Improvement of the method of geometric design of gear segments of a planetary rotary hydraulic machine

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Abstract. Until now, planetary rotary hydraulic machines with floating planetary gears were rarely produced and remained poorly studied. One of the main problems was the lack of a simple and affordable method for geometric design of non-circular gear rims of such machines. The article discusses an engineering method for profiling non-circular gear rims of volumetric planetary rotary hydraulic machines. The proposed method begins with the choice of the geometric parameters of the round-link planetary mechanism - the prototype of the designed hydraulic machine. Next, the number of waves of the sun gears and the form of the cyclic function characterizing the paths of the planetary gear center in the coordinates associated with the sun gear and the epicycle are selected. The preliminary calculation of the angle of rotation of the planetary gear and the polar coordinates of its center on the path relative to the given sun gear in the variety of the planetary gear positions is performed at the next stage. In this case, the positions of the planetary gear relative to the one of the sun gears correspond to its rolling along the calculated centroid of this gear at a constant angular velocity of the imaginary carrier. Next, the difference between the obtained angles of rotation of the planetary gear is calculated in the coordinates associated with the imaginary carrier. This difference is halved and distributed as a correction between the rotation angles of the planetary gear relative to the sun gear and the epicycle. In conclusion, the profile of the rim of each non-circular sun gear is obtained graphically as an envelope of the family of profile curves of the planetary gear in a variety of its positions. The proposed method provides an accurate solution to the problem and is available for the designers of any machine-building enterprise.

1. Introduction

Planetary rotary hydraulic machines (PRHM) with floating planetary gears (see, for example, figure 1) are the volumetric machines. PRHMs were not produced and remained poorly studied until now mainly due to the difficulties in manufacturing of non-circular gear segments. Difficulties arise in the process of the geometric design of the segments of such machines.

The theoretic framework of the synthesis of non-circular gears were established by F.L. Litvin [1]. In the later works of the other authors [3; 7; 8] the matters of the profiling of the non-circular gear wheels using CNC systems (machine tools) were considered. The design of the planetary mechanisms with non-circular sun gears leads to a number of additional conditions and difficulties [2; 3; 4; 5; 6].
Figure 1. Diagram of the PRHM 2×3. 1 – sun gear, 2 – epicycle, 3 – planetary gear.

The problem of geometric synthesis of the working mechanism of the PRHM was fully solved earlier only in the works of An I-Kan [2]. According to An I-Kan, the synthesis of centroids is performed at the first stage. In this case, the centroid of one of the sun gears and two centroids of the planetary gear are first set. Then the dependencies are set that ensure the fulfillment of the necessary kinematic relations and conditions for the closure of the centroids. The centroid of the second sun gear is obtained as the envelope of the second centroid of the planetary gear. Further, the tooth profiles are applied to these centroids using the methods of the theory of gearing. The disadvantage of the method developed by An I-Kan is the complexity of its implementation.

2. Method to be improved

In our previous works [9, 10] an engineering method for the geometric synthesis of PRHM was proposed.

1. At the first stage the parameters of the prototype of the designed working mechanism of the hydraulic machine are selected - the initial, calculated round-link mechanism. The calculation is carried out according to the standard method [11]. The output of the calculation: all the numbers of teeth \( Z_1; Z_2; Z_3 \) and displacement factors \( \chi_1; \chi_2; \chi_3 \), as well as a module \( m \) and axle spacing \( a_w \). In the example shown in figure. 1: \( Z_1=46; Z_2=69; Z_3=10; \chi_1=1; \chi_2=-0.04; \chi_3=0.3; m=1; a_w=29.144 \).

2. At the second stage, the numbers of waves of the sun gears \( M, N \) and interdependent (the same type) cyclic functions characterizing the path of the center of the planetary gear in the coordinates associated with each of the sun gears 1 and 2 are specified. In the general case, the equations of these paths in polar coordinates associated with each of the sun gears 1 and 2 are the following:

\[
\begin{align*}
r_1(\varphi_1) &= r_0 \cdot (1 + k \cdot F(M \cdot \varphi_1)); \\
r_2(\varphi_2) &= r_0 \cdot (1 + k \cdot F(N \cdot \varphi_2)),
\end{align*}
\]

where \( r_1(\varphi_1) \) and \( r_2(\varphi_2) \) – radius vectors of the planetary gear center paths; \( \varphi_1 \) and \( \varphi_2 \) – current angles of rotation of the imaginary carrier in polar coordinates associated with the corresponding segments;

\( k \) – coefficient of “non-circularity” of the paths;

\( r_0 = a_w \) – the radius of the calculated circle (into which both paths are degenerate at \( k = 0 \)).

In the simplest cases, a cyclic function is used: \( F(\varphi) = \cos(\varphi) \)

\[
\begin{align*}
r_1 &= r_0 \cdot (1 + k \cdot \cos(M \cdot \varphi_1)); \\
r_2 &= r_0 \cdot (1 + k \cdot \cos(N \cdot \varphi_2)).
\end{align*}
\]
In the example under consideration, the paths of the center of the planetary gear correspond to equations (3) and (4) with the coefficient $k = 0.08$.

3. Further, the variety of positions of the center of the planetary gear on the path given by equation (3) or (4), and the angles $\phi_c$ with its rotation relative to the given sun gear are calculated.

The angle $\phi_{c1}$ of rotation of the planetary gear relative to the corresponding sun gear 1 is determined by the equation:

$$\phi_{c1(2)} = \left(1 \pm \frac{Z_{1(2)}}{Z_3}\right) \cdot \xi_{1(2)} \cdot \int_{0}^{\varphi} \sqrt{(r_1(\varphi_{1(2)}))^2 + (r_1'(\varphi_{1(2)}))^2} \, d\varphi,$$

where $r_1'(\varphi_{1(2)})$ – the derivative of the corresponding function $r_1(\varphi_1)$ or $r_2(\varphi_2)$

$\xi_{1(2)}$ – the coefficient taking into account the change in the length of the corresponding center path in comparison with the length of the center circle of the original round link mechanism:

$$\xi_{1(2)} = \frac{2\pi}{\int_{0}^{\varphi} \sqrt{(r_1(\varphi_{1(2)}))^2 + (r_1'(\varphi_{1(2)}))^2} \, d\varphi}.$$

In the particular case, when the cyclic function $F(\varphi) = \cos(\varphi)$, the equations (5), (6) take the following form:

$$\phi_{c1} = \left(1 + \frac{Z_1}{Z_3}\right) \cdot \xi_1 \cdot \int_{0}^{\varphi} \sqrt{(1 + k \cdot \cos(M \cdot \varphi))^2 + (M \cdot k \cdot \sin(M \cdot \varphi))^2} \, d\varphi;$$

$$\phi_{c2} = \left(1 - \frac{Z_2}{Z_3}\right) \cdot \xi_2 \cdot \int_{0}^{\varphi} \sqrt{(1 + k \cdot \cos(N \cdot \varphi))^2 + (N \cdot k \cdot \sin(N \cdot \varphi))^2} \, d\varphi;$$

$$\xi_1 = \frac{2\pi}{\int_{0}^{\varphi} \sqrt{(1 + k \cdot \cos(M \cdot \varphi))^2 + (M \cdot k \cdot \sin(M \cdot \varphi))^2} \, d\varphi};$$

$$\xi_2 = \frac{2\pi}{\int_{0}^{\varphi} \sqrt{(1 + k \cdot \cos(N \cdot \varphi))^2 + (N \cdot k \cdot \sin(N \cdot \varphi))^2} \, d\varphi}.$$

For each of the non-circular sun gears the arrays of the parameters are created using mathematical software, for example, the program for the engineering calculations Mathcad: $\varphi$ - the current angle of rotation of the imaginary carrier relative to the given sun gear; $r_i$ - radius vector of the corresponding center path; $\phi_i$ - the rotation angle of the planetary gear in the coordinates associated with the given sun gear.

In the example in figure 1 the coefficients $\xi_1 = 0.9776$, $\xi_2 = 0.9520$. The calculation results using equations (7) and (8) are shown in table 1.

4. A planetary gear is designed in a variety of positions and the profile of the corresponding non-circular gear rim is found as an envelope of a family of curves using graphic programs.
Table 1. Parameters characterizing the law of motion of the planetary gear relative to the sun gears.

| Sun gear | Epicycle |
|----------|----------|
| $r_1$    | $\varphi_1$ | $\varphi_{c1}$ | $r_2$    | $\varphi_2$ | $\varphi_{c2}$ |
| 33.57389 | 0°       | 0°              | 33.57389 | 0°         | 0°              |
| 33.57292 | 0.4°     | 2.52284°        | …        | …          | …               |
| …        | …        | …               | 26.01177 | 30°        | -192.78387°     |
| 26.01177 | 45°      | 275.48405°      | …        | …          | …               |
| …        | …        | …               | …        | …          | …               |
| 33.57389 | 360°     | 2016°           | 59.5601  | 360°       | -22123.93°      |

This method is available to a wide range of users and is quite efficient in most practical cases. However, in some PRHM schemes, with extreme values of the coefficient $k$ in certain "critical" phases of the rotor rotation, there is a decrease in the side clearances between the teeth (up to the binding). The reason for this phenomenon is that the planetary gear cannot come to the calculated "critical" point $O_c$ with the angle of its rotation exactly corresponded to both gearings (figure 2). In order to assemble the mechanism, it is necessary to shift the center $X$ of the planetary gear in the circumferential direction by a distance $\delta$. However, with such a displacement, the planetary gear turns out to be clamped between the rims 1 and 2. The magnitude of the radial "interference" (deviation $\delta'$) is approximately equal to $\delta' \approx \delta \lambda$, where $\lambda$ is the angle of the planetary gear "retention".

3. Proposed improvement of the method
To design the profiles of the PRHM gears that are compatible with each other without an error in the angular position of the planetary gear, the rims of both sun gears must be obtained by bending around the planetary gear making one common movement. Such a problem can be solved by methods of the

Figure 2. Position of the planetary gear in the "critical" phase of movement.
gearing theory [1] in a theoretically rigorous formulation (through the transformation of coordinates). The simplest way to perform the necessary correction of the profile of the gear rim is given below. At first it is proposed to proceed from the rotation angles of the planetary gear \( \phi_{c1}, \phi_{c2} \) relative to both sun gears to the angles of their rotation relative to the imaginary carrier \( \phi_{c1,h} = (\phi_{c1} - \phi_{1}); \phi_{c2,h} = (\phi_{c2} - \phi_{2}) \).

If the condition \( \phi_{2} = M/N\phi_{1} \) is met, the difference \( D_{i} \) of the angles \( \phi_{c1,h} \) and \( \phi_{c2,h} \) represents the addition to the angle of rotation of the planetary gear relative to the epicycle in the process of profiling its rim, which will eliminate the undesirable phenomenon of "wedging" of the mechanism.

Now the necessary operations in the form of an algorithm that is convenient for a wide range of users will be presented.

1. The angles of rotation of the imaginary carrier corresponding to the "critical" points on the rims of the sun gears are determined. For the sun gear \( \phi_{1k} = 360°/4M \), for the epicycle \( \phi_{2k} = 360°/4N \).

2. The difference between the corresponding values of the rotation angles of the planetary gear \( \phi_{c2}, \phi_{c1} \) relative to the sun gear and the epicycle (Table 1) are calculated for the "critical" points and the maximum angle \( \Delta_{\text{max}} \) of the required "additional rotation" of the planetary gear is calculated:

\[
\Delta_{\text{max}} = (\phi_{c1} - \phi_{c2}) - (\phi_{1k} + \phi_{2k}).
\]

3. The values of the angles of rotation of the planetary gear used to construct the gear rims are specified.

a) The obtained angle \( \Delta_{\text{max}} \) can be used, for example, to correct the profile of the epicycle rim:

\[
\phi_{c2}^{\text{nev}} = -\phi_{c2} + \Delta_{\text{max}} \cdot \sin\left(N \cdot \phi_{2}\right).
\]

In this case, the profile of the other sun gear remains unchanged.

It can be seen (figure 3) that the corrected contour 2 of the epicycle rim and the original contour 1 are somewhat displaced relative to each other. The performed profile correction eliminates the "wedging" of the mechanism.

b) The opposite operation can be performed by adjusting the profile of the sun gear:

\[
\phi_{c1}^{\text{nev}} = \phi_{c1} - \Delta \cdot \sin\left(N \cdot \phi_{1}\right).
\]

The most rational option would be to distribute the "additional rotation" between the gear and the epicycle equally.

\[
\phi_{c2}^{\text{nev}} = -\phi_{c2} + \frac{\Delta \cdot \sin\left(N \cdot \phi_{2}\right)}{2};
\]

\[
\phi_{c1}^{\text{nev}} = \phi_{c1} - \frac{\Delta \cdot \sin\left(N \cdot \phi_{1}\right)}{2}.
\]

![Figure 3. The contours of the rims of the epicycles.](image)
4. Conclusions
The proposed method eliminates the previously obtained insignificant negative phenomenon of the convergence of the rims in the "critical" phase of their movement without sufficient complication of the process of profiling the PRHM gear rims.

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