Emergent universe with interacting fluids and the generalized second law of thermodynamics

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Abstract

We investigate the emergent Universe (EU) scenario in the presence of interacting fluids. The nonlinear equation of state (EoS) considered in the general theory of relativity for obtaining the EU is effectively a cosmological model with a composition of three fluids. In this paper, we consider two models to realize viable cosmological scenarios: (i) a two-fluid model with the interaction of a pressureless fluid, with the fluid having the nonlinear EoS needed for the EU, and (ii) a three-fluid model with interaction among the three fluids, which originate from the EoS of the EU. It is found that realistic cosmological models in accordance with observations are not ruled out for both of these cases. We further show that the generalized second law of thermodynamics is found to hold up well in the EU with interacting fluids.

Keywords: emergent Universe, interacting fluid, cosmological model without singularity

(Some figures may appear in colour only in the online journal)

1. Introduction

Astronomical and cosmological observations predict that we live in an expanding Universe. After the discovery of cosmic microwave background radiation [1], Big Bang cosmology became the standard model for cosmology, which presents the beginning of the Universe at some finite past. It is found that Big Bang cosmology does face some problems in
addressing issues of the observed Universe both in the early and late Universe. A number of problems such as the horizon problem, flatness problem, and singularity problem etc crop up when one probes the early Universe in the framework of the Big Bang model. These can, however, be resolved by evoking a phase of inflation [2–9] at a very early epoch. Furthermore, it is known that the large-scale structure formation of the Universe can be successfully addressed in this scenario. On the other hand, very recent observations predict that our Universe is passing through a phase of acceleration [10–12]. This phase of acceleration is believed to be a late time phase of the Universe and may be accommodated in the standard model by adding a positive cosmological constant to Einstein’s field equation.

In spite of its overwhelming success, modern Big Bang cosmology still has some unresolved issues. The physics of the inflation and the introduction of a small cosmological constant for late time acceleration are not completely understood [13, 14]. Moreover, various competing models exist that are not yet fully distinguished empirically from the currently available observational data. This is what motivates the search for an alternative cosmological model. In this context, Ellis and Maartens [15] considered the possibility of a cosmological model [16] in which there is no Big Bang singularity, no beginning of time, and the Universe effectively gets rid of a quantum regime for space-time by staying large at all times. The Universe started out in the infinite past in an almost static Einstein Universe, and subsequently it slowly entered into an expanding phase, eventually evolving into a hot Big Bang era. Later Ellis, Murugan, and Tsagas [17] constructed an emergent Universe (EU) scenario for a closed Universe with a minimally coupled scalar field ϕ, which has a special form for the interaction potential, V(ϕ). It was pointed out later [18], where the potential is similar to what one obtains from a modified gravitational action with a polynomial Lagrangian, \( L = R + \alpha R^2 \), after a suitable conformal transformation and identifying the field as \( \phi = -\sqrt{3} \ln(1 + 2\alpha R) \) with a negative \( \alpha \).

The EU scenario merits attention, as it promises to solve several conceptual and technical issues of the Big Bang model. A notable direction pertains to the cosmological constant problem [19]. Mukherjee et al [20] obtained an EU in the framework of the general theory of relativity in a flat Universe with a nonlinear equation of state (EoS) of the form

\[
p = A\rho - B\sqrt{\rho}
\]

where \( A \) and \( B \) are arbitrary constants. A spatially flat Universe is most likely as predicted from recent cosmological and astronomical observations. Such an EU scenario can also be realized in a modified theory of gravitation [21] by including a Gauss–Bonnet term in the presence of dilaton coupling [22], braneworld gravity [28, 29, 25], Brans–Dicke theory [26], the nonlinear sigma model [27], chiral cosmological fields in Einstein Gauss–Bonnet gravity [28], dark sector fields in a chiral model [29], and exact global phantom solution [30]. The EU accommodates a late time de-Sitter phase, and thus it naturally leads to the late time acceleration of the Universe as well. Such a scenario is promising from the perspective of offering unified early and late time dynamics of the Universe. Note, however, that the focal point of unification in such EU models lies in the choice of the EoS for the polytropic fluid, while several other models of unification rely more on the scalar field dynamics through the choice of field potentials [31–33].

The emergent model proposed by Mukherjee et al [20], in which a polytropic EoS is used, gives rise to a Universe with a composition of three different types of fluids determined by parameters \( A \) and \( B \). In the original EU model proposed by Mukherjee et al [20], it was assumed that noninteracting fluids and each of the three types of fluids identified satisfy conservation equations separately. Recently, using the observational prediction of WMAP7...
Planck2013, the permitted range of values of parameters $A$ and $B$ were determined. Recent cosmological observations from Planck2013 impose tight bounds on the EoS parameters in an EU. For a viable cosmological scenario, it is further necessary to consider a consistent model of the Universe that contains radiation-dominated, matter-dominated, and subsequently late accelerated phases of the Universe. The original EU model showed that the composition of the Universe is fixed once $A$ is fixed. A problem thus arises as to how a pressureless matter component could be accommodated within such a scenario. However, allowing interaction among the constituent fluids of the EU may open up richer physical consequences.

Interacting fluid cosmological models have previously been considered in the literature, and among the many reasons and motivations for such models, analyses of interactions within the dark sector are quite popular. In the present context, interaction among the constituent fluids is useful for obtaining a consistent evolutionary scenario of the Universe. Another important consistency condition is imposed through the thermodynamics of an expanding Universe. There has been significant recent interest in the study of the connection between thermodynamics and gravitational dynamics in the presence of horizons. Another aspect of the Hawking temperature of the apparent horizon in a Friedmann–Robertson–Walker (FRW) Universe was determined by Cai et al. Indeed, Einstein’s equations have been interpreted as a thermodynamical relation resulting from the displacement of the horizon. Since the EU scenario entails a phase of accelerated expansion, it is relevant to study the status of the second law of thermodynamics in the context of interacting fluids in the EU.

With the above motivations, in the present paper we consider two different cases in the EU scenario: (i) a two-fluid model, with interaction of the fluid having the nonlinear EoS given by equation (1) with another barotropic fluid, beginning at some time $t = t_i$ (Model I), and (ii) a three-fluid model with interaction among the various constituent fluids with different individual EoSs, starting at a time $t = t_o$ (Model II). The paper is organized as follows: in section 2, we set up the field equation for the EU in the general theory of relativity, and we obtain cosmological solutions. In section 3, we consider the above two cases, Model I and Model II. We show that the generation of pressureless matter fluids is possible within the EU scenario. In section 4, we demonstrate the consistency of the EU scenario with interacting fluids with the generalized second law of thermodynamics. Finally, we make some concluding remarks in section 5.

2. Field equation and cosmological solution

The Einstein field equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi GT_{\mu\nu}$$

where $R_{\mu\nu}$, $R$, $g_{\mu\nu}$, $T_{\mu\nu}$, and $G$ represent the Ricci tensor, Ricci scalar, metric tensor, matter–energy tensor, and Newton’s gravitational constant, respectively. Here we consider four dimensions for which $\mu$, $\nu$ runs from $(0, 1, 2, 3)$.

We consider a flat Robertson–Walker metric, which is given by

$$ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]$$

where $a(t)$ represents the scale factor of the Universe. We consider the energy momentum tensor as $T_{\nu}^{\mu} = \text{diagonal}(\rho, -p, -p, -p)$, where $\rho$ is the energy density and $p$ is the pressure.
Using the flat Robertson–Walker metric given by equation (3) in Einstein’s field equation, one obtains

\[ \rho = 3 \left( \frac{\dot{a}}{a} \right)^2, \quad (4) \]

\[ p = - \left[ 2\frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right], \quad (5) \]

where we set \( G = \frac{1}{8\pi} \) and \( c = 1 \). The conservation equation is given by

\[ \frac{d\rho}{dt} + 3H (p + \rho) = 0 \quad (6) \]

where \( H = \frac{\dot{a}}{a} \) represents the Hubble parameter. As mentioned earlier, Mukherjee et al [20] obtained an EU scenario with a polytropic EoS given by

\[ p = A\rho - B\rho^{1/2} \quad (7) \]

where \( A \) and \( B \) are arbitrary constants. Making use of the conservation equation and the EoS given by equations (4) and (5), in equation (7), one obtains a second-order differential equation given by

\[ 2\frac{\ddot{a}}{a} + (3A + 1) \left( \frac{\dot{a}}{a} \right)^2 - \sqrt{3}B\frac{\ddot{a}}{a} = 0. \quad (8) \]

The scale factor of the Universe is thus obtained by integrating equation (8), which is given by

\[ a(t) = \left[ \frac{3K(A + 1)}{2} \left( \sigma + \frac{2}{\sqrt{3}B} e^{\frac{\sigma t}{B}} \right) \right]^{\frac{2}{1 - 3}} \quad (9) \]

where \( K \) and \( \sigma \) are the two integration constants. It is interesting to note that \( B < 0 \) leads to a contracting Universe whereas \( B > 0 \) and \( A > -1 \) leads to a nonsingular solution that is expanding. The latter solution corresponds to an EU that was obtained by Mukherjee et al [20]. The energy density of the Universe in terms of the scale factor is obtained from equation (6), making use of the EoS given by equation (7), which is given by

\[ \rho(a) = \frac{1}{(A + 1)^2} \left( B + \frac{K}{a^{3(A+1)}} \right)^2. \quad (10) \]

Expanding the above expression, one obtains energy density as the sum of three terms that can be identified with three different types of fluids. Thus, the components of energy density and pressure can be expressed as follows:

\[ \rho(a) = \sum_{i=1}^{3} \rho_i \quad \text{and} \quad p(a) = \sum_{i=1}^{3} p_i \quad (11) \]

where we denote

\[ \rho_1 = \frac{B^2}{(A + 1)^2}, \quad \rho_2 = \frac{2KB}{(A + 1)^2 a^{3(A+1)}}, \quad \rho_3 = \frac{K^2}{(A + 1)^2 a^{3(A+1)}} \quad (12) \]
Comparing with the barotropic EoS given by $p_i = \omega_i \rho_i$, one obtains $\omega_1 = -1, \omega_2 = \frac{A-1}{2}$ and $\omega_3 = A$. Thus, parameter $A$ plays an important role in determining the composition of the fluids in the Universe. For example, $A = \frac{1}{3}$ leads to a Universe with radiation, exotic matter, and dark energy; $A = 0$ leads to dark energy, exotic matter, and dust. Thus, once the EoS parameter $A$ is fixed, the composition of the fluid in the Universe is determined. To obtain a viable scenario of the Universe, we consider the interacting fluids model in the next section, so that a transformation of one kind of fluid in the later epoch gives rise to the composition of matter that we observe today.

### 3. Cosmological models

In this section we consider two different models of interacting fluids in an EU scenario.

**Model I: the two-fluids model**

In this case, we consider two interacting fluids with densities $\rho$ and $\rho'$, respectively, which can exchange energies with each other. One of the fluids with energy density $\rho$ is dominated to begin with, satisfying a nonlinear EoS given by equation (1), which leads to an EU model with no interaction, as discussed above. The contribution of the other fluid in the energy density of the Universe is assumed to be important at a later epoch. The corresponding pressure of the former fluid is given by

$$p = \frac{B^2}{(A + 1)^2} \quad p_2 = \frac{KB(A - 1)}{(A + 1)^2} \frac{1}{a^{3(A+3)}},$$

$$p_3 = \frac{AK^2}{(A + 1)^2} a^{3(A+1)}.$$  \hspace{1cm} (13)

In this case, we consider a cosmological model where exchange of energy between two different fluids is allowed. There are many astrophysical and cosmological motivations for considering energy exchanges between the various components of the Universe [39–44]. Different phenomenological considerations dictate the onset of such interactions. For example, in various scalar field models of dark energy, such as in quintessence or k-essence, phase transitions arise during particular eras, resulting in both the decay of the cosmological vacuum energy, and in particle production. Similarly, there could be other cases of energy exchange—for example, due to the evolution of a population of primordial black holes whose evaporation time depends on a particular formation mechanism or formation era. In this paper, without assuming any specific mechanism for energy exchange, we assume that the interaction starts at some particular time, $t_i$. The two interacting fluids respect a total energy conservation equation, and their densities evolve with time as
where $\alpha$ represents a coupling parametrizing the energy exchange between the fluids. One may view the above interaction as a flow of energy from the first kind of fluid to the second kind (say, dark matter), beginning at the epoch considered here. Now, using equation (14) in equation (17), we get a first-order differential equation that can be integrated to obtain the behavior of energy density in terms of the scale factor of the Universe and the interaction coupling factor. Thus, the energy density and pressure for the first kind of fluid are given by

$$
\rho = \frac{B^2}{(A + 1 + \frac{a}{2})^2} + \frac{2KB}{(A + 1 + \frac{a}{2})^2 \frac{1}{a^{\frac{2}{3}(A+1)}}} + \frac{K^2}{(A + 1 + \frac{a}{2})^2 a^{3(A+1)}}.
$$

$$
p = -\frac{B^2}{(A + 1 + \frac{a}{2})^2} + \frac{KB}{(A + 1 + \frac{a}{2})^2} \frac{1}{a^{\frac{3}{2}(A+1)}} + \frac{(A + \frac{a}{2})K^2}{(A + 1 + \frac{a}{2})^2 a^{3(A+1)}}.
$$

If the interaction is with a pressureless dark fluid (i.e., $p' = 0$ (however, $\rho' \neq 0$)), equation (18) can be integrated using equations (16) and (19), which determine the total energy density and pressure as follows:

$$
\rho_{\text{total}} = \rho + \rho' = \frac{B^2}{A + 1} + \frac{2KB}{(A + 1)^2} \frac{1}{a^{\frac{3}{2}(A+1)}} + \frac{K^2}{(A + 1)^2 a^{3(A+1)}}.
$$

$$
p_{\text{total}} = p = -\frac{B^2}{A + 1} + \frac{KB(A - 1)}{(A + 1)^2} \frac{1}{a^{\frac{3}{2}(A+1)}} + \frac{AK^2}{(A + 1)^2 a^{3(A+1)}}.
$$

The EoS parameter for the second fluid is given by

$$
\omega' = \frac{p'}{\rho'} = \frac{p_{\text{total}} - p}{\rho_{\text{total}} - \rho}.
$$

In the limiting case as $\omega' \rightarrow 0$, we get $p_{\text{total}} = p$. An interesting case emerges when the coupling parameter $\alpha = 2$ and $A = \frac{1}{2}$. In this case, a Universe that begins with dark energy, exotic matter, and radiation (i.e., before the interaction sets in) made a transition to a matter-dominated phase. Thus, an EU that begins with radiation, dark energy (a component that is subdominant at early times), and exotic matter (for $A = \frac{1}{2}$) transits to a Universe with a matter-domination phase after an epoch after $t > t_i$ for the interaction coupling strength $\alpha = 2$. Hence, a consistent scenario of the observed Universe in the EU model may be realized in this case.

**Model II: the three-fluids model**

The original EU model was obtained in the presence of noninteracting fluids permitted by parameter $A$ in a flat Universe case. The corresponding densities and pressures are given by equations (12) and (13), respectively. For non-interacting fluids, the EoS parameters for the three fluids permitted above are given by $\omega_1 = -1$, $\omega_2 = \frac{1}{2}(A - 1)$, and $\omega_3 = A$. For $0 \leq A \leq 1$, it accommodates dark energy, exotic matter, and the usual barotropic fluid. The energy density and pressure of the exotic matter, and that of the barotropic fluids decreases with the expansion of the Universe. However, the rate of decrease is different, which is evident from equations (12) and (13). We assume an interaction among the components of the
fluid in the Universe, which is assumed to originate at a later epoch (such interactions could arise due to a variety of mechanisms [39–44]). Assuming an onset of interaction among the composition of the fluid at $t \geq t_o$, the conservation equations for the energy densities of the fluids now can be written as

$$\rho_1 + 3H\left(\rho_1 + p_1\right) = -Q', \quad (24)$$

$$\rho_2 + 3H\left(\rho_2 + p_2\right) = Q, \quad (25)$$

$$\rho_3 + 3H\left(\rho_3 + p_3\right) = Q' - Q, \quad (26)$$

where $Q$ and $Q'$ represent the interaction terms, which can have arbitrary form, $\rho_1$ represents the dark energy density, $\rho_2$ represents exotic matter, and $\rho_3$ represents normal matter. In this case, $Q < 0$ corresponds to energy transfer from the exotic matter sector to two other constituents, $Q' > 0$ corresponds to energy transfer from the dark energy sector to the other two fluids, and $Q' < Q$ corresponds to energy loss for the normal matter sector. The case $Q = Q'$ corresponds to the limiting case where dark energy interacts only with the exotic matter. It is important to see that although the three equations are different, the total energy of the fluids satisfies the conservation equation together. It is possible to construct the equivalent effective uncoupled model, described by the following conservation equations:

$$\dot{\rho}_1 + 3H\left(1 + \omega_1^{\text{eff}}\right)\rho_1 = 0 \quad (27)$$

$$\dot{\rho}_2 + 3H\left(1 + \omega_2^{\text{eff}}\right)\rho_2 = 0 \quad (28)$$

$$\dot{\rho}_3 + 3H\left(1 + \omega_3^{\text{eff}}\right)\rho_3 = 0 \quad (29)$$

where the effective EoS parameters are given below:

$$\omega_1^{\text{eff}} = \omega_1 + \frac{Q'}{3H\rho_1}, \quad (30)$$

$$\omega_2^{\text{eff}} = \omega_2 - \frac{Q'}{3H\rho_2}, \quad (31)$$

$$\omega_3^{\text{eff}} = \omega_3 + \frac{Q - Q'}{3H\rho_3}. \quad (32)$$

Now, if we consider the interaction as $Q - Q' = -\beta H\rho_3$, the effective state parameter for the normal fluid becomes

$$\omega_3^{\text{eff}} = \omega_3 - \frac{\beta}{3}. \quad (33)$$

In figure 1, we plot the variation of effective EoS parameter $\omega_3^{\text{eff}}$ with $\omega_3$ (which corresponds to $A$ of the EoS parameter) for different strengths of interaction determined by $\beta$. We note that as the strength of interaction is increased, the value of $\omega_3$ (i.e., $A$) for which $\omega_3^{\text{eff}} = 0$ (corresponds to matter-domination) is found to increase. Thus a Universe with any $A$ value is found to admit a matter-dominated phase at a late epoch, depending on the strength of the interaction that was not permitted in the absence of interaction in the EU model proposed by Mukherjee et al [20]. Note that in the very early era, a Universe is assumed to have a composition of three different fluids with no interaction in this picture; thus, the behavior of the Universe at early times remains unchanged, as was found in the original EU model.
Therefore, the EU scenario proposed by Mukherjee et al [20] can be realized in the early era, but at a later epoch the composition of matter changes in the present scenario from its original composition with the onset of interaction. This feature represents a clear improvement over the earlier cosmological scenario in an EU [20–22], where it is rather difficult to accommodate a pressureless fluid.

4. Generalized second law of thermodynamics in the EU model

In the above analysis, EU models have been discussed in the presence of different types of interaction among the fluids. In this present section, we investigate the consistency of thermodynamic properties, considering the Universe as a thermodynamical system. For a flat geometry, the apparent horizon coincides with the Hubble horizon, which is given by

\[ r_A = \frac{1}{H}. \]  

(34)

In general, the apparent horizon is a function of time. Thus, a change in the apparent horizon leads to a change in volume, and consequently, the energy and entropy will change by \( dE \) and \( dS \), respectively. The energy momentum tensors before and after the change are described by the same \( \mu\nu \) \( T \), and we can consider that the pressure and the temperature remain the same [45–49]. The first law of thermodynamics for the fluids considered here are given by

\[
\begin{align*}
&dS_1 = \frac{1}{T} (p_1 dV + dE_1) \\
&dS_2 = \frac{1}{T} (p_2 dV + dE_2) \\
&dS_3 = \frac{1}{T} (p_3 dV + dE_3)
\end{align*}
\]

(35)

where the volume of the system, \( V = \frac{4\omega^3}{3} \), is bounded by the apparent horizon, and thus \( dV = 4\pi r_A^2 dr_A \). Hence, the rates of change of entropy for the above fluids are given by
\[ S_1 = \frac{1}{T} \left( 4\pi r_A^2 \dot{r}_A \rho_1 + E_1 \right), \quad S_2 = \frac{1}{T} \left( 4\pi r_A^2 \dot{r}_A \rho_2 + E_2 \right), \quad S_3 = \frac{1}{T} \left( 4\pi r_A^2 \dot{r}_A \rho_3 + E_3 \right) \]  \tag{36}

where

\[ \dot{r}_A = \frac{1}{2} r_A^2 \left( (1 + \omega_1) \rho_1 + (1 + \omega_2) \rho_2 + (1 + \omega_3) \rho_3 \right), \]  \tag{37}

which is obtained by differentiating the equation

\[ \frac{1}{r_A} = \frac{1}{3} (\rho_1 + \rho_2 + \rho_3) \]  \tag{38}

and then making use of the conservation equations given by equations (27)–(29) with equation (34). The corresponding energy density and pressure are given by

\[ E_1 = \frac{4\pi}{3} r_A^3 \rho_1, \quad E_2 = \frac{4\pi}{3} r_A^3 \rho_2, \quad E_3 = \frac{4\pi}{3} r_A^3 \rho_3 \]  \tag{39}

and

\[ p_1 = \omega_1^{\text{eff}} \rho_1, \quad p_2 = \omega_2^{\text{eff}} \rho_2, \quad p_3 = \omega_3^{\text{eff}} \rho_3. \]  \tag{40}

Using the time derivative of the apparent horizon, we get

\[ \dot{S}_1 = \frac{4\pi r_A^2}{T} \rho_1 (\dot{r}_A - H r_A) \left( 1 + \omega_1^{\text{eff}} \right), \]
\[ \dot{S}_2 = \frac{4\pi r_A^2}{T} \rho_2 (\dot{r}_A - H r_A) \left( 1 + \omega_2^{\text{eff}} \right), \]
\[ \dot{S}_3 = \frac{4\pi r_A^2}{T} \rho_3 (\dot{r}_A - H r_A) \left( 1 + \omega_3^{\text{eff}} \right). \]  \tag{41}

According to the generalization of black hole thermodynamics [48–50] to a cosmological framework, the temperature of the horizon is related to its radius [51–53] as

\[ T_p = \frac{1}{2\pi r_A}, \]  \tag{42}

leading to the rate of change of entropy given by

\[ \dot{S}_A = 16\pi^2 r_A^2 r_A. \]  \tag{43}

Note that the Hawking radiation of the apparent horizon in a FRW Universe was computed by Cai et al [54]. The rate of change of total entropy becomes \( \dot{S}_{\text{total}} = \dot{S}_1 + \dot{S}_2 + \dot{S}_3 + \dot{S}_A \), which leads to

\[ \dot{S}_{\text{total}} = 4\pi^2 r_A^2 H \left[ (1 + \omega_1) \rho_1 + (1 + \omega_2) \rho_2 + (1 + \omega_3) \rho_3 \right]^2 \geq 0. \]  \tag{44}

The non-negativity of the time rate of change of \( S_{\text{total}} \) demonstrates the validity of the second law of thermodynamics in the context of the EU model.
5. Discussion

In this paper, we investigated the cosmology of the EU scenario in the presence of interacting fluids. This analysis aims to demonstrate the possibility of obtaining viable cosmological dynamics of the EU. Two different cosmological models have been presented here. In Model I, we considered the flow of energy from the fluids required to realize the EU to a pressureless fluid that sets in at an epoch, \( t = t_i \). The density of the pressureless fluid assumes importance as a matter component after the epoch, \( t_i \). In Model II, we considered interactions among the three fluids of the EU at time \( t = t_0 \). Before this epoch, the EU can be realized without an interaction among the fluids. The problem with earlier cosmological realizations of the EU was that once the EoS parameter \( A \) is fixed at a given value, the Universe is unable to come out of the phase with a given composition of fluids. In the present work, we overcame this problem by assigning an interaction among the fluids at the epoch \( t_0 \). A cosmological evolution of the observed Universe through unified dynamics of associated matter and dark energy components thus becomes feasible in the EU scenario. In figure (1), we plot variations of \( \omega_3^{\text{eff}} \) with EoS parameter \( \omega_3 = A \) for different interactions. It is evident that an early Universe with a radiation-dominated phase transits to a matter-dominated phase with all the features observable at the present moment with the onset of interaction considered here. We also showed the consistency of the interacting fluid EU scenario with the generalized second law of thermodynamics. Further work is needed on a comparative analysis of the interacting fluid EU cosmology with more popular current cosmological models. Detailed analyses of observational constraints pertaining to various eras of the Universe are expected to yield bounds on the parameters of the EU models considered here, leading to a firmer assessment of the viability of such models.

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