Chiral Dynamics of Baryons from String Theory

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We study baryons in an AdS/CFT model of QCD by Sakai and Sugimoto, realized as small instantons with fundamental string hairs. We introduce an effective field theory of the baryons in the five-dimensional setting, and show that the instanton interpretation implies a particular magnetic coupling. Dimensional reduction to four dimensions reproduces the usual chiral effective action, and in particular we estimate the axial coupling $g_A$ between baryons and pions and the magnetic dipole moments, both of which are proportional to $N_c$. We extrapolate to finite $N_c$ and discuss subleading corrections.

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Introduction:— Understanding baryons from the microscopic theory is a long standing problem, since it amounts to solving the low energy QCD, which is strongly coupled and highly nonlinear. Though limited successes were made such as in lattice QCD, it is far from complete. Recent discovery of string/gauge duality \cite{1}, however, enables us to address the problem, based on holographic models. One interesting and realistic model among them is the one by Sakai and Sugimoto \cite{2} (SS model for short). The model astutely implements chiral symmetry spontaneously broken, and describes the low-energy dynamics in a manner consistent with the hidden local symmetry (HLS) theory of the form developed some years ago with the rho meson \cite{3}.

In this letter, using the fully five-dimensional picture of baryons which naturally incorporates the infinite tower of vectors in construction of the baryon, we will show that chiral dynamics arises naturally in the large 't Hooft coupling limit $\lambda = g_N^2 / M N_c \rightarrow \infty$. It has been recognized since some time that the lowest-lying vector mesons as hidden local fields could play an important role in the soliton structure \cite{4} and dynamics \cite{5} of baryons, which was also recently reconsidered in the context of the SS model \cite{6}. Here we find that not just the lowest members but the \textit{whole tower} of the vector fields participate intricately in the dynamics of baryons, in such a manner that this effectively simplifies and also relates some four-dimensional interactions.

We start with a brief review of the SS model. In the model, the stack of D4 branes which carries the $SU(N_c)$ pure Yang-Mills theory is replaced by the dual geometry, when $\lambda \gg 1$, with the metric

$$G = \left( \frac{U}{R} \right)^{3/2} \left( ds^2 + f d\tau^2 + \frac{R^3}{U^3} \frac{dU^2}{f} + \frac{R^3}{U} d\Omega_4^2 \right)$$

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where $f(U) = 1 - U^3_{KK}/U^3$. The coordinate $\tau$ is periodic with the period $\delta \tau = 2 \pi / M_{KK}$ which defines the Kaluza-Klein (KK) mode scale $M_{KK}$ and sets the scale for massive vector mesons. Note that the parameters of dual QCD are mapped to the dimensionful parameters here as $R^3 = M^2_{KK}/2M_{KK}$ and $U_{KK} = 2 \lambda / M_{KK}^2$ with the string length scale $l_s$. The string coupling is related to $SU(N_c)$ Yang-Mills coupling as $2 \pi g_s = g^2_{YM}/M_{KK} l_s$.

The D8 branes, which share coordinates $x^{0,1,2,3}$ with D4 branes, are treated as probes and carry $U(N_f)$ Yang-Mills multiplets from D8-D8 open strings. The induced metric on D8 is

$$g_{8+1} = g_{4+1} + R^3/2U^{1/2} d\Omega_4^2$$

where the five-dimensional part is conformally equivalent to $R^{3+1} \times I$,

$$g_{4+1} = H(w) \left( dw^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right),$$

with $w = \int dU R^{3/2} / \sqrt{U^3 - U_{KK}^3}$ and $H = (U/R)^{3/2}$. This fifth coordinate is of finite range $[-w_{\max}, w_{\max}]$ with $w_{\max} \approx 3.64 / M_{KK}$. Near origin $w = 0$, we have the approximate relation, $U^3 \approx U_{KK}^3 (1 + M^2_{KK} w^2)$.

The main point of this model is that the D8 comes with two asymptotic regions (corresponding to UV) at $w \rightarrow \pm w_{\max}$, where $U(N_f)$ gauge symmetry of D8 can be each interpreted as $U(N_f)|_{L=0}$ chiral symmetry, respectively, of fermions from D4-D8 strings.

As D4’s are replaced by the geometry, these D4-D8 strings are connected and become D8-D8 strings. Thus, mode-expanding the $U(N_f)$ gauge fields on D8 along the fifth direction produces $SU(N_c)$ gauge singlets which are the pions and the infinite tower of massive vector mesons. Writing the chiral field as $\xi(x) = e^{i \pi(x)/F}$, we have in the $A_w = 0$ gauge

$$A(x; w) = i \alpha(x) \psi_0(w) + i \beta(x) + \sum_n \alpha^{(n)}(x) \psi^{(n)}(w)$$

with $\alpha(x) \equiv \{ \xi^{-1}, d\xi \}$ and $\beta(x) \equiv 1/2 \{ \xi^{-1}, d\xi \}$. The zero mode $\psi_0$ approaches $\pm 1/2$ at the two boundaries. Keeping the pions only results in the Skyrme Lagrangian for...
$U = \xi^2$, while including massive vectors as well produces a theory with hidden local symmetries [3].

**Baryons as small and hairy instantons:**— A baryon in this model corresponds to a D4 brane wrapping the compact $S^4$ [3], which is dissolved into D8 as an $U(N_F)$ instanton. Relation between this and the usual Skyrmeon picture was clarified in [2, 8].

In the present curved geometry, with $N_c$ flux in the background geometry, the compact D4 admits fundamental string tadpoles which have to be cancelled by $N_c$ fundamental strings attached to it. The other endpoints of the strings can only go to D8 and thus will behave as electric charge with respect to the trace part of $U(N_F)$. Thus, a D4 brane on $S^4$ tends to be pulled into D8 and becomes a finite-size instanton. On the other hand, the background geometry induces a position-dependent electric coupling for $U(N_F)$ gauge fields, which favors point-like instanton at $w = 0$. The competition between the two will determine the size of the instanton.

The 4 + 1 dimensional effective action of $U(N_F)$ Yang-Mills fields in the conformal coordinate system is

$$\frac{1}{4} \int d^4x dw \frac{8\pi^2 R^3 U(w)}{3(2\pi l_s)^2(2\pi g_s)} \text{tr} F_{mn} F^{mn}$$

(5)

from which we find the effective electric coupling

$$\frac{1}{e^2(w)} = \frac{8\pi^2 R^3 U_{KK} U(w)}{3(2\pi l_s)^2(2\pi g_s)} = \frac{\lambda N_c M_{KK} U(w)}{108\pi^3} U_{KK}.$$  

(6)

A point-like instanton that is localized at $w = 0$ would have the mass $m_B^{(0)} \approx 4\pi^2/e^2(0) = (\lambda N_c/27\pi) M_{KK}$ which is also the mass of D4 wrapping $S^4$ at $w = 0$.

If the instanton gets bigger, on the other hand, the configuration costs more energy, since $1/e^2(w)$ is an increasing function of $|w|$. For a very small instanton of size $\rho$, this additive correction to the instanton mass is found to be $\rho M_{KK}^2 \rho^2/6$, using the spread of the instanton density $D(x^i, w) \sim \rho^3/\lambda (x^2 + w^2 + \rho^2)^4$ [3]. The competing effects come from the energy cost of the electric charges, which arises due to a Chern-Simons term. One finds the electric charge density is proportional to $D(x^i, w)$, and the five dimensional Coulomb energy is readily estimated as [3]

$$\rho_M \sim \frac{\rho(0)^2 N_c^2}{20\pi^2 \rho^2},$$

(7)

provided that $\rho M_{KK} \ll 1$.

The size of the instanton localized at $w = 0$ is then determined by minimizing the sum. This gives the size of the baryon

$$\rho_{\text{baryon}} \sim \frac{2 \cdot 3 \cdot \pi^2/5^{1/4}}{M_{KK} \sqrt{\lambda}} \sim \frac{9.6}{M_{KK} \sqrt{\lambda}}.$$  

(8)

For a large 't Hooft coupling $\lambda$, the baryon size is then significantly smaller than the relevant scale of the dual QCD, and the mass correction to the baryon due to its 5-dimensional electric coupling is also suppressed by $1/\lambda$ compared to $m_B^{(0)}$. In making this estimate, we are ignoring the backreaction of the instanton to the geometry and the position-dependent coupling from the origin. This probably results in a slight underestimate of $\rho_{\text{baryon}}$, which is controlled by the inverse power of $\lambda$.

**Five-dimensional effective action of baryons:**— Our first task is to understand the effective action of the instanton soliton in five dimensions. The soliton by itself is a classical object. In order to treat it quantum mechanically, one first needs to quantize their collective coordinates and classify the resulting particles according to their spin content and the representations under other symmetries.

More subtleties come about because the instanton here is endowed with electric charges, which are remnant of fundamental strings attached to D4 on $S^4$. Having in mind an extrapolation to the real QCD, we restrict ourselves to the case of fermionic baryons with fundamental representation under $U(N_F = 2)$, denoted as $B$.

After a suitable rescaling of the $B$ field in the conformally flat coordinates $(x^i, w)$, we have

$$-i \int d^4x dw \left[ B_{\gamma}^{\mu} D_{\mu} B + B_{\gamma}^{5} \partial_{w} B + m_b(w) B B \right],$$

(9)

in the $A_w = 0$ gauge with $D_{\mu} = \partial_{\mu} - i A_{\mu}$ and $(-,+,+,+,+)$ convention. The gauge field $A$ here is that of $U(N_F)$ on D8, as before, which encodes the pions and the entire tower of massive vector mesons. $m(w)$ reflects the fact that the instanton costs more energy if it moves away from $w = 0$. This is the minimal set of terms consistent with the diffeomorphism and gauge invariance.

However, this cannot be the complete form of the baryon action at low energy. Since the baryon is represented by a small instanton soliton with a long-range tail of self-dual gauge field $F \sim \rho_{baryon}^2/\lambda$, there should be a coupling between a $B$ bilinear and the five-dimensional gauge field such that each $B$-particle generates the tail on $F$. There is a unique vertex that does the job,

$$\int d^4x dw \left[ g_5(w) \frac{\rho_{baryon}^2}{e^2} B_{\gamma}^{mn} F_{mn} B \right].$$

(10)

Writing the upper 2-component part of $B$ as $U e^{-iEt}$, and approximating $m_b$ by its central value, we find the on-shell condition is solved by

$$B = \left( \begin{array}{c} i \frac{U}{\pm iU} \\ e^{\mp im_b t} \end{array} \right),$$

(11)

where the two signs originate from the sign of $E/m_b$ and thus correspond to the baryon and the anti-baryon, respectively.

A static and localized spinor configuration sources the Yang-Mills field via

$$\bar{B}_{\gamma}^{mn} F_{mn} B = \pm \frac{1}{2} F_{jk}^{a} e^{kj} (\sigma_i \tau^a)_B + F_{5k}^{a}(\sigma_i \tau^a)_B$$

(12)
where \( \langle \sigma_1, \tau^a \rangle_B \equiv 2 \{ \mathcal{U}_1, \sigma_1, \tau^a, \mathcal{U}_1 \} \). In order to generate self-dual or anti-self-dual long-range fields, the spin index and the gauge index must be locked. For instance, one choice that gives a long-range self-dual field is \( \mathcal{U}_{1,a} = \frac{1}{2} e^{\alpha A} \) in which case \( \langle \sigma_1, \tau^a \rangle_B = -\delta^a_0 \) so that the source term (with the upper sign) is \( -F_{mn}^w \gamma^a_{l,mn}/2 \) with the anti-self-dual 't Hooft symbol \( \tilde{\eta} \) (\( m, n = 1, 2, 3, 5 \) and \( a = 1, 2, 3 \)).

Now assume that such a source appears in a localized form at the origin. The gauge field far away from the source obeys, after a gauge choice and ignoring \( w \)-dependence of the electric coupling,
\[
\nabla^2 A^a_{mn} = 2 g_5(0) \rho^a_{\text{baryon}} \gamma^a_{l,mn} \partial_n \delta^{(4)}(x) \tag{13}
\]
whose solution is
\[
A^a_{mn} = -\frac{g_5(0) \rho^a_{\text{baryon}}}{2m^2} \gamma^a_{l,mn} \partial_n \frac{1}{r^2 + m^2} \tag{14}
\]
The general shape of the long-range field is consistent with the identification of the baryon as the instanton. In order to fix \( g_5(0) \), we need to match the states in \( \mathcal{B} \) with quantized instanton. This means that the long range field of the instanton should be modified due to quantum fluctuation of the instanton along different global gauge directions. How to implement this quantum effect is explained in detail in [4]. Here we briefly sketch the reasoning.

Representing the global gauge rotation in (10) as
\[
S^I A^a_{MN}(\tau^a/2)S = A^a_{MN}(\tau^a/2) \left( \text{tr} \left[ S^I (\tau^a/2) S^J \right] \right) , \tag{15}
\]
the quantization replaces the quantity in the parenthesis by its expectation value. Following a reasoning mathematically identical to that used by Atkins et al. [10] for the Skyrme model, we obtain
\[
\langle \text{tr} \left[ S^I (\tau^a/2) S^J \right] \rangle_B = -\frac{1}{3} (\sigma_b \tau^a)_B = -\frac{1}{3} \delta^b_a . \tag{16}
\]
This allows us to arrive at
\[
g_5(0) = 2\pi^2 / 3 . \tag{17}
\]
We have ignored \( w \)-dependence of \( g_5(w) \) and \( e(w) \). This we believe is harmless for the very small-size baryon/instanton for the usual reason.

Four-dimensional effective action of baryons— After identifying the relevant five-dimensional action, we perform the KK expansion for the five-dimensional bulk fields along \( w \) to derive a four-dimensional Lagrangian. The four-dimensional nucleon arises as the lowest eigenmode of the five-dimensional bulk baryon along the \( w \) coordinate, which should be a mode localized near \( w = 0 \). We mode-expand \( B_{LR}(x^\mu, w) = B_{LR}(x^\mu) f_L(R)(w) \), where \( \gamma^5 B_{LR} = \pm B_{LR} \) are four-dimensional chiral components, with the profile functions \( f_{L,R}(w) \) satisfying
\[
\begin{align*}
\partial_w f_L(w) + m_b(w) f_L(w) &= m_B f_R(w), \\
-\partial_w f_R(w) + m_u(w) f_R(w) &= m_B f_L(w). \tag{18}
\end{align*}
\]
The four-dimensional Dirac field for the baryon is then reconstructed as \( B = (B_L, B_R)^T \). See Ref.[11] for a similar model.

We will use the mode expansion in Eq. (11) to obtain the baryon couplings to mesons. The eigenmode analysis done in [2] shows that \( \psi_{(2k+1)}(w) \) is even and \( \psi_{(2k)}(w) \) is odd under \( w \rightarrow -w \), corresponding to vector and axial-vector mesons respectively. Inserting this expansion into the action, we obtain an effective Lagrangian density for four-dimensional baryons,
\[
\mathcal{L}_4 = -i \bar{B} \gamma^\mu \beta_\mu B + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}} + \cdots , \tag{19}
\]
where the coupling to vector mesons \( a^{(2k+1)}_\mu \) is given by
\[
\mathcal{L}_{\text{vector}} = -i \bar{B} \gamma^\mu \beta_\mu B - \sum_{k=0}^\infty g_A^{(k)} B \gamma^\mu \gamma^5 a^{(2k+1)}_\mu B \tag{20}
\]
and the baryon couplings to axial mesons, including pions, as
\[
\mathcal{L}_{\text{axial}} = -\frac{i g_A}{2} \bar{B} \gamma^\mu \gamma^5 \beta_\mu B - \sum_{k=1}^\infty g_A^{(k)} B \gamma^\mu \gamma^5 a^{(2k)}_\mu B \tag{21}
\]
where various couplings constants \( g_A^{(k)} \) as well as the pion-nucleon axial coupling \( g_A \) are calculated by the overlap of wavefunctions.

For the couplings we have two contributions, one from the minimal coupling, denoted \( g_{A,n,n} \), which is calculable for a given \( \lambda N_c \) [3] and the other from the magnetic term [10]. However, in the large \( N_c \) limit, the contribution from the magnetic term is dominant for all axial couplings. For the leading axial coupling with the pion, in particular, the main contribution to \( g_A \) becomes
\[
g_A \simeq 0.18 N_c \int dw \left[ \frac{2U(w)g_5(w)}{g_5(0)U_{KK}} \right] |f_L|^2 \bar{\partial}_w \psi_0 \frac{1}{M_{KK}} , \tag{22}
\]
where we used the previous estimates to find
\[
g_5(0) \rho_{\text{baryon}} / e^2(0) \simeq 0.18 N_c / M_{KK} . \tag{23}
\]
For sufficiently localized \( f_{L,R} \), which is guaranteed by large \( \lambda N_c \), the leading axial coupling \( g_A \) from the magnetic term is approximately
\[
g_A \simeq 0.18 N_c \times (4/\pi) \simeq 0.7 (N_c/3) . \tag{24}
\]
We stress that this is independent of the ‘t Hooft coupling \( \lambda \) and the KK mass \( M_{KK} \) and consequently of the pion decay constant.

Let us now consider electromagnetic responses of the baryons. The simplest way to obtain the coupling is to include the electromagnetic field as a nonnormalizable mode of the gauge fields on D8 branes,
\[
A_\mu(x; w) = A_\mu(x) + i \alpha_\mu(x) \psi_0(w) + \cdots \tag{25}
\]
The five-dimensional gauge interaction of such a nonnormalizable mode gives the vertex \( \int d^5 x A_\mu f^\mu \), into which we embed the electromagnetic interaction.
Here we are interested in isolating the magnetic moment of the nucleon from the magnetic vertex we found above. For $N_F = 2$, for example, we find the isovector magnetic moment of the nucleon, $\Delta \mu \equiv \mu_p - \mu_n$, to be

$$\left( \frac{4g_s(0)\rho_{\text{baryon}}^2}{e(0)^2} \right) \int d^4x \ [U^\dagger B \cdot \sigma U]$$

where $B$ is the $SU(N_F)$ part of the magnetic field strength, embedded into $F \equiv dA + A^2$. Given the normalization, $\text{tr}U^\dagger U = 1/2$, one can identify $\text{tr}U^\dagger \sigma U$ as the spin operator $S$ of the baryon. Recall that the minimal coupling of the Dirac field to a vector field produces a universal magnetic moment $e_{EM}S/m_B$. It is easy to show that this latter contribution, smaller by relative factor of $1/N_c^2$, adds to the above leading contribution.

For $N_F = 2$ the $SU(2)$ part of the electromagnetic charge is given as $\text{diag}(1/2, -1/2)$. The “anomalous magnetic moment” of the nucleon in which strong-interaction dynamics is encoded is given by

$$\frac{\Delta \mu_{\text{an}}}{e_{EM}} = \left[ \frac{2g_s(0)\rho_{\text{baryon}}^2}{e(0)^2} \right] \approx \frac{0.36N_c}{M_{KK}}.$$  

**Discussions:** So far, all the computations were carried out in the large $\lambda$ and large $N_c$ limit, so direct comparisons with nature would be difficult to justify rigorously. Nevertheless there is a line of reasonings which suggests a particular form of the $O(1)$ correction for $g_A$ and $\mu_{\text{an}}$, in the $1/N_c$ expansion. We would like to close this letter with a brief description and the resulting comparison with experimental values.

It is based on the following set of observations: (1) The instanton baryon in five dimensions we obtain is related to a skyrmion in four dimensions [2], and shares the same symmetry structure; (2) in the large $N_c$ expansion, the skyrmion description is equivalent to the constituent quark model [12]; and (3) a simple group theoretic structure in constituent-quark and skyrmion models of the spin-flavor operators figuring in both $g_A$ and $\Delta \mu_{\text{an}}$ suggests that $N_c$ could be replaced by $N_c + 2$ [13]. Although we are ill-equipped to verify the above shift directly, let us assume in comparing with nature that such a shift of $N_c$ takes into account most, if not all, of the leading corrections.

To proceed we need to fix the parameters of the model. In doing this, we will adhere to the strategy adopted in baryon chiral perturbation theory, namely, the parameters are determined in the meson sector. We take the parameters $f_\pi$, $M_{KK}$ etc. as given in [2]. In particular we will take $M_{KK} \approx 0.94$ GeV and $\lambda N_c \approx 26$. Including the subleading correction $g_{\text{an}} \approx 0.15$ for the given $\lambda N_c$, we obtain $g_A \approx 1.32$ and $\Delta \mu_{\text{an}}/\mu_B \approx 3.6$, where $\mu_B$ is the Bohr-magneton. These should be compared with the experimental values $g_A^{\text{exp}} = 1.26$ and $\Delta \mu_{\text{an}}^{\text{exp}}/\mu_B = 3.7$.

Clearly much more study is needed to correct predictions in a more rigorous and systematic manner. What is intriguing is that even at this “crude” leading order, the chiral lagrangian, derived from the string/gauge duality, is found to describe baryons remarkably well, which indicates it certainly captures the correct physics of strong interactions.

More details as well as implications of the infinite tower of the vector mesons on the vector dominance structure of baryon electromagnetic form factors will be reported elsewhere [9].

**Note Added:** After this work was completed and has appeared, a related paper has appeared [14], with a partial overlap on the instanton size estimate in early part of our manuscript. This work was supported in part (D.K.H.) by KOSEF Basic Research Program with the grant No. R01-2006-000-10912-0, by the KRF Grants, (M.R.) KRF-2006-209-C00002, (H.U.Y.) KRF-2005-070-C00030, and (P.Y.) by the KOSEF through the Quantum Spacetime (CQUeST) Center of Sogang University with the grant number R11-2005-021.

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