I. INTRODUCTION

The nuclear shell model is one of the most popular theoretical frameworks which has been extensively used for understanding nuclear structural properties. Its success mainly depends on the two-nucleon effective interaction employed in it. There are many effective interactions available in pf-shell for shell model calculation, e.g., KB, KB3, KB3G, GXPF1, GXPF1A, GXPF1B, and so on[1–5]. Based on the experimental data, a constant improvement in these interactions have been made to find a reliable interaction which can describe nuclear structural properties accurately and systematically. The recent interaction GXPF1B is the upgraded version of GXPF1A. It is made by changing single-particle energies and two-body matrix elements of GXPF1A related to 1p1/2 and 1p3/2 orbitals in order to match with the recent experimental data for 51–54Ca. The GXPF1B interaction predicts ground and excited states of pf-shell nuclei very well. However, in the recent experimental interaction predicts ground and excited states of pf-shell nuclei very well. However, in the recent experimental data for 48,52Ca. The improved interaction is named as GX1R, and used to calculate the effective-single particle energies of \( \nu = 0f1p \) orbitals in Ca chain, and low-lying states of \( ^{52–54} \)Ca isotopes, consists of both even and odd-\( A \) isotopes. The excitation energies of \( 2^+_1 \) state of even Ti and Cr isotopes are also calculated. It is found that this new interaction satisfactorily describes \( N = 32 \) and 34 semi-magic shell gap in Ca isotopes, and fairly reproduces the excitation energy data of Ca, Ti and Cr isotopes.

The discussion on characteristic properties of tensor force is given in Refs.[11, 12]. The angular momentum averaged (monopole) two-body matrix elements of tensor force

\[
V_T^{jj'} = \sum_j (2J + 1) < jj' | V | jj' > J_T, \quad (1)
\]

are repulsive, if the nucleons are in either both spin up \((j_+ \text{ and } j'_+)\) or both spin down \((j_- \text{ and } j'_-)\) orbitals, whereas attractive if the nucleons are either \((j_+ \text{ and } j'_-)\) or \((j_- \text{ and } j'_+)\) orbitals. where \(j_+ = l + \frac{1}{2} \) (spin-up) and \(j_- = l - \frac{1}{2} \) (spin-down) partners. In addition to this, tensor force holds renormalization persistency [13, 14], i.e., bare tensor force is not much differing from the tensor force obtained from the effective interactions.

It has been shown in Refs.[13, 14] that tensor force matrix elements \( \bar{V}_T^{j=0} \) have the same property, and matrix elements \( \bar{V}_T^{j=0} \) hold their property against the renormalization procedure. Further, the numerical analysis shows that the tensor force of well-established shell model effective interaction, e.g., USDB, has the same property as for \( \bar{V}_T^{j=0} \). However, it has been pointed out by Wang et al. [13] that tensor force matrix elements \( \bar{V}_T^{j=1} \) and \( \bar{V}_T^{j=1} \) of widely used GXPF1A interaction do not have the property as unveiled by Otsuka [12]. Along the line, we have also carried out the spin-tensor decomposition of GX1B1 interaction, and calculated T=1 tensor force monopole matrix elements of GX1B1 interaction. In our calculation, it is found that the characteristic properties of tensor force are missing for some of the T=1 matrix elements of GX1B1. The discrepancies arising in the T=1 tensor force of effective interaction may be due to the lack of adjustment of many-body forces in effective interactions. Thus, in the present study, we have made an attempt to modify GX1B1 in such a way that it incorporates the desired characteristic properties for two nucleon interactions compatible with the observed tensor force [11, 12]. We have noticed that such changes correctly describe the feature of T=1 tensor force in the effective interaction, and produce quite satisfactory results as explored in this study.

---

* kanhaiya.jha@iitrpr.ac.in
This article is organized as follows. In Section II, we have discussed the discrepancies of T=1 tensor force monopole matrix elements present in the GX1B1 interaction. In Section III, a theoretical framework used for modification of interaction is presented. In Section IV, we have tested the modified interaction by performing shell-model calculations. The summary of this work is given in Section V.

II. T = 1, TENSOR FORCE MONOPOLE MATRIX ELEMENTS OF GX1B1

Spin-tensor decomposition method [14, 17] has been used to separate tensor force matrix elements of GX1B1. The T=1 tensor force monopole matrix elements (|V_{jj'}^T=1|) of GX1B1 are shown in Fig. 1. Based on the above discussion of characteristic properties of tensor force, |V_{7/2,7/2}^T=1|, |V_{5/2,5/2}^T=1|, |V_{7/2,5/2}^T=1|, |V_{5/2,5/2}^T=1|, |V_{7/2,3/2}^T=1|, |V_{5/2,3/2}^T=1| should be repulsive, and |V_{7/2,1/2}^T=1|, |V_{5/2,1/2}^T=1| should be attractive, but the observed trends are found to be opposite, respectively. It is found that the observed trends for |V_{jj'}^T=0| of GX1B1 are very well satisfying the basic feature of tensor force, see Fig. 2. It shows that the tensor force is well established in the T=0 channel, but large irregularity is present in the T=1 channel. This kind of discrepancies in T=1 tensor force matrix elements is also reported in Ref. [15]. The authors [15] showed that the uncertainties in the T = 1 tensor force obtained from different effective interactions may be due to the use of different adjustments parameters for different interactions. Also, as the many-body forces are the least known part of nuclear interactions, it could be the cause of the uncertainties in T = 1 tensor force. Whateover the reason of uncertainty, it is essential to incorporate T = 1 tensor force properly in the effective interaction, so that a fair judgment of its effect can be made in shell evolution and structure properties of the nuclei.

III. MODIFIED EFFECTIVE INTERACTION

In order to ameliorate the discrepancies of |V_{jj'}^T=1| of GX1B1, we have separately calculated the tensor force matrix elements for tensor force

\[ V_T = V(r)\sqrt{\frac{24\pi}{5}} |Y(2), (\sigma_1 X \sigma_2) (2)| (\tau_1, \tau_2) \]

and replaced them for the T=1 tensor force matrix elements of GX1B1. The radial dependency of two nucleon interactions in above expression of tensor force is treated with Yukawa potential

\[ V(r) = -V_0 e^{-r/a} \]

where, ‘a’ is Compton scattering length of pion given as 1.4157 fm for mass of pion m_π = 139.4 MeV, and V_0 is strength parameter, which, in our calculations is obtained by fitting |V_{jj'}^T=1| calculated for sd-shell with |V_{jj'}^T=1| of USDB [18] interaction. This fitting procedure was first attempted for T=0 matrix elements. The obtained V_0 was used to calculate |V_{jj'}^T=0| in pf-shell, and was found well reproducing |V_{jj'}^T=0| of GX1B1. Thus, it gave us the confidence to replace the T = 1 tensor force matrix elements of GX1B1 by separately calculated matrix elements. Fig. 2 shows the similarity among |V_{jj'}^T=0| of GX1B1 and calculated ones, which is quite satisfactory. In Fig. 3, |V_{jj'}^T=1| calculated in pf-shell are shown. These |V_{jj'}^T=1| have their characteristic features. After replac-

\[ V(r) = -V_0 e^{-r/a} \]

FIG. 1. Tensor force monopole matrix elements of GX1B1 interaction for isospin T = 1.

FIG. 2. Tensor force monopole matrix elements in fp-shell for isospin T = 0. Red and black colors are used for the calculated matrix elements and the matrix elements of GX1B1, respectively.
have been changed by -1.2 MeV, -0.4 MeV and 0.7 MeV respectively. The new SPEs’ are -9.8240 MeV, -6.0793 MeV and -0.6829 MeV respectively. The new interaction is named as GX1R.

IV. SHELL MODEL CALCULATION

A. Single-particle energy gaps in Ca isotopes

The modified GX1R interaction has been made by changing ninety-four $T=1$ two body matrix elements, and single-particle energies of $0f_{7/2}$, $0f_{5/2}$, and $1p_{3/2}$ orbitals of GX1B1. In order to clarify its applicability, we have calculated single-particle energies gaps at $N=28$, $32$ and $34$. The expression of the effective single-particle energy (ESPE’s) of neutron orbit $j'$ in Ca isotopes is given as

$$\epsilon_j^{\nu'}(A) = \epsilon_j^{\nu} + \sum_j n_j^\nu V^{\nu\nu'}_j(A),$$

where $V^{\nu\nu'}_j(A)$ is mass dependent monopole matrix elements, and $n_j^\nu$ is the number of neutron in valence orbitals $j$. Fig. 4, shows the variation of ESPEs’ of $\nu-0f1p$ orbitals for $^{40-54}$Ca isotopes. For $^{40}$Ca, ESPEs’ of all $\nu - 0f1p$ orbitals are equal to their unperturbed single-particle energies $\epsilon_j^{\nu}$ as there is no valence neutron present in pf-model space. The $\nu(1p_{3/2} - 0f_{7/2})$ gap first increases from $^{40}$Ca to $^{48}$Ca as the $\nu0f_{7/2}$ filled with neutron, and then decreases when the orbitals $\nu1p_{3/2}$ and $\nu1p_{1/2}$ are filled with neutrons. The $\nu(1p_{3/2} - 0f_{7/2})$ gap is maximum for $^{48}$Ca, and it is the signature of a conventional $N = 28$ magic shell gap. The spin-orbit partners gap $\nu (1p_{1/2} - 1p_{3/2})$ is nearly constant when $\nu0f_{7/2}$ orbit filled with neutron, while this gap increases from $^{48}$Ca to $^{52}$Ca indicating a sub-shell at $N=32$. Further, this gap slightly decreases when orbit $\nu1p_{3/2}$ filled with neutrons. The $\nu(0f_{5/2} - 1p_{1/2})$ gap first decreases, and then increases as the neutrons start filling in the orbitals $\nu0f_{5/2}$, and $\nu1p_{3/2}$ and $\nu1p_{1/2}$, respectively. The sizable gap between $\nu0f_{5/2}$ and $\nu1p_{1/2}$ orbitals at $N=34$ in Ca isotopes is the signature of sub-shell at $N=34$. In Table 1, we summarize the sensitivity of the orbital energies gaps $\nu(1p_{3/2} - 0f_{7/2})$, $\nu(1p_{1/2} - 1p_{3/2})$ and $\nu(0f_{5/2} - 1p_{1/2})$ present at $N = 28$, $32$ and $34$ to the different com-
nants of $\nu - \nu$ interaction of GX1R interaction. For $\nu(1p_{1/2} - 0f_{7/2})$ gap at $N = 28$, central force has dominant contribution in splitting of orbitals $\nu 0f_{7/2}$ and $\nu 1p_{3/2}$ apart, and spin-orbit force contribute almost half to the central force to increase this gap. The tensor force acts opposite to both central and spin-orbit forces to reduce this gap. The large gap between $\nu 0f_{7/2}$ and $\nu 1p_{3/2}$ orbitals at $N = 28$, is the signature of a conventional $N = 28$ magic shell gap. It can be seen from Fig. 4, large reduction of this gap from $N = 28$ to 34 when the orbitals $\nu 1p_{3/2}$ and $\nu 1p_{1/2}$ are filled with neutrons. The central and spin-orbit forces play an important role in lowering of this gap from $N = 28$ to 32, while central force is mainly responsible in lowering of this gap from $N = 32$ to 34. For spin-orbit partners $\nu(1p_{1/2} - 1p_{3/2})$ gap at $N = 28$, spin-orbit force has significant contribution, while both central and tensor forces nullify it’s contribution, resulting in nearly constant $\nu(1p_{1/2} - 1p_{3/2})$ gap at $N = 28$. The central and spin-orbit forces both equally contribute to increasing this gap from $N = 28$ to 32, whereas the tensor force contributes almost half to them towards decreasing this gap. Consequently, the gap $\nu(1p_{1/2} - 1p_{3/2})$ markedly increases by $\sim 0.44$ MeV at $N = 32$. The increasing of this gap from $N = 28$ to 32 indicating a sub-shell at $N = 32$. This gap decreases when $\nu 1p_{1/2}$ orbit filled with the neutrons. The central force has a large contribution for lowering this gap, while the tensor force is contributing in the opposite direction of central force to increase this gap. It is remarkable that the central, as well as non-central forces, acts in the same direction to decrease the gap between the $\nu 0f_{5/2}$ and $\nu 1p_{1/2}$ orbitals at $N = 28$. Further, the central force plays a vital role in increasing this gap from $N = 28$ to 34, and hence important for the formation of sub-shell closure in Ca isotopes. The tensor force, however, contributing small but reducing this gap from $N = 32$ to 34. The spin-orbit force, which contributes almost half to the tensor force, also work for reducing this gap from $N = 32$ to 34.

**B. Low lying states of Ca, Ti and Cr**

In order to test the validity of interaction GX1R, we have carried out shell model calculations for Ca, Ti and Cr isotopes. The calculations have been performed with

FIG. 5. Level structure of odd-A $^{43-53}$Ca isotopes. Theoretical calculations are performed with GX1B1 and GX1R. The experimental data is taken from Ref. [25].
shell-model code NUSHELLX@MSU [20]. We have calculated excitation energies for low-lying states of \( ^{43-54}\text{Ca} \) isotopes and \( 2^+_1 \) states of Ti and Cr isotopes.

Fig. 5 shows level structure of odd-\( A(43-53) \) Ca isotopes. For \( ^{43,45}\text{Ca} \), GX1R reasonably reproduce the experimental level structure. For \( ^{47-53}\text{Ca} \), there are only very few experimental states up to 2.5 MeV, and these states are fairly reproduced by GX1R. In \( ^{47}\text{Ca}, \frac{4}{2}^- \) state measured at 2.01 MeV is predicted at 2.33 MeV. This state is dominated by \( \nu p_{3/2} \otimes \nu f_{7/2} \) configuration with 92.37% contribution. Therefore, this state has effect of \( N = 28 \) magic shell gap which appears from its high excitation energy. In \( ^{49,51}\text{Ca}, \frac{5}{2}^- \) states measured at 2.02 MeV and 1.72 MeV are calculated at 2.18 MeV and 1.68 MeV, respectively. These states are dominated by \( \nu p_{1/2} \otimes \nu p_{3/2}^2 \nu f_{7/2}^8 \) configurations with 91.22% and 86.02% contribution, respectively. The high excitation energy of these states is a signature of \( N = 32 \) semi-magic shell gap between orbitals \( 1p_{3/2} \) and \( 1p_{1/2} \). In \( ^{53}\text{Ca}, \frac{2}{1}^- \) and \( \frac{8}{2}^- \) states measured at 1.75 MeV and 2.2 MeV are calculated at 1.61 MeV and 2.24 MeV, respectively. The \( \frac{5}{2}^- \) state of \( ^{53}\text{Ca} \) is dominated by \( \nu f_{5/2} \otimes \nu p_{1/2} \nu p_{3/2}^4 \nu f_{7/2}^8 \) configuration with 86.90% contribution. The high excitation energy of this state is a signature of \( N = 34 \) semi-magic shell gap between orbitals \( 1p_{3/2} \) and \( 0f_{5/2} \).

In Fig. 6, we have shown the level structures of even-\( A(44-54) \) Ca isotopes. For \( ^{44,46}\text{Ca} \), the experimental low-lying states are satisfactorily reproduced. For doubly-magic isotope \( ^{48}\text{Ca} \), the calculated and measured excitation energies of \( 2^+_1 \) state are found to be same. For \( ^{50}\text{Ca} \), the calculated energy difference between \( 2^+_1 \) and \( 2^+_2 \) state is 1.98 MeV, whereas, the measured energy difference is 2.12 MeV. In the experimental level structure of \( ^{50}\text{Ca} \), spin-parity of the third excited state is tentatively assigned. In the calculation, this state comes as \( 1^+ \). For semi-magic nuclei \( ^{52,54}\text{Ca} \), the calculated excitation energies of \( 2^+_1 \) are 2.59 MeV and 2.00 MeV, respectively, which are in good agreement with the experimental measured excitation energies, \( i.e., \), 2.56 MeV and 2.04 MeV. The calculations for Ca isotopes are also performed with GXPF1B1 interaction and obtained results are shown in Fig. 5 and Fig. 6. For \( ^{43-51}\text{Ca} \) and \( ^{44-52}\text{Ca} \), the prediction of GX1B1 is same as of GX1R. However, for \( ^{53,54}\text{Ca} \), it is slightly different. For \( ^{53}\text{Ca}, \) GX1B1 does
for $\nu = 34$ semi-magic has fragile nature, and it breaks down 68 semi-magic shell gap [21, 22]. But, this E(2$^{+}$) of open-shell isotopes 56 subsequently quadrupole collectivity enhances. As mentioned 56 of Ca, GXPF1B1 relative to GX1R, predicts 3$^{+}$ and 0$^{+}$ state approximately 0.4 52 of Ca is a signature of N = 34 semi-magic shell gap. However, the notable decrease in 56 MeV higher in energy. In Fig. 7, the evolution of the excitation energies of 2$^{+}$ state in Ca, Ti and Cr isotopes is shown. A good agreement is found between theory and experiment in this figure. Further, in Fig. 7, the first peak is the signature of a conventional N = 28 magic 52 shell gap. The second peak is the signature of N = 32 semi-magic shell gap. For 56Ca, GXPF1B1 relative to GX1R, predicts 3$^{+}$ and 0$^{+}$ state approximately 0.4 MeV higher in energy. In Fig. 7, the evolution of the excitation energies of 2$^{+}$ state in Ca, Ti and Cr isotopes is shown. A good agreement is found between theory and experiment in this figure. Further, in Fig. 7, the first peak is the signature of a conventional N = 28 magic 52 shell gap. The second peak is the signature of N = 32 semi-magic shell gap. However, the notable decrease in E(2$^{+}$) from $^{52}$Ca to $^{56}$Cr connotes that the N = 32 0 state approximately 0.4 MeV higher in energy. In Fig. 7, the evolution of the excitation energies of $^{2+}$ state in Ca, Ti and Cr isotopes is shown. A good agreement is found between theory and experiment in this figure. Further, in Fig. 7, the first peak is the signature of a conventional N = 28 magic shell gap. The second peak is the signature of N = 32 semi-magic shell gap. However, the notable decrease in E(2$^{+}$) from $^{52}$Ca to $^{56}$Cr connotes that the N = 32 semi-magic shell gap reduces with the increase in Z, and consequently, quadrupole collectivity enhances. As mentioned earlier, the high E(2$^{+}$) of $^{54}$Ca is a signature of N = 34 semi-magic shell gap [21, 22]. But, this E(2$^{+}$) reduces to half of its value for $^{56}$Ti, which is nearly equal to E(2$^{+}$) of open-shell isotopes $^{52,56}$Ti. Thus, it manifests that N = 34 semi-magic has fragile nature, and it breaks down for $^{56}$Ti. This situation is similar to N = 40 semi-magic shell gap whose signatures are seen in $^{68}$Ni, while, they are absent in $^{70}$Zn [23, 24]. However, in theoretical study 8, 26, it has been reported that N = 34 semi-magic gets strong for Z < 20 nuclei. Recent experimental results of 52Ar also support this fact 27.

There is quite a number of recent studies available for pf-shell nuclei 28, 38. These studies reasonably describe the structure and spectroscopic properties of pf-shell nuclei if three-nucleon force is included in the calculations. In the present, though, we have adopted an empirical method based on spin-tensor decomposition to modify an effective interaction, but, it seems to be righteous in all the dimensions. In the new interaction, the J-averaged tensor force matrix elements have their common features, and with the new interaction, the properties of pf-shell nuclei can be described satisfactorily. Therefore, it can be inferred that the effect of three-body force and other many-body effect have been adequately incorporated in this new interaction, which is assumed to be not present appropriately in GX1B1 interaction. Further, this interaction can be used as a supplementary part of the interaction for fp$g_{9/2}$d$5/2$ model space, like GXPF1B1 39, for precisely describing the properties of very neutron-rich pf-shell nuclei.

V. SUMMARY

In this work, the spin-tensor decomposition method has been used to investigate the properties of tensor force monopole matrix elements of GX1B1 interaction, and it is found that many of its T=1 matrix elements do not have the base features. Hence, to improve this discrepancy, the ninety-four T = 1 two-body matrix elements of GX1B1 interaction have been modified by making apt use of the spin-tensor decomposition method. The single-particle energies of 0$f_{7/2}$, 0$f_{5/2}$, and 1$p_{3/2}$ orbitals are also modified to reproduce the E(2$^{+}$) of $^{48,54}$Ca. The modified interaction have been named GX1R.

In GX1R interaction, tensor force monopole matrix elements have their common features, and this interaction satisfactorily describes the properties of Ca, Ti and Cr isotopes. It is found that both the central and spin-orbit forces are important to develop N = 28 magic shell gap between $\nu 1p_{3/2}$ and $\nu 0f_{7/2}$ orbitals, and N = 32 semi-magic shell gap between $\nu 1p_{1/2}$ and $\nu 1p_{3/2}$ orbitals. To develop N = 34 semi-magic shell gap between $\nu 0f_{5/2}$ and $\nu 1p_{1/2}$ orbital, the contribution of central force is found to be crucial. Further, the calculated excitation energies of the low-lying states of $^{43–54}$Ca isotopes, and the excitation energies of 2$^{+}$ state of Ti and Cr isotopes are found to be in good agreement with experimental data.

VI. ACKNOWLEDGMENTS

K. Jha acknowledges P. K. Rath, and S. K. Ghorui for their interest and useful scientific discussion, and Ministry of Human Resource and Development, Government of India for providing financial support.
[1] Kuo et al., Nucl Phys A, 114: 241279 (1968)
[2] A. Poves et al., Phys. Rep., 70: 235314 (1981)
[3] A. Poves, J. Sanchez-Solano et al., Nucl Phys A, 694: 157198 (2001)
[4] M. Homma, T. Otsuka, B. A. Brown et al., Phys Rev C 65, 061301(R) (2001), Phys Rev C 69, 034335 (2004)
[5] M. Homma, T. Otsuka, B. A. Brown et al., Eur. Phys. J. A 25, 499-502 (2005)
[6] M. Homma et al., Shell-model description of neutron-rich Ca isotopes, RIKEN Accel. Prog. Rep. 41, 32, (2008)
[7] D. Steppenbeck et al., Evidence for a new nuclear magic number from the level structure of $^{54}$Ca, Nature 502, 207, (2013)
[8] T. Otsuka, Rintaro Fujimoto et al., Magic Numbers in Exotic Nuclei and Spin-Isospin Properties of the NN Interaction, Phys. Rev. Lett., 87, 082502, (2001)
[9] T. Otsuka, Toshih Suzuki et al., Novel Features of Nuclear Forces and Shell Evolution in Exotic Nuclei, Phys. Rev. Lett. 104, 125201 (2010)
[10] P. Kumar et al., Proton-neutron force and Proton-single particle strength in Sc, F and Li isotopes, Phys. Rev. C, 100, 024328 (2019)
[11] A. Uneya et al., Roles of NN-interaction components in shell-structure evolution, Nucl. Phys. A 955, 194 (2016)
[12] T. Otsuka, Toshih Suzuki et al., Evolution of Nuclear Shells due to the Tensor Force, Phys. Rev. Lett. 95, 232502 (2005)
[13] Tsunoda, T. Otsuka et al., Renormalization persistency of the tensor force in nuclei, Phys. Rev. C 84, 044322 (2011)
[14] Tsunoda, T. Otsuka et al., Tensor force in effective interaction of nuclear force, Journal of Phy: Conf. Series 267 (2011)
[15] X. B. Wang and G X Dong, Revisiting the monopole components of effective interactions for the shell model, J. Phys. G: Nucl. Part. Phys. 42, 125101 (2015); A short revisit to Kuo-Brown effective interactions, Sci China-Phys Mech Astron, 58: 102001 (2015)
[16] M. W. Kirson, Spin-Tensor Decomposition of Nuclear Effective interactions, Phys. Lett. B. 47, 110 (1973)
[17] K. Klingenbeck et al., Central and noncentral components of the effective sd-shell interaction, Phys. Rev. C 15, 1483 (1977)
[18] B. A. Brown and W. A. Richter, New USD Hamiltonians for the sd shell, Phys. Rev. C 74, 034315 (2006)
[19] N. A. Smirnova, K. Heyde et al., Nuclear shell evolution and in-medium NN interaction, Phys. Rev. C 86, 034314 (2012)
[20] B. A. Brown and W. D. M. Rae, The Shell-Model Code NUSHELLX@MSU, Nucl. Data Sheets 120, 115 (2014)
[21] S. Michimasa et al., Magic Nature of Neutrons in $^{54}$Ca: First Mass Measurements of $^{55-57}$Ca Phys. Rev. Lett. 121, 022506 (2018)
[22] M. Rejmund, S. Bhattacharyya et al., Shell evolution and the N = 34 magic number, Phys. Rev. C 76, 021304(R) (2007)
[23] O. Perru et al., Enhanced Core Polarization in $^{70}$Ni and $^{72}$Zn, Phys. Rev. Lett. 96, 232501 (2006)
[24] M. Homma et al., New effective interaction for f$^5p^5g$-shell nuclei, Phys. Rev. C 80, 064323 (2009)
[25] [http://www.nndc.bnl.gov/ensdf/]
[26] S. N. Liddick, P. F. Manita et al., Lowest Excitations in $^{56}$Ti and the Predicted N=34 Shell Closure, Phys. Rev. Lett. 92, 072502 (2004)
[27] H. N. Liu, A. Obertelli et al., How Robust is the N = 34 Subshell Closure? First Spectroscopy of $^{52}$Ar, Phys. Rev. Lett. 122, 072502 (2019)
[28] Wienholtz, F. et al., Masses of exotic calcium isotopes pin down nuclear forces, Nature 498, 346349 (2013)
[29] H. L. Crawford et al., Unexpected distribution of $\nu 1f_{7/2}$ strength in $^{49}$Ca, Phys. Rev. C 95, 064317 (2017)
[30] R. F. Garcia Ruiz, M. L. Bissell et al., Ground-state electromagnetic moments of calcium isotopes, Phys. Rev. C 91, 041304(R) (2015); Unexpectedly large charge radii of neutron-rich calcium isotopes, Nat. Phys. 12, 594 (2016)
[31] E. Leistenschneider et al., Dawning of the N = 32 Shell Closure Seen through Precision Mass Measurements of Neutron-Rich Titanium Isotopes, Phys. Rev. Lett. 120, 062503 (2018)
[32] D. C. DincA, R. V. F. Janssens et al., Phys Rev C 71, 041302(R) (2005)
[33] G. Hagen, M. Hjorth-Jensen et al., Evolution of Shell Structure in Neutron-Rich Calcium Isotopes, Phys. Rev. Lett. 109, 032502 (2012)
[34] J. D. Holt, J. Menendez et al., Three-nucleon forces and spectroscopy of neutron-rich calcium isotopes, Phys. Rev. C 90, 024312 (2014)
[35] J. D. Holt, J. Menendez et al., The role of three-nucleon forces and many-body processes in nuclear pairing, J. Phys. G 40, 075105 (2013)
[36] E. Yuksel, N. Van Giai et al., Effects of the tensor force on the ground state and first $2^+$ states of the magic $^{54}$Ca nucleus, Phys. Rev. C 89, 064322 (2014)
[37] S. M. Lenzi, F. Nowacki et al., Island of inversion around $^{60}$Cr, Phys. Rev. C 82, 054301 (2010)
[38] S. R. Stroberg et al., Nucleus-Dependent Valence-Space Approach to Nuclear Structure, Phys. Rev. Lett. 118, 032502 (2017)
[39] Tomoaki Togashi, Noritaka Shimizu et al., Large-scale shell-model calculations for unnatural-parity high-spin states in neutron-rich Cr and Fe isotopes, Phys. Rev. C 91, 024320 (2015)