An effective thermal-parametrization theory for the slow-light dynamics in a Doppler-broadened electromagnetically induced transparency medium

Shih-Wei Su\textsuperscript{1}, Yi-Hsin Chen\textsuperscript{1}, Shih-Chuan Gou\textsuperscript{2} and Ite A Yu\textsuperscript{1}

\textsuperscript{1} Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan
\textsuperscript{2} Department of Physics, National Changhua University of Education, Changhua 50058, Taiwan

E-mail: scgou@cc.ncue.edu.tw and yu@phys.nthu.edu.tw

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Abstract
We model the effects of the atomic thermal motion on the propagation of a light pulse in an electromagnetically induced transparency medium by introducing a set of effectively temperature-dependent parameters, including the Rabi frequency of the coupling field, optical density and relaxation rate of the ground state coherence, into the governing equations. The validity of this effective theory is verified by the close agreement between the theoretical results and the experimental data.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
The ability of controlling light is crucial to the fulfillment of quantum communications and all their practical applications. It has been demonstrated recently that a light pulse can be slowed down \cite{1,2} and even stored in media \cite{3,4} by using the effect of electromagnetically induced transparency (EIT), a nonlinear optical phenomenon that renders an opaque medium transparent by irradiating it with an electromagnetic field \cite{5}. Subsequent studies indicate that the slow-light (SL) effect can greatly enhance optical nonlinearity at the low-light level \cite{6} and has potential applications in the development of low-light-level and single-photon devices, such as the all-optical switches \cite{7} and quantum phase gates based on the cross-phase modulation scheme \cite{8,9}. The novelty of EIT thus provides a new fashion to manipulate the behaviour of light and has attracted a great deal of research interest.

The prototype of EIT systems consists of a weak probe beam of central frequency $\omega_p$ and a strong coupling beam of frequency $\omega_c$ interacting with a three-level atom, as shown in figure 1. The dynamics of the slowly varying density matrix elements in the weak probe limit are described by the optical Bloch equation \cite{10}

\begin{equation}
\frac{\partial \rho_{12}}{\partial t} = -i[(\Delta_p - \Delta_c) + \gamma] \rho_{12} - \frac{i\Omega_p}{2} \rho_{13},
\end{equation}

\begin{equation}
\frac{\partial \rho_{13}}{\partial t} = -\left(\frac{i\Delta_p + \Gamma}{2}\right) \rho_{13} - \frac{i\Omega_p^*}{2} \rho_{12} - \frac{i\Omega_c^*}{2},
\end{equation}

where $\Omega_p$ and $\Omega_c$ denote the Rabi frequencies driving the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively, $\gamma$ is the relaxation rate of the ground state coherence and $\Gamma$ is the spontaneous decay rate of the excited state $|3\rangle$. The detunings $\Delta_p$ and $\Delta_c$ are defined by $\Delta_p = \omega_{31} - \omega_p$ and $\Delta_c = \omega_{32} - \omega_c$, where $\omega_{ij}$ denotes the transition frequency between the energy levels $|i\rangle$ and $|j\rangle$. The propagation of the probe pulse is described by the Maxwell–Schrödinger equation

\begin{equation}
\frac{1}{c} \frac{\partial \Omega_p(z,t)}{\partial t} + \frac{\partial \Omega_p(z,t)}{\partial z} = i\eta \rho_{31},
\end{equation}

where $\eta$ measures the opacity per unit length of the medium and is related to the optical density of the medium by $\Delta \rho_{opt} = 2L\eta / \Gamma$ ($L$ is the medium length). Here $\rho_{31}$ corresponds to the slowly varying amplitude of the optical coherence of the probe transition.
the effects of the atom’s external degrees of freedom are fully developed a numerical scheme to study the dynamics of SL and temperature-dependent parameters which are responsible for Bloch equations. The central idea of this work is to treat Doppler-broadened EIT based on the formalism of optical equations under the Galilean transformation, Su coherence and the Doppler effect arising from the atomic in a cold atomic cloud by solving the Maxwell–Liouville another method to solve the dynamics of stationary light pulses variable does not enter equations (1)–(3), finite temperatures. Since the temperature as an independent randomized Doppler shifts stemming from the thermal motion

\[ \rho_{31} \]

of an EIT medium at finite temperatures, it is desirable to take the atom’s motion into account by the statistical approach. Javan et al suggested that the macroscopic polarization, which serves as a response to the randomized frequency shift, should be obtained by taking the average over all possible velocities characterized by the Maxwell–Boltzmann distribution [15, 16]. It is expected that such an averaging process smears the atomic coherence and thus contributes largely to the relaxation of \( \rho_{31} \). More recently, the dynamics of SL, light storage and stationary light pulse in finite-temperature EIT media are studied in [17, 18]. Wu et al [17] have developed another method to study the dynamics of stationary light pulses in a cold atomic cloud by solving the Maxwell–Liouville equations which contain the higher order spin and optical coherence and the Doppler effect arising from the atomic thermal motion. By using the gauge-invariance of Schrödinger equations under the Galilean transformation, Su et al [18] have developed a numerical scheme to study the dynamics of SL and light storage in a Doppler-broadened EIT medium, in which the effects of the atom’s external degrees of freedom are fully considered.

In this paper, we investigate the dynamics of SL in a Doppler-broadened EIT based on the formalism of optical Bloch equations. The central idea of this work is to treat the atom as a stationary object but otherwise introduce some temperature-dependent parameters which are responsible for the manifestation of thermal effects. Specifically, we analyse the output probe pulse under the influence of the atomic thermal motion by endowing the coupling field, the optical density and the phenomenological decay constant with temperature dependence, namely, we let \( \Omega_\alpha \rightarrow \Omega_\alpha(T) \), \( n \rightarrow n(T) \), \( \gamma \rightarrow \gamma(T) \) for a stationary EIT medium. The derivation of these effective parameters is described in the following.

2. Formalism

To begin with, we assume that the shape distortion of the output probe pulse can be treated as a superposition of plane waves of different frequencies propagating through the highly dispersive and slightly absorptive EIT medium with length \( L \).

Each output plane wave attains a factor of \( e^{\pm iL(\Omega_\alpha + i\eta)}/\lambda \) while propagating inside the medium, where \( n_\eta \) and \( n_k \) are the real and imaginary parts of the refractive index of the medium, which lead to the attenuation and phase shift of the output plane waves, respectively. Note that the refractive index is related to \( n_\eta \) by \( 2\pi(n_k + i n_\eta)/\lambda \), \( \eta = \rho_{31}/\Omega_\alpha \) [19].

In the weak probe limit, the steady state solution of \( \rho_{31} \) of the \( \Lambda \)-type EIT system is given by [19]

\[
\rho_{31} = \frac{\Delta_\alpha - \Delta_\epsilon}{\Omega_p} + i \gamma
\]

which can be expanded as a power series of \( \omega_p \):

\[
\rho_{31} = \frac{\rho_{30}}{\Omega_p} \sum_i b^{(i)}(\omega - \omega_p)^i,
\]

where \( \omega_p \) is the centre frequency of the probe pulse. It has been shown that for an input Gaussian probe pulse with a width \( \tau_0 \),

\[
\Omega_{p, in}(t) = \Omega_{p0} e^{-t^2/\tau_0^2};
\]

the output probe pulse will take the form [19]

\[
\Omega_{p, out}(t) = \Omega_{p0} \frac{\tau_0}{\tau_d} e^{-\beta(t-\tau_d)^2/\tau_w^2},
\]

where \( \beta = L\eta \text{Im}[b^{(0)}] \), \( \tau_d = L\eta \text{Re}[b^{(1)}] \) and \( \tau_w = \sqrt{\tau_0^2 + 4L\eta \text{Im}[b^{(2)}]} \), corresponding to the attenuation constant, delay time and the broadened width of the output probe pulse, respectively. In deriving equation (7), we retain the power series in equation (5) merely up to the quadratic terms and set \( \Delta_\alpha = \Delta_\epsilon \). Under the approximations of a strong coupling field, \( \Omega^2_\alpha \gg \gamma \Delta_\alpha \) and \( \Omega^2_\epsilon \gg \gamma \gamma_\Gamma \), \( b^{(0)} \) and \( b^{(1)} \) are purely imaginary numbers and \( b^{(2)} \) is a real number. The above conditions for achieving equation (7) are well justified in the EIT-related experiments since the EIT bandwidth is proportional to \( \Omega^2_\alpha/\Gamma \) and the frequency bandwidth of the probe pulse limits the maximum value of \( (\omega - \omega_p) \). Note that equation (7) is obtained based on the assumption that all atoms are kept fixed. Realistically, the atoms move freely with a velocity distribution obeying the Maxwell–Boltzmann statistics at finite temperatures. To reconcile these two contradictory situations, equations (1) and (2) should be understood as the motion equations seen by any observer fixed on the moving atoms. To incorporate
the Doppler shift into equation (4) in an uncomplicated way, we assume that the probe pulse propagates along the major axis of the atomic cloud and the coupling field is applied with an angle π with respect to the major axis. We also assume that the centre frequencies of coupling and probe fields are on two-photon resonance in the laboratory frame, namely, Δp = Δc. Hence, for an atom moving with a velocity v = v_L + v_c in the non-relativistic limit, where v_c is the velocity along the major axis and v_L is the velocity in the transverse direction, it experiences the Doppler shifts so that Δp → Δp − k_p v_c and Δc → Δc − (k_c cos θ + k_p v_L sin θ) [20]. As a result, the condition of two-photon resonance does not hold generally, and the two-photon detuning, δ = Δp − Δc, is maximized when θ = π.

In the presence of Doppler shifts, all b(i) in equation (5) now turn out to be velocity dependent, i.e. b(i) → b(i)(v_L, v_c). Statistically, b(i) can be replaced by an effective coefficient obtained by taking the ensemble average of b(i)(v_L, v_c) over the Maxwell–Boltzmann velocity distribution at a given temperature T,

\[ \tilde{b}(i)(v) \rightarrow \tilde{b}(i)(T) = \frac{1}{\pi v_r^2} \int dv_L dv_c b(i)(v) e^{-v^2/v_r^2}, \] (8)

where v_r = \sqrt{2k_B T/m} is the one-dimensional root-mean-square velocity. It should be noted that the ensemble-averaging process in equation (8) is liable to the relaxation of ρ_21 owing to the atomic thermal motion. To be consistent, in each b(i)(v), we let γ = 0 and keep γ_0 in γ while evaluating the coefficients \( \tilde{b}(i) \). Our effective theory is formulated based on the above ensemble-averaging process, in which the analytical forms of \( \tilde{\Omega}(T), \tilde{\eta}(T) \) and \( \tilde{\gamma}(T) \) are derived. The basic idea relies on the presumption that either the stationary medium characterized by \( \tilde{\Omega}, \tilde{\eta} \) and \( \tilde{\gamma} \) should lead to the same attenuation constant, delay time and the broadened width of the output probe pulse, namely,

\[ \frac{\beta}{L} = \tilde{\eta} \tilde{b}^{(0)}(T)|_{\tilde{\gamma}, \tilde{\Omega}}, \] (9)

\[ \frac{\tau_2(T)}{\tilde{\gamma}} = \tilde{\eta} \tilde{b}^{(1)}(T)|_{\tilde{\gamma}, \tilde{\Omega}}, \] (10)

and

\[ \frac{\tau_2^2(T) - \gamma^2}{4L} = \tilde{\eta} \tilde{b}^{(2)}(T)|_{\tilde{\gamma}, \tilde{\Omega}}. \] (11)

We evaluate equations (9)–(11) by making use of equations (4) and (8), together with the assumption Δp = Δc = 0 for a stationary medium, and this yields

\[ \int dv_L dv_c \frac{1}{\tilde{\Omega}_c^2/2 + [2h(v, \theta) + i\gamma_0] G(v_z)} = \tilde{\eta} \frac{\pi v_r^2}{\tilde{\gamma}} \tilde{\Omega}_c^2/2 + \tilde{\gamma} \tilde{\gamma}, \] (12)

\[ \int dv_L dv_c \frac{1}{\tilde{\Omega}_c^2/2 + [2h(v, \theta) + i\gamma_0] G(v_z)^2} = \tilde{\eta} \frac{\pi v_r^2}{2 \tilde{\gamma}^2} \tilde{\Omega}_c^2/2 + 2 \tilde{\gamma}^2. \] (13)

while evaluating \( \tilde{\Omega} \), \( \tilde{\eta} \) and \( \tilde{\gamma} \) can be replaced by an effective coefficient \( \tilde{\eta} \), \( \tilde{\gamma} \) and \( k_p \approx k_c = k \) which are the typical conditions for the EIT experiments. The condition \( \sqrt{|\sin \theta|} \ll 1 \) is generally true since so far all EIT experiments have used either the nearly co-propagating (θ ≈ 0) or counter-propagating (θ ≈ π) geometry. In our experiments, the central wavelength of the laser beam is 780 nm, \( \gamma_0 \approx 5 \times 10^{-4} \Gamma \), \( \gamma \approx 2 \times 10^{-3} \Gamma \), \( \Delta p = 0 \) and 0.61 ≤ Ω_2 ≤ 0.97, where the spontaneous decay rate of the excited state is \( \Gamma = 2\pi \times 5.9 \text{ MHz} \). Obviously, the aforementioned assumptions are valid and thus the effective parameters can now be solved as

\[ \tilde{\Omega}_c(T) = \Omega_c \left( 1 + 4 \left( \frac{2 f(\theta) - 2}{\Gamma^2} \right)^2 \right) \tilde{\gamma}^2 + \cdots, \] (15)

\[ \tilde{\eta}(T) = \eta \left( 1 + 8 \left( \frac{2 f(\theta) - 2}{\Gamma^2} \right)^2 \right), \] (16)

and

\[ \tilde{\gamma}(T) = \gamma_0 + 4 f(\theta) \frac{\Omega_2^2}{\Gamma} x^2 + \cdots, \] (17)

where \( f(\theta) = 1 - \cos \theta \) and \( x = \sin(\theta/2) k v \Gamma / 2 \Omega_2 \), which is the ratio of the effective Doppler width to that of the EIT window. In equation (17), the leading term of \( \tilde{\gamma}(T) \) is the ground state relaxation rate originating from the intrinsic effects and the second term is resulted from the Doppler shifts. When the probe and coupling fields are nearly co-propagating, i.e. \( \theta \approx 0 \), the Doppler effect is considerably suppressed, and this is why the EIT experiments at room temperatures are always studied in the co-propagating geometry [13]. In contrast, when the probe and coupling fields are in the nearly counter-propagating geometry, i.e. \( \theta \approx \pi \), the decoherence mechanism is dominated by Doppler effects, and this is why the EIT-related experiments in the counter-propagating geometry are always studied in cold atoms [22].

3. Results and discussions

To demonstrate the availability of the current scheme, we use the experimental data of SL as an input for equations (15)–(17). To highlight the effect of the thermal motion of atoms, we consider the case when the probe and coupling beams are arranged with \( \theta \approx \pi \) along the major axis of a cigar-shaped cloud of laser-cooled ^87Rb atoms [21]. The experimental values of the coupling Rabi frequency, opacity and relaxation rate of the ground state coherence (denoted as \( \Omega_2^2, L \eta^* \) and \( \gamma_0^* \)) can be estimated by the method described.
We apply the numerical scheme of [17], which considers all velocity groups of the atoms in the calculations, to determine the temperature $T$ of the atomic medium and obtain the best fitting of $\bar{\Omega}_c$ and $L\eta$ as the grey dotted lines plotted in figures 2(a) and (b). To be generic, we consider two independent sets of experimental data of SL with the estimated values $(\bar{\Omega}^*_c, L\eta^*, \gamma^*_0) = (0.687\Gamma, 15\Gamma, 0.0005\Gamma)$ and $(0.69\Gamma, 20.5\Gamma, 0.0005\Gamma)$. The best fitting comes up correspondingly with $(T, \bar{\Omega}_c, L\eta) = (205 \mu K, 0.71\Gamma, 16.5\Gamma)$ and $(305 \mu K, 0.68\Gamma, 20.5\Gamma)$. Inserting the above best fitted parameters into equations (15)–(17), the effective parameters are found to be $(\bar{\Omega}_c, L\bar{\eta}, \bar{\gamma}) = (0.7\Gamma, 16.13\Gamma, 0.0143\Gamma)$ and $(0.67\Gamma, 19.98\Gamma, 0.0227\Gamma)$. Using $\bar{\Omega}_c, L\bar{\eta}$ and $\bar{\gamma}$ in equations (1)–(3), we numerically calculate the output probe pulse as a function of time and find it is in good agreement with the experimental data as the black lines plotted in figures 2(a) and (b). The validity of equations (15)–(17) is hence demonstrated.

The proximity of $\bar{\Omega}_c$ and $\bar{\eta}$ to their zero-temperature counterparts can be examined by defining the fractional changes, $\delta \bar{\Omega}_c = (\bar{\Omega}_c - \Omega_c) / \Omega_c$ and $\delta \bar{\eta} = (\eta - \bar{\eta}) / \eta$, which are shown in figure 3 as functions of $\bar{\Omega}_c$ and $\bar{\eta}$ within the experimentally accessible regime. Obviously, the small values of $\delta \bar{\Omega}_c$ and $\delta \bar{\eta}$ shown in figure 3 indicate that $\bar{\Omega}_c \approx \Omega_c$ and $\eta \approx \bar{\eta}$, a conclusion that is consistent with the perturbative results in equations (15) and (16). We thus expect that the atomic thermal motion affects the ground state relaxation rate mostly. In figure 4, we compare the measured ground state relaxation rate with the theoretical prediction, equation (17). Here the measured relaxation rate is determined by fitting the experimental data with the numerical results from equations (1)–(3). In figure 4, the theoretical predictions agree with the measured data very well and we see that the averaged temperatures of our experimental setup of $L\eta^* = 15\Gamma$ and 20.5$\Gamma$ are found to be $T = 200$ and 330 $\mu K$.
Figure 5. Contour plot of $\tilde{\gamma}$ with $\theta = \pi$. The vertical axis ranges from $T = 1 \mu K$ to 1000 $\mu K$ and the horizontal axis ranges from $\Omega_c = 0.41^{\circ}$ to 1.2$^{\circ}$. The values of the contour lines are from 0.12 (the upper left), 0.1, 0.08, 0.06, 0.04, 0.02, 0.01, 0.005 (the lower right).

Figure 6. The plot of $\tilde{\gamma}$ as a function of $\theta$ and $k_p v_1 / \Omega_c^2$, in the logarithm scale. The vertical axis ranges from $\theta = 1^{\circ}$ to $21^{\circ}$ and the horizontal axis ranges from $k_p v_1 / \Omega_c^2 = 0.01 \Gamma^{-1}$ to $\Gamma^{-1}$. The values of the contour lines are from $10^{-1}$ (the upper right) to $10^{-10}$ (the lower left) with a spacing of one order of magnitude.

Figure 7. The plot of $\tilde{\gamma}$ as a function of $\theta$ and $k_p v_1 / \Omega_c^2$. The vertical axis ranges from $\theta = 0.1^{\circ}$ to $2.1^{\circ}$ and the horizontal axis ranges from $k_p v_1 / \Omega_c^2 = 10^{-1} \Gamma^{-1}$ to $1000 \Gamma^{-1}$. The values of the contour lines are from $10^0$ (the upper right) to $10^{-5}$ (the lower left) with a spacing of one order of magnitude.

respectively. After demonstrating the validity of equation (17), we study how the decoherence rate $\tilde{\gamma}$ varies with $\Omega_c$, $T$ and $\theta$. Giving $\theta = \pi$, we first depict $\tilde{\gamma}$ as a function of $\Omega_c$ and $T$ in figure 5. Here we extend the ranges of $\Omega_c$ and $T$ so that the conditions of stationary light pulses and those using the counter-propagating beam geometry with laser-cooled Rb atoms can be included. We see that $\tilde{\gamma}$ decays with increasing intensity of the coupling field, as shown in figure 5. This is because the width of the transparency window depends on the intensity of the coupling field, and an increasing $\Omega_c$ can lower the absorption of the probe pulse, so that $\tilde{\gamma}$ would become smaller. The influence of the orientation of the coupling beam and the effect of the Doppler width on $\tilde{\gamma}$ is investigated by plotting $\tilde{\gamma}$ as a function of $\theta$ and $k_p v_1 / \Omega_c^2$ in figures 6 and 7, which were obtained based on the numerical calculations of equations (12)–(14). The conditions of most experiments utilizing a small separation angle between two propagating beams in laser-cooled atoms are met in the ranges, $1^{\circ} \leq \theta \leq 21^{\circ}$, $0.01 \Gamma^{-1} \leq k_p v_1 / \Omega_c^2 \leq \Gamma^{-1}$, as shown in figure 6. On the other hand, those experiments utilizing the nearly co-propagating beam geometry for hot atoms are mostly achieved in the ranges $0.1^{\circ} \leq \theta \leq 2.1^{\circ}$, $10^{-1} \leq k_p v_1 / \Omega_c^2 \leq 1000 \Gamma^{-1}$, as shown in figure 7. Taking our experimental conditions ($^{87}$Rb atoms and $\lambda = 780$ nm) as an example, if $T = 100 \mu K$, then we have $k_p v_1 / \Omega_c^2 \approx 0.03 \Gamma^{-1}$. Figures 5–7 thus provide useful information to determine the relaxation rate for those EIT-related experiments in which the Doppler-broadening cannot be ignored.

4. Concluding remarks

In conclusion, we have theoretically studied the effects of the atomic thermal motion on the dynamics of SL in a Doppler-broadened EIT medium. By extending the results in [19], we have obtained the attenuation constant, the delay time and the broadened width of the output probe field at finite temperatures. We also have derived a set of parameters, $\Omega_c$, $\eta$ and $\tilde{\gamma}$, which are temperature dependent and serve as effective thermal sources in the optical Bloch equation for a stationary EIT medium. Our approach provides an approximate and convenient way to determine the relaxation rate of SL instead of appealing to the more complicated multiple-velocity calculations. This effective theory is not only valid for the cold atomic medium but also applicable to the atomic medium at room temperatures.

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