Charge screening and confinement in the massive Schwinger model

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Abstract

Within the framework of Euclidean path integral and mass perturbation theory we compute the Wilson loop of widely separated external charges for the massive Schwinger model. From this result we show for arbitrary order mass perturbation theory that integer external charges are completely screened, whereas for noninteger charges a constant long-range force remains.

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1 Introduction

The Schwinger model is two-dimensional QED with one fermion flavour. For a massless fermion the model is exactly soluble and has been studied intensively in the last decades ([1] – [8]). In fact, it is equivalent to the theory of one massive, free scalar field (Schwinger boson), which may be interpreted as a fermion-antifermion bound state. When the fermion is massive (massive Schwinger model), the Schwinger boson turns into an interacting particle that may form bound states and undergo scattering processes ([9] – [13]).

Another feature of both massless and massive model is that the perturbative vacuum is not invariant under large gauge transformations, and, therefore, instanton-like gauge fields are present and a \( \theta \) vacuum has to be introduced as a new, physical vacuum ([2] – [7], [9, 11, 14]).

A further aspect that is present in both models and stimulated some investigations is confinement ([5, 15, 16, 17]). In both models there are no fermions in the physical spectrum, so ”confinement” is realized in a certain sense. When these confinement properties are further tested by putting widely separated external probe charges into the vacuum, the two models, however, behave differently. In the massless model the two charges are completely screened by vacuum polarization, and the ”quark-antiquark” potential approaches a constant for large distances. This may be interpreted as a dynamical Higgs mechanism, where the Schwinger boson acts as a massive gauge boson.

In [17] the following behaviour of the massive Schwinger model was shown to hold in first order mass perturbation theory: As long as the external charges are integer multiples of the fundamental charge, \( g = ne, n \in \mathbb{N} \), these charges are completely screened as in the massless model. On the other hand, when \( g \neq ne \), the potential between the probe charges rises linearly for large distances. So ”screening” is realized in the massless model, whereas true confinement takes place in the massive model.

In this article we want to generalize the first order result of [17] to arbitrary order mass perturbation theory (mass perturbation theory is discussed e.g. in [10, 11, 13]).

2 String tension from the Wilson loop

The Euclidean vacuum functional of the massive model, for general \( \theta \), has the following two equivalent representations

\[
Z(m, \theta) = \sum_{k=-\infty}^{\infty} \int DA_k^\mu D\bar{\Psi} D\Psi e^{\int dx \left( \Psi (i\partial - eA) + m\right) \bar{\Psi} + \frac{1}{2} F^2 + \theta F \bar{\Psi} \Gamma_5 \Psi + \frac{i}{2} \bar{\Psi} \gamma_5 \Psi + \frac{1}{2} F^2 \right)} 
\]

(1)

\[
Z(m, \theta) = \sum_{k=-\infty}^{\infty} \int DA_k^\mu D\bar{\Psi} D\Psi e^{\int dx \left( \Psi (i\partial - eA) + m\cos \theta \bar{\Psi} \Psi + im\sin \theta \bar{\Psi} \gamma_5 \Psi + \frac{1}{2} F^2 \right)} 
\]

(2)

\( (F = \frac{1}{2} \epsilon_{\mu\nu} F^{\mu\nu}, k \ldots \text{instanton number}) \), which are related by the chiral anomaly. \( Z(m, \theta) \) may be computed within mass perturbation theory by simply expanding the mass term.
It may be proved that $Z(m, \theta)$ exponentiates (as is, of course, expected),

$$Z(m, \theta) = e^{V\epsilon(m, \theta)}$$

$$\epsilon(m, \theta) = m\Sigma \cos \theta + \frac{m^2\Sigma^2}{4\mu_0^2}(E_+ \cos 2\theta + E_-) + o(m^3)$$

where $V$ is the space-time volume, $\Sigma$ is the fermion condensate of the massless model, $\mu_0$ is the Schwinger boson mass of the massless model, $\mu_0^2 = \frac{e^2}{2\pi}$, and $E_\pm$ are some numbers ($E_+ = -8.91, E_- = 9.74$); $\epsilon(m, \theta)$ is the vacuum energy density. A property of $\epsilon(m, \theta)$ that we need in the sequel is the fact that it is an even function of $\theta$,

$$\epsilon(m, \theta) = \sum_{l=0}^{\infty} \epsilon_l \cos l\theta,$$

where the instanton sectors $k = \pm l$ contribute to $\epsilon_l$ (the $\epsilon_l$, in principle, contain arbitrary orders of $m$). This property, equ. (5), may be seen most easily from the representation (2) of the vacuum functional. Indeed, suppose we expand the mass term in (2). There the perturbation term is $\int dx (m\cos \theta \bar{\Psi}\Psi + im\sin \theta \bar{\Psi}\gamma^5 \Psi)$, and the expansion is about a massless theory with vanishing vacuum angle, $\theta = 0$. As a consequence, parity is conserved in the theory we are expanding about, and only even powers of $P = \bar{\Psi}\gamma^5 \Psi$ may contribute in the perturbation series. Therefore, arbitrary powers of $\cos \theta$ but only even powers of $\sin \theta$ may occur, which we wanted to prove.

So let us turn to the determination of the confinement behaviour. A usual way to investigate confinement is the computation of the string tension from the Wilson loop. The Wilson loop for a test particle of arbitrary charge $g = qe$ is defined as (the additional factor $i$ in Stokes’ law is due to our Euclidean conventions, see e.g. [8])

$$W_D = \langle e^{ig\int_D A_\mu dx^\mu} \rangle = \langle e^{g\int_D F(x)d^2x} \rangle = \langle e^{2\pi i q\int_D \nu(x)d^2x} \rangle$$

where $\nu(x)$ is the Pontryagin index density, $\nu(x) = -\frac{i}{2\pi}F(x)$ and $\nu$ the Pontryagin index. Further $\partial D$ is the contour of a closed loop and $D$ the enclosed region of space-time. We are interested in the string tension for very large distances; further we are able to explicitly separate the area dependence, therefore we set $D \to V$ in the sequel.

For the VEV of an exponential the following formula holds,

$$\langle e^{2\pi i \nu} \rangle = \exp \left[ \sum_{n=1}^{\infty} \frac{(2\pi i q)^n}{n!} \langle \nu^n \rangle_c \right]$$

where $\langle \rangle_c$ denotes the connected part of the $n$-point function. These VEVs are given by

$$\langle \nu^n \rangle_c = V \int dx_2 \ldots dx_n \langle \nu(0)\nu(x_2) \ldots \nu(x_n) \rangle_c = V (-i)^n \frac{\partial^n}{\partial \theta^n} \epsilon(m, \theta)$$

as is obvious from the vacuum functional (1). Performing the derivatives (8) on the vacuum energy density (5) we have to separate even ($\sim \cos l\theta$) and odd ($\sim \sin l\theta$) powers
of derivatives. We find for the Wilson loop

\[ W = \exp \left[ V \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} (-1)^n l^{2n} \epsilon_l \cos l \theta + V \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} (-1)^n l^{2n-1} \epsilon_l \sin l \theta \right] \]

\[ = \exp \left[ V \sum_{l=0}^{\infty} \epsilon_l \cos l \theta \left( \frac{(2\pi ql)^{2n}}{(2n)!} \right) + V \sum_{l=0}^{\infty} \epsilon_l \sin l \theta \sum_{n=1}^{\infty} (-1)^n \frac{(2\pi ql)^{2n-1}}{(2n-1)!} \right] \]

\[ = \exp \left[ V \sum_{l=0}^{\infty} \epsilon_l \cos l \theta \left( \cos 2\pi ql - 1 \right) - V \sum_{l=0}^{\infty} \epsilon_l \sin l \theta \sin 2\pi ql \right]. \] (9)

The string tension is defined as

\[ \sigma := -\frac{1}{V} \ln W = \sum_{l=0}^{\infty} \epsilon_l \left( \cos l \theta \left( 1 - \cos 2\pi ql \right) + \sin l \theta \sin 2\pi ql \right) \] (10)

and may be interpreted as the force between two widely separated probe charges \( g = qe \), where all quantum effects are included.

As indicated in the introduction, we find that, whenever the probe charge is an integer multiple of the fundamental charge, \( q \in \mathbb{N} \), the charges are screened, and the Wilson loop does not obey an area law. Observe that this result is exact!

For noninteger probe charges there is no complete screening and the string tension may be computed perturbatively,

\[ \sigma = m \Sigma \left( \cos \theta (1 - \cos 2\pi q) + \sin \theta \sin 2\pi q \right) + \frac{m^2 \Sigma^2 E_+}{4\mu_0^2} \left( \cos 2\theta (1 - \cos 4\pi q) + \sin 2\theta \sin 4\pi q \right) + o(m^3) \] (11)

showing that in the massive Schwinger model and for noninteger probe charges there remains a constant force (linearly rising potential) for very large distances. So in the massive Schwinger model true confinement is realized instead of charge screening in the general case.

### 3 Summary

As claimed, we succeeded in generalizing the first order results of [14] to arbitrary order mass perturbation theory: integer external probe charges are completely screened, whereas a linearly rising potential is formed between widely separated noninteger probe charges. There has been some debate about this point, therefore we should perhaps add a comment. From equ. (8) it is obvious that only nontrivial instanton sectors may contribute to the formation of the string tension. Therefore this string tension is a strictly nonperturbative phenomenon in the sense of ordinary (electrical charge \( e \)) perturbation theory and may not be detected by conventional perturbative methods. On the other hand, the mass perturbation theory is an expansion about the true, physical vacuum and takes all the nontrivial structure of the model into account. Therefore, a property that
holds for arbitrary order mass perturbation theory should hold for the model as an exact property.

The behaviour of the massless model – complete charge screening for arbitrary probe charges – we find as a trivial byproduct of our result (e.g. equ. (11)).

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