Conformal Field Theory Correlators From sine-Gordon Model on AdS Spacetime

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Abstract

Using the proposed AdS/CFT correspondence, we calculate the correlators of operators of conformal field theory at the boundary of AdS\(d+1\) corresponding to the sine-Gordon model in the bulk.
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1 Introduction

The well-known Maldacena conjecture proposed two years ago[1][2] brought to the string theory a revolution: it unravels a relation between Yang-Mills theory and string theory and thus sheds light on the final unification of all interactions. Although the original conjecture referred to the type-IIB string theory on AdS\(_5 \times S^5\) and the \(N = 4, d = 3 + 1\) U(N) super-Yang-Mills theory, it seems valid in general: a quantum field theory in the bulk of \(d+1\) dimensional anti-de Sitter spacetime (AdS\(_{d+1}\)) with some boundary conditions corresponds to a conformal field theory (CFT) on the boundary. Though an exact proof of the conjecture is still lacking, its validity can be confirmed by a number of tests[3]-[11]. The mathematical scheme of the correspondence was formulated explicitly by Witten [4] and independently by Polyakov et al [5]. For a scalar field \(\phi\) in AdS\(_{d+1}\), it is

\[
Z_{\text{AdS}}[\phi_0] = \int_{\phi_0} D\phi \exp(-I[\phi]) \equiv Z_{\text{CFT}}[\phi_0] = < \exp(\int_{\partial\Omega} d^{d+1}\phi_0) >
\]

(1)

The path-integral on the l.h.s. is calculated under the restriction that the field \(\phi\) approaches to \(\phi_0\) on the boundary. The correspondence says that this path-integral is to be identified with the r.h.s.
which is the partition of a CFT on the boundary with $\phi_0$ playing the role of a current coupled to a conformal operator $O$. The r.h.s. thus enables us to calculate the correlators of $O$ of the CFT on the boundary. Since the conformal invariance determines the 2- and 3-point functions, the nontrivial ones are the cases for $n \geq 3$. (So far, the correspondence (1) is still a kind of guesswork which is only more explicit than the original Maldacena’s statement. There must be some field-theoretic reason behind this.

One may regard (1) as the dilaton sector of the type-IIB string theory and corresponding CFT is the corresponding sector of the full super-Yang-Mills theory.)

The free $\phi$-theory was considered in [3] and the general interacting cases were studied in [5]. Yet a careful investigation shows that the consideration of [5] is not enough for the case of the sine-Gordon(sG) theory. This is the very purpose of the present paper. The action of the sG theory in AdS spacetime is

$$I[\phi] = \int_\Omega d^{d+1}x \sqrt{g}[\frac{1}{2}(\nabla \phi)^2 - \frac{m^2}{\beta^2} (\cos \beta \phi - 1)]$$  

The classical equation of motion reads

$$\nabla^2 \phi - \frac{m^2}{\beta^2} \sin \beta \phi = 0$$  

or

$$\nabla^2 \phi - m^2 \phi = m^2 (\frac{1}{\beta} \sin \beta \phi - \phi) \equiv J(\phi)$$  

The free field case is a limit of $\beta \to 0$. If we write

$$\frac{m^2}{\beta^2} (1 - \cos \beta \phi) = \frac{1}{2} m^2 \phi^2 + \sum_{n \geq 3} \frac{\lambda_n}{n!} \phi^n$$  

we have

$$\lambda_{2n+1} = 0, \quad \lambda_{2n} = (-1)^{n} m^2 \beta^{2(n-1)}$$  

Eq(4) can also be cast into the form

$$(\nabla^2 - m^2) \phi = \sum_{n \geq 3} \frac{\lambda_n}{(n-1)!} \phi^{n-1}$$  

as the eq(3) in [4]. The difference is that here there is only one parameter: $\beta$. As usual, the dominant contribution to the path-integral in the l.h.s. of (1) comes from the classical path satisfying the equation of motion (3). So as an approximation, we are for the moment interested in the classical solution to (3) with a given Dirichlet boundary condition $\phi(x_0, x)|_{\partial \Omega} = \phi_0$. We would like to emphasize that the nonlinearity of the equation of motion renders it to have possibly more than one solutions to the Dirichlet problem. It is a nontrivial problem whether different solutions lead to the same correlators of the boundary CFT. Using the covariant Green’s function satisfying

$$(\nabla^2 - m^2)G(x, y) = \frac{\delta(x - y)}{\sqrt{g(x)}}$$  

and the boundary condition $G(x, y)|_{x \in \partial \Omega} = 0$. The classical equation of motion can be expressed equivalently as

$$\phi(x) = \int_{\partial \Omega} d^d y \sqrt{h} n^\mu \frac{\partial}{\partial y^\mu} G(x, y) \phi(y) + \int_\Omega d^{d+1} y \sqrt{g(y)} G(x, y) \sum_{n \geq 3} \frac{\lambda_n}{(n-1)!} \phi^{n-1}$$  

This integral equation can be employed to obtain approximate solutions by recursion.

In section 2 we give a review of the study of free $\phi$ case while presenting a detailed derivation of the connection of the result in [3] and that in [2] of the $\epsilon$-boundary problem. In section 3 we study the case for sG theory.

## 2 The free-$\phi$ case: a review

We use the representation of AdS$_{d+1}$ as the upper half-space ($x_0 > 0$) with the metric

$$ds^2 = \frac{1}{x_0^2} \sum_{i=0}^{d} dx_i^2$$

(10)

The scalar curvature $R = -d(d+1)$. The boundary of AdS is identified with $x = 0$ and the single point $x_0 = \infty$. The solution of the classical equation of motion

$$(\nabla^2 - m^2) \phi = \left[ x_0^2 \sum_{i=0}^{d} \partial_i^2 - x_0 (d-1) \partial_0 - m^2 \right] \phi = 0$$

(11)

can be obtained

$$x_0^{d/2} e^{-i k \cdot x} I_\alpha(k x_0); \quad x_0^{d/2} e^{-i k \cdot x} K_\alpha(k x_0)$$

(12)

where $\alpha = \sqrt{d^2 + m^2}$. $k$ is the momentum $d$-vector and $k = |k|$. $I_\alpha$ and $K_\alpha$ are the Bessel functions. The modes in (12 are linearly independent and constitute a complete basis of the Hilbert space. As in quantum mechanics, the Green’s function can be expressed as $G(x, y) = \sum_n \frac{\psi_n(x) \psi_n^*(y)}{\lambda_n}$, $\lambda_n$ are the eigenvalues corresponding to the eigenfunctions $\psi_n$, here we have

$$G(x, y) = \int \frac{d^d k}{(2\pi)^d} x_0^{d/2} e^{-i k \cdot (x-y)} (-y_0^{d/2}) I_\alpha(k x_0) K_\alpha(k y_0)$$

(13)

$$K_\alpha(k x_0) I_\alpha(k y_0)$$

for $x_0 < y_0$

$$K_\alpha(k x_0) I_\alpha(k y_0)$$

for $x_0 > y_0$

It can also be expressed as [3]

$$G(x, y) = -\frac{c}{2\alpha} \xi^{-\Delta} F\left(\frac{d}{2}, \Delta; \alpha + 1, \frac{1}{\xi^2}\right)$$

(14)

where $F$ denotes the hypergeometric function and

$$\xi = \frac{1}{2 x_0 y_0} \frac{1}{2} ((x - y)^2 + (x - y^*)^2) + \sqrt{(x - y)^2(x - y^*)^2}$$

(15)

in which $y^* = (-y_0, y)$. $\Delta = d/2 + \alpha, c = \Gamma(\Delta)(\pi^{d/2} \Gamma(\alpha))$.

Since the classical solution of $\phi$ in terms of the boundary value $\phi_0$ involves the determinant of metric at the boundary which is singular, the $\epsilon$-description of the asymptotic boundary is necessary: one first solve the problem at $x_0 = \epsilon$ and then take the limit $\epsilon \to 0$ in the very end. The corresponding Green’s function is

$$G_\epsilon(x, y) = G_0(x, y) + \int \frac{d^d k}{(2\pi)^d} (x_0 y_0)^{d/2} e^{-i k \cdot (x-y)} K_\alpha(k x_0) K_\alpha(k y_0) I_\alpha(k \epsilon) K_\alpha(k \epsilon)$$

(16)

Now we calculate the normal derivative of $G_\epsilon$ at the $\epsilon$-boundary. For $x_0 \geq y_0$ since

$$G_\epsilon(x, y) = \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot (x-y)} x_0^{d/2} K_\alpha(k x_0) y_0^{d/2} [K_\alpha(k y_0) I_\alpha(k \epsilon) K_\alpha(k \epsilon) - I_\alpha(k y_0)]$$

(17)
so

\[
\frac{\partial}{\partial y_0} G_\epsilon(x, y)|_{y_0=\epsilon} = \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot (x-y)} x_0^{d/2} K_{\alpha}(k x_0) e^{i d/2} \frac{1}{K_{\alpha}(k \epsilon)} (k K'_{\alpha}(ky) I_{\alpha}(k\epsilon) - k K_{\alpha}(k\epsilon) I'_{\alpha}(ky))|_{y_0=\epsilon}
\]

(18)

Using

\[
K_\alpha(z) = \frac{\pi}{2 \sin \alpha \pi } [I_\alpha(z) - I_\alpha(-z)]
\]

(19)

we have the Wronskian determinant

\[
\begin{vmatrix}
I_\alpha & K_\alpha \\
I'_\alpha & K'_\alpha
\end{vmatrix}
\]

(20)

Since \( I_\alpha(z) = e^{-\frac{\pi i}{2}} J_\alpha(iz) \) we have

\[
\begin{vmatrix}
I_\alpha & I_{-\alpha} \\
I'_\alpha & I'_{-\alpha}
\end{vmatrix}
\]

(21)

therefore

\[
(k K_{\alpha}'(k y) I_{\alpha}(k \epsilon) - k K_{\alpha}(k \epsilon) I'_{\alpha}(k y))|_{y_0=\epsilon} = -\epsilon^{-1}
\]

(22)

Thus

\[
\frac{\partial}{\partial y_0} G_\epsilon(x, y)|_{y_0=\epsilon} = -x_0^{d/2} e^{d/2-1} \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot (x-y)} \frac{K_{\alpha}(k x_0)}{K_{\alpha}(k \epsilon)}
\]

(23)

Now we study the asymptotic behavior of the l.h.s. of (23). Using

\[
\lim_{z \to 0} z^\alpha K_\alpha(z) = 2^{\alpha-1} \Gamma(\alpha)
\]

(24)

we have in the limit \( \lim z \to 0 \)

\[
\int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot r} K_{\alpha}(k x_0) = 2 \int_{-\infty}^{\infty} dt \int_0^{+\infty} \frac{dk_0}{2\pi} k_0^{\alpha+1} K_{\alpha}(k_0 x_0) e^{i k_0^2 t^2} \int \frac{d^d k}{(2\pi)^d} e^{-i k^2 - i k \cdot r}
\]

(25)

where \( r = x - y \). We write

\[
\int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot r} K_{\alpha}(k x_0) = 2 \int_{-\infty}^{+\infty} dt \int_0^{+\infty} \frac{dk_0}{2\pi} k_0^{\alpha+1} K_{\alpha}(k_0 x_0) e^{i k_0^2 t^2} \int \frac{d^d k}{(2\pi)^d} e^{-i k^2 - i k \cdot r}
\]

(26)

Using

\[
\int \frac{d^d k}{(2\pi)^d} e^{-i k^2 - i k \cdot r} = e^{i\frac{r^2}{2}} \frac{\pi^{d/2}}{(2\pi)^d} \left\{ e^{i\pi d/4} t < 0 e^{-i\pi d/4} t > 0 \right\}
\]

(27)

We have

\[
\int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot r} K_{\alpha}(k x_0) = 2 \frac{\pi^{d/2}}{(2\pi)^d} \int_{-\infty}^{+\infty} dt \int_0^{+\infty} \frac{dk_0}{2\pi} k_0^{\alpha+1} K_{\alpha}(k_0 x_0) t^{-d/2} [e^{-i\pi d/4} e^{i(k_0^2 + r^2)} + c.c]
\]

(28)

Using (formula 3.47-11 in [13])

\[
\int_0^{+\infty} x^{\nu-1} e^{i\beta x^2 x} dx = i\pi^{\nu} e^{-\nu\pi/2} H_{-\nu}^{(1)}(\beta \mu)
\]

(29)
(\(H^{(1)}_{\nu}(\beta \mu)\) denotes the Hankel functions), we have

\[
\int_{0}^{\infty} dt t^{-d/2} e^{i(k_0 + \frac{r}{2})} = \pi \left(\frac{r}{2k_0}\right)^{\nu} e^{-i\nu\pi/2} H^{(1)}_{d/2-1}(k_0 r) = \left(\int_{0}^{\infty} dt t^{-d/2} e^{-i(k_0 + \frac{r}{2})}\right)^* ~ (30)
\]

Therefore the integral

\[
\int_{0}^{\infty} dk_0 \frac{8d/2}{k_0} K_{\alpha}(k_0 x_0) H^{(1)}_{d/2-1}(k_0 r)
\]

is involved. Using (formula 6.576-1 in [13])

\[
\int_{0}^{\infty} dx x^{-\lambda} K_{\mu}(\alpha x) J_{\nu}(bx) = \frac{b^{\nu} \Gamma(\nu - \lambda + \mu + 1) \Gamma(\nu - \lambda - \mu + 1)}{2^{\nu+1} \Gamma(\nu + 1)} F\left(\frac{\nu - \lambda + \mu + 1}{2}, \frac{\nu - \lambda - \mu + 1}{2}; \nu + 1; \frac{b^2}{a^2}\right) ~ (32)
\]

we have

\[
\int_{0}^{\infty} dk_0 \frac{8d/2}{k_0} K_{\alpha}(k_0 x_0) J_{d/2-1}(k_0 r) = \frac{r^{d/2-1} \Gamma(\Delta)}{2 \pi 21^{d/2+\Delta} (1 + r^2)^{2/2}} ~ (33)
\]

where we have used the formula \(F(-\alpha, \beta; \beta; -z) = (1 + z)\). Similarly

\[
\int_{0}^{\infty} dk_0 \frac{8d/2}{k_0} K_{\alpha}(k_0 x_0) J_{d/2-1}(k_0 r) = 0 ~ (34)
\]

where we have used that \(\Gamma(1) = 0\). Therefore

\[
\int_{0}^{\infty} dk_0 \frac{8d/2}{k_0} K_{\alpha}(k_0 x_0) H^{(1)}_{d/2-1}(k_0 r) = \frac{i}{\sin(d/2 - 1)\pi} e^{-i\pi(d/2-1)} \frac{r^{d/2-1} \Gamma(\Delta)}{2 \pi 21^{d/2+\Delta} (1 + r^2)^{2/2}} ~ (35)
\]

Note that \(K_{\nu}\) is real, we have hence

\[
\int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot \varphi} K_{\alpha}(k x_0) = \frac{2^{d/2+\Delta} \pi^{d/2+1} \Gamma(\Delta)}{(2\pi)^d+1} \frac{x_0^{\Delta-d/2}}{(x_0^2 + r^2)^{\Delta}} ~ (36)
\]

Therefore in the limit \(\lim_{\epsilon \to 0}\)

\[
\frac{\partial}{\partial y_0} G_{\epsilon}(x, y)|_{y_0=\epsilon} = -\epsilon^{\Delta-1} c\left(\frac{x_0}{x_0^2 + r^2}\right)^{\Delta} ~ (37)
\]

We thus have the solution to the \(\epsilon\)-boundary problem

\[
\phi(x) = c \epsilon^{\Delta-d} \int d^d y \phi_{\epsilon}(y) \left(\frac{x_0}{x_0^2 + |x - y|^2}\right)^\Delta ~ (38)
\]

Defining the boundary value of \(\phi\) at \(\epsilon = 0\) as

\[
\phi_{\epsilon}(x) = \epsilon^{\Delta-d} \phi_{\epsilon}(x) ~ (39)
\]

we can arrive at the result of [3] obtained in a tricky way.

Using the expression for the \(\epsilon\)-boundary problem

\[
\partial_0 \phi|_{x_0=\epsilon} = 2\alpha c \epsilon^{2\alpha-1} \int d^d y \frac{\phi_{\epsilon}}{|x - y|^{2\Delta}} + ... ~ (40)
\]
we find the value of the free action as
\[ I_{\text{free}} = -\frac{1}{2} \int d^d x d^d y 2 \alpha c c^{2(\Delta - d)} \frac{\phi_0(x) \phi_0(y)}{|x - y|^{2\Delta}} + \ldots \] (41)

Obviously the limit \( \lim_{\epsilon \to 0} \) makes sense. We can readily get the 2-point correlator of the CFT on the boundary
\[ < O(x) O(y) > = \frac{2 \alpha c}{|x - y|^{2\Delta}} \] (42)

3 The sine-Gordon theory

Equation (9) can be written as
\[ \phi(x) = \int_{\partial \Omega} d^d y K(x_0, x; y) \phi(y) + \int_{\Omega} \sqrt{g(y)} G(x, y) J[\phi(y)] \] (43)

where
\[ K(x_0, x; y) = c \frac{x_0^d}{(x_0^2 + x^2)^d} \] (44)

We search for a series solution in terms of powers of \( \beta \) for the sG theory. Defining \( \varphi_n \) by
\[ \varphi_0(x) = \int_{\partial \Omega} d^d y K(x_0, x; y) \phi_0(y) \] (45)
\[ \varphi_1(x) = \int_{\Omega} d^{d+1} y \sqrt{g(y)} G(x, y) J[\varphi_0(y)] \] (46)
\[ \varphi_2(x) = \int_{\Omega} d^{d+1} y \sqrt{g(y)} G(x, y) J[\varphi_0(y) + \varphi_1(y)] \] (47)

et al. then
\[ \varphi(x) = \varphi_0 + \varphi_1 + \varphi_2 + \ldots \] (48)

We assume that this series converges. As an approximation, we take the first two terms
\[ \phi(x) \approx \varphi_0 + \varphi_1 \] (49)

Then the action corresponding to this classical path is
\[ I[\phi] = \int_{\Omega} d^{d+1} x \sqrt{g(x)} \left\{ \frac{1}{2} \nabla^\mu \varphi_0 \nabla_\mu \varphi_0 + \nabla^\mu \varphi_0 \nabla_\mu \varphi_1 + \nabla^\mu \varphi_1 \nabla_\mu \varphi_1 + \frac{m^2}{\beta^2} \left[ 1 - \cos \beta (\varphi_0 + \varphi_1) \right] \right\} \] (50)
\[ = \int_{\Omega} d^{d+1} x \sqrt{g(x)} \left\{ \frac{1}{2} \nabla^\mu \varphi_0 \nabla_\mu \varphi_0 + m^2 (\varphi_0 + \varphi_1)^2 \right\} + \nabla^\mu \varphi_0 \nabla_\mu \varphi_1 + \frac{1}{2} \nabla^\mu \varphi_1 \nabla_\mu \varphi_1 \] (51)
\[ + \frac{m^2}{\beta^2} \left[ 1 - \cos \beta (\varphi_0 + \varphi_1) \right] - \frac{m^2}{2} (\varphi_0 + \varphi_1)^2 \] (52)

We consider the second term
\[ \int_{\Omega} d^{d+1} x \sqrt{g(x)} \nabla^\mu \varphi_0 \nabla_\mu \varphi_1 = \int_{\partial \Omega} \sqrt{h} d^d x n^\mu \varphi_1 \partial_\mu \varphi_0 \] (53)
Since \( G(x, y)_{\partial \Omega_x} = 0 \), we see that \( \varphi_1|_{\partial \Omega_x} = 0 \). Therefore

\[
I[\phi] = I^{(0)}[\phi] + \int_{\Omega} d^{d+1}x \sqrt{g(x)} \{ m^2 \varphi_0 \varphi_1 + \frac{1}{2} m^2 \varphi_1^2 + \frac{1}{2} \nabla^\mu \varphi_1 \nabla_\mu \varphi_1 + \frac{m^2}{\beta^2} [1 - \cos \beta (\varphi_0 + \varphi_1)] - \frac{m^2}{2} (\varphi_0 + \varphi_1)^2 \} \tag{54}
\]

where

\[
I^{(0)}[\phi] = \int_{\Omega} d^{d+1}x \sqrt{g(x)} \frac{1}{2} (\nabla^\mu \varphi_0 \nabla_\mu \varphi_0 + m^2 \varphi_0^2) \tag{55}
\]

We expand the rest part of \( I[\phi] \) in terms of the powers of \( \beta \). Since

\[
\varphi_1(x) = \int_{\Omega} d^{d+1}y \sqrt{g(y)G(x, y)} \sum_{n=1}^\infty \frac{(-1)^n m^2 \beta^{2n}}{(2n + 1)!} \varphi_0(y) \tag{56}
\]

we see that \( \varphi_1 \sim \beta^2, \varphi_0 \varphi_1 \sim \beta^2, \varphi_1^2 \sim \beta^4 \). So the part of order \( O(\beta^2) \) of \( I[\phi] \) is

\[
I^{(2)}[\phi] = \int_{\Omega} d^{d+1}x \sqrt{g(x)} \{ m^2 \varphi_0(x) \int_{\Omega} d^{d+1}y \sqrt{g(y)G(x, y)} (-\frac{m^2}{3!} \varphi_0^3(y) + \frac{1}{4!} \frac{m^2 \beta^2}{3!} \beta^4 \varphi_0) \} \tag{57}
\]

\[
= \int_{\Omega} d^d x_1 d^d x_2 d^d x_3 d^d x_4 \phi_0(x_1) \phi_0(x_2) \phi_0(x_3) \phi_0(x_4) \tag{58}
\]

\[
\left[ \frac{c^4 m^2 \beta^2}{4!} I_4(x_1, x_2, x_3, x_4) - \frac{m^4 \beta^2}{3!} J_4(x_1, x_2, x_3, x_4) \right] \tag{59}
\]

where

\[
I_4(x_1, x_2, x_3, x_4) = \int_{\Omega} d^{d+1}y \frac{y_0^{4\Delta-(d+1)}}{[(y_0^2 + |y-x_1|^2)(y_0^2 + |y-x_2|^2)(y_0^2 + |y-x_3|^2)(y_0^2 + |y-x_4|^2)]^\Delta} \tag{60}
\]

introduced in [3] and

\[
J_4(x_1, x_2, x_3, x_4) := \int d^{d+1}x d^{d+1}y (x_0 y_0)^{-(1+d)} G(x, y) K(x_0, x; x_1) K(y_0, y; x_2) K(y_0, y; x_3) K(y_0, y; x_4) \tag{61}
\]

So we have

\[
< \mathcal{O}(x_1) \mathcal{O}(x_2) \mathcal{O}(x_3) \mathcal{O}(x_4) > = -c^4 m^2 \beta^2 I_4(x_1, x_2, x_3, x_4) + 4m^4 \beta^2 J_4(x_1, x_2, x_3, x_4) \tag{62}
\]

It is seen that the both terms in (62) are of the same order so are equally important. It is a pity that both functions \( I_4 \) and \( J_4 \) are not analytically integrable. The two-point correlator is the same as that for the free \( \phi \) theory and the 3-point correlator vanishes for the sG theory.

### 4 Discussions

In this paper we studied the correlators of CFT on the boundary of AdS_{d+1} corresponding to the sine-Gordon theory in the bulk by the AdS/CFT correspondence. It is found that apart from \( I_4 \) in [3], there is another term which makes a contribution of the same order. So the general consideration for interacting \( \phi \)-theories in [3] is not enough and a more careful consideration is necessary for different specific theories. We stress that in general, the interacting \( \phi \) theory may have more than one solutions to the classical Dirichlet problem and it is to be answered that they lead to the same correlators for
the boundary CFT. It seems that the answer is yes, since perturbatively, the answer is positive and the AdS/CFT correspondence seems to ensure the uniqueness.

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