Towards a more refined model of three-nucleon interaction

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Abstract. We derive the minimal form of the two-derivative three-nucleon contact interaction by imposing all constraints from discrete symmetries and Fierz identities. In order to comply with the requirements of Poincaré covariance, a basis of operators depending on relative momenta is used. The resulting interaction depends on 10 unknown low-energy constants and leads to a three-nucleon potential which we give in local form in coordinate space.

1. Introduction

Understanding the nuclear interaction from first principles remains a challenging task. At the two-nucleon (2\textit{N}) level, phenomenological potentials have reached a remarkable degree of accuracy, providing an excellent fit to the very large body of experimental data. Comparable accuracy is attained by a new class of potentials, so called chiral potentials, which are developed in the framework of an effective field theory, incorporating all symmetry properties of the underlying theory of the strong interactions (see Refs. [1, 2, 3, 4, 5] for reviews).

The effective theory is formulated in terms of nucleons and pions, whose interactions are constrained by the chiral symmetry of QCD. The effect of heavier degrees of freedom is encoded in the value of (an infinite set of) coupling constants, the low-energy constants (LECs). The theory is organized as a systematic low-energy expansion, in powers of small momenta or quark masses (or rather, pion masses), so that at each order of the chiral expansion, only a finite number of LECs is involved. The occurrence of new LECs, as one proceeds in the expansion, limits the predictive power of the theory, which is lower and lower as the scale of "new physics" (mass of particles not explicitly included in the theory) becomes closer and closer. Nevertheless, it can be said that the effective theory approach represents a solid link between nuclear physics and QCD. It allows, for instance, to justify the hierarchy of nuclear forces: \( n \)-nucleon forces are twice suppressed with respect to the (\( n - 1 \))-nucleon forces.

Parallel to these developments there has been tremendous progress in the numerical techniques for the determination of bound and scattering states of few-nucleon systems. The latter then constitute a privileged theoretical laboratory to test different models of the nuclear interaction. Presently the low-energy expansion for the 2\textit{N} potential has been carried up to the next-to-next-to-next-to-leading order (N3LO), yielding a \( \chi^2 \) per datum very close to one.

The status of the three-nucleon (3\textit{N}) interaction is not as satisfactory. While its presence is necessary if one wants to reproduce the 3- and 4-nucleon binding energies, all currently available...
3N potentials fail to simultaneously describe also the neutron-deuteron doublet scattering length [6]. As a matter of fact, several low-energy data in the nucleon-deuteron system remain still unexplained (e.g. the so-called $A_y$ puzzle). This is not really surprising, since the adopted 3N interaction models only contain a few free parameters, contrary to what happens for the 2N interaction, which is parametrized by more than 20 adjustable parameters. For example, the chiral 3N force at N2LO only contains 2 adjustable LECs [7, 8], and its extension to N3LO does not involve any new LEC [9, 10, 11].

In this context, starting with the observation that the discrepancies between theory and experiment in the 3N system arise at very low energy, we propose to refine that part of the interaction which stems from purely nucleonic vertices. Indeed, at very low energies, even the pions can be integrated out of the theory (their effect being subsumed in the values of the LECs), giving rise to the “pionless” effective theory, and the interaction among nucleons is due to contact vertices. It should be noticed that purely contact vertices are also part of the “pionful” version of the effective theory: a matching procedure could in principle be used to find the relationship between the values of the LECs in both theories. A 3N contact term is already part of the N2LO chiral (“pionful”) 3N interaction. Invariance under parity dictates that subleading 3N contact terms contain two (covariant) derivatives of nucleon fields; they would therefore contribute, in the pionful version of the effective theory, at N4LO: they are not considered in any of the currently available models. Our strategy to identify a minimal set of such operators is based on the use of Fierz-like relations, which are consequences of the anticommuting nature of fermion fields.

2. Preliminaries: the leading 3N contact interaction

Already at the leading order the most general Lagrangian density satisfying rotational, isospin and time-reversal invariance contains redundant operators. Indeed, it is written as

$$L_{3N}^{(0)} = \sum_{i=1}^{6} E_i O_i^{(0)},$$

where the operators $O_i^{(0)}$ are defined in table 1. By simultaneously changing two nucleon fields and the corresponding spin-isospin indices one can derive relations among the different operators, analogously to what was done in Ref. [12] for the parity-violating NN interaction. The properties under exchange of spin indices are expressed by the following relations

$$\begin{align*}
(1)[1] &= \frac{1}{2} (1)[1] + \frac{1}{2} (\sigma) \cdot (\sigma) \\
(\sigma^i)[1] &= \frac{1}{2} (\sigma^i)[1] + \frac{1}{2} (1)[\sigma^i] - \frac{i}{2} \epsilon^{ijk} (\sigma^j)[\sigma^k] \\
(\sigma^i)[\sigma^j] &= \frac{1}{2} \left\{ \delta^{ij} (1)[1] - \delta^{ij} (\sigma) \cdot (\sigma) + (\sigma^i)[\sigma^j] + (\sigma^j)[\sigma^i] + i \epsilon^{ijk} (\sigma^k)[1] - i \epsilon^{ijk} (1)[\sigma^k] \right\},
\end{align*}$$

Table 1. Leading (no-derivative) 3N contact operators satisfying rotational, isospin and time-reversal invariance. Subscripts refer to the nucleon bilinear on which the operators act. For instance, $O_i^{(0)} = N^i \sigma^i NN^i \tau^a NN^i \tau^a N$. 

| $i$ | $O_i^{(0)}$ |
|----|-------------|
| 1  | 1           |
| 2  | $\sigma_1 \cdot \sigma_2$ |
| 3  | $\tau_1 \cdot \tau_2$ |
| 4  | $\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$ |
| 5  | $\sigma_1 \cdot \sigma_2 \tau_2 \cdot \tau_3$ |
| 6  | $\sigma_1 \times \sigma_2 \cdot \sigma_3 \tau_1 \times \tau_2 \cdot \tau_3$ |
where $1$ is the identity operator in the one-particle spin space and $(.)$ and $[,]$ denote spin indices of the enclosed operator. Analogous relations hold in the one-particle isospin space for the identity operator and $\tau$ matrices. If we interchange the spin-isospin indices of the first two bilinears we get the following relations

$$
O^{(0)}_1 = -\frac{1}{4} \left( O^{(0)}_1 + O^{(0)}_2 + O^{(0)}_3 + O^{(0)}_4 \right)
$$

$$
O^{(0)}_2 = -\frac{1}{4} \left( 3O^{(0)}_1 - O^{(0)}_2 + 3O^{(0)}_3 - O^{(0)}_4 \right)
$$

$$
O^{(0)}_3 = -\frac{1}{4} \left( 3O^{(0)}_1 + 3O^{(0)}_2 - O^{(0)}_3 - O^{(0)}_4 \right)
$$

$$
O^{(0)}_4 = -\frac{1}{4} \left( 9O^{(0)}_1 - 3O^{(0)}_2 - 3O^{(0)}_3 + O^{(0)}_4 \right)
$$

$$
O^{(0)}_5 = -\frac{1}{2} \left( 3O^{(0)}_3 - O^{(0)}_5 \right)
$$

$$
O^{(0)}_6 = 2 \left( O^{(0)}_4 - O^{(0)}_5 \right)
$$

(3)

while the relations deriving from the remaining permutations are linearly dependent on the ones above. As a consequence of these relations, all six operators are proportional to each other, so that only one operator is left, in agreement with Ref. [8]. It is important to realize that the equivalence among the six operators is in general lost in the presence of a cutoff: in such a case the three nucleons do not occupy exactly the same point in space. However, in the framework of the effective theory, different choices of the operator are equivalent up to higher order corrections.

3. Constraining the subleading contact interaction

The subleading $3N$ contact interaction involves operators with 2 spatial derivatives, as dictated by parity symmetry. Momentum conservation implies that the only possible space-structures have the general form

$$
X_A^{ij} = (N^I \nabla_i N)(N^I \nabla_j N)(N^I N)
$$

$$
X_B^{ij} = \nabla_i (N^I N) \nabla_j (N^I N)(N^I N)
$$

$$
X_C^{ij} = i \nabla_i (N^I N)(N^I \nabla_j N)(N^I N)
$$

$$
X_D^{ij} = (N^I \nabla_i \nabla_j N)(N^I N)(N^I N).
$$

(4)

In the above expressions, and the following ones, the superscripts refer to the time-reversal properties, once appropriate factors of $i$ have been included in order to ensure hermiticity. We are only interested in the isospin symmetric limit, therefore the relevant structures are

$$
T^+ = 1, \quad \tau_1 \cdot \tau_2, \quad \tau_1 \cdot \tau_3, \quad \tau_2 \cdot \tau_3, \quad T^- = \tau_1 \times \tau_2 \cdot \tau_3,
$$

(5)

where the subscripts of the Pauli matrices refer to the nucleon bilinears they belong to. Even (odd) combinations of $X \otimes T$ structures under time-reversal are to be associated with spin structures containing even (odd) numbers of $\sigma$ matrices. Finally, the spin-space indices have to be contracted with Kronecker $\delta$’s of Levi-Civita tensors $\epsilon$’s. An overcomplete list of 146 operators has been presented in Ref. [13], constructed according to the above criteria. Here we present an alternative list of operators constructed in terms of relative momenta, defined as

$$
\nabla_a = \nabla_2 - \nabla_1, \quad \nabla_b = \nabla_3 - \frac{1}{2} (\nabla_1 + \nabla_2).
$$

(6)

The use of such variables allows us to automatically comply with the requirements of Poincaré covariance, which in the context of the non-relativistic low-energy effective theory reduce to Galilean covariance. Indeed the use of the above combinations of momenta will entail a $P$-independent interaction, $P$ being the total momentum of the three particles. Notice however...
that the full Poincaré covariance can be consistently implemented order by order in the effective theory, as done in Ref.[14]: as a result one would obtain the so called fixed terms, i.e. two derivative terms with fixed coupling constants. We will not take this kind of terms into account in the present work. The gradients in Eq. (6) can be either \( \vec{\nabla} \), acting of the whole nucleon bilinear and depending on the single-nucleon momentum transfer \( \mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i \), or \( \vec{\nabla} \), depending on the single-nucleon momentum combination \( \mathbf{Q}_i = \mathbf{p}_i' + \mathbf{p}_i \). However not all the combinations need to be taken into account, as one can interchange the labels of the three particles to express some combinations in terms of others, for instance

\[
\begin{align*}
\vec{\nabla}_a \vec{\nabla}_b f(1,2,3) & \rightarrow \vec{\nabla}_a \vec{\nabla}_a [f(1,3,2) - f(1,2,3)] + 2 \vec{\nabla}_a \vec{\nabla}_b f(1,3,2), \\
\vec{\nabla}_b \vec{\nabla}_a f(1,2,3) & \rightarrow -\frac{1}{4} \left\{ \vec{\nabla}_a \vec{\nabla}_a f(1,2,3) - 4 \vec{\nabla}_a \vec{\nabla}_a f(1,3,2) +4 \vec{\nabla}_a \vec{\nabla}_b [f(1,2,3) + f(1,3,2)] \right\}
\end{align*}
\]

\[
\begin{align*}
\vec{\nabla}_b \vec{\nabla}_b f(1,2,3) & \rightarrow -\frac{1}{4} \left\{ \vec{\nabla}_a \vec{\nabla}_a f(1,2,3) - 4 \vec{\nabla}_a \vec{\nabla}_a f(1,3,2) +4 \vec{\nabla}_a \vec{\nabla}_b [f(1,2,3) + f(1,3,2)] \right\}
\end{align*}
\]

\[
\begin{align*}
\vec{\nabla}_b \vec{\nabla}_b f(1,2,3) & \rightarrow -\frac{1}{4} \left\{ \vec{\nabla}_a \vec{\nabla}_a f(1,2,3) - 4 \vec{\nabla}_a \vec{\nabla}_a f(1,3,2) +4 \vec{\nabla}_a \vec{\nabla}_b [f(1,2,3) + f(1,3,2)] \right\}
\end{align*}
\]

where \( f \) denotes an arbitrary spin-isospin function of the three particles. Using these and similar identities, we are left with the operators listed in table 2. The list is overcomplete, since the operators are related to each other through Fierz rearrangements. Such relations leave only 10 independent operators, in agreement with the findings of Ref. [13]. In order to write the minimal subleading contact Lagrangian, it is convenient to go back to the operator basis used in Ref. [13],

\[
\begin{align*}
\mathcal{L}^{(2)}_{3N} &= E'_1 \vec{\nabla} (N^\dagger N) \cdot \vec{\nabla} (N^\dagger N) N^\dagger N + E'_2 \vec{\nabla} (N^\dagger \tau^a N) \cdot \vec{\nabla} (N^\dagger \tau^a N) N^\dagger N \\
&+ E'_3 \vec{\nabla} \cdot (N^\dagger \sigma \tau^a N) \vec{\nabla} \cdot (N^\dagger \sigma \tau^a N) N^\dagger N + E'_4 \vec{\nabla} \cdot (N^\dagger \sigma \tau^a N) \vec{\nabla} \cdot (N^\dagger \tau^a \sigma N) N^\dagger N \\
&+ E'_5 \vec{\nabla} \cdot (N^\dagger \sigma \tau^a \sigma N) \vec{\nabla} \cdot (N^\dagger \tau^a \sigma N) N^\dagger N + E'_6 \vec{\nabla} \cdot (N^\dagger \sigma \tau^a \sigma N) \vec{\nabla} \cdot (N^\dagger \tau^a \sigma N) N^\dagger N \\
&+ E'_7 \vec{\nabla} \cdot (N^\dagger \sigma \tau^a \sigma N) \vec{\nabla} \cdot (N^\dagger \tau^a \sigma N) N^\dagger N + E'_{10} \epsilon^{abc} \vec{\nabla} (N^\dagger \tau^a N) \times \vec{\nabla} (N^\dagger \tau^b N) \cdot N^\dagger \tau^c N.
\end{align*}
\]

4. Subleading 3N contact potential

Taking the matrix element of the interaction between 3N states we can obtain the 3N potential. If we define \( \mathbf{k}_i = \mathbf{p}_i - \mathbf{p}_i' \) and \( \mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}_i' \), where \( \mathbf{p}_i \) and \( \mathbf{p}_i' \) denote the initial and final momenta of nucleon \( i \), the potential in momentum space reads

\[
V = \sum_{i \neq j \neq k} \left[ -E_1 k_i^2 - E_2 k_j^2 \tau_i \cdot \tau_j - E_3 k_j^2 \sigma_i \cdot \sigma_j - E_4 k_i^2 \sigma_i \cdot \sigma_j \cdot \tau_i \cdot \tau_j \\
- E_5 (3k_i \cdot \sigma_i k_i \cdot \sigma_j - k_i^2) \right] \quad - E_6 (3k_i \cdot \sigma_i k_i \cdot \sigma_j - k_i^2) \right) \tau_i \cdot \tau_j \\
+ \frac{i}{2} E_7 k_i \times (Q_i - Q_j) \cdot (\sigma_i + \sigma_j) + \frac{i}{2} E_8 k_i \times (Q_i - Q_j) \cdot (\sigma_i + \sigma_j) \tau_j \cdot \tau_k \\
- E_9 k_i \cdot \sigma_i k_j \cdot \sigma_j - E_{10} k_i \cdot \sigma_i k_j \cdot \sigma_j \tau_i \cdot \tau_j \\
\right],
\]

(12)
**Table 2.** Complete list of 2-derivative three-nucleon contact operators compatible with rotational, isospin, parity and time-reversal invariance. Subscripts refer to the nucleon bilinear on which the operators act.

| Operator | Expression |
|----------|------------|
| $O_{1-3}$ | $\nabla_a \cdot \nabla_a [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{4-6}$ | $\nabla_a \cdot \nabla_a \sigma_1 \cdot \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{7-10}$ | $\tilde{\nabla}_a \cdot \nabla_a \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{11-13}$ | $\nabla_a \cdot \sigma_1 \nabla_a \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{14-17}$ | $\tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_a \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{18}$ | $\nabla_a \cdot \sigma_1 \nabla_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{19}$ | $\tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{20}$ | $\nabla_a \cdot \sigma_3 \tilde{\nabla}_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{21-23}$ | $\nabla_a \cdot \nabla_a [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{24-26}$ | $\nabla_a \cdot \nabla_a \sigma_1 \cdot \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{27-30}$ | $\nabla_a \cdot \nabla_a \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{31-33}$ | $\nabla_a \cdot \sigma_1 \nabla_a \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |
| $O_{34-37}$ | $\tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_a \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{38}$ | $\nabla_a \cdot \nabla_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{39}$ | $\tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{40}$ | $\nabla_a \cdot \sigma_3 \tilde{\nabla}_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{41-44}$ | $( \tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_b \cdot \sigma_3 - \nabla_a \cdot \sigma_3 \tilde{\nabla}_b \cdot \sigma_1 ) [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{45-48}$ | $( \tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_b \cdot \sigma_3 - \nabla_a \cdot \sigma_3 \tilde{\nabla}_b \cdot \sigma_1 ) [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{49}$ | $i \nabla_a \cdot \sigma_1 \nabla_a \sigma_2 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{50}$ | $i \tilde{\nabla}_a \cdot \sigma_1 \tilde{\nabla}_a \sigma_2 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{51}$ | $i \nabla_a \cdot \sigma_3 \nabla_a \sigma_1 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{52}$ | $i \tilde{\nabla}_a \cdot \sigma_3 \tilde{\nabla}_a \sigma_1 [ 1, \tau_1 \times \tau_2, \tau_3 ]$ |
| $O_{53-56}$ | $i \nabla_a \cdot \sigma_1 \nabla_a \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{57-60}$ | $i \nabla_a \cdot \nabla_a \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{61}$ | $i \nabla_a \cdot \sigma_1 \nabla_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_3 ]$ |
| $O_{62-65}$ | $i \nabla_a \cdot \sigma_1 \nabla_a \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{66-67}$ | $i \nabla_a \cdot \sigma_3 \nabla_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_3 ]$ |
| $O_{68-71}$ | $i \nabla_a \cdot \sigma_1 \nabla_a \sigma_2 \times \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{72-73}$ | $i \nabla_a \cdot \sigma_3 \nabla_a \sigma_1 \times \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_3 ]$ |
| $O_{74-77}$ | $i \nabla_a \cdot \nabla_a \sigma_1 \sigma_2 \cdot \sigma_3 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3 ]$ |
| $O_{78-80}$ | $i \nabla_a \cdot \nabla_a \sigma_3 \sigma_1 \sigma_2 [ 1, \tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3 ]$ |

continues...
with $E_{1-10}$ combinations of the low-energy constants $E'_i$. The potential in coordinate space is obtained as a Fourier transform. In order to do this we need to regularize the potential by a momentum cutoff. Using a cutoff which only depends on the momentum transfer $k_i$, e.g. $F(k^2_i; \Lambda)F(k^2_i; \Lambda)$, the coordinate-space potential assumes a local form,

$$
V = \sum_{i \neq j \neq k} (E_1 + E_2 \tau_i \cdot \tau_j + E_3 \sigma_i \cdot \sigma_j + E_4 \tau_i \cdot \tau_j \sigma_i \cdot \sigma_j) \left[ Z''_0(r_{ij}) + \frac{2Z'_0(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})
$$

$$
+ (E_5 + E_6 \tau_i \cdot \tau_k) S_{ij} \left[ Z''_0(r_{ij}) - \frac{Z'_0(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})
$$

$$
+ (E_7 + E_8 \tau_i \cdot \tau_k)(L \cdot S)_{ij} \frac{Z'_0(r_{ij})}{r_{ij}} Z_0(r_{ik})
$$

$$
+ (E_9 + E_{10} \tau_j \cdot \tau_k) \sigma_j \cdot \hat{r}_{ij} \sigma_k \cdot \hat{r}_{ik} Z'_0(r_{ij}) Z'_0(r_{ik})
$$

(13)

where $S_{ij}$ and $(L \cdot S)_{ij}$ are respectively the tensor and spin-orbit operators for particles $i$ and $j$ and the function $Z_0(r)$ (also depending on the cutoff $\Lambda$) is defined as

$$
Z_0(r) = \int \frac{dp}{(2\pi)^3} e^{ip \cdot r} F(p^2; \Lambda)
$$

(14)

With the above choice of basis, most terms in the potential can be viewed as an ordinary interaction of particles $ij$ with a further dependence on the coordinate of the third particle. In particular the terms multiplying $E_7$ and $E_8$ are of a spin-orbit character. Such kind of terms was suggested in Ref. [15] to solve the so-called $A_y$ puzzle (see also Ref. [16, 17]). Notice also that some of the spin-isospin structures implied by the leading contact interaction, which were equivalent up to cutoff effects, are resolved at the two-derivative level.

### 5. Concluding remarks

As we have outlined, the $3N$ contact interaction consists of a leading contribution, at the N2LO of the chiral expansion, and a subleading one, arising at the N4LO. Parity requires that no

### Table 3. Continuation of table 2

| $O_{ij}^k$ | $i \nabla_a \cdot \nabla_b [\tau_1 \times \tau_2 \cdot \tau_3]$ |
|------------|--------------------------------------------------|
| $O_{ij}^2$ | $i \nabla_a \cdot \nabla_b \bar{\sigma}_1 \cdot \bar{\sigma}_2 [\tau_1 \times \tau_2 \cdot \tau_3]$ |
| $O_{ij}^3$ | $i \nabla_a \cdot \bar{\sigma}_1 \nabla_b \cdot \bar{\sigma}_2 [\tau_1 \times \tau_2 \cdot \tau_3]$ |
| $O_{ij}^4$ | $i \nabla_a \cdot \bar{\sigma}_1 \nabla_b \cdot \bar{\sigma}_3 [\tau_1 \times \tau_2 \cdot \tau_3]$ |
| $O_{ij}^5$ | $i \nabla_a \cdot \bar{\sigma}_3 \nabla_b \cdot \bar{\sigma}_1 [\tau_1 \times \tau_2 \cdot \tau_3]$ |
| $O_{ij}^6$ | $i \nabla_a \cdot \nabla_b \bar{\sigma}_1 \cdot \bar{\sigma}_2 \cdot \bar{\sigma}_3 [\tau_1 \times \tau_2 \cdot \tau_3]$ |
| $O_{ij}^{7-9}$ | $i \nabla_a \times \nabla_b \cdot \bar{\sigma}_1 [\tau_1 \cdot \tau_2, \tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3]$ |
| $O_{ij}^{9-12}$ | $i \nabla_a \times \nabla_b \cdot \bar{\sigma}_3 [\tau_1 \cdot \tau_3, \tau_2 \cdot \tau_3]$ |

$\Lambda$ is a momentum cutoff.
3N contact interactions appear at N3LO or N5LO. On the other hand, the leading 4N contact interaction arises at N5LO, therefore we are lead to consider such terms at the same level of accuracy, at least in the framework of pionless EFT. It turns out [13] that there is only one independent 4N contact operator. Using cutoff functions depending only on momentum transfer $k$, as done in the 3N case, e.g. choosing $F(k^2; \Lambda)F(k^2; \Lambda)F(k^2; \Lambda)$, the 4-body contact potential is local in coordinate space,

$$V_{4N} = F \sum_{i \neq j \neq k \neq l} Z_0(r_{ij})Z_0(r_{ik})Z_0(r_{il}),$$

(15)

where $F$ is the associated low-energy constant. Therefore we can decouple, to a certain extent, the 3N sector from the 4N sector, in the course of fixing all low-energy constants from data: indeed, one can adjust the subleading 3N potential in the $A = 3$ systems without worrying about the consequences for the $\alpha$ particle binding energy, which could always be reproduced by adjusting the 4N contact term.

It should be stressed that the terms considered here start to contribute at N4LO and are therefore beyond the accuracy of the presently developed nuclear interactions. In particular, a complete EFT calculation should also address the problem of the $NN$ interaction at the same order. Notice that the four-derivatives 2N operators are already part of the N3LO interaction while the six-derivatives ones would start to contribute at N5LO, and have not been considered in the literature so far. Nevertheless, we can expect that, despite representing only part of the N4LO interaction, the terms derived here could play an important role in the accurate description of the three-nucleon systems, since they are completely unconstrained by symmetries. This is at variance with the 3N N3LO interaction, which contains no free parameters [9, 10, 11]. Moreover, the same terms will also appear in the pionless version of the effective theory as next-to-leading 3N interaction and leading 4N interaction. After a sensitivity study of the 10 subleading terms of the 3N potentials, the corresponding low-energy constants could be determined from a fit of several polarization observables in $N - d$ scattering at low energies. There are well established discrepancies between theoretical predictions and experimental data in some of the polarization observables as for example the vector analyzing powers $A_y$ and $T_{11}$ and the vector analyzing power $T_{21}$ in elastic scattering \cite{18, 19, 20}, and in several unpolarized breakup cross sections \cite{21, 20}. We expect, based on the results of Ref. \cite{15}, that some of the 10 operators of the 3N potential will have sufficient sensitivity to fix these discrepancies with reasonable values of their low-energy constants. Correspondingly, the 4N contact term could be used to reproduce the $^4$He binding energy, and the predictions of the derived potentials could be tested in the description of the four-body scattering states. In fact, also in this case there exist large and still unexplained discrepancies between theory and experiment \cite{22, 23, 24, 25}. Studies along these lines are in progress.

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