N → Δ QUADRUPOLE TRANSITION IN THE CONSTITUENT QUARK MODEL

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Information on the intrinsic deformation of the proton can be obtained by studying the electromagnetic $p \rightarrow \Delta^+$ quadrupole transition. Recent experiments have shown that the electric quadrupole ($E_2$) strength in $\gamma p \rightarrow \Delta^+$ is about 10 times larger than predicted by the simple quark model using only one-body currents. Our analysis provides evidence for the dominance of exchange currents in the $N \rightarrow \Delta$ quadrupole transition, and identifies the physical mechanism leading to the observed $E_2$ strength.

1 Introduction

The quadrupole moment of a particle measures the deviation of its internal charge distribution from spherical symmetry. However, as a particle with total angular momentum $J = 1/2$, the nucleon does not have a quadrupole moment in the laboratory frame. In order to learn something about the shape of the nucleon one has to electromagnetically excite it, e.g. to the $\Delta(1232)$ resonance, with total angular momentum $J = 3/2$, or to higher resonances.

There are three different ways to electromagnetically produce a $\Delta(1232)$. Aside from the dominant $N \rightarrow \Delta$ magnetic dipole ($M_1$) excitation mode, in which the spin of a single quark is flipped, transverse electric quadrupole ($E_2$), and longitudinal charge quadrupole ($C_2$) transitions are allowed by angular momentum and parity selection rules. The strengths of these electromagnetic multipoles at photon three-momentum transfer $q = 0$, called $G_{M_1}(0)$, $G_{E_2}(0)$, and $G_{C_2}(0)$, can be extracted from high precision photo-pion- and electro-pion-production experiments off the proton. The quadrupole transition amplitudes $G_{E_2}(0)$ and $G_{C_2}(0)$ are a measure of the intrinsic deformation of the nucleon. The empirical $E_2$ strength is with $G_{E_2}^{\text{exp}} = 0.133(20)$ about 10 times larger than predicted by the first quark model calculations using spatial single quark currents.

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2 Single Quark $N \to \Delta$ Quadrupole Transition

Early quark model calculations of the $N \to \Delta$ quadrupole transition used aside from a spin-independent confinement a one-gluon exchange potential $V^{OGEP}$ between constituent quarks. Without retardation and for equal quark masses it reads

$$V^{OGEP}(r_i, r_j) = \frac{\alpha_s}{4} \lambda_i \cdot \lambda_j \left\{ \frac{1}{r} - \frac{\pi}{m_q^2} \left(1 + \frac{2}{3} \sigma_i \cdot \sigma_j\right) \delta(r) \right. \\
\left. - \frac{1}{4m_q^2} \frac{1}{r^3} \left(3 \sigma_i \cdot \hat{r} \sigma_j \cdot \hat{r} - \sigma_i \cdot \sigma_j\right) \right. \\
\left. - \frac{1}{2m_q^2} \frac{1}{r^3} \left[3 \left(r \times \frac{1}{2}(p_i - p_j)\right) \cdot \frac{1}{2}(\sigma_i + \sigma_j) \right] \\
\left. \right. - \left(r \times \frac{1}{2}(p_i + p_j)\right) \cdot \frac{1}{2}(\sigma_i - \sigma_j) \right\} , \tag{1}$$

where $r = r_i - r_j$; $\sigma_i$ is the Pauli spin matrix, and $\lambda_i$ is the color operator of the $i$-th quark. Usually, a regularized one-pion exchange interaction between constituent quarks is introduced, in order to account for the chiral symmetry of QCD and its spontaneous breaking at low energies

$$V^{OPEP}(r_i, r_j) = \frac{g_{\pi qq}^2}{4\pi(4m_q^2)} \frac{\Lambda^2}{3} \frac{1}{\Lambda^2 - \mu^2} \frac{1}{r^3} \left[3 \sigma_i \cdot \hat{r} \sigma_j \cdot \hat{r} - \sigma_i \cdot \sigma_j\right] \left(\frac{3}{r^3} \frac{\mu e^{-\mu r}}{r} - 4\pi \delta(r)\right) \right. \\
\left. + \left(3 \sigma_i \cdot \hat{r} \sigma_j \cdot \hat{r} - \sigma_i \cdot \sigma_j\right) \left(\frac{3}{(\mu r)^2} + \frac{3}{(\mu r)^2} \right) \mu^2 e^{-\mu r} \right) \\
\left. \right. - (\mu \leftrightarrow \Lambda) \right] . \tag{2}$$

Here, $\mu$ is the pion mass, $\Lambda$ the cut-off mass, and $\tau_i$ denotes the isospin of the $i$-th quark.

The tensor terms in the effective gluon and pion exchange potentials induce $D$-wave admixtures in the three-quark $N$ and $\Delta$ wave functions. For clarity we restrict ourselves to a two-state harmonic oscillator model for the wave functions:

$$|N\rangle = a_S |(S = 1/2, L = 0)J = 1/2\rangle + a_D |(S = 3/2, L = 2)J = 1/2\rangle$$

$$|\Delta\rangle = b_S |(S = 3/2, L = 0)J = 3/2\rangle + b_D |(S = 1/2, L = 2)J = 3/2\rangle , \tag{3}$$

where the $D$-states are of mixed symmetric type. In Eq.\(\text{(3)}\), the inner spin $S$ couples with the orbital angular momentum $L$ to the total angular momentum.
Table 1. Admixture coefficients for the $S$ and mixed symmetric $D$ states in the nucleon and $\Delta$ ground state wave function as calculated by different authors.

| Ref. | $a_S$ | $a_D$ | $b_S$ | $b_D$ |
|------|-------|-------|-------|-------|
| Ref. 2 | 0.93  | -0.04 | 0.97  | 0.07  |
| Ref. 3 | 0.95  | -0.04 | 0.97  | 0.07  |
| Ref. 5 | 0.94  | -0.09 | 0.96  | 0.10  |
| Ref. 8 | 0.906 | -0.045| 0.994 | 0.056 |
| Ref. 9 | 0.934 | -0.047| 0.990 | 0.064 |

$J$ of the quarks. The $D$-wave probabilities $a_D^2$ and $b_D^2$ as calculated by different authors lie between 0.2% and 0.4% (see table 1). Ref. 5 obtains a larger $D$-state probability of $\sim 1\%$. In any case, the calculated $D$ state probabilities are much smaller than the 5% $D$-state admixture in the deuteron wave function. We will come back to this point.

In the single quark transition model, based on the spatial one-body current $J_{[1]}$, a quark moving in an excited $D$ state in the nucleon can absorb electromagnetic quadrupole radiation ($L_\gamma=2$), and thus fall to the $S$ state in the $\Delta$. The amplitude for this process is proportional to $a_D b_S$. In addition, a quark in an $S$ state in the nucleon can absorb quadrupole radiation ($L_\gamma=2$), and jump to an excited $D$-state in the $\Delta$. This happens with an amplitude $a_S b_D$. Both transitions, graphically displayed in Fig. 1, change the orbital angular momentum $L$, but leave the inner spin state $S$ of the quarks unchanged. The resulting single-quark $E2$ transition strength is

$$G_{E2}(J_{[1]}) = \frac{1}{\sqrt{5}} (a_D b_S + a_S b_D).$$

The $G_{E2}$ used here is dimensionless. The connection with the dimensionful transition quadrupole moment, or the helicity amplitudes is given in Ref. 1. With the $D$ state admixtures calculated with the various potentials based on Eq.(1) and Eq.(2) one obtains $G_{E2} = 0.003 - 0.01$, i.e. theoretical results that differ by an order of magnitude from the recent experimental results. This suggests that some important dynamical feature is still missing.
Figure 1. Conventional explanation of the $\gamma N \rightarrow \Delta$ quadrupole transition via the one-body current $J^{[1]}$ in Fig. 2(a) (impulse approximation). In this approximation the $E2$ transition amplitude is a coherent superposition of the two $L$ changing but $S$ conserving one-body transitions (upper and lower part of the figure). In a single-quark transition, the absorption of a $E2$ photon is therefore only possible if either the nucleon (left) or the $\Delta$ (right) contains a $D$ wave admixture (deformed valence quark orbit). The transition probability amplitude $G_{E2}(J^{[1]})$ of Eq. (4) is strongly suppressed due to the small $D$ wave admixtures in the $N$ und $\Delta$ wave function.

3 Exchange Currents and Siegert’s Theorem

From the theoretical point of view the conventional explanation outlined above is incomplete because it violates the continuity equation:

$$q \cdot J(q) = [H, \rho(q)].$$  \hspace{1cm} (5)

After a decomposition of the charge and current operators into one- and two-body terms, i.e., $\rho = \rho^{[1]} + \rho^{[2]}$, and $J = J^{[1]} + J^{[2]}$, one can show that the spatial two-body current $J^{[2]}$ satisfies the following consistency relation

$$q \cdot J^{[2]}(q) = [V^{[2]}, \rho^{[1]}(q)].$$ \hspace{1cm} (6)

Equation (6) connects the quark-quark interactions $V^{[2]}$, which determine the coefficients $a_S, a_D, b_S, b_D$ in Eq. (5) with the two-body currents $J^{[2]}$ of Fig. 2.
Figure 2. Feynman diagrams of the four vector current $J_\mu = (\rho, J)$: (a) one-body current $J_{[1]}^{\mu}$ and (b-d) two-body gluon and pion exchange currents $J_{[2]}^{\mu}$. If the quarks interact via pion and gluon exchange, the two-body exchange currents depicted in diagrams (b-d) must be taken into account. If only diagram (a) is considered, the continuity equation (5) is violated.

which determine the electromagnetic properties of the $N-\Delta$ system. Eq.(6) is violated if the potential contains momentum and/or isospin dependent terms but the electromagnetic current contains only one-body terms. In fact, the potentials of Eq.(1) and Eq.(2) do not commute with the one-body charge operator, thus implying a two-body current $J_{[2]}$.

An important theorem based on Eq.(5) is Siegert’s theorem. In the limit of small momentum transfers it relates the transverse electric $T_{E J}$ and longitudinal Coulomb multipoles $T_{C J}$:

$$\langle f | T_{E J} (|q| \to 0) | i \rangle = -\frac{\omega}{|q|} \sqrt{\frac{J+1}{J}} \langle f | T_{C J} (|q| \to 0) | i \rangle .$$

Thus, $G_{E2}$ can be calculated via the charge operator (right hand side). The result will be the same as the one based on the spatial current operator (left-hand side).

However, until recently, calculations of the $E2$ transition strength $G_{E2}$ using the spatial one-body current operator have differed considerably (in some calculations by an order of magnitude) from those using the one-body charge operator and Siegert’s theorem. This important observation was made in Ref.4 and has been confirmed by other authors5. Several improvements have been proposed, e.g. an increase of the number of harmonic oscillator states, and the inclusion of relativistic corrections to the one-body current, in order to remove this difference. However, the main reason for this discrepancy has not been recognized in previous works. We have recently shown that this difference is almost entirely explained by spatial two-body exchange currents $J_{[2]}$ required by Eq.(6) (see Table 2). Exchange currents were not explicitly
included in previous analyses of this problem.

Table 2. The transverse electric quadrupole form factor $G_{E2}(q^2 = 0)$ for the $\gamma + p \rightarrow \Delta^+$ transition calculated with (i) the one-body charge density $\rho_{[1]}$ using Siegert’s theorem (first row), (ii) with the spatial current density $J = J_{[1]} + J_{[2]}$ (last row) for various quark models. A comparison of the results in the first and last rows shows that the continuity equation is approximately satisfied provided that the spatial two-body exchange currents $J_{[2]}$ required by Eq. (6) are included. The remaining discrepancy between theory and recent experiment is removed by including $\rho_{[2]}$ (see Eq. (13)).

|                  | Ref. | Ref. | Ref. | Ref. | Ref. | Ref. | π |
|------------------|------|------|------|------|------|------|---|
| $G_{E2}(\rho_{[1]})$ | 0.0192 | 0.0203 | 0.0796 | 0.0177 | 0.0165 |
| $G_{E2}(J_{[1]})$    | 0.0118 | 0.0092 | 0.0076 | 0.0027 | 0.0058 |
| $G_{E2}(J_{[2]})$    | 0.0084 | 0.0114 | 0.0561 | 0.0127 | 0.0105 |
| $G_{E2}(J)$         | 0.0202 | 0.0206 | 0.0637 | 0.0154 | 0.0163 |

4 Double Spin Flip $N \rightarrow \Delta$ Quadrupole Transition

As we have seen, a calculation using the one-body charge operator $\rho_{[1]}$ and Siegert’s theorem yields substantially larger values for $G_{E2}$ as pointed out in Ref. 5. We understand now why this is so. Most of the $\rho_{[1]}$ contribution to $G_{E2}$ comes from the two-body currents $J_{[2]}$ (see Table 2 and Ref. 7 for further explanation). This finding suggests a different interpretation of the deformation of the nucleon in the quark model. However, there is still a large discrepancy with experiment when only $\rho_{[1]}$ is taken into account. We have shown that a double spin flip transition based on the two-body charge operator $\rho_{[2]}$ gives values for $G_{E2}$ in agreement with experiment 7. This is explained in the following.

The Coulomb quadrupole operator entering Eq. (5) is:

$$T^{C2}(|q|) = \frac{1}{4\pi} \int d\Omega_q \rho(q) Y^2_2(\hat{q}),$$  \hspace{1cm} (8)

where $\rho = \rho_{[1]} + \rho_{[2]}$ is the total charge operator. The quadrupole operator projects onto the $Y^2_2(\hat{q})$, i.e., the quadrupole component in $\rho(q)$. We discuss the quadrupole components in $\rho_{[1]}$ and $\rho_{[2]}$ separately. After a multipole expansion of the spin-independent one-body quark charge $\rho_{[1]}$ we see that the Coulomb quadrupole operator is proportional to a second rank spherical harmonic

$$T^{C2}(\rho_{[1]}) \propto Y^2(\hat{r}_i),$$  \hspace{1cm} (9)
where $r_i$ is a single-quark position coordinate. This is the only allowed tensor of rank 2 that can be constructed from a one-body operator. The operator in Eq. (10) has nonvanishing matrix elements between the wave functions of Eq. (3) only for the off-diagonal $S \rightarrow D$, $D \rightarrow S$, and the diagonal $D \rightarrow D$ transitions.

On the other hand, the two-body gluon and pion exchange charge densities $\rho^{q\bar{q}}$ contain, just like the corresponding potentials, a tensor in spin space, and the quadrupole operator is

$$T^{C2}(\rho^{q\bar{q}}) \propto [\sigma_i \times \sigma_j]^2. \tag{10}$$

Consequently, the operator in Eq. (10) has a nonvanishing matrix element also for an $S \rightarrow S$ transition. We stress that unlike the single-quark operator in Eq. (9), the two-body quadrupole operator in Eq. (10) does not change the angular momentum of the wave function. However, as a tensor of rank 2 in spin space it simultaneously flips the spin of two quarks. The probability amplitude for this double spin flip transition is proportional to $a_S b_S \sim 1$, i.e., two orders of magnitude larger than the orbital angular momentum changing one-body transition of Eq. (4). Morpurgo has anticipated the important role of the operator in Eq. (10) for the $\gamma p \rightarrow \Delta^+$ quadrupole transition.

The inclusion of two-body exchange currents leads to a heretofore un-
known relation between the mean square charge radii of the $N$ and $\Delta$:

$$r_p^2 - r_{\Delta+}^2 = r_n^2.$$  \hfill (11)

Dillon and Morpurgo recently derived Eq. (11) using a rather general QCD parametrization method and the assumption that three-index and loop terms are small. They also estimate that if three-index and loop terms are included, deviations from Eq. (11) amount to $10 - 20\%$.

With the help of Eq. (11) one obtains a parameter-independent relation between the neutron charge radius $r_n^2$ and the $N \rightarrow \Delta$ transition quadrupole moment $Q_{p \rightarrow \Delta^+} = 1/\sqrt{2} (r_p^2 - r_{\Delta+}^2) = 1/\sqrt{2} r_n^2$. \hfill (12)

Our parameter-independent prediction for transition quadrupole moment $Q_{p \rightarrow \Delta^+}^{\text{theory}} = -0.083$ fm$^2$ is in excellent agreement with the value $Q_{p \rightarrow \Delta^+}^{\text{MAMI}} = -0.086(13)$ fm$^2$ extracted from the MAMI data, and $Q_{p \rightarrow \Delta^+}^{\text{LEGS}} = -0.105(16)$ fm$^2$ extracted from the LEGS data. Furthermore, using the empirical charge radius of the neutron and Siegert’s theorem, we obtain using both $D$ waves and exchange currents:

$$G_{E2}(0) = G_{E2}(\rho_{[1]}) + G_{E2}(\rho_{[2]}) = 0.017 + 0.107 = 0.124.$$ \hfill (13)

This has to be compared to the experimental results which lie between $G_{E2}^{\text{exp}} = 0.133(20)$ and $G_{E2}^{\text{exp}} = 0.107(17)$. \hfill (14)

We also mention that the Z-diagrams in Fig. not only explain $G_{E2}$ but also the experimental charge radius of the neutron $r_n$. \hfill (15)

This indicates that both the neutron charge radius and the $N \rightarrow \Delta$ quadrupole transition moment are dominated by nonvalence quark degrees of freedom.

### 5 Double spin flip in the deuteron

The pion exchange induced double spin-flip term also contributes to the quadrupole moment of the deuteron. The quadrupole moment of the deuteron including the pion-pair current correction is given as

$$Q_d = \frac{1}{\sqrt{50}} \int_0^\infty dr r^2 u_2(r) \left( u_0(r) - \frac{1}{\sqrt{8}} u_2(r) \right) + \frac{f_{\pi NN}^2}{4\pi} \frac{1}{M_N} \int_0^\infty dr r Y_1(\mu r) \left( 2u_0^2(r) - \sqrt{2} u_0(r) u_2(r) - \frac{1}{5} u_2^2(r) \right).$$ \hfill (14)
The first term in (14) is due to the nonrelativistic one-body charge density $\rho_{[1]}$, while the second term is due to the isoscalar two-body pion pair charge density $\rho_{[2]}$, and $Y_1(x) = (e^{-x}/x)(1 + 1/x)$. Evidently, the one-body contribution is only nonzero if there is a nonzero $D$-state wave function $u_2(r)$. On the other hand, even for a pure (hypothetical) $S$-wave deuteron one obtains a nonvanishing quadrupole moment due to the pion-pair exchange current term. The $u_0^2$ in the integrand is the contribution of the double spin flip term.

Numerically, the exchange current contribution to $Q_d$ is with $0.01 \text{ fm}^2$ rather small. The dominant contribution of $0.28 \text{ fm}^2$ comes from the one-body charge density involving the $D$-state in the deuteron.

Can one understand this role reversal between $D$ states and exchange currents when going from a single baryon to the deuteron? For a particle of mass $m$ moving in a harmonic oscillator potential there is an inverse proportionality between the excitation energy $\omega$, the size of the system $b$, and the mass $m$, namely $\omega \propto 1/(mb^2)$. Because the average distance between two quarks in the nucleon is approximately 1.0 fm, and the average distance between the nucleons in the deuteron is 4 fm, one needs $2\hbar \omega \sim 600 \text{ MeV}$ to lift a quark in an excited $D$ state, whereas one needs only $2\hbar \omega \sim 4 \text{ MeV}$ to lift a nucleon into a $D$ state. Given that the pion tensor potential is similar in strength in both systems, one can qualitatively understand why the $D$-state probability in the nucleon is so small compared to the one in the deuteron. On the other hand, the amount of particle-antiparticle pairs in a system is enhanced if the system size is decreased. Thus, the interchanged role of exchange currents and $D$-waves in the deuteron as compared to a single baryon is mainly a consequence of the different size of these systems.

6 Quadrupole Moment of the $\Delta$

Tensor forces and exchange currents also lead to a nonvanishing quadrupole moment of the $\Delta$. With configuration mixing but no exchange currents (impulse approximation) one obtains neglecting the small $b_D^2$ contributions and with typical values for the admixture coefficients:

$$Q_{\Delta}^{imp} = b^2 \frac{4}{\sqrt{30}} \left( b_{S_S} b_{D_S} + \frac{2}{\sqrt{3}} b_{S_D} b_{D_S} \right) e_\Delta = -0.087 b^2 e_\Delta.$$  \hspace{1cm} (15)

For the quadrupole moment of the $\Delta$ the symmetric $D$ state amplitude $b_{D_S}$ is relevant. For $b = 0.61 \text{ fm}$ one obtains $Q_{\Delta}^{imp} = -0.032 \text{ fm}^2 e_\Delta$. Using $V^{OGE}P$ of Eq.(1), Richard and Taxil, and Krivoruchenko and Giannini found $Q_{\Delta}^{imp} = (2/5) r^2 \pi e_\Delta$, which for $b = 0.71 \text{ fm}$ coincides with Eq.(15).
On the other hand, with exchange currents but no configuration mixing we found:

\[ Q^{\text{exc}}_\Delta = r^2_n e_\Delta. \] (16)

Inserting the measured neutron charge radius into Eq. (16) gives \( Q^{\text{exc}}_\Delta = -0.119 \text{fm}^2 e_\Delta \). Due to the smallness of the admixture coefficients in the \( \Delta \) wave function, this result remains qualitatively unchanged if the two-body operator is evaluated with mixed wave functions. Thus, nonvalence quark degrees of freedom effectively described here by quark-antiquark pair currents provide the dominant contribution to the quadrupole moment of the \( \Delta \).

### 7 Deformation of the Nucleon

This section is somewhat speculative. Because the nucleon has \( J = 1/2 \), its quadrupole moment in the laboratory frame is zero. This is analogous to a strongly deformed \( J = 0 \) nucleus. All orientations of a deformed \( J = 0 \) nucleus are equally probable. This results in a spherical charge distribution in the ground state and a vanishing quadrupole moment \( Q \) in the laboratory. Nevertheless, one can obtain information on the intrinsic quadrupole moment \( Q_0 \) by measuring electromagnetic quadrupole transitions between the ground and excited states, or by measuring the quadrupole moment of an excited state with \( J > 1/2 \) of that nucleus.

In the collective model the relation between the observable quadrupole moment \( Q \) and the intrinsic quadrupole moment \( Q_0 \) is

\[ Q = \frac{3K^2 - J(J + 1)}{(J + 1)(2J + 3)} Q_0, \] (17)

where \( J \) is the total spin of the nucleus, and \( K \) is the projection of \( J \) onto the \( z \)-axis in the body fixed frame (symmetry axis of the nucleus). The intrinsic quadrupole moment \( Q_0 \) characterizes the deformation of the charge distribution in the ground state. A simple model for a nonspherical homogeneous charge distribution is that of a rotational ellipsoid with charge \( Z \), major axis \( a \) along, and minor axis \( b \) perpendicular to the symmetry axis:

\[ Q_0 = \frac{2Z}{5}(a^2 - b^2) = \frac{4}{5} Z R^2 \delta, \] (18)

with \( \delta = 2(a - b)/(a + b) \) and \( R = (a + b)/2 \).

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\(^a\)Here, intrinsic means with respect to a body-fixed coordinate system that rotates with the nucleus.
Can one use this model to determine the sign and magnitude of the deformation of the nucleon? Because the quadrupole moment of the $\Delta$ and the $N \to \Delta$ transition quadrupole moment are dominated by gluon and pion degrees of freedom, and not by single quark degrees of freedom, the collective model maybe valid. Assuming that the collective model is applicable, we consider the $\Delta$ with spin $J = 3/2$ as a collective rotation of the entire nucleon with an intrinsic angular momentum $K = 1/2$. One then finds from Eq. (14) and Eq. (17) $Q_0 = 0.565$ fm$^2$ for the intrinsic quadrupole moment of the nucleon. Furthermore, from the naive nucleon model of Eq. (18) one obtains with $R = \sqrt{5/3} r_p = 1.113$ fm a deformation parameter $\delta \approx 0.57$ and a ratio of major to minor semi-axes $a/b \approx 1.79$. This magnitude of deformation seems to be quite large. Yet, we speculate that the sign of the intrinsic quadrupole moment given by Eq. (17) is correct. If so, the nucleon is a prolate spheroid. The $\Delta$ is clearly an oblate spheroid with an observable quadrupole moment given by Eq. (15).

Although the nucleon wave function is not observable, one prefers to describe the deformation of the nucleon in terms of a $D$ state admixture in the quark wave function. If both $S$ and $D$ wave amplitudes have the same (opposite) sign, one obtains a positive (negative) intrinsic quadrupole moment, and a prolate (oblate) deformation, respectively. For example, the sign of the relevant symmetric $D_S$ wave amplitude $b_{D_S}$ in the $\Delta$ wave function is opposite to the dominant $S$-wave amplitude $b_{S_S}$ (see Refs. 2). This leads to a negative quadrupole moment of the $\Delta^+$ and an oblate deformation of the $\Delta$, as can be seen from Eq. (15). Because of the negative $D$ wave amplitude in the nucleon wave function, one could conclude that the nucleon has an oblate deformation. However, one would then miss the contribution of the nonvalence quark degrees of freedom. One can (via a unitary transformation) eliminate the two-body transition operator, as discussed by Friar 3 for the deuteron, and reexpress it in terms of a modified potential, or modified wave functions. The matrix element is thereby left invariant. If the nucleon has an intrinsic quadrupole moment $Q_0 = 0.565$ fm$^2$, the nucleon wave function should have a large and positive $D$ wave amplitude, in contrast to the small and negative $D$-state admixture obtained with the original potentials.

8 Summary

With the help of Siegert’s theorem we have shown that the largest part of the $E2$ transition strength $G_{E2}$ in $\gamma N \to \Delta$ comes from a two-body spin flip due to exchange currents. The effect of one-body currents on $G_{E2}$ is relatively small. Exchange currents also dominate the quadrupole moment.
of the $\Delta(1232)$ and are very important for the radiative $E2$ decays of all decuplet baryons. The quadrupole moment of the $\Delta$ together with the $N \rightarrow \Delta$ transition quadruple moment can be seen as an indication for an intrinsic nucleon deformation. We conclude that the intrinsic deformation of baryons lies mainly in the nonvalence quark degrees of freedom ($q \bar{q}$ pairs, pions, gluons). Our prediction of the $E2$ transition amplitude, which is based on the parameter-independent relation of Eq.(12) and Siegert’s theorem, is in very good agreement with experiment.

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