Entropy change in a single Leontiev tube
during energy separation of low-Prandtl gas mixture

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Abstract. The results of numerical studies of energy separation of a helium-xenon gas mixture
in a single Leontiev tube with a central cylindrical channel are presented. On the basis of the
simulation data, a T-s diagram for energy separation is constructed. The changes in the total
temperature depending on the entropy change in the central subsonic channel and in the annu-
lar supersonic nozzle are shown.

1. Introduction

Gas-dynamic energy separation is the total temperature redistribution between the parts of the gas
flowing through the energy separation device. The effect depends on the nature of temperature inho-
mogeneity occurrence in the flow (vortex effect, resonance effect, and dissipative effect, etc.) [1].
However, regardless of the nature of the effect, a decrease in the temperature of one of the parts of
the separated gas flow leads to a decrease in its entropy at the outlet of the device, compared with the en-
tropy of the gas at the inlet. In vortex tubes or Leontiev tubes that are adiabatic with respect to the sur-
rounding medium, a decrease in the entropy of a gas flow part is possible only by increasing the entropy
in the other, heated part of the flow. The decrease in the entropy of the flow part distinguishes the
energy separation devices from throttling devices and nozzles. As it is known, nozzles and throttles
have found wide application in engineering as devices for lowering the pressure of the working fluid
without performing technical work (throttles) and with technical work (nozzles and turbines). In the
thermodynamic analysis of the operation of power and refrigeration cycles of heat machines, T-s dia-
agrams are widely used to simply and clearly analyze the efficiency of a particular cycle of a heat ma-
chines. For common working bodies (water and water vapor, air, ammonia, freons), state diagrams
with plotted characteristics of specific nozzle apparatuses and throttles are constructed. This allows
selecting the equipment necessary for the cycle implementation by setting the cycle parameters at key
points. In contrast to the wide range of industrially produced nozzles and throttles with known charac-
teristics, power separation devices are represented to the acceptable extent by vortex tubes only.
A relatively new energy separation device – the Leontiev tube [2–4] is not commercially available.
For this device, the literature sources lack diagrams of changes in the state of the working fluid, the
most effective of which is a helium-xenon gas mixture with a low Prandtl number (0.2), which does
not allow using these devices as part of thermal machines operating in thermodynamic cycles.

This paper presents the results of numerical modeling of changes in the entropy of a He-Xe gas
mixture (mass concentration of He equal 5%) during energy separation in a single Leontiev tube.
2. Problem statement and solution method

The tube schema, the geometry of the computational domain and the boundary conditions are shown in Figure 1. The energy separation in a Leontiev tube with a central cylindrical channel is investigated. The supersonic flow is formed in an annular Laval nozzle with an external inlet diameter $d_{in}$ of 30 mm, a critical diameter $d_{cr}$ of 12.5 mm and an output diameter $d_{out}$ of 19 mm. The inner wall of the nozzle has a diameter equal to that of the cylindrical tube located on the axis of the nozzle, $d_3 = 10.4$ mm. The tapering part represents half of the sinus period ($L_2 = 20$ mm). The rectilinear input section of the channel has a length $L_1 = 30$ mm. The expanding part of the nozzle has a conical shape with a smooth coupling in the critical section along a cubic parabola ($L_3 = 50$ mm, $L_4 = 150$ mm). The opening angle of the cone is 1.15°. The diameter of the channel in the central tube is 3.5 mm, and the thickness of the separation wall is 3.45 mm. The total length of the Leontiev tube is 250 mm. The maximum Mach number in the core of a supersonic flow is 2.8 with the flow of a helium-xenon mixture ($\gamma = 1.68$). The expansion of the conical part allows compensating for the decrease in the hydraulic diameter that occurs due to the growth of the boundary layers, and maintaining an almost constant Mach number in the flow core along the length. The total pressure and temperature at the inlet to the Leontiev tube are 7.5 atm and 295 K, respectively.

The pressure at the outlet from the central subsonic channel decreases from 7.495 to 5.5 atm, thus ensuring a change in the gas flow rate in the channel. For the geometry under consideration, a pressure of 5.5 atm ensures the sound velocity of the flow from the central channel and the maximum flow rate. For the accepted pressure at the inlet to the tube, the maximum flow rate is 210 g/s. The pressure at the outlet of the supersonic channel is 1 atm. The properties of the mixture correspond to work [5], and the Prandtl number is 0.22. The relationship between pressure, temperature and density is set by the ideal gas equation of state.

The simulation is carried out using the ANSYS Fluent solver, a generator of computational grids and macros for processing GGN modeling data of proper design. The flow dynamics and heat transfer in the gas phase are analyzed on the basis of the RANS equations in a stationary two-dimensional axisymmetric formulation by an implicit method. Turbulence is described by the Menter $k – \omega\ SST$ model taking into account viscous heating, compressibility effects, with a modification for low Reynolds numbers.

The Kays-Crawford model of the turbulent Prandtl number is used to describe the turbulent heat transfer [6]. At the boundaries with the heat-conducting wall, the conditions of no-slip and coupling by temperature and heat fluxes are set. The calculated grid has a power-law thickening to all walls. The number of nodes is more than 400 thousand. Iterative convergence is provided for all variables with an
accuracy of $10^{-5}$. The mass flow rate of gas through supersonic and subsonic channels is determined with the same accuracy.

3. Testing the design mathematical model
The black line in Figure 2 shows the static pressure distribution on the wall of the supersonic nozzle at air outflow from the Leontiev tube, obtained as a result of modeling, in comparison with the experimental data of [7, 8] for the separation wall made of ebonite and copper. Pressure at the tube inlet $p_{00} = 7.5$ atm., and total temperature $T_{00} = 295$ K. The simulation data are in good agreement with the experiment. The static pressure along the length of the nozzle is reduced to 0.3 atm, which is lower than the outlet pressure. An outflow with flow over-expansion is realized, which leads to the formation of a system of pseudo-shock waves at the nozzle outlet, on which the flow slows down, increasing its pressure to atmospheric. The red line in Figure 2 presents the data for the He-Xe mixture. It may be noted that thermal processes have little effect on the dynamics of the flow from the Laval nozzle.

Figure 2. Static pressure distribution along the wall of the supersonic part of the channel: the black line is the air outflow, the red line is the outflow of the helium-xenon mixture (a), the field of Mach numbers and the total temperature in the longitudinal section of the tube (b).

4. Results and discussion
Figure 3 shows the calculated results for the local values of entropy, static and total flow temperature in the longitudinal section of the tube at the total pressure at the outlet of the central channel of 5.5 atm (a), 7.44 atm (b) and 7.48 atm (c). The entropy value at the inlet to the central channel is taken as its zero value. At low pressure, the flow rate of the mixture through the central channel is limited by reaching the speed of sound at its exit. At that, as can be seen from the total temperature distribution, the gas almost does not cool down. The mass average entropy at the outlet of the central channel increases due to the dissipation of the kinetic energy of the flow in the viscous boundary layer. The increase in entropy in the annular nozzle is significantly greater than in the central channel, since the friction is also significantly greater at supersonic outflow. Despite the decrease in the thermodynamic temperature in the core of the supersonic flow, the equilibrium temperature at the separation wall is
close to the temperature at the inlet to the tube. With an increase in the back pressure at the outlet of the central channel, the gas flow decreases, which leads to its cooling due to the transfer of heat to the supersonic flow in the annular nozzle. The conditions of energy separation are implemented [9]. The entropy of the gas in the central channel becomes less than at the inlet to the tube. In a supersonic annular nozzle, the entropy increases more strongly than in adiabatic conditions.

Figure 3. Longitudinal static entropy, temperature, and total temperature fields for central channel outlet pressure of 5.5 (a), 7.44 (b), and 7.48 (c) atm.

Figure 4 shows the calculated results for the distributions of the mass average total temperature and the change in the mass average specific entropy of the gas flow along the length of the tube. The sections in which the calculations are carried out are indicated in Figure 3 by vertical lines. A significant decrease in the gas temperature in the central channel can be achieved only when the back pressure at the outlet increases. The increase in entropy in the annular nozzle for the considered conditions is 230 J/kg K. The decrease in entropy in the central channel does not exceed 50 J/kg K. The linear nature of the entropy increase in the conical section of the annular nozzle may be noted. Combining the data of Figure 4a and 4b, it is possible to construct a T-s diagram of the energy separation of the helium-xenon mixture in the Leontiev tube under consideration. If we take the entropy value at the tube inlet as zero, then the T-s diagram can be represented by Figure 5. With a decrease in the gas flow through the central nozzle, the entropy gradually passes into the region of negative values. In this case, the relationship of negative entropy values with temperature becomes linear. It is noticeable that in this area all the data may be generalized. For an annular nozzle, an increase in entropy occurs with a small increase in the total temperature (about 2 degrees). For practical calculations, the process of outflow from a supersonic nozzle can be considered isothermal.
5. Conclusions

On the basis of the numerical simulation data, a T-s diagram of energy separation of the helium-xenon gas mixture with a low Prandtl number in a single Leontiev tube has been constructed. A decrease in temperature in the central channel is shown to be accompanied by a decrease in entropy, only when the gas flow rate is below a certain value. A decrease in the flow rate leads to a decrease in heat release due to viscous dissipation, and the effect of energy separation becomes prevailing. The dependence of the entropy change on the total temperature for the cooled gas is linear. For a supersonic flow, energy separation leads only to a slight increase in the mass-average total temperature. For practical calculations, this process can be considered isothermal.

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