Estimations of baryon asymmetry for different neutrino mass models

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Abstract

We present a comparison of numerical predictions on baryon asymmetry of the universe ($Y_B$) for different neutrino mass models. We start with a very brief review on the main formalism of baryogenesis via leptogenesis through the decay of heavy right-handed Majorana neutrinos, and then calculate the baryon asymmetry of the universe for known six neutrino mass models viz., three quasi-degenerate, two inverted hierarchical and one normal hierarchical models, which are generated from canonical seesaw mechanism. The corresponding mass matrices for the right-handed Majorana neutrino $M_{RR}$ as well as the Dirac neutrino $m_{LR}$ are fixed at the seesaw stage for generating correct light neutrino mass matrices $m_{LL}$ consistent with observations, and are again employed in the calculation of baryogenesis. This two-tier procedure removes possible ambiguity on the choices of $m_{LR}$ and $M_{RR}$, and fixes the values of input parameters at the seesaw stage. We find that the ranges of predictions from both normal hierarchical model (NHT3) and the degenerate model (DegT1A) having mass eigenvalues $(m_1, -m_2, m_3)$, are almost consistent with the observed baryon asymmetry of the universe. Other two degenerate models (DegT1B, DegT1C) and one inverted hierarchical model (InvT2A) lead to very small baryon asymmetry, $Y_B \leq 10^{-19}$, whereas inverted hierarchical model (InvT2B) gives larger $Y_B \geq 10^{-6}$. Combining the present result with other predictions such as neutrino masses and mixings, and stability under radiative corrections in MSSM, the normal hierarchical model appears to be the most favorable choice of nature. Possible sources of uncertainty in the estimations are pointed out.

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1 Introduction

Left-right symmetric GUT models such as SO(10)GUT, assume the existence of heavy right-handed Majorana neutrinos, and also predict small non-zero neutrino masses through the celebrated seesaw mechanism[1]. Various neutrino mass models describing neutrino mass patterns, have been proposed[2] within the framework of seesaw mechanism, and their predictions on neutrino masses and mixings have been studied thoroughly. In our earlier communication[2] we had studied the stability criteria of these models in the presence of left-handed triplet Higgs field in type-II seesaw formula, and also the stability under radiative corrections in MSSM. The above criteria had been used to discriminate the neutrino mass models describing different patterns of neutrino masses. As a continuation of our earlier effort in this direction, we are now interested in the present work to have a comparison of the different theoretical predictions of the cosmological baryon asymmetry derived from the above neutrino mass models. This information may be treated as an additional point in the process of further discrimination of neutrino mass models.

In SO(10)GUT, the role of heavy right-handed Majorana neutrinos is manifold. In addition to its role in seesaw mechanism, it also plays an important role to explain the observed baryon asymmetry of the universe[3],

$$Y_{B}^{CMB} = (6.1^{+0.3}_{-0.2})10^{-10}.$$  

Such baryon asymmetry can be dynamically generated if the particle interaction rate and the expansion rate of the universe satisfy Sakharov’s three famous conditions[4]: (i) baryon non-conservation, (ii) C and CP violation and (iii) departure from thermal equilibrium. Being Majorana particles the heavy right-handed neutrinos automatically satisfy Sakharov’s one of the above three conditions i.e., C and CP violation as they can have asymmetric decay to lepton and Higgs particles, and the processes occur at different rates for particles and antiparticles. Baryogenesis through leptogenesis by the electroweak sphaleron process has been widely accepted as a correct approach and a lot of work has been done in this direction[5,6,7,8,9,10]. The electroweak sphaleron process violates baryon as well as lepton number, and the process is found to be in equilibrium for a range of temperature i.e., $100 GeV \leq T \leq 10^{12} GeV$.

For a systematic estimation of baryon asymmetry, one has to correlate it with the seesaw mechanism where both Dirac neutrino mass matrix and right-handed Majorana mass matrix are figured. In this context we refer to our earlier work[2] where we had employed the seesaw mechanism to successfully generate the degenerate, inverted and normal hierarchical models of neutrino mass patterns, using the diagonal texture of $m_{LR}$ and non-diagonal texture of $M_{RR}$. In most of the left-right symmetric SO(10)GUT models, the Dirac neutrino mass matrix $m_{LR}$ can be either charged lepton mass matrix (referred to as case i) or up-quark mass matrix (referred to as case ii). However, we find that the seesaw mechanism alone fails to discriminate the above two possible choices of $m_{LR}$, as well as the correct choice of the neutrino mass model in question. In the present work, we will estimate the baryon asymmetry of the
universe using the same input values of the parameters which were already fixed for the predictions of masses and mixing angles at the seesaw stage. This in fact assures good predictions of neutrino masses and mixings. As emphasised before, such additional information on the calculations of the baryon asymmetry may be used to further discriminate the two choices of $m_{LR}$, and also the correct pattern of neutrino mass model in question.

In section 2, we present a brief review on the main formalism for estimating baryon asymmetry through the out-of-equilibrium decays of heavy right-handed neutrinos. Section 3 presents the numerical calculations of the baryon asymmetry for different neutrino mass models, and their comparison. Section 4 concludes with a summary and discussion. Important expressions related to $m_{LL}$ and $M_{RR}$ for all the neutrino mass models derived from seesaw formula, are relegated to Appendix A for ready reference in the present calculations.

2 A brief review on the estimation of baryon asymmetry

2.1 Type-I seesaw formula and lepton asymmetry

Type-I seesaw formula relates the left-handed Majorana neutrino mass matrix $m_{LL}$ and heavy right handed Majorana mass matrix $M_{RR}$ in a simple way

$$m_{LL} = -m_{LR}M_{RR}^{-1}m_{LR}^T$$

where $m_{LR}$ is the Dirac neutrino mass matrix. In some left-right symmetric theories such as SO(10)GUT, the left-handed Higgs triplet $\Delta_L$ acquires a vacuum expectation value and this leads two sources in the neutrino mass matrix:

$$m_{LL} = m_{LL}^I + m_{LL}^{II},$$

where the first term is the usual seesaw term and the second one can be expressed as $m_{LL}^{II} = \gamma (M_W/v_R)^2 M_{RR}$. In general $\gamma$ is a function of various couplings, and without fine tuning $\gamma$ is expected to be order of unity ($\gamma \simeq 1$). The modified seesaw formula (referred to as Type-II) can be written as

$$m_{LL} = -m_{LR}M_{RR}^{-1}m_{LR}^T + \gamma (M_W/v_R)^2 M_{RR}$$

Three different possibilities can arise between the two contributing terms i.e., $m_{LL}^I$ and $m_{LL}^{II}$. The situation where the second term is dominant over the first term, has the potential to connect [12,13,14,15] the large atmospheric neutrino mixing and b-$\tau$ unification in the context of minimal supersymmetric SO(10) models. In Type II seesaw formula, the lepton asymmetry $\epsilon_1$ can be written as:

$$\epsilon_1 = \epsilon^I + \epsilon^{II}$$

where $\epsilon^I$ and $\epsilon^{II}$ are the contributions of first term and second term of eq.(2) respectively. For our calculation of lepton asymmetry we consider the model[5,6,7] where the asymmetric decay of the lightest of the heavy right-handed Majorana neutrinos, is assumed. We confine our calculation to the Type I contribution only, assuming the extra contribution is relatively very small ($\gamma \sim 10^{-4}$).

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The physical Majorana neutrino $N_R$ decays into two modes: $N_R \to l_L + \phi^i$ and $\bar{N}_L + \phi$, where $l_L$ is the lepton and $\bar{l}_L$ is the antilepton. For CP-violating decay through one-loop radiative correction by Higgs particle, the branching ratio for these two decay modes is likely to be different. The CP-asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of lightest of heavy right-handed Majorana neutrino $N_1$ is defined by $\epsilon = \frac{\Gamma(N_1 \to l_L \phi^i) - \bar{\Gamma}(N_1 \to \bar{l}_L \phi)}{\Gamma(N_1 \to l_L \phi^i) + \bar{\Gamma}(N_1 \to \bar{l}_L \phi)}$ where $\Gamma = \Gamma(N_1 \to l_L \phi^i)$ and $\bar{\Gamma} = \Gamma(N_1 \to \bar{l}_L \phi)$. A perturbative calculation from the interference between tree level and vertex plus self-energy diagrams for $N_1$ decay gives $\epsilon_1 = \frac{1}{8\pi} \sum_j \frac{\text{Im}[(hh^\dagger)_{1j}^2]}{\sum_i |(h)_{ii}|^2} f(x_j)$ where Yukawa coupling of the Dirac neutrino mass matrix, is expressed by $h = m_{L,R}/v$, and $f(x_j) = f(M_2^2/M_1^2) = f_V(x_j) + f_S(x_j)$ with $j=2,3$. The function $f_V(x_j)$ represents the vertex contribution and $f_S(x_j)$ represents the self-energy contribution of the decay process of the right-handed Majorana neutrino. The value of $f(x_j)$ in MSSM is twice its value for SM. For $x_j >> 1$, the expression (4) for SM case, can be written in terms of neutrino mass matrix as: $\epsilon^I = \epsilon_1 = \frac{3M_1}{16\pi v^2} \frac{\text{Im}[(h^* m_{LL}^t h^\dagger)_{11}]}{(hh^\dagger)_{11}}$ For $x_j \sim 1$, we will have a very particular situation where $M_1 \simeq M_2 < M_3$, known as quasi-degenerate spectrum. The asymmetry is largely enhanced by a resonance factor which is given by: $R = \frac{|M_1|}{2(|M_2| - |M_1|)}$ In such situation, we use the following expression to calculate the lepton asymmetry, $\epsilon^I = -\frac{M_2}{8\pi v^2} \frac{\text{Im}[(h^* m_{LL}^t h^\dagger)_{11}]}{(hh^\dagger)_{11}} R$ 2.2 From lepton asymmetry to baryon asymmetry through sphaleron process

The (B-L) conserving and (B+L) violating electroweak sphaleron processes give a set of coupled equations for the evolution of lepton number and baryon number. At temperature $T$ above the electroweak phase transition temperature (EWPT) $T_C$, the baryon asymmetry can be expressed in terms of (B - L) number density as:

$$B(T > T_c) = \frac{24 + 4N_H}{66 + 13N_H}(B - L)$$
Again (B-L) asymmetry per unit entropy is just the negative of $n_L/s$, since the baryon number is conserved in the right-handed Majorana neutrino decays. At the electroweak phase transition temperature, any primordial (B+L) will be washed out and the relation (8) can be written as [21,25]:

$$\frac{n_B}{s} \approx \frac{24 + 4N_H}{66 + 13N_H} \frac{n_L}{s}$$  \hspace{1cm} (9)

where $N_H$ is the number of Higgs doublet and $n_L/s$ represents the lepton number to entropy ratio. For standard model (SM), $N_H = 1$; and eq.(9) reduces to

$$\frac{n_B}{s} \approx -\frac{28}{79} \frac{n_L}{s}$$  \hspace{1cm} (10)

For very slow B non-conserving interactions, the baryon to entropy ratio in a comoving volume is conserved. But the baryon to photon ratio does not remain constant due to the variation of photon density per comoving volume [8,26] at the different epoch of the expanding universe. Considering the cosmic ray microwave background temperature $T \approx 2.7K$, we have $s = 7.04n_\gamma$. For SM case we have

$$\left(\frac{n_L}{s}\right)_{SM} \approx 8.66 \times 10^{-3} \kappa_1 \epsilon_1$$

where $\kappa_1$ is the dilution factor. This leads to the observed baryon asymmetry of the Universe [21],

$$Y_B^{SM} \equiv \left(\frac{n_B}{n_\gamma}\right)_{SM} \approx d \kappa_1 \epsilon_1 = 0.0216 \kappa_1 \epsilon_1$$  \hspace{1cm} (11)

For our numerical estimation of baryon asymmetry for different neutrino mass patterns, we use the above expression.

### 2.3 Out-of-Equilibrium Decay

To get the resultant baryon asymmetry, an out-of-equilibrium situation has to be provided by the expanding universe. The desired condition is satisfied if the temperature is smaller than the mass $M_1$ of the decaying neutrino at the time of its decay. The inverse decay is blocked [6,8,9] if the expansion rate is greater than the interaction rate $\Gamma$. Thus,

$$\Gamma < \frac{1.66\sqrt{g^*T^2}}{M_{pl}}$$

where $g^*$ is the effective numbers of degrees of freedom available at the temperature $T$ and $M_{pl}$ is the Planck mass scale. Substituting the expression of $\Gamma$ for the decay of $M_1$, we have,

$$\frac{(hh^\dagger)_{11}M_1}{8\pi} < \frac{1.66\sqrt{g^*T^2}}{M_{pl}}$$  \hspace{1cm} (12)
For attaining out-of-equilibrium decay of $M_1$ we equate $T = M_1$, and we use a parameter\[27\] derived from eq(12) as,

$$K \equiv \left( \frac{\langle hh \rangle_{11} M_1}{8\pi} \right) \left( \frac{M_{pl}}{1.66\sqrt{g^*}M_1^2} \right) < 1 \quad (13)$$

How much the produced asymmetry is washed out is described by Boltzmann equation and can be parametrized by another parameter $\kappa$ known as dilution factor\[8,27\]:

$$\kappa_1 \simeq \begin{cases} 
\frac{0.3}{K((\ln K)^{3/2})} & \text{if } 10 \leq K \leq 10^6 \\
\frac{2\sqrt{K+9}}{2K^{3/2}} & \text{if } 0 \leq K \leq 10 
\end{cases} \quad (14)$$

An equivalent description for the out-of-equilibrium decay can also be expressed by the decay parameter

$$K = \tilde{m}_1/m^*$$

where the effective neutrino mass is defined as

$$\tilde{m}_1 = \left( \frac{\langle hh \rangle_{11} v^2}{M_1} \right)$$

and the equilibrium neutrino mass is given by\[28\]:

$$m^* = \frac{16\pi^2}{3\sqrt{3}} g^* v^2 \frac{v^2}{M_{pl}} \simeq 1.08 \times 10^{-3}eV \quad (15)$$

Here, $g^* = 106.75$, $M_{pl} = 1.2 \times 10^{19}$GeV, $v = 174$GeV. Thus, for $\tilde{m}_1 < m^*$, the heavy right-handed neutrino will satisfy the out-of-equilibrium condition.

The values of $K$ for different neutrino mass models under consideration, ranges from $3.7 \times 10^2$ to $10^{-1}$ as shown in table-3. Therefore, the dilution factor $\kappa_1$ which takes into account the washout process due to inverse decays and lepton number violating scattering, are calculated from expression(14).

### 3 Numerical calculation and results

To start with, we predict the left-handed Majorana neutrino mass $m_{LL}$ using the seesaw formula\[2\]. As emphasised earlier, we adopt the diagonal form of Dirac neutrino mass matrix $m_{LR}$ for case(i) where the Dirac neutrino mass matrix is taken as charged lepton mass matrix and case(ii) where the Dirac neutrino mass matrix is taken as up-quark mass matrix. Using the non-diagonal form of right-handed Majorana mass matrix $M_{RR}$ we obtain three patterns of neutrino mass models: degenerate, inverted hierarchical and normal hierarchical. The corresponding expressions for $m_{LL}$ and $M_{RR}$ for these models, are given in Appendix-A for ready reference to the present calculation.

In the next step, the physical right-handed Majorana neutrino $N'$ is defined in the basis where $M_{RR}$ is diagonal with real positive eigen-values,i.e.,

$$N' = U_R^T N \quad (16)$$
where $U_R^T$ is extracted from the relation,

$$M_{RR}^{diag} = U_R^T M_{RR} U_R = \text{Diag}(M_1, M_2, M_3)$$  \hspace{1cm} (17)

This amounts to the re-definition of Dirac neutrino mass matrix $m_{LR}$:

$$m_{LR} \rightarrow m'_{LR} = m_{LR} U_R$$  \hspace{1cm} (18)

In general $U_R$ is complex, and the new $m'_{LR}$ in eq.(18) is made complex. It can be emphasised that such transformation makes the seesaw term $m_{LL}$ invariant.

In most calculations the Majorana CP phases enter through the construction of $m_{LL}$ but in the present case such complex phases are originated from the diagonalising matrix $U_R$ of $M_{RR}$. However both approaches are equivalent as they are related by an inverse seesaw relation $M_{RR} = -m_{LR} m'^{-1}_{LL} m^T_{LR}$, which is specially true for diagonal form of mass matrix $m_{LR}$. In our calculation four neutrino mass models given in appendix viz., DegT1A, DegT1C, InvT2B, and NRT3, have complex diagonalising matrix $U_R$ containing complex Majorana phases. But for two models DegT1B and InvT2A, the diagonalising matrix $U_R$ is real, leading to zero lepton asymmetry. Since the non-zero lepton asymmetry requires complex Yukawa couplings i.e., complex phases, the following transformation is introduced by hand. Thus

$$U_R \rightarrow U_R D$$  \hspace{1cm} (19)

where,

$$D = \begin{pmatrix}
e^{i\alpha} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{i\beta}
\end{pmatrix}$$

contains the complex Majorana phases. For further calculation of lepton asymmetry for the above two models we take $\alpha = \beta = \pi/2$. It can be emphasised here that such extra phase transformation of $U_R$ does not change the seesaw term $m_{LL}$ and also the term $(hh^\dagger)$ in eq.(4).

For demonstration we first consider the normal hierarchical model as an example. The corresponding heavy right-handed neutrino mass matrix and light left-handed neutrino mass matrices are collected from ref.[2] where we use the diagonal form of Dirac neutrino mass matrix $m_{LR} = \text{Diag}(\lambda^n, \lambda^n, 1) v$.

**Normal Hierarchical Type3(NHT3):**

$$M_{RR} = \begin{pmatrix}
\lambda^{2m-1} & \lambda^{m+n-1} & \lambda^{m-1} \\
\lambda^{m+n-1} & \lambda^{m+n-2} & 0 \\
\lambda^{m-1} & 0 & 1
\end{pmatrix} v_0$$

$$-m_{LL} = \begin{pmatrix}
-\lambda^4 & \lambda & \lambda^3 \\
\lambda & 1-\lambda & -1 \\
\lambda^3 & -1 & 1-\lambda^3
\end{pmatrix} m_0$$

We take the input values $\lambda = 0.3$, $m_0 = 0.03eV$ leading to correct predictions of neutrino mass parameters and mixing angles:
\[ \Delta m^2_{13} = 9.04 \times 10^{-5}\text{eV}^2, \Delta m^2_{23} = 3.01 \times 10^{-3}\text{eV}^2 \]
\[ \tan^2 \theta_{12} = 0.55, \sin^2 2\theta_{23} = 0.98, \sin \theta_{13} = 0.074 \]

These are almost consistent with recent data[2]. The predictions on neutrino masses and mixing angles for all the models, are presented in Table-1.

For case(i): We use the charged lepton mass matrix, \( m_E = m_{LR}, \ (m, n) = (6, 2), \ v_0 = 1.01 \times 10^{15} \text{GeV}, \ m_0 = 0.03 \text{eV}, \ m_{LR} = m_E = \text{diag}(\lambda^6, \lambda^2, 1) v; \) with \( v = 174\text{GeV}. \) This leads to

\[
M_{RR} = \begin{pmatrix}
1.7887 \times 10^9 & 2.2083 \times 10^{11} & 2.4537 \times 10^{12} \\
2.2083 \times 10^{11} & 7.3610 \times 10^{11} & 0 \\
2.4537 \times 10^{12} & 0 & 1.0097 \times 10^{15}
\end{pmatrix}
\]

For real and positive eigen values[29] of \( M_{RR} \) the corresponding diagonalising matrix is

\[
U_R = \begin{pmatrix}
0.964044e^{-i\frac{\pi}{2}} & -0.265732 & 0.00242998 \\
-0.265732e^{-i\frac{\pi}{2}} & -0.964047 & 5.31822 \times 10^{-7} \\
-0.00234248e^{-i\frac{\pi}{2}} & 0.000646238 & 0.999997
\end{pmatrix}
\]

and the three heavy masses are

\[ M^{diag}_{RR} = \text{diag}(6.51 \times 10^{10}, 7.97 \times 10^{11}, 1.01 \times 10^{15}). \]

The Yukawa coupling ‘\( h \)’ is defined[23] by \( h = m_{LR}/v, \) where \( v \) is the electroweak vacuum expectation value. As the Dirac neutrino mass matrix is rotated in the basis where right-handed Majorana neutrino mass matrix is diagonal, the new Yukawa coupling becomes: \( h = m_{LR}U_R/v \)

\[
h = \begin{pmatrix}
-0.000702788e^{-i\frac{\pi}{2}} & -0.00193718 & 1.77146 \times 10^{-6} \\
0.0239159e^{-i\frac{\pi}{2}} & -0.0867642 & 4.7864 \times 10^{-8} \\
0.00234248e^{-i\frac{\pi}{2}} & 0.000646238 & 0.999997
\end{pmatrix}
\]

For case(ii): We again use the up-quark mass matrix \( m_u = m_{LR}, \ (m, n) = (8, 4), \ v_0 = 1.01 \times 10^{15} \text{GeV}, \ m_0 = 0.03 \text{eV}, \ m_{LR} = m_u = \text{diag}(\lambda^8, \lambda^4, 1) v. \) This leads to

\[
M_{RR} = \begin{pmatrix}
1.4495 \times 10^7 & 1.7895 \times 10^9 & 2.2092 \times 10^{11} \\
1.7895 \times 10^9 & 5.9648 \times 10^9 & 0 \\
2.2092 \times 10^{11} & 0 & 1.0102 \times 10^{15}
\end{pmatrix}
\]

The corresponding diagonalising matrix is:

\[
U_R = \begin{pmatrix}
0.964047e^{-i\frac{\pi}{2}} & -0.265732 & 0.0002187 \\
-0.265732e^{-i\frac{\pi}{2}} & -0.964047 & -3.87423 \times 10^{-10} \\
-0.00210837e^{-i\frac{\pi}{2}} & 0.000581162 & -1
\end{pmatrix}
\]

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Table 1: Predicted values of the solar and atmospheric neutrino mass-squared differences and three mixing parameters calculated from \(m^I_{LL}\) derived from seesaw formula in the Appendix A.

| Type   | \(\Delta m^2_{21}[10^{-5}eV^2]\) | \(\Delta m^2_{23}[10^{-3}eV^2]\) | \(\tan^2\theta_{12}\) | \(\sin^22\theta_{23}\) | \(\sin\theta_{13}\) |
|--------|----------------------------------|----------------------------------|------------------------|------------------------|------------------------|
| DegT1A | 8.80                             | 2.83                             | 0.98                   | 1.0                    | 0.0                    |
| DegT1B | 7.91                             | 2.50                             | 0.27                   | 1.0                    | 0.0                    |
| DegT1C | 7.91                             | 2.50                             | 0.27                   | 1.0                    | 0.0                    |
| InvT2A | 8.36                             | 2.50                             | 0.44                   | 1.0                    | 0.0                    |
| InvT2B | 9.30                             | 2.50                             | 0.98                   | 1.0                    | 0.0                    |
| NHT3   | 9.04                             | 3.01                             | 0.55                   | 0.98                   | 0.074                  |

\[
M^{diag}_{RR} = diag(5.2699 \times 10^8, 6.4571 \times 10^9, 1.01 \times 10^{15})
\]

The Yukawa coupling ‘\(h\)’ is:

\[
h = \begin{pmatrix}
-7.02787 \times 10^{-4} e^{-i\frac{\pi}{2}} & -1.93719 \times 10^{-4} & 1.59431 \times 10^{-7} \\
0.02391 e^{-i\frac{\pi}{2}} & -0.08676 & 3.4868 \times 10^{-11} \\
2.10837 \times 10^{-4} e^{-i\frac{\pi}{2}} & 5.81161 \times 10^{-5} & 1
\end{pmatrix}
\]

To calculate the dilution factor first we calculate the effective mass \(\tilde{m}\) for each case (Table-3). For normal hierarchical case we have for case(i): \(\tilde{m}_1 = 2.47 \times 10^{-4}\) eV and for case(ii): \(\tilde{m}_1 = 2.47 \times 10^{-4}\) eV The corresponding dilution factors are 0.17 and 0.17 for case(i) and case(ii) respectively. Considering the decay of \(N_1\), the baryon asymmetry is found to be:

for case(i) where \(m_{LR} = m_E\),

\[
Y^S_M = 2.17 \times 10^{-9}
\]

and for case(ii) where \(m_{LR} = m_{up}\), \(Y^S_M = 1.76 \times 10^{-11}\) Following the same procedure we calculate the baryon asymmetry for all the models given in Appendix-A. The results are presented in Tables 1-5. As emphasised earlier, we use only the values of the parameters fixed at the first stage where seesaw formula operates, in the estimation of baryon asymmetry in the present calculation.

4 Summary and Discussion

We summarise the main points in this work. We have attempted to give a comparison of the numerical estimations on baryon asymmetry of the universe for different neutrino mass models. We have investigated these neutrino mass models (degenerate, inverted hierarchical and normal hierarchical) at two consecutive stages: (i) from neutrino oscillation parameters (Table 1) using seesaw mechanism and (ii) from the observed baryon asymmetry (Table 4), within a unified framework. In a model where baryon asymmetry is explained via lepton asymmetry through sphaleron process, the out-of-equilibrium decay of the right-handed Majorana neutrino plays a decisive role. As the out-of-equilibrium decay of the lightest heavy Majorana neutrino \(M_1\) is essential for
Case (ii):

\[ Y_{\text{BM}} \approx 0.1 \times 10^{-3} \]

For case (ii) the expressions of \( m, n \) are collected from Appendix A.

| Type      | \( m_1 \) | \( K \) | \( \kappa_1 \) | \( m_1 \) | \( K \) | \( \kappa_1 \) |
|-----------|-----------|--------|---------------|-----------|--------|---------------|
| DegT1A    | \( 3.76 \times 10^{-3} \) | 3.48   | 0.11          | \( 3.74 \times 10^{-3} \) | 3.46   | 0.11          |
| DegT1B    | 0.40      | 370    | 2.79 \times 10^{-4} | 0.40      | 370    | 2.79 \times 10^{-4} |
| DegT1C    | 0.40      | 370    | 2.79 \times 10^{-4} | 0.40      | 370    | 2.79 \times 10^{-4} |
| InvT2A    | 0.05      | 46.3   | 2.9 \times 10^{-3}  | 0.05      | 46.3   | 2.9 \times 10^{-3}  |
| InvT2B    | 2.85 \times 10^{-4} | 0.26   | 0.17           | 2.83 \times 10^{-4} | 0.26   | 0.17           |
| NHT3      | 2.47 \times 10^{-4} | 0.23   | 0.17           | 2.47 \times 10^{-4} | 0.23   | 0.17           |

Table 2: The three right-handed Majorana neutrino masses in GeV for both case (i) and case (ii). The expressions of \( M_{RR} \) are collected from Appendix A.

| Type      | \( \epsilon^l \) (case i) | \( \epsilon^l \) (case ii) | \( Y_{\text{SM}}^B \) (case i) | \( Y_{\text{SM}}^B \) (case ii) |
|-----------|---------------------------|---------------------------|--------------------------------|--------------------------------|
| DegT1A    | \( 2.10 \times 10^{-6} \) | \( 1.71 \times 10^{-8} \) | \( 4.99 \times 10^{-9} \) | \( 4.06 \times 10^{-11} \) |
| DegT1B    | \( 2.66 \times 10^{-18} \) | \( 2.16 \times 10^{-20} \) | \( 1.60 \times 10^{-23} \) | \( 1.30 \times 10^{-25} \) |
| DegT1C    | \( 1.74 \times 10^{-18} \) | \( 1.69 \times 10^{-20} \) | \( 1.05 \times 10^{-23} \) | \( 1.02 \times 10^{-25} \) |
| InvT2A    | \( 1.59 \times 10^{-14} \) | \( 1.27 \times 10^{-16} \) | \( 9.94 \times 10^{-19} \) | \( 7.96 \times 10^{-21} \) |
| InvT2B    | \( 1.47 \times 10^{-2} \)  | \( 1.62 \times 10^{-4} \)  | \( 5.40 \times 10^{-5} \)  | \( 5.94 \times 10^{-7} \)  |
| NHT3      | \( 5.90 \times 10^{-7} \)  | \( 4.78 \times 10^{-9} \)  | \( 2.17 \times 10^{-9} \)  | \( 1.76 \times 10^{-11} \) |

Table 3: Calculation of values of effective mass parameter \( m_1 \) in eV, and dilution factor \( \kappa_1 \).

| (m, n)  | (8, 4) | (6, 2) | (4, 2) | (7, 2) |
|---------|--------|--------|--------|--------|
| \( M_1 \) (GeV) | \( 5.27 \times 10^8 \) | \( 6.51 \times 10^{10} \) | \( 1.16 \times 10^{12} \) | \( 1.87 \times 10^{10} \) |
| \( \epsilon_1 \) | \( 4.78 \times 10^{-9} \) | \( 5.9 \times 10^{-7} \) | \( 1 \times 10^{-5} \) | \( 1.7 \times 10^{-7} \) |
| \( Y_{\text{SM}}^B \) | \( 1.76 \times 10^{-11} \) | \( 2.17 \times 10^{-9} \) | \( 3.24 \times 10^{-8} \) | \( 6.24 \times 10^{-10} \) |

Table 5: Calculation of baryon asymmetry for normal hierarchical mass model for different choices of \((m, n)\) pair.
fruitful leptogenesis, we consider the effective mass parameter \( \tilde{m}_1 \) for all the models presented in Table 3 where only inverted hierarchical model (InvT2B) and normal hierarchical model (NRT3) satisfy the out-of-equilibrium decay condition. In these two models the effective neutrino mass is less than the equilibrium neutrino mass \( m^* (\approx 10^{-3}) \) i.e., \( \tilde{m}_1 < m^* \).

We present the numerical predictions on solar and atmospheric neutrino mass-squared differences and three mixing angles, which are calculated using seesaw formula in Table 1. All the input parameters in \( M_{RR} \) and \( m_{LR} \) are fixed at this stage. Table 1 indicates good predictions for normal hierarchical model and inverted hierarchical model (InvT2A). However, the inverted hierarchical (Type 2A) model is not stable under radiative corrections in MSSM unlike normal hierarchical model (NRT3). In Table 2 we present the masses of the physical right-handed Majorana neutrinos, which are extracted through the diagonalisation of \( M_{RR} \). Further, the calculated values of baryon asymmetry for all neutrino mass models are shown in Table 4 which clearly shows the competitive nature of degenerate (Type IA) model [30] and Normal hierarchical model. For both models, the produced baryon asymmetry is slightly lesser than the experimental value in case (ii) and slightly larger in case (i). This possibly implies that the actual form of Dirac neutrino mass matrix may lie between these two extreme cases. A result of our investigations in this regards for normal hierarchical mass model, is presented in Table-5 for different values of \((m, n)\) pair in \( m_{LR} \) [31]. As the mass of the lightest right-handed Majorana neutrino \( M_1 \) is concerned, the model agrees with the famous Ibarra-Davidson bound [32] i.e., \( M_1 > 4 \times 10^8 \) GeV. Considering the stability criteria under radiative corrections in MSSM as well as good predictions on oscillation mass parameters and mixing angles, the normal hierarchical model is quite favourable. Future neutrino oscillation experiments may confirm this conjecture. Finally we also point out some ambiguities on the choice of the value of coefficient \( d = 0.216 \). The present value is found to be 2.2 times larger than the value appeared in refs.[16,19,23,33] and 0.74 times smaller than the value appeared in ref.[34].

**Appendix A**

Here we collect the various right-handed Majorana mass matrices \( M_{RR} \) and the light left-handed neutrino mass matrix \( m_{LL} \) from ref.[2] for ready reference in the text. Here we use diagonal form of Dirac neutrino mass matrix \( m_{LR} = \text{Diag}(\lambda^m, \lambda^n, 1)v \). The nomenclature of the different neutrino mass models has been changed here.

**Degenerate Type1A (DegT1A) with mass eigenvalues \( (m_1, -m_2, m_3) \):**

\[
M_{RR} = \begin{pmatrix}
-2\delta_2 \lambda^m & (\frac{\sqrt{2}}{2} + \delta_1) \lambda^{m+n} & (\frac{\sqrt{2}}{2} + \delta_1) \lambda^m \\
(\frac{\sqrt{2}}{2} + \delta_1) \lambda^{m+n} & (1/2 + \delta_1 - \delta_2) \lambda^{2n} & (-1/2 + \delta_1 - \delta_2) \lambda^n \\
(\frac{\sqrt{2}}{2} + \delta_1) \lambda^m & (-1/2 + \delta_1 - \delta_2) \lambda^n & (-1/2 + \delta_1 - \delta_2) \lambda^m \\
\end{pmatrix} v_0
\]

\[-m_{LL}^I = \begin{pmatrix}
(-2\delta_1 + 2\delta_2) & (\frac{\sqrt{2}}{2} - \delta_1) & (\frac{1}{2} - \delta_1) \\
(\frac{\sqrt{2}}{2} - \delta_1) & (1/2 + \delta_2) & (-1/2 + \delta_2) \\
(\frac{1}{2} - \delta_1) & (-1/2 + \delta_2) & (1/2 + \delta_2) \\
\end{pmatrix} m_0
\]
Here, $\delta_1 = 0.0061875$, $\delta_2 = 0.0031625$, $\lambda = 0.3$, $m_0 = 0.4\text{eV}$ and $v_0 = 7.57 \times 10^{13}\text{GeV}$

Degenerate Type1B(DegT1B) with mass eigenvalues $(m_1, m_2, m_3)$:

$$M_{RR} = \begin{pmatrix}
(1 + 2\delta_1 + 2\delta_2)\lambda^{2n} & \delta_1\lambda^{m+n} & \delta_1\lambda^n \\
\delta_1\lambda^{m+n} & (1 + \delta_2\lambda^{2n}) & \delta_2\lambda^n \\
\delta_1\lambda^n & \delta_2\lambda^n & (1 + \delta_2)
\end{pmatrix} v_0$$

$$-m_{LL}^1 = \begin{pmatrix}
(1 - 2\delta_1 - 2\delta_2) & -\delta_1 & -\delta_1 \\
-\delta_1 & (1 - \delta_2) & -\delta_2 \\
-\delta_1 & -\delta_2 & (1 - \delta_2)
\end{pmatrix} m_0$$

Here, $\delta_1 = 7.2 \times 10^{-5}$, $\delta_2 = 3.9 \times 10^{-3}$, $\lambda = 0.3$, $m_0 = 0.4\text{eV}$ and $v_0 = 7.57 \times 10^{13}\text{GeV}$.

Degenerate Type1C(DegT1C) with mass eigenvalues $(m_1, m_2, -m_3)$:

$$M_{RR} = \begin{pmatrix}
(1 + 2\delta_1 + 2\delta_2)\lambda^{2n} & \delta_1\lambda^{m+n} & \delta_1\lambda^n \\
\delta_1\lambda^{m+n} & (1 + \delta_2\lambda^{2n}) & \delta_2\lambda^n \\
\delta_1\lambda^n & \delta_2\lambda^n & (1 + \delta_2)
\end{pmatrix} v_0$$

$$-m_{LL}^1 = \begin{pmatrix}
(1 - 2\delta_1 - 2\delta_2) & -\delta_1 & -\delta_1 \\
-\delta_1 & (1 - \delta_2) & -\delta_2 \\
-\delta_1 & -\delta_2 & (1 - \delta_2)
\end{pmatrix} m_0$$

Here, $\delta_1 = 7.2 \times 10^{-5}$, $\delta_2 = 3.9 \times 10^{-3}$, $\lambda = 0.3$, $m_0 = 0.4\text{eV}$ and $v_0 = 7.57 \times 10^{13}\text{GeV}$.

Inverted Hierarchical Type2A(InvT2A) with mass eigenvalues $(m_1, m_2, m_3)$:

$$M_{RR} = \begin{pmatrix}
\eta(1 + 2\epsilon)\lambda^{2n} & \eta\epsilon\lambda^{m+n} & \eta\epsilon\lambda^n \\
\eta\epsilon\lambda^{m+n} & 1/2\lambda^{2n} & -(1/2 - \eta)\lambda^n \\
\eta\epsilon\lambda^n & -(1/2 - \eta)\lambda^n & 1/2
\end{pmatrix} \frac{v_0}{\eta}$$

$$-m_{LL}^1 = \begin{pmatrix}
1 - 2\epsilon & -\epsilon & -\epsilon \\
-\epsilon & \frac{1}{2} & \frac{1}{2} \\
-\epsilon & \frac{1}{2} & \eta
\end{pmatrix} m_0$$

Here, $\eta = 0.0045$, $\epsilon = 0.0055$, $\lambda = 0.3$, $m_0 = 0.05\text{eV}$ and $v_0 = 6.06 \times 10^{14}\text{GeV}$.

Inverted Hierarchical Type2B(InvT2B) with mass eigenvalues $(m_1, -m_2, m_3)$:

$$M_{RR} = \begin{pmatrix}
\lambda^{2m+7} & \lambda^{m+n+4} & \lambda^{m+4} \\
\lambda^{m+n+4} & \lambda^{2n} & -\lambda^n \\
\lambda^{m+4} & -\lambda^n & 1
\end{pmatrix} v_0$$

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\[ -m^I_{LL} = \left( \begin{array}{ccc}
0 & -1 & -1 \\
1 & -(\lambda^3 - \lambda^4)/2 & -(\lambda^3 + \lambda^4)/2 \\
1 & -(\lambda^3 + \lambda^4)/2 & -(\lambda^3 - \lambda^4)/2 \\
\end{array} \right) m_0 \]

Here, \( \lambda = 0.3, m_0 = 0.035 \text{eV} \) and \( v_0 = 5.34 \times 10^{16} \text{GeV} \)

Normal Hierarchical Type3 (NHT3) with mass eigenvalues \((m_1, m_2, m_3)\):

\[ M_{RR} = \left( \begin{array}{ccc}
\lambda^{m-1} & \lambda^{m+n-1} & \lambda^{m-1} \\
\lambda^{m+n-1} & \lambda^{m+n-2} & 0 \\
\lambda^{m-1} & 0 & 1 \\
\end{array} \right) v_0 \]

\[ -m^I_{LL} = \left( \begin{array}{ccc}
-\lambda^4 & \lambda & \lambda^3 \\
\lambda & 1 - \lambda & -1 \\
\lambda^3 & -1 & 1 - \lambda^3 \\
\end{array} \right) m_0 \]

Here, \( \lambda = 0.3, m_0 = 0.03 \text{eV} \) and \( v_0 = 1.01 \times 10^{15} \text{GeV} \).

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