MAGNETIC SCREENING AT FINITE TEMPERATURE

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Abstract

It is shown that at finite temperature and in the presence of magnetic sources magnetic fields are screened. This is proven within the framework of classical transport theory both for the Abelian and non-Abelian plasmas. Magnetic screening arises in this formalism as a consequence of polarization effects occurring in the plasmas, and it is proportional to the inverse of the gauge coupling constant. It is then discussed whether this mechanism could be relevant in realistic quantum gauge field theories, such as QCD.

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I. INTRODUCTION

The studies of gauge field theories at high temperature $T$ have attracted much attention in the recent past. The behavior of certain gauge theories, such as QCD, changes dramatically in their low or high temperature regimes. Thus, it is generally believed that QCD is in a unconfined phase above a critical temperature $T_c \sim 200$ MeV. In principle, due to asymptotic freedom, perturbation theory in the high $T$ regime of QCD could be naively expected to be valid, but this turns out not to be the case.

In the high $T$ regime of QCD, thermal excitations produce a plasma of charged particles which screens color electric fields. The electric thermal mass can be computed at one-loop order in perturbation theory, and it is proportional to $gT$, where $g$ is the gauge coupling constant. A self-consistent inclusion of the color electric mass in the Feynman loop computations requires the use of resummed perturbation theory.

Color magnetic fields are not screened at the same order of perturbation theory. Due to this fact several infrared (IR) divergences are encountered in the computations of different physical quantities. Those divergences even appear in the absence of quark matter in the computation of the perturbative partition function.

There has been a lot of discussions in the literature about the possible mechanism that could cure the IR problems in the magnetic sector of a pure Yang-Mills theory. It is generally believed that non-perturbative effects generate a color magnetic mass of order $g^2 T$, which would arise only in the non-Abelian theory but not in the Abelian case. The arguments to reach to that conclusion are the following. The infrared limit of QCD at high $T$ is governed by the spatial vector gauge field $A_i$. Using imaginary time formalism, the high $T$ limit of QCD is equivalent to an Euclidean three dimensional Yang-Mills theory, with an effective coupling constant $g' = g^2 T$. Those theories are believed to generate a dynamical mass gap proportional to the (dimensional) coupling constant, which is not computable in perturbation theory.

The above arguments are rather qualitative, but give no clue about what kind of mechanism could be responsible for the generation of the magnetic mass gap. They also do not explain how those effects could eliminate the IR problems found in specific computations.

Some attempts have been made in the literature to give an effective action that describes a magnetic mass term and such that could be used in specific Feynman loop diagrams computations. However, no conclusive evidence about the proposed Ansätze has been reached.

On the other hand, some discussions have been raised in the literature about whether the high $T$ limit of QCD could be correctly described within the classic rather than quantum field theory. In Ref. it has been realized that both the color electric and magnetic screenings are detected in lattice classical gauge field computations. This fact could be a clear indication that the magnetic screening could be understood in terms of classical but non-perturbative physics.

The purpose of this article is to investigate under which circumstances static magnetic screening can emerge in a hot non-Abelian theory using exclusively classical (or semiclassical) physics. I do not attempt here to solve the infrared problem of a Yang-Mills theory at finite temperature, but I just try to find a classical mechanism that could generate a thermal color
A study of the classical gauge field equations reveals that the absence of color magnetic screening in a hot plasma is entirely due to the absence of static magnetic sources. Therefore, in order to derive a thermal magnetic screening, I postulate the existence of magnetic charges. It will be shown in this article that in the presence of magnetic charges magnetic fields are naturally screened at finite $T$. The mechanism responsible for this screening is actually the same one as that generating the screening of electric fields in the QED and QCD plasmas. In hot plasmas of electric charges, polarization phenomena screens electric fields. The same kind of polarization effects would generate the magnetic screening if magnetic charges existed.

Classical transport theory will be used to prove that the existence of (non-) Abelian magnetic charges at finite $T$ implies the screening of (non-) Abelian magnetic fields. This formalism has already been used to derive the screening of (non-) Abelian electric fields, and those effects are reproduced exactly in the corresponding quantum field theory at high $T$. The generalized set of (non-) Abelian Vlasov equations in the presence of magnetic sources will be written, and from them magnetic screening will be derived. As it will be shown, the magnetic screening turns out to be proportional to the inverse gauge coupling constant.

This paper is structured as follows. In Sec. II, the Abelian plasmas are first studied. In Subsec. II A it is recalled how Debye screening is obtained from the Vlasov equations. In Subsec. II B the proposed “magnetic” or dual Vlasov equations are written, and from them the static screening of magnetic fields is derived. It is stressed there that the duality symmetry of electromagnetism allows to derive the magnetic screening from the electric one. In Sec. III the same study is reproduced for non-Abelian plasmas, when duality is not a symmetry of the theory. Subsec. IV A is devoted to review some static magnetic monopole fields solutions in the vacuum. In Subsec. IV B solutions to the gauge field equations in hot plasmas, which reproduce the screened magnetic fields, are found. Sec. V ends with a discussion of the results. Let us finally mention that throughout this paper a system of units where $\hbar = c = k_B = 1$ will be used.

II. THE ULTRARELATIVISTIC ABELIAN PLASMAS

A. Static Electric Screening in the Plasma of Electric Charges

In this subsection the derivation of the electric Debye screening effects from classical kinetic equations is reviewed [6], [7]. This will teach us how to derive the corresponding magnetic screening in the presence of magnetic charges.

Let us first recall the dynamical evolution of a charged point particle. A particle carrying an electric charge $e$, with mass $m$, and transversing a worldline $y^\mu(\tau)$, where $\tau$ is the proper time, obeys the equations (neglecting the effects of spin)

$$m \frac{dy^\mu(\tau)}{d\tau} = p^\mu(\tau), \quad (2.1a)$$

$$m \frac{dp^\mu(\tau)}{d\tau} = e F^{\mu\nu}(y(\tau)) p_\nu(\tau), \quad (2.1b)$$
where the electromagnetic field $F^\mu\nu$ is evaluated on the particle worldline. In our conventions $F^{0i} = -E^i$, and $F^{ij} = -\epsilon^{ijk}B^k$, where $E^i$ and $B^i$ are the electric and magnetic fields, respectively.

In a self-consistent picture the electromagnetic fields obey the Maxwell’s equations which have as sources the electric currents obtained from each charged particle of the system. Thus

$$\partial_\nu F^{\nu\mu}(x) = J^\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} j^\mu(x) , \quad (2.2)$$

where (helicity and species indexes are implicit)

$$j^\mu(x) = e \int d\tau \frac{dp^\mu(\tau)}{d\tau} \delta^{(4)}(x - y(\tau)) . \quad (2.3)$$

The above current is conserved, $\partial_\mu j^\mu(x) = 0$, as may be checked by using the equations of motion (2.1). This is required as a compatibility condition, as can be easily recognized by applying a partial derivative $\partial_\mu$ to (2.4), since $F^{\mu\nu}$ is antisymmetric in their indices.

The electromagnetic field tensor obeys the Bianchi identity

$$\partial_\nu * F^{\nu\mu}(x) = 0 \ , \quad (2.4)$$

where the dual field is $* F^{\nu\mu} = \frac{1}{2} \epsilon^{\nu\mu\rho\sigma} F_{\rho\sigma}$, and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita antisymmetric tensor in four dimensions, with $\epsilon^{0123} = 1$. In our conventions $* F^{0i} = -B^i$, and $* F^{ij} = \epsilon^{ijk}E^k$. The Bianchi identity (2.4) guarantees the existence of the vector gauge field $A_\mu(x)$, such that $\overrightarrow{E}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which is not unique. The ambiguity in defining the vector gauge field is that corresponding to gauge transformations.

The statistical description of the plasma of charged particles is given by the distribution function of its components in their phase-space. In the collisionless case the one-particle distribution function $f(x,p)$ of finding a particle in the state $(x,p)$ evolves in time via a transport equation

$$\frac{df(x,p)}{d\tau} = 0 \ . \quad (2.5)$$

Using the equations of motion (2.1), it becomes the Boltzmann equation

$$p^\mu \left[ \frac{\partial}{\partial x^\mu} - e F_{\mu\nu}(x) \frac{\partial}{\partial p^\nu} \right] f(x,p) = 0 . \quad (2.6)$$

In a self-consistent picture, the mean electromagnetic fields obey the Maxwell’s equations

$$\partial_\nu F^{\nu\mu}(x) = J^\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} j^\mu(x) , \quad (2.7)$$

$$\partial_\nu * F^{\nu\mu}(x) = 0 , \quad (2.8)$$

where now each particle species electric current is obtained from the corresponding distribution function as

$$j^\mu(x) = e \int dP P^\mu f(x,p) . \quad (2.9)$$
The momentum measure in (2.9) is defined as
\[ dP = \frac{d^4 p}{(2\pi)^3} \frac{2}{\pi^2} \theta(p_0) \delta(p^2 - m^2), \] (2.10)
so that it guarantees positivity of the energy and on-shell evolution.

The equations (2.6)-(2.9) are known as Vlasov equations [6].

The effects of static screening in the plasma of electrons and ions can be deduced from the Vlasov equations as follows [6], [7]. Let us consider a neutral plasma, that is, composed by the same number of positive and negative charges. The plasma, initially at equilibrium, is disturbed by a weak electromagnetic field. We look for a distribution function of the form
\[ f(x, p) = f^{(0)}(p_0) + e f^{(1)}(x, p) + \ldots, \] (2.11)
where \( f^{(0)}(p_0) \) is, up to a normalization constant, the Fermi-Dirac equilibrium function
\[ n_F(p_0) = \frac{1}{e^{p_0/T} + 1}. \] (2.12)

Neglecting second order terms, \( f^{(1)} \) obeys the following equation
\[ p^\mu \frac{\partial}{\partial x^\mu} f^{(1)}(x, p) = p^\mu F_{\mu 0}(x) \frac{d}{dp_0} f^{(0)}(p_0). \] (2.13)
Notice that only the electric field enters into the r.h.s. of (2.13), but not the magnetic field. This is actually the reason why there is only static electric screening but not static magnetic screening in this approach.

A total electric current density \( J^\mu(x, p) \) is defined such that the total current \( J^\mu(x) \) is found just by integrating over the momenta \( p \), using the momenta measure (2.10). The induced electric current density \( J^\mu(x, p) \) obeys the equation
\[ p \cdot \partial J^\mu(x, p) = e^2 p^\rho p^\nu F_{\rho \nu}(x) \frac{d}{dp_0} f^{(0)}(p_0). \] (2.14)

In the ultrarelativistic limit, that is, taking the fermion mass \( m = 0 \) in (2.10), the induced current in momentum space reads
\[ J^\mu(k) = -im_D^2 \int \frac{d^3 \varphi}{4\pi} \frac{\nu^\mu}{v \cdot k + i\epsilon} \mathbf{v} \cdot \mathbf{E}(k), \] (2.15)
where \( m_D^2 = e^2 T^2 / 3 \) is the Debye mass squared. Retarded boundary conditions have been imposed in (2.13), with the prescription \( i\epsilon \). The four vector \( \nu^\mu = (1, \mathbf{\hat{v}}) \) is the four velocity of the particles of the plasma, and in the ultrarelativistic situation considered here, it is light-like. The angular integral in (2.13) is defined over all possible directions of the three dimensional unit vector \( \mathbf{\hat{v}} \).

In the static situation \( J^i = 0 \), while the induced electric density is
\[ J^0(x) = -im_D^2 \int \frac{d^3 k}{(2\pi)^3} \frac{\mathbf{k} \cdot \mathbf{E}(k)}{k^2} e^{i\mathbf{k} \cdot \mathbf{x}}. \] (2.16)
The Maxwell’s equations which have as source the current \((2.15)\) are also known as Kubo equations. In the static limit they read

\[
\nabla \cdot \mathbf{E}(\mathbf{x}) = J^0(\mathbf{x}) \, , \quad \nabla \times \mathbf{B}(\mathbf{x}) = 0 \, , \quad (2.17a) \\
\nabla \cdot \mathbf{B}(\mathbf{x}) = 0 \, , \quad \nabla \times \mathbf{E}(\mathbf{x}) = 0 \, . \quad (2.17b)
\]

These equations describe the static screening of electric fields inside the plasma. In the static situation the magnetic fields obey the same equations as in the vacuum. In the non-static situation, magnetic fields also suffer a dynamical screening in the plasma, but we will not be concerned in this article about dynamical effects.

Let us finally recall that the static screening effects described in this subsection have been reproduced in the context of perturbative QED in the high temperature limit.

**B. Static Magnetic Screening in the Plasma of Magnetic Charges**

In this subsection the effects of static screening in a plasma of magnetic charges are derived.

Let us first recall the classical equations of motion of a particle carrying a magnetic charge \(\tilde{e}\), with mass \(m\), and transversing a worldline \(y^\mu(\tau)\) \[[8]\]

\[
m \frac{dy^\mu(\tau)}{d\tau} = p^\mu(\tau) \, , \quad (2.18a) \\
m \frac{dp^\mu(\tau)}{d\tau} = \tilde{e} \ast F^{\mu\nu}(y(\tau)) p_\nu(\tau) \, . \quad (2.18b)
\]

In a self-consistent picture, these are augmented with the field equations

\[
\partial_\nu F^{\nu\mu}(x) = 0 \, , \quad (2.19) \\
\partial_\nu \ast F^{\nu\mu}(x) = \tilde{j}^\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} \tilde{j}^\mu(x) \, . \quad (2.20)
\]

The magnetic current is computed for each particle species as

\[
\tilde{j}^\mu(x) = \tilde{e} \int d\tau \frac{dy^\mu(\tau)}{d\tau} \delta^{(4)}(x - y(\tau)) \, , \quad (2.21)
\]

and it is conserved \(\partial_\mu \tilde{j}^\mu(x) = 0\).

Comparing Eqs. \((2.1)-(2.4)\) and \((2.18)-(2.21)\), we see that they are symmetric under the interchange of electric and magnetic fields \((\mathbf{E}, \mathbf{B}) \rightarrow (\mathbf{B}, -\mathbf{E})\), and electric charges by magnetic ones. This is the so called duality symmetry of electromagnetism.

In the presence of a magnetic charge, and due to the absence of the Bianchi identity \((2.4)\), it is not ensured that the electromagnetic field can be derived globally from a vector gauge field as \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\). However, one can still define a vector gauge field which obeys that condition outside the worldline of the magnetic charge. Therefore, it is possible to define a vector gauge field \(A_\mu\) locally. For the case of the point magnetic charge, it is
enough to define two different vector gauge fields $A_\mu$ in different space-time regions, the two solutions being related in their common domain of definition by a gauge transformation\textsuperscript{1}. However, it would be possible to derive the dual electromagnetic field from a dual vector gauge field $\tilde{A}_\nu$, such that $*F_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$, since $\partial_\mu F_{\mu\nu} = 0$, and $F^{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} * F_{\alpha\beta}$.

The above equations show the exact duality of electromagnetism when electric and magnetic degrees of freedom are interchanged. Due to this fact, one naturally expects that the propagation properties of electric fields in the plasma of electric charges are the same as those of magnetic fields in the plasma of magnetic charges. In particular, this is true for static screening. Although from the duality symmetry arguments this is obvious, let us show how one would derive the static screening of magnetic fields.

Let $\tilde{f}(x, p)$ be the probability distribution function of finding a particle with mass $m$ carrying a magnetic charge $\tilde{e}$ in the state $(x, p)$. The corresponding set of dual Vlasov equations can be derived from the equations of motion (2.18), and it reads

\begin{align}
  p^\mu \left[ \frac{\partial}{\partial x^\mu} - \tilde{e} * F_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} \right] \tilde{f}(x, p) &= 0 , \\
  \partial_\nu F^{\nu\mu}(x) &= 0 , \\
  \partial_\nu * F^{\nu\mu}(x) &= \tilde{j}^\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} \tilde{j}^\mu(x) .
\end{align}

(2.22)

(2.23)

(2.24)

Now each particle species magnetic current is obtained from the corresponding distribution function as

\[ \tilde{j}^\mu(x) = \tilde{e} \int dP \; p^\mu \tilde{f}(x, p) . \]

(2.25)

Let us consider a neutral plasma of magnetic charges which is initially at equilibrium. If the system is perturbed by a weak field, then $\tilde{f}$ can be found in the form

\[ \tilde{f}(x, p) = \tilde{f}^{(0)}(p_0) + \tilde{e} \tilde{f}^{(1)}(x, p) + ... , \]

(2.26)

where $\tilde{f}^{(0)}$ is, up to a normalization constant, the Fermi-Dirac equilibrium distribution function $n_F(p_0)$. The equation obeyed by $\tilde{f}^{(1)}$ reads

\[ p^\mu \frac{\partial}{\partial x^\mu} \tilde{f}^{(1)}(x, p) = p^\mu * F_{\mu\nu}(x) \frac{d}{dp_\nu} \tilde{f}^{(0)}(p_0) . \]

(2.27)

Notice that only the magnetic field enters into the r.h.s. of (2.27), since $* F_{i0} = -B_i$.

In the ultrarelativistic limit, i.e. taking $m = 0$, the conserved magnetic current in Fourier space is given by

\[ \frac{1}{\Lambda} \text{Alternatively, one could work with an unique vector gauge field } A_\mu \text{ with a Dirac string attached to the monopole. We prefer the Wu-Yang construction which eliminates references to the Dirac string (see Subsec. [3]).} \]
\[ \tilde{J}^\mu(k) = -i\tilde{m}_D^2 \int \frac{d\Omega_q}{4\pi} \frac{v^\mu}{v \cdot k + i\epsilon} \mathbf{v} \cdot \mathbf{B}(k), \]  

(2.28)

where \( \tilde{m}_D^2 = \tilde{e}^2 T^2 / 3 \) is the magnetic Debye mass squared. The notation that has been used above is the same as in Eq. (2.13), that is, retarded boundary conditions have been implemented, and \( v^\mu \) is the four light-like velocity of the particle.

In the static situation \( \tilde{J}^i = 0 \), and the dual Kubo equations read

\[
\begin{align*}
\nabla \cdot \mathbf{E}(x) &= 0, \\
\nabla \times \mathbf{B}(x) &= 0, \\
\n\nabla \cdot \mathbf{B}(x) &= \tilde{J}^0(x), \\
\n\nabla \times \mathbf{E}(x) &= 0,
\end{align*}
\]

(2.29a, 2.29b)

where

\[ \tilde{J}^0(x) = -i\tilde{m}_D^2 \int \frac{d^3k}{(2\pi)^3} \frac{k \cdot B(k)}{k^2} e^{ik \cdot x}. \]

(2.30)

These equations describe the static screening of magnetic fields. Notice that they could have been obtained from Eqs. (2.17) just by using duality symmetry arguments.

Since the plasma is composed of quantum particles, the magnetic charge \( \tilde{e} \) is subject to the Dirac quantization condition

\[ e \tilde{e} = 2\pi n, \quad n \in \mathbb{Z}. \]

(2.31)

Therefore, the magnetic Debye mass can be written in terms of the electric charge as

\[ \tilde{m}_D = \frac{T}{\sqrt{3}e} 2\pi n. \]

(2.32)

Let us notice that in the perturbative regime \( e \ll 1 \), the expansion performed in Eq. (2.26) would have failed.

### III. THE ULTRARELATIVISTIC NON-ABELIAN PLASMA

#### A. Static Screening of Color Electric Fields in the Plasma of Non-Abelian Charges

Consider a particle bearing a non-Abelian \( SU(N) \) color charge \( Q^a, \ a = 1, \ldots, N^2 - 1 \), traversing a worldline \( y^\mu(\tau) \). The Wong equations describe the dynamical evolution of the variables \( x^\mu, p^\mu \) and \( Q^a \) (we also neglect here the effect of spin):

\[
\begin{align*}
\left. m \frac{dy^\mu(\tau)}{d\tau} \right|_{y^\mu(\tau)} &= p^\mu(\tau), \\
\left. m \frac{dp^\mu(\tau)}{d\tau} \right|_{y^\mu(\tau)} &= g Q^a(\tau) F^\mu_\nu(y(\tau)) p_\nu(\tau), \\
\left. m \frac{dQ^a(\tau)}{d\tau} \right|_{y^\mu(\tau)} &= -g f^{abc} p^\mu(\tau) A^b_\mu(y(\tau)) Q^c(\tau),
\end{align*}
\]

(3.1a, 3.1b, 3.1c)
where \( f^{abc} \) are the structure constants of the group, \( F^a_{\mu\nu} \) denotes the field strength, which is defined as

\[
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu .
\] (3.2)

The color fields obey the Yang-Mills equation

\[
[D_\nu F^{\nu\mu}]^a(x) = J^{\mu a}(x) = \sum_{\text{species}} \sum_{\text{helicities}} j^{\mu a}(x) ,
\] (3.3)

where the color current associated to each colored particle is

\[
j^{\mu a}(x) = g \int d\tau p_\mu(\tau) Q^a(\tau) \delta^{(4)}(x - y(\tau)) .
\] (3.4)

As a consequence of the definition (3.2), the color fields obey the non-Abelian Bianchi identity

\[
[D_\nu \ast F^{\nu\mu}]^a(x) = 0 ,
\] (3.5)

where \( \ast F^{\nu\mu} = \frac{1}{2} \epsilon^{\nu\mu\rho\sigma} F^a_{\rho\sigma} \).

The main difference between the equations of electromagnetism (2.1) and the Wong equations (3.1), apart from their intrinsic non-Abelian structure, comes from the fact that color charges precess in color space, and therefore they are dynamical variables. Equation (3.1c) guarantees that the color current associated to each colored particle is covariantly conserved

\[
(D_\mu j^{\mu})^a(x) = \partial_\mu j^{\mu a}(x) + gf_{abc} A^b_\mu(x) j^{\mu c}(x) = 0 ,
\] (3.6)

therefore preserving the consistency of the theory. Notice that (3.6) is required as a compatibility condition, since in applying a covariant derivative \( D_\mu \) to (3.3) the l.h.s of the equation should vanish.

Let us formulate the statistical description of a plasma of colored particles in their phase-space. The usual \((x,p)\) phase-space is enlarged to \((x,p,Q)\) by including color degrees of freedom for colored particles. Physical constraints are enforced by inserting delta-functions in the phase-space volume element \(dx\,dp\,dQ\). The momentum measure, chosen as in (2.10), guarantees positivity of the energy and on-shell evolution. The color charge measure enforces the conservation of the group invariants, e.g., for \(SU(3)\),

\[
dQ = d^8Q \, \delta(Q_a Q^a - q_2) \, \delta(d_{abc} Q^a Q^b Q^c - q_3) ,
\] (3.7)

where the constants \(q_2\) and \(q_3\) fix the values of the Casimirs and \(d_{abc}\) are the totally symmetric group constants. The color charges which now span the phase-space are dependent variables. These can be formally related to a set of independent phase-space Darboux variables [11]. For the sake of simplicity, the standard color charges will be used in the remaining part of this Section.

The one-particle distribution function \(f(x,p,Q)\) denotes the probability for finding the particle in the state \((x,p,Q)\). In the collisionless case, it evolves in time via a transport
equation \( \frac{df}{dt} = 0 \). Using the equations of motion (3.1), it becomes the Boltzmann equation \[ p^\mu \left[ \frac{\partial}{\partial x^\mu} - g Q_a F^a_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} - g f_{abc} A^b_\mu(x) Q^c \frac{\partial}{\partial Q_a} \right] f(x, p, Q) = 0 . \] (3.8)

A complete, self-consistent set of non-Abelian Vlasov equations for the distribution function and the mean color field is obtained by augmenting the Boltzmann equation with the Yang-Mills equations:

\[ [D_\nu F^{\nu\mu}]^a(x) = J^a_\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} j^\mu a(x) , \] (3.9)

where the color current \( j^\mu a(x) \) for each particle species is computed from the corresponding distribution function as

\[ j^\mu a(x) = g \int dP dQ p^\mu UQU^{-1} f(x, p, Q) . \] (3.10)

Notice that if the particle’s trajectory in phase-space would be known exactly, then Eq. (3.10) could be expressed as in Eq. (3.4). Furthermore, the color current (3.10) is covariantly conserved, as can be shown by using the Boltzmann equation [11].

The Wong equations (3.1) are invariant under the finite gauge transformations (in matrix notation)

\[ \bar{x}^\mu = x^\mu , \quad \bar{p}^\mu = p^\mu , \quad \bar{Q} = UQU^{-1} , \quad \bar{A}_\mu = U A_\mu U^{-1} - \frac{1}{g} U \frac{\partial}{\partial x^\mu} U^{-1} , \] (3.11)

where \( U = U(x) \) is a group element.

It can be shown [11] that the Boltzmann equation (3.8) is invariant under the above gauge transformation if the distribution function behaves as a scalar

\[ \tilde{f}(\bar{x}, \bar{p}, \bar{Q}) = f(x, p, Q) . \] (3.12)

To check this statement it is important to note that under a gauge transformation the derivatives appearing in the Boltzmann equation (3.8) transform as [11]:

\[ \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial \bar{x}^\mu} - 2 \text{Tr} \left( \left[ \frac{\partial}{\partial \bar{x}^\mu} U^{-1} , \bar{Q} \right] \frac{\partial}{\partial \bar{Q}} \right) , \quad \frac{\partial}{\partial p^\mu} = \frac{\partial}{\partial \bar{p}^\mu} , \quad \frac{\partial}{\partial \bar{Q}} = U^{-1} \frac{\partial}{\partial Q} U , \] (3.13)

that is, they are not gauge invariant by themselves. Only the specific combination of the spacial and color derivatives that appears in (3.8) is gauge invariant.

The color current (3.10) transforms under (3.11) as a gauge covariant vector:

\[ \tilde{j}^\mu(\bar{x}) = g \int dP dQ p^\mu UQU^{-1} f(x, p, Q) = U j^\mu(x) U^{-1} . \] (3.14)

This is due to the gauge invariance of the phase-space measure and to the transformation properties of \( f \).
Let us consider now a gluon plasma, that is, a plasma of particles carrying a non-Abelian charge in the adjoint representation, and which is initially at equilibrium. The system is disturbed by a weak color electromagnetic field, and one looks for the response of the plasma. The distribution function can be expanded in powers of $g$ as:

$$f = f^{(0)} + gf^{(1)} + \ldots,$$

(3.15)

where $f^{(0)}$ is, up to a normalization constant, the Bose-Einstein equilibrium distribution function

$$n_B(p_0) = \frac{1}{e^{p_0/T} - 1}.$$  

(3.16)

The Boltzmann equation (3.8) for $f^{(1)}$ reduces to [11]

$$p^\mu \left( \frac{\partial}{\partial x^\mu} - g f^{abc} A^b_\mu(x) Q_c \frac{\partial}{\partial Q^a} \right) f^{(1)}(x, p, Q) = p^\mu Q_a F^a_\mu(x) \frac{d}{dp_0} f^{(0)}(p_0).$$

(3.17)

Notice that a complete linearization of the equation in $A^a_\mu$ would break the gauge invariance of the transport equation, which is preserved in this approximation. But notice as well that this approximation tells us that $f^{(1)}$ also carries a $g$-dependence.

One can get the equation that the color current density $J^a_\mu(x, p)$ obeys by multiplying (3.17) by $p^\mu$ and $Q_a$ and then integrating over the color charges. For gluons in the adjoint representation

$$\int dQ Q_a Q_b = N \delta_{ab},$$

(3.18)

one finally gets, after summing over helicities,

$$\left[ p \cdot D J^\mu(x, p) \right]^a = 2 g^2 N p^\mu p^\nu F^a_\mu(x) \frac{d}{dp_0} f^{(0)}(p_0).$$

(3.19)

Notice that only the color electric field enters in the r.h.s. of the above equation. Thus, only the color electric field is screened in the static situation.

The induced color current can be expressed in terms of the parallel transporter $\Phi$ as [13]

$$J^\mu_a(x) = m_D^2 \int \frac{d\Omega}{4\pi} v^\mu \int_0^\infty du \Phi_{ab}(x, x - vu) v \cdot E_b(x - vu)$$

(3.20)

where $m_D^2 = g^2 T^2 N/3$ is the Debye mass squared, and $\Phi$ obeys the equation

$$\frac{\partial}{\partial u} \Phi_{ab}(x, x - vu) = g \Phi_{ac}(x, x - vu) f_{cde} v^\mu A^d_\mu(x - vu)$$

(3.21)

with the initial boundary condition $\Phi_{ab}(x, x) = \delta_{ab}$. The four vector $v^\mu$ is the velocity vector of the particles of the plasma, and in the ultrarelativistic limit is light-like. Retarded boundary conditions have also been implemented in Eq. (3.20).

Alternatively, in momentum space $J^\mu_a(k)$ may be expressed as an infinite power series in the vector gauge field $A^a_\mu(k)$ [14].
In the static situation the color current simplifies drastically and may be expressed as
\[ \mathbf{J}_\mu(x) = -m_D^2 \delta_{\mu 0} \mathbf{A}_0(x) \] \[ 15 \].

The non-Abelian Kubo equations were first derived in Ref. \[ 14 \]. In the static limit they read
\[ (\mathbf{D} \cdot \mathbf{E})_a = J_a^0(x) , \quad (\mathbf{D} \times \mathbf{B})_a + g f_{abc} A_b^0 \mathbf{E}^c = 0 , \quad (3.22) \]
\[ (\mathbf{D} \cdot \mathbf{B})_a = 0 , \quad (\mathbf{D} \times \mathbf{E})_a - g f_{abc} A_b^0 \mathbf{B}^c = 0 . \quad (3.23) \]

The static non-Abelian Kubo equations have been studied in Ref. \[ 15 \] and they describe the static screening of color electric fields. Furthermore, the color electric screening effects described by these equations are reproduced in the context of resummed perturbative QCD in the high temperature limit \[ 2 \].

B. Static Screening of Color Magnetic Fields in the Plasma of Non-Abelian Magnetic Charges

In this subsection the screening effects in a plasma of non-Abelian magnetic charges are derived. Those particles are the natural non-Abelian analogues of the magnetic Dirac monopoles that were studied in Subsec. \[ 1B \].

As opposed to what happened in the Abelian theory, one cannot appeal to duality symmetry arguments to describe the dynamics of non-Abelian magnetic charges. It is a well-known fact that a pure Yang-Mills theory without matter is not symmetric under the interchange of color electric and magnetic degrees of freedom \[ 10 \]. This may be understood as follows. In the absence of matter, the Yang-Mills equation and the non-Abelian Bianchi identity both involve a covariant derivative, but not a “dual” covariant derivative. Thus, in terms of the dual field \( *F_{\mu \nu}^a \) the Yang-Mills equation cannot be interpreted as the non-Abelian Bianchi identity for \( *F_{\mu \nu}^a \). In general, it is not possible to find a dual vector gauge field \( \hat{A}_\mu^a \) which is related to \( *F_{\mu \nu}^a \) as \( A_\mu^a \) is related to \( F_{\mu \nu}^a \) in (3.2). It has been realized in the literature that the dual of a Yang-Mills theory is a Freedman-Townsend like theory \[ 17 \], \[ 18 \]. The fundamental object of that theory is an antisymmetric two index tensor, \( \bar{F}_{\mu \nu}^a \). One can define in that theory a vector gauge field potential \( V_\mu^a \) which acts as a parallel transport for the phases of the charged particles, but which is not related to \( \bar{F}_{\mu \nu}^a \) as in a Yang-Mills theory.

The equations of motion of a particle of mass \( m \), carrying a non-Abelian magnetic charge of \( SU(N) \) \( \tilde{Q}_a \), with \( a = 1, ..., N^2 - 1 \), and transversing a worldline \( y^\mu(\tau) \) have been derived in Ref. \[ 17 \]. The derivation was made there using a variational principle in loop space. The equations written in terms of the color gauge fields are
\[ m \frac{d y^\mu(\tau)}{d \tau} = p^\mu(\tau) , \quad (3.24a) \]
\[ m \frac{d p^\mu(\tau)}{d \tau} = \tilde{g} \tilde{Q}^a(\tau) *F_{\mu \nu}^a(y(\tau)) p^\nu(\tau) , \quad (3.24b) \]
\begin{equation}
\frac{m}{\tau} \ddot{Q}^a(\tau) = -g f^{abc} p^\mu(\tau) A^b_\mu(y(\tau)) \dot{Q}^c(\tau) .
\end{equation}

These are augmented with the field equations

\begin{equation}
[D_\nu F^{\nu\mu}]_a(x) = 0 , \tag{3.25}
\end{equation}

\begin{equation}
[D_\nu * F^{\nu\mu}]_a(x) = \tilde{J}_a^\mu(x) = \sum_{\text{species}} \sum_{\text{helicities}} \tilde{j}^\mu_a(x) , \tag{3.26}
\end{equation}

where

\begin{equation}
\tilde{j}^\mu_a(x) = \tilde{g} \int d\tau p^\mu(\tau) \dot{Q}^a(\tau) \delta^{(4)}(x - y(\tau)) . \tag{3.27}
\end{equation}

The above current is covariantly conserved $[D_\mu \tilde{J}_a^\mu]_a(x) = 0$, which is required by consistency, as may be realized in applying a covariant derivative to (3.26).

Notice that a new coupling constant $\tilde{g}$ appears in the equations as coupling between the color electromagnetic fields and the non-Abelian magnetic charges. In principle, this is a new variable in the system. However, there is a Dirac quantization condition relating $g$ and $\tilde{g}$. We will come back to this point later on.

The above equations are the dual of the Wong equations written in Subsec. III A. Notice that they are not symmetric under the interchange of color magnetic and electric degrees of freedom, since the same covariant derivative $D_\mu$ appears in both of them. Some caution should be taken at this stage. Exactly as it happened in the Abelian situation, in the absence of the non-Abelian Bianchi identity one cannot define globally a vector gauge field $A^a_\mu$ obeying the field equations. It could be defined locally, outside the monopole worldline. It is possible to define different $A^a_\mu$ in different space-time regions, the different solutions or patches being related in their common domain of definition by a gauge transformation.

Let us stress that the non-Abelian electric and magnetic charges $Q_a(\tau)$ and $\dot{Q}_a(\tau)$ live in the same group manifold, and obey the same kind of dynamical evolution. Both transform under gauge transformations in the same way.

With the above classical equations of motion it is possible to derive the dual non-Abelian Vlasov equations. They read

\begin{equation}
p^\mu \left[ \frac{\partial}{\partial x_\mu} - \tilde{g} \dot{Q}_a * F^a_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} - g f^{abc} A^b_\mu(x) \dot{Q}^c \frac{\partial}{\partial \dot{Q}_a} \right] \tilde{f}(x, p, \dot{Q}) = 0 , \tag{3.28a}
\end{equation}

\begin{equation}
[D_\nu F^{\nu\mu}]_a(x) = 0 , \tag{3.28b}
\end{equation}

\begin{equation}
[D_\nu * F^{\nu\mu}]_a(x) = \tilde{J}^{\mu a}(x) = \sum_{\text{species}} \sum_{\text{helicities}} \tilde{j}^{\mu a}(x) , \tag{3.28c}
\end{equation}

where

\begin{equation}
\tilde{j}^{\mu a}(x) = \tilde{g} \int dPd\dot{Q} p^\mu \dot{Q}^a \tilde{f}(x, p, \dot{Q}) . \tag{3.29}
\end{equation}

The color magnetic charge measure $d\dot{Q}$ is defined as in Eq. (3.7).
The gauge transformation properties of the above equations will not be discussed here, as they turn out to be exactly the same as their dual partners.

Let us derive now the response of the non-Abelian magnetic plasma to a weak field disturbance. Let us suppose that the neutral plasma of non-Abelian magnetic charges in the adjoint representation is initially at equilibrium. Then one can look for a solution of the form

\[ \tilde{f} = \tilde{f}^{(0)} + \tilde{g}\tilde{f}^{(1)} + \ldots, \]  

(3.30)

where \( \tilde{f}^{(0)} \) is, up to a normalization constant, the Bose-Einstein equilibrium distribution function. Then \( \tilde{f}^{(1)} \) obeys the equation

\[ p^\mu \left( \partial_{x^\mu} - g f^{abc} A^b_{\mu}(x) \partial_{Q^c} \right) \tilde{f}^{(1)}(x, p, \tilde{Q}) = p^\mu \tilde{Q}^a \ast F^a_{\mu 0}(x) \frac{d}{dp_0} \tilde{f}^{(0)}(p_0). \]  

(3.31)

A weak coupling expansion in \( \tilde{g} \), the natural coupling constant of the problem, has been performed above. To preserve the gauge symmetry of the above Boltzmann equation, the term linear in \( A^a_\mu \) in the l.h.s. of (3.31) has to be kept. This tells us that \( \tilde{f}^{(1)} \) has a dependence on the coupling constant \( g \). The two constants \( g \) and \( \tilde{g} \) are related by a Dirac quantization condition that will be discussed later on.

From Eq. (3.31), one can derive the static screening of color magnetic fields. In order to do that, one should follow the same procedure as in the previous subsections. Notice that now only the color magnetic field enters in the r.h.s of (3.31).

The equation obeyed by the current density is obtained after integrating over the magnetic charges \( \tilde{Q}_a \), and summing over helicities, and it reads

\[ [p \cdot D \tilde{J}^\mu(x, p)]^a = 2 \tilde{g}^2 N p^\mu p^\nu \ast F^a_{\nu 0}(x) \frac{d}{dp_0} \tilde{f}^{(0)}(p_0). \]  

(3.32)

The solution of the above equation may be written as

\[ \tilde{J}^\mu_a(x) = \tilde{m}_D^2 \int \frac{d\Omega_2}{4\pi} v^\mu \left( \int_0^{\infty} du \Phi_{ab}(x, x - vu) v \cdot B^b(x - vu) \right), \]  

(3.33)

where \( \Phi \) is the parallel transporter, which obeys also Eq. (3.21). Here \( \tilde{m}_D^2 = \tilde{g}^2 T^2 N/3 \) is the magnetic Debye mass squared.

The dual Non-Abelian Kubo equation read in the static limit, therefore

\[ (D \cdot E)_a = 0, \quad (D \times B)_a + g f_{abc} A^b_0 E^c = 0, \]  

(3.34)

\[ (D \cdot B)_a = \tilde{J}^0_a(x), \quad (D \times E)_a - g f_{abc} A^b_0 B^c = 0, \]  

(3.35)

and they describe the static screening of color magnetic fields.

There is a Dirac quantization condition relating the two coupling constant \( g \) and \( \tilde{g} \), which depends on the Lie group under consideration. For the case considered here, where all the matter is in the adjoint representation, then the gauge field theory which is based upon the Lie algebra \( su(N) \) has as a global Lie group \( SU(N)/Z_N \) and not \( SU(N) \). Here \( Z_N \) is the \( N \)-element finite group consisting in the \( N \)-th roots of unity, that is, the integral powers of
exp \((2\pi i/N)\). This is due to the fact that two \(SU(N)\) matrices that differ only by a factor belonging to \(Z_N\) will be represented by the same matrix in the adjoint representation. For a \(SU(N)/Z_N\) theory the Dirac quantization condition reads \([22] , [19] , [21]\)

\[
g \tilde{g} = \frac{2\pi n}{N} \quad n \in \mathbb{Z} .
\]

Therefore, the dual Debye mass is written in terms of \(g\) as

\[
\tilde{m}_D = \frac{T}{g\sqrt{3N}} 2\pi n
\]

Therefore, for small values of \(N\) the static magnetic screening effects could not have been reproduced by using perturbation theory in \(g\), as then the expansion of Eq. (3.30) would not hold. However, notice that for \(N \to \infty\), both \(g\) and \(\tilde{g}\) can both be small, and therefore perturbative expansions in \(g\) and \(\tilde{g}\) can both be valid.

\section*{IV. STATIC MAGNETIC MONOPOLE FIELDS}

In this section exact solutions to the dual Kubo equations are found. Those solutions describe screened magnetic fields in the plasma of magnetic charges. Some known results on how to construct magnetic monopole fields in the vacuum for the Abelian and non-Abelian theories are first reviewed.

\subsection*{A. Monopole Fields in the Vacuum}

The magnetic field created by a magnetic charge \(\tilde{e}\) which is at rest at the origin of coordinates is given by

\[
B = \frac{\tilde{e}}{4\pi r^3} .
\]

As already explained in Subsec. [11], it is not possible to construct a global vector gauge field \(A_\mu(x)\) which generates the above magnetic field. However, it is possible to find a local vector gauge field which is defined everywhere except on a “Dirac string”. That was the original construction of magnetic monopole fields due to Dirac. Wu and Yang [11] showed that it is also possible to construct vector gauge fields without references to Dirac strings as follows. Let us denote by \(R\) the space region surrounding the magnetic monopole. Dividing \(R\) into two regions, \(R_+\) and \(R_-\) defined as (in spherical coordinates \((r, \theta, \phi)\)),

\[
\begin{align*}
R_+ : & \quad 0 \leq \theta < \pi/2 + \delta, \quad r > 0, \quad 0 \leq \phi < 2\pi , \\
R_- : & \quad \pi/2 - \delta < \theta \leq \pi, \quad r > 0, \quad 0 \leq \phi < 2\pi ,
\end{align*}
\]

with an overlap region \(\pi/2 - \delta < \theta < \pi/2 + \delta\), where \(0 < \delta \leq \pi/2\). In each region one may take [11]
\[
A_+ = \frac{\tilde{e}}{4\pi r} \frac{(1 - \cos \theta)}{\sin \theta} \hat{e}_\phi ,
\]
\[
A_- = -\frac{\tilde{e}}{4\pi r} \frac{(1 + \cos \theta)}{\sin \theta} \hat{e}_\phi .
\]

These two vector gauge potentials reproduce the magnetic field (4.4) in their respective domain of definition. Furthermore, in the overlap region the two vector gauge fields are related by a gauge transformation

\[
A_+ - A_- = \frac{\tilde{e}}{2\pi r} \hat{e}_\phi = \nabla \left( \frac{\tilde{e} \phi}{2\pi} \right).
\]

In a quantum description, this gauge transformation implies a change in the wavefunction \( \psi \) of a particle, \( \psi \rightarrow \exp \left( -ie\tilde{e} \phi /2\pi \right) \psi \). After requiring the wavefunction be single-valued, one arrives at the Dirac quantization condition \( \tilde{e} = 2\pi n \).

One can easily generalize the previous results to the non-Abelian case. Let us consider first for definiteness the \( SU(2) \) Lie group and take as its infinitesimal generators \( t_a = -\frac{i}{2} \sigma_a \), where \( \sigma_a \) are the Pauli matrices. A non-Abelian magnetic monopole field may be constructed from the Abelian one by taking the same value (4.1) in a particular direction in color space. For example,

\[
B_3 = \frac{\tilde{g}}{4\pi} \frac{r}{r^3} , \quad B_1 = B_2 = 0 .
\]

The vector gauge fields which reproduce the above color magnetic field are obtained from taking in the \( a = 3 \) direction in color space the Abelian vector gauge fields (4.4), while \( A_1^\mu = A_2^\mu = 0 \).

In general, for a simple Lie group, the corresponding magnetic monopole fields can be constructed from their Abelian counterparts, just by multiplying the Abelian fields by a constant matrix \( \tilde{Q} \) living in a specific representation of the Lie group. Those are solutions to the Yang-Mills equations in the presence of a static point magnetic source \( \tilde{J}_\mu = \delta_{\mu 0} \tilde{g} \tilde{Q} \delta^{(3)}(r) \).

Due to the Abelian character of these non-Abelian monopoles, the Dirac quantization condition can be easily worked out, and it reads

\[
\exp \left( 2\pi \frac{\tilde{g} \tilde{Q}}{2\pi} \right) = 1 .
\]

The above quantization condition is sensitive to the global structure of the Lie group under consideration, and it could be different for different Lie groups sharing the same Lie algebra. For example, for \( SU(2) \) and \( SO(3) \), Eq. (4.7) has different implications.

There is a dynamical [21], as well as topological classification [22] of these non-Abelian monopoles. The topological classification of the monopoles associates each class of monopole to each different element of the first homotopy group \( \pi_1(G) \) of the Lie group \( G \) under consideration. A stability analysis of the non-Abelian magnetic monopole fields was performed in [20], finding that there is one stable non-Abelian monopole field for each topological class.
B. Static Magnetic Monopole Fields in the Hot Plasmas

In this subsection some exact solutions to the static dual Abelian and non-Abelian Kubo field equations are presented. In order to find those solutions, rather than solving the corresponding differential equations, we will take profit of the following facts. First, in the Abelian situation, the solutions to the “magnetic” Kubo equations can be found from those of the “electric” ones, just by making use of the duality symmetry of electromagnetism. Second, solutions to the non-Abelian Kubo equations can always be constructed from the Abelian ones, as we have already explained. Using these facts, one can easily construct solutions to the dual non-Abelian Kubo equations which have an Abelian character.

Let us first recall how to find exact solutions to the static Kubo equations (2.17). Those can be easily solved in terms of the electric potential $A_0$, with $E = -\nabla A_0$. Then, the equation obeyed by $A_0$ is

$$\left( \nabla^2 - m_D^2 \right) A_0(x) = 0 ,$$

(4.8)

With spherical symmetry, and in spherical coordinates $(r, \theta, \phi)$, Eq. (4.8) can be easily separated. The angular part of the solution is given in terms of spherical harmonics, and the radial part is expressed in terms of modified spherical Bessel functions. For the first harmonic, or equivalently, the monopole term in a multipole expansion, and discarding the solution growing exponentially, one then finds

$$A_0 = a_0 e^{-m_D r} , \quad A_i = 0 .$$

(4.9)

which corresponds to a screened radial electric field.

The non-Abelian Kubo equations have been studied in the literature using different Ansätze [23]. The static case for the $SU(2)$ group with spherical symmetry was considered in Ref. [15]. Two particular solutions were found there. The first one corresponds to the Yang-Mills vacuum. The second one can actually be constructed by taking the Abelian solution in a specific direction of isospin space. Thus

$$A_0^a = r^a a_0 e^{-m_D r} , \quad A_i^a = 0 ,$$

(4.10)

which describes a screened color electric field. Pure non-Abelian solutions were also studied in Ref. [15], but it was shown there that they tend asymptotically to either the Yang-Mills vacuum or to (4.10).

Let us discuss now the magnetic Kubo equations. In the Abelian case, it is possible to find solutions to the Eq. (2.29), just by writing the magnetic field in terms of a “magnetic” potential, $B = -\nabla \tilde{A}_0$. This is possible since in this case $\nabla \times B = 0$, as we have already mentioned. Therefore, in terms of $\tilde{A}_0$, the field equation becomes

$$\left( \nabla^2 - \tilde{m}_D^2 \right) \tilde{A}_0(x) = 0 ,$$

(4.11)

which is exactly the same equation as (4.8), and therefore, has the same solutions. In particular, with spherical symmetry, the monopole solution is
\[ \tilde{A}_0 = \tilde{a}_0 \frac{e^{-\tilde{m}_D r}}{r} , \quad \tilde{A}_i = 0 , \]  

(4.12)

which gives the screened radial magnetic field

\[ \mathbf{B} = -\tilde{a}_0 \frac{r}{r^3} (1 + \tilde{m}_D r) e^{-\tilde{m}_D r} \]  

(4.13)

Let us consider now the non-Abelian case, for the group \( SO(3) \equiv SU(2)/Z_2 \). In this case it is possible to construct easily solutions from the Abelian ones. It is enough to consider vanishing vector gauge field configurations in all except one specific direction in color space. Then the dual non-Abelian Kubo equations reduce to the dual Abelian ones. For example, one solution is given by taking in the third direction of isospin space

\[ B_3 = -\tilde{a}_0 \frac{r}{r^3} (1 + \tilde{m}_D r) e^{-\tilde{m}_D r} , \quad B_1 = B_2 = 0 , \]  

(4.14)

which describes a screened color magnetic field.

V. DISCUSSION

The purpose of this article has been finding a classical mechanism which generates the thermal screening of magnetic fields. It has been shown that at finite \( T \), and in the presence of magnetic charges, magnetic fields are screened. This effect can be easily understood in the Abelian theory. The duality symmetry of electromagnetism allows to derive the magnetic screening from the electric one, without further complications. A non-Abelian theory is not symmetric under a duality transformation, but still it is possible to show that also color magnetic fields can be screened. In both cases, the effects of magnetic screening are proportional to the inverse of the gauge coupling constant.

The question which remains to be answered is whether the polarization effects that have been described in this article could be relevant in realistic quantum gauge theories, such as QED or QCD. The question can be actually reduced to answer if there are magnetic monopoles in those theories.

It seems obvious that QED does not possess magnetic monopoles. Thus, it is not expected that the mechanism described in this article takes place in QED. A pure Abelian gauge theory does not suffer from IR problems. The IR divergences of QED that arise in the magnetic sector of the theory at finite \( T \) can actually be cured in the same way than at zero \( T \) [24].

In a pure Yang-Mills theory 't Hooft [25] showed that in the maximal Abelian gauge that theory has magnetic monopoles. Actually this occurs in several different Abelian projections of the non-Abelian theory. The idea of describing the QCD ground state as a condensate of magnetic monopoles has deserved much attention [26], [27], since then one could understand the confinement of QCD as a dual Meissner effect. Although these ideas are very appealing, there is not yet a complete gauge independent analytical proof of this confinement scenario, except for the case of some supersymmetric non-Abelian models [28]. However, lattice computations have shown the magnetic monopole dominance for the string tension of QCD at zero temperature [29].
A natural expectation is that if magnetic monopoles explain the confinement of QCD because they condensate at zero temperature, they should also play an important role in explaining the magnetic screening at finite temperature. I have shown that this screening would occur by considering a plasma of magnetic charges. Let me stress that the real situation corresponding to the real QCD plasma would be much more complicated than the simple models that I have described, as then both electric and magnetic polarization phenomena should occur simultaneously. There is also a mismatch between the magnetic mass of order $T/g$ that has been derived here, and an expected one of order $g^2T$. Our result has been obtained on the assumption that the equilibrium density of monopoles is of order $T^3$. However this does not have to be so necessarily. To recover the expected magnetic mass of order $g^2T$, the equilibrium density of magnetic monopoles should be of order $(g^2T)^3$. This would naturally imply that magnetic monopoles are not elementary particles, but that they should be dynamically generated.

Lattice computations could check whether the mechanism of magnetic screening that I have described takes place or not in a pure Yang-Mills theory at finite temperature.

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