Analysis of the quark sector in the 2HDM with a four-zero Yukawa texture using the most recent data on the CKM matrix

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(Dated: December 9, 2013)
Abstract

In this letter we analyse, in the context of the general 2-Higgs Doublet Model, the structure of the Yukawa matrices, $\tilde{Y}^q_{1,2}$, by assuming a four-zero texture ansatz for their definition. In this framework, we obtain explicit and exact expressions for $\tilde{Y}^q_{1,2}$. These expressions are similar to the Cheng and Sher ansatz with the difference that here they are obtained naturally as a direct consequence of the functional structure of the invariants of the fermion mass matrices. Furthermore, the latter provide analytical and exact expressions for the parameters $(\chi^q_{ij})_{kl}$, the couplings entering the model predictions. We perform a $\chi^2$-fit based on current experimental data on the quark masses and the Cabibbo-Kobayashi-Maskawa mixing matrix $V_{\text{CKM}}$. Hence, we obtain the allowed ranges for the parameters $\tilde{Y}^q_{1,2}$ at 1$\sigma$ for several values of $\tan\beta$. The results are in complete agreement with the bounds obtained taking into account constraints on Flavour Changing Neutral Currents reported in the literature.

PACS numbers: 12.15.-y,12.60.-i,12.60.Fr

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Now that a Higgs particle has been discovered at the Large Hadron Collider (LHC) [1–3], with properties in very good accordance with the minimal version of Standard Model (SM) [4, 5], it becomes important to look for extensions of the Higgs sector beyond the SM structure that contain a neutral Higgs boson similar to the one found at the CERN machine. One of the most restrictive experimental results on extensions of the SM is that Flavour Changing Neutral Currents (FCNCs) must be controlled. The highly experimental suppression for FCNCs should be a test for models with more than one Higgs multiplet. In particular, in the 2-Higgs Doublet Model (2HDM) [6–8], FCNCs could be avoided through a discrete symmetry $Z_2$. It is well known that there are several versions of this model, known as Type I, II X and Y (2HDM-I [9, 10], 2HDM-II [11], 2HDM-X and 2HDM-Y [12–17]) or inert [18–21]. The most general version of the 2HDM contains non-diagonal fermionic couplings in the scalar sector implying the generation of unwanted FCNCs. Different ways to suppress FCNCs have been developed, giving rise to a variety of specific implementations of the 2HDM [22–26]. In particular, as it is done in Ref. [27], it is possible to analyze the Yukawa matrices through the quark sector phenomenology. The Cheng and Sher ansatz [28] has been successfully used to describe the Yukawa couplings. Several Yukawa textures proposed in literature [29, 30] have yielded the right description of the Yukawa couplings depending on fermion masses.

In this paper, we are interested in the Yukawa sector in the context of the general version of the 2HDM considering a four-zero texture fermionic mass matrix [29, 30]. This Yukawa texture has been studied in Refs. [31–35], which obtained interesting phenomenological results in both charged and neutral Higgs sectors. In this framework, we propose an alternative way to the one commonly used in the literature to determine the allowed range in the parameter space for the Yukawa matrix elements in the mass space, $(\tilde{\chi}_{q,j})_{kl}$. Then, we compute the Yukawa matrices in an explicit and exact way and, by considering the current experimental data on masses and mixing (as embedded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix) in the quark sector, we perform a $\chi^2$ fit over the $(\tilde{\chi}_{q,j})_{kl}$ parameters. We extract the allowed ranges for $(\tilde{\chi}_{q,j})_{kl}$ as a function $\tan\beta$ for each like-2HDM (i.e., each of the aforementioned types). The Cheng and Sher structure of the Yukawa couplings is obtained as a natural feature of our fermion mass matrices invariant. Finally, the allowed ranges for the parameters $(\tilde{\chi}_{q,j})_{kl}$ are in accordance with those given in Refs. [36–38].

The Yukawa Lagrangian for the quark fields is given by

$$L_Y = Y^u_1 \bar{Q}_L \tilde{\Phi}_1 u_R + Y^u_2 \bar{Q}_L \tilde{\Phi}_2 u_R + Y^d_1 \bar{Q}_L \Phi_1 d_R + Y^d_2 \bar{Q}_L \Phi_2 d_R + h.c.,$$  \hspace{1cm} (1)

where $\Phi_j = (\phi^+_j, \phi^0_j)^T$ with $j = 1, 2$ denoting the Higgs doublets, $\tilde{\Phi}_j = i\sigma_2 \Phi^*_j$, and $Y^q_j$ with $q = u, d$ are the $3 \times 3$ complex Yukawa matrices [39]. The Yukawa Lagrangian in eq. (1) has a
great deal of free parameters associated with the Yukawa interactions and five Higgs bosons, two of them charged \((H^\pm)\), two neutral CP-even (scalar) ones \((h^0\) and \(H^0)\, in increasing order of mass) and one neutral CP-odd (pseudoscalar) state \((A^0)\). The mechanism through which the FCNCs are controlled defines the version of the model and the specific emerging phenomenology that can be contrasted with experiments. Before we discuss how FCNCs are controlled in the 2HDM though, we analyse the mass matrix.

In flavour space, the mass matrix, in general, can be written as

\[
M_q = \frac{1}{\sqrt{2}} (v_1 Y^q_1 + v_2 Y^q_2), \quad q = u, d,
\]

(2)

where \(v_j\) are the Vacuum Expectation Values (VEVs) of the two Higgs doublet fields. There is no physical restriction on the structure of the mass matrix beyond the fact that the quark masses of different families differ by several orders of magnitude. Consequently, there is no restriction on either Yukawa matrix. The mass matrices \(M_q\) are diagonalysed by a biunitary transformation \([40]\),

\[
U_{qL} M_q U_{qR}^\dagger = \Delta_q,
\]

(3)

where \(\Delta_q = \text{diag}\{m_{q1}, m_{q2}, m_{q3}\}\). This transformation connects the flavour space and the mass space. Conversely, the mass matrices \(M_q\) in the mass space take the form

\[
U_{qL} M_q U_{qL}^\dagger = \frac{1}{\sqrt{2}} (v_1 \tilde{Y}^q_1 + v_2 \tilde{Y}^q_2) = \Delta_q,
\]

(3)

where \(\tilde{Y}^q = U_{qL} Y^q U_{qR}^\dagger\) are Yukawa matrices in mass space. The new Yukawa matrices \(\tilde{Y}^q_j\) are dependent upon each other and this dependence is pointed out by rewriting eq. (3) in components,

\[
(\Delta_q)_{kl} = \frac{v \cos \beta}{\sqrt{2}} \left[ (\tilde{Y}^q_1)_{kl} + \tan \beta (\tilde{Y}^q_2)_{kl} \right], \quad k, l = 1, 2, 3,
\]

(4)

where \(v^2 = v_1^2 + v_2^2 = (246.22\ \text{GeV})^2\) and \(\tan \beta = \frac{v_2}{v_1}\) \([7, 41]\). The off-diagonal elements of the Yukawa matrices in mass space, \(\tilde{Y}^q_j\), obey the following relation:

\[
(\tilde{Y}^q_j)_{kl} = -\tan \beta (\tilde{Y}^q_j)_{kl}, \quad k \neq l,
\]

(5)

From this expression we can observe that off-diagonal elements are proportional to each other. Furthermore, such proportionality is given by \(\tan \beta\). Eq. (5) also implies a restriction between corresponding arguments given that, if we require \(\tan \beta\) to be real and positive definite \([7, 39]\), the arguments must satisfy:

\[
(\varphi^q_j)_{kl} = (2n + 1) \pi + (\varphi^q_j)_{kl} \quad k \neq l,
\]

(6)

where \((\varphi^q_j)_{kl} = \text{arg}\left\{ (\tilde{Y}^q_j)_{kl} \right\}\). The condition in eq. (5) also means that the off-diagonal elements of the Yukawa matrices in mass space, \(\tilde{Y}^q_j\), are parallel or anti-parallel to each other, in the complex
plane. In other words, in the mass space the phases of the off-diagonal elements of the Yukawa matrices are (anti)aligned.

In the SM eq. (5) is trivially satisfied, because there exists an alignment between the mass matrix and the corresponding Yukawa matrix. In all 2HDM realisations (Type I, II, X and Y), wherein a discrete symmetry $Z_2$ is imposed, one of the Yukawas is zero. This implies an alignment between the mass matrix and the corresponding Yukawa one. In the Aligned 2HDM (A-2HDM), both Yukawas are aligned in flavour space, which in turn implies an alignment among the mass and Yukawa matrices. In the Minimal Flavour Violating 2HDM (MFV-2HDM), where a (non-discrete) flavour symmetry is imposed, an alignment between the mass matrix and the corresponding Yukawa one is obtained too. In the 2HDM Type III (2HDM-III), with a particular texture form, eq. (5) is satisfied by construction. The aim of this paper is that of proposing a hierarchical ansatz for the mass matrix and, by means of eq. (2), the simplest case is to consider that both Yukawas $Y_j^q$ possess the same structure (without anomalous cancellations of any of the elements of the matrices). In particular, we use an Hermitian 4-texture form and, indeed because of eq. (2), the complete mass matrix inherits this structure:

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B_q & \tilde{B}_q \\ 0 & B_q^* & A_q \end{pmatrix} = \frac{v \cos \beta}{\sqrt{2}} \begin{pmatrix} 0 & C_1^q & 0 \\ C_1^{q*} & B_1^q & B_2^q \\ 0 & B_1^{q*} & A_1^q \end{pmatrix} + \tan \beta \begin{pmatrix} 0 & C_2^q & 0 \\ C_2^{q*} & B_2^q & B_2^q \\ 0 & B_2^{q*} & A_2^q \end{pmatrix}. \quad (7)$$

In the polar form, the off-diagonal elements of the matrices in eq. (7) are: $C_q = |C_q| e^{i\phi_q}$, $B_q = |B_q| e^{i\phi_b}$, $C_j^q = |C_j^q| e^{i\phi_j}$, and $B_j^q = |B_j^q| e^{i\phi_j^q}$. The Hermitian mass matrices $M_q$ may be written in terms of a real symmetric matrix $\tilde{M}_q$ and a diagonal matrix of phases $P_q = \text{diag} \left[ 1, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3} \right]$ as follows: $M_q = P_q^\dagger \tilde{M}_q P_q$, see Ref. [42]. Now, the real symmetric matrix may be brought to a diagonal form by means of an orthogonal transformation, $\tilde{M}_q = O_q \text{diag} \left[ m_{q1}, -m_{q2}, 1 \right] O_q^\dagger$, where $m_{q1} = m_{q1}/m_{q3}$ and $m_{q2} = |m_{q2}|/m_{q3}$ are the ratios of the quark masses. Hence, while $\tilde{M}_q = M_q/m_{q3}$ are the normalised mass matrices, $O_q$ are real orthogonal matrices. In the same way as the mass matrices, the Yukawa matrices can be written in polar form as follows: $Y_j^q = P_j^\dagger \tilde{Y}_j^q P_j^q$, where $P_j^q = \text{diag} \left[ 1, e^{i\phi_j}, e^{i(\phi_j + \phi_{j'2}/2)} \right]$ and $\tilde{Y}_j^q$ is a real symmetric matrix.

The unitary matrices in eq. (3) satisfy the condition $U_{qL} = U_{qR} = U_q$ and can be written as $U_q = O_q^\dagger P_q$ [43][44]. From invariants of the real symmetric mass matrices, $\text{Tr} \left\{ \tilde{M}_q \right\}$, $\text{Tr} \left\{ \tilde{M}_q^2 \right\}$ and $\text{Det} \left\{ \tilde{M}_q \right\}$, we have

$$c_q^2 a_q = m_{q1} m_{q2}, \quad b_q + a_q = 1 + m_{q1} - m_{q2}, \quad c_q^2 + b_q^2 - \tilde{b}_q a_q = m_{q1} m_{q2} - m_{q1} + m_{q2}, \quad (8)$$

where $a_q = A_q/m_{q3}$, $b_q = B_q/m_{q3}$, $q_q = |B_q|/m_{q3}$, and $c_q = |C_q|/m_{q3}$. Thus the mass matrices
\( \hat{M}_q \) can be parameterised as \[43\]:

\[
\hat{M}_q = \begin{pmatrix}
0 & \sqrt{\frac{m_{q_1} m_{q_2}}{(1 - \delta_q)}} \hat{m}_{q_1} - \hat{m}_{q_2} + \delta_q & 0 \\
\sqrt{\frac{m_{q_1} m_{q_2}}{(1 - \delta_q)}} \hat{m}_{q_1} - \hat{m}_{q_2} + \delta_q & 0 & \sqrt{\frac{\delta_q}{(1 - \delta_q) f_1 f_2}} \\
0 & \sqrt{\frac{\delta_q}{(1 - \delta_q) f_1 f_2}} & 1 - \delta_q
\end{pmatrix},
\]

where \( f_{q_1} = (1 - \hat{m}_{q_1} - \delta_q) \) and \( f_{q_2} = (1 + \hat{m}_{q_2} - \delta_q) \). The free parameters \( \delta_q \) assume values in the following range: \( 0 < \delta_q < 1 - \hat{m}_{1q} \) \[43\] \[45\].

It is also important to mention that in this work the quark mass matrices have been re-parameterised in terms of the mass ratios, since the numerical values of the quark masses themselves are too much dependent upon the energy scale at which they are measured. However, these can be normalised with respect to the heaviest quark, obtaining as a result that the mass ratios are more stable under changes in the probing energy scale \[41\] \[46\].

The orthogonal real matrices \( O_q \) may also written in terms of a ratios of eigenmasses of \( M_q \) as \[43\]:

\[
O_q = \begin{pmatrix}
\left[ \frac{\hat{m}_{q_2} f_{q_1}}{D_{q_1}} \right]^\frac{1}{2} & - \left[ \frac{\hat{m}_{q_1} f_{q_2}}{D_{q_2}} \right]^\frac{1}{2} & \left[ \frac{\hat{m}_{q_1} \hat{m}_{q_2} \delta_q}{D_{q_3}} \right]^\frac{1}{2} \\
\left[ \frac{\hat{m}_{q_1} (1 - \delta_q) f_{q_1}}{D_{q_1}} \right]^\frac{1}{2} & \left[ \frac{\hat{m}_{q_2} (1 - \delta_q) f_{q_2}}{D_{q_2}} \right]^\frac{1}{2} & \left[ \frac{(1 - \delta_q) \delta_q}{D_{q_3}} \right]^\frac{1}{2} \\
- \left[ \frac{\hat{m}_{q_1} f_{q_2} \delta_q}{D_{q_1}} \right]^\frac{1}{2} & \left[ \frac{\hat{m}_{q_2} f_{q_1} \delta_q}{D_{q_2}} \right]^\frac{1}{2} & \left[ \frac{f_{q_1} f_{q_2}}{D_{q_3}} \right]^\frac{1}{2}
\end{pmatrix},
\]

with the definitions; \( D_{q_1} = (1 - \delta_q) (\hat{m}_{q_1} + \hat{m}_{q_2}) (1 - \hat{m}_{q_1}) \), \( D_{q_2} = (1 - \delta_q) (\hat{m}_{q_1} + \hat{m}_{q_2}) (1 + \hat{m}_{q_2}) \) and \( D_{q_3} = (1 - \delta_q) (1 - \hat{m}_{q_1}) (1 + \hat{m}_{q_2}) \).

Due to the fact we are working in a general \[2\]HDM structure, it is natural to have FCNCs induced by the terms \( \tilde{Y}^q_j \). We are however ready to compute the form of the FCNC terms. When the Yukawa matrices are represented by a four-zero texture, these matrices in mass space have the following form

\[
\tilde{Y}^q_j = m_{q_3} O_q^\dagger Q^q_j \tilde{Y}^q_j Q^q_j O_q,
\]

where \( Q^q_j = P^q_j P^q_q = \text{diag} \left[ 1, e^{i \phi_b^q}, e^{i(\phi_c^q + \phi_e^q)} \right] \) with \( \phi_b^q = \phi_b^q - \phi_b \) and \( \phi_c^q = \phi_c^q - \phi_c \) and where \( \tilde{Y}^q_j = Y^q_j / m_{q_3} \) is a real symmetric matrix normalised with respect to the heaviest quark. The matrices \( \tilde{Y}^q_j \), eq. \[11\], can now be written in the following generic form

\[
\left( \tilde{Y}^q_j \right)_{kl} = \frac{\sqrt{m_{q_1} m_{q_2}}}{v} \left( \tilde{Y}^q_j \right)_{kl} + \left( M^q_j \right)_{kl} c^q_j + \left( N^q_j \right)_{kl} c^q_j,
\]

where \( c^q_j = v \left| C^q_j \right| / m_{q_3} e^{-i \phi_{c_j}} \) whereas \( M^q \) and \( N^q \) are only functions of the mass ratios \( \hat{m}_{q_j} \) and the free parameter \( \delta_q \). The first term of eq. \[12\] corresponds to the Chen-Sher ansatz for the
coupling Fermion-Fermion-Higgs boson \((FFH \sim \sqrt{m_f m_H})\) while the second term does not have this structure and can thus be considered as a correction. In the limit where \(\left|C_{ij}^q\right| \propto m_q1/m_q3\), the latter term vanishes and we recover the Cheng-Sher structure and the \((\tilde{\chi}_j^q)_{kl}\)s have the same form as given in eq. (14) of [31].

In order to constraint the Fermion-Fermion-Higgs boson coupling that it contains all contributions, we rewrite eq. (12) as

\[
\left(\tilde{Y}^q_j\right)_{kl} = \frac{\sqrt{m_qk m_ql}}{v} \left(\tilde{\chi}_j^q\right)_{kl}, \quad k, l = 1, 2, 3.
\]

where

\[
(\tilde{\chi}_j^q)_{11} = \frac{f_{s1}}{D_{k1}^q} \sqrt{\frac{m_{s2}}{m_{s1}}} \eta_j (c_j^q + b_j^q) + \eta_j \frac{f_{s1}}{D_{k1}^q} \left(b_j^q + b_j^q\right) + \frac{\delta_{s2}}{D_{k2}^q} a_j^q,
\]

\[
(\tilde{\chi}_j^q)_{12} = -\frac{\eta_j f_{s1}}{D_{k1}^q D_{k2}^q} \left(\sqrt{\frac{m_{s1}}{m_{s2}}} c_j^q - \sqrt{\frac{m_{s2}}{m_{s1}}} a_j^q\right) + \frac{\eta_j f_{s1}}{D_{k1}^q} a_j^q + \frac{\delta_{s1}}{D_{k1}^q} b_j^q,
\]

\[
(\tilde{\chi}_j^q)_{13} = \frac{f_{s2}}{D_{k2}^q} \sqrt{\frac{m_{s1}}{m_{s2}}} \eta_j (c_j^q + b_j^q) + \frac{\eta_j f_{s2}}{D_{k2}^q} a_j^q + \frac{\delta_{s2}}{D_{k2}^q} b_j^q,
\]

\[
(\tilde{\chi}_j^q)_{22} = \frac{\eta_j f_{s1}}{D_{k1}^q D_{k2}^q} \left(b_j^q + b_j^q\right) + \frac{\delta_{s1}}{D_{k1}^q} a_j^q,
\]

\[
(\tilde{\chi}_j^q)_{23} = \frac{\delta_{s2}}{D_{k2}^q} \sqrt{\frac{m_{s1}}{m_{s2}}} \eta_j (c_j^q + b_j^q) + \frac{\eta_j f_{s2}}{D_{k2}^q} a_j^q + \frac{\delta_{s2}}{D_{k2}^q} b_j^q,
\]

\[
(\tilde{\chi}_j^q)_{33} = \frac{\delta_{s2}}{D_{k2}^q} \sqrt{\frac{m_{s1}}{m_{s2}}} \eta_j (c_j^q + b_j^q) + \frac{\eta_j f_{s2}}{D_{k2}^q} a_j^q + \frac{\delta_{s2}}{D_{k2}^q} b_j^q.
\]

With \(\eta_j \equiv (1 - \delta_q)\),

\[
a_j^q = \frac{v}{m_{q3}} A_j^q, \quad b_j^q = \frac{v}{m_{q3}} B_j^q, \quad \bar{b}_j^q = \frac{v}{m_{q3}} \bar{B}_j^q, \quad \text{and} \quad c_j^q = \frac{v}{m_{q3}} C_j^q e^{-i \varphi c_j^q}.
\]

The dependence of the Fermion-Fermion-Higgs couplings with respect to the fermion masses that are involved, see eq. (13), is a direct consequence of the functional structure of the invariants of the fermionic mass matrices, see eq. (8). Now, in order to determine the numerical value of the coefficients of the Yukawa parameters \(a_j^q, b_j^q, \bar{b}_j^q\) and \(c_j^q\) given in eq. (14), we consider that the values for the quark mass ratios at the \(M_Z\) scale are the following [44]:

\[
\tilde{m}_u = (1.73 \pm 0.75) \times 10^{-5}, \quad \tilde{m}_c = (3.46 \pm 0.43) \times 10^{-3},
\]

\[
\tilde{m}_d = (1.12 \pm 0.007) \times 10^{-3}, \quad \tilde{m}_s = (2.32 \pm 0.84) \times 10^{-2}.
\]

The values of the running quark masses at \(M_Z\) were calculated with the RunDec program [47], for more details see [44]. The free parameters \(\delta_q\) can be determined through flavour mixing with the help of the CKM matrix. The \(V_{CKM}\) matrix is defined as

\[
V_{CKM}^{th} = U_u U_d^\dagger = O_q^\dagger P^{(u-d)} O_d,
\]
where $O_{u,d}$ are the real orthogonal matrices given in eq. (10) and $P^{(u-d)\text{diag}}[1,e^{i\phi_1},e^{i(\phi_1+\phi_2)}]$ with $\phi_1 = \phi_c^u - \phi_c^d$ and $\phi_2 = \phi_b^u - \phi_b^d$. Hence, we make a $\chi^2$ fit in which the $\chi^2$ function is defined as [34, 38]:

$$\chi^2 = \left(\frac{|V_{ud}^\text{th}| - |V_{ud}|}{\sigma_{V_{ud}}^2}\right)^2 + \left(\frac{|V_{us}^\text{th}| - |V_{us}|}{\sigma_{V_{us}}^2}\right)^2 + \left(\frac{|V_{ub}^\text{th}| - |V_{ub}|}{\sigma_{V_{ub}}^2}\right)^2 + \left(\frac{J_{q}^\text{th} - J_q}{\sigma_{J_q}^2}\right)^2,$$

where the terms with super-index “th” are given in eq. (17) and the quantities without super-index are the following experimental data with uncertainty $\sigma_{V_{kl}}$ [34]:

$|V_{ud}| = 0.97427 \pm 0.00015$, $|V_{us}| = 0.2253 \pm 0.007$, $|V_{ub}| = 0.00351 \pm 0.00015$, $J_q = (2.96 \pm 0.18) \times 10^{-5}$. (19)

Therefore, without loss of generality, we can consider $\Phi_1 = \pi/2$ and $\Phi_2 = 0$ [33, 35, 48]. Thus, the resulting values for the free parameters $\delta_u$ and $\delta_d$ have the following range, at $1\sigma$:

$$\delta_u = (5.14^{+4.0}_{-2.2}) \times 10^{-2} \quad \text{and} \quad \delta_d = (3.36^{+3.32}_{-1.71}) \times 10^{-2}.$$ (20)

Furthermore, the moduli of the entries of the quark mixing matrix and the Jarlskog invariant take the follows values, at $1\sigma$:

$$|V_{\text{CKM}}^{\text{th}}|_{1\sigma} = \begin{pmatrix}
0.97427 \pm 0.00023 & 0.22533^{+0.0010}_{-0.00096} & 0.00351^{+0.0072}_{-0.00023} \\
0.22520 \pm 0.00100 & 0.97324^{+0.00058}_{-0.00023} & 0.0458^{+0.0033}_{-0.010} \\
0.00894^{+0.0016}_{-0.00069} & 0.0451^{+0.048}_{-0.010} & 0.998944^{+0.00042}_{-0.00016}
\end{pmatrix}.$$ (21)

and

$$J_q^{\text{th}} = (2.96 \pm 0.28) \times 10^{-5}.$$ (22)

Correspondingly, the numerical values of the normalised $d$- and $u$-type quark mass matrices, at $1\sigma$, are:

$$\widehat{M}_d = \begin{pmatrix}
0 & (5.18^{+0.30}_{-0.056}) \times 10^{-3} & 0 \\
(5.18^{+0.30}_{-0.056}) \times 10^{-3} & (1.16^{+3.32}_{-1.71}) \times 10^{-2} & 0.183^{+0.070}_{-0.054} \\
0 & 0.183^{+0.070}_{-0.054} & 0.966^{+0.018}_{-0.033}
\end{pmatrix},$$ (23)

and

$$\widehat{M}_u = \begin{pmatrix}
0 & (2.45^{+0.63}_{-0.335}) \times 10^{-4} & 0 \\
(2.45^{+0.63}_{-0.335}) \times 10^{-4} & (4.82^{+3.96}_{-2.18}) \times 10^{-2} & 0.221^{+0.067}_{-0.050} \\
0 & 0.221^{+0.067}_{-0.050} & 0.949^{+0.022}_{-0.040}
\end{pmatrix}.$$ (24)

Now we take advantage of the results obtained in the $\chi^2$ fit in order to perform a numerical analysis which constrains the Fermion-Fermion-Higgs boson couplings. Thus, the numerical values
of the parameters \((\chi_j^q)_{kl}\) of the Fermion-Fermion-Higgs boson couplings given in eq. (13) are as follows for both quark sectors.

i) For \(u\)-type quarks:

\[
\begin{align*}
(\tilde{\chi}_j^u)_{11} &= 4062^{+643}_{-837} \left( c_j^u + c_j^u \right) + 287^{+32}_{-55} \tilde{b}_j^u - 67^{+27}_{-23} \left( b_j^u + b_j^u \right) + 16^{+14}_{-8} a_j^u, \\
(\tilde{\chi}_j^u)_{12} &= -295^{+28}_{-51} \left( 7.25^{+1.8}_{-0.58} \times 10^{-2} c_j^u - 13.79^{+1.11}_{-2.74} c_j^u \right) + 287^{+31}_{-54} \tilde{b}_j^u \\
&\quad - 70^{+34}_{-25} \left( 0.9518^{+0.0218}_{-0.0394} b_j^u + 0.9485^{+0.0212}_{-0.0393} b_j^u \right) + 15.56^{+1.14}_{-7.87} a_j^u, \\
(\tilde{\chi}_j^u)_{13} &= 54_{-25}^{+25} \left( (1.73 \pm 0.75) \times 10^{-5} c_j^u + c_j^u \right) + 3.84^{+1.36}_{-1.11} \tilde{b}_j^u \\
&\quad - 17.40^{+0.73}_{-1.40} \left( (5.14^{+4.0}_{-2.2}) \times 10^{-2} b_j^u - 0.9485^{+0.0212}_{-0.0393} b_j^u \right) - 3.85^{+1.37}_{-1.10} a_j^u, \\
(\tilde{\chi}_j^u)_{22} &= -21.38^{+7.82}_{-1.43} \left( c_j^u + c_j^u \right) + 287^{+32}_{-55} \tilde{b}_j^u - 67^{+27}_{-23} \left( b_j^u + b_j^u \right) + 15.51^{+1.14}_{-7.81} a_j^u, \\
(\tilde{\chi}_j^u)_{23} &= 0.2857^{+0.1133}_{-0.00927} \left( (3.46 \pm 0.43) \times 10^{-3} c_j^u - c_j^u \right) + 3.84^{+1.36}_{-1.10} \tilde{b}_j^u \\
&\quad - 17.34^{+0.73}_{-1.42} \left( (5.14^{+4.0}_{-2.2}) \times 10^{-2} b_j^u - 0.9518^{+0.0218}_{-0.0394} b_j^u \right) - 3.87^{+1.33}_{-1.13} a_j^u, \\
(\tilde{\chi}_j^u)_{33} &= (1.26^{+1.43}_{-0.62}) \times 10^{-5} \left( c_j^u + c_j^u \right) + (5.13^{+3.91}_{-2.11}) \times 10^{-2} \tilde{b}_j^u + 0.2206^{+0.0664}_{-0.050} \left( b_j^u + b_j^u \right) \\
&\quad + 0.9487^{+0.0211}_{-0.0392} a_j^u.
\end{align*}
\]

(25)

ii) For \(d\)-type quarks:

\[
\begin{align*}
(\tilde{\chi}_j^d)_{11} &= 184^{+3}_{-10} \left( c_j^d + c_j^d \right) + 40^{+0.50}_{-4.0} \tilde{b}_j^d - 7.51^{+2.89}_{-2.20} \left( b_j^d + b_j^d \right) + 1.42^{+1.40}_{-0.73} a_j^d, \\
(\tilde{\chi}_j^d)_{12} &= -40^{+0.8}_{-2.81} \left( 0.220^{+0.0004}_{-0.0008} c_j^d - 4.55^{+0.18}_{-0.02} c_j^d \right) + 40^{+0.46}_{-4.07} \tilde{b}_j^d \\
&\quad - 7.59^{+3.30}_{-2.32} \left( 0.9895^{+0.0167}_{-0.0331} b_j^d + 0.9652^{+0.0172}_{-0.0331} b_j^d \right) + 1.38^{+1.37}_{-0.70} a_j^d, \\
(\tilde{\chi}_j^d)_{13} &= 5.30^{+2.19}_{-1.60} \left( (1.12 \pm 0.007) \times 10^{-3} c_j^d + c_j^d \right) + 1.15^{+0.44}_{-0.34} \tilde{b}_j^d \\
&\quad - 6.42^{+0.01}_{-0.21} \left( (3.36^{+3.32}_{-1.31}) \times 10^{-2} b_j^d - 0.9652^{+0.0172}_{-0.0331} b_j^d \right) - 1.17^{+0.45}_{-0.34} a_j^d, \\
(\tilde{\chi}_j^d)_{22} &= -8.88^{+0.11}_{-0.99} \left( c_j^d + c_j^d \right) + 39^{+1.5}_{-3.5} \tilde{b}_j^d - 7.33^{+2.82}_{-2.15} \left( b_j^d + b_j^d \right) + 1.35^{+1.34}_{-0.60} a_j^d, \\
(\tilde{\chi}_j^d)_{23} &= 0.256^{+0.11}_{-0.08} \left( (2.32 \pm 0.84) \times 10^{-2} c_j^d - c_j^d \right) + 1.14^{+0.45}_{-0.33} \tilde{b}_j^d \\
&\quad - 6.26^{+0.01}_{-0.21} \left( (3.36^{+3.32}_{-1.31}) \times 10^{-2} b_j^d + 0.9895^{+0.0167}_{-0.0331} b_j^d \right) - 1.15^{+0.44}_{-0.34} a_j^d, \\
(\tilde{\chi}_j^d)_{33} &= (1.71^{+1.85}_{-0.88}) \times 10^{-4} \left( c_j^d + c_j^d \right) + (3.29^{+3.25}_{-1.67}) \times 10^{-2} \tilde{b}_j^d + 0.1784^{+0.067}_{-0.053} \left( b_j^d + b_j^d \right) \\
&\quad + 0.9671^{+0.0167}_{-0.0325} a_j^d.
\end{align*}
\]

(26)

In the particular case where the free parameters \((\chi_j^q)_{kl}\) are real, the phases in eq. (13) satisfy the conditions: \(\phi_{bj}^d = \phi_{bj}^u - \phi_b^q = 0\) and \(\phi_{bj}^q = \phi_{cj}^q - \phi_b^q = 0\). In other words, we have an alignment between the phases of the mass matrix and those of the Yukawa one. However, the precedent
conditions do not imply that we should have zeros for the phases of the mass matrix entries or those of the Yukawa matrix.

From eqs. (25) and (26), keeping only the leading order terms, the entries of the Yukawa matrices, $a_j^q, b_j^q, \tilde{b}_j^q$ and $c_j^q$ may be written in terms of the parameters $(\chi_j^q)_{kk}$ and $(\chi_j^q)_{23} (k = 1, 2, 3)$ as:

### iii) For u-type quarks:

$$a_j^u = (3.85^{+0.09}_{-0.16}) \times 10^{-3} \left(\chi_j^u\right)_{33} - (1.03^{+0.43}_{-0.27}) \times 10^{-4} \left(\chi_j^u\right)_{23},$$

$$b_j^u = (8.96^{+2.69}_{-2.01}) \times 10^{-4} \left(\chi_j^u\right)_{33} + (2.10^{+0.29}_{-0.26}) \times 10^{-4} \left(\chi_j^u\right)_{23},$$

$$\tilde{b}_j^u = (1.27^{+0.27}_{-0.15}) \times 10^{-5} \left(\chi_j^u\right)_{22} + (2.08^{+1.59}_{-0.86}) \times 10^{-4} \left(\chi_j^u\right)_{33} + (1.03^{+0.43}_{-0.27}) \times 10^{-4} \left(\chi_j^u\right)_{23},$$

$$c_j^u = (5.00^{+1.26}_{-0.73}) \times 10^{-7} \left(\chi_j^u\right)_{11} - (4.96^{+1.30}_{-0.63}) \times 10^{-7} \left(\chi_j^u\right)_{22} + (1.33^{+1.08}_{-0.56}) \times 10^{-8} \left(\chi_j^u\right)_{23}. $$

### iv) For d-type quarks:

$$a_j^d = (3.93^{+0.07}_{-0.13}) \times 10^{-3} \left(\chi_j^d\right)_{33} - (2.26^{+0.97}_{-0.66}) \times 10^{-4} \left(\chi_j^d\right)_{23},$$

$$b_j^d = - (1.72^{+0.83}_{-0.80}) \times 10^{-5} \left(\chi_j^d\right)_{22} + (7.24^{+2.79}_{-2.12}) \times 10^{-4} \left(\chi_j^d\right)_{33} + (5.92^{+2.11}_{-0.44}) \times 10^{-4} \left(\chi_j^d\right)_{23},$$

$$\tilde{b}_j^d = (9.10^{+0.57}_{-0.33}) \times 10^{-5} \left(\chi_j^d\right)_{22} + (1.33^{+1.32}_{-0.68}) \times 10^{-4} \left(\chi_j^d\right)_{33} + (2.26^{+0.98}_{-0.66}) \times 10^{-4} \left(\chi_j^d\right)_{23},$$

$$c_j^d = (1.05^{+0.06}_{-0.02}) \times 10^{-5} \left(\chi_j^d\right)_{11} - (1.06^{+0.07}_{-0.01}) \times 10^{-5} \left(\chi_j^d\right)_{22} + (6.05^{+3.45}_{-1.91}) \times 10^{-7} \left(\chi_j^d\right)_{23}. $$

Now, for each quark sector, the Yukawa parameters $a_j^q, b_j^q, \tilde{b}_j^q$ and $c_j^q$ are restricted to satisfy eq. (7). Hence, in order to find the allowed regions for the parameters $(\chi_j^q)_{kk}$ and $(\chi_j^q)_{23}$, we define a new $\chi^2$ function:

$$\chi^2_{M_q} = \sum_k \left[ \frac{(M_{fit}^q)_{kk} - (M_q^{th})_{kk}}{\sigma(M_q)_{kk}} \right]^2 + \sum_{k \neq l} \frac{3}{2} \left[ \frac{(M_{fit}^q)_{kl} - (M_q^{th})_{kl}}{\sigma(M_q)_{kl}} \right]^2, $$

where the $M_{fit}^q$ matrices are given in eqs. (23) and (24) whereas the $M_q^{th}$ matrices are defined in eq. (7). In a preliminary analysis of eq. (29), we determined that the parameters most sensitive to variations are $(\chi_2^q)_{22}, (\chi_2^q)_{33}$ and $(\chi_2^q)_{23}$. Therefore, without loss of generality, we eventually considered the following allowed regions for the other parameters $(\chi_1^q)_{kk}, (\chi_3^q)_{kk}$:

$$-1.5 \leq (\chi_1^q)_{kk} \leq 1.5, \quad -1.5 \leq (\chi_3^q)_{11} \leq 1.5, \quad -12 \leq (\chi_1^q)_{23} \leq 12, \quad k = 1, 2, 3. $$

Also, the only free parameters in the function are $(\chi_2^q)_{22}, (\chi_2^q)_{33}$ and $(\chi_2^q)_{23}$. We performed in fact several $\chi^2$ fits, with $\tan \beta = 2, 6, 15, 30$ and our results, at 80% Confidence Level (CL), are shown in Tab. The same parameter regions at 80% CL are shown in Fig.
FIG. 1. The allowed regions, at 80% CL, of the $\tilde{\chi}^q_2$ parameters, where the yellow region is for $\tan \beta = 2$, the orange region is for $\tan \beta = 6$, the green region is for $\tan \beta = 15$ and the black region is for $\tan \beta = 30$. Also, the diagonal parameters are in the range $-1.5 \leq (\tilde{\chi}^q_1)_{kk} \leq 1.5$, $-1.5 \leq (\tilde{\chi}^q_2)_{11} \leq 1.5$ and $-12 \leq (\tilde{\chi}^q_1)_{23} \leq 12$ with $k = 1, 2, 3$.

| $\tan \beta$ | $(\tilde{\chi}^q_2)_{22}$ | $(\tilde{\chi}^q_2)_{33}$ | $(\tilde{\chi}^q_2)_{23}$ | $(\tilde{\chi}^q_2)_{22}$ | $(\tilde{\chi}^q_2)_{33}$ | $(\tilde{\chi}^q_2)_{23}$ |
|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2            | -1.20$^{+2.0}_{-4.40}$ | 0.98$^{+1.24}_{-0.19}$ | -1.80$^{+6.9}_{-3.0}$ | -2.40$^{+2.0}_{-2.40}$ | -1.60$^{+0.80}_{-0.90}$ | -2.70$^{+4.20}_{-0.30}$ |
| 6            | -2.0$^{+2.0}_{-3.2}$   | 1.35$^{+0.30}_{-0.20}$ | -0.90$^{+3.62}_{-1.8}$ | -1.60$^{+0.80}_{-2.80}$ | -1.25$^{+0.50}_{-0.10}$ | -0.30$^{+1.20}_{-1.50}$ |
| 15           | -1.60$^{+1.20}_{-3.0}$  | 1.36$^{+0.19}_{-0.10}$ | -0.42$^{+2.51}_{-1.70}$ | -1.60$^{+0.40}_{-2.40}$ | -1.40$^{+0.15}_{-0.60}$ | -0.30$^{+1.20}_{-0.60}$ |
| 30           | -1.20$^{+0.90}_{-0.30}$ | 1.40$^{+0.10}_{-0.10}$ | -1.12$^{+2.04}_{-1.91}$ | -1.60$^{+0.40}_{-2.40}$ | -1.45$^{+0.05}_{-0.15}$ | -0.20$^{+1.80}_{-0.40}$ |

TABLE I. Numerical values for the parameters $(\tilde{\chi}^q_2)_{22}$, $(\tilde{\chi}^q_2)_{33}$ and $(\tilde{\chi}^q_2)_{23}$ at 80% CL. We also consider: $-1.5 \leq (\tilde{\chi}^q_1)_{kk} \leq 1.5$, $-1.5 \leq (\tilde{\chi}^q_2)_{11} \leq 1.5$, $-12 \leq (\tilde{\chi}^q_1)_{23} \leq 12$ with $k = 1, 2, 3$.

In conclusion, we have presented the correlations between the Yukawa matrices in the framework of the 2HDM, originating from a four-zero Yukawa texture and the current data on the CKM matrix through a $\chi^2$ fit. Firstly, we presented the diagonalisation of the Yukawa matrices, eventually showing that the Cheng and Sher ansatz is a particular case of our general study. Here, the complete analytical expressions of the diagonalisation procedure were obtained. Secondly, we have performed a numerical analysis via a $\chi^2$ fit to the Yukawa matrices with respect to the measured entries of the CKM matrix. As a consequence, we have obtained bounds for the parameters of
the Yukawa texture, in particular for its off-diagonal terms. We have obtained results that are in complete agreement with the bounds obtained in our previous work in which we studied the flavour-violating constraints. As an outlook, we deem our current parameterisation a more easily implementable one with respect to our previous ones, thus we recommend its use for numerical analyses.

ACKNOWLEDGMENTS

This work has been supported in part by SNI-CONACYT (México) and by PROMEP (México) under the grant “Red Temática: Física del Higgs y del sabor”. FGC acknowledges the financial support received from PROMEP through a postdoctoral scholarship under contract 103.5/12/2548. SM is financed in part through the NExT Institute and is grateful to the University of Puebla for kind hospitality while parts of this work were being carried out.

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