Hidden-charmonium decays of $Z_c(3900)$ and $Z_c(4025)$ in intermediate meson loops model

Gang Li

Department of Physics, Qufu Normal University, Qufu 273165, People’s Republic of China

(Dated: May 11, 2014)

The BESIII collaboration reported an observation of two charged charmonium-like structure $Z_c^+(3900)$ and $Z_c^+(4025)$ in $e^+e^-\to (J/\psi\pi)^\pm\pi^+$ and $e^+e^-\to (D^*D^*)^{\pm}\pi^+$ at $\sqrt{s}=4.26$ GeV recently, which could be an analogue of $Z_b(10610)$ and $Z_b(10650)$ claimed by the Belle Collaboration. In this work, we investigate the hidden-charmonium transitions of $Z_c^+(3900)$ and $Z_c^+(4025)$ via intermediate $D^{(*)}D^{(*)}$ meson loops. Reasonable results for the branching ratios by taking appropriate values of $\alpha$ in this model can be obtained, which shows that the intermediate $D^{(*)}D^{(*)}$ meson loops process may be a possible mechanism in these decays. Our results are consistent with the power-counting analysis, and comparable with the calculations in the framework of nonrelativistic effective field theory to some extent. We expect more experimental measurements on these hidden-charmonium decays and search for the decays of $Z_c\to DD^*+c.c.$ and $Z_c\to D^*D^*$, which will help us investigate the $Z_c^{(*)}$ decays deeply.

PACS numbers: 13.25.GV, 13.75.Lb, 14.40.Pq

I. INTRODUCTION

Recently, a new charged state $Z_c^+(3900)$ (abbreviated to $Z_c^+$ in the following) is observed in the $J/\psi\pi^\pm$ invariant mass spectrum of $Y(4260)\to J/\psi\pi^\mp\pi^\mp$ decay by the BESIII Collaboration [1]. The reported mass and width are $M_{Z_c^+}=3899.0\pm3.6\pm4.9$ MeV and $\Gamma_{Z_c^+}=46\pm11.3\pm12.6$ MeV [1]. Belle Collaboration also observed a new charged charmonium-like structure in the $J/\psi\pi^\pm$ invariant mass spectrum with $5.2\sigma$ significance, with mass $M_{Z_c^+}=3894.5\pm6.6\pm4.5$ MeV and width $\Gamma_{Z_c^+}=63\pm24\pm26$ MeV [2]. The observation was confirmed later on by an analysis based on the CLEO data at the energy of 4.17 GeV [3]. Very recently, the BESIII Collaboration reported another new charged structure $Z_c^+(4025)$ (abbreviated to $Z_c^{'+}$ in the following) in $e^+e^-\to (D^*D^*)^{\pm}\pi^\pm$ at $\sqrt{s}=4.26$ GeV [4]. Different from the other hidden-charmonium-like states, such as $X(3872)$, $Y(4260)$ etc., $Z_c$ is an electric charged state. Such a state, if it exists, need at least four quarks as minimal constituents, which makes them ideal candidates for exotic hadrons beyond the conventional $qq\bar{q}$ mesons. On the one hand, this chain decay mode reminds us of the observations of $Z_b(10610)$ and $Z_b(10650)$ in $\Upsilon(5S)\to Z_b(Z_b^\prime)\pi\to \Upsilon(nS)\pi\pi$ [5,6]. On the other hand, the mass of $Z_c$ and $Z_c^\prime$ are in the vicinity of $DD^*+c.c.$ and $D^*D^*$, respectively. This similar phenomenon (mechanism) shows that $Z_c^+$ and $Z_c^{'+}$ may be an analogue of $Z_b(10610)$ and $Z_b(10650)$ claimed by the Belle Collaboration.

Recently, many investigations have been carried out to explain this exotic states [5,13]. The results of Ref. [7] show that it is necessary to include explicit $Z_c(3900)$ poles, i.e. a resonance structure as an isovector partner of $X(3872)$ in order for a more detailed description of the data. Applying heavy quark spin symmetry and heavy flavor symmetry [8], the authors found a promising isovector $1^{-+}DD^*$ virtual state near threshold that might very well be identified with the newly discovered $Z_c(3900)$ [13,14].

The intermediate meson loop (IML) transitions, or known as final state interactions, have been one of the important non-perturbative transition mechanisms in many processes [13-34]. In the energy region of charmonium masses, with more and more data from Belle, BaBar, CLEO and BESIII, it is widely recognized that intermediate hadron loops may be closely related to a lot of non-perturbative phenomena observed in experiment [22-37], e.g. apparent OZI-rule violations, sizeable non-$DD$ decay branching ratios for $\psi(3770)$, and the helicity selection rule violations in charmonium decays. Recently, the IML transitions are also applied to bottomonium decays [38-42]. By applying the on-shell approximation, the bottom meson loops were suggested to play an important role in the $\Upsilon(5S)$ transitions to the lower $\Upsilon$ states with the emission of two pions [38] or one $\eta$ [39]. This mechanism seems to explain many unusual properties that make the $\Upsilon(5S)$ different from $\Upsilon(4S)$. Similar approach was also applied to the study of $Z_b$ and $Z'_b$ by Liu et al. [41]. Within a nonrelativistic effective field theory (NREFT), the decays of the $Z_b(10610)$ and the $Z_b(10650)$ to $\Upsilon(nS)\pi$ and $h_b(mP)\pi$ are investigated in Ref. [41]. The power-counting analysis in Ref. [41] shows that the triangle transition $Z_b\to h_b(mP)\pi$ is not suppressed compared to $Z_b\to \Upsilon(nS)\pi$, although the decay is via a P-wave. In Ref. [42], we investigated the transitions from the $Z_b(10610)$ and $Z_b(10650)$ to bottomonium states with emission of a pion via intermediate $BB^*$ meson loops in the effective Lagrangian approach (ELA). The results show
that the intermediate $BB^*$ meson loops are crucial for driving the transitions of $Z_b/Z_b' \to \Upsilon(nS)\pi$ with $n = 1, 2, 3$, and $h_b(mP)\pi$ with $m = 1$ and 2.

Since the $Z_{c}^\pm$ and $Z_{c}'^\pm$ are very close to the $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds, the IML should be a possible mechanism in their decays. In this work, we will investigate the decays of $Z_{c}^{(*)}\to J/\psi\pi$, $\psi'\pi$ and $h_b\pi$ via intermediate charmed meson loops in an effective Lagrangian approach (ELA) with quantum numbers $I^G(J^{PC}) = 1^+(1^{-+})$ for the $Z_{c}/Z_{c}'$. The paper is organized as follows. In Sec. II we will introduce the formulas for the ELA. In Sec. III the numerical results are presented. A summary will be given in Sec. IV.

![FIG. 1: Schematic picture for the decay of $Z_{c}^\pm \to \psi\pi^\pm$ via $D^{(*)}D^{(*)}$ intermediate charmed meson loops. Similar diagrams for $Z_{c}'$ and $Z_{c}'^{(*)}$ states decays.](image)

![FIG. 2: Schematic picture for the decay of $Z_{c}^{(*)} \to h_b\pi^\pm$ via $D^{(*)}D^{(*)}$ intermediate charmed meson loops. Similar diagrams for $Z_{c}'$ and $Z_{c}'^{(*)}$ states decays.](image)

**II. TRANSITION AMPLITUDE**

The IML transitions can be schematically illustrated in Figs. [1](image) and [2](image). In order to calculate the leading contributions from the charmed meson loops, we need the leading order effective Lagrangians for the couplings. Based on the heavy quark symmetry \cite{43,44}, the relevant effective Lagrangians used in this work are as follows,

$$\mathcal{L}_1 = ig_1 Tr[P_{c\bar{c}}^{\mu} \bar{H}_{2i} \gamma_{\mu} \bar{H}_{1i} + h.c.]$$

$$\mathcal{L}_2 = ig_2 Tr[R_{c\bar{c}} \bar{H}_{2i} \gamma^{\nu} \partial_{\nu} \bar{H}_{1i} + h.c.]$$

where the spin multiplets for these four $P$-wave and two $S$-wave charmonium states are expressed as

$$P_{c\bar{c}}^{\mu} = \left( \frac{1 + \not{\gamma}}{2} \right) \left( \chi_{c2}^{\mu\alpha} \chi_{1 \nu} + \frac{1}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} v_{\alpha} \chi_{c1 \nu} + \frac{1}{\sqrt{3}} (\gamma_{\mu} - \gamma_{\nu}) \chi_{c0} + h_{c0}^{\mu} \gamma_{5} \right) \left( \frac{1 - \not{\gamma}}{2} \right),$$

$$R_{c\bar{c}} = \left( \frac{1 + \not{\gamma}}{2} \right) (\gamma_{\mu} - \eta_{c5}) \left( \frac{1 - \not{\gamma}}{2} \right).$$

The charmed and anti-charmed meson triplet read

$$H_{1i} = \left( \frac{1 + \not{\gamma}}{2} \right) [D^{*\mu}_{c} \gamma_{\mu} - D_{c} \gamma_{5}],$$

$$H_{2i} = [D^{*\mu}_{c} \gamma_{\mu} - D_{c} \gamma_{5}] \left( \frac{1 - \not{\gamma}}{2} \right),$$

where $D$ and $D^*$ denote the pseudoscalar and vector charmed meson fields, respectively, i.e. $D^{(*)} = (D^{0(*)}, D^{+(*)}, D_{s}^{+(*)}).$
Explicitly, the Lagrangians for the S-wave \( (J/\psi \text{ and } \psi') \) and P-wave \( (h_c) \) charmonia couplings to \( D \) and \( D^* \) become
\[
\mathcal{L}_{\psi D^*(\psi') D^*} = -ig_{\psi D^* D} \left\{ \psi^\mu (\partial_\mu D^{\mu^*} D^*_\mu - D^{\mu^*} D^*_\mu \partial_\mu) + (\partial_\mu \psi^\nu \partial_\nu D^{\nu^*} - \partial_\mu \psi^\nu \partial_\nu D^{\nu^*} D^*_\mu - D^{\mu^*} \partial_\mu \psi^\nu D^*_\nu) \right\}
+ ig_{\psi DD} \psi^\mu (\partial_\mu D^* D - D \partial^\mu D^*) - g_\psi D^* - D^* \epsilon^{\mu \nu \alpha \beta} \partial_\mu \psi^\nu (\partial_\alpha D^* \beta D + D \partial_\alpha D^*),
\]
\[
\mathcal{L}_{h_c D^*(\psi') D^*} = g_{h_c D^* D} h_c^\mu (D^* \mu + D^* \nu D) + ig_{h_c D^* D^*} \epsilon^{\mu \nu \alpha \beta} \partial_\mu h_c \epsilon_{\mu \nu \alpha \beta} D^*_\beta D^*.
\]
The relevant Lagrangians for \( Z_c \) and \( Z_c^\prime \) couplings to a pair of charmed mesons can be expressed as
\[
\mathcal{L}_{Z_c^\prime(D^* \psi') D^*} = g_{Z_c^\prime(D^* \psi') D^*} \left( D^* \mu + D^* \nu D \right) + ig_{Z_c^\prime(D^* \psi') D^*} \epsilon^{\mu \nu \alpha \beta} \partial_\mu Z_c^\prime(D^* \nu D^*),
\]
and the Lagrangian relevant to light pseudoscalar pion meson is
\[
\mathcal{L}_{D^* \pi} = -ig_{D^* \pi} (D_i \partial_\mu P_{ij} D_j^* - D_i^* \partial_\mu P_{ij} D_j) + \frac{1}{2} g_{D^* \pi} \epsilon^{\mu \nu \alpha \beta} D_i^* \partial_\mu P_{ij} \epsilon_{\mu \nu \alpha \beta} D_j.
\]
The coupling constants will be determined in the following.
The loop transition amplitudes for the transitions in Figs. 1 and 2 can be expressed in a general form in the effective Lagrangian approach as follows:
\[
M_{fi} = \int \frac{d^4q_2}{(2\pi)^4} \sum_{D^* \pi} V_1 V_2 V_3 \prod_i F_i(m_i, q_i^2)
\]
where \( V_i \) (\( i = 1, 2, 3 \)) are the vertex functions; \( a_i = q_i^2 - m_i^2 \) (\( i = 1, 2, 3 \)) are the denominators of the intermediate meson propagators. We adopt the form factor, \( \prod_i F_i(m_i, q_i^2) \), which is a product of monopole form factors for each of the internal mesons, i.e.
\[
\prod_i F_i(m_i, q_i^2) = \prod_i \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2},
\]
where \( \Lambda_i = m_i + \alpha \Lambda_{QCD} \) and the QCD energy scale \( \Lambda_{QCD} = 220 \text{ MeV} \). This parameter scheme has been applied extensively in other works.\(^{20, 30, 33, 12}\) This form factor is supposed to offset the off-shell effects of the exchanged mesons.\(^{12, 43, 44}\) In this approach the local couplings for a charmonium to charmed mesons, or a light meson to charmed mesons, are the same as used in NREFT,\(^{33}\) while the form factor parameter will be determined by comparison to experimental information. Thus, it is assumed here that all (at least the dominant part) of the short range physics related to meson loops can be parameterized in the form of Eq. (12).

Based on the above Lagrangians, the explicit amplitudes in Figs. 1 and 2 can be obtained, which are given in the Appendix A.

### III. NUMERICAL RESULTS

Before proceeding to the numerical results, we first discuss the parameters, such as the coupling constants, in the formulation given in Section.\(^{11}\) In Eq. (7), the following coupling constants are adopted in the numerical calculations,
\[
g_{\psi DD} = 2g_2 \sqrt{m_\psi m_D}, \quad g_{\psi D^* D} = \frac{g_{\psi DD}}{\sqrt{m_D m_{D^*}}}, \quad g_{\psi D^* D^*} = g_{\psi D^* D} \sqrt{\frac{m_{D^*}}{m_D}} \sqrt{\frac{m_D}{m_{D^*}}},
\]
In principle, the coupling \( g_2 \) must be computed by non-perturbative methods. It shows that vector meson dominance (VMD) would provide an estimate of these quantities.\(^{43}\) The coupling \( g_2 \) can be related to the \( J/\psi \) leptonic constant \( f_\psi \) which is defined by the matrix element \( \langle 0 | \bar{c} \gamma_\mu c | J/\psi(p, \epsilon) \rangle = f_\psi m_\psi \epsilon^\mu \), and \( g_2 = \sqrt{m_\psi / (2m_D f_\psi)} \), where \( f_\psi = 405 \pm 14 \text{ MeV} \), and we have applied the relation \( g_{\psi DD} = m_\psi / f_\psi \).

The ratio of the coupling constants \( g_{\psi DD} \) to \( g_{\psi DD} \) is fixed as in Ref.\(^{31}\):
\[
g_{\psi DD} / g_{\psi DD} = 0.9.
\]

In addition, the coupling constants in Eq. (8) are determined as
\[
g_{h_c D^* D} = -2g_1 \sqrt{m_{h_c} m_D m_{D^*}}, \quad g_{h_c D^* D^*} = 2g_1 \sqrt{m_{D^*}} / \sqrt{m_{h_c}},
\]

\( g_1 \) is fixed as in Ref.\(^{31}\)
FIG. 3: (a) The $\alpha$-dependence of the branching ratios of $Z_c^+ \to J/\psi\pi^+$ (solid line) and $\psi'\pi^+$ (dashed line). (b) The $\alpha$-dependence of the branching ratios of $Z_c'^+ \to J/\psi\pi^+$ (solid line), $\psi'\pi^+$ (dashed line).

FIG. 4: (a) The $\alpha$-dependence of the branching ratios of $Z_c^+ \to h_c\pi^+$. (b) The $\alpha$-dependence of the branching ratios of $Z_c'^+ \to h_c\pi^+$.

with $g_1 = -\sqrt{m_{\chi_{c0}}/3/f_{\chi_{c0}}}$, where $m_{\chi_{c0}}$ and $f_{\chi_{c0}} = 510 \pm 40$ MeV are the mass and decay constant of $\chi_{c0}(1P)$, respectively [47].

In analogy to what is known about the $Z_b$ states, we assume that the total widths of the $Z_c^+$ and $Z_c'^+$ are saturated by the $DD^*$ and $D^*D^*$. The coupling constants are obtained with the following relations

$$g_{Z_c^+DD^*} = -2z\sqrt{m_{Z_c^+}m_Dm_{D^*}}, \quad g_{Z_c'^+D^*D^*} = 2z\frac{m_{D^*}}{\sqrt{m_{Z_c'^+}}},$$

with $z = (0.85^{+0.07}_{-0.26})$ GeV$^{-1/2}$ and $z' = (0.33^{+0.06}_{-0.07})$ GeV$^{-1/2}$.

FIG. 5: The $\alpha$-dependence of the ratios $R_{Z_c}$ (solid line) and $R_{Z_c'}$ (dashed line) defined in Eq. (20).
The coupling constants relevant to the pion interactions in Eq. (9) are

\[ g_{D^*D} = \frac{2g}{\pi} \sqrt{m_{D^*}m_D}, \quad g_{D^*D^+} = \frac{g_{D^*D}}{\sqrt{m_{D^*}m_{D^+}}}, \tag{17} \]

where \( f_\pi = 132 \text{ MeV} \) is the pion decay constant and \( g = 0.59 \) \cite{38}.

In Ref. \cite{50}, the NREFT method was introduced to study the meson loop effects in \( \psi' \to J/\psi\pi^0 \) transitions. And a power-counting scheme was proposed to estimate the contribution of the loop effects, which is helpful to judge how important the coupled-channel effects are. This power-counting scheme was analyzed in detail in Ref. \cite{33}. Before giving the explicit numerical results, we will follow a similar power-counting scheme to qualitatively estimate the contributions of the coupled-channel effects discussed in this work. Corresponding to the diagrams Figs. 1 and 2, the amplitudes for \( Z_c^+ / Z_c^{'+} \to J/\psi\pi^+ (\psi'\pi^+) \) and \( Z_c^+ / Z_c^{'+} \to h_c\pi^+ \) scale as

\[ \frac{v^5}{(v^2)q^2} \sim \frac{q^2}{v} \tag{18} \]

and

\[ \frac{v^5}{(v^2)q^2} \sim \frac{q}{v}, \tag{19} \]

respectively. There are two scaling parameters \( v \) and \( q \) appeared in the above two formulas. As illustrated in Ref. \cite{51}, \( v \) is understood as the average velocity of the intermediate charmed meson. \( q \) denotes the momentum of the outgoing pseudoscalar meson. According to Eqs. (18) and (19), it can be concluded that the contributions of the coupled-channel effects would be significant here since the amplitudes scale as \( \mathcal{O}(1/v) \). And the branching ratio of \( Z_c^+ / Z_c^{'+} \to h_c\pi^+ \) is expected to be larger than that of \( Z_c^+ / Z_c^{'+} \to J/\psi\pi^+ \), because the corresponding amplitudes scale as \( \mathcal{O}(q) \) and \( \mathcal{O}(q^2) \), respectively. However, the momentum \( q \) in \( Z_c^+ / Z_c^{'+} \to J/\psi\pi^+ \) is larger than that in \( Z_c^+ / Z_c^{'+} \to h_c\pi^+ \), which may compensate this discrepancy to some extent.

Since there are no experimental data for the hidden-charmonium decays of \( Z_c^\pm \) and \( Z_c^{\pm'} \), we cannot determine the cutoff parameter \( \alpha \) for each channels. However, due to the similarity to the hidden-bottom decays of \( Z_b \), it is also possible to find an appropriate range of \( \alpha \) values for each decay channels that can account for the data via the intermediate charmed meson loops \cite{12}. And the future experimental measurements can help us test this point. So in this work, we only present the \( \alpha \)-dependence of the hidden-charmonium decays of \( Z_c^\pm \) and \( Z_c^{\pm'} \).

In Fig. 3(a), we plot the \( \alpha \)-dependence of the branching ratios of \( Z_c^+ \to J/\psi\pi^+ \) (solid line) and \( \psi'\pi^+ \) (dashed line), respectively. The \( \alpha \)-dependence is not drastically sensitive at the commonly accepted \( \alpha \) range. As shown in this figure, at the same \( \alpha \), the intermediate D-meson loop effects turn out to be more important in \( Z_c^+ \to J/\psi\pi^+ \) than in \( Z_c^+ \to J/\psi\pi^+ \). This is understandable since the mass of \( \psi' \) is closer to the thresholds of \( D D^* \) or \( D^* D^+ \) than \( J/\psi \). Thus, it gives rise to important threshold effects in \( Z_c^+ \to \psi'\pi^+ \).

One also notices that the \( \alpha \)-dependence of the branching ratios for \( Z_c^+ \to \psi'\pi^+ \) are less sensitive than that for \( J/\psi\pi^+ \). This indicates that the enhanced branching ratios are not from the off-shell part of the loop integrals and the enhanced (but rather stable in terms of \( \alpha \)) branching ratios for \( Z_c^+ \to \psi'\pi^+ \) suggest that more stringent dynamic constraints are presumably needed to describe the near-threshold phenomena where the local quark-hadron duality has been apparently violated. What makes this process different from e.g. \( \psi' \to h_c\pi^0 \) in Ref. \cite{32} is that there is no cancelations between the charged and neutral meson loops. As a consequence, the subleading terms in Refs. \cite{32,33} become actually leading contributions. In Fig. 3(b), we plot the \( \alpha \)-dependence of the branching ratios of \( Z_c^{'+} \to J/\psi\pi^+ \) (solid line) and \( \psi'\pi^+ \) (dashed line), respectively. At the commonly accepted \( \alpha \) range, the \( \alpha \)-dependence of the branching ratios is not dramatically sensitive.

The \( \alpha \)-dependence of the branching ratios of \( Z_c^+ \to h_c\pi^+ \) and \( Z_c^{'+} \to h_c\pi^+ \) are shown in Figs. 4(a) and (b). From this figure, we can see that the intermediate meson loop contributions are more important in \( Z_c^+ \to h_c\pi^+ \) than that in \( Z_c^{'+} \to h_c\pi^+ \).

It would be interesting to further clarify the uncertainties arising from the introduction of form factors by studying the \( \alpha \) dependence of the ratios between different partial decay widths. For the decays of \( Z_c^+ / Z_c^{'+} \to J/\psi\pi^+ \), we define the following ratios to the partial decay widths of \( Z_c^+ / Z_c^{'+} \to \psi'\pi^+ \):}

\[ R_{Z_c} = \frac{\Gamma(Z_c^+ \to J/\psi\pi^+)}{\Gamma(Z_c^+ \to \psi'\pi^+)}, \quad R_{Z_c'} = \frac{\Gamma(Z_c^{'+} \to J/\psi\pi^+)}{\Gamma(Z_c^{'+} \to \psi'\pi^+)} \tag{20} \]

which are plotted in Fig. 5. The ratios are relatively insensitive to the cutoff parameter, which is because the involved loops are the same. Since the first coupling vertices are the same for those decay channels when taking the ratio, also the mass of \( \psi' \) is closer to \( D(D^{(*)}) \) thresholds than \( J/\psi \), so the ratio only reflects the open threshold effects via the
TABLE I: The calculated branching ratios in NREFT approach. The uncertainties are from the experimental measurements of the total widths of $Z^+_c$ and $Z^+_c$.

| $Z^+_c$ | $Z^+_c$ |
|----------|----------|
| Branching ratios | Branching ratios |
| $J/\psi \pi^+$ | $49.89^{+22.88}_{-22.88}$% | $3.21^{+1.24}_{-1.24}$% |
| $\psi' \pi^+$ | $13.80^{+2.47}_{-2.47}$% | $1.99^{+0.77}_{-0.77}$% |
| $h_c \pi^+$ | $5.56^{+2.28}_{-2.28}$% | $6.37^{+2.46}_{-2.46}$% |

intermediate charmed meson loops. The future experimental measurements of $Z^+_c/Z^+_c \rightarrow J/\psi \pi^+$ and $\psi' \pi^+$ can help us investigate this issue deeply.

In order to understand this, the following analysis is carried out. First, one notices that we have adopted the couplings for the $h_c$ and $\psi$ to $D\bar{D}^*$ or $D^*\bar{D}^*$ in the heavy quark approximation. Since the physical masses for $D$ and $D^*$ are adopted in the loop integrals, the form factor will introduce unphysical pole contributions of which the interferences with the nearby physical poles would lead to model-dependent uncertainties. By assuming $M_{D^*} = M_D = 1869$ MeV and $M_{D^*} = M_D = 2010$ MeV, namely, by making the spin symmetry exactly, we calculate the branching ratios of $Z^+_c$ and $Z^+_c \rightarrow J/\psi \pi^+$, $\psi' \pi^+$ and $h_c \pi^+$. We expect that the exact spin symmetry will significantly lower the branching ratios since there will be only one physical pole in the loop and the unphysical one can be easily isolated away from the physical one. This is a rather direct demonstration of the sensitivity of the meson loop behavior when close to open threshold and when the dispersive part becomes dominant.

As a cross-check, we also calculate the branching ratios of the decays in the framework of NREFT and the relevant transition amplitudes are given in Appendix B. The numerical results in NREFT are listed in Table I, the uncertainties are due to the experimental measured uncertainties of the total widths of $Z^+_c$ and $Z^+_c$. As shown in this table, the results calculated in NREFT are in good agreement with the results in ELA at the commonly accepted range except for the case of $Z^+_c \rightarrow J/\psi \pi^+$, which indicates the availability of our model to some extent and shows that the higher order effects are important in $Z^+_c \rightarrow J/\psi \pi^+$. In fact, following the NREFT power counting for higher loops in Ref. [41], one sees that this is exactly the channel where higher order loops can be important. However, since there are still several uncertainties coming from the undetermined coupling constants, and the cutoff energy dependence of the amplitude is not quite stable, the numerical results would be lacking in high accuracy. Especially, since the kinematics and off-shell effects arising from the exchanged particles are different, the cutoff parameter can also be different in different decay channels. We expect more experimental measurements on these hidden-charmonium decays in the near future.

IV. SUMMARY

In this work, we investigate hidden-charm decays of the newly discovered resonances $Z^+_c$ and $Z^+_c$ via intermediate charmed meson loops. In this calculation, the quantum numbers of the neutral partners of these two resonances are fixed to be $I^G(J^{PC}) = 1^+(1^-)$, which has the same favored quantum number of $Z_b(10610)$ and $Z_b(10650)$. For $Z^+_c$ and $Z^+_c$ decays, our results show that the $\alpha$ dependence of the branching ratios are not dramatically sensitive. Our results show that the meson loop contributions are much more important when the final state mass threshold are close to the intermediate meson thresholds. Namely, the effects from the unphysical pole introduced by the form factors would interfere with the nearby physical poles from the internal propagators and lead to model-dependent uncertainties. It is also a consequence of the violation of spin symmetry and such a phenomenon has been discussed in Ref. [32]. Our results are in good agreement with the results in the framework of NREFT except for the case of $Z^+_c \rightarrow J/\psi \pi^+$, which indicates the availability of our model to some extent. However, since there are still several uncertainties, for example, the kinematics and off-shell effects arising from the exchanged particles are different, the cutoff parameter can also be different in different decay channels, so we expect more experimental measurements on these hidden-charmonium decays and search for the decays of $Z_c \rightarrow D\bar{D}^* + c.c.$ and $Z_c^* \rightarrow D^*\bar{D}^*$, which will help us investigate the $Z_c^{(*)}$ decays deeply.

Acknowledgements

Author thanks F.-K. Guo, X.-H. Liu, Q. Wang and Q. Zhao for useful discussions. This work is supported, in part, by the National Natural Science Foundation of China (Grant No. 11275113).
where $p_i$, $p_f$, $p_\pi$ are the four-vector momenta of the initial $Z_{c}^{(i)}$, final state charmonium and pion, respectively, and $q_1$, $q_2$, and $q_3$ are the four-vector momenta of the intermediate charmed mesons as defined in Figs. 1 and 2.

**Appendix B: Amplitudes in NREFT Approach**

The basic three-point loop function worked out using dimensional regularization in $d = 4$ is

$$I(q,m_1,m_2,m_3) = -\frac{i}{8} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^0 - f_1 + i\epsilon][l^0 - b_12 + i\epsilon][l^0 + b_12 - b_23 - (l^2 - q^2)/m_2^2 + i\epsilon]}$$

where $m_i (i = 1,2,3)$ are the masses of the particles in the loop; $\mu_{ij} = m_i m_j/(m_i + m_j)$ are the reduced masses; $b_{12} = m_1 + m_2 - M$ and $b_{23} = m_2 + m_3 + q_0 - M$ with $M$ being the mass of the initial particle; and

$$a = \left(\frac{\mu_{23}}{m_3}\right)^2 q^2, \quad c = 2\mu_{12} b_{12}, \quad c' = 2\mu_{23} b_{23} + \frac{\mu_{23}}{m_3} q^2.\hspace{1cm}(B2)$$

The vector loop integrals are defined as

$$q^j I^{(j)}(q,m_1,m_2,m_3) = -\frac{i}{8} \int \frac{d^d l}{(2\pi)^d} \frac{l^j}{[l^0 - f_1 + i\epsilon][l^0 - b_12 + i\epsilon][l^0 + b_12 - b_23 - (l^2 - q^2)/m_2^2 + i\epsilon]}$$

and we get

$$I^{(1)}(q,m_1,m_2,m_3) = \frac{\mu_{23}}{am_3} [B(c' - a) - B(c) + \frac{1}{2} (c' - c) I(q)],\hspace{1cm}(B4)$$

where the function $B(c)$ is

$$B(c) = -\frac{\mu_{12} \mu_{23} \sqrt{c - i\epsilon}}{16\pi}.\hspace{1cm}(B5)$$

In terms of the loop functions given above, the transition amplitudes for the intermediate meson loops listed in Figs. 1 and 2 in the framework of NREFT,

(i) $Z_{c}^{(i)} \rightarrow J/\psi \pi^+$ and $\psi' \pi^+$

$$\mathcal{M}(Z_{c}^{(i)} \rightarrow \psi \pi^+) = \frac{2\sqrt{2} g_{1} Z_{c}(i)}{f_{\pi}} \sqrt{M_{Z_{c}^{(i)}} M_{\psi} \langle \bar{q} \cdot \bar{\xi}(Z_{c}) \bar{q} \cdot \bar{\xi}(\psi) \rangle} [2 I^{(1)}(q,M_{D^*},M_{D},M_{D}) - I(q,M_{D^*},M_{D},M_{D})$$

$$-2 I^{(1)}(q,M_{D^*},M_{D},M_{D}) + I(q,M_{D^*},M_{D},M_{D^*}) + 2 I^{(1)}(q,M_{D^*},M_{D^*},M_{D^*})]$$

$$+ q^2 \bar{\xi}(Z_{c}) \cdot \bar{\xi}(\psi) [2 I^{(1)}(q,M_{D^*},M_{D},M_{D^*}) - I(q,M_{D^*},M_{D},M_{D^*})]$$

$$+ 2 I^{(1)}(q,M_{D^*},M_{D^*},M_{D^*}) - I(q,M_{D^*},M_{D^*},M_{D^*}) - 2 I^{(1)}(q,M_{D^*},M_{D^*},M_{D^*})]$$

$$+ I(q,M_{D^*},M_{D^*},M_{D}) - 2 I^{(1)}(q,M_{D^*},M_{D},M_{D^*}) + I(q,M_{D^*},M_{D^*},M_{D^*})] \hspace{1cm}(B6)$$

(ii) $Z_{c}^{(i)} \rightarrow h_{c} \pi^+$

$$\mathcal{M}(Z_{c}^{(i)} \rightarrow h_{c} \pi^+) = \frac{2\sqrt{2} g_{1} Z_{c}(i)}{f_{\pi}} \sqrt{M_{Z_{c}^{(i)}} M_{h_{c}} \epsilon^{ijk} q^i \epsilon^{j}(Z_{c}) \epsilon^{k}(h_{c})} [I(q,M_{D^*},M_{D^*},M_{D}) + I(q,M_{D^*},M_{D^*},M_{D'})$$

$$- I(q,M_{D^*},M_{D^*},M_{D}) + I(q,M_{D^*},M_{D^*},M_{D'})] \hspace{1cm}(B7)$$

[1] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 110, 252001 (2013) [arXiv:1303.5949] [hep-ex].
