Effect of reheating on electroweak baryogenesis

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The latent heat released during the expansion of bubbles in the electroweak phase transition reheats the plasma and causes the bubble growth to slow down. This decrease of the bubble wall velocity affects the result of electroweak baryogenesis. Since the efficiency of baryogenesis peaks for a wall velocity $v_w \sim 10^{-2}$, the resulting baryon asymmetry can either be enhanced or suppressed, depending on the initial value of the wall velocity. We calculate the evolution of the phase transition taking into account the release of latent heat. We find that, although in the SM the baryon production is enhanced by this effect, in the MSSM it causes a suppression to the final baryon asymmetry.

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Generating the baryon asymmetry of the universe (BAU) at the electroweak phase transition is a very attractive idea. Although in the minimal Standard Model (SM) it fails to explain the observed baryon abundance, such a proposal can be successful in an extension of the model. It has been shown, for instance, that electroweak baryogenesis is quantitatively possible in the Minimal Supersymmetric Standard Model (MSSM), provided that the Higgs boson and the lightest stop are sufficiently light.

According to the standard mechanism, baryogenesis occurs near the walls of expanding bubbles that form during the phase transition. After a bubble is nucleated and begins to grow, its wall quickly reaches a terminal velocity in the hot plasma. Due to $CP$ violating interactions of particles with the walls, different densities of left-handed quarks and their antiparticles are built up in front of the walls. This left-handed asymmetry biases the anomalous baryon number violating sphaleron interactions in the symmetric phase. As a consequence, a net baryon asymmetry is generated in front of the walls and immediately caught by them. In the broken symmetry region inside the bubbles, sphaleron processes are suppressed, so a subsequent washout of the baryon asymmetry is avoided.

The generated BAU has a strong dependence on the bubble wall velocity. If the latter is too large, the left-handed density perturbation will pass so quickly through a given point in space that sphaleron processes will not have enough time to produce baryons. Thus the resulting BAU will be small. On the other hand, for very small velocities thermal equilibrium will be restored, and the baryon asymmetry will be erased by sphalerons and the BAU will be small again. Consequently, the baryon production has a maximum at an intermediate wall velocity. Comparison of the baryon number violation time scale with the time of passage of the chiral asymmetry gives a wall velocity $v_w \sim 10^{-2}$ for maximum baryon asymmetry. Recent numerical calculations confirm that the BAU tends to peak for such a small value of $v_w$.

On the other hand, recent calculations of the friction of the plasma indicate that the wall velocity can be of that order of magnitude. However, these estimates of $v_w$ do not take into account the hydrodynamics of the phase transition. For such small velocities the only effect of hydrodynamics is a homogeneous reheating of the plasma during the expansion of bubbles. As $v_w$ is much less than the speed of sound in the plasma, the latent heat liberated in the expansion of a bubble is quickly distributed throughout space by a shock front that precedes the propagating wall of the bubble. As a consequence of this uniform reheating the bubble expansion slows down. It was shown by Heckler that the decrease of $v_w$ can dramatically affect the result of electroweak baryogenesis. The importance of reheating can be estimated by comparing the latent heat $L \equiv T (d (\Delta V) / dT)_{T=T_c}$, where $\Delta V (T)$ is the free energy difference between the symmetric and broken symmetry phases, with the energy needed to bring the plasma back to the critical temperature $T_c$ from the temperature $T_0$, at which nucleation of bubbles begins, $\Delta \rho = (\pi^2 g_*/30) (T_c^4 - T_0^4)$, where $g_* \simeq 107$ is the number of degrees of freedom of the plasma. In the SM, $L$ is at least one order of magnitude less than $\Delta \rho$, but in the MSSM the two quantities are of the same order, so the effect of reheating can be important in that case.

In this paper we compute the evolution of the phase transition including this effect in a simple model with a one-Higgs effective potential. Using an analytical approximation for the dependence of the BAU on $v_w$, we evaluate the effect of the decrease of the bubble wall velocity on baryogenesis. It is known that in the SM the reheating enhances electroweak baryogenesis. On the contrary, since in the MSSM the wall velocity is initially in the range of maximum BAU, its decrease will cause a suppression to the baryon asymmetry.

We use an effective potential of the form

$$V (\phi, T) = D \left( T^2 - T_0^2 \right) \phi^2 - E T \phi^3 + \frac{\lambda}{4} \phi^4 ,$$

(1)
which possesses a first-order phase transition between the critical temperature \( T_c = T_0 / \sqrt{1 - E^2 / \lambda D} \) and \( T_0 \). When the universe cools below \( T_c \approx 100 GeV \), bubbles of the broken symmetry phase begin to nucleate with a probability per unit volume and time \( \Gamma \approx T^4 e^{-S_3(T)/T} \), where \( S_3[\phi (r)] \) is a three-dimensional instanton action that coincides with the energy of the nucleated bubble \( S_3 \). The configuration \( \phi (r) \) of the bubble is obtained by finding an extremum of \( S_3 \). It can be calculated numerically to obtain the nucleation rate \( \Gamma \) and the radius \( r_0 \) and wall width \( l_w \) of the nucleated bubble \( S_3 \) as functions of temperature.

For the terminal velocity of the bubble wall in the plasma we use the formula \( \frac{\Delta V(T)}{\eta v(T)} \), (2)

\[
v_w(T) \approx \frac{20 T l_w(T) \Delta V(T)}{\eta v(T)},
\]

where \( \eta \) is a dimensionless friction coefficient accounting for the viscosity of the plasma, and \( v(T) \) is the minimum of \( V(\phi, T) \). The main effect of the reheating will be to decrease the free energy difference \( \Delta V(T) \) \( \eta \) and hence the value of \( v_w \). The progress of the phase transition is determined by the fraction of space that is still in the symmetric phase \( S_3 \),

\[
f(t) = \exp \left\{ -\frac{4 \pi}{3} \int_{t_c}^{t} \Gamma(T') r(t', t)^3 dt' \right\}, \tag{3}
\]

where \( r(t', t) = r_0 (T') + \int_{t_c}^{t} v_w(T') dt' \) is the radius at time \( t \) of a bubble created at \( t' \). The variation of temperature with time is given by

\[
\frac{dT}{dt} = \frac{TV'(T) - V(T) \eta v(T) dt}{(2\pi^2 g_*/15) T^3} - \left( \frac{8\pi^3 g_*}{90M_{Pl}^2} \right)^{1/2} T^3, \tag{4}
\]

where \( M_{Pl} = 1.22 \times 10^{19} GeV \) is the Plank mass. The first term is the contribution of reheating. It describes the increase of energy density of the plasma due to the release of latent heat during the phase transition. The second term is just \( -HT \); it accounts for the decrease of energy density caused by the expansion of the universe. Before and after the phase transition \( df/dt = 0 \), and Eq. (4) gives the well known relation \( t = \frac{\xi M_{Pl}/T^2}{\xi} \), with \( \xi \approx 0.03 \). During the phase transition, the coupled integro-differential equations \( \frac{dV}{dt} \) must be solved numerically.

To evaluate the effect of a changing wall velocity on baryogenesis we need to know how the baryon production depends on \( v_w \). As the bubble wall sweeps through space, it leaves behind a baryon density \( \Gamma \)

\[
n_B = \frac{9\Gamma_{ws}}{v_w} \int_0^\infty n_L e^{-\Gamma_{ws} x/v_w} dx, \tag{5}
\]

where \( \Gamma_{ws} \approx 200 \xi T \) \( \xi \) is the weak sphaleron rate, and \( n_L \) is the net left handed density in front of the wall. The exponential accounts for sphaleron relaxation of the baryon asymmetry for small velocities. The coefficient \( \xi \) depends on the quark spectrum and is \( \approx 10 \).

The left-handed density can be assumed to be of the form \( n_L(x) = A e^{-x/v_w} / D \), where \( D \approx 100 / T \) is an effective diffusion constant for the chiral asymmetry \( \xi \). The constant \( A \) depends on the CP violating force at the bubble wall that sources the asymmetry. It is in general proportional to the wall velocity, \( A \propto v_w \), so integration of Eq. (3) yields

\[
n_B = \frac{C}{v_w + c \Gamma_{ws} D/v_w}, \tag{6}
\]

where \( C \) does not depend on \( v_w \). This analytic approximation describes qualitatively the dependence of \( n_B \) on \( v_w \). It has a peak at \( v_{peak} \equiv \sqrt{c \Gamma_{ws} D} \approx 0.02 \). A calculation of the coefficient \( C \) is outside of the scope of this paper, since we will only be interested in relative values \( n_B(v_w)/n_B(v_0) \). Note that if the initial wall velocity is \( v_0 \gg v_{peak} \), then a decrease of \( v_w \) will produce an enhancement of the BAU, since in that case \( n_B \sim v_w^{-1} \). On the contrary, for small velocities \( n_B \approx v_w \), so a velocity decrease will cause a suppression of the final BAU.

As \( v_w \) varies during the phase transition, \( n_B(v_w(t)) \) gives only a local baryon density, generated in a volume \( dV(t) = -V_{total} f(t) dt \). The final baryon density is the average over the expansion of bubbles \( B = \frac{1}{V_{total}} \int n_B(v_w) dV \). According to Eq. (3), this is related to the result obtained with a constant wall velocity \( v_0 \) by \( B = S n_B(v_0) \), where the factor

\[
S = -\int \frac{df}{dt} \frac{v_0 + v_{peak}^2/v_0}{v_w(t)} dt. \tag{7}
\]

gives the enhancement or suppression due to the effect of reheating.

We consider three sets of parameters for the electroweak phase transition. We take as case A the SM with an unrealistically small value of the Higgs mass, which allows for a sufficiently strong first-order phase transition. Therefore, we set the values \( D = 0.2 \) and \( E = 0.006 \), and we choose \( \lambda = 2E \) in order to fulfill the condition \( v(T_c)/T_c \gtrsim 1 \), which is required for avoiding the washout of the BAU. For the friction of the plasma in the SM we assume the rough value \( \xi \approx 1 \).

For the MSSM, the one-Higgs potential \( \lambda \) can be used as an approximation in the case in which only one Higgs boson is light \( \xi \). In this scenario the parameter \( E \) can be at most an order of magnitude larger than in the SM. Although this limiting situation is hardly achieved in practice, we can use it to simulate, without departing from reasonable values of the parameters, an extension of the SM that is most favorable for baryogenesis. We thus choose for case B the values \( E = 0.06 \) with \( \lambda = 2E \), and \( D \approx 1 \). In this case we assume a friction coefficient \( \xi \approx 70 \), in accordance with recent calculations for the MSSM with a light right-handed stop \( \xi \).
As we use it only to compute the dynamics of the phase transition, \( V(\phi, T) \) need not be a real perturbative effective potential. Instead, we can regard the field \( \phi \) as an effective order parameter, and use Eq. (1) to model the phase transition dynamics \([1, 3]\). In that way the parameters \( D, E, \) and \( \lambda \) can be chosen so that the free energy \( V \) has the same thermodynamical properties of the theory we wish to study. The relevant quantities are the latent heat \( L \) defined above, the surface tension of the bubble wall, \( \sigma \equiv \int (d\phi/dx)^2 dx|_{T_c} \), and the correlation length, given by \( \xi^{-2} \equiv \partial^2 V/\partial \phi^2|_{\langle \phi(T), T \rangle} \). In our model these parameters are given by \( L/T_c^4 \simeq 8D(E/\lambda)^2 (1 - E^2/\lambda D) \), \( \sigma/T_c^3 \simeq 2\sqrt{2}E^3/3\lambda^{5/2} \), and \( \xi T_c \simeq \sqrt{2}/E \). We thus choose the parameters of case C in accordance with recent non-perturbative lattice simulations of the MSSM in the light right-handed stop scenario, which give \( L/T_c^4 \simeq 0.4 \), \( \sigma/T_c^3 \simeq 0.01 \), and \( \xi T_c \simeq 5 \) \([22]\). This case represents a more physical situation, as it allows electroweak baryogenesis for experimentally viable values of the Higgs mass.

In Fig. 1 we have plotted the evolution of the phase transition for case A. We have defined a dimensionless temperature \( \alpha \), fraction of volume in the symmetric phase \( f \), and bubble wall velocity \( v_w \), as functions of dimensionless time \( \tau \), for the parameters of case A, with and without including the release of latent heat. The vertical lines delimit approximately the phase transition interval.

\[ \alpha = (T - T_0) / (T_c - T_0), \]
\[ \tau = (t - t_c) / (t_c - t_0), \]
\[ t_0 = \xi_{\text{Pl}} / T_0^2 \]

is the time at which the universe reaches the temperature \( T_0 \) if reheating is ignored. The result obtained when neglecting the reheating is plotted with dashed lines. In that case the duration of the phase transition is so small that all the parameters that enter the BAU can be approximated by their values at the temperature \( T_n \) of the onset of nucleation \([15]\). When the effect of latent heat is included, the plasma heats up after the beginning of nucleation because the expansion of the universe does not remove energy fast enough to compensate the release of latent heat. The bubble wall velocity thus decreases, and the phase transition slows down until an equilibrium temperature is reached, at which all the released latent heat goes into expanding the universe. Finally, the phase transition completes and the temperature decreases again.

The plasma heats up very close to \( T_c \) and the temperature remains constant for a long period of time (see Fig. 2). However, the BAU turns out to be suppressed in this case. Since the initial wall velocity is less in the MSSM due to the larger friction of the plasma, the decrease of \( v_w \) occurs in the region on the left of the peak. Eq. (8) gives a suppression factor \( S \simeq 0.15 \). Note that the slow growth period, in which \( v_w \simeq 2 \times 10^{-4} \sim 10^{-2} v_0 \), gives only a contribution \( \sim 10^{-2} \) to \( S \). The baryon asymmetry is essentially generated in the initial stage in which bubbles fill a 20\% of the volume of the universe. There exists an additional suppression with respect to the maximum BAU, since the initial wall velocity \( v_0 \) is a little below \( v_{\text{peak}} \). The total suppression factor is \( 2 (v_0 / v_{\text{peak}} + v_{\text{peak}}/v_0)^{-1} S \simeq 0.1 \).

The results of case C are plotted in Fig. 3. Here the latent heat is smaller than in the previous case, so the final velocity is larger, \( v_w \simeq 5 - 7 \times 10^{-4} \), and the phase
transition occurs in a shorter time. As a consequence, the suppression of the baryon asymmetry due to reheating is less severe, \( S \simeq 0.4 \).

Finally, it is important to notice that in some cases the simplifying approximation \( f \) may fall short of describing the behavior of \( n_B (v_w) \), especially for small \( v_w \). For such cases, further suppression may arise in a more rigorous calculation. For example, in Ref. [10] a crossing through zero of \( n_B \) occurs at \( v_w \sim 10^{-3} \). That happens because the \( CP \) violating force changes sign near the bubble wall, and so does the chiral asymmetry. This gives an opposing contribution to \( n_B \), which becomes important for small wall velocities. Hence, in our last two cases baryon number densities of opposite sign would be generated in different regions of the universe. The negative contribution of the slow growth period to Eq. (6) can be roughly estimated by multiplying the ratio \( n_B (10^{-4}) / n_B (10^{-2}) \) by the fraction of space \( \Delta f \) that is filled during this stage. According to the results of Ref. [10], that ratio depends strongly on the bubble wall width and on the gaugino mass parameters. If we take for instance a value \( \sim -1/3 \), then in case B, with \( \Delta f \simeq 0.8 \), this contribution is of the same order of the previously generated baryon asymmetry. In case C, with \( \Delta f \simeq 0.4 \), the suppression factor would decrease a 30%.

To summarize, we have calculated the decrease of the bubble wall velocity that occurs during the electroweak phase transition as a consequence of reheating, as well as its effect on baryogenesis. In the MSSM this effect tends to suppress the baryon production. For the light stop scenario we have found a suppression factor \( S \simeq 0.4 \). Although this is not severe, we must stress that \( S \) can be smaller if the baryon density changes sign for small wall velocities.

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