A New Approach to Classify Growth Patterns Based on Growth Function Selection and K-means Method

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Abstract: While every growth behaviour can be described by a growth function with longitudinal independent variables, modeling longitudinal growth behaviour is complex and requires a strategic search for an appropriate growth function. Existing literature shows that several growth functions have been derived and applied to suitable growth behaviour. This implies that there is always the possibility of the existence of a unique growth function that fits a given growth data. In this paper, we propose a new statistical procedure to seek the “best” growth function as well as the “best” clusters of growth patterns. This new procedure involves a mixture of growth function selection and k-means method. Although our experiments were limited, using a real data set of sugi (Cryptomeria japonica) tree growth and hypothetical observations, our results show that the proposed method is promising. It allowed us to choose the “best” growth function that matches the “best” growth patterns at the same time.

Keywords: clustering, growth function, k-means, selection of growth function

1. Introduction

Modeling longitudinal forest growth behaviour is complex and requires a strategic search for an appropriate growth function, which correctly mimics the growth behavior. Generally, there are three stages of longitudinal growth behaviour. The first stage describes what occurs immediately after birth or plant establishment. At this early stage, growth is described as being slower and exponential in nature. This early stage is superseded by the mid stage, also referred to as the juvenile stage. At the juvenile stage trees are well established in their environment and grow at their most rapid pace. Growth at the juvenile stage can therefore be described as rapid. The third and final stage is the old age or mature stage. Here, growth continues at a slower, but at a steady pace until growth gradually attains an upper bound (Yang et al. 1978). These aforementioned three growth stages can be described as being “S-shaped” or “sigmoid”, and can be represented mathematically by a growth function with a longitudinal independent variable, for example, individual tree age. However, these longitudinal growth functions do not capture exogenous factors such environmental or competing factors.

Generally, growth functions are developed under several assumptions or mathematical constraints, resulting in a variety of growth functions with various growth patterns. Different growth patterns are often observed as a result of different geographic, climatic, genetic and other micro-environmental conditions. Thus, classifying the growth patterns plays an important role in finding an appropriate growth function to a given growth pattern. In statistics, “clustering” is a preferred approach to classify growth patterns from a given growth data. Yoshimoto et al. (2012) introduced a clustering method for classifying growth behaviour. They first assumed Richards-Chapman growth function (Richards 1959), then applied the “k-means” method (MacQueen 1967) to the set of derived estimated parameters for classifying the growth patterns. It is important to note that if an inappropriate growth function is assumed for the analysis, the results would be misleading. Kamo and Yoshimoto (2013a,b) proposed a method to select the “best” growth function based on information criterion without considering the classification of growth patterns. In this paper, we propose a new statistical procedure to select the “best” growth function as well as the “best” growth patterns to overcome the above shortcomings. The proposed approach consists of growth function selection and k-means clustering to classify the growth behavior. To validate this approach, we used a real data set of sugi (Cryptomeria japonica) tree growth and numerical examination.
2. Methods

2.1. Clustering of growth behavior

In this paper, we propose a two-step procedure for growth clustering. The first step is to select the “best” growth function among the candidates, and the second step is to apply cluster growth patterns by the k-means method. Let us describe growth behaviour by

\[ E[y_i(t_j)] = f_i(t_j | \beta) \quad (i = 1, 2, ..., n, j = 1, 2, ..., p) \]

where \( y_i(t_j) \) is the growth of the \( i \)th tree at time \( t_j \), \( E[\cdot] \) denotes the expected value, \( n \) is the number of sample trees, and \( p \) is the number of observations over time. Here \( f_i(\cdot) \) is the growth function describing growth behaviour of the \( i \)th tree, and \( \beta \) is a set of unknown parameters in the growth function.

For the selection of the growth function, because of the nonlinearity of the growth function, we use a cross-validation (CV) criterion (Stone 1974), since other criterion such as information criterion implicitly assumes model linearity. CV is defined as:

\[ CV = \sum_{k=1}^{K} (y_i - \hat{y}_{(-i)})^2 \]

where \( \hat{y}_{(-i)} \) is the \( i \)th jackknifed estimator of \( y_i \) based on the growth function applied. A smaller CV value means, the better the growth function. The concept of CV is based on the idea to evaluate the predictive error, which is not affected by the model linearity or nonlinearity.

After the first step of growth function selection, the next step is to apply k-means clustering (MacQueen 1967) to the set of the estimated parameters in the growth function. We investigated the situation that the same growth function can be used, however, the range of the parameter values was large. For example, in Yoshimoto et al. (2012), Richards-Chapman growth function was assumed, superseded by the application of k-means clustering to classify the growth patterns. This is based on the idea that the set of parameters included in the growth function characterizes the growth behaviour differently. Let \( \hat{\beta}_k \) as in vector expression, and cluster as \( \{C_1, C_2, ..., C_K\} \), where \( K \) denotes the number of clusters. Then the centroid vector \( \hat{\mu}_k \) and covariance matrix \( \hat{\Sigma}_k \) of cluster \( C_k \) are, respectively, estimated by

\[ \hat{\mu}_k = \frac{1}{n_k} \sum_{i \in C_k} \hat{\beta}_i \quad \text{and} \quad \hat{\Sigma}_k = \frac{1}{n_k} \sum_{i \in C_k} (\hat{\beta}_i - \hat{\mu}_k)(\hat{\beta}_i - \hat{\mu}_k)' \]

where \( n_k \) denotes the number of trees included in the cluster \( C_k \). Since k-means method is based on the fixed number of clusters, the optimal number of clusters is needed to be determined. We use information criteria using the estimate of risk function based on predicted Kullback-Leibler values (Kullback and Leibler 1951). Although the Akaike’s information criterion (AIC) (Akaike 1973) is often applied in such a case, we applied the corrected version of AIC (CAIC), that is,

\[ CAIC(K) = np \log 2\pi + \sum_{k=1}^{K} n_k \log |\hat{\Sigma}_k| + \frac{K}{n_k} \frac{np(n_k + 1)}{n_k - p - 2} \]

where \( K \) denotes the number of cluster. As in AIC and CV, the number \( K \) minimizing CAIC is regarded as the best number. This is because AIC tends to select the complicated model largely affected by the asymptotic bias. The concept or derivation of CAIC can be found in Sugiura (1978), Hurvich and Tsai (1989) and Fujikoshi and Satoh (1997). Figure 1 (a) shows the total flow of the procedure of our method, developed in this paper.
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Figure 1. Flowchart of two-step growth clustering.
2.2. Candidates for growth function

In this paper, we prepare the following twelve candidates of growth functions as in Zeide (1993):

1. Bertalanffy: \( Bert(t: \alpha, \beta) = \alpha(1 - e^{-\beta t})^3 \),
2. Gompertz: \( Gomp(t: \alpha, \beta, \gamma) = \alpha \exp(-\beta t^{-\gamma}) \),
3. Hossfeld-4: \( Hos4(t: \alpha, \beta, \gamma) = t^\gamma + t^{\gamma+1} \),
4. Korf: \( Korf(t: \alpha, \beta, \gamma) = \alpha \exp(-\beta t^{-\gamma}) \),
5. Levacovic-3: \( Lev3(t: \alpha, \beta, \gamma) = \alpha \left( t^2 + t^{\gamma+1} + t^{2\gamma+1} \right) \),
6. logistic: \( Logi(t: \alpha, \beta, \gamma) = 1 + \alpha \exp(-\beta t^{-\gamma}) \),
7. Monomolecular: \( Mono(t: \alpha, \beta, \gamma) = \alpha(1 - \beta e^{-\gamma t}) \),
8. Richards: \( Rich(t: \alpha, \beta, \gamma) = \alpha(1 - e^{-\gamma t})^\gamma \),
9. Weibull: \( Weib(t: \alpha, \beta, \gamma) = \alpha(1 - e^{-\beta t^\gamma}) \),
10. Levacovic-1: \( Lev1(t: \alpha, \beta, \gamma, \delta) = \alpha(\frac{t^\delta}{\beta+\gamma t})^\gamma \),
11. Sloboda: \( Slob(t: \alpha, \beta, \gamma, \delta) = \alpha \exp(-\beta e^{-\gamma t}) \),
12. Yoshida-1: \( Yosh1(t: \alpha, \beta, \gamma, \delta) = \frac{t^\delta}{\beta+\gamma t} + \gamma \),

where \( t \) denotes the independent variable related to the time element. All of the parameters \((\alpha, \beta, \gamma, \delta)\) included in the functions are restricted to be positive. The behaviour or derived curve of each function is shown in Figure 2, where the derived curve from each estimated function is generated from the individual tree growth data of DBH (diameter at breast height). Note section 3 uses these results. For the number of parameters in the growth function, equation (1) has two parameters; equations (2)-(9) have three parameters; and equations (10)-(12) have four parameters.

Among the candidates, the following are the similarities between the above twelve candidates.

First, they have four relationships for inclusion as follows (left hand side equates right hand side):

- \( Rich(t: \alpha, \beta, 3) = Bert(t: \alpha, \beta) \)
- \( Slov(t: \alpha, \beta, \gamma, 1) = Gomp(t: \alpha, \beta, \gamma) \)
- \( Lev1(t: \alpha, \beta, 1, \gamma) = Hos4(t: \alpha, \alpha \beta, \gamma) \)
- \( Lev1(t: \alpha, \beta, \gamma, 2) = Lev3(t: \alpha, \beta, \gamma) \)

Next, two relationships for intersection are

- \( Rich(t: \alpha, \beta, 1) = Mono(t: \alpha, 1, \beta) = Weib(t: \alpha, \beta, 1) \)
- \( Hos4(t: \alpha, \beta, 2) = Lev3(t: \alpha, \alpha \beta, 1) \)

Lastly, the following is the relationship for asymptotic intersection.

- \( Lev1(t: \alpha, \beta, 1, 2) = Lev3(t: \alpha, \beta, 1) = Yosh1(t: \alpha, \beta, \gamma, 2) \) as \( \gamma \to 0 \)

Based on these observations, it is expected that the twelve candidates of growth functions are broadly summarized into five main groups as follows:

- **Group 1:** Bertalanffy, Monomolecular, Richards and Weibull,
- **Group 2:** Gompertz and Sloboda,
- **Group 3:** Hossfeld-4, Levacovic-1, Levacovic-3 and Yoshida-1,
- **Group 4:** logistic,
- **Group 5:** Korf.

Note that it is important to prepare enough candidates to reduce the risk for the nonexistence of appropriate function among whole candidates.
Figure 2. Twelve growth functions fitted to individual tree growth of DBH. The solid line denotes the derived curve from the estimated growth function.
Table 1. Results of clustering using individual tree growth data of DBH.

| Tree ID | Selection of growth function | Group | Cluster |
|---------|-----------------------------|-------|--------|
| 7       | Korf                        | 5     | 5-2    |
| 12      | Korf                        | 5     | 5-2    |
| 15      | Korf                        | 5     | 5-2    |
| 31      | Levacovic-1                 | 3     | 3      |
| 37      | Korf                        | 5     | 5-2    |
| 52      | Korf                        | 5     | 5-2    |
| 59      | Korf                        | 5     | 5-1    |
| 62      | Korf                        | 5     | 5-1    |
| 72      | Korf                        | 5     | 5-1    |
| 73      | Korf                        | 5     | 5-1    |
| 76      | Korf                        | 5     | 5-1    |
| 77      | Korf                        | 5     | 5-1    |
| 78      | Korf                        | 5     | 5-1    |
| 83      | Korf                        | 5     | 5-2    |
| 86      | Korf                        | 5     | 5-1    |
| 87      | Levacovic-3                 | 3     | 3      |
| 93      | Korf                        | 5     | 5-2    |
| 96      | Korf                        | 5     | 5-2    |
| 100     | Korf                        | 5     | 5-2    |
| 102     | Korf                        | 5     | 5-2    |
| 104     | Korf                        | 5     | 5-2    |
| 107     | Korf                        | 5     | 5-2    |
| 111     | Korf                        | 5     | 5-2    |
| 116     | Levacovic-1                 | 3     | 3      |
| 120     | Korf                        | 5     | 5-2    |
| 126     | Levacovic-3                 | 3     | 3      |
| 127     | Levacovic-1                 | 3     | 3      |
| 131     | Korf                        | 5     | 5-2    |
| 133     | Levacovic-3                 | 3     | 3      |
| 135     | Levacovic-1                 | 3     | 3      |
| **Total** | **Korf: 23**              | **3:7** | **3:7**  |
|         | **Levacovic-1: 4**         | **5:23** | **5-1:8** |
|         | **Levacovic-3: 3**         |         | **5-2:15** |

Note:
The second column: the results of growth function selection from 12 candidates
The third column: the group of growth functions
The fourth column: the final result of clustering
3. Real data analysis

In this section, we present the results of applying the above procedures to a data of 23 years old sugi (Cryptomeria japonica) individual tree growth in Japan sampled in 2003. From a population of 136 individual trees in the research site, discs of 30 trees were collected. Therefore, the number of trees in the growth data is 30. The observed growth behaviour (DBH over tree age) is shown in Figure 2. For details on the data collection and measurement, readers can refer to Yoshimoto et al. (2012), as well as ancillary to this book. The whole data is available at http://www.asakura.co.jp/books/isbn/978-4-254-12817-8/(website is written in Japanese).

As previously stated, the first step is to select the “best” growth function for each tree from 12 candidates listed in section 2.2. Of the 30 trees, 23 trees were selected for the Korf functions, 4 trees were selected for Levacovic-1 function, and 3 trees were selected for the Levacovic-3 function. As seen in section 2.2, Levacovic-1 and Levacovic-3 functions belong to the same group (group 3) of growth functions.

The next step was to determine which function was a better representation of the functions in group 3 by applying CV to the seven trees. When this was carried out, Levacovic-3 function was selected. Therefore, Levacovic-3 function was chosen to represent the functions in group 3. Against this background, for the first step, the growth behaviour was divided into 2 functions; the Korf and the Levacovic-3.

After the growth function selection, the next step was to cluster individual growth pattern using the k-means method in each function selected, based on the set of the estimated parameters. Here, the optimal number of clusters was decided by CAIC. The optimal solution showed 2 clusters in Group 5 (Korf) and 1 in Group 3 (Levacovic-3).

From steps 1 and 2, the procedure resulted in three clusters (2 in Group 5 (i.e. Korf) and 1 in Group 3) from a sample of 30 trees. These results are presented in Table 1 and Figure 3. Figure 1(b) also shows the resultant selection for this data. In Table 1, each cluster is denoted 3, 5-1 and 5-2, the first number represents the group of the growth function defined in section 2.2 and the second number represents the order of the cluster within the same group. The average growth function of each cluster became 
\[ 15.18 \times (t^2 + t + 1)^{1.623} \] in cluster 3. It was 
\[ 38.03 \exp(-15.12t^{-0.96}) \] in cluster 5-1 and 
\[ 17.30 \exp(-29.96t^{-1.67}) \] in cluster 5-2.

4. Numerical examination

In this section, the performance of the procedure presented in section 2.1 is examined using hypothetical observations. It starts with the creation of hypothetical observations from the three growth functions already defined, that is,

(B-1) \[ Bert(t : 30, 0.3) = 30(1 - e^{-0.3t})^3 \]
(B-2) \[ Bert(t : 40, 0.2) = 40(1 - e^{-0.2t})^3 \]
(Lev) \[ Lev_3(t : 40, 40, 1.5) = 40(t^2 + t + 1)^{1.5} \]

Although (B1) and (B2) are the same growth functions (Bertlanffy), the parameters of the 2 equations are different. The hypothetical observations were created as 10 individuals in each group, this was then followed by the construction of the whole data using 30 observations. That is, the hypothetical observation is created in (B-1), (B-2) and (Lev) as

\[ \alpha(1 - e^{-\beta_1})^3 + \varepsilon, \quad \alpha \sim N(30, 4), \quad \beta \sim N(0.3, 0.0001), \quad \varepsilon \sim N(0, 0.01), \]
\[ \alpha(1 - e^{-\beta_1})^3 + \varepsilon, \quad \alpha \sim N(40, 4), \quad \beta \sim N(0.2, 0.0001), \quad \varepsilon \sim N(0, 0.01), \]
\[ \alpha(t^2 + t + 1)^{\gamma} + \varepsilon, \quad \alpha \sim N(40, 4), \quad \beta \sim N(0.4, 0.01), \quad \gamma \sim N(1.5, 0.0001), \quad \varepsilon \sim N(0, 0.01), \]

respectively. The results of the hypothetical observations are presented in Figure 4.
To classify the 30 observations, we implemented three different approaches:

(i) Function selection only,

(ii) k-means method only,

(iii) Function selection and k-means (as proposed in this paper).

Table 2 and Figure 5 show the results using our method and Figure 1 (c) shows the procedure of the numerical examination. By applying method (i), six functions were selected (16 Bertlanffy; 7 Levacic-3; 3 Richards; 2 Levacic-1; and 1 Sloboda). For method (ii) under a priori function, using Bertlanffy, CAIC chose 5 clusters. Finally, with the application of the third or proposed approach, it led to the selection of 4 clusters. The average function of each cluster became 40.06(1 − e^{0.20t})^3 for Bertlanffy in cluster 1; 30.04(1 − e^{0.30t})^3 for Bertlanffy in cluster 2; 40.13(\frac{t^2}{39.87+t^2})^{1.51} for Levacic-3 in cluster 3; and 30.87 exp(−6.15e^{0.55t^{0.82}}) for Sloboda in cluster 4. Each result using (iii) approach showed high coincidence probability to true cluster setting. This revealed that, under our experimental evaluation, the proposed two-step clustering approach was as “best” approach than the other two approaches.
Table 2. Results of clustering using numerical examination.

| Data ID | True | (i) Function selection | (ii) k-means | (iii) Mixture of (i) and (ii) |
|---------|------|------------------------|--------------|-----------------------------|
| 1       | Bertlanffy-(1) | Bertlanffy | 5             | 1                          |
| 2       | Bertlanffy    | 1          | 1              |
| 3       | Bertlanffy    | 1          | 1              |
| 4       | Bertlanffy    | 5          | 1              |
| 5       | Bertlanffy    | 1          | 1              |
| 6       | Sloboda       | 1          | 4              |
| 7       | Bertlanffy    | 1          | 1              |
| 8       | Bertlanffy    | 5          | 1              |
| 9       | Richards      | 1          | 1              |
| 10      | Richards      | 1          | 1              |
| 11      | Bertlanffy-(2) | Bertlanffy | 3             | 2                          |
| 12      | Bertlanffy    | 4          | 2              |
| 13      | Bertlanffy    | 3          | 2              |
| 14      | Bertlanffy    | 2          | 2              |
| 15      | Bertlanffy    | 4          | 2              |
| 16      | Richards      | 2          | 2              |
| 17      | Bertlanffy    | 3          | 2              |
| 18      | Bertlanffy    | 3          | 2              |
| 19      | Bertlanffy    | 3          | 2              |
| 20      | Bertlanffy    | 3          | 2              |
| 21      | Levacovic-3   | Levacovic-1 | 2            | 3                          |
| 22      | Levacovic-3   | 2          | 3              |
| 23      | Levacovic-3   | 2          | 3              |
| 24      | Levacovic-3   | 2          | 3              |
| 25      | Levacovic-3   | 2          | 3              |
| 26      | Levacovic-3   | 2          | 3              |
| 27      | Hossfeld-4    | 2          | 3              |
| 28      | Levacovic-3   | 2          | 3              |
| 29      | Levacovic-3   | 3          | 3              |
| 30      | Levacovic-1   | 2          | 3              |
| **Total** | Bertlanffy: 20 | Bertlanffy: 16 | 1:7          | 1:9                      |
|         | Levacovic-3: 10 | Hossfeld-4: 1 | 2:11         | 2:10                    |
|         | Levacovic-3: 7 | 3:7          | 3:10         |
|         | Richards: 3   | 4:2          | 4:1          |
|         | Levacovic-1: 2 | 5:3          |
|         | Sloboda: 1    |              |              |

Note:
The column with “True”: the setting of numerical examination
The column with (i): function selection only
The column with (ii) k-means clustering only
The column with (iii): the proposed approach (mixture of (i) and (ii)).
Figure 4. Hypothetical observations of 30 samples in the numerical examination. (20 Bertlanify and 10 Levacovic-3). The three thick lines denote the predefined growth function and over-plotted thin lines are hypothetical observations. The same symbol implies the same cluster.
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Figure 5. Results of clustering in the numerical examination (a) Entire clustering results, (b) Bertlanffy (1) $30.04(1 - e^{0.30t})^3$, (c) Bertlanffy (2) $40.06(1 - e^{0.20t})^3$, (d) Levacovic-3 $40.13\left(\frac{t^2}{39.57+77}\right)^{1.51}$, (e) Sloboda $30.87 \exp(-6.15e^{-0.55t^{0.85}})$
5. Conclusion

In this paper, a new clustering procedure for growth patterns was proposed and applied to a real data set of sugi tree growth. The proposed procedure is the mixture of growth function selection and k-means method. Our previous works on clustering growth behaviour used either growth function selection (Kamo and Yoshimoto 2013a,b) or the k-means method (Yoshimoto et al. 2012). In the data analysis of sugi (in section 3) for individual tree growth, our approach produced 3 clusters. However, we were unable to guarantee the accuracy of the results since the true structure is unknown. As mentioned in the introduction, it is expected that the individuals under similar condition will belong to the same cluster. Unfortunately, we lack information like the geographical information on each tree. By guessing that closer ID numbers of tree may grow under similar conditions, we could deduce the relationship between cluster and tree ID number. The study revealed that high number of tree ID belongs to cluster-3. By guessing this way, it suggests that our clustering process might be successful.

In the numerical examination, the true cluster structure is assumed to be known, therefore we were able to investigate the performance of our approach. Three approaches were compared. They were: i) growth function selection only, ii) k-means only, and iii) the mixture or proposed approach. In the function selection, 6 clusters were selected and in the k-means, 5 clusters were selected. On the other hand, the mixture of function selection and k-means resulted in the selection of 4 clusters, and the results were very close to the assumed structure. We can therefore conclude that the proposed procedure may function appropriately.

It is important to note that there remain some unresolved problems associated with the proposed approach. The first problem is operational, since there is the additional burden of one more calculation step. This problem can be solved through the programing process. The second problem arises by looking through the methodological lens. This is because excess clusters tend to increase the computational difficulty and solution time. For the analysis in this paper, we formed groups of growth functions manually as described in section 2.2. Although this may be reasonable in practice, it is not the best approach. Without grouping growth function a priori, the number of clusters should be determined automatically using some established standards. It can be information-criterion type judgment. The third problem associated with our approach depends on the structure of the data. Our approach considers longitudinal behaviour only. However, growth is affected by exogenous factors such as the environment. In such a situation, the exogenous factors should be considered in the growth model. That is, the clustering method should be constructed by taking into account the exogenous factors.

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Appendix (R-script)

We used the statistical software “R” for analyzing the real data as well as the hypothetical observations. Since the growth functions are nonlinear functions, nonlinear regression approach is often used for growth function fitting. In “R”, the function “nls()” is prepared for nonlinear regression analysis. This function needs the initial value of the parameters for iterative estimation. The setting for initial value is so delicate that in some situations it is difficult to converge to a finite estimator. Actually, Levacovic-I function cannot converge even though the initial value was set as suitable, for example, the estimation was done using “optim()” function. Since the optim function is more robust than nls function, we show the script using optim function for future reference. Here we show only for Bertlanfy function due to space limitation.

Since all of the parameters should be restricted as positive, the exponential transformation to the parameters is applied as follows:

$$\mu(t; \alpha, \beta) = e^{\alpha}(1 - \exp(1 - e^{\beta}t))^3$$

The value of this function is obtained by the following self-made function:

```r
ExpBert <- function(pars, t) {exp(pars[1])*(1-exp(-exp(pars[2])*t))^3}
```

Here arguments “pars” and “t” denotes parameter and observed time, respectively. We can then estimate the parameters by the following self-made functions:

```r
ExpBert.Est <- function(IC, t, y) {
  RSS.func <- function(b) {sum((y - ExpBert(b, t))^2)}
  optim(IC, RSS.func)$par
}
```

where arguments “IC”, “t” and “y” denotes initial value of parameter, observed time and growth amount, respectively.