INVESTIGATION OF ANOMALOUS AXIAL QED

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Abstract

Although axial QED suffers from a gauge anomaly, gauge invariance may be maintained by the addition of a nonlocal counterterm. Such nonlocal counterterms, however, are expected to ruin unitarity of the theory. We explicitly investigate some relevant Feynman diagrams and show that, indeed, unitarity is violated, contrary to recent claims.

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I. INTRODUCTION

We want to investigate axial QED in four dimensions, i.e. the theory of one massless fermion coupled to a gauge field $A_\mu$ via the axial current $J^5_\mu = \bar{\Psi} \gamma^\mu \gamma^5 \Psi$. The Lagrangian reads (we use the conventions $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ and $\epsilon_{0123} = 1$)

$$L = \bar{\Psi}(i\partial / + eA /\gamma^5)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$ (1)

The generally accepted point of view is that this theory may not be quantized consistently because of the axial anomaly [1] – [4] that spoils gauge invariance [5]. It is, however, wellknown that the effective gauge field action

$$e^{iS_{\text{eff}}[A_\mu]} = \int D\bar{\Psi} D\Psi e^{iS[\Psi, \bar{\Psi}, A_\mu]}$$ (2)

may be made gauge invariant by the addition of a nonlocal counterterm (see e.g. [6,7])

$$S_{\text{ct}} = \frac{e^3}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \int d^4x d^4y \partial_x A_\lambda(x) \square^{-1}(x - y) F_{\mu\nu}(y) F_{\alpha\beta}(y)$$ (3)

Via the axial anomaly

$$\partial_\mu J^5_\mu = \frac{e^2}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$ (4)

this counterterm changes the interaction to

$$S_1 = e \int d^4x A^\mu(x)(J^5_\mu(x) - \frac{\partial^\mu \partial^\nu}{\square} J^5_\nu(x))$$ (5)

which makes the gauge invariance obvious.

So one could ask if by the inclusion of this counterterm into the original action (1) a reasonable quantum theory may be obtained. Usually such nonlocal terms are rejected because the propagator $\square^{-1}(x - y)$ (the Feynman propagator of a massless boson) is expected to produce additional contributions to imaginary parts of e.g. scattering amplitudes, thereby spoiling unitarity. But in a recent paper [8] (see also [7]) it is argued that the net contribution of this counterterm to physical processes vanishes, and therefore the inclusion of the counterterm leads to an acceptable quantum theory. (Actually, in [8] a more general model, including both vector and axial vector gauge coupling, was investigated. It is, however, the axial coupling that suffers from a gauge anomaly.)

In this paper we explicitly investigate some Feynman diagrams and show that there exist physical processes where the counterterm does contribute, and therefore unitarity is spoiled, at least perturbatively. Our conventions are those of [6].

II. COMPUTATION OF THE COUNTERTERM

We study the model
\[ L = \bar{\Psi}(i\partial + eA_{\gamma 5})\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^3}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\int dy\partial^\lambda_x A_\lambda(x)\Box^{-1}(x-y)F_{\mu\nu}(y)F_{\alpha\beta}(y) \quad (6) \]

In the theory without counterterm the anomaly stems from the lowest order contribution to the three-point function

\[ T_{\mu\nu\lambda}(k_1, k_2, k_3) := FT(\langle J^5_\mu(x)J^5_\nu(y)J^5_\lambda(0)\rangle) \quad (7) \]

\[ k_1 + k_2 + k_3 = 0 \]

i.e. \( T_{\mu\nu\lambda} \) is given by the triangle diagram of Fig. 1 (the momentum routing in Fig. 1 is chosen in such a way as to ensure Bose symmetry, see e.g. [5])

\[ \text{Fig. 1} \]

An explicit expression for \( T_{\mu\nu\lambda} \) (which may be found e.g. in [6]) is

\[ T_{\mu\nu\lambda}(k_1, k_2, k_3) = -\frac{1}{3\pi^2}\epsilon_{\mu\nu\alpha\beta}[((k_1^\alpha - k_2^\alpha)k_3^2I_{1,2} + (k_2^\alpha - k_3^\alpha)k_1^2I_{2,3} + (k_3^\alpha - k_1^\alpha)k_2^2I_{3,1}) - \]

\[ \frac{1}{\pi^2}[\epsilon_{\alpha\beta\mu\nu}k_1^\alpha k_2^\alpha k_3^\beta k_3 I_{1,2} + \epsilon_{\alpha\beta\nu\lambda}k_2^\alpha k_3^\beta k_1^\lambda I_{2,3} + \epsilon_{\alpha\beta\lambda\mu}k_3^\alpha k_1^\beta k_2^\mu I_{3,1}] \quad (8) \]

\[ I_{i,j} := \int_0^1 dx_1dx_2dx_3 \frac{x_ix_j\delta(1-x_1-x_2-x_3)}{k_1^2x_2x_3 + k_2^2x_3x_1 + k_3^2x_1x_2} \quad (9) \]

It may be easily shown to fulfill the anomalous Ward identity

\[ k_3^\lambda T_{\mu\nu\lambda}(k_1, k_2, k_3) = -\frac{1}{6\pi^2}\epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \quad (10) \]

and analogous relations for \( k_1^\mu, k_2^\nu \) due to Bose symmetry.

Now let us have a look at the counterterm. It is of order \( e^3 \) and contains three gauge fields. Therefore it will occur within a Feynman diagram precisely in the same positions where the triangle graph occurs. Because of this, we may take the counterterm \( C_{\mu\nu\lambda}(k_1, k_2, k_3) \) into account by defining a new, "gauge invariantly regularized" triangle amplitude

\[ T^g_{\mu\nu\lambda}(k_1, k_2, k_3) = T_{\mu\nu\lambda}(k_1, k_2, k_3) + C_{\mu\nu\lambda}(k_1, k_2, k_3) \quad (11) \]
An explicit expression for the counterterm (without charges at the three vertices) reads

\[ C_{\mu\nu\lambda}(k_1, k_2, k_3) = \frac{1}{6i\pi^2} \left[ \epsilon_{\alpha\beta\mu\nu} k_1^\alpha k_2^\beta k_3^\gamma + \epsilon_{\alpha\beta\nu\lambda} k_2^\alpha k_3^\beta k_1^\gamma + \epsilon_{\alpha\beta\lambda\mu} k_3^\alpha k_1^\beta k_2^\gamma \right] \]  

(12)

Here the poles \( \frac{1}{k_i^2} \) that occur in \( C_{\mu\nu\lambda} \) are described by the usual \( i\epsilon \) description, i.e. they are Feynman propagators of a massless, scalar “ghost” field. It is easy to see that the new triangle amplitude \( T^g_{\mu\nu\lambda} \) fulfils the naive Ward identity

\[ k_1^\mu T^g_{\mu\nu\lambda}(k_1, k_2, k_3) = k_2^\mu T^g_{\mu\nu\lambda}(k_1, k_2, k_3) = k_3^\mu T^g_{\mu\nu\lambda}(k_1, k_2, k_3) = 0 \]  

(13)

Now it has to be checked whether the poles that are present in the counterterm give actually contributions to e.g. scattering amplitudes and thereby spoil unitarity of the theory (6). First observe that each term in \( C_{\mu\nu\lambda} \) is transverse at two vertices and longitudinal at the third vertex,

\[ C_{\mu\nu\lambda}(k_1, k_2, k_3) = A_{\mu\nu}(k_1, k_2)k_{3\lambda} + A_{\nu\lambda}(k_2, k_3)k_{1\mu} + A_{\lambda\mu}(k_3, k_1)k_{2\nu} \]  

(14)

\[ k_i^\mu A_{\mu\nu}(k_1, k_2) = 0 \quad \text{etc.} \]  

(15)

For a further discussion we need the gauge field propagator. The theory (6) is gauge invariant, therefore we have to introduce a gauge fixing term as usual and get the propagator

\[ D^\xi_{\mu\nu}(k) = \frac{1}{k^2} (g_{\mu\nu} - \xi \frac{k^\mu k^\nu}{k^2}) \]  

(16)

where \( \xi \) is the gauge fixing parameter. We will keep \( \xi \) arbitrary for most of our discussion, and only shortly discuss that we can rederive our conclusions for a special choice of \( \xi \).

So let us investigate Feynman graphs where triangle diagrams occur as subdiagrams and try to find what happens with the counterterm. First, whenever a vertex (e.g. \( \lambda \)) of \( T^g_{\mu\nu\lambda} \) is connected to an external photon, the part of the counterterm that is longitudinal at \( \lambda \) vanishes, because of the transversality of the external photon,

\[ \epsilon^\lambda(k_3)A_{\mu\nu}(k_1, k_2)k_{3\lambda} = 0 \]  

(17)

When a gauge field propagator \( D^\xi_{\mu\nu} \) is connected to the regularized triangle \( T^g_{\mu\nu\lambda} \), the \( \xi \)-dependent part of \( D^\xi_{\mu\nu}(k) \) does not contribute because of the Ward identity (13),

\[ D^\xi_{\mu\nu}(k_1)T^g_{\mu\nu\lambda}(k_1, k_2, k_3) = D^0_{\mu\nu}(k_1)T^g_{\mu\nu\lambda}(k_1, k_2, k_3) \]

\[ D^0_{\mu\nu}(k) = \frac{g_{\mu\nu}}{k^2} \]  

(18)

Now the gauge field propagator may connect a vertex of \( T^g_{\mu\nu\lambda} \) to different types of subgraphs:

1. When e.g. \( D^0_{\lambda\lambda}(k_3) \) connects \( T^g_{\mu\nu\lambda} \) to an external fermion-antifermion pair, the corresponding part \( A_{\mu\nu}(k_1, k_2)k_{3\lambda} \) of the counterterm does not contribute because of the equations of motion for the on-shell spinors \( (k_3 = p_1 + p_2) \)

\[ A_{\mu\nu}(k_1, k_2)k_{3\lambda} \frac{g^{\lambda\gamma}}{k_3^2} \bar{u}(p_1)\gamma^\gamma\gamma^5v(p_2) = A_{\mu\nu}(k_1, k_2) \frac{1}{k_3^2} \bar{u}(p_1)(p_1 + p_2)\gamma^5v(p_2) = 0 \]  

(19)
2. When e.g. the vertex $\lambda$ of $T_{\mu\nu\lambda}^g$ is connected to a closed fermion loop that is not a triangle, the counterterm $A_{\mu\nu}(k_1, k_2)k_{3\lambda}$ does not contribute because of the Ward identity for the closed fermion loop,

$$A_{\mu\nu}(k_1, k_2)k_{3\lambda} g^{\lambda\lambda'} \frac{1}{k_3^2} \langle J_{\lambda'}^5 J_{\rho_1}^5 \cdots J_{\rho_n}^5 \rangle (k_3, p_1, \ldots, p_n) = 0 \quad (20)$$

3. When e.g. the vertex $\lambda$ of $T_{\mu\nu\lambda}^g$ is connected to a fermion line that starts and ends at an external, on-shell fermion, the contribution of the counterterm to an individual diagram does not vanish. However, there are several diagrams of this type (see Fig. 2 for an example, where we denote the counterterm by a double line)

Here it is understood that, apart from the different positions where $k_{3\lambda}$ enters, all four diagrams (e.g. the connections of $\rho_1, \rho_2, \rho_3$ to the rest of the diagram) are completely identical. The contribution of $A_{\mu\nu}(k_1, k_2)k_{3\lambda}$ in this sum cancels, as may be computed easily from the expression for Fig. 2,

$$\bar{u}(p - p_1 - p_2 - p_3 - k_3),$$

$$\cdot \left[ \gamma_{\rho_5} \gamma_5 \frac{1}{\not{p} - \not{p}_1 - \not{p}_2 - \not{k}_3} \gamma_{\rho_2} \gamma_5 \frac{1}{\not{p} - \not{p}_1 - \not{k}_3} \gamma_{\rho_1} \gamma_5 \frac{1}{\not{p} - \not{k}_3} k_3 \gamma_5 \cdots \right] u(p) \quad (21)$$

by frequently using the identity

$$\frac{1}{\not{p} - \not{k}} \frac{1}{\not{k}} = \frac{1}{\not{p} - \not{k}} - \frac{1}{\not{p}} \quad (22)$$

and the equations of motion for $u, \bar{u}$.

This cancellation holds for an arbitrary string of fermion propagators beginning and ending at an external fermion. Further, this cancellation does not depend on the remainder of the diagram that is connected to the other vertices $\rho_i$ of the fermion line, therefore this cancellation continues to hold when some other triangle diagrams or some other vertices of the same triangle diagram (including the counterterm) are connected to the given fermion line.
Therefore we found so far that the contribution of the \( C_{\mu\nu\lambda} \) cancels completely as long as each vertex \( \mu, \nu, \lambda \) of the triangle diagram is connected either to an external photon, or to a closed fermion loop that is not a triangle, or to an “open” fermion line that begins and ends at an external fermion.

The last case we have to investigate is a triangle \( T_{\mu\nu\lambda} \) that is connected to another triangle, e.g.

\[
T_{\mu\nu\lambda}(k_1, k_2, k_3) \frac{g^{\lambda\lambda'}}{k_3^2} T_{\mu'\nu'\lambda'}(k_1', k_2', -k_3) \tag{23}
\]

We assume that the vertices \( \mu, \nu, \mu', \nu' \) are not connected to triangles, therefore the corresponding parts of the counterterms vanish, and we get \( (k_3 = -k_1 - k_2 = k_1' + k_2') \)

\[
\frac{1}{k_3^2} \left( T_{\mu\nu\lambda}(k_1, k_2, k_3) + A_{\mu\nu}(k_1, k_2)k_{3\lambda} \right) \left( T_{\mu'\nu'\lambda'}(k_1', k_2', -k_3) - A_{\mu'\nu'}(k_1', k_2')k_{3\lambda}' \right) \tag{24}
\]

Because \( T_{\mu\nu\lambda} \) obeys the Ward identity (13) we may again cancel one of the two counterterms and find e.g.

\[
\frac{1}{k_3^2} \left( T_{\mu\nu\lambda}(k_1, k_2, k_3) + A_{\mu\nu}(k_1, k_2)k_{3\lambda} \right) T_{\mu'\nu'\lambda'}(k_1', k_2', -k_3) =
\]

\[
\frac{1}{k_3^2} T_{\mu\nu\lambda}(k_1, k_2, k_3) T_{\mu'\nu'\lambda'}(k_1', k_2', -k_3) + \frac{1}{6\pi^2 k_3^2} \epsilon_{\alpha\beta\mu\nu} k_1^\alpha k_2^\beta \epsilon_{\alpha'\beta'\mu'\nu'} k_1'^\alpha k_2'^\beta' \tag{25}
\]

and the counterterm \( A_{\mu\nu}k_{3\lambda} \) does not vanish. This term contributes e.g. to fermion-antifermion scattering via Fig. 3

and the counterterm certainly contributes to the imaginary part of this scattering amplitude (remember that the \( \frac{1}{k_3^2} \) are Feynman propagators) and, therefore, violates unitarity.

Obviously, whenever two triangle diagrams are connected to each other within a larger graph, they will give a contribution like in (25) and therefore, in general, violate unitarity.

[Remark: there is a kind of cancellation that may occur in (24) for very special values of \( k_i^2 \). E.g. for \( k_1^2 = k_2^2 = k_3^2 \) it holds that \( T_{\mu\nu\lambda}(k_1, k_2, k_3) = -C_{\mu\nu\lambda}(k_1, k_2, k_3) \) and, therefore, the counterterm in (24) is cancelled by the triangle amplitude itself. However, this kind of cancellation may only occur for very specific values of the \( k_i^2 \), where the analytical structure of \( T_{\mu\nu\lambda} \) simplifies to the pole structure of \( C_{\mu\nu\lambda} \). For general \( k_i^2 \) the analytical structure of \( T_{\mu\nu\lambda} \) is much more complicated, no poles like in \( C_{\mu\nu\lambda} \) occur, and there is no cancellation (more details on the analytical structure of the triangle diagram may be found e.g. in \[10,11\]).]

Next we want to show that we find precisely the same violation of unitarity when we fix the gauge to be Landau gauge \( (\xi = 1) \) from the very beginning,

\[
D_{1\mu\nu}^L(k) = \frac{1}{k^2} (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \tag{26}
\]
where we see from (3), (5) that the gauge condition $\partial^\mu A_\mu = 0$ eliminates the counterterm.

First, when an external photon is connected to a vertex $\lambda$ of $T^g_{\mu\nu\lambda}$, the same cancellation of $A_{\mu\nu} k_{3\lambda}$ as above occurs.

Secondly, when all three vertices of $T^g_{\mu\nu\lambda}$ are connected to gauge field propagators (26), the counterterm $C_{\mu\nu\lambda}$ cancels completely because of the transversality of $D^1_{\mu\nu}$, as we already noticed. Therefore, a violation of unitarity must be due to the second term of $D^1_{\mu\nu} - \frac{k_\mu k_\nu}{k^4}$.

Again, as long as the triangle is connected to a subdiagram that is not a triangle, the term $-\frac{k_\mu k_\nu k_\lambda}{k^4}$ does not contribute, because the second momentum (e.g. $k_\nu$) meets a transverse vertex.

On the other hand, when $D^{1\lambda\lambda'}(k_3)$ connects two triangles, we find the contribution

$$T_{\mu\nu\lambda}(k_1, k_2, k_3) - \frac{k_3^{\lambda} k_3^{\lambda'}}{k_3^4} T_{\mu'\nu'\lambda'}(k'_1, k'_2, -k_3) =$$

$$\left(\frac{1}{6\pi^2 k_3^2}\right)^2 \epsilon_{\alpha\beta\mu\nu} k_1^\alpha k_2^\beta \epsilon_{\alpha'\beta'\mu'\nu'} k_1^{\alpha'} k_2^{\beta'}$$

and, therefore, the same unitarity violating term as before.

### III. ALTERNATIVE FORMULATION

Finally, we want to discuss an alternative, equivalent formulation of the theory (6) that was given in [8], too, and show how the violation of unitarity occurs there. The alternative formulation of the theory is given by the Lagrangian

$$L = \bar{\Psi}(i \partial - e A_5) \gamma_5 \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^3}{96\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} (a + b) - \frac{\mu^2}{2} (\partial_\mu a - A_\mu)^2 + \frac{\mu^2}{2} (\partial_\mu b - A_\mu)^2$$

(28)

Here $a$ and $b$ are scalar fields that only occur as internal lines in Feynman diagrams ($\mu$ is a dimensionful parameter on which the theory does not depend).

This theory is gauge invariant provided that, in addition to the usual gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \lambda , \quad \Psi \to e^{i e \gamma_5 A_5} \Psi , \quad \bar{\Psi} \to \bar{\Psi} e^{-i e \gamma_5 A_5}$$

(29)

the fields $a$, $b$ transform according to

$$a \to a + \lambda , \quad b \to b + \lambda$$

(30)

The original, nonlocal action (6) may be recovered from (28) by performing the (Gaussian) path integration over $a$ and $b$.

In [8] it was claimed that the “ghosts” $a$, $b$ will give no net contribution to physical amplitudes. There is indeed a partial cancellation of the $a$ and $b$ field contributions, because their kinetic terms have opposite signs and lead therefore to propagators with opposite signs.
The $a, b$ in (28) have two interaction terms. They couple to the index density $\epsilon_{\mu\nu\alpha\beta} F^{\mu
u} F_{\alpha\beta}$ with equal couplings $\frac{e^2}{6\pi^2}$, and to the gauge field (more precisely, to $\partial^\mu A_\mu$) with opposite couplings $\mp \mu^2$. Therefore, a cancellation between $a$ and $b$ propagators occurs as long as the $a$ ($b$) propagators connect either two index densities or two gauge fields. On the other hand, when the $a$ ($b$) propagators connect one index density and one gauge field, the opposite signs of the propagators are compensated by the opposite signs of the coupling to the gauge field, and the two contributions have equal sign. When these two terms are taken into account within a Feynman diagram, they lead precisely to the unitarity violating terms that we found in the first formulation of the theory.

\[
D_b(k) = \frac{\pm 1}{\mu^2 k^2}
\]  

where we inserted the propagator (31) for the contraction of the two $a$ fields and took into account all the possibilities to contract the three gauge fields in (32) with three fixed gauge fields of the remainder of the diagram where (32) is inserted. We recover precisely the counterterm (12).

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Note added: In a recent answer [12] to our paper it has been claimed that the counterterm in (25) is cancelled by a contribution of the first term (the triangle part) and, therefore, a violation of unitarity does not occur. Although this point has already been discussed in the literature [9], we nevertheless want to comment on it briefly. For a clear distinction between gauge field propagator and counterterm (and to be close to the notation in [12]) we introduce a small gauge field mass $m$ and rewrite our eq. (25)

\[
\frac{1}{k_3^2 - m^2} T_{\mu\nu\lambda}(k_1, k_2, k_3) T_{\mu'\nu'\lambda'}(k_1', k_2' - k_3) + \frac{1}{(6\pi^2)^2} \frac{1}{(k_3^2 - m^2)k_3^2} \epsilon_{\alpha\beta\mu\nu} k_1^\alpha k_2^\beta k_3^\mu k_3^\nu \epsilon_{\alpha'\beta'\mu'\nu'} k_1'^\alpha k_2'^\beta'.
\]

Now the claim in [12] is that the first term, too, contains a pole $\frac{1}{k_3^2}$ that precisely cancels the second term (the counterterm). This is indeed the case at the symmetric point $k_1^2 = k_2^2 = k_3^2$, where each triangle is exactly equal to minus the counterterm, $T_{\mu\nu\lambda} = -C_{\mu\nu\lambda}$. Therefore, at $k_1^2 = k_2^2 = k_3^2 = 0$ the imaginary part of the counterterm ($\sim \delta(k_3^2)$) is cancelled in the unitarity relations.

However, this cancellation does not hold in the general case. For $k_1^2, k_2^2 \neq k_3^2$ the triangle graph does not contain a $\frac{1}{k_3^2}$ pole and, therefore, its imaginary part does not contain a $\delta(k_3^2)$.
term, as may be shown by a straightforward but tedious calculation. For the analogous case of the VVA triangle graph the computation of the imaginary part of $T^{\nu\nu\lambda}_{\mu\nu\lambda}$ has been performed e.g. in [11,13]. The explicit expression for the imaginary part of $T^{\nu\nu\lambda}_{\mu\nu\lambda}$ (which is a rather complicated function) does not contain a $\delta(k^2_3)$ term in the general case; only in the limit $k^2_1, k^2_2 \to 0$ such a $\delta(k^2_3)$ term is produced, see [11,13].

[The physical reason for this behaviour of the imaginary part, which stems from the cutting of the triangle graph, may be easily understood: when all momenta squared entering the triangle are equal, the fermions of the triangle have to be collinear after the cutting, giving thereby rise to a $\delta(k^2_3)$ imaginary part; on the other hand, once not all of the $k^2_i$ are equal, the fermions need not be collinear after the cutting and, therefore, produce the usual discontinuity above the real particle production threshold of the complex $k^2_3$ variable, see [13,14].]

Therefore, there is no cancellation of the counterterm for $k^2_1, k^2_2 \neq k^2_3$. Further, there are, of course, contributions to the unitarity relations of some scattering processes where $k^2_1, k^2_2 \neq 0$ (e.g. for our Fig. 3). In all such cases the $\delta(k^2_3)$ term of the counterterm cannot be cancelled by a contribution from the triangle graph, and unitarity is violated.
REFERENCES

[1] S. Adler, Phys. Rev. 177, 2426 (1969).
[2] J. S. Bell and R. Jackiw, Nuovo Cim. A 60, 47 (1969).
[3] S. Adler, in Lectures in Elementary Particle Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, 1970).
[4] R. Jackiw, in Current Algebra and Anomalies, edited by S. B. Treiman, R. Jackiw, B. Zumino, and E. Witten (World Scientific, Singapore, 1985).
[5] D. J. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972).
[6] R. A. Bertlmann, Anomalies in Quantum Field Theory (Clarendon Press, Oxford, 1996).
[7] T. D. Kieu, Phys. Rev. D 44, 2548 (1991).
[8] P. Federbush, The Axial Anomaly Revisited, hep-th 9606110.
[9] A. Andrianov, A. Bassetto, and R. Soldati, Phys. Rev. D 44, 2602 (1991).
[10] J. Horejsi, Czech J. Phys. 42, 241, 345 (1992).
[11] J. Horejsi, Czech J. Phys. B 35, 820 (1985).
[12] P. Federbush, Comments on “Investigation of Anomalous Axial QED”, hep-th 9703173.
[13] Y. Frishman et al, Nucl. Phys. B177, 157 (1981).
[14] S. Coleman and B. Grossman, Nucl. Phys. B203, 205 (1982).