Parametric Analysis of the Interaction between Angular and Translational Vibrations of Vibration-Sensitive Systems

I. N. Shardakov*, I. O. Glot**, A. P. Shestakov***, K. V. Sobyanin****, and D. V. Gubskiy*****

* Institute of Continuous Media Mechanics, Ural Branch, Russian Academy of Sciences, Perm, Russia
** e-mail: shardakov@icmm.ru
*** e-mail: glot@icmm.ru
**** e-mail: shap@icmm.ru
***** e-mail: k.sobyanin@gmail.com

Received December 19, 2019; revised December 24, 2019; accepted December 24, 2019

Abstract—Modern radio-engineering complexes, computers, and navigation systems placed on moving objects (aircrafts, ships, cars, etc.) may be subjected to significant pulsed and vibration mechanical loads during exploitation, such as impacts, vibrations, linear overloads, and acoustic noise. They may distort the parameters of electric signals, introduce additional errors into instrument readings, and destroy elements of equipment. Therefore, there is a need to minimize undesirable motions of these devices. An efficient way to do this is to organize passive vibration isolation of device, based on the use of inertial, elastic, dissipative, and other passive elements. The object of this study is a block of electronic devices fixed (using a system of four dampers) on a rigid platform of a supporting structure, which is subjected to translational vibrations in three mutually orthogonal directions. As a result, angular vibrations are excited in the insulated block. Mathematical modeling of the block response to external disturbances is performed in the framework of the classical theory of rigid body dynamics. A series of numerical experiments is performed to determine the response of the kinematic characteristics of insulated block to an external periodic action at different values of the stiffness coefficient and energy dissipation coefficients of dampers and different positions of the center of mass of the system. It is shown that a deviation of the position of the center of mass from that of the center of rigidity, as well as a change in the stiffness and energy dissipation coefficients of dampers within the spread of their mean values, cause a significant increase in the angular oscillations of insulated block.

Keywords: passive vibration isolation, mathematical model, vibration tests, center of stiffness, center of mass, angular vibrations, dampers, stiffness coefficient, energy dissipation coefficient

DOI: 10.1134/S0021894420070111

1. INTRODUCTION

Currently, various-purpose electronic devices (control units, navigation systems, measurement devices, sensors, etc.) are being developed and manufactured for use in moving objects. These devices are very often subjected to significant pulsed and vibrational mechanical effects during exploitation, such as impacts, vibrations, linear overloads, and acoustic noise, which distort unacceptably the technical characteristics of the equipment, introduce additional errors in readings of devices, and sometimes cause their mechanical destruction. The most widespread packaging of such devices is aggregation of electronic elements into a single unit fixed on the equipment housing by means of dampers. If the sensor unit is much more rigid than the dampers, it can be considered as a rigid body, which has six degrees of freedom: three translational and three angular. The relationship between the translational and angular vibrational modes is most pronounced in the case where coupled oscillations (combining angular and translational motions) are implemented [1].

Thus, an important problem for many electronic devices is the limitation of their kinematic characteristics: displacements, rotation angles, and the corresponding velocities and accelerations. The most urgent issue for many electronic devices is the limitation of angular kinematic characteristics [2–5]. This problem
is especially urgent for modern navigation devices, for example, fiber-optic gyroscopes, whose operation is based on the Sagnac effect [6–9].

An efficient way to solve this problem is to apply passive vibration isolation [10–14]. To implement an efficient vibration isolation for a specific electronic unit, one must possess the following information: how the dampers must be organized in space in order to minimize the number of controlled kinematic parameters; how the directions of damper response axes must be correlated with the position of the rigid-block center of mass in order to minimize the relationship between the angular and translational vibrations, etc. One can answer these questions by carrying out mathematical modeling of the dynamic behavior of sensor unit and subsequent analysis of the numerical solution.

The object of our study is a rigid-body electronic block, fixed on a rigid platform using four dampers. The platform, mounted on a bearing support, is subjected to translational vibrations in three mutually orthogonal directions. The block vibrational response to these disturbances is investigated. The simulation is performed in the framework of the general theory of rigid body motion. Numerical realization is performed using the MATLAB software.

2. MATHEMATICAL MODEL

A schematic of the passive vibration isolation of electronic devices is shown in Fig. 1a. Vibration-isolated block 1 is mounted on rigid platform 2, which is in turn installed on a bearing support. The block is connected to the platform via four dampers (A, B, C, D), whose stiffness is much lower than that of the vibration-isolated block. The motion in three mutually orthogonal directions is transferred to the device through the translational motions of platform 2.

Since the dampers have a much lower stiffness as compared with the block, the latter can be assumed to be absolutely rigid, whereas the dampers can be considered as weightless elements having elastic and dissipative properties. The calculation model is shown in Fig. 1b. The damped part is presented as point mass 1 with specified characteristics: the position of the center of mass, coinciding with the origin of coordinates; the directions of principal axes; and the main moments of inertia. The point mass is connected with dampers A, B, C, and D via weightless rigid rods, oriented along the damper centerlines. The dampers are arranged so that their centerlines 3 intersect at the same point, and this point is referred to as the center of rigidity of the system. Dampers are modeled by hinges with six degrees of freedom. The stiffness and dissipative properties of dampers are set by two corresponding coefficients for each degree of freedom. The dampers are attached to rigid weightless platform 2 via rigid vertical rods 4. One can set displacements, velocities, and accelerations of the platform to describe different loading modes.
Mathematical modeling of the mechanical response of insulated block to external force factors is performed within the classical theory of rigid body dynamics [15]. Correspondingly, the equations of motion describing the translational motion of the center of mass of rigid block have the form

\[
m \frac{d^2 U_\zeta}{dt^2} = F_\zeta, \quad m \frac{d^2 U_\eta}{dt^2} = F_\eta, \quad m \frac{d^2 U_\zeta}{dt^2} = F_\zeta .
\]  

(1)

Here, \( \zeta, \eta, \zeta \) are the axes of immobile coordinate system; \( U_\zeta, U_\eta, U_\zeta \) are displacements of the block center of mass; \( m \) is the block mass; and \( F_\zeta, F_\eta, F_\zeta \) are the forces applied to the block center of mass.

The equations describing the rigid block rotational motion have the form

\[
I_x \frac{d\omega_x}{dt} + (I_z - I_y)\omega_y\omega_z = M_x , \quad I_y \frac{d\omega_y}{dt} + (I_x - I_z)\omega_z\omega_x = M_y ,
\]

\[
I_z \frac{d\omega_z}{dt} + (I_y - I_x)\omega_x\omega_y = M_z ,
\]

(2)

where \( t \) is time, \( x, y, z \) are the axes of mobile coordinate system, which coincide with the main axes of inertia of rigid block; \( I_x, I_y, I_z \) are the main moments of inertia; \( \omega_x, \omega_y, \omega_z \) are the angular velocities; and \( M_x, M_y, M_z \) are the torques applied to the block.

The position of mobile coordinate system is determined using Tait–Bryan angles [16], which are related to the angular velocities as follows:

\[
\frac{d\psi}{dt} = \omega_z \tan(\theta) \omega_x + \sin(\theta) \tan(\theta) \omega_y ,
\]

\[
\frac{d\theta}{dt} = \omega_x - \cos(\phi) \tan(\theta) \omega_x + \sin(\phi) \tan(\theta) \omega_z ,
\]

\[
\frac{d\phi}{dt} = \omega_y - \cos(\phi) \sin(\theta) \omega_y - \sin(\phi) \sin(\theta) \omega_z .
\]

(3)

The initial conditions have the form

\[
\omega_x |_{t=0} = \omega_x^0 , \quad \omega_y |_{t=0} = \omega_y^0 , \quad \omega_z |_{t=0} = \omega_z^0 , \quad \psi |_{t=0} = \psi^0 , \quad \theta |_{t=0} = \theta^0 , \quad \phi |_{t=0} = \phi^0 ,
\]

\[
U_\zeta |_{t=0} = U_\zeta^0 , \quad U_\eta |_{t=0} = U_\eta^0 , \quad U_\zeta |_{t=0} = U_\zeta^0 , \quad U_\zeta |_{t=0} = U_\zeta^0 , \quad U_\eta |_{t=0} = U_\eta^0 , \quad U_\zeta |_{t=0} = U_\zeta^0 ,
\]

(4)

where \( U_\zeta^0, U_\eta^0, U_\zeta^0 \) are the initial velocities of the center of mass of rigid block.

The system of equations (1)–(4) is formulated below in two bases: the first basis sets an immobile coordinate system: \( \{e_1^0, e_2^0, e_3^0\} \rightarrow \{\xi, \eta, \zeta\} \); the second sets a mobile coordinate system related to the rigid block: \( \{e_1^m, e_2^m, e_3^m\} \rightarrow \{x, y, z\} \). These bases coincide at the initial instant. Direct and inverse transformations of bases are performed using a rotation matrix \( R \), which depends on the Tait–Bryan angles \( \psi, \theta, \phi \):

\[
R = \begin{bmatrix}
\cos(\psi) \cos(\theta) & \sin(\psi) \sin(\theta) & -\cos(\phi) \\
\sin(\psi) \sin(\theta) & \cos(\psi) \sin(\theta) & \cos(\phi) \\
-\sin(\phi) \cos(\psi) & \sin(\phi) \sin(\psi) & \cos(\phi)
\end{bmatrix}.
\]

With the aid of matrix \( R \), any vector \( V^m \) defined in the mobile basis \( \{e_i^m\} \) can be written in the initial basis \( \{e_i^0\} \) as \( V^0 = R \cdot V^m \), and, vice versa, \( V^m = R^T \cdot V^0 \) (\( R^T \) is the transposed matrix \( R \)). Hereinafter, the vector superscripts (0 and m) indicate their affiliation to immobile and mobile bases, respectively.

The vibration-isolated rigid block is connected via dampers to the device external housing. The orientation of the damper is set by the system of vectors \( V_1, V_2, V_3 \) (Fig. 2), and their position relative to the rigid block is set by the radius vector \( V_0 \), which is directed from the immobile coordinate system origin \( O \) to the point \( O_1 \), located at the geometric center of damper. The system of vectors \( V_1, V_2, V_3 \) is rigidly connected to rigid part \( I \) of the damper and, therefore, moves jointly with it. At an arbitrary instant \( t \) the system of these vectors and the vector \( V_0 \) in the immobile coordinate system are determined as follows:

\[
[V_1^0, V_2^0, V_3^0] = R \cdot [V_1^0, V_2^0, V_3^0] , \quad V_0^0 = R \cdot V_0^0 + U_0^0 ,
\]

JOURNAL OF APPLIED MECHANICS AND TECHNICAL PHYSICS  Vol. 61  No. 7  2020
PARAMETRIC ANALYSIS OF THE INTERACTION BETWEEN ANGULAR

where $\mathbf{U}^0_g$ is the displacement vector of the center of mass of rigid block. These vectors retain their initial values in the mobile coordinate system:

$$[\mathbf{V}_0^m, \mathbf{V}_1^m, \mathbf{V}_2^m, \mathbf{V}_3^m] = [\mathbf{V}_0^m, \mathbf{V}_1^m, \mathbf{V}_2^m, \mathbf{V}_3^m].$$

To determine the forces arising in a damper during deformation, one must find the values of the following parameters:

(i) displacement of the damper center;
(ii) rotation angles of damper axes relative to the initial state;
(iii) angular and linear velocities of damper relative to the initial state.

The displacement of the center of gravity of damper is calculated from the formula $\mathbf{U}^0_g = \mathbf{V}_0^0 - \mathbf{V}_0^0$.

When determining the rotation of the damper axes relative to the initial state, the values to be calculated first are as follows: $\mathbf{V}_0^0 = \mathbf{V}_0^0 \times \mathbf{V}_0^0$ (the rotation vector translating the initial vector $\mathbf{V}_0^0$ to its finite state $\mathbf{V}_0^0$), $A_{\text{rot}} = \arcsin(|\mathbf{V}_{\text{rot}}|)$ (the rotation angle); $\mathbf{V}_{\text{rot},n} = \mathbf{V}_{\text{rot}}/|\mathbf{V}_{\text{rot}}|$ (the unit rotation vector);

$\mathbf{V}_{2tr}^0 = \mathbf{M}_{\text{rot}} (\mathbf{V}_{\text{rot},n} - A_{\text{rot}}) \cdot \mathbf{V}_0^2$, where $\mathbf{V}_0^2$ is the vector $\mathbf{V}_0^2$ in the position occupied by it at an arbitrary instant and $\mathbf{V}_{2tr}^0$ is its projection on the plane formed by the vectors $\mathbf{V}_2^0$ and $\mathbf{V}_3^0$ in the initial position; and $\mathbf{M}_{\text{rot}} (\mathbf{V}, A)$ (the matrix describing the rotation around the unit vector $\mathbf{V}$ by angle $A$). The components of this matrix have the form

$$
\begin{bmatrix}
\cos(A) + (1 - \cos(A)) V_1 V_1 (1 - \cos(A)) V_2 V_2 - \sin(A) V_3 V_3 (1 - \cos(A)) V_1 V_3 + \sin(A) V_2 V_3 \\
(1 - \cos(A)) V_1 V_2 + \sin(A) V_3 V_2 \cos(A) + (1 - \cos(A)) V_2 V_2 (1 - \cos(A)) V_2 V_3 - \sin(A) V_1 V_3 \\
(1 - \cos(A)) V_1 V_3 - \sin(A) V_2 V_3 V_2 (1 - \cos(A)) V_3 V_3 + \sin(A) V_1 V_3 \cos(A) + (1 - \cos(A)) V_3 V_3
\end{bmatrix}
$$

Using the found values, one can write the desired angles as follows:

$$A_1 = \arcsin(V_1^0 \cdot (V_2^0 \times V_3^0)),$$

$$A_2 = A_{\text{rot}}(\mathbf{V}_{\text{rot},n} \cdot \mathbf{V}_2^0),$$

$$A_3 = A_{\text{rot}}(\mathbf{V}_{\text{rot},n} \cdot \mathbf{V}_3^0).$$

The angular and linear velocities of the rigid part of any damper relative to its initial position can be calculated using the formulas

$$\omega_g^m = [V_1^0 V_2^0 V_3^0]^T \cdot \omega_g^m,$$

for the angular velocity ($\omega_g^m$ and $\omega_g^m$ are, respectively, the angular velocities of the damper and the center of mass of rigid block in the mobile coordinate system) and

$$\mathbf{v}_g^0 = \mathbf{v}_g^0 + \omega_g^0 \times \mathbf{v}_0^0,$$
for the linear velocity \( \omega^0_s = R \cdot \omega^m_s \) is the angular velocity of the center of mass of rigid block in the immobile coordinate system.

With allowance for the calculated deformation characteristics of dampers, the force reactions arising in them are as follows:

- the force caused by the damper displacement,
  \[
  F^0_u = C_{1u}(U^0_d \cdot V^0_y)V^0_1 + C_{2u}(U^0_d \cdot V^0_2)V^0_2 + C_{3u}(U^0_d \cdot V^0_3)V^0_3,
  \]

where \( C_{1u}, C_{2u}, C_{3u} \) are the stiffnesses at a displacement in directions of the \( V^0_1, V^0_2, V^0_3 \) axes;

- the moment of the force reactions to the damper displacement,
  \[
  M^m_u = V^0_0 \times F^m_u,
  \]

where \( F^m_u = R^T \cdot F^0_{du} \) is the force vector in the mobile coordinate system;

- the moment due to the damper rotations,
  \[
  M^m_v = C_{1v}A_1V^0_1 + C_{2v}A_2V^0_2 + C_{3v}A_3V^0_3,
  \]

where \( C_{1v}, C_{2v}, C_{3v} \) are the stiffnesses at rotation around the \( V^0_1, V^0_2, V^0_3 \) axes, respectively;

- the force caused by the damper displacement in a viscous medium,
  \[
  F^0_v = K_{1v}(v^0_d \cdot V^0_y)V^0_1 + K_{2v}(v^0_d \cdot V^0_2)V^0_2 + K_{3v}(v^0_d \cdot V^0_3)V^0_3,
  \]

where \( K_{1v}, K_{2v}, K_{3v} \) are the coefficients of viscous friction at displacements in directions of the \( V^0_1, V^0_2, V^0_3 \) axes, respectively; \( v^0_d \) is the velocity of the damper fixing point;

- the moment caused by the force reactions to damper displacement in a viscous medium,
  \[
  M^m_v = V^0_0 \times (R^T \cdot F^0_v);
  \]

and the moment caused by the force reactions to the damper rotation in a viscous medium,

\[
M^m_{vv} = \{K_{1v}\omega^0_{1d}, K_{2v}\omega^0_{2d}, K_{3v}\omega^0_{3d}\}^T,
\]

where \( K_{1v}, K_{2v}, K_{3v} \) are the coefficients of viscous friction at rotation around the \( V^0_1, V^0_2, V^0_3 \) axes, respectively.

Thus, the resulting responses in the form of force and moment in any damper have the form

\[
F^0 = F^0_u + F^0_v, \quad M^m = M^m_u + M^m_v + M^m_v + M^m_{vv}.
\]

The total force and total moment from \( N \) dampers are equal to the sum of reactions of all dampers:

\[
F^0_{\text{sum}} = \sum_{i=1}^{N} F^0_i, \quad M^m_{\text{sum}} = \sum_{i=1}^{N} M^m_i.
\]

The total reactions \( \{F^0_1, F^0_2, F^0_3\}^T = F^0_{\text{sum}} \) and \( \{M^m_x, M^m_y, M^m_z\}^T = M^m_{\text{sum}} \) are then substituted into Eqs. (1) and (2).

The thus derived system of ordinary differential equations was solved numerically by the finite difference method using the MATLAB software.

3. RESULTS AND DISCUSSION

The numerical experiments based on the developed mathematical model allowed us to find the influence of different factors on the motion of a damped rigid block. The stiffness and dissipative characteristics of dampers, as well as the position of the center of mass of device with respect to its center of rigidity, were considered as parameters potentially capable of disturbing the block motion.

The range of variation in both the stiffness and dissipation coefficients of dampers was estimated based on the results of vibrational tests of dampers. The stiffness and dissipation coefficients were determined in the axial \( (V^0_1) \) and radial \( (V^0_2) \) directions from the amplitude—frequency dependences obtained in physical
It was also established in physical experiments that the position of the center of mass of device does not coincide with the position of its center of rigidity for the specific configuration of rigid block. The coordinates of the center of mass in the coordinate system with an origin at the center of rigidity were found to be (‒1.5, ‒1.4, ‒1.6) mm. A numerical experiment was performed for this position of the center of mass, in which the dynamic response of the rigid block to a stationary external periodic impact on the device in the frequency range from 150 to 350 Hz with an acceleration of $g$, directed along the $z$ axis, was modeled. Different ratios of stiffnesses in the dampers were set in the experiment in correspondence with the following algorithm: at first, the mean stiffness and dissipation values for all four dampers were taken at each fixed frequency, and then the values of parameters in one of the dampers were successively changed by an rms deviation. The deviations of the stiffness and dissipative characteristics from the mean values and the maximum amplitudes of the rigid-block angular velocities corresponding to this combination for each step of numerical experiment are given in Table 2.

The maximum amplitudes of angular accelerations in directions of the $x$, $y$, $z$ axes for the case where the center of mass of the device coincides with its center of rigidity, and the stiffness and dissipation coefficients of all four dampers are equal to mean values, were obtained in a numerical experiment as well. These data are also presented in Table 2. The response of the angular acceleration amplitudes of the rigid block to a deviation of its center of mass from the center of rigidity and to the deviation from the nominal values of damper stiffness coefficients was investigated in the same way (by carrying out a numerical
The average values of the experimentally found characteristics (see Table 1) were considered to be nominal. The deviations of the position of the center of mass along the $x$, $y$, and $z$ axes and the changes in the stiffness coefficients of dampers A, B, C, and D in the longitudinal and radial directions ($V_1$ and $V_2$ in Fig. 2) were modeled.

The plots presented in Fig. 3 illustrate how the maximum values of angular accelerations of the rigid block change when the center of mass of the system is displaced along the $x$, $y$, and $z$ axes. The dampers have the same stiffness. A comparison of the plots shows that the system is most sensitive to a displacement of the center of mass in the direction coinciding with that of the external effect. The change in the angular acceleration at a displacement along the $z$ axis is greater by five orders of magnitude than for a similar displacement along the $x$ or $y$ axes.

Figure 4 demonstrates the sensitivity of the system to a change in the damper stiffness. The change in the angular acceleration of the rigid block with a change in the stiffness of dampers A and B by $\pm 2\%$ from the mean value is twice as small as that at a similar change in the stiffness of dampers C and D.

A comparison of the plots in Figs. 3 and 4 shows that the most significant factor provoking angular oscillations is specifically the displacement of the center of mass of the system along the $z$ axis, which coincides with the external effect direction. The accelerations arising in this case exceed significantly the values observed for all other factors.

Fig. 3. Dependences of maximum angular accelerations on the displacement of the center of mass of the system along the (a) $x$ and $y$ axes and (b) the $z$ axis.

Fig. 4. Dependences of the maximum angular accelerations on change in the stiffness of dampers (a) A, B and (b) C, D.
4. CONCLUSIONS

Our physical experiments revealed that the position of the center of mass of device does not coincide with its center of rigidity for a specific configuration of rigid block. The coordinates of the center of mass are (‒1.5, ‒1.4, ‒1.6) mm in the coordinate system with an origin at the center of rigidity.

The statistical processing of the data of physical experiments revealed that the stiffness and dissipation coefficients are different for the four dampers of the system; the rms deviations of the stiffness coefficients in the axial and radial directions are, respectively, 8.1 and 6.5% of the mean; for the dissipation coefficients, the corresponding values are 1.2 and 4.3%.

The numerical experiments based on the use of the developed mathematical model of passive vibration isolation of system revealed that the maximum angular velocity of the damped fragment is 5.6 deg/s for the design arrangement of the system with the coordinates of the center of mass (‒1.5, ‒1.4, ‒1.6) mm and average values of damper characteristics.

The maximum amplitude of angular velocity increases by a factor of almost 5 at a successive change in the characteristics of dampers A, B, C, and D by their statistical variance (±2% of their mean). The angular acceleration changes by a factor of 40–80.

The maximum angular velocity is 0.025 deg/s when the center of mass coincides with the center of rigidity (and the origin of coordinates) in the numerical experiment. Thus, the maximum angular velocity changes ~200 times when the center of mass is displaced relative to the balanced position of centers by ~1.5 mm (provided that the dampers have identical characteristics). A displacement of the center of mass leads to a significant rise in the angular accelerations of damped block. The angular accelerations reach maximum values when the center of mass is displaced in the application direction of external vibrations.

It follows from the analysis of the presented results that the device providing passive vibration isolation of electronic tools is a balanced mechanical system in the arrangement considered above. Any deviation from equilibrium for its main parameters (the position of the center of mass of damped part, the position and orientation of dampers, and the stiffness and dissipation coefficients of dampers) leads to a significant increase in the amplitudes of angular oscillations and may cause incorrect operation of the equipment.

FUNDING

This study was supported by the Complex Program of Fundamental Research of the Ural Branch of the Russian Academy of Sciences within project no. 18-11-1-10 “Study of the Vibrational Processes in Vibration-Sensitive Devices and Development of Approaches and Tools for Their Vibration Isolation.”

REFERENCES

1. Lee, J. and Okwudire, C.E., Reduction of vibrations of passively-isolated ultra-precision manufacturing machines using mode coupling. Precis. Eng., 2016, vol. 43, pp. 164–177. https://doi.org/10.1016/j.precisioneng.2015.07.006

2. Savage, P.G., Strapdown inertial navigation integration algorithm design. Part 1: Attitude algorithms, J. Guid. Contr. Dyn., 1998, vol. 21, pp. 19–28. https://doi.org/10.2514/2.4228

3. Lin, Y., Zhang, W., and Xiong, J., Specific force integration algorithm with high accuracy for strapdown inertial navigation system, Aero. Sci. Tech., 2015, vol. 42, pp. 25–30. https://doi.org/10.1016/j.ast.2015.01.001

4. Zhuravlev, V.P. and Klimov, D.M., Global motion of the celt, Mech. Solids, 2008, vol. 43, pp. 320–327. https://doi.org/10.3103/S0025654408030023

5. Crocker, M.J., Handbook of Noise and Vibration Control, Hoboken: Wiley, 2007.

6. Bohnert, K., Gabus, P., Nehring, J., and Brandle, H., Temperature and vibration insensitive fiber-optic current sensor, J. Lightwave Technol., 2002, vol. 20, pp. 267–276. https://doi.org/10.1109/50.983241

7. Wang, W., Wang, X., and Xia, J., The nonreciprocal errors in fiber optic current sensors, Opt. Laser Technol., 2011, vol. 43, pp. 1470–1474. https://doi.org/10.1016/j.optlastec.2011.05.002

8. Zhang, Y. and Gao, Z., Fiber optic gyroscope vibration error due to fiber tail length asymmetry based on elastic-optic effect, Opt. Eng., 2012, vol. 51, p. 124403. https://doi.org/10.1117/1.OE.51.12.124403

9. Kurbatov, A.M. and Kurbatov, R.A., The vibration error of the fiber-optic gyroscope rotation rate and methods of its suppression, J. Commun. Technol. Electron., 2013, vol. 58, pp. 840–846. https://doi.org/10.1134/S1064226913070085
10. Il’inskiy, V.S., Zashchita REA i pretsizionnogo oborudovaniya ot dinamicheskikh vozdeistvii (Protection of REA and Precision Equipment from Dynamic Effects), Moscow: Radio i Svyaz’, 1982.
11. Lee, J. and Okwudire, C.E., Reduction of vibrations of passively-isolated ultra-precision manufacturing machines using mode coupling, Precis. Eng., 2016, vol. 43, pp. 164–177.
https://doi.org/10.1016/j.precisioneng.2015.07.006
12. Verbaan, K., van der Meulen, S., and Steinbuch, M., Broadband damping of high-precision motion stages, Mechatronics, 2017, vol. 41, pp. 1–16.
https://doi.org/10.1016/j.mechatronics.2016.10.014
13. Eliseev, S.V., Khomenko, A.P., and Logunov, A.S., Dinamicheskii sintez v obobshchennykh zadachakh vibrozashchity i vibroizolyatsii tekhnicheskikh ob’ektov (Dynamic Synthesis of the Generic Problems of Vibration Protection and Vibration Insulation of Technical Objects), Irkutsk: Irkut. Gos. Univ., 2008.
14. Frolov, K.V. and Furman, F.A., Prikladnaya teoriya vibrozashchitnykh sistem (Applied Vibration Protection Theory), Moscow: Mashinostroyeniye, 1980.
15. Ganiev, R.F. and Kononenko, V.O., Kolebaniya tverdykh tel (Oscillations of Solids), Moscow: Nauka, 1976.

Translated by Yu. Sin’kov