Noise study in condensed matter physics
-Towards extension to surrounding fields-

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Abstract. I briefly review noise studies in condensed matter physics, such as the shot noise measurement in metals, the dynamic-coherent-volume investigation in charge-density waves, the macroscopic quantum tunneling in superconductors, and the experimental investigation of dynamic phase diagram of driven vortices in high-$T_c$ superconductors. With these examples, one finds that the noise studies have played many crucial roles in condensed matter physics. I also discuss a recent theoretical suggestion that noise measurements in Josephson junction may clarify the origin of the dark energy in the universe.

1. Introduction
For measurements in physics using electronics, noise is usually thought to be a nuisance. This motivated researches of the physics of noise in early days[1, 2, 3, 4], so that one can reduce the noise in measurements. Well known conventional noises are classified as follows.

(1) thermal noise[1, 2]: This is caused by the photon number fluctuation in a resistor in an equilibrium, and the voltage fluctuation at the frequency, $f$, $< (\Delta V)^2 > (f)$ in the frequency window $f \sim f + \Delta f$ was found to be

\[
< (\Delta V)^2 > (f) = 4k_B T R \Delta f,
\]

where $k_B$ is Boltzman constant, $T$ is the temperature, $R$ is the resistance of the resistor, and $\Delta f$ is the bandwidth. Conventionally, it is expressed in terms of the power spectral density, $S(f) \equiv < (\Delta V)^2 > (f)/\Delta f$, which is the Fourier transformation of the autocorrelation function, $\Phi(t) \equiv < V(t_0)V(t_0+t)> (V(t)$ is the fluctuating voltage at the time $t)[5]$. This Nyquist formula provides the lower limit of the noise level one can achieve. It should also be added that this frequency-independent spectrum resulted from the Maxwell-Boltzman statistics plus the one dimensionality of the problem. If the quantum effect becomes essential, it changes into the one dimensional version of “the Planck black-body radiation formula”:

\[
S_V(f) = 2k_B T R \left( \frac{hf}{k_B T} \right) \frac{1}{\exp(hf/k_B T) - 1},
\]

where $h$ is the Planck constant.

(2) shot noise[3]: This is caused by the particle nature of the current flow. In contrast to the thermal noise, shot noise is a purely non-equilibrium phenomenon. For the current flow emitted
in vacuum tube, the power spectral density of the current is expressed as

\[ S_I(f) = 2QI, \]  

where \( Q \) and \( I \) are the unit charge of the current, the average current, respectively. Thus, the shot noise provides the information on the unit charge, which can be used to investigate many novel charge excitations in contemporary condensed matter physics.

(3) excess noise[4]: In resistors, excess noises become prominent at low frequencies. Mostly, this excess noise has the spectrum which is inversely proportional to the frequency. Thus, it is often called as the \( 1/f \) noise. Since the \( 1/f \) noise was observed in wide range of systems, the origin of the \( 1/f \) noise has been a long standing problem[6, 7], and the self-similar nature of the \( 1/f \) noise has been interested also in terms of the nonlinear dynamics.

Thus, noises have played very important roles in condensed matter physics from very old days. In this article, I will introduce some examples where the noise measurement provided key information in a broad range of investigations in condensed matter physics. I will also comment on a recent interesting theoretical suggestion to use the noise experiments in condensed matter for clarifying the mystery of the universe.

2. Shot noise measurement in quantum Hall systems and metals

As was already mentioned, the shot noise provides the information on the unit charge of the current flow. In condensed matter physics, the unit charge of the elementary charge excitation is sometimes not \(-e\), the most typical example of which is the fractional quantum Hall effect[8], where the charge excitation by a fractional charge, such as \( e/3, e/5 \) etc. has been considered[9]. The first direct experimental evidence for such exotic charge excitations was provided by two independent shot noise experiments[10, 11].

Shot noise measurements in solids are very difficult. Even in ordinary metals, theoretical calculation showed that the full shot noise (\( 2eI \)) could not be observed in principle, because of the correlation effects between electrons[12, 13, 14, 15]. Another difficulty is that it is very hard to prepare the non-equilibrium condition in ordinary metals. Only very well designed experiments in micro-fabricated samples can achieve the non-equilibrium condition for electrons in metals. Thus, it is rather recently that the shot noise was measured successfully in metals[16, 17], which showed good agreements with the theoretical expectations.

In the experiments in the fractional quantum Hall systems[10, 11], point contact tunnel junctions were utilized to realize the non-equilibrium condition, and they succeeded in observing the full shot noise by the fractional charge \( e/3 \) etc.

Another interesting candidates are high-\( T_c \) cuprate superconductors, where various possibilities of exotic charge excitations have been considered. However, micro-fabrication technique (down to submicrons) of the cuprate superconductors has not been developed well. Quite recently, Bonetti et al.[18] succeeded in fabricating submicron wires of a high-\( T_c \) cuprate, \( \text{YBa}_2\text{Cu}_3\text{O}_y \), and observed many random-telegraph like jumps in the resistivity vs temperature curve. In such wires, shot noise might be measured in near future.

3. Dynamic correlation length of the sliding conduction of charge-density waves

In quasi-one dimensional metals, an ordinary metallic state is usually unstable, and they undergo a phase transition called as the Peierls transition at some temperature, \( T_c \), below which a static charge density modulation is formed. This is the charge-density wave (CDW) (Fig. 1). Since the period of the CDW is \( \pi/k_F \) (\( k_F \) is the Fermi wave number determined solely by the electron density), which is incommensurate to the period of the host lattice, the CDW can move freely if there is no disorder. This novel type of the collective electric conduction is originally proposed by Fröhlich[19]. Although the finite pinning by disorders leads to a finite threshold electric field
Figure 1. One dimensional array of atoms with uniform electron charge density above $T_c$ and the charge-density-wave state below $T_c$.

Figure 2. Current dependence of the conduction noise of monoclinic TaS$_3$ (taken from ref. [25]).

For the collective motion of the CDW to take place, this type of the novel electric conduction was observed in many quasi-one dimensional materials[20, 21], details of which were studied extensively. Since the CDW possesses an intrinsic periodicity, the collective motion of the CDW (sliding conduction) exhibits a periodic modulation of the velocity. This shows up as a periodic component in the current carried by the CDW, which was originally discovered as a periodic component in the conduction noise spectrum in the frequency domain (narrow-band noise, washboard noise)[22]. When the coherence was improved, it was observed as a current oscillation in the time domain, as is naturally expected[23, 24]. Then, it is interesting how coherent the sliding motion of the CDW is. Again, the noise study played a crucial role to determine the dynamical correlation length of the moving CDW[25]. Figure 2 shows the conduction noise generated by the sliding CDW in monoclinic TaS$_3$ as a function of current. Above some threshold value, large noise appeared. At the current where the noise became maximum, the power spectrum was found to be the $1/f$ type. With further increasing current, washboard noise appeared, suggesting the highly coherent nature. Indeed, at the noise maximum, we estimated the dynamical correlation length of the CDW based on a simple model[25]. The current carried by the CDW, $I_{CDW}$, is expressed as

$$I_{CDW} = A \times \frac{1}{V} \left[ \sum_i N_i ev_i(t) \right]$$

(4)

$$= \frac{1}{L} \left[ \sum_i N_i ev_i(t) \right],$$

(5)
where \( e \) is the electronic charge, \( N_i \) is the number of electrons in the \( i \)-th domain of the CDW, \( A, L, V \) are the cross section, the length, and the volume of the sample, respectively. \( v_i(t) \) is the stochastic variable, which takes 0 or \( v_0 \), alternatively. The average of \( v_i(t) \) is assumed to be

\[
<v_i(t)> = v_0 P. \tag{6}
\]

This formalism leads to the relative fluctuation of the current carried by the CDW as

\[
\frac{\left< (\Delta I)^2 \right>}{\left< I_{CDW} \right>^2} = \frac{1}{NAL} \frac{1 - P}{P}, \tag{7}
\]

where \( N \equiv 1/V_D \) is the domain density of the CDW segments (\( V_D \) is the domain volume). At the noise maximum, \( P \) is considered to be 1/2. Thus, we can estimate \( N \) from the experimental data. At low temperatures, \( N \) becomes \( 10^{-7} \) cm\(^{-3} \), which corresponds to \( NV \sim 1 \), and the CDW motion is highly coherent with almost one domain. Assuming an appropriate anisotropy, this number means the correlation length of 100 \( \mu \)m in the moving direction. As was already mentioned, for such a highly coherent motion, the washborad noise can be seen as the current oscillation in the time domain.

Since the local charge denstiy, \( \rho(r, t) \), of the CDW can be expressed as

\[
\rho(r, t) = \rho_0 + \rho_1 \cos(Qx + \phi(x, t)), \quad (Q = \pi n) \tag{8}
\]

where \( n \) is the condensed electron density in one dimension, the CDW motion can be microscopically described by the equation of motion for the phase of the CDW, \( \phi(x, t) \). If we make a rigid body approximation as the first step, the equation of motion of the CDW becomes identical to the dynamic equation for the phase of the Josephson tunnel junction in superconductors. We discuss this issue in the next section.

4. Josephson junction in superconductors
The Josephson effect is the tunneling of the supercurrent across the barriers (Fig. 4). Since the phase of the macroscopic wave function is constant in a superconductor, the Josephson current
flows depending on the phase difference, $\theta$, between different superconductors, as

$$I = I_c \sin \theta. \quad (9)$$

Above the critical current, $I_c$, zero-voltage state cannot exist, and finite voltage appears. Application of the finite voltage across the junction develops the phase as a function of time.

$$\hbar \frac{d}{dt} \theta = 2eV. \quad (10)$$

A typical $I - V$ characteristics of the Josephson junction is shown in Fig. 4 schematically. The $I - V$ characteristics of the Josephson junction was well expressed in terms of the simple equivalent circuit model; resistivity($R$) (and capacitance($C$)) shunted Josephson junction (RSJ or RCSJ) model[26, 27]. With an appropriate normalization, we obtained a 2nd class nonlinear differential equation for the phase, $\theta$.

$$\frac{d^2 \theta}{d\tau^2} + \frac{1}{Q} \frac{d\theta}{d\tau} + \sin \theta = i, \quad (11)$$

where $\tau = \omega_p t$, $i = I/I_c$, $\omega_p \equiv (2eI_c/\hbar C)^{1/2}$, $Q = \omega_p RC$. This is identical to the equation of the motion of a particle in a tilted periodic potential. As was already mentioned, this is also identical to the equation of motion for the sliding CDW within a rigid body approximation.

At finite temperatures, stochastic aspects are introduced by the thermal fluctuation, and a finite distribution should be considered for $I_c$. This phenomenon has been studied theoretically[28] and experimentally[29] for a long time. Experimentally, the distribution of $I_c$ was found to become narrower with decreasing temperature, in a good agreement with the theory. However, at very low temperatures ($\sim 100$ mK), novel features appeared. There, the distribution became temperature independent[30]. This has been considered due to the quantum effect, leading to a finite probability of the escape from the potential well even at very low temperatures. Since the tunneling takes place for the phase of the macroscopic wave function, this is called as the macroscopic quantum tunneling (MQT), where the energy dissipation becomes important for the quantum tunneling[31]. In situations where the MQT is remarkable, quantized levels are formed in the potential well[32], which can be used as quantum bits (qubits)[33]. It is expected that the use of the high-$T_c$ cuprates increases the operation temperature of qubits. Some recent experiments reported the observation of the temperature independent distribution of $I_c[34, 35]$, suggesting the occurrence of the MQT in the high-$T_c$ cuprates. In particular, Inomata et al.[35] reported that the temperature independent distribution was observed up to 1 K in the intrinsic Josephson junction of Bi$_2$Sr$_2$CaCu$_2$O$_y$. However, it is also suggested that the intrinsic Josephson junctions exhibit complicated natures at low temperatures[36]. Thus, further investigation is necessary to establish the occurrence of the MQT in the hig-$T_c$ cuprates.

5. Dynamic phase diagram of driven vortices

When the magnetic field, $B$, is applied on superconductors, the magnetic flux penetrates the superconductor in a quantized form, with a unit $\Phi_0 = \hbar/2e$ ($h$ is the Planck constant). These flux quanta, accompanying circulating current flow, forms a triangular lattice (Fig. 5). Application of the finite current density, $j$, moves the flux lattice under the driving force of $F = j \times B$. On the other hand, each fluxion can easily be pinned by random disorders, because the superconductivity order becomes weakened in the central part of the fluxion, which favors the position of disorders. Therefore, again, a similar nonlinear equation of motion to the CDW can be obtained for the motion of the driven vortices. The problem, however, looks like more interesting and complicated when compared with the CDW, since the vortex lattice is formed...
in the 2 dimension. This makes the possible internal degrees of freedom more complex. Thus, other than basic similarities among the phenomena, many novel features can be expected in the dynamical phase diagram of the driven vortex lattice[37]. In particular, the driven vortex lattice of the high-$T_c$ cuprate provides many interesting possibilities, since the large thermal energy and very strong two dimensionality introduce new aspects even for the equilibrium properties of superconductivity under magnetic fields. In fact, contrary to rather simple pictures established in conventional superconductors[42, 43, 44], many novel features have been proposed theoretically[45, 46, 47, 48, 49] on the dynamics of the driven vortices of high-$T_c$ superconductors. We approach this problem experimentally, again, by the noise measurements[38, 39, 40, 41]. The vortex motion contains two basically independent, different fluctuations (noises); the local density fluctuation of the magnetization, $\langle (\delta n)^2 \rangle$, and the velocity fluctuation, $\langle (\delta v)^2 \rangle$. Thus, we studied both of these independently, and also measured these simultaneously in the same sample. For the density noise measurement, the local Hall probe made of the 2 dimensional electron gas (2DEG) was used, whereas the velocity noise was measured across the voltage leads on the samples, as was similar to the CDW case. One merit is that we can discuss the spatial correlation of the locally generated density noise directly by taking the cross spectrum. We can also discuss the spatial correlation relative to the flow direction by changing the direction of the Hall probe array. Below, I list up our achievements obtained by these techniques. (1) We observed large density noises very close to the resistivity onset. This density noise possesses strong directivity. That is, the noise has large spatial correlation only in the direction of the vortex flow. This strongly suggests that the large density noise is due to the plastic stochastic motion of the vortex lattice. (2) In some samples, we observed the washboard conduction noise,
as was in the CDW. Surprisingly, our observation is the first example of the observation of the washboard noise in all superconductors. This might exclude theories that argued that the coherently moving state is the smectic phase[49], since these theories based on the previous situations where there was no reports of the observation of the washboard noise. Our results rather support the existence of the moving Brag glass[48]. (3) We also observed the interference effect between the intrinsic periodicity of the moving vortex lattice (washboard frequency) and the externally applied alternating driving force. However, this interference effect was observed in a limited region of the current vs magnetic field plane. (4) We investigated the density noise, conduction noise and the interference effect, in the same sample. By detailed comparisons, we clarified the relationship among these phenomena. Almost at the resistivity onset, the density noise appeared first. With increasing driving force, conduction noise became prominent. In the same region, the interference effect was observed. With further increasing driving force, the interference effect disappeared, whereas the conduction noise still survived, until the melting of the vortex lattice would be reached. (5) Based on all of these observations, we consider the following scenario for the development of the dynamical phases of the driven vortices in high-$T_c$ superconductors: (pinned Brag glass) $\rightarrow$ (plastic flow) $\rightarrow$ (coherent flow) $\rightarrow$ (less coherent “phase”) $\rightarrow$ (moving liquids). According to our experimental results, we did not observe any dynamic phase transition[45] which was argued in conventional superconductors[50, 51].

6. Comments on a recent theoretical suggestion to use noise experiment in Josephson junction to investigate the origin of dark energy

Recently, Beck[52] proposed that the noise measurement in Josephson junction might clarify the origin of the dark energy in the universe. The basic idea is as follows. If the dark energy is related to the zero-point photon number fluctuation, there must be a finite cut-off frequency above which no thermal noise (zero-point part) is observed, since the total number of the dark energy was already estimated. The existence of the zero-point fluctuation was already insisted based on the noise measurements in Josephson junction, where Koch et al.[53] showed the noise “spectrum” up to $6\times10^{11}$ Hz, which is very close to the cut-off frequency of $1.69\times10^{12}$ Hz, estimated by Beck. Since this frequency dependence data was obtained by converting the voltage to the frequency through the relationship, $h\omega = 2eV$, the original data in ref. [53] was the noise measured at very low frequencies as a function of applied voltage across the Josephson junction in Nb. Thus, in principle, the above theory can be checked if the measurement can be continued up to higher voltages. However, this might be difficult because of the Joule heating etc.. Physically, the measured noise at such low frequencies originated from the noise at the washboard frequency ($\omega_p$) down converted because of the strong nonlinearity of the system[54]. Thus, if the above argued mechanism is valid, it is not necessarily restricted on Josephson junctions. Other systems with both the intrinsic periodicity and the nonlinearity, such as the sliding CDW, driven vortices etc., can also be good candidates to check the above mentioned theoretical suggestion. These studies should be made in near future.

7. Conclusion

Some of the important noise studies in condensed matter physics were introduced, such as the shot noise measurements in metals, the dynamic coherent volume investigation in the CDW, the MQT in superconductors, and the experimental investigation of dynamic phase diagram of driven vortices in high-$T_c$ superconductors. With these examples, I tried to show that the noise studies have played many crucial roles in condensed matter physics. The recent theoretical suggestion by Beck must be investigated in different nonlinear systems. This must convince the importance of the noise studies in condensed matter not only within the condensed matter physics field, but also for surrounding different fields.
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