Stabilization for Networked Control Systems with Limited Communication

Liu Yingying¹,a and Chu Yunkai²,b

¹School of Information Engineering, Shenyang University, Shenyang, China
²Shenyang Institute of Automation Chinese Academy of Sciences, Shenyang, China

a e-mail: lyy3636@126.com
b e-mail: chuyunkai@sia.cn

Keywords: Networked control systems, Delay, Quantization

Abstract. This paper studies the stabilization problems of networked control systems (NCSs) with dynamical quantizers. A new model is proposed that takes into consideration the effect of the network induced delay, the quantization levels, and based on this model, dynamical quantization scheme is introduced. The relationship between the delay bound, the quantization range and stability is given by using Lyapunov stability theory and linear matrix inequalities (LMIs) approach, and convex condition of the stabilization controller is presented. A simulation example shows the effectiveness of the proposed method.

Introduction

Networked Control Systems (NCSs) have many advantages over conventional control systems because of low cost, power requirements, simple installation and maintenance, so it has received increasing research interests. When measurements to be used for feedback are transmitted by a digital communication channel, data are quantized before transmission. Therefore, to achieve better performance of the considered systems, the effect of data quantization on the network should be taken into consideration. The quantization problems have been paid considerable attention in recent years. [1] studied the stability and stabilization problems of discrete-time NCSs with state signal quantization. [2] studied the stability and $H_\infty$ control problems of discrete-time NCSs. [3] studied guaranteed cost control design for NCSs with state and control input quantization. In these results, the adopted quantizers were all static scheme. [4] studied the dynamical quantization control of discrete-time NCSs. [5] considered continuous-time NCSs with state quantization $H_\infty$ control problem. This paper studies the problems of dynamical control for NCSs. Taking consideration into delay, signal quantization, the stability condition in LMIs by Lyapunov functional approach and introducing relax variables technique is given, furthermore, the relationship of the delay bound, quantization level and the stability is presented. The controller design which enables the system stability is proposed.

Problem formulation:

Consider the following system:

\begin{align}
\dot{x}(t) &= Ax(t) + Bu(t) \\
u(t) &= Kx(t)
\end{align}

where $x(t) \in \mathbb{R}^n$ is the system state, $x(0) = x_0$, $u(t) \in \mathbb{R}^m$ is the control input, $A$, $B$ are some constant matrices of appropriate dimensions.

In this section, the definition of a quantizer is given with general form as in [6]. We assume that there exist positive real numbers $M$ and $\Delta$, such that the following condition hold:

\begin{equation}
\text{If } |z| \leq M \text{, then } |q(z) - z| \leq \Delta.
\end{equation}
(3) gives a bound on the quantization error when the quantizer does not saturate. Assume that \( q(z) = 0 \) for \( z \) in some neighborhood of the origin, i.e., the origin lies in the interior of the set \( \{ z : q(z) = 0 \} \). In the control strategy to be proposed below, the quantized measurements is presented with the following form:

\[
q_{\mu}(z) = \mu q\left(\frac{z}{\mu}\right),
\]

(4)

For the system (1)–(2), quantized controller with state feedback is as following:

\[
u(t) = q_{\mu_2}(Kq_{\mu_1}(x)) = \mu_{2k} q_2 \left( \frac{K \mu_{1k} q_1(x(i_k))}{\mu_{2k}} \right) \quad t \in [i_k h + \tau_k, i_k h + \tau_{k+1})
\]

(5)

The quantizers \( q_{\mu_1}(\bullet) \) and \( q_{\mu_2}(\bullet) \) are all dynamical quantizers, in (4). The quantizers \( q_{1}(\bullet) \) and \( q_{2}(\bullet) \) are all static quantizers, in (3). Denote the ranges of the quantizers \( q_{1}(\bullet) \) and \( q_{2}(\bullet) \) as \( M_1 \) and \( M_2 \). For this quantized controller, we consider a strategy always satisfying

\[
\mu_{2k} = \theta \mu_{ik},
\]

(6)

where \( \theta > 0 \) is a positive constant and should be adjusted in real applications. Applying (5) to (1), the following quantized closed-loop system is obtained:

\[
\dot{x}(t) = Ax(t) + B \mu_{2k} q_2 \left( \mu_{2k}^{-1} K \mu_{1k} q_1(x(t-\eta(t))) \right) t \in [i_k h + \tau_k, i_k h + \tau_{k+1})
\]

(7)

where \( \eta(t) = t - i_k h \), and \( \eta(t) \leq \eta \). Let \( t_k = i_k h + \tau_k \), then, we can obtain:

\[
\dot{x}(t) = Ax(t) + BKx(t-\eta(t)) + \eta \quad t \in [t_k, t_{k+1}),
\]

(8)

where \( F = B \mu_{2k} q_2(K \mu_{1k} q_1(x(t-\eta(t)))) - \mu_{2k}^{-1} K \mu_{1k} q_1(x(t-\eta(t))) \)

Main results:

Theorem 1: For given scalars \( \eta > 0, \theta > 0, 0 < \alpha < 1 \), matrices \( Q \) and \( K \), when delay and quantization parameter satisfy that:

\[
(1) \quad (i_{k+1} - i_k) h + \tau_{k+1} \leq \eta; \quad (2) \quad M_i \geq 2 \theta \| P B \| \| Q^T \|, \quad M_i \geq \frac{1}{\theta} \| K \| (\Delta + M_i),
\]

\[
\Delta = \Delta_2 + \| K \| \Delta_1, \quad \text{if there exist matrices} \quad P = P^T \geq 0, \quad T > 0, \quad S_i \quad (i = 1, 2, 3), \quad \text{such that}
\]

\[
\Phi + Q \quad S
\]

\[
* \quad -\frac{1}{h} M
\]

\[
< 0,
\]

(9)

\[
\Phi = \begin{bmatrix}
T + N_i A + A^T N_i^T + S_i + S_i^T & -S_i^T + A^T N_i^T + N_i B K & S_i^T + P + A^T N_i^T - N_i \\
* & -1(1-\mu) T - S_i^T + N_i B K + K^T B^T N_i^T - S_i^T & -S_i + K^T B^T N_i^T - N_2 \\
* & * & h M - N_i - N_i^T
\end{bmatrix}
\]

(10)

then the closed-loop system (1) and (2) is asymptotically stable.

Proof: \( \frac{x(i_k h)}{\mu_{ik}} \) is quantized before feedback, and when \( \frac{x(i_k h)}{\mu_{ik}} < \mu \), then

\[
\left| q_1 \left( \frac{x(i_k h)}{\mu_{ik}} \right) - \frac{x(i_k h)}{\mu_{ik}} \right| \leq \Delta_i \Rightarrow \left| q_1 \left( \frac{x(i_k h)}{\mu_{ik}} \right) \right| \leq M_i + \Delta_i,
\]

(11)

\[
\left| K \mu_{ik} q_1 \left( \frac{x(i_k h)}{\mu_{ik}} \right) \right| \leq \left\| K \| \mu_{ik} \right\| q_1 \left( \frac{x(i_k h)}{\mu_{ik}} \right) \leq \mu_{ik} \left\| K \| (\Delta_i + M_i) = \frac{1}{\theta} \left\| K \| (\Delta_i + M_i) = M_2,
\]

(12)
where $\theta \mu_k = \mu_{2k}$, and for $q_2(\bullet)$, there exists

$$
\left| \begin{array}{c}
\left( K\mu_k q_i(x_ih) \right)_{\mu_k} - K\mu_k q_i(x_ih) \\
\left( K\mu_k q_i(x_ih) \right)_{\mu_k} + K\mu_k q_i(x_ih) - (x_ih)
\end{array} \right| \leq q_2 \left( \begin{array}{c}
\left( K\mu_k q_i(x_ih) \right)_{\mu_k} - K\mu_k q_i(x_ih)
\left( K\mu_k q_i(x_ih) \right)_{\mu_k} + K\mu_k q_i(x_ih) - (x_ih)
\end{array} \right)
$$

(13)

\[ \leq \Delta_k + \|K\| \Delta = \Delta \]

using (8), then $\|F\| \leq B\mu_{2k}\Delta$. Consider the following Lyapunov-Krasovskii functional:

$$
V(t) = x^T(t)Px(t) + \int_{t-\eta}^{t} x(s)^T Tx(s) ds + \int_{t-\eta}^{t} \int_{t-\beta}^{t} \bar{x}^T(s) M \bar{x}(s) ds d\beta.
$$

(14)

For $t_k < t < t_{k+1}$, calculating the derivative of $V(t)$ with respect to $t$ along the solutions of the system

\[ \dot{V}(t) \leq 2x(t)Px(t) + x^T(t)Tx(t) - (1-\alpha)x^T(t-\eta(t))Tx(t-\eta(t)) + \eta \bar{x}^T(t)M \bar{x}(t) - \int_{t-\eta}^{t} \bar{x}^T(s) M \bar{x}(s) ds, \]

(15)

Define $S = \left[ S_1^T \ S_2^T \ S_3^T \right]^T$, $N = \left[ N_1^T \ N_2^T \ N_3^T \right]^T$, $\xi(t) = \left[ x^T(t) \ x^T(t-\eta(t)) \ \bar{x}^T(t) \right]^T$, then,

\[ \dot{V}(t) \leq 2x(t)Px(t) + x^T(t)Tx(t) - (1-\alpha)x^T(t-\eta(t))Tx(t-\eta(t)) + \eta \bar{x}^T(t)M \bar{x}(t) + 2\xi^T(t)N (Ax(t) + BKx(t-\eta(t)) + F - \bar{x}(t)) - \int_{t-\eta}^{t} \bar{x}^T(s) M \bar{x}(s) ds + 2\xi^T(t)S(x(t) - x(t-\eta(t))) \] $$\]

\[ + 2\xi^T(t)N (Ax(t) + BKx(t-\eta(t))) + 2\xi^T(t)NF - 2\xi^T(t)N \bar{x}(t) + 2\xi^T(t)SM^{-1} S \bar{x}(t) \]

combine (9), then

\[ \dot{V}(t) \leq - \frac{1}{\|Q^{-1}\|} \| \xi(t) \|^2 + 2\mu_{2k} \| N \| \| B \| \| \Delta \| \| Q^{-1} \| \leq - \frac{1}{\|Q^{-1}\|} \| \xi(t) \| (\| \xi(t) \| - 2\mu_{2k} \| N \| \| B \| \| \Delta \| \| Q^{-1} \|) \]

It can be seen that if $\| \xi(t) \| \geq 2\mu_{2k} \| N \| \| B \| \| \Delta \| \| Q^{-1} \|$, then $\dot{V}(t) \leq 0$, for $t_k < t < t_{k+1}$. So the system is asymptotically stable, when $M_1 \geq 2\theta \| N \| \| B \| \| \Delta \| \| Q^{-1} \|$. End.

Based on Theorem 1, stabilization controller design scheme for NCSs (1)-(2) will be given.

Theorem 2: For given scalars $\theta > 0$, $\eta > 0$, $\alpha_i (j = 1,2,3)$, and matrix $Q$, when delay and quantization parameter satisfy that:

(1) \[ (i_{k+1} - i_k)h + \tau_{k+1} \leq \eta \], and $\bar{\eta}(t) \leq \mu$;

(2) $M_1 \geq 2\theta \| N \| \| B \| \| \Delta \| \| Q^{-1} \|$, $M_1 \geq \frac{1}{\theta} \| K \| (\Delta_i + M_i)$, $\Delta = \Delta_2 + \| K \| \| \Delta \|$, if there exists matrix $Y$, $X = X^T$, $\bar{T} > 0$ and $\bar{S}_i (i = 1,2,3)$, satisfy:

\[ \begin{bmatrix}
\Phi & \bar{R} \\
\bar{R}^T & 0 \\
0 & -\frac{1}{\eta} \bar{M}
\end{bmatrix} < 0
\]

(16)

\[ \Phi = \begin{bmatrix}
\bar{T} + AX + XA^T + \bar{S}_3^T - \bar{S}_2^T + \beta_1 \bar{X}_2 - BY \\
* & -\frac{1}{\eta} \bar{M} \\
0 & * & -\frac{1}{\eta} \bar{M}
\end{bmatrix}
\]

\[ \bar{R} = \text{diag} \{X, X, X\}, \]

\[ \bar{S} = \begin{bmatrix}
\bar{S}_1^T \\
\bar{S}_2^T \\
\bar{S}_3^T
\end{bmatrix}
\]

then the closed-loop system (1) and (2) is asymptotically stable., and the state feedback controller $K = YX^{-1}$.

Proof: The proof is omitted.
Numerical Examples

Consider the following system [7]:
\[
\dot{x}(t) = \begin{bmatrix}
-0.8 & -0.01 \\
1 & 0.1
\end{bmatrix} x(t) + \begin{bmatrix}
0.4 \\
0.1
\end{bmatrix} u(t)
\]

Let \(\theta = 1.1\), \(Q = I\), quantization error \(\Delta_1 = \Delta_2 = 0.1\), using Theorem 2, we can obtain stabilization controller \(K = \begin{bmatrix}
-0.0918 \\
0.4449
\end{bmatrix}\), furthermore, \(M_1 = 2.272 \times 10^5\), \(M_2 = 0.4130\). It can be seen from the simulation results, we obtain the desired quantization range, on the other hand the maximum allowable delay of the network is 1.499, while [8], the maximum delay is 1.000, indicating that when the network introduces a dynamical quantizer, the system performance has been improved, and also shows the effectiveness of the controller design method presented in this paper. Fig 1 shows the state of the system response curve.

Conclusions

This paper considers the dynamical quantization control problems of the NCSs with state and control input signal quantization. By using Lyapunov stability theory and LMIs approach, the stability of the NCSs under dynamical quantization scheme is given, and convex condition of the stabilization controller is presented. Simulation examples show the effectiveness of the proposed method.

References

[1] X L Zhu, G H Yang. Quantized H∞ controller design for networked control systems [A], Proc. of 2008 Chinese Conference on Decision and Control[C], 2008: 293-298.
[2] H J Gao, T W Chen. A new approach to quantized feedback control systems[J]. Automatica, 2008, 44: 534-542.
[3] D Yue, C Peng, G Y Tang. Guaranteed cost control of linear systems over networks with state and input quantisations[J]. Control Theory and Applications, 2006, 153(6):658 - 664.
[4] E Tian, D Yue, X Zhao. Quantized control design for networked control systems[J]. IET Control Theory and Applications, 2007, 1(6): 1693-1699.
[5] C Peng, Y C Tian. Networked H∞ control of linear systems with state quantization[J]. Information Sciences, 2007, 177: 5763-5774.
[6] D Liberzon. On stabilization of linear systems with limited information[J], IEEE Transactions on Automatic Control, 2003, 48(2): 304-307.
[7] L S Hu, J Lam, Y Y Cao, H H Shao. LMI approach to robust H2 sampled-data control for linear uncertain systems[J]. IEEE Transactions of System, Man and Cybernet, 2003, Part B, 33(1): 149-155.
[8] L S Hu, T Bai, P Shi, Z M Wu. Sampled-data control of networked linear control systems [J], Automatica, 2007, 43: 903-911.