Theoretical Models of Dark Energy

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Motivation

- The composition of the universe
  - Dark Energy: ~73%
  - Dark Matter: ~23%
  - Other nonluminous components:
    - Intergalactic gas: 3.6%
    - Neutrinos: 0.1%
    - Supermassive BHs: 0.04%
  - Luminous matter:
    - Stars and luminous gas: 0.4%
    - Radiation: 0.005%

- Supernovae as a standard candle

Let’s see the sky!
Motivation

- Observational evidences on Dark Energy

Figure 1: There is strong evidence for the existence of dark energy. Plotted are $\Omega_m - \Omega_\Lambda$ (left panel) and $\Omega_m - \omega$ (right panel) confidence regions constrained from the observations of SN Ia, CMB and BAO. From Ref. Astrophys.J. 686 (2008) 749(778), [arXiv:0804.4142]

- accelerated expansion of the Universe, when $-1 \leq w < -\frac{1}{3}$
Motivation

- Cosmological constant

\[ \Lambda = \text{Einstein’s biggest blunder?!} \]
\[ = \text{vacuum energy?} \]

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \]

\[ S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + 2\Lambda) + S_M. \]
Motivation

- Cosmological constant problem

### Fine-tuning problem

**Theoretical expectation:**

\[ \rho_\Lambda \sim (10^{18} \text{GeV})^4 \sim 2 \times 10^{110} \text{erg/cm}^3 \]

**Observation:**

\[ \rho_\Lambda^{\text{obs}} \leq (10^{-12} \text{GeV})^4 \sim 2 \times 10^{-10} \text{erg/cm}^3 \]

"The worst theoretical prediction in the history of physics!"

Why small?

### Coincidence problem

Why now?

Discrepancy: \(10^{120}\)
Modify Matter vs. Modify Gravity

\[ G_{\mu\nu} = 8\pi G \ T_{\mu\nu} \]

\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \] describes geometry of spacetime
Modified Matter model

- Quintessence

$G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Scalar field ($\Phi$)?

$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_M.$

$w_Q = \frac{p_Q}{\rho_Q} = \frac{1}{2} \frac{\dot{\phi}^2}{\phi^2} - V(\phi)$

$\frac{1}{2} \frac{\dot{\phi}^2}{\phi^2} + V(\phi)$
Modified Matter model

- K-essence

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

The full action including a k-essence term is given by

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R + p(\phi, X) \right] + S_M \]
Modified Matter model

- Coupled dark energy and matter

Modified energy conservation equations

\[ \dot{\rho}_m + 3H(\rho_m) = \delta, \]
\[ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\delta. \]

Finding an appropriate form of the coupling \( \delta \)

\[ \delta = \kappa Q \rho_m \dot{\phi}, \]
\[ \delta = \alpha H(\rho_m + \rho_\phi), \]

Chameleon mechanism

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + \int d^4x \mathcal{L}_m(\psi_m^{(f)} g^{(f)}_{\mu\nu}) \]

\[ g^{(f)}_{\mu\nu} = e^{2\kappa \beta \phi} g_{\mu\nu}. \]

\[ |F| = \frac{GM_1 M_2}{r^2 (1 + 2\beta_1 \beta_2)}, \]

\[ G_{eff} = G(1 + 2\beta_1 \beta_2). \]
Modified Matter model

- Unified dark energy and matter

**Chaplygin gas**

\[ p = -\frac{A}{\rho^{\alpha}} \]

\[ \rho(t) = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \]

In the early epoch, \( a \ll 1 \), the Chaplygin gas energy density behaves as \( \rho \propto a^{-3} \) which corresponds to the matter dominated Universe. In the late epoch, \( a \gg 1 \), the energy density behaves as \( \rho \approx A^{1/(a+\alpha)} = \text{const.} \) which corresponds to the de Sitter Universe.

Thus the Chaplygin gas behaves as dark matter in the early epoch and dark energy in the later epoch. It explains **both dark sectors in terms of a single component**.
Modified Gravity model

- f(R) gravity

\[ G_{\mu\nu} = 8\pi G \, T_{\mu\nu} \]

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + \int d^4x \sqrt{-g} \mathcal{L}_M \]

\[ w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = \frac{2A\dot{H} + 3AH^2}{3AH^2 - 8\pi G \rho_m}. \]

\[ G_{eff} \equiv \frac{G}{f_R} \frac{1 - 4mk^2/(a^2 R)}{1 - 3mk^2/(a^2 R)} \cdot m \equiv \frac{Rf_{,RR}}{f_R}. \]
Modified Gravity model

- DGP model

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

5th Dimension!

\[ S = -\frac{M_5^3}{2} \int d^5x \sqrt{-g} \bar{R} - \frac{M^2_{pl}}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_m \]

\[ \tilde{G}_{AB} \equiv \tilde{R}_{AB} - \frac{1}{2} \tilde{g}_{AB} = 0 \]

\[ G_{\mu\nu} - \frac{1}{r_c} (K_{\mu\nu} - K g_{\mu\nu}) = 8\pi G T_{\mu\nu} \]

\[ H^2 + \frac{k}{a^2} = \left( \sqrt{\frac{\rho}{3M^2_{pl}} + \frac{1}{4r_c^2} + \frac{\epsilon}{2r_c}} \right)^2 \]
Non Dark Energy model

- Inhomogeneous LTB model

We are living in large underdense void!
Throw away the Friedmann-Lemaître-Robertson-Walker (FLRW) metric!!
(homogeneous and isotropic universe)

\[
\text{Metric for a spherically symmetric inhomogeneous Universe}
\]

\[
\begin{align*}
\text{Einstein equations for the dust dominated LTB Universe} & \\
H_+^2 + 2H_+H_\perp - \frac{\beta}{R^2} - \frac{\beta'}{RR'} &= 8\pi G\rho, \\
6\frac{\ddot{R}}{R} + 2H_\perp^2 - 2\frac{\beta}{R^2} - 2H_\perp H_\perp + \frac{\beta'}{RR'} &= -8\pi G\rho,
\end{align*}
\]
What to do?

- Dark Energy projects

| Stage I | Probes | SN Ia |
|---------|--------|-------|
| Higher-Z Team | 10, 33 | |
| SNLS | 34, 35, 39 | |
| ESSENCE | 37, 38 | |
| NSF | 39, CSP | 10, 41 |
| LOSS | 42, 43, 44 | |
| SDSS | 45, 46, SCP | 21, 47, 48, CfA |
| QUEST Survey | Palomar | |

| Stage I | CMB |
|---------|-----|
| COBE | 52 |
| TOCO | 53 |
| BOOMERanG | 54, Maxima |
| WMAP | 55 | |

| Stage I | BAO |
|---------|-----|
| 2dFGRS | 56 |
| SDSS | 57 |
| 6dFGRS | 58 |
| WiggleZ | 59 |

| Stage I | WL |
|---------|-----|
| CFHTLS | 60 |
| 61 | |

| Stage II | Probes |
|-----------|--------|
| Pan-STARRS1 | 62 |
| HST | 63 |
| KAIT | 64 |

| Stage II | Planck |
|-----------|--------|
| 65, 66 | |
| 67, SPT | 68 |
| ACT | 70 |

| Stage II | SDSS II |
|-----------|---------|
| 71 | |

| Stage II | SDSS III |
|-----------|---------|
| 72, BOSS | 73 |
| LAMOST | 75, WEAVE |
| 76 |

| Stage III | DES, HETDEX |
|------------|-------------|
| 81, BigBOSS | 85 |
| 86, ALPACA, SuMIRe | |

| Stage III | Pan-STARRS4, ALPACA, ODI |
|------------|--------------------------|

| Stage IV | LSST, WFIRST |
|-----------|--------------|
| EPIC | 89, 91, LiteBIRD, 92, B-Pol |
| LSST, SKA, WFIRST, Euclid | 95 |

Table 1: Dark energy projects. Classification is taken from ref. Report of the Dark Energy Task Force, astro-ph /0609591.

Stage I: completed projects that already released data
Stage II: on-going projects
Stage III: intermediate-scale, near-future projects
Stage IV: large-scale, longer-term future projects
What to do?

- Effective Newton's constant $G_{eff}$
- Eötvös parameter $\eta$

From Einstein's biggest blunder to the Nobel Prize in Physics 2011

- Now we are ready to meet the most mysterious cosmological discoveries!
THANK YOU!

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