Two-dimensional dipolar bosons with weak disorder

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We consider two-dimensional dipolar bosonic gas with dipoles oriented perpendicularly to the plane in a weak random potential. We investigate analytically and numerically the condensate depletion, the one-body density-matrix, the ground state energy, the sound velocity and the superfluid fraction. Concentrating on the regime where a rotonlike excitation spectrum forms, our results show that the superfluidity disappears below a critical value of disorder strength yielding the transition to a non-trivial quantum regime.

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Recently, ultracold dipolar gases in two-dimensional (2D) geometry have been the subject of intense experimental and theoretical investigations. What renders such systems particularly intriguing is the presence of the low-lying roton minimum in the excitation spectrum and the possibility of the crystallization of solid bubble into a lattice superstructure, resulting in a global supersolid phase. However, such supersolids require a dense regime with several particles within the interaction range, which can be difficult to achieve. The same holds for supersolids discussed for 2D dipolar Bose gases near the gas-solid phase transition. It was found also that this state appears in B-t-J model of two-component bosons as a result of the long-range DDI.

A more complicated situation arises if these phenomena are studied in a random environment. Bose gases in the random medium attracted a great deal of interest because it connects two central ideas of the condensed matter theory: the Bose-Einstein condensation (BEC) and localization. This latter occurs both for fermions and bosons and results in the existence of an insulating phase called Bose glass. The existence of a phase characterized by simultaneous glassiness and superfluidity and thus constitutes a glassy counterpart to the supersolid phase, was first observed in numerical Quantum Monte Carlo simulations of solid $^4$He samples by Boninsegni et al. This phase which is featured (at the same time) by superfluidity and a metastable amorphous structure called "superglass". Superglass may also have realizations in interacting bosons at very low temperatures and high density. In this context, Zamponi et al. have shown that quantum fluctuations can stabilize the superglass phase in a self-disordered environment induced by geometrical frustration.

On the other hand, many interesting works on BEC and superfluidity (See, e.g., ) have been reported for disordered cold atomic gases with pure contact interaction in continuum, the study on dipolar boson systems is still inadequate. In the present paper, for the first time to our knowledge, we investigate the properties of a quasi-2D homogeneous dipolar Bose gas in a weak random potential with delta correlated disorder. Our study is based on the Bogoliubov approach, this method which marked an important step towards a quantitative description of dirty dipolar Bose systems, allows for accurate determination of the condensed depletion, one-body density matrix, ground state energy, sound velocity and superfluid fraction. It is found that the presence of a disordered potential in the regime where the roton develops in the excitations spectrum strongly enhances fluctuations and thermodynamics quantities. We demonstrate that both BEC and superfluidity are depressed due to the competition between disorder and DDI yielding, the transition to an unusual quantum phase.

We consider a dilute Bose-condensed gas of dipolar bosons in an external random potentials. These particles can be confined to quasi-2D, by means of an external harmonic potential in the direction perpendicular to the motion (pancake geometry) and all dipoles are aligned perpendicularly to the plane of their translational motion, by means of a strong electric (or magnetic) field. In this quasi-2D geometry, at large interparticle separations the interaction potential is $V(r) = d^2/r^3 = h^2 r_s/mr^3$, with $d$ being the dipole moment, $m$ the particle mass, and $r_s = md^2/h^2$ the characteristic dipole-dipole distance. The disorder potential is described by vanishing ensemble averages $\langle U(\vec{r}) \rangle = 0$ and a finite correlation of the form $\langle U(\vec{r})U(\vec{r}') \rangle = R(\vec{r}, \vec{r}')$.

In the ultracold limit where the particle momenta satisfy the inequality $kr_s \ll 1$, the scattering amplitude is given by (see e.g. )

$$f(\vec{k}, \vec{k}') = g(1 - C|\vec{k} - \vec{k}'|),$$

where the 2D short-range coupling constant is $g = g_{3D}/\sqrt{2l_0}$ with $l_0 = \sqrt{h/m\omega}$, $\omega$ is the confinement frequency and $C = 2\pi\hbar^2 r_s/mg = 2\pi d^2/g$. Employing this result in the secondly quantized Hamiltonian, we obtain
where \( S \) is the surface area, \( E_k = \hbar^2 k^2/2m \), and \( \hat{a}^\dagger_k, \hat{a}_k \) are the creation and annihilation operators of particles. At zero temperature there is a true BEC in 2D, and we may use the standard Bogoliubov approach. Assuming the weakly interacting regime where \( mg/2\pi\hbar^2 \ll 1 \) and \( v_s \ll \xi \), with \( \xi = h/\sqrt{mng} \) being the healing length. We may reduce the Hamiltonian \( \hat{H} \) to a bilinear form, using the Bogoliubov transformation \( \hat{a}_k = u_k \hat{b}_k - v_k \hat{b}^\dagger_{-k} - \beta \), where \( \hat{b}_k^\dagger \) and \( \hat{b}_k \) are operators of elementary excitations. The Bogoliubov functions \( u_k, v_k \) are expressed in a standard way: \( u_k, v_k = (\sqrt{\varepsilon_k/E_k} \pm \sqrt{E_k/\varepsilon_k})/2 \), \( \beta_n = \sqrt{n}/SU_k/(\sqrt{\varepsilon_k})^2 \), and the Bogoliubov excitation energy is given by \( \varepsilon_n = \sqrt{E_n^2 + 2ngE_k(1 - Ck)} \).

To zero order the chemical potential is \( \mu = ng \). If \( C/\xi < \sqrt{3/\delta} \), \( \varepsilon_n \) is a monotonic function of \( k \). However, it shows a roton-maxon structure for the constant \( C/N \) in the interval \( \sqrt{3/\delta} \leq C/\xi \leq 1 \). It is then convenient to represent \( \varepsilon_n \) in the form \( 3 \).

\[
\varepsilon_n = \frac{\hbar^2}{2m} \sqrt{(k - k_r)^2 + k^2_\Delta},
\]

where \( k_r = 2C/\xi^2 \) and \( k_\Delta = \sqrt{4/\xi^2 - k_r^2} \). If the roton is close to zero, then \( k_r \) is the position of the roton, and \( \Delta = \hbar^2 k_r k_\Delta/2m = 2ngC\sqrt{mng/\hbar^2 - C^2(mng/\hbar^2)^2} \), is the height of the roton minimum. At \( C/\xi = 1 \), the roton minimum touches zero and for \( C/\xi > 1 \), the uniform Bose condensate becomes dynamically unstable and the uniform superfluid is no longer the ground state. Importantly, the spectrum \( 3 \) is independent of the random potential. This independence holds in fact only in zeroth order in perturbation theory; conversely, higher order calculations render the spectrum dependent on the random potential due to the contribution of the anomalous terms \( \langle \hat{a}_k \hat{a}_k \rangle \).

The diagonal form of the Hamiltonian of the dirty dipolar Bose gas \( 2 \) can be written in the usual form \( \hat{H} = \hat{E} + \sum_k \varepsilon_k \hat{b}_k^\dagger \hat{b}_k \), where \( \hat{E} = \hat{E}_0 + \delta \hat{E} + \hat{E}_R \) with \( \hat{E}_0 = Sg \varepsilon_n/2 \) and \( \delta \hat{E} = 1/2 \sum_k [\varepsilon_k - E_k - ng(1 - Ck)] \) being the ground-state energy correction due to quantum fluctuations.

\[
E_R = -\sum_k n|\langle U_k \rangle|^2 E_k \varepsilon_k = -\sum_k nR_k E_k \varepsilon_k,
\]

where \( \langle U_k \rangle = \sum_{k,q} U_{k-q} \hat{a}_{k-q} \hat{a}_k \hat{a}_{k+q} \hat{a}_{-q} \) and \( R_k \) is the height of the roton minimum. At \( C/\xi = 1 \), the roton minimum influences also the phenomenon of superfluidity \( 27 \).

The noncondensed density is defined as \( \tilde{n} = \sum_k \langle \hat{a}_k^\dagger \hat{a}_k \rangle \).

Then invoking for the operators \( \hat{a}_k \) the preceding Bogoliubov transformation, setting \( \langle \hat{b}_k^\dagger \hat{b}_n \rangle = \delta_{kk} N_k \) and putting the rest of the expectation values equal to zero, where \( N_k = \exp(\varepsilon_k/T) - 1 \) are occupation numbers for the excitations. Using the fact that \( 2N(x) + 1 = \coth(x/2) \) \( 26 \), we obtain:

\[
n^r = \frac{1}{2S} \sum_k \left[ \frac{E_k + ng(1 - Ck)}{\varepsilon_k} \coth \left( \frac{\varepsilon_k}{2T} \right) - 1 \right] + n_R,
\]

where

\[
n_R = \frac{1}{S} \sum_k (\beta_n^2) = \frac{1}{S} \sum_k nR_k \frac{E_k^2}{\varepsilon_k^2},
\]

is the condensate depletion due to the external random potential.

In order to investigate in a simple way how the random potential affects the behavior of the system, we will often make the white-noise assumption in which the external potential is described by a single parameter \( R(\vec{r}, \vec{r}') = R_0 \delta(\vec{r}, \vec{r}') \) \( 22 \), where \( R_0 \) denotes the disorder strength which has dimension \( (\text{energy})^2 \times \text{(length)}^2 \).

Let us now assume that the roton is close to zero and the roton energy is \( \Delta \ll ng \), we have the coefficient \( C \) close to \( \xi \), and \( k_r \approx 2/\xi \). Then, using Eq. \( 3 \), for the contribution of momenta near the roton minimum at \( T = 0 \), we obtain:

\[
\frac{n_R}{n} = \frac{mgR}{4\hbar^2} \left( \frac{2ng}{\Delta} \right)^3,
\]

where \( R = R_0/ng^2 \) is a dimensionless disorder strength. The noncondensed density is calculated via \( 5 \) as

\[
\frac{n^r}{n} \approx \frac{mg}{\pi \hbar} \left[ \ln \left( \frac{2ng}{\Delta} \right) + \frac{\pi}{4} R \left( \frac{2ng}{\Delta} \right)^3 \right]; \quad \Delta \ll ng. \tag{8}
\]

The leading term in Eq. \( 8 \), which comes from the quantum fluctuations was first obtained in our recent work \( 6 \), while the second term which grows faster than the first one, represents the disorder correction to the condensate depletion. The condensed fraction can be written as \( n_c/n = 1 - n^r/n \).

To check the result of Eq. \( 8 \), we solve Eqs. \( 5 \) and \( 6 \) numerically using Monte Carlo method. Figure \( 1 \) shows that in the absence of the random external potential i.e. \( R = 0 \), the noncondensed fraction grows logarithmically (see Eq. \( 8 \)) when the roton energy \( \Delta \) goes to zero yielding the transition to a supersolid state \( 6, 8, 9 \). It is worth stressing that in the context of the liquid helium, the position of the roton minimum influences also the phenomenon of superfluidity \( 27 \).

In the presence of the external disordered potential and for \( C \approx \xi \), the condensate depletion becomes more significant and diverges when \( \Delta = 0 \) indicating the transition to a novel quantum phase in dilute 2D dipolar bosons.
The same factors of Eq. (9) appear in the one-body density matrix $g_1(\vec{r}) = \langle \hat{\Psi}^\dagger(\vec{r}) \hat{\Psi}(0) \rangle$, where $\hat{\Psi}(\vec{r})$ is the field operator. The correction due to the disorder effects to the correlation function is $g_1^R(\vec{r}) = \int n_n e^{i\vec{k}\cdot\vec{r}} d^2k/(2\pi)^2$. Assuming that the roton minimum reaches zero and taking into account only the contribution of momenta near this minimum we have for $\Delta \ll ng$:

$$g_1(\vec{r}) = 1 + mg/\pi h^2 \left[ \ln \left(2ng/\Delta \right) + \pi^2 R \left(\frac{2ng}{\Delta} \right)^3 \right] J_0(2r/\xi),$$

where $J_0(x)$ is the Bessel function.

The numerical calculation of the one-body density matrix well agrees with the analytical result obtained in Eq. (10) and shows that $g_1(\vec{r})$ decays at large distance when $C \approx \xi$ as is depicted in Fig. 2. This signals the non-existence of the off-diagonal long-range order (i.e., BEC) in the disordered 2D dipolar bosons.

As we see from Eq. (10), for the roton minimum close to zero a small condensate depletion requires the inequality

$$\frac{mg}{\hbar^2} R \left(\frac{2ng}{\Delta} \right)^3 \ll 1.$$

We thus conclude that at $T = 0$, the validity of the Bogoliubov approach is guaranteed by the presence of the small parameter $\left(\frac{\Delta}{ng} \right)$.

However, the situation changes in the calculation of the correction to the ground-state energy due to the external random potential. When the roton minimum is approaching to zero, we get from (11)

$$\frac{E_R}{E_0} = -\frac{mg}{\hbar^2} R \left(\frac{2ng}{\Delta} \right); \ \Delta \ll ng.$$

Equation (11) shows that $E_R$ grows linearly with $ng/\Delta$, and has a negative value which leads to reduce the total energy of the system.

The correction to the ground-state energy due to quantum fluctuations can be given as

$$\frac{\delta E}{E_0} \approx 1 + \frac{2mg}{\pi \hbar^2} \frac{\pi^2}{\Delta} \ln \left(\frac{2ng}{\Delta} \right); \ \Delta \ll ng. \ \ \ (12)$$

The disorder correction to the chemical potential can be calculated easily through $\partial E_R/\partial N$

$$\frac{\mu_R}{\mu} \approx -\frac{mg}{\hbar^2} R \left(\frac{2ng}{\Delta} \right)^3 = -4n_R g; \ \Delta \ll ng. \ \ \ (13)$$

Note that quantum fluctuations corrections to the chemical potential had already obtained in our recent paper [6].

One can also show that the shift of the sound velocity is consistent with the change in the compressibility $mc_s^2 = n\partial\mu/\partial n$ [26, 28] and is given by

$$\frac{c_s^2}{c_{s0}^2} = 1 + \frac{mg}{\pi \hbar^2} \left[ 2 \ln \left(\frac{2ng}{\Delta} \right) + \left(\frac{2ng}{\Delta} \right)^2 - \frac{3\pi^2}{2} R \left(\frac{2ng}{\Delta} \right)^5 \right],$$

where $c_{s0} = \sqrt{\mu/m}$ is the zeroth order sound velocity. The second and the third terms originate from quantum fluctuations while the last term comes from the disorder contribution.

On the other hand, in an infinite uniform 2D fluid thermal fluctuations at any nonzero temperature are strong enough to destroy the fully ordered state associated with BEC, but are not strong enough to suppress superfluidity in an interacting system at low, but non-zero temperatures.
at low temperatures leads to an interesting interplay between superfluidity and condensation in all experimentally relevant finite-size systems. In this quasi-condensate, the phase coherence governs only regimes of a size smaller than the size of the condensate, characterized by the coherence length $l_\phi$ \cite{29,30}. Thermodynamic properties, excitations, and correlation properties on a distance scale smaller than $l_\phi$ are the same as in the case of a true BEC. Upon utilizing the previous definitions we find that the correction to the condensate depletion, the correlation function and thermodynamic quantities due to thermal fluctuations is given by the factor $(2mg/h^2)T/\Delta$ \cite{6}.

The superfluid fraction $n_s/n$ can be found from the normal fraction $n_n/n$ which is determined by the transverse current-current correlator $n_s/n = 1 - n_n/n$. We apply a Galilean boost with the total momentum of the moving system $\vec{P}_v = \vec{P} + mvN$, where $\vec{P} = \sum_k \hbar \vec{a}_k^\dagger \vec{a}_k$ and $v$ is the liquid velocity. In $d$-dimensional case, the superfluid fraction reads

$$n_s/n = 1 - 2 \frac{dT}{dT_n} \int \frac{d^d k}{(2\pi)^d} \left[ \frac{E_k}{4\sinh^2(\varepsilon_k/2T)} + \frac{nR_0E_k^2}{\varepsilon_k} \coth \left( \frac{\varepsilon_k}{2T} \right) \right].$$

At very low temperature we can put $\coth(\varepsilon_k/2T) = 2T/\varepsilon_k$. Thus, Eq. (15) reduces to

$$n_s/n = 1 - 4 \frac{n_R}{d} \frac{d}{dN} \int \frac{d^d k}{(2\pi)^d} \left[ \frac{E_k}{4\sinh^2(\varepsilon_k/2T)} \right].$$

Interestingly, the ratio between the normal fluid density and the corresponding condensate depletion increases to $2$ in $2D$ and to $4$ in $1D$, in contrast to the familiar $4/3$ in $3D$ geometry obtained earlier in \cite{21,22}. Another important remark is that the superfluid fraction (16) is no longer a tensorial quantity as in the case of a $3D$ dirty dipolar Bose gas \cite{24,25} since the dipoles are assumed to be perpendicular to the plane. However, if the dipoles would be tilted slightly, superfluidity would acquire an anisotropy and thus becomes a tensorial quantity.

Assuming now that the roton minimum is close to zero and $\Delta \ll T$, then the momenta near the roton minimum are the most important, and the use of Eq. (7) yields:

$$n_s/n = 1 - \frac{mg}{2h^2 \Delta} \left( \frac{2ng/\Delta}{\Delta} \right)^3 - 2\frac{mgT}{h^2 \Delta}.$$
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