Optical Null Medium: Experimental Verification of Super Scatterer in Broadband Continues Frequencies

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Abstract: We experimentally show the results of the super scatterer designed by the transformation optics (TO) technique. The coordinate transformation maps a finite region in physical space to a zero thickness in virtual space, yielding extreme material properties called optical null media (ONM). As a result, an object placed inside the core media will look larger from an outside observer. ONM media is realized using metal-dielectric layered structures in cylindrical coordinates. Closed form solutions for the scattering field from a cylindrical object is given in detail. ONM media is realized using air and iron layers in cylindrical layers using effective medium theory. We experimentally fabricate the ONM and show the super scattering effect using the ONM. For achieving super scattering validation, a metal pipe has been inserted into the center of the core medium made of Plexiglas and surrounded by ONM.

Keyword: experimental illusion perception, optical null medium, transformation optics, slit array, scattering theory.

1- Introduction

Relying on the form invariance of Maxwell's equations, transformation optics (TO) provides a methodology to control and manipulate the electromagnetic waves in order to design novel devices such as invisibility cloaks, optical black holes, negative index lenses, concentrators, rotators, PEC reshaper, super scatterer, reflectionless waveguide bends, waveguide miniaturization including many other devices. These devices, typically, are restricted by their anisotropic characteristics, narrow bandwidth, and high losses associated with resonant metamaterial elements. With the advance in metamaterial technology, however, some of these difficulties can be elevated, and these devices can be realized with increased capabilities.

One media that has recently attracted great attention is the optical null media (ONM). ONM media can be derived by transforming zero thickness into a finite region. As a result, the derived transformation media has extreme material properties (i.e., $[\varepsilon] = \{\infty, 0,0\}$ONM has two important characteristics. First, phase accumulation inside the transformation media is zero; thus, ONM effectively nullifies the occupying space. Second, the wave propagation is limited to one particular direction only. The latter is particularly used to realize interesting surface transformation devices. Many useful devices have been designed using ONM, including hyper-lens, magnetic hoses, and field concentrators. Metal-dielectric layered structures have been extensively used to realize
anisotropic effective material properties. By properly arranging the ratio of metallic and dielectric layer thickness, extreme material properties can be derived, such as ONM.

In this paper, we design a super scatterer device using the TO approach. In the coordinate transformation, zero thickness in the virtual space is mapped to a finite thickness in physical space, yielding extreme material properties. Any object that is placed inside the core region of the transformation device will look like a larger from an outside observer. We provide analytical calculations for the scattered field and show the equivalence between the original and equivalent problems. ONM is realized using air and iron metal layers in cylindrical coordinates. We show that the proposed structure approximates the effective medium parameters closely. We have also experimentally fabricated the transformation media and showed the superscatterer effect. The proposed device is realized using isotropic air and iron to realize ONM and does not require resonant metamaterial structures. As a result, the super scattering effect is not limited to a narrow frequency band.

2- Super scattering effects of ONM in wave expansion form solution

At first, we study CPs and functionalities of ONM. To design a cylindrical ONM using TO we must provide transformation relation between real space and virtual space with an extreme extension in one or more coordinate. Here we use radial extension in cylindrical coordinates as shown in Fig.1a and 1b with following radial transformation function,

\[
\begin{align*}
    r &= \begin{cases} 
        \frac{R_2 - \Delta}{R_2 - \delta} r' & r \in [0, R_1), \text{we name this region core medium} \\
        \frac{\Delta}{\delta} r' - \left(\frac{\Delta}{\delta} - 1\right) R_2 & r \in [R_1, R_2), \text{we name this region shell medium} \\
        r' & r \in (R_2, \infty), \text{outside of device}
    \end{cases}
\end{align*}
\]

Figure 1. (a) and (b) show the transformation relation between virtual space and real space, respectively. (c) numerical simulation of energy density of gaussian beam in 2D ONM.
Where \( r \) and \( r' \) represent virtual and real space, respectively. In this relation \( \delta \), a very thin region (a region between cylinders with radiiuses \( R_2 - \delta \) and \( R_2 \)) extremely extended to a finite region, \( \Delta = R_2 - R_1 \). The CPs of such an ONM can be calculated by TO,

\[
\text{For } r \in [0, R_1), \epsilon = \mu = \text{diag} \left( 1,1, \left( \frac{R_2 - \delta}{R_2 - \Delta} \right)^2 \right) \Rightarrow \text{diag} \left( 1,1, \left( \frac{R_2}{R_1} \right)^2 \right)
\]  

(2)

And for \( r \in [R_1, R_2] \), \( \epsilon = \mu = \text{diag} \left( P, \frac{1}{P}, \left( \frac{\delta}{\Delta} \right)^2 P \right) \Rightarrow \text{diag}(\infty, 0,0)
\]  

(3)

Where \( P = 1 + \left( \frac{\Delta}{\delta} + 1 \right) \frac{R_2}{r} \). Eq. (2) and (3) show that this extension resulted in a CP with extreme large value along radial direction and nearly zero in other directions. If we consider the relation,

\[
\frac{k_r^2}{\epsilon_r} + \frac{k_\phi^2}{\epsilon_\phi} = \omega^2,
\]

with extreme large value of \( \epsilon_r \), we can understand why wave propagate in \( r \) direction in this ONM. This is the most interesting properties of ONMs that has been simulated in Fig. 1c for 2D ONM.

Now we want to calculate and illustrate the scattering effect in 2D ONM quantitively. For producing scattering effects, we inserted a metallic pipe with radius \( d \) in core medium and we provide exact solution of wave equation in all region of ONM device in expansion series form.

We consider incident TM-z plane wave incident from left to right. Hence, for outside of device in \( r > R_2 \), the total wave including scattered term can be calculated as follow,

\[
H_z^2 = \sum_n [i^{-n} J_n(K_0r) + b_n H_n(K_0r)] e^{i\vartheta}
\]  

(4)

Where \( b_n \) and \( K_0 \) are scattered coefficient and wave vector in host medium, respectively.

For ONM region, \( R_1 < r < R_2 \),

\[
H_z^2 = g J_0(K_0r) + h Y_0(K_0r)
\]  

(5)

Where \( g \) and \( h \) are unknown coefficients. For inside the core medium \( d < r < R_1 \), we have the wave solution as follow,

\[
H_z^2 = \sum_n \left[ c_n J_n \left( \frac{R_2}{R_1} K_0 r \right) + d_n H_n \left( \frac{R_2}{R_1} K_0 r \right) \right] e^{i\vartheta}
\]  

(6)

Where \( c_n \) and \( d_n \) are unknown coefficients and \( d \) is the radius of cylindrical pipe sample that placed in center of core medium. Finally, for inside the cylindrical pipe sample, \( 0 < r < d \), we can calculate the wave as follow,

\[
H_z^2 = \sum_n f_n J_n \left( m \frac{R_2}{R_1} K_0 r \right) e^{i\vartheta}
\]  

(7)

In this equation if the sample was made of dielectric, \( f_n \) is unknown coefficient of wave and \( m \) is refractive index of dielectric cylinder. If sample is a metallic cylinder, \( f_n = 0 \).

Now using boundary condition for electric and magnetic fields at all interfaces we can calculate the scattering coefficient,
\[ b_n = \frac{i^{-n} \left[ A_n J_0(K_0R_2) - B_n Y_0(K_0R_2) \right] J'_0(K_0R_2) + B_n Y_0'(K_0R_2) - A_n J'_0(K_0R_2) J_n(K_0R_2) + B_n Y_0(K_0R_2) - A_n J_0(K_0R_2) H'_n(K_0R_2) \right]}{\left[ A_n J'_0(K_0R_2) - B_n Y'_0(K_0R_2) H_n(K_0R_2) + B_n Y_0(K_0R_2) - A_n J_0(K_0R_2) H'_n(K_0R_2) \right]} \]  

(8)

Where \( A_n = M_n(K_0R_2)Y'_0(K_0R_1) - M'_n(K_0R_2)Y_0(K_0R_1) \), \( B_n = M_n(K_0b)J'_0(K_0R_1) - M'_n(K_0R_2)J_0(K_0R_1) \) and \( M_n(K_0R_2) = \beta_1 J_n(K_0R_2) + \beta_2 H_n(K_0R_2) \).

Now, we can plot the distribution pattern of the total field outside the ONM in \( r > R_2 \). In the same calculation procedure, we can calculate scattering coefficient from bare metal pipe with radius \( d' = \frac{R_2}{R_1} d \) and refractive index \( m' = \frac{R_2}{R_1} m \) (calculated by TO) as follow,

For outside the cylinder, \( r > d' \),

\[ H_Z = \sum \left[ i^{-n} J_n(K_0r) + b_n H_n^{(1)}(K_0r) \right] e^{in\theta} \]  

(9)

And for inside of cylinder, \( 0 < r < d' \),

\[ H_Z = \sum [f_n J_n(m'K_0r)] e^{in\theta} \]  

(10)

using boundary condition for electric and magnetic fields at \( r = d' \), we can calculate the scattering coefficient outside the pipe, in region \( r > d' \),

\[ b_n = i^{-n} \frac{J_n'(k_0R_2)J_n(mk_0R_2) - J_n(k_0R_2)J_n'(mk_0R_2)}{H_n'(k_0R_2)J_n(mk_0R_2) - H_n(k_0R_2)J_n'(mk_0R_2)} \]  

(11)

We plot total Hz field in all region in both cases, metal inside the ONM and bare metal. In this numerical plot we set the working frequency in both plots, \( 1e9 \) Hz. In ONM device we set \( R_1 = 0.1m \), \( R_2 = 0.3m \) and \( d = 0.05m \). Also, for the bare metallic pipe we set \( d' = 0.15m \).

Figure 2 (a) Magnetic field pattern of ONM (b) magnetic field pattern of metallic cylinder
As we can see in Fig. 2a and Fig. 2b the scattered field outside the device, region \( r > R_2 \), are exactly the same as outside the same region, \( r > R_2 \), of bare pipe (remember that \( R_2 \) is bigger than radius of bare pipe, \( R_2 > d' \). In other word the proposed device produce larger scattering field pattern for the object with radius \( d \) than its actual size with radius \( d' \). Hence, we proved the perfect supper scattering effect in size of pipe sample using ONM.

3- Experimental validation

In this section we propose a radial alternative structure for ONM, as shown in Fig. 3a. this structure composed of slices made by two materials, air and iron. In this figure A and B represent air and iron media, respectively. In order to illustrate correspondence between the CPs of this structure and ONMs, we use effective medium theory (EMT). For simplicity we consider the thickness of iron slices so thin that it can be approximated as iron sheets. In experiment we set the thickness of iron sheets 0.4mm, in rectangular shape, 32mm × 500mm and fabricated the ONM by 100 pieces in radial alternative form fixed around the cylindrical Plexiglas core medium with radius 50mm, as shown in Fig.3b. Hence the filling factor of iron sheet and air slices in annual media approximately are \( f_B = 0.097 \) and \( f_A = 0.903 \), respectively. We also set \( \varepsilon_A = 1 \), and for iron \( \varepsilon_B = 100000 \) in microwave frequencies. Hence, based on EMT [40], \( \varepsilon_r = f_A\varepsilon_A + f_B\varepsilon_B \) and \( \varepsilon_r^{-1} = f_A/\varepsilon_A + f_B/\varepsilon_B \), we have \( \varepsilon_r = 9644 \) and \( \varepsilon_\phi = 1.1 \). In this structure we have an extreme large permittivity along \( r \) direction. Although in contract of perfect ONM parameter we don’t have near zero CP in \( \phi \) direction but in simulation, as shown in Fig.3c, we have extreme media that wave propagate along \( r \) direction similar to Fig.1c. So, this structure is a good candidate of ONM. In fact, the thinner iron sheets, the more perfect ONM parameters.

![Figure 3. (a)periodic structure in radial alternative form comprised of air and metallic slits. (b) electric energy density in medium with have \( \varepsilon_r = 9644 \) and \( \varepsilon_\phi = 1.1 \).](image-url)
Now we come to see how the slit array can produce super scattering effect in experiment. For this end we made a cylindrical hole with radius $d = 12.5 \text{mm}$ exactly in center of core medium and insert a metallic pipe, with exactly the same radius, inside the cylindrical hole as shown in Fig.4a and 4b.

Figure 4 (a) core region of device made by plexiglass with permittivity $\varepsilon = 2.7$ and a central hole with radius $d=12.5\text{mm}$ (b) core region of device including a metallic pipe in the central hole (c) fabricated device with annual slits media

In experiment set up, as shown in Fig. 5, we placed the sample vertically in front horn-shaped antenna located 100 cm away from the fabricated device. In this experiment we set the horn antenna to transmit H-field polarized along vertical direction. We name this direction, z axis.

Figure 5. schematic experimental set up.
To scan the spatial distribution of Hz component we used a split ring detecting antenna in coaxial cable with circular shape with radius 4mm and split 1mm. the split ring antenna can move in horizontally in x-y plane. We set the scanning range 300mmx300mm in dashed region of fig. 5 with spatial resolution 2mm × 2mm. we connect the horn antenna and detector to an s-parameter network analyzer in order to measure real $H_z$ field. Fig. 6a shows the real part of scanned $H_z$ field for ONM device including metal pipe (device depicted in Fig. 4c). Likewise, we set the experiment for a bare metallic cylinder with radius $d' = 20.5 \text{mm}$ in the same location and scanning range as ONM device. Fig.6b shows the measured real part of $H_z$ field distribution for bare metallic case. For the both experiment we set the frequency 9.86e9 HZ.

![Figure 6](image) (a) Hz-field of the ONM media including metallic cylinder with radius $d = 12.5 \text{mm}$ and working frequency 9.86 GHz (b) Hz-field of the bare metallic cylinder with radius $d' = 20.5 \text{mm}$ and working frequency 9.86 GHz.

4- Conclusion

In this paper we presented super scattering device extreme transformation function, ONM, in analytically approach and experimental verification. we have shown ONMs can be used for super scattering effects by wave expansion calculation. In continue we realize the ONM device using simple materials, air and Iron, in radial alternative structure that can mimic the perfect functionality of ONM. well functionality results achieved in both analytical and experimental parts. show the

5- References

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**Author contributions**: Mohammad Mehdi Sadeghi conceived the idea and did the theoretical calculations and the numerical simulations. Mohammad Mehdi Sadeghi fabricated the samples and did the experimental measurements. Mohammad Mehdi Sadeghi supervised the experimental and theoretical parts. Mohammad Mehdi Sadeghi and Hayrettin Odabasi wrote and revised the manuscript.