Two-point functions on deformed space-time

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Abstract: We present a review of one-loop photon (Π) and neutrino (Σ) two-point functions in a covariant and deformed U(1) gauge-theory on d-dimensional noncommutative spaces, determined by a constant antisymmetric tensor $\theta^{\mu\nu}$, and by a parameter-space $(\kappa_f, \kappa_g)$, respectively. For the general fermion-photon $S_f(\kappa_f)$ and photon self-interaction $S_g(\kappa_g)$ the closed form results reveal two-point functions with all kind of pathological terms: the UV divergence, the quadratic UV/IR mixing terms as well as a logarithmic IR divergent term of the type $\ln(\mu^2(\theta p)^2)$. In addition, the photon-loop produces new tensor structures satisfying transversality condition by themselves. We show that the photon two-point function in four-dimensional Euclidean spacetime can be reduced to two finite terms by imposing a specific full rank of $\theta^{\mu\nu}$ and setting deformation parameters $(\kappa_f, \kappa_g) = (0, 3)$. In this case the neutrino two-point function vanishes. Thus for a specific point $(0, 3)$ in the parameter-space $(\kappa_f, \kappa_g)$, a covariant $\theta$-exact approach is able to produce a divergence-free result for one-loop quantum corrections, having also both well-defined commutative limit and point-like limit of an extended object.

Keywords: Non-Commutative Geometry, Photon and Neutrino Physics, Nonperturbative Effects.
1. Introduction

Prior to the late 1990s, the possibility of experimentally testing the nature of quantum gravity was not seriously contemplated because of the immensity of the Planck scale ($E = 1.2 \times 10^{19}$ GeV). Now this view has been significantly altered: A possibility to have a string scale significantly below the Planck scale in a braneworld scenario become the core of current experimental protocols searching for quantum-gravity phenomena (notably production of black holes) at the Large Hadron Collider at CERN. Almost simultaneously, another possibility arose, where it was point out that distant astrophysical objects with rapid time variations could provide the most sensitive opportunities to probe very high energy scales, i.e., almost the near-Planck scale physics.

Another route to search for quantum-gravity effects involves a spontaneous breaking of Lorentz symmetry in string theory, when a tensor field acquires a vacuum expectation value (vev). Unlike the case of scalars, these tensor vevs do carry spacetime indices, causing the interaction represented by the standard-model fields coupled to these vevs to depend on the direction or velocity of the said fields. Stated differently, these background vevs brings about the breakdown of Lorentz symmetry. This entails a distinctive fact of most of Lorentz violating theories on the existence of ”preferred reference frames”, where
the equations of motion take on the simplest form. In contrast with the notion of the "motionless aether" from the end of the 19th century, we have a rather unique example of such a frame in modern cosmology today: the frame in which the Cosmic Microwave Background Radiation (CMBR) looks isotropic. From the determination of the detailed spectrum of the CMBR dipole (generally interpreted as a Doppler shift due to the Earth’s motion), our velocity with respect to that frame, of order of $10^{-3}c$, can be inferred.

An eligible way to infer the preferred reference frame predicted by generic quantum gravity frameworks, is to study dispersion relations for propagating particles. Instead of propagating (in a vacuum) with the speed of light, in Lorentz violating theories one expects an energy-dependent velocity $v(E)$ for massless particles. This is a consequence of the loss of Lorentz covariance in the dispersion relations for propagating particles, with the implication that a specific form $v(E)$ can be at best valid only in one specific reference frame. Thus, a preferred reference frame is singled out, in which the equation of motions possess the simplest form. This opens up a unique possibility to study constraints on violations of Lorentz invariance. The modification of the photon velocity of the form $v(E)$ would induce time lag for photons of different energies, which could be subsequently detected if such particles can propagate at cosmological distances. Such an alternation of the photon velocities has already been obtained in Loop quantum gravity (being another popular approach to quantum gravity) as well as in heuristic models of space-time foam inspired by string theory.

It is important to stress the invariance of the theory under coordinate changes, i.e., the invariance under an observer transformation (where the coordinates of the observer are boosted or rotated). This transformation is not related to the concept of Lorentz violation since in this transformation the properties of the background fields transforms to a new set of coordinates as well. On the other hand, an invariance under active or particle transformation, where both fields and states are being transformed, is now broken by the background fields themselves, leading to the concept of Lorentz violation.

One of the most striking observation regarding spontaneous Lorentz breaking via tensor vevs in the string theory framework is that it can be formulated as deformed field theories. Specifically, a low-energy limit is identified where the entire boson-string dynamics in a Neveu-Schwartz condensate is described by a minimally coupled supersymmetric gauge theory on noncommutative space such that the mathematical framework of noncommutative geometry/field theory does apply. In such a scenario, noncommutative Dirac-Bonn-Infeld (DBI) action is realized as a special limit of open strings in a background $B^{\mu\nu}$ field, in which closed string (i.e. gravitational) modes are decoupled, leaving only open string interactions. Since in string theory $B^{\mu\nu}$ field is a rather mild background, the antisymmetric tensor $\theta^{\mu\nu}$ governing spacetime noncommutative deformations is not specified, and therefore the scale of noncommutativity could, in principle, lie anywhere between the weak and the Planck scale. It is thus of crucial importance to set a bound on this scale from experiments.

In a simple model of NC spacetime we consider coordinates $x^\mu$ as the hermitian operators $\hat{x}^\mu$, $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$, $|\theta^{\mu\nu}| \sim \Lambda_{\text{NC}}^{-2}$, (1.1)
where $\theta^{\mu\nu}$ is a constant real antisymmetric matrix of dimension $\text{length}^2$, and $\Lambda_{\text{NC}}$ being the scale of noncommutativity. It is straightforward to formulate field theories on such noncommutative spaces as a deformation of the ordinary field theories. The noncommutative deformation is implemented by replacing the usual pointwise product of a pair of fields $\phi(x)$ and $\psi(x)$ by a $\star$-product in any action:

$$
\phi(x)\psi(x) \longrightarrow (\phi \star \psi)(x) = \phi(x)\psi(x) + \mathcal{O}(\theta, \partial\phi, \partial\psi).
$$

(1.2)

The specific Moyal-Weyl $\star$-product is relevant for the case of a constant antisymmetric noncommutative deformation tensor $\theta^{\mu\nu}$ and is defined as follows:

$$
(\phi \star \psi)(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu \partial_\nu \phi(x + \eta) \psi(y + \xi)}|_{\eta, \xi \to 0} = \phi(x) e^{\frac{i}{2} b_\mu \theta^{\mu\nu} \partial_\nu \psi(x)}.
$$

(1.3)

(The $\star$-product has also an alternative integral formulation, making its non-local character more transparent.) The coordinate-operator commutation relation (1.1) is then realized by the star($\star$)-commutator of the usual coordinates

$$
[x^\mu, \hat{x}^\nu] = [x^\mu \star x^\nu] = i\theta^{\mu\nu},
$$

(1.4)

implying the following spacetime uncertainty relations

$$
\Delta x^\mu \Delta x^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|.
$$

(1.5)

The above procedure introduces in general the field operators ordering ambiguities and breaks ordinary gauge invariance.

Since commutative local gauge transformations for the D-brane effective action do not commute with $\star$-products, it is important to note that the introduction of $\star$-products induces field operator ordering ambiguities and also breaks ordinary gauge invariance in the naive sense. However both the commutative gauge symmetry and the deformed noncommutative gauge symmetry describe the same physical system, therefore they are expected to be equivalent. This disagreement is remedied by a set of nonlocal and highly nonlinear parameter redefinitions called Seiberg-Witten (SW) map [3]. This map promotes not only the noncommutative fields and composite operators of the commutative fields, but also the noncommutative gauge transformations as the composite operators of the commutative gauge fields and gauge transformations. Through this procedure, deformed gauge field theories can be defined for arbitrary gauge groups/representations. Consequently, building semi-realistic deformed particle physics models are made much easier.

It is reasonable to expect that the new underlying mathematical structures in the NC gauge field theories (NCGFT) could lead to profound observable consequences for the low energy physics. This is realized by the perturbative loop computation first proposed by Filk [7]. There are famous examples of running of the coupling constant in the U(1) NCGFT in the $\star$-product formalism [8], and the exhibition of fascinating dynamics due to the celebrated ultraviolet/infrared (UV/IR) phenomenon, without [27, 33], and with the Seiberg-Witten map (SW) [36, 37, 38, 39, 40] included. Precisely, in [34, 35] it was shown for the first time how UV short distance effects, considered to be irrelevant, could alter
the IR dynamics, thus becoming known as the UV/IR mixing. Some significant progress on UV/IR mixing and related issues has been achieved [11, 12, 13, 14, 15] while a proper understanding of loop corrections is still sought for.

More serious efforts on formulating NCQFT models with potential phenomenological influence have started for about a decade ago. Strong boost came from the Seiberg-Witten (SW) map [3] based enveloping algebra approach, which enables a direct deformation of comprehensive phenomenological models like the standard model or GUTs [8, 9, 10]. It appeared then relevant to study ordinary gauge theories with the additional couplings inspired by the SW map/deformation included [8, 9, 10].

To include a reasonably relevant part of all SW map inspired couplings, one usually calls for an expansion and cut-off procedure, that is, to expansion of the action in powers of θμν [8, 9, 10, 11, 12, 13]. Next follows theoretical studies of one loop quantum properties [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25], as well as studies of some new physical phenomena, like breaking of Landau-Yang theorem, [26, 27, 28, 29, 31, 32], etc. It was also observed that allowing a deformation-freedom via varying the ratio between individual gauge invariant terms could improve the renormalizability at one loop level [19, 20].

The studies on phenomenology (possible experimental signal/bounds on noncommutative background) started parallel to the pure theoretical developments of NCQFTs. The majority of the accelerator processes had been surveyed up to the second power in θμν [28, 29, 30, 31]. The processes involving photons in noncommutative U(1) gauge theory now involve corrections to the known processes, since the new couplings, of which the most distinctive being the various photon self couplings, now emerge in the noncommutative background even at tree level. Such couplings might give rise to novel processes (normally forbidden in the standard theory) or to provide new channels in the already known processes.

The formulation of the SW-mapped actions is then recently made exact with respect to the noncommutative parameter θμν, offering thus an opportunity to compute various processes across wider energy scales [16, 17]. Accordingly, one should no longer rely on the expansion in powers in θ, which could be especially beneficial in case when the quantum gravity scale is not so tantalizing close to the Planck scale. Thus, in this and several prior work(s) we formulate the θ-exact action-model employing formal powers of fields [16, 17, 48, 49, 50, 51], aiming, at the same time, at keeping the nonlocal nature of the modified theory. Introduction of a nonstandard momentum dependent quantity of the type sin²(θpk/2k²)/sin²(θpk/2) into the loop integrals makes these theories drastically different from their θ-expanded cousins, being thus interesting for pure field theoretical reasons. The deformation-freedom parameters (ratios) are found to be compatible with the θ-exact action therefore included to study their possible effects on divergence cancelation(s).

In this review we present a closed forms for fermion-loop and photon-loop corrections to the photon and the neutrino self-energies using dimensional regularization technique and we combine parameterizations of Schwinger, Feynman, and modified heavy quark effective theory parameterization (HQET) [52]. Both two-point functions were obtained as a function of unspecified number of the integration dimensions D. Next we specify gauge field theory dimension d, and discuss the limits D → d = 4.
The paper is structured as follows: In the following section we describe generalized deformation freedom induced actions, and we give the relevant Feynman rules. Sections 3 and 4 are devoted to the computation of photon and neutrino two-point functions containing the fermion and the photon loop. Sections 5 and 6 are devoted to discussion and conclusions, respectively.

2. The model construction

The main principle that we are implementing in the construction of our \( \theta \)-exact noncommutative model is that electrically neutral matter fields will be promoted via hybrid SW map deformations \[53\] to noncommutative fields that couple to photons and transform in the adjoint representation of \( U_*(1) \). We consider a \( U(1) \) gauge theory with a neutral fermion which decouples from the gauge boson in the commutative limit. We specify the action and deformation as a minimal \( \theta \)-exact completion of the prior first order in \( \theta \) models \[8, 9, 19, 26, 27\], i.e. the new (inter-)action has the prior tri-particle vertices as the leading order.

In the tree-level neutrino-photon coupling processes only vertices of the form \( \bar{\psi}\psi \) contribute, therefore an expansion to lowest nontrivial order in \( \mu \) (but all orders in \( \theta \)) is enough. There are at least three known methods for \( \theta \)-exact computations: The closed formula derived using deformation quantization based on Kontsevich formality maps \[48\], the relationship between open Wilson lines in the commutative and noncommutative picture \[36, 49\], and direct recursive computations using consistency conditions. For the lowest nontrivial order a direct deduction from the recursion and consistency relations

\[
\delta\Lambda A_\mu \equiv \imath[\Lambda \hat{A}_\mu] = A_\mu[\hat{a}_\mu + \delta_\lambda a_\mu] - A_\mu[a_\mu] + O(\lambda^2),
\]

\[
\delta\Lambda \Psi \equiv \imath[\Lambda \hat{\Psi}] = \Psi[a_\mu + \delta_\lambda \hat{a}_\mu, \hat{\psi} + \delta_\lambda \psi] - \Psi[a_\mu, \psi] + O(\lambda^2),
\]

\[
\Lambda[[\lambda_1, \lambda_2], a_\mu] = [\Lambda[\lambda_1, a_\mu] \hat{\Lambda}[\lambda_2, a_\mu]] + i\delta_\lambda_1 \Lambda[\lambda_2, a_\mu] - i\delta_\lambda_2 \Lambda[\lambda_1, a_\mu],
\]

with the ansatz

\[
\Lambda = \hat{\Lambda}[a_\mu] \lambda = (1 + \hat{\Lambda}^1[a_\mu] + \hat{\Lambda}^2[a_\mu] + O(a^3))\lambda,
\]

\[
\Psi = \hat{\Psi}[a_\mu] \psi = (1 + \hat{\Psi}^1[a_\mu] + \hat{\Psi}^2[a_\mu] + O(a^3))\psi,
\]

is already sufficient. Here \( \hat{\Psi}[a_\mu] \) and \( \hat{\Lambda}[a_\mu] \) are gauge-field dependent differential operators that we shall now determine: Starting with the fermion field \( \Psi \), at lowest order we have

\[
\imath[\Lambda \hat{\Psi}] = \hat{\Psi} [\partial \lambda] \psi.
\]

Writing the star commutator explicitly as

\[
[\phi \hat{\psi}] = \phi(x)(e^{\frac{i\partial_\mu \theta_\mu}{2}} - e^{-rac{i\partial_\mu \theta_\mu}{2}})\psi(y) \Big|_{x=y} = 2i\phi(x)\sin\left(\frac{\partial_\mu \theta_\mu}{2}\right)\psi(y) \Big|_{x=y}
\]

\[
= i\theta^{ij} \left( \frac{\partial \phi(x)}{\partial x^i} \right) \sin\left(\frac{\partial_\mu \theta_\mu}{2}\right) \left( \frac{\partial \psi(y)}{\partial y^j} \right) \Big|_{x=y},
\]

\[
\text{Notation: Capital letters denote noncommutative objects, small letters denote commutative objects, hatted capital letters denote differential operator maps from the latter to the former.}
we observe that
\[ \hat{\Psi}[a_\mu] = -\theta^{ij} a_i \star_2 \partial_j \] (2.8)
will fulfill the consistency relation.

The gauge transformation \( \Lambda \) can be worked out similarly, namely
\[ 0 = [\lambda_1 \star \lambda_2] + i\hat{\Lambda}[\partial\lambda_1]\lambda_2 - i\hat{\Lambda}[\partial\lambda_2]\lambda_1 \]
\[ = \frac{1}{2}([\lambda_1 \star \lambda_2] - [\lambda_2 \star \lambda_1]) + i\hat{\Lambda}[\partial\lambda_1]\lambda_2 - i\hat{\Lambda}[\partial\lambda_2]\lambda_1, \tag{2.9} \]
and hence
\[ \hat{\Lambda}^1 = -\frac{1}{2}\theta^{ij} a_i \star_2 \partial_j. \tag{2.10} \]
The gauge field \( a_\mu \) requires slightly more work. The lowest order terms in its consistency relation are
\[ -\partial_\mu(\frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda) - i[\lambda \star a_\mu] = A_\mu^2[a_\mu + \partial_\mu \lambda] - A_\mu^2[a_\mu], \tag{2.11} \]
where \( A^2 \) is the \( a^2 \) order term in the expansion of \( A \) as power series of \( a \). The left hand side can be rewritten as \[-\frac{1}{2}\theta^{ij} \partial_\mu a_i \star_2 \partial_j \lambda - \frac{1}{2}\theta^{ij} \partial_\mu a_i \star_2 \partial_j \lambda - \theta^{ij} \partial_\mu \lambda \star_2 \partial_j a_\mu, \]
where the first term comes from \[-\frac{1}{2}\theta^{ij} \partial_\mu a_i \star_2 \partial_j a_\mu, \]
while the third one comes from \[-\theta^{ij} \partial_\mu a_i \star_2 \partial_j a_\mu. \]
After a gauge transformation, the sum of the first and third terms equals the second term. Ultimately, we obtain SW map solutions up to the \( O(a^2) \) order:
\[ A_\mu = a_\mu - \frac{1}{2}\theta^{\mu\rho} a_\rho \star_2 (\partial_\rho a_\mu + f_{\rho\mu}) + O(a^3), \]
\[ \Psi = \psi - \theta^{\mu\nu} a_\mu \star_2 \partial_\nu \psi + O(a^2)\psi \]
\[ \Lambda = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda + O(a^2)\lambda, \tag{2.12} \]
with \( f_{\mu\nu} \) being the abelian commutative field strength \( f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \).

The generalized star-product \( \star_2 \), appearing in (2.12), is defined, respectively, as \cite{36,37,38}:
\[ \phi(x) \star_2 \psi(x) = \sin \frac{\partial_1 \theta \partial_2 \phi(x_1)\psi(x_2)}{2 \partial_1 \partial_2} \bigg|_{x_1 = x_2 = x}, \tag{2.13} \]
where \( \star \)-product (1.3) is associative but noncommutative, while \( \star_2 \) is commutative but nonassociative. The resulting expansion defines in the next section the one-photon-two-fermion and the three-photon vertices, \( \theta \)-exactly.

2.1 Actions
We start with the following minimal NC model of a SW type \( U(1) \) gauge theory on Euclidean spacetime:
\[ S = \int \left( -\frac{1}{2} F^{\mu\nu} \star F_{\mu\nu} + i\bar{\Psi} \star \slashed{D}\Psi \right), \] (2.14)
with definitions of the non-Abelian NC covariant derivative and the field strength, respectively:
\[ D_\mu \Psi = \partial_\mu \Psi - i[A_\mu \star \Psi] \quad \text{and} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \star A_\nu]. \tag{2.15} \]
All the fields in this action are images under (hybrid) Seiberg-Witten maps of the corresponding commutative fields \(a_\mu\) and \(\psi\). In the original work of Seiberg and Witten and in virtually all subsequent applications, these maps are understood as (formal) series in powers of the noncommutativity parameter \(\theta^{\mu\nu}\). Physically, this corresponds to an expansion in momenta and is valid only for low energy phenomena. Here we shall not subscribe to this point of view and instead interpret the noncommutative fields as valued in the enveloping algebra of the underlying gauge group. This naturally corresponds to an expansion in powers of the gauge field \(a_\mu\) and hence in powers of the coupling constant \(e\). At each order in \(a_\mu\) we shall determine \(\theta\)-exact expressions. In the following we discuss the model construction for the photon and the massless fermion case. In the following we discuss the model construction for the massless case, and set \(e = 1\). To restore the coupling constant one simply substitutes \(a_\mu\) by \(ea_\mu\) and then divides the gauge-field term in the Lagrangian by \(e^2\). Coupling constant \(e\), carries (mass) dimension \((4 - d)/2\) in \(d\) dimensions.

The expansion in powers of the commutative (gauge) field content is motivated from the obvious fact that in perturbative quantum field theory one can sort the interaction vertices by the number of external legs and this is equivalent to the number of field operators in the corresponding interacting terms. For any specific process and loop order there exists an upper limit on the number of external legs. So if one expands the noncommutative fields with respect to the formal power of the commutative fields which are the primary fields in the theory up to an appropriate order, the relevant vertices in a specific diagram will automatically be exact to all orders of \(\theta\).

The minimal gauge invariant nonlocal interaction (2.14) includes the gauge boson self-coupling as well as the fermion-gauge boson coupling, denoted here as \(S_g\) and \(S_f\), respectively:

\[
S = S_{U(1)} + S_g + S_f. \tag{2.16}
\]

In the next step we expand the action in terms of the commutative gauge parameter \(\lambda\) and fields \(a_\mu\) and \(\psi\) using the U(1) SW map solutions (2.12). This way, the photon self-interaction up to the lowest nontrivial order are obtained:

\[
S_g = \int i \tilde{f}^{\mu\nu} \star [a_\mu \star a_\nu] + \partial_\mu (\theta^{\rho\sigma} a_\rho \star (\partial_\sigma a_\nu + f_{\sigma\nu})) \star f^{\mu\nu} + O(a^4)
= \int \theta^{\rho\tau} f^{\mu\nu} \left( \frac{1}{4} f_{\rho\tau} \star f_{\mu\nu} - f_{\mu\rho} \star f_{\nu\tau} \right) + O(a^4), \tag{2.17}
\]

with \(f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu\) being Abelian commutative U(1) field strengths. The the lowest order photon-fermion interaction reads as follows:

\[
S_f = \int \bar{\psi} \gamma^\mu [a_\mu \star \psi] + i (\theta^{ij} \partial_i \bar{\psi} \star a_j) \phi \psi - i \bar{\psi} \star \bar{\phi} (\theta^{ij} a_i \star \partial_j \psi) + \bar{\psi} \phi O(a^2) \psi
= - \int i \theta^{\rho\tau} \bar{\psi} \gamma^\mu \left( \frac{1}{2} f_{\rho\tau} \star \partial_\mu \psi - f_{\mu\rho} \star \partial_\tau \psi \right) + \bar{\psi} O(a^2) \psi. \tag{2.18}
\]

Note that actions for gauge and matter fields obtained above, (2.17) and (2.18) respectively, are nonlocal objects due to the presence of the (generalized) star products.
2.2 General deformed actions: $S_f$ and $S_g$

It is easy to see that each of the interaction (2.17) and (2.18) contains two U(1) gauge
invariant terms, therefore one could vary the ratio between them without spoiling the gauge
invariance. For this propose we introduce two further deformation parameters $\{\kappa_f, \kappa_g\}$.

The deformation parameter $\kappa_f$ in the photon-gauge boson interaction can be so chosen
that it realizes the linear superposition of two possible nontrivial noncommutative defor-
mations of a free neutral fermion action proposed in [39, 47]. Its existence was already
hinted in the $\theta$-expanded expressions in [26] but not fully exploited in the corresponding
loop computation before.

The pure gauge action $S_g$ deformation $\kappa_g$ was first presented in the non-Abelian gauge
sector action of the NCSM and NC SU(N) at first order in $\theta$, $S_g^{\theta}$ [19, 20], which could also
be realized by generalizing the standard SW map expression for gauge field strength into
the equivalent form [54]:

$$F_{\mu\nu}(\kappa_g) = f_{\mu\nu} + \theta^{\rho\tau} \left(f_{\mu\rho} f_{\nu\tau} - \kappa_g a_{\rho} \partial_{\tau} f_{\mu\nu}\right) + O(a^3).$$

(2.19)

We have observed in prior studies [39, 55, 56] that the above deformation can be made
$\theta$-exact and adopted it here:

$$F_{\mu\nu}(\kappa_g) = f_{\mu\nu} + \theta^{\rho\tau} \left(f_{\mu\rho} \star_2 f_{\nu\tau} - \kappa_g a_{\rho} \star_2 \partial_{\tau} f_{\mu\nu}\right) + O(a^3).$$

(2.20)

Thus, starting with (2.14), (2.20) and (2.18) we can finally write the generalized man-
ifestly gauge invariant actions:

$$S_{U(1)} = \int \left(-\frac{1}{2} f_{\mu\nu} f^{\mu\nu} + i \bar{\psi} \gamma^\mu \psi\right),$$

(2.21)

$$S_{g}(\kappa_g) = \int \theta^{\rho\tau} f^{\mu\nu} \left(\frac{\kappa_g}{4} f_{\rho\tau} \star_2 f_{\mu\nu} - f_{\mu\rho} \star_2 f_{\nu\tau}\right),$$

(2.22)

$$S_{f}(\kappa_f) = -\int i \theta^{\rho\tau} \bar{\psi} \gamma^\mu \left(\frac{1}{2} f_{\rho\tau} \star_2 \partial_{\mu} \psi - \kappa_f f_{\mu\rho} \star_2 \partial_{\tau} \psi\right).$$

(2.23)

Since $S_{g}(\kappa_g)$ and $S_{f}(\kappa_f)$ are both gauge invariant by themselves, one can incorporate
either or both of them into the full Lagrangian. The above action were obtained by a $\theta$-
exact gauge-invariant truncation of a $U_s(1)$ model up to tri-leg vertices. Such operation is
achievable because the $U(1)$ gauge transformation after deformation preserves the number
of fields within each term.

Motivation to introduce deformation parameters $\kappa_g$ and $\kappa_f$ was, besides the general
gauge invariance of the action, to help eliminating one-loop pathologies due to the UV
and/or IR divergences in both sectors. The parameter-space $(\kappa_f, \kappa_g)$ represents a measure
of the deformation-freedom in the matter $S_{f}(\kappa_f)$ and the gauge $S_{g}(\kappa_g)$ sectors, respectively.
We should clarify, that we are interested in the general gauge invariant interactions induced
by the $\theta^{\mu\nu}$ background instead of the strictly Moyal-Weyl star-product deformation of the
commutative gauge theories and its Seiberg-Witten map extension. We loose the constraint
that a deformation should be Moyal-Weyl type for the hope that such variation could
provide certain additional control on the novel pathologies due to the noncommutativity,
which had indeed occurred in the $\theta$-expanded models studied before, and as we will discuss later, in our $\theta$-exact model as well. We still constraint our model building by requiring that each of the gauge invariant interaction terms arises within a Seiberg-Witten map type deformation, only their linear combination ratios $\kappa_g$ and $\kappa_f$ are allowed to vary. Each parameter bears the origin from the corresponding $\theta$-expanded theory too.

By straightforward reading-out procedure from $S_g$ (2.22) we obtain the following Feynman rule for the triple-photon vertex in momentum space:

$$\Gamma^\mu_{\kappa_g}(p, k, q) = F(k, q) V^\mu_{\kappa_g}(p, k, q); \quad F(k, q) = \sin \frac{k\theta q}{k\theta q}, \quad (2.24)$$

with momenta $p, k, q$ are taken to be incoming satisfying the momentum conservation $p + k + q = 0$. The deformation freedom ambiguity $\kappa_g$ is included in the vertex function:

$$V^\mu_{\kappa_g}(p, k, q) = -(p\theta k) \left[ (p - k)^\rho g^{\mu\rho} + (k - q)^\mu g^{\nu\rho} + (q - p)^\nu g^{\mu\rho} \right]$$

$$- \theta^{\mu\nu} \left[ p^\rho (kq) - k^\rho (pq) \right] - \theta^{\nu\rho} \left[ k^\mu (pq) - q^\mu (pk) \right] - \theta^{\rho\mu} \left[ q^\nu (pk) - p^\nu (kq) \right]$$

$$+ (\theta p)^\nu \left[ g^{\mu\nu} q^2 - q^\nu q^\rho \right] + (\theta q)^\nu \left[ g^{\mu\nu} k^2 - k^\mu k^\nu \right] + (\theta k)^\nu \left[ g^{\mu\nu} q^2 - q^\nu q^\rho \right]$$

$$+ (\kappa_g - 1)(\theta p)^\mu \left[ g^{\nu\rho} (kq) - q^\nu k^\rho \right]$$

$$+ (\kappa_g - 1)(\theta k)^\nu \left[ g^{\mu\rho} (qp) - q^\rho k^\nu \right]$$

$$+ (\kappa_g - 1)(\theta q)^\rho \left[ g^{\mu\nu} (kp) - k^\mu p^\nu \right]. \quad (2.25)$$

The above vertex function (2.25) is in accord with corresponding Feynman rule for triple neutral gauge-boson coupling in [32].

From $S_f$ (2.23) the fermion-photon vertex reads as follows:

$$\Gamma^\mu_{\kappa_f}(k, q) = F(k, q) V^\mu_{\kappa_f}(k, q) = F(k, q) \left[ \kappa_f \left( \hat{k}(\theta q)^\mu - \gamma^\mu (k\theta q) \right) - (\theta k)^\mu \hat{\gamma} \right], \quad (2.26)$$

where $k$ is the photon incoming momenta, and the fermion momentum $q$ flows through the vertex, as it should. The above Feynman rules (2.24-2.26) are given with slightly different notations, with respect to previous ones from subsections 2.2 and 2.3, to become more transparent. However they do contain the deformation parameters $\kappa_g$ and $\kappa_f$, as they should.

### 3. Photon two-point function

#### 3.1 Computing photon two-point function using dimensional regularization

Employing the parametrization given in this section we illustrate the way we have performed the computation of the integrals which differ from regular ones by the existence of
a non-quadratic $kθp$ denominators. The key point was to introduce the HQET parametrization [52], represented as follows

\[
\frac{1}{a_1^{n_1}a_2^{n_2}} = \frac{\Gamma(n_1 + n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \frac{i^{n_1}y^{n_1-1}dy}{(ia_1y + a_2)^{n_1+n_2}}. \tag{3.1}
\]

To perform computations of our integrals, we first use the Feynman parametrization on the quadratic denominators, then the HQET parametrization help us to combine the quadratic and linear denominators. For example

\[
\frac{1}{k^2(p+k)^2} \frac{1}{kθp} = 2i \int_0^1 dx \int_0^\infty dy \left[ (k^2 + iε)(1 - x) + ((p + k)^2 + iε)x + iy(kθp) \right]^{-3}. \tag{3.2}
\]

After employing the Schwinger parametrization, the phase factors from (2.26) can be absorbed by redefining the $y$ integral. This way we obtain

\[
\frac{2 - e^{ikθp} - e^{-ikθp}}{k^2(p+k)^2(kθp)} \cdot \{\text{numerator}\} = 2i \int_0^1 dx \int_0^\infty dy \left[ dλλ2e^{-λ(\theta^2x(1-x)p^2 + \frac{4}{x^2}(θp)^2)} \right] \cdot \{y \text{ odd terms of the numerator}\}, \tag{3.3}
\]

with loop-momenta being $l = k + xp + \frac{i}{2}y(θp)$. By this means the $y$-integral limits take the places of planar/nonplanar parts of the loop integral. For higher negative power(s) of $kθp$, the parametrization follows the same way except the appearance of the additional $y$-integrals which lead to finite hypergeometric functions [57]. Following [53], we are enabled to follow the general procedure of dimensional regularization in computing one-loop two point functions. We first present the results with respect to general integration dimension $D$, then in the next sections we will discuss the behavior in different $D \to d = 4$ limits.

**Photon two-point function: Fermion-loop**

The fermion-loop contribution is read out from Fig.[1]

\[
Π_{κ_f}(p)_D = -\text{tr} \; Π_d \{ \frac{d^Dk}{(2π)^D} Γ_{κ_f}(−p, p + k) \frac{i(\hat{p} + \hat{k})}{(p + k)^2} Γ_{κ_f}(p, k) \frac{i\hat{k}}{k^2} \}, \tag{3.4}
\]

where the momentum structure and dependence on the parameter κf is encoded in

\[
\text{tr} \; V_{κ_f}^{\mu}(−p, p + k)(\hat{p} + \hat{k})V_{κ_f}^{ν}(p, k)\hat{k}. \tag{3.5}
\]

After considerable amount of computations we have found the following structure:

\[
Π_{κ_f}^{μν}(p)_D = \frac{1}{(4π)^2} \left[ \left( g^{μν}p^2 - p^μp^ν \right) F_1^{κ_f}(p) + (θp)^μ(θp)^ν F_2^{κ_f}(p) \right]. \tag{3.6}
\]
Figure 1: Fermion-loop contribution to the photon two-point function

with the loop-coefficients $F_{i}^{κ_f}(p)$

$$F_{1}^{κ_f}(p) = -4\text{Dim}(Cl[[d]])(4\pi)^{2-D} \mu^{d-D} κ_f^2 \left[ \Gamma \left( 2 - \frac{D}{2} \right) \frac{\Gamma(D)}{\Gamma(D)} \frac{1}{(θp)^2} \int_{0}^{1} dx x(1 - x) \frac{D}{4} K_{\frac{D}{2}}(X) \right],$$

(3.7)

$$F_{2}^{κ_f}(p) = \text{Dim}(Cl[[d]])(4\pi)^{2-D} \mu^{d-D} κ_f \left[ (κ_f - 1) \left( \frac{4}{(θp)^2} \right) \frac{D}{2} \Gamma \left( \frac{D}{2} \right) \frac{2}{D - 1} - κ_f \frac{D}{2} \left( \frac{θp)^2}{p^2} \right) \frac{D}{4} \int_{0}^{1} dx x(1 - x) \frac{D}{4} K_{\frac{D}{2}}(X) \right],$$

(3.8)

where $\text{Dim}(Cl[[d]])$ is the dimension of Clifford algebra and $X$ is a new dimensionless variable,

$$X = \sqrt{x(1 - x)p^2(θp)^2}. \quad (3.9)$$

The single finite term in (3.8), presenting an additional correction from the SW map induced deformation, vanishes only for $κ_f = 1$. All of the divergences arising from the fermion-loop (Fig.1) could be removed by the choice $κ_f = 0$, as in that case the whole general amplitude (3.6) vanishes for any integration dimensions $D$.

It is important to stress that there is an additional tadpole diagram contribution to the photon 2-point function, arising from 2-photon-2-fermion $(\bar{\psi}a^2\psi)$ interaction vertices [39, 47]. However, it was shown in [37], that this tadpole diagram vanishes due to the internal Lorentz structure.

It is straightforwardly to see that the tensor structure (3.6) does satisfy the Ward (Slavnov-Taylor) identity by itself, therefore $p_μΠ_{κ_f}^{μν}(p)_D = p_νΠ_{κ_f}^{μν}(p)_D = 0.$
Figure 2: Photon-loop contribution to the photon two-point function

Photon two-point function: Photon-loop

The photon-loop computation involves a single photon-loop integral contribution to photon self-energy from Fig. 2 in $D$ dimensions

$$\Pi_{\kappa_g}^{\mu\nu}(p) = \frac{1}{2} \mu^{d-D} \int \frac{d^D k}{(2\pi)^D} \Gamma_{\kappa_g}^{\mu\rho\sigma}(-p; -k, p + k) \frac{-ig_{\rho'\sigma'}}{k^2} \Gamma_{\kappa_g}^{\rho'\sigma'}(p; k, -k - p) \frac{-ig_{\sigma\sigma'}}{(p + k)^2}$$

$$= -\frac{1}{2} \mu^{d-D} \int \frac{d^D k}{(2\pi)^D} \frac{F^2(p, k)}{k^2(p + k)^2} V_{\kappa_g}^{\mu\rho\sigma}(-p; -k, p + k) (V_{\kappa_g})_{\rho\sigma}(p; k, -k - p),$$

(3.10)

and as a function of deformation freedom $\kappa_g$ ambiguity correction. The initial task is to evaluate contractions $V_{\kappa_g}^{\mu\rho\sigma}(-p; -k, p + k) (V_{\kappa_g})_{\rho\sigma}(p; k, -k - p)$.

After a lengthy computation we obtained the following compact form of the photon-loop contribution to the photon two-point function in D-dimensions,

$$\Pi_{\kappa_g}^{\mu\nu}(p) = \frac{1}{4\pi^2} \left\{ g^{\mu\nu} p^2 - p^\mu p^\nu \right\} B_1^{\kappa_g}(p) + (\theta p)^\mu (\theta p)^\nu B_2^{\kappa_g}(p)$$

$$+ \left[ g^{\mu\nu} (\theta p)^2 - (\theta \theta)^{\mu\nu} p^2 + p^{\mu}(\theta \theta p)^{\nu} \right] B_3^{\kappa_g}(p)$$

$$+ \left[ (\theta \theta)^{\mu\nu} (\theta p)^2 + (\theta \theta p)^{\mu}(\theta \theta p)^{\nu} \right] B_4^{\kappa_g}(p) + (\theta p)^{\mu}(\theta \theta p)^{\nu} B_5^{\kappa_g}(p) \right\},$$

(3.11)

Clearly the above structure is much more richer with respect to earlier $\theta$-exact without SW map results \[60, 62\]. Each of $B_i^{\kappa_g}$ the momentum structures satisfies Ward identities by itself, i.e. $p_\mu \Pi_{\kappa_g}^{\mu\nu}(p)_D = p_\nu \Pi_{\kappa_g}^{\mu\nu}(p)_D = 0$.

All coefficients $B_i^{\kappa_g}$ can be expressed as sum over integrals over modified Bessel and generalized hypergeometric functions. A complete list of coefficients $F_i^{\kappa_g}(p)$ and $B_i^{\kappa_g}(p)$ as a functions of dimension $D$ is given next.

3.2 Loop integral coefficients $F_i^{\kappa_g}$ and $B_i^{\kappa_g}$

Employing the aforementioned parametrization we observe that all loop integrals we have computed can be expressed using two series of integrals in addition to the usual planar
dimensional regularization formulas. These integrals, denoted as $K$ and $W$, are defined as follows:

\[
K[\nu; a, b] = 2^{-\nu}(\theta p)^\nu \int_0^1 dx x^a (1-x)^b X^{-\nu} K_\nu[X],
\]

\[
W[\nu; a, b] = \int_0^1 dx x^a (1-x)^b W_\nu[X],
\]

where $K_\nu[X]$ is the modified Bessel function of second kind, while

\[
W_\nu[X] = (\theta p)^{-2\nu} \left( X^{2\nu} \Gamma [-\nu] \left[ 1_F \frac{1}{2}, \nu+1 ; \frac{X^2}{4} \right] - \frac{2^{2\nu}}{1-2\nu} \Gamma [\nu] \left[ \frac{1-2\nu}{2} ; 1-\nu, \frac{3-2\nu}{2} ; \frac{X^2}{4} \right] \right).
\]

The variable $X$ is defined in (3.9).

Loop coefficients $F_{\kappa}^{\kappa_f}(p)$ involves integral $K$'s only:

\[
F_1^{\kappa_f}(p) = -4\text{Dim}(Cl[[d]])(4\pi)^2 \mu^{d-D} \kappa_f^2 \left( \Gamma \left( 2 - \frac{D}{2} \right) \frac{\Gamma(D)}{\Gamma(D)} \left( p^2 \right)^{\frac{D}{2}-2} - 2K \left[ \frac{D}{2} - 2; 1, 1 \right] \right),
\]

\[
F_2^{\kappa_f}(p) = \text{Dim}(Cl[[d]])(4\pi)^2 \mu^{d-D} \kappa_f \left( ( \kappa_f - 1 ) \left( \frac{4}{(\theta p)^2} \right)^{\frac{D}{2}} \frac{2\Gamma(D)}{D-1} - 2\kappa_f K \left[ \frac{D}{2} ; 0, 0 \right] \right),
\]

while the loop coefficients $B_{\kappa}^{\kappa_f}(p)$'s contain both integrals, the $K$'s and the $W$'s, respec-
\[ B_1^{\kappa_g}(p) = (4\pi)^{2-D} \mu^D \left\{ -\frac{2^{2-D} \pi^{\frac{3}{2}} \csc \frac{D\pi}{2} (p^2)^{\frac{D-2}{2}}}{\Gamma \left( \frac{D+1}{2} \right)} \right. \]

\[
\cdot \left\{ D^2(\kappa_g - 3)^2 - D \left( \kappa_g(3\kappa_g - 22) + 37 \right) \\
- 2\left( \kappa_g(\kappa_g + 2) - 11 \right) + \left( (D - 2)\kappa_g^2 + 2D\kappa_g + 3D - 4 \right)(\text{tr} \theta \theta) \frac{p^2}{(\theta p)^2} \\
+ 2\left( (D - 2)\kappa_g^2 + 2D\kappa_g + (D - 2) \right)(\theta p)^2 \frac{p^2}{(\theta p)^4} \right\} \\
- 8(\kappa_g - 2)^2 \mathcal{K} \left[ \frac{D}{2} - 2; 0, 0 \right] \\
+ 8 \left( (D - 3)^2 + 3(\kappa_g^2 - 2\kappa_g - 1) \right) \mathcal{K} \left[ \frac{D}{2} - 2; 1, 1 \right] \\
+ \left( -2\kappa_g^2 + 8\kappa_g + D - 11 \right)(\theta p)^2 \mathcal{W} \left[ \frac{D}{2} - 1; 0, 0 \right] \\
+ 2\left( 3(\kappa_g - 1)^2 + D(\kappa_g^2 - 6\kappa_g + 7) \right)(\theta p)^2 \mathcal{W} \left[ \frac{D}{2} - 1; 1, 1 \right] \\
+ (\text{tr} \theta \theta) \frac{p^2}{(\theta p)^2} \left\{ 4(\kappa_g + 2)^2 \mathcal{K} \left[ \frac{D}{2} - 2; 0, 0 \right] \\
- \frac{8(D + 1)(\kappa_g - 1)^2}{D - 1} \mathcal{K} \left[ \frac{D}{2} - 2; 1, 1 \right] + \frac{8(D + 1)(\kappa_g - 1)^2}{D - 1} p^{-2} \mathcal{K} \left[ \frac{D}{2} - 1; 0, 0 \right] \right\} \\
+ (\kappa_g^2 + 2)(\theta p)^2 \mathcal{W} \left[ \frac{D}{2} - 1; 0, 0 \right] - \frac{2(D + 1)(\kappa_g - 1)^2}{D - 1} (\theta p)^2 \mathcal{W} \left[ \frac{D}{2} - 1; 1, 1 \right] \left\{ \right\} \\
+ (\theta p)^2 \frac{p^2}{(\theta p)^4} \left\{ 8(\kappa_g^2 + 1) \mathcal{K} \left[ \frac{D}{2} - 2; 0, 0 \right] - \frac{16D(\kappa_g - 1)^2}{D - 1} \mathcal{K} \left[ \frac{D}{2} - 2; 1, 1 \right] \\
+ \frac{8D(\kappa_g - 1)^2}{D - 1} p^{-2} \mathcal{K} \left[ \frac{D}{2} - 1; 0, 0 \right] + (\kappa_g^2 + 1)(\theta p)^2 \mathcal{W} \left[ \frac{D}{2} - 1; 1, 1 \right] \right\} \bigg\}.
\]

(3.17)
\[ B_2^\kappa(p) = (4\pi)^{2-D} \mu^{D-d} p^2(\theta p)^2 \left\{ \frac{2^{2-D} \pi^{\frac{D}{2}} \csc \frac{\pi}{2} (p^2)^{\frac{D-d}{2}}}{\Gamma \left( \frac{D+1}{2} \right)} \right\} \]

\[
\cdot \left\{ -2D^3(\kappa - 1)^2 + D^2 \left( \kappa(7\kappa - 6) - 5 \right) \\
+ D \left( 19 - \kappa(9\kappa - 2) \right) + 2 \left( \kappa(\kappa + 2) - 7 \right) \\
- 2 \left( 3 - 2D + \kappa(\kappa - 2) \right) \left( \tr \theta \theta \right) \frac{p^2}{(\theta p)^2} - 8(\kappa - 1)^2(\theta \theta p)^2 \frac{p^2}{(\theta p)^4} \right\} \\
+ 8 \left( (D - 3)^2 + (12 - 4D)\kappa + 3D - 8 \right) K \left[ \frac{D}{2} - 2; 0, 0 \right] \\
+ 8 \left( -2D^3(\kappa - 1)^2 + D^2(\kappa_0^2 + 14\kappa - 17) + D(7\kappa_0^2 - 46\kappa + 37) \\
- 12\kappa_0^2 + 44\kappa - 42 \right) \frac{1}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \\
+ 4 \left( \kappa_0^2(D - 2)(D - 7) - 2\kappa_0^2(D - 2)(D - 9) + 2D^2 - 9D^2 - 7D + 38 \right) \\
\cdot \frac{1}{p^2(D - 1)} K \left[ \frac{D}{2} - 1; 0, 0 \right] \\
+ \left( 2(D + 1)\kappa_0^2 - 2(4D + 2)\kappa_0 + 8D + 13 \right)(\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] \\
- 2 \left( 2D^3(\kappa - 1)^2 + D^2(\kappa_0^2 - 22\kappa + 25) + 2D(\kappa_0^2 + 6\kappa - 2) + (\kappa - 1)^2 \right) \\
\cdot \frac{(\theta p)^2}{D - 1} W \left[ \frac{D}{2} - 1; 1, 1 \right] \\
+ (\tr \theta \theta) \frac{p^2}{(\theta p)^2} \left\{ 8K \left[ \frac{D}{2} - 2; 0, 0 \right] - \frac{8(\kappa - 1)^2}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \right\} \\
+ \frac{4D(\kappa - 1)^2}{D - 1} p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right] \\
+ (\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] - \frac{2(\kappa - 1)^2}{D - 1} (\theta \theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \right\} \\
\right. \\
+ (\theta \theta p)^2 \left\{ \frac{8(D - 4)}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \\
+ \frac{4D(D + 2)}{D - 1} p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right] - \frac{6}{D - 1} (\theta \theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \right\}, \right. \\
\left. \right. \\
(3.18)
\[ B_3^{\kappa_g}(p) = (4\pi)^2 \frac{D}{2} \mu^d D p^4 (\theta p)^{-2} \left\{ \frac{2^{2-D} \pi^3/2 \csc \frac{D\pi}{2} (p^2)^{D-2}}{\Gamma \left( \frac{D+1}{2} \right)} \right\} \]

\[ \times \left( 1 - 3\kappa_g (\kappa_g - 2) + D(\kappa_g + 1)(\kappa_g - 3) \right) \]

\[- 8K \left[ \frac{D}{2} - 2; 0, 0 \right] - \frac{8(D - 3)\kappa_g^2 - 2(D - 5)\kappa_g - D - 3}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \]

\[ + 4 \frac{(D - 3)\kappa_g^2 + (14 - 6D)\kappa_g + 2D^2 - 3D - 3}{D - 1} p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right] \]

\[- (4\kappa_g - D + 2)(\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] \]

\[ + 4 \left( \frac{\kappa_g^2 - 2(D + 1)\kappa_g + D^2 + 1}{D - 1} \right) \left( \frac{(\theta p)^2}{D - 1} \right) W \left[ \frac{D}{2} - 1; 1, 1 \right] \}

\[ B_4^{\kappa_g}(p) = (4\pi)^2 \frac{D}{2} \mu^d D p^4 (\theta p)^{-4} \left\{ \frac{2^{3-D} \pi^3/2 \csc \frac{D\pi}{2} (p^2)^{D-2}}{\Gamma \left( \frac{D+1}{2} \right)} (D - 1)(\kappa_g + 1)^2 \right\} \]

\[ + \left( 8(1 - D) + 32\kappa_g \right) K \left[ \frac{D}{2} - 2; 0, 0 \right] \]

\[- \frac{8(2D - 1)\kappa_g^2 - 6\kappa_g + 2D - 1}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \]

\[ + 4D(\kappa_g^2 - 6\kappa_g + 2D - 1) \frac{p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right]}{D - 1} \]

\[- (4\kappa_g + 1 - D)(\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] \]

\[ - \frac{2D(\kappa_g^2 - 6\kappa_g + 2D - 1)}{D - 1} (\theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \}

\[ B_5^{\kappa_g}(p) = (4\pi)^2 \frac{D}{2} \mu^d D p^4 (\theta p)^{-4} \left\{ \frac{2^{4-D} \pi^4/2 \csc \frac{D\pi}{2} (p^2)^{D-2}}{\Gamma \left( \frac{D+1}{2} \right)} \left( \kappa_g + (D - 3)\kappa_g + D \right) \right\} \]

\[ + 16\kappa_g K \left[ \frac{D}{2} - 2; 0, 0 \right] + \frac{16(2D - \kappa_g)(\kappa_g - 1)}{D - 1} K \left[ \frac{D}{2} - 2; 1, 1 \right] \]

\[ + \frac{8D(\kappa_g - 1)(\kappa_g - 2)}{D - 1} p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right] + 2\kappa_g(\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] \]

\[ + \frac{(D + 1 - \kappa_g)(\kappa - 1)}{D - 1} (\theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \}

3.3 Photon two-point function in four dimensions

**Fermion-loop:** In the limit \( D \to 4 - \epsilon \), the loop-coefficients can be expressed in the
following closed forms:

\[
F_1^{\kappa_f}(p) = -\kappa_f^2 \frac{8}{3} \left[ \frac{2}{\epsilon} + \ln \pi e^{\gamma} + \ln \left( \mu^2 (\theta p)^2 \right) \right] \\
+ 4\kappa_f^2 p^2 (\theta p)^2 \sum_{k=0}^{\infty} \frac{(k+2)(p^2 (\theta p)^2)^k}{4^k \Gamma [2k+6]} \\
\cdot \left[ (k+2) \left( \ln (p^2 (\theta p)^2) - \psi(2k+6) - \ln 4 \right) + 2 \right], \\
\]

\[
F_2^{\kappa_f}(p) = \kappa_f^2 \frac{8}{3} \left[ \kappa_f - 8(\kappa_f + 2) \frac{1}{p^2 (\theta p)^2} \right] - 4\kappa_f^2 p^4 \sum_{k=0}^{\infty} \frac{(p^2 (\theta p)^2)^k}{4^k \Gamma [2k+6]} \\
\cdot \left[ (k+1)(k+2) \left( \ln (p^2 (\theta p)^2) - 2\psi(2k+6) - \ln 4 \right) + 2k+3 \right]. \\
\]

The above expressions for \( F_{1,2}^{\kappa_f}(p) \) contain both contributions, from the planar as well as from the non-planar graphs.

**Photon-loop:** Using each \( B_i^{\kappa_g}(p) \) from (3.11) and (3.17-3.21) in the \( D \to 4 - \epsilon \) limit, we found expressions similar to the fermion-loop. Next we concentrate on the divergent parts in the IR regime

\[
B_1^{\kappa_g}(p) \sim \left( \frac{2}{3} (\kappa_g - 3)^2 + \frac{2}{3} (\kappa_g + 2)^2 \frac{p^2 (\theta p)^2}{(\theta p)^2} \right) - \frac{2}{3} \left( \kappa_g - 1 \right)^2 \frac{1}{(\theta p)^4} \left( \ln (\mu^2 (\theta p)^2) \right), \\
\]

\[
B_2^{\kappa_g}(p) \sim \left( \frac{8}{3} (\kappa_g - 1)^2 \frac{p^4 (\theta p)^2}{(\theta p)^6} + \frac{2}{3} (\kappa_g^2 - 2\kappa_g - 5) \frac{p^4 (\theta p)^2}{(\theta p)^4} + \frac{2}{3} \left( 25\kappa_g^2 - 86\kappa_g + 73 \right) \frac{p^2 (\theta p)^2}{(\theta p)^2} \right) - \frac{16}{3} (\kappa_g - 3) (3\kappa_g - 1) \frac{1}{(\theta p)^4} \\
+ \frac{32}{3} (\kappa_g - 1)^2 \frac{1}{(\theta p)^8} \left( \ln (\mu^2 (\theta p)^2) \right), \\
\]

\[
B_3^{\kappa_g}(p) \sim -\frac{1}{3} (\kappa_g^2 - 2\kappa_g - 11) \frac{p^2 (\theta p)^2}{(\theta p)^2} \left[ \frac{2}{\epsilon} + \ln (\mu^2 (\theta p)^2) \right] - \frac{8}{3} \frac{1}{(\theta p)^4} (\kappa_g^2 - 10\kappa_g + 17), \\
\]

\[
B_4^{\kappa_g}(p) \sim -2(\kappa_g + 1)^2 \frac{p^4 (\theta p)^2}{(\theta p)^4} \left[ \frac{2}{\epsilon} + \ln (\mu^2 (\theta p)^2) \right] - \frac{32p^2}{3(\theta p)^6} (\kappa_g^2 - 6\kappa_g + 7), \\
\]

\[
B_5^{\kappa_g}(p) \sim 4 \frac{1}{3} (\kappa_g^2 + \kappa_g + 4) \frac{p^4 (\theta p)^2}{(\theta p)^4} \left[ \frac{2}{\epsilon} + \ln (\mu^2 (\theta p)^2) \right] + \frac{64p^2}{3(\theta p)^6} (\kappa_g - 1)(\kappa_g - 2). \\
\]

Note that all \( B_i^{\kappa_g}(p) \) coefficients are computed for arbitrary \( \kappa_g \) and the notation \( \sim \) means that in the above equations we have neglected all finite terms. We observe here the presence
of the UV divergences as well as quadratic UV/IR mixing in all $B_4^{\kappa g}$'s. The logarithmic IR divergences from planar and nonplanar sources appear to have identical coefficient and combine into a single $\ln \mu^2 (\theta p)^2$ term. Finally no single $\kappa_g$ value is capable of removing all novel divergences.

3.4 Photon-loop with a special $\theta^\mu\nu$ in four dimensions

In our prior analysis we have found that in the $D \rightarrow 4 - \epsilon$ limit the general off-shell contribution of photon self-interaction loop to the photon two-point function contains complicated non-vanishing UV and IR divergent terms with existing and new momentum structures, regardless the $\kappa_g$ values we take. To see whether there exists certain remedy to this situation we explore two conditions which have emerged in the prior studies. First we tested the zero mass-shell condition/limit ($p^2 \rightarrow 0$) used in $\theta$-expanded models [23]. Inspection of Eq's (3.22,3.23) and (3.24-3.28) show some simplification but not the full cancelation of the pathological divergences. Such condition clearly appears to be unsatisfactory.

Next we have turned into the other one, namely the special full rank $\theta^\mu\nu$ choice

$$\theta^\mu\nu \equiv \theta^\mu\nu_2 = \frac{1}{\Lambda_{\text{NC}}^2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \frac{1}{\Lambda_{\text{NC}}^2} \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} \equiv \frac{1}{\Lambda_{\text{NC}}^2} i\sigma_2 \otimes I_2,$$ (3.29)

with $\sigma_2$ being famous Pauli matrix. This condition was used in the renormalizability studies of 4d NCGFT without SW map [13, 14]. Note also that this $\theta^\mu\nu_2$ is full rank and thus breaks in general the unitarity if one performs Wick rotation to the Minkowski spacetime.

This choice, in 4d Euclidean spacetime, induces a relation ($\theta\theta$)

$$\theta^\mu\nu \theta^\mu\nu \equiv \theta^\mu\nu_2 \theta^\mu\nu_2 \equiv \frac{1}{\Lambda_{\text{NC}}^4} i\sigma_2 \otimes I_2,$$ (3.29)

The tensor structures (3.11), with restored coupling constant $\epsilon$ included then simplifies into two parts

$$\Pi^\mu\nu_{\kappa_g}(p) \big|_{\theta^\mu\nu_2} = \frac{e^2}{(4\pi)^2} \left\{ g^{\mu\nu} p^2 - p^\mu p^\nu \right\} B_1^{\kappa g}(p) + (\theta p)^\mu (\theta p)^\nu B_1^{\kappa g}(p) \right\}$$

$$= \frac{e^2}{(4\pi)^2} \left\{ \left[ g^{\mu\nu} p^2 - p^\mu p^\nu \right] \left( B_1^{\kappa g} + 2 \frac{B_3^{\kappa g}}{\Lambda_{\text{NC}}^4} - \frac{B_4^{\kappa g}}{\Lambda_{\text{NC}}^8} \right) \right\}$$

Solving $B_1^{\kappa g}(p)$ and $B_1^{\kappa g}(p)$, and neglecting the IR safe terms, revolves the divergent parts:

$$B_1^{\kappa g}(p) \sim \frac{4(\kappa - 3)^2}{3} \left( \frac{2}{\epsilon} + \ln \left( \mu^2 (\theta p)^2 \right) \right) + \frac{16}{3} \frac{(\kappa - 3)(\kappa + 1)}{(\theta p)^2},$$ (3.31)

$$B_1^{\kappa g}(p) \sim 2 \frac{(\kappa - 3)(\theta p)^2}{(\theta p)^2} \left( \frac{2}{\epsilon} + \ln \left( \mu^2 (\theta p)^2 \right) \right) - \frac{16}{3} \frac{(\kappa - 3)(\theta p)^4}{(\theta p)^4}.$$ (3.32)
We observe immediately that point $\kappa_g = 3$ eliminates all divergences. A careful evaluation of the full photon-loop at this point revolves a simple structure

$$B^{\kappa_g=3}_I(p) = 2 \left[ \frac{56}{3} + \mathcal{I} \right], \quad B^{\kappa_g=3}_{II}(p) = -\frac{p^2}{(\theta p)^2} \left( 9 \mathcal{I} \right),$$

where

$$\mathcal{I} = 8 \left( \mathcal{K}[0; 0, 0] - 6 \mathcal{K}[0; 1, 1] \right) + (\theta p)^2 \left( 3 \mathcal{W}[1; 0, 0] - 16 \mathcal{W}[1; 1, 1] \right) = 0,$$

with $\mathcal{K}[\nu; a, b]$ and $\mathcal{W}[\nu; a, b]$ taken from Subsection 3.2. Thus, for special choice (3.29) in the $D \to 4 - \epsilon$ limit, and at $\kappa_g = 3$ point, we have found

$$B^{\kappa_g=3}_I(p) = \frac{112}{3}, \quad B^{\kappa_g=3}_{II}(p) = -72 \frac{p^2}{(\theta p)^2},$$

as the only one-loop-photon self-interaction corrections to the photon two-point function.

**The detailed proof for the vanishing identity $\mathcal{I}$**

We need to evaluate the equation (3.34) and verify that the integral identity $\mathcal{I}$ equals to zero. First we compute each of the special function integrals

$$\mathcal{K}[0; 0, 0] = \int_0^1 dx \ K_0[X]$$

$$= \int_0^1 \left( - \right) \sum_{k=0}^{\infty} \frac{x^k(1-x)^k}{(\Gamma[k+1])^2} \left( \frac{p^2(\theta p)^2}{4} \right)^k \left( \frac{1}{2} \ln \frac{x(1-x)p^2(\theta p)^2}{4} - \psi(k+1) \right)$$

$$= - \sum_{k=0}^{\infty} \frac{1}{\Gamma[2k+2]} \left( \frac{p^2(\theta p)^2}{4} \right)^k \left( \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} - \psi(2k+2) \right),$$

(3.36)

$$\mathcal{K}[0; 1, 1] = \int_0^1 dx \ x(1-x)K_0[X]$$

$$= \int_0^1 \left( - \right) \sum_{k=0}^{\infty} \frac{x^{k+1}(1-x)^{k+1}}{(\Gamma[k+1])^2} \left( \frac{p^2(\theta p)^2}{4} \right)^k \left( \frac{1}{2} \ln \frac{x(1-x)p^2(\theta p)^2}{4} - \psi(k+1) \right)$$

$$= - \sum_{k=0}^{\infty} \frac{(k+1)^2}{\Gamma[2k+4]} \left( \frac{p^2(\theta p)^2}{4} \right)^k \left( \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{2} \psi(k+1) - \psi(2k+4) \right),$$

(3.37)
\[
W[1; 0, 0] = \int_0^1 dx W_1[X]
\]
\[
= \int_0^1 -4(\theta p)^{-2} + 4(\theta p)^{-2} \sum_{k=0}^{\infty} \frac{2x^k(1-x)^k}{\Gamma[k+1]\Gamma[k+2](2k+1)} \left( \frac{p^2(\theta p)^2}{4} \right)^{k+1}
\cdot \left( \frac{1}{2} \ln \frac{x(1-x)p^2(\theta p)^2}{4} + \frac{1}{2} \psi(k+1) + \frac{1}{2} \psi(k+2) - \frac{1}{2k+1} \right)
= -4(\theta p)^{-2} + 4(\theta p)^{-2} \sum_{k=0}^{\infty} \frac{k+1}{\Gamma[2k+4](2k+1)} \left( \frac{p^2(\theta p)^2}{4} \right)^{k+1}
\cdot \left( \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{k+1} + \frac{2}{k+2} - \frac{2}{2k+1} - 2\psi(2k+4) \right),
\]

and

\[
W[1; 1, 1] = \int_0^1 dx x(1-x)W_1[X]
\]
\[
= \int_0^1 -4x(1-x)(\theta p)^{-2} + 4(\theta p)^{-2} \sum_{k=0}^{\infty} \frac{2x^{k+1}(1-x)^{k+1}}{\Gamma[k+1]\Gamma[k+2](2k+1)}
\cdot \left( \frac{p^2(\theta p)^2}{4} \right)^{k+1} \ln \frac{(1-x)p^2(\theta p)^2}{4} + \frac{1}{2} \psi(k+1) + \frac{1}{2} \psi(k+2) - \frac{1}{2k+1} \right)
= -\frac{2}{3}(\theta p)^{-2} + (\theta p)^{-2} \sum_{k=0}^{\infty} \frac{4(k+1)(k+2)^2}{\Gamma[2k+6](2k+1)}
\cdot \left( \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{k+1} + \frac{2}{k+2} - \frac{2}{2k+1} - 2\psi(2k+6) \right).
\]

Here \(\psi(z)\) is the polygamma function \(\psi(z) = d_z \ln \Gamma[z]\), which satisfies the recurrence
relation $\psi(z + 1) = \psi(z) + z^{-1}$. Now we have

$$\mathcal{I} = -8 \left[ \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} - \psi(2) \right] - \frac{6}{\Gamma[4]} \left( \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} + 1 - \psi(4) \right) - \left( 3 \cdot 4 \right)$$

$$- 16 \cdot \frac{2}{3} + \sum_{k=0}^{\infty} \left( \frac{p^2(\theta p)^2}{4} \right)^{k+1} \left[ \frac{-8}{\Gamma[2k + 4]} \left( \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} - \psi(2k + 4) \right) + \frac{3 \cdot 4(k + 1)}{\Gamma[2k + 4](2k + 1)} \right.$$  

$$+ 8 \cdot \frac{(k + 2)^2}{\Gamma[2k + 6]} \left( \frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{k + 2} - \psi(2k + 6) \right) + \frac{3 \cdot 4(k + 1)}{\Gamma[2k + 4](2k + 1)}$$

$$\left. \cdot \left( \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{k + 1} - \frac{2}{2k + 1} - 2\psi(2k + 4) \right) - \frac{16 \cdot 4(k + 1)(k + 2)^2}{\Gamma[2k + 6]} \right.$$  

$$\left. \cdot \left( \ln \frac{p^2(\theta p)^2}{4} + \frac{1}{k + 1} + \frac{2}{k + 2} - \frac{2}{2k + 1} - 2\psi(2k + 6) \right) \right]$$  

$$= \sum_{k=0}^{\infty} \left( \frac{p^2(\theta p)^2}{4} \right)^{k+1} \left[ \left( \ln \frac{p^2(\theta p)^2}{4} - 2\psi(2k + 4) \right) \left( - \frac{4}{\Gamma[2k + 4]} + \frac{24(k + 2)^2}{\Gamma[2k + 6]} \right) \right.$$

$$+ \frac{12(k + 1)}{\Gamma[2k + 4](2k + 1)} - \frac{64(k + 1)(k + 2)^2}{\Gamma[2k + 6](2k + 1)} + \frac{12(k + 1)}{\Gamma[2k + 4](2k + 1)}$$

$$\left. \cdot \left( \frac{1}{k + 1} - \frac{2}{2k + 1} - \frac{12(k + 1)(k + 2)^2}{\Gamma[2k + 6](2k + 1)} \right) \left( \frac{1}{k + 1} + \frac{1}{k + 2} - \frac{2}{2k + 1} \right. \right.$$  

$$\left. - \frac{2}{2k + 5} \right) + \frac{48(k + 2)^2}{\Gamma[2k + 6]} \left( \frac{1}{2k + 4} - \frac{1}{2k + 5} \right). \right]$$

One can then see that

$$- \frac{4}{\Gamma[2k + 4]} + \frac{24(k + 2)^2}{\Gamma[2k + 6]} + \frac{12(k + 1)}{\Gamma[2k + 4](2k + 1)} - \frac{64(k + 1)(k + 2)^2}{\Gamma[2k + 6](2k + 1)}$$

$$= \frac{4}{\Gamma[2k + 4]} \left( -1 + \frac{3(k + 2)}{2k + 5} + \frac{3(k + 1)}{2k + 1} - \frac{8(k + 1)(k + 2)}{(2k + 1)(2k + 5)} \right) = 0, \tag{3.41}$$

$$\frac{12(k + 1)}{\Gamma[2k + 4](2k + 1)} \left( \frac{1}{k + 1} - \frac{2}{2k + 1} \right) - \frac{64(k + 1)(k + 2)^2}{\Gamma[2k + 6](2k + 1)}$$

$$\cdot \left( \frac{1}{(k + 2)(2k + 5)} - \frac{1}{(k + 1)(2k + 1)} \right) + \frac{48(k + 2)^2}{\Gamma[2k + 6]} \left( \frac{1}{2k + 4} - \frac{1}{2k + 5} \right) \tag{3.42}$$

$$= \frac{1}{\Gamma[2k + 4]} \left[ -12 + \frac{96(2k + 3)}{(2k + 1)^2(2k + 5)^2} + \frac{12}{(2k + 5)^2} \right] = 0,$$

thus

$$\mathcal{I} = 0. \tag{3.43}$$

4. Neutrino two-point function

One-loop contributions as a function of $\kappa_f$ receive the same spinor structure as in [39]. We now reconfirm that by using the action (2.14) together with the Feynman rule (2.26), out of four diagrams in Fig.2 of [39], only the (non-vanishing) bubble graph is considered in this manuscript (Fig.3 in this manuscript). In the present scenario its contribution reads
Figure 3: Bubble-graph contribution to the neutrino two-point function

\[ \Sigma_{\kappa_f}(p)_D = \frac{-1}{(4\pi)^2} \left[ \gamma_\mu p^\mu N_{1,2}^{\kappa_f}(p) + \gamma_\mu (\theta \theta p)\mu N_{1,2}^{\kappa_f}(p) \right]. \quad (4.1) \]

Loop-coefficients \( N_{1,2}^{\kappa_f}(p) \), as a functions of an arbitrary dimensions \( D \), are:

\[
N_{1}^{\kappa_f}(p) = (4\pi)^{2-D} \mu^{D-1} \left\{ \frac{\pi^2 2^{3-D} \csc \frac{D\pi}{2} p^{D-4}}{\Gamma \left[ \frac{D-1}{2} \right]} \right. \\
\cdot \left\{ (D-3)(\kappa_f - 1) - (\theta \theta p) \frac{p^2}{(\theta p)^2} - 2(\theta \theta p)^2 \frac{p^2}{(\theta p)^4} \right\} \\
- 4(\kappa_f - 1) \left( (2D-3)\kappa_f - D \right) K \left[ \frac{D}{2} - 2; 1, 0 \right] \\
+ 8(D-1)(\kappa_f - 1)^2 K \left[ \frac{D}{2} - 2; 2, 0 \right] - (D-3)\kappa_f^2 (\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 0 \right] \\
- \left( (D+1)\kappa_f^2 + (1 - 3D)\kappa_f + 1 + D \right) (\theta p)^2 W \left[ \frac{D}{2} - 1; 1, 0 \right] \\
+ 2D(\kappa_f - 1)^2 (\theta p)^2 W \left[ \frac{D}{2} - 1; 2, 0 \right] - (\theta \theta p) \frac{p^2}{(\theta p)^2} \left\{ 4K \left[ \frac{D}{2} - 2; 0, 1 \right] \right. \\
+ \frac{8D(\kappa_f - 1)}{D-1} K \left[ \frac{D}{2} - 2; 1, 1 \right] + \frac{4(\kappa_f - 1)}{D-1} p^2 K \left[ \frac{D}{2} - 1; 0, 0 \right] \\
- (\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 1 \right] + \frac{2D(\kappa_f - 1)}{D-1} (\theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \right\} \\
\left. \right. \\
+ (\theta p)^2 W \left[ \frac{D}{2} - 1; 0, 1 \right] + \frac{2D(\kappa_f - 1)}{D-1} (\theta p)^2 W \left[ \frac{D}{2} - 1; 1, 1 \right] \right\} \right\}, \quad (4.2)
and they contain both integrals, the $K$'s and the $W$'s.

For arbitrary $\kappa_f$ in the limit $D \to 4 - \epsilon$ we have obtained the following loop-coefficients:

\[
N_2^{\kappa_f}(p) = (4\pi)^{2-D} \mu^{d-D} \left\{ - 2\kappa_f (\kappa_f - 1) B \left[ \frac{D}{2}, \frac{D}{2} - 1 \right] \Gamma \left[ 2 - \frac{D}{2} \right] p^{D-4} \frac{p^2}{(\theta p)^2} \right. \\
- \kappa_f (\kappa_f - 1) \left\{ \frac{4(D + 1)}{D - 1} K \left[ \frac{D}{2} - 2; 1, 0 \right] + \frac{8}{D - 1} K \left[ \frac{D}{2} - 2; 2, 0 \right] \right. \\
- \frac{4(D - 2)}{D - 1} p^{-2} K \left[ \frac{D}{2} - 1; 0, 0 \right] \right\} (4.3) \\
- \kappa_f (\theta p)^2 \left\{ - 2 W \left[ \frac{D}{2} - 1; 0, 0 \right] + \left( 1 - 3D \right) \kappa_f + 5D - 3 W \left[ \frac{D}{2} - 1; 1, 0 \right] \right. \\
+ \frac{2D (\kappa_f - 1)}{D - 1} W \left[ \frac{D}{2} - 1; 2, 0 \right] \right\},
\]

\[
N_1^{\kappa_f}(p) = \kappa_f \left\{ \left[ 2\kappa_f + (\kappa_f - 1) \left( \frac{2}{\epsilon} + \ln \pi e^{\gamma_E} + \ln (\mu^2 (\theta p)^2) \right) \right] 
- \frac{p^2 (\theta p)^2}{4} \sum_{k=0}^{\infty} \frac{(p^2 (\theta p)^2)^k}{4^k k(k+1)(2k+1)^2(2k+3) \Gamma[2k+4]} \left( k(k+1)(2k+1)(2k+3) \left( \kappa_f (2k+3) - 1 \right) \left( \ln (p^2 (\theta p)^2) - 2 \psi(2k) - \ln 4 \right) \right. \\
+ 3 + 28k + 46k^2 + 20k^3 - \kappa_f (2k+3)^2 (1 + 8k + 8k^2) \right. \\
+ (\text{tr} \theta \theta) \left\{ \frac{p^2}{(\theta p)^2} \left[ \frac{2}{\epsilon} + 2 + \gamma_E + \ln \pi + \ln (\mu^2 (\theta p)^2) + \frac{8(\kappa_f - 1)}{3 \kappa_f (\theta p)^2 \epsilon^2} \right] \right. \\
- \frac{p^4}{4} \sum_{k=0}^{\infty} \frac{(p^2 (\theta p)^2)^k}{4^k k(k+1)(2k+1)^2(2k+3) \Gamma[2k+4]} \left[ k(k+1)(2k+1)(2k+3) \right. \\
\left. \cdot \left( \ln (p^2 (\theta p)^2) - 2 \psi(2k) - \ln 4 \right) - 2k \left( 14 + k(23 + 10k) \right) - 3 \right] \right\} \\
+ (\theta p)^2 \left\{ 2 \frac{p^2}{(\theta p)^4} \left[ \frac{2}{\epsilon} + 1 + \gamma_E + \ln \pi + \ln (\mu^2 (\theta p)^2) + \frac{16(\kappa_f - 1)}{3 \kappa_f (\theta p)^2 \epsilon^2} \right] \\
+ \frac{p^4}{2 (\theta p)^2} \sum_{k=0}^{\infty} \frac{k(p^2 (\theta p)^2)^k}{(k+1)(2k+1)^2(2k+3) \Gamma[2k+4]} \left[ (k+1)(2k+1)(2k+3) \right. \\
\left. \cdot \left( \ln (p^2 (\theta p)^2) - 2 \psi(2k) - \ln 4 \right) + 16k^2 - 34k - 17 \right] \right\}, \]
\]

(4.4)
In the expressions for $N_{1,2}^\kappa(p)$ contributions from both the planar as well as the non-planar graphs are present. For any $\kappa_f \neq 1$ our neutrino self energy receive UV, and power as well as logarithmic UV/IR mixing terms.

**First we analyze the choice** $\kappa_f = 1$.

For $D = 4 - \epsilon$ in the limit $\epsilon \to 0$, we obtain the final expression for the self-energy as

$$\Sigma_{\kappa_f = 1}(p) = -e^2/(4\pi)^2 \gamma_\mu \left[ p^\mu N_1 + (\theta p)^\mu \frac{p^2}{(\theta p)^2} N_2 \right],$$  \hfill (4.6)

with restored coupling constant $e$ included. Here $N_i$ coefficients are as follows,

\begin{align*}
N_1 &= p^2 \left( \frac{\text{tr}\theta}{(\theta p)^2} + 2 \frac{(\theta p)^2}{(\theta p)^4} \right) A + \left[ 1 + p^2 \left( \frac{\text{tr}\theta}{(\theta p)^2} + \frac{(\theta p)^2}{(\theta p)^4} \right) \right] B, \hfill (4.7) \\
A &= \frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) + \ln(\pi e^{\gamma_E}) + \sum_{k=1}^{\infty} \left( \frac{p^2(\theta p)^2}{\Gamma(2k + 2)} \right)^k \ln \left( \frac{p^2(\theta p)^2}{4} + 2\psi_0(2k + 2) \right), \hfill (4.8) \\
B &= -\frac{(4\pi)^2}{2}N_2 = -2 \\
&+ \sum_{k=0}^{\infty} \frac{(\theta p)^2}{(2k + 1)(2k + 3)\Gamma(2k + 2)} \left( \ln \frac{p^2(\theta p)^2}{4} - 2\psi_0(2k + 2) - \frac{8(k + 1)}{(2k + 1)(2k + 3)} \right), \hfill (4.9)
\end{align*}

with $\gamma_E \simeq 0.577216$ being Euler’s constant. It is to be noted here that the spinor structure proportional to $\gamma_\mu(\theta p)^\mu$ is missing in the final result. This conforms with the calculation of the neutral fermion self-energy in the $\theta$-expanded SW map approach \[58\].

The $1/\epsilon$ UV divergence could in principle be removed by a properly chosen counterterm. However (as already mentioned) due to the specific momentum-dependent coefficient in front of it, a nonlocal form for it is required. It is important to stress here that amongst other terms contained in both coefficients $A_1$ and $A_2$, there are structures proportional to

$$(p^2(\theta p)^2)^{n+1} \left( \ln(p^2(\theta p)^2) \right)^m, \quad \forall n \text{ and } m = 0, 1.$$  \hfill (4.10)

The numerical factors in front of the above structures are rapidly-decaying, thus series are always convergent for finite argument, as we demonstrate in \[39\].
Turning to the UV/IR mixing problem, we recognize a soft UV/IR mixing term represented by a logarithm,

\[ \Sigma_{\kappa_f=1}^{UV/IR} = -\frac{e^2}{(4\pi)^2} \bar{p} p^2 \left( \frac{\text{tr}\theta\theta}{(\theta p)^2} + 2 \frac{(\theta p)^2}{(\theta p)^4} \right) \cdot \ln |\mu^2(\theta p)^2|. \]  

(4.11)

Thus, we see that the second naive expectation about NC gauge field theories does hold here even without invoking supersymmetry.

Instead of dealing with nonlocal counterterms, we take a different route here to cope with various divergences besetting (4.6). Since \( \theta^{0i} \neq 0 \) makes a NC theory nonunitary \cite{59}, we can, without loss of generality, choose \( \theta \) to lie in the (1, 2) plane

\[ \theta_{\mu\nu}^{\text{spec}} = \frac{1}{\Lambda^2_{NC}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \]  

(4.12)

Automatically, this produces

\[ \frac{\text{tr}\theta\theta}{(\theta p)^2} + 2 \frac{(\theta p)^2}{(\theta p)^4} = 0, \forall p. \]  

(4.13)

With (4.13), \( \Sigma_{\kappa_f=1}^{\text{spec}} \), in terms of Euclidean momenta, receives the following form:

\[ \Sigma_{\kappa_f=1}^{\text{spec}}(p) = -\frac{e^2}{(4\pi)^2} \gamma_\mu \left[ p^\mu \left( 1 + \frac{\text{tr}\theta\theta}{2} \frac{p^2}{(\theta p)^2} \right) - 2(\theta p)^2 \right] N_2. \]  

(4.14)

By inspecting (4.9) one can be easily convinced that \( A_2 \) is free from the \( 1/\epsilon \) divergence and the UV/IR mixing term, being also well-behaved in the infrared, in the \( \theta \to 0 \) as well as \( \theta p \to 0 \) limit. We see, however, that the two terms in (4.14), one being proportional to \( p^\mu \) and the other proportional to \( (\theta p)^\mu \), are still ill-behaved in the \( \theta p \to 0 \) limit. If, for the choice (4.12), \( P \) denotes the momentum in the (1, 2) plane, then \( \theta p = \theta P \). For instance, a particle moving inside the noncommutative plane with momentum \( P \) along the one axis, has a spatial extension of size \( |\theta P| \) along the other. For the choice (4.12), \( \theta p \to 0 \) corresponds to a zero momentum projection onto the (1, 2) plane. Thus, albeit in our approach the commutative limit \( (\theta \to 0) \) is smooth at the quantum level, the limit when an extended object (arising due to the fuzziness of space) shrinks to zero, is not. We could surely claim that in our approach the UV/IR mixing problem is considerably softened; on the other hand, we have witnessed how the problem strikes back in an unexpected way.

This is, at the same time, the first example where this two limits are not degenerate.

**Next we analyze the choice** \( \kappa_f = 0 \).

Using the Feynman rule (2.26) for \( \kappa_f = 0 \) and for general \( \theta \), we find the following closed form contribution to the neutrino self-energy from diagram \( \Sigma_1 \)

\[ \Sigma_{\kappa_f=0}(p) = \frac{e^2}{(4\pi)^2} \bar{p} \left[ 8 \frac{1}{3} \frac{\text{tr}\theta\theta}{(\theta p)^2} + 4 \frac{(\theta p)^2}{(\theta p)^4} \right]. \]  

(4.15)
The detailed computation is presented in Appendix B of ref. [39]. From (??) one gets the relation,
\[ g_{\mu\nu} V_{3alt}^{\mu\nu}(p, -p, q, q) = 0, \]
showing that \( \Sigma_2 = 0 \), while \( \Sigma_3 \) and \( \Sigma_4 \) vanish due to charge conjugation symmetry. Therefore we have again \( \Sigma_{1-loopalt} = \Sigma_{1alt} \). There is no alternative dispersion relation in degenerate case (4.12), since the factor that multiplies \( \phi \) in (4.15), does not depend on the time-like component \( p_0 \) (energy).

5. Discussion

In this review we present a \( \theta \)-exact quantum one-loop contributions to the photon (\( \Pi \)) and neutrino (\( \Sigma \)) self-energies and analyze their properties. Our method, extending the modified Feynman rule procedure [7], yields the one-loop quantum corrections for arbitrary dimensions in closed form, as function of the deformation-freedom parameter-space \((\kappa_f, \kappa_g)\), as well as momentum \( p^\mu \) and noncommutative parameter \( \theta^{\mu\nu} \). Full parameter-space freedom is kept in our evaluation here. Following the extended dimensional regularization technique we expressed the diagrams as \( D \)-dimensional loop-integrals and identify the relevant momentum structures with corresponding loop-coefficients. We have found that total contribution to photon two-point function satisfies the Ward-(Slavnov-Taylor) identity for arbitrary dimensions \( D \) and for any point \((\kappa_f, \kappa_g)\) in parameter-space:

\[
p_\mu \Pi_{(\kappa_f, \kappa_g)}^{\mu\nu}(p)_D = p_\mu \left( \Pi_{\kappa_f}^{\mu\nu}(p)_D + \Pi_{\kappa_g}^{\mu\nu}(p)_D \right) = 0. \quad (5.1)
\]

We observe the following general behavior of one-loop two-point functions in the \( D \to 4 - \epsilon \) limit: The total expressions for both the photon and the neutrino self-energy contain the \( 1/\epsilon \) ultraviolet term, the celebrated UV/IR mixing power terms as well as the logarithmic (soft) UV/IR mixing term. The \( 1/\epsilon \) divergence is always independent of the noncommutative scale. The logarithmic terms from the \( \epsilon \)-expansion and the modified Bessel function integral sum into a common term \( \ln(\mu^2 (\theta p)^2) \), which is divergent both in the IR limit \(|p| \to 0\), as well as in the vanishing noncommutativity \( \theta \to 0 \) limit.

Our evaluation of the four dimensional \( \theta \)-exact fermion-part contribution to the photon self-energy, i.e. the fermion-loop photon two-point function (3.6) yields two already known tensor structures [38, 31, 32]. The loop-coefficients \( F_{1/2}^{\kappa_f}(p) \), on the other hand, exhibit nontrivial \( \kappa_f \) dependence. Namely, in the limit \( \kappa_f \to 0 \implies F_{1}^{\kappa_f} = F_{2}^{\kappa_f} = 0 \), thus the photon self-energy (3.6) vanishes, while \( \kappa_f = 1 \) appears to be identical to the non SW-map model. Fermion-loop contains UV and logarithmic divergence in \( F_{1}^{\kappa_f} \) for \( \kappa_f \neq 0 \), while the quadratic UV/IR mixing could be removed by setting \( \kappa_f = 0, 2 \) in \( F_{2}^{\kappa_f}(p) \).

The photon-loop contribution to the photon two point function contains various previously unknown new momentum structures with respect to earlier \( \theta \)-exact results based on \( \star \)-product only. These higher order in \( \theta \) (\( \theta \theta \theta \theta \theta \) types) terms suggest certain connection to the open/closed string correspondence [3] [48] (in an inverted way). We consider such connection plausible given the connection between noncommutative field theory and quantum gravity/ string theory.

The four dimension expressions of the photon-loop contribution (3.11) to self-energy, contains the UV terms, a logarithmic IR singularity as well as quadratic UV/IR mixing terms. This reflects the fact that, up to the \( 1/\epsilon \) terms, the UV divergence is at most
logarithmic, i.e. there is a logarithmic ultraviolet/infrared term representing a soft UV/IR mixing. The results (3.24-3.28) in four dimensions for arbitrary \( \kappa_g \) show power type UV/IR mixing, therefore diverge at both the commutative limit \( (\theta \to 0) \) and the size-of-the-object limit \( (|\theta p| \to 0) \). Inspecting (3.24) to (3.28) together with general structure (3.11) we found decouplings of UV and logarithmic IR divergences from the power UV/IR mixing terms. The latter exists in all \( B_i^{\kappa_g} \)'s.

To simplify the tremendous divergent structures in \( B_i^{\kappa_g} \)'s at \( D \to 4 - \epsilon \), we have probed two additional conditions: One which appears to be ineffective is the zero mass-shell condition/limit \( p^2 \to 0 \), due to the uncertainty on its own validity when quantum corrections present. The other condition, namely setting \( \theta_{\mu\nu} \) to a special full ranked value \( \theta_{\mu\nu} \equiv \Pi_{\mu\nu}^{(0,3)}(p) \equiv \left( \frac{e^2 p^2}{\pi^2} \right) \left( \frac{7}{3} (g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}) - \frac{9}{2} \frac{(\theta p_{\mu})^{\nu} (\theta p)^{\mu}}{(\theta p)^2} \right) \), as the only one-loop-finite contribution/correction to the photon two-point function.

Considering neutrino self-energy (4.1), our results extends the prior works \[38, 39\] by completing the behavior for general \( \kappa_f \). Here we discuss some novel behaviors associated with general \( \kappa_f \). The neutrino self-energy does posses power UV/IR mixing phenomenon for arbitrary values of \( \kappa_f \), except \( \kappa_f = 1 \). In the limit \( \kappa_f \to 0 \) all UV, IR divergent terms as well as constant terms in \( N_{1,2}^{\kappa_f}(p) \) vanish; what remains are only the power UV/IR mixing terms. The UV divergence can be localized using the special \( \theta \) value \[38, 39\] in \( N_{1,2}^{\kappa_f} \) but not in \( N_{2}^{\kappa_f} \). The UV and the power IR divergence in \( N_{2}^{\kappa_f} \) can be removed by setting \( \kappa_f = 1 \).

Summing up, choice \( \kappa_f = 1 \) eliminates some of divergences, but not all of them. Imposing the special \( \theta_{\mu\nu} \) reduces the contribution to quadratic UV/IR mixing into a single term from \( N_{2}^{\kappa_f}(p) \), which has two zero points \( \kappa_f = 0,1 \). Only \( \kappa_f = 0 \) can induce full divergence cancelations, by removing the whole \( \Sigma(p) \), i.e. we have

\[
\Sigma_{\kappa_f=0}(p) \bigg|_{\theta_{\mu\nu}} = 0.
\]

The general existences of UV/IR mixings for both, photons and neutrinos respectively, in 4d spaces deformed by spacetime noncommutativity at low energies, suggests that the relation of quantum corrections to observations \[63, 64, 65, 66, 67, 68\] is not entirely clear. However, in the context of the UV/IR mixing it is very important to mention a complementary approach \[53, 56\] where NC gauge theories are realized as effective QFT’s, underlain by some more fundamental theory such as string theory. It was claimed that for a large class of more general QFT’s above the UV cutoff the phenomenological effects of the UV completion can be quite successfully modeled by a threshold value of the UV cutoff. So, in the presence of a finite UV cutoff no one sort of divergence will ever appear since the problematic phase factors effectively transform the highest energy scale (the UV cutoff) into the lowest one (the IR cutoff). What is more, not only the full scope of noncommutativity is experienced only in the range delimited by the two cutoffs, but for
the scale of NC high enough, the whole standard model can be placed below the IR cutoff [65]. Thus, a way the UV/IR mixing problem becomes hugely less pressing, making a study of the theory at the quantum level much more reliable.

6. Conclusion

The quantum corrections in NCQFTs are extremely profound, revealing a structure of pathological terms far beyond that found in ordinary field theories. For practical purposes, perturbative loop computation was the most intensively studied for the Moyal-Weyl (constant $\theta^{\mu\nu}$) type deformation [4, 5, 6, 7, 8], for its preservation of translation invariance allows a (modified) Feynman diagrammatic calculation. Much efforts went in taming divergences related to the ultraviolet/infrared (UV/IR) connection. This new principle, built-in in all NCQFT models, and closely related to the Black Hole Complementarity, does reverse the well-established connection (via the uncertainty principle) between energy and size. The existence of such pathological terms in NCQFTs raises a serious concern on the renormalizability/consistency of the theory. Although no satisfactory resolution for this issue had been achieved so far, very recently it has been observed that certain control over novel divergences may be obtained by certain gauge invariant variation of the SW mapped action [40]. The anomalous structures in the two point function further suggests possible modifications to obtain trouble-free and meaningful loop-results, necessary for studying the particle propagation [39]. Such effects were largely left untouched in literature so far, mostly due to the prior concern on the consistency/renormalizability from a purely theoretical viewpoint.

After having defined and explained the full noncommutative action-model origin of the deformation parameters $\kappa_f, \kappa_g$, we obtained the relevant Feynman rules. The one-loop photon self-energy in four dimensions contains the UV divergence and UV/IR mixing terms dependent on the freedom parameters $\kappa_f$ and $\kappa_g$. The introduction of the freedom parameters univocally has a potential to improve the situation regarding cancellation of divergences, since certain choices for $\kappa_f$ and $\kappa_g$ could make some of the terms containing singularities to vanish.

We have demonstrated how quantum effects in the $\theta$-exact Seiberg-Witten map approach to NC gauge field theory reveal a much richer structure for the one-loop quantum correction to the photon and fermion two-point functions (and accordingly for the UV/IR mixing problem) than observed previously in approximate models restricting to low-energy phenomena. Our analysis can be considered trustworthy since we have obtained the final result in an analytic, closed-form manner. We believe that a promising avenue of research would be using the enormous freedom in the Seiberg-Witten map to look for other forms which UV/IR mixing may assume. Two alternative forms have been already found [39]. Finally, our approach to UV/IR mixing should not be confused with the one based on a theory with UV completion ($A_{UV} \prec \infty$), where a theory becomes an effective QFT, and the UV/IR mixing manifests itself via a specific relationship between the UV and the IR cutoffs [63, 64, 65, 66, 67].
In conclusion, our main result in four-dimensional space is that we have all pathological terms under full control after the introduction of the deformation-freedom parameter-space \((\kappa_f, \kappa_g)\) and a special choice for \(\theta^{\mu\nu}\). Namely, working in the 4d Euclidean space with a special full rank of \(\theta_{\mu\nu}^{\sigma2}\) and setting \((\kappa_f, \kappa_g) = (0, 3)\), the fermion plus the photon-loop contribution to \(\Pi^{\mu\nu}_{(\kappa_f, \kappa_g)}(p)_4\) contain only two finite terms, i.e. all divergent terms are eliminated. In this case the neutrino two-point function vanishes.

7. Acknowledgment

J.T. would like to acknowledge J. Erdmenger and W. Hollik for discussions, and Max-Planck-Institute for Physics, Munich, for hospitality. A great deal of computation was done by using Mathematica 8.0 [69] and tensor algebra package xAct [70].

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, The Hierarchy problem and new dimensions at a millimeter, Phys. Lett. B 429 (1998) 263 [hep-ph/9803315].

[2] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Tests of quantum gravity from observations of gamma-ray bursts, Nature 393 (1998) 763 [astro-ph/9712103].

[3] N. Seiberg and E. Witten, String theory and noncommutative geometry, JHEP 09 (1999) 032, [hep-th/9908142].

[4] R. Jackiw and S. Y. Pi, Covariant coordinate transformations on noncommutative space, Phys. Rev. Lett. 88 (2002) 111603, [arXiv:hep-th/0111122].

[5] A. Connes, Noncommutative Geometry, Academic Press London (1994).

[6] J. Madore, An Introduction to Noncommutative Differential Geometry and its Physical Applications, 2nd Edition (Cambridge University Press, 1999).

[7] T. Filk, Divergencies in a field theory on quantum space, Phys. Lett. B376 (1996) 53–58.

[8] X. Calmet, B. Jurco, P. Schupp, J. Wess, and M. Wohlgenannt, The standard model on non-commutative space-time, Eur. Phys. J. C23 (2002) 363–376, [hep-ph/0111115].

[9] W. Behr, N. Deshpande, G. Duplančić, P. Schupp, J. Trampetić, and J. Wess, The \(Z \rightarrow \gamma\gamma, gg\) decays in the noncommutative standard model, Eur. Phys. J. C29 (2003) 441–446, [hep-ph/0202121].

[10] P. Aschieri, B. Jurco, P. Schupp, and J. Wess, Non-commutative GUTs, standard model and \(C, P, T\), Nucl. Phys. B651 (2003) 45–70, [hep-th/0205214].

[11] I. Hinchliffe, N. Kersting, and Y. L. Ma, Review of the phenomenology of noncommutative geometry, Int. J. Mod. Phys. A19 (2004).

[12] J. Trampetić, Renormalizability and Phenomenology of theta-expanded Noncommutative Gauge Field Theory, Fortsch. Phys. 56 (2008) 521.

[13] C. P. Martin, The Minimal and the New Minimal Supersymmetric Grand Unified Theories on Noncommutative Space-time, Class. Quant. Grav. 30 (2013) 155019, [arXiv:1302.3732 [hep-th]].
[14] A. A. Bichl, J. M. Grimstrup, L. Popp, M. Schweda and R. Wulkenhaar, Perturbative analysis of the Seiberg-Witten map, Int. J. Mod. Phys. A 17 (2002) 2219, [hep-th/0102044].
[15] A. Bichl, J. Grimstrup, H. Grosse, L. Popp, M. Schweda, and R. Wulkenhaar, Renormalization of the noncommutative photon self-energy to all orders via Seiberg-Witten map, JHEP 0106, 013 (2001), [arXiv:hep-th/0104097].
[16] J. M. Grimstrup and R. Wulkenhaar, Quantisation of theta-expanded non-commutative QED, Eur. Phys. J. C 26 (2002) 139 [arXiv:hep-th/0205153].
[17] R. Banerjee and S. Ghosh, Seiberg Witten map and the axial anomaly in noncommutative field theory, Phys. Lett. B 533, 162 (2002), [arXiv:hep-th/0110177].
[18] C. P. Martin, The gauge anomaly and the Seiberg-Witten map, Nucl. Phys. B652 (2003) 72–92, [hep-th/0211164].
[19] M. Buric, V. Radovanovic, and J. Trampetic, The one-loop renormalization of the gauge sector in the noncommutative standard model, JHEP 03 (2007) 030, [hep-th/0609073].
[20] D. Latas, V. Radovanovic, and J. Trampetic, Non-commutative SU(N) gauge theories and asymptotic freedom, Phys. Rev. D76 (2007) 085006, [hep-th/0703013].
[21] M. Buric, D. Latas, V. Radovanovic and J. Trampetic, The absence of the 4ψ divergence in noncommutative chiral models, Phys. Rev. D 77 (2008) 045031, [arXiv:0711.0887 [hep-th]].
[22] C. P. Martin, C. Tamarit, Noncommutative GUT inspired theories and the UV finiteness of the fermionic four point functions, Phys. Rev. D80, 065023 (2009), [arXiv:0907.2464 [hep-th]].
[23] C. P. Martin and C. Tamarit, Renormalisability of noncommutative GUT inspired field theories with anomaly safe groups, JHEP 0912 (2009) 042, [arXiv:0910.2677 [hep-th]].
[24] M. Buric, D. Latas, V. Radovanovic and J. Trampetic, Chiral fermions in noncommutative electrodynamics: renormalisability and dispersion, Phys. Rev. D 83 (2011) 045023, arXiv:1009.4603 [hep-th].
[25] M. Buric, D. Latas, B. Nikolic and V. Radovanovic, The role of the Seiberg-Witten field redefinition in renormalization of noncommutative chiral electrodynamics, Eur. Phys. J. C 73 (2013) 2542, [arXiv:1304.4451 [hep-th]].
[26] P. Schupp, J. Trampetic, J. Wess, and G. Raffelt, The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds, Eur. Phys. J. C36 (2004) 405–410, [hep-ph/0212292].
[27] P. Minkowski, P. Schupp, and J. Trampetic, Neutrino dipole moments and charge radii in non-commutative space-time, Eur. Phys. J. C37 (2004) 123–128, [hep-th/0302175].
[28] T. Ohl and J. Reuter, Testing the noncommutative standard model at a future photon collider, Phys. Rev. D70 (2004) 076007, [hep-ph/0406098].
[29] A. Alboteanu, T. Ohl, and R. Ruckl, Probing the noncommutative standard model at hadron colliders, Phys. Rev. D74 (2006) 096004, [hep-ph/0608155].
[30] A. Alboteanu, T. Ohl, and R. Ruckl, The Noncommutative standard model at $O(θ^2)$, Phys. Rev. D76 (2007) 105018, [0707.3595].
[31] A. Alboteanu, T. Ohl, and R. Ruckl, The Noncommutative standard model at the ILC, Acta Phys. Polon. B38 (2007) 3647, [0709.2359].
32. M. Buric, D. Latas, V. Radovanovic, and J. Trampetic, Nonzero Z \to \gamma\gamma decays in the renormalizable gauge sector of the noncommutative standard model, Phys. Rev. D75 (2007) 097701, hep-ph/0611293.

33. C.P. Martin, D. Sanchez-Ruiz, The One-loop UV Divergent Structure of U(1) Yang-Mills Theory on Noncommutative R^4, Phys. Rev. Lett. 83 (1999) 476–479, [hep-th/9903077].

34. S. Minwalla, M. Van Raamsdonk and N. Seiberg, Noncommutative perturbative dynamics, JHEP 0002, 020 (2000), [arXiv:hep-th/9912072].

35. A. Matusis, L. Susskind, and N. Toumbas, The IR/UV connection in the non-commutative gauge theories, JHEP 12 (2000) 002, hep-th/0002075.

36. T. Mehen and M. B. Wise, Generalized *-products, Wilson lines and the solution of the Seiberg-Witten equations, JHEP 12 (2000) 008, [hep-th/0010204].

37. P. Schupp and J. You, UV/IR mixing in noncommutative QED defined by Seiberg-Witten map, JHEP 08 (2008) 107, 0807.4888.

38. R. Horvat, A. Ilakovac, J. Trampetic and J. You, On UV/IR mixing in noncommutative gauge field theories, JHEP 12 (2011) 081, arXiv:1109.2485 [hep-th].

39. R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic and J. You, Neutrino propagation in noncommutative spacetimes, JHEP 1204 (2012) 108 [arXiv:1111.4951 [hep-th]].

40. R. Horvat, A. Ilakovac, J. Trampetic and J. You, Self-energies on deformed spacetimes, JHEP 1311 (2013) 071, [arXiv:1306.1239 [hep-th]].

41. H. Grosse and R. Wulkenhaar, Renormalization of \phi^4 theory on noncommutative R^4 in the matrix base, Commun. Math. Phys. 256 (2005) 305–374, hep-th/0401128.

42. J. Magnen, V. Rivasseau and A. Tanasa, Commutative limit of a renormalizable noncommutative model, Europhys. Lett. 86 (2009) 11001, [arXiv:0807.4093 [hep-th]].

43. D. N. Blaschke, H. Grosse, E. Kronberger, M. Schweda and M. Wohlgenannt, Loop Calculations for the Non-Commutative U^*(1) Gauge Field Model with Oscillator Term, Eur. Phys. J. C 67 (2010) 575 [arXiv:0912.3642 [hep-th]].

44. D. N. Blaschke, A New Approach to Non-Commutative U_\ast(N) Gauge Fields, Europhys. Lett. 91 (2010) 11001, [arXiv:1005.1578 [hep-th]].

45. S. Meljanac, A. Samsarov, J. Trampetic and M. Wohlgenannt, Scalar field propagation in the \phi^4 kappa-Minkowski model, JHEP 12 (2011) 010, arXiv:1111.5553 [hep-th].

46. R. Horvat, D. Kekez, J. Trampetic, Spacetime noncommutativity and ultra-high energy cosmic ray experiments, Phys. Rev. D83 (2011) 065013, [arXiv:1005.3209 [hep-ph]].

47. R. Horvat, D. Kekez, P. Schupp, J. Trampetic, J. You, Photon-neutrino interaction in theta-exact covariant noncommutative field theory, Phys. Rev. D84 (2011) 045004, arXiv:1103.3383 [hep-ph].

48. B. Jurco, P. Schupp, and J. Wess, Nonabelian noncommutative gauge theory via noncommutative extra dimensions, Nucl. Phys. B604 (2001) 148–180, hep-th/0102129.

49. Y. Okawa and H. Ooguri, An exact solution to Seiberg-Witten equation of noncommutative gauge theory, Phys. Rev. D64 (2001) 046009, hep-th/0104038.

50. G. Barnich, F. Brandt, and M. Grigoriev, Local BRST cohomology and Seiberg-Witten maps in noncommutative Yang-Mills theory, Nucl. Phys. B677 (2004) 503–534, hep-th/0308092.
[51] J. Zeiner, *Noncommutative quantumelectrodynamics from Seiberg-Witten Maps to all orders in Theta(\(\mu\nu\)).* (Wurzburg U.). Jul 2007. 139 pp. PhD thesis.

[52] A. G. Grozin, *Lectures on perturbative HQET. I.* [hep-ph/0008300]

[53] R. Horvat, A. Ilakovac, P. Schupp, J. Trampetić, and J. You, *Yukawa couplings and seesaw neutrino masses in noncommutative gauge theory,* Phys. Lett. B715, 340-347 (2012), [arXiv:1109.3085]

[54] J. Trampetic, *Signal for space-time noncommutativity: The Z \(\rightarrow\) gamma gamma decay in the renormalizable gauge sector of the theta-expanded NCSM,* SFIN A 1 (2007) 379, [arXiv:0704.0559 [hep-ph]].

[55] H. Grosse and G. Lechner, *Noncommutative Deformations of Wightman Quantum Field Theories,* JHEP 0809 (2008) 131, [arXiv:0808.3459 [math-ph]].

[56] R. Horvat, A. Ilakovac, D. Kekez, J. Trampetic and J. You, *Forbidden and Invisible Z Boson Decays in Covariant theta-exact Noncommutative Standard Model,* arXiv:1204.6201 [hep-ph].

[57] [http://functions.wolfram.com/HypergeometricFunctions/](http://functions.wolfram.com/HypergeometricFunctions/)

[58] M. M. Ettefaghi and M. Haghighat, *Massive Neutrino in Non-commutative Space-time,* Phys. Rev. D77 (2008) 056009, [0712.4032].

[59] J. Gomis and T. Mehen, *Space-time noncommutative field theories and unitarity,* Nucl. Phys. B 591, 265 (2000), [arXiv:hep-th/0005129].

[60] M. Hayakawa, *Perturbative analysis on infrared aspects of noncommutative QED on \(R^{**4}\),* Phys. Lett. B 478, 394 (2000), [arXiv:hep-th/9912094].

[61] M. Hayakawa, *Perturbative analysis on infrared and ultraviolet aspects of noncommutative QED on \(R^{4}\),* [hep-th/9912167].

[62] F. T. Brandt, A. K. Das, J. Frenkel, *General structure of the photon self-energy in noncommutative QED,* Phys. Rev. D65 (2002) 085017, [hep-th/0112127].

[63] L. Alvarez-Gaume and M. A. Vazquez-Mozo, *General properties of noncommutative field theories,* Nucl. Phys. B 668, 293 (2003).

[64] J. Jaeckel, V. V. Khoze and A. Ringwald, *Telltales traces of U(1) fields in noncommutative standard model extensions,* JHEP 0602, 028 (2006), [arXiv:hep-ph/0508075].

[65] S. Abel, C. -S. Chu and M. Goodsell, *Noncommutativity from the string perspective: Modification of gravity at a mm without mm sized extra dimensions,* JHEP 0611 (2006) 058, [hep-th/0606248].

[66] S. A. Abel, J. Jaeckel, V. V. Khoze and A. Ringwald, *Vacuum birefringence as a probe of Planck scale noncommutativity,* JHEP 0609, 074 (2006), [arXiv:hep-ph/0607188].

[67] R. J. Szabo, *Quantum Gravity, Field Theory and Signatures of Noncommutative Spacetime,* Gen. Rel. Grav. 42 (2010) 1-29, [arXiv:0906.2913 [hep-th]].

[68] R. Horvat, J. Trampetic, *Constraining noncommutative field theories with holography,* JHEP 1101 (2011) 112, [arXiv:1009.2933 [hep-ph]].

[69] Wolfram Research, Inc., *Mathematica, Version 8.0,* Champaign, IL (2010).

[70] J. Martin-Garcia, *xAct,* [http://www.xact.es/].

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