Nucleon Resonance Transition Couplings to Vector Mesons

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Abstract

Recent heavy ion experiments indicate modifications of the $\rho$--meson mass \textit{in medium}. In the CERES experiments $\rho$--mesons are produced at $\sim$ normal nuclear matter density, where hadrons are more appropriate constituents than quarks. A collective "nuclear $\rho"$, in which every nucleon is excited to the $N(1520)$ state, with equal amplitude, enters in this description. At the higher densities reached by future experiments constituent quarks become the appropriate variables. Here the $\rho$ and $\omega$ transition couplings to the nucleon resonances up to 1700 MeV, including the $N(1520)$, are derived by means of the chiral quark model. The relevant coupling constants are expressed in terms of the corresponding vector coupling constants to nucleons. The quality of the model relations is tested by a calculation of the corresponding pion-resonance coupling constants, which are known from the empirical pion decay widths.
1 Introduction

The observed enhancement \cite{1, 2, 3, 4} of low mass dilepton production in relativistic nucleon-nucleon collisions has stimulated considerable theoretical activity. The consensus in the theoretical treatment is that the dileptons seen in the CERES experiments \cite{1} to date originate with densities close to the density $\rho_0$ of normal nuclear matter. At these densities hadronic variables are more suitable than quarks in the theoretical treatment. A rather convincing description is that based on the coupling of nucleon isobar-hole excitations was initiated by Peters et al. \cite{5} and later systematized by Rapp and Wambach \cite{6}. A relation between this approach and the overall scaling relations proposed by Brown and Rho \cite{7}, formulated in quark language and possibly applicable at higher densities has been given in ref.\cite{8}. More recently the empirical data have been used to fix the parameters in an effective field theory approach \cite{9, 10}. In this relatively model independent treatment a good description of the dilepton excess seen by in the CERES experiments is given.

It has been argued that at higher densities, in the region between nuclear matter density, and the critical density of chiral symmetry restoration, hadronic variables should be replaced by quark variables \cite{11}. The example of the Nambu-Jona-Lasinio model, in which the constituent quark mass is the order parameter, suggests that hadron masses go to zero in the limit of bare current quark masses. In ref.\cite{8} this is brought about by replacing the $\rho-$meson mass $m_{\rho}$, which sets the scales of the denominator by the Rapp-Wambach Lagrangian \cite{6}, by the effective $\rho-$meson mass $m^*_{\rho}$. While this result may be obtained on the basis of self-consistency, the issue of the most appropriate treatment at densities above that of normal nuclear matter remains largely open. The chiral quark model, which implies relations between the vector meson transition couplings to the nucleon resonances and the vector meson couplings to nucleons, may impose useful constraints on the theoretical treatment.

The chiral quark model, in which constituent quarks couple directly to mesons, is known to describe the properties of the ground state octet and decuplet baryons fairly well \cite{12}. In particular it gives expressions for the $N-\Delta(1232)$ transition couplings, which are good at the level of $\sim 25\%$ accuracy, or better. Moreover, when augmented with a linear confining interaction, and two-pion and vector meson interactions between the quarks, it describes
the whole empirical baryon spectrum satisfactorily [13, 14].

We here use this model to calculate the $\rho$- and $\omega$-meson transition couplings to the nucleon resonances up to 1700 MeV. These coupling constants cannot be determined directly from empirical decay widths, as they lie below the thresholds for vector meson decay. The quality of the model is tested by a calculation of the corresponding pion-resonance transition coupling constants in the impulse approximation, which are then compared to the values that are determined from the pionic decay widths. That the single quark coupling model for pion decays should provide a fair overall description is indicated by the fact that it implies that the ratios of the decay widths for $\Xi^* \rightarrow \Xi \pi$, $\Sigma^* \rightarrow \Sigma \pi$ and $\Delta \rightarrow N \pi$ should be 1:4:9, which compares well to the empirical ratio 1:3.9:12.6 (that the number for the $\Delta$ exceeds 9 is due to the larger phase space volume available). For the excited $S-$ and the $P-$shell resonances the quark model values, which are calculated here in the impulse approximation, fall within factors 1.5 – 2 of the empirically extracted transition couplings. The situation for the $D-$shell resonances is less satisfactory. Improved agreement requires taking into account higher order contributions from two-quark operators.

The vector meson transition coupling constants to nucleon resonances are defined in terms of Lagrangian densities for the transition couplings, which involve generalized Rarita-Schwinger vector spinors. Comparison of the matrix elements of these Lagrangians to the corresponding matrix elements in the quark model makes it possible to express the transition coupling constants in terms of the corresponding vector meson coupling constants to the nucleons. The latter are determined - albeit within a liberal uncertainty range - by fits to nucleon-nucleon scattering data with phenomenological boson exchange interaction models. These expressions involve $SU(2)$ Clebsch-Gordan coefficients as well as orbital matrix elements of quark wave functions, which connect the $P-$ and $D-$shell and the excited $S-$shell states to the ground states. The latter depend on the Hamiltonian model for the 3-quark system. We here employ a simple covariant harmonic oscillator model based on linear confining interaction with a flavor-spin dependent hyperfine interaction, which describes the empirical spectrum very well [15].

There is some freedom in the choice of the resonance transition coupling Lagrangians. This freedom is constrained by comparison with the corresponding quark model expressions, especially because of the orthogonality of the resonance and nucleon wave functions in the quark model. The match-
ing of matrix elements of covariant Rarita-Schwinger type Lagrangians \[16\] and quark model matrix elements will here be made for off-mass shell vector mesons with zero energy. This choice of kinematics is made with application of resonance propagation in nuclear matter in mind. The coupling of spin-isospin modes that propagate in matter to the \(P\)-shell resonances has recently been shown to be both significant and intricate \([5, 9, 10]\). A key aim of the present study is to determine the strength of this coupling.

Only rough correspondences can be made between the couplings in the the chiral quark model and in the hadronic model. Nevertheless, the coherence in the \(\omega\)-meson coupling to the nucleon, where the factor 3 in the \(SU(3)\) relation \(g_{\omega NN} \approx 3g_{\rho NN}\) arises in the sum over the three quarks in the nucleon, seems to disappear as the coupling \(g_{\omega NN}^{(1520)}\) in the quark model is roughly equal to \(g_{\rho NN}\) in the hadronic model, which lacks coherence in the quark sum. In the quark model we find that \(g_{\omega NN}^{(1520)} \sim 1/2g_{\rho NN}^{(1520)}\). The same ratio of \(\omega\) to \(\rho\) transition couplings is found in hadronic resonance models as discussed below. A simple explanation for this is still wanting.

This paper is divided into 4 sections. In section 2 we derive the pion-
resonance couplings and compare them to empirical data. The \(\rho\)-meson and \(\omega\)-meson resonance transition couplings are derived in section 3. A summa-
izing discussion is given in section 4.

## 2 Pion coupling constants

Before deriving the expressions for the vector meson transition couplings to nucleon resonances it is instructive to derive the corresponding pion transition coupling constants within the quark model. Under the assumption that the pion transitions are described by single quark operators one may also derive these coupling strengths directly from experiment. Overall the single quark operator approximation for the pion coupling to nucleon resonances underestimates the pion decay widths of the resonances, and therefore mainly has qualitative value \([7, 8]\). It does however determine the phases of the coupling constants, and, as shown below, if these are multiplied by about a factor 2, the decay widths are within a few ten percent of the empirical values.

The coupling of pions to constituent \(u\) and \(d\) quarks may be described by
the pseudovector coupling

$$\mathcal{L}_{\pi qq} = i \frac{f_{\pi qq}}{m_\pi} \bar{\psi}_q \gamma_5 \gamma_\mu \partial_\mu \vec{\phi}_\pi \cdot \vec{\tau} \psi_q. \quad (2.1)$$

Here $\vec{\phi}_\pi$ is the isovector pion field operator, $\psi_q$ is the constituent quark field and $m_\pi$ is the pion mass. The pseudovector pion-quark coupling constant may be determined from the corresponding pion-nucleon coupling constant by comparison to the $\pi NN$ coupling:

$$\mathcal{L}_{\pi NN} = i \frac{f_{\pi NN}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma_\mu \partial_\mu \vec{\phi}_\pi \cdot \vec{\tau} \psi_N. \quad (2.2)$$

Comparison of the matrix elements of the Lagrangian (2.1) and (2.2) for the case of a proton with spin up, using the $SU(6)$ quark model wave functions in the case of the former, yields

$$< p, \frac{1}{2} | \mathcal{L}_{\pi NN} | p, \frac{1}{2} > = -i f_{\pi NN} \frac{k_3}{m_\pi},$$

$$< p, \frac{1}{2} | \sum_{q=1}^3 \mathcal{L}_{\pi qq} | p, \frac{1}{2} > = -i \frac{5}{3} f_{\pi qq} \frac{k_3}{m_\pi}, \quad (2.3)$$

where $k_3$ is the third component of the pion momentum. This gives the relation

$$f_{\pi qq} = \frac{3}{5} f_{\pi NN}. \quad (2.4)$$

As $f_{\pi NN} \approx 1$ it follows that $f_{\pi qq} \approx 0.6$. This result for the pion-quark coupling constant is close to that, which is obtained from the Goldberger-Treiman relation for quarks:

$$f_{\pi qq} = \frac{g_A^q m_\pi}{2 f_\pi}. \quad (2.5)$$

With $g_A^q \approx 0.88$ and the value $f_\pi = 93$ MeV for the pion decay constant this relation gives $f_{\pi qq} \approx 0.66$. This coupling model does give a reasonably satisfactory account of the pion decay widths of the $D$-meson resonances, for which the single quark current model should be adequate [21].

The chiral quark model may be employed to express the transition coupling constant of pions to nucleon resonance in terms of the pion quark coupling constant $f_{\pi qq}$. Since this is given in terms of the pion-nucleon coupling
constant $f_{\pi NN}$ by (2.3), it thus becomes possible to express all pion-resonance transition coupling constants in terms of the pion-nucleon coupling constant.

These relations will depend on the orbital wave functions of the 3 constituent quarks that form the baryons. We shall here use the covariant harmonic oscillator model for the mass operator of the 3 quarks constructed in ref. [15] to generate the 3 quark wave functions. That model is formed by an, in effect, linear confining interaction with a spin, flavor and orbital angular momentum dependent hyperfine interaction. It describes the baryon spectrum up to $\simeq 1700$ MeV to an accuracy of a few percent, which is quite adequate for the present application.

The 3-quark wave functions will be bilinear combinations of spin-flavor and orbital wave functions. The latter are products of harmonic oscillator functions of the two Jacobi coordinates of the 3-quark system:

$$\vec{r} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2),$$  

$$\vec{\rho} = \sqrt{\frac{2}{3}}(\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}).$$

The 3-quark wave functions in the model of ref. [15] are eigenfunctions of the mass operator

$$\mathcal{M}_0 = \sqrt{3(\vec{r}^2 + \vec{k}^2 + \omega^4(\vec{\rho}^2 + \vec{r}^2))},$$

where $\omega$ is a parameter that determines the strength of the confining interaction. Jacobi momenta $\vec{r}$ and $\vec{k}$ are the canonical momentum operators that are conjugate to the Jacobi coordinates (2.5). The numerical value for the oscillator parameter $\omega$ is 311 MeV, as determined from the baryon spectrum. This value will be used here. In the impulse approximation the empirical value of the rms radius of the proton (0.8 fm) obtains with $\omega = 245$ MeV.

The eigenfunctions of the mass operator (2.6) are products of harmonic oscillator functions of $\vec{r}$ and $\vec{\rho}$: $\varphi_{n_1l_1m_1}(\vec{r})\varphi_{n_2l_2m_2}(\vec{\rho})$. The appropriate symmetrized combinations of these with spin and isospin wave functions for the $S-$, $P-$, $D-$ and lowest excited $S-$state resonances are listed in Table 1. In the table $|\frac{1}{2}, s >_{\pm}$ and $|\frac{1}{2}, t >_{\pm}$ denote spin and isospin wave functions of mixed symmetry, which are symmetric (+)(”(112)” or antisymmetric (−)(”(121)” under exchange of the spins or isospins of the first two
quarks. The states $|\frac{3}{2}, s >$ and $|\frac{3}{2}, t >$ denote spin and isospin states with total spin and isospin $\frac{3}{2}$, which are symmetric under exchange of any set of two coordinates.

The explicit expressions for the states of mixed symmetry with spin-$z$ projection $s_z$ are

$$|\frac{1}{2}, s_z >_+ = \sum_{abc} (\frac{1}{2}, \frac{1}{2}, a, b|1, m)(\frac{1}{2}, 1, c, m|\frac{1}{2}, s_z)|a, b, c >, \quad \text{(2.8a)}$$

$$|\frac{1}{2}, s_z >_- = \sum_{ab} (\frac{1}{2}, \frac{1}{2}, a, b|0, 0)|a, b, s_z >. \quad \text{(2.8b)}$$

Here $|a, b, c >$ represent product states of three spins with $z-$projections $a, b$ and $c$ respectively. The corresponding isospin states with isospin-$z$ projection $t_z$ are readily constructed by analogy [19]. The rule for construction of symmetric combinations of product states of spin, isospin and spatial wave functions is based on the outer products of $S_3$.

Given the 3 quark wave functions for the nucleon resonances in Table 1, it becomes possible to calculate the matrix elements,

$$< p, \frac{1}{2} | \sum_{q=1}^{3} L_{\pi qq} | N^*+, \frac{1}{2} > = -\frac{if_{\pi qq}}{m_\pi} < p, \frac{1}{2} | \sum_{q=1}^{3} \bar{\sigma}^q (\vec{k} - \omega_\pi \vec{v}_q) \tau_3^q e^{-i\vec{k} \cdot \vec{r}_q} | N^*+, \frac{1}{2} >, \quad \text{(2.9)}$$

of the pion-quark Lagrangian (2.1). Here $N^*$ represents a nucleon or a $\Delta$ resonance, and \( \vec{k} \) is the momentum and \( \omega_\pi \) the energy of the pion. The velocity operator for quark $q$ is denoted $\vec{v}_q$. These matrix elements, with the overall factor $-if_{\pi qq}/m_\pi$ divided out, are listed in Table 2 for the resonances in Table 1.

In Table 2 the orbital matrix elements have been calculated with the eigenfunctions of the mass operator (2.7). These are harmonic oscillator functions, with the explicit forms

$$\varphi_{000}(\vec{\rho}) = (\frac{\omega_\pi^2}{\pi})^{3/4} e^{-\rho^2/2},$$

$$\varphi_{01m}(\vec{\rho}) = \sqrt{2} \omega \vec{\rho}_m \varphi_{000}(\vec{\rho}),$$

$$\varphi_{200}(\vec{\rho}) = \sqrt{\frac{2}{3}} \omega^3 (\rho^2 - \frac{3}{2\omega^2}) \varphi_{000}(\vec{\rho}),$$

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\[ \varphi_{02m}(\vec{\rho}) = \sqrt{\frac{4}{15}} \omega^2 \rho^2 \varphi_{000}(\vec{\rho}) \sqrt{4\pi} Y_{2m}(\hat{\rho}). \] (2.10)

In the Table 2 values of the following overlap integrals have been used:

\[ (\varphi_{000}(\vec{\rho}), e^{-i\sqrt{\frac{2}{3}} \vec{\rho} \cdot \vec{\chi}} \varphi_{000}(\vec{\rho})) = e^{-k^2/6\omega^2}, \] (2.11a)

\[ (\varphi_{000}(\vec{\rho}), e^{-i\sqrt{\frac{3}{5}} \vec{\rho} \cdot \vec{\chi}} \varphi_{200}(\vec{\rho})) = -\frac{\sqrt{6}}{18 \omega^2} k^2 e^{-k^2/6\omega^2}, \] (2.11b)

\[ (\varphi_{000}(\vec{\rho}), e^{-i\sqrt{\frac{3}{5}} \vec{\rho} \cdot \vec{\chi}} \varphi_{01m}(\vec{\rho})) = -i \frac{\sqrt{3} k_m}{3 \omega} e^{-k^2/6\omega^2}, \] (2.11c)

\[ (\varphi_{000}(\vec{\rho}), e^{-i\sqrt{\frac{3}{5}} \vec{\rho} \cdot \vec{\chi}} \varphi_{02m}(\vec{\rho})) = -\frac{\sqrt{3}}{9 \omega^2} k^2 e^{-k^2/6\omega^2} \sqrt{\frac{4\pi}{5}} Y_{2m}(\hat{k}). \] (2.11d)

The goal here is not, however, these quark model relations per se, but expressions for the pion transitions couplings to the resonances, when these are described as (generalized) Rarita-Schwinger field operators [16]. These couplings may be described by the following Lagrangians:

\[ \mathcal{L}^{(1232)}_{\pi N \Delta} = \frac{f_{\pi N \Delta}}{m_{\pi}} \bar{\psi} \gamma^\mu \partial_\mu \vec{\phi} \cdot \vec{\chi} \psi, \] (2.12a)

\[ \mathcal{L}^{(1440)}_{\pi N N^*} = i \frac{f_{\pi N N^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \vec{\phi} \cdot \vec{\chi} \psi_{N^*}, \] (2.12b)

\[ \mathcal{L}^{(1600)}_{\pi N \Delta^*} = \frac{f_{\pi N \Delta^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \partial_\mu \vec{\phi} \cdot \bar{\chi} \psi_{\Delta^*}, \] (2.12c)

\[ \mathcal{L}^{(1535)}_{\pi N N^*} = i \frac{f_{\pi N N^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \vec{\phi} \cdot \bar{\chi} \psi_{N^*}, \] (2.12d)

\[ \mathcal{L}^{(1520)}_{\pi N N^*} = \frac{f_{\pi N N^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \vec{\phi} \cdot \bar{\chi} \psi_{N^*}, \] (2.12e)

\[ \mathcal{L}^{(1620)}_{\pi N \Delta^*} = i \frac{f_{\pi N \Delta^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \vec{\phi} \cdot \bar{\chi} \psi_{\Delta^*}, \] (2.12f)

\[ \mathcal{L}^{(1700)}_{\pi N \Delta^*} = \frac{f_{\pi N \Delta^*}}{m_{\pi}} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \vec{\phi} \cdot \bar{\chi} \psi_{\Delta^*} + h.c., \] (2.12g)
\( \mathcal{L}^{(1650)}_{\pi NN^*} = i \frac{f^{(1650)}_{\pi NN^*}}{m_\pi} \bar{\psi} \chi^\dagger \gamma_\mu \partial_\mu \vec{\phi} \cdot \vec{\tau} \chi \psi_{N^*} + h.c., \quad (2.12h) \)

\( \mathcal{L}^{(1700)}_{\pi NN^*} = \frac{f^{(1700)}_{\pi NN^*}}{m_\pi} \bar{\psi} \chi^\dagger \gamma_5 \partial_\mu \vec{\phi} \cdot \vec{\tau} \chi \psi_\mu + h.c., \quad (2.12i) \)

\( \mathcal{L}^{(1675)}_{\pi NN^*} = \frac{f^{(1675)}_{\pi NN^*}}{m_\pi^2} \bar{\psi} \chi^\dagger \partial_\mu \vec{\phi} \cdot \vec{\tau} \chi \psi_{\mu \nu} + h.c., \quad (2.12j) \)

\( \mathcal{L}^{(1720)}_{\pi NN^*} = \frac{f^{(1720)}_{\pi NN^*}}{m_\pi} \bar{\psi} \chi^\dagger \partial_\mu \vec{\phi} \cdot \vec{\tau} \chi \psi_\mu + h.c., \quad (2.12k) \)

\( \mathcal{L}^{(1680)}_{\pi NN^*} = i \frac{f^{(1680)}_{\pi NN^*}}{m_\pi^2} \bar{\psi} \chi^\dagger \gamma_5 \partial_\mu \vec{\phi} \cdot \vec{\tau} \chi \psi_{\mu \nu} + h.c. \quad (2.12l) \)

Here \( \psi, \psi_\mu \) and \( \psi_{\mu \nu} \) represent spin \( 1/2, 3/2 \) and spin \( 5/2 \) Rarita-Schwinger spinor fields respectively, and \( \chi \) and \( \vec{\chi} \) represents isospin \( 1/2 \) spinor and \( 3/2 \) vector-spinors respectively.

The Rarita-Schwinger spinor field operators are defined as

\( \psi^M_\mu = \Sigma(1, \frac{1}{2}, m, s | \frac{3}{2}, M) \epsilon_\mu(m) u_s, \quad (2.13a) \)

\( \psi^M_{\mu \nu} = \Sigma(\frac{3}{2}, 1, r, n | \frac{5}{2}, M)(1, \frac{1}{2}, m, s | \frac{3}{2}, r) \epsilon_\mu(m) \epsilon_\nu(n) u_s. \quad (2.13b) \)

The spin \( 5/2 \) spinor \( \psi_{\mu \nu} \) is symmetric in the two 4-vector indices. Above \( u_s \) represents a Dirac spinor with \( s_z = s \).

The resonance couplings (2.12) have been written in the chiral symmetry mandated form, which requires that the couplings vanish with pion 4-momentum. For baryons on their mass shell the couplings to the negative parity resonances may be simplified by use of the Dirac and corresponding Rarita-Schwinger equations.

The matrix elements of the transition couplings (2.12) between resonance states with charge \( +e \) and spin-z projection \( +\frac{1}{2} \) and the proton with spin up are listed in Table 3. Direct comparison with the quark model couplings is possible only for those terms in (2.12) that have the corresponding dependence on pion momentum. Comparison of the terms, which depend on pion energy, would require the corresponding quark model couplings to be spelled out explicitly.
With this qualification comparison of the matrix elements in Tables 2 and 3 yield the sought for expressions for the resonance transition couplings to pions, \( f_{\pi NN^*} \), in terms of the pion-quark coupling constant, \( f_{\pi qq} \), and then by (2.4) in terms of the \( \pi N \) pseudovector coupling constant \( f_{\pi NN} \). The resulting expressions are listed in Table 4. In order to have real coupling constants in the Rarita-Schwinger formalism the phase factors \((-i)^l\) that appear in the quark model matrix elements have been dropped. These calculated values for the \( \pi NN^* \) coupling constants may be compared to the corresponding values determined from the empirically known widths for \( N\pi \) decay of these resonances.

The empirical decay widths for \( N\pi \) decay of the positive parity \( \Delta \) resonances \( \Delta(1232) \) and \( \Delta(1600) \) are obtained as

\[
\Gamma = \frac{1}{3} \frac{f_{\pi N\Delta}^2}{4\pi} \frac{E' + m_N}{m_{\Delta}} \frac{k^3}{m_{\pi}^2} \tag{2.14}
\]

Here \( E' \) is the energy of the final nucleon. Insertion of the empirical decay widths and the corresponding kinematical factors then yields the value \( f_{\pi N\Delta}^{(1232)} = 2.2 \pm 0.04 \) and \( f_{\pi N\Delta}^{(1600)} = 0.51 \pm 0.07 \). These values exceed the quark model values (1.55 and 0.47) in Table 4 by factors 1.5 and 1.1 respectively. These underestimates are typical of the quark model for the pion decays in the single quark approximation.

For the \( N(1440) \) the decay width for \( N(1440) \to N\pi \) is obtained as

\[
\Gamma = 3 \frac{(f_{\pi NN^*})^2}{4\pi} \frac{E' - m_N}{m^*} \frac{k}{m_{\pi}^2} (m^* + m_N)^2 \tag{2.15}
\]

Here \( m^* \) is the resonance mass. From the empirical decay width for \( N\pi \) decay \( 227 \pm 0.65 \) MeV of the \( N(1440) \) one obtains \( f_{\pi NN^*}^{(1440)} = 0.39 \pm 0.06 \). In this case the quark model value 0.26 again represent an underestimate of about a factor 1.5. This underestimate is a consequence of the fact that the \( \pi NN(1440) \) coupling vanishes at \( k = 0 \) in the quark model.

The \( N\pi \) decay width for the spin \( \frac{1}{2}^- \) resonances are obtained as

\[
\Gamma = \frac{f_{\pi NN^*}^2}{4\pi} \frac{E' + m_N}{m^*} \frac{k}{m_{\pi}^2} (m^* - m_N)^2. \tag{2.16}
\]

Here the factor \( \alpha \) is 3 for isospin 1/2 resonances and 1 for isospin 3/2 resonances with spin \( \frac{1}{2}^- \). For the \( N(1535) \), \( N(1650) \) and the \( \Delta(1620) \) this
expression yields the values \( f_{\pi NN_1}^{(1525)} = 0.36 \pm 0.05 \), \( f_{\pi NN_2}^{(1650)} = 0.31 \pm 0.03 \) and \( f_{\pi N\Delta}^{(1620)} = 0.34 \pm 0.06 \). The quark model results for these coupling constants in Table 4 are within 30% of these values.

The \( N\pi \) decay widths for the spin \( \frac{3}{2}^- \) resonances are obtained as

\[
\Gamma = \frac{1}{3} \frac{f_{\pi NN_1}^2}{4\pi} \frac{E' - m_N}{m^*} \frac{k^3}{m^2},
\]

From this expressions and the empirical \( N\pi \) decay widths we obtain the values \( f_{\pi NN_1}^{(1520)} = 1.56 \pm 0.06 \), \( f_{\pi NN_2}^{(1700)} = 0.36 \) and \( f_{\pi N\Delta}^{(1700)} = 1.31 \). The quark model values in Table 4 are fairly close to the first and the last of these values, but falls below that for the \( N(1700) \).

Finally the width for the decay \( N(1675) \to N\pi \) is obtained as

\[
\Gamma = \frac{2}{5} \frac{(f_{\pi NN_1}^{(1675)})^2}{4\pi} \frac{E' + m_N}{m^*} \frac{k^5}{m^4},
\]

The empirical decay width fraction \( \sim 67 \text{ MeV} \) yields the value \( f_{\pi NN_1}^{(1675)} = 0.10 \). This is close to the corresponding quark model value (0.09) in Table 4. Note that the empirical decay width does not determine the phase of the pion resonance transition coupling constants.

The expressions for the pionic decay widths of the \( D^- \) shell resonances \( N(1720) \) and \( N(1680) \) are \[16\]

\[
\Gamma(N(1720) \to N\pi) = \frac{1}{3} \frac{(f_{\pi NN_1}^{(1720)})^2}{4\pi} \frac{E' + m_N}{m^*} \frac{k^3}{m^2},
\]

\[
\Gamma(N(1680) \to N\pi) = \frac{2}{5} \frac{(f_{\pi NN_1}^{(1680)})^2}{4\pi} \frac{E' - m_N}{m^*} \frac{k^5}{m^4},
\]

respectively. The fractional widths for \( N\pi \) decay of these two resonances are 22 \( \pm \) 10 MeV and 84 \( \pm \) 10 MeV respectively \[20\]. Given these widths we obtain the following coupling constant magnitudes: \( |(f_{\pi NN_1}^{(1720)})| \sim 0.25 \pm 0.06 \) and \( |(f_{\pi NN_1}^{(1680)})| \sim 0.42 \pm 0.04 \). Comparison of the calculated coupling constants in Table 4 shows that the quark model, in the present approximation, over-estimates the former one of these coupling constants by about a factor 5 and underestimates the latter one by almost a factor 4. This problem with the pion decays of the \( D^- \) shell resonances has been noted before \[22\].
The overall situation that emerges is that with the one-quark transition operators, the quark model mostly underpredicts the resonance transition couplings by factors 1 - 1.5 with the present wave function model. The conclusion is that two-quark operators have to be significant for the description of pionic transitions of the baryon resonances \([23]\). If neglected these have to be compensated for by multiplication of the \(\pi NN^*\) couplings by factors of the order 2-3.

3 Vector meson coupling constants

A universal \(SU(2)\) symmetric model for the vector meson couplings to constituent quarks would be the following:

\[
\mathcal{L}_{V qq} = ig_{\rho qq} \bar{\psi} \gamma_\mu \vec{\rho}_\mu \psi + ig_{\omega qq} \bar{\psi} \gamma_\mu \omega_\mu \psi. \tag{3.1}
\]

Here \(\vec{\rho}_\mu\) and \(\omega_\mu\) are the \(\rho\)-meson and \(\omega\)-meson field operators respectively.

The vector meson coupling constant \(g_{V qq}\) may be determined from either the \(\omega\)- or \(\rho\)-nucleon coupling constants by writing the vector meson-nucleon coupling in the conventional form

\[
\mathcal{L}_{V NN} = ig_{\omega NN} \bar{\psi}_N [\gamma_\mu + i \kappa_\omega \sigma_{\mu\nu} \partial^\nu] \omega_\mu \psi_N + ig_{\rho NN} \bar{\psi}_N [\gamma_\mu + i \kappa_\rho \sigma_{\mu\nu} \partial^\nu] \rho_\mu \cdot \vec{\tau} \psi_N. \tag{3.2}
\]

Comparison of the matrix elements of the charge components of these Lagrangians for e.g. protons with spin up to the same matrix elements of the quark coupling operator (3.2) yields the relations

\[g_{\omega NN} = 3g_{V qq}, \tag{3.3a}\]

\[g_{\rho NN} = g_{V qq}. \tag{3.3b}\]

The tensor couplings \(\kappa_\omega\) and \(\kappa_\rho\) in (3.3) may be determined by comparing the matrix elements of the transverse part of the current couplings in (3.1) and (3.3). This yields the relations

\[
\frac{g_{\omega qq}}{m_q} = \frac{1}{m_N} g_{\omega NN}(1 + \kappa_\omega), \tag{3.4a}
\]
\[ \frac{5}{3} g_{\rho qq} = \frac{1}{m_N} g_{\rho NN}(1 + \kappa_\rho). \]  

Boson exchange models for the nucleon-nucleon interaction indicate that \( \kappa_\omega \) is small. From (3.4a) it then follows that \( m_q = m_N / 3 = 313 \text{ MeV} \), in agreement with conventional quark model phenomenology. Equations (3.3b) and (3.3b) then imply that \( \kappa_\rho = 4 \). This is close to the value \( \kappa_\rho = 4.22 \) in a recent realistic boson exchange model for the nucleon-nucleon interaction [24], but somewhat smaller than the value 6.6 indicated in earlier interaction models [23]. The values for the \( \omega NN \) coupling constants differ between different potential models. In the recent Nijmegen model [24] it is \( g_{\omega NN} = 10.35 \), while in the Bonn model it is as big as \( g_{\omega NN} = 15.85 \). The values for the \( \rho NN \) vector coupling constant are more stable, ranging from \( g_{\rho NN} = 2.97 \) [24] to 3.19 [25]. These uncertainties are commensurate with the expected uncertainties in the quark model.

These numbers are consistent with assuming equality between the \( \rho \) and \( \omega \) couplings to constituent quarks, and with taking

\[ g_{\rho qq} = g_{\omega qq} \simeq 3. \]  

The anomalous tensor couplings of the constituent quarks are so small that they may be taken to be 0. We shall use these values here.

In Table 5 the explicit matrix elements of the charge and transverse current components of the \( \omega \)-quark coupling (3.1) are given for all the nucleon and \( \Delta \) resonances in Table 1 are listed. Here only the non-vanishing terms of lowest order in the \( v/c \) expansion have been included. The corresponding matrix elements of the \( \rho \)-quark coupling are listed in Table 6.

As the aim here is to calculate the vector meson transition couplings to nucleon resonances by means of the quark model, the effective coupling Lagrangians will have to be expressed in a form, which has the same momentum dependence as the corresponding transition matrix elements in the quark model. The standard form of the generalized Rarita-Schwinger vector current couplings in ref. [10] does not meet this criterion. The form of the transition couplings below have been chosen to have have the same momentum dependence as the quark model couplings. As a consequence the overall meson momentum factors drop out in the expressions for the transition couplings in terms of the corresponding vector meson coupling constants to nucleons.
The \( \omega \)-meson transition couplings to the nucleon resonances may be described by the following effective Lagrangians:

\[
L_{\omega NN^*}^{(1440)} = -i \frac{g_{\omega NN^*}^{(1440)}}{m_\omega^2} \bar{\psi}_N [\gamma_\mu - \frac{m^* - m_N}{m_\omega^2} \partial_\mu] \partial^2 \omega_\mu \psi_{N^*} + h.c.,
\]

\[
L_{\omega NN^*}^{(1535)} = -i \frac{g_{\omega NN^*}^{(1535)}}{m_\omega^2} \bar{\psi}_N [\gamma_\mu \partial^2 - (m^* + m_N) \partial_\mu] \omega_\mu \psi_{N^*} + h.c.,
\]

\[
L_{\omega NN^*}^{(1520)} = i \frac{g_{\omega NN^*}^{(1520)}}{m_\omega^2} \bar{\psi}_N \sigma_\mu \partial_\nu \partial_\kappa \omega_\mu \psi_\kappa + h.c.,
\]

\[
L_{\omega NN^*}^{(1650)} = -i \frac{g_{\omega NN^*}^{(1650)}}{m_\omega^2} \bar{\psi}_N \gamma_5 [\gamma_\mu \partial^2 - (m^* + m_N) \partial_\mu] \omega_\mu \psi_{N^*} + h.c.,
\]

\[
L_{\omega NN^*}^{(1700)} = i \frac{g_{\omega NN^*}^{(1700)}}{m_\omega^2} \bar{\psi}_N \sigma_\mu \partial_\nu \partial_\kappa \omega_\mu \psi_\kappa + h.c.,
\]

\[
L_{\omega NN^*}^{(1675)} = i \frac{g_{\omega NN^*}^{(1675)}}{m_\omega^2} \epsilon_{\mu \nu \alpha \beta \delta} \bar{\psi}_N \partial_\alpha \partial_\nu \omega_\delta \partial_\mu \psi_\beta + h.c.,
\]

\[
L_{\omega NN^*}^{(1720)} = \frac{g_{\omega NN^*}^{(1720)}}{m_\omega^2} \bar{\psi}_N \gamma_5 [\delta_\mu_\nu - \frac{1}{m^* + m_N} \gamma_\mu \partial_\nu] \partial^2 \omega_\mu \psi_\nu + h.c.,
\]

\[
L_{\omega NN^*}^{(1680)} = i \frac{g_{\omega NN^*}^{(1680)}}{m_\omega^2} \bar{\psi}_N [\gamma_\mu - \frac{m^* - m_N}{m_\omega^2} \partial_\mu] \partial_\alpha \partial_\beta \omega_\mu \psi_{\alpha \beta} + h.c.. \]

The matrix element of the charge and transverse current coupling terms of these Lagrangians between resonances with charge state \(+e\) and the proton with spin up are listed in Table 7. As these matrix elements relate to virtual vector meson production, we have here considered the vector mesons as having zero energy. For the comparison with the quark model operators, we have in addition dropped terms of order \((m^* - m_N)/(m^* + m_N)\). For the heavier resonances this introduces a theoretical uncertainty range of almost 30%, which however is inherent in the mismatch between the non-relativistic quark model expressions and the covariant Rarita-Schwinger formalism. Numerical estimates for the \( \omega \) meson transition couplings to the nucleon resonances are given in Table 8. In the calculation of the numerical values, the quark wave function factors \(\exp\{-\vec{k}^2/6\omega^2\}\) were set to unity.
The $\rho$-meson transition coupling to the nucleon and $\Delta$-resonances are described by the following coupling Lagrangians in the generalized Rarita-Schwinger formalism:

\[
\mathcal{L}_{\rho N\Delta}^{(1232)} = g_{\rho N\Delta}^{(1232)} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\Delta + m_N} \gamma_\mu \partial_\nu \bar{\rho}_\mu \cdot \bar{\chi} \psi_\nu + h.c., \quad (3.7a)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1440)} = -i \frac{g_{\rho N N^*}^{(1440)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{m^* - m_N}{m_\rho^2} \partial_\mu + i \frac{K_{\rho N N^*}^{(1440)}}{m^* - m_N} \sigma_\mu_\nu \partial_\nu \bar{\rho}_\mu \cdot \bar{\rho}_\mu \chi \psi_{N^*} + h.c., \quad (3.7b)
\]

\[
\mathcal{L}_{\rho N\Delta^*}^{(1600)} = -g_{\rho N\Delta^*}^{(1600)} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\Delta^* + m_N} \gamma_\mu \partial_\nu \bar{\rho}_\mu \cdot \bar{\chi} \psi_\nu + h.c., \quad (3.7c)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1535)} = -i \frac{g_{\rho N N^*}^{(1535)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{m^* - m_N}{m_\rho^2} \partial_\mu \bar{\rho}_\mu \chi \psi_{N^*} + h.c., \quad (3.7d)
\]

\[
\mathcal{L}_{\rho N\Delta^*}^{(1520)} = i \frac{g_{\rho N\Delta^*}^{(1520)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \sigma_\mu_\nu \partial_\nu \partial_\kappa \bar{\rho}_\mu \cdot \bar{\chi} \psi_\kappa + h.c., \quad (3.7e)
\]

\[
\mathcal{L}_{\rho N\Delta^*}^{(1620)} = -i \frac{g_{\rho N\Delta^*}^{(1620)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \partial_\mu \bar{\rho}_\mu \cdot \bar{\chi} \psi_{\Delta^*} + h.c., \quad (3.7f)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1700)} = i \frac{g_{\rho N N^*}^{(1700)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \sigma_\mu_\nu \partial_\nu \partial_\kappa \bar{\rho}_\mu \cdot \bar{\chi} \psi_\kappa + h.c., \quad (3.7g)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1650)} = -i \frac{g_{\rho N N^*}^{(1650)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \partial_\mu \bar{\rho}_\mu \chi \psi_N + h.c., \quad (3.7h)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1700)} = i \frac{g_{\rho N N^*}^{(1700)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \sigma_\mu_\nu \partial_\nu \partial_\kappa \bar{\rho}_\mu \chi \psi_\kappa + h.c., \quad (3.7i)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1675)} = i \frac{g_{\rho N N^*}^{(1675)}}{m_\rho^2} \varepsilon_{\alpha_\beta_\gamma_\delta} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \partial_\alpha \bar{\rho}_\beta \bar{\gamma}_\delta \psi_{\mu_\nu} + h.c., \quad (3.7j)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1720)} = \frac{g_{\rho N N^*}^{(1720)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \gamma_\mu \partial_\nu \bar{\rho}_\mu \cdot \bar{\chi} \psi_\nu + h.c., \quad (3.7k)
\]

\[
\mathcal{L}_{\rho N N^*}^{(1680)} = i \frac{g_{\rho N N^*}^{(1680)}}{m_\rho^2} \bar{\psi} \gamma^5 \gamma_5 \rho \gamma_\mu \gamma_\nu \frac{1}{m_\rho^2} \frac{m^* - m_N}{m_\rho^2} \partial_\mu \bar{\rho}_\mu \partial_\beta \bar{\gamma}_\delta \psi_{\alpha_\beta} + h.c.. \quad (3.7l)
\]
The matrix elements of these transition couplings are listed in Table 9 for resonances with charge state $e$ and protons with spin $1/2$. The explicit expressions for the $\rho$–meson transition couplings in terms of the corresponding $\rho NN$ coupling constant are listed in Table 10, along with numerical estimates. These expressions are obtained by comparison of the quark model transition matrix elements in Table 6 with the corresponding matrix elements of the transition couplings in Table 9.

Of the resonances considered only the $N(1720)$ lies above threshold for $N\rho$ decay, for which the decay branch is in fact large ($\sim 80\%$). This makes it possible to determine magnitude of the transition coupling constant $g^{(1720)}_{\rho NN^*}$ directly from its decay width. For a real $\rho$ meson the coupling Lagrangian (3.7k) simplifies as the differential operator $\partial^2$ may be replaced by $m^2_\rho$. To lowest order in $v/c$ the Lagrangian (3.7c) then reduces to that of ref. [26], with the identification

$$f_{N^*N\rho} = -\frac{2\mu}{m_\rho} g^{(1720)}_{\rho NN^*},$$

where $f_{N^*N\rho}$ is the transition coupling constant defined in ref. [26]. This was determined from the partial decay width for $N(1720) \rightarrow N\rho$ to be $f_{N^*N\rho} \simeq 7.2$ in ref. [26]. This is somewhat smaller than the value 13.8 that is obtained from eqn. (3.8) with the value for $g^{(1720)}_{\rho NN^*}$ obtained with the quark model in Table 9. The fact that the quark model leads to an overprediction for the $\rho NN(1720)$ transition strength in clearly related to the corresponding overprediction of the $\pi NN(1720)$ coupling constant noted above.

4 Discussion

The fact that the pion resonance transition couplings are underestimated by factors 1.5–2 by the single quark operator approximation suggests that the quark model, in the same approximation, may also lead to similar underestimates for the vector meson transition couplings to nucleon resonances. For the subthreshold resonance transition couplings considered here, and which are required in a dynamical treatment of nuclear matter, there is however no alternative to calculation based on a dynamical model.

The expressions for the vector meson transition coupling constants were derived here by comparing the matrix elements of the transverse components of the vector meson transition currents to the corresponding quark model
matrix elements. The generalized Rarita-Schwinger coupling Lagrangians in eqs. (3.6) and (3.7) are, however, invariant and may be applied for virtual vector mesons of arbitrary momentum and energy in nuclear matter. As an example, consider the coupling (3.7e) of the \( \rho^- \) meson to the \( N(1520) \) \( 3/2^- \) resonance, which admits an interpretation as a \( \rho^- \)–nucleon resonance.

For non-zero \( \rho^- \) meson energy this coupling, to lowest order in the inverse baryon masses takes the form

\[
L_{\rho NN^*} \simeq \frac{g_{\rho NN^*}}{m_\rho^2} \psi^\dagger \chi^\dagger \left\{ \frac{\vec{k}^2}{2\mu} \rho_0^a + i \left( 1 - \frac{\omega}{2\mu} \vec{\sigma} \times \vec{\rho}^a \right) \vec{\tau}^a \vec{k} \cdot \vec{\psi} \chi + h.c. \right\} \tag{4.1}
\]

Here \( \mu \) is defined as the baryon mass combination \( \mu = 2m^*m_N/(m^* + m_N) \).

This form of the \( \rho NN(1520) \) coupling, which takes into account the \( L = 1 \) aspect of the 3 quark description of the \( N(1520) \) resonance differs from the form commonly employed for the same coupling \[5, 9\]:

\[
L_{\rho NN^*}^{(1520)} = \frac{f_{\rho NN^*}^{(1520)}}{m_\rho} \psi^\dagger \chi^\dagger (\omega \vec{\rho}^a - \vec{\rho}_0^a \vec{k}) \vec{\tau}^a \vec{\psi} \chi + h.c. \tag{4.2}
\]

If in (4.1) one sets \( \vec{k}^2 = k^2 \), as appropriate for zero energy vector mesons, and then imposes the on-shell condition \( k^2 = -m^2_\rho \), along with the relation \( \vec{\psi} = i \vec{\sigma} \times \vec{\psi} \), a formal equivalence between the expressions (4.1) and (4.2) obtains, provided that

\[
f_{\rho NN^*}^{(1520)} = \frac{m_\rho}{2\mu} g_{\rho NN^*}^{(1520)}. \tag{4.3}
\]

Comparing numbers, with the value 4.5 for \( g_{\rho NN^*}^{(1520)} \) given in Table 10, we obtain \( f_{\rho NN^*}^{(1520)} = 1.5 \), which is about half of the value 3.2 obtained in ref.\[9\]. Given the fact that the quark model underestimates the pion resonance transition couplings by factors 1.5–2 in the single quark operator approximation, this small quark model value is not unexpected.

This comparison may also be extended to the case of the \( \omega NN(1520) \) transition coupling. By comparing the isospin independent versions of the transition Lagrangians (4.1) and (4.2), we obtain

\[
f_{\omega NN^*}^{(1520)} = \frac{m_\omega}{2\mu} g_{\omega NN^*}^{(1520)}. \tag{4.4}
\]

With the value \( g_{\omega NN^*}^{(1520)} = 7.7 \) obtained by means of the quark model in Table 7, one obtains \( f_{\omega NN^*}^{(1520)} = 2.6 \). This value is somewhat less than one half of that
obtained in ref.\[9\]. The smaller value may be a consequence of the fact that the chiral quark model result should apply to higher densities as discussed above. It roughly corresponds to about 1/3 of $g_{\omega NN}$, which is the $\omega$–nucleon coupling at zero density. The factor 1/3 in the $SU(3)$ relation

$$g_{\omega NN} = 3g_{\rho NN},$$

arises from coherence in the sum of the couplings of the three nonstrange quarks in the $\omega$. We would expect this coherence to disappear at higher density or temperature scales. We see no obvious simple reason for why for this resonance $g_{\rho NN^*}$ is only about 1/2 of $g_{\omega NN^*}$, however.

This work should only be viewed as a first attempt at calculating the transition couplings for virtual vector mesons, and therefore the numerical values obtained should be viewed as suggestive, rather than as definite quantitative predictions. In an earlier study \[14\] we found that two-pion exchange between constituent quarks furnished a significant contribution to the hyperfine interaction between constituent quarks, which, when combined with the one-pion exchange interaction, provides a dynamical basis for the effective spin-flavor structure that is required for a satisfactory description of the empirical spectra, when combined with a linear confining interaction. A study of how such higher order corrections in the chiral quark model affects the problem at hand is now being undertaken. The goal is a better understanding of the change from meson to quark variables, given the general view that there is a region where the two descriptions overlap \[27\].

The present method for calculating the resonance transition couplings to the nucleon and $\Delta$ resonances in the $P$– and $SD$– shells may be directly generalized to the higher lying resonances. The explicit quark model wave functions for all the resonances in the $SD$–shell may be constructed by reference to the symmetry classification for the higher resonances in ref.\[12\]. The construction of the corresponding transition couplings in the generalized Rarita-Schwinger formalism may be carried out with the methods outlined in ref.\[16\] once care is taken to match the momentum dependence of the quark model matrix elements.

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| Resonance   | Explicit Wave Function                                                                 |
|------------|----------------------------------------------------------------------------------------|
| $p, n, \frac{1}{2}^+$ | $\frac{1}{\sqrt{2}}\varphi_{000}(\vec{\rho})\varphi_{000}(\vec{r})\{\left|\frac{1}{2}, t_3 > + \right| \left|\frac{1}{2}, s_3 > + \right| \left|\frac{1}{2}, t_3 > - \right| \left|\frac{1}{2}, s_3 > - \right| \}$ |
| $\Delta(1232), \frac{3}{2}^+$ | $\varphi_{000}(\vec{\rho})\varphi_{000}(\vec{r})|\frac{3}{2}, t_3 > \left|\frac{3}{2}, s_3 > \right| \}$ |
| $N(1440), \frac{1}{2}^+$ | $\frac{1}{\sqrt{2}}\{\varphi_{200}(\vec{\rho})\varphi_{000}(\vec{r}) + \varphi_{000}(\vec{\rho})\varphi_{200}(\vec{r})\}$ $\{\left|\frac{1}{2}, t_3 > + \right| \left|\frac{1}{2}, s_3 > + \right| \left|\frac{1}{2}, t_3 > - \right| \left|\frac{1}{2}, s_3 > - \right| \}$ |
| $\Delta(1600), \frac{3}{2}^+$ | $\frac{1}{\sqrt{2}}\{\varphi_{200}(\vec{\rho})\varphi_{000}(\vec{r}) + \varphi_{000}(\vec{\rho})\varphi_{200}(\vec{r})\}|\frac{3}{2}, t_3 > \left|\frac{3}{2}, s_3 > \right| \}$ |
| $N(1535), \frac{1}{2}^-$ | $\frac{1}{\sqrt{2}}\sum_{m,s}(1, \frac{1}{2}, m, s|J, s_3)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})\}$ |
| $N(1520), \frac{3}{2}^-$ | $\{\left|\frac{1}{2}, t_3 > + \right| \left|\frac{1}{2}, s > + \right| \left|\frac{1}{2}, t_3 > - \right| \left|\frac{1}{2}, s > - \right| \}$ $\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})|\left|\frac{1}{2}, t_3 > + \right| \left|\frac{1}{2}, s > - + \right| \left|\frac{1}{2}, t_3 > - \right| \left|\frac{1}{2}, s > + \right| \}$ |
| $\Delta(1620), \frac{1}{2}^-$ | $\frac{1}{\sqrt{2}}\sum_{m,s}(1, \frac{1}{2}, m, s|J, s_3)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})|\frac{3}{2}, t_3 > \left|\frac{1}{2}, s > + \right| \}$ |
| $\Delta(1700), \frac{3}{2}^-$ | $+\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})|\frac{3}{2}, t_3 > \left|\frac{1}{2}, s > - \right| \}$ |
| $N(1550), \frac{1}{2}^-$ | $\frac{1}{\sqrt{2}}\sum_{m,s}(1, \frac{3}{2}, m, s|J, s_3)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})|\frac{1}{2}, t_3 > + \}$ |
| $N(1700), \frac{3}{2}^-$ | $+\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})|\frac{1}{2}, t_3 > - \}|\frac{3}{2}, s > \}$ |
| $N(1675), \frac{5}{2}^-$ | $\{\left|\frac{1}{2}, t_3 > + \right| \left|\frac{1}{2}, s_3 > + \right| \left|\frac{1}{2}, t_3 > - \right| \left|\frac{1}{2}, s_3 > - \right| \}$ |
Table 2. Transition matrix elements of the quark operator $O = \sum_{q=1}^{3} \bar{\sigma}^q \cdot \vec{k} \tau^q e^{-i\vec{k} \cdot \vec{r}_q}$ between the nucleon resonances and the nucleon for charge states $+1$ with $s_z = +\frac{1}{2}$.

| $|p, \frac{1}{2} \rangle \langle O |p', \frac{1}{2} \rangle$ | Value |
|---|---|
| $\langle p, \frac{1}{2} | O | p, \frac{1}{2} \rangle$ | $\frac{5}{3} k_3$ |
| $\langle p, \frac{1}{2} | O | \Delta(1232)^+ \rangle$ | $\frac{4\sqrt{2}}{3} k_3 e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1440)^+ \rangle$ | $\frac{5\sqrt{2}}{2\pi} k_3 (\frac{k^2}{\omega^2} + \frac{3\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | \Delta(1600)^+ \rangle$ | $\frac{3\sqrt{6}}{2\pi} k_3 (\frac{k^2}{\omega^2} + \frac{3\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1535)^+ \rangle$ | $-i\frac{2\sqrt{2}}{9} \omega (\frac{k^2}{\omega^2} + \frac{9\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1520)^+ \rangle$ | $i\frac{2}{9} \frac{k^2 - k_3^2}{\omega} e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | \Delta(1620)^+ \rangle$ | $i\frac{\sqrt{2}}{9} \omega (\frac{k^2}{\omega^2} + \frac{9\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | \Delta(1700)^+ \rangle$ | $-i\frac{3}{3} \frac{k^2 - k_3^2}{\omega} e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1650)^+ \rangle$ | $-i\frac{\sqrt{2}}{9} \omega (\frac{k^2}{\omega^2} + \frac{9\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1700)^+ \rangle$ | $-i\frac{\sqrt{15}}{90} \frac{3k^2 - k_3^2}{\omega} e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1675)^+ \rangle$ | $i\frac{\sqrt{15}}{18} \frac{3k^2 - k_3^2}{\omega} e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1720)^+ \rangle$ | $-\frac{\sqrt{15}}{27} k_3 (\frac{k^2}{\omega^2} + \frac{15\omega_3}{2m_q}) e^{-k^2/6\omega^2}$ |
| $\langle p, \frac{1}{2} | O | N(1680)^+ \rangle$ | $\frac{\sqrt{15}}{18} k_3 (5k^2 - 3k_3^2) \frac{1}{\omega^2} e^{-k^2/6\omega^2}$ |
Table 3. Transition matrix elements $\langle p, \frac{1}{2} | \mathcal{L}_{\pi NN^*} | N^{*+}, \frac{1}{2} \rangle$ of the pion transition couplings in eqs. (2.12). Here $m^*$ denotes the mass of the corresponding resonance. The expressions after the vertical bars correspond to zero energy pions.

| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1232)} | \Delta(1232)^+, \frac{1}{2} \rangle$ | $-\frac{2}{3} i \frac{f_{\pi NN^*}}{m_e} k_3$ |
|---|---|
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1440)} | N(1440)^+, \frac{1}{2} \rangle$ | $-i \frac{f_{\pi NN^*}}{m_e} k_3$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1600)} | \Delta(1600)^+, \frac{1}{2} \rangle$ | $-i \frac{f_{\pi NN^*}}{m_e} k_3$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1535)} | N(1535)^+, \frac{1}{2} \rangle$ | $i f_{\pi NN^*} \frac{m^* - m}{m_e} \left\{ \frac{i}{4} \right\} \frac{f_{\pi NN^*}}{m_e} \frac{m^* + m}{m^*} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1520)} | N(1520)^+, \frac{1}{2} \rangle$ | $i \frac{f_{\pi NN^*}}{4\sqrt{6}} \frac{m^* - m}{m^* m} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1620)} | \Delta(1620)^+, \frac{1}{2} \rangle$ | $i \sqrt{\frac{3}{2}} \frac{f_{\pi NN^*}}{m_e} \frac{m^* + m}{m^* m} \left\{ \frac{1}{4} \sqrt{\frac{3}{2}} \right\} \frac{f_{\pi NN^*}}{m_e} \frac{m^* - m}{m^*} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1700)} | \Delta(1700)^+, \frac{1}{2} \rangle$ | $i \frac{f_{\pi NN^*}}{12} \frac{m^* - m}{m^* m} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1650)} | N(1650)^+, \frac{1}{2} \rangle$ | $i f_{\pi NN^*} \frac{m^* - m}{m_e} \left\{ \frac{i}{4} \right\} \frac{f_{\pi NN^*}}{m_e} \frac{m^* + m}{m^*} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1700)} | N(1700)^+, \frac{1}{2} \rangle$ | $i \frac{f_{\pi NN^*}}{4\sqrt{6}} \frac{m^* - m}{m^* m} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1675)} | N(1675)^+, \frac{1}{2} \rangle$ | $-i \frac{f_{\pi NN^*}}{10} \frac{m^* - m}{m_e} \left( \frac{3k_3^2}{m_e} - \vec{k}^2 \right)$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1720)} | N(1720)^+, \frac{1}{2} \rangle$ | $-i \sqrt{\frac{3}{5}} \frac{f_{\pi NN^*}}{m_e} \frac{k_3}{m_e}$ |
| $\langle p, \frac{1}{2} | \mathcal{L}_{\pi N^0}^{(1680)} | N(1680)^+, \frac{1}{2} \rangle$ | $i \frac{1}{3} \sqrt{\frac{5}{2}} \frac{f_{\pi NN^*}}{m_e} \frac{m^* + m}{m^* m} k_3 \left( \frac{5k_3^2}{m_e} - 3k^2 \right)$ |
Table 4. The resonance transition coupling constants in terms of the pion-nucleon pseudoscalar coupling constant $f_{\pi NN}$.

| Resonance Energy (MeV) | Coupling Constant |
|------------------------|-------------------|
| $f_{\pi N\Delta}^{(1232)}$ | $\frac{6\sqrt{2}}{5}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 1.55f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1440)}$ | $\frac{\sqrt{3}}{18}\frac{k^2}{\omega^2} + \frac{3\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.26f_{\pi NN}$ |
| $f_{\pi N\Delta}^{(1600)}$ | $\frac{\sqrt{6}}{10}\frac{k^2}{\omega^2} + \frac{3\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.47f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1535)}$ | $\frac{2\sqrt{2}}{15}\frac{\omega}{(m^*-m_N)}\frac{k^2}{\omega^2} + \frac{9\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.49f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1520)}$ | $-\frac{8\sqrt{6}}{15}(m^*-m_N)\omega e^{-k^2/6\omega^2}f_{\pi NN} \simeq -1.71f_{\pi NN}$ |
| $f_{\pi N\Delta}^{(1620)}$ | $-\frac{\sqrt{3}}{15}\frac{\omega}{(m^*-m_N)}\frac{k^2}{\omega^2} + \frac{9\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq -0.34f_{\pi NN}$ |
| $f_{\pi N\Delta}^{(1700)}$ | $\frac{12}{5}(m^*-m_N)\omega e^{-k^2/6\omega^2}f_{\pi NN} \simeq 2.6f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1650)}$ | $\frac{\sqrt{2}}{15}\frac{\omega}{(m^*-m_N)}\frac{k^2}{\omega^2} + \frac{9\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.28f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1700)}$ | $\frac{4\sqrt{15}}{75}(m^*-m_N)\omega e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.22f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1675)}$ | $\frac{10}{9}\frac{m^*}{m_N}e^{-k^2/6\omega^2}f_{\pi NN} \simeq 0.09f_{\pi NN}$ |
| $f_{\pi NN^*}^{(1720)}$ | $\frac{\sqrt{10}}{30}\frac{k^2}{\omega^2} + \frac{15\omega}{2m_q}e^{-k^2/6\omega^2}f_{\pi NN} \simeq -1.05$ |
| $f_{\pi NN^*}^{(1680)}$ | $-\frac{\sqrt{7}}{6}\frac{m^*}{(m^*+m_N)}\frac{(m_N)}{m_N}e^{-k^2/6\omega^2}f_{\pi NN} \simeq -0.12$ |
Table 5. Transition matrix elements of the isoscalar quark charge \( \sum_q e^{-i\vec{k}\cdot\vec{r}_q} \) and transverse spin current \( \sum_q \vec{\sigma}^q \cdot (\vec{k} \times \vec{e}) e^{-i\vec{k}\cdot\vec{r}_q} \) operators between the proton and nucleon resonances for charge states \(+1\) with \( s_z = +1/2\).

| transition | charge | spin current |
|------------|--------|--------------|
| \(< p, \frac{1}{2}|O|p, \frac{1}{2} >\) | 3 | \((\vec{k} \times \vec{e})_3\) |
| \(< p, \frac{1}{2}|O|N(1440)^+, \frac{1}{2} >\) | \(\frac{3\sqrt{3}}{18} \frac{k^2}{\omega^2} e^{-k^2/6\omega^2}\) | \(\frac{\sqrt{3}}{18} \frac{k^2}{\omega^2} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3\) |
| \(< p, \frac{1}{2}|O|N(1535)^+, \frac{1}{2} >\) | 0 | \(-\frac{\sqrt{3}}{3} \frac{k^2}{\omega} e^{-k^2/6\omega^2} \epsilon_3\) |
| \(< p, \frac{1}{2}|O|N(1520)^+, \frac{1}{2} >\) | 0 | \(i \frac{2}{3} \frac{1}{\omega} e^{-k^2/6\omega^2} [\vec{k}_3 (\vec{k} \times \vec{e})_3 + k_+ (\vec{k} \times \vec{e})_-]\) |
| \(< p, \frac{1}{2}|O|N(1650)^+, \frac{1}{2} >\) | 0 | \(\frac{\sqrt{3}}{6} \frac{k^2}{\omega} e^{-k^2/6\omega^2} \epsilon_3\) |
| \(< p, \frac{1}{2}|O|N(1700)^+, \frac{1}{2} >\) | 0 | \(i \frac{\sqrt{30}}{15} \frac{1}{\omega} e^{-k^2/6\omega^2} [k_3 (\vec{k} \times \vec{e})_3 + k_+ (\vec{k} \times \vec{e})_- + 3i\epsilon_3 \vec{k}_3]\) |
| \(< p, \frac{1}{2}|O|N(1675)^+, \frac{1}{2} >\) | 0 | \(-i \frac{3\sqrt{30}}{15} \frac{k^2}{\omega} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3\) |
| \(< p, \frac{1}{2}|O|N(1720)^+, \frac{1}{2} >\) | 0 | \(-\frac{\sqrt{30}}{180} \frac{k^2}{\omega^2} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3\) |
| \(< p, \frac{1}{2}|O|N(1680)^+, \frac{1}{2} >\) | \(-\frac{\sqrt{30}}{20} \frac{3k^2-k^2}{\omega^2} e^{-k^2/6\omega^2}\) | \(-i \frac{3\sqrt{30}}{45} \frac{k^4}{\omega^2} e^{-k^2/6\omega^2}\) |
| | | \(\sum_q (3, 1, q, -q|3, 0) \frac{1}{\sqrt{7}} Y_{3q}(\hat{k}) \vec{e}_q\) |
### Table 6. Transition matrix elements of the isovector quark charge $\sum_q \tau_3^q e^{-ik \cdot r_q}$ and transverse current operator $\sum_q \tau_3^q \vec{\sigma} \cdot (\vec{k} \times \vec{e}) e^{-ik \cdot r_q}$ between the proton and nucleon resonances with charge state $+1$ with $s_z = +1/2$.

| transition | charge | spin current |
|------------|--------|--------------|
| $< p, \frac{1}{2} | O | p, \frac{1}{2} >$ | 1 | $\frac{5}{3}(\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | \Delta(1232)^+, \frac{1}{2} >$ | 0 | $\frac{4\sqrt{3}}{3} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | N(1440)^+, \frac{1}{2} >$ | $\sqrt{3} \frac{k^2}{18 \omega^2} e^{-k^2/6\omega^2}$ | $\frac{5\sqrt{3}}{54} \frac{k^2}{\omega^2} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | \Delta(1600)^+, \frac{1}{2} >$ | 0 | $\frac{3\sqrt{6}}{4\omega} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | N(1535)^+, \frac{1}{2} >$ | $-i \frac{\sqrt{2}}{3} \frac{k_3}{\omega} e^{-k^2/6\omega^2}$ | $-\frac{2\sqrt{2}}{9} \frac{k^2}{\omega} e^{-k^2/6\omega^2} e_3$ |
| $< p, \frac{1}{2} | O | N(1520)^+, \frac{1}{2} >$ | 0 | $i \frac{4}{9} \frac{k}{\omega} e^{-k^2/6\omega^2} [k_3 (\vec{k} \times \vec{e})_3 + k_+ (\vec{k} \times \vec{e})_-]$ |
| $< p, \frac{1}{2} | O | \Delta(1620)^+, \frac{1}{2} >$ | $i \frac{\sqrt{2}}{9} \frac{k_3}{\omega} e^{-k^2/6\omega^2}$ | $\frac{\sqrt{7}}{18} \frac{\hat{k}^2}{\omega} e^{-k^2/6\omega^2} e_3$ |
| $< p, \frac{1}{2} | O | \Delta(1700)^+, \frac{1}{2} >$ | 0 | $i \frac{1}{9} \frac{k}{\omega} e^{-k^2/6\omega^2} [k_3 (\vec{k} \times \vec{e})_3 + k_+ (\vec{k} \times \vec{e})_-]$ |
| $< p, \frac{1}{2} | O | N(1650)^+, \frac{1}{2} >$ | $-i \frac{\sqrt{3}}{6} \frac{k_3}{\omega} e^{-k^2/6\omega^2}$ | $-\frac{\sqrt{7}}{18} \frac{\hat{k}^2}{\omega} e^{-k^2/6\omega^2} e_3$ |
| $< p, \frac{1}{2} | O | N(1700)^+, \frac{1}{2} >$ | 0 | $-i \frac{\sqrt{3}}{45} \frac{k}{\omega} e^{-k^2/6\omega^2} [k_3 (\vec{k} \times \vec{e})_3 + k_+ (\vec{k} \times \vec{e})_- + 3i e_3 \hat{k}^2]$ |
| $< p, \frac{1}{2} | O | N(1675)^+, \frac{1}{2} >$ | 0 | $i \frac{\sqrt{7}}{10} \frac{g_{NNN}^{(1675)}}{m_N^2} k_3 (\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | N(1720)^+, \frac{1}{2} >$ | 0 | $-\frac{\sqrt{30}}{108} \frac{k^3}{\omega^2} e^{-k^2/6\omega^2} (\vec{k} \times \vec{e})_3$ |
| $< p, \frac{1}{2} | O | N(1680)^+, \frac{1}{2} >$ | $\frac{\sqrt{3}}{60} \frac{3k^2_3 - \hat{k}^2}{\omega^2} e^{-k^2/6\omega^2}$ | $-i \frac{\sqrt{30}}{27} \frac{k^3}{\omega^2} e^{-k^2/6\omega^2}$ |

\[ \sum_q (3, 1, q, -q|3, 0) \sqrt{\frac{4\pi}{3}} Y_{3q} (\vec{k}) \vec{\epsilon} - q \]
Table 7. Transition matrix elements $< p, \frac{1}{2} | \mathcal{L}_{\omega NN^*} | N^{*+}, \frac{1}{2} >$ of the $\omega$-meson transition couplings in eqs. (3.6). Here $\mu$ is defined as $\mu = 2m_N m^*/(m_N + m^*)$.

| transition | charge                  | spin current                                      |
|------------|-------------------------|---------------------------------------------------|
| $< p, \frac{1}{2} | \mathcal{L}^{(1440)}_{\omega NN^*} | N(1440)^+, \frac{1}{2} >$ | $i \frac{1}{2} \frac{\vec{E}^2}{m_N^2} g^{(1440)}_{\omega NN^*}$ | $-i \frac{1}{2} \frac{\vec{E}^2}{m_N^2} (\vec{k} \times \vec{\epsilon})_3 g^{(1440)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1535)}_{\omega NN^*} | N(1535)^+, \frac{1}{2} >$ | 0 | $i \frac{\vec{E}^2}{m_N^2} \epsilon_3 g^{(1535)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1520)}_{\omega NN^*} | N(1520)^+, \frac{1}{2} >$ | 0 | $i \frac{\sqrt{6}}{3m_N^2} [k_3(\vec{k} \times \vec{\epsilon})_3 + k_+ (\vec{k} \times \vec{\epsilon})_-] g^{(1520)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1650)}_{\omega NN^*} | N(1650)^+, \frac{1}{2} >$ | 0 | $i \frac{\vec{E}^2}{m_N^2} \epsilon_3 g^{(1650)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1700)}_{\omega NN^*} | N(1700)^+, \frac{1}{2} >$ | 0 | $i \frac{\sqrt{10}}{10} \frac{1}{m_N^2} k_3(\vec{k} \times \vec{\epsilon})_3 g^{(1700)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1675)}_{\omega NN^*} | N(1675)^+, \frac{1}{2} >$ | 0 | $-i \frac{\sqrt{10}}{10} \frac{1}{m_N^2} k_3(\vec{k} \times \vec{\epsilon})_3 g^{(1675)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1720)}_{\omega NN^*} | N(1720)^+, \frac{1}{2} >$ | 0 | $i \frac{\sqrt{7}}{3m_N^2} \epsilon_3 \frac{\vec{E}^2}{m_N^2} g^{(1720)}_{\omega NN^*}$ |
| $< p, \frac{1}{2} | \mathcal{L}^{(1680)}_{\omega NN^*} | N(1680)^+, \frac{1}{2} >$ | $\frac{\sqrt{10}}{10} \frac{3k_2 - \vec{E}^2}{m_N^2} \frac{\vec{E}^2}{m_N^2} g^{(1680)}_{\omega NN^*}$ | $-i \frac{\sqrt{30}}{\mu} \frac{\vec{E}^2}{m_N^2} g^{(1680)}_{\omega NN^*}$ |
|                          |                          | $(3, 1, q, -q| 3, 0) \frac{4\pi}{\sqrt{3}} Y_{3q}(\hat{k}) \epsilon_{-q}$ |
Table 8. The \( \omega \)-meson transition coupling constants in terms of the \( \omega NN \) coupling constant \( g_{\omega NN} \). The numerical values correspond to \( |\vec{k}| = 0 \).

| \( g_{\omega NN} \) | \( g_{\omega NN} = \frac{\sqrt{3} m^2}{18 \omega^2} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx 5.5 \) |
|------------------|---------------------------------------------------------------|
| \( (1440) \)     |                                                                 |
| \( (1535) \)     | \( g_{\omega NN} = -\frac{\sqrt{2} m^2}{18 \omega m_q} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx -4.5 \) | |
| \( (1520) \)     | \( g_{\omega NN} = \frac{\sqrt{6} m^2}{18 \omega m_q} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx 7.7 \) | |
| \( (1650) \)     |                                                                 |
| \( (1700) \)     | \( g_{\omega NN} = \frac{\sqrt{15} m^2}{90 \omega m_q} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx 2.4 \) | |
| \( (1675) \)     | \( g_{\omega NN} = \frac{1 m^2}{6 \omega m_q} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx 9.4 \) | |
| \( (1720) \)     | \( g_{\omega NN} = -\frac{\sqrt{5} \mu m^2}{180 m_q \omega^2} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx -2.7 \) | |
| \( (1680) \)     | \( g_{\omega NN} = -\frac{\mu m^2}{18 m_q \omega^2} e^{-\vec{k}^2/6\omega^2} g_{\omega NN} \approx -12.2 \) | |
Table 9. Transition matrix elements $\langle p, \frac{1}{2} | L_{\rho N N^*} | N^{*+}, \frac{1}{2} \rangle$ of the $\rho$-meson transition couplings in eqs. (3.7) Here $\mu$ is defined as

$\mu = 2m^* m_N / (m^* + m_N)$. 

| transition | charge | spin current |
|------------|--------|--------------|
| $\langle p, \frac{1}{2} | L_{\rho N \Delta} | \Delta(1232)^+, \frac{1}{2} \rangle$ | 0 | $i \frac{1}{3 \mu} (\vec{k} \times \vec{\epsilon})_3 g_{\rho N \Delta}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1440)^+, \frac{1}{2} \rangle$ | $\frac{F_{\pi}^2}{m_\pi^2} g_{\rho N N^*}^{(1440)}$ | $-i \frac{1}{2 \mu} \frac{F_{\pi}^2}{m_\pi^2} (\vec{k} \times \vec{\epsilon})_3 g_{\rho N N^*}^{(1440)} (1 + \kappa_{\rho N N^*})$ |
| $\langle p, \frac{1}{2} | L_{\rho N \Delta^*} | \Delta(1600)^+, \frac{1}{2} \rangle$ | 0 | $i \frac{1}{3 \mu} \frac{\vec{k} \times \vec{\epsilon}}{m_\pi^2}_3 g_{\rho N \Delta^*}^{(1600)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1535)^+, \frac{1}{2} \rangle$ | 0 | $\frac{F_{\pi}^2}{m_\pi^2} \epsilon_3 g_{\rho N N^*}^{(1535)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1520)^+, \frac{1}{2} \rangle$ | 0 | $i \frac{1}{3 m_\rho^2} [k_3 (\vec{k} \times \vec{\epsilon})_3 + k_4 (\vec{k} \times \vec{\epsilon})_-] g_{\rho N N^*}^{(1520)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N \Delta^*} | \Delta(1620)^+, \frac{1}{2} \rangle$ | 0 | $\sqrt{\frac{2}{3}} \frac{\vec{k}^2}{m_\pi^2} g_{\rho N \Delta^*}^{(1620)} \epsilon_3$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1700)^+, \frac{1}{2} \rangle$ | 0 | $i \frac{1}{3 m_\rho^2} [k_3 (\vec{k} \times \vec{\epsilon})_3 + k_4 (\vec{k} \times \vec{\epsilon})_-] g_{\rho N N^*}^{(1700)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1650)^+, \frac{1}{2} \rangle$ | 0 | $\frac{F_{\pi}^2}{m_\pi^2} \epsilon_3 g_{\rho N \Delta^*}^{(1650)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1700)^+, \frac{1}{2} \rangle$ | 0 | $i \frac{\sqrt{6}}{3 m_\rho^2} [k \times \vec{\epsilon}]_3 + k_4 (\vec{k} \times \vec{\epsilon})_-] g_{\rho N N^*}^{(1700)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1675)^+, \frac{1}{2} \rangle$ | 0 | $-i \frac{3 \sqrt{10}}{10} \frac{1}{m_\rho^2} k_3 (\vec{k} \times \vec{\epsilon})_3 g_{\rho N N^*}^{(1675)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1720)^+, \frac{1}{2} \rangle$ | 0 | $\frac{1}{2 \mu} \sqrt{\frac{2}{3}} \frac{\vec{k}^2}{m_\pi^2} (\vec{k} \times \vec{\epsilon})_3 g_{\rho N N^*}^{(1720)}$ |
| $\langle p, \frac{1}{2} | L_{\rho N NN^*} | N(1680)^+, \frac{1}{2} \rangle$ | $\frac{\sqrt{10}}{10} \frac{3 k_3^2 - \vec{k}^2}{m_\pi^2} g_{\rho N N^*}^{(1680)}$ | $-i \frac{3 \sqrt{10}}{15} \frac{\vec{k}^2}{m_\pi^2} g_{\rho N N^*}^{(1680)}$ |

$| (3, 1, q, -q|3, 0) \sqrt{\frac{45}{7}} Y_{3q}(\vec{k}) \epsilon_q$
Table 10. The $\rho$-meson transition coupling constant in terms of the $\rho NN$ coupling constants $g_{\rho NN}$. The numerical values correspond to $|\vec{k}| = 0$.

| Energy (MeV) | $g_{\rho NN}$ | $g_{\rho NN}$' | $g_{\rho NN}$'' | $g_{\rho NN}$''' | $g_{\rho NN}$'''' |
|-------------|---------------|----------------|----------------|----------------|------------------|
| 1440        | $g_{\rho NN} = \frac{\sqrt{3}}{18} \frac{m_{\pi}^2}{\omega^2} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 8.7$ |
|             | $g_{\rho NN}^{(1440)} = \frac{\sqrt{3}}{18} \frac{m_{\pi}^2}{\omega^2} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 1.76$, $\kappa_{\rho NN}^{(1440)} = 4$ |
| 1600        | $g_{\rho NN} = \frac{\sqrt{6}}{10} \frac{m_{\pi}^2}{\omega^2} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 17.0$ |
| 1535        | $g_{\rho NN} = -\frac{\sqrt{3}}{9} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -2.9$ |
| 1520        | $g_{\rho NN} = \frac{\sqrt{6}}{9} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 4.5$ |
| 1620        | $g_{\rho NN} = \frac{\sqrt{3}}{36} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 0.88$ |
| 1700        | $g_{\rho NN} = \frac{1}{12} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq 1.5$ |
| 1650        | $g_{\rho NN} = -\frac{\sqrt{3}}{36} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -0.72$ |
| 1700        | $g_{\rho NN} = -\frac{\sqrt{3}}{90} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -0.45$ |
| 1675        | $g_{\rho NN} = -\frac{1}{6} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -3.0$ |
| 1720        | $g_{\rho NN} = -\frac{5}{18} \frac{m_{\pi}^2}{\omega m_q} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -4.4$ |
| 1680        | $g_{\rho NN} = -\frac{\sqrt{5}}{18} \frac{m_{\pi}^2}{\omega^2} e^{-\vec{k}^2/6\omega^2}$ $g_{\rho NN} \simeq -19.6$ |