The effect of supersymmetry breaking in the Mass Varying Neutrinos

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We discuss the effect of the supersymmetry breaking on the Mass Varying Neutrinos(MaVaNs) scenario. Especially, the effect mediated by the gravitational interaction between the hidden sector and the dark energy sector is studied. A model including a chiral superfield in the dark sector and the right handed neutrino superfield is proposed. Evolutions of the neutrino mass and the equation of state parameter are presented in the model. It is remarked that only the mass of a sterile neutrino is variable in the case of the vanishing mixing between the left-handed and a sterile neutrino on cosmological time scale. The finite mixing makes the mass of the left-handed neutrino variable.

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I. INTRODUCTION

In recent years, many cosmological observations have provided the strong evidence that the Universe is flat and its energy density is dominated by the dark energy component whose negative pressure causes the cosmic expansion to accelerate [1]−[8]. In order to clarify the origin of the dark energy, one has tried to understand the connection of the dark energy with particle physics.

In a dynamical model proposed by Fardon, Nelson and Weiner (MaVaNs), relic neutrinos could form a negative pressure fluid and cause the cosmic acceleration [9]. In this model, an unknown scalar field which is called “acceleron” is introduced and neutrinos are assumed to interact through a new scalar force. The acceleron sits at the instantaneous minimum of its potential, and the cosmic expansion only modulates this minimum through changes in the neutrino density. Therefore, the neutrino mass is given by the acceleron, in other words, it depends on the number density and changes with the evolution of the Universe. The equation of state parameter w and the dark energy density also evolve with the neutrino mass. Those evolutions depend on a model of the scalar potential and the relation between the acceleron and the neutrino mass strongly. Typical examples of the potential have been discussed in ref. [10].

The variable neutrino mass was considered at first in ref. [11], and was discussed for neutrino clouds [12]. Ref. [13] considered coupling a sterile neutrino to a slowly rolling scalar field which was responsible for the dark energy. Ref. [14] considered coupling of the dark energy scalar, such as the quintessence to neutrinos and discuss its impact on the neutrino mass limits from Baryogenesis. In the context of the MaVaNs scenario, there have been a lot of works [15]−[35]. The origin of the scalar potential for the acceleron was not discussed in many literatures, however, that is clear in the supersymmetric MaVaNs scenario [36, 37].

In this work, we present a model including the supersymmetry breaking effect mediated by the gravity. Then we show evolutions of the neutrino mass and the equation of state parameter in the model.

The paper is organized as follows: in Section II, we summarize the supersymmetric MaVaNs scenario and present a model. Sec. III is devoted to a discussion of the supersymmetry breaking effect mediated by the gravity in the dark sector. In Sec. IV, the summary is given.

II. SUPERSYMMETRIC MAVANS

In this section, we discuss the supersymmetric Mass Varying Neutrinos scenario and present a model.

The basic assumption of the MaVaNs with supersymmetry is to introduce a chiral superfield A in the dark sector, which is assumed to be a singlet under the gauge group of the standard model. It is difficult to build a viable MaVaNs model without fine-tunings in some parameters when one assumes one chiral superfield in the dark sector, which couples to only the left-handed lepton doublet superfield [36]. Therefore, we assume that the superfield A couples to both the left-handed lepton doublet superfield L and the right-handed neutrino superfield R.

In this framework, we suppose the superpotential

\begin{equation}
W = \frac{\lambda_1}{6} A^3 + \frac{M_A}{2} AA + m_D L A + M_D L R + \frac{\lambda_2}{2} A R^2 + \frac{M_R}{2} R R,
\end{equation}

where \(\lambda_i (i = 1, 2)\) are coupling constant of \(\mathcal{O}(1)\) and \(M_A, M_D, M_R\) and \(m_D\) are mass parameters. The scalar and spinor component of A are \((\phi, \psi)\), and the scalar component is assumed to be the acceleron which cause the present cosmic acceleration. The spinor component is a sterile neutrino. The Helium-4 abundancy gives the

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most accurate determination of a cosmological number of neutrinos and does not exclude a fourth thermalized neutrino at $T \sim \text{MeV}$ [38]. The third term of the right-hand side in Eq. (4) is derived from the Yukawa coupling such as $y L A H$ with $y < H = m_D$, where $H$ is the Higgs doublet.

In the MaVaNs scenario, the dark energy is assumed to be the sum of the neutrino energy density and the scalar potential for the acceleron:

$$\rho_{\text{DE}} = \rho_{\nu} + V(\phi).$$

(2)

Since only the acceleron potential contributes to the dark energy, we assume the vanishing expectation values of sleptons, and thus the effective scalar potential is given as

$$V(\phi) = \frac{\lambda^2}{4} |\phi|^4 + M_2^4 |\phi|^2 + m_D^2 |\phi|^2.$$  

(3)

We can write down a lagrangian density from Eq. (1):

$$L = \lambda_1 \phi \bar{\nu} \nu + M_A \bar{\nu} \nu + m_D \bar{\nu} L \nu + M_D \bar{\nu} L \nu + M_R \bar{\nu} R \nu + h.c.$$  

(4)

It is noticed that the lepton number conservation in the dark sector is violated because this lagrangian includes both $M_A \bar{\nu} \nu$ and $m_D \bar{\nu} L \nu$. After integrating out the right-handed neutrino, the effective neutrino mass matrix is given by

$$\mathcal{M} \approx \left( \begin{array}{c} -\frac{m_D^2}{M_R} + \frac{\lambda_2 M_2^2}{M_R} \frac{m_D}{M_A + \lambda_1 \phi} \\ m_D^2 \frac{m_D}{M_A + \lambda_1 \phi} \end{array} \right),$$

(5)

in the basis of $(\bar{\nu}_L, \psi)$, where we assume $\lambda_1 \phi \ll M_D \ll M_R$. The first term of the $(1,1)$ element of this matrix corresponds to the usual term given by the seesaw mechanism [39, 40, 41] in the absence of the acceleron. The second term is derived from coupling between the acceleron and the right-handed neutrino but the magnitude of this term is negligible small because of the suppression of $O(1/M_R)$. Therefore, we can rewrite the neutrino mass matrix as

$$\mathcal{M} \approx \left( \begin{array}{c} c \\ m_D^2 \frac{m_D}{M_A + \lambda_1 \phi} \end{array} \right),$$

(6)

where $c \equiv -M_2^2/M_R$. It is remarked that only the mass of a sterile neutrino is variable in the case of the vanishing mixing ($m_D = 0$) between the left-handed and a sterile neutrino on cosmological time scale. The finite mixing ($m_D \neq 0$) makes the mass of the left-handed neutrino variable. We will consider these two cases of $m_D = 0$ and $m_D \neq 0$ later.

In the MaVaNs scenario, there are two constraints on the scalar potential. The first one comes from observations of the Universe, which is that the magnitude of the present dark energy density is about $0.74 \rho_c$, $\rho_c$ being the critical density. Thus, the first constraint turns to

$$V(\phi^0) = 0.74 \rho_c - \rho_{\nu}^0,$$

(7)

where “0” represents a value at the present epoch.

The second one is the stationary condition. In this scenario, the neutrino mass is assumed to be a dynamical field which is a function of the acceleron. Therefore, the dark energy density should be stationary with respect to the variation of the neutrino mass:

$$\frac{\partial \rho_{\text{DE}}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi(m_\nu))}{\partial m_\nu} = 0.$$  

(8)

If $\partial m_\nu/\partial \phi \neq 0$, this condition is equivalent to the usual stationary condition stabilized by an ordinary scalar field. Eq. (8) is rewritten by using the cosmic temperature $T$:

$$\frac{\partial V(\phi)}{\partial m_\nu} = -T^4 \frac{\partial F(\xi)}{\partial \xi},$$

(9)

where $\xi \equiv m_\nu/T, \rho_\nu = T^4 F(\xi)$ and

$$F(\xi) \equiv \frac{1}{\pi^2} \int_0^\infty \frac{dy y^2 \sqrt{y^2 + \xi^2}}{e^y + 1}.$$  

(10)

We can get the time evolution of the neutrino mass from Eq. (9). Since the stationary condition should be always satisfied in the evolution of the Universe, this one at the present epoch is the second constraint on the scalar potential:

$$\frac{\partial V(\phi)}{\partial m_\nu} \bigg|_{m_\nu = m_\nu^0} = -T^4 \frac{\partial F(\xi)}{\partial \xi} \bigg|_{m_\nu = m_\nu^0, T = T_0}.$$  

(11)

In addition to two constraints for the potential, we also have two relations between the acceleron and the neutrino mass at the present epoch:

$$m_{\nu_L}^0 = \frac{c + M_A + \lambda_1 \phi^0}{2} + \frac{\sqrt{[c \nu (M_A + \lambda_1 \phi^0)]^2 + 4m_D^2}}{2},$$

(12)

$$m_{\psi}^0 = \frac{c + M_A + \lambda_1 \phi^0}{2} - \frac{\sqrt{[c \nu (M_A + \lambda_1 \phi^0)]^2 + 4m_D^2}}{2}.$$  

(13)

Next, we will consider the dynamics of the acceleron field. In order that the acceleron does not vary significantly on distance of inter-neutrino spacing, the acceleron mass at the present epoch must be less than $O(10^{-4} eV)$ [9]. Here and below, we fix the present acceleron mass as

$$m_\phi^0 = 10^{-4} eV.$$  

(14)

Once we adjust parameters which satisfy five equations [4, 14] and [11] ~ [14], we can have evolutions of the neutrino mass by using the Eq. 9.
equation in the Robertson-Walker background and the stationary condition Eq. (9):

\[ w + 1 = \frac{4 - h(\xi)}{3\rho_{\text{DE}}}, \]

(15)

where

\[ h(\xi) \equiv \frac{\Delta F(\xi)}{F(\xi)}. \]

(16)

It seems that \( w \) in this scenario depend on the neutrino mass and the cosmic temperature. This means that \( w \) varies with the evolution of the Universe unlike the cosmological constant.

In the last of this section, it is important to discuss the hydrodynamic stability of the dark energy from MaVaNs. The speed of sound squared in the neutrino-acceleron fluid is given by

\[ c_s^2 = \frac{\dot{p}}{\rho_{\text{DE}}} = \frac{\dot{\rho}_{\text{DE}} + w\rho_{\text{DE}}}{\rho_{\text{DE}}}, \]

(17)

where \( p \) is the pressure of the dark energy. Recently, it was argued that when neutrinos are non-relativistic, this speed of sound squared becomes negative in this scenario [27]. The emergence of an imaginary speed of sound shows that the MaVaNs scenario with non-relativistic neutrinos is unstable, and thus the fluid in this scenario cannot acts as the dark energy. However, finite temperature effects provide a positive contribution to the speed of sound squared and avoid this instability [33]. Then, a model should satisfy the following condition,

\[ \frac{\partial m_\nu}{\partial z} \left( 1 - \frac{5aT^2}{3m_\nu^2} \right) + \frac{25aT_\nu^2(z + 1)}{3m_\nu} > 0, \]

(18)

where \( z \) is the redshift parameter, \( z \equiv (T/T_0) - 1 \), and

\[ a \equiv \int_0^\infty \frac{dy y^4}{e^{y+1}} \simeq 6.47. \]

(19)

Actually, some models satisfy this condition [28, 29].

III. THE EFFECT OF SUPERSYMMETRY BREAKING

In order to consider the effect of supersymmetry breaking in the dark sector, we assume a superfield \( X \) which breaks supersymmetry, in the hidden sector, and the chiral superfield \( A \) in the dark sector is assumed to interact with the hidden sector only through the gravity. This framework is shown graphically in FIG. 1. Once supersymmetry is broken at TeV scale, its effect is transmitted to the dark sector through the following operators:

\[ \int d^4\theta \frac{X^\dagger X}{M_{p^l}^2} A^\dagger A, \quad \int d^4\theta \frac{X^\dagger + X}{M_{p^l}^2} A^\dagger A, \]

(20)

where \( M_{p^l} \) is the Planck mass. Then, the scale of the soft terms \( F_X(\text{TeV})^2/M_{p^l} \sim \mathcal{O}(10^{-3}-10^{-2}\text{eV}) \) is expected. Such a framework was discussed in the “acceleressence” scenario [42]. Now, we consider only one superfield which breaks supersymmetry for simplicity. If one extend the hidden sector, one can consider a different mediation mechanism between the standard model and the hidden sector from one between the dark and the hidden sector. We will return to this point later.

In this framework, taking supersymmetry breaking effect into account, the scalar potential is given by

\[ V(\phi) = \frac{\lambda^2}{4} |\phi|^4 - \frac{\kappa}{3} (\phi^3 + \text{h.c.}) + M_A^2 |\phi|^2 + m_D^2 |\phi|^2 - m^2 |\phi|^2 + V_0, \]

(21)

where \( \kappa \) and \( m \) are supersymmetry breaking parameters, and \( V_0 \) is a constant determined by the condition that the cosmological constant is vanishing at the true minimum of the acceleron potential. This scalar potential is the same one presented in [42]. We consider two types of the neutrino mass matrix in this scalar potential. They are the cases of the vanishing and the finite mixing between the left-handed and a sterile neutrino.

A. The Case of the Vanishing Mixing

When the mixing between the left-handed and a sterile neutrino is vanishing, \( m_D = 0 \) in the neutrino mass matrix [35]. Then we have the mass of the left-handed and a sterile neutrino as

\[ m_{\nu_L} = c, \]

(22)

\[ m_\psi = M_A + \lambda_1 \phi. \]

(23)
FIG. 2: Evolution of the mass of a sterile neutrino \((0 \leq z \leq 2000)\)

In this case, we find that only the mass of a sterile neutrino is variable on cosmological time scale due to the second term in Eq. (23).

Let us adjust parameters which satisfy Eqs. (7) and (11) \(\sim (14)\). In Eq. (7), the scalar potential Eq. (21) is used. Putting typical values for four parameters by hand as follows:

\[
\begin{align*}
\lambda_1 &= 1, \quad m_D = 0, \\
m^0_{\nu_L} &= 2 \times 10^{-2} \text{ eV}, \quad m^0_\psi = 10^{-2} \text{ eV},
\end{align*}
\]

we have

\[
\begin{align*}
\phi^0 &\simeq -1.31 \times 10^{-5} \text{ eV}, \quad c = 2 \times 10^{-2} \text{ eV}, \\
M_A &\simeq 10^{-2} \text{ eV}, \quad m \simeq 10^{-2} \text{ eV}, \\
\kappa &\simeq 4.34 \times 10^{-3} \text{ eV}.
\end{align*}
\]

There is a tuning between \(M_A\) and \(m\) in order to satisfy the constraint on the present acceleron mass of Eq. (14).

Now, we can calculate evolutions of the mass of a sterile neutrino and the equation of state parameter \(w\) by using these values. The numerical results are shown in FIG. 2-4. Especially, the behavior of the mass of a neutrino near the present epoch is shown in FIG. 3. We find that the mass of a sterile neutrino have varied slowly in this epoch. This means that the first term of the left hand side in Eq. (18), which is a negative contribution to the speed of sound squared, is tiny. We can also check the positive speed of sound squared in a numerical calculation. Therefore, the neutrino-acceleron fluid is hydrodynamically stable and acts as the dark energy.

**B. The Case of the Finite Mixing**

Next, we consider the case of the finite mixing between the left-handed and a sterile neutrino \((m_D \neq 0)\). In this case, the left-handed and a sterile neutrino mass are given by

\[
m_{\nu_L} = \frac{c + M_A + \lambda_1 \phi}{2}
\]

We can expect that both the mass of the left-handed and a sterile neutrino have varied on cosmological time scale due to the term of the acceleron dependence.

Taking typical values for four parameters as

\[
\begin{align*}
\lambda_1 &= 1, \quad m_D = 10^{-3} \text{ eV}, \\
m^0_{\nu_L} &= 2 \times 10^{-2} \text{ eV}, \quad m^0_\psi = 10^{-2} \text{ eV},
\end{align*}
\]

we have

\[
\begin{align*}
\phi^0 &\simeq -1.31 \times 10^{-5} \text{ eV}, \quad c \simeq 1.99 \times 10^{-2} \text{ eV}, \\
M_A &\simeq 1.01 \times 10^{-2} \text{ eV}, \quad m \simeq 1.02 \times 10^{-2} \text{ eV}, \\
\kappa &\simeq 4.34 \times 10^{-3} \text{ eV}.
\end{align*}
\]

where we required that the mixing between the active and a sterile neutrino is tiny. In our model, the small present value of the acceleron is needed to satisfy the constraints on the scalar potential in Eqs. (7) and (11).
When we add following terms of the supersymmetry breaking effects to the scalar potential,
\[ \lambda_1 m_3/2 \phi^3, \quad M_A m_3/2 |\phi|^2, \] (30)
a small gravitino mass \((m_3/2 < \mathcal{O}(10 \text{ eV}))\) is favored. Such a small gravitino mass has been given in the gauge mediation model of Ref. [43]. Therefore, a mediation mechanism between the standard and the hidden sector which leads to a small gravitino mass is suitable for our framework.

Values of parameters in (29) are almost same as the case of the vanishing mixing (25). However, the mass of the left-handed neutrino is variable unlike the vanishing mixing case. The time evolution of the left-handed neutrino mass is shown in FIG. 5. The mixing does not affect the evolution of the sterile neutrino mass and the equation of state parameter, which are shown in FIGs. 6, 7. Since the variation in the mass of the left-handed neutrino is not vanishing but extremely small, the model can also avoid the instability of speed of sound. The value of the sum of the left-handed and a sterile neutrino mass is within the limit provided by analyses from WMAP three-years date [44, 45].

Finally, we comment the smallness of the evolution of the neutrino mass at the present epoch. In our model, the mass of the left-handed and a sterile neutrino include the constant part. A variable part is a function of the acceleron. In the present epoch, the constant part dominates the neutrino mass because the present value of the acceleron should be small. This smallness of the value of the acceleron is required from the cosmological observation and the stationary condition in Eqs. (7) and (11).

IV. SUMMARY

We presented a supersymmetric MaVaNs model including the effects of the supersymmetry breaking mediated by the gravity. Evolutions of the neutrino mass and the equation of state parameter have been calculated in the model. Our model has a chiral superfield in the dark sector, whose scalar component causes the present cosmic acceleration, and the right-handed neutrino superfield. In our framework, supersymmetry is broken in the hidden sector at TeV scale and the effect is assumed to be transmitted to the dark sector only through the gravity. Then, the scale of soft parameters are \(\mathcal{O}(10^{-3}-10^{-2})\text{eV}\) is expected.

We considered two types of model. One is the case of the vanishing mixing between the left-handed and a sterile neutrino. Another one is the finite mixing case. In the case of the vanishing mixing, only the mass of a sterile neutrino had varied on cosmological time scale. In the epoch of \(0 \leq z \leq 20\), the sterile neutrino mass had varied slowly. This means that the speed of sound squared in the neutrino acceleron fluid is positive, and thus this fluid can act as the dark energy. In the finite mixing case, the mass of the left-handed neutrino had also varied. However, the variation is extremely small and the effect of the mixing does not almost affect the evolution of the sterile neutrino mass and the equation of state. Therefore, this model can also avoid the instability.

FIG. 5: Evolution of the mass of the left-handed neutrino \((0 \leq z \leq 2000)\)

FIG. 6: Evolution of the mass of a sterile neutrino \((0 \leq z \leq 2000)\)

FIG. 7: Evolution of \(w (0 \leq z \leq 50)\)
