Abstract—The complex frequency response is a complete
description of a tracking filter’s behavior at steady state. For
example: 1) The flatness order at zero frequency determines
the kinematic state dimension, thus the polynomial degree
of trackable target trajectories; 2) The magnitude and phase error
at a given non-zero frequency determine the radial and angular
(bias) errors for a target on a circular orbit of a given angular
velocity; and 3) The integral of squared magnitude determines
the (random) measurement-noise attenuation. A procedure for
the design of fixed-gain tracking filters, using an augmented-state
observer with signal and interference subspaces, is proposed as a
way of satisfying these frequency-response requirements. The
signal subspace incorporates an integrating Newtonian model
and a second-order maneuver model that is matched to a
sustained constant-g turn; the deterministic interference model
attenuates high frequencies for smoother track estimates. The
selected models provide a simple means of shaping the steady-
state response of tracking-filters of elevated order, when noise
statistics are unknown and non-Gaussian. Pole placement is used
to tune the transient response as an alternative to process noise.

Keywords— IIR filters, Kalman filters, Noise shaping,
Observers, State estimation, Tracking loops

I. INTRODUCTION

The exploitation of prior knowledge by the Kalman filter
(KF) gives rise to a variable gain and adaptive behavior, which
is highly desirable in automatic target-tracking systems.
Knowing the parameters of (Gaussian) measurement-noise and
process-noise statistics, and the covariance of the state estimate
that follows, provides additional leverage that may be used to:
accelerate the removal of bias errors during track
establishment; handle changing sensor/target characteristics;
accommodate variable revisit intervals; and manage
measurement-to-track assignment ambiguities that arise in the
presence of clutter, multiple closely-spaced targets, and
missing measurements.

For measurement statistics, the use of a Gaussian prior is
usually a reasonable approximation; however, for process
statistics it may be unreasonable, particularly for agile targets
that execute rare, extreme, and diverse, maneuvers. When noise
parameters are unknown and very non-Gaussian, maintaining
the covariance matrix and evaluating the resulting filter gain
on each update is an expensive overhead that does not necessarily
improve tracking accuracy [1]. In these cases, fixed-gain filters
that do not incorporate prior knowledge of second-order
statistical moments, i.e. the variance of noise distributions, are
more appropriate. The resulting filter structures make it easier
to incorporate more detailed process models that would
otherwise be difficult to parameterize, analyze, and realize,
within a stochastic framework.

A popular way of designing basic fixed-gain filters begins
with a KF for position-velocity(-acceleration) states; the
steady-state (SS) Kalman gain vector is then determined via the
Riccati equations. For a SS-KF of order \( K \), with noise matrices
\( Q \) and \( R \) of a specified form, this method yields closed-form
expressions for \( \alpha - \beta (K = 2) \) or \( \alpha - \beta - \gamma (K = 3) \) filter
coefficients, as a function of the dimensionless
\( \lambda_{SS,KF} = T_s^2 \sigma_o/\sigma_c \) quantity, which combines the sampling period \( T_s \)
and noise parameters, where \( \sigma_o \) & \( \sigma_c \) are the standard deviations
of the process-noise and measurement-noise distributions,
respectively [1]. This parameter is commonly referred to as the
tracking index [2]. General procedures for integrating models
of arbitrary order, for a wider filter bandwidth, are
available [1],[3]; however, this is likely to degrade tracking performance
without additional measures to attenuate noise and interference.

An alternative class of methods begins by specifying some
aspect of the transient response and/or the SS response (e.g.
signal bandwidth or noise gain), then proceeds with the
discrete-time transfer-function/frequency-response of the
tracking filter as the design object, rather than a set of assumed
statistical distributions [4]-[13]. From these requirements, the
\( \alpha - \beta (-\gamma) \) parameters of a fixed-gain filter [5]-[11], or
the process-noise parameter of a variable-gain KF are then derived
[12],[13].

The merit of using deterministic analysis to design
stochastic (i.e. Kalman) filters is questioned in [9] and the
possibility of using state observers (e.g. Luenberger) with
arbitrarily placed poles, is mooted; however, details of how
such a design procedure might be adapted for this specific
purpose are not provided. Posing the tracking problem in the
transform domain (i.e. \( z \) or \( \omega \)) simplifies the algebraic
manipulations required to derive tracking filter coefficients
[14],[15] and performance metrics [11]. It also opens the
door to other design and analysis possibilities that have not hitherto
been utilized in target-tracking problems.

Discrete-time shaping-filters with augmented states and a
variable gain were originally formulated for handling colored
noise in communications systems [16],[17]; they have since
been extended to incorporate online system identification and
applied to a variety of problems, such as speech processing
[18]-[20]. In this paper, a steady-state variant is used to solve
the Newtonian target-tracking problem: A deterministic
process model – with signal and interference subspaces – and
without random-noise terms is utilized, to provide a way of
balancing conflicting bandwidth/interference-noise-gain requirements in a
filter of elevated order.

Unlike conventional methodologies described in the
literature, that are based on the parameters of random process-
and measurement-noise statistics, the tracker design and analysis procedure presented in this paper revolves around the tracking filter’s frequency response. It is shown that deterministic process models may be crafted to shape the response, for improved maneuver handling at specified rates of turn and/or increased track smoothness at steady state. Pole-placement – standard in controller design but unusual in tracker design – is then used to determine the convergence behavior after track initiation and input discontinuities (e.g. steps or ramps), i.e. the transient response. The frequency response of the tracking filter determines its behavior at steady-state, whereas the position of the (repeated and real) poles determines its behavior as it approaches steady-state.

The problem of tracking a point target in a Cartesian coordinate system is considered here – a distant target in a high-resolution imaging radar, for example. Target and measurement dynamics are assumed to be approximately linear over the spatial and temporal scales of the sensor and the states are assumed to be independent and separable in both Cartesian dimensions. The derivation that follows, deals with a single scalar measurement from one of the dimensions and measurement-to-track assignment is unambiguous. Extensions that incorporate data association will be described in a future publication. A constant time interval ($\tau$) between measurements is also assumed.

II. FORMULATION

The sequence of measurements collected by the sensor is assumed to be generated by an endogenous linear-time-invariant (LTI) process with signal and interference subspaces. The signal subspace is further assumed to contain target and maneuver terms. The target model governs baseline dynamics; the maneuver process model is perfect; the impact of modelling errors is discussed in the section that follows.

A. Target process model

The linear state-space (LSS) representation, of this continuous-time process, in controller-canonical form (CCF), is as follows:

$$w_{\text{tgt}}(t) = A_{\text{tgt}}w_{\text{tgt}}(t), \ y_{\text{tgt}}(t) = C_{\text{tgt}}w_{\text{tgt}}(t)$$

with

$$A_{\text{tgt}} = \begin{bmatrix} 0(\kappa_{\text{tgt}} - 1)_{\times 1} & I(\kappa_{\text{tgt}} - 1) \end{bmatrix}_{\kappa_{\text{tgt}} \times \kappa_{\text{tgt}}}$$

and

$$C_{\text{tgt}} = \begin{bmatrix} 1 & 0_{\times (\kappa_{\text{tgt}} - 1)} \end{bmatrix}_{1 \times \kappa_{\text{tgt}}}$$

(1)

where $y_{\text{tgt}}(t)$ and $w_{\text{tgt}}(t)$ are the output signal and the process states; furthermore, $I_N$ is an $N \times N$ identity matrix and $0_{M \times N}$ is an $M \times N$ matrix of zeros; thus $A_{\text{tgt}}$ is a zero matrix with ones along the 1st upper diagonal. The corresponding discrete-time model, for a constant sampling period of $T_s$, is found via the state transition matrix $G_{\text{tgt}}$ as:

$$G_{\text{tgt}} = L^{-1} \{ \Phi_{\text{tgt}}(s) \} = I_{\kappa_{\text{tgt}}} - A_{\text{tgt}}^{-1}$$

(2a)

where $\Phi_{\text{tgt}}(s) = (sI - A_{\text{tgt}})^{-1}$ and $L^{-1}$ is the inverse Laplace transform ($t \leftrightarrow s$). The state-transition matrix $G_{\text{tgt}}$, is an upper-triangular Toeplitz matrix with the elements along the $k$th off-diagonal (for $k = 0 \ldots K_{\text{tgt}} - 1$) equal to $G(k; T_s) = T_s^k / k!$. As the diagonal elements of this triangular matrix are equal to unity, the discrete-time model of the target signal has $K_{\text{tgt}}$ repeated poles in the complex $z$-plane at $z = 1$, for a singularity at dc, i.e. at $\omega = 0$, where $\omega$ is the angular frequency (radians per sample).

B. Maneuver process model

This second-order term is defined in CCF as follows:

$$w_{\text{man}}(t) = A_{\text{man}}w_{\text{man}}(t), \ y_{\text{man}}(t) = C_{\text{man}}w_{\text{man}}(t)$$

with

$$A_{\text{man}} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ C_{\text{man}} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

(3a)

where $\Omega$ is the angular velocity (rad/s), or maneuver turn rate, for a constant-speed orbit of radius $R$; thus

$$C_{\text{man}} = \begin{bmatrix} \cos(\Omega T_s) & \sin(\Omega T_s) / \Omega \\ -\Omega \sin(\Omega T_s) & \cos(\Omega T_s) \end{bmatrix}$$

(3b)

which has poles in the $z$-plane at $z = e^{\pm i\Omega T_s}$ where $i^2 = -1$. Note that $A_{\text{man}}$ and $G_{\text{man}}$ are for the continuous-time and discrete-time processes, respectively. The latter representation is derived from the former via the same standard procedure used to discretize the target process model in (2).

A second-order damped oscillator is used in [23] to model “wind sway or platform roll”. It is an extension of the Singer model, which shifts a pole at the origin of the $s$-plane, left along the real axis [15],[24]. Although, this result is derived via the introduction of first-order auto-correlation in the noise input, its effect may be physically interpreted as a drag force...
acting on the target, for a second-order process, with position and velocity states, an acceleration input and a position output.

The undamped one-dimensional (1-D) oscillator used here in (3a), with poles on the imaginary axis of the s-plane at ±Ω, may seem somewhat unrealistic for a target’s motion in 1-D; however, when two such models are placed in parallel, to represent independent motion in two orthogonal Cartesian axes, they may be used to perfectly model circular motion, when their internal states fall into antiphase. Unlike the 2-D coordinated turn models discussed in [25], the relative phase when their internal states fall into antiphase. Unlike the 2-D axes, they may be used to perfectly model circular motion, however, when two such models are placed in parallel, to provide sufficient flexibility and control over tracking behavior in typical problems.

E. Observer design

As the process has \( K \) poles on the unit circle, it is marginally stable. We now seek a stable observer, i.e. all poles inside the unit circle, to estimate the states of the augmented-state vector, placed in series with the process, that has the following discrete-time LSS representation:

\[
\begin{align*}
\hat{\mathbf{x}}_\text{obs}(n) &= \mathbf{H}_\text{obs} \hat{\mathbf{x}}_\text{obs}(n) + \mathbf{K} \mathbf{e}_\text{obs}(n) \\
\mathbf{e}_\text{obs}(n) &= \mathbf{y}_\text{obs}(n) - \mathbf{H}_\text{obs} \hat{\mathbf{x}}_\text{obs}(n)
\end{align*}
\]

Note that the signal and interference models used here are just examples of possible processes that may be relevant in a tracking problem. Other process definitions and permutations are possible; however, this particular combination was found to provide sufficient flexibility and control over tracking behavior in typical problems.

\( K = K_{\text{tgt}} + 2K_{\text{man}} + K_{\text{int}} \), with \( K_{\text{man}} \leq 1 \)

(4)

D. Process model

The discrete-time model of the process that generates the sampled measurement sequence (i.e. “the process”) may now be defined as

\[
\begin{align*}
\mathbf{w}_\text{prec}(n) &= \mathbf{G}_\text{prec} \mathbf{w}_\text{prec}(n - 1) \\
y_{\text{prec}}(n) &= \mathbf{C}_\text{prec} \mathbf{w}_\text{prec}(n)
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{w}_\text{prec}(n) &= \begin{bmatrix} \mathbf{w}_\text{tgt}(n) \\ \mathbf{w}_\text{man}(n) \\ \mathbf{w}_\text{int}(n) \end{bmatrix}_{K \times 1} \\
\mathbf{G}_\text{prec} &= \begin{bmatrix} \mathbf{G}_\text{tgt} & \mathbf{0}_{K_{\text{g}} \times K_{\text{man}}} & \mathbf{0}_{K_{\text{g}} \times K_{\text{int}}} \\ \mathbf{0}_{K_{\text{man}} \times K_{\text{g}}} & \mathbf{G}_{\text{man}} & \mathbf{0}_{K_{\text{man}} \times K_{\text{int}}} \\ \mathbf{0}_{K_{\text{int}} \times K_{\text{g}}} & \mathbf{0}_{K_{\text{int}} \times K_{\text{man}}} & \mathbf{G}_{\text{int}} \end{bmatrix}_{K \times K} \\
\mathbf{C}_\text{prec} &= \begin{bmatrix} \mathbf{C}_\text{tgt} & \mathbf{C}_{\text{man}} & \mathbf{C}_{\text{int}} \end{bmatrix}_{1 \times K}
\end{align*}
\]

In the above definitions, \( q \) is the delay parameter (an integer, in samples) and \( \mathbf{G}_{\text{sig}}(q) \) is a signal state-transition matrix, for a time-step of \( -q \tau_s \) seconds, i.e.

\[
\mathbf{G}_{\text{sig}}(q) = \Phi_{\text{sig}}(t)\big|_{t=-q\tau_s} = \mathbf{G}_{\text{sig}}^{-1} = \mathbf{G}_{\text{sig}}^{-q}.
\]

Note that \( \mathbf{C}_{\text{g}} \) (for \( \hat{x} \)) involves the signal and noise subspaces, whereas \( \mathbf{C}_{\text{obs}} \) (for \( y_{\text{obs}} \)) considers only the signal subspace.

The gain vector \( \mathbf{K} \) is found in the usual way, via a coordinate transform, which reduces the process equations to a canonical form for the observable \( \mathbf{C}_{\text{g}} \mathbf{G}_{\text{prec}} \) pair [22]. The transform required is readily found using the observability matrices of both coordinate systems; alternatively, the discrete-time versions of the Ackermann or the Bass-Gura formulae may be used to find the gain vector directly [21],[22].
system modelling errors (i.e. $|\lambda_k| \to 0$). Repeated real poles, i.e. $\lambda_k = p$, for all $k$, with $0 \leq p < 1$, are sufficient for low-frequency signals with a narrow bandwidth, or for signals that are sampled at a sufficiently high rate, and they are used exclusively here. However, complex pairs of poles may be required in very-wideband filters. Using $p = 0$ yields a so-called “deadbeat” observer, with an FIR. For a complex argument (arg), $|\text{arg}|$ and $\angle \text{arg}$ are the magnitude and angle operators, respectively.

**F. Filter design**

The state observer accepts a scalar-valued input $x(n)$, and produces a vector-valued output $w_{\text{obs}}(n)$. In prediction, filtering, and smoothing, problems (using $q < 0$, $q = 0$ and $q > 0$, respectively), the position element ($k = 0$) of the state, is of primary importance. To analyze the response of this single-input/single-output (SISO) system, and to yield a minimal-complexity realization of the discrete-time transfer function $\mathcal{H}(z)$, which links the position measurement inputs to the position estimate outputs $y(n)$, it is convenient to determine a new coordinate transform that reduces (5) to a canonical form [21],[22]. The $b$ and $a$ coefficients of the canonical filter realization of $\mathcal{H}(z)$, where $\mathcal{H}(z) = \mathcal{B}(z)/\mathcal{A}(z)$ and $\mathcal{A}(z) = (z - p)^K$, are then readily extracted from the observer equations in canonical form, such that

$$y(n) = \sum_{k=0}^{\infty} b(k)x(n-k) - \sum_{k=1}^{\infty} a(k)y(n-k).$$

(7)

**G. Tracker analysis**

Bespoke closed-form expressions for the white-noise gain (WNG) of $a - \beta(-\gamma)$ filters are available for various filter configurations [5]-[11]; however, for general cases (i.e. arbitrary order and lead/lag configurations), it may be simply evaluated for any stable transfer function $\mathcal{H}(z)$, using its frequency-response $H(\omega)$, or more conveniently (via a loop until convergence) using its impulse response $h(n)$, as a consequence of Parseval’s theorem [26], i.e.

$$WNG = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(\omega)|^2 d\omega = \frac{1}{2\pi} \|H(\omega)\|_2^2$$

$$= \sum_{n=0}^{\infty} |h(n)|^2 = \|h(n)\|_2^2$$

(8)

where $\|\|_2$ is the $L_2$-norm. The gain of the maneuver error signal (MESG) is an analogous and complementary narrowband metric:

$$\text{MESG} = |H_d(\omega_{\text{man}})| - H(\omega_{\text{man}})|^2$$

(9)

where $\omega_{\text{man}} = \pm \Omega T_e$ or any other frequency of interest in the passband. Now let $\sigma_{\text{tgt}}^2$ and $\sigma_{\text{man}}^2$ be the expected squared-distance errors at SS in a Cartesian space with two position coordinates $(\bar{x}, \bar{y})$ – i.e. $\sigma_{\text{tgt}}^2 = (\epsilon_x^2 + \epsilon_y^2)$, where $(\cdot)^{\dagger}$ is the expectation operator and where bar accents are used to distinguish these coordinate variables from the input and output variables – for a signal produced by the target process model and a target executing a sustained constant-g turn, respectively. Furthermore, assume that white measurement-noise is added with a variance of $\sigma_{\text{sms}}^2$ in the former case. Then

$$\sigma_{\text{tgt}}^2 = 2WNG \cdot \sigma_{\text{sms}}^2$$

and

$$\sigma_{\text{man}}^2 = \text{MESG} \cdot R^2.$$
throughout, of \( v = 25 \) pix/s = 1 pix/frm @ 25 Hz, where \( T_s = 0.04 \) s. Gaussian measurement noise, with a mean of zero and a standard deviation of \( \sigma_R = 1 \) pix is added to the target position in each frame.

The KF was tuned using prior knowledge of the simulation parameters (\( \sigma_R, \Omega \) & \( R \), see Table I). A variable-gain KF was also implemented. The squared target speed was used to initialize the rate elements of the covariance matrix. The increased initial gain of this filter helped to quickly remove bias errors during track establishment. As no other prior information is supplied on-the-fly, the position element of the Kalman gain vector converges to within 0.1% of the steady-state value (of 0.36) after just 15 updates (i.e. 0.6 s). Events 1, 3 & 4 are the only instances of KF model mismatch in this scenario.

| TABLE I: FILTER PARAMETERS* |
|-----------------------------|
| Filter A\(^1\): K = 2, q = 2, with \( a = 0.36 \) and \( b = 0.08 \) in | \( b = [a - \beta q, \beta(1 + q) - a, 0] \) and \( a = [1, a + \beta - 2, 1 - a] \) |
| Filter B\(^1\): K = 3, \( K_{eq} = 2, K_{man} = 0 \), K_int = 1, q = 2 with | \( b = [0.046, 0.004 - 0.042, 0] \) and |
| Filter C\(^1\): K = 5, \( K_{eq} = 2, K_{man} = 1 \), K_int = 1, q = 2 with | \( b = [0.0899 - 0.1532, -0.0232, 0.1534 - 0.0666, 0] \) and |

\(^1\)Independent filters designed for each spatial dimension.

\(^2\)A 2nd-order SS-KF. This reference implementation was tuned using \( \sigma_R \) = 1.0 pix and \( \sigma_q = R H^2 = 625 \) pix/s\(^2\), respectively.

\(^3\)Designed with \( a \) extracted from \( A(z) \). Same pole radius as the SS-KF (i.e. \( p = 0.8 \)), to facilitate comparison.

B. Response analysis

The frequency response of the tracking filters is provided in Fig. 2 below.

![Filter frequency response](image)

Fig. 2. Filter frequency response, as a function of \( f = \omega / 2\pi \) (cycles per frame). Maneuver frequency \( \omega_{man} = T_s / \Omega / 2\pi \) and ideal passband linear phase response (for \( q = 2 \)) shown (dash-dot line). Insets show the low frequency (magnitude and phase-error) response around \( \omega_{man} \) (dashed line).

Prior knowledge of \( \sigma_R \) & \( \sigma_q \) in Filter A (the SS-KF) sets its bandwidth so that the response is approximately flat up to and around the maneuver frequency. The gain and phase error are small at \( \omega_{man} \). Equation (11) indicates that the SS tracking error for a circular turn of this frequency should also be small, which is indeed the case (see event 2 in Fig. 1). The filter gain rolls off slowly outside the passband (WNG = 0.156) and at \( f = 0.5 \) the gain is \(-20 \) dB or a factor of 0.1 on a linear scale. This indicates that the SS error during the jitter event, with an input magnitude of 10 pix should be approximately \( \pm 1 \) pix, which is indeed the case (see event 4 in Figure 1).

Use of the interference model in Filter B places a null at \( f = 0.5 \); therefore, the SS error should be zero during the jitter event, in the absence of white noise. This severe high-frequency cut reduces the overall noise gain (WNG = 0.125), but it distorts the response at low frequencies around \( \omega_{man} \). The negative phase error indicates that the track lags behind \( (\sim 11^\circ) \) the target executing a circular maneuver at a constant turn rate of \( \omega_{man} \), at SS. The positive gain of approximately 1 dB at \( \omega_{man} \) indicates that the track radius is larger than the target radius, at SS. Using \( R = 10 \) pix, \( H(\omega_{man}) = 1.122H_0(\omega_{man}) = 1.0 \) in (11a) yields a radial error of \( \mu_R = 1.22 \) pix at SS, in the absence of noise, which agrees with the overshoot for event 2 in Fig. 1.

The low-frequency distortion caused by the interference model is removed by the introduction of the maneuver model in Filter C. In the absence of noise, this model eliminates SS bias errors for turns at the model frequency, and it flattens the response around \( \omega_{man} \), but it shifts the gain “bulge” to higher frequencies (and increases the noise gain, WNG = 0.188). At those frequencies, this distortion corresponds to: an increase in the SS bias errors (i.e. a leading track with a larger radius) for mismatched circular maneuvers; and an increase in the colored-noise gain (i.e. “wobble”), in the absence of circular maneuvers.

Up to a point, almost all aspects of a tracking filter’s performance are improved by increasing \( q \). These gains are of course made at the expense of the system’s latency; indeed, it is reasonable to expect greater accuracy if estimates are deferred. Introducing a delay of \( q \) samples results in a modulation of the response by \( e^{-i\omega t} \) in the frequency domain; its effect on tracking behavior in the time domain is somewhat subtler but readily quantified and appreciated using the material presented in Section II.G.

The complex frequency response is a complete representation of a tracking filter’s steady-state behaviour: Integrating the squared magnitude over a band of interest yields its response to random noise inputs, in the absence of manoeuvres; Evaluating it at a frequency of interest yields its response to deterministic sinusoidal inputs, e.g. due to interference or circular manoeuvres, in the absence of noise. The target process model guarantees flatness at \( \omega = 0 \) for unbiased estimates of position derivatives; the interference process model ensured zero gain at \( \omega = \pi \) for improved high-frequency noise attenuation; and the manoeuvre process model ensures unity magnitude and zero phase-error at \( \omega = \omega_{man} \) for perfect tracking of circular turns at the specified angular rate. Each model constrains the response to the desired (complex) value at the specified frequency, but it also shapes or distorts the response elsewhere, which may improve or degrade overall performance.

In the design of optimal infinite-impulse-response (IIR) filters for digital signal processing applications, the pole radius
is typically a free parameter, subject to a stability constraint, thus the effective duration of the impulse response is unconstrained. This is reasonable because the signal parameters of interest are approximately stationary over the timescales considered. However, in target tracking problems this is usually not the case, thus the pole radius is fixed (i.e. placed arbitrarily) in the procedure described here, to ensure that the transient response is satisfactory. Constraints on the frequency response are then satisfied, for reasonable SS behaviour – e.g. accurate kinematic state estimation, high-frequency noise attenuation and circular manoeuvre tracking – by the appropriate placement of filter zeros. In this paper, they are placed indirectly through the selection of physical process models and the observer pole position (0 ≤ p < 1). In [27] & [28] they are derived from the linear coefficients that combine a set of basis functions in a way that satisfies derivative constraints in the ω-domain while (optionally) minimizing the WNG. Process models and pole placement are used here to derive the filter coefficients, primarily to highlight the relationship between the frequency response and the tracking properties of an estimator.

IV. CONCLUSION

When the statistical distributions assumed in a Kalman filter are unreasonable (e.g. for targets that execute non-Gaussian maneuvers) the intended adaptive behavior is lost, and it must be empirically tuned for the desired transient and steady-state response. Alternatively, if parametric uncertainty is accepted, and variance priors are ignored, the filter design and implementation is greatly simplified, which allows more detailed process models (e.g. maneuver and interference) to be considered for improved steady-state tracking. Observer poles are then placed to set the effective duration of output transients on input discontinuities.

The relationship between a tracking filter’s frequency response and its tracking behavior has been discussed and quantified; furthermore, an ab-initio design method that specifically addresses (steady-state and transient) response requirements is described in this paper. The simulation results and analysis indicate that the proposed augmented-state observer provides a simple and intuitive way of balancing the various performance trade-offs associated with the design of fixed-gain tracking filters, that would otherwise be difficult to achieve using a standard SS-KF with a filter order equal to the Newtonian state dimension.

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