Nonclassical properties of Hermite polynomial excitation on squeezed vacuum and its decoherence in phase-sensitive reservoirs

Shi-You Liu¹, Ya-Zhou Li¹, Li-Yun Hu¹,², Jie-Hui Huang¹,², Xue-Xiang Xu¹ and Xiang-Yang Tao¹

¹ Center for Quantum Science and Technology, Jiangxi Normal University, Nanchang 330022, People’s Republic of China
² Beijing Computational Science Research Center, Beijing 100084, People’s Republic of China

E-mail: hlyun2008@126.com

Received 20 December 2014, revised 7 January 2015
Accepted for publication 6 February 2015
Published 24 February 2015

Abstract

We introduce Hermite polynomial excitation squeezed vacuum (SV) and then investigate analytically the nonclassical properties according to Mandel’s Q parameter, second correlation function, squeezing effect and the negativity of the Wigner function (WF). It is found that all these nonclassicalities can be enhanced by Hermite polynomial operation and adjustable parameters (μ and ν). In particular, the optimal negative volume (NV) of WF can be achieved by modulating μ and ν for higher excitation. The decoherence effect of phase-sensitive environment on this state is examined. It is shown that the NV with higher order diminishes more quickly than that with a lower one, which indicates that single-photon subtraction SV presents more robustness. The parameter of reservoirs can be effectively used to improve the nonclassicality.

Keywords: nonclassical property, Hermite polynomial excitation, decoherence, squeezed vacuum

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonclassical light fields play a critical role in quantum optics and quantum information processes [1]. The generation and manipulation of these states have attracted much attention to obtain a more effective quantum processing, such as teleportation, dense coding and quantum cloning. There are many schemes proposed to realize this purpose. Non-Gaussian operation, namely photon addition and photon subtraction, has been widely employed to enhance the nonclassical properties of the input states [2–6]. For instance, a quantum-to classical transition has been realized experimentally through single-photon-added coherent states of light [7]. For any photon-addition coherent state in the dissipative channel, its nonclassical properties are examined theoretically [8] by the analytical expression of the Wigner function (a Laguerre–Gaussian function). As another example, photon subtraction or addition has been used to improve entanglement between Gaussian states and the average fidelity of quantum teleportation [9, 10].

On the other hand, superposition of operators, such as \( a^\dagger a, \) \( t a^\dagger + ra, a^2 + b^2, a^{12} + b^{12}, \) are applied to generate nonclassical states [11–16]. For example, the \( t a^\dagger + ra \) operator is used to realize quantum state engineering and improve quantum entanglement or non-Gaussian entanglement distillation or the effect of quantum teleportation [11, 12]. In addition, this superposition operation is even employed to enhance the degree of entanglement of entangled coherent state [17]. Thus, it will be interesting to investigate the different combination of elementary non-Gaussian operations to manipulate...
nonclassical quantum states. As kinds of polynomial states, for instance, the squeezed Hermite states are found to be the minimum uncertain states for amplitude-squared squeezing [18]. In addition, the squeezed two-variable Hermite polynomial states are shown to be the minimum uncertain states for amplitude-squared squeezing, which is referred to as the sum-frequency squeezing states [19].

Recently, the Hermite polynomial coherent state $H_n(Q)\langle \alpha \rangle$ was introduced [20], where $Q = (a + a^\dagger)/\sqrt{2}$ is the coordinate operator and $|\alpha\rangle = \exp\{a a^\dagger - a^\dagger a\}|0\rangle$ is the Glauber coherent state. Then some nonclassical properties are discussed in detail. In this letter, we shall introduce another kind of non-Gaussian state, which can be generated by operating Hermite polynomials of superposition of coherent photon-subtraction and addition (HPS), i.e. $H_n(\mu a + \nu a^\dagger)$ on single-mode squeezed vacuum (SV) $S(r)|0\rangle$. Single photon subtraction/addition SV, Hermite polynomial subtraction $H_n(\mu a)$ and addition $H_n(\nu a^\dagger)$ SV can be considered as special cases of the HPS. It is interesting to note that the HPS-SV can be generated by superposing some photon-addition and photon-subtraction SVs. As far as we know, there have been no previous reports on this in the literature.

This letter is arranged as follows. In section 2, we shall derive the normalization factor $N_{\mu,\nu,}$ for the non-Gaussian states. It is shown that $N_{\mu,\nu}$ is just the Legendre polynomial, which is needed for clearly discussing the statistical properties of the HPS-SV. In section 3, we shall discuss nonclassical properties of the HPS-VS by analytically deriving Mandel’s $Q$ parameter, second correlation function, photon-number distribution, and squeezing effect. In section 4, the Wigner function (WF) of the HPS-SV is obtained by using the property of the Weyl-ordered operators’ invariance under similar transformations. In particular, the nonclassical property is presented according to the negativity of the WF. Section 5 is devoted to discussing the effect of phase-sensitive reservoirs on the nonclassical HPS-SV in terms of the negativity of WF. The last section is used to draw a conclusion.

2. The HPS-SV and its normalization

The HPS-SV can be generated by operating Hermite polynomial operator $H_n(\mu a + \nu a^\dagger)$ on single-mode squeezed vacuum $S(r)|0\rangle$, 

$$|\Psi\rangle_H = N_{\mu,\nu}H_n(\hat{\alpha})S(r)|0\rangle, \hat{\alpha} \equiv \mu a + \nu a^\dagger, \tag{1}$$

where $N_{\mu,\nu}$ is the normalization factor to be determined, and $S(r) = \exp\{r/2(a^2 - a^\dagger^2)\}$ is the squeezing operator with $r$ being the squeezing parameter, and $H_n(\hat{\alpha})$ is the single-variable Hermite polynomial. $a$ and $a^\dagger$ are the Bose annihilate and creation operator, respectively, satisfying communicative relation $[a, a^\dagger] = 1$.

In order to calculate $N_{\mu,\nu}$, using the transformation relation of single-mode squeezed operator $[21]$, $S'(r) a S'(r) a^\dagger S'(r) = a c\cosh r - a^\dagger \sinh r$, $S'(r) a S'(r) = a^\dagger c\sinh r - a \cosh r$, we can get

$$\langle \Psi\rangle_H = H_n(\hat{\alpha})S(r) = H_n(\hat{\alpha}_r), \hat{\alpha}_r \equiv \mu a + \nu a^\dagger, \tag{2}$$

where $\mu_i = \mu \cosh r - \nu \sinh r, \nu_1 = \nu \cosh r - \mu \sinh r$. Thus the factor $N_{\mu,\nu}$ can be calculated according to the normalization

$$N_{\mu,\nu}^{-2} = \langle 0 \mid H_n(\hat{\alpha}_r^\dagger)H_n(\hat{\alpha}_r)|0\rangle = \frac{\partial^2 n}{\partial r^2} \exp\{-A(r^2 + 2r) + 4e^2 tr\} |_{r=r=0} \tag{3}$$

$$= 2^n! B^Pn(2e^2 r / B),$$

where $A = 1 - 2\mu_1, B = \sqrt{4e^2 + A^2}, P_n$ is the Legendre polynomial, and we used the formula [22]

$$\frac{\partial^2 n}{\partial r^2} \exp\{-t^2 - r^2 + 2xt\} |_{r=r=0} \tag{4}$$

$$= \frac{2^m n!}{(x^2 - 1)^{m/2}} P_n(x),$$

and the generating function of single-variable Hermite polynomial,

$$H_n(x) = \frac{\partial^n}{\partial x^n} e^{-x^2 + 2x} |_{x=0}. \tag{5}$$

In particular, when the squeezing parameter $r = 0$ is leading to $\mu_1 = \mu, \nu_1 = \nu$, i.e. the HPS vacuum ($|\Psi\rangle_H \rightarrow |\Psi\rangle$), we see

$$|\Psi\rangle \equiv N_{\mu,\nu}H_n(\hat{\alpha})|0\rangle, N_{\mu,\nu} = \left| N_{\mu,\nu}^{-2} \right|^{1/2} \rightarrow \mu a + \nu a^\dagger. \tag{6}$$

In the state $|\Psi\rangle$, we can get the average of $\langle a^\dagger a^\dagger \rangle$: 

$$\langle a^\dagger a^\dagger \rangle = \frac{N_{\mu,\nu}^2}{\lambda^2} \frac{2^\nu (\nu + 1)!}{\nu!(n - k)!} F_{-\nu,\nu-k}(\lambda^2), \tag{7}$$

where $\lambda = \nu / \sqrt{1 - 2\mu \nu}$ and we defined a special function whose mother function is given by

$$F_{\nu,\nu-k}(\lambda^2) \equiv \frac{\partial^n}{\partial \nu^n} e^{-\nu^2 - \nu \lambda^2} |_{\nu=0}. \tag{8}$$

From equation (6) one can see that $\langle a^\dagger a^\dagger \rangle = \langle a^\dagger a^\dagger \rangle^\ast$, $\langle a^\dagger \rangle = \langle a^\dagger \rangle^\ast$ and $\langle a \rangle = 0$, as expected. Equation (6) shall be useful for further calculations.

3. Nonclassical properties of the HPS-SV

In this section, we study the nonclassical properties of the HPS-SV according to Mandel’s $Q$ parameter, photon-number distribution, and squeezing effect.

3.1. Mandel’s $Q$ parameter and second-order correlation function

We examine the sub-Poissonian photon statistics by using Mandel’s $Q$ parameter [23], which is defined as

$$Q_M = \frac{\langle a^\dagger a^\dagger \rangle_H}{\langle a^\dagger a^\dagger \rangle_H} - \langle a a^\dagger \rangle_H. \tag{9}$$

Super-Poissonian, Poissonian, and sub-Poissonian statistics correspond to $Q_M > 0$, $Q_M = 0$, and $Q_M < 0$, respectively. In order
to obtain the result (8), it will be convenient to derive some average values: $\langle a^2 \rangle$, $\langle a^2 \rangle$, $\langle a^4 \rangle$, $\langle a^6 \rangle$ and $\langle a^8 \rangle$ under the state $|Ψ\rangle$ (5). These averages are obtained from equation (6).

Thus under the state $|Ψ\rangle$, these corresponding average values $(...,\rangle$ are given by $\langle S^\dagger ... S \rangle_{S \rightarrow H \rightarrow S}$. Substituting equation (6) into equation (8), we can get the Mandel’s $Q$ parameter. We can get the second-order correlation function $g_2^{(2)}=(a^2 a^2)^2_H(a^2 a^2)^2_H$ in a similar way. In particular, when $n=0$ (the SV), the Mandel’s $Q$ parameter and the second-order correlation function are given by $Q_M=\cosh 2r>1$, and $g_2^{(2)}=3+\sinh^2 r>3$, respectively.

In order to clearly see the effects of the Hermite polynomial on the SV, the numerical calculation results of squeezing parameter $r$ for several different values of $n$ and $(\mu, \nu) = (1, 1)$; $(a) n=2$.

3.2. Photon-number distribution

Now, we discuss the photon-number distribution (PND) of the HPS-SV. In this field, the PND of finding $m$ photons is given by

$$P_m = \left| N_{\mu, \nu, m} (m|H_S(\hat{O})S(r)|\rangle \right|^2$$

$$= N_{\mu, \nu, m}^2 (m! (n)^2 \tanh r)$$

$$\times \left| \frac{\partial^n \hat{O}}{\partial x^n} \right|_{x=a^2} \left[ e^{-a^2}-2(a^2)^2+2a^2G \right]_{x=0}^2 ,$$

where we have set $A^2 = 1 + 2\mu^2 \tanh r - 2\mu r$. $B^2 = \frac{1}{2} \tanh r$, $C_1 = \mu - \mu \tanh r$. It is easy to see that equation (9) just reduces to the PND of the SV when $n=0$.

In figure 3, the PND is plotted for different values of $(\mu, \nu)$, $r$ and $n$, from which one can see that (i) by modulating the order of Hermite polynomials, one is able to change the position of the peak (see figures 3(a) and (d)); (ii) for a small squeezing (say $r = 0.3$), the peak of PND is mainly located at $n$ (see figures 3(a), (b) and (d)); (iii) for a large squeezing (say $r = 0.9$), the peak moves to the small photon-number region (see figure 3(b)); (iv) in addition, the PND can be modulated by the parameters (see figures 3(a) and (c)), especially for $n \geq 2$.

3.3. Squeezing effects

In this subsection, we consider the squeezing effects of the HPS-SV, particularly from the Hermite polynomial operation.

Figure 1. Mandel’s $Q$ parameter $Q_M$ as a function of squeezing parameter $r$ for several different values of $n$ and $(\mu, \nu) = (1, 1)$; $(a) n=2$.

Figure 2. The second-order correlation function $g_2^{(2)}$ as the function of squeezing parameter $r$ for several different values of $n$ and $(\mu, \nu)$. (a) $(\mu, \nu) = (1, 1)$; $(b) n=2$. negative feature of $Q_M$ than $H_2(a)$ and $H_2(a + a^\dagger)$. In addition, from figure 2 one can get similar results for the second-order correlation function. For instance, the HPS-SV appears to exhibit an anti-bunching effect in a small region due to the Hermite operation (except for $n=0$).
First, we examine the wave function which can reflect the squeezing effect of the quantum state to some extent. Using the natural expression of single-mode squeezing operator in the momentum representation $|p\rangle$ [24], which leads to $S'(r)|p\rangle = 1/\sqrt{u} |p/u\rangle$, thus the distribution of the quadrature $p$ is given by

$$S(r) = \sqrt{u} \int_{-\infty}^{\infty} dp |u p\rangle \langle p|, u = e^r, \quad (10)$$
which is just the Hermite-Gaussian function.

In order to clearly see the squeezing effect, we present the distribution in figure 4 where the distributions are plotted for different values of $n$ and $(\mu, \nu)$. Different from the Gaussian distribution of the SV ($n = 0$), the HPS-SV has several different peak distributions with different $n (\neq 0)$ values. In addition, the amplitude values of peaks are affected by the parameters $\mu, \nu$ (see figure 4(b)).

Next, we further discuss the squeezing property of the HPS-SV by using the standard analysis of quadrature squeezing, i.e. $(\Delta Q)^2 < 1$ or $(\Delta P)^2 < 1$ which indicates the squeezing or sub-Possonian statistics. Here, we introduce a quadrature operator $Q_{\theta} = a_{\theta}e^{-i\theta} + a_{\theta}^*e^{i\theta}$. Thus, the squeezing can be characterized by the minimum value $\langle \Delta^2 Q_{\theta} \rangle < 1$ with respect to $\theta$, or by the normal ordering form $\langle \Delta^2 Q_{\theta} \rangle < 0$ [25]. Upon expanding the terms of $\langle \Delta^2 Q_{\theta} \rangle$, one can minimize its value over the whole angle $\theta$, which is given by [26] $S_{\text{opt}} = -2\langle (a^a)^2 \rangle + 2\langle a^a \rangle - 2\langle (a^a)^2 \rangle$, then its negative value in the range $[-1, 0)$ indicates squeezing (or nonclassical). For the HPS-SV, $\langle a^a \rangle = 0$, the degree of squeezing of the HPS-SV can be obtained as $S_{\text{HPS}} = 2\langle (a^a)^2 \rangle < 0$. In particular, when $n = 0$ (the case of squeezed vacuum), $S_{\text{HPS}} = -2e^{-r} \sinh r$, as expected.

The degree of squeezing of the HPS-SV is shown in figure 5 for several different values of parameters $n$ and $(\mu, \nu)$. It is found that the degree of squeezing of the HPS-SV increases with $r$. In figure 5(a) with a given $(\mu, \nu) = (1, 1)$, compared with the SV ($n = 0$), the HPS-SV can present squeezing only when the squeezing parameter $r$ exceeds a certain threshold value (say 0.5); the degree of squeezing can be enhanced by Hermite polynomial superposition operation (say $n = 2, 4$) in a larger region of squeezing parameter.

In figure 5(b) with $n = 2$, it is shown that (i) the Hermite photon-subtraction operation $H_2(a^a)$ on the SV can be used to improve the degree of squeezing in a small region ($r \leq 0.6$), while for $H_2(a^a)$ the case is not true; the coherent superposition operation ($H_2(a + a^a)$), rather than the $H_2(a)$ or $H_2(a^a)$ operations, can improve the degree of squeezing in a large region. This implies that the coherent operation $\mu a + \nu a^a$ achieves better squeezing than the mere photon-subtraction (+-addition) in a large region of $r$. In addition, the maximum degree of squeezing of the HPS-SV is $-1$.
Thus, the WF can be derived as
\[ W \propto e^{-\frac{1}{2}b^*a^2 - \frac{1}{2}a^*b^2} \mathbb{g}(a^*b^2 - a - a^*)^\dagger. \] (13)

where \( a = \alpha \cosh r + \alpha^* \sinh r \) and the WF can be defined as
\[ W(\alpha, \alpha^*) = \mathbb{g}(\alpha \cosh r + \alpha^* \sinh r)^2 = \mathbb{g}(\alpha \cosh r + \alpha^* \sinh r)^2. \] (14)

Obviously, the WF \( W(\alpha, \alpha^*) \) in equation (14) is a real function and non-Gaussian in phase space due to the presence of differential form. In particular, when \( n = 0 \), equation (14) just reduces to the WF of the SV \( W(\alpha, \alpha^*) = \mathbb{g}(\alpha \cosh r + \alpha^* \sinh r)^2 \), as expected.

In addition, when \( n = 1 \) corresponding to the single-photon-subtraction (addition) SV, \( W(\alpha, \alpha^*) = \mathbb{g}(\alpha \cosh r + \alpha^* \sinh r)^2 - 1 \), which indicates that there is always negative region at the center of phase space \( \alpha = 0 \) (independent of the two parameters \( \mu, \nu \)). In figure 6, the Wigner distributions are depicted in phase space for several different parameter values \( n, (\mu, \nu) \), from which it can clearly be seen that there are some obvious negative regions of the WF in the phase space which is an indicator of the nonclassicality of the state. In addition, these negative areas are modulated not only by \( n \) (see figure 6(a)–(c)), but also by the parameters \( (\mu, \nu) \) (see figures 6(b) and (d)). For instance, there is an obvious difference in WF distribution between figures 6(b) and (d). That is to say, for higher order with \( n \geq 2 \), the negative area depends on the two parameters. In order to clearly see this point, we can quantify the negative volume of the WF, defined by \( \delta = \frac{1}{2} \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq W(q, p) - 1 \) [29].

In figure 7, the negative volume (NV) of WF as a function of \( r \) or \( \nu \) on applying the Hermite coherent supposition operation \( H_\nu(\mu a + \nu a^*) \) for several different \( n \). From figure 7(a), one can find that the NV \( \delta \) increases with the order \( n \) (in a certain region of \( r \leq 0.45 \)) and decreases with \( r \). In particular, for the case of \( n = 1 \) (a superposition between single-photon addition/subtraction SV and SV), the NV is independent of parameters \( r \) and \( (\mu, \nu) \) and is kept unchanged \( (\delta = 0.213) \). In fact, one can calculate that the NV of WF with \( n = 1 \) is \( \delta = 2 \sqrt{e} - 1 \approx 0.213 \). In figure 7(b), we optimize the NV for different \( r \) and \( n \), where \( \mu, \nu \) are taken as \( \mu = 0.1, \nu = 1 \). From figure 7(b), it is found that (i) when \( \mu = \nu = 1/\sqrt{2} \), the NV \( \delta \) decreases with the order \( n \) for a given small \( r \) and decreases with \( r \) for a given \( n \) (see the vertical dotted line at the point of \( \nu = 1/\sqrt{2} \)); (ii) for a given parameter \( r = 0.1 \), the NV \( \delta \) increases with \( n \) when \( n \) exceeds a certain threshold \( (\nu = 0.41) \); (iii) for a given \( n = 2 \), \( \delta \) decreases with \( r \) when \( r \) exceeds a certain threshold \( (\nu = 0.45) \); (iv) the NV \( \delta \) does not monotonously increase with \( \nu \); in particular, one can find the maximum NV \( \delta \) in a bigger region of \( \nu (\nu \geq 0.41) \) for \( n = 2, 3, 4 \). This optimal value of \( \delta \) can be achieved at neither \( \nu = 0 \) nor \( \nu = 1 \).

### 4. Wigner distribution of the HPS-SV

As a kind of quasi-probability function, the WF is a powerful tool to describe the nonclassicality of optical fields, whose partial negativity implies the highly nonclassical properties of quantum states. In addition, the negativity is often used to present the decoherence of quantum states. In this section, we derive the analytical expression of WF for the HPS-SV by using the the Weyl-ordered operators’ invariance under similar transformations [27]. For a single-mode quantum system, the WF can be calculated as \( W = \text{tr}[\rho \Delta(\alpha)] \), where \( \Delta(\alpha) \) is a single-mode Wigner operator [27, 28],

\[ \Delta(\alpha) = \frac{1}{2} \delta(a - a^*) \delta(a^* - a) \mathbb{1}. \] (12)

Here \( \alpha = (q + ip)/\sqrt{2} \) and the symbol \denotes Weyl ordering.

The merit of Weyl ordering lies in the Weyl-ordered operators’ invariance proven under similar transformations, which
For instance, these points are $\nu = 0.71, 0.74, 0.78$ for different values of $n = 2, 3, 4$ and $r = 0.1$. These indicate that the effects of the coherent operation $H_n(\alpha a + \nu a^\dagger)$ with higher order $n \geq 2$ are more prominent than those of the mere photon subtraction $H_n(\nu a^\dagger)$ and the addition $H_n(\nu a^\dagger)$ particularly in the larger region of parameter $\nu$, whereas the optimal operation is not the photon subtraction or the photon addition in this region. This result is different from that in [11].

5. Decoherence of the HPS-SV in phase-sensitive reservoirs

In this section, we shall examine the time evolution of the HPS-SV in the presence of phase-sensitive reservoirs. In the interaction picture and the Born and Markow approximation, the time evolution of the density matrix is governed by the master equation [30]:

$$\frac{d}{dt} \rho(t) = \kappa \mathcal{L} [a^\dagger] \rho + \kappa (\bar{n} + 1) \mathcal{L} [a] \rho + \kappa M \mathcal{D} [a] \rho + \kappa M^* \mathcal{D} [a^\dagger] \rho,$$

and $\mathcal{L} [O] \rho = 2O^\dagger \rho O - O^\dagger O \rho - \rho O^\dagger O, \mathcal{D} [a] \rho = 2a^\dagger \rho a - a^\dagger a \rho - a^\dagger a^\dagger \rho - \rho a^\dagger a^\dagger$, where $\kappa$ and $\bar{n}$ are the dissipative coefficient and the average thermal photon number of the environment, respectively. Here $M$ is the complex correlation parameter between modes symmetrically displaced about center frequency. In fact, the ME in equation (15) includes two special cases: (1) for an uncorrelated reservoir, i.e. $M = 0$, equation (15) becomes the ME describing the interaction between a system and a thermal environment at finite temperature; (2) when $M = \bar{n} = 0$, equation (15) reduces to the one describing the photon-loss channel. For an (non)ideally squeezed reservoir, the constraint condition $|M|^2 = \bar{n}(\bar{n} + 1)$ is required.

In [31], we derived the Kraus operator-sum representation of density operator $\rho$ and the time evolution of some distribution functions by using the thermally entangled state representation ($\eta$). The evolution of the Wigner function is given by

$$W(\alpha, t) = \frac{2\mu_\infty}{T} \int \frac{d^2 \beta}{\pi} e^{-2\beta_\alpha} \Sigma(\alpha, \alpha^*),$$

where $\Sigma(\alpha, \alpha^*)$ is defined as $\Sigma(\alpha, \alpha^*) = (\alpha^2 \alpha^*) \sigma_\infty \left( \alpha \alpha^* \right)$ and $\sigma_\infty = \left( M^* \bar{n} + 1 - \bar{n}^2 / 2 M \right)$. Noting the differential expression of the WF equation (14), we can finally obtain $W(\alpha, t) = W(\alpha, t)$ $F_\eta(\alpha, t)$, where $W_\eta(\alpha, t)$ is the evolution of WF of the SV in the phase sensitive reservoir, and $F_\eta(\alpha, t)$ is a non-Gaussian item due to the presence of Hermite excitation. Here, the analytical expressions of $W_\eta(\alpha, t)$ and $F_\eta(\alpha, t)$ have, for simplicity, not been given.

In particular, at the center of phase space $\alpha = 0$, the existence of NV of WF leads to $\kappa t < \kappa t_c = \frac{1}{2} \ln(\mu_\infty + 1)$, which is independent of squeezing parameter $r$. It is easy to see that for any $|M|^2$ ranging from 0 to $\bar{n}(\bar{n} + 1)$,

$$\frac{1}{2} \ln \left( \frac{2\bar{n} + 2}{2\bar{n} + 1} \right) \leq \frac{1}{2} \ln(\mu_\infty + 1) \leq \frac{1}{2} \ln 2,$$

which indicates that the characteristic time of decoherence of single-photon added SV state in phase sensitive reservoirs is larger than that in the thermal environment and smaller than that in the photon-loss channel.

In order to measure the degree of nonclassicality for the evolved state, we consider the negative area and the NV in phase space. As shown in figure 8, the negative area gradually disappears with the incasement of $\kappa t$, while it increases with parameter $M$. To clearly see the effects of the decoherence and parameter $M$ on the nonclassical properties, the evolution of NV with time and squeezing parameter $M$ are plotted in figure 9 for given $n = 1, r = 0.3$ and $\bar{n} = 0.5$. From figure 9(a) one can see that the NV monotonically diminishes with $\kappa t$, and there is a more rapid attenuation for a big $n$ than a small one; this leads to a smaller NV for a big $n$ than a small one when $\kappa t$ exceeds a certain value. From this point, one can draw a conclusion that single-photon subtraction/addition SV presents a much stronger robustness against the reservoirs than a higher-order photon subtraction/addition which could have a bigger...
of the squeezing effect can be achieved by the Hermite photon-subtraction operation $H_2(a)$ and the coherent superposition operation $(H_2(a + a^\dagger))$ in a small region ($r \lesssim 0.6$) and a large region, respectively.

In addition, the numerical calculation of NV $\delta$ of WF showed that $\delta$ increases with the order $n$ for $r \lesssim 0.45$ and decreases with $r$. It is interesting to note that the NV $\delta$ is given $\delta = 2/\sqrt{e} - 1 \approx 0.2131$ independent of $\mu$ and $\nu$, as well as $r$. For high-order excitation ($n \geq 2$), the NV can be optimized by modulating parameters $\nu (\mu = \sqrt{1 - \nu^2})$ and $n$ for a given $r$. It is found that the NV $\delta$ may increase with $n$ and decrease with $r$ when $\nu$ exceeds a certain threshold. In particular, the optimal value of $\delta$ can be obtained in a bigger region of $\nu (1 > \nu \geq 0.41)$ not at $\nu = 0$ or $\nu = 1$. This implies that the effects of the coherent operation $H_n(\mu a + \nu a^\dagger)$ with higher order $n \geq 2$ are more prominent than those of the mere photon subtraction $H_n(\nu a^\dagger)$ and the addition $H_n(\nu a^\dagger)$ particularly in the larger region of parameter $\nu$, whereas the optimal operation is not the photon subtraction or the photon addition in this region, which is a new result.

Furthermore, we have considered the decoherence effects of the HPS-SV in phase-sensitive reservoirs according to the analytically derived WF distribution. It is shown that the negative area and volume diminish gradually with the evolution of time and disappear eventually. However, the NV of the HPS-SV with higher order excitation decays more rapidly with time, which implies that single-photon subtraction/addition SV has a much stronger robustness than a higher-order photon subtraction/addition although the latter has a bigger NV initially. In addition, the parameter $M$ describing the squeezing characteristic of reservoirs can be effectively used to enhance the nonclassicality.

Acknowledgments

This project was supported by the National Natural Science Foundation of China (Grant No.11264018), the Research Foundation of the Education Department of Jiangxi Province of China (No. GJJ14274) and the Degree and postgraduate education teaching reform project of Jiangxi province (No. JXYJG-2013-027).

6. Conclusions

In this letter, we have introduced a new non-Gaussian state, which is generated by applying Hermite-polynomial excitation on the SV. Single-photon addition/subtraction and Hermite polynomial addition/subtraction can be seen as its special cases. Its normalized factor is found to be a Legendre polynomial. Then we investigated its nonclassicality according to the Mandel’s $Q$ parameter, second-order correlation function, PND, squeezing effect and the negativity of WF in phase space. It is shown that all these nonclassical properties can be obviously improved by the HPS operation and can be remarkably modulated by superposition parameters $\mu$ and $\nu$. The degree of squeezing of the HPS-SV increases with $r$. In particular, compared with the SV, an observable improvement

References

[1] BouwmeesterD, Ekert A and Zeilinger A 2000 The Physics of Quantum Information (New York: Springer)
[2] Agarwal G S and Tara K 1991 Phys. Rev. A 43 492
[3] Marek P, Jeong H and Kim M S 2008 Phys. Rev. A 78 063811
[4] Kim M S 2008 J. Phys. B 41 133001
[5] Lee S Y, Ji S W and Lee C W 2013 Phys. Rev. A 87 052321
[6] Yang Y and Li F L 2009 Phys. Rev. A 80 022315
[7] Zavatta A, Viciani S and Bellini M 2004 Science 306 660
[8] Hu L Y and Fan H Y 2009 Phys. Scr. 79 035004
[9] Ourjoumtsev A, Dantan A, Tuia-Corrant R and Grangier Ph 2007 Phys. Rev. Lett. 98 030502
[10] Hu L Y and Zhang Z M 2013 J. Opt. Soc. Am. B 30 518
[11] Lee S Y and Nha H 2010 Phys. Rev. A 82 053812
[12] Lee S Y, Ji S W, Kim H J and Nha H 2011 Phys. Rev. A 84 042302
[13] Fiurasek J 2002 Phys. Rev. A 65 053818
[14] Kok P, Lee H, and Dowling J P 2002 Phys. Rev. A 65 052104
[15] Lee S Y and Nha H 2012 Phys. Rev. A 85 043816
[16] Zhang G P, Zheng K M, Liu S Y and Hu L Y 2014 Chin. Phys. B 23 050301
[17] Hu L Y, Zhang H L, Hu Y Q and Zhang Z M 2014 OPTIK 125 61
[18] Bergou J A, Hillery M and Yu D 1991 Phys. Rev. A 43 515
[19] Fan H Y and Ye X 1993 Phys. Lett. A 175 387
[20] Ren G, Du J M, Yu H J and Xu Y J 2012 J. Opt. Soc. Am. B 29 3412
[21] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)
[22] Hu L Y, Xu X X, Wang Z S and Xu X F 2010 Phys. Rev. A 82 043842
[23] Mandel L and Wolf E 1955 Optical Coherence and Quantum Optics (Cambridge: Cambridge University Press)
[24] Fan H Y 1997 Representation and Transformation Theory in Quantum Mechanics (Shanghai: Shanghai Scientific and Technical Publisher)
[25] Hong C K and Mandel L 1985 Phys. Rev. A 32 974
[26] Lee J, Kim J and Nha H 2009 J. Opt. Soc. Am. B 26 1363
[27] Fan H Y 2008 Am. Phys. 323 500
[28] Fan H Y and Zaidi H R 1987 Phys. Lett. A 124 303
[29] Kenfack A and Zyczkowski K 2004 J. Opt. B 6 396
[30] Gardiner C W and Zoller P 2000 Quantum Noise (Berlin: Springer)
[31] Hu L Y, Wang Q, Wang Z S and Xu X X 2012 Int. J. Theor. Phys. 51 331
[32] Xu Q 2014 Laser Phys. Lett. 11 075201