Size effects on generation-recombination noise

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We carry out an analytical theory of generation-recombination noise for a two level resistor model which goes beyond those presently available by including the effects of both space charge fluctuations and diffusion current. Finite size effects are found responsible for the saturation of the low frequency current spectral density at high enough applied voltages. The saturation behaviour is controlled essentially by the correlations coming from the long range Coulomb interaction. It is suggested that the saturation of the current fluctuations for high voltage bias constitutes a general feature of generation-recombination noise.

Generation recombination (GR) noise is due to fluctuations in the number of free carriers inside a device and is associated with random transitions of charge carriers between states in different energy bands. Accordingly, it represents an important noise source in semiconductor devices where carrier concentration can vary over many orders of magnitude. Typical examples of transitions are those between the conduction band and localized levels in the energy gap, conduction and valence bands, etc. GR noise can be detected as an excess noise when measuring current or voltage fluctuations.

The calculation of the number fluctuations, and hence of the current and voltage fluctuations, due to GR processes constitutes in general a rather complicated problem. Usually, one has to resort to simplifying assumptions as for instance the local space charge neutrality approximation and/or the neglect of the diffusion current. The physical implications of these approximations remain still an unsolved problem, since an exact theory including simultaneously both effects is lacking in the current literature.

The aim of the present paper is to address this issue by providing an exact analytical solution of the low frequency GR noise properties of a two-level resistor model within the framework of the drift-diffusion transport theory. The main result of the exact solution is the evidence that size effects on GR noise can lead to the saturation of the associated current spectral density at high enough applied bias. Remarkably, present results confirm unexpected findings of recent numerical studies by Bo-nani and Ghione concerning uniformly n—doped silicon samples with band-to-band and trap assisted generation-recombination processes, where the low frequency spectral density of the voltage fluctuations was shown to become independent of the applied bias for moderately strong applied bias.

We consider a two terminal structure consisting of a uniformly n—doped semiconductor sample of length \( L \) terminated by (metallic) ohmic contacts with a single trap (donor) level controlling the electron concentration. We assume that the transversal dimensions of the sample are much greater than the longitudinal dimension, thus allowing for a one dimensional electrostatic treatment. Without loss of generality, only GR noise sources are considered. The physical model necessary to describe the noise properties of this system consists of the continuity equations for the free electrons and the ionized traps, the current equation for the electrons, the Poisson equation and appropriate boundary conditions. The continuity equations for free carriers and ionized traps densities read

\[
\frac{\partial n(x, t)}{\partial t} = \frac{1}{q} \frac{\partial J_n(x, t)}{\partial x} - r_n(x, t) + g_n(x, t) + \gamma(x, t), \quad (1)
\]

\[
\frac{\partial N^+_n(x, t)}{\partial t} = -r_n(x, t) + g_n(x, t) + \gamma(x, t), \quad (2)
\]

where \( n(x, t) \) and \( N^+_n(x, t) \) refers to the free electron and ionized trap densities at point \( x \) and time \( t \), respectively, \( q \) is the positive electron charge, \( J_n(x, t) \) the conduction electron current, \( r_n(x, t) \) the recombination rate and \( g_n(x, t) \) the generation rate. Furthermore, \( \gamma(x, t) \) is the Langevin noise source related to GR processes which has zero mean and low frequency spectral density

\[
2 \int_{-\infty}^{+\infty} dt \gamma(x, t) \gamma(x, t') = \frac{2}{A} [\overline{\gamma_n(x)} + \overline{\tau_n(x)}] \delta(x - x'), \quad (3)
\]

with \( A \) being the sample cross-sectional area. The bar denotes average with respect to fluctuations. In the drift-diffusion approach, the conduction current writes:

\[
J_n(x, t) = q \mu n(x, t) E(x, t) + q D_n \frac{\partial n(x, t)}{\partial x}, \quad (4)
\]

where \( \mu \) is the electron mobility, \( E(x, t) \) the electric field and \( D_n \) the diffusion coefficient satisfying Einstein relation, \( D_n = \mu_n k_B T / q \), with \( k_B \) being Boltzmann’s constant, \( T \) the temperature and where non-degenerate statistics is assumed. To perform a self-consistent calculation, we take into account Poisson equation.
\[
\frac{\partial E(x,t)}{\partial x} = \frac{q}{\epsilon} \left[ N_t^+(x,t) - n(x,t) \right],
\]
where \( \epsilon \) is the semiconductor dielectric permittivity. The total electric current is thus given by

\[
I(t) = \frac{A}{L} \int_0^L dx \left[ J_n(x,t) + \epsilon \frac{\partial E(x,t)}{\partial t} \right].
\]

Finally, as appropriate boundary conditions we take ohmic contacts made of ideal metal-semiconductor interfaces in the diffusion approximation, i.e. we assume that the diffusion velocity is much smaller than the contact recombination velocity, thus allowing to neglect any contribution from thermionic emission processes. Accordingly, we take

\[
\pi(0) = \pi(L) \equiv n_c; \quad \delta n(0,t) = \delta n(0,t) = 0,
\]
where the carrier density at the contact is

\[
n_c = N_C e^{-\phi_{tn}/kT},
\]
with \( N_C \) being the effective density of states in the conduction band and \( \phi_{tn} \) the contact barrier height. Here, \( \delta n(x,t) \) refers to the free electron density fluctuation.

The model presented above admits an homogeneous stationary solution in the form

\[
\pi(x) = N_t^0(x) = n_c; \quad E(x) = \frac{T}{qA\mu n}; \quad \tau_n = \frac{2}{\nu n}\]

Note that being the density of traps \( N_t \) and the contact density \( n_c \) two independent variables, one can always find for them values which satisfy simultaneously Eq. (8) and \( \tau_n(\pi, N_t^+) = \tau_n(\pi, N_t^-) \). Under these homogeneous conditions the system behaves as a resistor with current-voltage characteristics satisfying Ohm’s law, \( I = V/R \), with resistance \( R = L/(qA\mu n) \).

To calculate the low frequency number fluctuations

\[
S_N(0) = 2 \int_{-\infty}^{+\infty} \delta N(t) \delta N(t') dt',
\]
we use that

\[
\delta N(t) = A \int_0^L dx \delta n(x,t),
\]
where \( \delta n(x,t) \) for low frequencies satisfies

\[
\frac{\partial^2 \delta n}{\partial x^2} + \frac{1}{L_E} \frac{\partial \delta n}{\partial x} - \frac{1}{L_D^2} \delta n = -\frac{\tau}{L_D^2} \gamma.
\]

Equation (12) can be derived by combining Eqs. (1), (2), (8) and (9) after linearization around the homogeneous stationary state and by neglecting the time derivatives due to the low frequency assumption. The parameters in Eq. (12) are defined as

\[
\frac{L_E}{L} = \frac{k_B T}{qV}; \quad \frac{L_D}{L} = \sqrt{\frac{k_B T e}{q^2 (n + \frac{2}{\tau n})}}; \quad \frac{1}{\tau} = \frac{1}{\tau_t} + \frac{1}{\tau_n},
\]

Here, \( L_D \) is a renormalized Debye screening length and

\[
\begin{align*}
\frac{1}{\tau_t} &= \frac{\partial r_n(n, N_t^+)}{\partial n} - \frac{\partial g_n(n, N_t^+)}{\partial n} + \frac{1}{\tau_n} = \frac{\partial r_n(n, N_t^+)}{\partial N_t^+} - \frac{\partial g_n(n, N_t^+)}{\partial N_t^+}. 
\end{align*}
\]

From the solution of the second order differential equation (12) we calculate \( \delta N(t) \) through Eq. (11), and after some lengthy but straightforward algebra we arrive at the following expression for \( S_N(0) \),

\[
S_N(0) = S_N^N(0) - S_N^\pi(0),
\]
where

\[
S_N^N(0) = 4AL\bar{x}_n \tau^2 = 4 \langle \Delta N^2 \rangle \tau,
\]
and

\[
S_N^\pi(0) = 4 \langle \Delta N^2 \rangle \tau \left( e^{\lambda_1 L} - 1 \right) \left( e^{\lambda_2 L} - 1 \right) \left( \lambda_1 - \lambda_2 \right)^2 \times \left[ \lambda_1 \left( 1 - e^{(\lambda_1 + \lambda_2) L} - 3e^{\lambda_2 L} + 3e^{\lambda_1 L} \right) \right] \times \left[ \lambda_2 \left( 1 - e^{(\lambda_1 + \lambda_2) L} - 3e^{\lambda_1 L} + 3e^{\lambda_2 L} \right) \right].
\]

In Equation (18) \( \lambda_1 \) and \( \lambda_2 \) refer to the eigenvalues of the differential operator in Eq. (12) and are given by

\[
\lambda_{1,2} = \frac{1}{2L_E} \left( -1 \pm \sqrt{1 + 4 \frac{L_E^2}{L_D^2}} \right); \quad (19)
\]

According to standard results of GR noise theory, \( S_I(0) \) is conveniently written as

\[
S_I(0) = \left( \frac{T}{N} \right)^2 \left( S_N^N(0) - S_N^\pi(0) \right). \quad (20)
\]

Equation (20), together with Eqs. (17) and (18), constitutes a fully analytical and exact solution for the low frequency current spectral density of the problem in subject and represents the main result of the present paper. In deriving it we have not made any assumption regarding either the condition for local space charge neutrality or the relevance of the diffusion current. We note, that in spite of the minus sign in Eq. (20), \( S_I(0) \) is a positive definite quantity, as should be.

To investigate the physical properties of the solution, we note that the term \( S_N^\pi(0) \) vanishes for an infinitely long sample thus implying

\[
S_I^\pi(0) = 4 \langle \Delta N^2 \rangle \tau \left( \frac{T}{N} \right)^2.
\]
as known from simpler theories. Therefore, \( S_N^\infty (0) \) constitutes a size effect related to the finite nature of the sample. This size effect is analyzed below by investigating the dependence of the current spectral density upon applied voltage. The results are summarized in Fig. 1 where we find convenient to normalize the current spectral density \( S_I(0) \) to the spectral density corresponding to an infinite sample with thermal current \( I_T = k_BT/(qR) \), i.e. \( S_I = 4\langle \Delta N^2 \rangle \tau (I_T/N)^2 \), and the applied voltage \( V \) to the thermal voltage \( V_T = k_BT/q \). With these renormalizing factors, \( S_I(0)/S_T(0) \) as a function of \( Vq/(k_BT) \), depends only on the dimensionless length, \( L/L_D \).

The essential result we want to stress from Fig. 1 is that the spectral density saturates for high applied voltages to the value

\[
S_I^{\text{sat}}(0) = \frac{1}{3} \left( \frac{L}{L_D} \right)^4 4\langle \Delta N^2 \rangle \tau \left( \frac{k_BT}{qRN} \right)^2 .
\]

(22)

The critical voltage for saturation \( V_{\text{sat}} \) satisfies

\[
V_{\text{sat}} = \begin{cases} \sqrt{\frac{40k_BT}{q}}; & \text{for } \frac{L}{L_D} < 1 \\ \left( \frac{L}{L_D} \right)^2 \frac{k_BT}{q} & \text{for } \frac{L}{L_D} \gtrsim 10 \end{cases}
\]

(23)

(and an intermediate behavior for \( 1 < L/L_D < 10 \)). The low bias behavior of \( S_I(0) \) displays also interesting features: it equals \( S_I^\infty (0) \) for \( L/L_D > 10 \) and

\[
0 < V < \left( \frac{L}{L_D} \right)^2 k_BT/q,
\]

but it is suppressed to \( 1/120 \left( \frac{L}{L_D} \right)^4 S_I^\infty (0) \) for \( L/L_D < 1 \) and \( 0 < V < \sqrt{\frac{40k_BT}{q}} \) (and an intermediate behavior in the range \( 1 < L/L_D < 10 \)).

The saturation of \( S_I(0) \) and the dependence of the results on the ratio \( L/L_D \) can be explained under the assumption that both space charge fluctuations and diffusion current play a relevant role. Indeed, although the system is neutral in average, i.e. \( \overline{\pi(x)} = N_t^+ (x) \), instantaneous space charge fluctuations can appear for which \( \delta n(x,t) \neq \delta N_t^+ (x,t) \). The relevance of these fluctuations is determined by their interaction with the rest of charges in the system through the long range Coulomb interaction. In this line of reasoning for \( L < L_D \) the long range Coulomb interaction does not play any role at all, and hence, as soon as the applied voltage is higher than the thermal value, the low frequency current spectral density saturates. On the other hand, when \( L > L_D \) the long range Coulomb interaction is able to restor space charge neutrality for low applied bias. However, as long as the applied bias is high enough as to make the transit time \( \tau_T = L^2/(qV) \) shorter than the renormalized dielectric relaxation time \( \tau_d = \epsilon/(q\mu\overline{\pi}(1 + \frac{\tau_d}{\tau_n})) \), the long range Coulomb correlations vanish and the low frequency spectral density saturates.

Furthermore, the fact that the diffusion current is playing a relevant role is seen on the fact that no shot noise is obtained in the high bias regime as it is obtained when the diffusion current is neglected.

It is worth noting that the results presented in the present paper are qualitatively similar to the numerical results recently reported by Bonani and Ghione for a more complex GR model. This fact indicates that saturation at high applied bias can be a general feature of GR noise. The present work, then, offers a more complete understanding of the finite size effects on generation-recombination noise in semiconductor devices.

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FIG. 1. Spectral density of current fluctuations associated with GR noise.