Role of lepton flavor violating (LFV) muon decay in Seesaw model and LSND

M.Jamil Aslam and Riazuddin
National Centre for Physics, Quaid-i-Azam University,
Islamabad, Pakistan.

The aim of the work is to study LFV in a newly proposed Seesaw model of neutrino mass and to see whether it could explain LSND excess. The motivation of this Seesaw model was that there was no new physics beyond the TeV scale. By studying \( \mu \to 3e \) in this model, it is shown that the upper bound on the branching ratio requires Higgs mass \( m_h \) of a new scalar doublet with lepton number \( L = -1 \) needed in the model has to be about 9 TeV. The predicted branching ratio for \( \mu \to e \nu \bar{\nu} \) is too small to explain the LSND.

PACS: 11.30.Hv, 14.60.Pq

I. INTRODUCTION

The problem of physics beyond the Standard model (SM) has been studied for a considerable length of time. In the past few years some progress has been made to understand this new physics, among them LFV is the most promising candidate. At present, we have rigorous bounds on LFV \( \mu \) decay, e.g. \[ \mathcal{B}(\mu \to 3e) \leq 10^{-12} \] (1)

Using experimental bounds on these three body decays the corresponding bounds on two body decays are calculated in [2,3],

\[ \mathcal{B}(Z \to \mu e) \leq 1.7 \times 10^{-13} \] (2)

and [3]

\[ \mathcal{B}(J/\psi \to \mu e) \leq 4 \times 10^{-13} \] (3)
\[ \mathcal{B}(\Upsilon \to \mu e) \leq 2 \times 10^{-9} \] (4)
\[ \mathcal{B}(\Phi \to \mu e) \leq 4 \times 10^{-17} \] (5)

At present, the best experimental limit on the branching ratio of \( Z \to \mu e \) decay is (95%C.L.)

\[ \mathcal{B}(Z \to \mu e) \leq 1 \cdot 7 \times 10^{-6} \] (6)

The possible source of the suppression of the bounds found in Eqs. (2)-(5) are discussed in [2,3,4,5].

The most alluring issue in the present day physics is whether or not the neutrinos have non zero mass. In the minimal standard model of particle interactions, neutrinos are massless. To generate a small neutrino mass in this model, there is an effective dimension five operator

\[ \mathcal{L}_{\text{eff}} = \frac{f_{ij}}{\Lambda} L_i L_j \Phi \Phi \] (7)

where \( L_i = (\nu_i, l_i)_L \) is the usual left-handed lepton doublet, \( \Phi = (\phi^+, \phi^0) \) is the usual scalar Higgs doublet, and \( \Lambda \) is an effective large mass scale. This operator has different tree-level realizations: (I) the canonical seesaw mechanism with right handed neutrino [7]; (II) the model having Higgs triplet [8]; and (III) the model having heavy Majorana fermion triplet [9]. These new interactions exist at higher mass scale. In the usual Seesaw mechanism, in order to have a very small mass for left handed neutrino, the corresponding mass for the right handed neutrino has to be very large, i.e. of order 10^3 TeV. Recently a new Seesaw model of neutrino mass is proposed with the motivation that there is no new physics beyond the TeV scale [10]. In this model the smallness of mass for right handed neutrino does not require a very heavy right handed neutrino. This mechanism requires \( m_N \sim 1 \) TeV. However in this model, a new Higgs doublet \( \eta \) with lepton number \( -1 \) is also necessary. The right handed neutrino \( N \) and Higgs \( \eta \) can give rise to LFV processes. We identify an effective operator in Standard Model and show that the scale of new physics \( \Lambda \) must be \( \Lambda \geq 5 \) TeV. In the Ma’s model, \( \mu \to 3e \) can proceed through \( N \) and \( \eta \) exchange at loop level. Using experimental limit on this process, the box diagrams provide the most stringent limit on mass of Higgs \( \eta \) \( (m_h \geq 9 \text{ TeV}) \). We also show by constructing an effective \( Z \to \mu e \) vertex and its realization through \( \eta \) and \( N \) exchange that no limit is put on \( m_h \).
The experimental evidence of the neutrino masses comes, from three anomalous effects; LSND excess \[11\] , atmospheric neutrino anomaly \[13,14,15\] and the solar neutrino deficit \[13,17,18,19\].

In addition to the three neutrinos, a sterile neutrino is needed to explain the three effects in terms of the neutrino flavor oscillations. But still the problem is unresolved \[20\]. Attempts to explain all the data in terms of the three massive neutrinos is excluded by the latest data \[21\].

Atmospheric anomaly and the solar neutrino deficit can be explained in terms of neutrino flavor oscillations, but LSND excess can not be explained on these lines, because of its small transition probability \(P_{\bar{\nu}_e \rightarrow \bar{\nu}_s} = (2.5 \pm 0.6 \pm 0.4) \times 10^{-3}\) \[24\]. It either requires a sterile neutrino or some mechanism other than neutrino flavor oscillation and LFV is one of the candidates \[23\]. We analyze the consequences of small LFV interactions to explain LSND excess and show that the branching ratio for \(\mu^+ \rightarrow e^+ \nu_l \bar{\nu}_l\) turns out to be too small to explain the LSND excess.

II. LFV IN SEESAW MODEL OF NEUTRINO MASS AND BOUNDS ON NEW HIGGS MEASON MASS

A new Seesaw model of neutrino mass has been proposed \[14\], where right handed fermion singlets \(N_i\) with lepton number \(L = 0\) are added to the minimal SM together with a second scalar doublet \((\eta^+, \eta^0)\) with lepton number \(L = -1\). The fermion singlet is allowed to have Majorana mass \(m_N\) with the effective interaction \(f_{ij} \bar{N}_{iL} \eta^0 - l_{jL} \eta^+\). The smallness of Seesaw neutrino mass \((m_\nu = \frac{m}{m_N^2})\) can be explained by a rather small value of \(m_N\), if \(m_D\) comes from \(\langle \eta^0 \rangle\) instead of \(\langle \phi^0 \rangle\) because \(\langle \eta^0 \rangle < \langle \phi^0 \rangle\), and is of the order 1 TeV and as such can be observed experimentally. As the motivation of the model is that there is no new physics beyond the TeV scale, therefore masses of the new Higgs scalar doublet, which are necessary in this model, should not be larger than a few TeV. We study this question via the experimental bounds on \(\mu \rightarrow 3e\) and \(Z \rightarrow \mu\epsilon\).

First we note that an effective operator invariant under the symmetries of the Standard Model that can induce the decay \(\mu \rightarrow 3e\) is

\[
\frac{1}{\Lambda^4} (\bar{L}_\mu e_R \Phi) (\bar{L}_e e_R \Phi)
\]

which is of dimension 8 and here \(\Phi\) is the Standard Model Higgs. On inserting the vacuum expectation value \(\langle \phi^0 \rangle = v\) it would generate the four-fermion interaction

\[
\frac{v^2}{\Lambda^4} (\bar{\mu}_L e_R) (e_L e_R).
\]

This operator leads to \(\mu \rightarrow 3e\) with the branching ratio

\[
\mathcal{B}(\mu \rightarrow 3e) = \left(\frac{v^2}{4\sqrt{2}G_F \Lambda^4}\right)^2
\]

Inserting \(v = 174\) GeV, the branching ratio \(\left[\frac{v^2}{4\sqrt{2}G_F \Lambda^4}\right]\) gives \(\Lambda \geq 4.7\) TeV. However, the above effective operator can not be induced in the Ma’s model at tree level. In this model \(\mu \rightarrow 3e\) occurs in the one loop shown in Fig. 1. The couplings at each vertex can be taken from \[14\].

The amplitude can be written as follows \[22\]:

\[
iT(\mu \rightarrow 3e) = 2 \sum_{i,j} \left( f_{\mu i}^* f_{e i} f_{e j}^* f_{e j} \right) \int \frac{d^4k}{(2\pi)^4} \bar{v}(p_1) \ k \left(\frac{1 - \gamma_5}{2}\right) v(p_2)
\]

\[
\times \bar{u}(p_4) \ k \left(\frac{1 - \gamma_5}{2}\right) v(p_4) \left[ \frac{1}{k^2 - m^2} \right] \left[ \frac{1}{k^2 - m^2} \right]
\]

\[
\left[ \frac{1}{k^2 - m^2} \right]
\]

(11)

It is assumed that the loop momenta is very high, i.e. \(k \rightarrow \infty\) so that the external momenta are neglected. After calculating the loop integration Eq. \(\left[\frac{v^2}{4\sqrt{2}G_F \Lambda^4}\right]\) becomes \[we take \(m_i = m_j = m\]

\[
T(\mu \rightarrow 3e) = \frac{1}{(4\pi)^2 \times 4m^2_h} \times \sum_{i,j} \xi_i \xi_j \left\{ \bar{v}(p_1) \gamma^\alpha (1 - \gamma_5) v(p_2) \bar{u}(p_4) \gamma_\alpha (1 - \gamma_5) v(p_4) \right\} A(x)
\]

(12)

where, \(x = \left(\frac{m^2}{m^2_h}\right)\) and \(f_{\mu i}^* f_{e i} = \xi_i, f_{e j}^* f_{e j} = \xi_j\), giving the decay width
\[ \Gamma (\mu \rightarrow 3e) = \left[ \frac{1}{(4\pi)^2 \times 4m_h^2} \right]^2 \sum_{i,k} \xi_i \xi_k^* \sum_{j,l} \xi_j \xi_l^* A(x) A(x) \times \frac{m_h^5}{192 \times \pi^3}, \]  

(13)

where,

\[ A(x) = \left\{ \frac{1 - x^2 + x \ln (x^2)}{2(x - 1)^3} \right\}. \]  

(14)

This gives the branching ratio

\[ \mathcal{B} (\mu \rightarrow 3e) = \left[ \frac{1}{4(4\pi)^2} \right]^2 \left( \frac{2}{G_F^2} \right) \sum_{i,k} \xi_i \xi_k^* \sum_{j,l} \xi_j \xi_l^* \left( \frac{x^2}{m^4} \right) [A(x)]^2. \]  

(15)

Assuming that all the Yukawa couplings are of the order unity and taking \( m = 1 \text{ TeV} \) the experimental bound (1) is obtained for \( x = 1.12 \times 10^{-2} \) giving \( m_h = 9 \text{ TeV} \).

We now consider \( Z \rightarrow \mu e \) for which the effective operator is

\[ g_{Z\mu e} \bar{L}_\mu \gamma^\mu D_\mu L_\mu \]  

(16)

or

\[ \tilde{g}_{Z\mu e} \bar{L}_R \gamma^\mu D_\mu e_R \]  

(17)

which are renormalizable operators of dimension 4 so that the effective coupling constants are dimensionless. \( D_\mu \) is the covariant derivative. The effective operator (16) is induced in the present model by renormalizable interaction represented by the Feynman diagrams shown in Fig. 2. Each of the above diagrams is logarithmically divergent; but this divergence cancels in the sum if we note that the first diagram involves \( Z \)- lepton coupling in the form

\[ (g_V + g_A) \frac{g_2}{2 \cos \theta_w} \]

\[ = \left\{ -\frac{1}{2} \left( 1 - 4 \sin^2 \theta_w \right) + \left( -\frac{1}{2} \right) \right\} \frac{g_2}{2 \cos \theta_w} \]

\[ = -\left( 1 - 2 \sin^2 \theta_w \right) \frac{g_2}{2 \cos \theta_w} \]  

(18)

while in the second diagram \( Z \rightarrow \eta \bar{\eta} \) gauge coupling is \( \frac{g_2}{2 \cos \theta_w} \left( 1 - 2 \sin^2 \theta_w \right) \). In fact the two diagrams exactly cancel in the limit \( m_Z = 0 \) and leptons mass= 0. Using the dimensional regularization these diagrams together give

\[ i T = \frac{g_2 \left( 1 - 2 \sin^2 \theta_w \right)}{2 \cos \theta_w} \frac{f^2}{(4\pi)^2} \gamma^\mu (1 - \gamma_5) \frac{v(k)}{m_h^2} \frac{m_Z^2}{m_h^2} I(x) \]  

(19)

where with \( x = m_e^2 / m_h^2 \) and where we have kept only term linear in \( m_Z^2 / m_h^2 \). \( I(x) \) is given by

\[ I(x) = -\frac{1}{36 (1 - x)} \left\{ 2 - \frac{3x}{1 - x} + \frac{6x^2}{(1 - x)^2} + \frac{6x^3}{(1 - x)^3} \ln x \right\}. \]  

(20)

In obtaining the final result we have neglected a convergent contribution from the second diagram which is proportional to the lepton mass. Using \( \sin^2 \theta_w \simeq \frac{1}{4} \), Eq. (19) gives

\[ \Gamma (Z \rightarrow \mu e) = \frac{G_F}{\sqrt{2}} \frac{m_e^3}{6\pi} \frac{1}{2} \left( \frac{1}{4\pi} \right) f^4 \left( \frac{m_Z^2}{m_h^2} \right)^2 \]  

(21)

while

\[ \Gamma_{Z}^{\text{tot}} = \frac{8G_F m_Z^3}{\sqrt{2}} \]  

(22)

Thus the branching ratio is given by
where we have taken the Yukawa couplings \( f \simeq 1 \). Taking the two extreme limits \( x \to 1 \) and \( x \to 0 \), \( I(x) \) is respectively \( \frac{1}{x} \) and \( \frac{1}{x^3} \); the branching ratio bound \( \[2\] \) can be satisfied for \( m_h \geq m_t \simeq 500 \text{ GeV} \). Thus no limit is put on \( m_h \).

### III. THE ANOMALOUS MUON DECAY AND LSND

The standard muon decay \( \mu^+ \to e^+ \nu_e \bar{\nu}_\mu \) has no \( \bar{\nu}_e \) which is found at LSND. Instead of the neutrino flavor oscillation this excess can be found through the LFV muon decay \( \mu^+ \to e^+ \nu_l \bar{\nu}_l \). The anomalous muon decay \( \mu^+ \to e^+ \nu_l \bar{\nu}_l \) (where \( l = e, \mu, \text{ or } \tau \) ) can occur via \( Z \)-exchange as shown in Fig. 3, where the effective Lagrangian \( Z\mu e \) vertex can be written as \( \[3\] \)

\[ \mathcal{L}_{\text{eff}} = g_{Z\mu e} \bar{\mu} L \gamma^\alpha \gamma_L Z^\alpha + \text{h.c.} \]  

(24)

Here, \( g_{Z\mu e} \) is the effective coupling of the LFV vertex which is constrained from the experimental bound on the branching ratio of \( \mu \to 3\nu \). The coupling through the diagram of Fig. 3, contributes to the \( A (\mu \to e\nu_l \bar{\nu}_l) \) amplitude a term

\[ A (\mu \to e\nu_l \bar{\nu}_l) = \frac{g_{Z\mu e} g_{Z\nu\bar{\nu}}}{M_Z^2 - s} \left[ \tilde{v}(p_1) \gamma^\alpha \gamma_L (p_2) \gamma_L (p_3) (1 - \gamma_5) v(p_3) \right] \]  

(25)

The corresponding decay width becomes

\[ \Gamma (\mu \to e\nu_l \bar{\nu}_l) = \left( \frac{g_{Z\mu e} g_{Z\nu\bar{\nu}}}{M_Z^2} \right)^2 \frac{m_\mu^5}{192\pi^3} G_F^2 \]  

\[ = \frac{g_{Z\mu e}^2}{2} \left( \frac{G_F}{\sqrt{2}} \right)^2 \frac{m_\mu^5}{192\pi^3} \]  

(26)

where we have used that \( g_{Z\nu\bar{\nu}} = \left( \frac{g_0}{2 \cos \theta_W} \right) \frac{1}{2} \). This gives the branching ratio

\[ \mathcal{B} (\mu \to e\nu_l \bar{\nu}_l)_{Z-\text{exch.}} = g_{Z\mu e}^2 \]  

\[ = \left( 4\sqrt{2} G_F m_Z^2 \right) \mathcal{B} (Z \to \mu e) \]  

\[ = (0.55) \mathcal{B} (Z \to \mu e) \]  

(27)

where we have used Eq. \( \[23\] \). If we use the bound \( \[8\] \), we obtain

\[ \mathcal{B} (\mu \to e\nu_l \bar{\nu}_l)_{Z-\text{exch.}} \leq 10^{-13}. \]  

(28)

which is much too small compared to the needed branching ratio for \( \mu \to e\nu_l \bar{\nu}_l \) implied by

\[ P_{\nu_\mu \to \nu_e} = (2.5 \pm 0.6 \pm 0.4) \times 10^{-3}. \]  

(29)

Even if one uses the direct limit \( \[8\] \) it still remains small \( \simeq 10^{-6} \).

Since there is no \( \nu_R \) in the Standard Model, one cannot write an operator of the form \( \[8\] \) with \( e_R \) replaced by \( \nu_R \). One can however write two dimension 9 operators as in \( \[23\] \) which would generate \( \Delta L = 2 \) decay \( \mu^+ \to e^+ \nu_l \bar{\nu}_l \) with a branching ratio which could explain LSND excess with out any conflict with \( \Delta L = 0 \) processes like \( \mu \to 3\nu \) with a scale of new physics at a rather low value \( \Lambda \simeq 360 \text{ GeV} \). However, such operators cannot be induced in the present model at tree level. If one considers the box diagrams for \( \mu \to e\nu_l \bar{\nu}_l \), which gives the same result as in Eq. \( \[13\] \), namely

\[ \mathcal{B} (\mu \to e\nu_l \bar{\nu}_l) = \left( \frac{1}{64\pi^2 m_t} \right)^2 \frac{2}{G_F^2} \left( 81 f^8 \right) \left[ x A(x) \right]^2 \]  

(30)

which gives the branching ratio to be \( \approx 10^{-12} \) for \( f \approx 1 \) and \( m_h \approx 9 \text{ TeV} \) as previously found. This is much too small compared to the required value \( \simeq 2.5 \times 10^{-3} \).
IV. CONCLUSION

By studying process $\mu \to 3e$ at the loop level, we have put a bound on the mass of the new Higgs boson $m_h$ needed in the Seesaw model of neutrino masses [10]. Taking the Yukawa-couplings $f$ to be order 1 and mass of the heavy neutrino to be 1 TeV as in [10], we found the bound on $m_h \geq 9$ TeV. No limit is put on $m_h$ from the experimental bound on $Z \to \mu e$. The LSND neutrino anomaly in terms of the decay $\mu^+ \to e^+ \nu_l \bar{\nu}_l$ which requires the branching ratio of the new decay to be about $(1.5 - 3) \times 10^{-3}$ can not be explained in the present model. Lastly we wish to discuss the sensitivity of the above bound on $f$. First of all we note that atmospheric anomaly required $m_\nu \geq 5 \times 10^{-2}$ eV which in Ma’s model implies that $f^2 \geq 5 \times 10^{-2}$ keeping $m_i \simeq 1$ TeV. Then one obtains no bound on $m_h$.

Acknowledgments

One of us (Jamil) like to thank Professor Fayyazuddin for stimulating discussions and Gilani for ever ready help. This work was supported by a grant from Pakistan Council of Science and Technology. A part of work of (Jamil) was also supported by Mumtaz Riazuddin fellowship.

[1] Particle Data Group, C. Caso et. al., Eur. Phys. J. C 15, (2000) 1.
[2] David Delepine and Francesco Vissani, INFN/TH-01/02.
[3] S. Nissinov, R. D. Peccei and X. M. Zhang, Phys. Rev. D63, (2000) 016003-1.
[4] J. I. Illana, M. Jack and T. Riemann, hep-ph/0001274.
[5] J. I. Illana and T. Riemann, hep-ph/0010193.
[6] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); E. Ma and U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998)
[7] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[8] E. Ma and U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998).
[9] R. Foot, H. Sew, X. F. He, and G. C. Joshi, Z. Phys. C44, 441 (1998).
[10] Ernest Ma, Phys. Rev. Lett. 86, (2001) 2502.
[11] C. Athanassopoulous et. al. [LSND Collaboration], [nucl-ex/9605001]; 9605003.
[12] C. Athanassopoulous et. al. [LSND Collaboration], [nucl-ex/9706000]; 9709006.
[13] S. J. Freedman et al., Phys. Rev. D47, 811 (1993).
[14] S. Fukuda et al. (Super-Kamiokande Collaboration), hep-ex/0009001 (2000)
[15] W.W.M. Allison et al. (Soudan 2 Collaboration), Phys. Lett. B449, 137 (1999).
[16] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998).
[17] Kamiokande, Y. Fukuda et al. Phys. Rev. Lett. 77, 1683 (1996).
[18] GALLEX, W. Hampel et al. Phys. Lett. B447 127 (1999).
[19] SAGE, J. N. Abdurashitov et al., astro-ph/9907113.
[20] Y. Fukuda et al. (Kamiokande Collaboration), Phys. Lett. B335, 237 (1994).
[21] C. Y. Cardall and G. M. Fuller, Phys. Rev. D53, 4421 (1996); A. Acker and S. Pakvasa, Phys. Lett. B397, 207 (1997); R. P. Thun and S. McKee, Phys. Lett. B439, 123 (1998).
[22] G. Mills [for the LSND Collaboration], Talk presented at Neutrino 2000, Sudbury, Canada,16-21 June 2000.
[23] A. Bueno, M. Campanelli, M. Laveder, J. Rico, A. Rubbia, November 1, 2000.
[24] M. K. Gaillard and Benjamin W. Lee, Phys. Rev. D10, 897 (1974).
[25] Seven Bergmann and Yuval Grossman, hep-ph/9809524.
[26] K. S. Babu and Sundip Pakvasa, hep-ph/0204236.

Figure Captions:

1. Box diagrams for $\mu \to 3e$
2. One loop diagrams for $Z \to \mu e$
3. Anamolous muon decay
Figure 1
Figure 2
