Remarks on the renormalization of gauge invariant operators in Yang-Mills theory

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Abstract

A simplified proof of a theorem by Joglekar and Lee on the renormalization of local gauge invariant operators in Yang-Mills theory is given. It is based on (i) general properties of the antifield-antibracket formalism; and (ii) well-established results on the cohomology of semi-simple Lie algebras.
1 Introduction

The BRST transformation in Yang-Mills theory was originally defined without reference to the equations of motion and incorporated only the gauge symmetry [1, 2]. However, it was subsequently found necessary to take the equations of motion into account. This can be achieved by introducing one “antifield” for each field appearing in the path integral, and by defining the BRST variation of the antifields in such a way that the BRST differential implements the equations of motion $D_\mu F_{\mu\nu}^a = 0$ in cohomology. These developments were first pursued in the context of the renormalization of the Yang-Mills field [3]. They then turned out to be crucial for the BRST formulation of arbitrary gauge theories with “open” algebras [4], for which one cannot separate the gauge symmetry from the dynamics. The key role played by the equations of motion and the concept of covariant phase space [1, 2, 3, 4] in the antifield formalism was emphasized in J[9, 10, 11] (see also [12] and references therein for a related but different point of view).

In the case of the pure Yang-Mills field, the complete BRST transformation reads, in the minimal sector containing the vector potential $A_\mu^a$, the ghosts $C^a$, and their respective antifields $A^{*\mu}_a$, $C^{*}_a$,

$$s = \delta + \sigma,$$

where $\delta$ is defined by

$$\delta A_\mu^a = 0, \quad \delta C^a = 0, \quad \delta A^{*\mu}_a = D_\nu F^{\nu\mu}_a, \quad \delta C^{*}_a = D_\mu A^{*\mu}_a,$$

and $\sigma$ is the original BRST differential of [1, 2],

$$\sigma A^a_\mu = D_\mu C^a, \quad \sigma C^a = \frac{1}{2} C^{abc}_b C^b C^c,$$

extended to the antifields so as to anticommute with $\delta$,

$$\sigma A^{*\mu}_a = -C^{\nu}_c A^{*\nu}_a C^c, \quad \sigma C^{*}_a = -C^{\nu}_c C^{*}_a C^c.$$

Here, $C^{\nu}_c$ are the structure constants of the Lie algebra $G$ of the gauge group, which we assume to be semi-simple. One has

$$\delta^2 = 0, \quad \sigma^2 = 0, \quad \delta \sigma + \sigma \delta = 0,$$
so that
\[ s^2 = 0. \]

In order to fix the gauge, one introduces also the antighosts \( \overline{C}_a \) and the auxiliary fields \( b_a \), together with their respective antifields \( \overline{b}^*_a \). In this “non minimal” sector, the BRST differential is defined by
\[ s\overline{C}_a = b_a, \quad sb_a = 0, \quad s\overline{b}^*_a = 0, \quad sb^*_a = -\overline{b}^*_a. \]

The fields and antifields have ghost number given by
\[ ghA^a = 0, \quad ghC^a = 1, \quad ghA^*_a = -1, \quad ghC^*_a = -2, \]
\[ gh\overline{C}_a = -1, \quad ghb_a = 0, \quad gh\overline{b}^*_a = 0, \quad ghb^*_a = -1. \]

We shall denote by \( A \) the algebra of polynomials in \( A^a_{\mu}, C^a, A^*_a, C^*_a, \overline{C}_a, b_a, \overline{b}^*_a, b^*_a \) and their derivatives up to a finite order; and by \( B \) the smaller algebra of polynomials in \( A^a_{\mu}, C^a, A^*_a, C^*_a \) and their derivatives up to a finite order.

The nilpotency of \( s \) enables one to define cohomological groups \( H^*(s) \) in the standard manner. The following question concerning the cohomology of \( s \) in \( A \) is of central interest in the analysis of the renormalization of local gauge invariant operators \([13, 14, 15, 16, 17, 18, 19, 20, 21]\): given a polynomial in \( A \) that is (i) BRST-invariant and (ii) of ghost number zero, can it be written as the sum of a gauge invariant polynomial and a BRST variation? In other words, does one have
\[ sP = 0, \quad ghP = 0, \quad P \in A \implies P = M + sQ \]
where \( M \) is a gauge invariant polynomial in the field strengths \( F^a_{\mu\nu} \) and their derivatives up to a finite order, and where \( Q \in A \)? Differently put, can one find in each equivalence class of \( H^0(s) \) a representative that does not involve the ghosts and that is strictly gauge invariant? The answer to this question was conjectured in \( J \) \([17]\) to be the affirmative. A proof of (10) has been given in \([18]\) following earlier work of \([16]\). The purpose of this letter is to provide an alternative and somewhat shorter proof, which is based on known results on the antifield formalism and the cohomology of Lie algebras.
2 Digression

The question (10) was actually formulated originally in terms of the gauge-fixed action. In order to make contact with the original formulation, let us denote collectively all the fields $A_\mu^a, C^a, \overline{C}_a$ and $b_a$ by $\phi^A$ and all the antifields $A_*^\mu, C_*^a, \overline{C}_*^a$ and $b_*^a$ by $\phi_*^A$. The algebra of polynomials in the fields $\phi^A$ and their derivatives up to a finite order is denoted by $\mathcal{F}$. The gauge fixed action is the integral of an element of $\mathcal{F}$ obtained by eliminating the antifields $\phi_*^A$ through $\phi_*^A = \delta \Psi / \delta \phi^A$, where $\Psi$ is the so-called gauge fixing fermion $[4]$.

The gauge-fixed BRST symmetry $s_\Psi$ is defined in the algebra $\mathcal{F}$ through $s_\Psi \phi^A = s \phi^A(\phi^B, \phi_*^B = \delta \Psi / \delta \phi^B)$ and is a symmetry of the gauge-fixed action. In the case of the Yang-Mills theory considered here, $s_\Psi$ does not depend on the gauge fixing since $s \phi^A$ does not depend on the antifields, $s_\Psi \phi^A = s \phi^A$. Thus, $s_\Psi$ is nilpotent off-shell (although, in general, $s_\Psi$ is only nilpotent on-shell). The gauge-fixed BRST cohomology $H^*_\Psi(s)$ is defined to be the set of equivalence classes of weakly BRST invariant elements of $\mathcal{F}$,

$$sR \approx 0, \ R \in \mathcal{F}$$

modulo weakly BRST exact ones,

$$R \sim R' \text{ iff } R \approx R' + sT, \ R, R', T \in \mathcal{F}. \quad (12)$$

One says that two polynomials in $\mathcal{F}$ are “weakly equal” (for the gauge conditions under consideration) if they coincide when the equations of motion following from the gauge-fixed action hold. They then differ by a combination of these equations of motion and their spacetime derivatives.

As shown in $[10, 11]$, the gauge-fixed BRST cohomology $H^*_\Psi(s)$ defined in the algebra $\mathcal{F}$ of the fields is isomorphic with the BRST cohomology $H^*(s)$ defined in the algebra $\mathcal{A}$ of the fields and the antifields. Furthermore, if $P \in \mathcal{A}$ defines an element of $H^*(s)$, then $R = P(\phi, \phi^* = \delta \Psi / \delta \phi) \in \mathcal{F}$ defines the corresponding element of the gauge-fixed BRST cohomology $H^*_\Psi(s)$. The question (10) is thus equivalent to : does

$$sR \approx 0, \ R \in \mathcal{F} \quad (13)$$

imply

$$R \approx M + sT, \ T \in \mathcal{F} \quad (14)$$
where $M$ is a gauge-invariant polynomial in the field strengths and their
derivatives up to a finite order? In some gauges ("linear gauges"), Eqs.
(13) and (14) can be further simplified because the equation of motion of the
antighost can be easily handled. However, even though $H(s)$ refers explicitly
to a definite gauge fixation through the gauge-fixed equations of motion used
in Eq. (12), it does not depend on the choice of gauge since it is isomorphic to
$H(s)$. Hence, its calculation can be carried out independently of the gauge
fixation. This leads to the formulation (10) in terms of the antifields. The
gauge independence of the cohomological questions behind renormalization
theory has been particularly stressed in [20, 22]. After this brief digression,
we can return to the proof of Eq. (10).

3 Removing the non-minimal sector

The first step in the proof of Eq. (10) consists in removing the non-minimal
sector.

**Theorem 1:** In each equivalence class of $H(s)$, one can find a representa-
tive that does not depend on the variables of the non minimal sector. That
is, if $sP = 0, P \in A$, then $P = R + sQ'$ with $R \in B, sR = 0, Q' \in A$.

**Proof:** See [9, 10, 11] (the BRST transformation in the non-minimal secto-

We shall thus assume from now on that $P$ in Eq. (10) belongs to the
polynomial algebra $B$ generated by the variables of the minimal sector and
their derivatives. In that algebra, we introduce a second grading, called the
“antighost number” through

$$
antigh A^a = 0, \ antigh C^a = 0, \ antigh A^a = 1, \ antigh C_a = 2, \ antigh \partial^a = 0.
$$

The splitting (1) of the differential $s$ simply corresponds to the splitting
according to definite antighost number,

$$
antigh \delta = -1, \ antigh \sigma = 0.
$$

(15)

As shown in [1, 10, 11], the differential $\delta$ is the Koszul-Tate differential as-
associated with the gauge-invariant equations of motion $D_\mu F^{aJ\mu} = 0$. One
has

$$
H_k(\delta) = 0, \ for \ k \neq 0.
$$

(17)
both in the algebra of all functionals \([9, 10, 11]\) and in the algebra \(B\) of local polynomials \([23]\), while \(H_0(\delta)\) is given by the equivalence classes of polynomials in \(B\) that coincide when \(D_\mu F^{aJ\mu} = 0\).

4 Cohomology of \(\sigma\)

The cohomology of \(\sigma\) - the usual BRST differential - has been computed by various authors \([24, 25, 26, 27]\) in the polynomial algebra generated by the potentials \(A^a_\mu\), the ghosts \(C^a\) and their derivatives. One can easily include the antifields \(A^{a*}_\mu\), \(C^{a*}\) and their derivatives as follows. The algebra \(B\) is the sum of two algebras, \(B = C \oplus D\), one of which, namely \(C\), is contractible.

The algebra \(C\) is the polynomial algebra in the components \(A^a_\mu\) of the vector potential, their symmetrized derivatives \(\partial_{(\mu_1\mu_2...\mu_k)}A^a_\mu\) and their \(\sigma\)-variations \([27]\). It is contractible because it takes the standard form \(\sigma x_i = y_i, \sigma y_i = 0\).

The algebra \(D\) is the polynomial algebra generated by the components of the field strengths \(F^{a\mu\nu}_\mu\), their covariant derivatives \(D^{\mu_1}_{\mu_2...\mu_k}F^{a\mu\nu}_{\mu\nu}\), the components of the antifields \(A^{a*}_\mu\), their covariant derivatives \(D^{\mu_1}_{\mu_2...\mu_k}A^{a*}_{\mu}\), the antifields \(C^{a*}\), their covariant derivatives \(D^{\mu_1}_{\mu_2...\mu_k}C^{a}\) and the ghosts \(C^a\) (without their derivatives, which are in \(C\)). One has

\[
\sigma \chi^{a}_\Delta = C^{a}_{bc} \chi^{b}_\Delta C^{c} \tag{18}
\]
\[
\sigma C^a = \frac{1}{2} C^a_{bc} C^{bc} \tag{19}
\]

where \(\chi^{a}_\Delta\) stands for \(D^{\mu_1}_{\mu_2...\mu_k}F^{a\mu\nu}_{\mu\nu}, D^{\mu_1}_{\mu_2...\mu_k}A^{a}_{\mu}\) and \(D^{\mu_1}_{\mu_2...\mu_k}C^{a}\) \((k = 0, 1, 2...\) and where the internal indices are raised with the Killing metric. Furthermore,

\[
H^*(\sigma, B) = H^*(\sigma, D) \tag{20}
\]

since \(C\) is contractible.

Now, each \(\chi^{a}_\Delta\) belongs to a copy of the adjoint representation of the Lie algebra \(G\). The algebra \(V\) of polynomials in \(\chi^{a}_\Delta\) provides therefore a representation \(\rho : G \rightarrow V\) of \(G\). Since \(G\) is semi-simple, \(V\) splits as a direct sum of finite-dimensional irreducible representations,

\[
V = V_0 \oplus (\oplus_{k>0} V_k), \quad V_0 = \oplus_{a} V_{0Ja} \tag{21}
\]

(the space of polynomials of degree \(\leq n\) with derivatives up to order \(k\) is invariant and finite-dimensional for arbitrary \(n\) and \(k\)). In (21), \(V_0\) stands
for the sum of the one-dimensional trivial representations $V_{0\alpha}$, i.e., contains all the invariant polynomials in $\chi^a_{\Delta}$, while $V_k$ denotes the irreducible non-trivial representations. Note that the trivial representation occurs an infinite number of times (one can form an infinite number of linearly independent invariant polynomials in the $\chi^a_{\Delta}$), so that $V_0$ is infinite-dimensional.

The differential $\sigma$ is nothing but the coboundary operator for the cohomology $H^\ast(G, V)$ of the Lie algebra $G$ in the representation $V$. According to Whitehead theorem, only the invariant subspace of the trivial representation contributes to the cohomology and so

$$H^\ast(\sigma) = H^\ast(G, V_0). \quad (22)$$

As it follows from the work of [28], the cohomology $H^\ast(G, V_0)$ is the tensor product of $V_0$ by the cohomology $H^\ast(G)$ of the Lie algebra $G$. $H^\ast(G)$ is isomorphic to the algebra of the invariant cocycles on $G$. Both $V_0$ and $H^\ast(G)$ are free graded-commutative algebras, and the most general element of $H^\ast(\sigma) = H^\ast(G, V_0)$ can be taken to have as representative

$$\sum_{i,J} \alpha_{iJ} P_i(\chi^a_{\Delta}) E^J(C^a) \quad (23)$$

where (i) $P_i(\chi^a_{\Delta})$ are a set of independent generators of $V_0$; and (ii) $E^J(C^a)$ are a set of independent generators of $H^\ast(G)$, which are known to be in number equal to the rank of $G$ ("primitive forms"). The independence of the generators in cohomology means that if $\sum_{i,J} \alpha_{iJ} P_i(\chi^a_{\Delta}) E^J(C^a) = \sigma(\text{something})$, then $\alpha_{iJ} = 0$ for all $i,J$. That is, if an invariant polynomial of the type (23) is $\sigma$-exact, then it is identically zero.

5  Cohomology of $\delta$ in $V_0$

Let $P(\chi^a_{\Delta})$ be an invariant polynomial that is $\delta$-closed and of positive antighost number,

$$\delta P = 0, \text{antigh}P > 0, \ P \in V_0. \quad (24)$$

By (17), one has

$$P = \delta T \quad (25)$$

but there is a priori no guarantee that $T$ belongs also to $V_0$.

**Theorem 2**: One may choose $T$ in (25) to be in $V_0$. 

6
Proof: the linear operator $\delta$ is defined in the space $\mathcal{V}$ of the polynomials in $\chi^a$ and commutes with the representation $\rho$ of $G$ in $\mathcal{V}$,

$$\delta \rho = \rho \delta.$$  \hspace{1cm} (26)

Hence it maps the invariant subspaces $\mathcal{V}_k$ of (21) on invariant subspaces. Since $\mathcal{V}_k$ is irreducible for $k \neq 0$, the subspace $\delta \mathcal{V}_k$ is either isomorphic to $\mathcal{V}_k$ and yields an equivalent representation, or is equal to 0. This implies that $\delta X$ has no component along the subspace $\mathcal{V}_0$ of the trivial representation if $X \in \mathcal{V}_k$, $k \neq 0$, i.e. $\delta \mathcal{V}_k \cap \mathcal{V}_0 = 0$ for $k \neq 0$. Let $T = T_0 + T_1 + T_2 + ...$ be the decomposition of $T$ along the subspaces of (21). Since $\delta T = P$ belongs to $\mathcal{V}_0$, it follows that $\delta(\sum_{k>0} T_k) = 0$, so that one may choose $T = T_0 \in \mathcal{V}_0$ in (25). This proves the theorem.

6 Cohomology of $s$

We can now prove the theorem

**Theorem 3**: Let $P$ be a polynomial in the algebra $\mathcal{A}$ which is (i) BRST-invariant; and (ii) of ghost number zero. Then

$$P = M + sQ$$  \hspace{1cm} (27)

where $M$ is a gauge invariant polynomial in the field strengths $F^a_{\mu\nu}$ and their derivatives up to a finite order, and where $Q \in \mathcal{A}$.

**Proof**: From Theorem 1, one has $P = R + sQ'$ where $R \in \mathcal{B}$ and $sR = 0$. Let us decompose $R$ according to the antighost number,

$$R = R_0 + R_1 + R_2 + ... + R_L; \quad \text{antigh}R_k = k, \quad k = 0, 1, ..., L.$$  \hspace{1cm} (28)

In virtue of Eqs (1) and (16), the equation $sR = 0$ is equivalent to the chain of equations

$$\sigma R_0 + \delta R_1 = 0, \ldots, \sigma R_{L-1} + \delta R_L = 0, \sigma R_L = 0.$$  \hspace{1cm} (29)

If $L = 0$, then $R = R_0$ does not contain the antifields. Since its ghost number vanishes, it does not contain the ghosts either. Our analysis of the $\sigma$-cohomology implies then that $R_0$ is an invariant polynomial in the field
strengths and their derivatives ($\sigma R_0 = 0$), which establishes the theorem. So let us assume that $L > 0$. From $\sigma R_L = 0$, one gets, as explained above

$$R_L = \sum_J P^J(\chi^a_\Delta)E^J(C^a) + \sigma Q_L,$$

where $P^J \in V_0$ and $E^J(C^a)$ are the primitive forms. By a redefinition of $R_{L-1}$ if necessary, one can absorb $Q_L$ in a $s$-variation. So, let us take $Q_L = 0$.

The $\delta$-variation of $R_L$ is an invariant polynomial of the type (23), $\delta R_L = \sum_J \delta P^J(\chi^a_\Delta)E^J(C^a)$ which, by (29), must be $\sigma$-exact. By our analysis of the cohomology of $\sigma$, it must thus vanish,

$$\delta R_L = 0,$$

i.e., $\delta P^J = 0$. By Theorem 2, this implies $P^J = \delta T^J$ with $T^J \in V_0$ and thus

$$R_L = \delta T_{L+1}, \quad T_{L+1} = \sum_J T^J(\chi^a_\Delta)E^J(C^a),$$

with $\sigma T_{L+1} = 0$. Accordingly,

$$R_L = sT_{L+1}$$

and one can remove $R_L$ from $R$ by adding a $s$-variation. Once this is done, $R$ contains only components of antighost numbers 0, 1... up to $L - 1$. Going on in the same fashion, one then removes successively $R_{L-1}$, $R_{L-2}$... up to $R_1$. This yields

$$R = R'_0 + sT'$$

where $R'_0$ is an invariant polynomial in the field strengths and their derivatives. Thus one gets

$$P = M + sQ$$

with $M = R'_0$ and $Q = Q' + T'$. This proves the theorem.
7 Comments

(1) The above analysis proves that in each equivalence class of \( H^0(s) \), one can find a representative that does not involve the ghosts or the antifields and that is strictly gauge invariant. This representative, of course, is not unique since there exist invariant polynomials in the field strengths and their derivatives which vanish when the equations of motion \( D_\mu F^{a\mu\nu} = 0 \) hold, and which can thus be written as \( s \)-variations. The theorem 3 strengthens the results of \([\text{I, 23}]\) in that (i) the work of \([\text{I, 23}]\) indicates that the local BRST cohomology in degree zero is isomorphic with the set of on-shell gauge invariant local polynomials in the field variables and their derivatives for a general gauge theory (with the identification of two such polynomials that coincide on-shell), but does not guarantee that one can find strongly gauge invariant polynomials in each equivalence class; (ii) the work of \([\text{I, 23}]\) shows that the equations \( sU = 0 \), \( antighU > 0 \) imply \( U = sV \) for some \( V \) but does not guarantee, as done in this letter, that \( V \) has a finite expansion in the antighost number \( (V \text{ could a priori contain an infinite number of terms of arbitrarily high antighost number, and so, not be a polynomial}).

Although it is the strong version expressed by Theorem 3 that has been invoked in renormalization theory, it appears that the isomorphism of \( H^0(s) \) with the set of on-shell gauge invariant polynomials is just sufficient on physical grounds. Indeed, the physical matrix elements of BRST-exact operators (with \( s \) given by the sum (1)) vanish and so, these operators are physically irrelevant \([\text{10, 11}]\). Accordingly, provided the BRST symmetry is not anomalous so that BRST invariant operators mix only with BRST invariant operators, and BRST exact operators mix only with BRST exact ones, then, the mixing is well defined in cohomology. By the isomorphism mentioned above, this means that only gauge-invariant operators are physically relevant. The isomorphism of \( H^*(s) \) with the set of gauge invariant functionals admits an extension to non-local operators \([\text{I, 10, 11}]\).

(2) Theorem 3 can be extended to other values of the ghost number. One shows along the same lines the existence, in each equivalence class of \( H^*(s) \), of a representative annihilated by \( \sigma \).

(3) Finally, our analysis does not cope with the more complicated question of computing \( H(s = \delta + \sigma \mid d) \). The cohomology of \( H(\sigma \mid d) \) has been the subject of various works \((\text{24, 25, 26, 27} \text{ and references therein})\). It would be of interest to extend those results to \( H(s \mid d) \). We plan to return to this
question elsewhere.

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