Research on Design and Mechanism of Sound Insulating Structure Based on Acoustic Metamaterials

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Abstract. In view of the difficult problems of low-frequency noise reduction, based on the clamping boundary-type acoustic metamaterial theory with negative effective mass density, a structure combining membrane-type and clamping boundary-type is designed, which can realize good sound insulation performance in the low-frequency region, and the theoretical calculation and numerical simulation are in good agreement. At the same time, the transmission property of the structure at zero refractive index and impedance matching is studied. The research suggests that the structure has good low-frequency broadband sound insulation performance, and can achieve high transmission performance of sound waves through structural regulation.

1. Introduction

From a physiological point of view, noise is the sound that people do not need. It can annoy people, destroy tranquility, affect physical health, interfere with conversation, etc. The harm of noise is expanding with the improvement of industrialization. In large and medium-sized cities with clustering of industries and relatively developed transportation, noise has become one of the major public hazards endangering people's physical and mental health. At the same time, in the field of aerospace, particularly, the control of low-frequency noise in cabins of large aircraft has become an important factor affecting aircraft performance. As people have a deeper understanding of the damage and control of noise, sound insulation is the most common way of noise control [1-3]. At present, sound insulation technology is mainly divided into two types. One is the traditional sound insulation technologies, which mainly reduce the propagation of noise through the reflection and absorption of sound waves by barriers, and control the noise by the structural parameters and material types of barriers such as designed composite materials and double-layer materials [4]. The other is to use the phononic crystal’s band-gap function for sound insulation designs, mainly including two mechanisms of Bragg band-gap [5-9] and local resonance band-gap [10-13], which regulates the band-gap of phononic crystals by changing the periodic acoustic medium constant.

Low-frequency noise mainly refers to the sound with a frequency in the range of 10~400 hertz. As the wavelength of the low-frequency sound wave is long, when it propagates in the air, air molecules have slight vibration, are slow in friction and consume less energy, so it goes far and has good transmission capacity. Therefore, sound waves caused by earthquakes, tsunamis and the like can propagate very far. Traditional sound insulation barriers have little effect on low-frequency sound waves. As a result, it is necessary to study the control of low-frequency noise based on new mechanisms. The team led by professor Hu Gengkai of Beijing Institute of Technology studied that under the clamping boundary condition, phononic crystals have negative effective mass below the cut-off frequency, which can cause rapid acoustic attenuation [14]. The team led by professor Liu
Zhengyou of Wuhan University studied the insertion of rigid inclusions into two-dimensional solid-based phononic crystals. Theoretical and experimental results have proved that there is a wide acoustic band-gap below the cut-off frequency \[15\]. Liang Z. X. and Li Y. et al. used curling space to design phononic crystals to obtain low-frequency band-gaps at low frequencies \[16-18\]. The team led by professor Wu Jiuhui of Xi'an Jiaotong University used dual-core resonance to obtain better low-frequency sound insulation effect \[18\].

In the light of simple structure and good low-frequency sound insulation effect in actual projects, this paper designed and studied the variation of sound insulation quantity of multilayer thin plates under different conditions such as central point constraint, edge point constraint and alternately fixed constraint, and analyzed the sound insulation mechanisms. Meanwhile, it conducted numerical simulation and theoretical analysis on the transmission performance and mechanism of impedance matching zero index metamaterials.

2. Equivalence Theory Model

Sound insulation \(TL\) of unrestrained single-layer thin plate can be calculated according to the law of mass action, as shown in Formula 1:

\[
TL = -42 + 20\log_{10} f + 20\log_{10} M
\]

Here, \(f\) refers to the incident wave frequency, and \(M\) is the mass of the partition wall per unit area.

Thin plates with clamping boundary at both ends can be regarded as rigid-elastic phononic crystals with thin waveguides. \[16\] The equivalent model is shown in Figure 1.

![Figure 1 Equivalent model of thin plates with clamping boundary at both ends](image1)

Here, \(m\) refers to the equivalent mass, and \(K\) and \(G\) are Young’s modulus and shear modulus of the materials respectively. So based on the literature \[16\], we can obtain the horizontal cut-off frequency \(\omega_c = \sqrt{\frac{2G}{m}}\).

The equivalent model of plates with clamping boundary at the center is shown in Figure 2, and the cut-off frequency \(\omega_c = \sqrt{\frac{G}{m/2}} = \sqrt{\frac{2G}{m}}\).

![Figure 2 Equivalent model of plates with clamping boundary at the center](image2)

The equivalent model of alternately clamping plates is shown in Figure 3, and the cut-off frequency \(\omega_c = \sqrt{\frac{2G}{m/2}} = \sqrt{\frac{4G}{m}}\).
According to the theoretical calculation of the cutoff frequencies of the above-mentioned three models, it is found that the cutoff frequency of the two models of clamping at the center and at both ends are equal. And after the two models are periodically arranged, it is found that the cutoff frequencies are doubled in the same direction. This provides a theoretical basis for us to design a new type of low-frequency broadband sound insulation material.

3. Model Numerical Simulation & Theoretical Calculation and Analysis

3.1. Establishment of Simulation Model

According to the theoretical calculation and analysis above, we designed a two-dimensional periodic structure with alternately clamping at the center and both ends, and the material is steel (elastic modulus $E= 21.6 \times 10^{10} \text{N/m}^2$, density $\rho=7.8 \times 10^3 \text{kg/m}^3$, Poisson's ratio $\sigma =0.28$). Thickness of each piece of steel is 1mm, which consists of 10 units, with the spacing of 1mm and height of 200mm. In the air medium, the schematic diagram of the model is shown in Figure 4. The rectangle represents the steel plate, and the black solid square represents the clamping boundary.

3.2. Comparative Analysis of Simulation Results and Theoretical Calculation

The plane wave is under normal incidence from the left side. Under the boundary condition of hard acoustic field, the acoustic-solid coupling module in the software COMSOL is used for numerical calculation and simulation at the frequency range of 10~1000Hz, and the reflection and transmission coefficient method\(^\text{[19]}\) is adopted to calculate the equivalent parameters, which covers two models of clamping at the center and at both ends. Meanwhile, the sound insulation quantity of the three models is compared. The sound insulation quantity $TL= 10\log_{10}(1/t_i)$. Here, $t_i$ is the transmission coefficient of sound intensity, as shown in Figure 5. As shown in the figure, it can be seen that in the frequency range of 10~400Hz, the sound insulation of the cross-connection structure is significantly higher than that of the other two structures. In this range, if the sound insulation is greater than120dB, it shows that the structure has high-strength sound insulation effect. By comparison, the sound insulation of single-point connection and double-points connection structures is obviously reduced. In the frequency range of 10~200Hz, the first two structures also have relatively good sound insulation effect and in the frequency range of 20~400Hz, the sound insulation of these two structures decreases obviously, indicating that the cut-off frequency is around 200Hz. For the third structure, it keeps high sound insulation in the frequency range of 10 ~ 400Hz, which is in good agreement with the theoretical calculation results in the first part. As for sudden changes in sound insulation of cross-connection and
single-point connection in the frequency range of 400–600Hz, it can be explained by mass density, as shown in Figure 6 (a). When the incident frequency is lower than 400Hz, the effective mass density of the structure’s real part is negative, and the equivalent elastic modulus is close to 0, as shown in Figure 6 (b). The single negative structure inevitably leads to a pure imaginary index of refraction, and the incident sound wave presents the exponential decay, so there is large quantity of sound insulation when the frequency is lower than 400Hz. At the frequency range of 400–600Hz, the effective mass density of the structure first increases rapidly, then decreases sharply to less than 0, and again slowly grows to greater than 0 at the frequency around 600 Hz. Therefore, at the frequency range of 400–600Hz, the sound insulation quantity fluctuates, and when the incident frequency is greater than 600Hz, the equivalent mass is greater than 0, resulting in a rapid decline in sound insulation. For single-point connection structure, when the incident frequency is less than 400Hz, the equivalent mass is negative, and the sound insulation quantity is large, but when the frequency is greater than 400Hz and less than 550Hz, the effective mass density is suddenly less than 0, as shown in Figure 6 (c). So the structure has good sound insulation in the frequency range of 400–550Hz. When the frequency of incident plane wave is greater than 550Hz, the effective mass density is greater than 0, causing the sound insulation of the structure to decrease rapidly.

Figure 5 Sound insulation of three structures

(a) Effective mass density of cross-connection  (b) Equivalent elastic modulus of cross-connection
In order to further clarify the sound insulation advantages of this cross-connection structure through comparison, we selected steel plates and rubber with the same thickness and height (elastic modulus $E = 0.3 \times 10^6$ N/m$^2$, density $\rho = 1.2 \times 10^3$ kg/m$^3$, Poisson's ratio $\sigma = 0.4$) for the simulation calculation of sound insulation quantity and the results are shown in Figure 7. As shown in the figure, it can be seen that in the frequency range of 0~600Hz, among the three structures, the cross-connection periodic structure has the best sound insulation effect. Although the pure steel plate has larger sound insulation quantity in the range of 600~1000Hz, the weight of this solid steel plate greatly increases and is severely restricted in practical application. Especially in the low-frequency range of 0~600Hz, the cross-connection periodic structure has the advantages of light weight and good sound insulation. Therefore, the cross-connection periodic structure shows excellent low-frequency acoustic wave control performance, and potential value in practical engineering applications. Excellent sound insulation performance is closely related to the cross-connection structure and the material properties. The future studies can focus on the sound insulation performance and other related performance of different cross-connection structures or materials. According to the theoretical basis of the first part, based on the formula $\rho_{\text{eff}} = \rho \left(1 - \frac{\omega^2}{\omega_n^2}\right)$, we conducted theoretical calculation of the relationship between effective mass density of cross-connection structures $\rho_{\text{eff}}$ and incident wave frequency $\omega_n$ and compared it with the simulation results, as shown in Figure 8. As can be seen from the figure, the simulation results are basically consistent with the theoretical calculation results, indicating that the simulation results are reliable.
3.3. Zero Index Impedance Matching Material

Figure 9 shows the sound intensity transmission coefficient of the cross-connection structure. It can be seen from the figure that the structure has high refractive index at the frequency of around 950Hz. Based on the studies of literature [16], the relative impedance and effective mass density of the structure are simulated and calculated. The left of Figure 10 is the relative impedance diagram of the structure, and the right figure is an enlarged view of the relative impedance at the frequency of around 950Hz. From the figure, it can be seen that when the frequency of the incident wave is around 953Hz (as shown by the dashed line), the relative impedance of the structure’s real part is 1, that of the imaginary part is 0 and the relative impedance is 1. Figure 11 is an enlarged view of the effective mass density of the cross-connection structure at the incident wave frequency range of 940–980Hz. From the figure, it can be seen that when the incident wave frequency is 953Hz, the effective mass density is 0. Through comparative analysis of the transmission coefficient diagram, relative impedance diagram and effective mass density diagram of the structure, it can be found that when the incident wave frequency is 953Hz, the relative impedance of the structure is 1 and the effective mass density is 0, which is consistent with that of the impedance matching zero index material given by the literature. Thus, the structure is an impedance matching zero index material at the incident wave frequency of 953Hz, and it also shows that the transmission coefficient can be adjusted by the structural parameters of the structure.
4. Conclusion

Based on the clamping boundary phononic crystal theory, this paper designed a cross-connection sound insulation structure. Through numerical simulation and theoretical calculation, it shows that the cross-connection structure designed in this paper has good low-frequency broadband sound insulation performance due to its wide band of effective negative mass density in the low-frequency range, and has the advantages of light weight, simple structure and ease of manufacturing and assembly, which can facilitate large-scale engineering applications. At the same time, it also analyzed the reason for the sudden change of the structure’s refractive index as the frequency is in the vicinity of the 953Hz. By theoretical analysis, the structure belongs to impedance matching zero index metamaterial at the frequency of around 953Hz. The future studies can focus on changing the materials and structural parameters, and further optimize the design of the sound insulation structure. At the same time, impedance matching zero index metamaterials are also of important research value.

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