I Prefer not to Say: Operationalizing Fair and User-guided Data Minimization

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Abstract

To grant users greater authority over their personal data, policymakers have suggested tighter data protection regulations (e.g., GDPR, CCPA). One key principle within these regulations is data minimization, which urges companies and institutions to only collect data that is relevant and adequate for the purpose of the data analysis. In this work, we take a user-centric perspective on this regulation, and let individual users decide which data they deem adequate and relevant to be processed by a machine-learned model. We require that users who decide to provide optional information should appropriately benefit from sharing their data, while users who rely on the mandate to leave their data undisclosed should not be penalized for doing so. This gives rise to the overlooked problem of fair treatment between individuals providing additional information and those choosing not to. While the classical fairness literature focuses on fair treatment between advantaged and disadvantaged groups, an initial look at this problem through the lens of classical fairness notions reveals that they are incompatible with these desiderata. We offer a solution to this problem by proposing the notion of Optional Feature Fairness (OFF) that follows from our requirements. To operationalize OFF, we derive a multi-model strategy and a tractable logistic regression model. We analyze the effect and the cost of applying OFF on several real-world data sets.

1 Introduction

Data-driven models are being increasingly deployed to support critical decision-making or even make such decisions independently without human oversight. Such crucial scenarios include hiring decisions [4, 34, 38], loan assignments [8, 42, 1], judicial parole decisions [3, 9], and disease risk prediction or diagnosis [31, 26]. While the day-to-day impact of automated data processing is steadily growing, modern regulations such as the European Union’s General Data Protection Regulation (GDPR) [13] or the California Consumer Privacy Act (CCPA) [30] strive to give individuals more control over their personal data. In particular, the principle of data minimization stated in Article 5 of the GDPR demands that personal data shall be “adequate, relevant and limited to what is necessary in relation to the purposes for which they are processed”[13]. In this work, we take a user-centric perspective on this regulation. Therefore, we let individuals decide which data they deem adequate and relevant for processing. To this end, we consider the scenario of machine learned classifiers, where users can voluntarily choose to provide one or many optional features.

To fix ideas, we consider the use-case of college admissions as a running example. Suppose that all applicants are asked to fill out an application form where they enter certain base features, for instance
information on advanced high-school courses and their grade point average (GPA). Additionally, prospective students can provide results of standardized tests, for instance the SAT. To be more specific, during the recent pandemic, many US colleges chose to make this feature optional.\(^1\) This means that applicants can decide whether to provide a relevant result by themselves; alternatively, they can also leave this box empty. We will refer to such an input as an *optional feature* and show an example of this kind of data set in Table 1.

| adv. courses    | GPA   | test score | avail. | admitted |
|-----------------|-------|------------|--------|----------|
| coding, math    | 3.6   | 87%        | 1      | Yes      |
| chemistry, math | 3.7   | N/A        | 0      | Yes      |
| coding, math    | 3.6   | 92%        | 1      | Yes      |
| coding, math    | 3.6   | N/A        | 0      | No       |
| biology         | 3.0   | 56%        | 1      | No       |

Table 1: Fictional samples for the college admissions use-case. We suppose two base features \(b\) and an optional feature \(z^*\), which is either equivalent to an unobserved \(z\), or takes a value of N/A. We introduce an artificial variable \(a \in \{0, 1\}\) to indicate the availability of the feature. The goal is to predict the label \(y\).

We are now facing a dilemma. Applicants with subpar test scores may be less inclined to provide this information, as they think it might lower their odds of acceptance. In this case, it is possible for a statistical model trained on such data to infer some information on the label only from the fact that an optional feature was observed (or not). A vanilla classifier trained on this data may thus systematically assign a lower score to individuals that did not share their data. Later in Section 6, we empirically show that the average rating of individuals that do not provide data can be up to 50\% worse than justified by their base features.

This opens the door to discrimination against the group of users that make use of their mandate to not provide the – often very personal – additional information. However, we argue that this is diametrically opposed to the goal of the colleges that opted to make SAT scores voluntary precisely to grant students that could not take the test a fair chance.

To summarize, we study how machine learning models in the presence of optional features can be learned under several key requirements. First, these models should not penalize users for leaving their data undisclosed (Non-Penalization). This requirement forbids the model to discriminate purely based on the availability of a feature. Second, we also require the model to incentivize users to provide optional data by appropriately leveraging information in the optional features if they are provided (Incentivization). Finally, we desire the models to maintain or improve the performance over a model relying only on the base features (Predictive Non-Degradation). Otherwise, the benefit of including the optional information could be negative.

Perhaps surprisingly, the question of fair treatment between individuals that provide additional information and those that choose to keep their data undisclosed has not been thoroughly considered in previous literature. Therefore, we aim to bridge this gap by making the following contributions:

- We formalize the problem of fair modeling with optional information. Further, we study common notions of group-based fairness in the optional feature setting and find that they fail to fulfill our set of desiderata.
- To solve this problem, we propose the notion of Optional Feature Fairness (OFF) that follows from the requirements of Non-Penalization, Incentivization and Predictive Non-Degradation
- To implement OFF, we derive a general *multi model* strategy that may however require exponential computational complexity. We study model classes under which OFF becomes tractable and analyze the effect and the costs of applying OFF on real-world data sets.

## 2 Related work

**Classification with Missing Values or Labels.** Classification models that can handle missing data have been studied in the previous literature with the goal of minimizing costs or increasing performance [44, 2], obtaining uncertainty estimates [20], or fulfilling classical fairness notions [45, 19, 40, 11]. In a related line of work, classification with noisy [12] or missing labels [22, 35] has been investigated, where the missingness is often a result of *selection bias*. The setting considered

\(^1\)https://www.usnews.com/education/best-colleges/articles/how-the-coronavirus-is-pushing-colleges-to-go-test-optional

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in this work is different in the sense that we are not concerned with fulfilling a fairness notion with respect to sensitive information, but between subjects with and without optional information.

**Data Minimization and the Right to be Forgotten.** The principle of Data Minimization is anchored in the GDPR [13]. A particular case of Data Minimization is the “right to be forgotten”, which enables individuals to submit requests to have their data deleted. In response to this regulation, several works consider the problem of updating a ML model without the need of retraining the entire model [41, 14, 18, 15, 5] or the effect of removals on model explanations [36, 32]. Our work differs from these lines of work as certain feature values may be unavailable from the data collection process. The task at hand is to train a fair model and not to remove influence of certain training samples or values.

**Algorithmic Fairness.** A multitude of formal fairness definitions have been put forward in the literature [39]. Examples include predictive parity [6], equalized odds, equality of opportunity [17], and individual fairness [10]. However, they are still a topic of discussion, for instance because these definitions are known to be incompatible [23] and it has recently been argued that even fair models may be prone to disparate treatment [25]. Additionally, there are several definitions that rely on causal mechanisms to assess fairness, e.g., counterfactual fairness [24] and the notion of unresolved discrimination [21]. While causal approaches to fairness might be preferable, they require information about the causal structure of the data generating process. Moreover, it has recently been shown that causal definitions may lead to adverse consequences, such as lower diversity [29]. For these reasons, we take a non-causal perspective in this work. We will discuss how existing fairness definitions could possibly be applied to the setting with optional features, but we find that none of the fairness definitions conforms with our three desiderata (see Sec. 3.2).

### 3 Problem formulation

For the scope of this work, we suppose to be given data samples of the following form: Every data instance contains a realization of a number of base features $b \in \mathcal{X}^b$, where $\mathcal{X}^b \subseteq \mathbb{R}^n$ is the space of the base features. Furthermore, let there be some optional information $z \in \mathcal{X}^z$, where $\mathcal{X}^z \subseteq \mathbb{R}$ is the value space of the optional feature. However, it is the user’s choice to decide if they want to disclose $z$ to the system, which results in an availability variable $a \in \{0, 1\}$. Accordingly, only imputed samples $z^* = \{z \text{ if } a=1, \text{ else } N/A\}$ are observed, where a value of N/A indicates that a user did not reveal the optional information, e.g., the test score in the admissions case. In summary, the data observations are tuples $x = (b, a, z^*)$ that reside in $\mathcal{X} = \mathcal{X}^b \times \{0, 1\} \times (\mathcal{X}^z \cup \{N/A\})$. For each training sample, we are also provided with a label $y \in \mathcal{Y}$. We suppose there is a data generating distribution $p$ with support $\mathcal{X} \times \mathcal{Y}$ and we have access to an i.i.d. training sample $(x, y) \sim p$. A data sample with the notation is shown in the previous Table 1. We denote the random variables for the respective quantities by $B, A, Z, Z^*$.

In many applications, the goal is to find a function $f : \mathcal{X} \rightarrow \mathcal{Y}$ that models the observed data. In particular, $f : \mathcal{X} \rightarrow [0, 1]$ may predict a probability of a positive outcome or $f : \mathcal{X} \rightarrow \mathbb{R}$ may return a numerical score. We suppose that the test data for which the model will be used come from the same distribution $p$, though with the label $y$ unobserved, and that the information provided is always correct. We additionally consider a loss function $\mathcal{L} : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}$. We want to minimize the expected loss over the entire data distribution. In the following, we restrict ourselves to the common mean squared error loss with $\mathcal{L}(f(x), y) = (f(x) - y)^2$. To minimize this loss, the best possible prediction $f(x)$ would be the conditional expectation:

$$f^*(x) = \arg\min_f \mathbb{E}_p \left[(f(x) - Y)^2\right] = \mathbb{E}[Y|x].$$

### 3.1 Desiderata

Our goal is to learn models $f : \mathcal{X} \rightarrow \mathcal{Y}$ that fulfill the desiderata of Non-Penalization, Incentivization and Non-Degradation. In this section, we crisply formalize these notions.

**Definition 3.1 (Non-Penalization)** For individuals that choose not to provide the optional feature $(a = 0)$, only the provided data should be used in the decision process to compute the outcome.

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2We extend our definitions to integrate multiple optional features in Section 4.

3Supposing $p(Y|x)$ is square-integrable for all $x$. 

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\[ f_{\lambda=0}(b, a, z^*) \], i.e., \( f \) restricted to the domain \( a = 0 \). Information on the availability of certain features may not be used to compute the score in this case.

Intuitively, the principle of Non-Penalization requires that when features remain undisclosed, the information encoded in the unavailability may not be used to compute the prediction score. Moreover, we define Incentivization to enforce that individuals who share optional data receive the most accurate prediction score possible in the decision-making process.

**Definition 3.2 (Incentivization)** Individuals that provide optional features should obtain the best estimate of the outcome given all their information, i.e., the outcome with minimal loss. For the MSE-loss, this is the conditional expectation, so we desire:

\[
E[Y|B = b, A = 1, Z^* = z^*].
\]

Finally, while Non-Penalization explicitly restricts the information that can be used, we do not wish to obtain models that can perform arbitrarily bad. In particular, we demand a model using the optional information to improve, or at least match, the performance of an optimal model not relying on the optional feature.

**Definition 3.3 (Predictive Non-Degradation)** For any density \( p \), the expected loss of a model \( f \) should not increase over the loss of the optimal model \( f_{\text{base}}^* \) relying only on the base features. In particular, for the MSE-loss, \( f_{\text{base}}^*(b) = E[Y|b] \) this requires:

\[
E[(Y - f_{\text{base}}^*(B))^2] = E[(Y - E[Y|B])^2] \geq E[(Y - f(B, A, Z^*))^2].
\]

### 3.2 Common fairness notions are not applicable

As many definitions of fair treatment between an advantaged and a disadvantaged group exist, we investigate whether existing definitions can readily be applied or easily adapted to the optional feature setting considered in this work. In the conventional fairness literature, the protected attribute is usually restricted to not have disparate impact on the prediction model. However, in the optional feature setting the point of departure is different since the optional feature may contain discriminative information that we explicitly want to use. If not stated otherwise, we consider the availability feature \( a \) to be the sensitive attribute. We denote the predicted label by \( \hat{Y} \) and discuss binary labels \( Y \in \{0, 1\} \) as in the original definitions.

**Fairness through Unawareness** [16, 24]. Fairness through unawareness demands that the protected feature \( a \) is not used as an explicit input in the decision-making process. Removing explicit information on the availability can be done easily by dropping the feature \( a \). This makes “Fairness through Unawareness” very easy to implement. However, the information is still implicitly contained in the imputed optional feature through the value \( N/A \). A sufficiently complex classifier is then able to infer this information and include it into its decision-making (cf. experiments in Section 6). Therefore, this fairness notion is not applicable because it violates the requirement of Non-Penalization.

**Predictive Parity** [6]. This notion of fairness constrains the False Discovery Rates to be equal across groups, i.e., \( p(Y = 0|\hat{Y} = 1, A = 0) = p(Y = 0|\hat{Y} = 1, A = 1) \). We argue that this definition and other error rate-based ones will not be overly useful in our setup because they do not allow for Incentivization: the prediction will desirably be more accurate with the feature \( z \) present \( (A = 1) \) because the information in \( z \) should explicitly be used in the decision-making process if users decide to share their data on the optional features. One can make an analogous argument for the notions of equalized odds and equal opportunity [17] (Appendix A.2).

**Conditional Statistical Parity** [10, 7]. As statistical parity is known to be notoriously unfair on an individual level [10, 43], Corbett-Davies et al. [7] define the notion of conditional statistical parity (CSP). It is an extension of statistical parity, where certain attributes are allowed to affect the decision. If we allow all base features \( b \), the resulting definition expressed in expectations would be \( E[Y|B = b, A = 0] = E[Y|B = b, A = 1] \). This definition would require individuals that do not provide additional information to be treated like the average of the population that provided these features. While this definition comes close to our goals of Incentivization and Non-Penalization, we show that the performance of a model with this constraint can be worse than that of a model trained only on the base features. This is the case even under idealized conditions (i.e., known expectations) and when Incentivization is perfectly fulfilled.
Fairness (1D-OFF) with perfect estimators is given by:

\[
\hat{f}_{\text{OFF}}(b, a, z^*) = \begin{cases} 
\mathbb{E}[Y|b, A = 1] & \text{if } a = 0 \\
\mathbb{E}[Y|b, A = 1, Z^* = z^*] & \text{if } a = 1
\end{cases}
\]

with \((Y, B, A, Z^*) \sim p\) that follows from perfect CSP and Incentivization can lead to higher expected MSE losses than the model that only uses the base features, \(f_{\text{base}}^*(b) = \mathbb{E}[Y|b]\), i.e.,

\[
\mathbb{E}[(Y - \hat{f}_{\text{base}}(B))^2] < \mathbb{E}[(Y - \hat{f}_{\text{OFF}}(B, A, Z^*))^2].
\]

We provide a derivation and a density \(p\) that serves as such a counterexample in Appendix C.1.

4 A Definition of Fairness with Optional Features

We have established that common fairness definitions fail to conform to our desiderata of Non-Penalization, Incentivization and Predictive Non-Degradation. Therefore, we suggest a novel notion of fairness for the optional feature setting named Optional Feature Fairness (OFF).

4.1 One-Dimensional Optional Feature Fairness

We first discuss what should happen in the case of \(a = 0\), i.e., an individual has not submitted optional information. To fulfill the requirement of Non-Penalization, we can only use the information in the base features \(b\). The estimate with the lowest error would be \(f_{\text{base}}^*(b) = \mathbb{E}[Y|b]\). In the case \(a = 1\), by Incentivization the outcome \(f_{i=1}^*(b, a, z^*)\) is determined to be \(f_{i=1}^*(b, a, z^*) = \mathbb{E}[Y|b, A = 1, Z^* = z^*]\), the justified outcome. Combining these two cases results in a novel fairness notion that we term Optional Feature Fairness (OFF):

**Definition 4.1 (One-dimensional Optional Feature Fairness, 1D-OFF)** Let \(f : X \rightarrow Y \subseteq \mathbb{R}\) be a model that can handle optional features. The model \(f\) that fulfills one-dimensional Optional Feature Fairness (1D-OFF) with perfect estimators is given by:

\[
f_{\text{OFF}}^*(b, a, z^*) = \begin{cases} 
\mathbb{E}[Y|b] & \text{if } a = 0 \\
\mathbb{E}[Y|b, A = 1, Z^* = z^*] & \text{if } a = 1
\end{cases}
\]

This definition encodes an intuitive notion of fairness with respect to users that opt not to provide their data \((a = 0)\): Their score under \(f\) is constrained to the best estimate for a user with the same base characteristics, no matter if additional data was provided. Contrarily, when the additional information through the optional feature is provided, we return the best estimate using the available information.

We now show that 1D-OFF fulfills all our three desiderata: (i) Non-Penalization is fulfilled because information on the non-availability is not used in the case \(a = 0\). The outcome is only derived from the base features. (ii) Incentivization is fulfilled by definition. (iii) We establish the following theorem to show that 1D-OFF also obeys Predictive Non-Degradation (proof shown in Appendix C.2).

**Theorem 4.1 (OFF obeys Predictive Non-Degradation)** A classifier that fulfills the notion of 1D-OFF with perfect estimators of the conditional mean conforms to Predictive Non-Degradation, i.e., for any density \((Y, B, A, Z^*) \sim p\) with square integrable conditional means,

\[
\mathbb{E}[(Y - f_{\text{OFF}}^*(B))^2] \geq \mathbb{E}[(Y - f_{\text{OFF}}^*(B, A, Z^*))^2].
\]

4.2 Optional Feature Fairness with multiple optional statements

While we have previously considered only a single optional feature, one can easily construct examples where multiple features can be provided voluntarily and independently of each other. For example, the admissions office might voluntarily accept results of another standardized assessment test such as the ACT. Therefore, let there now be \(r\) optional features such that \(z \in \mathcal{X}^r \times \cdots \times \mathcal{X}^r\) and \(\alpha \in \{0, 1\}^r\), where \(\mathcal{X}^r\) are the respective supports of each optional feature. By \(\mathcal{I} \subseteq [r] = \{1, \ldots, r\}\), we denote an index set that contains all feature indices present, i.e., \(\mathcal{I}(\alpha) = \{i \mid \alpha_i = 1, i = 1, \ldots, r\}\). When we index vectors with this set, e.g., \(Z_{\mathcal{I}}\), we refer to the subvector that only contains the indices in \(\mathcal{I}\).
Definition 4.2 (Optional Feature Fairness, OFF) Let \( f : \mathcal{X} \rightarrow \mathcal{Y} \subseteq \mathbb{R} \) be a model that can handle multiple optional features. The model \( f_{\text{OFF}} \) fulfills Optional Feature Fairness, if the following equality holds:

\[
f_{\text{OFF}}(b, a, z^*) = \mathbb{E}_{(B, A, Z^*) \sim p}[Y | B = b, A_{I(a)} = 1, Z_{I(a)} = z_{I(a)}^*],
\]

where \( A_{I(a)} = 1 \) means that each element that is set to 1 in \( a \) needs to be one in \( A \) as well.

For a single feature \( r = 1 \), the index set can either be \( I = \emptyset \) or \( I = \{1\} \) and the definition corresponds to 1D-OFF. The conditional expectation with \( A_{I(a)} = 1 \) effectively constrains the features in \( I \) to be available, but marginalizes over samples with or without further information.

5 Practical approaches for Optional Feature Fairness

5.1 A Brute Force Multi Model Approach

To implement a strategy that conforms to Definition 4.2 in practice, we propose a class of classification functions \( f \) which we term multi models. These models call a different ML model (e.g., Logistic Regression, DNNs, ...) for each combination of available features. They are defined as follows:

\[
f(b, a, z^*) := f^I(b, z^*_I), \text{ if } I = I(a) := \{i | a_i = 1, i = 1, \ldots, r\}.
\]

In this setting \( \{f^I\}_{I \subseteq [r]} : f^I : \mathcal{X} \times \left(\mathcal{X}_{i \in I} \mathcal{X}_i^T\right) \rightarrow \mathcal{Y} \) is a family of ML models that can each specifically handle a combination of available features. By the indexed \( \mathcal{X} \), we denote the cartesian product of the optional features’ spaces. To fulfill OFF, by Definition 4.2 we need each model to return:

\[
f^I(b, z^*_I) = \mathbb{E}_p(Y | B = b, Z^*_I = z^*_I, A_I = 1).
\]

Until now, we have supposed access to perfect estimators of the conditional expectations. However, in practice the conditional expectations can be well approximated by fitting a model \( f^I(b, z^*_I) \) to each subset of data that has at least the features in \( I \) while minimizing a suitable loss such as the MSE. Note that this amounts to training a total number of \( 2^r \) models. For large \( r \), this approach is computationally intractable. Motivated by this observation, we show in the following section that the number of parameters can be reduced to \( O(rn) \) or even to \( O(n + r) \) under reasonable conditions.

5.2 Logistic Regression Models with Fair Handling of Optional Features

As a realistic example, we choose logistic regression models. These models are well-understood and commonplace in ranking applications such as loan assignments [8]. Let us consider a binary classification task with true labels \( y \in \{0, 1\} \). We allow the availability and the optional information content to depend on the label and the base features \( b \), which corresponds to the graphical model in Figure 1. In Appendix D.2, we derive a realistic family of Gaussian feature distributions conforming to this graphical model that will result in posterior distributions of logistic form for each subset of available optional features. Hence, we propose the following condition:

**Condition 5.1 (Subset models are logistic)** Suppose for each \( I \subseteq [r] \), there exists a \( w \in \mathbb{R}^n \), \( \beta \in \mathbb{R}^{|I|} \), and \( s \in \mathbb{R} \) that allow to represent the odds \( (Y = 1 | b, z^*_I, A_I = 1) \) in the form:

\[
\begin{align*}
\mathbb{P}(Y = 1 | B = b, Z^*_I = z^*_I, A_I = 1) &= \exp \left[ w(I)^\top b + \beta(I)^\top z^*_I + s(I) \right],
\end{align*}
\]

This condition permits to represent each of the conditional densities required by OFF with a logistic model. Note that it still allows \( w(I), \beta(I), s(I) \) to freely depend on one out of the \( 2^r \) subsets \( I \) chosen, requiring an exponential number of fits. With the dependency structure in Figure 1, the following theorem shows that we can fit fair models with fewer parameters.
Theorem 5.1 Under the factored distribution corresponding to the graphical model in Fig. 1 and logistic subset models (condition 5.1), the odds\((Y = 1|\mathbf{b}, \mathbf{z}_I^+, A_I = 1)\) are given by

\[
\text{odds}(Y = 1|\mathbf{b}, \mathbf{z}_I^+, A_I = 1) = \exp \left[ \mathbf{w}^\top \mathbf{b} + t + \left( \sum_{i \in I} \omega_i^+ \mathbf{b}_i + \beta_i z_i^+ + s_i \right) \right],
\]

with parameters \(\mathbf{w} \in \mathbb{R}^n, t \in \mathbb{R}, \omega_i \in \mathbb{R}_+, \beta_i \in \mathbb{R}, s_i \in \mathbb{R} \) for \(i = 1, \ldots, r\), that do not depend on the subset \(I \subseteq [r]\).

The proof can be found in Appendix C.3. As this theorem shows, under the above conditions only \(nr + n + 2r + 1 \in \mathcal{O}(rn)\) values need to be estimated to be able to represent all \(2^r\) combinations of missing features where \(n\) is the number of base features and \(r\) denotes the number optional features. This allows for remarkable efficiency improvements: We provide a constructive algorithm to learn such models that only requires fitting \(r + 1\) logistic regression models and discuss how the number of parameters can be further reduced to \(\mathcal{O}(r + n)\) in a full Naive Bayes setting in Appendix D.

6 Experimental Evaluation

Data sets. We investigate the implications of applying the notion of Optional Feature Fairness (OFF) on four real-world data sets. Details regarding preprocessing, data sets and model hyperparameters for all experiments can be found in Appendix E.1. To link to our college admission example, we use the law school data set (“law”), where we predict whether certain individuals will obtain above-average grades in law school depending on features such as their GPA, test results, and gender. The HELOC data set contains data of individuals applying for a loan and the task is to predict their risk of default. The COMPAS data set comprises data from offenders of Broward Country, Florida, whose chance of recidivism in the following two years is to be estimated. It is commonly used in the fairness literature. In the domain of healthcare, the Diabetes data set (“diab.”) requires predicting how likely individuals are to exhibit diabetes from medical data such as blood pressure or glucose level.

Availability. We introduce stochastic availability dependent on a feature’s value to numerical features in the data sets. We compute a probability of unavailability \(p(A_i = 0|z_i)\) by applying a sigmoid function centered at the feature mean and sample the availability \(a\) from the respective conditional distribution. This results in feature values equivalent to the mean having a 50% chance to be unavailable that increases as the feature value continues to grow above the mean. However, we invert the probability if higher values are beneficial to obtain the positive outcome.

Unfair treatment on real-world data sets. First, we would like to stress the effect that non-compensation for unavailable features can have on real data. Therefore, we train a model (Random Forest, see Appendix E.2 for more models) on all features on the data set where stochastic availability has been introduced into one feature according to the sigmoidal strategy and unavailable values are filled with zeros (“imputed”). We subsequently train another model that only uses the base features by fully dropping the optional feature from the data (“base model”). We consider the subset of individuals with unavailable feature values and report the average (probabilistic) prediction of the negative class for both models in Table 2. We observe that the average prediction of a negative outcome is up to 53% higher for this group than that of the base model, which only uses the information that was intentionally provided. This impressively shows how the model used the availability information to drastically worsen the scores for this group and how it violates the principle of Non-Penalization.

The costs of fairness with respect to optional information. In the next experiment, we investigate the effects of applying OFF on the performance of the models. Commonly, the costs of classifiers trained with additional fairness constraints are higher than that of an unconstrained model [7]. We follow the same setup as in the previous experiment. However, we also include a multi model for OFF (with a single optional feature, this requires only two models) and a model constrained by the notion of conditional statistical parity in the comparison. We use the average 0-1-loss as a cost function which corresponds to the misclassification rate.\(^4\) We report another possible performance measure in Appendix E.3. The results in Table 3 confirm that OFF improves over the base model and thus allows to benefit from optional information, which aligns with our theoretical results in Theorem 4.1. Moreover, the costs using the model derived from CSP can sometimes even be higher than those of

\(^4\)We verified that all our data sets are roughly balanced.
We show that it allows to leverage the optional information through a fair performance.

Table 4: Fairness gap (OFF-Gap, %) and misclassification rate for different data sets. When using OFF, the costs are lower that that of the base feature model (dropping the optional features) and when using the fairness notion derived from conditional statistical parity.

Multiple Optionality with Logistic Regression Models for OFF. In a last experiment, the impact of making multiple features optional is considered. We select the maximum number \( r \) of discriminative features to be optional that still allows multi models on the subsets to be estimated from sufficient sample sizes (details provided in Appendix E.4). In this setting, we put our efficient logistic regression model with the OFF-constraint to the test (“OFF-LR”). We therefore study the average absolute gap of a real model \( f \) to the analytical or fitted multi model \( f_{\text{OFF}}^* \) that perfectly fulfills OFF, which is given by

\[
\text{OFF-Gap}(f) = \mathbb{E}_p \left[ |f(B, A, Z^*) - f_{\text{OFF}}(B, A, Z^*)| \right].
\]

After we verified that the gap converges to zero when the conditions are met (Appendix E.4), we are interested in the sensitivity of the derived model to small violations found in real data sets. Because no ground truth data is available, we use a multi model estimator built on a logistic regression model as the reference. We list the respective OFF-Gaps and performance indicators in Table 4. The results demonstrate that a small gap to the reference OFF remains present that is however drastically reduced compared to the base model’s gap. The same holds for the unfair model, which was trained on imputed data (Appendix E.4). We observed that the performance of the considered OFF-LR model increased over the base model’s score and is on par with its multi model equivalent while requiring significantly fewer model fits.

7 Conclusion and Future Work

In this paper, we formalized the problem of fair treatment in a setting where users can choose to provide additional information. We proposed three desiderata to obtain discrimination-free and incentivizing models. After ruling out common fairness notions, we define Optional Feature Fairness (OFF). We show that it allows to leverage the optional information through a fair performance increment over models not considering these inputs. However, OFF-compliant models still grant users that keep their data private with an appropriate chance to obtain the positive outcome.

For future work, we would like to investigate how the notion of OFF can be operationalized with more complex models such as decision trees or neural networks and with other loss functions apart from the MSE. Additionally, we are interested in deriving meaningful worst-case guarantees for models that confirm to OFF either analytically or approximately.
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A Related Work and Notions

A.1 Additional Related Work

Estimation of causal effects in the presence of missing data. The works by Mohan et al. [28, 27] introduce graphical models for incomplete data and study the consistent estimation of causal effects amidst missing values. Our work differs as we are not concerned with estimating true causal effects but focus on building a definition of fairness in the presence of optional data.

A.2 Additional Fairness Notions

In this section, we provide arguments for why more notions of fairness also cannot be applied to fulfill our desiderata in the setting of optional information.

Equalized Odds and Equal Opportunity [17]. Equalized odds requires the predicted label $\hat{Y}$ and the the protected attribute $A$ to be conditionally independent given the true label $Y$. Formally, this means $p(\hat{Y}|A,Y) = p(\hat{Y}|Y)$ for all values for $Y, A, \hat{Y}$. This effectively constrains the true and false positive rates to be equal across groups. However, by the Goal of Incentivization, it is required to use discriminative information in the optional feature, which will lead necessarily lead to lower missclassification rates for subjects with $A=1$. Equal opportunity is a relaxation of Equalized odds that only demands $p(\hat{Y}|A=1, Y=1) = p(\hat{Y}|A=0, Y=1)$, thus constraining the true positive rates across groups. To fulfill this notion, for $A=1$, the true positive rate would have to be kept artificially low to match that of the case $A=0$, with less information. This would thus result in a lower $p(\hat{Y}=1|A=0, Y=1)$, than possible. Supposing $Y=1$ to be the desirable outcome (e.g., admission into college), this means that less subjects are rewarded with the justified positive outcome. We argue that this kind of behavior is not desirable in practice.

Statistical Parity [10, 24]. This definition is satisfied by a classifier if subjects in both protected and non-protected groups have an equal probability of getting a positive classification outcome, $p(\hat{Y}=1|M=0) = p(\hat{Y}=1|M=1)$. This definition may lead to different thresholds where people that choose to provide information are getting a lower score to achieve parity. In cases where the missingness is strongly correlated to a class-discriminative base feature, this definition would even forbid using this base features’ full distinctive power, because one has to equalize over both missingness classes. Furthermore, statistical parity is known to be notoriously unfair on an individual level [10, 43].

Individual fairness [10]. Fairness definitions in this category use a distance metric $m$ to define similarities $m(x_i, x_j)$ between individuals $x_i$ and $x_j$. Considering the application in mind, the sensitive attributes should not play a role in determining the distance. The classifier output distributions for $f(x_j)$ and $f(x_i)$ that are compared by some divergence $D$ should not differ more than the distance between these individuals, i.e., $D(f(x_j), f(x_i)) \leq m(x_i, x_j)$ [10]. In the considered setting, following the proposition by Verma et al. [39], we could define the distance to be 0, if individuals have the same base features $b$. This would effectively constrain the the classification outcome to be identical independently of the optional feature specified, effectively prohibiting its use. Even when defining other distance metrics, the classification outcome will still be constrained to a certain range, again contradicting our ideal of Incentivization.

B Intuition and Additional Examples

In this section, we provide a simple example to show the problem of possible unfairness and provide more intuition for our notion of Optional Feature Fairness.

B.1 Standard losses may lead to unfair treatment

We revisit the example of college admission, to show how imputation leads to possibly unfair treatment. Suppose we are given the samples $\{ (x', \hat{y}') \}_{i=1}^N$ with $N = 5$ from Tab. 1. Using the standard Mean-Squared Error (MSE) loss, we solve the following empirical risk minimization
We argue that in the case of candidate 4, the availability information was implicitly used to compute when marginalizing over $Z$ would have been trained on the data set $F$ divided the optional features and those that did not. We now consider a non-probabilistic prediction function, we want $f^*$ to match the average output of the individuals that provided features. By our requirement of Incentivize-a-points with $N/A \hat{Y}$ for a single optional feature with random availability $A$, to be $N/A \hat{Y}_{\text{full}}$, the likelihood of unavailability can be entirely accounted for by the features itself should not be used in the determination of the model outcome when no additional information is available.

We argue that in the case of candidate 4, the availability information was implicitly used to compute the score and resulted in a lower outcome. If only the base features had been available, i.e., $f^*$ would have been trained on the data set $\{(b^i,y^i)\}_{i=1,k}$, the model outcome would be $f^*(b^4) = \frac{1}{|\{b^4=b\}|} \sum_{\{b^i=b\}} y^i$ with $f^*(b^4) = \frac{2}{3}$ as two out of three students with a GPA of 3.6 and a master’s in computer science have been admitted. In this work, we argue that the unavailability of certain features itself should not be used in the determination of the model outcome when no additional information is available.

B.2 Example: Missing at random (MAR) data.

For missing at random data [37], the likelihood of unavailability can be entirely accounted for by the observed features $b$ and is not affected by the partially observed $z$ and the label $y$. Formally, for a single optional feature with random availability $A$, $p(A = 0|b) = p(A = 0, b, z, y)$ for every $z \in \mathcal{Z}, y \in \mathcal{Y}$. Therefore,

$$p(y|b, A = 0) = \frac{p(y,b,A = 0)}{p(b,A = 0)} = \frac{p(y)p(b|y)p(A = 0|b, y)}{\sum_{y'} p(y')p(b|y')p(A = 0|b, y')}$$

(5)

$$= \frac{p(y)p(b|y)p(A = 0|b)}{p(A = 0|b) \sum_{y'} p(y')p(b|y')} = \frac{p(y)p(b|y)p(A = 0|b)}{p(A = 0|b)} \sum_{y'} p(y')p(b|y')$$

(6)

$$= \frac{p(y)p(b|y)}{\sum_{y'} p(y')p(b|y')} = p(y|b).$$

(7)

Therefore, we also have $E[Y|B = b, A = 0] = E_{p}[Y|B = b]$, indicating that the missingness does not affect the expected value of the label over the entire data distribution. Therefore, a perfect discriminative model with $f(x) = E_{p}[Y|x]$ will fulfill Definition 4.1 right away.

C Proofs

C.1 Counterexample: Conditional statistical parity can be inferior to the base model

The notion of conditional statistical parity phrases in expectations $E[\hat{Y}|B = b, A = 0] = E[\hat{Y}|B = b, A = 1]$ requires the prediction averages conditioned on $b$ to be equal among groups that provided the optional features and those that did not. We now consider a non-probabilistic prediction function $\hat{Y} = f(b, a, z^*)$. Plugging in the functional form would result in the following definition: $f_{\text{incl}}(b, a = 0, z^*) = E_{Z^* \sim p(Z^*|b, a=1)} [f(b, a = 1, Z^*)], \forall b$. In the case $a = 0$, $z^*$ is constrained to be 0/1 so we can ignore its value. The subscript is used to indicate the restriction of $f$ on the set of points with $a=0$. This definition constrains the output $f_{\text{incl}}$, when no additional features provided, to match the average output of the individuals that provided features. By our requirement of Incentivize-a-points, we want $f_{\text{incl}}$ to be the best approximation of $E[Y|B, A = 1, Z^*]$. Thus, we would have to set $f_{\text{incl}}$ to be $f_{\text{incl}}(b, a = 0, z^*) = E_{Z^* \sim p(Z^*|b, a=1)} [E[Y|B = b, A = 1, Z^*]] = E[Y|B = b, A = 1]$ when marginalizing over $Z^*$. Overall, this derivation results in the function $f_{\text{exp}}$ presented in Lemma 3.1,

$$f_{\text{exp}}(b, a, z^*) = \begin{cases} E[Y|b, A = 1] & \text{if } a = 0 \\ E[Y|b, A = 1, Z^* = z^*] & \text{if } a = 1 \end{cases}$$

(8)
In this section we present a simple example to show that this function $f_{\text{csp}}$ derived from notion of conditional statistical parity may lead to an increased Mean-Squared-Error Loss compared to the base model (not using the optional feature) even when the estimators of the conditional means are perfect.

For the example, we take any value $b$ and suppose $p(y|b, A, z)$ depends on $A$ but that $Z$ is useless and does not contribute any new information, i.e., $\forall z \in X^z; A \in \{0, 1\} : p(y|b, A, z) = p(y|b, A)$. Furthermore, we set the outcome to be deterministic of $A$:

$$p[Y = 0|b, A = 0] = 1$$

$$p[Y = 1|b, A = 1] = 1.$$  \hspace{1cm} (9)

(10)

In this case, the base model would predict

$$f_{\text{base}}(b) = E[Y|b] = E[Y|b, A = 0]p(A = 0|b) + E[Y|b, A = 1]p(A = 1|b)$$

$$= p(A = 1|b) \triangleq \alpha$$

independently of the realization of $A$ (because it is not allowed to use this information). The expected MSE Loss is given by:

$$L_{\text{base}} = p(A = 0|b)(\alpha - 0)^2 + p(A = 1|b)(\alpha - 1)^2$$

$$= (1 - \alpha)\alpha^2 + \alpha(1 - \alpha)^2 = (1 - \alpha)\alpha + (1 - \alpha) = (1 - \alpha)\alpha.$$  \hspace{1cm} (13)

If the notion derived from Conditional statistical parity is used, we would use $f_{\text{csp}}(b) = E[Y|b, A = 1] = 1$ to predict in both cases and obtain:

$$L_{\text{csp}} = p(A = 0|b)(0 - 1)^2 + p(A = 1|b)(1 - 1)^2 = p(A = 0|b) = 1 - \alpha.$$  \hspace{1cm} (15)

We see that in every case where $\alpha = p(A = 1|b) < 1$ this results in

$$L_{\text{csp}} = 1 - \alpha > (1 - \alpha)\alpha = L_{\text{base}}.$$  \hspace{1cm} (16)

We have now shown that for an arbitrary $b$, the loss can be higher that of the base feature model. We can complete the example to the overall loss over a distributions of $b$'s by supposing $p(B = b) = 1$, which would however be a degenerate distribution. As a broader alternative, one can conduct the above derivations for a set of $b \in B$ and suppose any probability distribution with support in $B$, i.e., $p(B \notin B) = 0$.

C.2 Proof of Theorem 4.1: OFF obeys Predictive Non-Degradation

In the case of the base model with as perfect estimator $f_{\text{base}}(b) = E[Y|b]$, the expected MSE loss is the expected conditional variance,

$$L_{\text{base}} = E[(Y - f_{\text{base}}(B))^2] = E[(Y - E[Y|B])^2]$$

$$= E[\text{Var}[Y|B]]$$

$$= p(A = 0)E[(Y - E[Y|B])^2|A = 0] + p(A = 1)E[(Y - E[Y|B])^2|A = 1]$$

$$= (1 - \alpha)\alpha^2 + \alpha(1 - \alpha)^2 = (1 - \alpha)\alpha + (1 - \alpha) = (1 - \alpha)\alpha.$$  \hspace{1cm} (17)

where the last decomposition follows from the law of total expectation. For the model that fulfills OFF with optimal estimators of the conditional expectation, i.e.,

$$f_{\text{OFF}}(b, \alpha, z^*) = \begin{cases} 
E[Y|b], & \text{if } A = 0 \\
E[Y|b, A = 1, Z^* = z^*], & \text{if } A = 1 
\end{cases}$$

the loss will be equal to

$$L_{\text{OFF}} = E[(Y - f_{\text{OFF}}(B, \alpha, Z^*))^2]$$

$$= p(A = 0)E[(Y - E[Y|B])^2|A = 0] + p(A = 1)E[(Y - E[Y|B, A = 1, Z^*])^2|A = 1]$$

$$= (1 - \alpha)\alpha^2 + \alpha(1 - \alpha)^2 = (1 - \alpha)\alpha + (1 - \alpha) = (1 - \alpha)\alpha.$$  \hspace{1cm} (18)

The difference is given by

$$L_{\text{base}} - L_{\text{OFF}} = p(A = 1)\left( E[(Y - E[Y|B])^2|A = 1] - E[(Y - E[Y|B, A = 1, Z^*])^2|A = 1] \right).$$  \hspace{1cm} (19)
We conduct the following reformulations that hold for any specific $b \in \mathcal{X}$:

$$
\mathbb{E}[(Y - \mathbb{E}[Y|b])^2|b, A = 1] = \mathbb{E}[(Y - \mathbb{E}[Y|b] + \mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b, A = 1])^2|b, A = 1]
$$

(25)

$$
= \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1]) + (\mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b])^2|b, A = 1]
$$

(26)

$$
= \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1])^2|b, A = 1]
$$

(27)

$$
+ \mathbb{E}[2(Y - \mathbb{E}[Y|b, A = 1])(\mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b])|b, A = 1]
$$

(28)

$$
+ \mathbb{E}[\mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b]^2|b, A = 1]
$$

(29)

$$
= \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1])^2|b, A = 1] + (\mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b])^2|b, A = 1]
$$

(30)

$$
\geq \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1])^2|b, A = 1] + \mathbb{E}[\mathbb{E}[Y|b, A = 1] - \mathbb{E}[Y|b]^2|b, A = 1]
$$

(31)

where in Eqn. (31) the middle term cancels out, because $\mathbb{E}[2(Y - \mathbb{E}[Y|b, A = 1])|b, A = 1] = 0$ (and the second factor is a constant). The final term in this line is constant and the expectation operator can be omitted. We can then apply the conditional law of total variance to introduce the variable $Z$ and get

$$
\mathbb{V}[\mathbb{E}[Y|b, A = 1] = \mathbb{E}[\mathbb{V}[Y|b, A = 1, Z^*]|b, A = 1] + \mathbb{E}[\mathbb{E}[Y|Z, b, A = 1]|b, A = 1] = \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1, Z^*])^2|b, A = 1]
$$

(32)

In summary, we have shown that for every $b \in \mathcal{X}$

$$
\mathbb{E}[(Y - \mathbb{E}[Y|b])^2|b, A = 1] \geq \mathbb{E}[(Y - \mathbb{E}[Y|b, A = 1, Z^*])^2|b, A = 1]
$$

(33)

from which follows the same for the expected value over the random variable $B$

$$
\mathbb{E}[(Y - \mathbb{E}[Y|B])^2|A = 1] \geq \mathbb{E}[(Y - \mathbb{E}[Y|B, A = 1, Z^*])^2|A = 1].
$$

(34)

\[\square\]

### C.3 Proof of Theorem 5.1. Less parameters are required with logistic subset models

By the graphical model shown in Fig. 1, we have that

$$
\text{odds}(Y = 1|b, z_{x_k}^*, A_{x_k} = 1) = \frac{p(Y = 1, b, z_{x_k}, A_{x_k} = 1)}{p(Y = 0, b, z_{x_k}, A_{x_k} = 1)} = \frac{p(b|Y = 1)}{p(b|Y = 0)} \left( \prod_{i \in \mathcal{I}} \frac{p(z_{i}, A_{i} = 1|b, Y = 1)}{p(z_{i}, A_{i} = 1|b, Y = 0)} \right) \frac{p(Y = 1)}{p(Y = 0)}
$$

(37)

(38)

for any set $\mathcal{I} \subseteq [r]$. By the condition of logistic subset models, we know (by setting $\mathcal{I} = \emptyset$), that

$$
\text{odds}(Y = 1|b) = \frac{p(b|Y = 1)}{p(b|Y = 0)} \frac{p(Y = 1)}{p(Y = 0)} = \exp \left[ \mathbf{w}(\emptyset)^\top \mathbf{b} + s(\emptyset) \right].
$$

(39)

Furthermore, for any $k \in 1, \ldots, r$, plugging in $\mathcal{I} = \{k\}$ in Eqn. (38) results in

$$
\text{odds}(Y = 1|b, z_{k}^*, A_{k} = 1) = \frac{p(b|Y = 1)}{p(b|Y = 0)} \frac{p(z_{k}, A_{k} = 1|b, Y = 1)}{p(z_{k}, A_{k} = 1|b, Y = 0)} \frac{p(Y = 1)}{p(Y = 0)}
$$

(40)

$$
= \exp \left[ \mathbf{w}(\emptyset)^\top \mathbf{b} + s(\emptyset) \right] \frac{p(z_{k}, A_{k} = 1|b, Y = 1)}{p(z_{k}, A_{k} = 1|b, Y = 0)}
$$

(41)

Note that $Z_{\{k\}} = Z_{k}$ and $A_{\{k\}} = A_{k}$.

However, we can also express the same odds by a logistic model with different parameters, i.e.,

$$
\text{odds}(Y = 1|b, z_{k}^*, A_{k} = 1) = \exp \left[ \mathbf{w}([k])^\top \mathbf{b} + \beta([k]) z_{k}^* + s([k]) \right].
$$

(42)
We can also consider a Naïve Bayes models with binary features which can possibly be unavailable as we furthermore suppose the features are binary, what is left to do is to estimate the ratios to be estimated to cover all feature availability combinations which is tractable.

A graph representation of this model can be found in Figure 2. In this case, we can express the odds ratio

$$\text{odds}(Y = 1|b, z_i^+, A_i) = \frac{p(Y = 1, b, z_i^+, A_i)}{p(Y = 0, b, z_i^+, A_i)} = \frac{p(Y = 1) p(b|Y = 1) p(z_i|Y = 1)p(A_i|z_i, Y = 1)}{p(Y = 0) p(b|Y = 0) p(z_i|Y = 0)p(A_i|z_i, Y = 0)}.$$  

Plugging the result for this density ratio and the one for the base model obtained in Eqn. (39) back into the general factorization Eqn. (38) and setting $w := w(0)$ and $t := s(0)$ we arrive at the claimed form

$$\text{odds}(Y = 1|b, z_i^+, A_i) = \frac{p(Y = 1, b, z_i^+, A_i)}{p(Y = 0, b, z_i^+, A_i)} = \exp \left[ w^T b + \sum_{i \in I} \omega_i b + \beta_i z_i^+ + s_i \right].$$  

### D Tractable Models with Optional Feature Fairness

#### D.1 Naïve Bayes models revisited

We can also consider a Naïve Bayes models with binary features which can possibly be unavailable as in Poole et al. [33]. Suppose that we have a Naïve Bayes model with independent availability mechanisms, i.e., the availability of feature $i$ is only dependent on the label $y$ and the corresponding feature value $z_i$ and thus $p(b, a, z, y) = (\prod_{i=1}^n p(b_i|y))(\prod_{i=1}^n p(z_i|y)p(a_i|z_i, y))p(y)$. A graphical representation of this model can be found in Figure 2. In this case, we can express the odds ratio as

$$\text{odds}(Y = 1|b, z_i^+, A_i) = \frac{p(Y = 1, b, z_i^+, A_i)}{p(Y = 0, b, z_i^+, A_i)} = \frac{\prod_{i \in I} p(b_i|Y = 1)}{\prod_{i \in I} p(b_i|Y = 0)} \frac{\prod_{i \in I} p(z_i|Y = 1)p(A_i|z_i, Y = 1)}{\prod_{i \in I} p(z_i|Y = 0)p(A_i|z_i, Y = 0)} \frac{p(Y = 1)}{p(Y = 0)}.$$  

As we furthermore suppose the features are binary, what is left to do is to estimate the ratios $p(z_i|Y = 1)p(A_i|z_i, Y = 1)$ for $b_i \in \{0,1\}$ and $p(z_i|Y = 0)p(A_i|z_i, Y = 0)$ for $z_i \in \{0,1\}$. This requires only $2r + 2n$ parameters to be estimated to cover all feature availability combinations which is tractable.
D.2 A parametric family of distributions with logistic subset models

In this section, we describe a set of conditions that can be used to construct a family of densities that have logistic subset models. We suggest the following assertions and show that they will result in logistic subset models:

C1: Base model is logistic:
\[ p(Y = 1|b) = \sigma(w^\top b + t) \]

C2: Availability is cond. independent
\[ \forall I \subseteq [r]: p(A_I = 1|b, y) = \prod_{i \in I} p(A_i = 1|b, y) \]

C3: Mut. independence when present:
\[ \forall I \subseteq [r]: p(z_I|b, y, \mathcal{A}_I = 1) = \prod_{i \in I} p(z_i|b, y, A_i = 1) \]

C4: Availability is sigmoidal:
\[ p(A_i = 1|b, Y = 1) = N(b)\sigma(u_i^\top b + \lambda_i) \]
\[ p(A_i = 1|b, Y = 0) = N(b) - p(A_i = 1|b, Y = 1) \]

C5: Base-dependent Normal distributions:
\[ (z_i|b, y, A_i = 1) \sim N(v_i^\top b + \tau_i(y), \eta^2) \]

Intuitively, after ensuring that the base feature model has a logistic form (C1), the next two assumptions follow directly from the graphical dependency model (C2, C3), see Figure 1. C4 suggests the availability should sigmoidally depend on the base features with a different offset for each class. The last condition (C5) allows the \( z_i \) to depend on the base features \( b \) with the same \( v_i \) for both classes \( y \). However, a different offset by the coefficient \( \tau_i \) can be added for each class.

In this special case, we can show that each of the models required will have the form of logistic regression again.

We start by determining some density ratios that will arise later:
\[ \frac{p(A_i = 1|b, Y = 1)}{p(A_i = 1|b, Y = 0)} = \frac{N(b)\sigma(u_i^\top b + \lambda_i)}{N(b)\left(1 - \sigma(u_i^\top b + \lambda_i)\right)} = \exp(u_i^\top b + \lambda_i) \]  

(49)

where the identity \( \frac{\sigma(x)}{1 - \sigma(x)} = \exp(x) \) was used. Furthermore,
\[ \frac{p(z_i|b, Y = 1, A_i = 1)}{p(z_i|b, Y = 0, A_i = 1)} = \frac{\exp\left(-\frac{(z_i - v_i^\top b - \tau_i)^2}{2\eta^2}\right)}{\exp\left(-\frac{(z_i - v_i^\top b - \tau_0)^2}{2\eta^2}\right)} \]

(50)

\[ = \exp\left[-\frac{(z_i - v_i^\top b - \tau_i)^2}{2\eta^2} - \frac{(z_i - v_i^\top b - \tau_0)^2}{2\eta^2}\right] \]

(51)

\[ = \exp\left[-\frac{2(v_i^\top b)(\tau_i - \tau_0) + \tau_i^2 - \tau_0^2}{2\eta^2}\right] \]

(52)

\[ = \exp\left[\frac{2(\tau_i - \tau_0)(z_i - v_i^\top b) + \tau_i^2 - \tau_0^2}{2\eta^2}\right] \]

(53)

\[ = \exp\left[\frac{\eta^{-2}(\tau_i - \tau_0)z_i - \eta^{-2}(\tau_{i1} - \tau_{01})v_i^\top b + \frac{1}{2} \eta^{-2}(\tau_{01}^2 - \tau_{11}^2)}{\beta_i}\right] \]

(54)

Let \( I \subseteq [r] \) be the index set of the present features once again. We can insert the previous results and obtain
When the parameters are estimated, inference is performed differently depending on each combination of available features according to the equation in Theorem 5.1. 

\[
\text{oDDS}(Y = 1|b, Z_T, A_T=1) = \frac{p(Y = 1, b, Z_T, A_T=1)}{p(Y = 0, b, Z_T, A_T=1)} = \frac{p(Y = 1|b) \ p(A_T = 1|b, Y=1) \ p(Z_T|b, Y=1, A_T=1)}{p(Y = 0|b) \ p(A_T = 1|b, Y=0) \ p(Z_T|b, Y=0, A_T=1)}
\]

\[= \frac{p(Y = 1|b) \ \prod_{i \in I} p(z_i|b, Y=1, A_i=1)p(A_i=1|b, Y=1)}{p(Y = 0|b) \ \prod_{i \in I} p(z_i|b, Y=0, A_i=1)p(A_i=1|b, Y=0)}
\]

\[= \exp(w^\top b + t + \sum_{i \in I} (u_i - \gamma_i) z_i + \beta_i z_i + \lambda_i + \theta_i)
\]

As this derivation shows, each subset model will again be of the logistic form.

### D.3 Estimating parameters for the logistic regression model with a fairness constraint

With the results of Theorem 5.1, estimating these parameters can easily be done by fitting \( r + 1 \) logistic regression models. The fit is inspired by the proof of the initial theorem and is conducted in the following steps.

1. Fit a model on the base features (corresponding to \( I = \emptyset \)) and use its fitted coefficients \( w(\emptyset), s(\emptyset) \) as estimates of \( w \) and \( t \) respectively.

2. Fit models on base features plus one optional feature at a time (i.e. use the index set \( I = \{k\} \) for each \( k = 1, \ldots, r \)). All records that do not have the optional feature are dropped for the fit. Use the fitted coefficients \( w(\{k\}), \beta(\{k\}), s(\{k\}) \) to estimate \( \omega_k, \beta_k, s_k \) according to Equation (44).

When the parameters are estimated, inference is performed differently depending on each combination of available features according to the equation in Theorem 5.1.

### E Additional Experimental Results and Details

#### E.1 Data Sets and Preprocessing

The COMPAS data set\(^7\) was originally collected by ProPublica and contains features describing criminal defendants in Broward County, Florida. It also contains their respective recidivism score provided by the COMPAS algorithm and whether or not they reoffended within the following two years. For our analysis, we only kept features relevant for the prediction of recidivism within the next two years and dropped irrelevant features such as name or date. Furthermore, we turned the categorical features race, sex and charge degree into numerical features by encoding the categories with integers.

The diabetes data set\(^6\) was collected by the National Institute of Diabetes and Digestive and Kidney Diseases. It contains diagnostic measurements of female patients that are at least 21 years old. The target variable "Outcome" describes whether or not a person has diabetes.

The Home Equity Line of Credit (HELOC) data set\(^5\) is a large collection of HELOC applications from anonymized homeowners, collected by the financial services provider FICO. The target variable RiskPerformance is "Bad" if the applicant was at least 90 past due within the two years after opening the credit account. "Good" and "Bad" were encoded as 1 and 0, respectively. We dropped the feature "ExternalRiskEstimate" from the data set since it seemed heavily correlated with all other important features (most likely it is a function of the other features) and thus minimized the effect of introducing stochastic availability.

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\(^5\)https://www.kaggle.com/s/danofer/compass

\(^6\)https://www.kaggle.com/s/mathchi/diabetes-data-set

\(^7\)https://community.fico.com/s/explainable-machine-learning-challenge?tabset=155d9=3
The Law School Admission data set contains information on students from law schools across the United States. Features are collected prior to their entry to law school and include race, sex, entrance exam scores (LSAT), grade-point average (GPA) and regional group. The predicted variable is the z-score of the first year average grade (ZFYA). Note that we use this dataset in a binary classification manner and only predict if the z-score is above the average.

We provide an overview of the characteristics of the different data sets in Table 6.

| Data Sets   | Label         | Num. features | Num. samples (N) |
|-------------|---------------|---------------|------------------|
| heloc       | RiskPerformance | 22            | 10459            |
| compas      | two_year_recid | 9             | 7192             |
| law school  | ZFYA          | 5             | 21791            |
| diabetes    | Outcome       | 8             | 768              |

Table 6: Characteristics of the data sets studied in this work.

Availability: We make values available by the following scheme over continuous features $z_i \in \mathcal{X}$:

$$p(A_i = 0 | z_i) = \text{sigmoid}(\lambda_i(z_i - \bar{z}_i)) = \frac{1}{1 + \exp(-\lambda_i(z_i - \bar{z}_i))}$$  \hspace{1cm} (59)

where we denote the empirical feature mean by $\bar{z}_i$ and $\lambda_i \in \mathbb{R}$ denotes a parameter that specifies how quickly the probability of unavailability ($A_i = 0$) increases with higher feature values (for positive values of $\lambda_i$). For negative values of $\lambda_i$, values of the feature that are lower than the mean are more likely to be unavailable. We show examples of the probabilities used in Figure 3.

Figure 3: Value distribution of the respective optional features per data set and corresponding function $p(A_i = 0 | z_i)$ with parameter $\lambda$ used to introduce stochastic availability.

E.2 Experiment 1: Unfair treatment on real-world data sets

This section provides additional results for Experiment 1. Tables 7 – 9 show the results for the same experiment, but with different probabilistic classification models or different hyperparameter settings.

*https://github.com/mkusner/counterfactual-fairness*
While the extent of change differs, for every model and data set the individuals that do not have feature values are rated worse.

| data   | opt. feature | imputed | base model | change     |
|--------|--------------|---------|------------|------------|
| law    | LSAT         | 31.49   | 25.38      | -6.11 ±2.99|
| heloc  | AvgMInFile   | 63.35   | 60.22      | -3.14 ±0.34|
| compas | priors_count | 80.32   | 35.60      | -44.72 ±2.02|
| diab.  | Glucose      | 57.74   | 37.77      | -19.98 ±2.83|

Table 7: Using a Random Forest, but with depth=8: Average predictions (in %) of negative outcome for samples without optional information.

| data   | opt. feature | imputed | base model | change     |
|--------|--------------|---------|------------|------------|
| law    | LSAT         | 29.19   | 23.99      | -5.20 ±2.22|
| heloc  | AvgMInFile   | 60.89   | 58.60      | -2.19 ±0.78|
| compas | priors_count | 79.13   | 38.09      | -41.04 ±1.13|
| diab.  | Glucose      | 54.60   | 39.18      | -15.42 ±3.15|

Table 8: Using AdaBoost: Average predictions (in %) of negative outcome for samples without optional information.

| data   | opt. feature | imputed | base model | change     |
|--------|--------------|---------|------------|------------|
| law    | LSAT         | 31.60   | 25.62      | -5.99 ±1.82|
| heloc  | AvgMInFile   | 62.16   | 59.79      | -2.37 ±0.35|
| compas | priors_count | 81.83   | 38.77      | -41.93 ±1.66|
| diab.  | Glucose      | 53.10   | 37.63      | -15.47 ±2.20|

Table 9: Using gradient boosting: Average predictions (in %) of negative outcome for samples without optional information.
E.3 Experiment 2: The cost of fairness with respect to optional information

This section contains additional results for Experiment two. Table 10 shows the results when using the area under the ROC-curve as a cost function. Tables 11 – 13 show the results for the same experiment when using different models, again using the missclassification rate as cost function. We did not observe large qualitative differences depending on the specific model or cost function used. Once (AdaBoost), we observed a slightly better performance for the base feature model than of the OFF-compliant one. However, these results were non-significant, which means they can be due to random effects.

| Data Sets | Base model | Cond. parity | OFF | Fair models | No fairness |
|-----------|------------|--------------|-----|-------------|-------------|
| heloc     | 20.90 ±0.11| 22.85 ±0.78  | 20.77 ±0.11| 20.60 ±0.12|
| compas    | 33.80 ±0.13| 38.30 ±1.15  | 28.82 ±0.41| 33.57 ±0.17|
| diabetes  | 23.14 ±0.50| 23.76 ±2.13  | 17.85 ±1.10| 17.03 ±0.62|
| law school| 38.27 ±0.09| 36.89 ±0.53  | 36.57 ±0.18| 36.52 ±0.35|

Table 10: Using Random Forest, max_depth=4: Costs of OFF (1 – area under the ROC curve) on different data sets.

| Data Sets | Base model | Cond. parity | OFF | Fair models | No fairness |
|-----------|------------|--------------|-----|-------------|-------------|
| heloc     | 26.92 ±0.31| 26.88 ±0.78  | 26.66 ±0.28| 26.58 ±0.30|
| compas    | 38.27 ±0.31| 40.73 ±0.59  | 35.39 ±0.43| 37.77 ±0.31|
| diabetes  | 30.36 ±1.30| 31.04 ±2.69  | 26.40 ±1.68| 23.31 ±0.79|
| law school| 40.86 ±0.14| 40.38 ±0.39  | 40.24 ±0.30| 40.12 ±0.32|

Table 11: Using Random Forest, max_depth=8: Costs of OFF (misclassification percentages) on different data sets.

| Data Sets | Base model | Cond. parity | OFF | Fair models | No fairness |
|-----------|------------|--------------|-----|-------------|-------------|
| heloc     | 27.49 ±0.99| 27.38 ±1.02  | 27.46 ±0.32| 27.24 ±0.29|
| compas    | 38.71 ±0.89| 42.12 ±1.37  | 34.83 ±0.48| 37.30 ±0.55|
| diabetes  | 27.53 ±2.41| 34.97 ±5.49  | 25.42 ±1.71| 25.55 ±1.54|
| law school| 40.56 ±0.71| 42.02 ±1.18  | 40.07 ±0.19| 40.02 ±0.24|

Table 12: Using gradient boosting: Costs of OFF (misclassification percentages) on different data sets.

E.4 Experiment 3: Logistic Regression Models for OFF

In this section, we provide additional details regarding the experiment with the derived logistic regression model.

**Synthetic data experiment.** We initially conduct a synthetic data experiment to verify our theory by using the class of distributions derived in Appendix D.2. We create two normally distributed base features and three optional features to test interesting dependency combinations by using the parameters in Table 16. This distribution includes cases where:

- the availability distribution depends on the base features ($u \neq 0$, feature 1)
Table 13: Using AdaBoost: Costs of OFF (misclassification percentages) on different data sets.

| Data Sets     | Base model | Cond. parity | OFF    | No fairness |
|---------------|------------|--------------|--------|-------------|
| heloc         | 28.06 ± 0.92 | 28.78 ± 0.80 | 28.17 ± 0.35 | 28.38 ± 0.36 |
| compas        | 38.22 ± 1.11 | 45.28 ± 0.19 | 34.37 ± 0.45 | 37.40 ± 0.34 |
| diabetes      | 27.92 ± 3.85 | 35.91 ± 6.54 | 25.16 ± 1.55 | 25.36 ± 1.44 |
| law school    | 40.74 ± 0.49 | 43.23 ± 2.02 | 40.15 ± 0.22 | 39.87 ± 0.33 |

Figure 4: Fairness gap to the theoretical OFF using different models based on logistic regression.

- the availability distribution depends on the class value ($\lambda \neq 0$, feature 1, feature 2)
- the feature value depends on the base features and the class value ($v \neq 0$, $\tau(0) \neq \tau(1)$, feature 1, feature 2, feature 3)

We draw increasing numbers of samples from the known distribution and fit the OFF-LR model to this data. As Figure 4 shows, the OFF-Gap (this time using the analytical expectations as a reference) approaches zero with increasing sample sizes for the multi model (logistic regression) and the OFF-LR model, whereas an unfair LR model fails to reach OFF.

**Real data experiment.** For the real data experiment, we apply the following preprocessing steps to induce stochastic availability:

- We identify the most discriminative features by dropping each feature from the data set and reporting the decline in predictive performance of a model trained without the feature with respect to a model trained on all features. We rank the features starting with the one resulting in the highest performance loss.
- We select the $r$ most discriminative features, such that on average, each submodel has at least 150 samples out of the initial data set size of $N$ to be fitted with, i.e.,

  $$r = \inf \left\{ r' \in \mathbb{N} : \frac{N}{2^{r'}} > 150 \right\}.$$

- In the case of the law school data set, the allowed size would be 6, but there are only 3 features with a meaningful order (required to induce stochastic availability by the sigmoidal approach), so we use $r = 3$ in this case. The optional features are listed in Table 14.
- We independently induce stochastic availability into each feature using the sigmoidal strategy. We use a $\lambda_i = \pm \frac{1}{\sqrt{\text{Var}(f_i)}}$, which is effectively equivalent to applying a sigmoid over normalized feature values. The signs are determined by the context such that negative indicators are more likely to be not provided and are also reported in Table 14.

Additional results including the unfair model can be found in Table 15. We observed this model to exhibit a higher OFF-gap than the OFF-LR model.
Table 14: Features made optional in the experiment with multiple optional features. Direction: (+) means higher values more likely to be unavailable, (-) indicates lower values to be more likely to be unavailable. The direction was chosen such that feature values that lead to more negative outcomes tend to be undisclosed more frequently.

Table 15: Fairness gap (OFF-Gap) and area under the receiver-operator characteristic (ROC-AUC) for different models based on Logistic regression. We include two additional unfair baselines: a single model fitted on zero-imputed data, and a multi model fit on subsets of the data for each availability combination \( a \).

Table 16: Parametric distribution parameters used in the synthetic data experiment. The used density covers all possible dependencies between availability, feature values and the base features that are allowed by the graphical model.