Bottom-tau unification in supersymmetric SU(5) models with extra matters

So Chigusa* and Takeo Moroi

Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

*E-mail: chigusa@hep-th.phys.s.u-tokyo.ac.jp

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We consider $b$-$\tau$ unification in supersymmetric SU(5) grand unified theories (GUTs) with extra matters. The renormalization group runnings of $b$ and $\tau$ Yukawa coupling constants may be significantly affected by the existence of extra matters. If the extra matters interact with the standard-model particles (and their superpartners) only through gauge interaction, the ratio of the $b$ to $\tau$ Yukawa coupling constants at the GUT scale becomes suppressed compared to the case without extra matters. This is mainly due to the change of the renormalization group running of the SU(3)$_C$ gauge coupling constant. If the extra matters have Yukawa couplings, on the contrary, the (effective) $b$ Yukawa coupling at the GUT scale can be enhanced due to the new Yukawa interaction. Such an effect may improve the $b$-$\tau$ unification in supersymmetric GUTs.

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1. Introduction

The unification of the standard model (SM) gauge groups into a larger group, like in SU(5) grand unified theories (GUTs) (Refs. [1–3]), is an attractive possibility of a new physics beyond the SM. One of the important check points of GUTs is the gauge coupling unification that predicts that the gauge coupling constants of the SM become equal at the unification scale up to threshold corrections. It is well known that, in the SM, there is no strong indication of the unification of the gauge coupling constants. In supersymmetric (SUSY) models, on the other hand, the situation changes because of the existence of the superparticles as well as up- and down-type Higgses. In particular, with the renormalization group equations (RGEs) of the minimal SUSY SM (MSSM), 3 gauge coupling constants more or less meet at the GUT scale $M_{GUT} \sim 10^{16}$ GeV if the mass scale of the superparticles is O(1–10) TeV (Refs. [4–9]).

In simple GUT models based on SU(5), quarks and leptons are embedded into full multiplets of SU(5). In particular, the right-handed down-type quarks and the left-handed lepton doublets are embedded into the anti-fundamental representations of SU(5), resulting in the unification of the down-type and charged lepton Yukawa coupling constants. In particular, the unification of the Yukawa coupling constants of $b$-quark and $\tau$-lepton is an interesting check point of (SUSY) GUTs. Indeed, the $b$-$\tau$ unification based on SUSY GUTs has been extensively studied in many literatures (Refs. [10–21]).

The renormalization group behaviors of the coupling constants are sensitive to the existence of new particles. If full multiplets of SU(5) are added at a single scale, the unification of the gauge coupling constants is unaffected (at least at the one-loop level), although the values of the gauge coupling...
constants depend on the particle content. Contrary to the gauge coupling unification, the unification of the $b$ and $\tau$ Yukawa coupling constants is expected to be significantly affected by new particles, because the renormalization group runnings of Yukawa coupling constants are strongly dependent on the behaviors of the gauge coupling constants. Importantly, there are various candidates for such new particles, like new fermions (as well as their superpartners) to realize Peccei–Quinn symmetry (Ref. [22]), extra chiral superfields in gauge mediation models (Refs. [23–25]), and so on. In addition, the existence of extra matters at the mass scale of the superparticles is required if there exists a nonanomalous discrete $R$-symmetry (Refs. [26,27]). Thus, their effects on the renormalization group runnings of the $b$ and $\tau$ Yukawa coupling constants are of great interest, in particular from the point of view of the Yukawa unification based on SUSY GUTs.

In this paper, we study $b$-$\tau$ unification in SUSY models with extra matters that have gauge quantum numbers under the SM gauge group. We will see that the existence of the extra matters may significantly affect the renormalization group running of the $b$ and $\tau$ Yukawa coupling constants, and hence modify the $b$-$\tau$ unification. As we will discuss, the ratio of the $b$ to $\tau$ Yukawa coupling constants may become very close to 1 if the extra matters have Yukawa couplings with MSSM particles, even though in a large fraction of the parameter space of the MSSM, the Yukawa coupling constant of $b$ becomes sizably smaller than that of $\tau$ at the GUT scale. We will also see that the ratio of the $b$ to $\tau$ Yukawa coupling constants at the GUT scale becomes smaller in models with extra matters if they do not have Yukawa interactions.

2. Model: Brief overview

We first introduce the model we consider. The present analysis is based on Ref. [21], in which $b$-$\tau$ unification in the MSSM was studied considering proper effective theories. In the present study, we include the effects of extra matters in the analysis of Ref. [21]. We study the model with extra matters that can be embedded into complete SU(5) representations. We concentrate on the case where the extra matters are embedded into $5 + \bar{5}$ or $10 + \bar{10}$ representations. The notation for the chiral superfields is summarized in Table 1.

In the model of our interest, the superpotential can be denoted by

$$W = W_{\text{Yukawa}} + \mu H_uH_d + W_5 + W_{10}, \quad (2.1)$$

with $H_u$ and $H_d$ being the up- and down-type Higgses, respectively. Here, $W_{\text{Yukawa}}$ is for Yukawa interaction, while $W_5$ and $W_{10}$ are SUSY invariant mass terms for extra matters. If the Yukawa interactions of the extra matters are negligible, the relevant part of $W_{\text{Yukawa}}$ is given by

$$W_{\text{Yukawa}} = y_b H_d q_L b_R^c + y_\tau H_d l_L^c \tau_R^c + y_t H_u q_L^c t_R^c + \cdots. \quad (2.2)$$

When there exist Yukawa couplings involving extra matters, $W_{\text{Yukawa}}$ is modified as described below in Sect. 3.2. We consider $N_5$ pairs of $5 + \bar{5}$ (or $N_{10}$ pairs of $10 + \bar{10}$), and hence

$$W_5 = \sum_{i=1}^{N_5} (\mu_D D_i^c D_i^c + \mu_L L_i^c L_i^c), \quad (2.3)$$

$$W_{10} = \sum_{i=1}^{N_{10}} (\mu_Q Q_i^c Q_i^c + \mu_U U_i^c U_i^c + \mu_E E_i^c E_i^c), \quad (2.4)$$

1 Hereafter, the SU(3)$_C$ and SU(2)$_L$ indices are omitted for notational simplicity.
As for the SUSY invariant masses of extra matters, we assume that the SUSY breaking bilinear terms are universal for extra matters with the same standard-model representations. (The effects of the trilinear couplings of the extra matters are discussed in Sect. 3.2.) The terms relevant parts of the soft SUSY breaking terms are given by

$$
\mathcal{L}^{(\text{soft})} = \mathcal{L}_{\text{scalar mass}}^{(\text{soft})} + \mathcal{L}_{\text{trilinear}}^{(\text{soft})} + \left( -B\mu H_u H_d - \frac{1}{2} M_1 \tilde{B}\tilde{B} - \frac{1}{2} M_2 \tilde{W}\tilde{W} - \frac{1}{2} M_3 \tilde{g}\tilde{g} + \text{h.c.} \right) + \mathcal{L}_{\tilde{5}}^{(\text{soft})} + \mathcal{L}_{\tilde{10}}^{(\text{soft})} + \cdots ,
$$

where $\tilde{B}$, $\tilde{W}$, and $\tilde{g}$ are Bino, Wino, and gluino, respectively. (The “tilde” is used for SUSY particles.) Here, $\mathcal{L}_{\text{scalar mass}}^{(\text{soft})}$ represents soft SUSY breaking scalar mass terms. Furthermore, $\mathcal{L}_{\text{trilinear}}^{(\text{soft})}$ denotes trilinear couplings; when the trilinear couplings of the extra matters are negligible, it is given by

$$
\mathcal{L}_{\text{trilinear}}^{(\text{soft})} = -A_h H_u \tilde{q}_L \tilde{b}_R - A_t H_d \tilde{t}_L \tilde{t}_R^c - A_t H_u \tilde{q}_L \tilde{t}_R^c + \text{h.c.} + \cdots .
$$

(The effects of the trilinear couplings of the extra matters are discussed in Sect. 3.2.) The terms $\mathcal{L}_{\tilde{5}}^{(\text{soft})}$ and $\mathcal{L}_{\tilde{10}}^{(\text{soft})}$ contain bilinear terms of extra matters,

$$
\mathcal{L}_{\tilde{5}}^{(\text{soft})} = \sum_{i=1}^{N_5} (B_{D_i} \mu_D \tilde{D}_i^c \tilde{D}_i^c + B_{L_i} \mu_L \tilde{L}_i^c \tilde{L}_i^c) + \text{h.c.},
$$

$$
\mathcal{L}_{\tilde{10}}^{(\text{soft})} = \sum_{i=1}^{N_10} (B_{\tilde{D}_i} \mu_D \tilde{\tilde{D}}_i^c \tilde{\tilde{D}}_i^c + B_{\tilde{U}_i} \mu_U \tilde{\tilde{U}}_i^c \tilde{\tilde{U}}_i^c + B_{\tilde{E}_i} \mu_E \tilde{\tilde{E}}_i^c \tilde{\tilde{E}}_i^c) + \text{h.c.}
$$

As for the SUSY invariant masses of extra matters, we assume that the SUSY breaking bilinear terms are universal for extra matters with the same standard-model representations.
Below the mass scale of the SUSY particles, the effective theory contains only the SM particles (as well as the extra matter fermions if the SUSY invariant masses of extra matters are smaller than the masses of SUSY particles). We denote the Lagrangian of such an effective theory as

$$\mathcal{L} = \mathcal{L}^{(SM)}_{\text{kin}} + \left( \bar{\tilde{y}}_b \tilde{H}_{SM} q_L b_R + \bar{\tilde{y}}_\tau \tilde{H}_{SM} l_L \tau^c_R + \bar{\tilde{y}}_l H_{SM} q_L \ell^c_R + \text{h.c.} \right)$$

$$- m^2_{H_{SM}} H^+_{SM} - \frac{\lambda}{2} (H^+_{SM})^2 + \mathcal{L}^{(\text{extra})} + \mathcal{L}^{(\tilde{G})} + \cdots,$$

(2.9)

where $\mathcal{L}^{(SM)}_{\text{kin}}$ represents the kinetic terms of SM fields and $H_{SM}$ is the SM-like Higgs doublet (with $\tilde{H}_{SM} \equiv \epsilon H^2_{SM}$). Yukawa coupling constants for the effective theories below the mass scale of the SUSY particles are denoted by $\bar{\tilde{y}}_b, \bar{\tilde{y}}_\tau$, and $\bar{\tilde{y}}_l$. Furthermore,

$$\mathcal{L}^{(\text{extra})} = \mathcal{L}^{(\text{extra})}_{\text{kin}} - \sum_{i=1}^{N_{\tilde{g}}} (\mu_D \tilde{D}'_i \tilde{D}'_i + \mu_L \tilde{L}'_i \tilde{L}'_i + \text{h.c.})$$

$$- \sum_{i=1}^{N_{\tilde{g}}} (\mu_Q \tilde{Q}'_i \tilde{Q}'_i + \mu_U \tilde{U}'_i \tilde{U}'_i + \mu_E \tilde{E}'_i \tilde{E}'_i + \text{h.c.}),$$

(2.10)

and

$$\mathcal{L}^{(\tilde{G})} = \mathcal{L}^{(\tilde{G})}_{\text{kin}} - \frac{1}{2} \left( M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g} + \text{h.c.} \right),$$

(2.11)

where $\mathcal{L}^{(\text{extra})}_{\text{kin}}$ and $\mathcal{L}^{(\tilde{G})}_{\text{kin}}$ are kinetic terms of extra matter fermions and gauginos, respectively. In Eq. (2.9), $\mathcal{L}^{(\text{extra})}$ and $\mathcal{L}^{(\tilde{G})}$ should be omitted below the mass scale of the extra matter fermions and gauginos, respectively.

Some of the Lagrangian parameters are related to each other at the GUT scale $M_{\text{GUT}}$. (In our analysis, we define $M_{\text{GUT}}$ as the scale at which $U(1)_Y$ and $SU(2)_L$ gauge coupling constants become equal.) For simplicity, we assume that the SUSY breaking scalar mass parameters are degenerate at the GUT scale for scalars with the same $SU(5)$ representations. For the bilinear terms and the soft SUSY breaking parameters, we neglect the threshold corrections at the GUT scale. Then, we parameterize the Lagrangian parameters at $Q = M_{\text{GUT}}$ (with $Q$ being the renormalization scale) as

$$\mu_D(M_{\text{GUT}}) = \mu_L(M_{\text{GUT}}) \equiv \mu_5,$$

(2.12)

$$\mu_Q(M_{\text{GUT}}) = \mu_U(M_{\text{GUT}}) = \mu_E(M_{\text{GUT}}) \equiv \mu_{10},$$

(2.13)

$$m^2_D(M_{\text{GUT}}) = m^2_L(M_{\text{GUT}}) \equiv m^2_5,$$

(2.14)

$$m^2_D(M_{\text{GUT}}) = m^2_L(M_{\text{GUT}}) = m^2_{10},$$

(2.15)

$$m^2_Q(M_{\text{GUT}}) = m^2_U(M_{\text{GUT}}) = m^2_E(M_{\text{GUT}}) \equiv m^2_{10},$$

(2.16)

$$m^2_Q(M_{\text{GUT}}) = m^2_U(M_{\text{GUT}}) = m^2_E(M_{\text{GUT}}) = m^2_{10},$$

(2.17)

$$m^2_{H_u}(M_{\text{GUT}}) \equiv m^2_{H_5},$$

(2.18)

$$m^2_{H_d}(M_{\text{GUT}}) \equiv m^2_{H_5},$$

(2.19)

\(^2\)We use same notation for the SM\text{ex} fermions and the corresponding superfields.
Table 2. Effective theories used in our analysis.

| Effective theories | Particle content | RGEs   |
|--------------------|------------------|--------|
| SM                 | SM particles     | two-loop |
| SM_{ex}            | SM particles and extra fermions | two-loop |
| GSM                | SM particles and gauginos | two-loop |
| GSM_{ex}           | SM particles, gauginos, and extra fermions | two-loop |
| MSSM               | MSSM particles   | two-loop |
| MSSM_{ex}          | MSSM particles and extra matters | two-loop |

\[
A_b(M_{\text{GUT}}) = A_t(M_{\text{GUT}}) \equiv a_d, \tag{2.20}
\]

\[
A_t(M_{\text{GUT}}) \equiv a_u, \tag{2.21}
\]

\[
B_D(M_{\text{GUT}}) = B_L(M_{\text{GUT}}) = m_\xi, \tag{2.22}
\]

\[
B_Q(M_{\text{GUT}}) = B_U(M_{\text{GUT}}) = B_E(M_{\text{GUT}}) = m_{10}, \tag{2.23}
\]

where \(m^2_D\), \(m^2_L\), \(m^2_Q\), \(m^2_U\), and \(m^2_E\) are soft SUSY breaking mass squared parameters of \(\tilde{b}_R^c\), \(\tilde{l}_L\), \(\tilde{q}_L\), \(\tilde{t}_R\), and \(\tilde{\tau}_R\), respectively. In addition, we impose the same boundary condition for \(m^2_\Phi\) as \(m^2_\Phi\) (\(\Phi = D, L, Q, U, E\)). For gaugino masses, we adopt the simple GUT relation

\[
M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) \equiv m_{1/2}. \tag{2.24}
\]

The mass spectrum of the SUSY particles (including those in the extra matter sector) is determined by solving the RGEs with the boundary conditions given above. Importantly, the masses of the scalars are comparable to or larger than the gaugino masses in the model of our interest because of the renormalization group effects. We also comment here that the gaugino masses may be much smaller than the scalar masses, as suggested by several models of SUSY breaking (Refs. [28–30]). Thus, we do not exclude the possibility that the gaugino masses are hierarchically smaller than the scalar masses.

In order to calculate the renormalization group running of coupling constants from the weak scale to the GUT scale \(M_{\text{GUT}}\), we consider several effective theories; the particle contents of all the effective theories used in our analysis are summarized in Table 2. In the present analysis, there are three important mass scales, i.e., the gaugino mass scale \(M_\tilde{G}\), the sfermion mass scale \(M_S\), and the extra fermion mass scale \(M_{\text{ex}}\), at which the effective theory changes from one to another. As we have mentioned, the scalar masses are comparable to or larger than the gaugino masses, and hence \(M_{\tilde{G}} < M_S\). In addition, the masses of the scalars in the extra matter sector have two contributions, i.e., SUSY invariant mass parameters and soft SUSY breaking masses (denoted by \(\mu_\Phi\) and \(m^2_\Phi\), respectively); the scalar masses in the extra matter sector are \(\sim (\mu^2_\Phi + m^2_\Phi)^{1/2}\). Thus, we assume that the scalars in the extra matter sector decouple from the effective theory at the renormalization scale \(Q = \text{max}(M_S, M_{\text{ex}})\). The relevant effective theory for each renormalization scale depends on \(M_{\tilde{G}}\), \(M_S\), and \(M_{\text{ex}}\), as summarized in Table 3.

For all the effective theories mentioned above, we use two-loop RGEs; for SUSY models, we use the Susyno package (Ref. [31]), while the RGEs for non-SUSY theories are calculated based on Refs. [32–34]. In addition, at each energy threshold, one effective theory is matched to another, taking into account one-loop threshold corrections to Lagrangian parameters. In the following, we summarize the important effects.
the mass of the pseudo-scalar Higgs, which is a component of the heavy Higgs multiplet

$$\delta \lambda_{MS}$$

approximation, these contributions are given by

sbottom-gluino and stop-chargino diagrams (Refs. [35–37]); at the leading order of the mass-insertion for the Higgs quartic coupling

$$\tilde{\lambda}$$

where

$$\tan \beta$$

is the ratio of the vacuum expectation value of $$H_u^0$$ to that of $$H_d^0$$. The boundary condition for the Higgs quartic coupling $$\lambda$$ at $$M_S$$ is

$$\lambda(M_S) = \frac{g_2^2(M_S) + g_3^2(M_S)}{4} \cos^2 2\beta + \delta \lambda,$$

(2.26)

where $$\delta \lambda$$ is the threshold correction due to heavy scalar particles (in particular, stops). In addition, the mass of the pseudo-scalar Higgs, which is a component of the heavy Higgs multiplet $$H_{\text{heavy}} = H_u \cos \beta - H_d^+ \sin \beta$$, is determined at this scale as

$$m_A^2 = [m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 - m_{H_{\text{SM}}}^2]_{Q=M_S},$$

(2.27)

where $$\mu^2$$ is determined from the following radiative electroweak symmetry breaking condition:

$$\mu^2 = -m_{H_u}^2 - m_{H_d}^2 \sin^2 \beta - m_{H_d}^2 \cos^2 \beta + B\mu \sin 2\beta,$$

(2.28)

$$B\mu = \frac{1}{2}(m_{H_u}^2 - m_{H_d}^2) \tan 2\beta.$$  

(2.29)

The Yukawa coupling constants $$y_f$$ (with $$f = t, b, and \tau$$) are matched to $$\tilde{y}_f$$ at $$Q = M_S$$, using the mixing angle $$\beta$$. In our analysis, the threshold correction to the bottom Yukawa coupling constant at $$Q = M_S$$ is important. The correction $$\Delta_b$$ is defined by

$$\tilde{y}_b(M_S) = y_b(M_S) \cos \beta (1 + \Delta_b),$$

(2.30)

where $$\tilde{y}_b$$ and $$y_b$$ are the bottom quark Yukawa coupling constants in the effective theory used just below and just above $$M_S$$, respectively. The most important contributions to $$\Delta_b$$ come from the sbottom-gluino and stop-chargino diagrams (Refs. [35–37]); at the leading order of the mass-insertion approximation, these contributions are given by

$$\Delta_b \simeq \left[ g_3^2 \frac{3}{16\pi^2} M_3 I(m_{b_1}^2, m_{b_2}^2, M_3^2) + \frac{y_t}{16\pi^2} A_1 I(m_{t_1}^2, m_{t_2}^2, \mu^2) \right] \mu \tan \beta,$$

(2.31)

where $$m_{b_1}$$ and $$m_{b_2}$$ (and $$m_{t_1}$$ and $$m_{t_2}$$) are masses of lighter (heavier) stop and sbottom, respectively, and

$$I(a, b, c) = -\frac{ab \ln(a/b) + bc \ln(b/c) + ca \ln(c/a)}{(a-b)(b-c)(c-a)}.$$  

(2.32)

Table 3. Effective theories for each renormalization scale $$Q$$.

| Effective theories | $$M_{ex} < M_G < M_S$$ | $$M_G < M_{ex} < M_S$$ | $$M_S < M_G < M_{ex}$$ | $$M_S < M_G < M_{ex}$$ |
|--------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| SM                 | $$m_t < Q < M_{ex}$$     | $$m_t < Q < M_S$$        | $$m_t < Q < M_{ex}$$     | $$m_t < Q < M_{ex}$$     |
| SM$^*$             | $$M_{ex} < Q < M_G$$     | $$M_{ex} < Q < M_S$$     | $$M_{ex} < Q < M_G$$     | $$M_{ex} < Q < M_S$$     |
| GSM                | $$M_G < Q < M_{ex}$$     | $$M_G < Q < M_S$$        | $$M_S < Q < M_{ex}$$     | $$M_S < Q < M_{ex}$$     |
| GSM$^*$            | $$M_G < Q < M_S$$        | $$M_S < Q < M_{ex}$$     | $$M_S < Q < M_{ex}$$     | $$M_S < Q < M_{ex}$$     |
| MSSM               | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    |
| MSSM$^*$           | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    | $$M_S < Q < M_{GUT}$$    |

At the sfermion mass scale $$M_S$$, two Higgs doublets in MSSM(ex) are matched to the SM-like Higgs as

$$H_{\text{SM}} = H_u \sin \beta + H_d^+ \cos \beta,$$

(2.25)

where $$\tan \beta$$ is the ratio of the vacuum expectation value of $$H_u^0$$ to that of $$H_d^0$$. The boundary condition for the Higgs quartic coupling $$\lambda$$ at $$M_S$$ is

$$\lambda(M_S) = \frac{g_2^2(M_S) + g_3^2(M_S)}{4} \cos^2 2\beta + \delta \lambda,$$

(2.26)

where $$\delta \lambda$$ is the threshold correction due to heavy scalar particles (in particular, stops). In addition, the mass of the pseudo-scalar Higgs, which is a component of the heavy Higgs multiplet $$H_{\text{heavy}} = H_u \cos \beta - H_d^+ \sin \beta$$, is determined at this scale as

$$m_A^2 = [m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 - m_{H_{\text{SM}}}^2]_{Q=M_S},$$

(2.27)

where $$\mu^2$$ is determined from the following radiative electroweak symmetry breaking condition:

$$\mu^2 = -m_{H_u}^2 - m_{H_d}^2 \sin^2 \beta - m_{H_d}^2 \cos^2 \beta + B\mu \sin 2\beta,$$

(2.28)

$$B\mu = \frac{1}{2}(m_{H_u}^2 - m_{H_d}^2) \tan 2\beta.$$  

(2.29)

The Yukawa coupling constants $$y_f$$ (with $$f = t, b, and \tau$$) are matched to $$\tilde{y}_f$$ at $$Q = M_S$$, using the mixing angle $$\beta$$. In our analysis, the threshold correction to the bottom Yukawa coupling constant at $$Q = M_S$$ is important. The correction $$\Delta_b$$ is defined by

$$\tilde{y}_b(M_S) = y_b(M_S) \cos \beta (1 + \Delta_b),$$

(2.30)

where $$\tilde{y}_b$$ and $$y_b$$ are the bottom quark Yukawa coupling constants in the effective theory used just below and just above $$M_S$$, respectively. The most important contributions to $$\Delta_b$$ come from the sbottom-gluino and stop-chargino diagrams (Refs. [35–37]); at the leading order of the mass-insertion approximation, these contributions are given by

$$\Delta_b \simeq \left[ g_3^2 \frac{3}{16\pi^2} M_3 I(m_{b_1}^2, m_{b_2}^2, M_3^2) + \frac{y_t}{16\pi^2} A_1 I(m_{t_1}^2, m_{t_2}^2, \mu^2) \right] \mu \tan \beta,$$

(2.31)

where $$m_{b_1}$$ and $$m_{b_2}$$ (and $$m_{t_1}$$ and $$m_{t_2}$$) are masses of lighter (heavier) stop and sbottom, respectively, and

$$I(a, b, c) = -\frac{ab \ln(a/b) + bc \ln(b/c) + ca \ln(c/a)}{(a-b)(b-c)(c-a)}.$$  

(2.32)
In our numerical analysis, we use the full one-loop expression of $\Delta_b$. The important point is that $\Delta_b$ is approximately proportional to $\mu \tan \beta$, resulting in the large correction to the bottom Yukawa coupling constant in the models with heavy Higgsinos or those with large $\tan \beta$.

We also include threshold corrections to the Wino and Bino masses at $Q = M_S$ due to the Higgs–Higgsino loop diagram (Ref. [38]):

$$
\delta M_1 = \frac{g_1^2(M_S)}{16\pi^2} L, \quad \delta M_2 = \frac{g_2^2(M_S)}{16\pi^2} L,
$$

where

$$
L = \mu \sin 2\beta \frac{m_A^2}{\mu^2 - m_A^2} \ln \frac{\mu^2}{m_A^2}.
$$

At $Q = M_{\tilde{G}}$ and $Q = M_{\text{ex}}$, we take into account one-loop threshold corrections to gauge coupling constants, gaugino masses, and scalar masses due to loop diagrams involving gauginos and extra matters. Then, at $Q = m_t$ the SM-like Higgs mass is evaluated as

$$
m_h^2 = 2\lambda(m_t)v^2 + \delta m_h^2,
$$

where $v \simeq 174$ GeV is the vacuum expectation value of the SM-like Higgs boson and $\delta m_h^2$ is the threshold correction.

3. Numerical results

In this section, we show the results of our numerical study. In addition to the SM parameters, the present model contains 10 new parameters: $\tan \beta$, $m_{\tilde{t}_2}^2$, $m_{\tilde{t}_10}^2$, $m_{\tilde{H}_S}^2$, $m_{\tilde{H}_S}^2$, $m_{1/2}$, $\mu$, $B$, $\mu_5$, and $\mu_{10}$, ignoring the Yukawa and the trilinear couplings related to the extra matters. Among them, $\mu$ and $B$ are determined at the sfermion mass scale $M_S$ to fix the vacuum expectation value of the SM-like Higgs boson $v$ and $\tan \beta$.

We numerically solve RGEs from the weak scale to the GUT scale. Our numerical calculation is based on the SOFTSUSY package (Ref. [39]), in which three-loop RGEs for the effective theory below the electroweak scale and two-loop RGEs for the MSSM are implemented. We have implemented into SOFTSUSY package two-loop RGEs for the other effective theories listed in Table 3, i.e., SM, SM$_{\text{ex}}$, $\tilde{G}$SM, $\tilde{G}$SM$_{\text{ex}}$, and MSSM$_{\text{ex}}$. In addition, one-loop threshold corrections due to the diagrams with SUSY particles or extra matters in the loop are included at relevant thresholds.

In our numerical calculation, $M_S$ is taken to be the geometric mean of the stop masses, while we take $M_{\tilde{G}} = M_3$; $M_{\text{ex}}$ is set to the mass of the bottom-like extra fermion mass $\mu_{1D}$ for models with $N_5 > 0$, and is set to the geometric mean of top-like extra fermion masses for models with $N_{10} > 0$. The gauge and Yukawa coupling constants are determined based on Ref. [40]. In particular, we use the bottom quark mass of $m_b^{(\overline{MS})}(m_b) = 4.18$ GeV, the top quark mass of $m_t = 173.21$ GeV, and $\alpha_3(M_Z) = 0.1185$ (with $\alpha_3 = g_3^2/4\pi$).

3.1. Extra matters without Yukawa couplings

Let us now study the effects of extra matters on the $b$-$\tau$ unification in SUSY GUT. We first consider the case where the extra matters interact with the MSSM particles only through gauge interactions.

Because the boundary conditions for the Yukawa coupling constants are fixed by using the fermion masses, $y_b(M_{\text{GUT}})$ and $y_\tau(M_{\text{GUT}})$ may differ in the present analysis. To quantify the difference, we
define
\[ R_{bt} \equiv \frac{y_b(M_{\text{GUT}})}{y_t(M_{\text{GUT}})}. \] (3.1)

We calculate \( R_{bt} \) as a function of model parameters, and study how it is affected by extra matters. If there is no source of GUT-scale threshold corrections other than the splitting of the masses of GUT-scale particles, then \( (R_{bt} - 1) \sim O(1) \)%. Thus, if \( R_{bt} \) is (much) larger than \( \sim O(1) \)% it indicates a sizable threshold correction at the GUT scale and/or a nontrivial flavor physics at the GUT scale or below.

If the Yukawa interactions of the extra matters are negligible, the main effect of extra matters on the Yukawa unification is through the enhancement of the gauge coupling constants at a high energy scale. With extra matters, the coupling constants of SU(3)\(_C\) × SU(2)\(_L\) × U(1)\(_Y\) become larger. This can be understood by examining the RGEs of the gauge coupling constants; in SUSY models, they are given by
\[
\frac{d}{d \ln Q} g_a = \frac{g_a^3}{16\pi^2} \left[ b_a + (N_5 + 3N_{10}) \right] + \cdots, \quad (3.2)
\]
where \((b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)\), and \( \cdots \) in the above equation denotes higher-order effects. One can see that, with nonvanishing \( N_5 \) or \( N_{10} \), the beta-function coefficients become larger, resulting in the enhancement of the gauge coupling constants at higher scale. In particular, the enhancement of \( g_3 \) is the most important because of the largeness of the coupling constant \( g_3 \) itself. The enhanced gauge coupling constants affect the renormalization group running of \( y_b \) and \( y_t \), whose RGEs are given by
\[
\frac{d}{d \ln Q} y_b = \frac{y_b}{16\pi^2} \left( y_t^2 + 6y_b^2 + y_t^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right) + \cdots, \quad (3.3)
\]
\[
\frac{d}{d \ln Q} y_t = \frac{y_t}{16\pi^2} \left( 3y_b^2 + 4y_t^2 - \frac{9}{5}g_1^2 - 3g_2^2 \right) + \cdots, \quad (3.4)
\]
where the mixings between different generations are neglected. With the low-scale values of the Yukawa coupling constants being fixed to realize the observed fermion masses, the above equations indicate that the Yukawa coupling constants at \( M_{\text{GUT}} \) are more suppressed as the gauge coupling constants become larger. Due to this effect, \( y_b \) is more suppressed than \( y_t \) because \( g_3 \) affects only the running of \( y_b \).

In Fig. 1, in order to investigate how these effects affect \( R_{bt} \), we show \( R_{bt} \) as a function of the mass scale of extra matters. The red, green, and blue lines correspond to the models with \( (N_5, N_{10}) = (1, 0), (2, 0), \) and \((0, 1)\), respectively. (Thus, the horizontal axis corresponds to \( \mu_5 \) for red and green lines and \( \mu_{10} \) for blue lines.) The dotted and solid lines are for models with \( \mu > 0 \) and \( \mu < 0 \), respectively. For the left figure, we take mSUGRA-like boundary conditions, \( m_{\tilde{5}} = m_{10} = m_{1/2} = 100 \) TeV, \( m_{H5} = m_{H\tilde{5}} = 80 \) TeV, and \( a_d = a_u = 0 \). Here, \( \tan \beta \) is determined so that the SM-like Higgs mass is given by the observed value \( m_h = 125.09 \) GeV; then, it takes values in the range \( 2.9 < \tan \beta < 3.1 \). The right figure shows the results for the model with gaugino mediation boundary conditions (Refs. [41,42]), taking \( \tan \beta = 50 \), \( m_{\tilde{5}} = m_{10} = m_{H5} = m_{H\tilde{5}} = 0 \), and \( a_d = a_u = 0 \). In this case, \( m_{1/2} \) is tuned so that \( m_h \) is equal to the observed Higgs mass, which gives \( 4.5 \) TeV \( < m_{1/2} < 6.5 \) TeV.

8/14
With such new interaction, the renormalization group runnings of the coupling and mass parameters effects of extra matters make case that high tan $\beta$ may help to make the extra matter in the case that this is indeed the case. Among several possibilities, we introduce the Yukawa interaction for the extra matters may couple to MSSM chiral multiplets via Yukawa couplings. In this subsection, we show small. However, the extra matters may couple to MSSM chiral multiplets via Yukawa couplings. So far, we have considered the case where the Yukawa interactions of the extra matters are negligibly small. However, the extra matters may couple to MSSM chiral multiplets via Yukawa couplings. With such new interaction, the renormalization group runnings of the coupling and mass parameters may be changed, affecting the unification of Yukawa coupling constants. In this subsection, we show that this is indeed the case. Among several possibilities, we introduce the Yukawa interaction for the extra matter in the $10$ representation of SU(5). We will see that, in such a model, the extra matters may help to make the $b-\tau$ unification successful.

Fig. 1. We show $R_{bt}$ as a function of the mass scale of the extra matters for models with low tan $\beta$ (left) and high tan $\beta$ (right), taking $(N_S, N_{10}) = (1, 0)$ (red), $(2, 0)$ (green), and $(0, 1)$ (blue). The horizontal axis denotes the value of $\mu_S$ for the red and green lines and that of $\mu_{10}$ for the blue lines. We consider both signs of the SUSY invariant Higgs mass: $\mu > 0$ (dotted) and $\mu < 0$ (solid). The boundary conditions used in the left figure are $m_3 = m_{10} = m_{1/2} = 100$ TeV, $m_H = m_{H^0} = 80$ TeV, and $a_d = a_u = 0$. The value of tan $\beta$ is determined so that $m_h = 125.09$ GeV, which results in $2.9 < \tan \beta < 3.1$. The boundary conditions for the right figure are $\tan \beta = 50$, $m_3 = m_{10} = m_{1/2} = 0$, and $a_d = a_u = 0$. The value of $m_{1/2}$ is determined so that $m_h = 125.09$ GeV, which results in $4.5$ TeV $< m_{1/2} < 6.5$ TeV.

As can be easily understood, the effects of extra matters on the runnings of $y_b$ and $y_\tau$ are more enhanced as the masses of the extra matters become smaller. We can see that $R_{bt}$ is suppressed by $\sim 10\%$ when the mass scale of the extra matters is at around the TeV scale, while $R_{bt}$ approaches the MSSM value when the mass scale is at around the TeV scale, while $R_{bt}$ becomes smaller. However, the suppression of $R_{bt}$ due to the enhancement of $g_3$ at a higher scale is more significant; consequently, $R_{bt}$ becomes smaller as $M_{ex}$ decreases, as shown in Fig. 1.

3.2. Extra matters with Yukawa couplings

So far, we have considered the case where the Yukawa interactions of the extra matters are negligibly small. However, the extra matters may couple to MSSM chiral multiplets via Yukawa couplings. With such new interaction, the renormalization group runnings of the coupling and mass parameters may be changed, affecting the unification of Yukawa coupling constants. In this subsection, we show that this is indeed the case. Among several possibilities, we introduce the Yukawa interaction for the extra matter in the $10$ representation of SU(5). We will see that, in such a model, the extra matters may help to make the $b-\tau$ unification successful.

Here, we consider the case with $(N_S, N_{10}) = (0, 1)$, and study the effect of the Yukawa interactions of extra matters. We concentrate on the case where the extra matter scale is comparable to or higher than $M_S$ so that all the extra matters (i.e., fermions and scalars) simultaneously decouple from the effective theory at a single scale $M_{ex}$.

For the study of such a case, it is instructive to use the fact that the Yukawa interaction above the GUT scale can be written in the following form:
\[ W_{\text{Yukawa}}^{(SU(5))} = \eta_{b,Y} \tilde{5}_H 10_Y \tilde{F} + 5_H \left( \begin{array}{cc} 10_Y & 10_0 \\ \end{array} \right) \left( \begin{array}{c} \eta_{t}^{(1)} \\ \eta_{t}^{(2)} \end{array} \right) \left( \begin{array}{c} 10_Y \\ 10_0 \end{array} \right), \quad (3.5) \]

where \( \eta \)'s are coupling constants. Here, \( 10_Y \) and \( 10_0 \) are chiral multiplets in the \( 10 \) representation of \( SU(5) \), and are given by linear combinations of \( T \) and \( T' \). Notice that, in this basis, only \( 10_Y \) couples to \( \bar{F} \) though the Yukawa interaction. In addition, \( 5_H \) and \( \bar{5}_H \) are chiral multiplets containing up- and down-type Higgses, respectively. In order to make our point clearer, we take \( \eta_{t}^{(1)} = \eta_{t}^{(2)} = 0 \) in the following analysis. With such an assumption, the Yukawa interaction below the GUT scale is given by

\[ W_{\text{Yukawa}} = y_b^{'} H_d Q_Y b^c + y_t^{'} H_d l_L E_Y + y_t^{'} H_0 Q_Y U_Y, \quad (3.6) \]

where \( Q_Y, U_Y, \) and \( E_Y \) are chiral superfields embedded into \( 10_Y \). (Chiral multiplets embedded into \( 10_0 \) are denoted by \( Q_0, U_0, \) and \( E_0 \).) Denoting the SUSY invariant mass terms for the extra matters by

\[ W_{10} = \sum_{\Phi=Q,U,E} \left( \mu_{\Phi} \Phi \bar{\Phi} + \mu_{\Phi_0} \Phi_0 \bar{\Phi} \right), \quad (3.7) \]

we obtain

\[ \left( \begin{array}{c} Q_Y \\ Q_0 \end{array} \right) = \left( \begin{array}{cc} \cos \theta_Q & \sin \theta_Q \\ -\sin \theta_Q & \cos \theta_Q \end{array} \right) \left( \begin{array}{c} q_L \\ Q' \end{array} \right), \quad (3.8) \]

where

\[ \cos \theta_Q = \frac{\mu_{Q_0}}{\mu_Q}, \quad \sin \theta_Q = \frac{\mu_{Q_Y}}{\mu_Q}, \quad (3.9) \]

with \( \mu_Q = (\mu_{Q_Y}^2 + \mu_{Q_0}^2)^{1/2} \). Similar relations hold for \( (U_Y, U_0) \) and \( (E_Y, E_0) \), with the mixing angles

\[ \theta_{U,E} = \cos^{-1} (\mu_{U_0,E_0}/(\mu_{U_Y,E_Y}^2 + \mu_{U_0,E_0}^2)^{1/2}). \]

At the mass scale of the extra matters, MSSM Yukawa coupling constants are given by

\[ y_b(M_{\text{ex}}) = y_b^{'}(M_{\text{ex}}) \cos \theta_Q, \quad (3.10) \]
\[ y_t(M_{\text{ex}}) = y_t^{'}(M_{\text{ex}}) \cos \theta_Q \cos \theta_U, \quad (3.11) \]
\[ y_t(M_{\text{ex}}) = y_t^{'}(M_{\text{ex}}) \cos \theta_Q \cos \theta_U. \quad (3.12) \]

In the present setup, the Yukawa structure is like that of the MSSM as Eq. (3.6) is obtained from the Yukawa interaction of the MSSM by replacing \( y_{t,b,\tau} \rightarrow y_{t,b,\tau}^{'} \) and \( (q_L, t_R, \tau_R) \rightarrow (Q_Y, U_Y, E_Y) \). Numerically, however, such a replacement may give significant effects on the Yukawa unification. This is because, as shown in Eq. (3.12), \( y_t^{'} \) can be significantly larger than \( y_t \) because \( \cos \theta_Q \cos \theta_U < 1 \).
Such an enhancement of the coupling constant may have the following consequences:

1. \( y'_b (M_{\text{GUT}}) \) is enhanced through the renormalization group effect while \( y'_t (M_{\text{GUT}}) \) is not (see Eqs. (3.3) and (3.4)).
2. \( m^2_{H_0} (M_S) \) is suppressed through the renormalization group effect. This can be understood from the RGE of \( m^2_{H_0} \), which is given by

\[
\frac{d}{dt} m^2_{H_0} = 6 y'_t (m^2_{H_0} + m^2_Q + m^2_U + A^2_t) + \cdots ,
\]

(3.13)

where only the \( y'_t \)-dependence of the beta-function at the one-loop level is shown in the above equation. This may result in the enhancement of \( |\mu| \) and \( |\Delta_b| \).

In order to study the effects of the Yukawa couplings of the extra matters on the \( b-\tau \) unification, we solve the RGEs numerically, taking into account the effects of extra Yukawa couplings. We assume that the threshold correction to the SUSY invariant masses of extra matters are negligible, and that they are unified at the GUT scale; we parameterize their boundary conditions as

\[
\mu_{Q_b} (M_{\text{GUT}}) = \mu_{U_0} (M_{\text{GUT}}) = \mu_{E_0} (M_{\text{GUT}}) \equiv \mu_{10},
\]

(3.14)

\[
\mu_{Q_t} (M_{\text{GUT}}) = \mu_{U_t} (M_{\text{GUT}}) = \mu_{E_t} (M_{\text{GUT}}) \equiv X \mu_{10}.
\]

(3.15)

Here, \( X^{-1} \) is approximately equal to \( \cos \theta_\Phi \) (with \( \Phi = Q, U, E \)), although they slightly differ because of the renormalization group effects from the GUT scale to the extra matter scale. (In our numerical analysis, we have taken into account the effects of the renormalization group running of SUSY invariant mass parameters.) In addition, for \( Q > M_{\text{ex}} \), the scalars in the extra matter sector have trilinear interactions. We assume that, at the GUT scale, the trilinear couplings are proportional to the corresponding Yukawa coupling constants, and that the trilinear interactions above the extra matter scale are given by

\[
\mathcal{L}^{(\text{soft}) \text{trilinear}} = -A'_i H_d \tilde{Q}_Y \tilde{b}_R - A'_i H_d \tilde{l}_L \tilde{E}_Y - A'_i H_u \tilde{\bar{Q}}_Y \tilde{U}_Y,
\]

(3.16)

with the boundary conditions \( A'_i (M_{\text{GUT}}) = A'_i (M_{\text{GUT}}) = A'_d \) and \( A'_i (M_{\text{GUT}}) = A'_u \). Furthermore, the SUSY breaking mass parameters above the extra matter scale can be written as

\[
\mathcal{L}^{(\text{soft}) \text{scalar mass}} = \sum_{\Phi = Q, U, E} (m^2_{\Phi_Y} |\tilde{\Phi}_Y|^2 + m^2_{\Phi_0} |\tilde{\Phi}_0|^2) + \cdots .
\]

(3.17)

Notice that, with the present choice of parameters, there is no mixing term between \( \tilde{\Phi}_Y \) and \( \tilde{\Phi}_0 \). Soft SUSY breaking parameters defined above and below the extra matter scale are matched at \( Q = M_{\text{ex}} \) using Eq. (3.8) (as well as the mixing angles for \( (U_Y, U_0) \) and \( (E_Y, E_0) \)).

We show examples of the running of Yukawa coupling constants in Fig. 2, which demonstrates the possibility of a successful unification of the Yukawa coupling constants due to the effects of the Yukawa interactions of extra matters. Solid lines show the results for the present model, while dotted lines denote the results for the MSSM as a reference. Here, we take \( \tan \beta = 27, m_{\tilde{g}} = m_{\tilde{\mu}} = m_{\tilde{\mu}} = m_{1/2} = 3 \text{ TeV}, a_d = a_u = 0, \mu_{10} = 10^{10} \text{ GeV}, \) and \( X = 1.4 \). The SM-like Higgs mass in each model is adjusted to be the observed value using \( m_{10} \), which gives \( m_{10} = 14 \text{ TeV} \) for the model with \( (N_5, N_1) = (0, 1) \), and \( 12 \text{ TeV} \) for the MSSM. The vertical dotted lines denote the matching scales in the model with extra matters: from left to right, they correspond to \( Q = m_t, M_{\tilde{G}}, M_S, M_{\text{ex}}, \) and
Fig. 2. Runnings of \( \bar{y}_t \) and \( y_t^{(1)} \) sin \( \beta \) (left) and those of \( \bar{y}_b, y_b^{(1)} \) cos \( \beta \), \( \bar{y}_t \), and \( y_t^{(1)} \) cos \( \beta \) (right) in the model with \((N_5, N_{10}) = (0, 1)\) (solid lines) and in the MSSM (dotted lines). Here, we take \( \tan \beta = 27, m_t = m_{\tilde{t} \ell} = m_{\tilde{b} \ell} = m_{1/2} = 3 \text{ TeV}, a_d = a_u = 0, \mu_{10} = 10^{10} \text{ GeV}, \) and \( X = 1.4 \). Here, \( m_{10} \) is tuned to adjust the SM-like Higgs mass to be \( m_h = 125.09 \text{ GeV} \), which gives \( m_{10} = 14 \text{ TeV} \) for the model with \((N_5, N_{10}) = (0, 1)\) and \( m_{10} = 12 \text{ TeV} \) for the MSSM. The vertical dotted lines denote the matching scales in the model with extra matters: \( Q = m_1, M_G, M_5, M_8, \) and \( M_{\text{GUT}} \) from left to right. For \( M_5 < Q < M_8 \) \((Q > M_8)\), the solid lines denote \( y_t \) sin \( \beta \) \((y_t' \sin \beta)\) or \( y_t \cos \beta \) \((y_t' \cos \beta)\) with \( f = b, \) or \( \tau \).

Fig. 3. Mixing parameter \( X \) dependence of \( R_{b\tau} \) defined in Eq. (3.18). The parameters used are the same as Fig. 2. The value of \( m_{10} \) is again used to adjust the SM-like Higgs mass to be \( m_h = 125.09 \text{ GeV} \) at each value of \( X \). The red and the green lines denote the model with \( \mu_{10} = 10^{10} \text{ GeV} \) and \( \mu_{10} = 10^6 \text{ GeV} \), respectively.

\( M_{\text{GUT}} \), respectively. The large “jumps” of solid lines in Fig. 2 at \( Q = M_S \) are due to the threshold corrections, while those at \( Q = M_{\text{ex}} \) are mainly due to the mixing effect represented in Eqs. (3.10) and (3.11) since we plot \( y_b' \cos \beta \) and \( y_t' \cos \beta \) instead of \( y_b \cos \beta \) and \( y_t \cos \beta \) with solid lines in the range \( Q > M_{\text{ex}} \). We can see from the figure that the enhancement of \(|\Delta_{b\tau}| \) significantly modifies the prediction for Yukawa unification. Together with the change in the running of \( y_b \) as we discussed before, the mixing \( X > 1 \) enlarges the prediction for \( R_{b\tau}' \), which is defined as

\[
R_{b\tau}' \equiv \frac{y_b(M_{\text{GUT}})}{y_{b\tau}(M_{\text{GUT}})}.
\]

It becomes almost 1 in the present choice of parameters, while \( R_{b\tau} \simeq 0.91 \) in the case of the MSSM with the same choice of GUT-scale boundary conditions for SUSY breaking parameters.

In Fig. 3, we show the \( X \) dependence of \( R_{b\tau}' \), taking \( \tan \beta = 27, m_5 = m_{\tilde{b} \ell} = m_{\tilde{b} \ell} = m_{1/2} = 3 \text{ TeV}, \) and \( a_d = a_u = 0 \). The value of \( m_{10} \) is tuned for each value of \( X \) to adjust the SM-like Higgs mass to be the observed value; as a result, \( m_{10} \) takes values in the range 11.5 to 14 TeV.
The red line shows the result for the case with $\mu_{10} = 10^{10} \text{ GeV}$ and the green one shows that with $\mu_{10} = 10^{4} \text{ GeV}$. We can understand from the figure that $R'_{b\tau}$ is enhanced with larger $X$. In the present choice of parameters with $\mu_{10} = 10^{10} \text{ GeV}$, $R'_{b\tau} = 1$ is possible. On the other hand, for $\mu_{10} = 10^{6} \text{ GeV}$, the enhancement is not so important since in this case, the suppression of $y_b$ due to the enhancement of the gauge coupling constants is so large (see Fig. 1) that it cancels the advantage of the mixing effect. Notice that the lines in Fig. 3 terminate at some value of $X$. This is because, for larger values of $X$, $m_{10}$ becomes larger in order to fix the SM-like Higgs mass, which in turn may make the right-handed sbottom mass squared or the left-handed slepton mass squared a negative value at $Q = M_S$, causing the tachyonic sfermion problem.

4. Summary

In this paper, we have studied the $b$-$\tau$ unification in SUSY SU(5) models with extra matters. We have assumed that the extra matters are embedded into full SU(5) multiplets. We have seen that the extra matters may significantly affect the $b$-$\tau$ unification, in particular when the mass scale of the extra matters is much lower than the GUT scale.

We first considered the case where the extra matters interact with the MSSM particles only through gauge interaction. In such a case, the ratio of $y_b$ and $y_\tau$ at the GUT scale, which we called $R_{b\tau}$, becomes suppressed as the mass scale of the extra matters becomes smaller. This is because, with the extra matters, the SU(3)$_C$ gauge coupling constant is enhanced at higher scales, resulting in the suppression of the bottom Yukawa coupling constant at the GUT scale. The suppression of $R_{b\tau}$ has been found to be $\sim 10\%$.

We have also studied the effects of the Yukawa couplings of the extra matters with MSSM particles. In the case we have studied, the Yukawa couplings above the mass scale of the extra matters are effectively enhanced, resulting in the change of the ratio of the (effective) $b$ and $\tau$ Yukawa coupling constants. In particular, we have shown that a simple Yukawa unification (i.e., $R'_{b\tau} = 1$) can be realized via the effects of extra matters with Yukawa interaction even though $R_{b\tau}$ is significantly smaller than 1 for the case without extra matters with the same GUT-scale boundary conditions.

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