The uses of FR and QWL for site effect

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Abstract. For an engineering site without strong motion records, the effect of full resonance (FR) and quarter-wavelength (QWL) to evaluate site response is discussed by comparing the predicted ground motion at each interface with the analytical solution of the frequency-wavenumber (F-K) method. We analyse the differences of time histories and amplitude spectra, with the inputs of full wave and upward wave on the buried bedrock. By comparing with the F-K result, the input of upward wave is reasonable. A solution to extract upward wave from full wave is presented and proved.

1. Introduction
Local site condition is important to the studies on strong ground motion. Attempts to evaluate site response have a long history and much effort has been expended. The most common theoretical methods are linear full resonance (FR), quarter-wavelength (QWL), equivalent linear and nonlinear[1]. FR and QWL are widely used, the former is the theoretical prediction of site amplification that accounts for the constructive and destructive interference of all reverberations in layered media, and the latter is an approximation. A few comparisons of SRI amplifications and FR amplifications have appeared in literatures[2,3]. In this paper, we compare the site response from these two methods, in the case of the 1994 Northridge Earthquake. We focus on the responses at different depths, with the inputs of upward waves and full waves, from the frequency-wavenumber (F-K) method[4].

2. Source model and crustal velocity structure
On January 17, 1994, the Mw6.8 Northridge Earthquake occurred, and the epicenter (34.213°N, 118.537°W) was near Northridge in the San Fernando Valley, a populated urban area northwest of Los Angeles, in southern California. After the event, many source models were published, based on observations of strong ground motions and deformations of the land surface. As the detailed investigation by U.S. Geological Survey[5], the earthquake began as a rupture on a hidden fault at a depth of about 17.5 km beneath the San Fernando Valley. Then, the rupture propagated upward and northwestward along the fault plane and spread out. Eventually, the rupture covered an area of approximately 15 km×20 km and terminated at a depth of about 5-6 km. To inverse the source process, Zeng and Anderson used digital records at 10 CSMIP strong-motion stations, with epicentral distance less than 33 km[6]. They modelled the slip area as 15 km×18 km, which covered three asperities, with strike, dip, and rake angles of 123°, 44° and 109°. The rupture initiated a weaker asperity near the lower southeast corner in 0-3s, and then ruptured toward two other larger asperities located 5 to 15km west of the hypocenter in 3-5s and 5-7s, with the rupture velocity of 2.7 km/s. Thio and Kanamori used body wave (P and SH wave) records from IRIS and IDA/IRIS networks, with epicentral
distances between 30° and 90°\cite{7}. The rupture was represented by three subevents with the interval of about 2s and strike, dip, and slip of 130°, 42°, and 116°. The first one occurred at a depth of about 19 km, followed by the second and largest one at 17 km and the third at about 13 km. Shen et al. used global positioning system (GPS) data from 62 observation sites. They preferred model included two fault surfaces: the main fault of 30 km×30 km with strike 122° and dip south 38°, a second rectangular fault of 12 km×10 km with dip south 58°, and these two hinged at a depth of 9.1 km\cite{8}. The maximum slip on the former was about 2.2 m at depth of 12.4 km.

Wald et al. determined the source model from the joint inversion of strong ground motion records, P and SH teleseismic body waves, GPS data and leveling-line displacements\cite{9}. The model size was 18 km×24 km, which covered three subfaults, with strike, dip, and rake angles of 122°, 40° and 101°. The rupture began at the hypocenter with depth of 17.5 km, proceeded up-dip to the largest subevent about 12 km away, then to the northwest, and terminated at a depth of about 6 km, with the rupture velocity of 3 km/s. We use this model in the following ground motion synthesis. The regional velocity structure is from Graves and Pitarka\cite{10} and listed in table 1.

| Layer | Depth of each layer (km) | P wave velocity (km/s) | S wave velocity (km/s) | Density (g/cm³) | Qp | Qs |
|-------|-------------------------|------------------------|------------------------|-----------------|----|----|
| 1     | 0.942                   | 3.70                   | 2.10                   | 2.500           | 210.0 | 105.0 |
| 2     | 2.000                   | 4.40                   | 2.40                   | 2.600           | 240.0 | 120.0 |
| 3     | 2.000                   | 5.10                   | 2.80                   | 2.700           | 280.0 | 140.0 |
| 4     | 1.000                   | 5.60                   | 3.15                   | 2.750           | 315.0 | 157.0 |
| 5     | 5.000                   | 6.15                   | 3.60                   | 2.825           | 360.0 | 180.0 |
| 6     | 5.000                   | 6.32                   | 3.65                   | 2.850           | 365.0 | 182.5 |
| 7     | 5.000                   | 6.55                   | 3.70                   | 2.900           | 370.0 | 185.0 |
| 8     | 10.000                  | 6.80                   | 3.80                   | 2.950           | 380.0 | 190.0 |
| 9     | -                       | 7.80                   | 4.50                   | 3.200           | 450.0 | 225.0 |

3. Methods of FR and QWL

For the case of vertical input of SH wave, shown as figure 1, in which $E_n$ and $F_n$ are the amplitude coefficients of upward and downward waves in the $n$th soil layer, $h_n$ is the thickness of the $n$th layer, $\rho_n$ is density, $v_{SN}$ is shear wave velocity, $d_n$ is damping ratio, $z_n$ is depth.

![Figure 1. Layered soil model.](image)

The transfer relationship of upward and downward wave amplitudes between adjacent layers can be expressed as equation (1).

$$H_{n+1} = T_n H_n$$

where, $T_n$ denotes transfer matrix between adjacent layers $n$ and $n+1$, as equation (2), $H_n$ and $H_{n+1}$ denote the vectors of wave amplitudes, equation (3).
where, \( \alpha_n = \frac{\rho_n v_s^* n}{\rho_{n+1} v_s^* n+1} \) is the impedance ratio of the adjacent layers, \( v_s^* \) denotes complex shear wave velocity of \( n^{th} \) layer, \( k_n^* \) denotes complex wavenumber.

By introducing unit matrix \( T_0 \), the transfer relationship of wave amplitudes between 1\(^{st}\) and \( n^{th} \) layers can be expressed as equation (4).

\[
H_n = T_{n-1} \cdots T_0 H_1
\]

QWL is a result of energy conservation of waves propagating through the media with gradually changing velocity, which approximately estimates the average amplification of soil layers from the surface to a depth \( z \) as the square root of the impedance ratio in equation (5)\(^{[2,11,12]}\).

\[
A(f) = \sqrt{\frac{\rho_s \beta_s}{\rho_s(f) \beta_s(f)}}
\]

where, \( \rho_s \) and \( \beta_s \) are the density and the shear wave velocity near the source; \( \bar{\rho}_z \) and \( \bar{\beta}_z \) are the average values from the surface to the depth \( z \), for a particular frequency corresponding to a quarter wavelength, as in equation (6).

\[
\bar{\rho}_z(f) = \frac{1}{z(f)} \int_0^z \rho_s \, dz
\]

\[
\bar{\beta}_z(f) = \frac{1}{z(f)} \int_0^z \beta_s \, dz
\]

4. Synthesize the ground motion at different interfaces by F-K method

F-K method can be regarded as a simplified analytical method to synthesize ground motion. Crustal velocity structure is simplified as horizontal layered medium, and Green's function can accurately express the influence of the crustal structure, the wave velocity, density and quality factor of each medium layer, and the detailed soil structure near the surface can be considered. Through the convolution of the rupture process of each sub-source on the fault plane and the Green's function, the ground motion on surface and at different interfaces can be synthesized.

The shear wave velocity structure of GVDA station (Garner Valley Downhole Array) in California is collected from Bonilla\(^{[13]}\) and listed in table 2. The shear wave velocity \( V_s \) at the depth of 219 m reaches 3000 m/s, and that at the depth of 942 m is 3050 m/s, which is larger than the average value of Northridge region in table 1. To emphasize the influence of the first four subsurface layers, we overlay those layers on the first layer in table 1.

| Layer | Depth of each layer (m) | Density (kg/m\(^3\)) | \( V_p \) (m/s) | \( V_s \) (m/s) | \( Q_p \) | \( Q_s \) |
|-------|------------------------|-----------------------|----------------|----------------|--------|--------|
| 1     | 6                      | 2000                  | 1225           | 175            | 15     | 10     |
| 2     | 9                      | 2000                  | 1525           | 200            | 15     | 10     |
| 3     | 7                      | 2200                  | 1600           | 320            | 15     | 10     |
| 4     | 36                     | 2400                  | 2000           | 550            | 20     | 15     |
| 5     | 29                     | 2800                  | 2150           | 650            | 20     | 15     |
| 6     | 132                    | 2800                  | 2820           | 1632           | 50     | 30     |
| 7     | 381                    | 2800                  | 5190           | 3000           | 100    | 50     |
| 8     | 4400                   | 2800                  | 5250           | 3050           | 1000   | 500    |
| 9     | 2800                   |                      | 6220           | 3490           | 1000   | 500    |
With the help of the F-K method\cite{4}, full-wave acceleration time histories on the surface, at the bottoms of the first layer, the second layer and the third layers, and on the buried bedrock are synthesized, respectively, on the fault-parallel (FP) and fault-normal (FN) directions. These time histories involve the incident (transmitted) waves from the bedrock and the reflected (transmitted) waves from the overlying interfaces, which can be divided into upward wave and downward wave, according to propagating direction. FR is based on the layered medium model in half space, in which it is assumed that there is no upward propagation of reflected wave, and only incident wave from buried bedrock, that is, the input of upward wave, is considered. Upward wave and downward wave are considered separately in the F-K method. The comparison between full wave, summed by the upward and downward waves on buried bedrock, and incident upward wave is shown in figure 2.

![Figure 2. Full wave and upward wave on buried bedrock, synthesized by the F-K method.](image)

It is shown in figure 2 that the amplitudes of full waves are much larger than those of the incident upward waves.

5. Site response predicted by FR, QWL and F-K methods

The bottom of the fourth layer in table 2 is deemed as the top of buried bedrock, on which FP ground motion synthesized by the F-K method is taken as input. Ground motions on four interfaces, including the surface, the first layer, the second layer and the third layer, are predicted by FR and QWL, respectively. With the help of FR, the ground motion on each interface can be calculated directly, however, QWL can only be used for single layer. First of all, we calculate the amplification spectra $QWL_s, QWL_3, QWL_2$ and $QWL_1$ of the surface ground motions to those on the bedrock, the third layer, the second layer and the first layer. From the ratio of $QWL_s$ to the last three, we then obtain the amplification spectra of ground motions at three interfaces to those on the bedrock, which are multiplied by the Fourier spectra of the ground motions on the top of bedrock to predict the ground motions at four interfaces. In FR, damping ratio model is adopted and the high frequency cutoff model of $\exp(-\pi \kappa_0\omega)$ is adopted in QWL. By inputting full wave, ground motions predicted by FR and QWL are shown in figure 3a and figure 3b, respectively, and compared with FP ground motion synthesized by the F-K method at the same depth.

![Figure 3a. FR and F-K results (full wave).](image)

![Figure 3b. QWL and F-K results (full wave).](image)
It is shown in figure 3 that when we input full wave of ground motion on the buried bedrock, the waveforms of ground motions predicted by FR and QWL are similar to that predicted by the F-K method, the amplitudes are larger, and the difference decreases with the increase of depth. By inputting upward wave, the results are shown in figure 4a and figure 4b, respectively.

As shown in figure 4, when we input upward wave of ground motion on the buried bedrock, both waveforms and amplitudes of the ground motion predicted by the two methods are similar to those predicted by the F-K method. Compared with the PGA predicted by the F-K method, the relative error from FR inputted by full wave is 29.24%–52.14%, that inputted by upward wave is 2.3%–8.91%; the relative error from QWL inputted by full wave is 4.49%–62.47%, and that inputted by upward wave is 7.87~40.82%.

The acceleration Fourier amplitude spectra from FR and QWL, at four interfaces, are shown in figure 5a and figure 5b, both full wave and upward wave are inputted on the buried bedrock.

When we input full wave, the amplitude spectrum at the surface from FR is close to that from QWL, which is higher than that from the F-K method. As depth increases, the amplitudes from QWL decrease to that from the F-K method, since QWL underestimates site amplification. When we input upward wave, the amplitude spectra at the surface from three methods are close. As depth increases,
the amplitudes from FR and F-K are almost the same and remain stable, while the amplitudes from QWL decrease. In the method of traditional spectral ratio, the ground motions on surface and at borehole are inputs, which are full wave involving reflected wave at different interfaces. However, site response is excited by incident wave, that is, upward wave, rather than full wave. Figure 6 shows the spectral ratios from TF and the F-K method with the inputs of full wave and upward wave.

![Figure 6. Spectral ratios from TF and the F-K method.](image)

It is shown that the spectral ratios from the F-K method with upward wave input on the bedrock are more close to FR than that with full wave input, and they are essentially in agreement at the frequencies of 1-10Hz.

The predicted ground motions and amplification factors will be very different, when we input full wave or upward wave on the bedrock. The input of upward wave is reasonable. Full wave on the buried bedrock can be transmitted to upward wave by equation (7), which is the relationship between the acceleration Fourier amplitude spectra of full wave \( A_{\text{full}} \) and upward wave \( A_{\text{up}} \).

\[
\frac{A_{\text{up}}}{A_{\text{full}}} = \frac{E_N}{E_N + F_N}
\]  

(7)

The full wave on the buried bedrock, predicted by the F-K method, is transmitted to upward wave by FR and shown as figure 7. It is shown that the upward waves transmitted by FR are consistent with the upward waves synthesized by the F-K method.
6. Conclusions
Predicted ground motion at four interfaces by FR and QWL, with inputs of full wave and upward wave, are compared with the analytical solution of the F-K method, respectively, for the example of GVDA station in the 1994 Northridge Earthquake. The results show that the waveforms and amplitudes of the ground motions predicted by these two methods are similar to those by the F-K method. The amplitudes with full wave input are larger, and the difference decreases with depth increases. It is pointed out that the ground motion on surface will be overestimated if we input full wave on buried bedrock, thus, the transmitted upward wave should be inputted for the prediction, and a method is provided as well.

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