Modeling charge density in AlGaN/AlN/InGaN/GaN-based double heterostructures including InGaN layer strain relaxation

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Abstract
An analytical model is presented to calculate the two-dimensional electron gas (2DEG) density and barrier height of bare surface AlGaN/AlN/InGaN/GaN double heterostructures, which use InGaN as the conducting layer. The basic model is derived from electrostatic analysis of the different material interfaces. The effect of strain relaxation in the InGaN layer is also incorporated here. Further, the impact of a two-dimensional hole gas at the InGaN/GaN interface, formed when the InGaN layer thickness is high, has been considered. The presented results are seen to agree with the available experimental results. Thus, this model can be a useful tool in the design and modeling of InGaN-based III-nitride heterostructures.

1. Introduction
AlGaN/GaN-based high electron mobility transistors (HEMTs) which use GaN as the channel material are now well-developed for high-power and high-frequency applications. The most useful feature of these heterostructures is the presence of a high-mobility and high-density two-dimensional electron gas (2DEG) with a sheet density \( N_s \) of approximately \( 10^{13} \text{ cm}^{-2} \) at the AlGaN/GaN heterointerface, even in the absence of both AlGaN barrier layer doping and a gate metal (bare surface) [1, 2]. Distributed donor states present at the AlGaN surface are understood to be the source of electrons forming the 2DEG at the AlGaN/GaN interface [2–7].

AlGaN/InGaN/GaN double heterostructures (DHs) employ InGaN as the conducting layer and GaN as a back barrier. These DHs improve the device performance because of their stronger carrier confinement and higher mobility, reduced current collapse, and higher saturation current than conventional AlGaN/GaN heterostructures [8–11]. On the other side, the use of an AlN spacer between the conducting channel and AlGaN barrier is suggested to enhance the mobility in GaN-based devices [12]. Numerical simulations have indicated even superior confinement characteristics of 2DEG in the presence of an AlN spacer [13, 14]. AlN possesses very high spontaneous polarization, and a pseudomorphic growth of AlN on GaN will cause strong piezoelectric polarization due to the large lattice mismatch. Therefore, AlGaN/AlN/InGaN/GaN-based DH-HEMTs are supposed to be more advantageous than conventional GaN channel HEMTs. An analytical model of the carrier density and bare surface barrier height in these DHs is, therefore, urgently needed.

The 2DEG density in a thin-InGaN conducting layer AlGaN/AlN/InGaN/GaN DH significantly increases with increasing In content as may be observed from the experimental Hall data [15–21] (the formal definition of ‘thin’ will be provided in the following sections). One intuitively expects that this increment in the 2DEG density is due to a modulation of the polarization charge at the AlGaN/InGaN interface caused by the In content of the InGaN layer. Further, besides the In content, experiments [19] have shown that the 2DEG density of such a DH with an In\(_{0.06}\)GaN conducting layer also increases when its thickness increases from 3 to 15 nm. In this case, the Al\(_{0.3}\)GaN barrier thickness is fixed at 24 nm. Additionally, Simin et al [8, 9] have reported a much lower 2DEG density in a 5 nm InGaN layer DH compared to that in a 10 nm InGaN layer DH [19] of similar composition and geometry. These results indicate that, apart from the material compositions and the barrier thickness, the InGaN layer thickness impacts the carrier concentration of thin-channel AlGaN/InGaN/GaN field-effect transistors (FETs) as well. Indeed, Chu et al [22] and Hwan et al [23] have suggested that, the 2DEG density in these DHs
with a fixed In composition is sensitive to the strain relaxation ratio of the InGaN layer. Clearly, the strain relaxation ratio would be sensitive to the InGaN layer thickness. A similar trend that the 2DEG density significantly increases with InGaN layer thickness has also been experimentally observed when the conducting layer is relatively thicker. For example, Wang et al. [20] have reported that in Al$_{0.32}$GaN/In$_{0.1}$GaN/GaN heterostructures, the sheet concentration increases when the InGaN layer thickness is increased from 10 to 50 nm (AlGaN thickness is 19 nm). Again, Wang et al. [24] have reported sufficiently high 2DEG density in an Al$_{0.2}$GaN/In$_{0.1}$GaN/GaN heterostructure for InGaN layer thickness of 20 nm (AlGaN thickness is 25 nm). It will be shown in the following sections that apart from the aforementioned strain relaxation effects, an additional effect that impacts the 2DEG density needs to be considered in the case of thick InGaN layers.

Recently, an analytical model is presented for the 2DEG density and bare surface barrier height in AlGaN/InGaN/GaN DHs, which considers AlGaN surface donor states as the source of electrons [25]. In the present work, the previous model has been significantly augmented to enable analysis of the AlGaN/AlN/InGaN/GaN DHs. This model also quantitatively incorporates InGaN layer-thickness-dependent strain relaxation effects on the polarization charge, and thereby, the 2DEG density. Additionally, the possible formation of a two-dimensional hole gas (2DHG) [26] in the case of thick InGaN layers has been analytically modeled.

### 2. Model

Figure 1 shows the conduction band (CB) and valence band (VB) diagram of an AlGaN/AlN/InGaN/GaN DH. Here, $E_F$ is the difference between the Fermi level and the CB minimum at the AlN/InGaN heterointerface. The dashed line shows the reshaped band diagram when the InGaN layer thickness becomes sufficiently high so that the valence band can cross the Fermi level energy. The polarization charges at different interfaces are also schematically shown. The charge neutrality condition is maintained throughout.

\[ EF = \Delta E_{C1} + \Delta E_{C2} + \Delta E_{C3} + \Delta E_{C4} + \epsilon \cdot q \cdot B \]

\[ P_{\text{pr}}(\text{AlN}) + P_{\text{pr}}(\text{InGaN}) + P_{\text{pr}}(\text{GaN}) \]

\[ P_{\text{pr}}(\text{AlGaN}) + P_{\text{pr}}(\text{AlN}) + P_{\text{pr}}(\text{InGaN}) + P_{\text{pr}}(\text{GaN}) \]

\[ P_{\text{pr}}(\text{AlGaN}) + P_{\text{pr}}(\text{AlN}) + P_{\text{pr}}(\text{InGaN}) + P_{\text{pr}}(\text{GaN}) \]

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the 2DEG density \( N \) is given by the following approximation based on the case of AlGaAs/GaAs heterointerfaces: \( E_F = \frac{N}{D} \) with \( D \) as the effective density of states of electrons \([27, 28]\). Now, from the results of electrostatic analysis (cf figure 1 straight line)

\[
q \cdot \Phi_R - F_1 d_{AlGaN} + \Delta E_{C1} - \Delta E_{C2} + E_F = F_2 d_{AlN},
\]

(1)

Here, \( F_1 (F_2) \) is the electric field at the AlGaN (AlN) layer. From the continuity of displacement at the AlGaN/AlN, AlN/InGaN, and InGaN/GaN interfaces

\[
\varepsilon F_1 = \varepsilon F_2 = - q(\sigma_{AlN} - \sigma_{AlGaN}),
\]

(2a)

\[
\varepsilon F_2 + \varepsilon F_3 = q\sigma_{AlN} - qN,
\]

(2b)

\[
\varepsilon F_3 = q\sigma_{InGaN}.
\]

(2c)

Here, \( F_1 \) represents the electric field at the InGaN layer. For simplicity in the model, \( \varepsilon \) is defined as the average dielectric constant of AlGaN, InGaN, and GaN layers. The GaN buffer is relaxed, while the InGaN, AlGaN, and AlN layers experience compressive, tensile, and tensile strain respectively. The polarization charges \( \sigma_{AlGaN}, \sigma_{AlN}, \) and \( \sigma_{InGaN} \) are modeled in the following way.

\[
\sigma_{AlGaN} = |P_{pe}(AlGaN) - P_{pe}(InGaN) + P_{pe}(AlGaN) - P_{pe}(InGaN)|,
\]

(3a)

\[
\sigma_{AlN} = |P_{pe}(AlN) - P_{pe}(InGaN) + P_{pe}(AlN) - P_{pe}(InGaN)|,
\]

(3b)

\[
\sigma_{InGaN} = |P_{pe}(InGaN) - P_{pe}(GaN) + P_{pe}(InGaN)|.
\]

(3c)

Here, \( P_{pe} \) (\( P_{pc} \)) denotes corresponding spontaneous (piezoelectric) polarization charge \([1]\).

\[
P_{pe}(AlGaN) = -0.029 - 0.052 \cdot m,
\]

(4a)

\[
P_{pe}(InGaN) = -0.029 - 0.003 \cdot n.
\]

(4b)

The piezoelectric polarization charge is calculated from the corresponding piezoelectric coefficients, elastic constants, and bulk and strained lattice parameters respectively \([1]\).

\[
P_{pe}(AlGaN) = 2 \left( \frac{a_r - a(AlGaN)}{a(AlGaN)} \right) \cdot \left( e_{s1} - e_{33} \frac{C_{s13}}{C_{33}} \right),
\]

(5a)

\[
P_{pe}(AlN) = 2 \left( \frac{a_r - a(AlN)}{a(AlN)} \right) \cdot \left( e_{s1} - e_{33} \frac{C_{s13}}{C_{33}} \right),
\]

(5b)

\[
P_{pe}(InGaN) = 2 \left( \frac{a_r - a(InGaN)}{a(InGaN)} \right) \cdot \left( e_{s1} - e_{33} \frac{C_{s13}}{C_{33}} \right).
\]

(5c)

Here, \( e_{s1} \) and \( e_{33} \) (\( e_{s1}^r \) and \( e_{33}^r \)) denote piezoelectric coefficients of AlGaN (InGaN), and \( C_{s13} \) and \( C_{33} \) \((C_{s13}^r \) and \( C_{33}^r \)) are the elastic constants of AlGaN (InGaN). The index \( a \) is used to denote the bulk lattice constant. If one assumes the InGaN layer to be partially relaxed, its uniform average lattice constant is denoted by \([10, 19, 29]\)

\[
a_r = (1 - r) \cdot a(AlGaN) + r \cdot a(InGaN).
\]

(6)

Then, a fully strained (relaxed) InGaN layer will have the lattice constant of GaN (InGaN) material. The parameter \( r \), known as the degree of strain relaxation, is supposed to depend on the InGaN layer thickness \( d_{InGaN} \). The upper AlN and AlGaN layers are considered completely strained against the InGaN layer. Figure 1 also schematically shows all the polarization charges at heterointerfaces.

Note that the number of empty surface donor states above the Fermi level is given by \( n_0 \cdot (q \cdot \Phi_R - E_d) \), which corresponds to the 2DEG density at the AlN/InGaN interface \( N \). Then, combining equations (1) and (2) with this expression, one obtains

\[
N = \frac{\sigma_1 d_{AlGaN} + \sigma_2 d_{AlN} + \frac{\Delta E_{C1} - E_d}{q}}{d_{AlGaN} + d_{AlN} + \frac{q}{D} \cdot \left( \frac{1}{D} + \frac{1}{m} \right)}.
\]

(7)

Here, \( \Delta E_{C1} \) is the CB offset at an AlGaN/InGaN heterointerface, \( \sigma_1 = \sigma_{AlGaN} - \sigma_{InGaN} \) and \( \sigma_2 = \sigma_{AlN} - \sigma_{InGaN} \). Then, SBH is given by \( q \cdot \Phi_R = \frac{N}{n_0} + E_F \). It may be noted that if one sets the In mole fraction \( n = 0 \) and the effect of \( E_d \) is marginalized, equation (7) reverts to the expression developed for the bare AlGaN/AlN/GaN heterostructures \([30]\). Moreover, if one sets \( d_{AlN} = 0 \), equation (7) becomes the expression developed for the bare AlGaN/InGaN/GaN heterostructures \([25]\). The AlGaN surface state parameters \( n_0 \) and \( E_d \) are given by Goyal et al \([31]\).

When the InGaN layer thickness increases beyond a certain value, the VB crosses the Fermi level to form a 2DHG at the InGaN/GaN interface. The modified band diagram is also depicted in figure 1 as a dashed line. In fact, the existence of such a high-density 2DHG in a GaN/AlGaN/GaN heterostructure is experimentally shown \([32]\). Modeling the effect of the 2DHG in a GaN/AlGaN/GaN is reported \([33, 34]\). Again, Chen et al \([26]\) have
Table 1. The variation of the threshold InGaN layer thickness (above which the 2DHG forms at the InGaN/GaN interface) with the In content is tabulated. The interface polarization charge is also shown.

| n (%) | 1.5 | 3  | 4.5 | 6   | 9   | 12  | 15  | 18  | 21  |
|-------|-----|----|-----|-----|-----|-----|-----|-----|-----|
| \(d_{th, InGaN} \) (nm) | 139 | 68 | 44.6| 32.8| 21  | 15.2| 11.7| 9.3 | 7.7 |

experimentally demonstrated a hole accumulation at the heterointerface due to a strong piezoelectric polarization effect in the InGaN/GaN heterostructure. The generated holes facilitate the formation of an equal density of electrons at the AlN/InGaN heterointerface. One can define a threshold InGaN layer thickness term \(d_{th, InGaN} \) above which there will be two sources of electrons in the AlN/InGaN interface, one source is the AlGaN surface donor states (as discussed before), and the other source is the formation of such a 2DHG contributing to equal amount of electrons. By denoting effective density of states of holes as \(D_h \), one can reformulate the continuity of displacement equation at the InGaN/GaN interface (cf equation (2c)) as

\[
\varepsilon F_3 = q\sigma_{InGaN} - qD_h(F_3d_{InGaN} - E_{F1} - E_g).
\]

Here, \(E_g \) is the band gap of the InGaN material, and \(E_{F1} \) is the difference between the Fermi level and the new CB minimum at the AlN/InGaN heterointerface. Then, the total 2DEG density is written as

\[
N_s = N_s + D_h(F_3d_{InGaN} - E_{F1} - E_g),
\]

where \(N_s \) is as in equation (7). Note that \(E_{F1} \) will be related to \(N_s; \ E_{F1} = \frac{N_s}{\beta} \). One must solve equations (8) and (9) to find \(N_s; \ N_s \) is derived to be

\[
N_s = \frac{N_s + D_h(q\sigma_{InGaN} - \varepsilon F_3)}{1 + \frac{D_h}{\varepsilon} - \frac{q\sigma_{InGaN}D_h}{\varepsilon N_s}}.
\]

By considering \(D_h = (1.5 \cdot \frac{q\gamma_{InGaN}}{\varepsilon N_s}) \) where \(n_p \) is the electron rest mass [28, 33], the term \(qD_h \ d_{InGaN} \) is evaluated to be in the order of \(10^{-9} \) F m⁻¹. As \(\varepsilon \) is in the order of \(10^{-11} \) F m⁻¹, one can safely write \(\varepsilon + qD_h \ d_{InGaN} \approx qD_h \ d_{InGaN} \). After simplifications, one finds the expression for \(N_s \) as shown below.

\[
N_s = N_s + N_{sh}, \tag{11a}
\]

\[
N_{sh} = \sigma_{InGaN} - \frac{\varepsilon F_3}{qD_{InGaN}}. \tag{11b}
\]

The term \(N_{sh} \) is the 2DHG density at the InGaN/GaN interface. Following this equation, one can obtain

\[
d_{th, InGaN} = \frac{\varepsilon F_3}{q\sigma_{InGaN}}. \tag{12}
\]

This means, 2DHG appears when \(d_{InGaN} > d_{th, InGaN} \). In such a case, equation (2b) should be rewritten as

\[
\varepsilon F_2 + \varepsilon F_3 = q\sigma_{InGaN} - qN_{sh}.
\]

Henceforth, the formation of a 2DHG will also modify the magnitude of the electric field \(F_1, F_2, \) and \(F_3 \).

3. Results and discussion

The threshold InGaN conducting layer thickness \(d_{th, InGaN} \) depends on the polarization charge in the InGaN layer and its band gap, thereby on \(n \). Table 1 gives an estimate of \(d_{th, InGaN} \) depending on \(n \). The wide variation of the polarization charge \(\sigma_{InGaN} \) with \(n \) is also shown. This table delineates that the 2DHG at the InGaN/GaN interface will form at a much lower InGaN layer thickness when its In composition increases.

3.1. Thin conducting layer: \(d_{InGaN} \leq d_{th, InGaN} \)

Consider the case when the InGaN layer thickness is less than its threshold value. First, 2DEG density in an AlGaN/InGaN/GaN DH is analyzed (i.e. \(d_{AlN} = 0 \)). By following equations (3a) and (3c), one can observe that \(\sigma_3 \) (and thereby \(N_3 \) in equation (7)) has no dependence on the InGaN polarization-charge (which, in turn, depends on \(n \)). Now, if the InGaN layer grown on the relaxed GaN buffer is assumed to be fully strained (i.e. \(r = 0 \), reference: equation (6)), the polarization charge term \(P_{pe} \) (AlGaN) becomes independent of \(n \) as well (cf equation (5a)). In this condition, \(\sigma_3 \) and \(N_3 \) lose their dependency on \(n \). Thus, introducing the In material in the conducting layer cannot have any impact on the 2DEG density. On the contrary, all available experimental data strongly predict a substantial increment of the 2DEG density with increasing \(n \) as discussed in the section 1, even
when the InGaN layer thickness is as low as 3 to 5 nm. These observations can only be explained if one takes into account a thickness-dependent strain relaxation effect in the InGaN layer. The non-zero $r$ which must increase with increasing $d_{\text{InGaN}}$ modulates the term $P_{\text{pe}}(\text{AlGaN})$, and thereby finally results in the 2DEG density modulation. Note that this analysis holds true even if the AlN layer is present ($r$ will impact both $P_{\text{pe}}(\text{AlGaN})$ and $P_{\text{pe}}(\text{AlN})$). Indeed, $r$ can be extracted by fitting the 2DEG density model with the available experimental data [8, 11, 19], where $m$, $n$, and $d_{\text{AlGaN}}$ parameters are known. How the 2DEG density in an AlGaN/InGaN/GaN DH increases with $n$ and $d_{\text{AlGaN}}$ is described in an earlier work [25]. Here, the model and experimental data of the 2DEG density along with a list of extracted $r$ (depending on $d_{\text{InGaN}}$) is shown in table 2.

Figure 2. The predicted variation of the 2DEG density in an Al$_m$GaN/AlN/In$_n$GaN/GaN heterostructure with barrier thickness is shown. A limited comparison to the available experimental data points is given in table 3.

Table 2. Comparison of the 2DEG density between the model prediction (cf. equation (7)) and the experimentally reported data [8, 11, 19] is shown ($d_{\text{AlN}} = 0$). The strain relaxation parameter $r$ is extracted.

| $m$  | $n$  | $d_{\text{AlGaN}}$ (nm) | $d_{\text{InGaN}}$ (nm) | 2DEG exp. data ($10^{13}$ cm$^{-2}$) | 2DEG model ($10^{13}$ cm$^{-2}$) | $r$ (approx.) |
|------|------|-------------------------|--------------------------|-------------------------------------|-------------------------------|--------------|
| 0.25 | 0.1  | 25                      | 5                        | 1.1                                 | 1.15                          | 0.3          |
| 0.3  | 0.06 | 24                      | 10                       | 1.52                                | 1.5                           | 0.7          |
| 0.3  | 0.1  | 24                      | 10                       | 1.8                                 | 1.75                          | 0.7          |
| 0.3  | 0.06 | 24                      | 5                        | 1.38                                | 1.32                          | 0.3          |
| 0.25 | 0.06 | 25                      | 5                        | 1.0                                 | 1.05                          | 0.3          |

Table 3. Comparison of the 2DEG density between the model prediction and the experimentally reported data [21, 30, 35] is shown ($d_{\text{AlN}} = 1$ nm). The parameter $r$ is extracted for $d_{\text{InGaN}} = 12$ nm.

| $m$  | $n$  | $d_{\text{AlGaN}}$ (nm) | $d_{\text{InGaN}}$ (nm) | 2DEG exp. data ($10^{13}$ cm$^{-2}$) | 2DEG model ($10^{13}$ cm$^{-2}$) | $r$ (approx.) |
|------|------|-------------------------|--------------------------|-------------------------------------|-------------------------------|--------------|
| 0.25 | 0    | 25                      | 0                        | 1.1                                 | 1.07                          | 0            |
| 0.25 | 0.08 | 25                      | 5                        | 1.24                                | 1.27                          | 0.3          |
| 0.25 | 0.13 | 25                      | 5                        | 1.41                                | 1.4                           | 0.3          |
| 0.25 | 0.19 | 25                      | 5                        | 1.57                                | 1.55                          | 0.3          |
| 0.25 | 0.04 | 17                      | 12                       | 1.0                                 | 1.04                          | 0.83         |

The 2DEG density in AlGaN/AlN/InGaN/GaN DHs is now explored with $d_{\text{AlN}} = 1$ nm. Figure 2 shows how the increasing In content in the InGaN layer noticeably increases the 2DEG density in these DHs. Table 3 shows a comparison among this model and available experimental data. Consider the case where $d_{\text{InGaN}} = 5$ nm, $m = 0.25$, $d_{\text{AlGaN}} = 25$ nm, and the In component is gradually increased. The respective experimental data for $r$ as high as 0.19 is also shown for the model validation. The previously extracted value of $r = 0.3$ for the samples with $d_{\text{InGaN}} = 5$ nm (ref. table 2) has been used here. As all the experimental data reported by Peng et al [21] fit well with the model predictions, one infers that the effect of strain relaxation in the
InGaN layer has been properly included. For the sample which has $d_{\text{InGaN}} = 12$ nm, $r$ is extracted to be as high as 0.83 from the experimental data [3]. Note in all cases, the InGaN layer thickness is lower than its threshold value.

The surface barrier height SBH in these DHs is discussed. It is already known how the use of an AlN spacer in an AlGaN/AlN/GaN heterostructure increases its SBH [30]. In an AlGaN/AlN/InGaN/GaN DH ($d_{\text{AlN}} = 1$ nm) and corresponding to ($d_{\text{AlGaN}} = 17$ nm, $m = 0.25$, and $n = 0.04$) [35], the SBH is calculated to be 2.84 eV. For ($d_{\text{AlGaN}} = 25$ nm, $m = 0.25$, and $n = 0.08$), ($d_{\text{AlGaN}} = 25$ nm, $m = 0.25$, and $n = 0.13$), and ($d_{\text{AlGaN}} = 25$ nm, $m = 0.25$, and $n = 0.19$) [21], the SBH is calculated to be 2.92, 3.12, and 3.35 eV, respectively. This means, increasing the In content in the InGaN layer further increases the SBH.

3.2. Thick conducting layer: $d_{\text{InGaN}} > d_{\text{th, InGaN}}$

Now, carrier densities in the DHs are investigated where the InGaN layer is comparatively thick, so as to facilitate the formation of a 2DHG at the InGaN/GaN interface. Table 4 shows the 2DEG density as a function of $m$, AlGaN barrier, and InGaN layer thickness ($d_{\text{AlN}} = 0$). The InGaN layer thickness is minimum 20 nm, and expected to be near fully relaxed. As $n = 0.1$ in all the cases, one can calculate $d_{\text{th, InGaN}} = 18.6$ nm. Thus, $d_{\text{InGaN}} > d_{\text{th, InGaN}}$ condition is ensured. As the predicted 2DEG density (cf equation (11a)) in the table matches with the available experimental data, one can conclude that the 2DHG density at the InGaN/GaN interface is correctly modeled by equation (11b). How the 2DHG density grows up with the increasing InGaN layer thickness is shown in figure 3, where $n$ is used as a parameter.

Figure 4 shows the variation of the extracted $r$ with $d_{\text{InGaN}}$ values {5, 10, 12, 20, 30, 50} nm. These are obtained from tables 2–4. The importance of this model is that it analytically predicts 2DEG density in bare surface AlGaN/AlN/InGaN/GaN double heterostructures. Its accuracy has been shown here to the extent of the experimental data available. This model allows us to calculate the electric fields $F_1$, $F_2$, and $F_2$ in the material layers too. Several RF applications demand different $d_{\text{AlGaN}}, d_{\text{AlN}}, d_{\text{InGaN}}, m$, and $n$ values for a certain charge density, which is easily enabled by the present approach. Analytical expression for the 2DHG density, formed when the InGaN layer thickness increases beyond a threshold value, is developed. This model takes the first
theoretical attempt to investigate the InGaN layer strain relaxation effects, and more theoretical and experimental reports are asked for. In spite of some minor deviations that can be related to experimental uncertainties, the proposed model is able to match the available experimental results well. Being generic in nature, this model also applies to InAlN/InGaN/GaN heterostructure designs [36–39]. This is because the surface state parameters of an InAlN barrier layer is already available [40].

4. Conclusion

A physics-based analytical model is presented for the 2D electron gas density and bare surface barrier height in AlGaN/AlN/InGaN/GaN double heterostructures. The thickness-dependent strain relaxation effect in the InGaN conducting layer has been evaluated, and the possible presence of a 2D hole gas at the InGaN/GaN interface has been calculated. The developed model agrees well with available experimental data and confirms that the presence of the In content in the InGaN layer improves both the electron density as well as surface barrier height in such heterostructures. Therefore, this model can be used for analyzing electrical characterization data as well as for predictive design of epitaxial AlGaN/AlN/InGaN/GaN double heterostructure stacks. It could be further refined with the availability of more experimental data on these double heterostructures.

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![Figure 4. The variation of the degree of InGaN layer strain relaxation with its thickness is shown. The line is a guide to the eye.](image-url)
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