Standard Model-like D-brane models and gauge couplings

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Abstract

We systematically search intersecting D-brane models, which just realize the Standard Model chiral matter contents and gauge symmetry. We construct new classes of non-supersymmetric Standard Model-like models. We also study gauge coupling constants of these models. The tree level gauge coupling is a function of compactification moduli, string scale, string coupling and winding number of D-branes. By tuning them, we examine whether the models can explain the experimental values of gauge couplings. As a result, we find that the string scale should be greater than $10^{14-15}$ GeV if the compactification scale and the string scale are the same order.
1 Introduction

The Standard Model (SM) is one of the greatest achievements of particle physics. It is consistent with all of the experimental results by tuning about 19 free parameters and succeeded in predicting new physics. The discovery of the Higgs scalar [1, 2] is the latest one. However, many questions still remain in particle physics. What is quantum gravity theory? How does the mysterious flavor structure of the SM appear? What is the origin of neutrino masses, inflation, dark matter and so on?

From the viewpoint of quantum gravity, superstring theory is the most promising candidate of its theory and almost only known candidate of it. Furthermore, superstring theory is also a unified theory of other interactions and matter fields. Superstring theory naturally has gauge symmetry. There appear gravitons, gauge bosons, matter fermions, and scalars in its massless spectrum. Thus, it is important to construct stringy origin theory explaining the SM.

The intersecting D-brane models are one of the interesting techniques to realize four-dimensional (4D) chiral gauge theory as low-energy effective theory from superstring theory [3, 4, 5, 6, 7] (see for review [8, 9] and references therein). In these models, chiral matter fermions are realized as the R-sector of open string stretching between D-branes at angles, while gauge bosons are realized as open strings on the same set of D-branes. It is amazing that simple compactification models realize the SM spectrum or supersymmetric SM spectrum as zero modes. For example, in [7], the intersecting D-brane model with just the SM spectrum was constructed which we call the IMR model in this paper. Similarly, supersymmetric SM-like models have been constructed (see e.g. [10, 11, 12]).

In addition to the massless spectrum, it is quite important to explain quantitative structure of the SM, i.e. the gauge couplings, Yukawa couplings and the Higgs potential parameters as well as maybe neutrino Majorana masses. In this paper, we focus on the gauge couplings. In 4D low-energy effective field theory derived from heterotic string theory, the gauge couplings at tree level are unified up to Kac-Moody levels $\kappa_a$ at the string scale [13], which is of $\mathcal{O}(10^{17})$ GeV [14]. This prediction is very strong. In order to explain the experimental values, we may need some corrections, e.g. stringy threshold corrections [15, 16, 17]. (See for numerical studies e.g. Refs. [18, 19].)

On the other hand, the gauge coupling is a function of D-brane volume in D-brane models. In intersecting D-brane models, gauge groups of the SM are originated from different D-branes, which have volumes independent of each other. Thus, at first sight, it seems always possible to explain the three gauge couplings of the SM by tuning volume moduli, because the number of parameters, moduli, is sufficiently larger than three. However, in an explicit model, the values of volume moduli are constrained by other conditions. For example, tachyonic modes may appear for some values of moduli in non-supersymmetric models. Also, the string coupling $g_s$ may be required to be strong for some values of moduli to derive the realistic values of the SM gauge couplings. However, our theory is reliable at the weak string coupling. Then, it is non-trivial to explain the

\[ 1 \text{In Ref. [20], a specific relation among the three gauge couplings is shown in a certain class of supersymmetric models.} \]
three SM gauge couplings under the above conditions.

In this paper, we study systematically the model construction of intersecting D-brane models. We construct new classes of non-supersymmetric SM-like models, which have the same gauge symmetry and chiral matter contents as those of the SM but no exotics except right-handed neutrinos. We show three classes of the SM-like models. We study their gauge couplings as well as those of the IMR model under the above constraints.

This paper is organized as follows. In section 2, we briefly review the intersecting D-brane models. In section 3, we construct new classes of SM-like models. We calculate gauge couplings in section 4. Section 5 is our conclusion. In Appendix A, we show systematic search of the SM-like models.

2 Intersecting D-brane model building

In this section, we briefly review the toroidal orientifold models with intersecting D6-branes. We first consider Type IIA superstring theory compactified on factorized six dimensional tori \( T^6 = T_1^2 \times T_2^2 \times T_3^2 \) including intersecting D6-branes, where \( T_i^2 \) is the \( i \)-th two-dimensional torus. It is the two-dimensional Euclidean space modded by a lattice,

\[
T_i^2 = \mathbb{C}/L(\tau_i),
\]

\[
L(\tau_i) = \{ z_k \in \mathbb{C} | z_k = m_i + n \tau_i, \quad n, m \in \mathbb{Z} \},
\]

(2.1)

where \( \tau_i \in \mathbb{C} \).

D6\(_a\)-branes wrap 3-cycles \( [\Pi_a] \) on \( T^6 \). Here, we restrict ourselves to the D-brane system in which all D6-brane’s 3-cycles \( [\Pi_a] \) are factorized, \( [\Pi_a] = [\Pi_1^a] \times [\Pi_2^a] \times [\Pi_3^a] \), where \( [\Pi_i^a] \) is a 1-cycle of \( T_i^2 \). Then we can specify the 3-cycles by using 6 integer winding numbers \( (n_i^a, m_i^a) \). \( n_i^a \) is the winding number along the \( \tau_i \) direction and \( m_i^a \) is the winding number along the imaginary axis of \( z_i \). The intersecting number between the D6\(_a\)-brane and D6\(_b\)-brane is denoted by \( I_{ab} \) which is determined by winding numbers,

\[
I_{ab} = [\Pi_a] \circ [\Pi_b] = \prod_{i=1}^3 (n_i^a m_i^b - n_i^b m_i^a) .
\]

(2.2)

The open string stretching between D6\(_a\)-brane and D6\(_b\)-brane has the following boundary conditions,

\[
\text{Re} \frac{\partial}{\partial \sigma} e^{-i \theta_a^i z_i} |_{\sigma=0} = 0, \quad \text{Im} \frac{d}{dt} e^{-i \theta_a^i z_i} |_{\sigma=0} = 0 ,
\]

(2.3)

\[
\text{Re} \frac{\partial}{\partial \sigma} e^{-i \theta_b^i z_i} |_{\sigma=\pi} = 0, \quad \text{Im} \frac{d}{dt} e^{-i \theta_b^i z_i} |_{\sigma=\pi} = 0,
\]

(2.4)

where

\[
\theta_a^i = \tan^{-1} \left( \frac{m_i^a + n_i^a \text{Im} \tau_i}{n_i^a \text{Re} \tau_i} \right),
\]

(2.5)

is the angle of D6\(_a\)-brane in the \( i \)-th torus. These boundary conditions resolve degeneracies of the ground states in the R-sector. The resultant ground state corresponds to a 4D
massless chiral fermion. Scalars appear in the NS sector. The ground state in the NS-sector depends on the intersecting angle $\theta_{ab}^0 = (\theta_{ba}^1 - \theta_{ba}^0)/\pi$. Assuming $1 > \theta_{ab}^0 > 0$, the masses squared of four candidates for the lightest state are shown in Table 1. They would be massive, massless or tachyonic depending on the angles. If there are any massless states, a part of supersymmetry is recovered. For example, when $\theta_{ba}^1 + \theta_{ba}^2 - \theta_{ba}^3 = 0$, the first state in Table 1 is the massless ground state and the others are massive.

| State | Mass $^2$ |
|-------|-----------|
| 1     | $\frac{1}{\alpha} (\theta_{ba}^1 + \theta_{ba}^2 - \theta_{ba}^3)$ |
| 2     | $\frac{1}{\alpha} (\theta_{ba}^1 - \theta_{ba}^2 + \theta_{ba}^3)$ |
| 3     | $\frac{1}{\alpha} (-\theta_{ba}^1 + \theta_{ba}^2 + \theta_{ba}^3)$ |
| 4     | $\frac{1}{\alpha} (1 - \frac{1}{2} (\theta_{ba}^1 + \theta_{ba}^2 + \theta_{ba}^3))$ |

Table 1: The masses squared of the light scalar states.

In this way, each intersecting point has a 4D massless chiral fermion as well as scalars. Also, a stack of $N_a$ D6$_a$-branes has gauge symmetry $U(N_a)$. The open strings ending at the D6$_a$-brane have Chan-Paton charges, which correspond to the fundamental representation of $U(N_a)$. This class of models lead to 4D chiral $U(N_a)$ Yang-Mills theory as low energy effective theory. This is the attractive fact to derive the SM at low energy.

Now, we introduce the orientifold. Torus orientifold is obtained by modding $T^6$ by reflection operator $\mathcal{R}$,

$$\mathcal{R} : \text{Im}z_{1,2,3} \rightarrow -\text{Im}z_{1,2,3}. \quad (2.6)$$

To define this operator $\mathcal{R}$ well, $\text{Im}\tau_i$ in $L(\tau_i)$ must be either 0 or 1/2. The torus is rigid for $\text{Im}\tau_i = 0$, while the torus is tilted for $\text{Im}\tau_i = 1/2$. It is useful to define new "winding number" $(\tilde{n}_a^i, \tilde{m}_a^i)$, where $\tilde{n}_a^i = n_b^i$ and $\tilde{m}_a^i = m_b^i + \text{Im}\tau_i n_a^i$. Hereafter, we use this $(\tilde{n}_a^i, \tilde{m}_a^i)$ as a winding number of D6$_a$-brane in the $i$-th torus.

In this setup, we can construct perturbative vacua which have several stacks of $N_a$ D6$_a$-branes wrapping whole 4D Minkowski spacetime and factorized 3-cycles $[\Pi_a]$ of $T^6$. In addition to D6$_a$-branes, we need their orientifold mirror D6$_{a*}$-branes such that the system is $\mathcal{R}$-invariant. D6$_{a*}$-brane’s winding numbers must be $(\tilde{n}_a^i, -\tilde{m}_a^i)$.

In the presence of orientifold, the gauge symmetry $G_a$ which a stack of $N_a$ D-branes has depends on whether D$_a$-branes lie on top of its orientifold mirror D$_{a*}$-branes or not. If D6$_a$-branes are apart from D6$_{a*}$-brane, the gauge group is $U(N_a)$. Otherwise the gauge group is $Sp(2N_a)$ or $SO(2N_a)$. The intersecting points between D6$_a$-branes and D6$_{a*}$-branes have a massless 4D chiral fermion transforming as bifundamental representations under $G_a \times G_b$. For example, if $G_{a,b} = U(N_a,N_b)$, it transforms as $(N_a, \overline{N}_b)$ under $U(N_a) \times U(N_b)$.

The number of intersecting points $I_{ab}$ is written by

$$I_{ab} = \Pi_{i=1}^3 (\tilde{n}_a^i \tilde{m}_b^i - \tilde{m}_a^i \tilde{n}_b^i). \quad (2.7)$$

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$^2$ We need the orientifold projection in order to obtain just the SM massless spectrum even if we do not consider supersymmetric models [9].
$I_{ab}$ is the number of chiral fermions having same representation appearing in 4D low-energy effective theory, which means that $I_{ab}$ is the generation number. Using this D-brane system, we can realize a lot of patterns of chiral (super) Yang-Mills theory as effective theory, but not all theories.

Next, let us discuss the constraints on intersecting D-brane models. D-branes have RR charges which must be canceled in compact space. This constraint is derived from D-brane kinematics, and it is the same as Gauss’s law of electromagnetism in compact space. This is called the RR tadpole cancellation condition. Since the RR charge is proportional to the D-brane homology, the constraint is written by

$$\sum_{a=1,\ldots,N} N_a[\Pi_a] - 4[\Pi_{O6}] = 0,$$

where $[\Pi_{O6}]$ is a cycle of O6-planes.

In general, the gauge symmetry includes several $U(1)$ factors. Some of them become massive by the generalized Green-Schwartz mechanism. That is, $U(1)$ gauge bosons have non zero couplings with RR-forms, especially $C_5$ and have non-perturbative St"uckelberg mass. The coupling between $U(1)_a$ gauge boson and $C_5$ is obtained by the Chern-Simons term,

$$S_{CS} = \sum_a N_a \int_{D_{6a}} C_5 \wedge \text{tr} F_a + \cdots.$$  \hspace{1cm} (2.9)

We introduce $[\alpha_k]$ as the basis of 3-cycles and its dual basis $[\beta_l]$, where $[\alpha_k] \circ [\beta_l] = \delta_{kl}$. We define

$$B^k_2 = \int_{[\alpha_k]} C_5.$$  \hspace{1cm} (2.10)

Then the coupling between $U(1)$ gauge bosons and $B^k_2$ can be written by

$$S_{4D-CS} = N_a Q_{ak} \int_{M^4} B^k_2 \text{tr} F_a + \cdots,$$  \hspace{1cm} (2.11)

where $Q_{ak} = [\Pi_a] \circ [\beta_k]$. This coupling induces masses of $U(1)$ gauge bosons. The $U(1)$ gauge boson corresponding to $U(1)_X = \sum_a c_a U(1)_a$ is massless if and only if $\sum_a c_a N_a[\Pi_a] \circ [\beta_k] - \sum_{a^*} c_a N_{a^*}[\Pi_{a^*}] \circ [\beta_k] = 0$ for any $k$. Otherwise, the $U(1)$ gauge boson becomes massive even if it is anomaly-free.

In the next section, we will construct intersecting D-brane models which have the same gauge group as that of the SM. We will show that we can get the exact SM gauge group by using above mechanism to make extra gauge bosons massive.

### 3 SM-like model

Our aim is to construct perturbative vacua which have the SM-like effective theory by using type IIA orientifold. For such a purpose, we systematically search vacua satisfying the following conditions:
- Gauge symmetry is the same as that of the SM up to the hidden sector, $SU(3) \times SU(2) \times U(1)_Y \times G_{\text{hidden}}$.

- The chiral massless spectrum is the same as that of the SM with three right-handed neutrinos up to the hidden sector.

For the RR tadpole cancellation, we need right-handed neutrinos and the $G_{\text{hidden}}$ sector. The matter fields in the hidden sector are singlets under the SM gauge.

There are two methods to realize the $SU(2)$ gauge symmetry. One is to use two D6$_a$-branes separating from their orientifold mirror D6$_{a*}$-branes. The theory in D6$_a$-brane worldvolume is $U(2)$ Yang-Mills theory which contains $SU(2)$ group as subgroup. We call this class of models $SU(2)$ models. In this scenario, we must use tilted tori to cancel $U(2)$ anomaly. There are many models using $SU(2)$ method, see for the model satisfying the above condition, e.g. [7]. The other is to use one D6$_a$-brane whose orientifold mirror D6$_{a*}$-brane is coincident with the D6$_a$-brane. In this case, the gauge group can be enhanced from $U(1)$ to $Sp(2)$. $Sp(2)$ is homomorphic to $SU(2)$ as Lie algebra. Then, we can get the $SU(2)$ gauge symmetry. We call this class of models $Sp(2)$ models.

We concentrate on the latter models in the following:

- We construct $Sp(2)$ models where $SU(2)$ gauge symmetry is realized by one brane and its orientifold mirror.

We can satisfy these conditions by using four stacks of branes, D6$_{a,b,c,d}$-branes. The multiplicity of D6$_a$-branes $N_a$ is equal to three, and the others are one. D6$_b$-brane is parallel (or perpendicular) to O6-planes and gauge symmetry is enhanced to $Sp(2)$. The intersection numbers of these branes are required as follows,

\[
I_{ab} = 3; \quad I_{ac} = -3; \quad I_{ac^*} = -3; \quad I_{ad} = 0; \quad I_{ad^*} = 0,
\]
\[
I_{bc} = 0; \quad I_{db} = 3; \quad I_{dc} = -3; \quad I_{dc^*} = -3,
\]
\[
I_{aa^*} = 0; \quad I_{cc^*} = 0; \quad I_{dd^*} = 0,
\]

such that the chiral spectrum of this model realizes the SM matter contents. For desired zero mode, we require D6$_{a,c,d}$-branes to be parallel to O-plane in at least one torus, too. The hypercharge $U(1)_Y$ corresponds to the following linear combination of $U(1)$s,

\[
U(1)_Y = \frac{1}{6} U(1)_a - \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d.
\]

There are some arbitrariness of the definition of $U(1)_Y$, but we can absorb it by renaming branes. In Table 2, we summarize the chiral spectrum of this model, quantum numbers of non-Abelian and Abelian gauge symmetries, and their names in the SM.

We carry out a systematic analysis on all the possible D-brane configurations, see Appendix A for the details. As a result, it is found that general solutions realizing Eq. (3.1) are classified into two classes of models.
Table 2: Chiral matter contents. All the SM chiral fields appear in intersecting points as zero modes of the open string R sector.

| Intersection name | SU(3) × SU(2) | Qa | Qc | Qd | Hypercharge |
|-------------------|---------------|----|----|----|-------------|
| (ab)              | Q_L           | 3(3,2) | 1  | 0  | 0           | \(\frac{1}{6}\) |
| (ac)              | U_R           | 3(3,1) | -1 | 1  | 0           | \(-\frac{2}{3}\) |
| (ac*)             | D_R           | 3(3,1) | -1 | -1 | 0           | \(-\frac{1}{3}\) |
| (db)              | L             | 3(1,2) | 0  | 0  | 1           | \(-\frac{1}{2}\) |
| (dc)              | N_R           | 3(1,1) | 0  | -1 | 1           | 0            |
| (dc*)             | E_R           | 3(1,1) | 0  | -1 | -1          | 1            |

Table 3: General solutions of the Sp(2) models. \(\beta_{2,3} = 1 - \text{Im}\tau_{2,3} \in \{1, 1/2\}\) and \text{Im}\(\tau_1\) is always zero. \(\rho \in \{1, 3\}\). The \(n, m\)s are integer parameters and satisfy \(n_a^1 m_a^2, n_d^1 m_d^2\) are divisors of \(\rho\) and \(m_b^1 n_b^2, m_c^1 n_c^2\) are divisors of \(3/\rho\). \(a, d, m_a^3\) are arbitrary integers and \(n_{a,d}^2 = \frac{a_d}{m_d^2} - 1\).

Both of them have the desired chiral spectrum. However, one of them can not make extra \(U(1)\) gauge boson massive through the Green-Schwartz mechanism with remaining \(U(1)\) gauge boson massless (see Appendix A). This extra \(U(1)\) corresponds to \(U(1)_{B-L}\). That is, both \(U(1)_Y\) and \(U(1)_{B-L}\) gauge bosons are massless or massive at the same time in that class of models. The other can make \(U(1)_{B-L}\) gauge boson massive with remaining \(U(1)_Y\) gauge boson massless. Thus, this class of models can reproduce the SM chiral spectrum and gauge symmetry. It is shown in Tables 3. There are no other solutions satisfying the conditions. Note that gauginos and adjoint scalars appear in the gauge sector of our models which would be massive by loop correction [7].

For later calculation, we classify further the class of models to three new classes as Table 4, Table 5, and Table 6. We refer to the class of models in Table 4 as 0til-SM, because they have no tilted torus. Also we refer the class of models in Table 5 and Table 6 as 1til-SM and 2til-SM, respectively. As we show in Table 3, we can not construct the SM-like models using three tilted tori since they always lead to even number of generations.

The Higgs bosons correspond to the open string in the NS sector stretching between D6\(_b\)-branes and D6\(_c\)-brane. These branes are parallel in \(T_2^1\) and \(T_2^2\). This situation is the same as that in the IMR model [7]. The Higgs mass is determined by the distance of D6-branes and the intersecting angle. Note that we need fine tuning to get light Higgs
(0, \tilde{S}M) models. All of the tori \( T_2^a \) are rigid. The integer parameters satisfy \( \rho \in \{1, 3\} \), \( n^a_1, m^a_1, n^a_2 \in \mathbb{Z} \), \( m^a_1, n^a_2, n^b_1 \) are divisors of \( \rho \). \( m^b_1, n^b_2, m^c_1, n^c_2 \) are divisors of \( 3/\rho \).

| D-brane | \( T_1^2 \) | \( T_2^2 \) | \( T_3^2 \) |
|---------|------------|------------|------------|
| a       | \( (n_a^1, 0) \) | \( (n_a^2, m_a^3) \) | \( (-\rho m_a^2, m_a^3) \) |
| b       | \( (0, m_b^1) \) | \( (n_b^2, 0) \) | \( (0, \frac{3}{\rho} m_b^1 n_b^2) \) |
| c       | \( (-\frac{2}{\rho} m_c^1 n_a^1 m_a^3, n^2_a + \frac{3 m_a^2}{m_d^2} n^2_d, m_c^1) \) | \( (n_c^2, 0) \) | \( (0, -\frac{3}{\rho} m_c^1 n_c^2) \) |
| d       | \( (n_d^1, 0) \) | \( (n_d^2, m_d^2) \) | \( (-\rho n_d^2 m_d^3, -3 n_d^2 m_d^3 m_a^3) \) |

Table 4: \( \tilde{S} \)M models. All of the tori \( T_2^a \) are rigid. The integer parameters satisfy \( \rho \in \{1, 3\} \), \( n^a_1, m^a_1, n^a_2 \in \mathbb{Z} \), \( m^a_1, n^a_2, n^b_1 \) are divisors of \( \rho \). \( m^b_1, n^b_2, m^c_1, n^c_2 \) are divisors of \( 3/\rho \).

| D-brane | \( T_1^2 \) | \( T_2^2 \) | \( T_3^2 \) |
|---------|------------|------------|------------|
| a       | \( (n_a^1, 0) \) | \( (a/\beta + 1, \beta m_a^2) \) | \( (-\rho n_a^2 m_a^2, m_a^3) \) |
| b       | \( (0, m_b^1) \) | \( (n_b^2/\beta, 0) \) | \( (0, \frac{3}{\rho} m_b^1 n_b^2) \) |
| c       | \( (n_c^1, m_c^1) \) | \( (n_c^2/\beta, 0) \) | \( (0, m_c^2) \) |
| d       | \( (n_d^1, 0) \) | \( (d/\beta + 1, \beta m_a^2) \) | \( (-\rho n_d^2 m_d^3, 3 n_d^2 m_d^3 m_a^3) \) |

Table 5: \( \tilde{S} \)M models, \( \beta \in \{1, 1/2\} \). If \( \beta = 1/2 \), \( T_2^a \) is tilted torus and the others are rigid. If \( \beta = 1/2 \), \( T_2^a \) is tilted torus and the others are rigid. The integer parameters satisfy \( \rho \in \{1, 3\} \), \( n^a_1, m^a_1, n^a_2 \) are divisors of \( \rho \) and \( m^b_1, n^b_2, m^c_1, n^c_2 \) are divisors of \( 3/\rho \). \( a, d, n^c_1 \) are arbitrary integer numbers.

mass.

The D-brane configurations in Tables 4, 5, and 6 do not satisfy the RR tadpole condition yet, but it is always possible by adding extra D6-branes which are parallel to O6-planes. Since D6\(_{a,b,c,d}\) branes and its orientifold mirrors have no intersecting point with O6-plane, there are no intersecting points between extra D-branes and D6\(_{a,b,c,d}\) branes. Thus, the introduction of these extra D6-branes does not change the chiral spectrum in the visible sector. In this sense, extra D6-branes correspond to the completely hidden sector.

These models have characteristic winding numbers. D6\(_{a}\)-brane and D6\(_{c}\)-brane are parallel to the O6-plane in \( T_2^a \) and \( T_2^c \). D6\(_{b}\)-brane and D6\(_{d}\)-brane are parallel to the O6-plane in \( T_2^b \). The charge of \( U(1)_a \) is 3 times baryon number and the \( U(1)_d \) is the lepton number. The intersecting numbers of D6\(_{a,c}\)-brane and D6\(_{b,d}\)-brane in \( T_2^a \) and \( T_2^d \) are same. Thus, the flavor structure of the quarks and leptons are exactly the same at perturbative level. (See for discrete flavor symmetries [21, 22].) However, if we take non-perturbative effects into account, these structure must be broken and, for example, right-handed Majorana neutrino masses might be generated [23, 24, 27]. At any rate, the study on the flavor sector is beyond our scope, and we would study elsewhere.

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3 Similarly flavor symmetries are obtained in heterotic orbifold models [24]. See also [21].
| D-brane | $T^2_1$ | $T^2_2$ | $T^2_3$ |
|---------|---------|---------|---------|
| a       | $(n^1, 0)$ | $(n^2, m^2_2/2)$ | $(-\rho n^1, m^3_3/m^3_a)$ |
| b       | $(0, m^1_b)$ | $(2n^2_b, 0)$ | $(0, -\rho n^1)$ |
| c       | $(\frac{2}{3} n^1_c m^1_n^3_c, \frac{3}{2} n^2_c m^2_d)$ | $(2n^2_c, 0)$ | $(0, -\frac{3}{\rho} m^1_c n^2_b)$ |
| d       | $(n^1_d, 0)$ | $(n^2_d, m^2_2/2)$ | $(-\rho n^1_d, -3n^3_d/m^3_a)$ |

Table 6: 2til-SM models. $T^2_{2,3}$ are tilted tori and the other is rigid. The integer parameters satisfy $\rho \in \{1, 3\}$, $n^1_a m^2_a, n^1_d m^2_d$ are divisors of $\rho$ and $m^1_b n^2_b, m^1_c n^2_c$ are divisors of $3/\rho$. $n^2_a, n^2_d$ are arbitrary odd number.

4 Gauge couplings

4.1 Constraints on model

We have found three classes of SM-like models in section 3. In these models, the gauge symmetry is the exactly same as that of the SM up to the hidden sector. Now, let us study the gauge sector quantitatively. That is, we study the question, whether it is possible to make all gauge couplings consistent with their experimental values. At first sight, it looks possible because there are a lot of parameters in these classes of models. For example, all classes of models have more than 3 integer winding numbers as free parameters and all models have torus moduli as free parameters\textsuperscript{4}. However, it is not simple when we take into account other constraints. One constraint is to avoid the tachyonic configuration and the other is a constraint of the string coupling.

The R-sector of the open string stretching between $D6_k$-brane and $D6_\ell$-brane has chiral fermionic zero-mode, while the corresponding NS-sector has the light scalar spectrum of Table 1. These NS-sector modes are superpartners of chiral fermions and some of them could be tachyonic in non-supersymmetric models. If a configuration has tachyons, it is unstable and decays to other configuration quickly. We must tune parameters to avoid such tachyons. This condition constrains the parameters significantly. In $Sp(2)$ models, there are six chiral fermion modes and each of them has superpartners at intersecting points. To make these scalars massive or massless, the models must satisfy 24 inequalities.

The other constraint is the perturbativity of theory. The tree level gauge coupling $\alpha_k = g_k^2/4\pi$ at the string scale is given by \begin{equation} \frac{1}{\alpha_k} = \frac{M^3 V_k}{(2\pi)^3 g_s \kappa_k}, \end{equation}

\textsuperscript{4}Precisely speaking, we need to consider the stabilization of the moduli. However, this issue is beyond scope of this paper and we treat the moduli as free parameters.
where $V_k$ denotes the $D6_k$-brane’s 3-cycle volume in the compact space, $M_s$ is the string scale and $g_s$ is the string coupling. $\kappa_k$ is obtained as $\kappa_k = 1$ for $U(N_k)$ and $\kappa_k = 2$ for $Sp(2N_k)/SO(2N_k)$. In this way, we can calculate all the gauge couplings, $\alpha_{a,b,c,d}$. For $U(1)_Y$, we must normalize the gauge field and $\alpha_Y$ is written by,

$$\frac{1}{\alpha_Y} = \frac{1}{6} \alpha_a + \frac{1}{2} \alpha_c + \frac{1}{2} \alpha_d.$$  \hfill (4.2)

On the other hands, by performing dimensional reduction of the type IIA supergravity action, one can write the Planck mass $M_p$ by string parameters as,

$$M_p^2 = \frac{8M^8sV_6}{(2\pi)^6g_s^2},$$ \hfill (4.3)

where $V_6$ is the volume of the compact space. From (4.1), (4.3), we can write the string coupling in terms of gauge couplings,

$$g_s = \frac{\alpha_4^4}{8^{3/2}(2\pi)^3\kappa_k^4} \left(\frac{V_k^2}{V_6}\right)^2 \frac{V_1}{M_p} M_p^3.$$ \hfill (4.4)

We have concentrated on perturbative vacua and its effective theory, but when $g_s > \mathcal{O}(1)$, perturbative theory is broken down and our models no longer make sense. To get sufficiently small $g_s$, there are constraints on parameters.

It is natural to assume $V_6 \sim 1/M_s^6$. The $\alpha_k$ in Eq. (4.4) is gauge coupling at the string scale, then we evaluate

$$g_s \sim 2 \times 10^{-4} \frac{\alpha_k(M_s)^4}{\kappa_k^4} \left(\frac{V_k^2}{V_6}\right)^2 \frac{V_aV_b}{V_6} \frac{M_p}{M_s}.$$ \hfill (4.5)

Naively, if $M_s$ is very small, $g_s$ is very large and perturbativity of the theory is violated.

Using the renormalization group equations and the experimental values of $\alpha_k(M_Z)$, we can evaluate $\alpha_k(M_s)$ in Eq. (4.5). The models obtained in the previous section have almost the same field contents as those of the SM, but include gauginos and adjoint scalars in the gauge sector. We assume that such gauginos and adjoint scalars gain masses around $M_s$ and neglect their threshold corrections. Hence, we can evaluate $\alpha_k(M_s)$ by using beta-functions of the SM. We find $\alpha_{3,2}(M_s) > 1/50$ for $M_s \leq 10^{18}$ GeV. Then, $V_{a,b}/(V_6)^{1/2}$ must be small to get sufficiently small $g_s$. This means that the direction which is perpendicular to the a,b-brane is large and $V_{a,b}/V_6$ is suppressed. However, in our models, we have $I_{ab} \neq 0$ and there is no direction which is perpendicular to a-brane and b-brane at the same time. Hence, generally we get $V_aV_b/V_6 > 1$. When $V_aV_b/V_6 > 1$ and $\alpha_3, \alpha_2 > 1/50$, we obtain

$$g_s \sim 2 \times 10^{-4} \alpha_3(M_s)^2 \alpha_2(M_s)^2 \left(\frac{V_aV_b}{V_6}\right)^2 \frac{M_p}{M_s}.$$ \hfill (4.6)
That is, it is required $M_s \gtrsim 10^{15}$GeV. When there is a large hierarchy between $V_6$ and $1/M_s^6$, this estimation would change. For $V_6M_s^6 = \gamma$, we have the constraint $M_s \gtrsim \gamma^{1/6}10^{15}$GeV. For example, we find $M_s \gtrsim 10^{16}$GeV for $\gamma = \mathcal{O}(10^6)$ and $M_s \gtrsim 10^{14}$GeV for $\gamma = \mathcal{O}(10^{-6})$. We comment on the effect of the gauginos and adjoint scalars on above argument. We have assumed that all of the gauginos and adjoint scalars have masses around $M_s$. If they are lighter, $\alpha_3$ and $\alpha_2$ become larger because they give positive contributions to beta-functions. Therefore, the lighter gauginos and adjoint scalars strengthen the constraint.

As mentioned above, the string scale is constrained. On the other hand, winding numbers and moduli are also constrained. As a concrete example, we study the 0til-SM models. In this class of models, the ratio of tree level gauge couplings are given by,

$$\frac{1}{\alpha_3} : \frac{1}{\alpha_2} = n_1^aRe\tau_1^a(\frac{\rho}{n_1^a m_a}Re\tau_2 + (m_a^2)\frac{2}{\rho}Re\tau_2)^2 + (m_a^3)^2 : 3Re\tau_2/\rho, \quad (4.7)$$

$$= n_1^aRe\tau_1^a(\frac{\rho}{n_1^a m_a}Re\tau_3)^2 + (m_a^3)^2 : 3/\rho, \quad (4.8)$$

where $\tau_i$ is the $T^2$ torus modulus. The renormalization group flows from the experimental values show that $\alpha_2(\mu)$ is similar to $\alpha_3(\mu)$ unless the running scale $\mu$ is very low. To realize $\alpha_2(M_s) \sim \alpha_3(M_s)$, it is required that $|n_1^a \tau_1|$ is less than $\mathcal{O}(1)$. In this way, the winding numbers and the value of the moduli are constrained.

### 4.2 Numerical analysis

We plot the gauge coupling ratios of our models in Figures 1, 2 and 3 for $M_s = 10^{16}$, $M_s = 10^{15}$ and $10^{14}$ GeV, respectively. For comparison, we also show the gauge coupling ratios of the IMR model in these figures. The blue dots correspond to the gauge coupling ratios, which are calculated by Eqs. (4.1) and (4.2) for the parameters to satisfy $g_s < 1$ assuming $V_6 = 1/M_s^6$ and to avoid tachyonic modes. We vary winding numbers from 1 to 100 and torus moduli from $10^{-2}$ to $10^2$. We neglect stringy threshold corrections \[29\].

Note that the ratios $\alpha_k/\alpha_l$ given by Eqs. (4.1) and (4.2) are independent of $M_s$. Thus, if we do not impose other constraints, the same blue dots would appear for $M_s = 10^{14}$, $10^{15}$ and $10^{16}$ GeV. However, the constraint $g_s < 1$ depends on $M_s$. The constraint becomes severe for the lower $M_s$. That is, the difference of these figures only comes from the perturbativity condition. Obviously, it is more constrained in Figures 2 and 3 and the number of the blue dots are less than in Figure 1. The red dots correspond to the $\overline{\text{MS}}$ renormalized gauge coupling ratios of the SM computed by using the experimental values, i.e. $\alpha_3(\mu)/\alpha_Y(\mu)$ and $\alpha_2(\mu)/\alpha_Y(\mu)$. From top to bottom, the dots represent $\mu = 10^3$, $10^4$, $\cdots 10^{19}$ GeV. The model can fit the gauge couplings if the blue dots overlap with the red dot corresponding

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to \( \mu = M_s, \mu = 10^{16} \text{ GeV} \) in Figure 1, \( \mu = 10^{15} \text{ GeV} \) in Figure 2 and \( \mu = 10^{14} \text{ GeV} \) in Figure 3.

![Figure 1: Distributions of the ratio of gauge couplings.](image)

Figure 1: Distributions of the ratio of gauge couplings. The blue dots are the gauge couplings ratios of \( Sp(2) \) models and the model in [7] and the red dots are renormalized gauge couplings of the SM. Right upper red dot is the renormalized gauge coupling at \( 10^{19} \text{ GeV} \) and lower left dot is at \( 10^{16} \text{ GeV} \). Winding numbers are from 1 to 100 and torus moduli are from \( 10^{-2} \) to \( 10^{2} \). We set \( M_s \) to \( 10^{16} \text{GeV} \) and non-perturbative configurations are eliminated. Tachyon configurations are eliminated too.

There are some characteristic features in these figures. In all models, the ratio of gauge couplings \( \alpha_3/\alpha_Y \) is less than 6. This is because \( U(1)_Y \) is linear combination of \( U(1)_{a,c,d,s} \) and \( \alpha_Y \) is function of \( \alpha_3 \). It makes upper bound on \( \alpha_3/\alpha_Y \). \( Sp(2) \) models tend to have larger \( \alpha_2 \) than \( U(2) \) model. It is because the b-brane must be parallel or perpendicular to the O6-plane in \( Sp(2) \) and its volume can not be so large. The \( Sp(2) \) models have larger allowed region than the IMR model. This is because the \( Sp(2) \) models have more parameters than the IMR model.
The IMR model

Figure 2: Distributions of the ratio of gauge couplings. Winding numbers and torus moduli are not changed from Figure 1. In this figure, we set $M_s$ to $10^{15}$ GeV.

Figure 1 shows that we can tune parameters to fit gauge couplings in all models to the experimental values if $M_s$ is greater than $10^{16}$ GeV. For $M_s = 10^{15}$ GeV, we can realize the gauge couplings in $Sp(2)$ models. For the IMR model, there are no blue dots overlapping red dots, but we would find good parameters explaining the experimental values by more dense parameter search. For $M_s = 10^{14}$ GeV, we can explain experimental values in 2til-SM models and it would be possible in the other $Sp(2)$ models. We checked that blue dots disappear in this region for $M_s = 10^{13}$ GeV and we can not tune parameters to fit the gauge couplings for weak $g_s$ in all of these models. The critical string scale is $10^{14-15}$ GeV. These results are consistent with Eq. (4.6).

We have analyzed assuming $V_6M_s^6 = 1$. Similarly, we can analyze gauge couplings for other values of $V_6M_s^6 = \gamma$. Unless there is a large hierarchy between them, we obtain almost the same results. Furthermore, even when $\gamma$ is very small or large, we would have the lower bound on $M_s$. In some case, the one-loop threshold corrections would be
The IMR model

Figure 3: Distributions of the ratio of gauge couplings. Winding numbers and torus moduli are not changed from Figure 1. In this figure, we set $M_s$ to $10^{14}$ GeV.

4.3 Explicit example

In this subsection, we give an explicit example of the models. As shown in Figure 1 there are a lot of winding numbers and moduli which realize the renormalized SM gauge couplings at the string scale. Table 7 shows one example.

In this model, the string scale is set to be $10^{18}$ GeV and the ratios of the gauge couplings in the model are given as,

$$\alpha_3/\alpha_Y = 1.2,$$
$$\alpha_2/\alpha_Y = 1.2.$$  \hspace{1cm} (4.9)

From the experimental values, the ratios of renormalized gauge couplings at $10^{18}$ GeV
| D-brane | $T_1^2 (1/\text{Re}\tau_1 = 10^{2/3})$ | $T_2^2 (1/\text{Re}\tau_2 = 10^{14/9})$ | $T_3^2 (1/\text{Re}\tau_3 = 10^{2/3})$ |
|--------|----------------|----------------|----------------|
| a      | (1,0)         | (3,1/2)        | (−3,1/2)       |
| b      | (0,1)         | (2,0)          | (0,1)          |
| c      | (−4,1)        | (2,0)          | (0,−1)         |
| d      | (1,0)         | (13,3/2)       | (−1,−1/2)      |

Table 7: The explicit example of winding numbers and moduli realizing the SM gauge coupling ratio in 2til-SM model.

To get the realistic gauge couplings, the string coupling should be $5 \times 10^{-3}$, which means that the theory is weak coupling.

5 Conclusion and discussion

We have studied SM-like intersecting D-brane models. We have constructed and classified the simplest class of models using $Sp(2)$ which realizes the SM gauge symmetry and chiral spectrum including three right-handed neutrinos as open string zero modes. These models are very simple and attractive. They have only four stacks of D-branes. The three generations of leptons and quarks are just realized by intersecting numbers of D-branes, and each generation is originated from the same type of the intersecting points. This is different from the IMR model, where one generation of quark doublet is originated from the intersecting point between $D6_a$-brane and $D6_b$-brane, while the other two generations are originated from the intersecting point between $D6_a$-brane and $D6_b^\ast$-brane. Thus, our models have very large flavor symmetry. Its proper breaking might be helpful to realize the flavor structure in nature.

We have studied gauge coupling constants of our models. At first sight, it seems always possible to fit the gauge couplings to the experimental values in most of models, because there are lots of free parameters. However, it is non-trivial to reproduce the SM gauge couplings because the two conditions, the absence of the tachyon and perturbativity, put the strong constraints on model parameters. Our calculation has shown that the string scale must be greater than $10^{14−15}\text{GeV}$ to get realistic gauge couplings when there is no large hierarchy between $V_6$ and $M_s$. Low energy string is disfavored in these models. This tendency may not be model-dependent. One reason is that $\alpha_Y$ must depend on $\alpha_3$ and $\alpha_3/\alpha_Y$ has some limits in intersecting D-brane models. When we try to reconstruct the SM, the values of gauge coupling constants are similar values.

In order to fit the gauge couplings to the experimental values, we have used moduli parameters as free parameters. However, moduli should be stabilized and their stabilized
values are important to realize the gauge couplings. All of our models have the hidden sector. Some dynamics in the hidden sector would play a role in moduli stabilization. Also the hidden sector may include dark matter. These topics are quite interesting, but beyond our scope. We would study elsewhere.

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A Systematic analysis on D-brane configurations

We study systematically all the possible D-brane configurations of four stacks of $D6_{a,b,c,d}$ leading to the gauge group $SU(3) \times Sp(2) \times U(1)_Y \times G_{\text{hidden}}$ and the following intersecting numbers:

\begin{align}
I_{ab} &= n; \quad I_{ac} = -n; \quad I_{ac^*} = -n; \quad I_{ad} = 0; \quad I_{ad^*} = 0, \\
I_{bc} &= 0; \quad I_{db} = n; \quad I_{dc} = -n; \quad I_{dc^*} = -n, \\
I_{aa^*} &= 0; \quad I_{cc^*} = 0; \quad I_{dd^*} = 0,
\end{align}

where $n$ is the generation number and obviously we are interested in $n = 3$, especially.

Since $I_{aa^*} = 0$ and $D6_a$-branes are parallel with O-plane in one brane to avoid extra zero mode, we can write,

\[(n^1_a, m^1_a) = (n^1_a, 0),\]

without loss of generality. Because $I_{ab} = n$ and $D6_b$-brane is parallel or perpendicular to O-plane, all the possible $D6_b$ configurations are classified as follows,

1. $(n^1_b, m^1_b) = (0, m^1_b), (n^2_b, m^2_b) = (n^2_b, 0), (n^3_b, m^3_b) = (n^3_b, 0)$,
2. $(n^1_b, m^1_b) = (0, m^1_b), (n^2_b, m^2_b) = (0, m^2_b), (n^3_b, m^3_b) = (0, m^3_b)$,
3. $(n^1_b, m^1_b) = (0, m^1_b), (n^2_b, m^2_b) = (n^2_b, 0), (n^3_b, m^3_b) = (0, m^3_b)$,

where $n^i_a$s and $m^i_a$s are integers. Since $I_{ab}$ is proportional to $n^1_a \cdot m^1_b$ and $|n^1_a|$ is even with $\text{Im} \tau_1 = 1/2$, we get $\text{Im} \tau_1 = 0$ to obtain odd generation. Thus, we can not construct three tilted torus models.

Let us study the case (1). Since $I_{dd^*} = 0, I_{db} = n$ and $I_{cc^*} = 0, I_{ac} = -n$, we find that $(n^1_d, m^1_d) = (n^1_d, 0)$ and $(n^2_c, m^2_c) = (n^2_c, 0)$. Then, we have

\[
I_{ac} = n^1_a m^1_c \cdot (-m^2_a n^2_c) \cdot (n^3_a m^3_c - m^3_a n^3_c) = -n,
\]

(A.2)
\[ I_{ac} = -n_a^1m_c^1 \cdot (-m_a^2n_2^2) \cdot (-n_a^3m_3^3 - m_a^3n_3^3) = -n, \] (A.3)

which reduce to \(-n_a^1m_c^1 \cdot m_a^2n_2^2 \cdot n_a^3m_3^3 = -n \) and \(m_a^3n_3^3 = 0\). On the other hand, the RR tadpole condition requires

\[ \sum_{x \in a,b,c,d} N_xm_x^1n_x^2m_x^3 = m_c^1 \cdot n_c^2 \cdot m_c^3 = 0. \] (A.4)

That leads to \( n = 0 \), and we can not obtain non-trivial solutions. Similarly, we can show that the case (2) does not lead to non-trivial solutions.

Next, let us discuss the case (3). In this case, all the possible \( D6_{c,d} \)-brane configurations are classified as follows,

(3a) \( (n_c^2, m_c^2) = (n_c^2, 0), (n_d^1, m_d^1) = (n_d^1, 0) \),

(3b) \( (n_c^2, m_c^2) = (n_c^2, 0), (n_d^3, m_d^3) = (n_d^3, 0) \),

(3c) \( (n_c^3, m_c^3) = (n_c^3, 0), (n_d^1, m_d^1) = (n_d^1, 0) \).

In the case (3a), the condition on intersecting numbers \( (A,1) \) and the tadpole conditions require

\[-n_a^1m_c^1 \cdot m_a^2n_2^2 \cdot n_a^3m_3^3 = n, \quad n_a^1n_a^1n_a^3 = n_a^1m_a^3d_a^3, \quad m_a^3n_3^3 = 0, \]
\[ m_a^3n_3^3 = 0, \quad m_a^1n_a^2m_2^3 + m_a^1n_a^2m_3^2 = 0, \quad 3n_a^1m_a^2m_3^3 + n_d^3m_d^3m_3^3 = 0. \] (A.5)

These results are shown in Table 8. For \( n = 3 \), this result leads to the models in Table 3

| D brane | \( T_1^2 \) | \( T_2^2 \) | \( T_3^2 \) |
|---------|---------|---------|---------|
| a       | \( (n_a^1, 0) \) | \( (n_a^2, m_a^2) \) | \( (-\rho/n_a^3m_a^3, m_a^3) \) |
| b       | \( (0, m_b^1) \) | \( (n_b^2, 0) \) | \( (0, n/\rho m_b^1m_b^2) \) |
| c       | \( (n_c^1, m_c^1) \) | \( (n_c^2, 0) \) | \( (n_c^3, -n/\rho m_c^1m_c^2) \) |
| d       | \( (n_d^1, 0) \) | \( (n_d^2, m_d^2) \) | \( (-\rho/n_d^3m_d^3, m_d^3) \) |

Table 8: The SM-like models with \( n \) generations. \( n_k^1, m_k^1, \rho \) are integer parameters satisfying \( m_a^3n_3^3 = 0, m_a^3n_3^3 = 0 \) and \( 3n_a^1m_a^2m_3^3 + n_d^3m_d^3m_3^3 = 0 \).

Similarly, we can discuss the other cases. As a result, we find that the case (3b) is allowed only for \( n = \text{even} \), and the case (3c) does not have non-trivial solutions.

As a result, only the case (3a) has non-trivial solutions with \( n = 3 \), and they are the models with the SM chiral matter fields as shown in Table 3 for \( n = 3 \). However, at this stage the gauge symmetry of our models is \( SU(3) \times SU(2) \times U(1)_a \times U(1)_c \times U(1)_d \). The hypercharge \( U(1)_Y \) corresponds to the linear combination, \( U(1)_1 = \frac{1}{3}U(1)_a - \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d \). We require the other two extra \( U(1) \) gauge bosons become massive by couplings with \( B_2 \). Here, we examine it. As the basis \([\alpha_k]\), we set

\[ [\alpha_1] = (1, 0) \times (0, 1) \times (0, 1), \]
\[ [\alpha_2] = (0, 1) \times (1, 0) \times (0, 1), \]
\[ [\alpha_3] = (0, 1) \times (0, 1) \times (1, 0). \] (A.6)
Each of $B^k_2$ has the coupling with $U(1)s$ as

\[
B^1_2 \wedge m^1_c n^2_c n^3_c F_c, \\
-\rho B^2_2 \wedge (3F_a + F_d), \\
B^3_2 \wedge (3n^1_a n^2_a m^3_a F_a - \frac{n^1_c n}{\rho m^1_c} F_c + n^1_d n^2_d m^3_d F_d).
\]

(A.7)

The condition to remain the $U(1)_Y$ gauge boson massless is given by

\[
n^3_c = 0, \quad 3n^1_a n^2_a m^3_a - \frac{n^1_c n}{\rho m^1_c} + n^1_d n^2_d m^3_d = 0.
\]

(A.8)

Satisfying this condition, $U(1)_Y$ gauge boson is massless. If $n^1_c$ is not zero, the extra gauge bosons become massive.

References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235 [hep-ex]].

[3] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480, 265 (1996) [hep-th/9606139].

[4] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP 0010, 006 (2000) [hep-th/0007024].

[5] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. 42, 3103 (2001) [hep-th/0011073]; JHEP 0102, 047 (2001) [hep-ph/0011132].

[6] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 489, 223 (2000) [hep-th/0007090].

[7] L. E. Ibanez, F. Marchesano and R. Rabadan, JHEP 0111, 002 (2001) [hep-th/0105155].

[8] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, Phys. Rept. 445, 1 (2007) [hep-th/0610327].

[9] L. E. Ibanez and A. M. Uranga, “String theory and particle physics: An introduction to string phenomenology,” Cambridge University Press (2012).

[10] M. Cvetic, G. Shiu and A. M. Uranga, Nucl. Phys. B 615, 3 (2001) [hep-th/0107166].
[11] G. Honecker and T. Ott, Phys. Rev. D 70, 126010 (2004) [Erratum-ibid. D 71, 069902 (2005)] [hep-th/0404055].

[12] D. Cremades, L. E. Ibanez and F. Marchesano, hep-ph/0212048.

[13] P. H. Ginsparg, Phys. Lett. B 197, 139 (1987).

[14] V. S. Kaplunovsky, Nucl. Phys. B 307, 145 (1988) [Erratum-ibid. B 382, 436 (1992)] [hep-th/9205068].

[15] L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B 355, 649 (1991).

[16] I. Antoniadis, K. S. Narain and T. R. Taylor, Phys. Lett. B 267, 37 (1991).

[17] J. P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 372, 145 (1992).

[18] L. E. Ibanez, D. Lust and G. G. Ross, Phys. Lett. B 272, 251 (1991) [hep-th/9109053]; L. E. Ibanez and D. Lust, Nucl. Phys. B 382, 305 (1992) [hep-th/9202046].

[19] H. Kawabe, T. Kobayashi and N. Ohtsubo, Nucl. Phys. B 434, 210 (1995) [hep-ph/9405420]; T. Kobayashi, Int. J. Mod. Phys. A 10, 1393 (1995) [hep-ph/9406238]; R. Altendorfer and T. Kobayashi, Int. J. Mod. Phys. A 11, 903 (1996) [hep-ph/9503388].

[20] R. Blumenhagen, D. Lust and S. Stieberger, JHEP 0307, 036 (2003) [hep-th/0305146].

[21] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B 820 (2009) 317 [arXiv:0904.2631 [hep-ph]]; H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, Phys. Rev. D 80 (2009) 126006 [arXiv:0907.5274 [hep-th]]; Phys. Rev. D 81 (2010) 126003 [arXiv:1001.1788 [hep-th]]; H. Abe, T. Kobayashi, H. Ohki, K. Sumita and Y. Tatsuta, JHEP 1406, 017 (2014) [arXiv:1404.0137 [hep-th]].

[22] M. Berasaluce-Gonzalez, P. G. Camara, F. Marchesano, D. Regalado and A. M. Uranga, JHEP 1209, 059 (2012) [arXiv:1206.2383 [hep-th]]; F. Marchesano, D. Regalado and L. Vazquez-Mercado, JHEP 1309, 028 (2013) [arXiv:1306.1284 [hep-th]].

[23] T. Kobayashi, S. Raby and R. J. Zhang, Nucl. Phys. B 704, 3 (2005) [arXiv:hep-ph/0409098]; T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768, 135 (2007) [arXiv:hep-ph/0611020]; P. Ko, T. Kobayashi, J. h. Park and S. Raby, Phys. Rev. D 76, 035005 (2007) [Erratum-ibid. D 76, 059901 (2007)] [arXiv:0704.2807 [hep-ph]]; F. Beye, T. Kobayashi and S. Kuwakino, arXiv:1406.4660 [hep-th].

[24] T. Higaki, N. Kitazawa, T. Kobayashi and K. -j. Takahashi, Phys. Rev. D 72, 086003 (2005) [hep-th/0504019].
[25] R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B 771, 113 (2007) [hep-th/0609191].

[26] L. E. Ibanez and A. M. Uranga, JHEP 0703, 052 (2007) [hep-th/0609213]; L. E. Ibanez, A. N. Schellekens and A. M. Uranga, JHEP 0706, 011 (2007) [arXiv:0704.1079 [hep-th]]; S. Antusch, L. E. Ibanez and T. Macri, JHEP 0709, 087 (2007) [arXiv:0706.2132 [hep-ph]].

[27] Y. Hamada, T. Kobayashi and S. Uemura, JHEP 1405, 116 (2014) [arXiv:1402.2052 [hep-th]].

[28] I. R. Klebanov and E. Witten, Nucl. Phys. B 664, 3 (2003) [hep-th/0304079].

[29] D. Lust and S. Stieberger, Fortsch. Phys. 55, 427 (2007) [hep-th/0302221].