I. INTRODUCTION

Over the last years we have witnessed a technological trend towards miniaturization of electronic circuits. This tendency has been accompanied by a growing research activity focused on achieving a better understanding of the mechanisms for heat dissipation and energy flow in mesoscopic systems. However, the motivation for the research in this area is not only technological, because the very fundamental concepts of standard statistical mechanics and thermodynamics are put into test when studying these systems, even more when the process under consideration corresponds to an out-of-equilibrium situation.

Several efforts have been made towards the extension of standard thermodynamical concepts to the out-of-equilibrium evolution of different systems. Some well-known examples are the aging regime of glassy systems, sheared glasses, granular materials and colloids. A very successful achievement in the characterization of such nonequilibrium states has been the identification of an effective temperature, i.e., a parameter with the same properties of the temperature of a system at equilibrium that is useful to describe the evolution of nonequilibrium systems. For instance, for glassy systems the definition of effective temperature was introduced by means of a generalization of the equilibrium fluctuation-dissipation relations (FDR) and the physical meaning of this concept was supported by showing that such a temperature would coincide with the one measured by a thermometer. The definition of an effective temperature from a FDR was introduced for quantum glassy systems in Ref. 2 and later analyzed in electronic systems. More recently, these temperatures were studied in an Ising chain after a sudden quench.

The physics of the mesoscopic scale is ruled by the quantum coherence of the particle propagation. This originates non-trivial interference mechanisms and surprising effects. Well known examples are the violation of the Fourier’s Law in low-dimensional phononic systems as well as the $2k_F$ oscillations of the local voltage and the negative electrical resistance. In a previous work we defined local and effective temperatures in electronic quantum systems driven out of equilibrium by external ac potentials. Examples of such systems are quantum dots with ac voltages acting at their walls (quantum pumps) and quantum capacitors. In that work we presented two concepts, which are the local and the effective temperatures. The local temperature was introduced following a procedure inspired in a work by Engquist and Anderson. The idea is to include a thermometer in the microscopic description of the system. On the other hand the effective temperature is defined from a local FDR involving single-particle Green’s functions. We showed that for low driving fre-
quencies both ways of defining the temperature coincide. In a more recent work, we slightly generalized the definition of the thermometer to consider the possibility of simultaneously sensing the local temperature and the local chemical potential of the sample. We showed that the new local temperature determined by this new definition coincides with the previous one. Even more, we showed that such a parameter verified the thermodynamical properties of a temperature, meaning that its gradient signals the direction for heat flow at the contacts.

The aim of this work is to analyze the role of effective temperatures within the context of a FDR for current-current correlation functions. The motivation is twofold. On one hand we are interested in testing the robustness of the definition of an effective temperature from a FDR, at the level of a correlation function different from the one we have considered in our previous work. On the other hand, current-current correlation functions are particularly appealing quantities since they are related to noise, which can be experimentally measured and contain valuable information on the nature of the elementary particles that take part in the transport process. The zero-frequency noise is usually used to characterize the correlations between particles in mesoscopic systems. Additionally in quantum pumps, noise is related to the correlations between particles in mesoscopic systems. Zero-frequency noise is usually used to characterize the particles that take part in the transport process. The noise, which can be experimentally measured and contains information about the temperature addressed in recent works. In Ref. 27 and since then an extensive theoretical literature on noise in mesoscopic systems analyzed within the scattering matrix formalism has been developed.

We use here another approach, which is based Keldysh formalism. For non-interacting systems both treatments were proved to coincide at the level of the description of the current for dc and ac-driven systems. In the present work we show that this is also the case for the current fluctuations correlations. The main goal of this work is to show that the effective temperature obtained from a fluctuation-dissipation relation for current-current correlation functions coincides with the local temperature defined using a thermometer and thus verifies the same thermodynamical properties of the latter.

This paper is organized as follows. In Sec. II we present the model and summarize the theoretical treatment. In Sec. III we review three definitions of temperature addressed in recent works. In Sec. IV we derive general expressions for current-current correlation functions and an explicit expression for the zero-frequency noise within the Keldysh Green’s functions formalism. In Sec. V we present numerical results for a particular system. Section VI is devoted to discussion and conclusions. We give some details of the calculation in the Appendix.

II. MODEL AND THEORETICAL TREATMENT

In Fig. 1 we display the same setup as in Refs. 19 and representing a quantum driven system, with the Hamiltonian $H_{sys}(t)$, connected to a probe characterized by $H_P$. The total system is then described by

$$H(t) = H_{sys}(t) + H_{cP} + H_P,$$

with $H_{cP}$ implementing the local coupling between the system and the probe. The Hamiltonian corresponding to the driven system can in turn be written as

$$H_{sys}(t) = H_L + H_{cL} + H_C(t) + H_{cR} + H_R,$$

where $H_C(t)$, $H_L$ and $H_R$ stand for the Hamiltonians of the central part and left and right reservoirs, coupled among themselves via the Hamiltonians $H_{cL}$ and $H_{cR}$.

The Hamiltonian describing the central system $(C)$ contains the ac fields and can be written as $H_C(t) = H_0 + H_V(t)$. We assume that $H_0$ is a Hamiltonian for non-interacting electrons while $H_V(t)$ is harmonically time dependent with a fundamental driving frequency $\Omega_0$. We leave further details of the model undetermined as much of the coming discussion is model independent

All three reservoirs (left, right and the probe) are modeled by systems of non-interacting electrons with many degrees of freedom, i.e., $H_\alpha = \sum_k \varepsilon_{k\alpha} c_{k\alpha}^\dagger c_{k\alpha}$, where $\alpha = L, R, P$. The corresponding contacts are described by $H_{ca} = w_\alpha \sum_k (c_{k\alpha}^\dagger c_{l\alpha} c_{l\alpha}^\dagger + c_{l\alpha}^\dagger c_{k\alpha} c_{k\alpha})$, where $\alpha$ denotes the coordinate of $C$ where the reservoir $\alpha$ is connected.

As in previous works, we consider non-invasive probe and we treat $w_{cP}$ at the lowest order of perturbation theory when necessary.

We will analyze the out-of-equilibrium dynamics of this system within the Schwinger-Keldysh Green’s functions formalism. Within this formalism, instead of the usual time-ordering operator used in equilibrium theory a contour-ordering operator which orders time-labels according to their order on the Keldysh contour is introduced. The single particle propagator reads

$$iG_{j,j'}(t,t') = \langle T_C [c_j(t)c_{j'}^\dagger(t')] \rangle.$$

The contour-ordered Green’s function contains four different functions depending on where the times $t$ and $t'$ are over the Keldysh contour. It is easy to see that they are not all independent. We then consider the lesser, greater and retarded Green’s functions,

$$iG^<_{j,j'}(t,t') = -\langle c_{j'}^\dagger(t')c_j(t) \rangle,$$

$$iG^>_j(t,t') = \langle c_j(t)c_{j'}^\dagger(t') \rangle,$$

$$iG^R_{j,j'}(t,t') = \Theta(t-t') \langle [c_j(t), c_{j'}^\dagger(t')]_+ \rangle,$$

where $[,]_+$ denote the anticommutator of the fermionic operators, $\langle \ldots \rangle$ is the quantum statistical average and the indexes $j, j'$ denote spatial coordinates of the system.
In general the *Keldysh* and *retarded* Green’s functions, can be expressed in terms of the *less*er and *greater* Green’s functions via

\[
A_{j,j'}^G(t,t') = A_{j,j'}^{\text{K}}(t,t') + A_{j,j'}^{\text{R}}(t,t'),
\]

\[
A_{j,j'}^{\text{R}}(t,t') = \Theta(t-t') \left[ A_{j,j'}^{\text{K}}(t,t') - A_{j,j'}^{\text{R}}(t,t') \right].
\]  

(9)

From the definition given in Eq. (8) it is straightforward to see that the Floquet-Fourier components of these functions can be written as

\[
A_{j,j'}^{\text{K}}(k,\omega) = A_{j,j'}^{\text{R}}(k,\omega) + A_{j,j'}^{\text{K}}(k,\omega),
\]

\[
A_{j,j'}^{\text{R}}(k,\omega) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{A_{j,j'}^{\text{K}}(k,\omega') - A_{j,j'}^{\text{K}}(k,\omega')}{\omega - \omega' + i0^+}.
\]  

(10)

### III. DEFINING THE TEMPERATURE

#### A. Local temperature determined by a probe

In Ref. [19] we defined the local temperature \(T_{lP}\) of the site \(lP\) of the system as the value of the temperature of the probe such that the time-averaged heat exchange between the central system and the probe vanishes. It can be shown\[^{20}\] that, given \(H_{C}(t)\) without many-body interactions, the dc component of the heat current flowing from the central system to the thermometer can be expressed as (\(h = k_B = c = 1\))

\[
J_{lP}^Q = \sum_{\alpha=L,R,P} \sum_{k} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ f_{\alpha}(\omega) - f_P(\omega_k) \right\} \langle \hat{\Gamma}_P(\omega) \hat{\Gamma}_{lP}(\omega_k) \rangle^2,
\]  

(11)

where \(\omega_k = \omega + k\Omega\), while \(\Gamma_{\alpha}(\omega) = -2\pi|w_\alpha|^2 \sum_{k} \delta(\omega - \gamma_{ka})\) are the spectral functions characterizing the reservoirs \((\alpha = L, R, P)\), and \(f_{\alpha}(\omega) = 1/[e^{\beta_{\alpha}(\omega)\mu_{\alpha}} + 1]\) is the Fermi function, which depends on \(T_\alpha = 1/\beta_\alpha\) and \(\mu_{\alpha}\) respectively the temperature and the chemical potential of the reservoir \(\alpha\). Thus, the local temperature \(T_{lP}\) corresponds to the solution of the equation

\[
J_{lP}^Q(T_{lP}) = 0.
\]  

(12)

In general, Eq. (12) must be solved numerically, but under certain conditions, an analytical expression can be found. In particular, for the low temperature weak-driving adiabatic regime, which corresponds to small amplitudes and frequencies of the driving potential, and for \(\Omega_0 \ll T_{lP}\)

\[
T_{lP} = T \left[ 1 + \lambda^{(1)}_{lP}(\mu)\Omega_0 \right],
\]  

(13)

where

\[
\lambda^{(n)}_{lP}(\omega) = \frac{1}{\sum_{k=-\infty}^{\infty} \varphi_{lP}(k,\omega)} \sum_{k=-\infty}^{\infty} (k)^n + 2d^n \frac{\varphi_{lP}(k,\omega)}{d\omega^n},
\]  

(14)
\[ \varphi_l(k,\omega) = \sum_{\alpha=L,R} \left| G^R_{l,l,\alpha}(k,\omega) \right|^2 \Gamma_\alpha(\omega). \] (15)

An alternative definition of local temperature was discussed in Ref. 23 where the fact that the heat current is related to the charge current was taken into account. Then, the local temperature \((T_{lP})\) and the local chemical potential \(\{\mu_{lP}\}\) were defined from the condition of simultaneously vanishing of the time-averaged charge and heat currents between the probe and the system. That is

\[
\begin{cases}
J^R_P(T_{lP},\mu_{lP}) = 0, \\
J^F_P(T_{lP},\mu_{lP}) = 0,
\end{cases}
\] (16)

where (see Refs. 32,33)

\[
J^R_p = \sum_{\alpha=L,R} \sum_{k=-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ f_\alpha(\omega) - f_P(\omega_k) \right\} \times \Gamma_P(\omega_k) \Gamma_\alpha(\omega) \left| G^R_{lP,l,\alpha}(k,\omega) \right|^2,
\] (17)

is the dc component of the charge current flowing through the contact between the system and the probe. The simultaneous equations given in Eq. (16) can be solved numerically for any situation, but an analytical expression can be found within the low temperature weak-driving adiabatic regime, when \(\Omega_0 \ll T\), and leads to \(T_{lP} = T_{IP}\), given in Eq. (13).

**B. Effective temperature from a single-particle fluctuation-dissipation relation (FDR)**

For systems in equilibrium, the fluctuation-dissipation theorem establishes a relation between the Keldysh (correlation) and the retarded Green’s functions. In Ref. 19 we defined a local FDR involving single-particle Green’s functions from which an effective temperature for the site \(l\) \((T^{eff}_l = 1/\beta^{eff}_l)\) can be extracted,

\[
iC^K_{\alpha\alpha}(0,0) - C^K_{\alpha\alpha}(0,\omega) = \coth \left[ \frac{\beta^{eff}_l(\omega - \mu)}{2} \right] C^K_{\alpha\alpha}(0,0),
\] (20)

with \(C^K_{\alpha\alpha}(0,0) = -2 \text{Im}[G^K_{\alpha\alpha}(0,0)] = \sum_k \varphi_k(k,\omega_k)\). In general, Eq. (18) defines an effective temperature that might depend on \(\omega\), so the limit \(\omega \rightarrow \mu\) is taken. An extrm term is added to the lhs of Eq. (18) because the rhs is always zero at \(\omega = \mu\) but \(G^K_{\alpha\alpha}(0,\mu)\) is not necessarily zero in an arbitrary out-of-equilibrium situation.

Within the low temperature weak-driving adiabatic regime, when \(\Omega_0 \ll T\), we showed\(^19\) that \(T^{eff}_l = T_{IP}\). Then, the conclusion of our previous investigations is that for the weak driving adiabatic regime the effective temperature defined from a single-particle FDR coincides with that determined by a thermometer.

**IV. CURRENT-CURRENT CORRELATION FUNCTIONS AND EFFECTIVE TEMPERATURE**

**A. A non-equilibrium fluctuation-dissipation relation**

We analyze the role of effective temperatures \((T^{eff}_l)\) from a FDR in the framework of two-particle correlation functions. As a priori they are not necessarily the same as the effective temperatures defined above we use an asterisk to refer to them. We will focus on current-current correlation functions since they are more easily accessible from an experimental point of view. We are particularly interested in a local relation, that is both currents evaluated at the same point.

As in the case of the single-particle FDR we focus on the dc components of the correlation functions to define the effective temperature. This corresponds to assuming that an equilibrium-like FDR holds for the temperature, \(\beta^{eff}_l\) playing the role of the inverse of temperature,

\[
C^{R\alpha\alpha}(0,0) = i \int_{-\infty}^{\infty} d\omega' C^{K\alpha\alpha}(0,\omega') \left[ \frac{\beta^{eff}_l(\omega')}{2} \right] .
\] (19)

An equivalent expression for the FDR given in Eq. (19) is obtained by considering the imaginary part, which leads to

\[
iC^K_{\alpha\alpha}(0,0) = \coth \left[ \frac{\beta^{eff}_l(\omega)}{2} \right] C^K_{\alpha\alpha}(0,0),
\] (20)

where \(C^K_{\alpha\alpha}(0,\omega) = -2 \text{Im}[C^{R\alpha\alpha}(0,\omega)]\). (Notice that the real part is simply derived by means of Kramers-Kronig relations.) As in the case of the single-particle FDR in Eq. (18), Eq. (20) defines an effective temperature that might depend on \(\omega\), so the limit \(\omega \rightarrow 0\) is taken.

It is important to notice the similarity of this expression with the one shown in Eq. (18) for single-particle Green’s functions (fermionic operators). In this case the hyperbolic tangent is replaced by an hyperbolic cotangent due to the bosonic statistic of current operators.

**B. Current-current correlation and noise**

Although we are more interested in the case of local current correlations, let us start by considering the more general case of correlation at different points. If we consider two reservoirs (\(\alpha\) and \(\beta\)) and two times (an absolute time \(t\) and a relative time \(\tau\)) we can define the correlation function of currents as

\[
P_{\alpha\beta}(t,t-\tau) = \frac{1}{2} (\Delta \tilde{J}_\alpha(t) \Delta \tilde{J}_\beta(t-\tau) + \Delta \tilde{J}_\beta(t-\tau) \Delta \tilde{J}_\alpha(t)),
\] (21)
With the definition of the contour-ordered current-current correlation function given in Eq. (9), the correlation function of currents given in Eq. (21) can be expressed as

\[ P_{\alpha\beta}(t, t - \tau) = \frac{i}{2} \left( C_{\alpha\beta}^>(t, t - \tau) + C_{\alpha\beta}^<(t, t - \tau) \right) = \frac{i}{2} C_{\alpha\beta}^K(t, t - \tau). \] (22)

If instead of a symmetrized current-current correlation we are interested in a nonsymmetrized one,

\[ P_{\alpha\beta}^n(t, t - \tau) = \langle \Delta \dot{J}_\alpha(t) \Delta \dot{J}_\beta(t - \tau) \rangle, \] (23)

the correlation becomes

\[ P_{\alpha\beta}^n(t, t - \tau) = i C_{\alpha\beta}^S(t - \tau, t). \] (24)

In this work we will give results for the symmetrized correlation only but it is straightforward to obtain the results for the nonsymmetrized one.

Since experimentally the noise spectrum is averaged over the absolute time t, the relevant quantity here is

\[ P_{\alpha\beta}(\omega) = 2 \int d\tau \langle P_{\alpha\beta}(t, t - \tau) \rangle e^{i\omega\tau} \] (25)

where \( \langle \ldots \rangle_t \) denotes the time average. From the definition of the Floquet-Fourier components given in Eq. (25) it is easy to see that

\[ P_{\alpha\beta}(\omega) = i C_{\alpha\beta}^K(0, \omega). \] (26)

Hence, the only relevant Floquet-Fourier component is the one with \( k = 0 \).

As we are considering non-interacting electrons the contour-ordered propagator given in Eq. (6) can be exactly evaluated in terms of single-particle propagators (48). Using Wick’s theorem (see the Appendix for the details), this contour-ordered function can be written as

\[ i C_{\alpha\beta}(t, t') = -w_{\alpha\alpha} w_{\beta\beta} \sum_{k, k'} \{ G_{\beta, k' \alpha}(t', t) G_{\alpha, k \beta}(t, t') - G_{k, k' \alpha}(t', t) G_{\beta, k' \alpha}(t, t') - G_{k, k \alpha}(t', t) G_{\alpha, k' \beta}(t, t') + G_{k, k' \alpha}(t', t) G_{\alpha, k \beta}(t, t') \}. \] (27)

In the Appendix we show the detailed calculation leading from this expression to the Floquet-Fourier components \( C_{\alpha\beta}^S(0, \omega) \). Here we only reproduce the results for two cases of particular interest.

The first case is the zero-frequency limit of \( C_{\alpha\beta}^K(0, \omega) \), which reads

\[ P_{\alpha\beta} = \frac{i}{2} C_{\alpha\beta}^K(k = 0, \omega = 0) = \delta_{\alpha\beta} P_\alpha + P_{\alpha\neq\beta}, \] (28)

where

\[ P_\alpha = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \Gamma_\alpha(\omega') \sum_{k = -\infty}^{\infty} \sum_{\gamma} \Gamma_\gamma(\omega'_k) f_{\alpha\gamma}(\omega', \omega'_k) \times |G_{\alpha, k \gamma}^R(-k, \omega'_k)|^2, \]

\[ P_{\alpha\neq\beta} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega' d\omega''}{2\pi} \Gamma_\alpha(\omega') \sum_{k = -\infty}^{\infty} \Gamma_\beta(\omega'_k) \left\{ f_{\alpha\gamma}(\omega', \omega'_k) \times \text{Re} \left[ G_{\beta, k \gamma}^R(k, \omega'_k) G_{\beta, k \gamma}^R(-k, \omega'_k) \right] \right. \]

\[ -2 \sum_{k = -\infty}^{\infty} \sum_{\gamma} \Gamma_\gamma(\omega'_k) f_{\alpha\gamma}(\omega', \omega'_k) \text{Im} \left[ G_{\beta, k \gamma}^R(k, \omega'_k) \right] \times \left. G_{\beta, k \gamma}^R(-k', \omega'_k) G_{\beta, k \gamma}^R(k' - k', \omega'_k)^* \right\} + G_{\beta, k \gamma}^R(k, \omega'_k) G_{\beta, k \gamma}^S(-k, \omega'_k)^* \right\} + \{\text{same with } \alpha \leftrightarrow \beta\}, \] (29)

being

\[ f_{\alpha\gamma}(\omega, \omega') = f_{\alpha}(\omega)(1 - f_{\beta}(\omega')) + f_{\beta}(\omega')(1 - f_{\alpha}(\omega)). \] (30)

It is important to notice that this is a general result for multiterminal quantum driven systems and the sum over \( \gamma \) extends over all reservoirs connected to the central system. For this work we chose a two terminal system, but this result is completely general as no assumption concerning the reservoirs was made in the calculation.

At this point it is interesting to compare with previous results obtained within the scattering matrix formalism (see Ref. 29). In order to do so, we need to assume that all reservoirs are at equal temperature and chemical potential (unbiased pump). We split the zero frequency noise into two contributions

\[ P_{\alpha\beta} = P_{\alpha\beta}^{(th)} + P_{\alpha\beta}^{(sh)} \] (31)

where

\[ P_{\alpha\beta}^{(th)} = \int_{-\infty}^{\infty} d\omega' \Gamma(\omega') \sum_{k = -\infty}^{\infty} \Gamma(\omega'_k) \times \delta_{\alpha\beta} \left\{ f(\omega') - f(\omega'_k) \right\} \]

\[ \times |G_{\alpha, k \gamma}^R(k, \omega'_k)|^2 - f(\omega')^2 \left( |G_{\alpha, k \gamma}^R(k, \omega'_k)|^2 + |G_{\alpha, k \gamma}^R(-k, \omega'_k)|^2 \right) \]

\[ \times \left( \Gamma_{\alpha, k \gamma}^R(k, \omega'_k) \right)^2 - f(\omega')^2 \left( f(\omega'_k) G_{\alpha, k \gamma}^R(k, \omega'_k) \right)^2 \]

\[ \times \left( f(\omega'_k) G_{\alpha, k \gamma}^R(k, \omega'_k) \right)^2 \times \left( \text{Re} \left[ f(\omega') G_{\beta, k \gamma}^R(k, \omega'_k) \right] \right)^2 \]

\[ - f(\omega'_k) G_{\alpha, k \gamma}^R(-k, \omega'_k)^* G_{\alpha, k \gamma}^S(-k, \omega'_k)^* \right\} \]

\[ \times \left( \text{Re} \left[ f(\omega') G_{\beta, k \gamma}^R(k, \omega'_k) \right] \right)^2 \] (32)

The first term \( P_{\alpha\beta}^{(th)} \) is the Nyquist-Johnson noise while \( P_{\alpha\beta}^{(sh)} \) is the shot noise. Using the relation between the
Floquet S-matrix and Green’s functions

\[
S_{F,\alpha\beta}(\omega_m,\omega_n) = \delta_{\alpha\beta}\delta_{n,m} - i\sqrt{\Gamma(\omega_m)\Gamma(\omega_n)}
\]
\[
\times G^R_{\alpha\beta}(m-n,\omega_n),
\]
(33)

it is easy to show that the result given in Eq. (32) coincides with the one obtained using the Floquet S-matrix formalism in Ref. 29.

The other case of interest is the one in which \(\alpha = \beta = P\), i.e. we concentrate in current fluctuations of the probe. Using the fact that the probe is noninvasive, we only keep terms to the lowest order in the coupling \(w_{cP}\) between the system and the thermometer,

\[
iC_{PP}^K(0,\omega) = \Gamma_P \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \sum_{k=-\infty}^{\infty} \sum_{\gamma=L,R} \Gamma_\gamma(\omega') \left\{ f_\gamma(\omega') \times \left[ 2 - f_P(\omega'_k + \omega) - f_P(\omega'_k - \omega) \right] + f_P(\omega'_k + \omega) + f_P(\omega'_k - \omega) \right\} \left| G^R_{\gamma P,\gamma P}(k,\omega') \right|^2.
\]
(34)

On the other hand we need \(\overline{\gamma}_P(\omega)\), which is

\[
\overline{\gamma}_P(\omega) = \Gamma_P \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \sum_{k=-\infty}^{\infty} \sum_{\gamma=L,R} \Gamma_\gamma(\omega') \times \left[ f_P(\omega'_k - \omega) - f_P(\omega'_k + \omega) \right] \times \left| G^R_{\gamma P,\gamma P}(k,\omega') \right|^2.
\]
(35)

The functions entering Eqs. (34) and (35) are the ones involved in the definition of effective temperature given in Eq. (20).

V. RESULTS

In this section we present results for a central device consisting of non-interacting electrons in a one-dimensional lattice:

\[
H_0 = -w \sum_{l,l'} \langle c^\dagger_l c_{l'} + H.c. \rangle,
\]
(36)

where \(w\) denotes a hopping matrix element between neighboring positions \(l,l'\) on the lattice. The driving term is chosen as

\[
H_V(t) = \sum_{j=1}^{2} \epsilon V_j(t) c^\dagger_j c_j,
\]
(37)

with \(V_j(t) = E_B + V_0 \cos(\Omega_0 t + \delta_j)\), \(l_j\) being the positions where two oscillating fields with frequency \(\Omega_0\) and phase-lag \(\delta\) are applied. This defines a simple model for a quantum pump where two ac gate voltages are applied at the walls of a quantum dot.\[20,32,37\]

A. Equivalence between effective and local temperature at weak driving

As in Refs. 19 and 23 we are interested in the weak driving regime, which corresponds to a situation where the ac voltage amplitudes are lower than the kinetic energy of the electrons in the structure and the driving frequency is much smaller than the inverse of the dwell time of these electrons. We have shown that in this regime the local temperature defined from Eq. (12), with the chemical potential of the probe fixed, is identical to the local temperature defined from Eq. (10), where the chemical potential of the probe has to be determined in order to satisfy both equations, and it is also identical to an effective temperature defined from a local fluctuation-dissipation relation of single-particle Green’s functions (see Eq. (18)).

We now turn our attention to the effective temperature \(T^{eff}_{fs}\) defined in Eq. (20), involving current-current correlation functions. The correlation functions given in Eqs. (34) and (35) depend on the temperature \(T_P\) and the chemical potential \(\mu_P\) of the probe via the Fermi function \(f_P\). Thus, the effective temperature \(T^{eff}_{fs}\), so calculated, also depends on \(T_P\) and \(\mu_P\). There are many possible reasonable choices for the latter quantities. In this subsection we will concentrate in only one choice and leave for the next subsection the analysis of other possibilities. We choose \(\mu_P\) equal to the chemical potential \(\mu\) of the reservoirs and \(T_P\) equal to the local temperature \(T_{IP}\), i.e. the one for which the heat flow between the system and the probe vanishes.

In Fig. 2 we show a typical plot for \(iC_{PP}^K(0,\omega)\), \(\overline{\gamma}_P(\omega)\) and their ratio as a function of \(\omega\). According to the definition of effective temperature given in Eq. (20), the derivative of this ratio at \(\omega = 0\) corresponds to \(\beta^{eff}_{fs}/2\). This derivative is calculated numerically. In the same figure we plot \(\tanh \left( \beta^{eff}_{fs}/\omega/2 \right)\) and we see that the quotient \(\overline{\gamma}_P(\omega)/iC_{PP}^K(0,\omega)\) is well fitted by a FDR-type relation for a reasonably large frequency interval.

In Fig. 3 we show the behavior of the effective temperature \(T^{eff}_{fs}\) and the local temperature \(T_{IP}\), calculated for the site connected to the left reservoir, as a function of the driving frequency \(\Omega_0\) for two different temperatures of the reservoirs. This analysis can be done for any site of the central system but we chose this particular site because its local temperature determine the heat current that flows into the left reservoir. Results for any other site of the central system are similar. In Fig. 3 the upper panel corresponds to \(T = 0.016\), while the lower corresponds to \(T = 0.005\). We see that both ways of defining the temperature coincide at low frequencies. This supports the idea that, for a given temperature \(T\) of the reservoirs, \(T^{eff}_{fs}\) is a bona fide temperature within the low driving regime. As we can see from Fig. 3, the higher the temperature \(T\) of the reservoirs, the broader the region of low driving frequency \(\Omega_0\) in which the two definitions of the temperature agree.

As we mentioned earlier, the definition given in Eq.
The effective temperature $T_{IP}^{\text{eff}}$ depends on the values of $T_P$ and $\mu_P$ (respectively the temperature and chemical potential of the probe). The choice analyzed in the previous section was $\mu_P = \mu_L = \mu_R = \mu = 0.2$ and a particular low value of $\Omega_0 = 0.001$. We can see that there is a good agreement between the two temperatures along the whole structure and an almost perfect agreement within the “Left” and “Right” regions (defined in Fig. 1), which are the ones from where we can determine the heat flow between the system and each one of the reservoirs (see Refs. 19,23). It is also important to notice the existence of $2k_F$ Friedel-like oscillations, $k_F$ being the Fermi vector of the electrons leaving the reservoirs. These oscillations are an indication of quantum interference. They were previously reported for exactly such a setup we study in this work and also predicted in other mesoscopic systems under a stationary driving.

### B. Different choices of $T_P$ and $\mu_P$

In this work we will only deal with these three possibilities.

If Fig. 4 we show the three different effective temperatures corresponding to the above mentioned cases together with the local temperature as a function of the driving frequency $\Omega_0$ for a given temperature of the reservoirs. As we can see, all three cases give a good estimate of the local temperature in the regime of interest (i.e. low driving frequencies). This behavior supports the robustness of the definition of the local temperature from a FDR.

FIG. 2: (Color online) Current-current correlation functions $\Phi^{\text{IP}}(\omega)$ (dotted red), $iC^{\text{IP}}(0,\omega)$ (dashed blue), their quotient (green diamonds), and $\tanh[\beta^{\text{IP}}\omega/2]$ (solid black) as a function of $\omega$. The reservoirs have chemical potential $\mu = 0.2$ and temperature $T = 0.025$. The driving frequency is $\Omega_0 = 0.01$, the amplitude is $V_0 = 0.05$ and $E_B = 0.2$.

FIG. 3: (Color online) Local temperature $T_{IP}$ (dashed black) and effective temperature $T_{IP}^{\text{eff}}$ (solid red) for the site $lP = lL$ (i.e. the site connected to the left reservoir) as a function of driving frequency $\Omega_0$. The reservoirs have chemical potential $\mu = 0.2$. The upper panel corresponds to $T = 0.016$, while the lower panel corresponds to $T = 0.005$.

FIG. 4: (Color online) Local temperature $T_{IP}$ (black diamonds) and effective temperature $T_{IP}^{\text{eff}}$ (red circles) along a one-dimensional model of $N = 30$ sites with two ac fields operating with a phase lag of $\delta = \pi/2$ at the positions indicated by dotted lines. The system is in contact with reservoirs with chemical potentials $\mu = 0.2$ and temperature $T = 0.02$. The driving frequency is $\Omega_0 = 0.001$, the amplitude is $V_0 = 0.05$ and $E_B = 0.2$.
In order to define the local temperature of a driven sample, using current-current correlation in real experiments, in particle propagators. This result opens the possibility of a thermometer and from a FDR at the level of single-cyces it coincides with the local temperature defined with correlation functions and showed that for low frequen-
ty of interacting systems.

VI. SUMMARY AND CONCLUSIONS

In this work we have calculated the current-current correlation functions for quantum driven systems and found an explicit expression for the zero-frequency noise within the Schwinger-Keldysh Green’s functions formalism. In the particular case of multiterminal unbiased quantum driven systems our result is in agreement with previous results obtained within the scattering matrix approach. For non-interacting systems both descriptions agree, while the Green’s functions has the advantage of providing a systematic framework for the study of interacting systems.

We have also defined an effective temperature from a local fluctuation-dissipation relation for current-current correlation functions and showed that for low frequencies it coincides with the local temperature defined with a thermometer and from a FDR at the level of single-particle propagators. This result opens the possibility of using current-current correlation in real experiments, in order to define the local temperature of a driven sample.

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Appendix: Analytical expressions for current-current correlation functions

Using Eq. (4), the connected contour-ordered current-current correlation (see Eq. (19)) can be written in terms of electron operators as

$$iC_{\alpha\beta}(t, t') = -w_{c\alpha}w_{c\beta} \sum_{k_\alpha,k_\beta} \left( D_{k_\alpha,l_\alpha,k_\beta,l_\beta}(t, t') - D_{k_\alpha,l_\alpha,l_\beta,k_\beta}(t, t') - D_{l_\alpha,k_\alpha,k_\beta,l_\beta}(t, t') + D_{l_\alpha,k_\alpha,l_\beta,k_\beta}(t, t') \right),$$

(A.1)

where

$$D_{i,j,k,l}(t, t') = \langle T_{\overline{c}}[\hat{c}_i(t)\hat{c}_j(t')\hat{c}_k(t')\hat{c}_l(t')] \rangle - \langle \hat{c}_i(t)\hat{c}_j(t) \rangle \langle \hat{c}_k(t')\hat{c}_l(t') \rangle.$$

(A.2)

Using Wick’s theorem and the definition of the contour-ordered Green’s function (Eq. (3)) we can rewrite Eq. (A.1) as

$$D_{i,j,k,l}(t, t') = G_{i,k}(t', t)G_{j,l}(t, t').$$

(A.3)

The substitution of Eq. (A.3) into Eq. (A.1) gives the result shown in Eq. (27).

The next step is to calculate the lesser and greater Green’s functions. In order to have the lesser (greater) Green’s function we need $t \in C_1$ and $t' \in C_2$ ($t \in C_2$ and $t' \in C_1$). If $t \in C_1$ and $t' \in C_2$, then

$$D_{i,j,k,l}^{<}(t, t') = G_{i,k}^{\Delta\gamma}(t', t)G_{j,l}^{\gamma}(t, t').$$

(A.4)

Using Eq. (A.4) we can write the lesser Green’s function as

$$iC_{\alpha\beta}^{<}(t, t') = w_{c\alpha}w_{c\beta} \sum_{k_\alpha,k_\beta} \left\{ \left( G_{k_\alpha,l_\alpha,k_\beta,l_\beta}^{\Delta\gamma}(t, t')G_{i_\alpha,k_\beta}^{\gamma}(t, t') \right) - \left( G_{k_\alpha,l_\alpha,l_\beta,k_\beta}^{\Delta\gamma}(t, t')G_{i_\alpha,k_\beta}^{\gamma}(t, t') \right) - \left( G_{i_\alpha,l_\alpha,k_\beta,l_\beta}^{\Delta\gamma}(t, t')G_{k_\alpha,k_\beta}^{\gamma}(t, t') \right) + \left( G_{i_\alpha,l_\alpha,l_\beta,k_\beta}^{\Delta\gamma}(t, t')G_{k_\alpha,k_\beta}^{\gamma}(t, t') \right) \right\}. $$

(A.5)

where the property $G_{j,k}^{\Delta\gamma}(t', t) = -G_{k,j}^{\Delta\gamma}(t', t)^*$ was used. Notice that the last term is equal to the first interchanging $<$ with $>$ and conjugating. The same situation arises with the second and third terms. The result for the greater Green’s function is obtained by switching $<$ with $>$.

We are interested in the Floquet-Fourier components of this Green’s functions. So, according to Eq. (5) we need to calculate

$$C_{\alpha\beta}^{<}(0, \omega) = i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \langle 0 | \hat{c}_{\alpha}^{\Delta\gamma}(-\omega)\hat{c}_{\beta}^{\gamma}(\omega) + \hat{c}_{\beta}^{\gamma}(\omega)\hat{c}_{\alpha}^{\Delta\gamma}(-\omega) \rangle. $$

(A.6)

Substituting with the Floquet-Fourier expansions of the Green’s functions (see Eq. (5)) into Eq. (A.5) and rewriting into the form of Eq. (A.6) we can obtain the $k = 0$ Floquet-Fourier component, which is

$$C_{\alpha\beta}^{<}(0, \omega) = i \left[ A_{\alpha\beta}^{\Delta\gamma}(\omega) + A_{\alpha\beta}^{\gamma}(\omega) + B_{\alpha\beta}^{\gamma}(\omega) + B_{\alpha\beta}^{\Delta\gamma}(\omega) \right].$$

(A.7)
where
\[
A_{\alpha\beta}^<(\omega) = \sum_{k=\alpha}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} w_{ca} \sum_{k} G_{\alpha\beta,k\alpha}(k,\omega_k)
\]
\[
\times w_{cb} \sum_{k} G_{\alpha\beta,k\beta}(k,\omega_k + \omega),
\]
\[
B_{\alpha\beta}^<(\omega) = \sum_{k=\alpha}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G_{\alpha\beta,k\beta}(k,\omega')^*
\]
\[
\times w_{ca} w_{cb} \sum_{k} G_{\alpha\beta,k\beta}(k,\omega' + \omega),
\]
and \(X^>\) is obtained from \(X^<\) by switching \(<\) with \(>\) \((X = A, B, C, D)\). Since the greater Floquet-Fourier component is obtained from Eq. \((A.5)\) by switching \(<\) with \(>\), it can be written as:
\[
C_{\alpha\beta}^>(0,\omega) = i \left[ A_{\alpha\beta}^<(\omega) + A_{\alpha\beta}^<(\omega)^* + B_{\alpha\beta}^<(\omega) + B_{\alpha\beta}^<(\omega)^* \right].
\]
(A.8)
where \(X^>\) is obtained from \(X^<\) by switching \(<\) with \(>\) \((X = A, B, C, D)\).

We want to obtain an expression of the previous quantities in terms of \(G_{\alpha\beta}^\lambda(t, t')\) with \(l, l'\) in the central system. In order to do so we need to calculate the Floquet-Fourier components of
\[
\sum_{k=\alpha}^{\infty} G_{\alpha\beta,k\alpha}(t, t') = \frac{1}{w_{ca}} \int dt_1 \left[ G_{\alpha\beta,k\alpha}(t, t_1) \Sigma_{\alpha}(t_1, t') + G_{\alpha\beta,k\alpha}^<(t, t_1) \Sigma_{\alpha}(t_1, t') \right],
\]
(A.9)
and
\[
\sum_{k=\alpha}^{\infty} G_{\alpha\beta,k\beta}(t, t') = \frac{1}{w_{ca} w_{cb}} \left[ \delta_{\alpha\beta} \Sigma_{\alpha}(t, t') + \int dt_1 dt_2 \right.
\]
\[
\times \left[ G_{\alpha\beta,k\beta}(t, t_1) G_{\alpha\beta,k\beta}(t, t_2) \Sigma_{\alpha}(t_2, t') + \Sigma_{\alpha}^R(t, t_1) G_{\alpha\beta,k\beta}(t, t_2) \Sigma_{\alpha}^A(t_2, t') + \Sigma_{\alpha}^<_R(t, t_1) G_{\alpha\beta,k\beta}(t, t_2) \Sigma_{\alpha}^A(t_2, t') \right].
\]
(A.10)

where
\[
\Sigma_{\alpha}(t, t') = |w_{ca}|^2 \sum_{k=\alpha}^{\infty} g_{\alpha\beta,k\alpha}(t, t'),
\]
\[
\Sigma_{\alpha}^R(t, t') = |w_{ca}|^2 \sum_{k=\alpha}^{\infty} g_{\alpha\beta,k\alpha}(t, t'),
\]
\[
\Sigma_{\alpha}^A(t, t') = \Sigma_{\alpha}^R(t', t'^*),
\]
(A.11)
with their respective Fourier transforms
\[
\Sigma_{\alpha}^<(\omega) = if_{\alpha}(\omega) \Gamma_{\alpha}(\omega),
\]
\[
\Sigma_{\alpha}^R(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \Gamma_{\alpha}(\omega') \Gamma_{\alpha}(\omega - \omega' + i0^+),
\]
\[
\Sigma_{\alpha}^A(\omega) = \Sigma_{\alpha}^R(\omega)^*.
\]
(A.12)

Using the Fourier transforms of \(\Sigma^<\) and \(\Sigma^R\) together with the Floquet-Fourier expansion for the Green’s functions (see Eq. \((A.3)\)) we obtain
\[
\sum_{k=\alpha}^{\infty} G_{\alpha\beta,k\alpha}(t, t') = \frac{1}{w_{ca}} \left\{ G_{\alpha\beta,k\alpha}(t, t') \Sigma_{\alpha}^<(\omega) + \Sigma_{\alpha}^R(\omega) \right\}.
\]
(A.13)

where
\[
G_{\alpha\beta,k\alpha}(t, t') = \sum_{m=-\infty}^{\infty} \Gamma_{\alpha}(m, m') \Sigma_{\alpha}^R(\omega_m) \Sigma_{\alpha}^A(\omega_m)^* \times \Sigma_{\alpha}^<(\omega),
\]
(A.14)

with \(\Sigma_{\alpha}^R(\omega) = \Gamma_{\alpha}(\omega) \lambda_{\alpha}^R(\omega), \text{ and}
\]
\[
\lambda_{\alpha}^R(\omega) = if_{\alpha}(\omega),
\]
\[
\lambda_{\alpha}^A(\omega) = -i(1 - f_{\alpha}(\omega)).
\]
(A.15)

On the other hand,
\[
\sum_{k=\alpha}^{\infty} G_{\alpha\beta,k\beta}(t, t') = \frac{1}{w_{ca} w_{cb}} \left\{ \delta_{\alpha\beta} \Sigma_{\alpha}^<(\omega) + G_{\alpha\beta,k\beta}(t, t') \Sigma_{\alpha}^R(\omega_k) + \Sigma_{\alpha}^R(\omega) \right\}.
\]
(A.16)

By substituting Eqs. \((A.13)\) and \((A.16)\) into Eq. \((A.8)\) and then into Eq. \((A.7)\) we obtain an expression for \(C_{\alpha\beta}^>(0, \omega)\). The expression for the greater Green’s function is obtained by switching \(<\) with \(>\).

We are interested in the case where \(\alpha = \beta = P\), i.e. we are interested in fluctuations in the current flowing to the probe. As we are dealing with a non-invasive probe, we only keep terms up to the lowest order in the coupling between the system and the probe. Since \(\Gamma_{\alpha}, H_{\alpha} \propto |w_{ca}|^2\) it is easy to see that
\[
\mathcal{A}_{pP}^R(\omega) = O\left(|w_{pP}|^4\right),
\]
\[
\mathcal{B}_{pP}^R(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} G_{\alpha\beta,\alpha\beta}(0, \omega') \lambda_{\alpha}^R(\omega') \Gamma_{\alpha}(\omega + \omega') + O\left(|w_{pP}|^4\right).
\]
(A.17)
Thus, \(\mathcal{B}_{pP}^R(\omega)\) and \(\mathcal{B}_{pP}^R(\omega)\) are the only terms with contributions of order \(|w_{pP}|^2\). Using Eqs. \((A.14)\) and \((A.15)\) we can write, up to this order,
\[
\mathcal{C}_{pP}^K(\omega) = -i \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \sum_{\gamma=\alpha\beta} \sum_{l, r} |G_{\alpha\beta,l\gamma}(l, \omega')|^2
\]
\[
\times \left\{ f_{\gamma,P}(\omega', \omega + \omega') \Gamma_{\gamma}(\omega + \omega') + f_{\gamma,P}(\omega', \omega') \Gamma_{\gamma}(\omega + \omega') \right\},
\]
(A.18)
To obtain an expression for $\mathcal{F}(\omega)$ we can use the following identity, easily deduced from Eq. (10),
\[
\mathcal{F}(\omega) \equiv -2i \mathrm{Im}[C_{\rho p}(0, \omega)] = i \left( C_{\rho p}^>(0, \omega) - C_{\rho p}^<(0, \omega) \right),
\]
which leads to
\[
\mathcal{F}(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \sum_{k=-\infty}^{\infty} \sum_{\gamma=L,R} \Gamma_{\gamma}(\omega_n'(\omega_n'))^2 \left\{ \left( f_{\gamma}(\omega_n') - f_{p}(\omega' + \omega) \right) \Gamma_{\rho}(\omega' + \omega) \\
- \left( f_{\gamma}(\omega_n') - f_{p}(\omega' - \omega) \right) \Gamma_{\rho}(\omega' - \omega) \right\}. \tag{A.20}
\]
If we additionally consider a constant density of states in the probe to obtain a result independent of any particular probe, we arrive to the expressions given in Eqs. \cite{34} and \cite{35}.

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