Roughness of a Tilted Anharmonic String at Depinning

T. Goodman and S. Teitel

aDepartment of Electrical Engineering, Bucknell University, Lewisburg, PA 17837
bDepartment of Physics and Astronomy, University of Rochester, Rochester, NY 14627

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We consider the discretized model of a driven string with an anharmonic elastic energy, in a two dimensional random potential, as introduced by Rosso and Krauth. Using finite size scaling, we numerically compute the roughness of the string in a uniform applied force at the critical depinning threshold. By considering a string with a net average tilt, we demonstrate that the anharmonic elastic energy crosses the model over to the quenched KPZ universality class, in agreement with recent theoretical predictions.

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Recently, Rosso and Krauth (RK) reported new simulations of roughening at the depinning threshold of a driven one dimensional string in a two dimensional (2d) random potential. Introducing higher order anharmonic terms to a quadratic elastic energy for the string, RK found a value for the roughness exponent $\zeta = 0.63$, in contrast to the values $\zeta \sim 1.2$ found in earlier simulations, and recently theoretically, using a purely quadratic energy. RK noted that the value $\zeta = 0.63$ had previously been found in some cellular automata models for depinning.

In a subsequent work RK (with Hartmann) noted that that when an average tilt is applied to the string, the anharmonic terms break the rotational invariance present in the quadratic model, thus suggesting that the anharmonic terms might cross the model into the quenched KPZ universality class, previously introduced by Kardar to explain the anisotropic depinning observed in the automata models. Simulations of a continuum model with the quenched KPZ term found $\zeta \approx 0.61 \pm 0.06$, consistent with the automata models and with the anharmonic model of RK. Most recently, a functional renormalization group calculation by Le Doussal and Wiese argued that the quenched KPZ term can indeed be generated, not only by the anisotropic disorder considered by Kardar, but also by the anharmonic elastic energy terms introduced by RK.

A key prediction of Kardar’s for anisotropic depinning is that the roughness exponent for a tilted interface will differ from that of an untitled one; for a tilted string in 2d, he predicted the exact value of $\zeta_{\text{tilt}} = 1/2$. In this report, by computing the $\zeta_{\text{tilt}}$ of RK’s model for the first time, we offer a direct numerical demonstration that their model does indeed belong in the quenched KPZ universality class.

Our model is the same as that of RK. We take for the energy of the string,

$$E[h_i] = \sum_{i=0}^{L-1} \{ V(i, h_i) - f h_i + E_{\text{el}}(h_{i+1} - h_i) \} \quad (1)$$

where $h_i$, is the integer height of the string at position $i = 0, \ldots, L$ on a discretized lattice, $V(i, j)$ is an uncorrelated random Gaussian potential with zero average and unit variance, $f$ is a uniform external driving force, and $E_{\text{el}}$ is the elastic energy of deforming the string. $V(i, j)$ is taken periodic on an $L \times L$ system size. In their work, RK used periodic boundary conditions. Here, to model a tilted interface with net slope $s$, we use boundary conditions $h_L = h_0 + sL$. Defining the height relative to a uniformly tilted line, $\delta h_i \equiv h_i - si$, so that $\delta h_L = \delta h_0$, we can rewrite Eq. (1) in terms of the $\delta h_i$ and recover the same model as RK except that the elastic term now has the form,

$$\sum_{i=0}^{L-1} E_{\text{el}}(\delta h_{i+1} - \delta h_i + s) \quad (2)$$

We now carry out simulations of Eq. (1), using the elastic term of Eq. (2). Using the same algorithm as RK, we consider slopes $s = 0$ and $s = 1$ for one of the specific cases studied in RK,

$$E_{\text{el}}(\Delta) = \Delta^4/16 \quad (3)$$

We compute the interface roughness $W$ for a system of length $L$,

$$W^2 \equiv \frac{1}{L} \sum_{i=0}^{L-1} \left[ (\delta h_i - \overline{\delta h})^2 \right] \sim L^{2\zeta} \quad (4)$$

where $\overline{\delta h}$ is the average height at site $i$ of the critical string at depinning, $\overline{\delta h}$ is the average relative height of the critical string, and $\zeta$ is the roughness exponent. We also compute the disorder average of the critical force, $f_c$.

Our results for string roughness $W^2$ vs. $L$, averaged over 500 disorder samples (for $L = 2048$ we use only 200 samples), are plotted in Fig. 1. For $s = 0$, our numerical values agree with those in Ref. 1. The straight lines on the log-log plot indicate the power law relation, $W^2 \sim L^{2\zeta}$, and the difference in slopes indicate clearly different
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