Radiative Corrections as Origin of Tiny Fermion Masses

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February 27, 2015

Abstract

The fermion masses in the standard model are introduced as arbitrary parameters and there is no understanding of their origin. In this letter it is suggested that small non zero neutrino masses may be a reflection of broken stochastic supersymmetry that guarantees the equivalence of Parisi Wu stochastic quantization scheme to standard quantum field theory.

All predictions of the standard model of electroweak unification have been beautifully confirmed by experiments so far. With the discovery of Higgs boson, the last missing link has been found. For a long time, until the discovery of neutrino oscillations, it had been widely believed that the neutrinos are massless and are adequately described by two component Weyl spinors. Renormalizability of the theory has been ensured by presence of axial anomaly cancellation mechanism between different fermion sectors. Subsequent confirmation of neutrino oscillations have changed this scenario completely. The observation of neutrino oscillations is universally seen as a signal of new physics beyond standard model. For a review and references on models beyond the standard model see references. Going beyond the standard model, one may have to introduce either Majorana neutrinos or four component Dirac neutrinos. In case of Dirac neutrinos right handed
components of neutrinos must be included in the theory. In either case, the neutrino masses are not constrained by the symmetries of the model and the masses have remained free parameters.

Theoretical models have been suggested to explain why neutrino masses could be small. Possible theoretical explanations, such as see-saw mechanism introduce new particles which will be seen at a higher energy scale. Review of models proposed and references can be found in [6, 7]. These models do not offer any insight into the fermion mass problem and do not predict the neutrino masses, and do not offer any clues as to why neutrino masses are so tiny. Most of the present efforts to understand the neutrino masses are centred around determining the masses from the experimental data.

In this article we suggest a possible mechanism for explanation of origin of tiny neutrino masses staying as close to the standard electroweak model as possible. It is suggested that the neutrino masses at the tree level are zero and non zero masses arise purely out of radiative corrections. Of course having non-zero mass requires a four component Dirac neutrino. As already remarked this suggestion goes beyond the standard model. Whether a model, with Dirac neutrinos, eventually explains the masses can be decided only by building a detailed model and comparing detailed theoretical predictions with experiments.

Instead of going beyond the standard model and looking for new interactions responsible for neutrino masses, it is suggested to look for quantization schemes beyond the standard quantization scheme, formally equivalent to canonical quantization. This scheme should have features that it should in some limit reproduce all the known results and should offer a possibility of tiny neutrino masses arising out of radiative corrections. Such an approach will predict the masses without any free parameters and can be tested against the experiments.

The conventional quantum field theory (CQFT) formalism, formulated using canonical quantization, treats a massless fermion with four components as if the theory has a left handed and a right handed massless fermion and a consistent formalism is also possible only with two components. This is primary the reason that the CQFT is equally well equipped to describe two
component Weyl, Majorana neutrino, or a four component Dirac neutrino. In absence of any other restriction in the standard model, this means that the fermion masses become free parameters. In this letter it is suggested that this situation will change when we go to Parisi Wu stochastic quantization method (SQM) \cite{8, 9, 10, 11}. Though it is widely believed that SQM is equivalent to the CQFT, this equivalence has been demonstrated mostly at a formal level.

For models with bosons only stochastic supersymmetry, found in Parisi Wu formulation of SQM, plays a crucial role in proof of equivalence of SQM and conventional formulations \cite{12, 13}. In absence of stochastic supersymmetry the equivalence proofs will not be applicable. The initial investigations in SQM have been driven by the fact that SQM offered an scheme for gauge theories without gauge fixing. Though the presence of Zwanziger gauge fixing, suggested later, does not respect the stochastic supersymmetry, the equivalence of pure Yang Mills in SQM has been investigated, and a proof of equivalence of gauge invariant amplitudes has been given in \cite{14} and renormalization of pure Yang Mills theory has been studied in \cite{15}. However, a detailed investigation and a proof of equivalence of gauge theories including fermions is not available.

The earliest formulation of fermionic theories in SQM was given in \cite{16}. It turns out the SQM of gauge theories in presence of fermions is not always equivalent to CQFT. In context of electroweak interactions, a proof of equivalence of SQM and CQFT at perturbative level has not been given. A physical field theory is completely defined by the Lagrangian, a way of handling divergences and by rules used to extract finite answers. Even though the equivalence between Parisi Wu SQM and conventional formulations of quantum field theory has been a subject of intense investigation the actual situation could turn out to be different from expected behaviour for reasons outlined below.

A closer look at the fermionic theories several reveals differences in the SQM and CQFT formalism. To begin with, a consistent formulation of SQM for fermions appears to require introduction of all components. A consistent SQM formulation of four component Dirac fermion always violates chiral
symmetry even at tree level. Also it has almost gone unnoticed that a renormalized theory of Dirac fermions based on SQM [19, 20] has some features very different from renormalized theory based on CQFT. In particular, new counter terms may be required which destroy the stochastic supersymmetry through the radiative corrections.

The three features of the SQM formalism, requiring use of four component Dirac fermions, violation of chiral symmetry even at tree level, and the renormalized theory not preserving stochastic supersymmetry will be demonstrated taking example of Yukawa scalar coupling and of a model with axial vector coupling respectively.

The main features of SQM formulation will be first summarized. The CQFT of a fermion is described by the Lagrangian

\[ L = \bar{\psi}(i\gamma_\mu \partial_\mu + M)\psi + L' \]  

where \( L' \) is part of the Lagrangian describing other fields which may be coupled to the fermion. So for a scalar field with Yukawa coupling \( L' = L_1 \) where

\[ L_1 = \frac{1}{2}[(\partial_\mu \phi)(\partial_\mu \phi) + m^2 \phi^2] + \lambda \bar{\psi}\psi\phi + \frac{g}{4!} \phi^4. \]  

The SQM formulation makes use of Euclidean action \( S \) corresponding to the Lagrangian \( L \). The SQM formulation of scalar field theory with Yukawa coupling and renormalization has been discussed in detail in [17]. Here we will recall basic equations and important features only and for details we refer to the original articles. The basic Langevin equations of SQM are given by

\[ \frac{\partial \phi(x,t)}{\partial t} = -\gamma^{-1} \frac{\delta S}{\delta \phi} + \eta(x,t) \]  

\[ \frac{\partial \psi(x,t)}{\partial t} = -\int dx' K(x,x') \frac{\delta S}{\delta \bar{\psi}(x')} + \theta(x,t) \]  

\[ \frac{\partial \bar{\psi}(x,t)}{\partial t} = \int dx' \frac{\delta S}{\delta \psi(x')} K(x,x') + \bar{\theta}(x,t). \]

Here \( x \) collectively stands for all the components of the Euclidean four vector \( x_\mu \) and \( t \) denotes the fifth time or the stochastic time. The Gaussian white
noises $\eta(x, t), \theta(x, t), \bar{\theta}(x, t)$ are assumed to have averages

$$\langle \eta(x, t)\eta(x', t') \rangle = 2\gamma^{-1}\delta(x - x')\delta(t - t')$$

$$\langle \theta(x, t)\bar{\theta}(x', t') \rangle = 2K(x, x')\delta(t - t')$$

In the operator formalism [18] of SQM, this theory is equivalent to a five dimensional field theory described by a five dimensional stochastic action given by

$$\Lambda = \int dxdt \left( \pi \frac{\partial \phi}{\partial t} + \frac{\partial \bar{\psi}}{\partial t} \omega + \omega \frac{\partial \psi}{\partial t} - H \right)$$

where

$$H = T\psi \left[ 2\bar{\omega}K\omega - \tilde{\omega}K\delta S \delta \bar{\psi} + \delta S \delta \bar{\psi} \right] + H_1$$

and $\pi, \omega$ are the stochastic momentum fields conjugate to the scalar field $\phi$ and the fermion field $\psi$.

A simple, but important feature of the stochastic theory, that the above equations bring out, is that, for every choice of a kernel $K$, different terms involving the fermion, $\frac{\partial \bar{\psi}}{\partial t} \omega + \omega \frac{\partial \psi}{\partial t}, 2\bar{\omega}K\omega$ and $\bar{\omega}K\frac{\delta S}{\delta \bar{\psi}} + \frac{\delta S}{\delta \psi} \tilde{K}\omega$ have different behaviour under axial transformations. This in turn means a necessary mix up the left and right components, independent of the interaction Lagrangian chosen in four dimensional CQFT. At the tree level the stochastic action $\Lambda$ is not invariant under axial transformations even if the underlying CQFT preserves the chiral symmetry.

The SQM cannot be formulated for a two component fermions because with two component fermion, the Langevin equations will be inconsistent with the requirement that the kernel $K(x, x')$ be invertible.

For the scalar Yukawa coupling, the structure of counter terms and the renormalized theory described by the five dimensional action $\Lambda$ has been discussed in detail in [17]. In this case it was shown that the stochastic supersymmetry, crucial to equivalence with CQFT, can be maintained. However it turns out that this is not always the case for fermions [19, 20]. A concrete support for this statement can be seen by taking a specific example. Let us
consider the case of and axial vector $A^\mu$ coupled to the fermion. In this case $L'$ would be equal to $L_2$ where

$$L_2 = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda \bar{\psi} i \gamma^\mu \gamma_5 \psi A_\mu$$

(11)

A straightforward exercise in power counting reveals that the stochastic theory requires finite number of counter terms. Due to divergent triangle diagram, a new counter term of the form $\epsilon_{\mu\nu\lambda\sigma} A_{\mu} \partial_\lambda A_\sigma$ is needed and this term preserves axial gauge invariance but violates stochastic supersymmetry. The appearance of this counter term is similar to appearance of $\phi^4$ term in CQFT of scalar field $\phi$ coupled with a fermion, in absence of a scalar self interactions at the classical level.

Even though the underlying the classical Lagrangian of a model may be the same, an anomaly free CQFT for vector and axial vector gauge coupling with four component massless fermions is likely to be very different from the renormalized theory within SQM framework. It is possible to preserve zero mass for a fermion in an anomaly free gauge theory with vector and axial vector couplings, but SQM of the same model, even at the tree level, does not preserve axial symmetry for massless fermions and does not guarantee a theory equivalent to that CQFT.

Barring some accidental cancellations, absence of both axial symmetry and stochastic supersymmetry in SQM will mean non zero radiative corrections to the fermion masses and also calculable effects beyond those present the suitably extended standard model in CQFT formalism.

The aim of this paper has been only to point out the possibility that the neutrino masses could come purely out of radiative corrections. In order to realize the mechanism suggested here a possible strategy will be as follows. We first start by adding right handed neutrinos, and ensure a renormalizable CQFT with zero mass neutrino at tree level by symmetry considerations with chiral symmetry ensuring zero masses for the neutrinos at all orders. This will require inclusion of some new features in the standard model. Having done this, SQM of such a model breaks the chiral symmetry at the tree level itself, and the stochastic supersymmetry in higher orders. Thus using SQM,
while the theory remains renormalizable, it will give different predictions. As
the equivalence of CQFT and SQM is lost in higher orders, the deviations in
SQM from CQFT predictions will come from radiative corrections and will
be calculable.

Whether nature chooses this scheme or something else can be determined
solely by a detailed computation and its comparison with the experiments.

A realistic model is most likely be complicated to analyse and is beyond
the scope of short communication like present letter and a complete analysis
needs to be taken up separately.

I thank Bindu Bambah for an illuminating discussion of present status of
research in neutrino physics which prompted this work and to H.S. Mani for
critical comments on an earlier draft of the manuscript.

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