Influence of the electron intrinsic magnetic moment on the transverse dielectric permittivity of degenerate electron gas

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Abstract. Using the linear response theory, the transverse dielectric permittivity of a homogeneous and isotropic system of charged particles is considered. In the ideal gas approximation for the polarization function, an explicit analytical expression for the transverse permittivity of a degenerate electron plasma, which takes into account electron spin, is found. This result describes both the Landau diamagnetism and Pauli paramagnetism in electron plasma. The influence of the electron intrinsic magnetic moment on the spatial and frequency dispersion of the transverse dielectric permittivity of degenerate electron plasma is numerically studied, that is crucial for determining the optical characteristics of plasma.

1. Introduction
In this paper, we analyzed the spatial and frequency dispersion of the transverse dielectric permittivity (DP) \( \varepsilon_{tr}(k, \omega) \) of the degenerate electron plasma with the influence of electron spin. The value of the transverse DP, which describes the response of charged particles on the transverse electromagnetic field, is necessary for many applications. In particular, the transverse DP of the degenerate electron gas is used for calculation of equilibrium and optical properties (including skin effect) of liquid and solid metals, surface waves, etc [1–4], where electrons are usually considered as quasifree, although the typical value of the interaction parameter \( r_s \approx 1–5 \). In modern theory, the exchange and correlations between electrons are well studied for both the longitudinal \( \varepsilon_{l}(k, \omega) \) (which is the response on the longitudinal electromagnetic field) and transversal dielectric functions [1, 5, 6]. At the same time, consideration of the transverse DP is not completed even for the case of quasifree electrons, since the influence of the electron intrinsic magnetic moment on the spatial and frequency dispersion of the transverse dielectric permittivity of degenerate electron plasma still was not known [7]. The main purpose of this paper is the consideration of this influence in the approximation similar to the random phase approximation (RPA) for the longitudinal DP. This kind of approximation has a limited applicability for a large values of the parameter \( r_s \). Nevertheless, the explicit description of the transverse DP for weakly interacted electron system and the respective numerical calculations which take into account electron spin are the first and necessary steps for development of a theory (see, e.g., [8, 9]) for strongly interacted electrons and for clarification of the existing experiments [10]. It is useful to mention the recent papers (see [11] and references therein), where the transverse
DP for weakly interacted degenerate electron system has been considered in the framework of \( \tau \) approximation for collisions. However, the existence of the electron intrinsic magnetic moment was not taken into account. For the case of collisionless electron plasma the DP in [11] coincides with the classical Lindhard result [12].

2. General relations

Sequential account the electron spin in calculation of the transversal DP for ideal quantum electron plasma is possible in the method of the kinetic equation (see., e.g., [13]), and on the basis of the linear response theory (see., e.g., [14]). Below we use the representation of the transversal DP for a weakly non-ideal quantum electron plasma at arbitrary degeneracy obtained in [15] (see also [16–19]). For arbitrary degeneracy the DP equals

\[
\varepsilon^t(k, \omega) = 1 - \frac{\omega^2}{\omega_c^2} - \frac{4\pi}{\omega_c^2}[\varphi_d(k, \omega) + \varphi_p(k, \omega)],
\]

where the diamagnetic and paramagnetic terms are defined by the expressions

\[
\varphi_d(k, \omega) = \frac{(2s + 1)\hbar^2e^2}{2m^2} \int \left( p^2 - \frac{(pk)}{k^2} \right) \frac{\Delta f(p, k)}{h(\omega + i0) + \Delta E(p, k)} \frac{d^3p}{(2\pi)^3},
\]

\[
\varphi_p(k, \omega) = \frac{(2s + 1)s(s + 1)}{3} \frac{\mu c}{2} k^2 \int \frac{\Delta f(p, k)}{h(\omega + i0) + \Delta E(p, k)} \frac{d^3p}{(2\pi)^3},
\]

\[
\Delta f(p, k) = f(p - k/2) - f(p + k/2), \quad \Delta E(p, k) = E(p - k/2) - E(p + k/2).
\]

Here \( \mu = -|e|/hmc \) and \( s \) are the electron magnetic moment, and electron spin, \( f(p) = 1/\{\exp[(E(p) - \zeta)/T] + 1\}^{-1}, E(p) = p^2/(2m) \) and \( \zeta \) are the Fermi–Dirac distribution function, the spectrum of free electron and the chemical potential of ideal plasma, respectively. The chemical potential is determined by the normalization relation

\[
n(T, \zeta) = (2s + 1) \int \frac{d^3p}{(2\pi)^3} f(p).
\]

We will show that the presence of the magnetic moment of electrons results in paramagnetic properties plasma for certain wave lengths in the respective frequency range. Plasma of liquid metals is characterized by a weak interaction of ions and quasifree electrons [1], so as an initial approximation we use a single-component model of the electron plasma–electron quantum system. Below we consider the electron subsystem of metallic plasma for the Fermi degeneracy condition \( T \ll E_F = p_F^2/(2m) \), i.e., the electron temperature is much lower than the Fermi energy. The results are applicable to metallic plasma (at first to liquid metals) and also has a general significance, as the demonstration of the explicit calculation and influence of the spin on degenerate electron system. For the degenerate electron gas the general form of the transverse DP can be rewritten, taking into account the concrete values of electron spin and magnetic moment and introducing the dimensionless variables

\[
\varepsilon^t(Q, \omega) = 1 - \frac{1}{W^2} - \frac{4\pi}{W^2}[\Phi_d(Q, W)) + \Phi_p(Q, W)],
\]

\[
\Phi_d(Q, W) = \frac{3}{4\pi^2} \int_{-\infty}^{\infty} \left( p^2 - \frac{P \cdot Q}{Q^2} \right) \frac{\Delta f(P, Q)}{2\alpha\Delta(W + i0) + \Delta E^*(P, Q)} d^3P,
\]

\[
\Phi_p(Q, W) = \frac{3}{8\pi^2} Q^2 \int_{-\infty}^{\infty} \frac{\Delta f(P, Q)}{2\alpha\Delta(W + i0) + \Delta E^*(P, Q)} d^3P.
\]
where the dimensionless variables $W = \omega/\omega_0$, $P = p/(\hbar k_F)$, $Q = q/(\hbar k_F)$, $E^r(p) = E(p)/E_F$, $r_s = \sqrt{3/(4\pi n a_0^3)}$ are used. The functions $\Phi_{d,p}(Q,W) = \varphi_{d,p}(k,\omega)$ and $a_0$ is the Bohr radius. We also introduced the constant $\alpha_\Delta = [4r_s/(3\pi)]^{1/2}[4/(9\pi)]^{1/6}$.

3. Calculations

Using the Fermi distribution function

$$f(|p| < p_F) = 1, \quad f(|p| > p_F) = 0,$$

it is possible to integrate equations (7),(8) [8] and to arrive at the expression

$$\text{Re} \Phi_d(Q,W) = \frac{3}{32\pi} \left\{ 3 \left( \frac{\alpha_\Delta W}{Q} \right)^2 + Q^2/4 - \frac{5}{3} + \frac{1}{2Q} \left[ 1 - \left( Q\Delta^-(Q,W) \right)^2 \right] \right\} \times \ln \left[ 1 + Q\Delta^-(Q,W) \right] - \frac{1}{2Q} \left[ 1 - \left( Q\Delta^+(Q,W) \right)^2 \right] \times \ln \left[ 1 + Q\Delta^+(Q,W) \right],$$

(11)

$$\text{Re} \Phi_p(Q,W) = -\frac{3Q^2}{32\pi} \left\{ 1 - \frac{1}{2Q} \left[ 1 - \left( Q\Delta^-(Q,W) \right)^2 \right] \ln \left[ 1 + Q\Delta^-(Q,W) \right] + \frac{1}{2Q} \left[ 1 - \left( Q\Delta^+(Q,W) \right)^2 \right] \ln \left[ 1 + Q\Delta^+(Q,W) \right] \right\},$$

(12)

$$\text{Im} \Phi_d(Q,W) = \begin{cases} 0, & |\Delta^-(Q,W)| \geq 1/Q, \\ -\frac{3}{16} \frac{\alpha_\Delta W}{Q} \left[ 1 - \left( \frac{\alpha_\Delta W}{Q} \right)^2 - \frac{Q^2}{4} \right], & |\Delta^+(Q,W)| \leq 1/Q, \\ -\frac{3}{64} \left[ 1 - \left( \frac{\alpha_\Delta W}{Q} \right)^2 \right]^2, & \end{cases}$$

(13)

$$\text{Im} \Phi_p(Q,W) = \begin{cases} 0, & |\Delta^-(Q,W)| \geq 1/Q, \\ -\frac{3}{32} \frac{\alpha_\Delta}{Q} W, & |\Delta^+(Q,W)| \leq 1/Q, \\ -\frac{3Q}{64} \left[ 1 - \left( \frac{\alpha_\Delta W}{Q} \right)^2 \right]^2, & \end{cases}$$

(14)

where $\Delta^{(-/+)}(Q,W) = (\alpha_\Delta W/Q^2)^{\mp 1/2}$.

According to equation (1), the expression (10–13) for the real and imaginary parts define the space-frequency dispersion of transverse plasma DP. It was found that the frequency dispersion essentially different for $Q < 2$ and $Q > 2$. Indeed, when $Q < 2$ by substituting formulas (12–13) into equation (1) we obtain

$$\text{Im} \varepsilon_{tr}(Q,W) = \begin{cases} 0, & W \geq \frac{2Q + Q^2}{2\alpha_\Delta} \\ \frac{3\alpha_\Delta W}{16Q} \left[ 1 - \left( \frac{2\alpha_\Delta W}{Q} \right)^2 + Q^2/4 \right], & |\Delta^+(Q,W)| \leq 1/Q, \\ \frac{3\pi}{16W^2Q} \left[ 1 - \left( Q\Delta^-(Q,W) \right)^2 \right]^2 + \frac{3\pi Q}{16W^2} \left[ 1 - \left( Q\Delta^-(Q,W) \right)^2 \right], & W \leq \frac{2Q + Q^2}{2\alpha_\Delta}. \end{cases}$$

(15)

As it is follows from the above relations and calculations for dimensionless frequency of

$$W \leq \frac{2Q + Q^2}{2\alpha_\Delta}$$

the imaginary part of the transversal DP $\text{Im} \varepsilon_{tr}(Q,W) \simeq 1/W$, i.e., for fixed
Figure 1. The dependence $\text{Im} \varepsilon^\text{tr}(Q, W)$ from $W$ for $Q = 0.5$ (curves 1) and $Q = 1$ (curves 2). Left side $r_s = 0.1$, right side $r_s = 1$.

values of $Q > 0$ the function $\text{Im} \varepsilon^\text{tr}(Q, W) \to \infty$ when $W \to 0$ (figure 1). This result shows a significant increase in plasma resistance at low frequency for the case $Q < 2$. With increasing frequency the imaginary part of the transversal DP decreases and, starting from a certain threshold value is zero (figure 1). For decreasing electron density $n$ and, consequently, for the increasing parameters $r_s$ and $\alpha_\Delta$, the imagine part $\text{Im} \varepsilon^\text{tr}(Q, W)$ tends to zero for lower threshold frequencies.

If $Q > 2$ frequency dependence has another behavior

$$\text{Im} \varepsilon^\text{tr}(Q, W) = \begin{cases} 0, & W \leq \frac{Q^2 - 2Q}{2\alpha_\Delta} \text{ or } W \geq \frac{Q^2 + 2Q}{2\alpha_\Delta}; \\ \frac{3\pi}{16W^2Q} \left[ 1 - \left( \frac{Q\Delta^{-1}}{2\alpha_\Delta} \right)^2 \right]^2 + \frac{3\pi Q}{16W^2} \left[ 1 - \left( \frac{Q\Delta^{-1}}{2\alpha_\Delta} \right)^2 \right], & \frac{Q^2 - 2Q}{2\alpha_\Delta} \leq W \leq \frac{Q^2 + 2Q}{2\alpha_\Delta}. \end{cases}$$

(16)

As is easy to see, at low wavelengths (corresponding to $Q > 2$) the imaginate part $\text{Im} \varepsilon^\text{tr}(Q, W)$ does not tends to infinity in contrast with the region of wave vectors $Q < 2$. Rather, the plasma is transparent for electromagnetic waves at large and for sufficiently low frequencies (figure 2). In average, there is a tendency of decrease $\text{Im} \varepsilon^\text{tr}(Q, W)$ with increase of wave vectors $Q$, which is due to the higher penetrating ability of shorter wavelengths.

Figure 3 shows plots of the real part of DP $\text{Re} \varepsilon^\text{tr}(Q, W)$ for different values of $Q$ and $r_s$. When $Q$ is very low ($Q \to 0$) the frequency dispersion coincides with the universal high-frequency behavior of the DP for collisionless plasma $\text{Re} \varepsilon^\text{tr}(Q \to 0, W) \to 1 - 1/W^2$ (see, e.g., [14]), i.e., plasma displays diamagnetic properties for a wide range of values of $W$ and $r_s$. The asymptotic behavior of the expressions $\text{Re} \Phi_{d,p}(Q, W)$ when $Q \to 0$ and fixed $W$ has the form

$$\text{Re} \Phi_{d}(Q, W) \to \frac{Q^2}{20\pi (\alpha_\Delta W)^2} + O(Q^4), \quad \text{Re} \Phi_{p}(Q, W) \to \frac{Q^4}{16\pi (\alpha_\Delta W)^2} + O(Q^6).$$

(17)

Therefore, for a fixed $W$ and $Q \to 0$ the functions $\text{Re} \Phi_{d}(Q, W) \to 0$ and $\text{Re} \Phi_{p}(Q, W)$ are the same order. In the case of non-zero wave numbers, the magnetic moment of the electron affects the electromagnetic characteristics of plasma. If for $Q \to 0$ the real part of the transversal DP takes large negative values, for the case of sufficiently large values of the $Q$ (at least for $Q \geq 0.5$
Figure 2. The dependence of Im$\varepsilon^{tr}(Q, W)$ from $W$ for $Q = 3$: left side $r_s = 0.1$, right side $r_s = 1$.

as follows from figure 3) we find that Re $\Phi_d(Q, W) \to \infty$ for $W \to 0$. The explicit consideration for the change of asymptotic behavior can be determined in more detail calculations. This means that depending on the wavelength, the electron plasma may possess diamagnetic or paramagnetic properties.

In average there is tendency of increase of the function Re$\varepsilon^{tr}(Q, W)$ with increase of $Q$. If the real part of the transverse DP does not exceed unit in a certain frequency range, in the case of $Q = 3$ the function Re$\varepsilon^{tr}(Q = 3, W) > 1$ for any $W$ even at a high electron density (small $r_s$) (figure 3).

In the case of $Q \to \infty$ the real part Re$\varepsilon^{tr}(Q, W)$ tends to unit, as follows from the asymptotic behavior of equations (10), (11) for large $Q$:

$$
\text{Re}\Phi_d(Q, W) \to -\frac{1}{5\pi Q^2} + O\left(\frac{1}{Q^3}\right), \quad \text{Re}\Phi_p(Q, W) \to -\frac{1}{4\pi} + O\left(\frac{1}{Q^2}\right).
$$

The respective plots are presented in figure 4.

4. Concluding remarks

So, the dependence of the transverse DP of the degenerate electron plasma as the function of frequency and wavelength of electromagnetic field is investigated in the framework of the linear response theory. The influence of the electron intrinsic magnetic moment is explicitly taken into account for small and moderate values of the interaction parameter $r_s$. It is found that Im$\varepsilon^{tr}(Q, W \to 0)$ for moderate $Q$ takes the large positive values (see figure 1). At the region of $Q \geq 3$ at low as well as at high values $W$, the function Im$\varepsilon^{tr}(Q > 3, W)$ vanishes. For the case of high $Q$ at both high and low frequencies Im$\varepsilon^{tr}(Q, W) = 0$. The real part Re$\varepsilon^{tr}(Q, W)$ at $Q \ll 1$ is not more than unit for all $W$ and different $1 < r_s < 5$. When $Q$ increases the region of values Re$\varepsilon^{tr}(Q, W) > 1$ appears and increases when the wave length decreases. This behavior is valid even for small values of the interaction parameter $r_s$.

As it follows from figure 5 the influence of the term $\Phi_p$ which takes into account the interaction of the electron intrinsic magnetic moment with a weak magnetic field results to crucial change in behavior of the DP already for moderate values $r_s$. In particular, the diamagnetic response (which manifests itself as a negative value of Re$\varepsilon^{tr}(Q, W) < 0$ for a finite values of $Q$ and
Figure 3. The plots $\text{Re } \epsilon^{tr}(Q, W)$. For $Q = 0.01$ (the plot for $r_s = 0.1$ (a)); and for different $Q$: $r_s = 0.1$ (b), $r_s = 1$ (c), $r_s = 5$ (d). The curves 1.2.3 correspond to values $Q=0.5, 1, 3$ respectively.

$W \to 0$ changes to the paramagnetic one (positive value of $\text{Re } \epsilon^{tr}(Q, W) > 0$ for a finite value of $Q$ and $W \to 0$) in electron system, see, e.g., figure 5 (a).

The consideration of the transverse DP for an ideal electron gas in this paper is equivalent to considering of the longitudinal DP in the well-known approximation of chaotic phases (RPA), which in the vast majority of papers on the theory of electron plasma is considered as the initial (zero) approximation. In this case, dependence of the transverse DP on the parameter $r_s$ is due to the interaction of electrons with an external electromagnetic field, rather than the interaction between electrons and electron–ion interaction (see, e.g., [20, 21], where the exchange and correlations between electrons have been taken into account, but the influence of the intrinsic magnetic moment on the DP was neglected). As follows from the presented results, the accounting of the intrinsic magnetic moment is necessary for a consistent description of the transverse DP in contrast to the longitudinal DP. This is because the magnetic field is transverse. Note, that the contribution to the transverse DP associated with the intrinsic magnetic moment of an electron is often associated in the literature with the so-called spin susceptibility. At the same time, in future, it is proposed to take into account the exchange-correlation effects on the
Figure 4. The dependence of $\text{Re} \varepsilon^{\text{tr}}(Q,W)$ as function of $Q$ for different $r_s$ ($r_s = 0.1$—curves 1; $r_s = 1$—curves 2; $r_s = 5$—curves 3): $W = 0.75$ (left side); $W = 2$ (right side).

Figure 5. (a) the plots $\text{Re} \varepsilon^{\text{tr}}(W)$ for $Q = 1$, $r_s = 1$, (b) the $\text{Re} \varepsilon^{\text{tr}}(Q)$ for $W = 2$, $r_s = 1$: 1—neglecting the function $\Phi_p(W,Q)$ (that corresponds to the Lindhard result [12], and to the result [11] for the collisionless case), 2—taking into account $\Phi_p(W,Q)$.

magnitude of the transverse DP (by analogy with consideration of the longitudinal DP). Within the framework of such an analogy, we can assume that such effects are small only under the condition $r_s \ll 1$. Therefore, the curves with $r_s \gg 1$ in this paper should be considered as a background for the future calculations which will take into account the exchange and correlation effects.

We emphasize that the effects of interaction between electrons, due to the presence of the intrinsic magnetic moment of the electron, which is directly related to the electron spin, can have an important influence on the properties of real systems, primarily magnetic materials. However, the exchange-correlation effects associated with the Coulomb interaction and the effects of statistics (electron identity) traditionally considered in the theory of longitudinal DP
are not related with the presence of an intrinsic electron magnetic moment. The influence of the intrinsic magnetic moment on the interaction between electrons can be taken into account either in the relativistic description of electrons or in taking into account the intrinsic quantized electromagnetic field in the system, which is beyond the scope of this work. In our paper, we consider a simple model which allows correctly take into account the interaction with an external magnetic field due to the presence of the intrinsic magnetic moment of electron. As already noted above, this does not eliminate the need for a consistent consideration of the effects of the Coulomb interaction of electrons on the magnitude of the transverse DP, in analogy with the description of the longitudinal DP.

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