The dark matter as a light gravitino

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Abstract. We address the question of gravitino dark matter in the context of gauge mediated supersymmetry breaking models. A special emphasis is put on the role played by the MSSM singlet messenger in the case of SO(10) grand unification.

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1 Introduction

In some instances, the requirement that supersymmetric particle dark matter scenarios ought to be the simplest to handle cosmologically and the least model-dependent, seems occasionally to take over the more fundamental question for supersymmetry, namely the origin of supersymmetry breaking. Since there is to date no particularly compelling susy breaking mechanism/model to be preferred to all the others, one should also consider the dark matter issue from a particle physics standpoint which offers different classes of susy breaking mechanisms, irrespective of whether the ensuing cosmological context is “simple” or not.

Recent developments\cite{1,2} stressing the existence of metastable susy breaking vacua, have renewed the interest in gauge-mediated susy breaking (GMSB) scenarios opening new possibilities for the model-building\cite{3}, and appear to be very interesting from the early Universe point of view as well\cite{4}. On the other hand, the gravitational interactions which play a minor role for susy breaking in GMSB models remain physically through the coupling to supergravity, at least in order to absorb the unphysical goldstino component, to adjust the cosmological constant to a small value and to avoid a massless R-axion. In such scenarios where supersymmetry breaking is triggered by non-perturbative dynamics of some (secluded) gauge sector and communicated to the MSSM by a messenger sector through perturbative gauge interactions, the susy breaking scale $\sqrt{F}$ and the mass scale $A$ of the secluded gauge sector can be well below the Planck scale. Moreover, if these two scales combine to trigger the electroweak symmetry breaking yielding $G_F^{-1/2} \sim (\alpha/4\pi) kF/A$, where $G_F$ is Fermi’s constant (and 0 < $k \leq 1$ measures the secludedness of the secluded sector), then the gravitino mass $m_{3/2} \approx F/(\sqrt{3} m_{P1}) \approx (4\pi/\alpha)(A/\sqrt{3} m_{P1}) G_F^{-1/2}$ where $m_{P1}$ is the reduced Planck mass, is expected to be very small (\lesssim O(1) GeV) and is the lightest supersymmetric particle (LSP). The question then arises as to which particle can be a good candidate for the cold dark matter (CDM) in this case? To answer this question requires an unconventional treatment as compared to the Neutralino “vanilla” candidate or even to the heavy gravitino candidate in the context of gravity mediated susy breaking models. Indeed, in contrast with the latter where the hidden sector is typically too heavy to be produced at the end of inflation, the secluded and messenger sectors of GMSB provide stable particles that may be present in the early Universe for a sufficiently heavy reheat temperature $T_{RH}$. We consider hereafter such configurations assuming that only the messenger (including the spurion) sector can be produced and illustrate its relevance to the issue of the CDM.

2 Curing a Messenger Problem

The mass degeneracy within a supermultiplet of messenger fields is lifted by susy breaking leading to a lighter and a heavier scalar messengers with masses $M_{A\pm} = M_X (1 \pm kF/M_X^{1/2})$ and a fermionic partner with mass $M_X$ (where $F$ and $M_X$ are related to the dynamical scale $A$). Thus $kF/M_X < 1$. Moreover, one has to require $kF/M_X \lesssim 10^6$ GeV to ensure an MSSM spectrum $\lesssim O(1)$ TeV. One then expects typically $M_X \gtrsim 10^6$ GeV. In GMSB models the lightest messenger particle (LMP) with mass $M_-$ is stable due to the conservation of a messenger quantum number. If present in the early Universe the messenger particles are thermalized through their gauge interactions with the thermal bath. The corresponding LMP relic density is calculable similarly to the case of Neutralino LSP albeit an extended particle content and couplings.

\textsuperscript{a} based on work in collaboration with K. Jedamzik (LPTA-Montpellier), M. Lemoine (IAP-Paris)\cite{7,9}, and work in progress, M. Kuroda (Meiji-Gakuin), M. Lemoine (Paris), M. Capdequi-Peyranre (Montpellier).

\cite{1,2}
However, it turns out to be typically too large to account for the CDM even in the most favorable case of the electrically neutral component of a $5 + \overline{5}$ representation of $SU(5)$ where it is found to scale as $\Omega_M h^2 \approx 10^2 \left( M_\chi / 10^3 \text{TeV} \right)^2$ with the LMP mass $\tilde{T}_d$. The situation is even worse in the case of $SO(10)$ where the LMP is a MSSM singlet with a suppressed annihilation cross-section leading to a very large relic density. One possible cure to this messenger overcloser problem is to allow the LMP to decay to MSSM particles. This can be easily achieved by adding renormalizable but messenger number violating operators to the superpotential, however, such low-scale operators would tend to ruin the nice FCNC suppression of GMSB models. A more appealing approach is to insist on the messenger number conservation as a consequence of a discrete accidental symmetry at low energy and invoke the typical violation of such a (non gauge) symmetry by gravitational interactions $\gamma$ once the GMSB model is coupled to supergravity. The LMP decay would then occur via Planck mass suppressed operators in the Lagrangian, which can originate either directly from effective non-renormalizable operators in the Kähler or the superpotential, or indirectly after susy breaking through the renormalizable renormalizable operators in the Kähler potential. In the latter case the suppression is controlled by the gravitino mass. In the case of $SU(5)$ an exhaustive study of these operators was carried out in $[7]$ for messengers transforming as $5 + \overline{5}$ or $10 + \overline{10}$. In $SO(10)$ with one set of messengers transforming as $16 + \overline{16}$, the LMP decay can be induced by non-renormalizable operators in the Kähler potential, e.g. $K \supset 16_\chi \overline{16}_{\chi M} 10_H / m_{\pi_1}$, or in the superpotential, e.g. $W \supset 16_\chi 10_F 10_H / m_{\pi_1}^{-1}$, leading respectively to 2- and 3-body decays, where $16_M$, $10_F$ and $10_H$ denote respectively the messenger, the standard matter and the electroweak Higgs supermultiplets. We assume a typical width $\Gamma_{\text{LMP}} = (1/16\pi) f'/M_{\chi}^2 / m_{\pi_1}^{-1}$ where $f'$ parameterizes our ignorance of the couplings and possible further phase space suppression. For couplings of $O(1)$, $f' \approx 1(3 \times 10^{-3})$ for 2- (3-body) decays into essentially massless particles. On the one hand, such suppressed decays would not upset the FCNC suppression, and on the other, will turn out actually to be a blessing regarding a solution to the gravitino overproduction in the early universe, eventually allowing the gravitino to account for the cold dark matter in the context of GMSB models.

3 Gravitino abundance

3.1 the cosmological set-up

In favorable parts of the parameter space, the LMP late decay into MSSM particles can release enough entropy to dilute the initial gravitino relic density down to a level which can account for the CDM in the Universe even for very high $T_{RH}$. $[8]$, $[7]$, $[9]$. For this to work, though, the LMP should dominate the Universe energy density before it decays, and should decay after the gravitino has freezed-out from the thermal bath. The necessary conditions $T_d < T_{MD}, T_{3/2}$ [where $T_d, T_{MD}, T_{3/2}$ denote respectively the LMP decay and messenger matter domination temperatures, and the gravitino freeze-out temperature] is then determined by the particle properties and annihilation cross-section and decay width of the LMP, delineating the favorable parts of the parameter space. We have studied this scenario in detail for the case of $SU(5)$ $[7]$ and $SO(10)$, $[9]$, $[10]$. Here we concentrate on the latter case with one set of messengers transforming as $16 + \overline{16}$. The entropy release $\Delta S \equiv S_{\text{after}} / S_{\text{before}},$ diluting the initial gravitino density, is determined by the temperatures before and after LMP decay and can be approximated to $T_{MD} / T_d$. $T_{MD}$ is given by the LMP yield and mass ($T_{MD} \approx (4/3) M_\chi \times Y_{\text{LMP}}$) and $T_d$ is determined by the LMP width ($\Gamma_{\text{LMP}} \approx H(T_d)$). The LMP yield $Y_{\text{LMP}}$ is controlled by the LMP annihilation into MSSM particles which enters the corresponding Boltzmann equation. Since in our case the LMP is an $SU(5)$ singlet $[8]$, $[9]$ this annihilation proceeds via loop effects of virtual messengers ($A_M, \psi_M$) and spurion ($S$) exchange. We consider here the leading annihilation cross-section into 2 gluons, fig.1, and parameterize its thermal averaged as $\langle \sigma v \rangle \sim f \times (\alpha_s / 4\pi)^2 k^4 / s$ where $k$ is the spurion-messenger coupling ($W \supset \kappa \overline{16}_M \overline{10}_M$), $\alpha_s$ the strong coupling constant, $\sqrt{s}$ the C.M. energy and $f$ a form factor depending on the internal masses and couplings. Neglecting the very heavy GUT sector contributions which typically decouple, one finds

$$f = \frac{32}{\pi} (3/8) g^2 + M_A^2 C_- + (3/4) g^2 - 1) M_{A_+}^2 + 2 M_{\chi}^2 (M_A^2 C_- + M^2 A_+ C_+ + (s - 4 M_{\chi}^2) C_X) - 1) D[s]^2$$

where $D[s]$ denotes the spurion propagator $(s - M_\chi^2 + i f_S MS)^{-1}$, $C_{\pm, X}$ are $C_0$ functions scaling as $s^{-1}$, and $g^2 \equiv 4\pi \alpha_s / k^2$. Since the messengers in the loops carry color charges, a substantial mass splitting occurs between the contributing $A_-$ states and the LMP due to RGE running from the GUT scale down to $Q = M_\chi^2$, as well as from genuine loop corrections $[11]$, $[9]$, leading typically to $M_{A_-} \approx 3 M_\chi$. Such effects, as well as the running of $\alpha_s$ should be taken into account for a precise determination of $Y_{\text{LMP}}$. On the other hand, the contribution of the scalar spurion depends on its mass and width. The spurion can be either heavier or lighter than the messenger sector. Here we consider only the former case, where the spurion decays at tree-level into pairs of messengers or at one-loop into MSSM particles $[8]$. We find that the decay into MSSM particles dominates irrespective of the value of the coupling $\kappa$, the total width $\Gamma_S$ remaining small though $\Gamma_S / M_S \approx (1 - 4) \times 10^{-3}$. A careful treatment (beyond the relative velocity expansion) of the

1 If the spurion is lighter than the LMP, an efficient tree-level annihilation of the latter into a pair of spurions would lead to a too small $Y_{\text{LMP}}$. 

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thermally averaged annihilation cross-section is thus required close to this narrow resonance, i.e. typically when $M_- \approx M_S/2$ and assuming a non-relativistic decoupling of the LMP from the thermal bath.

### 3.2 relic gravitinos

When the necessary temperature conditions discussed in the previous section are met, the final gravitino relic density is given by

$$\Omega_{\text{grav}} = \Omega_{\text{grav}}^h / \Delta S + \Omega_{\text{grav}}^{\text{NLSP}} + \Omega_{\text{grav}}^{\text{NLSP}} + \Omega_{\text{grav}}^{\text{NLSP}}$$

where the last two contributions denote non-thermal production through late decays or scattering. One should also consider various cosmological constraints (hotness/warmness, BBN, species dilution, etc...). Let us illustrate the case first with some effective fixed values for $f$ and $f'$. This is shown in fig.2 taking $T_{RH} \approx 10^{12}$ GeV, see also \[7\].

The red horizontal shading shows the theoretically excluded region where $k > 1$; the other red shading indicates the region excluded by BBN constraints. If the spurion is heavier than the LMP, gravitino cold DM (green region) occurs for relatively light LMPs and $m_{3/2} \sim 1$ keV – 10 MeV. Note that without the LMP induced entropy dilution, the reheat temperature would have been constrained to be several orders of magnitude lower than $10^{12}$ GeV in order to avoid overcloser for the range of gravitino masses found here.

More generally, in the models of ref. \[12\] one finds \[9\]

$$\Omega_{\text{grav}} h^2 \approx 3 f^3 f^3 / \Delta S / (10^{12} \text{GeV})^{0.3} \times (m_{3/2} / 1 \text{MeV})$$

for non-relativistic LMP freeze-out, putting the gravitino relic abundance in the ballpark of WMAP results. Considering now the specific one-loop form-factor $f$ given in the previous section and assuming for the sake of the illustration a spurion much heavier than the messenger sector, we still find regions consistent with WMAP results for $\Omega h^2$ at the 99% confidence level, e.g. for $M_X = 10^6 - 10^8$ GeV, one has $1.1 \text{MeV} < m_{3/2} < 4 \text{MeV}$ for a three body decay LMP, and $65 \text{keV} < m_{3/2} < 230 \text{keV}$ for a two body decay LMP.

For heavier spurions the annihilation into 2 gravitinos through tree-level gravitational interactions (see fig.1) becomes significant as its cross-section scales like

$$(\sigma v) \approx (1/24\pi) k^2 M_Z^2 / (m_{3/2}^2 m_{1/2})^2.$$  

It can dominate the 1-loop annihilation, eventually saturating the unitarity limit (the black dashed line in fig.2), thus disfavouring gravitino CDM solutions for $M_- \gtrsim 10^6$ GeV. reduce a thermally overproduced gravitino to a cosmologically acceptable level. Moreover, various constraints (e.g. on $T_{RH}$, \[13\], or on the gravitino mass \[14\]) simply do not apply in the scenarios we have illustrated, thus escaping possible tension with thermal leptogenesis. Finally, let us mention that such scenarios can potentially allow to avoid the recently studied cosmological gravitino problem due to inflaton decay \[15\].

### 4 Conclusion

Light gravitino can account for CDM irrespective of $T_{RH}$, making it a good DM candidate in GMSB: typically if $T_{RH} \lesssim 10^6$ GeV then the messengers are not produced and thermal gravitinos with $m_{3/2} \lesssim 1$ MeV provide the right CDM density, while for $T_{RH} \gtrsim 10^6$ GeV the messenger can be present and should be unstable, thus providing a source of entropy production that can

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Fig. 1. Feynman diagrams of the leading LMP annihilation into 2 gluons (a-e) or 2 gravitinos (f-i).

Fig. 2. Contours of $\Omega_{1/2}$ in the plane $M_X = M_{\perp} - m_3/2$ for one pair of messengers sitting in $16 + \overline{16}$ representations of $SO(10)$; the LMP is a singlet under $SU(3) \times SU(2) \times U(1)$. We take for illustration $\kappa^2 \approx \alpha_s/4\pi$, $f \sim O(1)$ and $f' \approx 5 \times 10^{-2}$ and a bino NLSP with $M_{NLSP} = 150 \text{ GeV}$, $M_{gluino} = 1 \text{ TeV}$ and $kF/M_\perp \approx 10^6 \text{ GeV}$; blue (hot), red (warm), green (cold) DM with $0.01 < \Omega_{\text{grav}} < 1$; yellow ($\Omega_{\text{grav}} < 0.01$), white ($\Omega_{\text{grav}} > 1$). In the right (left) panel the spurion is lighter (heavier) than the messenger. (taken from [7].)