Hadronic weak interaction in the two-nucleon system with effective field theories

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Abstract

Weak interactions in two-nucleon system at low energies are explored in the framework of effective field theory. We review our recent calculations of parity-violating observables in radiative neutron capture on a proton at threshold where both pionful and pionless theories are employed.

Key words: Parity violation, Effective field theory

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1. Introduction

Exchanges of $\pi$, $\rho$ and $\omega$ mesons have long been a standard way to describe the weak interactions of the baryon at the hadronic level. One of the important issues with the one-meson exchange (OME) weak interaction is the value of the weak coupling constants. A benchmark calculation for the weak coupling constants was done with a quark model \footnote{1}. Calculation of the weak coupling constants since then employed various models and theories, and the results as a whole turned out to be consistent to the "best values" in Ref. \footnote{1}. However, experimental determinations are yet quite controversial, so a revealing true values of the weak coupling constants still remains as an open question.

In the present situation, one can cast a question: Is the OME picture sufficient to describe the hadronic weak dynamics? It is quite recent that people started to consider the effective field theory (EFT), which already achieved good successes in the description of strong interactions and electromagnetic (EM) transitions in the low energy two-nucleon systems, in search for an answer to the question in the hadronic weak interaction problem \footnote{2, 4, 5, 7, 8}. With the counting rules of EFT, we can expand the two-nucleon weak potential systematically and perturbatively. Interestingly, EFT gives terms that are absent in the OME potential, but are more important than the heavy-meson terms in it.

In this work we consider parity-violating (PV) observables in radiative capture of thermal neutron on a proton. In the polarized neutron capture, it is known that the PV asymmetry, $A_\gamma$, is dominated by the pion-exchange term in the weak OME potential. We thus employ an EFT that explicitly includes pions while heavier degrees of freedom are integrated out, and the short range...
dynamics are represented by two nucleon contact terms. In the unpolarized neutron capture, on the other hand, PV polarization, $P_{\gamma}$, is dependent on the $\rho$- and $\omega$- exchange terms in the OME weak potential. Since heavy mesons are integrated out in the EFT, interactions relevant to $P_{\gamma}$ are only the two-nucleon contact terms. Therefore we are naturally lead to consider $P_{\gamma}$ with a theory where all the interactions are represented only with contact terms, so called the pionless theory.

In the next sections theories are briefly reviewed. In the following section, we present the results and discussion, and in the last section we summarize the work.

2. Theory

2.1. Pionful theory

Order of a contribution (equivalently a Feynman diagram) is counted in powers of $Q/\Lambda_{\chi}$, where $Q$ is the momentum scale of the incoming and outgoing particles, and $\Lambda_{\chi}$ is the chiral symmetry breaking scale. We count the order of $Q/\Lambda_{\chi}$ of a diagram with the counting rules of the heavy baryon chiral perturbation theory [9]. Weak pion-nucleon Lagrangian is given by

$$L_{p\nu} = -\frac{1}{\sqrt{2}} h_1^\pi \bar{N} (\tau \times \pi) \tau^3 N,$$

where $h_1^\pi$ is a weak pion-nucleon coupling constant. At the leading order (LO) ($\propto Q^{-1}$), only the one-pion-exchange (OPE) diagram contributes to the weak potential. There is no diagram at the order of $Q^0$, and the next-to-next leading order (NNLO) consists of the two-pion-exchange (TPE) contributions and a two-nucleon contact term (CT) which has an unknown low energy constant (LEC). Diagrams at LO and NNLO are shown in Fig. 1. Weak potentials in Fig. 1 in coordinate space can be written in the form

$$V_i(r) = i(\tau_1 \times \tau_2) \cdot (\sigma_1 + \sigma_2) \cdot [\mathbf{p}, V_i(r)].$$

where $\mathbf{p}$ is the momentum operator conjugate to the relative coordinate $r = r_1 - r_2$. In the Fourier transformation from momentum space to coordinate one, we encounter ultraviolet divergence for the TPE and contact terms. In order to regularize the divergence, we introduce the monopole cutoff function $\Lambda^2/(\Lambda^2 + q^2)$ for TPE and contact terms. Thus the regularized OPE, TPE and CT terms $v_i^\Lambda(r)$ in Eq. (2) read

$$v_{1\pi}^\Lambda(r) = \frac{g_A h_1^\pi}{2 \sqrt{2} f_\pi 4\pi r} \left( e^{-m_\pi r} - e^{-\Lambda r} \right),$$

$$v_{2\pi}^\Lambda(r) = \sqrt{2\pi} \frac{h_1^\pi}{\Lambda_{\chi}} \left[ g_A L^\Lambda(r) - g_A^3 \left[ 3L^\Lambda(r) - H^\Lambda(r) \right] \right],$$

$$v_{CT}^\Lambda(r) = -C_\xi^g A^g \frac{e^{-\Lambda r}}{4\pi r}.$$
Analytic expressions of the functions $L^\Lambda(r)$ and $H^\Lambda(r)$ can be found in Ref. [7], and $C_{\nu}^R$ is the renormalized LEC in the contact term. The LEC $C_{\nu}^R$ can be determined model independently with experimental data, but error bars of the data for the hadronic weak interaction are too large to constrain the LEC with reasonable uncertainty. In the theoretical calculation, $C_{\nu}^R$ is dependent on the regularization schemes, and several choices are possible. In this work we consider the minimal subtraction scheme, in which the renormalized LEC takes the form

$$C_{\nu}^R = C_{\nu}(\mu) - h_1^R \frac{\pi R^A}{\sqrt{2} \Lambda^3} \left[ \frac{1}{\epsilon} - \frac{1}{\Gamma} + \ln(4\pi) \right],$$

where $C_{\nu}(\mu)$ is the bare LEC, a function of the renormalization point $\mu$ [7], and $\Gamma = 0.5772.$ When the parity-odd potential is added to the parity-conserving (PC) potential, it causes parity mixture in the wave function, and this parity admixture can contribute to EM transitions that are forbidden in the pure parity states.

2.2. Pionless theory with dibaryon fields

With dibaryon fields in the pionless EFT, it has been shown in recent works [10, 11, 12] that low energy observables can be calculated with good convergence and accuracy. Details for the strong and EM interactions in the pionless dibaryon formalism can be found in those papers, and in this work we discuss the PV terms in the theory.

At sufficiently low energies, pion mass can be treated as a large scale. In this case, just as heavy mesons are integrated out in the pionful theory, the contribution of pion exchange can be subsumed in the two-nucleon contact terms. Then since we don’t have mesons dynamically in the theory, all the interactions are represented by the contact terms. Effective weak Lagrangian for the low energy in the pionless EFT have been obtained in Ref. [5]. Pionless Lagrangian with dibaryon fields can be obtained by replacing a two-nucleon field with a dibaryon one of the same quantum numbers. We obtain Lagrangian for the PV dibaryon-nucleon-nucleon ($dNN$) interaction as

$$L_{PV}^0 = \frac{h_{0s}^{PV}}{2 \sqrt{2} P_{4d} m_N^{5/2}} N^T \sigma_2 \tau_2 \tau_3 \bar{i} \left( \nabla - \nabla \right)_{\mu} N + h.c. \quad (7)$$

$$+ \frac{h_{0t}^{PV}}{2 \sqrt{2} P_{4d} m_N^{5/2}} i_0^T \sigma_3 \tau_2 \bar{i} \left( \nabla - \nabla \right)_{\mu} N + h.c., \quad (8)$$

where $h_{0s}^{PV}$ and $h_{0t}^{PV}$ denote weak $dNN$ coupling constants for parity mixing for the $^{1}S_0$ and $^{3}S_1$ dibaryon states, respectively. Spin-isospin operator $\sigma_2 \sigma_3 \tau_2 \tau_3$ in Eq. (7) projects two-nucleon system to $^3P_0$ state. PV vertex given by Eq. (7) therefore mixes $^3P_0$ in the $^1S_0$ state. Similarly, $\sigma_3 \tau_2$ in Eq. (8) is projection operator to $^1P_1$ state, and thus the Lagrangian mixes $^1P_1$ state in the $^3S_1$ state. We note that the terms in Eqs. (7,8) are relevant to the PV polarization in unpolarized neutron capture. They are part of the general Lagrangian that accounts for all the possible parity-mixing modes [13].

3. Result and Discussion

3.1. Asymmetry in polarized neutron capture

Parity-violating asymmetry $A_y$ in $\vec{n}p \rightarrow d\gamma$ is defined as

$$\frac{d\sigma}{d\Omega} \propto 1 + A_y \cos \theta, \quad (9)$$

where $\theta$ is an angle of the photon momentum with respect to the neutron polarization. We employ a hybrid scheme in the calculation, where only weak potentials are obtained from EFT, while strong potential is described with a phenomenological model, Argonne v18 and the EM operators are assumed to satisfy Siegert’s theorem. Weak pion-nucleon coupling constant $h^\pi_1$ is of the order of $10^{-7}$ so it is sufficient to work at its linear order. PV asymmetry is then proportional to $h^\pi_1$, and thus we write the result as

$$A_\gamma = a_\gamma h^\pi_1. \tag{10}$$

The coefficient $a_\gamma$ depends on various inputs such as strong interaction, weak interaction, EM operators, cutoff value. The results in Table 1 show the contribution to $a_\gamma$ from OPE and TPE terms, and their dependence on the cutoff $\Lambda$ and renormalization scale $\mu$. Dependence on the cutoff $\Lambda$ is non-negligible for both OPE and total value (OPE and TPE), and it is more drastic for the NNLO (total subtracted by OPE). Difference is relatively large between $\Lambda = 500$ MeV and $1000$ MeV, i.e. about 11% increase in magnitude for LO and about 54% for NNLO. However, the values become quite stable when $\Lambda$ is more than 1000 MeV, and the change reduces to about 1% for LO and 18% for NNLO. Dependence on the renormalization scale $\mu$ is already sizable, and the largest gap is about 15% of the value of the smallest magnitude, $-0.0966$. However, the cutoff dependence almost disappears when the renormalization scale is two times the pion mass, $\mu = 2m_\pi$. These values can be regarded as fixed-point results up to NNLO.

### 3.2. Polarization in unpolarized neutron capture

Parity-violating polarization $P_\gamma$ in $np \rightarrow d\gamma$ is defined as

$$P_\gamma = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \tag{11}$$

where $\sigma_+$ and $\sigma_-$ are the total cross section for the photons with right and left helicity, respectively. While we calculate $A_\gamma$ in $\bar{n}p \rightarrow d\gamma$ in a hybrid way, $P_\gamma$ is calculated self-consistently in the pionless dibaryon formalism. Both PC and PV amplitudes are calculated with the same counting rules, and the PC and PV diagrams are truncated at the same order. For the PC amplitude for $np \rightarrow d\gamma$, we employ the result in Ref. [10],

$$Y = \frac{\sqrt{2\pi}}{m_N^2} \sqrt{\frac{\gamma}{1 - \eta \mu_d}} \left[ (1 + \kappa_\gamma)(1 - \gamma a_s) - \gamma^2 a_s A_1 \right]. \tag{12}$$

Feynman diagrams that contribute to the PV amplitude at LO are shown in Fig. 2. Taking the
zero momentum approximation, we obtain the PV amplitude at LO as
\[
Z = \frac{1}{m_N^{*2}} \sqrt{\frac{\gamma}{1 - \gamma d}} \left[ h_{\text{ren}}^{0r} \left( \frac{1}{2} - \frac{1}{3} \gamma a_s \right) + \frac{1}{6} h_{\text{ren}}^{0s} \gamma d a_s \right],
\]
and the PV polarization \(P_{\gamma}\) at LO reads
\[
P_{\gamma} = -2 \frac{\text{Re}(YZ^*)}{|Y|^2} = -\frac{2}{\pi m_N^{*2}} \left( \frac{1}{2} - \frac{1}{6} \gamma a_s \right) h_{\text{ren}}^{0r} + \frac{1}{6} \gamma a_s h_{\text{ren}}^{0s} \sqrt{\frac{\gamma}{1 - \gamma d}} (1 + \kappa)(1 - \gamma a_s) - \gamma^2 a_s L_3.
\]
Since the parameters other than \(h_{\text{ren}}^{0r}\) and \(h_{\text{ren}}^{0s}\) are well fixed, measurements of \(P_{\gamma}\) provides a relation that can constraint the values of \(h_{\text{ren}}^{0r}\) and \(h_{\text{ren}}^{0s}\).

### 4. Summary

We considered weak interaction at the hadron level with effective field theories. With the pionful theory, we derived weak two-nucleon potential up to NNLO. Parity-violating asymmetry \(A_{\gamma}\) calculated with the weak potential shows that the perturbative expansion converges reasonably, and the results are non negligibly dependent on the cutoff value and renormalization scale. However, the LO contribution is sufficiently dominant that we can determine the first significant digit of the weak pion-nucleon coupling constant \(h_{1\pi}^{1}\) if \(A_{\gamma}\) is measured precisely.

With the pionless theory, we calculated the parity-violating polarization \(P_{\gamma}\) in \(np \rightarrow d\gamma\). In the hybrid calculation adopted in the calculation of \(A_{\gamma}\), orders of the strong interaction and EM operators are not countable because they are quoted from the formulations where perturbative expansion is absent. Therefore, order mismatch could be unavoidable in the hybrid calculation. On the other hand, \(P_{\gamma}\) is calculated in a single theory, and the order of the diagrams is fixed to a single order. The result is dependent on the weak coupling constants for \(1S_0 - 3P_0\) mixing and \(3S_1 - 1P_1\) one. Measurement of \(P_{\gamma}\) is expected to provide a constraint to reduce the possible space for these weak coupling constants.

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