COOLING MODELS FOR OLD WHITE DWARFS

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ABSTRACT

We present new white dwarf cooling models that incorporate an accurate outer boundary condition based on new opacity and detailed radiative transfer calculations. We find that helium-atmosphere dwarfs cool considerably faster than has previously been claimed, while old hydrogen-atmosphere dwarfs will deviate significantly from blackbody appearance. We use our new models to derive age limits for the Galactic disk. We find that the Liebert, Dahn, & Monet luminosity function yields an age of only 6 Gyr if it is complete to stated limits. However, age estimates of individual dwarfs and the luminosity function of Oswalt et al. are both consistent with disk ages as large as ~11 Gyr. We have also used our models to place constraints on white dwarf dark matter in the Galactic halo. We find that previous attempts using inadequate cooling models were too severe and that direct detection limits allow a halo that is 11 Gyr old. If the halo is composed solely of helium-atmosphere dwarfs, the lower age limit is only 7.5 Gyr. We also demonstrate the importance of studying the cooling sequences of white dwarfs in globular clusters.

Subject headings: Galaxy: fundamental parameters — Galaxy: halo — solar neighborhood — stars: evolution — stars: fundamental parameters

1. INTRODUCTION

The usefulness of white dwarf stars as stellar chronometers was recognized many years ago (Schmidt 1959), but only in the last decade have the observational data and theoretical models reached the level of sophistication necessary to provide meaningful constraints on their parent populations. The determination of the luminosity function of white dwarfs in the Galactic disk (Liebert et al. 1979; Liebert, Dahn, & Monet 1988, hereafter LDM) showed the presence of a turnover at faint magnitudes. The interpretation of this turnover as being a consequence of the finite Galactic age has been used as a constraint on the history of local star formation (Winget et al. 1987; Wood 1992; Hernanz et al. 1994), as have studies of the luminosity function of white dwarfs in common proper-motion binaries (Oswalt et al. 1996).

Tamanaha et al. (1990) used the LDM proper-motion sample to constrain the number density of white dwarfs in the Galactic halo. Motivation for this work was provided by the suggestion (Larson 1986; Silk 1991) that some or all of the dark matter in Galactic halos may be in the form of white dwarfs. This suggestion has recently received a fresh impetus (Adams & Laughlin 1996; Chabrier, Segretain, & Mera 1996; Graff, Laughlin, & Freese 1998) from the results of studies of gravitational microlensing toward the Large Magellanic Cloud (Paczynski 1986; Alcock et al. 1997; Aubourg et al. 1995). The use of white dwarfs to constrain the ages of open (Von Hippel, Gilmore, & Jones 1995; Richer et al. 1998) and globular clusters (Renzini et al. 1996; Richer et al. 1997) has also been pursued with some success.

In addition to “traditional” white dwarfs, the peculiar class of helium core dwarfs that result from binary evolution has received increasing observational (Kulkarni 1986; Van Kerkwijk 1996; Landsman et al. 1997) and theoretical (Vennes, Fontaine, & Brassard 1993; Benvenuto & Althaus 1998; Hansen & Phinney 1998a, 1998b) attention. In particular, they have been used to constrain the ages and properties of millisecond pulsars (Kulkarni 1986; Hansen & Phinney 1998b; Hansen 1998a).

The usefulness of old white dwarfs is not restricted simply to age determination. The cosmologically relevant timescales for white dwarf evolution provide a testing ground for such esoterica as the variation of fundamental constants (Garcia-Berro et al. 1995) and the flux of magnetic monopoles (Freese 1984; Freese & Krasteva 1998).

Given the above list of interesting applications, it becomes important to examine the accuracy of the white dwarf models used in these analyses. In particular, the recent advances in the treatment of white dwarf atmospheres (Bergeron, Wesemael, & Fontaine 1991; Bergeron, Saumon, & Wesemael 1995a, hereafter BSW; Bergeron, Wesemael, & Beauchamp 1995b) suggest the need for an improvement in the treatment of the outer boundary conditions in the cooling models, previously based on the gray atmosphere approximation. The cooling of old white dwarfs is very sensitive to the outer boundary condition (see, e.g., D’Antona & Mazzitelli 1990), so this is an important concern. The primary aim of this paper is to present a detailed set of cooling models based on a proper radiative transfer treatment of the outer boundary condition and to consider the astrophysical implications thereof. In addition, these models represent the first comprehensive set of models to cover the full mass range encompassing both carbon/oxygen and helium core white dwarfs appropriate for cosmochronological purposes.

In § 2 we discuss our atmospheric treatment and the consequences for white dwarf cooling. In § 3 we describe our treatment of the physics in the white dwarf core (in particular crystallization). Section 4 describes a comprehensive set of cooling curves appropriate for the determination of stellar ages, and § 5 discusses the consequences for several of the astrophysical problems outlined above.

2. ATMOSPHERIC TREATMENT

The importance of the atmospheric treatment stems from the strongly constrained thermal profile of an old white dwarf. In the degenerate core the energy transfer is domi-
nated by electron conduction, a very efficient mechanism that keeps the core essentially isothermal. In the outer parts convection dominates, so the temperature profile is determined by the equation of state. The convection extends to the photosphere (Böhm et al. 1977), so there is no “radiative buffer” where adjustments may occur to compensate for changes in atmospheric parameters and thereby keep the core temperature unaffected. Similar considerations arise in giant-planet studies (Guillot et al. 1995). Thus, changes in atmospheric parameters are reflected directly in changes in core temperature and, since white dwarf cooling is driven largely by the slow leakage of the thermal reservoir stored in the core, directly in the cooling rate.

Over the last decade, most white dwarf cooling calculations have used (entirely or in part) the results of Wood (1992, 1995), an offspring of the code of Lamb & Van Horn (1975). The atmospheric treatment used there is based on gray atmospheres and Rosseland mean opacities, using results from the OPAL group (Rogers & Iglesias 1992) where applicable and, for cooler models, the tabulated results of Lenzuni, Chernoff, & Salpeter (1991). The accuracy of the gray approximation is determined by the photospheric opacity. If the opacity is approximately constant over the appropriate wavelength range, then it may be well described by a mean opacity (either Rosseland or Planck) and the emergent spectrum will resemble a blackbody. Once the opacity becomes strongly peaked in some wavelength region, the approximation fails and a proper radiative transfer calculation is required. Another independent cooling code (Benvenuto & Althaus 1999) has recently appeared, but it suffers from the same lack of accurate opacities and radiative transfer at low temperatures.

A simple test of the gray atmosphere approximation is to compare the values of the Rosseland and Planck means (for definitions, see, e.g., Mihalas 1970; Rybicki & Lightman 1979). When the opacity \( \kappa \) is approximately constant with wavelength, the two means are similar in magnitude. If they deviate significantly, then a proper radiative transfer treatment is required. Figure 1 shows the comparison between the two mean opacities as a function of temperature at fixed density (\( \rho = 10^{-9} \text{ g cm}^{-3} \)) for a pure hydrogen atmosphere. The deviation for \( T < 5000 \text{ K} \) is a consequence of the formation of molecular hydrogen.

For the effective temperatures appropriate for the oldest white dwarfs (\( T_{\text{eff}} < 5000 \text{ K} \)), the atmospheric constituents are neutral. The large gravities and associated pressure and temperature gradients of white dwarfs lead to rapid separation of elements in the atmosphere, so the atmospheric constituents are either hydrogen or helium. The distribution of helium and hydrogen layer masses in old white dwarfs is a subject with a long and controversial history (e.g., Kepler & Bradley 1995) and is one of the prime motivations behind the field of white dwarf asteroseismology (Nather 1993). For our purposes, the processes that determine the chemical composition are largely immaterial, as the changes in the relative fractions of DA and non-DA stars (terminology of Sion et al. 1983) occur at temperatures greater than \( 10^4 \text{ K} \), whereas white dwarfs spend most of their cooling ages below this temperature. The relative proportions of hydrogen and helium-atmosphere dwarfs are equal for cool white dwarfs (although Bergeron, Ruiz, & Leggett 1997 suggest that the hydrogen fraction may be higher than previously thought). The actual values of the hydrogen and helium layers are important however, as is the possibility of trace constituents.

Asteroseismological observations of DA stars suggest that many have “thick” hydrogen envelopes, i.e., mass fractions \( \sim 10^{-4} \) (Bradley 1998), and the results are consistent with all DA stars having similar structures (Clemens 1993) although not conclusively so as yet. This value is that expected from stellar evolution calculations (Iben 1984; Schönberner 1983; D’Antona & Mazzitelli 1979; Wood & Faulkner 1986). The mass fractions of helium layers are in the range \( 10^{-6} \) to \( 10^{-2} \) based on asteroseismological (Bradley & Winget 1994; Nitta & Winget 1998) or chemical dredge-up (Pelletier et al. 1986; MacDonald, Hernanz, & Jose 1998) considerations. The composition of the atmospheres inferred from spectroscopic comparisons (BSW; Bergeron et al. 1995b) suggests that a small admixture of helium in hydrogen atmospheres is possible. We shall see that hydrogen atmospheres are insensitive to small admixtures of helium, but that helium atmospheres are very sensitive to small admixtures of hydrogen (essentially, a small amount of hydrogen in a helium atmosphere increases the opacity dramatically, while the converse is not true).

### 2.1. Radiative Transfer

The formation of molecular hydrogen leads to absorption in wide bands centered on the collisionally induced rovibrational transitions of the \( \text{H}_2 \) molecule for \( \lambda > 1 \mu \text{m} \) (for a general review see Borysow & Jorgensen 1999). The result of such strong infrared absorption is to drive much of

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1 Benvenuto & Althaus have also criticized the mass-radius relation of Hansen & Phinney (1998a). Although the level of disagreement is immaterial for the cooling evolution, we note that the mass-radius relations of the models presented here agree with those of Wood at the 1% level.

2 We will show later that the non-DA gap near 5000 K identified by Bergeron et al. (1997) is a simple consequence of the different cooling rates of the two populations.
the emergent flux blueward, causing large deviations from blackbody appearance and significant departures from gray atmosphere behavior (see Fig. 2). Detailed calculations of these effects for white dwarf atmospheres have been performed by Bergeron and collaborators (Bergeron et al. 1991; BSW), and their effects on observational appearance have been demonstrated. However, the cooling calculations have not kept pace with these developments, so the subsequent attempts to derive cooling ages (Bergeron, Ruiz, & Leggett 1997, hereafter BRL; Leggett, Ruiz, & Bergeron 1998, hereafter LRB) are not completely self-consistent.

To properly treat the cooling behavior of white dwarfs with nongray atmospheres, we have performed our own set of radiative transfer calculations. As in the case of our calculation of mean opacities (see Hansen & Phinney 1998a), we have used the opacity microphysics from Lenzuni et al. (1991) with additional collisionally induced absorption cross sections from Zhang & Borysow (1995). Using these opacities, we solve the radiative transfer equation using the Feautrier and Avrett-Krook methods (see, e.g., Mihalas 1970) and including a mixing-length prescription for convection. The numerical treatment of convection can produce convergence problems (BSW; Saumon, Chabrier, & Van Horn 1995) caused by a local minimum in the opacity (as a function of temperature at fixed density) near $T_D \approx 5000$ K. This occurs where the dominant opacity changes from that due to $\text{H}~$to that due to $\text{H}_2$ molecular absorption. This local minimum can cause discontinuous jumps in temperature as a function of optical depth. However, we note from the results of BSW that such discontinuities occur at low optical depth, where the temperature profile is essentially isothermal. Thus, to avoid these convergence problems, we enforce isothermal atmospheres below optical depths $\tau < 0.05$ or above the convection region (if the convective region extends to optical depths that small). This means that the outer part of the grid acts as a single point in the temperature correction procedure but as a full grid when calculating the flux through the atmosphere. Comparison with the results of BSW indicates that this is a robust procedure. For the purposes of comparison between different atmospheres, we use the Rosseland photosphere, i.e., the position in the atmosphere where the Rosseland mean opacity is 2/3.

For a pure hydrogen atmosphere, the photospheric density increases as the white dwarf cools. This trend continues down to $T_{\text{eff}} \approx 3000$ K, after which the photosphere moves again to smaller densities. This occurs because, at these temperatures, the Planck function peaks at precisely those wavelengths where the $\text{H}_2$ absorption is strongest. The resultant increase in the Rosseland mean opacity means a smaller column and, hence, a lower photospheric density are required. For pure helium atmospheres, molecules do not form upon recombination, so the photospheric density continues to increase. This trend is halted only when the densities get high enough that pressure ionization becomes important as a source of electrons and hence opacity. As can be seen from Figure 3, the photospheric densities of hydrogen and helium atmospheres differ dramatically, a consequence of the differences in opacity, as shown in Figure 2.

These results provide the outer boundary condition for the cooling models in the form of tables of temperature and pressure at a specified point in the atmosphere as a function of luminosity and radius (or effective temperature). The exact location of the point at which the condition is

![Fig. 2.—Main opacity contributions at the photosphere for both pure hydrogen $(\rho \approx 10^{-2}$ g cm$^{-3}$) and pure helium $(\rho \approx 100$ g cm$^{-3}$) atmospheres at $T_{\text{eff}} = 4000$ K. Top panel: Large change in opacity for $\lambda > 1$ $\mu$m demonstrates the powerful influence of molecule formation. Middle panel: Planck function at this temperature, which indicates the distribution of flux with wavelength before modification by the radiative transfer.

![Fig. 3.—Positions of the photosphere for pure hydrogen (solid line) pure helium (dotted line) atmospheres and for mixed H/He atmospheres with the indicated mass fractions of hydrogen (dashed lines). The closed and open circles indicate the corresponding quantities from BSW. The label “PPT” indicates the location of the plasma phase transition of Saumon & Chabrier (1992), which corresponds to the pressure ionization of hydrogen. Thus, a correct treatment of pressure ionization is not critical to the calculation of hydrogen atmospheres, although it is very important for cool helium atmospheres.](image-url)
enforced is not very important, usually taken to be the point at which the Rosseland opacity $\tau_R = 100$. Thus, the variations in density resulting from atmospheric composition translate directly into the marked differences in cooling shown in the subsequent sections.

The radiative transfer calculations also provide the emergent spectrum of radiation from these cool dwarfs. As the hydrogen atmospheres cool below 5000 K, they show progressively larger deviations from blackbody appearance (Saumon et al. 1994 and Fig. 4). Helium atmospheres do not show the same dramatic differences but do show some deviations resulting from the fact that a major opacity source at short wavelengths in these atmospheres is Rayleigh scattering from neutral helium atoms (see Fig. 2). Scattering, as opposed to true absorption, simply redistributes radiation in angle, rather than thermalizing it, leading to deviations from blackbody appearances in atmospheres with significant scattering contributions (see, e.g., Mihalas 1970).

The deep convection zones that extend all the way to the photosphere raise the possibility of mixed hydrogen and helium compositions in the atmosphere. As expected, the photospheres lie somewhere in between the pure hydrogen and helium mixtures. The observational appearance, however, deviates quite significantly from either (as noted in BRL), as shown in Figure 4. The strongly wavelength-dependent molecular absorption is strongly affected by the higher densities in atmospheres with reduced amounts of hydrogen, so that mixed hydrogen/helium atmospheres deviate more from a blackbody than pure hydrogen atmospheres. Based on these considerations, BRL find that the vast majority of cool white dwarfs are consistent with pure hydrogen or helium compositions.

At this point it is worth noting what of the above analysis we may consider on solid ground and what is still subject to some uncertainty. The primary purpose of this calculation is to establish the true boundary condition applicable to old white dwarfs whose atmospheres deviate strongly from the gray approximation. For this purpose the results presented here may be considered robust (as confirmed recently by Saumon & Jacobsen 1999) because they essentially rest on establishing the appropriate photospheric densities and temperatures. Even in the most uncertain case, namely, that of very cool helium atmospheres whose atmospheres are dominated by pressure ionization, the results are robust, because pressure ionization occurs only in a limited density range near $\rho \sim 100$ cm$^{-3}$. Note also these results are not sensitive to trace amounts of hydrogen (helium) in otherwise pure helium (hydrogen) atmospheres, because the amounts allowed by the model atmospheres (BRL) are not enough to alter the photospheric densities much.

We will also present detailed calculations of the optical and infrared appearance of these objects, in a similar fashion to the calculations of BSW and BRL. Note that these results are not intended to supplant the previous atmosphere calculations but are rather presented to provide a comprehensive and self-consistent description of the white dwarf cooling models presented below. In particular, we have not treated line formation and broadening in these atmospheres, and thus the determination of individual white dwarf parameters is not possible to the accuracy obtained by BRL. However, we have extended our results to lower temperatures than BSW. The lower temperature limit of 4000 K used by BSW was due to the looming specter of hydrogen pressure ionization at lower temperatures. However, we have noted that the photosphere moves to lower density again below 3000 K because the blackbody peak is located in regions of high molecular opacity (see above).

The calculations of observational appearance are somewhat less certain than the cooling calculations themselves. The general character of the solutions is robust but residual uncertainties regarding convection, conduction, and pressure ionization do introduce some uncertainty. Bergeron et al. (1991) find that atmospheric calculations are insensitive to different parameterizations of mixing length convection. The influence of electron conduction (Kapranidis 1983) is small (BSW) but may become increasingly important at lower temperatures, as are the contributions to the opacity of transitions to higher rotovibrational states in the hydrogen molecules (Zhang & Borysow 1995).

3. CRYSTALLIZATION AND QUANTUM CORRECTIONS

The application of these new boundary conditions represents the primary purpose of this paper. However, some issues regarding the treatment of the core physics also require attention in order to make comparisons with extant models in the literature.

Our code was originally developed to describe low-mass helium core white dwarfs, and the basic details and tests of the code can be found in Hansen & Phinney (1998a). The Coulomb interactions in a helium core are not strong enough to cause crystallization, so no detailed description of this process was included in the original code. However, for stars with carbon and oxygen interiors, crystallization is an important factor in the evolution, and below I describe its implementation in the cooling code.

Crystallization is important for two reasons. The first is that it provides a source of extra energy. Apart from the
release of $\sim kT$ per ion of latent heat of crystallization there may also be an additional energy release associated with the chemical fractionation between solid and liquid phases in a mixed fluid (Stevenson 1980; Garcia-Berro et al. 1988; Segretain & Chabrier 1993). Original estimates of the importance of minor species such as $^{22}$Ne (Segretain et al. 1994; Hernanz et al. 1994) have been reduced with further calculation (Segretain 1996), but the fractionation of the primary constituents carbon and oxygen may prove important, introducing delays of $\sim 1$–2 Gyr in the cooling of the faintest white dwarfs. The exact contribution (or even if there is any at all) is still debatable, however. This is because the continued operation of the separation requires that the oxygen-depleted liquid region remain well mixed, possibly by Rayleigh-Taylor instabilities (Mochkovitch 1983; Isern et al. 1997). Furthermore, the amount of energy released will depend on the chemical stratification of material in the proto–white dwarf core during previous stellar evolution stages, which is itself somewhat uncertain because of uncertainties in the cross section for the $^{12}$C$(\alpha, \gamma)^{16}$O nuclear-burning cross section (see Salaris et al. 1997 and references therein). Thus, we shall compare cooling models with and without the chemical separation energy contribution. It will also be shown that the importance of the above effects is dependent somewhat on the aforementioned boundary conditions, because faster cooling lessens the effect of a given energy release.

The second important effect of core crystallization is the change in the heat capacity due to the formation of a Coulomb lattice. As the star cools further, the heat capacity drops rapidly as the crystal enters the Debye regime where fewer of the normal modes of the lattice are excited. The influence of quantum effects on the heat capacity of the liquid state (Chabrier, Ashcroft, & De Witt 1992) are also included, although, as shown by Chabrier (1993), this correction affects only the more massive white dwarfs.

To implement crystallization (in particular the release of latent heat) with a Heney code (which converges poorly when the physical inputs are discontinuous), we release the latent heat between $\Gamma = 165$ and $\Gamma = 185$, i.e., we consider crystallization to occur at $\Gamma = \Gamma_c = 175$ (Slattery, Doolen, & De Witt 1982; Farouki & Hamaguchi 1993), where $\Gamma$ is the familiar Coulomb coupling parameter. Since the separation energy is also released upon crystallization, we may consider it as an extra latent heat, albeit a function of local composition. The energy per ion released is taken to be (see, e.g., Chabrier 1998)

$$\frac{\Delta u}{kT} = -0.9\Gamma_c \Delta \chi \left( \frac{Z_1^{5/3} - Z_2^{5/3}}{A_1 - A_2} \right) ,$$

where $\Gamma_c = \Gamma/\langle Z \rangle^2$ and $\Delta \chi$ is the difference in composition between the newly crystalline material and the instantaneous liquid composition (which contain ions of mass and charge $A_{1,2}$ and $Z_{1,2}$ respectively). Dividing this by the latent heat yields an “enhancement factor”

$$q = 1 + 1.526 \left( \frac{\Delta \chi}{\langle Z \rangle/6} \right) \left( \frac{\Gamma}{175} \right),$$

where $\langle Z \rangle$ is the mean charge in the precrystallization liquid. Thus, the energy released is approximately 30% more than that released by pure latent heat. We have used the crystallization curves of Segretain & Chabrier (1993) to calculate the crystallization of C/O mixtures assuming uniform mixing in the liquid core region. Although the energy is, in fact, released over much of the core (since the origin is the gravitational binding energy released in the rearrangement of the density profile), approximating it as a localized extra latent heat will not provide a significant source of error because of the efficient heat transport properties of the degenerate, isothermal core.

4. RESULTS

Using the above modifications to the code of Hansen & Phinney (1998a), we have calculated a series of cooling models appropriate to the study of faint white dwarfs. In addition to the major modifications mentioned above, we have included hydrogen burning by the pp-process for the hot models with hydrogen envelopes. This is important for the more massive hydrogen envelope white dwarfs because it sets an upper limit on the mass of hydrogen allowed on the surface of the white dwarf (an effect not included in Wood’s models). This could be important if some of the chemically peculiar white dwarfs owe their compositional idiosyncrasies to dredge-up by deep convection zones. Figure 5 shows the magnitude of this effect. We have also taken account of whether the convection zone gets deep enough to dredge up material directly from deeper layers of different composition. This can serve to limit the allowed thicknesses of helium layers that show small enhancements of carbon.

4.1. Comparison with Other Models

The models of Wood (1992, 1995) are the standard that has been adopted over the last decade for the study of cool white dwarfs. The envelope $L-T_c$ relations from these

![Figure 5](image-url)

Fig. 5.—Hydrogen mass fraction $q_H$ as a function of time for models of several masses, all starting with $q_H = 10^{-4}$. The more massive stars burn more hydrogen because of larger temperatures and pressures at the base of the hydrogen layer.

This limit comes from steady burning only. The limits could be even more stringent if the self-induced novae of Iben & MacDonald (1986) occur.
models are also used in the series of cooling curves published by the European group beginning with Hernanz et al. (1994) and culminating in the paper by Salaris et al. (1997, hereafter SDG). Thus, these models serve as a convenient template with which to analyze the changes introduced by our modifications.

Figure 6 shows a comparison of our models and Wood’s for a standard DA model. The envelope consists of a hydrogen layer of mass fraction $q_H = 10^{-4}$ atop a helium layer of mass fraction $q_{He} = 10^{-2}$. We have calculated models for 0.6 $M_\odot$ and core compositions of pure carbon, pure oxygen, and mixed C/O according to the appropriate profile of SDG. These are compared with the corresponding pure carbon and pure oxygen models of Wood (1995). The inclusion of proper outer boundary conditions leads to a significant ($\sim 2$ Gyr) increase in the cooling ages at low luminosities. The mixed C/O model lies midway between the C and O curves. It should be remembered, however, that this model is $\sim 90\%$ oxygen in the center after crystallization.

Comparison in the helium-atmosphere case is more difficult. The standard Wood model of this type has a helium mass fraction $q_{He} = 10^{-4}$. This was consistent with the determinations of helium envelope mass by Pelletier et al. (1986), who studied the dredge up of carbon from a diffusion tail to make DQ white dwarfs. Recently, MacDonald et al. (1998) found thicker envelope masses of $q_{He} = 10^{-3}$ to $10^{-2}$ in a similar analysis, the deeper envelopes resulting from deeper convection zones because of the use of the OPAL opacities. In our case, our convection zone is even deeper because of the larger photospheric opacity and consequent change in outer boundary condition. Thus, we use as our thin helium envelope a mass fraction $q_{He} = 10^{-3.25}$. As expected, cooling is significantly more rapid for helium atmospheres, because of their lower opacity. These comparisons are shown in Figure 7, which also shows the 0.61 $M_\odot$ cooling curve from SDG. The model is supposedly based on the standard helium-atmosphere models of Wood but resembles much more closely our hydrogen-atmosphere cooling behavior. This incongruity exists for all the papers in the series, suggesting that it is not only the separation energy contribution (which has decreased along the sequence of papers because of the inclusion of progressively more realistic interior models) but rather a mismatch of atmospheric models that lies at the root of the discrepant ages. This is important, because the significantly longer ages found by the European group have been ascribed to their more complete treatment of the crystallization process. It appears that much of the difference are actually due to an inaccurate envelope model.

A more direct comparison of envelope models (largely independent of core physics) is to compare the $L$-$T_c$ (luminosity–central temperature) relations. This is shown in Figure 8 for the 0.6 $M_\odot$ models detailed above. The H and He sequences initially diverge as the atmospheres become neutral but start to converge again when the hydrogen opacities become dominated by H$_2$. We see that the Wood hydrogen models turn over sooner than the nongray models. This is due to both the use of a Rosseland mean opacity instead of a full radiative transfer calculation and also because of the use of Lenzuni et al. (1991) opacities, which are not pure hydrogen. To illustrate the difference we also include in Figure 8 a calculation using the same opacity tables as in our full hydrogen-atmosphere calculation, but using only the Rosseland mean opacities. We see that the turnover is determined primarily by the radiative transfer treatment, while the improved opacity tables become more important at later times and lower luminosities. The $L$-$T_c$ relation used in the series of papers culminating in SDG is shown as the long-dashed curve and can be found in Garcia-Berro et al. (1996) (a factor 0.6 has been applied to
that formula to be appropriate for this comparison). Here we can see why, although it purports to represent a helium-atmosphere model, the cooling behavior looks more like that of our hydrogen atmospheres (this fit was based on the models used by Winget et al. 1987, which used opacities from Cox & Stewart 1965). Recall that $L \propto dT_c/dt$, so the deviation shown here results in slower cooling. Thus cooling models based on this fit are increasingly less accurate for luminosities $\log L/L_\odot < -4$.

Why are the helium atmospheres so discrepant? The problem lies with the extreme transparency of neutral helium. Any approximation that results in an atmosphere containing a small contribution from another, more opaque material will have dramatic consequences on the atmospheric opacities (Fig. 3 shows the effects of only 1% hydrogen by mass). Thus, our helium calculations, which reproduce the photospheric densities of the BSW atmosphere models, are the first that yield cooling ages appropriate for use with modern helium-atmosphere models.

In light of these concerns, we must reevaluate the importance of the release of separation energies. We calculate a 0.6 $M_\odot$ white dwarf with the C/O profile from SDG and both hydrogen and helium atmospheres. In each case we have calculated the model twice—once with the separation included and concomitant release of energy and once with the original profile throughout and no energy release. For the case of the hydrogen atmosphere, the delay is reduced to $\sim 0.2$ Gyr, and the delay is similar for the helium atmosphere. This conforms to what we expect for radiating $\sim 10^{46}$ ergs at $\log L/L_\odot \sim -3.5$. Since, at a given central temperature, our $L-T_c$ relation gives higher $L$ (see Fig. 8), we expect smaller delays than SDG for a fixed release of energy.

The reduced influence of separation energy on cooling time is because $\log L/L_\odot$ upon release is now between $-3.5$ and $-4$, as opposed to extending down to $-4.5$ (see, e.g., Chabrier 1998).

In summary, we need to explore a range of core compositions to constrain the uncertainties in the models. To this end we have calculated our models with hydrogen and helium atmospheres for cores composed of pure Oxygen, pure Carbon and C/O mixtures given by SDG. Figure 9 shows a sequence of such models for both helium and hydrogen atmospheres. The much faster cooling of the helium atmospheres is again evident.

### 4.2. Helium Core Models

Helium core white dwarfs are thought to originate from the truncation of normal stellar evolution by Roche lobe overflow due to the presence of a binary companion (Kippenhahn, Kohl, & Weigart 1967). The cooling of such objects has received little attention until the last few years, when the detections of apparently low-mass objects in large surveys (Bergeron, Saáfer, & Liebert 1992; Bragaglia et al. 1990; Bragaglia, Renzini, & Bergeron 1995; Saáfer, Livio, & Yungelson 1998) and low-mass companions to millisecond pulsars (see, e.g., Kulkarni 1986; Lorimer et al. 1995; Lundgren, Foster, & Camilo 1996) spurred theoretical efforts (Benvenuto & Althaus 1998; Hansen & Phinney 1998a, 1998b).

The cooling of helium core white dwarfs is conceptually similar but contains important differences. The first is that the heat content stored in the nondegenerate ions ($\propto kT M/Am_p$) is proportional to the total number of ions for fixed temperature. This means that helium cores ($mass number A = 4$) contain more heat than do carbon or oxygen cores ($A = 12$ and 16), and thus the helium core dwarfs are brighter at fixed age. Other more subtle differences result from the lower mass and hence lower gravities...
and concomitantly larger nondegenerate layers (and larger radii), which result in deeper convection zones and cooler atmospheres. Figure 10 shows the comparison between our CO and pure helium core sequences. The helium cores are distinctly brighter.

Observed helium white dwarfs usually reside in binaries (however, see Maxted & Marsh 1998) and thus may have potentially observable companions. If the helium dwarf results from the original secondary, then it will have an older C/O white dwarf as companion (or a millisecond pulsar if the original primary was massive enough). Although the helium core dwarfs are brighter, the presence of a fainter companion can still influence the observed parameters. Consider the example of a 0.3 \( M_\odot \) helium core dwarf cooling alongside a companion of 0.6 \( M_\odot \) that has a cooling age 1 Gyr older (being born from the more massive star). If the pair is unresolved then their fluxes are simply added. When the helium dwarf is older than 3 Gyr the effective temperatures of the two stars are similar (and remain so for ages up to \( \sim 7 \) Gyr). Thus, the combined object will appear to be at the same temperature (since the spectral shape is not sensitive to the gravity) although brighter than it should be, yielding an inferred single object mass \( \sim 0.21 \ M_\odot \). This illustrates the problem of interpreting the apparently “overluminous” dwarfs found in proper-motion surveys, where the presence of a fainter companion can bias the interpretation. Note that this problem does not arise for millisecond pulsar binaries, as addressed in Hansen & Phinney (1998b), because the dynamical and evolutionary constraints assure the presence of a single body.

The determination of ages for low-mass white dwarfs is further complicated by the possibility of much larger hydrogen masses on the lowest mass dwarfs (Webbink 1975; Driebe et al. 1998), although this is subject to significant uncertainties in the treatment of wind mass loss and shell flash burning (Iben & Tutukov 1986; Driebe et al. 1998).

One possible constraint on these hydrogen masses may be obtained by reversing the age arguments in millisecond pulsar binaries to constrain the ages of the white dwarfs using the pulsar timing ages (Hansen 1999).

5. APPLICATIONS

5.1. Disk White Dwarf Luminosity Function I

The existence of a long-suspected (Schmidt 1959) edge to the white dwarf luminosity function was demonstrated by Liebert et al. (1979) and Liebert et al. (1988). Winget et al. (1987) derived the first age limits using comparisons with theoretical models. Thereafter, increasingly sophisticated theoretical models were used by Iben & Laughlin (1989), Yuan (1989), Wood (1992), Hernanz et al. (1994) and Salaris et al. (1997) to further refine the age of the star-forming component of the Galactic disk, all using the luminosity function derived by LDM.

The above determinations of an age from the luminosity function are sensitive to several theoretical and observational uncertainties. The existence of the turnover is well established, but the theoretical interpretation is sensitive to the rather uncertain bolometric corrections for the cool white dwarfs at such faint magnitudes. These uncertainties have been the subject of recent theoretical (BSW) and observational (BRL; LRB) investigations that have reduced the errors in the LDM sample considerably. However, other proper-motion samples of Evans (1992) and Oswalt et al. (1996) have found different turnovers at faint magnitudes, which make the interpretation of the LDM turnover rather uncertain. Ongoing proper-motion surveys in the southern hemisphere (Ruiz & Takamiya 1995) also find higher densities of faint white dwarfs than expected from the LDM results. Nevertheless, the LDM sample is easily the best studied example and will form the basis of our analysis below.

There are also several theoretical uncertainties in the age determination, most comprehensively outlined by Wood (1992). To determine an age from the luminosity function requires a prescription for the star formation history, including the initial mass function, the main-sequence–white dwarf mass function, and the cooling curves (Wood 1992). It also requires an assumption about the relative populations of hydrogen and helium atmospheres in the white dwarf sample. Most authors assume the white dwarfs are either exclusively hydrogen atmosphere (Wood 1995) or exclusively helium (Wood 1992; Hernanz et al. 1994; SDG). Below we shall demonstrate how our cooling curves affect results under both assumptions and then examine how well justified such idealizations are.

We calculate the differential luminosity function (space density per unit luminosity)

\[
\frac{\partial \Phi}{\partial \log L/L_\odot} = \int_{M_t}^{M_M} \Psi(t) \xi(M) \frac{\partial \tau_{cool}}{\partial \log L/L_\odot} \frac{dM_{wd}}{dM} \ dM,
\]

where \( \Psi \) and \( \xi \) are the star formation rate and initial mass function, respectively. The white dwarf–main-sequence mass relation is \( dM_{wd}/dM \), and \( \partial \tau_{cool}/\partial \log L/L_\odot \) describes the cooling of the white dwarfs as a function of mass and composition. Our default values used below include a constant star formation rate, a Salpeter mass function, and a white dwarf–main-sequence mass relation based on that of Wood (1992), \( M_{wd} = 0.49 \ M_\odot \exp(0.096 M) \). The cooling time \( \tau_{cool} \) is related to the total stellar age by \( \tau_{cool} = \frac{\partial \tau_{cool}}{\partial \log L/L_\odot}. \)
Our default models do not include inflation of the stellar scale height with age. To compare with the data, this quantity must then be integrated over appropriate luminosity bins.

The LDM observational sample was chosen from the Luyten Half Second sample (Luyten 1979). The proper-motion limits are \( 0.8 \, \text{yr}^{-1} < \mu < 2.5 \, \text{yr}^{-1} \) and for \( R < 18 \). For Galactic disk white dwarfs, the observational sample is essentially proper-motion limited. The turnover in the luminosity function occurs near \( M_V \sim 16 \), while observations down to \( M_V \sim 19 \) were possible. However, the \( V/V_{\text{max}} \sim 0.37 < 0.5 \) for this sample (LRB), indicating that the sample is not complete. Based on the detectability of other types of stars down to \( M_V \sim 19 \), LDM claim that the incompleteness is not severe. Nevertheless, this makes quantitative estimates somewhat uncertain. The common proper-motion sample of Oswalt et al. (1996, hereafter OSWH) also has \( V/V_{\text{max}} < 0.5 \) but contains a correction for incompleteness not used by LDM. This results in a somewhat larger effective sample volume and a more gentle turnover.

Figure 11 shows the comparison of our computed luminosity functions (both for pure hydrogen and pure helium atmospheres) with the two observational data sets. We see that, for hydrogen atmospheres, the LDM sample yields an age of \( 8 \pm 1 \) Gyr and the OSWH sample yields \( 9.5 \pm 1.5 \) Gyr. The helium models yield \( 5.5 \pm 0.5 \) and \( 8 \pm 2 \) Gyr, respectively. These models were calculated with mixed C/O profiles. Pure oxygen models reduce the estimates by \( \sim 1 \) Gyr, and pure carbon core models increase them by \( \sim 0.5 \) Gyr. The agreement with the conclusions of OSWH is not surprising given that the nongray effects that distinguish the current models from those of Wood really make a difference only below \( T_{\text{eff}} \sim 5000 \, \text{K} \), corresponding to \( \log L/L_\odot < -4.1 \). However, for questions regarding dwarfs of greater age than that of the peak, the difference will become appreciable. The faster cooling of helium atmospheres results in a much broader peak in the luminosity function and a fit to much younger disk ages than previously derived. Thus, previous ages based on helium-atmosphere models are considerably overestimated.

We have seen that the choice of atmosphere composition makes a difference \( \sim 2 \) Gyr in the inferred ages. We know that both types are present in the number counts, so it makes sense to examine hybrid luminosity functions. As a prelude to that, we now examine the properties of the individual dwarfs that define the peak and turnover of the LDM luminosity function.

### 5.2. Individual Old White Dwarfs

The application of detailed atmospheric models has led to the determination of individual gravities and effective temperatures for many of the cooler dwarfs (BRL), including those in the LDM sample (LRB). The determination of masses and compositions allows us to investigate the ages of individual stars.

The most interesting stars are obviously those that define the turnover of the LDM luminosity function. We will consider the bolometric luminosity function given by LRB (the different bolometric corrections for hydrogen and helium atmospheres means that different stars may inhabit the

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**Fig. 11.** Left panels: Comparison of the Liebert et al. data with our (upper panel) hydrogen and (lower panel) helium atmosphere luminosity functions. The open circles are the data of Fleming et al. Right panels: Same comparison but with the common proper motion data of Oswalt et al. The curves for the hydrogen models are for disk ages of 6, 8, 10, and 12 Gyr. The helium curves include a 4 Gyr curve as well.
faintest luminosity bin for \( M_V \) and \( M_{bol} \) luminosity functions). The faintest luminosity bin defined by LDM contained three stars, 1300 + 263, 2251 – 070, and 2002 – 110. All three are classified as helium-atmosphere dwarfs, with masses determined by BRL and LRB to be
\[
M = 0.72 \pm 0.11 \, M_\odot, \quad M = 0.82 \pm 0.03 \, M_\odot, \quad \text{and} \quad M = 0.78 \pm 0.01 \, M_\odot
\]
respectively. The effective temperatures are all between 4500 and 5000 K, which yield ages of \( 4.8 \pm 0.2, 4.7 \pm 0.2, \) and \( 4.5 \pm 0.1 \) Gyr respectively. These are considerably less than those quoted by BRL and LRB because they used the helium-atmosphere models of Wood, whose outer boundary conditions are not consistent with the atmospheric determinations at these low temperatures.

The next coolest bin contains eight stars, five helium- and three-hydrogen atmosphere dwarfs. However, two of the hydrogen-atmosphere dwarfs have low inferred gravities, suggesting that they may be helium core white dwarfs with masses \( \sim 0.3–0.4 \, M_\odot \). Another possible interpretation is that they are unresolved binaries, which thus appear overluminous (and yield a lower gravity in the BRL and LRB analyses). Interpreting these two objects as either individual helium dwarfs or as an unresolved binary containing two identical more massive dwarfs (of same effective temperature as inferred) yields ages \( \sim 10 \) Gyr in both cases. The only “normal” hydrogen-atmosphere dwarf in this bin is 1108 + 207, with a mass \( 0.63 \pm 0.07 \, M_\odot \) and inferred age \( 6.3 \pm 0.8 \) Gyr. All the helium-atmosphere dwarfs in this bin have ages between 4 and 5 Gyr. Figures 12 and 13 show all the dwarfs in the LDM sample, with parameters taken from LRB, along with appropriate cooling curves and age curves. Table 1 shows the individual ages for the faintest objects.

Several features should be noted about Figures 12 and 13. The early crystallization and subsequent rapid cooling of the more massive models cause the isochrones to “bend over” at the top of the diagram, as noted by several authors before. Note also the discontinuous nature of the isochrones between C/O and helium cores, which is caused by the greater heat capacity of a core composed of smaller ions, thereby making the helium core dwarfs brighter than C/O dwarfs of comparable mass.

Figure 12 shows that the hydrogen-atmosphere dwarfs also show a concentration of stars with ages \( \sim 4 \) Gyr, with only 1108 + 207 of the normal dwarfs having an age greater than 5 Gyr. These stars were not mentioned above because they are brighter than the helium dwarfs; i.e., the fainter magnitude bins are dominated by helium-atmosphere dwarfs. Indeed, of the 11 stars in the faintest two bins, eight were helium-atmosphere dwarfs, despite the fact that the LDM sample contains approximately equal numbers of hydrogen- and helium-atmosphere dwarfs (22 hydrogen and 20 helium). The helium-atmosphere dwarfs in Figure 13 also show a concentration at similar ages, although at lower effective temperatures, because of their more rapid cooling. Thus, the “non-DA gap” identified by BRL is simply a result of the different cooling rates, rather than being due to any chemical evolution. These figures also suggest that the ages inferred from our helium model luminosity function are more accurate, primarily because the fainter magnitude bins contain more helium-atmosphere dwarfs than hydrogen dwarfs. The ages of the apparently overluminous hydrogen-atmosphere dwarfs are discrepant with this picture but have to be regarded with caution because they

### Table 1

| Name          | \( \pi \) | \( V \) | \( V-I \) | H/He | \( M (M_\odot) \) | \( T_{eff} \) | Age (Gyr) | \( \Delta \) (Gyr) |
|---------------|---------|--------|--------|------|----------------|-------------|------------|-----------------|
| 1300 + 263.   | 28 + 3  | 18.77  | 1.28   | He   | 0.72 \pm 0.11 | 4539 \pm 50 | 4.8 \pm 0.2 | -4.7            |
| 2251 – 070.   | 124 + 4 | 15.71  | 1.15   | He   | 0.82 \pm 0.03 | 4590 \pm 70 | 4.7 \pm 0.1 | -5.0            |
| 2002 – 110.   | 58 + 1  | 16.95  | 1.09   | He   | 0.78 \pm 0.01 | 4813 \pm 54 | 4.5 \pm 0.1 | -4.5            |
| 1247 + 551.   | 40 + 1  | 17.79  | 1.45   | H    | 0.33 \pm 0.01 | 4000 \pm 70 | 9.8 \pm 0.3^a | +5.7            |
| 1108 + 207.   | 38 + 1  | 17.70  | 1.07   | H    | 0.63 \pm 0.07 | 4640 \pm 160 | 6.3 \pm 1.0  | -1.0            |
| 0747 + 073A.  | 55 + 1  | 16.96  | 1.26   | H    | 0.39 \pm 0.01 | 4166 \pm 81 | 10.8 \pm 0.4^a | +6.1            |
| 2316 – 065.   | 32 + 4  | 18.15  | 1.16   | He   | 0.69 \pm 0.11 | 4747 \pm 53 | 4.7 \pm 0.1  | -3.9            |
| 0552 – 041.   | 155 + 2 | 14.47  | 0.98   | He   | 0.78 \pm 0.01 | 5080 \pm 60 | 4.3 \pm 0.1  | -3.9            |
| 0747 + 073B.  | 55 + 1  | 16.63  | 1.08   | He   | 0.60 \pm 0.01 | 4871 \pm 54 | 4.1 \pm 0.1  | -2.7            |
| 2054 – 050.   | 65 + 5  | 16.69  | 1.32   | He   | 0.63 \pm 0.08 | 4630 \pm 50 | 4.5 \pm 0.1  | -3.8            |
| 1444 – 175.   | 69 + 4  | 16.44  | 1.01   | He   | 0.83 \pm 0.05 | 4990 \pm 60 | 4.3 \pm 0.1  | -4.2            |

**Notes.**—These values assume mass fraction \( q_H = 10^{-4} \) for hydrogen atmospheres and \( q_{He} = 10^{-3.25} \) for helium atmospheres. The last column indicates the age difference we infer here with respect to that given in Table 2 of Leggett et al. 1998.

^a These ages assume that the observations are uncontaminated by companion light.

**Fig. 12.**—Hydrogen-atmosphere dwarfs in the LDM sample. The dotted lines are cooling curves for C/O core models, and the short dashed lines are for helium core models. The solid lines are isochrones while the two long dashed curves indicate the region over which core crystallization takes place. The concentration of stars in the region near 4 Gyr is apparent.
Once again, the dotted lines indicate cooling curves for C/O models and the solid lines are isochrones. The dashed lines indicate the region over which core crystallization takes place. The non-DA gap alluded to by BRL is evident here between 6000 and 5000 K.

may be unresolved binaries, and the combination of two different atmospheres is a nonlinear process and makes the inversion to determine the parameters rather difficult.

With individual mass determinations we may now simply use the oldest white dwarf as a constraint on the Galactic disk age. Neglecting the possibly overluminous dwarfs, the oldest dwarf in the LDM sample is 1108 ± 207, with an age 6.3 ± 0.8 Gyr. For individual dwarf constraints we need not bother with completeness concerns, and we can turn to the larger sample of BRL, in which the oldest star is the hydrogen-atmosphere dwarf 1310 ± 472, which has mass 0.63 ± 0.03 M☉ and age 8.4 ± 0.3 Gyr (white dwarf cooling age only). If one considers the apparent low-mass dwarfs at face value as helium core dwarfs, the oldest of these is 0747 ± 073A, with a mass ~0.4 M☉ and an age 10.8 ± 0.4 Gyr.

5.3. Disk White Dwarf Luminosity Function II

The above is a very simple constraint, and there is much more information to be gained from applying a detailed analysis to the full luminosity function. However, we have demonstrated that such an exercise really must be done for both hydrogen and helium atmospheres separately. The fact that both DA and DB samples have concentrations of white dwarfs at similar ages ~4.5 Gyr, although at different temperatures, supports the claim that these two populations of white dwarfs really are distinct and remain that way, cooling at different rates. This fact must also be taken into account when attempting to analyze the chemical evolution of white dwarfs. Thus, the theoretical luminosity function must take account of this differential cooling. Figure 14 shows the comparison of the data with a luminosity function in which it is assumed that 50% of all white dwarfs are born with hydrogen and 50% with helium atmospheres that then cool according to their respective cooling tracks. While this is obviously in conflict with observations of more luminous white dwarfs, it is consistent with the observations for white dwarfs with $T_{\text{eff}} < 10^4$ K, i.e., over the entire LDM sample (and most of the cooling age is spent in this range). In this case, comparison with the observations provides an estimated disk age of 6.5 ± 1 Gyr for the LDM sample. The more gradual turnover of the OSWH sample is consistent with a wide range of ages from 6 to 11 Gyr. The inclusion of both kinds of white dwarfs allows for a peak in the luminosity function at log $L/L_\odot$ ~ −4 (as observed) while maintaining a more gradual turnover than would occur with pure hydrogen models. This provides an acceptable fits over a wide range of ages.

Another test of the white dwarf models and stellar evolution history used to construct the luminosity function is to compare the observed fraction of helium to hydrogen white dwarfs in different magnitude bins. Using the simple 50/50 split with constant star formation rate assumed in Figure 14 we have calculated the “DB fraction” in four luminosity bins. This is shown in Figure 15. The theoretical curves of different ages are compared to the weighted fractions calculated for the LDM sample using the $M_{\text{bol}}$ and $1/v_{\text{max}}$ weights of LRB. The error bars are obtained by removing the single largest contributor to either helium and hydrogen-atmosphere fractions in each magnitude bin. We see that the general trend of rising helium fraction at low luminosities is consistent with the 6 Gyr curve, although the brightest point appears to be discrepant with all theoretical curves. The equivalent numbers from the OSWH sample are not shown for two reasons. The first is that the atmospheric compositions have not been studied in the same detail (making assignment to hydrogen and helium bins less certain), and the second is that the large completeness corrections used by Oswalt et al. (1996) result in each lumi-
nosity bin being dominated by a single object. Given the uncertainty in the completeness of the LDM sample, the significance of the agreement is not clear. However, we have seen that it is essential to take account of both hydrogen and helium cooling rates separately, and such a comparison offers the possibility of constraining the relative populations of hydrogen and helium in future analyses.

To conclude, the comparison of the models presented here with the most well-studied white dwarf proper-motion sample (LDM) suggests an age of only \( \sim 6.5 \pm 1 \) Gyr for the stellar populations in the solar neighborhood, somewhat less than other recent determinations. This difference is primarily due to our updated helium-atmosphere opacities, because the faintest stars in the LDM sample are of this type. However, the completeness of the LDM sample at faint magnitudes has been questioned (Ruiz & Takamiya 1995; Oswalt et al. 1996), so this should really be considered only a lower limit. Comparison with the OSWH sample of Oswalt et al. (1996) suggests a much larger age range. However, the white dwarfs in these systems have not been studied to the same detail as those in the LDM sample, and, in some cases, uncertainties exist about their exact luminosities and chemical compositions (see LRB). Nevertheless, this age is consistent with the inferred ages of individual helium core objects in the LDM sample as well as with the oldest hydrogen-atmosphere C/O core dwarfs studied by BRL. Clearly a larger proper-motion sample is needed to confirm these conclusions.

5.5. Halo White Dwarfs

White dwarfs have been considered as baryonic dark matter candidates for many years (Larson 1986; Silk 1991; Carr 1994) but have received special attention recently resulting from the observation of microlensing toward the Large Magellanic Cloud (Paczynski 1986; Alcock et al. 1997). This has resulted in several papers attempting to constrain the white dwarf halo scenario using various observations of the presumed local contribution (Adams & Laughlin 1996; Chabrier et al. 1996; Graff, Laughlin, & Freese 1998; Isern et al. 1997). Unfortunately, none of these papers uses white dwarf models that remain accurate to the ages required to produce a proper constraint. In particular, the last three used the results of Hernanz et al., which we have seen are very inaccurate for halo ages.

The two observational data sets commonly used are the halo contribution to the LDM white dwarf luminosity function and the Hubble Deep Field (Williams et al. 1996) point-source number counts (Flynn, Gould, & Bahcall 1996; Elson, Santiago, & Gilmore 1996; Mendez et al. 1996). As we have noted above, the completeness of the white dwarf luminosity function at faint magnitudes is somewhat uncertain, making inferences based on nondetections particularly troubling. Nevertheless, it is illustrative to examine the constraints.

As noted by Graff et al. (1998), the larger relative velocities of the halo sample make the upper cutoff in proper motion (2.5 yr\(^{-1}\)) quite important. As the white dwarfs fade, the magnitude limited distance starts to approach the minimum distance set by this upper proper-motion cutoff, leaving little volume available for detection. To examine the constraints imposed by the LDM luminosity function, let us first assume it is complete to the stated limit of \( R = 18 \). We shall adopt the velocity distribution used by Alcock et al. (1997) in deriving the characteristic MACHO mass, namely, a three-dimensional Maxwellian distribution with isotropic velocity dispersion \( \sigma = 150 \text{ km s}^{-1} \). Using a circular velocity of 220 km s\(^{-1}\), we determine the effective volume probed

The value quoted in Hansen & Phinney is 6.3 Gyr and was obtained using hydrogen-atmosphere models. Recent more sophisticated models (Hansen 1998a), including helium atmospheres, allow a slightly lower cooling age.

Graff et al. (1998) state that they use a dispersion \( \sigma = 270 \text{ km s}^{-1} \). However, this is their three-dimensional rms velocity, so that their model is actually the same as that used by the MACHO group (D. Graff, personal communication).
The dwarf sequence in the old globular cluster M4 to be consistent with $\sim 0.0078$ pc.

We will consider a population of white dwarfs that rotate with the circular velocity but have velocity dispersions of 50 and 135 km s$^{-1}$ which again suggests some kind of stellar relic. To place limits on such a population, we shall consider a population of white dwarfs that rotate with the circular velocity but have velocity dispersions of 50 and 135 km s$^{-1}$ (to match the Gates et al. scale heights of 1.5 and 2.5 kpc, respectively). These limits are also shown in Figure 16. Note that $f_{h}$ is actually the fraction of the local dark matter density in old white dwarfs, so the constraints can also be applied to this case. We see in Figure 16 that the 50 km s$^{-1}$ model is ruled out for ages less than 15 Gyr.

We will again argue that, given the uncertainties in the completeness, $f_{h}$ should be reduced by a factor of 5. This allows a somewhat weaker constraint on hydrogen white dwarfs $\sim 11.5$ Gyr. There are several uncertainties underlying these numbers, but they serve to illustrate that, given the uncertainties in the completeness of the LDM sample at faint magnitudes, it is quite possible to make a white dwarf halo model consistent with the data. Given the blueward shift of hydrogen white dwarfs at late times, one must also worry about color selection in these proper-motion samples. This is probably not a problem for studies of the disk luminosity function because the faintest magnitude bins are dominated by helium-atmosphere dwarfs (which remain red).

An alternative model for the microlensing results involves a dark thick disk or spheroid (Gates et al. 1998). The greater concentration toward the disk allows one to obtain the required optical depth to microlensing with a smaller fraction of the dynamical halo mass. The estimated mass of the lens is still $\sim 0.2-0.4 M_{\odot}$, which again suggests some kind of stellar relic. To place limits on such a population, we shall consider a population of white dwarfs that rotate with the circular velocity but have velocity dispersions of 50 and 135 km s$^{-1}$ (to match the Gates et al. scale heights of 1.5 and 2.5 kpc, respectively). These limits are also shown in Figure 16. Note that $f_{h}$ is actually the fraction of the local dark matter density in old white dwarfs, so the constraints can also be applied to this case. We see in Figure 16 that the 50 km s$^{-1}$ model is ruled out for ages less than 15 Gyr.

The constraints from the Hubble Deep Field (HDF; Williams et al. 1996) are weaker but more robust (Hansen 1998b). The HDF is a very deep but extremely narrow spatial sample. Since we have no proper motions for the detected objects, we have to rely on number counts and color selection to constrain the white dwarfs. Figure 17 shows the $V-I$ versus $V$ plot for all the point sources identified by Flynn et al. (1996), Elson et al. (1996), and Mendez et al. (1996). Also plotted are the expected positions of 0.6 $M_{\odot}$ white dwarfs of hydrogen and helium atmospheric compositions. We see that the HDF does not provide any meaningful constraints on helium-atmosphere dwarfs (because of their rapid cooling). Hydrogen-atmosphere dwarfs should be observable within $\sim 2$ kpc, and their colors in fact correspond well with the main group of faint blue point sources detected by Elson et al. (1996).

This population was originally ruled out because of the assumption that all old white dwarfs become redder with age, whereas we have shown above that hydrogen white dwarfs eventually turn toward the blue again because of the molecular hydrogen in their atmospheres.

However, this is far from conclusive. Richer et al. (1997) find the white dwarf sequence in the old globular cluster M4 to be consistent with 0.51 $M_{\odot}$.
Having determined that white dwarfs could be present, we also need to determine whether there are enough point sources. Helium dwarfs cool too rapidly to be of interest down to the magnitude limit, so we consider only hydrogen-atmosphere dwarfs. The HDF covers a field of view of $\Omega = 4.4$ arcmin$^2$. Thus, the volume probed out to a given distance $d$ determined by a magnitude limit of $m_V$ is

$$V = \frac{\Omega}{3} 10^{0.6(m_V - M_V) + 3} \text{ pc}^3,$$

where $M_V$ is the absolute magnitude of the population of objects. Given a local density $\rho$ of dark matter and a characteristic dwarf mass $m_{\text{wd}}$, we may predict a number

$$N \sim \frac{\rho}{m_{\text{wd}}} \frac{\Omega}{3} 10^{0.6(m_V - M_V) + 3}$$

of objects above a given magnitude limit (where $f$ is the fraction of the dark mass stored in such objects). Assuming $\rho \sim 0.0078 M_\odot \text{ pc}^{-3}$ as above, completeness limit $m_V \sim 28$, and a hydrogen white dwarf population of age $\sim 14$ Gyr ($M_V \sim 17.7$), we have $N \sim 2.1f$. Given that the microlensing results suggest $f \sim 0.5$ and that we are considering only the hydrogen atmospheres (estimated to be $\sim 0.5$ for disk dwarfs), this implies $N \sim 0.53$. Since several blue point sources were detected, the HDF does not place much of a constraint on very old hydrogen-atmosphere dwarfs either. A similar conclusion was reached by Graff et al. (1998), who found that the constraints from the LDM sample were more restrictive. However, as we saw above, concerns exist about the completeness of the LDM sample. The HDF results suggest that proper motions are essential to detect old hydrogen-atmosphere dwarfs, since there will be an abundance of extragalactic objects of similar magnitude and color in deep images (Elson et al. 1996; Hansen 1998b).

5.6. Globular Clusters

The existence of a 12 Gyr old white dwarf dark halo is still somewhat uncertain. However, the mean age of the Galactic globular cluster system is estimated to be $11.5 \pm 1.3$ Gyr (Chaboyer et al. 1998), so that the white dwarf sequences in these objects offer the potential to test the cooling curves derived above and verify the constraints they place on the detection of dark halo dwarfs as well as offering another method for globular cluster age determination. The large deviations from blackbody colors shown by old hydrogen-atmosphere white dwarfs suggests that sufficiently deep observations of the white dwarf sequence in globular clusters should make it possible to observe a splitting in the white dwarf cooling sequences near $M_V \sim 17$ into hydrogen and helium parts. Interestingly, the observations of M4 (Richer et al. 1997) reach almost this far (to white dwarf ages $\sim 9$ Gyr). With deeper observations, it should be possible to examine this splitting, which may provide a more tangible estimate of white dwarf age than a simple cutoff, whose veracity is a function of completeness (see Fig. 18). Furthermore, given the very small amount of gas found in globular clusters, the accretion history of globular cluster dwarfs is likely to be quite similar to that of halo dwarfs. If accretion of interstellar material has any effect on the relative abundances of hydrogen and helium in white dwarf atmospheres, the determination of relative proportions of hydrogen and helium atmospheres among the globular cluster dwarfs will be useful in comparing the disk population to the putative halo population.

However, the exact nature of the hydrogen cooling tracks will be sensitive to the main-sequence–white dwarf mass relation. For a population of coeval stars, more massive white dwarfs are born first and will thus have longer cooling ages. Thus, the white dwarf mass will vary along the cooling sequence in a globular cluster. For the purposes of current...
This paper describes the construction of a new set of white dwarf cooling models appropriate for the study of very old white dwarfs. The principal advance is the incorporation of new opacities and an accurate outer boundary condition based on proper radiative transfer calculations in the white dwarf atmosphere. These models allow us to study in detail the differences in cooling between dwarfs with hydrogen and helium atmospheres and to extend the cooling sequences to ages $\sim 15$ Gyr, suitable for studying the oldest white dwarfs in our Galaxy. Our models also incorporate a variety of core compositions, allowing us to compare cooling ages for low- and high-mass white dwarfs.

We have used the new models to study several important questions concerning white dwarf evolution and its consequences. The new cooling curves reduce the possible importance of the separation energy (Garcia-Berro et al. 1998; Segretain et al. 1994; SDG) because the energy is released at higher luminosities; hence, the delay introduced by a fixed energy release is smaller. We have also shown that the non-DA gap between 5000 and 6000 K identified by Bergeron et al. (1997) may simply be a consequence of the different cooling rates of hydrogen and helium atmospheres, because the hydrogen-atmosphere dwarfs above the gap are of the same age as the helium-atmosphere dwarfs below the gap.

We can place constraints on the age of the Galactic disk using the luminosity function of local proper-motion stars (Liebert et al. 1988; Leggett et al. 1998) and common proper-motion binaries (Oswalt et al. 1996) using the new models. The LDM sample yields a very small age $\sim 6.5 \pm 1$ Gyr if one assumes that the sample is complete. This is also consistent with the age determinations for individual stars in the sample, except for two overluminous hydrogen-atmosphere dwarfs, which may be low-mass helium-core dwarfs. The latter yield ages $\sim 10-11$ Gyr, which is larger than the LDM limits but consistent with the wider allowed age range we derive from OSWH luminosity function and individual ages for some dwarfs not in the LDM sample. Our models are consistent with ages considerably less than some derived recently in the literature, primarily because of our updated opacities and the recognition that helium-atmosphere dwarfs cool much faster than hydrogen-atmosphere dwarfs. However, the determination of disk ages from the white dwarf luminosity function will not be conclusive until several outstanding issues regarding the completeness of the luminosity function at the faint end have been addressed. This may also be linked to the kinematic evolution of the white dwarf population (Garcia-Berro et al. 1999).

Despite uncomfortable chemical evolution constraints (Gibson & Mould 1997; Fields, Mathews, & Schramm 1997), white dwarfs remain a favored candidate for halo dark matter. We have used our new models to demonstrate that previous attempts to constrain such populations directly (Adams & Laughlin 1996; Chabrier et al. 1996; Graff, Laughlin, & Freese 1998; Isern et al. 1997) were too severe in their constraints because of inaccuracies in the cooling models. We have presented conservative estimates of the constraints on such models and find that it is possible to make consistent white dwarf halo models with ages no larger than those of the globular clusters (based on direct detection criteria; chemical evolution constraints persist).

We have also presented predictions for the colors and luminosities of the putative population. For white dwarfs older than $\sim 9$ Gyr, a full radiative transfer treatment of the outer boundary condition is very important.

Deep proper-motion surveys for old white dwarfs can add much to our current understanding of Galactic structure and formation. However, to obtain the most information, it will be important to determine the white dwarf atmospheric composition through spectroscopy and/or multiwavelength photometry and to calculate cooling models that are consistent with the derived parameters. We also note that the photometric peculiarities of old hydrogen-atmosphere dwarfs must be accounted for in color selection of observational samples.

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