Single-loop realization of arbitrary non-adiabatic holonomic single-qubit quantum gates in a superconducting circuit

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Geometric phases are noise-resilient, and thus provide a robust way towards high-fidelity quantum manipulation. Here we experimentally demonstrate arbitrary non-adiabatic holonomic single-qubit quantum gates for both a superconducting transmon qubit and a microwave cavity in a single-loop way. In both cases, an auxiliary state is utilized, and two resonant microwave drives are simultaneously applied with well-controlled but varying amplitudes and phases for the arbitrariness of the gate. The resulting gates on the transmon qubit achieve a fidelity of 0.996 characterized by randomized benchmarking and the ones on the cavity show an averaged fidelity of 0.978 based on a full quantum process tomography. In principle, a nontrivial two-qubit holonomic gate between the qubit and the cavity can also be realized based on our presented experimental scheme. Our experiment thus paves the way towards practical non-adiabatic holonomic quantum manipulation with both qubits and cavities in a superconducting circuit.

High-fidelity quantum manipulation is essential for large scale quantum computation. However, as quantum systems are fragile under noises from either the surrounding environment or the control fields, error-resilient manipulations of quantum states are preferable. Geometric phases [1, 2] depend only on the global properties of the evolution trajectories, and thus have built-in noise-resilient features against certain local noises [3–5]. Therefore, they can naturally be used to achieve high-fidelity quantum gates. Consequently, considerable interests have been paid to various applications of geometric phases in quantum computation [6].

Due to the non-commutativity, non-Abelian geometric phases are natural for the so-called holonomic quantum computation [7]. Schemes based on the adiabatic evolution of the non-Abelian geometric phases have been proposed on a variety of systems for quantum computation [8–13]. However, these schemes are rather difficult for experimental realization as they rely on complicated control over multilevel systems. Meanwhile, the gates are based on the rather slow adiabatic quantum dynamics and thus decoherence can induce considerable errors. Therefore, it is desirable to implement quantum gates with non-adiabatic evolutions [14]. Recently, much attention has been paid to the non-adiabatic holonomic quantum computation with three-level systems [15, 16]. Compared to the adiabatic ones, this type of new schemes is fast and easy to realize, and has been experimentally demonstrated in superconducting circuits [17, 18], NMR [19], and nitrogen-vacancy centers in diamond [20–22].

More importantly, arbitrary single-qubit holonomic gates can be achieved in a single-loop evolution [23–25]. This will simplify gate sequences in practical quantum information processing compared with the original proposal [15], where two sequential gates are required for an arbitrary single-qubit gate. In the last year, the single-loop scheme has been experimentally demonstrated in NMR [26] and nitrogen-vacancy centers in diamond with off-resonance drives [27], which are experimentally difficult for arbitrary pulse shapes. Therefore, single-loop schemes with resonant drives are preferable as they are experimentally friendly.

Here, with a circuit quantum electrodynamics architecture [28–32] we experimentally demonstrate arbitrary non-adiabatic holonomic single-qubit gates for both a superconducting transmon qubit and a microwave photonic qubit in a single-loop way. This is realized by varying the amplitudes and phases of a two-tone resonant microwave drive [25]. Besides transmon qubits, photonic qubits in a microwave cavity are also desirable for quantum information processing because of their long coherence times [33, 34] and ease of realizing quantum error correction [35, 36]. In our realization, the gates on the transmon qubit achieve a fidelity of 0.996 characterized by randomized benchmarking (RB) and the ones on the cavity show an averaged fidelity of 0.978 based on a full quantum process tomography (QPT). The demonstrated holonomic gates are also compatible with and can be readily used in the recently realized distributed quantum information processing [37–41].

We first address the implementation of arbitrary single-qubit holonomic gates on a superconducting transmon qubit in the \(|g\rangle, |f\rangle\) subspace, as shown in Fig. 1a. Here, \(|g\rangle, |e\rangle, \text{and} |f\rangle\) denote the three lowest energy levels of the transmon qubit, as described by

\[
\mathcal{H}_1 = \Omega_{ge}(t)e^{i\phi_1} |g\rangle \langle e| + \Omega_{ef}(t)e^{i\phi_1} |f\rangle \langle e| + \text{H.c.}
\]

\[
= \Omega(t)e^{i(\phi_1 - \pi)} \left( \sin \frac{\theta}{2} e^{i\phi} |g\rangle - \cos \frac{\theta}{2} |f\rangle \right) \langle e| + \text{H.c.}(1)
\]

where \(\Omega_{ge}(t)\) and \(\Omega_{ef}(t)\) are the time-dependent amplitudes of the two microwave drives with the corresponding initial phases \(\phi_1, \phi = \phi_0 - \phi_1 + \pi\). \(\Omega(t) = \sqrt{\Omega_{ge}^2(t) + \Omega_{ef}^2(t)}\), and \(\tan(\theta/2) = \Omega_{ge}(t)/\Omega_{ef}(t)\). As seen in Eq. 1, the quantum dynamics is captured by the resonant coupling between the bright state \(|b\rangle = \sin(\theta/2)e^{i\phi}|g\rangle - \cos(\theta/2)|f\rangle\) and the auxiliary state \(|e\rangle\), while the dark state \(|d\rangle = \cos(\theta/2)|g\rangle + \sin(\theta/2)|f\rangle\).
FIG. 1: Single-loop single-qubit holonomic gates. (a) Two microwave fields are resonantly coupled to the $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions of a transmon qubit to generate arbitrary single-qubit holonomic gates in the $\{|g\rangle, |f\rangle\}$ subspace. The first excited state $|e\rangle$ is an auxiliary level. (b) Bloch sphere representation of the holonomic gate: a combination of two microwave fields, with two different sets of phases for the first and the second half of the gate operation, equivalently drives the bright state $|b\rangle$ to the auxiliary state $|e\rangle$ and then back with an additional phase, while the dark state $|d\rangle$ remains unchanged. (c) Simplified experimental setup. Two transmon qubits in two microwave trenches are coupled to two microwave cavities, one for readout and the other one for storage. The qubit holonomic gates are demonstrated with $Q_1$, while the cavity holonomic gates are realized with the Fock states $\{|0\rangle, |1\rangle\}$ in the storage cavity facilitated by both qubits (Fig. 4). (d) QPT is used to characterize the performance of the arbitrary holonomic gates. Arbitrary initial states are prepared with sequential pulses on $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions. Nine sequential pre-rotation pulses on the qubit are performed before the final measurement to obtain the transmon’s full state tomography with three levels.

$\sin(\theta/2)e^{-i\phi}|f\rangle$ is decoupled. Under the cyclic evolution condition, $\int_0^T \Omega(t)dt = \pi$, one can obtain a quantum gate depending on $\theta$ and/or $\phi$. Meanwhile, since there is no transition between $|d\rangle$ and $|b\rangle$ states when $\theta$ is time-independent and also no dynamical phases due to the on-resonance drives, the obtained gates are thus holonomic [15].

To achieve a universal set of single-qubit holonomic gates in a single-loop way [25], we divide the evolution time $T$ into two equal halves and choose $\phi_0 = \phi, \phi_1 = \pi$ for $t \in [0, T/2]$ and $\phi_0' = \phi - \gamma - \pi, \phi_1' = \gamma$ for $t \in [T/2, T]$, such that the Hamiltonians during these two halves are $\mathcal{H}_\theta = \Omega(t)\langle b|\langle + |e\rangle \langle |e\rangle + |e\rangle \langle b\rangle$ and $\mathcal{H}_{\phi} = -\Omega(t)(e^{i}\gamma|b\rangle \langle e| + e^{-i\gamma}|e\rangle \langle b\rangle)$, respectively. Geometrically, the two evolutions coincide at two poles in the Bloch sphere, and the cyclic geometric phase is illustrated as the red slice contour in Fig. 1b. Therefore, in the qubit subspace $\{|g\rangle, |f\rangle\}$, the obtained holonomic single-qubit gate is

$$U_1(\theta, \gamma, \phi) = \begin{pmatrix} \cos \frac{\gamma}{2} - i\sin \frac{\gamma}{2} \cos \theta & -i\sin \frac{\gamma}{2} \sin \theta e^{i\phi} \\ -i\sin \frac{\gamma}{2} \sin \theta e^{-i\phi} & \cos \frac{\gamma}{2} + i\sin \frac{\gamma}{2} \cos \theta \end{pmatrix} = \exp\left(-i\frac{\gamma}{2} \mathbf{n} \cdot \mathbf{\sigma}\right),$$

which describes a rotation operation around the axis $\mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ by an angle $\gamma$, up to a global phase factor $\exp\left(i\gamma/2\right)$.

In our experiment, two superconducting transmon qubits are dispersively coupled to two three-dimensional cavities [42–45], as shown in Fig. 1c. The $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transition frequencies of the two qubits $Q_1$ and $Q_2$ are $\omega_{gq1}/2\pi = 5.036$ GHz and $\omega_{q2}/2\pi = 4.782$ GHz, $\omega_{ge}/2\pi = 5.605$ GHz and $\omega_{de}/2\pi = 5.367$ GHz, respectively. One of the cavities with a transition frequency of $\omega_s/2\pi = 8.540$ GHz is connected to a Josephson parametric amplifier (JPA) for a fast and high-fidelity joint readout of the two-qubit states [46–49]. The other cavity with a transition frequency of $\omega_q/2\pi = 7.614$ GHz is utilized for storage and manipulation of the photonic states and for implementing the holonomic gates between Fock states $|0\rangle$ and $|1\rangle$ as will be discussed below. In the following, we ignore the readout cavity and the “cavity” refers to the storage cavity. More details about the device parameters can be found in Ref. [50].

We now demonstrate the realization of the arbitrary holonomic gates in a single-loop way with transmon qubit $Q_1$ based on the procedure discussed above. The envelopes of the two drives are truncated Gaussian pulses with a total width of $\sigma = 120$ ns. We characterize the holonomic single-qubit gates by a full QPT including all three levels, $|g\rangle, |e\rangle$, and $|f\rangle$ [50–52]. The experimental pulse sequence is shown in Fig. 1d. To evaluate the QPT, we have used both attenuated and unattenuated $\chi$ matrix fidelities, which are respectively defined as $F_{\text{att}} = \frac{1}{T} \text{Tr} \left( \chi \chi_{\text{th}} \right)$ and $F_{\text{unatt}} = \frac{1}{T} \left| \text{Tr} \left( \chi \chi_{\text{th}} \right) \right|^2 \left( \frac{1}{T} \text{Tr} \left( \chi \chi_{\text{exp}} \right) \frac{1}{T} \text{Tr} \left( \chi_{\text{th}} \chi_{\text{th}}^\dagger \right) \right)$ [19, 53, 54], where $\chi_{\text{exp}}$ is the experimental process matrix and $\chi_{\text{th}}$ is the corresponding ideal process matrix. The latter fidelity can ignore the errors due to signal loss, e.g., the errors in state preparations and measurements. Figure 2a shows $F_{\text{unatt}}$ of the gates as a function of both $\theta$ and $\gamma$ with $\phi = 0$, and the averaged fidelity $F_{\text{unatt}} = 0.994$. Energy relaxation and dephasing of both excited states and non-perfect microwave drives can cause a population leakage outside the computation subspace $\{|g\rangle, |f\rangle\}$ to the auxiliary $|e\rangle$ state. This leakage can be characterized by the trace of the reduced process matrix $\chi_{\text{e}}$, which describes the process only involving $|g\rangle$ and $|f\rangle$ and ignores any operators acting on the auxiliary state $|e\rangle$ [50]. Figure 2b shows the traces of $\chi_{\text{e}}$ originated from the measured $\chi_{\text{exp}}$ whose fidelities are shown in Fig. 2a. The high value (0.992) of the averaged trace indicates that there is nearly no leakage outside the computation subspace for the holonomic gates on the transmon qubit. $\chi_{\text{e}}$ of four example gates are shown in Fig. 2c with $F_{\text{unatt}} = 0.997, 0.996, 0.996, \text{and } 0.996$.
FIG. 2: QPT of the single-qubit holonomic gates. (a) Unattenuated \( \chi \) matrix fidelity \( F_{\text{unatt}} \) of the single-qubit holonomic gates \( U_1(\theta, \gamma, \phi) \) with different \( \theta \) and \( \gamma \) while \( \phi = 0 \). The averaged fidelity \( F_{\text{unatt}} = 0.994 \) while the averaged attenuated fidelity \( F_{\text{att}} = 0.975 \). (b) The traces of the reduced process matrix \( \chi R \) as a function of both \( \theta \) and \( \gamma \) with \( \phi = 0 \). The averaged trace is 0.992, indicating small leakage outside the computation subspace \( \{ |g\rangle, |f\rangle \} \). (c) Bar charts of the real and imaginary parts of \( \chi R \) of four specific gates: \( X_\pi = U_1(\pi/2, \pi, 0) \), \( X_{\pi/2} = U_1(\pi/2, \pi/2, 0) \), \( H = U_1(\pi/4, \pi, 0) \), and \( Z_\pi = U_1(0, \pi, 0) \), where \( R_\theta \) denotes a rotation of the qubit by an angle \( \phi \) along the axis \( R \) and \( H \) represents the Hadamard gate. The numbers in the \( x \) and \( y \) axes correspond to the operators in the basis set \( \{ I, X, -iY, Z \} \) of the \( \{ |g\rangle, |f\rangle \} \) subspace. The solid black outlines are for the ideal gates.

respectively (the corresponding \( F_{\text{att}} = 0.976, 0.980, 0.963, \) and 0.988).

Another regular way to extract gate fidelity only without relying on perfect state preparations and measurements is RB [55–59]. An agreement between \( F_{\text{unatt}} \) and the fidelity from RB should provide more confidence in the gate performance. We utilize the Clifford-based RB and the experimental sequences are shown in Fig. 3a, where we perform both a reference RB experiment and an interleaved RB experiment. The results of the four holonomic gates presented in Fig. 2c are shown in Fig. 3b. Each Clifford gate is realized by choosing specific parameters \( \theta, \gamma \), and \( \phi \). The reference RB experiment gives an average gate fidelity of the single-qubit holonomic gates in the Clifford group \( F_{\text{avg}} = 0.996 \). The measured gate fidelities of the four specific holonomic gates \( X_\pi, X_{\pi/2}, H, \) and \( Z_\pi \) are 0.998, 0.996, 0.997, and 0.995, respectively. These fidelities are consistent with the measured \( F_{\text{unatt}} \), thus validating \( F_{\text{unatt}} \) as a good measure of gate performance. The loss of fidelity is mainly from the decoherence of both \( |e\rangle \) and \( |f\rangle \) of the transmon qubit, as confirmed by numerical simulations based on QuTiP in Python [60, 61].

In addition to the implementation of holonomic gates on the transmon qubit, we also realize holonomic operations on the cavity Fock states following a similar scheme. As shown in Fig. 4a, the holonomic gates are implemented by using a selective two-photon transition drive \( \Omega_1 e^{i\omega_1 t} \) on qubit \( Q_1 \) conditional on only zero photon in the cavity and a cavity-assisted Raman transition drive \( \Omega_2 e^{i\omega_2 t} \) between \( |1g\rangle \) and \( |0f\rangle \) [37, 62]. Here the drive frequencies \( \omega_1 = (\omega_{e1} + \omega_{e1})/2, \omega_2 = \omega_{e1} + \omega_{e1} - \omega_s, \) and \( \Omega_1 \) and \( \Omega_2 \) are the corresponding drive strengths.

Similarly, the above two drives together generate the following effective Hamiltonian [25, 62]:

\[
\mathcal{H}_2 = \tilde{g}_1 |0g\rangle \langle 0f| + \tilde{g}_2 |1g\rangle \langle 0f| + \text{H.c.}
= \tilde{g} \left( \sin \frac{\theta}{2} e^{i\phi} |0g\rangle - \cos \frac{\theta}{2} |1g\rangle \right) \langle 0f| + \text{H.c.},
\]

where, \( \tilde{g}_1 \) and \( \tilde{g}_2 \) are the effective coupling strengths of the two drives, and \( \tilde{g} = \sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}, \tilde{g}_1/\tilde{g}_2 = \tan(\theta/2)e^{i\phi} \). To validate the above Hamiltonian, \( \tilde{g}_1 \) of the two-photon transition drive should be much smaller than the dispersive shifts between qubit \( Q_1 \) and the cavity. In the experiment, both the two-photon transition drive and the Raman transition drive are set to have the same pulse envelope (a wide square pulse with sine squared ramp-up and ramp-down edges) to keep the two drives synchronized and the ratio of two amplitudes fixed. Both \( \tilde{g}_1 \) and \( \tilde{g}_2 \) are carefully calibrated with square pulses with different amplitudes [50]. We have also carefully taken into account the AC-Stark shifts under the strong external drives to eliminate the possible dynamical phases [50]. Similarly to the transmon qubit holonomic gates, by adjusting the ratios of

FIG. 3: RB of the single-qubit holonomic gates. (a) Sequences of both a reference RB experiment and an interleaved RB experiment. (b) The sequence fidelity decay as a function of the gate length \( m \). The fidelity for each sequence length \( m \) is measured for \( k = 100 \) different random sequences with the standard deviation from the mean plotted as the error bars. Both curves are fitted with \( F = Ap^m + B \) with different sequence decays \( p = p_{\text{ref}} \) and \( p_{\text{gate}} \). The average gate fidelity per Clifford gate is \( F_{\text{avg}} = 1 - (1 - p_{\text{ref}})/2 = 0.996 \). The difference between the reference and the interleaved RB experiments gives the specific gate fidelity \( F_{\text{gate}} = 1 - (1 - p_{\text{gate}}/p_{\text{ref}})/2 \). The red dashed line indicates the threshold for exceeding gate fidelity 0.990.
gates $X_\pi$, $Y_\pi$, $H_1$, and $H_2$, and perform QPT of these gates. The experimental process matrices $\chi_{\text{exp}}$ are shown in Fig. 4c. The averaged experimental attenuated and unattenuated process fidelities of these holonomic gates are $F_{\text{att}} = 0.879$ and $F_{\text{unatt}} = 0.978$, respectively. The infidelities of the holonomic gates for Fock states are mainly limited by the encoding and decoding errors (including the initial state preparation and final measurement errors), decoherence process during the gate, and imperfections of the control pulses. As a reference, the process fidelity of the encoding and decoding processes only without any gate is 0.95 (an error of 5%). The dissipation and dephasing of the two excited states $|e\rangle$ and $|f\rangle$ during the gate process can induce an additional infidelity of 5-6% due to the long gate duration time. These two dominant error budgets are consistent with the measured attenuated fidelities. A higher fidelity gate can be achieved with a shorter gate operation time under a larger dispersive shift.

To achieve a universal quantum computation, two-qubit gates are necessary. A nontrivial two-qubit holonomic gate between the qubit and the cavity can in principle be realized in a similar way to the single-qubit gates, i.e., by using two resonant selective pulses on $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions respectively, conditional on only zero photon in the cavity. Then the effective Hamiltonian is $\mathcal{H}_3 = \Omega_{2g}(t) e^{i\phi} |0g\rangle \langle 0e| + \Omega_{2g} e^{i\phi} (t) |0f\rangle \langle 0e| + \text{h.c.}. A$ nontrivial two-qubit holonomic gate in the subspace $\{ |0g\rangle, |0f\rangle, |1g\rangle, |1f\rangle \}$ can then be realized with the form of $U_3(\theta, \gamma, \phi) = \begin{pmatrix} U_1(\theta, \gamma, \phi) & 0 \\ 0 & I \end{pmatrix}$. 

In conclusion, in a circuit quantum electrodynamics architecture we have experimentally demonstrated high-fidelity arbitrary non-adiabatic holonomic single-qubit gates for both a superconducting transmon qubit and a microwave cavity in a single-loop way. Moreover, our method can be generalized to achieve holonomic gates between the nearest Fock states $|n\rangle$ and $|n+1\rangle$, if the resonant drives are on $|n,g\rangle \leftrightarrow |n,f\rangle$ and $|n+1,g\rangle \leftrightarrow |n+1,f\rangle$ transitions. Combining these gates, we can realize full holonomic control of both the transmon qubit and the cavity mode [63–67], which has important applications in cavity-assisted quantum information processing and high-precision measurements [68]. Our experiment thus opens the door to implement holonomic manipulations of both qubits and cavities in a superconducting circuit.

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Supplementary Material for “Single-loop realization of arbitrary non-adiabatic holonomic single-qubit quantum gates in a superconducting circuit”

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I. EXPERIMENTAL DEVICE

Our experimental device contains two superconducting transmon qubits in two trenches strongly coupled to two three-dimensional (3D) waveguide cavities, as shown in Fig. S1. The qubits, each of which consists of a Josephson junction connected to two antenna pads with different lengths, are fabricated on two separate c-plane sapphire chips with a double-angle evaporation of aluminum after a single electron-beam lithography step. The two qubits are separated by 10 mm to suppress their direct crosstalk. The two cavities are machined from a block of high purity (5N5) aluminum and are chemically etched for a better coherence quality [1, 2]. One cavity is for a fast and high-fidelity joint readout of the two qubits, while the other one is for storage and manipulation of the photonic states. The input and output couplings of the cavities can be adjusted by tuning the lengths of the input and output coupler pins.

FIG. S1: The experimental device. (a) Optical image of the two 3D Al microwave cavities, housing two chips (depicted in red boxes). Each chip contains a superconducting transmon qubit. (b) Optical image of the qubit with a Josephson junction (depicted in a blue box) connected to two antenna pads. The thin line across the junction is used to protect the qubit during fabrication and is cut open before installing into the cavities. (c) Scanning electron microscope image of the Al/AlOx/Al Josephson junction fabricated with a double-angle evaporation.

II. EXPERIMENTAL SETUP

The experimental device is installed inside a magnetic shield and cooled down to $T \approx 10$ mK in a cryogen-free dilution refrigerator. The full wiring diagram is shown in Fig. S2. The qubit control pulses are generated directly from Tektronix arbitrary waveform generator (AWG) 70002A benefiting from its large bandwidth and sampling rate [3]. The pulses for the initial state preparations and the pre-rotations before measurement have a truncated Gaussian envelope with a width of $4\sigma = 40$ ns. The technique of “derivative removal by adiabatic gate” is applied to both $|e\rangle \leftrightarrow |f\rangle$ and $|g\rangle \leftrightarrow |e\rangle$ transition drives in order to remove the leakage to the undesired energy levels [4, 5]. The storage cavity drives are generated by IQ modulations with two analog channels of a Tektronix AWG 5014C for arbitrary cavity controls. Another two analog channels of the AWG 5014C are used to modulate the readout pulse. The readout signal is amplified by a Josephson parameter amplifier (JPA) at base temperature, a high electron mobility transistor (HEMT) at 4K stage, and a standard commercial amplifier at room temperature. Finally, the readout signal is mixed down to 50 MHz with a local oscillator (LO) before being digitized and recorded by the analog-to-digital converters (ADC).

III. SYSTEM HAMILTONIAN

In our device, two transmon qubits are dispersively coupled to two 3D cavity modes. Each transmon has a large anharmonicity and is considered as a three-level artificial atom. The whole system can be described by the following Hamiltonian

$$\mathcal{H}/\hbar = \omega_r (a_r^\dagger a_r + 1/2) + \omega_s (a_s^\dagger a_s + 1/2)$$
$$+ \omega_{ge1} |e_1\rangle \langle e_1| + (\omega_{ge1} + \omega_{sf1}) |f_1\rangle \langle f_1|$$
$$+ \omega_{ge2} |e_2\rangle \langle e_2| + (\omega_{ge2} + \omega_{sf2}) |f_2\rangle \langle f_2|$$
$$- \chi_{rq1}^{ge} |e_1\rangle \langle e_1| a_r^\dagger a_r - \left(\chi_{rq1}^{ge} + \chi_{rq1}^{ef}\right) |f_1\rangle \langle f_1| a_r^\dagger a_r$$
$$- \chi_{rq2}^{ge} |e_2\rangle \langle e_2| a_r^\dagger a_r - \left(\chi_{rq2}^{ge} + \chi_{rq2}^{ef}\right) |f_2\rangle \langle f_2| a_r^\dagger a_r$$
$$- \chi_{sq1}^{ge} |e_1\rangle \langle e_1| a_s^\dagger a_s - \left(\chi_{sq1}^{ge} + \chi_{sq1}^{ef}\right) |f_1\rangle \langle f_1| a_s^\dagger a_s$$
$$- \chi_{sq2}^{ge} |e_2\rangle \langle e_2| a_s^\dagger a_s - \left(\chi_{sq2}^{ge} + \chi_{sq2}^{ef}\right) |f_2\rangle \langle f_2| a_s^\dagger a_s$$
$$- K_c a_r^\dagger a_r a_t - K_s a_s^\dagger a_s a_t$$

where $\omega_r, \omega_s$ are the readout and the storage cavity frequency, respectively; $a_{r,s}$ are the corresponding ladder operators; $\omega_{gei}$

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and $\omega_{ef}$ are the transition frequencies among the lowest three levels $\{|g\rangle, |e\rangle, |f\rangle\}$ of the $i$-th qubit; $\chi$’s are the corresponding dispersive interactions between the two qubits and the two cavities; and $K_{r,s}$ are the self-Kerr of the readout and the storage cavity, respectively. All the relevant parameters in the Hamiltonian are listed in Table I. In our joint readout of the two qubits, since we only need to distinguish the $|gg\rangle$ state (both qubits in the ground state) from all others, $\chi_{ef}^{rq1}$, $\chi_{ef}^{rq2}$, and $K_r$ are irrelevant and thus not measured directly.

The coherence properties of the two qubits and the two cavities are also experimentally measured and listed in Table II. The relaxation times of each transmon are obtained by measuring the free evolutions of the populations of the three levels $P_g$, $P_e$, and $P_f$ with an initial $|f\rangle$ state, following the technique described in Ref. [6]. $P_g$ and $P_f$ are measured by comparing the population of $|g\rangle$ through a $\pi$ pulse and two sequential $\pi$ pulses, respectively. The experimental results are shown in Figs. S3a and S3b. The decay curves are globally fitted with the rate equation $d\vec{p}/dt = \Gamma \cdot \vec{p}$, where $\vec{p} = (P_g, P_e, P_f)^T$, and $\Gamma$ is the decay rate matrix given by

$$\Gamma = \begin{pmatrix} \Gamma_e & \Gamma_{fg} & \Gamma_{eg} \\ -\Gamma_e & \Gamma_f & \Gamma_{fe} \\ 0 & -\Gamma_f & \Gamma_{ef} \end{pmatrix},$$  

(2)

FIG. S2: Schematic of the full wiring of the experimental setup.

| Terms ($/2\pi$) | Measured | Terms ($/2\pi$) | Measured |
|-----------------|----------|-----------------|----------|
| $\omega_{r}$    | 8.540 GHz| $\omega_{s}$    | 7.614 GHz|
| $\omega_{ge1}$  | 5.036 GHz| $\omega_{ge2}$  | 5.605 GHz|
| $\omega_{ef1}$  | 4.782 GHz| $\omega_{ef2}$  | 5.367 GHz|
| $\chi_{ge}^{rq1}$ | 2.230 MHz| $\chi_{ge}^{sq1}$ | 0.942 MHz|
| $\chi_{ef}^{rq1}$ | -        | $\chi_{ef}^{sq1}$ | 0.843 MHz|
| $\chi_{ge}^{rq2}$ | 3.00 MHz | $\chi_{ge}^{sq2}$ | 1.436 MHz|
| $\chi_{ef}^{rq2}$ | -        | $\chi_{ef}^{sq2}$ | 1.193 MHz|
| $K_r$           | -        | $K_s$           | 3.7 kHz  |

TABLE I: Hamiltonian parameters. $\chi_{eq1}^{ef}$, $\chi_{eq2}^{ef}$, and $K_r$ are irrelevant for the joint readout and thus not measured directly. The numbers 1 and 2 in the subscript correspond to qubits $Q_1$ and $Q_2$ respectively.
where we ignore the negligible upward transition rates and only include the downward transition rates $\Gamma_{eg}$, $\Gamma_{fe}$, and $\Gamma_{f}$.

Since the non-sequential decay rate $\Gamma_{f}$ is much slower than the sequential decay rates $\Gamma_{eg}$ and $\Gamma_{fe}$, the corresponding coherence times $1/\Gamma_{eg}$ and $1/\Gamma_{fe}$ of both qubits are listed as $T_1$ in Table II.

The dephasing rates between $|g\rangle$ and $|e\rangle$ and between $|e\rangle$ and $|f\rangle$ of each qubit are measured with Ramsey interference experiments and the results are shown in Figs. S3c-f. The Ramsey fringes are fitted with an exponentially damped double sinusoidal function $\gamma = y_0 + e^{-t/T_2^*} \left[ A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2) \right]$ and the extracted $T_2^*$ are also listed in Table II. The coherence times $T_1$ and $T_2^*$ of the storage cavity are measured through the relaxation of Fock state $|1\rangle$ and the dephasing of $(|0\rangle + |1\rangle)/\sqrt{2}$, respectively [2]. Both initial states are generated with selective number-dependent arbitrary phase gates [7].

IV. QUANTUM PROCESS TOMOGRAPHY

The holonomic quantum gates are characterized by a full quantum process tomography of the three-level system. We first initialize the three-level transmon qubit with the following:

\[
\begin{align*}
|\psi\rangle &= a|g\rangle + b|e\rangle + c|f\rangle, \\
\langle\psi'| &= a^*|g\rangle + b^*|e\rangle + c^*|f\rangle,
\end{align*}
\]

\[
\rho = \begin{pmatrix} 1 & 0 & 0 \\ 0 & |a|^2 & b^*c \frac{\sqrt{1 - |b|^2}}{2} \\ 0 & b\frac{\sqrt{1 - |b|^2}}{2} & |c|^2 \end{pmatrix}
\]

The density matrix of the three-level qubit state can then be obtained by preparing the qubit in $|\psi\rangle$ and performing the state tomography measurement.

\[
\chi = \frac{1}{2} \sum_{k,l} \rho_{kl} \sigma^k \sigma^l
\]

where $\sigma^k$ are the Pauli operators acting on the three-level qubit. The density matrix of the three-level qubit state can then be reconstructed by the maximum likelihood estimation method [11]. With the nine initial states $\rho_i$, the experimental process matrix $\chi_{exp}$ can be extracted from the nine corresponding final states $\rho_f$ through $\rho_f = \sum_{i=0}^{n-1} \chi_{exp} \rho_i \sigma^k \sigma^l$.

For the holonomic gates on the transmon qubit, the state $|e\rangle$ serves as an auxiliary state. Therefore, we have calculated the reduced process matrix $\chi_{exp}$, which describes the process only involving $|g\rangle$ and $|f\rangle$ and ignores any operators acting on the auxiliary state. In order to compare with the process acting on a two-level system, the reduced process matrix $\chi_{exp}$ is obtained by a normalization factor of $3/2$. And the basis operators of the reduced process matrix are $\{I, X, -iY, Z\}$ as in the main text.

For the holonomic gates on the cavity Fock states, only four initial states $\{|g\rangle, |f\rangle, |g\rangle + |f\rangle\}/\sqrt{2}, \{|g\rangle - i|f\rangle\}/\sqrt{2}$ are prepared with qubit $Q_2$, and then they are mapped onto Fock states $\{|0\rangle, |1\rangle, |0\rangle + |1\rangle\}/\sqrt{2}, \{|0\rangle - i|1\rangle\}/\sqrt{2}$ in the cavity with the encoding process. After performing the holonomic gates on these Fock states, the cavity states are mapped back to qubit $Q_2$ with the decoding process and finally the state tomography of qubit $Q_2$ is performed.

---

**Table II: Coherence properties of the system.**

| Modes       | $T_1$ | $T_2$ |
|-------------|-------|-------|
| readout cavity | 84 ns | -     |
| storage cavity | 135 $\mu$s | 193 $\mu$s |
| $Q_1\ (|g\rangle \leftrightarrow |e\rangle)$ | 45.6 $\mu$s | 24.4 $\mu$s |
| $Q_1\ (|g\rangle \leftrightarrow |f\rangle)$ | 20.3 $\mu$s | 8.3 $\mu$s |
| $Q_2\ (|g\rangle \leftrightarrow |e\rangle)$ | 42.2 $\mu$s | 44.0 $\mu$s |
| $Q_2\ (|e\rangle \leftrightarrow |f\rangle)$ | 24.9 $\mu$s | 13.6 $\mu$s |
FIG. S4: Experimental full process matrices $X_{\text{exp}}$. (a) and (b) Bar charts of the real and imaginary parts of the holonomic gates $X_{\pi/2} = U_1(\pi/2, \pi/2, 0)$ and $H = U_0(\pi/4, \pi, 0)$ respectively. The numbers in the $x$ and $y$ axes correspond to the operators in the basis set $\{I_g, \sigma^x_g, -i\sigma^y_g, \sigma^z_g, -i\sigma^x_c, \sigma^y_c, -i\sigma^z_c, L_c\}$. Solid black outlines are the corresponding ideal process matrices $X_{\text{id}}$.

V. CALIBRATION OF THE TWO-PHOTON TRANSITION AND CAVITY-ASSISTED RAMAN TRANSITION DRIVES

To implement the holonomic gates on the cavity Fock states, the effective coupling strengths and the Stark-shifts of the two-photon transition and the cavity-assisted Raman transition drives are critical and thus fully calibrated.

The cavity-assisted Raman transition drives for both qubits $Q_1$ and $Q_2$ are calibrated separately as follows, since they are used for the implementation of the holonomic gates and the encoding/decoding processes, respectively. One of the qubits is initialized in $|f\rangle$ state by two sequential $\pi$ pulses while the other one remains in $|g\rangle$ state during the calibration. A microwave pulse at frequency $\omega_0$ with a square envelope at a fixed digital-analog converter (DAC) level is then applied. The coherent oscillations between the corresponding $|0f\rangle$ and $|1g\rangle$ as a function of the square pulse width and the drive frequency $\omega_0$ produce a chevron pattern as shown in Fig. S5a for $Q_2$. From a sinusoidal fit of the coherent oscillations, we obtain the Rabi frequency $\Omega_R$ as a function of the drive frequency as shown in Fig. S5b. The solid red line is fitted with the function $\Omega_R = \sqrt{\Delta^2 + (2\tilde{g}^2)^2}$, where $\Delta = \omega_0 - \omega$, giving the resonant frequency $\omega_0$ and the effective coupling strength $\tilde{g}_2$ for $Q_2$. Note that we choose $\tilde{g}_2$ ($\tilde{g}_2'$) for $Q_1$ ($Q_2$) to be consistent with the notation in the main text. Repeats of the same experiment at different amplitudes (DAC level) of the drive pulse give a calibration of the resonant frequency $\omega_0$ and the effective coupling strength $\tilde{g}_2$ ($\tilde{g}_2'$) as a function of the DAC level. The results for both qubits are shown in Figs. S5c and S5d, respectively.

The cavity-assisted Raman transition for qubit $Q_2$ is used for the encoding and decoding processes, and we choose DAC level $= 3000$ for the drive pulse which corresponds to an effective coupling strength $\tilde{g}^2 / 2\pi = 0.845$ MHz. Therefore, a 294 ns square pulse with gradual ramp-up and ramp-down edges (10 ns for each) can provide a fast encoding/decoding process. Because the encoding and decoding drives cause an additional phase due to the Stark shift, we have to calibrate this phase. This is achieved by preparing qubit $Q_2$ in $(|g\rangle + |f\rangle) / \sqrt{2}$ state, then performing the encoding and decoding processes sandwiched with variable waiting times, and finally measuring the state tomography of qubit $Q_2$. We then can counteract this Stark shift by choosing a proper rotation axis of the decoding pulse in the process tomography measurement.

The two-photon transition drive of qubit $Q_1$ is calibrated in a similar way. The holonomic gates for Fock states $|0\rangle$ and $|1\rangle$ are realized when both the two-photon transition and the cavity-assisted Raman transition drives are simultaneously on, therefore the resonant frequencies of the two drives change slightly. We optimize the resonant frequencies of the two drives in order to realize high-fidelity holonomic gates.

For the holonomic gates on $Q_1$, $X_{\pi} = U_2(\pi/2, 0)$ and $Y_{\pi} = U_2(\pi/2, \pi/2)$ with $\theta = \pi/2$, we choose the effective coupling strengths $\tilde{g}_1 / 2\pi = \tilde{g}_3 / 2\pi = 0.25$ MHz for both drives. Two square pulses (1420 ns long) with gradual ramp-up and ramp-down edges (10 ns for each) are used to implement these two holonomic gates. The only difference between these two gates is the phase difference between the two drives. For the two Hadamard gates $H_1 = U_2(\pi/4, 0)$ and $H_2 = U_2(\pi/4, \pi/2)$
with $\theta = \pi/4$, the effective coupling strengths are $\tilde{g}_1/2\pi = 0.25$ MHz and $\tilde{g}_2/2\pi = 0.60$ MHz for the two-photon transition drive and the cavity-assisted Raman transition drive respectively. The time for these two Hadamard gates are 779 ns, also including the gradual ramp-up and ramp-down edges. Since the phase difference between the two drives can be well-controlled, we could realize the holonomic gates $U_2(\theta, \phi)$ with arbitrary $\theta$ and $\phi$. With the same method for the transmon qubit, we can divide the evolution time into two intervals with different phase sets to realize arbitrary single-qubit holonomic gates $U_1(\theta, \gamma, \phi)$ on the Fock states.

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