Turbulent Effects on Fluid Flow through Disordered Porous Media

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Abstract

The influence of turbulent effects on a fluid flow through a (pseudo) porous media is studied by numerically solving the set of Reynolds-averaged Navier-Stokes equations with the $\kappa$-\$\epsilon$ model for turbulence. The spatial domains are two-dimensional rectangular grids with different porosities obtained by the random placing of rigid obstacles. The objective of the simulations is to access the behavior of the generalized friction factor with varying Reynolds number. A good agreement with the Forchheimer’s equation is observed. The flow distribution at both low and high Reynolds conditions is also analyzed.

1 Introduction

The flux of fluid through microscopically disordered and macroscopically homogeneous porous media is a function of the pressure gradient, which for low Reynolds conditions is well approximated by a linear relation known as Darcy’s law \[1\],

$$\nabla \bar{P} = -\frac{\mu \bar{u}}{k},$$ \hspace{1cm} (1)

where $\bar{P}$ and $\bar{u}$ are the averages of pressure and velocity, respectively, over cross sections perpendicular to the fluid flow, $\mu$ is the fluid viscosity and $k$ is the global permeability. This was formulated by the French engineer Henri Darcy based on experimental observations of the pressure driven flux of water through sand filters. However, experimental observations and numerical simulations have shown that the relation between pressure gradient and mean velocity is indeed nonlinear, and can be well fitted by the so-called Forchheimer’s equation \[2, 3, 4\]

$$- \nabla \bar{P} = \alpha \mu \bar{u} + \beta \rho \bar{u}^2,$$ \hspace{1cm} (2)

where the coefficient $\alpha$ corresponds to the inverse of the permeability and $\beta$ is usually called the inertial parameter. The transition from the linear (Darcy’s law) to the nonlinear regime occurs gradually \[2\] as the Reynolds number increases. In order to understand the connections between the pore morphology
and the macroscopic behavior of the system it is necessary to analyze the spatial pore distribution and relate it to the mechanisms of momentum transfer (inertial, viscous and turbulent). Previous studies have been conducted in this direction leading to successful prediction of permeability coefficients of real porous materials [6]. The role of inertial effects over such a transition has been analyzed through computational simulations in [5, 6] within laminar flow regime through the pore space.

The random aspect of the pore distribution induces a highly heterogeneous local flow which becomes turbulent at high Reynolds regimes. Following the settings of previous numerical works [5, 6], we are going to analyze the combined effects of inertia and turbulence in the behavior of flux of fluid through two-dimensional pseudo-porous systems, whose pore connectivity is based on the ideas of site percolation disorder. To set the geometry for the simulations we start with a square lattice of $64 \times 64$ obstacles from which cells are removed at random until we get a pseudo-porous medium with a prescribed void fraction. To minimize the end effects over the flow field in the pore region we attach a header region in the inlet and a recovery region in the outlet, both of them free of obstacles. The fluid is assumed to be incompressible and Newtonian and we adopt the \( \kappa-\epsilon \) model of turbulence. The velocity field at the inlet is assigned a constant value normal to the boundary. At the outlet we fix a null gradient velocity boundary condition.

One goal of our simulations is to analyze the exactness of Forchheimer’s equation eq.(2) in representing the behavior of turbulent fluid flow in porous media. We rewrite eq.(2) in a form that has been successfully used to correlated experimental data from a large variety of porous materials and flow conditions, viz.,

\[
f - 1 = \frac{1}{\text{Re}'},
\]

where \( f \equiv - \nabla \bar{P}/(\beta \rho \bar{u}^2) \) and \( \text{Re}' \equiv \beta \rho \bar{u}/(\alpha \mu) \). The flow localization over the pore space is quantified by the partition function [6, 8],

\[
\pi = \left( \frac{n}{\sum_{i=1}^{n} q_i^2} \right)^{-1}, \quad \frac{1}{n} \leq \pi \leq 1,
\]

where \( n \) is the total number of fluid cells and \( q_i = E_i/\sum_{j=1}^{n} E_j \), with \( E_i \) being the kinetic energy associated with the fluid cell \( i \). This is a measure of the uniformity of the flow through the available pore space. If the flow is evenly distributed over the entire number of cells, \( \pi \) attains the value 1. On the other hand, if the kinetic energy is concentrated over few cells the function \( \pi \) would be close to the value \( 1/n \).

### 2 Turbulent Flow and the \( \kappa-\epsilon \) model

To account for the turbulence we use the Reynolds averaging approach of decomposing the variables into a mean (ensemble-averaged or time-averaged) and
a fluctuating component

\[ \phi_i = \bar{\phi}_i + \phi'_i \]  

(5)

where \( \phi \) represents a generic variable. For the velocity components we have

\[ u_i = \bar{u}_i + u'_i. \]  

(6)

Substituting the decomposed expressions for the velocity and the pressure into the Navier-Stokes equations for incompressible fluid and dropping the overbar on the mean velocity and mean pressure we get the Reynolds-averaged Navier-Stokes equations

\[ \frac{\partial u_i}{\partial x_i} = 0, \]  

(7)

\[ \rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j}(-\rho u'_i u'_j). \]  

(8)

In order to close these equations, the Reynolds stress term \(-\rho u'_i u'_j\) must be modeled, and a common method employs the Boussinesq hypothesis to relate it to the mean velocity gradients:

\[ -\rho u'_i u'_j = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho \kappa \delta_{ij}. \]  

(9)

In the case of the \( \kappa-\epsilon \) models, two additional transport equations (for the turbulent kinetic energy, \( \kappa \), and the turbulent dissipation rate, \( \epsilon \)) are solved, and \( \mu_t \) is computed as a function of \( \kappa \) and \( \epsilon \),

\[ \mu_t = \rho C_{\mu} \frac{\kappa^2}{\epsilon}, \]  

(10)

with \( C_{\mu} = 0.09 \) a constant. The equations for \( \kappa \) and \( \epsilon \) are

\[ \rho \frac{D\kappa}{Dt} = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_\kappa} \right) \frac{\partial \kappa}{\partial x_i} \right] + G_\kappa - \rho \epsilon \]  

(11)

and

\[ \rho \frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_i} \right] + C_{1\kappa} \frac{\epsilon}{\kappa} G_\kappa - C_{2\kappa} \frac{\epsilon^2}{\kappa}, \]  

(12)

where

\[ G_\kappa = -\rho u'_i u'_j \frac{\partial u_j}{\partial x_i} \]

represents the generation of turbulent kinetic energy by the mean velocity gradients. To evaluate \( G_\kappa \) in a manner consistent with the Boussinesq hypothesis, the Fluent code is implemented with

\[ G_\kappa = \mu_t S^2, \]
where $S$ is the modulus of the mean rate-of-strain tensor, defined as

$$S \equiv \sqrt{2S_{ij}S_{ij}},$$

with the mean strain rate $S_{ij}$ given by

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

We use the Fluent default values for the constants appearing in the above equations, i.e., $C_1 = 1.44$, $C_2 = 1.92$, $\sigma = 1.0$ and $\sigma = 1.3$.

We use the software Fluent to solve these equations in our simulations, which has implemented the $\kappa-\epsilon$ model of turbulence.

The criteria for convergence we used in the simulations is defined in terms of the residuals which provide a measure of the degree to which each of the conservation equations are satisfied throughout the flow field. Residuals are computed by summing the imbalance in each equation for all cells in the domain. The residuals for each flow variable (e.g. velocity, pressure, etc.) give a measure of the error magnitude in the solution at each iteration. In general, a solution can be considered well converged if the normalized residuals are on the order of $10^{-3}$. In all of our simulations, convergence is considered to be achieved only when each of the normalized residuals fall below $10^{-5}$.

3 Experiments

We performed experiments with three values of porosity ($\epsilon = 0.7, 0.8, 0.9$) which are greater than the critical percolation porosity. For each value of $\epsilon$, we generated ten different grids as described above, and with each one we simulated the flows using 17 values of viscosity.

The velocity and pressure fields in the pore, header and recovery regions were numerically obtained through discretization by means of the volume finite-difference technique [7]. In each simulation, after we reached a converged flow, we did a post processing to compute the difference between the averaged values of the pressure on the cross sections at the beginning and at the end of the pore region. We computed also the participation function $\pi$. Then we computed the average over the ten realizations and construct the curves of $\mu \times \Delta P$ and $\mu \times \pi$. Using a linear regression on the graph of $\mu \times \Delta P$ we determined the constants $\alpha$ and $\beta$ and then computed the values of Re’ and $f$. The results are presented through the graphs of Re’ $\times (f - 1)$ and Re’ $\times \pi$.

4 Results and conclusion

Our simulations may be summarized at two graphs, one for Re’ $\times (f - 1)$ and the other for Re’ $\times \pi$. At both graphs we can see an agreement with previous simulations [3,4]. In the graph of Re’ $\times (f - 1)$ we can see that the Forchheimer’s
equation (dashed straight line) is indeed a good fit for the numerical results up to \( \text{Re}' \approx 1 \), for \( e = 0.7 \) and \( e = 0.8 \), and up to \( \text{Re}' \approx 10^4 \) for \( e = 0.9 \).

The inclusion of turbulence makes it possible to extend the simulations to a wider range of Reynolds number, including values that would possible present convergence problem if considering the laminar Navier-Stokes equations. This is due to the addition of the turbulent viscosity which increases the energy dissipation.

The higher the porosity the bigger is the participation function, which expresses the fact that the flow gets more uniform as the porosity increases. Also, we can see that the variation of \( \pi \) increases as the porosity gets higher. We can also identify two plateaus: one for low and the other for high values of \( \text{Re}' \). This is an indication that for low velocities (low \( \text{Re}' \)) conditions, the flux is mainly through a few preferential channels, the number of which increases with the porosity. As the velocity (\( \text{Re}' \)) increases more channels get to be used increasing the participation function \( \pi \), which however, has an upper limit smaller than unity.

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Figure 1: Plots in log-log scale showing the relation between the generalized friction factor $f - 1$ and the modified Reynolds number $Re'$ for the experiments with porosities $e = 0.7, 0.8$ and $0.9$. The dashed straight line represents Forchheimer’s equation (eq. (1)).

Figure 2: Log-linear plots of the modified Reynolds number versus the participation function $\pi$ for the porosity values $e = 0.7, 0.8$. 