Full major-shell calculation for states that were degenerate in a single-\(j\)-shell calculation

A. Escuderos\(^1\), S.J.Q. Robinson\(^2\), and L. Zamick\(^1\)

\(^1\)Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854 and 
\(^2\)Geology and Physics Department, University of Southern Indiana, Evansville, Indiana 47712

(Dated: December 24, 2021)

Abstract

A full \(fp\) calculation is performed for states which were degenerate in a single-\(j\)-shell calculation in which isospin-zero two-body matrix elements were set to zero energy. Most of the splitting in a complete shell calculation (but not all) comes from the \(T = 0\) part of the interaction.

PACS numbers:
In a previous work [1], we explained why certain states were degenerate in the single \( j \) shell for an interaction in which the isospin \( T = 0 \) two-body matrix elements were set to zero. The degeneracy splitting was also obtained by reintroducing the full interaction. In this work, we shall calculate the energy splittings in a full \( fp \)-shell calculation.

In our single-\( j \)-shell calculation with \( j = f_{7/2} \), we took the two-body matrix elements \( E(J) = \langle (f_{7/2}^2)^J | V | (f_{7/2}^2)^J \rangle \) from experiment, i.e., from the spectrum of \( ^{42}\text{Sc} \). The even-\( J \) states of the \( f_{7/2} \) configuration have isospin \( T = 1 \), while the odd-\( J \) states have isospin \( T = 0 \). The values of \( E(J) \) in MeV (with the \( J = 0 \) state taken to be at zero energy) are:

\[
\begin{array}{c|c|c}
T = 1 & T = 0 \\
\hline
J = 0 & 0.0000 & J = 1 & 0.6111 \\
J = 2 & 1.5863 & J = 3 & 1.4904 \\
J = 4 & 2.8153 & J = 5 & 1.5101 \\
J = 6 & 3.2420 & J = 7 & 0.6163 \\
\end{array}
\]

The interaction \( T0E(J) \) is obtained by setting the second column to zero. There will actually be no difference if we set the \( T = 0 \) matrix elements to a constant, as long as we are considering splittings of states of the same isospin, which indeed we are.

In the single-\( j \)-shell case, the degeneracies fall into two classes. In the first case, we have degeneracies of states in \( ^{43}\text{Sc} \) (\( ^{43}\text{Ti} \)) and \( ^{44}\text{Ti} \). For these, we have an explanation—a partial dynamical symmetry [2, 3]. We found that the degeneracies for \( T0E(J) \) occurred for states with angular momenta that could not exist for systems of identical particles, i.e., in \( ^{43}\text{Ca} \) and \( ^{44}\text{Ca} \), respectively. In the second case, we have all other degeneracies that are listed in Table I. For these, we could not find any symmetries related to the degeneracies and concluded that we indeed had degeneracy without symmetry.

For the full-\( fp \)-shell calculation, we use the FPD6 interaction [4]. The interaction obtained by setting the \( T = 0 \) matrix elements to zero but keeping the \( T = 1 \) ones unchanged is called \( T0FPD6 \).

We list the results in Table II for the full-\( fp \)-shell calculation. Note that with \( T0FPD6 \) the levels in question are no longer degenerate, but the splittings are, for the most part, much larger when the full FPD6 interaction is turned on.

As a comparison, we give in Table III the results for the shifts \( \Delta E \) in both the single-\( j \)-shell case and the full \( fp \) calculation. With two exceptions, the results in Table III show a
### Table I: Full-\(fp\)-shell excitation energies (MeV) for states that were degenerate in the single \(j\) shell with \(T0E(J)\). All experimental energies are taken from Ref. [5].

| \(J\) \(\text{FPD6}\) | \(\text{T0FPD6}\) | \(\text{Experiment}\) |
|---|---|---|
| \(43\text{Sc} \ (43\text{Ti})\) | | |
| \((1/2)\_1^-\) | 1.809 | 2.915 |
| \((13/2)\_1^-\) | 3.675 | 3.041 |
| \((13/2)\_2^-\) | 4.779 | 3.648 |
| \((17/2)\_1^-\) | 4.380 | 3.671 | 4.360 |
| \((19/2)\_1^-\) | 3.360 | 3.581 | 3.123 |
| \(44\text{Ti}\) | | |
| \(3^+_2\) | 8.176 | 5.531 |
| \(7^+_2\) | 9.207 | 6.635 |
| \(9^+_1\) | 8.828 | 6.454 |
| \(10^+_1\) | 7.614 | 6.360 | 7.671 |
| \(10^+_2\) | 9.929 | 7.194 | 8.984\(^a\) |
| \(12^+_1\) | 8.312 | 7.061 | 8.039 |
| \(45\text{Ti}\) | | |
| \((25/2)\_1^-\) | 8.652 | 6.955 |
| \((27/2)\_1^-\) | 7.697 | 6.850 | 7.143 |
| \(46\text{V}\) | | |
| \(12^+_1\) | 7.841 | 8.276 | 8.268 |
| \(12^+_2\) | 8.729 | 8.669 |
| \(13^+_1\) | 7.341 | 8.106 | 7.105 |
| \(13^+_2\) | 10.389 | 9.735 |
| \(15^+_1\) | 8.995 | 9.559 | 8.488 |
| \(47\text{V}\) | | |
| \((29/2)\_1^-\) | 11.848 | 9.413 | 10.7685\(^a\) |
| \((31/2)\_1^-\) | 11.068 | 9.340 | 10.003 |

\(^a\)Taken from the Experimental Unevaluated Nuclear Data List.

Continuity between single \(j\) and the full \(fp\) calculation. The exceptions, \((1/2\_1^- - 13/2\_1^-)\) in \(43\text{Sc}\) and \((3^+_2 - 7^+_2)\) in \(44\text{Ti}\), will soon be discussed.

Note that, aside from the above exceptions, the shifts \(\Delta E\) are much smaller for T0FPD6 than for FPD6. For example, the \((9^+_1 - 10^+_1)\) splitting is 1.214 MeV for FPD6, but it is only 0.094 MeV for T0FPD6. This supports the fact that most of the splitting comes from the
TABLE II: Splitting in energies (MeV) for states that were degenerate in the single $j$ shell with $T0E(J)$. All experimental energies are taken from Ref. [5].

|        | $\Delta E$ | Single $j$ | FPD6  | T0FPD6 | Exp.  |
|--------|------------|------------|-------|--------|-------|
| $^{43}$Sc ($^{43}$Ti) | $(1/2)_1^- - (13/2)_1^-$ | 0.816 | -1.866 | -0.126 |
|        | $(13/2)_2^- - (17/2)_1^-$ | 0.653 | 0.399  | -0.023 |
|        | $(17/2)_1^- - (19/2)_1^-$ | 0.653 | 1.020  | 0.090  | 1.237 |
| $^{44}$Ti | $3^+_2 - 7^+_2$   | 0.320 | -1.031 | -1.104 |
|        | $7^+_2 - 9^+_1$   | 0.391 | 0.379  | 0.181  |
|        | $9^+_1 - 10^+_1$  | 0.600 | 1.214  | 0.094  |
|        | $10^+_1 - 12^+_1$ | 1.203 | 1.617  | 0.133  | 0.945 |
| $^{45}$Ti | $(25/2)_1^- - (27/2)_1^-$ | 0.580 | 0.955  | 0.105  |
| $^{46}$V | $12^+_1 - 13^+_1$ | 0.863 | 0.500  | 0.170  | 1.163 |
|        | $13^+_2 - 15^+_1$ | 0.809 | 1.394  | 0.176  |
| $^{47}$V | $(29/2)_1^- - (31/2)_1^-$ | 0.229 | 0.780  | 0.072  | 0.765 |

$T = 0$ part of the two-body interaction.

The two exceptions mentioned above seem to occur for low angular momentum states: $1/2^-$ in $^{43}$Sc and $3^+$ in $^{44}$Ti. For these low-lying states, there tends to be much more configuration mixing; hence, some states having very little to do with the $f_{7/2}$ configuration must have slipped down in energy.

We have previously studied the effects of removing and then reinserting the $T = 0$ two-body matrix elements in nuclear structure calculations [6, 7]. In Ref. [6] we compared the yrast spectra of even–even nuclei in the $fp$ shell using FPD6 and T0FPD6. While the reintroduction of the $T = 0$ matrix elements causes the spectra to become more rotational, it is clear that the $T = 1$ part of the interaction dominates the spectrum. Thus, it is not easy to get a quantitative handle on the effects of the $T = 0$ two-body matrix elements by looking at the yrast spectrum alone.

Therefore, we are looking for various benchmarks that, in combination, will help to obtain the correct $T = 0$ effective interaction. In Ref. [6] it was noted that the $B(E2)$'s were enhanced by the reintroduction of $T = 0$ matrix elements, but this point is somewhat obscured by the uncertainty in what effective charges should be used for neutrons and
protons.

In Ref. [7] it was noted that, in some channels, the Gamow-Teller (GT) matrix elements were very sensitive to the $T = 0$ interaction and, perhaps most important, the isovector orbital transition strength (scissors mode) was greatly enhanced with FPD6 relative to T0FPD6.

In this work, we have focussed on measurements and calculations where the $T = 0$ two-body matrix elements are very important for obtaining energy splittings of states that would be degenerate in single-$j$-shell calculations using T0FPD6. Although there are some cases where the single-$j$ calculations are closer to experiment than the full calculation with FPD6, it is clear that the study of the correct $T = 0$ interaction will have to be carried out using a full $fp$ space.

From the results in Table II, it appears that the high-spin splittings are the simplest to put to the test. Note, however, that some experimental data are missing, e.g., the $(25/2^+ - 27/2^-)$ splitting in $^{45}$Ti and $(13^+_2 - 15^+_1)$ in $^{46}$V.

We would like to acknowledge support from a U.S. Dept. of Energy Grant No. DE-FG0105ER05-02 and from the Secretaría de Estado de Educación y Universidades (Spain) and the European Social Fund. We thank Ben Bayman for technical help and stimulating discussions.

[1] A. Escuderos, B.F. Bayman, L. Zamick, and S.J.Q. Robinson, Phys. Rev. C 72, 054301 (2005).
[2] S.J.Q. Robinson and L. Zamick, Phys. Rev. C 63, 064316 (2001).
[3] S.J.Q. Robinson and L. Zamick, Phys. Rev. C 64, 057302 (2001).
[4] W.A. Richter, M.G. Van Der Merwe, R.E. Julies, and B.A. Brown, Nucl. Phys. A 523, 325 (1991).
[5] National Nuclear Data Center (http://www.nndc.bnl.gov/).
[6] S.J.Q. Robinson, A. Escuderos, and L. Zamick, Phys. Rev. C 72, 034314 (2005).
[7] S.J.Q. Robinson and L. Zamick, Phys. Rev. C 66, 034303 (2002).