SEARCHING FOR LONG-WAVELENGTH NEUTRINO OSCILLATIONS IN THE DISTORTED NEUTRINO SPECTRUM OF GALACTIC SUPERNOVA REMNANTS

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ABSTRACT

We investigate the muon neutrino event rate in km$^3$ neutrino telescopes due to a number of galactic supernova remnants (SNRs) expected on the basis of these objects’ known γ-ray signals. We evaluate the potential of such instruments to detect breaks in the expected power-law behavior of these SNRs’ neutrino signals. Such breaks are, in particular, induced by the long-wavelength neutrino oscillations predicted by various neutrino mixing schemes including pseudo-Dirac scenarios and the exact parity model. With 10 years’ data, neutrino signals from Sgr A East, alone, should either exclude neutrino oscillations governed by a $\delta m^2$ parameter close to $10^{-15}$ eV$^2$ or, alternatively, discover such oscillations in the likely event that other explanations for any observed break can be excluded. If data from γ-ray observations are included in the analysis, then oscillations governed by a $\delta m^2$ in the approximate range from $10^{-13}$ to $10^{-14}$ eV$^2$ might be discovered or excluded. Terrestrial or solar system neutrino experiments do not have the capability to observe oscillations governed by such tiny $\delta m^2$ values.

Subject headings: acceleration of particles — cosmic rays — elementary particles — neutrinos — radiation mechanisms: nonthermal — supernova remnants

1. INTRODUCTION AND PLAN

In this work we present a novel extension of the idea that astrophysical neutrinos can probe tiny values of $\delta m^2$, the difference in the squared masses of relevant neutrino mass eigenstates. This is, namely, that through the observation of a deviation away from pure power-law scaling—in other words, a spectral distortion—in a particular galactic supernova remnant’s observed muon neutrino spectrum, an experimentalist can infer the existence of exactly such a tiny mass splitting, given the (likely) exclusion of other explanations for such distortion.

The plan of this work is as follows: In § 2 we briefly discuss the plausibility of galactic supernova remnants (SNRs) as sites for cosmic-ray acceleration up to at least $10^{15}$ eV. In § 3 we discuss neutrino production at SNRs through pion decay and the relationship between an SNR’s γ-ray signal and its expected neutrino flux at Earth. In § 4 we briefly review neutrino oscillations and the status of the various experiments purporting to demonstrate such oscillations. We also introduce here the theoretical motivations for tiny $\delta m^2$ values. In § 5 we consider the effect such $\delta m^2$ values might have on the phenomenology of SNR neutrinos. In § 6 we briefly review the extensive code we have written to model neutrino telescope detection of SNR neutrinos. Finally, we set out the results of this code in § 7, which goes on to demonstrate the empirical feasibility of the spectral distortion method.

2. PARTICLE ACCELERATION IN SNR SHELLS

Although the hoped-for TeV γ-ray signals from the six EGRET sources with compelling young-SNR associations (Esposito et al. 1996) have not been observed (Buckley et al. 1998; Rowell et al. 2000; Prosch et al. 1995; Allen et al. 1995), cosmic rays up to (and possibly exceeding) energies of $\sim 10^{15}$ eV (near the so-called knee in the distribution observed at Earth) are still widely believed to be produced by galactic SNRs. Rather surprisingly, the first strong empirical evidence for acceleration of cosmic rays to near such high energies at SNRs has instead come from recent CANGAROO (Collaboration of Australia and Nippon [Japan] for a Gamma Ray Observatory in the Outback) observations of SN 1006 (Tanimori et al. 1998) and SNR RX J1713.7–3946 (Muraishi et al. 2000), neither of which was detected by EGRET. (We present the high-energy γ-ray data for the EGRET SNR γ Cygni and SN 1006 in Fig. 1 for comparison.) In the case of SN 1006, for instance, analysis of the TeV γ-ray emission from its northeast rim indicates either super-50 TeV electrons inverse Compton scattering 2.7 K cosmic microwave background photons or the decay of $\pi^0$ mesons—particularly if the remnant is close by (Aharonian & Atoyan 1999) —or both. Note that these putative, high-energy electrons might be either directly shock-accelerated or decay products resulting from collisions of shock-accelerated protons and nuclei. Given that the γ-ray emission is from the rim, however, they may not be accelerated by electromagnetic processes associated with the remnant’s neutron star. In any case—whether the γ-rays are hadronic or leptonic in origin—SN 1006 (and SNR RX J1713.7–3946) present evidence for shock acceleration of protons to high energies (at least ~100 TeV), given that more significant loss processes act on shock-accelerated electrons than on protons in SNR environments. In the following, we will consider the potential of both SN 1006 and SNR RX J1713.7–3946 to act as super-TeV neutrino sources.

Another source likely to be of some considerable significance to neutrino astronomy is Sgr A East, an SNR-like object at the Galactic center (GC; Blasi & Melia 2000; Crocker, Melia, & Volkas 2000). Unlike SN 1006 and SNR RX J1713.7–3946, EGRET has detected γ-rays from the
1006 CANGAROO data points is the expected integral flux from this
SN 1006 data are from Tanimori et al. (1998)
one, those due to HEGRA are
the two CANGAROO sources can be expected to generate around five muon-like events per year in such a detector, and Sgr A East may generate 50 muon-like events, where we have factored in a flux attenuation of one half due to the averaged $\nu_\mu \to \nu_\tau$ oscillations we expect on the basis of the Super-Kamiokande atmospheric neutrino data (Fukuda et al. 1998a, 1998b, 1998c).

3.1. Power-Law Nature of SNR Neutrino Spectra

Given that our analysis is critically predicated on the power-law nature of SNR $\nu$ spectra, we briefly review here why such power-law scaling is a robust expectation. It is generally held that shell-produced high-energy emission in the remnants of powerful explosions results from the energizing effects of strong shocks. Gamma-ray observations, particularly of Sgr A East, seem to support this view. It is not clear how the compression ratio in these sites could be very different from four, unless a very strong magnetic field is present (see, e.g., Kirk et al. 2000), for which there is no observational evidence. As such, it is difficult to argue against an accelerated particle distribution with a spectral index near two. As previously mentioned, the neutrinos are produced by the decay of pions, which, themselves, result from proton-proton scatterings. The neutrinos, then, mirror the hadronic parent population and there is little reason to anticipate any deviation from a neutrino power-law spectrum at the source. Recent sophisticated simulations (see, e.g., Ellison, Berezhko, & Baring 2000) do show that nonlinear shocks may produce spectral features in the proton distribution, though these tend to be “up” breaks that enhance the number of high-energy particles relative to those at lower energy. We are not aware of any other possible source modifications that can affect the neutrino oscillation signature we are predicting in this work (see later), and all the evidence we have thus far, based on $\gamma$-ray observations, seems to support this view. Of course, there is always the possibility that an unexpected systematic effect, either in the source or along the line of sight, may produce its own spectral distortion. Even granted this caveat, however, note that one may conclude with certainty that the nonobservation of a spectral distortion (i.e., the observation of a pure power-law spectrum) rules out a well-defined neutrino oscillation parameter range, provided sufficient statistics can be accrued.

4. NEUTRINO OSCILLATIONS BETWEEN SNR AND EARTH

4.1. Distance Considerations

The distance between the neutrino source and detector is about 8 kpc $\simeq 2.5 \times 10^{22}$ cm in the case of the Sgr A East.
Distance determinations to SN 1006 lie between around 0.7 (Willingale & West 1996) and 2 kpc (Winkler & Long 1997). For SNR RX J1713.7–3946 they likewise range from 1 kpc (Koyama et al. 1997) to 6 kpc (Slane et al. 1999).

Given the range of distance determinations, all three SNRs have linear dimensions in the range of 5–50 pc. Source dimensions are relevant because we need to know how neutrino oscillation lengths compare with the size of the emitting object to determine whether the neutrino source is flavor coherent.

4.2. Introduction to Neutrino Oscillations

For the moment we consider a two-flavor oscillation mode $\nu_\alpha \leftrightarrow \nu_\beta$ for illustrative purposes. Suppose a beam of flavor $\alpha$ is produced at $x = 0$. Then at a point $x$ distant from the source the oscillation probability is

$$P(\alpha \rightarrow \beta) = \sin^2 2\theta \sin^2 \left(\frac{x}{L_{osc}}\right).$$

The parameter $\theta$ is the “mixing angle” that determines the amplitude of the oscillations. The value $\theta = \pi/4$, which leads to the largest possible amplitude, is termed “maximal mixing.” The parameter $L_{osc}$ is the “oscillation length”—the length scale over which oscillation between two mass eigenstates occur—and is given by $L_{osc} = (4\pi E/\delta m^2)hc$.

This works out to be

$$L_{osc} \approx 0.8 \frac{E/P \text{ eV}}{\delta m^2/10^{-10} \text{ eV}^2} \text{ kpc}. \quad (2)$$

Note that the oscillation length increases linearly with energy. The parameter $\delta m^2 \equiv |m_2^2 - m_1^2|$ is the squared-mass difference between the two mass eigenstates neutrinos.

Totally averaged oscillations see the $x$-dependent $\sin^2$ factor in equation (1) set equal to $1/2$, leading to

$$\langle P(\alpha \rightarrow \beta) \rangle = \frac{1}{2} \sin^2 2\theta. \quad (3)$$

Such averaging can be due to either distance (this is due only to the finite size of the $\nu$ source—on which scale the detector is tiny) or energy spread or both.

4.3. Atmospheric Neutrinos

The atmospheric neutrino anomaly detected by the Super-Kamiokande atmospheric neutrino experiment is convincingly resolved by positing oscillations governed by parameters in the range of (Fukuda et al. 1998a, 1998b, 1998c)

$$\delta m_{\text{ATM}}^2 = 10^{-2} \text{ eV}^2 \rightarrow 10^{-3}, \quad \sin^2 2\theta_{\mu\tau} = 1, \quad (4)$$

where the oscillations are $\nu_\mu \rightarrow \nu_\tau$, with $x \neq e$ and $x = \tau$ or $x = s$ (sterile) or a combination thereof. This experiment currently favors oscillations to $\nu_\tau$ over oscillations to a sterile neutrino at the 2 $\sigma$ level (Fukuda et al. 2000, though note some have cast doubt over this result; Foot 2000; Kobayashi & Lim 2000), so for definiteness we will take it that the resolution of the atmospheric neutrino anomaly lies in maximal $\nu_\mu \rightarrow \nu_\tau$ oscillations over the relevant length scale, $L_{\text{ATM}}$.

Substituting $\delta m_{\text{ATM}}^2$ into equation (2), we see that the $\nu_\mu \rightarrow \nu_\tau$ oscillation length is orders of magnitude less than the size of a typical SNR shell for the entire neutrino spectrum. This means that the oscillations will be distance averaged, and hence at Earth we expect a 50/50 mixture of $\nu_\mu$ and $\nu_\tau$ from an SNR.

4.4. Solar Neutrinos

The solar neutrino problem can be solved by $\nu_e \rightarrow \nu_s$ oscillations, where $y = \mu, \tau, s$, or a combination thereof, are all allowed, with two important provisos: (1) if the Los Alamos Liquid Scintillator Neutrino Detector (LSND) experiment is correct, then $\nu_\mu \rightarrow \nu_\mu$ oscillations, with parameters that cannot solve the solar neutrino problem, have already been detected (White 1999). So, if the still-controversial LSND result is correct, then pure $y = \mu$ is ruled out; (2) somewhat at variance with the LSND result (given that, as previously mentioned, Super-Kamiokande analysis currently favors the notion that $\nu_{\mu}$’s are maximally mixed with $\nu_s$’s over atmospheric scales), the recently announced results from Sudbury Neutrino Observatory (SNO) on solar neutrino charged-current event rates (Ahmad et al. 2001), when combined with the latest Super-Kamiokande solar data (Fukuda et al. 2001), disfavor pure $\nu_e \rightarrow \nu_s$ oscillations. In this work we set aside the LSND result, though it could be easily accommodated in the six-neutrino mass eigenstate models discussed below.\(^5\)

The precise oscillation parameter space required to account for the solar data depends on which of the solar neutrino experiments are held to be correct $\nu_e \rightarrow \nu_s$ oscillation, however, with a very large mixing angle (LMA) $\sin^2 2\theta_{\mu\tau} \simeq 1$ is broadly consistent with the data for the rough parameter range

$$\delta m_{\text{LMA}}^2 = 10^{-3} \rightarrow 10^{-10} \text{ eV}^2. \quad (5)$$

The previously determined small mixing angle solution that operated through the Mikheyev-Smirnov-Wolfenstein effect is now disfavored by Super-Kamiokande data (Fukuda et al. 2001), and we disregard this possibility. The lower end of the solar $\delta m^2$ parameter space, $\delta m_{\text{LMA}}^2 \leq 10^{-9} \text{ eV}^2$, defines “just-so” oscillations where the oscillation length for solar neutrinos is of the order of 1 AU. For larger $\delta m_{\text{LMA}}^2$ values, completely averaged oscillations, with a flux suppression factor of $\frac{1}{2} \sin^2 2\theta_{\mu\tau}$, result. Maximal mixing explains almost all of the data with averaged oscillations (excepting the Homestake result; Cleveland et al. 1998). Values of $\delta m_{\text{LMA}}^2 > 10^{-3} \text{ eV}^2$ are ruled out by the non-observation of $\nu_e$ disappearance from reactors (CHOOZ; Bemporad 1999 and Palo Verde experiments; Boehm 1999).

Note that though there are potentially interesting phenomena associated with SNR $\nu_s$’s, we ignore these because of the difficulties predicted for the detection of electron-type $\nu_s$ at SNR energies (see Crocker et al. 2000 and references therein for more detail on the chances for SNR-energy $\nu_s$ detection). Given that double-bang detection of $\nu_s$ also only becomes practicable in the uppermost decade of the SNR neutrino energy spectrum (the ANTARES Collaboration

\(^5\) Also note that the SNO data still do not rule out the existence of light sterile neutrinos. Indeed, given (see later) that the existence of maximally mixed light sterile does enjoy some strong theoretical motivations, if it comes to be firmly established that oscillations to such sterile do not operate over terrestrial and solar system neutrino experiment baselines, then they must be split from active species by exactly the kind of (tiny) $\delta m^2$’s we suggest might be probed by SNR neutrino signals—or by still smaller amounts.
1999), in this work we will only consider possibilities for SNR $\nu_\mu$ detection.

### 4.5. Atmospheric and Solar Data Combined

Any neutrino mixing scheme that seeks to accommodate both the atmospheric and solar neutrino anomalies must possess at least three distinct neutrino mass eigenstates. Furthermore, with just the minimal three-neutrino mass eigenstates, if we demand maximal $\nu_e \rightarrow \nu_\tau$ oscillations over the atmospheric scale and maximal mixing also in the solar case, we find that $\nu_e$ is forced to be maximally mixed with both $\nu_\mu$ and $\nu_\tau$ over solar length scales. In other words, we have bimaximal mixing (Visani 1997; Barger et al. 1998; Baltz, Goldhaber, & Goldhaber 1998; Jezabek & Sumino 1998; Altarelli & Farghio 1998; Mohapatra & Nussinov 1999).

That close-to-maximal mixing is demanded by the atmospheric neutrino anomaly (Fukuda et al. 1998a, 1998b, 1998c; Apollonio et al. 1998) and is also favored by the most recent Super-Kamiokande solar neutrino data (Fukuda et al. 2001) raises hope for extracting interesting particle physics from astrophysical neutrino phenomenology. This is because the poor statistics of the proposed neutrino telescopes mean that in practice only modes with large mixing angles, $\theta$, can be probed.

#### 4.6. Finding Oscillations

Generically, neutrino oscillations may be evidenced in three ways, viz:

1. By a difference between the neutrino flavor ratios at point of generation to those found at point of detection;
2. Via an observed discrepancy between detected and expected flux of a particular neutrino species, given one has an accurate fix on the absolute flux of this species expected on the basis of an SNR’s $\gamma$-ray signal;
3. Through the observation of a spectral anomaly, i.e., a distortion of a particular neutrino flavor’s energy distribution away from its expected shape. In the case of an SNR’s $\nu$ signal, this would mean a deviation away from pure power-law scaling of neutrino flux with energy (given that it is expected a priori that the region of the distribution under investigation be governed by a single spectral index—see § 3.1). The great advantages of this third method over the former two are that
   a) it only requires observation of a single neutrino species: $\nu_\mu$ for SNRs in practice. One is not obliged to positively identify the $\nu$ flavor to which the $\nu_\mu$’s might oscillate to uncover oscillation evidence, nor is one required to know precisely what the expected flux of the $\nu_\mu$’s is;
   b) it can give a range for the $\Delta m^2$ governing the oscillations, not just a lower bound.

Note that as a well-justified variant on this diagnostic, one can restrict the fitted function not only to be a power law, but further fix its spectral index to be predetermined by and equal to that of the super-10 GeV $\gamma$-ray spectrum of the object under question (see § 3). Only the overall normalization, therefore, remains here a free parameter. Long-wavelength oscillations would then show up as an unacceptable fit of such a function to the observed neutrino spectrum. Now, this mode of analysis would employ information beside neutrino event rate determinations, but it can therefore draw a stronger conclusion than the previous variant.

### 4.7. Long-Wavelength Oscillations

Now we have already seen that for the entire allowable $\Delta m^2_{\text{ATM}}$ and $\Delta m^2_{\text{S}}$ regimes we pragmatically expect totally averaged oscillations in SNR signals, i.e., precisely half the $\nu_\mu$’s and $\nu_\tau$’s generated in an SNR should oscillate to something else on their journey to Earth given bimaximal mixing (though the actual number of $\nu_e$’s detected would not vary from the naive expectation because of oscillations from the maximally mixed $\nu_\mu$-$\nu_\tau$ subsystem to $\nu_e$; see, e.g., Bento, Keränen, & Maalampi 2000; Athar, Jezabek, & Yasuda 2000). In other words, the three oscillation diagnostics outlined above, when applied to SNR neutrino signals, would only ever act as confirmatory to existing terrestrial neutrino oscillation experiments, albeit over vastly different energy and distance regimes.

Note, however, from equation (2) that if an extra $\Delta m^2$ significantly smaller than $10^{-10}$ eV$^2$ were operating in astrophysical neutrino oscillations—thereby introducing an extra, longer oscillation length scale—an energy-dependent spectral distortion might be observable in the SNR’s $\gamma$-ray signal. We label this new $\Delta m^2_{\text{L}}$, $\Delta m^2_{\text{LONG}}$, where “LONG” denotes long-wavelength oscillations, i.e., long in respect of existing terrestrial and solar-system scale neutrino oscillation experiments. Note that such oscillations are, in the context of these experiments, also labeled “subdominant.”

Of course, any scenario invoking this new oscillation scale demands one or more new neutrino mass eigenstates. The theory will then contain additional weak eigenstates that are obliged to be sterile neutrinos.

Consider the range of $\Delta m^2_{\text{L}}$ potentially probed by an SNR neutrino signal. To see the spectral distortion mentioned above, we require that it occurs for a particular SNR at some distance $L_{\text{SNR}}$, at an energy below the maximum that the SNR’s neutrinos are observed to reach (which might either be determined by the physics of the source or lack of statistics given the expected tail-off of high-energy events with a power-law spectrum) and above the energy at which the SNR’s signal becomes invisible because of atmospheric neutrino background. As mentioned previously, our calculations show that this energy is around 1 TeV for all nine SNRs (i.e., Sgr A East, SNR RX J1713.7–3946, SN 1006, and the six EGRET SNRs) so far discussed. The $\Delta m^2_{\text{LONG}}$ ranges, with such a threshold energy, are approximately

$$10^{-10} \rightarrow 10^{-14} \text{ eV}^2 \quad \text{for 1 kpc ,}$$
$$10^{-11} \rightarrow 10^{-15} \text{ eV}^2 \quad \text{for 10 kpc .}$$

One may note immediately that the $\Delta m^2_{\text{LONG}}$ ranges discussed, given they are so tiny, are not ruled out by any existing neutrino oscillation experiment (even with the largest $\Delta m^2_{\text{LONG}}$ at an energy of 1 MeV, $L_{\text{LONG}}$ is still over a million kilometers). Put another way, if such $\Delta m^2_{\text{LONG}}$ scales operate in nature, we can only probe them with astrophysical neutrinos. These values for $\Delta m^2_{\text{LONG}}$ are tiny numbers. We now go on to discuss how we might motivate them.

#### 4.7.1. Maximal Mixing and Sterile Neutrinos

As in the case of solar and atmospheric scale oscillations, we require that mixing be large in amplitude for there to be
an observational consequence of long-wavelength oscillations for astrophysical neutrinos. Indeed, the most favorable situation for interesting phenomenology is close-to-maximal or maximal mixing over the new long-wavelength oscillation scale. There are two scenarios we know of that naturally incorporate this. These are the exact parity model (EPM: Foot, Lew, & Volkas 1991, 1992; Foot 1994; Foot & Volkas 1995) and the generic, active \( \leftrightarrow \) sterile, pseudo-Dirac scenario (Wolfenstein 1981; Bilenkii & Pontecorvo 1983; Bilenkii & Petcov 1987; Kobayashi, Lim, & Nojiri 1991; Giunti, Kim, & Lee 1992; Bowes & Volkas 1998; Geiser 1999; Kobayashi & Lim 2000).

Note that the EPM and the pseudo-Dirac scenarios require that every active neutrino be maximally mixed and close-to-maximally mixed (respectively) with a sterile partner. There is quite some freedom within these scenarios, though, regarding how oscillations over atmospheric and solar length scales are accommodated, given the freedom that exists in determining the mass splitting between each active neutrino and its sterile partner. In this paper we examine two broad variants of the EPM and pseudo-Dirac scenarios, viz:

1. **EPM/pseudo-Dirac with active-sterile oscillations in the solar sector.**—In this variant—"six neutrino mixing"—we arrange for close-to-maximal mixing between \( \nu_e \) and \( \nu_s \) to explain the atmospheric anomaly (Yoon & Foot 2000; Kobayashi & Lim 2000), and \( \nu_x \rightarrow \nu_e \) (i.e., active to sterile) oscillations to explain the solar anomaly. Demanding here that the scale of the mass splitting between \( \nu_x \) and \( \nu_s \) does not interfere with atmospheric neutrino experiment results only constrains \( m_{\text{osc}}^2 \) to be somewhat less than \( 10^{-3} \text{eV}^2 \).

2. **EPM/pseudo-Dirac with bimaximal mixing.**—Alternatively, if pure \( \nu_x \rightarrow \nu_e \) oscillations are phenomenologically untenable, as the SNO+Super-Kamiokande data would suggest (Ahmad et al. 2001), one can construct EPM or pseudo-Dirac variants that incorporate bimaximal mixing in the active sector. As particular instantiations of such variants we examine below a number of what we label "four neutrino models." These involve—for the purposes of the phenomenology we model—oscillations between three active neutrinos and a sterile one. From the theoretical point of view, however, these models should be seen as belonging to the particular region of parameter space for EPM/pseudo-Dirac+bimaximal mixing, wherein a hierarchy of active-sterile mass splittings exists. This hierarchy is such that the largest splitting affects \( \nu_s \) oscillation phenomenology on galactic length scales, whereas the other active-sterile mass splittings are too small to be evidenced even over these distances. To reiterate, even though such models can be said to contain three sterile neutrinos, only one affects parsec-scale oscillation phenomenology.

For the purposes of oscillation phenomenology then, introducing a fourth, light neutrino mass eigenstate with a mass very close to one of the existing states will result in long-wavelength oscillations of both \( \nu_x \) and \( \nu_s \) to a new \( \nu_s \) over long length scales. Again if this mass difference is in the \( m_{\text{osc}}^2 \) range described above and the mixing is maximal, then there will be phenomenological consequences for astrophysical neutrino signals from SNI.

With three mass splittings, \( m_{\text{osc}}^2_{\text{ATM}} \), \( m_{\text{osc}}^2_{\text{LONG}} \), and \( m_{\text{osc}}^2_{\text{SNI}} \), and four mass eigenstates, there are six possible arrangements of the latter. These can be broken down into two "double-doublet" (arrangements of two pairs of mass eigenstates defining \( m_{\text{osc}}^2 \) and \( m_{\text{osc}}^2_{\text{LONG}} \) split by \( m_{\text{osc}}^2_{\text{SNI}} \)) and four "mixed" arrangements.

5. OBSERVATIONAL CONSEQUENCES OF OSCILLATION SCENARIOS

5.1. In Theory

The phenomenological consequences of the maximal, long-wavelength four- and six-mass eigenstate scenarios mentioned can be gauged by noting the energy dependence that they introduce to \( F_{\nu_s} \), the fraction of the total initial neutrino flux from an SNR that arrives at Earth with flavor \( \nu_s \):

\[
F_{\nu_s} = \frac{1}{2} \times P(\nu_x \rightarrow \nu_s) + \frac{3}{2} \times P(\nu_x \rightarrow \nu_s)
\]

\[
\equiv \rho - \sigma \sin^2 \Delta(E)_{\text{LONG}},
\]

where we define \( \Delta(E)_{\text{LONG}} \equiv \frac{\delta m_{\text{osc}}^2 - L_{\text{SNR}}}{4E} \text{hc} \) and \( \Delta(E)_{\text{SNI}} \equiv \{ \text{ATM}, \, \varnothing, \, \text{LONG} \} \). Here \( \rho \) is \( \frac{1}{2} \) for all the long-wavelength oscillation scenarios under investigation, whereas \( \sigma \) is \( \frac{1}{2} \) in the case of the six-mass eigenstate scenario, \( \frac{1}{3} \) for the two scenarios of four-mass eigenstate double-doublet, and \( \frac{1}{4} \) in the case of the four mixed scenarios of four-mass eigenstates.

\( F_{\nu_s} \) is, in principle, dependent on all three oscillation scales—\( \Delta_{\text{SNI}}, \Delta_{\text{ATM}}, \) and \( \Delta_{\text{LONG}} \). In practice, no dependence of \( F_{\nu_s} \) on \( \Delta_{\text{ATM}} \) is evident because, over the entire energy range of any SNR, atmospheric oscillations will be averaged as already discussed. More interestingly, any dependence on \( \Delta_{\text{SNI}} \) actually cancels out between \( \frac{1}{2} \times P(\nu_x \rightarrow \nu_s) \) and \( \frac{3}{2} \times P(\nu_x \rightarrow \nu_s) \) essentially because \( \nu_x \) and \( \nu_s \) are maximally mixed over the solar and atmospheric scales, respectively.

The energy dependence in \( \rho - \sigma \sin^2 \Delta(E)_{\text{LONG}} \) will show up in the latter two oscillation diagnostics provided sufficient statistics can be accrued and \( m_{\text{osc}}^2_{\text{LONG}} \) falls within the range defined by equation (6): over the energy range of an SNR’s detected \( \nu \) spectrum, \( \Phi_{\nu_x}/\Phi_{\text{theor}} \) will go from some constant fraction \( (\frac{1}{2}) \) well below \( E_{\text{crit}} \) to exhibiting oscillatory behavior around \( E = E_{\text{crit}} \) to a constant value of \( \frac{1}{2} \) well above \( E_{\text{crit}} \). Also, in these three regimes (over increasing energy) a plot of the SNR’s differential \( \nu \) flux versus energy on a log-log scale produces a line that at first has some constant slope, \( \alpha \), goes through some oscillatory regime around \( E = E_{\text{crit}} \), and then resumes along the initial constant slope, \( \alpha \). By \( E_{\text{crit}} \) we denote the energy at which \( L_{\text{LONG}}(E_{\text{crit}}) = L_{\text{SNR}} \), where \( L_{\text{LONG}}(E) = \left( 4\pi E^2 / \sin^2 \frac{\Delta(E)_{\text{SNI}}}{\text{hc}} \right) \) and we have employed equation (1) setting \( \theta = \pi/4 \).

Note that because \( \sigma \) is constant within each categorization (six mass eigenstate and four eigenstate, either "double-doublet" or "mixed"), different examples within each of these categorizations produce the same phenomenological consequences in terms of the latter two oscillation diagnostics.

The first oscillation diagnostic does not identify long-wavelength oscillations to a \( \nu_s \); however, because with \( \nu_s \) and \( \nu_x \) already mixed over the atmospheric scale, we simply
expect the only practically measurable ratio, $\Phi^{\text{obs}}_{\nu,\tau} / \Phi^{\text{obs}}_{\nu,\mu}$, to equal $\frac{1}{2}$, independent of energy.

5.2. In Practice

The efficacy of the techniques we have described in finding long-wavelength oscillations in SNR $\nu$ signals may be limited by statistics. In particular, we must determine whether event rates are large enough that deviations from pure power-law scaling due to long-wavelength oscillations might be positively identified against unavoidable statistical fluctuations. The derivation of fluxes from event rates also requires that a detector’s energy-dependent response function be sufficiently well characterized and we assume this is the case.

There are plans afoot for the construction of km$^3$ scale neutrino telescopes in both the Antarctic ice (AMANDA and its planned extension IceCube; Spiering 2001) and the deep Mediterranean sea (the ANTARES and NESTOR projects; see the ANTARES Collaboration 1999 and Botai et al. 2000, respectively).

6. SIMPLE MODELING OF DETECTOR OPERATION

We have written a FORTRAN code that, from a particular SNR’s 10 GeV $\gamma$-ray flux, extracts event rates in a neutrino telescope of 1 km$^3$ volume, assuming that the $\gamma$-ray emission is hadronic in origin. The code takes into account Earth shadowing effects relevant to a particular SNR given its declination and the detector’s latitude. These we estimate on the basis of the work of Naumov & Perrone (1999). The code also employs the energy-dependent detection probability parametrization given by Halzen (1998). The event rates are given per decade energy bin from $10^2$ to $10^8$ GeV to $10^5$ to $10^6$ GeV. The code determines these event rates given various values for $dm^2_{\text{LONG}}$ and for the minimum and maximum determined distances to each SNR. The code does not assume that the neutrino signal has to be coming for it to be detected. Rather, for nadir angle bins from $0^\circ$ to $10^\circ$ to $170^\circ$ to $180^\circ$ it compares the signal to the atmospheric background over an assumed detector resolution. This background has two components: atmospheric $\nu_\mu$’s and, for nadir angles greater than $90^\circ$, atmospheric $\mu$’s. We employ the zenith angle-dependent parametrization of the sea-level $\nu_\mu$ and $\mu$ flux given by Lipari (1993) and also employ the results of Lipari & Stanev (1991) to account for the attenuation of muons with their propagation through the Earth. In the muon-attenuation subroutine, we have made the approximation that each detector is located at a single, well-defined depth below the Earth’s surface (1.6 km in the case of a South Pole detector and 2.4 km in the case of a Mediterranean detector). The code starts to record events in a particular $10^\circ$ angle bin only when the signal rises above the background. Because the atmospheric $\nu_\mu$ and $\mu$ backgrounds go with spectral index $\alpha = 3.7$ in the energy ranges of concern, whereas the sources go with index $\alpha \approx 2$, the background quickly drops away from the signal once it has been surpassed.

We estimate the detector angular resolution by employing the parameterization suggested by the ANTARES Collaboration (1999):

$$\Theta = \frac{0.7}{(E_\nu \text{ TeV}^{-1})^{0.6}} + 0.1. \quad (8)$$

The AMANDA project (which will hopefully evolve into IceCube) has to contend with the short scattering length of the Čerenkov light in ice, 24 m, as compared to sea water at greater than 200 m. Despite this, IceCube will achieve an angular resolution of less than $1^\circ$ and perhaps as low as $0.4^\circ$ (F. Halzen 2000, private communication), and we adopt the same parameterization of detector resolution—equation (8)—independent of detector medium.

The binning of event rate data into decade energy bins is forced upon us by two factors: the expected limits to the energy resolution of the proposed detectors and low statistics. The ANTARES Collaboration (1999) has determined that they can gauge a muon neutrino’s energy to within a factor of 3 for $E_\nu > 1$ TeV, so binning simulated data into decades of energy is reasonable.

7. RESULTS

Because of their locations in southern skies, the three SNR-like objects that we have determined should produce detectable neutrino fluxes at the Earth—the Sgr A East, SN 1006, and SNR RX J1713.7—3946—must exceed the atmospheric $\nu_\mu$ background before they can be visible to IceCube. This means that, whereas all three emerge above background at or below $\sim 1$ TeV for a detector at Mediterranean latitudes, at the South Pole their signals only emerge above $\sim 10$ TeV. The 10 yr, no-oscillation event rates in a km$^3$ detector, per decade energy bin that we determine for these three objects are presented in Table 1. (The numbers here should be halved to incorporate the effect of averaged $\nu_\mu \rightarrow \nu_\tau$ oscillations.) We have taken it that the maximum energy attained by an SNR $\nu$ is 1 PeV $[(1/12) \times 10^{16} \text{ eV}]$.

Note that, above background and below the maximum neutrino energy, there are only two decade energy bins operating in a South Polar detector for all three neutrino sources. IceCube might provide evidence for long-wavelength oscillations only via diagnostic $2 -$observation of energy

| Detector Location | Object     | $10^2 \rightarrow 10^3$ (GeV) | $10^3 \rightarrow 10^4$ (GeV) | $10^4 \rightarrow 10^5$ (GeV) | $10^5 \rightarrow 10^6$ (GeV) |
|-------------------|------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| Mediterranean ....| Sgr A East | 108                            | 491                            | 277                            | 118                            |
|                   | SN1006     | 0                              | 18                             | 16                             | 8                              |
|                   | SNR RX     | 0                              | 23                             | 20                             | 10                             |
| South Pole ........| Sgr A East | 0                              | 0                              | 156                            | 152                            |
|                   | SN1006     | 0                              | 0                              | 1                              | 11                             |
|                   | SNR RX     | 0                              | 0                              | 3                              | 14                             |
dependence in the value of $\Phi^{\text{obs}}/\Phi^{\text{theor}}$. We repeat, however, that positive identification of such variation requires that $\Phi^{\text{theor}}$ be well determined, which, in turn, necessitates accurate determinations of the normalizing differential photon flux at 10 GeV and the photon spectral index.

On the other hand, Sgr A East shows up in four energy bins for a Mediterranean detector and the other two sources in three. Thus, the possibility that the third oscillation diagnostic might be brought into play is held out. Whether this diagnostic can be made to work in practice depends on statistical error, granted that potential confounds like energy dependence in the detector response are well-enough pinned down.

Figure 2 illustrates the expected, 10 yr event rates in a km$^3$ detector at the Mediterranean for the most promising oscillation scenario—six—mass eigenstate mixing—and the best astrophysical neutrino source, Sgr A East. The figure details event rates in decade energy bins with no oscillations and also (atmospheric and) long-wavelength oscillations governed by $\delta m^2_{\text{LONG}}$’s from $10^{-14}$ to $10^{-9}$ eV$^2$. The vertical error bars indicate the statistical error ($\sqrt{n/n}$) in each “datum.”

For the case of Sgr A East, $\delta m^2_{\text{LONG}}$’s larger than $\sim 10^{-11}$ eV$^2$ are not discernible one from another. This is because, with the increase of the $\delta m^2_{\text{LONG}}$ parameter, the energy at which the source might exhibit oscillatory behavior is pushed into the highest energy bin where the expected event rate is low. At $\delta m^2_{\text{LONG}}$ values of $10^{-3}$ oscillation are completely averaged over Sgr A East’s observable neutrino spectrum.

Figure 3 illustrates a way to test the hypothesis that a neutrino spectrum obeys a power law. The $E^2$ weighted differential flux at each particular energy $E$ is related to the integral flux from $E$ to $10E$, $N(E, 10E)$, one would infer from the event rate (in that particular decade energy bin) by the relation

$$\frac{dN(E)}{dE} = \frac{1 - \alpha}{10^\alpha - 1} \frac{N(E, 10E)}{E}. \quad (9)$$

The figure shows the differential flux at the minimum energy of each of the four decade energy bins one might extrapolate from actual observation over 10 years of Sgr A East with a km$^3$ neutrino telescope at Mediterranean latitude, assuming a power-law spectrum. The error bars are calculated from the relative statistical error in the event rate for the same energy bin. If the plotted points form a straight line within statistical error, then the power-law hypothesis is consistent with the “data.” From Figure 3, however, one may see by eye that for a $\delta m^2_{\text{LONG}}$ value of $10^{-15}$ eV$^2$, a constant power law (i.e., straight line) fit to the differential flux is not possible within the vertical error bars. On the other hand, a straight line fit to the “no-osc” points (denoting the differential neutrino flux expected from Sgr A East if no neutrino oscillations whatsoever were to take place) is very good because there are no oscillations acting over propagation to distort the neutrino spectrum away from its power-law nature at the source. Likewise, as mentioned above, for a $\delta m^2_{\text{LONG}}$ of $10^{-11}$ eV$^2$ the long-wavelength oscillations are averaged over the detectable spectrum, excluding the last bin, which has the least statistical weight. As expected, the flux in this situation is approximately one-quarter that found for the “no-osc” situation also illustrated because the oscillations are averaged over the atmospheric scale and, again, almost completely, over the long-wavelength scale (i.e., $\frac{1}{4} = \frac{1}{4} \times \frac{1}{4}$), so that the spectrum, once more, is close to a constant power law. Finally, for a $\delta m^2_{\text{LONG}}$ of $10^{-16}$ eV$^2$, $L_{\text{LONG}}$ only becomes equal to $L_{\text{SNR}}$ for energies below the atmospheric threshold and, again as expected, the detectable flux is averaged (only) over the atmospheric scale and, therefore, half that for the “no-osc” situation in all active energy bins.

The $\chi^2$ and $\chi^2/d.o.f.$ values for least-squares fits to the differential flux data for the various values of $\delta m^2_{\text{LONG}}$ with a two-free-parameter function, $f(E) = a \times E^{-b}$, are presented in Table 2. The hypothesis that the neutrino differential flux is governed by a pure power law can be excluded with fair certainty (i.e., $2 \sigma$ level) by this method if $\delta m^2_{\text{LONG}}$
such methods (for the defined range of
is near $10^{-15}$ eV$^2$, confirming what can be seen directly by
out the oscillation signature. In fact, the no-oscillation case
may be only just distinguished from all the separate oscillation
examples. The neutrino signal from SN 1006 (and also
may be only just distinguished from all the separate oscillation
elements. The neutrino signal from SN 1006 (and also
with Spectral Index Fixed to 2.1) to Modeled
Differential Fluxes of Sgr A East $\nu$'s
for Different Values of $\delta m^2_{\text{LONG}}$

| $\delta m^2_{\text{LONG}}$ (eV) | $\chi^2$ | $\chi^2$/d.o.f. |
|-----------------------------|---------|----------------|
| $10^{-11}$                   | ~0      | ~0             |
| $10^{-12}$                   | 1.7     | 0.9            |
| $10^{-13}$                   | 2.7     | 1.4            |
| $10^{-14}$                   | 3.0     | 1.5            |
| $10^{-15}$                   | 6.2     | 3.1            |
| $10^{-16}$                   | ~0      | ~0             |
| No osc                      | ~0      | ~0             |

8. CONCLUSION

We have described a calculation that determines whether
deviation away from pure power-law scaling might be dis-
covered in the muon neutrino signal from three galactic
SNRs. The identification of such a deviation constitutes, in
some circumstances, an efficacious technique for uncovering
ultralong-wavelength neutrino oscillations. This technique
does not require either observation of $\nu_e$'s or $\bar{\nu}_e$'s or unreal-
istically constrained measurements of these objects’ high-
energy $\gamma$-ray signals. We have determined through careful
modeling that when applied to the Sgr A East neutrino sig-
nal the technique will allow for the discovery or exclusion of
long-wavelength neutrino oscillations governed by a $\delta m^2$
parameter with a value near $10^{-15}$ eV$^2$. If a further con-
straint is added—that the fitted neutrino spectrum must, as
theory predicts, have the same spectral index as that of the
super-10 GeV $\gamma$-ray spectrum of the same object—then a
value of $\delta m^2$ in the approximate range $10^{-13}$ to $10^{-15}$ eV$^2$
can be discovered or excluded. Such values cannot be
probed by any conceivable terrestrial or solar system neu-
trino experiment.

TABLE 2
The $\chi^2$ and $\chi^2$/d.o.f. Values for the Fitting of a
Power Law with Variable Normalization
and Spectral Index to Modeled
Differential Fluxes of Sgr A East $\nu$'s
for Different Values of $\delta m^2_{\text{LONG}}$

| $\delta m^2_{\text{LONG}}$ (eV) | $\chi^2$ | $\chi^2$/d.o.f. |
|-----------------------------|---------|----------------|
| $10^{-11}$                   | ~0      | ~0             |
| $10^{-12}$                   | 1.7     | 0.9            |
| $10^{-13}$                   | 2.7     | 1.4            |
| $10^{-14}$                   | 3.0     | 1.5            |
| $10^{-15}$                   | 6.2     | 3.1            |
| $10^{-16}$                   | ~0      | ~0             |
| No osc                      | ~0      | ~0             |

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super-10 GeV $\gamma$-ray spectrum of the same object—then a
value of $\delta m^2$ in the approximate range $10^{-13}$ to $10^{-15}$ eV$^2$
can be discovered or excluded. Such values cannot be
probed by any conceivable terrestrial or solar system neu-
trino experiment.

TABLE 3
The $\chi^2$ and $\chi^2$/d.o.f. Values for the Fitting of a
Power Law with Variable Normalization (but
with Spectral Index Fixed to 2.1) to Modeled
Differential Fluxes of Sgr A East $\nu$'s
for Different Values of $\delta m^2_{\text{LONG}}$

| $\delta m^2_{\text{LONG}}$ (eV) | $\chi^2$ | $\chi^2$/d.o.f. |
|-----------------------------|---------|----------------|
| $10^{-11}$                   | ~0      | ~0             |
| $10^{-12}$                   | 3.2     | 1.1            |
| $10^{-13}$                   | 16.1    | 5.4            |
| $10^{-14}$                   | 19.3    | 6.4            |
| $10^{-15}$                   | 12.0    | 4.0            |
| $10^{-16}$                   | ~0      | ~0             |
| No osc                      | ~0      | ~0             |
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