Application of MADM Methods for the Synthesis of Alumina/Lanthanum Hexa-aluminate Powders by Chemical Precipitation and Filtration Method

Y N V Sai Ram¹,²*, C Tara Sasanka³ and J Prabakaran⁴

¹ Research Scholar, Annamalai University, India
²,³ Assistant Professor, R.V.R. & J.C. College of Engineering, India
⁴ Associate Professor, Annamalai University, India

* Corresponding author e-mail: sairamynv@gmail.com

Abstract. In product design, the selection of apt material plays a vital role as each material has individual uniqueness that add numerous aspects to suit the specific application. Picking of wrong material selection promotes product failure and also high-cost contribution. Consequently, it becomes hard to pick a specific one among the different materials available. For the choice of content, a better methodology is increasingly appreciated. MADM methods are the most looming concepts in today’s world which facilitate the decision maker to select best strategies among the different variable alternatives. Multi-Attribute Decision Making (MADM) strategies based on mathematical models include a ranking of the various alternatives, making it easier to make decisions. As a thermal barrier coating (TBC), Lanthanum Hexa-aluminate (LHA) with a magneto plumbite structure is a promising competitor to yitria partially stabilized zirconia (Y-PSZ) and is synthesized via a chemical precipitation and filtration process. LHA is proved to be an efficient material however the correct composition reflecting better mechanical properties is a challenging task. Therefore, this paper presents the selection of best composition of Lanthanum Hexa-aluminate ceramic powder using AHP and TOPSIS methods and also the impact of weighting factors has been addressed as well.

Keywords: Lanthanum Hexa-aluminate (LHA), Multi-Attribute Decision Making (MADM), Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Entropy based weighting (EBW), measure of output (MOP),

1. Introduction
Materials with elevated temperature capability are imperative in many sectors like manufacturing including processing of materials, chemical engineering, power generation, transport and aerospace [1-3]. The work capacity at elevated temperatures is therefore a vital factor for industrial efficiency. The material from which the part is produced basically meets the mechanical requirements of the process and provides a ounce of environmental resistance. Nevertheless, to provide resistance to environment a coating that is physically and chemically friendly with the matrix material may be desired. However,
Selection of material is characterized as a factor that influences the choice of a material for a specific application, and it is typically done by many young engineers [4-5]. It should be mentioned that there may not be always a single objective to select the elite material, so designers and engineers must weigh a diverse range of materials selection criteria. Initial screening, production and comparison of alternatives and determination of the best solution are the three stages of material selection [6-7]. Many aspects must be considered when selecting a material for agreeable economic output in a specific application. Multi-Criteria Decisions Making (MCDM) [8-10] divides decision making in the presence of multiple competing criteria into two categories: Multi-Attribute Decision Making (MADM) approaches and Multi-Objective Decision Making (MODM) [11]. Recently, MADM methods have become more commonly available to assist researchers and practitioners in assessing and picking the best compromise choices. These MADM models are capable of classifying materials based on a range of properties (attributes) [12]. Analytical Hierarchy Process (AHP) [13–16] and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [17–19] are two common MADM methods that have earned a lot of attention for dealing with serious problems with friction factors. TOPSIS and AHP methods are used in this analysis to rate competing materials for their applications.

Lanthanum Hexaaluminate (LaAl$_{11}$O$_{18}$) is truncated as LHA which belongs to the rare-earth aluminate family with magneto plumbite structure with a unique stoichiometry and it is the key compound formed in La$_2$O$_3$–Al$_2$O$_3$ system and it is used as a material in thermal barrier coating [20-21]. This paper focuses on application of MADM methods to opt best condition for synthesis of LHA. Problem formulation and methodology details are mentioned in section 2 & 3 while the results are discussed in the subsequent sections.

2. Problem formulation

Three different compositions of Alumina/LHA powder ranging from 20 to 60 vol% at 20 vol% interval were synthesized using chemical precipitation and filtration technique [22]. But the economical production of Lanthanum Hexaaluminate nano particles is a major challenge. Experiments are designed based on the Taguchi L9 orthogonal array by considering the three parameters at three levels as in Table 1. From Table 2, the decision making to pick the specific combination is complex as it is based on the application perspective. The present problem involves identification of best combination of parameters among the nine runs available trying to get the preeminent combination fulfilling the requirements thereby decision maker has to judge the best one.

3. Methodology

3.1. Determining the weightage of each attribute

A subsection

The problem proposed here is the implementation entropy based weighting factors for each attribute and to take an accurate decision.

3.2. Employment of different techniques

When the decision maker relies on the weightages, a variety of different methodologies may be used. The MADM methods are discussed in simple terms below:

- Note down the attributes, alternatives.
- Pick out relative value weightages for each attribute.
- The table should be normalized based on beneficiary and non-beneficiary variables.
- Using various MADM methodologies, the measure of output (MOP) of each alternate is determined.
- On the basis of measure of output (MOP), ranking is allotted to various alternates.

Despite the fact that there are more MADM approaches, only a few have a greater potential for solving problems in the world of manufacturing. Among all MADM methods, the Simple Additive
Weighing Method (SAW method) and the Weighted Product Method (WPM) are the most common. To observe the preference, AHP and TOPSIS procedure and its variant modifications are used.

### Table 1. Investigated parameters and their levels.

| Parameters                        | Level I | Level II | Level III |
|-----------------------------------|---------|----------|-----------|
| % of Composition                  | 1:20    | 1:40     | 1:60      |
| Calcination Temperature (°C)      | 750     | 900      | 1000      |
| Time (Hr)                         | 1       | 2        | 3         |

### Table 2. Experimental measured values for Hardness and Ultimate compressive strength (Taguchi Orthogonal array L9).

| Runs | % of Composition | Calcination Temperature (°C) | Time (Hr) | Hardness (VHR) | Ultimate Compressive strength (N/mm²) |
|------|------------------|-----------------------------|-----------|----------------|--------------------------------------|
| 1    | 1:20             | 750                         | 1         | 151            | 12.0                                 |
| 2    | 1:20             | 900                         | 2         | 164            | 5.5                                  |
| 3    | 1:20             | 1000                        | 3         | 152            | 5.7                                  |
| 4    | 1:40             | 750                         | 2         | 147            | 12.0                                 |
| 5    | 1:40             | 900                         | 3         | 158            | 12.0                                 |
| 6    | 1:40             | 1000                        | 1         | 142            | 12.0                                 |
| 7    | 1:60             | 750                         | 3         | 145            | 12.0                                 |
| 8    | 1:60             | 900                         | 1         | 159            | 12.0                                 |
| 9    | 1:60             | 1000                        | 2         | 148            | 5.7                                  |

3.3. **AHP Method**

In multiple domains of science and technology decision making plays a vital role to hash out that Satty [23] developed the AHP (Analytic Hierarchy Process) in 1981. AHP is mostly used to solve multi-criteria and uncertain decision problems [24-25]. Because of its flexibility, ease of use, and simplicity this tool has a wide range of applications.

3.4. **TOPSIS Method**

To spot out solutions from a finite set of available alternatives Hwang et.al developed the TOPSIS method. The selection criteria for the best possible alternative using TOPSIS method is as follows.

a) By considering both beneficial and non-beneficial variables a standardized matrix is prepared that meets the objective on the basis of decision table.

b) Obtain normalized decision matrix,

\[
R_{pq} = \frac{m_{pq}}{\left[\sum_{q=1}^{M} m_{pq}^2\right]^{1/2}}
\]

(1)

Where \(m_{pq}\) is the element on decision table of \(p^{th}\) alternative and \(q^{th}\) attribute.

c) Find the weighted Normalized matrix,

\[
U_{pq} = w_q \times R_{pq}
\]

(2)
d) Obtain Ideal and Negative Ideal solutions by the expressions given below.
\[ U^+ = \{ \left( \sum_{p}^{\text{max}} U_{pq} / q \in Q \right), \left( \sum_{p}^{\text{min}} U_{pq} / q \in Q \right) / p \quad \forall \ p = 1, 2, 3, ... M \} \]

\[ U^- = \{ \left( \sum_{p}^{\text{min}} U_{pq} / q \in Q \right), \left( \sum_{p}^{\text{max}} U_{pq} / q \in Q \right) / p \quad \forall \ p = 1, 2, 3, ... N \} \]

The positive ideal best and negative ideal worst values of the considered attribute among the available values of the same attribute in various alternatives are \( U^+ \) and \( U^- \). In case of non-beneficiary variables \( U^+ \) is the lower and \( U^- \) is the higher value.

e) For each alternative, Euclidian distance is calculated.

\[ S_p^+ = \left\{ \sum_{q=1}^{M} (U_{pq} - U^+_{q})^2 \right\}^{0.5} \quad \forall \ p = 1, 2, ..., N \]

\[ S_p^- = \left\{ \sum_{q=1}^{M} (U_{pq} - U^-_{q})^2 \right\}^{0.5} \quad \forall \ p = 1, 2, ..., N \]

f) For each alternative \( X_p \) relative closeness is calculated by using the formula

\[ X_p = S_p^- / (S_p^+ + S_p^-) \]

g) For each alternative, ranking is allotted based on \( S_p^+ \) and \( S_p^- \).

4. Results and Discussion

4.1. Entropy based weighting (EBW) method

It's not always appropriate to use randomized weights and replay the experiment until the decision maker is pleased. The EBW method is a very much accurate information calculation method for enacting accurate weights [12]. When subjective weights are unreliable, objective weighting is especially useful. Reduction in decision makers experiments as much as possible for obtaining the weights in EBW method to solve a MADM problem. These additionally supportive in distinguishing the weights to attributes in a less amount of time. In EBW method, the attributes with performance rating vary with each other has privileged significance in the problem. For decision making process, bigger the entropy value corresponding to the particular attribute signifies the smaller attribute’s weight and the less power of that attribute in decision making process. The bigger the entropy values referring to a specific attribute, the lower the attribute's weight and the less influence it has in the decision-making process. Table 3 shows the weighting given to each attribute using EWB method.

| Attribute                      | Weighting |
|--------------------------------|-----------|
| Hardness                       | 0.9806    |
| Ultimate compression strength  | 0.0194    |

4.2. AHP method

Since the weights have already been determined, the alternatives are equated one by one to gain a better understanding of how each attribute compares to the others. The Relative mode is used to rank alternatives in order of choice. Table 4 shows the AHP score and ranking structure. As a result, the AHP method selection priority is \( S2 \) – \( S3 \) – \( S9 \) – \( S8 \) – \( S5 \) – \( S1 \) – \( S4 \) – \( S7 \) – \( S6 \).

4.3. TOPSIS method

Using MATLAB programme formulae of the method have been developed to calculate the attributes (\( U^+, U^- \)), alternatives (\( S^+, S^- \)) which are based on entropy weights and tabulated in Table 5. For each alternative performance score is evaluated using TOPSIS method and tabulated in Table 6.
Table 4. Performance score and ranking based on AHP method with EBW method

| Sample | % of Composition | Calcination Temperature (°C) | Time (Hr) | AHP | Ranking Structure for AHP |
|--------|------------------|-----------------------------|-----------|-----|--------------------------|
| S1     | 1:20             | 750                         | 1         | 0.0813 | 6                        |
| S2     | 1:20             | 900                         | 2         | 0.1750 | 1                        |
| S3     | 1:20             | 1000                        | 3         | 0.1687 | 2                        |
| S4     | 1:40             | 750                         | 2         | 0.0812 | 7                        |
| S5     | 1:40             | 900                         | 3         | 0.0814 | 5                        |
| S6     | 1:40             | 1000                        | 1         | 0.0811 | 9                        |
| S7     | 1:60             | 750                         | 3         | 0.0812 | 8                        |
| S8     | 1:60             | 900                         | 1         | 0.0814 | 4                        |
| S9     | 1:60             | 1000                        | 2         | 0.1687 | 3                        |

Table 5. Calculated Values of Attributes and Alternatives

| Sample | ATTRIBUTES | ALTERNATIVES |
|--------|------------|--------------|
|        | $U^+$      | $U^-$        | $S^+$      | $S^-$      |
| S1     | 0.3799     | 0.0064       | 0.0019     | 0.0004     |
| S2     | 0.1741     | 0.0070       | 1.0000     | 0.2058     |
| S3     | 0.1805     | 0.0065       | 0.9691     | 0.1995     |
| S4     | 0.3799     | 0.0063       | 0.0010     | 0.0002     |
| S5     | 0.3799     | 0.0067       | 0.0033     | 0.0007     |
| S6     | 0.3799     | 0.0060       | 0           | 0          |
| S7     | 0.3799     | 0.0062       | 0.0006     | 0.0001     |
| S8     | 0.3799     | 0.0068       | 0.0035     | 0.0007     |
| S9     | 0.1805     | 0.0063       | 0.9691     | 0.1995     |

Table 6. Performance score and ranking based on TOPSIS method with EBW method

| Sample | % of Composition | Calcination Temperature (°C) | Time (Hr) | TOPSIS | Ranking Structure for TOPSIS |
|--------|------------------|-----------------------------|-----------|--------|-----------------------------|
| S1     | 1:20             | 750                         | 1         | 0.0019 | 6                           |
| S2     | 1:20             | 900                         | 2         | 1      | 1                           |
| S3     | 1:20             | 1000                        | 3         | 0.9691 | 3                           |
| S4     | 1:40             | 750                         | 2         | 0.001  | 7                           |
| S5     | 1:40             | 900                         | 3         | 0.0033 | 5                           |
| S6     | 1:40             | 1000                        | 1         | 0      | 9                           |
| S7     | 1:60             | 750                         | 3         | 0.0006 | 8                           |
| S8     | 1:60             | 900                         | 1         | 0.0035 | 4                           |
| S9     | 1:60             | 1000                        | 2         | 0.9691 | 2                           |

As a result, the TOPSIS method’s selection priority is S2 – S9 – S3 – S8 – S5 – S1 – S4 – S7 – S6.

5. Conclusions
This paper successfully implemented the AHP and TOPSIS methods for opting the best combination for the synthesis of Lanthanum Hexa-Aluminate Powders (LHA). These methods provided simple and
powerful ranking criteria for decision making. Out of all highest preference is for 1:20, 900°C, 2hrs (Sample 2) and least preference is for 1:40, 1000°C, 1hr in both the methods. It is detected that the powder prepared with 1:20, 900°C, 2hrs has yielded good mechanical properties among the conditions compared in this work. The same problem can be extended further to measure the mechanical behaviour using Taguchi method and GRA analysis as well.

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