Quantum secret sharing using pseudo-GHZ states

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We present a setup for quantum secret sharing using pseudo-GHZ states based on energy-time entanglement. In opposition to true GHZ states, our states do not enable GHZ-type tests of nonlocality, however, they bare the same quantum correlations. The relatively high coincidence count rates found in our setup enable for the first time an application of a quantum communication protocoll based on more than two qubits.

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Entangled particles play the major role both as candidates for entangled states of fundamental physics [1–4] as well as in the whole field of quantum communication [5]. Until recently, most work has been focussed on two-particle correlations. Since a couple of years, however, the interest in multi-particle entanglement – which we identify in this article with n>2 – is growing rapidly. From the fundamental side, particles in so-called GHZ states enable new tests of nonlocality [3]. From the side of quantum communication, more and more ideas for applications, like quantum secret sharing [6], emerge. A major problem still is the lack of multi-photon sources. Nonlinear effects that enable to "split" a pump photon into more than two entangled photons are extremely low efficient, and experiments still lie in the future. Recently Bouwmeester et al. could demonstrate a different approach where they started with two pairs of entangled photons and transformed them via a clever measurement into three photons in a GHZ state and a fourth independant trigger photon [8]. In this letter we present another method to create what we term pseudo-GHZ states. It is based on a recently developed novel source for quantum communication, creating entangled photons in energy-time Bell-states [9,10]. In opposition to "true" GHZ states, the three photons being in the pseudo-GHZ state do not consist of three downconverted photons but only of two downconverted ones plus the pump photon. We will comment on similarities and differences compared to true GHZ states and demonstrate a first application of our states for quantum secret sharing. In this case, the difference to true GHZ states is not only of no importance, but enables for the first time to realize a multi-particle application of quantum communication.

Entangled states of more than two qubits, so-called GHZ states, can be described in the form

$$\left| \psi \right\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left( |0\rangle_1 |0\rangle_2 |0\rangle_3 + |1\rangle_1 |1\rangle_2 |1\rangle_3 \right)$$  

where $|0\rangle$ and $|1\rangle$ are orthogonal states in an arbitrary Hilbert space and the indices label the particles (in this case three). As shown by Greenberger, Horne and Zeilinger in 1989 [4], the attempt to find a local model able to reproduce the quantum correlations faces an inconsistency. In the multi-particle case, the contradiction occurs already when trying to describe the perfect correlations. Thus, demonstrating these correlations directly shows that nature can not be described by local theories. However, since it will never be possible to experimentially demonstrate perfect correlations, the question arises whether there is some kind of threshold, similar to the one given by Bell inequalities for two-particle correlations [4], that enables to separate the "non-local" from the "local" region. Indeed, the generalized Bell inequality for the three-particle case [12]

$$S_3^\alpha = \left| E(\alpha', \beta, \gamma) + E(\alpha, \beta', \gamma) + E(\alpha, \beta, \gamma') - E(\alpha', \beta', \gamma') \right| \leq 2$$  

with $E(\alpha, \beta, \gamma)$ the expectation value for a correlation measurement with analyzer settings $\alpha, \beta, \gamma$ can be violated by quantum mechanics, the maximal value being

$$S_3^{exp} = 4.$$

For instance, finding a correlation function of the form $E(\alpha, \beta, \gamma) = \cos(\alpha + \beta + \gamma)$ with visibility $V$ above 50% shows that the correlations under test can not be described by a local theory. Note that this value is much lower than in the two-particle case where the threshold visibility is $\approx 71\%$.

Quantum secret sharing [6,8] is an expansion of the “traditional” quantum key distribution to more than two parties. In this new application of quantum communication, a sender, usually called Alice, distributes a secret key to two other parties, Bob and Charly, in a way that neither Bob nor Charly alone have any information about the key, but that together they have full information. Moreover, an eavesdropper trying to get some information about the key creates errors in the transmission data and thus reveals his presence. The motivation for secret sharing is to guarantee that Bob and Charly must cooperate – one of them might be dishonest – in order to do some task, one might think for instance of accessing classified information. As pointed out by Hillery et al. [7], this protocol can be realized using GHZ states. Assume three photons in a GHZ state of the form [12] with $|0\rangle$ and $|1\rangle$ being different modes of the particles (Fig. 1). After combining the modes at beamsplitters located at Alice’s, Bob’s and Charly’s, respectively, the probability to find the three photons in any combination of outputs ports depends on the settings $\alpha, \beta, \gamma$ of the phase shifters:
\[ P_{i,j,k} = \frac{1}{2}(1 + ijk \cos(\alpha + \beta + \gamma)) \] (4)

with \(i,j,k = \pm 1\) labeling the different output ports. Before every measurement, Alice chooses randomly one out of two phase values \((0, \pi/2)\), Bob applies a phase shift of \(-\pi/2\) or \(0\), and Charly chooses between \(\pi/2\) and \(3\pi/2\). After a sufficient number of runs, they publicly identify the cases where all detected a photon. All three then announce the phases chosen and single out the cases where the sum adds up either to 0 or to \(\pi\). Note that the probability function (Eq. 4) yields 1 for these cases. Denoting \(l = \cos(\alpha' + \beta + \gamma) = \pm 1\) and using \(P_{i,j,k} = 1\), Eq. 4 leads to

\[ ijk = 1. \] (5)

At this point, each of them knows two out of the values \(i,j,k,l\). If now Bob and Charly get together and join their knowledge, they know three of the four parameters and can thus determine the last one, which is also known to Alice. Identifying "–1" with bitvalue "0" and "+1" with "1", the correlated sequences of parameter values can then be turned into a secret key. Note that this scheme is completely symmetric. Any of the three can force the two other to collaborate in order to get information about his key, which in turn enables to read his confidential message. Like in two-party quantum cryptography, the security of quantum secret sharing using GHZ states is guaranteed by the fact that the measurements are made in noncommuting bases \(A B C\). An eavesdropper, including a dishonest Alice, Bob or Charly, is thus forced to guess about the bases that will be chosen. The fact that she will guess wrong in half of the cases then leads to detectable errors in the transmission data which reveal her presence.

![FIG. 1. Schematics for quantum secret sharing using GHZ states.](image1)

![FIG. 2. Principle setup for quantum secret sharing using energy-time entangled pseudo-GHZ states. Here shown is a fiberoptical realization.](image2)

We now explain how to implement secret sharing using our source (see Fig. 2). A short light pulse emitted at time \(t_0\) enters an interferometer having a path length difference which is large compared to the duration of the pulse. The pulse is thus split into two pulses of smaller amplitude, following each other with a fixed phase relation. The light is then focussed into a nonlinear crystal where some of the pump photons are downconverted into photon pairs. The pump energy is assumed to be such that the possibility to create more than one pair from one initial pump pulse can be neglected. This first part of the setup is located at Alice’s. The downconverted photons are then separated and send to Bob and Charly, respectively. Both of them are in possession of a similar interferometer as Alice, introducing exactly the same difference of travel times. We assume the transmission probabilities via the different arms of any of the three interferometers to be alike. The two possibilities for the photons to pass through any device lead to three time differences between emission of the pump pulse at Alice’s and detection of signal or idler photon, as well as between the detection of one downconverted photon at Bob’s and the correlated one at Charly’s (Fig. 2). Looking for example at the possible time differences between detection at Bob’s and emission of the pump pulse \((t_B - t_0)\), we find three different terms. The first one is due to "pump pulse travelled via the short arm and Bob’s photon travelled via the short arm" to which we refer as \(|s\rangle_A; |s\rangle_B\). Please note that this notation considers the pump pulse as being a single photon (now termed "Alice’s photon"), stressing the fact that only one pump photon is annihilated to create one photon pair. Moreover, the fact that this state is not a product state is taken into account by separating the two kets by ",". The second time difference is either due to \(|s\rangle_A; |l\rangle_B\), or to \(|l\rangle_A; |s\rangle_B\), and the third
one to $|l\rangle_A |l\rangle_B$. Similar time spectra arise when looking at the time differences between emission at Alice’s and detection at Charly’s ($t_C - t_B$), as well as between the detections at Bob’s and Charly’s ($t_C - t_B$). Selecting now only processes, leading to the central peaks, we find two possibilities: Either Alice’s photon traveled via the long arm and Bob’s as well as Charly’s took the short ones, or Alice’s photon choose the short arm and Bob’s and Charly’s the both the long ones. If both possibilities are indistinguishable, the process is described by

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |l\rangle_A |s\rangle_B |s\rangle_C + e^{i(\alpha+\beta+\gamma)} |s\rangle_A |l\rangle_B |l\rangle_C \right),$$

with phases $\alpha, \beta, \gamma$ in the different interferometers. The maximally entangled state is similar to the GHZ state given in Eq.1, the difference being that the three photons do not exist at the same time (remember the “;”). Therefore, our state is obviously of no significance concerning GHZ-type tests of nonlocality. To stress this difference, we call it pseudo-GHZ state. However, the probability-function describing the triple coincidences (Eq.3) – in our case between emission of a pump pulse and detection at Bob’s and Charly’s – is the same as the one originating from a true GHZ state. To avoid the complication of switching the pump laser randomly between one of the two input ports – equivalent to detecting a photon in one or the other output port –, we let Alice chose between one of four phase values $\psi’(0, \pi/2, 3\pi/2)$. To map the choice of phases to the initial scheme where the information of Alice, Bob and Charly is given by a phase setting and a detector label, we assign a different notation to characterize Alice phases (table 1). Using this convention, we can implement the same protocol as given above, the advantage being the fact that our setup circumvents creation and coincidence detection of photon triples. The emission of the bright pump pulse can be considered as detection of a photon with 100% efficiency. Moreover, no low efficient triple photon generation is necessary. This leads to much higher triple coincidence rates, enabling the demonstration of a multi-photon applications of quantum communication. One might question the security of our setup, the weak point being the channel leading from Alice’s interferometer to the crystal. Here, the light is classical and the phase could be measured without modifying the system. However, since this part is controlled by Alice and the parts, physically accessible to an eavesdropper carry only quantum systems, our realization does not incorporates any loophole.

The experimental setup is described in [11], where it is used to demonstrate two-party quantum key distribution using energy-time Bell states. We will thus give only a brief outline. To generate the short pump pulses, we use a pulsed diode laser (PicoQuant PDL 800), emitting 600ps (FWHM) pulses of 655 nm wavelength at a repetition frequency of 80 MHz. The light is channelled through a fiberoptical Michelson interferometer (path length difference corresponding to 1.2 ns travel time difference) and focussed into a 4x3x12 mm $KNbO_3$ crystal, producing photon pairs at 1310 nm wavelength. The average power before the crystal is $\approx$ 1 mW, and the energy per pulse – remember that each initial pump pulse is now split into two – $\approx$ 6 pJ. After absorption of the red pump light, the downconverted photons are separated and are guided to fiberoptical Michelson interferometers, located at Bob’s and Charly’s, respectively. To access the second output port, usually coinciding with the input port for this kind of interferometer, we implement 3-port optical circulators. The interferometers incorporate equal path length differences, and the travel time difference is the same than the one introduced by the interferometer acting on the pump pulse. The output ports are connected to single-photon counters – passively quenched germanium avalanche photodiodes, operated in Geiger-mode and cooled to 77 K. We operate them at dark count rates of 30 kHz, leading to quantum efficiencies of $\approx$ 5% and single photon detection rates of 4-7 kHz. The electrical output from each detector is fed into a fast AND-gate, together with a signal, coincident with the emission of a pump pulse. We condition the detection at Bob’s and Charly’s on the central peaks $(|s\rangle_P|l\rangle_A$ and $|l\rangle_P|s\rangle_A$, and $|s\rangle_P|l\rangle_B$ and $|l\rangle_P|s\rangle_B$, respectively). Looking at coincident detections between two AND-gates – equivalent to triple coincidences –, we finally select only the interfering processes for detection.

FIG. 3. Result of the measurement when changing the phase in Alice interferometer. The different mean values are due to non-equal quantum efficiencies of the single photon detectors. The values for the case $i=-1$ have been found corresponding to table I. For global phase zero, we find a mean QBER of $(3.9 \pm 0.4)\%$. If Bob and Charly both detect a photon in the “+”-labeled detectors in this case ($l=+1$), they know that Alice value $i$ must be $+1$ as well.

Since the stability of our interferometers is not sufficient to maintain stable phases over a long time, we demonstrate that our source can be used for quantum secret sharing by continuously changing the phases in Alice’s as well as in Bob’s interferometer. We observe sinusoidal fringes in the triple coincidence rates with maximum count rates around 1600 in 100 sec and minimum
ones around 70 (see Fig. 3). Visibilities are inbetween 89.3 and 94.5% for the different detector combinations, leading to a mean visibility of 92.2±0.8% and a quantum bit error rate BER – the ratio of errors to detected events – of (3.9±0.4)%. The critical visibility above which the information that might have been obtained by an eavesdropper can be made arbitrarily small using classical error correction and privacy amplification is not known yet. In case of two-party quantum key distribution, it corresponds exactly to a violation of two-particle Bell inequalities [13]. It is thus reasonable to compare the mean visibility to the value given by generalized Bell inequality (Eq.2), even if our setup does not incorporate GHZ-type nonlocality: The found visibility of 92.2±0.8% is more than 50 standard deviations (σ) higher than the threshold visibility for the three-particle case. Moreover, it is more than 25 σ above 71%, the value given by standard (two-particle) Bell inequalities. Within this respect, it is also interesting to calculate $S_{\exp}$: We find $S_{\exp}=3.69$, well above $S_{\lambda}=2$ (Eq.3). Therefore, the performance of our source is good enough to detect any eavesdropping and to ensure secure key distribution. Moreover, the bit-rate of $\approx 16$ Hz underlines its potential for real applications. To compare our coincidence rate to an experiment using true GHZ states [6], Bouwmeester et al. found one GHZ-state per 150 sec. However, in order to really implement our setup for quantum secret sharing, an active phase stabilization and a fast switch still have to be incorporated [13].

Like in all experimental quantum key distribution, the QBER is non-zero, even in the absence of any eavesdropping. The observed 4% can be traced back to wrong counts from accidentally correlated event at the single-photon counters, non-perfect localization of the pump pulse, limited resolution of the single-photon detectors and non-perfect interference. Note that the number of errors due to the last mentioned points decrease with distance (caused by higher transmission losses) and thus do not engender an increase of the QBER. In opposition, the number of errors due to accidental coincidences stays almost constant since it is mostly due to detector noise. However, it causes only 10% of the total errors in our laboratory demonstration. Therefore, the QBER will increase only at a small rate, enabling quantum secret sharing over tens of kilometers. In conclusion, we demonstrated quantum secret sharing using energy-time entangled pseudo-GHZ states in a laboratory experiment. We found bit-rates of around 16 Hz and quantum bit error rates of 4%, low enough to ensure secure key distribution. The advantage of our scheme is the fact that neither triple-photon generation nor coincidence detection of three photons is necessary, enabling for the first time an application of a multiparticle quantum communication protocol. Moreover, since energy time entanglement can be preserved over long distances [1], our results are very encouraging for realizations of quantum secret sharing over tens of kilometers.

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\[ \begin{align*}
\alpha' & | \quad 0 \quad \pi/2 \quad \pi \quad 3\pi/2 \\
\alpha & | \quad 0 \quad \pi/2 \quad 0 \quad \pi/2 \\
\iota & | \quad 1 \quad 1 \quad -1 \quad -1
\end{align*} \]

| TABLE I. | Mapping of the four possible phases $\alpha'$ at Alice’s to two phase values $\alpha$ and the parameter $\iota$. |

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