Research on Strapdown Inertial Navigation and Laser Doppler Combination Navigating Algorithm

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Abstract. In the current strapdown inertial navigation system, there are errors in the calculation model set by the airborne computer and the accuracy of related equipment, and the errors accumulate over time, which will lead to lower navigation accuracy and larger speed deviation. In order to solve the above problems, a new integrated navigation algorithm is proposed in this paper, which takes strapdown inertial navigation system as the main body and calculates the velocity and attitude of the aircraft. Laser Doppler assisted velocity measurement system is used as the auxiliary navigation system to correct and eliminate the velocity error of strapdown inertial navigation system. At the same time, Kalman filter technology is used to modify the velocity data of integrated navigation system. The simulation results show that the integrated navigation algorithm can reduce the speed error and improve the navigation accuracy, which verifies the effectiveness and feasibility of the integrated navigation algorithm.

1. Introduction
The strapdown inertial navigation system obtains the angular acceleration and acceleration data of the aircraft by using inertial sensitive elements such as accelerometers and rate gyroscopes, and uses the on-board computer to calculate the relevant solutions, so as to obtain navigation information such as speed and attitude, so that the strapdown inertial guidance system can navigate without the help of external equipment and even complete the navigation process independently. However, because of the independence of the strapdown inertial navigation system, it is difficult to correct the error without the relevant reference, and the error will accumulate and increase with time.

It is precisely because the advantages and disadvantages of strapdown inertial navigation system exist at the same time, so it is necessary to make use of the combination navigator to compensate for the shortcomings of the strapdown inertial guidance system, and the strapdown inertial navigation system becomes the main body of the combined navigation, using other navigation methods for auxiliary navigation. The laser Doppler velocity measurement system uses the Doppler frequency shift to calculate the speed of the vehicle motion, realizes the real-time velocity calculation, and the properties of the beam itself makes it difficult to be disturbed too much, which has good application value.

In this paper, it is based on the speed to establish the mathematical model of strapdown inertial navigation and laser Doppler speedometer, and the navigation algorithm suitable for this kind of combined navigation system is proposed. The measurement equation is established by using the velocity difference as the observation amount of Kalman filter. The state equations are established by using the state amount of strapdown inertial navigation and laser Doppler correlation, and the simulation model and filter are established by MATLAB. With the help of the specific force and angular velocity data obtained, Kalman filter is used to complete the data processing of combined
navigation, and obtain the relevant speed, attitude and position data to draw the corresponding images, so as to verify the feasibility of strapdown inertial navigation combined with laser Doppler guidance.

2. Design of integrated navigation system

2.1 Overview
With the navigation parameter error as the main state, combination navigation system makes use of speed deviation to build the measurement equation, which forms the basis of Kalman filter. In this paper, Kalman filter technology is used to establish the relevant state equation and measurement, so as to make the data processing and correction.

![Diagram of Combined Navigation Structure](image)

**Figure 1. Diagram of Combined Navigation Structure**

2.2 Integrated navigation system Model

2.2.1. Measurement equation
Measurement equation established by the combined navigation system:

\[ Z = HX + U \]  \hspace{1cm} (1)

\[ X = \begin{bmatrix} \delta l, \delta L, \delta h, \delta V_x, \delta V_n, \delta V_a, \phi_e, \phi_n, \phi_a, \delta V_{bx}, \delta V_{by}, \delta V_{bz}, \delta k_x, \delta k_y, \delta k_z \end{bmatrix}^T \]  \hspace{1cm} (2)

Among them, the latitude error and longitude error are expressed respectively by \( \delta l, \delta L \), \( \delta h \) represent the height error, \( \delta V_x, \delta V_n, \delta V_a \) respectively represent the velocity error of the eastward direction, the northward direction and the sky direction of the strapdown inertial navigation, and \( \phi_e, \phi_n, \phi_a \) respectively represent the error angle of the eastward direction, the northward direction and the sky direction (the error angle of the platform system and the navigation system caused by the calculation), \( \delta V_{bx}, \delta V_{by}, \delta V_{bz} \) respectively represent longitudinal, transverse and heavenly velocity of the laser Doppler velocity measurement system, and \( \delta k_x, \delta k_y, \delta k_z \) respectively represent the tick factor deviation of the laser Doppler velocity measurement on three axes.

Firstly, the measurement equation is established under the carrier coordinate system as follows:

\[ Z = \begin{bmatrix} \delta V_x \\ \delta V_y \\ \delta V_z \\ u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} \delta V_{bx} - \delta V_{ix} \\ \delta V_{by} - \delta V_{iy} \\ \delta V_{bz} - \delta V_{iz} \\ V_{bx} - V_{ix} \\ V_{by} - V_{iy} \\ V_{bz} - V_{iz} \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \]  \hspace{1cm} (3)

\( \delta V_{bx}, \delta V_{by}, \delta V_{bz} \) respectively represent the speed components of the strapdown inertial navigation system in the longitudinal, transverse and heavenly direction of the aircraft, and \( u_x, u_y, u_z \) represent the measured noise for the corresponding directions, \( \psi \) are related to the yaw angle of the...
carrier, and because $\phi_u$ is a very small angle, then $\sin(\phi_u) \approx \phi_u$, $\cos(\phi_u) \approx 1$, and because $\delta V_e$, $\delta V_n$ are also the small amounts, the two product of the small quantity is ignored.

The following is a decomposition calculation formula for the relevant calculation amount:

The calculation process for $\delta V_x$ is as follows

$$V_{bx} = (V_x + \delta V_x)\sin(\psi + \phi) + (V_n + \delta V_n)\cos(\psi + \phi)$$

(4)

$$V_x = V\sin(\psi) + V_n\cos(\psi)$$

(5)

Therefore, the formula can be reduced to:

$$V_x + \delta V_x\sin(\psi) + V_n\cos(\psi) = \delta V_x$$

(6)

$$\delta V_x = \delta V_x\sin(\psi) + V_n\cos(\psi) - V_n\sin(\psi)\phi - \delta V_n\cos(\psi) - \delta V_n\sin(\psi)\phi$$

(7)

The calculation process for $\delta V_y$ is as follows

$$V_{by} = (V_y + \delta V_y)\cos(\psi + \phi) - (V_n + \delta V_n)\sin(\psi + \phi)$$

(8)

$$V_y = V\cos(\psi) - V_n\sin(\psi)$$

(9)

Therefore, the formula can be reduced to:

$$V_y + \delta V_y\cos(\psi) = \delta V_y$$

(10)

$$\delta V_y = \delta V_y\cos(\psi) - V_n\sin(\psi)\phi - \delta V_n\cos(\psi) - \delta V_n\sin(\psi)\phi$$

(11)

The calculation process for $\delta V_z$ is as follows

$$V_{bc} = V_z + \delta V_u$$

(13)

$$V_z = (1 + \delta k_z)(V_z + \delta V_z)$$

(14)

$$\delta V_z = \delta V_z - V_z\delta k_z - \delta V_z$$

(15)

According to the above calculation formula can get:

$$Z = \begin{bmatrix} 0 & 0 & \sin(\psi) & \cos(\psi) & 0 & 0 & 0 & V_y & 1 & 0 & 0 & -V_x & 0 & 0 \\ 0 & 0 & \cos(\psi) & -\sin(\psi) & 0 & 0 & -V_y & 0 & -1 & 0 & 0 & -V_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & -V_y \end{bmatrix} + \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

(16)

2.2.2. State equation

In order to obtain more accurate navigation parameters and simplify the calculation equation, the selected state amount is as follows:

Variable expressions: $R_n = R(1-e^2)/(1-e^2\sin^2 L)^{3/2}$, $R_E = R/(1-e^2\sin^2 L)^{1/2}$, $R = 6378137m$, $f = 1/298.257$, $e = [(2-2f)^{1/2}$

$$\delta \hat{v} = \frac{V_\tan L\sec L}{R_E + h} \delta L - \frac{V_\sec L}{(R_E + h)^2} \delta h + \frac{\sec L}{R_E + h} \delta V_e$$

(17)
\[
\delta L = \frac{1}{R_N + h} \delta V_u - \frac{V_u}{(R_N + h)^2} \delta h
\]  
\[
\delta h = \delta V_u
\]  
\[
\begin{align*}
\delta V_e &= \left(2\Omega \cos LV_u + \frac{V_e}{R_e + h} \sec^2 L + 2\Omega \sin LV_u \right) \delta L + \left(\frac{V_u}{R_N + h} \tan L - \frac{V_u}{R_N + h} \right) \delta V_e + \\
&\quad \left(2\Omega \sin L + \frac{V_e}{R_e + h} \tan L \right) \delta V_u - \left(2\Omega \cos L + \frac{V_e}{R_e + h} \right) \delta V_u - f_e \phi_e + f_u \phi_u
\end{align*}
\]  
\[
\begin{align*}
\delta V_u &= -\left(2\Omega \cos LV_e + \frac{V^2}{R_e + h} \sec^2 L \right) \delta L - \left(2\Omega \sin L + \frac{2V_e}{R_N + h} \tan L \right) \delta V_e - \\
&\quad \frac{V_u}{R_N + h} \delta V_u - \frac{V_u}{R_N + h} \delta V_u + f_e \phi_e - f_u \phi_u
\end{align*}
\]  
\[
\begin{align*}
\dot{\phi}_e &= -\frac{1}{R_N + h} \delta V_u + \left(\Omega \sin L + \frac{V_e \tan L}{R_e + h} \right) \dot{\phi}_u - \left(\Omega \cos L + \frac{V_e}{R_e + h} \right) \phi_u
\end{align*}
\]  
\[
\begin{align*}
\dot{\phi}_u &= -\Omega \sin L \delta L + \frac{1}{R_e + h} \delta V_u - \left(\Omega \sin L + \frac{V_e \tan L}{R_e + h} \right) \dot{\phi}_u - \frac{V_u}{R_N + h} \phi_u
\end{align*}
\]  
\[
\dot{x} = \begin{bmatrix}
\Omega \cos L + \frac{V_e}{R_e + h} \sec^2 L \\
\end{bmatrix} \delta L + \frac{1}{R_e + h} \delta V_e + \left(\Omega \cos L + \frac{V_e}{R_e + h} \right) \dot{\phi}_u + \frac{V_u}{R_N + h} \phi_u
\]  
\[
\begin{align*}
\delta \dot{V}_x &= -\mu_x \delta V_{\dot{x}} + w_x \\
\delta \dot{V}_y &= -\mu_y \delta V_{\dot{y}} + w_y \\
\delta \dot{V}_z &= -\mu_z \delta V_{\dot{z}} + w_z \\
\delta \dot{k}_x &= \delta \dot{k}_y = \delta \dot{k}_z = 0
\end{align*}
\]  

The relevant equations established by using the state quantity selected above and \( W \) are expressed in accordance with the above analysis:

\[
W = \begin{bmatrix}
0 & 0 & 0 & a_e & a_n & a_u & \omega_e & \omega_n & \omega_u & \omega_x & \omega_y & \omega_z & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T
\]  

The interference in the upper formula, \( a_e, a_n, a_u \) express as the relevant inertial sensitive element under the carrier coordinate system, and \( \omega_e, \omega_n, \omega_u \) express the amount of interference caused by the external influence of the other related inertial sensitive element

2.2.3. Kalman Filter

In general, Kalman filter is mostly used in linear systems, in order to achieve optimal estimation, using the state amount of the previous moment to predict the relevant state amount of the next moment, and state equation established is shown in the following form:

\[
x_{k+1} = \phi_k x_k + \nu
\]  

Correlation measurement equations established:
\[ y_{k+1} = H_{k+1}x_{k+1} + n \]  

(32)

In the equations listed above: \( x_k \) is the relevant state amount of the previous moment \( (t_k) \), \( w \) is the related interference noise, \( \phi_k \) is the change matrix according to the State related relations, \( H_{k+1} \) is based on the relationship between measurement and demand to establish the relationship calculation in the time \( t_{k+1} \), \( n \) is the noise because the measurement is affected by the corresponding interference is the time \( t_{k+1} \). Because of the uncertainty of the added noise, the weighted average value is generally recorded as zero, and the covariance matrix is generally used and expressed by \( Q \) and \( R \).

If the optimal state quantity \( x_k \) of the time \( t_k \) is estimated to be substituted by \( x_{k/k} \), and because the weighted average value of the disturbed noise of the system is zero, the state variable at the next moment is estimated according to the state of the previous moment and the relevant optimal estimate is expressed as:

\[ x_{k+1/k} = \phi_k^T x_{k/k} \]  

(33)

For precise arithmetic values, the covariance of the previous moment can be used to estimate the covariance at the next moment as:

\[ P_{k+1/k} = \phi_k P_{k/k} \phi_k^T + \Delta_k Q \Delta_k^T \]  

(34)

According to the measurement equation established above and the measured value \( y_{k+1} \) of the next moment obtained by measuring, and comparing it with the predicted value predicted by using the calculated conversion equation based on the obtained amount, the exact proportion between the two is determined, and the data are updated in comparison to obtain more accurate estimates as follows:

\[ x_{k+1/k+1} = x_{k+1/k} - K_{k+1} [H_{k+1} x_{k+1/k} - y_{k+1}] \]  

(35)

The exact covariance of the updated moment is recorded as:

\[ P_{k+1/k+1} = P_{k+1/k} - K_{k+1} H_{k+1} P_{k+1/k} K_{k+1}^T \]  

(36)

And the gain is expressed as follows:

\[ K_{k+1} = P_{k+1/k} H_{k+1}^T [H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1}]^{-1} \]  

(37)

Through the above analysis, we can see that as long as the initial value is given, the corresponding data can be calculated by the state equation, and the optimal estimation can be formed by comparing with the measured value.

3. Simulation and analysis of combinatorial navigation algorithm

In order to verify the correctness of combinatorial navigation, the real route data is obtained by using the whole route simulation, and the acceleration and angular acceleration signals which can be measured by inertial elements are provided for the combined navigation system. In standard routes, the northward and sky initial speeds are approximately 0.0m/s, the initial longitude is 180 degrees, the initial latitude is 30 degrees, the gravitational acceleration ignored subtle changes is recorded 9.8 \( m/s^2 \), the sampling interval is 0.5s, and the initial flight height is 5000m.

At the initial moment, aircraft flies along the southeast direction, in the vertical channel. Firstly, the aircraft maintain a 5000m height of the flat fly, in the no.400 second or so climbs from 5000m height to 8000m height. During 1000s to 1100s, it drops from 8000m to 4000m, followed by a flat flight to no.1500 second. In the heading channel, the aircraft turns 30 degrees at around no.400 second and keeps the state flying about 200s, following by a cycle of 10 degrees or 5 degrees per 200s, and maintains a new heading to fly about 200s, with a final heading angle of about 75 degrees. In the rolling channel, the aircraft continuously maintains a 30-degree rolling corner flight at the 0~400s, reducing the rolling angle to 20 degrees at no.400 second and continuing to fly about 200s, reducing the rolling angle to 10 degrees at no.600 second and continuously flying about 200s, increasing the rolling angle to 20 degrees at no.800 second and holding the flat flying about 200s. At no.1000
second, the rolling angle of aircraft increases to 30 degrees and keeps 200 seconds. After no.1000 second, the rolling angle decreases from 30 degrees to 10 degrees in according to the regular between no.200 second and no.700 second, till no.1500 second.

The combined navigation simulation indicates the following:

![Figure 2. Northward Velocity Indication of Combined Navigation System](image)

In the figure above, the red curve (C curve) represents the change of integrated navigation speed, the blue curve (A curve) represents the real speed curve, and the green curve (B curve) represents the speed curve of strapdown inertial navigation system. From the image, it can be seen that after 1500 sampling points, the advantages of integrated navigation system are gradually manifested, which not only reduces the speed measurement error, but also restrains the accumulation of strapdown inertial navigation system error with time to a certain extent. The errors of the two navigation modes are shown in Table 1.

| Sampling point serial number | Strapdown inertial navigation speed error (m⋅s⁻¹) | Combination navigation speed error (m⋅s⁻¹) |
|-----------------------------|-----------------------------------------------|------------------------------------------|
| 2000                        | 0.198                                         | 0.0                                      |
| 2500                        | 0.286                                         | 0.01                                     |
| 3000                        | 0.302                                         | 0.04                                     |

### 4. Conclusion

In this paper, the state equation of SINS and the measurement equation based on the velocity deviation of SINS and LDV are established. The algorithm of SINS and LDV integrated navigation is constructed, and its feasibility is verified by simulation experiments. The simulation results show that the strapdown inertial navigation and laser Doppler integrated navigation not only reduce about 90% of the velocity measurement error, but also restrain the accumulation of errors over time to a certain extent, which has great effect and value for improving navigation accuracy, and has a good application prospect.

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