SELECTION AND IMPACT OF DECISION MODE OF ENCROACHMENT AND RETAIL SERVICE IN A DUAL-CHANNEL SUPPLY CHAIN

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Abstract. Consider a supply chain consisting of one manufacturer and one retailer. The manufacturer may open direct channels through ex-ante or ex-post encroachment, and the retailer can provide consumers with ex-ante or ex-post service. We investigate the effects of encroachment and services on the optimal strategy for two members in three decision modes: MR mode (ex-ante encroachment), MRM mode (ex-post encroachment and ex-post service), and MRMR mode (ex-post encroachment and ex-ante service). The results show that in the MRM mode, both the wholesale and retail prices may become higher with encroachment. Improving the service efficiency may hurt the retailer, and increasing the operating cost for direct channels harms the retailer, while benefits the manufacturer. In addition, only in the MRM mode, the retailer maybe benefits from encroachment under certain conditions. We further study the equilibrium mode and the result shows as follows. The MR mode, widely adopted by the literature on manufacturer encroachment, always is worst for the manufacturer. Only when both the operating cost for direct channels and the service efficiency are low, the equilibrium decision mode is the MRMR mode, otherwise the MRM mode is the equilibrium decision mode.

1. Introduction. With the rapid advancement of E-commerce, consumers online shopping have continued to grow, which induced many manufacturers open direct channels to sell products to consumers in addition to retail channels [3]. This phenomenon is regarded as manufacturer encroachment, and such distribution system is called a dual-channel supply chain [1]. Examples include electronic product manufacturers (e.g., Apple and Dell), apparel and fashion manufacturers (e.g., Adidas and Nike), and soft-drink manufacturers (e.g., soft-drink and Coca-Cola).

Undoubtedly in a dual-channel environment, direct channels split up a partial of retail channels’ market and channel conflict occurs. Thus, the manufacturer must carefully handle the relationship with the retailer. As a specific problem, how should the manufacturer introduce a direct channel to maximize its own profit, and how should the retailer adjust its strategy to beat back encroachment?

In the practice of dual-channel management, different manufacturers have adopted different encroachment mode to maximize their profits. Many manufacturers use the dominant role and adopt ex-ante encroachment, i.e., the decision on the sale

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quantity/pricing for the direct channel is made before the retailer’s decision. These researches have stated that ex-ante encroachment may force retailers to lower pricing for retail channels, which can weaken double marginalization and benefits the manufacturer and the retailer [7, 10]. However, [14] and [20] indicate that ex-ante encroachment may aggravate double marginalization and hurts the retailer. To appease the retailer, the manufacturer usually gives up ex-ante encroachment and adopts ex-post encroachment, i.e., the decision on the sale quantity/pricing for the direct channel is made after the retailer’s decision. For example, though IBM may take orders for PCs over the Web, it gives priority to the sales of its distributors in an attempt [26]. When the Air Jordan 2011 shoe was first launched, Nike sells it on the official website after selling through traditional retail stores for several months [4]. [18, 19] adopt ex-post encroachment under asymmetric demand information and show that ex-post encroachment maybe hurts both the manufacturer and retailer under certain conditions. Therefore, a question naturally arises which encroachment strategy the manufacturer should choose to introduce direct channels.

On the other hand, encroachment brings channel conflict and hurts retailers [14, 20]. In response to encroachment, retailers provide retail services to attract consumers and cope with disadvantage factors from channel conflicts to help increase revenues. In practice, different retailers usually adopt different service strategy. For example, in the film industry, many cinemas, behaving as the retailers, spend lots of time and cost on the marketing efforts before the premiere of a new film, which indicates that the film retailers make ex-ante service efforts [13]. In a sharp contrast, as for PC retailing, Sundan Co., Ltd adopts ex-post service strategy, i.e., simultaneously makes service efforts and pricing decisions after the manufacturer’s decisions. [17] compare the three service effort strategy: no service, ex-ante service and ex-post service, and find ex-post service strategy is best and enables the retailer to benefit the most. Therefore, which service strategy the retailer should choose to maximize its own profit when facing encroachment.

To address the above-mentioned questions, we consider a dual-channel supply chain consisting of a manufacturer (she) and a retailer (he). The manufacturer introduce a direct channel through ex-ante encroachment or ex-post encroachment, and the retailer provides consumers with ex-ante service or ex-post service. According to the two partner’s selection, there exist three decision modes between the manufacturer and retailer (The sequence of events under ex-ante encroachment and ex-ante service is the same as that under ex-ante encroachment and ex-post service, so these two decision modes is actually identical). The first mode (“MR mode”) is ex-ante encroachment and ex-ante service (or ex-post service). This is a two-stage game and the sequence of events is as follows. The manufacturer simultaneously sets the wholesale price and direct sale quantity (ex-ante encroachment), and the retailer then determines the service level and retail sale quantity (regardless of ex-ante service or ex-ante service). The second mode (“MRM mode”) is ex-post encroachment and ex-post service. This is a three-stage game where the manufacturer first declares the wholesale price, and the retailer then simultaneously sets the service level and retail sale quantity, and the manufacturer finally determines

1 see https://www.sundan.com.

2 If the manufacturer chooses ex-ante encroachment, i.e., she simultaneously announces the wholesale price and direct sale quantity to the retailer, the ex-ante service (i.e., first determining the service level and then setting retail sale quantity) is the same as ex-post service (simultaneously determines the service level and retail sale quantity).
The third mode (“MRMR mode”) is ex-post encroachment and ex-ante service. This is a four-stage game where the manufacturer first sets the wholesale price and the retailer then sets the service level. After the wholesale price and the service level are determined, the manufacturer sets the direct quantity and the retailer then sets the retail quantity.

We discuss the optimal sale quantity and pricing for the two channels under in the three modes. In the MR mode, the wholesale price keeps the same as that without encroachment, but both the price and sale quantity for the retail channel decrease. As a result, the retailer becomes worse off. In addition, Improving the service efficiency benefits the retailer as well as the manufacturer, which means that the manufacturer free rides services. In the MRMR mode, the wholesale price decreases while the retail sale quantity increases, and the retailer can become worse off since the retail price decreases. The manufacturer free-riding services also occurs. However, in the MRM mode, both the wholesale price and sale quantity for the retail channel may become higher than that without encroachment. The retailer can benefit from encroachment. As a result, both the manufacturer and retailer can benefits from encroachment. Besides, under certain conditions, improving the service efficiency may hurt the retailer, and increasing the operating cost for the direct channel may benefit the manufacturer, which is inconsistent with our intuition.

In addition, we analyze the equilibrium decision mode between the manufacturer and retailer. The result is as follows. From the manufacturer’s perspective, ex-post encroachment is always better than ex-ante encroachment, though ex-ante encroachment has been widely adopted by lots of researchers in a dual-channel supply chain. That is, the MR mode is worst for the manufacturer. From the retailer’s perspective, given the manufacturer choosing ex-post encroachment, the retailer’s choice depends on the service efficiency and the operating cost for the direct channel. Specifically, in the equilibrium mode, when both the unit operating cost and the service efficiency are low, the equilibrium mode is the MRMR mode, otherwise, it is the MRM mode.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the basic model and benchmark without encroachment. Sections 4 and 5 discuss the optimal outcomes and the equilibrium decision mode between the manufacturer and retailer. Section 6 concludes the paper.

2. Literature review. This paper is mainly related to three streams of the literature: manufacturer encroachment, retail services, and the sequence of decisions in a supply chain.

In the recent two decades, many researches have focused on dual-channel management issues and have achieved fruitful results [6, 7, 9, 10, 16, 20, 22, 23, 24, 26]. [10, 7, 26, 5, 9] state that manufacturer encroachment can benefit both the manufacturer and the retailer due to weakening the double marginalization. [10] shows that encroachment forces retailers to lower the retail price, which weakens the double marginalization, even if no sales occur in the direct channel. As a result, encroachment benefits both the manufacturer and the retailer. [7] and [26] suggest that encroachment can benefit both sides if the prices for the two channels keep consistent. [5] shows that the retailer may benefit from manufacturer encroachment when the competitiveness of retail channel is very strong. [9] focuses on the coordination of a dual-channel supply through a two-part tariff and a profit-sharing contract, and find that the retailer may be better off. On the other hand, many studies on encroachment have come to a contrary result that encroachment may
hurts the retailer [20, 22]. [20] considers the interaction between encroachment and personalized pricing and show encroachment hurts the retailer. [22] indicates that encroachment may aggravate double marginalization and hurt the retailer. All the above literature adopt ex-ante encroachment, i.e., the decision on the pricing for the direct channel is made before the retailer’s decision. Such the sequence of events is also adopted in MR mode. Different with the existing literature, we discuss the quantity competition for the two channel, and analyze the effects of retail services in MR mode. The result shows that encroachment benefits the manufacturer while always hurts the retailer. In addition, as the response to encroachment, the retailer improves the service level, but the manufacturer always free rides services.

Besides ex-ante encroachment, There exits the literature that adopts ex-post encroachment strategy, i.e., the price/sale quantity for the direct channel is determined after the retailer’s decision [1, 19, 18, 32]. [1] establishes a quantity competition model between the two channels in ex-post encroachment, and find that manufacturer encroachment may benefit the retailer only when the unit operating cost for the direct channel is high. [18, 19] extend the research of [1] by considering a retailer with privately knowing the demand information. They find that encroachment under asymmetric information may hurt the manufacturer while benefit the retailer in certain conditions. In MRM mode, we adopt ex-post encroachment and find that the manufacturer always benefits from encroachment, while the retailer may become either better off or worse off.

This paper is related to the literature on the retailer’s service effort in a dual-channel supply chain. The service effort strategies, including counseling service, product advertising, and exhibition halls, indeed effect consumers’ shopping decisions [28, 30, 29, 11, 8, 2, 17, 32]. The existing literature show that encroachment can bring channel competition, which motivates the retailer to improve the service level to beat back encroachment. [25] and [33] point out that the manufacturer free rides retail services. Such free-riding behavior may restrain the retailer’s service effort. Our result shows that the free-riding behavior indeed occurs. Few literature has considered the selection problem of service decision sequence. Different from the previous literature, this paper focuses on which service decision, ex-ante service strategy and ex-post service strategy, the retailer should adopt when facing encroachment. The result shows that improving the service efficiency may hurts the retailer under certain conditions. The recent literature [17] considers the pricing and profit for the members in the three service decision sequences. Different with [17], we discuss the choice of sequence of decisions on encroachment and services and analyze the equilibrium decision mode between the manufacturer and retailer.

Finally, this paper is also related to the choice of decision sequence in supply chain management. In supply chain management, the strategic sequence of decisions has received attention in the literature on competitive firms. The sequence of decisions is usually regarded as simultaneous (e.g., Bertrand game) and sequential (e.g., Stackelberg game). In a dual-channel supply chain, the existing literature mainly focuses on the pricing/quantity for the direct channel is determined before the retailer’s decision [10, 7, 5], or after the retailer’s decisions [1, 19, 18, 32]. The issue of whether the dual-channel manufacturer should lead the channel in simultaneously setting the wholesale price and the direct sale quantity or in sequential setting both is significant because the retailer can observe each of the manufacturer’s decisions before making his own. The issue on whether the the retailer should simultaneously set the retail sale quantity and the service level or should sequential
set both also is significant because the retailer’s decision sequence can affect the manufacturer’s strategy. Thus, which information (only the wholesale price or both wholesale price and the direct sale quantity) should the manufacturer announce to the retailer, and the latter will react by choosing different service strategy (ex-ante service or ex-post service) which impact the demands and profits for each of the two channels. This issue has not received attention in literature on a dual-channel supply chain. This paper focuses on which encroachment strategy the manufacturer should choose. Our result shows that ex-ante encroachment strategy, which has been widely adopted by the vast majority of literature, is worst for the manufacturer. That is, the manufacturer should adopt ax-post encroachment strategy, i.e., she should make the direct sale quantity decision after the retailer makes decisions.

3. The basic model. We present the assumptions and settings used for this paper in this section. Table 1 summarizes the notations in the model.

| Notation | Description |
|----------|-------------|
| \(a\) | The maximum potential market size |
| \(c\) | The unit operating cost for the direct channel |
| \(\lambda\) | The efficacy of the service level |
| \(\eta\) | The coefficient for the service cost |
| \(S\) | The service efficiency, where \(S = \frac{\lambda^2}{\eta}\) |
| \(p_r\) | The retail price for the retail channel, where \(p_n = u_n + w_n\) |
| \(p_d\) | The retail price for the direct channel |
| \(\pi_M/\pi_R/\pi_C\) | Manufacturer’s /Retailer’s/Whole chain’s profit |
| \(w\) | The wholesale price |
| \(Q_d\) | The sale quantity for the direct channel |
| \(Q_r\) | The sale quantity for the retail channel |
| \(s\) | The service level for the retail channel |
| \(MR/\)RM/RMR | The optimal outcome in the MR/MRM/MRMR mode |
| \(B\) | The optimal outcome in the benchmark without encroachment |

This paper considers that a manufacturer produces and sells a product to consumers through an independent retailer (retail channel). In addition to the retail channel, she has an option to open a direct channel and sells directly to consumers. In line with the literature [1, 19], we normalize the unit production cost to zero. Besides, in order to consider the case that the retailer has an advantage in retail operations, we assume that the manufacturer need to pay a unit cost \(c > 0\) for selling directly to consumers (hereafter “operating cost for the direct channel”).

The retailer provides retail services to attract consumers and increase sales for the retail channel. We assume that only the retailer can offer retail services because he can communicate with consumers face to face. Let \(s\) be the service level provided by the retailer, and denote \(\eta s^2\) as the corresponding service cost of providing the service level \(S\) (see [26, 31, 6, 15]), where \(\eta > 0\) measures the cost coefficient.

In line with [1, 18, 19], we assume that the price-demand relationship is linear and specifically as follows:

\[
p_r = a - Q_r - Q_d + \lambda s, \quad p_d = a - Q_d - Q_r. \tag{1}
\]

In Equ. (1), \(a > 0\) indicates the maximum potential market size, \(s\) is the service level, and \(\lambda\) measures the efficacy of the retail service in stimulating the retail channel’s demand. Note that \(p_r > p_d\) due to \(s > 0\), which means that the retail
price is higher than the direct price since consumers enjoy retail services. In practice, consumers need to bear higher price when enjoying professional and excellent services.

According to (1), the profits for the two partners are shown as follows

$$\pi_M(w, Q_d) = wQ_r + Q_d(a - Q_r - Q_d) - cQ_d. \quad (2)$$

$$\pi_R(s, Q_r) = (a - Q_r - Q_d + \lambda s)Q_r - wQ_r - \eta s^2. \quad (3)$$

3.1. The sequence of events. This paper considers a manufacturer-Stackelberg game where the manufacturer is the leader and the retailer is the follower. As the leader, the manufacturer first chooses one of ex-ante encroachment and ex-post encroachment. Under ex-ante encroachment, the manufacturer simultaneously sets both the wholesale price and the sale quantity for the direct channel before the retailer makes decisions on the sale quantity for the retail channel and the service level. Under ex-post encroachment, the manufacturer only sets the wholesale price before the retailer makes decisions while determines the direct sale quantity according to the retailer making decisions. As the follower, the retailer may also choose either ex-ante service or ex-post service to provide services. Under ex-ante service, the retailer first determines only the service level and then sets the retail sale quantity. Under ex-post service, the retailer simultaneously determines both the service level and the retail sale quantity after observing the manufacturer’s action. Thus, the interaction between the manufacturer and retailer can bring three different decision modes, namely, MR mode, MRM mode and MRMR mode. Figure 1 shows the sequence of events in the three decision modes.

**Figure 1.** The sequence of the events under three decision modes

Under the MR mode (i.e., ex-ante encroachment), the manufacturer simultaneously sets the wholesale price and direct sale quantity before the retailer’s decision. Under such case, there is no difference in the sequence of events between ex-ante and ex-post service. Obviously, the MR mode is a two-stage game and has been widely adopted in lots of the literature on a dual-channel supply chain [10, 26, 7, 21, 27, 33, 17]. Different with the MR mode, the MRM and MRMR modes (i.e., ex-post encroachment) consider that the manufacturer sets only the wholesale price before the retailer’s decision while determines the direct sale quantity after observing the retailer’s response. If the retailer chooses ex-post service, i.e., setting simultaneously the service level and the sale quantity for the retail channel, the manufacturer determines the direct sale quantity after observing the retailer’s action. This is a three-stage game and is denoted by the MRM mode. If
the retailer chooses ex-ante service, i.e., setting only the service level after observing the wholesale price, the manufacturer determines the direct sale quantity after observing the service level, and finally the retailer sets the retail sale quantity. This is a four-stage game, denoted by the MRMR mode.

3.2. The benchmark without encroachment. In order to compare the affects of encroachment on decisions for the manufacturer and retailer, we first present the benchmark where the manufacturer does not open a direct channel. In such case, the manufacturer first sets the wholesale price, and the retailer then sets the service level and the retail sale quantity (ex-ante service is no different with ex-post service). Through the backward induction, the optimal outcomes for the two partners can be easily obtained and are shown in Table 2.

In Table 2, \( S = \frac{\lambda^2}{\eta} \), consisting of \( \lambda \) and \( \eta \), reflects the efficiency of retail service on influence of the price for the retail channel. Given \( \eta \), a large \( \lambda \) inducing a large \( S \) means that the service level has a significant impact on the price for the retail channel. Similarly, Given \( \lambda \), a small \( \eta \) inducing a large \( S \) means that the service cost has not a significant impact on the price for the retail channel. As is shown in the rest of this paper, \( \lambda \) and \( \eta \) always appear in the form of \( S = \frac{\lambda^2}{\eta} \). Thus, we regard \( S \) as the service efficiency in this paper. Similar to the literature \([6, 12, 26]\), we assume the service efficiency \( S \leq 1 \). This assumption means that the cost in providing retail services is not very cheap due to labor costs.

**Proposition 1**: \( \frac{dw^0}{dS} = 0, \frac{dp^0}{dS} > 0, \frac{dQ^0}{dS} > 0, \frac{d\pi_M^0}{dS} > 0, \frac{d\pi_R^0}{dS} > 0, \) and \( \frac{d\pi_C^0}{dS} > 0 \).

Proposition 1 shows that improving the service efficiency does not affect the wholesale price, while can increase both the price and the retail sale quantity, as well as the profits for the manufacturer, the retailer and the whole supply chain. This means that retail services offered by the retailer has a positive effect on the retail channel and benefits the two partners, which means that the manufacturer free rides retail services.

4. The optimal outcomes under three decision modes. In this section, we first discuss the optimal strategy after encroachment in three decision modes, and then analyze the impact of encroachment on the optimal strategy.

4.1. The MR mode. The MR mode is a two-stage game between the manufacturer and retailer. The sequence of events is as follows. The manufacturer first sets the wholesale price \( w \) and the direct sale quantity \( Q_d \), and the retailer then sets the retail sale quantity \( Q_r \) and the service level \( s \). Through the backward induction, the optimal outcomes can be obtained and are shown in Table 3.

| \( c \) | \( w^{MR} \) | \( Q_r^{MR} \) | \( s^{MR} \) | \( p_r^{MR} \) | \( \pi_M^{MR} \) | \( \pi_R^{MR} \) | \( \pi_C^{MR} \) |
|---|---|---|---|---|---|---|---|
| \( c < \frac{(2-S)a}{4-S} \) | \( \frac{a}{2} \) | \( \frac{\lambda a}{2(2-S)} \) | \( \frac{(6-S)a}{2(4-S)} \) | \( \frac{a^2}{4} + \frac{(a-\epsilon)^2}{4} + \frac{a^2}{4-S} \) | \( \frac{(4-S)c^2}{4(4-S)} \) |
| \( c \geq \frac{(2-S)a}{4-S} \) | \( \frac{a}{2} \) | \( \frac{a}{2} \) | \( \frac{\lambda a}{2(2-S)} \) | \( \frac{(6-S)a}{2(4-S)} \) | \( \frac{a^2}{4} \) | \( \frac{(4-S)c^2}{4(4-S)} \) |

From Table 3, we can obtain the condition of encroachment in the MR mode.
Theorem 1. In the MR mode, the manufacturer introduces a direct channel when \( c < \frac{(2-S)s}{4-S} \); otherwise, she does not introduce a direct channel.

Theorem 1 indicates that in the MR mode, if the operating cost \( c \) is very high, encroachment hurts the manufacturer, so she does not introduce a direct channel, otherwise, she introduces a direct channel. Improving the service efficiency \( S \) reduces \( \frac{(2-S)s}{4-S} \), so the feasibility of introducing a direct channel also decreases. This means that improving the service efficiency can prevent encroachment.

In the MR mode, when \( c < \frac{(2-S)s}{4-S} \), the manufacturer introduces a direct channel. After encroachment, we compare the optimal outcomes in the MR mode with that in the benchmark without encroachment and obtain as follows.

Proposition 2. (1) \( w^{MR} = w^{B}, p^{MR} < p^{B}, Q^{MR} < Q^{B} \); (2) \( \frac{d\pi_{M}}{dc} > 0, \frac{d\pi_{R}^{MR}}{dc} > 0, \frac{d\pi_{R}^{MR}}{ds} > 0 \).

Proposition 2(1) shows that the wholesale price remains unchanged, but the retail price becomes lower, which weakens the double marginalization problem in a decentralized supply chain, which is in line with [10] and [26]. Proposition 2(2) shows that increasing the operating cost benefits the retailer but hurts the manufacturer, which is consistent with our instincts. In addition, increasing the service efficiency benefits both the retailer and manufacturer. This means that the manufacturer free rides retail services.

4.2. The MRM mode. When the manufacturer chooses ex-post encroachment and the retailer chooses ex-post service, the decision mode is the MRM mode. This is a three-stage game and the sequence of events is as follows. The manufacturer first only sets the wholesale price, and the retailer then simultaneously sets the retail price. The manufacturer then chooses whether to introduce a direct channel. In the MR mode, the manufacturer introduces a direct channel when \( c < \frac{(2-S)s}{4-S} \), otherwise, she does not introduce a direct channel.

From Table 4, we can obtain the following result.

Theorem 2. In the MRM mode, the manufacturer introduces a direct channel when \( c < \frac{(3-S + \sqrt{4-S})s}{4-S + \sqrt{4-S}} \), otherwise, she does not introduce a direct channel.

Theorem 2 states as follows. The boundary \( \frac{(3-S + \sqrt{4-S})s}{4-S + \sqrt{4-S}} \) shows that a high service efficiency reduces the operating cost threshold for manufacturer encroachment,
and and vice versa. This implies that an effective strategy for the retailer to counter encroachment is to improve the service efficiency.

When \( c < \frac{(3-S)a}{4+S+\sqrt{4-S}} \), the manufacturer introduces a direct channel in the MRM mode. In such case, the effects of manufacturer encroachment on the optimal outcomes and profits for the two partners are shown in Proposition 3.

**Proposition 3.** (1) \( w_{RM} > w_B \) for \( c > \frac{(2-S)a}{4-S} \), \( p_{r_{RM}} > p_{rB} \) for \( c > \frac{(3-S)a}{4-S} \), \( Q_{r_{RM}} > Q_{rB} \) for \( \frac{(3-2S)a}{2(4-S)} < c < \frac{(3-S)a}{4-S} \); (2) \( \pi_{M_{RM}} > \pi_{M_{B}} \), \( \pi_{R_{RM}} > \pi_{R_{B}} \) when \( \frac{(3-2S)a}{2\sqrt{(4-S)(2-S)} < c < \frac{(3-S)a}{4-S} \).

Proposition 3(1) shows that both the wholesale price and retail price for the retail channel may become larger with manufacturer encroachment in the MRM mode, which does not occur in the MR mode. This means that manufacturer encroachment in the MRM mode may aggravate double marginalization. Proposition 3(2) shows that manufacturer encroachment benefits the retailer when the operating cost is medium \( \frac{(3-2S)a}{2\sqrt{(4-S)(2-S)}} < c < \frac{(\sqrt{(4-S)(2-S)}-1)a}{\sqrt{(4-S)(2-S)}} \). The reason is that for the medium operating cost, the retail sale quantity becomes larger even though the wholesale price becomes higher. The retailer becomes better off since the benefit from the increase of sale quantity outweighs the loss from the increase of wholesale price. This means that manufacturer encroachment in the MRM mode can lead to a “win-win” outcome for the manufacturer and retailer. However, a “win-win” outcome does not occur in the MR mode where only a “win-lose” outcome for the manufacturer and retailer occurs in the MR mode.

![Figure 2](image-url)  
**Figure 2.** The effects of \( c \) on the optimal outcomes in the MRM mode

To gain a more intuitive understanding, we conduct a numerical example to reveal all possible outcomes. Given \( a = 1 \) and \( S = 0.4 \), Figure 2 plots the wholesale price, sale quantities for the two channels, and the profits for the manufacturer, the retailer, and the supply chain changing with the operating cost. The left subfigure shows as follows. As the operating cost increasing, the sale quantity always decreases, even decreases to zero. However, the wholesale price first decreases and then increases, and the sale quantity for the retail channel first increases and then decreases. Both the wholesale price and the sale quantity for the retail channel is higher than that in the benchmark. The right subfigure shows that the manufacturer always benefit from encroachment, while the retailer and the whole supply chain benefit only if the operating cost is low.
chain maybe become worse off, any maybe become better off. This means that encroachment may largen double marginalization, and the whole supply chain becomes inefficiency.

In the MRM mode there exist the following conclusions, which is not consistent with intuition, and does not occur in the MR mode.

**Property 1.** When \((3-2S)a < \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}\), the manufacturer introduces a direct channel even though the direct sale quantity is zero.

Property 1 shows that when \((3-2S)a < \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}\), even if the direct sale quantity is zero, i.e., introducing a direct channel is inefficient, which is mentioned by [10], introducing a direct channel is a profitable strategy for the manufacturer because introducing a direct channel can increase the manufacturer’s profit.

**Property 2.** (1) \(\frac{d\pi_{RM}}{dS} < 0\) and \(\frac{d\pi_{RM}}{dc} < 0\) when \((3-2S)a < \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}\), (2) \(\frac{d\pi_{RM}}{dS} > 0\), \(\frac{d\pi_{RM}}{dc} > 0\) when \((3-2S)a < \frac{(2-S)a}{3-S}\).

Property 2(1) shows that the service efficiency has a negative impact on the retailer when \((3-2S)a < \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}\). The reason is that as the service efficiency increases, only the wholesale price increases, both the retail sale quantity and price remain unchanged. In addition, the operating cost has a negative impact on the retailer, i.e., the higher the operating cost is, the less the retailer’s profit will be. Intuitively, the higher the operating cost is, the weaker the direct channel’s competitiveness will be, so the retailer benefits from the high operating cost. However, this result is quite contrary to intuition. The reason is that as the operating cost increases, the retail quantity decreases which induces the wholesale and retail price to increase. The loss from the decreasing sale quantity and increasing wholesale price outweighs the benefit from the increasing retail price, so the retailer becomes worse off from the high operating cost. Property 2(2) shows that the manufacturer benefits from services, i.e., she free rides services. However, the operating cost has a positive impact on the manufacturer, which does not occur in the MR mode. The reason that as the operating cost increases, the wholesale price increases but the retail sale quantity decreases. The manufacturer can become better off from the high operating cost because the benefit from an increase of the wholesale price outweighs the loss from the decrease of the retail sale quantity.

![Figure 3. Impacts of \(\frac{c}{a}\) and \(S\) on the profits in the MRM mode](image-url)
For intuitive reflecting how the parameters $a$, $c$ and $S$ affect the profits for the manufacturer and retailer, Figure 3 shows that the effects of the service efficiency and the operating cost on the profits for the two partners. Figure 3 indicates as follows. When the operating cost is high, i.e., $\frac{S}{a}$ is close to one, the manufacturer becomes worse off when encroachment, so she does not introduce a direct channel. When the operating cost is not high, the manufacturer can benefit from encroachment, so she introduces a direct channel. In such case, when the operating cost is medium, the retailer can benefit from encroachment, so the “win-win” outcome between the manufacturer and retailer occurs.

4.3. The MRMR mode. When the manufacturer chooses ex-post encroachment and the retailer chooses ex-ante service, the decision mode between the manufacturer and the retailer becomes the MRMR mode. This is a four-stage game and the sequence of events is as follows. In the stage 1, The manufacturer sets only the wholesale, and in the stage 2, the retailer sets only the service level, and in the stage 3, the manufacturer determines the direct sale quantity, and in the stage 4, the retailer determines the retail sale quantity. The optimal outcomes are shown in Table 5 through the backward induction.

Table 5. The optimal outcomes in the MRMR mode

| $c$ | $\omega_{RMR}^{M}$ | $\omega_{RMR}^{R}$ | $Q_{RMR}^{M}$ | $Q_{RMR}^{R}$ | $P_{RMR}^{M}$ | $P_{RMR}^{R}$ | $\pi_{RMR}^{M}$ | $\pi_{RMR}^{R}$ |
|-----|------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\omega_{RMR}^{M}$ | $\left(\frac{128-96S+9S^2}{256-144S+9S^2}\right)a$ | $\left(128-96S+9S^2\right)a$ | $\left(\frac{128-96S+9S^2}{256-144S+9S^2}\right)a$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $\omega_{RMR}^{R}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $Q_{RMR}^{M}$ | $\left(128-96S+9S^2\right)a-\left(256-144S+9S^2\right)c$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $Q_{RMR}^{R}$ | $\left(128-96S+9S^2\right)a-\left(256-144S+9S^2\right)c$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $P_{RMR}^{M}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $P_{RMR}^{R}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $\pi_{RMR}^{M}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |
| $\pi_{RMR}^{R}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ | $\frac{a}{2}$ |

From Table 5, we can draw the following conclusion.

Theorem 3. In the MRMR mode, the manufacturer introduces a direct channel only when $c < \frac{(128-96S+9S^2)a}{256-144S+9S^2}$, otherwise, she does not introduce a direct channel.

Theorem 3 indicates that the manufacturer would introduce/not introduce a direct channel when $\frac{41a}{121} / c > \frac{a}{2}$ regardless of the service efficiency. When $\frac{41a}{121} \leq c \leq \frac{a}{2}$, the service efficiency starts to play a negative role on introducing the direct channel. The boundary $\frac{(128-96S+9S^2)a}{256-144S+9S^2}$ shows that a high service efficiency reduces the operating cost threshold for the encroachment, and vice versa. The result is similar with that in the MR mode and the MRM mode.

After the manufacturer introduces a direct channel in the MRMR mode (i.e., $c < \frac{(128-96S+9S^2)a}{256-144S+9S^2}$), we can obtain the following conclusion.

Proposition 4. (1) $\omega_{RMR}^{M} < \omega_{R}^{B}$, $Q_{RMR}^{R} > Q_{R}^{R}$ when $c > \frac{(128-96S+9S^2)a}{256-144S+9S^2}$, $P_{RMR}^{R} > P_{R}^{R}$ when $c > \frac{(128-96S+9S^2)a}{512-224S-128S+9S^2}$, (2) $\pi_{RMR}^{M} > \pi_{M}^{R}$, $\pi_{RMR}^{R} < \pi_{R}^{R}$.

Proposition 4(1) shows that when the manufacturer introduces the direct channel in the MRMR mode, the wholesale price becomes lower. Though the lower wholesale price can increase the retail sale quantity, the lower operating cost can increase the
direct sale quantity, which means that the demand for the retail channel is serious cannibalized by the direct channel. As a result, the sale quantity for the retail channel becomes larger only when the operating cost is relative high. In addition, the retail price maybe becomes higher when the operating cost is high due to the direct sale quantity decreasing with the operating cost. The phenomenon that both the sale quantity and the retail price for the retail channel become larger also occurs in the MRM mode, but not occur in the MR mode. Proposition 4(2) shows that the retailer always becomes worse off in the MRMR mode, even though the wholesale price becomes lower and the retail sale maybe becomes larger, which is similar with that in the MR mode. That is, only the “win-lose” outcome for the manufacturer and the retailer occurs in the MRMR mode and the MR mode, while the “win-win” outcome only occurs in the MRM mode.

![Figure 4](image-url)

**Figure 4.** The effects of $c$ on the optimal outcomes in the MRMR mode

To gain a more intuitive understanding, we further conduct a numerical example to reveal all possible outcomes. Given $a = 1$ and $S = 0.4$, Figure 4 plots the wholesale price, sale quantities for the two channels, and the profits for the manufacturer, the retailer, and the supply chain changing with the operating cost. The left sub-figure shows as follows. As the operating cost increasing, the sale quantity always decreases, even decreases to zero. However, the wholesale price first decreases and then increases, and the sale quantity for the retail channel first increases and then decreases. Both the wholesale price and the sale quantity for the retail channel is higher than that in the benchmark without encroachment. The left subfigure shows that the manufacturer always benefit from encroachment, while the retailer and the whole supply chain maybe become worse off, any maybe become better off. This means that encroachment may largen double marginalization, and the whole supply chain becomes inefficiency.

**Property 3.** When $c < \frac{(128 - 96S + 9S^2)a}{256 - 144S + 9S^2}$, (1) $\frac{d\pi_{RMR}}{dS} > 0$, (2) $\frac{d\pi_{RMR}}{dc} > 0$; (2) $\frac{d\pi_{RMR}}{dc} > 0$ when $c < \frac{(128 - 96S + 9S^2)a}{256 - 96S + 9S^2}$.

Property 3 shows that how the service efficiency and the operating cost affect the profits for the manufacturer and retailer. Property 3(1) means that the service efficiency has a positive impact on both the retailer and manufacturer, i.e., improving the service efficiency benefits not only the retailer but also the manufacturer. This is similar with that in the MR mode, but different with that in the MRM mode where improving the service efficiency maybe hurts the retailer. Property 3(2) shows that the operating cost has a positive impact on the retailer, which is similar with that
in the MR mode, but different with that in the MRM mode where increasing the operating cost maybe hurts the retailer. Besides, the low/high operating cost has a negative/positive effect on the manufacturer, which is similar with that in the MRM mode, but different with that in the MR mode where the operating cost always has a negative impact on the manufacturer.

5. The equilibrium decision mode. In the above section, we discuss the optimal strategies and profits for the manufacturer and retailer in three decision modes, i.e., the MR mode, MRM mode and MRMR mode, respectively. In this section, we discuss the equilibrium decision mode between the manufacturer and retailer. From Theorem 1/Theorem 2/Theorem 3, we can obtain the MR mode/MRM mode/MRMR mode is a feasible mode when
\[ c < \frac{(2-S)\alpha}{4-S} < \frac{(3-S+\sqrt{4-S})\alpha}{4-S+\sqrt{4-S}} < \frac{(128-96S+9S^2)\alpha}{256-144S+9S^2}. \]
It is easy to prove \( \frac{(2-S)\alpha}{4-S} < \frac{(128-96S+9S^2)\alpha}{256-144S+9S^2} < \frac{(3-S+\sqrt{4-S})\alpha}{4-S+\sqrt{4-S}}. \) Therefore, all the three decision modes are feasible modes when \( c < \frac{(2-S)\alpha}{4-S}. \) We first compare the profits for the manufacturer and retailer in the three decision modes, and then discuss the equilibrium decision mode between the manufacturer and retailer.

**Proposition 5.** Decision mode preferences for the manufacturer and the retailer are given in decreasing order as follows: (1) the manufacturer: MRM\( \succ \)MRMR\( \succ \)MR; (2) the retailer: MR\( \succ \)MRMR\( \succ \)MRM for \( S \leq 0.3763 \), MR\( \succ \)MRM\( \succ \)MRMR for \( 0.3763 < S \leq 0.6096 \), and MRM\( \succ \)MR\( \succ \)MRMR for \( S > 0.6096 \).

Proposition 5(1) states that the manufacturer prefers the MRM mode to the MRMR mode, and prefers the MRMR mode to the MR mode. This means that the traditional decision mode adopted by the lot of researches in a dual-supply chain (i.e., the MR mode) is worst for the manufacturer. Though the manufacturer is the leader in a supply chain, she should give up the first-move right and adopt ax-post encroachment strategy. However, Proposition 5(2) indicates that the MR mode, i.e., the traditional decision mode, is optimal for the retailer when the service efficiency is not high, and the MRM mode is optimal for the high service efficiency.

![Figure 5. The effects of S on the profits in three decision modes](image)

Figure 5 plots how the profits for the manufacturer and retailer vary with the service efficiency \( S \) given \( \alpha = 1 \) and \( c = 0.3 \). All the three decision modes are feasible modes when \( S \) varies from 0 to 1 given \( \alpha = 1 \) and \( c = 0.3 \). From Figure 5, we can obtain as follows. The MRM mode is always best for the manufacturer, and the MR mode is always worst. For the retailer, the MRMR mode always is not optimal. The optimal decision mode depends on the service efficiency. When the
service efficiency is not high, the MR mode is best, while when it is high, the MRM mode is optimal. These results are consistent with Proposition 5.

We have discussed the optimal decision mode from the manufacturer’s view and the retailer’s view, respectively. Next, we discuss the equilibrium mode between the manufacturer and retailer. Notice that if the manufacturer declares to encroach, the choice of the decision mode between the manufacturer and the retailer is as follows. If the manufacturer chooses ex-ante encroachment, the decision mode between the manufacturer and the retailer is the MR mode regardless of the retailer choosing ex-ante service and ex-post service. According to Proposition 5, the MR mode is worst for the manufacturer, so the MR mode is not an equilibrium decision mode. If the manufacturer chooses ex-post encroachment, the decision mode is the MRM mode when the retailer chooses ex-post service, and is the MRMR mode when the retailer chooses ex-ante service. So the retailer decides to choose either the MRM mode or the MRMR mode. According to Proposition 5, we can obtain the retailer’s choice between the MRM mode and the MRMR mode depends on the service efficiency. Thus, the equilibrium decision mode between the manufacturer and the retailer depends on the service efficiency, shown in Theorem 4.

**Theorem 4.** The equilibrium decision mode is as follows: (1) if $S \leq 0.3763$, the MRM mode when 
$$c < \frac{\sqrt{(4-S)(2-S)}(3-2S)}{2\sqrt{4-S}}$$ 
and the MRMR mode when $c < \frac{(128-96S+9S^2)a}{(256-144S+9S^2)}$; (2) if $0.3763 \leq S < 0.4355$, the MRM mode when 
$$c < \frac{\sqrt{(4-S)(2-S)}(3-2S)}{2\sqrt{4-S}} \text{ or } c < \frac{\sqrt{(4-S)(2-S)}(3-2S)}{2\sqrt{4-S}}$$ 
or $c < \frac{(128-96S+9S^2)a}{(256-144S+9S^2)}$; (3) if $S \geq 0.4355$, the MRM mode when $c < \frac{\sqrt{(4-S)(2-S)}(3-2S)}{2\sqrt{4-S}}$.

Theorem 4 indicates that when the operating cost is high, which results in the competitiveness of direct channel is weak, the manufacturer does not introduce a direct channel. When the operating cost is medium, the manufacturer chooses ex-ante encroachment to introduce a direct channel and the retailer chooses ex-post service, so the equilibrium decision mode the MRM mode. When the operating cost is low, the manufacturer inclines to introduce a direct channel. In addition, Theorem 4 shows that the retailer prefers to choose the MRMR mode for a low service efficiency and the MRM mode for the high service efficiency.

**Figure 6.** The impact of $\frac{c}{a}$ and $S$ on the equilibrium decision mode
Figure 6 illustrates the impacts of the service efficiency and the operating cost on the equilibrium decision mode and profits for the two partners. From Figure 6, we can infer as follows. (i) The operating cost plays a critical role in the introduction of a direct channel. Specifically, the manufacturer introduced/does not introduce a direct channel under low/high operating cost regardless of the service efficiency. Under a relatively medium operating cost, whether to introduce a direct channel depends on the service efficiency. The service efficiency has a negative impact on manufacturer encroachment. (ii) When the operating cost is medium, the equilibrium mode is the MRM mode (a light-grey subarea in Figure 6). In such case, introducing a direct channel benefits both the manufacturer and the retailer, which means that a “win-win” outcome occurs in a dual-channel supply chain. (iii) When the parameter pair \((S,c/a)\) drops into the dark-grey area, the retailer would choose the MRMR mode and the direct sale quantity is zero even if the manufacturer introduces a d-channel, so the profits for the manufacturer and retailer is equal to that without a d-channel. This means that introducing a d-channel is quite meaningless for the manufacturer. Thus, the manufacturer does not introduce a direct channel when \((S,c/a)\) drops into the dark-grey area.

6. Conclusion. This paper considers a dual-channel supply chain consisting of one manufacturer and one retailer. We focus on the sequence of two strategic decisions related to encroachment and retail services. The existing literature predominantly assumes that the manufacturer, as the leader, simultaneously sets the wholesale price for the retail channel and the direct sale quantity, and the retailer, as the follower, simultaneously sets the sale quantity and the service level for the retail channel. In this paper, we investigate how different decision sequences affect manufacturer encroachment. We develop a game theoretic model and assume that the manufacturer may choose ex-ante encroachment or ex-post encroachment to introduce a direct channel through paying a unit operating cost for selling a unit product through the direct channel. The retailer can provide customers with ex-ante services or ex-post services to stimulate the retail channel’s demand. Thus, for the manufacturer and retailer, there exist three decision modes: MR mode (ex-ante encroachment), MRM mode (ex-ante encroachment and ex-post service) and MRMR mode (ex-ante encroachment and ex-ante service). We investigate how manufacturer encroachment affects the optimal strategies and profits for the two partners in the three decision modes, and then analyze the equilibrium decision mode for the manufacturer and the retailer.

The main results show that the sequence of decisions has a significant effects on channel members’ decisions and profits. In the MR mode and the MRMR mode, manufacturer encroachment always benefits the manufacturer while hurts the retailer. That is, only the “win-lose” outcome for the manufacturer and retailer occurs. However, in the MRM mode, manufacturer encroachment may benefit the retailer under certain conditions. That is, in the MRM mode, besides the “win-lose” outcome for the manufacturer and retailer, the “win-win” outcome maybe occurs. In addition, in the MRM mode, even if the direct sale quantity is zero, the manufacturer regards a direct channel as a strategic method to force the retailer to increase the retail sale quantity, and the manufacturer increases the wholesale price. Thus, the manufacturer can benefit from encroachment even if the direct sale quantity is zero. In such case, there exist the results which are not consistent with our intuition. Generally, channel competition induced by encroachment can
reduce the pricing for the retail channel, but the wholesale price becomes higher in the MRM mode. In addition, both the service efficiency and the direct-channel operating cost have a negative effects on the retailer. That is, improving the service efficiency hurts the retailer, and increasing the operating cost also harms him, which does not occur in the MR mode and the MRMR mode. In the MRM mode and the MRMR mode, increasing the operating cost maybe benefits the manufacturer, while increasing the operating cost always hurts the manufacturer in the MR mode.

We further discuss the equilibrium decision mode between the manufacturer and retailer. From the manufacturer’s perspective, the MR mode, which is widely adopted by the existing literature on a dual-channel supply chain, is worst for the manufacturer. The result indicates that the manufacturer should give up the first-move right, and should adopt ex-post encroachment strategy. From the retailer’s view, the service efficiency play a significant role on the optimal decision mode, and the retailer can benefit from the manufacturer’s first move. That is, when the service efficiency is not high, the MR mode is best for him. In equilibrium mode, the operating cost for the direct channel is a major factor in manufacturer encroachment and the equilibrium decision mode. When the operating cost is high, the manufacturer only sells through the retail channel, i.e., she does not introduce a direct channel. When the operating cost is medium, the manufacturer chooses the MRM mode to introduce a direct channel. When the operating cost is low, the manufacturer chooses the MRMR/MRM mode for the low/high service efficiency.

This paper has a few limitations. We assume that manufacturer encroachment is endogenous while retail service is exogenous. In practice, there exist many retailers that do not provide retail services. In this paper, the result shows that increasing the service efficiency hurts the retailer under certain conditions. This means that no offering retail services benefits the retailer. Thus, Considering an endogenous retail service is worthy to further research. In addition, this paper assumes that the information about the market demand, the operating cost, and the service efficiency are symmetric for the two members. Generally, the manufacturer/the retailer does not exactly know the information about the service efficiency/the operating cost. Moreover, there exist competitions between the upstream manufactures and/or the downstream retailers. It would be interesting to incorporate these elements into the model in the future research.

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**Appendix. The derivation of Table 3.** Under the MR mode, given $w$ and $Q_d$, the retailer sets $Q_r$ and $s$ to maximize his profit, i.e., solves

$$\max_{Q_r, s} \pi_R(Q_r, s) = (a - Q_r - Q_d + \lambda s - w)Q_r - \eta s^2. \quad (A.1)$$

It is easy to prove that $\pi_R(Q_r, s)$ is strictly jointly concave in $(Q_r, s)$. Thus, the retailer’s optimal strategy is as follows,

$$Q_r^{MR} = \begin{cases} \frac{2(a-w-Q_d)}{4-\lambda}, & w + Q_d < a, \\ 0, & w + Q_d \geq a. \end{cases}, \quad s^{MR} = \lambda Q_r^{MR}. \quad (A.2)$$
Substituting (A.2) into (2) and simplifying, we can get the manufacturer’s profit as
\[
\max_{w, Q_d} \pi_M(w, Q_d) = \begin{cases} 
\frac{2w(a-w-Q_d)+[(4-S)(a-c)-2(a-w)-(2-S)Q_d]Q_d}{4-S}, & w + Q_d < a, \\
(a - Q_d - c)Q_d, & w + Q_d \geq a.
\end{cases}
\] (A.3)

Because \(\pi_M(w, Q_d)\) is jointly concave in \((w, Q_d)\), from the first-order optimality condition, we can obtain
\[
w_{MR}^* = \frac{a}{2}, \quad Q_{d}^{MR} = \begin{cases} 
\frac{(4-S)(a-c)-2a}{4-2S}, & c < \frac{(2-S)a}{4-S}, \\
0, & c \geq \frac{(2-S)a}{4-S}.
\end{cases}
\] (A.4)

From (A.1)-(A.4), the optimal outcomes are summarized in Table 3. \(\square\)

**Proof of Proposition 2.** It is easy to obtain Proposition 2 from Table 3, omitted. \(\square\)

**The derivation of Table 4.** In the MRM mode, given \(w, Q_r\) and \(s\), the manufacturer sets \(Q_d\) to maximize her profit, i.e., solves the following optimization problem:
\[
\max_{Q_d} \pi_M(w, Q_d) = wQ_r + Q_d(a - c - Q_r - Q_d).
\] (A.5)

From (A.5), we can obtain the optimal direct sale quantity is
\[
Q_{d}^{RM} = \begin{cases} 
\frac{a-c}{2}, & Q_r < a - c, \\
0, & Q_r \geq a - c.
\end{cases}
\] (A.6)

where the superscript \(RM\) represents the optimal outcomes in the MRM mode. After anticipating the direct sale quantity \(Q_{d}^{RM}\) in (A.6), the retailer sets \(Q_r\) and \(s\) to maximize his profit,
\[
\max_{Q_r, s} \pi_R(Q_r, s) = \begin{cases} 
(a - Q_r - \frac{a-c}{2}Q_r + \lambda s - w)Q_r - \eta s^2, & Q_r < a - c, \\
(a - Q_r + \lambda s - w)Q_r - \eta s^2, & Q_r \geq a - c.
\end{cases}
\] (A.7)

From (A.7), the retail sale quantity and the service level are, respectively,
\[
Q_{r}^{RM}(w) = \begin{cases} 
\frac{2a-2w+\frac{a-c}{2}}{2-S}, & 2a-(3-S)(a-c) < w, \\
\frac{2a-(4-S)(a-c)}{2}, & 2a-(4-S)(a-c) \leq w \leq \frac{2a-(3-S)(a-c)}{2}, \quad s^{RM}(w) = \frac{\lambda Q_{d}^{RM}(w)}{2\eta}.
\end{cases}
\] (A.8)

The manufacturer sets the wholesale price \(w\) to maximize her profit, i.e.,
\[
\max_w \pi_M(w) = \begin{cases} 
\frac{(w(a+c-2w)}{2-S} + \frac{(3-S)(a-c)-2a+2w}{2(2-S)} \geq w \leq \frac{2a-(3-S)(a-c)}{2}, & \frac{2a-(4-S)(a-c)}{2} < w < \frac{a+c}{2}, \\
\frac{2w(a-w)}{4-S}, & w \leq \frac{2a-(4-S)(a-c)}{2}.
\end{cases}
\] (A.9)

From (A.9), the optimal wholesale price is
\[
w_{MR}^* = \begin{cases} 
\frac{a}{2}, - \frac{(3-S)c-(1-S)a}{2(2-S)} \leq c < \frac{(3-S)a}{5-2S}, \\
\frac{3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}, & \frac{(3-S)a}{5-2S} \leq c < \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}, \\
\frac{(3-S)c-(1-S)a}{2}, & c \geq \frac{(3-S+\sqrt{4-S})a}{4-S+\sqrt{4-S}}.
\end{cases}
\] (A.10)

From (A.5)-(A.10), the outcomes can be obtained, showed in Table 4. \(\square\)
Proof of Theorem 2. The proof is straightforward, omitted. □

Proof of Proposition 3. It is easy to obtain Proposition 3 from Table 4, omitted. □

The derivation of Table 5. In the MRMR mode, given \( w, Q_d \) and \( s \), the retailer sets \( Q_r \) to maximize his profit, i.e., solves the following optimization problem:

\[
\max_{Q_r} \pi_r(s, Q_r) = (a - Q_r - Q_d + \lambda s - w)Q_r - \eta s^2.
\] (A.11)

The optimal retail sale quantity is

\[
Q^{RMR}_r = \begin{cases} \frac{a - w - Q_d + \lambda s}{2}, & Q_d < a - w + \lambda s, \\ 0, & Q_d \geq a - w + \lambda s. \end{cases}
\] (A.12)

where the superscript \( RMR \) represents the optimal outcomes in the MRMR mode. After anticipating the retail sale quantity \( Q^{RMR}_r \) in (A.12), the manufacturer sets \( Q_d \) to maximize her profit

\[
\max_{Q_d} \pi_M(w, Q_d) = \begin{cases} \frac{w(a - w + \lambda s)}{2} + \frac{(a - 2c - \lambda s - Q_d)Q_d}{2}, & Q_d < a - w + \lambda s, \\ (a - c - Q_d)Q_d, & Q_d \geq a - w + \lambda s. \end{cases}
\] (A.13)

From (A.13), the direct sale quantity is

\[
Q^{RMR}_d(w, s) = \begin{cases} \frac{a - 2c - \lambda s}{2}, & a + 2c - 2w + 3\lambda s > 0, \\ \frac{a - c}{2}, & a + 2c - 2w + 3\lambda s \leq 0. \end{cases}
\] (A.14)

Obviously, if \( a + 2c - 2w + 3\lambda s \leq 0 \), then the retail sale quantity is zero, which means that the retailer will exist the market. In such case, the manufacturer’s profit is \( \pi_M = \frac{(a - c)^2}{4} \). If \( a + 2c - 2w + 3\lambda s > 0 \), then the retailer sets the service level \( s \) to maximize his profit is

\[
\max_s \pi_M(w) = \frac{(a + 2c - 2w + 3\lambda s)^2}{16} - \frac{\eta s^2}{2(a - w + 3\lambda s)}, a + 2c - 2w + 3\lambda s > 0.
\] (A.15)

It is easy to obtain \( s^{RMR} = \frac{3a(2c + 2w)}{\eta(16 - 9\lambda)} \). Thus, the manufacturer maximizes profit is

\[
\pi_M(w) = \frac{w[(16 - 6S)a + 6Sc - (16 - 3S)w]}{2(16 - 9S)} + \frac{[8 - 6S]a - (16 - 6S)c + 3Sw)^2}{2(16 - 9S)^2}.
\] (A.16)

From (A.16), the optimal wholesale price is \( w^{MRM} = \frac{a}{2} - \frac{9S}{2(128 - 96S + 9S^2)} \). It is easy to obtain \( Q^{RMR}_d \) is

\[
\frac{(128 - 96S + 9S^2)a - (256 - 144S + 9S^2)c}{2(128 - 96S + 9S^2)}. \] (A.17)

Thus, (i) when \( c < \frac{(128 - 96S + 9S^2)a}{256 - 144S + 9S^2}, Q^{RMR}_d > 0 \). In such case, \( \pi^{RMR}_M > \pi_M = \frac{(a - c)^2}{4} \). (ii) when \( c \geq \frac{(128 - 96S + 9S^2)a}{256 - 144S + 9S^2}, Q^{RMR}_d \leq 0 \). In such case, the manufacturer only sells through the retail channel and her profit is \( \pi^{RMR}_M \leq \pi_M = \frac{(a - c)^2}{4} \). To sum up, we can obtain Table 4. □

Proof of Theorem 3. It is easy to obtain Theorem 3 from Table 5. □

Proof of Proposition 4. It is easy to obtain Proposition 4 from Table 5. □

Proof of Proposition 5. From Tables 3, 4 and 5, it is easy to obtain Proposition 5. □

Proof of Theorem 4. From Proposition 5, it is easy to obtain Theorem 4. □
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