Dynamics of particle deposition on a disordered substrate: II. Far-from Equilibrium behavior.

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Abstract

The deposition dynamics of particles (or the growth of a rigid crystal) on a disordered substrate at a finite deposition rate is explored. We begin with an equation of motion which includes, in addition to the disorder, the periodic potential due to the discrete size of the particles (or to the lattice structure of the crystal) as well as the term introduced by Kardar, Parisi, and Zhang (KPZ) to account for the lateral growth at a finite growth rate. A generating functional for the correlation and response functions of this process is derived using the approach of Martin, Sigga, and Rose. A consistent renormalized perturbation expansion to first order in the non-Gaussian couplings requires the calculation of diagrams up to three loops. To this order we show, for the first time for this class of models which violates the fluctuation-dissipation theorem, that the theory is renormalizable.

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We find that the effects of the periodic potential and the disorder decay on very large scales and asymptotically the KPZ term dominates the behavior. However, strong non-trivial crossover effects are found for large intermediate scales.

I. INTRODUCTION

1. General.

A few years ago the near equilibrium dynamics of a growing crystalline surface was elucidated \[1,2\]. It was found that a roughening transition occurs at \( T = T_r \) between a high-temperature rough phase and a phase with a flat surface for \( T < T_r \). The mobility of the growing surface drops from a finite value to zero as \( T \to T_r^+ \). In the low-temperature phase the growth is "activated" with the formation of higher "islands" on top of the flat surface. Similar behavior occurs in deposition of cubic particles with diffusion on a flat substrate. The transition in this case is as function of the inherent noise due to spatial and temporal fluctuations in the deposition.

In view of the existence of a low temperature (or low noise) flat phase, the question of how disorder in the substrate will change the behavior had to be addressed. We have initiated a comprehensive study of the related questions. A short letter which announces the novel and surprising results was published elsewhere \[3\]. In a previous full paper \[4\] (denoted by I in the following) we have presented the detailed calculation and analysis for the near-equilibrium dynamics. In this regime the averaged growth rate is small. The equation of motion is derived from the Hamiltonian of the system. Detailed-balance and the Fluctuation-Dissipation Theorem (FDT) \[5\] both hold. Yet we found very non-trivial results: super-rough
correlations, temperature-dependent dynamics exponent, and a non-linear relation between the average growth rate and the driving force, were found below a super-roughening transition temperature $T_{sr}$.

In the present paper, the second in the series, we present our detailed calculations and results for the dynamics far-from equilibrium. In this case the equation-of-motion cannot be derived from a Hamiltonian. Detailed-balance and the FDT are both violated. That situation represents a much more serious theoretical challenge since even the very renormalizability of the process is questionable.

The equation of motion we analyze describes the deposition of cubic particles on a random substrate. It will also apply to the surface of a crystal if the solid is very rigid. Since the effects of the disorder in the substrate will be felt only up to a height $h^*$ (which is larger if the solid is more rigid), our theory will apply as long as $h < h^*$.

The width of a growing surface $w$ follows generally the scaling form [6,7]::

$$w(L,t) \sim L^{\alpha} f(t/L^z),$$

(1)

where $t$ is the time and $L$ is the linear size of the system. $\alpha$ is the roughening exponent and $z$ is the dynamic exponent.

The leading difference between near- and far-from equilibrium is due to the lateral growth. If the growth rate is finite the lateral growth add a term proportional to $(\nabla h)^2$ to the equation of motion. This term was derived by Kardar, Parisi and Zhang (KPZ) [8] as the most relevant term in the expansion in term of $\nabla h$.

As the RG analysis shows, the KPZ nonlinearity is marginally relevant in 2+1 dimensions. The asymptotic behavior is controlled by this nonlinearity, which violates the FDT.
Consequently, the roughness of the growing surface is also affected by the nonlinearity. Several simulations showed $\alpha \sim 0.4$ [9,10] which is larger than that of the near-equilibrium case ($\alpha = 0$). Many variants of KPZ-related models have been studied in recent years [6,7,11-22].

The effects of such a term on the growing crystalline surface on a flat substrate was considered by Hwa, Kardar, and Paczuski (HKP) [23]. We review and revise their results in the next section of this Introduction (Ch.I). Ch. II will be devoted to the derivation of the generating functional, and the description of the RG procedure. Ch. III, IV and V are devoted to the outline of the calculations of the different renormalization factors. Extensive details of these calculation are relegated to the appendices. In Ch. VI the recursion relations are derived. Ch. VII is devoted to the analysis of the asymptotic and crossover behaviors which follows from these recursion relations. Our main conclusions are summarized in the last chapter VIII.

2. Review of previous works

The system we shall study in this paper has three important and non-trivial ingredients:

(i) The periodic potential, (ii) The disorder in the substrate, and (iii) The KPZ non-linearity arising from the lateral growth in the presence of a finite driving force (or deposition rate).

Paper (I) discussed the case in which (i)+(ii) are present. The major physical consequences, namely the existence of a super-rough ”glassy phase” for $T < T_{sr}$ with intriguing static and dynamic properties, were discussed there and will not be elaborated further here. Our goal here is to see how the addition of (iii) (i.e. the KPZ term) modifies the behavior.

However, we can also take another point of view and ask how the addition of (ii) (i.e. the disorder in the substrate) is modifying the behavior found in presence of (i)+(iii). The model which discussed the non-equilibrium growth on a flat surface in presence of both the periodic potential and the KPZ non-linearity was analyzed by HKP [23]. The equation of motion which
describes the growing surface under these calculation is:

\[ \bar{\mu}^{-1} \frac{\partial h(\vec{x}, t)}{\partial t} = F + \nu(\nabla^2 h(\vec{x}, t)) + \frac{\lambda}{2}(\nabla h)^2 + \frac{\gamma y_2}{a^2} \sin[\gamma(h(\vec{x}, t))] + \zeta(\vec{x}, t), \tag{2} \]

In this equation \( h(\vec{x}, t) \) is the local height at time \( t \), \( \vec{x} = (x, y) \) are the coordinates in the 2d basal plane; \( \mu \) is the microscopic mobility; \( F \) is the driving force; \( \nu \) is the surface tension; \( \lambda \) is the coefficient of the KPZ non-linearity (which is proportional to \( F \)); \( y_2 \) is the coefficient of the leading harmonics (higher harmonics are irrelevant and the lattice spacing is taken to be unity for simplicity); \( \gamma = \frac{2\pi}{b} \), where \( b \) is the vertical lattice spacing ;\( a \) is the horizontal lattice spacing; and \( \eta(\vec{x}, t) \) is the noise in the deposition (or due to thermal fluctuations) which obeys:

\[ < \eta(\vec{x}, t)\eta(\vec{x}^{\prime}, t^{\prime}) > = 2D\delta(\vec{x} - \vec{x}^{\prime})\delta(t - t^{\prime}) \tag{3} \]

They obtained a set of recursion relations from which they reached several conclusions. They found a critical temperature \( T_c \). For \( T > T_c \), \( y_2 \) decays slowly to zero and the large scale behavior is determined by the KPZ coupling. Approaching \( T_c \) from above the linear-response macroscopic mobility (Namely the ratio \( v/F \) where \( v = <\frac{\partial h}{\partial t}> \) is the averaged growth rate in the limit \( F \to 0 \)) vanishes as \((\ln |T - T_c|)^{-\eta}\).

In the low temperature phase \( T < T_c \) they have found that \( y_2 \) grows indefinitely large and therefore they concluded that the surface is flat.

While reanalyzing their work we have discovered a term that was overlooked in their calculations and which might affect their latter conclusion. Indeed it may be shown that a term of the form \( y_1 \cos(2\pi h) \) is generated under renormalization from the contraction of the terms \( \lambda(\nabla h)^2 \) and \( y_2 \sin(2\pi h) \). This term also feeds back into the renormalization of \( y_2 \).

The recursion relations to lowest order are:
\[
\frac{dy_1}{dl} = (2 - \frac{\pi \mu D}{\nu}) y_1 - \rho \frac{\lambda}{\nu} y_2, 
\]
\[
\frac{dy_2}{dl} = (2 - \frac{\pi \mu D}{\nu}) y_2 + \rho \frac{\lambda}{\nu} y_1,
\]

where \( \rho = \frac{1}{2} [4\pi^2 \ln(4/3)(\frac{\mu D}{\nu})^2 + (\frac{\mu D}{\nu})] \). The two harmonic terms may be combined into a single term:

\[
|y| \sin[2\pi h + \vartheta(l)]
\]

with \( y^2 = y_1^2 + y_2^2 \), and \( \vartheta(l) = tg^{-1}y_1/y_2 \).

If we look at the flow of \( |y| \) and \( \vartheta \) we find that indeed \( |y| \to \infty \) for \( T < T_c \) which would indicate a flat surface if \( \vartheta \) was to remain constant. However, the recursion relations imply that the phase shift angle is rotating with \( l \) like: \( \vartheta(l) = \omega l \) with an angular velocity \( \omega \sim \lambda/\nu \). Since \( l = \ln \tilde{b} \) where \( \tilde{b} \) is the rescaling factor that means an ever changing \( \vartheta \sim \omega \ln \tilde{b} \), with rescaling.

Therefore it is not obvious that the surface is flat. Hopefully, higher-order terms in the recursion relations of \( y_1, y_2 \) and \( \nu \) will help to identify with more confidence the nature of low-temperature phase.

3. Introducing the disorder:

As we explained in I, and as is clear from Fig. 1, the disorder will shift the origin of the periodic potential by a random and uncorrelated amount at every point \( \vec{x} \) in the basal plane \( \vec{x} \).

We take the deposited particle to have a rectangular shape with a square base of linear extent \( a \) and height \( b \) in the growth direction. Then the periodic potential becomes proportional to \( \sin[\frac{2\pi}{b}(h(\vec{x}, t) + d(\vec{x}))] \), where \( d(\vec{x}) \) are the local deviation in the height of the substrate. Let us denote the associate phase \( \Theta(\vec{x}) = 2\pi d(\vec{x})/b \). It obeys:

\[
< e^{-\Theta(\vec{x})} e^{-\Theta(\vec{y})} > = a^2 \delta^2(\vec{x} - \vec{y}).
\]
We have assumed that $d(\vec{x})$ are typically of order $b$ (or larger) and that if correlations exist in $d(\vec{x})$ at different $\vec{x}$ they are at most short range (in which case they fall in the same universality class as the $\delta$ correlated disorder we study here) The equation of motion we need to study is therefore:

$$\tilde{\mu}^{-1} \frac{\partial h(\vec{x}, t)}{\partial t} = F + \nu (\nabla^2 h(\vec{x}, t)) + \lambda a^2 \nabla h(\vec{x}, t) + \gamma a^2 \sin[\gamma h(\vec{x}, t) + \Theta(x)] + \zeta(\vec{x}, t).$$  (8)

II. GENERATING FUNCTIONAL AND BASIC DIAGRAMS.

The Martin, Sigga, and Rose (MSR) [24–26] method is utilized to obtain the generating functional for the correlation and response theory. An auxiliary field $i\tilde{h}(\vec{x}, t)$ is introduced to enforce the equation of motion through an integral representation of the delta function. Then the thermal noise and the disorder are averaged upon to yield the following generating functional:

$$\langle Z_{\Theta}[\tilde{J}, J] \rangle_{\text{disorder}} = \int \mathcal{D}\tilde{h} \mathcal{D}h \exp\left\{ \int d^2x dt [\tilde{D}\tilde{\mu}^2 \tilde{h}^2 - \tilde{h} (\frac{\partial}{\partial t} h - \tilde{\mu} \nu \nabla^2 h - \tilde{\mu} \lambda a^2 \nabla^2 h)] 
+ \frac{\tilde{\mu}^2 \gamma^2 \tilde{g}}{2a^2} \int \int d^2x dt dt' \tilde{h}(\vec{x}, t) \tilde{h}(\vec{x}, t') \cos(\gamma(h(\vec{x}, t) - h(\vec{x}, t'))). \right\}.$$  (9)

a. The Basic diagrams:

As explained in details in I the Gaussian (quadratic) part of the ”action” gives rise to the bare response function

$$\langle \phi(q, \omega) \tilde{\phi}(-q, -\omega) \rangle = \frac{1}{\mu(q^2 + m^2) + i\omega},$$  (10)

which is depicted in Fig. 2.
The bare correlation function is:

\begin{equation}
\langle \phi(\vec{q}, \omega)\phi(-\vec{q}, -\omega) \rangle = \frac{2D\mu^2}{[\mu(q^2 + m^2)]^2 + \omega^2},
\end{equation}

where \(m\), the mass of the field \(\phi\), is introduced to control the infrared divergences in the Feynman integrals. (In the two-dimensional regularization, it is notorious [25] that their infrared and ultra-violet divergences will mingle together without introducing masses for the fields.)

In the momentum and time representation, they are given by [26]:

\begin{equation}
\langle \phi(\vec{q}, t)\tilde{\phi}(-\vec{q}, t') \rangle = \theta(t - t')e^{-\mu(q^2 + m^2)(t-t')},
\end{equation}

\begin{equation}
\langle \phi(\vec{q}, t)\phi(-\vec{q}, t') \rangle = \frac{D\mu}{q^2 + m^2}e^{-\mu(q^2 + m^2)|t-t'|},
\end{equation}

where \(\theta(t) = 1\) for \(t > 0\) and \(\theta(t) = 0\) for \(t < 0\). The bare vertex \(\phi\phi'\cos(\phi - \phi')\) is drawn in Fig. 3.

b. The renormalization group procedure:

We follow the minimal subtraction scheme. The renormalization parameters are related to the bare ones by the so-called \(Z\) factors. Minimal scheme consists in extracting from the diagrams only the divergent parts which are all expressed in terms of functions of the dimensionality of the system.

The bare and renormalized vertex functions can be related by factors of \(Z_\phi, Z_{\tilde{\phi}}\). For instance,

\begin{equation}
\Gamma_{N,L}^R(q, \omega; \varsigma_R, m_R, \kappa) = (Z_{\tilde{\phi}})^\frac{N}{2}(Z_\phi)^\frac{L}{2}\Gamma_{N,L}(q, \omega; \varsigma_0, m_0, a),
\end{equation}

where \(\varsigma_R\) and \(\varsigma_0\) label renormalized parameters \((g, \mu, \cdots)\) and bare parameters \((g_0, \mu_0, \cdots)\), respectively. \(q\) and \(\omega\) are the external momentum and frequency, respectively. In the corresponding vertex function, \(a\) is a short-distance cutoff, and \(\kappa\) is a mass scale. Here \(\Gamma_{N,L}\) stands
for the vertex function with \( L \) external \( \phi \) lines and \( N \) external \( \tilde{\phi} \) lines. The factors, \( Z_\phi \) and \( Z_{\tilde{\phi}} \), are set to remove the divergent parts of the vertex function \( \Gamma \).

The following renormalization constants are defined through the relations between the bare and the renormalized couplings \[25,27–29\]

\[
D_0 = Z_D D, \quad g_0 = Z_g g, \quad \lambda_0 = Z_\lambda \lambda, \quad (15)
\]

\[
m_0^2 \phi^2 = m^2 \phi^2_R, \quad \gamma_0^2 \tilde{\phi}^2 = \gamma^2 \phi^2_R, \quad \phi^2 = Z_\phi \phi^2_R, \quad \tilde{\phi}^2 = (Z_{\tilde{\phi}}) \tilde{\phi}^2_R, \quad (16)
\]

where \( Z_{\tilde{\phi}} = Z_{\phi}^2 \).

In the next chapter we concentrate on the procedures to calculate the \( Z \) factors. The details are relegated to the appendices.

The renormalized perturbation theory even to lower non-trivial orders in \( g \) and \( \lambda \) has to be consistent order by order in \( \gamma \). It also requires the calculations of Feynman diagrams \[25,29\] with up to three loops.

**III. CALCULATIONS OF \( Z_D \) AND \( Z_\mu \)**

The renormalization of \( \mu \) is not affected in the presence of the KPZ nonlinearity since the associated vertex function \( \Gamma_{1,1} \) comes with one external \( \tilde{\phi} \) and one external \( \phi \) and the basic vertex \( \lambda \) contains derivatives on its two \( \phi \) legs. Thus the factor \( Z_\mu \) remains the same as in paper I (equilibrium dynamics), and so does the recursion relation for \( \mu \). On the other hand, the parameter \( D \) suffers additional renormalization of order \( \lambda^2 \). Basically, the renormalization of \( D \) from \( \lambda^2 \) is the same as that of \( D \) encountered in the KPZ model. Here, we still focus on the same vertex function \( \Gamma_{2,0} \) as we did in the previous paper (I).
Obviously, the first nontrivial contribution begins from the second order in \( \lambda \). As shown in Fig. 5, the vertex function is modified by the associated integral with \( \lambda^2 \). The corresponding integral is given by:

\[
\frac{(\mu_0 \lambda_0)^2}{2(2\pi)^2} \int_{-\infty}^{\infty} d^2 \vec{p} \int_{-\infty}^{\infty} d\Omega \frac{1}{2\pi} \frac{(2D_0 \mu_0^2)^2 p^2 \cdot p^2}{[\mu_0^2(p^2 + m^2)^2 + \Omega^2][\mu_0^2(p^2 + m^2)^2 + \Omega^2]}
\]

\[
= \frac{1}{4} \lambda_0^2 (\mu_0^3 D_0^2) \int_{-\infty}^{\infty} d^2 \vec{p} \frac{1}{(2\pi)^2} \frac{1}{(p^2 + m^2)}
\]

\[
= \frac{1}{4} \lambda_0^2 (D_0^2 \mu_0^3) \left[- \frac{1}{4\pi} \ln(cm^2 a^2) \right].
\]  

Therefore, to find out \( Z_g \) is just to calculate the renormalization of \( \Gamma_{2,0} \).

The contributions to \( Z_g \) we need to sum are of order \( g^2 \) (as in paper I) and of order \( \lambda^2 g \).
The combination of $g$ and $\lambda^2$ leads to two types of diagrams, 2-loop and 3-loop diagrams. Some of the associated diagrams are canceled by each other as shown in Fig. 6 and Fig. 7. The other non-vanishing diagrams, including six 2-loop and two 3-loop diagrams, are shown in Fig. 8 – Fig. 15. The detailed calculations of 2-loop integrals are given in Appendix A, where we also explain the cancellation of sub-divergences of some diagrams. The sum of the leading and sub-leading divergences contributing to $Z_g$ are listed in Eq. 22 (see the third and fourth terms).

In the Appendix B, we present the detailed calculation of the leading divergences in the 3-loop diagrams and also show that they do not contain any sub-divergence.

Now, what remains to complete the 3-loop results is just to sum up the leading divergent terms in $\ln(cm^2a^2)$, which essentially contribute to the recursion relations. The $\ln(cm^2a^2)$ contribution of the diagram in Fig. 14 will be

$$
-\frac{1}{2}[(3.1-1) - 2(3.1-2) - 2(3.1-3)]
$$

$$
= \frac{3}{2}I_B + 2I_D + 4I_E + 20\frac{1}{2}\ln\left(\frac{3}{4}\right) - 6(\ln\left(\frac{4}{3}\right))^2 - 4\ln(2) - \frac{5}{2}(\ln 2)^2 + 22\ln\left(\frac{3}{2}\right)
$$

$$
+ 3\Phi(1, 2) - 3\Phi\left(\frac{1}{2}, 2\right) - \Xi\left(\frac{1}{4}, 2\right)
$$

(20)

The contribution from the diagram in Fig. 15 is:

$$
\begin{align*}
&\left[-(3.2-1) + 2(3.2-2) + 2(3.2-3)\right] \\
&= \frac{1}{-\frac{3\epsilon}{2}}\left[\frac{1}{4}I_C + I_D - 2I_E + \frac{1}{2}\ln\left(\frac{3}{2}\right) - \frac{3}{4}\ln\left(\frac{4}{3}\right) - \frac{1}{2}\ln\left(\frac{4}{3}\right)\right].
\end{align*}
$$

(21)

Now we are in a position to calculate $Z_g$. Inserting the above calculation results into the self energy in reference [28], we obtain:

$$
Z_g = 1 - \delta_0 \ln(cm^2a^2) + \frac{\lambda^2\gamma^2(D\mu)^2}{8} \left[\ln(cm^2a^2)\right]^2 - \frac{(-5 + 11\ln\left(\frac{4}{3}\right))}{16}\gamma^2(\mu D)^2\lambda^2
$$

11
\[
\frac{\ln(cm^2a^2)}{(4\pi)^2} + (-90.5)\gamma^4(D\mu)^3\lambda^2 \frac{\ln(cm^2a^2)}{(4\pi)^3}. \tag{22}
\]

By using \(\delta_0 = \delta + \frac{1}{4} \lambda^2(D\mu)^2 \gamma^2 \frac{\ln(cm^2a^2)}{(4\pi)^2}\), we can calculate the recursion relation of \(g\), as will be shown in the next section. As we will see the term \(\ln(cm^2a^2)^2\) is canceled when we derive the Callan-Symanzik (CS) [25,29] equation. Thus the scaling equation is consistent.

V. THE CALCULATION OF \(Z_{\lambda}\)

For the calculation of \(Z_{\lambda}\), one should consider the vertex function \(\Gamma_{1,2}\). For the diagrams in Fig. 16 and Fig. 17, one can write down the associated integrals as:

\[
\text{nla} = \int_{-\infty}^{\infty} d^2\vec{k} \int_{-\infty}^{\infty} d\Omega \frac{2D\mu^2 \vec{p} \cdot (\vec{p} - \vec{k})}{\left\{\mu \left[(\vec{p} - \vec{k})^2 + m^2\right] - i\Omega\right\}\left\{\mu \left[(\vec{p} - \vec{k})^2 + m^2\right] + \Omega^2\right\}} f(\Omega) \tag{23}
\]

and

\[
\text{nlb} = \int_{-\infty}^{\infty} d^2\vec{k} \int_{-\infty}^{\infty} d\Omega \frac{2D\mu^2 \vec{p} \cdot (\vec{p} + \vec{k})}{\left\{\mu \left[(\vec{p} + \vec{k})^2 + m^2\right] - i\Omega\right\}\left\{\mu \left[(\vec{p} + \vec{k})^2 + m^2\right] + \Omega^2\right\}} f(\Omega), \tag{24}
\]

where

\[
f(\Omega) = \gamma^2 \int_{-\infty}^{\infty} dt e^{i\Omega t} [R_0(0,t)e^{\gamma^2 C_0(0,t)}]
= - \int_{-\infty}^{\infty} dt e^{i\Omega t} \frac{1}{\mu^2 D} [R_0(0,t)\gamma^2]e^{\gamma^2 C_0(0,t)}
= \frac{1}{\mu^2 D} (i\Omega) \int_{0}^{\infty} dt e^{i\Omega t} (e^{\gamma^2 C_0(0,t)} - 1). \tag{25}
\]

For simplicity, let \(x = (\vec{p} + \vec{k})^2\), and \(y = (\vec{p} - \vec{k})^2\). The summation of nla in Eq. (23) and nlb in Eq. (24) is proportional to \(\int_{-\infty}^{\infty} d\Omega \frac{2x}{(x^2 + \Omega^2)(y^2 + \Omega^2)} f(\Omega)\).

With the help of Eq. (25), the frequency dependent part can be integrated out first:

\[
\int_{-\infty}^{\infty} d\Omega \frac{(x)(i\Omega)e^{i\Omega t}}{(x^2 + \Omega^2)(y^2 + \Omega^2)} k' \sim \frac{e^{-xt}}{x^2 - y^2} - \frac{e^{-yt}}{x^2 - y^2} \tag{26}
\]
We then have:

\[
\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d^2k \frac{\vec{p} \cdot (\vec{k} - \vec{k})}{4\left(\frac{\vec{p}^2}{4} + k^2 + m^2\right)\mu^2} \{e^{-\frac{\mu}{4\left(\vec{p}^2 + \vec{k}^2 + m^2\right)}t} - e^{-\frac{\mu}{4\left(\vec{p}^2 - \vec{k}^2 + m^2\right)}t}\}
\]

\[
\times \mu\left[\left(\frac{\vec{p}}{2} + \vec{k}\right)^2 + m^2\right]
\]

\[
\vec{p} \cdot (\vec{p} - \vec{k})(-2) e^{-\mu\left(\vec{k}^2 + m^2\right)t} \left(\frac{\vec{p} \cdot \vec{k}}{\vec{p}^2}ight) + \vec{p} \left(\frac{\vec{p}}{2} - \vec{k}\right) e^{-\mu\left(\vec{k}^2 + k^2 + m^2\right)t} \left(\frac{-\vec{p} \cdot \vec{k}}{\vec{p}^2 + k^2 + m^2}\right)(-2)(\vec{p} \cdot \vec{k})t
\]

(27)

In the hydrodynamic (long wave-length) limit, \(\vec{p} \rightarrow 0\), the relevant term in the first term in Eq. (27) can be easily found as:

\[
\int_{-\infty}^{\infty} d^2k(-2) \frac{\vec{p}^2}{2} e^{-\mu\left(\vec{k}^2 + m^2\right)t} = -\vec{p}^2 \frac{1}{2\mu} e^{-\mu m^2t}.
\]

(28)

The relevant term in second term of Eq. (27) is:

\[
p^2(2) \int_{0}^{2\pi} d\theta \cos^2 \theta \int_{0}^{\infty} \frac{k^2 e^{-\mu\left(k^2 + m^2\right)t}}{k^2 + m^2} = p^2 \frac{1}{2\mu} e^{-\mu m^2t}.
\]

(29)

Thus there are no contributions to the renormalization of \(\lambda\) due to their mutual cancellation.

Other possible diagrams arise, but result in no contributions. In Fig. 18. those two diagram will not contribute the renormalization when one impose the long-time prescription (\(\Omega_{\text{ext}} = 0\)).

The diagrams in Fig. 19. do not contribute neither since the interaction \(g\) is local in space and therefore there is no \(p\)-dependent part of the vertex \(\Gamma_{1,2}\).

To sum up, \(\lambda\) suffers no renormalization within the perturbative expansion to order of \(g\), and therefore \(Z_{\lambda} = 1\).

VI. RECURSION RELATIONS

Once the \(Z\)-factors are known to the leading order in \(g\), the recursion relations are obtained via the so-called \(\beta\)-functions [25,26,29]:
\[ \beta_\mu = \kappa \left( \frac{\partial \mu}{\partial \kappa} \right)_b = \mu \kappa \left( \frac{\partial \ln Z_\tilde{\phi}}{\partial \kappa} \right)_b, \] (30)

\[ \beta_D = \kappa \left( \frac{\partial D}{\partial \kappa} \right)_b = -D \kappa \left( \frac{\partial \ln Z_D}{\partial \kappa} \right)_b, \] (31)

\[ \beta_g = \kappa \left( \frac{\partial g}{\partial \kappa} \right)_b = -g \kappa \left( \frac{\partial \ln Z_g}{\partial \kappa} \right)_b \] (32)

\[ \beta_v = \kappa \left( \frac{\partial \bar{\nu}}{\partial \kappa} \right)_b \] (33)

where subscript \( b \) means that all bare parameters are fixed when one performs the differentiations \cite{25, 26, 29} and \( \kappa \) is a mass scale. The renormalization of the couplings may also be related to the same \( \beta \) functions.

The renormalization \( Z \) factors are the ratios between the corresponding renormalized and bare parameters. Therefore it is a standard procedure to extract from their dependence on the momentum scale \( \kappa \) (or the bare mass \( m_0 \)) the flow of the renormalized couplings under rescaling of all length scales by \( \tilde{b} = \exp(l) \). The first step is to compute the so-called beta functions which when subtracted from the naive (engineering) dimension of the couplings yield the flow equations. Following this procedure, rescaling length scales \( x \to \tilde{b} x \), momenta \( k \to \tilde{b}^{-1} k \), time \( t \to \tilde{b}^\gamma t \) and frequencies \( w \to \tilde{b}^{-\gamma} w \), we find the following recursion relations:

\[
\frac{d \nu}{d t} = 0 \quad (34)
\]

\[
\frac{d \bar{\nu}}{d t} = \frac{\pi \gamma^2}{4 \nu (D \mu)^3} g^2 \quad (35)
\]

\[
\frac{d F}{d t} = 2 F + \pi \lambda \quad (36)
\]

\[
\frac{d D}{d t} = \left( \frac{\lambda^2}{8\pi} D \mu + \frac{\gamma^2 \sqrt{cg}}{D \mu} \right) D \quad (37)
\]

\[
\frac{d \mu}{d t} = \left( -\frac{\gamma^2 \sqrt{cg}}{D \mu} \right) \mu \quad (38)
\]
\[
\frac{dg}{dl} = (2 - \frac{D\mu \gamma^2}{2\pi} - \frac{\lambda^2 c'}{\gamma^2})g - \frac{2\pi}{(D\mu)^2}g^2
\]

(39)

\[
\frac{d\lambda}{dl} = 0.
\]

(40)

\(\gamma\) is not renormalized because \(Z_\phi = 1\) and keeps its bare value \(\gamma = 2\pi\). For the same reason \(\nu\) is not renormalized and may be chosen as \(\nu = 1\).

The two constants are \(c = \frac{1}{4}e^{2E} = 0.7931\) where \(E\) is the Euler Constant. and \(c' \sim 180.08\), which is derived from the sum of the terms contributing to \(Z_\phi\) in Eq. (22).

**VII. THE ASYMPTOTIC BEHAVIOR**

In this chapter we proceed with the analysis of the physical implications of the recursion relations. We begin, in the next section by looking at the asymptotic destination of the flows which will yield the physical properties on very large scales of time and space. In the following subsection will analyze the crossover behavior which determines the properties on intermediate scales.

1. The asymptotic behavior

The analysis of the recursion relations may be facilitated by the introduction of a "temperature" like variable (temperature is not well-defined far away from equilibrium where the Einstein relation does not hold). Here we define it by \(T = D\mu\) (it is not the thermodynamic temperature). Its recursion relation is obtained from Eq. 37 and Eq. 38. It obeys:

\[
\frac{dT}{dl} = \frac{\lambda^2}{8\pi^2} T,
\]

(41)

where the critical value \(D\mu = 1/\pi\) is substituted. Since \(\lambda\) and \(\gamma\) are kept constant, this equation may be integrated:
\[ T(l) = T_0 e^{\frac{\lambda^2}{8\pi^2}}. \]  

(42)

We see that no matter how small \( T_0 \) is \( T(l) \) will grow indefinitely with \( l = \ln b \) such that:

\[ T(l) = T_0 \left( \frac{L}{a} \right)^{\lambda^2/8\pi^2}. \]  

(43)

So the effective "temperature" becomes higher on longer length scales. Asymptotically the system is always at a high "temperature". The growth of \( T \) is, however, quite slow. Therefore crossover effects discussed below plays an important role.

What is the effect of high \( T \)? For that we have to look at the flow of \( g \):

\[ \frac{dg(l)}{dl} = [2 - \frac{T(l)}{2\pi} \gamma^2 - \frac{\lambda}{\gamma^2}] g(l) - \frac{\gamma^4 g^2}{8\pi^2}. \]  

(44)

It is clear that if \( T(l) \) grows very large it will cause \( g(l) \) to decay to zero, no matter what are the bare values \( g_0, T_0 \) and \( \lambda \). Once \( g \to 0 \) the asymptotic behavior becomes equivalent to that of the KPZ equation.

We, thus, conclude that asymptotically on very large scales and very long time the scaling properties are these associated with a far-from equilibrium growth without the disorder and the periodic potential. The behavior will be determined by the effect of the lateral growth alone. The KPZ properties in 2 + 1 dimensions (dominated by an inaccessible fixed point) will be the asymptotic ones for the system under consideration.

**The Crossover Behavior** As we have found in the previous section, the "temperature" \( T(l) \) rises with the scale quite slowly. Hence the decay of \( g \) to zero might also be slow. As long as \( g \) is not vanishing the effects of the disorder and the periodic potential are still felt. Hence we should expect a slowing down of the dynamics. This slowing down will be observable on larger scales as well because the mobility obeys the equation:
\[
\frac{\partial \mu}{\partial l} = -\gamma^2 \sqrt{\gamma} g(l) \frac{1}{T(l)} \mu(l),
\]  
(45)

and therefore

\[
\mu(l) = \mu_0 e^{-\gamma^2 \sqrt{\gamma} \int_0^l dl' [g(l')/T(l')]}.
\]  
(46)

The ratio \(\mu(l)/\mu_0\) does depend on the integral:

\[
J(l) = \int_0^l \frac{g(l')}{T(l')} dl'.
\]  
(47)

and clearly \(J(l)\) is sensitive to \(g(l)\) on small scales as well.

To evaluate \(J(l)\) we need to know \(T(l)\) given in Eq. [12] and \(g(l)\) which we calculate next. Given \(T(l)\), \(g(l)\) is found by integration of its recursion relation:

\[
\frac{1}{g(l)} = \frac{1}{g(0)} e^{s(l)} - \frac{\gamma^4}{8\pi^2} \int_0^l dx e^{-s(x)}
\]  
(48)

where

\[
s(x) = \left[\frac{\lambda^2 c'}{\gamma^2} - 2\right]x + T_0 \frac{\gamma^4}{\pi \lambda^2} (e^{\lambda^2 x/2\gamma^2} - 1).
\]  
(49)

Clearly the second term dominates for large \(x\). Hence we see that for large \(l\):

\[
g(l) \sim \exp\{-\exp(Al)\}
\]  
(50)

with \(A \sim \frac{\lambda^2}{8\pi^2}\).

It is easy to see that for large enough \(l\), \(g(l)\) decays to zero faster than exponential. Since \(T(l)\) diverges, the most important contribution to \(J(l)\) comes from small (or at most intermediate) values of \(l\). At large \(l\), \(J(l)\) approaches asymptotically a constant and its dependence on \(l\) becomes much weaker.

This asymptotic value of \(J(l)\) depends mostly on the bare values of the parameters. To summarize, the mobility decays fast on initial small scales and then saturates to an almost constant value on large scales.
VIII. CONCLUSIONS

In this work we have investigated the behavior of a class of growth systems in which three different effects play important roles: Periodicity, disorder and lateral growth. Our model applies to the situations in which all three effects are present in deposition processes or solidification of rigid crystals.

In I we looked at the system near-equilibrium when the lateral growth is negligible. There we found a continuous transition from a rough phase at high temperature into a super-rough and glassy phase for $T < T_g$.

The main conclusion of the present work is that far-from equilibrium the KPZ term prevent this transition. The ultimate asymptotic behavior is dominated by the KPZ non-linearity while the second non-linear term, (obtained upon averaging the periodic potential over the disorder) is irrelevant.

We have seen, however, that while this term is decaying it still affects the behavior on intermediate scales. The effect of the disorder in the periodic potential is to slow the dynamics. In particular the mobility decays from its bare value as:

$$\frac{\mu(l)}{\mu(0)} = \exp[-4\pi^2 \times 1.78 J(l)],$$

where $J(l)$ is given by Eq. [47].

It is clear that the behavior on intermediate scales drastically depends on the bare values of the parameters.

We have also shown that the theory is consistently renormalizable to first order in $\lambda$ and $g$. To obtain the correct renormalization we had to keep diagrams up to and including three non-trivial loops. As far as we are aware of no other calculation have shown the renormalizability
up to this order for a dynamic system for which the fluctuation-dissipation theorem is not satisfied. It is reassuring to see that the renormalization group can be successfully applied to the dynamics far-from equilibrium.

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APPENDIX A: 2-LOOP CALCULATION FOR $Z_g$

As explained in Chapter IV, the calculation of $Z_g$ is based on the evaluation of the vertex function $\Gamma_{2,0}$.

For clarity, we neglect the prefactors and symmetry factors. Here we have 6 2-loop and 2 3-loop Feynman diagrams. The 2-loop diagrams will be denoted by FD 2l#, and 3-loop diagrams will be denoted by FD 3l#, where # stands for the sequel. First we look at the 2-loop diagrams. The basic rules [26,30] for the calculations of these diagrams have been described in the Appendix of our previous paper I. Here we shall simply write down the corresponding integral for each diagram.

We employ the momentum-time representation for correlation and response functions, in terms of which the corresponding integrals will be easily handled.

Feynman diagram (FD) 2l1 is shown in Fig. 8. The corresponding integral over time is given by:

\[
\text{FD 2l1} = \int_0^\infty dt_y \int_0^{t_y} dt_x \frac{[\vec{q} \cdot (\vec{q} - \vec{p})][\vec{p} \cdot \vec{q}]}{(q^2 + m^2)(p^2 + m^2)} e^{-|q^2 + (\vec{p} - \vec{q})^2 + 2m^2|t_x} e^{-(p^2 + m^2)t_y} e^{-(p^2 + m^2)(t_y - t_x)}. \tag{A1}
\]

The integration of the time dependent sectors gives:

\[
\int_0^\infty dt_y \int_0^{t_y} dt_x e^{-|q^2 + (\vec{p} - \vec{q})^2 + 2m^2|t_x} e^{-2(p^2 + m^2)t_y} = \int_0^\infty dt_y e^{-2(p^2 + m^2)t_y} \frac{-1}{[q^2 + (\vec{p} - \vec{q})^2 + m^2 - p^2]} [e^{-|q^2 + (\vec{p} - \vec{q})^2 + m^2|t_y} - 1] = \frac{1}{2(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \tag{A2}
\]

By decomposing $\vec{p} \cdot \vec{q}$ into...
\[
\vec{p} \cdot \vec{q} = -\frac{1}{2} p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2 - 2p^2 - 2q^2 - 3m^2 \]  
(A3)

and \( \vec{q} \cdot (\vec{q} - \vec{p}) \) into

\[
(q^2 - \vec{p} \cdot \vec{q}) = \frac{1}{2} |p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2 - 2p^2 - 3m^2|  
(A4)

we obtain

\[
\frac{(\vec{q} \cdot \vec{p})}{(p^2 + m^2)(q^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} 
\begin{align*}
&= -\frac{1}{2}\frac{1}{(p^2 + m^2)(q^2 + m^2)} - \frac{2}{(p^2 + m^2)(q^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \\
&\quad - \frac{2}{(p^2 + m^2)^2[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} + \text{irrelevant terms}.
\end{align*}
(A5)

Since we impose the minimal subtraction (MS) scheme, the unwanted nonsingular parts (finite parts) will be ignored in all calculations in this paper. In this whole paper, an irrelevant term means a term which does not contribute to the singular part of the integral. We sometimes use equal sign to represent the equality of the singular parts on both sides.

We substitute Eq. (A4) and Eq. (A5) into Eq. (A1), and obtain:

Eq. (A1)

\[
\begin{align*}
&= -\frac{1}{4} \left\{ \frac{(q^2 - \vec{p} \cdot \vec{q})}{(p^2 + m^2)^2(q^2 + m^2)} - \frac{[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2 - 2p^2 - 3m^2]}{(p^2 + m^2)(q^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \\
&\quad - \frac{[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2 - 2p^2 - 3m^2]}{(p^2 + m^2)^2[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \right\} \\
&= -\frac{1}{4}\frac{1}{(p^2 + m^2)^2} + \frac{\vec{p} \cdot \vec{q}}{(p^2 + m^2)(q^2 + m^2)} - \frac{1}{(p^2 + m^2)(q^2 + m^2)} \\
&\quad + \frac{2}{(q^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} - \frac{1}{(p^2 + m^2)^2} \\
&\quad + \frac{2}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \\
&= \left[ \frac{1}{4} \tilde{B} - \tilde{C} \right],
\end{align*}
(A6)

where
\[
\begin{align*}
\hat{B} &= \frac{1}{(p^2 + m^2)(q^2 + m^2)} \quad \text{(A7)} \\
\hat{C} &= \frac{1}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \quad \text{(A8)}
\end{align*}
\]

and the second term in Eq. (A6) is discarded because it vanishes after the integration over the momentum variables.

In the same way, the associated integral for Fig. 9 reads:

\[
\begin{align*}
\text{FD 2l2} &= \int_0^\infty dt_x \int_{-\infty}^{t_x} dt_y [\vec{q} \cdot (\vec{p} - \vec{q})] [\vec{q} \cdot (\vec{p} - \vec{q})] e^{-(p^2 + m^2)t_x} \\
&\quad e^{-[(\vec{p} - \vec{q})^2 + q^2 + 2m^2]} e^{-(p^2 + m^2)(t_x - t_y)}. \quad \text{(A9)}
\end{align*}
\]

To begin with, we integrate over the time variables, and that yields:

\[
\begin{align*}
\int_0^\infty dt_x \int_{-\infty}^{t_x} dt_y &e^{-2(p^2 + m^2)t_x} e^{-(p^2 + m^2)t_y} [\vec{q} \cdot (\vec{p} - \vec{q})] [\vec{q} \cdot (\vec{p} - \vec{q})] \\
&\quad e^{-(\vec{p} - \vec{q})^2 + q^2 + 2m^2} = \frac{1}{(p^2 + m^2)(p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2)}. \quad \text{(A10)}
\end{align*}
\]

By inserting Eq. (A10) into Eq. (A9), we obtain:

\[
\begin{align*}
\frac{[\vec{q} \cdot (\vec{p} - \vec{q})][\vec{q} \cdot (\vec{p} - \vec{q})]}{(q^2 + m^2)(p^2 + m^2)[(\vec{p} - \vec{q})^2 + m^2][p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \\
= \frac{3}{4} \frac{q^2}{(p^2 + m^2)[(\vec{p} - \vec{q})^2 + m^2][p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} \\
= \frac{1}{2} \frac{1}{[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2][q^2 + m^2]} \\
= \frac{3}{4} \hat{A} - \frac{1}{2} \hat{C}, \quad \text{(A11)}
\end{align*}
\]

where

\[
\hat{A} = \frac{q^2}{(p^2 + m^2)[(\vec{p} - \vec{q})^2 + m^2][p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]}. \quad \text{(A12)}
\]

For the Feynman diagram in Fig. 10, the integral with the time variables is written as:
FD 2l3 = \frac{(\vec{q} \cdot \vec{p})(\vec{p} \cdot \vec{p})}{(p^2 + m^2)(q^2 + m^2)} \int_0^\infty dt_x \int_0^{t_x} dt_y e^{-(p^2 + m^2)(t_x - t_y)} e^{[(\vec{p} - \vec{q})^2 + q^2 + 2m^2]t_y}. \hspace{1cm} (A13)

Again, the time dependent sectors are integrated out in advance.

\int_0^\infty dt_x \int_0^{t_x} dt_y e^{-2(p^2 + m^2)t_x} e^{-[q^2 + (\vec{p} - \vec{q})^2 + 2m^2]t_y} e^{-(p^2 + m^2)t_y}
\quad = \frac{1}{2} \frac{1}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]}.
\hspace{1cm} (A14)

With the help of Eq. (A14), Eq. (A13) is simplified into:

\frac{(\vec{p} \cdot \vec{q})(\vec{p} \cdot \vec{q})}{(p^2 + m^2)(q^2 + m^2)} \frac{1}{2} \frac{1}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} = \frac{1}{4} [\tilde{A} \tilde{C} - \tilde{B}]. \hspace{1cm} (A15)

Now we turn to the diagram in Fig. 11. The associated integral is represented by:

FD 2l4 = -\frac{p^2[(\vec{p} - \vec{q}) \cdot \vec{q}]}{(q^2 + m^2)[(\vec{p} - \vec{q})^2 + m^2]} \int_0^\infty dt_x \int_{-\infty}^{t_x} dt_y e^{-(p^2 + m^2)t_x}
\quad e^{-[q^2 + (\vec{p} - \vec{q})^2 + 2m^2]t_y} e^{-(p^2 + m^2)(t_x - t_y)}. \hspace{1cm} (A16)

The time dependent sectors in Fig. 11 is identical to Eq. (A10), so we will not repeat the calculation here.

In the same fashion, the integrand takes the form:

\frac{[q^2 - \vec{p} \cdot \vec{q}]}{(q^2 + m^2)[(\vec{p} - \vec{q})^2 + m^2][p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} = \frac{1}{2} \tilde{B} - \tilde{A}. \hspace{1cm} (A17)

For the diagram in Fig. 12, we have the associated integral as:

FD 2l5
\quad = -\frac{[\vec{q} \cdot (\vec{p} - \vec{q})](p^2)}{(q^2 + m^2)(p^2 + m^2)} \int_0^\infty dt_x \int_0^{t_x} dt_y e^{-[q^2 + (\vec{p} - \vec{q})^2 + 2m^2]t_y} e^{-(p^2 + m^2)(t_x - t_y)} e^{-(p^2 + m^2)t_x}.
\hspace{1cm} (A18)
The time dependent term is integrated out first:

\[
\int_0^\infty dt_x \int_0^{t_x} dt_y e^{-[q^2+(\vec{p} - \vec{q})^2+2m^2]t_y} e^{-(p^2+m^2)(t_x-t_y)} e^{-(p^2+m^2)t_x}
= \frac{1}{2} \frac{1}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]}.
\]

(A19)

Then we obtain:

\[
- \frac{[\vec{q} \cdot (\vec{p} - \vec{q})](p^2)}{(q^2 + m^2)(p^2 + m^2)} \frac{1}{2} \frac{1}{(p^2 + m^2)[p^2 + q^2 + (\vec{p} - \vec{q})^2 + 3m^2]} = \frac{1}{4} (\bar{B} - 2 \bar{C}).
\]

(A20)

We turn to the simplest figure among 2-loop diagrams, Fig. 13, of which the related integral is evaluated as:

\[
\{ \int_{-\infty}^{\infty} \int_{0}^{\infty} dtp^2 R_0(\vec{p}, t) C_0(\vec{p}, t) \}^2 = \frac{1}{4} [\int_{-\infty}^{\infty} d^d \vec{p} \frac{1}{(p^2 + m^2)}]^2.
\]

(A21)

\[
\text{FD 216} = -\frac{1}{4} \int_{-\infty}^{\infty} d^d \vec{p} \frac{1}{(p^2 + m^2)} \int_{-\infty}^{\infty} d^d \vec{p} \frac{1}{(p^2 + m^2)}
= -\frac{1}{4} B
\]

(A22)

Now we go on to perform the integration over the momentum variables. Before the evaluation of the singular parts of integrals, it will be helpful to present some identities, which will play important roles in later calculations and have been frequently employed in these types of calculations. The first one is the Feynman parameterization formula, which reads:

\[
\frac{1}{A^\alpha B^\beta \cdots E^\sigma} = \frac{\Gamma(\alpha + \beta + \gamma + \cdots + \epsilon)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)\cdots\Gamma(\sigma)} \int_0^1 \cdots \int_0^1 dx dy dz \cdots \delta(1 - x - y - z \cdots) \times
\frac{x^{\alpha-1}y^{\beta-1}\cdots z^{\sigma-1}}{(Ax + By + \cdots + Ez)^{\alpha+\beta+\gamma+\cdots+\sigma}}.
\]

(A23)

A set of integral formulas is also valuable and is shown as below:

\[
J_0 = \int_{-\infty}^{\infty} d^d \vec{k} \frac{1}{(k^2 + 2k \cdot \vec{p} + M)^{\alpha}} = \frac{\pi^{d/2}}{\Gamma(\alpha)}(M - p^2)^{d/2 - \alpha}\Gamma(\alpha - \frac{d}{2}).
\]

(A24)
With these formulas, one can evaluate the singular parts of the integrals. Let \( X = A, B \) or \( C \), and \( d = 2 + \epsilon \). The evaluations of \( B, C \) and \( A \) are carried out as below:

\[
B^{1/2} = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{(k^2 + 2k \cdot \vec{p} + M)^\alpha} = -p^\nu J_0. \tag{A25}
\]

With these formulas, one can evaluate the singular parts of the integrals. Let \( X = A, B \) or \( C \), and \( d = 2 + \epsilon \). The evaluations of \( B, C \) and \( A \) are carried out as below:

\[
B^{1/2} = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{(k^2 + 2k \cdot \vec{p} + M)^\alpha} = -p^\nu J_0. \tag{A25}
\]

\[
C = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{d^d \vec{q}}{(p^2 + q^2 + (p^2 - q^2 + 3m^2) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{(p^2 + q^2 + (p^2 - q^2 + 3m^2)\nonumber \text{non-singular terms}} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{\Gamma(1 - d/2) \Gamma(1)} (\frac{1}{4p^2 + \frac{3}{4}m^2})^{-\epsilon/2} \tag{A26}
\]

\[
A = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{d^d \vec{q}}{(p^2 + q^2 + (p^2 - q^2 + 3m^2)} = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{d^d \vec{q}}{(p^2 + q^2 + (p^2 - q^2 + 2m^2)} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{\Gamma(1 - d/2) \Gamma(1)} (\frac{1}{4p^2 + \frac{3}{4}m^2})^{-\epsilon/2} \tag{A27}
\]

\[
A = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{d^d \vec{q}}{(p^2 + q^2 + (p^2 - q^2 + 3m^2)} = \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{d^d \vec{q}}{(p^2 + q^2 + (p^2 - q^2 + 2m^2)} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d^d \vec{P}}{(p^2 + m^2)} \frac{1}{\Gamma(1 - d/2) \Gamma(1)} (\frac{1}{4p^2 + \frac{3}{4}m^2})^{-\epsilon/2} \tag{A28}
\]
The singular parts of integral in Eq. (A28) can be evaluated by the changes of variables, \( x = st, \ y = s(1 - t) \). Let \( A' \) denote the \( x, y \) dependent part in Eq. (A28). We obtain

\[
A' = \int_0^1 \int_0^1 dx dy \frac{1}{[x + y] - [(x + \frac{y}{2})^2]^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds dt \frac{s}{[s - s^2(1 - \frac{t}{2})]^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}[1 - (1 - \frac{t}{2})s]^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}} \left\{ \frac{1}{[1 - (1 - \frac{t}{2}s)]^{1+\epsilon/2}} - 1 \right\} + \int_0^1 ds \frac{1}{s^{1+\epsilon/2}}. \tag{A29}
\]

From the form of Eq. (A29), it is not hard to see that the singular term as \( \epsilon \to 0 \) is the manifestation of the singular behavior of the pole \( s = 0 \) in the integral. To simplify the expressions of equations, the term \( 1 - \frac{t}{2} \) is symbolized by \( \alpha \). Eq. (A29) can be rewritten as:

\[
\text{Eq. (A29)} = \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}} \frac{2\alpha s - \alpha^2 s^2}{(1 - \alpha s)^{2+\epsilon/2}} + \int_0^1 ds \frac{1}{s^{1+\epsilon/2}}. \tag{A30}
\]

The analysis of two terms in Eq. (A30) will be carried out in order. The first term in Eq. (A30) is recast into:

\[
\int_0^1 \int_0^1 ds dt \frac{\alpha}{s^{1+\epsilon/2}(1 - \alpha s)^{2+\epsilon/2}} + \int_0^1 \int_0^1 ds dt \frac{\alpha}{s^{1+\epsilon/2}(1 - \alpha s)^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds dt \frac{\alpha}{(1 - \alpha s)^{2+\epsilon/2}} + \epsilon \int_0^1 \int_0^1 ds dt \frac{[\ln(s)] \alpha}{(1 - \alpha s)^{2+\epsilon/2}}
\]

\[
+ \int_0^1 \int_0^1 ds dt \frac{\alpha}{(1 - \alpha s)^{1+\epsilon/2}} + O(\epsilon). \tag{A31}
\]

The second term in Eq. (A31) is of order \( \epsilon \) and therefore is discarded. The first term in Eq. (A31) is evaluated as:

\[
\int_0^1 \int_0^1 ds dt \frac{\alpha}{(1 - \alpha s)^{2+\epsilon/2}} = \frac{1}{1 + \epsilon/2} \left\{ \int_0^1 dt \frac{1}{[1 - \alpha]^{1+\epsilon/2}} - 1 \right\}
\]

\[
= \frac{1}{1 + \epsilon/2} \left\{ \int_0^1 dt \frac{1}{(t - \frac{\epsilon}{2})^{1+\epsilon/2}} - 1 \right\}, \tag{A32}
\]

where
\[ \int_0^1 dt \frac{1}{(t - \frac{c^2}{4})^{1+\epsilon/2}} = \int_0^1 dt \frac{1}{t^{1+\epsilon/2}} \left[ \frac{1}{(1 - \frac{t}{4})^{1+\epsilon/2}} - 1 \right] + \int_0^1 dt \frac{1}{t^{1+\epsilon/2}} \]

\[ = \int_0^1 dt \frac{1}{t^{1+\epsilon/2}} \left( \frac{1}{4} + (1 - \frac{t}{4}) \ln \left( 1 - \frac{t}{4} \right) + \cdots \right) + \int_0^1 dt \frac{1}{t^{1+\epsilon/2}} \]

\[ = \int_0^1 dt \frac{1}{4t^{1+\epsilon/2}} \left( \frac{1}{(1 - \frac{t}{4})^{1+\epsilon/2}} \right) + O(\epsilon) + \frac{1}{-\epsilon/2} \]

\[ = \frac{1}{4} \int_0^1 dt \frac{1}{(1 - \frac{t}{4})} - \frac{2}{\epsilon} + O(\epsilon) \]

\[ = \ln \left( \frac{4}{3} \right) - \frac{2}{\epsilon} + O(\epsilon). \quad (A33) \]

The third term in Eq. (A31) is:

\[ \int_0^1 \int_0^1 ds dt \frac{\alpha}{(1 - \alpha s)^{1+\epsilon/2}} = \int_0^1 \int_0^1 ds dt \frac{\alpha}{(1 - \alpha s)} + O(\epsilon) \]

\[ = - \int_0^1 dt \ln \left[ 1 - (1 - \frac{t}{2})^2 \right] + O(\epsilon) \]

\[ = 3 \ln \left( \frac{4}{3} \right) + O(\epsilon) \quad (A34) \]

The second term in Eq. (A30) equals to \(- \frac{2}{\epsilon} \).

With the combination of the prefactors in Eq. (A28) and the results obtained above, the singular part of \( A \) is given by:

\[ A = (\pi)^4 \left[ \frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \left( \frac{4}{3} \right) - \frac{2\gamma}{\epsilon} + \frac{1}{\epsilon} \right] + \text{finite terms}, \quad (A35) \]

where \( \gamma \) is the Euler number. The finite part of the subdivergence diagram can be neglected, if the scale equation is well defined. One can always scale it away. In the short distance cutoff, they appears as:

\[ A = \frac{1}{16\pi^2} \left[ \frac{1}{2} (\ln(cm^2a^2))^2 - \ln \left( \frac{4}{3} \right) \ln(cm^2a^2) + \frac{1}{2} \ln(cm^2a^2) \right] \quad (A36) \]

\[ B = \frac{1}{16\pi^2} \left[ (\ln(cm^2a^2))^2 \right] \quad (A37) \]

\[ C = \frac{1}{16\pi^2} \left[ \frac{1}{4} (\ln(cm^2a^2)) + \frac{1}{4} \ln \left( \frac{3}{4} \right) \ln(cm^2a^2) \right] \quad (A38) \]

Now we retrieve the symmetry factors for each 2-loop diagrams (see Table A).
TABLE I. Symmetry factors and integrals of 2-loop diagrams

| Feynman diagrams | Symmetry factors | Integrals          |
|------------------|------------------|--------------------|
| FD.2l1           | 16               | $(\frac{1}{4}B-C)$ |
| FD.2l2           | 8                | $(\frac{3}{4}A-\frac{1}{2}C)$ |
| FD.2l3           | 32               | $(C-\frac{1}{4}B)$ |
| FD.2l4           | 16               | $(\frac{1}{4}B-A)$ |
| FD.2l5           | 32               | $(\frac{1}{4}B-\frac{1}{2}C)$ |
| FD.2l6           | 16               | $\frac{1}{4}B$     |
As one can see from Fig. 8 – Fig. 13, only Fig. 9, Fig. 12, and Fig. 13 contain sub-divergent diagrams. One also can verify this from the results listed in Table A. The leading terms in the diagrams without sub-divergences, such as Fig. 8, Fig. 10, and Fig. 11, are of order \( \ln(cm^2a^2) \). On the other hand, the leading terms of the diagrams with subdivergence as mentioned above are of order \( [\ln(cm^2a^2)]^2 \). Furthermore, the leading divergences of the diagrams in Fig. 12 and Fig. 13 are canceled out by each other. They have the same type of sub-divergences, (see the subdiagrams enclosed by the boxes in their own figures) which are not present in the lower order (1-loop) calculation. Another diagram in Fig. 9 includes the sub-divergent diagram (see the subdiagram enclosed by the box in Fig. 9) which occurs in the 1-loop calculation for \( D \).

As usual, it will be canceled when one calculates the recursion relations, eventhough the \( Z_g \) factors contain some terms like \( [\ln(cm^2a^2)]^2 \). As we will show in the next Appendix, the 3-loop diagrams do not consist of any sub-divergent diagrams. Thus, the subdivergence in the present expansion only occurs in 2-loop diagrams. The cancellation of sub-divergences in Fig. 12 and Fig. 13, and that of Fig. 9 and Fig. 3 ensure the renormalizability of this theory (at least at this order). The sum of leading divergences and sub-leading divergences are summed up to contribute to \( Z_g \) (see the third and fourth terms in Eq. 22).

**APPENDIX B: 3-LOOP CALCULATION FOR \( Z_g \)**

In this Appendix, we shall present the calculations for 3-loop diagrams, which are mentioned in Chapter IV.

The diagram shown in Fig. 14 representing the integral is given by:

\[
\text{FD 311}
\]
\begin{align*}
&= \int_{-\infty}^{\infty} d^d \vec{p} \int_{-\infty}^{\infty} d^d \vec{q} \int_{-\infty}^{\infty} d^d \vec{k} \frac{-[\vec{p} \cdot \vec{q}] [(\vec{p} - \vec{k}) \cdot \vec{k}]}{(q^2 + m^2)(k^2 + m^2) [(\vec{p} - \vec{k})^2 + m^2]} \times \\
&\int_0^\infty dt_y \int_0^t_y dt_x e^{-[(q^2 + m^2) + (p^2 + m^2)] t_y} (t_y - t_x) e^{-(a^2 + m^2) t_x} e^{-[(k^2 + m^2) + (p^2 + m^2)] t_x}.
\end{align*}

(B1)

To simplify the calculation, we denote $a = (q^2 + m^2)$, $b = [(\vec{p} - \vec{q})^2 + m^2]$, $c = (k^2 + m^2)$, $d = [(\vec{p} - \vec{k})^2 + m^2]$, and $e = (p^2 + m^2)$. Along the same line as our preceding calculations of 2-loop diagrams, we integrate out the time dependent term first.

\begin{align*}
&\int_0^\infty dt_y e^{-[(q^2 + m^2) + (p^2 + m^2)] t_y} \int_0^t_y dt_x \\
e^{-[(p^2 + m^2) - (q^2 + m^2)] t_x} &
\int_0^\infty dt_y e^{-(a + b + c + d) t_y} [e^{-(e - a - b) t_y} - 1] \frac{1}{b + c - e} \\
&= \int_0^\infty dt_y e^{-(e + c + d) t_y} e^{-(a + b + c + d) t_y} \frac{1}{b + c - a} \\
&= \frac{1}{(c + d + e)(a + b + c + d)}.
\end{align*}

(B2)

Therefore

\begin{align*}
\text{Eq. (B1)} & \\
&= \int_{-\infty}^{\infty} d^d \vec{p} \int_{-\infty}^{\infty} d^d \vec{q} \int_{-\infty}^{\infty} d^d \vec{k} \frac{-[\vec{p} \cdot \vec{q}] [(\vec{p} - \vec{k}) \cdot \vec{k}]}{(q^2 + m^2)(k^2 + m^2) [(\vec{p} - \vec{k})^2 + m^2]} \times \\
&\frac{1}{(c + d + e)(a + b + c + d)} \\
&= \left(-\frac{1}{2}\right) \int_{-\infty}^{\infty} d^d \vec{p} \int_{-\infty}^{\infty} d^d \vec{q} \int_{-\infty}^{\infty} d^d \vec{k} \frac{\vec{p} \cdot \vec{q}}{(q^2 + m^2)(k^2 + m^2) [(\vec{p} - \vec{k})^2 + m^2]} \\
&\frac{1}{(q^2 + m^2) [(\vec{p} - \vec{k})^2 + m^2]} \frac{1}{(k^2 + m^2) + (p^2 + m^2) + [(\vec{p} - \vec{k})^2 + m^2]} \\
&- \frac{2\vec{p} \cdot \vec{q}}{(q^2 + m^2) [(\vec{p} - \vec{k})^2 + m^2]} \frac{1}{(k^2 + m^2) + (p^2 + m^2) + [(\vec{p} - \vec{k})^2 + m^2]} \\
&\times \frac{1}{(q^2 + m^2) + (k^2 + m^2) + [(\vec{p} - \vec{k})^2 + m^2]} \\
&- \frac{2\vec{p} \cdot \vec{q}}{(q^2 + m^2)(k^2 + m^2) [(p^2 + m^2) + (k^2 + m^2) + (\vec{p} - \vec{k})^2 + m^2]} \\
&\times \frac{1}{(q^2 + m^2) + (k^2 + m^2) + [(\vec{p} - \vec{k})^2 + m^2]} \\
&\times \frac{1}{(q^2 + m^2) + (k^2 + m^2) + (p^2 + m^2) + (\vec{p} - \vec{k})^2 + m^2]} \\
&\times \frac{1}{(c + d + e)(a + b + c + d)}.
\end{align*}
\[
\frac{1}{((q^2 + m^2) + (k^2 + m^2) + [(\vec{p} - \vec{q})^2 + m^2] + [(\vec{p} - \vec{k})^2 + m^2])},
\]
(B3)

where we have used the identities below to simplify the expression of the equation:

\[
\frac{s \cdot \vec{k}}{c + d + e} = \frac{1}{2} \left[ 1 - \frac{2k^2 - 2s^2 - 3m^2}{c + d + e} \right] \frac{1}{a + b + c + d},
\]
(B4)

where \( s = \vec{p} - \vec{k} \). To simplify the calculation, we treat Eq. (B3) as a linear combination of 3 integrals, 3.1-1, 3.1-2, and 3.1-3. Namely,

\[
\text{Eq. (B3)} = \frac{-1}{2} [(3 - 1.1) - 2(3 - 1.2) - 2(3 - 1.3)].
\]
(B5)

The term denoted by 3.1 – 1 yields:

3.1 – 1

\[
\begin{align*}
&= \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{-\infty}^{\infty} d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(q^2 + m^2)(k^2 + m^2)[(\vec{p} - \vec{k})^2 + m^2]} \\
&= \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{-\infty}^{\infty} d^4 \vec{k} \frac{1}{2} \frac{\Gamma(1 + 1 + 1)}{\Gamma(1)\Gamma(1)\Gamma(1)} \int_{0}^{1} \int_{0}^{1} dxdy \int_{-\infty}^{\infty} \vec{p} \cdot \vec{q} \\
&\quad \cdot \frac{1}{k^2 + \vec{k} \cdot \vec{p}(-2x - y) + p^2(x + y) + \vec{p} \cdot \vec{q}(-y) + yq^2 + m^2]^3} \\
&= (\pi)^{d/2} \frac{\Gamma(2 - \epsilon/2)}{2} \frac{\Gamma(3)}{\Gamma(3)} \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{0}^{1} \int_{0}^{1} d\vec{k} \cdot \vec{p} \cdot \vec{q} \\
&\quad \cdot \frac{1}{(q^2 + m^2)y^{2-\epsilon/2}[q^2 - \vec{p} \cdot \vec{q} + \frac{\Delta}{y}p^2 + \frac{1}{y}m^2]^{2-\epsilon/2}} \\
&= (\pi)^{d/2} \frac{\Gamma(2 - \epsilon/2)}{2} \frac{\Gamma(3)}{\Gamma(3)} \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{0}^{1} \int_{0}^{1} d\vec{k} \cdot \vec{p} \cdot \vec{q} \\
&\quad \cdot \frac{1}{y^{2-\epsilon/2}[q^2 - \vec{p} \cdot \vec{q} + \frac{\Delta}{y}p^2 + \frac{1}{y}m^2]^{2-\epsilon/2}} \\
&= (\pi)^{d/2} \frac{\Gamma(2 - \epsilon/2)}{2} \frac{\Gamma(3)}{\Gamma(3)} \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{0}^{1} \int_{0}^{1} d\vec{k} \cdot \vec{p} \cdot \vec{q} \\
&\quad \cdot \frac{1}{y^{2-\epsilon/2}[(\Delta/\gamma - (\frac{m}{\gamma})^2)p^2 + \frac{1}{\gamma}m^2]^{2-\epsilon/2}} \frac{1}{(\pi)^{d/2}} \frac{\Gamma(2 - \epsilon)}{\Gamma(3 - \epsilon/2)} \\
&= \frac{1}{(\pi)^{d/2}} \frac{\Gamma(2 - \epsilon/2)}{\Gamma(3 - \epsilon/2)} \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{0}^{1} \int_{0}^{1} \frac{1}{y^{2-\epsilon/2}[(\Delta/\gamma - (\frac{m}{\gamma})^2)p^2 + \frac{1}{\gamma}m^2]^{2-\epsilon/2}} \frac{1}{(\pi)^{d/2}} \frac{\Gamma(2 - \epsilon)}{\Gamma(3 - \epsilon/2)}
\end{align*}
\]
31
\[
= (\pi)^d \Gamma(2 - \epsilon) \int_0^1 \int_0^1 \int_0^1 dx dy dz \int_{-\infty}^{\infty} d\vec{p} \frac{z^{2-\epsilon/2}}{y^{2-\epsilon/2}(\Box)^{2-\epsilon} [p^2 + \frac{m^2}{y^2}]^{2-\epsilon}}
\]
\[
= \frac{\pi^d \Gamma(2 - \epsilon)^4}{4 \Gamma(2 + \epsilon/2) \Gamma(1 + \epsilon/2) \Gamma(2 - \epsilon)} \int_0^1 \int_0^1 \int_0^1 \int_0^1 dx dy dz \frac{z^{2+\epsilon}}{y^{2+\epsilon}(\Box)^{2+\epsilon/2}},
\]

(B6)

where \( \Delta = (x + y) - (x - \frac{y}{2})^2 \) and \( \Box = \frac{z^2}{y} \Delta - \frac{z^4}{4} \).

The working principles of extracting the singularity of the integral is based on the separation of the singular contributions from different points. All the calculation here follow this scenario. However, one should be able to keep track of those highly nested procedures. In the last line of Eq. (B6), the prefactors before the integral contains a leading singular-pole of order \( \frac{1}{\epsilon} \), and thus we should extract the contribution up to the zero order in \( \epsilon \) from this integral. The extraction of the poles and finite parts of this integral proceeds as below:

\[
\int_0^1 \int_0^1 \int_0^1 dx dy dz \frac{z^{2+\epsilon}}{y^{2+\epsilon}(\Box)^{2+\epsilon/2}}
\]
\[
= \int_0^1 dz \int_0^1 \int_0^1 dx dy \frac{z^{\epsilon/2}}{y^{\epsilon/2}} [\left\{(x + y) - (x + \frac{y}{2})^2\right\} - \frac{z^2}{4} + \frac{z^4}{4}]^{2+\epsilon/2}
\]

(B7)

Rewriting \( x, y \) as \( y = st, x = s(1 - t) \), we inherit the simplified equation:

\[
\text{Eq. (B7)} = \int_0^1 dz \int_0^1 \int_0^1 ds dt \frac{z^{\epsilon/2}}{s^{\epsilon/2} t^{\epsilon/2} [s - s^2(1 - t)^2 - \frac{z^2}{4} st]^{2+\epsilon/2}}
\]
\[
= \int_0^1 dz \int_0^1 \int_0^1 ds dt \frac{z^{\epsilon/2}}{t^{\epsilon/2} s^{1+\epsilon} [1 - s(1 - t)^2 - \frac{z^2}{4}]^{2+\epsilon/2}}.
\]

(B8)

Recast Eq. (B8) into:

\[
\text{Eq. (B8)} = \int_0^1 dz \int_0^1 \int_0^1 ds dt \frac{z^{\epsilon/2} s^{\epsilon/2}}{t^{\epsilon/2} (1 - \frac{z^2}{4})^{2+\epsilon/2} s^{1+\epsilon}(1 - s\bar{\alpha})^{2+\epsilon/2}},
\]

(B9)
where \( \tilde{\alpha} = \frac{(1-t/2)^2}{(1-zt/4)} \). Consider the integration over \( s \) first,

\[
\int_0^1 ds \frac{1}{s^{1+\epsilon}(1-s\tilde{\alpha})^{2+\epsilon/2}}
= \int_0^1 ds \frac{s\tilde{\alpha}[2-s\tilde{\alpha}] + (1-s\tilde{\alpha})^2 \frac{\epsilon}{2} \ln(1-s\tilde{\alpha}) + \cdots}{s^{1+\epsilon}(1-s\tilde{\alpha})^{2+\epsilon/2}} + \int_0^1 ds \frac{1}{s^{1+\epsilon}}
= A_1 + A_2.
\] (B10)

\( A_1 \) can be represented as the sum of \( A_{11} \) and \( A_{12} \).

\[
A_1 = A_{11} + A_{12}
= \int_0^1 ds \frac{\tilde{\alpha}}{s^{1+\epsilon}(1-s\tilde{\alpha})^{2+\epsilon/2}} + \int_0^1 ds \frac{\tilde{\alpha}}{s^{1+\epsilon}(1-s\tilde{\alpha})^{1+\epsilon/2}}
\] (B11)

Furthermore, the term \( A_{11} \) can be decomposed into the following:

\[
A_{11} = A_{111} + A_{112}
= \int_0^1 ds \frac{\tilde{\alpha}}{(1-s\tilde{\alpha})^{2+\epsilon/2}} + \epsilon \int_0^1 ds \frac{\ln s\tilde{\alpha}}{(1-s\tilde{\alpha})^{1+\epsilon/2}}.
\] (B12)

\( A_{112} \) should not concern us because it contains terms of at least first order in \( \epsilon \). We only calculate \( A_{111} \) as:

\[
A_{111} = \left[ \frac{1}{(1+\epsilon/2)(1-s\tilde{\alpha})^{1+\epsilon/2}} \right]_0^1
= \frac{1}{(1+\epsilon/2)} \left[ -1 + \frac{1}{(1-\tilde{\alpha})^{1+\epsilon/2}} \right].
\] (B13)

Substituting \( A_{111} \) into Eq. (B9), we obtain:

\[
\int_0^1 dz \int_0^1 dt \frac{z^{\epsilon/2}}{(1+\epsilon/2)t^{\epsilon/2}((1-\frac{zt}{4})^{2+\epsilon/2})} \left[ \frac{1}{(1-\tilde{\alpha})^{1+\epsilon/2}} - 1 \right]
= \int_0^1 dz \int_0^1 dt \frac{z^{\epsilon/2}}{(1+\epsilon/2)t^{\epsilon/2}((1-\frac{zt}{4})^{2+\epsilon/2})} \left[ 1 - \frac{(1-t/2)^2}{(1-zt/4)} \right]^{1+\epsilon/2/2}
\]

\[
- \int_0^1 dz \int_0^1 dt \frac{z^{\epsilon/2}}{t^{\epsilon/2}(1+\epsilon/2)((1-\frac{zt}{4})^{2+\epsilon/2})}
= B_{1111} - B_{1112}.
\] (B14)
where we separate the integrand into two parts:

\[
B_{1111} = \frac{1}{1 + \epsilon/2} \int_0^1 \int_0^1 dt \, dz \frac{z^{\epsilon/2}}{t^{1+\epsilon/2}(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})^{1+\epsilon/2}}
\]

\[
= \frac{1}{1 + \epsilon/2} \int_0^1 dz \, z^{\epsilon/2} \int_0^1 dt \, \frac{z^{\epsilon/2}}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})^{1+\epsilon/2}} \tag{B15}
\]

The integral over the variable \( t \) in Eq. (B15) is given by:

\[
\int_0^1 dt \, \frac{z^{\epsilon/2}}{t^{1+\epsilon}(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})^{1+\epsilon/2}} = \int_0^1 dt \, \frac{t^{1+\epsilon}(1 - \frac{z}{4})(1 - \frac{t}{4})^{\epsilon/2}}{z^{\epsilon/2}} \]

\[
= \int_0^1 dt \, \frac{\frac{1}{4} z \times z^{\epsilon/2}}{t(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})^{1+\epsilon/2}} \quad \text{as } 0 < t < 1
\]

\[
= C_{1111} + C_{1112} \tag{B16}
\]

The first term \( C_{1111} \) is substituted into Eq. (B15), and the contribution is denoted by \( B_{11111} \), which reads:

\[
B_{11111} = \int_0^1 \int_0^1 dt \, dz \, \frac{\frac{1}{4} z}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})} + \cdots \text{irrelevant terms}
\]

\[
= 4 \left[ 9 \ln \left( \frac{3}{4} \right) + 4 \ln 2 - \frac{1}{2} (\ln 2)^2 + \Phi(1, 2) - \Phi \left( \frac{1}{2}, 2 \right) - \Xi \left( \frac{1}{4}, 2 \right) \right] \tag{B17}
\]

The evaluation of the integral is quite straightforward although tedious. One can find the basic integrals in Appendix C, whose compositions will be used to represent those complex integrals encountered in Eq. (B15).

Here we take the evaluation of \( B_{11111} \) as an example and set aside the rest of similar calculations.

\[
B_{11111} = \int_0^1 \int_0^1 dt \, dz \, \frac{\frac{1}{4} z}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})} - \frac{t}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})}
\]

\[
= \int_0^1 \int_0^1 dt \, dz \, \left[ \frac{\frac{1}{4} z}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})} - \frac{1}{(1 - \frac{z}{4})(1 - \frac{z}{4} - \frac{t}{4})} \right]
\]

\[
= -4 \int_0^1 dt \left[ \frac{(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} \ln \left( \frac{3}{4} - \frac{t}{4} \right) - \frac{\ln(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} (1 - \frac{t}{4}) - \frac{\ln(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} \right],
\]

\[
= -4 \int_0^1 dt \left[ \frac{(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} \ln \left( \frac{3}{4} - \frac{t}{4} \right) - \frac{\ln(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} (1 - \frac{t}{4}) - \frac{\ln(1 - \frac{t}{4})}{(1 - \frac{t}{2})^2} \right].
\]

(B18)
where $\tilde{\Delta} = 1 - t - t^2$. Let $u = 1 - t^2$. Eq. (B18) turns into:

\[
\text{Eq. (B18)} = -2 \times 1/2 \int_1^{1/2} du \left[ \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u^2} + \frac{\ln\left(\frac{1}{4} + \frac{u}{2}\right)}{u} - \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u^2} - \frac{\ln\left(\frac{1}{4} + \frac{u}{2}\right)}{u} - \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u^2} + \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{1 - u} \right].
\] (B19)

Each term in Eq. (B18) can be easily represented in terms of the basic integrals listed in Appendix C. For example,

\[
\int_1^{1/2} \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u^2} = 2 \ln(2) + 3 \ln\left(\frac{3}{4}\right)
\] (B20)

\[
\int_1^{1/2} du \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u} = \left[ 2 \ln^2(2) - \frac{1}{2} \ln^2(2) + \Phi\left(\frac{1}{2}, 2\right) - \Phi(1, 2) \right]
\] (B21)

\[
\int_1^{1/2} du \left[ \frac{\ln(1 + u)}{u^2} - \frac{\ln(2)}{u^2} \right] = -3 \ln\left(\frac{3}{2}\right) + 2 \ln(2)
\] (B22)

\[
\int_1^{1/2} du \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{u^2} = \ln^2(2) + \Phi\left(\frac{1}{2}, 2\right) - \Phi(1, 2)
\] (B23)

\[
\int_1^{1/2} du \frac{\ln\left(\frac{1}{2} + \frac{u}{2}\right)}{1 - u} = \Xi\left(\frac{1}{4}, 2\right)
\] (B24)

\[
\int_1^{1/2} du \left(\frac{1 + 2u}{u}\right) = -\frac{1}{2} \ln^2(2) - \Phi(1, 2) + \Phi\left(\frac{1}{2}, 2\right)
\] (B25)

\[
\int_1^{1/2} du \left(\frac{1 + u}{u}\right) = \Phi\left(\frac{1}{2}, 2\right) - \Phi(1, 2)
\] (B26)

where $\Phi$ and $\Xi$ are defined in the Appendix C.

Now we turn to the calculation of $B_{11112}$, which is defined as $B_{11112} = \int_0^1 dt C_{1112}$.

\[
B_{11112} = \int_0^1 dz \int_0^1 dt \frac{z^{\epsilon/2}}{(1 - \frac{z}{4} - \frac{t}{2})^{1+\epsilon/2}}
\]

\[
= \int_0^1 dz \frac{z^{\epsilon/2}}{(1 - \frac{z}{4})^{1+\epsilon/2}} \int_0^1 dt \frac{1}{t^{1+\epsilon/2}} \left[ 1 - \frac{t}{4(1-z/4)} \right]^{1+\epsilon/2},
\] (B28)

where the $t$-dependent integral can be separated into:

\[
\int_0^1 dt \frac{1}{t^{1+\epsilon}(1 - \beta t)^{1+\epsilon/2}}
\]
\[ = \int_0^1 dt \frac{1}{t^{1+\epsilon}} \left[ 1 - (1 - \tilde{\beta} t)^{1+\epsilon/2} \right] + \int_0^1 dt \frac{1}{t^{1+\epsilon}} \]

\[ = \int_0^1 dt \frac{1}{t^{1+\epsilon}} \frac{\tilde{\beta}}{(1 - \tilde{\beta})^{1+\epsilon/2}} + O(\epsilon) + \int_0^1 dt \frac{1}{t^{1+\epsilon}} \]

\[ = D_1 + D_2 + O(\epsilon) \quad (B29) \]

with \( \tilde{\beta} \) being \( \frac{1}{4-z} \). The term \( D_1 \) in Eq. (B29) is

\[ D_1 = \int_0^1 dt \frac{\tilde{\beta}}{(1 - \tilde{\beta} t)} = -\ln(1 - \tilde{\beta}). \quad (B30) \]

After substituting the above equation into Eq. (B28), one has:

\[ \int_0^1 dz (-1) \frac{z^{\epsilon/2} \ln(1 - \frac{1}{4-z})}{(1 - \frac{z}{4})^{1+\epsilon/2}} = \int_0^1 dz \frac{\ln(4 - z) - \ln(3 - z)}{(1 - \frac{z}{4})} + O(\epsilon) \]

\[ = I_A - I_B + 4 \ln^2(\frac{4}{3}). \quad (B31) \]

Inserting \( D_2 \) in Eq. (B29) into Eq. (B28), we have:

\[ = \int_0^1 dt \frac{1}{t^{1+\epsilon}} \times \int_0^1 dz \frac{z^{\epsilon/2}}{(1 - \frac{z}{4})^{1+\epsilon/2}} \]

\[ = -\frac{1}{\epsilon} \times \left[ \int_0^1 dz \frac{1}{(1 - \frac{z}{4})} - \frac{\epsilon}{2} \int_0^1 dz \frac{\ln(1 - \frac{z}{4})}{(1 - \frac{z}{4})} + \int_0^1 dz \frac{\ln(z)}{(1 - \frac{z}{4})} \right] + O(\epsilon) \]

\[ = (-\frac{1}{\epsilon})[4 \ln(\frac{4}{3}) - \frac{\epsilon}{2} I_A + \frac{\epsilon}{2} I_C] \quad (B32) \]

Now we move on to calculate the contribution of \( A_2 \) in Eq. (B10). By inserting \( A_2 \) into Eq. (B3), one has:

\[ \int_0^1 \int_0^1 \int_0^1 dz dt ds \frac{z^{\epsilon/2}}{s^{1+\epsilon}(1 - \frac{z}{4})^{2+\epsilon/2}} \]

\[ = \int_0^1 \int_0^1 \int_0^1 dz dt ds \frac{1}{s^{1+\epsilon}(1 - \frac{z}{4})^2} + \int_0^1 \int_0^1 dz dt ds \]

\[ \frac{1}{s^{1+\epsilon}(1 - \frac{z}{4})^2} \left[ (-\frac{\epsilon}{2}) \ln(1 - \frac{z}{4}) + \frac{\epsilon}{2} \ln(z) - \frac{\epsilon}{2} \ln(t) \right] + \cdots \]

\[ = 4 \ln(\frac{4}{3}) - \frac{1}{\epsilon} + \int_0^1 \int_0^1 dz dt (\frac{\epsilon}{2}) \]

\[ \left[ -\ln(1 - \frac{z}{4}) + \ln(z) - \ln(t) \right] \quad (B33) \]
Here we decompose the second term in Eq. (B33) into three parts, $P_1$, $P_2$, and $P_3$.

\[ P_1 = \int_0^1 \int_0^1 dzdt \frac{\ln(1 - \frac{zt}{4})}{(1 - \frac{zt}{4})^2} = \int_0^1 dt \left( -\frac{4}{t} \ln(1 - \frac{zt}{4}) - \int_0^1 dzdt \frac{(1 - \frac{zt}{4})}{(1 - \frac{zt}{4})^2} \right) = \int_0^1 dt \left( -\frac{4}{t} \ln(1 - \frac{zt}{4}) - \frac{4\ln(4)}{3} \right) \]

\[ P_2 = \int_0^1 \int_0^1 dtdz \frac{\ln(z)}{(1 - \frac{zt}{4})^2} = \int_0^1 \ln(z) \frac{1}{(1 - \frac{zt}{4})} = I_C \]

\[ P_3 = \int_0^1 \int_0^1 dtdz \frac{\ln(t)}{(1 - \frac{zt}{4})^2} = I_C. \]

Now we go back to finish the calculation of $A_{12}$ in Eq. (B11).

\[ A_{12} = \int_0^1 \int_0^1 dzt \frac{\ln(1 - (\frac{1}{t} + \frac{1}{z}))}{(1 + \frac{1}{z})^2} = -\int_0^1 \int_0^1 dzt \ln(1 - \frac{zt}{4}) + \int_0^1 \int_0^1 dzt \ln(1 - \frac{zt}{4}) \]

\[ = [4I_D + I_A + 4\ln(\frac{4}{3})] + I_C + \int_0^1 dtdz \frac{4\ln(1 - \frac{zt}{4})}{(1 - \frac{zt}{4})^2} = [4I_D + I_A + 4\ln(\frac{4}{3})] + I_C + L_1 - M_1 + M_2, \]

where $L_1$, $M_1$ and $M_2$ are analyzed below.

\[ L_1 = \int_0^1 dt \frac{4}{t} \ln \left( \frac{\frac{1}{4} - \frac{t}{4}}{\frac{1}{4} - \frac{t}{4}} \right) - \ln(1 - \frac{t}{4}) = \int_0^1 dt \left\{ \frac{4}{t} \ln \left( \frac{\frac{3}{4} - \frac{t}{4}}{\frac{3}{4} - \frac{t}{4}} \right) - \frac{4}{t} \ln(1 - \frac{t}{4}) \right\} = \int_0^1 dt \frac{4}{t} \ln \left( \frac{\frac{3}{4} - \frac{t}{4}}{\frac{3}{4} - \frac{t}{4}} \right) + [-4\ln(\frac{3}{4}) + I_B - 4I_D]. \]
The first term in Eq. (B38) is divergent. As we will show later, it will be canceled by a term in $M_2$.

\[
M_1 = 8 \int_0^1 dt \frac{1}{t} \left( \frac{1}{\Delta} + 1 \right) \ln \left( 1 - \frac{t}{4} \right) - 8 \int_0^1 dt \frac{1}{t} \ln \left( 1 - \frac{t}{4} \right)
= 8 \int_1^{1/2} du \left[ \frac{\ln \left( \frac{1}{2} + \frac{u}{2} \right)}{u^2} + \frac{\ln \left( \frac{1}{2} + \frac{u}{2} \right)}{u} \right] - 8 \int_0^1 dt \frac{1}{t} \ln \left( 1 - \frac{t}{4} \right)
= 8 \left[ 2 \ln(2) - 3 \ln \left( \frac{3}{2} \right) + \ln \left( \frac{3}{2} \right)^2 + \Phi \left( \frac{1}{2}, 2 \right) - \Phi(1, 2) \right] - 8I_D,
\]

where $\Delta = (1 - \frac{t}{2})^2$ and $u = 1 - \frac{t}{2}$.

\[
M_2 = \int_0^1 dt \frac{4}{t} \ln \left( \frac{3}{4} - \frac{1}{4}t \right) \left( \frac{1}{\Delta} + 1 \right) - \int_0^1 dt \frac{4}{t} \ln \left( \frac{3}{4} - \frac{t}{4} \right)
= 4 \int_1^{1/2} du \frac{1 + u}{u^2} \ln \left( \frac{1}{4} + \frac{u}{2} \right) - \int_0^1 dt \frac{4}{t} \ln \left( \frac{3}{4} - \frac{t}{4} \right)
= 4 \left[ 2 \ln(2) + 3 \ln \left( \frac{3}{4} \right) + 2 \ln^2(2) - \frac{1}{2} \ln^2(2) + \Phi \left( \frac{1}{2}, 2 \right) - \Phi(1, 2) \right]
- \int_0^1 dt \frac{4}{t} \ln \left( \frac{3}{4} - \frac{t}{4} \right),
\]

where, as mentioned above, the first term in Eq. (B38) is canceled by the last term in Eq. (B40).

The term $B_{1112}$ in Eq. (B14) is calculated below.

\[
B_{1112} = \frac{1}{1 + \epsilon/2} \int_0^1 dzdt \frac{z^{t/2}}{t^{1/2}(1 - \frac{zt}{4})^{2+\epsilon/2}}
= \int_0^1 dzdt \frac{1}{(1 - \frac{zt}{4})^2} + O(\epsilon)
= 4 \ln \left( \frac{4}{3} \right) + O(\epsilon).
\]

The second term $3.1 - 2$ in Eq. (B3) is calculated as follows:

\[
3.1 - 2
= \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{-\infty}^{\infty} d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(q^2 + m^2)[(\vec{p} - \vec{k})^2 + m^2][(k^2 + m^2) + [k^2 + (\vec{p} - \vec{k})^2 + 2m^2]]}
\]

\[
\frac{1}{\{[q^2 + (\vec{p} - \vec{q})^2 + 2m^2] + [k^2 + (\vec{p} - \vec{k})^2 + 2m^2]\}}
\]

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\[ \Gamma(3) \int_{0}^{1} \int_{0}^{1} dxdy \frac{y}{(\Delta^2 + \frac{m^2}{2})} \left( \frac{1}{\Delta^3 - \epsilon} \right)^{1/2} \]

\[ \pi \frac{\Gamma(1 - \epsilon)}{8} \int_{0}^{1} \int_{0}^{1} dxdy \frac{1}{(\Delta^2 - \epsilon)} \left( \frac{1}{\Delta^3} \right)^{1/2} \]

\[ \frac{\pi^{d/2}}{8} \frac{\Gamma(-\frac{3}{2}\epsilon)}{\Delta^{3+\epsilon}} \left( \frac{y}{\Delta^2} \right)^{1+\epsilon/2} \]

where \( \Delta = (x + y)^2 - \frac{1}{4}(x + y)^2, \quad \Box = \frac{y}{\Delta} - \frac{y}{\Delta} \).

The integral in Eq. (B42) is analyzed below.

\[ \int_{0}^{1} \int_{0}^{1} dxdy \frac{y}{(\Delta + \frac{y}{2})^{1+\epsilon/2}} \frac{1}{(\Delta - \frac{y}{2})^{1+\epsilon/2}} \]

\[ = \int_{0}^{1} \int_{0}^{1} dsdt \frac{s \times st}{s(1 - \frac{s}{4})s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \]

\[ = \int_{0}^{1} \int_{0}^{1} dsdt \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \left( \frac{1}{1 - \frac{s}{4}} - 1 \right) \]

\[ + \int_{0}^{1} \int_{0}^{1} dsdt \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \]

\[ = S + R_1 + R_2 + O(\epsilon) \]

where the transformations of \( y = st \) and \( x = s(1 - t) \) have been used, and the calculations for \( S, R_1 \) and \( R_2 \) will be presented successively.
\[
\int_0^1 ds \frac{1}{1 - \frac{s}{4}} \left[ - \ln(\frac{3}{4} - \frac{s}{4}) + \ln(1 - \frac{s}{4}) \right] = [I_A + 4 \ln^2(\frac{3}{4}) - I_B] \tag{B44}
\]

The reason for the rise of \( R_1 \) and \( R_2 \) can be revealed by the following decomposition:

\[
R_1 + R_2 = \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 ds \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2} - 1}
\]

\[
+ \int_0^1 \int_0^1 ds dt \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 ds \frac{1}{s^\epsilon}, \tag{B45}
\]

where

\[
R_1 = \int_0^1 \int_0^1 ds \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 ds \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2}}
\]

\[
= \int_0^1 \int_0^1 ds \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 ds \frac{1}{s^\epsilon(1 - \frac{s}{4})^{1+\epsilon/2} + O(\epsilon)}
\]

\[
= \int_0^1 \int_0^1 ds \frac{\ln(4 - t) - \ln(3 - t)}{(1 - \frac{s}{4})} = 4 \ln^2(\frac{4}{3}) + I_A - I_B, \tag{B46}
\]

and

\[
R_2 = \int_0^1 \int_0^1 ds \frac{1}{s^{1+\epsilon/2}(1 - \frac{s}{4})^{1+\epsilon/2}} \int_0^1 ds \frac{1}{s^{1+\epsilon}}
\]

\[
= \int_0^1 \int_0^1 ds \frac{1}{s^{1+\epsilon}} \ln(\frac{1}{1 - \frac{s}{4}}) + \int_0^1 \int_0^1 ds \frac{\ln(t) + 1/2 \ln(1 - \frac{4}{t})}{(1 - \frac{4}{t})} + O(\epsilon)
\]

\[
= 4 \ln(\frac{4}{3})(\frac{1}{1 - \epsilon}) + I_C + \frac{1}{2} I_A. \tag{B47}
\]

The term \( 3 - 1.3 \) will be equal to \( 3 - 2.2 \), which will be shown in next two subsections.

For the other 3-loop diagram, we can still employ the same technique. To begin with, we write down the corresponding integral for Fig. 15.

\[
\text{FD 3l2} = \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{-\infty}^{\infty} d^4 \vec{k} \frac{(-\vec{p} \cdot \vec{k}) \times (-\vec{p} \cdot \vec{q})}{(k^2 + m^2)(p^2 + m^2)(q^2 + m^2)} \int_0^\infty dt_x \int_{-\infty}^{t_x} dt_y \nonumber
\]

\[
e^{-[k^2 + m^2 + (\vec{p} - \vec{k})^2 + m^2](t_x - t_y)} e^{-[q^2 + m^2 + (\vec{q} - \vec{p})^2 + m^2]t_x} e^{-(p^2 + m^2)|t_y|.} \tag{B48}
\]
Again, the time dependent part of the integral in Eq. (B48) can be integrated first. For convenience, let \( a = (p^2 + m^2), b = (\vec{p} - \vec{q})^2 + m^2, c = (k^2 + m^2), d = (\vec{p} - \vec{k})^2 + m^2 \), and \( e = (p^2 + m^2) \). Then we have:

\[
\int_0^\infty dt_x e^{-[c+d+a+b]t_x} \int_0^{t_x} dt_y e^{(c+d-e)t_y} + \int_{-\infty}^0 dt_y e^{(c+d+e)t_y} = \int_0^\infty dt_x e^{-(a+b+c+d)tx} \left[ \frac{1}{c + d - e} (e^{(c+d-e)t_x} - 1) + \frac{1}{c + d + e} \right] = \frac{2}{(c + d + e)(a + b + c + d)}. \tag{B49}
\]

In the same spirit of the calculation as performed in the previous case, we separate the integrals into several pieces, each of which only contains an isolated pole, and then extract the corresponding singular parts.

One can start with decomposing \( \vec{p} \cdot \vec{k} \) into:

\[
\vec{p} \cdot \vec{k} = \frac{1}{2} [-(p^2 + k^2 + (\vec{p} - \vec{k})^2 + 3m^2) + 2(p^2 + m^2) + 2(k^2 + m^2)] \tag{B50}
\]

Then one can rewrite Eq. (B48) as \(-[3.2 - 1] + 2(3.2 - 2) + 2(3.2 - 3)\), where

\[
3.2 - 1 = \int_{-\infty}^\infty d^4 \vec{p} \int_{-\infty}^\infty d^4 \vec{q} \int_{-\infty}^\infty d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(k^2 + m^2)(q^2 + m^2)(p^2 + m^2)} \left[ \frac{1}{k^2 + q^2 + (\vec{p} - \vec{q})^2 + (\vec{p} - \vec{k})^2 + 4m^2} \right] \tag{B51}
\]

\[
3.2 - 2 = \int_{-\infty}^\infty d^4 \vec{p} \int_{-\infty}^\infty d^4 \vec{q} \int_{-\infty}^\infty d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(k^2 + m^2)(q^2 + m^2)(p^2 + k^2 + (\vec{p} - \vec{k})^2 + 3m^2)} \left[ \frac{1}{k^2 + q^2 + (\vec{p} - \vec{q})^2 + (\vec{p} - \vec{k})^2 + 4m^2} \right] \tag{B52}
\]

\[
3.2 - 3 = \int_{-\infty}^\infty d^4 \vec{p} \int_{-\infty}^\infty d^4 \vec{q} \int_{-\infty}^\infty d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(p^2 + m^2)(q^2 + m^2)(p^2 + k^2 + (\vec{p} - \vec{k})^2 + 3m^2)} \left[ \frac{1}{k^2 + q^2 + (\vec{p} - \vec{q})^2 + (\vec{p} - \vec{k})^2 + 4m^2} \right]. \tag{B53}
\]

Again, we have three different types of the integrals to handle.

The calculation for the term 3.2 - 1 is shown as below.
\[ 3.2 - 1 \]

\[
\begin{align*}
&= \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_{-\infty}^{\infty} d^4 \vec{k} \frac{\vec{p} \cdot \vec{q}}{(k^2 + m^2)(q^2 + m^2)(p^2 + m^2)} \times \\
&\quad \frac{1}{k^2 + q^2 + (\vec{q} - \vec{p})^2 + (\vec{p} - \vec{k})^2 + 4m^2} \\
&= \frac{1}{2} \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \frac{\vec{p} \cdot \vec{q}}{(p^2 + m^2)(q^2 + m^2)} \int_0^1 dx \int_{-\infty}^{\infty} d^4 \vec{k} \\
&\quad \frac{1}{[k^2 - x\vec{p} \cdot \vec{k} + xq^2 - x\vec{p} \cdot \vec{q} + m^2]^2} \\
&= \frac{1}{2} \int_{-\infty}^{\infty} d^4 \vec{q} \frac{\vec{p} \cdot \vec{q}}{(p^2 + m^2)(q^2 + m^2)} \\
&\quad \int_0^1 dx \frac{\Gamma(1-\epsilon/2)}{\Gamma(2)} (x - \frac{x^2}{4})^1 \frac{(x - \frac{x^2}{4})p^2 - x\vec{p} \cdot \vec{q} + xq^2 + m^2)^{1-\epsilon/2} \\
&= \frac{\pi^{d/2}}{2} \Gamma(1-\epsilon/2) \Gamma(2-\epsilon/2) \int_{-\infty}^{\infty} d^4 \vec{p} \int_{-\infty}^{\infty} d^4 \vec{q} \int_0^1 dx dy \frac{1}{(q^2 + m^2)\Delta^{1-\epsilon/2}} \\
&\quad \frac{\vec{p} \cdot \vec{q}}{(p^2 - x\vec{q}^2 + \frac{x}{\Delta}q^2 + \frac{y}{\Delta}m^2)^{2-\epsilon/2}} \\
&= \frac{\pi^{d}}{2} \Gamma(1-\epsilon) \int_0^1 dx dy \int_{-\infty}^{\infty} d^4 \vec{q} \frac{y^{-\epsilon/2}}{(q^2 + m^2)\Delta^{1-\epsilon/2}} \\
&\quad \frac{\vec{q} \cdot (\frac{x}{\Delta} + \frac{y}{\Delta})}{\{(\frac{x}{\Delta} \Delta) - (\frac{y}{\Delta})^2\}^2 + \frac{y}{\Delta}m^2} \Delta^{1-\epsilon} \\
&= \frac{\pi^{d}}{2} \Gamma(1-\epsilon) \int_0^1 dx dy \int_{-\infty}^{\infty} d^4 \vec{q} \frac{(xy)y^{-\epsilon/2}}{2\Delta^{1-\epsilon} \Delta^{1-\epsilon/2} (q^2 + \frac{y}{\Delta}m^2)^{1-\epsilon}} \\
&= \frac{\pi^{3d/2}}{4} \Gamma(-\frac{3\epsilon}{2}) (m^2)^{3\epsilon/2} \int_0^1 dx dy \frac{xy^{1+\epsilon/2}}{\Delta^{1+\epsilon/2} \Delta^{2+\epsilon}}, \quad \text{(B54)}
\end{align*}
\]

where \( \Delta = x - \frac{x^2}{4} \) and \( \Box = \frac{xy}{x - \frac{x^2}{4}} \). The integration over \( x \) and \( y \) in Eq. (B54) is carried out as below.

\[
\begin{align*}
&\int_0^1 dx dy \frac{xy^{1+\epsilon}}{(xy)^{1+\epsilon/2}(\Delta - \frac{xy}{4})^{1+\epsilon/2}} \\
&= \int_0^1 dx dy \frac{y^{\epsilon/2}}{x^{1+\epsilon}(1 - \frac{x}{4} - \frac{y}{4})^{1+\epsilon/2}}
\end{align*}
\]
\begin{equation}
\int_0^1 dy \frac{y^{\epsilon/2}}{(1 - \frac{y}{4})^{1+\epsilon/2}} \int_0^1 dx \frac{1}{x^{1+\epsilon}(1 - \frac{x}{4y})^{1+\epsilon/2}}.
\end{equation}

Rewrite Eq. (B55) as:

\begin{equation}
\text{Eq. (B55)} = \int_0^1 dy \frac{y^{\epsilon/2}}{(1 - \frac{y}{4})^{1+\epsilon/2}} \left[ \int_0^1 dx \frac{1}{x^{1+\epsilon}} \left( 1 - \frac{x}{4y} \right)^{1+\epsilon/2} - 1 \right] + \int_0^1 \frac{1}{x^{1+\epsilon}}
\end{equation}

\begin{equation}
= \int_0^1 dy \frac{y^{\epsilon/2}}{(1 - \frac{y}{4})^{1+\epsilon/2}} \left[ - \ln(1 - \frac{x}{4y})^{1+\epsilon/2} + \frac{1}{\epsilon} + O(\epsilon) \right].
\end{equation}

One also expands the terms in Eq. (B55) up to the zero order in \( \epsilon \). The finite part of the first term in Eq. (B56) is found to be:

\begin{equation}
\int_0^1 dy \frac{1}{(1 - \frac{y}{4})} \left[ \ln(4 - y) - \ln(3 - y) \right] = 4 \left( \frac{\ln 4}{3} \right)^2 + I_A - I_B.
\end{equation}

The second part is extracted in the same manner:

\begin{equation}
\int_0^1 dy \frac{y^{\epsilon/2}}{(1 - \frac{x}{4y})^{1+\epsilon/2}} = \left\{ \int_0^1 dy \frac{1}{(1 - \frac{y}{4})} + \frac{\epsilon}{2} \left[ \frac{\ln(y)}{(1 - \frac{y}{4})} - \frac{\ln(1 - \frac{y}{4})}{1 - \frac{y}{4}} \right] \right\} + O(\epsilon)
\end{equation}

\begin{equation}
= [4 \ln(\frac{4}{3}) + \frac{\epsilon}{2} (I_C - I_A)].
\end{equation}

Now we turn to the term 3.2 - 2.

3.2-2

\begin{equation}
= \int_{-\infty}^{\infty} d^4\bar{p} \int_{-\infty}^{\infty} d^4\bar{q} \int_{-\infty}^{\infty} d^4\hat{k} \int_0^1 \int_0^1 \int_{x+y<1} dxdy \frac{1}{4(q^2 + m^2)} \frac{\bar{p} \cdot \bar{q}}{(q^2 + m^2) \Gamma(2 - \epsilon/2)} \Gamma(3)
\end{equation}

\begin{equation}
\frac{\Delta^{2-\epsilon/2} [\bar{p}^2 + \frac{y}{\Delta} q^2 - \frac{y}{\Delta} \bar{p} \cdot \bar{q} + \frac{m^2}{\Delta}]}{\Gamma(1) \Gamma(2 - \epsilon/2) \Gamma(1 - \epsilon) \Gamma(3)}
\end{equation}

\begin{equation}
\bar{q} \cdot (\frac{y}{\Delta^2} \bar{q}) \Delta^{2-\epsilon/2}[\frac{y}{\Delta^2} - (\frac{y}{\Delta^2})^2 q^2 + \frac{m^2}{\Delta}]
\end{equation}

\begin{equation}
= \int_{-\infty}^{\infty} d^4\bar{q} \int_0^1 \int_0^1 \int_{x+y<1} dxdy \frac{1}{4(q^2 + m^2)} \frac{\bar{q} \cdot \bar{q}}{\Gamma(2 - \epsilon/2) \Gamma(3)} \Gamma(2 - \epsilon/2) \Gamma(1 - \epsilon)
\end{equation}

\begin{equation}
\frac{\Delta^{2-\epsilon/2} [\frac{y}{\Delta} - (\frac{y}{\Delta^2})^2 q^2 + \frac{m^2}{\Delta}]}{1-\epsilon}
\end{equation}
\[
\pi^d \Gamma(1 - \epsilon) \frac{q^2}{8} \int_{-\infty}^{\infty} dq^\alpha \Delta^3 - \epsilon^2 \Box^{1 - \epsilon} \left( q^2 + \frac{m^2}{\Box} \right)^{1 - \epsilon} \left( q^2 + m^2 \right) \\
(\pi)^{d/2} \frac{\Gamma(2 + \epsilon/2) \Gamma(-3\epsilon/2)}{\Gamma(1 + \epsilon/2) \Gamma(1 - \epsilon)}
\]
\[= \pi^{3d/2} \Gamma(-3\epsilon/2) \int_0^1 \int_0^1 dx dy \frac{\Gamma(2 + \epsilon/2)}{8 \Gamma(1 + \epsilon/2) \Delta^{3 + \epsilon} \Box^{1 + \epsilon/2}}. \tag{B59}
\]

Here \( \Delta = (x + y) - \frac{1}{4}(x + y)^2 \) and \( \Box = \frac{y^2}{\Delta} - \left( \frac{y}{\Delta} \right)^2. \)

The integration over \( x \) and \( y \) in Eq. (B59) is performed as below.

\[
\int_0^1 \int_0^1 dx dy \frac{y}{x} \frac{(\frac{y}{\Delta} - (\frac{y}{\Delta})^2)^{1 + \epsilon/2} \Delta^{3 + \epsilon}}{s \times st} = \int_0^1 \int_0^1 ds dt \frac{y}{st} (st)^{1 + \epsilon/2} \Delta (\Delta - \frac{y}{4})^{1 + \epsilon/2} \]
\[= \int_0^1 \int_0^1 ds dt \frac{1}{s^{1 + \epsilon/2} (1 - \frac{x}{4}) (1 - \frac{x}{4} - \frac{x}{4})^{1 + \epsilon/2}}. \tag{B60}
\]

The \( s \)-dependent part in Eq. (B60) can be separated into:

\[
\int_0^1 ds \frac{1}{s^{1 + \epsilon}(1 - \frac{4}{1 - t})^{1 + \epsilon/2}} \left[ \frac{1}{1 - \frac{4}{1 - t}} - 1 \right] + \int_0^1 ds \frac{1}{s^{1 + \epsilon}(1 - \frac{4}{1 - t})^{1 + \epsilon/2}} \]
\[= \int_0^1 ds \frac{1}{s^{1 + \epsilon}} \left( 1 - \frac{4}{1 - t} \right) \left( 1 - \frac{4}{1 - t} \right) + \int_0^1 ds \frac{1}{s^{1 + \epsilon}} \left( 1 - \frac{4}{1 - t} \right)^{1 + \epsilon/2} \]
\[+ \int_0^1 ds \frac{1}{s^{1 + \epsilon}} \ln(1 - \frac{t}{4}) - \ln(1 - \frac{t}{3})]. \tag{B61}
\]

The first term in Eq. (B61) can be integrated out as:

\[
\int_0^1 ds \frac{1}{s^{1 + \epsilon}} \left( 1 - \frac{4}{1 - t} \right) = \ln(1 - \frac{t}{4}) - \ln(1 - \frac{t}{3}). \tag{B62}
\]

Inserting it into Eq. (B60), we inherit:

\[
\int_0^1 dt \frac{4 - t}{t} \left[ \ln(1 - \frac{t}{4}) - \ln(1 - \frac{t}{3}) \right] = 4(I_D - I_E) + O(\epsilon) \tag{B63}
\]

Again, inserting the second term in Eq. (B61) into Eq. (B60), one has:

\[
\int_0^1 dt \frac{1}{1 - \frac{t}{4}} \left[ - \ln(1 - \frac{1}{1 - \frac{4}{1 - t}}) \right] = I_A - I_B + 4 \ln^2(\frac{4}{3}). \tag{B64}
\]
After substituting the 3nd term in Eq. (B61) into Eq. (B60), one obtains:

\[
\int_0^1 ds \frac{1}{s^{1+\epsilon}} \cdot \int_0^1 dt \left[ \frac{1}{1 - \frac{t}{4}} - \frac{\epsilon [\ln(t) + \ln(1 - \frac{t}{4})]}{1 - \frac{t}{4}} \right] = \left( \frac{1}{-\epsilon} \right) \left[ 4 \ln\left( \frac{4}{3} \right) + \frac{\epsilon}{2} (-I_C - I_A) \right].
\]  

(B65)

Here we evaluate the integral of 3.2 – 3.

\[3.2 - 3\]

\[
= \frac{1}{4} \int_{-\infty}^{\infty} d^d\tilde{p} \int_{-\infty}^{\infty} d^d\tilde{q} \int_{-\infty}^{\infty} d^d\tilde{k} \frac{\tilde{p} \cdot \tilde{q}}{(p^2 + m^2)(q^2 + m^2)} \int_0^1 dx \frac{1}{(k^2 - \tilde{p} \cdot \tilde{k} + p^2 + xq^2 - x\tilde{p} \cdot \tilde{q} + m^2)^2} \\
= \frac{1}{4} \int_{-\infty}^{\infty} d^d\tilde{p} \int_{-\infty}^{\infty} d^d\tilde{q} \int_{-\infty}^{\infty} d^d\tilde{q} \frac{\tilde{p} \cdot \tilde{q}}{(p^2 + m^2)(q^2 + m^2)} \int_0^1 dx \pi^{d/2} \frac{\Gamma(1 - \epsilon/2)}{\Gamma(2)} \\
= \frac{1}{4} \pi^{d/2} \frac{\Gamma(1 - \epsilon/2)}{(\frac{3}{4})^{1-\epsilon/2}} \int_0^1 dx \int_{-\infty}^{\infty} d^d\tilde{p} \int_{-\infty}^{\infty} d^d\tilde{q} \frac{\tilde{p} \cdot \tilde{q}}{(p^2 + m^2)(q^2 + m^2)} y^{-\epsilon/2} \\
= \frac{1}{4} \pi^{d/2} \frac{\Gamma(2 - \epsilon/2)}{(\frac{3}{4})^{1-\epsilon/2}} \int_0^1 dx \int_{-\infty}^{\infty} d^d\tilde{q} \frac{\tilde{q} \cdot (\frac{2}{3}xy\tilde{q})}{(q^2 + m^2)^{1-\epsilon}} \\
= \frac{1}{4} \pi^{d/2} \frac{\Gamma(1 - \epsilon)}{(\frac{3}{4})^{1-\epsilon/2}} \int_0^1 dx \int_{-\infty}^{\infty} d^d\tilde{q} \frac{y^{-\epsilon/2}(2xy\tilde{q})}{(q^2 + m^2)^{1-\epsilon}} \\
= \frac{1}{4} \pi^{d/2} \frac{\Gamma(1 - \epsilon)}{(\frac{3}{4})^{1-\epsilon/2}} \int_0^1 dx \int_{-\infty}^{\infty} d^d\tilde{q} \frac{y^{-\epsilon/2}xy}{(\frac{4}{3}xy)^{1-\epsilon}}. 
\]  

(B66)

Here \(\Box = \frac{4}{3}xy - (\frac{2}{3}xy)^2\). The integration over \(x\) and \(y\) in Eq. (B66) is obtained as:

\[
\int_0^1 \int_0^1 dx dy \frac{(4y)^{3\epsilon/2}}{(3\Box)^{\frac{3\epsilon}{2}}} xy^{1+\epsilon} \\
\int_0^1 \int_0^1 dx dy \frac{4\epsilon/2}{(3\Box)^{\frac{3\epsilon}{2}}} xy^{1+\epsilon}.
\]  

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Finally, we summarize the results of two 3-loop calculations. One should observe that there are not any sub-divergences in 3-loop diagrams and therefore no such a term as $\ln(cm^2a^2)^2$ exists. In the following summation of both 3-loop calculations results, we demonstrate this observation by explicit calculations. By collecting all previous results, the contributions of FD.3l1 and FD.3l2 in $\varepsilon^{-2}$ are given by:

\[
\begin{align*}
\text{FD.3l1} : & \quad - \frac{1}{2} \left[ \frac{1}{4} \Gamma(-\frac{3\varepsilon}{2}) \ln \left( -\frac{8}{\varepsilon} \right) - 2 \frac{1}{8} \Gamma(-\frac{3\varepsilon}{2}) \ln \left( -\frac{4}{\varepsilon} \right) - 
\right. \\
& \quad \left. 2 \frac{1}{8} \Gamma(-\frac{3\varepsilon}{2}) 4 \ln \left( \frac{4}{3} \right) \frac{-1}{\varepsilon} \right] = 0 \\
\text{FD.3l2} : & \quad - \frac{1}{4} \left[ \Gamma(-\frac{3\varepsilon}{2}) 4 \ln \left( \frac{4}{3} \right) \frac{-1}{\varepsilon} + 2 \frac{1}{8} \Gamma(-\frac{3\varepsilon}{2}) 4 \ln \left( \frac{4}{3} \right) \frac{-1}{\varepsilon} \right] = 0
\end{align*}
\]

These confirm our observation mentioned above. The consequent results of the leading divergences which contributes to $Z_g$ are used in Eq. [22].

\text{APPENDIX C: BASIC INTEGRALS}

We define $\Xi(x, s) = \sum_{n=1}^{\infty} \frac{x^n}{n^s}$, and $\Phi(x, s) = \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^s}$.

\[
\begin{align*}
I_A. \quad & \int_0^1 du \frac{\ln(1-\frac{u}{4})}{(1-\frac{u}{4})} = -2 \ln^2 \left( \frac{3}{4} \right) \\
I_B. \quad & \int_0^1 du \frac{\ln(1-\frac{u}{4})}{u} = 2 \ln^2 \left( \frac{1}{4} \right) - 4 [\Xi \left( \frac{1}{3} \right) 2 - \Xi \left( \frac{1}{4} \right) 2] \\
I_C. \quad & \int_0^1 du \frac{\ln u}{(1-\frac{u}{4})} = -4 \Xi \left( \frac{1}{4} , 2 \right) \\
I_D. \quad & \int_0^1 du \frac{\ln(1-\frac{u}{4})}{u} = -\Xi \left( \frac{1}{4} , 2 \right) \\
I_E. \quad & \int_0^1 du \frac{\ln(1-\frac{u}{4})}{u} = -\Xi \left( \frac{1}{3} , 2 \right)
\end{align*}
\]
\[ I_F. \int_0^1 \int_0^1 dudv \frac{\ln(1 - \frac{uv}{u})}{u} = -[\Xi(\frac{1}{4}, 2) + 3 \ln(\frac{1}{3}) - 1] \]

\[ I_G. \int_0^1 du \ln(u) \ln(1 - \frac{u}{4}) = 2 - 3 \ln(\frac{1}{3}) - 4 \Xi(\frac{1}{4}, 2) \]

\[ I_H. \int_0^1 \int_0^1 dudv \frac{\ln(1 - \frac{uv}{u})}{u} = 4 \ln(\frac{4}{3}) - \frac{5}{2} \ln(\frac{3}{2}) + \frac{1}{2} \]

\[ I_J. \int_0^1 du \ln(1 - \frac{u}{4}) = 3 \ln(\frac{4}{3}) - 1 \]
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Figures Caption

Fig. 1
A two-dimensional cut (along a lattice plane perpendicular to the disordered substrate) of the three-dimensional system.

Fig. 2
The Feynman diagram for the correlation function and response function.

Fig. 3
The Feynman diagram representing $\tilde{\phi}(\vec{x}, t)\tilde{\phi}(\vec{x}, t')\cos(\phi(\vec{x}, t) - \phi(\vec{x}, t'))$.

Fig. 4
The basic diagram for $\lambda$.

Fig. 5
The Feynman diagram for $\Gamma_{2,0}$ up to order $\lambda^2$.

Fig. 6
Two mutually canceled two-loop Feynman diagrams.

Fig. 7 Two mutually canceled two-loop Feynman diagrams.

Fig. 8
A 2-loop Feynman diagram: FD. 2l1

Fig. 9
A 2-loop Feynman diagram: FD. 2l2

Fig. 10 A 2-loop Feynman diagram: FD. 2l3

Fig. 11
A 2-loop Feynman diagram: FD. 2l4
Fig. 12
A 2-loop Feynman diagram: FD. 2l5

Fig. 13
A 2-loop Feynman diagram: FD. 2l6

Fig. 14
A 3-loop Feynman diagram: FD. 3l1

Fig. 15
A 3-loop Feynman diagram: FD. 3l2

Fig. 16
A Feynman diagram: FD. nla

Fig. 17
A Feynman diagram: FD. nlb

Fig. 18
Two Feynman diagrams contributing to $\Gamma_{1,2}$.

Fig. 19
The other two Feynman diagrams contributing to $\Gamma_{1,2}$. 
Fig. 1
Fig. 2
Fig. 3
Fig. 12

Fig. 13
Fig. 14

Fig. 15
Fig. 16

Fig. 17
Fig. 18
