Off-shell initial state effects, gauge invariance and angular distributions in the Drell-Yan process

Maxim Nefedov\textsuperscript{a,b}, Vladimir Saleev\textsuperscript{b,*}

\textsuperscript{a}Samara National Research University, Moskovskoe Shosse 34, 443086 Samara, Russia
\textsuperscript{b}II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

Abstract

We discuss production of Drell-Yan lepton pairs at hadron colliders in the framework of the parton Reggeization approach, which includes off-shell initial state effects in a gauge-invariant way. Other possible prescriptions to restore gauge-invariance of hard-scattering coefficient with off-shell initial-state partons are also investigated and significant differences for the resulting structure functions are found, especially for the $F_{UU}^{\cos 2\phi}$. We compare our numerical results for $q_T$-spectra of the lepton pair with experimental data, obtained by E-288 collaboration ($\sqrt{S} = 19.4$ and 23.8 GeV) and find a good agreement. Also we perform predictions for the Drell-Yan structure functions at NICA $pp$-collider ($\sqrt{S} = 24$ GeV).

Keywords: Drell Yan process, angular distributions, Collins-Soper frame, TMD factorization, Boer-Mulders function, gauge invariance, multi-Regge kinematics, parton Reggeization approach

1. Introduction

The Drell-Yan(DY) process of production of lepton pairs with large invariant mass in hadronic collisions is one of the most important tests of perturbative quantum chromodynamics (QCD), as well as the unique source of information about partonic structure of hadrons. Apart from the inclusive cross-section, differential w.r.t. squared invariant mass ($Q^2$), transverse momentum ($q_T$) and rapidity ($Y$) of the lepton pair or some equivalent variable, such as momentum fraction in the Collinear Parton model (CPM) ($x_{A,B}$), also structure functions or angular coefficients, which parametrize the angular distribution of leptons in the rest frame of the lepton pair are often under consideration. Behavior of the latter class of observables in the region of relatively small $q_T \leq Q$ will be the main subject of the present paper.

*Corresponding author
At low $q_T \ll Q$, already the prediction of inclusive cross-section, integrated over all directions of lepton momentum in the center-of-mass frame of the pair, presents a considerable difficulty for the conventional CPM, since at any fixed order of perturbation theory the cross-section diverges as $1/q_T^2$ at $q_T \to 0$. These un-physical divergence is regulated through the resummation of higher-order corrections in $\alpha_s$ enhanced by $\log^2(q_T/Q)$ and $\log(q_T/Q)$ through Collins-Soper-Sterman formalism [1], which later has been reformulated in a form of Transverse Momentum Dependent (TMD) factorization theorem [2].

In TMD-factorization, the hard-scattering coefficient (HSC) doesn’t depend explicitly on the transverse momenta of colliding partons. Instead, it is calculated with on-shell initial-state partons and corresponding partonic tensor automatically satisfies the QED Ward identity. However, it is possible to develop a complementary approach to TMD factorization, starting not from the collinear limit but from Multi-Regge limit for QCD scattering amplitudes, i.e. from the limit when all final-state particles are highly-separated in rapidity. We call such scheme of calculations – the Parton Reggeization Approach (PRA). It’s logic is outlined below in the Sec. 2.

In PRA, the HSC, although being gauge-invariant, nevertheless explicitly depends on the transverse momenta of initial-state partons. Below we demonstrate, that this dependence is important for the calculation of the angular structure functions, since alternative prescriptions which one could propose to naively “restore” the gauge-invariance of HSC with off-shell initial-state partons lead to significantly different numerical values for them. Most importantly, for the structure function $F_{UU}^{(\cos 2\phi)}$ the difference starts already at leading power in $q_T/Q$.

The present paper has following structure: In Sec. 2 main ideas of Parton Reggeization Approach are outlined. In Sec. 3 the analytic results for angular structure functions in PRA are listed. The same quantities in the alternative gauge-invariant TMD-factorization scheme, which we call quasi-on-shell scheme are derived in the Sec. 4 and in the Sec. 5 the numerical results for structure functions in both schemes are presented. We perform our numerical computations for the planned energy of $pp$-collisions at NICA collider: $\sqrt{S} = 24$ GeV. Comparison with experimental data of E-288 Collaboration for a very close energies, is also presented in the Sec. 5 to justify the extension of PRA to this domain of relatively low energies.

2. Parton Reggeization Approach

More detailed introduction to the PRA and derivation of our factorization formula is presented in the Ref. [3]. Here we only briefly outline the main ideas. Factorization formula of PRA is based on modified-MRK approximation for QCD matrix elements. This approximation smoothly interpolates between well-known collinear and Multi-Regge asymptotics (see e.g. Ref. [4] for the review of the latter) of matrix element of ordinary CPM hard subprocess with emission of two additional partons. In the collinear limit, additional partons
have \(|k_T| \ll \mu\) where \(\mu\) is the hard scale \((\mu \sim Q\) in the case of DY process), while in the Multi-Regge limit, additional partons are highly separated in rapidity from the system of interest \((t^+t^-\) for DY process), while their typical \(|k_T| \sim \mu\).

In both limits, QCD matrix element can have the \(t\)-channel-factorized form, however in the MRK case the partons, propagating in the \(t\)-channels are not ordinary QCD quarks and gluons, but special gauge-invariant degrees of freedom of high-energy QCD, called Reggeized quarks \((Q)\) and gluons \((R)\). Due to the \(t\)-channel-factorized form of the mMRK-approximation, the cross section of lepton of high-energy QCD, called Reggeized quarks \((Q)\) and gluons \((R)\). Due to the \(t\)-channel-factorized form of the mMRK-approximation, the cross section of lepton pair production in proton-proton collisions, \(p(P_1) + p(P_2) \rightarrow l^+(k_1) + l^-(k_2) + X\), can be presented in \(k_T\)-factorized form:

\[
d\sigma = \frac{1}{0} \frac{dx_1}{x_1} \int \frac{d^2q_{T1}}{\pi} \Phi_q(x_1, t_1, \mu^2) \frac{1}{0} \frac{dx_2}{x_2} \int \frac{d^2q_{T2}}{\pi} \Phi_{\bar{q}}(x_2, t_2, \mu^2) \cdot d\sigma_{PRA}, \tag{1}
\]

where \(x_1 = q_1^+ / p_1^+\), \(x_2 = q_2^- / p_2^-\), four-momenta of partons in the initial-state of the leading order (LO) PRA hard-scattering subprocess \(Q(q_1) + \bar{Q}(q_2) \rightarrow l^+ + l^-\) are parametrized as \(q_1 = \frac{1}{2n_q} n_+ + q_{T1}\), \(q_2 = \frac{1}{2n_q} n_+ + q_{T2}\), \(t_{1,2} = q_{T1,2} = -q_{T1,2}^2\), and light-cone vectors are defined as \(n_\mu = 2P_i^+ / \sqrt{S}\), \(n_\mu^- = 2P_i^- / \sqrt{S}\) where \(S = (P_1 + P_2)^2 = 2P_1 P_2\). For any four-vector the light-cone components are \(k^\pm = (kn^\pm)\), so that \(k^2 = k^+ k^- - k_T^2\), and we do not distinguish between upper and lower light-cone indices \(k^\pm = k_\pm\).

The partonic cross-section \(d\sigma_{PRA}\) is:

\[
d\sigma_{PRA} = \frac{|A_{PRA}|^2}{2Sx_1x_2} \cdot (2\pi)^4 \delta^{(4)} (q_1 + q_2 - k_1 - k_2) \cdot d\Phi(k_1, k_2), \tag{2}
\]

where \(d\Phi(k_1, k_2)\) is the element of Lorentz-invariant phase space for final-state leptons, \(2x_1x_2S\) is the appropriate flux-factor for initial state off-shell partons (see discussion in Ref. [3]).

The LO unintegrated PDF (unPDF) \(\Phi_q(x, t, \mu^2)\) in Eq. (1) is related with ordinary PDFs of CPM as follows:

\[
\Phi_q(x, t, \mu^2) = \frac{T_q(t, \mu^2)}{t} \frac{\alpha_s(t)}{2\pi} 1 - \Delta \int_0^\infty dz \frac{x}{z} \left[ P_{qq}(z) f_q \left( \frac{x}{z}, \mu^2 \right) + P_{qg}(z) f_g \left( \frac{x}{z}, \mu^2 \right) \right], \tag{3}
\]

where \(f_{q,g}(x, \mu^2)\) are relevant collinear PDFs, and the Kimber-Martin-Ryskin cut condition [5][6], \(\Delta = \frac{\sqrt{t}}{\sqrt{\mu^2 + \sqrt{t}}}\), follows from the rapidity ordering between the last emission and the hard subprocess. In Eq. (3), \(T_q(t, \mu^2)\) is well known Sudakov factor with boundary conditions \(T_q(\mu^2, \mu^2) = T_q(0, \mu^2) = 1\), which lead to the following normalization for unintegrated PDF: \(\int dt \cdot \Phi_q(x, t, \mu^2) = x f_q(x, \mu^2)\).

In the PRA, the squared amplitude of the subprocess \((Q\bar{Q} \rightarrow l^+l^-)\) can be presented as convolution of standard lepton tensor \(L^\mu_\nu = 2[-Q^2 g^\mu_\nu + 2(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu)]\).
\[ k_1^\nu (k_2^\nu) \] and the partonic tensor \( w_{\mu\nu}^{\text{PRA}} \):

\[
|A(Q\bar{Q} \rightarrow l^+l^-)|^2 = \frac{16\pi^2}{N_c Q^4} \alpha_s^2 e^2 q_{\mu \nu} w_{\mu\nu}^{\text{PRA}},
\]

where \( N_c = 3 \) and partonic tensor reads

\[
w_{\mu\nu}^{\text{PRA}} = \frac{1}{4} \text{tr} \left[ \left( \frac{q_2\cdot \hat{n}^+}{2} \right) \Gamma_{\mu}(q_1, q_2) \left( \frac{q_1\cdot \hat{n}^-}{2} \right) \Gamma_{\nu}(q_1, q_2) \right],
\]

where factor 1/4 stands for the averaging over spins of the quark and antiquark, \( \hat{k} = k_{\mu}\gamma^\mu \) and \( \Gamma_{\mu}(q_1, q_2) \) is the Fadin-Sherman \( Q\bar{Q}\gamma \) vertex \([7, 8, 9]\):

\[
\Gamma_{\mu}(q_1, q_2) = \gamma^\mu - q_1\cdot \hat{n}_\mu q_1 - q_2\cdot \hat{n}_\mu q_2.
\]

The QED Ward identity \((q_1 + q_2)^\mu \Gamma_{\mu}(q_1, q_2) = 0\) is satisfied by this vertex for any \(q_1\) and \(q_2\).

The first term in Eq. (6) corresponds to the usual \( t \)-channel quark-antiquark annihilation diagram (a) in the Fig. 1. While other two ("eikonal") terms in Eq. (6), contain factors \(1/q_1^+\) and \(1/q_2^-\). These factors can be understood as a remnants of \( s \)-channel propagators in the diagrams where photon interacts with particles highly separated in rapidity from the lepton pair. More rigorously, the common lore in high-energy QCD (see e.g. Ref. [4, 10] and references therein) is, that particles in the central rapidity region interact with other particles, highly separated from them in rapidity, as with Wilson lines stretched along the light-cone. The “eikonal” terms in Eq. (6) correspond to the coupling of photon with these Wilson lines. Corrections to this approximation are suppressed by powers of \( e^{-\Delta y} \), where \( \Delta y \) is the rapidity gap. In other words, inclusion of the second and the third terms in Eq. (6) is the simplest possible way to effectively take into account the diagrams (b) and (c) in the Fig. 1 where photon interacts directly with the proton and it’s remnants. This approximation assumes only that the systems \( X_1 \) and \( X_2 \) are highly separated in rapidity from the central region. Rapidity gap between collinear subgraphs exists for \( q_T \ll Q \) at the level of leading region for the Drell-Yan process. This rapidity gap is filled by soft particles emitted from the Glauber gluon exchanges between collinear subgraphs, which does not lead to violation of factorization (See e.g sec. 14.2 and 14.3 in [2]).

3. Structure functions for DY process in PRA

In the notation of Ref. [11] differential cross section of DY pair production in collision of non-polarized protons can be written as the combination of helicity structure functions (SFs):

\[
\frac{d\sigma}{dxd\phi d^2q_T d\Omega} = \frac{\alpha^2}{4Q^2} \left[ F^{(1)}_{UU} \cdot (1 + \cos^2 \theta) + F^{(2)}_{UU} \cdot (1 - \cos^2 \theta) + \right]
\]

\[
+ F^{(\cos \phi)}_{UU} \cos(2\theta) \cos \phi + F^{(\cos 2\phi)}_{UU} \sin^2 \theta \cos(2\phi),
\]

where
Figure 1: Feynman diagrams for the $t$-channel quark-antiquark annihilation subprocess (a), which leads to the usual parton-model picture, and direct interaction subprocesses (b,c) which are necessary to restore QED gauge invariance of diagram (a).

were $x_{A,B} = Q e^{\pm Y}/\sqrt{S}$, angles $\theta$ and $\phi$ are defined in the Collins-Soper frame \[12\] and $F_{UU}^{1,2,\cos \phi}$ are the helicity SFs at some fixed values of $S, q_T = |q_{T1} + q_{T2}|, x_A, x_B$. With the help of factorization formula \[1\] SFs can be represented as:

$$F_{UU}^{(1,\ldots)} = \frac{S}{6\pi^2 Q_T^2} \int dt_1 \int d\phi_1 \sum_q \Phi_q^p(x_1,t_1,\mu^2)\Phi_q^\bar{p}(x_2,t_2,\mu^2) \cdot e_q^2 f^{(1,\ldots)}, \quad (7)$$

where $t_2 = (q_T - q_{T1})^2$, $Q_T^2 = Q^2 + q_T^2$ and $e_q$ is the quark electric charge in units of electron charge. Projecting the partonic tensor \[5\] on transverse, longitudinal, single spin-flip and double spin-flip helicity states of the virtual photon, one obtains the following expressions for partonic SFs in PRA \[13\]:

$$f_{PRA}^{(1)} = Q^2 + \frac{q_T^2}{2}, \quad f_{PRA}^{(2)} = (q_T - q_{T2})^2,$$

$$f_{PRA}^{(\cos \phi)} = \sqrt{\frac{Q_T^2}{q_T^2}}(q_{T1} - q_{T2}), \quad f_{PRA}^{(\cos 2\phi)} = q_T^2. \quad (8)$$

In the case of collisions of identical target and projectile (e.g. in $p p$-collisions), the SF $F_{UU}^{\cos \phi}$ is equal to zero in PRA, due to the factor $(t_1 - t_2)$ in Eq. \[8\]. However for collisions of different particles we expect nonzero value of $F_{UU}^{\cos \phi}$ due to the difference of transverse-momentum distributions of quarks and antiquarks in the projectile and in the target\[1\].

4. Quasi-on-shell schemes

From the point of view of standard TMD factorization \[2\] terms in Eq. \[1\] which restore the Ward identity for $t_{1,2} \neq 0$ can be viewed as corrections sub-

\[1\]In the Ref. \[13\] partonic coefficient $w_A$ corresponding to the $\cos \phi$ harmonic has been erroneously put to zero. This has no effect on the plots published in \[13\], however now we predict small but nonzero value of angular coefficient $\mu$, which is still compatible with NuSea \[14\] experimental data for $pD$ collisions within uncertainties. The erratum is in preparation and will be submitted to the Phys. Rev. D.
leading in powers of $q_T/Q$. Therefore it is not obvious that these terms have 
significant numerical effect on the SFs at moderate $q_T < Q$ and especially for 
$q_T \ll Q$. It is tempting to say, that the scheme of restoration of gauge-invariance 
of partonic tensor is not unique, and all of them should lead to the same results 
for SFs at $q_T \ll Q$.

The simplest way to restore gauge-invariance, retaining the transverse 
momentum of initial-state partons, is to artificially put their virtuality to zero on 
the level of hard-scattering coefficient. Such hard-scattering coefficient is just 
an amplitude of scattering of on-shell partons, which satisfies the Ward identity 
automatically. We call such an approach — quasi-on-shell (QOS) scheme.

Below we will compare the results of PRA with two versions of QOS-scheme. 
In the Ref. [2] (Sec. 14.5.2) the hard-scattering coefficient does not depend 
explicitly on $q_{T1}$ and $q_{T2}$, and four-momenta of initial-state partons, which has 
been used for the calculation of the partonic tensor, has been chosen as follows:

$$
\begin{align*}
(q_1^{(\text{QOS-1})})^{\mu} &= \frac{1}{4\kappa} \left( q^+ (\kappa + 1) n^- + q^- (\kappa - 1) n^+ \right) + \frac{q^{\mu}_T}{2}, \\
(q_2^{(\text{QOS-1})})^{\mu} &= \frac{1}{4\kappa} \left( q^+ (\kappa - 1) n^- + q^- (\kappa + 1) n^+ \right) + \frac{q^{\mu}_T}{2},
\end{align*}
$$

(9)

where $\kappa = \sqrt{Q_T^2/Q^2}$ and $q^{\pm} = Q_T e^{\pm Y}$, so that $\tilde{q}_1 + \tilde{q}_2 = q$ while $\tilde{q}_{1,2} = 0$. In 
the QOS-approximation, the partonic tensor reads:

$$
w^{\text{QOS}}_{\mu\nu} = \frac{1}{4} \text{tr} \left[ \tilde{q}_1 \gamma_\mu \tilde{q}_1 \gamma_\nu \right].
$$

and the only nonzero partonic SF, corresponding to the choice [9], is $f_{\text{QOS-1}}^{(1)} = Q^2$ while $f_{\text{QOS-1}}^{(2)} = f_{\text{QOS-2}}^{(\text{cos} \phi)} = f_{\text{QOS-1}}^{(\text{cos} 2\phi)} = 0$ like in CPM.

To do better, one can try to re-introduce the $q_{T1,2}$-dependence into the 
QOS-scheme. To this end, one adds the “small” light-cone components $q_1^-$ and 
$q_2^-$ to put vectors $\tilde{q}_1$ and $\tilde{q}_2$ on-shell:

$$
\begin{align*}
(q_1^{(\text{QOS-2})})^{\mu} &= \frac{1}{2} \left( q_1^+ n_- + \frac{q_{T1}}{q_1^+} n_+ \right) + q^{\mu}_T, \\
(q_2^{(\text{QOS-2})})^{\mu} &= \frac{1}{2} \left( q_2^+ n_- + q_2^- n_+ \right) + q^{\mu}_T,
\end{align*}
$$

(10)

where “large” light-cone components are determined from the condition $\tilde{q}_1 + \tilde{q}_2 = q$ to be 
$q_1^- = (Q_T^2 + t_1 - t_2 + \sqrt{D})/(2q^-)$ and $q_2^- = (Q_T^2 - t_1 + t_2 + \sqrt{D})/(2q^+)$ 
where $D = (Q_T^2 - t_1 - t_2)^2 - 4t_1 t_2$. Partonic SFs in the new QOS scheme are equal to:

$$
\begin{align*}
 f_{\text{QOS-2}}^{(1)} &= Q^2 \frac{(q_{T1} - q_{T2})^2}{2Q_T^2} + \frac{(q_{T1}^2 - q_{T2}^2)^2}{2Q_T^2}, \\
f_{\text{QOS-2}}^{(2)} &= (q_{T1} - q_{T2})^2 - \frac{(q_{T1}^2 - q_{T2}^2)^2}{Q_T^2}, \\
f_{\text{QOS-2}}^{(\text{cos} \phi)} &= \frac{Q^2}{Q_T^2} \frac{q_{T1}^2 - q_{T2}^2}{q_{T1}^2}, \\
f_{\text{QOS-2}}^{(\text{cos} 2\phi)} &= -\frac{(q_{T1} - q_{T2})^2}{2Q_T^2} + \frac{Q^2 + Q_T^2 (q_{T1}^2 - q_{T2}^2)^2}{2Q_T^2}.
\end{align*}
$$

(11)
In this version of QOS-scheme, the coefficients $f_{QOS}^{(1)}$, $f_{QOS}^{(2)}$ and $f_{QOS}^{(\cos \phi)}$ are equal to PRA results at leading power in $|q_{T1,2}|/Q$, however the coefficient $f_{QOS}^{(\cos 2\phi)}$ is completely different from the PRA result. At small $q_T = q_{T1} + q_{T2}$ the first term dominates and this coefficient is negative.

5. Numerical results and discussion

To justify the use of PRA at relatively low $\sqrt{S} = 24$ GeV, which is expected to be achieved during the operation of NICA collider in the $pp$-collider mode (see e.g. [15]), we compare our numerical results for the differential cross-section $d\sigma/d^3q$ as a function of $q_T$ and $Q$ with experimental data of E-288 Collaboration [16], obtained in the collisions of the proton beam with platinum fixed target at $\sqrt{S} = 19.4$ and 23.8 GeV (Fig. 2). The KMR unPDF is generated from the LO PDFs MSTW-2008 [17]. We use the factorization scale-choice $\mu_F = \xi Q_T$ and vary $\xi$ in the range $1/2 \leq \xi \leq 2$ to obtain the scale-uncertainty band. The “$\pi^2$-resummation” K-factor (see Eq. (53) in Ref. [13]) is applied to the cross-section. From the Fig. 2 one can see, that LO PRA calculation describes the E-288 data at all values of $Q$ and $Y$ reasonably well.

Comparison of LO PRA predictions for the $q_T$-dependence of polarization parameters $\lambda$ and $\nu$ with experimental data of NuSea Collaboration [14] obtained in the $pp$-collisions with $\sqrt{S} = 39$ GeV is presented in the Ref. [13] and also demonstrates a good agreement with data. This agreement justifies our attempt to provide the predictions for helicity SFs below.

Figure 2: Transverse momentum spectra of DY pairs. The histogram corresponds to calculation in PRA. The data are from the E288 Collaboration [16], left panel: $\sqrt{S} = 19.4$ GeV, $0.1 < Y < 0.7$; right panel: $\sqrt{S} = 23.8$ GeV, $-0.09 < Y < 0.51$.

In the Fig. 3 the PRA predictions for helicity SFs $F_{UU}^{(1,2,\cos 2\phi)}$ are plotted for the case of $pp$-collisions with $\sqrt{S} = 24$ GeV for two bins in the invariant mass of the pair: $2 \leq Q \leq 5$ GeV and $5 \leq Q \leq 10$ GeV. Also, the central lines of predictions of the QOS-scheme, obtained with the same KMR unPDFs but using the partonic SFs (11) are plotted in the Fig. 3 together with PRA predictions.
Figure 3: Predictions for unpolarized Drell-Yan SFs $F^{(1)}_{UU}$, $F^{(2)}_{UU}$ and $F^{(\cos 2\phi)}_{UU}$ in $pp$-collisions at $\sqrt{s} = 24$ GeV. Solid lines with uncertainty bands – PRA predictions. Dashed lines – predictions in the QOS-scheme for the default scale-choice. Short-dashed line – plot of the $(-F^{(\cos 2\phi)}_{UU})$ in the QOS scheme, since this SF in QOS scheme is negative at low $q_T$.

As expected from the comparison of partonic SFs in the Sec. 4, the PRA and QOS predictions for $F^{(1)}_{UU}$ and $F^{(2)}_{UU}$ agree for $q_T \ll Q$, however the SFs $F^{(\cos 2\phi)}_{UU}$ in two approaches differ by more than factor 3 for $q_T > 2$ GeV and have different signs for $q_T \to 0$.

In “parton-model style” TMD-factorization [11], based solely on the $q\bar{q}$-annihilation picture (diagram (a) in the Fig. 1), the TMD quark correlators for the case of unpolarized protons are parametrized in terms of unpolarized quark distribution $f_{1q}^q(x, q_T^2)$ and Boer-Mulders [18] function $h_{-\perp q}^{1q}(x, q_T^2)$. While former is responsible for the $(1 + \cos^2 \theta)$ angular dependence and contributes mostly to $F^{(1)}_{UU}$ function, the latter leads to nonzero $F^{(\cos 2\phi)}_{UU}$. The $(1 - \cos^2 \theta)$ angular dependence does not arise in TMD-factorization at leading power in $q_T^2/Q^2$ [11]. In agreement with this, in PRA the SF $F^{(2)}_{UU}$ is suppressed by factor $Q^2$ w.r.t. $F^{(1)}_{UU}$.

As we have shown above, numerical value of $F^{(\cos 2\phi)}_{UU}$ even at $q_T^2 \ll Q^2$ strongly depends on the details of the procedure of restoration of gauge-invariance of the hard-scattering coefficient. On the other hand, in the TMD-factorization, based on the diagram (a) in the Fig. 1 the hadronic tensor (e.g. Eq. (73) in [11]) does not satisfy Ward identity for $q_T \neq 0$. This raises serious doubts about the Boer-Mulders function as well-defined physical quantity in this approach.

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