Imaginary part of the electromagnetic lepton form factors

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Abstract

The charge $F_1(0)$ and the magnetic $F_2(0)$ form factors of heavy charged leptons have been shown in the framework of the perturbation theory to have imaginary part. The imaginary parts of the form factors for muon and tau lepton have been calculated at the two-loop level in the Standard Model. The effects where these imaginary parts could manifest themselves are discussed.

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1 Introduction

Recent high precision experiments to verify the Standard Model of electroweak interactions require, on the side of the theory, higher precision calculation of various physical quantities. The charge and the magnetic form factors of the photon-lepton-lepton vertex belong, in particular, to the class of such quantities. These form factors are the fundamental quantities in the elementary particles physics. Their importance derives from the fact that these quantities can be measured very precisely, and at the same time are calculable from first principles.

Let $\Gamma_\mu(p_1, p_2)$ be the vertex amplitude of the photon-lepton-lepton process. If $u_1, \bar{u}_2$ are the spinors describing the on-shell initial and final lepton states with mass $m$, the most general form of the matrix element is

$$\bar{u}_2 \Gamma_\mu u_1 = -ie \bar{u}_2 \left[ F_1(t) \gamma_\mu + \frac{i}{4m} F_2(t) (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) q^\nu + \frac{1}{m} F_3(t) q^\nu + \cdots \right] u_1, \quad (1)$$

where "\cdots" denote the terms proportional to $\gamma_5$; $q^\mu = p_2^\mu - p_1^\mu$ is the photon momentum; $t = q^2$; $p_1^2 = p_2^2 = -m^2$, the space-time we are working in is the $N$-dimensional Euclidean one. As can be verified by using the Gordon decomposition, the $t$-dependent form factors can be interpreted in a standard way for $t = 0$ and with the lepton on the mass shell, at the same time, i.e., $F_1(0)$ is the electric charge of the lepton, $F_2(0)$ is the static anomalous magnetic moment of the lepton. Using the method described in Ref. [1], $F_1(0)$ and $F_2(0)$ can be directly extracted from the vertex amplitude. Expanding the amplitude $\Gamma_\mu(p, q)$ up to the first order in $q$

$$\Gamma_\mu(p, q) = \Gamma_\mu(p, 0) + q^\nu \frac{\partial}{\partial q^\nu} \Gamma_\mu(p, q)|_{q=0} \equiv V_\mu(p) + q^\nu T_{\nu\mu}(p),$$

we have:

$$F_1(0) = -\frac{1}{4m^2 e} Sp \left[ (i \hat{p} - m) p_\mu V_\mu(p) \right],$$

$$F_2(0) = -\frac{1}{8m (N - 2) (N - 1) e} Sp \left[ (i \hat{p} - m) (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) (i \hat{p} - m) T_{\nu\mu} \right]$$

$$+ \frac{i}{4m^2 (N - 1) e} Sp \left[ (m^2 \gamma_\mu + N \hat{p} p_\mu + im (N - 1) p_\mu) V_\mu(p) \right].$$

Therefore, the calculation of the charge and magnetic form factors of the lepton reduces, after differentiation and contractions with projection operators, to diagrams of the propagator type with external momentum on the lepton mass shell. All the $F_i(0)$ must be real thanks to hermicity of the electromagnetic current. However, this statement is fully correct only for QED. In the framework of the Standard Model all the $F_i(0)$ for unstable leptons have imaginary parts. This paper is aimed at calculation of the imaginary parts of the corresponding form factors, and at discussion of their identification in real experiments.
Our plan is the following: In Sect.2 we consider the toy model as an example and show that the instability of fermions leads to the imaginary parts of the electromagnetic form factors. Sect.3 represents the calculation of the imaginary parts of the electromagnetic form factors of leptons at the two-loop level in the Standard Model. Sect.4 is devoted to discussion of their physical meaning.

2 The toy model

Let us consider a “toy” model where heavy and massless charged spinors (Ψ and E, respectively), the photon \( A_\mu \), and a light neutral scalar field \( \Phi \) are included. The scalar has the Yukawa coupling \( y \) to the heavy and massless spinors. The Lagrangian of this model can be written (in the Euclidean space-time) as

\[
L = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} m^2 \Phi^2 + \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \bar{\Psi} \left( \hat{\partial} + M \right) \Psi + \bar{E} \hat{\partial} E + ie \left( \bar{\Psi} \hat{A} \Psi + \bar{E} \hat{A} E \right) + y\Phi \left( \bar{\Psi} E + \bar{E} \Psi \right),
\]

where \( e \) is the electric charge; \( \xi \) is the gauge parameter. In this model the leptons’ instability displays itself in the electromagnetic form factors even at the one-loop level – which significantly facilitates the calculation.

\[
\gamma \quad \Phi \quad \Psi
\]

\[ \begin{array}{c}
\text{E} \\
\text{E}
\end{array} \]

\( \Psi \quad \Phi \quad \Psi \)

Figure 1: The Yukawa contribution to the electromagnetic form factors of the heavy spinor within the toy model.

The contribution of the diagram given in Fig.1 to the electromagnetic form factors of the heavy spinor can be easily found:

\[
F_1(0) = \frac{y^2}{32\pi^2} \left\{ \frac{1}{\varepsilon} - \ln \left( \frac{m^2}{\mu^2} \right) - \ln \left( 1 - \frac{M^2}{m^2} \right) \right. \\
+ \left. \frac{m^2}{M^2} \left[ 3 - 2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right] + 3 \left( \frac{m^2}{M^2} \right)^2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right\},
\]

\[
F_2(0) = -\frac{y^2}{16\pi^2} \left[ 1 + 2 \frac{m^2}{M^2} + 2 \left( \frac{m^2}{M^2} \right)^2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right],
\]

so that

\[
F_1(0) = \frac{y^2}{32\pi^2} \left\{ \frac{1}{\varepsilon} - \ln \left( \frac{m^2}{\mu^2} \right) - \ln \left( 1 - \frac{M^2}{m^2} \right) \right. \\
+ \left. \frac{m^2}{M^2} \left[ 3 - 2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right] + 3 \left( \frac{m^2}{M^2} \right)^2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right\},
\]

\[
F_2(0) = -\frac{y^2}{16\pi^2} \left[ 1 + 2 \frac{m^2}{M^2} + 2 \left( \frac{m^2}{M^2} \right)^2 \ln \left( 1 - \frac{M^2}{m^2} \right) \right],
\]
\[ ImF_1(0) = -i \frac{y^2}{32\pi} \left[ 1 + 2 \frac{m^2}{M^2} - 3 \left( \frac{m^2}{M^2} \right)^2 \right], \] (4)

\[ ImF_2(0) = -i \frac{y^2}{8\pi} \left( \frac{m^2}{M^2} \right)^2, \] (5)

where the sign of the imaginary part is defined by the "causal" \( i0 \)-prescription, \( \ln(-m^2) = \ln(m^2) + i\pi \). Since we are interested only in the imaginary parts of the form factors, the diagram with the virtual photon is omitted. It is obviously why the imaginary parts of the form factors may appear. The vertex diagram reduces, after differentiation and contractions with projection operators, to the sum over the propagator type diagrams with external momentum on the mass shell of the heavy particle. The last type of diagrams generates imaginary part proportional to the decay width of the heavy particle. The relations between the imaginary part of the form factors and the corresponding decay width of the heavy fermion may be naturally suggested to exist and to be simple

\[ ImF_i(0) = C_i \Gamma, \] (6)

where \( \{C_i\} \) are the constants. This relation can be easily checked. Let us consider the one-loop-particle irreducible two-point function \( \Sigma(\hat{p}, m) \) of the heavy fermion with the massless fermion and the light neutral scalar particle inside the loop. The decay width \( \Gamma \) of the particle is proportional to the imaginary part of the propagator at the point \( \hat{p}^2 = -M_P^2 \), where \( M_P \) is the pole mass of the particle. Solving the equation \( i\hat{p} + m - \Sigma(\hat{p}, m) \), we obtain

\[ \Gamma \sim Im\Sigma(M_P, M_P) = i \frac{y^2}{32\pi} \left( 1 - \frac{m^2}{M^2} \right)^2. \] (7)

The comparison of the Eqs.(4), (5), and (7) shows that the relation (6) does not work in the general case. It is natural to expect that the imaginary parts of the corresponding form factors are related to the decay width in a more complicated nonlinear way.

An additional contribution into the form factors \( F_i(t) \) like \( t \ln(-t) \) appears at small non-zero \( q^2 = t \). In the space-time region \( t > 0 \) that contribution results in imaginary part due to zero threshold with respect to \( t \) (see Fig.1). However, this value is proportional to \( i\pi t \), that is why it disappears at \( t = 0 \).

### 3 The imaginary part of the lepton’s form factors

Now we concentrate on the charge and the magnetic form factors of leptons in the framework of the Standard Model. The imaginary parts of given form factors arise only at the two-loop level. If we restrict ourselves to the perturbation theory only,
Figure 2: The two-loop diagrams contributing to the imaginary part of the electromagnetic form factors of a heavy lepton within the Standard Model.

the light quarks should be considered as fermions with the corresponding masses. The corresponding diagrams are presented in Fig. 2. Since we work in the Feynman gauge, the would-be-Goldstone $\phi$ has the same mass $M_W$ as the charge boson $W$. All the rest dimensional parameters in these diagrams are small in compared with $M_W$. So, the asymptotic expansion method is wholly suitable for calculation of the diagrams under consideration. The rules of purely Euclidean asymptotic expansions are as follows. The expansion is a sum over 'ultraviolet' subgraphs of the diagram. An ultraviolet subgraph must contain all lines with large masses, the points where large external momenta (if any) flow in/or out. The large momenta ought to go only through the ultraviolet subgraph, and obey the momentum conservation law. And last, an ultraviolet subgraph should be one-particle irreducible with respect to lines with small and zero masses, although may consist of several disconnected parts. An ultraviolet subgraph is Taylor-expanded in its small parameters (external momenta and internal masses), and then shrunk to a point and inserted in the numerator of the remaining Feynman integral. The two consecutive expansions are carried out to calculate the diagrams given in Fig. 2. The first of them is the large-mass expansion with respect to the heavy mass $M_W$. The set of the corresponding subgraphs is shown in Fig. 3. This expansion says that only the last subgraph consists of an imaginary part. The large-mass expansion leads to
Figure 3: The structure of large-mass expansion. Bold and thin lines correspond to heavy-mass and light-mass (massless) propagators, respectively. Dashed lines indicate the lines omitted in the original graph to yield the subgraph.

Figure 4: The structure of large-momentum expansion. The notations are the same as in Fig. 3.

\[
F = \sum_{l=0}^{\infty} \left( \frac{1}{M_{\phi}^2} \right)^{l} \sum_{k=0}^{2} \Phi_{l,k}(m_u^2, m_d^2, m^2) \ln \frac{M_{\phi}^2}{\mu^2},
\]

where \( F \) is the initial Feynman integral, \( m_u^2, m_d^2 \) are the fermion masses in the loop, and \( m^2 \) is the mass of an external lepton. \( \Phi_{l,k}(m_u^2, m_d^2, m^2) \) are some complicated functions of their arguments in the general case. The maximum power of logarithm is defined by the highest degree of divergences (ultraviolet, infrared, collinear) in the subgraphs (equals 2 in our cases); \( \mu^2 \) is the subtraction point. As a result of the asymptotic expansion with respect to the heavy mass, the two-loop bubble integrals and the propagator-type integral with the different masses \( (m_u^2, m_d^2) \), with the external momentum \( (p^2 = -m^2) \) and with the reduced number of internal lines arise. The second type of the integrals mentioned can be calculated by using the asymptotic expansion with respect to the large external momentum, which is the particular case of Euclidean expansion. The structure of the asymptotic expansion in this case is given in Fig. 4, so that
\[
\Phi_{t,k}(m_1^2, m_2^2, m^2) = (m^2)^t \sum_{a,b=0}^{\infty} \left( \frac{m_1^2}{m^2} \right)^a \left( \frac{m_2^2}{m^2} \right)^b \sum_{p=0}^{2-k} \left( A_{p}^{ab} \ln^p \frac{m_1^2}{\mu^2} + B_{p}^{ab} \ln^p \frac{m_2^2}{\mu^2} + C_{p}^{ab} \ln^p \frac{m^2}{\mu^2} \right),
\]

where \( \{A_{p}^{ab}, B_{p}^{ab}, C_{p}^{ab}\} \) are the numbers. All the calculations are performed by means of the package TLAMM \[3\]. Since the imaginary part of the corresponding form factors does not include divergences, no additional renormalization is required.

4 Discussion and conclusions

The imaginary parts of the charge and the magnetic form factors of leptons in the leading order of the Standard Model have the following form:

\[
\text{Im} F_1(0) = i \frac{G_F^2 m^4}{8\pi^3} N_c \sum_f \left[ (Q_d - Q_u) \left( \frac{5}{48} - \frac{r_1^2 + r_2^2}{2} + O(r_k^4 \ln r_k) \right) + \frac{m^2}{M_W^2} \left( Q_d - Q_u \right) \left( \frac{1}{30} - \frac{r_1^2 + r_2^2}{6} \right) - \frac{13}{240} + \frac{17}{48} (r_1^2 + r_2^2) + O(r_k^4 \ln r_k) \right] + O(m^4/M_W^4),
\]

\[
\text{Im} F_2(0) = i \frac{G_F^2 m^4}{8\pi^3} N_c \sum_f \left[ -\frac{Q_d}{12} + \frac{r_1^2 + r_2^2}{3} (Q_d + 2Q_u) + O(r_k^4 \ln r_k) + \frac{m^2}{M_W^2} \left( \frac{Q_d - Q_u}{40} - \frac{1}{40} + \frac{r_1^2 + r_2^2}{4} + Q_d \left( -\frac{5}{36} r_1^2 + \frac{r_2^2}{2} - \frac{r_1^2}{3} \ln r_1 \right) + Q_u \left( \frac{r_1^2}{6} - \frac{17}{12} r_2^2 \ln r_2 \right) + O(r_k^4 \ln r_k) \right] + O(m^4/M_W^4),
\]

where all the fermions with \( T_3 = 1/2 \) are called as u-fermions with the electric charge \( Q_u = 2/3 \) (in units of the positron charge) and with the mass \( m_u \). Correspondingly, the fermions with \( T_3 = -1/2 \) are d-fermions with the charge \( Q_d = -1/3 \) and the mass \( m_d \); \( m \) is the mass of external lepton; \( \sum_f \) is the sum over all the fermions with \( m_f^2 < m^2 \); \( r_1 = m_d/m \) and \( r_2 = m_u/m \). To extract the contribution of lepton with the mass \( m_l \), we should accept that \( m_u = 0, Q_u = 0, Q_d = -1, m_d = m_l \); \( N_c \) is the color factor which is equal to 3 for quarks and 1 for leptons, respectively; \( G_F/\sqrt{2} = g^2/8/M_W^2 \), and the corresponding Cabibbo-Kobayashi-Maskawa matrix is equal to \( I \).

When considering light quarks as internal fermions it should be pointed out that the perturbative quantum field theory leads to incorrect results. In particular, the muon has no hadronic mode of decay. So, at evaluation of the imaginary part of the muon electromagnetic form factors we take into account only the lepton contribution:
The tau-lepton is the only presently known lepton massive enough to decay into hadrons \cite{6}. Taking the decay mode \( \tau^- \rightarrow \nu_{\tau} d \bar{u} \) into account, we obtain the following evaluation for the imaginary parts of the corresponding form factors of the tau-lepton:

\[
Im F_1^\tau(0) \geq -i \frac{25}{384} \frac{G_{\text{F}}^2 m_{\tau}^4}{\pi^3}, \quad Im F_2^\tau(0) \geq -i \frac{G_{\text{F}}^2 m_{\tau}^4}{24\pi^3}. \tag{10}
\]

The leading, over the mass, contribution into imaginary parts of electromagnetic form factors can be found in the Fermi theory of electromagnetic interaction. Since we are interested in the renormalization of the corresponding form factors in the on-shell scheme, we choose the renormalizable theory GWS for the calculation.

![Figure 5: The diagrams contributing to the imaginary part of the lepton’s wave-function renormalization constant in the Standard Model.](image-url)

It is worthy to discuss the physical meaning of the imaginary parts of the corresponding form factors. For this purpose let us first consider the charge form factor. In QED the electric charge of electron is usually fixed in the Thompson limit of Compton scattering. This definition ensures that \( e \) indeed represents the electric charge which describes the interaction between electron and electromagnetic fields in the classic electrodynamics. In the on-shell renormalization scheme \cite{4} the charge of electron is formally defined as the coupling of a photon with zero momentum to an on-shell lepton. This statement is commonly assumed to be valid also in the Standard Model, and has been recently generalized to the case of arbitrary charged particle \cite{5}. There are reasons to suppose that this statement works also for any charged lepton at arbitrary order of the perturbation theory. This condition is usually written in the on-shell scheme as follows:

\[
F_1^{\text{ren}}(0) = 1, \tag{11}
\]

where

\[
\Gamma_\mu^{\text{ren}} = -i \left[ F_1^{\text{ren}}(t) \gamma_\mu + \frac{i}{4m} F_2^{\text{ren}}(t) (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) q^\nu \right] = \sqrt{Z_\gamma} \gamma_0 \sqrt{Z_f} \gamma_0 \Gamma_\mu \sqrt{Z_f}. \]

Here \( Z_f \) and \( Z_\gamma \) are the renormalization constants of the on-shell photon wave functions, respectively; \( \Gamma_\mu \) is the bare vertex photon-lepton-lepton where \( \gamma - Z \).
mixing is taken into account. At the two-loop level the wave function renormalization constant $2Z_f = Z_R + Z_L + \gamma_5(Z_L - Z_R)$ is the complex value. The corresponding finite imaginary part arises from the diagrams depicted in Fig. 3. As was shown in Refs. [7], the only hermitian (real) part of the wave function renormalization constant contributes to physical observables. The antihermitian (imaginary) part is related to transformation of the matrices of fermion fields from the weak interaction eigenstates to the mass eigenstates. The renormalization conditions, which fix these rotation matrices, are independent of all the other on-shell renormalization conditions. It is noteworthy to say also that physical observables cannot be constructed from these matrices in the lepton sector of the Standard Model (in the quark sector, the Cabibbo-Kobayashi-Maskawa matrix is such an observable). Therefore, the finite antihermitian (imaginary) part of the lepton wave function renormalization constant can be omitted from the renormalization of the charge and the magnetic form factors. So, at the two-loop level the only $F_1(0)$ gives an imaginary contribution in $\Gamma_{\text{ren}}$. The definition (11) allows one to find the relation between the bare charge of a fermion and the physical charge $e$. In the Standard Model at the one-loop level, this relation does not depend on the on-shell lepton – which in turn means ”charge universality” [8]. We understand ”charge universality” as the fact that the relation between the bare charge and the physical one is independent of the on-shell lepton. The presence of imaginary part in the electromagnetic form factor $F_1^{\text{ren}}(0)$ leads to the breakdown of ”charge universality”. Certainly, this problem might be related with inconsistency of treatment of instable particles in the standard perturbative quantum field theory [9]. In the framework of such a self-consistent theory (if it will be created), the $F_1^{\text{ren}}(0)$ of a heavy lepton is expected not to consist of imaginary part at all, and the ”charge universality” principle will be restored. Although there is still no such a self-consistent theory, we should find a way to treat with imaginary parts arisen. Since the amplitude for fermion scattering from an electric field is proportional to the $F_1^{\text{ren}}(0)$, it is naturally to use the following condition:

\[
|F_1^{\text{ren}}(0)| = 1, \tag{12}
\]

instead of (11). If the renormalization condition (12) is taken into account, it is then easy to show that the two-loop imaginary part is equivalent to the finite four-loop correction into the relation between the bare and the physical charge of a lepton. Therefore, the ”charge universality” saves, up to the four-loop level.

Now, let us concentrate on the magnetic form factor. It is commonly assumed that $\frac{e_2}{2} = F_2(0)$. However, this statement is correct only for stable particles. So, the anomalous magnetic moment of instable particle is not defined good enough, both from theoretical point of view and experimental one (it is difficult to measure this value due to the finite life time of such a particle). When analyzing a fermion scattering from a static vector field we obtain that the matrix element is proportional to the sum $F_1(0) + F_2(0)$. So, we should suppose that the following relation can be used for the numeric estimation of the magnetic moment for unstable particle:

\[
|F_1(0) + F_2(0)| = \frac{g}{2}, \tag{13}
\]
where $g$ is the Landé-factor. If Eq. (12) is taken into consideration, we obtain that the imaginary parts of the charge and the magnetic form factors are equivalent to the four-loop contribution to the anomalous magnetic moment of lepton, and can be omitted in the two-loop results [10].

The presence of imaginary parts in electromagnetic form factors of heavy leptons is a subject of only theoretical studies at the present time, because their influence on electroweak processes is extremely small [11]. In fact, their effect is equivalent to four-loop radiative corrections – which is beyond the modern experimental precision.

In the Standard Model it is commonly believed that the charged leptons are identical in all respect, excepting for their masses and their distinct and conserved lepton numbers. This statement turns out to be incorrect at the two-loop level where the instability of heavy leptons results in the imaginary parts of the electromagnetic form factors. The problem of how the particles instability should be correctly taken into account at calculation of physical quantities in the Standard Model is still open [14]. Therefore, additional relations between the arisen imaginary parts and physical observables should be found (see, for example, [13]). This paper deals with the calculation of the imaginary parts of the two-loop electromagnetic form factors of charged leptons in the Standard Model. The leading part of such contributions comes from the diagrams with $W$-exchange. The contribution of the rest diagrams is generated by additional powers of $(m/M_W)^2$ in accordance with the Feynman’s rules (note that imaginary part enters all the diagrams in $\mathbf{2}$). As was demonstrated, the imaginary parts of the form factors $F_i(0)$ are related with the leptons instability, and there are no trivial relations between the corresponding imaginary parts and the decay width. To take the imaginary parts of the corresponding form factors into account, the conditions (12) and (13) have been suggested as a generalization of the standard relations between the electric (magnetic) form factors and the electric charge (the anomalous magnetic moment) of a lepton, respectively.

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