Study the deformation in some even krypton isotopes (88-92) using IBM-1 model

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Abstract. This research is being studied the low lying energy levels structure of some even-even krypton isotopes and possibility of electromagnetic transmission for them using the Interacting Boson Model-IBM-1 to estimate the nuclear structure for ⁸⁸-⁹²Kr isotopes. The results were obtained using IBM program with Fortran language and the values of parameters in this calculations indicated that Krypton isotopes have a vibration U(5) and unstable O(6) properties. These properties are clear from the ratios and from the potential energy surface. The calculated results are in a good agreement with recent experimental data for theses isotopes.

1. Introduction
The single particle shell model is characterized by its success in calculation of spin, parity, beta and gamma decay in ground levels, as well as the calculation of magnetic Moments, but failed to determine and calculate quadrupole moments and transition rates, especially in transition zones between closed shells. The lack of success led to the emergence of geometric models. In these models, a large number of nucleons were assumed to be moving in a cooperative collective. A description of a set of nuclei properties is capable to describe the installation of vibrating and rotating beams [1].

One of these models is the IBM studies the low –lying nuclear spectra where assuming that the nucleus with an even – even mass number and atomic number ,consists of an inert core with some valence particles. Furthermore, the valence particles merge together to form bosons with angular momentum L=0 (s-boson ) and L=2 (d-boson[2]. These bosons are interpreted as correlated pairs of protons and correlated pairs of neutrons in the valence shell. This interpretation places restriction on the boson number which is determined by counting the number of particle pairs (separately for protons and neutrons) if the shell is less than half filled and by counting the number of hole pairs if the shell is more than half filled. If the bosons of neutrons and the bosons of neutrons were considered identical then the IBM is in its simplest form which is called IBM-1[3-5].The second version of IBM is IBM-2, which is a modification of IBM-1, through distinguishes between the neutrons and protons[6,7]. This made it possible to distinguish between wave function for protons and wave function for neutrons.

Energy levels and electric transitions are produced from this model by modify a certain number of parameters in the model to fit the results with experimental data[2]. We have been studied analytic description of nuclei shape for ⁸⁸-⁹²Kr isotopes with IBM-1 and geometric studied with potential surface energybecause they have many unknown energy levels, and have electric transition 2⁺ to 0⁺ only which studied byK. Nomura and T. Rz, aca-Urban [8,9].
2. IBM-1

In the IBM-1 it is assumed that the Hamiltonian contains only one-body and two-body terms, thus introducing creation (s↑, d↑,m) and annihilation (s, d, m) operators, where m=0,±1,±2. The most general Hamiltonian, which includes one-body terms in boson-boson interaction is[6,10]:

\[ H = \varepsilon n_d + a_0 \hat{P} \hat{P} + a_1 \hat{L}^2 + a_2 \hat{Q}^2 + a_3 \hat{T}_3^2 + a_4 \hat{T}_4^2 \] (1)

where \( \varepsilon, P, L, Q, T \) and \( T_4 \) are the energy, pairing, angular momentum, quadrupole, octopole, and hexadecapole operators respectively, and \( a_0, a_1, a_2, a_3, \) and \( a_4 \) are the parameters that we determined in this model to get the output levels.

In the IBM, axially symmetric rotors and spherical vibrators are schematically described in the IBM by the analytically solvable dynamical symmetries SU(3), U(5) and O(6) with schematically describes \( \gamma \) soft nuclei[11].

We estimated the limit of \(^{88}\text{Kr}\) isotope in the \( \gamma \)–soft unstable region, from the ratio between \( E2 \), \( E2 \) with Hamiltonian[12]:

\[ H = a_0 \hat{P} \hat{P} + a_1 \hat{L}^2 + a_2 \hat{T}^2 \] (2)

\(^{88}\text{Kr}\) and \(^{92}\text{Kr}\) are in transition region U(5) to O(6), because the ratio equal 2.1 and 2.3 respectively, was compared with Casten Triangle, which its Hamiltonian is:

\[ H = \varepsilon n_d + a_0 \hat{P} \hat{P} + a_1 \hat{L}^2 + a_3 \hat{T}^2 \] (3)

2.1. Electric transition

In order to calculate the transition rates one must specify the transition operators in the simplest form of the IBM the one body transition operator which has the second quantized form is [13]:

\[ T_m^{(L)} = \alpha_L [d^\dagger \times s + s^\dagger \times d]^2_m + \beta_L [d^\dagger \times d]^2_m \] (4)

\( \alpha_L \) and \( \beta_L \) are the coefficients of the various terms in the operator, \( d \) and \( s \) are d-boson and s-boson respectively. This equation yields transition operators for \( E0 \), \( M1 \), \( E2 \), \( M3 \) and \( E4 \) transition with appropriate values of corresponding parameters.

The \( T_m^{E2} \) operator which has enjoyed a widespread application in the analysis of \( \gamma \)-ray transition can thus take the form[14]:

\[ T_m^{E2} = \alpha_2 [d^\dagger \times s + s^\dagger \times d]^2_m + \beta_2 [d^\dagger \times d]^2_m \] (5)

It is clear that for the \( E2 \) multipolarity two parameters \( \alpha_2 \) and \( \beta_2 \) are need in addition to the wave function of initial and final states.

The electric quadrupole reduced transition probabilities B (E2) is defined as:

\[ B(E2), L_i \rightarrow L_f = \frac{\langle L_f |T(E2)|L_i \rangle^2}{2L_i + 1} \] (6)

where \( L_i \) and \( L_f \) are angular moment of the initial and final states respectively. The E2 transition operator \( T(E2) \) is given by:

\[ T(E2) = eQ \] , \( e \) = effective charge.

2.2. Potential Energy

The geometric properties of interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables. It is more convenient to use in the discussion of the geometric properties of the interacting boson model another set of coherent states the projective states. These were introduced by Bore and Mottelson [15], Gnocchio and Kirson [16] and Dieperink, Schollton and Iachello [17].
The essential concepts for nuclear structure defines which the so called potential energy surface PE which represents potential energy of nucleon as a function for given factors $\beta, \gamma$ and boson number $N$ for the relations:

$$PE(N; \beta, \gamma) = E_0 + \frac{N}{(1+\beta^2)}(e_5 + e_4 \beta^2) + \frac{N(N-1)}{(1+\beta^2)^2}(t_1 \beta^4 + t_2 \beta^3 \cos 3\gamma + t_3 \beta^2 + \frac{1}{2} \nu_0)$$

3. Results and discussion
The energy levels for the isotopes were calculated using IBM program written in fortran language and the Hamiltonian as equation (1) with the parameters in table (1):

| Isotopes | $N$ | $\varepsilon$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|----------|-----|---------------|-------|-------|-------|-------|-------|
| $^{88}$Kr | 5   | 1.1611        | 0.16  | 0.0029| 0     | -0.0861| 0     |
| $^{90}$Kr | 6   | 0             | 0.3908| 0.0496| 0     | 0.27   | -0.15 |
| $^{92}$Kr | 7   | 0.7342        | 0.053 | 0.0283| 0     | 0.0142 | 0     |

Mev is mega electron volt which is the unit of energy.
From these parameters we get the energy levels for $^{88}$Kr, $^{90}$Kr and $^{92}$Kr as in figure(1), (2) and (3) respectively, there are good agreement with experimental results[20]. We see that there are many energy levels unsure or unknown or its party also.

**Figure1.** the energy levels of $^{88}$Kr compression with experiment (Exp[20]) results and IBM-1
Figure 2. The energy levels of $^{90}$Kr compression experiment (Exp[20]) results and IBM-1.

Figure 3. The energy levels of $^{92}$Kr compression experiment (Exp[20]) results and IBM-1.

We can notes from the results of energy levels, that $^{88}$Kr locate in U(5) $\rightarrow$ O(6) region, because the ratio of $E_4^+ / E_2^+$ = 2.12 so for $^{92}$Kr equal 2.3 and it’s the same region, this is matching with the experimental values, while $^{90}$Kr have unknown levels except $0^+$ but we depended on the electric transition from ref.[20].

The electric quadrupole reduced transition probabilities from IBMT program has been compared with experiment data as in table (2) for transition from initial level (i) to final one (f), with the values of E2SD and E2DD which getting applying equation (5). These transitions important to get the probability of emission or absorption gamma ray with any energy.

E2SD equal effective charge and it is very important to get the fit electric transition and E2DD = $-0.7 / (5)^{1/2} \alpha_2$ where $\alpha_2$ depended on the limit the region of the isotopes.
Finally, we got the potential energy surface (PE), which is the geometric measurements and confirm the limit of the isotopes in our study, this is clear in figure (4). These results had been calculated using equation (7) in IBMP program and the diagram represent the relation between the deformation $\beta$ and the potential energy as in figure (4-b), which we can see the minus values of energy in $^{88}$Kr and $^{92}$Kr, but the puzzle diagram for $^{90}$Kr because there is breaking in symmetry since the Hamiltonian for O(6) region plus hexadecapole term to correct the bands of this isotopes which the 02 level after 22 level in experimental results. Figure (4-a) is the relation between the deformation, potential energy surface and the angle of symmetry.

Table 2. Electric quadrupole reduced transition probabilities in ($eb)^2$ with $\alpha_2$ and $\beta_2$ in $eb$ unit (all party is positive)

| transition | $^{88}$Kr | $^{90}$Kr | $^{92}$Kr |
|------------|-----------|-----------|-----------|
|            | $\alpha_2$= 0.06 | $\alpha_2$= 0.063 | $\alpha_2$= 0.077 |
|            | $\beta_2$= -0.0188 | $\beta_2$= -0.0198 | $\beta_2$= -0.024 |
| E2SD       | Exp       | IBM-1     | Exp       | IBM-1     | Exp | IBM-1 |
| 2$_1$ → 0$_1$ | 0.018 | 0.02 | 0.024 | 0.024 | 0.042 | 0.0457 |
| 2$_1$ → 0$_2$ | - | 0.00002 | - | 0.0008 | - | 0.0115 |
| 2$_1$ → 2$_2$ | - | 0.0298 | - | 0.031 | - | 0.074 |
| 2$_2$ → 0$_1$ | - | 0.00007 | - | 0 | - | 0.00008 |
| 2$_2$ → 0$_2$ | - | 0.0063 | - | 0 | - | 0.002 |
| 4$_1$ → 2$_1$ | - | 0.0298 | - | 0.031 | - | 0.074 |
| 4$_1$ → 2$_2$ | - | 0.0005 | - | 0 | - | 0.00008 |
| 4$_2$ → 2$_2$ | - | 0.0164 | - | 0.0162 | - | 0.0465 |
| Q$_{21}$ | - | -0.09 | - | 0 | - | -0.1154 |

e is electron charge and b is the unit of area (barn).

4. conclusions
We have been estimated many energy levels for three Krypton isotopes ($^{88}$-92), and getting unknown and unsure levels from the results using the framework of IBM-1 model with a good agreement with the experimental results. Some of these levels are 2$_2$, 3$_1$, 4$_2$, 5$_1$, 6$_2$, 7$_1$ and 8$_2$ all of them with positive party for $^{88}$Kr, while $^{90}$Kr has only 2$_1^+$ and we calculate others levels which agreement with the experimental values[21]. The last isotope $^{92}$Kr has ground band agreement with experimental and calculate 0$_2^+$ for beta band with two levels 2$_2^+$ and 4$_2^+$ were being sure.

The transition probability increases with increase of boson number with small values. The transitions from ground band to beta band increases with increase boson number and for gamma band too. There are phase transition from vibration U(5) to $\gamma$-unstable O(6) for $^{88}$Kr and $^{92}$Kr while $^{90}$Kr has breaking that’s clear from the result of energy levels and from potential energy surface. From figure(4), $^{88}$Kr and $^{92}$Kr, the minimum value of potential energy surface occur in $\beta=\pm 0.2$, which closure from vibrational nuclei ($\beta=0$ in U(5)), but for $^{92}$Kr the minimum value in $\beta=\pm 0.6$, which transition to O(6) ($\beta=\pm 1$ in O(6)).
Figure 4. The relation between β and: (a) the potential energy surface in Mev. (b) the contour plot with the energy surface and the angle γ.

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