On the Generalized Metric Structure of Space-Time: Finslerian Anisotropic Gravitational Field

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Abstract. The study of Finslerian, locally anisotropic gravitational field extends the framework of the Riemannian approach to general relativity. The introduction of an independent internal variable \( y \left( \frac{dx}{dt} \right) \) provides the basic framework for the description of a Finslerian gravitational field. Notably, its intrinsic behavior can be considered as a property of the field itself. A possible anisotropic structure in a perfect-fluid universe is investigated on the basis of geometrical concepts of Finsler geometry. Finally, we deal with Einstein-type equations and we prove that they are associated with a cosmological constant in a Finslerian space-time.

1. Introduction

In some theories of anisotropy, the basic approach to study an anisotropic space-time is constrained to perturbing a spatially homogeneous, isotropic universe. This is achieved in terms of small variations of the curvature [1].

If we accept that the universe was exactly homogeneous and isotropic at an earlier stage, then we have to show how it evolved to a non-homogeneous and anisotropic structure as that which we now clearly observe on certain scales. One certain possibility is to assume that perturbations or fluctuations were present and that the former grew into the present clumps. The attempts in this direction are summarized in an effort to model the evolution of the universe [2].

In such models, the universe initially has Friedmann geometry. The anisotropy is hidden in the higher moments of the particle distribution function, which do not affect the geometry. As the universe expands the particle energies and momenta will eventually move into a regime where non-trivial interactions take place and the particle distribution anisotropy is then expressed to the space-time geometry. Einstein field equations play an important role and alter that geometry.

The above mentioned consideration retains the geometrical concepts of the homogeneous isotropic case. Finsler geometry on the other hand has fundamentally different geometrical approach to the study of locally anisotropic cosmological phenomena, i.e. it intrinsically incorporates the anisotropy in the geometry of space. For this reason it appears logically as valid candidate for the construction of such a theory [3] [4]. A Finslerian geometrical model which corresponds to the anisotropic structure of regions of space-time (radius \( \leq 10^8 \) light years) [5] can be introduced.

In \( \S 2 \) we give the connection of Finsler geometry with some fundamental principles of Riemannian general relativity. We also study Finsler-Randers (F-R) spaces of constant curvature. This type of spaces is connected with the Riemannian space of constant curvature
Lagrangian that includes the anisotropy. Its form is

\[ L \equiv \sqrt{g - \sigma^2} \]

\( r \) description of the field between the external (e.g. Riemannian description of the gravitational field) and the internal elements (e.g. Finslerian field may be regarded as a gravitational field spanned by the line-metric tensor \( f \)).

In this equation we observe the additional term \( \sigma \) that represents the direction of the observed anisotropy of the microwave background radiation. The intrinsic behavior of the internal vector \( \phi(x) \hat{\ell}_a = \hat{\ell}_a \), in which the matter density is hidden, can be considered as a property of the field itself: it expresses the spin density tensor \( S_{ab} = \sqrt{\frac{2}{3}} \epsilon_{abc} \hat{\ell}^c(x) \) (cf. Eq. (45) in [4]).

The equation of geodesics in a space with metric function of the form of Eq. (1) is

\[ \frac{d^2x^\ell}{ds^2} + \Gamma_{ij}^{(r)} y^i y^j + \sigma r^{lm} ( \partial_j \hat{\ell}_m - \partial_m \hat{\ell}_j ) y^j = 0 \]

In this equation we observe the additional term \( \sigma r^{lm} ( \partial_j \hat{\ell}_m - \partial_m \hat{\ell}_j ) y^j \), where \( \sigma = \sqrt{|r_{ij} y^i y^j|} \) and \( \Gamma_{ij}^{(r)} \) are the Riemannian Christoffel symbols. This term expresses a rotation of the anisotropy.

2. Finsler geometry associated with the gravity

As it is well known from general relativity, Einstein used the principle of equivalence as the basis for a geometrical description of gravity. In a four-dimensional world space-time, the trajectory of a particle falling freely in a gravitational field is a certain fixed curve. Its direction at any point depends on the velocity of the particle. The equivalence principle implies that there is a preferred set of curves in space-time: at any point, pick any direction and there is a unique curve in that direction that will be the trajectory of any particle starting with that velocity [5], [7].

On the other hand in the Finslerian framework the gravitational field is defined in a space that is spanned by the vectors \( y \), with \( y^i = \frac{dx^i}{dt} \) (\( y^i / i = 0, 1, 2, 3 \)) which are attached to each point \( x \) (\( x^k / k = 0, 1, 2, 3 \)) as an independent internal variable. Namely the independent variables become \( (x^k, y^i) \). In terms of E. Cartan the couple \( (x^k, y^i) \) (position-velocity or direction) is called an element of support.

The intrinsic behavior of the internal variable \( y \) (\( y = v = \frac{dx}{dt} \)) of the field in a space-time can be considered as a property of the field itself. Consequently it is reasonable to consider a Finsler space as the basic structure for the study of the gravitational field. In a Finslerian space-time the concept of anisotropy is physically incorporated to the internal variable as the basic structure for the study of the gravitational field. In a Finslerian space-time the gravitational field is defined in a space that is spanned by points \( \ell \) (e.g. Riemannian description of the gravitational field) and the internal \( y \)-field spanned by vectors \( y \).

An application of the above mentioned consideration is given in [4]. In that study we used a Lagrangian that includes the anisotropy. Its form is

\[ L(x, y) = \sqrt{|r_{ij} y^i y^j|} + \phi(x) \ell_a y^a \]

where \( r_{ij} \) a pseudo-Riemannian metric, \( \phi(x) \) is related to the mass density and \( \ell_a \) a unit vector that represents the direction of the observed anisotropy of the microwave background radiation. The intrinsic behavior of the internal vector \( \phi(x) \hat{\ell}_a = \hat{\ell}_a \), in which the matter density is hidden, can be considered as a property of the field itself: it expresses the spin density tensor \( S_{ab} = \sqrt{\frac{2}{3}} \epsilon_{abc} \hat{\ell}^c(x) \) (cf. Eq. (45) in [4]).

Finally, in §4 a cosmological constant is studied in an F-R type space by using Einstein field equations in the Finslerian framework.
axis. In our case (cf. Eq. (1)), the equation of geodesics of the Riemannian space-time is
generalized, a fact that is closely related to the equivalence principle.

A Finsler space is constructed by a differentiable manifold and a fundamental smooth metric
function $F(x, y)$ on its tangent bundle $TM$ which depends on the variables, $x \in M$ of position
and $y = \frac{dx}{d\tau}$ of direction in which $F$ is homogeneous of first degree with respect to $y$ [10].

Suppose $(F^4, f_{ij}(x, y))$ is a four dimensional differentiable manifold and $f_{ij}(x, y)$ the
anisotropic Finslerian metric is considered to have signature $(+, -, -, -)$ for any $(x, y)$. The
square of the length of an arbitrary contravariant vector $X^i$ is to be defined by the quadratic
form

$$|X|^2 = f_{ij}(x, y)X^iX^j$$

The motion of a particle in a Finslerian space-time $F^4$ is described by a pair $(x, V)$ where
$x \in F^4$ and $V = \frac{dx}{d\tau}$ is the 4-velocity of the particle ($\tau$ is the proper time) which represents
the tangent of its world-line expressing the motion of typical observers in the Finslerian anisotropic
universe. The time-like, null and space-like curves can be defined in the Finslerian framework
by the following relations [11]:

- time-like $f_{ij}(x, y)V^iV^j > 0$
- null-like $f_{ij}(x, y)V^iV^j = 0$
- space-like $f_{ij}(x, y)V^iV^j < 0$

Finslerian geodesics satisfy the Euler-Lagrange equations

$$\frac{d}{dt} \left( \frac{\partial F(x, y)}{\partial y^i} \right) - \frac{\partial F(x, y)}{\partial x^i} = 0 \quad (3)$$

This is the equation of Finslerian geodesics associated with the Lagrangian $F(x, y)$. In the case
where the vector $y^i = \dot{x}^i = \frac{dx^i}{ds}$ has unit Finslerian length, then the Finslerian geodesics take on
the form

$$\frac{d^2 x^i}{ds^2} + \gamma_{jk}^i(x, \dot{x})\dot{x}^j\dot{x}^k = 0 \quad (4)$$

where $\gamma_{jk}^i(x, \dot{x})$ are the Finslerian Christoffel symbols. This equation shows the dependence
of geodesic equations on velocity, which is incorporated by the principle of equivalence. This
means that the velocity or the geometry of tangent space (fiber), influences the equations of
background geodesics [12]. This approach extends the Riemannian general relativity without
any contradiction with its basic principles.

In addition the above mentioned consideration reveals a profound relation between the
principle of equivalence of the Riemannian framework and the paths (geodesics) in the Finslerian
space-time. This can be seen, for example, if we consider a Finsler-Randers (F-R) space,
$(M^n, F(x, y))$. The metric function of this space has the form

$$F(x, y) = \sqrt{r_{ij}(x, y)y^iy^j} + \kappa A_i(x)y^i \quad (5)$$

where $(M^n, F(x, y))$ is a Riemannian manifold with metric $r_{ij}$, $A_i(x)$ represents an
electromagnetic potential and $\kappa$ is some constant. The electromagnetic field is intrinsically
incorporated in the geometry [13]. The equation of geodesics in this space satisfies the Lorentz
equation and the curvature tensor of space is then written as

$$H_{hjk}(x, y) = R^i_{hjk} + E_{hjk} \quad (6)$$
where $R^i_{hjk}$ represents the Riemannian curvature and $E^i_{hjk}$ the "electromagnetic curvature" [8], [13]. In many cases we consider a convenient Finsler metric for the research on gravitation theories. This metric is connected with a Riemannian one, $a_{ij}(x)$, which is called osculating Riemannian metric and is defined over a region $U$ of $F^n$

$$a_{ij}(x) = f_{ij}(x, y(x))$$  \hspace{1cm} (7)

A generalized Finslerian metric tensor can be defined as well, by the underlying Riemannian metric

$$f_{ij}(x, v) = r_{ij}(x) + \frac{\partial f_{ij}}{\partial v^k} v^k + O(v^2)$$  \hspace{1cm} (8)

where $r_{ij}(x)$ is the Riemannian metric and $v^i(x) = y^i(x) = \frac{dx^i}{ds}$. If we take into account the anisotropic torsion tensor of Cartan, $C_{ijk}(x) = \frac{1}{2} \frac{\partial f_{il}}{\partial x^j} \frac{\partial f_{jm}}{\partial x^l} - f^{lh}(x, v)C_{klh}(x, \xi) \frac{\partial \xi^l}{\partial x^h}$, the equation of geodesics becomes

$$\frac{dv^i}{ds} + a_{kj}^i(x)v^k = 0$$  \hspace{1cm} (9)

where

$$a_{kj}^i(x) = \gamma_{jk}^i(x, v) + C_{kl}^i \frac{\partial \xi^l}{\partial x^k} + C_{lj}^i \frac{\partial \xi^l}{\partial x^j} - f^{ih}(x, v)C_{khi}(x, \xi) \frac{\partial \xi^l}{\partial x^h}$$  \hspace{1cm} (10)

This approach has been studied for a static gravitational field in [8].

In virtue of (4), (5) a weak metric can be introduced when studying gravitational waves in a Finslerian space-time by assuming a weak gravitational field as in the Riemannian framework, where the weak Riemannian metric is analyzed to the form

$$r_{ij}(x) = \eta_{ij} + h_{ij}, \hspace{1cm} \|h_{ij}\| \ll 1$$  \hspace{1cm} (11)

By inserting the form of $r_{ij}$ from Eq. (11) to Eq. (8) we get the Finslerian weak metric

$$f_{ij}(x, v) = (\eta_{ij} + h_{ij}) + C_{ijkl} v^k + O(v^2)$$

This metric can be considered as a Finslerian perturbation of the weak Riemannian metric.

Weak gravitational models of this type have been studied in [14]. This analysis can be used for the anisotropic description of the polarization of waves. It is closely connected to the equation of deviation of geodesics, that can be used to detect gravitational waves [7].

In order to study the weak field of a F-R space we relate it to the deviation of two moving charged particles. It is necessary to take into account the relation

$$\frac{\delta^2 z^i}{\delta u^2} + E^i_{jkl} v^j v^k = 0$$  \hspace{1cm} (12)

where $z^i$ expresses the deviation vector between two charged particles and $v^k$ their 4-velocity. The operator $\frac{\delta}{\delta u}$ represents the covariant derivative in the framework of Finsler geometry [10]. This is reasonable since in order to detect a gravitational wave at least two particles are needed. So the deviation of geodesics of the weak F-R space is written in the form

$$\frac{D^2 \xi^i}{d\tau^2} + (\epsilon^i_{ljm} + h^i_{ljm}) \xi^j \frac{dx^l}{d\tau} \frac{dx^m}{d\tau} = 0$$  \hspace{1cm} (13)

where $\epsilon^i_{ljm}$ expresses the linearized Riemannian curvature tensor and $h^i_{ljm}$ are functions of the electromagnetic field and velocities [13].
The Eq. (13), in a first order approximation of \( h_{ij} \), takes the form

\[
\frac{\partial^2 \xi^i}{\partial t^2} = - \left( \frac{1}{2} \frac{\partial^2 \epsilon^i_{jk}}{\partial t^2} + 2F^i_0F^0_j + u_j \frac{\partial F^i_0}{\partial t} \right) \xi^j
\]  

(14)

The Eq. (14) coincides with the corresponding equation for a weak field of the Riemannian space-time [15]. The difference between these two is that in the Riemannian case the electromagnetic field has been introduced ad hoc. In Eq. (12) the electromagnetic field is incorporated in the geometry and the two particles are moved in geodesics (potential lines) of the Finsler space. In virtue of Eq. (13) their relative acceleration is governed by the curvature of the electromagnetic field that is produced by one part of the energy-momentum tensor (cf. Eq. (6)).

3. F-R type spaces of constant Riemannian curvature. Anisotropic perfect fluids.

The geometrical concept of constant curvature spaces of Riemannian geometry is generalized in a Finslerian framework. Riemannian spaces of constant curvature play an important role in cosmological models of general relativity. A point \( x \) in a Finsler space \( F^n \) is called isotropic point of \( F^n \) if the Riemannian scalar curvature \( R(x, y, X) \) of the space at that point with respect to a 2-direction \((y, X)\) is independent of the choice of the vector \( X \), where \( y = \frac{dx}{dt} \). We shall denote the corresponding value \( R(x, y, X) \) by \( R(x, y) \). In Riemannian geometry the definition requires the \( R(x, y, X) \) to be independent of \( y \) and \( X \) [10].

If a Finsler space is isotropic at each point in a region and if the scalar \( R(x, y) \) is independent of each directional arguments \( y^i \), then the Riemannian curvature is constant through that region. The relation that characterizes isotropic points in Finsler geometry is analogous to that of Riemannian geometry. Its form is given by

\[
L_{ijkl} = K(f_{ik}f_{jl} - f_{il}f_{jk}), \quad K : \text{constant}
\]  

(15)

where \( L_{ijkl} \) represents the third curvature of a Finsler space with metric tensor \( f_{ij} \).

In a Finsler space \( F^n \) the geodesic deviation equations are given by

\[
\frac{\delta^2 z^i}{\delta s^2} + L^i_{jkl}y^jy^kz^l = 0
\]  

(16)

where \( z^i \) is the deviation vector between two neighboring parameter curves \( C, C' \) and \( y^j \) represents the tangent vector field of the fundamental geodesic \( C \) on the 2-dimensional subspace \( F^2 \) of \( F^n \). For a 2-dimensional case the classical equation of Jacobi is directly generalized in the form of:

\[
z'' + zR(x, y) = 0
\]  

(17)

where \( z(s) \) is a scalar function of an affine parameter \( s \).

Taking into account the space-time of the metric (1) with constant negative curvature, we get the Ricci curvature \( L_{ij} = 3k f_{ij} \), where

\[
f_{ij} = \frac{\mathcal{L}}{\sigma} a_{ij} + \frac{\phi(x)}{2\sigma} (y_i \hat{\ell}_j + y_j \hat{\ell}_i) - \frac{\beta \phi(x)}{\sigma^2} y_i y_j + \frac{\phi^2(x)}{\sigma^3} \hat{\ell}_i \hat{\ell}_j
\]

with \( \beta = \hat{\ell}_a y^a \) and \( \sigma = \sqrt{a_{ij} y^i y^j} \).

The Einstein field equations in vacuum are given by

\[
\tilde{E}^{ij} = L^{ij} - \frac{1}{2} \tilde{R} f^{ij} = 0
\]  

(18)
where
\[ \tilde{E}_{ij} = -\tilde{R} \left[ \frac{\mathcal{L}}{\sigma} a_{ij} + \frac{\phi(x)}{2\sigma} (y_i \dot{\ell}_j + y_j \dot{\ell}_i) - \frac{\beta \phi(x)}{\sigma^2} y_i y_j + \phi^2(x) \dot{\ell}_i \dot{\ell}_j \right] \]

The condition
\[ \tilde{E}_{ij}^j = 0 \quad (19) \]
holds, where the symbol “\(|\)" denotes covariant differentiation in a Finsler space.

In the context of cosmology, there is a preferred family of world lines representing the motion of typical observers in the universe (fundamental observers). There are often referred as “fluid flow lines" with a 4-velocity vector tangent to these world lines
\[ u^\alpha = \frac{dx^\alpha}{ds}, \]
where \( s \) is the proper time. By considering this point of view we define the length scale \( S(s) \) along anisotropic flows of the Robertson-Walker scale parameter in the form of
\[ \dot{S} S = \frac{1}{3} \Theta, \]
where \( S(s) \) is the average distance behavior of the fluid determining \( S \) up to a constant factor along each world line and \( \Theta \) is the expansion [16].

In the Finslerian framework of general relativity the form of a perfect fluid has been given by
\[ T_{ij}(x, V(x)) = (\mu + p)V_i(x)V_j(x) + p a_{ij} \quad (20) \]
where \( p = p(x), \mu = \mu(x) \) represent the pressure and the density of the fluid respectively, \( T_{ij} \) is the energy-momentum tensor associated with the osculating Riemannian metric tensor \( a_{ij}(x) = f_{ij}(x), V(x) \) is the 4-velocity. The Einstein equations
\[ L_{ij}(x, V(x)) = K \left( T_{ij}(x, V(x)) - \frac{1}{2} T_k^k a_{ij} \right), \quad K : \text{constant} \quad (21) \]
determine the Ricci tensor \( L_{ij} \) directly from the matter energy-momentum tensor \( T_{ij} \) at each point, associated with the osculating Riemannian metric tensor. In virtue of Eq. (20), (21) we have
\[ L_{ij}V^iV^j = \frac{1}{2} K (\mu + 3p) \quad (22) \]
We may apply the geodesic deviation equation (16) in the case of perfect fluids along the two neighboring world lines by substituting Eq. (22) in Eq. (16). Thus, we have
\[ \frac{\delta^2 z^i}{\delta s^2} + L_{jk} V^j V^k z^i = 0 \quad (23) \]
or
\[ \frac{\delta^2 z^i}{\delta s^2} = -L_{jk} V^j V^k z^i \]
This relation shows that the term \( L_{jk} V^j V^k \) corresponds to the anisotropic gravitational influence of matter along world lines of the fluid with density \( \mu \) and pressure \( p \). This term expresses the tidal force of the field.

In analogy to the Riemannian framework of general relativity we may define a representative scale factor along each world line in Finslerian anisotropic models by the relation
\[ \tilde{H} = \frac{\dot{a}}{a} = \frac{1}{3} \tilde{\Theta} \quad (24) \]
where \( \tilde{a}(\xi^i(s)) \) and \( \dot{a}(s) = \frac{da}{ds} = \frac{\partial a}{\partial \xi^i} \xi^i(s) \), \( \xi^i(s) \) is the unit tangent vector in the flow lines. The term \( \tilde{\Theta} \) is the expansion in the Finslerian space-time which is defined by \( \tilde{\Theta} = V_i^i - C_{im} V^m \). The symbol “\(|\)" denotes the Riemannian covariant derivative associated
with the osculating Riemannian metric $a_{ij}(x)$, $C^i_{jk}$ represent the Cartan connection coefficients $(C_{ijk} = \frac{1}{2} \frac{\partial f_{ij}(x,V)}{\partial V^k}, \ V^k : \text{velocity})$ [10]. The intrinsic anisotropic character of $\tilde{H}$ is revealed by Cartan’s connections. Therefore, the volume of any fluid elements varies as $\tilde{a}^3$ along the anisotropic world lines of the fluid.

In the Riemannian approach of general relativity, when the energy-momentum tensor has the form of that of a perfect fluid the function $a(s)$ represents the separation of neighboring geodesic flow lines (fundamental observers, $s$: proper time) that is of “nearby” galaxies [17]. This standpoint is closely connected to the deviation vector between neighboring observers. Similarly, we can consider an analogous generalized relation to $\tilde{a}(s)$ in the case of perfect fluids in the Finslerian space-time, in virtue of Eq. (16) and (17).

The motion and the rate of change of the deviation vector, $z^i$ (cf. Eq. (16)), plays a significant part to the behavior of the fluid. If $z^i$ is rotated in the direction of motion of the geodesics, then a vorticity $\tilde{\omega}_{ab}$ arises in the congruences of geodesics. Also, the density of geodesics is measured by the shear, $\tilde{\sigma}_{ab}$. Raychauduri’s equation, which is the key for the theorems of singularities, was generalized in the framework of Finsler geometry in [18]. In addition, the Finslerian-perturbed, Robertson-Walker model takes on the form

$$\text{diag}(a_{ij}) = \left(1, -\frac{\tilde{a}^2(\xi(s))}{1-kr^2}, -\tilde{a}^2r^2, -\tilde{a}^2r^2 \sin^2 \theta \right)$$ (25)

where $s$ denotes the proper time, if we use the scale factor along its world line in the form $\tilde{a}(\xi(s)) = \tilde{a}(s)$.

### 4. Finslerian models with a cosmological constant

The cosmological constant problem of general relativity can be extended to locally anisotropic spaces with Finslerian structure. According to S. Weinberg [19] everything that contributes to the energy density at the vacuum acts just like a cosmological constant. In the Finslerian framework of space-time the anisotropic form of the microwave background radiation may contribute to that content, if we consider a metric of the form of Eq. (1).

The field equations in a Finslerian space-time are to be obtained from a variational principle [8], [20]. In the case where we get a F-R space of constant curvature, Einstein’s field equations take the form:

$$\tilde{E}_{ij} = kT_{ij}$$ (26)

where $\tilde{E}_{ij}$ and $T_{ij}$ are given by Eqs. (18) and (20) and $k$ is a constant ($k = 8\pi$). By inserting the cosmological constant $\lambda$ in the field equations we have

$$\tilde{E}_{ij} - \lambda a_{ij} = -kT_{ij}$$ (27)

From Eq. (15) we get

$$\tilde{E}_{ij} = 3Ka_{ij}$$ (28)

where we have substituted the metric $f_{ij}$ by the osculating metric $a_{ij}$. Therefore the energy-momentum tensor in an F-R space of constant curvature is

$$T_{ij} = k^{-1}(\lambda - 3K)a_{ij}$$ (29)

A relation between the cosmological constant $\lambda$ and the constant curvature $K$ can be given by

$$\lambda = 3K$$ (30)
This makes $T_{ij} = 0$ and then we get the empty De-Sitter space satisfying the field equations

$$\tilde{E}_{ij} - \lambda a_{ij} = 0$$

This equation is analogous to that of the Riemannian approach of general relativity in which the constant is $4K$ by Yasuda-Shimada theorem [6]. In the general case of a Finsler space, the field equations yield

$$L^{ij} - \frac{1}{2} a^{ij} \tilde{R} = -8\pi G \left( T^{ij} - \frac{1}{2} a^{ij} T^k_k \right)$$

where $\tilde{R} = R + S$ is the scalar curvature in the Finsler framework and it is explicitly given in [20].

Taking into account the Eq. (19) and the conservation law for the perfect fluids (cf. Eq. (20)), we have

$$T_{ij} = 0$$

We get the covariant derivative of Eq. (32) with respect to $x^k$ and we obtain

$$\tilde{R}_k = 8\pi G T^\nu_{\nu k}$$

or

$$\tilde{R} - 8\pi G T^\nu_{\nu} = -\lambda, \quad \lambda: \text{constant}$$

From Eqs. (32) and (34) we obtain

$$L^{ij} - \frac{1}{2} a^{ij} \tilde{R} - \lambda a^{ij} = -8\pi T^{ij}$$

The last field equation recovers the Einstein field equations with a cosmological constant.

5. Conclusion

An anisotropic direction-dependent expansion of the universe may be present if its underlying geometry is anisotropic. We are concerned here with the study of the anisotropic gravitational field which is emerged by a Finslerian structure of space-time.

The principle of equivalence of Riemannian general relativity can be set in the framework of a generalized metrically relativistic field theory. Finslerian geodesics satisfy the Euler-Lagrange equations of geodesics (cf. Eq. (2)) in a special type of an F-R space space-time is derived, including additional terms than these of the Riemannian prototype. The anisotropy vector of the microwave background radiation is present in the geodesics. As a consequence, it creates a rotation in the form of the geodesics, which is caused by the direction of a Finslerian anisotropic universe. The intrinsic behavior of the internal vector $\hat{\ell}_\alpha$ (spin density tensor), in which the matter density is hidden, can be considered as a property of the field itself. A weak F-R space-time is proposed for the study and detection of gravitational waves, in virtue of the equation of deviation of geodesics (cf. Eq. (12)).

In addition, we have considered an interesting class of F-R spaces with constant curvature. A correlation of these spaces with perfect fluids gives rise to the form of an anisotropic gravitational influence of matter on world lines (cf. Eq. (23)). Also, the scale factor in a Finslerian anisotropic model, can provide a useful tool in the study of a Finslerian-perturbed Robertson-Walker metric. Finally, the cosmological constant problem of general relativity is considered and extended for an F-R space of constant curvature.
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