Θ⁺ baryon, $N^\ast(1685)$ resonance, and πN sigma term in the context of the LEPS and DIANA experiments

Ghil-Seok Yang and Hyun-Chul Kim

1Center for High Energy Physics and Department of Physics, Kyungpook National University, Daegu 702-701, Republic of Korea
2Department of Physics, Inha University, Incheon 402-751, Republic of Korea
3Department of Physics, University of Connecticut, Storrs, CT 06269, U.S.A.
4School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Republic of Korea

(Dated: April, 2012)

We reexamine properties of the baryon antidecuplet Θ⁺ and $N^\ast$, and the πN sigma term within the framework of a chiral soliton model, focusing on their dependence on the Θ⁺ mass. It turns out that the measured value of the $N^\ast$ mass, $M_{N^\ast} = 1686$ MeV, is consistent with that of the Θ⁺ mass $M_{Θ⁺} = 1524$ MeV by the LEPS collaboration. The $N^\ast \rightarrow N\gamma$ magnetic transition moments are almost independent of the Θ⁺ mass. The ratio of the radiative decay width $Γ_{n\gamma}$ to $Γ_{pp}$ turns out to be around 5. The decay width for $Θ⁺ \rightarrow NK$ is studied in the context of the LEPS and DIANA experiments. When the LEPS value of the Θ⁺ mass is employed, we obtain $Γ_{n\gamma} / Γ_{pp}$ turns out to be almost independent of the Θ⁺ mass. In addition, we derive a new expression for the πN sigma term in terms of the isospin mass splittings of the hyperon octet as well as that of the antidecuplet $N^\ast$.

Keywords: Baryon antidecuplet Θ⁺ and $N^\ast$, chiral soliton model, πN sigma term

I. INTRODUCTION

The baryon antidecuplet is the first excitation consisting of exotic pentaquark baryons. Since the LEPS collaboration reported the first measurement of the pentaquark baryon Θ⁺, the pentaquark baryons have attracted much attention, before a series of the CLAS experiments announced null results of the Θ⁺, which casted doubt on the existence of the pentaquarks. On the other hand, the DIANA collaboration has pursued searching for the Θ⁺ and observed the formation of a narrow $pK^0$ peak with mass of 1538 ± 2 MeV/c² and width of $γ = 0.39 \pm 0.10$ MeV in the $K^+n \rightarrow K^0p$ reaction with higher statistical significance ($6σ - 8σ$). The decay width was more precisely measured in comparison with the former DIANA measurement, the statistics being doubled. The SVD experiment has also found a narrow peak with the mass, $(1523 \pm 2_{\text{stat}} \pm 3_{\text{syst}}) \text{ MeV}$ in the inclusive reaction $pA \rightarrow pK^0 + X$. In 2009, the LEPS collaboration has again announced the evidence of the Θ⁺. The mass of the Θ⁺ is found at $M_{Θ⁺} = 1524 \pm 2 \pm 3$ MeV/c² and the statistical significance of the peak turns out to be 5.1σ. The differential cross section was estimated to be $(12 \pm 2) \text{ nb/sr}$ in the photon energy ranging from 2.0 GeV to 2.4 GeV in the LEPS angular range. Note that the statistics of the new LEPS data has been improved by a factor of 8 over the previous measurement. Very recently, Amaryan et al. have reported a narrow structure around 1.54 GeV in the process $\gamma + p \rightarrow pK_S K_L$ via interference with $\phi$-meson production with the statistical significance 5.9σ, based on the CLAS data.

In addition to the Θ⁺ baryon, Kuznetsov et al. observed a new nucleon-like resonance around 1.67 GeV from $\eta$ photoproduction off the deuteron in the neutron channel. The decay width was measured to be around 40 MeV without the effects of the Fermi motion excluded. On the other hand, this narrow resonant structure was not seen in the quasi-free proton channel. The finding of Ref. is consistent with the theoretical predictions of non-strange exotic baryons. Moreover, its narrow width and isospin asymmetry in the initial states, also called as the neutron anomaly, are the typical characteristics for the photo-excitation of the non-strange antidecuplet pentaquark. New analyses of the free proton GRAAL data have revealed a resonance structure with a mass around 1685 MeV and width $Γ \leq 15$ MeV, though the data of Ref. do not agree with those of Ref. For a detailed discussion of this discrepancy, we refer to Ref. The CB-ELSA collaboration has also confirmed an evidence for this $N^\ast$ resonance in line with those of GRAAL. Very recently, Kuznetsov and Polyakov...
have extracted the new result for the narrow peak: $M_{N^*} = 1686 ± 7 \pm 5$ MeV with the decay width $\Gamma \approx 28 \pm 12$ MeV [33]. All these experimental facts are compatible with the results for the transition magnetic moments in the chiral quark-soliton model (χQSM) [23, 24] and phenomenological analysis for the non-strange pentaquark baryons [34]. The $\gamma N \rightarrow p N$ reaction was studied within an effective Lagrangian approach [35, 36] that has described qualitatively well the GRAAL data. The present status of the $N^*(1685)$ is summarized in Ref. [37] in which the reason was discussed why the $N^*(1685)$ can be most probably identified as a member of the baryon antidecuplet in detail.

In the present work, we want to examine the relation between the $\Theta^+$ mass and other observables such as the mass of the $N^*$ ($M_{N^*}$), $N^* \rightarrow N \gamma$ transition magnetic moments ($\langle \mu_{NN^*}\rangle$), the decay width of the $\Theta^+$ ($\Gamma_{\Theta^+}$), and $\pi N$ sigma term ($\sigma_{\pi N}$), in the context of the LEPS and DIANA experiments. In particular, we will regard the $N^*(1685)$ resonance with the narrow width as a member of the antidecuplet in this work. The mass splittings of the SU(3) baryons within a chiral soliton model (χSM) were reinvestigated with all parameters fixed unequivocally [38]. Since the mass of the $\Theta^+$ observed by the LEPS collaboration is different from that by the DIANA collaboration, it is of great importance to examine carefully the relevance of the analysis in Ref. [38] with regard to the LEPS and DIANA experiments. We will show in this work that the decay width $\Gamma_{\Theta^+}$ obtained from the χSM [38] is consistent with these two experiments. We will also study the dependence of the $N^*$ mass on the $M_{\Theta^+}$, which turns out to be compatible with the LEPS data. In addition, we also investigate the dependence of the $N^* \rightarrow N$ magnetic transition moment that is shown to be almost insensitive to the $\Theta^+$ mass. Finally, the $\sigma_{\pi N}$ will be examined, which becomes one of essential quantities in the physics of dark matter [39, 40]. Motivated by its relevance in dark matter, a great amount of efforts was put on the evaluation of the $\sigma_{\pi N}$. For example, there are now various results from the lattice QCD [41, 42]. However, the value of $\sigma_{\pi N}$ still does not converge, but is known only with the wide range of uncertainties: $35 - 75$ MeV. Thus, we will discuss the $\sigma_{\pi N}$ in connection with the baryon antidecuplet and will show that it is rather stable with respect to the $\Theta^+$ mass. Moreover, its predicted value is smaller than that used in previous analyses [2, 45, 46].

The present work is organized as follows: In Section II, the pertinent formulae for the baryon antidecuplet within a chiral soliton model are compiled. In Section III, we discuss the results. Final Section is devoted to summary and conclusion.

II. BARYON ANTIDECEUPLET FROM A CHIRAL SOLITON MODEL

We first recapitulate briefly the formulae of the mass splittings, the magnetic moments, and the axial-vector constants within the framework of the χSM. We begin with the collective Hamiltonian of chiral solitons, which have been thoroughly studied within various versions of the χSM such as the chiral quark-soliton model [47, 48], the Skyrme model [49], and the chiral hyperbag model [50]. The most general form of the collective Hamiltonian in the SU(3) χSM can be written as follows:

$$H = M_{cl} + H_{rot} + H_{sb},$$

where $M_{cl}$ denotes the classical soliton mass. The $H_{rot}$ and $H_{sb}$ respectively stand for the $1/N_c$ rotational and symmetry-breaking corrections with the effects of isospin and SU(3) flavor symmetry breakings included [51]:

$$H_{rot} = \frac{1}{2I_1} \sum_{i=1}^{3} \tilde{j}_i^2 + \frac{1}{2I_2} \sum_{p=4}^{7} \tilde{j}_p^2,$$

$$H_{sb} = (m_d - m_u) \left( \frac{\sqrt{3}}{2} \alpha D_{38}^{(8)}(A) + \beta \bar{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^{3} D_{3i}^{(8)}(A) \tilde{J}_i \right)$$

$$+ (m_s - \bar{m}) \left( \alpha D_{18}^{(8)}(A) + \beta \bar{Y} + \frac{1}{\sqrt{3}} \gamma \sum_{i=1}^{3} D_{8i}^{(8)}(A) \tilde{J}_i \right)$$

$$- (m_u + m_d + m_s)(\alpha + \beta),$$

where $I_{1,2}$ represent the soliton moments of inertia that depend on dynamics of specific formulations of the χSM. The $J_i$ denote the generators of the SU(3) group. The $m_u$, $m_d$, and $m_s$ designate the up, down, and strange current quark masses, respectively. The $\bar{m}$ is the average of the up and down quark masses. The $D_{ab}^{(R)}(A)$ indicate the SU(3) Wigner $D$ functions. The $\bar{Y}$ and $\bar{T}_3$ are the operators of the hypercharge and isospin third component, respectively. The $\alpha$, $\beta$, and $\gamma$ are given in terms of the $\sigma_{\pi N}$ and soliton moments of inertia $I_{1,2}$ and $K_{1,2}$ as follows:

$$\alpha = -\left( \frac{\sigma_{\pi N}}{3\bar{m}} - \frac{K_2}{I_2} \right), \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right).$$
Since $\alpha$, $\beta$, and $\gamma$ depend on the moments of inertia and $\sigma_{\pi N}$, they are also related to details of specific dynamics of the $\chi$SM. Note that $\alpha$, $\beta$, and $\gamma$ defined in the present work do not contain the strange quark mass, while those in Refs. 2, 45 include it.

In the $\chi$SM, we have the following constraint for $J_8$

$$J_8 = -\frac{N_c}{2\sqrt{3}} B = -\frac{\sqrt{3}}{2}, \quad Y'' = \frac{2}{\sqrt{3}} J_8 = -\frac{N_c}{3} = -1,$$

where $B$ represents the baryon number. It is related to the eighth component of the soliton angular velocity that is due to the presence of the discrete valence quark level in the Dirac-sea spectrum in the SU(3) $\chi$SM [47, 52], while it arises from the Wess-Zumino term in the SU(3) Skyrme model [53-55]. Its presence has no effects on the chiral symmetry breaking [46], due to the presence of the discrete valence quark level in the Dirac-sea spectrum in the SU(3) Skyrme model [53-55].

The baryon collective wavefunctions of $H$ are written as the SU(3) Wigner $D$ functions in representation $R$:

$$\langle A|R, B(Y T T_3, Y' J J_3)\rangle = \Psi^{(R_8; Y; T T_3)}(\mathcal{R}, T, T_3) \delta^{(R_8, T, T_3)}(-Y', J, J_3)(A),$$

$$\langle A|\mathcal{R}, B(Y T T_3, Y' J J_3)\rangle = \sqrt{\dim(R)} (-)^{|J_0 + Y'/2|} D^{(R_8)}(Y, T, T_3)(-Y', J, J_3)(A),$$

where $\mathcal{R}$ stands for the allowed irreducible representations of the SU(3) group, i.e. $R = 8, 10, \overline{10}, \cdots$ and $Y, T, T_3$ are the corresponding hypercharge, isospin, and its third component, respectively. The constraint of the right hypercharge $Y'' = 1$ selects a tower of allowed SU(3) representations: The lowest ones, that is, the baryon octet and decuplet, coincide with those of the quark model. This has been considered as a success of the collective quantization and as a sign of certain duality between a rigidly rotating heavy soliton and a constituent quark model. The third lowest representation is the antidecuplet [2] that includes the $\Theta^+$ and $N^*$ baryons.

Different SU(3) representations get mixed in the presence of the symmetry-breaking term $H_{ab}$ of the collective Hamiltonian in Eq. (3), so that the collective wave functions are no longer in pure states but are given as the following linear combinations [47, 50]:

$$|B_{8}\rangle = \left|8_{1/2}, B\right\rangle + c_{\overline{10}}^{B} \left|10_{1/2}, B\right\rangle + c_{27}^{B} \left|27_{1/2}, B\right\rangle,$$

$$|B_{10}\rangle = \left|10_{3/2}, B\right\rangle + a_{27}^{B} \left|27_{3/2}, B\right\rangle + a_{\overline{27}}^{B} \left|\overline{27}_{3/2}, B\right\rangle,$$

$$|B_{1}\rangle = \left|1_{0}, B\right\rangle + d_{27}^{B} \left|27_{1/2}, B\right\rangle + d_{\overline{27}}^{B} \left|\overline{27}_{1/2}, B\right\rangle.$$

The detailed expressions for the coefficients in Eq. (4) can be found in Refs. 45, 47.

Since we take into account the effects of isospin symmetry breaking, we also have to introduce the EM mass corrections to the mass splitting of the SU(3) baryons, which are equally important. The EM corrections to the baryon masses can be derived from the baryonic two-point correlation functions. The corresponding collective operator was already derived in Ref. 57.

$$M_{B}^{EM} = \langle B|O^{EM}|B\rangle,$$

where

$$O^{EM} = c^{(1)} D_{\Lambda\Lambda}^{(1)} + c^{(8)} \left(3D_{\Sigma\Sigma}^{(8)} + D_{\Lambda\Lambda}^{(8)}\right) + c^{(27)} \left(\sqrt{3} D_{\Sigma\Sigma}^{(27)} + \sqrt{3} D_{\Sigma\Sigma}^{(27)} + D_{\Lambda\Lambda}^{(27)}\right).$$

The unknown parameters $c^{(8)}$ and $c^{(27)}$ are determined by the experimental data for the EM mass splittings of the baryon octet, while $c^{(1)}$ can be absorbed in the center of baryon masses. The values of $c^{(8)}$ and $c^{(27)}$ were obtained as

$$c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39$$

in units of MeV [57].

The final expressions for the masses of $\Theta^+$ and $N^*$ are given as

$$M_{\Theta^+} = M_{10} + \frac{1}{4} \left(c^{(8)} - \frac{4}{21} c^{(27)}\right) - 2(m_s - \bar{m})\delta,$$

$$M_{N^*} = M_{10} + \frac{1}{4} \left(c^{(8)} - \frac{32}{63} c^{(27)}\right) T_3 + \frac{1}{4} \left(c^{(8)} + \frac{8}{63} c^{(27)}\right) \left(T_3^2 + \frac{1}{4}\right) - (m_d - m_u)T_3 \delta - (m_s - \bar{m})\delta,$$

where $M_{10}$ denotes the center of the mass splittings of the baryon antidecuplet and $\delta$ is a parameter defined as

$$\delta = -\frac{1}{8} \alpha - \beta + \frac{1}{16} \gamma.$$
The collective operators for the magnetic moments and axial-vector constants can respectively be parameterized by six parameters that can be treated as free \[\hat{\mu}, \hat{\gamma}, \hat{\Theta}, \hat{\theta}, \hat{\gamma}, \hat{\beta}\]:

\[
\hat{\mu} = w_1 D_X^{(8)} + w_2 d_{pq3}^2 D_{Xp}^{(8)} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D_{Xs}^{(8)} \hat{J}_3 \\
+ \frac{w_4}{\sqrt{3}} d_{pq3}^2 D_{Xp}^{(8)} \cdot \hat{J}_q + w_5 \left(D_{X3}^{(8)} D_{88}^{(8)} + D_{X8}^{(8)} D_{88}^{(8)} \right) + w_6 \left(D_{X3}^{(8)} D_{88}^{(8)} - D_{X8}^{(8)} D_{88}^{(8)} \right),
\]

\[
\hat{\gamma}_A = a_1 D_{X3}^{(8)} + a_2 d_{pq3}^2 D_{Xp}^{(8)} \cdot \hat{J}_q + \frac{a_3}{\sqrt{3}} D_{Xs}^{(8)} \hat{J}_3 \\
+ \frac{a_4}{\sqrt{3}} d_{pq3}^2 D_{Xp}^{(8)} D_{8q}^{(8)} + a_5 \left(D_{X3}^{(8)} D_{88}^{(8)} + D_{X8}^{(8)} D_{88}^{(8)} \right) \\
+ a_6 \left(D_{X3}^{(8)} D_{88}^{(8)} - D_{X8}^{(8)} D_{88}^{(8)} \right), \tag{13}
\]

where \(\hat{J}_q, (\hat{J}_3)\) stand for the \(q\)-th (third) component of the spin operator of the baryons. The parameters \(w_i\) and \(a_i\) can be unambiguously fixed by using the magnetic moments and semileptonic decay constants of the baryon octet. We refer to Refs. \[61, 62\] for the detailed expressions for the \(\Theta^+\) magnetic moment and axial-vector constants for the \(\Theta \to KN\) decay.

III. RESULTS AND DISCUSSION

In order to find the masses of the baryon antidecuplet, we need to fix the relevant parameters. There are several ways to fix them. For example, Diakonov et al. \[2\] use the mass splittings of the baryon octet and decuplet, \(\pi N\) sigma term, and the octet \[61, 62\]. We refer to Refs. \[61\] for the detailed expressions for the \(\Theta^+\) mass and axial-vector constants for the \(\Theta \to KN\) decay.

On the other hand, Ellis et al. \[45\] carried out the analysis for the mass splittings of the baryon antidecuplet, based on the then experimental data of the \(\Theta^+\) and \(\Xi^--\) masses together with those of the baryon octet and decuplet. They predicted the \(\pi N\) sigma term \(\sigma_{\pi N} = 73\) MeV from the fitted values of the parameters:

\[
I_2 = 0.49\text{ fm, } m_s \alpha = -218\text{ MeV, } m_s \beta = -156\text{ MeV, } m_s \gamma = -107\text{ MeV.} \tag{14}
\]

Very recently, Ref. \[38\] reanalyzed the mass splittings of the SU(3) baryons within a \(\chi\)SM, employing isospin symmetry breaking. An obvious advantage of including the effects of isospin symmetry breaking is that one can fully utilize the whole experimental data of the octet masses to fix the parameters. Using the baryon octet masses, \(\Omega^-\) mass \((1672.45 \pm 0.29)\) MeV \[63\], and \(\Theta^+\) mass \((1524 \pm 5)\) MeV \[10\], both of which are the isosinglet baryons the key parameters were found to be:

\[
I_2 = (0.420 \pm 0.006)\text{ fm, } m_s \alpha = (-262.9 \pm 5.9)\text{ MeV, } m_s \beta = (-144.3 \pm 3.2)\text{ MeV, } m_s \gamma = (-104.2 \pm 2.4)\text{ MeV.} \tag{16}
\]

In addition, the \(\pi N\) sigma term was predicted as \(\sigma_{\pi N} = (36.4 \pm 3.9)\) MeV. Since \(\delta\) defined in Eq. (12), let us compare its values from each work mentioned above. The corresponding results are given, respectively, as follows:

\[
m_s \delta = 177\text{ MeV (Diakonov et al.), } m_s \delta = 108\text{ MeV (Ellis et al.), } m_s \delta = 171\text{ MeV (present work),} \tag{17}
\]

with isospin symmetry breaking switched off. If we use the LEPS experimental data \[10\] for \(M_{\Theta^+}\), we can immediately obtain the corresponding masses of the \(N^*\), respectively:

\[
M_{N^*} = 1700\text{ MeV (Diakonov et al.), } M_{N^*} = 1631\text{ MeV (Ellis et al.), } M_{N^*} = 1694\text{ MeV (present work).} \tag{18}
\]

If one employs the DIANA data \[12\], the \(N^*\) mass is yielded as

\[
M_{N^*} = 1715\text{ MeV (Diakonov et al.), } M_{N^*} = 1646\text{ MeV (Ellis et al.), } M_{N^*} = 1708\text{ MeV (present work).} \tag{19}
\]

The comparison made above already indicates that the predicted masses of the \(N^* (1685)\) resonance from the previous analyses are deviated from the experimental data. Moreover, it is essential to take into account the effects of isospin symmetry breaking, in order to produce the mass of the \(N^*\) resonance quantitatively \[38\]. Since there are, however, two different experimental values of the \(\Theta^+\) mass from the LEPS and DIANA collaborations, it is necessary to examine
carefully the dependence of the relevant observables on that of the \( \Theta^+ \) baryon rather than choosing one specific value of \( M_{\Theta^+} \) to fit the parameters. Thus, in the present Section, we discuss the dependence of relevant observables on \( M_{\Theta^+} \), taking it as a free parameter.

In Fig. 1 we draw the \( N^* \) mass as a function of \( M_{\Theta^+} \). The vertical shaded bars bounded with the solid and dashed lines denote the measured values of the \( \Theta^+ \) mass with uncertainties by the LEPS and DIANA collaborations, respectively. The horizontal shaded region denotes the values of the \( N^* \) mass with uncertainty taken from Ref. [22]. The sloping shaded region shows the dependence of the \( N^* \) mass on \( M_{\Theta^+} \). The \( N^* \) mass increases monotonically, as \( M_{\Theta^+} \) increases. This behavior can be easily understood from Eq. (11): the mass of the \( N^* \) resonance depends linearly on the parameter \( \delta \). Interestingly, if we take the \( M_{\Theta^+} \) value of the LEPS experiment, i.e., \( M_{\Theta^+} = 1524 \) MeV, we obtain \( M_{N^*} \approx 1690 \) MeV, which is in good agreement with the experimental data: \( M_{N^*} = (1685 \pm 12) \) MeV [22]. On the other hand, if we use the value of \( M_{\Theta^+} \) measured by the DIANA collaboration, the \( N^* \) mass turns out to be larger than 1690 MeV. It implies that the \( \Theta^+ \) mass reported by the LEPS collaboration [16] is consistent with that of \( N^*(1685) \) from recent experiments [18, 30–33], at least, at the present framework of a \( \chi SM \) with isospin symmetry breaking [38].

The parameters \( w_i \) in Eq. (13) can be fitted by the magnetic moments of the baryon octet [58, 60]. However, since the mixing coefficients appearing in Eq. (7) depend explicitly on \( \alpha \) and \( \gamma \), the parameters \( w_i \) are also given as functions of \( \sigma_{\pi N} \) through \( \alpha \) and \( \gamma \) as shown in Ref. [24]. As previously mentioned, since the mass parameters \( \alpha \) and \( \gamma \) as well as \( \sigma_{\pi N} \) were unambiguously fixed in Ref. [38], we can derive the transition magnetic moments for the \( N^* \to N\gamma \) decay unequivocally. Explicitly, the transition magnetic moments \( \mu_{pp^*} \) and \( \mu_{nn^*} \) are recapitulated, respectively, as follows [24]:

\[
\begin{align*}
\mu_{pp^*}^{(0)} &= 0, \\
\mu_{pp^*}^{(op)} &= -\frac{1}{27\sqrt{5}} w_4 - \frac{1}{18\sqrt{5}} \left( w_5 + \frac{3}{2} w_6 \right), \\
\mu_{pp^*}^{(wf)} &= -\frac{5}{24\sqrt{5}} \left( w_1 + \frac{5}{2} w_2 - \frac{1}{2} w_3 \right) c_\pi - \frac{35}{72\sqrt{5}} \left( w_1 - \frac{11}{14} w_2 - \frac{3}{14} w_3 \right) c_{27} \\
&\quad + \left[ \frac{1}{2\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 + \frac{1}{6} w_3 \right) - \frac{7}{6\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 - \frac{1}{14} w_3 \right) \right] d_{27}, \\
\mu_{nn^*}^{(0)} &= \frac{1}{6\sqrt{5}} \left( w_1 + w_2 + \frac{w_3}{2} \right),
\end{align*}
\]
\[ \mu_{nn^*}^{(op)} = -\frac{1}{54\sqrt{5}} w_4 + \frac{1}{18\sqrt{5}} \left( w_5 + \frac{3}{2} w_6 \right), \]
\[ \mu_{nn^*}^{(w)} = \frac{7}{36\sqrt{5}} \left( w_1 - \frac{11}{14} w_2 - \frac{3}{14} w_3 \right) c_{27} + \frac{1}{2\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 + \frac{1}{6} w_3 \right) d_{27}. \]

As already discussed in Ref. [24], \( \mu_{pp^*} \) vanishes in the SU(3) symmetric case. Thus, \( \mu_{pp^*} \) is only finite with the effects of SU(3) symmetry breaking included.

![Figure 2: The dependence of the transition magnetic moments for the \( N^* \rightarrow N \gamma \) decay on \( M_{\Theta^+} \).](image)

Table I: The results of the decay widths in unit of keV. The mass to be larger than that of DIANA collaborations, respectively. The horizontal shaded regions stands for the present results of the transition magnetic moments \( \mu_{pp^*} \) and \( \mu_{nn^*} \).

| \( M_{\Theta^+} \) [MeV] | \( \mu_{NN^*} \) [\( \mu_N \)] | \( \mu_{NN^*}^{(op)} \) | \( \mu_{NN^*}^{(w)} \) | \( \mu_{NN^*}^{(old)} \) | \( \Gamma_{NN^*} \) [keV] |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 1500           | 0              | 0.272 \pm 0.051 | -0.125 \pm 0.013 | 0.146 \pm 0.053 | 17.7 \pm 3.2 |
| 1520           | 0              | -0.159 \pm 0.042 | 0.107 \pm 0.003 | -0.304 \pm 0.089 | 77.1 \pm 11.3 |
| 1540           | 0              | -0.252 \pm 0.077 | 0.107 \pm 0.003 | -0.304 \pm 0.089 | 77.1 \pm 11.3 |
| 1560           | 0              | -0.252 \pm 0.077 | 0.107 \pm 0.003 | -0.304 \pm 0.089 | 77.1 \pm 11.3 |
| 1580           | 0              | -0.252 \pm 0.077 | 0.107 \pm 0.003 | -0.304 \pm 0.089 | 77.1 \pm 11.3 |

In Table I we list each contribution to \( \mu_{NN^*} \) as well as the radiative decay widths for \( N^* \rightarrow N \gamma \) with the mass of the \( \Theta^+ \) from the LEPS experiment used. Note that the sign of \( \mu_{nn^*} \) is negative whereas that of \( \mu_{pp^*} \) is positive. However, the previous result for \( \mu_{nn^*} \) was positive [24]. The reason can be found in the different values of \( w_i \). Let us compare closely the present results with those of Ref. [24], considering only the SU(3) symmetric part without loss of generality. In fact, \( w_i \) derived in Ref. [24] depends on \( \sigma_{\pi N} \):

\[ w_1^{old} = -3.736 - 0.107 \sigma_{\pi N}, \quad w_2^{old} = 24.37 - 0.21 \sigma_{\pi N}, \quad w_3^{old} = 7.547. \]

If one takes the value of the \( \pi N \) sigma terms as \( \sigma_{\pi N} \approx 40 \) MeV (70 MeV), one gets

\[ w_1^{old} = -8.14 (-11.44), \quad w_2^{old} = 15.97 (9.67), \quad w_3^{old} = 7.547, \]

while the results in this work use the newly obtained values of \( w_i \),

\[ w_1 = -12.95 \pm 0.10, \quad w_2 = 5.388 \pm 0.933, \quad w_3 = 8.354 \pm 0.861. \]
Thus, the magnitude of the present $w_1$ is larger than those of $w_1^{\text{old}}$, whereas that of $w_2$ turns out to be smaller than those of $w_2^{\text{old}}$. Since $w_1$ and $w_2$ have different sign as in Eq. (20), they destructively interfere each other, so that the sign of $\mu_{nn^*}$ becomes negative in the present case but it is positive in Ref. [24]. However, magnetic properties of the octet and decuplet baryons are almost intact because of the constructive interference of $w_1$ and $w_2$, even though we have the different values of $w_i$. The ratios of the transition magnetic moments and of the radiative decay widths are obtained as

$$\left| \frac{\mu_{nn^*}}{\mu_{pp^*}} \right| = 2.08 \pm 0.97, \quad \frac{\Gamma_{nn^*}}{\Gamma_{pp^*}} = 4.36 \pm 1.02. \quad (24)$$

![Figure 3](image)

Figure 3: The dependence of the decay width $\Gamma_{N\Theta^+}$ for the $\Theta^+ \rightarrow KN$ decay on $M_{\Theta^+}$. The vertical shaded bars bounded with the solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties by the LEPS and DIANA collaborations, respectively. The horizontal shaded region draws the values of the $N^*$ mass with uncertainty taken from Ref. [22]. The sloping shaded region represents the present results of the $M_{\Theta^+}$ dependence of $\Gamma_{N\Theta^+}$.

The narrowness of the decay width is one of the peculiar characteristics of the pentaquark baryons. For example, the decay width of the $\Theta^+ \rightarrow KN$ vanishes in the nonrelativistic limit [59]. The decay width $\Gamma_{\Theta NK}$ was already studied in chiral soliton models with SU(3) symmetry breaking taken into account. We refer to Refs. [68, 69] for details. In Fig. 3, we examine the dependence of the decay width $\Gamma_{N\Theta}$ for $\Theta^+ \rightarrow KN$ on the $\Theta^+$ mass. Being different from the $N^*$ mass and the transition magnetic moments, the decay width $\Gamma_{\Theta N}$ increases almost quadratically as $M_{\Theta^+}$ increases. This can be understood from the fact that the decay width is proportional to the square of the $g_{NK\Theta^+}$ coupling constant which depends linearly on $M_{\Theta^+}$. When the $\Theta^+$ mass is the same as the value measured by the LEPS collaboration, $\Gamma_{N\Theta^+}$ turns out to be about 0.5 MeV. However, at the value $M_{\Theta^+} \approx 1540$ MeV corresponding to that of the DIANA experiment, the decay width $\Gamma_{N\Theta^+}$ is close to 1 MeV. We want to emphasize that the decay width of the $\Theta^+$ is still below 1 MeV in the range of $M_{\Theta^+}: 1520 - 1540$ MeV. When we use the measured value of $M_{\Theta^+}$ by the LEPS collaboration, we obtain $\Gamma_{\Theta^+ \rightarrow NK} = 0.5 \pm 0.1$ MeV.

Figure 4 depicts predicted values of the $\pi N$ sigma term as a function of the $\Theta^+$ mass. At first sight, the result is rather surprising. Firstly, it is almost insensitive to the $\Theta^+$ mass. Secondly, the value of $\sigma_{\pi N}$ is pretty smaller than those known from the previous works on the baryon antidecuplet [15, 16]. In order to understand the reason of this difference, we want to examine in detail the $\pi N$ sigma term in comparison with those discussed in previous works, in particular, with Ref. [16], where the $\pi N$ sigma term was extensively studied within the same framework. Since $\sigma_{\pi N}$ is expressed as

$$\sigma_{\pi N} = -3\tilde{m}(\alpha + \beta), \quad (25)$$

we need to scrutinize the dependence of $\tilde{m}\alpha$ and $\tilde{m}\beta$ on $M_{\Theta^+}$. Figure 5 depicts the results of the parameters $-3\tilde{m}\alpha$ and $-3\tilde{m}\beta$ as functions of $M_{\Theta^+}$. Interestingly, while $-3\tilde{m}\alpha$ increases monotonically as $M_{\Theta^+}$ increases, $-3\tilde{m}\beta$ decreases almost at the same rate as $-3\tilde{m}\alpha$. Consequently, $\sigma_{\pi N}$ remains rather stable. On the other hand, Schweitzer [16] expressed $\sigma_{\pi N}$ in terms of the mass splittings of each representation:

$$\frac{m}{\tilde{m}}\sigma_{\pi N} = 3(4M_2 - 3M_\Delta - M_N) + 4(M_{1\Omega} - M_{3\Delta}) - 4(M_{3\Sigma/2} - M_{\Theta^+}). \quad (26)$$
and determined it to be \( \sigma_{\pi N} = (74 \pm 12) \text{ MeV} \), taking the experimental values of \( M_{\Theta^+} = 1540 \text{ MeV} \) and \( M_{\Xi_{3/2}} = 1862 \text{ MeV} \) \([67]\) for granted at that time, and using the ratio of the current quark mass \( m_s/\bar{m} = 25.9 \). However, using the predicted value of \( M_{\Xi_{3/2}} \approx 2020 \text{ MeV} \) in Ref. \([38]\), we get \( \sigma_{\pi N} \approx 45 \text{ MeV} \). Thus, the present result is not in contradiction with that of Ref. \([46]\).

Taking the effects of isospin symmetry breaking into account, however, we can rewrite \( \sigma_{\pi N} \) in terms of the mass splittings of the isospin multiplets

\[
\sigma_{\pi N} = \frac{3\bar{m}}{m_d - m_u} \left[ \frac{10}{3} (M_{\Sigma^0} - M_{\Sigma^+}) + \frac{5}{3} (M_{\Xi^-} - M_{\Xi^0}) - 4(M_{n^*} - M_{p^*}) \right].
\]  

Plugging the ratio \( (m_d - m_u)/(m_u + m_d) = 0.28 \pm 0.03 \) \([70]\) into Eq. (27), considering the experimental data for the corresponding baryon octet masses \([65]\), and using the values of the \( M_{n^*} \) and \( M_{p^*} \) predicted in Ref. \([38]\), we obtain \( \sigma_{\pi N} \approx 34 \text{ MeV} \), which is almost the same as that of Ref. \([35]\).
IV. SUMMARY AND CONCLUSION

In the present work, we aimed at investigating various observables of the baryon antidecuplet $\Theta^{+}$ and $N^{*}$, emphasizing on their dependence on the $\Theta^{+}$ mass within a chiral soliton model. We utilized the mass parameters $\alpha$, $\beta$, and $\gamma$ derived unequivocally in Ref. [22]. We first compared the present result of the $N^{*}$ mass with those predicted by the previous analyses [2, 45]. We then examined the dependence of the $N^{*}$ mass on the $\Theta^{+}$ one. We found that the measured value of the $\Theta^{+}$ mass by the LEPS collaboration turned out to be consistent with that of the $N^{*}$ mass by Kuznetsov and Polyakov [22] within the present framework. We then scrutinized the transition magnetic moments of the radiative decay $N^{*} \rightarrow N\gamma$. While $\mu_{pp^{0}}$ is almost independent of the $\Theta^{+}$ mass, $\mu_{nn^{0}}$ decreases slowly as $M_{\Theta^{+}}$ increases. We also discussed the results of the $N^{*} \rightarrow N\gamma$ transition magnetic moments with those of previous works. The decay width of the $\Theta^{+}$ was studied and was found to be $0.5 \pm 0.1$ MeV when the LEPS data of $M_{\Theta^{+}}$ was employed, which is compatible with the corresponding measured decay width by the DIANA collaboration. Finally, we analyzed the $\pi N$ sigma term within the present framework. It turned out that $\sigma_{\pi N}$ was almost independent of the $\Theta^{+}$ mass. We explained the reason why it was rather smaller than those in previous analyses, in particular, in Ref. [46]. In addition, we found a new expression for the $\pi N$ sigma term in terms of the isospin mass splittings of the hyperon octet as well as that of the antidecuplet $N^{*}$.

Acknowledgments

We are grateful to T. Nakano for suggesting the analysis of the $\Theta^{+}$ mass dependence of relevant observables. The present work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (grant number: 2010-0016265).

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