A Supersymmetric Contribution to the Neutrino Mass Matrix
and Breaking of \( \mu - \tau \) Symmetry

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Supersymmetry (SUSY) is the best motivated candidate for beyond the Standard Model (SM) physics, since it for instance provides Dark Matter candidates or solves the gauge hierarchy problem. Phenomenologically, the requirement of SUSY breaking is obvious, its origin and mechanism are however still a mystery. One possibility is the anomaly mediated SUSY breaking (AMSB)\(^1\), which always exists in supergravity frameworks. It avoids the presence of sizable flavor changing neutral currents, the so-called SUSY flavor problem, but has a well-known drawback, namely the presence of tachyonic slepton masses. One simple solution for this dilemma is to introduce an additional \( \tau \) slepton masses. One possible solution to this problem results in “decoupling”, i.e., the first two generations of sfermions are much heavier than the third one. We note that in this scenario a sizable loop-induced contribution to the neutrino mass matrix results. As an application of this scenario we take advantage of the fact that the decoupling evidently not obeys 2–3 generation exchange symmetry. In the neutrino sector, this 2–3 symmetry (or \( \mu - \tau \) symmetry) is a useful Ansatz to generate zero \( \theta_{13} \) and maximal \( \theta_{23} \). The induced deviations from these values are given for some examples, thereby linking SUSY breaking to the small parameters (including possibly the solar mass splitting) of the neutrino sector.

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Supersymmetry broken by anomaly mediation suffers from tachyonic slepton masses. A possible solution to this problem results in “decoupling”, i.e., the first two generations of sfermions are much heavier than the third one. We note that in this scenario a sizable loop-induced contribution to the neutrino mass matrix results. As an application of this scenario we take advantage of the fact that the decoupling evidently not obeys 2–3 generation exchange symmetry. In the neutrino sector, this 2–3 symmetry (or \( \mu - \tau \) symmetry) is a useful Ansatz to generate zero \( \theta_{13} \) and maximal \( \theta_{23} \). The induced deviations from these values are given for some examples, thereby linking SUSY breaking to the small parameters (including possibly the solar mass splitting) of the neutrino sector.

Here \( \psi_L \) is the lepton doublet of flavor \( i \), \( H \) the up-type Higgs doublet and \( M_R \) the scale of neutrino mass generation. After electroweak symmetry breaking this operator is the neutrino mass matrix \( m_\nu = \kappa \langle H \rangle^2 / M_R \). In the present work we will show explicitly how in the AMSB-decoupling framework a sizable and flavor-dependent contribution to Eq. \( (1) \) arises and will also discuss some interesting applications.

The structure of \( \kappa \) defines the neutrino mass and mixing phenomena\(^1\) and explaining their peculiar scheme is one of the most interesting problems of particle physics. An interesting Ansatz is to implement a 2–3 or \( \mu - \tau \) exchange symmetry\(^2\) in \( \tau \). This yields zero \( \theta_{13} \) and maximal \( \theta_{23} \), which are the best-fit points of global analyzes of neutrino data\(^3\). Deviations from these values are achieved by breaking of \( \mu - \tau \) symmetry, which however is usually done by hand\(^4\). We propose here to take advantage of the AMSB-decoupling contribution to a neutrino mass matrix with \( \mu - \tau \) symmetry. The fact that the first two sfermion families are much heavier than the third one apparently violates 2–3 exchange symmetry. Assuming that \( \kappa \) conserves \( \mu - \tau \) symmetry and taking the \( \mu - \tau \) violating AMSB correction to \( \kappa \) into account leads to testable corrections to the initial mixing parameters. The size of the contribution to Eq. \( (1) \) depends on the SUSY masses. Hence, the implied small values of \( \theta_{13} \) and \( \theta_{23} - \pi / 4 \) are also sensitive to the SUSY masses. In addition, it is also possible to generate the small solar neutrino mass squared difference via the AMSB-decoupling term.

Now let us estimate the 1-loop correction for the neutrino dimension 5 operator in Eq. \( (1) \). As an effect of
quantum corrections, the neutrino mass term is obtained by replacing every entry \( ij \) as follows:

\[
(m_\nu)_{ij} \rightarrow (m_\nu)_{ij} (1 + \Delta_{ij}) \quad \text{for all } i, j.
\]

(2)

Here the factor \( \Delta_{ij} \) is the correction from the (neutral) wino and bino dressed 1-loop diagram (see Fig. 1):

\[
\Delta_{ij} \simeq \frac{1}{2} \left( \frac{\alpha_2}{4\pi} \right) F_\phi M_2^2 f^{(2)}_{ij} + \frac{3}{10} \left( \frac{\alpha_1}{4\pi} \right) F_\phi M_1^4 f^{(1)}_{ij} \\
\simeq \frac{|M_2|^2}{2} (f^{(1)}_{ij} + f^{(2)}_{ij}).
\]

(3)

We defined the loop-function

\[
f(M_I, a, b) = f(M_I, \tilde{m}_{L_i}, \tilde{m}_{L_j}) = \frac{1}{a^2 - b^2} \left[ \frac{a^2}{a^2 - |M_I|^2} \ln \frac{a^2}{|M_I|^2} - (a \leftrightarrow b) \right].
\]

(4)

where \( \alpha_{1,2} \) are the gauge couplings, \( M_{1,2} \) the wino (\( \tilde{W}_3 \)) and bino (\( \tilde{b} \)) masses, and \( \tilde{m}_{L_i} \) denotes the mass of the i-th generation slepton. \( F_\phi \) is the \( F \)-component of the compensating multiplet of the AMSB sector and is of the order of the gravitino mass. The second equation in Eq. (3) uses the AMSB relation \( M_I = (\alpha_I / 4\pi) b_I F_\phi \) (\( I = 1, 2 \) and \( b_1 = 33 / 5 \) and \( b_2 = 1 \) are the beta functions), which implies \( M_2 / M_1 \simeq 0.3 \). In the decoupling mass spectrum under consideration, the 1-3 and 2-3 generation sfermion mixings are small, so that we can safely neglect their mixings at the vertices. Note that we did not specify the origin of the dimension 5 operator \( \kappa \). Our analysis is therefore independent of the leading mechanism of neutrino mass generation.

The contribution to \( \Delta_{ij} \) is of order \( \alpha_I / (4\pi) M_I F_\phi f^{(1)}_{ij} \), with \( f^{(1)}_{ij} / M_2 \), where \( M_{ij} \) is here some soft SUSY breaking mass. In other SUSY breaking scenarios there are also diagrams as the one displayed in Fig. 1. However, such scenarios typically predict that \( F_\phi \), the gravitino, gaugino and sfermion masses are of the same order of magnitude. As a consequence, the loop-diagram gives a contribution to \( \kappa \) strongly suppressed by a loop factor of \( 10^{-2} \). The crucial point we wish to make here occurs when there is only the AMSB breaking effect in which case \( F_\phi \) is larger by a factor of \( 10^2 \) than the gaugino masses. Then this factor \( 10^2 \) cancels with \( \alpha_I / (4\pi) \) and the contribution of \( \Delta_{ij} \) to \( \kappa_{ij} \) is sizable.

Let us show two typical examples for the magnitude of \( \Delta_{ij} \). When we take \( M_2 = 220 \text{ GeV}, \tilde{m}_{L_3} = 300 \text{ GeV}, \) and \( \tilde{m}_{L_{1,2}} \geq 4.5 \text{ TeV (“small slepton masses”), the dimensionless matrix} \Delta_{ij} \text{ is estimated as}

\[
\Delta_{ij} \simeq \begin{pmatrix}
\lambda^4 & \lambda^4 & \lambda^3 \\
\cdot & \lambda^4 & \lambda^3 \\
\cdot & \cdot & \lambda^2
\end{pmatrix} \simeq \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

(5)

We introduced here the small parameter \( \lambda = 0.22 \), which is of the order of the Cabibbo angle, and proves very useful in the estimates. The heavier the 1st and 2nd generation slepton masses become, the smaller the elements except for the (3, 3) entry become. It is the peculiar mass ordering of the sfermions that makes the (3, 3) entry dominate in \( \Delta_{ij} \).

A similar hierarchy in \( \Delta_{ij} \), but with a smaller (3, 3) entry is obtained by choosing \( M_2 = 200 \text{ GeV}, \tilde{m}_{L_3} = 760 \text{ GeV}, \) and \( \tilde{m}_{L_{1,2}} \geq 8.8 \text{ TeV (“large slepton masses”), for which}

\[
\Delta_{ij} \simeq \begin{pmatrix}
\lambda^5 & \lambda^5 & \lambda^4 \\
\cdot & \lambda^5 & \lambda^4 \\
\cdot & \cdot & \lambda^2
\end{pmatrix} \simeq \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

(6)

We can therefore adjust the magnitude of the correction to the neutrino mass matrix by choosing the slepton and gaugino mass spectrum. The sizable differences of the slepton masses and the light gauginos (as typical for AMSB scenarios) might be demonstrated at future colliders such as LHC or the ILC.

As obvious from Eqs. (3, 4), the correction to \( m_\nu \) violates \( \mu - \tau \) symmetry and can serve as a perturbation to such mixing scenarios. We will illustrate this in what follows. A general \( \mu - \tau \) symmetric neutrino mass matrix leaves the neutrino mass spectrum and the solar neutrino mixing angle unconstrained. To be definite, we assume here first a normal hierarchy and bimaximal mixing (8), for which \( \theta_{12} = \pi/4 \). The mass matrix (with the smallest neutrino mass \( m_1 \geq 0 \)) can be written as

\[
m_\nu \simeq \frac{m_3}{2} \begin{pmatrix}
\epsilon & \epsilon/\sqrt{2} & \epsilon/\sqrt{2} \\
\cdot & 1 + \epsilon/2 & -1 + \epsilon/2 \\
\cdot & \cdot & 1 + \epsilon/2
\end{pmatrix},
\]

(7)

with \( \epsilon \simeq \sqrt{\Delta m^2_{31} / \Delta m^2_{\chi}} \), where the square root lies at 3 (1)\( \sigma \) between 0.15 and 0.25 (0.17 and 0.21), with a best-fit value of 0.19 (8). The effect of the AMSB-decoupling contribution in Eq. (3) is now basically just multiplying the (3, 3) entry of Eq. (3) with \( (1 + \lambda) \). We can then
predict the modified neutrino observables:

\[ R \equiv \frac{\Delta m^2_{32}}{\Delta m^2_{41}} \simeq \frac{4\epsilon^2 + \epsilon \lambda + \lambda^2/8}{4 + 2\lambda} \], \quad |U_{e3}| \simeq \frac{\epsilon \lambda}{8} \quad (8) \]

\[ \tan \theta_{23} \simeq 1 - \lambda/2 \], \quad \tan 2\theta_{12} \simeq \frac{4\epsilon}{\lambda} , \]

which for \( \lambda = 0 \) reproduces bimaximal mixing with \( \Delta m^2_{32} = \epsilon^2 \Delta m^2_{41} \). For \( \lambda \ll \epsilon \simeq \sqrt{R} \) the required large solar neutrino mixing (observation indicates \( \tan 2\theta_{12} \simeq 2\sqrt{2} \)) is easily achieved. Note that the deviation from maximal atmospheric mixing is large, namely of order \( \sqrt{R} \), whereas \( |U_{e3}| \) is of order \( R \). Both quantities depend on the SUSY breaking parameters as encoded in \( \lambda \).

Another possibility for bimaximal mixing and the normal hierarchy is when before breaking one chooses \( m_1 = -m_2 \), i.e., we start with vanishing solar \( \Delta m^2 \). The mass matrix reads

\[ m_\nu \simeq \frac{m_3}{2} \left( \begin{array}{ccc} 0 & \sqrt{2}\epsilon & \sqrt{2}\epsilon \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{array} \right) , \quad (9) \]

with \( \epsilon = m_2/m_3 \ll 1 \). Note that here the effective mass as measurable in neutrino-less double beta decay (the (1,1) element of \( m_\nu \)) is much smaller than in the previous example. Taking now the perturbation from Eq. (8) into account yields

\[ R \simeq \frac{2\epsilon \lambda}{4 + 2\lambda} \], \quad |U_{e3}| \simeq \frac{\epsilon \lambda}{4} \], \quad \tan 2\theta_{12} \simeq \frac{8\epsilon}{\lambda} \]

and an identical results for \( \theta_{23} \) as in Eq. (8). We stress that in this scenario there is a link between the small solar mass splitting and the breaking of supersymmetry. We can also perturb tri-bimaximal mixing \([10]\), which is \( \mu-\tau \) symmetry with \( \sin^2 \theta_{12} = \frac{1}{3} \). This requires a smaller perturbation to the mass matrix, i.e., Eq. (9), and therefore larger slepton masses. Initial tri-bimaximal mixing with \( m_1 = 0 \) corrected by Eq. (9) yields

\[ \sin^2 \theta_{12} \simeq \frac{1}{3} + \frac{1}{9} \lambda^2/\epsilon , \quad (10) \]

very small \( |U_{e3}| \simeq \epsilon \lambda^2/\sqrt{12} \) and moreover close-to-maximal \( \tan \theta_{23} \simeq 1 - \lambda^2/2 \). \( R \) is of order \( \epsilon^2 \). All deviations from the initial values are much smaller than for initial bimaximal mixing. In principle it would be possible that the correction to the mass matrix stems from Eq. (9), i.e., is larger. To accommodate the observed value of \( \sin^2 \theta_{12} \simeq \frac{1}{3} \), however, the contribution proportional to \( \lambda \) should be suppressed by fine-tuned values of possible \( CP \) phases.

Consider now the inverted hierarchy. Since in the inverted hierarchy one can not perturb the bimaximal mixing to accommodate the observed large but non-maximal solar mixing (unless one accepts fine-tuning), we start with free \( \theta_{12} \):

\[ m_\nu \simeq m \left( \begin{array}{ccc} A & B \\ \cdots & D \end{array} \right) \]

\[ m_\nu \simeq m \left( \begin{array}{ccc} A & B \\ \cdots & D \end{array} \right) \]

\[ \begin{array}{ccc} A & B & B \\ \cdots & D & D \\ \cdots & D & D \end{array} \]

with \( A \equiv s^2_{12} - (1 - \epsilon) c^2_{12} \), \( B \equiv \frac{1}{2\sqrt{2}} (2 - \epsilon) \sin 2\theta_{12} \), \( D \equiv \frac{2}{\epsilon} + \frac{1}{4} (2 - \epsilon) \cos 2\theta_{12} \)

and \( \epsilon \equiv (m_2 - m_1)/m \), where \( m_2 \simeq m_1 \simeq m \simeq \sqrt{\Delta m^2_{41}} \).

The two non-zero masses \( m_1 \) and \( m_2 \) have opposite \( CP \) parities here. As in the normal hierarchy with tri-bimaximal mixing, we require a small perturbation to the mass matrix: if we would add Eq. (6) to this matrix, then the ratio of solar and atmospheric \( \Delta m^2 \) would be of order \( \lambda + 2\epsilon \), which is too large (this will also hold for the unstable case of equal \( CP \) parities of the masses \( m_1 \) and \( m_2 \)). Hence, we are lead to use Eq. (8), corresponding to large slepton masses. Choosing \( \theta_{12} \) such that \( \sin^2 \theta_{12} = \frac{1}{3} \), i.e., again tri-bimaximal mixing \([10]\), one finds

\[ R \simeq \frac{\lambda^2 + 2\epsilon}{1 + \frac{8}{9} \lambda^2 + \frac{8}{9} \epsilon} \], \quad \tan^2 \theta_{23} \simeq 1 + \frac{2}{3} \lambda^2 , \]

\[ |U_{e3}| \simeq \frac{\sqrt{2}\lambda^2}{3} \], \quad \sin^2 \theta_{12} \simeq \frac{1}{3} - \frac{1}{9} \lambda^2 + \frac{2}{27} \epsilon . \quad (11) \]

The order of magnitude of \( \theta_{13} \) is the same as in the case of normal hierarchy and initial bimaximal mixing, namely of order \( R \). The deviation from maximal atmospheric mixing is of order \( R \). Starting with \( \epsilon = 0 \), i.e., with vanishing \( \Delta m^2_{41} \), we have \( R \simeq 3 |U_{e3}|/\sqrt{2} \) and

\[ \sin^2 \theta_{12} \simeq \frac{1}{3} - \frac{1}{3\sqrt{2}} |U_{e3}| \simeq \frac{1}{2} - \frac{1}{6} \tan^2 \theta_{23} . \quad (12) \]

Again, a large contribution to the mass matrix corresponding to smaller slepton masses is in principle possible but requires fine-tuned \( CP \) phases.

Up to now it is obvious that the \( \mu-\tau \) symmetry was only obeyed by the neutrino mass matrix, while the charged lepton mass matrix was real and diagonal. We choose now an example in which both the neutrinos and the charged leptons conserve \( \mu-\tau \) symmetry. Naively, one would expect that even after breaking the symmetry the PMNS matrix would contain only a sizable 12 mixing, since the (close-to-)maximal 23 angle cancels in the product of the matrices diagonalizing the neutrinos and charged leptons, respectively. This can change for quasi-degenerate light neutrinos \([8]\). The mass matrices are:
charged leptons (choosing for simplicity the term of Eq. (5), we have in the basis of diagonal same results. Perturbing the neutrino mass matrix with to choose a $\mu\tau$ symmetric $m_\ell m_\nu^\dagger$ where only the leading terms are shown. Large atmo-
matically. Note that $m_\ell$ is symmetric. That does not affect our analysis, since for non-symmetric $m_\ell$ one would have to choose a $\mu\tau$ symmetric $m_\ell m_\nu^\dagger$, which would give the same results. Perturbing the neutrino mass matrix with the term of Eq. (5), we have in the basis of diagonal charged leptons (choosing for simplicity $\theta_{13} \ll 1$)

$$m_\nu \simeq \begin{pmatrix} A_\nu & \sqrt{2}B_\nu & 0 \\ \cdot & 2E_\nu & \frac{1}{2}(D_\nu + E_\nu) \lambda \\ \cdot & \cdot & 2D_\nu \end{pmatrix} ,$$

(13)

where only the leading terms are shown. Large atmospheric neutrino mixing is only possible for $D_\nu \simeq E_\nu$. This corresponds to a quasi-degenerate spectrum and is exactly the scenario put forward in [8] to accommodate $\mu\tau$ symmetry in both the charged lepton and the neutrino mass matrix. Focusing on the 23 sector and denoting the common mass scale with $m_0$, close-to-maximal $\theta_{23}$ is achieved for $\lambda (D_\nu + E_\nu) \gg |D_\nu - E_\nu|$, which leads to $\cos 2\theta_{23} \simeq 4m_0 |D_\nu - E_\nu|/\Delta m^2_\text{sol}$. The parameter $\lambda$ is related to the masses:

$$\lambda \simeq \frac{\Delta m^2_\text{sol}}{2m_0^2} \simeq \begin{cases} 0.11 & \text{for } m_0 = 0.1 \text{ eV} , \\ 0.012 & \text{for } m_0 = 0.3 \text{ eV} . \end{cases} \quad (14)$$

Such values of the neutrino masses are in agreement even with the tightest cosmological constraints [11]. Light neutrino masses around $0.1$ eV require therefore a per-
turbation with small slepton masses and larger neutrino masses need larger slepton masses.

Numerically, for $m_0 \simeq 0.1$ eV we have $\cos 2\theta_{23} \simeq |D_\nu - E_\nu|/0.0055$ eV), which shows the required fine-
tuning in such a scenario. If $\theta_{23} = \pi/4 - 0.1$, then $|D_\nu - E_\nu| \simeq 0.0011$ eV and for $\theta_{23} = \pi/4 - 0.01$ one has $|D_\nu - E_\nu| \simeq 0.0001$ eV. For a larger mass of $m_0 \simeq 0.3$ eV one finds that for $\theta_{23} = \pi/4 - 0.1$ one requires $|D_\nu - E_\nu| \simeq 0.0004$ eV, while for $\theta_{23} = \pi/4 - 0.01$ one has $|D_\nu - E_\nu| \simeq 0.00004$ eV.

In summary, anomaly mediated supersymmetry breaking with connection to decoupling of the sfermions can gen-
erate a sizable contribution to the neutrino mass matrix. This can serve as a natural explanation for a perturbation of neutrino mixing scenarios. Noting that the decoupling scenario violates 2-3 exchange symmetry, it is natural to take $\mu\tau$ symmetry as an example. We showed in various examples that interesting and testable [12] correlations between the neutrino mixing parameters result. We stress again that in the scenarios presented here the breaking of supersymmetry is intimately related to small $U_{e3}$, $\cos 2\theta_{23}$ and even to the small ratio of the solar and atmospheric $\Delta m^2$.

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[13] Note that the form of the mass matrix Eq. (13) is generated at a scale of order \( \text{TeV} \). Hence, the running of the mass and mixing parameters to low scale \( m_Z \) is rather moderate, in particular for small neutrino masses.