CAD/CAM for Double Woven Fabric: Center Warp Stitching

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Abstract
To bond the two layers in double fabrics, various types of stitching can be distinguished such as self-stitching, double stitching, center warp stitching, center weft stitching, and so on. In this article, a mathematic model based on a software program has been developed to automatically generate a double fabric stitched by additional warp called center warp. Each layer has been represented in a 2D binary matrix, and a new matrix called warps order matrix has been defined to demonstrate the modality of position of the center warps in relating to the top and bottom fabrics’ warps. After insertion of all warps in the extended weave matrix, the lifter conditions has been discussed and the stitching points have been determined.

Keywords
double woven fabric, CAD/CAM, stitching, multilayer fabric, mathematical modeling

Introduction
Technical textiles are reported to be the fastest growing sector in textile industry. The development of all techniques and equipments is very necessary to decrease the production time especially with the use of electronically equipped modern looms and communicated with computers. The automatization of woven design is one of the ways to facilitate the work and to increase the production, especially in the weaving of complex structure, due to the difficulties involved in the manual design (e.g., the combination of weaves, seeking of stitches and introducing it, generating of lifting plan, doing cross-sections; Chen & Potiyaraj, 1999a).

However, a number of researches dealing with the CAD/CAM fabric weaves have been represented in the form of 2D matrices. A lot of CAD/CAM softwares were developed for complicated woven structures. Some of these researches depended on mathematical functions to describe and generate automatically the 2D and 3D weaves (Chen, 2011; Chen & Potiyaraj, 1998, 1999b). In other works, the algorithm of Kronecker Product was applied to describe weaves (Ping & Lixin, 1997, 1999).

In addition, other CAD/CAM softwares that deal with geometric modeling of woven structures have been developed to visualize the fabric appearance before weaving (Liao & Adanur, 1998; Lomov, Perie, Ivanov, Verpoest, & Marsal, 2011; Smith & Chen, 2009). However, they do not discuss all the problems due to the wide variety of weaves and stitching ways.

This article carries out the research related to complex woven fabrics and deals with one of the stitching ways (center warps stitching) used in double woven fabric. Moreover, based on some mathematical functions presented by Chen and Potiyaraj (1998, 1999a) and by Chen, Knox, McKenna, and Mather (1996), this article describes the programmable mathematic module to automatically generate such a fabric.

Double Woven Fabrics Construction
Double cloth or double weave is a type of a compound woven structure in which two or more sets of warps and two or more sets of weft or filling yarns are interlaced to form a two-layer cloth. The movement of threads between the layers allows complex patterns and surface textures to be created.

One set of each kind of yarns forms the upper layer of the fabric and the other forms the lower layer of the fabric. The stitching of these two layers together forms one of the main features of double cloth construction (Goerner, 1989).

Symbolic and Mathematical Representation of a Weave
The pattern representing the warp and weft interlacement is called the weave diagram or fabric weave (Adanur, 2000), as illustrated in Figure 1. Besides this graphical representation, a mathematical representation of weave in the form of a W binary matrix of 0 and 1 can be used (Lourie, 1973;
In this representation, each element is expressed by \( w(x, y) \) which provides the interweaving between a warp and a weft at the crossover \((x, y)\). The element \( w(x, y) = 0 \) if a weft goes over a warp and \( w(x, y) = 1 \) if a warp is over a weft as represented in Figure 1.

Figure 2 shows double fabric weave composed of two single layer weaves (the upper layer and the lower layer), each has own specifications.

To obtain stitched double fabric, these two layers must be bonded together by a certain number of points called stitches. Various types of stitching can be distinguished (Elnashar & Dobnik-Dubrovski, 2008; Goerner, 1989) as follows:

- **Self-stitching and double stitching:** These two ways were studied by Chen and Potiyaraj (1999a) and in a different way by Ping and Lixin (1999), and mathematic and programmatic modules have been developed.
- **Center warp stitching:** This method was studied by Ping and Lixin (1999) based on the application of Kronecker product, and will be described differently in this article and a programmable mathematic module will be done.
- **Other stitching ways** may be used for binding the two layers as center weft stitching, using an intermediate fabric, warp or weft interchange with interlacing, and so on.

Indeed, except in technical fabrics, the purpose of the stitching points are not always to join together two fabrics, but they also play a decorative role.

It can be noticed that the warp and weft arrangements control the positioning modality of threads of each layer in the weave according to the fabric specifications. Figure 3 illustrates a non-stitched double weave with warp arrangement 2:1 and with weft arrangement 1:1; meaning that the upper and lower layer warps are arranged in the proportion of two upper layer ends to one lower layer end, and the upper and lower layer wefts are placed alternately in the ratio of one pick for each layer. When the two weaves are combined, repeating design will be produced on the warps in the ratio of two repeats of the upper layer weave and one repeat of the lower layer weave. The weave produced from the combination of the two single layer weaves is called extended double fabric weave.

It can be noticed that if only the double fabric face and back have technical or visual significance, it is necessary to seek the possible stitching points and plot them according to this condition.

In this article, the described programmable mathematic module takes into account the case where the fabric face and back weaves are previously defined where the stitches are introduced by extra warps.

### Center Warp Stitching

In case of double fabric stitched by center warps, we can distinguish two types of stitches, the first is called an **upper stitch** symbolized by \((X)\) where the center warp lift over the upper layer weft, and the second is called a **lower stitch** represented by means of the symbol \((O)\) produced when the center warp is under the lower layer weft. The center threads stay in a horizontal position between upper and lower layers when not used for stitching (Goerner, 1989).

Figure 4 illustrates a double fabric stitched by center warps, constructed of 2/2 Z twill upper layer weave with an arrangement of 2 warps/1weft and 2/2 Z twill lower layer weave with an arrangement of 1warp/1weft and stitched by tow center warps located on ends 6 and 13. Lifters (\(\mathcal{T}\)) indicate upper layer ends over lower layer picks and center ends over lower layer picks. The transferring of weave is shown in Table 1.

### Extended Weaves Generation

As mentioned above, the first step consists in defining the upper and lower layer weaves as follows:

\[
A_{w1}, A_{f1}, r_{w1}, r_{f1}: \text{the warp and weft arrangement and the warp and weft repeats, respectively, for the upper layer.}
\]

\[
A_{w2}, A_{f2}, r_{w2}, r_{f2}: \text{the warp and weft arrangement and the warp and weft repeats, respectively, for the lower layer.}
\]

Then, identifying the center warps used for stitching by determining the center thread repeat and arrangement \((r_c, A_c)\). The total number of \(R_c\) center threads can be calculated as follows:

\[
R_{wc} = r_{wc} \cdot A_{wc}. \tag{1}
\]

Based on Chen and Potiyaraj (1998, 1999a) the extended weaves of each layer, can be calculated by the following equations:

\[
M_{wn} = M_{w} \cdot A_{wn} / r_{wn} \tag{2}
\]
where 

\[ n = 1 \text{ for the upper fabric and } n = 2 \text{ for the lower layer,} \]

\( M_{w1}, M_{w2} \): the numbers of upper and lower layer weaves repeats, respectively, in the warp direction,

\( M_{f1}, M_{f2} \): the numbers of upper and lower layer weaves repeats, respectively, in the weft direction,

\( M \): the smallest number that makes this relation equal to zero:

\[ (M_w \cdot A_{w1}) \mod r_{w1} = (M_w \cdot A_{w2}) \mod r_{w2} = 0 \quad (4) \]

\( M_f \): the smallest number that makes this relation equal to zero:

\[ (M_f \cdot A_{f1}) \mod r_{f1} = (M_f \cdot A_{f2}) \mod r_{f2} = 0 \quad (5) \]

and “\( \text{Mod} \)” is the remainder operator for division.

The warp and weft repeats of the extended upper and lower layer weaves can be calculated from these equations:

\[ R_{wn} = M_{wn} \cdot r_{wn}, \quad (6) \]

\[ R_{fn} = M_{fn} \cdot r_{fn}, \quad (7) \]
where, $R_{w1}, R_{f1}$ are the warp and weft repeats of the extended upper weave, respectively. $R_{w2}, R_{f2}$ are the warp and weft repeats of the extended lower weave, respectively.

As 2D binary matrix, these two extended upper and lower layer weaves can be denoted by $ET$ and $EB$, respectively, and the elements of these matrices are $t'_{(x,y)}$ and $b'_{(x,y)}$, respectively. The warp and weft repeats of each extended weave represent the corresponding matrix size.

Inserting the upper and lower layer weaves matrix elements $t_{(i,j)}$ and $b_{(i,j)}$ into the corresponding extended matrices is achieved by

$$
t_{(i,j)} \rightarrow ET \Leftrightarrow t'_{(i,j)} = t_{(i,j)} \text{ for } i = 1 \text{ to } r_{f1} \text{ and } j = 1 \text{ to } r_{w1}.
$$

(8)

$$
b_{(i,j)} \rightarrow EB \Leftrightarrow b'_{(x,y)} = b_{(i,j)} \text{ for } i = 1 \text{ to } r_{f2} \text{ and } j = 1 \text{ to } r_{w2}.
$$

(9)

All binaries $(x,y)$ values formed from each binary $(i,j)$ are calculated by the following formulas:

$$
x = i + R_{fn} - hR_{fn} \text{ for } h = 1, 2, \ldots, M_{fn}.
$$

(10)

$$
y = j + R_{an} - aR_{an} \text{ for } a = 1, 2, \ldots, M_{an}.
$$

(11)

---

**Table 1. Weave Transferring.**

|                      | 2  | 2  | .  | 2  | 2  | .  |
|----------------------|----|----|----|----|----|----|
| Top fabric warps     | 2  | 2  | .  | 2  | 2  | .  |
| Bottom fabric warps  | 1  | .  | 1  | 1  | .  | 1  |
| Centre warps         | .  | 1  | .  | .  | 1  | .  |

= 8

= 4

= 2

---

**Figure 4.** Double woven fabric stitched by center warps.
In the example shown in Figure 4, the two single layers have the following parameters:
Upper layer weave: \( n = 1, ~ r_{u1} = 4, ~ r_{f1} = 4, ~ A_{w1} = 2, ~ A_{f1} = 1 \)
Lower layer weave: \( n = 2, ~ r_{u2} = 4, ~ r_{f2} = 4, ~ A_{w2} = 1, ~ A_{f2} = 1 \)

From Equations 2 to 7, the following results are obtained:

\[
(M_w \cdot A_{w1}) \mod r_{w1} = (M_w \cdot A_{w2}) \mod r_{w2} = 0 \quad \iff \quad M_w = 4
\]
\[
M_{w1} = M_w \cdot A_{w1} / r_{w1} \quad \iff \quad M_{w1} = 2
\]
\[
M_{w2} = M_w \cdot A_{w2} / r_{w2} \quad \iff \quad M_{w2} = 1
\]
\[
(M_f \cdot A_{f1}) \mod r_{f1} = (M_f \cdot A_{f2}) \mod r_{f2} = 0 \quad \iff \quad M_f = 4
\]
\[
M_{f1} = M_f \cdot A_{f1} / r_{f1} \quad \iff \quad M_{f1} = 1
\]
\[
M_{f2} = M_f \cdot A_{f2} / r_{f2} \quad \iff \quad M_{f2} = 1
\]
\[
R_{u1} = M_{u1} \cdot r_{u1} \quad \iff \quad R_{u1} = 8
\]
\[
R_{u2} = M_{u2} \cdot r_{u2} \quad \iff \quad R_{u2} = 4
\]
\[
R_{f1} = M_{f1} \cdot r_{f1} \quad \iff \quad R_{f1} = 4
\]
\[
R_{f2} = M_{f2} \cdot r_{f2} \quad \iff \quad R_{f2} = 4
\]

Based on Equations 8 to 11, that is, to put the element \( t_{(2,3)} \) into the matrix \( ET \) we have to find all \((x,y)\) binaries values resulting from the \((i = 2, j = 3)\) binary value as follows:

\[
x = i + R_{f1} - h \cdot r_{f1} \quad \Rightarrow \quad x = 2 + 4 - (h) \cdot (4) = 6 - 4 \cdot (h), \text{for} \quad h = 1 \quad \text{where:} \quad M_{f1} = 1
\]

\[
x = 6 - 4 \cdot (1) = 2
\]

\[
y = j + R_{u1} - a \cdot x_{u1} \quad \Rightarrow \quad y = 3 + 8 - (a) \cdot (4) = 11 - 4 \cdot (a), \text{for} \quad a = 1, 2 \quad \text{where:} \quad M_{u1} = 2
\]

\[
\text{for} \quad a = 1, \quad y = 3 + 8 - (1) \cdot (4) = 7
\]

\[
\text{for} \quad a = 2, \quad y = 3 + 8 - (2) \cdot (4) = 3
\]

The \((x,y)\) binary values resulting from the \((i = 2, j = 3)\) binary value are \((2, 7)\) and \((2, 3)\), which represent the elements \( t^e_{(2,7)} \) and \( t^e_{(2,3)} \) and consequently gives

\[
t^e_{(2,7)} = t^e_{(2,3)} = t_{(2,3)} = 1
\]

In a similar method, we can complete all elements of the extended matrices \((ET, EB)\) as follows:

\[
T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad \rightarrow \quad ET = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad \rightarrow \quad EB = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}
\]

To find the dimensions of the extended weave matrix \( W_{(x,y)} \), these formulas can be used:

\[
R_w = R_{u1} + R_{u2} + R_{uc}.
\]

\[
R_f = R_{f1} + R_{f2}.
\]

To insert all the upper and lower layer threads and center warps into the extended weave \( W \), a matrix called \textbf{warps order matrix} \( D \) have to be identified.
In Figure 4, it can be noted that the warp ends can be numbered from 1 to 14. Designating each warp from the upper fabric \( T \), lower fabric \( B \), and Center warp \( C \), the warps numeration table can be deduced (Table 2).

In the warps numeration table, we can distinguish that the center warp is between the upper and lower warps. Hence, warps order \( T.C.B \) can be deduced. Then warps numeration table may be transferred into warps order diagram as shown in Figure 5.

This warps order diagram consists in three rows and \( M_w \) columns, where \( M_w \) are calculated from Equation 3. The first row indicates the upper layer warps and the second row specifies the center warps whereas the third row is for the lower layer warps.

The warps order diagram can be transferred into a binary matrix \( D \) of 0 and 1 having the same diagram dimensions, where each element of this matrix is represented by \( d_{(c,m)} \).

\[
D = \begin{bmatrix}
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

Warps Order Matrix

Another warps order \( T.B.C \) may be observed in double fabric stitched by center warps. In the warps order diagram corresponding to \( T.B.C \) order, the first row indicates the upper layer warps and the second row is specified for the lower layer warps whereas the third row is for the center warps, as illustrated in Figure 6. In this case, the warps numeration table is shown in Table 3, and the warps order matrix is:

\[
D = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0
\end{bmatrix}
\]

Threads Insertion in Extended Weave \( W \)

Upper and Lower Layer Threads Insertion

Each element from the extended upper and lower weave matrices \( ET \) and \( EB \) will be inserted in the matrix \( W \) as follows:

\[
t^e_{(i,j)} \rightarrow W \Leftrightarrow w^e_{(x,y)} = t^e_{(i,j)} \quad (14)
\]

\[
h^c_{(i,j)} \rightarrow W \Leftrightarrow w^c_{(x,y)} = h^c_{(i,j)} \quad (15)
\]

\[
x = i + \left[ \left( i - 1 \right) A_{fn} + \left( n - 1 \right) \right] \left( A_{f1} + A_{f2} - A_{fn} \right) \quad \text{for } i = 1 \, \text{to} \, R_{fn}. \quad (16)
\]

Each warp \( j \) will be put in the extended weave \( W \) as follows:
Figure 6. Double fabric stitched by center warps in T.B.C order.

Table 3. Warps Numeration Table for Order T.B.C.

|   |   |   |   |   |   | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| T | B | T | B | C |   | T | B | C | T  | B  |    |    |    |

The parameter K is calculated from the following equation:

\[ y = j + \left[ \left[ \left( (j-1) \backslash A_{wn} \right) + (n-1) \right] \cdot \left( A_{w1} + A_{w2} - A_{wn} \right) \right] + \sum_{m=1}^{k} (d_{(c,m)} A_{wc}). \]  

(17)

The parameter K is calculated from the following equation:

\[ k = \left\{ \left( (j-1) \backslash A_{wn} \right) + (n \backslash c) \right\}, \]  

(18)

where:

\[
\begin{cases}
\text{for } j = 1 \text{ to } R_{wn} \\
0 \leq k \leq M_w \\
0 = 1 \text{ for upper fabric} \\
0 = 2 \text{ for lower fabric}
\end{cases}
\]

It can be noted that in all equations, \( c = 2 \) if warps are in T.C.B order and \( c = 3 \) if warps are in T.B.C order.

Center Warps Insertion

To insert each center warp \( j \) (where \( j = 1, 2 \ldots R_{wc} \)) in the matrix \( W \), the smallest number(s) which carries out the following inequality has to be found:

\[ \sum_{k=1}^{s} (d_{(c,k)} A_{wc}) \geq j \text{ for } 1 \leq s \leq M_w. \]  

(19)

Then, the following equation is applied to put in the warp \( j \) into the matrix \( W \):
Based on Figure 4, the extended upper and lower layer matrices (\( E_T \) and \( E_B \)) were calculated based on T.C.B order and the D matrix was found. Other parameters were deduced as follows:

**Upper layer weave:**
- \( n = 1 \), \( r_{u1} = 4 \), \( r_{f1} = 4 \), \( A_{u1} = 2 \), \( A_{f1} = 1 \), \( R_{u1} = 8 \), \( R_{f1} = 4 \)
- \( c = 2 \) if warps in T.C.B order
- \( c = 3 \) if warps in T.B.C order

**Lower layer weave:**
- \( n = 2 \), \( r_{u2} = 4 \), \( r_{f2} = 4 \), \( A_{u2} = 1 \), \( A_{f2} = 1 \), \( R_{u2} = 4 \), \( R_{f2} = 4 \)

**Centre warps:**
- \( r_{wc} = 2 \), \( A_{wc} = 1 \), \( R_{wc} = 2 \)

The size of extend matrix \( W \) is calculated from Equations 12 and 13 as follows:

\[
R_u = R_{u1} + R_{u2} + R_c \quad \Rightarrow \quad R_u = 8 + 4 + 2 \quad \Rightarrow \quad R_u = 14
\]
\[
R_j = R_{f1} + R_{f2} \quad \Rightarrow \quad R_j = 4 + 4 \quad \Rightarrow \quad R_j = 8
\]

**Insertion of the element** \( t'_{(2,6)} = 1 \) **in the matrix** \( W \) **is done according to Equations 16, 17, and 18 as follows:**

\[
x = i + \left( (i-1)A_{f1} + (n-1)(A_{f1} + A_{f2} - A_{f1}) \right) = 2 + \left( (2-1)1 + (1-1)(1+1-1) \right) = 3
\]

\[
y = j + \left( (j-1)A_{u1} + (n-1)(A_{u1} + A_{u2} - A_{u1}) \right) + \sum_{m=1}^{j} (d_{(m)} \cdot A_{uc}) = 6 + \left( (6-1)2 + (1-1)(2+1-2) \right) + \sum_{m=1}^{2} (d_{(m)} \cdot (1)) = 6 + 2 + 1 = 8 + 0 + 1 = 9
\]

Then we find that \( w_{(x,j)} = t'_{(i,j)} \Leftrightarrow w_{(3,9)} = t'_{(2,8)} = 1 \).

**Insertion of the element** \( b_{(4,3)} = 0 \) **in the matrix** \( W \) **is done according to Equations 16, 17, and 18 as follows:**

\[
x = i + \left( (i-1)A_{f1} + (n-1)(A_{f1} + A_{f2} - A_{f1}) \right) = i + \left( (i-1)A_{f2} + 1 \right)(A_{f1}) = 4 + \left( ((4-1)1 + 1) \right) = 8
\]

\[
y = j + \left( (j-1)A_{u1} + (n-1)(A_{u1} + A_{u2} - A_{u1}) \right) + \sum_{m=1}^{j} (d_{(m)} \cdot A_{uc}) = 6 + \left( (6-1)2 + (1-1)(2+1-2) \right) + \sum_{m=1}^{2} (d_{(m)} \cdot (1)) = 6 + 2 + 1 = 8 + 0 + 1 = 9
\]

Then we find that \( w_{(x,j)} = t'_{(i,j)} \Leftrightarrow w_{(3,9)} = t'_{(2,8)} = 1 \).

**Insertion of the element** \( b_{(4,3)} = 0 \) **in the matrix** \( W \) **is done according to Equations 16, 17, and 18 as follows:**
\( n = 1, j = 3 \) and \( k = 3 \) \( \Rightarrow \)

\[
y = j + \left[\left( (j-1) \cdot A_{w1} \right) + (n-1) \cdot (A_{w1} + A_{w2} - A_{w1}) \right] + \sum_{m=1}^{k} (H_{(c,m)} \cdot A_{wc})
\]

\[
y = j + \left[\left( (j-1) \cdot A_{w2} + 1 \right) \cdot (A_{w1}) \right] + \sum_{m=1}^{k} (H_{(2,m)} \cdot A_{wc})
\]

\[
y = 3 + \left[\left( (3-1) \cdot A_{w2} + 1 \right) \cdot (2) \right] + \sum_{m=1}^{k} (H_{(2,m)} \cdot (1))
\]

\[
y = 3 + [6] + (1) \cdot \left( H_{(2,1)} + H_{(2,2)} + H_{(2,3)} \right) = 3 + [6] + (1) \cdot (0 + 1 + 0) = 10
\]

Then we find that \( w_{(x,y)} = h_{(c,i)}^e \Leftrightarrow w_{(8,10)} = h_{(4,3)}^e = 0 \). Now, we insert the center warp \( j = 2 \) into the matrix \( W \) according to Equations 19 and 20 where the number (s) has to be first found:

\[
\sum_{k=1}^{s} \left( d_{(c,k)} \cdot A_{wc} \right) \geq j \quad \Leftrightarrow \quad \sum_{k=1}^{s} \left( d_{(c,k)} \cdot A_{wc} \right) \geq 2
\]

for \( s = 1 \) we have : \( \sum_{k=1}^{s} \left( d_{(2,k)} \cdot (1) \right) = 0 \geq 2 \) rejected

for \( s = 2 \) we have : \( \sum_{k=1}^{s} \left( d_{(2,k)} \cdot (1) \right) = 1 \geq 2 \) rejected

for \( s = 3 \) we have : \( \sum_{k=1}^{s} \left( d_{(2,k)} \cdot (1) \right) = 1 \geq 2 \) rejected

for \( s = 4 \) we have : \( \sum_{k=1}^{s} \left( d_{(2,k)} \cdot (1) \right) = 2 \geq 2 \) accepted.

From Equation 20, we find \( y = j + s \cdot A_{w1} + \left[ s - (3 \mod c) \right] \cdot A_{w2} \)

\[
y = 2 + (4) \cdot (2) + \left[ 4 - (3 \mod 2) \right] \cdot (1) = 2 + 8 + 3 = 13
\]

In the same way, the whole \( W \) matrix can be formed given as follows:

\[
W = \begin{bmatrix}
- & - & - & - & 0 & - & - & 0 & - & - & 1 \\
1 & 0 & - & 0 & 1 & - & - & 1 & 0 & - & 0 & 1 & - & - \\
- & - & 0 & - & - & 0 & - & - & 1 & - & - & 1 \\
0 & 0 & - & 1 & 1 & - & - & 0 & 0 & - & 1 & 1 & - & - \\
- & - & 0 & - & - & 1 & - & - & 1 & - & - & 0 \\
0 & 1 & - & 1 & 0 & - & - & 0 & 1 & - & 1 & 0 & - & - \\
- & - & 1 & - & - & 1 & - & - & 0 & - & - & 0 \\
1 & 1 & - & 0 & - & - & 1 & 1 & - & 0 & 0 & - & - \\
\end{bmatrix}
\]

**Lifters**

Lifters in a double fabric stitched by center warp indicate that upper layer ends or center threads are over lower layer picks. Identifying the lifters and placing them in the \( W \) extended weave matrix require the determination of layers to which \( x \) and \( y \) of the \( w_{(x,y)} \) belong.

Therefore, for each warp \( y \), the first step is to find the smallest number \( m \) which verifies the following inequality:

\[
m \cdot (A_{w1} + A_{w2}) + \sum_{k=1}^{m} (d_{(c,k)} \cdot A_{wc}) \geq y
\]

where:

\[
\left\{ \begin{array}{l}
1 \leq m \leq M_w \\
1 \leq y \leq R_w
\end{array} \right.
\]

(21)
Then, we calculate the warp layer identifier \( p \) from the following equation:

\[
p = y - (m - 1) \left( A_{w1} + A_{w2} \right) - A_{w1} - \sum_{k=1}^{m-1} \left( d_{(c,k) - A_{we}} \right).
\]

(22)

The second step is to calculate weft layer identifier \( q \) as follows:

\[
q = \left[ (x \text{Mod} \left( A_{f1} + A_{f2} \right)) \right] + \left[ x \setminus \left( A_{f1} + A_{f2} \right) \right] \left( x / \left( A_{f1} + A_{f2} \right) \right) \left( A_{f1} + A_{f2} \right) \text{ for } x = 1 \text{ to } R_f.
\]

(23)

Five layer identifier conditions have to be imposed for \( p \) and \( q \) to determine the layer to which \( x \) or \( y \) belongs, they are

1. If \( 0 < q \leq A_{f1} \) then \( x \in \text{upper layer} \) and \( n(x) = 1 \).
2. If \( A_{f1} < q \leq (A_{f1} + A_{f2}) \) then \( x \in \text{lower layer} \) and \( n(x) = 2 \).
3. If \( -A_{w1} < p \leq 0 \) then \( y \in \text{upper layer} \) and \( n(y) = 1 \).
4. If \( \left( \left( 3 \text{ mod } c \right) \cdot (d_{(c,m) - A_{we}}) \right) \leq p \leq \left( (c - 3) \cdot A_{w2} + (d_{(c,m) - A_{we}}) \right) \) then \( y \in \text{lower fabric} \) and \( n(y) = 2 \).
5. If \( \left( (c - 3) \cdot A_{w2} \right) < p \leq \left( (c - 3) \cdot A_{w2} + (d_{(c,m) - A_{we}}) \right) \) then \( y \in \text{centre warps} \) and \( n(y) = 3 \).

According to the lifter’s definition, if \( x \) belongs to the lower layer and \( y \) belongs to the upper layer or to the center warp, then the intersection \( W_{(x,y)} \) represents a lifter giving \( w_{(x,y)} = 1 \).

Weaving the upper layer separately (i.e., without stitching between the two layers) requires keeping the lower and center ends without raising during the insertion of the upper wefts. Then, if \( x \) belongs to upper layer and \( y \) belongs to lower layer or center warps, the element \( w_{(x,y)} = 0 \). These conditions are summed up in Table 4.

For example, to complete the matrix \( W \) produced above, taking into account the elements \( W_{(x,y)} = W_{(5,7)} \) and \( W_{(x,y)} = W_{(6,13)} \), lifters have to be verified according to lifter conditions shown in Table 4.

First: for element \( W_{(5,7)} \) where \( x = 5 \) and \( y = 7 \), from Equation 21, the smallest number \( m \) must be found:

\[
m \cdot (A_{w1} + A_{w2}) + \sum_{k=1}^{m} \left( d_{(c,k) - A_{we}} \right) \geq y
\]

\[
m \cdot (2 +1) + \sum_{k=1}^{m} \left( d_{(2,k) - A_{we}} \right) \geq 7
\]

for \( m = 1 \Rightarrow 3 + \sum_{k=1}^{2} \left( d_{(2,k) - A_{we}} \right) = 3 + \left( d_{(2,1)} - A_{we} \right) = 3 + 1 = 4 \) \( \geq 7 \) rejected

for \( m = 2 \Rightarrow 6 + \sum_{k=1}^{2} \left( d_{(2,k) - A_{we}} \right) = 6 + \left( d_{(2,1)} - A_{we} \right) = 6 + (1 +1) = 8 \) \( > 7 \) accepted

Then, this inequality is verified when \( m = 2 \).

The layer identifier conditions mentioned previously have to be found according to the fabric parameters. The conditions are:

1. If \( 0 < q \leq 1 \). Then, the current \( x \) belongs to upper layer and \( n_x = 1 \).
2. If \( 1 < q \leq 2 \). Then, the current \( x \) belongs to lower layer and \( n_x = 2 \).
3. If \( -2 < p \leq 0 \). Then, current warp \( y \) belongs to upper layer and \( n_y = 1 \).
4. If \( 1 < p \leq 2 \). Then, the warp \( y \) belongs to lower layer, and \( n_y = 2 \).
5. If \( 0 < p \leq 1 \). Then, this warp \( y \) belongs to center warps and \( n_y = 3 \).
For the warp $y = 7$, the warp layer identifier $p$ is calculated from Equation 22:

$$p = 7 - (2 - 1). (2 + 1) - 2 - \sum_{k=1}^{1} (d_{(2,k)})(1) = 2.$$ 

For the weft $x = 5$, weft layer identifier $q$ can be calculated from Equation 23:

$$q = \left[ (5 \text{Mod} \ (1 + 1)) + \left\lceil \left( 5 \left( 1 + 1 \right) \right) \right\rceil / \left( 1 + 1 \right) \right]. (1 + 1) = 1.$$ 

In comparison with the layer identifier conditions, it can be noted that the warp $y = 7$ belongs to the lower fabric and $n_{(y)} = 2$, while the weft $x = 5$ belongs to the upper fabric and $n_{(x)} = 1$.

Then, according to lifter conditions, we find that $n_{(x)} = 1, n_{(y)} = 2$ and $n_{(y)} \neq n_{(x)}$ then $W_{(5,7)} = 0$ and no lifter is made in this intersection.

**Second:** for element $W_{(6,13)}$ where $x = 6$ and $y = 13$, following the same steps, we find as follows: $m = 4, p = 1, q = 2, n_{(x)} = 2$ and $n_{(y)} = 3$.

In comparison with the lifter conditions, we find $n_{(x)} = 2, n_{(y)} = 3$ and $n_{(y)} \neq n_{(x)}$, then $W_{(6,13)} = 1$ and lifter is made in this intersection.

The following matrix $W$ can be found from lifter determination for all matrix elements discussed above:

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$ 

This matrix represents the upper and lower layer weaves and the center warps. The stitching of the two layers by center warps is not yet made.

To determine to which layer the element $x$ belongs ($n_{(x)}$ th) and its position ($i$th) in that layer, the following equation can be used:

$$i = \left[ x \left( A_{f1} + A_{f2} \right) \right] A_m + \left[ q - (n_{(x)} - 1).A_{f1} \right].$$

(24)

In a similar method, $y$ warp represents the $j$th warp of the $n_{(y)}$th layer and $j$ can be calculated as follows:

$$j = P + (1 \left( n_{(y)} \right) A_{w1} + \left[ (m - 1) \left( 1 - (n_{(y)} \cdot 3) \right) \right] A_m - \left[ (n_{(y)} \cdot 3) \cdot (c - 2) \right] A_{w2} + \left( (n_{(y)} \cdot 3). \sum_{k=1}^{n_{(y)} - 1} d_{(c,k)} \right) - (3 \text{ mod } n_{(y)}). (3 \text{ mod } c) \right] A_{wc},$$

(25)

where $A_{w3} = A_{c}$ and the parameter $m$ are calculated from Equation 21.

For verifying the above equations, based on the $W_{(5,7)}$ element calculated previously, where:

$$n_{(x)} = 1, \ n_{(y)} = 2, \ q = 1, \ \ p = c = m = 2, \ A_{w1} = 2, \ A_{f1} = A_{w2} = A_{f2} = A_{wc} = 1$$

$x = 5 \Rightarrow$ from equation 24 $\Rightarrow i = \left[ 5 \left( 1 + 1 \right) \right].1 + \left[ 1 - (1 - 1).1 \right] \Rightarrow i = 3$

$y = 7 \Rightarrow$ from equation 25 $\Rightarrow j = 2 + (1 \left( 2 \right) .2 + \left[ (2 - 1) \left( 1 - (2 \cdot 3) \right) \right] \cdot 2$

$- \left[ (2 \cdot 3). (2 - 2) \right].1 + \left( (2 \cdot 3). \sum_{k=1}^{2} d_{(2,k)} \right) - (3 \text{ mod } 2) . (3 \text{ mod } 2) \right].1$

$\Rightarrow j = 2 + (0) .2 + \left[ (1).1 - 0] \right] .1 - \left[ (0). (0) \right] + \left( (0) . d_{(2,0)} \right) - (1) . (1) \right].1 = 2.$
Then, the fifth row of the matrix \( W \) represents the third weft of the upper layer and the seventh column of the matrix \( W \) represents the second warp of the lower layer.

### Stitches determination

To obtain a double fabric stitched, stitches have to be introduced among the two layers by means of center warps. The upper stitch takes place on the upper layer (center warp over upper layer weft), represented in the weave diagram as (\( x \)). In this case, the element \( w(x,y) \) takes the value 1 in the \( W \) matrix, inducing \( w_{xy} = 1 \) and \( x \) belongs to upper layer (\( n(x) = 1 \)).

Contrarily, the lower stitch is always on the lower layer (lower layer weft is over the center warp). The symbol (\( O \)) is used to indicate this stitch in the weave diagram and the value (0) is given to the element \( w(x,y) \) in the \( W \) matrix, inducing \( w_{xy} = 0 \) and \( x \) belongs to lower layer (\( n(x) = 2 \)). This stitch can be obtained by canceling the lifter corresponding to these coordinates.

To make stitches, we have to put some conditions. Considering that \( y_c \) is a center warp of the matrix \( W \) is to be bonded to the \( n \)th layer (upper or lower layer) with the assumption that the stitch is to be made at the crossover corresponding to the \( w(x,y_c) \) element. From the warp point of view, stitch have to be after or before adjacent \( w(x,y_1) \) or \( w(x,y_2) \) warp ends, or between two adjacent \( w(x,y_1) \) and \( w(x,y_2) \) warp ends on the same layer \( n \) to which the weft \( x \) belongs.

All conditions cited previously can be illustrated in Table 5, in which the following parameters and symbols have to be defined:

- \( y_c \) represents the center warp presented in the matrix \( W \), it is calculated from Equations 19 and 20 for \( j = 1 \) to \( R_{wc} \).
- \( n(x) \) is the layer to which the weft \( x \) belongs. It can be identified from Equation 23 and the layer identifier conditions.
- \( y_1 \) and \( y_2 \) are two consecutives warps from the same layer, between them a stitch may be produced by the \( y_c \) center warp. These two warps are calculated from the following equations:

\[
y_1 = y_c + \alpha - \varepsilon \cdot (j \ Mod \ A_{wc}) \cdot \left[ (A_{wc}) \cdot \left[ (j \ Mod \ A_{wc}) \cdot (j / A_{wc}) \right] \right].
\]

\[
y_2 = \left( y_c + \alpha + \gamma + R_w - (j \ Mod \ A_{wc}) \cdot \left[ (A_{wc}) \cdot \left[ (j \ Mod \ A_{wc}) \cdot (j / A_{wc}) \right] \right] \right) \ Mod \ R_w
\]

\[
\begin{align*}
  \text{if } y_2 & = 0 \text{ then we take: } y_2 = R_w \\
  \text{if } y_1 & = 0 \text{ then we take: } y_1 = R_w
\end{align*}
\]

where \( \alpha, \beta, \gamma \) are parameters depending on the warps arrangements, warps order, \( n(x) \) and \( n(y_1) \), they can be found from Table 5.

- \( n(y_1) \) is the layer to which the warp \( y_1 \) belongs. Equation 22 can be applied to identify this layer, knowing that \( y_2 \) belongs to same layer of \( y_1 \).
Example: To carry out the stitching by the center warp \( j = 2 \) (where \( j \) ranges from 1 to \( R_{wc} = 2 \)), we have to insert this warp in the matrix \( W \) using Equations 19 and 20. We deduce that \( y_c = 13 \).

Now, taking the intersection \( w_{(x,y)} = w_{(5,13)} \), where \( x = 5 \), and from Equation 23, we can find that \( n_{(x,y)} = 1 \).

Knowing that the warps in T.C.B order and \( n_{(x,y)} = 1 \), we find from Table 5 that \( \alpha = 1, \ v = 1, \ \gamma = \beta \) and \( \beta = 2 + 1 + 1 - 2 = 2 \).

Then, from Equation 26, we calculate \( y_1 \):

\[
y_1 = 13 + 1 \mod 2 - (2 \mod 1) - \left[ (1) \cdot \left( (2/1)(2/1) \right) \right] = 12.
\]

From Equation 22, we find that the layer to which the warp \( y_1 = 12 \) belongs is: \( n_{y_1} = 1 \), and then

\[
A_{w_1} = A_{w_{(y_1)}}.
\]

Then, we calculate \( y_2 \) according to Equation 27:

\[
y_2 = \left( 13 + 1 + 2 + 14 - \left( 2 \mod 1 \right) - \left[ (1) \cdot \left( (2/1)(2/1) \right) \right] \right) \mod 14 = 29 \mod 14 \Rightarrow y_2 = 1.
\]

We find that the intersection \( w_{(x,y_2)} = w_{(5,12)} = 1 \) and \( w_{(x,y_1)} = w_{(5,1)} = 0 \).

Then all the intersections \( w_{(x,y_2)} \) for \( j = 1, \ldots, R_{wc} \) have to be equal to 1, that is, \( w_{(5,13)} = 1 \), this intersection represents an upper stitch.

In the same way, we can find all possible stitches in the \( W \) matrix for \( x = 1 \) to \( R_{wc} \).

This \( W \) matrix represents the double woven fabric shown in Figure 4 with all possible stitch points represented in the following intersections:

Upper stitches: \( w_{(1,6)} = 1, \ w_{(5,6)} = 1, \ w_{(7,6)} = 1, \ w_{(1,13)} = 1, \ w_{(5,13)} = 1, \ w_{(7,13)} = 1 \)
Lower stitches: \( w_{(4,6)} = 0, \ w_{(6,6)} = 0, \ w_{(8,6)} = 0, \ w_{(2,13)} = 0, \ w_{(4,13)} = 0, \ w_{(8,13)} = 0 \)

\[
W = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

The \( W \) matrix gives the maximum stitch points available. But, the user could choose the stitches according to the density of stitch points required in the final weave. For example, we choose two upper stitches in the intersection \( w_{(2,13)} = 1 \) and \( w_{(1,6)} = 1 \) and two lower stitches represented by the elements \( w_{(2,13)} = 1 \) and \( w_{(4,6)} = 1 \) and the other intersections automatically come back to non-stitching position:

\[
W_{(1,6)} = 1, \ w_{(5,6)} = 0, \ w_{(7,6)} = 0, \ w_{(1,13)} = 0, \ w_{(5,13)} = 0, \ w_{(7,13)} = 1
\]
\[
W_{(4,6)} = 0, \ w_{(6,6)} = 1, \ w_{(8,6)} = 1, \ w_{(2,13)} = 0, \ w_{(4,13)} = 1 \ w_{(8,13)} = 1
\]

\[
W = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Description of the Programmatic Module

In the implanted program, the first step is to define all fabric parameters such as threads arrangements and upper and lower weave repeats, in addition to the center warps arrangement and repeat. The weaves diagrams and warps order diagram will be automatically created according to the inserted parameters. To obtain the upper and lower weaves, the designer has to click on the square for filling it if warp is to be over weft and leave a square blank if the intersection represents the warp under the weft. Warps order diagram has to be marked according to the threads' position required in the final weave. Figure 7 represents the interface for identifying and designing the double fabric given in Figure 4.

The weave will be automatically calculated and the double fabric weave with all possible stitches will be generated and shown in another window (Figure 8). After the selection of required stitch points, the final double fabric weave will be generated and visualized in a separate window as shown in Figure 9.

Conclusion

Double woven fabrics is a type of compound woven structure composed of upper and lower layers stitched between them by various ways such as self-stitching, double stitching, center warp stitching, center weft stitching and so on. The double cloth stitched by center threads has been mathematically described and automatically generated by developing of mathematical programmable module. In this program, after identification of this double fabric (warp and weft repeat, threads arrangement, warps order matrix), an extended weave will be created, showing all lifters and all possible stitch points. The designer will select the stitch points required to automatically create the final weave diagram which is ready to be transferred to the weaving machine after the calculation of draft and lifting plans.

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