THE ORIGIN OF SOLAR ACTIVITY IN THE TACHOCLINE

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ABSTRACT

Solar active regions, produced by the emergence of tubes of strong magnetic field in the photosphere, are restricted to within 35° of the solar equator. The nature of the dynamo processes that create and renew these fields, and are therefore responsible for solar magnetic phenomena, are not well understood. We analyze the magnetorotational stability of the solar tachocline for general field geometry. This thin region of strong radial and latitudinal differential rotation, between the radiative and convective zones, is unstable at latitudes above 37°, yet is stable closer to the equator. We propose that small-scale magnetorotational turbulence prevents coherent magnetic dynamo action in the tachocline except in the vicinity of the equator, thus explaining the latitudinal restriction of active regions. Tying the magnetic dynamo to the tachocline elucidates the physical conditions and processes relevant to solar magnetism.

Subject headings: hydrodynamics — instabilities — MHD — Sun: activity — Sun: magnetic fields — Sun: rotation

Online material: color figure

1. INTRODUCTION

Magnetic fields are a leading actor throughout a star’s life—from accreting protostar to degenerate dwarf, main-sequence star to magnetar (Mestel 1999). They shape jets in some supernovae and drive a pulsar’s radiative engine. Yet we still lack a coherent theory of how even our own star generates and sustains its field, and so are ignorant of the wellspring of much of this evolutionary richness (Parker 1979). Since the 1950s, much effort has been expended on models which locate the dynamo in the convective zone. There are several known difficulties with this scenario (e.g., Parker 1975; Petrovay 2000). Recently, attention has turned to the lower convective zone and the tachocline, its boundary with the stably stratified radiative zone, in part because stratification can anchor the strong emerging fields (Parker 1975; Petrovay 2000; Dikpati & Gilman 2005; Browning et al. 2006; Hughes et al. 2007). Dynamical considerations suggest the presence of a large-scale magnetic field in the upper radiative zone (Gough & McIntyre 1998), of plausible strength ~1 G.

Here we propose that the solar magnetic dynamo is intimately tied to the tachocline, where latitudinal differential rotation amplifies the toroidal field and, together with radial differential rotation, controls the limiting role of the magnetorotational instability (MRI). This instability creates small-scale turbulence that tends to prevent large-scale, ordered field growth, which leads to surface emergence and sunspots.

The hydrodynamic (e.g., Charbonneau et al. 1999; Dikpati & Gilman 2001; Arlt et al. 2005) and magnetohydrodynamic (e.g., Gilman & Fox 1997; Miesch et al. 2007; Miesch 2007) stability of the tachocline have been the subject of extensive study. Here we consider stability with respect to the diffusive MRI, which has not been previously examined. Naively, one would expect that since \( \partial \Omega / \partial \theta > 0 \) the tachocline would be stable to the MRI, in the presence of the strong radial entropy stratification that suppresses fluid motions in the radial direction. However, we show that the inclusion of thermal diffusion enables the radial movement of fluid on small scales, allowing the negative radial gradient of angular velocity to be tapped, which results in instability at large heliolatitudes.

2. STABILITY ANALYSIS

The tachocline is a boundary layer imposed on the solid-body-rotating radiative zone by the convective zone, whose global angular momentum is extracted by the magnetic solar wind (e.g., Spiegel & Zahn 1992; Hughes et al. 2007). Differential rotation in the tachocline, created by external convective torques, is not necessarily reduced by instabilities operating within it. The rotational gradients result from the different means of energy and momentum transport in the radiative and convective zones, so that a tachoclinic dynamo could in principle feed on the central nuclear energy source. Using helioseismic constraints (Thompson et al. 2003), we consider a thin shell confined to the tachocline with differential rotation given by an angular velocity profile

\[
\Omega(r, \theta) = \Omega_{\text{rad}} + \frac{r - r_t + w}{2w} (1 - \alpha_3 \cos^2 \theta - \alpha_4 \cos^4 \theta) \delta \Omega_{\text{eq}}
\]

where \( \Omega_{\text{rad}} = 2.69 \times 10^{-6} \text{ rad s}^{-1} \) is the angular velocity of the radiative zone, \( \delta \Omega_{\text{eq}} = 1.08 \times 10^{-7} \text{ rad s}^{-1} \) is the differential in angular velocity across the tachocline at the equator, \( r = 0.7 R_\odot \) is the radius of the tachocline’s midpoint, and \( w = 0.02 R_\odot \) is its width (Basu & Antia 2001). We use both spherical coordinates \((r, \phi, \theta)\), where \( \theta \) is the colatitude angle measured from the pole, and cylindrical coordinates \((R, \phi, Z)\) in the following analysis. This model implies \( q = \alpha \ln \Omega / \alpha \ln r = -10.2 \) near the pole, \( q = +1.4 \) at the equator, and \( q = 0 \) at \( \theta = 62^\circ \). Calculations are performed at \( r = r_t \). The constants \( \alpha_3 = 3.56 \) and \( \alpha_4 = 4.21 \) are derived from GONG data (Basu & Antia 2001).

We do not have direct knowledge of the radial structure of the entropy stratification in the tachocline, because it is not resolved by helioseismology. Using density and pressure profiles from a standard solar model (Bahcall et al. 2005), we calculate an approximate stratification profile for the tachocline, and vary the stratification strength in our stability analysis within the range of plausible values. We find that our results...
are not strongly dependent on the assumed level of stratification.

The MRI is a local, linear, weak-field instability present in systems with radially decreasing angular velocity (Balbus & Hawley 1991, 1998). The most general form (Acheson 1978; Menou et al. 2004) includes the effects of entropy stratification, viscosity $\nu$, and thermal and magnetic diffusivities ($\xi$ and $\eta$, respectively). Thermal diffusion allows a small plasma element to radiate photons and entropically equilibrate with its surroundings, which reduces the effective stratification. Stability is determined by the rotational gradients (including their signs) and not by the presence of (direction-independent) shear, making the MRI qualitatively different from shear instabilities.

Necessary and sufficient linear stability criteria for the adiabatic MRI ($\nu = \xi = \eta = 0$) can be derived via a generalized axisymmetric Solberg-Hoiland analysis (Balbus 1995):

$$N^2 + \left( r \sin^2 \theta \frac{\partial}{\partial r} + \sin \theta \cos \theta \frac{\partial}{\partial \theta} \right) \Omega^2 > 0,$$

$$N^2 \cos \theta \frac{\partial \Omega^2}{\partial \theta} > 0,$$

where $N_r$ is the radial component of the Brunt-Väisälä (buoynancy) frequency. In the limit of strong thermal diffusion the plasma is able to come to entropic equilibrium with its surroundings arbitrarily quickly, so that the stratification is effectively removed. Therefore by setting $N_r$ to a vanishingly small constant we can use the adiabatic criteria to determine magnetorotational stability in the idealized case of infinite thermal diffusivity. Calculated using our rotation profile and $N_r \to 0$, equations (2) and (3) and (3) indicate that the tachocline can be magnetorotationally unstable for all $\theta < 62^\circ$ (Fig. 1). We vary $r$ and $\omega$ independently and find that the extent of the unstable region is insensitive to details of the radial structure. The linear radial profile of angular velocity assumed in our tachocline model is that which is least magnetorotationally unstable, because any nonlinear profile with the same angular velocities at the boundaries must have a greater radial angular velocity gradient at some point. For example, we find the same region of instability for an exponential profile (Basu & Antia 2001).

These stability criteria (with $N^2 \to 0$) determine the maximum extent of the instability, because they are inclusive of every relevant local length scale, field strength, and geometry, subject to several assumptions (see Balbus 1995). In particular the magnetic field must be “weak,” i.e., having an Alfven velocity less than both the rotational velocity and the sound speed. To investigate whether instability exists for physically reasonable scales and fields, and to confirm that it is not stabilized by a realistic entropy stratification, we numerically solve the triple-diffusive dispersion relation (Menou et al. 2004) for axisymmetric modes of frequency $\omega$:

$$\omega^2 + \left( \frac{k^2}{k_Z^2} + \omega_0^2 \right) \omega - \frac{1}{\gamma \rho} \left( DP \right) D \ln \frac{P}{\rho}^{-1} = 0,$$

$$R^2 \left( D R^2 \Omega^2 \right) - 4 \Omega^2 \left( k \cdot v_\lambda \right)^2 \omega = 0,$$

where

$$v_\lambda = B/\sqrt{4\pi \rho}, \quad k^2 = k_r^2 + k_z^2,$$

$$\omega^2 = \omega_r^2 - \left( k \cdot v_\lambda \right)^2,$$

$$\omega_r = \omega + i \eta k^2, \quad \omega_r = \omega + i \eta k^2,$$

For each magnetic field configuration, we solve for 2 million wavevectors distributed in $k$ phase space (guaranteeing convergence with respect to phase-space resolution) and choose the fastest growing mode which, under exponential growth, maximally unstable region, as determined by the Balbus stability criteria. The arrows above the plot delimit the areas potentially subject to Tayler instability, for radial and latitudinal fields. Active regions are found to the right of the vertical long-dashed line. [See the electronic edition of the Journal for a color version of this figure.]

![Fig. 1.](image)

3. RESULTS

We find growth rates on the order of the rotation frequency for $|B| = 1$ G and $N^2/\Omega_{\text{rad}}^2 = 5 \times 10^{-4}$ (see Fig. 1), appropriate values for the strongly stratified lower tachocline (e.g., Gough & McIntyre 1998). The diffusive instability extends to $\theta \approx 53^\circ$. For this value of $N_r$, radial fields are relatively insensitive to field strength; at 870 G the growth rate peaks at a value $\sigma_{\text{max}}/\Omega_{\text{rad}} \approx 2 \times 10^{-3}$ while stabilization occurs at 880 G and above. With latitudinal fields, the growth rates decrease more rapidly as the field strength is increased: the peak value of $\sigma_{\text{max}}/\Omega_{\text{rad}} \approx 10^{-3}$ at 50 G and $\sim 10^{-6}$ at 200 G. Instability persists for stratifications up to and including that present at the top of the radiative zone, $N^2/\Omega_{\text{rad}}^2 \approx 1.6 \times 10^{-1}$. Radial fields give higher growth rates than latitudinal ones, with mixed geometries falling in between. As expected (Menou & Le Meur 2006),
at very small field values (i.e., \(\sim 10^{-2} \text{ G}\)) diffusion causes the MRI modes to have smaller growth rates than the hydrodynamic modes. Our survey indicates that a large region of the parameter space is magnetorotational unstable for \(\theta \leq 53^\circ\), in the manner shown in Figure 1, for field strengths from a small fraction of a gauss to several tens or hundreds of gauss and for a range of stratifications.

Diffusive instability does not extend as far in colatitude as in the maximally unstable case discussed above; the radial gradient of angular velocity becomes less negative as the point of inflection, \(\theta = 62^\circ\), is approached from the pole and its destabilizing influence is more easily counteracted by the positive latitudinal gradient \((\partial \Omega / \partial \theta > 0)\) and stratification \((N_c^2 > 0)\). For definiteness, we only show values of \(B_{\text{rev}}\) larger than \(10^{-3}\). As we now discuss, very small values may allow linear winding of the field before it is disrupted by significant MRI exponential growth, possibly introducing other instabilities.

Surveying relevant processes, it appears that two other weak-field instabilities could be active in the tachocline (e.g., Acheson 1978; Spruit 1999). Both should be secondary to the exponentially growing MRI when it is present with a sufficiently large growth rate, however, since they initially rely on linear amplification of a toroidal field by azimuthal stretching of the poloidal component. Tayler instability is of a pinch or interchange type, to which a toroidal field is most unstable by or perturbations (Tayler 1973). Stability is determined solely by the values of \(\partial B_x / \partial \theta\) and \(\theta\) (e.g., Spruit 1999). We test the Tayler stability of toroidal fields created by azimuthal winding of four distinct poloidal field configurations, having poloidal components \(B_p\) strictly along \(\hat{r}\), \(\hat{\theta}\), \(\hat{\phi}\), and \(\hat{Z}\), using (Spruit 1999)

\[
B_x = N \hat{\Omega} \cdot B_p, \tag{5}
\]

where \(N\) is the number of windings and \(\hat{\Omega} = \nabla \Omega / |\nabla \Omega|\) is calculated from equation (1). We find that the instability should be largely contained within \(62^\circ\) or less of the pole, although the \(62^\circ \leq \theta \leq 68^\circ\) region could be unstable for some field geometries (Fig. 1). This indicates a limited relevance of the Tayler process for the tachocline, consistent with the general expectation that this process should be active primarily in polar regions. For instance, if the tachocline’s rotational structure is related to magnetic stresses, the weakness of \(|\partial \Omega / \partial \theta|\) relative to \(|\partial \Omega / \partial r|\) suggests that \(B_r \gg B_x\), because the rates at which Maxwell stresses transport angular momentum radially and latitudinally are proportional to \(B_r B_x\) and \(B_r B_p\) respectively (Spruit 2002). If so, Tayler instability would only be possible for \(\theta < 62^\circ\), largely within the region where the exponentially growing MRI dominates. Unlike the MRI, we find that the Tayler instability depends strongly on the magnetic field geometry. In the absence of active constraints on \(B\) in the tachocline, we conclude that there may be a role for the Tayler instability in the transitional latitudes between MRI dominance and active region formation, \(50^\circ \leq \theta \leq 60^\circ\).

A second weak-field instability occurs if \(\phi = \partial \ln B / \partial \ln r < 0\), when gas is supported against gravity by magnetic pressure. Given a sufficiently strong radial gradient, free energy can be liberated by field-line buckling (e.g., Parker 1955; Acheson 1978). As with other weak-field instabilities, stratification (reduced by thermal diffusion) will be stabilizing.

In the tachocline, at the equator, the instability criterion is \(|p|B^2 \sim 10^{11} \text{ G}^2\) (Acheson 1978; Spruit 1999); assuming a nonsingular \(p \sim -1\) value, diffusive buoyancy instability would require \(|B| \geq 10^4 \text{ G}\), which is also the field strength at which large-scale adiabatic buoyant instability has been shown to cause flux emergence, resulting in active region formation (e.g., D’Silva & Choudhuri 1993; Schüssler et al. 1994). Therefore the small-scale diffusive buoyant instability appears to be of little relevance.

To summarize, we infer that the tachocline is magnetorotationally unstable for \(\theta \leq 53^\circ\) and stable closer to the equator. In the unstable region we expect the small-scale turbulence produced by the diffusive MRI, which is a narrow salt-finger-type instability (Korycansky 1991; Menou et al. 2004), to prevent the ordered growth of magnetic field on large scales. We surmise that in the magnetorotationally stable region, latitudinal differential rotation is free to wind the poloidal field in the azimuthal direction, creating a linearly amplified toroidal component as in the original Babcock-Leighton dynamo (Babcock 1961; Leighton 1964). When the toroidal field reaches a strength of order \(10^5 \text{ G}\) it is subject to buoyant instability and emerges. The Babcock-Leighton model is consistent with the dearth of active regions within \(8^\circ\) of the equator, since \(\partial \Omega / \partial \theta \to 0\) as the equator is approached (giving no field winding), and can account for the phase dependence of the flux-emergence latitude (Babcock 1961; Dikpati & Charbonneau 1999).

4. DISCUSSION AND CONCLUSION

Strong radial shear exists in both equatorial and polar regions of the tachocline, yet active regions are confined to \(\theta \geq 55^\circ\). At approximately this colatitude the negative radial differential rotation is just strong enough to overcome stratification and trigger the diffusive MRI, while closer to the equator the rotational gradient is of the wrong sign. The \(\theta = 55^\circ\) point does not appear to be a critical colatitude for any other weak-field instability, which is simple circumstantial evidence that the diffusive MRI is a discriminant for the observed latitudinal cutoff. We have confirmed this expectation with a rigorous stability theorem and a detailed numerical stability analysis.

Our study is chiefly limited by its linear character, whereby it is not possible fully to determine the effect of the saturated MRI on the tachocline. The instability will likely generate an effective anisotropic “turbulent viscosity,” which could be important in explaining the tachocline’s radial thinness. Interestingly, observational constraints on the tachocline’s width imply a latitudinal thickness variation consistent with two separate processes in action at high and low latitudes, as in our scenario (Basu & Antia 2001).

The identification of the tachocline as a critical region for solar dynamo action creates opportunities to further our understanding of solar magnetism. A particularly interesting question concerns the complex relationship that may exist between the tachocline’s generation of magnetic fields, which emerge to form active regions that impact the solar magnetic wind, and the effect that this same wind has in modifying the differential rotation in the tachocline by applying torque at the solar surface.

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REFERENCES

Acheson, D. J. 1978, Philos. Trans. R. Soc. London A, 289, 459
Arlt, R., Sule, A., & Rüdiger, G. 2005, A&A, 441, 1171
Babcock, H. W. 1961, ApJ, 133, 572
Bale, J. N., Serenelli, A. M., & Basu, S. 2005, ApJ, 621, L85
Balbus, S. A. 1995, ApJ, 453, 380
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
———. 1998, Rev. Mod. Phys., 70, 1
Basu, S., & Antia, H. M. 2001, MNRAS, 324, 498
Browning, M. K., Miesch, M. S., Brun, A. S., & Toomre, J. 2006, ApJ, 648, L157
Charbonneau, P., Dikpati, M., & Gilman, P. A. 1999, ApJ, 526, 523
Dikpati, M., & Charbonneau, P. 1999, ApJ, 518, 508
Dikpati, M., & Gilman, P. A. 2001, ApJ, 551, 536
———. 2005, ApJ, 635, L193
D’Silva, S., & Choudhuri, A. R. 1993, A&A, 272, 621
Gilman, P. A., & Fox, P. A. 1997, ApJ, 484, 439
Gough, O. D., & McIntyre, M. E. 1998, Nature, 394, 755
Hughes, D., Rosner, R., & Weiss, N. 2007, The Solar Tachocline (Cambridge: Cambridge Univ. Press)
Korycansky, D. G. 1991, ApJ, 381, 515
Leighton, R. B. 1964, ApJ, 140, 1547
Menou, K., Balbus, S. A., & Spruit, H. C. 2004, ApJ, 607, 564
Menou, K., & Le Meur, J. 2006, ApJ, 650, 1208
Mestel, L. 1999, Stellar Magnetism (Oxford: Clarendon Press)
Miesch, M. S. 2007, ApJ, 658, L131
Miesch, M. S., Gilman, P. A., & Dikpati, M. 2007, ApJS, 168, 337
Parker, E. N. 1955, ApJ, 121, 491
———. 1975, ApJ, 198, 205
———. 1979, Cosmical Magnetic Fields (New York: Oxford Univ. Press)
Petrovay, K. 2000, in The Solar Cycle and Terrestrial Climate, ed. A. Wilson (ESA SP-463; Noordwijk: ESA), 3
Schüssler, M., Caligari, P., Ferriz-Mas, A., & Moreno-Insertis, F. 1994, A&A, 281, L69
Spiegel, E. A., & Zahn, J.-P. 1992, A&A, 265, 106
Spruit, H. C. 1999, A&A, 349, 189
———. 2002, A&A, 381, 923
Tayler, R. J. 1973, MNRAS, 161, 365
Thompson, M. J., Christensen-Dalsgaard, J., Miesch, M. S., & Toomre, J. 2003, ARA&A, 41, 599