Scalar glueball in a holographic model of QCD

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Summary. — I describe scalar glueballs in the soft wall model of holographic QCD (AdS/QCD).

1. – Introduction

Recently, a lot of interest has grown around the possibility of applying string inspired techniques to the non-perturbative regime of QCD. The starting point is the AdS/CFT correspondence [1], a conjectured duality between a maximally supersymmetric strongly coupled conformal field theory and the supergravity limit of type IIB string theory, which involves theories different from QCD. Further developments [2] have tried to apply the correspondence to QCD, induced by the evidence of the existence of a window of energy in which QCD shows an approximate conformal behaviour [3]. These developments have taken different directions. The framework through which I move here is the so-called soft wall model of AdS/QCD [4], a phenomenological model originally built to holographically describe chiral symmetry breaking and then adapted to several strong interaction processes. For a list of other approaches the reader can refer to [5].

In the following, I discuss the scalar glueball sector and how the spectrum and the two-point correlation function are represented in the soft wall model. Then, I comment on the results, comparing them with current phenomenology and lattice data.

2. – Framework

The considered model is defined in a 5d curved space (the bulk) with metric:

\[ ds^2 = g_{MN} dx^M dx^N = R^2 \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \]

(1)

with \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \); \( R \) is the AdS curvature radius, and the coordinate \( z \) runs in the range \( 0 \leq z < +\infty \). QCD is supposed to live on the boundary \( z = 0 \), where the element \( \eta_{\mu\nu} dx^\mu dx^\nu \) describes a flat Minkowski space.
In addition to the AdS metric, the model is characterized by the presence of a background dilaton field:

\[ \Phi(z) = (cz)^2 \]

exponentially coupled to the fields, whose functional form is chosen in such a way to have linear Regge trajectories for light vector mesons [4]; \( c \) is a dimensionful parameter setting the scale of QCD quantities and it is of \( \mathcal{O}(\Lambda_{\text{QCD}}) \). It is the responsible of the breaking of conformal symmetry and it is fixed by the experimental slope of the rho mesons trajectory.

3. – Scalar glueballs

0\(^{++} \) glueballs can be described in QCD by the dimension four operator \( \mathcal{O}_S(x) = \text{Tr} \{ \beta(\alpha_s) G^a G^{a\mu
u} \} \). In the five dimensional theory its dual field is a massless scalar \( Y(x, z) \) [6], whose action is given by:

\[ S = -\frac{1}{2k} \int d^5x \sqrt{-g} e^{-\phi} g^{MN} (\partial_M Y)(\partial_N Y) \]

where \( k \) is a parameter introduced to give the correct dimension to the action. The AdS/CFT dictionary states that this action is equivalent to the QCD partition function, in which the source of \( \mathcal{O}_S(x) \) is the boundary value \( Y_0(x) \) of the field \( Y(x, z) \). The following relation can be written:

\[ Y(x, z) = \int d^4x' K(x - x', z)Y_0(x') \]

where the function \( K(x - x', z) \) is called bulk-to-boundary propagator, since it links the fields in the bulk with the sources on the boundary.

It is possible to obtain QCD correlation functions functionally deriving the action (3) with respect to \( Y_0(x) \). The two-point function obtained in this way is, in the limit \( q^2 \to +\infty \) [7, 8]:

\[ \Pi_{AdS}(q^2) = \frac{R^3}{k} \left\{ q^4 \cdot \frac{1}{8} \left[ 2 - 2\gamma_E + \ln 4 - \ln(q^2/\nu^2) \right] + q^2 \left[ -\frac{\nu^2}{2} + \frac{\alpha_s^2}{4} \left( 1 - 4\gamma_E + 2\ln 4 - 2\ln(q^2/\nu^2) \right) \right] + \right\} \]

\[ -\frac{5\alpha_s^4}{6} + \frac{2\alpha_s^6}{3q^2} + \mathcal{O} \left( \frac{1}{q^4} \right) \]

(5)

to be compared with the QCD result [9]:

\[ \Pi_{QCD}(q^2) = 2 \left( \frac{\beta_1}{\pi} \right)^2 \left( \frac{\alpha_s}{\pi} \right)^2 q^4 \ln \left( \frac{\nu^2}{q^2} \right) + 2 - \frac{1}{\epsilon'} + 4\beta_1^2 \left( \frac{\alpha_s}{\pi} \right) \langle C_4 \rangle \]

\[ + 8\beta_1^2 \left( \frac{\alpha_s}{\pi} \right)^2 \frac{\langle C_6 \rangle}{q^2} + \mathcal{O} \left( \frac{1}{q^4} \right) \]

(6)
Matching (5) with (6) \( \left( \frac{R_3}{m} \right)^2 = 2(\beta_1/\pi)^2(\alpha_s/\pi)^2 \) one gets the fully analytic form of the correlator. By casting it in the form:

\[
\Pi_{\text{AdS}}(q^2) = \sum_{n=0}^{\infty} \frac{f_n^2}{q^2 + m_n^2 + i\varepsilon}
\]

it is possible to find the poles \( m_n^2 = 4c^2(n+2) \) and the related residues \( f_n^2 = \langle 0|\mathcal{O}_S(0)|n \rangle^2 = \frac{8R_3^3c^3}{k}(n+1)(n+2) \), corresponding to the mass spectrum and the decay constants of the scalar glueballs. The results for the lowest lying state are:

- \( M_{0^{++}} = 1.089 \text{ GeV} \)
- \( f_{0^{++}} = \langle 0|\mathcal{O}_S(0)|n = 0 \rangle = 0.763 \text{ GeV}^3 \).

The \( 0^{++} \) glueball turns out to be heavier than the \( \rho \) meson, as expected by phenomenology, but slightly lighter with respect to the results from lattice simulations [10].

Another point is that in the large \( q^2 \) expansion there is a dimension two condensate, absent in QCD since there are no ways to construct scalar local gauge invariant quantities with that dimension [11].

Finally, the dimension four condensate \( \langle \mathcal{C}_4 \rangle \) turns out to be negative in this picture, at odds with the commonly used value \( \langle \alpha_s/\pi \rangle G^2 \approx 0.012 \text{ GeV}^4 \) [12].

A discussion on how to fix some of the problems exposed above can be found in [7, 13].

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REFERENCES

[1] J. M. Maldacena, Adv. Theor. Math. Phys. 2, (1998) 231; E. Witten, Adv. Theor. Math. Phys. 2, (1998) 253; S. S. Gubser et al., Phys. Lett. B 428, (1998) 105; E. Witten, Adv. Theor. Math. Phys. 2, (1998) 505.

[2] I. R. Klebanov and E. Witten, Nucl. Phys. B 556 (1999) 89; J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88 (2002) 031601.

[3] See e.g. S. J. Brodsky and G. F. de Teramond, Phys. Rev. D 96, (2006) 201601.

[4] O. Andreev, Phys. Rev. D 73, (2006) 107901; A. Karch et al., Phys. Rev. D 74, (2006) 015005.

[5] U. Gursoy and E. Kiritsis, JHEP 0802 (2008) 032; U. Gursoy et al., JHEP 0802 (2008) 019; J. Erdmenger et al., Eur. Phys. J. A 35 (2008) 81.

[6] P. Colangelo et al., Phys. Lett. B 652 (2007) 73.

[7] P. Colangelo et al., arXiv:0711.4747 [hep-ph].

[8] H. Forkel, arXiv:0711.1179 [hep-ph].

[9] V. A. Novikov et al., Nucl. Phys. B 165 (1980) 67; Nucl. Phys. B 191 (1981) 301; P. Pascual and R. Tarrach, Phys. Lett. B 113 (1982) 495; C. A. Dominguez and N. Paver, Z. Phys. C 31 (1986) 591; S. Narison, Nucl. Phys. B 509 (1998) 312.

[10] C. J. Morningstar and M. J. Peardon, Phys. Rev. D 60 (1999) 034509.

[11] Discussions about the possible existence of a dimension two condensate can be found in: V. I. Zakharov, AIP Conf. Proc. 964 (2007) 143.

[12] P. Colangelo and A. Khodjamirian, in ‘At the Frontier of Particle Physics / Handbook of QCD’, ed. by M. Shifman (World Scientific, Singapore, 2001) vol. 3* 1495-1576, arXiv:hep-ph/0010175.

[13] P. Colangelo, F. De Fazio, F. Gianuzzo, F. Jugeau and S. Nicotri, arXiv:0807.1054 [hep-ph].