The slope of the hadron spin-flip amplitude and the determination of $\rho(s, t)$

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Abstract

We re-examine the extraction of $\rho(s, t)$, the ratio of the real part to the imaginary part of the scattering amplitude, and of the spin-flip amplitude, from the existing experimental data in the Coulomb-hadron interference region. We show that it is not possible to find reasonable assumptions about the structure of the scattering amplitude of proton-proton and proton-antiproton elastic scattering at high energy that would lead, in proton-antiproton scattering for $3.8 < p_L < 6.0$ GeV/c, to an agreement between data and an analysis based on dispersion relations.

1 Introduction

The calculation via dispersion relations of the ratio of the real part to the imaginary part of the forward spin-non-flip amplitude, $\rho(s, t)$, does not agree with the data until one gets to high energies, and it misses all the interesting intermediate-energy structures.

On the theory side, the situation is very complex and uncertain. Analyticity showed that one could not do without a real part, while polarization data proved that it was not possible to ignore spin complications, as the real part of the spin-non-flip amplitude has a zero, around which the contribution of the spin-flip amplitude, which decreases quite slowly with energy, cannot be ignored.

On the experimental side, the situation is not entirely clear cut either [1], and one of the difficulties is due to the lack of experimental data at high energies and small momentum transfer.

In this talk, we consider in great detail the situation concerning $\rho(s, t)$. The model we propose takes into account all known features of the near-forward proton-proton and proton-antiproton data, i.e. different slopes for the spin-non-flip and the spin-flip amplitudes, the value of total cross sections and of $\rho(s, t)$, the relative phase of the Coulomb and hadron amplitudes and the form factors of the nucleons.

2 Impact of the Coulomb-hadron phase

Let us first compare different approximations for the Coulomb-hadron interference used in fits to the experimental $p\bar{p}$-scattering data [2]. First, we use the simple West-Yennie

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Figure 1: $\rho(s,0)$ - the ratio of the real part to the imaginary part of the elastic scattering amplitude for proton-antiproton scattering at low energies. The curve shows the dispersion relation description for $p\bar{p}$ scattering [8], and the stars are the result of our analysis.

form of the relative phase [3]. This leads to values for $\rho(s,0)$ shown in the second column of Table 1. The results show the distribution of the values of $\rho(s)$ extracted from the experiments. In two cases, they lie slightly above $\rho_{\text{exp}}$ (at $p_L = 4.066$, 5.603, 5.94 GeV/c); in three cases they lie considerably higher than $\rho_{\text{exp}}$ (at $p_L = 5.72$, 6.23 GeV/c) and in one case they lie below (at $p_L = 3.7$ GeV/c).

If we take the slightly more complicated phase proposed by Cahn [4], the results are almost the same (see the third column of Table 1). Finally, if we use the expression derived by one of us [5, 6], taking into account the two-photon amplitude and using a dipole form factor, the fit gives different values for $\rho(s)$ (see the last column of Table 1): the results lie above $\rho_{\text{exp}}$ for all the considered energies, so that the difference with the predictions of the dispersion analysis gets worse, as shown in Fig. 1.

### 3 Impact of the spin-flip amplitude

In most analyses, one assumes that the imaginary and real parts of the spin-non-flip amplitude have an exponential behaviour with the same $t$ slope, and that the imaginary and real parts of the spin-flip amplitudes, without the kinematic factor $\sqrt{|t|}$, are proportional to the corresponding spin-non-flip parts of the amplitude, with a proportionality constant independent of $s$. In [7] it was shown that if the slope of the spin-flip amplitude is bigger than that for spin non-flip, $B_{sf} = 2B_{nf}$, the contribution of the spin-flip amplitude can be felt in the differential cross sections of elastic hadron scattering at small $|t|$. As it is not possible to calculate the hadronic amplitudes from first principles, we have to resort to some assumptions about their $s$ and $t$ dependencies [9, 10].

Here, we use this simple model for the spin-flip amplitude and study its impact on the determination of $\rho(s,t)$. We take the spin-non-flip and spin-flip amplitudes in the simplest exponential form

$$F_{nf}^h = h_{nf} [i + \rho(s,0)] e^{B_{nf} t/2};$$  \hfill (1)

$$F_{sf}^h = \sqrt{-t} / m_p h_{sf} [i + \rho(s,0)] e^{B_{sf} t/2},$$  \hfill (2)
Table 1: The dependence of $\rho(s,0)$ on the model used for the Coulomb-hadron phase in proton-antiproton scattering. $N$ is the number of data points.

| $p_L$(GeV/c) | N | $\rho_{\text{exper.}}$ | $\rho(\text{phase }[3])$ | $\rho(\text{phase }[4])$ | $\rho(\text{phase }[5,6])$ |
|--------------|---|----------------|------------------|----------------|------------------|
| 3.702        | 34 | +0.018 ± 0.03 | +0.0077 ± 0.02 | +0.0078 ± 0.08 | +0.028 ± 0.08 |
| 4.066        | 34 | −0.015 ± 0.03 | +0.0377 ± 0.02 | +0.0378 ± 0.08 | +0.0324 ± 0.08 |
| 5.603        | 215| −0.047 ± 0.03 | +0.035 ± 0.02  | +0.036 ± 0.08  | −0.0017 ± 0.08  |
| 5.724        | 115| −0.051 ± 0.03 | +0.0139 ± 0.02 | +0.014 ± 0.08  | −0.0088 ± 0.08  |
| 5.941        | 140| −0.063 ± 0.03 | −0.0003 ± 0.02 | −0.004 ± 0.08  | −0.0055 ± 0.08  |
| 6.234        | 34 | −0.06 ± 0.03  | +0.0162 ± 0.02 | +0.0162 ± 0.08 | −0.0216 ± 0.08  |

with $B_{sf} = 2B_{nf}$. The differential cross section in this case will be

$$\frac{d\sigma}{dt} = 2\pi \left[ |F_{nf}|^2 + 2|F_{sf}|^2 \right], \quad (3)$$

where the amplitudes $F_{nf}$ and $F_{sf}$ will include the corresponding electromagnetic parts and the Coulomb-hadron phase factors as mentioned previously.

The results of our new fits of the proton-antiproton experimental data for $p_L$ in [3.7, 6.2] GeV/c are presented in Table 2. The changes of $\chi^2$ after the inclusion of the spin-flip amplitude are measured by the coefficient

$$R_\chi = \frac{\chi^2 \text{ without } sf. - \chi^2 \text{ with } sf.}{\chi^2 \text{ without } sf.} \quad (4)$$

We again obtain values of $\rho$ close to zero and prevalently positive. Once again, as seen from Fig. 1, the results do not agree with the prediction by the dispersion analysis [8].

### 4 Conclusion

The present analysis, which includes the contributions of Coulomb interference and spin effects, shows a contradiction between the extracted value of $\rho(s,0)$ and the predictions from the analysis based on dispersion relations.

If such a situation is confirmed by future new data from the LHC experiments, it could reveal new effects such as, for example, a fundamental length of the order of 1 TeV.

It is likely, however, that the theoretical analysis can be further developed, to include additional corrections connected with possible oscillations in the scattering amplitude and with the $t$-dependence of the spin-flip scattering amplitude. We hope that the forward experiments at NICA will also give valuable information for the improvement of our theoretical understanding of this question.
Table 2: Spin dependence of proton-antiproton elastic scattering

| $p_L$(GeV/c) | N  | $\rho_{exp.}$ | $R_X$ | $\rho_{model}$ | $h_{sf}$, GeV |
|------------|----|----------------|------|----------------|----------------|
| 3.702      | 34 | $+0.018 \pm 0.03$ | 8%   | $+0.057 \pm 0.02$ | $49.8 \pm 1.4$ |
| 4.066      | 34 | $-0.015 \pm 0.03$ | 25%  | $+0.052 \pm 0.009$ | $48.9 \pm 0.7$ |
| 5.603      | 215| $-0.047 \pm 0.03$ | 3.5% | $+0.014 \pm 0.005$ | $35.6 \pm 4.0$ |
| 5.724      | 115| $-0.051 \pm 0.03$ | 6.5% | $+0.023 \pm 0.004$ | $38.2 \pm 4.5$ |
| 5.941      | 140| $-0.063 \pm 0.03$ | 4.5% | $+0.007 \pm 0.003$ | $43.2 \pm 0.4$ |

References

[1] R. Fiore, L. L. Jenkovszky, R. Orava, E. Predazzi, A. Prokudin and O. Selyugin, Int. J. Mod. Phys. A 24 (2009) 2551 [arXiv:0810.2902 [hep-ph]].

[2] S. Trokenheim, Precision measurements of anti-proton - proton elastic scattering at small momentum transfers, Fermilab-Thesis-1995-40 (1995); Durham HepData Project, M.R. Whalley, [http://durpdg.dur.ac.uk/hepdata/reac.html](http://durpdg.dur.ac.uk/hepdata/reac.html)

[3] G. B. West and D. R. Yennie, Phys. Rev. 172 (1968) 1413.

[4] R. Cahn, Z. Phys. C 15 (1982) 253.

[5] S. V. Goloskokov, S. P. Kuleshov and O. V. Selyugin, Mod. Phys. Lett. A 9, 1207 (1994) [arXiv:hep-ph/9312244](http://arxiv.org/abs/hep-ph/9312244).

[6] O. V. Selyugin, Phys. Rev. D 60 (1999) 074028.

[7] O.V. Selyugin, O. V. Selyugin, Mod. Phys. Lett. A 14 (1999) 223.

[8] P. Kroll and W. Schweiger, Nucl. Phys. A 503 (1989) 865.

[9] E. Predazzi and O. V. Selyugin, Eur. Phys. J. A 13 (2002) 471 [arXiv:hep-ph/0111367](http://arxiv.org/abs/hep-ph/0111367).

[10] J. R. Cudell, E. Predazzi and O. V. Selyugin, Eur. Phys. J. A 21 (2004) 479 (2004) [arXiv:hep-ph/0401040](http://arxiv.org/abs/hep-ph/0401040).