Formation of salinity and density vertical stratification of the Black sea

G K Korotaev
FSBIS Marine Hydrophysical Institute Russian Academy of Sciences, 2 Kapitanskaya Str., Sevastopol, 299011, Russian Federation
E-mail: gkorotaev@gmail.com

Abstract. A simple model of the haline and density stratification evolution in the cylindrical rotating basin under the influence of buoyancy fluxes through the side border is proposed in the work based on the dimension analysis. The model is based on the assumption that the sharp pycnocline and two different circulation cells in the vertical plane above and below pycnocline. The model shows that the typical time of vertical circulation cell modification is determined by the inertia of the main body of waters in the basin. Also under fixed ambient conditions, one should expect pycnocline sharpening and increased intensity of vertical circulation cells over time. The model application to describe the climatic changes of the Black Sea haline stratification is discussed. It is asserted that the response of a marine basin to changes in the external conditions depends significantly on the inertia of a main water body. It is noted that this result is of importance not only for the Black Sea, but also for the World Ocean. This effect explains the slow change in the meridional heat transport even in case of considerable variations of deep convection at high latitudes.

1. Introduction
One of the topical problems of Geophysical Fluid Dynamics is a description of meridional heat transport in the atmosphere and ocean. In view of the high temperature difference of upper and deep oceans, circulation cell in the vertical plane makes the dominant contribution to the formation of the meridional heat transport. The theory of the main oceanic thermocline [1] and the scheme of deep-water circulation, which built in [2] give a base for the understanding of the vertical cell structure. The theory [1] states the rise of deep water in the temperate latitudes. Their formation according to [2] is in localized areas in circumpolar regions. It indicates that in the Atlantic cold deep-water moves to the South, while in the Indian and Pacific oceans, it moves to the North. Accordingly, in the upper layers of the Atlantic warm water moves north, and south in the Indian and Pacific oceans. It was noticed in [3] that this circulation cell provides meridional heat transport from the south to the north across the Atlantic, which was subsequently confirmed by integrating the observed heat fluxes at the boundary of ocean and atmosphere [4]. Further study slightly clarified the scheme of heat transport in the oceans due to meridional overturning circulation, which sometimes is named as the ocean conveyor belt.

Another example of the efficiency of the circulating cells in vertical plane is the Black Sea, where it plays a decisive role in shaping haline and density stratification. High-salinity and dense water comes in the Black Sea from the Marmara Sea through the Bosporus Strait. The river runoff provides the freshening of the surface waters of the Black Sea. Additional effects on the distribution of the density of seawater are exerted by precipitation and evaporation on the surface of the sea. Under the influence of
these factors in the basin vertical movements are developing, forming circulation cells in the vertical plane.

Investigation of the vertical circulation cell formation in the Black Sea was conducted in [5, 6, 7] on the basis of laboratory and numerical modelling. The rotating cylindrical basin was suggested there as a conceptual model. The research has found that buoyancy fluxes through the lateral borders lead to formation of pycnocline and two cells of circulation in the radial section of the cylinder. The further development of the above-mentioned research was carried out in [8] considering temporal evolution of haline stratification in the Black Sea. It was shown there that the haline stratification evolves quasi-stationary after an adaptation period, which for the Black Sea has an order of 50-100 years.

The model of a rotating cylindrical basin is used in the present work for the study of the temporal evolution of density stratification under the influence of buoyancy fluxes. It is based on the findings of [8] about quasi-stationary character of this process, as well as features of pycnocline structure and circulation in the radial cross-section of the cylinder, derived in [7]. The first section discusses the system of equations that describes the evolution of density stratification in a rotating cylindrical basin under the influence of buoyancy fluxes on its borders. The second section interprets the pycnocline as an internal boundary layer. The third section provides an analysis of the scaling of core variables within the pycnocline. The closed system of equations describing the slow evolution of density is presented in the section four. The obtained results are used to discuss the process of formation and evolution of haline stratification in the Black Sea in the last section. Also, their significance for understanding of meridional overturning circulation changes under the influence of climatic processes is mentioned.

2. Basic equations of the theory

The use of a rotating cylindrical pool as a conceptual model that reproduces the main features of circulation and stratification of the Black Sea was proposed in [5,6]. The imposition of the radially symmetric boundary conditions reduces the dimension of the problem, which makes it much easier to study.

The first detailed investigation of water circulation in a rotating cylindrical basin based on comparison of the analytical solution of linearized equations with the results of a laboratory experiment was carried out in [9]. This work demonstrated an excellent agreement of observations and theory. A more complete study of the problems was presented in [7], where the laboratory observations were compared with numerical simulations. The important outcome from [7] is the ability to use the following system of equations to simulate the density stratification and circulation in a rotating cylindrical basin, heated and cooled through a side border with parameters that are appropriate to the conditions in the Black Sea basin:

\[ \frac{\partial u}{\partial t} - fv = -\frac{\partial p}{\partial r} + A_x \frac{\partial^2 u}{\partial z^2} + A \left( r \frac{1}{\partial r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) \]  \hspace{1cm} (2.1)

\[ \frac{\partial v}{\partial t} + fu = A_x \frac{\partial^2 v}{\partial z^2} + A \left( r \frac{1}{\partial r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) \]  \hspace{1cm} (2.2)

\[ -g\rho = -\frac{\partial p}{\partial z} \]  \hspace{1cm} (2.3)

\[ \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0 \]  \hspace{1cm} (2.4)

\[ \frac{\partial \rho}{\partial r} + u \frac{\partial \rho}{\partial r} + w \frac{\partial \rho}{\partial z} = \kappa_x \frac{\partial^2 \rho}{\partial z^2} + \kappa_t \frac{1}{r} \frac{\partial \rho}{\partial r} \]  \hspace{1cm} (2.5)

Equation (2.1) to (2.5) are written in cylindrical coordinates by assuming axial symmetry. The radial coordinate is designated as \( r \), the vertical coordinate \( z \) is directed downward, and \( t \) is the time. The following notation is adopted in equations (2.1) to (2.5): \( u, v, w \) are the radial azimuth and vertical components of current velocity, \( P \) is the pressure, normalized to the reference density of seawater, and \( \rho \) is the density of sea water, depending linearly on its temperature or salinity and normalized to the reference density. The coefficients of turbulent exchange of momentum and viscosity are introduced in
equations (2.1), (2.2) and (2.5). Typically, in oceanography different vertical and horizontal turbulent exchange coefficients $A_z$ and $A$ are introduced due to the large difference between horizontal and vertical scale. Similarly entered coefficients of vertical and horizontal turbulent diffusion $\kappa$ and $\kappa_1$. Horizontal turbulent exchange and diffusion coefficients at that substantially exceed vertical ones. However, in the works [6, 8] they were equal because laminar motion modeled in laboratory experiment.

The system of equations solved with the following boundary conditions:

$$\frac{\partial u}{\partial r} = \frac{\partial v}{\partial r} = 0,$$  
$$u = v = w = 0, \quad \text{at } z = 0$$  
$$u = v = w = 0, \quad \text{at } r = 0$$  
$$u = v = 0, \quad \rho = \rho_0(z) \quad \text{at } r = R$$  

where $H_c$ is the height of the cylinder, $R$ is its radius. Function $\rho_0(z)$, as well as in laboratory experiment was piecewise with equal values, but different sings in the top and bottom halves of the pool.

Specific of the equations (2.1) to (2.5) consists in the use of the linearized momentum equations balance in the radial and azimuthal directions and hydrostatic approximation. However, unlike [9] in equation (2.5) saved nonlinear terms. The system of equations (2.1) to (2.5) is similar to the one used in the oceanic thermocline theory. As it was mentioned above, its numerical solution with presented above boundary conditions well reproduced the results of laboratory observations. Because of that, we will build on it our further considerations.

3. Pycnocline as an internal boundary layer

Observations obtained in laboratory experiments and numerical simulations presented in [7] characterize density and current velocity fields, formed under the influence of heating and cooling on the border of the pool only when it reaches a steady state. The density of water at this stage changes strongly vertically near the middle of the pools, at the level of the transition from heating to cooling. The vertical density gradient outside this relatively thin layer is insignificant. Thus, schematically the basin consists of two layers almost homogeneous fluids separated by sharp pycnocline. The scale of the density changes within pycnocline is significantly less than the height of the vertical cylinder that allows consider the pycnocline as an internal boundary layer.

Numerical simulations presented in [7], show that two cells of circulation are formed at the upper and lower halves of the basin. Water circulates counter-clockwise in the upper half of the pool and clockwise in the lower one. The most intense movement concentrated in the pycnocline and near the border of the basin. Thus, the analysis of the density field and current velocity shows that two boundary layers are formed in the basin. One is near the border, where the density of water changes slightly, and radial velocity is reduced to zero. Another one is internal and associated with large vertical gradients of the density. We assume that both boundary layers layer will remain during the quasi-stationary evolution of the density and current velocity fields explained at [8].

Assumption about quasi-stationary evolution and the existence of two boundary layers allows one to write a simplified system of equations, describing the distribution of density and current velocity fields within pycnocline away from the boundaries of the basin. Conditions of quasi-stationarity permits omit the time derivatives in equations (2.1), (2.2) and (2.5). Terms with vertical turbulent exchange in equations (2.1) and (2.2) may also be omitted, since for typical conditions they are significant only immediately near the top and bottom boundaries of the basin. Terms responsible for horizontal turbulent exchange and turbulent diffusion in equations (2.1) and (2.5) away from the sidewall of the basin are also insufficient. As a result, within the pycnocline and outside the boundary layer near the wall equations (2.1) to (2.5) are transformed as follow:

$$-fv = \frac{\partial \rho}{\partial r},$$  
$$fu = A \left( r \frac{\partial}{\partial r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right),$$  
$$-g\rho = \frac{\partial \rho}{\partial z}.$$  

3
The equations (3.1)-(3.5) should be solved within unrestricted vertically area to either side of the pycnocline center. The density of water should tends to some constant values at an infinite distance from the mid of pycnocline.

4. Deduction of the equation of a temporary evolution of basin stratification

Let us transform now equations (3.1) - (3.5) to dimensionless variables. Choose radius of the basin $R$ as a scale of radial variable. Vertical scale within pycnocline let us denote as $H$. Then according to the equation (3.5) find the scale of the radial and vertical components of current velocity:

$$u_{sc} = \frac{kR}{H^2} u_m$$

$$w_{sc} = \frac{k}{gH^4}$$

After that using equations (3.2), (3.1) and (3.3) we define consequently

$$v_{sc} = f \frac{kR^3}{A H^2} P_{sc} = \frac{f^2 k R^4}{A g H^4}$$

Let introduce also the scaling of a buoyancy flux

$$\Gamma_{sc} = \frac{k^2 \kappa^2 v}{H^4}$$

Note that the vertical scale within pycnocline is not defined yet. However, we can fix the scale for the density of water, associating it with density difference between fluids layers above and below the pycnocline. Denote the difference of density as $\Delta \rho$ and let $\rho_{sc} = \Delta \rho$. Now let express the vertical scale within the pycnocline through the density difference:

$$H = \frac{\frac{1}{2} \frac{2}{3} \frac{4}{3}}{g^2 \Delta \rho}$$

Accordingly, the buoyancy flux scale has following expression

$$\Gamma_{sc} = \frac{\frac{1}{2} \frac{2}{3} \frac{4}{3} \Delta \rho}{f^3}$$

Thus, buoyancy flux is expressed through the dimensionless function $\Gamma(r,z)$, depending on the dimensionless coordinates $r = \frac{r}{R}$ and $z = \frac{z}{H}$ where $H_p$ is the depth of pycnocline.

$$\Gamma (r,z) = \frac{1}{f^2} \frac{2}{3} \frac{2}{3} \frac{4}{3} \frac{\Delta \rho}{R^2}$$

Next, calculate the buoyancy flux through the entire surface of the pycnocline. Integrating expression (4.7) at the depth of the pycnocline over the cylinder cross-section, we find

$$\Gamma_p = 2 \pi \int_0^R r \Gamma (r,0) dr = \text{const} \frac{1}{f^2} \frac{2}{3} \frac{2}{3} \frac{4}{3}$$

Thus, the total diffusive buoyancy flux is proportional to the density differential between the layer above and below the pycnocline in degree 4/3. Dimensionless constant of proportionality in the formula (4.7) certainly is not possible to find within the proposed approach.

The utility of expression (4.8) consists in possibility apply it to the building of a closed approximate system of equations describing the evolution of stratification in a rotating cylindrical basin. Denote the average density of the water in the layer below pycnocline as $\rho_+$, and $\rho_-$ in the layer above it. Note that if the pycnocline and boundary layer near the wall thin enough then with high precision $\Delta \rho = \rho_+ - \rho_-$. Let also $\Gamma_+$ is net buoyancy flux through part of the lateral border of the cylinder below pycnocline, and $\Gamma_-$ is similar buoyancy flux through the remainder of the sidewall located above it (value of fluxes is assumed to be positive). Suppose also that part of the volume cylinder below pycnocline is denoted as $V_+$ and $V_-$ is the remainder of the volume (of course, $V_+ + V_- = \pi R^2 H_p$). Then

$$V_+ \frac{d \rho_+}{dt} = \Gamma_+ - \Gamma_p$$

(4.8)
If fluxes on the border of the basin are known functions of time, then the system of equations (4.8), (4.9) is closed and can be integrated. Simple case, when fluxes on the border of the basin are constants, gives an opportunity to receive qualitative insight into the behavior of the density difference across the pycnocline through time. Indeed, combining equations (4.8) and (4.9) get in this case

\[
\frac{d\Delta \rho}{dt} = \frac{\Gamma_+}{V_+} + \frac{\Gamma_-}{V_-} - \left(\frac{1}{V_+} + \frac{1}{V_-}\right) \Gamma_p
\]  

(4.10)

Bearing in mind the relationship (4.7), we see that equation (4.10) contains a single variable. Do not dwell on finding an exact solution of the equation (4.10). Note only that for any small initial value \(\Delta \rho\) the difference of density across the pycnocline monotonically increases over time until the limit value at which the right-hand side of equation (4.10) becomes zero.

The system of equations (4.8), (4.9) will also be closed, if one knows that

\[
\Gamma_+ = \lambda_+ (\rho_+^b - \rho_+) \\
\Gamma_- = \lambda_- (\rho_- - \rho_-^b)
\]  

(4.11)  

(4.12)

where \(\rho_+^b\) and \(\rho_-^b\) are known constants or functions of time. The last case corresponds to the conditions of forming haline stratification in the Black Sea under the influence of river runoff, precipitation, evaporation and water exchange through the Bosporus Strait.

5. Discussion

Obtained above results provide a basis for qualitative descriptions of evolution haline stratification of the Black Sea on the climatic scales. The Black Sea according to hypothesis presented at [10] was a freshwater lake, isolated from the ocean until about 7500 years ago. Then, the Bosporus Strait opened up due to tectonic processes and the Black Sea basin began to receive salty oceanic water. It is convenient to carry out the study of climatic variability haline stratification and Black Sea circulation in the framework of this hypothesis. Indeed, in accordance with the simulations [8] in this case, after the harmonization of the Black and Marmara sea levels in the Black Sea basin should be installed quasi-stationary state with well-presented haloklin. Then, salinity difference across haloklin will be monotonically increasing up to the equilibrium (see discussion of the equation (4.10) in previous section), Since salinity difference increase in time together with the total salt content in the basin, model (4.8) and (4.9) explains the results of the simulations presented in [8]. Moreover, the model (4.8) and (4.9) can give absolute timing haline stratification of the Black Sea basin.

The model also gives a qualitative description of the stratification and circulation changes over time. The thickness of the halocline \(H\) according (4.5) is proportional to the density differential between the layer above and below the pycnocline in degree \(-1/3\. Thus, the haloklin should be rather broad at an early stage of the climatic evolution and sharpening in time. Characteristic scale of vertical velocity is inversely proportional to \(H\) and therefore it increases over time. This means that circulation cell in the vertical plane is enhanced with sharpening stratification.

Let discuss also the characteristic time scale of liquid particles rotation within haloklin:

\[
T = \frac{R}{u_{sc}} = \frac{H}{w_{sc}} = \frac{\mu^2}{\kappa} = \frac{f^2 \Delta \rho^b}{g^2 A \lambda \kappa \Delta \rho^b}
\]  

(5.1)

Choosing parameters relevant to generally accepted for the Black Sea: \(g \Delta \rho = 3 \text{ cm/s}^2\), \(A = 5 \cdot 10^7 \text{ cm}^2\text{s}^{-1}\), \(f = 10^{-4} \text{s}^{-1}\), \(\kappa = 0.03 \text{ cm}^2\text{s}^{-1}\). Radius of the cylinder \(R = 250 \text{ km}\) is selected in such a way that its cross-section area corresponds to the area of the Black Sea deep part. Then find \(H = 90 \text{ m}\) and \(T = 85 \text{ years}\).

The resulting score scale slightly higher, but generally close to the thickness of the haloklin obtained in [8]. Evaluation of the characteristic time in 85 years turns out to be consistent with the adjustment time for quasi-steady mode, also priced for the Black Sea in [8]. In this regard, it is possible to identify the adjustment time introduced in [8] with the time of haloklin adjustment to the salinity difference across of it. Based on this assumption it is possible to describe a response of the Black Sea basin on the drastic changes of external conditions. Let, for example, salinity changes rapidly in the upper layer of
the sea. Then one can expect to see at first adjustments of haloklin for several decades and after that a slow evolution of stratification following changing salt and fresh water fluxes at the borders of the basin.

However, despite the progress in understanding the evolution of the Black Sea haline stratification, the proposed model may not qualify for high accuracy. The main problem is that the upper layer of the sea is affected by wind and heat exchange with the atmosphere. Further research is needed to understand how these processes will transform the upper circulation cell in the vertical plane.

Note also that these patterns described by this article can be useful for understanding the reaction of meridional overturning circulation to changes in climatic conditions. The presented above analysis indicates that the inertia of the heat transport in the temperate latitudes is largely determined by the difference in temperature of surface and deep-ocean. Even significant variation of conditions in the areas of deep waters formation do not lead to changes in the intensity of upwelling on the bottom of the oceanic thermocline if the temperature difference between the surface and deep layers of the ocean does not change. Then the thermohaline circulation with rise deep waters near the equator and sink at higher latitudes will remain at least partially. Time of its transformation will depend on the speed of changes the surface and abyssal water masses properties.

Acknowledgements
This work was fulfilled in FSBIS Marine Hydrophysical Institute Russian Academy of Sciences with the support of the Russian Science Foundation, grant No. 17-77-30001

References
[1] Robinson A R and Stommel H 1959 Tellus 11 (3) 295–308
[2] Stommel H and Arons A B 1960 Deep-Sea Res. 6 217–33
[3] Stommel H 1980 Proc/ Natl. Acad. Sci. USA 77 (5) 2377–81
[4] Hastenrath S 1982 J. Phys/ Oceanogr. 12 (2) 368–76
[5] Bulgakov S N, Korotaev G K and Whitehead J A 1996 Izvestia – Atmospheric and oceanic physics 32 (4) 548–56
[6] Bulgakov S N, Korotaev G K and Whitehead J A 1996 Izvestia – Atmospheric and oceanic physics 32 (4) 557–64
[7] Whitehead J A, Korotaev G K and Bulgakov S N 1998 Geophys. Astrophys. Fluid Dynamics 89 169–203
[8] Senderov M V and Mizyuk A I 2017 Sevastopol: Ecologicheskaya bezopasnost pribrejnoj i shelfovoj zon morya 2 82–9
[9] Pedlosky J, Whitehead J A and Veitch G 1997 Fluid Mech. 339 391–411
[10] Ryan W B and Pitman W C 1999 Noah’s Flood: The New Scientific Discoveries About the Event That Changed History (New York: Simon and Schuster) p 319