Abstract: This paper considers the vibrations of cylindrical shells when exposed internal explosive loads. Explosive load applied to the axis of symmetry of the cylindrical body. The problem is reduced to the study of the bending of the transverse oscillations relative to the element sheath. An analytical expression for the radial displacement and the corresponding numerical results.

Key words: Shell, explosive load, transverse vibrations, displacement, normal force, element, problem, vibrations, transverse.

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Introduction
The study sewn construction in the form of rods and shells is considered in [1, 2]. Calculation sewn construction is often based on field and laboratory experiments [3].

In this paper we consider the cylindrical protective structures [4, 12]. Under dynamic stress in the wall of the cylindrical protective structure having transverse vibrations of bending. Explosive load applied to the axis of symmetry of the cylindrical body. The problem is reduced to the study of the bending of the transverse oscillations relative to the element sheath.

II. Statement of the problem.
Fig. 1 shows an element with the current in it internally. The figure shows the bending moments along the generator shell $M_x$, annular $M_0$, normal ring of force $N_0$ and transverse forces $Q_x$, and the directions of the axes of coordinates and transverse forces $Q_x$, and the directions of the axes of coordinates and the corresponding displacement. The equilibrium equation shell element is the sum of the projections of all efforts on the axis Z. It can be written in the form

$$ m a d \phi d x \frac{\partial^2 w}{\partial x^2} + \frac{\partial Q_x}{\partial x} dx a d \phi + 2 N \phi d x \sin \frac{d \phi}{2} = p(x, t) a d \phi d x $$

Given, that
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\[
\sin \frac{d\phi}{2} \equiv \frac{d\phi}{2},
\]

have

\[
\frac{\partial Q_x}{\partial x} = \frac{\partial^2 M_x}{\partial x^2}.
\]

We denote by \(\sigma, h, \rho, u, p\) circumferential compressive stress, respectively, wall thickness, density, and external pressure; then the equation of the transverse vibrations can be written as:

\[
m \frac{\partial^2 w}{\partial t^2} + \frac{\partial M_x}{\partial x} + \frac{1}{a} N_x = p(x,t)
\]

(1.1)

Where \(N_{x0} = N_{0x}\); the torqueses \(M_{x0} = M_{0x}\) in both sections are zero.

By the same reason, annular an effort \(M_0\) and \(N_0\) must be the same along the entire perimeter shell; \(w\) - transverse movements; \(M_x\) - bending moment along the generator shell; \(N_0\) - normal ring force, \(m\) - mass per unit length; \(p(x,t)\) - external load, which is attached inside the cylindrical body; \(\nu\) - the Poisson coefficient

\[
N_x = \frac{Eh}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = 0,
\]

(1.2)

\[
N_\phi = \frac{Eh}{1-\nu^2} (\epsilon_\phi + \nu \epsilon_x)
\]

(1.3)

From (1.2) we obtain \(\epsilon_x = -\nu \epsilon_\phi\); substituting this into (1.3) and taking into account that

\[
\epsilon_\phi = -\frac{w}{t_0}
\]

have

\[
N_\phi = -\frac{Eh}{t_0} w
\]

(1.4)

Because of symmetry conditions can be seen that the curvature of the circumference of the enclosure should be constant. Then it follows from the theory of plates, that:

\[
M_x = D \frac{\partial^2 w}{\partial x^2}.
\]

Substituting the expressions (1.4) and (1.5) into equation (1.1), we obtain the equation of forced vibrations of a closed cylindrical shell [4,5,6,7,8].

\[
w^{IV} + 4\beta^4 w + \lambda^2 w^2 = p(x,t)/D
\]

(1.6)

Here

\[
\beta^4 = \frac{Eh}{4R^2D} = \frac{3(1 - \nu^2)}{E^2 h^2}; D = \frac{Eh^3}{12(1 - \nu^2)}; \lambda^2 = \frac{vh}{gD}
\]

As the origin of time \(t\) will take the start of the deformation. Thus, the problem of deformation of the cylindrical shell of the explosion is reduced to finding the solution of equation (6) satisfying the zero initial conditions and the corresponding boundary conditions.

The solution of this problem is reduced to the determination \(w(x,t)\), since the forces and stresses after determining \(w(x,t)\) are located.

Fig.1. Settlement scheme elementary area of a closed cylindrical shell with transverse vibrations.
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Fig. 2. Settlement scheme of a cylindrical shell.

With all the variety of loads, resulting in the explosion, the nature of deformation of the shell provided with stiffening rings can be found by solving for a single load instantly attached at a distance \( x = L_0 \) (Fig. 2.a).

Let us find the solution, i.e., define the movement of the shell wall at the instantaneous application of a load of the form

\[
P(x, t) = \begin{cases} 1, & n_p u x = L_0 \\ 0, & n_p u x \neq L_0 \end{cases}\]  

(1.7)

The solution of equation (1.7) at the right side of (1.8) in the form of a number of

\[w = \sum_{k=1}^{\infty} B_k(t) W_k(\beta_k x),\]  

(1.8)

Where \( w(\beta_k, x) \) - fundamental functions satisfying the equation

\[W_k''(\beta_k x) - \beta_k^4 W_k(\beta_k x) = 0\]  

(1.9)

and appropriate boundary conditions; \( \beta_k \) - unknown coefficients to be determined. As is known, the function \( U_k \) for the boundary conditions \( W_k = W_k'' = 0 \) at \( x = 0 \) and \( x = L \) have the following form;

\[W_k = \sin \beta_k x - sh \beta_k x - \frac{\sin \beta_k L - sh \beta_k L}{\cos \beta_k L - ch \beta_k L} (\cos \beta_k x - ch \beta_k x),\]

(1.10)

To find the unknown coefficients \( A_k \) series (1.8), in the form we are looking for solution of the problem, put the number as well as the expression (1.11) in equation (1.1). As a result of this substitution, taking into account the equality (1.11) we obtain the following differential equation for \( B_k \)

\[\ddot{B}_k + \frac{\beta_k^6}{x^2} B_k - \frac{A_k}{\lambda^2 d^2} = 0\]  

(1.12)

Fig. 3. Settlement scheme. Cylindrical protective structure.
A solution of these equations for zero initial conditions 
the following
$$B_k = \frac{C_n(1 - \cos q_k t)}{(\beta_k^2 + 4\beta^4)t}$$

Where $q_k = \sqrt{\beta_k^2 + 4\beta^4}$. 

Thus, the solution of equation (1.1) we have the following form:
$$W = \sum_{k=1}^{\infty} \frac{W_k(\beta_k x)}{D(\beta_k^2 + 4\beta^4)} \sin q_k x$$

Wake assumes that $t = T_g$ load removed. Solution for $t > T_g$ we find the principle of superposition, suggesting that at the time $t_1$ applied to the system unit load directed in the opposite direction. In this case the decision is determined by the formula
$$W = 2\sum_{k=1}^{\infty} \frac{W_k(\beta_k L_0) \sin q_k x}{D(\beta_k^2 + 4\beta^4)} \int_0^T U_k(\beta_k x) dx$$

The last formula can be obtained movement caused by the instantaneous unit impulse, i.e. when $t_1 \to 0$ and $T = 1$

$$W_m(e_0, x, t) = \sum_{k=1}^{\infty} \frac{W_k(\beta_k L_0) \sin q_k x}{D(\beta_k^2 + 4\beta^4)} \int_0^T U_k(\beta_k x) dx$$

Now consider the load $P(t)$ as a set of pulses. The action of the force $F(t)$ at time $T$ for a short time interval $dt$ can be considered as a pulse $R(t)dt$. Moving the shell wall at time $t(t > T)$, caused by this pulse is equal
to
$$dW = \sum_{k=1}^{\infty} \frac{W_k(\beta_k e_0) \sin q_k x}{D(\beta_k^2 + 4\beta^4)} \int_0^T R(t) dt \frac{\sin q_k x}{\sqrt{T^2 + 4\beta^4}}$$

Let us consider some particular cases:
1. Pressure $P(t) = P = const$. Pressure is applied instantaneously, and act on the membrane for a time $t$. At the time $t_1$ pressure instantaneously is removed. We find the solution for the given case load applied to the shell having boundary conditions $W = W' = 0$ at $x = 0$ and $x = L$.

For time interval $0 \leq t_1$, using (1.13) we obtain

$$Q_2(t) = \left( 0, 0 \right) + 0.72 \int_0^T x^2 W_k(\beta_k L_0) dx$$

3. The pressure $P(x) = \sum_{n=1}^{N_0} p_n \sin \frac{\pi x}{L} \text{ applied instantaneously and is valid for the time } 0 < T$. At time $T$ pressure instantaneously withdrawn. We find the solution for the given case load, taking boundary conditions $W = W' = 0$ at $x = 0$ and $x = L$. For the data the boundary conditions at $N = 1$:

$$W = \int_0^T x^2 W_k(\beta_k L_0) dx$$

III. The numerical results and their analysis are given below.

When calculating take the following initial data:

Where.

Some solutions of particular problems identified natural frequencies, which are listed in Table 1. The results of calculations are compared with the results presented in [4,10,11]. The discrepancy between the results of up to 20% Calculation of cylindrical shell on the effect of dynamic load.
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From Figure 5 it is clear that the time reaches its maximum value at the initial time, and then gradually decreases.

4. Dynamic stress-strain state of an infinitely long cylindrical shell when exposed to explosive load.

Consider a cylindrical shell (Fig. 3) clamped around the edges made of reinforced concrete and having the following dimensions and physical constants: $D = 4m$, the height $h = 4m$; elastic modulus $E = 2,1 \cdot 10^5 \text{ kg/m}^2$, $v = 0.25$; volumetric weight of the material of the dome $\gamma = 2,1 \cdot 10^3 \text{ kg/m}^2$.

On the inner cylinder acts uniformly distributed load, time-varying linearly (Fig. 2). Figure 4 shows the chain and bending stresses in the shell when exposed to pulsed loads as $P(x, \phi, t) = \sigma e^{-t/\tau}$, ($D = 4m$ - amplitude load).

From Figure 5 it is clear that the time reaches its maximum value at the initial time, and then gradually decreases.

4. Dynamic stress-strain state of an infinitely long cylindrical shell when exposed to explosive load.

Moving wall infinitely long cylindrical shell with instant annexed thereto unit load found by solving for the ultimate shell passing to the limit $l \to \infty$. Form this case, the solution (2.14) takes the form:
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\[ W = \frac{1}{\pi} \int_{0}^{\infty} \frac{\cos y_1 x_1}{(y^4 + 4\beta^4)} D \left( 1 - \cos \frac{1}{x} \sqrt{y^4 + 4\beta^4 t} \right) \]

This expression coincides with the expression obtained [5] with the cosine - Fourier transformation.

Table 2. Estimated efforts in the wall.

| X./ H | X   | W, мм | X" | Mx, кн. |
|-------|-----|-------|-----|---------|
| -     | 0   | 0     | -11,011 | -1030.68 |
| 0.1   | 1.773 | 0.063 | -7.294 | -682.76 |
| 0.2   | 6.136 | 0.216 | -3.661 | -342.69 |
| 0.3   | 11.856 | 0.419 | -0.296 | -27.71 |
| 0.4   | 17.706 | 0.625 | 2.582 | 241.69 |
| 0.5   | 22.635 | 0.779 | 4.727 | 442.47 |
| 0.6   | 25.88 | 0.914 | 5.934 | 555.45 |
| 0.7   | 27.029 | 0.964 | 6.072 | 568.37 |
| 0.8   | 26.024 | 0.919 | 5.083 | 475.79 |

| X./ H | Mx, кн. | Nx, кн/м | X" | Qx, кн/м |
|-------|---------|-----------|-----|---------|
| -     | 171.78  | 0         | 6.192 | 0       |
| 0.1   | 113.79  | 318.8     | 6.15 | 575.67  |
| 0.2   | 57.11   | 1093.1    | 5.885 | 550.87  |
| 0.3   | 4.62    | 2120.4    | 5.267 | 493.02  |
| 0.4   | 40.28   | 30162.9   | 4.24 | 396.89  |
| 0.5   | 73.75   | 4043.5    | 2.835 | 265.37  |
| 0.6   | 92.58   | 4625.5    | 1.138 | 106.52  |
| 0.7   | 94.73   | 4827.9    | -0.705 | -65.99 |
| 0.8   | 79.3    | 4650.8    | -2.555 | -239.16 |
| 0.9   | 47.24   | 4149.8    | -4.27 | -399.69 |
| 1     | 0       | 3456.5    | -5.75 | -538.23 |

If the applied load instantaneously after time T will be charged, the decision in this case, was found by a superposition, will have the following form

\[ W = \frac{1}{\pi} \int_{0}^{\infty} \frac{\cos y_1 x_1 \sin \frac{1}{2\lambda} \sqrt{y^4 + 4\beta^2 T}}{D_1} Y_1(t) \]

Fig. 6. Schedule to determine the values of the dynamic coefficients At t < T where

\[ Y_1(t) = \sin \frac{1}{\lambda} \sqrt{y^4 + 4\beta^2 \left( t - \frac{T}{2} \right)} \]

\[ D_1 = D(y^4 + 4\beta^4) \]
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Fig. 7. Stresses in cylindrical body with a pulse Speed and load.

Figure 5 and 6 shows the stress in the cylindrical body by the action of impulse and step load. Subsequent waves effects are less and less energy, so for practical purposes is sufficient to apply no more than two - three waves. Calculated efforts shell wall are given in Table 5.

Fig. 8 Change displacement versus time.

In view of the required accuracy of the results of calculations are presented in Figure 3.8, with

\[ v = 0.25; E = 2 \cdot 10^4 \cdot \frac{k g \cdot h}{s m^2 \cdot R} = 0.1 \]
\[ R = 1; 2; 3; N = 10^{-4}; 1, 2, 3… 5 \]

At \( N = 5 \) and \( N = 6 \) value \( w \) differs from the previous fifth decimal place. Change \( w \) depending on \( t \) shown in Figure 8.

It can be seen that with increasing time (> 0.03 sec) movement reaches its maximum value, and then approaches zero. There results are presented in Figure 8.

V. Conclusions.

1. An algorithm and a program is to address the problem of the impact of shock waves on the cylindrical shell. Numerical results and to analyze their error. This technique is not very important in terms of structural strength. In the axially symmetric case, the effects of reflection are extended mainly

2. The results show that the effect of the reflected waves is significant at relatively small scale charge. The largest county deformation concentrated in the central zone of the cylinder, near the line of the meeting, and the highest - longitudinal related to edge effects - in the vicinity of the ends. Predominant among the are district deformation.

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