Energy loss due to field fluctuations in a two-stream QCD plasma

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Abstract

We derive the expression for the collisional energy loss in two stream plasma induced by the fluctuating chromoelectric field. It is revealed that the main contribution here comes from the unstable modes which grow exponentially with time. A strong direction dependence of the energy loss has also been demonstrated.

One of the goals of the ongoing relativistic heavy ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) and the experiments at CERN Large Hadron Collider (LHC) is to produce quark gluon plasma (QGP) and study its properties. According to the prediction of lattice quantum chromodynamics, QGP is expected to be formed when the temperature of nuclear matter is raised above its critical value, $T_c \sim 170$ MeV, or equivalently the energy density of nuclear matter is raised above $1$ GeV/$f m^3$ [1]. The possibility of QGP formation at RHIC experiment, with initial density of $5$ GeV/$f m^3$ is supported by the observation of high $p_T$ hadron suppression. This phenomena commonly known as jet quenching actually is related to the energy loss of the fast moving partons in the plasma [2]. Apart form jet-quenching, several possible probes have been studied in order to characterize the properties of QGP.

The estimation of energy loss in quark gluon plasma is thus essential to understand high $p_T$ hadron production in relativistic plasma. The first such estimation was performed by Bjorken [2] long back. Since then, lot of progress has been made to perfect such estimates under various circumstances including both the collisional and radiative losses [3]. Most of these calculations are performed in situations where the distributions of soft partons providing the thermal background are assumed to have isotropic momentum distribution. In realistic scenario, due to rapid initial expansion the system in the longitudinal direction cools faster than the transverse direction leading to $\langle p_T^2 \rangle_L \ll \langle p_T^2 \rangle_T$. Such momentum anisotropy might lead to collective modes having characteristic behavior distinct from what happens in isotropic plasma. This has been vividly exposed in [4, 5] where it is shown that these modes might have several branches and some of those might even be unstable. The number of unstable modes, as revealed in [4] depends on whether the system is stretched or squeezed along the direction of anisotropcity. The most important among those are the modes which grow exponentially in time. These, as we shall see, contribute to the energy loss when the random statistical fluctuations of the fields are considered. This particular aspect of energy loss has remained unexplored so far.

In all the calculations except [6] the contribution due to field fluctuations have been ignored or eikonal approximation is assumed. Interestingly, however, in [6] it has been shown that such a field fluctuation might lead to an energy gain of a moving quark in isotropic QGP irrespective of its velocity. It would therefore be interesting to see what happens in situations where the ground state is anisotropic and the random behavior of the chromodynamic fields are considered. This is precisely what we consider in this letter. This contribution is time dependent and dominates at the early stage of the collision when the anisotropcity exists and appear as corrections to the polarization loss. The latter, i.e. the polarization loss however, remain unaffected by the unstable modes for reasons explained in [7, 8].

The calculations of fluctuation spectrum of chromodynamic fields even for an equilibrium system is a daunting task because of numerous terms to be taken into account. For anisotropic system to be realized in relativistic heavy ion collisions, it becomes quite difficult to calculate the field correlations. Instead of considering momentum distribution corresponding to a QGP likely to be produced in relativistic heavy ion collisions we concentrate here on a two-stream plasma for the sake of tractability of our calculations and to bring the underlying dynamics to clearer relief. The two stream plasma is unstable with respect to both electric and magnetic interactions. The effect of these unstable modes in a two stream plasma has been studied recently [9, 11, 12, 13]. In [13] they have shown that the transport
coefficient grows exponentially in time due to these unstable modes. In the present work, we shall be concentrating on the contributions to the energy change of a fast moving parton due to chromoelectric field fluctuations which, as indicated before, remained unaddressed so far even in the calculations of quark energy loss in anisotropic medium. In particular, we shall show that the unstable modes lead to an exponentially growing energy change due to fluctuations in a two-stream plasma.

When a charge particle moves through a plasma it loses part of its energy due to its interaction with medium particles \[14, 15\]. The energy loss of a particle is determined by the breaking forces acting on the particle due to the chromoelectric field produced by the particle itself while moving. The energy loss of the moving particle per unit time:

$$\frac{dE}{dt} = Q^a v \mathbf{E}^a |_{r=vr}.$$  \hspace{1cm} (1)

If we take into account the fluctuation of the field in the plasma and also the change of velocity of the particle, that is, the average change in energy of the moving particle due to the correlation between the fluctuation in the velocity of friction due to the space-time correlation of the fluctuations in the electric field \[14, 15\]. The third part determines the energy loss of a particle is determined by the breaking forces acting on the particle due to the chromoelectric field which is a random function of position and time.

The classical equation of motion of the particles in the electromagnetic field has the form

$$\frac{dp}{dt} = Q^a [\mathbf{E}^a(r(t), t) + v \times \mathbf{B}^a(r(t), t)].$$  \hspace{1cm} (3)

Integrating the above equation, we obtain

$$v(t) = v_0 + \frac{1}{E_0} \int_0^t dt_1 Q^a \mathbf{F}^a(r(t_1), t_1),$$

$$r(t) = v_0 t + \frac{1}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 Q^a \mathbf{F}^a(r(t_2), t_2).$$  \hspace{1cm} (4)

where \(r_0\) and \(v_0\) are the radius vector and the velocity at the initial stage of the particle. \(E_0\) is the initial parton energy and \(F = E + v \times B\).

We choose a time interval \(\Delta t\) sufficiently large compared with the period of the random fluctuation of the particle field in the plasma but small compared to the time during which the motion of the particle changes appreciably. During this time interval \(\Delta t\) the particle trajectory differs little from straight line. So we can write only the leading order terms for the particle velocity and the field acting upon the particle at this time interval:

$$v(t) = v_0 + \frac{1}{E_0} \int_0^t dt_1 Q^a \mathbf{F}^a(r(t_1), t_1),$$

$$\mathbf{E}^a(r(t), t) = \mathbf{E}^a(r_0(t), t) + \frac{Q^a}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 \mathbf{E}^a_j(r_0(t_2), t_2) \frac{\partial}{\partial r_{0j}} \mathbf{E}^a(r_0(t), t)$$  \hspace{1cm} (5)

where, \(r_0(t) = v_0 t\). Substituting Eq. (5) into Eq. (3) and using \(\langle E_i^a E_j^a \rangle_\beta = 0 \quad [16]\), we find,

$$\frac{dE}{dt} = \langle Q^a v_0 - \mathbf{E}^a(r_0(t), t) \rangle_\beta + \frac{Q^a Q^b}{E_0} \int_0^t dt_1 \mathbf{E}^b_j(r_0(t_1), t_1, t) \mathbf{E}^a(r_0(t), t) \rangle_\beta$$

$$+ \frac{Q^a Q^b}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 \mathbf{E}^a_j(r_0(t_2), t_2) \times \frac{\partial}{\partial r_{0j}} v_0 \mathbf{E}^a(r_0(t), t) \rangle_\beta.$$  \hspace{1cm} (6)

The average value of the fluctuating part of the field vanishes so that the quantity \(\langle \mathbf{E}(r(t), t) \rangle\) is the chromoelectric field produced by the particle itself in the plasma. Thus, the first term in Eq. (6) is the polarization energy loss of the moving parton calculated in Ref. \[14, 15, 17\]. The second part corresponds to the statistical friction due to the space-time correlation of the fluctuations in the electric field \[14, 15\]. The third part determines the average change in energy of the moving particle due to the correlation between the fluctuation in the velocity of
the particle and the fluctuation in the electric field in the plasma. The presence of such correlations lead to additional change in the energy of the moving particle. The calculation of polarization energy loss in anisotropic plasma to be realized in relativistic heavy ion collision has been calculated in Ref. [7, 8]. As mentioned earlier, in this work, we shall consider a two stream plasma which permits us to obtain results in closed form revealing the physical mechanism into clearer focus. Hence we take,

\[ f(p) = (2\pi)^3 \delta^3(p-q) + \delta^3(p+q) \]

where, \( n \) is the effective parton density in a single stream. Such a system is extremely unstable and leads to exponentially growing collective modes for the plasma excitation. As mentioned earlier, the first term in Eq.(6) corresponds to the polarization loss which can be calculated in such a system and the presence of unstable modes can be taken care of as in Ref. [7, 8]. Our main concern here is to calculate the contribution due to statistical part where the unstable modes play important role. It is to be noted that in non-equilibrium situation it is extremely difficult to obtain the chromodynamics field [10, 11, 12, 13] although in the present case only the chromoelectric field is involved. We consider only the longitudinal part of the chromoelectric contribution. The corresponding modes are obtained by setting the dielectric function, \( \varepsilon_\perp(\omega, k) = 0 \) giving four roots, \( \pm \omega_\perp(k) \), where [11, 13]

\[ \omega_\perp^2(k) = \frac{1}{k^\perp} \left[ k^\perp(k^\perp)^2 + \mu^2(k^\perp)^2 - (k^\perp)^2 \right] \pm \sqrt{(k^\perp)^2 - (k^\perp)^2(4k^\perp(k^\perp)^2 + \mu^2(k^\perp)^2 - (k^\perp)^2))} \]

Note that, \( 0 < \omega_\perp(k) \in R \) for any value of \( k \) but \( \omega_\perp(k) > \omega_\perp(k) \) for \( k^\perp(k^\perp)^2 < 2\mu^2(k^\perp)^2 - (k^\perp)^2 \) when it represents the two-stream electrostatic instability generated due to the mechanism analogous to the Landau damping. For \( k^\perp(k^\perp)^2 \geq 2\mu^2(k^\perp)^2 - (k^\perp)^2, \) the mode is stable, \( 0 < \omega_\perp(k) \in R \). Let us now consider the domain of wave vectors obeying \( k^\perp(k^\perp)^2 < 2\mu^2(k^\perp)^2 - (k^\perp)^2 \) when \( \omega_\perp(k) \) is imaginary and it represents the unstable electrostatic mode. In this case we write down \( \omega_\perp(k) \) as \( \gamma_\perp(k) \) with \( 0 < \gamma_\perp(k) \in R \).

Considering the correlations between the electric field fluctuations and between the velocity and field fluctuations coming from the contributions of the unstable mode (which are fastest growing function of \( (t_1 + t_2) \)), the second and the third terms in Eq.(6) can be combined to obtain [18]

\[ \frac{dE}{dt} = \frac{Q^2Q^8 e^{-2}}{E_0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^\perp} \left[ \gamma_\perp^2 (k^\perp)^2 \right] \left[ \gamma_\perp^2 (k^\perp)^2 \right] \frac{\omega_\perp^2}{(\omega_\perp^2)^2} \]

where, we have only considered the fastest growing modes and dropped the other terms.

As can be seen from Eq.(10) the expression for the change in energy, in the limit \( \gamma_\perp \to 0 \), diverges. However, for non-zero \( \gamma_\perp \), as is the case here, the effect of the growth of unstable modes in a two-stream plasma is clearly revealed. This contribution to the energy change due to presence of unstable modes in an expanding plasma should be added to the usual polarization loss. For numerical estimate we integrate Eq.(10) for light quark. Since we are interested in the relativistic streams, we use \( u = 1 \) for subsequent calculation. The anisotropy is determined by a single vector, here the flow velocity \( u \) which is assumed to be in the \( z \)-direction. Two cases have been considered : (i) \( v_0 \parallel u \) and (ii) \( v_0 \perp u \).

The results are shown in Fig.(1) for a typical plasma temperature, \( T \approx 400 \text{ MeV} \). Here, we find that the light parton loses energy in a two-stream plasma due to chromoelectric field fluctuations unlike what has been observed in case of heavy quark in isotropic plasma [6]. It is also observed that the energy loss grows with time. We also show that the energy loss strongly depends on the direction of propagation with respect to the anisotropy vector, \( u \) in this case as reflected in Fig.(1) and in Eq.(10). For a parton, the energy loss for a given time is more in the anisotropy direction than in the perpendicular direction. Similar behaviour is observed in Refs. [7, 8] while calculating the polarization loss in anisotropic plasma.

In the present work we estimate the parton energy loss in two-stream plasma due to chromoelectric field fluctuations. In particular, we expose the role of the unstable modes those are present in such a scenario which changes the results both qualitatively and quantitatively. In fact, we show that these modes lead to an exponential growth of the energy change in presence of the fluctuating fields. For simplicity, only the longitudinal chromoelectric fields have been considered here. It is found that light partons lose energy in a two-stream plasma, which further, depends on the
direction of propagation with respect to the anisotropy vector (direction of the stream velocity in the present case). Note that the fields corresponding to the QGP produced in heavy ion collisions are difficult to obtain with a realistic anisotropic momentum distribution. However, it will be interesting to extend the present calculation in the context of relativistic heavy ion collisions with realistic anisotropic momentum distributions of the bath particles. Inclusion of such corrections, to the usual polarization energy loss, might prove to be an important mechanism for the parton to lose energy into the plasma, at least initially, when large anisotropy might exists.

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