Photon-induced tunneling in optomechanical systems

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In contrast to recent studies [Rabl, Phys. Rev. Lett. 107, 063601 (2011); Nunnenkamp et al., Phys. Rev. Lett. 107, 063602 (2011)] on photon blockade that prevents subsequent photons from resonantly entering the cavity in optomechanical systems, we study the photon-induced tunneling that increases the probability of admitting subsequent photons in those systems. In particular, we analytically and numerically show how two- or three-photon tunneling can occur by avoiding single-photon blockade. Our study provides another way on photon control using a single mechanical resonator in optomechanical systems.

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I. INTRODUCTION

Optomechanical systems [1, 2] have attracted extensive attention in the past years because of their potential application in high-precision measurements and quantum information processing. To realize these benefits, the mechanical resonator has to be in its ground state; and the optomechanical radiation-pressure interaction strength should be bigger than the decay rates of the cavity field and the mechanical oscillator. The ground state cooling of the mechanical resonator has been experimentally studied in these systems (e.g., in Refs. [3–10]. Although the strong coupling is not easily achieved in standard optomechanical systems, the experimentalists have obtained an effectively strong coupling, by applying a classical driving field to the cavity mode, which has led to observations of normal-mode splitting (e.g., in Refs. [11, 12]) and optomechanically induced transparency (e.g., in Refs. [13–15]). However, the resulting effective coupling resembles two linearly coupled harmonic oscillators [11], and the coupling strength is proportional to the square root of the mean cavity photon number; thus it does not really describe the nonlinear effect at the single-photon level.

Two recent proposals [16, 17] showed that the single-photon effect or photon blockade [18] can occur when the optomechanical systems are approaching the single-photon strong coupling [19–21]. This is because the mechanical resonator parametrically modulates the frequency of the cavity field and results in the photon-photon interaction [22–27]. If the strong optomechanical interaction makes the photon-photon coupling strength bigger than the decay rate of the cavity field, then the photons can prevent the subsequent photons from resonantly entering the cavity. The photon blockade has been demonstrated experimentally in cavity QED systems for microwave [28, 29] and optical [30, 31] photons. Meanwhile, the experimentalists [30, 31] also observed the photon induced tunneling, that is, the probability of admitting subsequent photons is increased when there is one photon inside the cavity. Moreover, absorption and emission of resonant photons in pairs have been observed [32, 33].

Motivated by recent works [16–33] and also in contrast to the photon blockade [16–17], here we study photon-induced tunneling in optomechanical systems. In Sec. II, the theoretical model and the master equation are introduced. The effect of the mechanical resonator on the mean photon numbers is discussed. In Sec. III, the two-photon tunneling is discussed via the normalized second-order correlation function of the cavity field. In Sec. IV, we use the three-photon tunneling as an example to show the multiphoton tunneling phenomena. A summary is finally given in Sec. V.

II. THEORETICAL MODEL

We study an optomechanical system (e.g., review in Ref. [1]) in which a cavity field is coupled to a mechanical resonator through the radiation pressure with the Hamiltonian

\[ H = \hbar \omega_0 a^\dagger a + \hbar \omega_m b^\dagger b + \hbar G a^\dagger a \left( b^\dagger + b \right), \]

where \( a \) (and \( a^\dagger \)) are the annihilation and creation operators of the cavity field (mechanical resonator) with the frequency \( \omega_0 \) (\( \omega_m \)). The coupling strength between the cavity field and the mechanical resonator is \( G \). By applying a unitary transform \( U = \exp \left[ -G a^\dagger a \left( b^\dagger - b \right) / \omega_m \right] \) to Eq. (1), we obtain an effective Hamiltonian \( H' = U H U^\dagger \) with

\[ H' = \hbar \omega_m b^\dagger b + \hbar \left( \omega_0 - \frac{G^2}{\omega_m} \right) a^\dagger a - \hbar \omega_m a^\dagger a, \]

which has eigenstates \( |n, \tilde{m} \rangle \) and corresponding eigenvalues

\[ E_{n, m} = \hbar \left( n \omega_0 - \frac{G^2}{\omega_m} + m \omega_m \right), \]

where \( |n, \tilde{m} \rangle \equiv U |n, m \rangle \). \( |n, m \rangle \) represents a state of \( n \) photons and \( m \) phonons. A unitary transform does not change the eigenvalues of the Hamiltonian; thus the eigenvalues of the Hamiltonian in Eq. (1) are the same as in Eq. (3).

We now assume that the cavity field is driven by a weak probe field with the frequency \( \omega_c \) and the coupling strength...
is shifted to Eq. (2), this is because the transition frequency of the cavity field can be obtained as the mean photon number inside the cavity can be obtained as a physical quantity of the system can be obtained. For example, the formal solution of the normalized detuning \( \Delta = \Delta_0 \) where \( \Delta = \omega_0 - \omega_c \) is the detuning between the cavity field and the probe field. When the environmental effect is taken into account, the dynamical evolution of the reduced density operator \( \rho(t) \) for the cavity field and the mechanical reso- nator can be described via the master equation \[34\]

\[
\frac{d\rho}{dt} = \frac{1}{\hbar} \left[ \hat{H}, \rho \right] + \frac{\gamma}{2} \left( 2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a \right) \\
+ \frac{\gamma_m}{2} \left( 2b\rho b^\dagger - b^\dagger b \rho - \rho b^\dagger b \right) \\
+ \gamma_m \bar{n}_m \left( b\rho b^\dagger + b^\dagger b \rho - b^\dagger b \rho + \rho b^\dagger b \right),
\]

(5)

with the decay rates \( \gamma \) and \( \gamma_m \) of the cavity field and mechanical mode. \( \bar{n}_m = \exp(\hbar \omega_m/k_B T) - 1 \) is the thermal phonon number of the mechanical resonator with the Boltzmann constant \( k_B \) and the environmental temperature \( T \) of the mechanical resonator. Here, the frequency of the cavity field is assumed to be high enough; thus the thermal photon effect can be neglected. In the basis of the states \( |n, m\rangle \), the formal solution of \( \rho(t) \) in Eq. (5) can be given by

\[
\rho(t) = \sum_{n,m,n',m'} \rho_{n,m,n',m'}(t) |n, m\rangle \langle n', m'|.
\]

(6)

If all elements \( \rho_{n,m,n',m'}(t) \) in Eq. (6) are given, then any physical quantity of the system can be obtained. For example, the mean photon number inside the cavity can be obtained as

\[
\langle n \rangle = \langle a^\dagger a \rangle = \text{Tr}[\rho(t) a^\dagger a] = \sum_{n,m} n \rho_{n,m,n,m}(t).
\]

(7)

Using Eqs. (5)-(7), \( \langle n \rangle \) is plotted in the steady state as a function of the normalized detuning \( \Delta/\Delta_0 \) with \( \Delta_0 = G^2/\omega_m \) for (i) different \( \gamma_m \) in Fig. 1(a) and (ii) different \( T \) in Fig. 1(b). We find that \( \langle n \rangle \) is maximum at \( \langle \Delta/\Delta_0 \rangle = 1 \). As shown in Eq. (2), this is because the transition frequency of the cavity field from the one-photon state \( |1\rangle \) to the ground state \( |0\rangle \) is shifted to \( \omega_0 - (G^2/\omega_m) \) when the mechanical resonator is coupled to the cavity field. Moreover, Fig. 1 shows that when the decay rate \( \gamma_m \) or the environmental temperature \( T \) of the mechanical resonator increases, the full width at half maximum in the curve of \( \langle n \rangle \) becomes broad for other given parameters. This means that \( \gamma_m \) and \( T \) significantly affect the lift time of the cavity photons.

III. TWO-PHOTON TUNNELING

The statistical properties of the photons can be characterized by the normalized \( n \)-th order correlation function

\[
g^{(n)}(0) = \frac{\langle a^\dagger a \rangle^n}{\langle a^\dagger a \rangle} \equiv \text{Tr}(\rho a^{n\dagger} a^n) / \text{Tr}(\rho a^\dagger a),
\]

(8)
at the zero time delay. Here, the \( \rho \) of the optomechanical system can be obtained by solving the master equation in Eq. (5).

We now study the simplest photon tunneling in pairs. Similar to the photon blockade, this phenomenon can be characterized by \( g^{(2)}(0) \). When \( g^{(2)}(0) \) is less than one, the photon blockade happens and single photons come out of the cavity. However, when \( g^{(2)}(0) \) is bigger than one, photons inside the cavity enhance the resonantly entering probability of subsequent photons; this can make photons come out of the cavity in pairs under certain condition. In Refs. \[12, \[13. \[14\], the second-order differential correlation function

\[
C^{(2)}(0) = \langle a^{\dagger 2} a^2 \rangle - \langle a^\dagger a \rangle^2 \equiv [g^{(2)}(0) - 1] \langle n \rangle^2,
\]

(9)
at the zero time delay is introduced to characterize the probability of creating photon pairs simultaneously in the cavity.

To explore two-photon tunneling and compare it with the photon blockade, \( g^{(2)}(0) \) and \( C^{(2)}(0) \) are plotted in Figs. 2(a) and 2(b) as the functions of \( \Delta \) and \( G \) for given \( T \) in the steady state. Each figure has two curves, corresponding to single-\( \Delta = \Delta_0 \) and two-photon \( \Delta = 2\Delta_0 \) resonant excitations from the ground state to the first- and the second-excited states of the cavity field, respectively. Figures 2(a) and 2(b) clearly show \( g^{(2)}(0) < 1 \) and \( C^{(2)}(0) < 0 \) for the single-photon resonant excitation when \( G \) is bigger than \( \gamma \), that is, the single-photon phenomenon or photon blockade occurs as shown in Refs. \[12, \[13. \[14\]. However, when the frequency \( \omega_c \) of the probe field equals half of the transition frequency from the ground state to the second excited state, i.e., \( \Delta = 2\Delta_0 \), we find \( g^{(2)}(0) > 1 \) and \( C^{(2)}(0) > 0 \) as shown in Figs. 2(a) and 2(b). This means that the single-photon transition from the ground state to the first excited state is suppressed, but the second photon can enter the cavity and make resonant transition from the ground state to the second excited state together with the first photon. That is, the photon-induced tunneling happens and photons can be absorbed in pairs simultaneously.

We further show the effect of \( T \) on the photon tunneling via \( g^{(2)}(0) \) and \( C^{(2)}(0) \) in Figs. 2(c) and 2(d). We find that the low \( T \) makes photon-induced tunneling and photon blockade easily observed. It is easily understood because the higher \( T \) corresponds to the bigger decay rate \( \gamma \) of the cavity field as shown in Fig. 1(b). Thus, with the increase of \( T \), \( g^{(2)}(0) \) and \( C^{(2)}(0) \) increase under the single-photon \( \Delta = \Delta_0 \) resonant driving, but decrease under the two-photon \( \Delta = 2\Delta_0 \) resonant driving. Moreover, there is an additional peak in Fig. 2(c)
then the resonant transition between two eigenstates $|\tilde{m}\rangle$ and $|\tilde{n}\rangle$ as shown in Fig. 3(a). We find that the height of the resonant peak at $\Delta = 3\Delta_0$ increases when $T$ becomes higher. Satisfactorily, this is because $T$ changes the population distribution, especially it enhances the population in higher energy levels, and then the transition from $|\tilde{1}, \tilde{m}\rangle$ to $|\tilde{2}, \tilde{m}\rangle$ is also enhanced. All this results in the increase of $g^{(2)}(0)$ at $\Delta = 3\Delta_0$ with the increase of $T$.

The effect of the phonon states on the photon blockade has been studied in Refs. [16, 26]. It was found that the two-phonon resonant transition between $|0, \tilde{0}\rangle$ and $|2, \tilde{m}\rangle$ can also occur when $\Delta_0 = m\omega_m/2$ under the photon blockade condition $\Delta = \Delta_0$. Thus there are resonant peaks located at $G/\omega_m = \sqrt{m/2}$ for $g^{(2)}(0)$ with different phonon states $|\tilde{m}\rangle$ (e.g., $m = 1, 2, 3$) as shown in Fig. 3(b). Similarly, we find that the phonon states also affect the photon-induced tunneling. For example, Fig. 3(c) shows five peaks for $g^{(2)}(0) \gg 1$ in the regime $0 < G < \omega_m$ under the condition $\Delta = 0$ (i.e., $\omega_0 = \omega_c$). As shown Fig. 3(a), these five peaks correspond to two types of the resonant conditions: (i) if $\Delta_0 = m\omega_m/4$, then the resonant transition between $|0, \tilde{0}\rangle$ and $|2, \tilde{m}\rangle$ can occur with the peaks at $G/\omega_m = \sqrt{m}/4$ ($m = 1, 2, 3$); (ii) when $\Delta_0 = m\omega_m/3$, another resonant transition between $|1, \tilde{0}\rangle$ and $|2, \tilde{m}\rangle$ is allowed with the peaks at $G/\omega_m = \sqrt{m}/3$ ($m = 1, 2$). As the transitions $|1, \tilde{0}\rangle \rightarrow |2, \tilde{m}\rangle$ are enhanced with the increase of $T$, the peak at $G/\omega_m = \sqrt{m}/3$ ($m = 1, 2$) increases when $T$ is increased.

To approximately obtain the conditions of the photon blockade and the photon-induced tunneling, we analyze the relation between $g^{(2)}(0)$ and the probabilities $P(n)$ of $n$-photon distribution. $P(n)$ corresponding to the state in Eq. (6) can be given by $P(n) = \sum_m \rho_{m,n;m,n}(t)$; then we have

$$g^{(2)}(0) = \frac{\text{Tr}[(\rho a^2 a^\dagger)]}{[\text{Tr}(\rho a a^\dagger)]^2} = \frac{\sum_n (n-1) P(n)}{\left[ \sum_n n P(n) \right]^2}, \quad (10)$$

In the limit of the weak probe field (e.g., $\varepsilon_c = 0.1\gamma$), $P(n) \gg P(n+1)$, $g^{(2)}(0)$ can be given approximately by

$$g^{(2)}(0) \approx \frac{2P(2)}{P(1)^2} = \frac{\gamma + i2(\Delta - \Delta_0)}{\gamma + i2(\Delta - 2\Delta_0)^2}, \quad (11)$$

in the steady state. Here we have assumed that the photon is in its ground state to obtain the second approximated expression. This assumption has also been made in the following derivations in Eq. (13) and Eq. (16). Equation (11) means that $g^{(2)}(0)$ is proportional to the ratio between the probability to prepare a two-photon state and that to prepare two single-photon states independently. In Fig. 3(a), $g^{(2)}(0)$ and its approximated expression in Eq. (11) versus the normalized detuning $\Delta$ are numerically simulated in the steady state. We find that the approximated expression fits well with the exact solution of $g^{(2)}(0)$ in the limit of the weak probe field. Thus $g^{(2)}(0) > 1$ or $g^{(2)}(0) < 1$ means that the probability to excite the two-photon state is bigger (smaller) than that to excite two single-photon states independently, and then photon-induced tunneling (photon blockade) happens. To quantitatively show the relation between the height of Fig. 3(c) and also Fig. 4(a) and system parameters, we derive an approximated expression,

$$g^{(2)}(0) \approx (\gamma^2 + 4\Delta_0^2)/\gamma^2, \quad (12)$$
just use the three-photon case as an example to show multi-
be studied via Eq. (8). However, for the sake of simplicity, we
create a three-photon state and that to create three single pho-
tons independently. If we introduce a quantity

\[
g(2) = \frac{\gamma}{(\gamma^2 + 4\Delta_0^2)},
\]

for \( \Delta = \Delta_0 \), which corresponds to the deep in Fig. 4 (a)
and Fig. 2 (c). It is clear that the condition to observe photon
blockade or photon-induced tunneling is \( \Delta_0 > \gamma/2 \).

**IV. MULTI-PHOTON TUNNELING**

In principle, the multiphoton tunneling (more than two) can
be studied via Eq. (3). However, for the sake of simplicity, we
just use the three-photon case as an example to show multi-
photon tunneling. Using the similar discussions as for Eq. (10)
and Eq. (11), \( g(3) \) can be approximately written as

\[
g(3) = \frac{\gamma^2}{(\gamma^2 + 4\Delta_0^2)^2},
\]

in the steady state with the weak probe field. It is clear that
\( g(3) \) is proportional to the ratio between the probability to
create a three-photon state and that to create three single pho-
tons independently. If we introduce a quantity

\[
g_2(3) = \frac{g(3)}{g(2)} = \frac{\text{Tr} (\rho a^{13}a^3)}{\text{Tr} (\rho a^{13}a^3)\text{Tr} (\rho a^{13}a^2)},
\]

to characterize the ratio between the normalized third-order
and second-order correlation functions, then we can obtain

\[
g_2(3) \approx \frac{3P(3)}{P(1)P(2)^2} \approx \frac{\gamma + i2(\Delta - \Delta_0)^2}{\gamma + i2(\Delta - 3\Delta_0)^2},
\]

in the steady state. \( g_2(3) \) is proportional to the ratio be-
tween the probability to create a three-photon state and the
joint probability to create a single- plus a two-photon state in-
dependently, which is another way to create three photons.

In Fig. 4 (b), \( g(3) \) and its approximated expression
\( 6P(3)/[P(1)]^3 \) as well as \( g_2(3) \) and its approximated
expression \( 3P(3)/[P(1)P(2)]^2 \) are plotted as functions of
\( \Delta/\Delta_0 \). We find that the approximation in both Eq. (14)
and Eq. (16) is valid for the weak probe field. Under the condition
that \( \Delta_0 > \gamma/2 \), we can further find

\[
g(3) \approx \frac{(\gamma^2 + 8\Delta_0^2)^2}{\gamma^2 + 4\Delta_0^2},
\]

g_2(3) \approx \frac{(\gamma/\Delta_0)^2}{(\gamma/\Delta_0)^2},
\]

for the resonant condition \( \Delta = \Delta_0 \), and

\[
g(3) \approx \frac{2\Delta_0/\gamma}{2\Delta_0/\gamma},
\]

for the resonant condition \( \Delta = 3\Delta_0 \). Equations (13) and (19)
show that the heights of the resonant peaks are approximately
determined by the ratio of \( \Delta_0 \) and \( \gamma \) as for two-photon tunneling.

Figure 3(b) and Eqs. (17)–(19) show the following. (i) For
the single-photon resonant excitation (\( \Delta = \Delta_0 \)), \( g(3) \) \( < 1 \)
and \( g_2(3) \) \( < 1 \), which means that the photon inside the cavity is antibunching
or the subsequent photons will be blocked by the photons inside
the cavity. (ii) For the three-photon resonant excitation (\( \Delta = 3\Delta_0 \)), \( g(3) \) \( > 1 \)
and \( g_2(3) \) \( > 1 \), that is, the photon inside the cavity is bunching or the cavity
can absorb three photons simultaneously. (iii) For the two-
photon resonant excitation (\( \Delta = 2\Delta_0 \)), \( g(3) \) \( > 1 \) which
means that the probability of three-photon absorption is bigger
than that of three single-photon absorption independently.

We also find \( g_2(3) \approx 1 \) at the point \( \Delta = 2\Delta_0 \), which means
that the joint probability of the photon absorption in pairs after
or before single-photon absorption is approximately equal to
that of three-photon absorption by the cavity. Therefore, the
necessary condition for absorbing three photons by the cavity simultaneously (or three-photon tunneling) is \( \Delta = 3\Delta_0 \). In
this condition, \( g_2(3) \) \( > 1 \) and \( g_2(3) \) \( > 1 \), the probability of
generating three photons simultaneously is bigger than those
generating three single-photons independently and generating
a single-photon after or before generating a photon pair.

**V. CONCLUSIONS**

In summary, we have studied the photon induced tunneling
and compared it with the photon blockade in optomechanical systems with the strong optomechanical coupling. Our study shows that the cavity field can exhibit photon antibunching when the probe field is resonant with the transition from the ground state to the first excited state of the cavity field. However, the two-photon tunneling occurs when the frequency of the probe field equals half of the transition frequency from the ground state to the second excited state of the cavity field. Moreover, we find that three-photon tunneling occurs when the frequency of the probe field satisfies the condition of the three-photon resonant excitation from the ground state to the
third excited state. We further show that the phonon states greatly affect the multiphoton resonance in the certain condition. Our studies can be easily generalized to the $n$-photon ($n > 3$) case. Our results show that the photon-induced tunneling can be experimentally observed when the optomechanical system approaches the strong-coupling limit.

Note added. We note a related paper that appeared recently [36].

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[1] T. J. Kippenberg and K. J. Vahala, Science 321, 1172 (2008), and the references therein.
[2] F. Marquardt and S. M. Girvin, Physics 2, 40 (2009).
[3] O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, Nature 444, 71 (2006).
[4] D. Kleckner and D. Bouwmeester, Nature 444, 75 (2006).
[5] S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Bäuerle, M. Aspelmeyer, and A. Zeilinger, Nature 444, 67 (2006).
[6] A. Schliesser, P. Del’ Heye, N. Nooshi, K. J. Vahala, and T. J. Kippenberg, Phys. Rev. Lett. 97, 243905 (2006).
[7] J. D. Teufel, J. W. Harlow, C. A. Regal, and K. W. Lehnert, Phys. Rev. Lett. 101, 197203 (2008).
[8] J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin, and J. G. E. Harris, Nature 452, 72 (2008).
[9] T. Rocheleau, T. Ndukum, C. Macklin, J. Hertzberg, A. Clerk, and K. Schwab, Nature 463, 72 (2009).
[10] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature 475, 359 (2009).
[11] S. Groblacher, K. Hammerer, M. R. Vanner, and M. Aspelmeyer, Nature 460, 724 (2009).
[12] J. D. Teufel, D. Li, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, and R. W. Simmonds, Nature 471, 204 (2011).
[13] G. S. Agarwal and Sumei Huang, Phys. Rev. A 81, 041803(R) (2010).
[14] S. Weis, R. Riviere, S. Deleglise, E. Gavartin, O. Arcizet, A. Schliesser, and T. J. Kippenberg, Science 330, 1520 (2010).
[15] A. H. Safavi-Naeini, T. P. Mayer Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. E. Chang, and O. Painter, Nature 472, 69 (2011).
[16] P. Rabl, Phys. Rev. Lett. 107, 063601 (2011).
[17] A. Nunnenkamp, K. Borjke, and S. M. Girvin, Phys. Rev. Lett. 107, 063602 (2011).
[18] A. Imamoglu, H. Schmidt, G. Woods, and M. Deutsch, Phys. Rev. Lett. 79, 1467 (1997).
[19] S. Gupta, K. L. Moore, K. W. Murch, and D. M. Stamper-Kurn, Phys. Rev. Lett. 99, 213601 (2007).
[20] F. Brennecke, S. Ritter, T. Donner and T. Esslinger, Science 322, 235 (2008).
[21] M. Eichenfield, J. Chan, R. M. Camacho, K. J. Vahala, and O. Painter, Nature 462, 78 (2009).
[22] P. Meystre, E. M. Wright, J. D. McCullen, and E. Vignes, J. Opt. Soc. Am. B 2, 1830 (1985).
[23] S. Mancini and P. Tombesi, Phys. Rev. A 49, 4055 (1994).
[24] Z. R. Gong, H. Ian, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. A 80, 065801 (2009).
[25] J. Q. Liao, H. K. Cheung, and C. K. Law, Phys. Rev. A 85, 025803 (2012).
[26] J. Q. Liao and C. K. Law, Phys. Rev. A 87, 043809 (2013).
[27] B. He, Phys. Rev. A 85, 063820 (2012).
[28] C. Lang, D. Bozyigit, C. Eichler, L. Steffen, J. M. Fink, A. A. Abdumalikov, Jr., M. Baur, S. Filipp, M. P. da Silva, A. Blais, and A. Wallraff, Phys. Rev. Lett. 106, 243601 (2011).
[29] A. J. Hoffman, S. J. Srinivasan, S. Schmidt, L. Spievj, J. Aumentado, H. E. Türeci, and A. A. Houck, Phys. Rev. Lett. 107, 053602 (2011).
[30] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Nature 436, 87 (2005).
[31] A. Farao, I. Fushman, D. Englund, N. Stoltz, P. Petro, and J. Vuckovic, Nature Phys. 4, 859 (2008).
[32] A. Kubanek, A. Ourjoumtsev, I. Schuster, M. Koch, P. W. H. Pinkse, K. Murr, and G. Rempe, Phys. Rev. Lett. 101, 203602 (2008).
[33] M. Koch, C. Sames, M. Balbach, H. Chibani, A. Kubanek, K. Murr, T. Wilk, and G. Rempe, Phys. Rev. Lett. 107, 023601 (2011).
[34] H. J. Carmichael, An Open Systems Approach to Quantum Optics, (Springer-Verlag, Berlin, 1993).
[35] A. Majumdar, M. Bajcsy, and J. Vuckovic, Phys. Rev. A 85, 041801(R) (2012).
[36] A. Kronwald, M. Rudwig, and F. Marquardt, Phys. Rev. A 87, 013847 (2013).