Approximate Flavor Symmetries in the Lepton Sector

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Abstract

Approximate flavor symmetries in the quark sector have been used as a handle on physics beyond the Standard Model. Due to the great interest in neutrino masses and mixings and the wealth of existing and proposed neutrino experiments it is important to extend this analysis to the leptonic sector. We show that in the see-saw mechanism, the neutrino masses and mixing angles do not depend on the details of the right-handed neutrino flavor symmetry breaking, and are related by a simple formula. We propose several ansätze which relate different flavor symmetry breaking parameters and find that the MSW solution to the solar neutrino problem is always easily fit. Further, the $\nu_\mu - \nu_\tau$ oscillation is unlikely to solve the atmospheric neutrino problem and, if we fix the neutrino mass scale by the MSW solution, the neutrino masses are found to be too small to close the Universe.

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1 Introduction

The smallness of the Yukawa couplings has been related to the approximate flavor symmetries ([1], [2]). The Yukawa couplings can be understood as being naturally small [3], since by putting them equal to zero new global symmetries appear in the theory. These are the flavor symmetries under which each of the fermions transforms separately but scalars do not. In the low energy theory these symmetries are broken by different amounts as is evident from the nonzero masses of fermions. The lack of knowledge of the exact mechanism by which the symmetries were broken was parametrized by a set of small numbers (denoted by $\epsilon$). Each Yukawa coupling is then given approximately as the product of small symmetry breaking parameters $\epsilon$ for all flavor symmetries broken by that coupling. So for example the coupling of a scalar doublet $H$ to the $i$th generation of left-handed doublet quarks $Q_i$ and the $j$th generation of right-handed up quark $U_j$ is given by

$$\lambda_{i j}^U \approx \epsilon_{Q_i} \epsilon_{U_j}.$$ (1)

Whereas this gives the right order of magnitude for the couplings, the exact relation of couplings to the $\epsilon$s may be off by a factor of 2 or 3. This is because the underlying theory (possibly a GUT) by which the flavor symmetries are broken is unknown. Therefore all estimates of the possible new flavor changing interactions should be taken to have at least the same uncertainty.

The flavor symmetry breaking parameters $\epsilon$ can be estimated in several ways. They can be postulated by ansätze [2] which are consistent with the known values of fermion masses and mixings. Alternatively, one may use the experimental results to constrain the $\epsilon$s. This was done in the quark sector [4] using the known values of quark masses and Kobayashi-Maskawa mixing matrix elements.
The important result so obtained was that flavor changing scalar interactions are possible even for the masses of the new scalars as low as a few hundred GeV to 1 TeV, and numerous estimates for different flavor changing interactions were given \[2, 4\]. Hall and Weinberg \[4\] also noticed that allowing for complex Yukawa couplings in this scheme, the observed smallness of CP violation constrains the phases to be small (i.e of the order of \(10^{-3}\)).

In the lepton sector, the masses are small indicating that the approximate flavor symmetries are preserved to a high degree of accuracy. Therefore lepton flavor changing interactions would be extremely hard to test \[2\]. Further, since the neutrino masses and mixing angles have not been measured yet, one cannot estimate the corresponding \(\epsilon_s\) without additional assumptions. Nevertheless we find it important to address the question of approximate flavor symmetries in the lepton sector, because many experiments which aim to measure or set better limits on neutrino masses and mixings are under way or being planned for the near future. Indeed, in this paper we find that statements can be made about the lepton sector, regardless of additional assumptions about the \(\epsilon_s\). For example, while the MSW solution to the solar neutrino problem can be easily fit, the predicted neutrino masses are unlikely to close the Universe.

This paper is organised as follows. In section 2 we express the lepton sector Yukawa couplings in terms of flavor symmetry breaking parameters. Here we study two cases of neutrino masses. In the see-saw mechanism \[5\] case we show that the neutrino masses and mixings are independent of the right-handed neutrino flavor symmetry breaking mechanism. We also include for completeness the case of Dirac neutrino masses only (as in the case of charged fermions). In section 3 we provide several ansätze for the lepton flavor symmetry breaking parameters and list their predictions in terms of ratios of neutrino masses, and mixings. We study their relevance to the solar neutrino problem, atmospheric neutrino problem, closure of the Universe, etc. General features are noted
which are independent of the particular ansatz used.

2 Approximate Flavor Symmetries in the Lepton Sector

By adding the right-handed neutrinos \( N_i, i = 1, 2, 3 \) to the particle content of the standard model we can allow for Dirac type masses. Under the action of approximate flavor symmetries, whenever an \( N_i \) enters a Yukawa interaction, the corresponding coupling must contain the symmetry breaking parameter \( \epsilon_{N_i} \).

A natural way to justify the smallness of neutrino masses is to use the see-saw mechanism in which the smallness of the left-handed neutrino masses is explained by the new scale of heavy right-handed neutrinos. The mass matrices will have the following structure

\[
\begin{align*}
m_{N_Dij} &\approx \epsilon_{Li}\epsilon_{Nj}v_{SM}, \\
m_{N_Mij} &\approx \epsilon_{N_i}\epsilon_{Nj}v_{Big}, \\
m_{E_{ij}} &\approx \epsilon_{Li}\epsilon_{Ej}v_{SM},
\end{align*}
\]

where \( m_{N_D} \) and \( m_E \) are the neutrino and charged lepton Dirac mass matrices, \( m_{N_M} \) is the right-handed neutrino Majorana mass matrix, \( v_{SM} = 174 \text{ GeV} \) and \( v_{Big} \) is the new large mass scale. The generation indices \( i \) and \( j \) run from 1 to 3. In the following we assume a hierarchy in the \( \epsilon \)s (i.e. \( \epsilon_{L1} << \epsilon_{L2} << \epsilon_{L3} \), etc.) as suggested by the hierarchy of quark and charged lepton masses. Then the diagonalization of the neutrino mass matrix will give a heavy sector with masses \( m_{N_Hi} \approx \epsilon_{N_i}^2v_{Big} \) and a very light sector with mass matrix

\[
m_{N_Lij} \approx (m_{N_D}m_{N_M}^{-1}m_{N_D}^T)_{ij} \approx \epsilon_{Li}\epsilon_{Li}\frac{v_{SM}^2}{v_{Big}},
\]

where the number \( \text{Tr}(\epsilon_N(\epsilon_N\epsilon_N^{-1})^{-1}\epsilon_N) \) is assumed to be of order one. We have the expected result: the heavy right-handed neutrino decouples from the theory leaving behind a very
light left-handed neutrino. The masses and mixing angles are independent of the right-handed symmetry breaking parameters $\epsilon_{N_i}$:

\begin{align}
    m_i^N &\approx \epsilon_{N_i}^2 \frac{v_{SM}^2}{v_{\text{Big}}}, \\
    m_i^E &\approx \epsilon_{N_i} \epsilon_{E_i} v_{SM} \quad (\text{no sum on } i), \\
    V_{ij} &\approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i < j).
\end{align}

Therefore, besides the unknown scale $v_{\text{Big}}$, only two sets of $\epsilon$s are needed: $\epsilon_{L_i}$ and $\epsilon_{E_i}$. In fact, the neutrino masses and mixings depend only on $\epsilon_{L_i}$ and they are approximately related through

\begin{equation}
    V_{ij} \approx \sqrt{\frac{m_i^N}{m_j^N}}. \quad (7)
\end{equation}

Equation (7) reduces the number of parameters needed to describe neutrino masses and mixings by three; for example, given two mixing angles and one neutrino mass, we can predict the third mixing angle and the other two neutrino masses. These results are extremely general. They follow simply from the factorization of the Dirac masses, regardless of the specific form of $m^{-1}_{N_M}$, which only contributes to set the scale.

For completeness, we note that in the case of Dirac masses only, the neutrinos and the charged leptons mass matrices become

\begin{align}
    m_{N_{Dij}} &\approx \epsilon_{N_i} \epsilon_{N_j} v_{SM}, \\
    m_{E_{ij}} &\approx \epsilon_{L_i} \epsilon_{E_j} v_{SM}. \quad (9)
\end{align}

Their diagonalization yields

\begin{align}
    m_i^N &\approx \epsilon_{L_i} \epsilon_{N_i} v_{SM} \quad (\text{no sum on } i), \\
    m_i^E &\approx \epsilon_{L_i} \epsilon_{E_i} v_{SM} \quad (\text{no sum on } i). \quad (10)
\end{align}
whereas the lepton mixing matrix is approximately diagonal with off diagonal elements of the order of

\[ V_{ij} \approx \frac{\epsilon_{Li}}{\epsilon_{Lj}} \ (i < j) . \] (11)

Therefore in this case we need three sets of \( \epsilon \)s to explain masses and mixings (cf. Eqns. (10)-(11)): \( \epsilon_{Li} \), \( \epsilon_{Ni} \) and \( \epsilon_{Ei} \).

In addition to having one more set of (unknown) \( \epsilon \)s than the see-saw case, the Dirac only case has no natural explanation for the very low scale associated with the neutrino masses, i.e. that the flavor symmetries of right-handed neutrinos are much better preserved than for other fermions. It turns out that one typically gets the \( \epsilon_{Ni} \) of order \( 10^{-12} \) or so, compared to quark and charged lepton \( \epsilon \)s which are typically \( 10^{-3} \) and larger. Therefore, in the following we will mostly concentrate on the see-saw case.

3 Ansätze and Predictions

While it was possible to determine the flavor symmetry breaking parameters in the quark sector from quark masses and mixings [4], in the lepton sector the situation is much more difficult. At this time direct laboratory experiments provide only upper limits on neutrino masses and mixings, although some indirect sources like solar neutrinos or cosmology point to some specific allowed ranges. Therefore to estimate the sizes of lepton flavor symmetry breaking parameters \( \epsilon \)s additional assumptions are needed.

Our strategy is as follows: we list several plausible or GUT motivated ansätze which relate \( \epsilon \)s of different fields. This will enable us to estimate ratios of neutrino masses and mixings. If the mixings are consistent with the allowed range for the MSW solution [8] of the solar neutrino problem, we take this as a hint to fix the mass scale and predict all neutrino masses. We then look at further predictions. As is the case with any calculation based on these approximate flavor symmetries ([1], [2], [3]), the factors of two or three
may contribute coherently factors of an order of magnitude or so. In addition, the numerical results depend on the specific ansatz. What we seek are the general features of those results rather than the detailed numerical results themselves.

I. As our first ansatz we assume that $\epsilon_{L_i} = \epsilon_{E_i}$ [2]. This ansatz can be justified as follows [7]. Assume that in the lepton sector the only combination of symmetry which is broken is the axial flavor symmetry. This means that, instead of breaking separately left(L) and right(R) flavor symmetries, only the combination L+R is broken. Therefore we need only one set of $\epsilon$s, which are then determined from $\epsilon_{L_i} \approx \sqrt{m^{\nu}_E \nu_{SM}}$. The $\nu_e - \nu_\mu$ mixing found in this way is consistent with the small mixing angle region [8] of the MSW explanation for the Solar Neutrino Problem (SNP). The mixing angles for this and the other ansätze can be found in Table 1. We checked that for these mixing angles the third flavor does in fact decouple (see [9]). If this is indeed the solution for that problem, the mass of the muon neutrino must be around $3 \times 10^{-3} eV$. This then sets the scale for the new physics and the other neutrino masses at

$$v_{Big} \approx \frac{\epsilon_{L_2}^2 \nu_{SM}^2}{m_{\nu_{\mu}}} \approx 10^{13} GeV,$$

(12)

$$m_{\nu_e} \approx \frac{\epsilon_{L_2}^2}{\epsilon_{L_1}} m_{\nu_{\mu}} \approx 10^{-5} eV,$$

$$m_{\nu_e} \approx \frac{\epsilon_{L_3}^2}{\epsilon_{L_2}} m_{\nu_{\mu}} \approx 10^{-2} eV.$$  (13)

Taking into account the excluded regions due to the IMB experiment [10], the predicted muon to tau neutrino mixing ($\Delta m^2 \approx 10^{-3} eV^2, \sin^2(2\theta_{\mu\tau}) \approx 0.2$) is around a factor of five away from the parameters consistent with a $\nu_\mu - \nu_\tau$ oscillation explanation of the Atmospheric Neutrino Problem (ANP) [11], and still two orders of magnitude away from the laboratory limits for this mixing. Finally, the smallness of all three neutrino masses in this ansatz suggests that neutrinos cannot be responsible for closing the Universe.

II. Another interesting ansatz is suggested by the fact that in the quark sector $\epsilon_{Q_i} \approx$
\( \epsilon_{U_i}, i = 1, 2, 3; \) as found by Hall and Weinberg [4]. Inspired by an \( SU(5) \) type of unification we are led to look at an ansatz in which,

\[
\epsilon_{L_i} \propto \epsilon_{D_i}, \quad \epsilon_{E_i} \propto \epsilon_{Q_i} \approx \epsilon_{U_i}.
\]

(14)

In this ansatz we predict (using the numerical values of \( \epsilon_Q, \epsilon_U \) and \( \epsilon_D \) from [4]) the ratios of charged lepton masses to be within factors of three of the measured values. We consider this an interesting result. Further, the mixing angles are consistent with the three flavour mixing explanations of the SNP [9] for squared masses of \( \nu_\mu \) and \( \nu_\tau \) of order \( 10^{-4} \text{eV}^2 \). Therefore, the \( \nu_\mu - \nu_\tau \) oscillation explanation of the ANP is unlikely. In addition this mass scale cannot be tested in the laboratory nor provide an explanation for Dark Matter.

**III.** Finally, one might look for inspiration in the breaking of \( SO(10) \) into \( SU(4) \otimes SU(2)_L \otimes SU(2)_R \). We know that at the renormalization scale of \( 1\text{GeV} \) we have been working at, the \( SU(2)_R \) symmetry must be badly broken since \( m_t >> m_b \) and \( m_e >> m_{\nu_e} \). Further, assuming the ansatz, \( \epsilon_{L_i} \propto \epsilon_{Q_i}, \epsilon_{E_i} \propto \epsilon_{D_i}, \) and \( \epsilon_{N_i} \propto \epsilon_{U_i} \) would lead to \( m_e/m_\mu \approx 4.8 \times 10^{-2} \), which is wrong by an order of magnitude. Assuming that the \( SU(4) \) might still provide some useful information for the \( SU(2)_L \) singlets we look at the ansatz,

\[
\epsilon_{E_i} \propto \epsilon_{D_i}, \quad \epsilon_{N_i} \propto \epsilon_{U_i}.
\]

(15)

In this ansatz we again predict a \( \nu_e - \nu_\mu \) mixing angle consistent with the small angle MSW solution to the SNP. Again, assuming that this is indeed the solution for that problem fixes the see-saw neutrino masses at

\[
m_\nu_\mu \approx 10^{-3} \text{eV} ,
m_\nu_e \approx \left( \frac{\epsilon_{L_1}}{\epsilon_{L_2}} \right)^2 m_\nu_\mu \approx 10^{-6} \text{eV} ,
m_\nu_\tau \approx \left( \frac{\epsilon_{L_3}}{\epsilon_{L_2}} \right)^2 m_\nu_\mu \approx 0.5 \text{eV}.
\]

(16)
We again find it unlikely that the values obtained can close the Universe or solve the ANP.

In conclusion, we extended the concept of approximate flavor symmetries to the lepton sector. In particular we considered the see-saw mechanism as a source of the neutrino masses and showed that the predictions do not depend on the neutrino flavor symmetry breaking parameters. This yields a simple relation (cf. eq. (7)) between neutrino masses and mixing angles which reduces the number of parameters needed to describe the lepton sector. The lack of information on the neutrino masses and mixing angles led us to propose several ansätze. These exhibit the following common features. They are consistent with the MSW solution of the SNP. The ANP is unlikely to be explained through $\nu_\mu - \nu_\tau$ oscillations and the scale of neutrino masses is too small to close the Universe.

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Table Captions

Table 1: Neutrino mixing angle predictions in the three ansätze introduced. As noted in the text, these results are meant as estimates rather than precise calculations.
\[
\sin^2(2\theta_{e\mu}) \quad \sin^2(2\theta_{e\tau}) \quad \sin^2(2\theta_{\mu\tau})
\]

| ansatz | \( \sin^2(2\theta_{e\mu}) \) | \( \sin^2(2\theta_{e\tau}) \) | \( \sin^2(2\theta_{\mu\tau}) \) |
|--------|-------------------------------|-------------------------------|-------------------------------|
| I      | \(2 \times 10^{-2}\)         | \(10^{-3}\)                  | 0.2                           |
| II     | 0.2                           | 0.1                           | 0.8                           |
| III    | \(2 \times 10^{-3}\)         | \(8 \times 10^{-6}\)         | \(2 \times 10^{-2}\)         |