Can dileptons reveal the in-medium properties of vector mesons?

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Abstract

Dilepton production from both pion-pion and kaon-antikaon annihilation in heavy-ion collisions is studied using the relativistic transport model. The formation of a rho meson from pion-pion annihilation and a phi meson from kaon-antikaon annihilation, their propagation in the medium, and their decay into dileptons are explicitly treated. Including the medium modifications of the masses and widths of vector mesons as predicted by the QCD sum-rule calculations, we study their effects on the dilepton invariant mass spectra from heavy-ion collisions at SIS/GSI energies.
I. INTRODUCTION

Hadron properties are believed to be modified in the nuclear medium due to the partial restoration of chiral symmetry. A well-known example is the reduced nucleon mass in nuclear matter, as shown in the Walecka model [1], in the Dirac-Brueckner calculation [2–3], and more recently in the QCD sum-rule calculations [3,7]. Based on the scaling properties of QCD, Brown and Rho [8] have argued that the masses of non-strange vector mesons in nuclear matter should also be reduced at about the same rate as the nucleon mass. This intriguing suggestion has led to many theoretical studies of hadron in-medium properties based on the QCD sum rules [8–11] and the effective field theory [12–14].

Phenomenologically, the enhanced cross section in kaon-nucleus scattering and the suppression of the electromagnetic longitudinal response in electron-nucleus scattering have been considered as evidence for the decrease of vector meson masses in the nuclear medium [15,16]. However, the density involved in these reactions is small, i.e., below or at most equal to the normal nuclear matter density. On the other hand, heavy-ion collisions at intermediate energies provide a unique way to form in the laboratory a piece of dense matter with density up to $2-3\rho_0$ ($\rho_0 \approx 0.16$ fm$^{-3}$) and thus make it possible to study experimentally the properties of hadrons in a dense medium. Unfortunately, the dense matter formed in heavy-ion collisions exists only as an intermediate stage for a very short period. Direct experimental studies of the dense medium is thus impossible. A theoretical transport model has to be used in order to deduce the properties of the intermediate stage from the known initial conditions and the measured final observables. Among transport models, the relativistic transport model based on the Walecka model offers a consistent description of changing hadron masses in the dense matter formed in intermediate-energy heavy-ion collisions [17–20]. In this model, the nucleon mass in a dense medium is reduced by the attractive scalar field, and the energy is thus stored in the scalar field energy. As the density decreases, the scalar field energy diminishes and is converted back to the nucleon mass, which eventually returns to its free mass at freeze-out. In addition to the propagation of nucleons (and deltas) in the mean-field
potential, the relativistic transport model also incorporates elastic and inelastic two-body collisions among nucleons, deltas, and pions.

In order to determine unambiguously the properties of hadrons in a dense matter, one needs also to find suitable observables that carry the information of the intermediate stage and are not significantly affected by final-state interactions. The electromagnetic signals, especially dileptons, due to their weak interaction with hadrons, have been considered as good observables to probe the properties of the intermediate stage of heavy-ion collisions. There have already been studies of dilepton production in heavy-ion collisions using both fireball models [21–26] and transport models [27–30]. In these studies, it has been shown that dileptons are indeed mostly produced from the hot and dense stage of heavy-ion collisions.

The possibility of studying in-medium pion dynamics through dilepton production from pion-pion annihilation was first pointed out in Ref. [21] in a schematic model and later confirmed in Ref. [23] using the delta-hole model. Because of the softening of the pion dispersion relation, an enhanced yield of dileptons with invariant masses around twice the pion mass was predicted. However, more refined investigations indicate that this enhancement is largely cancelled by vertex corrections [25,26]. Furthermore, for dileptons with invariant masses around twice the pion mass, the background from the eta and delta Dalitz decays as well as the neutron-proton bremsstrahlung are more significant than that from the pion-pion annihilation [30]. It is thus very difficult, if not impossible, to study the in-medium pion dispersion relation from measuring the dilepton invariant mass spectrum.

Since the electromagnetic form factor of a pion is dominated by the rho meson according to the vector dominance model, the dilepton spectrum may also reveal information about the property of rho meson in dense matter. For large dilepton invariant masses, the background from Dalitz decays and bremsstrahlung is small, it should therefore be easy to identify the rho meson. Calculations using the free rho-meson mass and width in the pion electromagnetic form factor indeed reveal a broad but visible peak in the dilepton spectrum at invariant masses around the free rho-meson mass (≈ 770 MeV) [28,29]. In Ref. [31], the medium modification of the pion electromagnetic form factor due to the delta-hole polariza-
tion has been studied. As a result of the softened pion dispersion relation in a medium, the pion electromagnetic form factor is broadened. Its effect on the dilepton spectra has been investigated in Ref. [30], and it has been found that the peak at the rho-meson mass almost disappears because of the increase of the rho-meson width in the medium. However, the calculation of Ref. [31] neglects the medium effects on the vacuum, which, when included as in the QCD sum-rule [9,10] and effective field theory calculations [13], lead to the reduction of the rho-meson mass in dense matter. The rho-meson peak in the dilepton spectrum is thus expected to shift to lower invariant masses.

Another vector meson whose properties in a dense medium have been under extensive discussion is the phi meson [9,10,32–36]. The QCD sum-rule calculation shows that the phi-meson mass also decreases with increasing density, but at a smaller rate than that of the rho meson [9,10]. The width of the phi meson also changes with density, mainly due to the medium modification of kaon and antikaon masses, as well as the phi-meson mass, as will be discussed later. Since the kaon-antikaon annihilation to a dilepton mainly proceeds through the phi meson, dilepton production in heavy-ion collisions thus allows one to learn about the properties of the phi meson in the dense medium.

The main purpose of this paper is to study dilepton production from both pion-pion and kaon-antikaon annihilation in heavy-ion collisions to see whether one can learn about the in-medium properties of the rho and phi mesons from the dilepton spectrum. For the purpose of illustration we shall use in our calculation the in-medium masses of the rho and phi mesons as predicted by the QCD sum-rule calculations of Refs. [9,10]. It should be mentioned that due to ambiguities in the four-quark condensate and in the strangeness content of a nucleon, the present QCD sum-rule predictions for the in-medium masses of the rho and phi mesons are not without uncertainty.

In all previous calculations of dilepton production from the pion-pion (kaon-antikaon) annihilation [21–30], the effects of the intermediate rho (phi) meson are included through the pion (kaon) electromagnetic form factor using the vector dominance model. In this form factor approach, the time delay between the formation of the rho (phi) meson and
its decay into a dilepton is neglected. Physically, dilepton production from the pion-pion (kaon-antikaon) annihilation proceeds in three steps. First, a rho (phi) meson is formed from the pion-pion (kaon-antikaon) annihilation, which then propagates in the hadronic medium, and finally decays into a dilepton with an appropriate branching ratio. In this dynamical approach, the formation and decay of vector mesons are treated explicitly. If there are no medium effects, the two approaches give essentially the same results. Since we are interested in possible medium effects on the vector mesons in the dilepton spectra, the dynamical approach is more realistic. Neglecting the time delay between formation and decay of vector mesons (i.e., neglecting dilepton formation time) in the form factor approach would overestimate the medium effects.

This paper is organized as follows. In section 2, we briefly review the relativistic transport model of Ref. [17–19]. The necessary formalism for calculating dilepton production from both pion-pion and kaon-antikaon annihilation is discussed in section 3. The results are presented in section 4. Finally, a short summary is given in section 5.

II. RELATIVISTIC TRANSPORT MODEL

Our study is based on the relativistic transport model developed in Refs. [17–19]. At energies considered in this work, the colliding system consists mainly of nucleons, deltas and pions. Pions are produced from decays of deltas that are excited in nucleon-nucleon inelastic interactions. While pions are treated as free particles, nucleons and deltas are propagated in their mean-field potentials according to the following equations of motion

\[
\frac{dx}{dt} = \frac{p^*}{E^*}, \quad \frac{dp}{dt} = -\nabla_x [E^* + (g_\omega/m_\omega)^2 \rho_B],
\]

with \( E^* = (m^2 + p^{*2})^{1/2} \). The effective mass and kinetic momentum of a baryon (nucleon or delta) are given, respectively, by \( m^* = m + \Sigma_S \), and \( p^* = p - \Sigma_V \), where the scalar \( \Sigma_S \) and the vector \( \Sigma_V \) self-energies are determined from the non-linear \( \sigma-\omega \) model. We use the so-called soft equation-of-state, as determined in Ref. [37], in the present calculation.
Elastic ($NN \rightarrow NN, N\Delta \rightarrow N\Delta, \Delta\Delta \rightarrow \Delta\Delta$) and inelastic ($NN \leftrightarrow N\Delta, \Delta \leftrightarrow N\pi$) reactions among nucleons, deltas and pions are also included. The standard Cugnon parametrizations [38,39] and the proper detailed-balance prescription [40] are used for describing these reactions. In Fig. 1, we show the time evolution of the average central density (dotted curve) and the pion multiplicity for a Ni+Ni collisions at an incident energy of 2.0 GeV/nucleon and an impact parameter of 0 fm. The final pion multiplicity is about 39, which gives a pion-to-nucleon ratio of about 0.33, and is in good agreement with the experimental data [41]. We also see that pions materialize at a later stage of the collision when the system has already started to expand.

For heavy-ion collisions at incident energies considered here, kaon and antikaon production are treated perturbatively as their production probabilities are very small. We refer to Refs. [42,43] for details on the elementary production cross section and the treatment of final-state interactions. In Refs. [42,43], we have found that an attractive scalar potential derived from the chiral Lagrangian [44–47] is needed for both the kaon and the antikaon in order to explain quantitatively the experimental data from SIS/GSI [48,49]. In the mean-field approximation to the chiral Lagrangian, the effective masses of the kaon and the antikaon in a nuclear medium are

$$m_{K,K^*} = m_K \left[ 1 - \frac{\Sigma_{KN}}{f_K^2 m_K^2} \rho_S + \left( \frac{3}{8} \frac{\rho_B}{f_K^2 m_K} \right)^2 \right]^{1/2} \pm \frac{3}{8} \frac{\rho_B}{f_K^2},$$

where the plus and the minus sign correspond to the kaon and the antikaon, respectively. With a kaon decay constant $f_K \approx 93$ MeV and a $KN$ sigma term $\Sigma_{KN} \approx 350$ MeV, we find that the kaon mass increases with density while the antikaon mass decreases with density, as shown in Fig. 2. In Fig. 1, we also show the time evolution of the kaon and antikaon multiplicities. Kaons and antikaons are produced in the early stage of the collision when baryon-baryon collisions are most energetic. Their abundance reaches a maximum when the system is most compressed. The final kaon multiplicity is about 0.33, and that of the antikaon is about 0.009. We note that if the attractive scalar potential for the kaon and the antikaon is neglected, the kaon yield is reduced by about 40% to 0.22 (dashed curve), while
the antikaon yield is reduced by about a factor of 3 to 0.003 (dashed curve).

III. DILEPTON PRODUCTION: FORMALISM

A. Pion-pion annihilation

According to the vector dominance model, dilepton production from pion-pion annihilation proceeds through a rho meson which then converts to a virtual photon and decays into a dilepton. This is shown by the diagram in Fig. 3. Including the medium modification of the rho-meson mass and width which will be specified later, the invariant matrix element for this process is

\[ \mathcal{M} = \frac{e^2 m_{\rho}^2}{q^2 [q^2 - (m_{\rho}^* - \frac{i}{2} \Gamma_{\rho}^*)^2]} (q_1 - q_2) \bar{u}(k_1, s_1) v(k_2, s_2), \]  

(3)

where we have used the relation, \( g_{\rho \gamma} = \frac{e m_{\rho}^*}{g_{\rho \pi \pi}} \), between the \( \rho_{\pi \pi} \) coupling constant and the \( \rho_\gamma \) coupling constant \[11].

From Eq. (3) we have

\[ \tilde{\mathcal{M}}^2 = \frac{2 e^4 m_{\rho}^*}{m_{\pi}^* [q^2 - (m_{\rho}^* - \frac{i}{2} \Gamma_{\rho}^*)^2]} \left[ \frac{1}{4} q^4 + m_{\pi}^2 q^2 + 4 (k \cdot q)^2 \right]. \]  

(4)

The dilepton production cross section from pion-pion annihilation is then given by

\[ \sigma_{\pi^+ \pi^- \to \rho^0 \to e^+ e^-}(M) = \frac{8 \pi \alpha^2 k}{3 M^3} \frac{m_{\rho}^*}{(M^2 - m_{\rho}^* \Gamma_{\rho}^*)^2} \]  

(5)

where \( M \) is the invariant mass of the \( \pi^+ \pi^- \) pair, and \( \alpha \) is the fine structure constant. The three momentum \( k \) is given by

\[ k = \frac{1}{2} (M^2 - 4 m_{\pi}^2)^{1/2}. \]  

(6)

Theoretical studies based on the QCD sum rules \[9,10\] and the effective field theory \[13\] indicate that the rho-meson mass decreases with increasing nuclear density. In this work, we use the results from the QCD sum-rule calculation \[9,10\] which can be parametrized by
\[ \frac{m^*_\rho}{m_\rho} \approx 1 - 0.18(\rho/\rho_0) \approx \frac{1}{1 + 0.18(\rho/\rho_0)}. \]  

(7)

The first parametrization has been given in Refs. [9,10]. We rewrite it as the second form to extend to higher densities. The in-medium rho-meson mass is seen to decrease with increasing density, as shown in Fig. 2. At twice the normal nuclear matter density, it decreases by about 25%. We note that in the QCD sum-rule calculations, the density dependence of the rho-meson mass is largely determined by the density dependence of the four-quark condensate which has been, however, only approximately calculated.

The width of the rho meson in a medium is given by

\[ \Gamma^*_\rho = \frac{g^2_{\rho\pi\pi}}{4\pi} \frac{1}{12m^2_\rho} (m^*_{\rho} - 4m^2_\pi)^{3/2}, \]

(8)

where the \( \rho\pi\pi \) coupling constant \( g_{\rho\pi\pi} \) is determined from the decay width of the rho meson in free space. Using \( m_\rho=768.1 \text{ MeV}, m_\pi=139.6 \text{ MeV} \) and \( \Gamma_\rho=151.5 \text{ MeV} \) [50], we have \( g^2_{\rho\pi\pi}/4\pi \approx 2.9 \). Because of the reduction of the rho-meson mass in the medium, the width of the rho meson decreases with increasing density, as shown in Fig. 4. In Eq. (8), the effect on the rho-meson width due to the medium modification of the pion dynamics has not been included, as it has been shown to be less important than the effect from the decrease of the rho-meson mass in the medium [11]. Also, the possible medium modification of the \( \rho\pi\pi \) coupling constant has been neglected.

In order to treat explicitly the formation, propagation and decay of the rho meson, we express the dilepton production cross section, Eq. (5), as the product of the rho-meson formation cross section and the branching-ratio for its decay into a dilepton, i.e.,

\[ \sigma_{\pi^+\pi^-\to\rho^0\to e^+e^-}(M) = \sigma_{\pi^+\pi^-\to\rho^0}(M) \frac{\Gamma_{\rho\to e^+e^-}(M)}{\Gamma_{\rho}(M)}, \]

(9)

where \( \Gamma_{\rho\to e^+e^-}(M) \) is the rho-meson decay width into a dilepton. Using the following expressions [51]

\[ \Gamma_{\rho\to e^+e^-}(M) = \frac{1}{3} \alpha^2 M \left( \frac{4\pi}{g^2_{\rho\pi\pi}} \right), \]

(10)
\[ \Gamma_{\rho}(M) = \frac{2 g_{\rho \pi \pi}^2}{3} \frac{k^3}{4\pi M^2}, \]  

(11)

and comparing Eq. (9) with Eq. (5), we have

\[ \sigma_{\pi^+ \pi^- \rightarrow \rho^0}(M) = \frac{12\pi}{k^*_{\rho}^2} \frac{(m_{\rho}^* \Gamma_{\rho}^*)^2}{(M^2 - m_{\rho}^*)^2 + (m_{\rho}^* \Gamma_{\rho}^*)^2} \left( \frac{k}{k^*_{\rho}} \right)^4 \left( \frac{m_{\rho}^*}{M} \right)^6, \]

(12)

where \( k^*_{\rho} \) is given by Eq. (6) with \( M = m_{\rho}^* \). This is the rho-meson formation cross section used in our numerical calculation.

The propagation of the rho meson is treated as that of a free particle, i.e., we neglect the effect of the mean-field potential during its propagation. We include, however, approximately the change of the rho-meson mass as it propagates from one place (with density \( \rho_1 \)) to another (with density \( \rho_2 \)):

\[ M_2 = M_1 + m_{\rho} \left[ \frac{0.18(\rho_1/\rho_0)}{1 + 0.18(\rho_1/\rho_0)} - \frac{0.18(\rho_2/\rho_0)}{1 + 0.18(\rho_2/\rho_0)} \right]. \]

(13)

This is similar to the change of the nucleon effective mass as it propagates through matter via the change of the scalar field.

Denoting, at time \( t \), the differential multiplicity of rho mesons with mass \( M \) by \( P_{\rho}(M, t) \), then the differential dilepton production probability is given by

\[ P_{e^+ e^-}(M) = \int_0^T P_{\rho}(M, t) \Gamma_{\rho \rightarrow e^+ e^-}(M) \, dt + P_{\rho}(M, T) \frac{\Gamma_{\rho \rightarrow e^+ e^-}(M)}{\Gamma_{\rho}(M)}, \]

(14)

where \( \Gamma_{\rho \rightarrow e^+ e^-}(M) \) and \( \Gamma_{\rho}(M) \) are given by Eqs. (10) and (11), respectively. In our numerical simulation, \( T \) in Eq. (14) is taken to be 32 fm/c. At that time, the central density of the system is about 0.01 \( \rho_0 \), thus any further change of the medium effects can be neglected, and the contribution of any remaining rho mesons to dileptons is calculated using its branching ratio as shown by the second term in Eq. (14).

In the form factor approach, the effect of the intermediate rho meson is included through the pion electromagnetic form factor,

\[ |F_{\pi}(M)|^2 = \frac{m_{\rho}^4}{(M^2 - m_{\rho}^*)^2 + m_{\rho}^* \Gamma_{\rho}^*}. \]

(15)
The dilepton production cross section in this approach is then given by

$$\sigma_{\pi^+\pi^+\rightarrow e^+e^-} (M) = \frac{8\pi\alpha^2 k}{3M^3}|F_\pi(M)|^2. \quad (16)$$

This is the same as Eq. (5). Neglecting medium effects, Eq. (16) reduces to the familiar formula used in Refs. [21–30].

**B. Kaon-antikaon annihilation**

Dilepton production from kaon-antikaon annihilation proceeds mainly through a phi meson which couples to a virtual photon and decays into a dilepton, as shown by the diagram in Fig. 5. Including the medium modifications of the kaon, antikaon and phi meson masses and the phi width, the invariant matrix element for this process is

$$\mathcal{M} = \frac{e^2 m_\phi^2}{3 q^2 [q^2 - (m_\phi^* - \frac{1}{2} \Gamma_\phi^*)^2]} (q_1 - q_2)\bar{u}(k_1, s_1) v(k_2, s_2), \quad (17)$$

where the relation, $g_{\phi\gamma} = \frac{e m_\phi^2}{g_{\phi K\bar{K}}}$, between the $\phi K\bar{K}$ coupling constant $g_{\phi K\bar{K}}$ and the $\phi\gamma$ coupling constant $g_{\phi\gamma}$ has been used [51].

From Eq. (17), we have

$$\mathcal{M}^2 = \frac{e^4 m_\phi^4}{9 m_\phi^2 q^4 [q^2 - (m_\phi^*)^2 + (m_\phi^* \Gamma_\phi^*)^2]} \cdot \left[-\frac{1}{2} q^4 + (m_K^* + m_{\bar{K}}^*) q^2 + 8 (k \cdot q)^2 - \frac{1}{2} (m_K^* - m_{\bar{K}}^*)^2 \right]. \quad (18)$$

The dilepton production cross section from the kaon-antikaon annihilation is then given by

$$\sigma_{K\bar{K}\rightarrow\phi\rightarrow e^+e^-} (M) = \frac{8\pi\alpha^2 k}{27M^3} \frac{m_\phi^4}{(M^2 - m_\phi^*)^2 + (m_\phi^* \Gamma_\phi^*)^2}, \quad (19)$$

where $M$ is the invariant mass of the kaon-antikaon pair, and the three momentum $k$ is given by

$$k = \frac{1}{2M} \left[(M^2 - (m_K^* + m_{\bar{K}}^*)^2)(M^2 - (m_K^* - m_{\bar{K}}^*)^2) \right]^{1/2}. \quad (20)$$
The QCD sum-rule calculations\cite{9,10} indicate that the phi meson mass is also modified in the medium. The results of Refs.\cite{9,10}, obtained with a nucleon strangeness content of \( y = 2 < ss >_N / < \bar{u}u + \bar{d}d >_N \approx 0.17 \), can be parametrized by the simple expression

\[
\frac{m_\phi^*}{m_\phi} \approx 1 - 0.0255 (\rho/\rho_0) \approx 1 + 0.0255 (\rho/\rho_0),
\]

(21)

which shows that the phi-meson mass decreases slowly with increasing density, as shown in Fig. 2. At twice the normal nuclear matter density, the phi-meson mass decreases by about 5%. We note that in the QCD sum-rule calculations, the in-medium mass of the phi meson depends on the nucleon strangeness content which is, however, not well determined.

The decay width of the phi meson into a kaon and an antikaon is given by

\[
\Gamma_{\phi}^* = \frac{g_{\phi KK}^2}{4\pi} \frac{1}{6m_{\phi}^*} \left[ \left( m_{\phi}^* - (m_K^* + m_{\bar{K}}^*)^2 \right) \left( m_{\phi}^* - (m_K^* - m_{\bar{K}}^*)^2 \right) \right]^{3/2}.
\]

(22)

As in the case of the rho meson, the coupling constant \( g_{\phi KK} \) is assumed not to be modified in the medium and is determined from the decay width of the phi meson in free space into a kaon and an antikaon. Using \( m_K=495.5 \) MeV, \( m_{\phi}=1019.4 \) MeV and \( \Gamma_{\phi \to KK}=3.7 \) MeV\cite{50}, we obtain \( g_{\phi KK}^2/4\pi=1.69 \). We note that in the present calculation the small decay width of the phi meson into a pion and a rho meson is neglected. The density dependence of the phi decay width calculated from Eq. (22) is shown in Fig. 3 as a function of density. Although the phi meson mass is reduced in the medium, its width increases with density as a result of the larger reduction of the antikaon in-medium mass than the increase of the kaon in-medium mass and hence an increase of the available phase space. At twice the normal nuclear matter density, the phi-meson decay width is about 30 MeV.

Similarly, to treat explicitly the formation, propagation and decay of the phi meson, we express Eq. (19) as the product of the phi-meson formation cross section and the branching ratio for its decay into the dilepton, i.e.,

\[
\sigma_{KK \to \phi \to e^+e^-}(M) = \sigma_{KK \to \phi}(M) \frac{\Gamma_{\phi \to e^+e^-}(M)}{\Gamma_{\phi}(M)},
\]

(23)

where \( \Gamma_{\phi \to e^+e^-}(M) \) is the phi-meson decay width into the dilepton. Using the following expressions
\[
\Gamma_{\phi \to e^+e^-}(M) = \frac{1}{27} \alpha^2 M \left(\frac{4\pi}{g_{\phi KK}^2}\right),
\] (24)

\[
\Gamma_\phi(M) = \frac{4 \, g_{\phi KK}^2}{3} \frac{k^3}{4\pi} M^2,
\] (25)

and comparing Eq. (23) with (19), we get

\[
\sigma_{KK \to \phi}(M) = \frac{6\pi}{k_{\phi}^2} \left(\frac{m_\phi^* \Gamma_\phi^*}{\Gamma_\phi \Gamma_{\phi KK}}\right)^2 \left(\frac{k_{\phi}}{M}\right)^4 \left(\frac{m_\phi^*}{M}\right)^6,
\] (26)

where \(k_{\phi}^*\) is given by Eq. (20) with \(M = m_\phi^*\).

The propagation of the phi meson is also treated as that of a free particle. The change of its mass in the medium as it propagates from one place to another is approximately described by

\[
M_2 = M_1 + m_\rho \left[\frac{0.0255(\rho_1/\rho_0)}{1 + 0.0255(\rho_1/\rho_0)} - \frac{0.0255(\rho_2/\rho_0)}{1 + 0.0255(\rho_2/\rho_0)}\right],
\] (27)

as in the case of the rho meson.

The differential dilepton production probability is then given by an equation similar to Eq. (14), with \(\Gamma_\rho(M)\), \(\Gamma_{\rho \to e^+e^-}(M)\), and \(P_\rho(M,t)\) replaced, respectively, by \(\Gamma_\phi(M)\), \(\Gamma_{\phi \to e^+e^-}(M)\), and \(P_\phi(M,t)\), where \(P_\phi(M,t)\) is the differential multiplicity at time \(t\) of phi mesons with mass \(M\).

In the form factor approach, the effect of the intermediate phi meson is included through the kaon electromagnetic form factor \[22,52\],

\[
|F_K(M)|^2 \approx \frac{1}{9} \frac{m_\phi^4}{(M^2 - m_\phi^* 2 + m_\phi^* 2 \Gamma_\phi^2).}
\] (28)

The cross section for the production of dileptons with an invariant mass \(M\) from the kaon-antikaon annihilation is then given by

\[
\sigma_{KK}^{e^+e^-}(M) = \frac{8\pi \alpha^2 k}{3M^3} |F_K(M)|^2.
\] (29)

This is the same as Eq. (19) and reduces to the formula used in Ref. \[22\] when the medium effects are neglected.
IV. DILEPTON PRODUCTION: RESULTS AND DISCUSSIONS

In obtaining the dilepton invariant mass spectra, we have used a bin size of 10 MeV for dileptons from pion-pion annihilation and a bin size of 4 MeV for those from kaon-antikaon annihilation. We have used 100 test particles for each run of the simulation, and repeated the simulation 40 and 10 times for calculating the pion-pion and the kaon-antikaon contribution, respectively. For pion-pion annihilation, we have thus more than 1000 events in each low invariant mass bin, and about 100 events in each high invariant mass bin. For kaon-antikaon annihilation, we have a few thousand events in each low invariant mass bin, and a few hundred in each high invariant mass bin. We note that in our perturbative treatment of kaon and antikaon production, we allow a quasi-kaon or quasi-antikaon to be produced whenever the available energy of a baryon-baryon collision is above the kaon or the antikaon production threshold. This quasi-kaon or quasi-antikaon carries a probability which is given by the ratio of its production cross section to the baryon-baryon total cross section. We find that at the time when the system is most compressed, there are about 150 quasi-kaons and 30 quasi-antikaons, while the (physical) pion number is about 20. In this way, we obtain good statistics for both pion-pion and kaon-antikaon annihilation.

We first carry out a calculation in which the medium effects on kaons, antikaons, rho and phi mesons are neglected, i.e., we use free masses and widths in Eqs. (12) and (26). Also, in calculating the kaon and antikaon production probabilities the attractive scalar potential is neglected. The total dilepton spectrum from both pion-pion and kaon-antikaon annihilation is shown in Fig. 6. Since in our numerical simulation a bin size of 10 MeV is used for dileptons from pion-pion annihilation, it has been converted to a new bin size of 4 MeV by interpolations in order to add it to the kaon-antikaon contribution.

For pion-pion annihilation we observe a broad but visible peak around the rho-meson mass ($\approx 770$ MeV), which is in agreement with earlier results obtained in non-relativistic transport models [28,29]. Since most pion-pion collisions are of low invariant masses, a bump is thus seen in this region. As shown in Refs. [29,30], the background from the eta and delta
Dalitz decays and the neutron-proton bremsstrahlung is much smaller than the pion-pion contribution around the rho-mass region. Therefore, if the rho meson mass is not modified in the medium, one should observe a broad peak around 770 MeV in the dilepton spectrum.

For kaon-antikaon annihilation we observe a very sharp peak around the phi-meson mass (≈1.02 GeV). This peak is about a factor of 4 above the background from pion-pion annihilation. At these invariant masses, the background from other sources, e.g., Dalitz decays and bremsstrahlung, are again small or at most comparable to that from pion-pion annihilation [29,30]. Thus, if the properties of the kaon, the antikaon and the phi meson are not modified in the medium, one should see a peak in the dilepton spectrum around 1020 MeV. Of course, a detector of excellent resolution is needed to see this peak due to its very narrow width (≈ 4MeV).

Our main point, however, is to see whether we can learn from dilepton spectra about medium modifications of hadron properties. We show in Fig. 7 the dilepton spectrum obtained by including in the calculation all medium effects discussed above. We find that the maximum in the pion-pion contribution shifts from the free rho-meson mass to around 550 MeV as a result of the decrease of the rho-meson mass in dense matter. The height of this peak is about four times the height of the peak at the free rho-meson mass when the medium effects are not included. This very broad peak results from the overlap of a number of peaks with different positions and widths (depending on the density where the pion-pion annihilation occurs). The small shoulder at low invariant masses is again due to the fact that most pion-pion collisions are of such invariant masses. According to Ref. [30], the background from Dalitz decays and bremsstrahlung around 550 MeV is, at most, of the same magnitude as that of the free rho-meson peak obtained without the medium effects. Since the rho-meson peak at 550 MeV after including the medium effects is enhanced by about a factor of four as compared to the original peak without the medium effects, we expect that if the rho-meson mass is indeed reduced in the medium as predicted by the QCD sum-rule calculations, a broad peak in the dilepton invariant mass spectrum around 550 MeV will be observed. Although to detect such a broad peak is difficult experimentally,
the study of the in-medium properties of the rho meson from the dilepton spectra is still possible if the background from Dalitz decays and bremsstrahlung can be removed.

The dilepton spectrum from kaon-antikaon annihilation is also modified when the medium effects are included. The peak now shifts to a slightly lower invariant mass and is about a factor of five broader than the original peak at the free phi-meson mass when no medium effects are included. The broadening of the phi peak is mainly due to the increase of its width in the medium and the overlap of a number of peaks with different widths and positions. More interestingly, a raised shoulder appears around 950 MeV as a result of the decrease of the phi-meson mass at high densities. This shoulder is about a factor of two above the background from pion-pion annihilation. The background from other sources of dileptons at these invariant masses, such as Dalitz decays and bremsstrahlung, is small. It is thus feasible to study the in-medium properties of the phi meson by measuring dilepton spectra in heavy-ion collisions. Finally, because of the decrease of the antikaon mass in the medium, we have dileptons with invariant masses as low as 800 MeV from the kaon-antikaon annihilation.

The change of the dilepton spectrum due to the medium effects can be more clearly seen by examining the pion-pion and the kaon-antikaon contribution separately. In Fig. 8, we compare the dilepton spectra from pion-pion annihilation with and without medium effects on the rho meson. The shapes of the spectra are clearly different. The peak shifts to lower invariant masses and becomes broader. In Fig. 9, a similar comparison is shown for the dilepton spectra from kaon-kaon annihilation. The appearance of a shoulder around 950 MeV and the broadening of the peak around the free phi-meson mass are clearly observed.

To test the reliability of the form factor approach that has been used often in previous calculations, we have also carried out two calculations using Eqs. (16) and (29). The results are shown in Fig. 10 and Fig. 11 for the cases without medium effects and with medium effects, respectively. Comparing Fig. 10 with Fig. 6, we see that if the medium effects are neglected, the form factor approach gives essentially the same results as those obtained from the dynamical approach for both pion-pion and kaon-antikaon annihilation. When the
medium effects are included, the two methods of calculations lead to similar predictions for the dilepton spectra from pion-pion annihilation, i.e., the shift of the peak from the free rho-meson mass to a lower invariant mass and the broadening of the peak when the medium effects are included. This similarity can be easily understood, as the decay width of a rho meson is quite large (see Fig. 4 where the decay width corresponding to the peak mass $m_{\rho}^*$ is shown) and the rho meson propagates thus for only a very short distance before it decays. The neglect of the formation time of dileptons in the form factor approach is then not a bad approximation. However, there do exist some quantitative differences between the results of the two calculations, namely, the peak shifts to a slightly larger invariant mass and its width is almost doubled in the case of the dynamical approach as compared to the results from the form factor approach.

The situation is quite different for kaon-antikaon annihilation. In the form factor approach, we see in Fig. 11 a peak around 950 MeV which is well above the background from pion-pion annihilation, whereas in the dynamical approach only a raised shoulder is seen to be just above this background. On the other hand, in the form factor approach we also have a small peak around the free phi-meson mass that comes from kaon-antikaon annihilation at the later stage of heavy-ion collisions when the density is low. In the dynamical approach, there appears a strong peak just slightly below the free phi-meson mass. The apparent difference between the results of the two calculations goes back to the fact that the phi-meson width is small (see Fig. 4). A phi meson, when formed from the kaon-antikaon annihilation at high densities, does not decay right away as assumed in the form factor approach, but rather propagates in the medium to lower densities as the system expands. Therefore, for those intermediate vector mesons with long life times, the explicit treatment of their formation, propagation and decay is necessary if we want to study their in-medium properties by measuring the dilepton invariant mass spectra.
V. SUMMARY

In summary, we have studied dilepton production from both pion-pion and kaon-antikaon annihilation in central heavy-ion collisions at SIS/GSI energies. The collision dynamics have been described by the generalized relativistic transport model which includes the medium effects on kaons and antikaons in the mean-field approximation to the chiral Lagrangian.

We have studied in particular the effects of medium modifications of the vector meson properties on the dilepton invariant mass spectra. Dilepton production has been calculated in two ways. In the form factor approach, the formation time of the dileptons is neglected and the effects of the intermediate vector mesons are included through the electromagnetic form factors. In the dynamical approach, the formation, propagation and decay of the intermediate vector mesons are explicitly treated, and the change of the properties of these vector mesons in the medium is included. It is found that for vector mesons with large decay widths (e.g., rho meson), the form factor approach gives results qualitatively similar to the dynamical approach. However, for mesons with small decay widths (e.g. phi meson), an explicit treatment of their formation, propagation and decay is necessary, otherwise the medium effects will be overestimated.

Differences are seen between dilepton spectra with and without medium effects. These include the shift of the vector meson peaks to lower invariant masses and the broadening of their widths. For pion-pion annihilation, the peak shifts to around 550 MeV, and its height increases by about a factor of four. This peak is expected to be above the background from Dalitz decays and bremsstrahlung which can in principle be subtracted from the data. We thus conclude that the in-medium properties of the rho meson can be studied through the dilepton spectra. For kaon-antikaon annihilation, the situation is more subtle. The peak shifts only slightly to a lower invariant mass, so the experimental detection of this shift may be difficult. In addition, a shoulder appears around 950 MeV, which is unfortunately only slightly above the background from pion-pion annihilation.

As is well known, the kaon-to-pion ratio increases with increasing incident energies (at
SIS energies, the $K^+ / \pi^+$ ratio is about 1%, while at AGS energies, this ratio increases to about 20%). It is thus expected that at higher incident energies (e.g., at AGS energies), the shoulder in the kaon-antikaon contribution will be well above the pion-pion contribution and therefore be easily detected and more accurately analysed. To study this theoretically, we need, however, first to extend the relativistic transport model to include other degrees of freedom so that it will be suitable for the description of heavy-ion collisions at these energies.

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Figure Caption

Fig. 1: Time evolution of the average central density $\rho/\rho_0$ (dotted curve), the pion, kaon, and antikaon multiplicities. Dashed curves are the kaon and antikaon multiplicities without medium effects.

Fig. 2: Effective masses of kaon, antikaon, rho meson and phi meson as a function of density.

Fig. 3: Feynman diagram for dilepton production from pion-pion annihilation.

Fig. 4: Effective widths of rho and phi meson as a function of density.

Fig. 5: Feynman diagram for dilepton production from kaon-antikaon annihilation.

Fig. 6: Dilepton invariant-mass spectrum from pion-pion and kaon-antikaon annihilation without medium effects in the dynamical approach.

Fig. 7: Same as Fig. 6 with medium effects.

Fig. 8: Comparison of dilepton spectra from pion-pion annihilation with and without medium effects.

Fig. 9: Same as Fig. 8 for kaon-antikaon annihilation.

Fig. 10: Same as Fig. 6 in the form factor approach.

Fig. 11: Same as Fig. 7 in the form factor approach.
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