Observable Features of QED Cascades in Collisions of GeV Electrons with Intense Laser Pulses

Arseny Mironov, Alexander Fedotov and Nikolay Narozhny

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe sh. 31, 115409 Moscow, Russian Federation
E-mail: mironov.hep@gmail.com

Abstract. We consider interaction of multi-GeV electron beam with electromagnetic field formed by intense focused laser pulses. At intensity $\sim 10^{24} \div 10^{25}$ W/cm$^2$ such a scheme can be used to produce both S(Shower)-type and A(Avalanche)-type (or “self-sustained”) QED cascades. The proposed experimental design provides a solution to the problem of injection of seed particles into the strong field region. Here we are focusing on discussion of some peculiarities of cascade dynamics and evaluation of such quantities as final angular and spectral particle distributions, that could be observed in a realistic experiment. It turns out that one can unambiguously attribute some features of such distributions to either an S- or A-type cascade. We also discuss the possibility to initiate A-type cascades by collision of GeV electrons with a single tightly focused laser pulse. As a byproduct, it would be possible to produce in this way bright collimated $\gamma$-ray beams of GeV energies.

1. Introduction
A-type or self-sustained electron-positron-photon (QED) cascades have become a rather widely discussed phenomena since their first predictions [1–3]. Most authors assume that a slow electron is somehow placed in a strong field region formed by a few laser pulses [1–12]. Examples of discussion of more realistic scenarios, like irradiation of solid or gas targets by strong laser pulses, are also known [13–17], but such studies require rather involved models and simulations. We discuss another realistic option of injecting seed electrons into the strong field region, which is based on the effect of collapse and revival of cascades caused by collision of GeV electron beam with focused laser pulses [18, 19]. Though having access to the whole simulation data it is typically evident whether or not an A-type cascade occurs, we are interested in developing criterions that rely on such features as final distributions, which should be directly observable in a realistic experimental setup.

2. Two laser pulses setup
Assume that two circularly polarized laser pulses synchronously counter-propagate along the same axis as shown in Fig. 1, and that laser field satisfies the conditions of A-type cascade initiation by a slow $e^-$ in the focus. This condition is satisfied e.g. if lasers are optical ($h\omega \approx 1$ eV), and the total intensity is $I \sim 10^{25}$ W/cm$^2$ (corresponding to electric field strength $E_0 \sim 10^{-3}E_{cr}$, where $E_{cr} = m^2c^3/eh$). Now suppose that $e^-$-beam with energy $\varepsilon_0 \sim 1 \div 10$ GeV is injected into the focus. We study cascade dynamics using 3D Monte Carlo code, which
Figure 1. Scheme of experiment. Red shapes sketch the laser field and blue dot represents the initial GeV $e^{-}$.

Figure 2. Evolution of the density $d^2N_{e^-e^+}/dt dz$ of $e^-e^+$ creation events along $z$-axis (in a.u., on top), and the electron creation rate $dN_{e^-}/dt$ (in a.u., on bottom) for $\varepsilon_0 = 3 \, \text{GeV}$, $E_0 = 3.2 \times 10^{-3} E_S$ (total $I \approx 4.8 \times 10^{21} \, \text{W/cm}^2$). Thin solid lines indicate the structure of electric field distribution.

was developed and tested in Refs. [18,19]. We neglect all plasma effects since we study cascades near their formation threshold when plasma density remains rather low, and assume that each laser pulse is tightly focused and of finite duration $\tau_L = 10 \, \text{fs}$.

Under the above conditions an S-type cascade is initiated first. Assume the cascade is induced by electrons moving along the focal $z$-axis towards the focal center, see Fig. 1. As can be seen from Fig. 2, as they approach the strong field region, the particle creation rate tends to grow. As S-type cascade develops, the emitted $\gamma$ and secondary $e^\pm$ propagate along $z$-axis at almost speed of light. However the charged particles are losing energy and longitudinal momentum $p_\parallel$ randomly, hence their distribution after some time is broadened, particle creation rate is decreased and the S-cascade starts to collapse, as is seen for $t \gtrsim -0.1 \tau_L$ in Fig. 2. Though according to Pomeranchuk’s theorem initial electrons cannot reach the focal center, photons still propagate freely, and by reaching the focus have a chance for creating a pair of slower $e^\pm$, which further gain large transverse momentum $p_\perp \gg p_\parallel$ from the field. These secondary particles act as seeds for an A-type cascade which takes place at the central antinode of the field. Particle creation rate is growing again after $t > 0$, i.e. cascade revives and continues until laser field in the focal plane remains strong enough.

Though the assumption that direction of the initial $e^-\text{-beam}$ is aligned along the optical axis of laser pulses is not pleasurable from experimental point of view, the mechanism of collapse and revival does not change qualitatively even if electrons hit the focus at some nonzero tilt angle. Moreover, when the initial direction of $e^-\text{-beam}$ is set orthogonal to focal axis, almost all particles of the initial S-type cascade fall into the central antinode of the field. As the number of seeds increases, this results in enhancement of the subsequent A-type cascade, see Fig. 3 and further details in Ref. [19].

Evidence for subdivision of a cascade into S- and A- stages and occurrence of the latter is clear from evolution of spatial distributions presented in Fig. 2. However it is hard to imagine that such instant distributions could be ever resolved in a real experiment. Therefore one should rather study final distributions of $\gamma$ and $e^\pm$ trying to identify signatures of both cascades. For an S-type cascade, multiplicity depends on both the initial electron energy and laser field intensity, and its dynamics strongly depends on initial direction of the particles, whereas properties of
Figure 3. Number of $e^-e^+$ pairs produced in a cascade normalized to the initial number of electrons in the beam for $\varepsilon_0 = 3 \text{ GeV}$, $E_0 = 3.2 \times 10^{-3} E_S$. $\Theta$ is the tilt angle of $e^-$-beam with the focal axis.

Figure 4. Normalized spectra of photons emitted at angles $\theta = 0$ (top) and $\theta = 90^\circ$ (bottom) within spread $\Delta\theta \approx 10^\circ$ for different intensities and $\varepsilon_0 = 3 \text{ GeV}$.

an A-type cascade depend solely on the laser field structure, and in addition for them there exists some threshold in laser intensity. Bearing all that in mind, let us imagine that we can vary the intensity from low (when A-cascade does not occur) to higher (when it prevails) values, looking for those changes in final particle distributions that may allow to identify occurrence of an A-type cascade as well as to distinguish it from the S-type one.

The simplest quantity to measure is the total number $N$ of $e^\pm$ created in a cascade. Multiplicity of S-type cascade is proportional to $\propto \varepsilon_0 E_0$ [18], while for A-type cascade it depends exponentially on $E_0$ [4]. This means that when intensity is gradually increased, the slope of the dependence $N(E_0)$ should change above the threshold $E_{th}$, which is exactly the case observed in our simulations for both longitudinal and transverse incidence of $e^-$ beam at $E_{th} \approx 2.5 \div 3.0 \times 10^{-3} E_S$ (or $I \approx 4 \times 10^{24} \text{ W/cm}^2$), see Fig. 3.

More information can be extracted from the final photon spectra. For example, as discussed above, for the setup in Fig. 1 almost all the particles of an S-type cascade are propagating along $z$-axis, while A-type cascade develops approximately in an orthogonal central antinode plane $z = 0$. Hence photons of these cascades are emitted in different directions. Let $\theta$ be the angle between $z$-axis and the direction of propagation of $\gamma$. If the detectors measure photons emitted along directions $\theta = 0$ and $\theta = \pi/2$, they count photons from S- and A-type cascade, respectively. Spectra of photons emitted in these directions is shown in Fig. 4. When the intensity is not enough for initiation of an A-type cascade, almost all $\gamma$ are emitted along $\theta = 0$, but once one increases $E_0$ opening the A-type “channel”, more photons are detected at $\theta = \pi/2$. At the same time spectrum of photons from S-type cascade becomes narrower since less energy is needed for a photon to create a pair, and hence smaller amount of energetic photons survive up to the end. Spectrum of photons from A-type cascade behaves in the opposite manner: since $e^\pm$ are accelerated by the field, as $E_0$ increases, they tend to emit more energetic photons contributing to the final distribution. Such diverse changes in photon spectra can be considered as another evidence of collapse and revival of a cascade.
Figure 5. Distribution of particles and laser field at different moments of time, from left to right: $t = -0.16\tau_L$, $t = 0.20\tau_L$, $t = 1.40\tau_L$. Dark red and black dots correspond to $e^-$ and $e^+$, yellow to $\gamma$. Blue distribution depicts laser field (in a.u.) of intensity $I \approx 1.2 \times 10^{26}$ W/cm$^2$. Initial $e^-$ with energy $\varepsilon_0 = 2$ GeV counter propagate the laser pulse so that in absence of the field they would reach center of the focus at $t = 0$.

Figure 6. Pair creation rate in a cascade at intensities $I \approx 3 \times 10^{25}$ W/cm$^2$ (blue), $I \approx 7 \times 10^{26}$ W/cm$^2$ (green), and $I \approx 1.2 \times 10^{26}$ W/cm$^2$ (black) for $\varepsilon_0 = 2$ GeV.

Figure 7. Normalized final spectra of photons at intensities $I \approx 3 \times 10^{25}$ W/cm$^2$ (magenta), $I \approx 7 \times 10^{26}$ W/cm$^2$ (black-dashed) and $I \approx 1.2 \times 10^{26}$ W/cm$^2$ (red) for $\varepsilon_0 = 2$ GeV.

3. Single pulse setup
Up to now A-type QED cascades have been almost always considered in standing waves formed by a couple or more of laser pulses, mainly in order to lower the required laser intensity per single pulse. Nevertheless, it is worth studying whether self-sustained cascades can occur in a single laser pulse. Obviously, for that laser field should be first of all tightly focused, since a weakly focused, i.e. plane wave-like pulse is not capable for particle acceleration in the sense of increasing the key parameter $\chi$. Secondly, while in case of several colliding laser pulses there are some alternative scenarios of injecting seed electrons into the focus, for a single laser pulse there is no alternative other than head-on collision of GeV $e^-$ - or photon beam with the laser pulse.

Particle distributions at different moments of time and the corresponding pair creation rate for such a scheme are shown in Fig. 5 and Fig. 6, respectively. We assume that laser pulse propagates along $z$-axis, and a bunch of $e^-$ of the same initial position and momentum propagates in the opposite direction. S-type cascade starts immediately after the electron bunch reaches the strong field region, as indicated by a drastic increase of particle production rate. The high-energy $\gamma$ are emitted towards the center of the focus, and create $e^-e^+$ pairs close to the focal region, which is non-accessible for initial electrons. As particles approach the center of the focus, they lose their energy. In addition, the field contorts their trajectories and accelerates them in
opposite directions (at least on average). As a result, the initial S-type cascade starts collapsing and the particle creation rate reduces substantially. But after that the laser pulse reaccelerates them in the direction of its own propagation carrying away along $z$-axis. Since the pulse is tightly focused, in a proper frame of the focal center it looks as a standing spherical wave, hence the particles moving along with the pulse are under optimal conditions for seeding an A-type cascade, and the cascade starts over again. The second long hump on Fig. 6 corresponds to such a secondary A-type cascade. We note that its occurrence is suppressed for $I \lesssim 3 \times 10^{25}$ W/cm$^2$, while S-cascade has almost the same multiplicity for all the values of $I$ in our simulations. Such behavior confirms that the mechanism of collapse and revival is the same as in case of two laser pulses considered above.

The main observed difference with the case of cascades in a two pulses setup is sharper separation of the S- and A-type cascade stages. This happens since S-type cascade develops as long as particles are capable for penetration inside the field, while A-type cascade cannot start until they turn around starting to propagate along with the pulse. Though such a scheme requires laser intensity $I \sim 10^{25} \div 10^{26}$ W/cm$^2$, one order of magnitude higher than the two pulses setup, sharper separation may be useful to study properties of each stage more thoroughly. As in two pulses setup, it makes sense to study e.g. distributions of photons emitted at different angles.

The final spectrum of photons in case of single laser pulse, which is presented in Fig. 7, shares some common features with the spectra of photons from the A-type cascade in Fig. 4. In particular, the spectrum becomes broader as intensity is growing. But unlike the two-pulse setup, where the photons are emitted uniformly in all directions orthogonal to the focal axis, here they form a narrow bunch of collimated $\gamma$-quanta, of few $\mu$m length, propagating along with the laser pulse. The energies of photons in such a bunch are in GeV range. This implies that QED cascades produced by a single laser pulse may serve as a source of short collimated hard $\gamma$-rays.

4. Conclusion
Collision of high-energy GeV electrons with laser pulses of intensity $10^{24} \div 10^{26}$ W/cm$^2$ (depending on laser field configuration) is a realistic experimental scenario for observation of A-type cascades. Though one could expect A-type cascade to be shaded by a preceding S-type cascade, according to our results final particle distributions provide enough information to separate them, in particular to verify occurrence of an A-type cascade. Besides, we demonstrate that the collapse and revival scenario reveals even with a single laser pulse. Even though such simplified scheme requires higher laser intensity, it is still favorable in certain aspects, for example provides sharper separation of the S- and A-type stages. It can also serve as sources of short collimated bunches of $\gamma$-rays or high-energy electrons and/or positrons.

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