Diluting Cosmological Constant via Large Distance Modification of Gravity

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Abstract

We review a solution (Ref. [1]) of the cosmological constant problem in a brane-world model with infinite-volume extra dimensions. The solution is based on a nonlinear generally covariant theory of a metastable graviton that leads to a large-distance modification of gravity.

From the extra-dimensional standpoint the problem is solved due to the fact that the four-dimensional vacuum energy curves mostly the extra space. The four-dimensional curvature is small, being inversely proportional to a positive power of the vacuum energy. The effects of infinite-volume extra dimensions are seen by a brane-world observer as nonlocal operators.

From the four-dimensional perspective the problem is solved because the zero-mode graviton is extremely weakly coupled to localized four-dimensional sources. The observable gravity is mediated not by zero mode but, instead, by a metastable graviton with a lifetime of the order of the present-day Hubble scale. Therefore, laws of gravity are modified in the infrared above the Hubble scale. Large wave-length sources, such as the vacuum energy, feel only the zero-mode interaction and, as a result, curve space very mildly. Shorter wave-length sources interact predominantly via exchange of the metastable graviton. Because of this, all standard properties of early cosmology, including inflation, are intact.

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1 Introduction

The cosmological constant problem can be cast into two questions. First, there is an old question:

(i) Why is the vacuum energy determined by a momentum scale much smaller than any reasonable cut-off scale in effective field theory of particle interactions?

This is sometimes referred to as the “old” cosmological constant problem. In view of the present astrophysical observations [2] one should also find an answer to the second question:

(ii) How come that the vacuum energy and the matter energy are comparable today? Do we live in a special epoch?

This is the so-called “cosmic coincidence” problem. To our knowledge the only existing framework, which can address both questions simultaneously, is the anthropic approach [3, 4]. Below we will concentrate on a dynamical solution of the “old” cosmological constant problem suggested in [1] which is based on a nonlinear and generally covariant model of metastable graviton proposed in Refs. [5, 6]. We have nothing to say about the “cosmic coincidence” problem.

Before reviewing the solution of Ref. [1] let us formulate the question properly. As it stands, the question (i) is ill-posed for our purposes. Let us first recall why the vacuum energy in the universe is normally assumed to be small.

In general relativity the cosmological expansion of the universe is described by a standard metric

\[ ds^2 = dt^2 - a^2(t) \, d\vec{x}^2. \]

Here \( a(t) \) is the scale factor and \( t \) is time in the co-moving coordinate system (we assume for definiteness that three-dimensional curvature is zero). In the presence of the vacuum energy density \( E \), the scale factor \( a(t) \) obeys the Friedmann equation

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{E}{3 M_{Pl}^2}, \]

where \( \dot{a} \equiv da/dt \) (for simplicity we set \( E > 0 \)).

We have no direct experimental way to measure \( E \). Instead, we measure space-time curvature by cosmological observations, and then determine \( E \) through Eq. (2). Thus, claiming that \( E \) should be small we implicitly assume that Eq. (2) is valid for arbitrarily large length scales. This assumption does not need to be true.

Many approaches to the cosmological constant problem were designed to dynamically cancel the right-hand side of Eq. (2). Within four-dimensional local field theories with a finite number of degrees of freedom dynamical cancellation is ruled out by Weinberg’s no-go theorem [7]. We take an alternative route. Following [1], we accept that the vacuum energy in our world can be large, but we question the validity of Eq. (2) in the extreme low-energy approximation.

We use the framework of Refs. [5, 6] where gravity in general, and the Friedmann equation, in particular, is modified for wavelengths larger than a certain critical
value. The cosmological constant problem is then remedied in the following way: Due to large-distance modification of gravity the energy density $\mathcal{E} \gtrsim (1 \text{ TeV})^4$ does not curve the space as it would in the conventional Einstein gravity. Therefore, the observed space-time curvature is small, despite the fact that $\mathcal{E}$ is huge (as it comes out naturally). This is the most crucial point of the approach of Ref. [1] — the point where we depart from the previous investigations.

Before we come to details, let us briefly discuss the Weinberg no-go theorem adapted to the present case (for details see Ref. [4]). The theorem states that $\mathcal{E}$ cannot be canceled (without fine-tuning) in any effective four-dimensional theory that satisfies the following conditions:

(a) General covariance;
(b) Conventional four-dimensional gravity is mediated by a massless graviton;
(c) Theory contains a finite number of fields below the cut-off scale;
(d) Theory contains no negative norm states.

Since we do not try to cancel $\mathcal{E}$ but rather intend to suppress the space-time curvature induced by it, one could have argued that the theorem is inapplicable to begin with. However, this argument is incomplete and unsatisfactory. The point is that in any theory obeying conditions (a-d) Eq. (2) is valid. Therefore, in any such theory small $H^2$ would require small $\mathcal{E}$.

We conclude that a successful solution must violate at least one condition in (a-d). The solution of Ref. [1] does violate (b) and (c). In particular, the zero-mode graviton, that mediates gravity in the far-infrared, has a coupling which is many orders of magnitude smaller than the Newton coupling. That is why large value of $\mathcal{E}$ does not induce huge curvature.

The mode that mediates gravity between shorter wave-length sources, such as most of the sources in the observable part of the universe, is coupled to the sources with the usual Newton constant. This gives rise to conventional interactions for observable sources in the universe.

Note that the division in two different modes is purely for convenience. In general one could just drop the mode expansion language altogether and state that the strength of gravity depends on the wave length of the source. Although in the present case condition (c) is violated, in other possible realizations of the present idea this may not be necessary. At the same time, violation of condition (b) seems inevitable in any model that solves the cosmological constant problem through large-distance modification of gravity.

In four-dimensional general relativity gravitational interactions are mediated (at least at distances $r \gtrsim 0.1 \text{ mm}$) by a massless spin-2 particle, the graviton $h_{\mu\nu}$. General covariance (and the absence of ghosts and tachyons) requires the universality of the graviton coupling to matter

$$h_{\mu\nu} T^{\mu\nu}. \quad (3)$$

General covariance also fixes uniquely the low-energy effective action to be the
Einstein-Hilbert action
\[ S = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{g} \left( R - 2\Lambda \right), \]

where the cosmological constant \( \Lambda = -\mathcal{E}/M_{\text{Pl}}^2 \) is included. The universal coupling to all sorts of energy, including the vacuum energy, is the reason for the emergence of the cosmological constant problem.

If the measured four-dimensional gravity were not mediated by an exactly massless state, the universality of coupling could be avoided. Thus, one may hope that in such theories very large wave-length sources (such as the vacuum energy) may effectively decouple from four-dimensional gravity, eliminating the cosmological constant problem. This is what happens in theories with infinite-volume extra dimensions. This phenomenon can be understood from the point of view of a four-dimensional brane observer as a modification of the equation of motion for the graviton. In conventional linearized general relativity the graviton obeys the following free field equation:
\[ \nabla^2 h_{\mu\nu} = 0, \]

while in the present case \[8, 1\] (see also Sec. 4) the graviton obeys a modified linearized equation
\[
\left\{ 1 + \frac{M_*^{N-2}}{c_1(N) M_*^{N-2} + c_2(N) \left( \nabla^2 \right)^{N/2}} \frac{1}{r_c^2 \nabla^2} \right\} \nabla^2 h_{\mu\nu} = 0, \]

where \( M_* \) is the scale of higher dimensional theory, \( c_{1,2} \) are some constants, and \( N \) denotes the number of extra dimensions. \( r_c = M_{\text{Pl}}/M_*^2 \) is the critical crossover distance beyond which gravity is modified, as described by Eq. (6). The effective strength of gravity is set by the operator in the braces, which tends to unity for the short wave length (\( \ll r_c \)) gravitons. As a result, Eq. (6) becomes indistinguishable from Eq. (5). Thus, in this region, the theory reproduces predictions of ordinary gravity with the standard \( G_N \) coupling. However, in the deep-infrared region, where the momenta are smaller than \( r_c^{-1} \), new terms in (6) dominate over the conventional term (5). Hence, infrared physics is modified. It is this modification that serves as a loophole in the no-go theorem for solving the cosmological constant problem. It is remarkable that such a modification takes place in a manifestly generally covariant theory.

2 A brief overview of the model

The effective low-energy action in these theories takes the form \[8, 9\]
\[ S = M_*^{2+N} \int d^4x d^N \rho \sqrt{G} \mathcal{R} + \int d^4x \sqrt{g} \left( \mathcal{E} + M_{\text{ind}}^2 R + \mathcal{L}_{\text{SM}} \right). \]
Here $G_{AB}$ stands for a $(4 + N)$-dimensional metric $(A, B = 0, 1, 2, \ldots, 3 + N)$, while $\rho$ are “perpendicular” coordinates. For simplicity we do not consider brane fluctuations. Thus, the induced metric on the brane is given by

$$\bar{g}_{\mu\nu}(x) \equiv G_{\mu\nu}(x, \rho_n = 0), \quad (n = 4, \ldots, 3 + N).$$  \hspace{1cm} (8)

The first term in (8) is the bulk Einstein-Hilbert action for $(4 + N)$-dimensional gravity, with the fundamental scale $M_*$. The expression in (7) has to be understood as an effective low-energy Lagrangian valid for graviton momenta smaller than $M_*$. We imply that, in addition, there are an infinite number of gauge-invariant high-dimensional bulk operators suppressed by inverse powers of $M_*$. The $M_*^2 \ln R$ term in (7) is the four-dimensional Einstein-Hilbert (EH) term of the induced metric. This term plays a crucial role. It ensures that the laws of four-dimensional gravity are reproduced at observable distances on the brane despite the fact that there is no localized zero-mode graviton. Its coefficient $M_{\text{ind}}$ is another parameter of the model. $\mathcal{E}$ denotes the brane tension. Thus, the low-energy action, as it stands in (7), is governed by three parameters $M_*$, $M_{\text{ind}}$ and $\mathcal{E}$. Both parameters $M_*$ and $M_{\text{ind}}$ are perturbatively-stable under quantum corrections. The parameter $\mathcal{E}$ plays the role of the vacuum energy.

For $\mathcal{E} = 0$ the gravitational dynamics on the brane is as follows. The infinite extra space notwithstanding, a brane observer measures four-dimensional gravitational interaction up to a certain critical distance $r_c$. The potential between two static sources on the brane scales as

$$V(r) \propto -\frac{1}{M_{\text{ind}}^2 r},$$  \hspace{1cm} (9)

for distances in the interval

$$M_*^{-1} \lesssim r \lesssim r_c,$$  \hspace{1cm} (10)

where

$$r_c \sim \begin{cases} 
M_{\text{ind}}^2/M_*^3 & \text{for } N = 1 \\
M_{\text{ind}}/M_*^2 & \text{for } N > 1
\end{cases}.$$  \hspace{1cm} (11)

However, at distances smaller than $M_*^{-1}$ and larger than $r_c$ gravity changes. An important requirement is that gravity must be soft above the scale $M_*$. The expression in (9) fixes $M_{\text{ind}}^2 = M_{\text{Pl}}^2/2$. In order for the late-time cosmology to be standard we require that $r_c \sim H_0^{-1} \sim 10^{28}$ cm. This restricts the value of the bulk gravity scale to lie in the ballpark

$$10^{-3} \text{ eV} \lesssim M_* \lesssim 100 \text{ MeV} \quad \text{for } N = 1,$$

$$M_* \sim 10^{-3} \text{ eV} \quad \text{for } N > 1.$$  \hspace{1cm} (12)
Inclusion of non-vanishing $E \gg M_4^4$ triggers inflation on the brane. The inflation rate is rather peculiar and is given by [1]

$$H^2 \sim M_4^2 \left( \frac{M_4^4}{E} \right)^{\frac{2}{N-2}}.$$  \hfill (13)

(This formula is not applicable to the $N = 2$ case; see Ref. [10] for a discussion of induced gravity in this case.) Therefore, a large vacuum energy density $E$ causes a small rate of inflation for $N > 2$.

The following question immediately come to one’s mind: How can such a behavior be understood from the standpoint of a four-dimensional observer on the brane? We will answer this question using the linearized analysis of [5].

The effect can be best understood in terms of the four-dimensional mode expansion. From this perspective a high-dimensional graviton represents a continuum of four-dimensional states and can be expanded in these states. Below we will be interested only in spin-2 components for which the KK decomposition can schematically be written as follows:

$$h_{\mu \nu}(x, \rho_n) = \int d^N m \epsilon_{\mu \nu}^m(x) \sigma_m(\rho_n),$$ \hfill (14)

where $\epsilon_{\mu \nu}^m(x)$ are four-dimensional spin-2 fields of mass $m$ and $\sigma_m(\rho_n)$ are their wave-function profiles in extra dimensions. The strength of individual mode coupling to a brane source is given by the value of the wave function at the position of the brane, that is $\sigma_m(0)$. Four-dimensional gravity on the brane is mediated by exchange of all the above modes. Each of these modes gives rise to a Yukawa type gravitational potential. The net result is

$$V(r) \propto \frac{1}{M_4^{2+N}} \int_0^\infty dm \, m^{N-1} |\sigma_m(0)|^2 \frac{e^{-m}}{r}.$$ \hfill (15)

It is a crucial property of the model [7] that four-dimensional gravity on the brane is recovered for $r \ll r_c$ due to the fact that modes with $m > 1/r_c$ have suppressed wave-functions [11, 12] and, therefore, the above integral is effectively cut-off at the upper limit at $m \sim 1/r_c$. Most easily this can be seen from the propagator analysis. Gravitational potential (15) on the brane is mediated by an “effective” 4D graviton which can be defined as

$$h_{\mu \nu}(x, 0) = \int d^N m \epsilon_{\mu \nu}^m(x) \sigma_m(0).$$ \hfill (16)

The Green’s function for this state can be defined in a usual way. Using (16) and orthogonality of the $\epsilon_{\mu \nu}^m(x)$ states we obtain

$$G(x - x', 0)_{\mu \nu, \gamma \delta} = \langle h_{\mu \nu}(x, 0) h_{\gamma \delta}(x', 0) \rangle = \int d^N m |\sigma_m(0)|^2 \langle \epsilon_{\mu \nu}^m(x) \epsilon_{\gamma \delta}^m(x') \rangle.$$ \hfill (17)
From now on we will suppress the tensor structure, which is not essential for this discussion. Passing to the Euclidean momentum space we get the following expression for the scalar part of the propagator

\[ G(p, 0) = \int dm \frac{m^{N-1} |\sigma_m(0)|^2}{m^2 + p^2}. \]  

This is the spectral representation for the Green’s function

\[ G(p, 0) = \int ds \frac{\rho(s)}{s + p^2}, \]  

with \( s \equiv m^2 \) and

\[ \rho(s) = \frac{1}{2} s^{\frac{N-2}{2}} |\sigma_{\sqrt{s}}(0)|^2. \]  

Therefore, the spectral representation can be simply understood as the Kaluza-Klein mode expansion (14). Then, the wave-function suppression of the heavy modes can be read off from Eqs. (18) and (19) by using the explicit form of the propagator \( G(p) \) [6, 8],

\[ G(p, 0) = \frac{1}{M_{Pl}^2 p^2 + M_{*}^{2+N} D^{-1}(p, 0)}, \]  

where \( D^{-1}(p, 0) \) is the inverse Green’s function of the bulk theory with no brane. For the purposes of the present discussion it is enough to note that at large momenta \( p \gg r_c^{-1} \) the above propagator behaves as [5, 6]

\[ G(p, 0) \approx \frac{1}{M_{Pl}^2 p^2}, \]  

which is the propagator of a massless four-dimensional graviton with the coupling \( 1/M_{Pl}^2 \). Substituting (22) in the left-hand side of (19) we find that the function \( \rho(s) \) must be suppressed at \( s \gg r_c^{-2} \). If so, the relation (20) implies that the wave functions of the heavy modes must be vanishingly small as well.

For the \( N = 1 \) case both the propagator [6] and the wave-function profiles can be evaluated analytically [12],

\[ G(p, 0) = \frac{1}{M_{Pl}^2 p^2 + 2M_{*}^3 p}, \]  

and

\[ |\sigma_m(0)|^2 = \frac{4}{4 + m^2 M_{Pl}^4/M_{*}^6}. \]  

This shows that all the modes which are heavier than \( r_c^{-1} = M_{*}^3/M_{Pl}^2 \) are suppressed on the brane. Substituting (24) into (15) we derive the usual Newton potential (9) at distances \( r \ll r_c \).

One can interpret the above Green’s function as describing a metastable state that decays into the bulk states with the lifetime \( \tau_c \sim r_c \). A remarkable fact is that the existence of such a metastable state is perfectly compatible with the exact four-dimensional general covariance.
3 Dilution of the cosmological constant

It is instructive to rederive the above-mentioned results with extra dimensions being compactified at very large cosmological distances. For non-vanishing $\mathcal{E}$ the compactification is not trivial. If we were to set the compactification radius $L$ smaller than the gravitational radius of the brane $[13, 14]$,

$$
\rho_g \sim M_s^{-1} \left( \frac{\mathcal{E}}{M_*^4} \right)^{-\frac{1}{N-2}} \,,
$$

(25)

this would distort the brane gravitational background very strongly. Therefore, $L$ has to be at least somewhat larger than $\rho_g$. For realistic values of $\mathcal{E}$ and $M_*$ this leads to an estimate $L \gtrsim H_0^{-1} \sim 10^{28}$ cm $[1]$. In what follows for simplicity we will not distinguish the values of $L$, $\rho_g$ and $H_0^{-1}$, in spite of the fact that there should be an order of magnitude difference between these scales ($L \gtrsim \rho_g \gtrsim H_0^{-1}$). We simply set all these values in the ball-park of $H_0^{-1}$. Let us now address the question: what does a 4D observer see on the brane?

Because the space is compactified, there is a mass gap in the KK modes. Start from the zero-mode massless graviton. This mode couples universally to the brane matter and vacuum energy. However, because the compactification scale is huge $L \sim \rho_g \sim H_0^{-1}$ the coupling of the zero mode $G_{zm}$ is tiny. Indeed,

$$
G_{zm} \sim \frac{1}{M_*^{2+N} L^N + M_{Pl}^2} \,.
$$

(26)

The first term in the denominator is due to the compactness of the extra space and the second term is due to the induced EH term in (7). In particular, $M_*^{2+N} L^N \gg M_{Pl}^2$ and, therefore,

$$
G_{zm} \ll G_N \,.
$$

(27)

Besides the zero mode, there is a tower of massive KK modes, with masses quantized in the units of $H_0$. \textit{A priori} each of these modes is important for interactions at observable distances $\ll L$, for instance in the solar system. However, due to the presence of the induced EH term on the brane (7) the wave functions of these modes are suppressed on the 4D world volume. The net result due to the KK modes can now be summarized as a single metastable graviton with the lifetime $\sim L \sim H_0^{-1}$. The two descriptions — one in terms of the KK modes, and the other one in terms of the metastable graviton — are complimentary to each other.

The coupling of this metastable graviton to matter is determined by the coefficient in front of the induced EH term in (7). Since the latter is set to be $M_{Pl}$, the metastable mode couples to matter with the Newton coupling $G_N$. Therefore, at observable distances, that are somewhat smaller than $10^{28}$ cm, the laws of 4D gravity are enforced by a metastable graviton which at these scales can be treated as a (almost) stable spin-2 state interacting with $G_N$. For these scales the presence of the
zero mode is irrelevant because of its tiny coupling $G_{zm}$. This provides conventional gravity and cosmology for any time scale up until today.

Let us now turn to distances somewhat larger that $10^{28}$ cm. There, the metastable mode does not mediate interactions (since its lifetime is less than $H_0^{-1}$). Instead, one should think in terms of the massive KK modes. Since the mass of the lightest KK mode is $\sim H_0$, the interactions due to the KK modes are exponentially suppressed at distances larger than $H_0^{-1}$. Therefore, one is left with the zero mode interactions only. Any source which has characteristic wave lengths larger than $H_0^{-1}$, (i.e., the characteristic momenta smaller than $H_0$) will feel gravity only due to the zero mode. Because the coupling of the zero mode is tiny (26), the curvature $R$ produced by this source will also be small

$$R \sim G_{zm} \mathcal{E} \sim \frac{\mathcal{E}}{M_s^{2+N} L^N} \ll \frac{\mathcal{E}}{M_{Pl}^2}. \quad (28)$$

Since $R \sim H^2$ and $L \sim \rho_g$ where $\rho_g$ is defined in (25) we find from (28)

$$H^2 \sim M_s^2 \left( \frac{M_s^4}{\mathcal{E}} \right)^{\frac{2}{N-2}}. \quad (29)$$

This coincides with (13). Therefore, the small inflation rate (small curvature) is due to the fact that the zero mode is very weakly coupled to vacuum energy. Summarizing, compactification of extra dimensions with the ultra-large radius (25) produces essentially the same physical picture as in the model with infinite extra dimensions.

### 4 Four-dimensional picture on the brane

In this section we return to the uncompactified case. Four-dimensional gravitational interactions are mediated by a gapless infinite tower of Kaluza-Klein modes the wave functions of which are suppressed on the brane. Alternatively, these interactions can be thought of to be mediated by a single metastable four-dimensional graviton. The expression for the two-point Green’s function on the brane [5] leads to the following equation for this four-dimensional “effective” graviton [15]:

$$\left( \nabla^2 + m_c \sqrt{\nabla^2} \right) h_{\mu \nu} = 8 \pi G_N \left( T_{\mu \nu} - \frac{1}{3} \eta_{\mu \nu} T \right). \quad (30)$$

Here $T \equiv T^{\nu}_\nu$ and $m_c \equiv r_c^{-1} \sim M_s^3/M_{Pl}^2$. This refers to one extra dimension\footnote{Two branches of the square root in (30) lead to the standard and self-inflationary cosmological solutions [16].}, $N = 1$. As before, $M_s$ denotes an ultraviolet scale at which gravity breaks down as an effective field theory. On phenomenological grounds, this can be any scale in the interval $10^{-3} \text{eV} \lesssim M_s \lesssim M_{Pl}$. As we discussed before, in the five-dimensional...
model we have $10^{-3} \text{eV} \ll M_* \ll 100 \text{ MeV}$, so that $m_c \ll H_0 \sim 10^{-42} \text{ GeV}$ is less than the Hubble scale $H_0$.

We would like to provide a nonlinear completion of Eq. (30). Let us start with the right-hand side (r.h.s.) of the equation. It is known that in the massive (metastable) graviton theory the tensorial structure of the graviton propagator is affected by *nonlinear* corrections [17, 18]. The expression which takes these nonlinear effects into account can be parametrized as follows:

$$\text{r.h.s.} = 8 \pi G_N \left\{ T_{\mu\nu} - K \left( \frac{m_c}{\mu} \right) \eta_{\mu\nu} T \right\},$$

(31)

where $K(m_c/\mu)$ is a function of the physical scale $\mu$ set by the source $T$ and of the distance from the source [17, 18]. For large (unobservable) distances it gives rise to $K(m_c/\mu) \simeq 1/3$, i.e., the result in (30). However, at measurable distances (e.g. solar system) we have $K(m_c/\mu) \simeq 1/2$ due to nonlinear effects. Hence, the discontinuity in the mass parameter $m_c$ [19, 20, 21] is in fact absent in nonlinear theory of massive gravity [17] and, in particular, in the model of metastable graviton [5], as was shown in [18, 22, 23, 24] (see also [25]). Therefore, for the distances of practical interest

$$\text{r.h.s.} = 8 \pi G_N \left( T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \right),$$

(32)

which is a correct tensorial structure of the conventional Einstein equation.

The next step is to perform a nonlinear completion of the left-hand side (l.h.s.) of Eq. (31). The latter procedure is somewhat arbitrary from the point of view of pure 4D theory, however it is uniquely fixed by the higher dimensional theory [24]. Here we are interested in qualitative features which are independent of the form of the completion. To this end one can use the substitution

$$h_{\mu\nu} \to \frac{1}{\sqrt{2}} R_{\mu\nu},$$

(33)

where $R_{\mu\nu}$ denotes the Ricci tensor. Furthermore, performing simple algebra we find the following nonlinear completion of the Eq. (31):

$$G_{\mu\nu} + \frac{m_c}{\sqrt{2}} G_{\mu\nu} = 8 \pi G_N T_{\mu\nu},$$

(34)

where $G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R$. This should be compared with the conventional Einstein equation

$$G_{\mu\nu} = 8 \pi G_N T_{\mu\nu}.$$  

(35)

Note that the second term on the l.h.s in Eq. (34) does not appear in the Einstein equation. In fact, this is the very same term that dominates in the infrared region. The Bianchi identity imposes a new constraint on possible gravitational
backgrounds \( m_c \nabla^\mu (\nabla^2)^{-1/2} G_{\mu\nu} = 0 \) (in the linearized approximation the latter reduces to a gauge fixing condition). One can also note that from the standpoint of four-dimensional theory a violation of unitarity takes place. This corresponds to the fact that the metastable graviton can decay into KK modes. It is clear that the full five-dimensional unitarity is preserved while it can be violated in any given four-dimensional subspace.

As we have already mentioned, the procedure of the nonlinear completion used above is not unique. From the 4D point of view there is no guiding principle for this completion. In general, an infinite number of nonlinear terms with arbitrary coefficients can be added to the r.h.s. of (33). This would lead to an infinite number of new terms on the l.h.s. of Eq. (34). However, what is critical is that there is a unique set of these terms which complete the 4D theory to a higher dimensional theory that we started from. The purpose of the exercise performed above is to demonstrate that the presence of these nonlocal terms leads to modification of gravity in the infrared.

Similar arguments apply to higher co-dimensions. In a model with \((4 + N)\) dimensions the equation analogous to (33) takes the form

\[
\left\{ \nabla^2 + \frac{M^N \pi^2 / M^2_{\text{Pl}}}{c_1(N) M_s^{N-2} + c_2(N) (\nabla^2)^{N-2}} \right\} h_{\mu\nu} = 8 \pi G_N \left\{ T_{\mu\nu} - K \left( \frac{m_c}{\mu} \right) \eta_{\mu\nu} T \right\}. \tag{36}
\]

where \(c_{1,2}(N)\) are some constants that depend on \(N\) (we do not discuss here the \(N = 2\) case for which there are logarithmic functions of \(\nabla^2\) on the l.h.s. in (36)). Using the same method as above we propose a nonlinear completion of the equation which looks as follows:

\[
G_{\mu\nu} + \left\{ \frac{M^N \pi^2 / M^2_{\text{Pl}}}{c_1(N) M_s^{N-2} + c_2(N) (\nabla^2)^{N-2}} \right\} \frac{1}{\nabla^2} G_{\mu\nu} = 8 \pi G_N T_{\mu\nu}. \tag{37}
\]

One observes the same pattern: the second term on the l.h.s. dominates in the infrared. Hence, infrared gravity is modified. This is a necessary condition for the present approach to solve the cosmological constant problem. However, it is not sufficient, generally speaking. As we argued in Ref. [1], and in the previous sections, only \(N > 2\) models can solve the cosmological constant problem.

### 5 Killing cosmological constant does not kill inflation

The present scenario solves the cosmological constant problem due to modification of 4D gravity at very low energies. Due to this modification a strictly constant vacuum energy curves four-dimensional space extremely weakly. As we have seen, the contribution to the effective Hubble expansion rate from the vacuum energy \(E\) is
set by an inverse power of $E$, see Eq. (13). A question to be discussed in this section is whether the dilution of cosmological constant kills inflation. Naively this seems inevitable. According to the inflationary scenario, our 4D universe underwent the period of an exponentially fast expansion (for a review see [20]). In the standard case this is achieved by introducing a slow-rolling scalar field, the inflaton. During the slow-roll epoch the potential energy dominates and mimics an approximately constant vacuum energy. This leads to an accelerated growth of the scale factor.

However, as we have just argued, in the present case any constant vacuum energy is diluted according to Eq. (13). If so, it seems that the same should happen to any inflationary energy density. In other words, one would naively expect that standard inflation with the energy density $E_{\text{inf}}$ should generate the acceleration rate $H \sim M_\ast (M_\ast^4/\mathcal{E}_{\text{inf}})^{1/N-2}$. If such a relation indeed took place during the slow-roll period this would eliminate inflation and all the benefits one gets with it. Fortunately, this is not the case as we will now argue.

The relation (13) is only true for energy sources with wave lengths $\gg r_c \sim H_0^{-1}$. Brane sources of shorter wave lengths gravitate according to the conventional 4D laws since they are coupled to the resonance graviton. Furthermore, let us recall that $\mathcal{E}_{\text{inf}}$ is time-dependent during slow-roll inflation, with a typical time scale that is smaller than $r_c$ by many orders of magnitude. For instance, consider inflation with $\mathcal{E}_{\text{inf}} \sim (1 \text{ TeV})^4$. For such inflation to solve the horizon and flatness problems and generate density perturbations, $\mathcal{E}_{\text{inf}}(t)$ must be approximately constant on the time scale of the inflationary Hubble time $\sim M_P/\sqrt{\mathcal{E}_{\text{inf}}} \ll \text{mm}$. This is at least 30 orders of magnitude smaller than our $r_c$. That is why inflation in our scenario proceeds in the same manner as in the conventional setup. Below we will consider this issue in more detail.

Let us first consider the simplest nonlinearly-completed effective 4D equation (37). Although it is equivalent to the high-dimensional model (7) only in the linearized approximation, nevertheless, it captures the most essential feature — the large-distance modification of gravity. Taking the trace of the above equation we get the following relation between the curvature and stress tensor

$$[1 + \frac{M_\ast^{N-2}}{c_1 M_\ast^{N-2} + c_2 \nabla^{N-2} (r_c \nabla)^2}] \frac{1}{(r_c \nabla)^2} R = 8\pi G_N T^\mu_\mu.$$

Let us consider a constant vacuum energy $\mathcal{E}$ as the source in the r.h.s. of this equation. In the conventional Einstein gravity this would induce curvature $R \sim G_N \mathcal{E}$.

However, this cannot be a solution in our case since the second term on the l.h.s. diverges for a constant curvature. This divergence is just an artifact of our simplified non-linear completion in which we keep only finite number of the $\nabla^{-1}$ operators. Careful resummation of all such terms should give rise to a small but finite curvature as it is apparent from the high-dimensional description (see Eq. (13)).

What is important, however, is the fact that the nonlocal terms dominate for constant curvature, while they are negligible for time-dependent sources with char-
acteristic wavelengths $\ll r_c$. For instance, let us assume $T_\mu^\mu$ is the energy density of a slowly-rolling inflaton field. For the standard inflation, a typical time scale of change in the energy density is by many orders of magnitude smaller than any reasonable value of $r_c$. For a typical slow-roll inflation with $H \sim 10^{12}$ GeV we get

$$R \sim 10^{-108} R.$$  \hspace{1cm} (39)

From this estimate it is clear that the nonlocal terms are negligible, and the standard relation is intact.

Were we able to calculate all nonlocal terms in 4D world volume, we would recover the series of the form

$$R = \frac{1}{(r_c \nabla)2} \left[ 1 + \frac{M_s^{N-2}}{c_1 M_s^{N-2} + c_2 \nabla^{N-2}} \frac{1}{(r_c \nabla)^2} + \sum_n a_n \frac{(r_c \nabla)^n}{(r_c \nabla)^n} \right] 8 \pi G N T_\mu^\mu,$$  \hspace{1cm} (40)

where the coefficients $a_n$ are, generally speaking, functions of metric invariants that die away in the linearized flat-space limit. For the time independent curvature, such as the one induced by a constant vacuum energy, each term diverges. Hence, explicit summation must be performed in (40). In each particular case the summation should be possible, in principle. However, in practice it may be much easier to solve directly the high-dimensional equations. This, as we know, indicates that the resulting curvature is given by Eq. (13) and is tiny.

The proportionality of the curvature to the negative power of vacuum energy is a peculiar property of large wave-length sources. Nothing of the kind happens for shorter wave-length sources. For the slow-roll inflationary curvature the contribution from nonlocal terms is completely negligible. This is why we expect that inflation, as well as all “short distance” physics, obey the conventional laws. A simple dimensional analysis shows that deviations from the conventional behavior must be suppressed by powers of $\lambda/r_c$, where $\lambda$ is a typical wave length (or time scale) of the system. Thus, our solution predicts measurable deviations from predictions of the Einstein gravity for sufficiently large objects. In fact this was explicitly demonstrated in Ref. [23] in the five-dimensional example where the deviation for the Jupiter orbit might be potentially observable.

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References
[1] G. Dvali, G. Gabadadze and M. Shifman, *Diluting cosmological constant in infinite volume extra dimensions*, hep-th/0202174.

[2] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [astro-ph/9805201]; S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999) [astro-ph/9812133].

[3] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).

[4] J. Garriga, M. Livio, A. Vilenkin, Phys. Rev. D **61**, 023503 (2000) [astro-ph/9906216]; J. Garriga and A. Vilenkin, Phys. Rev. D **61**, 083502 (2000) [astro-ph/9908115]; Phys. Rev. D **64**, 023517 (2001); [hep-th/0011262]. For the recent review, see, A. Vilenkin, Talk given at XVIIIth IAP conference “Nature of dark energy”, Paris, July 1-5, 2002.

[5] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [hep-th/0005016].

[6] G. R. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [hep-th/0008054].

[7] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[8] G. Dvali, G. Gabadadze, X. Hou and E. Sefusatti, *See-saw modification of gravity*, hep-th/0111266.

[9] G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, hep-th/0106058; Phys. Rev. D **64**, 084004 (2001) [hep-ph/0102216].

[10] O. Corradini, A. Iglesias, Z. Kakushadze and P. Langfelder, Phys. Lett. B **521**, 96 (2001) [hep-th/0108053].

[11] M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros and C. E. Wagner, Nucl. Phys. B **609**, 499 (2001) [hep-ph/0102172].

[12] G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D **64**, 084004 (2001) [hep-ph/0102216].

[13] R. Gregory, Nucl. Phys. B **467**, 159 (1996) [hep-th/9510202].

[14] C. Charmousis, R. Emparan and R. Gregory, JHEP **0105**, 026 (2001) [hep-th/0101198].

[15] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [astro-ph/0105068].

[16] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [hep-th/0010186].
[17] A. I. Vainshtein, Phys. Lett. 39B, 393 (1972).

[18] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Phys. Rev. D 65, 044026 (2002) [hep-th/0106001].

[19] Y. Iwasaki, Phys. Rev. D 2 (1970) 2255.

[20] H. van Dam and M. Veltman, Nucl. Phys. B22, 397 (1970).

[21] V. I. Zakharov, JETP Lett. 12, 312 (1970).

[22] A. Lue, *Cosmic strings in a brane world theory with metastable gravitons*, hep-th/0111168.

[23] A. Gruzinov, *On the graviton mass*, astro-ph/0112246.

[24] M. Porrati, Phys. Lett. B 534, 209 (2002) [hep-th/0203014].

[25] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, Phys. Lett. B 503, 173 (2001) [hep-th/0011138]; M. Porrati, Phys. Lett. B 498, 92 (2001) [hep-th/0011152].

[26] A.D. Linde, *Particle physics and inflationary cosmology*, Harwood Academic, Switzerland (1990).