EFFECTS OF THE NON-RADIAL MAGNETIC FIELD ON MEASURING MAGNETIC HELICITY TRANSPORT ACROSS THE SOLAR PHOTOSPHERE

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ABSTRACT

It is generally believed that the evolution of magnetic helicity has a close relationship with solar activity. Before the launch of the Solar Dynamics Observatory (SDO), earlier studies had mostly used Michelson Doppler Imager/SOHO line of sight (LOS) magnetograms and assumed that magnetic fields are radial when calculating the magnetic helicity injection rate from photospheric magnetograms. However, this assumption is not necessarily true. Here we use the vector magnetograms and LOS magnetograms, both taken by the Helioseismic and Magnetic Imager on SDO, to estimate the effects of the non-radial magnetic field on measuring the magnetic helicity injection rate. We find that: (1) the effect of the non-radial magnetic field on estimating tangential velocity is relatively small; (2) when estimating the magnetic helicity injection rate, the effect of the non-radial magnetic field is strong when active regions are observed near the limb and is relatively weak when active regions are close to disk center; and (3) the effect of the non-radial magnetic field becomes minor if the amount of accumulated magnetic helicity is the only concern.

Key words: magneto-hydrodynamics (MHD) – Sun: activity – Sun: corona – Sun: magnetic fields – Sun: photosphere

1. INTRODUCTION

Magnetic helicity is a physical quantity that describes the topology and complexity of a magnetic field. It is believed that magnetic helicity plays an important role in solar activity (Zhang & Low 2005). Theoretical studies have suggested that the accumulation of magnetic helicity in the solar atmosphere provides free magnetic energy; since the total magnetic helicity in a force-free magnetic field has an upper bound, once this limit is exceeded a non-equilibrium situation will then result in a coronal mass ejection (CME)expulsion (e.g., Zhang et al. 2006, 2012; Zhang & Flyer 2008).

Observations also suggest that impulsive changes of the magnetic helicity injection rates have occurred during solar flares (Moon et al. 2002a, 2002b, 2003; Zhang et al. 2008). In a survey of 393 active regions, Labonte et al. (2007) suggested that X-class flares occur when the peak magnetic helicity injection rate is greater than $6 \times 10^{36}$ Mx$^2$ s$^{-1}$. Nindos & Andrews (2004) suggested that it is the amount of stored magnetic helicity that will determine whether a large flare will be eruptive or confined. Park et al. (2010) studied the occurrence of the X3.4 flare in NOAA 10930 and concluded that the flare may be initiated by a helicity injection into a system of oppositely signed helicity. Similarly, Vemareddy et al. (2012) studied the helicity injection in two active regions and suggested that flux motions and the spatial distribution of helicity injection are important ingredients in the understanding of the favorable conditions for solar eruptions. All of these indicate that magnetic helicity plays an important role in solar activities and hence it is important to have an accurate measurement of the magnetic helicity injection rate.

The rate of magnetic helicity flux across a solar surface $S$, that is, the helicity injection rate, can be estimated by the following equation (Berger 1984)

$$\frac{dH_F}{dt} = 2 \int_s \left[ (A_p \cdot B_n) V_n - (A_p \cdot V_n) B_n \right] ds,$$  \hspace{1cm} (1)

where $B_n$ and $B_p$ are the tangential and normal magnetic fields, $V_n$ and $V_p$ are the corresponding velocities, and $A_p$ is a unique vector potential satisfying the conditions

$$\nabla \times A_p \cdot \hat{n} = B_p, \quad \nabla \cdot A_p = 0, \quad A_p \cdot \hat{n} = 0.$$  \hspace{1cm} (2)

To apply Equation (1) to the observational data, we need to know $B_n$, $B_p$, $V_n$, and $V_p$. However, before the launch of the Solar Dynamics Observatory (SDO), the only available data with good temporal resolution and continuous, uniform observations were longitudinal Michelson Doppler Imager (MDI)/SOHO magnetograms. To make use of these data to estimate the helicity injection rate, two hypotheses are frequently invoked.

The first was suggested by Démoulin & Berger (2003). They argued that the horizontal motions ($U$), deduced by tracking the footpoints of magnetic flux tubes, already include the effect of both the emergence and shearing motions. This is to say that the horizontal velocity $U$, obtained by tracking the movement of magnetic footpoints, is actually

$$U = V_n - V_p \cdot \frac{B_t}{B_n}.$$  \hspace{1cm} (3)

With this, Equation (1) becomes simplified as

$$\frac{dH_F}{dt} = -2 \int_s (A_p \cdot U) \cdot B_n ds.$$  \hspace{1cm} (4)
This approach has since then become a standard procedure in calculating the helicity injection rate, although $U$ can be obtained by various methods such as local correlation tracking (LCT; e.g., Chae 2001), the differential affine velocity estimator (DAVE; e.g., Schuck 2006) and nonlinear affine velocity estimator (NAVE; e.g., Schuck 2005) etc. Recent work (Schuck 2008; Liu & Schuck 2012; Kazachenko et al. 2014; Liu et al. 2014) has called into question the validity of the Démoulin & Berger conjecture in which the apparent horizontal motions determined by, e.g., LCT flows, include both the twisting and flux emergence terms. These studies find instead that the LCT- or DAVE-derived flows correspond mainly with the true horizontal motions and are insensitive to flux emergence. On the other hand, Liu & Schuck (2012) and Liu et al. (2014) find that in a variety of active regions, the horizontal (twisting) term dominates the flux emergence term, even in cases where magnetic flux is observed to be emerging. The end result is that a straightforward application of Equation (4), in which the velocity $U$ used includes the results of horizontal velocities returned by LCT or DAVE, should include the dominant contribution to the magnetic helicity flux.

The second hypothesis comes from assuming that magnetic fields on the photosphere are predominantly vertical. With this, the vertical magnetic field strength $B_e$ in Equations (1)–(4) can then be estimated as the observed line of sight magnetic field strength, $B_t$ times $1/\cos \theta$, where $\theta$ is the heliocentric angle of the region. Using active region NOAA 10365 as an example, Chae et al. (2004) argued that this method can be more generally applied, even to regions with inclined magnetic fields. Similar to the approach proposed by Démoulin & Berger (2003), this has also become a standard approach and many studies have been carried out based on these two hypotheses.

With the launch of SDO, we now have full disk vector magnetograms with good time cadence and continuity (Bobra et al. 2014; Hoeksema et al. 2014). Now we can use Equation (1) to calculate the full terms of the helicity injection rate. In order to better understand the previous results that have been obtained based on the two hypotheses, it may also be the time to find the degree of inaccuracy that our above two hypotheses have brought.

In this paper we intend to investigate the influence of the second hypothesis, that is, the effect of the existence of the non-radial magnetic field on measuring magnetic helicity transport. We organize our paper as follows. In Section 2 we present the sample and data reduction. Our analysis and results are given in Section 3. Our conclusion and discussion are presented in Section 4.

### Table 1

| NOAA No. | Start of Observation | End of Observation | Sunspot Classification |
|----------|----------------------|--------------------|-----------------------|
| 11072    | 2010.05.20 16:24:00  | 2010.05.27 14:48:00 | $\beta$               |
| 11084    | 2010.06.28 00:24:00  | 2010.07.06 23:48:00 | $\alpha$              |
| 11158    | 2011.02.10 22:00:00  | 2011.02.18 14:12:00 | $\beta\gamma\delta$  |

Figure 1. Histogram of the ratio between the strengths of the tangential field and normal field in one magnetogram of AR 11072, obtained at 16:36 UT on 2010 May 25. The horizontal axis is the value of the ratio; the vertical axis is the number of pixels.

### 2. SAMPLE AND DATA REDUCTION

The magnetograms we used in this study are taken by the Helioseismic and Magnetic Imager (HMI)/SDO. The HMI instrument (Scherrer et al. 2012; Schou et al. 2012a, 2012b) observes the full solar disk at 6173 Å with a 4096 × 4096 CCD detector to study the oscillations and magnetic fields on the photosphere. The magnetograms are obtained using a Milne–Eddington based inversion code (Borrero et al. 2011) and the 180° ambiguity problem is resolved using a “minimum energy” algorithm (Metcalf 1994; Metcalf et al. 2006; Leka et al. 2009).

Two streams of active-region vector data have been produced and provided (Sun 2013), namely, hmi.sharp_720s and hmi.sharp_clea_720s. The series of hmi.sharp_720s preserves vector data in the native coordinate, i.e., a 2D array as measured at each CCD pixel. The field vectors are expressed as field strength ($B$), inclination ($\gamma$), and azimuth ($\phi$). The series of hmi.sharp_clea_720s uses a cylindrical equal area (CEA) projection. The standard CEA coordinates ($x, y$) relate to the heliographic longitude and latitude, with each pixel representing an equal area, making them suitable for the computation of total flux. The field vectors in hmi.sharp_clea_720s are presented as $(B_x, B_y, B_\phi)$ in heliocentric spherical coordinates, and are obtained by mathematical methods, called “remapping” and “vector transformation,” from the hmi.sharp_720s data.

We use both streams of the active-region vector data in this study. We use the series of hmi.sharp_clea_720s to obtain $B_i$,
that is, $B_n$ in Equations (1)–(4). We use the series of hmi.sharp_720s to get the longitudinal magnetic field $B_n$, the type of data used in most previous studies. To make the two series of data comparable on a pixel-to-pixel basis, we have remapped the hmi.sharp_720s data to CEA coordinates. Since we are primarily interested in testing the second hypothesis, we will focus on comparing the difference between using the “real” $B_n$ from vector data and the “estimated” $B'_n$ by assuming that the magnetic field is radial. We will use Equation (4) to calculate the magnetic helicity injection rate.
The vector potential $A_p$ and the horizontal velocity field $U$ are derived using the methods of a fast Fourier transform and the DAVE (Schuck 2006), respectively. The accumulated magnetic helicity is also calculated as

$$H(t) = \int_0^t \frac{dH_R(t)}{dt} dt,$$

where $t = 0$ is the starting time of our calculation.

We studied three active regions, NOAA 11072, NOAA 11084, and NOAA 11158. Both NOAA 11072 and NOAA 11158 are emerging active regions and have been studied by Liu & Schuck (2012) with a purpose that is different from ours. NOAA 11072 has a bipolar magnetic field structure, with no C-class or above flares having occurred during its passage across the disk. NOAA 11158 is a multipolar complex active region that has produced several major flares. For a comparison, NOAA 11084 is selected as a mature and simple active region with no significant flaring activity. Compared to Liu & Schuck (2012), our sample contains data extending closer to the solar limb, ending when the three active regions are 60° in longitude near the western limb. Table 1 lists some properties of these three active regions.

3. ANALYSIS AND RESULTS

Magnetic fields in active regions are not strictly radial, especially in regions such as sunspot penumbra and around magnetic neutral lines. Figure 1 presents the histogram...
of the ratio between the tangential field ($B_t$) and the normal field ($B_n$) in a magnetogram of NOAA 11072. The magnetogram was obtained at 16:36 UT on 2010 May 25 when the active region was located at S14W33 and only data points whose normal magnetic field strengths were stronger than 100 G were used. We see that in a significant fraction of the active regions near the disk center and close to the limb.

Figure 5. Scatter plots of $B_t$ vs. $B_n$ for NOAA 11158 (top panels), NOAA 11072 (middle panels), and NOAA 11084 (bottom panels). Left panels are active regions near the disk center and right panels are regions close to the limb.
of the active region the tangential field strength is larger than that of the radial field. This shows that the tangential field cannot be ignored. Note that this is a general situation for most active regions and that the study of how these tangential magnetic fields affect helicity transport is important.

3.1. On Obtaining the Radial Magnetic Field

As mentioned before, when using MDI/SOHO magnetograms to calculate the helicity injection rate, a common practice is to assume that the magnetic field is radial and then obtain the strength of the radial magnetic field \( B_r \) from the longitudinal magnetic field \( B_l \), that is, \( B_r = B_l / \cos \theta \), where \( \theta \) is the heliocentric angle of the observed region. However, this “estimated” \( B_r \), denoted as \( B'_r \) hereafter, could be significantly different from the “real” \( B_r \).

Figures 2–4 show a comparison of the estimated \( B'_r \) and the real \( B_r \) for active regions NOAA 11158, 11072, and 11084, respectively. Here we see that when the active region is observed near the disk center, \( B'_r \) and \( B_r \) look quite similar to each other. This indicates that the influence of the tangential field on estimating the radial magnetic field is not severe. However, when the active region is observed far away from the disk center, the estimated radial field \( B'_r \) could be significantly different from the real \( B_r \). In some areas (outlined by the red squares in Figures 2 and 3) even the sign of the estimated radial magnetic field \( B'_r \) could be different from that of the real \( B_r \). This will certainly influence the calculation of helicity transport.

To show this difference more quantitatively, we present the scatter plots of \( B'_r \) versus \( B_r \) in Figure 5 for the three active regions studied. Left panels are for active regions near the disk center and right panels for regions near the limb. The correlation coefficients between \( B'_r \) and \( B_r \) are all higher than 0.95, which means that the \( B'_r \) maps are very similar to the \( B_r \) maps. When the active regions are at 45° in the west, the correlation coefficients drop below 0.85. This means that the \( B'_r \) maps are already different from the \( B_r \) maps.

3.2. Effect on Obtaining the Tangential Velocity Field

Figure 6 shows the tangential velocity field estimated using the DAVE method (Schuck 2006) for NOAA 11158. On using the DAVE code, we have followed Liu & Schuck (2012) and used 19 pixels (8.5′ for HMI data) for the window size. The right panels in Figure 6 show the tangential velocities obtained by using \( B_r \) magnetograms, and the left panels by using \( B'_r \) magnetograms. Similar to Figure 2, we also show the calculation in two representative moments, one when the active region is near the disk center (top panels) and one when the active region is 45° from the disk center.
It is possibly not surprising to see from the top panel that using $B_r$ or $Br$ will give relatively similar maps of the tangential velocities; we already know from Figure 2 that the influence of the tangential field on estimating the radial magnetic field is not severe when the active region is near the disk center. However, it is interesting to determine from the bottom panels that the tangential velocity maps also look similar, even though the magnetograms, $B_r$ or $B$, depending on which the velocities are obtained, are quite different. This means that the effect of the non-radial magnetic field on estimating the tangential velocity is small, no matter if the active region is near to or away from the disk center.

Figure 7. Correlation between the two types of velocity maps shown in Figure 5, one obtained using $B_r'$ and the other using $B_r$ magnetograms. Left panels are the moment when the active region is near the disk center, and right panels are when the active region is $45^\circ$ in the west. Top panels are $V_x$ (positive in the direction of solar rotation), middle panels are $V_y$ (positive from south to north), and bottom panels are $V$ (the magnitude of the tangential velocity). The top-left corner of each panel shows the correlation coefficient between the two quantities plotted in each panel.
To show this more quantitatively, Figure 7 presents the correlation between the two types of velocity maps, one obtained from $B_r$ magnetograms and the other from $B'_r$ magnetograms. The top panels are for the component $V_x$ and are positive in the direction of solar rotation. The middle panels are for the component $V_y$, and are positive from the disk. The bottom panels are for the component $V_x$, and are positive from the disk.

Figure 8. Correlation coefficients between velocities obtained using $B_r$ magnetograms and those obtained using $B'_r$ magnetograms, during the entire passage across the disk, for NOAA 11158 (top panel), NOAA 11172 (middle panel), and NOAA 11084 (bottom panel). Red lines are for $V_x$, blue are for $V_y$, and black are for $V_z$. The Astrophysical Journal, 804:102 (12pp), 2015 May 10

Song & Zhang
south to the north. The bottom panels show the correlation between the magnitudes of the tangential velocities. We see here that in all six panels the correlation is high. The correlation coefficients are all larger than 0.97 when the active region is near the disk center. When the active region is 45° in the west, the correlation coefficients are all above 0.72, still high considering the large differences of the magnetograms from which they are obtained.

Similarly, the correlation coefficients between the two types of tangential velocities are calculated and presented in Figure 8 for the three active regions during the entire passage across the disk. We see here that the correlation coefficients are all larger than 0.97 when the active region is near the disk center. When the active region is 45° in the west, the correlation coefficients are all above 0.72, still high considering the large differences of the magnetograms from which they are obtained.

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Table 2 lists some statistics of these correlation coefficients. We see that for the two emerging active regions (NOAA 11158 and NOAA 11072) the range where CC > 0.6 extends to 54° in the west. Even for the simple active region NOAA 11084 the regions where CC > 0.6 cover from E50 to W41. All these indicate that the effect of non-radial magnetic fields on obtaining tangential velocities is relatively small, as long as the active region is not too far away from the disk center, for example, within 40° of the disk center.

In addition, we should note that from obtaining the tangential velocity by the DAVE code we used the constant area map projections as mentioned in Section 2. Welsch et al. (2009) pointed out that such projections do not preserve direction, in which case the velocities $V_x$ and $V_y$ probably contain distortions. Welsch et al. (2009) argued that one should use a conformal map, such as the Mercator projection, to ensure that the velocity directions are determined correctly. On the other hand, Liu & Schuck (2012) argue that if the active region being analyzed represents a small part of the surface, these distortions...
are probably not important. Since we are analyzing active regions as Liu & Schuck (2012) did, we expect the distortions on the obtained velocity maps to be small.

3.3. Effect on Measuring the Magnetic Helicity Transport

3.3.1. On the Magnetic Helicity Injection Rate

Using Equation (4) we can calculate the helicity injection rate. Again, we are interested in comparing the two types of helicity injection rates, one obtained using $B_r$ magnetograms, denoted as $dh/dt$ hereafter, and one obtained using estimated $B'_r$ magnetograms, denoted as $dh'/dt$ hereafter. As an example, Figure 9 gives the distributions of the obtained magnetic helicity injection rates for NOAA 11158. The top panels still show the distributions when the active region is near the disk center and the bottom panels when the active region is 45° from the disk center.

From the top two panels of Figure 9 we see that the two distributions look very similar. The total magnetic helicity injection rate for the top-left panel is $\text{ }7.10 \times 10^{37} \text{ Mx}^2 \text{ s}^{-1}$. For the top-right panel $dh'/dt$ is $7.10 \times 10^{37} \text{ Mx}^2 \text{ s}^{-1}$. The difference is $(7.10 - 6.59)/7.10 = 7\%$. However, when the active region is 45° from the disk center, the difference between $dh/dt$ and $dh'/dt$ become evident, as shown in the bottom two panels of Figure 9. The two red squares show where they differ most. We see in this subregion even the signs of the helicity injection rate are different. This reminds us that we need to be careful in interpreting the distribution of helicity injection rate map, particularly if we are using estimated $B'_r$ magnetograms. The total helicity injection

Figure 10. Temporal profiles of unsigned magnetic flux (top panel), magnetic helicity injection rate (middle panel), and accumulated magnetic helicity (bottom panel) of NOAA 11158. Solid lines are for quantities obtained using $B_r$ magnetograms, dotted lines are for those obtained using $B'_r$.

Figure 11. Same as Figure 10, but showing temporal profiles of unsigned magnetic flux (top panel), magnetic helicity injection rate (middle panel), and accumulated magnetic helicity (bottom panel) for NOAA 11072.

Figure 12. Same as Figure 10, but showing temporal profiles of unsigned magnetic flux (top panel), magnetic helicity injection rate (middle panel), and accumulated magnetic helicity (bottom panel) for NOAA 11084.
rate in the bottom-left panel is $dH/dt = 3.90 \times 10^{37} \text{ Mx}^2 \text{ s}^{-1}$ and $dh/dt = 2.75 \times 10^{37} \text{ Mx}^2 \text{ s}^{-1}$ for the bottom-right panel. The difference between these two is now $(3.90 - 2.75)/2.75 = 42\%$.

This phenomenon exists for all three active regions, as can be seen from the middle panels in Figure 10 (for NOAA 11158), Figure 11 (NOAA 11072), and Figure 12 (NOAA 11084). We see there that when the active regions are near the disk center, the values of $dH/dt$ (presented by dotted lines) and $dh/dt$ (presented by solid lines) are close to each other. But when the active regions are far away from the disk center, even the signs of $dH/dt$ and $dh/dt$ could be different.

Similar to Figure 8, in order to show the correlation between $dH/dt$ and $dh/dt$ more quantitatively, we have calculated the correlation coefficients between the two helicity injection rates for the three active regions during the entire passage across the disk. The results are presented in Figure 13. We see that, unlike the correlation coefficients between the tangential velocities, the correlation coefficients between the two helicity injection rates decrease very quickly as the active regions move away from the disk center. The value of the correlation coefficient can even become negative when the active region is around 60° from the disk center.

Similar to Table 2, we list some statistics on the correlation coefficients between $dH/dt$ and $dh/dt$ in Table 3. We see here that for the helicity injection rates the longitudes where $CC > 0.6$ shrink to within about 25° from the disk center, which means that the effect of the non-radial magnetic field on the correlation of the magnetic helicity injection rate is relatively strong.

3.3.2. On the Amount of Accumulated Magnetic Helicity

Using Equation (5) we can calculate the amount of accumulated magnetic helicity, from the starting point of observation, for the three active regions. The results are presented in the bottom panels of Figures 10–12 for NOAA 11158, NOAA 11072, and NOAA 11084, respectively. Here $H$ denotes for the amount of accumulated magnetic helicity calculated using $B_r$ magnetograms and $H'$ is the amount of accumulated magnetic helicity calculated using $B'_{r}$ magnetograms.

We see from these panels that the differences between $H$ and $H'$ are small, except for NOAA 11072 when locating 45° further to the west. This shows that the effect of the non-radial magnetic field on the measurement of the accumulated magnetic helicity becomes less significant, as the amount of accumulated magnetic helicity comes mostly from the moments when the active regions are not far away from the disk center.

A more quantitative comparison is given in Table 4. We see that at the position of W40, even though the helicity injection rates of $dH/dt$ and $dh/dt$ are obviously different, the difference between $H$ and $H'$ are not very large. The largest difference between $H$ and $H'$ at W40 is 16%. Interestingly is that for active regions NOAA 11158 and NOAA 11084, even at 60° from the disk center, the differences between $H$ and $H'$ are still smaller than 6%.

These indicate that if the observation is done not far away from the disk center, for example within 45° or 45° from the disk center, the effect of the non-radial magnetic field on the calculation of the accumulated magnetic helicity is not severe.

![Figure 13](image)

**Figure 13.** Similar to Figure 8 but for correlation coefficients between the helicity injection rates during the entire passage across the disk for the three active regions.

| NOAA No. | Maximum | Minimum | Longitudes where $CC > 0.6$ |
|----------|---------|---------|-----------------------------|
| 11158    | 0.98    | 0.10    | ~W41                        |
| 11072    | 0.96    | ~0.01   | ~W34                        |
| 11084    | 0.85    | ~0.10   | E41–W24                     |

Table 3: Statistics on the Correlation Coefficients between $dH/dt$ and $dh/dt$

| NOAA No. | Maximum | Minimum | Longitudes where $CC > 0.6$ |
|----------|---------|---------|-----------------------------|
| 11158    | 41.6    | 19.4    | 24.6 20.7 16% |
| 11072    | 0.29    | ~4.54   | ~1.80 ~2.03 13% |
| 11084    | 1.29    | ~0.43   | 2.46 2.48 1% |

Note. The unit for $dH/dt$ and $dh/dt$ is $10^{37} \text{ Mx}^2 \text{ s}^{-1}$, and for $H$ and $H'$ is $10^{42} \text{ Mx}^2$.

Table 4: Comparison of the Two Types of Helicity Calculations near the Western Limb

| NOAA No. | W40 | W50 | W60 |
|----------|-----|-----|-----|
| 11158    | 41.6 | 24.6 | 20.7 16% |
| 11072    | 0.29 | ~1.80 | ~2.03 13% |
| 11084    | 1.29 | ~0.43 | 2.46 2.48 1% |

Note. The unit for $dH/dt$ and $dh/dt$ is $10^{37} \text{ Mx}^2 \text{ s}^{-1}$, and for $H$ and $H'$ is $10^{42} \text{ Mx}^2$. 
4. CONCLUSION AND DISCUSSION

We have used HMI/SDO magnetic field data to investigate the effect of the non-radial magnetic field on measuring the magnetic helicity transport across the solar photosphere. Three active regions are studied: NOAA 11072, NOAA 11084, and NOAA 11158.

First, we compared the differences between the true radial magnetic field $B_r$ and the estimated radial field obtained by $B'_r = B_r / \cos \theta$ based on the assumption that the magnetic field is radial. The comparison shows that the radial assumption is not valid and can bring in significant distortion to the magnetic structure when the active region is far away from the disk center.

Then we studied the effect of the non-radial magnetic field on estimating tangential velocity using the DAVE method. Interestingly, we found that the effect is relatively small. The main reason might be that, though the structure of the estimated radial magnetic field obtained based on the radial assumption is obviously different from that of the true radial field when the active region is away from disk center, the information on the field-line footpoint movement is still preserved.

Finally, we discussed the effect of the non-radial magnetic field on estimating the magnetic helicity transport. When the active region is far away from the disk center, the radial assumption can bring much distortion to the distribution of the helicity injection rate and even the sign of the injection rate can be opposite of the true one. However, if we only consider the amount of accumulated magnetic helicity, the effect of the non-radial magnetic field then becomes minor, as long as our calculation is done within 40° or 45° from the disk center.

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