Suspension system vibration analysis with regard to variable type ability to smooth road irregularities

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Abstract. The paper aims to analyze vibrations of the dynamic system equivalent of the suspension system with regard to tyre ability to smooth road irregularities. The research is based on static dynamics for linear systems of automated control, methods of correlation, spectral and numerical analysis. Input of new data on the smoothing effect of the pneumatic tyre reflecting changes of a contact area between the wheel and road under vibrations of the suspension makes the system non-linear which requires using numerical analysis methods. Taking into account the variable smoothing ability of the tyre when calculating suspension vibrations, one can approximate calculation and experimental results and improve the constant smoothing ability of the tyre.

1. Introduction
Elastic-tyred wheel is one of the most significant components of the dynamic system (a car, an aircraft, a tractor) determining its quality. The tyre is a transfer member which links a carrying frame of the car with the road and transfers the kinematic impact of road irregularities. Tyre characteristics and transfer properties will determine smooth ride and vibration protection, the vibration level and dynamic loading of minor components. In addition, converting properties of the tyre determine extreme traffic conditions which cause significant degradation of pulling and dynamic properties, road holding abilities and even loss of control.

Differences in calculation and experimental results might be due to absorption and smoothing abilities. In studies on smooth ride and vibration loading of the car and tractor which take into account smoothing abilities of the tyre, the constant smoothing model is applied. The tyre is presented as an ideal rectangular filter with the length equal of the length of the tyre print formed on the road surface under nominal loading [6].

When the tyre is really interacting with road irregularities, vibration system parameters, including the length of the contact area, vary. If one ignores that fact, suspension response will become zero under two or three frequencies of impact in a spectral line formed by the microprofile of different roads. It is certainly at odds with experimental results.
2. **Suspension vibration analysis with regard to variable tyre smoothing of road irregularities.**

Let us assume that [1, 2, 4]:

1) pneumatic tyre is a thin elastic cover which tightly covers all road irregularities in a contact area of a variable length (Fig.1);

2) pneumatic tyre has properties of a lumped element with elastic and damping characteristics (Fig.2);

3) length of the contact area between the tyre/road surface varies when travelling across road regularities and is dependent on the parameters of a vibration system which the tyre is a part of.

The first assumption means that in that position of the wheel, determined by its travelling along the road axis kinematic effect of road irregularities along the length of the contact area, can be expressed as a variable average value of the function, which smooths changes of microprofile ordinates.

![Figure 1. Interaction of the elastic-tyred wheel and road irregularities: 1 is wheel position at the contact are between the tyre/road; 2 is wheel position at the maximum tyre deformation; \( \xi(l) \) is vertical movement of the wheel axis; \( q(l) \) are ordinates of the profile of irregularities.](image)

The averaging can be expressed by a moving average operator with an interval which can vary from peak to peak under the load of length of the contact area:

\[
q_{av}(l) = \frac{1}{a(l)} \int_{l-a(l)/2}^{l+a(l)/2} q(l') \, dl',
\]

where the length of the contact \( a(l) \) is the function of the wheel travelling along the road axis \( l \) (Fig.1).

Figure 2 shows the scheme of the vibration system equivalent of the suspension system with an added element for variable smoothing. Thus, the traditional vibration system equivalent of the suspension system is supplied with a new element – smoothing element \( J \) which is located between the road and the tyre. It averages an input impact of the road at a variable interval. In addition, in the traditional system, there is a feedback from all elements of the vibration system to a smoothing element. That relation reflects the functional dependence of the averaging parameter (length of the contact area) on an output parameter of the system (normal tyre deflection).

Mathematical description of the dynamic system, except for two linear second-order differential equations reflecting the dynamics of the two-mass vibration system, involves integral equation with variable integration limits reflecting the dynamics of a smoothing element, and non-linear algebraic feedback equation for a parameter of the smoothing element which reflects relation between tyre deflection and length of the tyre print.
Figure 2. A functional diagram of the vibration system equivalent of the suspension system with regard to absorption and smoothing properties of the tyre: $M$ - sprung mass of the suspension; $2C_p$ - total coefficient of normal stiffness of elastic suspension elements; $\eta_n$ - total suspension resistance coefficient; $m$ - unsprung mass; $2C_z$ - total coefficient of the normal tyre stiffness; $\eta_{she}$ - equivalent coefficient of viscous tyre resistance; $H_{sh}, n$ - parameters of an elliptic power model; $z, \dot{z}$ - vertical movements of sprung and unsprung masses; $q$ - current value of ordinates of the road surface microprofile under the wheel axis; $J$ - smoothing element.

An equation system describing movements of vibration system masses and operation of a smoothing element is as follows:

\[
\begin{cases}
M\ddot{z} + \eta_n(z - \dot{z}) + 2C_p(z - \dot{z}) = 0, \\
m\ddot{\xi} + \eta_n(\dot{\xi} - q_{ot}) + 2C_z(\xi - q_{ot}) - \eta_n(z - \dot{z}) - 2C_p(z - \dot{z}) = 0, \\
q_{ot}(l) = \frac{1}{a(l)} \int_{-\delta(l)/2}^{\delta(l)/2} q(l) \, dl,
\end{cases}
\]

\[
a(l) = F(h_z)
\]

In terms of automatic control, the vibration system is a closed loop automated control system consisting of four elements: an element of variable smoothing, an element of argument variation, a second-order vibration element, and a feedback element. Feedback is from the system output to an averaging parameter of smoothing. A random stationary signal with normal distribution proportional to the shift of ordinates of a road microprofile is sent to the system input.

A structural scheme of the automated control system is shown in Fig. 3.
Figure 3. A structural scheme of the automated control system equivalent of the vibration system of the front (rear) suspension system based on the model of variable smoothing: $J$ – element of variable smoothing; $l/t$ – element of argument variation; $H_k(p)$ – second-order vibration element; $F(h_z)$ – feedback element; $q(l)$, $q_c(l)$, $q_c(t)$ – functions of primary and smoothed microprofiles; $h_z(t)$ – tyre deformation function; $a(h_z)$ – smoothing parameter function.

For linear stationary systems which are acted upon by a random stationary function with known static characteristics (correlation function or spectral density), one can obtain analytical expressions of transfer functions and frequency characteristics of the system, including the smoothing element, and statistical characteristics of output signals [5].

Those methods are not suitable for non-linear automated control systems. Non-linear dynamics methods have not been fully developed for engineering applications. For this reason, to analyze the model of variable smoothing and its impact on output characteristics of the vibration system equivalent of the suspension system, numerical analysis was applied. The studies involved numerical random test road microprofile simulation, numerical solution of the integral equation (1) and system of differential equations and algebraic equation (2), calculation of statistical characteristics of input and output signals, transfer functions and frequency characteristics of elements and entire system using standard and original software.

Figure 4 shows the scheme of software which implements the target tasks.
Figure 4. A diagram of software for numerical simulation of vibration systems equivalent of the suspension system based on a new variable smoothing model.

The first unit «ROAD» simulates a random road microprofile based on the targeted correlation function [3]:

\[
\rho(\tau) = A_1 e^{-a_1 \tau} + A_2 e^{-a_2 \tau} \cos \beta |\tau|,
\]

where \(\rho(\tau)\) is the normalized correlation function; \(A_1, A_2\) are the impact coefficients for components for which \(A_1 + A_2 = 1\) should be observed; \(a_1, a_2\) are the coefficients of the random process irregularity degree; \(\beta\) is the frequency of a periodical component with random amplitude and phase; \(\tau\) is the time correlation interval.

The issue of simulating a random microprofile of a real road surface is to determine an impulse transfer function of the generating dynamic system which would transform the signal sort of 'white noise' sent to the input of the system into a random process with a correlation function (3). Thus, there is a dynamic system with an unknown transfer function and an impulse transition characteristics, a random process with known spectral characteristics at its input, and a process with known spectral characteristics at its output (Fig. 5).

To determine the values of the impulse transition characteristics, one can apply a factorization method.

\[
S_x(\lambda) = 1
\]

Figure 5. Scheme of the generating filter: \(H_f(\lambda)\) – transfer function of the generating filter; \(h_f(l)\) – impulse transfer characteristics of the filter; \(x(l)\) – white noise; \(q(l)\) – target random process (function of a random road microprofile); \(\Delta l\) – length correlation interval; \(\lambda\) - wavelength frequency.
In "TYRE", vibration movement of suspension elements was simulated based on the variable ability of the tyre to smooth irregularities of a simulated road. In "TEST", the authors calculated and constructed statistical characteristics of input and output processes, frequency characteristics of elements and entire system.

Simulation results are shown in Fig. 6 and 7.

**Figure 6.** Amplitude and frequency characteristics of the smoothing element when simulating the system equivalent of the front suspension of the car travelling along an asphalt road (a) and a cobbled road (b): 1 – at a constant length of the contact area \( a = \text{const} \); 2, 3, 4 – at a variable length \( a = \text{var} \); \( a = V_a = 10 \text{ km/h} \); 3 - \( V_a = 80 \text{ km/h} \); 4 - \( V_a = 120 \text{ km/h} \); 6 - 2 - \( V_a = 5 \text{ km/h} \); 3 - \( V_a = 10 \text{ km/h} \); 4 - \( V_a = 30 \text{ km/h} \).

The purpose for the first research stage was to assess the element of variable smoothing based on the graphical analysis of amplitude and frequency characteristics (AFC). It identified that so called “zeros” of characteristics disappeared.

At the second stage, the authors calculated spectral densities of vertical vibration accelerations of the executive-class car travelling along a cobbled road and compared them with field measurements (Fig. 8).

**Figure 7.** Amplitude and frequency characteristics of the vibration system equivalent of the front suspension of the car travelling along a good cobbled road at a rate of 30 km/h: a – by normal tyre deflection \( h \); b – by sprung mass acceleration \( z \); 1, 3 – by variable smoothing model; 2, 4 – by constant smoothing model.
Figure 8. Spectral densities of vertical accelerations of ZIL-4102 travelling along a paved road at a rate of 100 km/h: а – for the rear left seat; b – under the seat; ——— calculation; - - - - experiment.

3. Conclusion
Implementation of a new model enables us to change methods for forming disturbance effect of the road which now takes place when solving differential equations of vibrations of a simulated car based on the dynamics of variation of the length of the contact area. Firstly, it enabled us to reduce RAM capacity for no additional running time. Secondly, it enables improving convergence of calculation and experiment results for evaluative springing parameters. Thirdly, it enables assessing impact of variable smoothing on ride comfort parameters and compare them with experimental ones.

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