Neutron stars with spin polarized self-interacting dark matter

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Abstract

Dark matter, one of the important portion of the universe, could affect the visible matter in neutron stars. An important physical feature of dark matter is due to the spin of dark matter particles. Here, applying the piecewise polytropic equation of state for the neutron star matter and the equation of state of spin polarized self-interacting dark matter, we investigate the structure of neutron stars which are influenced by the spin polarized self-interacting dark matter. The behavior of the neutron star matter and dark matter portions for the stars with different values of the interaction between dark matter particles and spin polarization of dark matter is considered. In addition, we present the value of the gravitational redshift of these stars in different cases of spin polarized and self-interacting dark matter.

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Key words: Neutron stars; Dark matter; Spin polarization

1 Introduction

Because of the compactness and high density of compact objects, the accretion of dark matter (DM) particles can take place on compact stars [12]. The compact objects are sensitive probes of DM and they set constraints on the properties of DM particles and its density [1]. It has been shown that the heating due to the accretion of WIMPs onto cool white dwarf stars could be detected [1]. Self-annihilating neutralino WIMP DM accreted onto neutron stars results in a mechanism to seed compact objects with long-lived lumps of

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strange quark matter, indicating a possible conversion of most of the star into a strange star [3]. This self-annihilation can affect their kinematical properties such as velocity kicks and rotation patterns [4]. Since even neutron stars located at regions of low DM density can accrete WIMPs which lead to collapse and form a mini black hole [5,6], the constraints on the WIMP self-interactions which are stricter than the ones from the bullet cluster have been derived [7]. The accretion of millicharged DM onto a neutron star can result in the expulsion of extra electric charge from the poles of the star and impede further the rotation of the star yielding braking indices consistent with the observational results [8]. A limit on the neutron star survival rate against transitions to more compact objects has been suggested according to the amount of decaying DM accumulated in the central regions in neutron stars [9]. This limit also sets constraints on the DM particle decay time. The collapse of neutron stars due to capture and sedimentation of DM within their cores presents a solution for the problems of non-detection of pulsars within the galaxy inner region ∼ 10 pc and the sources of fast radio bursts [10]. Considering the number of self-interacting but not self-annihilating DM particles that a neutron star accumulates over its lifetime, it has been confirmed that the DM self-interactions have a significant role in the rapid accumulation of DM in the core of neutron stars [11].

The compact stars made of fermionic DM have been studied applying the free and interacting Fermi gas model for the DM [12]. The results show that a unique mass-radius relation for compact stars made of free fermions exists which is independent of the fermion mass. Besides, the mass-radius relation for compact stars with strongly interacting fermions indicates that the radius remains constant for a wide range of compact star masses. The repulsion of DM in neutron star leads to form DM halo [13]. This repulsion results in disappearing the density dependencies of nuclear symmetry energy which this also leads to difference in particle number density distributions in DM admixed neutron stars and consequently in star radii [13]. Since the DM halo of star may extend around the star, the mass of halo can result in the alternative gravitational effects [14,15]. The stellar structure [16,17,13], temperature [18], linear and angular momentum [4] of neutron star are affected by DM.

The spin state of DM particles and the properties of DM related to the spin polarization of these particles have been explored in the last decade. The spin of the mother particle and the DM particle in the center of momentum system of a decaying particle can be specified [19]. Theoretically, it has been confirmed that a spin one half matter field with mass dimension one and the dominant interaction via Higgs can be a candidate for DM [20,21]. A DM candidate particle of spin 3/2 with neutrino-like Standard Model strength interactions has been introduced [22]. This particle can couple to the nucleon via Z-exchange and it may lead to large spin-independent and spin-dependent cross sections for a Dirac or Majorana particle, respectively.
A model independent analysis considering the spin-dependent interactions with both protons and neutrons has been studied [23]. In the detection of DM, $^{19}F$ is the most useful particle to detect the spin-dependent interactions [24]. The spin-dependent WIMP interactions on $^{19}F$ have been searched by the PICASSO experiment at Sudbury Neutrino Observatory LAB using the superheated droplet technique [25]. The cold DM has been examined by the direct detection of WIMPs based on the spin-dependent interactions with nuclei [26]. There exist some mechanisms of interferences that lead to suppression of the spin-independent interaction in the scattering of scalar DM with nucleus compared to the spin-dependent interaction [27]. It has been argued that by considering the Maxwellian speed distribution, the spin-dependent interactions become the dominant source of scattering around the interference regions. In addition, for a collision with a known speed, the dominance of the spin interaction presents stringent limits of the WIMP mass around the interference point. By modeling three stars including DM energy transport and applying asteroseismic diagnostics, the indications limiting the effective spin-dependent DM-proton coupling for masses of a few GeV was found [28]. In that work, using the observational data and the results of stellar models including DM energy transport, it has been tried to constrain the properties of low-mass asymmetric DM with an effective spin-dependent coupling. The spin contribution on the cross section of natural DM candidate from supersymmetry has been investigated [29]. Models which predict a substantial fraction of higgsino lead to a relatively large spin induced cross section due to the Z-exchange [29]. The dependency of DM scattering on the intrinsic spin of DM particles has been studied [30]. The general formulas for spin-dependent cross sections for the scattering of WIMPS with intrinsic spin 0, 1/2, 1, and 3/2 have also been considered in Ref. [30]. The sensitivity of the MIMAC-He3 detector for supersymmetric DM search to neutralinos ($M_\tilde{\chi} \gtrsim 6 \text{ GeV} c^{-2}$) has been studied via spin-dependent interaction with $^3He$ [31]. This sensitivity leads to complementarity of MIMAC-He3 with ongoing experiments. The isotope $^{73}Ge$ can provide probe to the spin-dependent couplings of WIMPs with the neutrons [32]. The improved limits on spin-independent and spin-dependent couplings of low-mass WIMP DM with a germanium detector have been presented [32]. The models for the direct DM detection which are detectable via spin-dependent interactions have been explored [33]. The findings verify that most models with detectable spin-dependent interactions generate detectable spin-independent interactions. Different elastic spin-dependent operators have been studied in the detection and solar capture of WIMP [34]. It has been concluded that the efficiency of the detection strategies depends on the spin-dependent operators.

The size of the bound WIMP population for the DM bound to the solar system by solar capture depends on the WIMP mass $m$, spin-independent cross section, and spin-dependent cross section [35]. The central temperature of the Sun and the resulting $^8B$ neutrino flux decrease in the Models of DM with
large spin-dependent interactions and an intrinsic asymmetry that prevents post freeze-out annihilations [36]. The constraints on the spin-dependent cross section of asymmetric fermionic DM WIMPs based on the existence of compact stars in globular clusters have been investigated [6]. It has been confirmed that asymmetric WIMP candidates with only spin-dependent interactions trapped during the lifetime of the progenitor can thermalize inside the white dwarf. Moreover, the characteristics of the DM capture rate by stars are very different for the spin-dependent and spin-independent DM particle-nucleon scattering cross sections [37].

Some other studies have been considered to explore the spin-dependent and magnetic properties of DM particles. Considering a neutral Dirac fermion as a DM candidate, it has been shown that the elastic scattering is due to a spin-spin interaction [38]. The models of inelastic DM for the DM direct detection experiments in which iodine with its large magnetic moment is used have been investigated [39]. In that study, the dipole moments for the WIMP have been applied with the conventional magnetism and also dark magnetism and both magnetic-magnetic and magnetic-electric scattering. Moreover, a model of multi-component DM with magnetic moments has been presented to describe the 130 GeV gamma-ray line hinted by the Fermi-LAT data [40]. Another aspects of the magnetic properties of DM particles has been suggested in Ref. [41]. In that work, it has been argued that the unconventional properties of DM may create the galactic magnetic fields. The effect of supernova explosions on magnetized DM halos has been explored applying a set of high resolution simulations [42]. In an interesting recent research, the detection of magnetized quark nuggets as a candidate for DM has been investigated [43]. They have calculated the flux of electromagnetic radiation from electrons which is swept up by the magnetic field of quark-nuggets and their synchrotron radiation from a magnetized quark nugget interaction with the galactic magnetic field.

On the other hand, it has been argued that for the ordinary matter, the spin-dependent interactions can result in the spontaneous spin polarized systems [44,45,46,47]. In nuclear matter, for non-localized protons, there exists a threshold value of the spin interaction above which the system can develop a spontaneous polarization [44]. Besides, it has been found that strong interactions between electrons lead to a ferromagnetic ground state in a certain range of electron densities [45]. The phase transition with spontaneous breaking the spin symmetry due to exchange interaction in electron systems has been also reported in Ref. [46]. In addition, the spin polarization increases with increasing the interaction parameter [46]. Moreover, it has been shown that a spin polarized electron system localizes at electron densities higher than a spin unpolarized one is a result of the exchange correlation effects [47]. Noting these results and supposing a similarity between ordinary matter and dark matter properties, one can assume that the spin-dependent interactions can also lead...
to a spontaneous spin polarization in a system of dark matter particles. Therefore, it is possible to take a system of DM particles into account which are spin polarized. The spin polarization of DM particles could be a result of the spin-dependent interactions and couplings to the nucleon [22,23,24,28,32,37], the spin-dependent interactions with nucleus [25,26,27,31], the spin-spin interaction between DM particles [38], the dipole moments of the WIMP with the conventional magnetism and also dark magnetism [39], the unconventional properties of DM which create the galactic magnetic fields [41], and the magnetic field of quark-nuggets [43]. As we explain in the following, we focus on the influence of the spin polarized DM on the structure of neutron stars. In this regard, one can anticipate that with a system of DM which are spin polarized, the particles occupy the Fermi sphere in a way that the Fermi momentum of the fermions is larger than the case of spin unpolarized one. This effect can result in the more stiffening of the DM equation of state (EOS) and larger masses of neutron star.

Regarding the above discussions, the effects of DM particle spin on the physical properties of star can be considerable. Hereof, the importance of spin-dependent interaction in interferences [27], the spin-dependent interactions as the dominant source of scattering [27], the large spin induced cross section due to the Z-exchange [29], and the decrease of central temperature in the Sun due to DM with large spin-dependent interactions [30] are some cases. In this work, we are interested in the properties of neutron stars which are affected by the spin polarized self-interacting DM. Considering a system of DM particles with spin one half which can be spin polarized in the neutron star, it is possible to understand the spin nature of DM particles, the bulk properties related to the spin polarization of particles, and the strength of the interaction via the influence on the observational results. Here we show how the spin and strength of interaction between DM particles affect the properties of neutron stars.

2 Spin polarized self-interacting dark matter equation of state

We treat the DM in neutron star as an interacting spin polarized Fermi gas at zero temperature. A system composed of \( N \) particles with mass \( m \) and spin \( \frac{1}{2} \) is considered. The internal energy per particle for this system is given by,

\[
E_{\text{tot}} = E_1 + E_2, \tag{1}
\]

in which \( E_1 \) and \( E_2 \) denote the one-body and interaction two-body energies. For the spin polarized system, the one-body term is as follows,
Fig. 1. Equation of state for the spin polarized DM with the mass $m = 1$ GeV and different values of interaction between particles, $m_I$, and spin polarization parameter, $\eta$. 

$$E_1 = \frac{1}{N} \sum_{i=+,-} \sum_{k \leq k_F^{(i)}} \sqrt{\hbar^2 c^2 k^2 + m^2 c^4},$$

where $k_F^{(i)}$ is the Fermi momentum of a DM particle with spin projection $i$. It is easy to show that the one-body term takes the form,

$$E_1 = \frac{m^4 c^5}{2\pi^2 \hbar^3} \rho \sum_{i=+,-} \frac{1}{8} \{x_F^{(i)} \sqrt{1 + x_F^{(i)} 2 (1 + 2 x_F^{(i)}^2)} - \sinh^{-1}(x_F^{(i)}) \}. \tag{3}$$

In the above equation, $\rho$ is the total number density of spin polarized DM particles and $x_F^{(i)} = \frac{\hbar k_F^{(i)}}{m c}$. The interaction two-body energy in Eq. (1) can be expressed in the form $[12]$,

$$E_2 = \frac{u}{\rho}, \tag{4}$$

in which $u$ is the interaction energy density of the particles. It should be noted that we have considered the spin-independent interaction between DM particles. In the lowest order approximation, the interaction energy density is presented by $[12]$,

$$u = \frac{\rho^2}{m_I^2}. \tag{5}$$

In the last equation, the value of $m_I$ shows the energy scale of the interaction between DM particles $[12]$. Using this approximation, Eq. (4) leads to

$$E_2 = \frac{\rho}{m_I^2}. \tag{6}$$
To quantify the amount of bulk spin polarization of DM, we introduce the parameter $\eta$ as follows,

$$\eta = \frac{\rho^{(+)} - \rho^{(-)}}{\rho},$$

(7)

in which $\rho^{(i)}$ is the number density of DM particles with spin projection $i$.

Using this definition, the internal energy, Eq. (1), takes the form

$$E_{\text{tot}} = \frac{m}{16\pi^2\rho} \left( \frac{m^2 (3\pi^2 \rho (1 + \eta))^{1/3}}{m}\sqrt{1 + \frac{(3\pi^2 \rho (1 + \eta))^{2/3}}{m^2}} \right)$$

$$+ \frac{m^2 (3\pi^2 \rho (1 - \eta))^{1/3}}{m}\sqrt{1 + \frac{(3\pi^2 \rho (1 - \eta))^{2/3}}{m^2}}$$

$$+ 6\pi^2 \rho (1 + \eta) \sqrt{1 + \frac{(3\pi^2 \rho (1 + \eta))^{2/3}}{m^2}}$$

$$+ 6\pi^2 \rho (1 - \eta) \sqrt{1 + \frac{(3\pi^2 \rho (1 - \eta))^{2/3}}{m^2}} - m^3 \sinh^{-1}\left(\frac{(3\pi^2 \rho (1 + \eta))^{1/3}}{m}\right)$$

$$- m^3 \sinh^{-1}\left(\frac{(3\pi^2 \rho (1 - \eta))^{1/3}}{m}\right) + \frac{\rho}{m^2}.$$

(8)

In the next step, applying the first law of thermodynamics, $P = \rho \left(\frac{\partial E_{\text{tot}}}{\partial \rho}\right)$, the pressure of spin polarized DM is obtained. Fig. 1 presents the DM EOS related to the interacting spin polarized DM for the cases with different strength of interaction, $m_I$, and spin polarization parameter, $\eta$. We can see that for each strength of the interaction, the stiffness of the EOS increases with the increase in the spin polarization parameter. This increase in the stiffening of the DM EOS is due to the fact that with a system of more spin polarized DM, the particles occupy the Fermi sphere so that the Fermi momentum of the fermions is larger. The effects of spin polarizability of DM are more significant at higher densities. This Figure also confirms that the increase in the value of $m_I$, which is corresponding to the decrease in the interaction between DM particles, leads to the softening of the EOS.

3  Piecewise polytropic equation of state for the neutron star matter

To describe the neutron star matter (NSM) in our calculations, we apply a parameterized piecewise-polytropic EOS [48,49]. For four segments of polytropes,
the EOS for the NSM with the rest-mass density $b_i \leq b \leq b_{i+1}$, $(0 \leq i \leq 3)$, is as follows,

\[ P = K_i b^{\Gamma_i}, \]  

in which $P$ denotes the pressure, $K_i$ shows the polytropic constant, and $\Gamma_i$ presents the adiabatic index. In this model, the pressure is continuous at the boundaries of the piecewise polytropes, $b_i$. Similar to the parameters in [48], we set $\Gamma_0 = 1.3562395$ and $K_0 = 3.594 \times 10^{13}$ (cgs units). The values of the boundary density are also chosen as $b_2 = 10^{14.7} \text{ g/cm}^3$ and $b_3 = 10^{15.0} \text{ g/cm}^3$. Therefore, with the free parameters $(P_2, \Gamma_1, \Gamma_2, \Gamma_3)$ and the continuity of the pressure, the piecewise-polytropic EOS is completely determined. It should be noted that $P_2$ is the NSM pressure at $b = b_2$. In our calculations, we apply the piecewise-polytropic EOS in APR4 model with $\log(P_2(\text{dyn/cm}^2)) = 34.269$, $\Gamma_1 = 2.830$, $\Gamma_2 = 3.445$, and $\Gamma_3 = 3.348$ [50,51]. Moreover, using the first law of thermodynamics we have the following expression for the NSM energy density with the rest-mass density $b_i \leq b \leq b_{i+1}$ [48],

\[ \epsilon(b) = (1 + a_i)b + \frac{K_i}{\Gamma_i - 1} b^{\Gamma_i}, \]  

with

\[ a_i = \frac{\epsilon(b_i)}{b_i} - 1 - \frac{K_i}{\Gamma_i - 1} b_i^{\Gamma_i}. \]  

Fig. 2 presents the piecewise-polytropic NSM EOS in APR4 model. For the NSM with the number densities higher than $0.05 \text{ fm}^{-3}$, we apply the above
EOS. Besides, for the number densities lower than 0.05 \(fm^{-3}\), the EOS calculated by Baym [52] is used.

4 Structure of neutron star with spin polarized self-interacting dark matter

In order to study the structure of neutron stars which are admixed by the DM, we apply the two-fluid formalism [16,17] for the NSM and DM. In this model, a system is composed of NSM and DM particles which interact with each other just through gravity. We investigate the properties of a neutron star with two concentric spheres. One of the spheres contains NSM, and the other is formed by the spin polarized self-interacting DM. Considering the static and spherically symmetric space-time with the line element, (together with the units in which \(G = c = 1\)),

\[
d\tau^2 = e^{2\nu(r)}dt^2 - e^{2\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),
\]

and the energy-momentum tensor of a perfect fluid,

\[
T^{\mu\nu} = -pg^{\mu\nu} + (p + \varepsilon)u^\mu u^\nu,
\]

we obtain the structure of the neutron star. We denote the total pressure and total energy density by \(p\) and \(\varepsilon\), respectively, which are related to the pressure and energy density of NSM and DM by

\[
p(r) = p_N(r) + p_D(r)\quad \text{and} \quad \varepsilon(r) = \varepsilon_N(r) + \varepsilon_D(r).
\]

In this model, the Einstein field equations lead to [16,17,13],

\[
e^{-2\lambda(r)} = 1 - \frac{2M(r)}{r},
\]

\[
\frac{d\nu}{dr} = \frac{M(r) + 4\pi r^3p(r)}{r[r - 2M(r)]},
\]

\[
\frac{dp_N}{dr} = -[p_N(r) + \varepsilon_N(r)]\frac{d\nu}{dr},
\]

\[
\frac{dp_D}{dr} = -[p_D(r) + \varepsilon_D(r)]\frac{d\nu}{dr},
\]

(12)

in which \(M(r) = \int_0^r dr4\pi r^2\varepsilon(r)\) shows the total mass inside a sphere with radius \(r\). The above relations are the result of the assumption that two fluids interact just via gravity. These two-fluid TOV equations can be applied to calculate the neutron star structure and the properties of the NSM and DM spheres. The radius, \(R_N\), and mass, \(M_N\), of NSM sphere are obtained with the condition \(p_N(R_N) = 0\). In addition, the radius, \(R_D\), and mass, \(M_D\), of the DM sphere are given by the condition \(p_D(R_D) = 0\). The total mass of the neutron star is determined by the sum of the masses of NSM and DM spheres, i.e. \(M = M_N + M_D\). It is important to note that the pressure and density profiles of two fluids are different with each other.
Fig. 3. Total mass of neutron star, $M$, with DM and without DM (normal neutron star) versus the radius of NSM sphere, $R_N$. In addition, the curves which show the permitted region are presented, see the text.

4.1 Total mass and visible radius

Fig. 3 presents the total mass of neutron star with DM and without DM (Normal neutron star) versus the visible radius, i.e. the radius of the NSM sphere. We have also shown the permitted region by presenting the curves from the general relativity $M > \frac{c^2 R}{2 G}$ (GR), finite pressure $M > \frac{4}{3} \frac{c^2 R}{G}$ (Finite P), causality $M > \frac{10}{20} \frac{c^2 R}{G}$ (Causality), and rotation of 716 Hz pulsar J1748-2446ad (Rotation) [53]. It is clear that the existence of DM in the neutron star leads to the reduction of neutron star size. In addition, the maximum mass of neutron star with DM is lower than this quantity for the normal neutron star. In the most cases, for each interaction strength, the visible size of neutron star with a specific mass reduces by increasing the polarization of DM. Therefore, the more polarized the DM, the more compact the neutron star. In addition, we can see that the neutron stars with full polarized DM could have lower size. This is while by decreasing the polarization of DM, the smaller neutron stars are not acceptable. For stars with a special size, the mass of neutron star grows when the spin polarization parameter increases. This is a consequence of the higher Fermi momentum of the fermions and more stiff DM EOSs in the cases with the more spin polarized DM. Comparing the neutron stars with DM and without DM indicates that with DM, regardless of its polarization and strength of the interaction, the behavior of the total mass versus the visible radius, i.e. $M - R_N$ relation, is essentially different from this relation for the normal neutron stars.
4.2 Total mass versus the radius of dark matter sphere

Fig. 4 presents the total mass of neutron star, $M$, versus the radius of DM sphere, $R_D$. In all cases, the DM sphere is smaller for more massive stars. We can see that for $m_I = 1$ GeV and $m_I = 300$ GeV, the increase in the spin polarization parameter of DM leads to the larger size of the DM sphere. The increase in the size of DM sphere due to the polarization of DM is more significant in the case with $\eta = 1$. The increase in the mass of the stars with the same size DM sphere is clear when the spin polarization parameter grows; another result of the more stiff DM EOS. Figs. 3 and 4 confirm that for more massive stars, the radius of NSM sphere is larger than the radius of DM sphere. This shows that the visible matter surrounds the DM sphere. However, in the stars with lower masses, the DM sphere has a size bigger than the NSM sphere. In the cases of massive stars, the radii of NSM sphere and DM sphere can be 11 km and 4 km, respectively. However, for the low mass stars, these radii can reach 7 km and 18 km. Thus, the DM sphere can vary in size more than the NSM sphere.

4.3 Neutron star matter sphere mass versus the visible radius

Fig. 5 presents the contribution of NSM portion in the mass of star, $M_N$, versus the radius of NSM sphere. For the stars with larger NSM sphere, the mass of NSM sphere is higher. It is clear that with a mass for NSM sphere, the NSM sphere radius decreases by increasing the spin polarization parameter. Fig. 5 also confirms that for the stars with the same visible size, the mass of NSM sphere grows by increasing the spin polarization parameter of DM. It means that the stiffer DM EOS shifts the mass of NSM sphere to higher values. Figs. 3 and 5 verify that the behavior of the NSM sphere mass and total mass versus the size of NSM sphere are similar.
4.4 Mass and radius of dark matter sphere

Fig. 5 shows the mass of DM sphere versus its radius. For the stars with larger DM sphere, the contribution of this sector in the total mass is more considerable. The range of mass of this sphere is $0.06 M_\odot \lesssim M_D \lesssim 0.68 M_\odot$ which is smaller than the range $0.01 M_\odot \lesssim M_N \lesssim 2.08 M_\odot$ related to the NSM sphere (see Fig. 5). Fig. 6 indicates that by increasing the value of $m_I$, the $M_D - R_D$ relation becomes more flattened and the mass contribution of DM decreases. This is while for each value of the interaction strength, the contribution of DM in the total mass grows by increasing the spin polarization parameter; another consequence of stiffer DM EOSs. This indicates that the star with more spin polarized DM have more massive halo of DM. The radius of DM sphere lies between $4 \text{ km} \lesssim R_D \lesssim 18 \text{ km}$ which is a wider range in comparison with $7 \text{ km} \lesssim R_N \lesssim 11 \text{ km}$ for the NSM sphere. This phenomenon, along with the one related to the mass of spheres results in the less accumulation of DM in comparison with the visible matter.
Fig. 7. Gravitational redshift at the surface of neutron star, $Z_S$, with DM and without DM (normal neutron star).

4.5 Gravitational redshift versus the total mass

We have shown the gravitational redshift at the surface of neutron star with DM and without DM (normal neutron star) in Fig. 7. For a special mass, the gravitational redshift of neutron star with DM is higher compared to the normal neutron star. In most cases, with a certain mass of neutron star, the gravitational redshift increases with the increase in the spin polarization parameter. In addition, the strength of interaction between DM particles affects the gravitational redshift. Fig. 7 indicates that the influence of spin polarizability on the gravitational redshift is more significant for the case with $m_I = 300 \text{ GeV}$, i.e. with lower self-interaction of DM particles. Since the gravitational redshift is obtained using the observational data, it is possible to use our results to estimate the interaction and also polarization of DM.

5 Conclusion

Employing the piecewise polytropic equation of state for the neutron star matter and the equation of state of spin polarized self-interacting dark matter, the properties of neutron stars affected by the spin polarized self-interacting dark matter have been considered. The general relativistic formalism and the spin polarized self-interacting dark matter equation of state have been employed. The increase in the spin polarization parameter of the dark matter results in the more stiffening of the EOS. Our results show that the visible size of the neutron star decreases by increasing the spin polarization of DM. This is while, the size of the dark matter sphere increases with the growth of spin polarization parameter. Because of the stiffer DM EOSs in the cases with more spin polarized DM, the total mass of neutron star, the mass of NSM sphere, and the mass of DM sphere, grow when the spin polarization parameter increases. It has been confirmed that the relations $M - R_N$, $M - R_D$, $M_N - R_N$, and $M_D - R_D$ are influenced by the spin polarization parameter of DM. We have
shown that the behavior of the NSM sphere mass and total mass versus the size of NSM sphere are similar. Moreover, the gravitational redshift of neutron star grows when the spin polarization parameter increases.

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