Secrecy Performance of Terahertz Wireless Links in Rain and Snow

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Abstract

Wireless communication technique at terahertz (THz) frequencies is regarded as the most potential candidate for future wireless networks due to its wider frequency bandwidth and higher data capacity when compared to that employing radio frequency (RF) and millimeter wave (mmWave). Besides, a THz link can achieve higher security at physical layer when it propagates in clear weather due to its higher directionality, which reduces the possibility of eavesdropping attacks. However, under adverse weather conditions (such as water fog, dust fog, rain and snow), the link degradation due to weather particles and gaseous molecules will affect the link secrecy performance seriously. In this work, we present theoretical investigations on physical layer security of a point-to-point THz link in rain and snow with a potential eavesdropper locating outside of the legitimate link path. Signal degradation due to rain/snow, gaseous attenuation and beam divergence are included in a theoretical model to estimate the link performance. Secrecy capacity of the link with carriers at 140, 220 and 340 GHz is calculated and compared. Insecure regions are also presented and the secrecy performance is analyzed. We find that the THz link suffers least eavesdropping attacks in rain and the maximum data transmission rate decreases for higher carrier frequencies in rain and snow.

Keywords: Terahertz wireless communications, physical layer security, rain, snow, Mie scattering theory, secrecy capacity, maximum data transmission rate
1. Introduction
The fifth-generation (5G) wireless networks, targeted IMT Vision for IMT 2020, promise to satisfy requirements for low latency data transmissions, enhanced system capacity, and ultra-reliable communications [1]. Peak download/upload data rates of 20/10 Gbps are the most specific requirement, which would not be achieved except the use of millimeter wave frequencies. It has been recognized that higher carrier frequencies with wider bandwidth is now under imperious demand to support a continuous improvement in wireless data capacity [2, 3]. So terahertz communication technique has become one of the promising candidates for the next generation of wireless networks due to its advantages of large data capacity and high physical layer security [3, 4] corresponding to wide bandwidth and high directivity [5]. It also provides a solution to achieve reliable data transmission in some adverse application scenarios, such as in rain, fog, snow, dust and atmospheric turbulence, when a wireless link at higher frequencies (such as infrared) is prohibited or restricted [6]. Ma et. al. demonstrated THz link performances in rain and snow [7-10], and observed scintillation effects caused by the statistical distribution of rain/snow particles.

Unlike wire communications, the broadcasting nature of wireless links could make it to be vulnerable to eavesdropping attacks. One alternative solution for that is employing carriers with higher beam directionality to improve security at physical layer [11]. A line-of-sight (LOS) link with carriers at THz frequencies is proved to be able to provide an additional layer of security when an eavesdropper appears between two legitimate peers [4] due to its inherent narrower beam width compared to the almost broadcast nature of RF signals [12], which leads to a high directionality and makes it much harder and even impossible to intercept the signals in clear weather. However, at the case of adverse weather conditions, due to the serious scattering effect by weather particles, it may be necessary to investigate the possibility of signal eavesdropping by a non-line-of-sight (NLOS) path scattered away the legitimate link path.

In this work, considering our previous researches on outdoor THz link performance [7-10], we propose a further study on the link secrecy performance in rain and snow, and find out the influencing mechanisms by scattering and inherent beam directionality. This has never been investigated before, but a comprehensive study on this topic is definitely required for the future applications of THz wireless communications.

2. Physical Characteristics of Rain and Snow
The description of link performance in rain and snow requires detailed physical properties including particle shape, dielectric constant, and raindrop/snowdrop size distributions, which have been demonstrated in several publications [8, 10, 13, 14]. In this section, we would just take a brief introduction on that and employ rainfall/snowfall rate instead of visibility to express the fall intensity because of the incorrect indication of actual fall intensity usually provided by visibility due to the variation in rain/snow type and the differences in the nature of visibility targets during day and night [15].

2.1 Physical Characteristics of Rain

Raindrops are usually formed due to the precipitation of water vapor in atmosphere and the forming mechanisms are different in warm and cold cloud due to the influence of different temperatures [16]. Researches in [17, 18] find that the shape of raindrops usually depends on the falling speed, and falling raindrops with a diameter larger than 2 mm become oblate spherical in shape due to the pressure difference it suffered between the top and bottom surfaces. However, most published theoretical models are under the approximation of spherical raindrops for simplicity, and can give a realistic scattering profile successfully [8].

Raindrop size distribution is the most important parameter in theoretical predictions, which is defined as the number concentration of raindrops with diameter \( D \) is in a given volume space. Many distributions, such as negative exponential [19, 20], gamma [21] and log-normal distribution [22], etc., have been demonstrated and employed. Joss [23] distribution was first obtained by measuring the raindrop size distribution using a distrometer in Switzerland. Marshall-Palmer model is a negative exponential function which reaches maximum at 0 mm raindrop diameter. So it overestimates the number of small droplets, which may no longer be applicable for rain in the terahertz band. In the log-normal distribution, a fewer number of small raindrops are included which makes it have more flexibility in representing different drop distributions for a given rainfall rate [16]. Weibull distribution was found to agree well with measured data acquired in Japan [24] and U.K. [25], which would be considered in our calculations.

The dielectric constant of raindrops can be calculated as pure water. Currently, the double-debye dielectric model (D3M) is the most accurate model for computing the dielectric constant of water [26]. It was first developed for sea water based on experimental measurements [27-30] and can also be reduced to pure water [31] when the water salinity (\( S \)) is set to zero.
2.2 Physical Characteristics of Snow

Unlike raindrops, snow particles, having complex and varied shapes, cannot be thought of simply as homogeneous spheres. They usually do not have any preferred dimensions, but different dimensions were found have almost identical influence on the absorption and scattering attenuation efficiencies [32]. Besides, an average ratio (the maximum horizontal dimension to the maximum vertical dimension) of near unity for falling snow particles was measured in [33]. Hence, a spherical approximation is usually assumed for the purpose of simplifying computations.

Snow particles are usually regarded as a mixture of ice, air and water. The dielectric constant of snow can therefore be related to the dielectric constant and volume fraction of each individual component. Dry snow can be treated as a mixture of ice and air, and its dielectric properties could be obtained by an empirical formula in [34]. Wet snow is a mixture of ice, air and free water. A two-phase Polder-Van Santen model with -ellipsoidal water inclusions is proposed to obtain the complex dielectric constant of wet snow and shows good agreement with measurements in [35]. A modified Debye-like model, with a higher accuracy, is more commonly used for frequencies above 15 GHz [36] when dielectric constants of water and ice are available.

In the Debye model [37], the real part of dielectric constant of pure ice is essentially independent of frequency till 1 THz and depends on temperature weakly [38], so it can be simplified to a constant as 3.1884. The imaginary part was calculated in [39] and can be extended to frequencies above 1 THz based on the far-infrared work in [40] by including a new term [41].

Snowdrop size distribution can be affected by various microphysical and dynamic processes inside and below cloud layers. In practical applications, empirical mathematical formulas derived from the observed size spectra have been used to approximate natural snow size distributions. Unlike raindrops following negative exponential [19, 20], Gamma [21] and Log-normal distribution [22], etc., snow size distribution is widely described by a negative exponential function, such as Marshall-Palmer [M-P], Sekhon-Srivastava [S-S] and Gunn-Marshall [G-M] distribution function. The first negative exponential distribution function was reported by Gunn and Marshall based on ground observations of snow [43] and an assessment method used for raindrop size distribution in [20]. Based on results in [42, 43] and parameters in [19], a modification to M-P model was developed by Scott [44] with actual snow particle size \( r_m \) used. Sekhon and Srivastava [S-S] demonstrated an updating [45] by analyzing the data set in [46] with additional snowflake size
distribution measurements.

3. Attenuation by Falling Rain and Snow

Mie scattering theory can be used to calculate the signal loss due to absorption and scattering by rain/snow particles with known complex refractive index when the ordinary size of particles in the range from mm to cm [47], which is usually comparable to or larger than the THz wavelength. For very small particles, solution also given by the Mie theory approaches the Rayleigh approximation. This method systematically describes the scattering mechanism of THz waves by particles of various sizes in the atmosphere. Under the assumption of single scattering and scatter independence, the attenuation suffered by a THz wave traveling along a path in rain/snow can be obtained [48] by

\[
\alpha = 4.343 \cdot 10^3 \int_0^\infty \sigma_{\text{ext}} (m, r, \lambda) N(r) \, dr,
\]

where \( m \) is the refractive index of the particle and \( N(r) \) is the rain/snow size distribution we mentioned before. \( \sigma_{\text{ext}} \) is extinction cross section. The following relationship exists between the absorption cross-section \( \sigma_{\text{abs}} (m, r, \lambda) \) and the scattering cross-section \( \sigma_{\text{sca}} (m, r, \lambda) \) as \( \sigma_{\text{ext}} (m, r, \lambda) = \sigma_{\text{abs}} (m, r, \lambda) + \sigma_{\text{sca}} (m, r, \lambda) \), because the signal attenuation by rain/snow is mainly due to absorption and scattering effects.

The transmission of THz waves in rain/snow also suffers attenuation due to gaseous molecules in atmosphere. Because of the small volume of gaseous molecules, the scattering effect could be neglected over a short distance, but the absorption effect is obvious and should always be considered. There are two main absorption components in atmosphere which affects the propagation of THz waves significantly - oxygen and water vapor. Oxygen plays a major role in the absorption of terahertz waves below 100 GHz. However, when the frequency is greater than 100 GHz, the absorption spectrum of oxygen is submerged in the continuous absorption and spectral absorption of water vapor, and the transmission attenuation of terahertz waves is mainly caused by water vapor. Here, we would employ a theoretical model provided by the ITU Recommendation Sector (ITU-R) [49] based on the physical model MPM93 [50]. This method is proved to be valid at frequencies below 500 GHz [51].

3.1 Attenuation by Falling Rain

The attenuation of THz waves caused by falling rain is considered in the near-surface atmosphere, which is difficult to analyze due to the physical nature of the rain particles, as there is a lot of weather dependent variation in type, shape, dielectric constants and size distribution [52]. In addition
to Mie scattering, there is another way to predict attenuation of THz waves in rain below 1 THz [53]. The specific attenuation $\alpha_{\text{rain}} = a R^b$ (dB/km) is proposed by ITU-R with parameters $a$ and $b$ being frequency-dependent [53]. Although ITU model gives relatively good average results, the actual attenuation depending on the actual raindrop size distribution can produce differences. So we would employ the Mie scattering theory to calculate the attenuation under different raindrop size distribution models, which has been used in [54] to estimate THz spectral attenuation and the result is in good agreement with measured data.

The spectral attenuation by rain is shown in Fig. 1 with several commonly used raindrop size distribution models (M-P, JOSS and Weibull) considered. We set temperature and pressure at 25°C and 1013 hPa, respectively. Relative humidity (RH) in the air is set at 97%. The total attenuation combining Mie scattering theory and gaseous attenuation is calculated. We can see that the total attenuation in rain increases at higher frequencies due to large gaseous absorption. When no gaseous attenuation included, this curve should keep at the same level for frequencies above 100 GHz as measured in [8]. We could also observe that the signal loss due to scattering is much smaller than that due to absorptiption because of much water contained in rain. The scattering loss does not change for frequencies above 100 GHz, which makes the difference between losses due to absorption and scattering increases significantly with the increasing of carrier frequencies. When we compare the signal degradation under three different raindrop size distributions, same evolution in total attenuation, absorption and scattering could be observed, which indicates that the THz link under such rain conditions would own identical or almost identical link performance.
Attenuation by rain under a rainfall rate of 10 mm/hr (heavy rain [25]) when (a) M-P, (b) Joss and (c) Weibull distributions are employed. ($T = 25^\circ C$, $P = 1013$ hPa and RH = 97%)

3.2 Attenuation by Falling Snow

The total signal attenuation by dry and wet snow is calculated by employing Mie scattering theory with G-M and S-S snowdrop size distributions considered. Detailed distribution parameter definition should be found in [10]. Atmospheric pressure is set at 1013hPa, relative humidity (RH) in the air is 97% and snowfall rate is 10 mm/hr. For dry snow in Fig. 2(a) and Fig. 2(c) when temperature $T = -1^\circ C$, the attenuation by scattering (red curve) on THz waves is close to the total attenuation (black curve, consisting of absorption and scattering) at lower frequencies because there is almost no absorption by dry snow particles [10] with a dielectric constant of 3.1884 over the frequency range. The total attenuation increases significantly at higher carrier frequencies due to the large gaseous absorption it suffered. The scattering loss also increases, but it becomes smaller than the absorption loss and the difference between both becomes wider.

In the calculation of wet snow as shown in Fig. 2(b) and (d), the parameter settings are identical to that in Fig. 2(a) and (c) except a 0$^\circ$C temperature. We can see that the large water contained in wet
snow makes the absorption loss always higher than the scattering loss, which would reduce the contribution of scattering to the total attenuation. Like that in rain, the scattering loss in wet snow keeps almost identical for frequency from 100 GHz to 500 GHz. This means the scattered power by snow would not be frequency-dependent in that frequency range.

**Figure 2** Total attenuation (black), absorption (blue) and scattering (red) by dry snow at -1°C under (a) G-M and (c) S-S distributions, and by wet snow (wetness 25%) at 0°C under (b) G-M and (d) S-S distributions with a snowfall rate (equivalent rainfall rate) of 10 mm/hr. (P=1013hPa, RH=97%)

### 4. Link Secrecy Performance Analysis

In this section, we assume there is a point-to-point outdoor THz link with the configuration as shown in Fig.3 (a). A transmitter (Alice) sends information to a legitimate receiver (Bob) by a LOS link which suffers absorption and scattering effects in rain/snow. Meanwhile, an eavesdropper (Eve) exists outside but near the legitimate link path and aims to capture the information through a NLOS channel scattered away from the legitimate (LOS) path. The link distance $d$ is set at $d=1$ km in all the calculations and the positions of Alice and Bob are always fixed, while Eve can change its position and adjust its pointing direction to obtain optimal received signal.
In the presence of rain/snow, the LOS link suffers atmospheric attenuation $G_A$ and divergence attenuation $G_D = 4A/(\pi d^2 \alpha_A^2)$ with $\alpha_A$ as the full divergence angle of Alice and $A$ as the effective receiving area of Bob. The atmospheric attenuation including attenuation only by rain/snow (with coefficient as $\alpha_t$) and gaseous attenuation by air (with coefficient as $\alpha_g$) can be obtained from the previous section. So the atmospheric attenuation coefficient is $\alpha_{atm} = \alpha_t + \alpha_g$ and the atmospheric attenuation can be obtained by $G_A = \exp(-\alpha_{atm}d)$.

Combining both atmospheric attenuation and divergence attenuation, we can get the total LOS link gain as

$$G_{LOS} = G_A G_D = \frac{4Ae^{-\alpha_{atm}d}}{\pi d^2 \alpha_A^2}.$$ (2)

To obtain the link gain of the eavesdropping (NLOS) link achieved by scattering, a widely used model, called single-scattering model [55, 56], is employed. In the link geometry shown in Fig. 1(b), we set Alice at the origin of the coordinate $(0, 0)$, Bob at $(d, 0)$ and Eve at $(x, y)$ with $x$ and $y$ as variables. The signal is transmitted along the positive direction of the $x$-axis between Alice and Bob. The NLOS link gain $G_{NLOS}$ can be obtained as in [57] by

$$G_{NLOS} = \int_{L_a}^{L_b} \Omega(l) p(\mu) \alpha_{atm} e^{-\alpha_{ang}(x-l)^2+y^2} dl,$$ (3)

with integral variable $l$ as the signal transmission distance before scattering occurs. The lower bound $L_a$ and upper bound $L_b$ can be expressed as

$$L_b = \min \left\{ \max \left\{ x - \frac{y}{\tan(\alpha - \beta/2)}, 0 \right\}, d \right\}.$$ (4)

and
\[ L_c = \min \left\{ \max \left\{ x - \frac{y}{\tan(\alpha + \beta/2)}, 0 \right\}, d \right\}, \tag{5} \]

which divides the scattering region. Here, \( \alpha \) is the angle between the pointing direction of Eve and the positive \( x \) axis. \( \beta \) is the field-of-view (FOV) full angle of Eve. We assume that Eve tries to optimize the pointing direction to maximize the link gain of the link gain of the NLOS channel while its position and \( \beta \) are fixed. So the optimal scattering angle, corresponding to maximum NLOS link gain, can be expressed as \( \alpha^* = \arctan \left( \frac{y}{x} \right) + \beta/2 \) for \( \alpha < \pi/2 - \beta/2 \) \cite{58}. \( \Omega(l) \) denotes the solid angle from receiving area to the scattering point as

\[ \Omega(l) = \frac{A}{\left[ (x-l)^2 + y^2 \right]^{3/2}} \frac{(x-l) + y \tan \alpha}{\sqrt{1 + \tan^2 \alpha}}. \tag{6} \]

The phase function \( p(\mu) \) can be expressed as

\[ p(\mu) = \frac{1 - g^2}{4\pi} \left[ \frac{1}{\left( 1 + g^2 - 2g\mu \right)^{3/2}} + f \frac{3\mu^2 - 1}{2\left( 1 + g^2 \right)^{3/2}} \right], \tag{7} \]

where generalized Henyey-Greenstein function is adopted, and it is defined as scattering phase function indicating the probability distribution of scattering angle. \( \mu = (x-l)/\sqrt{(x-l)^2 + y^2} \) represents the cosine of scattering angle in \((l, 0)\) with \( g \) as an asymmetry factor related to wavelength, scattering particle radius and refractive index \cite{59}.

Secrecy capacity is defined as the maximum data rate from Alice to Bob when perfect secrecy performance is maintained \cite{60} and can be expressed as

\[ C_s = \left[ I(X;Y) - I(X;Z) \right]^+, \tag{8} \]

with \( \left[ x^+ \right] = \max \{ 0, x \} \) indicating that the secrecy capacity can never be less than 0. Parameters \( X, Y \) and \( Z \) represents the signals of the Alice, Bob and Eve, respectively. \( I(X;Y) \) and \( I(X;Z) \) denote the mutual information of LOS and NLOS links \cite{61}, respectively, and can be expressed as

\[ I(X;Y) = q(\lambda_L + \lambda_B) \log(\lambda_L + \lambda_B) + \lambda_B \log(\lambda_B) - (q\lambda_L + \lambda_B) \log(q\lambda_L + \lambda_B) \]

\[ I(X;Z) = q(\lambda_N + \lambda_B) \log(\lambda_N + \lambda_B) + \lambda_B \log(\lambda_B) - (q\lambda_N + \lambda_B) \log(q\lambda_N + \lambda_B). \]

In Eqs.(9) and (10), an on-off keying modulation format with a duty cycle \( q \) and a Poisson distribution, which is a common assumption for stochastic links, are assumed. \( \lambda_L = \tau \eta G_{\text{LOS}} P/E_p \)
and $\lambda_N = \tau \eta G_{\text{NLOS}} P / E_p$ represent the mean numbers of detected photoelectrons of signal component in each slot for the LOS and NLOS links, respectively. $P$ is the output power from Alice and $\eta$ is the receiver efficiency, which are identical for Bob and Eve and are always set at $P = 1$ W and $\eta = 0.1$ in the calculations. $E_p$ is the energy of one THz photon and $\tau$ is integration time of the receivers of Bob and Eve. $\lambda_b$ represent the mean number of detected photoelectrons of background radiation component in each slot. In experimental measurement, the THz radiation is converted to direct current (DC) electrical power using a rectifying diode and an antenna, which is based on the conversion of photoelectron to current. So here, we take the photoelectron in consideration in our calculation and the signal to noise ratio (SNR) of receiver for LOS channel can be obtained by dividing $\lambda_L$ by $\lambda_b$.

### 4.1 Secrecy Performance in Rain

In the calculation, outdoor links with carriers at 140, 220 and 340 GHz are considered (which has been achieved in [62-65]) and assumed to be collimated with the same beam width. The FOV angles of Eve for both links are set to be 15° and the receiving areas for Bob and Eve are all $1\text{cm}^2$, which are all chosen in the following calculations. To calculate the atmosphere (gaseous) attenuation, we set temperature $T = 25^\circ\text{C}$, pressure $P = 1013$ hPa, and relative humidity $\text{RH} = 97\%$. When Eve is set at a position of $(500\text{m, 10m})$, the link gain is calculated and shown in Fig. 4. Solid curves represent the evolution of link gain $G_{\text{LOS}}$ for the LOS link as a function of rainfall rate. It decreases as the increasing of rainfall rate due to the increasing of attenuation by rain. With the increasing of carrier frequencies, the LOS link gain decreases due to the higher atmospheric attenuation as indicated in Fig. 1. The dashed lines represent the variation of NLOS link gain $G_{\text{NLOS}}$. When at 140 GHz, it is smaller than the $G_{\text{LOS}}$ at small rainfall rate corresponding to small scattering effect. However, when the rainfall rate increases to 27 mm/hr, both link gain $G_{\text{LOS}}$ and $G_{\text{NLOS}}$ intersect ($G_{\text{LOS}} = G_{\text{NLOS}}$). And the $G_{\text{NLOS}}$ becomes larger than the $G_{\text{LOS}}$ at higher rainfall rate $Rr > 27$ mm/hr due to the larger signal attenuation suffered by the LOS link. For the carriers at 220 GHz, the links gains are smaller than that at 140 GHz and the intersection can be achieved at 4 mm/hr rainfall rate, which indicates a reduced link security because of the larger atmospheric attenuation and scattering effect. For the carrier at a higher frequency (340 GHz), the link gain for LOS and NLOS links at 340 GHz is much smaller than -200 dB and cannot be shown in the plot. So here, we can say that the link at lower carrier frequencies is less vulnerable to attacks in rain when an eavesdropper is positioned at
Figure 4 LOS (solid) and NLOS (dashed) link gain with respect to rainfall rate with carriers at 140GHz (black), 220GHz (blue) with Eve at a position of (100m, 10m).

The secrecy capacity distributions with respect to arbitrary positions of Eve are calculated and shown in Fig. 5 when the rainfall rate $R_r = 10$ mm/hr. The color bar on the right denotes the safe transmission rate $C_s$ in Gbps, which increases gradually as the color changes from blue to yellow. In Fig. 5(a) for the 140 GHz link, the dark blue region represents $C_s = 0$, which means the secure transmission cannot be guaranteed if Eve is located in this region. We would call it as insure region.

For the 220 GHz link in Fig. 5(b), the area of insecure region becomes smaller due to the much higher signal attenuation suffered by the LOS link in rain, which reduces the signal-to-noise-ratio (SNR) and further lower possibility of eavesdropping attacks. This has been demonstrated in [66] for satellite networks. The degradation of SNR could also lead to the decreasing of the maximum safe data transmission rate to 90 Gbps, which is lower than that at 140 GHz (around 145 Gbps) in Fig. 5(a). When we set the carrier frequency to 340 GHz as in Fig. 5(c), the insecure region is further reduced and the maximum safe data transmission rate becomes 60 Gbps.
Figure 5 Secrecy capacity distribution for 2-D positions of Eve when (a) 140 GHz link, (b) 220 GHz link, and (c) 340 GHz link with rainfall rate $R_r = 10$ mm/hr (The color bar denotes the safe transmission rate in Gbps).

4.2 Secrecy Performance in Dry Snow

With the same parameter settings as in Fig. 4, the link gain for LOS and NLOS links in dry snow is calculated and shown in Fig. 6 with Eve set at a position of (500m, 10m), temperature $T= 0^\circ$C,
pressure $P=1013\text{hPa}$ and relative humidity RH= 97%. We choose G-M model as snowdrop size distribution. Solid and dashed links represent evolution of link gain $G_{\text{LOS}}$ and $G_{\text{NLOS}}$, respectively, which are higher than that in rain due to the smaller signal loss in dry snow. Similar to the trend in Fig. 4, the LOS link gain $G_{\text{LOS}}$ decreases as the increasing of snowfall rate due to the increasing of attenuation by snow, but there is no intersection between $G_{\text{LOS}}$ and $G_{\text{NLOS}}$ when carrier at 140 GHz, which means the link is always secure over the snowfall rate range of 0-50 mm/hr. However, the intersection (where, $G_{\text{LOS}} = G_{\text{NLOS}}$) appears when $R_r = 50$ mm/h for the 220 GHz link and $R_r = 10$ mm/hr for the 340 GHz link due to the more serious link degradation suffered by higher carrier frequencies. This is consistent with the results in rain.

![Figure 6](image.png)

**Figure 6** LOS(solid) and NLOS (dashed) link gain with respect to snowfall rate with carriers at 140GHz (black), 220GHz (blue), 340GHz (red) with Eve at a position of (100m, 10m).

Fig. 7 shows the secrecy capacity distributions with respect to arbitrary positions of Eve in dry snow when the snowfall rate (equivalent rainfall rate) $R_r = 10$ mm/hr. In Fig. 7(a) for the 140 GHz link, the dark blue (insecure) region is much larger than that in Fig. 5(a), even though the maximum safe data transmission rate is still at 145 Gbps. We attribute this to the larger snow particle density under the same fall rate. The snowfall rate is usually represented by equivalent rainfall rate as in [10], so, under the same fall rate of 10 mm/hr, there should be much more dry snow particles than rain particles dropping down, which would lead to more serious scattering effect in dry snow. This can explain why a worse secrecy in dry snow is observed, even though the link signal loss is less as the comparison between Figs. 1 and 2. When the carrier frequency is set at 220 GHz in Fig. 7(b), the insecure region increases and the maximum safe data transmission rate is reduced to 90 Gbps. This is
due to the more scattered power from the 220 GHz link than the 140 GHz link which is confirmed in Fig 2(a) and demonstrated in [67]. However, when we increase the carrier frequency to 340 GHz, the area of insecure region is reduced because of the increasing of signal absorption on the LOS link, which leads to a much lower SNR at Bob.

Figure 7 Secrecy capacity distribution for 2-D positions of Eve when (a) 140 GHz link, (b) 220 GHz link and (d) 340 GHz link with snowfall rate $Rr = 10$ mm/hr (The color bar denotes the safe transmission rates in Gbps).
4.2 Secrecy Performance in Wet Snow

In wet snow with temperature $T=0^\circ$C, pressure $P=1013$ hPa and relative humidity $RH=97\%$, the link gains for Bob and Eve are calculated and shown in Fig. 8 when the G-M raindrop size distribution model is considered again. For carriers at 140 GHz, the LOS link gain $G_{LOS}$ is always higher than the NLOS link gain $G_{NLOS}$ with the snowfall rate up to 50 mm/hr. This means the link at 140 GHz is secure when an eavesdropper set at (500m, 10m). However, this secrecy is decreased when the carrier frequency is changed to 220 GHz, where an intersection ($G_{LOS} = G_{NLOS}$) appears with snowfall rate at $R_r = 41$ mm/hr, which indicates that the link security attack comes when snowfall rate $R_r > 41$ mm/hr. The intersection point for the 340 GHz link comes at an even smaller snowfall rate $R_r = 16$ mm/hr and the link secrecy is much worse.

![Figure 8 LOS (solid) and NLOS (dashed) link gain with respect to snowfall rate with carriers at 140GHz (black), 220GHz (blue) and 340GHz (red) with Eve positioned at (100m, 10m).](image)

When we set the snowfall rate $R_r = 10$ mm/hr, the secrecy capacity distributions with respect to arbitrary positions of Eve in wet snow are calculated and shown in Fig. 9. For the 140 GHz link, the insecure region is smaller than that in dry snow in Fig. 7(a) under the same snowfall rate due to less scattering it suffered in wet snow. We also attribute this to the more snow particles in dry snow than in wet snow (where more water contained inside) when under the same snowfall rate. This phenomenon is also observed for the 220 GHz and 340 GHz links as in Fig. 9(b) and (c), which indicates that the link secrecy performance in wet snow is better than that in dry snow. Besides, with the increasing of carrier frequency from 140 GHz to 220 and 340 GHz, the insecure region decreases
but the maximum safe data transmission rate decreases from 145 Gbps to 90 Gbps and 60 Gbps, respectively.

![Figure 9: Secrecy capacity distribution for 2-D positions of Eve when (a) 140 GHz link, (b) 220 GHz link and (c) 340 GHz link with snowfall rate $R_r = 10$ mm/hr (The color bar denotes the safe transmission rates in Gbps).](image)

5. Conclusions
In this work, we investigate the secrecy performance of a LOS THz link in rain and snow when an eavesdropper locates outside of the legitimate link path and tries to collect and decode the scattered signal. A theoretical model combining Mie scattering theory, stochastic models and gaseous attenuation is proposed to estimate the link gain (of legitimate and eavesdropping links) and secrecy performance. The calculation results show that the absorption loss in rain is much larger than the scattering loss, while the difference between both is much smaller in wet snow and the scattering loss is even larger and contributes more to the total attenuation in dry snow. The link secrecy capacity distribution in rain and snow is plotted and insecure region is obtained. A smallest insecure region is observed in rain which indicates the best secrecy performance, while in dry snow, the serious scattering effect leads to more eavesdropping attacks. The maximum safe data transmission rate in rain and snow decreases with the increasing of carrier frequencies because of larger link signal loss exerted on the legitimate link.

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Disclosures
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