Simultaneously static and dynamic layout designs of stiffening structures

Zhijie Feng¹, Kai Wu², Xuan Liang² and Jianbin Du³,³

¹ AVIC Aerospace Life-Support Industries, LTD, Aviation Key Laboratory of Science and Technology on Life-Support Technology, Xiangyang 441000, P.R. China;
² School of Aerospace Engineering, Tsinghua University, Beijing 100084, P.R. China
³ Email: dujb@tsinghua.edu.cn

Abstract. In the present paper, the topological design problem of lightweight stiffening structures are studied. Multiple design criteria like static compliance, dynamic compliance, eigenfrequency/eigenfrequency gaps and weight are considered simultaneously as sub-objectives in the optimization model. A bound formulation is developed and implemented to deal with the problem concerned, by which the difficulties of non-continuity of the multiple objective functions can be got over. Comparing with the empirical adjustment design or more elaborated parametric design, the topological design may introduce more degrees of design freedom, and thus is possible to produce more innovative conceptual design. The method of sequential convex programming is employed to find the solution of the optimization problem. Numerical examples show that, using the presented approach, the optimized layout of the stiffening structures with enhanced static and dynamic mechanical properties can be obtained.

1. Introduction

Monolithic stiffened thin-walled plate structures are widely used currently for high efficiency and integrity in modern industry designs, e.g. in the designs of car structures, train structures and aircraft structures, which provide work and life space for the pilots and passengers and also functions as installation and running platform for a variety of crucial devices. The mechanical properties of the structure, including the weight, stiffness, vibration and noise level and so on, are very often concerned by the designers for the safety and reliability of the structure.

As a traditional way of design of the stiffened structure, several candidate stiffening patterns, e.g. the cross orthogonal and/or triangle stiffening ribs, are normally first selected according to the experiences of the designer. Then a design procedure based on the cycle of analysis-comparison-adjustment or more elaborate method of parameter optimization is performed to improve the initial design. However, the layout of the structure cannot be changed automatically during the design process, i.e. the degrees of freedom of the traditional way of design are small and the gain obtained from the design is limited.

Different from the traditional way of design like adjustment design, or more elaborate method concerning shape or size optimization, topology optimization method may generate new connections or new holes in the design domain and is thus regarded as a powerful way of changing the layout of the structure in the most effective manner [1-2]. During the last two decades, topology design methods have been developed to a great extent. As a powerful tool in the conceptual design stage [3-5],
topology optimization method has been widely applied to different fields of engineering designs, such as frequency designs [6-7], dynamic designs [8-10], vibro-acoustic designs [11-13], material/microstructural designs [14-19], multi-scale designs [20-21] and so on. However, up to now studies on multiple objective topological design problems of the reinforcement structures considering simultaneously different indices including static stiffness, dynamic stiffness, weight and eigenfrequency or eigenfrequency gap are far from sufficient in both the literatures and engineering applications, and thus will be discussed in this paper.

In the present paper, problem on multiple objective topology design will firstly be addressed in detail. Then a bound formulation will be developed and utilized to solve problem concerned. Finally, several selected numerical examples will be presented and some interesting features will be revealed and discussed.

2. Bound formulation for multiple objective topological design

In general, the formulation of multiple objective design problem can be presented as:

$$\min_{\mathbf{x} \in G} \{f_1(x), f_2(x), \ldots, f_n(x)\}$$  \tag{1}

Where $f_i$ represents the sub-objective function corresponding to the $i$th index and $G$ the admissible set of the design variables $\mathbf{x}$. Effective solutions of multi-objective optimization problem may refer to the Pareto solution, which may be obtained using numerical optimization method for large-scale problem.

In order to avoid the numerical instability during optimization, which is induced by non-continuity of the multiple objective functions due to exchange of the orders of the sub-objective values, a bound formulation for multi-objective topology optimization model is proposed as follows:

$$\min_{\rho, \beta} \{\beta\}$$

Subject to:

$$\sum_{e=1}^{N_e} \rho_e V_e \leq V^* \leq \sum_{e=1}^{N_e} \rho_e V_e \leq V^*$$
$$0 \leq \rho_e \leq \rho_e \leq 1$$
$$w_i \frac{f_i}{f_i} \leq \beta, \quad (i = 2, \ldots, 5)$$  \tag{2}

Where the scalar variable $\beta$ is the upper limit of each sub-objective and also plays the roles of the design variable. The SIMP model is employed to implement the 0-1 topology design, where $\rho$ is the relative volume density of material and is also topological design variable here. $\rho_e$ is a prescribed lower bound of the material volume density. $V^*$ denotes the upper bound of the total volume of the given materials. $w_i$ is the weighting coefficient of the $i$th sub-objective, whose values may vary between 0 and 1, and satisfy $\sum_{i=1}^{s} w_i = 1$. $\overline{f}_i$ is the reference value of the $i$th sub-objective. This way the objective function is continuous and smooth so that the computation process is stable.

In the present paper, both the static and dynamic properties of the structure are concerned, thus the sub-design objectives considered include

$$f_1 \sim V = \sum_{e=1}^{N_e} \rho_e V_e$$  \tag{3}
$$f_2 \sim C = \mathbf{P}^T \mathbf{U}$$  \tag{4}
\[
f_3 \sim \frac{1}{\lambda_i} = \frac{1}{\omega_i^2} \quad \text{or} \quad f_3 \sim \frac{1}{(\lambda_{i+1} - \lambda_i)} = \frac{1}{(\omega_{i+1}^2 - \omega_i^2)} \quad (5)
\]
\[
f_4 \sim C_d = \left| P_d^T U_d \right| \quad (6)
\]
\[
f_5 \sim \Pi = \int_S \frac{1}{2} \text{Re} \left( p v_n^* \right) dS \quad (7)
\]

Where \( C \) and \( C_d \) are the static and dynamic compliance of the structure, \( V \) the material volume, \( \lambda_i \) and \( \lambda_{i+1} - \lambda_i \) denote the \( i \)th eigenfrequency and the eigenfrequency gap between the adjacent eigenfrequencies. \( \Pi \) represents the sound power radiated from the vibrating structure to its surrounding acoustic medium.

3. Sensitivity analysis by adjoint method and solution

The gradient-based algorithm is utilized to solve the optimization problem in this paper, and thus the sensitivity analysis should be considered which involves the derivatives of the sub-objectives with respect to the design variable \( \rho_e \). Starting from the equilibrium equations of the structure for statics, free vibration, forced vibration and vibro-acoustic analysis, the sensitivity analysis of each sub-objective function with respect to the design variable may be performed using the adjoint method, the results of sensitivity analysis are given as follows:

\[
\frac{\partial C}{\partial \rho_e} = 2U^T \frac{\partial P}{\partial \rho_e} - U^T \frac{\partial K}{\partial \rho_e} U \quad (8)
\]
\[
\frac{\partial \lambda_i}{\partial \rho_e} = \phi_i \left( \frac{\partial K}{\partial \rho_e} - \lambda_i \frac{\partial M}{\partial \rho_e} \right) \phi_i \quad (9)
\]
\[
\frac{\partial C_d}{\partial \rho_e} = \text{sign}(C_d) \left( 2U_d^T \frac{\partial P_d}{\partial \rho_e} - U_d^T \frac{\partial K_d}{\partial \rho_e} U_d \right) \quad (10)
\]
\[
\frac{\partial \Pi}{\partial \rho_e} = \rho_c c_0 \omega_p^2 \left( U_s^* \frac{\partial P}{\partial \rho_e} - U_s^* \frac{\partial K}{\partial \rho_e} U_d \right) \quad (11)
\]

The symbol \( \phi_i \) is the \( i \)th eigenmode normalized to the mass matrix. Equation (9) is specific to the case of single eigenfrequency. Sensitivity results for multiple eigenfrequencies can be found in [7]. Equation (11) corresponds to the weak coupling case between the structure and the acoustic medium, and a high frequency approximation is also used to simplify the computation [11]. Sensitivity results for strong coupling case may be found in [13]. The symbols \( K \) and \( M \) are the global stiffness and mass matrices of the structure, and \( K_d \) is the dynamic stiffness matrix which is defined by \( K_d = K + i \omega_p C - \omega_p^2 M \), where \( C \) is the structural damping matrix, and \( \omega_p \) is the prescribed excitation frequency. The symbols \( \rho_0 \) and \( c_0 \) are the specific mass density of the acoustic medium and the sound speed. The symbol \( U_s \) may be interpreted as the virtual displacement vector, which may be solved by the equation \( K_s U_s = S S_d U_d \), where \( S_s \) is known as the surface normal matrix [11].

Based on the sensitivity results, the optimization problem may be solved by the famous sequential convex programming method - the Method of Moving Asymptotes (MMA) [22].
4. Numerical examples

4.1. Topology optimization of reinforcement plate

In Figure 1, a four edges fixed square plate (0.5 × 0.5 × 0.01m) is constructed by two layers with the same thickness: a base layer and a reinforcement layer. The plate is subjected to uniformly distributed unit pressure loading on its upper surface. The admissible design domain is the reinforcement layer which implies that the base layer is unchanged in the design. The design objectives include minimization of the static compliance and minimization of the weight of the structure. Here, we transfer the second sub-objective to a constraint of the total material volume (30% volume fraction of the reinforcement layer). Two prescribed materials are employed in the design: (1) aluminum alloy with Young’s modulus $E = 71$GPa, mass density $\gamma = 2700$kg/m$^3$ and Poisson’s Ratio $\nu = 0.3$; (2) high-strength carbon fiber treated as isotropic material approximately with $E = 83$GPa, $\gamma = 2180$kg/m$^3$, $\nu = 0.3$. Figure 2(a) shows the optimum topologies of the reinforcement layer. Figure 2(b) and Table 1 compare the static compliance results between four traditional reinforcement designs and the optimum design from which we can see that the optimum design gives the best value of the compliances.

![Figure 1. Reinforcement plate with two layers: the base layer and the reinforcement layer.](image1)

![Figure 2. Optimum reinforcement layout: (a) Carbon fiber in the reinforcement layer (black) and Aluminum in the base layer (grey); (b) Static compliances of the four conventional reinforcement designs (1~4) and the optimum reinforcement layouts (5).](image2)
### Table 1. Comparison of static compliance between 5 different stiffening designs in Figure 2(b).

| Design 1 | Design 2 | Design 3 | Design 4 | Design 5 (Optimum design) |
|----------|----------|----------|----------|---------------------------|
| 1.039E-9 | 0.747E-9 | 1.009E-9 | 0.677E-9 | 0.502E-9                  |

#### 4.2. Static and dynamic multiple objective topological design of the reinforcement plate with non-structure mass

Figure 3 shows a four edges fixed rectangular reinforcement plate (0.5×0.5×0.015m). the thickness of the reinforcement layer and the base layer are 0.01m and 0.005m, respectively. Four concentrated masses $m_A = 10$kg, $m_B = 5$kg, $m_C = 2$kg and $m_D = 1$kg are fixed at coordinates $A(0.375, 0.375)$, $B(0.125, 0.375)$, $C(0.125, 0.125)$ and $D(0.375, 0.125)$ on the surface of the base layer, respectively. The material used in both the reinforcement layer and the base layer is the same one, i.e. Aluminum Alloy 5A06 with $E = 70$Gpa, $\gamma = 2750$kg/m³ and $\nu = 0.35$. The upper bound of the material volume fraction is set as 30% of the reinforcement layer.

The sub-objectives are: 1) Minimization of the total volume of the materials (as constraint); 2) Minimization of the static compliance with weighting coefficient $w_1$; 3) Minimization of the dynamic compliance ($\omega_p = 350$rad/s) with weighting coefficient $w_2$; 4) Maximization of the gap between the 1st and 2nd eigenfrequencies with weighting coefficient $w_3$; 5) Maximization of the fundamental frequency with weighting coefficient $w_4$.

Figure 4(a-d) show the optimum layouts of the reinforcement layer obtained from the designs for four different combinations of the weighting coefficients. Significant differences among the four designs may be observed from the results which implies that weighting coefficients have big influences on the solutions of multiple objective designs. Comparisons of the sub-objective values between the optimum design (Figure 4(a)) and the initial uniform design are given in Table 2. It can be seen that all the values of the sub-objectives in the optimum design are improved greatly in comparison with those of the initial design.

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**Figure 3.** Four-edges fixed stiffened plate structure with four non-structural concentrated masses.
Figure 4. Optimum topologies of the reinforcement layer by multiple objective designs corresponding to four different combinations of the weighting coefficients. (a) $w_1 = w_2 = w_3 = 1/3$, $w_4 = 0$; (b) $w_1 = 0.1$, $w_2 = 0.3$, $w_3 = 0.6$, $w_4 = 0$; (c) $w_1 = w_2 = w_3 = 0$, $w_4 = 1$; (d) $w_1 = w_2 = 0$, $w_3 = 1$, $w_4 = 0$.

Table 2. Sub-objective values between the optimum design (cf. Figure 4) and the initial uniform design.

| Stiffening layer          | Sub-objectives |
|---------------------------|----------------|
|                           | $C$ (W) | $C_d$ (W) | $\Delta \omega_{1,2}$ (rad/s) | $\omega_1$ (rad/s) |
| Initial uniform design    | 2.87E-9 | 4.83E-9  | 197                             | 477                 |
| Optimum design Figure 4(a)| 0.70E-9 | 0.75E-9  | 992                             | 856                 |

5. Conclusion
In the present paper, the problem of structural topology optimization of the stiffening plate is studied. A multi-objective topology optimization model and the corresponding solution method are developed to improve simultaneously different design indices including weight, stiffness, eigenfrequencies and vibration/noise level. Numerical examples show the effectiveness of the above-mentioned model and methods. The optimum layouts of the stiffening layer of the macrostructure may be obtained, and excellent vibrational characteristics combined with other good mechanical properties may be achieved by the designs. The presented work may provide some new ideas and relevant theoretical basis for the structural design of industrial equipment. Moreover, we have developed preliminarily a set of computer programs to implement multi-objective and multi-material size and topology optimization for more general and complex objectives, such as 3D irregular structures or mechanism which may provide a straightforward guide for the concept design of practical engineering structures to some extent.

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References
[1] Cheng G, Olhoff N 1982 Regularized formulation for optimal design of axisymmetric plates Int. J. Solids Struct 17 305-323
[2] Bendsøe M. P., Kikuchi N 1988. Generating Optimal Topologies in Structural Design Using a Homogenization Method Computer Methods in Applied Mechanics and Engineering 71(2) 197-224
[3] Du J.B. 2015. Structural optimization and its application in vibro-acoustic design (in Chinese). Beijing: Tsinghua University Press

[4] Eschenauer H., Olhoff N 2001 Topology Optimization of Continuum Structures: a Review Appl Mech Rev 54(4) 331-389

[5] Guo X, Cheng GD 2010 Recent Development in Structural Design and Optimization Acta Mech Sin 26(6) 807-823

[6] Ma ZD Kikuchi N, Cheng HC 1995 Topological design for vibrating structures Comput. Methods Appl. Mech. Engrg. 121 259-280

[7] Du J, Olhoff N 2007 Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and eigenfrequency gaps Struct. Multidisc. Optim. 34(2) 91-110

[8] Kang Z, Zhang X, Jiang S, Cheng G 2011 On topology optimization of damping layer in shell structures under harmonic excitations Struct. and Multidisc. Optim. 46(1) 51-67

[9] Olhoff, N., Du J 2009 On Topological Design Optimization of Structures Against Vibration and Noise Emission In book Computational Aspects of Structural Acoustics and Vibration 5 217-276

[10] Du J, Huang Z, Yang R 2015 Optimization of the motion control mechanism of the hatch door of airliner Struct Multidisc Optim 51 1173-1186

[11] Du J, Olhoff N 2007 Minimization of Sound Radiation From Vibrating Bi-Material Structures Using Topology Optimization Structural and Multidisciplinary Optimization 33(4-5) 305-321

[12] Du J, Olhoff N 2010 Topological Design of Vibrating Structures with Respect to Optimum Sound Pressure Characteristics in a Surrounding Acoustic Medium Structural and Multidisciplinary Optimization 42(1) 43-54

[13] Du JB, Song XK, Dong LL 2011 Design on material distribution of acoustic structure using topology optimization Chinese Journal of Theoretical and Applied Mechanics 43(2) 306-315

[14] Sigmund, O 1994 Materials with Prescribed Constitutive Parameters: An Inverse Homogenization International Journal of Solids and Structures 31(17) 2313-2329

[15] Sigmund, O 2000. A New Class of Extremal Composites Journal of the Mechanics and Physics of Solids 48(2) 397-428

[16] Gibiansky, L. V., Sigmund O. 2000 Multiphase Composites with Extremal Bulk Modulus, Journal of the Mechanics and Physics of Solids 48(3) 461-498

[17] Fujii, D., Chen BC, Kikuchi N 2001. Composite Material Design of Two-Dimensional Structures Using the Homogenization Design Method, International Journal for Numerical Methods in Engineering 50(9) 2031-2051

[18] Yang R, Du J 2013 Microstructural topology optimization with respect to sound power radiation Struct Multidisc Optim 47(2): 191-206

[19] Du J, Yang R. 2015 Vibro-acoustic design of plate using bi-material microstructural topology optimization Journal of Mechanical Science and Technology 29(4) 1-7

[20] Du J, Taylor JE 2002 Application of an energy-based model for the optimal design of structural materials and topology Struct Multidisc Optim 24 277-292

[21] Rodrigues, H., Guedes J. M., Bendsoe, M. P. 2002 Hierarchical Optimization of Material and Structure Structural and Multidisciplinary Optimization 24(1) 1-10

[22] Svanberg, K. 1987 The Method of Moving Asymptotes - A New Method for Structural Optimization Int. J. Numer. Methods Eng. 24(2) 359-373