Is the standard singlet Higgs a true massive field?

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Abstract

The phenomenon of spontaneous symmetry breaking admits a physical interpretation in terms of the Bose-condensation process of elementary spinless quanta. In a cutoff theory, this leads to a picture of the vacuum as a condensed medium whose excitations might deviate from exact Lorentz covariance in both the ultraviolet and infrared regions. For this reason, the conventional singlet Higgs boson, the shifted field of spontaneous symmetry breaking, rather than being a purely massive field, might possess a gapless branch describing the long-wavelength fluctuations of the scalar condensate. To test this idea, that might have substantial phenomenological implications, I compare with a detailed lattice simulation of the broken phase in the 4D Ising limit of the theory. The results are the following: i) differently from the symmetric phase, the single-particle energy spectrum is not reproduced by the standard massive form ii) for the value of the hopping parameter $\kappa = 0.076$, increasing the lattice size from $20^4$ to $32^4$, the mass gap is found to decrease from the value 0.392(1) reported by Balog et al. (see Nucl. Phys. B714 (2005) 256) to the value 0.366(5). Both results confirm that, in the infrared region, the standard singlet Higgs cannot be considered as a simple massive field. Several arguments indicate that, approaching the continuum limit of the lattice theory, the observed volume dependence of the mass gap might require larger and larger lattice sizes before to show up.
1. Introduction

The idea of a ‘condensed vacuum’ is generally accepted in modern elementary particle physics. Indeed, in many different contexts, one introduces a set of elementary quanta whose perturbative empty vacuum state $|\phi\rangle$ is not the true ground state of the theory. In the physically relevant case of the Standard Model, where the condensation process corresponds to spontaneous symmetry breaking (SSB) through the $\lambda\Phi^4$ sector, the situation can be summarized saying [1] that "What we experience as empty space is nothing but the configuration of the Higgs field that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have Bose condensed". The translation from field jargon to particle jargon can be obtained, for instance, along the lines of Ref.[2] where the substantial equivalence between the one-loop effective potential of quantum field theory and the energy density of a dilute particle gas was established.

With these premises, it was argued in Ref.[3] that such a physical representation of the vacuum might run into a contradiction with the conventional picture where the standard singlet Higgs boson, the shifted field of SSB, is represented as a purely massive field. In fact, in the hydrodynamical regime of disturbances whose wavelengths are much larger than the mean free path of the elementary constituents, quite independently of the Goldstone phenomenon and even for a spontaneously broken one-component $\lambda\Phi^4$ theory, one would expect that the lowest excitations of the system correspond to small displacements of the condensed quanta that already preexist in the ground state. In this case, for momenta $p \to 0$, the energy spectrum should end with an acoustic "phonon" branch $\sim c_s|p|$ as it happens in superfluids and in all known condensed media. The existence of such a regime is, in fact, a very general result [4] that does not depend on the details of the short-range interaction and even on the nature of the elementary constituents. For instance, the same coarse-grained description is found in superfluid fermionic vacua [5] that, as compared to the Higgs vacuum, bear the same relation as superfluid $^3$He has to superfluid $^4$He.

In this sense, as discussed in Ref.[6], the idea of a massive excitation spectrum of the form $\sqrt{p^2 + M_h^2}$ down to $p = 0$ can be considered equivalent to the incompressibility limit where there is no phonon branch and $c_s = +\infty$. This can be a valid representation of the vacuum for the continuum theory where, as in axiomatic quantum field theory, one can derive the exact Lorentz covariance of the energy spectrum. However, in the condensed phase of a cutoff theory one is faced with ”reentrant violations of special relativity in the low-energy corner” [7] that only disappear in the continuum limit where the ultraviolet cutoff $\Lambda \to \infty$.

In a condensate of spinless particles, these violations would be simple density fluctuations extending over a small region $|p| < \delta$. Lorentz covariance can become exact if the shell $\delta$ vanishes, in units of the scale $M_h$ associated with the massive part of the spectrum, in the limit $\Lambda \to \infty$. Introducing dimensionless units, $\epsilon \equiv \frac{\delta}{M_h}$ and $t \equiv \frac{\Lambda}{M_h}$, the "reentrant" nature of the Lorentz violating infrared effects implies the equivalence of the $\epsilon \to 0$ and $t \to \infty$ limits as, for instance, embodied in the single relation [6]

$$ \delta \sim \frac{M_h^2}{\Lambda} $$

In the cutoff theory, the very long-wavelength phonons of the Bose condensate would
produce a very weak $1/r$ potential [8]. Thus one can consider [6] possible viable phenomenological frameworks to constrain the relative magnitude of $\Lambda$ and $\delta$, while keeping $M_h$ at the Fermi scale. For instance, for $M_h = 250$ GeV and $\Lambda \sim 10^{19}$ GeV one gets $\delta \sim 10^{-5}$ eV and a ratio $\epsilon = \delta/M_h \sim 4 \cdot 10^{-17}$ that might well represent the physical realization of a formally infinitesimal quantity. If this were the right order of magnitude, the non-Lorentz covariant density fluctuations of the vacuum would start to show up from wavelengths larger than a few millimeters up to infinity. These lengths are actually infinitely larger than the Fermi scale but, nevertheless, have a physical meaning.

Now, one might ask if there is any "first-principle" support for the intuitive physical scenario sketched above. To this end, two arguments were given in Ref.[3] to motivate a possible failure of the standard perturbative picture in the far infrared region of the broken-symmetry phase. The first argument starts from the separation of the full scalar field into a constant background $\phi$ and a shifted fluctuation field $h(x)$

$$\Phi(x) = \phi + h(x)$$

(2)

Here, in order the separation to be unambiguous, $\phi$ denotes the average field in a large four-volume $\Omega$, i.e.

$$\phi = \frac{1}{\Omega} \int d^4x \Phi(x)$$

(3)

and the limit $\Omega \to \infty$ has to be taken at the end. In this way, the full functional measure can be expressed as

$$\int [d\Phi(x)] = \int_{-\infty}^{+\infty} d\phi \int [dh(x)]$$

(4)

where the functional integration on the right-hand side is over all quantum modes with four-momentum $p_\mu \neq 0$.

In the standard approach, one describes the shifted field $h(x)$ as a purely massive field whose mass is related to the quadratic shape of a non-convex semiclassical effective potential $V_{\text{NC}}(\phi)$ at one of its absolute minima. This identification is based on introducing, for any given $\phi$, the zero-momentum two-point function $\Gamma_2(p = 0)$, i.e. the inverse zero-momentum connected propagator $D^{-1}(p = 0)$, through the relation

$$D^{-1}(p = 0) = \frac{d^2V_{\text{NC}}}{d\phi^2}$$

(5)

and defining $M_h^2$ from the value of $D^{-1}(p = 0)$ at $\phi = \pm v$, the absolute minima of $V_{\text{NC}}(\phi)$.

In this series of steps one treats $\phi$, the zero-momentum mode of the full scalar field $\Phi(x)$, as a purely classical object without taking into account that, on the base of Eq.(4), there should be one more functional integration over $d\phi$. In an alternative approach, one can start from the generating functional in the presence of a constant source $J$

$$Z(J) = \int_{-\infty}^{+\infty} d\phi \exp[-\Omega(V_{\text{NC}}(\phi) - J\phi)]$$

(6)

where $V_{\text{NC}}(\phi)$ is the non-convex effective potential obtained, order by order in the loop expansion, after integrating out all modes with $p_\mu \neq 0$.

Further, introducing the generating functional for connected Green’s functions $w(J)$ through

$$\Omega w(J) = \ln \frac{Z(J)}{Z(0)}$$

(7)
the vacuum field and the zero-momentum propagator are defined from the relations

\[ \varphi(J) = \frac{dw}{dJ} \]  

(8)

and

\[ G(p = 0) = \frac{d^2 w}{dJ^2} \]  

(9)

in the double limit \( J \to 0 \) and \( \Omega \to \infty \).

Now, it is easy to show that in order to have SSB, i.e.

\[ \lim_{J \to 0^\pm} \varphi(J) \neq 0 \]  

(10)

the four-volume \( \Omega \) has to diverge in such a way that the dimensionless quantity \( x = \Omega J v \) tends to a non-zero limit. However, even assuming that \( x \) tends to \( \pm \infty \), so that the vacuum field \( \varphi(J) \) tend to \( \pm v \), still the zero-momentum inverse propagator \( G^{-1}(p = 0) \) does not uniquely tend to \( M^2_h \). In fact \( G^{-1}(p = 0) \) is the second derivative of the exact effective potential that, as defined from the Legendre transform of the generating functional, is not an infinitely differentiable function in the infinite-volume limit [9]. For this reason, in the broken-symmetry phase, there are two solutions

\[ G^{-1}_a(p = 0) = M^2_h \quad G^{-1}_b(p = 0) = 0 \]  

(11)

as if there were two different particles in the theory and not just one.

In this sense, the situation of the broken phase is reminiscent of what happens in superfluid \(^4\)He. Also there the spectrum is considered to arise from the combined effect of two different types of excitations, phonons and rotons, whose separate energy spectra match giving rise to a complicated pattern. Just following this analogy, one can try to use quantum hydrodynamics [10] to obtain a physical interpretation of the parameter \( M_h \) associated with the massive part of the spectrum in terms of the energy-gap for vortex formation in a superfluid medium possessing the same constituents and the same density as the scalar condensate. In this picture, where phonons would be physically cut off above those infinitesimal momenta that correspond to the inverse-millimeter range, the branch of the spectrum \( \sim \sqrt{p^2 + M^2_h} \) covers in practice the full range that is relevant for particle physics.

The above derivation of two solutions for the zero-momentum propagator is non-perturbative and independent of any diagrammatic expansion. In Ref.[3], however, it was suggested that the same conclusion is also obtained taking into account the effect of the one-particle reducible, zero-momentum tadpole graphs. These enter the diagrammatic expansion for the propagator in the presence of a constant background field and can be considered a manifestation of the quantum nature of the scalar condensate. These effects are usually neglected in the standard procedure where, at the minima of \( V_{NC}(\phi) \), one defines the connected propagator from a Dyson sum of 1PI graphs alone. The reason is that the one-particle reducible tadpole graphs are proportional to the one-point function

\[ J(\phi) = \frac{dV_{NC}}{d\phi} \]  

(12)

that vanishes at \( \phi = \pm v \). However, these zero-momentum graphs are attached to the other parts of the diagrams through zero-momentum propagators. Therefore, ignoring
their contribution at $\phi = \pm v$ (or including their effect in a pure perturbative way with a form of the zero-momentum propagator $G(p = 0) = \frac{1}{M^2} + ...$) is equivalent to assume the regularity of $G(p = 0)$ when $J \to 0$. Instead, if one relaxes this assumption and looks for the general form of $G^{-1}(p = 0)$, one finds again two solutions as in Eq.(11).

Addressing to Ref.[3] for more details, the tacit assumption at the base of the standard approach is better illustrated with a very simple example. Consider the quadratic equation

$$f^{-1}(x) = 1 + x^2 - g^2 x^2 f(x)$$  (13)

for $g^2 \ll 1$. The analogy with the problem of the one-particle reducible zero-momentum tadpole graphs is established when comparing $f(x)$ with $G(0)$, at a given value of $\phi$, and the limit $x \to 0$ with the limit $J(\phi) \to 0$. Standard perturbation theory is based on the iterative structure $f^{\text{reg}} = 1/(1 + x^2) + O(g^2)$ that provides a class of solutions that are regular for $x \to 0$ where $f^{\text{reg}}(0) = 1$. For this class of solutions, that corresponds to the massive propagator as defined from the one-particle irreducible graphs only, the third term in the r.h.s. of Eq.(13) vanishes identically for $x \to 0$. On the other hand, Eq.(13) has also a singular solution $f^{\text{sing}} \sim 1/g^2 x^2$ for $x \to 0$ and this corresponds to a divergent zero-momentum propagator when $\phi \to \pm v$. This can only be discovered by retaining the full non-linearity of the problem where the zero-momentum propagators joining to the vacuum sources are not approximated perturbatively.

I realize that the various arguments given above cannot be considered a "proof" that there is a gapless branch in the excitation spectrum of the broken phase. They represent, however, convergent indications that the standard perturbative derivation of a massive spectrum, which is based on the quadratic part of the shifted classical lagrangian after simply replacing $\phi = \pm v$ in Eq.(2), may be too naive. A deeper understanding of the ground state might be needed to obtain a proper description of the far infrared region of the condensed phase. For instance, exploiting the superfluid analogy, the methods of quantum hydrodynamics (see e.g. Ref.[11]) might represent a natural alternative.

After this general introduction, I will provide in the rest of the paper two more arguments that also indicate an unconventional infrared behaviour. The first new argument, presented in Sect.2, is based on the results of Ref.[12]. It uses the numerical solution of the coupled RG equations for the effective potential and field strength at various values of the infrared cutoff $k$. Expressing the full scalar field as in Eq.(2), the numerical results indicate that, in the broken phase, the fluctuation field $h(x)$ cannot be represented in terms of purely massive states in the $k \to 0$ limit.

The second new argument is based on a numerical simulation of the broken phase in the Ising limit. If the shifted fluctuation field were a purely massive field, the single-particle energy spectrum should be reproduced by (the lattice version of) the standard massive form $\sqrt{\mathbf{p}^2 + \text{const}}$ in the $\mathbf{p} \to 0$ limit. At the same time, there should be no observable change in the mass-gap increasing the linear lattice size above a typical length scale associated with 7-8 correlation lengths. As discussed in Sect.3 (details in the Tables at the end of the paper), both expectations are not consistent with the lattice data.

Finally, Sect.4 will contain a summary and the conclusions.

2. The RG equations for $V(\phi)$ and $Z(\phi)$.

The aim of Ref.[12] was to study the effective potential $V(\phi)$ and the field strength $Z(\phi)$, as functions of the background field $\phi$, at various values of the infrared cutoff. This is a
widely accepted technique where one starts from a bare action defined at some ultraviolet cutoff $\Lambda$ and effectively integrates out shells of quantum modes down to an infrared cutoff $k$. This procedure generates a $k$--dependent effective action $\Gamma_k[\Phi]$ that evolves into the full effective action $\Gamma[\Phi]$ in the $k \to 0$ limit \cite{13–17}. The $k$--dependence of $\Gamma_k[\Phi]$ is determined by a differential functional flow equation that is known in the literature in slightly different forms. In particular, with the flows discussed in detail in Ref.\cite{18} one starts from first principles and obtains a class of functionals that interpolate between the classical bare Euclidean action and the full effective action of the theory. However, some features, such as the basic convexity property of the effective action for $k \to 0$ \cite{19–22}, are independent of the particular scheme.

In this approach, the relevant quantities are the $k$--dependent effective potential $V_k(\phi)$ and field strength $Z_k(\phi)$, which naturally appear in a derivative expansion of $\Gamma_k[\Phi]$ around a space-time constant configuration $\Phi(x) = \phi$. They are governed by two coupled equations derived and discussed in Refs.\cite{22–25}.

Introducing dimensionless variables: $t = \ln(\Lambda/k)$, $x = k^{1-D/2}\phi$, $V(t, x) = k^{-D}V_k(\phi)$ and $Z(t, x) = Z_k(\phi)$ (where $D$ indicates the number of space-time dimensions) and defining the first derivative of the effective potential $f(x, t) = \partial_x V(t, x)$, these coupled equations can be expressed in the form \cite{12} $(f' = \partial_x f(t, x), Z' = \partial_x Z(t, x),...)$

\begin{align*}
\frac{\partial f}{\partial t} &= \frac{(D + 2)}{2} f + \frac{(2 - D)}{2} x \frac{\partial f}{\partial x} - \frac{1}{(4\pi)^{D/2}} \frac{\partial}{\partial x} e^{-f'/Z} \\
\frac{\partial Z}{\partial t} &= \frac{(2 - D)}{2} x \frac{\partial Z}{\partial x} + \frac{1}{(4\pi)^{D/2}} \frac{\partial}{\partial x} \left( \frac{Z'}{Z} e^{-f'/Z} \right) \\
&\quad - \frac{e^{-f'/Z}}{(4\pi)^{D/2}} \left( \frac{f'(Z')^2}{Z^3} + \frac{18D - D^2 - 20 (Z')^2}{24} + \frac{(4 - D)Z'^2}{6Z^2} - \frac{(f'')^2}{6Z^2} \right)
\end{align*}

(14)

(15)

It is easy to show that these two coupled equations can be transformed into the structure ($i,j=1-3$)

\begin{equation}
\sum_{j} P_{ij} \frac{\partial U_i}{\partial t} + Q_i = \frac{\partial R_i}{\partial x}
\end{equation}

(16)

where the components of the vector $U_i(x, t)$ are the unknown functions of the problem and where $P_{ij}, Q_i$ and $R_i$ can depend on $x, t, U_i, \frac{\partial U_i}{\partial x}$. In this way, the numerical solution was obtained with the help of the NAG routines.

The analysis of Ref.\cite{12} was focused on the quantum-field theoretical case $D = 4$ assuming standard boundary conditions at the cutoff scale: i) a renormalizable form for the bare, broken-phase potential

\begin{equation}
V_\Lambda(\phi) = -\frac{1}{2}M^2\phi^2 + \lambda\phi^4
\end{equation}

(17)

and ii) a unit normalization condition for the derivative term in the bare action

\begin{equation}
Z_\Lambda(\phi) = 1
\end{equation}

(18)

It was also assumed a weak-coupling limit $\lambda = 0.1$, fixing $M = 1$ and $\Lambda = 10$. In this way, there is a well defined hierarchy of scales where the infrared region corresponds to the limit $k \ll M \ll \Lambda$. 


The numerical results can be summarized as follows. For not too small values of the infrared cutoff $k$, the effective potential $V_k(\phi)$ remains a smooth, non-convex function of $\phi$ as in the loop expansion. In this region of $k$ one also finds a field strength $Z_k(\phi) \sim 1$ for all values of $\phi$.

However, a tiny scale $\delta \sim 0.15$ exists such that for $k < \delta$ the effective potential $V_k(\phi)$ starts to flatten in an inner region of $|\phi|$ while still matching with an outer, asymptotic shape of the type expected in perturbation theory. The flattening in the inner $|\phi|$-region, while reproducing the expected convexity property of the effective potential, does not correspond to a smooth behaviour.

For such small values of $k$ there are large departures of $Z_k(\phi)$ from unity in the inner $|\phi|$-region with a strong peaking at the end point $|\hat{\phi}| = |\hat{\phi}(k)|$ of the flattening region. On the base of the general convexification property, the $k \to 0$ limit of such end point, $\hat{\phi}(0)$, coincides with one of the minima $\pm v$ of a suitable semiclassical, non-convex effective potential and is usually taken as the physical realization of the broken phase.

To interpret these results, let us start from the usual point of view where SSB is described in terms of a classical potential with perturbative quantum corrections. These corrections, with the choice of the bare parameters of Ref.[12], are typically small for all quantities. In particular $Z$, in perturbation theory, is a non-leading quantity since its one-loop correction is ultraviolet finite. Therefore, perturbation theory predicts tiny deviations of $Z$ from unity, $\sim 10^{-2}$. This is also in agreement with the assumed exact "triviality" of the theory [26] that requires $Z \to 1$ in the continuum limit.

This prediction fits well with the profile of $Z_k(\phi)$ for not too small values of the infrared cutoff. However, for $k < \delta$, there are large deviations from unity with the mentioned strong peaking phenomenon.

If we express the full scalar field $\Phi(x)$ as in Eq.(2), the above results indicate that the higher frequency components of the fluctuation field $h(x)$, those with 4-momentum $p_\mu$ such that $\delta \leq |p| \leq \Lambda$, represent genuine quantum corrections for all values of the background field $\phi$ in agreement with their perturbative representation as weakly coupled massive states.

On the other hand, the components with a 4-momentum $p_\mu$ such that $|p| \leq \delta$, are non-perturbative for values of the background field in the range $0 \leq \phi \leq \hat{\phi}(|p|)$. In particular, the very low-frequency modes with $|p| \to 0$ behave non-perturbatively for all values of the background in the full range $0 \leq |\phi| \leq v$ and thus cannot be represented as standard massive states. In fact, non-perturbative infrared phenomena cannot occur in a genuine massive theory.

Notice that the unexpected effects show up in connection with the convexification process, precisely as discussed in Ref.[3] and reviewed in the Introduction. In this sense, one can say that the convexification process is induced by the infinitely long-wavelength modes that, so to speak, "live" in the full region $0 \leq |\phi| \leq v$.

Notice also that, by itself, the existence of a non-perturbative infrared sector in a region $0 \leq |p| \leq \delta$ might not be in contradiction with the assumed exact "triviality" property of the theory if, in the continuum limit, the infrared scale $\delta$ vanishes in units of the physical parameter $M_h$ associated with the massive part of the spectrum. This means to establish a hierarchy of scales $\delta \ll M_h \ll \Lambda$ such that $\frac{\delta}{M_h} \to 0$ when $\frac{M}{\Lambda} \to 0$, as discussed in the Introduction.

If this happens, the region $0 \leq |p| \leq \delta$ would just shrink to the zero-measure set $p_\mu = 0$, for the continuum theory where $M_h$ sets the unit mass scale, thus recovering the
exact Lorentz covariance of the energy spectrum since the point $p_\mu = 0$ forms a Lorentz-invariant subset. In this limit, the RG function $Z_k(\phi)$ would become a step function which is unity for all finite values of $k$ (and $\phi$) and is only singular for $k = 0$ in the range $0 \leq |\phi| \leq v$. In this way, one is left with a massive, free-field theory for all non-zero values of the momentum, and the only remnant of the non-trivial infrared sector is the singular re-scaling of $\phi$ (the projection of the full scalar field $\Phi(x)$ onto $p_\mu = 0$).

3. Lattice simulation of the broken phase in the Ising limit

In this section I will present the results of a numerical simulation of the theory. The main goal of the numerical experiment is to check the standard assumption that for $k \to 0$ the excitation spectrum of the broken phase is the same as in a weakly coupled massive theory. Quite independently of the theoretical arguments given in the Introduction and in Sect.2, the idea of deviations from the simple massive behaviour of perturbation theory is supported by the lattice results of Ref.[27]. There, differently from what happens in the symmetric phase, the connected scalar propagator was found to deviate significantly from (the lattice version of) the massive single-particle form $1/(p^2 + \text{const})$ for $p_\mu \to 0$. In particular, looking at Figs. 7, 8 and 9 of Ref.[28], one can clearly see that, approaching the continuum limit of the lattice theory, these deviations become more and more pronounced but also confined to a smaller and smaller region of momenta near $p_\mu = 0$.

As in Ref.[27], the test has been performed in the Ising limit that traditionally has been chosen as a convenient laboratory for the numerical analysis of the theory. In this limit, a one-component $\Phi_4^4$ theory becomes governed by the lattice action

$$S_{\text{Ising}} = -\kappa \sum_x \sum_\mu \left[ \phi(x + \hat{e}_\mu) \phi(x) + \phi(x - \hat{e}_\mu) \phi(x) \right]$$

(19)

where $\phi(x)$ takes only the values $\pm 1$. The broken-symmetry phase corresponds to values of the hopping parameter $\kappa > \kappa_c$ with $\kappa_c \sim 0.07484$ [29].

Observables include the bare magnetization:

$$v_B = \langle |\phi| \rangle \quad , \quad \phi \equiv \sum_x \phi(x)/L^4$$

(20)

(where $\phi$ is the average field for each lattice configuration) and the bare zero-momentum susceptibility:

$$\chi_{\text{latt}} = L^4 \left[ \langle |\phi|^2 \rangle - \langle |\phi| \rangle^2 \right].$$

(21)

The equivalence of these two definitions with other standard lattice definitions that are found in the literature has been discussed in Ref. [30].

Let us now consider the mass gap of the theory, say $m_{\text{TS}}(k = 0)$, that is traditionally extracted at zero 3-momentum from the exponential decay (TS=’Time Slice’) of the connected two-point correlator. This is a well known strategy (see for instance Ref. [31]) that, however, for the sake of clarity it might be useful to review in some detail.

The Fourier transform of the connected two-point correlator can be expressed as

$$C_1(t, 0; k) \equiv \langle S_c(t; k)S_c(0; k) + S_s(t; k)S_s(0; k) \rangle_{\text{conn}}$$

(22)

where

$$S_c(t; k) \equiv \frac{1}{L^3} \sum_x \phi(x, t) \cos(k \cdot x),$$

(23)
Here, 0 ≤ t ≤ L is the Euclidean time; x is the spatial part of the site 4-vector \( x^\mu \); \( k \) is the lattice 3-momentum \( k = (2\pi/L)(n_x, n_y, n_z) \), with \( (n_x, n_y, n_z) \) non-negative integers; and \( \langle ... \rangle \) conn denotes the connected expectation value with respect to the lattice action, Eq. (19).

To obtain the time-slice mass, one starts from the general expression (see Ref. [31])

\[
C_1(t, 0; k) = \sum_\alpha |A_\alpha|^2 e^{-E_\alpha(k) t} \tag{25}
\]

where the sum is over the eigenstates of the lattice Hamiltonian corresponding to the given value of \( k \). In perturbation theory, these are represented in the Fock space as massive \( \alpha \)-particle states coupled to total momentum \( k \) and one predicts \( E_1(k) \leq E_2(k) \leq ... \). Therefore, using the lattice dispersion relation

\[
m_{TS}^2(k) = 2(\cosh E_1(k) - 1) - 2 \sum_{\mu=1}^{3} (1 - \cos k_\mu) \tag{26}
\]

one can extract the mass from the single-particle energy \( E_1(k) \).

Now, in an interacting theory, a model-independent approach to the energy spectrum would require to extract the leading term \( E_1(k) \) from the exponential decay at asymptotic \( t \). However, in a real numerical simulation the vanishing of the correlator can hardly be studied for asymptotic times with enough statistics. Thus, in order to extract the leading single-particle energy, one is forced to introduce some model-dependent assumptions, such as the shape of \( E_1(k) \), its relation with the higher excitations \( E_2(k), E_3(k), ... \) and so on.

On the other hand, in a weakly coupled theory (as in a ”trivial” theory close to the continuum limit), there is a simple strategy to extract the single particle energy that has the advantage of being free of uncontrolled theoretical assumptions. It can be applied once the ratio

\[
R = \frac{|A_1|^2}{\sum_\alpha |A_\alpha|^2} \leq 1 \tag{27}
\]

is expected to be very close to unity. In this regime, by further noticing that the residual corrections to a single-particle correlator are of order

\[
(1 - R) e^{-\Delta_n(k) t} \tag{28}
\]

with \( \Delta_n(k) = E_n(k) - E_1(k) > 0 \) and \( n=2,3, ... \), and thus are suppressed by additional phase-space factors, one can start parameterizing the full correlator (with periodic boundary conditions) in terms of an effective single-particle spectrum

\[
C_1(t, 0; k) = A \left[ \exp(-E(k) t) + \exp(-E(k)(L - t)) \right] \tag{29}
\]

Whenever higher excited states were really needed to describe the lattice data, one should detect appreciable differences, both in the value of the normalized chi-square and in the fitted value of \( E(k) \), by simply varying the range of \( t \) where the fit is performed. This means to look for the stability of the results finding a characteristic ‘plateau’ \( E^{(p)}(k) \)
(p=‘plateau’) for the fitted energy where the different indications converge and that can be used to safely extract the time-slice mass.

After these preliminaries, let us now consider the numerical results. The simulations were performed with a numerical code written by Paolo Cea and Leonardo Cosmai. It employs the Swendsen-Wang cluster algorithm [32], using the cluster improved estimator [33] to compute the time slice correlations. The statistical analysis is performed using the jackknife method [34].

To illustrate with an example the determination of the energy plateau, let us first look at the symmetric phase for \( \kappa = 0.074 \). The raw data for the connected correlator are reported in Tables 1, 2 and 3 for three values of the lattice 3-momentum \( k^2 = 0, 0.375 \) and \( k^2 = 0.922 \) and two lattice sizes. I also show the quality of the fits and the resulting time-slice mass in various ranges of \( t \). As one can see, after simply skipping the first one or two time slices, one gets very good stability with mass values that are independent of the spatial momentum and in excellent agreement with each other. This confirms to a very high precision that, in the symmetric phase, the single-particle energy spectrum of the theory is well reproduced by a standard massive spectrum.

Let us now consider the broken phase. This was simulated choosing the value of the hopping parameter \( \kappa = 0.076 \). At this value of \( \kappa \), in fact, published data by Balog et al. [30] from a 20\(^4\) lattice provide the estimate \( m_{\text{TS}}(0) = 0.392(1) \). A check that this is a real mass gap can be obtained in two ways. First, as done for the symmetric phase, one can study the stability of \( m_{\text{TS}}(k) \) at various values of the spatial momentum. Second, one can check the dependence on the lattice size. In fact, if \( m_{\text{TS}}(0)=0.392(1) \) were a real mass gap, a 20\(^4\) lattice contains already \( \sim 8 \) correlation lengths and, thus, there should be no significant change by further increasing the lattice size.

The simulations were first performed on 20\(^4\) lattices at different values of \( k^2 \). In Table 4, I report the results of a simulation of 6 Msweeps for \( k = 0 \). As one can check, the magnetization and the susceptibility are in excellent agreement with those reported by Balog et al. for the same lattice size (see the corresponding entries for \( \kappa = 0.076 \) in Table 3 of Ref.[30]).

Concerning the time-slice mass, the value \( m_{\text{TS}}(0) = 0.3920(24) \) is also in excellent agreement with the result of Ref.[30]. In particular, using all data in the \( t \)-range 1-19 the fit with Eq. (29) goes through all central values with a total chi-square which is less than 0.1. This shows that, for \( k = 0 \), the deviations of the correlator data from a pure single exponential are confined to the first time slice.

The results for \( k \neq 0 \) are reported in Tables 5-9. The number of sweeps is such to provide statistical errors of the time-slice mass that are approximately equal to those reported in Table 4 for \( k = 0 \). As one can see, differently from the symmetric phase, there is a distinct dependence of \( m_{\text{TS}}(k) \) on the spatial momentum (that was already observed in Ref.[27]).

Let us first compare with the data in Tables 5 and 6. These refer to the lowest two momenta of a 20\(^4\) lattice. Again, as for \( k = 0 \), the deviations from a pure single exponential are limited to the first time slice since the fit in the full \( t \)-range 1-19 with Eq. (29) goes through all central values to a very high precision. Using the results of the fit in this range one obtains time-slice masses \( m_{\text{TS}}(k) = 0.3992(21) \) and 0.4039(21) for the integer assignments of the 3-momentum \( (1,0,0) \) and \( (1,1,0) \) respectively. These values are not consistent within 3-5 \( \sigma \) with the corresponding value 0.3920(24) reported in Table 4. Therefore, the lattice data of the broken phase are not well reproduced by the standard
massive form in the $k \to 0$ limit.

Let us now consider the data at the higher spatial momenta reported in Tables 7-9. From the substantial equivalence of the fits in the ranges 2-18 and 3-17, one can deduce that, in this momentum range, the deviations from a pure single exponential affect now the first two time slices. However, notice the difference with the symmetric phase. From Table 3, using the fit results in the t-range 2-18, one obtains $m_{TS}(k) = 0.2133(33)$ in very good agreement with the mass values reported in Table 1. From Tables 7-9, on the other hand, one obtains the corresponding entries $m_{TS}(k) = 0.4122(15), 0.4127(18), 0.4129(20)$ that are not consistent within 7-8 $\sigma$ with the mass value 0.3920(24) of Table 4. Using the fit results in the t-range 3-17, statistical errors become larger but the discrepancy remains at the level of 3-5 $\sigma$.

On the basis of the previous results for $k \neq 0$, it should be clear that interpreting the value $m_{TS}(0) = 0.392(1)$ as a real mass gap is not so obvious. Actually, if one looks for stability with respect to changes of the spatial momentum, it is only at higher momenta that one gets consistency with the standard concept of ”mass” as a $k$-independent quantity. In fact, looking at Tables 7-9, if one requires both a good-quality fit and a substantial independence of the chi-square per degree of freedom on the sample of data (as it happens for the fits 2-18 and 3-17) one obtains a remarkable momentum independence for a mass value $\sim 0.412(3)$. This confirms the basic idea of the Introduction, namely that the shifted fluctuation field is not purely massive and that the end point of the spectrum at $k = 0$ cannot be used to extract the ”mass”.

The idea that $m_{TS}(0)$ might not be a real, physical mass gap is also confirmed by its volume dependence. To this end, I report, in Tables 10-13 the raw correlator data from $32^4$ lattices. The total statistics is 6 Msweeps as for the simulation at $k = 0$ on the $20^4$ lattice reported in Table 4. To shorten the total time of the numerical experiment, however, the full statistics, for the $32^4$ case, was collected using different computers. In the $32^4$ case, I only report data for which the S/N ratio is larger than unity.

Notice that the magnetization and the susceptibility are completely consistent with those measured on the $20^4$ lattice. On the other hand, looking at the results of the global fit to the correlator data, one finds a serious discrepancy in the value of the mass gap. In fact both from Table 4 and Table 14, there are clean indications for the existence of an energy plateau. However the two values are substantially different. In particular, the $32^4$ value $E^{(p)}(0) \sim 0.364(5)$ gives a time slice mass $m_{TS}(0) = 0.366(5)$ that is not consistent within 5 $\sigma$ with the value 0.392(1) reported by Balog et al. for the $20^4$ lattice.

As an additional check, a fit with 2 exponentials was also performed along the lines indicated in Ref.[35]. In this case, Eq.(29) is replaced by a constrained 2-mass formula with $E_2(0) = 2E_1(0)$. The results of this other type of fit are shown in Tables 15 and 16. The entries that are unchanged correspond to fits where the normalization of the 2-particle term is set to zero by the fit routine. For other entries, the statistical errors become much larger. However, the discrepancy between the mass value from the $20^4$ lattice and that obtained from the $32^4$ lattice remains.

Notice that, differently from $m_{TS}(0)$ the susceptibility $\chi_{latt}$ remains almost unchanged when increasing the lattice size. Since the susceptibility is nothing but the zero-four-momentum connected propagator, one can express the numerical stability of the result in the form $\chi_{latt} = G_a(p = 0)$ thus relating $\chi_{latt}$ to the a-type, massive solution for the zero-momentum propagator in Eq.(11). On the other hand, the lattice results suggest also that, if there were a gapless branch of the shifted field, the lattice definition of the b-type
of solution might be identified through the relation $m_{TS}^2(0) = G_b^{-1}(p = 0)$. To prove this conjecture, one should check that increasing the lattice size one gets smaller and smaller values of the mass gap. To get a clean indication, for the same value $\kappa = 0.076$, this will require the non-trivial task to compute the connected two-point correlator, with the same Swendsen-Wang cluster algorithm and a statistics of $\sim 6$ Msweeps, on a substantially larger lattice, say on a $48^4$.

Before ending this section, I observe that from the results of Sect.2 the non-perturbative infrared sector of the broken phase emerges as a threshold phenomenon starting at a momentum scale $\delta \ll M_h$. As anticipated, a massive free-field limit for the continuum theory implies a hierarchical structure of scales where the ratio $\epsilon = \frac{\delta}{M_h} \rightarrow 0$ when $t = \frac{L}{M_h} \rightarrow \infty$. Therefore, the discrepancy in the values of the mass gap, that is already observed at $\kappa = 0.076$ by simply increasing $L$ from 20 to 32, would show up on larger and larger lattice sizes $L > 1/\delta$ for $\kappa$ approaching the critical value $\kappa_c \sim 0.07484$ [29].

For instance assuming a hierarchical relation of the type in Eq.(1), the minimum lattice size would be $L > \Lambda/M_h^2$. Thus, using the relation [36] $\Lambda \sim \frac{1.5\pi}{a}$, to express the ultraviolet cutoff of a $\lambda\Phi^4$ theory in units of the inverse lattice spacing, one should expect

$$\frac{L}{a} > \frac{1.5\pi}{(M_h a)^2}$$

This gives $\frac{L}{a} > 29, 52, 118, 471$ for $M_h a = 0.4, 0.3, 0.2, 0.1$ respectively. In this sense, the range close to $\kappa = 0.076$, where $M_h a \sim 0.4$, is uniquely singled out since it lies in the scaling region but, at the same time, the required lattice sizes are not too large. For the same reason, the stability of a mass gap such as 0.205 when increasing the lattice size from $\frac{L}{a} = 36$ to $\frac{L}{a} = 48$ (see Table 4 of Ref.[30]) might be consistent with the results presented here. In this case, in fact, to detect the same discrepancy found for $M_h a \sim 0.4$ a minimum value $\frac{L}{a} \sim 112$ might be needed.

4. Summary and conclusions

In this paper, following the original idea of Ref.[3], I have presented additional evidences that the conventional perturbative picture of SSB might miss an important infrared phenomenon: the standard singlet Higgs boson might not be a purely massive field. In fact, as reviewed in the Introduction, there are arguments to expect a non-perturbative infrared sector corresponding to the long-wavelength excitations of the scalar condensate.

On a more formal ground, as a consequence of the convexification process of the effective potential, the inverse zero-momentum connected propagator should be considered a two-valued function that, besides the standard massive solution $G_a^{-1}(p = 0) = M_h^2$, includes the value $G_b^{-1}(p = 0) = 0$ as in a gapless theory. As shown in Ref.[12] and illustrated in Sect.2, the convexification process is a threshold phenomenon that starts when the infrared cutoff $k$ is below a tiny scale $\delta$. In this regime, where the effective potential $V_k(\phi)$ deviates from the smooth semiclassical form of perturbation theory, the field strength $Z_k(\phi)$ starts to exhibit large deviations from unity with a strong peaking phenomenon that extends to the boundary of the flatness region in the limit $k \rightarrow 0$. Such a behaviour could hardly be explained if the infrared region were that of a simple massive theory.

In Sect. 3 (and in the Tables at the end of the paper) I have produced detailed numerical results from a lattice simulation of the broken phase in the 4D Ising limit of
the theory. They point to the following conclusions:

i) differently from the symmetric phase, the single-particle energy spectrum is not well reproduced by (the lattice version of) the standard massive form $\sqrt{p^2 + \text{const.}}$ in the limit $p \to 0$ and a $|p|$-independent mass parameter is only found at higher momenta.

ii) for the value $\kappa = 0.076$, the mass gap reported by Balog et al.[30] on a $20^4$ lattice is $m_{TS}(0) = 0.392(1)$. On the other hand increasing the lattice size up to $32^4$ the data provide $m_{TS}(0) = 0.366(5)$, contrary to the expectation that there should be no significant change. At the same time, since the susceptibility is practically unchanged, this might represent one more evidence for the existence of two solutions for the zero-momentum propagator and for the subtle nature of the zero-momentum limit in the broken-symmetry phase.

For this reason, the observed volume dependence of $m_{TS}(0)$, with its possible interpretation in terms of ('non-Goldstone') collective excitations of the scalar condensate, poses interesting questions and would deserve to be checked by other groups with new systematic investigations of the $L \to \infty$ limit of the lattice theory. To this end, one should also take into account that the deviations from a simple massive spectrum $\sqrt{p^2 + M_h^2}$ might be confined to a region of momenta $|p| < \delta$ where $\delta$ vanishes in units of $M_h$ in the infinite-cutoff limit. This means that, approaching the continuum limit of the lattice theory, the volume dependence of the mass gap might require larger and larger lattice sizes before to show up.

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| $t$  | $C_1(t, 0; k = 0)$ | statistical error |
|------|------------------|-------------------|
| 0    | 0.46445456707497D-03 | 0.46037664989336D-06 |
| 1    | 0.37571910789012D-03 | 0.46116594635711D-06 |
| 2    | 0.30382577238347D-03 | 0.45018527889238D-06 |
| 3    | 0.2458206021523D-03  | 0.43438322180999D-06 |
| 4    | 0.1990504016424D-03  | 0.41560747498003D-06 |
| 5    | 0.16136544145195D-03 | 0.39542065913881D-06 |
| 6    | 0.13104967814461D-03 | 0.3753132727689D-06 |
| 7    | 0.10671304049363D-03 | 0.3557753750683D-06 |
| 8    | 0.08238915133100D-04 | 0.33706071111197D-06 |
| 9    | 0.07138673750074D-04 | 0.32019883688822D-06 |
| 10   | 0.05950209789982D-04 | 0.3057426422856D-06 |
| 11   | 0.049970839560653D-04 | 0.294339022413D-06 |
| 12   | 0.042700968213871D-04 | 0.2977239849853D-06 |
| 13   | 0.03736513141293D-04 | 0.29930693571799D-06 |
| 14   | 0.033719361396463D-04 | 0.2993065059117D-06 |
| 15   | 0.03159279983943D-04 | 0.29950600189874D-06 |
| 16   | 0.030893472344528D-04 | 0.3000572524679D-06 |
| 17   | 0.03159279983943D-04 | 0.29950600189874D-06 |
| 18   | 0.032719361396463D-04 | 0.2993065059117D-06 |
| 19   | 0.03736513141293D-04 | 0.29930693571799D-06 |
| 20   | 0.042700968213871D-04 | 0.2977239849853D-06 |
| 21   | 0.049970839560653D-04 | 0.294339022413D-06 |
| 22   | 0.05950209789982D-04 | 0.3057426422856D-06 |
| 23   | 0.07138673750074D-04 | 0.32019883688822D-06 |
| 24   | 0.08238915133100D-04 | 0.33706071111197D-06 |
| 25   | 0.10671304049363D-03 | 0.3557753750683D-06 |
| 26   | 0.13104967814461D-03 | 0.3753132727689D-06 |
| 27   | 0.16136544145195D-03 | 0.39542065913881D-06 |
| 28   | 0.1990504016424D-03  | 0.41560747498003D-06 |
| 29   | 0.2458206021523D-03  | 0.43438322180999D-06 |
| 30   | 0.30382577238347D-03 | 0.45018527889238D-06 |
| 31   | 0.37571910789012D-03 | 0.46165946357112D-06 |
| 32   | 0.46445456707497D-03 | 0.46037664989336D-06 |

| $t$ – range | $\chi^2$/d.o.f. | $k^2$ | $E(k)$ | $m_{TS}(k)$ |
|------------|----------------|------|--------|-------------|
| 0-32       | 0.072          | 0.000| 0.2129 (2) | 0.2133 (2) |
| 1-31       | 0.075          | 0.000| 0.2128 (2) | 0.2133 (2) |
| 2-30       | 0.052          | 0.000| 0.2128 (3) | 0.2132 (3) |
| 3-29       | 0.035          | 0.000| 0.2127 (3) | 0.2131 (3) |

Table 1: The raw correlator data obtained for a spatial momentum $k = 0$, on a $32^4$ lattice, for the value $\kappa = 0.074$ in the symmetric phase. The statistics is 135Ksweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained by using Eqs. (29) and (26) in various ranges of $t$. 
Table 2: The raw correlator data obtained for a spatial momentum $k^2 = 0.375$ (corresponding to the integer assignment $(n_x, n_y, n_z) = (0,3,1)$ on a $32^4$ lattice), for the value $\kappa = 0.074$ in the symmetric phase. The statistics is 135K sweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained by using Eqs.(29) and (26) in various ranges of $t$. 

$$
\begin{array}{cccccc}
 t & C_1(t, 0; k) & \text{statistical error} & \\
 0 & 0.14623108565057D-03 & 0.14165730212821D-07 & \\
 1 & 0.77214443895274D-04 & 0.13288355211454D-07 & \\
 2 & 0.40796525737581D-04 & 0.11455026260932D-07 & \\
 3 & 0.2155838760062D-04 & 0.95795734647824D-08 & \\
 4 & 0.11389647365225D-04 & 0.8167891276023D-08 & \\
 5 & 0.60169117411735D-05 & 0.7854995762898D-08 & \\
 6 & 0.31785227732011D-05 & 0.64601886340349D-08 & \\
 7 & 0.16760427054163D-05 & 0.57641678003130D-08 & \\
 8 & 0.88228124131541D-06 & 0.53085501360843D-08 & \\
 9 & 0.46368972631045D-06 & 0.46940187418747D-08 & \\
10 & 0.2430232660048D-06 & 0.4236763037242D-08 & \\
11 & 0.12847759060431D-06 & 0.38866243669447D-08 & \\
12 & 0.6853590339602D-07 & 0.37374075184433D-08 & \\
13 & 0.3776826515379D-07 & 0.37369789003130D-08 & \\
14 & 0.22751593190996D-07 & 0.39349497056091D-08 & \\
15 & 0.15386378604173D-07 & 0.42589381877184D-08 & \\
16 & 0.13380756871513D-07 & 0.4421054740855D-08 & \\
17 & 0.15386378604173D-07 & 0.42589381877184D-08 & \\
18 & 0.22751593190996D-07 & 0.39349497056091D-08 & \\
19 & 0.3776826515379D-07 & 0.37369789003130D-08 & \\
20 & 0.6853590339602D-07 & 0.37374075184433D-08 & \\
21 & 0.12847759060431D-06 & 0.38866243669447D-08 & \\
22 & 0.2430232660048D-06 & 0.4236763037242D-08 & \\
23 & 0.46368972631045D-06 & 0.46940187418747D-08 & \\
24 & 0.88228124131541D-06 & 0.53085501360843D-08 & \\
25 & 0.16760427054163D-05 & 0.57641678003130D-08 & \\
26 & 0.31785227732011D-05 & 0.64601886340349D-08 & \\
27 & 0.60169117411735D-05 & 0.7854995762898D-08 & \\
28 & 0.11389647365225D-04 & 0.8167891276023D-08 & \\
29 & 0.2155838760062D-04 & 0.95795734647824D-08 & \\
30 & 0.40796525737581D-04 & 0.8167891276023D-08 & \\
31 & 0.77214443895274D-04 & 0.13288355211454D-07 & \\
32 & 0.14623108565057D-03 & 0.14165730212821D-07 & \\
\end{array}
$$
| $t$ | $C_1(t, 0; \mathbf{k})$ | statistical error |
|-----|----------------------|------------------|
| 0   | 0.37264797340762D-03 | 0.39868629339350D-07 |
| 1   | 0.14423703753117D-03 | 0.35128421137473D-07 |
| 2   | 0.55852686582681D-04 | 0.29204927503163D-07 |
| 3   | 0.21643050470909D-04 | 0.25324204561198D-07 |
| 4   | 0.8391190169690D-05  | 0.23695231347215D-07 |
| 5   | 0.32553421665971D-05 | 0.2375227353389D-07 |
| 6   | 0.12625860175981D-05 | 0.25644772176930D-07 |
| 7   | 0.48959050096638D-06 | 0.23531686001271D-07 |
| 8   | 0.19263769009245D-06 | 0.16045711212737D-07 |
| 9   | 0.6945906205410D-07  | 0.17742752623037D-07 |
| 10  | 0.30668878114950D-07 | 0.19019731595239D-07 |
| 11  | 0.6945909077044D-07  | 0.17742753057175D-07 |
| 12  | 0.19263769455122D-06 | 0.16045711766908D-07 |
| 13  | 0.48959049739535D-06 | 0.2353168326186D-07 |
| 14  | 0.1262586091612D-05  | 0.25644774746016D-07 |
| 15  | 0.3253421642504D-05  | 0.2375229326030D-07 |
| 16  | 0.83911901631759D-05 | 0.23695234653985D-07 |
| 17  | 0.21643050344165D-04 | 0.25324219112844D-07 |
| 18  | 0.55852686485936D-04 | 0.2920490946865D-07 |
| 19  | 0.14423703585060D-03 | 0.35128469121253D-07 |
| 20  | 0.37264797340762D-03 | 0.39868629339350D-07 |

| $t$ − range | $\chi^2$/d.o.f. | $k^2$ | $E(\mathbf{k})$ | $m_{TS}(\mathbf{k})$ |
|-------------|----------------|-------|-----------------|-------------------|
| 0-20        | 0.451          | 0.922 | 0.9489 (1)      | 0.2186 (6)        |
| 1-19        | 0.274          | 0.922 | 0.9485 (3)      | 0.2162 (14)       |
| 2-18        | 0.244          | 0.922 | 0.9479 (6)      | 0.2133 (33)       |
| 3-17        | 0.282          | 0.922 | 0.9476 (16)     | 0.2120 (83)       |

Table 3: The raw correlator data obtained for a spatial momentum $k^2 = 0.922$ (corresponding to the integer assignment $(n_x, n_y, n_z) = (3,1,0)$ on a $20^4$ lattice), for the value $\kappa = 0.074$ in the symmetric phase. The statistics is 215 Ksweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained by using Eqs.(29) and (26) in various ranges of $t$. 


Table 4: The raw correlator data, at $k = 0$ and for $\kappa = 0.076$ in the broken phase. The statistics is 6 Msweeps on a $20^4$ lattice. I also show the quality of the fits with Eq.(29), the effective single particle energy and the time-slice mass obtained in various ranges of $t$. The vacuum expectation value was $\langle |\phi| \rangle = 0.30157(3)$ and the susceptibility $\chi_{\text{latt}} = 37.87(9)$.
| $t$ | $C_1(t,0;k)$ | Statistical error |
|-----|--------------|-------------------|
| 0.25 | 0.71526729630726D-03 | 0.13615402429507D-05 |
| 0.5 | 0.43018990381477D-03 | 0.13115919558418D-05 |
| 0.76 | 0.26023053854951D-03 | 0.12910546778340D-05 |
| 1.01 | 0.15766111080779D-03 | 0.12742350485637D-05 |
| 1.27 | 0.95697663109043D-04 | 0.12054623450829D-05 |
| 1.53 | 0.58281796633920D-04 | 0.11691774644051D-05 |
| 1.78 | 0.35714557269430D-04 | 0.10649585762136D-05 |
| 2.04 | 0.22277393587883D-04 | 0.99046217772169D-06 |
| 2.3 | 0.14385158394354D-04 | 0.10404673826236D-05 |
| 2.57 | 0.10544242495683D-04 | 0.1162241471229D-05 |
| 2.83 | 0.94058521329496D-05 | 0.13030286785723D-05 |
| 3.09 | 0.10544242398532D-04 | 0.1162241580206D-05 |
| 3.35 | 0.14385158341362D-04 | 0.10404674051829D-05 |
| 3.61 | 0.2227739346926D-04 | 0.99046212649583D-06 |
| 3.87 | 0.35714557356030D-04 | 0.1064958354061D-05 |
| 4.14 | 0.582817963961D-04 | 0.11691777666978D-05 |
| 4.4 | 0.95697663247686D-04 | 0.1205462164341D-05 |
| 4.66 | 0.15766111116237D-03 | 0.12742352704367D-05 |
| 4.92 | 0.26023053826732D-03 | 0.1291054561123D-05 |
| 5.18 | 0.43018990454806D-03 | 0.13115923352524D-05 |
| 5.45 | 0.71526729630726D-03 | 0.13615402429507D-05 |

| $t$ – Range | $\chi^2$/d.o.f. | $k^2$ | $E(k)$ | $m_{TS}(k)$ |
|-------------|----------------|------|--------|-------------|
| 0-20        | 0.235          | 0.098| 0.5044 (10)| 0.4024 (13) |
| 1-19        | 0.011          | 0.098| 0.5019 (16)| 0.3992 (21) |
| 2-18        | 0.007          | 0.098| 0.5014 (25)| 0.3985 (32) |
| 3-17        | 0.009          | 0.098| 0.5014 (38)| 0.3985 (49) |

Table 5: The raw correlator data for $\kappa = 0.076$ in the broken phase, on a $20^4$ lattice, for the integer assignment $(1,0,0)$. The statistics is 230 Ksweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of $t$. 
| \( t \) | \( C_1(t, 0; k) \) | statistical error |
|---|---|---|
| 0 | 0.60134431492201D-03 | 0.72278071460665D-06 |
| 1 | 0.33121256195148D-03 | 0.67224614280968D-06 |
| 2 | 0.18340753552086D-03 | 0.5700195246459D-06 |
| 3 | 0.1016903856086D-03 | 0.5812629776147D-06 |
| 4 | 0.56548852939487D-04 | 0.6260100175195D-06 |
| 5 | 0.31288412716245D-04 | 0.6185213006675D-06 |
| 6 | 0.17273284231833D-04 | 0.6184780066169D-06 |
| 7 | 0.98445753758052D-05 | 0.6061958406242D-06 |
| 8 | 0.58757929618968D-05 | 0.6407187806801D-06 |
| 9 | 0.40294379323518D-05 | 0.6899457132939D-06 |
| 10 | 0.3420227682787D-05 | 0.7474648254179D-06 |
| 11 | 0.40294380412313D-05 | 0.6899457456547D-06 |
| 12 | 0.5875792936507D-05 | 0.6407187623713D-06 |
| 13 | 0.98445753579537D-05 | 0.6061958035858D-06 |
| 14 | 0.1727328428419D-04 | 0.6184780060206D-06 |
| 15 | 0.31288412838347D-04 | 0.6185209771265D-06 |
| 16 | 0.56548852916696D-04 | 0.6260098461147D-06 |
| 17 | 0.10169038565092D-03 | 0.5812628812842D-06 |
| 18 | 0.18340753431478D-03 | 0.5700195246459D-06 |
| 19 | 0.33121256136276D-03 | 0.6722458201855D-06 |
| 20 | 0.60134431492201D-03 | 0.72278071460665D-06 |

| \( t - \text{range} \) | \( \chi^2/\text{d.o.f.} \) | \( k^2 \) | \( E(k) \) | \( m_{TS}(k) \) |
|---|---|---|---|---|
| 0-20 | 0.358 | 0.196 | 0.5932 (8) | 0.4081 (12) |
| 1-19 | 0.036 | 0.196 | 0.5905 (14) | 0.4039 (21) |
| 2-18 | 0.036 | 0.196 | 0.5899 (23) | 0.4030 (35) |
| 3-17 | 0.042 | 0.196 | 0.5902 (42) | 0.4034 (63) |

Table 6: The raw correlator data for \( \kappa = 0.076 \) in the broken phase, on a \( 20^4 \) lattice, for the integer assignment \((1,1,0)\). The statistics is 230 Ksweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of \( t \).
| $t$ | $C_1(t, 0; k)$ | statistical error |
|-----|----------------|-------------------|
| 0   | 0.34053202765797D-03 | 0.25796702821673D-07 |
| 1   | 0.12939893707440D-03 | 0.22644567310486D-07 |
| 2   | 0.49445741636059D-04 | 0.18855019661045D-07 |
| 3   | 0.18941908006539D-04 | 0.1896579348131D-07 |
| 4   | 0.72475836118000D-05 | 0.18733026319633D-07 |
| 5   | 0.27696225381887D-05 | 0.19177920168895D-07 |
| 6   | 0.10512924615448D-05 | 0.18318892429225D-07 |
| 7   | 0.4175728168673D-06 | 0.18213926845448D-07 |
| 8   | 0.17889655811547D-06 | 0.21653248311599D-07 |
| 9   | 0.75160519493844D-07 | 0.26073999719728D-07 |
| 10  | 0.37277101746356D-07 | 0.31392903787941D-07 |
| 11  | 0.75160519493844D-07 | 0.26073999719728D-07 |
| 12  | 0.17889655811547D-06 | 0.21653248311599D-07 |
| 13  | 0.4175728168673D-06 | 0.18213926845448D-07 |
| 14  | 0.10512924615448D-05 | 0.18318892429225D-07 |
| 15  | 0.27696225381887D-05 | 0.19177920168895D-07 |
| 16  | 0.72475836118000D-05 | 0.18733026319633D-07 |
| 17  | 0.18941908006539D-04 | 0.1896579348131D-07 |
| 18  | 0.49445741636059D-04 | 0.18855019661045D-07 |
| 19  | 0.12939893707440D-03 | 0.22644567310486D-07 |
| 20  | 0.34053202765797D-03 | 0.25796702821673D-07 |

| $t - range$ | $\chi^2$/d.o.f. | $k^2$ | $E(k)$ | $m_{TS}(k)$ |
|-------------|----------------|------|-------|-------------|
| 0-20        | 25.51          | 0.824| 0.9656(1) | 0.4275 (2) |
| 1-19        | 0.794          | 0.824| 0.9614 (2) | 0.4163 (6) |
| 2-18        | 0.278          | 0.824| 0.9599 (6) | 0.4122 (15) |
| 3-17        | 0.272          | 0.824| 0.9609 (15) | 0.4150 (40) |

Table 7: The raw correlator data for $\kappa = 0.076$ in the broken phase, on a $20^4$ lattice, for the integer assignment (3,0,0). The statistics is 10 Msweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of $t$. 
Table 8: The raw correlator data for $\kappa = 0.076$ in the broken phase, on a $20^4$ lattice, for the integer assignment (3,1,0). The statistics is 10 Msweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of $t$. 

| $t$     | $C_1(t, 0; k)$ | statistical error |
|---------|----------------|-------------------|
| 0       | 0.32191840093250D-03 | 0.28215740200667D-07 |
| 1       | 0.11716187697558D-03 | 0.24420632370780D-07 |
| 2       | 0.42874548504809D-04 | 0.19461125127975D-07 |
| 3       | 0.15726715346676D-04 | 0.1710759460693D-07 |
| 4       | 0.57725432712762D-05 | 0.17245120570366D-07 |
| 5       | 0.21143021635064D-05 | 0.17637391271899D-07 |
| 6       | 0.77592919860017D-06 | 0.17545895751891D-07 |
| 7       | 0.29550904531463D-06 | 0.17673166191011D-07 |
| 8       | 0.12817308964179D-06 | 0.18301057235521D-07 |
| 9       | 0.57731193876899D-07 | 0.23297882913256D-07 |
| 10      | 0.2948792232530D-07 | 0.25758084311071D-07 |
| 11      | 0.57731193876899D-07 | 0.23297882913256D-07 |
| 12      | 0.12817308964179D-06 | 0.18301057235521D-07 |
| 13      | 0.29550904531463D-06 | 0.17673166191011D-07 |
| 14      | 0.77592919860017D-06 | 0.17545895751891D-07 |
| 15      | 0.21143021635064D-05 | 0.17637391271899D-07 |
| 16      | 0.57725432712762D-05 | 0.17245120570366D-07 |
| 17      | 0.15726715346676D-04 | 0.1710759460693D-07 |
| 18      | 0.42874548504809D-04 | 0.19461125127975D-07 |
| 19      | 0.11716187697558D-03 | 0.24420632370780D-07 |
| 20      | 0.32191840093250D-03 | 0.28215740200667D-07 |

| $t - range$ | $\chi^2$/d.o.f. | $k^2$ | $E(k)$ | $m_{TS}(k)$ |
|-------------|----------------|-------|--------|--------------|
| 0-20        | 19.00          | 0.922 | 1.0087 (1) | 0.4295 (3) |
| 1-19        | 0.858          | 0.922 | 1.0046 (3) | 0.4178 (7) |
| 2-18        | 0.309          | 0.922 | 1.0028 (6) | 0.4127 (18) |
| 3-17        | 0.351          | 0.922 | 1.0022 (17) | 0.4113 (48) |

Table 8: The raw correlator data for $\kappa = 0.076$ in the broken phase, on a $20^4$ lattice, for the integer assignment (3,1,0). The statistics is 10 Msweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of $t$. 

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Table 9: The raw correlator data for $\kappa = 0.076$ in the broken phase, on a $20^4$ lattice, for the integer assignment $(3,1,1)$. The statistics is 10 Msweeps. I also show the quality of the fits, the effective single-particle energy and the time-slice mass obtained in various ranges of $t$.

| $t$   | $C_1(t,0;k)$     | statistical error |
|-------|------------------|-------------------|
| 0     | 0.30566681793518D-03 | 0.24505540691796D-07 |
| 1     | 0.10682582826270D-03 | 0.20658932104437D-07 |
| 2     | 0.37545987045166D-04 | 0.15460405900989D-07 |
| 3     | 0.13225370098169D-04 | 0.14067447764431D-07 |
| 4     | 0.465770441941379D-05 | 0.15233371727808D-07 |
| 5     | 0.16509226154241D-05 | 0.1696416047581D-07 |
| 6     | 0.58929799490177D-06 | 0.1741230984827D-07 |
| 7     | 0.20719049454559D-06 | 0.15254671198613D-07 |
| 8     | 0.69295505341081D-07 | 0.15879409375053D-07 |
| 9     | 0.22006955714087D-07 | 0.17865514829934D-07 |
| 10    | 0.1678649400784D-07 | 0.20544843933658D-07 |
| 11    | 0.22006955714087D-07 | 0.17865514829934D-07 |
| 12    | 0.69295505341081D-07 | 0.15879409375053D-07 |
| 13    | 0.20719049454559D-06 | 0.15254671198613D-07 |
| 14    | 0.58929799490177D-06 | 0.1741230984827D-07 |
| 15    | 0.16509226154241D-05 | 0.1696416047581D-07 |
| 16    | 0.465770441941379D-05 | 0.15233371727808D-07 |
| 17    | 0.13225370098169D-04 | 0.14067447764431D-07 |
| 18    | 0.37545987045166D-04 | 0.15460405900989D-07 |
| 19    | 0.10682582826270D-03 | 0.20658932104437D-07 |
| 20    | 0.30566681793518D-03 | 0.24505540691796D-07 |

| $t$ range | $\chi^2$/d.o.f. | $k^2$ | $E(k)$     | $m_{TS}(k)$ |
|-----------|-----------------|-------|------------|-------------|
| 0-20      | 21.23           | 1.020 | 1.0493 (1) | 0.4308 (3)  |
| 1-19      | 0.667           | 1.020 | 1.0450 (2) | 0.4183 (7)  |
| 2-18      | 0.135           | 1.020 | 1.0432 (6) | 0.4129 (20) |
| 3-17      | 0.131           | 1.020 | 1.0422 (20)| 0.4099 (61) |
Table 10: The raw correlator data at $k = 0$ obtained from a simulation on a $32^4$ lattice, for $\kappa = 0.076$ in the broken phase. The statistics is 3500 Ksweeps. Only data with $S/N > 1$ are reported. The vacuum expectation value was $\langle |\phi| \rangle = 0.30158(2)$ and the susceptibility $\chi_{\text{latt}} = 37.70(11)$. 

| $t$ | $C_1(t, 0; k = 0)$ | statistical error |
|-----|-------------------|--------------------|
| 0   | 0.18825024647556D-03 | 0.16941891466779D-05 |
| 1   | 0.14042050173424D-03 | 0.16936167168555D-05 |
| 2   | 0.98762414233733D-04 | 0.16927933487534D-05 |
| 3   | 0.68288037480761D-04 | 0.16919086215327D-05 |
| 4   | 0.46981340861454D-04 | 0.16913027902220D-05 |
| 5   | 0.32320623399664D-04 | 0.16907071684861D-05 |
| 6   | 0.22322431762012D-04 | 0.16901776642593D-05 |
| 7   | 0.15515610279490D-04 | 0.16897524471058D-05 |
| 8   | 0.10880836739844D-04 | 0.16894000480709D-05 |
| 9   | 0.77275164550180D-05 | 0.16893186514232D-05 |
| 10  | 0.55921686139766D-05 | 0.16891971059864D-05 |
| 11  | 0.41522126343909D-05 | 0.16891265663505D-05 |
| 12  | 0.31962571843670D-05 | 0.1689317502024D-05 |
| 13  | 0.25821007487241D-05 | 0.16897826971614D-05 |
| 14  | 0.25821007487241D-05 | 0.16897826971614D-05 |
| 15  | 0.31962571843670D-05 | 0.1689317502024D-05 |
| 16  | 0.41522126343909D-05 | 0.16891265663505D-05 |
| 17  | 0.55921686139766D-05 | 0.16891971059864D-05 |
| 18  | 0.77275164550180D-05 | 0.16893186514232D-05 |
| 19  | 0.10880836739844D-04 | 0.16894000480709D-05 |
| 20  | 0.15515610279490D-04 | 0.16891971059864D-05 |
| 21  | 0.22322431762012D-04 | 0.16897524471058D-05 |
| 22  | 0.32320623399664D-04 | 0.16907071684861D-05 |
| 23  | 0.46981340861454D-04 | 0.16919086215327D-05 |
| 24  | 0.68288037480761D-04 | 0.16913027902220D-05 |
| 25  | 0.98762414233733D-04 | 0.16927933487534D-05 |
| 26  | 0.14042050173424D-03 | 0.16936167168555D-05 |
| 27  | 0.18825024647556D-03 | 0.16941891466779D-05 |
| $t$  | $C_1(t, 0; k = 0)$ | statistical error |
|------|-------------------|-------------------|
| 0    | 0.1915658349118E-03 | 0.42089126114641E-05 |
| 1    | 0.14376486787758E-03 | 0.42064742520006E-05 |
| 2    | 0.10210247213207D-03 | 0.4196516984451D-05 |
| 3    | 0.715796915195D-04 | 0.41737044386843D-05 |
| 4    | 0.50219314163541D-04 | 0.41429253869347D-05 |
| 5    | 0.3550394123410D-04 | 0.4096377585245D-05 |
| 6    | 0.25544524377202D-04 | 0.40649605084329D-05 |
| 7    | 0.18747584437939D-04 | 0.4036321221907D-05 |
| 8    | 0.14150039767838D-04 | 0.40174857064957D-05 |
| 9    | 0.11028271382665D-04 | 0.40038139588244D-05 |
| 10   | 0.88824632819617D-05 | 0.39929623745097D-05 |
| 11   | 0.74336822982801D-05 | 0.39881786206720D-05 |
| 12   | 0.64935569222907D-05 | 0.40024501405502D-05 |
| 13   | 0.5891701270399D-05 | 0.40348389009236D-05 |
| 14   | 0.5891701270399D-05 | 0.40348389009236D-05 |
| 15   | 0.64935569222907D-05 | 0.40024501405502D-05 |
| 16   | 0.74336822982801D-05 | 0.39881786206720D-05 |
| 17   | 0.88824632819617D-05 | 0.39929623745097D-05 |
| 18   | 0.11028271382665D-04 | 0.40038139588244D-05 |
| 19   | 0.14150039767838D-04 | 0.40174857064957D-05 |
| 20   | 0.18747584437939D-04 | 0.4036321221907D-05 |
| 21   | 0.25544524377202D-04 | 0.40649605084329D-05 |
| 22   | 0.3550394123410D-04 | 0.4096377585245D-05 |
| 23   | 0.50219314163541D-04 | 0.41429253869347D-05 |
| 24   | 0.715796915195D-04 | 0.41737044386843D-05 |
| 25   | 0.50219314163541D-04 | 0.41429253869347D-05 |
| 26   | 0.3550394123410D-04 | 0.4096377585245D-05 |
| 27   | 0.50219314163541D-04 | 0.41429253869347D-05 |
| 28   | 0.715796915195D-04 | 0.41737044386843D-05 |
| 29   | 0.10210247213207D-03 | 0.4196516984451D-05 |
| 30   | 0.14376486787758D-03 | 0.42064742520006D-05 |
| 31   | 0.1915658349118E-03 | 0.42089126114641E-05 |
| 32   | 0.25544524377202D-04 | 0.40649605084329D-05 |

Table 11: The raw correlator data at $k = 0$ obtained from a simulation on a $32^4$ lattice, for $\kappa = 0.076$ in the broken phase. The statistics is 1250K sweeps. Only data with $S/N > 1$ are reported. The vacuum expectation value was $\langle |\phi| \rangle = 0.30156(4)$ and the susceptibility $\chi_{\text{latt}} = 37.72(18)$. 
Table 12: The raw correlator data at $k = 0$ obtained from a simulation on a $32^4$ lattice, for $\kappa = 0.076$ in the broken phase. The statistics is 820K sweeps. Only data with $S/N > 1$ are reported. The vacuum expectation value was $\langle |\phi| \rangle = 0.30157(3)$ and the susceptibility $\chi_{\text{latt}} = 37.74(20)$. 

| $t$  | $C_1(t, 0; k = 0)$ | statistical error |
|------|-----------------|-------------------|
| 0    | 0.18830816108376D-03 | 0.36295412301977D-05 |
| 1    | 0.14049039900582D-03 | 0.36347914715019D-05 |
| 2    | 0.98868629477251D-04 | 0.36448907002102D-05 |
| 3    | 0.68418079591984D-04 | 0.36545903093631D-05 |
| 4    | 0.47126928040045D-04 | 0.36613388157817D-05 |
| 5    | 0.32499368020916D-04 | 0.36675738973308D-05 |
| 6    | 0.22525563622636D-04 | 0.36739990211847D-05 |
| 7    | 0.15786687380170D-04 | 0.36845952758579D-05 |
| 8    | 0.11231440928165D-04 | 0.37072940633664D-05 |
| 9    | 0.81636421519316D-05 | 0.3727196019070D-05 |
| 10   | 0.60622792191850D-05 | 0.37404570700649D-05 |
| 11   | 0.46039724904828D-05 | 0.37442837585148D-05 |
| 21   | 0.46039724904828D-05 | 0.37442837585148D-05 |
| 22   | 0.60622792191850D-05 | 0.37404570700649D-05 |
| 23   | 0.81636421519316D-05 | 0.3727196019070D-05 |
| 24   | 0.11231440928165D-04 | 0.37072940633664D-05 |
| 25   | 0.15786687380170D-04 | 0.36845952758579D-05 |
| 26   | 0.22525563622636D-04 | 0.36739990211847D-05 |
| 27   | 0.32499368020916D-04 | 0.36675738973308D-05 |
| 28   | 0.47126928040045D-04 | 0.36613388157817D-05 |
| 29   | 0.68418079591984D-04 | 0.36545903093631D-05 |
| 30   | 0.98868629477251D-04 | 0.36448907002102D-05 |
| 31   | 0.14049039900582D-03 | 0.36347914715019D-05 |
| 32   | 0.18830816108376D-03 | 0.36295412301977D-05 |

| $t$  | $C_1(t, 0; k = 0)$ | statistical error |
|------|-----------------|-------------------|
| 0    | 0.18830816108376D-03 | 0.36295412301977D-05 |
| 1    | 0.14049039900582D-03 | 0.36347914715019D-05 |
| 2    | 0.98868629477251D-04 | 0.36448907002102D-05 |
| 3    | 0.68418079591984D-04 | 0.36545903093631D-05 |
| 4    | 0.47126928040045D-04 | 0.36613388157817D-05 |
| 5    | 0.32499368020916D-04 | 0.36675738973308D-05 |
| 6    | 0.22525563622636D-04 | 0.36739990211847D-05 |
| 7    | 0.15786687380170D-04 | 0.36845952758579D-05 |
| 8    | 0.11231440928165D-04 | 0.37072940633664D-05 |
| 9    | 0.81636421519316D-05 | 0.3727196019070D-05 |
| 10   | 0.60622792191850D-05 | 0.37404570700649D-05 |
| 11   | 0.46039724904828D-05 | 0.37442837585148D-05 |
| 21   | 0.46039724904828D-05 | 0.37442837585148D-05 |
| 22   | 0.60622792191850D-05 | 0.37404570700649D-05 |
| 23   | 0.81636421519316D-05 | 0.3727196019070D-05 |
| 24   | 0.11231440928165D-04 | 0.37072940633664D-05 |
| 25   | 0.15786687380170D-04 | 0.36845952758579D-05 |
| 26   | 0.22525563622636D-04 | 0.36739990211847D-05 |
| 27   | 0.32499368020916D-04 | 0.36675738973308D-05 |
| 28   | 0.47126928040045D-04 | 0.36613388157817D-05 |
| 29   | 0.68418079591984D-04 | 0.36545903093631D-05 |
| 30   | 0.98868629477251D-04 | 0.36448907002102D-05 |
| 31   | 0.14049039900582D-03 | 0.36347914715019D-05 |
| 32   | 0.18830816108376D-03 | 0.36295412301977D-05 |
Table 13: The raw correlator data at $k = 0$ from a simulation on a $32^4$ lattice, for $\kappa = 0.076$ in the broken phase. The statistics is 620Ksweeps. Only data with $S/N > 1$ are reported. The vacuum expectation value was $\langle |\phi| \rangle = 0.30157(3)$ and the susceptibility $\chi_{\text{latt}} = 37.72(28)$.

| $t$ | $C_1(t, 0; k = 0)$ | statistical error |
|-----|-------------------|--------------------|
| 0   | 0.18662641483139D-03 | 0.42499108352562D-05 |
| 1   | 0.13880114074366D-03 | 0.42356693269278D-05 |
| 2   | 0.97166753677375D-04 | 0.42277156923244D-05 |
| 3   | 0.6681534205013D-04 | 0.4213078274730D-05 |
| 4   | 0.45350298378516D-04 | 0.42041206560544D-05 |
| 5   | 0.30654197711880D-04 | 0.4212487342185D-05 |
| 6   | 0.20607943469257D-04 | 0.42459030224462D-05 |
| 7   | 0.1375407397773D-04 | 0.42864246337440D-05 |
| 8   | 0.90869249084038D-05 | 0.43060940267945D-05 |
| 9   | 0.5932798437900D-05 | 0.43120683752934D-05 |
| 23  | 0.5932798437900D-05 | 0.43120683752934D-05 |
| 24  | 0.90869249084038D-05 | 0.43060940267945D-05 |
| 25  | 0.1375407397773D-04 | 0.42864246337440D-05 |
| 26  | 0.20607943469257D-04 | 0.42459030224462D-05 |
| 27  | 0.30654197711880D-04 | 0.4212487342185D-05 |
| 28  | 0.45350298378516D-04 | 0.42041206560544D-05 |
| 29  | 0.6681534205013D-04 | 0.42277156923244D-05 |
| 30  | 0.97166753677375D-04 | 0.42277156923244D-05 |
| 31  | 0.13880114074366D-03 | 0.42356693269278D-05 |
| 32  | 0.18662641483139D-03 | 0.42499108352562D-05 |

Table 14: The results of the global fit to the correlator data reported in Tables 10-13, for $\kappa = 0.076$ in the broken phase. The total statistics is 6.19 Msweeps on $32^4$ lattices. I report the quality of the fits with Eq.(29), the effective single particle energy and the time-slice mass obtained in various ranges of $t$. These results should be compared with those in Table 4 from the $20^4$ lattice.

| $t$ range | $\chi^2$/d.o.f. | $k^2$ | $E(k)$ | $m_{\text{TS}}(k)$ |
|-----------|-----------------|------|-------|-----------------|
| 0-32      | 0.779           | 0.000| 0.3443 (25) | 0.3460 (25) |
| 1-31      | 0.255           | 0.000| 0.3624 (37) | 0.3644 (37) |
| 2-30      | 0.254           | 0.000| 0.3655 (54) | 0.3675 (54) |
| 3-29      | 0.256           | 0.000| 0.3622 (77) | 0.3642 (77) |
| 4-28      | 0.252           | 0.000| 0.3546 (110)| 0.3565 (110)|
| $t$ − range | $\chi^2$/d.o.f. | $k^2$ | $E_1(k)$ | $m_{TS}(k)$ |
|-------------|----------------|-------|-----------|-------------|
| 0-20        | 1.573          | 0.000 | 0.3804 (17) | 0.3827 (17) |
| 1-19        | 0.002          | 0.000 | 0.3880 (71) | 0.3904 (71) |
| 2-18        | 0.001          | 0.000 | 0.3866 (113) | 0.3890 (113) |
| 3-17        | 0.000          | 0.000 | 0.3861 (159) | 0.3885 (159) |

Table 15: The results of a fit to the correlator data reported in Table 4 from the $20^4$ lattice. The fit is performed using a constrained 2-mass formula with $E_2(0) = 2E_1(0)$ as suggested in Ref.[35]. I report the quality of the fits, $E_1(0)$ and the time-slice mass obtained in various ranges of $t$. These results should be compared with those in Table 4 obtained fitting the correlator data to Eq.(29).

| $t$ − range | $\chi^2$/d.o.f. | $k^2$ | $E_1(k)$ | $m_{TS}(k)$ |
|-------------|----------------|-------|-----------|-------------|
| 0-32        | 0.787          | 0.000 | 0.3443 (25) | 0.3460 (25) |
| 1-31        | 0.258          | 0.000 | 0.3624 (37) | 0.3644 (37) |
| 2-30        | 0.236          | 0.000 | 0.3416 (198) | 0.3433 (198) |
| 3-29        | 0.225          | 0.000 | 0.3143 (324) | 0.3156 (324) |

Table 16: The results of a fit to the correlator data reported in Tables 10-13 from the $32^4$ lattice. The fit is performed using a constrained 2-mass formula with $E_2(0) = 2E_1(0)$ as suggested in Ref.[35]. I report the quality of the fits, $E_1(0)$ and the time-slice mass obtained in various ranges of $t$. These results should be compared with those in Table 14 obtained fitting the same correlator data with Eq.(29) and with those in Table 15 obtained from the $20^4$ lattice data using the same 2-mass fitting function.