TYPE I PLANET MIGRATION IN NEARLY LAMINAR DISKS

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ABSTRACT

We describe two-dimensional hydrodynamic simulations of the migration of low-mass planets (≤ 30 M⊕) in nearly laminar disks (viscosity parameter α < 10⁻³) over timescales of several thousand orbit periods. We consider disk masses of 1, 2, and 5 times the minimum mass solar nebula, disk thickness parameters of H/r = 0.035 and 0.05, and a variety of α values and planet masses. Disk self-gravity is fully included. Previous analytic work has suggested that Type I planet migration can be halted in disks of sufficiently low turbulent viscosity, for α ∼ 10⁻⁴. The halting is due to a feedback effect of breaking density waves that results in a slight mass redistribution and consequently an increased outward torque contribution. The simulations confirm the existence of a critical mass (Mₗ.cr ∼ 10 M⊕) beyond which migration halts in nearly laminar disks. For α ≳ 10⁻³, density feedback effects are washed out and Type I migration persists. The critical masses are in good agreement with the analytic model of Rafikov. In addition, for α ≲ 10⁻⁴ steep density gradients produce a vortex instability, resulting in a small time-varying eccentricity in the planet’s orbit and a slight outward migration. Migration in nearly laminar disks may be sufficiently slow to reconcile the timescales of migration theory with those of giant planet formation in the core accretion model.

Key words: accretion, accretion disks – hydrodynamics – methods: numerical – planetary systems: formation – planetary systems: protoplanetary disks – solar system: formation

1. INTRODUCTION

The standard theory of Type I (low planet mass) migration presents a challenge for understanding planet formation. According to the core accretion model, the growth time from planetesimals to gas giant planets is dominated by a phase that occurs when a newly formed solid core with mass ∼ 10 M⊕ accretes gas (Pollack et al. 1996; Hubickyj et al. 2005). The duration of this slow phase, ∼ 10⁶ yr, is determined by a thermal bottleneck that prevents the gaseous envelope from contracting until it achieves sufficient mass. On the other hand, the standard theory of Type I migration (Tanaka et al. 2002) predicts that planets in the slow growth phase would migrate into the disk center in ∼ 10⁵ yr for the minimum solar mass nebula. These migration timescales have been confirmed by multidimensional hydrodynamical simulations (Bate et al. 2003; D’Angelo & Lubow 2008). Recently, Ida & Lin (2008) and Schlaufman et al. (2008) have studied the effect of the ice line on the disk surface density profile and consequently on the Type I migration. They obtained much better agreement with the observed extrasolar planet mass–semimajor axis distribution if the Type I migration is reduced by an order of magnitude from the linear theory values.

The shortness of the standard migration timescale has motivated investigations of possible effects to slow or even reverse migration. These include magnetic fields (Terquem 2003), magnetorotational instability (MRI) turbulent fluctuations (Nelson & Papaloizou 2004), and density traps (Menou & Goodman 2004). Recently, protoplanet migration in the non-isothermal disks has been investigated (Paardekooper & Mellema 2006; Baruteau & Masset 2008; Paardekooper & Papaloizou 2008; Kley & Crida 2008). These simulations show indications of slowing migration due to co-orbital torques for certain ranges of gas diffusivity and turbulent viscosity. In this Letter, we discuss another mechanism that naturally occurs when the disk turbulent viscosity is sufficiently small.

The Tanaka et al. (2002) Type I migration rates were derived under the assumption that the disk density distribution is unaffected by the presence of the planet. Numerical simulations commonly adopt turbulent viscosity parameter values α ≳ 10⁻³. Such values are suggested by considering the observationally inferred disk masses and accretion rates for T Tauri stars (e.g., Hartmann et al. 1998). With such α values, turbulent diffusion suppresses disk disturbances for planets of mass less than 0.1 M⊕. The numerical simulations then satisfy the Tanaka et al. (2002) assumptions and yield migration rates that are in close agreement.

The various models of planet formation by core accretion typically involve lower α values, α ≲ 10⁻⁵ (see Cuzzi & Weidenschilling 2006), which we refer to as nearly laminar values. To form planetesimals via gravitational instability from small dust particles (Safronov 1969; Goldreich & Ward 1973) requires the dust layer to be very thin ∼ 10⁻⁴ H, suggesting α ≪ 10⁻⁴. For the solids to be dynamically decoupled from the gas requires a dust layer disk of thickness ≲ 10⁻² H, again suggesting nearly laminar conditions. Cuzzi & Weidenschilling (2006) estimate that α ≲ 2 x 10⁻⁴ in order that meter size solids avoid destructive effects of collisions due to turbulent motions.

In the planet-formation regions of disks, considerations of the MRI (Balbus & Hawley 1991) suggest that the disk may be unstable only in surface layers, due to the low levels of ionization below these layers (Gammie 1996). A major uncertainty is the abundance of small grains that can suppress the instability. The disk may be nearly laminar for the purposes of planet formation. However, surface layer turbulent fluctuations may propagate disturbances to the disk midplane. They may provide some effective turbulence in that region as well (Fleming & Stone 2003; Turner & Sano 2008).

In nearly laminar disks, density waves launched by a planet at various Lindblad resonances can redistribute disk mass as they damp. The disk turbulent viscosity needs to be sufficiently small for the density perturbation to not diffuse away. The
2. NUMERICAL METHOD AND INITIAL SETUP

We assume that the protoplanetary disk is thin and can be described by the two-dimensional isothermal Navier-Stokes equations in a cylindrical \([r, \phi]\) plane centered on the star with vertically integrated quantities. The differential equations are the same as given in Kley (1999). Simulations are carried out using a hydro code developed at Los Alamos (Li et al. 2005). We also use the local comoving angular sweep as proposed in the FARGO scheme of Masset (2000) and modified in Li et al. (2001). The equations of motion of the planets are the same as given in D’Angelo et al. (2005), which we adapted to the polar coordinates with a fourth-order Runge–Kutta solver. During each hydrodynamics time step, the motion of the planet is divided into several substeps so that the planet always moves within 0.05 local grid spacing, \(\delta = [(\Delta r)^2 + (\Delta \phi)^2]^{1/2}\), in one substep. The disk gravitational force on the planet is assumed to evolve linearly with time between two hydrodynamics time steps. Furthermore, we have implemented a full two-dimensional self-gravity solver on our uniform disk grid (Li et al. 2008). This solver uses a mode cut-off strategy and combines fast Fourier transform in the azimuthal direction and direct summation in the radial direction. The algorithm is sufficiently fast so that the self-gravity solver costs less than 10% of the total computation cost in each run. This code has been extensively tested on a number of problems. With our pseudo-three-dimensional treatment (see Li et al. 2005 for details) and a small (a few grid size) softening distance in the planet’s potential, migration rates from simulations with sufficient viscosity (dimensionless kinematic viscosity \(\nu \simeq 10^{-6}\)) agree well (within a few percent) with the three-dimensional linear theory results of Tanaka et al. (2002). As the softening distance increases to \(r_H\), the migration rates from such simulations are \(\sim 30\%\) slower than the three-dimensional linear theory result. The runs presented here use \(r_H\) as the softening distance.

The two-dimensional disk is modeled between 0 and 4. The planet is initially located at \(r = 1\), which corresponds to a physical distance of Jupiter’s orbital radius (5.2 AU), and orbits about a 1\(M_\odot\) star. The unit of time is the initial orbit period \(P\) of the planet, which is about 12 yr. A corotating frame that rotates with the initial angular velocity of the planet is used. The coordinate plane is centered on the central star \((r, \phi) = (0, 0)\) (acceleration due to frame rotation is also included, the so-called indirect term). The disk is assumed to be isothermal throughout the simulated region, having a constant sound speed \(c_s\). The dimensionless disk thickness is scaled by the initial orbital radius of the planet \(h = c_s/v_\phi(r = 1)\), where \(v_\phi\) is the Keplerian velocity. We consider values \(h = 0.035\) or 0.05 in the simulations. We have also made runs using a constant disk aspect ratio \(H/r\), and found that our main conclusions are not changed.

Our numerical schemes require two ghost cells in the radial direction (the angular direction is periodic). Holding these ghost cells at the initial steady-state values produced the weakest boundary reflections among all boundary conditions we investigated. We choose an initial surface density profile normalized to the minimum mass solar nebular model (Hayashi 1981) as \(\Sigma(r) = 152 f (r/5\text{ AU})^{-3/2} \text{ g cm}^{-2}\), where \(f\) ranges from 1 to 5 in our simulations. The initial rotational profile of the disk is calculated so that the disk will be in equilibrium with the disk self-gravity and pressure (without the planet). The mass ratio between the planet and the central star is \(\mu = M_p/M_\star\), which ranges from \(3 \times 10^{-6}\) to \(10^{-4}\). The planet’s Hill (Roche) radius is \(r_H = r_p (\mu/3)^{1/3}\). The dimensionless kinematic viscosity \(\nu\) (normalized by \(\Omega^2 r\) at the planet’s initial orbital radius) is taken to be spatially constant and ranges between 0 and 10\(^{-5}\). For \(h = 0.05\), the effective Shakura and Sunyaev \(\alpha = \nu/h^2\) at the initial planet radius ranges between 0 and 4 \times 10^{-3}. We have performed various tests to show that when \(\nu = 0\) the effective numerical viscosity in our simulations is \(\nu < 10^{-9}\) or \(\alpha < 4 \times 10^{-7}\). We typically evolve the disk without the planet for 10\(P\). Subsequently, the planet’s gravitational potential is gradually “turned-on” over a 30- orbit period, allowing the disk to respond to the planet potential gradually. Note that the time shown in all the figures in this Letter starts at the time of the planet release. Runs are made typically using a radial and azimuthal grid of \((nr \times n_\phi) = 800 \times 3200\), though we have used higher resolution to ensure convergence on some runs. Simulations typically last several thousand orbit periods at \(r = 1\).

3. RESULTS

Figure 1 shows the influence of the imposed disk viscosity on the migration for a planet with \(\mu = 3 \times 10^{-5}\) or 10\(M_\odot\), \(h = 0.035\), and \(f = 5\). For relatively large viscosity \((\nu = 10^{-6}, \alpha = 8 \times 10^{-4})\), the migration rates agree well with the Type I rates given by Tanaka et al. (2002), as discussed above. At early times the migration rates are largely independent of the disk viscosity. As viscosity decreases, after about 100\(P\), the migration is drastically slowed or completely halted. The rapid oscillations with modest amplitude at \(r \sim 800P\) are due to the excitation of vortices from a secondary instability (Koller et al. 2003; Li et al. 2005; see also Li et al. 2001), which will be a subject for future studies. Figure 2 reveals the reason for the slow-down. A partial gap in the disk around the planet has formed at \(t = 500P\) and the density profile deviates significantly from the initial power law. The asymmetry in the density distribution interior and exterior to the planet has reduced the contribution from the outer Lindblad torque so that the net torque is approximately zero. We verified that the slow-down shown in Figure 1 is largely caused by the density redistribution. In principle, the torque distributions per unit disk mass \((dT/dM(r))\), see Figure 1 in D’Angelo & Lubow 2008) could also be affected. But we find these changes only slightly modify (at the few percent level) the net migration torque.

In the case of \(\nu = 10^{-6}\) in Figure 2, the profile is qualitatively similar to the expectations of steady-state theory (Ward 1997; Rafikov 2002). In particular, there is a density peak at \(r < r_p\) and a trough at \(r > r_p\). The torque that the disk exerts on the planet is localized to a region of a few times the disk thickness or about \(\pm 4r_H\) from radius \(r_p\). In Figure 2 for \(\nu = 10^{-6}\), we
we determine apply over the time range of several thousand single value above which the migration will be halted. The reduction in migration rates is gradual, so it is difficult to define a

distribution (see Figure 2) that ensures a much reduced (or zero) net total torque.

4. DISCUSSION

The local wave damping model of Ward & Hourigan (1989) suggests critical planet masses of $M_{cr} \sim \Sigma_p^2 h^3$, which evaluates to $0.006 f M_\oplus$ and $0.02 f M_\oplus$ for $h = 0.035$ and 0.5, respectively. These values differ from Table 1 by a factor of more than 100. However, the scaling of the critical mass with disk thickness is close to $h^2$ as given by this theory. The scaling of $M_{cr}$ with surface density in Table 1 is, however, weaker than that suggested by this theory. In another local wave damping model, Ward (1997) suggest that $M_{cr} \sim 0.2 \Sigma_p^2 h$, which evaluates to $1.1 f M_\oplus$ and $1.6 f M_\oplus$ for $h = 0.035$ and 0.5, respectively. Although these values are numerically closer to the simulation values in Table 1, the predicted linear scaling in both $f$ and $h$ does not agree with the trends in Table 1. The analytic model of Rafikov (2002) includes the effects of nonlocal damping by means of shocks. In that case, the critical masses are given by

$$M_{cr} = \frac{2c_s^3}{3\Omega G} \min[5.2Q^{-5/7}, 3.8(Q/h)^{-5/13}],$$

(1)

where $Q = \Omega c_s/(\pi G \Sigma)$. All the simulated cases correspond to the strong feedback branch of $M_{cr}$ that is given by the second argument of the “min” function. Values for these critical masses are also given in Table 1. We see that the agreement between the simulation and theory is quite good.

The critical masses are in the range of the core masses during the slow phase of gas accretion in the core accretion model of planet formation. This result suggests that planet migration might not be a limiting factor in planet formation. The phase of run-away mass accretion follows the slow evolution phase. Previous studies suggest that run-away mass accretion
Although this picture is suggestive of a resolution of the migration problem, the longer term evolution of nearly laminar disk–planet systems requires further exploration. The slowly migrating planet will continue to create a deeper gap over time. The steepening density gradients should lead to the vortex instability (Koller et al. 2003; Li et al. 2005). The consequences of the vortex instability should be explored.

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Figure 3. Migration history for several planet masses in nearly inviscid ($\nu = 0$) disks. As the planet mass increases, its migration transitions from the Type I to being much slower. The horizontal axis is the planet orbital radius in units of the initial orbital radius. The Planet Orbital Radius

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