Mathematical Reconstruction of a Traffic Crash

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Abstract
The objective is to find the impact speeds of two motor vehicles involved in a traffic crash. The calculations take into account the approach (pre-collision) and departure (post collision) angles, as well as weights and post collision speeds of both vehicles. The data is provided by Timothy Sleyzack who investigated this traffic collision. The conclusions confirm the validity of the use of Conservation of Linear Momentum in the field of traffic crash reconstruction.

Keywords
traffic collision, Conservation of Linear Momentum, impact speed, post collision speed, pre-collision and post collision angles, coefficient of restitution

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PROBLEM STATEMENT

Use Conservation of Linear Momentum to reconstruct a traffic crash mathematically and accurately.

MOTIVATION

In the field of traffic crash reconstruction, one of the most commonly asked questions of the investigator is: What is the speed of each vehicle, especially the striking vehicle, at the time of the collision? It is very important for the investigator to have an accurate account of speeds of vehicles as it helps to adequately analyze and conclude on events and causes in a collision. Investigators commonly find witness accounts as to a vehicle’s speed to be conflicting. To address this problem effectively, a mathematical approach is preferred to witness testimony.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The solution approach is based on Conservation of Linear Momentum. Linear momentum is a product of the mass and velocity of an object. Momentum is a vector which points in the same direction as velocity. The total amount of momentum never changes, and this property is called conservation of momentum.

While reconstructing the traffic crash, I first determine the approach (pre-collision) and departure (post collision) angles for each vehicle from a scale diagram of the crash scene which is provided by Timothy Sleyzack who investigated the crash (see Appendices). So the first thing that needs to be done is to find all four of the angles (α,ψ,θ,ϕ) that are required for the mathematical equations (A) and (B) below. The next thing is to obtain the weights of the vehicles and their post collision speeds. After that, it is a matter of plugging in the variables to get the desired values for the pre-collision speeds of the vehicles. The final step is to calculate the coefficient of restitution.

After determining each vehicle’s approach and departure angle, I calculate the sine and cosine values for each angle. I then use the weight of both vehicles and their post collision energy loss into the following mathematical equations.

\[
(A) \quad S_2 = \frac{(W_1)(S_3)(\sin\theta)}{(W_2)(\sin\psi)} + \frac{(S_4)(\sin\phi)}{\sin\psi}
\]

For equation (A), the pre-collision speed of vehicle-2 (S_{2}) is equal to the weight of vehicle-1 (W_{1}), multiplied by the post collision speed of vehicle-1 (S_{3}), multiplied by the sine of the departure angle of vehicle-1 (\theta); divided by the product of the weight of vehicle-2 (W_{2}) and the
sine of the approach angle of vehicle-2 (ψ). That calculation is then added to the product of the post collision speed of vehicle-2 (S₄) and the sine of the departure angle of vehicle-2 (φ), divided by the sine of the approach angle of vehicle-2 (ψ).

\[
(B) \quad S₁ = (S₃)(\cosθ) + (W₂)(S₄)(\cosφ) - (W₂)(S₂)(\cosψ)
\]

For equation (B), the pre-collision speed of vehicle-1 (S₁) is equal to the post collision speed of vehicle-1 (S₃), multiplied by the cosine of the departure angle of vehicle-1 (θ); divided by the cosine of the approach angle of vehicle-1 (α). That calculation is then added to the product of the weight of vehicle-2 (W₂), the post collision speed of vehicle-2 (S₄) and the cosine of the departure angle of vehicle-2 (φ); divided by the weight of vehicle-1 (W₁) multiplied by the cosine of the approach angle of vehicle-1 (α). That calculation is then subtracted by the product of the weight of vehicle-2 (W₂), the pre-collision speed of vehicle-2 (S₂) and the cosine of the approach angle of vehicle-2 (ψ); divided by the product of the weight of vehicle-1 (W₁) and the cosine of the approach angle of vehicle-1 (α).

\[
(C) \quad e = \frac{(S₃)-(S₄)}{(S₁)-(S₂)}
\]

For equation (C), the coefficient of restitution (e) is the difference of the post collision speeds of vehicle-1 (S₃) and vehicle-2 (S₄) divided by the difference of the pre-collision speeds of vehicle-1 (S₁) and vehicle-2 (S₂).

**DISCUSSION**

I calculated the impact speed of the target vehicle (vehicle-2) to be 21.96 miles-per-hour and that of the striking vehicle (vehicle-1) to be 40.85 miles-per-hour. To ensure the accuracy of results, two mathematical approaches are used. The first, Coefficient of Restitution, with a desired value between zero and one, produces a result of 0.15. The second mathematical approach, Vector Sum Analysis, where a similar value for each vehicles’ change in momentum (ΔM) is desired, results in identical values.

To further ensure the accuracy of my calculations, two electronic software calculations are used. The first one is Rec-Tec Professional Accident Reconstruction Software. Rec-Tec requires the input of each vehicles’ weight, approach angle, departure angle and post collision speed to calculate an impact speed of 40.97 miles-per-hour for vehicle-1 and 22.35 miles-per-hour for vehicle-2. The second electronic software that is used to ensure the accuracy of my results is the review of each vehicle’s Crash Data Retrieval System Report (Bosch), provided by Timothy Sleyzack. The Crash Data Retrieval System is often referred to as the Airbag Control Module or
the vehicle’s Black Box. This provides data such as a vehicle’s speed, the time that an airbag
deployed, up to five seconds prior to the deployment of the airbags or collision. The Crash Data
Retrieval System denotes an impact speed of 37 miles-per-hour for vehicle-1 and a speed of 14
miles-per-hour for vehicle-2 (see below).

While reconstructing the traffic collision, my project objective, which is to calculate each
vehicles’ impact speed, is met. It confirms that the use of Conservation of Linear Momentum in
the field of traffic crash reconstruction is a useful and accurate tool for a crash investigator.

CONCLUSION AND RECOMMENDATIONS

While the impact speed calculated for vehicle-1 is close (within 3 miles-per-hour) to the impact
speed denoted in the Electronic Data Retrieval Report, the impact speed calculated for vehicle-2
is further (7 miles-per-hour) from the impact speed denoted in the Electronic Data Retrieval
Report. This discrepancy can be attributed to the fact that vehicle-2’s approach angle, which
relies on physical evidence such as tire marks and swerve marks, can be susceptible to error,
whereas the approach angle for the striking vehicle is always zero degrees.

For someone looking to do the same project I have a few recommendations. To obtain accurate
results, when taking the angles into consideration, the angles should be measured multiple times.
This will ensure that the angles have been measured effectively.

In conclusion, the importance of accurately determining approach and departure angles is
significant when using Conservation of Linear Momentum.
NOMENCLATURE

Variables used in Conservation of Linear Momentum

| W   | Weight               | Pounds (lbs.) |
|-----|----------------------|---------------|
| α   | Approach Angle Vehicle-1 | Degrees      |
| ψ   | Approach Angle Vehicle-2 | Degrees      |
| θ   | Departure Angle Vehicle-1 | Degrees     |
| φ   | Departure Angle Vehicle-2 | Degrees     |
| S_1 | Pre-Collision Speed Vehicle-1 | Miles-per-hour (mph) |
| S_2 | Pre-Collision Speed Vehicle-2 | Miles-per-hour (mph) |
| S_3 | Post Collision Speed Vehicle-1 | Miles-per-hour (mph) |
| S_4 | Post Collision Speed Vehicle-2 | Miles-per-hour (mph) |
| e   | Coefficient of Restitution | N/A          |

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### APPENDICES

#### Momentum Worksheet

|                      | Degrees | Sin  | Cos  | Weights and Speeds |
|----------------------|---------|------|------|--------------------|
| Approach Angle of V1 | $\alpha$ | 0    | 0    | $W_1 - 66.52$     |
| Approach Angle of V2 | $\Psi$  | 134  | 0.6560, 0.7547 | $W_2 - 26.78$     |
| Departure Angle of V1| $\Theta$| 6.5  | 0.1133, 0.9935 |                   |
| Departure angle of V2| $\Phi$  | 17   | 0.2923, 0.9563 |                   |
| Impact Speed of V-1 |         |      |      | $S_1 - 40.75$     |
| Impact Speed of V-2 |         |      |      | $S_2 - 21.48$     |
| Post Collision Speed of V-1 |       |      |      | $S_3 - 24.02$     |
| Post Collision Speed of V-2 |       |      |      | $S_4 - 26.93$     |

#### Non - Collinear Crashes (Angular)

$$S_2 = \frac{W_1 S_2 \sin \Theta + S_2 \sin \Phi}{W_2 \sin \Psi}$$

$$S_1 = \frac{S_2 \cos \Theta + W_2 S_4 \cos \Phi}{W_1 \cos \alpha} - \frac{W_2 S_2 \cos \Psi}{W_1 \cos \alpha}$$

#### Collinear Crashes (In Line)

**Rear End:**

$$S_1 = S_3 + \frac{W_2 S_4}{W_1} - \frac{W_2 S_2}{W_1}$$

**Head On:**

$$S_1 = S_3 + \frac{W_2 S_4}{W_1} + \frac{W_2 S_2}{W_1}$$
Crash Diagram
1. Pre-collision speed vehicle - 2

\[ S_2 = \frac{(W_2)(S_1)(\sin \theta)}{(W_1)(\sin \alpha)} + \frac{(S_1)(\sin \phi)}{\sin \gamma} \]

\[ S_2 = \frac{(6682)(24.05)(0.1132)}{(6678)(0.6560)} + \frac{(26.05)(0.2923)}{0.6560} \]

\[ S_2 = \frac{18141.4777}{1756.7860} + \frac{7.6144}{0.6560} \]

\[ S_2 = 10.3544 + 11.6073 = 22.96 \text{ mph} \]

2. Pre-collision speed vehicle - 1

\[ S_1 = \frac{(S_2)(\cos \theta)}{\cos \alpha} + \frac{(W_2)(S_2)(\cos \phi)}{(W_1)(\cos \alpha)} - \frac{(W_1)(S_1)(\cos \alpha)}{(W_1)(\cos \alpha)} \]

\[ S_1 = \frac{(24.05)(0.4935)}{(6678)(0.9543)} + \frac{(26.05)(6.43)(0.9543)}{(6682)(1)} - \frac{(26.05)(21.96)(-0.747)}{(6682)(1)} \]

\[ S_1 = \frac{23.8936}{6682} + \frac{68966.9548}{6682} - \frac{44383.0617}{6682} \]

\[ S_1 = 23.8936 + 10.3213 + 6.6421 = 41.85 \text{ mph} \]
3. Coefficient of Restitution

\[ e = \frac{S_3 - S_4}{S_1 - S_2} \]

\[ e = \frac{24.05 - 26.93}{40.85 - 31.96} \]

\[ e = \frac{-2.88}{18.89} = \frac{2.88}{18.89} = 0.15 \]

**Vector Sum Analysis My Calculations**

\[ \Delta M_2 = 3.3^\circ \]

\[ \Delta M_1 = 3.3^\circ \]
Vector Sum Analysis Computer Analysis

Rec-Tec Data
Vehicle-1 Crash Data Retrieval

| Time Stamp (sec) | Pre-Crash Recorder Status | Speed, Vehicle Indicated (MPH, Kmph) | Accelerator Pedal, % Full | Engine Brake, % Full | Service Brakes | Engine RPM | ABS Activity | Stability Control | Steering Input (deg) |
|------------------|---------------------------|--------------------------------------|--------------------------|---------------------|---------------|------------|--------------|------------------|----------------------|
| -5.0             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,460         | No         | On           | No               | 6                   |
| -4.9             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,459         | No         | On           | No               | 9                   |
| -4.8             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,455         | No         | On           | No               | 10                  |
| -4.7             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,452         | No         | On           | No               | 10                  |
| -4.6             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,454         | No         | On           | No               | 10                  |
| -4.5             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,452         | No         | On           | No               | 11                  |
| -4.4             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,457         | No         | On           | No               | 11                  |
| -4.3             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,458         | No         | On           | No               | 11                  |
| -4.2             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,449         | No         | On           | No               | 11                  |
| -4.1             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,450         | No         | On           | No               | 11                  |
| -4.0             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,447         | No         | On           | No               | 11                  |
| -3.9             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,449         | No         | On           | No               | 11                  |
| -3.8             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,451         | No         | On           | No               | 11                  |
| -3.7             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,444         | No         | On           | No               | 11                  |
| -3.6             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,453         | No         | On           | No               | 10                  |
| -3.5             | Complete                  | 66 [108]                             | SNA                      | Off                 | 1,447         | No         | On           | No               | 10                  |
| -3.4             | Complete                  | 65 [105]                             | SNA                      | Off                 | 1,443         | No         | On           | No               | 10                  |
| -3.3             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,448         | No         | On           | No               | 10                  |
| -3.2             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,441         | No         | On           | No               | 11                  |
| -3.1             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,450         | No         | On           | No               | 11                  |
| -3.0             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,454         | No         | On           | No               | 11                  |
| -2.9             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,452         | No         | On           | No               | 11                  |
| -2.8             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,447         | No         | On           | No               | 15                  |
| -2.7             | Complete                  | 65 [105]                             | SNA                      | Off                 | 1,441         | No         | On           | No               | 16                  |
| -2.6             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,451         | No         | On           | No               | 15                  |
| -2.5             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,456         | No         | On           | No               | 15                  |
| -2.4             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,450         | No         | On           | No               | 15                  |
| -2.3             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,449         | No         | On           | No               | 18                  |
| -2.2             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,444         | No         | On           | No               | 20                  |
| -2.1             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,448         | No         | On           | No               | 20                  |
| -2.0             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,454         | No         | On           | No               | 18                  |
| -1.9             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,455         | No         | On           | No               | 18                  |
| -1.8             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,456         | No         | On           | No               | 18                  |
| -1.7             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,451         | No         | On           | No               | 18                  |
| -1.6             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,449         | No         | On           | No               | 11                  |
| -1.5             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,451         | No         | On           | No               | 9                   |
| -1.4             | Complete                  | 66 [106]                             | SNA                      | Off                 | 1,441         | No         | On           | No               | 3                   |
| -1.3             | Complete                  | 65 [105]                             | SNA                      | Off                 | 1,434         | No         | On           | No               | -6                  |
| -1.2             | Complete                  | 63 [103]                             | SNA                      | On                  | 1,382         | No         | Engaged     | -25               |
| -1.1             | Complete                  | 68 [107]                             | SNA                      | On                  | 1,319         | Yes         | 19          |                  |
| -1.0             | Complete                  | 57 [111]                             | SNA                      | On                  | 1,232         | Yes         | 9           |                  |
| -0.9             | Complete                  | 56 [89]                              | SNA                      | On                  | 1,195         | Yes         | 12          |                  |
| -0.8             | Complete                  | 54 [87]                              | SNA                      | On                  | 1,227         | Yes         | 13          |                  |
| -0.7             | Complete                  | 51 [83]                              | SNA                      | On                  | 1,118         | Yes         | 97          |                  |
| -0.6             | Complete                  | 49 [79]                              | SNA                      | On                  | 1,031         | Yes         | 51          |                  |
| -0.5             | Complete                  | 45 [73]                              | SNA                      | On                  | 1,025         | Yes         | 28          |                  |
| -0.4             | Complete                  | 43 [69]                              | SNA                      | On                  | 1,036         | Yes         | 4           |                  |
| -0.3             | Complete                  | 44 [71]                              | SNA                      | On                  | 1,078         | Yes         | 32          |                  |
| -0.2             | Complete                  | 42 [87]                              | SNA                      | On                  | 948           | Yes         | 46          |                  |
| -0.1             | Complete                  | 37 [60]                              | SNA                      | On                  | 789           | Yes         | 35          |                  |

Skid mark on scene supports drive tire may be traveling slower than vehicle actually is.
### Pre-Crash Data -5 to 0 sec [2 samples/sec] (First Record)

| Times (sec) | Speed vehicle indicated MPH [km/h] | Accelerator pedal, % full | Service brake, on/off | Engine RPM | ABS activity (engaged, non-engaged) | Brake Powertrain Torque Request | Driver Gear Selection | Traction Control via Brakes | Wheel Torque |
|-------------|------------------------------------|---------------------------|-----------------------|------------|-------------------------------------|---------------------------------|----------------------|------------------------|-------------|
| -5.0        | 29 [46]                            | 0.0                       | On                    | 1,228      | non-engaged                         | No                              | Drive                | non-engaged            | -282        |
| -4.5        | 27 [44]                            | 0.0                       | On                    | 1,173      | non-engaged                         | No                              | Drive                | non-engaged            | -232        |
| -4.0        | 26 [42]                            | 0.0                       | On                    | 1,123      | non-engaged                         | No                              | Drive                | non-engaged            | -206        |
| -3.5        | 24 [39]                            | 0.0                       | On                    | 1,026      | non-engaged                         | No                              | Drive                | non-engaged            | -204        |
| -3.0        | 23 [37]                            | 0.0                       | On                    | 956        | non-engaged                         | No                              | Drive                | non-engaged            | -202        |
| -2.5        | 21 [35]                            | 0.0                       | Off                   | 934        | non-engaged                         | No                              | Drive                | non-engaged            | -184        |
| -2.0        | 19 [33]                            | 0.0                       | On                    | 854        | non-engaged                         | No                              | Drive                | non-engaged            | -88         |
| -1.5        | 17 [31]                            | 0.0                       | On                    | 820        | non-engaged                         | No                              | Drive                | non-engaged            | -44         |
| -1.0        | 16 [29]                            | 0.0                       | On                    | 806        | non-engaged                         | No                              | Drive                | non-engaged            | -58         |
| -0.5        | 14 [27]                            | 0.0                       | Off                   | 796        | non-engaged                         | No                              | Drive                | non-engaged            | -54         |
| 0.0         | 14 [27]                            | 0.0                       | Off                   | 742        | non-engaged                         | No                              | Drive                | non-engaged            | -50         |

There is a (.5) second delay from last data point and impact, therefore Vehicle could have accelerated to a higher speed.