7. DEVELOPMENTAL PARADIGMS *

7.1 Introduction.

Consider the real line. If you believe that time is the ordinary continuum, then the entire real line can be your time line. Otherwise, you may consider only a subset of the real line as a time line. In the original version of this section, the time concept for the MA-model was presented in a unnecessarily complex form. As shown in [3], one can assume an absolute substratum time within the NSP-world. It is the infinitesimal light-clock time measures that may be altered by physical processes. In my view, the theory of quantum electrodynamics would not exist without such a NSP-world time concept.

Consider a small interval \([a, b)\), \(a < b\) as our basic time interval where as the real numbers increase the time is intuitively considered to be increasing. In the following approach, one may apply the concept of the persistence of mental version relative to descriptions for the behavior of a Natural (i.e. physical) system at a moment of time within this interval. An exceptionally small subinterval can be chosen within \([a, b)\) as a maximum subinterval length = \(M\). “Time” and the size of a “time” interval as they are used in this and the following sections refer to an intuitive concept used to aid in comprehending the notation of an event sequence. [See below.] First, let \(a = t_0\). Then choose \(t_1\) such that \(a < t_1 < b\). There is a partition \(t_1, \ldots, t_m\) of \([a, b)\) such that \(t_0 < t_1 < \cdots < t_m < b\) and \(t_{j+1} - t_j \leq M\). The final subinterval \([t_m, b)\) is now separated, by induction, say be taking midpoints, into an increasing sequence of times \(\{t_q\}\) such that \(t_m < t_q < b\) for each \(q\) and \(\lim_{q \to \infty} t_q = b\).

Assume the prototype \([a, b)\) with the time subintervals as defined above. Let \([t_j, t_{j+1})\) be any of the time subintervals in \([a, b)\). For each such subinterval, let \(W_i\) denote the readable sentence

- This||frozen||segment||gives||a||description||for||the||time||interval||that||has||as||its||leftmost||endpoint||the||time||\(\lceil t_i \rceil\)||that||corresponds||to||the||natural||number||\(i\).

Let \(T_i = \{xW_i \mid x \in W\}\). The set \(T_i\) is called a totality and each member of any such \(T_i\) is called a frozen segment. Notice that since the empty word is not a member of \(W\), then the cardinality of each member of \(T_i\) is greater than that of \(W_i\). Each \(T_i\) is a (Dedekind) denumerable set, and if \(i \neq j\), then \(T_i \cap T_j = \emptyset\).

It is obvious that the concept of “time” need not be the underlying interpretation for these intervals. Time simply refers to an external event ordering concept. For other purposes, simply call these intervals “event intervals.” In the above descriptions for \(W_i\), simply replace “time||interval” with “event||interval” and replace the second instance of the word “time” with the word “event.” If this interpretation is made, then other compatible interpretations would be necessary when applying a few of the following results.

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I point out two minor aspects of the above constructions. First, within certain descriptions there are often "symbols" used for real, complex, natural numbers etc. These objects also exist as abstract objects within the structure $\mathcal{M}$. No inconsistent interpretations should occur when these objects are specifically modeled within $\mathcal{M}$ since to my knowledge all of the usual mathematical objects used within physical analysis are disjoint from $\mathcal{E}$ as well as disjoint from any finite Cartesian product of $\mathcal{E}$ with itself. If for future research within physical applications finite partial sequences of natural numbers and the finite equivalence classes that appear in $\mathcal{E}$ are needed and are combined into one model for different purposes than the study of descriptions, then certain modifications would need to be made so that interpretations would remain consistent. Secondly, I have tried whenever intuitive strings are used or sets of such strings are defined to use Roman letter notation for such objects. This only applies for the intuitive model. Also $W_i$ is only an identifier and may be altered.

7.2 Developmental Paradigms

It is clear that if one considers a time interval of the type $(-\infty, +\infty)$, $(-\infty, b)$ or $[a, +\infty)$, then each of these may be considered as the union of a denumerable collection of time intervals of the type $[a, b)$ with common endpoint names displayed. Further, although $[a, b)$ is to be considered as subdivided into denumerably many subintervals, it is not necessary that each of the time intervals $[t_j, t_{j+1}) \subset [a, b)$ be accorded a corresponding description for the appearance of a specific Natural system that is distinct from all others that occur throughout the time subinterval. Repeated descriptions only containing a different last natural number $i$ in the next to last position will suffice. Each basic developmental paradigm will be restricted, at present, to such a time interval $[a, b)$.

Where human perception and descriptive ability is concerned, the least controversial approach would be to consider only finitely many descriptive choices as appropriate. A finite set is recursive and such a choice, since the result is such a set, would be considered to be the simplest type of algorithm. You "simply" check to see if an expression is a member of such a finite set. If we limited ourselves to finitely many human choices for Natural system descriptions from the set of all totalities and did not allow a denumerable or a continuum set to be chosen, then the next result establishes that within the Nonstandard Physical world (i.e. NSP-world) such a finite-type of choice can be applied and a continuum of descriptions obtained.

The following theorem is not insignificant even if we are willing to accept a denumerable set of distinct descriptions — descriptions that are not only distinct in the next to the last symbol, but are also distinctly different in other aspects as well. For, if this is the case, the results of Theorem 7.2.1 still apply. The same finite-type of process in the NSP-world yields such a denumerable set as well.

The term "NSP-world" will signify a certain second type of interpretation for nonstandard entities. In particular, the subtle logics, unreadable sentences, etc.
This interpretation will be developed throughout the remainder of this book. One important aspect of how descriptions are to be interpreted is that a description correlates directly to an assumed or observed real Natural phenomenon, and conversely. In these investigations, the phenomenon is called an *event*.

In order to simplify matters a bit, the following notation is employed. Let $\mathcal{T} = \{ T_i \mid i \in \mathbb{N} \}$. Let $F(\mathcal{T})$ be the set of all nonempty and finite subsets of $\mathcal{T}$. This symbol has been used previously to include the empty set, this set is now excluded. Now let $A \in F(\mathcal{T})$. Then there exists a finite choice set $s$ such that $x \in s$ iff there exists a unique $T_i \in A$ and $x \in T_i$. Now let the set $C$ denote the set of all such finite choice sets. As to interpreting these results within the NSP-world, the following is essential. Within nonstandard analysis the term “hyper” is often used for the result of the * map. For example, you have $^*\mathbb{R}$ as the hyperreals since $\mathbb{R}$ is termed the real numbers. For certain, but not all concepts, the term “hyper” or the corresponding * notation will be universally replaced by the term “ultra.” Thus, certain purely subtle words or *-words become “ultrawords” within the developmental paradigm interpretation. [Note: such a word was previously called a superword.] Of course, for other scientific or philosophical systems, such abstract mathematical objects can be reinterpreted by an appropriate technical term taken from those disciplines.

As usual, we are working within any enlargement and all of the above intuitive objects are embedded into the G-structure. Recall, that to simplify expressions, we often suppress within our first-order statements a specific superstructure element that bounds a specific quantifier. The alphabet $A$ is now assumed to be countable.

[Note 2 MAY 1998: The material between the [[ and the ]] has been altered from the original that appeared in the 1993 revision.] [[Although theorem 7.2.1 may be significant, it is no longer used for the other portions of this research. The set of all developmental paradigms corresponds to the set of all choice functions define on $\mathcal{T}$. Also see http://arxiv.org/abs/math/0605120]]

**Theorem 7.2.1** Let $\emptyset \neq \zeta \subset \mathbb{N}$ and $\tilde{\mathcal{T}} = \{ T_i \mid i \in \zeta \}$. There exists a set of sets $S$ determined by hyperfinite set $Q$ and hyper finite choice defined on $Q$ such that:

(i) $s' \in S$ iff for each $T \in \tilde{\mathcal{T}}$ there is one and only one $[g] \in ^*T$ such that $[g] \in s'$, and if $x \in s'$, then there is some $T \in \tilde{\mathcal{T}}$ and some $[g] \in ^*T$ such that $x = [g]$. (If $^*[g] \in ^*T$, then $[g] = [f] \in T$.)

Proof. (i) Let $A \in F(\tilde{\mathcal{T}})$. Then from the definition of $\tilde{\mathcal{T}}$, there exists some $n \in \mathbb{N}$ such that $A = \{ T_{j_i} \mid i = 0, \ldots, n \wedge j_i \in \mathbb{N} \}$. From the definition of $T_k$, each $T_k$ is denumerable. Notice that any $[f] \in T_k$ is associated with a unique member of $A_{j_i}$. Simply consider the unique $f_0 \in [f]$. The unique member of $A_1$ is by definition $f_0(0)$. Thus each member of $T_k$ can be specifically identified. Hence, for each $T_i$, there is a denumerable $M_i \subset \mathbb{N}$ and a bijection $h_i : M_i \rightarrow T_i$ such that $a_i \in T_i$ iff there is a $k_i \in M_i$ and $h_i(k_i) = a_i$. Consequently, for each $i = 0, \ldots, n$ and $a_{j_i} \in T_{j_i}$, we have that $h_{j_i}(k_{j_i}) = a_{j_i}$, and conversely for each $i = 0, \ldots, n$ and $k_{j_i} \in M_{j_i}$, $h_{j_i}(k_{j_i}) \in T_{j_i}$. Obviously, $\{ h_{j_i}(k_{j_i}) \mid i = 0, \ldots, n \}$ is a finite choice set.
All of the above may be translated into the following sentence that holds in $\mathcal{M}$.

(Note: Choice sets are usually considered as the range of choice functions. Further, “bounded formula simplification” has been used.)

$$\forall y(y \in F(\tilde{T}) \rightarrow \exists s((s \in \mathcal{P}(\mathcal{E})) \land \forall x((x \in y) \rightarrow \exists z((z \in x) \land (z \in s) \land$$

$$\forall w(w \in \mathcal{E} \rightarrow ((w \in s) \land (w \in x) \leftrightarrow (w = z))))))$$

(7.2.1)

$$\forall u(u \in \mathcal{E} \rightarrow ((u \in s) \leftrightarrow \exists x_1((x_1 \in y) \land (u \in x_1))))$$

For each $A \in F(\tilde{T})$, let $S_A$ be the set of all such choice sets generated by the predicate that follows the first $\rightarrow$ formed from (7.2.1) by deleting the $\exists$s and letting $y = A$. Of course, this set exists within our set theory. Now let $\mathcal{C} = \{S_A | A \in F(\tilde{T})\}$.

Consider $^\ast \mathcal{C}$ and $^\ast(S_A)$. Then $s \in (S_A)$ iff $s$ satisfies (7.2.1) as interpreted in $^\ast \mathcal{M}$. Since we are working in an enlargement, there exists an internal $Q \in (F(\tilde{T}))$ such that $^\ast \mathcal{C} \subset Q \subset ^\ast \mathcal{T}$. Recall that $^\ast \mathcal{T} = \{^\ast T | T \in \mathcal{T}\}$. Also $^\ast \mathcal{T} \subset ^\ast \mathcal{C}$ for each $T \in \mathcal{T}$. From the definition of $^\ast \mathcal{C}$, there is an internal set $S_Q$ and $s \in S_Q$ iff $s$ satisfies the internal defining predicate for members of $S_Q$ and this set is the set of all such $s$. ($\Rightarrow$) Consequently, since for each $T \in \mathcal{T}$, $^\ast T \in Q$, then the generally external $s' = \{s \cap ^\ast T | T \in \mathcal{T}\}$ satisfies the $\Rightarrow$ for (i). Note, however, that for $^\ast T, T \in \mathcal{T}$, it is possible that $s \cap ^\ast T = \{^\ast [f]\}$ and $^\ast [f] \in ^\ast T$. In this case, by the finiteness of $[f]$ it follows that $[f] = ^\ast [f]$ implies that $s \cap ^\ast T = \{[f]\}$. Now let $\mathcal{S} = \{s' | s \in S_Q\}$. In general, $\mathcal{S}$ is an external object.

($\Leftarrow$) Consider the internal set $S_Q$. Let $s'$ be the set as defined by the right-hand side of (i). For each internal $x \in s'$ and applying, if necessary, the $^\ast$-axiom of choice for $^\ast$-finite sets, we have the internal set $A_x = \{y | (y \in S_Q) \land (x \in y)\}$ is nonempty. The set $\{A_x | x \in s'\}$ has the finite intersection property. For, let nonempty internal $B = \{x_1, \ldots, x_n\}$. Then the set $A_B = \{y | (y \in S_Q) \land (x_1 \in y) \land \cdots \land (x_n \in y)\}$ is internal and nonempty by the $^\ast$-axiom of choice for $^\ast$-finite sets. Since we are in an enlargement and $s'$ is countable, then $D = \bigcap\{A_x | x \in s'\} \neq \emptyset$. Now take any $s \in D$. Then $s \in S_Q$ and from the definition of $\mathcal{S}$, $s' \in \mathcal{S}$. This completes the proof.

[Note: Theorem 7.2.1 may be used to model physical developmental paradigms associated with event sequences.]

Although it is not necessary, for this particular investigation, the set $\mathcal{S}$ may be considered a set of all developmental paradigms. Apparently, $\mathcal{S}$ contains every possible developmental paradigm for all possible frozen segments and $\mathcal{S}$ contains paradigms for any $^\ast$-totality $^\ast T$. There are $^\ast$-frozen segments contained in various $s'$ that can be assumed to be unreadable sentences since $^\ast T \neq ^\ast T$.]

Let $A \in F(\tilde{T})$ and $M(A)$ be a subset of $S_A$ for which there exists a written set of rules that selects some specific member of $S_A$. Obviously, this may be modeled by means of functional relations. First, $M(A) \subset S_A$ and it follows, from the difference in cardinalities, that there are infinitely many members of $^\ast(S_A)$ for which there does not exist a readable rule that will select such members. However, this does not preclude the possibility that there is a set of purely unreadable sentences that
do determine a specific member of $\ast S_A - \sigma M(A)$. This might come about in the following manner. Suppose that $H$ is an infinite set of formal sentences that is interpreted to be a set of rules for the selection of distinct members of $M(A)$. Suppose we have a bijection $h: M(A) \to H$ that represents this selection process. Let $\ast M$ be at least a polysaturated enlargement of $M$, and consider $\sigma f : \sigma(M(A)) \to \ast H$. The map $\sigma f$ is also a bijection and $\sigma f : \sigma(M(A)) \to \ast H$. Since $|\sigma(M(A))| < |M|$, it is well-known that there exists an internal map $h: A' \to \ast H$ such that $h | \sigma(M(A)) = \sigma f$, and $A'$, $h[A']$ are internal. Further, for internal $A' \cap \ast (S_A) = B$, $\sigma(M(A)) \subset B$. However, $\sigma(M(A))$ is external. This yields that $h$ is defined on $B$ and $B \cap (\ast S_A - \sigma(M(A))) \neq \emptyset$. Also, $\sigma H \subset h[B] \subset \ast H$ implies, since $h[B]$ is internal, that $\sigma H \neq h[B]$. Consequently, in this case, $h[B]$ may be interpreted as a set of $\ast$-rules that determine the selection of members of $B$. That is to say that there is some $[g] \in h[B] - \sigma H$ and a $[k] \in \ast S_A - \sigma(M(A))$ such that $([k], [g]) \in h$. As it will be shown in the next section, the set $H$ can be so constructed that if $[g] \in h[B] - \sigma H$, then $[g]$ is unreadable.

### 7.3 Ultrawords

Ordinary propositional logic is not compatible with deductive quantum logic, intuitionistic logic, among others. In this section, a subsystem of propositional logic is investigated which rectifies this incompatibility. I remark that when a standard propositional language $L$ or an informal language $P$ isomorphic to $L$ is considered, it will always be the case that the $L$ or $P$ is minimal relative to its applications. This signifies that if $L$ or $P$ is employed in our investigation for a developmental paradigm, then $L$ or $P$ is constructed only from those distinct propositional atoms that correspond to distinct members of $d$, etc. The same minimizing process is always assumed for the following constructions.

Let $B$ be a formal or, informal nonempty set of propositions. Construct the language $P_0$ in the usual manner from $B$ (with superfluous parentheses removed) so that $P_0$ forms the smallest set of formulas that contains $B$ and such that $P_0$ is closed under the two binary operations $\land$ and $\to$ as they are formally or informally expressed. Of course, this language may be constructed inductively or by letting $P_0$ be the intersection of all collections of such formula closed under $\land$ and $\to$.

We now define the deductive system $S$. Assume substitutivity, parenthesis reduction and the like. Let $d = \{F_i \mid i \in \mathbb{N}\} = B$ be a development paradigm, where each $F_i$ is a readable frozen segment and describes the behavior of a Natural system over a time subinterval. Let the set of axioms be the schemata

1. $(A \land B) \to A, A \in B$
2. $(A \land B) \to B$
3. $A \land (B \land C) \to (A \land B) \land C,$
4. $(A \land B) \land C \to A \land (B \land C)$.

If $P_0$ is considered as informal, which appears to be necessary for some applications, where the parentheses are replaced by the concept of symbol strings
being to the "left" or "right" of other symbol strings and the concept of strengths of connectives is used (i.e. $A \land B \rightarrow C$ means $(A \land B) \rightarrow C$), then axioms $3-4$ and the parentheses in (1) and (2) may be omitted. The one rule of inference is Modus Ponens (MP). Proofs or demonstrations from hypotheses $\Gamma$ contain finitely many steps, hypotheses may be inserted as steps and the last step in the proof is either a theorem if $\Gamma = \emptyset$ or if $\Gamma \neq \emptyset$, then the last step is a consequence of (a deduction from) $\Gamma$. Notice that repeated application of (4) along with (MP) will allow all left parentheses to be shifted to the right with the exception of the (suppressed) outermost left one. Thus this leads to the concept of left to right ordering of a formula. This allows for the suppression of such parentheses. In all the following, this suppression will be done and replaced with formula left to right ordering.

For each $\Gamma \subset P_0$, let $S(\Gamma)$ denote the set of all formal theorems and consequences obtained from the above defined system $S$. Since hypotheses may be inserted, for each $\Gamma \subset P_0$, $\Gamma \subset S(\Gamma) \subset P_0$. This implies that $S(\Gamma) \subset S(S(\Gamma))$. So, let $A \in S(S(\Gamma))$. The general concept of combining together finitely many steps from various proofs to yield another formal proof leads to the result that $A \in S(\Gamma)$. Therefore, $S(\Gamma) = S(S(S(\Gamma)))$. Finally, the finite step requirement also yields the result that if $A \in S(\Gamma)$, then there exists a finite $F \subset \Gamma$ such that $A \in S(F)$. Consequently, $S$ is a finitary consequence operator and observe that if $C$ is the propositional consequence operator, then $S(\Gamma) \subseteq C(\Gamma)$. Of course, we may now apply the nonstandard theory of consequence operators to $S$.

It is well-known that the axiom schemata chosen for $S$ are theorems in intuitionistic logic. Now consider quantum logic with the Mittelstaedt conditional $i_1(A, B) = A^+ \lor (A \land B)$. [1] Notice that $i_1(A \land B, B) = (A \land B)^+ \lor ((A \land B) \land B) = (A \land B)^+ \lor (A \land B) = I$ (the upper unit.) Then $i_1(A \land B, A) = (A \land B)^+ \lor ((A \land B) \land A) = (A \land B)^+ \lor (A \land B) = I$; $i_1((A \land B) \land C, A \land (B \land C)) = ((A \land B) \land C)^+ \lor (A \land (B \land C)) = I = i_1((A \land (B \land C)), (A \land B) \land C)$. Thus with respect to the interpretation of $A \rightarrow B$ as conditional $i_1$ the axiom schemata for the system $S$ are theorems and the system $S$ is compatible with deductive quantum logic under the Mittelstaedt conditional.

Recall that $d = \{F_i \mid i \in \mathbb{N}\}$ is a development paradigm, where each $F_i$ is a readable frozen segment, and describes the behavior of a Natural system at each moment of a time interval. For the next construction a formal language that is, of course, isomorphic to the informal language is employed. Each $\land$ [resp. $F_i$] corresponds to a specific $(|||\text{and}|||$ [resp. a propositional atom that corresponds to a specific word] when embedded. This eliminates confusion, when $(|||\text{and}|||$ appears in the $F_i$. Let $M_0 = d$. Define $M_1 = \{F_0|||\text{and}|||F_1\}$. Assume that $M_n$ is defined. Define $M_{n+1} = \{x|||\text{and}|||F_{n+1} \mid x \in M_n\}$. From the fact that $d$ is a developmental paradigm, where the last two symbols in each member of $d$ is the time indicator "i."

It follows that no member of $d$ is a member of $M_n$ for $n > 0$. Now let $M_d = \bigcup\{M_n \mid n \in \mathbb{N}\}$. Intuitively, $(|||\text{and}|||$ behaves as a conjunction and each $F_i$ as an atom within our language. Notice the important formal demonstration fact that for an hypothesis consisting of any member of $M_n$, $n > 0$, repeated applications of (1), (MP), (2), (MP) will lead to the members of $d$ appearing in the proper time
ordering at increasing (formal) demonstration step numbers.

Theorem 7.3.1 For \( d = \{ F_i \mid i \in \mathbb{N} \} \), there exists an ultraword \( w \in *M_d - *d \) such that \( F_i \in *S(\{w\}) \) (i.e. \( w \models_S F_i \)) for each \( i \in \mathbb{N} \).

Proof. Consider the binary relation \( G = \{(x, y) \mid (x \in d) \land (y \in M_d - d) \land (x \in S(\{y\}))\}. Suppose that \( \{(x_1, y_1), \ldots, (x_n, y_n)\} \subseteq G \). For each \( i = 1, \ldots, n \) there is a unique \( k_i \in \mathbb{N} \) such that \( x_i = F_{k_i} \). Let \( m = \max\{k_i \mid (x_i = F_{k_i}) \land (i = 1, \ldots, n)\} \). Let \( b \in M_{m+1} \). It follows immediately that \( x_i \in S(\{b\}) \) for each \( i = 1, \ldots, n \) and, from the construction of \( d \), \( b \notin d \). Thus \( \{(x_1, b), \ldots, (x_n, b)\} \subseteq G \). Consequently, \( G \) is a concurrent relation. Hence, there exists some \( w \in *M_d - *d \) such that \( *F_i = F_i \in *S(\{w\}) \) for each \( i \in \mathbb{N} \). This completes the proof. [See note 4.]

Observe that \( w \) in Theorem 7.3.1 has all of the formally expressible properties of a readable word. For example, \( w \) has a hyperfinite length, among other properties. However, since \( d \) is a denumerable set, each ultraword has a very special property.

Recall that for each \( [g] \in \mathcal{E} \) there exists a unique \( m \in \mathbb{N} \) and \( f' \in T^m \) such that \( [f'] = [g] \) and for each \( k \) such that \( m < k \in \mathbb{N} \), there does not exist \( g' \in T^k \) such that \( [g'] = [g] \). The function \( f' \in T^m \) determines all of the alphabet symbols, the symbol used for the blank space, and the like, and determines there position within the intuitive word being represented by \( [g] \). Also for each \( j \) such that \( 0 \leq j \leq m \), \( f'(j) = i(a) \in i[W] = T \), where \( i(n) \) is the “encoding” in \( T \) of the symbol “a”.

For each \( m \in \mathbb{N} \), let \( P_m = \{ f \mid (f \in T^m) \land (\exists z((z \in \mathcal{E}) \land (f \in z) \land \forall x((x \in \mathbb{N}) \land (x > m) \rightarrow \neg \exists y((y \in T^x) \land (y \in z))))\} \). An element \( n \in *T \) is a subtle alphabet symbol if there exists \( m \in \mathbb{N} \) and \( f \in *P_m \) such that \( \forall \delta \in \mathbb{N}_\infty \) and \( f \in *P_\delta \), and some \( j \in *\mathbb{N} \) such that \( f(j) = n \). A symbol is a pure subtle alphabet symbol if \( f(j) = n \notin i[W] \). Subtle alphabet symbols can be characterized in \( \mathcal{E} \) for they are singleton objects. A \([g] \in \mathcal{E} \) represents a subtle alphabet symbol iff there exists some \( f \in (*T)^0 \) such that \([f] = [g] = [(0, f(0))] \), \( f = \{(0, f(0))\} \).

Theorem 7.3.2 Let \( d = \{ F_i \mid i \in \mathbb{N} \} \) be a denumerable developmental paradigm and use \( M_1 \) of 9.1. For each ultraword \( w \), that yields \( d \) via \( *S \), as in Theorem 7.3.1, the external cardinality of the collection of all pure subtle alphabet symbols represented in each \( w \) is greater than or equal to \( 2^{\aleph_0} \).

Proof. Consider the conceptual Kleene “tick” notation for the natural numbers (i.e. \(| |, ||, |||, \ldots \)). For this proof, let \(| | \) correspond to 0. Every member of \( d = \{ F_i \mid i \in \mathbb{N} \} \) contains a distinct symbol-string \( b_i \) that represents the natural number followed by the “period” symbol that appears as the last two symbols in a member of \( d \). Consider the single \( W_n \in M_n \), \( n > 0 \). Then \( n + 2 \) of these distinct symbol-strings, the Kleene symbols and a “period” symbol, appear in \( W_n \) along with other alphabet systems. Hence, in \( W_n \), there are more than \( n + 2 \) alphabet symbols.

For the embedding \( \mathcal{E} \), there is a \( W_n \) representation \([g] \in \mathcal{E} \) and two unique mappings \( f_k \sim f_0 \sim g \), where the inverse of the embedding \( i \) yields the entire word for \( f_0 \) and, for \( f_k \in P_k \), yields the entire word as it is join constructed from individual symbols (eq. 1.2.4). In this case, \( k > n + 2 \).
Consider the *-transform. Let \( w = [g] \) be an ultrawords such that for each \( i \in \mathbb{N}, F_i \in \ast \mathcal{S}(\{w\}) \). Theorems 7.3.1 shows that such ultrawords exist. From the definition of \( S, w \in \ast \mathcal{M}_d - \ast \mathcal{M}_d \). Hence, there is a \( \nu, \delta \in \mathbb{N}_\infty \) and \( \ast \mathcal{M}_\nu \in \{ \ast \mathcal{M}_x \mid x \in \ast \mathbb{N} \} \) such that \( [f_\nu] = [g] \in \ast \mathcal{M}_\nu, \delta > \nu + 2, f_\delta \in \ast \mathbb{P}_\delta \).

Let \( K = \{ [1, n + 2] \mid n \in \mathbb{N} \} \). Then there exists a mapping \( C: K \to \mathbb{N} \) such that \( C([1, n + 2]) = n + 2 \). The mapping \( C \) is considered as yielding the intuitive cardinality of \([1, n + 2]\). Hence, \( \ast C([1, \nu + 1]) = \nu + 2 \). To get an idea as to the external cardinality \( \|1, \nu + 2\| \) of \([1, \nu + 2]\), consider Theorem 3.1 in [16, p. 201], where it is shown that \( \|1, \nu + 2\| \geq 2^{\aleph_0} \). Since \( \|1, \delta\| \geq \|1, \nu + 2\| \), and the set of all subtle alphabet symbols that yields members of \( W \) is denumerable, then it follows that for \( w \) the set of all pure subtle alphabet symbols also has an external cardinality great than or equal to \( 2^{\aleph_0} \). This completes the proof.

With respect to the proof of Theorem 7.3.2, the function \( f_\delta \) determines the alphabet composition of the ultraword \( w \). The word \( w \) is unreadable not only due to its infinite length but also due to the fact that it is composed of infinitely many purely subtle alphabet symbols.

The developmental paradigm \( d \) utilized for the two previous theorems is composed entirely of readable sentences. We now investigate what happens if a developmental paradigm contains countably many unreadable sentences. Let the nonempty developmental paradigm \( d' \) be composed of at most countably many members of \( \mathcal{E} - \mathcal{E} \) and, for countable \( B \), let \( d' \subset \ast B \subset \ast \mathcal{P}_0 \). Construct, as previously, the set \( \mathcal{M}_B \) from \( B \), rather than from \( d \) and suppose that \( B \cap \mathcal{M}_i = \emptyset, i \neq 0 \). [See Note 2 on page 82.] Let \( \lambda \neq \emptyset \subset \mathbb{N} \).

**Theorem 7.3.3** Let \( d' = \{ [g_i] \mid i \in \lambda \} \). Then there exists an ultraword \( w \in \ast \mathcal{M}_B - \ast B \) such that for each \( i \in \mathbb{N}, [g_i] \in \ast \mathcal{S}(\{w\}) \).

Proof. Consider the internal binary relation \( G = \{ (x, y) \mid (x \in \ast B) \land (y \in \ast \mathcal{M}_B - \ast B) \land (x \in \ast \mathcal{S}(\{y\})) \} \). Note that members of \( d' \) are members of \( \varphi \mathcal{E} \) or, at the most, denumerably many members of \( \mathcal{E} - \varphi \mathcal{E} \). From the analysis in the proof of Theorem 7.3.1, for a finite \( F \subset B \), there exists some \( y \in \mathcal{M}_B - B \) such that \( F \subset \mathcal{S}(\{y\}) \). It follows by \( \ast \)-transfer that if \( F \) is a finite or \( \ast \)-finite subset of \( \ast B \), then there exists some \( y \in \ast \mathcal{M}_B - \ast B \) such that \( F \subset \ast \mathcal{S}(\{y\}) \). As in the proof of Theorem 7.3.1, this yields that \( G \) is at least concurrent on \( \ast B \). However, \( d' \subset \ast B \) and \( |d'| \leq \aleph_0 \). From \( \aleph_1 \)-saturation, there exists some \( w \in \ast \mathcal{M}_B - \ast B \) such that for each \( [g_i] \in d', [g_i] \in \ast \mathcal{S}(\{w\}) \). This completes the proof.

Let \( \theta \neq \lambda, \gamma \subset \mathbb{N}, j \in \gamma, D_j = \{ d_{ij} \mid i \in \lambda \} \), and for each \( j \in \gamma, i \in \lambda, d_{ij} \subset \ast B \) is considered to be a developmental paradigm either of type \( d \) or type \( d' \) and \( B \cap \mathcal{M}_i = \emptyset, i \neq 0 \). Notice that \( D_j \) may be either a finite or denumerable set and Theorem 7.3.1 holds for the case that \( d \subset B \), where \( w \in \ast \mathcal{M}_B - \ast B \). For each \( d_{ij} \in D_j \), use the Axiom of Choice to select an ultraword \( w_{ij} \in \ast \mathcal{M}_B - \ast B \) that exists by Theorems 7.3.1 (extended) or 7.3.3. Let \( \{ w_{ij} \mid i \in \lambda \} \) be ultrawords.
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Theorem 7.3.4 For \( j \in \gamma \), there exists an ultimate ultraword \( w'_j \in * M_B - * B \) such that for each \( i \in \lambda \), \( w_{ij} \in * S(\{w'_j\}) \) and, hence, for each \( d_{ij} \in D_j \), \( d_{ij} \subset * S(\{w_{ij}\}) \subset * S(\{w'_j\}) \).

Proof. For each finite \( \{F_1, \ldots, F_n\} \subset M_B - B \) there is a natural number, say \( m \), such that for each \( i = 1, \ldots, n \), \( F_i \in M_j \) for some \( j \leq m \). Hence, taking \( b \in M_{m+1} \), we obtain that each \( F_i \in S(\{b\}) \). Since \( b \notin B \), by *-transfer, it follows that the internal relation \( G = \{(x, y) \mid (x \in * M_B - * B) \land (y \in * M_B - * B) \land (x \in * S(\{y\})) \} \) is concurrent on internal \( * M_B - * B \) and \( \{w_{ij} \mid i \in \lambda \} \subset * M_B - * B \).

Consider \( \gamma \)-saturated yields that there is some \( w'_j \in * M_B - * B \) such that for each \( i \in \lambda \), \( w_{ij} \in * S(\{w'_j\}) \). The last property is obtained from \( d_{ij} \subset * S(\{w_{ij}\}) \subset * S(\{w'_j\}) \) since \( \{w_{ij}\} \) is an internal subset of \( * P_0 \). This completes the proof.]

Corollary 7.3.4.1 There exists an ultimate ultraword \( w' \in * M_B - * B \) such that for each \( j \in \gamma \), \( w'_j \in * S(\{w'\}) \) and, hence, for each \( d_{ij} \in D_j \), \( d_{ij} \subset * S(\{w'_j\}) \subset * S(\{w'\}) \).

The same analysis used to obtain Theorem 7.3.2 can be applied to the ultrawords of Theorems 7.3.3 and 7.3.4. (See note [5].)

7.4 Ultracontinuous Deduction

In 1968, a special topology on the set of all nonempty subsets of a given set \( X \) was constructed and investigated by your author. We apply a similar topology to subsets of \( \mathcal{E} \).

Suppose that nonempty \( X \subset \mathcal{E} \). Let \( \tau \) be the discrete topology on \( X \). In order to topologize \( \mathcal{P}(X) \), proceed as follows: for each \( G \in \tau \), let \( N(G) = \{A \mid (A \subset X) \land (A \subset G)\} = \mathcal{P}(G) \). Consider \( B = \{N(G) \mid G \in \tau\} \) to be a base for a topology \( \tau_1 \) on \( \mathcal{P}(X) \). Let \( A \in N(G_1) \cap N(G_1) \). The discrete topology implies that \( N(A) \) is a base element and that \( N(A) \subset N(G_1) \cap N(G_2) \). There is only one member of \( B \) that contains \( X \) and this is \( \mathcal{P}(X) \). Thus if \( \mathcal{P}(X) \) is covered by members of \( B \), then \( N(X) = \mathcal{P}(X) \) is one of these covering objects. Thus \( \langle \mathcal{P}(X), \tau_1 \rangle \) is a compact space.

Further, since \( N(\emptyset) \subset N(G) \) for each \( G \in \tau \), the space \( \langle \mathcal{P}(X), \tau_1 \rangle \) is connected. The topology \( \tau_1 \) is a special case of a more general topology with the same properties.

[2] Suppose that \( D \subset X \). Let \( D \in N(G) = \mathcal{P}(G), G \in \tau \). Then \( D \in N(D) \subset N(G) \). This yields that the nonstandard monad is \( \mu(D) = \bigcap \{N(G) \mid N(G) \in B\} = * (\mathcal{P}(D)) = * \mathcal{P}(\{D\}) \).

Theorem 7.4.1 Any consequence operator \( C: (\mathcal{P}(X), \tau_1) \rightarrow (\mathcal{P}(X), \tau_1) \) is continuous.

Proof. Let \( A \in \mathcal{P}(X) \) and \( H \in * C[\mu(A)] \). Then there exists some \( B \in \mu(A) \) such that \( * C(B) = H \). Hence, \( B \in * \mathcal{P}(\{A\}) \). By *-transfer of a basic property of our consequence operators, \( * C(B) \subset * C(\{A\}) = * \mathcal{C}(A) \). Thus \( * (\mathcal{C}(B)) \in * (\mathcal{C}(A)) \) implies that \( * C(B) \subset \mu(A) \). Therefore, \( * C[\mu(A)] \subset \mu([C(A)]) \). Consequently, \( C \) is continuous. [4]
Corollary 7.4.1.1 For any \( X \in \mathcal{E} \), and any consequence operator \( C: \mathcal{P}(X) \to \mathcal{P}(X) \), the map \( *C: *(\mathcal{P}(X)) \to *(\mathcal{P}(X)) \) is ultracontinuous.

Corollary 7.4.1.2 Let \( d \) [resp. \( d' \), \( d \) or \( d' \)] be a developmental paradigm as defined for Theorem 7.3.1 [resp. Theorem 7.3.3, 7.3.4]. Let \( w \) be a ultraword that exists by Theorem 7.3.1 [resp Theorem 7.3.3, 7.3.4]. Then \( d \) [resp. \( d' \), \( d \) or \( d' \)] is obtained by means of a ultracontinuous subtle deductive process applied to \( \{ w \} \).

Recall that in the real valued case, a function \( f: [a, b] \to \mathbb{R} \) is uniformly continuous on \([ a, b ]\) iff for each \( p, q \in *[ a, b ] \) such that \( p - q \in \mu(0) \), then \( f(p) - f(q) \in \mu(0) \).

For \( D \subset [a, b] \) is compact, then \( p, q \in *D \) and \( p - q \in \mu(0) \) imply that there is a standard \( r \in D \) such that \( p, q \in \mu(r) \). Also, for each \( r \in D \) and any \( p, q \in \mu(r) \), it follows that \( p - q \in \mu(r) \). Thus, if compact \( D \subset [a, b] \), then \( f: D \to \mathbb{R} \) is uniformly continuous iff for every \( r \in D \) and each \( p, q \in \mu(r) \), \( *f(p), *f(q) \in \mu(f(r)) \).

With this characterization in mind, it is clear that any consequence operator \( C: \mathcal{P}(X) \to \mathcal{P}(X) \) satisfies the following statement. For each \( A \in \mathcal{P}(X) \) and each \( p, q \in \mu(A), \) \( *C(p), *C(q) \in \mu(C(A)) \).

From the above discussion, one can think of ultracontinuity as being a type of ultrauniform continuity.

### 7.5 Hypercontinuous Gluing

There are various methods that can be used to investigate the behavior of adjacent frozen segments. All of these methods depend upon a significant result relative to discrete real or vector valued functions. The major goal in this section is to present a complete proof of this major result and to indicate how it is applied.

First, as our standard structure, consider either the intuitive real numbers as atoms or axiomatically a standard structure with atoms \( \text{ZFR} = \text{ZF} + \text{AC} + A_1(\text{atoms}) + A(\text{atoms}) + |A| = c \), where \( A \) is isomorphic to the real numbers and \( A_1 \cap A = \emptyset \). Then, as done previously, there is a model \( \langle C, \varepsilon, = \rangle \) within our \( \text{ZF} + \text{AC} \) model for \( \text{ZFR} \), where \( A \) has all of the ordered field properties as the real numbers. A superstructure \( \langle \mathcal{R}, \varepsilon, = \rangle \) is constructed in the usual manner, where the superstructure \( \langle \mathcal{N}, \varepsilon, = \rangle \) is a substructure. Proceeding as in Chapter 2, construct \( \mathcal{M}_1 = \langle *\mathcal{R}, \varepsilon, = \rangle \) and \( Y_1 \). The structure \( Y_1 \) is called the Extended Grundlegend Structure — the EGS. The Grundlegend Structure is a substructure of \( Y_1 \).

It is important to realized in what follows that the objects utilized for the G-structure interpretations are nonempty finite equivalence classes of partial sequences. Due to this fact, the following results should not lead to ambiguous interpretations.

As a preliminary to the technical aspects of this final section, we introduce the following definition. A function \( f: [a, b] \to \mathbb{R}^n \) is differentiable-C on \([a, b] \) if it is continuously differentiable on \((a, b) \) except at finitely many removable discontinuities. This definition is extended to the end points \( \{ a, b \} \) by application of one-sided derivatives. For any \( [a, b] , \) consider a partition \( P = \{ a_0, a_1, \ldots, a_n, a_{n+1} \} , \) \( n \geq \)
1, \( a = a_0, b = a_{n+1} \) and \( a_{j-1} < a_j, \ 1 \leq j \leq n+1 \). For any such partition \( P \), let the real valued function \( g \) be defined on the set \( D = [a_0, a_1) \cup (a_1, a_2) \cup \cdots \cup (a_n, a_{n+1}] \) as follows: for each \( x \in [a_0, a_1) \), let \( g(x) = r_1 \in \mathbb{R} \); for each \( x \in (a_{j-1}, a_j) \), let \( g(x) = r_j \in \mathbb{R}, \ 1 < j \leq n \); for each \( x \in (a_n, b] \), let \( g(x) = r_{n+1} \in \mathbb{R} \). It is obvious that \( g \) is a type of simple step function. Notationally, let \( \mathcal{F}(A, B) \) denote the set of all functions with domain \( A \) and codomain \( B \).

**Theorem 7.5.1** There exists a function \( G \in *\mathcal{F}([a, b], \mathbb{R}) \) with the following properties.

(i) The function \( G \) is *-continuously *-differentiable and *-uniformly *-continuous on \(*[a, b],\)

(ii) for each odd \( n \in *\mathbb{N}, \ (n \geq 3) \), \( G \) is *-differentiable-C of order \( n \) on \(*[a, b],\)

(iii) for each even \( n \in *\mathbb{N}, \ G \) is *-continuously *-differentiable in \(*[a, b]\) except at finitely many points,

(iv) if \( c = \min\{r_1, \ldots, r_{n+1}\}, \ d = \max\{r_1, \ldots, r_{n+1}\} \), then the range of \( G = *[c, d], \text{st}(G) \) at least maps \( D \) into \([c, d]\) and \((\text{st}(G))D = g\).

Proof. First, for any real \( c, d \), where \( d \neq 0 \), consider the finite set of functions

\[
  h_j(x, c, d) = (1/2)(r_{j+1} - r_j)\left(\sin((x-c)\pi/(2d)) + 1\right) + r_j,
\]

\( 1 \leq j \leq n \). Each \( h_j \) is continuously differentiable for any order at each \( x \in \mathbb{R} \). Observe that for each odd \( m \in \mathbb{N} \), each \( m \)th derivative \( h_j^{(m)} \) is continuous at \((c + d)\) and \((c - d)\) and \( h_j^{(m)}(c + d) = h_j^{(m)}(c - d) = 0 \) for each \( j \).

Let positive \( \delta \in \mu(0) \). Consider the finite set of internal intervals \( \{(a_0, a_1 - \delta), (a_1 + \delta, a_2 - \delta), \ldots, (a_n + \delta, b]\} \) obtained from the partition \( P \). Denote these intervals in the expressed order by \( I_j, \ 1 \leq j \leq n + 1 \). Define the internal function

\[
  G_1 = \{(x, r_j)|x \in I_j\} \cup \cdots \cup \{(x, r_{n+1})|x \in I_{n+1}\}.
\]

(7.5.2)

Let internal \( I_j^1 = [a_j - \delta, a_j + \delta], \ 1 \leq j \leq n, \) and for each \( x \in I_j^1 \), let internal

\[
  G_j(x) = (1/2)(r_{j+1} - r_j)\left(\sin((x-a_j)\pi/(2\delta)) + 1\right) + r_j.
\]

(7.5.3)

Define the internal function

\[
  G_2 = \{(x, G_1(x))|x \in I_j^1\} \cup \cdots \cup \{(x, G_n(x))|x \in I_{n+1}^1\}.
\]

(7.5.4)

The final step is to define \( G = G_1 \cup G_2 \). Then \( G \in *\mathcal{F}([a, b], \mathbb{R}) \).

By *-transfer, the function \( G_1 \) has an internal *-continuous *-derivative \( G_1^{(1)} \) such that \( G_1^{(1)}(x) = 0 \) for each \( x \in I_1 \cup \cdots \cup I_{n+1} \). Applying *-transfer to the properties of the functions \( h_j(x, c, d) \), it follows that \( G_2 \) has a unique internal *-continuous *-derivative

\[
  G_2^{(1)} = (1/(4\delta))(r_{j+1} - r_j)\pi\left(\cos((x-a_j)\pi/(2\delta))\right).
\]

(7.5.5)
for each \( x \in I_1^1 \cup \cdots \cup I_n^1 \). The results that the \(*\)-left limit for the internal \( G_1^{(1)} \) and the \(*\)-right limit for internal \( G_1^{(2)} \) at each \( a_j - \delta \) as well as the \(*\)-left limit of \( G_2^{(1)} \) and \(*\)-right limit of \( G_1^{(1)} \) at each \( a_j + \delta \) are equal to 0 and \( 0 = G_1^{(1)}(a_j - \delta) = G_2^{(1)}(a_j + \delta) \) imply that internal \( G \) has a \(*\)-continuous \(*\)-derivative \( G^{(1)} = G_1^{(1)} \cup G_2^{(2)} \) defined on \([a, b]\).

A similar analysis and \(*\)-transfer yield that for each \( m \in \mathbb{N} \), \( m \geq 2 \), \( G \) has an internal \(*\)-continuous \(*\)-derivative \( G^{(m)} \) defined at each \( x \in [a, b] \) except at the points \( a_j \pm \delta \) whenever \( r_{j+1} \neq r_j \). However, it is obvious from the definition of the functions \( h_j \) that for each odd \( m \in \mathbb{N} \), \( m \geq 3 \), each internal \( G^{(m)} \) can be made \(*\)-continuous at each \( a_j \pm \delta \) by simply defining \( G^{(m)}(a_j \pm \delta) = 0 \) and with this parts (i), (ii), and (iii) are established.

For part (iv), assume that \( r_j \leq r_{j+1} \). From the definition of the functions \( h_j \), it follows that for each \( x \in I_j \cup I_j^1 \cup I_{j+1} \), \( r_j \leq G(x) \leq r_{j+1} \). The nonstandard intermediate value theorem implies that \( G\left( [\ast a_j, a_{j+1}] \right) = \ast [r_j, r_{j+1}] \) and in like manner for the case that \( r_{j+1} < r_j \). Hence, \( G\left( [\ast a, b] \right) = \ast [c, d] \). Clearly, \( \text{st}(\ast D) = [a, b] \). If \( p \in D \) and \( x \in \mu(p) \cap D \), then \( G(x) = r_j = g(p) \) for some \( j \) such that \( 1 \leq j \leq n + 1 \). This completes the proof.

The nonstandard approximation theorem 7.5.1 can be extended easily to functions that map \( D \) into \( \mathbb{R}^m \). For example, assume that \( F: D \to \mathbb{R}^3 \), the component functions \( F_1 \), \( F_2 \) are continuously differentiable on \([a, b] \); but that \( F_3 \) is a \( g \) type step function on \( D \). Then letting \( H = (\ast F_1 \ast F_2, G) \), on \( \ast [a, b] \), where \( G \) is defined in Theorem 4.1, we have an internal \(*\)-continuously \(*\)-differentiable function \( H: \ast [a, b] \to \ast \mathbb{R}^3 \), with the property that \( \text{st}(H)|D = F \).

With respect to Theorem 7.5.1, it is interesting to note that if \( h_j \) is defined on \( \mathbb{R} \), then for even orders \( n \in \mathbb{N} \),

\[
| h_j^{(n)}(c + d) | = \left| \frac{(r_{j+1} - r_j)\pi^n}{2^{n+1}d^n} \right| = 0
\]

for \( r_{j+1} = r_j \) but not 0 otherwise. If \( r_{j+1} - r_j \neq 0 \), then \( G_2^{(n)}(a_j \pm \delta) \) is an infinite nonstandard real number. Indeed, if \( m_i \) is an increasing sequence of even numbers in \( \ast \mathbb{N} \) and \( r_{j+1} \neq r_j \), then \( |G_2^{(m_i)}(a_j \pm \delta)| \) forms a decreasing sequence of nonstandard infinite numbers. The next result is obvious from the previous result.

**Corollary 7.5.1.1** For each \( n \in \ast \mathbb{N} \), then internal \( G^{(n)} = G_1^{(n)} \cup G_2^{(n)} \) is \(*\)-bounded on \([a, b] \).

Let \( D(a, b) \) be the set of all bounded and piecewise continuously differentiable functions defined on \([a, b] \). By considering all of the possible (finitely many) subintervals, where \( f \in D(a, b) \), it follows from the Riemann sum approach that for each real \( \nu > 0 \), there exists a real \( \nu_1 > 0 \) such that for each real \( \nu_i \), \( 0 < \nu_i < \nu_1 \), a sequence of partitions \( P_i = \{a = b_0^i < \cdots < b_k_i = b\} \) can be selected such that the mesh\( (P_i) \leq \nu_i \) and

\[
| (f(b) - f(a)) - \sum_{n=1}^{k_i} f'(t_n)(b_n^i - b_{n-1}^i) | < \nu
\]
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for any \( t_n \in (b_{n-1}^i, b_n^i) \).

Moreover, for any given number \( M \), the sequence of partitions can be so constructed such that there exists a \( j \) such that for each \( i > j \), \( k_i > M \), where \( P_i \) and \( P_j \) are partitions within the sequence of partitions. By \(*\)-transfer of these facts and by application of Theorem 7.5.1 and its corollary we have the next result.

**Corollary 7.5.1.2** For each \( n \in \mathbb{N} \) and each internal \( G^{(n)} \), the difference \( G^{(n)}(b) - G^{(n)}(a) \) is infinitesimally close to an (externally) infinity \(*\)-finite sum of infinitesimals.

A developmental paradigm is a very general object and, therefore, can be used for numerous applications. At present, developmental paradigms are still being viewed from the substratum or external world. For what follows, it is assumed that a developmental paradigm \( d \) traces the evolutionary history of a specifically named natural system or systems. In this first application, let each \( F_i \in d \) have the following property (\( P \)).

\( F_i \) describes “the general behavior and characteristics of the named natural system \( S_1 \) as well as the behavior and characteristics of named constituents contained within \( S_1 \) at time \( t_i \).”

Recall that for \( F_i \), \( F_{i+1} \in d \), there exist unique functions \( f_0 \in F_i = [f], g_0 \in F_{i+1} = [g] \) such that \( f_0, g_0 \in T^0 \) and \( \{(0, f_0(0))\} \in [f], \{(0, g_0(0))\} \in [g] \). Thus, to each \( F_i \in d \), correspond the unique natural number \( f_0(0) \). Let \( D = [t_{i-1}, t_i] \cup (t_i, t_{i+1}] \) and define \( f_1: D \to \mathbb{N} \) as follows: for each \( x \in [t_{i-1}, t_i] \), let \( f_1(x) = f_0(0) \); for each \( x \in (t_i, t_{i+1}] \), let \( f_1(x) = g_0(0) \). Application of theorem 7.5.1 yields the internal function \( G \) such that \( G|D = f_1 \). For these physical applications, utilize the term “substratum” in the place of the technical terms “pure standard.” [Note: Of course, elsewhere, the term “pure NSP-world” or simply the “NSP-world” is used as a specific name for what has here been declared as the substratum.] This yields the following statements, where the symbols \( F_i \) and \( F_{i+1} \) are defined and characterized by the expression inside the quotation marks in property (\( P \)).

(A): There exists a substratum hypercontinuous, hypersmooth, hyperuniform process \( G \) that binds together \( F_i \) and \( F_{i+1} \).

(B): There exists a substratum hypercontinuous, hypersmooth, hyperuniform alteration process \( G \) that transforms \( F_i \) into \( F_{i+1} \).

(C): There exists an ultracontinuous subtle force-like (i.e. deductive) process that yields \( F_i \) for each time \( t_i \) within the development of the natural system.

In order to justify (A) and (B), specific measures of physical properties associated with constituents may be coupled together. Assume that for a subword \( r_i \in F_i \in d \), the symbols \( r_i \) denote a numerical quantity that aids in characterizing the behavior of an object in a system \( S_1 \) or the system itself. Let (\( M_1 \)) be the statement:
“There exists a substratum hypercontinuous, hypersmooth, hyper-
uniform functional process \( G_i \) such that \( G_i \) when restricted to the
standard mathematical domain it is \( f_i \) and such that \( G_i \) hypercon-
tinuously changes \( r_i \) for system \( S_1 \) at time \( t_i \) into \( r_{i+1} \) for system
\( S_1 \) at time \( t_{i+1} \).”

This modeling procedure yields the following interpretation:

(D) If there exists a continuous or uniform [resp. discrete] func-
tional process \( f_i \) that changes \( r_i \) for \( S_1 \) at time \( t_i \) into \( r_{i+1} \) for \( S_1 \)
at time \( t_{i+1} \), then (M1).

At a particular moment \( t_i \), two natural systems \( S_1 \) and \( S_2 \) may interface. More
generally, two very distinct developmental paradigms may exist one \( d_1 \) at times prior
to \( t_i \) (in the \( t_i \) past) and one \( d_2 \) at time after \( t_i \) (in the \( t_i \) future). We might refer
to the time \( t_i \) as a standard time fracture. Consider the developmental paradigm
\( d_3 = d_1 \cup d_2 \). In this case, the paradigms may be either of type \( d \) or \( d' \). For the type
\( d' \), the corresponding system need not be considered a natural system but could be
a pure substratum system.

At \( t_i \) an \( F_i \in d_3 \) can be characterized by statement (P) (with the term natural
removed if \( F_i \) is a member of a \( d' \)). In like manner, \( F_{i+1} \) at time \( t_{i+1} \) can be
characterized by (P). Statements (A), (B), (C) can now be applied to \( d_3 \) and
a modified statement (D), where the second symbol string \( S_1 \) is changed to \( S_2 \).
Notice that this modeling applies to the actual human ability that only allows for
two discrete descriptions to be given, one for the interval \( [t_{i-1}, t_i) \) and one for the
interval \( (t_i, t_{i+1}] \). From the modeling viewpoint, this is often sufficient since the
length of the time intervals can be made smaller than Planck time.

Recall that an analysis of the scientific method used in the investigation of
natural system should take place exterior to the language used to describe the
specific system development. Suppose that \( \mathcal{D} \) is the language accepted for a scientific
discipline and that within \( \mathcal{D} \) various expressions from mathematical theories are
used. Further, suppose that enough of the modern theory of sets is employed so
that the EGS can be constructed. The following statement would hold true for \( \mathcal{D} \).

\[
If \text{ by application of first-order logic to a set of non-mathematical}
\text{ premises taken from } \mathcal{D} \text{ it is claimed that it is not logically possible}
\text{ for statements such as (A), (B), (C) and (D) to hold, then the set}
\text{ of premises is inconsistent.}
\]
CHAPTER 7 REFERENCES

1 Beltrametti, E. G.and G. Cassinelli, The logic of quantum mechanics, in Encyclopedia of Mathematics and its Application, Vol. 15, Addison-Wesley, Reading, 1981.

2 Herrmann, R. A., Some Characteristics of Topologies on Subsets of a Power Set, University Microfilm, M-1469, 1968.

3 Herrmann, R. A., The Theory of Infinitesimal Light-clocks, (1993) http://arxiv.org/abs/math/0312189

NOTES

[1] Note that this last requirement for B can be achieved as follows: construct a special symbol not originally in \( A \). Then this symbol along with \( A \) is considered the alphabet. Next only consider a B that does not contain this special symbol within any of its members. Now using this special symbol in place of the \( \land \), construct \( M_i^0, i \neq 0 \). Of course, \( \land \) is interpreted as this special symbol in the axiom system \( S \).

[2] The actual members, \( F_i \), of a developmental paradigm \( d \) need not be unique. However, the specific information contained in each readable word used for a specific \( F_i \in \mathbb{N} \) is unique. Other readable sentences can be used in place of a specific \( F_i \) as long as they are “equivalent” in the sense that the specific information being displayed by each is the same information.

[3] Depending upon the application, a single standard word may also be termed as an ultraword.

[4] For a method to obtain an ultraword for a refined developmental paradigm, see pages 4 - 7 in http://arxiv.org/abs/math.0605120

[5] (Added 9/20/2009.) A concurrent relation is not needed to obtain important “ultrawords.”

**Theorem 7.3.5** For \( d = \{F_n \mid n \in \mathbb{N}\} \) and each infinite \( \lambda \in \mathbb{N}_\infty \), there exists one and only one \( w_\lambda \in \ast M_\lambda \) and hyperfinite \( d_\lambda \) such that \( d \subset d_\lambda \subset \ast S(\{w_\lambda\}) \), and \( d_\lambda \subset \ast d \).

Proof. For each, \( n \in \mathbb{N} \), let \( G(n) = \{F_i \mid 0 \leq i \leq n\} \subset d \). Thus, \( G: \mathbb{N} \to \mathcal{F}(d) \) the set of all finite subsets of \( d \). Let \( n > 0 \). Then \( M_n \) has one and only one member and by definition \( w_n \in M_n \) has the property that \( G(n) \subset S(\{w_n\}) \). Hence, by \( \ast \)-transfer, for the function \( \ast M \), and each \( \lambda \in \mathbb{N}_\infty \), there is one and only one \( w_\lambda \in \ast M_\lambda \) such that hyperfinite \( \ast G(\lambda) \subset \ast S(\{w_\lambda\}) \). Finally, by definition of \( G \), \( d \subset \ast G(\lambda) \subset \ast d \). \( \blacksquare \)

Note that theorems that generate or use ultrawords may need to be trivially modified or not used depending upon the definition for \( d \). For example, for the ordering used in [1], then the ultraword used and its location would be an ultimate ultraword as generated in theorem 7.3.4, where each \( w_{ij} \) is an ultraword or ultimate ultraword that, upon application of \( \ast S \), yields the developmental paradigm for each interval \( [c_i, c_{i+1}) \).
[1] The GGU-model and Generation of the Developmental Paradigms
http://arxiv.org/abs/math/0605120
8. A SPECIAL APPLICATION

8.1 A Neutron Altering Process.

The purpose of this chapter is to justify the interpretations utilized in reference [1]. Let $B$ be the set of all nondecreasing bounded real valued functions defined on $D = [a, t') \cup (t', T]$. Let $Q \in B$ and $Q(t) = 2$ for each $t \in [a, t')$; $Q(t) = 3$ for each $t \in (t', T]$ be the discrete neutron altering process. Application of Theorem 7.5.1 implies that there exists internal $G$: *$[a, T] \rightarrow * \mathbb{R}$ such that $\mathfrak{e}(G)|D = G|D = Q$, and $G$ is hypercontinuous, hypersmooth, hyperaltering process defined on the hyperinterval *$[a, T]$. Hence, $G$ satisfies statement (A) in section (2) of [1]. Theorem 7.5.1 also implies that $G$ is hyperuniformly continuous on *$[a, T]$.

Recall how a *-special partition for *$[a, T]$ is generated. Let $0 < \Delta t \in \mathbb{R}^+$. Then $P(\Delta t) = \{a = t_0 \leq \cdots t_n \leq t_{n+1} = T\}$, where $n$ is the largest natural number such that $a + n(\Delta t) \leq T$ and for $i = 0, \ldots, (n-1)$; $t_{i+1} - t_i = \Delta t$, and $t_{n+1} - t_n = b - (a + n(\Delta t)) < \Delta t$. It is possible that $t_{n+1} = t_n$. If $P$ is the set of all special partitions, then letting $dt \in \mu(0)^+$ (the set of all positive infinitesimals) it follows that $P(dt) \in *P$ and $P(dt)$ has the same first-order properties as does $P(\Delta t)$.

**Theorem 8.1.1** Let internal $G$ be hypercontinuous on *$D$ such that $|x - y| < dy$ it follows that $|G(x) - G(y)| < dx$, and there is a hyperfinite partition $\{a = t'_0 < \cdots < t'_{\nu+1} = T\}$ such that for $i = 0, \ldots, \nu+1$; $j = 1, \ldots, m$ we have $t'_j \neq z_j$, $G(t'_{i+1}) - G(t'_i) \in \mu(0)$, $t'_{i+1} - t'_i \in \mu(0)$ and $G(T) - G(a) = \sum_{i=0}^{\nu}(G(t'_{i+1}) - G(t'_i))$.

Proof. Since internal $G$ is *-uniformly continuous, it follows that for any $dy \in \mu(0)^+$ there exists some $\delta$ such that $0 < \delta \in *\mathbb{R}$ and for each $x, y \in *[a, T]$ such that $|x - y| < \delta$, it follows that $|G(x) - G(y)| < dx$. Now let $dy < \delta$ and $dy \in \mu(0)^+$ and consider the *-special partition $P(dy/3)$. Let $y \in [t_i, t_{i+1}]$, $x \in [t_{i+1}, t_{i+2}]$, $i = 0, \ldots, \nu - 1$ and $x, y \neq t'$. Then $|y - x| < dy$ and each *-closed interval is nonempty. By means of internal first-order statements that imply the existence of certain objects and the choice axiom, select $t'_0 = a$, $t'_{\nu+1} = T$ and if $t_{\nu+1} = t'$, then for $i = 1, \ldots, \nu - 2$ select some $t'_i \in [t_i, t_{i+1}]$ such that $t'_i \neq z_j$ for $j = 1, \ldots, m$; or if $t_{\nu+1} \neq t'$, then for $i = 1, \ldots, \nu - 1$ select some $t'_i \in [t_i, t_{i+1}]$ such that $t'_i \neq z_j$ for $j = 1, \ldots, m$. This yields a hyperfinite internal partition with the properties listed in the hypothesis and by*-transfer of the properties of a finite telescoping series, we have that $G(T) - G(a) = \sum_{i=0}^{\nu}(G(t'_{i+1}) - G(t'_i))$, and $|G(t'_{i+1}) - G(t'_i)| < dx$ for $i = 0, \ldots, \nu$ implies that $G(t'_{i+1}) - G(t'_i) \in \mu(0)$ and each $t'_i \in *D$ has the property that $|t'_{i+1} - t'_i| < dy$. This complete the proof.}

We now apply Theorem 8.1.1 to the discrete altering function $Q$. Let $Q$ be the set of all finite partitions of $D$. Then, for $n > 0$ and the partition \{a = t_0 < t_1 < \cdots < t_n \leq t_{n+1} = T\}, consider the partial sequence $S_n: [0, n+1] \rightarrow \mathbb{R}$ defined by $S(i) = t_i$, $i = 0, \ldots, n + 1$. Define $T_i = [t_i, t_{i+1}]$, $i = 0, \ldots, n$. Consider the set
$H = \{ T_i \mid i = 0, \ldots, n \}$. Then $H \in \mathcal{P}(C)$, where $C = [a, T]$. There is an $N \in \mathbb{R}_{\mathcal{P}(C)-\emptyset}$ such that $N(T_i) = \mathcal{Q}(t_{i+1}) - \mathcal{Q}(t_i)$, \( t_i = r_i, i = 0, \ldots, n \). The function $N$ is a resolving process for the function $\mathcal{Q}$ and each $r_i$ is a degree for the constituent $N(T_i)$. Let $M(\mathcal{Q})$ be the set of all such resolving processes generated by the infinite set of finite partitions of $D$ for a fixed $\mathcal{Q}$. Consider the *-finite partition $P(dy/3)$ of $D$ generated in the proof of Theorem 8.1.1. Now modify this *-finite partition in the following manner. Consider the standard finite partition generated by $S: [0,n+1] \to \mathbb{R}$. Let $T_i = [t_i, t_{i+1}] = [S(i), S(i+1)]$, $t_i \neq t'$, $i = 0, \ldots, n$; $H = \{ T_i \mid i = 0, \ldots, n \}$ and, for the fixed $\mathcal{Q}$, $N(t_i) = \mathcal{Q}(t_{i+1}) - \mathcal{Q}(t_i) = G(t_{i+1}) - G(t_i)$, $i = 0, \ldots, n$, where $G$ is in the statement of Theorem 8.1.1. This sequence $S$ extends in the usual manner to $^*S: [0,\nu+1] \to ^*\mathbb{R}$.

Since $0 < |t_i - t_j|$, where $i \neq j$, for each $t_i$ there exists a *-closed interval $[v_j, v_{j+1}]$ generated by $P(dy/3)$ such that $t_i \in (v_j, v_{j+1})$, or $t_i = v_j$ or $v_{j+1}$ not both. In the case that $t_i \in (v_j, v_{j+1})$, the interval is unique. Moreover, there are only finitely many such $t_i$ in the standard partition. Hence for these finitely many real number cases, where $t_i \in (v_j,v_{j+1})$, modify the partition by subdividing $[v_j, v_{j+1}]$ into two intervals $[v_j, t_i] \cup [t_i, v_{j+1}]$. This process can, obviously, be defined by a finite set of first-order statements. This adds an additional finite number of intervals to our hyperfinite partition and yields a partition number $\lambda \in ^*\mathbb{N}$ to replace $\nu$. Since the infinitesimal length of these adjoined intervals is $< dy/3$, Theorem 8.1.1 still holds with $\lambda$ replacing $\nu$. This yields an internal sequence $S': [0,\lambda+1] \to ^*\mathbb{R}$ such that $S'(i) = t'_i$ as defined in the proof of Theorem 8.1.1 with the condition that finitely many of the $t'_i$ correspond to the standard partition elements $t_i$. (Notice that, for all of this construction, the assumed $n$ is fixed.)

From the above, we have the *-closed intervals $T'_i = [t'_i, t'_{i+1}]$, $i = 0, \ldots, \lambda$ as well as the internal $H' = \{ z \mid (z \in \mathcal{P}(^*[a, T])) \land (\exists i \in [0, \lambda]) \land (x \in z \leftrightarrow S'(i) \leq x \leq S'(i+1)) \}$. Obviously, $H' \in \mathcal{P}(C)$. Thus there is in "$M(\mathcal{Q})$" an internal hyperresolving process $N'$ such that $N'(T'_i) = \mathcal{Q}(t'_{i+1}) - \mathcal{Q}(t'_i) = G(t'_{i+1}) - G(t_i)$, where $T' = [S'(i), S'(i+1)] \subset H'$ and $i = 0, \ldots, \lambda$.

Technically, it is not true that $H \subset H'$. Thus define the standard restriction of $N'$ to $N$, where $N$ is generated by the standard sequence $S: [0,n+1] \to \mathbb{R}$ that is obtained as follows: consider the set $\{ S'(i) \mid i = 0, \ldots, \lambda + 1 \} \cap [a, T] = P_0$. Since $P_0$ is a finite standard set, it can be ordered by the $< \subset$ of the reals and let $P_0 = \{ a = t_0 < t_1 < \cdots < t_n \leq T = T \}$ This yields a sequence $S'': [0, n+1] \to \mathbb{R}$, $S''(i) = t_i$, $i = 0, \ldots, n+1$. Let $S'' = S$. Utilizing $S''$, generate the original resolving process $N$ from $N'$.

Application of Theorem 8.1.1 yields the following description. There exists a hyperpartition (generated by) $S'$ for the hyperinterval "$[a, T]$" and $S''$ (generates) the hyperresolution $N'$ for the hyperaltering process $G$. The hyperresolution $N'$ is defined on the hyperfinitely many internal subintervals of "$[a, T]$" and the range of $N'$ is composed of hyperfinitely many hyperconstituents $G(t'_{i+1}) - G(t'_i)$ that, by *-transfer of the standard supremum function defined on nonempty finite sets of real numbers, yields a maximum degree among all of the de-
degrees of the hyperconstituents. This maximum degree is infinitesimal, by Theorem 8.1.1, and since \( G(T) - G(a) \in \mathbb{R}^+ \) and taking \( G \) as nondecreasing, this maximum degree is a positive infinitesimal. By the above restriction process, \( N \) is the restriction of \( N' \) to the standard world. Consequently, \( N' \) satisfies statement (B) in section 4 of [1].

Finally, the length function \( L \) defined on the set of all closed intervals extends to the set of all *-closed intervals that are subsets of *\( \mathbb{R} \). Then \( *L( *[a, T]) = T - a = L([a, T]) \). Thus (C) of section 4 in [1] holds. (D) in section 4 of [1] follows from the unused conclusions that appear in Theorem 8.1.1, among others.

For the nondecreasing bounded classical neutron altering process \( CQ \), there is assumed to exist a standard smooth function \( f \) defined on \([a, t]\) such that \( f|D = CQ \). Now define standard \( G: [a, T] \to \mathbb{R} \) as follows: let \( G_0(t) = f(t), \ t \in [a, t'); \ G_1(t) = f(t), \ t \in (t', T] \). Then since \( G_0(t_0) \leq G_1(t_1) \), for \( t_0 \in [a, t') \) and \( t_1 \in (t', T] \), it follows that \( h = \sup\{G_0(t) \mid t \in [a, t')\} \) exists, and we can let \( G = G_0 \cup G_1 \cup \{(t', h)\} \). Obviously, \( G(t_0) \leq G(t') \leq G(t_1), \ t_0 \in [a, t'), \ t_1 \in (t', T], \) and \( G|D = CQ \). It follows from left and right limit considerations that \( G = f \). (Note: \( G \) is defined in this manner only to conform to the discrete case.) Theorem 8.1.1 holds for *\( G \) and, in this case, we simply repeat the entire discussion that appears after that statement of Theorem 8.1.1 and replace the \( G \) that appears in that discussion with *\( G = f \). This yields a model for statements (E), (F), (G) and (H) in section 4 of reference [1].

**CHAPTER 8 REFERENCE**

1 Herrmann, R. A. Mathematical philosophy and developmental processes, *Nature and System*, 5(1/2) (1983), 17—36.
9. **NSP-WORLD ALPHABETS**

9.1 An Extension.

Although it is often not necessary, we assume when its useful that we are working within the EGS. Further, this structure is assumed to be $|\mathcal{M}_1|^\dagger$-saturated, and a polyn enlargement [5, p. 35], where $\mathcal{M}_1 = \langle \mathbb{R}, +, \cdot, = \rangle$, (or $\mathcal{M}_1 = \langle \mathbb{Q}, +, \cdot, = \rangle$, where $\mathbb{Q}$ is the set of rational numbers). Referring to the paragraph prior to Theorem 7.3.3, it can be assumed that the developmental paradigm $d' \subseteq \star \mathbb{B} \subseteq \star \mathbb{P}_0$. It is not assumed that such a developmental paradigm is obtained from the process discussed in Theorem 7.2.1, although a modification of the proof of Theorem 7.2.1 appears possible in order to allow this method of selection.

**Theorem 9.1.1** Let $d' = \{ [g_i] | i \in \lambda \}$, $|\lambda| < |\mathcal{M}_1|^\dagger$. There exists an ultraword $w \in \star \mathbb{M}_B - \star \mathbb{B}$ such that for each $i \in \lambda$, $[g_i] \in \star \mathcal{S}(\{w\})$.

**Proof.** The same as Theorem 7.3.3 with the change in saturation.

Let $\mathcal{D} = \{ d_i | i \in \lambda \}$, $|\lambda| < |\mathcal{M}_1|^\dagger$, $|d_i| < |\mathcal{M}_1|^\dagger$ and each $d_i \subseteq \star \mathbb{B}$ is considered to be a developmental paradigm either of type $d$ or type $d'$. For each $d_i \in \mathcal{D}$, use the Axiom of Choice to select an ultraword $w_i \in \star \mathbb{M}_B - \star \mathbb{B}$ that exists by Theorems 9.1.1. Let $\{ w_i | i \in \lambda \}$ be such a set of ultrawords.

**Theorem 9.1.2** There exists an ultraword $w' \in \star \mathbb{M}_B - \star \mathbb{B}$ such that for each $i \in \lambda$, $w_i \in \star \mathcal{S}(\{w'\})$ and, hence, for each $d_i \in \mathcal{D}$, $d_i \subseteq \star \mathcal{S}(\{w'\})$.

**Proof.** The same as Theorem 7.3.4 with the change in saturation.

9.2 **NSP-World Alphabets.**

First, recall the following definition. $P_m = \{ f | (f \in T^m) \land (\exists z ((z \in \mathcal{E}) \land (f \in z)) \land \forall x ((x \in \mathbb{N}) \land (x > m) \rightarrow \neg \exists y ((y \in T^2) \land (y \in z)))) \}$. The set $T = i[W]$. The set $P_m$ determines the unique partial sequence $f \in [g] \in \mathcal{E}$ that yields, for each $j \in \mathbb{N}$ such that $0 \leq j \leq m$, $f(j) = i(a)$, where $i(a)$ is an “encoding” in $A_1$ of the alphabet symbol “a” used to construct our intuitive language $W$. The set $[g]$ represents an intuitive word constructed from such an alphabet of symbols.

Within the discipline of Mathematical Logic, it is assumed that there exists symbols — a sequence of variables — each one of which corresponds, in a one-to-one manner, to a natural number. Further, under the subject matter of generalized first-order theories [2], it is assumed that the cardinality of the set of constants is greater than $\aleph_0$. In the forthcoming investigation, it may be useful to consider an alphabet that injectively corresponds to the real numbers $\mathbb{R}$. This yields a new alphabet $\mathcal{A}'$ containing our original alphabet. A new collection of words $\mathcal{W}'$ composed of nonempty finite strings of such alphabet symbols may be constructed. It may also be useful to well-order $\mathcal{R}$. The set $\mathcal{E}$ also exists with respect to the set of words $\mathcal{W}'$. Using the ESG, many previous results in this book now hold with respect to $\mathcal{W}'$ and for the case that we are working in a $|\mathcal{M}_1|^\dagger$-saturated polyn enlargement.

With respect to this extended language, if you wish to except the possibility, a definition as to what constitutes a purely subtle alphabet symbol would need to
be altered in the obvious fashion. Indeed, for $T$ in the definition of $P_m$, we need to substitute $T' = i[W']. Then the altered definition would read that $r \in ^{*}A_1 \simeq ^{*}\mathbb{R}$ is a pure subtle alphabet symbol if there exists an $m \in \mathbb{N}$ and $f \in ^{*}(P_m)$, or if $m \in ^{*}\mathbb{N} - \mathbb{N}$ an $f \in P_m$, and some $j \in ^{*}\mathbb{R}$ such that $f(j) = r \notin i[W']$. Further, some of the previous theorems also hold when the proofs are modified.

Although these extended languages are of interest to the mathematician, most of science is content with approximating a real number by means of a rational number. In all that follows, the cardinality of our language, if not denumerable, will be specified. All theorems from this book that are used to establish a result relative to a denumerable language will be stated without qualification. If a theorem has not been reestablished for a higher language but can be so reestablished, then the theorem will be termed an extended theorem.

9.3 General Paradigms.

There is the developmental paradigm, and for nondetailed descriptions the general developmental paradigm. But now we have something totally new — the general paradigm. It is important to note that the general paradigm is considered to be distinct from developmental paradigms, although certain results that hold for general paradigms will hold for developmental paradigms and conversely. For example, associated with each general paradigm $G_A$ is an ultraword $w_g$ such that the set $G_A \subset ^{*}S(\{w_g\})$ and all other theorems relative to such ultrawords hold for general paradigms. The general paradigm is a collection of words that discuss, in general, the behavior of entities and other constituents of a natural system. They, usually, do not contain a time statement $W_i$ as it appears in section 7.1 for developmental paradigm descriptions. Our interest in this section is relative to only two such general paradigms. The reader can easily generate many other general paradigms.

Let $c'$ be a symbol that denotes some fixed real number and $n'$ a symbol that denotes a natural number. [Note: what follows is easily extended to an extended language.] Suppose that you have a theory which includes each member of the following set (i suppressed).

\[
G_A = \{ An || \text{elementary} || \text{particle} || \alpha(n') || \text{with} || \}
\]

(9.3.1) kinetic || energy || $c' + 1/(n')$. \( n \in G \land n \neq 0 \},
\]

where $G$ is, at the least, a denumerable subset of the real numbers.

Of particular interest is the composition of members of $^*G_A - G_A$. Notice that $|G_A| = |G|$ since $z_1, z_2 \in G_A$ and $z_1 \neq z_2$ iff $[x_1] = z_1, [x_2] = z_2, x_1(30) = x_1(2) \neq x_2(30) = x_2(2), x_1(2), x_2(2) \in G$. Now consider the bijection $K: G_A \rightarrow G$. 
Theorem 9.3.1 The set \([g] \in {}^{*}G_{A} - G_{A}\) iff there exists a unique \(f \in {}^{*}(P_{55})\) and \(\nu \in {}^{*}G - G\) such that \([g] = [f]\), and \(f(55) = i(A), f(54) = i(n), f(53) = i(|||), \ldots, f(30) = f(2), \ldots, f(3) = i(), f(2) = \nu \in {}^{*}G - G \subset {}^{*}\mathbb{R} - \mathbb{R}, f(1) = i(), f(0) = (\cdot)\).)

Proof. From the definition of \(G_{A}\) the sentence

\[
\forall z(z \in E \to (\exists \exists w((w \in G) \land (x \in P_{55}) \land (x \in z) \land (55, i(A)) \in x) \land ((54, i(n)) \in x) \land \cdots \land (x(30) = x(2)) \land \cdots \land ((3, i(\cdot)) \in x) \land (x(2) = w) \land (K(z) = w) \land ((1, i(\cdot)) \in x) \land ((0, i(\cdot)) \in x))].
\]

(9.3.2)

holds in \(\mathcal{M}\), hence in \(^{*}\mathcal{M}\). From the fact that \(K\) is a bijection, it follows that

\(^{*}K[ \cdot G_{A} - G_{A}] = {}^{*}G - G \subset {}^{*}\mathbb{R} - \mathbb{R}\). The result now follows from \(*\)-transfer.

Using Theorem 9.3.1, each member of \(^{*}G_{A} - G_{A}\), when interpreted by considering \(i^{-1}\), has only two positions with a single missing object since positions 30 and 2 do not correspond to any symbol string in our language \(W\). This interpretation still retains a vast amount of content, however. For a specific member, you could substitute a new constructed symbol, not in \(W\), into these two missing positions. Depending upon what type of pure nonstandard number this inserted symbol represents, the content of such a sentence could be startling. Let \(\Gamma'\) be a nonempty set of new symbols disjoint from \(W\) and assume that \(\Gamma'\) is injectively mapped by \(H\) into \(^{*}G - G\).

Although human ability may preclude the actual construction of more than denumerably many new symbols, you might consider this mapping to be onto if you accept the ideas of extended languages with a greater cardinality. As previously, denote these new symbols by \(\zeta'\). Now let

\[
G'_{A} = \{An|||\text{elementary}|||\text{particle}|||\alpha(\zeta')|||\text{with}|||\text{kinetic}|||\text{energy}|||c' + 1/(\zeta')| | H(\zeta') \in {}^{*}G - G\},
\]

(9.3.3)

This leads to the following interpretation stated in terms of describing sets for the extended language.

(1) The describing set \(G_{A}\) (mathematically) exists iff the describing set \(G'_{A}\) (mathematically) exists.

9.4 Interpretations

Recall that the Natural world portion of the NSP-world model may contain \textit{undetectable} objects, where “undetectable” means that there does not appear to exist human, or humanly constructible machine sensors that directly detect the objects or directly measure any of the objects physical properties. The rules of the scientific method utilized within the micro-world of subatomic physics allow all such undetectable Natural objects to be accepted as existing reality.[1] The properties of such objects are indirectly deduced from the observed properties of gross matter. In order to have indirect evidence of the objectively real existence of such objects, such indirectly obtained behavior will usually satisfy a specifically accepted model.
Although the numerical quantities associated with these undetectable Natural (i.e. standard) world objects, if they really do exist, cannot be directly and exactly measured via any known instrumentation, these quantities are still represented by standard mathematical entities. By the rules of correspondence for interpreting pure NSP-world entities, the members of $G_A'$ must be considered as undetectable pure NSP-world objects, assuming any of them exist in this background world. On the other hand, when viewed within the EGS, any finite as well as many infinite subsets of $G_A'$ are internal sets. Consequently, some finite collects of such objects may be assumed to indirectly effect behavior in the Natural world.

The concept of realism often dictates that all interpreted members of a mathematical model be considered as existing in reality. The philosophy of science that accepts only *partial realism* allows for the following technique. One can stop at any point within a mathematically generated physical interpretation. Then proceed from that point to deduce an intuitive physical theory, but only using other not interpreted mathematical formalism as auxiliary constructs or as catalysts. With respect to the NSP-world, another aspect of interpretation enters the picture. Assuming realism, then the question remains which, if any, of these NSP-entities actually indirectly influence Natural world processes? This interpretation process allows for the possibility that none of these pure NSP-world entities has any effect upon the standard world. These ideas should always be kept in mind.

If you accept that such particles as described by $G_A$ can exist in reality, then the philosophy of realism leads to the next interpretation.

(2) *If there exist elementary particles with Natural system behavior described by $G_A$, then there exist pure NSP-world objects that display within the NSP-world behavior described by members of $G_A'$.*

The concept of absolute realism would require that the acceptance of the elementary particles described by $G_A$ is indirect evidence for the existence of the $G_A'$ described objects. I caution the reader that the interpretation we apply to such sets of sentences as $G_A$ are only to be applied to such sets of sentences.

The EGS may, of course, be interpreted in infinitely many different ways. Indeed, the NSP-world model with its physical-type language can also be applied in infinitely many ways to infinitely many scenarios. I have applied it to such models as the MA-model— and the GGU-model among others. In this section, I consider another possible interpretation relative to those Big Bang cosmologies that postulate real objects at or near infinite temperature, energy or pressure. These theories incorporate the concept of the *initial singularity(ies).*

One of the great difficulties with many Big Bang cosmologies is that no meaningful physical interpretation for formation of the initial singularity is forthcoming from the theory itself. The fact that a proper and acceptable theory for creation of the universe requires that consideration not only be given to the moment of zero cosmic time but to what might have occurred “prior” to that moment in the nontime period is what partially influenced Wheeler to consider the concept of a
It is totally unsatisfactory to dismiss such questions as “meaningless” simply because they cannot be discussed in your favorite theory. Scientists must search for a broader theory to include not only the question but a possible answer.

Although the initial singularity for a Big Bang type of state of affairs apparently cannot be discussed in a meaningful manner by many standard physical theories, unless one adjoins to the theory an ad hoc quantum field, it can be discussed by application of our NSP-world language. Let $c'$ be a symbol that represents any fixed real number. Define

$$G'_{\text{B}} = \{ \text{An} \mid \text{elementary} \mid \text{particle} \mid \alpha(n') \mid \text{with} \mid \text{total} \mid \text{energy} \mid c' + n'. \mid n \in \mathbb{N} \}, \quad (9.4.1)$$

Application of Theorem 9.3.1 to $G_{\text{B}}$ yields the set

$$G'_{\text{B}} = \{ \text{An} \mid \text{elementary} \mid \text{particle} \mid \alpha(\zeta') \mid \text{with} \mid \text{total} \mid \text{energy} \mid c' + \zeta'. \mid \zeta \in \ast \mathbb{N} - \mathbb{N} \}, \quad (9.4.2)$$

(3) If there exist elementary particles with Natural system behavior described by $G_{\text{B}}$, then there exist pure NSP-world objects that display within the NSP-world behavior described by members of $G'_{\text{B}}$.

The particles being described by $G'_{\text{B}}$ have various infinite energies. These infinite energies do not behave in the same manner as would the real number energy measures discussed in $G_{\text{B}}$. As is usual when a metalanguage physical theory is generated from a formalism, we can further extend and investigate the properties of the $G'_{\text{B}}$ objects by imposing upon them the corresponding behavior of the positive infinite hyperreal numbers. This produces some interesting propositions. Hence, we are able to use a nonstandard physical world language in order to give further insight into the state of affairs at or near a cosmic initial singularity. This gives one solution to a portion of the pregeometry problem. I point out that there are other NSP-world models for the beginnings of our universe, if there was such a beginning. Of course, the statements in $G'_{\text{B}}$ need not be related at all to any Natural world physical scenario, but could refer only to the behavior of pure NSP-world objects.

Notice that Theorems such as 7.3.1 and 7.3.4 relative to the generation of developmental paradigms by ultrawords, also apply to general paradigms, where $M, M_{\text{B}}, P_0$ are defined appropriately. The following is a slight extension of Theorem 7.3.2 for general paradigms. Theorem 9.4.1 will also hold for developmental paradigms.

**Theorem 9.4.1** Let $G_{\text{C}}$ be any denumerable general paradigm. Then there exists an ultraword $w \in \ast P_0$ such that for each $F \in G_{\text{C}}$, $F \in \ast S(\{w\})$ and there exist infinitely many $[g] \in \ast G_{\text{C}} - G_{\text{C}}$ such that $[g] \in \ast S(\{w\})$.

Proof. In the proof of Theorem 7.3.2, it is shown that there exists some $\nu \in \ast \mathbb{N} - \mathbb{N}$ such that $\ast h[[0, \nu]] \subset \ast S(\{w\})$ and $\ast h[[0, \nu]] \subset \ast G_{\text{C}}$. Since $|\ast h[[0, \nu]]||M_1|_1^+$, then $|\ast h[[0, \nu]] - h[\mathbb{N}]| \geq |M_1|_1^+$, for $h$ is a bijection. This completes the proof. \[\square\]
Corollary 9.4.1.1 Theorem 9.4.1 holds, where $G_C$ is replaced by a developmental paradigm.

(4) Let $G_C$ be a denumerable general paradigm. There exists an intrinsic ultranatural process, $^*S$, such that objects described by members of $G_C$ are produced by $^*S$. During this production, numerously many pure NSP-objects as described by statements in $^*G_C - G_C$ are produced.

9.5 A Barrier To Knowledge.

Our final discussion in this chapter deals with the use of $|M|^+\text{-saturated models}$ and our ability to analyze sets of sentences such as $G_A'$. It is a very strange property of the human mind that it often produces an infinite cause and effect sequence.

Consider the following comprehensible and potentially infinite set of sentences \{I think, I think about my thinking, I think about my thinking about my thinking, I think about my thinking about my thinking about my thinking, \ldots\}. As far as comprehension is concerned, one can ask what would be the “first cause” for our thinking? For the Natural sciences, we have Engel’s biological sequence of evolutionary causes and effects and, of course, our previously mentioned initial singularity problem or as Misner, Thorne and Wheeler write, “No problem of cosmology digs more deeply into the foundations of physics than the question of what ‘proceeded’ the ‘initial’ state....” [3] A Natural science question and one which contains some logical difficulties might be “What precedes that which precedes?” Regardless of whether or not this strange mental behavior persists when we analyze Natural system behavior, the next result shows the existence of a possible Natural barrier to human knowledge.

Each of our previous investigations is done with respect to a specific NSP-world structure $^*M$ based upon an infinite standard set $H$ (with a cardinality usually equal to $\aleph_0$) into which is mapped the symbols and words for all languages. The requirement that $H$ be a standard set is relative to the standard universe in which we function. Although there are infinitely many distinct nonisomorphic NSP-world structures, each of our results is with respect to members of a subclass of the class of all such structures. In particular, $|M|^+$-saturated polyenlargement, where $\mathcal{M}$ is based upon a standard set $H$, where $\mathbb{N} \subset H$.

In order to analyze general paradigms $G_A', G_B'$ and the like, we need to start, I believe, with a comprehensible set of sentences, such as $G_A, G_B$, with nonempty content and insert new symbols but retain some of the content of the original sentences. What is shown next is that if we use any of our models based on $H$ and require them to be $|\mathcal{M}|^+$-saturation polyenlargement, then we cannot embed our new alphabet into the standard set $H$ and, thus, we cannot fully analyze sets of sentences such as $G_A', G_B'$ using our embedding procedures.
Theorem 9.5.1 Let $\Gamma'$ be a set of symbols adjoined to a countable alphabet $\mathcal{A}$, which is disjoint from $\mathcal{A}$, and such that it is used to obtain the set of sentences in $G'_B$. Let $\ast \mathcal{M} = \langle \ast \mathcal{H}, \in, = \rangle$ be any $|\mathcal{M}|^+$-saturated polyenlargement of a superstructure based on the ground set $H$, where here $\mathbb{N} \subset H \subset \mathbb{R}$. There does not exist an injection from $\Gamma' \cup \mathcal{A}$ into $L$, where $L \in \mathcal{H}$.

Proof. Suppose that there exists an injection $i : (\Gamma' \cup \mathcal{A}) \to L$. Since the model is a polyenlargement, then $|\Gamma' \cup \mathcal{A}| \geq |\mathcal{M}|^+$. However, $|L| < |\mathcal{M}|^+$. But under the assumption $|\Gamma' \cup \mathcal{A}| \leq |L|$. This contradiction implies that the injection does not exist and this completes the proof. 

CHAPTER 9 REFERENCES

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10. LAWS, RULES AND OTHER THINGS

10.1 More About Ultrawords.

Previously, we slightly investigated the composition of an ultraword $w \in \mathcal{M}_d - d$. Using the idea of the minimum informal language $P_0 \subset P$, where $d$ is denumerable and $P$ is a propositional language, our interest now lies in completely determining the composition of $\mathcal{S}(\{w\})$. [Note: since our language is informal axiom (3) and (4) are redundant in that superfluous parentheses have been removed.]

First, two defined sets.

\[ A = \{ x \mid x \in P_0 \text{ is an instance of an axiom for } S \} \quad (10.1.1) \]
\[ C = \{ x \mid x \in P_0 \text{ is a finite } (\geq 1) \text{ conjunction of members of } d \} \quad (10.1.2) \]

Notice that it is also possible to refine the set $C$ by considering $C$ to be an ordered conjunction with respect to the ordering of the indexing set used to index members of $d$. Further, as usual, we have that $A$, $C$, $d$ are mutually disjoint.

**Theorem 10.1.1** Let $w \in \mathcal{M}_d - d$ be an ultraword for infinite $d \subset \mathcal{S}(\{w\})$. Then $\mathcal{S}(\{w\}) = \mathcal{A} \cup Q_1 \cup d_1'$, where for internal *-finite $d_1'$, $d \subset d_1' \subset \mathcal{S}(\{w\})$ and internal $Q_1 \subset \mathcal{C}$ is composed of *-finite $(\geq 1)$ conjunctions (i.e., $i(\{|||\} \cup \{||\})$) of distinct members of $d_1'$ and $w \in Q_1$. Further, each member of $d_1'$ and no other *-proposition is used to form the *-finite conjunctions in $Q_1$, the only *-propositions in $\mathcal{S}(\{w\})$ are those in $w$, and $\mathcal{A}$, $Q_1$, and $d_1'$ are mutually disjoint.

Proof. The intent is to show that if $w \in \mathcal{M}_d - d$, then $S(\{w\}) = A \cup Q \cup d'$, where $Q \subset C$, finite $d' \subset d$ and $Q$ is composed of finite $(\geq 1)$ conjunctions of members of $d'$, each member of $d'$ is used to form these conjunctions and no other propositions.

Let $J$ be the set of propositional atoms in the composite $w$. (0) Then $J \subset S(\{w\})$. If $K$ is the set of all propositional atoms in $S(\{w\})$, then $J \subset K$. Let $b \in K - J$. It is obvious that $b \notin S(\{w\})$ since otherwise $\{w, b\} \subset S_0(\{w\})$ but $\not\in S_0 \rightarrow b$. Thus, $J = K$. Consequently, $J \subset S(\{w\})$, $J \subset d$ and there does not exist an $F \in d - J$, such that $F \notin S(\{w\})$. (1) Let $J = d'$. The only propositional atoms in $S(\{w\})$ are those in $w$. Obviously $A \subset S(\{\emptyset\})$.

Assume the language $P_0$ is inductively defined from the set of atoms $d$. Recall that for our axioms $\mathcal{X} = D \rightarrow F$, the strongest connective in $\mathcal{X}$ is $\rightarrow$. While in $\mathcal{D}$, or $\mathcal{F}$ when applicable, the strongest connective is $\land$. Since $\emptyset \subset \{w\}$, it follows that $S(\{w\}) = S(\emptyset) \cup S(\{w\})$. Let $b \in S(\emptyset)$. The only steps in the formal proof for $b$ contain axioms or follows from modus ponens. Suppose that step $B_k = b$ is the first modus ponens step obtained from steps $B_i$, $B_j$, $i, j < k$, where $B_i = A \rightarrow b$, $B_j = A$. The strongest connective for each axiom is $\rightarrow$. However, since $A \rightarrow b$ is an axiom, the strongest connective in $A$ is $\land$. This contradicts the requirement that $\land$ must also be an axiom with strongest connect $\rightarrow$. Thus no modus ponens step can occur in a formal proof for $b$. Hence, (2) $A = S(\emptyset)$. (No modus ponens step can occur using two axioms.)
Let $B_k = b_1 \in P_0$ and suppose (a) that $b_1 = w$, or (b) $b_1 \neq w$ and is the first nonaxiom step that appears in a formal demonstration from the hypothesis $w$. Assume (b). Then all steps $B_i \in \{w\} \cup A$, $0 \leq i < k$. Then the only way that $b_1$ can be obtained is by means of modus ponens. However, all other steps, not including that which is $w$, are axioms. No modus ponens step can occur using two axioms. Thus one of the steps used for modus ponens must not be an axiom. The only nonaxiom that occurs prior to the step $B_k$ is the step $B_m = w$. Hence, one of the steps required for $B_k$ must be $B_m = w$. The other step must be an axiom of the form $w \rightarrow b_1$ and $b_1 \neq w$. Thus, from the definition of the axioms (3) $b_1$ is either a finite (≥ 1) conjunction of atoms in $d'$, or a single member of $d'$. Assume strong induction. Hence, for $n > 1$, statement (3) holds for all $r$, $1 \leq r \leq n$. A similar argument shows that (3) holds for the $b_{n+1}$ nonaxiom step. Thus by induction, (3) holds for all nonaxiom steps.

Hence, there exists a $Q \subset C$ such that each member of $Q$ is composed of finitely many (≥ 1) distinct members of $d'$ and the set $G(Q)$ of all the proposition atoms that appear in any member of $Q = d' = J$ since $w \in Q$. Moreover, (4) $S(\{w\}) = A \cup d' \cup Q$ and (5) $A$, $d'$, $Q$ are mutually disjoint.

\[
\forall x(x \in M_d - d \rightarrow \exists y \exists z((y \in F(d)) \land (z \subset C) \land (S(\{x\})) = (A \cup y \cup z) \land (A \cap y = \emptyset) \land (A \cap z = \emptyset) \land (x \in z)
\]

(10.1.3)

holds in $M$, hence also in $^*M$. So, let $w$ be an ultraword. Then there exists internal $Q_1 \subset ^*C$, $w \in Q_1$ and *-finite $d'_1 \subset d$ such that $d \subset ^*S(\{w\}) = ^*A \cup d'_1 \cup Q_1$; $^*A$, $d'_1$, $Q_1$ are mutually disjoint and $^*G(Q_1) = d'_1 = J$. Hence, $d \subset d'_1$.

Now to analyze the objects in $Q_1$. Let $d = \{F_i \mid i \in \mathbb{N}\}$. Consider a bijection $h: \mathbb{N} \rightarrow d$ defined by $h(n) = F_n = \{f\}$, where $f \in T^0$ is the special member of $F_n$ such that $f = \{0, f(0)\}$, $f(0) = i(F_n) = q_n \in i[d]$. From the above analysis, (A) $[g] \in S(\{w\}) - A - d$, $(w \in M_d - d)$, if there exist exist $k, j \in \mathbb{N}$ such that $k < j$ and $f'_1 = i[P_0]|^{2(j-k)}$ such that $[f'_1] = [g]$, and this leads to (B) that for each even $2p$, $0 \leq 2p \leq 2(j - k)$; $f'_1(2p) = q_{k+p} \in i[P_0] \subset A_1$, $\{(0, q_{k+p})\} \in d'$, all such $q_{k+p}$ being distinct. For each odd $2p+1$ such that $0 \leq 2p+1 \leq 2(j - k)$, $f'_1(2p+1) = i[[|||and|||]]$. Also (C) $h(p) = h([k, j]])$ if there exists an even $2p$ such that $0 \leq 2p \leq 2(j - k)$ and $f'_1(2p) = h(p) = q_{k+p} \in i[P_0]$. [Note that 0 is considered to be an even number.]

By *-transfer of the above statements (A), (B) and (C), $[g] \in Q_1$ iff there exists some $j, k \in ^*\mathbb{N}$, $k < j$, and $f' \in ((i[P_0])^{2(j-k)})$ such that $[f'] = [g]$ and $h([k, j]) \subset d$. Moreover, each $^{*}h(r)\subseteq [k, j]$ is a distinct member of $d$. The conjunction "codes" for $i[[|||and|||]] \in A_1$ that are generated by each odd $2p + 1$ are all the same and there are *-finitely many of them. Hence, $Q_1$ is the *-finite (≥ 1) conjunctions of distinct members of $d'_1$, no other *-propositions are utilized and since $^*G(Q_1) = d'_1$, all members of $d'_1$ are employed for these conjunctions. This completes the proof. \[\]
Corollary 10.1.1.1 Let \( w \in \text{ }^*M_d = \text{ }^*d \) be an ultraword for denumerable \( d \) such that \( d \subset \text{ }^*S\{(w)\} \). Then \( \text{ }^*S\{(w)\} \cap P_0 = A \cup Q \cup d \) and \( A, Q, d \) are mutually disjoint. The set \( Q \) is composed of finite \( \geq 1 \) conjunctions of members of \( d \) and all of the members of \( d \) are employed to obtain these conjunctions.

Proof. Recall that due to the finitary character of our standard objects \( \sigma A = A = \text{ }^*A \cap P_0 \). In like manner, since \( d \subset d'_1, d'_1 \cap P_0 = d \). Now \( P_0 \cap Q_1 \) are all of the standard members of \( Q_1 \). For each \( k \in \text{ }^*\mathbb{N} \), \( \text{ }^*h(k) = F_k \in \text{ }^*d \) and conversely. Further, \( F_k \in d \) iff \( k \in \mathbb{N} \). Restricting \( k, j \in \mathbb{N} \) in the above theorem yields standard finite \( \geq 1 \) conjunctions of standard members of \( d'_1 \); hence, members of \( d \). Since ultraword \( w \in Q_1 \), we know that there exists some \( \eta \in \text{ }^*\mathbb{N} - \mathbb{N} \) and \( f'_1 \in \text{ }^*(i[P_0])^{2\eta} \), where \( f'_1 \) satisfies the \( ^* \)-transfer of the properties listed in the above theorem. Since finite conjunctions of standard members of \( d'_1 \) are \( ^* \)-finite conjunctions of members of \( d'_{1j} \) and \( d = d_1' \cap d_{1j}' \), it follows that all possible finite conjunctions of members of \( d \) that are characterized by the function \( f'_1 \in i[P_0]^{2(j-k)} \) are members of \( Q_1 \) for each such \( j, k < \eta \). Also for such \( j, k \) the values of \( f'_1 \) are standard. On the other hand, any value of \( f'_1 \) is nonstandard iff it corresponds to a member of \( d'_1 - d \). Thus \( Q_1 \cap P_0 = Q \) and this completes the proof. 

If it is assumed that each member of \( d \) describes a Natural event (i.e. N-event) at times indicated by \( X_i \), dropping the \( X_i \) may still yield a denumerable developmental paradigm without specifically generated symbols such as the “\( i \)”.

Noting that \( d'_1 \) is \( ^* \)-finite and internal leads to the conclusion that we can have little or no knowledge about the word-like construction of each member of \( d'_1 - d \). These pure nonstandard objects can be considered as describing pure NSP-world events, as will soon be demonstrated. Therefore, it is important to understand the following interpretation scheme, where descriptions are corresponded to events.

Standard or internal NSP-world events or sets of events are interpreted as directly or indirectly influencing N-world events. Certain external objects, such as the standard part operator, among others, are also interpreted as directly or indirectly influencing N-world events.

Notice that standard events can directly or indirectly affect standard events. In the micro-world, the term indirect evidence or verification is a different idea than indirect influences. You can have direct or indirect evidence of direct or indirect influences when considered within the N-world. An indirect influence occurs when there exists, or there is assumed to exist, a mediating “something” between two events. Of course, indirect evidence refers to behavior that can be observed by normally accepted human sensors as such behavior is assumed to be caused by unobserved events. However, the evidence for pure NSP-world events that directly or indirectly influence N-world events must be indirect evidence under the above interpretation.

In order to formally consider NSP-world events for the formation of objective standard reality, proceed as follows: let \( \mathcal{O} \) be the subset of \( \mathcal{W} \) that describes those
Natural events that are used to obtain developmental or general paradigms and the like. Let $E_j \in \mathcal{O}$. Linguistically, assume that each $E_j$ has the spacing symbol $|||$ immediately to the right. Thus within each $T_i$, there is a finite symbol string $F_i = E_j \in \mathcal{O}$ that can be joined by the justaposition (i.e. join) operation to other event descriptions. Assume that $W_1$ is the set of nonempty symbol strings (with repetitions) formed from members of $\mathcal{O}$ by the join operation. These finite strings of symbols generate the basic elements for our partial sequences.

Obviously, $W_1 \subset W$. Consider $T'_i = \{XW_i \mid X \in W_1\}$ and note that in many applications the time indicator $W_i$ need not be of significance for a given $E_j$ in some of the strings. Obviously, $T'_i \subset T_i$ for each $i$. For our isomorphism $i$ onto $A_1$, the following hold.

\begin{equation}
\forall y (y \in \mathcal{E} \rightarrow (y \in T'_i \leftrightarrow \exists x \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \land (x \in \mathbb{N}) \land (f(0) = i[W_j]) \land (f \in P) \land \forall z((z \in \mathbb{N}) \land (0 < z \leq x) \rightarrow f(z) \in w)))).
\end{equation}

\begin{equation}
\forall x (x \in \mathbb{N} \rightarrow \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \land (f \in P) \land \forall z((z \in \mathbb{N}) \rightarrow (0 < z \leq x \rightarrow f(z) \in w)))).
\end{equation}

\begin{equation}
\forall w((\emptyset \neq w \in F(i[\mathcal{O}]) \rightarrow \exists x \exists f \exists y((f \in P) \land (x \in \mathbb{N}) \land (y \in T'_i) \land (f(y) \land \forall z((z \in \mathbb{N}) \rightarrow (0 < z \leq x \rightarrow f(z) \in w)))))).
\end{equation}

Since each finite segment of a developmental paradigm corresponds to a member of $T'_i$, each nonfinite hyperfinite segment should correspond to a member of $^*(T'_i) - T'_i$ and it should be certain individual segments of such members of $^*(T'_i) - T'_i$ that correspond to the ultranatural events produced by an ultraword; UN-events that cannot be eliminated from an NSP-world developmental paradigms. [Note: For a scientific language, 10.1.4 - 10.1.6 and other such statements correspond to a $W'$ as generated by, at least, a denumerable alphabet as used in 9.2, 9.3.]

### 10.2 Laws and Rules.

One of the basic requirements of human mental activity is the ability to recognize the symbolic differences between finitely long strings of symbols as necessitated by our reading ability and to apply linguistic rules finitely many times. Gödel numberings specifically utilize such recognitions and the rules for the generation of recursive functions must be comprehended with respect to finitely many applications. Observe that Gödel number recognition is an “ordered” process while some fixed intuitive order is not necessary for the application of the rules that generate recursive functions.

In general, the simplest “rule” for ordered or unordered finite human choice, a rule that is assumed to be humanly comprehensible by finite recognition, is to simply list the results of our choice (assuming that they are symbolically representable in some fashion) as a partial finite sequence for ordered choice or as a finite set of finitely long symbol strings for an unordered choice. Hence, the end result for a finite choice can itself be considered as an algorithm “for that choice only.” The next application of such a finite choice rule would yield the exact same partial
sequence or choice set. Another more general rule would be a statement which would say that you should “choose a specific number of objects” from a fixed set (of statements). Yet, a more general rule would be that you simply are required to “choose a finite set of all such objects,” where the term “finite” is intuitively known. Of course, there are numerous specifically described algorithms that will also yield finite choice sets.

From the symbol string viewpoint, there are trivial machine programmable algorithms that allow for the comparison of finitely long symbols with each member of a finite set of symbol strings B that will determine whether or not a specific symbol string is a member of B. These programs duplicate the results of human symbol recognition. As is well-known, there has not been an algorithm described that allows us to determine whether or not a given finite symbol string is a member of the set of all theorems of such theories as formal Peano Arithmetic. If one accepts Church’s Thesis, then no such algorithm will ever be described.

Define the general finite human choice relation on a set A as \( H_0(A) = \{(A, x) \mid x \in F_0(A)\} \), where \( F_0 \) is the finite power set operator (including the empty set = no choice is made). Obviously, the inverse \( H_0^{-1} \) is a function from \( F(A) \) onto \( \{A\} \). There are choice operators that produce sets with a specific number of elements that can be easily defined. Let \( F_1(A) \) be the set of all singleton subsets of \( A \). The axioms of set theory state that such a set of singleton sets exists. Define \( H_1(A) = \{(A, x) \mid x \in F_1(A)\} \), etc. Considering such functions as defined on sets \( X \) that are members of a superstructure, then these relations are subsets of \( \mathcal{P}(X) \times \mathcal{P}(X) \) and as such are also members of the superstructure.

Let \( A = P_0 \). Observe that \( H_0(A) = \{(*A, x) \mid x \in F_0(A)\} \) and \( H_1(A) = \{(*A, x) \mid x \in F_1(A)\} \) for each \( i \geq 1 \). With respect to an ultraword \( w \) that generates the general and developmental paradigms, we know that \( w \in P_0 - P_0 \) and that \( \{(*P_0, \{x\}) \} \in H_1(P_0) \). The actual finite choice operators are characterized by their set-theoretic second projector operator \( P_2 \) as it is defined on \( H_1(A) \). This operator embedded by the injection \( \theta \) is the same as \( P_2 \) as it is defined on \( H_1(A) \). Thus, when \( h = (A, x) \in H_1(A) \), then we can define \( x = P_2(h) = C_i(h) = C_i(a, b) \). The maps \( C_i \) and \( C_i \), formally defined below, are the specific finite choice operators. For consistency, we let \( C_i \) and \( C_i \) denote the appropriate finite choice operators for \( H_1(A) \) and \( H_1(A) \), respectively.

Since the \( *P_2 \) defined on say \( H_1(A) \) is the same as the set-theoretic second projection operator \( P_2 \), it would be possible to denote \( *C_i \) as \( C_i \) on internal objects. For consistency, the notation \( *C_i \) for these special finite choice operators is retained. Formally, let \( C_i: H_i(A) \to F_i(A) \). Observe that \( *C_i = \{(*a, b) \mid (a, b) \in C_i\} \) for \( i \geq 1 \) and, for \( b \in F_i(A) \), \( C_i(((A, b)) = b \) implies that \( *C_i(((A, b)) = \{a \mid a \in b\} = b \) from the construction of \( E \). Thus in contradistinction to the consequence operator, for each \( (A, b) \in H_1 \), the image \( (*C_i)\((A, b)) = (*)C_i(((A, b))) \) is \( *C_i(((A, b))) \) for \( (A, b) \in H_1 \). Consequently, the set map...
$\sigma C_i : \sigma H_i \rightarrow F_i(A) = \sigma(F_i(A))$ and $* C_i | \sigma H_i = \sigma C_i$. Finally, it is not difficult to extend these finite choice results to general internal sets.

In the proofs of such theorems as 7.2.1, finite and other choice sets are selected due to their set-theoretic existence. The finite choice operators $C_i$ are not specifically applied since these operators are only intended as a mathematical model for apparently effective human processes — procedures that generate acceptable algorithms. As is well-known, there are other describable rules that also lead to finite or infinite collections of statements. Of course, with respect to a Gödel encoding $i$ for the set of all words $W$ the finite choice of readable sentences in $E$ is one-to-one and effectively related to a finite and, hence, recursive subset of $\mathbb{N}$.

From this discussion, the descriptions of the finite choice operators would determine a subset of the set of all algorithms (“rules” written in the language $W$) that allow for the selection of readable sentences. Notice that before algorithms are applied there may be yet another set of readable sentences that yields conditions that must exist prior to an application of such an algorithm and that these application rules can be modeled by members of $E$.

In order to be as unbiased as possible, it has been required for N-world applications that the set of all frozen segments be infinite. Thus, within the proof of Theorem 7.2.1, every N-world developmental, as well as a general paradigm, is a proper subset of a *-finite NSP-world paradigm, and the *-finite paradigm is obtained by application of the *-finite choice operator $* C_0$. As has been shown, such *-finite paradigms contain pure unreadable (subtle) sentences that may be interpreted for developmental paradigms as pure refined NSP-world behavior and for general paradigms as specific pure NSP-world ultranatural events or objects.

Letting $\Gamma$ correspond to the formal theory of Peano Arithmetic, then assuming Church’s Thesis, there would not exist a N-world algorithm (in any human language) that allows for the determination of whether or not a statement $F$ in the formal language used to express $\Gamma$ is a member of $\Gamma$. By application of the *-finite choice operator $* C_0$, however, there does exist a *-finite $\Gamma'$ such that $\sigma \Gamma = \Gamma \subset \Gamma'$ and, hence, within the NSP-world a “rule” that allows the determination of whether or not $F \in \Gamma'$. If such internal processes mirror the only allowable procedures in the NSP-world for such a “rule,” then it might be argued that we do not have an effective NSP-world process that determines whether or not $F$ is a member of $\Gamma$ for $\Gamma$ is external.

As previously alluded to at the beginning of this section, when a Gödel encoding $i$ is utilized with the N-world, the injection $i$ is not a surjection. When such Gödel encodings are studied, it is usually assumed, without any further discussion, that there is some human mental process that allows us to recognize that one natural number representation (whether in prime factored form or not) is or is not distinct from another such representation. It is not an unreasonable assumption to assume that the same effective (but external) process exists within the NSP-world. Thus within the NSP-world there is a “process” that determines whether or not an object is a member of $* \mathbb{N} - \mathbb{N} = \mathbb{N}_\infty$ or $\mathbb{N}$. Indeed, from the ultraproduct con-
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struction of our nonstandard model, a few differences can be detected by the human mathematician. Consequently, this assumed NSP-world effective process would allow a determination of whether or not $F = [f_m]$ is a member of $\Gamma$ by recalling that $f_m \in P_m$ signifies that $[f_m] \in \ast \Gamma - \Gamma$ implies $m \in \mathbb{N}_\infty \simeq \ast \mathcal{W} - \mathcal{W} = \ast A_1 - A_1$.

The above NSP-world recognition process is equivalent, as defined in Theorem 7.2.1, to various applications of a single (external) set-theoretic intersection. Therefore, there are internal processes, such as $\ast C_0$, that yield pure NSP-world developmental paradigms and a second (external) but acceptable NSP-world effective process that produces specific N-world objects. Relative to our modeling procedures, it can be concluded that both of these processes are intrinsic ultranatural processes.

With respect to Theorem 10.1.1, the NSP-world developmental or general paradigm generated by an ultraword is $\ast$-finite and, hence, specifically NSP-world obtainable prior to application of $\ast S$ through application of $\ast C_0$ to $\ast d$. However, this composition can be reversed. The NSP-world (IUN) process $\ast C_1$ can be applied to the appropriate $\ast M_d$ type set and an appropriate ultraword $w \in \ast M_d$ obtained. Composing $\ast C_1$ with $\ast S$ would yield $d'_1$ in a slightly less conspicuous manner. Obviously, different ultrawords generate different standard and nonstandard developmental or general paradigms.

To complete the actual mental-type processes that lead to the proper ordered event sequences, the above discussion for the finite choice operators is extended to the human mental ability of ordering a finite set in terms of rational number subscripts. New choice operators are defined that model not just the selection of a specific set of elements that is of a fixed finite cardinality but also choosing the elements in the required rational number ordering. The ultrawords $w$ that exist are $\ast$-finite in length. By application of the inverses of the $f$ and $\tau$ functions of section 7.1, where they may be considered as extended standard functions $\ast f$ and $\ast \tau$, there would be from analysis of extended theorem 7.3.2 a hyperfinite set composed of standard or nonstandard frozen segments contained in an ultraword. Further, in theorem 7.3.2, the chosen function $f$ does not specifically differentiate each standard or nonstandard frozen segment with respect to its “time” stamp subscript. There does exist, however, another function in the $\ast$-equivalence class $[g] = w$ that will make this differentiation. It should not be difficult to establish that after application of the ultralogic $\ast S$, there is applied an appropriate mental-like hyperfinite ordered choice operator (an IUN-selection process) and that this would yield that various types of event sequences. Please note that each event sequence has a beginning point of observation. This point of observation need not indicate the actual moment when a specific Natural system began its development.

Various subdevelopmental (or subgeneral) paradigms $d_i$ are obtained by considering the actual descriptive content (i.e. events) of specific theories $\Gamma_i$ that are deduced from hypotheses $\eta_i$, usually, by finitary consequence operators $S_i$ (the inner logics) that are compatible with $S$. In this case, $d_i \subset S_i(\eta_i)$. It is also possible to include within $\{d_i\}$ and $\{\eta_i\}$ the assumed descriptive chaotic behavior that seems
to have no apparent set of hypotheses except for that particular developmental paradigm itself and no apparent deductive process except for the identity consequence operator. In this way, such scientific nontheories can still be considered as a formal theory produced by a finitary consequence operator applied to an hypothesis. Many of these hypotheses \( \eta \) contain the so-called natural laws (or first-principles) peculiar to the formal theories \( \Gamma_i \) and the theories language, where it is assume that such languages are at least closed under the informal conjunction and conditional.

Consider each \( \eta \) to be a general paradigm. For the appropriate \( M \) type set constructed from the denumerable set \( B = \{ \bigcup \{ d_i \} \cup (\bigcup \{ \eta_i \} ) \} \), redefine \( M_B \) to be the smallest subset of \( P_0 \) containing \( B \) and closed under finite (\( \geq 0 \)) conjunction. (The usual type of inductively defined \( M_B \).) Then there exist ultrawords \( w_i \in \ast M_B - \ast B \) such that \( \eta_i \subset \ast S(\{ w_i \}) \) (where due to parameters usually ultranatural laws exist in \( \ast S(\{ w_i \}) - \eta_i \) and \( d_i \subset \ast S(\{ w_i \}) \)). Using methods such as those in Theorem 7.3.4, it follows that there exists some \( w^* \in \ast M_B - M_B \) such that \( w_i \in \ast S(\{ w^* \}) \) and, consequently, \( \eta_i \cup d_i \subset \ast S(\{ w^* \}) \). Linguistically, it is hard to describe the ultraword \( w^* \). Such a \( w^* \) might be called an ultimate ultranatural hypothesis or the ultimate building plain.

**Remark.** It is not required that the so-called Natural laws that appear in some of the \( \eta_i \) be either cosmic time or universally applicable. They could refer only to local first-principles. It is not assumed that those first-principles that display themselves in our local environment are universally space-time valid.

Since the consequence operator \( S \) is compatible with each \( S_i \), it is useful to proceed in the following manner. First, apply the IUN-process \( "S" \) to \( \{ w^* \} \). Then \( d_i \cup \eta_i \subset \ast S(\{ w^* \}) \). It now follows that \( d_i \cup \eta_i \subset \ast S(\{ w_i \}) \subset \ast S_i(\ast S(\{ w_i \})) \subset \ast S_i(\ast S_i(\{ w_i \})) = \ast S_i(\{ w_i \}) \). Observe that for each \( a \in \Gamma_i \) there exists some finite \( F_i \subset \eta_i \) such that \( a \in S_i (F_i) \). However, \( F_i \subset \eta_i \) for each member of \( F(\eta_i) \) implies that \( a \in \ast S_i (F_i) \subset \ast S_i (\ast S_i (\{ w_i \})) \). Consequently, \( \Gamma_i \subset \ast S_i (\ast S_i (\{ w_i \})) \).

We now make the following observations relative to “rules” and deductive logic. It has been said that science is a combination of empirical data, induction and deduction, and that you can have the first two without the last. That this belief is totally false should be self-evident since the philosophy of science requires its own general rules for observation, induction, data collection, proper experimentation and the like. All of these general rules require logical deduction for their application to specific cases — the metalogic. Further, there are specific rules for linguistics that also must be properly applied prior to scientific communication. Indeed, we cannot even open the laboratory door — or at least describe the process — without application of deductive logic. The concept of deductive logic as being the patterns our “minds” follow and its use exterior to the inner logic of some theory should not be dismissed for even the (assumed?) mental methods of human choice that occur prior to communicating various scientific statements and descriptions.

Finally, with respect to the hypothesis rule in [9], it might be argued that we
can easily analyze the specific composition of all significant ultrawords, as has been previously done, and the composition of the nonstandard extension of the general paradigm. Using this assumed analysis and an additional alphabet, one might obtain specific information about pure NSP-world ultranatural laws or refined behavior. Such an argument would seem to invalidate the cautious hypothesis rule and lead to appropriate speculation. However, such an argument would itself be invalid.

Let $W_1$ be an infinite set of meaningful readable sentences for some description and assume that $W_1$ does not contain any infinite subset of readable sentences each of which contains a mathematically interpreted entry such as a real number or the like. Since $W_1 \subset W$ and the totality $T_i = \{XW_i \mid X \in W\}$ is denumerable, the subtotality $T'_i = \{XW_i \mid X \in W_1\}$ is also denumerable. Hence, the external cardinality of $*T'_i \geq |M|^+.$

Consider the following sentence

$$\forall z (z \in i[W_1] \rightarrow \exists y \exists x ((y \in A^{[0,1]}_1) \land (x \in T'_i) \land (y \in x) \land ((0, i[W_i]) \in y) \land ((1, z) \in y))). \quad (10.2.1)$$

By $*-\text{transfer}$ and letting “$z$” be an element in $* (i[W_1]) - i[W_1]$ it follows that we can have little knowledge about the remaining and what must be unreadable portions that take the “X” position. If one assumes that members of $W_1$ are possible descriptions for possible NSP-world behavior at the time $t_i$, then it may be assumed that at the time $t_i$ the members of $*T'_i - T_i$ describe NSP-world behavior at NSP-world (and N-world) time $t_i$. Now as $i$ varies over $* \mathbb{N}$, pure nonstandard subdevelopmental paradigms (with or without the time index statement $W_i$) exist with members in $*T$ and may be considered as descriptions for time refined NSP-world behavior, especially for a NSP-world time index $i \in \mathbb{N}_\infty$.

**CHAPTER 10 REFERENCES**

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17 Note that the NSP-world model is not a local hidden variable theory.
11. "Things"

11.1 Propertons (Subparticles).

What is a properton? Or, what is an infant? Or, better still, what is a thing? I first used the name infant for these strange objects. I then coined the term "things." These three names do not convey the exact intuitively mean and are prone to incorrect mental images. As of 11 July 2012, the term "properton" is employed. As discussed, they carry physical or physical-like "properties" in a coded form. (As stated in [9], these objects are not to be described in terms of any geometric configuration. These multifaceted things, these propertons, are not to be construed as either particles nor waves nor quanta nor anything that can be represented by some fixed imagery. Propertons are to be viewed only operationally. Propertons are only to be considered as represented by a *-finite sequence \( \{a_i\}_{i=1}^n \), \( n \in \mathbb{N} \), of hyperreal numbers. Indeed, the idea of the n-tuple \((a_1, a_2, \ldots, a_i, \ldots)\) notation is useful and we assume that \( n \) is a fixed member of \( \mathbb{N}_\infty \). The language of coordinates for this notation is used, where the i’th coordinate means the i’th value of the sequence. Obviously, 0 is not a domain member for our sequential representation.

The first coordinate \( a_1 \) is a “naming” coordinate. The remaining coordinates are used to represent various real numbers, complex numbers, vectors, and the like physical qualities needed for different physical theories. For example, \( a_2 = 1 \) might be a counting coordinate. Then \( a_i, \ 3 \leq i \leq 6 \) are hyperreal numbers that represent NSP-world coordinate locations of the properton named by \( a_1 \cdots a_7 \), \( a_8 \) represent the positive or negative charges that can be assigned to every properton — \( a_9, a_{11}, a_{13} \) hyperreal representations for the inertial, gravitational and intrinsic (rest) mass. For vector quantities, continue this coordinate assignment and assign specific coordinate locations for the vector components. So as not to be biased, include as other coordinates hyperreal measures for qualities such as energy, apparent momentum, and all other physical qualities required within theories that must be combined in order to produce a reasonable description for N-world behavior. For the same reason, we do not assume that such N-world properties as the uncertainty principle hold for the NSP-world. (See note (2) on page 116.)

It is purposely assumed that the qualities represented by the coordinate \( a_i, \ i \geq 3 \) are not inner-related, in their basic construction, by any mathematical relation since it is such inner-relations that are assumed to mirror the N-world laws that govern the development of not only our present universe but previous as well as future developmental alterations. The same remarks apply to any possible and distinctly different universes that may or not occur. Thus, for these reasons, we view the properton as being totally characterized by such a sequence \( \{a_i\} \) and always proceed cautiously when any attempt is made to describe all but the most general properton behavior. Why have we chosen to presuppose that propertons are characterized by sequences, where the coordinates are hyperreal numbers?

For chapters 11, 12 assume EGS. Let \( r \) be a positive real number. The number \( r \)
can be represented by a decimal-styled number, where for uniqueness, the repeated 9s case is used for all terminating decimals. From this, it is seen that there is a sequence $S_i$ of natural numbers such that $S_i/10^i \to r$. Consequently, for any \( \omega \in \mathbb{N}_\infty = \mathbb{N} - \mathbb{N} \), it follows that $\pm S_\omega/10^\omega \in \mu(\pm r)$, where $S_\omega \in \mathbb{N} \text{ and } \mu(\pm r)$ is the monad about $\pm r$. In [9], it is assumed that each coordinate \( a_i, i \geq 3 \) is characterized by the numerical quantity $\pm 10^-\omega$, \( \omega \in \mathbb{N}_\infty \). Obviously, we need not confine ourselves to the number $10^-\omega$.

**Theorem 11.1.1** For each $0 < i \in \mathbb{N}$, let $0 < m_i \in \mathbb{N}$ and $m_i \to \infty$. Let any \( \omega, \lambda \in \mathbb{N}_\infty \). Then, for each $r \in \mathbb{R}$, there exists a $b/ \omega_{\omega} \in \{x/ \omega_{\omega} | (x \in \mathbb{Z}) \land (|x| < \lambda \omega_{\omega})\}$, where $\omega_{\omega} \in \mathbb{N}_\infty$, and $b/ \omega_{\omega} \approx r$ (i.e. $b/ \omega_{\omega} \in \mu(r)$). If $r \neq 0$, then $|b| \in \mathbb{N}_\infty$.

Proof. For $r \in \mathbb{R}$, there exists a unique integer $n \in \mathbb{Z}$ such that $n \leq r < n + 1$. Partition $[n, n + 1)$ as follows: for each $0 < i \in \mathbb{N}$, and $0 < m_i \in \mathbb{N}$, consider $[n, n + 1/m_i), \ldots, [n + (m_i - 1)/m_i, n + 1]$. Then there exists a unique $c_i \in \{0, 1, \ldots, m_i - 1\}$ such that $r \in [n + c_i/m_i, n + (c_i + 1)/m_i]$. Let $S_i = (m_i n + c_i)/m_i = f_i/m_i$. Since $0 \leq r - S_i < 1/m_i$ and $m_i \to \infty$, then $S_i \to r$. This yields two sequences $S: \mathbb{N} \to \mathbb{Q}$ and $f: \mathbb{N} \to \mathbb{Z}$, where, for each $\omega \in \mathbb{N}_\infty$, $S_\omega = f_\omega/\omega_{\omega} \approx r$ and $f_\omega \in \mathbb{Z}$. Observe that $f_\omega/\omega_{\omega}$ is a finite (i.e. limited) number and $\omega_{\omega} \in \mathbb{N}_\infty$. Hence, $f_\omega/\omega_{\omega} \lt \lambda$ entails that $|f_\omega| < \lambda \omega_{\omega}$. Therefore, $f_\omega/\omega_{\omega} \in \{x/\omega_{\omega} | (x \in \mathbb{Z}) \land (|x| < \lambda \omega_{\omega})\}$. If $\omega_{\omega} \in \mathbb{Z}$, then $f_\omega/\omega_{\omega} \approx 0$.

**Corollary 11.1.1.1** For each $0 < i \in \mathbb{N}$, let $0 < m_i \in \mathbb{N}$ and $m_i \to \infty$. Let any \( \omega, \lambda \in \mathbb{N}_\infty \). Then, for each $r \in \mathbb{R}$, there is a sequence $f: \mathbb{N} \to \mathbb{Z}$ such that $f_\omega/\omega_{\omega} \in \mu(r)$. There are unique $n \in \mathbb{Z}$, $c_\omega = 0$ or $c_\omega \in \mathbb{N}_\infty$ such that $c_\omega \leq \omega_{\omega} - 1$ and $f_\omega/\omega_{\omega} = \omega_{\omega}n + c_\omega$.

For the ultra-properton, each coordinate $a_i = 1/10^\omega$ $i \geq 3$ and odd, $a_i = -1/10^\omega$ $i \geq 4$ and even, $\omega \in \mathbb{N}_\infty$. From the above theorem, the choice of $10^-\omega$ as the basic numerical quantity is for convenience only and is not unique except in its infinitesimal character. Of course, the sequences chosen to represent the ultra-properton are pure internal objects and as such are considered to directly or indirectly affect the N-world. Why might the *-finite “length” of such propertons (here is where we have replaced the NSP-world entity by its corresponding sequence) be of significance?

First, since our N-world languages are formed from a finite set of alphabets, it is not unreasonable to assume that NSP-world “languages” are composed from a *-finite set of alphabets. Indeed, since it should not be presupposed that there is an upper limit to the N-world alphabets, it would follow that the basic NSP-world set of alphabets is an infinite *-finite set. Although the interpretation method that has been chosen does not require such a restriction to be placed upon NSP-world alphabets, it is useful, for consistency, to assume that descriptions for substratum processes that affect, in either a directly or indirectly detectable manner, N-world events be so restricted. For the external NSP-world viewpoint, all such infinite *-finite objects have a very significant common property. Note: in what follows \( \mathcal{M}_1 \) is the extended superstructure constructed on page 70.
Theorem 11.1.2 All infinite *-finite members of our (ultralimit) model $*\mathcal{M}_1$ have the same external cardinality which is $\geq |\mathcal{M}_1|$.

Proof. Hanson [8] and Zakon [16] have done all of the difficult work for this result to hold. First, one of the results shown by Henson is that all infinite *-finite members of our ultralimit model have the same external cardinality. Since our model is a comprehensive enlargement, Zakon’s theorem 3.8 in [16] applies. Zakon shows that there exists a *-finite set, $A$, such that $|A| \geq |\mathcal{M}_1| = |\mathcal{R}|$. Since $A$ is infinite, Hanson’s result now implies that all infinite *-finite members of our model satisfy this inequality.

For an extended infinite standard set $*A$ it is well-known that $|*A| \geq |\mathcal{M}_1|^+$. One may use these various results and establish easily that there exist more than enough propertons to obtain all of the cardinality statements relative to the three substratum levels that appear in [9] even if we assume that there are a continuum of finitely many properton qualities that are needed to create all of the N-world.

Consider the following infinite set of statements expressed in an extended alphabet.

$$G_A = \{\text{An}||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c'+1/(n') . \ | (i,j,n) \in \mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+ \} \land (1 \leq k \leq m)\}, \quad (11.1.1)$$

where $\mathbb{N}^+$ is the set of all nonzero natural numbers and $m \in \mathbb{N}^+$. Applying the same procedure that appears in the proof of Theorem 9.3.1 and with a NSP-world alphabet, we obtain

$$G_A' = \{\text{An}||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c'+1/(n') . \ | (i,j,n) \in *\mathbb{N}^+ \times *\mathbb{N}^+ \times *\mathbb{N}^+ \} \land (1 \leq k \leq m)\}, \quad (11.1.2)$$

Assume that there is at least one type of elementary particle with the properties stated in the set $G_A$. It will be shown in the next section that within the NSP-world there may be simple properties that lead to N-world energy being a manifestation of mass. For $c = 0$, we have another internal set of descriptions that forms a subset of $G_A'$.

$$\{\text{An}||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c'+1/(10^\zeta) . \ | (i,j,\zeta) \in *\mathbb{N}^+ \times *\mathbb{N}^+ \times *\mathbb{N}^+ \} \land (1 \leq k \leq m)\}, \quad (11.1.3)$$

For our purposes, (11.1.3) leads immediately to the not ad hoc concept of propertons with infinitesimal proper mass. As will be shown, such infinitesimal proper mass can be assumed to characterize any possible zero proper mass N-world entity. The set $G_A'$ has meaning if there exists at least one natural entity that can possess the energy expressed by $G_A$, where this energy is measured in some private unit of measure.
Human beings combine together finitely many sentences to produce comprehensible descriptions. Moreover, all N-world human construction requires the composition of objectively real N-world objects. We model the idea of finite composition or finite combination by an N-world process. This produces a corresponding NSP-world intrinsic ultranatural process ultrafinite composition or ultrafinite combination that can either directly or indirectly affect the N-world, where its effect is indirectly inferred.

Let the index $j$ vary over a hyperfinite interval and fix the other indices. Then the set of sentences

$$G''_A = \{\text{An||elementary||particle||k'(i',j')||with||}
\text{total||energy||c'}+1/(\Omega'). \mid (j \in {}^*\mathbb{N}^+) \wedge (1 \leq j \leq \lambda)\},$$

(11.1.4)

where $\lambda \in {}^*\mathbb{N}^+$, $3 \leq i \in {}^*\mathbb{N}$, $n \in \mathbb{N}_{\infty}$ and $1 \leq k \leq m$, forms an internal linguistic object that can be assumed to describe a hyperfinite collection of ultranatural entities. Each member of $G''_A$ has the $i$'th coordinate that measures the proper mass and is infinitesimal (with respect to NSP-world private units of measure). In the N-world, finite combinations yield an event. Thus, with respect to such sets as $G''_A$, one can say that there are such N-world events iff there are ultrafinite combinations of NSP-world entities. And such ultrafinite combinations yield a NSP-world event that is an ultranatural entity.

Associated with such ultrafinite combinations for the entities described in $G''_A$ there is a very significant procedure that yields the $i$'th coordinate value for the entity obtained by such ultrafinite combinations. Such entities are called intermediate propertons. Let $m_0 \geq 0$ be the N-world proper mass for an assumed elementary particle denoted by $k'$. If $m_0 = 0$, then let $\lambda = 1$. Otherwise, from Theorem 11.1.1, we know that there is a $\lambda \in {}^*\mathbb{N}$ such that $\lambda/(10^\omega) \in \mu(m_0)$, where $\omega \in \mathbb{N}_{\infty}$ and since $m_0 \neq 0$, $\lambda \in \mathbb{N}_{\infty}$. Consequently, for $b_n = 10^{-\omega}$, the $^\ast$-finite sum

$$(11.1.5) \quad \sum_{n=1}^{\lambda} b_n = \sum_{n=1}^{\lambda} \frac{1}{10^\omega} = \frac{\lambda}{10^\omega}$$

has the property that $\operatorname{st}(\sum_{n=1}^{\lambda} 1/(10^\omega)) = m_0$. (Note the special summation notation for a constant summand.) The standard part operator $\operatorname{st}$ is an important external operator that is a continuous [11] NSP-world process that yields N-world effects. The appropriate interpretation is that

_ ultrafinite combinations of ultra-propertons yield an intermediate properton that, after application of the standard part operator, has the same effect as an elementary particle with proper mass $m_0$._

An additional relevant idea deals with the interpretation that the $^\ast$-finite set $G''_A$ exists at, say, nonstandard time, and that such a set is manifested at standard time when the operator $\operatorname{st}$ is applied. The standard part operator is one of those
external operators that can be indirectly detected by the presence of elementary particles with proper mass \(m_0\).

The above discussion of the creation of intermediate propertons yields a possible manner in which ultra-propertons are combined within the NSP-world to yield appropriate energy or mass coordinates for the multifaceted propertons. But is there an indication that all standard world physical qualities that are denoted by qualitative measures begin as infinitesimals?

Consider the infinitesimal methods used to obtain such things as the charge on a sphere, charge density and the like. In all such cases, it is assumed that charge can be infinitesimalized. In 1972, it was shown how a classical theory for the electron, when infinitesimalized, leads to the point charge concept of quantum field theory and then how the *-finite many body problem produced the quasi-particle. [15] Although this method is not the same as the more general and less ad hoc properton approach, it does present a procedure that leads to an infinitesimal charge density and then, in a very ad hoc manner, it is assumed that there are objects that when *-finitely combined together entail a real charge and charge density. Further, it is the highly successful use of the modeling methods of infinitesimal calculus over hundreds of years that has lead to our additional presumption that all coordinates of the basic sequential properton representation are a ± fixed infinitesimal.

In order to retain the general independence of the coordinate representation, independent *-finite coordinate summation is allowed, recalling that such objects are to be utilized to construct many possible universes. [This is the same idea as *-finitely repeated simple affine or linear transformations.] Thus, distinct from coordinatewise addition, *-finitely many such sequences can be added together by means of a fixed coordinate operation in the following sense. Let \(\{a_i\}\) represent an ultra-properton. Fix the coordinate \(j\), then the sequence \(\{c_i\}, c_i = a_i, \ i \neq j\) and \(c_j = 2a_j\) forms an intermediate properton. As will be shown, it is only after the formation of such intermediate propertons that the customary coordinatewise addition is allowed and this yields, after the standard part operator is applied, representations for elementary particles. Hence, from our previous example, we have that ultrafinite combinations of ultra-propertons yield propertons with “proper mass” \(\lambda/(10^\omega) \approx m_0\) while all other coordinates remain as \(\pm 10^\omega\). This physical-like process is not a speculative ad hoc construct, but, rather, it is modeled after what occurs in our observable natural world. Intuitively, this type of summation is modeled after the process of inserting finitely many pieces of information (mail) into a single “postal box,” where these boxes are found in rectangular arrays in post offices throughout the world.

Now other ultra-propertons are ultrafinitely combined and yield for a specific coordinate the \(\pm\) unite charge or, if quarks exist, other N-world charges, while all other coordinates remain fixed as \(\pm 1/(10^\omega)\), etc. Rationally, how can one conceive of a combination of these intermediate propertons, a combination that will produce entities that can be characterized in a standard particle or wave language?

Recall that a finite summation is a *-finite summation within the NSP-world.
Therefore, a finite combination of intermediate propertons is an allowed internal process. [Note that external processes are always allowed but with respect to our interpretation procedures we always have direct or indirect knowledge relative to application of internal processes. Only for very special and reasonable external processes do we have direct or indirect knowledge that they have been applied.]

Let \( \zeta_i \in \mu(0) \), \( i = 1, \ldots, n \). Then \( \zeta_1 + \cdots + \zeta_n \in \mu(0) \). The final stage in properton formation for our universe — the final stage in particle or wave substratum formation — would be finite coordinatewise summation of finitely many intermediate propertons. This presupposes that the N-world environment is characterized by but finitely many qualities that can be numerically characterized. This produces the following type of coordinate representation for a specific coordinate \( j \) after \( n \) summations with \( n \) other intermediate propertons that have only infinitesimals in the \( j \) coordinate position.

\[
(11.1.6) \quad \sum_{i=1}^{\lambda} \left( \frac{1}{10^{\omega}} \right) + \sum_{i=1}^{n} \zeta_i.
\]

Assuming \( \lambda \) is one of those members of \( \mathbb{N}_\infty \) or equal to 1 as used in (11.1.5), then the standard part operator can now be applied to (11.1.6) and the result is the same as \( \text{st}(\sum_{i=1}^{\lambda} \left( \frac{1}{10^{\omega}} \right)) \).

The process outlined in (11.1.6) is then applied to finitely many distinct intermediate propertons — those that characterize an elementary particle. The result is a properton each coordinate of which is infinitely close to the value of a numerical characterization or an infinitesimal. When the standard part operator is applied under the usual coordinatewise procedure, the coordinates are either the specific real coordinatewise characterizations or zero. Therefore, N-world formation of particles, the dense substratum field, or even gross matter may be accomplished by a ultrafinite combination of ultra-propertons that leads to the intermediate properton; followed by finite combinations of intermediate propertons that produce the N-world objects. Please note, however, that prior to application of the standard part operator such propertons retain infinitesimal nonzero coordinate characterizations in other noncharacterizing positions. (See note (1) on 116.)

We must always keep in mind the hypothesis law [9] and avoid unwarranted speculation. We do not speculate whether or not the formed particles have point-like or “spread out” properties within our space-time environment. These additional concepts may be pure catalyst type statements within some standard N-world theory and could have no significance for either the N-world or NSP-world.

With respect to field effects, the cardinality of the set of all ultra-propertons clearly implies that there can be ultrafinite combinations of ultra-propertons “located” at every “point” of any finite dimensional continuum. Thus the field effects yielded by propertons may present a completely dense continuum type of pattern within the N-world environment although from the monadic viewpoint this is not necessarily how they “appear” within the NSP-world.
There are many scenarios for quantum transitions if such occur in objective reality. The simplest is a re-ultrafinite combination of the ultra-propertons present within the different objects. However, it is also possible that this is not the case and, depending upon the preparation or scenario, the so-called “conservation” laws do not hold in the N-world.

As an example, the neutrino could be a complete fiction, only endorsed as a type of catalyst to force certain laws to hold under a particular scenario. Consider the set of sentences

\[
G_B = \{ \text{An \{} ||| \text{elementary} ||| \text{particle} ||| k'(i', j') ||| \text{with} \} ||| \text{total} \} ||| \text{energy} ||| c' + n'. \ \big| \ ((i, j, n) \in \mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \wedge (1 \leq k \leq m) \}. \tag{11.1.7}
\]

It is claimed by many individuals that such objects as being described in \(G_B\) exist in objective reality. Indeed, certain well-known scenarios for a possible cosmology require, at least, one “particle” to be characterized by such a collection \(G_B\). By the usual method, these statements are \(\ast\)- transferred to

\[
G'_B = \{ \text{An \{} ||| \text{elementary} ||| \text{particle} ||| k'(i', j') ||| \text{with} \} ||| \text{total} \} ||| \text{energy} ||| c' + n'. \ \big| \ ((i, j, n) \in \ast \mathbb{N}^+ \times \ast \mathbb{N}^+ \times \ast \mathbb{N}^+) \wedge (1 \leq k \leq m) \}. \tag{11.1.8}
\]

Hence, letting \(n \in \mathbb{N}_\infty\) then various “infinite” NSP-world energies emerge from our procedures. With respect to the total energy coordinate(s), ultra-propertons may also be ultrafinitely combined to produce such possibilities. Let \(\lambda = 10^{2\omega} \) [ resp. \(\lambda = \omega^2\)] and \(\omega \in \mathbb{N}_\infty\). Then

\[
\sum_{n=1}^{\lambda} \frac{1}{10^n} = 10^\omega \ [\text{resp.} \ \sum_{n=1}^{\lambda} \frac{1}{\omega} = \omega] \in \mathbb{N}_\infty. \tag{11.1.9}
\]

Of course, these numerical characterizations are external to the N-world. Various distinct “infinite” qualities can exist rationally in the NSP-world without altering our interpretation techniques. The behavior of the infinite hypernatural numbers is very interesting when considered as a model for NSP-world behavior. A transfer of finite energy, momentum and, indeed, all other N-world characterizing quantities, back and forth, between these two worlds is clearly possible without destroying NSP-world infinite conservation concepts.

Further, observe that various intermediate propertons carrying nearstandard coordinate values could be present at nearstandard space-time coordinates, and application of the continuous and external standard part operator would produce an apparent not conserved N-world effect. These concepts will be considered anew when we discuss the Bell inequality.

Previously, ultrawords were obtained by application of certain concurrent relations. Actually, basic ultrawords exist in any elementary nonstandard superstructure model, as will now be established for the general paradigm.

Referring back to \(G_A\) equation (11.1.1), for some fixed \(k, 1 \leq k \leq m, \) let
h_κ: \mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+ \rightarrow \mathbb{G}_A \text{ be defined as follows: } h_κ(i,j,n) = A_n||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c' + 1/(n'). Since the set \( F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \) is denumerable, there exists a bijection \( \mathbb{H}: \mathbb{N} \rightarrow F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \). For each \( 1 \leq \lambda \in \mathbb{N} \) and fixed \( i, n, \lambda \in \mathbb{N}^+ \), let \( \mathbb{G}_A(\lambda) = \{A_1||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c' + 1/(n'). | (1 \leq j' \leq \lambda) \land (j' \in \mathbb{N}^+) \} \). Let \( p \in \mathbb{N} \). If \( |\mathbb{H}(p)| \geq 2 \), define finite \( \mathbb{M}(h_κ[\mathbb{H}(p)]) = \{A_1||\text{and}||A_2||\text{and}|| \cdots ||\text{and}||A_m\}, \) where \( A_j \in h_κ[\mathbb{H}(p)], m = |\mathbb{H}(p)| \). If \( |\mathbb{H}(p)| \leq 1 \), then define \( \mathbb{M}(h_κ[\mathbb{H}(p)]) = \emptyset \). Let \( \mathbb{M}_0 = \bigcup\{\mathbb{M}(h_κ[\mathbb{H}(p)]) | p \in \mathbb{N}\} \). Please note that the \( k' \) represents the “type” or name of the elementary particle, assuming that only finitely many different types exist, \( i' \) is reserved for other purposes, and the \( j' \) the number of such elementary particles of type \( k' \).

**Theorem 11.1.3** For any \( i, n, \lambda \in \mathbb{N}^+ \), such that \( 2 \leq \lambda \), there exists \( w \in \mathbb{M}_0 - \mathbb{G}_A, \mathbb{G}_A(\lambda) \subset \mathbb{S}\{\{w\}\} \) and if \( A \in \mathbb{G}_A - \mathbb{G}_A(\lambda),\) then \( A \notin \mathbb{S}\{\{w\}\} \).

Proof. Let \( i, j, \lambda \in \mathbb{N}^+ \) and \( 2 \leq \lambda \). Then there exists some \( r \in \mathbb{N} \) such that \( h_κ[\mathbb{H}(r)] = \mathbb{G}_A(\lambda) \). From the construction of \( \mathbb{M}_0 \), there exists some \( r' \in \mathbb{N} \) such that \( w(r') = A_n||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c' + 1/(n').||\text{and}||A_n||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c' + 1/(n').||\text{and}|| \cdots ||\text{and}||A_n||\text{elementary}||\text{particle}||k'(i',j')||\text{with}||\text{total}||\text{energy}||c' + 1/(n'). \in \mathbb{M}[h_κ[\mathbb{H}(r')]]. Note that \( w(r') \notin \mathbb{G}_A, h_κ[\mathbb{H}(r')] \subset \mathbb{S}\{\{w(r')\}\} \) and if \( A \in \mathbb{G}_A - h_κ[\mathbb{H}(r)], \) then \( A \notin \mathbb{S}\{\{w(r')\}\} \). The result follows by our embedding and *-transfer.

The ultrawords utilized to generate various propertons, whether obtained as in Theorem 11.1.3 or by concurrent relations, are called *ultramixtures* due to their applications. The ultrafinite choice operator \( \mathbb{C}_1 \) can select them, prior to application of \( \mathbb{S} \). Moreover, application of the ultrafinite combination operator entails a specific intermediate properton with the appropriate nearestandard coordinate characterizations. Please notice that the same type of sentence collections may be employed to infinitesimalize all other quantities, although the sentences need not have meaning for certain popular N-world theories. Simply because substitution of the word “charge” for “energy” in the above sentences \( \mathbb{G}_A \) does not yield a particular modern theory description, it does yield the infinitesimal charge concept prevalent in many older classical theories.

Using such altered \( \mathbb{G}_A \) statements, one shows that there does exist ultramixtures \( w_i \) for each intermediate properton and, thus, a single ultimate ultramixture \( w \) such that \( \mathbb{S}\{\{w_i\}\} \subset \mathbb{S}\{\{w\}\} \). Each elementary particle may, thus, be assumed to originate from \( w \) through application of the ultralogic \( \mathbb{S} \).

Recall that if a standard \( A \subset \mathbb{N} \) is infinite, then it is external, and if \( B \) is internal, \( A \subset B \), then \( B \neq A \). Therefore, there exists some \( \eta \in B \) such that \( \eta \notin A \). This simple fact yields many significant nonstandard results. For example, as the next theorem shows, if \( \eta \in \mathbb{N}_\infty \), then there exists some \( \lambda \in \mathbb{N}_\infty \) such that \( 10^{2\lambda} < \eta \).
Theorem 11.1.4 Let $f: \mathbb{N} \to \mathbb{N}$ and $f[\mathbb{N}]$ be infinite. If $\eta \in \mathbb{N}_\infty$, then there exists some $\lambda \in \mathbb{N}_\infty$ such that $^*f(\lambda) < \eta$.

Proof. For $\eta \in \mathbb{N}_\infty$, consider the nonempty internal set $B = \{ ^*f(x) \mid ( ^*f(x) < \eta) \land (x \in ^*\mathbb{N}) \} \subset ^*f[^*\mathbb{N}]$. Let $n \in \mathbb{N}$. Then $f(n) \in \mathbb{N}$ and $f(n) < \eta$ imply that $( ^*f)[\mathbb{N}] = ^*(f[\mathbb{N}]) = f[\mathbb{N}] \subseteq B$. Since $f[\mathbb{N}]$ is infinite, it is external. Thus $^*f: ^*\mathbb{N} \to ^*\mathbb{N}$ implies that there exists some $\lambda \in ^*\mathbb{N}$ such that $^*f(\lambda) \in B \setminus f[\mathbb{N}]$. However, $^*f$ is a function. Hence, $\lambda \in \mathbb{N}_\infty$ and $^*f(\lambda) < \eta$.

Theorem 11.1.4 has many applications and can be extended to other functions not just those with domain and codomain $\mathbb{N}$, and other $B$ type relations.

11.2 Ultraenergetic Propertons (Subparticles).

There is a possibility that propertons can have additional and unusual properties when they are generated by statements such as $G_B$. With respect to the translated $G'_B$ statements, we have coined the term ultraenergetic to discuss propertons that have various infinite energies. I again note that such ultraenergetic propertons may be considered as under the control of our previously discussed ultralogics and ultranatural choice operators (i.e. hyperfinite choice). Further, it is possible to place these ultraenergetic propertons into pools of infinite energy that are mathematically termed as “galaxies” and that have interesting mathematical properties. However, these properties will not be discussed in this present book.

If our universe or any portion of it began or exists at this present epoch in a state of “infinite” energy, then the ultraenergetic propertons could play a critical role. I point out that general developmental paradigms indicate, as will be shown, that the actual state of affairs for any beginnings of our universe cannot be known by the present methods of the scientific method. Now, any such singularity that might exist cosmologically, even in our local environment, may owe its existence to various ultralogically generated ultraenergetic propertons. Of course, this is pure speculation, but these NSP-world alternative explanations for assumed quantum physical phenomena yield indirect evidence for the acceptance of the NSP-world model.

Quantum mechanics has now become highly positivistic in character although certain previous states of affairs have been partially accepted. This important possibility was stated by Bernard d’Espagnet with respect to one of our preliminary investigations — the experimental disproof of the Bell inequality and the local variable concept [17] — that “seems to imply that in some sense all of the objects [particles or aggregates] constitute an indivisible whole” [2]. One aspect of the MA-model, (11.4.5) of section 11.4, can be used as an aid to model this statement.

The ideas developed within our theory of developmental paradigms do not contradict d’Espagnet’s (weak) definition of realism. He simply requires that if we can describe a relation between physical entities produced by some experimental process, a relation that is not observed and, thus, not described prior to the experiment, then their must be a cause that has produced this new relation. I don’t believe that it is necessary, under his definition, that this cause be describable.
In May 1984, this author became aware of the Bell inequality and d’Espagnat’s discussion of local realism [3]. In particular, we discovered that d’Espagnat may, to some degree, embrace the statements that appear in (11.4.5) section 11.4 as an explanation for this experimental disproof. “Perhaps in such a world the concept of an independent existing reality can retain some meaning, but it will be altered and one remote from everyday experience” [4]. But is there a NSP-world cause, indeed, a mechanism that for such behavior?

There are many scenarios as to how “instantaneous” informational signals may be transmitted within the NSP-world without violating Einstein separability in the N-world (i.e. no influence of any kind within the N-world can propagate faster than the speed of light [5].) The basic N-world interpretation for any verified effect of the Special Theory would imply that the only reason for Einstein separability is a relation between those propertons that create the N-world and the NSEM field propertons. But ultraenergetic propertons are not of either of these types and need not interact with the NSEM field for many reasons. The most obvious is that the NSEM field is not dense from the NSP-world viewpoint but is scattered. Obviously ultraenergetic propertons may be used for this purpose. Recall that we should be very careful when speculating about the NSP-world due to the difficulty of describing refined behavior. However, this should not completely restrain us, especially when the general paradigm method states that such things as these exist logically.

It is possible to describe a mechanism and a possible new type of properton that can send N-world instantaneous informational signals between all standard material particles, field objects or aggregates and not violate N-world Einstein separability. One possibility is that these influences would be imparted by means of independent coordinate summation to the propertons that comprise these objects and yet in doing so these new entities could not be humanly detected, not detectable except as far as the instantaneous state change indicates, since the total energy (in this case classical kinetic) utilized by this NSP-world mechanism would be infinitesimal. If it is an instantaneous energy change, then, as will be shown, only that specific energy change would appear in the N-world.

From the methods employed to construct propertons, it is immediately clear that there exists a very “large” quantity of propertons that are not used for standard particle and field effect construction. This can be seen by allowing the i’th symbol is such statements as \( G_A^i \) to vary from 1 to some value in \( \mathbb{N}_\infty \). The cardinality of such collections of statements would be great than or equal to \( |\mathcal{M}|^+ \). We simply pass this external cardinality statement to the propertons being described.

It is a basic tenet of infinitesimal reasoning that without further justification the only properties that we should associate with such unutilized objects are of the simplest classical type. The logic of particle physics allows us to logically accept the existence of such propertons without any additional justification. Let \( \lambda \in \mathbb{N}_\infty \). Then \((1/\lambda)^4 \in \mu(0)\). Let a pure NSP-world properton, not one used to construct a universe, have mass coordinate of the value \( m = (1/\lambda)^4 \). Call this properton P
and have P increase its velocity over a finite NSP-time interval from zero to $\lambda$. Propertons that attain such velocities are called ultrafast propertons.

Extending the classical idea of kinetic energy to the NSP-world it follows that the (kinetic) energy attained by this properton, when it reaches its final velocity, is $(1/2)(1/\lambda)^2 \in \mu(0)$. Suppose that there is a continuum or less of positions within our universe. Since $c < \lambda$, the kinetic energy used to accelerate enough of these propertons to the $\lambda$ velocity so that they could effect every position in our universe would be less than $(1/2)(1/\lambda) \in \mu(0)$, assuming the energy is additive in the NSP-world.

Each of these ultrafast propertons besides altering some other specific coordinate would also add its total kinetic energy to the intermediate properton since the new intermediate properton simply includes this new one. But then suppose that our universe has existed for less than or equal to a continuum of time. Then since $2c < \lambda$, once again the amount of energy that would be added to our universe over such a time period, if each of these informational propertons combined with one member of an intermediate, would be infinitesimal. All the state alterations give the N-world appearance of being instantaneously obtained although the existence of the $G$ function of Theorem 7.5.1 clearly states that in the NSP-world such alterations are actually hypercontinuous and hypersmooth.

What if the state change itself depends upon the velocity of such an ultrafast properton? We use kinetic energy as an example. Say the change is in the kinetic energy coordinate in the standard amount of $\hbar$. Then all one needs to consider is an ultrafast properton with infinitesimal mass $m = 2\hbar(1/\lambda)^2$. Moving with a velocity of $\lambda$, such an ultrafast properton has the requisite kinetic energy. Things can clearly be arranged so that all other coordinates of such ultrafast propertons are infinitesimal. Independent coordinate summation for any finite number of alterations will leave all other nonaltered coordinates of the intermediate properton infinitely close to the original values for the alteration is but obtained by the addition of a finite number of new propertons to the collection.

We acknowledge that the N-world inner coordinate relations have been used to obtain these alterations. This need not be the way it could be done. Can we describe the method of capture and other sorts of behavior? Probably too much has already been described in the language of this book. One should not forget that descriptions may exist for such NSP-world behavior but not in a readable language.

The state of affairs described above lends credence to d’Espagnat’s explanation of why the Bell inequality is violated and gives further evidence for the acceptance of the NSP-world model. “The basic law that signals cannot travel faster than light is demoted from a property of external [N-world] reality to a feature of mere communicatable human experience. ....the concept of an independent or external reality can still be retained as a possible explanation of observed regularities in experiments. It is necessary, however, that the violation of Einstein separability be included as a property, albeit a well-hidden and counterintuitive property....”[6]
11.3 More on Propertons (Subparticles),

The general process for construction of all N-world fundamental entities from propertons can be improved upon or achieved in an alternate fashion. One of the basic assumptions of subatomic physics is that in the Natural-world two fundamental subatomic objects, such as two electrons, cannot be differentiated one from another by any of its Natural-world properties. One of the conclusions of what comes next is that in the NSP-world this need not be the case. Of course, this can also be considered but an auxiliary result and need have no applications. At a particular instant of (universal) time, it is possible to associate with each entity a distinct “name” or identifier through properton construction. This is done through application of the properton naming coordinate \( a_1 \). As will be shown, the concept of independent \(*\)-finite coordinate summation followed by \( n\)-tuple vector addition can be accomplished by means of a simple linear transformation. However, by doing so, the concept of the \(*\)-finite combinations or the gathering together of propertons as a NSP-world physical-like process is suppressed. Further, a simple method to identify each N-world entity or Natural system is not apparent. Thus, we first keep the above two processes so as to adjoin to each entity constructed an appropriate identifier.

A standard properton is modeled by a finite collection of numerical or coded descriptive physical characteristics. These characteristics are represented by coordinates within \( n\)-tuples. Other identifiers can also be included as specific coordinates. Included within these propertons are those of the following special type.

Informally, consider the denumerable set all prime numbers \( P \), a bijection \( h: \mathbb{N}' \rightarrow P \), \( \mathbb{N}' = \mathbb{N} - \{0\} \), and the sequence \( g: \mathbb{N} \rightarrow Q \), the set of rational numbers, where \( g(n) = 1/10^n \). Let \( \omega \in \ast \mathbb{N}' - \mathbb{N}' = \ast \mathbb{N} - \mathbb{N} = \mathbb{N}_\infty \). Since \( g \to 0, n \to \infty \), then \( \ast g(\omega) = 1/10^\omega \in \mu(0) \), the set of all infinitesimals.

Definition 11.3.1, Ultra-propertons. Let even \( K > 2, K \in \mathbb{N} \) and \( f: [1,K] \rightarrow \{1/10^\omega\} \). Then \( C = \{(\ast h(i), 1, -f(1), f(2), \ldots, -f(K-1), f(K)) \mid i \in \mathbb{N}'\} \). Each member of the set \( C \) represents an ultra-properton.

For Definition 11.3.1, it is assumed that there is no more than \( K \) physical or physical-like numerical or coded descriptive characteristics for the any elementary entity.

Theorem 11.3.1. Consider any nonempty internal \( D \) and \( A \subset D \) such that \( |A| < |\mathcal{M}_1|^+ \). Then there exists a hyperfinite \( B_A \) such that \( A \subset B_A \subset D \).

Proof. Let \( \mathcal{F} \) be the finite power set operator. That is for any set \( X \), \( \mathcal{F}(X) \) is the set of all finite subsets of \( X \), where a set \( Y \) is finite if it is empty or there exists an \( n \in \mathbb{N}' = \mathbb{N} - \{0\} \) and a bijection \( f': [1,n] \rightarrow Y \). In our structure, there is a least \( n \in \mathbb{N}' \), such that internal \( D \in \ast X_n \). If \( x \in A \), then \( x \in \ast X_{n-1} \), \( \ast \mathcal{F}(D) \in \ast X_{n+2} \). If \( y \in \ast \mathcal{F}(D) \), then \( y \in \ast X_{n+1} \).

Consider the internal binary relation \( C = \{(x,y) \mid (x \in y) \land (y \in \ast X_{n+1}) \land (x \in D) \land (x \in \ast X_{n-1}) \land (y \in \ast \mathcal{F}(D))\} \). Let \( \{(x_1,y_1), \ldots, (x_m,y_m)\} \subset C \). Then
Consider any nonempty hyperfinite $A \subset A'$. Then there exists a hyperfinite $B_A$ such that $A \subset B_A \subset A'$. This complete the proof.

**Corollary 11.3.1.1** Consider any $*E$ and $A \subset *E$ such that $|A| < |\mathcal{M}_1|^+$. Then there exists a hyperfinite $B_A$ such that $A \subset B_A \subset *E$.

**Theorem 11.3.2** Consider any nonempty hyperfinite $A \subset *\mathbb{N}'$. Then there exists a $\gamma \in *\mathbb{N}$ such that $A \subset [1, \gamma]$.

Proof. Every nonempty finite subset $F$ of $\mathbb{N}'$ has a greatest member $M_F \in \mathbb{N}$. That is if $x \in F$, then $x \in [1, M_F]$. By *transfer, $A$ has a *greatest member $\gamma \in *\mathbb{N}'$ such that if $x \in A$, then $x \in [1, \gamma]$.

Let $r_1 \in \mathbb{R}$. By Theorem 11.1.1 in Herrmann (1979-93), there is a $\lambda_1 \in \mathbb{N}_\infty$ such that $\lambda_1/10^\omega \in \mu([r_1])$. Hence, $\text{st}((\lambda_1/10^\omega)) = |r_1|$. Then there are $K, \lambda_i, i \in [1, K]$ that yield the $K$ characteristics. For an elementary entity $e_j$, some characteristics can be 0, meaning that the measure has value 0. Throughout the combining processes, if a coordinate retains its infinitesimal value $\pm 1/10^\omega$, this indicates that the characteristic has no meaning for $e_j$. In order to indicate these differences, any characteristic that has measure 0 is obtained from a combination of two ultra-propertons. The standard part physical realization operator $\text{St}$ is only applied to coordinates of the intermediate properton representations with the form $\pm \lambda/10^\omega$, where $\lambda \geq 2$.

There are other characteristics such as spin, where the 0 takes on a different meaning. However, such coding is rather arbitrary and can be replaced with non-zero numbers or non-zero codings for the characteristics so as to not confuse them with a 0 measurement. For the needed intermediate properton $e_1$, with a third coordinate characteristic under independent coordinate addition, the set of ultra-propertons $\{(\ast h(i), 1, -1/10^\omega, \ldots, 1/10^\omega) \mid i \in [1, \lambda_1]\}$ is employed. Hence, the first intermediate properton is $(\Pi_{i=1}^{\lambda_1}, \lambda_1, -\lambda_1/10^\omega, 1/10^\omega, \ldots, 1/10^\omega)$. For a forth coordinate intermediate properton for value $r_2$, consider $\{(\ast h(i), 1, -1/10^\omega, \lambda_2/10^\omega, 1^\omega, \ldots, 1/10^\omega) \mid i \in [\lambda_1 + 1, \lambda_1 + \lambda_2]\}$. Continue these definition for each member of $[1, K]$. Thus the entire collection of ultra-propertons used to obtain one of the $e_1$ entities is $\lambda_1 + \cdots + \lambda_K = \delta_1 \in \mathbb{N}_\infty$.

It is assumed that there are a nonempty countable (i.e non-zero finite or denumerable) collection of $\{e_i\}$ needed. Thus there is a non-zero finite or denumerable set $\{e_i\}$ and in the finite case, consider $\sum \delta_i \in \mathbb{N}_\infty$. Next consider $\{e_i \mid i \in \mathbb{N}'\}$. Then $\{e_i \mid i \in \mathbb{N}'\} \subset *\mathbb{N}$. The $\{|\{e_i \mid i \in \mathbb{N}'\}| < |\mathcal{M}_1|^+\}$. Hence, there is a $\gamma_1 \in \mathbb{N}_\infty$ such that $\{\delta_i \mid i \in \mathbb{N}'\} \subset [1, \gamma_1]$ by application of Corollary 11.3.1.1 and Theorem 11.3.2. Thus, in both cases, there is a $\Gamma_1 \in \mathbb{N}_\infty$ such that $\{\delta_i\} \subset [1, \Gamma_1]$. This shows that there are “enough” ultra-propertons to produce the set $\{e_i\}$. For another type of elementary particle, simply repeat this for the identifiers $h(i), i > \Gamma_1$. Then continue by induction.

For this application, it appears unnecessary to consider more than $H$, where $1 \leq H \in \mathbb{N}$, different types of elementary entities. The set of ultra-propertons $\{e_i\}$
\{ ( ^n h(i), 1, -1/10^ω, \ldots, 1/10^ω ) \mid i \in * \mathbb{N}^* \} = C \) is an internal set and as such the hyperfinite operator \( *F \) is defined for it. For properton generation, a universe can be considered as a collection of physical-systems. Hence application of a finite iteration \( *F^i \) to \( C \) yields \( \bigcup \{ *F^i(C) \mid (0 \leq i \leq n) \wedge (i \in \mathbb{N}) \} \), an internal collection that is sufficient to generate the physical-systems for any of the presently considered cosmologies. To accommodate the formation of the physical-like systems, internal \( X \) that is disjoint from \( \bigcup \{ *F^i(C) \mid (0 \leq i \leq n) \wedge (i \in \mathbb{N}) \} \) is adjoined to \( \bigcup \{ *F^i(C) \mid (0 \leq i \leq n) \wedge (i \in \mathbb{N}) \} \).

Relative to the GGU-model and generation of a universe, a hyperfinite \( *I^q(i, j) \) yields a universe-wide frozen-frame. (See the appendix prior to the symbols page.) Each instruction \( x \in *I^q(i, j) \), yields a physical or physical-like system. The physical-systems are disjoint. Each collection of ultra-propertons that yields a specific physical-system is distinct from the set of ultra-propertons that yields any other physical-system. Hence, each physical-system within a universe-wide frozen-frame has a distinct identifier via the collection of all of the identifiers for the ultra-propertons or the intermediate propertons employed to produce the physical-system.

11.4 MA-Model.

In this section, we look back and gather together various observations relative to formal theorems that yield the concept I have described as the Metaphoric-Anamorphosis (i.e. MA) model. The different types of developmental paradigms that can be selected by ultrafinite (i.e. ultranatural) choice and a few of our previous results leads immediately to the following logically acceptable possibilities. [Of course, as is the case with all mathematical modeling, simply because a possibility exists it need not be utilized to describe an actual scenario and if it is used, then it need not be an objectively real description.]

**11.4.1** Entire microscopic, macroscopic or large scale natural systems can apparently appear or disappear or be physically altered suddenly.

**11.4.2** Theorem 7.3.1 shows clearly that the suddenly concept in (11.4.1) is justifiable. Ultralogics, ultrawords, the intermediate properton and the ultrafast properton concept are possible mechanisms that can yield the behavior described in (11.4.1).

**11.4.3** All such alterations may occur in an ultracontinuous manner.

**11.4.4** None of these NSP-world concepts are related to the notion of hidden variables.

**11.4.5** Any numerical quantity associated with any elementary particle, field effect or aggregate is associable with every numerical quantity associated with every other elementary particle or aggregate by means of hypercontinuous, hyperuniform and hypersmooth
pure NSP-world functions. These functions may be interpreted as representing the IUN-altering process of utilizing ultrafinite composition (i.e. ultranatural composition) in order to “change” any elementary particle, field effect or aggregate into any type of elementary particle, field effect or aggregate.

Statement (11.4.5) is particular significant in that it may be coupled with ultralogics and ultrafinite choice operators and entails an additional manifestation for the possibility that there is no N-world independent existing objective reality. Further, notice that depending upon the space-time neighborhood, statements such as (11.4.1) need not be humanly verifiable (i.e. they may be undetectable).

CHAPTER 11 REFERENCES

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(1) For propertons, only two possible intrinsic properties for elementary particle formation are here considered. Assuming that there are such things as particles or elementary particles, then they would be differentiated one from the other by their intrinsic properties that are encoded within properton coordinates. When there are particle interactions, these intrinsic properties can be altered or even changed to extrinsic properties. How the alteration from intrinsic to extrinsic occurs probable cannot be known since it most likely is an ultranatural event. For further results on this subject, see http://arxiv.org/abs/quant-ph/9909078

(2) To conceive of propertons properly, quantum theory is viewed as an approximation. Moreover, in terms of physically determined units, the numerical characteristics produced by applications of the standard part operator are considered as exact.
1. Logic-System Generation for Instructions

As is customary, the nonstandard model used in all of the articles on the GGU-model is a polysaturated polyenlargements (Lobe and Wolff, 2000; Stroyan and Bayod, 1986). In this paper, \( q = 1, 2, 3, 4 \). These numbers denote the four primitive-time intervals (Herrmann 2006) employed for the GGU-model. The ultraword approach to generate a universe is replaced with an ultra-logic-system. This is a hyperfinite logic-system where, after application of the extended logic-system algorithm, generates each member of the hyperfinite instruction paradigm \( d^q_x \) in the proper \( \leq d^q_x \) order such that \( d_q \subset d^q_x \subset \ast d_q \), where \( q = 1, 2, 3, 4 \) and \( x = \lambda, \nu\lambda, \mu\lambda, \nu\gamma\lambda \) respectively. Finally, in this article, the term "subparticle" was previously used. To prevent incorrect mental images as to models for subparticles, the term "properton" replaces the term "subparticle." Without visualizing, a properton is an entity characterized only by a list of properties.

The primitive entity that yields physical reality for any GGU-model generated universe is dense collection of ultra-propertons. When first conceived this author had not investigated quantum field theory and did not base propertons upon any quantum theoretic approach. All of the GGU-model entities and processes can be considered as existing in a background universe or substratum world. This world can be considered as a physical-like world, where the rules that govern universe formation are distinct from those processes and rules that govern the development of any physical universe. They are simple rules that only refer to counting. This substratum world is also interpreted philosophically in other ways.

If necessary for a specific physical theory, any continuity requirement is satisfied by the properton field (Herrmann, 1983, 1989). For our universe, a collection of propertons has been shown to be closely associated with relativistic effects (Herrmann, 2003). No other known primitive entities, such as superstrings, will have any effect upon the application of propertons as the primitive entities that generate a universe. The processes used to obtain particles and all other physical entities from ultra-propertons need not correspond to the rules of quantum field theory or any additional rules like how quarks combine to form particles.

For our universe, quantum field theory contains descriptions (rules or instructions) that produce such particles from immaterial fields. Such fields are quantum mechanical systems and, when represented, have various degrees of freedom. These are but parameters that contribute to the overall state of the system. For various particles, parameters for physical measures or states are the characterizing features of propertons. The physical appearance and disappearance of particles are trivial applications of properton processes. For quantum field theory, one has the “creation” and “annihilation” operators that mathematically yield the same results.

For the GGU-model, quantum theory does not produce steps in a development since the method of production must be universe and physical law independent. For our universe, the development “satisfies” the predictions of accepted physical
Theories. I personally consider quantum theory as mostly a product of human imagination that predicts behavior, behavior that we cannot otherwise comprehend. That is, it is a model that mimics.

The GGU-model can be based upon observable human behavior and the mathematics predicts, for our universe, behavior that satisfies the behavior predicted by accepted physical theories. There is a vast amount of evidence for the predicted GGU-model processes. Whether such processes exist in some sort of reality is a philosophic choice. One can make this choice based upon various factors. One can choose to accept properton existence based upon the same philosophy expressed by those that accept that entities postulated in quantum field and particle theory exist.

The concept of instructions or rules is generalized to instructions that yield a physical reality from combinations of propertons. They are substratum laws. (So as not to confuse these with physical laws, they are called instructions. Further, in what follows, the events that correspond to each \( f^q(i, j) \) are denoted by \( E^q(i, j) \).)

This does not mean that the rules used in quantum theory (QT) actually yield each \( E^q(i, j) \). As mentioned, what this signifies is that the QT rules are verified via the production of event sequences that yield our universe. For the GGU-model, the physical realization of each \( f^q(i, j) \) is not the result of any of these physical theories. These theories are but verified by each realized \( f^q(i, j) \) and they allow us to predict what behavior occurred in or will occur within other realized \( f^q(p, k) \).

For the GGU-model, the “instructions” are rather simple ones that lead to all the characteristics that allow one to identify any material entity for any of the presently known cosmologies.

Rather than the \( f^q(i, j) \) being a general description, one considers instructions or rules \( I^q(i, j) \) - a nonempty finite subset of \( L \), which is equivalent to a single word in \( L \). These sets of instructions - instruction-sets - (also called instruction-information) are also indexed in the same way as the general descriptions and determine the instruction paradigm \( I_q \). Indeed, there is an injection \( H \) on \( d_q \) onto \( I_q \), where \( H(f^q(i, j)) = I^q(i, j) \) and \( (i, j) \) varies over the same set of integers and natural numbers. There is one instruction paradigm for each pre-designed universe and there can be a vast collection of such universes. Rather than simply applying this bijection as a means to reproduce each of the instruction paradigm results from the developmental paradigm results, what follows is a duplicate of these results and how they are obtained in terms of instruction paradigm notation.

Relative to the GGU-model and generation of a universe, a hyperfinite \( *f^q(i, j) \) yields a universe-wide frozen-frame. Each instruction \( x \in *f^q(i, j) \), yields a physical or physical-like system. The physical-systems are disjoint. Each collection of ultra-propertons that yields a specific physical-system is distinct from the set of ultra-propertons that yields any other physical-system. Hence, each physical-system within a universe-wide frozen-frame has a distinct identifier via the collection of all of the identifiers for the ultra-propertons or the intermediate propertons employed to produce the physical-system.
2. Logic-System Generation for the Type-1 Interval.

The notation in all that follows is from Herrmann (2006). Notice that there are two different t sequence notations. One t is in the informal world, while another t is in the formal standard superstructure. These two sequence are, of course, consider as equivalent since the set of objects that informally yield the informal t are also formally present within the standard superstructure. The informal composition \( f^q = I^q \circ t^q \), when embedded relative to \( E \) is denoted by \( f^q = I^q \circ t^q \) since the \( t^q \) is not embedded relative to \( E \) and it merely generates a rational number sequence for the embedded informal paradigm. These different notations are eliminated and only the math-italics font is employed. This is the customary practice throughout Herrmann (1979 - 1993). Notation for informal natural, rational and real numbers, if applicable, is usually the same for the informal and more formal superstructure objects. Each \( t^q(i, j) \) is a rational number. Each \( f(i, j) \) is a nonempty instruction-set.

Each member of \( I_q \) is now considered as determined by a function defined on a set \( R_q \) of rational numbers, \( Q \). The members of \( R_q \) carry the restricted rational simple order and the order \( \leq_I \) for the members of \( I_q \) (the lexicographic order) is order isomorphic to \( R_q \) in the obvious way. Each interval partition is of the form \([c_i, c_{i+1})\) (with a closed interval in two cases), where \( i \in \mathbb{Z} \) and \( \mathbb{Z} \) is the set of integers, and \( t^q(i, 0) = i, t^q(i + 1, 0) = i + 1 \). Each member of \((c_i, c_{i+1})\) is a defined rational number \( t^q(i, j) \), where \( i < j < i + 1 \). For example, consider \([c_2, c_3)\). Then \( t^q(2, 1) = 3 - 1/2, t^q(2, 2) = 3 - 1/4, t^q(2, 3) = 3 - 1/8 \), then, in general, \( t^q(2, j) = 3 - 1/2^j \). Hence, \( f^q(2, 0) <_{t^q} f^q(2, 1) <_{t^q} f^q(2, 2) <_{t^q} \cdots <_{t^q} f^q(3, 0) \). (The order \( \leq_{t^q} \) is lexicographic and is isomorphic to the rational number order for a specific set of rational numbers.)

Let \( I_1 \) be the standard instruction paradigm. An instruction paradigm is defined mathematically in the exact same manner as that of the developmental paradigm in Herrmann (2006) and is equivalent to the range of a sequence \( g': \mathbb{N} \rightarrow \mathcal{P}(L) \), where \( L \) is our denumerable general language. The first case illustrated for the GGU-model is for a developing universe starting with a frozen segments (frame) instruction-set \( g'(0) \). For the other three GGU-model cases, this sequence is appropriately modified. In all cases, the \((f^q(i, j), f^q(p, k))\) is equivalent to “If \( f^q(i, j) \), then \( f^q(p, k) \)). This notation will be simplified later.

For the type-1 case \([0, b], b > 0\), as indicated above, a denumerable instruction paradigm displays a refined form. For \( 1 < m \in \mathbb{N} \), \( I_1 = \{f^1(i, j) \mid (0 \leq i \leq m) \wedge (i \in \mathbb{Z}) \wedge (j \in \mathbb{N}) \} \). Using \( I_1 \), consider the following logic-system.

Due to the simplicity and special nature of the logic-systems used, a simplified algorithm is employed. The basic logic-system algorithm is re-defined for sets of two distinct objects \( \{A, B\} \). If a deduction yields C and C is a member of \( \{A, B\} \), then the “other” member is a deduction. Hence, if A is deduced, then from \( \{A, B\} \), B is deduced. This can be written as \( \{A, B\} = \{A\} \) is deduced. In general, this approach is only valid for these special collections of two element sets. This process mimics the proposition-logic modus ponens rule of inference.
characterize the doubleton set notion and can include all necessary bounds for the quantifiers.) Further, under the simplification used here, each member of containing n-set (i.e. a set of “n” members).

\[
q \text{ propositional tautology. Notice that } M = 1 \text{ called the “jump elements.” Also, each } \Lambda \text{ is a finite set.}
\]

In general, members in \( L^q \) can be characterized by a first-order sentence. When the deduction algorithm is applied to \( \Lambda^q(n) \) the result is an ordered set of words from \( L \) - the ordered instruction paradigm. In accordance with the juxtaposition join operator that yields words in \( L \), this ordered instruction paradigm is a word in \( L \). It can be obtained using the spacing symbol where each member of this paradigm is considered a sentence. For a multi-universe theory, each such universe is a portion of each of the original members of the instruction paradigm.

In order to make the notation as simple as possible for the next construction, notice that \( L^1 \) is denumerable. Let \( \mathbb{N} - \{0\} = \mathbb{N}' \). Thus, there is a bijection \( D^1: \mathbb{N}' \to L^1 \). We use the subscript notation for this bijection. Thus, consider \( L^1 = \{D^1_i \mid i \in \mathbb{N}'\} \). For each \( n \in \mathbb{N}' \), define \( M^1_n = \{D^1_1, \ldots, D^1_n\} \). Let \( \mathcal{M}^1 = \{M^1_n \mid n \in \mathbb{N}'\} \). The set \( \mathcal{M}^1 = \{D^1_1, \ldots, D^1_n\} \), as before, can be considered as a single word-like object.

(There are a few typographic errors in Herrmann (2006) and (2006a). For example, in Theorem 4.1, \( m > 0 \) should read \( m > 1 \), and \( *D \), should read \( *D_1 \). In Herrmann (2006a), page 12, in the first (4), the \( \nu \in *Z^{\geq 0} - Z \) should be replaced with \( \nu \in *Z^{\leq 0} - Z \), \( \gamma \in *Z^{\leq 0} - Z \) should be replaced with \( \gamma \in *Z^{\geq 0} - Z \).)

A finite consequence operator \( S \) is defined in Herrmann (1979 - 1993, p. 65). However, a new simplified logic-system \( S^q \), \( q = 1, 2, 3, 4 \) is defined. When a logic-system is applied, it generates a specific finite consequence operator. It is the logic-system algorithm that does this. In this article, this algorithm is explicitly noted since only logic-systems are used. In general, logic-systems are stated in terms of metamathematics n-tuples. If a set \{A, B, C, \ldots, D\} is used as an hypothesis, then it is word-like since the objects the logical deduction models via the algorithm yields words or word-like objects.

Define \( \mathcal{M}^q \), \( q = 2, 3, 4 \), in the same manner as \( \mathcal{M}^1 \), from members of \( L^q \). For each \( G^q \in \mathcal{M}^q \), there exists a unique \( n \in \mathbb{N}' \) such that \( G^q \in M^q_n \). This \( G^q = \{D^q_1, \ldots, D^q_n\} \), \( D^q_i \in L^q \), \( 1 \leq i \leq n \).

Define the logic-system that generates \( S^q \) as \( S^q = \{x, y \mid (\exists n(n \in \mathbb{N}')) \wedge (x \in M^q_n) \wedge (y \in L^q) \wedge (y \in x)\} \). (This definition can be further described in order to characterize the doubleton set notion and can include all necessary bounds for the quantifiers.) Further, under the simplification used here, each member of \( S^q \) is a propositional tautology. Notice that \( M^q \) is a function with values a singleton set containing an n-set (i.e. a set of “n” members).
Usually, such a logic-system would use ordered pairs to model the rules of inference. Within these rules, finite conjunctions are displayed as first coordinates via n-sets. Again the simplified doubleton-set approach is used here, where one of these sets is \{\{D_1\}, D_1\}.

Hypotheses are considered as members of a set (a 1-ary relation), when part of a logic-system. They are, usually, considered as a list of the members of this set. In general, a logic-system, when considered as an operator, is defined on subsets of the language employed.

From the definitions employed for the logic-systems used here, the properties of the logic-system algorithm \( \mathcal{A} \) can be explicitly described in set-theoretic notation. For these applications, \( \mathcal{A} \) is a function defined on various defined logic-systems and a set of hypotheses. For example, the entire set of deductions or the order in which the deductions are made, among a few other characteristics. In our application to a logic-system, the notation used signifies all of the “deduced” results the algorithm produces when the logic-system is applied to a set of hypotheses. This yields the same results as a corresponding finite consequence operator. What the notation indicates is that the finite consequence operator is being displayed in a more refined and explicit manner. Hence, the algorithm and its relation to the logic-system can be embedded into the formal structure via formalizable characteristics.

When the application characteristics are *-transferred, then the notation \( \mathcal{A} \) is employed. The process of applying the algorithm to the logic-system \( \mathcal{S}^q \), that is applied it to a set of hypotheses \( Y \), is denoted by \( \mathcal{A}((\mathcal{S}^q, Y)) \). Hence, \( \mathcal{A} \) is defined upon a set of ordered pairs. The result of \( \mathcal{A}((\mathcal{S}^q, Y)) \) is a set. An additional step can be included for this specific algorithm, where \( Y \) is removed. When this is done the algorithm is denoted by \( \mathcal{A}' \). The necessary informally and, hence, formally described properties are specifically displayed. In general, the \( q \) notion is not included as part of the \( \mathcal{A} \) notation unless confusion would result.

For the denumerable set \( \mathcal{L}^1 \), notice that for any \( \Lambda^1(k), k \in \mathbb{N} \) there exists a \( k' \in \mathbb{N} \) and \( X^1_{k'} \in M^1_{k'} \), such that \( \Lambda^1(k) \in \mathcal{A}'(\mathcal{S}^1, \{X^1_{k'}\}) \) and, in this case, finite choice yields the \( \Lambda^1(k) \) logic-system. Notice that the logic-system \( \Lambda^1(k) \) is considered as a set-theoretic set. Then the logic-system algorithm \( \mathcal{A} \) is applied to \( (\Lambda^1(k), \{f^1(0, 0)\}) \), where \( f^1(0, 0) \) is the only hypothesis contained in the logic-system. This yields \( f^1(i, j) \in \mathcal{I}_1 \) as a deduction from \( f^1(0, 0) \). Conversely, if \( f^1(i, j) \in \mathcal{I}_1 \), then there is an \( X^1_{k'} \in M^1_{k'} \) and a logic-system \( \Lambda(k) \in \mathcal{A}'(\mathcal{S}^1, \{X^1_{k'}\}) \) such that application of the logic-system algorithm \( \mathcal{A} \) to \( (\Lambda^1(k), \{f^1(0, 0)\}) \) yields \( f^1(i, j) \) as a deduction from \( f^1(0, 0) \).

The informal algorithm \( \mathcal{A} \) is defined on any logic-system that contains an hypothesis and, in this paper, such a logic-system is \( \Lambda^q(x) \) and application is on \( (\Lambda^q(x), Y) \) where \( Y \) is an hypothesis contained in the logic-system and containing but one member. Due to the construction of the \( \Lambda^q(x) \), this yields a partial sequence of members of \( \mathcal{I}_q \). This sequence is denoted by \( \mathcal{A}((\Lambda^q, Y)) \). This sequence represents the steps in the deduction and satisfies the \( \leq_{\mathcal{I}_q} \) order. Also, for this case, \( \mathcal{A}((\Lambda^q(x), Y)) = \mathcal{I}^q \subset \mathcal{I}_q \). Significantly, for \( n, k \in \mathbb{N} \), \( n \leq k, \mathcal{A}((\Lambda^1(n), Y)) \subset \mathcal{A}((\Lambda^1(n), Y)) \).
\( \mathcal{A}((\Lambda^1(k), Y)) \) and \( \mathcal{A}((\Lambda^1(k), Y))[1, n] = \mathcal{A}((\Lambda^1(n), Y)) \).

In the usual way, all of the above informally defined objects are embedded relative to \( \mathcal{E} \). When the informal set-theoretic expresses are considered as embedded into the standard superstructure, all of the bold font conventions defined in Herrmann (1979-1993) are observed. All other embedded symbols retain their math-italics form. Where script notation is used, an underline is used in place of the bold face font. All the following results are relative to our nonstandard model \( \mathcal{M} = ( \mathcal{Q}, \epsilon, =) \) or \( \mathcal{M} = ( \mathcal{R}, \epsilon, =) \) (Herrmann, 1979 - 1993).

**Theorem 2.1** Consider primitive time interval \( 1 = [0, b], b > 0 \). It can always be assumed that interval \( 1 \) is partitioned into two or more intervals \([c_0, c_1], \ldots, [c_{m-1}, c_m], c_m = b, m > 1, m \in \mathbb{Z} \). Let \( \mathcal{L}_1 \) be an instruction paradigm order isomorphic to the rational numbers \( R_1 \subset [0, b] \). For any \( \lambda \in \mathbb{N}_\infty \), there exists a unique hyperfinite \( \mathcal{A}^1(\lambda) \in *\mathcal{L}_1^\prime \) and a \( \lambda ' \in \mathcal{M} \) such that the ultra-word-like \( X^1_{\lambda '}, \mathcal{M}^1_{\lambda } \) and ultra-logic-system \( \mathcal{A}^1(\lambda) \in *\mathcal{A}((\mathcal{A}^1_{\lambda '}, \{ X^1_{\lambda '}\})) \) and \( \sigma \mathcal{L}_1 \subset *\mathcal{A}((\{ \mathcal{A}^1(\lambda), \{ f^1(0, 0)\}) = \mathcal{I}_1 \subset *\mathcal{L}_1 \). Also the \( *\mathcal{A}((\mathcal{A}^1(\lambda), \{ f^1(0, 0)\})) \) \( f \)-steps satisfy the \( \leq \mathcal{L}_1^\prime \) order and \( (\sigma \mathcal{L}_1 - *\mathcal{L}_1) \cap \{ \mathcal{A}((\mathcal{A}^1(\lambda), \{ f^1(0, 0)\})) \) = an infinite set.

Proof. This follows in the same manner as Theorem 4.1 in Herrmann (2006) by \( *\)-transfer of the appropriate first-order statements that precede this theorem statement. Also note that since for every \( n \in \mathbb{N} \), the \( \Lambda(n) \) is finite, then, via the identification process, \( \sigma \mathcal{A}^1(n) = \Lambda(n) \). It also follows that \( \mathcal{A}((\Lambda(n), \{ f^1(0, 0)\})) \subset \mathcal{A}((\Lambda(k), \{ f^1(0, 0)\})) \), from the above and, via \( *\)-transfer, it follows that \( *\mathcal{L}_1 \subset *\mathcal{A}((\L_1^\prime, \{ \mathcal{A}^1(\lambda), \{ f^1(0, 0)\}) = \mathcal{I}_1 \subset *\mathcal{L}_1 \). From the definition of \( \Lambda^1(n) \), these steps numbers are order isomorphic the set of rational numbers \( R_1 \). Hence, \( *\mathcal{A}((\mathcal{A}^1(\lambda), \{ f^1(0, 0)\})) \) is \( f \)-order isomorphic to a hyperfinite subset of \( \mathcal{Q} \). Since there are infinitely many \( \lambda ' < \lambda \) and \( \lambda ' \in \mathbb{N}_\infty \), there are infinitely many \( *f(i, j) \in *\mathcal{A}((\mathcal{A}^1(\lambda), \mathcal{Q}) = *\mathcal{L}_1, \) where \( *f(i, j) \in *\mathcal{L}_1 - \sigma \mathcal{L}_1 \). These are interpreted as ultranatural events but in some cases may differ from physical events only in their primitive time identifications. This completes the proof.

By considering the definition of \( \mathcal{L}_1 \), it follows that the given \( 1 < m \in \mathbb{N} \), \( \mathcal{A}^1(\lambda) \) is precisely \( \{ \mathcal{A}^1(0, 0)\} \cup \{ \{ \mathcal{A}^1(0, 0) \cup \{ \{ \mathcal{A}^1(0, 0) \} \) \cup \{ \{ \mathcal{A}^1(0, 0) \} \} \cup \{ \{ \mathcal{A}^1(0, 0) \} \} \). Of significance is the fact that the steps in the \( *\)-deduction \( *\mathcal{A}((\mathcal{A}^1(\lambda), \{ f^1(0, 0)\})) \) preserve the order \( \leq \mathcal{L}_1 \). Notice that \( \mathcal{A}^1(\lambda) \) is obtained by hyperfinite choice. Further, any \( \mathcal{A}^1(1, x) \in \{ \mathcal{A}^1(0, x) \} \cup \{ \mathcal{A}^1(0, y) \} \cup \{ \mathcal{A}^1(0, 0) \} \) is a hyperfinite \( *\)-deduction from \( f^1(0, 0) = \mathcal{A}^1(0, 0) \). And, it also follows that the set of all such \( *\)-deductions yields a hyperfinite set \( \mathcal{I}_1 \) such that \( *\mathcal{L}_1 \subset \mathcal{I}_1 \subset *\mathcal{L}_1 \).

For the GGU-model and each of the four cases, an internal nonempty set \( X \) disjoint from \( \mathcal{U} \{ \mathcal{B}(\mathcal{C}) \} \) is adjointed when info-fields are employed. It is assumed that a physical universe is a collection of many (\( > 1 \)) physical-systems. By the way each universe-wide frozen frame is constructed, each
unique hyperfinite $\ast 1_R^2$ of the logic-system algorithm $A$.\footnote{\label{footnote}Consider each $\ast f(i, j) \in *I_n$. There is a function $G_n^2(i, j) : \ast f(i, j) \rightarrow \mathcal{P}(\bigcup \ast f^i(C) \mid (0 \leq i \leq n) \land (i \in \mathbb{N}) \cup X)$. The image of $G_n^2(i, j)$ is an info-field. Each $\gamma \in \ast f(i, j)$ determines the properton composition for each of the physical-systems. For a $z \in \ast f(i, j)$ that is not equivalent to a $\gamma$, $G_n^2(i, j)(z)$ determines a physical-like system relative to $X$.}

For the type-2 case $\{0, +\infty\}$, a denumerable instruction paradigm displays a refined form. For this case, $I_2 = \{f^2(i, j) \mid (0 \leq i) \land (i \in \mathbb{Z}) \land (j \in \mathbb{Z})\}$. Using $I_2$, consider the following logic-system.

**Definition 3.1** Let $0 \leq i \in \mathbb{Z}$. For each $n \in \mathbb{N}$, let $k^2(n) = \{(f^2(i, j), f^1(i, j + 1)) \mid (0 \leq j \leq n) \land (j \in \mathbb{N})\}$. For $0 < m \in \mathbb{Z}$, let $K^2(m, n) = \bigcup(k^2(n) \mid (0 < i \leq m) \land (i \in \mathbb{Z})\}$. Finally, let $\Lambda^2(m, n) = \{f^2(0, 0)\} \cup K^2(m, n) \cup \{f^2(p, 0) \mid (0 < p \leq m) \land (p \in \mathbb{Z})\} \cup \{f^2(m, j), f^2(m, j + 1) \mid (0 \leq j < n) \land (j \in \mathbb{N})\}$. Let $A^2(x, y) = (0 \leq x \in \mathbb{Z}) \land (y \in \mathbb{Z})$. Notice that if $0 \leq i < j, i, k \in \mathbb{Z}$, then $x^A((\Lambda^2(i, j), \{f^2(0, 0)\})) \subset A((\Lambda^2(k, n), \{f^2(0, 0)\}))$ for any $j$, $n \in \mathbb{N}$. Also, each $\Lambda^2(m, n)$ is a finite set. (Notice that members in $\mathcal{L}_2$ can be characterized by a first-order sentence.)

Consider any $\Lambda^2(q, k)$. Then there exists an $q'k' \in \mathbb{N}'$ ($q'k'$ is a natural number in $\mathbb{N}'$) and the $q'k'$-set $X^2_{q'k'} \in M^2_{q'k'}$, such that $\Lambda^2(q, k) \subseteq S^2(\{X^2_{q'k'}\})$ and, in this case, finite choice yields the $\Lambda^2(q, k)$ logic-system. Then the logic-system algorithm $A$ applied to $(\Lambda^2(q, k), \{f^2(0, 0)\})$ yields $f^2(q, k)$ as a deduction from $f^2(0, 0)$. Further, $f^2(q, k) \in I_2$. Conversely, if $f^2(q, k) \in I_2$, then there exists an $q'k' \in \mathbb{N}'$ and an $X^2_{q'k'} \in M^2_{q'k'}$ and a logic-system $A(q, k) \in A((S^2, \{X^2_{q'k'}\}))$ such that application of the logic-system algorithm $A$ to $(\Lambda^2(q, k), \{f^2(0, 0)\})$ yields a deduction of $f^2(q, k)$ from $f^2(0, 0)$.

**Theorem 3.1** Consider primitive time interval $2 = [0, +\infty)$. It can always be assumed that interval 2 is partitioned into intervals $[c_0, c_1], \ldots, [c_{m-1}, c_m], m > 1, m \in \mathbb{Z}$. Let $d_2$ be an instruction paradigm order isomorphic to the rational numbers $R_2 \subset [0, +\infty)$. For any $\lambda \in \mathbb{N}_\infty$ and $\nu \in *\mathbb{Z} - \mathbb{Z}, \nu > \nu$, there exists a unique hyperfinite $\Lambda^2(\nu, \lambda) \in *\mathcal{L}_2$ and $\nu', \lambda' \in *\mathbb{N}$ such that the ultra-word-like $X^2_{\nu', \lambda'} \in *M_{\nu', \lambda'}^2$ and ultra-logic-system $\Lambda^2(\nu, \lambda) \in *A((S^2, \{X^2_{\nu', \lambda'}\}))$ and $\sigma_{I_2} \subset *A((\Lambda^2(\nu, \lambda), \{f^2(0, 0)\})) = I_{\nu', \lambda'} \subset *\mathcal{L}_2$. Also the $*A((\Lambda^2(\nu, \lambda), \{f^2(0, 0)\}))$ steps satisfy the $\leq_{I_2}$ order and $(\sigma_{I_2} - *I_2) \cap *A((\Lambda^2(\nu, \lambda), \{f^2(0, 0)\}))$ is an infinite set.

Proof. As in Theorem 2.1, the proof follows by *-transfer of the appropriate formally presented material that appears above in this section 3.
assumed that interval 3 is partitioned into intervals \ldots .

Theorem 4.1

For any \( I \) let \( \Lambda^3(I) \) be an instruction paradigm order isomorphic to the rational numbers \( \mathbb{Q} \). Further, any \( *f^2(i,j) \in \{ *f^2(x,y) \mid (0 \leq x < \nu) \wedge (0 \leq y \leq \lambda) \wedge (x \in *\mathbb{Z} \wedge y \in *\mathbb{N}) \} \) is a hyperfinite \(*\)-deduction from \( f^2(0,0) \). And, it also follows that the set of all such \(*\)-deductions yield a hyperfinite set \( I^3_{\nu} \), such that \( *I^3_{\nu} \subseteq I^3_{\nu} \).

4. Logic-System Generation for the Type-3 Interval

For the type-3 case \(( \infty, 0 ] \), a denumerable instruction paradigm displays a refined form. For this case, \( d_3 = \{ f^3(i,j) \mid (i \leq 0) \wedge (i \in *\mathbb{Z}) \wedge (j \in *\mathbb{N}) \} \) Using \( d_3 \), consider the following logic-system.

Definition 4.1

Let \( i \in \mathbb{Z}, i \leq 0 \). For each \( n \in \mathbb{N} \), let \( k^3_i(n) = \{(f^2(i,j), f^1(i,j+1)) \mid (0 \leq j \leq n-1) \wedge (j \in *\mathbb{N}) \} \). For \( m \in \mathbb{Z} \), \( m < 0 \), let \( K^3(m,n) = \{ f^3(m,n) \mid (m \leq i < 0) \wedge (i \in *\mathbb{Z}) \} \). Finally, let \( \Lambda^3(m,n) = \{ f^3(m,0) \} \cup K^3(m,n) \cup \{(f^3(p-1,n), f^3(p,0)) \mid (m < p \leq 0) \wedge (p \in *\mathbb{N}) \} \). And, \( \mathcal{L}^3 = \{ \Lambda^2(x,y) \mid (0 \leq x \leq \mathbb{Z}) \wedge (y \leq \mathbb{N}) \} \). Notice that if \( i < k \leq 0, i, k \in \mathbb{Z} \), then \( \mathcal{A}(\{ \Lambda^3(i,j) \}, \{ f^3(m,0) \}) \subseteq \mathcal{A}(\{ \Lambda^3(k,n) \}) \). For any \( n \in \mathbb{N} \). Also, each \( \Lambda^3(m,n) \) is a finite set. (Notice that members in \( \mathcal{L}^3 \) can be characterized by a first-order sentence.)

Consider any \( \Lambda^3(q,k) \). Then there exists an \( q'k' \in \mathbb{N} \) and \( X^3_{q'k'} \in M^3_{q'k'} \), such that \( \Lambda^3(q,k) \in \mathcal{A}((S^3, \{ X^3_{q'k'} \})) \). Finally, in this case, finite choice yields the \( \Lambda^3(q,k) \) logic-system. Then the logic-system algorithm \( \mathcal{A} \) applied to \( (\Lambda^3(q,k), \{ f^3(q,0) \}) \) yields \( f^3(q,k) \) as a deduction from \( f^3(q,0) \). Further, \( f^3(q,k) \in \mathcal{I}_3 \). Conversely, if \( f^3(q,k) \in \mathcal{I}_3 \), then there is an \( X^3_{q'k'} \in \mathcal{M}_q^k \) and a logic-system \( \Lambda(q,k) \in \mathcal{S}^3(\{ X^3_{q'k'} \}) \) such that application of the logic-system algorithm \( \mathcal{A} \) to \(( \Lambda^3(q,k), \{ f^3(q,0) \}) \) yields \( f^3(q,k) \) as a deduction from \( f^3(q,0) \).

Theorem 4.1

Consider primitive time interval 3 = \(( \infty, 0 ] \). It can always be assumed that interval 3 is partitioned into intervals \ldots . Let \( d_3 \) be an instruction paradigm order isomorphic to the rational numbers \( R_3 \subset ( \infty, 0 ] \).

For any \( \lambda \in \mathbb{N}_\infty, \mu \in *\mathbb{Z} \setminus \mathbb{Z}, \mu < 0, \) there exists a unique hyperfinite \( *\Lambda^3(\mu, \lambda) \in *\mathcal{L}^3 \) and \( \mu', \lambda' \in *\mathbb{N} \) such that the ultra-word-like \( X^3_{\mu', \lambda'} \in *M^3_{\mu', \lambda'} \) and ultra-logic-system \( \{ *\Lambda^3(\mu, \lambda) \in *\mathcal{A}((S^3, \{ X^3_{\mu', \lambda'} \})) \} \) and \( \mathcal{I}_3 \subseteq *\mathcal{A}(\{ *\Lambda^3(\mu, \lambda) \}, \{ *f^3(\mu, 0) \}) \). Also the \( *\mathcal{A}(\{ *\Lambda^3(\mu, \lambda) \}, \{ *f^3(\mu, 0) \}) \) \(*\)-steps satisfy the \( \leq \mathcal{I}_3^\lambda \) order and \( ( *\mathcal{I}^3_3 \setminus \mathbb{I}_3^3 \) \) \(*\mathcal{A}(\{ *\Lambda^3(\mu, \lambda) \}, \{ *f^3(\mu, 0) \}) \) \(*\)-steps is an infinite set.

Proof. As in Theorem 3.1, the proof follows by \(*\)-transfer of the appropriate formally presented material that appears above in this section 3.
5. Logic-System Generation for the Type-4 Interval

Theorem 5.1 Consider primitive time interval $4 = (-\infty, +\infty)$. It can always be assumed that interval $4$ is partitioned into intervals $\ldots, [c_{-2}, c_{-1}), [c_{-1}, c_0), \ldots$. Let $d_4$ be an instruction paradigm order isomorphic to the rational numbers $R_4 \subset (-\infty, +\infty)$. For any $\lambda \in \mathbb{N}_\infty$, $\nu, \gamma \in \mathbb{Z}$, such that $\nu \leq 0, \gamma \geq 0$, there exists a unique hyperfinite $^\ast \Lambda^4(\nu, \gamma, \lambda) \in \mathbb{Z}^4$ and $\nu, \gamma, \lambda' \in \mathbb{N}$ such that the ultra-word-like $X^4_{\nu, \gamma, \lambda'} \in \mathbb{M}^4_{\nu, \gamma, \lambda'}$ and ultra-logic-system $^\ast \Lambda^4(\nu, \gamma, \lambda) \in \mathbb{A}^4(\{s^4, \{X^4_{\nu, \gamma, \lambda'}\}\})$ and $^\ast \mathcal{L}_4 \subset \mathbb{A}^4(\{s^4\})$ is obtained by hyperfinite choice. Also the $^\ast \mathcal{A}(\{^\ast \Lambda^4(\nu, \gamma, \lambda), \{^\ast f^4(\nu, 0)\}\})$ steps satisfy the $\leq_{\nu, \gamma, \lambda}$ order and $(^\ast \mathcal{L}_4 - \sigma \mathcal{L}_4) \cap ^\ast \mathcal{A}(\{^\ast \Lambda^4(\nu, \gamma, \lambda), \{^\ast f^4(\nu, 0)\}\}) = \mathrm{an~infinite~set}.

By considering the definition of $\mathcal{L}_4$, it follows that the $^\ast \Lambda^4(\nu, \gamma, \lambda)$ is precisely $\{^\ast f^4(\nu, 0)\} \cup \{\{\{^\ast k^4 | (\nu \leq i < \gamma) \& (i \in \mathbb{Z})\}\} \cup \{\{^\ast k^4(p - 1, \lambda), \{^\ast f^4(p, 0)\} | (\nu < p \leq \gamma) \& (p \in \mathbb{Z})\}\} \cup \{\{^\ast f^4(\gamma, j), \{^\ast f^4(\gamma, j + 1)\} | (0 \leq j < \lambda) \& (j \in \mathbb{Z})\}\}$. Of significance is the fact that the steps in the $^\ast$-deduction $^\ast \mathcal{A}(\{^\ast \Lambda^4(\nu, \gamma, \lambda), \{^\ast f^4(\nu, 0)\}\})$ preserve the order $\leq_{\nu, \gamma, \lambda}$. Notice that $^\ast \Lambda^4(\nu, \gamma, \lambda)$ is obtained by hyperfinite choice. Further, any $^\ast f^4(i, j) \in \{^\ast f^4(x, y) | (\nu \leq x \leq \gamma) \& (0 \leq y \leq \lambda)\}$ is a hyperfinite $^\ast$-deduction from $f^4(\nu, 0)$. And, it also follows that the set of all such $^\ast$-deductions is a hyperfinite set $\mathcal{L}_4$ such that $\mathcal{L}_4 \subset \mathcal{I}_4 \subset \mathcal{L}_4$.

The above established theorems and appropriate definitions all hold for the development paradigms and yield ultra-logic-systems that can replace the ultralogic notion.

6. The Complete GGU-model Scheme

For the $(\mathcal{S}tG_q)$ is defined in Herrmann (2006a). The following scheme is not in composition notational form due to one application of choice and the step-by-step application of $(\mathcal{S}tG_q)$. It represents an ordered application of the GGU-model operators. For $q = 1, 2, 3, 4, x = \lambda, \nu, \gamma, \mu, \lambda, \nu, \gamma, \lambda$ with or without commas, respectively. The $a$, $b$, $c$ take the appropriate value for a specific $q$.

$$(\mathcal{S}tG_q)(\{^\ast \mathcal{A}(\{^\ast A^4(a), \{^\ast f^4(b, c)\}\})\}(\{^\ast \mathcal{A}(\{^\ast S^4, \{X^4_{\nu, \gamma, \lambda}\}\})\)).$$

The operators $\mathcal{A}$ and $\mathcal{A}'$ have characterizing first-order statements. These statements need not capture all of the intuitive statements that describe the algorithms. The results of application of $\mathcal{A}'$ as formalized can show major aspects of the algorithm’s selection process. For example,

$$\forall x \forall y \forall z \forall w ((w \in \mathcal{A}^q) \& (y \in \mathcal{F}(\mathcal{L}^q)) \& (y \in w) \& (x \in \mathcal{A}'(\mathcal{S}^q, y)) \rightarrow \exists p ((p \in \mathcal{S}^q) \& (y \in p) \& (x \in p) \& (y \neq x)).$$

Of course, the natural numbers and the embedded $D^q$ can also be employed.

7. The Participator Universe

For the GGU-model, one of the most difficult requirements is to include the concept of the “participator” universe. As stated at the May 1974 Oxford Symposium in Quantum Gravity, Patton and Wheeler describe how existence of human
beings alter the universe to various degrees. “To that degree the future of the universe is changed. We change it. We have to cross out that old term ‘observed’ and replace it with the new term ‘participant.’ In some strange sense the quantum principle tells us that we are dealing with a participator universe.” (Patton and Wheeler (1975, p. 562).) This aspect of the GGU-model is only descriptively displayed in section 4.8 in Herrmann (2002). It is now possible to obtain formally the collection of pre-designed universes that satisfies this participator requirement.

The previous notation is modified for finitely many (> 0) instruction paradigms as previously denoted by \( I_q \). From the construction of each instruction paradigm using \( L \), it follows that there is, at least, a sequence of possible alterations. An instruction paradigm is a nonempty subset of the subsets of \( L \). Hence, the collection of all such instruction paradigms is a member of \( \mathcal{P}(\mathcal{P}(L)) \). There can be infinitely many basic universes. These are universes prior to participator alterations. For each of these, there is a collection of ultra-word-like objects of the appropriate type. What follows next is for an arbitrary member of this collection of ultra-world-like objects and, hence, an arbitrary basic universe.

So as to include the type of universe being considered, let \( q : \mathbb{N}' \times [1, 4] \to \mathcal{P}(\mathcal{P}(L)) \). Then for a specific \( p \in [1, 4] \) an expression \( \{ x \mid (x = q(n, p)) \land (n \in \mathbb{N}') \} = I_p \) represents this denumerable set of instruction paradigms for type-p universes. (If \( n' \neq m \), then \( q(n', p) \neq q(m, p) \).) Let \( \{ x \mid \exists p (p \in [1, 4] \land (x = I_p)) \} = \mathcal{I} \). Then, a specific \( f^1(0, 0) \) is further identified relative to the sequences. As an example, \( f q^{(3,1)}(0, 0) \subset L \) represents a specific \( n = 3 \) type-1 instruction-set. Thus, as embedded into the formal structure this last expression reads \( f q^{(3,1)}(0, 0) \subset L \).

An original alteration can be miniscule and made in one or more of the necessary parameters that are satisfied by a specific cosmology. This can be done in such a way that only miniscule alterations in physical-system satisfy the alterations. On the other hand, a highly altered cosmology can also occur. An alteration is local prior to it being propagated during a universe’s development. Although not specifically included, and indeed the definition would need to be altered slightly, other sequences \( q' \) can be used where various values can be empty or repeated. Also, for a universe with infinitely many local alterations at the same moment, one can include the obvious change in the \( q \) sequence, where the sequence \( q \) is a sequence of type-4 except that each image is a “universe.”

For the GGU-model, the various members of \( I_p \) satisfy the “participator” requirements, when participators exist, for each of the known suggested cosmologies. When embedded into the formal structure, properties of \( q \) can be easily characterized, using various forms, in a first order language. For example,

\[
\forall x \forall p ((x \in \mathbb{N}') \land (p \in [1, 4]) \to \\
\exists y \exists z ((z \in \mathcal{I}) \land (y \in z) \land (q(x, p) = y)).
\]

Consider a specific \( p' \in [1, 4] \). Then

\[
\forall y ((y \in \mathbb{N}') \to \exists x ((x \in \mathbb{N}') \land (q(x, p') = y))).
\]
In *-transfer form, these two sentences read

\[ \forall x \forall p((x \in *\mathbb{N}') \land (p \in [1,4])) \rightarrow \exists y \exists z((z \in *\mathbb{I}) \land (y \in z) \land (\ast q(x,p) = y)). \]

\[ \forall y((y \in *\mathbb{I}') \rightarrow \exists x((x \in *\mathbb{N}') \land (\ast q(x,p') = y))). \]

The previous four theorems are all relative to a specific instruction paradigm and each holds for a collection of these instruction paradigms. Thus, the notion can be added and the additional statement that the results hold for each \( n \in *\mathbb{N}' \) and each \( p \in [1,4] \). The special processes noted in the scheme in section 6 are applied to each set of instruction paradigms.

Each member of In Herrmann (2002), hyperfast propertons are mentioned as mediators for the automatic selection of a particular member of \( \mathbb{I}_p \). Each not realized member of \( \{\mathbb{I}\}_p \) is termed as covirtual. Notice that from the transformed formal statements above, there exist a member of \( \mathbb{I}_p' \) for each \( \gamma \in *\mathbb{N}' - \mathbb{N}' \). These can be used for various interpretations using either a \( q, q' \), \( q \) type of sequence. Further, GGU-model predicted processes and entities can aid in comprehending the notion of the non-temporal and its relation to the temporal.

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Herrmann, R. A. (1989), Fractals and ultrasmooth microeffects, J. Math. Physics, 30(4), 805-808. (Note that there are typographical errors in this paper. In the proof of Theorem 4.1, in equations \( b(x,c,d), G_j(x), \) the \( + ) \) should be \( ) + \)). In \( G_j(x) \), the second \( c \) should be replaced with \( a_j \). On page 808, the second column, second paragraph, line six, \( st(D) \) should read \( st( *D) \) and, trivially, \( x \in \mu(p) \), should read \( x \in \mu(p) \cap *D \). In the proof of Theorem 3.1, first line \( *R^m \) and should read \( R^m \).

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| Symbol | Definition | Page no. |
|--------|------------|---------|
| $A_h$ | .......................... | 7. |
| $H_t$ | .......................... | 7. |
| $\mathcal{A}$ | .......................... | 7. |
| $\mathcal{W}$ | .......................... | 7. |
| ZFC | .......................... | 8. |
| ZFH | .......................... | 8. |
| $D$ | .......................... | 9. |
| $W_X$ | .......................... | 9. |
| $B_Y$ | .......................... | 9. |
| $\mathcal{P}(B_Y)$ | .......................... | 9. |
| $H^n = H^{[0,n]}$ | .......................... | 9. |
| $P_H$ | .......................... | 10. |
| $P_{[\mathcal{W}] = P}$ | .......................... | 10. |
| $f(k) \leq f(j)$ | .......................... | 10. |
| $\sim$ | .......................... | 10. |
| $[f]$ | .......................... | 11. |
| $| \cdot |$ | .......................... | 11. |
| $\mathcal{E}$ | .......................... | 11. |
| $C$ | .......................... | 11. |
| $F(A)$ | .......................... | 11. |
| $C : \mathcal{P}(A) \to \mathcal{P}(A)$ | .......................... | 11. |
| $k \subset F(A) \times A$ | .......................... | 12. |
| $C_k$ | .......................... | 12. |
| $k_c$ | .......................... | 13. |
| $\Theta : \mathcal{W} \to \mathcal{E}$ | .......................... | 13. |
| $\mathcal{X}$ | .......................... | 17. |
| $\theta : i[\mathcal{W}] \to \mathcal{E}$ | .......................... | 20. |
| $R_{\mathcal{A}}$ | .......................... | 21. |
| $\mathcal{M}$ | .......................... | 22. |
| $^*M = \langle ^*\mathcal{N}, \in, = \rangle$ | .......................... | 23. |
| $\sigma B$ | .......................... | 23. |
| $\mathcal{Y}$ | .......................... | 23. |
| $L_0$ | .......................... | 24. |
| $\{P_i \mid i \in \omega\}$ | .......................... | 25. |
| $B'$ | .......................... | 26. |

**Symbol Definitions**:

- $B$ to $E$: Various symbols and their definitions are listed.
- $\mathcal{A}$ and $\mathcal{W}$: Various constants and sets.
- $ZFC$ and $ZFH$: Mathematical symbols and notations.
- $D$ and $W_X$: Variables and functions.
- $B_Y$ and $\mathcal{P}(B_Y)$: Structures and functions.
- $H^n = H^{[0,n]}$: Functions and their applications.
- $P_H$ and $P_{[\mathcal{W}] = P}$: Variables and functions.
- $f(k) \leq f(j)$: Relations.
- $\sim$, $[f]$, and $| \cdot |$: Various notations.
- $\mathcal{E}$ and $C$: Mathematical sets.
- $F(A)$ and $C : \mathcal{P}(A) \to \mathcal{P}(A)$: Functions.
- $k \subset F(A) \times A$, $C_k$, and $k_c$: Various expressions.
- $\Theta : \mathcal{W} \to \mathcal{E}$, $\mathcal{X}$, and $\theta : i[\mathcal{W}] \to \mathcal{E}$: Functions and mappings.
- $R_{\mathcal{A}}$, $\mathcal{M}$, and $^*M = \langle ^*\mathcal{N}, \in, = \rangle$: Various expressions.

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\[\Gamma \] .................. 53.
\[(A)_R \] .................. 54.
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\[TC \] .................. 54.
\[SS \] .................. 54.
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