Friction in Gravitational waves: a test for early-time modified gravity

Valeria Pettorino and Luca Amendola

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg

Modified gravity theories predict in general a non standard equation for the propagation of gravitational waves. Here we discuss the impact of modified friction and speed of tensor modes on cosmic microwave polarization B modes. We show that the non standard friction term, parametrized by \( \alpha_M \), is degenerate with the tensor-to-scalar ratio \( r \), so that small values of \( r \) can be compensated by negative constant values of \( \alpha_M \). We quantify this degeneracy and its dependence on the epoch at which \( \alpha_M \) is different from the standard, zero, value and on the speed of gravitational waves \( c_T \). In the particular case of scalar-tensor theories, \( \alpha_M \) is constant and strongly constrained by background and scalar perturbations, \( 0 \leq \alpha_M < 0.01 \) and the degeneracy with \( r \) is removed. In more general cases however such tight bounds are weakened and the B modes can provide useful constraints on early-time modified gravity.

In Modified Gravity models, the equation for the tensor metric perturbations (gravitational waves) is affected in several ways. First, the speed \( c_T \) of the gravitational waves can be different from the speed of light [1, 2]. A second effect is instead related to a modification of the friction term in the tensor equation that depends on the evolution rate of the effective Planck mass or equivalently on the effective universal gravitational interaction of the cosmological model [3]. In more general modifications of gravity, for instance in bimetric models [4], two coupled tensor equations are present, corresponding to the two metrics of the theory and additional changes are possible [5]. Considering only the first two possible modifications, gravitational wave speed and friction, the linear generalized tensor equation for the amplitude \( \dot{h} \) in vacuum can be written in a Friedmann-Robertson-Walker (FRW) metric as [3, 6]:

\[
\dot{h} + (3 + \alpha_M) \frac{k^2}{a^2} \dot{h} = 0 ,
\]

(1)

where the dot represents derivative with respect to cosmic time, \( \alpha_M, c_T \) are time-dependent functions that vary with the specific model, \( k \) is the wavenumber, \( a \) is the scale factor and \( H \) the Hubble function. In the standard case one has \( \alpha_M = 0 \) and the speed of gravitational waves \( c_T \) equals the speed of light, \( c_T = 1 \). General models belonging to the so-called Horndeski Lagrangian [7] produce both effects, i.e. \( \alpha_M \neq 0 \) and \( c_T \neq 1 \). When neutrinos are present, a source term in [1] is also included, although it is typically negligible.

Any modification of the tensor wave equation can potentially lead to observable effects on the Cosmic Microwave Background (CMB), on both the temperature and the polarization spectra. The recent measurement of the B-modes at multipoles around \( \ell = 100 \) reported by the BICEP2 experiment [8], as well as follow up analysis on foreground contributions such as dust emission [9,11], has motivated a great interest in the information contained in the polarization B-mode signal, especially for as concerns the inflationary dynamics. As it is well known, in absence of vector sources, the primordial B-mode spectrum is generated exclusively by tensor waves and affects small to intermediate multipoles. Larger multipoles \( \ell \gtrsim 100 \) are mainly affected by CMB-lensing. In [1,2] it has been shown that the gravitational wave speed at the epoch of decoupling or before affects the position of the inflationary and of the reionization peak in the polarization B-modes so that a measurement of B-modes at \( \ell \approx 100 \) can be employed to set limits on the early-time speed of gravitational waves.

In many cases, the effects of these changes on the CMB can be safely neglected if one assumes that gravity deviates from the Einsteinian form only recently, as in several models proposed to explain the recent epoch of cosmic acceleration by non-standard gravity. In general, however, modification of gravity can occur at any time in the past; in some models, e.g. Brans - Dicke theories or some bimetric models [12], gravity is modified at all times. In this Letter we wish to study the impact and limits that the current observations of B-modes can set on the two modified gravity tensor parameters, \( \alpha_M \) and \( c_T \). Although they are both in general time-dependent quantities, we assume here for simplicity that they are constant or that they deviate from the standard case only at early time, i.e. before some epoch \( z_d \).

We have modified the tensor equation in CAMB [20] and combined it within CosmoMC [13] to include the \( \alpha_M, c_T \) parameters. We first consider the case in which \( c_T = 1 \) but \( \alpha_M \) is arbitrary and constant. All the other parameters are as in standard \( \Lambda \)CDM. In Fig.1 we show the effect of \( \alpha_M \) on the BB spectrum of the CMB both on the tensor modes only and on the total spectrum. As expected, a positive \( \alpha_M \) increases the friction term and therefore reduces the wave amplitude, while a negative \( \alpha_M \) has the opposite effect.

From Fig.1 we can expect a degeneracy between the tensor-to-scalar ratio \( r \) and \( \alpha_M \), as they both regulate the amplitude of the primordial peak. Comparing only with the BICEP2 data and fixing the optical depth to the Planck best fit value (\( \tau = 0.09 \) for Planck + WMAP polarization [14]), we obtain the allowed region for \((\alpha_M, r)\),
shown in Fig. (2), which clearly shows the degeneracy. Values of $r$ close to zero can be reconciled with the BICEP2 data if $\alpha_M$ is close to $-2$. This is the central result of this paper. In the same figure we also compare the results obtained when including all nine band powers of BICEP2 with the case in which only the first five are included. As expected from Fig. (1), a negative value of $\alpha_M$ also increases tensor modes at large multipoles, therefore smaller values of $\alpha_M$ are favoured if also the last four (higher multipole) band powers of BICEP2 are included.

Before drawing any conclusion from Fig. (2), one should consider however that the term $\alpha_M$, as already mentioned, is proportional to the time derivative of $G_{\text{eff}}$. It enters, therefore, into the scalar perturbation equations and contributes to the Sachs-Wolfe, Integrated Sachs-Wolfe and lensing signal, affecting both the temperature and the polarization spectra. In principle, one should therefore consider all the spectra at the same time, and also the background evolution which in general will be different from $\Lambda$CDM. However to a large extent the low-$\ell$ B modes are independent of all the other signals (T and E spectra, and high-$\ell$ B modes) since they depend uniquely on the tensor modes; on the other hand, tensor modes affect only marginally the other CMB spectra. Therefore, to a first approximation, we can just use the already available constraints on $\alpha_M$ in some specific model to see which fraction of the parameters space of Fig. (2) is allowed.

Let us consider for instance one of the simplest cases of modified gravity, the scalar-tensor theory. Perturbation equations for scalar-tensor theories with a scalar field $\phi$ have been calculated for example in $[15, 16]$ including tensor equations for the metric. The gravitational wave equation turns out to be

$$\ddot{h} + \left(3H + \frac{\dot{\phi}}{\phi}\right) \dot{h} + \frac{k^2}{a^2} h = 0$$

(2)

so that in scalar-tensor theories we can readily identify

$$\alpha_M = \frac{\dot{\phi}}{\phi H} = \frac{d \log \phi}{d \log a}$$

(3)

In the simplest form of scalar-tensor model, the original Brans-Dicke model, the evolution is controlled by the single observable parameter $\omega$. As long as the matter density is dominated by a component with constant equation of state $w_m$, the background expansion has a simple analytical solution $[17]$, $\phi \sim t^{2 - 6w_m}, \quad a \sim t^{2 + 2\omega - 2\omega w_m}$

(4)

In this case, we can relate the $\alpha_M$ parameter to $\omega$, such that:

$$\alpha_M = \frac{2 - 6w_m}{2 + 2\omega - 2\omega w_m}.$$  

(5)

During the matter dominated era, this becomes simply $\alpha_M = 1/(1 + \omega)$. Assuming a constraint $\omega > 100$ (see e.g. $[18]$), we get $0 \leq \alpha_M < 0.01$. Taking this constraint at face value, we should conclude that the effect of $\alpha_M$ on the tensor modes is practically negligible. However, this is only true for the particular case in which $\alpha_M = \text{const}$ at all times. If $\alpha_M$ varies in time (as expected in general) and in particular if $\alpha_M$ is very small after decoupling, the scalar effects can become arbitrarily weak since they are mostly due to post-decoupling physics (except of course
for the inflationary initial conditions, that we are assuming to be independent of the gravity modifications we are considering here). On the other hand, B modes depend on the evolution of gravitational waves before decoupling. To give an example, if we have an extreme case in which $\alpha_M$ suddenly decreases to zero just after decoupling then the B modes would be practically the same as if $\alpha_M$ were constant at all times (see Fig. [3]), while the scalar perturbations would be the same as in $\Lambda$CDM. In this case, $\alpha_M$ has effect on B modes up to decoupling, while it has no impact on secondary anisotropies such as integrated Sachs-Wolfe and CMB lensing. As a consequence, the constraints we obtain on $\alpha_M$ from B modes are also valid for all those models in which the modified gravity effects are due to an $\alpha_M$ that is a non zero constant only until the epoch of decoupling. Needless to say, assuming $\alpha_M$ to be exactly zero immediately after the decoupling serves merely as an illustrative example and should not be taken as a realistic model.

![Diagram](image)

**FIG. 3:** Posterior likelihood for $\alpha_M$ and $r_{0.05}$. Blue (top) contours are obtained using all nine bandpowers from BICEP2 and modifying $\alpha_M$ in the full $z$ range (same as previous figures). Gray contours correspond to the case in which $\alpha_M$ is only modified up to decoupling. We fix $c_T = 1$.

A larger friction term has two competing effects: on one side, it delays the horizon reenter and the subsequent damping, therefore momentarily enhancing the tensor amplitude; on the other, it increases the damping itself, quenching the “acoustic” oscillations more than in the standard case. This implies that if the epoch at which $\alpha_M$ goes to zero moves from the decoupling to an earlier epoch, e.g. $z_d = 2000$, the BB spectra change in a non trivial way. High multipoles, e.g. $\ell \gg 100$, which correspond to wavelengths that are well within the horizon at decoupling, move monotonically closer to the $\Lambda$CDM spectrum the higher is $z_d$, as expected since $\alpha_M$ vanishes during a longer part of the evolution. Modes that are crossing the horizon at decoupling, however, have a more complicate behavior since they are just beginning their oscillations: for these scales the trend is not monotonic with $z_d$ when $z_d$ is close to the decoupling epoch. Only for $z_d$ larger than 10,000 are they back to the $\Lambda$CDM amplitude. Since $r$ is related to the primordial peak in the BB spectrum, which changes non monotonically if $\alpha_M$ is set to zero before decoupling, this also means that the direction of the degeneracy between $r$ and $\alpha_M$ is not necessarily the one in Fig. [2] if $\alpha_M$ is not constant at all times until decoupling. In particular, even for negative $\alpha_M = -1$, the primordial B spectrum becomes smaller than $\Lambda$CDM if $z_d \approx 1500$. The degeneracy becomes then inverted (larger $r$ corresponding to smaller $\alpha_M$ if $z_d$ is larger than 1500, for instance fixed to matter-radiation equality).

Since $\alpha_M$ is related to $G_{\text{eff}}$, another caveat is that $G_{\text{eff}}$ influences the growth of scalar perturbations before decoupling. As a consequence, $r$ can also change indirectly through the Poisson equation and the modification of scalar perturbations, even if tensor spectra stay the same. This effect will have to be investigated more systematically implementing also the full set of scalar equations, which is beyond the scope of this paper. However we note that it will mainly play a role only after perturbations have entered the horizon, affecting scalar perturbations only for $\ell \gtrsim 100$ through a change in the Sachs Wolfe effect, given by the different gravitational potential. Moreover, a change in $G_{\text{eff}}$ could be compensated with a change in $\Omega_m$. If $z_d$ is set to a redshift before the equality, the effect should be negligible as matter perturbations only start growing when $\alpha_M$ is already zero.

We can now consider simultaneously both parameters, $\alpha_M$ and $c_T$. In this case, marginalizing over $c_T$, we obtain the contours in Fig. [4], which appear to be very similar to the case in which we fix $c_T = 1$. The contours of $\alpha_M$, $c_T$ and $r$, $c_T$ are in Fig. [5]. We find a mean and standard deviation for $c_T^2 = 1.3 \pm 0.5$ with a best-fit of $c_T^2 (\text{best fit}) = 0.8$ which is compatible with $\Lambda$CDM and, most of all, the same value found in [1] using also the temperature power spectra besides BICEP2. This confirms that the relevant epoch during which the tensor equation is affected mainly by B modes is the one before decoupling.

Finally, we note that the theoretical BB spectrum shows another peak at $\ell \approx 5$, still to be detected, due to the effects of tensor modes on the scattering during reionization. A non-zero $\alpha_M$ changes the amplitude of the reionization peak, similarly to what happens when $c_T$ is modified [1]. Its detection, for instance with the proposed satellite mission LiteBIRD [19] [21], could therefore put constraints on the friction and gravitational wave speed before and during reionization.

In conclusion, we have studied the impact on B modes
of a modified gravity tensor equation taking into account both the friction and the speed term. We have shown that a low value of the tensor-to-scalar ratio $r$ can be reconciled with the BICEP2 recent data if $\alpha_M$ is close to -2. In specific models, such as Brans Dicke scalar-tensor theories, the $\alpha_M$ parameter is already strongly constrained by the temperature spectra and the degeneracy with $r$ is removed. We argue however that in general this is not the case and therefore the BB spectrum is a useful test of early time modifications of gravity.

![Image of diagrams](image-url)

**FIG. 4:** Posterior likelihood for $\alpha_M$ and $r_{0.05}$. Blue (top) contours are obtained using all nine bandpowers from BICEP2 and fixing $c_T$ while orange contours marginalize over $c_T$.

**FIG. 5:** Posterior likelihood for $\alpha_M$ and $r_{0.05}$ vs $c_T^2$.

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[1] L. Amendola, G. Ballesteros, and V. Pettorino, ArXiv e-prints (2014), 1405.7004.
[2] M. Raveri, C. Baccigalupi, A. Silvestri, and S.-Y. Zhou, ArXiv e-prints (2014), 1405.7974.
[3] E. Bellini and I. Sawicki (2014), 1404.3713.
[4] S. Hassan and R. A. Rosen, JHEP 1107, 009 (2011), 1103.6055.
[5] I. D. Saltas, I. Sawicki, L. Amendola, and M. Kunz (2014), 1406.7139.
[6] A. De Felice and S. Tsujikawa, JCAP 1202, 007 (2012), 1110.3878.
[7] G. W. Horndeski, Int.J.Theor.Phys. 10, 363 (1974).
[8] P. Ade et al. (BICEP2 Collaboration) (2014), 1403.3985.
[9] C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith, R. S. Hill, B. Gold, M. Halpern, et al., ArXiv e-prints (2012), 1212.5225.
[10] H. Liu, P. Mertsch, and S. Sarkar (2014), 1404.1899.
[11] Planck Collaboration, P. A. R. Ade, M. I. R. Alves, G. Aniano, C. Armitage-Caplan, M. Arnaud, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday, et al., ArXiv e-prints (2014), 1405.0874.
[12] F. Könnig, Y. Akrami, L. Amendola, M. Motta, and A. R. Solomon (2014), 1407.4331.
[13] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002), arXiv:astro-ph/0205436.
[14] Planck Collaboration, P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday, et al., ArXiv e-prints (2013), 1303.5076.
[15] L. Amendola, M. Litterio, and F. Occhionero, Phys.Lett. B231, 43 (1989).
[16] J. C. Hwang, Class. Quant. Grav. 7, 1613 (1990).
[17] H. Nariai, Progress of Theoretical Physics 42, 544 (1969).
[18] V. Acquaviva, C. Baccigalupi, S. M. Leach, A. R. Liddle, and F. Perrotta, Phys. Rev. D 71, 104025 (2005), astro-ph/0412052.
[19] T. Matsumura, Y. Akiba, J. Borrill, Y. Chinone, M. Dobbs, H. Fuke, A. Ghribi, M. Hasegawa, K. Hattori, M. Hattori, et al., ArXiv e-prints (2013), 1311.2847.

[20] http://camb.info/

[21] http://litebird.jp/