INVESTIGATION OF NEUTRON–DEUTERON CHARGE-EXCHANGE REACTION AT SMALL TRANSFER MOMENTUM

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Analysis of the \( nd \to p(nn) \) reaction in a GeV-energy region is performed in the framework based on the multiple-scattering theory for the few-nucleon system. The special kinematic condition, when momentum transfer from neutron beam to final proton closes to zero, is considered. The possibility to extract the spin-dependent term of the elementary \( np \to pn \) amplitude from \( nd \)-breakup process is investigated. The energy dependence of the ratio \( R = \frac{d\sigma_{nd}}{dt}/\frac{d\sigma_{np}}{dt} \) is obtained taking account of the final-state interaction of the two outgoing neutrons in \( ^1S_0 \)-state.

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1. Introduction

The nucleon–deuteron charge-exchange reaction is the subject of the investigation in the set of the experiments, which are started in the VBLHE JINR at STRELA and DELTA SIGMA [1] setups and in COSY [2] at ANKE spectrometer. These experiments are performed in special kinematics, where transfer momentum from the initial nucleon to the outgoing fast nucleon is close to zero. The goal of these experiments is to extract the additional information about the spin-dependent part of the elementary $np \rightarrow pn$ amplitude from the nucleon–deuteron reaction. First this idea was suggested by Pomernanchuk [3] already in 1951. Later, it was shown that in the plane-wave impulse approximation (PWIA) the differential cross section and tensor analyzing power $T_{20}$ in the $dp$ charge-exchange reaction are actually fully determined by the spin-dependent part of the elementary $np \rightarrow pn$ amplitudes [2], [4].

However, the relative momentum of the two slow nucleons is very small under such kinematical conditions, where momentum of the emitted fast nucleon has the same direction and magnitude as the beam (in the deuteron rest frame). As a consequence, the final-state interaction (FSI) effects have to play very important role. How much is the FSI influence? It is the general question of our consideration in this paper.

We consider the $nd \rightarrow pnn$ reaction in kinematics of the DELTA SIGMA experiment, where the outgoing proton has the same direction as the projectile neutron and transfer momentum is close to zero. The neutron-beam kinetic energy changes from 0.8 up to 1.3 GeV. The theoretical approach is based on the Alt–Grassberger–Sandhas formulation of the multiple-scattering theory for the three-nucleon system. We apply the matrix inversion method to describe the FSI contributions. Earlier, this formalism has been employed for description of the deuteron–proton breakup in the GeV region [5]-[6]. All calculations have been performed in the deuteron rest frame. The results, which obtained in the present paper, are very interesting, from our point of view, and useful for further investigations of this problem.

The paper is organized as follows. The theoretical formalism is given in the Section 2 in
which we consider both the general description of the \( nd \) break-up reaction and the special case, where transfer momentum is close to zero. The results are presented in Section 3. The ratio of the \( nd \rightarrow pnn \) differential cross section to the free \( np \) scattering differential cross section at \( \theta_{\text{lab}} = 0 \) has been studied. It is shown that final-state interaction is very important, although the FSI contribution decreases with the energy increasing. In Section 4 the conclusions are given.

2. Theoretical formalism

In accordance with the Alt–Grassberger–Sandhas (AGS) formalism of the three-body collision theory \([7]-[8]\), the amplitude of the neutron–deuteron charge-exchange reaction,

\[
n(p) + d(0) \rightarrow p(p_1) + n(p_2) + n(p_3),
\]

is defined by the matrix element of the transition operator \( U_{01} \):

\[
U_{nd\rightarrow pnn} \equiv \sqrt{2} <123|(1 - (1, 2) - (1, 3))U_{01}|1(23)> = \delta(p - p_1 - p_2 - p_3)J. \tag{2}
\]

The state \( |1(23)> \) corresponds to the configuration of nucleons 2 and 3 forming the deuteron state and nucleon 1 being the projectile, whereas the state \( <123| \) represents the free motion of three nucleons after the reaction. The permutation operators for two nucleons \((i, j)\) appear in this expression due to particle identity both in initial and final states.

Iterating the AGS equations up to the second-order terms, we can obtain the expression for the transition operator \( U_{01} \):

\[
U_{01} = g_0^{-1} + t_2 + t_3 + t_1 g_0 t_2 + t_1 g_0 t_3 + t_2 g_0 t_3 + t_3 g_0 t_2 + O(t^3). \tag{3}
\]

By definition, \( t_3 = t_{12} \) is a two-nucleon transition operator (the others can be obtained via cyclic permutations). Here, \( g_0 \) is the free nucleon propagator,

\[
g_0 = [E + i0 - K]^{-1},
\]
with the kinetic-energy operator for the three-body system $K$ and the on-shell energy $E = E_1 + E_2 + E_3$. Obviously, the term $g_0^{-1}$ does not contribute to the on-shell amplitude (2).

Since we consider only such kinematical conditions, when outgoing proton carries away significant part of the initial neutron momentum, we can neglect the corrections due to the recoil reaction mechanism, where final fast proton leaves the deuteron without direct knock-out. In addition, we ignore terms, which correspond to the double scattering with participation of a fast nucleon. Then, the matrix element $U_{nd\rightarrow pnn}$ can be approximated by

$$U_{nd\rightarrow pnn} = \sqrt{2} <123|\left[1 - (2, 3)\right]|1 + t_{23}(E - E_1)g_{23}(E - E_1)|t_{12}^{\text{sym}}|123>,$$  

(4)

where the operator $g_{23}(E - E_1)$ is a free propagator for the $(23)$-subsystem and the scattering operator $t_{23}(E - E_1)$ satisfies the Lippmann–Schwinger (LS) equation with two-body force operator $V_{23}$ as driving term

$$t_{23}(E - E_1) = V_{23} + V_{23}g_{23}(E - E_1)t_{23}(E - E_1).$$  

(5)

The operator $t_{12}^{\text{sym}}$ is symmetrized NN-operator, $t_{12}^{\text{sym}} = [1 - (1, 2)]t_{12}$. 

The expression (4) can be schematically presented by the set of graphs (Fig.1). Here, two first terms correspond to the PWIA and the others represent the scattering with FSI of the slow nucleons. Let us rewrite the matrix element (4) indicating explicitly the particle quantum numbers,

$$U_{nd\rightarrow pnn} = \sqrt{2} <p_1m_1\tau_1, p_2m_2\tau_2, p_3m_3\tau_3|[1 - (2, 3)]\omega_{23}t_{12}^{\text{sym}}|p_m\tau, \psi_{1M_d,00}(23)>,$$  

(6)

where $\omega_{23} = [1 + t_{23}(E - E_1)g_{23}(E - E_1)]$ and the spin and isospin projections are denoted as $m$ and $\tau$, respectively.

In momentum representation the deuteron wave function (DWF) $\psi_{1M_d}(k)$ with spin projection $M_d$ is written as

$$|\psi_{1M_d}(k)> = \sum_{L=0,2} \sum_{M_L=-L}^{L} <LM_L1M_S|1M_d> u_L(k)Y^M_L(\hat{k})|1M_s>,$$  

(7)

4
with the spherical harmonics $Y^M_L(\hat{k})$ and the Clebsh–Gordon coefficients in the standard form. In our calculations, we use the pole parameterization of the deuteron wave function [9]:

$$u_0(p) = \sqrt{\frac{2}{\pi}} \sum_i \frac{c_i}{\alpha_i^2 + p^2} \quad u_2(p) = \sqrt{\frac{2}{\pi}} \sum_i \frac{d_i}{\beta_i^2 + p^2}. \quad (8)$$

Inserting in Eq.(6) the unity

$$1 = \int dp'^* |p'm'r' > < p'm'r'|,$$

and using the definition of the deuteron wave function, Eq.(7), we get the following expression for the reaction amplitude:

$$J = \frac{1}{2} < 1^2_2 m^3_1 | S M_S > < 1^2_2 m^3_1 | S M'_S > < L M_L 1 M_S | 1 M_d > \frac{1}{2} m''^1_2 m'_3^1 | 1 M_S > \times$$

$$\times \int dp'_0 \left\langle p_0', S M'_S \right| 1 + m_N t^{ST=1}(E - E_1) \left| p_0', S M'_S \right\rangle u_L(|p'_0 - q/2|) Y^{M_L}_L(\hat{p}'_0 - q/2) \times$$

$$\times < p_1 m_1, (p'_0 + q/2) m'_2 | t^{sym}_{T=0}(E - E'_3) - t^{sym}_{T=1}(E - E'_3) | p m, (p'_0 - q/2) m'' > -$$

$$- (2 \leftrightarrow 3), \quad (9)$$

where $m_N$ is the nucleon mass, $S$ corresponds to the spin of the two slow nucleons which participate in the final-state interaction. We have introduced here the momentum transfer $q = p - p_1$, relative momenta $p_0 = \frac{1}{2}(p_2 - p_3)$ and $p'_0 = \frac{1}{2}(p'_2 - p'_3)$. Henceforth, all summations over dummy discrete indices are implied.

The two slow neutrons interaction is described by the wave function

$$< \psi_{p_0 S M'_S T M'_T}^{(-)} | p'_0 S M'_S T M_T > = \delta(p_0 - p'_0) \delta_{M_3 M'_3} +$$

$$+ \frac{m_N}{p_0^2 - p'_0^2 + i0} < p_0 S M_S | t^{ST} | p'_0 S M'_S > . \quad (10)$$

This wave function contains the FSI part, which can be taken into account in different ways.

In the present paper we use the matrix inversion method (MIM) suggested in [10, 11] and applied to study the deuteron electro-disintegration [12, 13] and deuteron proton
breakup process \cite{5, 6}. As in ref. \cite{12}, we consider the truncated partial-wave expansion,

\[
< \psi_{p_0 S T M_T}^{(-)} | p_0' S M_T' T M_T > = \delta_{M_S M_T} \delta(p_0 - p_0') + \\
+ \sum_{J=0}^{J_{\text{max}}} \sum_{M_J = -J}^J Y_{l}^{\mu}(\hat{p}_0) < l \mu \psi_{l l}^{\alpha}(p_0') < \psi_{l l}'^{\mu'}(p_0') >,
\]

where $J_{\text{max}}$ is the maximum value of the total angular momentum in $nn$ partial waves and $\alpha = \{J, S, T\}$ is the set of conserved quantum numbers.

Under kinematical conditions, where transfer momentum $q = p - p_1$ is close to zero, one can anticipate that the FSI in the $^1S_0$-state is prevalent at comparatively small $p_0$-values. Then two-neutron wave function is

\[
< \psi_{p_0}^{(-)} | p_0' > = \delta(p_0 - p_0') + \frac{1}{4\pi} \psi_{001}^{001}(p_0').
\]

The radial part of this wave function $\psi_{001}^{001}(p_0)$ can be expressed by a series of $\delta$-functions:

\[
\psi_{001}^{001}(p_0') = \sum_{j=1}^{N+1} C^{001}(j) \frac{\delta(p_j - p_0)}{p_j^2},
\]

where $p_j (j = 1, \ldots, N)$ are the grid points associated with the Gaussian nodes over the interval $[-1, 1]$ with dimension equal $N$ and $p_{N+1} = p_0$. The coefficients $C(j)$ are determined from the solution of the linear algebraic equations system approximately equivalent to the Lippmann–Schwinger equation for two-neutron scattering \cite{13}. In such a way we get the following expression for the amplitude of the $nd$ charge-exchange process \cite{6}:

\[
\mathcal{J} = \mathcal{J}_{\text{PWIA}} + \mathcal{J}_{^1S_0},
\]

\[
\mathcal{J}_{\text{PWIA}} = \frac{1}{2} < LM_L 1 M_S | 1 M_D > \times \\
\times \left\{ < \frac{1}{2} m'_2 \frac{1}{2} m_3 | 1 M_S > < m_1 m_2, p_1, p_0 + q/2 | t^0 - t^1 | p_0 - q/2, m m_2 ' > \times \\
\times u_L(|p_0 - q/2|) Y_{LM}^M(p_0 - q/2) - \\
- < \frac{1}{2} m'_2 \frac{1}{2} m_2 | 1 M_S > < m_1 m_3, p_1, p_0 - q/2 | t^0 - t^1 | p_0 + q/2, m m_2 ' > \times \\
\times u_L(|p_0 + q/2|) Y_{LM}^M(p_0 + q/2) \right\},
\]

(13)
\[ J_{S_0} = \frac{(-1)^{1-m_2-m'_2}}{2 \cdot 4\pi} \delta_{m_2-m_3} <LM_L1M_S|1M_D> <\frac{1}{2}m''_1 \cdot \frac{1}{2} - m'_2|1M_S> \times \]
\[ \times \int dp_0' p_0'^2 <m_1m'_2, p_1, p_0' + q/2 | t^0 - t^1 | p, p_0' - q/2, mm'' > \psi_{001}(p_0') \times \]
\[ \times u_L(\overrightarrow{p_0' - q/2}) Y^M_L(\hat{p_0'} - q/2). \]

Since the \(q, p_0 \ll p, p_1\), we can neglect the \(q\)- and \(p_0\)- dependences of the high-energy \(np\) \(t\)-matrix in Eqs. (13)-(14). Besides, the integrand in Eq. (14) is suppressed at high \(p_0'\). It offers us the opportunity to consider \(np \rightarrow pn\) vertex as the free \(np\) scattering at angle \(\theta^* = \pi\) in the center-of-mass system (c.m.s.) (or \(np \rightarrow pn\) at \(\theta_{lab} = 0\)). As well known, the \(NN\) \(t\)-matrix is described in collinear geometry by the three independent amplitudes:

\[ t^{NN}_{cm}(\theta^* = \pi) = A(E) + [F(E) - B(E)](\sigma_1 \hat{p}^*)(\sigma_2 \hat{p}^*) + B(E)(\sigma_1 \sigma_2), \]

where \(\hat{p}^*\) is the unit vector in the beam direction in the c.m.s. and \(E\) is the on-shell energy of two nucleons. Note, that this description is correct only for on-energy shell \(t\)-matrix in the center-of-mass system. However, we need the off-energy shell \(t\)-matrix in the frame, which in our kinematics corresponds to the laboratory one. In order to relate two these \(t\)-matrices, we use some results of the relativistic potential theory [15, 16]. This procedure has been presented in details in [5]. Here we give only the final formula:

\[ <m_1m'_2, pp_0'|t|pp_0', mm'' > = NN'F <m_1|D^\dagger(u, p)|\mu_1 > \times \]
\[ \times <m'_2|D^\dagger(u, p'_0)|\mu'_2 > <\mu_1\mu'_2, p^*|t_{cm}|\mu\mu'', p^* > \times \]
\[ \times <\mu|D(u, p)|m > <\mu''|D(u, p'_0)|m'' >, \]

where \(D\) is a Wigner rotation operator in the spin space and \(u\) is a four-velocity. In our kinematical situation, where \(p = p_1 \gg p_0'\), the Wigner rotation operators can be considered as the unity. In the other words, the spin structure of the \(t_{NN}\)-matrix in the reference frame is the same as the spin structure of the \(t_{NN}\)-matrix in the c.m.s.

The normalization factors \(N\) and \(N'\) are determined by the Jacobians for the transformations between the reference frame and c.m.s. In our situation these factors are equal
to each other:
\[ N = N' = \left( \frac{m_N + E_p}{4m_N E_p} \right) \sqrt{2m_N(m_N + E_p)} \]  
(17)

Other coefficient \( F \) is the kinematical factor connected with the transition from the on-energy shell \( t \)-matrix to off-energy shell one
\[ F = \frac{m_N + E_p}{2E_p}. \]
(18)

Since we have neglected the \( p'_0 \) dependence of the high-energy \( np \) \( t \)-matrix, Eq.(14) can be integrated over \( p'_0 \) taking into account Eqs.(8), (12). In order to simplify final expressions, we consider here only \( S \)-component of the deuteron wave function. This assumption does not to influence on the results, since we do not consider in the present paper any polarization observables, for which \( D \)-wave contribution is very important.

\[ \mathcal{J} = \frac{m_N + E_p}{4E_p} \frac{1}{\sqrt{4\pi}} \times \]
\[ \times \left\{ \frac{1}{2} \frac{m'_2}{2} \frac{m_2}{2} 1M_d > < m_1m_2, p^* | t^0_{cm} - t^1_{cm} | p^*, mm'_2 > u_0(|p_0 - q/2|) - \right. \]
\[ - \left. \frac{1}{2} \frac{m'_2}{2} \frac{m_2}{2} 1M_d > < m_1m_3, p^* | t^0_{cm} - t^1_{cm} | p^*, mm'_2 > u_0(|p_0 + q/2|) + \right. \]
\[ \left. + (-1)^{m_2 - m_2} \delta_{m_2, m_3} \frac{1}{2} \frac{m'_2}{2} \frac{m_2}{2} 1M_d > \times \right. \]
\[ \times \sum_{j=1}^{N+1} \sum_{i} \frac{c_j}{q p_j} Q_0 \left( \frac{\alpha^2 + p_j^2 + q^2/4}{p_j q} \right) C^{001}(j) < m_1m'_2, p^* | t^0_{cm} - t^1_{cm} | p^*, mm'' > \} \]

Here \( Q_0(z) \) is the Legendre polynomials of the second kind.

The cross section of the \( nd \rightarrow pnn \) reaction is defined in the standard manner:
\[ \sigma = (2\pi)^4 \frac{E_p}{p} \frac{1}{6} \int d\mathbf{p}_1 d\mathbf{p}_2 \delta(M_d + E_p - E_1 - E_2 - E_3) |\mathcal{J}|^2, \]
(20)

where \( p_3 = p - p_1 - p_2 \) and \( E_3 = \sqrt{m_N^2 + (p - p_1 - p_2)^2} \). The squared amplitude can be obtained straightforwardly from Eq.(19):
\[ |\mathcal{J}|^2 = \frac{1}{2\pi} \left( \frac{m_N + E_p}{4E_p} \right)^2 \left\{ (2B^2(E) + F^2(E))[3u_0^2(|p_0 - q/2|) + 3u_0^2(|p_0 + q/2|) - \right. \]
\[ - \left. 2u_0(|p_0 - q/2|)u_0(|p_0 + q/2|) + \right. \]
\[ \left. + \sum_{j=1}^{N+1} \sum_{i} \frac{2c_i}{q p_j} Q_0 \left( \frac{\alpha^2 + p_j^2 + q^2/4}{p_j q} \right) C^{001}(j) \right\}^2 \]
(21)
\[ + \sum_i \frac{2c_i Q_0}{q p_j} \left( \frac{\alpha_i^2 + p_j^2 + q^2}{p_j q} \right) [C^{001}(j) + C^{001}(j)^*] \times \]
\[ \times \left( u_0(|p_0 - q/2|) + u_0(|p_0 + q/2|) \right) + \]
\[ + 3A^2(E) \left[ u_0(|p_0 + q/2|) - u_0(|p_0 - q/2|) \right]^2 \].

This expression contains both the spin-dependent part of the \( np \) amplitude, \( B \) and \( F \) terms, and the spin-independent one, \( A \) amplitude. However, \( A \) amplitude is multiplied by the difference of the two deuteron wave functions, which depend on the practically undistinguished arguments. The Eq.\&(21)\ can be significantly simplified by the assumption of \( q = 0 \):

\[ |J|^2 \approx \frac{1}{2\pi} \left( \frac{m_N + E_p}{2E_p} \right)^2 (2B^2(E) + F^2(E)) \left\{ u_0(p_0) + \sum_{j=1}^{N+1} u_0(p_j)C^{001}(j) \right\}^2. \quad \text{(22)} \]

In this expression the term corresponding to the spin-independent part of the \( np \rightarrow pm \) amplitude has vanished. Due to that we get the factorization of the squared \( nd \rightarrow pnn \) amplitude in the two parts. One of them depends on the deuteron and two slow-neutron wave functions. Other term corresponds to the spin-dependent component of the elementary \( np \rightarrow pm \) cross section. This result is very important, since it offers us the opportunity to extract the spin-dependent part of the neutron–proton charge-exchange cross section from more complicated reaction with the deuteron participation. But it should be noted, that such factorization is possible due to special kinematical conditions, when the transfer momentum is small in respect to the beam one. Moreover, we have used some model to take account of the two slow neutron final-state interaction. Therefore, the obtained result is model-dependent, what does not allow us to correctly extract the spin dependent part of the \( np \) cross section.

3. Results

In order to relate our calculation with the existing experimental data [1] we consider the ratio of the \( nd \) charge-exchange differential cross section to the free \( np \) scattering
differential cross section at $\theta_{\text{lab}} = 0$:

$$R = \frac{d\sigma(nd \rightarrow pnn)}{dp_1 d\Omega} \frac{d\sigma(np \rightarrow pn)}{d\Omega}.$$  \hspace{1cm} (23)

The recent energy-dependent phase shift analysis data (PSA) [17] have been used for the determination of the $np$ amplitudes, which are needed to define both the $nd$ and $np$ charge-exchange differential cross sections.

This ratio for the initial neutron kinetic energy $T_n = 1$ GeV is presented in Fig.2 as a function of the final-proton momentum $p_1$. The dashed and solid curves correspond to the PWIA and PWIA+FSI calculations, respectively. One can see, the behaviours of these curves are significantly distinguished. The solid line has a very sharp peak, when the momentum $p_1$ is close to the beam momentum $p$, or transfer momentum $q$ is close to zero, while we do not observe any peak for the dashed line. This peak indicates the FSI contribution to the $nd$ differential cross section. In this region the value of the $R$ ratio varies in 10 times, when transfer momentum changes on a few MeV/c. Since any experiment has the limited momentum resolution, we consider also the $R$ ratio integrated over $p_1$ in some region:

$$R_{\text{int}} = \int_{p-\Delta p}^{p} dp_1 R(p_1) = \int_{p-\Delta p}^{p} dp_1 \frac{d\sigma(nd \rightarrow pnn)}{dp_1 d\Omega} \frac{d\sigma(np \rightarrow pn)}{d\Omega}.$$  \hspace{1cm} (24)

Here we introduce a new variable $\Delta p$, which is a small difference between the initial neutron momentum and outgoing proton one. The integration limits change from $p - \Delta p$ up to maximal value of $p_1$ equal to $p$.

The integrated $R$ ratio is shown in Fig 3, in dependence on $\Delta p$ at the neutron kinetic energy $T_n = 1$ GeV. One can see, the PWIA curve is close to the PWIA+FSI one, when $\Delta p$ increases. As it follows from Fig.2, the FSI contribution at first increases and then decreases the $nd$ differential cross section in respect to the PWIA predictions. This influence of the final-state interaction on the $nd$ differential cross section has an effect on the behaviour of the integrated $R$ ratio. In fact, the difference between the PWIA and full calculation results is about 30% for $\Delta p$ equal to 10 MeV/c, about 15% for $\Delta p$ equal to 20 MeV/c and these lines are practically undistinguished, when $\Delta p$ is equal to 60 MeV/c.
The energy dependence of the integrated $R$ ratio is presented in Fig.4. The integration has been performed for $\Delta p$ equal to 30 MeV/c. We investigate the energy region between 800 and 1300 MeV. The upper limit is defined by the existing phase shift analysis data for $np$ scattering. The dash-dotted line is obtained using the result of [13] with NN amplitudes taken from energy-dependent PSA [17]. The difference between result, obtained taking into account FSI, and PWIA result is about 10% for kinetic energy 800 MeV and few per cent for kinetic energy 1300 MeV. Thus, the contribution of the FSI decreases, when the kinetic energy is increasing.

4. Conclusion

In this paper the $nd \rightarrow p(nn)$ reaction has been studied at the neutron kinetic energy $T_n = 0.8 - 1.3$ GeV in kinematics, where transfer momentum is close to zero. It was shown that the expression for the $nd \rightarrow pnn$ differential cross section is factorizable in the two parts. One of them is fully defined by the spin-dependent part of the elementary $np \rightarrow pn$ cross section, while the other part depends on the deuteron and two slow neutrons wave functions. This factorization allows us to extract the spin-dependent part of the $np$ charge-exchange squared amplitude, using the $nd \rightarrow p(nn)$ reaction. But obtained result will be dependent on the model, which was applied for FSI description, and the choice of the deuteron wave function. This fact does not offer the opportunity to get the precise value of the spin-dependent part of the free $np$ scattering amplitude by such method. However, it is possible to extract some useful information about $np$ charge-exchange process (for example, sign, approximate value, etc.).

The other important question, which has been studied, is the role of the final-state interaction of the two slow neutrons. We have considered the $R$ ratio of the $nd$ differential cross section to the elementary $np \rightarrow pn$ one. It was shown, that the final-state interaction gives a considerable contribution into the three-fold $nd \rightarrow pnn$ differential cross section, while the FSI influence on the integrated variables is small. Moreover, the contribution of the FSI into the integrated $R$ ratio decreases with the increasing energy.
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Figure captions

**Fig.1.** Graphic representation of the amplitude of the $nd \rightarrow pnn$ reaction.

**Fig.2.** $R$ ratio vs. the fast proton momentum $p_1$ at $T_n = 1$ GeV.

**Fig.3.** Integrated $R$ ratio as a function of $\Delta p$ at $T_n = 1$ GeV.

**Fig.4.** Energy dependence of the integrated $R$ ratio.
Figure 1: Graphic representation of the amplitude of the $nd \rightarrow pnn$ reaction.
Figure 2: $R$ ratio vs. the fast proton momentum $p_1$ at $T_n = 1$ GeV.
Figure 3: Integrated R ratio as a function of $\Delta p$ at $T_n = 1$ GeV.
Figure 4: Energy dependence of the integrated R ratio.