Review Article

Application of Calculus of Variation in the Optimization of Functional Parameters of Compacted Modified Soils: A Simplified Computational Review

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Fixed endpoint problems (FEPPs) in constrained systems like the effect of curing time or the effect of certain additives in soil stabilization operations have been reviewed illustratively for sustainability purposes in geotechnics. The calculus of variation (CoV) technique of Hamilton’s problem was demonstrated using a typical case in geotechnics; the effect of curing time on the unconfined compressive strength of expansive soils is utilized as foundation materials. The era of smart technologies is evolving, and to key into this fast-moving area to help the field of geotechnics, it is required that these new areas are deployed to study their usefulness. The use of CoV in modeling or simulating geotechnical properties of soil behavior is not prominent and has been played down due to the uncertainties surrounding it. However, this work has identified that if any geotechnical system can be demonstrated in graphs, then the use of CoV becomes easy with the mathematical concept that curves are elements of straight paths. The results of this work show that CoV is a powerful tool to achieving sustainable optimization of quality properties of stabilized for sustainable and optimal materials handling, design, and construction.

1. Introduction

The application of numerical and empirical techniques is an evolving method in geotechnical and geoenvironmental engineering. This has also evolved to the deployment of artificial intelligence algorithms in solving many of the everyday problems in earthworks and geotechnics in general as posited by Onyelowe et al. [1, 2]. These novel techniques have encouraged smart geotechnics design and the effectiveness of computational geotechnics. Calculus of variation (CoV) is one such smart and numerical method that has helped researchers and engineers to resolve complex engineering problems. Although this method is not commonly and presently evolving in soil mechanics and stabilization, it is the mission of this work to present the possibilities of utilizing this evolving mathematical method, which relies on the graphical behavior of a system to optimize its characteristic components. Fundamental approaches have been identified in the solutions of CoV, and they are known as Hamilton’s and Brachistochrone problems [3, 4]. While Hamilton’s method is commonly used in solving most engineering problems, the Brachistochrone method handles systems with the effects of velocity, gravity, friction, and so on. In this overview, Hamilton’s method will be used while the other methods will be extensively dealt with in future works. According to Hamilton’s method, solving problems in calculus of variations (CoV) is such that the local minima and maxima of the function \( f(x) \) of a complex system are identified:

\[
f'(x) = 0.
\]  

(1)

Further taking the second derivative of the above system of functions, \( f(x) \) gives
that the calculus of variation has hardly made it in problems inasmuch it can be seen and confirmed open a new innovative and novel approach towards solving the field of computational geotechnics, that promises to performance of engineering systems [1, 15–17].

logic, and ANN, and the results have also shown that these regression, back-propagation artificial neural network, fuzzy optimization, extreme vertices, analysis of variance, multiple stabilizations and civil earthworks in general, like Scheffe methods have been applied in different protocols of soil uncertainties of footings on slopes [13, 14]. Other smart algorithm was greatly useful in resolving the applied force slopy surface, and this parametric and nondimensional algorithm was on the bearing capacity problem of a foundation on a technique under consideration was applied in geotechnics was on the bearing capacity problem of a foundation on a...
following equation: 

\[ \frac{\partial y_{(x)}}{\partial \Delta S} = 0 + \frac{\partial \Delta S}{\partial \Delta S^x} + \Delta S \frac{\partial}{\partial \Delta S^x} \varepsilon_x. \] \tag{10} 

The function in equation (9) is minimized when the degree of variation (\( \Delta S \)) is 0; thus, 

\[ \frac{dI}{d\Delta S}(\Delta S = 0) = 0 = \int_{x_1}^{x_2} \left[ \frac{\partial F}{\partial y} \frac{\partial \varepsilon_x}{\partial \Delta S} + \frac{\partial F}{\partial y^1} \frac{\partial \varepsilon_x^1}{\partial \Delta S} \right] \cdot dx, \] \tag{11} 

where 

\[ \frac{\partial y}{\partial \Delta S} = \varepsilon_x, \] \tag{12} \]

Equation (11) is at the minimum point where the straight path of the system forms a stationary point and zero slope or zero degree of variation (\( \Delta S \)). However, equation (11) becomes equation (13) after substituting for equation (12) derivative parameters: 

\[ \frac{dI}{d\Delta S}(\Delta S = 0) = 0 = \int_{x_1}^{x_2} \left[ \frac{\partial F}{\partial y} \cdot \varepsilon_x + \frac{\partial F}{\partial y^1} \cdot \varepsilon_x^1 \right] \cdot dx. \] \tag{13} 

From the laws of integrals, equation (13) is estimated by integration by parts solution method by adopting the end conditions; \( x_1, x_2, 0 \), and it becomes 

\[ \left[ \frac{\partial F}{\partial y} - \frac{d}{dx} \left( \frac{\partial F}{\partial y^1} \right) \right] \varepsilon_x \cdot dx = 0. \] \tag{14} 

The minimized path to achieve the optimization (minima and maxima) of the system follows equation (14), where \( F \) is the Lagrange or the variation of the system. The Euler–Lagrange equation is the first part of equation (14); thus, 

\[ \frac{\partial F}{\partial y} - \frac{d}{dx} \left( \frac{\partial F}{\partial y^1} \right) = 0. \] \tag{15} 

This follows that for every engineering problem with system and material characteristics of the form illustrated in Figure 1, such system can be solved by optimizing its functional parts by the method of CoV. With this and from the foregoing, the Lagrange \( F \) is given by equation (8) as 

\[ F \sqrt{1 + (y_1^1)^2}. \] \tag{16} 

Following boundary conditions of equation (11),
the strength of treated soils and the utilization of construction materials to solve soil stabilization problems and to optimize the formulation of UCS to show the novel approach of using CoV compacted subgrade materials. In this illustrative review, the assessment of the suitability of soils utilized as compacted soils is a major engineering parameter that is pivotal to the management of the material components during and after design, construction, and use is of utmost importance. The application of computational geotechnics and of course artificial intelligence (AI) ensures that construction materials are managed for optimum application. Unconfined compressive strength (UCS) of compacted or treated and compacted soils is a major engineering parameter that is pivotal to the assessment of the suitability of soils utilized as compacted subgrade materials. In this illustrative review, the intention is to conduct the calculus of variation formulation of UCS to show the novel approach of using CoV to solve soil stabilization problems and to optimize the strength of treated soils and the utilization of construction materials as binders in this procedure. In this case, UCS is used interchangeably with C to designate the unconfined compressive strength. Figure 2 presents a typical case, which shows the effect of curing time (t) in days on the unconfined compressive strength (C) in kN/m². For clarity purposes, this pair of quality properties of the soil study could have been obtained with CBR, resilient modulus (MR), resistance value (r-value), compaction, durability, and son on of untreated or treated compacted soils for foundation design and construction. In this, C is maximized against a value of minimized t where A-B is admissible variational space (AVS) of the system with end conditions of $C(t_1)$ and $C(t_2)$. Apart from the natural curve ADB of the system, the system could have also taken another curve path ACB to arrive at B, which is the maximum point of the maximized C. However, within the AVS exists the stationary function which optimizes the system and gives the shortest path between A and B at minimum t. Figure 3 shows the degree of variation δ that exists between the natural curve path of the system A-C-B and A-D-B or A-E-B while the optimized path of AVS A-B is the path of stationary function upon which C is maximized at a minimized t.

To solve this optimization problem using Hamilton’s solution, the E-L equation of the system illustrated in Figures 2 and 3 has to be formulated and solved to prove that the variational solution of the system depends on the straight path between A and B. Once solved to that extent, the slope and intercept of the straight-line equation become natural boundary conditions for solving the entire CoV system. The variational solution of Figures 2 and 3 is as follows:

$$C(t) = C^1_{(t)} + \delta \epsilon_t,$$

where $C(t)$ is the function of the curve of typical compression-time system, $C^1_{(t)}$ is the first partial derivative ($\partial C/\partial t$) of the compression function, $\delta$ is the degree of variation from the natural curve of the system, and $\epsilon_t$ is the change in function due to variational shifts.

$$\delta(t) = \int_{t_1}^{t_2} 1 + \left(\frac{C^1(t)}{C(t)}\right)^2 \, dt.$$  

The distance function of the path A-B is given as

$$I[f] = I(\delta) = \int_{t_1}^{t_2} F(t, C(t), \frac{\partial C}{\partial t}) \, dt.$$  

Formulating the chain equation of the system from equation (21) and equation (22) gives

$$\frac{dI}{d\delta}(\delta = 0) = 0 = \int_{t_1}^{t_2} \left[ \frac{\partial F}{\partial C} \left( \frac{\partial C}{\partial \delta} \right) + \frac{\partial F}{\partial C^1} \left( \frac{\partial C^1}{\partial \delta} \right) \right] \, dt,$$

where

$$\frac{\partial C}{\partial \delta} = \epsilon_t,$$

$$\frac{\partial C^1}{\partial \delta} = \epsilon^1_t.$$
From the laws of integrals, equation (24) is substituted into equation (23) and estimated by integration by parts solution method by adopting the end conditions \( t_1 \leq t \leq t_2 \) as fixed endpoint problem of a constrained system, and it becomes
\[
\int_{t_1}^{t_2} \left[ \partial F/\partial C - d \left( \frac{\partial F}{\partial C^1} \right) \right] dt = 0. \tag{25}
\]

The E-L equation of the unconfined compressive system curing time system from equation (25) presented in Figure 4 becomes
\[
\frac{\partial F}{\partial C} - d \left( \frac{\partial F}{\partial C^1} \right) = 0, \tag{26}
\]
where \( F \) which is interchangeably used with \( L \) is the Lagrange and is given by equation (16) as
\[
F = \sqrt{1 + (C^1)^2}. \tag{27}
\]

Equation (27) is substituted in equation (26) applying boundary conditions and solving to achieve the straight-line path of the system which lies within the AVS to get
\[
C_1^1 = at + b, \tag{28}
\]
where \( a \) is the slope of the system path A-B and \( b \) is the intercept of the system path A-B on C axis in kN/m\(^2\). The values of \( a \) and \( b \) become the boundary conditions for re-solving the entire system and estimating the maximized \( C \) and minimized \( t \).

4. Conclusions

CoV has been demonstrated as an optimization tool which can simulate the behavior of geotechnical properties of construction materials in soft or expansive soil blends as stabilized and compacted earth materials utilized in the construction industry. Various computational and artificial intelligence procedures have been in use, but the use of CoV in the field of geotechnical and geoenvironmental engineering is evolving. This illustrative overview work has keyed in to demonstrate that the CoV can be used to model systems in geotechnics for sustainable construction materials utilization and design. This has further shown that as long as the quality properties of an engineering system can be illustrated graphically, the deployment of CoV becomes a powerful tool to simulate the input parameters for material utilization, design, and performance monitoring purposes. The typical effect of curing time on the compressive strength of untreated or treated compacted earth material has been used to demonstrate these possibilities in geotechnical engineering. The functional changes and degrees of variation of the systems under study are harmonized using Hamilton’s method for the fact that these factors are not affected by friction and gravity. Further research is suggested to study the employment of Brachistochrone solution to resolve geotechnics systems affected by friction and gravity, for example, compaction procedure where the compactive effort.
hammer drops from a measured height to effect densification of compacted soils and shear parameters evaluation.

Data Availability

The data were derived from the available literature and computational breakthrough evolved as a novel method where calculus of variations unlike before is deployed in solving geotechnical and construction material system problems.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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