Mass and chemical asymmetry in QCD matter

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Abstract

We consider two-flavor asymmetric QCD combined with a low-energy effective model inspired by chiral perturbation theory and lattice data to investigate the effects of masses, isospin and baryon number on the pressure and the deconfinement phase transition. Remarkable agreement with lattice results is found for the critical temperature behavior. Further analyses of the cold, dense case and the influence of quark mass asymmetry are also presented.

Key words: Deconfinement phase transition, Mass effects, Baryon and isospin chemical potentials
PACS: 25.75.Nq, 12.39.Fe

The phase diagram of strong interactions embraces at the same time several physical phenomena and theoretical open questions and has been intensively studied during the last years. This investigation involves not only the full usage of in-medium techniques but also the understanding of the non-perturbative regime of QCD, lattice simulations being the main tool to probe the full theory \cite{1}.

Despite the fact that Lattice QCD is increasingly providing results with realistic (small) quark masses and probing a larger domain at finite baryon chemical potential, it is still considerably restricted by current lattice sizes and the Sign Problem \cite{23}. QCD at finite isospin density and vanishing baryon chemical potential provides a useful framework to test alternative techniques to simulate finite density on the lattice, since it has a positive fermionic determinant and therefore no Sign Problem. In addition, it is also part of the physical phase diagram for strong interactions, and exhibits a very rich phenomenology \cite{4}. Theoretical predictions with finite quark masses, isospin and baryon densities are then crucial for both technical and phenomenological reasons, complementing in a natural way the full QCD thermodynamics probed by lattice simulations.

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Preprint submitted to Elsevier 15 October 2008
We investigate the effects of finite quark masses, isospin, and baryon number on the equation of state of QCD matter and on the deconfining phase transition within an effective model inspired by lattice results and chiral perturbation theory (χPT). Results for the critical temperature [5] are compared to lattice data showing remarkable agreement in contrast with previous model predictions. The influence of the quark-mass asymmetry on the deconfinement transition at finite temperature is also analyzed. Furthermore, we present results for the mass and isospin dependence of the critical baryon chemical potential [6]. While the high temperature regime is of interest to the phenomenology of high-energy heavy ion collisions, the findings for cold, dense matter can be relevant for compact stars and for the phenomenon of color superconductivity [7].

To study the influence of masses and chemical potentials on the deconfining phase transition, we constructed a simple effective model with explicit breaking of the chiral and isospin symmetries, including the relevant parameters (quark masses \( m_u \) and \( m_d \), and quark chemical potentials \( \mu_u = (\mu_B + \mu_I)/2 \) and \( \mu_d = (\mu_B - \mu_I)/2 \)) in the high and low energy regimes in a consistent manner [5]. This feature induces an interdependent implementation of variations of quark masses and chemical potentials in both sectors that percolates in a positive fashion to the critical parameters.

The low energy regime is described by a χPT-inspired effective model of dressed pions and nucleons. The quasi-pions satisfy effective dispersion relations which incorporate temperature and isospin chemical potential corrections, as well as the dependence on the quark masses, \( m_u/d = m + \delta m \). They were calculated for both phases \( \mu_I < m_\pi \) and \( \mu_I > m_\pi \) (in which the negatively charged pions Bose condense) in Refs. [8]. The nucleon effective masses \( M_{p/n} = M + \delta M \) are taken to be the leading-order result in zero-temperature Baryon-χPT as functions of the quark average mass, \( m \), and mass difference, \( \delta m \) [9].

For the high energy sector, we adopt 2-flavor QCD with massive quarks and explicit isospin symmetry breaking (\( m_d, m_u \neq 0, m_u \neq m_d \)) and complement phenomenologically the perturbative result with non-perturbative corrections through the fuzzy bag model [10]. Besides the usual MIT-type bag constant, the total QCD pressure in this model has also a non-perturbative thermal contribution \( \sim T^2 \) to account for the unusually flat behavior of the trace anomaly normalized to \( T^2 \) observed in lattice results above the critical temperature. Here we adopt a simple extension of the model introduced in [10] which includes finite masses and chemical potentials within the perturbative contribution.

All the coefficients of our effective model are either fixed to reproduce observed properties of the QCD vacuum or extracted from lattice simulations. For more details on the model, explicit expressions and parameter fixing through lattice data and vacuum QCD observables, the reader is referred to [5,6].

We computed the massive free gas contribution of the pQCD pressure in the fuzzy bag model at finite temperature, isospin and baryon number and the free gas pressure of quasi-pions and nucleons in the low energy regime [5]. The critical temperature and chemical potential for the deconfining phase transition are then extracted by maximizing the total pressure. It should be noticed that the results for very large pion masses are in principle outside the domain of validity of our approximations, since the low energy regime of the model is based on leading-order χPT. It is still interesting however to determine the qualitative tendency.

In Fig. 1, the critical temperature is plotted as a function of the isospin chemical potential. The results are in very good agreement with lattice computations [15], even
though the curves closer to the lattice points correspond to smaller vacuum pion masses, which is not the situation simulated on the lattice. In comparison with previous results, our model appears to agree with lattice data within a considerably larger interval of isospin chemical potential [5].

The pion mass dependence, or equivalently the quark average mass dependence, of the critical temperature is displayed in Fig. 2. The approximate mass independence observed in the lattice data [11] is well reproduced here, while previous treatments tended to generate a rather different behavior (cf. Fig. 1 in Ref. [5]). The effect of a finite $\mu_B$ is completely imperceptible for $\mu_B \leq 100$ MeV. We also computed the critical parameters by considering the usual bag model in the high energy regime, finding values systematically lower, but with the same qualitative behavior as with the fuzzy bag. On the other hand, the behavior of $T_c$ with $\mu_I$ for the effective model with the usual bag pressure does not reproduce lattice data as well as the fuzzy one: even for $m_\pi = 25$ MeV, the bag model curve is still between the $m_\pi = 400$ MeV and $m_\pi = 600$ MeV fuzzy results.

Fig. 3 displays the cold, dense case: the value of baryon chemical potential beyond which matter is deconfined increases with the vacuum pion mass. The outcome of a
finite, small isospin density is to decrease the critical baryon chemical potential. This property could in principle play an important role in astrophysics. It is interesting to notice that the critical baryon chemical potential seems to be significantly more sensitive to mass variations in comparison with the finite temperature deconfinement transition. The relevance of mass effects in the thermodynamics of cold, dense matter has already been shown in different related contexts, such as perturbative QCD \cite{12,13} and perturbative Yukawa theory \cite{14}.

The effect of considering a finite quark mass difference is presented in Fig. 4. It tends to increase the critical temperature, though by a quantitatively small amount, as expected.

We constructed a simple effective model that includes all the relevant ingredients with a clear and consistent connection between the high and low energy parameters, allowing for a systematic investigation of the influence of finite quark masses and isospin symmetry breaking on the QCD phase diagram at finite temperature and density. Despite the simplicity of the framework, a free gas equation of state yielded the critical deconfinement temperature as a function of the pion mass and the isospin chemical potential in surprisingly good agreement with different sets of lattice data. This success indicates that the model captures the essential features associated with nonzero masses and chemical potentials. For the mass dependence, the heavy nucleon content in the low energy sector is probably crucial, since the previous approaches that failed to render the approximate mass invariance of the critical temperature consisted essentially of chiral fields. On the other hand, the behavior of $T_c(\mu_I)$ suggests that the quarks and the quasi-particles at low energy represent the adequate degrees of freedom carrying isospin charge in each phase. Furthermore, this model can supply information about the cold and dense regime of QCD matter, which is not yet fully covered by lattice simulations.

Acknowledgements: This work was partially supported by ANPCyT, CAPES, CNPq, FAPERJ, and FUJB/UFRJ.

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