Penrose Limit of $AdS_4 \times V_{5,2}$ and Operators with Large $R$ Charge

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abstract

We consider M-theory on $AdS_4 \times V_{5,2}$ where $V_{5,2} = SO(5)/SO(3)$ is a Stiefel manifold. We construct a Penrose limit of $AdS_4 \times V_{5,2}$ that provides the pp-wave geometry. There exists a subsector of three dimensional $\mathcal{N} = 2$ dual gauge theory, by taking both the conformal dimension and $R$ charge large with the finiteness of their difference, which has enhanced $\mathcal{N} = 8$ maximal supersymmetry. We identify operators in the $\mathcal{N} = 2$ gauge theory with supergravity KK excitations in the pp-wave geometry and describe how the gauge theory operators made out of chiral field of conformal dimension $1/3$ fall into $\mathcal{N} = 8$ supermultiplets.
1 Introduction

The large $N$ limit of a subsector of $d = 4, \mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory is dual \cite{1} to type IIB string theory in the pp-wave background \cite{2, 3}. This subspace of the gauge theory is described by string theory in the pp-wave background. By taking a scale limit of the geometry near a null geodesic in $AdS_5 \times S^5$, it gives rise to the appropriate subspace of the gauge theory. The operators with large $R$-charge in the subsector of $\mathcal{N} = 4$ $SU(N)$ gauge theory were identified with the stringy states in the pp-wave background. There exists a particularly interesting model by replacing $S^5$ with a five-dimensional manifold, $T^{1,1}$ with lower supersymmetry. It was found that the Penrose limit of $AdS_5 \times T^{1,1}$ provides pp-wave geometry of $AdS_5 \times S^5$ \cite{4, 5, 6}. Using AdS/CFT correspondence, one can identify gauge theory operators with large $R$-charge with the stringy excitations in the pp-wave geometry. Moreover, the maximal $\mathcal{N} = 4$ multiplet structure hidden in the $\mathcal{N} = 1$ gauge theory can be predicted from both a chiral operator and semi-conserved operator with large $R$-charge. There are many papers \cite{8} on the work of \cite{1}.

It is natural to think about the subsector of $\mathcal{N} = 2$ gauge theory in $d = 3$ in the context of $AdS_4 \times X^7$ where $X^7$ is an Einstein seven manifold. Recently the operators with large $R$-charge in the boundary field theory were obtained from the complete spectrums of 11-dimensional KK compactifications on $AdS_4 \times Q^{1,1,1}$ \cite{9} and $AdS_4 \times M^{1,1,1}$ \cite{10} in pp-wave limit. In old days, all the supersymmetric $\mathcal{N} = 2$ homogeneous manifolds were classified in \cite{11}. There exist only three $\mathcal{N} = 2$ theories and they are $Q^{1,1,1}, M^{1,1,1}$ and $V_{5,2}$. The isometry of these manifolds corresponds to the global symmetry of the dual SCFT including $U(1)_R$ symmetry of $\mathcal{N} = 2$ supersymmetry.

In this paper, we consider a similar duality that is present between a certain three dimensional $\mathcal{N} = 2$ gauge theory and 11-dimensional supergravity theory in a pp-wave background with the same spirit as in \cite{9, 10, 5, 12}. This is a continuation of previous considerations \cite{9, 10}. We describe this duality by taking a scaling limit of the duality between 11-dimensional supergravity on $AdS_4 \times V_{5,2}$ where $V_{5,2}$ was found in \cite{11} and three dimensional superconformal field theory. The boundary theory is a gauge theory with gauge group $USp(2N) \times O(2N - 1)$ with chiral fields $S^i$ transforming in the $(2N, 2N - 1)$ color representation and transforming in the spinor representation of the flavor group $SO(5)$. The complete analysis on the spectrum of $AdS_4 \times V_{5,2}$ was found in \cite{12}. This gives the theory that lives on $N$ M2-branes at the conical singularity of a Calabi-Yau four-fold. The scaling limit is obtained by considering the geometry near a null geodesic carrying large angular momentum in the $U(1)_R$ isometry of the $V_{5,2}$ space which is dual to the $U(1)_R$ R-symmetry in the $\mathcal{N} = 2$ superconformal field theory.

In section 2, we consider the scaling limit around a null geodesic in $AdS_4 \times V_{5,2}$ from the explicit metric of $V_{5,2}$ given in terms of angular variables \cite{13} and obtain a pp-wave background.
In section 3, we identify supergravity excitations in the Penrose limit with gauge theory operators. What we observed is the presence of a semi-conserved field in the long vector multiplet II propagating in the $AdS_4$ bulk. In section 4, we summarize our results.

2 Penrose Limit of $AdS_4 \times V_{5,2}$

Let us start with the supergravity solution dual to the $\mathcal{N} = 2$ superconformal field theory \[12\]. By putting a large number of $N$ coincident M2-branes at the conifold singularity and taking the near horizon limit, the metric becomes that \[13\] of $AdS_4 \times V_{5,2}$(See also \[14\])

\begin{equation}
\begin{array}{c}
\nu^2_{11} = ds^2_{AdS_4} + ds^2_{V_{5,2},}
\end{array}
\end{equation}

where

\begin{align}
\nu^2_{AdS_4} &= L^2 \left( - \cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_2^2 \right), \\
\nu^2_{V_{5,2}} &= \frac{9L^2}{16} \left[ d\psi + \frac{1}{2} \cos \alpha \left( d\beta - \sum_{i=1}^2 \cos \theta_i d\phi_i \right) \right]^2 + \frac{3L^2}{8} \, d\alpha^2 \\
&\quad + \frac{3L^2}{32} \sin^2 \alpha \left( d\beta - \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 + \frac{3L^2}{32} \left( 1 + \cos^2 \alpha \right) \sum_{i=1}^2 \left( \sin^2 \theta_i d\phi_i^2 + d\theta_i^2 \right) \\
&\quad + \frac{3L^2}{16} \sin^2 \alpha \cos \beta \sin \theta_1 \sin \theta_2 d\phi_1 d\phi_2 - \frac{3L^2}{16} \sin^2 \alpha \cos \beta d\theta_1 d\theta_2 \\
&\quad + \frac{3L^2}{16} \sin^2 \alpha \sin \beta \left( \sin \theta_1 d\phi_1 d\theta_2 + \sin \theta_2 d\phi_2 d\theta_1 \right),
\end{align}

where $d\Omega_2$ is the volume form of a unit $S^2$ and the curvature radius $L$ of $AdS_4$ is given by \((2L)^6 = 32\pi^2 \ell_p^6 N\). The spherical coordinates $(\theta_i, \phi_i)$ parametrize two sphere $S^2_i$, as usual, and the angles vary over the ranges, $0 \leq \theta_i \leq \pi$, $0 \leq \phi_i \leq 2\pi$, $0 \leq \beta \leq 4\pi$ and $0 \leq \alpha \leq \pi/2$. The $SO(5) \times U(1)$ isometry group of $V_{5,2}$ consists of $SO(5)$ global symmetry and $U(1)_R$ symmetry of the dual superconformal field theory of \[12\].

Let us make a scaling limit around a null geodesic in $AdS_4 \times V_{5,2}$ that rotates along the $\psi$ coordinate of $V_{5,2}$ whose shift symmetry corresponds to the $U(1)_R$ symmetry in the dual superconformal field theory. Let us introduce coordinates which label the geodesic

\begin{align}
x^+ &= \frac{1}{2} \left[ t + \frac{3}{4} \left( \psi + \frac{1}{2} \beta - \frac{1}{2} \phi_1 - \frac{1}{2} \phi_2 \right) \right], \\
x^- &= \frac{L^2}{2} \left[ t - \frac{3}{4} \left( \psi + \frac{1}{2} \beta - \frac{1}{2} \phi_1 - \frac{1}{2} \phi_2 \right) \right],
\end{align}

and make a scaling limit around $\rho = 0 = \theta_1 = \theta_2 = \alpha$ in the above geometry \[14\]. By taking the limit $L \to \infty$ while rescaling the coordinates

\begin{align}
\rho &= \frac{r}{L}, \quad \theta_1 = \frac{2\zeta_1}{\sqrt{3}L}, \quad \theta_2 = \frac{2\zeta_2}{\sqrt{3}L}, \quad \alpha = \frac{\sqrt{2}\zeta_3}{\sqrt{3}L},
\end{align}

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with the fact that the quantities in the last two lines of (2) do not contribute, the Penrose limit of the $AdS \times V_{5,2}$ becomes

$$ds^2_{11} = -4dx^+dx^- + \sum_{i=1}^{3} \left( dr^i dr^i - r^i r^i dx^+ dx^+ \right) + \frac{1}{4} \left( d\zeta_3^2 + \zeta_3^2 d\phi_3^2 - 2\zeta_3^2 d\phi_3 dx^+ \right) + \frac{1}{4} \sum_{i=1}^{2} \left( d\zeta_i^2 + \zeta_i^2 d\phi_i^2 + 2\sqrt{3}\zeta_i^2 d\phi_i dx^+ \right)$$

$$= -4dx^+dx^- + \sum_{i=1}^{3} \left( dr^i dr^i - r^i r^i dx^+ dx^+ \right) + \frac{1}{4} \left( dwd\bar{w} + i \left( \bar{w} dw - w d\bar{w} \right) dx^+ \right) + \frac{1}{4} \sum_{i=1}^{2} \left( dz_i d\bar{z}_i - i\sqrt{3} \left( \bar{z}_i dz_i - z_i d\bar{z}_i \right) dx^+ \right) \ (3)$$

where we define $\phi_3 = \frac{1}{2} (\beta - \phi_1 - \phi_2)$ and in the last line we introduce the complex coordinate $w = \zeta_3 e^{i\phi_3}$ for $\mathbb{R}^2$ and a pair of complex coordinates $z_i = \zeta_i e^{i\phi_i}$ for $\mathbb{R}^4$. Since the metric has a covariantly constant null Killing vector $\partial/\partial x^+$, it is also pp-wave metric. The pp-wave has a decomposition of the $\mathbb{R}^9$ transverse space into $\mathbb{R}^3 \times \mathbb{R}^2 \times \mathbb{R}^4$ where $\mathbb{R}^3$ is parametrized by $r^i$, $\mathbb{R}^2$ by $w$ and $\mathbb{R}^4$ by $z_i$. The symmetry of this background is the $SO(3)$ rotation in $\mathbb{R}^3$. In the gauge theory side, the $SO(3)$ symmetry corresponds to the subgroup of the $SO(2, 3)$ conformal group. Note that the pp-wave geometry (3) in the scaling limit reduces to the maximally supersymmetric pp-wave solution of $AdS_4 \times S^7$ [13, 16]

$$ds^2_{11} = -4dx^+dx^- + \sum_{i=1}^{9} dr^i dr^i - \left( \sum_{i=1}^{3} r^i r^i + \frac{1}{4} \sum_{i=4}^{9} r^i r^i \right) dx^+ dx^+ . $$

The supersymmetry enhancement in the Penrose limit implies that a hidden $\mathcal{N} = 8$ supersymmetry is present in the corresponding subsector of the dual $\mathcal{N} = 2$ superconformal field theory. In the next section, we provide precise description of how to understand the excited states in the supergravity theory that corresponds in the dual superconformal field theory to operators with a given conformal dimension.

### 3 Gauge Theory Spectrum

The 11-dimensional supergravity theory in $AdS_4 \times V_{5,2}$ is dual to the $\mathcal{N} = 2$ gauge theory with gauge group $USp(2N) \times O(2N - 1)$ with chiral fields $S^i, i = 1, 2, 3, 4$ transforming in the $(2N, 2N - 1)$ color representation $\mathbf{1}$ and transforming in the spinor representation $\mathbf{4}$ of the flavor group $SO(5)$ [12]. The Stiefel manifold $V_{5,2}$ is a coset manifold $SO(5)/SO(3)$ where the embedding of $SO(3)$ in $SO(5)$ is the canonical one: the fundamental of $\mathbf{5}$ of $SO(5)$ breaks into $\mathbf{5} \to \mathbf{3} + \mathbf{1} + \mathbf{1}$ under $SO(3)$. At the fixed point, the chiral superfield $S^i$ has conformal

\[\text{We take the convention of } USp \text{ group such that } USp(2) = SU(2).\]
weight $1/3$ with $U(1)_R$ charge $1/3$. We identify states in the supergravity containing both short and long multiplets with operators in the gauge theory. In each multiplet, we specify a $SO(5)$ representation $\mathbf{\frac{2}{3}}$, conformal weight and $R$-charge.

- **Massless(or ultrashort) multiplets**

1) Massless graviton multiplet: $[0,0], \quad \Delta = 2, \quad R = 0$

There exists a stress-energy tensor superfield $T_{\alpha\beta}(x, \theta)$ satisfying the equation for conserved current $D^\pm_\alpha T^{\alpha\beta}(x, \theta) = 0$. This $T_{\alpha\beta}(x, \theta)$ expressed as quadratic in chiral field $S_i$ is a singlet with respect to the flavor group $SO(5)$ and its conformal dimension is 2 with vanishing $R$ charge. So this corresponds to the massless graviton multiplet that propagates in the $AdS_4$ bulk.

2) Massless vector multiplet: $[1,1], \quad \Delta = 1, \quad R = 0$

There exists a conserved vector current, a scalar superfield $J_{SO(5)}(x, \theta)$, to the generator of the flavor symmetry group $SO(5)$ satisfying the conservation equations $D^\pm_\alpha D^\pm_\beta J_{SO(5)}(x, \theta) = 0$. This $J_{SO(5)}(x, \theta)$ transforms in the $[1,1]$ representation of the flavor group and its conformal dimension is 1 with vanishing $R$-charge. This corresponds to the massless vector multiplet propagating in the $AdS_4$ bulk. Therefore massless multiplets 1) and 2) saturate the unitary bound and have a conformal weight related to the maximal spin: $\Delta = s_{\text{max}}$ where $s_{\text{max}}$ is 2, 1 for graviton and vector multiplet respectively.

- **Short multiplets**

It is known that the dimension of the scalar operator in terms of energy labels, in the dual SCFT corresponding $AdS_4 \times V_{5,2}$ is

$$\Delta = \frac{3}{2} + \frac{1}{2} \sqrt{1 + \frac{m^2}{4}} = \frac{3}{2} + \frac{1}{2} \sqrt{45 + \frac{E}{4} - 6\sqrt{36 + E}}. \quad (4)$$

The energy spectrum on $V_{5,2}$ exhibits an interesting feature which is relevant to superconformal algebra and it is given by

$$E = \frac{32}{9} \left(6M^2 + 9N + 3N^2 + 12M + 6MN - Q^2\right) \quad (5)$$

where the eigenvalue $E$ is classified by $SO(5)$ quantum numbers $M, N$ (totally we have $2M + N$ boxes) characterized by Young tableau notation $[M + N, M]$, and $U(1)$ charge $Q$: $M, N = 0, 1, 2, \cdots$ and $Q = 0, \pm 1, \cdots$. The $U(1)$ part of the isometry goup of $V_{5,2}$ acts by shifting $U(1)$ charge $Q$. The $R$-charge, $R$ is related to $U(1)$ charge $Q$ by $R = 2Q/3$. Let us take $R \geq 0$. One can find the lowest value of $\Delta$ is equal to $R$ corresponding to a mode scalar with

\[A\text{ representation of } SO(5) \text{ can be identified by a Young diagram and when we denote the Dynkin label } (a_2, a_1), \text{ the dimensionality of an irreducible representation is given by } N(a_1, a_2) = \frac{1}{2}(a_1 + a_2 + 2)(a_1 + 1)(a_2 + 1)(2a_1 + a_2 + 3). \text{ Although the representation with its dimensionality is possible, we use the same notation as } \cite{12} \text{ for comparison. That is, } SO(5) \text{ quantum number, } M \text{ and } N \text{ characterized by Young tableau } [M + N, M] \text{ such that there are } 2M + N \text{ boxes.}
$M = 0, N = 3R/2$ because $E$ becomes $16R(R + 3)$ and plugging back to (4) then one obtains $\Delta = R$.

Thus we find a set of operators filling out a $\left[ \frac{3R}{2}, 0 \right]$ multiplet of $SO(5)$. The condition $\Delta = R$ saturates the bound on $\Delta$ from superconformal algebra. The fact that the $R$-charge of a chiral operator is equal to the dimension was observed in [7].

1) Hypermultiplet:

$$\left[ \frac{3R}{2}, 0 \right], \quad \Delta = R.$$  

The information on the Laplacian eigenvalues allows us to get the spectrum of hypermultiplets of the theory corresponding to the chiral operators of the SCFT. This spectrum was given in [12] and the form of operators is

$$\text{Tr} \Phi_c \equiv \text{Tr} \left( S^t \Gamma^a S \right)^{3R/2}$$  \hspace{1cm} (6)

where the flavor indices are totally symmetrized, the chiral superfield $\Phi_c(x, \theta)$ satisfies $D_\alpha^+ \Phi_c(x, \theta) = 0$ and the $\Gamma$’s are the gamma matrices in five dimensions. The complex coordinates $z^a$ parametrized by a Calabi-Yau four-fold is given by $z^a = \text{Tr} \left( S^t \Gamma^a S \right)$, $a = 1, 2, 3, 4, 5$. The hypermultiplet spectrum in the KK harmonic expansions on $V_{5,2}$ agrees with the chiral superfield predicted by the conformal gauge theory. From this, the dimension of $S^t$ should be $1/3$ to match the spectrum. In fact, the conformal weight of a product of chiral fields equals the sum of the weights of the single components. This is due to the the relation of $\Delta = R$ satisfied by chiral superfields and to the additivity of the $R$-charge.

2) Short graviton multiplet: $\left[ \frac{3R}{2}, 0 \right], \quad \Delta = R + 2$

The gauge theory interpretation of this multiplet is obtained by adding a dimension 2 singlet operator with respect to flavor group into the above chiral superfield $\Phi_c(x, \theta)$. We consider $\text{Tr} \Phi_{a\beta} \equiv \text{Tr} \left( T_{a\beta} \Phi_c \right)$, where $T_{a\beta}(x, \theta)$ is a stress energy tensor and $\Phi_c(x, \theta)$ is a chiral superfield. All color indices are symmetrized before taking the contraction. It is easy to see this composite operator satisfies $D_\alpha^+ \Phi^{a\beta}(x, \theta) = 0$. When $R$-charge vanishes, it leads to a massless graviton multiplet we have discussed.

3) Short vector multiplet II:

$$\left[ \frac{3R}{2} + 1, 1 \right], \quad \Delta = R + 1.$$  

One can construct the following gauge theory object by recognizing that the above $SO(5)$ representation can be decomposed into $\left[ \frac{3R}{2}, 0 \right]$ and $[1, 1]$, corresponding to the short vector multiplet II, $\text{Tr} \Phi \equiv \text{Tr} \left( J_{SO(5)} \Phi_c \right)$, where $J_{SO(5)}(x, \theta)$ is a conserved vector current transforming in the $[1, 1]$ representation of $SO(5)$ flavor group and $\Phi_c(x, \theta)$ is a chiral superfield [3]. In this
case, we have \( D^+D^+_\alpha \Phi(x, \theta) = 0 \). This multiplet reduces to a massless vector multiplet when \( R = 0 \).

4) Short gravitino multiplet II:

\[
\left[ \frac{3(R-1)}{2}, 0 \right], \quad \Delta = R + 3/2.
\]

The corresponding gauge theory operator is identified with \( \text{Tr} \Phi_\alpha \equiv \text{Tr} \left[ X_\alpha (S^a \Gamma^a S)^{3(R-1)/2} \right] \) where \( X_\alpha(x, \theta) \) is a semi-conserved current transforming as a singlet of flavor group and satisfying the condition \( D^+ \alpha X_\alpha(x, \theta) = 0 \) by interpreting the \( \Delta \) as a sum of \( R - 1 \) and \( 5/2 \). By construction, we have \( D^+ \Phi_\alpha = 0 \). The operator of conformal dimension \( \Delta = 5/2 \) (which can be interpreted as the sum of 1, \( s = 1/2 \) and \( R = 1 \) according to the definition of a semi-conserved current) with \( R \)-charge 1 is given by contracting the \( SO(5) \) indices \([12]\)

\[
X_\alpha = \text{ST}_a S \left( \overline{\text{ST}}_b \overline{\text{ST}}^a D^+_\alpha \overline{\text{ST}}^b \overline{S} - 2 \overline{\text{ST}}_b D^+_\alpha \overline{\text{ST}}^a \overline{S} \right).
\]

Note that the conformal dimension and \( R \)-charge of fermionic coordinate \( \theta^+_\alpha \) are \( \Delta = 1/2 \) and \( R = 1 \) respectively. Therefore the counting of \( R = 1 \) for the \( X_\alpha \) field is due to the \( D^+_\alpha \) with the additivity of \( R \)-charge( the fact that there are equal numbers of \( S \) and \( \overline{S} \) does not contribute to the \( R \)-charge counting).

5) Short gravitino multiplet I:

\[
\left[ \frac{3R + 1}{2}, 1 \right], \quad \Delta = R + 3/2.
\]

In this case, one can think of the following gauge theory object corresponding this multiplet as \( \text{Tr} \Psi_\alpha \equiv \text{Tr} \left[ L_\alpha (S^a \Gamma^a S)^{3(R-1)/2} \right] \) where \( L_\alpha(x, \theta) \) is a semi-conserved current transforming in \([2, 1]\) representation of flavor group with \( D^+ L_\alpha(x, \theta) = 0 \) by writing the \( SO(5) \) representation in terms of \( \left[ \frac{3(R-1)}{2}, 0 \right] \) and \([2, 1]\). Similarly, one has \( D^+ \Phi_\alpha(x, \theta) = 0 \). The explicit form for the operator of conformal dimension \( \Delta = 5/2 \) with \( R \)-charge 1 being consistent with the definition of semi-conserved current is given by \([12]\)

\[
(L_\alpha)^{a(bc)} = \overline{\text{ST}}^a S D^+_\alpha \overline{\text{ST}}^b \overline{S} - D^+_\alpha \overline{\text{ST}}^a S \overline{\text{ST}}^b \overline{S}.
\]

Therefore the short \( OSp(2|4) \) multiplets 1), 2), 3), 4) and 5) saturate the unitary bound and have a conformal dimension related to the \( R \)-charge and maximal spin: \( \Delta = R + s_{\text{max}} \) where \( s_{\text{max}} \) is 2, 3/2 and 1 for graviton, gravitino and vector multiplet respectively.

• Long multiplets \([12]\)

Although the dimensions of nonchiral operators are in general irrational, there exist special integer values of \( m, n \) such that for

\[
[M + N, M] = \left[ m + n + \frac{3R}{2}, m \right],
\]
one can see the Diophantine [12] like condition (See also [17, 9, 10]),

\[ m^2 - n^2 - 2mn - 3n - m = 0 \]  \( (7) \)

make \( \sqrt{36 + E} \) be equal to \( 4R + 2(2m + 2n + 3) \). Furthermore in order to make the dimension be rational (their conformal dimensions are protected), \( 45 + E/4 - 6\sqrt{36 + E} \) in (3) should be square of something. It turns out this is the case without any further restrictions on \( m \) and \( n \).

Therefore we have \( \Delta = R + m + n \). This is true if we are describing states with finite \( \Delta \) and \( R \). Since we are studying the scaling limit \( \Delta, R \to \infty \), we have to modify the above analysis. This constraint (7) comes from the fact that the energy eigenvalue of the Laplacian on \( V_{5,2} \) for the supergravity mode (3) takes the form

\[ E = \frac{32}{3} \left[ 5m^2 + n^2 + (6R + 7) m + 3 (1 + R) n + 4mn + \frac{3}{2} R (R + 3) \right]. \]  \( (8) \)

One can show that the conformal weight of the long vector multiplet II below becomes rational if the condition (7) is satisfied.

1) Long vector multiplet II:

\[ \Delta = -\frac{3}{2} + \frac{1}{4} \sqrt{E + 36}. \]

However, as we take the limit of \( R \to \infty \), this constraint (4) is relaxed. The combination of \( \Delta - R \) is given by

\[ \Delta - R = m + n + O\left(\frac{1}{R}\right) \]  \( (9) \)

where the right hand side is definitely rational and they are integers. So the constraint (7) is not relevant in the subsector of the Hilbert space we are interested in. Candidates for such states in the gauge theory side are given in terms of semi-conserved super fields [12]. Although they are not chiral primaries, their conformal dimensions are protected. The ones we are interested in take the following form,

\[ \text{Tr} \Phi_{S.C.} \equiv \text{Tr} \left[ \left( J_{SO(5)} \right)^m \left( K_{SO(5)} \right)^n \left( S^a \Gamma^a S \right)^{3R/2} \right] \]  \( (10) \)

where the scalar superfields \( J_{SO(5)}(x, \theta) \) transform in the [1, 1] representation of flavor group \( SO(5) \) and satisfy \( D^{\pm a} D^\pm_a J_{SO(5)}(x, \theta) = 0 \) with conformal dimension 1 and zero \( U(1)_R \) charge. Also there exists a scalar superfield \( K_{SO(5)}(x, \theta) \) transforming in [1, 0] representation of the flavor group with same conformal dimension and \( R \)-charge. One can construct the following conserved flavor currents in the [1, 1] representation under \( SO(5) \) and the ones in the [1, 0] representation as follows:

\[ \left( J_{SO(5)} \right)^{ab} = S \Gamma^{ab} S, \quad \left( K_{SO(5)} \right)^a = S \Gamma^a S \]
where the color indices are contracted in the right hand side. Note that the conformal dimension of these currents is not the one of naive sum of $S^i$’s. The supergravity theory in $AdS_4 \times V_{5,2}$ acquires an enhanced $\mathcal{N} = 8$ superconformal symmetry in the Penrose limit. This implies that the spectrum of the gauge theory operators in this subsector should fall into $\mathcal{N} = 8$ multiplets. We expect that both the chiral primary fields of the form (6) and the semi-conserved multiplets of the form (10) combine into $\mathcal{N} = 8$ multiplets in the limit. Note that for finite $R$, the semi-conserved multiplets should obey the Diophantine constraint (7) in order for them to possess rational conformal weights. If $m = 1$ and $n = 0$, we see a shortening of the multiplet related to the massless vector multiplet or short vector multiplet II depending on $R = 0$ or not.

In the remaining multiplets we consider the following particular representations in the global symmetry group $SO(5)$: $[M + N, M] = [m + n + 3R, 2, m]$.

2) Long graviton multiplet:

$$\Delta = \frac{1}{2} + \frac{1}{4}\sqrt{E + 36}.$$ 

For finite $R$ with rational dimension, after inserting the $E$ into the above, we will arrive at the relation with same constraint (6) which is greater than (4) by 2:

$$\Delta - R = 2 + m + n + O\left(\frac{1}{R}\right).$$

The gauge theory interpretation of this multiplet is quite simple. If we take a semi-conserved current $\Phi_{s.c.}(x, \theta)$ defined in (10) and multiply it by a stress-energy tensor superfield $T_{\alpha\beta}(x, \theta)$ that is a singlet with respect to the flavor group, namely $\text{Tr} \left(T_{\alpha\beta}\Phi_{s.c.}\right)$, we reproduce the right $OSp(2|4) \times SO(5)$ representations of the long graviton multiplet with right conformal dimension.

Also one can expect that other candidate for this multiplet with different $SO(5)$ representation by multiplying a semi-conserved current with a quadratic conserved scalar superfield $J_{SO(5)}^2$: $\text{Tr} \left(J_{SO(5)}^2\Phi_{s.c.}\right)$. In this case, the constraint for finite $\Delta$ and $R$ is shifted as $m \to m + 2$. Similarly $\text{Tr} \left(K_{SO(5)}^2\Phi_{s.c.}\right)$ with the shift of $n \to n + 2$ and $\text{Tr} \left(J_{SO(5)}K_{SO(5)}\Phi_{s.c.}\right)$ with the shift of $m \to m + 1, n \to n + 1$. If $m = 0 = n$, then the conformal dimension reduces to the shortening condition related to the protected operator corresponding to massless and short graviton multiplets depending on $R = 0$ or not.

3) Long vector multiplet I:

$$\Delta = \frac{5}{2} + \frac{1}{4}\sqrt{E + 36}.$$ 

The combination of $\Delta - R$ with Penrose limit $R \to \infty$ in the gauge theory side becomes

$$\Delta - R = 4 + m + n + O\left(\frac{1}{R}\right).$$
By taking quadratic stress-energy tensor and multiplying it into a semi-conserved current, one obtains \( \text{Tr} \left( T_{\alpha \beta}^2 \Phi_{\text{s.c.}} \right) \). Or one can construct \( \text{Tr} \left( T_{\alpha \beta} J_{SO(5)}^2 \Phi_{\text{s.c.}} \right) \) corresponding to this vector multiplet and the constraint coming from the requirement of rationality of conformal dimension is also changed for finite \( \Delta \) and \( R \). Also one can do the similar things for \( \text{Tr} \left( T_{\alpha \beta} J_{SO(5)} K_{SO(5)} \Phi_{\text{s.c.}} \right) \) and \( \text{Tr} \left( T_{\alpha \beta} J_{SO(5)} K_{SO(5)} \Phi_{\text{s.c.}} \right) \). One can see there is no shortening condition for this vector multiplet.

4) Long gravitino multiplet I:

\[
\Delta = -\frac{1}{2} + \frac{1}{4} \sqrt{E + 24}.
\]

There exist special integer values of \( m,n \) such that for \( [M + N, M] = [m + n + \frac{3R}{2}, m] \), one can see the Diophantine like condition \( [12] \),

\[
m^2 - n^2 + 2m - 2mn = 0
\]

make \( \sqrt{24 + E} \) be equal to \( 4R + 2(2m + 2n) \). It turns out this is the case without any further restrictions on \( m \) and \( n \). Therefore we have \( \Delta = -\frac{1}{2} + R + m + n \). This is true if we are describing states with finite \( \Delta \) and \( R \). Since we are studying the scaling limit \( \Delta, R \to \infty \), we have to modify the above analysis. However, as we take the limit of \( R \to \infty \), this constraint (11) is relaxed. The combination of \( \Delta - R \) is given by

\[
\Delta - R = -\frac{1}{2} + m + n + O \left( \frac{1}{R} \right)
\]

where the right hand side is definitely rational. So the constraint (11) is not relevant in the subsector of the Hilbert space we are interested in. Although they are not chiral primaries, their conformal dimensions are protected. The ones we are interested in take the following form,

\[
\text{Tr} \Phi_{\text{s.c.}} \equiv \text{Tr} \left[ L_{\alpha} \left( J_{SO(5)} \right)^{m-1} \left( K_{SO(5)} \right)^{n-1} \left( S^a \Gamma^a S \right)^{3(R-1)/2} \right]
\]

by realizing that the conformal dimension can be written as \( \Delta = 5/2 + (m - 1) + (n - 1) + (R - 1) \).

If \( m = 1 = n \), we have short gravitino multiplet I we have seen before.

5) Long gravitino multiplet II:

\[
\Delta = \frac{3}{2} + \frac{1}{4} \sqrt{E + 24}.
\]

The combination of \( \Delta - R \) is given by

\[
\Delta - R = \frac{3}{2} + m + n + O \left( \frac{1}{R} \right).
\]

One constructs gauge theory operator corresponding to this multiplet as follows:

\[
\text{Tr} \Psi_{\text{s.c.}} \equiv \text{Tr} \left[ X_{\alpha} \left( J_{SO(5)} \right)^{m} \left( K_{SO(5)} \right)^{n} \left( S^a \Gamma^a S \right)^{3(R-1)/2} \right]
\]

by writing the conformal dimension \( \Delta \) as \( 5/2 + m + n + (R - 1) \). If \( m = 0 = n \), it is easy to see that there exists a shortening condition: it leads to a short gravitino multiplet II.
4 Conclusion

We described an explicit example of an $\mathcal{N} = 2$ superconformal field theory that has a subsector of the Hilbert space with enhanced $\mathcal{N} = 8$ superconformal symmetry, in the large $N$ limit from the study of $AdS_4 \times V_{5,2}$. The pp-wave geometry in the scaling limit produced to the maximally $\mathcal{N} = 8$ supersymmetric pp-wave solution of $AdS_4 \times S^7$. The result of this paper shares common characteristic feature of previous case of $AdS_4 \times Q^{1,1,1}$ [9] and $AdS_4 \times M^{1,1,1}$ [10]. This subsector of gauge theory is achieved by Penrose limit which constrains strictly the states of the gauge theory to those whose conformal dimension and $R$ charge diverge in the large $N$ limit but possesses finite value $\Delta - R$. We predicted for the spectrum of $\Delta - R$ of the $\mathcal{N} = 2$ superconformal field theory and proposed how the excited states in the supergravity correspond to gauge theory operators. In particular, both the chiral multiplets (8) and semi-conserved multiplets (10) of $\mathcal{N} = 2$ supersymmetry should combine into $\mathcal{N} = 8$ chiral multiplets. It would be interesting to find out a Penrose limit of other types [18] of M-theory compactification along the line of [19]. These examples have different supersymmetries and the structures of four-form field strengths are more complicated than what we have discussed so far.

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