,

$$\left(\alpha_1, \alpha_c, \alpha_{n+1}, \alpha, \alpha_b\right) = \left(0, 0, \frac{2}{m}, 2, 0\right).$$  (40)

Note that these configurations are a particular case of example (i), now with $\omega = 0$ and, accordingly, $\sigma = -\frac{1}{m-1}$, $\alpha_i = 0$ for $i = 1, 2, \ldots, n$ and $(\alpha_{n+1}, \alpha, \alpha_b)$ as given in equation (33). Also, note that either $\alpha_i = 0$ or

---

6The process of intersecting branes forming bound states, becoming fractional, and creating large entropy is a non local process extending over the brane volume, namely over the size of the compact directions $i = 1, 2, \ldots, n$ along which the branes wrap. In the asymptotic limit $t > t_c \gg 1$ considered here, there is sufficient time for this process to reach thermal equilibrium and, hence, the $w_i$’s for the anisotropic universe may be assumed to be those of the fractional branes obtained as in [7].
$w_i = 0$ for all directions and, hence, $\alpha_b = \sum w_i \alpha_i = 0$. It then follows from equation (27) that $X = 1$. Hence, the entropy $S$ for all of these 3 and 4 charge configurations is given by $S \simeq E$.

The entropy is not of the form expected from considerations given in the paragraph containing footnote 5, namely $\simeq E_3^2$ for 3 charge cases and $\simeq E^2$ for 4 charge cases. Given the importance of 4 charge fractional branes in understanding the early universe, it is important to study how to obtain the expected form for $S$. Hence, we now consider a few possible ways of achieving this.

(i) The entropy $S$ and the energy $E$ above are those contained in the volume within which causal contact is possible. It may be that the non local fractionation process extends not only over the brane volume, namely over the compact space, but over the entire transverse space also. Then it is the total entropy $S_{\text{tot}}$ and total energy $E_{\text{tot}}$ contained in the entire transverse space which should obey the expected relation.\footnote{This possibility is pointed out by a referee.} If the transverse space is non compact then $S_{\text{tot}}$ and $E_{\text{tot}}$ are infinite. Hence, consider a large but compact transverse space. Then $S_{\text{tot}}$ and $E_{\text{tot}}$ are given by $S$ and $E$ when $t > t_\perp$ and are constants for the present cases since $\alpha_b = 0$. However, as discussed in section 4 (ii), the relation between these constant quantities is likely to be $S_{\text{tot}} \simeq E_{\text{tot}}$ and not $S_{\text{tot}} \simeq E_{\text{tot}}^2$.

(ii) One may define a ‘holographic’ entropy $S_{\text{hol}}$ as the maximum possible entropy in the physical volume $V$. In the case of non compact transverse space, or for $t < t_\perp$ in the case of compact transverse space, $S_{\text{hol}} \simeq A$, the area bounding the volume $V$. From equations (25) and (26), it follows that $S_{\text{hol}}$ is given in terms of the energy $E$ as

$$S_{\text{hol}} \simeq E^{X_{\text{hol}}} , \quad X_{\text{hol}} = \frac{m + \alpha_c - 1}{m + \alpha_c - \alpha - \alpha_b} . \quad (41)$$

Note that for any $w_i$s for which $\alpha_c = 0$ and $\alpha + \alpha_b = 2$, we have $X_{\text{hol}} = X_1(m) = \frac{m-1}{m-2}$. Applied to 3 and 4 charge cases, one obtains the expected relation $S_{\text{hol}} \simeq E^{\frac{2}{N}}$. This relation is also likely to hold between the corresponding total $S_{\text{hol}}$ and $E_{\text{tot}}$, defined as in the possibility (i) above. Hence, may be it is the holographic entropy $S_{\text{hol}}$ which should obey the expected relation.
(iii) Another method of obtaining the expected relation may be the following. Let us view the 3 and 4 charge configurations as a particular case of example (i) with $\omega = 0$. The entropy $S \simeq E$ obtained for the brane configurations above are then a particular case of equation (34) with $\omega = 0$.

Seen as a particular case of example (i), it is now clear how to obtain the expected entropy for the fractional brane configurations: one assumes that $\sigma = \frac{m\omega - 1}{m - 1}$, as in example (i), and that $\omega \rightarrow 1$. Then the entropy $S$ is given by equation (34) with $\omega \rightarrow 1$, that is $S \simeq E^{X_1(m)}$ with $X_1(m) = \frac{m - 1}{m - 2}$.

Applied to 3 and 4 charge cases, one obtains the expected relation $S \simeq E^{\frac{m}{2}}$.

The possibility (iii) may be interpreted physically in the following way. In [7], the $w_i$’s are obtained assuming that the fractional brane and antibrane configurations, which wrap the compact directions, are not decaying or annihilating (or they do so very slowly and negligibly). Thus, the compact space has negative pressure, produced by the tension of the branes wrapping it, and the transverse space has no decay products. Then, as the calculations in [7] show, one gets $w_i = -\frac{1}{m - 1}$ for compact directions and $w_i = 0$ for transverse directions.

However, the branes and antibranes are expected to eventually decay and annihilate each other and emit radiation, massless scalars, etcetera as decay products. Thus, the compact directions will eventually be relieved of their negative pressure because the branes wrapping them decay and annihilate, and the space will be full of decay products. In particular, this implies that if the brane antibrane dynamics are fully taken into account then the $w_i$’s will be different from the ones used above. It is physically reasonable that massless scalars dominate the decay channels since they are generically present in string theory and are entropically favourable. It is then likely that $w_i = \omega \rightarrow 1$ for the transverse directions.

One also has to understand why $w_i = \sigma$ for the compact directions $i = 1, 2, \cdots, n$ and why $\sigma$ is related to $\omega$ as in the example (i). For this purpose, note that the 3 and 4 charge configurations of M theory considered here can be described equivalently by those of string theory ones, namely D1D5B and 3333 respectively, obtained by a series of S, T, and U dualities. This means that the physical quantities should be the same for these equivalent descriptions. Using the results of [7], which are applicable for these cases also, it is easy to show that $m, \sigma$, and $\alpha$’s remain the same as before but, for string theory, $n = 10 - m$ since $D = 10$.  

17
Considering 222 and 3333 configurations, it is clear that the compact space may be taken to be isotropic and, hence, \( w_i = \sigma \) for the compact directions. But, it is not clear why or how the relation \( \sigma = \frac{m\omega - 1}{m - 1} \) is enforced. This relation follows, as in the example (i), if one assumes that the sizes of the compact directions remain constant. But the physical reason for this assumption is not immediately clear.

Now, note that example (ii) is applicable here even if \( \sigma \neq \frac{m\omega - 1}{m - 1} \). From the results presented here, it is clear in particular that the exponent \( X \) that characterises the amount of entropy depends in general on \( n \), the dimension of the compact space. The only exception is the case of example (i).

Since the 3 and 4 charge configurations considered here can be described equivalently by M theory or by string theory, it follows that the physical quantities, in particular the entropy \( S \), should not depend on \( n \), the dimension of the compact space since the \( n = 10 - m \) in string theory and the \( n = 11 - m \) in M theory are different. This independence is possible if the fractional branes obey the relation \( \sigma = \frac{m\omega - 1}{m - 1} \) throughout their evolution in the universe, including their decay and annihilation. Since this follows from the requirements of S, T, U duality symmetries, it is likely that the relation between \( \sigma \) and \( \omega \) is enforced by these same symmetries.

The T duality symmetries of string theory also have another implication. In example (ii), \( \alpha_c < 0 \) if \( x \geq \frac{m}{m - 1} \). The compact space then contracts to zero size. For toroidal compactifications, and perhaps more generally also, the T duality symmetry ensures that the physical size of the compact space remains \( \gtrsim l_s \), the string length. This implies that T duality symmetry must also ensure that the physical \( w_i \)'s will be such that \( \alpha_i \geq 0 \) for the compact directions.

In the example (ii) then \( x \) must be \( \leq \frac{m}{m - 1} \) by T duality symmetry. Then \( \alpha_c \) remains \( \geq 0 \); hence, the exponent \( X \) which characterises the amount of entropy remains \( \leq X_1(m) = \frac{m - 1}{m - 2} \), in accord with one’s expectation that the most entropic object in an \( (m+1) \)–dimensional spacetime is a Schwarzschild black hole or an isotropic universe containing matter with \( \omega = 1 \).

7. Conclusion

We conclude by mentioning a few issues for further study. In this paper, we considered only two special sets of \( w_i \)'s which were sufficient for our
purposes here. It is desirable to consider a general set of $w_i$'s, in the range $-1 \leq w_i \leq 1$, and obtain the general conditions on $w_i$'s required to realise the expected entropy for the 3 and 4 charge configurations; the conditions required to ensure that their entropy, equivalently the exponent $X$ in equation (27), remains the same in the equivalent string and M theoretic descriptions; and also the conditions required to ensure that the compact sizes remain $\gtrsim l_s$ as required by T duality symmetry.

In this context, it may also be useful to study the implications of S, T, U duality symmetries at the level of equations of motion and their solutions.

As we have seen here, the expected entropy for the 3 and 4 charge fractional branes can be realised under some conditions which are likely to be enforced by S, T, U dualities of the string/M theory. Then the 4 charge configurations, having the highest entropy for a given energy, may indeed provide a detailed realisation of the maximum entropic principle proposed in [3] to determine the number $(3+1)$ of large spacetime dimensions. It is therefore important to study in detail the decay and annihilation processes in the 4 charge fractional brane configurations in the universe, and to understand the role of S, T, U duality symmetries of string/M theory in this setting.

It is also important to explore other ways of obtaining the expected form for entropy, besides those considered here.

Acknowledgement: We thank S. D. Mathur for a private correspondence and the referees for pointing out [6], for their comments, and for helpful suggestions which improved the paper.

References

[1] M. J. Bowick and L. C. R. Wijewardhana, “Superstring Gravity And The Early Universe,” Gen. Rel. Grav. 18 (1986) 59.

[2] S. Kalyana Rama, “A stringy correspondence principle in cosmology,” Phys. Lett. B 638 (2006) 100, arXiv: hep-th/0603216.

[3] S. Kalyana Rama, “A principle to determine the number (3+1) of large spacetime dimensions,” Phys. Lett. B 645 (2007) 365, arXiv: hep-th/0610071.
[4] J. Kripfganz and H. Perlt, “Cosmological Impact Of Winding Strings,” Class. Quant. Grav. 5 (1988) 453;
R. H. Brandenberger and C. Vafa, “Superstrings in the Early Universe,” Nucl. Phys. B 316 (1989) 391;
A. A. Tseytlin and C. Vafa, “Elements Of String Cosmology,” Nucl. Phys. B 372 (1992) 443, arXiv: hep-th/9109048;
R. Durrer, M. Kunz and M. Sakellariadou, “Why do we live in 3+1 dimensions?,” Phys. Lett. B 614 (2005) 125, arXiv: hep-th/0501163;
A. Karch and L. Randall, “Relaxing to three dimensions,” Phys. Rev. Lett. 95 (2005) 161601, arXiv: hep-th/0506053.

[5] There is an extensive literature on the subject of string/brane gas cosmology, only a few of which we mention in here. See, for example,
M. Sakellariadou, “Numerical Experiments in String Cosmology,” Nucl. Phys. B 468 (1996) 319, arXiv: hep-th/9511075;
S. Alexander, R. H. Brandenberger and D. Easson, “Brane gases in the early universe,” Phys. Rev. D 62 (2000) 103509, arXiv: hep-th/0005212;
R. Brandenberger, D. A. Easson and D. Kimberly, “Loitering phase in brane gas cosmology,” Nucl. Phys. B 623 (2002) 421, arXiv: hep-th/0109165;
D. A. Easson, “Brane gas cosmology and loitering,” arXiv: hep-th/0111055;
R. Easther, B. R. Greene, M. G. Jackson and D. Kabat, “Brane gas cosmology in M-theory: Late time behavior,” Phys. Rev. D 67 (2003) 123501, arXiv: hep-th/0211124;
R. Easther, B. R. Greene, M. G. Jackson and D. Kabat, “Brane gases in the early universe: Thermodynamics and cosmology,” JCAP 01 (2004) 006, arXiv: hep-th/0307233;
R. Easther, B. R. Greene, M. G. Jackson and D. Kabat, “String windings in the early universe,” JCAP 02 (2005) 009, arXiv: hep-th/0409121;
R. Danos, A. R. Frey and A. Mazumdar, “Interaction rates in string gas cosmology,” Phys. Rev. D 70 (2004) 106010, arXiv: hep-th/0409162;
R. H. Brandenberger, “Challenges for string gas cosmology,” arXiv: hep-th/0509099;
R. H. Brandenberger, “Moduli stabilization in string gas cosmology,”
[6] S. Watson and R. H. Brandenberger, “Isotropization in brane gas cosmology,” Phys. Rev. D 67 (2003) 043510, arXiv: hep-th/0207168; M. Berkooz, B. Pioline and M. Rozali, “Closed strings in Misner space,” JCAP 08 (2004) 004, arXiv: hep-th/0405126.

[7] B. D. Chowdhury and S. D. Mathur, “Fractional brane state in the early universe, arXiv: hep-th/0611330.

[8] W. Fischler and L. Susskind, “Holography and cosmology,” arXiv: hep-th/9806039; R. Bousso, “A Covariant Entropy Conjecture,” JHEP 07 (1999) 004, arXiv: hep-th/9905177; R. Bousso, “Holography in general space-times,” JHEP 06 (1999) 028, arXiv: hep-th/9906022; R. Bousso, “The holographic principle,” Rev. Mod. Phys. 74 (2002) 825, arXiv: hep-th/0203101.

[9] See, for example, T. Banks and W. Fischler, “An holographic cosmology,” arXiv: hep-th/0111142; T. Banks and W. Fischler, “Holographic cosmology 3.0,” Phys. Scripta T117 (2005) 56 arXiv: hep-th/0310288; T. Banks, W. Fischler and L. Mannelli, “Microscopic quantum mechanics of the p = rho universe,” Phys. Rev. D 71 (2005) 123514 arXiv: hep-th/0408076.