Dynamics of a dusty plasma with intrinsic magnetization

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Abstract. We consider a dusty plasma where dust particles have a magnetic dipole moment. A Hall-MHD type of model, generalized to account for the intrinsic magnetization, is derived. The model is shown to be energy conserving, and the energy density and flux are derived. The general dispersion relation is then derived, and we show that kinetic dust-Alfvén waves exhibit instability for a low dust and ion temperature and high dust density. We discuss the implication of our results.

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1. Introduction

Plasmas with impurity particles, so-called dusty or complex plasmas, have manifold applications, ranging from applied to basic research problems. Due to this wide range of applicability, there has been a steadily increasing interest in the physics of dusty plasmas. The above-mentioned spectrum of applications covers e.g. planetary rings, lightning discharges in smoke contaminated air, fusion plasmas, low-temperature laboratory plasmas and processing plasmas in the semiconductor industry (see [1] and references therein). To be specific, by a
dusty plasma one usually means a three-component plasma consisting of electrons, ions and dust, which is considered to be significantly heavier than the ions [2]. The charge of the dust is assumed to range from a few electron charges to thousands. For astronomical applications it is often important to also include the dynamics of neutral particles [3]. Dusty plasmas contain novel physical phenomena, such as dust acoustic waves [4], dust ion-acoustic waves [5], dusty plasma crystals [6, 7] and dust lattice waves [8], all of which have been experimentally verified, see [1, 9] and references therein. In fact, due to the (in general) relatively low phase velocity of dusty plasma waves, these plasmas are useful for probing basic properties of plasma excitations.

Lately there has also been increasing interest in quantum plasma physics [10]–[19], in particular the nonlinear aspects of such systems. Such plasmas are in general typical for condensed matter environments, where the density of the electron gas is high [13, 20], giving a considerable influence from the wave function structure of the electrons. Moreover, the spin properties of plasmas have been investigated recently by means of quantum hydrodynamical models [10, 21, 22] and also by the use of spin kinetic models [23]–[25]. Even in a regime considered as classical, the effects of spin may give a nontrivial influence on the dynamics of an electron plasma [26]. Furthermore, spin and the intrinsic magnetic moment of the constituent particles are an essential part of magnetic fluids or ferrofluids [27]. A ferrofluid is a mixture of nanosized magnetic particles suspended in a liquid. In this paper, we will consider a model where magnetic dust particles are suspended in an electron–ion plasma, which can be said to be the plasma analogue of a ferrofluid. Such systems have recently been investigated both theoretically, considering single particle dynamics [28]–[32], and experimentally [33, 34]. Here, we will consider sufficiently low-frequency phenomena so that we may use a hydrodynamical model to describe the dynamics of dust particles. Naturally, this ignores certain effects associated with the strong coupling regime, e.g. crystallization, and similarly all effects of wave–particle resonances are left out.

In section 2 we present the governing equation and show that the model satisfies an energy conservation law in which magnetization transport is included. We then linearize the equations in section 3 to obtain the general dispersion relation. For the case of a static external magnetic field, we consider modes propagating perpendicular to the magnetic field as well as kinetic Alfvén waves. We show that these modes exhibit instabilities. Finally, in section 4 we summarize and draw our conclusions.

2. A magnetized dust model

We here consider a three-component plasma consisting of electrons, positive ions and negatively charged dust particles (denoted by subscripts e, i and d, respectively). The dust is assumed to be magnetized and has a charge $-Ze$ (where $e$ is elementary charge). The magnetization of dust can be assumed to be due to quantum mechanical spin or from a macroscopic magnetization of the dust grains themselves. The size of dust particles in e.g. an astronomical environment ranges from a few nanometers to about $100 \mu m$ and the weight from about $10^{-15}$ to $10^{-5}$ g [1, 2]. In the laboratory for example, [34] uses grains with size $4.5 \mu m$ and mass $\sim 10^{-14}$ g. Hence the dust is much heavier than the ions, $m_d \gg m_i$.

In the framework of multi-fluid theory, the dynamics is governed by the continuity and momentum conservation equations. The dust continuity equation is given by

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0,$$  

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where \( n_d \) is dust number density and \( \mathbf{v}_d \) is dust fluid velocity. The momentum conservation for dust reads as

\[
m_d n_d \left( \frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = -e n_e (\mathbf{E} + \mathbf{v}_d \times \mathbf{B}) - k_B \nabla (T_d n_d) + M_d \nabla B_d, \tag{2}
\]

where \( n_d \) is the number density of dust, \( k_B \) is Boltzmann’s constant, \( T_d \) is dust temperature and \( \mathbf{M} = (M_1, M_2, M_3) \) is magnetization. The last term in the equation above is usually neglected when considering plasmas, but for micrometer-sized dust grains in a plasma the mutual magnetic dipole interaction can be of importance for laboratory conditions \([33]\). The last term accounts for the magnetic moment of particles, see \([10]\).

We will consider perturbations slow compared with the plasma frequencies of ions and electrons. Neglecting the momentum of these particles, we obtain

\[
0 = -en_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - k_B \nabla (T_e n_e), \tag{3}
\]

\[
0 = Z_i en_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - k_B \nabla (T_i n_i), \tag{4}
\]

where \( T_{e(i)} \) is the temperature of electrons (ions) and \( Z_i e \) is the charge of ions. Note that we neglect the spin of the electrons and protons. The magnetization of dust is assumed to be orders of magnitude larger. The equations above are coupled to Maxwell’s equations

\[
\nabla \cdot \mathbf{E} = -\frac{e}{\epsilon_0} (n_e - Z_i n_i + Z_d n_d), \tag{5}
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{6}
\]

and

\[
\nabla \times \mathbf{B} = \mu_0 \nabla \times \mathbf{M} - e \mu_0 (n_e \mathbf{v}_e - Z_i n_i \mathbf{v}_i + Z_d n_d \mathbf{v}_d). \tag{7}
\]

A closed system of equations for the dust can be derived. To do this we start by adding the momentum equations \((3)\) and \((4)\) and assume that quasi-neutrality holds \((Z_i n_i \approx n_e + Z_d n_d)\) to obtain

\[
-e Z_d n_d \mathbf{E} = -e(n_e \mathbf{v}_e - Z_i n_i \mathbf{v}_i) \times \mathbf{B} - k_B \nabla (T_i n_i + T_e n_e). \tag{8}
\]

Solving this for the electric field and inserting it into the momentum equation for dust \((2)\) yields

\[
m_d n_d \left( \frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = -e(n_e \mathbf{v}_e - Z_i n_i \mathbf{v}_i + Z_d n_d \mathbf{v}_d) \times \mathbf{B} - k_B \nabla (T_d n_d + T_i n_i + T_e n_e) + M_d \nabla B_d. \tag{9}
\]

Using equation \((7)\) we obtain

\[
m_d n_d \left( \frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = \left[ \nabla \times \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) \right] \times \mathbf{B} \]

\[
- k_B \nabla \left( T_e + \frac{T_i}{Z_i} \right) n_e + \left( T_d + \frac{Z_d T_i}{Z_i} \right) n_d \right] + M_d \nabla B_d, \tag{10}
\]

where the quasi-neutrality condition has been used to rewrite the thermal pressure terms and we will assume that \((T_e + T_i/Z_i)n_e \ll (T_d + Z_d T_i/Z_i)n_d\) in order to obtain a closed set of equations.
This condition can of course be relaxed, introducing new thermal effects. This will be at
the price of a much more complicated set of equations, however, introducing the full ion dynamics.
Since our main purpose is to demonstrate the effect of intrinsic magnetization on dust dynamics,
we will leave out this complication in the present paper. Whenever thermal effects are not
dominating, the picture derived from our model system will still be qualitatively correct. Using
some vector identities we can finally write the continuity and momentum equations for dust as
\[
\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0 \quad (11a)
\]
and
\[
m_d n_d \left( \frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = \mathbf{B} \cdot \nabla \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) - \nabla \left[ \frac{B^2}{2\mu_0} - \mathbf{M} \cdot \mathbf{B} + k_B \left( T_d + \frac{Z_d T_i}{Z_i} \right) n_d \right], \quad (11b)
\]
respectively. Using equations (6)–(8) we can derive the time evolution equation for the
magnetic field
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_d \times \mathbf{B}) + \nabla \times \left\{ \frac{[\nabla \times (\mathbf{B} - \mu_0 \mathbf{M})] \times \mathbf{B}}{\mu_0 Z_d e n_d} \right\}. \quad (11c)
\]
The magnetization is taken to be proportional to the density of dust particles and in the direction
of the magnetic field
\[
\mathbf{M} = \mu_0 (B, T_d) n_d \hat{\mathbf{B}}. \quad (11d)
\]
This model includes the case where particles have an intrinsic magnetic moment that will be
aligned with an applied magnetic field, which is what we have in mind. However, it also includes
the case where dust particles have no net magnetic moment. The occurrence of an external
magnetic field will induce magnetization in the dust particles, which yields macroscopic
magnetization of the fluid. Magnetization of the dust could also arise from spinning of the
dust particles (with charges attached to the surface), which yields a diamagnetic response to an
applied field. However, the magnetic dust–dust interaction due to this can often be neglected. See [31]
for a more detailed discussion. Our model can be compared with a ferrofluid that is a colloidal suspension of magnetic particles in a liquid. In an ionic ferrofluid the magnetic
particles are kept apart by repulsive electrostatic forces, see e.g. [27, 34], [35]–[37]. Due to
the high density and the correspondingly high collision frequency, we note that free currents
can typically be neglected in ionic ferrofluids. By contrast, the simultaneous existence of free
currents and magnetic dipole moment has been shown to be significant in dusty plasmas [34].
A more general assumption than equation (11d) would be to assume that magnetization is also
dependent on the electron and ion temperatures since collisions with these particles may change
the magnetization.

With the exception of the occurrence of the magnetization due to the magnetic moment of
the dust, \( \mathbf{M} \), these equations are the same as Hall-MHD theory [38, 39]. Moreover, it should be
noted that the structure of the system of equations (11a)–(11c) is the same as the one obtained
from the magnetized ideal MHD model of [10, 40], if that system is extended to include the
Hall current. In that case naturally dust density and velocity instead will refer to the ion density
and velocity. Although equations (11a)–(11c) can describe an electron–ion plasma, it should be
noted that the physics for that case is different in several respects. Firstly, in the case of an
electron–ion plasma, it is the lighter species, the electrons, that contribute to the magnetization,
due to their magnetic moments being larger than those of the ions. Secondly, we stress that the
validity conditions of the model have no simple correspondence between the dust dominated plasma and the electron–ion plasma case. In what follows, we will mainly be concerned with dusty plasma applications.

In order to show the soundness of the system of equations (11), we derive an energy conservation law. To do this we must specify an equation of state for the system. For laboratory plasma the dust particles are strongly influenced by their mutual Coulomb interaction and this must be taken into account in the equation of state, see [41, 42]. However, here we use the adiabatic model to simplify the calculations as our main concern is not the thermal effects. We hence assume that the pressure satisfies

$$\frac{P}{P_0} = \left( \frac{n_d}{n_{d0}} \right)^\gamma,$$

for the total pressure $P = k_B (T_d + Z_d T_i / Z_i) n_d$, where $P_0$ and $n_{d0}$ are equilibrium pressure and density, respectively, and are both assumed to be time independent and homogeneous. With this equation of state the energy conservation becomes

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{P} = 0,$$

where

$$W = \frac{m_d n_d v_d^2}{2} + \frac{P}{\gamma - 1} + \frac{B^2}{2 \mu_0} - \mathbf{B} \cdot \mathbf{M}$$

is the energy per volume and

$$\mathbf{P} = \frac{m_d n_d v_d^2}{2} \mathbf{v}_d + \frac{\gamma P}{\gamma - 1} \mathbf{v}_d - (\mathbf{B} \cdot \mathbf{M}) \mathbf{v}_d - \left[ \mathbf{v}_d \times \mathbf{B} + \frac{1}{Z_d e n_d} (\nabla \times \mathbf{H}) \times \mathbf{B} \right] \times \mathbf{H}$$

is the flow of energy out of the region. In equation (14) the first term is the kinetic energy per volume, the second term is the energy density from the pressure, the third term is the energy stored in the magnetic field and the last term is the energy in each volume element due to the magnetic moment of the dust particles. Similarly, in equation (15), the first three terms are the flow of kinetic, pressure and magnetic energy density that follows the flow of each volume element. The last term is the Poynting vector, which is modified by the inclusion of the Hall-term and magnetization.

3. The dispersion relation

We linearize equations (11) and Fourier decompose according to

$$n_d = n_{d0} + n_{d1} \exp[i \mathbf{k} \cdot \mathbf{x} - i \omega t],$$

$$\mathbf{v}_d = \mathbf{v}_{d1} \exp[i \mathbf{k} \cdot \mathbf{x} - i \omega t],$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 \exp[i \mathbf{k} \cdot \mathbf{x} - i \omega t],$$

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_1 \exp[i \mathbf{k} \cdot \mathbf{x} - i \omega t],$$

where the perturbed quantities are assumed to be small compared to the equilibrium counterparts ($n_{d1} \ll n_{d0}$, $|\mathbf{B}_1| \ll |\mathbf{B}_0|$ and $|\mathbf{M}_1| \ll |\mathbf{M}_0|$) and we neglect quadratic and higher order terms. The equilibrium quantities are assumed to be homogeneous in space and time independent.
Furthermore, the coordinate system is defined so that $B_0 = B_0 \hat{z}$ and $k = k_x \hat{x} + k_z \hat{z}$. For simplicity we choose an isothermal pressure model ($\nabla T_d = \nabla T_i = 0$). This gives the dispersion relation

$$\begin{bmatrix}
\omega^2 - k_x^2 \left( \bar{V}_{da}^2 - V_{db}^2 \right) - k_z^2 \bar{V}_{da}^2 & -i \frac{\omega}{\omega_{cd}} \left( k_z^2 \bar{V}_{da}^2 - k_x^2 V_{db}^2 \right) & -k_x k_z \bar{V}_{da}^2 \\
- \frac{\omega}{\omega_{cd}} k_z^2 \bar{V}_{da} & \omega^2 - k_z^2 \bar{V}_{da}^2 & -i \frac{\omega}{\omega_{cd}} k_z \bar{V}_{da} \ \\
-k_x k_z \bar{V}_{da}^2 & i \frac{\omega}{\omega_{cd}} k_x k_z \bar{V}_{da}^2 & \omega^2 - k_z^2 \bar{V}_{da}^2
\end{bmatrix} = 0, \quad (17)$$

where $\bar{V}_{dA} \equiv V_{dA}^2 - V_{dM}^2$, $\bar{V}_{da} \equiv V_{da}^2 - V_{dM}^2$, and we have defined

$$V_{da}^2 = \frac{k_B}{m_d} \left( T_d + \frac{Z_d}{Z_i} T_i \right), \quad (18a)$$

$$V_{dA}^2 = \frac{B_0^2}{\mu_0 m_d n_0}, \quad (18b)$$

$$V_{dM}^2 = \frac{\mu_0 B_0}{m_d}, \quad (18c)$$

$$V_{dB}^2 = \frac{\partial \mu_d}{\partial B} \bigg|_{B = B_0} \frac{B_0^2}{m_d} \quad (18d)$$

where $\mu_{d0} \equiv \mu_d(B_0, T_d)$. The velocity $V_{da}$ is a generalized thermal speed for the dust, $V_{dA}$ is the dust Alfvén speed and $V_{dM}$ and $V_{dB}$ are related to the magnetization of the dust. The frequency $\omega_{cd} = Z_d e B_0 / m_d$ is the cyclotron frequency of the dust. The dispersion relation, equation (17), is in general a third-degree polynomial in $\omega^2$. Specifically, for $\omega \ll \omega_{cd}$ the three roots to the dispersion relation are the fast and slow dust-magnetosonic modes, and the shear dust-Alfvén wave [43].

To see the implications of the derived dispersion relation, a couple of special cases are now considered. For a wave propagating perpendicular to the magnetic field ($k = k_z \hat{z}$), the dispersion relation is obtained from equation (17) and reads

$$\omega^2 = k_z^2 \left[ \bar{V}_{dA}^2 + \bar{V}_{da}^2 - V_{dB}^2 \right]. \quad (19)$$

To obtain a qualitative description of the instability condition for equation (19), we assume that the dust and ions are in thermal equilibrium so we may take $(T_d + Z_d T_i / Z_i) = NT_d$ where $N > 1$ is a constant. Typically, we have $Z_d / Z_i$ ranging from unity to a few thousands [1]. We note that the kinetic temperature of laboratory dusty plasma is currently under investigation (see e.g. [44]) and the assumption that $T_i \approx T_d$ may not hold for a majority of dusty plasma. Furthermore, we assume that the spins are thermally distributed (see e.g. [43]) and the effective magnetic moment function (see equation (11d)) can be written as

$$\mu_d = \tilde{\mu}_d \tanh \left( \frac{\mu_d B}{k_B T_d} \right). \quad (20)$$

In the case that dust particles have a high total spin number the tanh function in equation (20) corresponding to Fermi–Dirac statistics should be replaced by the Langevin function, corresponding to Maxwell–Boltzmann statistics. The difference between these functions is relatively small, however, and thus we will use equation (20) for the remainder of this paper. We note that the magnetic moment $\tilde{\mu}_d$ can be several orders of magnitude larger than the Bohr
magneton. Furthermore, we emphasize that equation (20) is a thermodynamical relation. Thus in order for it to hold in a time-dependent situation the magnetic dipole moment must align with the magnetic field faster than the dynamical timescale. The condition for instability, equation (19), then becomes

$$\frac{B_0}{\mu_0 n_0 \mu_d} + N a \frac{k_B T_d}{\mu_d B_0} - 2 \frac{\mu_{d0}}{\mu_d} - \frac{\mu_d B_0}{k_B T_d} \left[ 1 - \left( \frac{\mu_{d0}}{\mu_d} \right)^2 \right] < 0,$$

(21)

where $\mu_{d0} \equiv \mu_d(B_0)$ in accordance with equation (20). As pointed out above, for equation (20) to apply, the relaxation time to reach the lower energy state must be shorter than the wave period time. In case the opposite ordering holds, the fraction of particles in the different energy states remains constant during a wave period, and consequently the term proportional to $V_{dB}^2$ in equations (17) and (19) should be dropped, which corresponds to neglecting the fourth and last term in equation (21). The difference in the dispersion relation, depending on whether the magnetic dipoles have time to change during a wave period or not, turns out to be relatively small, however. For definiteness, we will stick to the case where the relaxation time to reach the lower energy state must be shorter than the wave period time. In case the opposite ordering holds, the fraction of particles in the different energy states remains constant during a wave period, and consequently the term proportional to $V_{dB}^2$ in equations (17) and (19) should be dropped, which corresponds to neglecting the fourth and last term in equation (21). The difference in the dispersion relation, depending on whether the magnetic dipoles have time to change during a wave period or not, turns out to be relatively small, however. For definiteness, we will stick to the case where the relaxation time to reach the lower energy state must be shorter than the wave period time.

Finally, we can estimate the maximum growth rate of the instability from equation (19) to be

$$\sim k_z \sqrt{2V_{dM}^2 + V_{dB}^2}.$$  

Next, we consider the kinetic Alfvén type of waves. In this case, the ordering $k_z \ll k_x$, $V_{da} \ll V_{dA}$ and $\omega \sim k_z V_{dA}$ applies. The dispersion relation can then be approximated by

$$\frac{\omega^2}{k_z^2 V_{dA}^2} = 1 + \frac{k_z}{\omega^2_{cd}} \left[ -k_z^2 \frac{k_z}{\omega^2_{cd}} \right] - \frac{k_d^2}{\omega^2_{cd}} V_{dA}^2 + V_{dM}^2 \left( \frac{\tilde{V}_{da}^2 + \tilde{V}_{dB}^2 - V_{dB}^2}{V_{dM}^2} \right).$$

(22)

For $V_{dM}^2, V_{dB}^2 \rightarrow 0$, this reduces to the well-known kinetic dust-Alfvén waves [45]. Analyzing equation (22) it is seen that the wave mode can be unstable provided the numerator of the second term of the right-hand side is negative, i.e. we obtain the instability condition

$$\left( \tilde{V}_{dA}^2 - V_{dB}^2 \right) \tilde{V}_{da}^2 + V_{dM}^2 \left( \tilde{V}_{da}^2 + \tilde{V}_{dB}^2 - V_{dB}^2 \right) < 0.$$  

(23)

Assuming once more that the dust and ions are in thermal equilibrium, we obtain

$$\frac{B_0}{\mu_0 n_0 \mu_d} - \frac{\mu_d B_0}{k_B T_d} - \frac{(N - 1) \mu_d B_0}{N} \frac{\mu_d B_0}{k_B T_d} \tanh^2 \left( \frac{\mu_d B_0}{k_B T_d} \right) < 0.$$  

(24)

The conditions for instabilities equations (21) and (24) have been plotted in figure 1. Note that in order to have an instability we need to have sufficiently high dust density and/or sufficiently low ion and dust temperatures.

We here give the following simplified picture of why instability of this type can occur. The volume elements of the plasma are electrically neutral since the electron and ion background will screen any excess electrical charge. Further, the magnetization of a volume element is in the direction of the magnetic field and the different volume elements will attract each other like small magnets. Consider now the magnetic flux through a surface with normal parallel to $\mathbf{B}$. If the oscillations have $k_z = 0$ then the magnetic flux through the surface will not change, and hence there will be no build-up of magnetization. If, on the other hand, the oscillations occur perpendicular to the field there can be a local build-up of the magnetic field, which contributes

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to a negative energy density through the $-\mathbf{B} \cdot \mathbf{M}$ term in equation (14). This can, for sufficiently low ion and dust temperatures and high dust density, cause the plasma to collapse similarly to the case of Jeans instability [43, 46].

4. Summary and conclusion

In the present paper, we have put forward a Hall-MHD type of model with intrinsic magnetization. We have shown that the magnetized Hall-MHD model is energy conserving, and have presented expressions for energy density and energy flux. A set of equations of this type could describe different types of systems: an ordinary electron–ion plasma, in which case magnetization would be due to electron spin, or—as emphasized here—a three-component plasma containing electrons, ions and heavy dust particles.

As the next step, we have investigated the linear modes and stability properties of the homogeneous system. The general dispersion relation has been derived, describing the fast and slow dust-magnetosonic modes and the shear dust-Alfvén waves, as modified by the Hall current and the magnetization of the system. Due to the magnetization, the homogeneous system may be unstable, as predicted already from a magnetized ideal MHD type of model [43]. The main new finding from the stability analysis in this paper is that inclusion of the effects due to the Hall current extends the unstable region of parameter space, as described by figure 1. The wave that first becomes unstable turns out to be the magnetized version of the kinetic dust-Alfvén wave.

For the effects of magnetization to be significant, we need relatively high-density dusty plasmas and/or low ion and dust temperatures. For the case of electron–ion plasmas, these can be found in astrophysical environments, such as the interior of white dwarf stars and pulsars.

We can make some numerical estimations for the parameters $X = \bar{\mu}_d B_0 / (k_B T_d)$ and $Y = B_0 / (\mu_0 n_d \bar{\mu}_d)$. From [33] we find that the magnetic moment per particle can be of the order of $10^{-12}$ m$^{-2}$ A$^{-1}$ and magnetic induction of the order of 0.1 T. Furthermore, we assume
that the dust temperature is low $T_d \sim 1$ K. The density of particles is taken to be $n_{d0} \sim 10^{12} \text{m}^{-3}$ as in [29]. This gives us $X \approx 10^{10}$ and $Y \approx 10^8$. Comparing this with figure 1, we see that the instabilities considered here are not possible to detect in current experiments. It should be stressed that the theory given by equations (11a)–(11d) is a rather simplified one, and it is not clear that experimental conditions can be created so as to allow a direct comparison. In particular, tailoring the dust properties to meet the criteria of the model constitutes a challenge. As shown e.g. by [33, 34], however, magnetic dipole effects can still be of significance in dusty plasmas.

The model developed here should be considered only as a first step since it does not account for potentially important effects of more elaborate models. These include the two-fluid model, where spin-up and spin-down populations are considered as different species [26], the kinetic description [25] and models including nearest-neighbor interactions [32, 33], which are important in the strongly coupled regime.

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