Prospect of Higgs physics at future $e^+e^-$ linear colliders is reviewed for the weakly-coupled Higgs sector. Several topics related to the determination of various couplings of the Higgs boson in the standard model as well as in the supersymmetric standard model are discussed.

1 Introduction

It has been more than twenty years since the basic idea of the standard model (SM) of elementary particle theory was proposed. This theory is based on two fundamental principles, i.e., the gauge principle and the Higgs mechanism. From recent experiments it is now clear that strong and electroweak interactions are described by an $SU(3) \times SU(2) \times U(1)$ gauge theory. On the other hand, little is known about mechanism of the electroweak symmetry breaking. To clarify dynamics behind this symmetry breaking in the SM is the primary remaining objective and the most important physics motivation of future experiments at both LHC and $e^+e^-$ linear colliders.

Exploring the Higgs sector is important not only because the Higgs particle is the only ingredient still missing from the SM, but also because this will be a key to the physics beyond the SM. In the minimal SM the Higgs sector consists of one Higgs doublet, and the only free parameter is the Higgs-boson mass. A heavy Higgs boson corresponds to a large self-coupling constant, and a light Higgs boson suggests that the dynamics of the Higgs sector is well described perturbatively. For many extensions of the Higgs sector a similar relationship holds between the Higgs-boson mass and the strength of the interaction which governs the symmetry-breaking dynamics. For example, if we require perturbative unification of the three gauge coupling constants as predicted in Grand Unified Theories (GUT), the Higgs particle cannot be heavier than about 200 GeV. On the other hand, models like Technicolor predict either a very heavy scalar particle or no particle at all which acts as the Higgs boson.
In this talk I will discuss the physics of the Higgs sector at future $e^+e^-$ linear-collider experiments with a center-of-mass (CM) energy ranging from 300 GeV to 1.5 TeV. I restrict myself to the “light Higgs case” where at least one Higgs particle exists below, say, 200 GeV, and the dynamics behind this particle is described by perturbation theory. The strategy to study the strongly coupled Higgs sector in future $e^+e^-$ linear-collider experiments is quite different from that in the light Higgs case, and it is covered elsewhere in this workshop.

From intensive discussions, including those reported in the last workshop of this series, it is now clear that the discovery of such a Higgs particle is easy at an $e^+e^-$ linear collider with $\sqrt{s} \sim 300 - 500$ GeV, if it exists with a mass below 200 GeV and with a production cross-section and decay branching ratios similar to the SM Higgs boson. Therefore I would like to stress subjects which become important after the Higgs particle is discovered. In other words, I would like to consider how various couplings related to the Higgs particle will be measured in future $e^+e^-$ experiments and what the impact of these measurements will be on the establishment of the SM and the search for physics beyond the SM. Many of the contributions in the Higgs session in this workshop are related to various Higgs couplings.

In this talk I will first give a short review of the SM Higgs sector and the Higgs sector in the Minimal Supersymmetric Standard Model (MSSM). Also I will comment on Higgs mass and properties for some extended versions of supersymmetric (SUSY) standard models. Then, I will report four topics discussed at this workshop, all of which are related to measurements of various Higgs couplings.

2 The SM Higgs

In the minimal SM the Higgs-boson mass ($m_h$) and the Higgs-boson self-coupling constant ($\lambda$) from the Higgs potential ($V_{\text{Higgs}} = m^2|\Phi|^2 + \lambda|\Phi|^4$, ($m^2 \leq 0$)) are related by $m_h^2 = 2\lambda \nu^2$, where $\nu(= 246$ GeV) is the vacuum expectation value. As a result, the theory behaves quite differently for small and large values of the Higgs-boson mass. This situation can be most easily understood by evaluating the running self-coupling constant, $\lambda$, using the renormalization group equations (RGE’s). Neglecting Yukawa coupling constants except for the one for the top quark, $y_t$, the RGE for $\lambda$ at the one-loop level is written as

$$\frac{d\lambda}{dt} = \frac{1}{(4\pi)^2} \{24\lambda^2 + 12y_t^2\lambda - 6y_t^4 - 12A\lambda + 6B\}, \quad (1)$$

where $A = \frac{1}{4}g_1^2 + \frac{3}{2}g_2^2$, $B = \frac{1}{16}g_1^4 + \frac{3}{8}g_1^2g_2^2 + \frac{3}{8}g_2^4$ for $U(1)$ and $SU(2)$ gauge coupling constants $g_1$ and $g_2$ respectively and $t = \ln \mu$ ($\mu$ is the renormalization scale). Since the input values of the gauge coupling constants and the top Yukawa coupling constant at the electroweak scale can be determined from recent experimental results, we can draw the flow of the running self-coupling constant for each value of $m_h$ as shown in Figure 1.
Figure 1: The flow of the Higgs self-coupling constant ($\lambda$) in the minimal SM for the several values of the Higgs boson masses. The top quark mass is assumed to be 170 GeV.

From this figure it is clear that the possible scenarios for new physics are different for two cases, i.e. (i) $m_h \gg m_{\text{top}}$ and (ii) $m_h \lesssim m_{\text{top}}$. In case (i) the coupling constant becomes very large at a relatively low energy scale, and therefore new physics is indicated well below the Planck scale ($\approx 10^{19}$ GeV). On the other hand, the theory can be weakly coupled up to approximately the Planck scale in case (ii), which is consistent with the idea of grand unification. Of course the flow of the coupling constants is different if we change the particle content in the low-energy theory, but the upper bound on the Higgs mass is believed to be about 200 GeV for most GUT models. This is also true for the SUSY GUT. As for the MSSM, however, a stronger bound on the Higgs mass is obtained independently of the assumption of grand unification.

3 The Higgs Sector in the MSSM

Supersymmetry is now considered as the most promising candidate for physics beyond the SM. Recent improvements in measurements of the three gauge coupling constants enable us to distinguish various GUT models, and it has become clear that the supersymmetric version is favored. The particle content of the SUSY GUT below the GUT scale is just that of the MSSM.

The Higgs sector of the MSSM consists of two Higgs doublets. The most important feature of this Higgs sector is that the Higgs-self-coupling constant at the tree level is completely determined by the $SU(2)$ and $U(1)$ gauge coupling constants. Af-
ter electroweak symmetry breaking, the physical Higgs states include two CP-even Higgs bosons \((h, H)\), one CP-odd Higgs boson \((A)\) and one pair of charged Higgs bosons \((H^\pm)\) where we denote by \(h\) and \(H\) the lighter and heavier Higgs bosons respectively. Although at the tree level the upper bound on the lightest CP-even Higgs boson mass is given by the \(Z^0\) mass, the radiative corrections weaken this bound.\[1\] The Higgs potential is given by

\[
V_{Higgs} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1 \cdot H_2 + \bar{H}_1 \cdot \bar{H}_2) \\
+ \frac{g_2^2}{8}(\bar{H}_1 \tau^a H_1 + \bar{H}_2 \tau^a H_2)^2 + \frac{g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 \\
+ \Delta V,
\]

where \(\Delta V\) represents the contribution from one-loop diagrams. Since the loop correction due to the top quark and its superpartner, the stop squark, are proportional to the fourth power of the top Yukawa coupling constant and hence are large, the Higgs self-coupling constant is no longer determined only by the gauge coupling constants. The upper bound on the lightest CP-even Higgs mass \((m_h)\) can significantly increase for a reasonable choice of the top-quark and stop-squark masses. Figure 2 shows the upper bound on \(m_h\) as a function of top-quark mass for several choices of the stop mass and the ratio of two Higgs-boson vacuum expectation values \((\tan \beta = \frac{<H_0^2>}{<H_0^1>})\). We can see that, in the MSSM, at least one neutral Higgs-boson should exist below 130 - 150 GeV depending on the top and stop masses.

Other Higgs states, namely the \(H, A, H^\pm\), are also important to clarify the structure of the model. Their existence alone is proof of new physics beyond the SM, but we may be able to distinguish the MSSM from a general two-Higgs model through the investigation of their masses and couplings. In the MSSM the Higgs sector is described by four independent parameters for which we take the mass of the CP-odd Higgs boson \((m_A)\), \(\tan \beta\), the top-quark mass \((m_t)\) and the stop mass \((m_{stop})\). The top and stop masses enter through radiative corrections to the Higgs potential. Speaking precisely, there are left- and right-handed stop states which can mix to form two mass eigenstates; therefore more than just one parameter is required to specify the stop sector. In Figure 3, the masses for the \(H, A, H^\pm\) are shown as a function of \(m_A\) for several choices of \(\tan \beta\) and \(m_{stop}=1\) TeV. We can see that, in the limit of \(m_A \rightarrow \infty\), \(m_h\) approaches a constant value which corresponds to the upper bound in Figure 2. Also in this limit the \(H, A\) and \(H^\pm\) become degenerate in mass.

The neutral Higgs-boson couplings to gauge bosons and fermions are determined by the ratio of vacuum expectation values \(\tan \beta\) and the mixing angle \(\alpha\) of the two CP-even Higgs particles defined as

\[
ReH_1^0 = \frac{1}{\sqrt{2}}(v \cos \beta - h \sin \alpha + H \cos \alpha)
\]
Figure 2: The upper bound on the lightest CP-even Higgs mass in the MSSM as a function of the top quark mass for various $\tan\beta$ and two large stop mass scales. The solid (dashed) line corresponds to $m_{\text{stop}}=1$ (10) TeV without left-right mixing of two stop states. These masses are calculated by the method with the renormalization group equation.

Figure 3: The light ($h$), heavy ($H$) CP-even Higgs masses and the charged Higgs ($H^\pm$) mass as a function of the CP-odd Higgs ($A$) mass. The top and stop masses are taken as $m_t = 170$ GeV and $m_{\text{stop}} = 1$ TeV.
\begin{equation}
ReH_2^0 = \frac{1}{\sqrt{2}}(v\sin\beta + h\cos\alpha + H\sin\alpha).
\end{equation}

For Higgs-boson production, the Higgs-bremsstrahlung process $e^+e^- \rightarrow Zh$ or $ZH$ and the associated production $e^+e^- \rightarrow Ah$ or $AH$ play complimentary roles. Namely $e^+e^- \rightarrow Zh$ ($ZH$) is proportional to $\cos(\beta - \alpha)(\sin(\beta - \alpha))$, and $e^+e^- \rightarrow Ah$ ($AH$) is proportional to $\sin(\beta - \alpha)(\cos(\beta - \alpha))$, so at least one of the two processes has a sizable coupling. It is useful to distinguish the following two cases when we discuss the properties of the Higgs particles in the MSSM, namely (i) $m_A \lesssim 150$ GeV, (ii) $m_A \gg 150$ GeV. In case (i), the two CP-even Higgs bosons can have large mixing, and therefore the properties of the neutral Higgs boson can be substantially different from those of the minimal SM Higgs. On the other hand, in case (ii), the lightest CP-even Higgs becomes a SM-like Higgs, and the other four states, $H, A, H^\pm$ behave as a Higgs doublet orthogonal to the SM-like Higgs doublet. In this region, $\cos(\beta - \alpha)$ approaches unity and $\sin(\beta - \alpha)$ goes to zero so that $e^+e^- \rightarrow Zh$ and $e^+e^- \rightarrow AH$ are the dominant production processes. Scenarios for the Higgs physics at a future $e^+e^-$ linear collider are different for two cases. In case (i) it is possible to discover all Higgs states with $\sqrt{s} = 500$ GeV, and the production cross-section of the lightest Higgs boson may be quite different from that of the SM so that it may be clear that the discovered Higgs is not the SM Higgs. On the other hand, in case (ii), only the lightest Higgs may be discovered at the earlier stage of the $e^+e^-$ experiment, and we have to go to a higher energy machine to find the heavier Higgs bosons. Also, since the properties of the lightest Higgs boson may be quite similar to those of the SM Higgs boson we need precision experiments on the production and decay of the particle in order to investigate possible deviations from the SM.

4 The Higgs-boson mass and production cross-section in extended versions of the SUSY SM

Although the MSSM is the most widely studied model, there are several extensions of the SUSY version of the SM. If we focus on the structure of the Higgs sector, the MSSM is special because the Higgs self-couplings at the tree level are completely determined by the gauge coupling constants. It is therefore important to know how the Higgs phenomenology is different for models other than the MSSM.

A model with a gauge-singlet Higgs boson is the simplest extension. This model does not destroy the unification of the three gauge coupling constants since the new light particles do not carry the SM quantum numbers. Moreover, we can include a term $W_\lambda = \lambda NH_1H_2$ in the superpotential where $N$ is a gauge singlet superfield. Since this term induces $\lambda^2|H_1H_2|^2$ in the Higgs potential, the tree-level Higgs-boson self-coupling depends on $\lambda$ as well as the gauge coupling constants. There is no definite upper-bound on the lightest CP-even Higgs-boson mass in this model unless a further assumption on the strength of the coupling $\lambda$ is made. If we require all dimensionless coupling constants to remain perturbative up to the GUT
scale we can calculate the upper-bound of the lightest CP-even Higgs-boson mass. In Figure 4, the upper bound of the Higgs-boson mass is shown as a function of the top-quark mass. In this figure we have taken the stop mass as 1 TeV and demanded that no dimensionless coupling constant may blow up below the GUT scale (∼10^{16} GeV). We can see that the upper bound is given by 130 ∼ 140 GeV for this choice of the stop mass. The top-quark-mass dependence is not significant compared to the MSSM case because the maximally allowed value of λ is larger (smaller) for a smaller (larger) top mass.

From this figure we can see that the lightest Higgs boson is at least kinematically accessible at an $e^+e^-$ linear collider with $\sqrt{s} \sim 300 – 500$ GeV. This does not, however, mean that the lightest Higgs boson is detectable. In this model the lightest Higgs boson is composed of one gauge singlet and two doublets, and if it is singlet-dominated its couplings to the gauge bosons are significantly reduced, hence its production cross-section is too small. In such a case the heavier neutral Higgs bosons may be detectable since these bosons have a large enough coupling to gauge bosons. In fact we can put an upper-bound on the mass of the heavier Higgs boson when the lightest one becomes singlet-dominated. By quantitative study of the masses and the production cross-section of the Higgs bosons in this model, we can show that at least one of the three CP-even Higgs bosons has a large enough production cross-section in the $e^+e^- \rightarrow Zh_i^0$ ($i = 1, 2, 3$) process to be detected at an $e^+e^-$ linear collider with $\sqrt{s} \sim 300 – 500$ GeV. For this purpose we define the minimal production cross-section, $\sigma_{\text{min}}$, as a function of $\sqrt{s}$ such that at least one
of these three $h_0^0$ has a larger production cross section than $\sigma_{\text{min}}$ irrespective of the parameters in the Higgs mass matrix. We can show that $\sigma_{\text{min}}$ is larger than 0.04 pb for $m_t = 120 - 180$ GeV and $m_{\text{stop}} = 1$ TeV at an $e^+e^-$ linear collider with $\sqrt{s} = 300$ GeV, and therefore the discovery of at least one neutral Higgs boson is guaranteed with an integrated luminosity of 10 fb$^{-1}$. More recently, a condition to give $\sigma_{\text{min}}$ is improved by a closer investigation, and $\sigma_{\text{min}}$ turns to be given by just one third of the SM production cross-section with the Higgs boson mass equal to the upper-bound value. In Figure 5 we show this $\sigma_{\text{min}}$ as a function of $\sqrt{s}$. If we include more singlets $\sigma_{\text{min}}$ is just given by $\frac{1}{n+2}$ times the SM production cross-section, where $n$ is the number of gauge-singlet Higgs bosons which mix with the doublets. Therefore, as long as the number of gauge-singlet Higgs bosons is not too large ($\lesssim 5$), it is possible to discover at least one neutral Higgs boson in the first stage of the $e^+e^-$ linear-collider experiment.

5 A ZZ$h$ anomalous coupling at a TeV linear collider

In order to establish the SM and to search for new physics beyond it, it is fundamentally important to investigate whether or not various couplings among gauge bosons and the Higgs boson are described by the SM. For example, the existence of the self-couplings of gauge bosons are an important feature of the non-Abelian nature of the gauge interaction.

If a Higgs boson is discovered at a relatively light mass scale ($\lesssim 200$ GeV), most probably the physics of electroweak symmetry breaking is described by an
interaction whose strength is not very much different from that of the electroweak interaction. In such a case the measurement of the couplings involving the light Higgs boson is interesting in order to look for effects from physics beyond the SM. Assuming that some new physics exists in the multi-TeV region, we can write down the general form of higher dimensional operators which are induced after integrating out the heavy fields.

\[ L = \sum_i \frac{f_i}{\Lambda^2} O_i + \ldots \]  

(4)

where the \( f_i \) are dimensionless couplings, and \( \Lambda \) is the new-physics scale. The \( O_i \) are gauge-invariant operators composed of gauge bosons and Higgs doublet fields as well as fermion fields.

In this workshop it was pointed out that the production cross-section in the Higgs-bremsstrahlung process, \( e^+e^- \rightarrow Z h \), is sensitive to one anomalous coupling of this type, \( i.e. \frac{1}{4} (D_{\mu} \Phi)^\dagger W^{\mu\nu} (D_{\nu} \Phi) \), for an \( e^+e^- \) linear collider with a center-of-mass energy over 1 TeV. In the SM in this energy region the dominant Higgs production process is WW fusion process rather than Higgs-bremsstrahlung. However, as long as the new-physics effect are concerned, the Higgs-bremsstrahlung process is more important. This is because the relevant energy scale of this process is \( \sqrt{s} \) and the anomalous coupling of \( ZZ h \) becomes large as the energy scale increases while the energy scale relevant to the fusion process is the Higgs mass scale, not \( \sqrt{s} \). Thus, in order to look for new-physics effects, the measurement of the production cross-section in \( e^+e^- \rightarrow Z h \) is more important than the process \( e^+e^- \rightarrow WW \bar{\nu}\nu \rightarrow h \bar{\nu}\nu \).

### 6 Determination of the heavy Higgs mass scale from branching measurements in the MSSM

It is generally accepted that the SM Higgs boson with a mass less than about 200 GeV will be discovered at the first stage of an \( e^+e^- \) linear collider experiment where the CM energy is \( \sim 300 – 500 \) GeV. This is sufficient to discover at least one CP-even Higgs boson of the MSSM. If a Higgs boson is discovered, we would like to determine whether or not this boson is the SM Higgs boson by studying its production and decays. It is therefore important to investigate to what extent the production cross-section and decay branching ratios can be determined and what the impact of these determinations will be on establishing the SM and searching for physics beyond the SM in the context of the MSSM, the question can be restated as whether the parameters in the Higgs sector are determined by various observable quantities related to the Higgs boson. Although it is possible to discover all five Higgs states at the first stage of the linear collider experiment, we may at first be able to find only one CP-even Higgs boson. In this situation it is important to determine in which mass region the other Higgs states exist so that these particles become targets of the second stage of the \( e^+e^- \) linear-collider experiments after the beam energy is increased.
This problem was addressed by Kamoshita’s talk in this workshop. The free parameters required to specify the Higgs sector in the MSSM can be taken to be the CP-odd Higgs-boson mass \(m_A\), the ratio of two vacuum expectation values \((\tan \beta)\) and masses of the top quark and the stop squark. The latter two parameters \((m_t, m_{stop})\) are necessary to evaluate the Higgs potential at the one-loop level. Suppose that the lightest CP-even Higgs boson is discovered such that its mass \((m_h)\) is precisely known. Then we can solve for one of the free parameters, for example, \(\tan \beta\), in terms of the other parameters. Assuming the top-quark mass is well determined by the time when the \(e^+e^-\) linear collider is under operation, the unknown parameters for the Higgs sector are then \(m_A\) and \(m_{stop}\). The question is, to what extent these parameters are constrained from observable quantities such as the production cross-section and the various branching ratios.

It has been pointed out that one particular ratio of two branching ratios,

\[
R_{br} = \frac{Br(h \to c\bar{c}) + Br(h \to gg)}{Br(h \to bb)} ,
\]

(5)

is especially useful to constrain the heavy Higgs mass scale. In the MSSM, each of the two Higgs doublets couples to either up-type or down-type quarks. Therefore, the ratio of the Higgs couplings to up-type quarks and to down-type quarks is sensitive to the parameters of the Higgs sector, i.e. the angles \(\alpha\) and \(\beta\) in Section 3. Since the gluonic width of the Higgs boson is generated by a one-loop diagram with an internal top-quark, the Higgs-gluon-gluon coupling is essentially proportional to the Higgs-top coupling. Then \(R_{br}\) is proportional to square of the ratio of the up-type and down-type Yukawa coupling constants. Since the up-type (down-type) Yukawa coupling constant contains a factor \(\frac{\cos \alpha}{\sin \beta}, \frac{-\sin \alpha}{\cos \beta}\) compared to the SM coupling constant, \(R_{br}\) is proportional to \((\tan \alpha \tan \beta)^{-2}\). In Figure 6 \(R_{br}\) is shown as a function of \(m_A\) for several choices of \(m_{susy} (\equiv m_{stop})\). From this figure we can see that \(R_{br}\) is almost independent of \(m_{stop}\). In fact, it can be shown that \(R_{br}\) in the MSSM, normalized by \(R_{br}\) in the SM, is approximately given by,

\[
\frac{R_{br}(MSSM)}{R_{br}(SM)} \approx \left(\frac{m_h^2 - m_A^2}{m_Z^2 + m_A^2}\right)^2
\]

(6)

for \(m_A \gg m_h \sim m_Z\). Measuring this quantity to a good accuracy is therefore important for constraining the scale of the heavy Higgs mass. Note that \(R_{br}\) approaches the SM value in the large \(m_A\) limit. We can see that \(R_{br}\) is reduced by 20% even for \(m_A = 400\) GeV.

In Nakamura’s talk the experimental determination of these branching ratios was discussed. Although it is very difficult to measure the charm and gluonic branching ratios separately with good accuracy, the sum of the two branching ratios can be determined reasonably well. The statistical error in the determination of \(R_{br}\) after two years at an \(e^+e^-\) linear collider with \(\sqrt{s} = 300\) GeV is 19%. We also need to know the theoretical ambiguity of the calculation of the branching ratios in
Figure 6: \( R_{br} = \frac{(Br(h \rightarrow c \overline{c}) + Br(h \rightarrow gg))}{Br(h \rightarrow bb)} \) as a function of \( m_A \) for several values of \( m_{susy} \) for the lightest CP-even Higgs mass \( m_h = 120 \text{ GeV} \). The following parameters are used for the calculation of the branching ratios: \( m_t = 170 \text{ GeV}, \ m_c(m_c) = 1.2 \text{ GeV}, \ m_b(m_b) = 4.2 \text{ GeV}, \ \alpha_s(m_Z) = 0.12. \)

\( h \rightarrow bb, c\bar{c}, gg \). Uncertainties in the charm-quark and bottom-quark masses as well as in the strong coupling constant are important. At the moment the theoretical error in the calculation of \( R_{br} \) is estimated to be larger than 20% and mainly comes from uncertainties in \( \alpha_s \) and \( m_c \). Both theoretical and experimental improvements are necessary to calculate the branching ratios more precisely.

7 Heavy Higgs decays to SUSY particles

The investigation of the properties of the heavy Higgs bosons (\( A, H, H^\pm \)) is one of main goals of a TeV linear collider. Since the MSSM is a special type of two-Higgs-doublet model, the discovery of these particles as predicted is strong evidence for the MSSM. Also, the determination of the parameters \( m_A \) and \( \tan \beta \) of the Higgs sector is important in exploring the whole structure of the SUSY model since these parameters are relevant to other sectors of the model in the context of the MSSM and/or supergravity models. If we assume that the mass of the heavy Higgs is larger than 200 \( \sim \) 300 GeV it is possible that some decay channels to SUSY particles are open. In Djouadi’s talk various decays of heavy Higgs bosons including SUSY modes are considered in the context of the SUSY GUT model with Yukawa unification.

Let us first summarize the dominant decay modes of heavy Higgs bosons if SUSY decay modes are not open. Since the coupling of the \( H \) and the \( A \) to down-type quarks is enhanced for large \( \tan \beta \), the \( H \) and \( A \) dominantly decay to \( bb \) or \( \tau^+\tau^- \) for \( \tan \beta > 10 \). The situation is different in the smaller \( \tan \beta \) region where, if the
The $t\bar{t}$ mode is open, this mode dominate over other modes. Below the $t\bar{t}$ threshold the $H \rightarrow hh, H \rightarrow WW$ and $A \rightarrow Zh$ modes can be dominant. For the charged Higgs boson, the $H^+ \rightarrow b\bar{t}$ mode is dominant if kinematically accessible, and otherwise $H^+ \rightarrow \tau^+\nu$ becomes the main decay mode.

If we allow SUSY decay modes, heavy Higgs bosons can decay to squark-pairs, slepton-pairs and charginos and neutralinos. Of course whether or not these decay modes are available depends on the mass spectrum of SUSY particles. Here a model based on minimal supergravity is considered with an assumption of Yukawa coupling unification. Requiring that the $m_b/m_\tau$ ratio is correctly reproduced from the SU(5) SUSY GUT assumption and that the electroweak symmetry breaking is induced from the renormalization effects on the Higgs mass term from the universal SUSY-breaking mass at the GUT scale (the radiative electroweak symmetry breaking scenario), we can reduce the number of free parameters of the model. There are two separate regions of $\tan\beta$ according to this scenario, but the so-called small $\tan\beta$ solution is interesting where $\tan\beta \simeq 1.75$. Essentially this model contains two free parameters for which we can take $m_A$ and $M_{1/2}$ (a gaugino mass parameter). Then, for a fixed heavy Higgs mass, all superparticle masses are determined as a function $M_{1/2}$ so that the decay widths including SUSY modes are calculable. The importance of the SUSY modes is quite different depending upon whether or not the $H(A) \rightarrow t\bar{t}$ mode is open. Below the top threshold the SUSY modes can dominate over the SM mode, and the total width can be enhanced by an order of magnitude. This is especially evident if the stop becomes light enough to be a dominant decay mode. On the other hand, above the top threshold the $t\bar{t}$ mode is almost always dominant, and the SUSY modes play minor roles. This is because the decay width to the $t\bar{t}$ pair is already large compared to other modes. Only when the stop is light enough can the SUSY modes give a sizable contribution to the total width.

8 Multi-Higgs production in the MSSM

There were two talks which covered multi-Higgs production in the MSSM; one deals with various double and triple Higgs production processes at a TeV collider, and the other focuses on the $e^+e^- \rightarrow Zhh$ process at an $e^+e^-$ linear collider with $\sqrt{s} \sim 300 - 500$ GeV.

In the SM the $e^+e^- \rightarrow Zhh$ process is especially important because this process depends on the triple Higgs coupling (Figure 7). Therefore, information on the Higgs potential is obtained from this process. The production cross-section is, however, not so large. For $m_h = 100$ GeV it is a few times $10^{-1}$ fb for $\sqrt{s} = 500 \sim 1$ TeV; therefore, we need more than a hundred fb$^{-1}$ to observe this process. In the MSSM case the situation changes in two ways. The $ZZh$ and the $h^3$ couplings are modified from the SM couplings, and additionally there are diagrams which contain heavy Higgs($H, A$) in an internal line as shown in Figure 8. In both talks it was noticed that the multi-Higgs production cross-section becomes large only when both
Figure 7: The relevant Feynman diagrams for $e^+e^- \rightarrow Zhh$ in the SM.

Figure 8: The additional Feynman diagrams for $e^+e^- \rightarrow Zhh$ in the MSSM.
The $m_A$ dependence of the $e^+e^- \rightarrow Zhh$ cross-section in the MSSM (solid) for $\tan \beta = 2$ and 10 at $\sqrt{s} = 500$ GeV. The top and stop masses are taken as $m_t = 170$ GeV and $m_{\text{stop}} = 1$ TeV. In this figure the Higgs mass is given as a function of $m_A$. For comparison, the SM (dashed) and the NT (dotted) cross-sections with the same Higgs mass are also shown.

Heavy Higgs ($H, A$) production and the subsequent decay through $H \rightarrow hh$ or $A \rightarrow hZ$ are kinematically allowed. In Figure 9, the production cross-section $\sigma(e^+e^- \rightarrow Zhh)$ is given as a function of $m_A$ for $\tan \beta = 2$ and 10 for the MSSM. Here the top-quark and stop-squark mass are taken as $m_t = 170$ GeV and 1 TeV respectively. In this figure the lightest Higgs mass varies as a function of $m_A$ (see Figure 3). For comparison the production cross-sections for the SM and for a model without a triple Higgs coupling (NT) calculated with the same Higgs mass are shown. We can see that, in the region of small $m_A$, the cross-section becomes very large compared to the SM value. This corresponds to a region of parameter space where $e^+e^- \rightarrow HZ$ and $H \rightarrow hh$ are possible. On the other hand, there are some regions of parameter space where the production cross-section is much reduced compared to the SM.

When the heavy Higgs boson is directly produced and multi-Higgs production becomes large, an interesting possibility arises for measuring the $H-h-h$ coupling constant. If $2m_h < m_H < 2m_t$, the heavy Higgs boson $H$ can have a sizable decay branching-ratio both in the $H \rightarrow hh$ and $H \rightarrow WW$ modes. In such a case the $H-h-h$ coupling constant can be extracted from the ratio of two branching ratios since the coupling of the $H$ to two gauge bosons is determined from the Higgs production.
cross section of the fusion process as well as the Higgs-bremsstrahlung process.

9 Conclusions

I have reviewed some aspects of the Higgs physics at future $e^+e^-$ linear colliders whose CM energy is ranging from 300 GeV to 1.5 TeV. At earlier stage of the experiment with $\sqrt{s} \sim$ 300 - 500 GeV, it is easy to find a light Higgs boson predicted in SUSY standard models or GUT. In particular, both in the MSSM and the SUSY SM with a gauge singlet Higgs, at least one of neutral Higgs bosons is detectable. This is important because it is known that there is a parameter space in the MSSM where no signal of Higgs bosons is obtained in the LHC experiment.

Advantage of linear collider experiments is, however, not only the discovery potential of the Higgs particle. More importantly, detailed study on properties of the Higgs boson is possible through measurements of various production cross-sections and branching ratios. Here several examples are discussed: anomalous coupling of $Z-Z-h$, Higgs couplings to $c\bar{c}/g/\bar{b}b$ in the MSSM, Higgs decays to superparticles and the measurement of triple-Higgs-couplings through the $e^+e^- \rightarrow Zhh$ process. Combining information obtained from the LHC experiment, we will be able to clarify the Higgs sector of the SM and explore physics beyond the SM.

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