Research Article

Robust Adaptive Control for a Class of Uncertain Nonlinear Systems with Time-Varying Delay

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We present adaptive neural control design for a class of perturbed nonlinear MIMO time-varying delay systems in a block-triangular form. Based on a neural controller, it is obtained by constructing a quadratic-type Lyapunov-Krasovskii functional, which efficiently avoids the controller singularity. The proposed control guarantees that all closed-loop signals remain bounded, while the output tracking error dynamics converge to a neighborhood of the desired trajectories. The simulation results demonstrate the effectiveness of the proposed control scheme.

1. Introduction

In the practical control process, control system is usually required to meet the stability and the corresponding performance index, which affects the system stability factors mainly including the uncertainties and time delays. On the study of the uncertain time-delay many scholars have achieved valuable fruits [1, 2]. Paper [3] has analyzed and designed the optimal $H_\infty$ feedback controller by the LMI method. In recent decades, the delay nonlinear systems with neural network research have received extensive attention [4–20]. Paper [4] has solved the problem of chaotic synchronization phenomenon by the neural network method. In [5–11], the study of the nonlinear continuous system and discrete nonlinear system is based on adaptive neural network control. The tracking and stabilization problem of nonlinear systems has been studied by neural network backstepping method [12, 13]. In [14], neural network control has been applied to a piece of triangle structure of multiple-input multiple-output nonlinear time-delay system, in which a dynamic system neural network is used mainly for unknown function approximation and separation. In multiple input and multiple output nonlinear system, [15] presents a new adaptive neural network controller design method but does not consider with external disturbance and time-varying delay. In [18], the problem of the adaptive neural networks control for a class of nonlinear state-delay systems with unknown virtual control coefficients is considered. In [19], a control scheme combined with backstepping, radial basis function (RBF) neural networks and adaptive control are proposed for the stabilization of nonlinear system with input and state delay.

This paper mainly aims at studying the simultaneous presence of uncertainties and time-varying delay MIMO nonlinear system. By defining the new quadratic Lyapunov-Krasovskii functionals, it has analyzed and designed the adaptive neural network controller by neural network approximation method in [15, 16].

2. Description of the Problem

Let us consider the following block-triangular structure with the disturbance of nonlinear MIMO systems with time-varying delays:
\[ \begin{align*}
\dot{x}_{ij} & = f_{ij}(x_{ij}) + g_{ij}(x_{ij})x_{ij} + \lambda_{ij}(x_{ij}) + \omega_{ij}(t), \\
\dot{\lambda}_{ij} & = S(U_j - \lambda_{ij}) + \beta_{ij}(t), \quad \dot{\beta}_{ij} = \alpha_{ij} - \dot{\lambda}_{ij} - \gamma_{ij},
\end{align*} \]

where \( x_{ij} \) and \( \lambda_{ij} \) are the state vector and the output of the network, respectively. \( f_{ij} \) and \( g_{ij} \) are the basis function and the weight vector, respectively. \( \omega_{ij} \) is the disturbance input. \( S(\cdot) \) is the activation function, and \( \beta_{ij} \) is the adaptive parameter. \( \alpha_{ij} \) and \( \gamma_{ij} \) are the design parameters.}

**3. Adaptive Neural Network Controller Design**

In this section, we will introduce a novel adaptive NN control design procedure. There are \( m_j \) design steps in the design procedure for the \( j \)-th subsystem. In each step, the unknown nonlinear function \( f_{ij}(x_{ij}) \) will be approximated by a radial neural network approximation function. Define an unknown constant as

\[ \alpha_j = \frac{1}{g_{j0}} \max \left\{ \|W_{ij}\|_2 : 1 \leq i_j \leq m_j \right\}, \quad (3) \]

where the constant \( g_{j0} \) is defined as in Assumption 2; function \( f_{ij} \) and vector \( Z_{ij} \) will be specified in each step. Furthermore, for \( j = 1, 2, \ldots, n \) and \( i_j = 1, 2, \ldots, m_j-1 \), choose the virtual control laws as follows:

\[ \lambda_{ij} = - \left( k_{ij} + 1 \right) \dot{z}_{ij}, \]

\[ \dot{\alpha}_{ij} = \frac{1}{2a_{ij}} \dot{\alpha}_{ij} - \frac{1}{4a^2_{ij}} \dot{\alpha}_{ij}^2, \quad (4) \]

where \( k_{ij} > 0 \) and \( a_{ij} > 0 \) are design parameters. \( \alpha_{ij} \) represent the estimation of the unknown constant \( \alpha_i \). Let \( \zeta_{ij} \) be the basis function vector, and define the variables \( \lambda_{ij} \) as follows:

\[ z_{ij} = x_{ij} - \dot{\lambda}_{ij}, \quad z_{ij} = x_{ij} - y_{dij}, \quad (5) \]

for \( j = 1, \ldots, n \), \( i_j = 2, \ldots, m_j-1 \). Choose the adaptive laws \( \dot{\alpha}_{ij} \) as follows:

\[ \dot{\alpha}_{ij} = \sum_{j=1}^{m_j-1} \frac{r_{ij}}{2a_{ij}} \dot{\alpha}_{ij} - b_{ij}, \quad (6) \]

where \( r_{ij} > 0 \) and \( b_{ij} > 0 \) are design parameters.}

Step 1 (1 \( \leq j \leq n \)). For the first differential equation of the \( j \)-th subsystem, choose the Lyapunov function candidate

\[ V_{ij} = \frac{1}{2} \zeta_{ij}^2 + \frac{g_{j0}}{2r_{ij}} \lambda_{ij}^2, \quad (7) \]

where \( \zeta_{ij} = x_{ij} - y_{dij}, \quad \alpha_{ij} = \alpha_j - \dot{\alpha}_{ij} \). Taking the time derivative of \( V_{ij} \), we obtain

\[ \dot{V}_{ij} = \zeta_{ij} \left( f_{ij} + g_{ij} \lambda_{ij} - y_{dij} + h_{ij} \left( x_{ij} \right) + \omega_{ij}(t) \right) \]

\[ + h_{ij} \left( x_{ij} \right) + \omega_{ij}(t), \quad (8) \]

With Lemma 3, existence of positive function \( Q_{ij}^{ij} \) such that

\[ \dot{h}_{ij} \left( x_{ij} \right) \leq \sum_{i=1}^{m_j} Q_{ij}^{ij} \left( x_{ij} \right), \quad (9) \]
Then, we have
\[ z_{j,i} h_{j,i} (\bar{x}_{j,i}) \leq |z_{j,i}| Q_{j,i}^{1/2} \leq \frac{1}{2} z_{j,i}^2 + \frac{1}{4} |Q_{j,i}^{1/2} (x_{j,i})|^2, \]
(10)
Substituting (10) into (8) yields
\[ \dot{V}_{z_{j,i}} \leq z_{j,i} \left( f_{j,i} + g_{j,i} \lambda_{j,i} - \dot{y}_{d_{j,i}} + \frac{1}{2} z_{j,i}^2 + \omega_{j,i} (t) \right) + z_{j,i} g_{j,i} z_{j,i}^2 + \frac{1}{2} |Q_{j,i}^{1/2} (x_{j,i})|^2 - \frac{g_{j,0}}{r_{j,i}} \dot{\lambda}_{j,i} \dot{\lambda}_{j,i}. \]
(11)
To overcome the time-varying delay terms of (11), consider the following Lyapunov-Krasovskii functional:
\[ V_{j,i} = V_{z_{j,i}} + V_{u_{j,i}}, \]
(12)
where
\[ V_{u_{j,i}} = \int_{t_{j,i}-\tau_{j,i} (t)}^{t} \frac{1}{2} \left( Q_{j,i}^{1/2} (x_{j,i} (s)) \right)^2 ds. \]
(13)
Take the time derivative of \( V_{u_{j,i}} \):
\[ \dot{V}_{u_{j,i}} = \frac{1}{2} \left( Q_{j,i}^{1/2} (x_{j,i} (t)) \right)^2 - \frac{1}{2} \left( Q_{j,i}^{1/2} (x_{j,i} (t - \tau_{j,i} (t))) \right)^2; \]
(14)
from (11) and (14), one has
\[ \dot{V}_{j,i} \leq z_{j,i} \left( f_{j,i} + g_{j,i} \lambda_{j,i} + \omega_{j,i} (t) - \frac{g_{j,0}}{r_{j,i}} \dot{\lambda}_{j,i} \dot{\lambda}_{j,i} \right) + z_{j,i} g_{j,i} z_{j,i}^2 + \frac{1}{2} \left( z_{j,i} \right)^2 \left( \frac{1 - 2 \tan^2 \left( \frac{z_{j,i}}{\eta_{j,i}} \right) \right) U_{j,i}, \]
(15)
where
\[ Z_{j,i} = \left[ x_{j,i}, y_{d_{j,i}}, \dot{y}_{d_{j,i}}, \dot{\lambda}_{j,i} \right]^T, \]
\[ U_{j,i} = \frac{1}{2} \left( Q_{j,i}^{1/2} (x_{j,i}) \right)^2, \]
\[ f_{j,i} (Z_{j,i}) = f_{j,i} - \dot{y}_{d_{j,i}} + \frac{1}{2} z_{j,i}^2 + 2 \tan \left( \frac{z_{j,i}}{\eta_{j,i}} \right) U_{j,i}, \]
\[ \bar{f}_{j,i} (Z_{j,i}) = f_{j,i} - \dot{y}_{d_{j,i}} + \frac{1}{2} z_{j,i}^2 + 2 \tan \left( \frac{z_{j,i}}{\eta_{j,i}} \right) U_{j,i}, \]
(16)
and \( \eta_{j,i} \) is a positive constant.

From Lemma 4, the function \((1/z) \tan h^2 (z/\eta)\) is defined at \( z = 0 \) and can be approximated by a neural network. Therefore the function \( \bar{f}_{j,i} \) will be approximated by the NN \( W_{j,i}^{T} S (Z_{j,i}) \), such that, for given \( \varepsilon_{j,i} > 0 \),
\[ \bar{f}_{j,i} (Z_{j,i}) = W_{j,i}^{T} S (Z_{j,i}) + \theta_{j,i} (Z_{j,i}), \quad |\theta_{j,i} (Z_{j,i})| \leq \varepsilon_{j,i}, \]
(17)
where \( \theta_{j,i} (Z_{j,i}) \) is the approximation error. Furthermore, a straightforward calculation shows that
\[ z_{j,i} \bar{f}_{j,i} (Z_{j,i}) \leq \frac{1}{2 a_{j,i}^2} g_{j,0} z_{j,i}^2 \alpha_{j,i} (Z_{j,i}) S (Z_{j,i}) \]
\[ + \frac{1}{2 a_{j,i}^2} g_{j,0} z_{j,i}^2 + \frac{1}{2} z_{j,i} \eta_{j,i - 1}. \]
(18)
In addition, from (6), we obtain that for any initial conditions \( \alpha_j (t_0) \geq 0, \alpha_j (t) > 0 \) for all \( t > t_0 \). Therefor
\[ z_{j,i} g_{j,i} \lambda_{j,i} \leq - \frac{g_{j,0}}{2 a_{j,i}^2} \dot{\alpha}_j z_{j,i}^2 S^T (Z_{j,i}) S (Z_{j,i}) \]
\[ - \frac{1}{2} g_{j,0} z_{j,i}^2, \]
(19)
\[ z_{j,i} \omega_{j,i} (t) \leq - \frac{1}{2} g_{j,0} z_{j,i}^2 + 2 \frac{d_j^2}{\eta_{j,i}} S_{j,i}^{-1}. \]
(20)
Substituting (18)–(20) into (15) yields
\[ \dot{V}_{j,i} \leq k_{j,i} g_{j,0} z_{j,i}^2 + \frac{1}{2} \left( a_{j,i}^2 + \epsilon_{j,i}^2 g_{j,0} + a_{j,i}^2 g_{j,0}^{-1} \right) \]
\[ + \frac{g_{j,0}}{r_{j,i}} \dot{\lambda}_{j,i} \left( \eta_{j,i} \right) \left( \frac{2 a_{j,i}^2}{r_{j,i}^2} z_{j,i}^2 S^T (Z_{j,i}) S (Z_{j,i}) - \dot{\lambda}_{j,i} \right) \]
\[ \leq k_{j,i} g_{j,0} z_{j,i}^2 + \frac{1}{2} \left( a_{j,i}^2 + \epsilon_{j,i}^2 g_{j,0} + a_{j,i}^2 g_{j,0}^{-1} \right) \]
\[ + z_{j,i} g_{j,i} \lambda_{j,i} + \left( 1 - 2 \tan \left( \frac{z_{j,i}}{\eta_{j,i}} \right) \right) \left( \frac{1 - 2 \tan \left( \frac{z_{j,i}}{\eta_{j,i}} \right) \right) U_{j,i}, \]
(21)
**Step j, i** \((i_j = 2, \ldots, m_j - 1)\). Define the Lyapunov-Krasovskii functional as
\[ V_{z_{j,m_j}} = \frac{1}{2} z_{j,m_j}^2; \]
(22)
differentiating \( V_{z_{j,m_j}} \) yields
\[ \dot{V}_{z_{j,m_j}} = z_{j,m_j} \left( f_{j,m_j} + g_{j,m_j} x_{j,i+1} - \dot{\lambda}_{j,i+1} \right) + h_{j,i} (\bar{x}_{j,i}) \]
\[ + h_{j,i} (\bar{x}_{j,i}) \]
(23)
From (10), we have
\[ z_{j,m_j} h_{j,i} \leq \sum_{k=1}^{i_j} \left( \frac{1}{2} z_{j,m_j}^2 + \frac{1}{2} \left( Q_{j,m_j}^{1/2} (x_{j,i}) \right)^2 \right) \]
\[ \lambda_{j,i+1} (Z_{j,i+1}) \]
(24)
\[ \lambda_{j,i+1} (Z_{j,i+1}) \]
(25)
Similar to (24), we can get
\[-z_{jj}^i \frac{1}{\partial x_{ij}} h_{ijk} (x_{\tau_{ij}}) \leq \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[(26)\]

Substituting (24)–(26) into (23) yields that
\[V_{x_{ij}} \leq z_{jj} \left( f_{jj} + g_{jj} x_{ij} + \omega_{jj} \right)
- \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} (f_{jk} + g_{jk} x_{jk} + \omega_{jk})
+ \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[V_{x_{ij}} \leq z_{jj} \left( f_{jj} + g_{jj} x_{ij} + \omega_{jj} \right)
- \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} (f_{jk} + g_{jk} x_{jk} + \omega_{jk})
+ \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[(27)\]

To overcome the delay terms in (27), let us consider the following Lyapunov-Krasovskii functional:
\[V_{x_{ij}} = V_{x_{ij}} + U_{x_{ij}}, \]
where
\[V_{x_{ij}} = \sum_{k=1}^{i-1} \int_{t-\tau_{jk}}^{t} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \left( f_{jk} + g_{jk} x_{jk} + \omega_{jk} \right) + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[V_{x_{ij}} = \sum_{k=1}^{i-1} \int_{t-\tau_{jk}}^{t} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \left( f_{jk} + g_{jk} x_{jk} + \omega_{jk} \right) + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[(29)\]

Differentiating \(V_{x_{ij}}\) yields
\[V_{x_{ij}} = \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} (f_{jk} + g_{jk} x_{jk} + \omega_{jk})
+ \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[V_{x_{ij}} = \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} (f_{jk} + g_{jk} x_{jk} + \omega_{jk})
+ \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} + \sum_{k=1}^{i-1} \frac{1}{2} \frac{\partial h_{ijk}}{\partial x_{ij}} \cdot \]
\[(30)\]
The NN $W_{j,i}^T S(Z_{j,i})$ is used to approximate $\mathcal{F}_{j,i}$ such that for given $\epsilon_{j,i} > 0$ we have

$$\mathcal{F}_{j,i} = W_{j,i}^T S(Z_{j,i}) + \theta_{j,i}(Z_{j,i}), \quad \lVert \theta_{j,i}(Z_{j,i}) \rVert < \epsilon_{j,i}, \quad (34)$$

where $\theta_{j,i}(Z_{j,i})$ represent the approximation error. Similar to (18) and (20), we have

$$V_{j,i} \leq -k_{j,i}g_{j0}z_{j,i}^2 + \frac{1}{2}(a_{j,i}^2 + \epsilon_{j,i}^2g_{j0} + d_{j,i}^2g_{j0})$$

$$+ \frac{g_{j0}}{r_j} r_j z_{j,i}^2 + \frac{z_{j,i}}{\alpha_{j,i}} (\theta_{j,i} - \frac{\partial l_{j,i}}{\partial y_{j,i}}) + g_{j,i}z_{j,i}^2$$

$$+ \left[ 1 - 2 \tan h^2 \left( \frac{z_{j,i}}{\eta_{j,i}} \right) \right] u_{j,i}, \quad (35)$$

Step $j \cdot m_j$ ($1 \leq j \leq n$). In the final step of the $j$th subsystem to construct the actual control law $u_j$, let us consider the following Lyapunov-Krasovskii function:

$$V_{j,i} = \frac{1}{2} \xi^2_{j,m_j} + V_{u_{j,m_j}}, \quad (36)$$

where

$$V_{u_{j,m_j}} = \sum_{j=1}^{m_j} \sum_{k=1}^{m_j} \int_{t_{-\tau_j}}^{t} \frac{1}{2(1-\tau_j)} \left[ Q_{j,k}^m \left( x_{j,k} (s) \right) \right]^2 ds$$

$$+ \sum_{k=1}^{m_j} \sum_{l=1}^{m_j} \int_{t_{-\tau_{j,k}}}^{t} \frac{1}{2(1-\tau_{j,k})} \left[ Q_{j,k}^m \left( x_{j,k} (s) \right) \right]^2 ds$$

and $z_{j,m_j} = x_{j,m_j} - \lambda_{j,m_j-1}$. Similar to (32) we get

$$V_{j,m_j} \leq z_{j,m_j} \left( q_{j,m_j} - \frac{\partial l_{j,m_j-1}}{\partial y_{j,i}} \right)$$

$$+ \left[ 1 - 2 \tan h^2 \left( \frac{z_{j,m_j}}{\eta_{j,m_j}} \right) \right] u_{j,m_j}$$

$$+ z_{j,m_j} \left( \mathcal{F}_{j,m_j} + g_{j,i}z_{j,m_j} + \omega_{j,m_j} \right), \quad (38)$$

where $\mathcal{F}_{j,m_j}(z_{j,m_j})$ can be defined by (33) with $i = m_j$.

We use the NN $W_{j,m_j}^T S(Z_{j,m_j})$ to approximate $\mathcal{F}_{j,m_j}$ such that, for given $\epsilon_{j,m_j} > 0$, we have

$$\mathcal{F}_{j,m_j} = W_{j,m_j}^T S(Z_{j,m_j}) + \theta_{j,m_j}(Z_{j,m_j}), \quad (39)$$

where $\theta_{j,m_j}(Z_{j,m_j})$ express the approximation error.

Choose the control law $u_j$ as

$$u_j = -\left( k_{j,m_j} + 1 \right) z_{j,m_j} + \frac{1}{2\epsilon_{j,m_j}} \alpha_j z_{j,m_j}^2 S^T(Z_{j,m_j}) S(Z_{j,m_j}). \quad (40)$$

Similar to (21) we have

$$V_{j,m_j} \leq \frac{1}{2} (a_{j,m_j}^2 + \epsilon_{j,m_j}^2g_{j0} + d_{j,m_j}^2g_{j0})$$

$$+ \frac{g_{j0}}{r_j} r_j z_{j,m_j}^2 + \frac{z_{j,m_j}}{\alpha_{j,m_j}} (\theta_{j,m_j} - \frac{\partial l_{j,m_j-1}}{\partial y_{j,i}})$$

$$+ \left[ 1 - 2 \tan h^2 \left( \frac{z_{j,m_j}}{\eta_{j,m_j}} \right) \right] u_{j,m_j}, \quad (41)$$

Let $V_{n,m_n} = \sum_{j=1}^{n} \sum_{k=1}^{m_n} V_{j,k}$. Combining (21), (35), and (41) gives that

$$V_{n,m_n} \leq - \sum_{j=1}^{n} \sum_{k=1}^{m_n} k_{j,k} g_{j0} z_{j,k}^2$$

$$+ \sum_{j=1}^{n} \sum_{k=1}^{m_n} \frac{1}{2} (a_{j,k}^2 + \epsilon_{j,k}^2g_{j0} + d_{j,k}^2g_{j0})$$

$$+ \sum_{j=1}^{n} \sum_{k=1}^{m_n} \frac{g_{j0}}{r_j} r_j z_{j,k}^2 + \frac{z_{j,k}}{\alpha_{j,k}} (\theta_{j,k} - \frac{\partial l_{j,k-1}}{\partial y_{j,k}})$$

$$+ \sum_{j=1}^{n} \sum_{k=2}^{m_n} \left[ 1 - 2 \tan h^2 \left( \frac{z_{j,k}}{\eta_{j,k}} \right) \right] U_{j,k}$$

$$+ \sum_{j=1}^{n} \sum_{k=2}^{m_n} z_{j,k} (q_{j,k} - \frac{\partial l_{j,k-1}}{\partial y_{j,k}}), \quad (42)$$

The control law design is thus completed.

**4. Stability Analysis**

Now, the main result in this paper can be presented as follows.

**Theorem 5.** Consider the nonlinear time-delay system (1) with the NN adaptation law (6) and the control law (40) satisfying Assumptions 1–2. All the closed-loop trajectories can guarantee boundedness if the unknown function $\mathcal{F}_{j,i}$ can be approximated by neural network and the approximating error $\theta_{j,i}$ is boundedness.
Proof. Define functions $\varphi_{j,k}$, such that

$$-\sum_{j=1}^{n} \sum_{k=2}^{m_j} z_{j,k} \left( \varphi_{j,k} - \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right) \leq 0. \quad (43)$$

Let $0 < S^T(\cdot)S(\cdot) < L$, where $L$ is the number of neural network weights.

From (6), we can get

$$- \sum_{k=2}^{m_j} z_{j,k} \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \leq \sum_{k=2}^{m_j} z_{j,k} \left( b_j \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right) \leq \sum_{k=2}^{m_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j}.$$ 

By choosing function $\varphi_{j,k}$ as

$$\varphi_{j,k} = - b_j \tilde{\alpha}_j \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} - \frac{r_j L}{2 \sigma_{j,k}^2} z_{j,k}^2 \sum_{l=1}^{k-1} \frac{z_{j,l} \partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} + \frac{\sigma_{j,k}^2}{2 \sigma_{j,k}^2} S^T(Z_{j,k}) S(Z_{j,k}) \quad (44)$$

$$+ \sum_{k=2}^{m_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j}.$$

(43) holds.

In a similar way, we can get

$$\sum_{j=1}^{n} \frac{g_{j,0} \tilde{\alpha}_j}{r_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j} \leq \sum_{j=1}^{n} \frac{g_{j,0} \tilde{\alpha}_j}{r_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j}.$$ 

Now, choose the Lyapunov function as $V = V_{n,m}$. Combining (42)–(46) gives that

$$\dot{V}_{n,m} \leq - \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{g_{j,0} \tilde{\alpha}_j}{r_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j} \leq \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left. \frac{\partial \lambda_{j,k-1}}{\partial \tilde{\alpha}_j} \right|_{\tilde{\alpha}_j}.$$ 

5. Simulation Examples

In this section, we will give one example to demonstrate the effectiveness of the proposed method in this paper. Let us consider the following example:

$$\dot{x}_{1,1} = -x_{1,1} + \left( 1 + \cos^2(x_{1,1}) \right) x_{1,2} + x_{2,1}^2 + \omega_{1,1}(t),$$

$$\dot{x}_{1,2} = x_{1,1} x_{1,2} + x_{2,1} + x_{2,2} + \left( 1 + 0.5 \cos^2(x_{2,2}) \right) u_1 + x_{r_{1,2}} + \omega_{1,2}(t),$$

$$\dot{x}_{2,1} = -x_{2,1} + x_{2,2} + x_{r_{2,1}} + \omega_{2,1}(t),$$

$$\dot{x}_{2,2} = (x_{1,2} + x_{2,1}) x_{2,2} - x_{1,1} u_1 + \left( 2 + \sin^2(u_1) \right) u_2 + x_{r_{2,2}} + \omega_{2,2}(t),$$

where $x_{r_{ij}} = x_{ij}(t - r_{ij})$, $j = 1, 2$, $i_j = 1, 2$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{System output $y_1(t)$ ("--") and the reference $y_{d1}(t)$ ("--".)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{System output $y_2(t)$ ("--") and the reference $y_{d2}(t)$ ("--".)}
\end{figure}
And the time delays are chosen as
\[
\begin{align*}
\tau_{1,1} &= 0.9 + 0.1 \sin(t), & \tau_{1,2} &= 1 + 0.5 \sin(t), \\
\tau_{2,1} &= 0.4 + 0.1 \cos(t), & \tau_{2,2} &= 2 + 0.1 \cos(t),
\end{align*}
\]
(50)
given the reference output signals as
\[
\begin{align*}
y_{d1}(t) &= 0.5(\sin(t) + \sin(0.5t)), \\
y_{d2}(t) &= 0.5 \sin(t) + \sin(0.5t).
\end{align*}
\]
The control law is given by (40). The NN adaptation law is given by (6). Choose the design parameters
\[
\begin{align*}
k_{1,1} &= k_{1,2} = k_{2,1} = k_{2,2} = 20, \\
a_{1,1} &= a_{1,2} = 2, & a_{2,1} &= a_{2,2} = 1, \\
r_1 &= r_2 = 400, \\
b_1 &= b_2 = 0.025.
\end{align*}
\]
(51)
Take the external disturbance as
\[
\begin{align*}
\omega_{1,1}(t) &= \omega_{1,2}(t) = 0.04 \sin(2\pi t), \\
\omega_{2,1}(t) &= \omega_{2,2}(t) = 0.04 \cos(2\pi t).
\end{align*}
\]
(52)
The simulation is run under the initial conditions
\[
x_{i,j}(0) = 0, \quad -\tau_0 \leq \theta \leq 0, \quad j = 1, 2, \quad i = 1, 2,
\]
and \([\hat{\alpha}_1(0), \hat{\alpha}_2(0)]^T = [0, 0]^T\). The result of control scheme is displayed in Figures 1–5. Figures 1 and 2 demonstrate the outputs of system and the reference signals. The responses of state variables \(x_{1,2}\) and \(x_{2,2}\) are shown in Figure 3. The control input signals \(u_1\) and \(u_2\) are illustrated in Figures 4 and 5 which depict the boundedness of adaptive parameters \(\hat{\alpha}_1\) and \(\hat{\alpha}_2\).

6. Conclusion

For a class of perturbed nonlinear MIMO time-varying delay systems in a block-triangular form, an adaptive neural control design is presented. Although there are some fluctuations of the systems and control output under the influence of interference, the required performance can be achieved in a short period of time by using the controller designed in this paper and guarantees the boundedness of all the signals in the closed-loop system. It is further extended on the bases in [14, 15], which makes it suitable for wider range of applications. The effectiveness of the proposed approach is provided by a simulation example.

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