ON THE TIME VARIABILITY OF THE STAR FORMATION EFFICIENCY

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Abstract

A star formation efficiency per free-fall time that evolves over the lifetime of giant molecular clouds (GMCs) may have important implications for models of supersonic turbulence in molecular clouds or for the relation between the star formation rate and H$_2$ surface density. We discuss observational data that could be interpreted as evidence of such a time variability. In particular, we investigate a recent claim based on measurements of H$_2$ and stellar masses in individual GMCs. We show that this claim depends crucially on the assumption that H$_2$ masses do not evolve over the lifetimes of GMCs. We exemplify our findings with a simple toy model that uses a constant star formation efficiency and, yet, is able to explain the observational data.

Key words: galaxies: evolution – stars: formation

1. INTRODUCTION

The lifetimes of giant molecular clouds (GMCs) have been at the center of a major debate for at least the last 40 years (Goldreich & Kwan 1974; Zuckerman & Evans 1974; Solomon et al. 1979; Elmegreen 2000, 2007). GMCs that live for many free-fall times need a mechanism that prevents them from gravitational collapse. Over the last couple of years the growing consensus is that the lifetimes of GMCs are likely a few free-fall times or even less (Elmegreen 2000; Ballesteros-Paredes & Hartmann 2007; Murray 2010), and the focus has shifted toward the challenge of explaining the low star formation efficiencies in GMCs. The star formation efficiency per free-fall time $\epsilon_{ff}$ is defined as the ratio of free-fall time $t_{ff}$ to gas depletion time $M_{H_2}/M_*$. In other words,

$$M_* = \frac{\epsilon_{ff}}{t_{ff}} M_{H_2},$$  

(1)

i.e., the instantaneous star formation rate (SFR) is proportional to the available amount of molecular hydrogen (H$_2$) via the proportionality factor $\epsilon_{ff}/t_{ff}$. The observed value $\epsilon_{ff} \approx 0.01-0.02$ (e.g., Krumholz & Tan 2007) means that only 1%-2% of the mass of a GMC is converted into stars over a free-fall time. If star formation is supported by supersonic turbulence (Krumholz & McKee 2005), $\epsilon_{ff}$ is expected to be only very weakly dependent on the Mach number of the turbulent flow and thus approximatively constant, but this may be an oversimplification (see, e.g., Vázquez-Semadeni et al. 2005; Li & Nakamura 2006).

On the other hand, if GMCs have lifetimes of the order of a free-fall time they do not need to be supported by turbulence. The star formation efficiency in such clouds may increase as the Mach number in the flow decreases and the cloud collapses, see, e.g., Bonnell et al. (2010). We note that a time-varying $\epsilon_{ff}$ should introduce additional scatter in the relation between SFR and H$_2$ surface density on small ($\leq 100$ pc) scales. This scatter should propagate up to $\sim$kpc scales (see, e.g., Feldmann et al. 2010), and hence would contribute to the scatter in the Kennicutt–Schmidt relation. This, at least in principle, could be used to test observationally the time dependence of $\epsilon_{ff}$.

In Section 2, we will discuss two common misconceptions that could give rise to the impression that $\epsilon_{ff}$ varies over the lifetimes of GMCs even if it is a constant. In Section 3, we present and analyze a toy model in order to exemplify and quantify our statements.

2. DO OBSERVATIONS CONFIRM A TIME-VARYING STAR FORMATION EFFICIENCY?

In Equation (1) we define the star formation efficiency per free-fall time $\epsilon_{ff}$. Another commonly used efficiency is the star formation efficiency of the GMC $\epsilon_{GMC}$, i.e., the fraction of H$_2$ mass of the cloud that is converted into stars over the lifetime of the cloud. In a picture where GMCs start with an initial reservoir of H$_2$, which is used in the subsequent star formation process, the final stellar mass $M_*(\text{final})$ is divided by the initial H$_2$ mass of the cloud. If the cloud accretes a substantial amount of H$_2$ over its lifetime, the definition has to be generalized. We will use

$$\epsilon_{GMC} = \frac{M_*(\text{final})}{\text{max}(M_{H_2})},$$  

(2)

where $\text{max}(M_{H_2})$ is the maximal H$_2$ mass of the GMC. By definition $\epsilon_{GMC}$ is a non-evolving quantity and it can be estimated, e.g., by comparing luminosity distribution of OB associations in the Milky Way with the mass spectrum of molecular clouds (Williams & McKee 1997). It cannot be directly measured on a cloud-to-cloud basis, because $M_*$ and $M_{H_2}$ must be known at two different times. Instead such observations (see, e.g., Myers et al. 1986) estimate the following quantity:

$$\eta_{GMC}(t) = \frac{M_*(t)}{M_{H_2}(t) + M_*(t)} \approx \frac{M_*(t)}{M_{H_2}(t)}.$$  

(3)

The latter term is due to the fact that for most observed GMCs $M_*$ is smaller than $M_{H_2}$. Obviously, $\eta_{GMC}(t)$ increases over the lifetime of a cloud and should not be confused with either $\epsilon_{GMC}$ or $\epsilon_{ff}$. From Equation (1) we can estimate $M_*(t) = \xi \epsilon_{ff}/t_{ff} \text{max}(M_{H_2})\eta_{GMC}$, hence $\epsilon_{GMC} = \xi \epsilon_{ff}/t_{ff} \text{max}(M_{H_2})$, where $\xi$ is a constant fudge factor of order unity that depends on the actual time evolution of the SFRs and $M_{H_2}$ (and $\epsilon_{ff}$ if it is time dependent). We will estimate $\xi$ for a simple toy model in Section 3. Combining this result with Equations (2) and (3) we obtain

$$\eta_{GMC}(t) \approx \xi \epsilon_{ff} \left[ \frac{t_{ff}}{M_*(t) M_{H_2}(t)} \right] \text{max}(M_{H_2}).$$  

(4)
There are several ways of creating large values of $\eta_{\text{GMC}}$ and they correspond to the various terms in this equation. First, $\epsilon_{\text{ff}}$ could be time dependent. For instance, it could smoothly increase as the cloud collapse advances or, alternatively, vary stochastically about some average value. A second possibility is that some clouds may live for many free-fall times, i.e., $t_{\text{free}}/t_{\text{ff}}$ is large in a subset of GMCs. The factor in the third bracket in Equation (4) explains why $\eta_{\text{GMC}}$ can also be smaller than $\epsilon_{\text{GMC}}$. Finally, $\eta_{\text{GMC}}$ can be boosted if the observed H$_2$ mass is significantly less than $\max(M_{\star})$, i.e., if GMCs lose (in one way or another) a large fraction of their molecular hydrogen over their lifetime. The latter scenario predicts that $\eta_{\text{GMC}}(t)$ should roughly scale $\propto M_{\text{H}_2}^{-1}$ over the lifetime of individual GMCs. An observational sample of an ensemble of GMCs shows this trend (Murray 2010). However, this trend can also be produced by a selection effect based on stellar mass, e.g., selecting GMCs with $M_{\star} > M_{\star,\text{limit}}$ excludes values of $\eta_{\text{GMC}}$ that are smaller than $M_{\star,\text{limit}}/M_{\text{H}_2}$ (see Equation (3)). In fact, Murray (2010) is selecting clouds based on ionizing luminosities, which roughly correspond to selecting clouds based on the total mass formed within the last 4 Myr. Such a selection effect could explain why a different study of $\sim 10^4$ M$_\odot$ GMCs finds much lower efficiencies (Lada et al. 2010). The existence of a selection effect is not an argument against or in favor of an evolving $\epsilon_{\text{ff}}$, rather it shows that the GMCs in the sample of Murray (2010) with large values of $\eta_{\text{GMC}}$ are likely a heavily biased subset. The way $\eta_{\text{GMC}}$ (and the upper boundary of the region excluded by the discussed selection effect) scales with the H$_2$ mass of the GMC$^4$ ($\propto M_{\text{H}_2}^{-1}$) implies that the GMCs in the sample of Murray (2010) should all have large, rather similar maximal H$_2$ masses $\max(M_{\text{H}_2})$.

A different issue can arise if one compares SFRs and H$_2$ masses in order to estimate $\epsilon_{\text{ff}}/t_{\text{ff}}$ via Equation (1). For example, let us assume that we measure SFRs and H$_2$ masses within small ($\lesssim 100$ pc) apertures around peaks of CO emission (tracing the H$_2$ mass) and peaks of H$_2$ emission (tracing SFRs; see, e.g., Schruba et al. 2010). If we observe that CO peaks have lower SFRs at given H$_2$ mass compared with peaks of H$_2$ emission, does this imply a time-varying $\epsilon_{\text{ff}}/t_{\text{ff}}$? The answer to this question depends on the way the SFRs are measured. SFRs that are derived from H$_2$ emission are effectively averaged over the past $5\ldots10$ Myr, which might well be a significant fraction of the lifetime of the molecular cloud. For SFRs that are based on H$_2 + 24$ $\mu$m emission, this averaging time span would be even longer. The star formation efficiencies per free-fall time that are estimated from such a time averaged SFR will be small initially (no stars have been formed over most of the time averaging interval simply because the GMC has only formed recently). The measured SFRs will increase until the age of the GMC is similar to the averaging time span. In addition, the H$_2$ mass of the cloud might decrease leading to an additional increase in the apparent value of $\epsilon_{\text{ff}}/t_{\text{ff}}$ with time. A recent study that measures SFRs with reasonably short averaging times (2 Myr; Lada et al. 2010) estimates star formation efficiencies per free-fall time of the order of 2% for most clouds in the sample, with the scatter mostly driven by the mass of molecular gas of relatively low density ($n < 10^4$ cm$^{-3}$) that does not participate in the star formation.

3. TOY MODEL

We will now discuss a toy model in order to both exemplify the points made in Section 2, but also to provide a framework in which we can make some quantitative predictions. We should stress that the statements made in the previous section are completely generic and do not depend on the specific assumption that go into the model that we are going to present. Our model is highly simplistic, and, given that, our aim is not to reproduce the full complexity in the evolution of GMCs or even to be consistent with any available observation. On the other hand the model offers a pragmatic approach to the mass evolution of GMCs and may be easily generalized to facilitate more complex scenarios.

The ansatz of the model is to supplement Equation (1) with an equivalent equation that describes the evolution of the H$_2$ mass:

$$M_{\text{H}_2} = \frac{\epsilon_{\text{ff}}}{t_{\text{ff}}} M_{\text{H}_2} - \alpha M_{\star} + \gamma.$$  

The extra term $\alpha M_{\star}$ is motivated by assuming that stellar feedback is limiting the lifetime of molecular clouds, e.g., via photoionization, thermal pressure, or radiation pressure (Williams & McKee 1997; Murray et al. 2010; Lopez et al. 2010). This feedback should therefore couple to the formed stellar mass via some efficiency factor $\alpha$ that sets the timescale for the destruction/removal of H$_2$ from the cloud.$^5$ The term $\gamma$ is the net “accretion” rate of H$_2$, which includes all processes that create and destroy H$_2$ and are not directly coupled to either $M_{\star}$ or $M_{\text{H}_2}$. Both $\alpha$ and $\gamma$ could in principle be time dependent. For simplicity we assume that they are constant. Our model is minimalistic (compared with, e.g., Matzner 2002; Tan et al. 2006; Huff & Stahler 2006; Krumholz et al. 2006), but it has the advantage that we can parameterize our ignorance of the relevant physical processes that destroy and disperse the cloud into the parameters $\alpha$ and $\gamma$. Together with appropriate initial conditions Equations (1) and (5) fully determine the evolution of the masses of molecular hydrogen and the stellar component in a GMC.

We will also make the simplifying assumption that the free-fall time does not evolve strongly over the history of the GMC, i.e., both the star formation efficiency per free-fall time and the star formation timescale are now fixed. This assumption is not crucial for the model, but we will use it for the following reasons. First, there is no clear systematic trend of free-fall time with mass over the range of GMCs that we are comparing to, see, e.g., Table 2 of Murray (2010). Second, assuming a non-evolving free-fall time allows for a convenient analytical solution of the problem. Third, we find that even with this assumption our model describes the observed data reasonably well. We stress that our main aim is to show that a simple model can produce an observational signal that could be misinterpreted as evidence for evolution of the star formation efficiencies. We do not try to model the precise properties of the ensemble of GMCs in the Galaxy.

With $t_{\text{ff}}$ fixed (and, of course, we assume that the star formation efficiency per free-fall time is a constant too) we can insert Equation (1) into Equation (5) and obtain a linear second-order differential equation for $M_{\text{H}_2}$, i.e., the equation of a damped harmonic oscillator.

$^4$ A linear regression of $\eta_{\text{GMC}}$ versus $M_{\text{H}_2}$ for the data presented in Murray (2010) gives a slope of $-0.59 \pm 0.19$. This is consistent with the prediction of our toy model ($\text{slope} \sim -0.75$; see Section 3) that takes into account that, in fact, not the total stellar mass has been measured, but only the stellar mass formed within the last $\sim 4$ Myr.

$^5$ Depending on the type of feedback $M_{\star}$ should refer to the total stellar mass times a weight parameter that takes into account that feedback is provided by stars which have a limited lifetime. For simplicity we will assume that $M_{\star}$ is the total amount of stellar mass formed within the cloud.
Equation (7). In the no accretion scenario we obtain mass that is formed during the lifetime of the cloud from and in the pure accretion scenario, respectively. The different lines correspond to the H$_2$ mass (solid blue line), total stellar mass (dashed red line), and stellar mass formed within 4 Myr (dot-dashed green line).

Solving the differential equation we obtain

$$M_{\text{H}_2}(t) = Ae^{-tb/2} \cos(\omega t + \phi), \quad (6)$$

$$M_*(t) = \frac{M_{\text{H}_2}}{\alpha} (\omega \tan(\omega t + \phi) - b/2) + \frac{\gamma}{\alpha}, \quad (7)$$

where $b = \epsilon_0/\tau_{ff}$ is the inverse of the star formation timescale and $\omega = \sqrt{ab - b^2/4}$ is the “oscillation” period.

Phase $\phi$ and amplitude $A$ depend on the initial conditions. In the following, we restrict ourselves to two special cases of the general model (Equations (6) and (7)).

1. No accretion scenario. It assumes $\gamma = 0$, $M_{\text{H}_2}(t = 0) = M_0 > 0$, and $M_*(t = 0) = 0$. It follows $\phi = \tan(\omega/2\omega)$ and $A = M_0/\cos(\phi)$.

2. Pure accretion scenario. It assumes that all H$_2$ is “accreted,” i.e., $M_{\text{H}_2}(t = 0) = 0$, $M_*(t = 0) = 0$, and $\gamma > 0$. In this case phase and amplitude are given by $\phi = -\pi/2$ and $A = \gamma/\alpha$.

We adopt the parameters $\epsilon_0 = 0.02$ and $\tau_{ff} = 6$ Myr, which are consistent with observations of $\epsilon_0$ over a range of density scales (Krumholz & Tan 2007), and with the free-fall times $6.1^{+0.8}_{-0.6}$ Myr measured in the sample of Murray (2010), respectively. We note that only the ratio $\epsilon_0/\tau_{ff} = 0.0033$ Myr$^{-1}$ enters our model. The $\alpha$ parameter is chosen such that the lifetime of the cloud, i.e., the time $t_{\text{final}}$ at which $M_{\text{H}_2}(t_{\text{final}}) = 0$, is $\sim 20$ Myr (Williams & McKee 1997). Hence, we use $\alpha = 2$ Myr$^{-1}$ in the no accretion scenario and $\alpha = 8$ Myr$^{-1}$ in the pure accretion scenario, respectively.

Assuming $\epsilon_0/\tau_{ff} \ll \alpha$ the lifetime of a GMC is given by

$$t_{\text{final}} \approx \frac{\pi}{2\sqrt{\alpha \epsilon_0/\tau_{ff}}} \quad \text{and} \quad t_{\text{final}} \approx \frac{\pi}{\sqrt{\alpha \epsilon_0/\tau_{ff}}}.$$  

The left (right) expression refers to the no accretion (pure accretion) scenario. We note that in both considered scenarios the lifetime does not depend on the initial cloud mass or the accretion rate, respectively. The evolution of the H$_2$ and the stellar mass, normalized to max($M_{\text{H}_2}$), is shown in Figure 1.

Assuming $\epsilon_0/\tau_{ff} \ll \alpha$ we can easily estimate the total stellar mass that is formed during the lifetime of the cloud from Equation (7). In the no accretion scenario we obtain

$$M_*(t_{\text{final}}) \approx M_0 \sqrt{\epsilon_0/\tau_{ff} \frac{\alpha}{M_0}} \left[ 1 - \frac{\epsilon_0}{2\tau_{ff}} \frac{t_{\text{final}}}{\tau_{ff}} \right] \approx M_0 \sqrt{\epsilon_0/\tau_{ff} \frac{\alpha}{M_0}},$$

while the pure accretion scenario predicts

$$M_*(t_{\text{final}}) \approx \frac{2\nu}{\alpha} \left[ 1 - \frac{t_{\text{final}}}{2\tau_{ff}} \frac{\epsilon_0}{\tau_{ff}} \right] \approx \frac{2\nu}{\alpha}.$$  

In the pure accretion scenario a GMC attains its maximum mass at $t \approx t_{\text{final}}/2$. The H$_2$ mass is then approximately $\gamma/\sqrt{\alpha \epsilon_0/\tau_{ff}}$. Combining these results we see that the star formation efficiency of a GMC is

$$\epsilon_{\text{GMC}} \approx \left[ \frac{\epsilon_0/\tau_{ff}}{\alpha} \right]^{1/2} \epsilon_{\text{GMC}} \approx 2 \sqrt{\epsilon_0/\tau_{ff} \frac{\alpha}{M_0}}.$$  

Again, the left (right) expression refers to the no accretion (pure accretion) scenario. Written in terms of the lifetime of the GMC both expression are identical, namely, $\epsilon_{\text{GMC}}/\epsilon_{\text{ff}} \approx (2/\pi) \epsilon_0/\tau_{ff}$, i.e., $\xi = 2/\pi$ (Section 2). In Figure 2, we show the predictions for $\eta_{\text{GMC}}$ and $\eta_{\text{ff}}$ of the two scenarios of our model, together with $\epsilon_0$ and $\epsilon_{\text{GMC}}$, and the observational data from Murray (2010) and Lada et al. (2010). To be consistent with Murray (2010), only the stellar mass that is formed within the last $t_{\text{avg}} = 4$ Myr is used to compute $\eta_{\text{GMC}}$, see Equation (3). However, when we compare with Lada et al. (2010), who measure stellar masses from counting young stellar objects, we use the stellar mass formed within the last $t_{\text{avg}} = 2$ Myr. In both cases $\epsilon_0$ is estimated via the expression $\eta_{\text{ff}} = \epsilon_{\text{GMC}}/\epsilon_{\text{ff}}$. One arrives at this expression by approximating the instantaneous SFRs in Equation (1) with the average SFR over the last $t_{\text{avg}}$. Another, equivalent, expression is

$$\eta_{\text{ff}} = \epsilon_0 \left[ \frac{M_{\text{H}_2}(t_{\text{final}})}{M_{\text{H}_2}(t)} \right] \bar{M}_{\text{H}_2}(t) = \frac{1}{t_{\text{avg}}} \int_{t - t_{\text{avg}}}^{t} M_{\text{H}_2}(t') dt'.$$  

Our model reproduces the trends of $\eta_{\text{GMC}}$ and $\eta_{\text{ff}}$ with GMC mass suggesting that these are maybe not solely due to selection effects. In both considered scenarios $\eta_{\text{GMC}}$ and $\eta_{\text{ff}}$ roughly scale as $M_{\text{H}_2}$ over the mass range $10^4$–$10^6 M_\odot$. The fact that our toy model explains the trends in $\eta_{\text{ff}}$ and $\eta_{\text{GMC}}$ simultaneously is not given per se. The smallness of $M_{\text{H}_2}/\max(M_{\text{H}_2})$ near the end of the lifetimes of GMCs can explain the large observed values of $\eta_{\text{GMC}}$ (see Equation (4)), but does not necessarily lead to large values of $\eta_{\text{ff}}$. As Equation (8) shows, the required ingredients are a decreasing H$_2$ mass with time and the existence of a star formation tracer with finite lifetime $t_{\text{avg}}$. Under those conditions the time-averaged SFRs are larger than the instantaneous SFRs (an equivalent statement holds for $M_{\text{H}_2}$) and $\eta_{\text{ff}} > \epsilon_0$.

With the chosen parameters our model predicts that $\eta_{\text{GMC}}$ is only significantly larger than $\epsilon_{\text{GMC}}$ for the last $\sim 1$ Myr in the life of a GMC, this includes most of the GMCs with masses less than $\sim 10^5 M_\odot$ in the sample of Murray (2010). In contrast, the clouds in the sample of Lada et al. (2010) are predicted to span a broad range of ages and are not necessarily in the last Myr of their lives.

Our model exemplifies that it is difficult to prove the existence of a time-varying star formation efficiency based on observational quantities such as $\eta_{\text{GMC}}$ or $\eta_{\text{ff}}$. This is not to say that such a time dependence does not exist, we merely conclude that current observational evidence for its existence is insufficient.

A potentially promising way to settle the question of a time-varying $\epsilon_0$ is to apply the method of Lada et al. (2010) to a larger sample of GMCs, including clouds as massive and star-

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6 The precise time does depend on the assumed lifetime of the cloud. Clouds with shorter lifetimes spend more time in a state in which $\eta_{\text{GMC}} > \epsilon_{\text{GMC}}$. 

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Figure 1. Evolution of the masses of the GMC components (normalized to the maximum H$_2$ mass of the GMC) according to the two scenarios: no accretion (left) and pure accretion (right). We assume $\epsilon_0/\tau_{ff} = 0.0033$ Myr$^{-1}$ and $\alpha = 2$ Myr$^{-1}$ ($\alpha = 8$ Myr$^{-1}$) in the no accretion (pure accretion) scenario. The different lines correspond to the H$_2$ mass (solid blue line), total stellar mass (dashed red line), and stellar mass formed within 4 Myr (dot-dashed green line).
forming as the ones discussed in Murray (2010). If $\epsilon_f$ is in fact non-evolving and the lifetimes of the clouds are much longer than the lifetime of the star formation tracer, then the estimates $\eta_G$ should strongly cluster around $\epsilon_f$ and excursions above and below that value should be rare. Other approaches, e.g., one that tries to measure the decrease in the $H_2$ mass by comparing $H_2$ and dynamical masses, are potentially possible, but hinge on uncertainties about molecular outflows from GMCs and the conversion factor between $H_2$ and CO.

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Figure 2. Estimators of the star formation efficiencies as function of molecular mass of the GMC. The quantity $\eta_{GMC}$ ($\eta_H$) is shown in the left (right) panel. The observational data presented by Murray (2010) and Lada et al. (2010) are indicated with empty squares and triangles, respectively. The overplotted solid and dot-dashed lines refer to the predictions of the toy model. Specifically, the three solid (dot-dashed) blue lines correspond to the no accretion scenario with $\epsilon_f = 0.02$, $t_f = 6$ Myr, $\alpha = 2$ Myr$^{-1}$ (pure accretion scenario with $\epsilon_f = 0.02$, $t_f = 6$ Myr, $\alpha = 8$ Myr$^{-1}$). They differ (from bottom to top) in the value of the initial $H_2$ mass $M_0/10^2 = 0.3, 1, 3 M_\odot$ (solid lines) and the accretion rate $\gamma = 0.3, 1, 3 M_\odot$ yr$^{-1}$ (dot-dashed lines), respectively. For consistency with Murray (2010), only the stellar mass formed within the last 4 Myr is considered in the computation of $\eta_{GMC}$ and $\eta_H$. Red lines (overlying the triangles) use (1) a factor two smaller $t_f$ (reflecting the fact that the clouds in the sample of Lada et al. 2010 are smaller), (2) a factor two larger $\alpha$ values (to keep the same $\epsilon_f$ and the same ratio between $t_{final}$ and $t_f$), and (3) use only the stellar mass formed within the last 2 Myr (Lada et al. 2010 derive stellar masses from counting young stellar objects). The solid (dot-dashed) red lines correspond to the no accretion scenario with $M_f/10^2 = 0.2, 1 M_\odot$ (pure accretion scenario with $\gamma = 0.01, 0.03 M_\odot$ yr$^{-1}$). Filled circles and filled stars indicate when the age of the modeled GMC is half its total lifetime and when the cloud is 1 Myr away from the end of its life, respectively. The diagonal dashed line indicates a slope of $-0.75$, which is approximately the slope predicted by our toy model. The observed data are consistent with this slope. A linear regression of $\eta_{GMC}$ and $\eta_H$ as function of GMC mass using all clouds with masses $> 10^4 M_\odot$ returns slopes of $-0.59 \pm 0.19$ and $-0.49 \pm 0.32$, respectively, at the 95% confidence limit. Each panel also contains a horizontal line that denotes the value of the star formation efficiency $\epsilon_{GMC} = 0.04$ and $\epsilon_f = 0.02$, respectively.