Hydraulic resistance and convective heat transfer within independent power generation micro sources (IPM) channels

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Abstract. The introduction of new structural materials and technologies contributes to the efficiency increase for the compact IPMs used in various branches of engineering. Use of a driving high-temperature (TIT>1600K), regenerative (the regeneration ratio is E>85%) micro gas turbine engine μGTE, major components which are made of structural ceramics, allows not only to maintain the effective efficiency at ηe=26-30%, but, also, sharply reduce the material consumption rate for the micro source as a whole.

Application of the laser prototyping technique to manufacture the air heater, which is a part of μGTE, increases the IPM compactness. Miniaturization of the air heater, manufactured by the structural ceramics laser fusion, can significantly reduce the hydraulic diameter (dh≤1.0 mm) of the channels, designed to transport the working media inside it. Reducing dh leads to a significant increase in the hydraulic resistance of the micro channels. The associated increase in the energy consumption for μGTE’s own needs is compensated by increasing the TIT, E, and heat transfer coefficients in micro channels, and by eliminating the need in cooling for high temperature IPM components.

Introduction
The continued growth in demand for compact mobile and efficient decentralized energy power systems, robotics, transportation, gas pipelines, aircraft micro hardware, energy and electronics cooling provision, etc., requires creation of independent power micro sources (IPM). As an electric drive for IPM, various types of heat engines are used, though only the micro gas turbine engine (μGTE) can dramatically increase the specific energy density, increase independency, power, and improve the environmental performance of IPM (Fig. 1).

However, small sizes of the electric generator (EG) gas turbine engine will inevitably reduce the efficiency of its basic components: compressor, turbine, combustion chamber (CC) and, consequently, the efficiency of IPM as a whole. It is known that the efficiency of μGTE working on a simple thermodynamic cycle does not exceed 3-6%, which makes the problem of improving its efficiency extremely urgent, especially for micro-engines of power Ne less than 10 kW.
Figure 1. Effect of technologies applied on the heat and electric efficiency of the existing IPMs [1].

Designations:  
- • – Stirling engines; ▲ – internal combustion technologies (ICE, GTE);  
- – high temperature solid oxide fuel cell (SOFC),  
- – low temperature FC; ● – Rankine cycle engines;  
- – Line showing the total efficiency, 85%.

The most promising way to increase the efficiency of μGTE, as shown in [2] (Fig. 2), is a combination of increasing the initial temperature (TIT) of the working media and using the heat recovery E in the thermodynamic cycle of the engine.

Figure 2. Effect of the initial cycle temperature, TIT, pressure ratio \( \pi_k \), regeneration ratio E and loading for μGTE \( N/N_n \) on the thermal efficiency \( \eta_e \) of μGTE [2].

The growth of the initial cycle temperature TIT limits the thermal strength of materials used to manufacture the basic IPM components.  
The thermal strength increase is achieved through:  
- the use of new high-temperature structural materials, including ceramics;  
- the development of laser fusion technologies, suitable to manufacture IPMs of structural ceramics.

The presence of the air heater in the IPM not only reduces the fuel consumption, but also imposes a positive effect on its environmental performance.
The ceramics density is several times less that of steel alloys, which dramatically reduces the material consumption of the micro source. Its high thermal stability allows abandoning cooling of the high-temperature IPM parts, reducing the energy consumption for its own needs, and improving the reliability of μGTE by:

- the reduction of the thermal stress in the furnace and the firing sections in the combustion chamber,
- the improvement of the flame stability due to a "hotter" air feeding to the combustion zone,
- the reduction of the temperature gradient in all the stator parts and components of the engine, etc.

In addition, the regenerative μGTE allows:

- to reduce significantly the pressure ratio \( \pi_\text{c} \) in its cycle [3],
- to simplify the design of the engine compressor through increasing its compactness,
- to improve the rigidity of the turbocharger rotor through shifting its critical speeds towards the zone of speeds which are significantly higher the operating values.

Recuperative heat exchangers are now widely used as μGTE air heaters. Given their high thermal efficiency, they are characterized by a low compactness and a relatively high material consumption rate. Significant improvements in terms of the weight and size parameters of the recuperative air heaters can be achieved by a miniaturization, which is realized through:

- having micro channels made in their matrices with \( d_h < 1.0 \text{ mm} \),
- using such a manufacturing technology as the laser fusion.

1. Hydraulic resistance at gas motion in microchannels.

Development of the air heaters with micro channels makes it necessary to establish the applicability of the "classical" equations of the thermal and hydrodynamic similarity to its heat hydraulic calculation. The careful analysis of experimental and theoretical works carried out by a number of researchers and dedicated to this problem is presented in [4]. The authors note the existence of significant variations regarding an impact of some factors on the hydrodynamics and heat transfer during the coolant motion in the micro channels; these factors are as follows:

- the absolute diameter of the channel,
- the relative roughness \( \Delta = \delta / d_h \) of its walls, where \( \delta \) is the absolute roughness,
- the cross-sectional shape and its relative dimensions,
- the turbulence ratio at inlet, etc.

An ambiguous and contradictory nature of the experimental results is largely due to the difference in performance of the design execution of operation areas for pilot plants, experiments conducting methodologies, and accuracy of the measurements performed.

The results of many experiments show that:

- correlations developed for the calculation of the convective heat transfer at the coolant motion in "conventional" tubes [5] are not fully suitable for calculation of the heat exchange in the micro channels;
- there is a need in more systematic researches to determine the mechanisms responsible for the flow structure and heat exchange variations in such channels;
- under the laminar condition in the micro channels, the friction coefficients are in good agreement with the dependencies of Hagen-Poiseuille only within the zone, where the Reynolds number does not exceed \( \text{Re} < 600 \);
- at \( \text{Re} > 600 \), the experimental data considered in the review [4] deviate from the Hagen-Poiseuille’s law in the direction of higher values of the friction coefficient \( \xi_f \);
- as in the case for the coolant flow in rough industrial tubes, the transition from the laminar to the turbulent flow in an isothermal flow in the micro channels occurs at \( 1,900 < \text{Re} < 2,500 \).
Therefore, some experimental data of studies over the hydraulic resistance and heat transfer at the coolant motion in the micro channels as applied to the matrix type heat exchangers, manufactured using innovative nonconventional technologies [6] are reported in the given paper.

Figure 3. Effect of the absolute size of the hydraulic diameter \( d_h \) of micro channels on the relative deviation \( \varepsilon_{Re} \) of the Re number from the critical \( Re_{c1} \) (a) and the friction coefficient \( \zeta_f \) (b) from the Hagen-Poiseuille’s dependency \( \frac{\zeta_f}{Re} = 64/Re \) \((1)\) [4].

Researchers of the transition processes taking place in micro channels indicate that with decreasing the channel diameter \( d_h \) it is required to change the heat exchanger design as a whole, since, to maintain constant the total pressure drop \( \Delta P_\Sigma \) in it at the heat output \( Q \), temperature differences \( \Delta T \) of the coolant mass flows \( G_i \), it would be necessary to change the number \( n \), the channel length \( L \), and the heat exchange surface area \( F \). For to the most probable laminar flow condition, it is possible to assess potential changes in the heat exchange design using graphics from the paper by Palm and Peng [6] (Fig. 4).

Figure 4. Effect of changes of hydraulic diameter \( d_h \) in micro channels on the structural and geometrical parameters of the heat exchanger: 1 - channel length, 2 - number of channels in the matrix; 3 – heat exchange surface, 4 – convective heat transfer coefficient in the heat exchanger [6].

Along with the conclusions reported in [4], studies conducted by Ruckes [7] have shown that there is no deviation from the "classical" theory [5] in channels with the diameter \( d_h = 0.123-2.0 \) mm for the isothermal air flow, and that there is no impact by the critical Reynolds numbers \( Re_{c1}, Re_{c2} \) on the absolute size of \( d_h \).
With the compressed air flowing in the capillary channels \((d_h=0.28-0.622 \text{ mm})\), M. Valdhagen revealed a shift of \(Re_{cp1}\) toward the lower values \((Re_{c1}=1,900)\) \[8\]. It was established in the same paper that the air flow turbulization in the channels of a smaller diameter begins at the lower Reynolds numbers. This is consistent with a deviation of the resistance coefficient from the Hagen-Poiseuille’s law established above; this law being at the same time valid for “conventional” engineering tubes with a non-uniform roughness. Results of an experimental study of isothermal flow of nitrogen in glass capillaries \((d_h=0.08-1.4 \text{ mm})\) \[6\] serve as a confirmation of this phenomena. It is stressed upon in the study that even at \(Re=1,500\) the coefficient of hydraulic resistance ceases to obey the Hagen-Poiseuille’s law. These experiments were performed under isothermal conditions without the flow turbulization by the convective forces.

A systematic study of hydrodynamics and heat exchange was carried out at the Institute for Thermal Power Engineering, Academy of Sciences of Ukraine \[9,10\] with the compressed air flow in capillaries of stainless steel \((d_h=0.39-2.0 \text{ mm})\) and in complicated cross-section slotted channels (slot lock joints of blades with the gas turbine disc, Fig. 5). The hydraulic diameters of the slotted channels are close to those of the capillary tubes studied earlier \[9,10\].

![Figure 5](image)

**Figure 5.** Micro channels of various configuration studied in \[9,10\].

**Designations:** a, b - mounting clearances of herringbone shanks, c - the same for shanks of GT-6-750 TMZ (Turbomotor Works) unit type; d - cooling channels for shanks of GT-6-750 TMZ.

The heating of the capillary tubes walls in the experimental study \[9\] is electric. Experiments were carried out at relatively low temperature gradients (temperature ratio \(T_f/T_w=0.9\)), so the impact of the free convection and the direction of heat flow is small. The air flow rate did not exceed 0.15 \(M\), where \(M\) is the Mach number, so there was no need in accounting for the air compressibility.

In all channels studied: flat gap thickness \(\delta_s=0.18-0.05 \text{ mm}, \delta_s=1.0-4.0 \text{ mm}\) at \(Re<2,300\), the channel length averaged resistance coefficient \(\xi_f\) is described by the known Hagen-Poiseuille’s relation, while at \(Re>2,300\) it is described by the Blasius dependence (2).

\[
\xi_f=0.3164/Re^{0.25}.
\]

Change of \(\xi_f=f(Re)\) for the air flow in the channels with a different cross-sectional configuration and different size of \(d_h\) is shown in Fig. 6.
Figure 6. The friction resistance coefficient $\zeta_f=f(\text{Re})$ with the air moving in the shank mounting clearances of rotor blades (1, 2) and in capillary tubes with diameter: $d_h=0.39$ mm (3); 0.51 mm (4) and 2 mm (5) for laminar, transitional and turbulent (A) conditions and under the turbulent condition for mounting clearances of rotor blades (1); capillary tubes with diameter 0.51 mm (2) and 2 mm (3) - (B) [10].

Designations: a – Hagen-Poiseuille’s dependence (1); b – Blasius dependence (2).

It is seen that at $\text{Re}<2,000$, the experimental data are placed near the line "a", while with increasing the Reynolds number they are displaced in the direction of the line "b", as it is peculiar to tubes with non-uniform wall roughness. At $\text{Re}=6,000$, the resistance coefficient obeys the Blasius law [11,12] (line b).

Results of experiments with micro channels $d_h=0.39$ and 2.0 mm are in good agreement with the "cold" test tube where $d_h=0.51$ mm.

The laminar gas flow in micro channels is maintained if $\text{Re}<2,000$, with the channel roughness imposing no effect on the friction coefficient $\zeta_f$, the latter determined by the Hagen-Poiseuille’s law

$$\zeta_f=A/\text{Re}$$

where A is a value dependent on the configuration and cross-sectional dimensions of the channel.

In the transition zone:
- for the first section ($\text{Re}=2,000...4,000$), $\zeta_f$ increases sharply with increasing the Reynolds number, regardless of the relative roughness walls:

$$\Lambda=\delta/d_h;$$

- for the second section, it obeys the Blasius law for smooth channels, which is valid in accordance with the following regularity: the greater the $\Lambda$, the smaller is the range of validity;
for the third section, $\xi_f$ deviates from the Blasius law (2) and with $\Delta$ increasing $\xi_f$ increases as well.

With a heat supply to the air flow and the increased turbulence at inlet, there is a smooth "crisis-free" transition from the Hagen-Poiseuille’s dependence (a) to the Blasius dependence (b) (Fig. 7).

![Figure 7](image)

**Figure 7.** Effect of initial gas flow turbulence (Tu) on the $\xi_f=f(Re)$ dependence [10].

Designations: 1, 2 – $Tu=1.0-1.5\%$, 3-5 – $Tu=5.0-18.0\%$.

The surface roughness of channels, posing a significant effect on the pressure drops and heat transfer in the micro channels differs in structure and its absolute size from the roughness of "conventional" tubes, for which $d_0>>\delta_k$ (thickness of the boundary layer).

However, for the micro channels with the non-uniform roughness as well as for "conventional" tubes, the calculation of the boundary values $Re_{c,1}$ and $Re_{c,2}$ can be done by the formulas reported in the Reference Book by I.E. Idelchik [11], according to which:

$$Re_{c,1}=1160(\delta/d_0)^{0.11} \text{ at } (\delta/d_0)>0.007;$$

$$Re_{c,2}=2090(\delta/d_0)^{0.0636} \text{ (for tubes with any roughness).}$$

In the Reynolds number transition zones reported by Celata [4], the following dependences are recommended to calculate $\xi_f$ in the micro channels:

$$Re_0<Re<Re_{c,1} \delta_0=4.4 Re^{-0.595} \exp(-0.00275/(\delta/d_0)), \text{ where } Re_0=1,900;$$

$$Re_{c,1}<Re<Re_{c,2} \xi_f=(\xi_2-\xi_1)\exp(-(Re_{c,2}-Re))^2+\xi_1.$$  

Here: at $(\delta/d_0)\leq0.007 \xi_1=0.032; \xi_2=7.244 Re^{-0.643},$

$$Re_{c,1}<Re<Re_{c,2} \xi_f=(\xi_2-\xi_1)\exp(-(Re_{c,2}-Re))^2+\xi_1.$$  

at $(\delta/d_0)>0.007 \xi_1=0.0758-0.0109(\delta/d_0)^{0.286}; \xi_2=0.145/(\delta/d_0)^{0.244}.$

With the self-similar turbulent condition, $\xi_f$ in the micro channels, as in “conventional” tubes, does not depend on the Reynolds number and is determined solely by the RR value for the channel walls.

For the micro channel in the transition zone, $\xi_f$ depends on the Re number and RR. At the transition to the turbulent condition, $\xi_f$ gradually decreases with increasing the Re number, reaching the lowest values at the quadratic (self-similar) mode. For the laminar flow in the channel with an equivalent RR over 0.7%, a deviation from the Hagen-Poiseuille’ law may occur in the direction of $\xi_f$ increasing. The greater is RR, with a smaller Re number the deviation occurs.

On the basis of the analysis of results of the experiments carried out [9,10,13,14], we can conclude that there is no influence of the absolute sizes of the microchannel on the transition from
the laminar to the turbulent flows, which allows to apply the criteria relations given in the reference literature [5,11,12,15] for the calculation of hydrodynamics in the capillary channels.

2. Convective heat transfer in micro channels

The use of lengthy ((l/dₜ)>100) channels of small diameter leads to generation of the laminar flow condition. Here, we see a relatively low flow rates and hence small values of the complex 〈Pe dₜ/l〉 (Pe dₜ/l<10, where Pe=RePr is the Peclet number; l is the length of the channel). As a result, the Nusselt number Nu ceases to depend on this complex and tends to a constant value Nu_{min}. That is, the convective heat transfer coefficient is independent of the inertial forces (working media flow rate) [16].

![Figure 8. Heat transfer in micro channels with Pe dₜ/l<10 for channels of rectangular cross-section with a different aspect ratio ψ=h/b (where h, b are thickness and width of channel [16]).](image)

Designations: 1.3 - ψ=1; 3 is the calculation result; 2.4 - ψ=1; 2.62 is the test result [17].

Here, the number of Nu_{min} and the hydraulic resistance coefficient of friction ζₕ is significantly affected by the cross-sectional shape of the channel. For the capillary tubes of circular cross-section Nu_{min,c}≈3.66, and the rectangular Nu_{min,r}=7.5-8.24, as a function of the ratio ψ=h/b. Here, Nu_{min,t} is the minimum Nusselt number at the constant wall temperature t_w. With reduction of ψ, Nu_{min,t} increases. The reduction of the complex "Pe dₜ/l" is promoted by the Prandtl number Pr, which lowers in a gas coolant (e.g. air) and becomes less 1.0. [16,18]. With respect to the slotted micro channel, to calculate ζₕ and Nu_{min,t}, you can use the following approximate formulas (9) and (10):

\[ \zeta_h \text{Re} = 57.736 \psi^2 - 95.072 \psi + 94.792; \tag{9} \]

\[ \text{Nu}_{min,t} = 8.7699 \psi^2 - 13.524 \psi + 7.8092. \tag{10} \]

At the laminar gas flow in the micro channel (opposite to conventional channels [19]), the convective heat transfer is strongly affected by the Mach number M (Fig. 9) [4].

It is seen that the effect of the Mach number M on the heat transfer in the micro channel is already apparent at M>0.2, and a situation is likely to take place when the heat flow is directed away from the wall towards the "hot" gas, i.e. T_{aw}>T_{w}.

Here:

\[ T_{aw} = T_w (1 + r(k-1)/2M^2), \tag{11} \]

T_{aw} is the adiabatic wall temperature,

T_{w} is the gas stream temperature;

r is the temperature recovery coefficient at the gas motion in the channel, equal to:

\[ r = Pr_{t}^{0.33} \Delta T_{x}. \tag{11a} \]

\[ \Delta T_{x} = 7.16 \times 10^4 \text{Re}^{0.4} f(x/d_b); \tag{11b} \]

x is the characteristic size;
f(x/d_h)=1 at x/d_h=0-15 (initial area); 

(11c)

f(x/d_h)=1+0.0413(x/d_h-15) at x/d_h=15-27 [20]. 

(11d)

Figure 9. Mach number effect on the convective heat transfer at gas motion in micro channels [4].

The cross-sectional shape of micro channels is very diverse (Fig. 10). It has a significant effect on the rate of heat transfer in channels, especially under the laminar condition.

Figure 10. Applied shape and sizes of cross section of micro channels in flat multi-channel tubes used to manufacture heat exchangers.

The difference in heat transfer is due to the secondary flows generated in such channels. They lead to significant differences in the coefficients of heat transfer in comparison with a round channel of the same hydraulic diameter. In publications devoted to the study of heat transfer in micro channels (d_h<1.0 mm), it is shown that the technology of manufacturing a trial channel with a noncircular cross section causes a number of effects leading to significant differences in heat transfer compared with the round section micro tubes. Available information on the heat transfer in
circular and noncircular micro channels is not enough for reliable recommendations for carrying out thermo-hydraulic calculations of compact heat exchangers with micro channels. More systematic researches are needed to consider each of the parameters that affect the transfer processes at the gas motion in micro channels. At the same time, you should not neglect the experimental information already available and summarized in the works by Palm and Peng [6] for heat transfer in noncircular micro channels with \( d_h = 133-367 \, \mu m \). The paper’s authors found a significant difference in the intensity of heat transfer in the micro channels and "normal" channels.

For the laminar flow in micro channels, the experimental data for a series of parallel rectangular channels are summarized by the equations

\[
\begin{align*}
\text{Nu} &= C_1 \text{Re}^{0.62} \text{Pr}^{0.3} \quad (12) \\
\text{или} \quad \text{Nu} &= 0.1165 \text{Re}^{0.62} \text{Pr}^{0.33} \left( \frac{d_h}{b} \right)^{0.81} \left( \frac{h}{b} \right)^{-0.79}, \quad (13)
\end{align*}
\]

which are applicable at \( 900 \geq \text{Re} \geq 80 \) and \( 367 \geq d_h \geq 133 \, \mu m \). Here, \( C_1 \) is the empirical coefficient dependent both on the hydraulic diameter \( d_h \) and the ratio of channel thickness \( h \) to its width \( b \) (Fig. 11).

**Figure 11.** Effect of relative height (h/b) and \( d_h \) on convective heat transfer in micro channels [6].

It is evident that there is an optimal height (h/b), which minimizes the thermal resistance in the rectangular channel. In Peng’s experiments [6], this height is \( h/b = 0.75 \).

**Table 1.** The geometrical parameters of investigated micro channels and empirical coefficients in the Peng’s criteria equations

| №  | Width b, mm | Height h , mm | Length L, mm | Hydraulic diameter \( d_h \), mm | Ratio h/b | \( C_1 \) |
|----|-------------|---------------|--------------|-------------------------------|----------|--------|
| 1  | 0.4         | 0.3           | 50           | 0.343                         | 0.75     | 0.058  |
| 2  | 0.3         | 0.3           | 50           | 0.3                           | 1.0      | 0.0384 |
| 3  | 0.4         | 0.2           | 50           | 0.267                         | 0.50     | 0.0426 |
| 4  | 0.3         | 0.2           | 50           | 0.24                          | 0.667    | 0.0472 |
| 5  | 0.2         | 0.2           | 50           | 0.20                          | 0.10     | 0.0468 |
| 6  | 0.3         | 0.1           | 50           | 0.15                          | 0.333    | 0.0104 |
| 7  | 0.2         | 0.1           | 50           | 0.133                         | 0.50     | 0.0285 |

For conditions in which \( \text{Re} \geq 1,000 \), Palm and Peng [6] suggested a correlation (13)

\[
\text{Nu} = C_1 \text{Re}^{0.8} \text{Pr}^{0.33} \quad (13)
\]
for the turbulent flow. Unfortunately, this correlation features a considerable scatter of experimental data.

On the basis of the analysis of experimental data on heat transfer in the micro channels under the turbulent flow condition, it is established that the hydraulic diameter for the micro channels is \( d_h \approx 1.2 \text{ mm} \). This is the limiting value of \( d_h \) at which you can use the "classical" equations of thermal similarity \([12,15,20]\) to calculate the convective heat transfer.

In N.M. Zotov's study \([21]\) over the air flow in micro slotted channels (h=0.048...0.18 mm), a stratification of the experimental data was revealed, depending on the relative roughness of the channel walls regardless of a relatively small height of irregularities (\( \delta_s \) not more than 1.2 \( \mu \text{m} \)). In some experiments, the air flow rate at the channel outlet reached the sound speed. Results of this work are consistent with the materials by Watts \([22]\), where the hydraulic resistance of friction was studied for \( \frac{\xi_f}{\delta} \) with nitrogen, argon and hydrogen moving in planar channels of glass and silicon of height \( h=0.028...0.065 \text{ mm} \), with the average height of roughness bulges on the surface being less \( \delta=0.004 \text{ mm} \) and RR ratio \( \Delta=0.143...0.0615 \). With such RR, greatly exceeding the limit value \( \Delta_{\lim}=15/\text{Re}=0.03...0.015 \) for any commercial-grade tubes, the stratification should take place as noted in the handbook by I.E. Idelchik \([11]\).

Influence of the isotropic surface roughness on the intensity of heat transfer was examined in the experimental study by E.P. Dyban et al \([13]\) at the compressed air motion (P=0.01...0.7 MPa) in micro slotted channels. Slotted channels of stainless steel and glass were employed. The absolute roughness of the channel walls was: \( \delta_s=0.1, 1, 2, 3, 4, 7, 10 \text{ mm} \), while the thickness of pads that form the channel is \( h=0.05; 0.06; 0.09 \text{ mm} \). The thickness of the pads was measured within the 1 \( \mu \text{m} \) accuracy. Channel width was \( b=3.1 \text{ mm} \), length \( L=13.5; 22.6 \text{ mm} \). To calculate \( \xi_f \) over the average length of the slotted channel, a relationship was employed applicable to an object that has no heat exchange with the environment, with the resistance coefficient being constant along the length of the channel.

On the basis of the analysis of results of the experiment, we can conclude that there is no influence of the absolute size of the micro channels on the transition from the laminar to turbulent flows, which allows for calculation of the heat transfer in the capillary channels of the conventional criteria relations \([11,12,15,20]\).

All the capillary tubes, where the air flow was investigated, are hydraulically smooth \([23]\), so no experimental data stratification under the turbulent condition was revealed. Increasing the level of the initial turbulence reduces \( \text{Re}_{c1} \) to 1,700...1,800, and practically does not change the friction resistance coefficient in the turbulent air flow. At the higher turbulence (\( Tu>5...7\% \)), a crisis-free transition from the laminar to turbulent flow occurs (Fig. 7), and at \( \text{Re}=1,000...1,900 \) \( \xi_f \) is 20...40% higher that which corresponds to the Hagen-Poiseuille’s law \( \xi_f=64/\text{Re} \) for circular cross-section tubes.

The experimental materials \([21-27]\) confirm the applicability of the classical equations of similarity to calculate the convective heat transfer at the motion of gases in the micro channels, and the applicability of the formulas of Hagen-Poiseuille and Blasius for determination of \( \xi_f \).

### 3. Heat exchange enhancement

To intensify the heat exchange within the micro channels, some known methods \([28]\) can be used, namely the coolant flow twisting \([29,30]\) and variation of the longitudinal channel configuration, the latter contributing to generation of the pressure fluctuations in the moving stream \([31-33]\).

At the gas/air motion along curvilinear channels, an uneven distribution of velocities and pressures is established in its sections causing a secondary vortex which is superimposed on the main stream. The stream lines are closed in the channel cross section.

The secondary flows consist of two streams, which are near the walls and are directed to the convex surface, while in the center of the channel they are directed to the concave surface and have a symmetrical – helical nature.
The structure of the secondary flow and the ensuing loss of energy is the function of the geometrical shape of the channel and the flow condition, defined by the Reynolds and Mach numbers. The secondary flows enhance the heat transfer in any mode of flow [29].

With the Dean number $De=Re\sqrt{d/D_{ben}}=26-7,000$ and $(d/D_{ben})=6.2-62.5$, where $D_{ben}$ is the bending diameter, the laminar flow occurs with the macro vortexes, at which:

$$Nu=0.0575 \ Re^{0.8} \ Pr^{0.43} \left(\frac{d}{D_b}\right)^{0.21} \left(\frac{Pr_f}{Pr_w}\right)^{0.25}.$$  \hspace{1cm} (14)

The scatter of the experimental data relative to this common equation is not above ±(12-15%). The $\xi_f$ calculation could be carried out using the Tschukin’s formula [29]:

$$\xi_f=0.0385(D_{ben}/d_h)^{0.5} +0.312Re^{0.25}.$$ \hspace{1cm} (15)

For the heat transfer enhancement, longitudinally profiled tubes with the configuration of the "convergent-divergent" type are applied. According to the Mighay’s data [32], their thermal efficiency at $Re>10^4$ is significantly higher that achieved by other methods of heat transfer improvement in tubes, which is confirmed by more recent systematic experimental studies of models of convergent-divergent and divergent-convergent channels, made by E.F. Kuznetsov and B.P. Ivanov [33] in a wide range of the Reynolds numbers $5.0 <Re <2\cdot10^5$.

4. Conclusions

1. Experimental data on the mass- and heat exchange at the coolant motion in micro channels ($d_h<1.0$) are ambiguous and often contradictory. These data do not provide specific guidance on the thermal-hydraulic calculation of miniature heat exchangers to which matrix the similar micro channels are applied.

2. For the thermal-hydraulic calculation of miniature heat exchangers, it is much to the point to use "classical" empirical equations of thermal and hydrodynamic similarity obtained for "conventional" type tubes and channels for which $d_h>>\delta_h$.

3. When using "classical" equations of similarity with respect to the channels $d_h<1.0$ mm, the accuracy of the calculated data will be significantly lower those for "conventional" tubes.

4. When you develop a micro heat exchanger for $\mu$GTE, there should be borne in mind that results of the calculation of "classical" equations of similarity are to be compared with the recommendations of individual researchers; their empirical equations are placed in this paper.

5. To improve the heat transfer coefficients in both paths of the miniature heat exchanger matrices, micro channels with slotted or flat-oval cross-sectional configuration should be used.

6. Reduction of the hydraulic resistance of micro channels of compact heat exchangers for IPMs is promoted by the use of convergent-divergent type channels.

7. Twisting of the coolant flow and a rational management of variations in the longitudinal configuration of micro channels for miniature heat exchangers lead to a significant enhancement of heat transfer in them.

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