Effective Field Theory for Neutron Stars with Genuine Many-body Forces

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The aim of our contribution is to shed some light on open questions facing the high density nuclear many-body problem. We focus our attention on the conceptual issue of naturalness and its role for the baryon-meson coupling for nuclear matter at high densities. As a guideline for the strengths of the various couplings the concept of naturalness has been adopted. In order to encourage possible new directions of research, we discuss relevant aspects of a relativistic effective theory for nuclear matter with natural parametric couplings and genuine many-body forces. Among other topics, we discuss in this work the connection of this theory with other known effective Quantum Hadrodynamics (QHD) models found in literature and how we can potentially use our approach to describe new physics for neutron stars. We also show some preliminary results for the equation of state, population profiles and mass-radius relation for neutron stars assuming local charge neutrality and beta equilibrium.

1 Introduction

Quantum Chromodynamics (QCD) with quark and gluon degrees of freedom is the underlying theory of the strong interaction physics. However, at energies relevant for most nuclear phenomena, hadron and meson fields provide a more appropriate and convenient description. In this domain, effective quantum field theory (EFT) and, more precisely, Quantum Hadrodynamics (QHD) represent efficient frameworks for describing the nuclear many-body problem as a relativistic system of baryons and mesons.

The motivation for using QHD in this domain can be synthesized in a few words: simplicity, efficiency, consistency. There are many arguments that vindicate this assertion: it is the simplest approach for describing nuclear matter and nuclear systems based on a local Lorentz-invariant lagrangian density; it represents an efficient way to parametrize an S matrix or other observables in nuclear matter; this approach is consistent with analyticity, unitarity, causality, cluster decomposition, parity conservation, spin and isospin symmetries, and spontaneously breaking of chiral symmetry for systems undergoing strong interaction.

Moreover, since the most relevant global phenomena in nuclear physics are confined to longer length scales, ie, in the interaction sector where less massive meson fields dominate, it is not necessary to explicitly include dynamics at significantly shorter length scales, where the presence of the most massive meson fields becomes crucial. Thus, in describing global properties of nuclear systems through effective field theory, one can ignore the latter by integrating out heavier degrees of freedom corresponding to shorter length scales. It is important to notice, however, the effects of these heavier degrees of freedom are still implicitly contained in the coupling parameters of the effective field theory (Serot & Walecka 1997). EFT and QHD contain on the other hand an infinite number of interaction terms and degrees of freedom. Thus, one needs an organizing principle to make sensible calculations.

As a conventional way of classifying and organizing the interaction terms in our EFT approach as well as a guideline for the strengths of the various couplings the concept of naturalness has been adopted.

1.1 Naturalness

We take an interaction Lagrangian, which involves the \( \sigma \) isoscalar-scalar meson and the \( \omega \) isoscalar-vector meson fields coupled to the nucleon field, defined in Serot &
Walecka (1997) as

\[
\mathcal{L} = \sum_{i,k} \frac{G_V}{f_{\pi}^2} \left( \frac{\sigma}{M} \right)^i \frac{\omega}{f_{\pi}^2}^k \left( \frac{\bar{\psi} \Gamma \psi}{f_{\pi}^2} \right)^k \frac{f_{\pi}^2 \Lambda^2}{M^2},
\]

with unknown expansion coefficients \( c_{i,k} \) (overall coupling constants). In the expression above, \( \psi \) represents a baryon field, \( \Gamma \) is a Dirac matrix, and derivatives are denoted by \( \partial \). The resulting expressions of the Lagrangian density also contain factorial counting factors, needed to accomplish normalization. A direct generalization of this expression may involve additional fields such as the \( \pi, g, \delta \) meson fields as well as the photon.

There is evidence from studies on properties of ordinary nuclear matter, that the expansion in the nonlinear meson coupling sector quickly converges. In fact, keeping only the cubic and quartic couplings of the \( \sigma \) meson one obtains, at saturation and intermediate densities, a semi-quantitative fit of nuclear matter properties (Boguta & Bodmer 1997; Serot & Walecka 1986). Nevertheless, a controlled expansion of the Lagrangian density to significantly higher densities \( (\rho \geq 5 \rho_0) \), as those found in the interior of neutron stars, requires some assumptions on a natural ordering of the expansion coefficients. To accomplish that goal, we may express equation (1) in the form

\[
\mathcal{L} = \left( \frac{\partial m_\pi}{M} \right) f_{\pi}^2 \Lambda^2 \sum_{i,k} \frac{c_{i,k}}{i! k!} \left( \frac{f_{\pi}^2}{M^2} \right)^i \left( \frac{f_{\pi}^2}{M^2} \right)^k \left( \frac{\bar{\psi} \Gamma \psi}{f_{\pi}^2} \right)^k ,
\]

where, in this expression, the Goldberger-Treiman relation \( (g_{\pi} f_{\pi} \sim M) \) was used to eliminate the \( f_{\pi} \) factor in the sum. The assumption of naturalness in the strong interaction physics means that, unless a more detailed explanation exists, all conceivable action terms that preserve the required fundamental symmetries should appear in the effective action of a theory with natural coupling coefficients (Georgi 1993; Manohar 1996). Thus, the naturalness condition, when applied to an effective theory of the strong interaction, establishes that once the appropriate dimensional scales have been extracted using the naive dimensional analysis proposed by Georgi (1993) and Manohar (1996), the remaining dimensionless coefficients appearing in the effective action should all remain of order unity.

If the naturalness assumption is valid, then the effective strong interaction Lagrangian density can be truncated, with an acceptable confidence, within the phenomenological physical domain of the theory. The naive dimensional analysis when applied in the formulation of an effective Lagrangian density involving nucleons and strongly interacting meson fields may be synthesized as follows: the amplitude of each strongly interacting field in the lagrangian, i.e. the meson fields, becomes dimensionless when divided by the pion decay weak constant; furthermore, to obtain the correct dimension ((energy)\(^3\)) for the Lagrangian density, an overall normalization scale \( f_{\pi}^2 \Lambda^2 \sim f_{\pi}^2 M^2 \), with \( M \) denoting the nucleon mass, has to be included; finally, for identical meson fields, self-interacting terms of power \( n \) and a symmetrization factor \( n! \) (for proper counting) should be included in the formalism. The overall dimensionless coefficients, after the dimensional factors and appropriate counting factors are extracted, are of order \( O(1) \) if naturalness holds. Of course, there is no general proof of the naturalness property, since no one knows how to derive the effective strong interaction Lagrangian density from Quantum Chromodynamics (QCD). Nevertheless, the validity of naturalness and naive power counting rules is supported by phenomenological studies.

At least two schemes allow a compact summation of the Lagrangian density \( \mathcal{L} \). For \( c_{i,k} = 1 \), we have

\[
\mathcal{L} = \left( \frac{\partial m_\pi}{M} \right) f_{\pi}^2 \Lambda^2 \exp \left( \frac{\sigma}{M} + \frac{\omega}{M} \right) \left( \bar{\psi} \Gamma \psi \right)^k \frac{f_{\pi}^2 \Lambda^2}{M^2} ;
\]

and for \( c_{i,k} = i! k! \) we obtain

\[
\mathcal{L} = \left( \frac{\partial m_\pi}{M} \right) f_{\pi}^2 \Lambda^2 \exp \left( \frac{1}{1 + \frac{\sigma}{M}} \right) \left( \frac{1}{1 + \frac{\omega}{M}} \right) \left( \bar{\psi} \Gamma \psi \right)^k \frac{f_{\pi}^2 \Lambda^2}{M^2} .
\]

### 1.2 Lagrangian Density

Our strategy in this work is to consider a phenomenological and more flexible parametrization of the QHD Lagrangian density which combines the two previous limits and an extension of the interaction phase space of baryon and meson fields. The complete expression of our QHD interaction Lagrangian exhausts the whole fundamental baryon octet \( (\eta, n, \Sigma^-, \Sigma^0, \Sigma^+, \Lambda, \Xi^-, \Xi^0) \) and includes many-body forces simulated by nonlinear self-couplings and meson-meson interaction terms involving scalar-isoscalar \( (\sigma, \sigma^+, \tau, \omega, \delta) \), vector-isoscalar \( (\omega, \phi) \), vector-isovector \( (g) \) and scalar-isovector \( (\delta) \).

The interaction term of our Lagrangian density, \( \mathcal{L}_{\text{int}} \), is defined as

\[
\sum_B \bar{\psi}_B \left[ i \gamma_\mu \partial^\mu \gamma_1 - \bar{\psi}_B \Sigma_{\mu \xi}^\mu (\sigma, \sigma^+, \delta, \omega) - \gamma_\mu \tau \Sigma_{\mu \delta} (\sigma, \sigma^+, \delta, \epsilon) - \gamma_\mu \Sigma_{\mu \epsilon} (\sigma, \sigma^+, \epsilon, \delta) \right] \ \bar{\psi}_B
\]

where one can identify the following self-energy insertions:

\[
\Sigma_{\mu \xi}^\mu (\sigma, \sigma^+, \delta, \omega) = g_{\omega B \epsilon} \epsilon^\mu ; \Sigma_{\mu B} (\sigma, \sigma^+, \delta, \epsilon) = \frac{1}{2} g_{\omega B \epsilon} \epsilon^\mu ; \Sigma_{B \epsilon} (\sigma, \sigma^+, \delta, \epsilon) = M \Sigma_{B \epsilon} .
\]

In these expressions, \( g^\mu \) with \( \Phi = \sigma, \sigma^+, \omega, \phi, \delta, \epsilon \), represents the effective parametrized baryon-meson coupling constants, defined as \( g_{\omega B} = g_{\omega B m_{B}} \), \( g_{\phi B} = g_{\phi B m_{B}} \), \( g_{\epsilon B} = g_{\epsilon B m_{B}} \), and \( \Sigma_{B \epsilon} = m_{B \epsilon} \), with

\[
m_{B \epsilon} \equiv \left( 1 + \frac{g_{\omega B}^2 + g_{\phi B}^2 \sigma^+ + \frac{1}{2} g_{\epsilon B}^2 \tau \cdot \delta}{\alpha M_B} \right)^{-\alpha} ;
\]

\( (\alpha = \zeta, \kappa, \eta, \varsigma) \) (Taurines et al. 2001; Vasconcellos et al. 2012). Properties of the fields considered in our formulation are presented in table (1)).
In what follows, we use the abbreviations: ISS: isoscalar-scalar; IVS: isovector-scalar; ISV: isoscalar-vector; IVV: isovector-vector.

Table 1 Properties of the fields considered in the formulation (9). In what follows, we use the abbreviations: ISS: isoscalar-scalar; IVS: isovector-scalar; ISV: isoscalar-vector; IVV: isovector-vector.

| Fields  | Classification | Particles | Coupling Constants | Mass (MeV) |
|---------|----------------|-----------|--------------------|------------|
| $\psi_B$ | Baryons | N, Λ, Σ, Ξ | N/A | 939, 1116, 1193, 1318 |
| $\psi_I$ | Leptons | $e^-, \mu^-$ | N/A | 0.5, 106 |
| $\sigma$ | ISS-meson | $g_{\sigma_{B}}$ | 550 |
| $\delta$ | IVS-meson | $g_{\delta_{B}}$ | 980 |
| $\omega_{\mu}$ | ISV-meson | $g_{\omega_{B}}$ | 782 |
| $\eta_{\mu}$ | IVV-meson | $g_{\eta_{B}}$ | 770 |
| $\phi_{\mu}$ | ISS-meson | $g_{\phi_{B}}$ | 975 |
| $\phi_{\mu}$ | ISV-meson | $g_{\phi_{B}}$ | 1020 |

The complete expression for the Lagrangian density in the mean field approximation, $\mathcal{L}_{\xi\eta\gamma}$, becomes

$$\mathcal{L}_{\xi\eta\gamma} = \frac{1}{2} m_{\sigma_{B}}^2 \sigma_{0}^2 + \frac{1}{2} m_{\sigma_{B}}^2 \sigma_{0}^2 + \frac{1}{2} m_{\omega_{B}}^2 \omega_{0}^2 + \frac{1}{2} m_{\phi_{B}}^2 \phi_{0}^2 + \sum_{B} \left( \overline{\psi}_{B} (i \gamma_{\mu} \partial_{\mu} - m_{B}) \psi_{B} \right)$$

$$- \sum_{B} \overline{\psi}_{B} \left( \frac{1}{2} g_{\sigma_{B}} m_{\sigma_{B}} \gamma_{0} \gamma_{0} + g_{\omega_{B}} m_{\omega_{B}} \gamma_{0} \gamma_{0} \right) \psi_{B}$$

with the effective baryon mass defined as $M_{B_{c}} = M_{B} m_{B_{c}}$.

The resulting expression for the Lagrangian density is known as the parameterized coupling model. Among the numerous possibilities of choice of parameters for the model we may focus our attention in variations of the $\xi$ parameter keeping $\eta = \kappa = \gamma = 0$ (scalar or S-model). This parametrization reduces to the Walecka’s QHD-I model (Serot & Walecka 1997) if $\xi = 0$ and $g_{\omega_{B}} = g_{\omega_{B}} = 0$. In case only $g_{\omega_{B}} \neq 0$, this parametrization reduces to the Walecka’s QHD-II model (Serot & Walecka 1997). Assuming $\xi = 1$, performing a binomial expansion of $m_{B_{c}}$, truncating the perturbative series to cubic and quartic self-interactions terms involving the scalar-isoscalar $\sigma$ meson (making $g_{\sigma_{B}} = g_{\omega_{B}} = 0$), this parametrization reduces to the model of Boguta & Bodmer (1997). The second choice contemplates variations of the parameters $\xi$ and $\kappa$ while keeping $\eta = \gamma = 0$ (scalar-isoscalar-vector or SISV-model). In the third choice we may consider variations of the coupling constants $\xi$, $\eta$, and $\kappa$, and fixing $\gamma = 0$ (scalar-isoscalar-vector-isovector or SISV-I-model), as already discussed in (Dexheimer, Vanconcellos & Bodmann 2008). Finally, in the fourth choice we may consider variations of the four parameters of the theory, $\xi$, $\eta$, $\kappa$, and $\xi$ (scalar-isoscalar-vector-isovector or SISV-VII-model). These examples of parameterizations of our approach are shown in Table 2.

We consider, as an example, the expansion of the term $g_{\omega_{B}} m_{\omega_{B}} \gamma_{0} \gamma_{0}$ in expression (9), in the particular case $\xi = 1$.

$$g_{\omega_{B}} m_{\omega_{B}} \gamma_{0} \gamma_{0} \sim g_{\omega_{B}} \left( \frac{g_{\sigma_{B}} \sigma_{0} + g_{\omega_{B}} \omega_{0} + g_{\phi_{B}} \phi_{0}}{M_{B}} \right)^{2} \gamma_{0} \gamma_{0}$$

$$+ g_{\omega_{B}} \left( \frac{g_{\sigma_{B}} \sigma_{0} + g_{\omega_{B}} \omega_{0} + g_{\phi_{B}} \phi_{0}}{M_{B}} \right)^{2} \gamma_{0} \gamma_{0}$$

$$- g_{\omega_{B}} \left( \frac{g_{\sigma_{B}} \sigma_{0} + g_{\omega_{B}} \omega_{0} + g_{\phi_{B}} \phi_{0}}{M_{B}} \right)^{2} \gamma_{0} \gamma_{0}$$

From this expression we can identify many-body attractive and/or repulsive coupling terms like for instance $\sigma \omega_{B}$, $\sigma \omega_{B}$, $\delta \omega_{B}$, $\delta \omega_{B}$, $\delta \omega_{B}$, $\delta \omega_{B}$, $\sigma \delta \omega_{B}$, $\sigma \delta \omega_{B}$, and many others. Similarly, the remaining terms of the Lagrangian density also exhibit additional many-body coupling terms. Again, the final selection of the contributions to be considered requires an analysis of the formal coherence of the theory. In this sense, it is important to remember that the theory must embody fundamental symmetries, conservation laws as well as physical properties which are relevant for strong interacting relativistic nuclear many-body systems, such as Lorentz covariance, microscopic causality, naturalness, analyticity, uniqueness, among others.

From expression (11), an extension of the model of Serot & Walecka (1997), which exhausts the whole fundamental baryon octet ($n$, $\rho$, $\sigma$, $\xi$, $\omega$, $\Omega$, $\Lambda$, $\Xi$, $\Xi$) and includes baryon-meson interaction terms involving scalar-isoscalar ($\sigma$, $\sigma$), vector-isoscalar ($\omega$, $\phi$), vector-isovector ($\delta$) and scalar-isovector ($\delta$) may be synthesized as

$$\mathcal{L} = \sum_{B} \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \frac{1}{M_{B}} \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \frac{d^{2}}{d x_{i}^{2}} \Lambda^{2}$$

$$\times \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \frac{d^{2}}{d x_{i}^{2}} \Lambda^{2}$$

with $\sigma_{B} = i, j, m, n, q$. The natural limit of this expression, namely, by assuming $c(\phi) \rightarrow 1$, is

$$\mathcal{L} = \sum_{B} \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \frac{1}{M_{B}} \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \left( \frac{\partial \sigma_{B} \sigma_{B}}{\partial x_{i}} \right) \exp \sum_{B} \frac{g_{B} \Phi_{B}}{M_{B}}$$

1 It is well known that in the realm of the mean field approximation, expectation values of meson fields may be treated as classical numbers in space-time despite the density dependence of these fields in Fermi space.
properties of neutron stars, as for instance the equation of
clear force on the structure of the star. We seek in this way to
model described above. In this version, we analyze the ef-

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density regime, the HM coupling constants have been fitted
to hypernuclear properties (Schaffner et al. 1994), with the
long-range attractive components contribute to stellar con-
short-range components of the strong interaction play a fundamental role in the stability of the star. In
a later contribution, we will specifically examine the correspond-
ing the short-range components of the strong interaction, which also play a fundamental role in the stability of the star. From the global point of view, the long-range attractive components contribute to stellar con-
short-range components of the strong interaction act in the opposite direction.

We consider a mean field scalar version of our model
with the explicit inclusion of an Yukawa attractive term in
the Lagrangian density:

\[ \mathcal{L}_\lambda = \frac{1}{2} m_\lambda^2 \phi^2 + \frac{1}{2} m_\omega^2 \phi^2 + \frac{1}{2} m_\rho^2 \phi^2 
+ \sum_B \bar{\psi}_B(i\gamma_\mu \partial^\mu - m_B) \psi_B 
+ \sum_B \bar{\psi}_B(i\gamma_\mu \partial^\mu - g_\omega \phi \gamma^0 \partial^0 - M^2_{B\lambda}) \psi_B 
- \sum_B \bar{\psi}_B \left( \frac{1}{2} g_\omega \phi \gamma^0 \partial^0 \right) \psi_B . \]

where the subscripts \( B \) and \( l \) label, respectively, the baryon octet (\( n, p, \Lambda^0, \Sigma^+, \Sigma^- \)) and lepton (\( e^-, \mu^- \)) species. For more details see (Vasconcellos et al. 2012).

Evidently, our model must be in agreement with the experimental data related to the saturation properties of
nuclear matter. We adopt for the saturation density of nuclear matter \( \rho_0 = 0.17 \text{ fm}^{-3} \) and for the binding energy \( \epsilon_B = -16.0 \text{ MeV} \). The \( \lambda \) parameter is constrained to describe the nucleon effective mass at saturation between
0.70 – 0.78 MeV. The isovector coupling constant \( g_\omega \), on the other hand, is chosen to describe the symmetry energy
coefficient \( a_{sym} = 32.5 \text{ MeV} \); for more details see for instance Haensel, Potekhin & Yakovlev (2007). Furthermore, we use the equilibrium properties of nuclear matter to introduce a coherent set of NM coupling constants in the saturation density range, \( g_{N}, g_{\Lambda N}, g_{\Sigma N}, g_{\rho N} \) (see table 3). In the high density regime, the HM coupling constants have been fitted to hypernuclear properties (Schaffner et al. 1994), with the scalar coupling constants fixed to the potential depth of the corresponding hyperon species: \( U_{\Lambda} = U_{\Sigma} = -30 \text{ MeV} \) and \( U_{\Xi} = 30 \text{ MeV} \).

The effective parameterized nucleon mass \( M^2_{B\lambda} \), for

\[ M^2_{B\lambda} \simeq M_B \left( 1 - \frac{g_{\omega B} \sigma_{0}}{M_B} \right) + \left( \frac{\lambda}{2} \right) \left( \frac{g_{\omega B} \sigma_{0}}{\lambda M_B} \right)^2 + \mathcal{O}(3), \]

with \( \left( \frac{\lambda}{2} \right) \) representing the generalized binomial coefficients of the expansion and where we emphasize the direct dependence of the effective nucleon mass on the \( \lambda \) parameter of the model. The dependence of the effective nucleon mass on the \( \lambda \) parameter at nuclear saturation is shown in Fig. 1.

![Fig. 1](image)

Dependence of the effective nucleon mass at nuclear saturation on the \( \lambda \) parameter.

The density dependence of the nuclear matter symmetry energy \( (E_{sym}) \) can be characterized in terms of a few bulk parameters by expanding it in a Taylor series around the nuclear saturation density \( \rho_0 \)

\[ E_{sym}(\rho) = E_{sym}(\rho_0) + L \left( \frac{\rho - \rho_0}{3 \rho_0} \right)^2 + \frac{K_{sym}}{2} \left( \frac{\rho - \rho_0}{3 \rho_0} \right)^4 + \mathcal{O}(3) \] + ...

where \( E_{sym}(\rho_0) = a_{sym} \) is the value of the symmetry nuclear energy at saturation. In this expression, \( L \) is the slope parameter and the curvature parameter \( K_{sym} \) is the isovector correction to the compression modulus, defined respectively as

\[ L = 3 \rho_0 \left( \frac{\partial a_{sym}}{\partial \rho} \right)_{\rho = \rho_0} ; K_{sym} = 9 \rho_0^2 \left( \frac{\partial^2 a_{sym}}{\partial \rho^2} \right)_{\rho = \rho_0} . \]

Fig. 2 shows the dependence of \( L \) on \( \lambda \) and on the symmetry coefficient \( a_{sym} \). The slope parameter \( L \) is related to \( P_0 \), the pressure from the symmetry energy for pure neutron matter at saturation density. The symmetry pressure \( P_0 \) provides the dominant baryon contribution to the pressure in neutron stars at saturation density.

3 Results and Conclusions
In the following, we determine the EoS and population pro-
profiles for neutron stars assuming local charge neutrality and
maximum mass of a neutron star is the one with the lower particle population, central density, and maximum mass. The dependence of the slope parameter on \( \lambda \) for different values of the \( \lambda \) parameter are illustrated in Figs. 3, 4, 5, 6, and 7. In the figures, each value of \( \lambda \) generates a sequence of neutrons stars, where each star will have a different EoS, particle population, central density, and maximum mass. The parametrization that generates the higher pressure and maximum mass of a neutron star is the one with the lower possible value of the \( \lambda \), i.e., \( \lambda = 0.06 \), which provides a mass of 1.97 \( M_\odot \), in very good agreement with recent observations (Demorest et al. 2010). This result corresponds to the parametrization which provides a smaller effective baryon mass and, hence, a more bound matter. Although smaller lambda values originate higher values for the maximum mass of neutron stars, this parameter should attain a minimum value that allows the model to reproduce nuclear saturation properties, like for example a compressibility modulus smaller than 300 MeV.

Fig. 2 Dependence of the slope parameter \( L \) on \( \lambda \) for different values of the symmetry energy \( \epsilon_{\text{sym}} \).

beta equilibrium for the scalar version of our approach. By solving the Tolman-Oppenheimer-Volkoff (TOV) equations (Tolman 1939; Oppenheimer & Volkoff 1939) we obtain the mass-radius relation for families of neutron stars with hyperon content. Our results for different values of the \( \lambda \) parameter are listed in Table 3. In this table, each value of \( \lambda \) generates a sequence of

| \( \lambda \) | \( M_\odot \) | \( K_0 \) (MeV) | \( \left( g_{B^+ M} \right)^2 \) | \( \left( g_{B^0 M} \right)^2 \) | \( \left( g_{B^- M} \right)^2 \) |
|---|---|---|---|---|---|
| 0.06 | 0.70 | 262 | 11.87 | 6.49 | 3.69 |
| 0.10 | 0.75 | 226 | 10.42 | 5.10 | 3.94 |
| 0.14 | 0.78 | 216 | 9.51 | 4.32 | 4.07 |

Table 3 Coupling parameters of our approach.

The analysis of these results demands first to remember that a stiffer, or equivalently, more rigid equation of state of nuclear matter is related to higher values of the internal pressure of the system and, accordingly, to higher values of the compressibility modulus \( |K_{\text{sym}}| \) of nuclear matter. This in turn requires stronger contributions from repulsive components of the nuclear force when compared to the attractive ones. In our general approach, however, many body forces lowers the intensities both of attractive and repulsive interaction terms due to shielding effects, which result in higher (lower) values of the compressibility modulus \( |K_{\text{sym}}| \) of nuclear matter in the case of higher (lower) relative reduction of the attractive (repulsive) contributions.

Our present results (scalar model) are consistent with the analysis above, since we have obtained a stiffer equation of state for lower values of the \( \lambda \) parameter (\( \lambda = 0.06 \)), which corresponds to the stronger relative reduction of the attractive contribution (see Eq. (13)). Similary, the results for the particle population show that the threshold equation for a given species (Glendenning 1996),

\[
\mu_n - q_B \mu_e - g_{\omega B} \omega_0 - g_{\sigma B} \sigma_0 \geq M_B (1 - g_{\sigma B} \sigma_0 / M_B)
\]

is also affected by the presence of many-body forces components in the strong interaction sector (see Eq. (16)):

\[
\mu_n - q_B \mu_e - g_{\omega B} \omega_0 - g_{\sigma B} \sigma_0 \geq M_B (1 - g_{\sigma B} \sigma_0 / M_B)
\]

where \( \mu_n \) and \( \mu_e \) represent respectively the neutron and electron chemical potential, and \( q_B \) is the baryon charge. According to Eq. (16) many body forces shift the critical density for hyperon saturation to higher densities and originate an anti-correlation between the amount of hyperons and the values of \( \lambda \); to the lowest values of \( \lambda \) correspond larger amounts of hyperons in the system, which may cause the softening of the EoS. However, our results show that the shielding effect of the long-range nuclear components of the nuclear force is predominant. The first hyperon species that appears is the \( \Sigma^- \), closely followed by the \( \Lambda \), since the negative charge of the \( \Sigma^- \) outweighs the 80 MeV mass difference, as a result of the more lenient conditions \( \mu_\Lambda = \mu_n \) and \( \mu_\Sigma^- \geq \mu_n + \mu_\Xi \). The \( \Sigma^- \) fraction saturates at about 0.3, while \( \Lambda \), free of isospin-dependent forces, continue to accumulate until short-range repulsion forces cause them to saturate. Other hyperon species follow at higher densities.

Fig. 3 Dependence of the equation of state (EoS) on \( \lambda \).

It is our understanding that the naturalness condition is equivalent to exhaust the phase space of the fundamental
interactions. This depletion of the phase space can be accomplished by including in nuclear matter the largest possible number of information, i.e., baryon and meson degrees of freedom and additionally considering many body forces between baryon and meson fields, as well as self-coupling terms involving meson fields. When the condition of naturalness is achieved, it is unnecessary to get rid of heavier degrees of freedom by integrating them out. In this case, the effects of heavier degrees of freedom are not anymore implicitly contained in coupling parameters of the effective field theory. An interesting aspect of our study is that apparently, many body forces occupy a larger role than originally thought in the description of properties of nuclear system at high density. Moreover, the condition of naturalness is achieved with only a small part of the parameter set. This result was expected and can be explained by the saturation property of nuclear matter: for parameter values less than 3, the model with parameterized couplings completely exhausts the phase space of many-body interactions.

The model with parameterized couplings shows promising results. However, the model needs to broaden its scope by the adoption of new parameterizations, expanding the set of parameters of the theory. Interesting issues for future studies will also be the role of finite temperature, neutrino trapping and strong magnetic effects in neutron stars. Work along these lines is in progress.

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