Effective field theories of non-equilibrium physics

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Non equilibrium effective field theory is presented as an inhomogeneous field theory, using a formulation which is analogous to that of a gauge theory. This formulation underlines the importance of structural aspects of non-equilibrium, effective field theories. It is shown that, unless proper attention is paid to such structural features, hugely different answers can be obtained for a given model. The exactly soluble two-level atom is used as an example of both the covariant methodology and of the conclusions.

I. SCALES AND INTERACTIONS

Our conventional wisdom about isolated systems is that processes which dominate at microscopic scales have no detailed effect on the behaviour of a physical system at much larger scales. This linear viewpoint is the basis of effective field theory. Put another way it says that, when the ratio of microscopic to macroscopic scales is very marked, one is justified in making a continuum approximation and blurring out the small details of the system.

This rule of thumb is not universal. A prolific exception, to name a single example, is the evolution of life. Biologically, very small changes at the molecular level (genes/proteins) induce large and very dramatic changes in the way a system develops macroscopically (phenotype). What starts as a few proteins bumping into one another, ends up as a person reflecting on the nature of the development at a physics conference. The transition is a dramatic one and clearly cannot be described by any simple field theory because it involves complex interactions with time varying boundary conditions coupling a whole hierarchy of scales. In contrast, when you look at a grown organism, you do not see the effects of every protein movement in the body: so there a continuum model seems to be a good idea. The difference is that there are no longer major structural changes taking place. So in equilibrium things are well described by a continuum hypothesis, but during development (far from equilibrium) that it not necessarily true.

What occurred in between was due to complicated time-dependent boundary conditions which caused processes from a hierarchy of scales to interact strongly. While each arbitrary part of a system follows microscopic laws in every detail, the totality of a complex system exceeds the sum of its parts because it involves structural information about how to put those parts together, i.e. the boundary conditions between neighbouring elements in a system. Cooperation and competition between neighbouring cells introduces huge complications to model builders and—even in the simplest case—we probably need an evolving effective theory, if not several, to describe even the simplest non-equilibrium development in a reasonable fashion.

This paper is about basic structural aspect of field theories with spacetime dependent external perturbations. The perturbations will be treated generically and therefore they can be thought of as effective interactions; they may be justified by any number of arguments: as the generators of resummations in interacting field theories, as renormalization counterterms, or as external influences whose form is dictated by coordinate invariance and unitarity. By using a generic Schwinger source theory, all of these possibilities are covered in a single abstraction. By taking a quadratic action, one considers the simplest idealization of a system interacting with its surroundings. Despite being the prototype for basically all interacting systems to low perturbative order, even this simple problem is difficult and not very well understood. I would like to illustrate how changes in the basic structural elements of non-equilibrium field theory can have huge consequences for the way in which a theory behaves and leave this both as an interesting direction for future investigation and as a ‘warning as to the unwar’ as to the kind of pitfalls which one might encounter in model building away from equilibrium.

II. SUMMARY

This paper contains three overlapping messages.

• That a proper understanding of non-equilibrium development involves all of the subtleties and structure which gauge theories introduce.

• That interesting similarities exist between the classical two level atom and Schwinger’s closed time path (CTP) formalism. Broadly speaking the upper and lower levels of the two-level system correspond to creation and annihilation in regular field theory.

• That small changes in theoretical structure can lead to large changes in the behaviour of solutions.

The latter two points are explored using an exactly soluble model of the two-level atom interacting with a strong radiation source. This talk is a summary of more detailed work contained in two papers [12]. The conventions are detailed in reference [1].
III. INHOMOGENEITY

The purpose of this section is to motivate a method for the analysis of non-equilibrium field theory, drawing structural features from gauge theories. The action is presented in generic form, in terms of general bi-local sources which represent the interactions which a system may have at quadratic order.

It might strike one as surprising that gauge theory ideas would crop up in the study of arbitrary non-equilibrium systems, but the reason is clear. Disequilibrium in a physical system is associated with some kind of inhomogeneity: either in space or in time, usually both. Uneven perturbations and variations in physical quantities lead to transport and relaxation. It is thus useful to think of field theory away from equilibrium as an inhomogeneous field theory. Gauge theory is about inhomogeneous, space-time dependent phases, or complex inhomogeneous field theory. Inhomogeneity crops up repeatedly in nonequilibrium development. The currents associated with sources $I, J, K^\mu$ are not necessarily conserved since their behaviour is not completely specified by the action, but the action is differentially reversible. The sum of rows and columns in this operator is zero, as required for unitarity and subsequent causality. The significance of the off-diagonal terms involving $K^\mu$ can be seen by writing out the coupling fully:

$$K^\mu(x, x') \cdot (\phi_1 D^K_{\mu} \phi_2 - \phi_2 D^K_{\mu} \phi_1).$$

The term in parentheses has the form of a current between components $\phi_1$ (the forward moving field) and $\phi_2$ (the backward moving field). When these two are in equilibrium there will be no dissipation to the external reservoir and these off-diagonal terms will vanish. This indicates that these off-diagonal components (which are related to off-diagonal density matrix elements, as noted earlier) can be understood as the mediators of a detailed balance condition for the field. When the term is nonvanishing, it represents a current flowing in one particular direction, pointing out the arrow of time for either positive or negative frequencies. The current is a ‘canonically conjugate’ and is clearly related to the fundamental commutator for the scalar field in the limit $+ \to -$.

The system can be analyzed by looking for the Green functions associated with this system. These can all be expressed in terms of the Wightman functions $G^{(\pm)}(x, x')$ using the relations

$$G^{(+))(x, x')} = \left[ G^{(-)}(x, x') \right]^* = -G^{(-)}(x', x).$$

The Green functions are the sum of all positive or negative energy solutions, satisfying the closed time path field equations, found by varying the action above. Thus they are the embodiment of the dispersion relation" between $k$ and $\omega = k^0$:

$$\tilde{G}(x, x') = G^{(+)}(x, x') + G^{(-)}(x, x')$$
$$\overline{G}(x, x') = G^{(+)')(x, x') - G^{(-)(x, x').}$$
The latter is the classical dynamical equivalent of the fundamental commutation relations in the quantum theory of fields. Other Green functions may be constructed from these to model the processes of emission, absorption and fluctuation respectively:

\[ G_e(x, x') = -\theta(t, t') \tilde{G}(x, x') \]
\[ G_a(x, x') = \theta(t', t) \tilde{G}(x, x') \]
\[ G_F(x, x') = -\theta(t, t') G^{(+)}(x, x') + \theta(t', t) G^{(-)}(x, x'). \]  

(11)

It may be verified that, since \( G^{(+)}(k, \overline{r}) \) depends only on the average coordinate, the commutation relations are preserved (see equation (10)) even with a time-dependent action. The general solution for the positive frequency Wightman function may be written

\[ G^{(+)}(x, x') = -2\pi i \int \frac{d^{n-1}k}{(2\pi)^{n-1}} e^{2ik_{\mu}x^\mu} \frac{1 + f(k_0, \overline{r})}{2|\omega|} \]

(12)

where \( f(k_0, \overline{r}) \) is an unspecified function of its arguments and it is understood that \( k_0 = |\omega| \) (this describes the dispersion of the plane wave basis). The ratio of Wightman functions describes the ratio of emission and absorption of a coupled reservoir (the sources). As observed by Schwinger [10], all fluctuations may be thought of as arising from generalized sources via the Green functions of the system. Thus a source theory is an effective description of an arbitrary statistical system.

In an isolated system in thermal equilibrium, we expect the number of fluctuations excited from the heat bath to be distributed according to a Boltzmann probability factor [13].

\[ \frac{\text{Emission}}{\text{Absorption}} = \frac{-G^{(+)}(\omega)}{G^{(-)}(\omega)} = e^{\hbar \beta |\omega|}. \]

(13)

\( \hbar \omega \) is the energy of the mode with frequency \( \omega \). This is a classical understanding of the well-known Kubo-Martin-Schwinger relation [11, 12] from quantum field theory. In the usual derivation, one makes use of the quantum mechanical time-evolution operator and the cyclic property of the trace to derive this relation for a thermal equilibrium. The argument given here is identical to Einstein’s argument for stimulated and spontaneous emission in a statistical two state system, and the derivation of the well-known A and B coefficients. It can be interpreted as the relative occupation numbers of particles with energy \( \hbar \omega \). This is a first hint that there might be a connection between heat-bath physics and the two level system.

Finally, it is useful to define quantities of the form

\[ F_\mu = \frac{\partial_\mu f}{f} = \frac{1}{2} \frac{\partial_\mu}{\omega} \ln(f) \]
\[ \Omega_\mu = \frac{\mu_\mu}{\omega} = \frac{1}{2} \frac{\partial_\mu}{\omega} \ln|\omega(\overline{r})|. \]

(14)

(15)

which occur repeatedly in the field equations and dispersion relations for the system and characterize the average rate of development of the system. Note the similarity in form to the connection term in the derivative of eqn. (8).

IV. INHOMOGENEOUS SCALING AND GAUGE FORMULATION

General covariance in an inhomogeneous environment requires one to acknowledge the existence of a connection which transforms, by analogy with a gauge theory.

Inhomogeneous field theory can be presented in a natural form by introducing a ‘covariant derivative’ \( D_\mu \) which commutes with the average development of the field and is physical in the sense of being Hermitian in the presence of the sources. This description parallels the structure of a gauge theory (in momentum space) with a complex charge. One may also speak of a generalized chemical potential or of a special case of quantum field theory in curved spacetime (see the local momentum space expansion approach of ref. [8] as well as the paper by A.G. Nicola in these proceedings). Consider the derivative

\[ D_\mu = \partial_\mu - a_\mu \]

(16)

and its square

\[ D^2 = \Box - \partial_\mu a_\mu - 2a_\mu \partial_\mu + a_\mu a_\mu. \]

(17)

Derivatives occur in the field equations and in the dispersion relation for the field and they are thus central to the dynamics of the field and the response (Green) functions. As with a gauge theory, the effect of derivatives on spacetime dependent factors may be accounted for in a number of equivalent ways, by redefinitions of the field. In a gauge theory, we call this a gauge transformation and we usually demand that the theory be covariant, if not invariant under such transformations. In a non-equilibrium field theory, we require only covariance, since it is normal to deal with partial systems in which conserved currents are not completely visible and thus invariance need not be manifest.
In order to solve the closed time path field equations, it is useful to solve the dispersion relation, giving the physical spectrum of quasi-particles in the system. In the Keldysh diagrammatic expansion of Schwinger’s closed time path generating functional, one expands around free particle solutions. By starting with a quasi-particle basis here we can immediately take advantage of resummations and renormalizations which follow from the unitary structure (specifically two-particle irreducible or daisy diagrams). It also allows one to track changes in the statistical distribution through the complex dispersion relation, instead of using real Vlasov equations coupled to real equations of motion. Without any approximation, it is straightforward to show that, in the general inhomogeneous case,

\[
(-\Box + m^2)G^{(+)}(x, x') = -2\pi i \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \frac{(1 + f)}{2|\omega|} e^{ik(x-x')}
\]

\[
\left[ -(ik_\mu + F_\mu - \Omega_\mu)^2 - \partial^\mu (ik_\mu + F_\mu - \Omega_\mu) \right] = 0. \quad (18)
\]

It is then natural to make the identification

\[
a_\mu = F_\mu - \Omega_\mu + \overline{K}_\mu \\
= -\partial_\mu S_E(k) + \overline{K}_\mu. \quad (19)
\]

This expression shows that the connection embodies the effect of changing statistical distributions and quasi-particle energies (ω is solved in terms of the sources through the dispersion relation). It also shows that the source Kμ plays essentially the same role as these effects and it thus capable of ‘resumming’ them. Furthermore, the field a_μ is related to the rate of increase of the entropy S_E. In terms of the covariant derivative, one now has:

\[
(D^2 + m^2)G^{(+)}(x, x') = -2\pi i \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \frac{(1 + f)}{2|\omega|} \\
\left[ -(ik_\mu - \overline{K}_\mu)^2 \right] - \partial^\mu (ik_\mu - \overline{K}_\mu). \quad (20)
\]

The differential equation satisfied by G^{(+)}(x, x') is thus

\[
\left[ -D^2 + m^2 + \overline{K}^2(k, \overline{\tau}) + T(k, \overline{\tau}) - \overline{J}(k) \right. \\
\left. + i\frac{1}{2} (\overline{\partial}_\mu \overline{J}) (T^\mu - v^\mu_\rho / \omega) \right] G^{(+)}(k, \overline{\tau}) = 0 \quad (21)
\]

where the appearance of the subscript k to the bracket serves to remind that the equation exists under the momentum integral. The positive frequency field may be ‘gauge transformed’ using the integrating factor (Wilson line)

\[
\phi(k) \rightarrow \phi(k)e^{\int a_\mu x^\mu} \quad (22)
\]

This shows the explicit decay (amplification) of the k-th field mode. This transformation also has a nice physical interpretation in terms of the entropy of the modes, defined above. The Wilson line is the negative exponential of the entropy, showing how the field decays as the energy of a mode becomes unavailable for doing work, i.e. as its entropy rises.

One should not confuse these transformations with similarity transformations on the closed time path action. Because of unitarity, the plus and minus components of closed time path fields satisfy a global O(1,1) symmetry, which allows a certain freedom in the way one chooses to set up the solution of the system. One can choose, for instance, to work with Feynman Green functions and Wightman functions, or with advanced and retarded functions, or with general mixtures of these. The only constraint imposed by unitarity is that the sum of rows and columns in the action (i.e. in the argument of the exponential in the generating functional) remains zero when plus and minus labels are removed. The transformations considered here are simply field rescalings. This need not even be a symmetry of the system. Symmetries do not have a monopoly on covariance, indeed covariance is especially important for changes which do not leave the system invariant. All reparameterizations of a theory, be they gauge transformations or field redefinitions demand this. The reason for the similarity in form between a gauge theory and a theory of field rescalings is that both are linked through the conformal group. This also makes the connection to the curved spacetime approach already referred to. General spacetime metrics are not of interest, but conformal rescalings are. Gauge theories bear the structure of the conformal group, not the Lorentz group and time dependent perturbations and changes of variable are also connected with inhomogeneous rescalings of the conformal group. One will not normally see a conformal symmetry in the original action because the effective field theories we are discussing are incomplete: they describe partial systems, in which we ignore heat baths and external influences etc. One can, of course, argue that there are reasons to generalize descriptions of non-equilibrium field theory such that they are fully covariant with respect to such transformations. That is a central observation of this paper.

It is, of course, natural to suppose that changes of variable, i.e. changes of perspective will lead to transformations analogous to gauge transformations. After all, we are perturbing a system with sources which vary in space and time. Methods of solution which rely on diagonalization of the action will also involve transformations which depend on space and time. All such transformations demand covariant derivatives and transforming auxiliary fields. The amplification of modes makes this a recipe for a kind of space-time dependent renormalization group. Some authors have suggested making coupling constants run with time, but there are canonical restrictions associated with making coupling constants depend on space-time coordinates [12]. The idea of completion by general covariance is also reminiscent of the Vilkovisky-DeWitt effective action [13]; perhaps this also has an interpretation in non-equilibrium physics.

An important question associated with this new gauge-
like formulation is therefore: should we introduce ‘gauge’ fields from the start? If so, what initial value should they have? To demonstrate the importance of this issue, as well as to provide an example of the approach, I would now like to turn to a directly analogous problem which is closely related to hard experimental data for the maser.

V. TWO STATE SYSTEM

The purpose of this section is to explore the meaning of the ‘connection’ in the context of an exactly soluble model, and to show that its presence has intriguing effects on the nature of the theory. The suggestion is that a more careful understanding of the role of this connection and its conformal structure could lead to improved calculational schemes in more general cases.

The two level atoms is not, in itself, important to the message of this paper. Any exactly soluble model would suffice, though such benisons are hard to come by. It does have several advantages however. In particular, the two level structure has a strikingly similar structure to the plus, minus labels of the closed time path generating two level structure has a strikingly similar structure to the diagonalization procedure requires a time-dependent unitary transformation and thus general covariance demands that this will transform in a different basis. The physics of the model depends on the initial value of this ‘connection’ and this is the key to the trivial solubility of the Jaynes-Cummings model.

In matrix form we may write the action for the matter fields

$$S = \int dV \bar{\psi}_A \mathcal{O}_{AB} \psi_B$$

where

$$\mathcal{O} = \begin{bmatrix} -\frac{\hbar^2 \nabla^2}{2m} - V_1 - i\hbar D_t & \bar{J}(t) - i\Gamma_{12} \\ \bar{J}(t) + i\Gamma_{12} & \frac{\hbar^2 \nabla^2}{2m} - V_2 - \frac{i\hbar}{2} \bar{D}_t \end{bmatrix}$$. (27)

The level potentials may be regarded as constants in the effective theory. They are given by

$$V_1 = E_1$$
$$V_2 = E_2 - \hbar \Omega_R$$

where $\hbar \Omega_R$ is the interaction energy imparted by the photon during the transition i.e. the continuous radiation pressure on the atom. In the effective theory we must add this by hand since we have separated the levels into independent fields which are electrically neutral, but it would follow automatically in a truly microscopic theory in which all electromagnetic forces were included. The quantum content of this model is now that this recoil energy during a transition is a quantized unit of $\hbar \Omega$, the energy of a photon at the frequency of the source. Also the amplitude of the source $J$ would be quantized and proportional to the number of photons on the field.

To solve the problem posed in eqn. (24), it is desirable to perform a unitary transformation on the action

$$\psi \rightarrow U \psi$$
$$\mathcal{O} \rightarrow U \mathcal{O} U^{-1}$$

which diagonalizes the operator $\mathcal{O}$. The connection $\Gamma$ transforms under this procedure by

$$\frac{\delta S}{\delta \Gamma_{AB}} = \frac{i}{2} (\psi^*_A \psi_B - \psi^*_B \psi_A)$$ (24)

which represents the amplitude for stimulated transition between the levels. The current generated by this connection is conserved only on average, since we are not taking into account any back-reaction. The conservation law corresponds merely to

$$\partial_t \left( \frac{\delta S}{\delta \Gamma_{AB}} \right) \propto \sin(2 \int X(t))$$ (25)

where $X(t)$ will be defined later. The potential $V(t)$ is time dependent and comprises the effect of the level splitting as well as a perturbation mediated by the radiation field. The ‘connection’ $\Gamma_{21} = -\Gamma_{12}$ is introduced since the diagonalization procedure requires a time-dependent unitary transformation and thus general covariance demands that this will transform in a different basis. The physics of the model depends on the initial value of this ‘connection’ and this is the key to the trivial solubility of the Jaynes-Cummings model.
\( \Gamma \to \Gamma + \frac{i\hbar}{2} \left( U(\partial_t U^{-1}) - (\partial_t U)U^{-1} \right) \) \hspace{1cm} (31)

since this requires a time-dependent transformation. For notational simplicity we define \( \hat{L} = -\frac{\hbar^2 \nabla^2}{2m} - \frac{i\hbar}{2} \hat{D}_t \), so that the secular equation for the action is:

\[
(\hat{L} - E_1 - \lambda(t))(\hat{L} - E_2 + \hbar\Omega - \lambda(t)) - (J^2(t) + \Gamma_{12}^2) = 0. \tag{32}
\]

Note that since \( J \rightarrow \hat{J} \), \( J = 0 \) there are no operator difficulties with this equation. The eigenvalues are thus

\[
\lambda_{\pm} = \hat{L} - \mathbf{E}_{12} + \hbar\Omega \pm \sqrt{\frac{1}{4}(\mathbf{E}_{21} - \hbar\Omega)^2 + J^2(t) + \Gamma_{12}^2} \tag{33}
\]

\[
\equiv \hat{L} - \mathbf{E}_{12} + \hbar\Omega \pm \sqrt{\hbar^2 \omega^2 + J^2(t) + \Gamma_{12}^2} \tag{34}
\]

\[
\equiv \hat{L} - \mathbf{E}_{12} + \hbar\Omega \pm \hbar\omega_R, \tag{35}
\]

where \( \mathbf{E}_{12} = \frac{1}{2}(E_1 + E_2) \) and \( \mathbf{E}_{21} = (E_2 - E_1) \). For notational simplicity we define \( \hat{\omega} \) and \( \omega_R \) by this relation. One may now confirm this procedure by looking for the explicit eigenvectors and constructing \( U^{-1} \) as the matrix of these eigenvectors. This is done in ref. [2] and takes the form

\[
U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \tag{36}
\]

In the diagonal basis, the centre of mass motion of the neutral atoms factorizes from the wave-function, since a neutral atom in an electromagnetic field is free on average. The two equations in the matrix above are unravelled by writing the ‘gauge transformation’

\[
\psi_{\pm}(x) = e^{\pm i \int_0^t X(t') dt'} \overline{\psi}(x), \tag{37}
\]

where \( X(t) \) is presently undetermined and the wave-function for the centre of mass motion in \( n = 3 \) dimensions,

\[
\overline{\psi}(x) = \int \frac{d\omega}{2\pi} \frac{d^n \mathbf{k}}{(2\pi)^n} e^{i(k \cdot x - \omega t)} \delta(\chi) \tag{38}
\]

is a general linear combination of plane waves satisfying the dispersion relation for centre of mass motion

\[
\chi = \frac{\hbar^2 \mathbf{k}^2}{2m} + \hbar(\Omega - \omega) - \mathbf{E}_{12} = 0. \tag{39}
\]

Substituting this form, we identify \( X(t) \) as the integrating factor for the uncoupled differential equations. The complete solution is therefore

\[
\psi_{\pm}(x) = e^{\mp i \int_0^t (\omega_R + i\hbar \theta) dt'} \overline{\psi}(x). \tag{40}
\]

Notice that this result is an exact solution in the sense of being in a closed form. In the language of a gauge theory this result is gauge dependent. This is because our original theory was not invariant under time dependent transformations. The covariant procedure we have applied is simply a method to transform the equations into an appealing form; it does not imply invariance of the results under a wide class of sources. On the other hand, it might be argued that the invariant extension of eqn. (23) should be considered.

The form of the action in eqn (23) seems arbitrary and unrelated to the earlier discussion but it may be placed in the context of ref. [6] by an understanding of the conformal nature of the perturbation. This includes an understanding of the value of the connection \( \Gamma_{12} \). The action is perturbed by a time-dependent source which one hopes would lead to a stable theory (the form of the action remaining the same over time). This suggests precisely a measure of conformal covariance. In order to understand this conformal connection in a familiar language, it is advantageous to write the above model as the limit of a pseudo-relativistic theory since the conformal group is an extension of the Lorentz group. This also makes direct contact with ref. [1]. The consistency of such an approach has been verified in ref. [14]. Beginning with the Lorentz covariant action

\[
S = \int dV \left\{ \frac{1}{2} (\partial_{\mu} \phi_A)(\partial^\mu \phi_A) + \frac{1}{2} m_A \phi_A^2 + J_{AB}(t) \phi_A \phi_B \right\}, \tag{41}
\]

where \( J_{AB} \) now stands in place of \( V_{AB} \) in the non-relativistic formulation, we consider a conformal rescaling by letting \( g_{\mu\nu} \rightarrow \Omega^2 \bar{g}_{\mu\nu} \). The action is not invariant under this rescaling; if it were, there would be no need for the connection \( \Gamma \), or indeed this paper. The volume element scales as the square root of the determinant of the metric, i.e. \( \sqrt{g} \rightarrow \Omega^2 \sqrt{\bar{g}} \) in 3 + 1 dimensions, but we shall keep this separate for now. Since the issue is not invariance but equivalence, this will not play a crucial role. The first term in the braces contains one inverse power of the metric, the second none and the third two. Choosing \( \Omega^2 = J_{AB} \), the off-diagonal, symmetric matrix with non-zero elements \( J(t) \), one can absorb the time dependent interaction by performing a generalized rescaling. Rescaling the fields by \( \phi \rightarrow \Omega \phi \), the action takes the form

\[
S = \int dV' \left\{ \frac{1}{2} (D_{\mu} \phi_A)(D^\mu \phi_A) + \frac{1}{2} m_A \phi_A^2 - K_{AB}'(D_{\mu} \phi_A) \right\} \tag{42}
\]

where \( D^\mu = \partial^\mu \delta_{AB} + K_{AB}' \), which is obtained by moving the scale factors through the derivatives, and

\[
\frac{1}{2} \frac{\partial_t J(t)}{J(t)} = \left( \frac{\partial_t \Omega}{\Omega} \right)_{AB} = K_{AB}. \tag{43}
\]

\( K \) is now analogous to \( \Gamma \). The familiar form of the conformal correction \( \partial_{\mu} \Omega/\Omega \) is replaced in eqn. (23) simply by \( K_{12} = \partial_t \Omega \), which makes the initial value of \( K_{12} \sim \partial_t J \).
clear: it is the connection required for the derivatives to commute with a conformal rescaling brought about by the perturbation.

The non-relativistic limit of the transformed action with a time-only dependent $\Omega(t)$ leads to eqn. (23), up to a Jacobian. Thus although these actions are not identical, they are related by an overall spacetime-dependent scale factor which behaves as though to view the system through a distorting glass, exactly analogous to very similar analysis of an effective non-equilibrium system in ref. [13]. This is the price one pays for considering partial systems. The reason why these two theories give essentially the same results is that they have the same structural elements. As indicated in the introduction, it is this feature of effective field theories which makes them robust and usable.

The field solutions to the two-level atom in the Jaynes-Cummings approximation, near resonance, are known to exhibit so-called Rabi oscillations, where the major populations oscillate between the upper and lower levels. The oscillations in the general system are dramatically different away from resonance, when one uses parameter values which are appropriate for the micro-maser. The outcome of this analysis is perhaps surprising: the result which one obtains by making the rotating wave approximation in a quantum mechanical formalism (the Jaynes-Cummings model), coincidentally corresponding to a connection $K_{12} \sim \sin(\Omega t)$, and this is a stable, steady state effective field theory. The same action solved fully without approximation $K_{12}$ corresponds to $K_{12} \sim 0$ and is not stable to re-scalings; indeed it leads to extremely complicated behaviour. That the stable model is identifiable with the Jaynes-Cummings model is due as much to the method of analysis in time dependent quantum mechanics as it is to do with the rotating wave approximation. The post Wilson renormalization group philosophy would tend to favour a stable theory and say that another theory with $K_{12} = 0$ was not usefully predictable. Here we have an example where the model can be solved, and indeed it does not appear to be give any comparable pattern of behaviour to the Jaynes-Cummings model at experimental values typical for the micro-maser, off resonance [16], although it is clearly deterministic and simple in structure.

VI. CONCLUSION

The central message of this paper is that correct results in non-equilibrium field theory are extremely sensitive to initial the conditions and to the covariant structure of the action. A gauge-like formulation might allow us to solve inhomogeneous systems more easily, but it also provides a conceptual perspective absent from the usual diagrammatic approaches, showing us that approximation methods can unwittingly change the effective structural elements of a theory and produce completely different answers. We make such approximations all the time in interacting field theory. Since the non-local sources (which include the connection $a_\mu$) implement ‘resummations’ [14] in interacting theories, this must be of central importance in QCD and self-interacting models.

This work is supported by NATO collaborative research grant CRG950018. I am grateful to Meg Carrying and Gabor Kunstatter for helpful discussions. Thanks also to Cliff Burgess for comments.

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