In-flight inertial navigation system alignment under non-controlled delays in aiding data

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Abstract. In the presented paper we consider a well-known in-flight alignment problem for strapdown inertial navigation system (INS) via position and velocity data provided by the reference inertial navigation system. Main challenges of the problem are:
• displacement of the reference navigation system relative to the navigation system being aligned;
• mutual misalignment of the relevant body frames;
• uncontrolled time delays in the positional and velocity aiding data. We propose in-flight alignment algorithm that was tested based on simulation. Typical vehicle's trajectories were derived from relevant telemetry data. The results of numerical experiments are also presented.

1. Introduction
Consider in-flight alignment problem for INS via position and velocity data provided by the reference inertial navigation system. Main challenges of the problem are:
• displacement of the reference navigation system relative to the navigation system being aligned;
• mutual misalignment of the relevant body frames;
• uncontrolled time delays in the position and velocity aiding data.
At the heart of the research is a thorough simulation, which included:
• coordinated trajectory and attitude parameters simulation of the reference INS and the INS being aligned;
• simulation of inertial sensor readings and their errors in accordance with the accuracy class;
• simulation using telemetric trajectory data;
• imitation of uncontrolled time delays in aiding data using pseudorandom number generator;
• alignment algorithm modeling with a pure estimation and a feedback design;
• simulation of inertial dead reckoning algorithm.

2. Alignment algorithm
2.1. Overview
Alignment algorithm is based on the Kalman filter. Numerical implementation of Kalman filter involves the use of Cholesky factorization of the covariance matrix. Two scenarios are considered: a pure estimation and a feedback design [4].
In a pure estimation design dead reckoning algorithm stays untouched. Estimation of INS position, velocity and attitude errors and their compensation are done at the output of the dead reckoning algorithm.

On the other hand, feedback design requires modifications inside dead reckoning algorithm: correction signals are added to the dynamic and kinematic equations of inertial navigation (in continuous form) to minimize current INS position, velocity and attitude errors.

Position and velocity data, provided by the reference INS, is used for aiding. Aiding models are close to ones mentioned in [2], [3].

2.2. Estimation problem

Alignment problem can be posed as a linear stochastic estimation problem of a general form:

$$\frac{dx}{dt} = Ax + q, \quad z = Hx + r.$$

Here $x$ is a state vector, $A$ is the matrix of dynamic system, $z$ is a vector of the linearized measurements, $H$ is the observation matrix, $q$ is a driving noise vector of the dynamic system, $r$ is a vector of non-modeled errors (noise) of measurements.

State vector $x$ consists of the position, velocity, attitude INS errors, gyroscopes and accelerometer’s constant biases, time delay and displacement parameters:

$$x = (\Delta y_E, \Delta y_N, \delta V_E, \delta V_N, \alpha_E, \alpha_N, \beta_3, v_0^N, v_0^E, \Delta f_2^0, \Delta f_3^0, \Delta f_4^0, \tau, l_x, l_y, l_z)^T,$$

where:

- $\Delta y_E, \Delta y_N$ are horizontal position errors in local level reference frame in East and North directions;
- $\delta V_E, \delta V_N$ are dynamical parts of relative horizontal velocity errors [4];
- $\alpha_E, \alpha_N$ are deflection of virtual horizon, $\beta_3$ is azimuth error [4];
- $v_0^N, \Delta f_2^0$ are gyroscopes and accelerometer’s constant biases in body frame respectively (for the INS being aligned);
- $\tau$ is time delay for aiding data;
- $l_x, l_y, l_z$ are displacement parameters in body frame.

The following INS error equations (in computed local level reference frame) hold [1]:

$$\Delta \dot{y}_E = \Omega_3^r \Delta y_N + \delta V_E + \beta_3 V'_N,$$

$$\Delta \dot{y}_N = -\Omega_3^r \Delta y_E + \delta V_N - \beta_3 V'_E,$$

$$\delta \dot{V}_E = (\Omega_3^r + 2u_3') \delta V_N - g \alpha_N + \Delta f_3^0,$$

$$\delta \dot{V}_N = -(\Omega_3^r + 2u_3') \delta V_E + g \alpha_E + \Delta f_3^0,$$

$$\dot{\alpha}_E = -\frac{\delta V_N}{a} + \omega_3^' \alpha_N - u_3' \beta_3 - u_3'' \frac{\Delta y_N}{a} + v_E,$$

$$\dot{\alpha}_N = -\frac{\delta V_E}{a} - \omega_3^' \alpha_E + u_3' \beta_3 - u_3'' \frac{\Delta y_E}{a} + v_N,$$

$$\dot{\beta}_3 = \omega_N^' \left( \alpha_E + \frac{\Delta y_N}{a} \right) - \omega_3^' \left( \alpha_N - \frac{\Delta y_E}{a} \right) + v_3.$$

Prime symbol «'» represents data from the INS, which performs the alignment, and will be used from now on. In the error equations $u$ is Earth rotation rate vector, $\Omega$ is relative angular rate of local level reference frame with respect to the Greenwich triad, $\omega$ is absolute angular rate of local level reference frame, $g$ is nominal magnitude of gravity acceleration, $a$ is Earth ellipsoid semi-major axis, $v, \Delta f$ are full errors of gyroscopes and accelerometers.
Let the reference INS provide position \((\lambda, \varphi)\) and velocity \((V_E, V_N)\) information for in-flight alignment problem.

2.3. Position measurements
First consider the case when time delay and displacement are not present.

Using aiding position data \((\lambda, \varphi)\) measurement vector \(z_{pos} = (\lambda, \varphi)^T\) is calculated as:

\[
\begin{align*}
z_\lambda &= (\lambda' - \lambda) R_E \cos \varphi, \\
z_\varphi &= (\varphi' - \varphi) R_N,
\end{align*}
\]

where \(R_E, R_N\) are local curvature radii of the Earth ellipsoid.

Thus, we obtain the following observation model for position measurement:

\[
\begin{align*}
z_\lambda &= \Delta y_E + r_\lambda, \\
z_\varphi &= \Delta y_N + r_\varphi.
\end{align*}
\]

2.3.1. Delay estimation
Now let the aiding position data have time delay \(\tau\). Using linear approximation for delayed data we get the observation model with the time delay as a state vector component:

\[
\begin{align*}
z_\lambda &= \left(\lambda'(t) - \lambda(t - \tau)\right) R_E \cos \varphi \approx \left(\lambda'(t) - \lambda(t) + \tau \dot{\lambda}(t)\right) R_E \cos \varphi = \Delta y_E + \tau V_E + r_\lambda, \\
z_\varphi &= \left(\varphi'(t) - \varphi(t - \tau)\right) R_N \approx \left(\varphi'(t) - \varphi(t) + \tau \dot{\varphi}(t)\right) R_N = \Delta y_N + \tau V_N + r_\varphi.
\end{align*}
\]

2.3.2. Displacement estimation
If aiding position data contains unknown displacement \(l\) of the reference INS relative to the INS being aligned, we can center aiding position data at the INS being aligned:

\[
\begin{align*}
\lambda' \rightarrow \lambda &= \frac{l_E}{R_E \cos \varphi} = \lambda - \frac{l_{z_1} a_{11} + l_{z_2} a_{12} + l_{z_3} a_{13}}{R_E \cos \varphi}, \\
\varphi' \rightarrow \varphi &= \frac{l_N}{R_N} = \varphi - \frac{l_{z_1} a_{21} + l_{z_2} a_{22} + l_{z_3} a_{23}}{R_N},
\end{align*}
\]

where \(a_{ij}\) are elements of attitude matrix \(A_{xoz}\) of local level reference frame with respect to body frame of the INS being aligned. Therefore, we obtain the observation model with displacements \(l_{z_1}, l_{z_2}, l_{z_3}\) as state vector components:

\[
\begin{align*}
z_\lambda &= \Delta y_E - \left(l_{z_1} a_{11} + l_{z_2} a_{12} + l_{z_3} a_{13}\right) + r_\lambda, \\
z_\varphi &= \Delta y_N - \left(l_{z_1} a_{21} + l_{z_2} a_{22} + l_{z_3} a_{23}\right) + r_\varphi.
\end{align*}
\]

2.4. Velocity measurements
Similarly to the position measurements, first consider the case when time delay and displacement are not present.

Using aiding velocity data \((V_E, V_N)\) measurement vector \(z_{vel} = (z_{V_E}, z_{V_N})^T\) is calculated as follows:

\[
\begin{align*}
z_{V_E} &= V_E' - V_E, \\
z_{V_N} &= V_N' - V_N.
\end{align*}
\]

Hence, we obtain the observation model for velocity measurement:

\[
\begin{align*}
z_{V_E} &= \delta V_E + (\Delta \lambda \sin \varphi + \beta_3) V_N + r_{V_E'}, \\
z_{V_N} &= \delta V_N - (\Delta \lambda \sin \varphi + \beta_3) V_E + r_{V_N'}.
\end{align*}
\]

Using the reference INS position data, we can execute the following compensation:

\[
\begin{align*}
z_{V_E} &\rightarrow z_{V_E} - (\lambda' - \lambda) \sin \varphi V_N, \\
z_{V_N} &\rightarrow z_{V_N} + (\lambda' - \lambda) \sin \varphi V_E.
\end{align*}
\]
Finally the observation model for velocity measurements has the form:
\[ z_{V_E} = \delta V_E + \beta_3 V_N + r_{V_E}, \]
\[ z_{V_N} = \delta V_N - \beta_3 V_E + r_{V_N}. \]

2.4.1. Delay estimation
Now let the aiding velocity data have time delay. In the same way we obtain the observation model with the time delay as a state vector component:
\[ z_{V_E} = V'_E(t) - V_E(t - \tau) \approx V'_E(t) - V_E(t + \tau V_E(t)), \]
\[ z_{V_N} = V'_N(t) - V_N(t - \tau) \approx V'_N(t) - V_N(t + \tau V_N(t)). \]

Derivatives \( \dot{V}_E, \dot{V}_N \) can be calculated using previous velocity values or dynamic equations of motion.

2.4.2. Displacement estimation
Let the aiding velocity data have unknown displacement \( l \) of the reference INS relative to the INS being aligned. In this case, relative velocity \( l \) of two systems is subtracted from the aiding velocity data:
\[ V_E \rightarrow V_E - \dot{l}_E, \quad V_N \rightarrow V_N - \dot{l}_N. \]

Relative velocity \( \dot{l} \) is calculated using formula for the time derivative of a vector \( l \) in a rotating frame of reference (time derivative of \( l \) in an inertial reference frame is equal to zero):
\[ \dot{l}_{x^0} = \tilde{\Omega}_{x^0}^0 l_{x^0}, \]
where \( \Omega_{x^0} \) is angular velocity of local level frame of reference with respect to the Greenwich triad and \( \tilde{\Omega}_{x^0} \) is skew-symmetric matrix
\[
\begin{pmatrix}
0 & -\Omega_{x^0}^3 & \Omega_{x^0}^2 \\
\Omega_{x^0}^3 & 0 & -\Omega_{x^0}^1 \\
-\Omega_{x^0}^2 & \Omega_{x^0}^1 & 0
\end{pmatrix}.
\]

Thus, we get the following observation model:
\[ z_{V_E} = \delta V_E + \beta_3 V_N - l_z d_{11} - l_z d_{12} - l_z d_{13} + r_{V_E}, \]
\[ z_{V_N} = \delta V_N - \beta_3 V_E - l_z d_{21} - l_z d_{22} - l_z d_{23} + r_{V_N}, \]
where matrix \( D = (d_{ij}) \) is equal to:
\[ D = \tilde{\Omega}_{x^0} A_{x^0 z}. \]

2.5. Delay mathematical model
To describe delay parameter \( \tau \) behavior several models can be used:
1. Constant delay model \( \dot{\tau} = 0 \).
2. Linear trend model \( \ddot{\tau} = 0 \).
3. Stochastic delay model \( \dot{\tau} = q_{\tau} \) or \( \ddot{\tau} = q_{\tau} \), where \( q_{\tau} \) is white noise process.

3. Inertial measurement unit errors simulation
3.1. Mathematical model
For accelerometers and gyroscopes we accept a well-known error model:
\[ \Delta f_z = \Delta f_z^0 + \Gamma f_z + \Delta f_z^\tau, \quad \Gamma = \begin{pmatrix}
\Gamma_{11} & 0 & 0 \\
\Gamma_{21} & \Gamma_{22} & 0 \\
\Gamma_{31} & \Gamma_{32} & \Gamma_{33}
\end{pmatrix}, \]
\[ v_z = v_0^z + \Theta \omega_z + v_s^z, \quad \Theta = \begin{pmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{pmatrix}. \]

where:
- \( \Delta f_z, v_z \) are full errors of accelerometers and gyroscopes respectively;
- \( \Delta f_z^0, v_z^0 \) are constant biases;
- \( \Gamma_{ii}, \Theta_{ii} \) are errors of scaling factors \((i = 1, 2, 3)\);
- \( \Gamma_{ij}, \Theta_{ij} \) are axial misalignments \((i, j = 1, 2, 3, i \neq j)\);
- \( f_z, \omega_z \) are simulated sensor readings;
- \( \Delta f_s^z, v_s^z \) are stochastic white noise terms.

Above model is given in a body frame of a vehicle.

Note: although errors of scaling factors and axial misalignments are used to simulate sensor errors, they aren’t present in the state vector for Kalman filter.

3.2. Typical values

During simulation, we considered the following order of magnitude for sensor error parameters:
- \( v_0^z \sim 0.1 ^\circ/\text{h}, \ v_z^0 \sim 0.1 ^\circ/\text{h}; \)
- \( \Theta_{ii} \sim 0.001\%, \Theta_{ij} \sim 1\', \ i, j = 1, 2, 3 \ (i \neq j); \)
- \( \Delta f_{z}^0 \sim 10'' \), \( \Delta f_{z}^s \sim 1''; \)
- \( \Gamma_{ii} \sim 0.01\%, \Gamma_{ij} \sim 1\', \ i, j = 1, 2, 3 \ (i \neq j). \)

Aiding data delay time, mutual misalignment and displacement of the reference INS relative to the INS being aligned had the following order of magnitude:
- mutual misalignment of the relevant body frames \( \sim 1^\circ; \)
- \( l_{z1}, l_{z2}, l_{z3} \sim 1 \text{ m}; \)
- \( \tau \sim 20 \div 10 \text{ ms}. \)

Fig. 1. Attitude angles for telemetric trajectory (yaw \( \psi \), pitch \( \theta \), roll \( \gamma \).
4. Simulation results

Vehicle’s trajectories were derived from the relevant telemetry data. The alignment period of 120 seconds was chosen as a standard.

During alignment, an aircraft can turn or perform maneuvers such as «scissors». After the alignment period ends, Kalman filter performs only prediction steps.

An example trajectory is shown in fig. 1. Simulation results for this trajectory are presented in fig. 2. Initial attitude errors for yaw, pitch and roll were equal to 60°, 30°, 30° respectively. After the alignment period of 120 seconds they were equal to 7.2′, 0.6′, 0.2′.

![Fig. 2. Estimation errors for yaw, pitch, roll (left) and estimated error covariances for $\alpha_E$, $\alpha_N$, $\beta_3$ (right).](image)

5. Conclusion

The observation models for delayed and spatially displaced aiding data have been described. Thorough simulation made it possible to determine a potential accuracy level of the alignment problem solution. A prototype of an onboard alignment algorithm was developed and tested.

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