Multiple glasses in asymmetric binary hard spheres

Th. Voigtmann\textsuperscript{(a)}

Institut für Materialphysik im Weltraum, Deutsches Zentrum für Luft- und Raumfahrt (DLR)
51170 Köln, Germany, EU,
Fachbereich Physik, Universität Konstanz - 78457 Konstanz, Germany, EU and
Zukunftskolleg, Universität Konstanz - 78457 Konstanz, Germany, EU

received 30 June 2011; accepted in final form 19 September 2011
published online 20 October 2011

PACS 64.70.Q- – Theory and modeling of the glass transition
PACS 64.70.pv – Glass transitions of specific systems: Colloids

Abstract – Multiple distinct glass states with different dynamic and mechanical properties occur in binary hard-sphere mixtures with constituents of very disparate sizes according to the mode-coupling theory of the glass transition (MCT), distinguished by considering whether small particles remain mobile or not, and whether small particles contribute significantly to perturb the big-particle structure or not. In the idealized case, the four different glasses are separated by well-defined glass and localization transitions that give rise to higher-order singularities involving logarithmic decay laws, and to anomalous power-law-like diffusion.

Contact:\textsuperscript{(a)}E-mail: thomas.voigtmann@dlr.de

Many glass formers and virtually all simple model systems for slow dynamics are mixtures of some sort, to allow fine tuning of material properties or simply to avoid unwanted crystallization [1]. Suspensions composed of different colloid species have allowed particularly detailed studies: close to a glass transition generic mixing effects appear, such as dramatic changes in viscosity induced by small composition changes [2,3], that are relevant for applications and may help to shed light on the microscopic processes driving glass formation.

Even more interesting is the formation of qualitatively distinct types of glass, depending on mixture composition. In mixtures with sufficiently disparate constituents, a glass can form where some (slow) species freeze, but a fast component is able to diffuse through the voids left in the amorphous packing. This scenario is particularly relevant for transport through heterogeneous disordered media [4–14] or glassy ion conductors [15–18]. The simplest model are binary hard-sphere mixtures with large size disparity, where experiments on colloidal suspensions indeed found, depending on the relative concentration, a partially frozen “single glass” with mobile small particles, separate from a “double glass” where both particle species freeze [19,20]. In mixtures of star polymers [21–23] yet another kind of glass emerged, argued to be a hallmark of ultra-soft interactions typical for this system. It was termed “asymmetric” as it is characterized by few big particles frozen in a small-particle matrix, rendering nearest-neighbor cages highly “nonspherical” if one tracks only the big particles.

Another kind of glass intuitively conceived is the “attractive glass” famous from colloid–polymer (CP) mixtures where free polymer induces depletion attraction among the colloids. If that attraction is weak, the colloidal-particle glass that forms is essentially hard-sphere like, while at sufficiently strong and short-ranged attraction, a new glass driven by bonding instead of hard-core repulsive caging appears. There, particles are localized to within the attraction range, much less than the usual 10% of a diameter suggested by the Linde mann criterion for freezing. Based on extensively tested predictions of the mode-coupling theory of the idealized glass transition (MCT) for a square-well model potential [24–31], one expects the two glasses to be separated by a glass-glass transition. Crossing it, e.g., the elastic moduli of the glass exhibit sharp changes. Considering the generality of the depletion-interaction mechanism [32], one may expect a similar glass–glass transition to be present in binary mixtures quite generically (lest the relevant parameter space cannot be explored). The ensuing endpoint singularities give rise to universal logarithmic decay laws for the time-dependent correlation functions even in the liquid [33]. Indeed, such laws have been reported in computer simulations of soft-sphere mixtures with sufficient size disparity [34]. A full discussion of the underlying transition diagram is however still lacking.
Here I demonstrate, that already the simplest glass-forming mixture model, the binary hard-sphere mixture, allows to identify four qualitatively different glassy states separated by well-defined transitions. The transition diagram lends itself to an intuitive classification: one has to distinguish i) whether small particles remain mobile in the glass or not, and ii) whether the structure of the big-particle glass is or is not significantly disturbed by the small particles. There are three distinct types of transitions: a) common liquid-glass transitions driven by slowing-down of collective dynamics at nearest-neighbor length scales; b) glass-glass transitions that typically arise when two collective mechanisms of arrest compete; c) localization-type transitions concerning the single-particle dynamics of the small species.

Calculations here are based on binary-mixture MCT [35,36] using the Percus-Yevick (PY) approximation for the static structure factor. Following our physically plausible classification, the results can be expected to hold rather generally for binary mixtures whose parameters can be tuned widely enough. Despite its known defects, the PY structure factors appear to be a sensible approximation even in the regime where we use it [37]. The possible interplay with equilibrium phases is a sensible approximation even in the regime where we use it [37]. The possible interplay with equilibrium phases is ignored here. Experiment or computer simulation may need to resort to more complicated interaction potentials or polydispersity to suppress them.

We discuss cuts through the parameter space at constant size ratio \( \delta = d_a/d_s \leq 1 \), where \( d_a \) are the hard-sphere diameters (\( \alpha = s,1 \) for small, large). The number densities \( \rho_s \) are conveniently expressed as a total packing fraction, \( \varphi = \sum \varphi_a = (\pi/6) \sum \rho_a d_a^3 \), and a concentration (by volume) of small spheres, \( \hat{x} = \varphi_s/\varphi \).

Numerical calculations follow the method presented earlier [36,38]; details on the equations can be found in refs. [24,35,36]. MCT takes partial static structure factors \( S_{\alpha\beta}(q) = \langle g_{\alpha\beta}^s(q) g_{\beta\alpha}(q) \rangle \) as input (Greek indices label species, and static triplet correlations are neglected). Here \( \langle \cdot \rangle \) is the canonical average, and \( g_{\alpha\beta}(q) = \sum \exp[ i \vec{q} \cdot \vec{r}_{\alpha\beta} ] \), the number density fluctuation of species \( \alpha \) (the sum runs over all particle positions \( \vec{r} \) of that type). MCT then calculates dynamical density correlation functions, \( \langle \cdot \rangle \) viz., collective, \( \Phi_{\alpha\beta}(q,t) = \langle g^s_{\alpha\beta}(\vec{q},t) g_{\beta\alpha}(\vec{q}) \rangle \), and tagged-particle ones, \( \phi^s_{\alpha\beta}(q,t) = \langle \exp[i\vec{q} \cdot (\vec{r}_{\alpha\beta}(t) - \vec{r}_{\alpha\beta}(0))] \rangle \). The time evolution is given by the implicit dependence of \( \Phi_{\alpha\beta}(q) \) on the trajectories.

The long-time limit of collective density correlations, \( F(q) = \lim_{t \to \infty} \Phi(q,t) \), distinguishes ergodic liquid states, \( F(q) = 0 \), from nonergodic ones (the idealized glass), where the correlation decays not to zero but to a positive definite matrix, \( F(q) > 0 \). These nonergodicity factors are determined by a nonlinear implicit equation, \( (S(q) - F(q))^{-1} = S^{-1}(q) + M[F,F](q) \), where \( M \) is the bilinear mode-coupling functional whose coupling vertices are fully determined by the static equilibrium structure of the system. It is always the largest positive definite solution of this equation that describes the long-time limit of the correlation functions [39]. Standard MCT liquid-glass transitions are identified as bifurcations in these implicit equations of the \( A_2 \) type where \( F(q) \) jumps discontinuously. Such jumps can also occur inside the glass, then called glass-glass transitions. Close to \( A_2 \) singularities, there hold the well-known asymptotic predictions of MCT: two-step relaxation of correlation functions involving two power laws, \( t^{-\alpha} \) and \( -t^\beta \), with nonuniversal exponents given by a state–point–dependent parameter \( \lambda \). Bifurcations of higher order, \( A_{2,2} \), can occur as endpoints of glass-glass transitions, and they can be observed through different asymptotic laws involving logarithms instead of the power laws close to the transition point [24].

The transition points are identified from a stability analysis of the equations determining \( F(q) \). For the tagged-particle correlators, a similar equation holds for \( f^s_{\alpha}(q) = \lim_{t \to \infty} \phi^s_{\alpha}(q,t) \), viz., \( (1 - f^s_{\alpha}(q))^{-1} = 1 + m^s_{\alpha}[F,F](q) \). The tagged-particle MCT functional is only linear in \( f^s_{\alpha} \). Thus the tagged-particle nonergodicity parameters typically inherit transitions from the collective \( F(q) \); for sufficiently weak coupling, they may however remain zero even if \( F(q) \neq 0 \), and show a continuous rise to \( f^s_{\alpha}(q) > 0 \) at increasing coupling strength. This transition scenario is qualitatively different from the one close to an \( A_2 \) singularity. For numerical calculations [36,39], we introduce a finite wave number grid, \( q_i = (i + 1/2) \Delta q \) with \( i = 1, \ldots, N = 500 \) and \( \Delta q = 0.4/d_1 \) for \( \delta > 0.2 \), and \( N = 1000 \) for \( \delta < 0.2 \). This discretization implies a long-wavelength cutoff that is unimportant for discussing the cage effect and glass transitions, but changes some features of the theory close to localization transitions [40,41]. We restrict ourselves to a discussion of quantities and state points that do not sensitively depend on such a cutoff.

Figure 1 illustrates the glass-transition diagram for size ratios 0.2 ≤ \( \delta \) ≤ 0.5; larger \( \delta \) are topologically the same as
\( \delta = 0.5 \) [36]. For all compositions at \( \delta \geq 0.42 \), the binary-mixture glass is separated from the liquid by a smooth line of ordinary (\( A_2 \)) liquid-glass transitions. Mixing typically favors the fluid region, although the opposite effect is predicted by MCT and seen in simulation for \( \delta \) close to unity both in 3D and 2D systems [3,36,42,43]. In contrast, the curve for, say, \( \delta = 0.35 \) demonstrates the emergence of a glass-glass transition: the liquid-glass transition splits into two lines that do no longer join smoothly, but cross discontinuously at some point, at which one of the lines stops. The other continues inside the glass as a glass-glass transition, creating a \( \lambda \)-shaped transition curve in fig. 1. It terminates at a higher-order transition point (\( A_\phi \)) and separates two distinct glasses that can be transformed into each other discontinuously (crossing the line) or smoothly (following a path in parameter space encircling the \( A_\phi \)).

The glass-glass transition signals two competing arrest mechanisms: caging can be dominated by large particles (\( \delta \to 0 \)) or by small ones (\( \delta \to 1 \)). Once the ratio of relevant length scales (essentially \( \delta \)) sufficiently differs from unity and both mechanisms compete (\( \delta \approx 0.5 \)), a sharp distinction between two glasses with different intrinsic scales (the localization length, or typical cage size, as discussed below) becomes possible. Compression favors formation of the more tightly localized glass, hence it is the large-\( \hat{x} \) part of the transition line that continues as a glass-glass transition. The structure of the large-\( \hat{x} \) glass resembles certain types of sweets such as Italian torroncino (big nuts frozen in small-particle paste). The distinction is only strict for size ratios \( \delta < \delta_c \). At \( \delta_c \), the glass-glass transition endpoint coincides with the crossing, resulting in a singularity of still higher order (\( A_\phi \)). In the present calculations, \( \delta_c \approx 0.41 \).

Mathematically, the higher-order singularities are identical to those in one-component descriptions of CP mixtures [33], and similar asymptotic expansions predicting the appearance of logarithmic decay laws and their precursors apply here, too. If the mixture parameters are close enough to the position of the \( A_\phi \) singularity, these laws can be seen already on the liquid side of the transition. Analyzing dynamic correlation functions in terms of the usual MCT power laws, their exponents decrease, and the MCT exponent parameter \( \lambda \) approaches unity.

MCT predicts that at the transitions discussed so far, all collective density correlation functions jointly become nonergodic [39]. This does not need to hold for the nonergodicity factors \( F_\alpha (q) = \lim_{t \to -\infty} \Phi_\alpha (q,t) \) associated with self-motion of species \( \alpha \). \( F_\alpha (q) \) can remain zero when crossing a glass transition, if the small particles are below a certain size \( \delta_\phi \) [44–46]. It indicates that the small particles can undergo confined-liquid-like long-range motion in a network of voids left by the frozen large particles [47,48]: Within MCT, \( \eta_\alpha (t) = \lim_{\delta \to 0} \frac{\partial}{\partial \delta} \Phi_\alpha (t) \) determines the long-time self-diffusion coefficient of particles of species \( \alpha \) via \( D_\alpha = 1/\int_0^\infty \eta_\alpha (t) \) dt. Thus, even if \( \Phi (t) \to F > 0 \) for \( t \to \infty \), \( f_\alpha = 0 \) renders the integral convergent, yielding a finite long-time diffusivity for that species. The collective small-small density correlation function remains nonergodic also in this partially frozen glass, in distinct disagreement with the theory of ref. [49]. This may appear counterintuitive, but can be rationalized by arguing that even the dynamics of mobile particles will pick up a nonergodic contribution, corresponding to an elastic scattering signal originating from the frozen background [5]. As a consequence, a precursor appears in the liquid: the structural relaxation of all collective density fluctuations (at least on length scales associated with caging) becomes orders of magnitude slower than that of tagged-particle density fluctuations of the species that remains mobile. This time scale separation has been seen in computer simulations of sodium silicates [15,17].

A dashed line in fig. 1 indicates for \( \delta = 0.2 \) the transition (as approximated in our discretized model [41]) that is expected on physical grounds: the partially frozen glass transforms into a fully frozen one if the small particles have too little void space to move, or are dense enough to form a glass on their own [4–7]. Thus, in fig. 1 it is bounded from below by the liquid-glass transition, from the high-\( \hat{x} \) side by the glass-glass transition, and from above by a localization transition (dashed line).

The localization transition is notably different from the \( A_2 \) glass transitions shown as solid lines: crossing it, the nonergodicity factor \( f_\phi (q) \) rises continually from zero, as opposed to exhibiting the finite jump typical for glass transitions. This reflects the different nature of the two transition mechanisms: ordinary glass transitions are driven by collective caging on local length scales. At the localization transition, the single-(small-)particle localization length diverges continuously.

The separate localization transition in our calculation occurs only below some size ratio \( \delta_l \) smaller than the \( \delta_c \), where the higher-order singularity first occurs. Its precursor is anomalous diffusion in the sense that the small-particle mean-squared displacement, \( \delta \tilde{r}^2 (t) = \langle (\tilde{r}_s(t) - \tilde{r}_s(0))^2 \rangle \), exhibits power-law growth \( \delta \tilde{r}^2 (t) \propto t^{\lambda} \) with an exponent \( \gamma < 1 \) [44]. This allows the “double-transition” scenario to be tested, as in recent simulations of a soft-sphere mixture [18], even if one cannot truly enter the glassy state.

So far, we have found three distinct glasses. The glass-glass transition is similar to the one connected to attractive glasses in CP mixtures: both arise from a competition of arrest on distinct length scales. Tempting as it may be [50], the analogy is flawed: in the CP mixture the small component is always assumed to remain mobile, whereas at the glass-glass transition in fig. 1 the small component itself becomes the main glass former. The analog of the attractive glass in the binary mixture has to be found inside the single-glass region.

Indeed, extending fig. 1 to smaller \( \delta \), fig. 2, a second glass-glass-transition emergence: a second \( A_\phi \) point at roughly \( \delta^* = 0.194 \) marks the onset of the higher-order singularity scenario for \( \delta < \delta^* \) at around \( \hat{x} \approx 0.3 \). Again,
it indicates the discontinuous change of a big-particle dominated glass to one where the small particles set a relevant length scale, but now in the sense that the small particles induce a strong depletion attraction while themselves remaining mobile. This glass-glass transition can be viewed as analogous to the one encountered in CP mixtures. In the latter, it is found together with a can be viewed as analogous to the one encountered in CP mixtures. In the latter, it is found together with a

Fig. 2: Transition diagram as in fig. 1, but for smaller δ. Open, filled, and shaded symbols correspond to the small-\( \hat{\delta} \), large-\( \hat{\delta} \), and intermediate-\( \hat{\delta} \) branches of \( \varphi_0(\delta, \hat{\delta}) \), respectively. Dashed lines indicate localization transitions \( \varphi_0^*(\delta, \hat{\delta}) \) for \( \delta = 0.2 \) and 0.18 (bottom to top).

Fig. 3: (a) Shear modulus \( G \) of the binary mixture along the cut \( (\delta, \hat{\delta}) = (0.18, 0.45) \) as a function of packing fraction \( \varphi_0 \) in units of \( \rho_k a T \). (b) Single-particle localization lengths \( r_{c,0}^* \) for large and small particles along the same cut. Dotted lines indicate the transitions shown in fig. 2.

by the frozen big particles only: note \( G/\rho k_B T \approx 140 G/\rho k_B T \approx 10 \). The jump by about a factor 10 in \( G \) upon entering (II) is comparable to the jump when entering the depletion-driven glass in CP mixtures [26]. Entering regime (IV), \( G/\rho k_B T \approx G/\rho k_B T = O(10) \) again, as now the small particles determine the mechanical properties of the glass.

Considering the localization lengths, fig. 3(b), all four glasses become apparent through singular behavior at four critical densities. As the glass is first entered, \( r_{c,1}^* \approx 0.1d_1 \), the Lindemann criterion for arrest of the big particles. This value drops discontinuously by about \( \delta \) entering region (II), as depletion forces bond the large particles. The small-particle \( r_{c,0}^* \) remains infinite up to the localization transition at \( \varphi_0^* \), and shows a continuous divergence in region (III) as \( \varphi \to \varphi_0^* \) from above (while \( r_{c,1}^* \) remains smooth). The details of this divergence are cutoff sensitive [41], and hence not shown in fig. 3(b). As region (IV) is entered, both \( r_{c,0}^* \) drop discontinuously again, and \( r_{c,0}^* \approx 0.1d_0 \) indicates a small-particle (repulsive) glass.

To highlight the intricate features inherited by the correlation functions in the vicinity of the four transitions, we show in fig. 4 the diagonal elements of the collective density correlators \( \Phi_{\alpha \alpha}(q, t) \) for an intermediate \( q \) representing dynamics on the nearest-neighbor scale (solid lines), together with their tagged-particle counterparts \( S_{\alpha \alpha}(q) \phi_{\alpha}^*(q, t) \) (dashed lines). Brownian short-time dynamics was chosen, with time in units of the free-diffusion time of the large particles, \( t_\ell \), and free-diffusion coefficients that obey the Stokes-Einstein relation, \( D_{\alpha \alpha} \sim 1/d_\alpha \). For small \( \delta \), this induces nontrivial \( \Phi_{\alpha \alpha}(q, t) \)

\( \rho_k \) is the number density of hard-sphere systems, \( G/\rho k_B T = O(10^3) \) [36]. The values for the repulsive partially frozen glass (I) at \( (\delta, \hat{\delta}) = (0.18, 0.45) \) are two orders of magnitude smaller. This illustrates that the mechanical properties of this glass are determined
Fig. 4: Collective density correlators $\Phi_{\alpha\alpha}(q,t)$ for $\alpha = 1$ (upper panel) and $\alpha = \pi$ (lower; solid lines), for $\delta = 0.18$, $\tilde{\varphi} = 0.45$, and $\varphi = 0.44, 0.48, 0.5, 0.515, 0.518, 0.52, 0.525, 0.528, 0.53, 0.55, 0.57, 0.585, 0.59, 0.592, 0.593, and 0.594 (bottom to top). Dashed lines: tagged-particle correlators as $S_{\alpha\alpha}(q)\Phi^t_{\alpha\alpha}(q,t)$. Wave number $q = 7.8/d_0$. Roman numerals indicate the distinct glasses of fig. 2 for the $\Phi_{\alpha\alpha}(q,t)$.

multiple glasses in asymmetric binary hard spheres

The tagged-particle correlators for the small particles follow a qualitatively different behavior. As seen in the lower panel of fig. 4, there is no indication of a two-step relaxation in regions (I) and (II), and a finite long-time limit is only attained in regions (III) and (IV). In regions (I) and (II), we thus recover the scenario indicated above: while a decaying $\Phi^t_{\alpha\alpha}(q,t)$ indicates long-range motion of the small particles, the collective $\Phi_{\alpha\alpha}(q,t)$ does not decay, indicating a nonergodic frozen matrix. There is a subtle difference between this description starting from a fully equilibrated binary mixture with a dynamically self-generated large-particle background, and the treatment developed for quenched-annealed mixtures where one component moves through a prescribed frozen matrix: in the latter theory, the nondecaying correlators discussed here are replaced by the so-called connected density correlations that indeed decay as long as the small particles remain mobile.

To summarize, four different glasses are predicted for the simplest model of glass-forming mixtures, viz., binary hard-sphere mixtures, if both composition and size ratio are changed. The four glasses come in two categories, one (“fully frozen”) where both species freeze simultaneously, and one (“partially frozen”) where the smaller component remains mobile inside the frozen environment. Each of these comes in two variants, depending on whether the structure is primarily driven by large-particle caging, or by the small-particle–induced forces (be they arrested or not). These variants can be both smoothly or discontinuously transformed into each other: the glass-glass transition endpoints may be understood as the dynamic-transition analogs of a liquid-gas critical point. Increasing the overall packing density at fixed size ratio $\delta$ and concentration, one traverses some or all four glasses in a sequence: starting from the liquid, one enters the partially frozen “repulsive” “single glass” (I), the depletion-induced “attractive glass” (II), the fully frozen “double glass” (III), and the small-particle–driven “torroncino” glass (IV). Transitions between those states are classified into glass-transitions (driven by collective caging) and localization ones. Besides purely entropic size effects, different energy scales in the interactions could come into play, as, for example, in sodium silicate melts [17].

MCT predicts that the four types of glass are accompanied by regions in parameter space where signatures of higher-order singularities or of anomalous diffusion may be seen already in the liquid. The latter was found in recent simulations [18]. Features of logarithmic decay that are likely the signature of the first set of $A_\delta$ singularities have been reported in computer simulation [34]. The possibility of depletion-induced “single” glasses emerging around $\delta \approx 0.2 \approx \delta^*$ has also been hinted at [53]. Further simulations and experiments are called for to conclusively test our predictions.
REFERENCES

[1] Wang W. H., Prog. Mater. Sci., 52 (2007) 540.
[2] Williams S. R. and van Megen W., Phys. Rev. E, 64 (2001) 041502.
[3] Foffi G., Götzte W., Sciortino F., Tartaglia P. and Voigtmann Th., Phys. Rev. Lett., 91 (2003) 085701.
[4] Krakoviack V., Phys. Lett., 94 (2005) 065703.
[5] Krakoviack V., J. Phys.: Condens. Matter, 17 (2006) S3565.
[6] Krakoviack V., Phys. Rev. E, 75 (2007) 031503.
[7] Krakoviack V., Phys. Rev. E, 79 (2009) 061501.
[8] Kurzidim J., Coslovich D. and Kahl G., Phys. Rev. Lett., 103 (2009) 138303.
[9] Kim K., Miyazaki K. and Saito S., EPL, 88 (2009) 36002.
[10] Kurzidim J., Coslovich D. and Kahl G., J. Phys.: Condens. Matter, 23 (2011) 234122.
[11] Kim K., Miyazaki K. and Saito S., J. Phys.: Condens. Matter, 23 (2011) 234123.
[12] Höfling F., Franosch T. and Frey E., Phys. Rev. Lett., 96 (2006) 165901.
[13] Höfling F. and Franosch T., Phys. Rev. Lett., 98 (2007) 140601.
[14] Höfling F., Munk T., Frey E. and Franosch T., J. Chem. Phys., 128 (2008) 164517.
[15] Horbach J., Kob W. and Binder K., Phys. Rev. Lett., 88 (2002) 125502.
[16] Meyer A., Horbach J., Kob W., Kargl F. and Schober H., Phys. Rev. Lett., 93 (2004) 027801.
[17] Voigtmann Th. and Horbach J., Europhys. Lett., 74 (2006) 459.
[18] Voigtmann Th. and Horbach J., Phys. Rev. Lett., 103 (2009) 205901.
[19] Imhof A. and Dhont J. K. G., Phys. Rev. Lett., 75 (1995) 1662.
[20] Imhof A. and Dhont J. K. G., Phys. Rev. E, 52 (1995) 6344.
[21] Zaccarelli E., Mayer C., Asteriadi A., Likos C. N., Sciortino F., Roovers J., Iatrou H., Hadjichristidis N., Tartaglia P., Löwen H. and Vlassopoulos D., Phys. Rev. Lett., 95 (2005) 268301.
[22] Mayer C., Stiakakis E., Zaccarelli E., Likos C. N., Sciortino F., Tartaglia P., Löwen H. and Vlassopoulos D., Rheol. Acta, 46 (2007) 611.
[23] Mayer C., Zaccarelli E., Stiakakis E., Likos C. N., Sciortino F., Munam A., Gauthier M., Hadjichristidis N., Iatrou H., Tartaglia P., Löwen H. and Vlassopoulos D., Nat. Mater., 7 (2008) 780.
[24] Götzte W., Complex Dynamics of Glass-Forming Liquids (Oxford University Press) 2009.
[25] Bergenholtz J. and Fuchs M., Phys. Rev. E, 59 (1999) 5706.
[26] Dawson K., Foffi G., Fuchs M., Götzte W., Sciortino F., Sperl M., Tartaglia P., Voigtmann Th. and Zaccarelli E., Phys. Rev. E, 63 (2001) 011401.
[27] Pham K. N., Puertas A. M., Bergenholtz J., Egelfaaf S. U., Moussaïd A., Pusey P. N., Schoffield A. B., Cates M. E., Fuchs M. and Poon W. C. K., Science, 296 (2002) 104.
[28] Pham K. N., Egelfaaf S. U., Pusey P. N. and Poon W. C. K., Phys. Rev. E, 69 (2004) 011503.
[29] Sciortino F., Tartaglia P. and Zaccarelli E., Phys. Rev. Lett., 91 (2003) 268301.
[30] Zaccarelli E., Foffi G., Sciortino F. and Tartaglia P., Phys. Rev. Lett., 91 (2003) 108301.
[31] Zaccarelli E. and Poon W. C. K., Proc. Natl. Acad. Sci. U.S.A., 106 (2009) 15203.
[32] Asakura S. and Oosawa F., J. Polym. Sci., 33 (1958) 183.
[33] Sperl M., Phys. Rev. E, 68 (2003) 031405.
[34] Moreno A. J. and Colmenero J., J. Chem. Phys., 125 (2006) 164507; Phys. Rev. E, 74 (2006) 021409.
[35] Götzte W., Comments on the mode coupling theory of the liquid glass transition, in Proceedings of Amorphous and Liquid Materials, edited by Lüscher E., Frisch G. and Jacucci G., NATO ASI Ser. E: Applied Physics, Vol. 118 (Nijhoff, Dordrecht) 1987, pp. 34-81.
[36] Götzte W. and Voigtmann Th., Phys. Rev. E, 67 (2003) 021502.
[37] Zaccarelli E., private communication.
[38] Foffi G., Götzte W., Sciortino F., Tartaglia P. and Voigtmann Th., Phys. Rev. E, 69 (2004) 011505.
[39] Franosch T. and Voigtmann Th., J. Stat. Phys., 109 (2002) 237.
[40] Leutheusser E., Phys. Rev. A, 28 (1983) 1762.
[41] Schnyder S., Höfling F., Franosch T. and Voigtmann Th., J. Phys.: Condens. Matter, 23 (2011) 234121.
[42] HAJNAL D., BRADER J. M. and SCHILLING R., Phys. Rev. E, 80 (2009) 021503.
[43] WEYSSER F. and HAJNAL D., Phys. Rev. E, 83 (2011) 041503.
[44] SJÖGREN L., Phys. Rev. A, 33 (1986) 1254.
[45] Bosse J. and Thakur J. S., Phys. Rev. Lett., 59 (1987) 998.
[46] Bosse J. and Kaneko Y., Phys. Rev. Lett., 74 (1995) 4023.
[47] Götzte W., Leutheusser E. and Yip S., Phys. Rev. A, 23 (1981) 2634.
[48] Götzte W., Leutheusser E. and Yip S., Phys. Rev. A, 24 (1981) 1008.
[49] JuÁREZ-MALDONADO R. and MEDINA-NOYOLA M., Phys. Rev. E, 77 (2008) 051503.
[50] JuÁREZ-MALDONADO R. and MEDINA-NOYOLA M., Phys. Rev. Lett., 101 (2008) 267001.
[51] Zaccarelli E., Löwen H., Wessels P. P. F., Sciortino F., Tartaglia P. and Likos C. N., Phys. Rev. Lett., 92 (2004) 225703.
[52] Fuchs M., Götzte W., Hofacker I. and Latz A., J. Phys.: Condens. Matter, 3 (1991) 5047.
[53] GERMAIN Ph. and AMOKRANE S., Phys. Rev. Lett., 102 (2009) 058301.