Abstract—Symbiotic radio has emerged as a promising technology for spectrum- and energy-efficient wireless communications, where the passive secondary backscatter devices (BDs) reuse not only the spectrum but also the power of the active primary users to transmit their own information. In return, the primary communication links can be enhanced by the additional multipaths created by the BDs. This is known as the mutualism relationship of symbiotic radio. However, due to the severe double-fading attenuation of the passive backscattering links, the enhancement of the primary link provided by one single BD is extremely limited. To address this issue and enable full mutualism of symbiotic radio, in this paper, we study multiple-input multiple-output (MIMO) symbiotic radio communication systems with massive BDs. We first derive the achievable rates of the primary active communication and secondary passive communication, and then consider the asymptotic regime as the number of BDs goes large, for which closed-form expressions are derived to reveal the relationship between the primary and secondary communication rates. Furthermore, the precoding optimization problem is studied to maximize the primary communication rate while guaranteeing that the secondary communication rate is no smaller than a certain threshold. Simulation results are provided to validate our theoretical studies.

Index Terms—MIMO symbiotic radio, backscattering, massive passive devices, active and passive communication, asymptotic analysis

I. INTRODUCTION

Symbiotic radio has emerged as a new paradigm to achieve both spectrum-efficient and energy-efficient wireless communications, for which the secondary user modulates its information over the radio frequency (RF) signals received from primary transmitter (PT) [1]–[3]. As such, the secondary backscatter device (BD) in symbiotic radio systems leverages not only the spectrum as in the extensively studied cognitive radio systems [4]–[6], but also the energy of the primary signals via passive backscattering technology for its own information transmission. To overcome the shortcoming of poor reliability suffered from the conventional passive ambient backscatter communication (AmBC) receiver [8]–[10], symbiotic radio introduces cooperation between the backscatter transmission and the primary transmission by using a joint receiver. Depending on the relations between the symbol durations of the primary and secondary signals, symbiotic radio can be classified into parasite symbiotic radio (PSR) and commensal symbiotic radio (CSR) [11]. In PSR, the secondary and primary signals have equal symbol durations, so that the information transmission of BD interferes with the primary transmission. By contrast, for CSR, the symbol duration of BD signals is much longer than that of the primary signals, so that the backscattering signal of the BD may create additional multipaths to enhance the primary communication links. This is known as the mutualism relationship of symbiotic radio [11]. The mutualism spectrum sharing and low power consumption nature of symbiotic radio render it an attractive massive access technology for the sixth-generation (6G) mobile communication networks [12]–[14], to realize spectrum- and energy-efficient Internet of Things (IoT). By riding over different types of primary networks, there are many emerging applications for symbiotic radio, such as e-health, smart home, and environmental monitoring [1].

Significant research efforts have been recently devoted to the theoretical analysis and performance optimization of symbiotic radio systems. For example, the performance analysis in terms of achievable rate [15], [16], channel capacity [17] and outage probability [18] are given in different setups. By exploiting multi-antenna techniques for performance enhancement, multiple-input-single-output (MISO) and multiple-input-multiple-output (MIMO) symbiotic radio systems are investigated in [15], [19], [20]. Furthermore, beamforming optimization problems have been studied to maximize various performance metrics, such as energy efficiency [15], sum capacity of the primary and secondary communications [19], and fairness of the secondary users [20]. Besides, energy efficiency (EE) studies for symbiotic radio systems have also received growing attentions recently [21]–[23]. For example, in [21] and [22], the EE of symbiotic radio is defined as the ratio of the total throughput of all links to the total energy consumption of the network, while [23] characterizes the EE region of symbiotic radio systems, which is defined as all the achievable EE pairs by the active PT and passive BD. In addition, the combination of symbiotic radio with other technologies has also been investigated. For example, systems integrating non-orthogonal multiple-access (NOMA) into symbiotic radio have been studied in [18], [24], [25].
In [26, 27], full-duplex technique is introduced into a symbiotic radio system, which enables a BD to transmit and receive information simultaneously. In [28], cell-free networking architecture is integrated with symbiotic radio transmission technology to realize passive communication with high macro-diversity.

Note that due to the severe double-fading attenuation, the strength of the backscattering link is usually much weaker than that of the direct link, which results in low data rates for the secondary communication and only marginal enhancement to the primary transmission by the backscattering link in the CSR setup. To address such issues, various techniques have been proposed, such as the active-load assisted [29–31] symbiotic radio by using negative resistances, or configurable intelligent surface (RIS) aided [32–34] symbiotic radio by configuring large scale passive reflecting elements. However, the use of active loads requires additional power supply to generate the negative resistances, and a large scale deployment in RIS may not be suitable for many IoT devices that have small form factors. Thus, such existing techniques for enhancing the secondary communication links may increase the cost and complexity of the passive BD, which undermines the initial motivation of symbiotic radio.

In this paper, we propose an alternative method to significantly enhance the secondary backscattering links and enable the full mutualism of symbiotic radio systems, by exploiting the potential gain brought by multiple BDs, which provides abundant multi-user diversities for secondary communications and multipath gains for primary transmission. This is motivated by the 6G visions to support ultra-massive connectivity, say 10 million devices per square kilometer [35], most of which are expected to be IoT devices. Our main contributions are summarized as follows:

- First, we present the mathematical model of MIMO symbiotic radio communication systems with multiple BDs in the CSR setup, where the mutualism relationship of symbiotic radio can be fully exploited. We derive the achievable rate expression of the primary communication, as well as the sum rate expression of the passive BDs, by noting that it corresponds to a multiple access channel (MAC) where minimum mean square error estimation (MMSE) with successive interference cancellation (SIC) is optimal [36].

- Next, to show how the achievable communication rates of symbiotic radio are affected by the number of BDs, we consider the asymptotic analysis as the number of BDs goes large. Closed-form expressions are derived for the primary active communication rate and secondary passive communication rate, both of which are shown to be increasing functions of the number of BDs. For the special case of single-input-multiple-output (SIMO) symbiotic radio setup, we derive the closed-form expression of the asymptotic primary communication rate as a function of the asymptotic secondary sum-rate, which is revealed to be an increasing function. This thus demonstrates that the mutualism relationship of symbiotic radio can be fully enabled with massive BD access.

- Furthermore, we formulate a precoding optimization problem to maximize the primary communication rate by taking into account the additional multipaths created by the BDs, while guaranteeing that the secondary communication rate is no smaller than a certain threshold. The problem is non-convex in its original form, and we propose an effective technique to transform it to an equivalent convex problem of optimizing the transmit covariance matrix. Two solutions with different trade-offs between complexity and performance are proposed, namely sample-average based solution and upper bound based solution. Numerical results are provided to demonstrate that the proposed optimization approaches are effective in MIMO symbiotic radio communication systems.

The rest of this paper is organized as follows. Section II presents the system model of MIMO symbiotic radio communication with multiple BDs. Section III derives the achievable rate of the primary communication and the sum rate of the secondary BDs. In Section IV, by assuming that the number of BDs goes large, asymptotic performance analysis is provided to reveal the relationship between primary active communication and secondary passive communication. In Section V, the precoding optimization problem is studied to maximize the primary communication rate while guaranteeing that the secondary communication rate is no smaller than a certain threshold. Section VI presents numerical results to validate our theoretical studies. Finally, we conclude the paper in Section VII.

Notations: In this paper, scalars are denoted by italic letters. Vectors and matrices are denoted by boldface lower-and upper-case letters, respectively. For a vector a, its transpose, Hermitian transpose, and Euclidian norm are respectively denoted as aT, aH and ∥a∥. The conjugate, transpose, Hermitian transpose and rank of a matrix A are denoted as A*, AT, AH and rank(A), respectively. vec(A) represents stacking the columns of matrix A into a column vector. For a square matrix B, tr(B), |B|, B−1, and B≥0 denotes its trace, determinant, inverse, and matrix square-root, respectively, while B≥0 represents that the matrix B is positive semidefinite. λ1(B), ..., λM(B) denote the eigenvalues of B. ⊗ refers to the Kronecker product. diag(x1, ..., xM) denotes an M × M diagonal matrix with x1, ..., xM being the diagonal elements. I_M denotes an M × M identity matrix. C^{M×N} denotes the space of M × N matrices with complex entries. E[X] denotes the expectation with respect to the random variable X. log2(·) denotes the logarithm with base 2. Furthermore, C/N(x, Σ) denotes the distribution of a circularly symmetric complex Gaussian (CSCG) random vector with mean x and covariance matrix Σ.

II. System Model

As shown in Fig. 1, we consider a MIMO symbiotic radio communication system, which consists of one PT, one access point (AP) and J BDs. We assume that the PT and AP have MPT and Mf antennas, respectively, and each BD is equipped with one antenna. The AP wishes to decode not only the primary information from the PT, but also the
secondary information from the $J$ BDs, which modulate their information via passively backscattering the incident signal from the PT. As such, the BDs reuse not only the spectrum but also the power of the PT. In return, their scattered signals may create additional channel paths to enhance the primary communication link, as long as their power rate is much lower than that of the primary signal. This is known as mutualism relationship of symbiotic radio [11]. Denote the MIMO direct-link channel from the PT to the AP as $\mathbf{H}_d \in \mathbb{C}^{M_t \times M_r}$. Further denote by $\mathbf{h}_j \in \mathbb{C}^{M_t \times 1}$ the MISO channel from the PT to BD $j$, and $\mathbf{g}_j \in \mathbb{C}^{M_r \times 1}$ the SIMO channel from BD $j$ to the AP, where $j = 1, \ldots, J$. Then the channel matrix for the cascaded backscattering link from the PT to AP via BD $j$ is $\mathbf{g}_j \mathbf{h}_j^H$.

We focus on the CSR setup [11], where the symbol duration of the BDs is $K \gg 1$ times of that of the PT. Let $c_j(n)$ denote the independent and identically distributed (i.i.d.) information-bearing symbols of BD $j$, and $s(k, n) \in \mathbb{C}^{M_r \times 1}$ denote the i.i.d. information-bearing symbols of the PT, which follows CSG distribution with normalized power, i.e., $s(k, n) \sim \mathcal{CN}(0, \mathbf{I}_{M_r})$, $k = 1, \ldots, K$. Note that $M_t \leq M_r$ denotes the number of data streams of the PT signal, which is one of the optimization variables to be determined later. Furthermore, let $P$ denote the transmit power by the PT. $\alpha \in [0, 1]$ be the fraction of the power backscattered by each BD, and $\mathbf{F} \in \mathbb{C}^{M_r \times M_s}$ denote the transmit precoding matrix of the PT, with $\text{tr} (\mathbf{F} \mathbf{F}^H) = 1$. Then the signal received by the AP during the $nt$ BD symbol duration is

$$y(k, n) = \sqrt{P} \mathbf{H}_d s(k, n) + \sum_{j=1}^J \sqrt{P} \alpha \mathbf{g}_j \mathbf{h}_j^H \mathbf{F} s(k, n) c_j(n) + z(k, n), \quad z(k, n) = 1, \ldots, K. \quad (1)$$

where $z(k, n) \in \mathbb{C}^{M_r \times 1}$ is the i.i.d. CSGC noise with zero mean and power $\sigma^2$, i.e., $z(k, n) \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{M_r})$.

### III. Achievable Rate Analysis

Since the BD symbols $c_j(n)$ remain unchanged for each block of $K$ PT symbols, the second term in (1) constitutes additional multi-path channel components for the primary signal. As a result, the equivalent MIMO channel for decoding $s(k, n) \in \mathbb{C}^{M_r \times 1}$ is dependent on the BD symbols $c(n) = [c_1(n), c_2(n), \ldots, c_J(n)]^T$, which is denoted as $\mathbf{H}_{eq}(c(n)) = \mathbf{H}_d + \sum_{j=1}^J \sqrt{\alpha} \mathbf{g}_j \mathbf{h}_j^H c_j(n)$. Therefore, (1) can also be expressed as

$$y(k, n) = \sqrt{P} \mathbf{H}_d s(k, n) + z(k, n), \quad k = 1, \ldots, K. \quad (2)$$

Note that (2) is essentially a block fading channel where the MIMO channel matrix $\mathbf{H}_{eq}(c(n))$ remains unchanged in each block $n$ of $K$ symbol durations, while varies across different blocks according to the BD symbols $c(n)$. We assume that the AP has the knowledge of the equivalent MIMO channel $\mathbf{H}_{eq}(c(n))$, e.g., via standard training-based MIMO channel estimation [37], while the PT only knows the information of $\mathbf{H}_d, \mathbf{h}_j$, and $\mathbf{g}_j$, but not $\mathbf{H}_{eq}(c(n))$, since the BD symbols $c(n)$ are unknown at the PT. In this case, with a sufficiently large $K$, the average primary communication rate for the input-output relationship (2) is

$$R_s = \mathbb{E}_{c(n)} \left[ \log_2 \left( \frac{1}{\mathbf{I}_{M_r} + \frac{P}{K} \mathbf{H}_{eq}(c(n)) \mathbf{F} \mathbf{F}^H \mathbf{H}_{eq}(c(n))} \right) \right], \quad (3)$$

where $\frac{P}{K} \mathbf{F}^H \mathbf{F}$ is defined as the transmit signal-to-noise ratio (SNR).

On the other hand, the symbols $c_j(n)$ for each BD $j$, by concatenating $y(k, n)$ in (1) for all $k = 1, 2, \ldots, K$, we have $Y(n) = [y(1, n), y(2, n), \ldots, y(K, n)] \in \mathbb{C}^{M_t \times K}$. Similarly, let $S(n) = [s(1, n), s(2, n), \ldots, s(K, n)] \in \mathbb{C}^{M_r \times K}$ and $Z(n) = [z(1, n), z(2, n), \ldots, z(K, n)] \in \mathbb{C}^{M_t \times K}$. Then (1) can be compactly written as

$$Y(n) = \sqrt{P} \mathbf{H}_d \mathbf{F} S(n) + \sum_{j=1}^J \sqrt{P} \alpha \mathbf{g}_j \mathbf{h}_j^H \mathbf{F} s(k, n) c_j(n) + Z(n). \quad (4)$$

After decoding $s(k, n)$ at the AP, $k = 1, \ldots, K$, the primary signal component $S(n)$ can be subtracted from (4) before decoding the BD signals, which yields

$$\hat{Y}(n) = \sum_{j=1}^J \sqrt{P} \alpha \mathbf{g}_j \mathbf{h}_j^H \mathbf{F} c_j(n) + Z(n). \quad (5)$$

Furthermore, with $S(n)$ decoded at the receiver, the temporal-domain matched filtering can be applied, by right multiplying $\hat{Y}(n)$ in (5) with $(1/\sqrt{K}) \mathbf{S}^H(n)$. For sufficiently large $K$, due to the law of large numbers and the fact that the information-bearing symbols $s(k, n)$ are i.i.d. with distribution $s(k, n) \sim \mathcal{CN}(0, \mathbf{I}_{M_r})$, we have $(1/K) \mathbf{S}(n) \mathbf{S}^H(n) \rightarrow \mathbf{I}_{M_r}$. Therefore, the resulting signal is

$$\tilde{Y}(n) = \frac{1}{\sqrt{K}} \hat{Y}(n) \mathbf{S}^H(n) = \sum_{j=1}^J \sqrt{K} \alpha \mathbf{g}_j \mathbf{h}_j^H \mathbf{F} c_j(n) + \tilde{Z}(n), \quad (6)$$

where $\tilde{Z}(n) = \frac{1}{\sqrt{K}} Z(n) \mathbf{S}^H(n) = [\tilde{z}_1(n), \tilde{z}_2(n), \ldots, \tilde{z}_{M_r}(n)] \in \mathbb{C}^{M_r \times M_r}$. It can be shown that $\tilde{z}_l(n), l = 1, \ldots, M_r$, are i.i.d. random vectors following distribution $\mathcal{CN}(0, \sigma^2 \mathbf{I}_{M_r})$. Let $\mathbf{H}_j = \sqrt{K} \alpha \mathbf{g}_j \mathbf{h}_j^H \mathbf{F}, \quad \mathbf{x}_j = \text{vec}(\mathbf{H}_j), \quad \mathbf{y}(n) = \text{vec}(\tilde{Y}(n)),$
and \( z(n) = \text{vec}(\mathbf{Z}(n)) \). The input-output relationship (6) can be equivalently written as

\[
y(n) = \sum_{j=1}^{J} x_{j} c_{j}(n) + z(n),
\]

where \( z(n) \sim \mathcal{CN}(0, \sigma^2 I_{M,M_s}) \). Note that (7) is essentially a SIMO MAC, where MMSE-SIC receiver is known to be capacity-achieving [36]. Specifically, the \( J \) BD users are ordered according to their channel strength \( ||x_j||^2 \), based on which the SIC decoding order is determined. Without loss of generality, assuming that \( ||x_1||^2 \geq ||x_2||^2 \geq \cdots \geq ||x_J||^2 \), then the SIC decoding order is 1, 2, \ldots, \( J \). Let’s focus on BD \( j \), where the signals for BDs 1, ..., \( j-1 \) have already been decoded and perfectly removed, and those for BDs \( j+1, \ldots, J \) are treated as noise. Denote the beamforming vector for BD \( j \) as \( \mathbf{w}_j \in \mathbb{C}^{M,M_s \times 1} \). Then the resulting signal can be written as

\[
y_j(n) = \mathbf{w}_j^H x_j c_j(n) + \mathbf{w}_j^H \sum_{i=j+1}^{J} x_i c_i(n) + \mathbf{w}_j^H \mathbf{z}(n). \tag{8}\]

The resulting SINR for BD \( j \) is

\[
\gamma_{c_j} = \frac{||\mathbf{w}_j^H x_j||^2}{\sum_{i=j+1}^{J} ||\mathbf{w}_j^H x_i||^2 + \sigma^2 ||\mathbf{w}_j||^2}. \tag{9}\]

The optimal linear MMSE beamforming that maximizes the SINR is

\[
\mathbf{w}_j = \left( \sum_{i=j+1}^{J} x_i x_i^H + \sigma^2 I_{M_s,M_s} \right)^{-1} x_j, \tag{10}\]

and the corresponding maximum SINR is

\[
\gamma_{c_j} = x_j^H \left( \sum_{i=j+1}^{J} x_i x_i^H + \sigma^2 I_{M_s,M_s} \right)^{-1} x_j. \tag{11}\]

Hence, the sum capacity of the \( J \) BDs can be written as

\[
R_{BD} = \frac{1}{K} \sum_{j=1}^{J} \log_2 \left( 1 + \gamma_{c_j} \right), \tag{12}\]

where the pre-log factor 1/\( K \) accounts for the fact that in the CSR setup, the symbol duration of the BD users is \( K \) times that of the primary users.

**Theorem 1:** The sum capacity of the BDs in (12) can be equivalently expressed as

\[
R_{BD} = \frac{1}{K} \log_2 \left| I_{M_s,M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H \right|. \tag{13}\]

**Proof:** Please refer to Appendix A. \(\blacksquare\)

By substituting \( x_j = \text{vec}(\mathbf{H}_j) = \text{vec}(\sqrt{K} \mathbf{P} \mathbf{g}_j \mathbf{h}_j^H \mathbf{F}) \), the sum capacity of the BDs can be expressed in terms of the precoding matrix \( \mathbf{F} \) of the PT, which leads to the following Lemma.

**Lemma 1:** The sum capacity of the BDs \( R_{BD} \) in (13) can also be expressed as

\[
R_{BD} = \frac{1}{K} \log_2 \left| I_{M_s,M_s} + K \mathbf{P} \sum_{j=1}^{J} \left( \mathbf{F}^H \mathbf{h}_j \mathbf{h}_j^H \mathbf{F} \otimes \left[ \mathbf{g}_j \mathbf{g}_j^H \right]^T \right) \right|. \tag{14}\]

**Proof:** Please refer to Appendix B. \(\blacksquare\)

**IV. ASYMPTOTIC PERFORMANCE ANALYSIS**

To show how the communication rates of symbiotic radio are affected by the number of BDs \( J \) and also to explicitly reveal the mutualism relationship between active and passive communications, in this section, we give the asymptotic performance analysis for massive BDs, i.e., as \( J \) goes sufficiently large. We first consider the general MIMO symbiotic radio setup for \( M_t \geq 1 \), and then consider the special SIMO case with \( M_t = 1 \) to get more insights.

**A. MIMO Symbiotic Radio: \( M_t \geq 1 \)**

To obtain tractable asymptotic performance analysis, we assume that the BD channels \( \mathbf{h}_j \) and \( \mathbf{g}_j \) are i.i.d. distributed for different BDs, with \( \mathbb{E} [\mathbf{h}_j \mathbf{h}_j^H] = \beta_0 \mathbf{I}_{M_t} \) and \( \mathbb{E} [\mathbf{g}_j \mathbf{g}_j^H] = \beta_0 \mathbf{I}_{M_s} \), \( \forall j = 1, \ldots, J \), where \( \beta_0 \) and \( \beta_0 \) are the large-scale channel gains. Under such assumptions, we first study the asymptotic behavior of the capacity of the BDs, for which we have the following Lemma:

**Lemma 2:** For symbiotic radio with massive BDs, i.e., \( J \gg 1 \), the sum capacity of the BDs in (14) approaches to

\[
\frac{M_t}{K} \log_2 \left| I_{M_t} + J K \mathbf{P} \alpha \beta_0 \beta_0 \mathbf{F} \mathbf{F}^H \right|. \tag{15}\]

**Proof:** Please refer to Appendix C. \(\blacksquare\)

Lemma 2 shows that for symbiotic radio with massive BDs, the sum capacity of the BDs increases monotonically with the number of BDs \( J \), thanks to the multi-user diversity gains.

If the objective is to maximize the asymptotic secondary passive communication rate without considering that of the primary active communication rate, then based on Lemma 2, we have the following optimization problem

\[
\max \frac{M_t}{K} \log_2 \left| I_{M_t} + J K \mathbf{P} \alpha \beta_0 \beta_0 \mathbf{F} \mathbf{F}^H \right| \tag{16}\]

s.t. \( \text{tr} (\mathbf{F} \mathbf{F}^H) = 1. \)

Define \( \mathbf{Q} \triangleq \mathbf{F} \mathbf{F}^H \) as the transmit covariance matrix of the PT, and denote the eigenvalues of \( \mathbf{Q} \) as \( \lambda_i(\mathbf{Q}) \geq 0 \), \( i = 1, \ldots, M_t \). Then the constraint (16) can be expressed as

\[
\sum_{i=1}^{M_t} \lambda_i(\mathbf{Q}) = 1. \tag{17}\]

Furthermore, the objective function in (15) can be expressed as

\[
\frac{M_t}{K} \log_2 \left| I_{M_t} + J K \mathbf{P} \alpha \beta_0 \beta_0 \mathbf{F} \mathbf{F}^H \right| = \frac{M_t}{K} \log_2 \left| I_{M_t} + J K \mathbf{P} \alpha \beta_0 \beta_0 \mathbf{Q} \right| \tag{18}\]

\[
= \frac{M_t}{K} \log_2 \left( \prod_{i=1}^{M_t} (1 + J K \mathbf{P} \alpha \beta_0 \beta_0 \lambda_i(\mathbf{Q})) \right). \]

According to inequality of arithmetic and geometric mean, we have

\[
\frac{M_t}{M_t} \prod_{i=1}^{M_t} (1 + J K \mathbf{P} \alpha \beta_0 \beta_0 \lambda_i(\mathbf{Q})) \leq \frac{1}{M_t} \sum_{i=1}^{M_t} (1 + J K \mathbf{P} \alpha \beta_0 \beta_0 \lambda_i(\mathbf{Q})), \tag{19}\]

Please refer to Appendix C.
where the equal sign holds when \( \lambda_1(Q) = \cdots = \lambda_{M_t}(Q) = \frac{1}{M_t} \). As a result, the optimal solution to (15) is achieved when \( Q = F F^H = \frac{1}{M_t} I_{M_t} \), and the optimal objective value is

\[
R_{BD} \rightarrow R_{BD}^{\text{asym}} = \frac{M_t M_s}{K} \log_2 \left( 1 + \frac{J K P_0 \beta_h \beta_d}{M_t} \right). \tag{20}
\]

In this case, the optimal number of data streams \( M_s \) is equal to \( M_t \), since rank \( (F) = M_t \).

Next, we study the asymptotic behavior of the achievable rate of the PT in (3), for which we have the following Lemma:

**Lemma 3:** For symbiotic radio with massive BDs, i.e., \( J \gg 1 \), the average rate of the PT in (3) approaches \( R_s \rightarrow \log_2 |I_{M_t} + \frac{P F F^H}{M_t} (H_d^H H_d + J M_t \beta_h \beta_d I_{M_t})| \).

**Proof:** Please refer to Appendix D.

If the design objective is to maximize the asymptotic primary active communication rate without considering that of the secondary passive communication rate, then based on Lemma 3, we have the following optimization problem

\[
\begin{align*}
\max & \quad \log_2 |I_{M_t} + \frac{P F F^H}{M_t} (H_d^H H_d + J M_t \beta_h \beta_d I_{M_t})| \tag{21} \\
\text{s.t.} & \quad \text{tr} (F F^H) = 1. \tag{22}
\end{align*}
\]

Define \( \bar{H}_{eq} \) as the equivalent MIMO channel matrix in (21), where \( \bar{H}_{eq}^H \bar{H}_{eq} = H_d^H H_d + J M_t \beta_h \beta_d I_{M_t} \). Let the singular value decomposition (SVD) of the matrix direct-link \( H_d = U_d H_d, \Sigma_d V_d^H \), with \( \Sigma_d \) a diagonal matrix containing the eigenvalues of \( H_d^H H_d \). \( U_d \in \mathbb{C}^{M_t \times M_t} \) and \( V_d \in \mathbb{C}^{M_t \times M_t} \) are both unitary matrices. \( \bar{H}_{eq}^H \bar{H}_{eq} \) can be accordingly expressed as

\[
\begin{align*}
\bar{H}_{eq}^H \bar{H}_{eq} &= \left( U_d \Sigma_d V_d V_d^H \right)^H \left( U_d \Sigma_d V_d V_d^H \right) \\
&= \Sigma_d \left( \Sigma_d^H \Sigma_d + J M_t \beta_h \beta_d I_{M_t} \right) \Sigma_d.
\end{align*}
\]

According to (33), the optimum precoder that maximizes the primary asymptotic rate in (21) is given by

\[
F = \frac{1}{\sqrt{M_t}} V_d P^{\frac{1}{2}}, \tag{24}
\]

where \( P \) is the diagonal power allocation matrix given by

\[
P = \text{diag} \left( P_0, \cdots, P_{M_t-1} \right), \tag{25}
\]

where \( M_s = M_t \) since the matrix \( \bar{H}_{eq}^H \bar{H}_{eq} \) is full rank, and the optimum \( P_0, \cdots, P_{M_t-1} \) is given by the waterfilling policy that can be obtained as the fixed point of the equations

\[
P_i^* = \frac{1}{M_t} - \text{MMSE}_i \left( P_i^* \right), \quad i = 0, \cdots, M_s - 1,
\]

\[
\text{MMSE}_i \left( P_i^* \right) = \frac{1}{1 + \frac{P}{M_t} \beta_h \beta_d \lambda_i (H_d^H H_d) + J M_t \beta_h \beta_d}.
\]

Then the resulting primary asymptotic rate is

\[
R_s \rightarrow R_s^{\text{asym}} = \frac{1}{M_t} \sum_{i=0}^{M_t-1} \log_2 \left( 1 + \frac{P}{M_t} \beta_h \beta_d \lambda_i (H_d^H H_d) + J M_t \beta_h \beta_d \right). \tag{27}
\]

It is observed from (23) that the asymptotic average primary communication rate also increases with the number of BDs \( J \), thanks to the multi-path diversity created to the primary communication link via passive backscattering. To more explicitly show the mutualism relationship between active and passive communications, we consider the special case of SIMO setup to gain more insights, as revealed in the following subsection.

**B. SIMO Symbiotic Radio: \( M_t = 1 \)**

When the transmitter has only one antenna, i.e., \( M_t = 1 \), the transmit precoding matrix \( F \) degenerates to a scalar \( f \), with \(|f|^2 = 1 \), and the MISO channel vector \( h_j \) from the PT to BD \( j \) degenerates to a channel coefficient \( h_j \). In this case, Lemma 2 degenerates to the following corollary.

**Corollary:** For SIMO symbiotic radio with massive BDs, i.e., \( J \gg 1 \), the sum capacity of the BDs in (14) approaches to

\[
R_{BD} \rightarrow R_{BD}^{\text{asym}} = \frac{M_t}{K} \log_2 \left( 1 + J K \bar{P} \alpha \beta_h \beta_g \right). \tag{29}
\]

Similarly, the asymptotic average rate of the PT in Lemma 3 for SIMO setup degenerates to the following result:

**Corollary:** For SIMO symbiotic radio with massive BDs, i.e., \( J \gg 1 \), \( R_s \) in Lemma 3 can be written as

\[
R_s \rightarrow R_s^{\text{asym}} = \log_2 \left( 1 + \bar{P} \left( ||h_d||^2 + J M_r \beta_h \beta_d \right) \right). \tag{30}
\]

It is not difficult to see that \( R_s \) in both MIMO symbiotic radio setup and SIMO symbiotic radio setup in Lemma 3 and (30) respectively increases monotonically with the number of BDs \( J \). Thus, with more BDs connected to the symbiotic radio system, the enhancement to the primary transmission becomes more significant. Based on (29) and (30), by eliminating the common variables \( J K \bar{P} \alpha \beta_h \beta_g \), we have the following result:

**Theorem 2:** For SIMO symbiotic radio with massive BDs, i.e., \( J \gg 1 \), the asymptotic primary communication rate \( R_{BD}^{\text{asym}} \) can be expressed in closed-form in terms of the asymptotic secondary sum capacity \( R_{BD}^{\text{asym}} \) as:

\[
R_{BD}^{\text{asym}} = \log_2 \left( 1 + \bar{P} ||h_d||^2 + \frac{M_t}{K} \left( 2 \bar{P} \beta_h \beta_d - 1 \right) \right). \tag{31}
\]

It is evident that \( R_{BD}^{\text{asym}} \) in (31) monotonically increases with \( R_{BD}^{\text{asym}} \), which clearly reveals the mutualism relationship of symbiotic radio with massive BDs. Furthermore, when the direct link from the PT to AP \( h_d \) is a line of sight (LoS) link, then \( ||h_d||^2 \) is proportional to the number of receive antennas \( M_r \) at the AP, i.e., \( ||h_d||^2 = \beta_d M_r \) for some \( \beta_d \). Then (31) can be expressed as

\[
R_{s}^{\text{asym}} = \log_2 \left( 1 + \bar{P} \beta_d M_r + \frac{M_t}{K} \left( 2 \bar{P} \beta_d - 1 \right) \right). \tag{32}
\]

Fig. 2 gives an example plot of (32) with \( \beta_d = -120 \) dB, \( M_t = 8 \) and \( K = 128 \), for four different transmit SNR values \( \bar{P} \). It is worth mentioning that while (32) was derived for asymptotic setup with \( J \gg 1 \), it is also applicable for the extreme case with \( J = 0 \) or \( R_{BD}^{\text{asym}} = 0 \), for which case the third term inside the logarithm of (32) vanishes.
which is difficult to be directly solved. Fortunately, it can be solved as the precoding matrix \( \mathbf{F} \). To this end, define the transmit covariance matrix \( \mathbf{T} \). Intuitively, \( \mathbf{F} \) vanishes as \( \mathbf{T} \) takes the derivative of \( \mathbf{R} \) to maximize the average primary communication rate in (3), which becomes sufficiently large, \( \mathbf{R} \) becomes sufficiently large, \( \mathbf{R} \) is still non-convex since the constraint (37) and (39) are non-convex. In the following, to find an equivalent convex transformation of (P1.1), we first consider the special case of one single BD, i.e., \( J = 1 \) and then consider the more general case with \( J \geq 1 \).

A. Single BD: \( J = 1 \)

For the special case of single BD with \( J = 1 \), the left hand side of (37) can be simplified as:

\[
R_1 = \frac{1}{K} \log_2 \left| \mathbf{I}_{M_r, M_s} + K \bar{P} \alpha (\mathbf{F}^\dagger \mathbf{h}_1^+ \mathbf{F} \otimes (\mathbf{g}_1 \mathbf{h}_1^\dagger)^T) \right|
\]

This shows that the mutualism of symbiotic radio can only be fully exploited for sufficiently large BDs. It is also interesting to observe that as \( R^{sym}_{BD} \) becomes sufficiently large, \( R^{sym} \) for different \( P \) values merge. This can be verified by taking the derivative of \( R^{sym} \) with respect to \( P \), which vanishes as \( R^{sym}_{BD} \) gets sufficiently large.

V. PRECODING OPTIMIZATION

In this section, for any given finite number of BDs \( J \) where the asymptotic results in the previous section no longer hold, we consider the transmit precoding optimization problem to maximize the average primary communication rate in (3), under the sum rate constraint of the BDs in (14). The following problem can be formulated:

\[
\begin{align*}
\max_{\mathbf{F} \in \mathbb{C}^{M_t \times M_s}, \mathbf{M}_s \preceq \mathbf{M}_t} & \quad \mathbb{E}_{c(n)} \left[ \log_2 \left| \mathbf{I}_{M_r} + \bar{P} \mathbf{H}_{eq} (\mathbf{c} (n)) \mathbf{F} \mathbf{F}^\dagger \mathbf{H}_{eq}^\dagger (\mathbf{c} (n)) \right| \right] \\
\text{s.t.} & \quad \frac{1}{K} \log_2 \left| \mathbf{I}_{M_r, M_s} + K \bar{P} \alpha \sum_{j=1}^{J} \left( (\mathbf{F}^\dagger \mathbf{h}_j^+ \mathbf{F}) \otimes (\mathbf{g}_j \mathbf{h}_j^\dagger)^T \right) \right| \geq r_{BD}, \\
& \quad \text{tr}(\mathbf{F} \mathbf{F}^\dagger) = 1,
\end{align*}
\]

which is the transmit covariance matrix of the PT. Then problem (P1.1) can be equivalently written as

\[
\begin{align*}
\max_{\mathbf{F} \in \mathbb{C}^{M_t \times M_s}, \mathbf{M}_s \preceq \mathbf{M}_t} & \quad \mathbb{E}_{c(n)} \left[ \log_2 \left| \mathbf{I}_{M_r} + \bar{P} \mathbf{H}_{eq} (\mathbf{c} (n)) \mathbf{Q} \mathbf{H}_{eq}^\dagger (\mathbf{c} (n)) \right| \right] \\
\text{s.t.} & \quad \frac{1}{K} \log_2 \left| \mathbf{I}_{M_r, M_s} + K \bar{P} \alpha \sum_{j=1}^{J} \left( (\mathbf{F}^\dagger \mathbf{h}_j^+ \mathbf{F}) \otimes (\mathbf{g}_j \mathbf{h}_j^\dagger)^T \right) \right| \geq r_{BD}, \\
& \quad \text{tr}(\mathbf{Q}) = 1, \\
& \quad \mathbf{Q} = \mathbf{F} \mathbf{F}^\dagger.
\end{align*}
\]

(P1.1) is still non-convex since the constraint (37) and (39) are non-convex. In the following, to find an equivalent convex transformation of (P1.1), we first consider the special case of one single BD, i.e., \( J = 1 \), and then consider the more general case with \( J \geq 1 \).
that are feasible to (P1.1), we may simply let $M_s = r$, and $F = U \Sigma^{1/2}$. Obviously, the corresponding solution is optimal to (P1.1) since it achieves the same optimal value as its relaxed problem (P2). Therefore, the remaining task is to solve (P2) by optimizing the transmit covariance matrix $Q$. It is not difficult to see that (P2) is a convex optimization problem. The details will be presented in Subsection IV B, since similar techniques will also be used for the general case with multiple BDs presented in the next subsection.

**B. Multiple BDs: $J \geq 1$**

In this subsection, we consider the general case of (P1.1) with multiple BDs. In this case, by using Theorem 1, the left hand side of (37) can be equivalently expressed as (13). Define $X = [x_1, x_2, \ldots, x_J]$, we have

$$R_{BD} = \frac{1}{K} \log_2 \left| I_{M_s, M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H \right|$$

$$= \frac{1}{K} \log_2 \left| I_{M_s, M_s} + \frac{1}{\sigma^2} XX^H \right|$$

where the second last equality follows from the Weinstein-Aronszajn identity $|I + AB| = |I + BA|$, and the last equality follows from the identity $|A^T| = |A|$. By using the identity $\text{vec}(A_1 A_2 A_3) = (A_3^T \otimes A_1) \text{vec}(A_2)$, we have

$$x_j = \text{vec} \left( \sqrt{K P} g_j H_j^H F \right)$$

$$= \sqrt{K P} \alpha (F^T \otimes g_j H_j^H) \text{vec}(I_{M_s})$$

Then $X$ can be expressed as (45) shown at the top of the next page.

For notational convenience, we let $H = [g_1 H^H, g_2 H^H, \ldots, g_J H^H] \in \mathbb{C}^{M_s \times M_s J}$ and

$$\Psi = \begin{bmatrix} \text{vec}(I_{M_s}) & 0 & \cdots & 0 \\ 0 & \text{vec}(I_{M_s}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \text{vec}(I_{M_s}) \end{bmatrix} \in \mathbb{R}^{M_s^2 J \times J},$$

so that $X = \sqrt{K P} \alpha (F^T \otimes H) \Psi$. Then $R_{BD}$ in (44) can be expressed as

$$R_{BD} = \frac{1}{K} \log_2 \left| I_J + \frac{1}{\sigma^2} X^T X^* \right|$$

$$= \frac{1}{K} \log_2 \left| I_J + K P \alpha (F^T H^T) (F^T H^T)^* \right|$$

$$= \frac{1}{K} \log_2 \left| I_J + K P \alpha (F F^H \otimes (H H^T) \Psi^* \right|$$

where the last equality follows from the identities $(A \otimes B)^T = A^T \otimes B^T$, $(A \otimes B)^* = A^* \otimes B^*$, $(A \otimes B)(C \otimes D) = AC \otimes BD$ and $\Psi \in \mathbb{R}$. When $J = 1$, $H = g_1 h_1^H$, $\Psi = \text{vec}(I_{M_s})$, (48) can be simplified to (49). By replacing $R_{BD}$ in (37) with (48) and letting $Q = F F^H$, we transform (P1.1) into

$$\max_Q \quad \mathbb{E}_{e(n)} \left[ \log_2 \left( I_{M_s} + \hat{P} H_{eq}(c(n)) Q H_{eq}(c(n)) \right) \right]$$

subject to

$$\frac{1}{K} \log_2 \left| I_J + K P \alpha (Q \otimes (H H^T) \Psi \right| \geq r_{BD},$$

$$\text{tr}(Q) = 1, Q \succeq 0.$$  

After solving (P3), we can obtain $F$ by using the same way as discussed in Subsection V A.

Note that similar to (P2), the transmit covariance matrix $Q$ in (P3) is optimized to maximize the expected primary communication rate, with the expectation taken with respect to the random BD symbols $e(n)$. In the following, we propose two solution approaches to (P3), termed sample-average based solution and upper bound based solution, which have different trade-offs on performance and computational complexity.

1) Sample-Average Based Solution: For the sample-average based solution, the expectation of the primary communication rate in (49) is approximated by its sample average. Specifically, let $c_s, s = 1, \ldots, S$ be $S$ independent realizations of $e(n)$ following its distribution $f(e(n))$. Then when $S$ is sufficiently large, based on the law of large numbers, (P3) can be approximated as

$$\max_Q \quad \frac{1}{S} \sum_{s=1}^{S} \log_2 \left| I_{M_s} + \hat{P} H_{eq}(c_s) Q H_{eq}(c_s) \right|$$

subject to

$$\frac{1}{K} \log_2 \left| I_J + K P \alpha (Q \otimes (H H^T) \Psi \right| \geq r_{BD},$$

$$\text{tr}(Q) = 1, Q \succeq 0.$$  

(P4) is a convex optimization problem, which can be optimally solved by using software tools like CVX [39].

2) Upper Bound Based Solution: Note that the above sample average based method needs to collect a large number of samples for the random state $e(n)$ before solving the stochastic optimization problem. Hence, it requires huge memory to store the samples and the corresponding computational complexity is also high. To address such issues, we substitute (49) with its upper bound and convert the problem into a more tractable convex problem. By using Jensen’s inequality, an upper bound of $R_s$ in (49) is given by

$$R_s = \mathbb{E}_{e(n)} \left[ \log_2 \left| I_{M_s} + \hat{P} Q H_{eq}(c(n)) H_{eq}(c(n)) \right| \right] \leq R_{sUB} \triangleq \log_2 \left| I_{M_s} + \hat{P} Q \mathbb{E}_{e(n)} \left[ H_{eq}(c(n)) H_{eq}(c(n)) \right] \right|,$$
The large-scale channel gains of PT-to-AP and PT-to-BDs links are modeled as the scale channel coefficients \( \beta \) that of the sample-average based solution. The transmit power is \( \sigma = 2.7 \) respectively. Furthermore, the large-scale channel coefficients \( \beta_{tg} \) of the cascaded PT-BDs-AP channels are modeled as \( \beta_{tg} = 0.01 \beta_{h} \), where \( \beta_{h} \) represents the large-scale coefficients of PT-to-BDs channels. The carrier frequency is 3.5GHz, the noise power is \( \sigma^2 = -110 \) dBm, the number of transmit antennas at the PT is \( M_t = 4 \), the number of receive antennas at the AP is \( M_r = 8 \), and the power reflection coefficient is \( \alpha = 1 \). Furthermore, we set the ratio between the symbol duration of the BD symbols and that of the PT symbols as \( \gamma = 128 \). For the sample-average based solution, the number of samples \( S \) is set as 1000.

Fig. 3 plots the average optimization time versus the number of BDs for the two proposed solutions, where the average is taken over 100 independent channel realizations and \( S = 1000 \) independent samples. The transmit power is \( P = 0 \) dBm. It is observed that the optimization time increases monotonically with the number of BDs \( J \), which is expected since the matrix dimension of the constraint term in (59) becomes larger as \( J \) increases. It can also be observed that the average optimization time of the upper bound based solution is much lower than that of the sample-average based solution.
Fig. 4 and Fig. 5 compare the performance of the average rate of primary active communication and secondary passive communication with three different precoding schemes, where the average is taken over $S = 1000$ independent samples. Besides our proposed sample average based solution and the upper bound based solution in subsection V B, we also consider the direct-link matching precoding scheme as a benchmark, where the PT ignores the multipath created by the BDs and simply sets the precoding matrix to match the direct link, i.e., $\mathbf{F} = \sqrt{\frac{1}{M_t}} \mathbf{V}_d \Sigma_d^{1/2} \mathbf{H}_d$, with $\mathbf{V}_d$ obtained based on the (reduced) SVD of the channel matrix, $\mathbf{H}_d = \mathbf{U}_d \Sigma_d \mathbf{V}_d^H$. The results in Fig. 4 and Fig. 5 are obtained for one realization of the channels, and the number of BDs is $J = 50$. It is observed that for all the three precoding schemes, the average primary active rate and secondary passive rate increase monotonically with the transmit power $P$, as expected. Furthermore, both our proposed solutions in subsection V B significantly outperform the benchmarking direct-link matching precoding scheme, thanks to the consideration of the effective channel constituted by both the direct link and the backscattered multipaths. Furthermore, it is observed that the sample-average based solution achieves slightly higher primary rate than the upper bound based solution, but at the cost of higher computation complexity, as illustrated in Fig. 3.

Next, we study the impact of the number of BDs $J$ on the average primary and secondary communication rates, where the average is taken over 100 independent channel realizations and $S$ independent samples. For each channel realization, the primary communication rate is obtained with the proposed upper bound based solution. Note that though sub-optimal in general, the upper bound based solution has much lower computational complexity than the sample-average based solution. Fig. 6 and Fig. 7 plot the average primary and secondary com-
munication rates versus the number of BDs $J$, respectively. It is firstly observed that the primary communication rate is in general much higher than the secondary rate. This is expected since the symbol rate of primary signals is $K = 128$ times of that of the secondary signals, and that the backscattered link for one single BD is in general much weaker than the primary communication link. Furthermore, it is observed that as $J$ increases, both primary and secondary rates increase, which corroborates our theoretical results in Section III.

VII. CONCLUSION

In this paper, a MIMO symbiotic radio system with a massive number of BDs was investigated. The achievable rates of both the primary active communication and secondary passive communication were derived. Furthermore, considering the asymptotic regime as the number of BDs goes large, closed-form expressions were derived for the general MIMO symbiotic radio setup and the special SIMO setup, both of which were shown to be increasing functions of the number of BDs. This thus demonstrated that the mutualism relationship of symbiotic radio can be fully exploited with massive BD access. In addition, the precoding optimization problem was studied to maximize the primary communication rate while guaranteeing that the secondary communication rate was no smaller than a certain threshold. Extensive simulation results were provided to verify the effectiveness of our proposed solutions.

APPENDIX A

PROOF OF THEOREM 1

Denote the right hand side of (13) in Theorem 1 as $C$, which can be expressed as

$$C = \frac{1}{K} \log_2 \left[ I_{M,M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H \right] = \frac{1}{K} \log_2 \left[ I_{M,M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H + \frac{1}{\sigma^2} x_1 x_1^H \right].$$

(60)

Let $A_i = I_{M,M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H$, then $C$ can be expressed as

$$C = \frac{1}{K} \log_2 \left[ A_0 \right] = \frac{1}{K} \log_2 \left[ A_1 + \frac{1}{\sigma^2} x_1 x_1^H \right] = \frac{1}{K} \log_2 \left[ A_1 \left( I_{M,M_s} + \frac{1}{\sigma^2} A_1^{-1} x_1 x_1^H \right) \right] = \frac{1}{K} \log_2 \left[ A_1 \right] + \frac{1}{K} \log_2 \left[ I_{M,M_s} + \frac{1}{\sigma^2} A_1^{-1} x_1 x_1^H \right] = \frac{1}{K} \log_2 \left[ A_1 \right] + \frac{1}{K} \log_2 \left( 1 + \frac{1}{\sigma^2} A_1^{-1} x_1 \right) + \frac{1}{K} \log_2 \left( 1 + \gamma_{c_1} \right),$$

(61)

where the second last equality follows from the Weinstein-Aronszajn identity $|I_n + AB| = |I_n + BA|$, and $\gamma_{c_1}$ is the maximum SINR of BD 1 given in (11). By applying the similar decomposition, $\frac{1}{K} \log_2 \left[ A_1 \right]$ can be expressed as

$$\frac{1}{K} \log_2 \left[ A_1 \right] = \frac{1}{K} \log_2 \left[ A_2 \right] + \frac{1}{K} \log_2 \left( 1 + \gamma_{c_2} \right).$$

(62)

By applying the above result recursively, $C$ can be expressed as a general form

$$C = \frac{1}{K} \log_2 \left| A_J \right| + \frac{1}{K} \sum_{i=1}^{J} \log_2 \left( 1 + \gamma_{c_i} \right).$$

(63)

By letting $j = J$, (63) can be expressed as

$$C = \frac{1}{K} \log_2 \left| A_J \right| + \frac{1}{K} \sum_{i=1}^{J} \log_2 \left( 1 + \gamma_{c_i} \right) = \frac{1}{K} \sum_{j=1}^{J} \log_2 \left( 1 + \gamma_{c_j} \right) = R_{BD}.$$  

(64)

This thus completes the proof of Theorem 1.

APPENDIX B

PROOF OF LEMMA 1

To show Lemma 1, by substituting $x_j = \sqrt{KP} \text{vec}(g_j h_j^H F)$, we have

$$x_j x_j^H = K P \text{vec}(g_j \cdot 1 \cdot h_j^H F) \text{vec}(g_j \cdot 1 \cdot h_j^H F)^H = KP \left( (h_j^H F)^T \otimes g_j \right) \left( (h_j^H F)^T \otimes g_j \right)^H = KP \left( F^T h_j^H h_j^H F^* \otimes (g_j g_j^H) \right),$$

(65)

where the second equality follows from the identity $\text{vec}(A_1 A_2 A_3) = (A_1^T \otimes A_1) \text{vec}(A_2)$, and the last equality follows from $(A \otimes B)^H = A^H \otimes B^H$ and $(A \otimes B)(C \otimes D) = AC \otimes BD$. Thus, $R_{BD}$ in (13) can be expressed as

$$R_{BD} = \frac{1}{K} \log_2 \left[ I_{M,M_s} + \frac{1}{\sigma^2} \sum_{j=1}^{J} x_j x_j^H \right] = \frac{1}{K} \log_2 \left[ I_{M,M_s} + KP \alpha \sum_{j=1}^{J} \left( (F^T h_j^H h_j^H F^*) \otimes g_j g_j^H \right)^T \right] = \frac{1}{K} \log_2 \left[ I_{M,M_s} + KP \alpha \sum_{j=1}^{J} \left( (F^T h_j^H h_j^H F) \otimes (g_j g_j^H)^T \right) \right].$$

(66)

This thus completes the proof of Lemma 1.

APPENDIX C

PROOF OF Lemma 2

By using the law of large numbers, for $J \gg 1$, we have:

$$J \sum_{j=1}^{J} \left( (F^H h_j h_j^H F) \otimes (g_j g_j^H)^T \right) \rightarrow J \mathbb{E} \left[ (F^H h_j h_j^H F) \otimes (g_j g_j^H)^T \right]$$

(67)

$$J \mathbb{E} \left[ (F^H h_j h_j^H F) \otimes (g_j g_j^H)^T \right] = J \mathbb{E} \left[ (F^H h_j h_j^H F) \right] \otimes \mathbb{E} \left[ (g_j g_j^H)^T \right] = J \beta h \beta_g (F^H F) \otimes I_{M_s}.$$
It then follows from (14) that

$$\begin{align*}
R_{BD} & \to \frac{1}{K} \log_2 |I_{M,M_s} + JK \hat{P} \alpha \beta \tilde{\alpha} \beta (F^H F \otimes I_{M_s})| \\
& = \frac{1}{K} \log_2 |(I_{M_s} + JK \hat{P} \alpha \beta \tilde{\alpha} \beta F^H F) \otimes I_{M_s}| \\
& = \frac{1}{K} \log_2 \left| I_{M_s} + JK \hat{P} \alpha \beta \tilde{\alpha} \beta (F^H F)^{M_r} I_{M_s} \right| \\
& = \frac{M_r}{K} \log_2 |I_{M_s} + JK \hat{P} \alpha \beta \tilde{\alpha} \beta F^H F| \\
& = \frac{M_r}{K} \log_2 |I_{M_s} + JK \hat{P} \alpha \beta \tilde{\alpha} \beta F^H F|.
\end{align*}$$

(68)

where the third last equality follows from the identity $|A \otimes B| = |A|^{\text{rank}(B)}|B|^{\text{rank}(A)}$.

This completes the proof of Lemma 2.

**APPENDIX D**

**PROOF OF LEMMA 3**

According to (3), we have

$$\begin{align*}
R_\alpha & = \mathbb{E}_{c(n)} \left[ \log_2 \left| I_{M_s} + \bar{P}H_{eq} \left( c(n) \right) H_{eq}^H \left( c(n) \right) \right| \right] \\
& = \mathbb{E}_{c(n)} \left[ \log_2 \left| I_{M_s} + \bar{P}F^H F H_{eq} \left( c(n) \right) H_{eq} \left( c(n) \right) \right| \right].
\end{align*}$$

(69)

Furthermore, we have

$$\begin{align*}
H_{eq}^H (c(n)) H_{eq} (c(n)) & = \left( H_d + \sum_{j=1}^{J} \sqrt{\alpha} g_j h_j^{HL} c_j (n) \right)^H \left( H_d + \sum_{j=1}^{J} \sqrt{\alpha} g_j h_j^{HL} c_j (n) \right) \\
& = H_d^H H_d + \sqrt{\alpha} \sum_{j=1}^{J} c_j^H (n) h_j^{HL} H_d + \sqrt{\alpha} \sum_{j=1}^{J} c_j (n) H_d^H h_j^{HL} \\
& + \alpha \sum_{j=1}^{J} \sum_{i=1}^{J} c_j^H (n) c_i (n) h_j^{HL} h_i^{HL} \\
& = H_d^H H_d + \sqrt{\alpha} \sum_{j=1}^{J} c_j^H (n) h_j^{HL} H_d + \sqrt{\alpha} \sum_{j=1}^{J} c_j (n) H_d^H h_j^{HL} \\
& + \alpha \sum_{j=1}^{J} \sum_{i=1}^{J} c_j^H (n) c_i (n) h_j^{HL} h_i^{HL}.
\end{align*}$$

(70)

Due to the law of large numbers, for $J \gg 1$, we have:

$$\alpha \sum_{j=1}^{J} \sum_{i=1}^{J} c_j^H (n) c_i (n) h_j^{HL} h_i^{HL} \to 0. \text{ Therefore,}$$

$$\begin{align*}
H_{eq}^H (c(n)) H_{eq} (c(n)) \to H_d^H H_d + J \sqrt{\alpha} \mathbb{E} \left[ c_j^H (n) h_j^{HL} H_d \right] + J \sqrt{\alpha} \mathbb{E} \left[ c_j (n) H_d^H h_j^{HL} \right] \\
+ J \alpha \mathbb{E} \left[ \| c_j (n) \|^2 \| g_j \|^2 \| h_j \|^2 \right] \\
= H_d^H H_d + J \alpha M_r \beta \tilde{\alpha} \beta I_{M_s}.
\end{align*}$$

(71)

By substituting (71) into (69), the proof of Lemma 3 thus completes.
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