Goldstone Bosons in the $^3P_2$ Superfluid Phase of Neutron Matter and Neutrino Emission

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Abstract

At the high densities present in the interior of neutron stars, the neutrons are condensed into the $^3P_2$ superfluid phase. While this condensation has little impact on the equation of state, it can have an important role in determining the low-temperature energy-momentum transport properties. The spontaneous breaking of baryon number by the condensate gives rise to the familiar Goldstone boson, but in addition, the spontaneous breaking of rotational invariance by the condensate gives rise to three Goldstone bosons, in general, one for each broken generator of rotations. These Goldstone bosons, which couple to the $Z^0$, provide a new mechanism for neutrino emission. Using a low-energy effective field theory to describe the dynamics of these Goldstone bosons we estimate the neutrino emissivity of dense neutron matter and show that their annihilation is the dominant energy-loss mechanism over a range of temperatures.

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I. INTRODUCTION

One of the greatest challenges facing nuclear physics today is to understand the behavior of nuclear matter away from nuclear matter density. This impacts not only our understanding of the interior of neutron stars, but also our search for the deconfined phase of QCD in relativistic heavy ion collisions. We have a good phenomenological description of the interaction between nucleons that extends up to momenta much greater than Fermi momentum in nuclear matter density \[1\]. However, when nuclear matter is significantly compressed this phenomenological description becomes unreliable as one moves toward the deconfined phase of QCD. In fact, a number of more “exotic” phases have been proposed as the ground state of nuclear matter at high densities, such as the color-flavor locked (CFL) phase \[2\] at asymptotically high densities.

At relatively low densities we have a good description of nuclear interactions which are dominated by the attractive S-waves, with higher partial waves suppressed by powers of the typical momentum. As the density is increased the repulsive nature of the S-waves at higher momenta becomes important and at \[\rho_{nm} \approx 1.5\] times nuclear matter density, the average \[^3P_2\] interactions are the most attractive suggesting the formation of a \[^3P_2\] neutron condensate \[3\]. This has been known for sometime, and considerable work has gone into determining the magnitude of the \[^3P_2\] gap as a function of density with the most sophisticated nuclear potentials \[3, 4\], and also with effective low-energy potentials \[5\]. In this work we point out that since a \[^3P_2\] condensate spontaneously breaks rotational invariance, there will be three Goldstone bosons (angulons). In addition, baryon number is also broken, as in any superfluid, leading to the existence of another, well known, Goldstone boson. These modes will dominate the low-energy, low-temperature properties of the system. In particular they provide an important new mechanism for neutrino emission that is not exponentially suppressed at temperatures below the critical temperature, \(T_c\).

We can make a rough estimate of the size of the contribution coming from angulon annihilation into a neutrino pair using dimensional analysis

\[\mathcal{E} \approx G_F^2 T^9,\]

where \(G_F\) is the Fermi constant and \(T\) the temperature. This estimate assumes that powers of dimensionful quantities like the Fermi momentum \(k_F\) or the value of the gap \(\bar{\Delta}\) are not relevant. We will argue that this is indeed justified. The temperature dependence does not have the characteristic exponential suppression \(\sim e^{-2\bar{\Delta}/T}\) found in processes involving gapped fermions but it is one power of \(T\) higher than the electron-electron scattering contribution. On the other hand it is not suppressed by the low electron density present in \(\beta\)-equilibrated matter. There are two caveats with the estimate in eq. (1). First, the size of the coupling between angulons and the weak neutral gauge boson is not at all obvious, if in fact one exists. Second, the annihilation process is proportional to two powers of the angulon density \(n(T)\), and since the Bose distribution function depends on the energy \(E = \nu p\) of the angulon, the density scales as \(n(T) \sim T^3/\nu^3\). If the angulon speed \(\nu\) is small, the number of angulons in a momentum interval is greatly enhanced, and therefore so is the emissivity. For this reason it is important to obtain an estimate of both \(\nu\) and the dependence of the emissivity on \(\nu\). We will estimate these factors using an effective theory to organize our arguments. A true model independent calculation is, unfortunately, not possible at the moment.
A. The effective theory

When neutrons condense in the $^3P_2$ superfluid phase the order parameter is given by

$$i \langle n^T \sigma_2 \sigma_1 \vec{\nabla}^k n \rangle = \Delta^{jk},$$

where $n$ are neutron field operators, $\vec{\nabla}^j = \vec{\nabla}^j - \vec{\nabla}^j$, and $\sigma_2 \sigma_j$ acts in spin-space. As the neutrons are coupled together in the $^3P_2$ state, the order parameter $\Delta^{jk}$ is a symmetric, traceless tensor. In general, a traceless, symmetric tensor is determined by two orthonormal frames that diagonalize its real and imaginary parts, and two of the eigenvalues (the third one follows from the tracelessness condition). Depending on the value of the eigenvalues different phases with different unbroken symmetries arise. It is the dynamics of the system that determines which one of those phases is the true ground state, symmetry arguments alone cannot determine it. Unfortunately, not much is known about the form of the order parameter in dense neutron matter. At temperatures close to the critical one, arguments based on the Landau-Ginsburg energy suggest that $\Delta^{ij}_0$ is real, but the relative sizes of the eigenvalues are hard to predict \cite{6}. At zero temperature those arguments fail and much less is known. Due to this uncertainty we will consider here a particular choice of the order parameter that makes some calculations feasible. The order of magnitude of the emissivity likely will not depend on this choice.

We consider the phase where the equilibrium value of the gap matrix $\Delta_0$ has eigenvalues equal to the cubic roots of the unity. That is, there is an orthonormal frame where it can be written as

$$\Delta_0 = \Delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i \frac{2\pi}{3}} & 0 \\ 0 & 0 & e^{-i \frac{2\pi}{3}} \end{pmatrix}.$$  \hspace{1cm} (3)

Rotational invariance is completely spontaneously broken down to a discrete subgroup and as a consequence there are three exactly massless Goldstone bosons, one for each rotation generator. In addition, as $\Delta \neq 0$ spontaneously breaks baryon number, there is a fourth (and well-known) Goldstone boson. Nuclear forces do not conserve spin and orbital angular momentum separately, due to the tensor and spin-orbit forces. The smallness of the $^3P_2$ gap, as well as its small mixing with the $^3F_2$ channel, suggests that the effective strength of those forces at the Fermi surface is small. For the sake of argument it will be convenient to consider the enlarged rotation symmetry group including independent spin and orbital rotations. This group is explicitly broken by the tensor and spin-orbit interactions down to the diagonal group of combined spin and orbital rotations. The formation of the gap further spontaneously breaks this group down to a discrete subgroup which depends on the particular form of the eigenvalues of $\Delta_0$. \footnote{For special cases, as when two of the eigenvalues are degenerate, a $O(2)$ subgroup of rotations is left unbroken.} The analysis that follows does not depend on the particular discrete group left unbroken and we will not discuss it further. This situation is represented by the diagram:
$SU_S(2) \otimes SO_L(3) \otimes U(1) \xrightarrow{\text{tensor/spin-orbit}} SO_J(3) \otimes U(1)$

\[
\begin{array}{c}
\langle mn \rangle \neq 0 \\
\text{discrete}
\end{array}
\xrightarrow{\text{tensor/spin-orbit}}
\begin{array}{c}
\langle mn \rangle \neq 0 \\
\text{discrete}
\end{array}
\]

where the horizontal arrows represent explicit breaking and the vertical arrows denote spontaneous breaking due to the pairing. Disregarding for the moment the effect of the tensor and spin-orbit forces we expect seven Goldstone bosons. We write the order parameter as $\Delta = U \Delta = U \xi_S \Delta_0 \xi_L$, where $U = e^{i2\phi/f_0}$ is a phase and $\xi_S$ and $\xi_L$ are orthogonal matrices.

The transformation rules under phase ($e^{i\theta}$), spin ($R_S$) or spatial ($R_L$) rotations are:

\[U \rightarrow e^{2i\theta} U, \quad \Delta \rightarrow R_S(\theta_S) \Delta R^T_L(\theta_L).\]

At low energies the system can be described by the most general Lagrangian containing the low energy degrees of freedom (the Goldstone bosons). The Lagrangian invariant under the full $SU_S(2) \times SO_L(3) \times U(1)$ group is

\[
\mathcal{L} = \frac{f^2}{8\Delta^2} \left[ \text{Tr}[\partial_0 \Delta \partial_0 \Delta^\dagger] - v^2 \text{Tr}[\partial_i \Delta \partial_i \Delta^\dagger] - w^2 \partial_i \Delta^\dagger_{ik} \partial_j \Delta_{kj} \right] + \frac{f_0^2}{8} \left[ \partial_0 U \partial_0 U^\dagger - v_0^2 \partial_i U \partial_i U^\dagger \right] + iH_V Z_0^0 (U \partial_0 U^\dagger - \partial_0 U U^\dagger) + iH_A Z_0^0 \text{Tr}[J^i (\Delta \partial_0 \Delta^\dagger - \partial_0 \Delta \Delta^\dagger)] + \cdots,
\]

where $Z_0^0$ and $Z_i^0$ are the time and spatial components of the $Z^0$ boson. Missing from eq. (5) are terms that break the non-diagonal part of the rotation group, terms that do not vanish for non-unitary $\Delta_0/\Delta$, as well as terms with more derivatives whose contribution to low energy observables are suppressed by powers of the typical energy divided by $\Delta$.

To the terms explicitly shown in eq. (5) we have to add terms that break separate spin and orbital rotations. Since the spin-orbit interaction drives the formation of the gap, its strength is suppressed by $\sim 1/\log(\Delta/\mu)$. One of its effects is to give a mass to three out of the seven Goldstone bosons, specifically, to the ones corresponding to opposite spin and orbital rotations $\xi_S = \xi_L^\dagger$. The remaining four are strictly massless, as they correspond to the breaking of the exact rotation and baryon number symmetry. Even though the size of this mass term is suppressed, for small enough temperatures, the number of these pseudo-Goldstone bosons is exponentially suppressed, and we will discard them. For the range of temperatures where it is relevant (if any), this extra degrees of freedom could contribute to the $\bar{\nu} \nu$ emissivity and our calculation should be considered as a lower bound. On the other hand, the effect of the symmetry breaking terms on the interactions should be suppressed and we disregard them in our order of magnitude estimate.

**B. Determination of Low-Energy Constants**

In order for the low-energy effective field theory discussed in the previous section to be predictive, the *a priori* unknown coefficients that enter $v$, $w$, $f$, $H_V$ and $H_A$, must be determined from QCD. Such a matching is not possible at this point in time but we are helped by the fact that all that is needed is information about neutron interactions close to
the Fermi surface. Thus we can imagine integrating out modes away from the Fermi surface and obtaining an effective theory valid for small excitations around the Fermi sphere. It is known that, to leading order, the resulting theory is very simple and contains, besides a kinetic term with a modified Fermi speed, only the interactions leading to the formation of the gap [12]. That means that any underlying theory resulting in the same Fermi speed and gap will lead to the same low energy properties of the system. We choose then a particularly simple one \( \mathcal{L}^N = \mathcal{L}^N_S + \mathcal{L}^N_W \) with

\[
\mathcal{L}^N_S = n^\dagger \left( i\partial_0 + \frac{\nabla^2}{2\tilde{M}} + \mu \right) n - g_{3P_2} \chi^{kl}_{ij} (n^T \sigma_2 \sigma^k \hat{V}_i^l n)^\dagger n^T \sigma_2 \sigma^l \hat{V}_j^k n ,
\]

\[
\mathcal{L}^N_W = C_V Z_0^0 n^\dagger n + C_A Z_i^0 n^\dagger \sigma^i n - g_{Z\pi} Z_\mu^0 \nabla \gamma^\mu (1 - \gamma_5) \nu ,
\]

where the tensor \( \chi^{kl}_{ij} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2/3\delta_{ij}\delta_{kl}) \) is a projector onto the \( ^3P_2 \) channel, \( \mu \) is the neutron chemical potential, and \( g_{3P_2} \) is an effective strong coupling constant in the \( ^3P_2 \) channel. Not much is known about the renormalization of the weak interactions by the process of integrating out modes away from the Fermi surface. In particular we do not know whether the form above is universal, in the same sense as the strong part of the Lagrangian is.

We have kept the same form of the interaction as in the vacuum and also retained only the leading order terms in the derivative and multi-body expansion of the weak interactions. The multi-body operators are analogous to \( L_{1,A} \) in the pionless effective field theory [10].

The fact that the strong interaction in eq. \( (6) \) occurs in the \( ^3P_2 \) channel is suggested by the vacuum value of the phase shifts for nucleon-nucleon scattering, and is supported by sophisticated nuclear models. This is an assumption underlying our work. The neutron mass is renormalized when modes far from the Fermi surface are integrated out of the theory and thus \( \tilde{M} \) is the renormalized mass or the “in-medium” effective mass. An analogous renormalization occurs for the coupling to the weak currents and the chemical potential \( \mu \).

The model in eq. \( (6) \) favors neutron spin-pairing and the formation of a gap in the \( ^3P_2 \)-channel. The strong coupling \( g_{3P_2} \) in eq. \( (6) \) is traded for the neutron gap, via \( (\tilde{\Delta}_0)_{ij} = -g_{3P_2} (n^T (-p) \sigma_2 \sigma_i p_j n(p)) \). The tilde is to denote the model-dependence of this gap. The propagator for neutrons in this condensed phase is

\[
i S(p_0, \mathbf{p}) = \frac{i}{p_0^2 - \epsilon_p^2 - p^j (\tilde{\Delta}_0)_{ij} p^i} \begin{pmatrix}
  p_0 + \epsilon_p & -i(\tilde{\Delta}_0)_{ij} \sigma_2 \sigma_i p^j \\
  i(\tilde{\Delta}_0)_{ij} \sigma_i p^j & p_0 - \epsilon_p
\end{pmatrix} ,
\]

where \( \epsilon_p = |\mathbf{p}|^2/(2\tilde{M}) - \mu \).

The neutral current couplings in eq. \( (6) \) are

\[
g_{Z\pi}^2 = \frac{G_F M_Z^2}{2\sqrt{2}} , \quad C_V^2 = \tilde{C}_V^2 \frac{G_F M_Z^2}{2\sqrt{2}} , \quad C_A^2 = \tilde{C}_A^2 \frac{G_F M_Z^2}{2\sqrt{2}} ,
\]

where \( \tilde{C}_V = -1 \), constrained by vector current conservation, and \( \tilde{C}_A = g_A + \Delta s \sim 1.1 \pm 0.15 \) [11]. \( g_A \sim 1.26 \) is the nucleon isovector axial coupling that is well measured in nuclear \( \beta \)-decay, while \( \Delta s \) is the matrix element of the strange axial-current in the proton that is measured in deep-inelastic scattering and neutrino-nucleon interactions.

C. Estimate of \( f_0 \) and \( v_0 \)

In order to determine the parameters for the phonon \( \phi \) it is convenient to introduce a fictitious \( U(1) \) gauge-symmetry into the theory. We then require that the low-energy
FIG. 1: The one-loop diagrams that determine the decay constants $f_0$ and $v_0$. The solid line denotes a neutron, while the wiggly line denotes the fictitious gauge field $A_0$.

effective field theory reproduce matrix elements of the underlying theory. At leading order in the derivative expansion, this amounts to replacing partial derivatives with covariant derivatives:

$$
\partial_\mu n \rightarrow D_\mu n = \partial_\mu n + i A_\mu n \quad , \quad \partial_\mu \Delta \rightarrow D_\mu \Delta = \partial_\mu N + 2 i A_\mu \Delta \quad ,
$$

where $A_\mu$ is the fictitious gauge field associated with the fictitious $U(1)$ gauge-symmetry. In the underlying theory $A_0$ couples to the neutron density and acts as a chemical potential. The correlation function $\langle A_0 A_0 \rangle$ is the linear response function determining how the density changes due to a change in chemical potential. In other words, it is the density of states at the Fermi surface $dN/d\mu = M k_F^2/\pi^2$ (up to corrections of order $\sim \Delta/\mu$), where $k_F$ is the Fermi momentum. In a diagrammatic calculation, $\langle A_0 A_0 \rangle$ is given by the two diagrams of Fig. (1). Using the propagator in eq. (7) we find that the two diagrams give equal contributions

$$
\langle A_0 A_0 \rangle = \frac{2}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{-(k_0 + \epsilon_k)^2}{(k^2 - \epsilon_k^2 - k_0 \Delta_0 \Delta_0^\dagger k_0)^2} = i \int \frac{d^3 k}{(2\pi)^3} \frac{k^2 \Delta^2}{(\epsilon_k^2 + k^2 \Delta^2)^{3/2}} \approx i \frac{M k_F}{\pi^2} + O(\Delta^2/\mu^2) ,
$$

where we used the fact that the integral is dominated by momenta around the Fermi surface. Similarly, one can compute $\langle A_i A_j \rangle$. In the underlying theory, in Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$, it is given by the same two diagrams in Fig. (1) plus a tadpole graph with a $-A_i A^i n^i n/(2M)$ vertex, as shown in Fig. (2). In contrast to the calculation of $\langle A_0 A_0 \rangle$ there is a relative minus sign for the anomalous graph in Fig. (2) due to the derivative coupling at the vertices. Consequently, these two graphs cancel, leaving the tadpole contribution

$$
i \langle A_i A_j \rangle = -i \frac{N}{M} \delta_{ij} \approx -i \frac{k_F^3}{3\pi^2 M} \delta_{ij} .
$$

In the effective theory $\langle A_0 A_0 \rangle$ and $\langle A_0 A_0 \rangle$ are given by the tree level contributions and we find

$$
i \langle A_0 A_0 \rangle = i \frac{M k_F}{\pi^2} = i f_0^2 ,
$$

$$
i \langle A_i A_j \rangle = -i \frac{k_F^3}{3\pi^2 M} \delta_{ij} = -i f_0^2 v_0^2 \delta_{ij} .
$$

We conclude that

$$f_0^2 = \frac{M k_F}{\pi^2} , \quad v_0 = \frac{v_F^2}{3} .
$$

The breakdown of rotational invariance plays no role in the propagation of $\phi$ and, consequently, the value of $v_0$ agrees with a general analysis which assumes rotational invariance (after a suitable generalization to the non-relativistic context) [13].
D. Estimate of \( f \) and \( v \)

To estimate \( f \) and \( v \) we consider the fictitious approximate gauge spin rotation symmetry \( \eta \rightarrow e^{i\frac{\pi}{2} \cdot \theta} n \). The microscopic theory possesses this symmetry (up to corrections due to tensor/spin-orbit forces) if the neutrons couple to the fictitious gauge field \( B_\mu \) through a covariant derivative

\[
D_\mu n = \partial_\mu n + iB_\mu^i \sigma^i n. \tag{14}
\]

The effective theory should then be written in terms of covariant derivatives of the \( \Delta \) field

\[
D_\mu \Delta = \partial_\mu \Delta + iB_\mu^i \bar{J}^i \Delta. \tag{15}
\]

We then match the quantities \( \langle B_0^i B_0^i \rangle \) and \( \langle B_1^i B_1^i \rangle \) in the microscopic and the effective theory. The graphs contributing to them in the microscopic theory are the same ones as in the case of the \( A \) field correlators. We find

\[
i\langle B_0^i B_0^i \rangle = \frac{1}{4} \int \frac{d^4k}{(2\pi)^4} \frac{-(k_0 + \epsilon_k)^2 \text{Tr}[\sigma^i \sigma^j] - (\Delta_0 \cdot k_a)(\Delta_0^\dagger k_b) \text{Tr}[\sigma^a \sigma^b \sigma^c \sigma^d]}{(k_0^2 - \epsilon_k^2)^2}
\]

\[
= i\delta_{ij} \frac{Mk_F}{6\pi^2} \tag{16}
\]

where we used \( \text{Tr}[\sigma^b \sigma^a \sigma^c \sigma^d] = 2(\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk} + \delta_{jk} \delta_{il}) \). Notice that the denominators of the fermion propagators are spherically symmetric, as \( k, |\Delta_0|^2, k = k^2 \Delta^2 \). We also match the spatial part:

\[
i\langle B_1^i B_1^i \rangle = -\frac{1}{4M^2} i^2 \int \frac{d^4k}{(2\pi)^4} \frac{-(k_0 + \epsilon_k)^2 \text{Tr}[\sigma^i \sigma^j] - (\Delta_0 \cdot k_a)(\Delta_0^\dagger k_b) \text{Tr}[\sigma^a \sigma^b \sigma^c \sigma^d]}{(k_0^2 - \epsilon_k^2)^2}
\]

\[
- \frac{i}{2M} \text{Tr}[\sigma^i \sigma^j] \delta_{kl} \int \frac{d^4k}{(2\pi)^4} \frac{i(k_0 - \epsilon_k)}{k_0^2 - \epsilon_k^2}
\]

\[
\cong i \frac{k_F^2}{4\pi^2 M} \left[ -\frac{4}{15} \delta^{ij} \delta_{kl} + \frac{1}{30 \Delta^2} (\Delta_{0k}^{ij} \Delta_{0l}^{ij} + \Delta_{0l}^{ij} \Delta_{0k}^{ij} + \Delta_{0k}^{ij} \Delta_{0l}^{ij} + \Delta_{0l}^{ij} \Delta_{0k}^{ij}) \right] \tag{17}
\]

where the integrals were computed up to corrections of orders \( O(\Delta/\mu) \) but the matching is valid only up to much larger terms, of order \( g \sim 1/\log(\Delta_0/\mu) \), since the spin rotation symmetry is only an approximate symmetry.

The same matrix elements are given in the effective theory by

\[
i\langle B_0^i B_0^i \rangle = i\frac{f^2}{2} \delta_{ij}, \quad i\langle B_1^i B_1^i \rangle = -i\frac{f^2 v^2}{2} \delta_{ij} \delta_{kl} - i\frac{f^2 w^2}{8} (\Delta_0^i (J^i J^j + J^j J^i) \Delta_0 + \text{c.c.})_{kl} \tag{18}
\]

and by matching to the expressions in the full theory using the relation

\[
(\Delta_{0k}^{ij} \Delta_{0l}^{ij} + \Delta_{0l}^{ij} \Delta_{0k}^{ij} + \Delta_{0k}^{ij} \Delta_{0l}^{ij} + \Delta_{0l}^{ij} \Delta_{0k}^{ij}) = 4\Delta^2 \delta_{ij} \delta_{kl} - (\Delta_0^i (J^i J^j + J^j J^i) \Delta_0 + \text{c.c.})_{kl} \tag{19}
\]

gives

\[
f^2 = \frac{Mk_F}{3\pi^2}, \quad v^2 = \frac{1}{5} v_F^2, \quad w^2 = \frac{1}{5} v_F^2. \tag{20}
\]
We parameterize the scalars fields as $U = e^{2i\phi/f_0}$, $\xi = e^{i\sqrt{3}/2}f\pi/\xi$ so the fields $\phi$ and $\pi$ will be canonically normalized. However, the space derivative terms mix the different components of $\pi^i$. The matrix for the quadratic of the $\pi^i$ Lagrangian reads

$$L_0 = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \pi_i(-p) G(p)_{ij} \pi_j(p),$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$. The kinetic part can be diagonalized by using the fields $\alpha^i$ defined by

$$\pi_i(p) = K_{ia}(p)\alpha_a \quad .$$

The explicit form of $K_{ia}(p)$ can be easily found but we will omit it here since is not very enlightening.

E. Estimate of the Weak Couplings $H_A, H_V$

Looking at the coupling of the fictitious fields $A_\mu$ and $B_\mu$ to the neutrons we see that we can identify

$$A_0 \to -C_V Z_0^0 \quad ,$$
$$B_0^i \to -2C_A (Z^0)^i \quad ,$$

from which we can determine $H_V$ and $H_A$ to be

$$H_V = -\frac{f^2}{4} C_V \quad , \quad H_A = -\frac{f^2}{4\Delta^2} C_A \quad .$$

Among other contributions, these weak coefficients receive corrections of order $\sim 1/\log(\bar{\Delta}/\mu)$ coming from the strong interactions that are not invariant under local spin rotations. In terms of the eigenmodes $\alpha^i$ we have the couplings

$$L_{Z^0} = -f_0 C_V Z_0^0 \partial_0 \phi - fC_A \sqrt{3} Z_i^0 K_{ia} \partial_0 \alpha_a + \frac{C_A}{2} Z_k^0 \epsilon^{ijk} K_{ia}(p) K_{jb}(k) \alpha_a(p) \partial_0 \alpha_b(k) + \cdots$$

FIG. 2: The one-loop diagrams that determine $f$ and the “speeds” $v$ and $w$ in eq. 5. The solid line denotes a neutron, while the wiggly line denotes the fictitious gauge field $B_i$. The solid circle denotes an insertion of the tadpole vertex.
\section{Neutrino Emissivity}

The Goldstone bosons identified in this work will contribute to the neutrino emissivity of dense nuclear matter at finite temperature. The main mechanism is the annihilation process $\alpha_i\alpha_j \rightarrow \bar{\nu}\nu$. The amplitude for the annihilation of $\alpha_i$ and $\alpha_j$ is given by

$$M_{ab} = \frac{G_F \hat{C}_A}{4\sqrt{2}} \left[ H_{abh}(k) + H_{bak}(p) \right] \bar{\nu}\gamma^k(1 - \gamma_5)\nu \quad (\text{no sum implied}), \quad (26)$$

where $E_i(p)$ is the energy of the angulon $\alpha_i$ with three momentum $p = |p|$ and $H_{abh} = \sum_{i,j=1}^3 e^{ijk}K_{ia}(p)K_{jb}(k)$.

The emissivity due to these annihilation processes is defined as the energy loss due to neutrino emissions per unit time per unit volume, and for $\alpha_i\alpha_j \rightarrow \bar{\nu}\nu$ it is given by

$$\mathcal{E}_{ab} = \int \frac{d^3p}{(2\pi)^32E_{\nu}} \frac{d^3\bar{\nu}}{(2\pi)^32E_{\bar{\nu}}} \frac{d^3p}{(2\pi)^32E_{a}(p)} \frac{d^3k}{(2\pi)^32E_{b}(k)} n(E_a(p)) n(E_b(k))$$

$$\left( E_{\nu} + E_{\bar{\nu}} \right) (2\pi)^4 \delta^{(4)}(p + k - \nu - \bar{\nu}) \sum_{s,s'} |M_{ab}|^2, \quad (27)$$

where the angulons $\alpha_a$, $\alpha_b$ carry incoming momenta $p$, $k$ respectively, as indicated. $n(E) = [\exp(E/T) - 1]^{-1}$ is the Bose distribution function and $\sum |M_{ab}|^2$ is the spin summed squared matrix element obtained from eq. \ref{26}. Using the Lorentz invariant quantity

$$I_{\mu\nu} = \int \frac{d^3p}{(2\pi)^32E_{\nu}} \frac{d^3\bar{\nu}}{(2\pi)^32E_{\bar{\nu}}} (2\pi)^4 \delta^{(4)}(q - p - \nu - \bar{\nu}) \frac{1}{\theta(q_0)} \theta(q^0) \theta(q^2) \left( \bar{\nu}\gamma^\mu \bar{\nu}\gamma^\nu(1 - \gamma_5) \right)$$

for the neutrino phase-space integration gives

$$\mathcal{E}_{ab} = \frac{1}{3\pi} \left( \frac{G_F \hat{C}_A}{4\sqrt{2}} \right)^2 \int \frac{d^3p}{(2\pi)^32E_{\nu}(p)} \frac{d^3k}{(2\pi)^32E_{b}(k)} (E_a(p) + E_b(k)) n(E_a(p)) n(E_b(k))$$

$$\left[ H_{abh}(k) + H_{bak}(p) \right] \left[ H_{ahl}(k) + H_{kal}(p) \right] (p + k)_a(p + k)_l + (E_a(p) + E_b(k))^2 \delta_{kl} - |p + k|^2 \delta_{kl}$$

$$\theta \left( (E_a(p) + E_b(k))^2 - |p + k|^2 \right) \theta(q_0) \theta(q^0) \theta(q^2) \right), \quad (28)$$

The phase space integral is complicated by the angular dependence of the vertex tensor $H_{abh}$ but we can extract its dependence on $T$ and $\nu$ ($= w$). First, redefine $E_i(p) = v\tilde{E}_i(p)$ in order to have the factors of $\nu$ explicit. Then expand the step function as $\theta(v^2(\tilde{E}_i(p) + \tilde{E}_j(k))^2 - |p + k|^2) \approx \theta(-|p + k|^2)) + v^2(\tilde{E}_i(p) + \tilde{E}_j(k))^2 \delta(v^2(\tilde{E}_i(p) + \tilde{E}_j(k))^2 - |p + k|^2) + \cdots$. The first term does not contribute to the integral and we are left with

$$\mathcal{E} \sim \nu^5 \int \frac{d^3p}{(2\pi)^32E_{\nu}(p)} \frac{d^3k}{(2\pi)^32E_{\nu}(k)} n(v\tilde{E}_a(p)) n(v\tilde{E}_b(k)) \left( \tilde{E}_a(p) + \tilde{E}_b(k) \right)^5$$

$$\left[ H_{abh}\tilde{E}_b(k) + H_{bak}\tilde{E}_a(p) \right] \left[ H_{ahl}\tilde{E}_l(k) + H_{kal}\tilde{E}_a(p) \right] \delta(-|p + k|^2))$$

$$\nu^3 \frac{79}{v^3}, \quad (30)$$
where in the last step we rescale the momenta as \( p \rightarrow T x / v, k \rightarrow T y / v \) and used the fact the delta function restricts the six-dimensional integral to back-to-back pairs with \( \vec{p} = -\vec{k} \) only. We have checked that in the analytically calculable case where the vertex is independent of the angle, the scaling above is obeyed.

Our estimate for the annihilation process is then

\[
\mathcal{E} = 6 \times \frac{1}{3\pi} \left( \frac{G_F C_A}{4\sqrt{2}} \right)^2 \frac{T^9}{v^3}
\]

\[
\cong 10^{17} T^9 \left( \frac{0.15}{v} \right)^3 \text{erg cm}^{-3} \text{s}^{-1},
\]  

(31)

where we used for the numerical estimates \( v = 0.15 \) (corresponding to \( k_F \sim 308 \text{ MeV} \), \( M = 940 \text{ MeV} \), neglecting the renormalization of the mass and chemical potential), \( T_9 = T / (10^9 \text{ K}) \) and the factor of six is due to the six possible combinations of angulon pairs annihilated.

At typical temperatures, e.g. \( T \sim 3 \times 10^8 K \sim T_c / 10 \), and densities the emissivity due to electron bremsstrahlung is of order \( \mathcal{E}_e \sim 10^{10} \text{ erg cm}^{-3} \text{s}^{-1} \), which is significantly less than that due to angulon annihilation of \( \mathcal{E}_{\alpha\alpha} \sim 10^{12} \text{ erg cm}^{-3} \text{s}^{-1} \), where we have used \( v_F \sim 0.33 \). Further, comparing with processes involving the neutrons near the Fermi surface, such as modified Urca and neutron bremsstrahlung, one finds that at temperatures much below the critical temperature, \( T_c \), where such processes are exponentially suppressed \([14, 15]\), the annihilation of angulons is likely to dominate the emissivity as it is power-law suppressed only.

### III. CONCLUSIONS

We have pointed out that in the \(^3P_2\) neutron condensed phase that is favored for densities greater than \( \sim 1.5 \rho_{nm} \), there are Goldstone modes that contribute to the neutrino emissivity and the energy-momentum transport properties. Using effective field theory arguments we estimated the neutrino emissivity from finite temperature, superfluid neutron matter for a particularly simple form for the gap, one that gives a unitary order parameter. We showed that the emissivity can be related to the Fermi speed and the parameters determining the anisotropy of the gap, but is not strongly dependent on the value of the gap itself. By using reasonable estimates of these parameters we estimate an emissivity larger than other processes involving neutrons, which are exponentially suppressed, and larger than that from electron bremsstrahlung, for the densities and temperatures relevant to neutron stars. Further, these Goldstone bosons will likely dominate other low temperature observables such as neutrino opacity and viscosities. Our calculation was dependent on a particular choice of the form of the gap parameter. This highlights the fact that a determination of the actual phase of cold neutron matter would have a larger impact on understanding the cooling processes of the \(^3P_2\) phase than a precise determination of the size of the gap. A better assessment of the impact of angulon annihilation in the cooling of neutron stars requires the rates computed in this paper be inserted into a realistic cooling code.
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