Probing the pairing interaction through two-neutron transfer reactions

M Grasso\textsuperscript{1}, D Beaumel\textsuperscript{1}, E Khan\textsuperscript{1}, D Lacroix\textsuperscript{1}, J Margueron\textsuperscript{2}, A Vitturi\textsuperscript{3}

\textsuperscript{1}Institut de Physique Nucléaire, IN2P3-CNRS, Université Paris-Sud, F-91406 Orsay Cedex, France
\textsuperscript{2}Institut de Physique Nucléaire, IN2P3-CNRS, Université Lyon 1, 69622 Villeurbanne, France
\textsuperscript{3}Dipartimento di Fisica, Università di Padova, IT-35131 Padova, Italy; Istituto Nazionale di Fisica Nucleare, Sezione di Padova, IT-35131 Padova, Italy

Abstract. We analyze two–neutron transfers in nuclei within a mean–field approach. We first show that the response associated with pairing excitations can be affected by the surface/volume localization of the employed effective pairing interaction. Then, ground–state–to–ground–state transitions are studied and some improvements on the currently used theoretical formulas are discussed. Finally, Cr isotopes at the drip line are analyzed. The analysis of pair–transfer reactions in these nuclei may improve our understanding of two aspects: the spatial distribution of pairing correlations in nuclei and the debated question of the persistence of pairing at the drip lines.

1. Introduction
The spatial localization properties of the pairing interaction are analyzed within the mean–field framework. A Skyrme interaction is used together with a zero–range density–dependent interaction for the pairing channel in the Hartree–Fock–Bogoliubov (HFB) model. Two–neutron transfers are studied within the quasiparticle random–phase approximation (QRPA) model from the ground state of a nucleus \( A \) to excited states of the nucleus \( A \pm 2 \): \((p, t)\) reactions for very neutron–rich Sn isotopes are suggested as good cases where the spatial localization (surface/volume) of the pairing interaction may be explored. Ground-state to ground-state transitions are also analyzed by using the HFB wave functions and different formulas are compared. Finally, the interplay of the spatial localization of the interaction and the surface properties of low-\( l \) states is investigated in weakly bound Cr isotopes located close to the drip line. The manuscript is organized as follows: In Sec. 2 the transitions from the ground state of the nucleus \( A \) to excited states of the nucleus \( A \pm 2 \) are studied. In Sec. 3 the ground-state to ground-state transitions are investigated. The interplay of surface effects at the drip line is illustrated in Sec. 4. Conclusions are drawn in Sec. 5.

2. From the ground state of the nucleus \( A \) to excited states of the nucleus \( A \pm 2 \)
A detailed analysis may be found in Ref. [1]. Similar studies have been also published in Refs. [2,3].

The pairing interaction employed in the HFB + QRPA calculations is the following:

\[
V(\vec{r}_1, \vec{r}_2) = V_0 \left[ 1 - x \left( \frac{\rho(\vec{r})}{\rho_0} \right)^\gamma \right] \delta(\vec{r}_1 - \vec{r}_2),
\]

where \( V_0 \) is the strength of the interaction to be tuned and \( \rho_0 \) is the saturation density. The value of \( \gamma \) is chosen equal to 1. A cutoff of 60 MeV is employed for the quasiparticle states.
and the parameter $x$, that controls the surface/volume mixing of the interaction, is chosen equal to 0.35, 0.65 (mixed interactions), and 1 (surface–peaked interaction). $V_0$ is adjusted to reproduce the two–neutron separation energy of Sn isotopes. In the HFB + QRPA calculations a box discretization is employed to treat the continuum states. The excited states associated with addition/removal two–neutron transfer reactions are analyzed by using different pairing interaction in the HFB-QRPA model. In the QRPA model, the particle–particle and the hole–hole sectors are employed to study the addition and the removal modes, respectively. The QRPA transition densities are used as form factors in zero–range DWBA reaction calculations to evaluate $(p, t)$ cross sections. The limitation of such reaction calculations is that the absolute cross sections cannot be calculated and that inelastic excitations and two–step processes (corresponding to sequential particle transfers) are not included. The reaction $^{136}\text{Sn}(p, t)^{134}\text{Sn}$ is investigated. Since absolute cross sections cannot be evaluated, we have analyzed the ratios of cross sections. We show in Fig. 1 the ratio of the cross sections associated with the transitions to the first two excited states, obtained by using different pairing interactions (different values of $x$) and for different energies of the incident proton.

We observe that, at low incident proton energy (15 MeV), some differences between the results obtained by using a surface–peaked pairing interaction and a mixed pairing interaction are visible. These results are an indication that, for very neutron–rich Sn isotopes, one may expect measurable effects in the cross sections related to the nature of the pairing interaction.

3. From the ground state of the nucleus $A$ to the ground state of the nucleus $A \pm 2$

The analysis of ground–state to ground–state transition probabilities from the nucleus $A$ to the nucleus $A \pm 2$ has also been discussed in Ref. [4]. In Ref. [5], we have improved the expressions to be used for the calculation of the transition probabilities. Within the approximation employed in Ref. [4], the removal and addition transfer probabilities are calculated as:

$$P_{GS}^{\text{rem}}(A) = P_{GS}^{\text{add}}(A) = | \int dr \sum_{nlj} (2j + 1) u_{nlj}^A(r)v_{nlj}^A(r)|^2, \quad (2)$$

where the HFB wave functions of the nucleus $A$ are used. This approximation is clearly not valid for closed–shell nuclei, where the transition probabilities calculated with Eq. (2) are equal to zero. By using a slightly improved approximation, we have introduced different expressions to calculate the probabilities, namely:

$$P_{GS}^{\text{rem}}(A) = | \int dr \sum_{nlj} (2j + 1) u_{nlj}^{A-2}(r)v_{nlj}^A(r)|^2, \quad (3)$$
Figure 2. Removal transfer probabilities for Sn isotopes obtained with a mixed surface/volume pairing interaction by using different levels of approximation. The black solid line is obtained with the formula using only the wave functions of the nucleus $A$. The red circles are obtained with the improved formula. The blue squares represent the results obtained with the particle number projection.

$$P^{add}_{GS}(A) = \int dr \sum_{nlj}^{A}(2j + 1)u^A_{nlj}(r)v^{A+2}_{nlj}(r)^2.$$  \hspace{1cm} (4)

It can be observed that now the wave functions of all implied nuclei ($A$ and $A \pm 2$) are used. In Ref. [5] effects associated with particle number projection are also studied. Some results are summarized in Fig. 2.

The black line corresponds to the results obtained with Eq. (2), the red circles to the results obtained with Eq. (3), and the blue squares are the results obtained by performing the particle number projection. One observes that the improved formula (red circles) provides enhanced probabilities with respect to the black line and, in particular, probabilities that are not equal to zero at shell closures. The particle number projection has the effect to reduce the probabilities at mid-shell, still keeping finite values at shell closures.

4. Interplay between surface effects
A large odd–even staggering has been found in reaction cross sections of Ne and Cr isotopes in recent studies [6,7]. This staggering is found for weakly bound nuclei located at the drip lines and is related to the presence of a low–l ($s$ state). In Ref. [8] this interplay between surface effects related to the presence of a weakly bound $s$ state and different spatial localizations of the pairing interaction is investigated for Cr isotopes in the transfer probabilities. With the employed Skyrme interaction SkM*, the drip line Cr isotope is $^{82}$Cr, where the last bound neutron state is the $3s_{1/2}$ state. In Fig. 3 we show the ground–state to ground–state transition probabilities for Cr isotopes by using Eq. (2) (dotted lines) and Eq. (4) (solid lines); $\eta$ corresponds to the parameter $x$ introduced above. Let us consider only the results obtained with the improved formulas. In the case of a surface–peaked pairing interaction (c) a change of slope is found when approaching the drip lines in the transition probabilities. This change of slope may be related to the enhancement of surface effects that takes place in the calculation of the probability for the nucleus $^{80}$Cr due to the wave function of the weakly bound neutron $s$ state that enters into play.
Figure 3. Addition transfer probabilities for Cr isotopes calculated with a volume (a), a mixed (b), and a surface–peaked (c) pairing interaction.

5. Conclusions
The spatial localization properties of the pairing interaction have been analyzed within the mean–field framework. Two–neutron transfers have been studied within the QRPA model from the ground state of a nucleus $A$ to excited states of the nucleus $A \pm 2$: $(p,t)$ reactions for very neutron–rich Sn isotopes have been suggested as good cases where the spatial localization (surface/volume) of the pairing interaction may be explored. Ground-state to ground-state transitions have been also analyzed by using the HFB wave functions and different formulas have been compared. Finally, the interplay of the spatial localization of the interaction and the surface properties of weakly bound $s$ states has been investigated for Cr weakly bound nuclei located at the drip line, where the last neutron bound state is the $3s_{1/2}$ state.

References
[1] E. Pllumbi, M. Grasso, D. Beaumel, E. Khan, J. Margueron, and J. Van de Wiele, Phys. Rev. C 83, 034613 (2011).
[2] M. Matsuo and Y. Serizawa, Phys. Rev. C 82, 024318 (2010).
[3] B. Avez, C. Simenel, and P. Chomaz, Phys. Rev. C 78, 044318 (2008).
[4] H. Shimoyama and M. Matsuo, Phys. Rev. C 84, 044317 (2011).
[5] M. Grasso, D. Lacroix, and A. Vitturi, Phys. Rev. C 85, 034317 (2012).
[6] K. Hagino and H. Sagawa, Phys. Rev. C 84, 011303 (R) (2011).
[7] K. Hagino and H. Sagawa, Phys. Rev. C 85, 014303 (2012).
[8] M. Grasso, Phys. Rev. C 87, 064308 (2013).