Renormalization group running of couplings and masses in the basic extension of the standard model

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Abstract. We discuss the range of validity of the basic two Higgs doublet model by numerically solving the renormalization group equations for the quartic couplings and masses. We consider parameter-values that satisfy the vacuum stability and triviality principles, some of them used in the literature.

1. Introduction
The theory that is associated with the sectors of the gauge bosons and fermions of the standard model (SM), is in excellent agreement with a large amount of experimental data on low energy phenomena [1, 2, 3]. Currently, the most interesting part of the SM is the Higgs boson, which generates the masses of fermions and bosons W\(\pm\) and Z\(^0\) [4, 5]. A minimal extension of the Higgs sector of the SM consists in the addition of one scalar doublet SU(2) with exactly the same quantum numbers as the original. Here we focus our attention on the simplest possible extension of the SM, the two Higgs doublet model (2HDM).

2. The 2HDM model
The Higgs sector of the 2HDM consists of two identical (hypercharge-one) scalar doublets \(\Phi_1\) and \(\Phi_2\). The 2HDM is determined by the choice of the Higgs potential and the Yukawa couplings of the two scalar doublets with the elements of the three generations of quarks and leptons. We denote the two complex Higgs doublet fields by

\[ \Phi_i = \left( \begin{array}{c} \phi^+_i \\ \phi^0_i \end{array} \right), \]

with \(i = 1, 2\). The most general Higgs potential, which can be constructed with two scalar doublets, which is gauge invariant and renormalizable is given by

\[ V = -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 - \left\{ \mu_{12}^2 \left( \Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right\} + \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) \\
+ \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{1}{2} \left[ \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \left( \Phi_1^\dagger \Phi_1 \right)^2 \right] + \left\{ \lambda_7 \left( \Phi_1^\dagger \Phi_1 \right) + \lambda_8 \left( \Phi_2^\dagger \Phi_2 \right) \right\}, \]

(2)
The number of parameters in eq. (2) is reduced to five $\lambda_i$ s by imposing the $Z_2$ discrete symmetry [6, 7, 8, 9, 10], preserving CP symmetry.

2.1. Constraints for the $\lambda_i$ parameters

The vacuum stability constraints arise from the requirement of a positive potential for large values of the fields in an arbitrary direction in the plane ($\Phi_1$, $\Phi_2$) imposed on the quartic potential. Furthermore, the masses are defined by the quartic couplings and the vacuum expectation values.

\[ M_{\Phi^0}^2 = \lambda_1 v^2_1 + \lambda_2 v^2_2 \pm [(\lambda_1 v^2_1 - \lambda_2 v^2_2)^2 + 4v^4_1 v^2_2 \Lambda^2)^{\frac{1}{2}}, \]

\[ M_{\Phi^0}^2 = -v^2\lambda_5, \quad M_{\Phi^0}^2 = -(1/2)v^2(\lambda_4 + \lambda_5), \]

where $\Lambda = (1/2)(\lambda_3 + \lambda_4 + \lambda_5)$, $v_1$ and $v_2$ are the vacuum expectation values of the scalar fields, which are related in the following way $v = v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$.

The restrictions on the quartic couplings may also be obtained directly from the mass spectrum of 2HDM. A rigorous way to obtain additional restrictions for the quartic couplings is through the use of the Lagrange multipliers method and by the implementation of the vacuum stability condition. The problem we consider involves constraints that are formulated by equality and inequality relations [11, 12, 13]. The Lagrange function we propose has the following form

\[ L(x, \Lambda, \Gamma) = F(x) + \sum_{i=1}^{n} \Lambda_i g_i(x) + \sum_{j=1}^{m} \Gamma_j h_j(x), \]

where $F(x)$ is the quartic potential, $\Lambda_i$ and $\Gamma_j$ are the Lagrange multipliers and the explicit form of the constraints $g_i(x)$ and $h_i(x)$ is

\[ g(x) = x_1 + x_2 - v^2 = 0, \quad h(x) = x_1 x_2 - x_3^2 - x_4^2 \geq 0, \]

where

\[ x_1 = \Phi_1^{\dagger} \Phi_1, \quad x_2 = \Phi_2^{\dagger} \Phi_2, \quad x_3 = \text{Re} (\Phi_1^{\dagger} \Phi_2), \quad x_4 = \text{Im} (\Phi_1^{\dagger} \Phi_2). \]

Applying the stability condition

\[ L(x_{\text{min}}) > 0, \]

and the Karush-Kuhn-Tucker restraints [12, 14], we obtain the following requirements to be satisfied by the parameters of the potential

\[ \lambda_1 > 0, \quad \lambda_5 > 0, \quad \lambda_5 < 0, \quad \lambda_4 + \lambda_5 < 0, \quad \lambda_3 + \lambda_4 \pm |\lambda_5| < \lambda_1 + \lambda_2, \quad -2\sqrt{\lambda_1 \lambda_2} < \lambda_3 + \lambda_4 \pm |\lambda_5|. \]

3. Results

The analysis based on the renormalization group equations [RGE] is of great importance in the study of the physics of the standard model and its extensions [15, 16]. The knowledge of the energy evolution of the mass and coupling parameters is essential to analyze electro-weak experiments. For the numerical evaluation we use data that has been published recently in relation with the charged Higgs mass [17, 18, 19]. Several scenarios are considered in the analysis.

In a first stage, considering [17], we have fixed the values of the Higgs masses as $M_{H^\pm} = 609$ GeV, $M_{A^0} = 622$ GeV. In this case, in which the masses are large, to obtain reasonable results
in the evolution of the couplings, the appropriate initial values for the scalars are $\lambda_1 = 12$, $\lambda_2 = 3$, $\lambda_3 = 11.5$, $\lambda_4 = -5.5$, $\lambda_5 = -6$ and $\tan \beta = v_3/v_1 = 1.4$ ($\beta$ is the mixing angle between the charged states). The behavior of these parameters may be observed in Figures 1 and 2. Here we can see that the range of validity of the model is limited, since the energy extent before reaching singularities is very short ($M_t < E < 292$ GeV, in terms of the variable $t$, $0 < t < 0.52$, where $t$ is defined as $t = \ln (\frac{E}{m_t^0})$, $E$ is the renormalization point energy and $m_t$ is mass of the top quark).

In a second stage rested on [18], we fixed the values of the masses as $M_{H^\pm} =$ 129.4 GeV, $M_{A^0} =$ 131.9 GeV. Here we consider the following values for the parameters $\lambda_1 = \lambda_2 = 0.18$, $\lambda_3 = 3$, $\lambda_4 = -0.25$, $\lambda_5 = -0.27$ and $\tan \beta = 41.2$. The behavior of quartic couplings and masses is exhibited in Figures 3 and 4. In this case we can see that the model is valid in the whole energy range, i.e. up to the energy scale of electroweak unification. For a more numerical detail see Table 1.

![Figure 1. Evolution of quartic couplings.](image1)

![Figure 2. Evolution of the masses.](image2)

In Table 1, we consider a column that is refereed to a 7 TeV energy in which the $M_{H^0} =$ 127.7.

| $E[\text{GeV}]$ | $E[\text{GeV}]$ | $E[\text{GeV}] = 1.2 \times 10^{13}$ |
|-----------------|-----------------|---------------------------------|
| $\tan \beta$   | 41.21           | 41.2                            |
| $\nu_1$         | 6.16            | 5.97                            |
| $\nu_2$         | 253.73          | 245.87                          |
| $\nu$           | 253.81          | 245.94                          |
| $g_t$           | 0.965           | 0.817                           |
| $g_b$           | 0.965           | 0.820                           |
| $\lambda_1$     | 0.18            | 0.135                           |
| $\lambda_2$     | 0.18            | 0.135                           |
| $\lambda_3$     | 0.3             | 0.19                            |
| $\lambda_4$     | -0.25           | -0.087                          |
| $\lambda_5$     | -0.27           | -0.32                           |
| $m_{h^0}$       | 2.92            | 1.93                            |
| $m_{H^0}$       | 152.26          | 127.65                          |
| $m_{A^0}$       | 129.42          | 110.20                          |
| $m_{H^\pm}$     | 131.88          | 138.01                          |

In Table 1, we consider a column that is refereed to a 7 TeV energy in which the $M_{H^0} =$ 127.7.
This value is very close to the one reported in [20, 21, 22, 23] as an evidence of the existence of the SM Higgs particle.

Figure 3. Evolution of quartic couplings.

In a third scenario, a unification of the Yukawa couplings associated with the quarks $t$ and $b$ ($g_t=g_b$) is considered at low energies. Figures 5, 6 and 7 show the behavior of the Yukawa couplings, quartic couplings and Higgs masses. The model is valid in the whole energy range. Table 2 shows the numerical data for this figures.

Figure 5. Evolution of the Yukawa couplings, with unification at low energy.

In the last case we consider a unification of the Yukawa couplings at high energies. Figures 8, 9 and 10 show the evolution of the parameters. As can be seen, the model is also valid in the whole energy range. The numerical values of the parameters for this case are presented in Table 3.

4. Conclusions

We have considered the scalar potential in the 2HDM model, which is the simplest extension of the standard model, in which the CP symmetry is preserved. Through the vacuum stability principle and the Lagrange multipliers method we have obtained conditions to be satisfied by the quartic couplings in the potential. We have solved numerically the renormalization group equations for the quartic couplings and masses in the 2HDM model. We have considered various scenarios for which, we have obtained the limits of validity of the model.
Figure 7. Evolution of quartic couplings.

Figure 8. Evolution of the masses.

Figure 9. Evolution of quartic couplings.

Figure 10. Evolution of the masses.

Table 2. Data for Figures 5, 7 and 8.

|       | $E[\text{GeV}] = 173.1$ | $E[\text{GeV}] = 1.2 \times 10^{13}$ |
|-------|--------------------------|-------------------------------------|
| $\tan \beta$ | 41.21                    | 41.02                               |
| $\nu_1$   | 6.156                    | 5.715                               |
| $\nu_2$   | 253.74                   | 234.465                             |
| $\nu$     | 253.81                   | 234.534                             |
| $g_t$     | 0.9647                   | 0.5082                              |
| $g_\theta$| 0.9647                   | 0.5238                              |
| $\lambda_1$| 0.2849                   | 3.0                                 |
| $\lambda_2$| 0.2852                   | 3.0                                 |
| $\lambda_3$| 0.2347                   | 1.0                                 |
| $\lambda_4$| -0.3487                  | -1.5                                |
| $\lambda_5$| -0.393                   | -2.0                                |
| $m_{h^0}$ | 2.1252                   | 13.9518                             |
| $m_{H^0}$ | 191.684                  | 574.32                              |
| $m_{A^0}$ | 159.115                  | 331.682                             |
| $m_{H^\pm}$| 154.569                  | 117.267                             |

Table 3. Data for figures 6, 9 and 10.

|       | $E[\text{GeV}] = 173.1$ | $E[\text{GeV}] = 1.2 \times 10^{13}$ |
|-------|--------------------------|-------------------------------------|
| $\tan \beta$ | 41.21                    | 41.43                               |
| $\nu_1$   | 6.156                    | 5.6869                              |
| $\nu_2$   | 253.735                  | 235.641                             |
| $\nu$     | 253.81                   | 235.709                             |
| $g_t$     | 0.9729                   | 0.516                               |
| $g_\theta$| 0.9567                   | 0.516                               |
| $\lambda_1$| 0.2915                   | 2.5                                 |
| $\lambda_2$| 0.2876                   | 2.5                                 |
| $\lambda_3$| 0.2043                   | 0.2                                 |
| $\lambda_4$| -0.2498                  | 1.0                                 |
| $\lambda_5$| -0.2735                  | -1.2                                |
| $m_{h^0}$ | 3.9232                   | 12.7164                             |
| $m_{H^0}$ | 192.458                  | 526.099                             |
| $m_{A^0}$ | 132.749                  | 258.207                             |
| $m_{H^\pm}$| 129.838                  | 74.5378                             |
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