On separate chemical freeze-outs of hadrons and light (anti)nuclei in high energy nuclear collisions

K. A. Bugaev, B. E. Grinyuk, A. I. Ivanytskyi, V. V. Sagun, D. O. Savchenko, G. M. Zinovjev, E. G. Nikonov, L. V. Bravina, E. E. Zabrodin, D. O. Savchenko, G. M. Zinovjev, E. G. Nikonov, L. V. Bravina, E. E. Zabrodin, D. O. Savchenko, G. M. Zinovjev, E. G. Nikonov, L. V. Bravina, E. E. Zabrodin

1 Bogolyubov Institute for Theoretical Physics of the National Academy of Sciences of Ukraine, 03680 Kiev, Ukraine
2 Department of Fundamental Physics, University of Salamanca, Plaza de la Merced s/n 37008, Spain
3 Centro de Astrofísica e Gravitação - CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal
4 Laboratory for Information Technologies, Joint Institute for Nuclear Research, Dubna 141980, Russia
5 Department of Physics, University of Oslo, PB 1048 Blindern, N-0316 Oslo, Norway
6 Skobeltsyn Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia
7 National Research Nuclear University “MEPhI” (Moscow Engineering Physics Institute), 115409 Moscow, Russia
8 Institute of Theoretical Physics, University of Wroclaw, pl. M. Borna 9, 50-204 Wroclaw, Poland
9 Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Joliot-Curie str. 6, 141980 Dubna, Russia
10 University of Nantes, Faculté des Sciences et des Techniques, 2, rue de la Houssinière, 44322 Nantes, France
11 SUBATECH, Ecole des Mines, 4 rue Alfred Kastler, 44307 Nantes, France

E-mail: Bugaev@th.physik.uni-frankfurt.de

Abstract. The multiplicities of light (anti)nuclei were measured recently by the ALICE collaboration in Pb+Pb collisions at the center-of-mass collision energy \( \sqrt{s_{_{NN}}} = 2.76 \) TeV. Surprisingly, the hadron resonance gas model is able to perfectly describe their multiplicities under various assumptions. For instance, one can consider the (anti)nuclei with a vanishing hard-core radius (as the point-like particles) or with the hard-core radius of proton, but the fit quality is the same for these assumptions. In this paper we assume the hard-core radius of nuclei consisting of \( A \) baryons or antibaryons to follow the simple law \( R(A) = R_b(A) \frac{A}{3} \), where \( R_b \) is the hard-core radius of nucleon. To implement such a relation into the hadron resonance gas model we employ the induced surface tension concept and analyze the hadronic and (anti)nuclei multiplicities measured by the ALICE collaboration. The hadron resonance gas model with the induced surface tension allows us to verify different scenarios of chemical freeze-out of (anti)nuclei. It is shown that the most successful description of hadrons can be achieved at the chemical freeze-out temperature \( T_h = 150 \) MeV, while the one for all (anti)nuclei is \( T_A = 168.5 \) MeV. Possible explanations of this high temperature of (anti)nuclei chemical freeze-out are discussed.
1. Introduction

Light (anti)nuclei production measured recently by the ALICE collaboration in Pb+Pb collisions at the center-of-mass collision energy $\sqrt{s_{NN}} = 2.76$ TeV [1] caught a considerable attention. Partly it is inspired by the fact that it looks rather surprising that light nuclei with binding energies of an order of a few MeV are produced at all in such violent collisions. Two main approaches, thermal production model [2, 3, 4, 5, 6] and coalescence one [7, 8, 9, 10, 11, 12] are able to explain this phenomenon equally well. Usually, the thermal model assumes a perfect chemical equilibrium above the chemical freeze-out (CFO) temperature $T_{CFO}$ and a sharp CFO of all hadrons at this temperature. After the CFO the yields are assumed to be unchanged, but particles can scatter elastically until the system reaches the moment of kinetic freeze-out, which at the ALICE experiment was reported to occur at temperature about 115 MeV [1], while the corresponding CFO temperature varies found in thermal model varies from 150 [5, 6] MeV to 160 MeV [2]. In contrast to these assumptions, the coalescence approach postulates that the light nuclei are formed only at late times of the nuclear-nuclear reaction via the binding of nucleons that move close in phase space.

The main problem with the existing formulations of thermal model is that none of them is suited to treat the excluded volume of (anti)nuclei and hadrons on the same footing. Indeed, in light (anti)nuclei the nucleons and hyperons are loosely bound and, hence, the radius of small nuclei of $A$ nucleons, or baryons in general, with $2 \leq A \leq 4$ is about 1.8-2 fm [13] i.e. it is essentially larger than the maximal double hard-core radius of hadrons (0.84 fm) [5, 6] even for a deuteron ($A = 2$). The total proper volume of $A$ baryons with the hard-core radius $R_b$ is $V_A = A^{1/3} \pi R_b^3$. Hence, the equivalent hard-core radius of a nucleus consisting of $A$ baryons is

$$R_A = A^{1/3} R_b,$$

since each baryon in a nucleus interacts with external particle individually and the other baryons do not affect them. Note that the recently developed hadron resonance gas model (HRGM) which is based on the induced surface tension (IST) concept [5, 6, 14] belongs to the class of HRGM (see, e.g., [15, 16]) which can handle any number of individual hadron excluded volumina corresponding to different hard-core radii (for a discussion and important applications see also [17]). Hence, in this work we report our preliminary results on the analysis of the ALICE hadronic and (anti)nuclei multiplicities measured at the collision energy $\sqrt{s_{NN}} = 2.76$ TeV with the hard-core radii of nuclei given by Eq. (1). Since the list of analyzed data is rather long, but is well-known, we refer to the original works [4, 5, 6] in which these ALICE data were thoroughly analyzed. Our treatment of hadronic data follows the line of Refs. [5, 6], i.e. the ratios will be analyzed, while the (anti)nuclei multiplicities are included into a fitting procedure in a way which allows us to verify two different hypotheses about their production.

The work is organized as follows. In Sect. 2 we describe the main features of the HRGM with IST. The results of the ALICE data analysis are presented in Sect. 3, while our conclusions are formulated in Sect. 4.

2. HRGM with multicomponent hard-core repulsion

The most advanced version of the HRGM with IST [5, 6] was originally formulated for the Boltzmann particles [14], but recently it is straightforwardly generalized to the cases of quantum particles [18, 19]. For the high CFO temperatures achieved in Pb+Pb reactions one can safely use the Boltzmann statistics for all hadrons and (anti)nuclei [5, 6]. Moreover, this provides an essential speed up of the fitting process, since the momentum integration can be done only once for each particle species.

In the grand canonical ensemble the HRGM with IST can be written as a system of two
coupled equations for the system pressure $p$ and the IST coefficient $\Sigma$:

$$p = T \sum_{k=1}^{N} \phi_k \exp \left[ \frac{-v_k p - s_k \Sigma}{T} \right], \quad \Sigma = T \sum_{k=1}^{N} R_k \phi_k \exp \left[ \frac{-v_k p - s_k \alpha \Sigma}{T} \right], \quad (2)$$

where $v_k = \frac{4}{3} \pi R_k^3$ is the proper volume of the particle with hard-core radius $R_k$, $s_k = 4 \pi R_k^2$ denotes its proper surface and $\alpha = 1.25 [5, 6]$. For the (anti)nuclei of $A$ (anti)baryons the hard-core radius is given by Eq. (1). In the system (2) all chemical potentials are set to zero, since at this value of collision energy there is almost no difference between particle and antiparticle [4, 5, 6]. The thermal density of the $k$-th particle species accounts for the Breit-Wigner mass attenuation of hadron resonances

$$\phi_k = g_k \int_{M_k^{Th}}^{\infty} \frac{dm}{N_k(M_k^{Th})} \left( \frac{m}{m_k} \right)^2 + \frac{\Gamma_k}{2} \int \frac{d^3p}{(2\pi)^3} \exp \left[ -\frac{\sqrt{p^2 + m^2}}{T} \right],$$

where $m_k$ and $\Gamma_k$ denote, respectively, the mass and width of the $k$-th particle species. The factor $N_k(M_k^{Th}) \equiv \int_{M_k^{Th}}^{\infty} \frac{dm \Gamma_k}{(m-m_k)^2 + \Gamma_k^2 / 4}$ denotes a corresponding normalization, while $M_k^{Th}$ corresponds to the decay threshold mass of the $k$-sort of hadrons in the leading channel. Although Eq. (3) is anapproximation for the wide resonances, it is known that such an approximation is sufficiently accurate [20]. It is necessary to stress that the mass attenuation in Eq. (3) for a mixture of hadron resonances can be rigorously obtained [21, 22] from the Phi-functional approach [23], when the Phi-functional is chosen from the class of two-loop diagrams only, see also [24].

In contrast to the usual multicomponent HRGM formulations to determine the particle number densities $\{\rho_k\}$ one needs to solve only a system of two equations of the system (2) (the total strange charge is zero by construction) irrespective to the number of different hard-core radii in the model. Hence, we believe that HRGM with IST is perfectly suited for the analysis of all hadronic multiplicities which will be measured in the nearest future at SPS, RHIC, NICA and FAIR.

Another great advantage of the HRGM with IST is its validity up to the packing fractions $\eta \equiv \sum_k \frac{4}{3} \pi R_k^3 \rho_k \approx 0.2 - 0.22 [5, 6]$, i.e. at the particle number densities for which the traditional HRGM based on the Van der Waals approximation is entirely wrong. Using the particle number density $\rho_k$ of $k$-th sort of hadrons one can determine the thermal $N_k^{Th} = V \rho_k$ ($V$ is the effective volume at CFO) and the total multiplicities $N_k^{tot}$. The latter should account for the hadronic decays after the CFO and, hence, the ratio of total hadronic multiplicities becomes

$$\frac{N_k^{tot}}{N_j^{tot}} = \frac{\rho_k + \sum_{l \neq k} \rho_l B_{rl \rightarrow k}}{\rho_j + \sum_{l \neq j} \rho_l B_{rl \rightarrow j}}, \quad (4)$$

where $B_{rl \rightarrow k}$ is the branching ratio, i.e. a probability of particle $l$ to decay strongly into a particle $k$. More details on the fitting procedure of experimental data with the HRGM can be found in [5, 6].

### 3. Results

The HRGM with IST outlined above is applied to describe the whole set of the ALICE data measured at $\sqrt{s_{NN}} = 2.76$ TeV. First we consider the traditional approach to CFO in which all particles, i.e. hadrons and (anti)nuclei, have a single CFO hyper-surface (Model I hereafter). The total $\chi^2_{tot}(V)$ of Models I and II is defined as

$$\chi^2_{tot}(V) = \chi^2_h + \chi^2_A = \sum_{k \in h} \left[ \frac{R_{k}^{theo} - R_{k}^{exp}}{\delta R_{k}^{exp}} \right]^2 + \sum_{A} \left[ \frac{\rho_A(T) V - N_A^{exp}}{\delta N_A^{exp}} \right]^2,$$

where $\rho_A(T)$ is the total $A$-particle density at the collision energy.
where $\chi^2_{\text{tot}}$ and $\chi^2_A$ denote the mean deviation squared for hadrons and (anti)nuclei, respectively. Since hadronic part of $\chi^2_{\text{tot}}(V)$ involves only the ratios of hadron multiplicities [5, 6], then it does not contain the CFO volume $V$. On a contrary, the (anti)nuclei part of $\chi^2_{\text{tot}}(V)$ depends on the thermal densities of (anti)nuclei of $A$ (anti)baryons and the CFO volume. The main assumption of the Model I is that the CFO volume is defined by the multiplicity of $\pi^+$-mesons as $V_I = \frac{N^{\pi^+}}{A^{\pi^+}}$. Hence, the Model I has two parameters to reproduce data, namely CFO temperature and $V_I$. We tried another way of fitting by introducing the ratios for (anti)nuclei to the multiplicity of $\pi^+$-mesons, like it is done for hadronic ratios. However, in these case one automatically increases the resulting error for the ratio, although we got practically the same results of fit like for the Model I. The only found difference between this prescription from the Model I is that the $\chi^2_{\text{tot}}(V)$ minimum is slightly broader and slightly lower, but the CFO temperature is the same as in the Model I.

The Model I provides a typical poor quality of the ALICE data description with $\min(\chi^2_{\text{tot}}(V_I)) \approx 30.77$, $\chi^2_{1h} \approx 17.69$ and $\chi^2_{1A}(V_I) \approx 13.08$ which results in $\min(\chi^2_{\text{tot}}(V_I)/\text{dof}) \approx 30.77/17 \approx 1.81$, if one uses the hard-core radius of (anti)nuclei according to Eq. (1). The CFO temperature $T_{CFO} \approx 157.1 \pm 6$ MeV and volume $V_I \approx 7467 \text{ fm}^3$ and the fit quality of the Model I are very similar to the results of Ref. [4]. A poor quality of the Model I description is rooted in the fact that the local minimum of the hadronic $\chi^2_{1h}$ is located at the temperature about 150 MeV, while the local minimum of $\chi^2_{\text{tot}}(V_I)$ for (anti)nuclei is located at the temperature about 161.5 MeV. Therefore the minimum of their sum is located in between them and provides essentially worse quality than either of its parts.

The Model II differs from the Model I by an assumption that CFO of (anti)nuclei occurs at the other hyper-surface. Note that a few years ago two groups independently suggested an idea that some hadronic flavors can experience the CFO separately from the other ones due to a different underlying mechanism of their freeze-out [25, 26, 27, 28, 29]. The Model II generalizes such a hypothesis to the (anti)nuclei and, hence, the CFO volume $V_{II}$ in this case is found from the minimum of $\chi^2_{\text{tot}}(V)$ with respect to $V$:

$$\frac{d\chi^2_{\text{tot}}(V)}{dV} = 0 \Rightarrow V_{II}(T) = \sum_A \frac{\rho_A(T)N^{exp}_A}{[\delta N_A^{exp}]^2} \cdot \left[ \sum_A \frac{\rho_A(T)\rho_A(T)}{[\delta N_A^{exp}]^2} \right]^{-1}.$$  

Substituting $V_{II}(T)$ of Eq. (6) into expression for $\chi^2_{A}(V)$ from Eq. (5), one can find the global minimum of $\chi^2_A(V_{II}(T))$ with respect to CFO temperature $T$. In this case the CFO temperature of (anti)nuclei is $T_{A} = 168.5$ MeV, i.e. it is essentially higher than the one for hadrons $T_{h} = 150$ MeV, whereas the CFO volume $V_{II} = 2725 \text{ fm}^3$ is smaller than the corresponding volume of hadronic CFO $V_h = 8965 \text{ fm}^3$. Another striking result of Model II is that the separate description of hadronic and (anti)nuclei data provides a sizably better quality than the one obtained by the Model I and other versions of HRGM, i.e. $\chi^2_{\text{tot}}(V_{II}(T)) \approx 13.27/16 \approx 0.83$ with a single additional fitting parameter compared to the Model I.

From the values of CFO volumes obtained by Model II one may deduce that the (anti)nuclei are produced from the quark gluon plasma bags earlier than hadrons. Such a conclusion may partly explain the fact that the (anti)nuclei can survive during the expansion of these bags and their subsequent hadronization. If the initial collective velocity of (anti)nuclei is sufficiently high, than the hadrons will be able to interact and destroy them at very late times of heavy ion collision reaction.

However, our educated guess is that the high CFO temperature of (anti)nuclei is caused by the fact that the quark gluon bags formed in Pb+Pb collisions have a mass spectrum of the Hagedorn model [30]. In this case the temperature of the hadrons produced from such bags depends not only on the masses of emitted particles, their number and the mass of quark gluon bag [31, 32], but also on their mass spectrum [32]. If the mass spectrum of quark gluon bags is not purely
Figure 1. Left panel: The hadronic ratios from Ref. [5] were fitted by the HRGM with IST for the set of hard-core radii shown in the left panel. The obtained CFO temperature for hadrons is $T_{CFO} \approx 150 \pm 7$ MeV. The quality of the fit is $\chi^2_{2h}/dof \approx 11.07/10 \approx 1.1$. The upper panel shows the fit of the ratios, while the lower panel shows the deviation between data and theory in the units of error. Right panel: The (anti)nuclei yields found by the ALICE experiment (symbols) are compared with the ones obtained by the HRGM with IST for the scenario of separate CFO of nuclei (bars). The fit quality (anti)nuclei yields is $\chi^2_{2A}/dof \approx 2.2/6 \approx 0.367$.

4. Conclusions

In this work we developed the hadron resonance model with the induced surface tension which allows us to treat the hard-core repulsion of hadrons and light (anti)nuclei on the same footing. In contrast to all previous studies here we employ the hard-core radius of $A$ nucleons in the form $R(A) = R_b(A)^{1/3}$, where $R_b = 0.365$ fm denotes the hard-core radius of (anti)baryons. Then we consider two models of chemical freeze-out of (anti)nuclei. The first of them, Model I, corresponds to the usual assumption that the hadrons and (anti)nuclei multiplicities are frozen at the same hyper-surface. In this case we obtain the typical value of the fit quality $\min\{\chi^2_{2h}(V_I)/dof\} \simeq 30.77/17 \simeq 1.81$ which is slightly better than the results of Ref. [4]. The Model II assumes that the chemical freeze-out of nuclei occurs separately from the one of hadrons. In this case we have one extra fitting parameter compared to the Model I, namely the temperature of (anti)nuclei $T_A = 168.5$ MeV, which is essentially higher than the one for hadrons $T_h = 150$ MeV, but is similar to the one found for the highest RHIC energies [5, 6]. Such a model provides $\min\{\chi^2_{2h}(V_{II})/dof\} 13.27/16 \simeq 0.83$ which is more than two times smaller than the corresponding value of the Model I. We believe that a possible explanation for such a high value of the chemical freeze-out temperature of (anti)nuclei may be related to the fact that the quark gluon bags producing the (anti)nuclei have not purely exponential mass spectrum $\exp(m/T_H)$, but the one which corresponds to the Hagedorn model, i.e. $m^{-3} \exp(m/T_H)$. In this case the temperature of light and heavy particles produced by such bags may differ by 10-15%, which is close to our results for the Model II.
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