Hawking Temperature in Taub-NUT (A)dS spaces via the Generalized Uncertainty Principle

Seyen Kouwn
Department of Physics and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746 Korea
seyen@skku.edu

Chong Oh Lee
Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
cohlee@sciborg.uwaterloo.ca

Phillial Oh
Department of Physics and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746 Korea
ploh@skku.edu

Abstract

Using the extended forms of the Heisenberg uncertainty principle from string theory and the quantum gravity theory, we derived Hawking temperature of a Taub-Nut-(A)dS black hole. In spite of their distinctive natures such as asymptotically locally flat and breakdown of the area theorem of the horizon for the black holes, we show that the corrections to Hawking temperature by the generalized versions of the Heisenberg uncertainty principle increases like the Schwarzschild-(A)dS black hole and give the reason why the Taub-Nut-(A)dS metric may have AdS/CFT dual picture.
1 Introduction

The Taub-NUT metric was suggested in \[1, 2\] in search for metrics with high symmetry, and as a natural generalization of the Schwarzschild metric. It was also investigated in \[3\] through the various analysis of the metric. The generalized version of the Taub-NUT-AdS metric in arbitrary even dimension was found in \[4\]. The metric with NUT charge has several special properties. For example, the solutions of the metric are not asymptotically flat (AF) but asymptotically locally flat (ALF) \[5, 6\]. In contrast to a common black hole, the entropy is not just a quarter area at the horizon \[5, 6\]. However, in spite of these distinctive natures, such a metric well satisfies the AdS-CFT correspondence \[7, 8, 9\].

The basic ideas of the generalized uncertainty principle (GUP) started from considering how to construct a gravity theory at quantum limitation \[10, 11\]. The GUP appeared in several contexts. In the context of string theory, the GUP appeared in \[12, 13, 14\] and in particular it was explicitly showed that the the Heisenberg uncertainty principle (HUP) needs some modifications due to a minimum physical length \[15\]. It revived many investigations in quantum gravity along these directions \[16, 17, 18, 19, 20\]. It was found that the space noncommutativity is as essentially similar concept as the GUP \[21, 22\], the gravitational interaction of the photon and the particle can lead to the GUP \[23\], and the GUP increases the Hawking temperature and decreases the entropy \[24\]. The GUP is also useful scheme to study the thermodynamic quantities of black hole. One can resolve the quantum corrections to black hole entropy without much difficulty \[25, 20, 27, 28\], test the stability in black hole \[29, 30\], and investigate thermal properties in quantum gravitational effects \[31, 32, 33, 34\].

It was pointed out in \[32\] that only Schwarzschild-(A)dS-like black hole seems to allow a holographic dual theory differently from Schwarzschild-like black hole since the generalized forms of the HUP due to quantum gravity theory lead to the thermodynamic quantities of Schwarzschild-(A)dS (S-(A)dS). There is an equivalence between a gravitational theory in the bulk and a conformal field theory on the boundary in the Taub-Nut-(A)dS spaces \[7, 8, 20\]. Thus one intriguing question is whether the thermodynamics of the Taub-Nut-(A)dS metric is obtained from the extended forms of the HUP and the other is whether the extended versions of the HUP increases its Hawking temperature as S-(A)dS-like black hole \[31, 24, 34\].

In this paper, we address these questions. we show that the thermodynamic of Taub-Nut-(A)dS metric can be explained with the generalized Heisenberg uncertainty principle and the correction to the Hawking temperature increasing like the S-(A)dS black hole.
2 Hawking Temperature of Taub-NUT and Taub-NUT-AdS in 3+1 dimension

In this section, we consider a four-dimensional Taub-NUT with or without a negative cosmological constant and show the Hawking temperature of the Taub-NUT black hole can be obtained from the conventional HUP but that of the Taub-NUT-(A)dS black hole can not be obtained from HUP.

2.1 Four-dimensional Taub-NUT

The metric of the Taub-NUT black hole in 3+1 dimension is given by \[2, 3\]
\[
ds^2 = -f(r)\left[dt + 2n \cos \theta d\phi \right]^2 + \frac{dr^2}{f(r)} + (r^2 + n^2)d\Omega^2_2 ,
\]
where \(f(r)\) and \(d\Omega^2_2\) are
\[
f(r) = \frac{r^2 - n^2}{r^2 + n^2} - \frac{2mr}{r^2 + n^2}, \quad d\Omega^2_2 = d\theta^2 + \sin^2 \theta d\phi^2 .
\]
Here, \(n\) denotes a NUT charge and \(m\) is a geometric mass. The Hawking temperature \(T_H\) arises from imposed condition in order to ensure regularity in the Euclidean time \(t_E\) and radial coordinate \(r\)
\[
\frac{1}{T_H} = \frac{4\pi}{\partial_r f(r)} \bigg|_{r=r_+} ,
\]
where \(1/T_H\) is the period of Euclidean time \(t_E\) and \(r_+\) is the radius of the event horizon. Then the Hawking temperature \(T_H\) becomes
\[
T_H = \frac{\hbar c}{4\pi r_+} .
\]
The HUP and thermal properties of black hole also lead to the Hawking temperature \[2,4\] \[31\] \[32\]. By modeling a black hole as a black box with linear size \(r_+\), the uncertainty in the energy of the emitted quanta by the Hawking effect is
\[
\Delta E \simeq c\Delta p \simeq \frac{\hbar c}{\Delta x} \simeq \frac{\hbar c}{r_+} .
\]
\(\Delta E\) can be identified as the characteristic temperature of the Hawking radiation. Employing an appropriate proportional constant \(1/4\pi\), we can reproduce the Hawking temperature \(T_H\) \[2,4\]. This implies the HUP in even the Taub-NUT space well suffices to derive the black hole temperature.
2.2 Taub-NUT-AdS$_4$

The Taub-NUT-AdS black hole line element of 3+1 dimension is given by

$$ds^2 = -g(r)\left(dt + 2n \cos \theta d\phi\right)^2 + \frac{dr^2}{g(r)} + (r^2 + n^2)d\Omega_2^2,$$  \hspace{1cm} (2.6)

where $g(r)$ is

$$g(r) = \frac{l^2r^2 - n^2l^2 + r^4 + 6n^2r^2 - 3n^4}{l^2(r^2 + n^2)} - \frac{2mr}{r^2 + n^2},$$  \hspace{1cm} (2.7)

and a negative cosmological constant $\Lambda$ is $\Lambda = -3/l^2$. Using the same method as in the previous subsection, the Hawking temperature with the black hole horizon $r_+$ is given by

$$T_{H(AdS)} = \frac{1}{4\pi} \left[ \frac{1}{r_+} + \frac{3(n^2 + r_+^2)}{l^2 r_+} \right] \hbar c,$$  \hspace{1cm} (2.8)

which has the feature that two thermodynamical limits may be realized. In the limit of the Taub-NUT, $l \gg 1$, since the radius of the event horizon is much smaller in comparison to the radius of curvature of the AdS space, the temperature of the Taub-NUT-AdS$_4$ black hole becomes,

$$T_{H(AdS)} \bigg|_{r_+ \ll 1} \approx \frac{\gamma \hbar c}{4\pi r_+},$$  \hspace{1cm} (2.9)

where $\gamma = 1 + 3n^2/l^2$, which is a dimensionless constant. Comparing this with the previous result (2.5) and this one can find that a vacuum is shifted due to the effect of the NUT charge in AdS space as

$$\frac{\hbar c}{r_+} \rightarrow \frac{\gamma \hbar c}{r_+}.$$  \hspace{1cm} (2.10)

In the AdS limit, $r_+ \gg 1$, the radius of black hole event horizon dramatically grows up in comparison to the radius of curvature of the AdS space so that the temperature of the (cosmological) horizon is obtained by

$$T_{H(AdS)} \bigg|_{r_+ \gg 1} \approx \frac{3\hbar c}{4\pi l^2 r_+},$$  \hspace{1cm} (2.11)

which cannot be explicitly derived from the HUP. However, the Hawking temperature (2.8) of the Taub-NUT-AdS$_4$ solution can be obtained by substituting the usual Heisenberg relation with its extended versions, like S-(A)dS-like black hole [32]. The details are discussed in following sections.
3 Generalized Uncertainty Principle

In this section we first briefly mention the various extended versions of HUP before they are applied to the Hawking temperature of the Taub-NUT-AdS space.

The GUP of the HUP is usually given by

$$\Delta x \Delta p \geq \hbar \left[1 + \alpha^2 l_p^2 \frac{\Delta p^2}{\hbar^2}\right], \quad (3.1)$$

where $l_p = (hG/c^3)^{1/2}$ is the Plank length and $\alpha$ is a dimensionless real constant of order one. However since a vacuum is shifted, the GUP with the Taub-NUT charge in AdS space is modified as

$$\Delta x \Delta p \geq \hbar \left[\gamma + \alpha^2 l_p^2 \frac{\Delta p^2}{\hbar^2}\right], \quad (3.2)$$

The second term in (3.2) gives rise to an absolute minimum in the position uncertainty

$$\Delta x \geq 2\alpha l_p \sqrt{\gamma}, \quad (3.3)$$

and the uncertainty in the momentum is given by

$$\frac{\hbar \Delta x}{2\alpha^2 l_p^2} \left[1 - \sqrt{1 - \frac{4\gamma \alpha^2 l_p^2}{(\Delta x)^2}}\right] \leq \Delta p \leq \frac{\hbar \Delta x}{2\alpha^2 l_p^2} \left[1 + \sqrt{1 - \frac{4\gamma \alpha^2 l_p^2}{(\Delta x)^2}}\right]. \quad (3.4)$$

Note that the GUP (3.1) goes back to the usual HUP as $2\alpha l_p \to 0$, i.e. when $\Delta x \gg l_p$ or $\alpha \to 0$. When one consider the dual form of the GUP, one get another generalized version of the HUP [21, 32], so-called the extended uncertainty principle (EUP)

$$\Delta x \Delta p \geq \hbar \left[\gamma + \beta^2 \frac{\Delta x^2}{l_p^2}\right], \quad (3.5)$$

where $\beta$ is a constant parameter. Then there exists an absolute minimum in the momentum uncertainty

$$\Delta p \geq \frac{2\hbar \beta \sqrt{\gamma}}{l_p}, \quad (3.6)$$

and the uncertainty in the position is

$$\frac{l_p^2 \Delta p}{2\hbar \beta^2} \left[1 - \sqrt{1 - \frac{4\gamma \beta^2 \hbar^2}{l_p^2 (\Delta p)^2}}\right] \leq \Delta x \leq \frac{l_p^2 \Delta p}{2\hbar \beta^2} \left[1 + \sqrt{1 - \frac{4\gamma \beta^2 \hbar^2}{l_p^2 (\Delta p)^2}}\right]. \quad (3.7)$$
By combining (3.1) and (3.5), the symmetric generalized uncertainty principle (SGUP) is obtained as

$$\Delta x \Delta p \geq \hbar \left[ \gamma + \alpha^2 l_p^2 \Delta p^2 + \beta^2 \Delta x^2 \right].$$  

(3.8)

Inverting (3.8), the inequalities are given by

$$\frac{l_p^2 \Delta p}{2 \hbar \beta^2} \left[ 1 - \sqrt{1 - \frac{4 \beta^2 \hbar^2}{l_p^2 (\Delta p)^2} \left[ \gamma + \alpha^2 l_p^2 \Delta p^2 \right]} \right] \leq \Delta x \leq \frac{l_p^2 \Delta p}{2 \hbar \beta^2} \left[ 1 + \sqrt{1 - \frac{4 \beta^2 \hbar^2}{l_p^2 (\Delta p)^2} \left[ \gamma + \alpha^2 l_p^2 \Delta p^2 \right]} \right],$$

(3.9)

and

$$\frac{\hbar \Delta x}{2 \alpha^2 l_p^2} \left[ 1 - \sqrt{1 - \frac{4 \alpha^2 l_p^2}{(\Delta x)^2} \left[ \gamma + \beta^2 (\Delta x)^2 \right]} \right] \leq \Delta p \leq \frac{\hbar \Delta x}{2 \alpha^2 l_p^2} \left[ 1 + \sqrt{1 - \frac{4 \alpha^2 l_p^2}{(\Delta x)^2} \left[ \gamma + \beta^2 (\Delta x)^2 \right]} \right],$$

(3.10)

which lead to the absolute minimum values with respect to $\Delta x$ and $\Delta p$, respectively

$$(\Delta x)^2 \geq \frac{4 \gamma \alpha^2 l_p^2}{1 - 4 \alpha^2 \beta^2}, \quad (\Delta p)^2 \geq \frac{4 \hbar^2 \gamma \beta^2 / l_p^2}{1 - 4 \alpha^2 \beta^2}. \quad (3.11)$$

The inequalities (3.9) and (3.10) are required to be real so that an additional condition is given by

$$\beta^2 < \frac{1}{4 \alpha^2}. \quad (3.12)$$

4 Four-dimensional Taub-NUT and Taub-NUT-AdS\textsubscript{4} Thermodynamics with GUP

In this section, we will show that the Hawking temperature of the four-dimensional Taub-NUT and the Taub-NUT-AdS\textsubscript{4} black hole can be calculated in the extended forms of HUP.

4.1 Hawking Temperature with GUP in four-dimensional Taub-NUT space

Here we apply GUP (3.1) to Taub-NUT space and check how it modify the Hawking Temperature from HUP.
Using (3.4) with $\gamma = 1$, we can infer that Hawking Temperature is given by

\[ T_{\text{GUP}} = \left( \frac{1}{4 \pi} \right) \frac{r_+}{2 \alpha^2 l_p^2} \left[ 1 - \sqrt{1 - \frac{4\alpha^2 l_p^2}{r_+^2}} \right] \hbar c, \]  

(4.1)

where we replaced $\Delta x$ by $r_+$ and choose left side inequality for the minimum energy and introducing the calibration factor $(d - 2)/4\pi$ in the temperature (4.1). The absolute minimum value in the position uncertainty (3.3) is

\[ r_+ \geq 2\alpha l_p. \]  

(4.2)

When $r_+ \gg \alpha l_p$ (the large black hole), the Hawking temperature is given by [34, 35, 36]

\[ T_{\text{GUP}} \approx \frac{1}{4 \pi} \left[ \frac{1}{r_+} + \frac{\alpha^2 l_p^2}{r_+^3} \right] \hbar c. \]  

(4.3)

Since the second term in (4.3) gives always a positive correction, the magnitude of Hawking temperature with GUP is always bigger than that of Hawking temperature from HUP as shown in Fig. 1. Therefore, the temperature of the Taub-NUT black hole due to the GUP increases like that of Schwarzschild black hole so that the Hawking temperature with GUP evaporates faster up to a minimum radius $r_{\text{min}} = 2\alpha l_p$ (see the solid line in Fig. 1). This lower bound $r_{\text{min}}$ comes from the second term in the GUP (3.1). Thus, the GUP can prevent the Taub-NUT black hole from complete evaporation which arises from emitting black body radiation at the Hawking temperature, i.e. the Taub-NUT black hole evaporation stops at $r_{\text{min}}$. In fact, using rather generic and model-independent considerations, the GUP with the similar result of string theory is found [12, 13, 14, 15]. The scale of minimum radius $r_{\text{min}}$ itself is not explicitly determined by the GUP. With the help of string theory, such scale is decided, i.e. this scale can be considered as a melting scale of the Taub-NUT black hole which is followed by the string phase [37, 34].

The GUP (3.1) gives a interpolation between the quantum mechanical limit and the quantum gravity limit [32]. When $\Delta p \ll \hbar/(\alpha l_p)$, we get the quantum mechanical limit. The quantum gravity limit is calculated when $\Delta p \simeq \hbar/(\alpha l_p)$.

### 4.2 Hawking Temperature with EUP in the Taub-NUT-AdS$^4$ space

Since the GUP does not have any upper bound on the maximum uncertainty in the position, the GUP can not be naively applied for the (A)dS limit [14, 15, 33, 34]. In this subsection, we will examine the corrections to the Hawking Temperature of the EUP in the Taub-NUT-AdS$^4$ space. The SGUP case will be discussed in following subsection.
The uncertainty in the energy of the emitted particle is given by,

$$\Delta E \simeq c \Delta p \simeq \left[ \frac{\gamma}{\Delta x} + \frac{\beta^2}{l_p^2} \Delta x \right] \hbar c.$$  (4.4)

When we identify the parameter $\beta$ with the second term in (2.8) as

$$\beta^2 \equiv \frac{3l_p^2}{l^2},$$  (4.5)

we reproduce Hawking temperature of the Taub-Nut-AdS$_4$ black hole (2.8) by the EUP (3.5).

The EUP (3.5) also gives an interpolation between the two different quantum mechanical and the quantum gravity limits [32]. The quantum mechanical limit is computed when $\Delta x \ll l_p/\beta$, which leads to $\beta \ll 1$. When $\Delta x \simeq l_p/\beta$, the quantum gravity limit is archived and the quantum gravitational effects give the manifest contribution at very large distance. Thus, the EUP (3.5) has an interpolation between two thermal limits.

As mentioned above, in contrast to four-dimensional Taub-NUT black hole, the Taub-NUT-AdS$_4$ solution has the two thermodynamical limits. In fact, they arise from the two limiting relations between position and momentum of the EUP, i.e. $\Delta p \simeq \hbar/\Delta x$ in quantum mechanical (low energy) limit and $\Delta p \simeq \hbar \Delta x/l_p^2$ in quantum gravity (high energy) limit. Furthermore, the EUP is a result of string theory and the Hawking temperature of the Taub-NUT-AdS$_4$ is obtained by the high energy limit of the EUP, which seems to reflect a quantum gravitational nature. However, even if the HUP suffices to derive the
four-dimensional Taub-NUT black hole thermodynamics, there does not exist consistent formulation of string theory in even Schwarzschild geometry as well as in the Taub-NUT space. Therefore, the Hawking temperature of the Taub-NUT-AdS$_4$ solution seems to have different derivation from that of the four-dimensional Taub-NUT metric. One may apprehend why only the Taub-Nut-(A)dS black hole may allow AdS/CFT correspondence in spite of their distinctive properties such as ALF and breakdown of the area theorem of the horizon.

4.3 Hawking temperature with SGUP in Taub-NUT-AdS$_4$ space

Substituting the parameter $\beta$ (4.5) in the inequality (3.10), we find the Hawking temperature with SGUP

$$T_{\text{AdS(SGUP)}} = \left(\frac{1}{4\pi}\right) \frac{r_+}{2\alpha^2l_p^2} \left[ 1 - \sqrt{1 - \frac{4\alpha^2l_p^2}{r_+^2} \left[ \gamma + \frac{3r_+^2}{l^2} \right]} \right] \hbar c. \quad (4.6)$$

Since we require the square root in (4.6) to be real, we get the minimum radius $r_+$

$$r_+ \geq \frac{2\alpha l_p \sqrt{\gamma}}{\sqrt{1 - 12\alpha^2 l_p^2/l^2}}. \quad (4.7)$$

When one keep the first leading power in $1/r_+$, i.e. semi-classical regime $\alpha l_p \ll r_+ \ll l$, the Hawking temperature with SGUP (4.6) goes back to the EUP result. When one keep the second leading power in $1/r_+$, the temperature (4.6) is written as

$$T_{\text{AdS(SGUP)}} \simeq \frac{1}{4\pi} \left[ \left( \gamma + \frac{6\gamma \alpha^2 l_p^2}{l^2} \right) \frac{1}{r_+} + \frac{3r_+^2}{l^2} + \frac{\gamma^2 \alpha^2 l_p^2}{r_+^3} \right] \hbar c. \quad (4.8)$$

Here, all terms in the bracket are positive, and the Hawking temperature of the Taub-NUT-AdS$_4$ with the SGUP increase (see the solid line in Fig. 2 (a)). Then, the black hole decays up to the minimum radius $r_{\text{min}}$ (4.7) as shown in Fig. 2 (a). Also the Hawking temperature grows up according to increasing of the NUT charge as shown in Fig. 2 (b).
5 Taub-NUT and Taub-NUT-(A)dS Thermodynamics in Arbitrary Even Dimensions

In this section, we first will show that the Hawking temperature with the extended versions of HUP can be obtained in arbitrary even dimensional Taub-NUT-AdS space. Next, we will take the limit of cosmological constant going to zero and get the temperature of the Taub-NUT black hole. Finally, we will obtain the Hawking temperature of the Taub-NUT-dS space through analytic continuation of the cosmological parameter.

5.1 Taub-NUT-AdS Thermodynamics with SGUP

The Taub-NUT-AdS metric in higher dimension has the following form \[4,8\]

\[
ds^2 = -f(r) \left( dt + 2n \sum_{i=1}^{k} \cos(\theta) d\phi_i \right)^2 + \frac{dr^2}{f(r)} + (r^2 + n^2) d\Omega_{d-2},
\]  
(5.1)
where \((d + 1) = 2k + 2\) is the total number of dimension and \(d\Omega_{d-2}\) is the area of the unit \(S^{d-2}\). The metric function \(f(r)\) has the general form
\[
f(r) = \frac{r}{(r^2 + n^2)^2} \int r \left[ \frac{(s^2 + n^2)^k}{s^2} + \frac{2k + 1}{l^2} \frac{(s^2 + n^2)^{k+1}}{s^2} \right] ds - \frac{2mr}{(s^2 + n^2)^k}. \tag{5.2}
\]
The Hawking temperature can be written using (2.3) as
\[
T_{\text{H(AdS)}} = \left( \frac{d - 2}{4\pi} \right) \left[ \frac{1}{r^2} + \left( \frac{d}{d - 2} \right) \frac{n^2 + r^2}{l^2 r^2} \right] \hbar c. \tag{5.3}
\]
Using EUP relation (3.5) and following the same method as in subsection 4.2, we find that we can reproduce the above Hawking temperature (5.3) from EUP by identifying
\[
\gamma_d \equiv 1 + \left( \frac{d}{d - 2} \right) \frac{n^2}{l^2}, \quad \beta^2 \equiv \left( \frac{d}{d - 2} \right) \frac{n^2}{l^2}, \tag{5.4}
\]
the Hawking temperature with the EUP can be expressed as (5.3). Similarly the Hawking temperature coming from SGUP (3.10) in the Taub-NUT-AdS space is given by
\[
T_{\text{AdS(SGUP)}} = \left( \frac{d - 2}{4\pi} \right) \frac{r_+}{2\alpha^2 l^2 p} \left[ 1 - \sqrt{1 - \frac{4\alpha^2 l^2 p}{r_+^2}} \left[ \gamma_d \left( \frac{d}{d - 2} \right) \frac{r_+^2}{l^2} + \gamma_d^2 \alpha^2 l^2 p \right] \right] \hbar c, \tag{5.5}
\]
and the absolute minimum \(r_+\)
\[
r_+ \geq \frac{2\alpha l p \sqrt{\gamma_d}}{\sqrt{1 - \left( \frac{d}{d - 2} \right) \frac{4\alpha^2 l^2 p}{r_+^2}}}. \tag{5.6}
\]
Keeping the second leading power in \(1/r_+\), the temperature with SGUP (5.5) is found as
\[
T_{\text{AdS(SGUP)}} \approx \left( \frac{d - 2}{4\pi} \right) \left\{ \gamma_d + \left( \frac{d}{d - 2} \right) \frac{2\alpha^2 l^2 p}{l^2} \right\} \frac{1}{r_+} + \left( \frac{d}{d - 2} \right) \frac{r_+}{l^2} + \gamma_d^2 \alpha^2 l^2 p \right\} \hbar c. \tag{5.7}
\]
This shows that the corrections due to the SGUP increase the Hawking temperature without depending on the dimensions of the Taub-NUT-AdS since all corrections in (5.7) are positive.

Now, taking the limit of cosmological constant approaching zero, we will obtain the Hawking temperature with the GUP in the arbitrary dimensional Taub-NUT space. When \(l \to \infty; \gamma_d \to 1\), the temperature (5.5) reaches to the Hawking temperature with the GUP
\[
T_{\text{GUP}} = \left( \frac{d - 2}{4\pi} \right) \frac{r_+}{2\alpha^2 l^2 p} \left[ 1 - \sqrt{1 - \frac{4\alpha^2 l^2 p}{r_+^2}} \right] \hbar c, \tag{5.8}
\]
and the absolute minimum $r_+$ has the same form as \(^1\text{12}\) independently on the dimension. Then, for the large black hole, the temperature \(^5\text{.8}\) approaches

$$T_{\text{GUP}} \simeq \left( \frac{d - 2}{4\pi} \right) \left[ \frac{1}{r_+} + \frac{\alpha_\text{GUP}^2 l_p^2}{r_+^3} \right] \hbar c. \quad (5.9)$$

Thus, one finds that the Hawking temperature with the GUP in the arbitrary dimensional Taub-NUT space also increases.

### 5.2 Taub-NUT-dS Thermodynamics with SGUP

Employing analytic continuation of the cosmological parameter $l \to il$, and substituting in \(^5\text{.3}\)–\(^5\text{.7}\) in the AdS space, one can obtain several results in the dS space as following: the Hawking temperature is given by

$$T_{\text{AdS}} = \left( \frac{d - 2}{4\pi} \right) \left[ \frac{\lambda_d}{r_+} - \left( \frac{d}{d - 2} \right) \frac{r_+}{l^2} \right] \hbar c, \quad (5.10)$$

where $\lambda_d \equiv 1 - \left( \frac{d}{d - 2} \right) \frac{n^2}{l^2}$, the $\beta$ and the Hawking temperature with SGUP become

$$\beta^2 \equiv -\left( \frac{d}{d - 2} \right) \frac{l_p^2}{l^2}, \quad (5.11)$$

and

$$T_{\text{dS}(SGUP)} = \left( \frac{d - 2}{4\pi} \right) \frac{r_+}{2\alpha_\text{SGUP}^2 l_p^2} \left[ 1 - \sqrt{1 - \frac{4\alpha_\text{SGUP}^2 l_p^2}{\lambda_d - \left( \frac{d}{d - 2} \right) \frac{r_+^2}{l^2}} \left[ \lambda_d - \left( \frac{d}{d - 2} \right) \frac{r_+^2}{l^2} \right] \right] \hbar c. \quad (5.12)$$

The absolute minimum $r_+$ is given by

$$r_+ \geq \frac{2\alpha l_p \sqrt{\lambda_d}}{\sqrt{1 + \left( \frac{d}{d - 2} \right) \frac{4\alpha_\text{SGUP}^2 l_p^2}{l^2}}}, \quad (5.13)$$

and the large $r_+$, we have

$$T_{\text{dS}(SGUP)} \simeq \left( \frac{d - 2}{4\pi} \right) \left[ \left\{ \lambda_d - \left( \frac{d}{d - 2} \right) \frac{2\alpha_\text{SGUP}^2 l_p^2}{l^2} \right\} \frac{1}{r_+} - \left( \frac{d}{d - 2} \right) \frac{r_+}{l^2} + \frac{\lambda_d^2 \alpha_\text{SGUP}^2 l_p^2}{r_+^3} \right] \hbar c. \quad (5.14)$$

The magnitude of Hawking temperature with SGUP is always bigger than that of Hawking temperature with EUP like the case of AdS space as shown in Fig. \(\text{3}\). (a). Therefore, black hole with SGUP temperature evaporates faster up to a minimum radius.
$r_{\text{min}}$ (see the solid line in Fig. 3 (a)). Since the minimum radius $r_{\text{min}}$ (5.13) must be real, the maximum value for a NUT charge is obtained by

$$n^2 \leq \left( \frac{d-2}{d} \right) l^2 \equiv n_{\text{max}}^2,$$

and the maximum value for $r_+$ is

$$r_+ \leq \sqrt{\left( \frac{d-2}{d} \right) l^2 - n^2} \equiv r_{\text{max}}.$$

In contrast to the Taub-NUT-AdS, the Hawking temperature of the Taub-NUT-dS decreases according to increasing of the NUT charge up to the upper bound $n_{\text{max}}$ (5.15) as shown in Fig. 3 (b).

Figure 3: (a) Plot of Hawking temperature with EUP (dashed line) (5.10) and Hawking temperature with SGUP (solid line) (5.12) in the Taub-NUT-dS space for $\lambda_{(3)} = 0.9375$. (b) Profiles of Hawking temperature with SGUP (5.12) for the various $\lambda_{(3)}$’s in the Taub-NUT-dS space. Solid line with $\lambda_{(3)} = 1$, dashed line with $\lambda_{(3)} = 0.9375$, and dotted-dashed line with $\lambda_{(3)} = 0.8775$. Here, we plot the case with $\alpha = 0.1, \hbar = l_p = 1, l = 2, d = 4, r_{\text{min}} \simeq 0.18$ (5.13), and $r_{\text{max}} \simeq 1.04$ (5.16).
6 Conclusion

Considering the Taub-NUT-(A)dS spaces and employing the extended versions of the HUP based on the string theory and quantum gravity theory, we showed these versions gives rise to their thermal quantities. We also found that two thermodynamical limits of the Taub-NUT-(A)dS metric are the quantum regime limit and the quantum gravity regime limit like S-(A)dS-like black hole. This seems to reflect that the thermodynamic origin of the Taub-NUT-(A)dS is different from that of the Taub-NUT metric. From this result one can apprehend why only the Taub-Nut-(A)dS metric may allow AdS/CFT duality even if the metric with NUT charge has distinctive properties such as ALF and breakdown of the area theorem of the horizon. We also obtained the result that the generalized forms of the HUP increase the Hawking temperature of Taub-NUT-(A)dS black as in the case of the S-(A)dS like black hole.

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