Distributed adaptive robust $H_\infty$ control of intelligent-connected electric vehicles platooning subject to communication delay

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Abstract
With an aim to improve the performances of intelligent-connected electric vehicle platoon with communication delay, we present a distributed adaptive robust $H_\infty$ control scheme in this paper. First, the longitudinal non-linear dynamic model of vehicle $i$ is established, and in order to reduce the difficulty of analysing such a non-linear system, a simplified uncertain dynamic model of vehicle $i$ is obtained via a feedback linearisation method. Second, a novel distributed state-feedback robust platoon controller is designed by using the relative information between neighbouring electric vehicles. Third, we derive the sufficient condition that the presented distributed robust platoon controller exists under the influence of communication delay. This condition is given in the form of a linear matrix inequality, which is low dimensional and whose dimension is independent of the length of the intelligent electric vehicle platoon. Furthermore, the closed-loop platoon system controlled by the presented controller is proved stable with respect to communication delay by using the Lyapunov stability theory. In addition, an adaptive control term is added to the robust platoon controller. We present the sufficient condition that such a modified controller exists and prove the platoon string stability. Finally, the effectiveness of the presented adaptive robust platoon control scheme is verified by conducting numerical simulation tests.

1 | INTRODUCTION

In recent years, the growing use of vehicles has brought convenience to people’s lives. But the current carriage facilities bore the pressure of the growing haul requirements, which has caused a series of issues such as energy consumption and traffic safety [1]. The intelligent electric vehicle platoon control can command the vehicles to work together efficiently to reach the desired configuration and consistent driving speed by automatically adjusting the longitudinal movement state of each vehicle based on the information of adjacent vehicles [2]. Vehicle platoon control has the advantage of improving traffic efficiency and reducing fuel consumption. Moreover, highly intelligent driving can avoid vehicle accidents caused by human factors, and hence enhance traffic safety [3, 4].

A large number of researches on vehicle longitudinal platoon control have been carried out, such as PATH in California [5], SARTRE in Europe, and Energy-ITS in Japan [6], and so forth. For linear dynamics vehicle platoon, Ploeg et al. [7] designed a $H_\infty$ controller to meet the performance requirements of the platoon. Aimed at an intelligent-connected vehicle platoon with dynamic uncertainties, Gao and Jiang [8] proposed a dynamic programming approach to realise the platoon suboptimal control. Kianfar et al. [9] constructed a vehicle platoon model predictive control scheme to ensure the internal chord stability of closed-loop dynamics. Gao et al. [10] proposed a distributed adaptive sliding mode control method. In this method, an adaptive mechanism is adopted to deal with the parameter uncertainties.
Wireless communication is usually employed to achieve the sharing of information in real time during the running of the vehicle platoon. Through using wireless communication to extend the ability of the platoon to perceive the environment, the platoon control effect is improved. The leader-predecessor following (LPF) topology is the most common communication topology using wireless communication, in which the leading vehicle sends its state information to every following vehicle by wireless communication. The predecessor follower (PF) and the LPF topologies were compared in [9], and the conclusion that LPF can bring more excellent string stability margin in the case of a great delay was drawn.

The information interaction among vehicles has the potential to reduce vehicle spacing and increase traffic flow [11]. However, the information interaction must be executed over a wireless communication network, which is non-immune to network factors such as communication delay. Such factors may cause the performance level of the platoon under the designed controller to deteriorate intolerably [12]. Considering the case that the delays are heterogeneous and time-varying, a distributed control medium was designed by Bernardo et al. [13], in which the platoon control problem was modelled as a consensus problem. Ge and Orosz [14] designed an optimal control scheme to solve the heterogeneous vehicles platoon control problem subject to communication delay. Aim at the platoon control system under homogeneous delays and indeterminate dynamics, Gao et al. [15] constructed an $H_\infty$ control scheme so as to ensure the stability of the platoon. However, this control method is not applicable for large-scale vehicle platoons, as the matrix inequality for deriving the controller gain is high dimensional, and its dimension increases with the platoon’s length. Furthermore, communication delays are generally heterogeneous rather than homogeneous in the actual situation.

When there are simultaneously uncertain dynamics, external disturbances and heterogeneous time-varying communication delays, the platoon control will be more knotty. However, few existing works have solved the platoon control problem that simultaneously takes into consideration the uncertain characteristics, external disturbances and heterogeneous time-varying communication delays. The purpose of this paper is to propose a distributed adaptive robust platoon control precept for intelligent-connected electric vehicles, which can achieve robust stability [16] to external disturbances, as well as guarantee desired tracking performance and string stability in the case of heterogeneous time-varying communication delays. The main contributions of the paper are as follows:

1. We present a distributed adaptive robust $H_\infty$ platoon control precept for intelligent-connected electric vehicles, in which an adaptive control term is introduced into a linear controller, subject to reduce the impact of uncertain dynamics, external disturbances and heterogeneous time-varying communication delays on vehicle platoon system.
2. The condition of guaranteeing the platoon stability under the proposed method is given by using the Lyapunov stability theory. The condition is given in terms of a linear matrix inequality and such inequality is low dimensional, whose dimension is independent of the length of the intelligent electric vehicle platoon.
3. To further improve the robust performance of the system, an adaptive control term is introduced into the robust platoon control system, and the sufficient condition that such a modified controller exists is presented.

The rest of the paper is arranged as follows. An uncertain model of vehicle platoon is described in Section 2. A distributed adaptive robust control scheme for intelligent-connected electric vehicles with communication delay is presented in Section 3. Section 4 proves the platoon string stability. The tests of the proposed platoon control method are carried out in Section 5. Last, Section 6 draws the conclusion of this paper.

2  |  SYSTEM DESCRIPTION

2.1  |  Notations

Let $\mathbb{R}^{N \times N}$ be the $N \times N$ real matrices. $I$ expresses an identity matrix, and $I_N$ is an $N \times N$ identity matrix. For square matrix $A$, $\text{He}(A) = A + A^T, A \otimes B$ expresses the Kronecker product between $A$ and $B$. The notation $s$ is the Laplace operator, and $j = \sqrt{-1}$ is the imaginary unit.

2.2  |  Intelligent electric vehicle platoon model

Figure 1 shows the vehicle platoon researched in this paper. The number of vehicles in the platoon is $N + 1$, in which the first one is the leader and the rest are followers. We label the leader as 0 and the followers from 1 to $N$ in series. The set of all following vehicle numbers is defined as $\mathbb{N}$, that is, $\mathbb{N} = \{1, \ldots, N\}$. We use the steady distance strategy to restrict the platoon. That is to say, the expected distance $d_{i,i-1}$ is set as a constant. The platoon control purpose is to keep the consistent speed to the leader and the desired interval to the previous vehicle:

\[
\begin{aligned}
\lim_{t \rightarrow \infty} \left| v_i(t) - n_i(t) \right| &= 0 \\
\lim_{t \rightarrow \infty} \left| p_{i,i-1}(t) - p_i(t) - d_{i,i-1} \right| &= 0, \quad i \in \mathbb{N}.
\end{aligned}
\]

We make some assumptions for the sake of gaining a succinct vehicle longitudinal dynamic model: (1) The vehicle body is considered to be symmetric and rigid; (2) the tire longitudinal slip is neglected; (3) the pitch and yaw motions are overlooked; (4) the vehicle dynamics system is lumped into a first-order inertial transfer functions. Then, we can simplify the non-linear longitudinal dynamics model of vehicle $i$ as [17]

\[
p_i = v_i,
\]

\[
v_i(t) = a_i(t) = \frac{1}{m_i} \left[ T_i(t) - c_i v_i^2(t) - m_i g f_i \right].
\]
FIGURE 1 Structure of the platoon

\[ \tau_i T_i (t) + T_j (t) = T_{ij} (t), i = 0, \ldots, N. \]  

(4)

For the DC hub motor of the electric vehicle, its dynamic model is [18]

\[ J_w (t) \omega (t) = T_{w} (t) - \mu T_j (t) \]  

(5)

This paper uses the inverse model compensation technology [19] to linearise the vehicle longitudinal dynamics model, and the desired torque \( T_{w} \) is designed as

\[ T_{ij} (t) = \epsilon \omega^2 (t) + m_j g f_j (t) + 2 \tau_i \tau_j v_j (t) v_i (t) + m_i u_i (t). \]  

(6)

Combining Equations (2) to (4) and (6), We can gain the linearised longitudinal dynamic model of intelligent-connected electric vehicle \( \hat{\xi} \):

\[ \dot{\xi}_i = -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i} u_i, \]  

(7)

The state vector of vehicle \( i \) is defined as

\[ \xi_i (t) = \begin{bmatrix} p_i (t) \\ v_i (t) \\ a_i (t) \end{bmatrix}. \]

Defining \( \xi_j + \Delta \xi_j = \frac{1}{\tau_j + \Delta \tau_j} \), we can formulate the dynamics state-space model of vehicle \( \hat{\xi} \):

\[ \dot{\xi}_i (t) = (A_i + \Delta A_i) \xi_j (t) + (B_i + \Delta B_i) u_i (t), i \in \mathbb{N}, \]  

(8)

where

\[ A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\xi_j \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \xi_j \end{bmatrix}, \]

\[ \Delta A_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\Delta \xi_j \end{bmatrix}, \Delta B_i = \begin{bmatrix} 0 \\ 0 \\ \Delta \xi_j \end{bmatrix}. \]

Assumption 1. [20]: We assume that the parameter uncertainties in this paper are unknown but bounded, which are in the following form:

\[ \Delta A_i, \Delta B_i = DF_i (t) [E_1 \ E_2], \]

where \( D, E_1 \) and \( E_2 \) are appropriate dimensioned known constant matrices, which describe the structural information of uncertainties; \( F_i (t), i \in \mathbb{N} \) are the time-varying unknown matrix that satisfy

\[ F_i^T (t) F_i (t) \leq I. \]

2.3 Communication topology

This paper uses the tool from the graph theory to describe the communication connection among vehicles [21]. Concretely, a directed graph \( G \) is used in the modelling of communication links. We define the adjacency matrix of \( G \) as \( M = (m_{ij}) \in \mathbb{R}^{N \times N} \). If the information of \( j \) can be obtained by vehicle \( i \), \( m_{ij} = 1 \); else \( m_{ij} = 0 \). The Laplacian matrix associated with \( G \) is defined as

\[ L_i = \begin{cases} 1 \sum_{j=1, j \neq i}^{N} m_{ij}, \\ -m_{ij}. \end{cases} \]

We introduce a pinning matrix to represent the connection between the followers and leader:

\[ R = \begin{bmatrix} r_1 & \cdots & r_N \end{bmatrix}. \]

where \( r_i \) if the information of leader can be obtained by vehicle \( i \); else, \( r_i = 0 \). Figures 2 and 3 show the PF and LPF topologies, respectively.

3 DESIGN OF THE ADAPTIVE ROBUST CONTROLLER

The purpose of intelligent-connected electric vehicle platoon control is to improve traffic efficiency and ensure safe driving. There are unavoidable factors in the platoon system such as external disturbances and parameter uncertainties, which may deteriorate the platoon control and even threaten the safety of vehicles. In addition, the wireless communication network is usually suffering from factors such as non-negligible
communication delay, which renders it difficult to realise the platoon control. So it is necessary to ensure that the vehicle platoon system is asymptotically stable when the system has no external disturbances and to improve the robustness of the platoon against the uncertain characteristics, external disturbances and heterogeneous time-varying communication delays.

This article assumes that the vehicles are homogeneous, which means $\zeta_i = \zeta$, $i \in \mathbb{N}$. We design a distributed state-feedback controller as follows:

$$u_i(t) = K \sum_{j=1}^{N} m_{ij} (\dot{x}_j(t) - x_i(t) - D_{ij}) + K_r (x_0(t) - \eta - x_i(t - \eta) - D_{0i}), i \in \mathbb{N}.$$  \hspace{1cm} (9)

where $K = [k_1 \ k_2 \ k_3]$ is the gain of the controller that needs to be designed; $D_{ij} = [d_{ij} \ 0 \ 0]^T$.

The tracking error of node $i$ is defined as

$$\tilde{x}_i(t) = x_0(t) - x_i(t) - D_{0i}, i \in \mathbb{N}. $$ \hspace{1cm} (10)

Taking the derivative of Equation (10) and considering the fact that $(A + \Delta A)D_{0i} = 0$ and $D_{ij} = D_{0i} - D_{00}$, we can obtain the dynamics of the vehicle tracking errors

$$\dot{\tilde{x}}_i(t) = (A + \Delta A)\tilde{x}_i(t) - (B + \Delta B)u_i(t) + (B + \Delta B)m_0(t), i \in \mathbb{N}. $$ \hspace{1cm} (11)

Substituting Equations (9) into (11) yields

$$\dot{\tilde{x}}_i(t) = (A + \Delta A)\tilde{x}_i(t) - (B + \Delta B)K \sum_{j=1}^{N} m_{ij} (\tilde{x}_j(t) - \tilde{x}_i(t)) - (B + \Delta B)K_r \tilde{x}_i(t - \eta) + (B + \Delta B)m_0(t), i \in \mathbb{N}. $$ \hspace{1cm} (12)

Let the leader control input be the interference imposed on it, that is, $u_0(t) = w(t)$. The tracking errors of all the followers in a vector form can be written as

$$\tilde{x}(t) = \begin{bmatrix} \tilde{x}_1(t) & \cdots & \tilde{x}_N(t) \end{bmatrix}^T.$$
and
\[
\Phi_j = \begin{bmatrix}
\Gamma_i & I_2 \\
\vdots & \ddots & \ddots \\
I_2 & \ddots & \Gamma_j
\end{bmatrix}, \quad q + 1 \leq i \leq q + r.
\]

Lemma 2. \cite{23}: M, H, \gamma and \gamma are matrices with appropriate dimensions, where \( M^T = M, F^T F \leq I \); then,
\[
M + HH^T + J^T F^T H < 0
\]
is satisfied, if and only if there exists \( \varepsilon > 0 \), such that
\[
M + \varepsilon HH^T + \varepsilon^{-1} J^T J < 0.
\]
In the following theorem, we present the condition guaranteeing that the platoon system is asymptotically stable while the system has no external disturbances.

**Theorem 1.** When \( w(t) = 0 \), if there exist symmetric positive definite matrices \( P, S \), such that
\[
\begin{bmatrix}
\varphi_1 & - (J_N \otimes P) \left[ R \otimes (B + \Delta B) \right] & \varphi_4 \\
* & - (J_N \otimes S) & \varepsilon
\end{bmatrix} \leq 0,
\]
(15)
with
\[
\varphi_1 = \text{He} \left\{ (J_N \otimes P) \left[ J_N \otimes (A + \Delta A) - \left[ L \otimes (B + \Delta B) K \right] \right] + J_N \otimes \left[ K^T S K \right] \right\}
\]
then there exists a controller in Equation (9) that can make the closed-loop dynamics model in Equation (13) asymptotically stable.

**Proof.** Consider the Lyapunov function as the following:
\[
V'(t) = \tilde{x}^T (t) (J_N \otimes P) \tilde{x}(t) + \int_{t-\eta}^{t} \tilde{x}^T (\tau) \left[ J_N \otimes (K^T S K) \right] \tilde{x}(\tau) \, d\tau.
\]
(16)
The derivative of \( V'(t) \) is
\[
\begin{align*}
V'(t) &= \left\{ J_N \otimes (A + \Delta A) \right\} \tilde{x}(t) - \left[ L \otimes (B + \Delta B) K \right] \tilde{x}(t) \\
&\quad - \left[ \left( R \otimes (B + \Delta B) K \right) \tilde{x}(t) - \left[ J_N \otimes P \right] \tilde{x}(t) \right]^T (J_N \otimes P) \tilde{x}(t) \\
&\quad + \tilde{x}^T (t) (J_N \otimes P) \left\{ J_N \otimes (A + \Delta A) \right\} \tilde{x}(t) \\
&\quad - \left[ L \otimes (B + \Delta B) K \right] \tilde{x}(t)
\end{align*}
\]
(17)
\[
\dot{V}(t) < 0 \text{ is satisfied when Equation (15) holds, and it indicates that the closed-loop model in Equation (13) is asymptotically stable. Therefore, Theorem 1 is proved.}
\]
The sufficient condition that the platoon system is asymptotically stable is given in Theorem 1. We now proceed to give a robust stability condition of the platoon when the external disturbances are present.

**Theorem 2.** When \( w(t) \neq 0 \), if there exist symmetric positive definite matrices \( P, S \), such that
\[
\begin{bmatrix}
\varphi_2 & \varphi_3 & \varphi_4 \\
* & - (J_N \otimes P) [R \otimes (B + \Delta B)] & 0 \\
* & * & - (J_N \otimes S)
\end{bmatrix} < 0,
\]
(18)
with
\[
\begin{align*}
\varphi_2 &= \text{He} \left\{ (J_N \otimes P) \left[ J_N \otimes (A + \Delta A) - \left[ L \otimes (B + \Delta B) K \right] \right] + J_N \otimes \left[ K^T S K \right] \right\} + I_N \otimes \left[ (1 - C^T C) \right] \\
\varphi_3 &= (J_N \otimes P) \left[ R \otimes (B + \Delta B) \right] \\
\varphi_4 &= - (J_N \otimes P) \left[ R \otimes (B + \Delta B) \right]
\end{align*}
\]
then there exists a controller in Equation (9) that makes the closed-loop model in Equation (13) robust \( H_{\infty} \) stable. That is, for a given scalar \( \gamma > 0 \), the following inequality is satisfied at zero initial condition \cite{24}:
\[
\|y\|_2 \leq \gamma \|w\|_2
\]
(19)

**Proof.** For all \( w(t) \neq 0 \), we have
\[
\begin{align*}
\int_{0}^{t} (\gamma^{-1} y \dot{y} - \gamma \dot{y} T w + V'(\tau)) \, d\tau &- V(t)
\end{align*}
\]
(20)
If for all eigenvalues \( \lambda_i, i = 1, ..., q + r \) of Laplacian matrix \( L \), there exist constants \( \xi_1 > 0, \xi_2 > 0, \xi_3 > 0 \), symmetric positive definite matrices \( X, Y \), matrix \( \tilde{Y} \), such that

\[
\dot{V}(t) + \gamma^{-1} y^T(t)y(t) - \gamma w^T(t)w(t)
\]

is positive definite matrices \( \Omega = \begin{bmatrix} \varphi_2 & \varphi_3 & \varphi_4 \\ * & -I_N \otimes \gamma \ I_3 & 0 \\ * & * & - (I_N \otimes \ S) \end{bmatrix} \)

When Equation (18) holds, we can obtain

\[
\int_0^t (\gamma^{-1} y^T - \gamma w^T w + \dot{V}(\tau)) \, d\tau < 0. 
\]

Equation (22) is equal to

\[
\int_0^t y^T \, y \, d\tau < \gamma^2 \int_0^t w^T \, w \, d\tau < \gamma^2 \int_0^\infty w^T \, w \, dt.
\]

Equation (23) holds for all \( t > 0 \). Therefore, the condition in Equation (19) is satisfied. This completes the proof.

Note that Equation (18) is a non-linear matrix inequality, which contains parameter uncertainty matrices. It is difficult to verify whether it holds for all allowed uncertainty matrices. In addition, Equation (18) is high dimensional, and its dimension increases with the number of platoon vehicles, which is not desirable for large-scale platoon vehicle systems. To deal with the above issues, this paper applies Lemma 1 to convert Equation (18) into some equivalent low-dimensional linear matrix inequalities, and then in the following theorem, we give the design method of the \( H_\infty \) controller.

**Theorem 3.** If for all eigenvalues \( \lambda_i, i = 1, ..., q + r \) of Laplacian matrix \( L \), there exist constants \( \xi_1 > 0, \xi_2 > 0, \xi_3 > 0 \), symmetric positive definite matrices \( X, Y \), matrix \( \tilde{Y} \), such that

\[
\begin{bmatrix}
\varphi_5 & X C^T & B & 0 & 0 \\
\ast & -I_N \otimes \gamma \ I_3 & 0 & 0 & 0 \\
\ast & * & -I_N \otimes \gamma \ I_3 & 0 & 0 \\
\ast & * & * & - (I_N \otimes \ S) & < 0
\end{bmatrix}
\]

with

\[
\varphi_5 = H_\infty (AX - \lambda_i BY) + \varepsilon_1 DD^T + \varepsilon_2 \lambda_i^2 DD^T + \varepsilon_3 BB^T,
\]

there exist a state-feedback \( H_\infty \) controller in Equation (9) so that the model in Equation (13) is robust \( H_\infty \) stable, and the controller gain is \( K = YX^{-1} \).

**Proof.** We can find a non-singular matrix \( \Psi \in \mathbb{R}^{N \times N} \) that satisfies \( \Psi^{-1} L \Psi = \Phi \) by using Lemma 1. Multiplying both sides of Equation (18) by diag[\( \Psi^{-1} \Psi^{-1} \Psi^{-1} \)] and diag[\( \Psi \Psi \Psi \)], respectively, yields

\[
\begin{bmatrix}
\varphi_7 & \varphi_5 & \varphi_4 \\
\ast & -I_N \otimes \gamma \ I_3 & 0 \\
\ast & * & - (I_N \otimes \ S) & < 0
\end{bmatrix}
\]
with
\[
\varphi_7 = \text{He} \left\{ (I_N \otimes \Phi) \left[ I_N \otimes (A + \Delta A) - \Phi \otimes ((B + \Delta B) K) \right] \right. \\
+ I_N \otimes (K^T SK) + I_N \otimes (\gamma^{-1}C^T C),
\]
where \( \Phi \) is the Jordan normal form of Laplacian matrix \( L \).

For all real eigenvalues \( \lambda_i, i = 1, \ldots, q \) of \( L \), Equation (26) holds only if
\[
\begin{bmatrix}
\varphi_8 & P(B + \Delta B) & -P(B + \Delta B) \\
* & -\gamma I & 0 \\
* & * & -S
\end{bmatrix} < 0,
\]
with
\[
\varphi_8 = \text{He} \left\{ P \left[ (A + \Delta A) - \lambda_i (B + \Delta B) K \right] \right. \\
+ K^T SK + \gamma^{-1}C^T C.
\]

Accordingly assumption 1 yields
\[
\begin{bmatrix}
\varphi_9 & PB & -PB \\
* & -\gamma I & \varepsilon_1^{-1} \\
* & * & -S
\end{bmatrix} + \begin{bmatrix}
PD \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
E_1 & E_2 & E_2 \\
F(\ell) & 0 & 0
\end{bmatrix} + \begin{bmatrix}
-\lambda_i PD \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
E_1 & E_2 & E_2 \\
F(\ell) & 0 & 0
\end{bmatrix}^T
\]
\[
+ \begin{bmatrix}
E_2K & 0 & 0
\end{bmatrix}^T \begin{bmatrix}
-\lambda_i PD \\
0 \\
0
\end{bmatrix}^T < 0,
\]
with
\[
\varphi_9 = \text{He} \left\{ P \left[ (A - \lambda_i PBK) \right] + K^T SK + \gamma^{-1}C^T C. \right. 
\]

Using Lemma 2, Equation (28) can be simplified as
\[
\begin{bmatrix}
\varphi_9 & PB & -PB \\
* & -\gamma I & \varepsilon_1^{-1} + \varepsilon_1 \\
* & * & -S
\end{bmatrix} \begin{bmatrix}
PD \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
PD \\
0 \\
0
\end{bmatrix}^T
\]
\[
+ \varepsilon_1^{-1} \begin{bmatrix}
E_1 & E_2 & E_2 \\
E_1 & E_2 & E_2
\end{bmatrix}^T \begin{bmatrix}
E_1 & E_2 & E_2 \\
E_1 & E_2 & E_2
\end{bmatrix} \\
+ \varepsilon_2 \begin{bmatrix}
-\lambda_i PD \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
-\lambda_i PD \\
0 \\
0
\end{bmatrix}^T < 0,
\]

with
\[
\varphi_9 = \text{He} \left\{ P \left[ (A - \lambda_i PBK) \right] + K^T SK + \gamma^{-1}C^T C. \right. 
\]

Applying the Schur complement lemma, Equation (29) is equivalent to
\[
\begin{bmatrix}
\varphi_{11} & C^T & PB & 0 & 0 & E_1^T \\
* & -\gamma I & 0 & 0 & 0 & 0 \\
* & * & -\gamma I & 0 & 0 & 0 \\
* & * & * & -S & 0 & 0 \\
* & * & * & * & -\varepsilon_1 I & 0 \\
* & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0,
\]

with
\[
\varphi_{11} = \text{He} \left\{ P \left[ (A - \lambda_i PBK) \right] + \varepsilon_1 PDD^T P + \varepsilon_2 \lambda_i^2 PDD^T P + \varepsilon_1 (E_2K)^T (E_2K) \right. 
\]

Multiplying both sides of Equation (30) by \( \text{diag}[P^T, I, I, S^{-1}, I, I, I] \) while letting \( X = P^{-1}, Y = K P^{-1}, T = S^{-1} \), we can get the matrix inequality in Equation (24).

For all complex eigenvalues \( \lambda_i, i = q + 1, \ldots, q + r \) of \( L \), the reasoning lines of Equation (25) is similar to that of Equation (24). Therefore, Theorem 3 is proved.

Theorem 3 gives the design method of state feedback \( H_\infty \) controller, which can realise the intelligent electric vehicle longitudinal platoon control even though there exist external disturbance, parameter uncertainties and communication delay.

\textbf{Remark 1.} We can observe from the models in Equations (8), (11) and (13) that the matrices \( A \) and \( B \) are low-dimensional matrices whose dimensionality is fixed while the dimensions of Laplacian matrix \( L \) and pinning matrix \( K \) vary with the number of vehicles in the platoon. The linear matrix inequalities in Theorem 3 does not contain the matrices \( L \) and \( K \). Therefore, the linear matrix inequalities given by Theorem 3 is low dimensional, and its dimensions will not increase with the count of vehicles.
In order to improve the platoon control performance, an adaptive term is added to the linear controller in Equation (9). The new controller is designed as

\[
\begin{bmatrix}
\varphi_{12} & X C^T & B & 0 & 0 & E_1 X & (E_2 Y)^T & (E_2 B K_2)^T & Y^T \\
* & -\gamma I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\gamma I & 0 & 0 & E_2^T & 0 & 0 \\
* & * & * & -T & T & (E_2 T)^T & 0 & 0 \\
* & * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\
* & * & * & * & * & -\varepsilon_i I & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_i I & 0 \\
* & * & * & * & * & * & * & -T \\
\end{bmatrix} < 0,
\]

Substituting the controller in Equations (31) into (11) gives a closed-loop dynamics model

\[
\begin{bmatrix}
\varphi_{13} & I_2 \otimes (X C^T) & I_2 \otimes B & 0 & 0 & I_2 \otimes (E_1 X)^T & I_2 \otimes (E_2 Y)^T & I_2 \otimes (E_2 B K_2)^T & I_2 \otimes Y^T \\
* & -I_2 \otimes (\gamma I) & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -I_2 \otimes (\gamma I) & 0 & 0 & I_2 \otimes E_2^T & 0 & 0 \\
* & * & * & -I_2 \otimes T & I_2 \otimes T & I_2 \otimes (E_2 T)^T & 0 & 0 \\
* & * & * & * & -I_2 \otimes (\varepsilon_i I) & 0 & 0 & 0 \\
* & * & * & * & * & -I_2 \otimes (\varepsilon_i I) & 0 & 0 \\
* & * & * & * & * & * & -I_2 \otimes (\varepsilon_i I) & 0 \\
* & * & * & * & * & * & * & -I_2 \otimes T \\
\end{bmatrix} < 0,
\]
Proof. Consider the Lyapunov function as the following:

\[
\dot{V}(t) = \ddot{x}^T(t) (J_N \otimes P) \ddot{x}(t) \\
+ \int_{t-\eta}^{t} \ddot{x}^T(s) K^T (J_N \otimes S) K \ddot{x}(s) \, ds + J_N \otimes K_2^T(t) K_2(t).
\]  

(37)

For all \( \eta(t) \neq 0 \), we have

\[
\dot{V}(t) + \gamma^{-1} \dot{y}(t)\dot{y}(t) - \gamma y^T(t)w(t)
\]

with

\[
\varphi_{15} = \text{He}\{ (J_N \otimes P) [J_N \otimes (A + \Delta A) - \Phi \otimes (B + \Delta B) K \\
+ I_N \otimes (B + \Delta B)B^T PK_2(t)] + K^T (J_N \otimes S) K \\
+ I_N \otimes \gamma^{-1}C^T C - I_N \otimes [2K_2(t) PBB^T P] \},
\]

(38)

According to Lemma 1, there exists a non-singular matrix \( \Psi \in \mathbb{R}^{N \times N} \) such that \( \Psi^{-1}L \Psi = \Phi \). Multiplying both sides of Equation (38) by \( \text{diag}[\Psi^{-1} \Psi^{-1} \Psi^{-1}] \) and \( \text{diag}[\Psi \Psi \Psi] \), respectively, yields

\[
\varphi_{15} = \text{He}\{ (J_N \otimes P) [J_N \otimes (A + \Delta A) - \Phi \otimes (B + \Delta B) K \\
+ I_N \otimes (B + \Delta B)B^T PK_2(t)] + K^T (J_N \otimes S) K \\
+ I_N \otimes \gamma^{-1}C^T C - I_N \otimes [2K_2(t) PBB^T P] \},
\]

(39)

The maximum of \( K_2(t) \) is set as \( \tilde{K}_2 \). Applying the Schur complement lemma, Equation (41) is equivalent to

\[
\varphi_{16} = \text{He}\{ [P (A + \Delta A) - \lambda \gamma I (B + \Delta B) K \\
+ (B + \Delta B)B^T PK_2(t)] + K^T (J_N \otimes S) K \\
- 2K_2(t) PBB^T P \},
\]

(41)

with

\[
\varphi_{17} = \text{He}\{ [P (A + \Delta A) - \lambda \gamma I (B + \Delta B) K \\
+ (B + \Delta B)B^T PK_2(t)] + K^T (J_N \otimes S) K \\
+ \gamma^{-1}C^T C - 2K_2(t) PBB^T P \},
\]

(42)
Multiply both sides of Equation (42) by \( \text{diag}[P^{-1} I I S^{-1} I I I I I] \) while \( X = P^{-1} \), \( Y = KP^{-1} \), \( T = S^{-1} \) give the matrix inequality in Equation (35).

For all complex eigenvalues \( \lambda_i \), \( i = q + 1, \ldots, q + r \) of \( L \), the reasoning lines of Equation (36) is similar to that of Equation (35). Therefore, Theorem 4 is proved.

Remark 2. Theorem 4 gives the solution method of the adaptive robust controller. The designed controller includes a constant gain and an adaptive gain terms. Compared with Theorem 3, the vehicle platoon controlled by the controller with an adaptive term in Theorem 4 has better performance. That is verified in the simulation in Section 5.

4 | STRING STABILITY

It is necessary to prove the platoon string stability to ensure that the vehicle transient spacing errors will not expand along the platoons. String stability is the main performance indicator of platoon control, which avoids the distance error between the vehicles gradually increasing backwards along the platoon because of the existence of disturbance. Moreover, satisfying the string stability has the potential of reducing traffic jams and rear-end collisions in the platoon.

Definition 1. If the transfer function between the distance error of the current vehicle and the error of the previous vehicle is less than 1 [25], that is,

\[
|G(\omega)| < 1, \forall \omega > 0,
\]

where \( G(\omega) \) is the frequency response of distance error transfer function \( G(i) = \frac{\delta_i(t)}{d_{i-1}(t)} \), then we say the platoon control system has string stability.

Consider the \( i \)-th follower vehicle under controller in Equation (9). With the spacing error vector \( \delta_i = p_{i-1} - p_i - d_{i-1} \), Equation (43) can be gain

\[
\ddot{\delta}_i = \ddot{u}_{i-1} - \dot{u}_i.
\]  
(43)

Substituting Equations (7) and (9) into (43) gives

\[
\tau \ddot{\delta}_i(t) = -\ddot{\delta}_i(t) - k_1 \ddot{\delta}_i(t) - k_2 \ddot{\delta}_i(t) - k_{1} \ddot{\delta}_i(t) \\
- k_1 \ddot{\delta}_i(t - \eta) - k_2 \ddot{\delta}_i(t - \eta) - k_3 \ddot{\delta}_i(t - \eta) \\
+ k_1 \ddot{\delta}_{i-1}(t) + k_2 \ddot{\delta}_{i-1}(t) + k_3 \ddot{\delta}_{i-1}(t).
\]  
(44)

We can derive the transfer function \( G_i(i) \) by taking the Laplace transform of Equation (44)

\[
G_i(i) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{k_1 + k_2 s + k_3 s^2}{\tau s^3 + \tau s^2 + (k_1 + k_2 s + k_3 s^2)(1 + e^\eta)}.
\]  
(45)

Substituting \( s = j \omega \) with \( \omega > 0 \) and Euler formula \( e^{j\omega} = \cos(\omega) + j \sin(\omega) \) into (45) yields

\[
|G(j\omega)| = \sqrt{\frac{a}{a + b}},
\]

where

\[
a = k_1^2 - 2k_1 k_2 \omega^2 + k_2^2 \omega^2 + k_3^2 \omega^4,
\]

\[
b = \tau k_2 \omega^6 + [-2k_1 \omega \sin(\eta) \omega^3] \omega^5 \\
+ \left(k_2^2 + 2k_3 - 2k_3 \omega + 1 + 2 (k_2^2 + k_3 - k_2 \tau) \cos(\eta) \omega^4 \\
+ 2 (k_2 \tau - k_2) \sin(\eta) \omega^5 \\
+ (k_2^2 - 2k_3 - 2k_3 \tau + 2 (k_2 - k_2 \tau - 2k_3 \tau) \cos(\eta) \omega^2 \\
+ k_3^2 + 2k_3^2 \cos(\eta) \omega).
\]

Note that \( a > 0 \). Then, we can get that \( |G(j\omega)| < 1 \) if \( b > 0 \).

Theorem 5. For the platoon spacing error, the transfer function \( G(i) \) satisfies \( |G(j\omega)| < 1 \) for any \( \omega > 0 \) if the communication delay \( \eta \) is less than the optimal solution of the following optimisation problem

\[
\max \eta \quad \text{s.t.} b > 0 \text{ for any } \omega > 0.
\]  
(47)

Proof. When \( \eta < \eta_{\text{max}} \), where \( \eta_{\text{max}} \) is the optimal solution of the optimisation problem in Equation (47), \( b > 0 \) is satisfied for any \( \omega > 0 \). So \( |G(j\omega)| < 1 \) for any \( \omega > 0 \). This completes the proof.

Theorem 5 gives the upper bound of communication delay, which satisfies the string stability. When the platoon delay is less than this upper bound, the vehicle transient spacing errors will not expand along platoons to avoid a collision.

5 | PERFORMANCE VERIFICATION

To validate the feasibility and effectiveness of the presented adaptive robust \( H_\infty \) control method (controller A), this section performs a series of simulation tests to compare the proposed control method with the robust \( H_\infty \) controller (controller B).

The vehicle platoons of the simulation consist of one leader and six followers. We assume that each vehicle can receive the information from its previous vehicle and the leader. Constant spacing strategy is utilised to formulate the platoon [26], with desired distance of 5 m. The initial distance error of adjacent vehicles, the initial speed error and the initial acceleration error of each vehicle relative to the leader vehicle are set as zero. The
leader vehicle speed is set as

\[
\tau_0 = \begin{cases} 
10 \text{m/s} & \leq t \leq 5 \text{s} \\
5 + t \text{m/s} & 5 \text{s} \leq t \leq 20 \text{s} \\
25 \text{m/s} & 20 \text{s} \leq t \leq 30 \text{s} \\
-5 + t \text{m/s} & 30 \text{s} \leq t \leq 40 \text{s} \\
35 \text{m/s} & 40 \text{s} \leq t \leq 50 \text{s} \\
85 - t \text{m/s} & 50 \text{s} \leq t \leq 70 \text{s} \\
15 \text{m/s} & 70 \text{s} \leq t \leq 100 \text{s}
\end{cases}
\]

In the simulations, we set the time constant \( \tau = 0.2 \). The other parameters used in the simulations are set as

\[
A = [0 \ 0 \ 1], \quad B = [0], \quad C = [0 \ 1 \ 0], \quad D = [0], \quad E_1 = [0 \ 0 \ 0 \ -5 \ 0 \ 5 \ 0 \ 0 \ 1 \ 0.5], \quad E_2 = 0.1, \quad F(t) = \sin(t).
\]

We can calculate that the string stability is satisfied as long as \( \eta < \eta_{\text{max}} = 0.1140 \) according to Theorem 5. So, without loss of generality, we set the communication delay of the first follower as 0.1 s, delay of 2–4 followers as \((0.09 + 0.01 \sin(t))\) s and delay of 5–6 followers as \((0.07 + 0.03 \sin(t))\) s.

By setting \( K_2 = 5 \), and solving the inequality in Equation (34) in Theorem 4 by using the LMI toolbox, we can calculate the controller gain of controller A:

\[
K = \begin{bmatrix} 5.7380 & 9.2839 & 3.0913 \end{bmatrix},
\]

\[
K_1(t) = \begin{bmatrix} 5.7380 & 9.2839 & 3.0913 \end{bmatrix} \tilde{x}(t).
\]

Similarly, we can get the gain of controller B by solving the inequality in Equation (23) in Theorem 3:

\[
K = \begin{bmatrix} 4.1521 & 6.7452 & 2.2956 \end{bmatrix}.
\]

The response results of distance error, speed error and acceleration of the follower vehicles are regulated through controller A and B, respectively, are shown in Figure 5. Apparently, the distance error and speed error both converge to zero under two controllers. It can be observed from Figures 5(a) and (b) that the absolute value of distance error is less than 0.13 m, and the distance error between the vehicles gradually reduces backwards along the platoon. Thus, the running of the platoon is safe and the collisions will not occur. In addition, the distance error under controller A presented in Figure 5(a) is smaller than that presented in Figure 5(b). Furthermore, as shown in Figures 5(c) and (d), the absolute maximum of speed error under controller A after 20 s is less than 0.05 m/s, which is half of the absolute maximum of speed error under controller B. These results show that the platoon under controller A has smaller distance and speed errors, and the control effect is better than that under controller B.

Moreover, if we focus on Figures 5(e) and (f) in the 20–23 s time period, the fluctuation of acceleration under controller B is nearly 0.1 m/s, while that under controller B is nearly 0.5 m/s. We can make out that controller A can more effectively suppress the overshoot and oscillation of the system state response, compared with controller B.

Therefore, the proposed controller in Equation (30) obtained by adding the adaptive term to the robust controller can significantly suppress the impact of parameter uncertainties and disturbances on the system so that the system can achieve the required control performance of distance and speed error tending to zero.

6 | CONCLUSION

A distributed adaptive robust platoon control scheme of intelligent-connected electric vehicles with parameter uncertainties and communication delay is proposed in this paper. This adaptive robust control scheme guarantees that the closed-loop control system is robust stable while also meeting the string stability. Through eigenvalue decomposition, the decoupling of the platoon system is realised so that the solution of the controller gain is transformed into multiple low-dimensional linear matrix inequality problems. Simulations show that the platoon system with communication delay, external disturbances and parameter uncertainties under the control of the presented method can converge to the equilibrium state within a limited period and has an excellent control performance.

One future work is to study the platoon control under the influence of other communication defects, such as communication interruption. In addition, since vehicles in actual traffic are heterogeneous, it is worth to study the heterogeneous platoon.

NOMENCLATURE

- \( d_i \): expected distance between adjacent vehicles
- \( \delta_p \): distance error
- \( \delta_v \): speed error
FIGURE 5  Simulation results (a) distance error of controller A, (b) distance error of controller B, (c) speed error of controller A, (d) speed error of controller B, (e) acceleration of controller A, (f) acceleration of controller B
\( \delta_p \) acceleration error
\( T_e \) desired driving torque
\( J_m \) rotational inertia of wheel and hub motor
\( \omega \) wheel speed
\( T_m \) motor torque
\( \mu \) mechanical efficiency of the transmission system
\( \tau \) first-order inertial element time constant
\( \eta \) communication delay
\( a \) vehicle acceleration
\( c \) aerodynamic drag coefficient
\( f \) resistance coefficient
\( g \) gravity constant
\( L \) vehicle length
\( m \) vehicle mass
\( p \) vehicle position
\( T \) driving torque
\( u \) control input
\( v \) vehicle speed

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