BEYOND THE GROSS-PITAEVSKII EQUATION: GROUND STATE AND LOW ENERGY EXCITATIONS FOR TRAPPED BOSONS

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1. INTRODUCTION

The recent experimental realization of Bose-Einstein condensation (BEC) of magnetically trapped alkali atoms has generated a huge amount of experimental and theoretical activity, with over twenty experimental groups in the world that can produce atomic condensates and more than one thousand articles on the subject published by now. The present status of the field has been recently reviewed by F. Dalfovo et al.[1].

The history of BEC gets back to 1924, when S. Bose derived the law for black-body radiation by considering the photons as a gas of identical particles. Immediately after, A. Einstein generalized Bose’s ideas to an ideal Bose gas and predicted that the bosons would condensate in the lowest quantum state of the system at sufficiently low temperature. It took seventy years of experimental efforts and technological progress until the first atomic Bose condensate was achieved in 1995. In this year, groups at the University of Colorado and at the Massachusetts Institute of Technology (MIT) using a laser cooled and magnetically trapped dilute gas of $^{87}$Rb [2] and $^{23}$Na [3] atoms respectively, were able to demonstrate unambiguously the occurrence of BEC.

In those early experiments, the number of trapped atoms was relatively small ($N \sim 10^3$ atoms) and the transition temperature was around 100 nK. The magnetic trap is well described by a harmonic oscillator potential, usually with cylindrical symmetry. However, throughout this paper we will consider a spherical potential well...
confining the atoms.

In order to have quantum effects, i.e., wave behaviour, we need a de Broglie wave length \( \lambda = (2\pi\hbar^2/mT)^{1/2} \) of the order of the distance between the atoms \((\rho)^3 \sim 1\). On the other hand, the system should be kept dilute, therefore the critical temperature will be extremely low, of the order of nanokelvins.

Up to now, the experimental conditions were such that the atomic gas was very dilute, i.e., the average distance between the atoms is much larger than the range of the interaction. As a consequence, the physics should be dominated by two-body collisions, generally well described in terms of the s-wave scattering length \( a \). The case of a positive scattering length is equivalent to considering a very dilute system of hard spheres, whose diameter coincides with the scattering length itself.

Typical scattering lengths are 53 \( A \) for \(^{87}\text{Rb} \) and 28 \( A \) for \(^{23}\text{Na} \). On the other hand, the size of the trap is defined by the harmonic oscillator length \( a_{\text{HO}} = (\hbar/m\omega)^{1/2} \) which is of the order of 10\(^4\)\( A \). The corresponding distance between the energy levels associated with this potential well is around 4 nK. For those initial experiments, a common \(^{87}\text{Rb} \) atom density in the trap was \( \rho \sim 10^{12} - 10^{14} \) atoms/cm\(^3\) giving an average inter-atom distance \( d \sim \rho^{-1/3} \sim 10^4 \)\( A \). Therefore, the effective atom size, defined by the scattering length is usually small compared to both the trap size and the inter-atom distance. The crucial parameter that defines the condition of diluteness is \( x = \rho a^3 \), which until very recently was kept rather small (i.e., \( x \sim 10^{-5} \)). Under these conditions, the Gross-Pitaevskii equation \([4]\), which assumes all the particles in the condensate, seems the logical tool to study those systems.

The situation is somehow different in homogeneous liquid \(^4\text{He} \). In this case, BEC manifests itself as a macroscopic occupation of the zero momentum state, measured by the condensate fraction, i.e., the fraction of the total number of particles in this state. However, there is only indirect evidence for this macroscopic occupation. Theoretical calculations and the analysis of inelastic neutron scattering data predict a condensate fraction of \( \sim 10\% \) \([5]\). Such a large depletion is an indication that \(^4\text{He} \) liquid is a strongly correlated system.

There are two ways to bring \( x \) outside the regime of validity of the mean field description. The first consists in increasing the density and the second in changing the effective size of the atoms. Recent experiments have explored both possibilities. On one side they have reached a very high number of atoms in the condensate, \( \sim 10^8 \), and on the other they have been able to change the scattering length of the atoms. This is the case of a recent experiment employing \(^{85}\text{Rb} \), where, by taking advantage of the presence of a Feshbach resonance at a magnetic field \( B \sim 155 \) Gauss, it was possible to vary the scattering length from negative to very high positive values. Under these conditions, effects beyond the mean field approximation are expected to be observable \([6,7]\).

We will start by discussing a homogeneous system of Bose hard spheres and using the results to determine the regime of validity of the Gross-Pitaevskii equation. We will then present an extension of the GP equation \([8]\), still in the framework of mean field theories, that allows for giving a first estimate of the expected corrections to the GP results in these new scenarios. Finally, we will briefly analyze the effects on the low collective excitations of the system.
2. Homogeneous hard-sphere Bose gas

We consider a system of \( N \) spinless bosons having mass \( m \) and described by the many-body Hamiltonian:

\[
H = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} V(r_{ij}).
\] (1)

The uniform system is studied in the thermodynamic limit, \( N \to \infty \) and \( \Omega \to \infty \) keeping the density, \( \rho = N/\Omega \), constant. The hard-spheres potential is defined as \( V(r < a) = \infty \) and \( V(r > a) = 0 \).

Correlated Basis Functions (CBF) theory provides a very efficient way to handle the correlations induced by the interactions between the particles (for a review, see Ref. [9]). In its simplest version, one takes a Jastrow correlated wave function [10]

\[
\Psi_J(1, \ldots, N) = \prod_{i<j} f(r_{ij}),
\] (2)

where the Jastrow correlation function, \( f(r) \), depends only on the interparticle distance. Once the trial function is defined, the variational principle ensures that, if we are capable to calculate the expectation value of the Hamiltonian,

\[
E_{\text{CBF}} = \frac{\langle \Psi_J \mid H \mid \Psi_J \rangle}{\langle \Psi_J \mid \Psi_J \rangle},
\] (3)

then \( E_{\text{CBF}} \) is an upper bound to the true ground state energy. The correlation function, \( f(r) \), is variationally determined by minimizing \( E_{\text{CBF}} \).

\( E_{\text{CBF}} \) may be calculated once the two-body distribution function, \( g(r) \), is known. In fact, the energy per particle can be written as

\[
e = \frac{1}{2} \rho \int d^3r \ g(r) \left[ V(r) - \frac{\hbar^2}{2m} \nabla^2 \ln f(r) \right].
\] (4)

In the particular case of hard spheres, the distribution function is strictly zero for \( r < a \) and the previous expression reduces just to the kinetic energy part

\[
e = -\frac{1}{2} \rho \int d^3r \ g(r) \frac{\hbar^2}{2m} \nabla^2 \ln f(r).
\] (5)

\( g(r) \) may be evaluated by using cluster expansion and Hypernetted Chain (HNC) theory. HNC is an integral equation method which allows for massive summations of the cluster diagrams associated with \( g(r) \).

The optimal choice for the Jastrow factor would be the one satisfying the Euler equation \( \delta E_{\text{CBF}}/\delta f = 0 \). However, a less demanding and often effective approach consists in choosing a parametrized functional form of \( f(r) \) and in minimizing the energy with respect to the parameters. We adopt here the correlation function minimizing the two–body cluster energy of a homogeneous Bose gas, with the healing
conditions at a distance $d$ ($f(r \geq d) = 1$ and $f'(d) = 0$). $d$ is taken as a variational parameter. For the hard-spheres case, $f(r < a) = 0$ and $f(r > a) = u(r)/r$, where $u(r)$ is the solution of the Schrödinger-like equation: $-u'' = K^2 u$. $f(r)$ has the form

$$f(r) = \frac{d \sin[K(r - a)]}{r \sin[K(d - a)]},$$

and the healing conditions are satisfied through the relation: $\cot[K(d - a)] = (Kd)^{-1}$.

An alternative calculation, based on perturbation theory in the expansion parameter $x = \rho a^3$, leads to the low-density expansion for the energy density[12]:

$$E = \frac{2\pi \rho^2 a^2 h^2}{m} \left[ 1 + \frac{128}{15} \sqrt{\frac{\rho a^3}{\pi}} + 8 \left( \frac{4}{3} \pi - \sqrt{3} \right) \rho a^3 \ln(\rho a^3) + O(\rho a^3) \right].$$

Up to these orders of the expansion, the details of the potential do not show up, and any potential with the same scattering length would give identical results. This universal behavior has recently been checked by a diffusion Monte Carlo calculation (DMC)[13], which provided the exact solution of the many–body Schrödinger equation.

Fig. 1. shows the energy per particle in units of $\hbar^2/2ma^2$ for homogeneous hard spheres as a function of $x$. The energies have been multiplied by $10^{3(2,1)}$ at $x =$
The agreement between the HNC/0 and the DMC results is excellent in most of the wide range of densities considered. However, there is a 5% disagreement at the highest $x$-value ($x = 10^{-2}$). From this value on, the contribution of the elementary diagrams and a better optimization of the correlation should be probably taken into account. The LD$_0$ results are only accurate at very low densities, while LD$_1$ gives also a good representation of the exact DMC results. On the contrary, the addition of the logarithmic term spoils the agreement already at intermediate densities.

In the next section, we will describe the trapped bosons by a local density approximation (LDA) employing the homogeneous gas results. The local value of the parameter $x$ in the trap will give an idea of the differences that we can expect by using the different energies reported in Fig. 1 as inputs to build the energy functional.

2. Ground state of trapped hard spheres

The energy functional associated with the Gross-Pitaevskii theory is simply obtained in the local-density approximation by keeping only the first term in the low density expansion (7):

$$E_{GP}[\Psi] = \int d\mathbf{r} \left[ \frac{\hbar^2}{2m} |\nabla \Psi(\mathbf{r})|^2 + \frac{m}{2} \omega^2 r^2 |\Psi(\mathbf{r})|^2 + \frac{2\pi \hbar^2 a}{m} |\Psi(\mathbf{r})|^4 \right],$$

(8)

where the wave function $\Psi(\mathbf{r})$, in which all the atoms belong to the condensate, is normalized to $N$. By a functional variation of $E_{GP}[\Psi]$ one finds the Gross-Pitaevskii equation,

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + \frac{m}{2} \omega^2 r^2 + \frac{4\pi \hbar^2 a}{m} |\Psi(\mathbf{r})|^2 \right] \Psi(\mathbf{r}) = \mu \Psi(\mathbf{r}),$$

(9)

where $\mu$ is the chemical potential. The GP equation has the form of a nonlinear stationary Schrödinger equation, and it has been solved for several types of traps using different numerical methods.

The next logical step, in the spirit of LDA, is to include into the energy functional the next terms of the correlation energy expansion for the uniform system. According to the behavior of the different terms, shown in Fig. 1, it seems clear that it is reasonable to consider only the first correction, LD$_1$. However, before proceeding further it is convenient to simplify the notation by expressing lengths and energies in harmonic oscillator (HO) units. The spatial coordinates, the energy, and the wave function are rescaled as $\mathbf{r} = a_{HO} \bar{\mathbf{r}}$, $E = \hbar \omega \bar{E}$, and $\Psi(\mathbf{r}) = (N/a_{HO}^3)^{1/2} \Psi_1(\bar{\mathbf{r}})$, where $\Psi_1(\bar{\mathbf{r}})$ is normalized to unity.

Using these new variables and taking into account the second term of the expansion, we obtain the modified Gross-Pitaevskii (MGP) energy functional for the
energy per atom, \( \bar{e}_{\text{MGP}} = \hat{E}_{\text{MGP}} / N \),

\[
\bar{e}_{\text{MGP}}[\Psi_1] = \int d\tilde{r} \left[ \frac{1}{2} |\nabla_\tilde{r} \Psi_1|^2 + \frac{1}{2} r^2 |\Psi_1|^2 + 2\pi \tilde{a} N |\Psi_1|^4 + \frac{256 \sqrt{\pi \tilde{a}^3 N^3}}{15} |\Psi_1|^5 \right],
\]

and the corresponding modified Gross-Pitaevskii equation,

\[
\left[ -\frac{1}{2} \nabla^2_{\tilde{r}} + \frac{1}{2} \tilde{r}^2 + 4\pi \tilde{a} N |\Psi_1(\tilde{r})|^2 + \sqrt{\pi \tilde{a}^3 N^3} \frac{128}{3} |\Psi_1(\tilde{r})|^3 \right] \Psi_1(\tilde{r}) = \mu_1 \Psi_1(\tilde{r}),
\]

where \( \tilde{a} = a / a_{\text{HO}} \) and \( \mu \) is the chemical potential in HO units.

In alternative, we have also used as local correlation energy the one provided by the uniform system HNC results. This option has the advantage that one is not limited to hard spheres, but, in principle, any type of potential for the two-body interaction can be considered. In this case, the local correlation energy \( V_{\text{corr}}^{LD} \), is given by

\[
V_{\text{corr}}^{LD} = \frac{1}{N} \int d\tilde{r} \rho_1(\tilde{r}) e_{\text{HNC}}^{\text{hom}}(\rho_1),
\]

where \( e_{\text{HNC}}^{\text{hom}}(\rho_1) \) is the HNC homogeneous gas energy per particle at density \( \rho_1 \). The minimization of the energy gives the HNC correlated Hartree equation (CH\(_{\text{HNC}}\)),

\[
\left[ -\frac{1}{2} \nabla^2_{\tilde{r}} + \frac{1}{2} \tilde{r}^2 + \bar{e}_{\text{HNC}}^{\text{hom}}(x_{\text{loc}}) + x_{\text{loc}} \frac{\partial e_{\text{HNC}}^{\text{hom}}(x_{\text{loc}})}{\partial x_{\text{loc}}} \right] \Psi_1(\tilde{r}) = \mu_1 \Psi_1(\tilde{r}),
\]

where we have introduced the scaled unities and the local gas parameter, \( x_{\text{loc}}(\tilde{r}) = \rho_1(\tilde{r}) a^3 = N \tilde{a}^3 |\Psi_1(\tilde{r})|^2 \).

The GP, MGP and CH\(_{\text{HNC}}\) equations have been solved by the steepest descent method for an isotropic harmonic oscillator trap.

Several relationships between the different contributions to the energy per atom or to the chemical potential exist, and are useful to check the accuracy of the numerical procedure. By direct integration of the GP equation, one finds

\[
\bar{\mu} = \bar{e}_{\text{kin}} + \bar{e}_{\text{HO}} + 2\bar{e}_{\text{int}}^{(1)},
\]

where \( \bar{e}_{\text{kin}} = -\frac{1}{2} \int d^3\tilde{r} \tilde{r} \nabla^2 \Psi_1, \bar{e}_{\text{HO}} = \frac{1}{2} \int d^3\tilde{r} |\Psi_1|^2 \tilde{r}^2 \) and \( \bar{e}_{\text{int}}^{(1)} = \int d^3\tilde{r} 2\pi \tilde{a} N |\Psi_1|^4 \) are the different terms contributing to the total energy. Further relationships can be obtained by means of the virial theorem,

\[
2\bar{e}_{\text{kin}} - 2\bar{e}_{\text{HO}} + 3\bar{e}_{\text{int}}^{(1)} = 0.
\]

It is also important to notice that the dimensionless parameter characterizing the effects of the interaction in the GP equation is given by \( \tilde{a} N \). This implies that one can get the same results with a proper rescaling of the variables \( N \) and \( \tilde{a} \). As it can be seen by a simple inspection of the equations, the scaling property is lost
Table 1

Chemical potential $\mu$, and ground state energy per particle $e$, of $N$ $^{87}$Rb atoms in an isotropic trap ($\omega/2\pi = 77.78\text{Hz}$) in different approaches. Energies are in units of $\hbar\omega$.

| $N$ | $\mu$ | $e$ |
|-----|-------|-----|
|     | TF    | GP  | MGP | CH$_{\text{HNC}}$ | TF    | GP  | MGP | CH$_{\text{HNC}}$ |
| $10^5$ | 16.75 | 16.85 | 16.99 | 16.94 | 11.96 | 12.10 | 12.19 | 12.20 |
| $10^6$ | 42.07 | 42.12 | 42.69 | 42.53 | 30.05 | 30.12 | 30.48 | 30.48 |
| $10^7$ | 105.68 | 105.70 | 107.97 | 107.20 | 75.49 | 75.52 | 76.94 | 76.85 |

in the MGP approach. Moreover, the relation between the different contributions to the chemical potential changes to

$$\mu = \bar{\varepsilon}_{\text{kin}} + \bar{\varepsilon}_{\text{HO}} + 2\bar{\varepsilon}_{\text{int}}^{(1)} + \frac{5}{2}\bar{\varepsilon}_{\text{int}}^{(2)},$$

(16)

where

$$\bar{\varepsilon}_{\text{int}}^{(2)} = \int d^3\bar{r} \frac{256}{15} \sqrt{\pi a^5 N^3} \Psi_1(\bar{r})^3 \Psi_1^*(\bar{r}).$$

(17)

In this case the relation implied by the virial theorem is

$$2\bar{\varepsilon}_{\text{kin}} - 2\bar{\varepsilon}_{\text{HO}} + 3\bar{\varepsilon}_{\text{int}}^{(1)} + \frac{9}{2}\bar{\varepsilon}_{\text{int}}^{(2)} = 0.$$  

(18)

A simple approach, valid for large $N$ and loosely called the Thomas-Fermi (TF) approximation, is obtained by neglecting the kinetic energy term in the GP equation. In the TF approximation it is possible to derive simple analytical expressions [14], useful to make quick estimates of several quantities. For instance, $\bar{\mu}_{\text{TF}} = 1/2(15aN)^{2/5}$, while the energy is related to the chemical potential by $\bar{e}_{\text{TF}} = 5\bar{\mu}_{\text{TF}}/7$. The local value of $x = Na^3 \rho_1(0)$ at the center of the density distribution of the trapped bosons is given by $x_{\text{TF}}(0) = (15^2 a^{12} N^2)^{1/5}/(8\pi)$.

As can be seen in Table I, for large number of particles the TF and GP results are practically identical and there is also a very good agreement between the MGP and CH$_{\text{HNC}}$ results. In the case $N = 10^7$, the contributions to the energy for the GP(MGP) equations are: $\bar{\varepsilon}_{\text{kin}} = 0.0294(0.0292)$, $\bar{\varepsilon}_{\text{HO}} = 45.306(46.57)$, $\bar{\varepsilon}_{\text{int}}^{(1)} = 30.184(28.96)$ and $\bar{\varepsilon}_{\text{int}} = 0(1.379)$. The virial theorem is well fulfilled in both cases.

By changing the number of trapped atoms in the range of the experimental availability, the average values of $x$ are such that the corrections to the GP equation are kept small and of the order of 2% in the case of $N = 10^7$ atoms. However, the recent experiments, where the scattering length can be largely manipulated, open
the door to explore higher values of \( x \). In fact, in order to vary \( x \), it is much more efficient to change the scattering length than the number of atoms. Experimental results are available for \(^{85}\text{Rb}\), whose scattering length is modulated by its Feshbach resonance. The number of trapped atoms is \( N \sim 10^4 \) and the trap is anisotropic. In order to estimate the corrections induced by the MGP equation, we have considered an isotropic trap characterized by the frequency \( \omega/2\pi = 10 \) Hz, corresponding to an average of the cylindrical trap frequencies used in the experiment. The \( \omega \) value is smaller than that employed for \(^{87}\text{Rb}\) and therefore \( a_{\text{HO}} \) is larger. We take \( \bar{a} = 0.1228 \), which is in the range considered by the experiments. It corresponds to \( a = 8000a_0 \), where \( a_0 \) is the Bohr radius of the hydrogen atom. In this case, \( x_{\text{TF}}(0) = 0.03 \), which is just beyond the range of the points plotted in Fig.1. The energies per atom turn out to be: \( \bar{e}_{\text{GP}} = 18.25 \) and \( \bar{e}_{\text{MGP}} = 21.85 \). For the chemical potential we have: \( \bar{\mu}_{\text{GP}} = 25.48 \) and \( \bar{\mu}_{\text{MGP}} = 31.09 \). As a consequence of the use of the MGP equation, the corrections are of the order of 20%. The actual cylindrical trap is currently under study [15].

4. COLLECTIVE EXCITATIONS

Information on the excitation spectrum of a system are contained in the dynamic structure function, which, for a given excitation operator \( F \), is given by

\[
S_F(E) = \sum_n |\langle n| F |0 \rangle|^2 \delta(E - (E_n - E_0)).
\]

In the case of inelastic neutron scattering (for instance against liquid \(^4\text{He}\)) the operator \( F \) corresponds to the density fluctuation operator, \( F = \sum_j e^{iqr_j} \), and the inclusive inelastic cross section is proportional to the dynamic structure function.

A useful tool to analyze \( S_F(E) \) is provided by the sum rules approach, extensively used in quantum many-body systems both in the context of nuclear [16] and condensed matter physics, in particular to analyze the excitation spectrum of quantum liquids [5]. The sum rules establish rigorous links among the energy momenta of \( S_F(E) \) and ground state properties. The energy weighted integrals are defined as:

\[
m_k = \int_0^\infty E^k S_F(E) dE,
\]

and \( m_k \) can be calculated, without an explicit knowledge of \( S_F(E) \), as a ground state expectation value of certain combinations of commutators of the excitation operator \( F \) and the Hamiltonian. So, quantities like \( m_{k+1}/m_k \) or \((m_{k+2}/m_k)^{1/2}\) supply rigorous upper bounds to the energy of the lowest excited state that can be connected to the ground state through the operator \( F \). The upper bounds are very reliable when the excited state is highly collective, i.e., when the strength distribution is almost exhausted by a single mode. We will concentrate here in the case of the compressional modes, or monopole excitations, whose associated excitation operator is \( F = \sum_i^N r_i^2 \).

In the present case, we will study the upper bound provided by \((m_3/m_1)^{1/2}\). Using the definition of the dynamic structure function and the completeness of the
eigenstates $|n\rangle$, the moments $m_1$ and $m_3$ can be expressed as

$$m_1 = \frac{1}{2} \langle 0 | [F^\dagger, [H, F]] | 0 \rangle$$

(21)

and

$$m_3 = \frac{1}{2} \langle 0 | [[F^\dagger, H], [H, [H, F]]] | 0 \rangle .$$

(22)

By explicitly calculating the commutators, $m_1$ is expressed in terms of the mean square radius,

$$m_1 = 2 \hbar^2 m \langle 0 | r^2 | 0 \rangle .$$

(23)

Obviously, we get the same expression of $m_1$ for both GP and MGP equations.

In the case of the monopole excitation, it is more efficient to calculate $m_3$ as

$$m_3 = \frac{1}{2} \left( \frac{2\hbar^2}{m} \right)^2 \frac{d^2 E(\lambda)}{d\lambda^2} |_{\lambda=1} ,$$

(24)

where $\lambda$ is the parameter of the scaling transformation $\rho(r) \rightarrow \lambda^3 \rho(\lambda r)$. In fact, for the GP equation, the energy for the scaled density is

$$E(\rho, \lambda) = N \left[ \lambda^2 e_{\text{kin}}(\rho) + \frac{e_{HO}}{\lambda^2} + \lambda^3 e_{\text{int}}^{(1)} \right] .$$

(25)

The condition $dE(\rho, \lambda)/d\lambda = 0$ at $\lambda = 1$ satisfies the virial theorem. $m_3$ is given by

$$m_3 = \left( \frac{2\hbar^2}{m} \right)^2 \hbar^2 N e_{HO} \left[ 5 - \frac{e_{\text{kin}}}{e_{HO}} \right] .$$

(26)

Finally

$$E_{ex}^{GP} \sim \sqrt{\frac{m_3}{m_1}} = \hbar \omega \left[ 5 - \frac{e_{\text{kin}}}{e_{HO}} \right]^{1/2} .$$

(27)

By a similar procedure for the MGP equation we get

$$E_{ex}^{MGP} = \hbar \omega \left[ 5 - \frac{e_{\text{kin}}}{e_{HO}} + \frac{27 e_{\text{int}}^{(2)}}{8 e_{HO}} \right]^{1/2} .$$

(28)

In the $^{85}$Rb case the estimates of the monopole excitation energies (in HO units) are 2.23 and 2.38 for the GP and MGP, respectively. So, the MGP correction to the excitation energy is about 7%. This correction is smaller than that to the energy itself. The whole spectrum is shifted but the separation between the excitation energy levels is less affected.

4. CONCLUSIONS
In conclusion, we find that the MGP equation induces corrections of 20% in the ground state properties of the condensate, when the conditions of the recent experiments for $^{85}$Rb are considered ($x \sim 10^{-2}$). MGP is still a mean field theory, since it tries to incorporate correlation effects into the average single particle potential, and it cannot predict the depletion of the condensate. However, we believe that the estimates of the energy, chemical potential, and density profile are surely indicative and can still be accurate. Moreover, it is legitimate at these densities to question the use of a simplified interaction, given in terms of hard spheres. In any case, it is clear that fully microscopic calculations, which might address the many–body wave function and take explicitly into account the depletion of the condensate, are urgently required [17].

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