Improved Factorization Method in Studying $B$-meson Decays

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$B$ decays are a subject of active research since they provide useful information on the dynamics of strong and electroweak interactions for testing the Standard Model (SM) and models beyond and are ideally suited for a critical analysis of CP violation phenomena. Within the standard model, there exist certain relations between CP violating rate differences in $B$ decays, as for example $\Delta(B^0 \rightarrow \pi^+ \pi^-) = -\Delta(B^0 \rightarrow \pi^+ K^-)$. The goal of this letter is to study the direct CP violation asymmetry in a class of processes where there has been recent theoretical progress, as for example the $B$ decays into two light pseudoscalars mesons and into a light pseudoscalar and a light vector meson. We identify relations between rate asymmetries which are valid in the SU(3) limit and we compute SU(3) breaking corrections to them, going beyond the naive factorization by using the QCD improved factorization model of Beneke et al. Finally, in some processes as for example $BR(B^- \rightarrow \eta' K^-)$, we claim that one has to add SUSY contributions to the Wilson coefficients. In these cases, we end with a $BR$ depending on three parameters, whose values are constrained by the experimental data.

I. INTRODUCTION

As it is known, in the Standard Model, the CP violation arises solely from the phase in the $3 \times 3$ unitary CKM matrix and any CP violating observable is proportional to $Im(V_{ij}V_{kl}^*)$, with $i \neq k$ and $j \neq l$. The SU(3) invariant amplitude for $B \rightarrow PP$ and $B \rightarrow PV$ decays, in terms of the tree and penguin contributions, are, for example,

$$A(B^0 \rightarrow \pi^+ \pi^-) = V_{ub}V_{ud}^* T + V_{ub}V_{cd}^* P,$$

$$A(B^0 \rightarrow \pi^+ K^-) = V_{ub}V_{us}^* T + V_{ub}V_{cs}^* P,$$

with $T$ and $P$ the same in the two processes. Even there are no simple relations among the branching ratios of these decays since the CKM factors in the $T$ and $P$ amplitudes are different, one has the following relations among the CRP violating rate differences, [6],

$$\Delta_{\pi^+ \pi^-} = -\Delta_{K^+ K^-} = \Delta_{K^+ \pi^-} = -\Delta_{\pi^+ K^-}.$$

and similarly for $B \rightarrow PV$, where

$$\Delta_{PP} = \Gamma(B \rightarrow PP) - \Gamma(B \rightarrow \bar{P}P).$$

while the CP asymmetry is defined as

$$A_{CP} = \frac{\Gamma(B \rightarrow PP) - \Gamma(B \rightarrow \bar{P}P)}{\Gamma(B \rightarrow PP) + \Gamma(B \rightarrow \bar{P}P)}.$$

The most important question now is to establish to what precision these relations hold within the standard model, or equivalently to estimate the corrections they might receive from different sources, as for example the SU(3) breaking effects.

In this respect, let us discuss the corrections in QCD improved factorization method developed by Beneke et al. [3, 4] and compare the results with the ones obtained within the so called Naive Factorization, [1], which will be briefly presented in the next section.

II. NAIVE FACTORIZATION

The effective weak Hamiltonian for $B \rightarrow PP$ decays,[1],

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i O_i + C_7 O_7 + C_8 O_8 \right] + h.c. \quad (4)$$

where $\lambda_p = V_{pb}V_{pq}^*$, with $p = u, c$ and $q = d$ (for $\Delta S = 0$ processes) and $q = s$ (for $\Delta S = 1$ processes),

in the case when $a_1 = a_2 = 0$, becomes
\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \lambda_u (C_1 O_1^u + C_2 O_2^u) + \lambda_p \left( \sum_{i=3}^{10} C_i O_i^p + C_7 \gamma \gamma + C_8 O_{8g} \right) \right] + \text{h.c.} \] (5)

It is expressed in terms of the Wilson coefficients \( C_i \) (evaluated at the renormalization scale \( \mu = m_B \)), the usual tree level left-handed current-current operators, the QCD and electroweak penguin operators, the electromagnetic and chromomagnetic dipole operators.

\[ \langle P_1 P_2 | H_{\text{eff}} | B \rangle = i \frac{G_F}{\sqrt{2}} \lambda_p \left( \frac{1}{N_c} C_i + C_j \right) f_{P_2} (m_B^2 - m_t^2) F_0^{B \to P_1} (m_B^2) + (1 \leftrightarrow 2), \] (6)

where \( N_c = 3 \) is the number of colors and we introduce the usual combinations of the Wilson coefficients

\[ a_i \equiv C_i + \frac{1}{3} C_{i+1} \text{ (for } i = \text{ odd}), \]
\[ a_i \equiv C_i + \frac{1}{3} C_{i-1} \text{ (for } i = \text{ even}). \] (7)

Within naive factorization, one is able to put the matrix element of the Hamiltonian in terms of the form factors and decay constant of the meson which is factorized as

\[ r_{\pi(K)} = \frac{2 m_{\pi(K)}^2}{(m_b - m_u)(m_u + m_{d(s)})} \approx 1.2. \] (10)

Using the unitarity relations and introducing the notations:
\[ \frac{v_u^d}{v_c^d} \equiv - R_d e^{-i \gamma}, \quad \frac{v_c^d}{v_u^d} \equiv R_s e^{-i \gamma}, \]
\[ \alpha \equiv a_4 + a_10 + r(a_6 + a_8), \quad \beta \equiv a_1 + \alpha \] (11)

the amplitudes (8) get the expressions

\[ A(B_0 \to \pi^+ \pi^-) = -i \frac{G_f}{\sqrt{2}} f_\pi F_0^{B \to \pi^+ \pi^-} (m_B^2 - m_\pi^2) v_u^d [R_d e^{-i \gamma} \beta + \alpha], \]
\[ A(B_0 \to \pi^+ K^-) = -i \frac{G_f}{\sqrt{2}} f_K F_0^{B \to \pi^+ \pi^-} (m_B^2 - m_\pi^2) v_c^s [R_s e^{-i \gamma} \beta + \alpha], \] (12)

which allow us to write down the CP violating amplitude difference

\[ |A|^2 - |\bar{A}|^2 \] (13)

where the quantity \( \delta = \text{Re}(\beta) \text{Im}(\alpha) - \text{Im}(\beta) \text{Re}(\alpha) \) is the same.

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relation, [8],
\[ \Delta_{\pi^+\pi^-} = -\frac{f_2^2}{f_K} \Delta_{\pi^+K^-}, \]  
(14)

which can be used to test the SM CP violation, or to predict one rate difference if the other one is known. In these assumptions, the relation between the CP asymmetries,
\[ A_{CP}(\pi^+\pi^-) = -\frac{f_2^2}{f_K} B r(\pi^+K^-) A_{CP}(\pi^+K^-), \]  
(15)

for the reported CP branching ratios [Babar]
\[ B r(\pi^+\pi^-) = (5.8 \pm 0.4 \pm 0.3) \times 10^{-6} \]
\[ B r(\pi^+K^-) = (19.4 \pm 0.6) \times 10^{-6}, \]
leads to the following result,
\[ A_{CP}(\pi^+\pi^-) \approx -2.2 A_{CP}(\pi^+K^-), \]
which does not agree with the experimental data that are just emerging from Babar,
\[ A_{CP}(\pi^+K^-) = -0.107 \pm 0.018 \]

and Belle,
\[ A_{CP}(\pi^+\pi^-) = 0.55 \pm 0.08 \pm 0.05. \]

### III. IMPROVED FACTORIZATION METHOD

Let us turn to the improved factorization method, (IFM), developed by Beneke et al. [3], which gives a systematic and model-independent calculation of two-body hadronic decays, in the heavy-quark limit. The factorization formula, presented in the previous section, is applicable, since the nonfactorizable corrections are included in the \( a_i \) parameters, which have imaginary parts coming from vertex corrections and penguin contributions. In this approach, the Wilson coefficients are calculated at the scale \( \mu = m_b \) using next-to-leading order modified scheme, the electroweak penguin contributions are considered as next-to-leading order and there is also a contribution coming from the hard scattering with the spectator.

The IFM formula when both \( M_1 \) and \( M_2 \) are light mesons is

\[ \langle M_1 M_2 | O_4 | B \rangle = F_{0}^{B - M_1} f_{M_2} \int dx T_{M_2}^{I} \phi_{M_2}(x) + (1 \leftrightarrow 2) \]
\[ + f_B f_{M_1} f_{M_2} \int dz dy dx T_{M_2}^{II}(x, y, z) \phi_{B}(z) \phi_{M_1}(y) \phi_{M_2}(x), \]  
(16)

where \( \phi \) are the leading-twist light-cone distribution amplitudes and the integration is over the momentum fractions inside the mesons, \( T_{M_2}^{I} \) includes tree diagrams plus corrections (hard gluon exchanges and penguins) and \( T_{M_2}^{II} \) expresses the hard gluon exchange with the spectator. For \( T_{M_2}^{I} = 1 \) and \( T_{M_2}^{II} = 0 \), we recover the naive factorization. The meson wave functions will be an important source of SU(3) breaking. For the light mesons, we have twist-2 and twist-3 distribution amplitudes respectively defined in the following bilocal operator matrix elements:

\[ \langle M(p)|\bar{q}(z_2)\gamma_{\mu}\gamma_5 q(z_1)|0 \rangle = -f_{M\mu} \int_{0}^{1} dx e^{i(xp \cdot z_2 + \bar{x}p \cdot z_1)} \phi(x) \]
\[ \langle M(p)|\bar{q}(z_2)\gamma_5 q(z_1)|0 \rangle = f_{M\mu} \int_{0}^{1} dx e^{i(xp \cdot z_2 + \bar{x}p \cdot z_1)} \phi_{\mu}(x), \]  
(17)

where \( \mu_M \) is expressed in terms of the quark masses as \( \mu_M = \frac{m_2^2}{m_1 + m_2} \) and \( \bar{x} = 1 - x \). In the momentum space, the light-cone projector operator of a light pseudoscalar meson described by both the
twist-2 and twist-3 amplitudes is:

\[ \Phi(M) = \frac{iF}{4N_c} \left\{ \hat{p} \gamma_5 \phi(x) - \mu M \gamma_5 \phi_p(x) \right\}, \]

(18)

where \( \hat{p} = p \cdot \gamma \).

We notice that in \( a_1, a_2, a_3, a_4, a_5, a_7, a_9, a_{10} \), where we have \((V - A)(V + A) \), only the twist-2 amplitude is taken, while in \( a_6, a_8 \) the maximum of the amplitude and is very small \( a \in [0.05 \div 0.1] \). However, since the momenta is almost carried by the heavy quark, one may work with a strongly peaked function around \( z_0 = \lambda_B/m_B \approx 0.066 \pm 0.029 \), for \( \lambda_B = 0.35 \pm 0.15 \) GeV.

In the this approach, \( [3] \), the \( a_i \) coefficients,

\[
\begin{align*}
    a_1(M_1M_2) &= C_1 + \frac{C_2}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right], \\
    a_2(M_1M_2) &= C_2 + \frac{C_1}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right], \\
    a_3(M_1M_2) &= C_3 + \frac{C_4}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right], \\
    a_4^p(M_1M_2) &= C_4 + \frac{C_3}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right] + \frac{C_F \alpha_s}{4\pi N_c} p^p_{M_2}, \\
    a_5(M_1M_2) &= C_5 + \frac{C_6}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (-12 - V_{M_2} - H) \right], \\
    a_6^p(M_1M_2) &= C_6 + \frac{C_5}{N_c} \left( 1 - 6 \frac{C_F \alpha_s}{4\pi} \right) + \frac{C_F \alpha_s}{4\pi N_c} p^p_{M_2}, \\
    a_7(M_1M_2) &= C_7 + \frac{C_8}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (-12 - V_{M_2} - H) \right], \\
    a_8^p(M_1M_2) &= C_8 + \frac{C_7}{N_c} \left( 1 - 6 \frac{C_F \alpha_s}{4\pi} \right) + \frac{\alpha}{9\pi N_c} p^p_{M_2}, \\
    a_9(M_1M_2) &= C_9 + \frac{C_{10}}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right], \\
    a_{10}^p(M_1M_2) &= C_{10} + \frac{C_9}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right] + \frac{\alpha}{9\pi N_c} p^p_{M_2}, \\
\end{align*}
\]

(21)

where \( C_F = (N_c^2 - 1)/2N_c \), include the vertex, the hard gluon exchange with the spectator and the penguin contributions, at \( \mu = m_b \), defined as

\[ \alpha = G_F \sqrt{\frac{2}{\sqrt{\pi}}}. \]
\[ V_M = -18 + \int_0^1 dx g(x) \phi_M(x), \]

\[ P_{M,2}^p = C_1 \left[ \frac{2}{3} + G_M(s_p) \right] + C_3 \left[ \frac{4}{3} + G_M(0) + G_M(3) \right] \]

\[ + (C_4 + C_6) \left[ (n_f - 2)G_M(0) + G_M(s_c) + G_M(1) \right] - 2C_{\pi \phi}^p \int_0^1 \frac{dx}{x} \phi_M(x), \]

\[ P_{M,2}^{p,EW} = (C_1 + N_c C_2) \left[ \frac{2}{3} + G_M(s_p) \right] - 3C_{\pi \gamma}^p \int_0^1 \frac{dx}{x} \phi_M(x), \]

\[ P_{M,3}^p = C_1 \left[ \frac{2}{3} + \hat{G}_M(s_p) \right] + C_3 \left[ \frac{4}{3} + \hat{G}_M(0) + \hat{G}_M(1) \right] \]

\[ + (C_4 + C_6) \left[ (n_f - 2)\hat{G}_M(0) + \hat{G}_M(s_c) + \hat{G}_M(1) \right] - 2C_{\pi \phi}^{p,EW}, \]

\[ P_{M,3}^{p,EW} = (C_1 + N_c C_2) \left[ \frac{2}{3} + \hat{G}_M(s_p) \right] - 3C_{\pi \gamma}^{p,EW}, \]

\[ H = \frac{4\pi^2}{N_c} \frac{f_{B} f_{M_1}}{m_B \lambda_B} \int_0^1 \frac{dx}{x} \phi_{M_2}(x) \int_0^1 \frac{dy}{y} \left[ \phi_{M_1}(y) + \frac{2\mu_{M_1}}{m_B} \frac{x}{x} \phi_{M_1}(y) \right], \quad (22) \]

where the parameter \(2\mu_M/m_b\) coincides with \(r\) and \(s_i = m_i^2/m_b^2\) are the mass ratios for the quarks involved in the penguin diagrams, namely \(s_u = s_d = s_s = 0\) and \(s_c = (1.3/4.2)^2\).

Putting everything together, the amplitudes (8) get the explicit form

\[ A(\bar{B}^0 \to \pi^+ \pi^-) = -\frac{G_F}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B \to \pi}(m_\pi^2) f_\pi [V_{ub} V_{ud}^* a_1(\pi \pi) \]

\[ + V_{pd} V_{pu}^* (a_1^n(\pi \pi) + a_1^{\alpha_2}(\pi \pi) + r_\pi (a_2^n(\pi \pi) + a_2^{\alpha_2}(\pi \pi))], \quad (23) \]

\[ A(\bar{B}^0 \to \pi^+ K^-) = -\frac{G_F}{\sqrt{2}} (m_B^2 - m_K^2) F_0^{B \to \pi}(m_K^2) f_K [V_{ub} V_{ud}^* a_1(\pi K) \]

\[ + V_{pd} V_{pu}^* (a_1^n(\pi K) + a_1^{\alpha_2}(\pi K) + r_K (a_2^n(\pi K) + a_2^{\alpha_2}(\pi K))], \quad (24) \]

where \(p\) is summed over \(u\) and \(c\), and consequently, the relation (14) turns into, [6],

\[ \frac{\Delta^{\bar{B}^0}_{\pi^+ \pi^-}}{\Delta^{\bar{B}^0}_{\pi^+ K^-}} \approx \frac{f_\pi^2}{f_K^2} \left[ \frac{1 - 0.748\alpha_1^0 - 0.109\alpha_2^0 - 0.0013H_{\pi \pi}}{1 - 0.748\alpha_1^K - 0.109\alpha_2^K - 0.0013H_{\pi K}} \right], \quad (25) \]

pointing out the following SU(3) breaking effects: the difference in the decay constants and form factors, the difference in the \(\alpha_1\) and \(\alpha_2\) coefficients that appear in the twist-2 distribution amplitudes (19) and the \(H_{\pi \pi}\) and \(H_{\pi K}\) contributions (defined in (22)).

The decay amplitudes for \(\bar{B}^0_s \to K^+ \pi^-\) and \(\bar{B}^0_s \to K^+ K^-\) can be obtained by using the appropriate transition form factor \(F_0^{B_s \to K}\) and by changing \(1/m_B^2 \lambda_B\) to \(1/m_B^2 \lambda_B\) in \(H_{M_1 M_2}\). One gets the same expression, (25), and thus we have come to the following relation:

\[ \frac{\Delta^{\bar{B}^0}_{\pi^+ \pi^-}}{\Delta^{\bar{B}^0}_{\pi^+ K^-}} \approx \frac{\Delta^{\bar{B}^0}_{\pi^+ \pi^-}}{\Delta^{\bar{B}^0}_{\pi^+ K^-}}. \quad (26) \]

These example shows that important SU(3) breaking effects arise from the light-cone distributions of mesons in addition to those already present in the decay constants. These effects can only be estimated with large uncertainty because the parameters \(\alpha_{1,2}^{B_s}\) are not well determined at present. Using the currently

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allowed ranges we find,
\[ A_{CP}(\pi^+\pi^-) \approx - (3.1^{+1.9}_{-0.9}) A_{CP}(\pi^+K^-), \tag{27} \]
which can also be used to test the SM and the IFM to some extent.

However, relations which are independent of \( \alpha_{1,2} \) parameters and decay constants, such as (26), are more reliable since they do not receive the main SU(3) breaking corrections that we have investigated.

A. \( B \rightarrow PV \) Decays

When the vector meson is factored out, as in
\[ \bar{B}^0 \rightarrow \pi^+\rho^- \quad \bar{B}^0_s \rightarrow K^+K^{*-}, \]
and
\[ \bar{B}^0 \rightarrow K^+\rho^- \quad \bar{B}^0_s \rightarrow \pi^+K^{*-}, \]
the decay amplitudes can be obtained by replacing the \( r_K \) factor with \( r_K = \frac{m_{\pi^+}}{m_{\rho^-}} f_\rho^{*} f_K \approx 0.3 \) (and similarly for \( r_\rho \)), and by removing the penguin terms \( P_{M_{1,2,3}}^{PV} \) in the expressions for \( \alpha_6 \) and \( \alpha_8 \) (the vector meson is described only by a twist-2 distribution amplitude).

With all these taken into account, we get, [6],
\[
\begin{align*}
\frac{\Delta \bar{B}^0_{\pi^+\rho^-}}{\Delta \bar{B}^0_{K^+K^{*-}}} & \approx - \frac{m_B}{m_{B_s}} \frac{f_\rho^2}{f_K^2} \times \left( \frac{F_{1,2}}{F_{1,2}} \right)^2 \frac{1 - 1.25\alpha_2^0 - 0.18\alpha_2^0}{1 - 1.25\alpha_1^0 - 0.18\alpha_1^0}. \tag{28} \end{align*}
\]

Using the central values of the ranges
\[ \alpha_1^0 = 0, \quad \alpha_2^0 = 0.15, \]
\[ \alpha_1^{K^+} = 0.04, \quad \alpha_2^{K^+} = 0.10 \]
and taking \( f_\rho \approx 0.96 f_{K^*} \), we find,
\[
\begin{align*}
\frac{\Delta \bar{B}^0_{\pi^+\rho^-}}{\Delta \bar{B}^0_{K^+K^{*-}}} & \approx - 0.95 \left( \frac{F_{1,2}}{F_{1,2}} \right)^2, \\
\frac{\Delta \bar{B}^0_{\pi^+\rho^-}}{\Delta \bar{B}^0_{\pi^+K^{*-}}} & \approx - 0.95 \left( \frac{F_{1,2}}{F_{1,2}} \right)^2. \tag{29} \end{align*}
\]

When the meson that picks up the spectator is a vector, as for example in
\[ \bar{B}^0 \rightarrow \rho^+\pi^- \quad \bar{B}^0_s \rightarrow K^{*-}K^- \]
\[ \bar{B}^0 \rightarrow K^+\pi^- \quad \bar{B}^0_s \rightarrow K^{*-}\pi^- \]
the corresponding decay amplitudes can be obtained by replacing the form factor \( F_{1,2}^{B \rightarrow V} \) with \( A_{1,2}^{B \rightarrow V} \) and \( r \) with \( -r \). In this case, the SU(3) breaking is large and estimates are unreliable since the analogue of (29), [6],
\[
\begin{align*}
\frac{\Delta \bar{B}^0_{\rho^+\pi^-}}{\Delta \bar{B}^0_{K^{*-}K^-}} & \approx - \frac{m_B}{m_{B_s}} \frac{f_\rho^2}{f_K^2} \times \left( \frac{A_{0}^{B \rightarrow \rho}}{A_{0}^{B \rightarrow K^-}} \right)^2 \frac{1 + 110\alpha_1^0 + 15.5\alpha_2^0}{1 + 110\alpha_1^K + 15.5\alpha_2^K} \tag{30} \end{align*}
\]
contains large coefficient of \( \alpha_1^0 \) in both the numerator and denominator making a prediction for this asymmetry impossible within this framework. On the other hand, this provides an opportunity to constrain (or even to determine) \( \alpha_1^K \) when the ratio in (30) is measured.

B. \( B^- \rightarrow K^-\eta' \)

Let us turn now to the \( B^- \rightarrow \eta'K^- \) decay which has become of a real interest after CLEO announced its large numerical value \( BR(B^- \rightarrow \eta'K^-) = (6.5^{+1.5}_{-0.9}) \times 10^{-5} \), which could not be explained by the existent theoretical models. As improved measurements followed, providing even larger values, \( (80^{+10}_{-9}) \times 10^{-6} \) (CLEO) and \( (76.9 \pm 3.5 \pm 4.4) \times 10^{-6} \) (BaBar), inclusion of new contributions for accommodating these data has quickly become a real theoretical challenge.

The relevant decay amplitude for \( B^- \rightarrow \eta'K^- \) is,
\[ A(B^- \rightarrow \eta' K^-) = -i \frac{G_F}{\sqrt{2}} (m_B^2 - m_{\eta'}^2) F_0^{B^{-\eta'}} (m_K^2) f_K [V_{ub} V_{us}^* a_1(X) \\
+ V_{sb} V_{us}^* (a_4^p(X) + a_4^p(X) + r_K(a_6^p(X) + a_6^p(X))) ] \\
- i \frac{G_F}{\sqrt{2}} (m_B^2 - m_{\eta'}^2) F_0^{B^{-\eta'}} (m_K^2) f_0^{\eta'} [V_{ub} V_{us}^* a_2(Y) + V_{sb} V_{us}^* [(a_5(Y) - a_5(Y)) (2 + \sigma) \\
+ \left[ a_5^p(Y) - \frac{1}{2} a_5^{10}(Y) + r'(a_6^p(Y) - \frac{1}{2} a_6^{10}(Y)) \right] \sigma + \frac{1}{2} (a_6(Y) - a_7(Y)) (1 - \sigma) ]] . \]

where \( X = \eta' K \) and \( Y = K \eta' \), \( r' = 2m_{\eta'}/(m_b - m_s) \) and \( \sigma = f_{\eta'}/f_{\eta'}^u \). As it can be noticed, the coefficients \( a_i \) are different for the \( X \) and \( Y \) final states, since they depend on the twist-2 and twist-3 wave functions of the \( M_2 \) meson, except for the hard contribution where the wave functions for both \( M_1 \) and \( M_2 \) are involved.

The twist-2 distribution amplitude \( \phi_{\eta'}(x) \) has the usual expansion in Gegenbauer polynomials, while the corresponding twist-3 amplitude, \( \phi_{\eta''}(x) \), is 1. In what it concerns the physical states \( \eta' \) and \( \eta'' \), these are mixtures of SU(3)-singlet and octet components \( \eta_0 \) and \( \eta_8 \) and the corresponding decay constants, in the two-angle mixing formalism, are given by, \([1] \), \( f_{\eta'}/f_{\eta'} = 63.5 \) MeV, \( f_{\eta''} = 141 \) MeV, while the relevant form factor for the \( B \rightarrow \eta' \) transition is \( F_0^{B^{-\eta'}} = 0.137 \). Even the \( \eta' \) flavor singlet meson has a gluonic content, \([10] \), which could bring a contribution to the wave function, this is supposed to be small and therefore we employ, in the calculation of \( V_{u'}/V_{u} \) and \( V_{d/b}^{EWM} \) in \( a_i(Y) \), only the leading twist-2 distribution amplitude \( \phi_{\eta'} = 6x \bar{\phi} \). Also, since the twist-3 quark-antiquark distribution amplitude does not contribute, due to the chirality conservation, the penguin parts in \( a_6^p(Y) \) and \( a_6^q(Y) \) are missing.

In IFM, we get for the \( B^- \rightarrow K^- \eta' \) decay the numerical value \( Br(B \rightarrow K \eta') = 3.65 \cdot 10^{-5} \) which is comparable to other theoretical estimations, \([1, 4, 10] \), but is only half of the averaged experimental data, suggesting that one has to incorporate take into account new contributions in order to increase the \( Br(B \rightarrow K \eta') \) numerical values.

In this respect, we have employed the Minimal Supersymmetric Standard Model (MSSM), by adding to the effective SM Hamiltonian the following SUSY contribution

\[ H_{\tau \gamma}^{SUSY} = -i \frac{G_F}{\sqrt{2}} (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \left( \phi_{\tau}^{SUSY} \mathcal{O}_{\tau\gamma} + \phi_{\tau}^{SUSY} \mathcal{O}_{\tau\gamma} \right) , \]

expressed in terms of the gluon and photon operators:

\[ O_{\tau\gamma} = \frac{g_s}{8 \pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b , \quad O_{\tau\gamma} = \frac{e}{8 \pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b . \]

The Wilson coefficients are given by, \([5] \),

\[ \phi_{\tau}^{SUSY} (M_{SU^2}) = - \frac{\sqrt{2} \pi \alpha_s}{G_F (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) m_b^2} \delta_{LR} \frac{m_b}{m_b} G_0(x) , \]

\[ \phi_{\tau}^{SUSY} (M_{SU^3}) = - \frac{\sqrt{2} \pi \alpha_s}{G_F (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) m_b^2} \delta_{LR} \frac{m_b}{m_b} F_0(x) , \]

where

\[ G_0(x) = \frac{x}{3 (1 - x)^4} \left[ 22 - 20 x - 2 x^2 + 16 x \ln(x) - x^2 \ln(x) + 9 \ln(x) \right] , \]

\[ F_0(x) = - \frac{4 x}{9 (1 - x)^4} \left[ 1 + 4 x - 5 x^2 + 4 x \ln(x) + 2 x^2 \ln(x) \right] . \]

In the above relations, \( x = m_B^2 / m_{\tilde{g}}^2 \), with \( m_{\tilde{g}} \) being the gluino mass and \( m_{\tilde{q}} \) an average squark mass,

Insert PSN Here
while the factor $\delta^{bs} = \Delta^{bs}/m_q^2$, where $\Delta^{bs}$ are the off-diagonal terms in the sfermion mass matrices, comes from the expansion of the squark propagator in terms of $\delta_q$ for $\Delta \ll m_q^2$. In principle, the dimensionless quantities $\delta^{bs}$, measuring the size of flavor changing interaction for the $\tilde{s}\tilde{b}$ mixing, are present in all the SUSY corrections to the Wilson coefficients and they are of four types, depending on the L or R helicity of the fermionic partners. However, one finds that $\{c_{i}^{SUSY}\}_{i=36}$, for $M_{SUSY} = m_{\tilde{q}} = 500$ GeV and $x \approx 1$, do not bring any significant contribution to the branching ratio. The situation looks different in what it concerns the SUSY Wilson coefficients (34) that, when $m_{\tilde{g}}$ is of order of few hundred GeV, will dominate the SM ones.

Thus, we replace the Wilson coefficients $c_{\gamma}^{eff}$ and $c_{\gamma}^{eff}$, by the total quantities

$$c_{\gamma}^{total}[x, \delta] = c_{\gamma}^{eff} + c_{\gamma}^{SUSY}(m_b),$$  
and

$$c_{\gamma}^{total}[x, \delta] = c_{\gamma}^{eff} + c_{\gamma}^{SUSY}(m_b),$$  

where $c_{\gamma}^{SUSY}(m_b)$ have been evolved from $M_{SUSY} = m_{\tilde{g}}$ down to the $\mu = m_b$ scale. For $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{g}} = m_{\tilde{q}}$ and $\delta^{bs}_{LR} \equiv \rho e^{i\phi}$, the total branching ratio can be expressed in terms of the parameters $\rho$ and $\phi$ as

$$BR_{total} = 10^{-5} (3.65 + 447\rho\cos\phi + 13670\rho^2 + 13.78\rho\sin\phi),$$  

pointing out, besides the (IFM)-value $3.65 \times 10^{-5}$, the SUSY contribution depending on $\rho$ and $\phi$.

A detailed analysis of this formula, suggests that one should take $\rho \in [0.005, 0.01]$ and $\phi \in [-\pi/4, \pi/2]$, for accommodating the range within the two extreme experimental data, $BR_{exp}(BaBar) = 7 \times 10^{-5}$ and $BR_{exp}(CLEO) = 8 \times 10^{-5}$. For $\rho$ close to the lowest limit of its interval, the predicted $BR_{total}$ values lie below the experimental data, while for $\rho$ moving to the central value and $-\pi/4 \leq \phi \leq \pi/4$, one gets $BR_{total} \in [7 \times 10^{-5}, 8 \times 10^{-5}]$.

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[1] A. Ali hep-ph/9804363.
[2] P. Ball et al., Nucl. Phys. B529 (1998) 323; P. Ball and V.M. Braun, hep-ph/9808229; P. Ball, V.M. Braun and A. Lenz, J. High Ener. Phys. 0605:004 (2006).
[3] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. B606 (2001) 245.
[4] M. Beneke and M. Neubert, hep-ph/0110085.
[5] A.J. Buras, et al., Nucl. Phys. B566 (2000) 3.
[6] M.A. Dariescu, N.G. Deshpande et al., hep-ph/0212333.
[7] M.A. Dariescu and C. Dariescu, Eur. Phys. J. C36 (2004) 215.
[8] N.G. Deshpande and X.G. He, Phys. Rev. Lett. 75, (1995) 1703; X.G. He, Eur. Phys. J. C9 (1999) 443.
[9] D.S. Du, D. Yang and G.H. Zhu, hep-ph/0103211.
[10] D.S. Du, C.S. Kim and Y. Yang, Phys. Lett. B426 (1998) 133; D.S. Du, D.S. Yang and G.H. Zhu, Phys. Rev. D59 (1999) 014007; M.Z. Yang and Y.D. Yang, Nucl. Phys. B609 (2001) 469.
[11] F. Gabbiani, et al., Nucl. Phys. B477, 321 (1996).