Neutral color-spin locking phase in neutron stars

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Received: date / Revised version: date

Abstract. We present results for the spin-1 color-spin locking phase (CSL) using a NJL-type model in two flavor quark matter for compact stars applications. The CSL condensate is flavor symmetric and therefore charge and color neutrality can easily be satisfied. We find small energy gaps $\Delta \gtrsim 1$ MeV, which make the CSL matter composition and the EoS not very different from the normal quark matter phase. We keep finite quark masses in our calculations and obtain no gapless modes that could have strong consequences in the late cooling of neutron stars. Finally, we show that the region of the phase diagram relevant for neutron star cores, when asymmetric flavor pairing is suppressed, could be covered by the CSL phase.

PACS. 24.85.+p Quarks, gluons and QCD in nuclei and nuclear processes – 26.60.+c Nuclear matter aspects of neutron stars

1 Introduction

The investigation of cold dense quark matter has received special attention due to the possible consequences for compact stars [1]. In particular, color superconducting quark matter phases enforcing color and charge neutrality has been widely studied [2]. Model calculations have shown that the intermediate density region of the neutral QCD phase diagram, where the quark chemical potential is not sufficiently large to have the strange quark deconfined, might be dominated by $u, d$ quarks [3]. If this is the case, two-flavor quark matter phases may occupy a large volume in the core of compact stars [4].

On the other hand, local charge neutrality disfavor the occurrence of phases with large gaps where quarks with different flavor pair in a spin-0 condensate, such as the 2SC phase [5], in which quarks pair in e.g. $(u, d_g)$ and $(d, u_g)$ diquarks leaving the $u_b, d_b$ unpaired. Therefore, while the occurrence of neutral 2SC pure phase is rather model dependent and might be unlikely for moderate coupling constants [6], other phases such e.g. with spin-1 pairings [7], could be relevant for neutron star phenomenology. Spécially, because these condensates with small energy gaps ($\Delta \simeq 1$ MeV) do not influence the equation of state (EoS) but they strongly affect the transport and thermal properties of quark matter [8] and consequently the neutron star cooling. The unpaired quarks in the core lead to rapid cooling via the direct Urca process, incompatible with the observations. Phases that present no gapless modes prevent the direct Urca to work uncontrolled suppressing the neutrino emissivities and could explain the observed data [9].

We consider in this work the color-antitriplet single-flavor spin-1 pairing in the Color-Spin-Locking (CSL) phase [7] and compare it with the 2SC phase. Our results for the CSL phase are obtained using the Nambu Jona-Lasinio (NJL) model [10] keeping finite quark masses and thus obtaining no gapless modes. Since these condensates are color neutral and single-flavor, neutron beta-equilibrated CSL quark matter is obtained easily. We present also the thermal behavior of the neutral CSL phase showing the phase diagram. Finally, we stress important features of the CSL phase that could give a consistent picture of a compact star with a superconducting quark matter core.

2 Flavor symmetric (CSL) vs flavor asymmetric (2SC) pairing

We consider two flavor ($f = u, d$) quark matter, assuming that the strange quark mass is large enough to appear only at higher densities. In the NJL-type [10] models the quarks interact locally by a 4-point vertex effective force. The NJL-Lagrangian $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{qq} + \mathcal{L}_{\bar{q}q}$ contains a free part $\mathcal{L}_0$, a quark-antiquark channel $\mathcal{L}_{qq}$ that causes spontaneous chiral symmetry breaking with condensates $\sigma = \langle \bar{q}q \rangle$ and a diquark channel $\mathcal{L}_{\bar{q}q}$ that describes color superconductivity with condensate $\Delta$. The constituent quark mass is

\begin{align*}
\text{We consider also nonlocal extensions of the NJL model: the quark interactions act over a certain range introducing momentum dependent form factors in the the current-current interaction terms. The inclusion of high momenta states beyond the NJL-cutoff reduces the diquark condensates and lowers the density for the chiral phase transition (for a discussion see [11]).}
\end{align*}
defined as $M = m - 4G\sigma$ and the energy gaps $\Delta$ are listed in Tab.1 for the phases: spin-0 2SC and spin-1 CSL.

| phase      | condensate $\Delta$ | diquarks | free          |
|------------|----------------------|----------|---------------|
| 2SC (spin-0) | $2G_1(\psi^z C \tau_5 \tau_2 \lambda_2 \psi)$ | $u_\mu, d_\mu, u_\nu, d_\nu$ | $u_b, d_b$ |
| CSL (spin-1) | $4H_v(\psi^z C \tau_5 \lambda_2 \psi)$ | $u_\mu u_\nu, u_\nu u_\mu, u_b u_b$ | $u \rightarrow d$ |

Linearizing $\mathcal{L}_{\text{eff}}$ in the presence of the condensates the thermodynamical potential $\Omega(T, \{\mu_i\})$ can be derived. For the quark sector, in the 2SC case we obtain

$$\Omega_q = 4G\sigma^2 + \frac{|\Delta|^2}{4G_1} - 2\sum_{\Delta, J = \nu}^3 \left[ E^{\pm}_{\nu} + 2T \ln \left(1 + e^{-E^\pm_{\nu}/T}\right) \right]$$

and for CSL, since the flavors decouple, $\Omega_q = \sum_f \Omega_f(T, \mu_f)$

$$\Omega_f = 4G\sigma^2 + \sum_{k = 1}^6 \left[ E^{\pm}_{k} + 2T \ln \left(1 + e^{-E^\pm_{k}/T}\right) \right],$$

where $E^{\pm}_{\nu}$ and $E^{\pm}_{k}$ are the corresponding dispersion relations and the integration is in the momentum space.

The lepton sector is modeled as an ideal electron gas with chemical potential $\mu_e$. At the mean-field level, the stationary points $\delta\Omega/\delta\Delta = \delta\Omega/\delta M = 0$ define a set of gap equations for $\Delta$ and $M$. The stable solution is the one which corresponds to the absolute minimum of $\Omega$.

The parameters (quark mass $m = 5$ MeV, the coupling $G = 4.66$ GeV$^{-2}$ and the cut-off $A = 664$ MeV) are chosen to fit the pion mass and the decay constant to a vacuum constituent quark mass equals 300 MeV.

**3 Results and Discussion**

We solve self-consistently the gap equations for the dynamical mass $M$ and the energy gap $\Delta$ for beta-equilibrated neutral matter: our solutions satisfy that the total quark electric charge $\mu_Q = -\mu_e$ and that the total charge density $n_Q - n_e = \frac{2}{3}n_u - \frac{1}{3}n_d - n_e$ vanishes. In Fig. 1 we show $M$ and $\Delta$ as a function of the baryon chemical potential $\mu_B$ for the flavor asymmetric spin-0 2SC phase on the left and for the flavor asymmetric spin-1 CSL phase on the right. While, for a fixed $\mu_B$, $M$ presents a similar magnitude for both phases, $\Delta$ differs by order of magnitudes: $\simeq 100$ MeV for 2SC, $\simeq 1$ MeV for CSL. Moreover, the strength of the coupling constant determines whether the 2SC phase occurs: for strong coupling $G_1/G = 1$, it presents large gaps $\Delta \simeq 150 - 200$ MeV that decrease as soon as the coupling does $\Delta \simeq 50 - 100$ MeV for the usual Fierz value $G_1/G = 3/4$ and vanishing for values lower than a critical one. The crucial point is that the coupling should be large enough to pair quarks of different flavors overcoming a large Fermi sea mismatch ($\simeq 60 - 80$ MeV). As a consequence, asymmetric flavor pairing might not be favorable unless the coupling is very strong (at least larger than $G_1/G = 3/4$). Therefore, flavor symmetric pairing becomes important when $G_1/G$ is not large enough to have 2SC superconductivity.

On the other hand, the occurrence of the flavor symmetric CSL pairing is not affected by the charge neutrality constraint. The CSL condensates, having small energy gaps in comparison to the free energy of the system, do not modify the thermodynamic properties respect to the normal phase. In Fig. 4 we show the number density for quarks, $n_u, n_d$, and for electrons, $n_e$, as a function of $\mu_B$ for the 2SC (on the left) and for the CSL phase (on the right). Different values of the coupling for the 2SC phase from $G_1/G = 1$ down to $G_1/G = 0$ (NQ phase) are considered. We clearly see that $n_i^{2SC, G_1/G=0} = n_i^{NQ} \simeq n_i^{CSL}$ for each particle specie $i$, so the two phases, CSL and NQ, are indistinguishable from the matter composition. This conclusion holds also for other thermodynamic quantities like pressure or energy density and therefore for the EoS.

Finally, we found that while the phase diagram for strong coupling is dominated by the 2SC phase (Fig. 4)

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Actually, the critical value of $G_1/G$ at which the 2SC condensate breaks down is model and parameterization dependent. NJL calculations have obtained no pure 2SC at intermediate densities ($\mu_B = 1200$ MeV) below $G_1/G = 3/4$, see [3,4].
for intermediate or weak coupling, the CSL phase is favorable (Fig. 4). Note that the low CSL critical temperatures \( T_{CS}^C \approx 5 \text{ MeV} \) in contrast to the 2SC case, for which \( T_{2SC}^C \approx 50 \text{ MeV} \). Thus, we expect that in the late cooling of a neutron star, when the temperature has fallen below the MeV scale, a CSL superconducting quark core could develop. Stable hybrid star configurations have been obtained with a relatively large NQ matter core[4], therefore, hybrid stars with a CSL superconducting core will be stable as well. Finally, a qualitative study of the interaction of the magnetic field with the CSL phase shows that a CSL-core is consistent with recent observations and models of magnetized neutron stars[12].

Acknowledgments: D.N.A. thanks J. A. Pons, D. Blaschke, N. N. Scoccola and J. A. Miralles for fruitful discussions.

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