Effect of Wave Collapse on Lyman and Balmer lines

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Abstract. The nonlinear coupling of Langmuir, ion sound and electromagnetic waves can create wave collapse conditions in a plasma, and modify the line shapes emitted by hydrogen atoms. Such conditions may appear in the presence of a strong source of energy, and create numerous regions where wave packets localize and the plasma density is decreased. We use a model of envelope solitons for the electric field acting on the emitter, and calculate the dipole autocorrelation function and the line shape with a numerical simulation. We investigate the role of the frequency of wave packets, and propose an approach retaining a dispersion relation $\omega(k)$.

1. Introduction

We examine conditions where the ratio $W$ of the wave energy density to the plasma thermal energy density is of the order of unity or larger, allowing the development of a modulational instability. This instability modulates an initially uniform plane wave and can separate it into wave packets [1]. The evolution of such wave packets may be described by a non-linear Schrödinger equation (NLSE) in the limit of small density fluctuations. Solving the NLSE shows that the solution is a stable soliton for one dimension, but that this stability is lost for a three-dimensional problem. In the latter case, wave packets undergo a wave collapse during a cycle where the electric field magnitude grows, collapses, dissipates and reforms. For such conditions, we have recently proposed a model for the electric field experienced by an emitter in a plasma. It consists of a sequence of solitons each one acting during a lifetime of the order of a wave packet cycle [2,3]. Wave collapse conditions are expected in different types of plasmas submitted to a source of energy like a beam of energetic particles. This situation is encountered in astrophysics, e.g. in active galactic nuclei, but also in laboratory plasmas in the presence of electron beams. The aim of the work is to provide a spectroscopic diagnostic of wave collapse using hydrogen Lyman and Balmer lines. We present in the following our model and discuss the effect of the oscillation frequency of the soliton, as well as the use of a dispersion relation for Langmuir waves.

2. Model for the dipole autocorrelation function

We use a sequence of envelope solitons of frequency $\omega_k$, with $k$ the wave number linked to the frequency by a dispersion relation. In the following each soliton has a Lorentzian envelope and the lifetime is sampled with an exponential law $\nu \exp(-\nu t)$, with $\nu$ the jumping frequency chosen as the inverse of the characteristic time $\tau$ for the wave collapse cycle [2]. This time can be estimated from simulations [4], and is expressed as $\tau \approx 40(W)^{-1}c\omega_p^{-1}$, where the brackets denote an average value and the plasma frequency is defined by $\omega_p = (N_e e^2/m\varepsilon_0)^{1/2}$, with $N_e$ the electronic plasma density, $e$ and $m$ the electron charge and mass, and $\varepsilon_0$ the permittivity of free space. The magnitude of the
electric field has been taken to obey a Gaussian probability density function as measured in some experiments [5]. We obtain the dipole autocorrelation function (DAF) for the emitter by solving numerically the Schrödinger equation for the atomic evolution operator. Line shapes are obtained by a Fourier transform of the DAF, and can be calculated for a convolution of the equilibrium Stark profile and wave collapse profile.

3. Effect of the soliton frequency
Calculations already presented for the DAF and line shapes [2,3] have mostly been made by using an oscillation at the plasma frequency. But even a single Langmuir wave satisfies the dispersion relation

\[ \omega_k \approx \omega_p (1 + 3k^2 \lambda_D^2 / 2), \]  

indicating that the soliton frequency can take a continuum of values larger than the plasma frequency. In this expression the Debye length is defined by \( \lambda_D = (\varepsilon_0 k_B T / N_e e^2)^{1/2} \), with \( k_B \) the Boltzmann constant, and the wave number \( k \) can take values between zero and a maximum value \( k_{\text{max}} \). Large values of the wave number correspond to short wavelengths, but as the wavelengths become of the order of the Debye length, such waves are strongly damped due to the resonance effect of electrons travelling at the phase velocity of the waves. One thus usually employs this dispersion relation assuming \( k \lambda_D \ll 1 \). We first study the effect of the soliton frequency on the \( L_\gamma \) and \( H_\gamma \) lines using a sequence of solitons with a single oscillation frequency. These lines are of interest for a plasma diagnostic since they are particularly sensitive to electric fields. We consider a hydrogen plasma with a density \( N = N_e = N_i = 10^{19} \) m\(^{-3} \), and temperature \( T = T_i = T_e = 10^5 \) K. The ratio \( W \) has been taken equal to 1.9, corresponding to an electric field magnitude \( E_L = 200 E_0 \), where \( E_0 \) is the average plasma microfield defined by \( E_0 = e / 4\pi \varepsilon_0 r_0^2 \), with the average distance between particles defined by \( r_0 = \sqrt{3/4\pi N_\varepsilon} \). We first calculate the DAF of \( L_\gamma \) for oscillation frequencies \( \omega_p, 2\omega_p, \) and \( 4\omega_p \). For short times the DAF first exhibit a strong decay which can be attributed to the large electric field magnitude reached around the peak of the envelope soliton. Smaller field magnitudes contribute to the weaker decay observed for times longer than \( \tau \). Satellites clearly appear on the \( L_\gamma \) DAF separated by a time equal to the period of the oscillation frequencies. A surprising feature is that the overall decay of the DAF is reduced as the frequency is increased. This can be explained by the contribution of non adiabatic transitions between Stark sublevels induced by the oscillating field. The field produces transitions back and forth between Stark sublevels without completely destroying the emission, a situation referred to as the problem of overlapping lines or phase memory effect [6]. The corresponding broadening mechanism is maximized as the oscillation frequency is equal to the average separation of the Stark sublevels. Here the average Stark splitting is smaller than the plasma frequency, and an increase of the oscillation frequency decreases the DAF decay, since we move away from resonance.

**Figure 1.** \( L_\gamma \) (figure (a)) and \( H_\gamma \) (figure (b)) DAF calculated with our model for a density \( N = 10^{19} \) m\(^{-3} \) and temperature \( T = 10^5 \) K. Effect of the oscillation frequency for \( \omega_p, 2\omega_p, \) and \( 4\omega_p \).
Hγ is a Balmer line (n=5→n=2) radiating in the visible, with a larger width than Lyγ, and a corresponding faster decay of the DAF. The decay is slightly underestimated in our calculations, since we neglect the broadening of the lower level. As for Lyγ but less numerous, satellites are also visible for oscillation frequencies ωp, 2ωp, and 4ωp, and are a response of the DAF in phase with the oscillation. Also similarly to Lyγ, an increase of the frequency reduces the decay of the DAF, but to a lesser extent than for Lyγ. The sensitivity of the DAF to the oscillation frequency raises the question of the effect of frequency dispersion. Assuming a dispersion relation given by equation (1), we have calculated the Hγ DAF for solitons with variable frequencies obtained by sampling wavenumbers k between 0 and a maximum value kmax chosen in an attempt to retain Landau damping. We have plotted the Hγ DAF for a fixed frequency ωp (solid line), and frequencies obtained from a dispersion relation with wavenumbers sampled from zero up to kmax=0.1/λD (dashed line), 0.5/λD (dotted line) and 1/λD (dash-dotted line). It can be seen on figure 2 (a) that a significant change of the decay occurs only if the wavelengths are chosen close to the Debye length. We can thus conclude that a model using a constant frequency ωp is adequate for such wave collapse conditions. We finally show on figure 2 (b) Hγ line profiles for the single effect of solitons (dotted line), equilibrium Stark effect (dashed line) and a convolution of Stark and soliton profiles (solid line). This convolution leads to a line shape about 60% broader than an equilibrium Stark profile, highlighting the possibility of spectra modifications in presence of wave collapse.

Figure 2. Effect of a dispersion relation for three different values of kmax λD on Hγ DAF (figure (a)). Hγ line shape for solitons effect, Stark effect and the convolution of both effects (figure (b)).

4. Conclusion
The single effect of a sequence of envelope solitons decorrelates the dipole radiation of hydrogen atoms in a plasma. We have studied the role of the frequency of oscillation due to a high frequency electrostatic wave. For practical calculations of wave collapse effects it is concluded that using a constant plasma frequency is a fair approximation for Lyman and Balmer lines.

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