Multi-surface phase demodulation based on the selective weighted multi-step algorithms

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Abstract. The transparent parallel plate plays a practical role in the optical system. The accurate measurement of its surface topography is a great importance of quantifying and evaluating the performance of the transparent plate. Based on the phase shifting interferometry, a measurement method using triple iteration and selective phase demodulation algorithms is presented, which can realize multi-surface interferometry at arbitrary positions. Through the design of the three-dimensional data matrix used to store errors, its element pixels can be read out a wealth of key parameter information, thus guiding the design of the algorithm, realizing the unity of the actual use of the algorithm and the measurement scheme, achieving the multi-surface interference measurement with the minimum residual errors with arbitrary cavity length, and expanding the limitations of the traditional weighted multi-step algorithm.

1. Introduction

Laser interferometry technology is widely used in surface measurement for obtaining the accurate morphology of the measured objects. The multi-surface transparent plate plays an important role in the optical system, and the distribution of its surface morphology can impact the optical performance. Therefore, the non-contact measurement for multi-surface plate by modern measurement methods has high application value and research value[1].

In recent years, the new algorithms by using the wavelength tuning phase shifting interferometer (WTPSI) are rapidly developed[2]. However, the interferograms captured by the interferometer are the superposed results of multi-surface signals, including the interference information between each reflection signals and the reference surface, as well as the background light intensity, error, defect and other harmonics, which cannot be directly resolved and measured[2]. To obtain the main surface information (the front surface, rear surface and thickness variation signals) of the tested object, a variety of algorithms can be applied to the processing[3-5], such as: the Fourier transform signal extraction technology, weighted multi-step sampling technology, least square technology and so on. For the Fourier transform signal extraction technology, in the actual calculation process, the calculation cost is high, which is not suitable for the fast multi-surface simultaneous measurement. The least square technique is limited by the characteristics of the least square equation matrix. When the coefficient matrix is near singular or ill conditioned, the difference of iterative results is very large,
and it is greatly affected by the initial value estimation, so it has great limitations in actual measurement.

The multi-step interference solution technology based on the discrete multi-step weighting calculation in the time domain has a great development prospect, but because of its own algorithm characteristics, the traditional multi-step sampling weighting technology can only solve the case of the specific cavity length (numerically equal to the geometric distance between the front surface and the reference surface) and the thickness of the measured plate optical path[6]. To describe clearly, the ratio is defined as the cavity length coefficient. In fact, it means that the traditional multi-step weighted sampling technology can only solve the specific and discrete position of the tested plate. When the position of the tested parts changes, a nearly designed algorithm cannot measure the current situation of the tested parts. At the same time, in the selection process of a variety of algorithms, their advantages and disadvantages are not considered, and it is impossible to achieve the unity of measurement under any cavity length and the actual measurement and algorithm design.

In this paper, an adaptive method is proposed to solve the problem of multi-surface measurement with arbitrary cavity length. Based on the principle of the weighted multi-step algorithm of DFT (discrete Fourier transform), the Hanning window, Hamming window and 2N-1 window functions are considered, and the phase-shift reference coefficients under different cavity lengths are calculated. Through the data matrix construction and addressing methods involved, rich data information can be obtained, and the detection of multi-surface transparent plate at any measurement positions is realized, which breaks through the weighted multi-step algorithm problem limited the fixed cavity length, and extends the application range of the algorithm. The results show that the maximum error of each surface is less than 0.5 nm, which proves that the proposed method can achieve accurate phase demodulation.

2. The principle of the weighted sampling algorithm

When using the WPSI to obtain the interferograms for the multi-surface plate, the interference signals of each surface will contribute signals to the results, due to the optical path differences of signals is different, the tuning frequencies of the interferometric signals are different[7]. Therefore, the superimposed intensity captured from CCD (Charge Coupled Device) is shown as follows.

\[
I(x, y) = I_0(x, y) + \sum_{m=1}^{\infty} r_m(x, y) \cos[\varphi_m(x, y) + \Delta \varphi_m(x, y)]
\]  

(1)

where \(I(x, y)\) is the intensity at \((x, y)\); \(I_0(x, y)\) is the background; \(r_m(x, y)\), \(\varphi_m(x, y)\), \(\Delta \varphi_m(x, y)\) are the contrast ratio, initial phase and phase shifting value of the \(m^{th}\) harmonic. Theoretically, any harmonic signal can be expressed as the superposition form of multiple trigonometric functions. According to the difference of optical path difference between the signals, it is the basis of signal separation and harmonic extraction. Through frequency estimation and discrete Fourier fitting of different frequencies, the initial phases of each signal can be obtained [8]. The optical path difference is specifically described as: the optical path difference between each surface of the tested plate and the rear surface of the reference plate (the optical wedge is adopted), and the optical path difference of self-interference between the front and rear surfaces of the tested plate, which has a certain mathematical relationship. Therefore, the frequencies of the target signals can be estimated through this laser interference characteristic. Consequently, the phase shifting values can be presented by the corresponding phase shifting frequency \(\nu_m(x, y)\) and the time variable \(t\).

\[
\Delta \varphi_m(x, y) = 2\pi \nu_m(x, y)t
\]  

(2)

By analyzing the optical path difference distribution of each signal, the numerical relationship of phase shifting frequency between different signals can be obtained, as shown in Figure 1.
In Figure 1. $v_f$, $v_r$, $v_{f-r}$, $v_{r-r}$ represents the tuning frequencies of the front surface, rear surface, thickness variation and secondary reflection signals respectively. And $h$ represents the length of the air gap between the reference surface and the front surface of the measured plate, which is equal to the interference cavity length. $T$ is the average thickness of the measured plate. In order to meet the actual measurement requirements, the front surface, rear surface and thickness variation signals of the tested plate are selected to reconstruct the wavefronts of the target surfaces[9].

Based on the DFT principle, the harmonics with different tuning frequencies can be extracted by the sampling weight algorithm in the time domain, which can be shown in equation (3)-(4).

$$\sum_{k=1}^{Z} a_m(k) I(k, x, y) \approx \gamma_m \cos[\varphi_m(x, y)]$$

$$\sum_{k=1}^{Z} b_m(k) I(k, x, y) \approx \gamma_m \sin[\varphi_m(x, y)]$$

$a_m$ and $b_m$ are the sampling weights of the harmonics. By these two terms, the sine term and cosine term of each harmonic signal containing the initial phase information can be obtained. After the arc tangent and the unwrapping calculation, the initial phase distribution of each signal can be known.

3. The algorithm design under arbitrary cavity length coefficient

Based on the principle of DFT and the harmonics distribution, a weighted multi-step sampling algorithm using characteristic polynomials for window function design and harmonic extraction can be developed[10]. The representation of characteristic polynomials are as follows.

$$P_s(x) = \sum_{k=1}^{Z} P_s(x) = \sum_{k=1}^{Z} \left[1 + x^2 + x^3 + \ldots + x^{N-1}\right]^{P_M} = \sum_{k=1}^{2N-1} w_{2N-1} x^{k-1}$$

The values of $2N-1$ sampling window is given by the coefficient distribution of the variance $x$ of characteristic polynomial $P_s(x)$, $x = \exp(j 2\pi n / N)$, $j$ is the imaginary unit, $N$ is the phase shifting coefficient, and $n$ is the octave number. $P_M$ is the highest order of the polynomial and $P_M=2$ is chosen, which can determine the required acquisition frame number to $2N-1$ and phase shifting interval to $2\pi / N$.

The value of the $2N-1$ sampling window can be obtained by the above conditions. When the optical path from the front surface of the tested plate to the reference surface is greater than the optical path of the thickness variation, the latter one is taken as the fundamental frequency, otherwise the former should be chosen. When the thickness change signal is used as the fundamental frequency, the octaves corresponding to the front surface signal, rear surface signal and thickness change signal are given by the cavity length coefficients to $M$, $M+1$ and 1 respectively (at this time, $M = h / n(T)$. When the front surface signal of the tested piece is taken as the fundamental frequency, the octave number corresponding to the front surface signal, rear surface signal and thickness change signal is cavity
length coefficient $1, M + 1$ and $m$ respectively. Carry in the following equations for weighting calculation to demodulate the phases[11].

$$a_i = w_k \cos \frac{2\pi}{N}(k - N)$$

$$b_i = w_k \sin \frac{2\pi}{N}(k - N)$$

In Eq. (6), $k$ is the frame sequence number of the interferograms; $Z$ is the minimum number of required acquisition frames of interferogram, and $Z = 2N - 1$ for the Hanning window and the $2N - 1$ characteristic polynomial window, $Z = 2N$ for the Hamming window. $i = f, r, f-r$ represent the front surface, rear surface and thickness variation signals, respectively [12-14].

$$h_f(x, y) = \frac{\varphi_f(x, y)}{4\pi}$$

$$h_r(x, y) = \frac{\varphi_r(x, y)}{4\pi}$$

$$T_w(x, y) = \frac{\varphi_{f-r}(x, y)}{4\pi}$$

The reconstruction of the front surface, rear surface and thickness variation of the tested plate can be solved by the above equations, corresponding to $h_f(x, y), h_r(x, y), T_w(x, y)$ respectively. $\lambda_0$ is the starting wavelength, $n_i$ is the refractive index of the measured plate.

Under the current condition (the phase shifting coefficient, the cavity length coefficient and the step selection algorithm), the iterative total step distance of cavity length coefficient $M$ is $0-N$ (the current value), and the step distance is 0.1. The total iterative step of $N$ is 8-12, and the single step is 1. According to the characteristics of the algorithm, the calculation cost of the algorithm can be reduced by repeat the periodic error distribution. If the maximum residual error (front surface, rear surface and thickness variation) between the reconstructed surfaces and the real surfaces is less than 15nm, it is judged that the current condition is available, and the error of this step is recorded for treatment.

The error of each step is calculated and recorded. The single maximum error in the available measurement conditions of the previous iteration should be stored in the data matrix, the structural characteristics of the matrix are designed to contain the cavity length information, and the ordinal and coordinate values for the phase shifting coefficient information, so the designed method can be used to obtain multiple information at one step. As mentioned, the design of the matrix shall contain rich information for the convenience of calculation. The design method is as follows. The Hanning window and the Hamming window are presented as equation (10)-(11).

$$w_{\text{hanning}} = \frac{2}{N} \cos(\pi \frac{k - N}{2N}) \quad k = 1, 2, \ldots, 2N - 1$$

$$w_{\text{hammimg}} = \frac{2}{N}[0.08 + 0.92 \cos(\pi \frac{k - N}{2N})] \quad k = 1, 2, 3 \ldots, 2N$$

The data matrix is a three-dimensional matrix, the first dimension and the second dimension are row number and column number respectively, and the third dimension is the ordinal number of the algorithms. Through the way of data addressing, the ordinal number ① of the third dimension can be set as the dimension of the Hanning window, and the ordinal number ② can be set as the dimension of the Hamming window, the ordinal number ③ is set as the dimension of the 2N-1 sampling window; The cavity length coefficient is distributed in the first row of all data, that is, the column ordinal number can interpret the current cavity length coefficient information. The line ordinal number can interpret the phase shifting coefficient, and the error addressing condition of iteration is that when the algorithm is available under the current measurement condition, the phase shifting parameter $N$ is
taken as the line ordinal number of error placement, that is, the error is addressed in the corresponding pair of $N$ lines under the current measurement conditions.

In the process of data location and extraction, the judgment method adopted can be presented as follows.

The first judgment condition is the minimum errors of surfaces; The second judgment condition is the minimum number of frames that must be collected when the first judgment condition is met, and the positioning information is taken as the addressing information by this step error of three algorithms (i.e. the maximum errors of each surface that meets the settings under the above different measurement conditions). According to the above interpretation method, the current optimal algorithm type and phase shifting coefficient $N$ are obtained. Take the front surface as an instance, the maximum error distribution of $M=0~6$ and $N=8~10$ for the $2N$-$1$ algorithm is shown in Figure 2.

![Figure 2](image)

**Figure 2.** Residual error distribution of $M=0~6$ and $N=8~10$ for the $2N$-$1$ algorithm.

After getting the optimal algorithm type and the optimal $N$, guide the formulation of the measurement scheme (the phase shifting value and the minimum number of frames that must be captured), and calculate the phases through the chosen algorithm, obtain the initial phases of the target signals, complete the signals separation and phase unwrapping operation, so as to get the required profile distribution, and finally achieve the measurement of multi-surface measurement.

As the preferred technical scheme, in the process of formulating iteration and value determination methods, the Zernike polynomials are used to simulate the interferograms during the multi-surface numerical analysis, and the superimposed interference signals are used as the simulation objects, so as to conduct the error analysis and calculation. Besides, the maximum errors of each surface obtained by three algorithms are stored in the third dimension of the data matrix, the value of the unusable element is 0, otherwise, the value is the current maximum error. In the data matrix constructed by the Hanning window, the cavity length coefficient $M = 1.9 ~ 2.7$ times is shown in Figure 3.
Based on the above analysis, the application process of the proposed method can be summarized as follows: first, locate the current $M$ according to the measurement condition, and then compare the data of three columns of $M$ to address the minimum errors, then take this value as the minimum available element. As mentioned, the column ordinal number of the minimum available element is the cavity length coefficient $M$ (the initial iteration value plus the current cumulative step distance can be obtained), the row ordinal number is the current available minimum $N$, and the third dimension order value is the current available optimal algorithm type. Note that only the iteration result of the current phase shifting coefficient needs to be obtained according to the characteristics of the sampling algorithm, that is, the result can be copied and, which means the results distribution under the next period can be obtained.

4. Experiment
To verify the feasibility of the proposed method, a Littman external cavity (TLB-6804, New Focus) is used as the light source. The spectral range within which the wavelength can be selected is between 632.67nm to 632.97nm. The minimum fine-tuning resolution is 0.1 GHz, corresponding to $\Delta \lambda = 5 \times 10^{-5}$ nm for $\lambda_0 = 632.8$ nm. The stability of the light source wavelength is less than 10-7nm during the recording time of 2 min.

In the experimental setup, the expanded and collimated laser beam is then directly projected onto both surfaces of the measured plate through a beam splitter and the reference surface. Consequently, the multi-surface interferometric signals are formed. It could be reflected through the same beam splitter and finally captured by the CCD camera with a resolution of $1024 \times 1024$ pixels.
It can be observed from the phase distribution results that the algorithm can perform better in the actual measurement process. For the phase distribution of surfaces, there is no accompanying harmonic noise, which further verifies the effectiveness of the proposed algorithm.

5. Conclusion
In this paper, we present a method for multi-surface interferometric measurement, which can be used in any measured position. By this method, the phase distribution of each surface can be obtained simultaneously, which lays a foundation for the reconstruction of wavefronts. Through the design of the Hanning window, the Hamming window, the 2N-1 window's weighted multi-step sampling function based on characteristic polynomials, and the iteration of the corresponding phase shifting reference coefficient under different cavity length coefficients, the acceptable error is stored by the constructed data matrix. When using the data addressing method, we can get rich algorithm information, so as to realize the measurement under any cavity length and the selection of the optimal algorithm and parameters. The experimental results show that this method can overcome the problem of any measured positions of the actual captured interferograms, widen the application scope of the traditional weighted multi-step algorithm, break through the cavity length limit condition, and adjust the algorithm through the setting of allowable error. Therefore, the proposed method realizes the setting of the optimal measurement scheme, unifies the design of the algorithm and the actual measurement scheme.

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References
[1] Hanayama R, Hibino K, Warisawa S and Mitsuishi M 2004 Optical Review 11 337–43.
[2] Dong B, Pan B, Zhang Y and Bai Y 2018 Polymer Testing 68 233–7.
[3] Sun T, Zheng W, Yu Y, Yan K, Asundi A and Valukh S 2019 Applied Sciences (Switzerland) 9.
[4] He Z, Zheng J, Garden R and De Groot P 2007 AD ’07 - Proceedings of Asia Display 2007 1 959–63.
[5] Kim Y, Hibino K, Sugita N and Mitsuishi M 2014 Optics Express 22 18203.
[6] Surrel Y 1993 Applied Optics 32 3598.
[7] Yu J 2010 Chinese Optics 3 605–15.
[8] Kim Y, Sugita N and Mitsuishi M 2017 Precision Engineering 48 67–74.
[9] Chen D, Wang C, Valyukh S, Wu X and Yu Y 2020 Measurement: Journal of the International Measurement Confederation 157 107626.
[10] Bai Y, Zhou Y, He Z, Ye S, Dong B and Xie S 2018 Optics and Laser Technology 98 229–33.
[11] Vikram Karwal 2009 Aspectos Generales De La Planificacion Tributaria En Venezuela 2009 31–47.
[12] Yingjie Y, Benhao Z and Yunfang J 2003 Optics and Precision Engineering 11 560–6.
[13] Yingjie Y and Liuxing S 2003 Journal of Astronautic Metrology and Measurement 23 1–8.
[14] Kim E-H, Hahn J, Kim H and Lee B 2009 Optics Express 17 7818.