Predicting the Seesaw Scale in a Minimal Bottom-Up Extension of MSSM

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(Dated: November, 2005)

Abstract

We analyze a minimal bottom-up seesaw scenario where we require the theory to satisfy three phenomenological conditions: (i) it is supersymmetric; (ii) it has a local $B-L$ symmetry as part of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory to implement the seesaw mechanism and (iii) $B-L$ symmetry breaking is such that it leaves R-parity unbroken giving a naturally stable dark matter. We show that in such a theory, one can predict the seesaw scale for neutrino masses to be $M_R \simeq \sqrt{M_{SUSY} M_{Pl}} \simeq 10^{11}$ GeV. We show that the ground state with this property is a stable minimum and is lower than possible electric charge violating minimum in this theory. Such models in their generic version are known to predict the existence of a light doubly charged Higgs boson and Higgsino which can be searched for in collider experiments. We give expressions for their masses in this minimal version. We then indicate how one can get different expectation values for the MSSM Higgs doublets in the theory required to have realistic quark masses.
I. INTRODUCTION

One of the simplest ways to understand the small neutrino masses is to use the seesaw mechanism where one adds a right-handed neutrino to the standard model. This allows a Higgs coupling that leads to a mass connecting the right- and the left-handed neutrinos, but this is much too large to be acceptable due to naturalness arguments that dictate this mass $m_D$ is of order of the known quarks and leptons. However, the fact that the right-handed neutrino is a standard model singlet allows one to write a large Majorana mass $M_R$ for it leading to a small effective Majorana mass for the left-handed neutrino given by $m_\nu \sim - \frac{m_D^2}{M_R} \ll m_{e,u,d}$. This is known as the seesaw mechanism \[1\].

The seesaw mechanism raises the following questions:

(i) is there a natural way for the right-handed neutrino to appear in the theory rather than just being added to the standard model by hand?

(ii) how large is the seesaw scale $M_R$? In particular, why is $M_R \ll M_{P\ell}$ as required by observations? Can we predict the value of $M_R$?

The answers to these questions are connected. To answer the second question, one may start with the observation that the Majorana masses of the right-handed neutrinos break the $B-L$ symmetry, and if $B-L$ is a gauge symmetry of Nature\[2\], then that will explain why $M_R \ll M_{P\ell}$. It is known that if the weak gauge group beyond the standard model is either $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ ($G_{211}$) or the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, then the right-handed neutrino appears naturally due to reasons of anomaly cancellation. So, $B-L$ naturally explains both the seesaw scale and the presence of $\nu_R$.

None of these considerations, however, tell us what the magnitude of the seesaw scale $M_R$ is. Observations tell us that if we want to understand the atmospheric neutrino data in a seesaw scheme we get $M_R \sim 10^{11} - 10^{15}$ GeV, depending on the magnitude of the Dirac mass of the neutrino for the third generation (between 1-100 GeV). The higher value is tantalizingly close to the conventional grand unification scale in supersymmetric theories. As a result, in GUT theories such as $SO(10)$, one can identify $M_R$ with the scale of grand unification. Yet such theories allow many different values for $M_R$ while remaining consistent with the grand unification of couplings \[3\], depending on how the symmetry is broken and the
choice of Higgs multiplets, which prevents this connection between \( M_R \) and grand unification from being unique. Nonetheless, simple one or two step symmetry breaking \( SO(10) \) models have provided a compelling class of models for studying the consequences of the seesaw mechanism for neutrino masses and mixings and need to be taken very seriously.

In this paper, we take an alternative point of view to the understanding of neutrino masses by making a minimal extension of the standard model to the supersymmetric left-right model (SUSYLR)\[4\] or \( G_{211} \). For the SUSYLR models we wish to consider, there are three defining phenomenological conditions:

(i) it is supersymmetric;

(ii) it has a local \( B-L \) symmetry as part of the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) so that one can implement the seesaw mechanism and

(iii) \( B-L \) symmetry breaking is such that it leaves R-parity unbroken so that there is a naturally stable dark matter.

We will show in this paper that if in addition to the above, we add to this model an R-symmetry (not to be confused with R-parity, which is a property of the model regardless of the R-symmetry assumed), then it will predict \( M_R \simeq \sqrt{M_{SUSY}M_{Pl}} \sim 10^{11} \text{ GeV} \) which is of the right order of magnitude to be the seesaw scale. This is the main result of our paper and we believe that this is of interest since it determines the seesaw scale from first principles without the assumption of grand unification.

One must be careful with both the R-symmetry and the R-Parity because R-symmetry restricts the number of parameters in the superpotential and R-parity conservation allows a stable charge violating vacuum; therefore, it is not obvious \( a \ priori \) that this model has a stable, electric charge conserving vacuum. In fact, it has been shown that if we take only renormalizable terms in the theory, the R-parity must break in the electric charge conserving vacuum\[10\]. We demonstrate that in our model (which includes non-renormalizable terms) there exists, within the parameter space, a charge conserving vacuum lower than the charge violating one.

Working, then, in the context of the charge conserving vacuum, our model contains light particles in addition to those found in the standard model. Among these are two unstable neutral scalar bosons and a neutral fermion in the 100 GeV mass range. Unfortunately these
particles have no experimental implications because they will be difficult to produce in the laboratory due to their weak coupling to standard model particles.

Yet there are light particles in our model that do have an experimental consequence: these are light doubly charged particles with masses in the 100 GeV to TeV range and they are present in all SUSYLR models \[5\] (even those without the R-symmetry we impose). Phenomenological implications of these particle has been studied in \[6\] and their experimental search has been conducted \[7\] at LEP, CDF and D0 experiments and can be searched at the LHC. Low energy consequences of the doubly charged Higgs bosons have also been studied extensively\[8\]. The present lower limits on their masses is around 115 GeV or so depending on which channel the particle has been searched for.

The details of the above discussion will be presented in the remaining of the paper in following order: in sec. II we present our minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model where the seesaw scale is predicted as the geometric mean of the weak scale and the Planck scale and analyze the ground state of the theory. We then proceed to find the effective low-energy theory, verify that it contains the minimal supersymmetric model, and address the breaking of $SU(2)_L \times U(1)_Y$. The end of section III verifies that the ground state of the theory conserves electric charge. In sec. III we present the masses of the Higgs bosons and check that there are no tachyonic states; additionally, we show that there are TeV scale doubly charged fermions and bosons in this theory—confirming earlier results derived for nonminimal models \[9\]. In sec. IV we discuss possible grand unification prospects for the model.

II. THEORETICAL MODEL AND THE SEESAW SCALE

The quarks and leptons in this model are assigned to the representations of the group as shown in Table II. We assume that parity symmetry is broken at a high scale so that the left handed partner of the $\Delta^c$ and $\bar{\Delta}^c$ are not included in the theory.
### TABLE I: Assignment of the fermion and Higgs fields to the representation of the left-right symmetry group.

For the sake of clarification, we note that the various fields transform under $SU(2)_L \times SU(2)_R$ as follows:

$$
Q \rightarrow U_L Q, \quad Q^c \rightarrow U_R Q^c, \quad L \rightarrow U_L L, \quad L^c \rightarrow U_R L^c
$$

$$
\Delta^c \rightarrow U_R \Delta^c U_R^\dagger, \quad \bar{\Delta}^c \rightarrow U_R \bar{\Delta}^c U_R^\dagger, \quad \Phi_a \rightarrow U_L \Phi_a U_R^\dagger
$$

Given those transformations, the fields may be written as

$$
Q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad Q^c = \begin{pmatrix} d^c \\ -u^c \end{pmatrix}, \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}, \quad L^c = \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix}
$$

$$
\Delta^c = \begin{pmatrix} \Delta^{c0} \\ \Delta^{-} \\ \Delta^{-} \\ \Delta^{-} \end{pmatrix}, \quad \bar{\Delta}^c = \begin{pmatrix} \bar{\Delta}^{c+} \\ \bar{\Delta}^{c+} \\ \bar{\Delta}^{c0} \\ \bar{\Delta}^{c0} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_0^d & \phi_0^u \\ \phi_d^- & \phi_u^+ \end{pmatrix}
$$

where the neutral fields can be split into their real and imaginary parts by

$$
\Delta^{c0} = \frac{1}{\sqrt{2}} (\text{Re} \Delta^{c0} + i \text{Im} \Delta^{c0}), \quad \bar{\Delta}^{c0} = \frac{1}{\sqrt{2}} (\text{Re} \bar{\Delta}^{c0} + i \text{Im} \bar{\Delta}^{c0})
$$

$$
\phi_u^0 = \frac{1}{\sqrt{2}} (\text{Re} \phi_u^0 + i \text{Im} \phi_u^0), \quad \phi_d^0 = \frac{1}{\sqrt{2}} (\text{Re} \phi_d^0 + i \text{Im} \phi_d^0)
$$

(1)

The superpotential is given by

$$
W = W_Y + W_H
$$

(2)
where
\[ W_Y = i h Q^T \tau_2 \Phi Q^c + i h' L^T \tau_2 \Phi L^c + i f_c L^c T \tau_2 \Delta^c L^c \]
\[ W_H = \frac{\lambda_A}{M_{P\ell}} (\text{Tr}(\Delta^c \tilde{\Delta}^c))^2 + \frac{\lambda_B}{M_{P\ell}} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\tilde{\Delta}^c \tilde{\Delta}^c) \]
\[ + \frac{\lambda_c}{M_{P\ell}} \text{Tr}(\Delta^c \tilde{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2) \]

Several comments regarding this superpotential are in order:

(i) There are no bilinear terms in the fields \( \Delta^c \) or \( \Phi \). This property is the one that permits the prediction of the seesaw scale, and the lack of such terms can always be accomplished by requiring an appropriate R-symmetry in the theory; e.g.,
\[ (\tilde{\Delta}^c, \Delta^c, \Phi) \rightarrow e^{i\pi/2}(\tilde{\Delta}^c, \Delta^c, \Phi); \ (L, L^c, Q, Q^c) \rightarrow e^{-i\pi/4}(L, L^c, Q, Q^c). \]

(ii) There is no distinct \( \text{Tr}(\Delta^c \tau_2 \Phi^T \tau_2 \Phi \tilde{\Delta}^c) \) term like the one that occurs in the non-supersymmetric theory. This is due to the fact that a superpotential must be holomorphic in the fields—excluding the appearance of a \( \Phi^\dagger \). Thus, the only invariant combination involving both the right-handed \( \Delta \)'s and the \( \Phi \)'s must be of the form \( \Phi^T \tau_2 \Phi \) to satisfy \( SU(2)_L \) invariance; the transpose then forces another \( \tau_2 \) to be involved. This, coupled with the fact that \( \Phi^T \tau_2 \Phi = 1 \cdot \det \Phi \), gives that
\[ \text{Tr}(\Delta^c \tau_2 \Phi^T \tau_2 \Phi \tilde{\Delta}^c) = \frac{1}{2} \text{Tr}(\Delta^c \tilde{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2) \]

From Eqs. (2) and (3) we can write down the Higgs potential taking the supersymmetry breaking terms into account as follows:
\[ V(\Phi, \Delta^c, \tilde{\Delta}^c) = V_F + V_D + V_{\text{SUSY}} \]
where

\[
V_F = \frac{4\lambda_2^2}{M_{\text{Pl}}^2} |\text{Tr} (\Delta^c \bar{\Delta}^c)|^2 \left( |\text{Tr} \Delta^c|^2 + |\text{Tr} \bar{\Delta}^c|^2 \right) + \frac{4\lambda_2^2}{M_{\text{Pl}}^2} \left[ |\text{Tr} (\Delta^c \bar{\Delta}^c)|^2 \text{Tr} |\Delta^c|^2 + |\text{Tr} (\Delta^c \bar{\Delta}^c)|^2 \text{Tr} |\bar{\Delta}^c|^2 \right] + \frac{\lambda_2^2}{M_{\text{Pl}}^2} \left[ \text{Tr} (\Phi^T \tau_2 \Phi \tau_2) \right]^2 \left( |\text{Tr} \Delta^c|^2 + |\text{Tr} \bar{\Delta}^c|^2 \right) + \frac{4\lambda_2^2}{M_{\text{Pl}}^2} \left| \text{Tr} (\Delta^c \bar{\Delta}^c) \right|^2 \text{Tr} |\Phi|^2 + \left[ \frac{4\lambda_A \lambda_B}{M_{\text{Pl}}^2} \text{Tr} (\Delta^c \bar{\Delta}^c) \left( \text{Tr} (\Delta^c \bar{\Delta}^c)^* \text{Tr} (\Delta^c \bar{\Delta}^c) + \text{Tr} (\Delta^c \bar{\Delta}^c)^* \text{Tr} (\Delta^c \bar{\Delta}^c)^* \right) \right] \\
+ \frac{2\lambda_A \lambda_A}{M_{\text{Pl}}^2} \text{Tr} (\Delta^c \bar{\Delta}^c) \text{Tr} (\Phi^T \tau_2 \Phi \tau_2)^* \left( |\text{Tr} \Delta^c|^2 + |\text{Tr} \bar{\Delta}^c|^2 \right) + \frac{2\lambda_B \lambda_A}{M_{\text{Pl}}^2} \text{Tr} (\Phi^T \tau_2 \Phi \tau_2)^* \left( \text{Tr} (\Delta^c \bar{\Delta}^c) \text{Tr} (\Delta^c \bar{\Delta}^c)^* \text{Tr} (\bar{\Delta}^c \Delta^c) \right) + \text{c.c.} \right] \tag{5}
\]

\[
V_D = \frac{g_R^2}{8} \sum_j \left[ \text{Tr} (2\Delta^c \tau_j \Delta^c + 2\bar{\Delta}^c \tau_j \bar{\Delta}^c + \Phi \tau_j \Phi^* \right] \left[ \text{Tr} (\Phi^T \tau_j \Phi) \right]^2 + \frac{g_L^2}{8} \sum_j \left[ \text{Tr} (\Phi^T \tau_j \Phi) \right]^2 \\
+ \frac{g_{BL}^2}{2} \left[ \text{Tr} (\Delta^c \bar{\Delta}^c - \bar{\Delta}^c \Delta^c) \right]^2 \tag{6}
\]

\[
V_{\text{SUSY}} = -m_{\Delta^c}^2 \text{Tr} (\Delta^c \bar{\Delta}^c) - m_{\bar{\Delta}^c}^2 \text{Tr} (\bar{\Delta}^c \Delta^c) - m_{\phi}^2 \text{Tr} (\Phi^T \Phi) \\
- \frac{Z_A m_{3/2}}{M_{\text{Pl}}} \left[ \left( \text{Tr} (\Delta^c \bar{\Delta}^c) \right)^2 + \text{c.c.} \right] + \frac{Z_B m_{3/2}}{M_{\text{Pl}}} \left[ \left( \text{Tr} (\Delta^c \bar{\Delta}^c) \text{Tr} (\bar{\Delta}^c \Delta^c) \right) + \text{c.c.} \right] \\
- \frac{Z_{\alpha} m_{3/2}}{M_{\text{Pl}}} \left[ \text{Tr} (\Delta^c \bar{\Delta}^c) \text{Tr} (\Phi^T \tau_2 \Phi \tau_2) + \text{c.c.} \right] \tag{7}
\]

with $V_F$ being the $F$-term contribution, $V_D$ the $D$-term, and $V_{\text{SUSY}}$ the SUSY breaking terms.

The minima equations, correct up to electroweak order (i.e. neglecting terms of order $v_{wk}/M_{\text{Pl}}$ and smaller), are:

\[
\frac{1}{2} \left( g_{BL}^2 + g_R^2 \right) \left( v_R^2 - \bar{v}_R^2 \right) - \frac{1}{4} g_R^2 \left( \kappa_u^2 - \kappa_d^2 \right) - m_{\Delta^c}^2 - \frac{Z_A m_{3/2}}{M_{\text{Pl}}} \bar{v}_R^2 + \frac{\lambda_3^2 v_R^2}{M_{\text{Pl}}^2} \left( 2v_R^2 + \bar{v}_R^2 \right) = 0 \tag{8}
\]

\[
- \frac{1}{2} \left( g_{BL}^2 + g_R^2 \right) \left( v_R^2 - \bar{v}_R^2 \right) + \frac{1}{4} g_R^2 \left( \kappa_u^2 - \kappa_d^2 \right) - m_{\bar{\Delta}^c}^2 - \frac{Z_A m_{3/2}}{M_{\text{Pl}}} v_R^2 + \frac{\lambda_3^2 v_R^2}{M_{\text{Pl}}^2} \left( v_R^2 + 2\bar{v}_R^2 \right) = 0 \tag{9}
\]
\[
\left[ -\frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) + \frac{1}{8} (g_R^2 + g_L^2) (\kappa_u^2 - \kappa_d^2) - m_a^2 + \frac{\lambda_a^2 v_R^2 \bar{v}_R^2}{M_{P\ell}^2} \right] \kappa_u \\
- v_R \bar{v}_R \left[ \frac{Z_A m_{3/2}}{M_{P\ell}} - \frac{\lambda_A \lambda_a (v_R^2 + \bar{v}_R^2)}{M_{P\ell}^2} \right] \kappa_d = 0 \tag{10}
\]
\[
\left[ \frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) - \frac{1}{8} (g_R^2 + g_L^2) (\kappa_u^2 - \kappa_d^2) - m_\Phi^2 + \frac{\lambda_a^2 v_R^2 \bar{v}_R^2}{M_{P\ell}^2} \right] \kappa_d
- v_R \bar{v}_R \left[ \frac{Z_A m_{3/2}}{M_{P\ell}} - \frac{\lambda_A \lambda_a (v_R^2 + \bar{v}_R^2)}{M_{P\ell}^2} \right] \kappa_u = 0 \tag{11}
\]
where we have taken the VEV’s to be the real part of the neutral field; that is
\[
v_R \equiv \langle \text{Re} \Delta^0 \rangle \quad \bar{v}_R \equiv \langle \text{Re} \bar{\Delta}^0 \rangle \quad \kappa_u \equiv \langle \text{Re} \phi_u^0 \rangle \quad \kappa_d \equiv \langle \text{Re} \phi_d^0 \rangle \tag{12}
\]
Considering only Eq. (8) and Eq. (9) for the moment, take
\[
v_R = v \sin \theta_R \quad \bar{v}_R = v \cos \theta_R \quad \kappa_u = \kappa \sin \beta \quad \kappa_d = \kappa \cos \beta \tag{13}
\]
Now, the difference of the squares of \(v_R\) and \(\bar{v}_R\) must be of order \(v_{wk}^2\) (subtracting Eq. (8) from Eq. (9) will reveal this), so \(\theta_R\) must be near \(\pi/4\). Therefore, let
\[
\theta_R = \frac{\pi}{4} + \frac{\epsilon}{2} \tag{14}
\]
and expand to first order in \(\epsilon\) (as we shall see, \(\epsilon \sim v_{wk}/M_{P\ell}\)—so \(\epsilon\) is quite small). The sum of equations (8) and (9) yields a quadratic for \(v^2\):
\[
-(m_{\Delta e}^2 + m_{\Delta e}^2) - \frac{Z_A m_{3/2}}{M_{P\ell}} v^2 + \frac{3 \lambda_A^2 v^4}{2 M_{P\ell}^2} = 0 \tag{15}
\]
and the difference gives an expression for \(\epsilon\):
\[
\epsilon = \frac{m_{\Delta e}^2 - m_{\Delta e}^2 - \frac{1}{2} g_R^2 \kappa^2 \cos 2 \beta}{(g_{BL}^2 + g_R^2) v^2} \tag{16}
\]
The solution to Eq. (15),
\[
v^2 = \left( \frac{Z_A m_{3/2} + \sqrt{\left( Z_A m_{3/2} \right)^2 + 6 \lambda_A^2 (m_{\Delta e}^2 + m_{\Delta e}^2)}}{3 \lambda_A^2} \right) M_{P\ell} \tag{17}
\]
gives our prediction of the right breaking scale. Since \(Z_A \sim \lambda_A \sim 1\) and \(m_{\Delta e} \sim m_{\Delta e} \sim m_{3/2} \sim v_{wk}\), we get the result \(v \sim \sqrt{v_{wk} M_{P\ell}}\). This is the major result of the paper, which shows that we can determine the seesaw scale in terms of two other commonly assumed and well motivated scales in the theory; i.e., the Planck scale in four dimensions and the
supersymmetry breaking scale (which is of the order of the weak scale to solve the gauge hierarchy problem). This gives for the seesaw scale $M_R \approx v \sim 10^{11}$ GeV. Using this and Eq. (16) then shows that $\epsilon \lesssim \frac{v}{M_{Pl}}$.

Now turn to Eq. (10) and Eq. (11)—again using Eq. (13) and expanding to first order in $\epsilon$, their sum yields an expression for $\sin 2\beta$:

$$\sin 2\beta = \left[ \frac{Z_\alpha m_{3/2}}{M_{Pl}} - \frac{\lambda_A \lambda_\alpha v^2}{2 M_{Pl}} \right] v^2$$  \quad (18)

and their difference—after using Eq. (16) and Eq. (18)—gives

$$\cos 2\beta = \frac{\frac{1}{4} g^2_R g^2_{BL+g^2_R}}{g^2_L + \frac{g^2_{BL+g^2_R}}{g^2_{BL+g^2_R}}} \frac{\lambda_\alpha v^4}{2 M_{Pl}} - 2 m^2_\Phi$$  \quad (19)

Both Eq. (18) and Eq. (19) are consistent with any value of $\beta$ and constrain the parameter space once a value of $\beta$ has been specified. It is also easy to make an analogy between them and the usual Minimal Supersymmetric Standard Model (MSSM) results as we now do.

**A. Correspondence to MSSM**

We begin our discussion of the effective low energy theory with the relationship between our parameters and those in the MSSM. When $SU(2)_R \times U(1)_{B-L}$ is broken the $\text{Tr} (\Delta^c \Delta^c) \text{Tr} (\Phi^T \tau_2 \Phi \tau_2)$ of Eq. (8) will yield a mass term for the $SU(2)_L$ doublets $\phi_u$ and $\phi_d$; since these are basically the $H_u$ and $H_d$ of the MSSM, this must be the usual $\mu$ term. We then have that

$$|\mu| = \left| \frac{\lambda_\alpha v^2}{2 M_{Pl}} \right| \sim v_{wk}$$  \quad (20)

which is of the desired order of magnitude without any extra assumptions.

Similar reasoning yields that the SUSY breaking bilinear term, $B$, will have a contribution resulting from the $Z_\alpha$ term in Eq. (4); however, it will also receive a contribution from the $F$-term of Eq. (5)—specifically the coefficient of $\lambda_A \lambda_\alpha$. Together these give

$$B = \left[ \frac{Z_\alpha m_{3/2}}{2 M_{Pl}} - \frac{\lambda_A \lambda_\alpha v^2}{2 M_{Pl}^2} \right] v^2$$  \quad (21)

Using the expressions for $B$ and $\mu$ and examining the minimization conditions given by
Eq. (10) and Eq. (11), we can read off $m^2_{H_u}$ and $m^2_{H_d}$:

$$m^2_{H_u} = -m^2_\Phi - \frac{1}{4} g^2_R \left(v^2_R - \bar{v}^2_R\right) + \frac{1}{8} \frac{g^4_R}{g^2_{BL} + g^2_R} \left(\kappa^2_u - \kappa^2_d\right)$$  \hspace{1cm} (22)

$$m^2_{H_d} = -m^2_\Phi + \frac{1}{4} g^2_R \left(v^2_R - \bar{v}^2_R\right) - \frac{1}{8} \frac{g^4_R}{g^2_{BL} + g^2_R} \left(\kappa^2_u - \kappa^2_d\right)$$  \hspace{1cm} (23)

Here it is noticed that $m^2_{H_u} \neq m^2_{H_d}$ despite the apparent symmetry of the superpotential and the soft-breaking mass term. This splitting is due to the $D$-terms, which is reflected in the fact that their difference is proportional to $g^2_R$. Specifically, it is the $D$-term involving $\tau_3$ (the ones involving $\tau_1$ and $\tau_2$ won’t contribute because when the VEVs are placed in for $\Delta c$ and $\bar{\Delta} c$ these are zero) that gives a positive $\bar{v}^2_R - v^2_R$ contribution to $m^2_{H_u}$ and a negative one to $m^2_{H_d}$.

Using Eq. (16) these expressions may be recast into the form

$$m^2_{H_u} = -m^2_\Phi - \frac{1}{4} g^2_R \frac{g^2_{BL}}{g^2_{BL} + g^2_R} \left(m^2_{\Delta c} - m^2_{\bar{\Delta} c}\right)$$  \hspace{1cm} (24)

$$m^2_{H_d} = -m^2_\Phi + \frac{1}{4} g^2_R \frac{g^2_{BL}}{g^2_{BL} + g^2_R} \left(m^2_{\Delta c} - m^2_{\bar{\Delta} c}\right)$$  \hspace{1cm} (25)

This form is advantageous because we now have that

$$m^2_{H_u} - m^2_{H_d} = \frac{1}{2} g^2_R \frac{g^2_{BL}}{g^2_{BL} + g^2_R} \left(m^2_{\Delta c} - m^2_{\bar{\Delta} c}\right)$$  \hspace{1cm} (26)

which, with the masses of the left-handed (standard model) particles

$$m^2_Z = \frac{1}{4} \left[g^2_L + \frac{g^2_{BL} g^2_R}{g^2_{BL} + g^2_R}\right] \kappa^2$$  \hspace{1cm} (27)

$$m^2_W = \frac{1}{4} g^2_L \kappa^2$$  \hspace{1cm} (28)

allows us to write Eq. (18) and Eq. (19) in the enticing form

$$\sin 2\beta = \frac{2B}{2 |\mu|^2 + m^2_{H_u} + m^2_{H_d}}$$  \hspace{1cm} (29)

$$\cos 2\beta = \frac{m^2_{H_u} - m^2_{H_d}}{m^2_Z + 2 |\mu|^2 + m^2_{H_u} + m^2_{H_d}}$$  \hspace{1cm} (30)

which are the usual expressions of the MSSM for breaking $SU(2)_L \times U(1)_Y$ down to $U(1)$_{em}.

The interesting aspect of this result is that, provided $m^2_{\Delta c} \neq m^2_{\bar{\Delta} c}$, this model provides a means for $\tan \beta \equiv \frac{\kappa_u}{\kappa_d} \gg 1$. This is an important feature because, as has already been noted for theories involving a single bidoublet\([11]\), it allows the quarks and leptons to get realistic masses and mixings. Since this model does not require additional particles to achieve $\tan \beta \gg 1$ (as opposed to those previously discussed\([12, 13]\)), it is truly a minimal scheme.
B. Charge Violation Consideration

The above model is based on vacuum expectation values (vevs) that are consistent with the charge conserving vacuum. However, it has been noted in earlier works that in SUSYLR models, the \( \Delta^c \) fields may have a vev that breaks electric charge conservation unless one breaks R-parity. In this model though, the existence of non-renormalizable terms allow for the charge conserving vacuum to have a much lower ground state energy than the charge conserving one for large regions of the parameter space. This ensures that the theory will spontaneously break into the phenomenologically viable vacuum, the charge conserving one.

To see this we can compare the ground state values of the two potentials, the charge violating one (CV) and the charge conserving (CC) one. The vevs for the CC case have already been discussed, their analogues in the CV case are:

\[
\langle \Delta^c \rangle = \begin{pmatrix} 0 \\ \frac{v_R}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad \langle \bar{\Delta}^c \rangle = \begin{pmatrix} 0 \\ \frac{v_R}{\sqrt{2}} \\ 0 \end{pmatrix}
\]

The resulting ground state expressions, to order \( v_R \), are:

\[
\langle V \rangle_{CV} = -\frac{1}{2} v^2 \left( m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2 \right) + \frac{(Z_B - Z_A) m_{3/2} v^4}{2M_{P\ell}} + \frac{(\lambda_A + \lambda_B)^2 v^6}{M_{P\ell}^2} \tag{31}
\]

\[
\langle V \rangle_{CC} = -v^2 \left( m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2 \right) - \frac{2Z_A m_{3/2} v^4}{M_{P\ell}} + \frac{8\lambda_A^2 v^6}{M_{P\ell}^2} \tag{32}
\]

Where \( v^2 \) for the CC case was given in Eq. (17) and \( v^2 \) for the CV case is:

\[
v^2 = \left( \frac{(Z_A - Z_B) m_{3/2} + \sqrt{((Z_A - Z_B) m_{3/2})^2 + 6 (\lambda_A + \lambda_B)^2 (m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2)}}{6 (\lambda_A + \lambda_B)^2} \right) M_{P\ell} \tag{33}
\]

The crucial point here is that the CV ground state expression has a dependence on both \( Z_B \) and \( \lambda_B \), which do not appear in the CC expression. This means that for sufficiently large values of these parameters, the CC ground state will be lower. In the numerical analysis conducted in a later section, this will be taken into account and the difference between the two ground state values will be compared.
III. MASS SPECTRUM AND NUMERICAL ANALYSIS

A. Mass Spectrum

Once the value of the minimization conditions and the values of the vevs have been determined, the mass spectrum can be explored to ensure that all the resulting physical Higgs bosons have positive mass squares. This is nontrivial because if too few terms are included in the superpotential, there is no a priori guarantee that there is a stable minimum instead of a flat direction or an unstable minimum.

We begin\(^1\) with \(\text{Im} \Delta^0, \text{Im} \Delta^0, \text{Im} \phi^0_u,\) and \(\text{Im} \phi^0_d\) (the imaginary components of the neutral fields) since two linear combinations of them are eaten by gauge bosons (so there are two zero modes). The four by four mass matrix resulting after the spontaneous symmetry breaking can be split into two by two matrices for the \(\Delta^c\)'s and the \(\Phi\)'s:

\[
V_{\text{mass}} \supset \frac{1}{2} \begin{pmatrix} \text{Im} \Delta^0 & \text{Im} \Delta^0 \\ \text{Im} \Delta^0 & \text{Im} \Delta^0 \end{pmatrix} M_{\Delta^c}^2 \begin{pmatrix} \text{Im} \Delta^0 \\ \text{Im} \Delta^0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \text{Im} \Phi^0_u & \text{Im} \Phi^0_d \\ \text{Im} \Phi^0_u & \text{Im} \Phi^0_d \end{pmatrix} M_{\Phi}^2 \begin{pmatrix} \text{Im} \Phi^0_u \\ \text{Im} \Phi^0_d \end{pmatrix}
\]

where

\[
M_{\Delta^c}^2 = \begin{pmatrix} Z_{\alpha} m_{3/2}^{3/2} v^2 M_{\ell} & Z_{\alpha} m_{3/2}^{3/2} v^2 M_{\ell} \\ Z_{\alpha} m_{3/2}^{3/2} v^2 M_{\ell} & Z_{\alpha} m_{3/2}^{3/2} v^2 M_{\ell} \end{pmatrix}
\]

\[
M_{\Phi}^2 = \begin{pmatrix} \frac{1}{2} \kappa_u & \left[ Z_{\alpha} m_{3/2}^{3/2} - \frac{\lambda_A \lambda_3 v^2}{M_{\ell}} \right] v^2 M_{\ell}^2 & v^2 & \frac{1}{2} \kappa_u \\ \frac{1}{2} \kappa_d & \left[ Z_{\alpha} m_{3/2}^{3/2} - \frac{\lambda_A \lambda_3 v^2}{M_{\ell}} \right] v^2 M_{\ell}^2 & \left[ Z_{\alpha} m_{3/2}^{3/2} - \frac{\lambda_A \lambda_3 v^2}{M_{\ell}} \right] v^2 M_{\ell}^2 & \frac{1}{2} \kappa_u \end{pmatrix}
\]

The above matrices each have determinant equal to zero, and the remaining non-zero eigenvalues are, respectively,

\[
m_{B^0}^2 = \frac{2Z_A m_{3/2}}{M_{\ell}^2} v^2
\]

\[
m_{A^0}^2 = \frac{\lambda^2 v^4}{2M_{\ell}^2} - 2m_{\Phi}^2
\]

where the latter value has been simplified using Eq. (18). Here we have introduced \(B^0\) as the axial Higgs boson associated with the \(\Delta^c\) fields and \(A^0\) is the usual MSSM axial Higgs boson.

\(^1\) In this section, as with the previous work, we will neglect terms of order \(v^3_{\text{ew}} / M_{\ell}\) and only retain first order in \(\epsilon\).
The mass of $B^0$ will always be positive provided $Z_A > 0$, which means that the minus
sign in front of the $Z_A$ term in Eq. (4) is crucial for a positive mass-square. The second
could easily be positive depending on the value of $\lambda_\alpha$ and the phase of $m_{\Phi}^2$.

Next we move on to the singly charged fields since they also have two zero masses. The
mass matrix for these fields can not be split apart; however, if we write

$$V_{mass} \supset \begin{pmatrix} \phi^+_u & \phi^-_d \\ \Delta^c+ & \Delta^c- \end{pmatrix} \begin{pmatrix} M^2_{SC} \\ \Delta^c- \end{pmatrix}$$

(39)

then $M^2_{SC}$ can be seen to be three distinct two by two matrices:

$$M^2_{SC} = \begin{pmatrix} M^2_\Phi & M^2_{\Phi\Delta^c} \\ (M^2_{\Phi\Delta^c})^\dagger & M^2_{\Delta^c} \end{pmatrix}$$

(40)

where

$$M^2_\Phi = \begin{pmatrix} \frac{1}{4}g^2_R \kappa_-^2 + \frac{1}{2} \kappa_+ \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} \\ \frac{1}{2}g^2_R \kappa_u \kappa_d + \frac{1}{2} \kappa_+ \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} \end{pmatrix} v^2 + \frac{1}{2} \kappa_+ \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} v^2$$

(41)

and

$$M^2_{\Phi\Delta^c} = \begin{pmatrix} \frac{1}{2}g^2_R v^2 (1 + \epsilon) + \frac{1}{2} g^2_R (\kappa_+ - \kappa_-) \\ \frac{1}{2} \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} \end{pmatrix} v^2 + \frac{1}{2} \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} v^2$$

(42)

$$M^2_{\Delta^c} = \begin{pmatrix} \frac{1}{4}g^2_R v^2 (1 - \epsilon) + \frac{1}{4} g^2_R (\kappa_+ - \kappa_-) \\ -\frac{1}{2} \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} \end{pmatrix} v^2 + \frac{1}{2} \frac{Z_{\alpha m3/2}}{M_{\ell}} - \frac{\Delta \kappa_+^2}{M_{\ell}^2} v^2$$

(43)

Checking the order of magnitude of each of these matrices, it can be seen that

$$\left| (M^2_{\Phi^+})_{ij} \right| \sim \epsilon v^2 \quad \left| (M^2_{\Phi\Delta^c})_{ij} \right| \sim \sqrt{\epsilon} v^2 \quad \left| (M^2_{\Delta^c})_{ij} \right| \sim v^2$$

(44)

so, $M^2_{SC}$ may be written as

$$\begin{pmatrix} \epsilon \Lambda_1 v^2 & \sqrt{\epsilon} \Lambda_2 v^2 \\ \sqrt{\epsilon} \Lambda_2 v^2 & \Lambda_3 v^2 \end{pmatrix}$$

(45)
where each element of each \( \Lambda \) matrix is of order one. This matrix structure is exactly that of the neutrino mass matrix in the Type II Singular Seesaw scenario with the associations

\[
\delta^2 m_L \rightarrow M_{\Phi}^2 \quad \delta m_D \rightarrow M_{2\Delta^c}^2 \quad M_R \rightarrow M_{\Delta^c}^2
\]  

(46)

Since the determinant of \( M_{2\Delta^c}^2 \) is zero, there is only one large eigenvalue given by

\[
m_{D^+}^2 = \frac{1}{2} g_R^2 v^2
\]  

(47)

the resulting mass matrix for the lighter fields is then read directly from the Seesaw formula:

\[
\begin{pmatrix}
\frac{1}{4} g_L^2 \kappa_d^2 - \frac{1}{2} \kappa_d & \frac{Z_{m3/2}}{M_{Pl}} - \frac{\lambda A_0 v^2}{M_{Pl}} & v^2 & 0 \\
\frac{1}{4} g_L^2 \kappa_u^2 - \frac{1}{2} \kappa_u & \frac{Z_{m3/2}}{M_{Pl}} - \frac{\lambda A_0 v^2}{M_{Pl}} & v^2 & 0 \\
0 & \frac{1}{4} g_L^2 \kappa_u^2 - \frac{1}{2} \kappa_u & \frac{Z_{m3/2}}{M_{Pl}} - \frac{\lambda A_0 v^2}{M_{Pl}} & v^2 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]  

(48)

Evidently zero is one of the eigenvalues, and the determinant of the remaining two by two is also zero. These correspond to the two modes that are eaten by the charged gauge bosons. The trace of the two by two is then the non-zero eigenvalue, which corresponds to the MSSM charged Higgs Boson \( h^+ \). So, after using Eq. 18

\[
m_{h^+}^2 = \frac{1}{4} g_L^2 \kappa^2 + \frac{1}{2} \frac{\lambda A_0 v^4}{M_{Pl}^2} - 2 m_{\Phi}^2
\]  

(49)

Note that the first term of the right-hand side is just \( m_W^2 \) and that the last two terms sum to the aforementioned \( m_{A_0}^2 \). We can therefore rewrite Eq. 19 as

\[
m_{h^+}^2 = m_W^2 + m_{A_0}^2
\]  

(50)

which matches the MSSM result and will be positive if \( m_{A_0}^2 \) is.

The remaining charged fields—the doubly charged Higgs bosons—can only consist of \( \Delta^c \) and \( \tilde{\Delta}^c \), so this mass matrix is a two by two. For these fields we have

\[
V_{mass} \supset \begin{pmatrix}
\Delta^{c++} & \Delta^{c--} \\
\tilde{\Delta}^{c++} & \Delta^{c--}^*
\end{pmatrix} M_{\Delta^{c\pm}}^2 \begin{pmatrix}
\Delta^{c++} & \Delta^{c--}
\end{pmatrix}
\]  

(51)

2 for a review of the Type II Singular Seesaw Mechanism see Appendix A.
where

\[
M_{\Delta_{\pm \pm}}^2 = \begin{pmatrix}
 g_R^2 v^2 \epsilon - \frac{1}{2} g_R^2 (\kappa_u^2 - \kappa_d^2) \\
+ \frac{1}{2} \left( \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{\lambda_A^2 v^2}{M_{P\ell}} \right) v^2 + \lambda_B (\lambda_A + \lambda_B) v^4 \\
\left( \left( \frac{Z_{Bm3/2}}{M_{P\ell}} + \frac{\lambda_A \lambda_B v^2}{M_{P\ell}} \right) - \frac{1}{2} \left( \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{\lambda_A^2 v^2}{M_{P\ell}} \right) \right) v^2 - \frac{g_R^2 v^2 \epsilon}{2} + \frac{1}{2} \left( \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{\lambda_A^2 v^2}{M_{P\ell}} \right) v^2 + \frac{\lambda_B (\lambda_A + \lambda_B) v^4}{M_{P\ell}}
\end{pmatrix} v^2
\]

(52)

The eigenvalues can be found, and they are given by

\[
m_{D_{++/d++}}^2 = \left[ \frac{1}{2} \Lambda_A + \frac{\lambda_B (\lambda_A + \lambda_B) v^2}{M_{P\ell}} \right] v^2 \pm \sqrt{\left( -g_R^2 \epsilon v^2 + \frac{1}{2} g_R^2 (\kappa_u^2 - \kappa_d^2) \right)^2 + \left( \Lambda_B - \frac{1}{2} \Lambda_A \right)^2 v^2}
\]

(53)

where

\[
\Lambda_A \equiv \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{\lambda_A^2 v^2}{M_{P\ell}} \quad \Lambda_B \equiv \frac{Z_{Bm3/2}}{M_{P\ell}} + \frac{\lambda_A \lambda_B v^2}{M_{P\ell}}
\]

These eigenvalues are of order \(v_{wk}^2\) as expected and are positive for sufficiently large \(\lambda_B\). It is worth noting that a large \(\lambda_B\) is consistent with a stable charge conserving vacuum as mentioned earlier.

Finally, we come to the real neutral fields. These fields, like the singly charged, require use of the seesaw mechanism (as discussed in Appendix A). If we write

\[
V_{mass} \supset \frac{1}{2} \begin{pmatrix}
\text{Re } \Phi^0_u \\
\text{Re } \Phi^0_d \\
\text{Re } \Delta^{c \mp} \\
\text{Re } \bar{\Delta}^{c \mp}
\end{pmatrix} M_{RN}^2 \begin{pmatrix}
\text{Re } \Phi^0_u \\
\text{Re } \Phi^0_d \\
\text{Re } \Delta^{c \mp} \\
\text{Re } \bar{\Delta}^{c \mp}
\end{pmatrix}
\]

(54)

where

\[
M_{RN}^2 = \begin{pmatrix}
M_{\Phi \Phi} & M_{\Phi \Delta} \\
(M_{\Phi \Delta})^\dagger & M_{\Delta \Delta}
\end{pmatrix}
\]

(55)

with

\[
M_{\Delta \Delta}^2 = \begin{pmatrix}
\frac{1}{2} \left( g_{BL}^2 + g_{BR}^2 \right) v^2 (1 - \epsilon) + \frac{\lambda_A^2 v^4}{M_{P\ell}} & -\frac{1}{2} \left( g_{BL}^2 + g_{BR}^2 \right) v^2 - \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{2 \lambda_A^2 v^2}{M_{P\ell}} \right) v^2 \\
-\frac{1}{2} \left( g_{BL}^2 + g_{BR}^2 \right) v^2 - \frac{Z_{Am3/2}}{M_{P\ell}} - \frac{2 \lambda_A^2 v^2}{M_{P\ell}} \right) v^2 & \frac{1}{2} \left( g_{BL}^2 + g_{BR}^2 \right) v^2 (1 + \epsilon) + \frac{\lambda_A^2 v^4}{M_{P\ell}}
\end{pmatrix}
\]

(56)
\[ M_{\Phi^2} = \left( \frac{1}{4} (g_L^2 + g_R^2) \kappa_u + \frac{1}{2} \kappa_d \left[ \frac{Z_{a m^3/2}}{M_{P\ell}} - \lambda A \lambda v^2 \right] \right) v^2 - \frac{1}{4} (g_L^2 + g_R^2) \kappa_u \kappa_d - \frac{1}{2} \left[ \frac{Z_{a m^3/2}}{M_{P\ell}} - \lambda A \lambda v^2 \right] v^2 \]

\[ M_{\Phi^2 \Delta^e} = \left( \frac{1}{2 \sqrt{2}} g_R^2 v K_u - \frac{1}{2 \sqrt{2}} g_R^2 v K_d \right) \]

and make the associations

\[ \delta^2 m_L \rightarrow M_{\Phi^2} \quad \delta m_D \rightarrow M_{\Phi^2 \Delta^e} \quad M_R \rightarrow M_{\Delta^e \Delta^c} \]

then the single large eigenvalue of \( M_{\Delta^e \Delta^c} \) is given by

\[ m_{d^0}^2 = (g_{BL}^2 + g_{R}^2) \frac{\alpha}{\alpha M_{P\ell}} v^2 \]

The resulting mass matrix for the lighter fields is

\[ \begin{pmatrix}
\frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_{R}^2}{g_{BL}^2 + g_{R}^2} \right] \kappa_u^2 + \frac{1}{2} \kappa_d \left[ \frac{Z_{a m^3/2}}{M_{P\ell}} - \lambda A \lambda v^2 \right] v^2 & -\frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_{R}^2}{g_{BL}^2 + g_{R}^2} \right] \kappa_u \kappa_d - \frac{1}{2} \left[ \frac{Z_{a m^3/2}}{M_{P\ell}} - \lambda A \lambda v^2 \right] v^2 & 0 \\
-\frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_{R}^2}{g_{BL}^2 + g_{R}^2} \right] \kappa_u \kappa_d & \frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_{R}^2}{g_{BL}^2 + g_{R}^2} \right] \kappa_d^2 + \frac{1}{2} \kappa_d \left[ \frac{Z_{a m^3/2}}{M_{P\ell}} - \lambda A \lambda v^2 \right] v^2 & 0 \\
0 & 0 & \left[ \frac{\lambda A \lambda v^2}{M_{P\ell}} \right] v^2
\end{pmatrix} \]

Clearly one of the eigenvalues can be read off, it is

\[ m_{d^0}^2 = -\frac{Z_{A m^3/2}}{M_{P\ell}} + \frac{3 \lambda^2 A v^2}{M_{P\ell}^2} \]

and is of electroweak order.

The remaining two by two matrix has the eigenvalues

\[ m_{H^0/H^0}^2 = \frac{1}{2} \left[ m_Z^2 + m_A^2 \pm \sqrt{(m_Z^2 + m_A^2)^2 - 4 m_Z^2 m_A^2 \cos 2\beta^2} \right] \]

where we have used Eq. (60) and Eq. (61) to simplify this expression. Note that these also match MSSM expressions.

That completes the Higgs spectrum analysis and we now briefly address additional fermionic content of the theory. Here we find that there are three light fermions in this model: two doubly charged and a neutral one. The neutral fermion has a mass in the electroweak range and is the superpartner of the \( d^0 \).
B. Numerics

The purpose of this subsection is to validate the above arguments with numerical analysis. Specifically, our purpose is simply to show that the general arguments about the positivity of the Higgs masses can be supported in the parameter space. Other values of interest are also reported including: \( v_R \), \( \tan \beta \), and the difference in the ground state values of the CC and CV potentials (as mentioned earlier we need \( \langle V \rangle_{CV} - \langle V \rangle_{CC} > 0 \), so this is verified in that last column of TABLE IV).

We will keep six of the dimensionful parameters constant (in GeV)

\[
\begin{align*}
    m_{\Delta_e} &= 350 \\
    m_{\Delta_e} &= 450 \\
    m_{3/2} &= 450 \\
    \kappa &= 250 \\
    M_{Pl} &= 2.44 \times 10^{18}
\end{align*}
\]

and three of the coupling constants at:

\[
\begin{align*}
    g_R &= 1.2 \\
    g_L &= .65 \\
    g_1 &= .38
\end{align*}
\]

We vary the remaining as follows:

| Case Number | \( \lambda_A \) | \( \lambda_B \) | \( \lambda_\alpha \) | \( Z_A \) | \( Z_B \) | \( Z_\alpha \) | \( m_\Phi^2 \) (GeV\(^2\)) |
|-------------|------------------|------------------|------------------|--------|--------|--------|------------------|
| Case 1      | 0.9              | 0.8              | 0.99             | 0.65   | 0.3    | 1.29   | 300\(^2\)        |
| Case 2      | 0.5              | 0.45             | 0.1              | 0.54   | 0.3    | 0.16   | -100\(^2\)       |
| Case 3      | 0.4              | 0.4              | 0.2              | 0.36   | 0.3    | 0.29   | 100\(^2\)        |
| Case 4      | 0.2              | 0.3              | 0.1              | 0.18   | 0.3    | 0.14   | 100\(^2\)        |
| Case 5      | 0.9              | 0.85             | 0.2              | 0.54   | 0.3    | 0.25   | -100\(^2\)       |

TABLE II: Points in parameter space used to evaluate the Higgs masses

These values yield the following masses for the Higgs Boson (in GeVs) and the vacuum defining parameters respectively:
TABLE III: Higgs masses based on parameters from TABLE II. The masses are given in GeV. As predicted previously, the doubly charged particles ($D^{++}$ and $d^{++}$) have masses in the electroweak range.

| Case Number | $D^{++}$ | $d^{++}$ | $H^+$ | $D^0$ | $d^0$ | $H^0$ | $h^0$ | $D^0$ | $A^0$ |
|-------------|---------|---------|------|------|------|------|------|------|------|
| Case 1      | 988     | 161     | 186  | 5.01 × 10^{10} | 917  | 167  | 93   | 619  | 167  |
| Case 2      | 1140    | 235     | 188  | 7.10 × 10^{10} | 983  | 170  | 90   | 796  | 169  |
| Case 3      | 1170    | 209     | 188  | 7.75 × 10^{10} | 953  | 170  | 90   | 718  | 169  |
| Case 4      | 1560    | 555     | 186  | 10.9 × 10^{10} | 948  | 167  | 93   | 706  | 167  |
| Case 5      | 986     | 166     | 186  | 4.91 × 10^{10} | 895  | 167  | 93   | 551  | 167  |

TABLE IV: Vacuum related parameters based on parameters from TABLE II. The second column shows $v_R$ and as can be seen is the correct order of the seesaw scale. The last column presents the difference in the ground state energy of the charge violating and the charge conserving vacuum. A positive value in this column indicates that the charge conserving vacuum is the stable one.

| Case Number | $v_R$ (GeV) | $\epsilon$ | $\tan \beta$ | $\langle V \rangle_{CV} - \langle V \rangle_{CC}$ (GeV^4) |
|-------------|------------|------------|-------------|-------------------------------------------------|
| Case 1      | 2.8 × 10^{10} | 5.0 × 10^{-17} | $\infty$ | 1.7 × 10^{27} |
| Case 2      | 4.0 × 10^{10} | 2.5 × 10^{-17} | 9.9 | 4.0 × 10^{27} |
| Case 3      | 4.4 × 10^{10} | 2.1 × 10^{-17} | 9.9 | 5.4 × 10^{27} |
| Case 4      | 6.1 × 10^{10} | 1.0 × 10^{-17} | 50 | 18 × 10^{27} |
| Case 5      | 2.8 × 10^{10} | 5.2 × 10^{-17} | 50 | 1.6 × 10^{27} |

C. Implications

The TeV scale theory in this model differ from MSSM in that we have several new particles in the 100 GeV–TeV range. These particles are: $\Delta^{c++}$, $\Delta^{c--}$, $d^0$, $\tilde{\Delta}^{c++}$, $\tilde{\Delta}^{c--}$ and $\tilde{d}^0$ where the Higgsinos are differentiated from Higgses by a tildes. The charged particles lead to spectacular signatures in colliders due to their decay modes: $\Delta^{c++} \rightarrow \ell^+ \ell^+$, $\tilde{\Delta}^{c++} \rightarrow \ell^+ \ell^+ \chi$ ($\chi$ being the lightest neutralino). On the other hand, the neutral particles will be hard to produce in the labartory because of their low coupling values to MSSM matter content. Their dominant decay channel is via $d^0 \rightarrow \chi \chi$ with decay lifetimes of the order $10^{-10}$ sec for generic values of the parameters. It is worthwhile to mention that $d^0$ and $\tilde{d}^0$ would have been
present in the early stages of the universe, but would have decayed away before the era of Big Bang nucleosynthesis and therefore do not alter our understanding of this period.

IV. GRAND UNIFICATION PROSPECTS

Since the effective TeV scale theory in our model is very different from MSSM (due to the presence of a pair of doubly charged fields), it is interesting to explore whether there is grand unification of couplings. This question was investigated in [14], where it was noted that if there are two pairs of Higgs doublets (corresponding to two bidoublets $\phi_1, \phi_2$), at the TeV scale, the gauge couplings unify around $10^{12}$ GeV or so. This raises an interesting point: if there is a grand unified theory at $10^{12}$ GeV, then this theory must be very different from conventional GUT theories. The reason this must be so is that GUTs violate baryon number and present limits on proton life time require that the scale of grand unification be $10^{15}$ GeV. Our GUT theory, should it exist, must conserve baryon number due to the low unification scale.

An example of such a theory is the $SU(5) \times SU(5)$ model discussed in [15], which embeds the left-right symmetric group we are discussing. We do not discuss the details of this theory here, but rather indicate the basic features: we envision $SU(5) \times SU(5)$ to be broken down to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ by a Higgs multiplet belonging to the representation $\Phi \equiv (5, \bar{5})$ with vacuum expectation value (vev) as follows: $\langle \Phi \rangle = \text{diag} (a, a, a, 0, 0)$. This is then subsequently broken to the standard model.

The fermions in this model belong to the $(\bar{5}, 1) \oplus (10, 1) \oplus (1, \bar{5}) \oplus (1, 10)$ representation as follows:

$$F_L = \begin{pmatrix} D^c_e \\ D^c_e \\ D^c_e \\ \nu_e \\ e \end{pmatrix}, \quad T_L = \begin{pmatrix} 0 & U^c_e & U^c_e & u & d \\ -U^c_e & 0 & U^c_e & u & d \\ -U^c_e & -U^c_e & 0 & u & d \\ -u & -u & -u & 0 & E^+ \\ -d & -d & -d & -E^+ & 0 \end{pmatrix}$$ (64)

and similarly for the right chiral fields.

Implementation of the seesaw mechanism in this model requires that we add the Higgs representation $(15, 1) \oplus (1, 15)$ along with their complex conjugate representations. The multiplet $b(1, 15)$ plays the role of $\Delta^c$ of the left-right model. When the $\nu^c \nu^c$ component of
(1, 15) acquires a vev, it gives mass to the right handed neutrino fields triggering the seesaw mechanism. The doubly charged Higgs fields are part of the right handed (1, 15) Higgs representation. Symmetry breaking and fermion masses in this model are briefly touched on in [15] and will be discussed in detail in a separate paper.

V. CONCLUSION

To summarize, we have considered a bottom-up extension of the MSSM based on the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) that explains small neutrino masses via the seesaw mechanism. We have also shown that if the superpotential of the model is assumed to obey an R-symmetry, then the \( B-L \) breaking scale (seesaw scale) can be predicted to be around \( 10^{11} \) GeV—a phenomenologically acceptable value for this scale. This model also solves the \( \mu \) problem of the MSSM and predicts two TeV scale doubly charged bosons and fermions which couple to like sign dileptons and like sign lepton-slepton respectively. Such particles have been searched for in various existing experiments and will be searched for at the LHC and other future colliders [16]. Additionally, the model predicts unstable neutral bosons and fermions which can not be easily probed by experiment, but which would have been produced in the early universe.

Finally we note that the conclusions of this paper can be equally applied to the group \( SU(2)_L \times U(1)_{I_3 R} \times U(1)_{B-L} \), with the mass spectrum being identical except for the lack of light doubly charged particles and a heavy singly charged.

VI. ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation grant no. Phy-0354401.

APPENDIX A: TYPE II SINGULAR SEESAW MECHANISM

We start with a mass matrix of the form

\[
\mathcal{M} = \begin{pmatrix}
\delta^2 m_L & \delta m_D \\
\delta m_D^\dagger & M_R
\end{pmatrix}
\] (A1)
where $\delta$ carries the relative order of magnitude of the elements of each of the three $(n \times n)$ matrices $m_L$, $m_D$, and $M_R$—i.e. there is a hierarchy which can be thought of as either $|(M_R)_{ij}| \gg |(\delta m_D)_{kl}| \gg |(\delta^2 m_L)_{pq}|$; or, alternatively, $\delta \ll 1$, $|(M_R)_{ij}| \sim |(m_D)_{kl}| \sim |(m_L)_{pq}| \equiv v$.

It is not assumed, however, that all the eigenvalues of $M_R$ are of this high scale $v$, so in the limit $\delta \to 0$, it is possible that $\det M_R = 0$. Therefore, to exploit this hierarchy it is necessary to extract those smaller eigenvalues. This is done as follows:

First, diagonalize $M_R$ through an $(n \times n)$ rotation $R$ via

$$\mathcal{R} = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}$$

so that

$$\mathcal{R} \mathcal{M} \mathcal{R}^T = \begin{pmatrix} \delta^2 m_L & \delta m_D R^T \\ \delta R m_D^\dagger & M_d \end{pmatrix}$$

with $M_d \equiv R M_R R^T$ which is a diagonal matrix. The matrix $R$ should be chosen so that the first $k$ eigenvalues of $M_d$ are the small ones—thus, for $1 \leq i \leq k$

$$(\delta^2 \mu_R)_{ii} \equiv (M_d)_{ii} = \delta^2 \lambda_i v^2$$

where $\lambda_i \sim 1$ and $(\delta^2 \mu_R)_{ij} = 0$ for $i \neq j$. The remaining (large) eigenvalues are then placed in a separate matrix:

$$\Delta_R \equiv \text{diag} \left( (M_d)_{k+1,k+1}, (M_d)_{k+2,k+2}, \ldots, (M_d)_{n,n} \right)$$

Also define

$$\delta \mu_1 \equiv \begin{pmatrix} 0 & \text{col}_1 (\delta m_D R^T) & \cdots & \text{col}_k (\delta m_D R^T) \\ \text{row}_1 (\delta R m_D^\dagger) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \text{row}_k (\delta R m_D^\dagger) & 0 & \cdots & 0 \end{pmatrix}$$

$$\delta^2 \mu_2 \equiv \begin{pmatrix} \delta^2 m_L & 0 \\ 0 & \delta^2 \mu_R \end{pmatrix}$$

$$\delta \mu_D \equiv \begin{pmatrix} \text{col}_{k+1} (\delta m_D R^T) & \text{col}_{k+2} (\delta m_D R^T) & \cdots & \text{col}_n (\delta m_D R^T) \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$
With those definitions we may write

\[ \mathcal{R} \mathcal{M} \mathcal{R}^T = \begin{pmatrix} \delta \mu_1 + \delta^2 \mu_2 & \delta \mu_D \\ \delta \mu_D^\dagger & \Delta_R \end{pmatrix} \]  

(A9)

Now a matrix \( P \) is chosen so that it block diagonalizes \( \mathcal{R} \mathcal{M} \mathcal{R}^T \). This \( P \) is implemented through \( \mathcal{P} \) which, to order \( \delta^2 \), is given by

\[ \mathcal{P} = \begin{pmatrix} 1 - \frac{1}{2} \delta^2 P P^\dagger & -\delta P \\ \delta P^\dagger & 1 - \frac{1}{2} \delta^2 P^\dagger P \end{pmatrix} \]  

(A10)

The matrix \( P \) is then determined by the requirement

\[ \mathcal{P} \mathcal{R} \mathcal{M} \mathcal{R}^T \mathcal{P}^\dagger = \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} \]  

(A11)

with \( m \) the \((n + k) \times (n + k)\) mass matrix of interest and \( M = \Delta_R + \mathcal{O}(\delta) \). The off-block-diagonal condition yields

\[ P = \mu_D \Delta_R^{-1} \]  

(A12)

and then using that \( P \), the mass matrix for the light eigenstates can be determined:

\[ m = \delta \mu_1 + \delta^2 \mu_2 - \delta^2 \mu_D \Delta_R^{-1} \mu_D^\dagger \]  

(A13)

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