Two-Phase System in a Rotating Cylindrical Cavity under the Transverse Vibrations

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Abstract. The effect of large-amplitude translational vibrations on the dynamics of a two-phase system (light cylinder in liquid or two immiscible liquids, placed in a rotating cylindrical cavity) is studied experimentally. The experiments are run at high rotation rate, when under the action of centrifugal force the light phase is located near the rotation axis. Vibrations are perpendicular to the rotation axis and their frequency is close to the rotation rate. When the frequencies coincide, the vibrations change the centrifugal field configuration. The light phase column shifts stationary in the rotating frame of reference.

1. Introduction

Vibrations are an efficient instrument of control of mechanical systems and make new technological solutions possible [1]. The effect of vibrations on density-inhomogeneous hydrodynamic systems leads to a number of averaged effects. One of the bright effects connected with vibrations is steady streaming [2], which finds its application in the most various areas: from geophysics and industrial technologies to the description of the principles of physiology of a human body. Separately, it is worth mentioning rotating systems, for which a large number of natural oscillations is specific that can be excited by variable force fields [3]. Examples of the rotating two-phase systems: a centrifuged liquid layer with a free surface [4–6], bodies in liquid [7, 8], systems of immiscible liquids of different density [9–11]. It is known that the field of gravity leads to a radial displacement of the phase inclusion in the rotating cavity [4, 8]. The oscillations of the liquid excited in this case lead to the differential rotation of the interface [5, 8]. Independently of the action of the gravity field, vibrations lead to the excitation of an additional differential rotation of the light phase due to the excitation of inertial oscillations [6, 7, 9, 10]. In the case of a light solid body, its averaged motion depends on the frequency ratio between cavity rotation and vibrations, and its dynamics is complex [9–11]. In the case of a centrifuged two-liquid system, the stability of the interface strongly depends on the relative radius of the surface, determined by the volume filling, and the relative length of the cavity [5, 8]. Interestingly, the effects described above also manifest themselves in rotating systems with a temperature distribution that is not uniform along the radius [12]. Experiments showed that resonant excitation of oscillations of the rotating hydrodynamic systems by means of vibrations with eigenfrequencies can serve as an efficient instrument of control of flows and of heat and mass transfer. In this work, we consider the case of vibrations applied to a rotating two-phase system with the frequency close to the frequency of rotation.

2. Experimental setup and methods

The experimental model is a transparent cylindrical cavity I (figure 1) made of acrylic glass tube with a transparent flange 2. In the first case, a light (whose density is less than that of the surrounding liquid) cylindrical body 3 is placed inside, and the space between it and the walls of the cavity is filled with a
working liquid. In the second case, the cavity is filled with a pair of immiscible liquids of different densities. Dimensions of the working cavity are the following: the length $L_1 = 26.5\, \text{cm}$ (with a body) and $L_1 = 13.5\, \text{cm}$ (with two liquids), the inner radius $R_1 = 3.5\, \text{cm}$. The solid body is made of polyamide and has the length $L_2 = 24.5\, \text{cm}$, the radius $R_2 = 2.0\, \text{cm}$. Its density is $0.88\, \text{g/cm}^3$. The length of the light liquid column is $L_3 = 13.5\, \text{cm}$, and its radius varies in the range $R_3 = (0.7 - 2.3)\, \text{cm}$. In experiments with a two-liquid system, a shorter cavity is used. This allows to weaken the effects associated with the occurrence of centrifugal waves at the interface of liquids during vibrations. The physical characteristics of the working liquids used in the experiments are given in Table 1. The cavity is fixed in pillars with bearings 4, which are tightened with metal studs 5 in order to prevent the poles from tilting relative to the cavity. The cavity is brought into rotation by a FL86STH80-4208A stepper motor 6, fixed on a textolite platform 7. The motor is controlled by an SMD-4.2 driver paired with a ZET 210 ADC module (8), and the power is supplied by a NES-350-48 power supply (9). On the motor, a fan 10 of the HAF-71M type is fixed, aimed to maintain the required thermal operation mode. Rotation from the motor is transmitted to the cavity through the coupling 11.

![Experimental setup](image)

**Figure 1.** Experimental setup (top view).

| Water       | 1       | 1       | 54   |
|------------|---------|---------|------|
| Oil 1-5A   | 0.83    | 10.3    | 27   |
| Water+glycerol (50%) | 1.12    | 5       | 37   |
| Water+glycerol (75%) | 1.2     | 21      | 44   |

**Table 1.** Physical characteristics of the working liquids.

The entire experimental model is fixed on an aluminum platform 12. Without rotation, the light cylindrical body (light liquid) is located in the upper part of the cavity. Experimental study is conducted for the case of centrifuged state when the light phase inclusion is located near the axis of cavity rotation (figure 2). In this case, the light liquid takes the form of an axisymmetric column, elongated parallel to the axis of rotation.
Figure 2. Centrifuged state of the light phase in the absence of vibrations: (a) the solid body; (b) the liquid column.

The experimental setup is located on a vibrator platform 13 which performs horizontal vibrations (Figure 1). The vibrator platform is installed on linear ball bearings, which slide on horizontal parallel rails 14. Rotational motion of the rotor of a motor 15 is transformed into translational motion of the vibrator platform by means of a crank-and-rod mechanism 16. The direction of vibrations is perpendicular to the cavity rotation axis. Observation of the body's behavior, as well as its photo and video recording, is carried out from the side of the transparent end of the cell under continuous illumination. High-speed video recording is made with an Optronis CamRecord CL600x2 (17) camera. Capturing images and processing the results of photo and video recordings is performed using a computer 18. To study the dynamics of the phase inclusion, frame-by-frame video recording is performed. The rotation rate of the body in the laboratory reference frame \( f_s \) is calculated from the displacement of a label stuck on its end and from the time between frames, which is known from the camera records. The rotation rate in the cavity reference system is \( \Delta f = f_s - f_r \), where \( f_r \) is the rotation rate of the cavity. In the case of liquid inclusion, the definition of \( \Delta f \) by this method is not possible. To find the trajectory of the light body and of the light-liquid column, the images are processed using the programme ImageJ. The coordinates of the cavity center and of the phase inclusion are obtained. The axis of the cavity retains a fixed position relative to the vibrator platform. Subtracting the coordinates of the cavity center from the coordinates of the center of inclusion, a transition is made to the reference system associated with the vibrator platform, relative to which the inclusion motion is considered.

In order to determine the working ranges of the frequency of cavity rotation and the frequency of vibrations, experiments were conducted in the gravity field in the absence of vibrations. Thus, the solid body goes from the cavity wall to the middle at \( f_s = 4.88 \) rps. For the solid phase inclusion, the parameters of experiments are: the frequency of rotation (vibrations) \( f_v = f_v = 5 - 7 \) Hz, the amplitude of vibrations is \( A_v = 0 - 3.0 \) cm. For a two-liquid system, the frequency of centrifugation depends on the parameters of the liquids: viscosity, density, relative filling of the cavity \( q = R^2 / R_1^2 \). The studied pairs of liquids and their relative characteristics are presented in Table 2. The parameters of experiments: \( f_v = f_v = 5.75 - 7.75 \) Hz, \( A_v = 0 - 1.5 \) cm. The amplitude of vibrations is lower than for the solid body, due to the appearance of waves at the interface and, as a result, destruction of the steady centrifuged state of the light-liquid column.

| Physical characteristics of liquid pairs. | Density ratio \( \rho_{rel} \) | Viscosity ratio \( \nu_{rel} \) | Coefficient of interfacial tension, dyn/cm |
|------------------------------------------|------------------|-----------------|-------------------------------------|
| Water – Oil I-5A                        | 0.83             | 10.3            | 31                                  |
| Water+glycerol (50%) – Oil I-5A         | 0.74             | 2.1             | 17                                  |
| Water+glycerol (75%) – Oil I-5A         | 0.69             | 0.49            | 8.6                                 |
3. Experimental results and discussion

The impact on the cylindrical phase inclusion in the rotating cavity is defined by the superposition of gravity and the oscillating force of inertia related to vibrations. Both of these forces excite oscillations in the cavity reference frame (with different frequencies), causing additive averaged effects. The intensity of vibrations can be characterized by the dimensionless vibration acceleration $a_{\text{vib}}$, describing the inertial forces in the reference frame of the vibrator platform due to the transverse horizontal vibrations relative to the rotation axis. If the unit of measurement is the characteristic centrifugal acceleration in the cavity $(2\pi f_r)^2 R^2$, then this parameter takes the form

$$a_{\text{vib}} = \frac{A_{\text{vib}} (f_{\text{vib}}/f_r)^2}{R^2}. \quad (1)$$

In the following section, the dynamics of a solid light body and a column of light liquid in a rotating cavity under vibrations with a frequency extremely close to the frequency of rotation, $f_{\text{vib}} = f_r$, is considered. In this regard, discussing the variation in the rotation frequency implies the variation in both of them.

3.1. Solid body dynamics

In the absence of vibrations under the action of gravity, the body shifts along the radius, and the differential rotation rate $\Delta f < 0$. The impact of vibrations with a frequency close to the frequency of rotation causes two-dimensional circular body oscillations, which are summed with the stationary displacement caused by the action of gravity. This is evidenced by the trajectory of the body (figure 3(a, b)). It performs circular oscillations (point 1) along a trajectory of radius $h_1$ relative to its average position (point 2), which in turn is displaced from the axis of cavity rotation by the distance $b_1$. This means that the body performs both azimuthal and radial motion in the cavity. At the same time, the body rotates around its axis.

Qualitatively, two types of body motion can be distinguished. The choice between them does not depend on the frequency of cavity rotation (vibrations) and is determined by the amplitude. The first type is characteristic for vibration amplitudes $A_{\text{vib}} < 1.5$ cm. The trajectory of the center of the body is characterized by a periodic change in its radius $h_1$ (figure 3(a)). The second type of body motion is observed with an increase in the amplitude of vibrations and, accordingly, an increase in the vibration acceleration. At the same time, the body moves along a path with the constant radius $h_2$ (figure 3(b)). Considering the trajectory shown in figure 3(b) in the reference frame rotating with the cavity, the light body occupies a stable position at the distance $h_2$ from the axis of rotation. Relative to it, the body oscillates at a small amplitude $b_2$. The angular coordinate of the body relative to the rotating cavity does not change with time. Comparing figures 3(a) and (b) reveals that $h_1$ increases with the amplitude of vibrations. In the extreme case, when $h_1 = R_1 - R_2$, the body is in contact with the wall of the cell (figure 3(c)).

Displacement and oscillations of the body relative to the cavity lead to oscillations of the liquid and, as a result, to the generation of an average mass force causing differential rotation of the body [9]. At a fixed frequency, the differential rotation rate of the body $\Delta f$ increases almost monotonically with the increase in vibration amplitude $A_{\text{vib}}$ (figure 4(a)). It should be noted that the series of experimental points split according to the rotation frequency; this is due to the contribution of the gravity field to the differential rotation of the body [11]. In the range $A_{\text{vib}} = 1.0 - 1.5$ cm, non-monotonous behavior of $\Delta f$ is observed. This phenomenon is under study. Presumably, it may be associated with a change in the type of body motion. When $A_{\text{vib}} = 3.0$ cm, the body is in contact with the wall of the cavity and the differential rotation stops, $\Delta f = 0$. With a fixed amplitude in the studied range of parameters, an increase in the rotation frequency leads to a slight increase in the differential rotation of the body (figure 4(b)). It should be noted that the dimensionless amplitude $a_{\text{vib}}$ does not change for some given value of $A_{\text{vib}}$.
Figure 3. The trajectory of the center of the body in the reference frame of the vibrator platform at $f_r = f_{vib} = 5$ Hz: (a) $A_{vib} = 0.53$ cm; (b) $A_{vib} = 2.05$ cm; (c) body position at $A_{vib} = 3$ cm.

The data from figures 4(a) and (b) is generalized in figure 4(c) on the plane of dimensionless parameters. As it is shown [10], the differential rotation rate of the body is a linear superposition of two components, caused independently by the action of gravity and vibrations: $\Delta f = \Delta f_{vib} + \Delta f_g$. Consequently, the vibration component of the differential rotation rate $\Delta f_{vib} = \left( \Delta f - \Delta f_g \right)$. Figure 4(c) shows the graph of dependence of $\Delta f_{vib} / f_r$ on $A_{vib} / R_z$.

Figure 4. Experimental dependence of $\Delta f$: (a) on $A_{vib}$ in the dimensional form; (b) on $f_r$ in the dimensional form; (c) on $A_{vib}$ in the dimensionless form.

The amplitude of vibrations is nondimensionalized by the radius of the body, and $\Delta f_{vib}$ — by the frequency of cavity rotation. The gravitational component of the body rotation rate, $\Delta f_g$, was found in
the absence of vibrations \((f_{\text{vib}} = 0 ~ \text{Hz}, A_{\text{vib}} = 0 ~ \text{cm})\). It can be noted that at \(A_{\text{vib}} = 0.75 ~ \text{cm}\) the response of the body to a disturbance with a different frequency is the same. In general, the three dependences are qualitatively consistent with each other and can be approximated by a quadratic dependence: 

\[
\frac{\Delta f - \Delta f_g}{f_r} \approx \left(\frac{A_{\text{vib}}}{R_s}\right)^2
\]

(solid line in figure 4(c)). As it can be seen from the graph, the vibrations excite the leading body rotation \((\Delta f_{\text{vib}} > 0)\) whose rate increases with the amplitude.

3.2. Light liquid column

In general, the dynamics of the light liquid column is similar to the case of the light cylinder: under vibrations, the column shifts along the radius as a whole and occupies a stable position in the rotating reference frame. However, the system of two liquids also has a number of features. First, this system is characterized by the regular motion along a closed circular trajectory, \(b_1\) does not oscillate (figure 5). The figure shows the trajectories for two cases: (a) \(v_{\text{rel}} = 2.1, \rho_{\text{rel}} = 0.74, \ R_{\text{rel}} = 0.36\); (b) \(v_{\text{rel}} = 0.49, \rho_{\text{rel}} = 0.69, \ R_{\text{rel}} = 0.49\). Here, \(R_{\text{rel}} = R_h / R_s\). All relative parameters are normalized to the properties of the heavy liquid (solution of glycerin in water). In the reference frame of the vibrator platform, the center of the liquid column performs circular oscillations relative to the average shifted position. This dynamics does not depend on such parameters as the relative radius of the column \(R_{\text{rel}}\) and the properties of liquids \(\rho_{\text{rel}}, v_{\text{rel}}\).

Since the interface between two liquids is deformable, at high-amplitude vibrations, the centrifuged state is destroyed, and the column is broken. However, this is preceded by the appearance of steady waves at the interface. Two characteristic modes of wave propagation can be distinguished. The first type is marked by the occurrence of an azimuthal wave (figure 6(a)). The liquid column is distorted, however, these disturbances are two-dimensional, and the wave crests are elongated parallel to the axis of the column. The second type is recognized by the simultaneous appearance of axial and azimuthal waves (figure 6(b)). These disturbances are already three-dimensional and precede the regime when the liquid column collapses. At the same time, in the lateral projection, a stable pattern of clusters of visualizer particles is observed, forming lines in the wave nodes (figure 6(c)). Despite the fact of the interface perturbation in both cases, the column of light liquid under the action of vibrations performs circular oscillations similar to figure 5.

3.3. Discussion

In [12], the dynamics of a rotating nonisothermal liquid in a field of transverse vibrations was experimentally and theoretically considered. It was shown for the case of small density gradients \((1 - \rho_{\text{rel}}) \sim 10^{-3}\) that, if the frequency of cavity rotation and the frequency of transverse vibrations in the rotating reference frame coincide, a stationary force field is created that breaks the symmetry of the centrifugal force field. In the present experimental study, the case of coincidence of the frequencies of rotation and vibrations in systems with an interface is considered, where the density profile is step-like and \((1 - \rho_{\text{rel}}) \sim 10^{-1}\). Despite this difference, we observe a qualitatively similar dynamics. The averaged effect of vibrations at \(f_r = f_{\text{vib}}\) is equivalent to a parallel translation of the rotation axis to a distance equal to half \(A_{\text{vib}}\). This is evidenced by the experimental values of the displacement relative to the axis of cavity rotation of the solid (figure 7(a) and the liquid column (figure 7(b)). The ratio \(b_1 / A_{\text{vib}} \approx 0.5\) is maintained in all experiments and does not depend on what kind of light inclusion is considered. This ratio does not depend on the values of \(f_r\) and \(f_{\text{vib}}\) provided that \(f_r / f_{\text{vib}} \approx 1\).
Figure 5. The trajectory of the center of the liquid column in the reference frame of the vibrator platform at
(a) $f_r = f_{vib} = 7.75 \text{ Hz}$, $A_{vib} = 0.55 \text{ cm}$;
(b) $f_r = f_{vib} = 7.5 \text{ Hz}$, $A_{vib} = 0.88 \text{ cm}$.

Figure 6. Wave visualization at the interface of two liquids at $f_r = f_{vib} = 7.75 \text{ Hz}$: (a) $A_{vib} = 0.75 \text{ cm}$;
(b,c) $A_{vib} = 1.1 \text{ cm}$.

Also, in the case of a two-liquid system, with the variation in the relative radius $R_{rel}$, relative density $\rho_{rel}$
and viscosity ratio $\nu_{rel}$, the value $b_1 / A_{vib} \approx 0.5$ remains constant. This confirms the result of the theory
[12] that the shift of the light inclusion from the symmetry axis of the cavity is caused by the averaged inertial field. The results also indicate that the theory [12] can be generalized for a wide class of rotating systems that are inhomogeneous in density. An interesting feature of the interface between liquids is that centrifugal waves propagate in the reference system associated with the column of light liquid. This means that the wave inertia axis is also shifted at $0.5 A_{vib}$ relative to the cavity axis.

Figure 7. The dependence of the dimensionless radial displacement on $A_{vib}$:
(a) the solid body, $f_r = f_{vib} = 5 \text{ Hz}$ (1);
(b) the light liquid column, $(f_r, \nu_{rel}, \rho_{rel}, R_{rel}) = (7.75 \text{ Hz, } 2.1, 0.74, 0.36)$ (1) and
$(7.5 \text{ Hz, } 0.49, 0.69, 0.49)$ (2).

4. Conclusion
At the coincidence of $f_r$ and $f_{vib}$, according to the conducted experimental study, translational vibrations lead to the creation of a stationary force field in the cavity reference system. In this case, the action of vibrations is equivalent to a parallel translation of the axis of rotation, both of the solid body
and the column of light liquid, to the distance equal to half the amplitude of perpendicular vibrations. A slight mismatch of the frequency of vibrations and the frequency of cavity rotation leads to an average azimuthal drift of the phase inclusion (light body or column of light liquid) relative to the cavity. The latter is determined by the azimuthal displacement of the direction of the averaged force field and can be both lagging and leading. The studies have shown that the vibrations perpendicular to the rotation axis with a frequency close to the frequency of rotation can be used for azimuthal and radial positioning of light phase inclusions in rotating cavities. The positioning is possible at varying parameters of the system.

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