Charging mechanisms of cloud particles in view of fractal medium

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Abstract. The paper proposes a model for charge dynamics on drops and ice particles in a unipolarly charged region of the cloud. The condensation, sublimation and coagulation mechanisms for drops and ice particles formation are considered in view of the fractal medium. Analytical solutions are presented and the results are compared with the results for the classical model. The peculiarity of the mechanisms of the charge increase in the cloud particles are discussed.

Introduction
A number of scientists have been engaged in the study of cloud charged particles. Among them Russian scientists Imyanitov I.M., Muchnik V.M., Frenkel Ya.I., Solov'ev V.A., Sergieva A.P., Wigand A., Krasnogorskaya N.V. and among their foreign colleagues are Aderaks O. Yu., Barakan N. B., Phillips B., Kintser J., Gunn R., Simpson J., Chalmers I., Workman E., Reynolds S., Weikman H., Kampe H. and et al.

Analyzing the current situation in this field and basing on the results obtained by Mareev E. A., Iudin D. I., Adzhiev A. Kh., Shapovalov A. V. we can argue that cloud simulation in view of electrical processes have been successfully developing in Russia till now [1-5].

As it has been find out during investigation of thunderclouds one of the major factors affecting various processes inside the cloud is electrically charged cloud particles. As is known, drops, hailstones, aerosols, cosmic dust, etc. have electric charges [6]. This fact results in changes of boundary conditions in electrodynamics of the cloud processes. The issue, therefore, relating to the charging of the cloud particles is very much to the point.

Another equally important element in the study of charged cloud particles is the fractality nature of cloud environment since clouds have fractal structure, i.e. self-affine structure. The clouds can be formed using fractal clusters.

Rice F. and Waldfogel A. were the first who noted in [7] that the larger portions of clouds had a nontrivial fractal structure. For the study of intra-cloud discharges a fractal-dynamic approach is employed [2], which in whole is expected to significantly develop cloud physics whose one of the extensively discussing subjects is the genesis of a thunderstorm foci [1].

The method of fractional integro-differentiation is widely used for investigation of various geological processes taking into account fractal medium [8, 9] allowing to do without numerical solution to complex systems. Currently one may note that it is not possible to describe processes inside the clouds without fractal definition.
This paper presents theoretical study for the charge dynamics on the cloud particles (drop, ice particle) considering fractality of a thunderstorm cloud and using fractional integro-differentiation apparatus.

Mathematical modeling for drops and ice particles charge dynamics

It is known that clouds are formed of the charged air and though the charges are the same their electric potential is different: it is less in clouds consisting of small droplets than in clouds with larger size drops resulted from fast condensation [10]. That is, as drops grow the charge is also increases. The opinions on signs of the charge on the droplets are contradictory and not the same.

As is known colloidal particles always acquire negative or positive charge; a compensating charge with opposite sign remains in the air. Cloud droplets and ice particles contained in a cloudy environment represent colloidal systems where drops and ice are the colloidal particles while the cloudy environment is a dispersion medium. Consequently, cloud drops and ice particles always acquire a certain charge due to the electrical double layer at the particle/air interface.

Ya. I. Frenkel associated the phenomenon of selective ion accumulation with the fact that water is polar liquid and an electrical double layer forms on its surface. The preferential orientation of the water molecules is observed with negative ends outward. Electric potential jump in this layer equals 0.3 V. Negative ions penetrate through the electrical double layer easier than positive ones. At equal ion concentrations of opposite signs the negative ions predominant capturing occurs until the jump in zeta potential is compensated by Coulomb field of the drop. If the cloud drops are in a weakly ionized air, the zeta potential is positive.

Basing on the Temkin’s experimental measurements Frenkel argue that it equals 0.25 V; while A. M. Frumkin and his colleagues [11] consider that the most likely positive value is around 0.1-0.2 V. Under these conditions, cloud drops acquire negative charges (One elementary charge is equilibrium at 5.10-7 cm, and many scientists had investigated this matter ignoring the fractal nature of the cloudy environment.

Fractal medium is a fractal material distributed in the space. Its mass dimension is less than the mass dimension of the space being filled. The acknowledgement of the fractality of the cloud environment fundamentally replaces the equation of motion for a charged particle with the fractional differential equations of motion of a charged particle.

Let us consider the growth of the charged cloud particle assuming that the process takes place in the unipolarly charged region of the cloud. Then the process of particles charging is defined by formula [12]

$$Q = \varphi r,$$

(1)

where \(Q\) is the particle charge, \(\varphi\) is zeta potential, \(r\) is the particle radius, may considered acceptable.

There are two mechanisms that may account for the growth of the cloud particles: condensation and coagulation for drops and sublimation and coagulation for ice particles [13].

We can write for drops the following expression

$$\frac{dR_d}{dt} = \left(\frac{dR_d}{dt}\right)_c + \left(\frac{dR_d}{dt}\right)_{ca},$$

(2)

where \(R_d\) is the drop size, \(\left(\frac{dR_d}{dt}\right)_c\) is the droplet growth by condensation, \(\left(\frac{dR_d}{dt}\right)_{ca}\) is the growth of ice particles by coagulation

$$\frac{dR_i}{dt} = \left(\frac{dR_i}{dt}\right)_s + \left(\frac{dR_i}{dt}\right)_{coa},$$

(3)
where $R_i$ is the size of the ice particle, $\left(\frac{dR_i}{dt}\right)_s$ is the growth of ice particles by sublimation, $\left(\frac{dR_i}{dt}\right)_{ca}$ is the growth of ice particles by coagulation.

Then changes in the charge resulted from particle size increase (2) and (3) in view of (1) can be written for droplet:

$$ \frac{dQ_d}{dt} = \varphi \left[ \left(\frac{dR_d}{dt}\right)_s + \left(\frac{dR_d}{dt}\right)_{ca} \right], \quad (4) $$

where $Q_d$ is the charge on the drop, for the ice particles:

$$ \frac{dQ_i}{dt} = \varphi \left[ \left(\frac{dR_i}{dt}\right)_s + \left(\frac{dR_i}{dt}\right)_{ca} \right], \quad (5) $$

where $Q_i$ is the charge on the ice particle.

Basing on cloud fractality and employing the Caputo derivative, $\mathcal{D}_a^\alpha u(t) = \text{sign}^n (a-t) D_{at}^{a-n} \frac{\partial^n u(t)}{\partial t^n}, \quad n-1 < \alpha \leq n, \quad n \in \mathbb{N},$ where $D_{at}^{a-n}$ is Riemann-Liouville fractional integro-differentiation operator, which is defined as follows:

$$ D_{at}^{a-n} u(t) = \begin{cases} \frac{\text{sign}(t-a) u(s)}{\Gamma (-\alpha)} \int_s^{t-a} \frac{ds}{(t-s)^{\alpha+1}}, & \alpha < 0, \\ u(t), & \alpha = 0, \\ \text{sign}^n (t-a) \frac{\partial^n}{\partial t^n} D_{at}^{a-n} u(t), & n-1 < \alpha \leq n, n \in \mathbb{N}, \end{cases} $$

where $\Gamma(z)$ is Euler’s Gamma function, $\alpha$ is the order of fractional integro-differentiation, as well as the concept for the effective rate for change in a certain physical quantity $Q$ defined as in [14-16]

$$ \left\langle \frac{dQ(t)}{dt} \right\rangle = \frac{1}{\tau} D_{\tau}^{\alpha-1} \frac{dQ(t)}{dt} = \frac{1}{\tau} \mathcal{D}_0^\alpha Q(t), \quad 0 < \alpha < 1, \quad (6) $$

equations (4) and (5) cast into the form for drops

$$ \mathcal{D}_0^\alpha Q_d(t) = \varphi \tau \left[ \left(\frac{dR_d}{dt}\right)_s + \left(\frac{dR_d}{dt}\right)_{ca} \right], \quad 0 < \alpha < 1, \quad (7) $$

for ice-particles

$$ \mathcal{D}_0^\alpha Q_i(t) = \varphi \tau \left[ \left(\frac{dR_i}{dt}\right)_s + \left(\frac{dR_i}{dt}\right)_{ca} \right], \quad 0 < \alpha < 1. \quad (8) $$

As is known condensation growth of droplets is determined by the formula

$$ \left(\frac{dR_d}{dt}\right)_s = \frac{\varepsilon D}{R_d}, \quad (9) $$
where $\varepsilon$ is a dimensionless absolute supersaturation, $D$ is a diffusion coefficient of water vapor in air and growth by coagulation

$$\left( \frac{dR_r}{dt} \right)_{ca} = \int_0^r W R_d dr,$$  \hspace{1cm} (10)

where $W = E \cdot s \cdot \Delta v$ is the probability of droplet collision, $E$ is the impact coefficient, $s$ is the effective impact area, $n$ is a number of particles having a radius $r$ per unit volume, $\Delta v$ is the fall velocity of the particle.

Then taking into account (9) and (10), уравнение equation (7) casts into the form

$$\frac{dQ_d}{dt} = Q_d \left[ \frac{\varepsilon D}{R_d^2} + W R_d \right], \hspace{1cm} 0 < \alpha < 1,$$  \hspace{1cm} (11)

Consider the Cauchy problem for equation (11) with the initial condition

$$Q_d (0) = Q_{d,0}.$$  \hspace{1cm} (12)

The solution to problem (11) - (12) is [9]

$$\frac{dQ_d}{dt} = D_{0,x}^{(0)} Q_d (t) - \sum_{k=1}^{n} \frac{t^{k-\alpha-1}}{\Gamma (k-\alpha)} Q_d^{(k-1)} (0),$$

$$D_{0,x}^{(0)} Q_d (t) - \beta Q_d (t) = \sum_{k=1}^{n} \frac{t^{k-\alpha-1}}{\Gamma (k-\alpha)} Q_d^{(k-1)} (0) f (t) \equiv F (t),$$

$$Q_d (t) = F * t^{\alpha-1} E_{\alpha,1} (\beta t^\alpha) + \sum_{k=1}^{n} t^{k-\alpha-1} \frac{t^{\alpha-1}}{\Gamma (k-\alpha)} E_{\alpha,\alpha-k+1} (\beta t^\alpha) =$$

$$= f * t^{\alpha-1} E_{\alpha,1} (\beta t^\alpha) + \sum_{k=1}^{\infty} Q_d^{(k-1)} (0) \frac{t^{k-\alpha-1}}{\Gamma (k-\alpha)} t^{\alpha-1} E_{\alpha,\alpha} (\beta t^\alpha) = Q_{d,0} E_{\alpha,1} (\beta t^\alpha), \hspace{1cm} 0 < \alpha < 1,$$ \hspace{1cm} (13)

where $E_{\alpha,1} (\beta t^\alpha) = \sum_{k=0}^{\infty} \frac{\beta^k t^{k\alpha}}{\Gamma (ak+1)}$, is a Mittag – Leffler type function $\beta = \tau \left[ \frac{\varepsilon D}{R_d^2} + W R_d \right].$

Equation (13) describes the droplets charge dynamics in view of the fractality of the cloud environment.

For the ice particles growth sublimation replaces with condensation. The particle growth by sublimation and condensation differ only by supersaturation values and substance density.

Let us write the formula characterizing the ice particles growth by sublimation

$$\frac{dR_r}{dt} = \frac{3c_i f B}{R_i^2 \rho},$$  \hspace{1cm} (14)

where $f$ is a wind multiplier, $c$ is electric capacity, $B$ is B. J. Mason’s value, $R_i$ is the ice-particle size, $\rho$ is the ice density.

The coagulation mechanism for coarsening of ice particles and droplets growth is basically the same, i.e.

$$\left( \frac{dR_r}{dt} \right)_{ca} = \int_0^r W R_d dr,$$  \hspace{1cm} (15)
Bearing in mind (14) and (15) rewrite the equation (8) as following
\[ \partial_{0}^{\alpha}Q_{i}(t) = Q_{i}\tau\left[\frac{3cfB}{R^{2}\rho} + WR_{i}\right], \quad 0 < \alpha < 1, \quad (16) \]

Like in the case with drops let us consider the Cauchy problem for equation (16) with the initial condition
\[ Q_{i}(0) = Q_{i0}. \quad (17) \]

The solution to (16)-(17) casts into the form
\[ Q_{i} = Q_{i0}E_{\alpha,1}(\gamma t^{2}), \quad 0 < \alpha < 1. \quad (18) \]

where \( E_{\alpha,1}(\gamma t^{2}) = \sum_{k=0}^{\infty} \frac{\gamma^{k}t^{k\alpha}}{\Gamma(\alpha k + 1)} \) is a Mittag–Leffler type function, \( \gamma = \tau\left[\frac{3cfB}{R^{2}\rho} + WR_{i}\right] \).

Equation (18) describes the charge dynamics on the ice particles with the fractality of the cloud environment in view.

The main results of the model calculations

The diffusion coefficient \( D \) for the droplet in all performed calculations is constant and equal to 0.2 \( \text{sm}^{2}\text{s}^{-1} \). This holds for heights ranging from 100 to 1000 m above a lower cloud boundary. For higher altitudes use formula [13]
\[ D = D_{0}\left(\frac{T_{w}}{T_{0}}\right)^{1.75}\left(\frac{P_{0}}{P}\right), \]

where \( D_{0} \) is the diffusion coefficient value as \( T_{0} = 273K \) and \( P_{0} = 1 \text{ atm} \).

Calculation of the charge increase on the droplet have been carried out as \( \Delta v = 3, 5, 7, 10 \text{ m/s} \), the drops and ice particle radius is the same and equal to \( R = 10^{-5} \text{m}, Q_{d_{0}} = 3.2 \cdot 10^{-16} \text{C}, W = 0.9 \). The absolute dimensionless supersaturation \( \varepsilon = 5 \cdot 10^{-9} \div 5 \cdot 10^{-8} \) which corresponds to relative supersaturation 0.1\% - 1.0\% [17].

Figure 1 shows the calculated curves of the relationship between charge dynamics on the cloud drops and fractional parameter in case of a general condensation and coagulation growth mechanism \( (0 < \alpha < 1) \). It’s clear for small parameter values \( \alpha \) a sharp increase occurs in the charge on the drop and once passed through the zone of the fractal effect it decreases. Fractal zone implies the zone where the graphs lying in the area \( 0 < \alpha < 1 \) intersect the graph as \( \alpha = 1 \). In this area a manifestation of fractal properties begins. Accordingly, we can assume that the processes that takes place in fractal cloud environments are much slower than in continuous ones.
Figure 1. Relationship curves for drops between charge dynamics and parameter $\alpha$

Figure 2. Relationship curves for ice particles between charge dynamics and parameter $\alpha$

Calculations for the ice particles of the charge sharp increase have also been carried out for $\Delta v = 3, 5, 7, \ 10 \ m/s$, with the radius of the ice particles equal to $R = 10^{-5} \ m \ Q_{de} = 1,2 \cdot 10^{-15} \ C \ c = 93.4 \cdot 10^{-12} \ F \ B = 1.3, \ f = 0.8, \ \rho = 900 \ kg/m^3$.

Fig. 2 presents relationship calculated curves for the cloudy ice particles of the charge dynamics and fractional parameters with the anomalous advection ($0 < \alpha < 1$). It is clear a sharp increase occurs in the charge at small values of the parameter $\alpha$ like in the case with drops, and once the fractal effect zone have been passed it decreases.

Whatever, what are the properties of these structures, how do they depend on the state of the medium, and how is the fractal dimension related to the derivative fractional index in the dynamic equation describing the charge on the cloud ice particles. These issues require separate studies.

Summary
A model of the charge dynamics on droplets and ice particles was constructed in view of the fractality of the cloudy environment using the apparatus of the theory of fractional differential calculus. The obtained formulas can be used for calculation changes in the charge acquired by the cloud particles under condensation, sublimation and coagulation with given parameters and in view of fractal
medium. Numerical experiments assessing the effect of the fractal medium on the charge acquired by cloud particles with microphysical parameters in different combinations showed a general relationship between the charge and fractal medium parameters. Under both mechanisms, the fractal and dynamic mode for the particle charge growth takes place after passing the fractal effect zone to a continuous medium where the growth process is diminishing.

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