Reaching to a featured formula to deduce the energy of the heaviest particles producing from the controlled thermonuclear fusion reactions

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Abstract. Thermonuclear fusion reaction plays an important role in developing and construction any power plant system. Studying the physical behavior for the possible mechanism governed energies released by the fusion products to precise understanding the related kinematics. In this work a theoretical formula controlled the general applied thermonuclear fusion reactions is achieved to calculating the fusion products energy depending upon the reactants physical properties and therefore, one can calculate other parameters governed a given reaction. By using this formula, the energy spectrum of ⁴He produced from T-³He fusion reaction has been sketched with respect to reaction angle and incident energy ranged from (0.08-0.6) MeV.

1. Introduction
A fusion reaction is a subatomic process where two light nuclei are obliged together, so they will fuse producing energy. Energy yield because of the mass of the sum of masses of the individual nuclei greater than the mass of the combination of fusion reaction. If the combined nuclear mass is less than that of iron at the binding energy curve, then the nuclear particles will be more tightly bound than they were in the lighter nuclei and that decrease in energy released from mass based on the Einstein’s equation [1].

Fusion has the ability to offer basic steady source of energy for the future, Fusion energy represented a clean source of energy with a basic fuel that is plentiful, inexpensive, and available to all human beings. It is presently being produced in the universe in stellar systems and on earth as a thermonuclear or hydrogen weapons.

To use fusion in producing energy, the fuel must be in the plasma state. In a usual state of matter, it is complicated to get fusion reactions. The reason is most of the kinetic energies of reactant are rapidly lost by means stopping power (processes like excitation and ionization of the constitutive atoms and
dissociation of the molecules)[2], so with the evolution of fusion researches, in particular the “ion-driven inertial confinement fusion” and “fast ignition”, the stopping power in plasmas has interest subject. The stopping data of different ion beams through the plasmas containing various materials is needed to the fusion research. Besides fusion fuels, some other materials such as C and Be are usually utilize in the design of fusion targets or fusion devices of fast ignition driven by ions. During the implosion process, these materials will mix with fusion fuels inevitably, which will affect the heating of fused ions. Fast charged particles passing through matter are lose their energy gradually in many small steps. Stopping power is defined as, the average energy loss of the particle per unit path length. Stopping power is very important for many parts of basic science, especially for all technological applicat [3, 4].

The cross section for atomic and molecular processes are larger by many times of magnitude than those for fusion reactions. These unwanted events can be obviated if the fuel is in the plasma state by the high-temperature plasma ions, which have energies high enough to transcend the Coulomb barrier [2].

It follows from the above discussion, in order to fuse nuclei, the electromagnetic repulsive force between protons must be transcend, oncoming nuclei come close enough with each other making the strong force to be active, On other hand the Coulomb barrier is effects and have minimum value for hydrogen. [5].

2. Tritium fuel and 3He
Tritium and 3He are extremely vital to nuclear science side by side to Deuterium, as they are some of the major isotopes used in most nuclear fusion reactions mentioned in the table (1). The two main types of reactions by Tritium and 3He is T-3He fusion reaction, Broken down, the T-3He fusion reaction looks like:

$$T + {^3}He \rightarrow D + {^4}He$$
$$T + {^3}He \rightarrow D + {^4}He$$
$$T + {^3}He \rightarrow p + {^5}He$$

As you can see from above, Appearance of more than one product formation, each of them release a different amount of energy. However, the reaction that releases the protons, which also released about the same amount of energy that the other have, rarely occurs. It is much less probable than the other reactions.

In a nuclear reaction, it is noted that the reaction rate is very important, in addition, it is the main part responsible for determining the factor of the releasing energy. This reaction rate is consists of three parts: The Maxwell-Boltzmann velocity, the cross-section and the Gamow factor.

Distribution of particle speeds described by the Maxwell Boltzmann velocity distribution, the velocities in this distribution given by [6]:

$$f(v) = A \nu^2 e^{-mv^2/(2kT)}$$

A is constant given by

$$A = 2\pi (mkT)^3$$

$$f(v)$$ is the relative probability that you can find a particle with speed between $\nu$ and $\nu + d\nu$ in a collection of particles that are all in a thermally equal state at a constant temperature $T$.

Since $E= 1/2 \, mv^2$, the Maxwell-Boltzmann probability distribution can also be written as a function of the kinetic energy as [6]:

$$f(E) = \left[ m\pi (mkT)^3 \right] E^{1/2} e^{-E/kT}$$
The first step toward determined the energy generated by T-\(^3\)He fusion reaction is to explore the microscopic physics of nuclear collisions. This perception is to relative moving to the macroscopic scale for determine the power releasing in magnetic fusion reactor. The microscopic concepts include the cross section etc. the probabilities of fusion reactions are quantitatively fixing as the cross sections, one of the essential physical quantities is the fusion cross section, it benefits parameter in the fusion engineering and design, the cross-section describes in quantitative frame the probability that a pair of T and \(^3\)He nuclei will comes to a nuclear fusion reaction. For more specifically, suppose the \(^3\)He nucleus is stationary and the T nucleus is come toward it with a velocity \(v\), considering that the \(^3\)He is the target particle while the T is the incident particle. The interactions like this called a hard-sphere collision. With a view to hard-sphere collision to occur, conservation of energy is needed with respect to the center of mass frame, the sum of initial kinetic energies of the T and \(^3\)He when they are far from each other must overcome the Coulomb energy calculated at the surface of the particle when they come close to each other. If this is not happen, the T and \(^3\)He are repelled away from each other before they collide. In mathematical treat, a collision occurs when:

\[
\frac{M_T}{2} \left( \frac{A_{\text{He}}}{M_T + A_{\text{He}}} V \right)^2 + \frac{A_{\text{He}}}{2} \left( \frac{M_T}{M_T + A_{\text{He}}} V \right)^2 \geq \frac{e^2}{4\pi\varepsilon_0 d}
\]

By observing Figure. 1 The T-\(^3\)He cross-section has a peak of about 0.1 barns at \(T \approx 1000\) KeV. [7, 8] Particle accelerators provide a good study of the fusion. Accelerated H, D, T, \(^3\)He, \(^4\)He particles are used to bombard targets of these nuclides [9]. Well known plots of these cross sections have been given by approximate sketch illustrated here.

![Figure 1](image_url)

**Figure 1.** Fusion reaction cross-sections as a function of the incident particle energy, for the nuclear fusion reactions. [2]

The energy released from nuclear fusion reactions is the sacrifice of the mass. Thus, the energy of fusion \(Q\) can considered as part of the reaction equation, so the equation of T-\(^3\)He fusion is:

\[
\text{T} + ^3\text{He} \rightarrow \text{D} + ^4\text{He} + 14.320\ \text{MeV}
\]
The release of (Q) 14.320 MeV per $^4$He nuclide formed is interesting because that both the reactants and products are at very high temperature, a considerable amount of fusion energy is released as short-wave X- or $\gamma$-ray radiation.

Taking all the reactions into account, fusion doesn't produce radioactive ash or waste. For this reason, fusion is considered clean and environmental friendly for energy generation [2].

3. Calculations and results
An important notation related to the products energy from any nuclear reaction are essentially depend upon the so called Q-value equation given below which can be written based on conservation of mass-energy and linear momentum laws

$$Q = \varepsilon_{\text{product}} \left(1 + \frac{M_3}{M_4}\right) - \varepsilon_{\text{incident}} \left(1 - \frac{M_1}{M_4}\right) - \frac{2 \cdot \frac{M_1}{M_4} \varepsilon_{\text{incident}} \varepsilon_{\text{product}}}{M_4} \cos \theta$$  \hspace{1cm} (5)

Q-value equation has independence concerning the reaction mechanisms (as compound nucleus, scattering, etc.). And here masses can be replaced without any intense mistakes by the corresponding integer-valued mass numbers A, For more accurate work, the neutral atomic masses are used.

The general fundamentals physicals equations for calculating the energies related with the released particles from any nuclear reaction based on the laws of energy and momentum conservations given by [10,11].

$$\sqrt{\varepsilon_{\text{product}}} = v \pm \sqrt{v^2 + w}$$  \hspace{1cm} (6)

Where $v = \frac{2 \cdot \frac{M_1}{M_4} \varepsilon_{\text{incident}}}{(M_3 + M_4)} \cos \varphi$, and $w = \frac{M_4 \varepsilon_{\text{incident}} (M_4 - M_1)}{(M_3 + M_4)}$.

The energy parameter $\varepsilon_{\text{product}}$ can be treated as the neutron, proton, and alpha particles energies etc. corresponding to the specific or given fusion reaction under study, Also The symbols M1, M2, M3, M4, present the masses for the reactant and product particles respectively.

In our work, we concentrated on the T-$^3$He thermonuclear fusion reaction, in which can present an important reaction in applied systems or devices, and all materials appear in any reaction play as a fuels for operating such small or mini scale devices or large scale devices, i.e.; (Dense Plasma Focus and Tokamak systems).

By substituting the values of the masses for the corresponding chosen fusion reactions, and taken into account some mathematical treatment, we simply bring to a general standard formula for calculating the energy of the heaviest emitted particles (Alpha) $\varepsilon_\alpha$ and other particles, given:

$$\varepsilon_\alpha = \frac{L_1 Q + L_2 \varepsilon_{\text{incident}}}{L_3} \left[ \sqrt{1 + S_1 G} - S_2 G \sin^2 \varphi + \sqrt{S_3 G} \cos \varphi \right]^2$$  \hspace{1cm} (7)

Where $G = \frac{P \varepsilon_{\text{incident}}}{L_1 Q + L_2 \varepsilon_{\text{incident}}}$

Referring that L1, L2, L3, P, S1, S2, and S3 are constants see (table 1), Q mean the Q- value for the specific fusion reaction and $\varepsilon_{\text{incident}}$, is the bombarding projectile energy in which have a wide range of energies corresponding to a specific or given experimental or theoretical studies. G represents a variable depending upon the bombarding projectile energies. While $\varphi$ is the reaction angles for heaviest products particles measured in lab coordinates with respect to a direction of the incident particle. Their corresponding values are completely described in the following table for common fusion reactions.
Table 1. The useful constants for common fusion reactions.

| Thermonuclear fusion reaction | probability | $L_1$ | $L_2$ | $L_3$ | $P$ | $S_1$ | $S_2$ | $S_3$ | MeV |
|------------------------------|-------------|-------|-------|-------|-----|-------|-------|-------|-----|
| $\frac{2}{3}D + \frac{2}{3}D \rightarrow ^3\text{He} + \text{neutron}$ | 50%         | 2     | 2/3   | 8     | 1   | 5/3   | 3     | 3     | 3.269 |
| $\frac{2}{3}D + \frac{2}{3}D \rightarrow \frac{3}{2}T + \text{proton}$ | 50%         | 2     | 2/3   | 8     | 1   | 5/3   | 3     | 3     | 4.033 |
| $\frac{2}{3}D + \frac{3}{2}T \rightarrow ^4\text{He} + \text{neutron}$ | 100%        | 5     | -10   | 25    | 1   | 13    | 8     | 8     | 17.589 |
| $\frac{2}{3}D + \frac{3}{2}\text{He} \rightarrow ^4\text{He} + \text{proton}$ | 100%        | 5     | -10   | 25    | 1   | 13    | 8     | 8     | 18.353 |
| $\frac{3}{2}T + ^3\text{He} \rightarrow ^4\text{He} + 2 \text{neutron}$ | 100%        | 2     | -3    | 6     | 2   | 2     | 1     | 1     | 11.332 |
| $\frac{3}{2}T + ^3\text{He} \rightarrow \frac{2}{3}D + ^4\text{He}$ | 43%         | 2     | -3    | 6     | 2   | 2     | 1     | 1     | 14.320 |
| $\frac{3}{2}T + ^3\text{He} \rightarrow ^3\text{He} + p + ^5\text{He}$ | 6%          | 6     | -24   | 36    | 1   | 27    | 15    | 15    | 14.320 |

‘Figure 2’, ‘figure 3’ and ‘figure 4’ described the variation for the fusion energies of $^4\text{He}$ versus bombarding projectiles energies of the T-$^3\text{He}$ thermonuclear fusion reaction.

![Figure 2](image-url)  
**Figure 2.** $^4\text{He}$ fusion products energy spectrum for T-$^3\text{He}$ fusion reaction
Figure 3. Variation of $^4$He fusion energies with the incident Triton energies for T-$^3$He fusion reaction.

![Figure 3](image)

Figure 4. Fusion product energy distribution as a function of both a) Reaction angle b) Incident Triton energy.

Table 2. Variation of reaction angles for heavies and lightest fusion product controlling the thermonuclear fusion reaction $^3_1T + ^3He \rightarrow ^2_1D + ^4He + 14.320$ MeV.

| Reaction angles in the laboratory system (deg) | $0.08$ [MeV] | $0.1$ [MeV] | $0.2$ [MeV] | $0.4$ [MeV] | $0.6$ [MeV] |
|---------------------------------------------|---------------|-------------|-------------|-------------|-------------|
| $\theta_\beta$ | $\varphi_{4He}$ | $D$ | $4He$ MeV | $D$ | $4He$ MeV | $D$ | $4He$ MeV | $D$ | $4He$ MeV | $D$ | $4He$ MeV |
| 0 | 180 | 10.2825 | 4.1175 | 14.4000 | 16.3773 | 4.0427 | 14.4200 | 10.7610 | 3.7590 | 14.5200 | 11.3366 | 3.8384 | 14.7200 | 11.8052 | 3.1148 | 14.9200 |
| 30 | 150 | 10.1797 | 4.2066 | 14.3803 | 16.2614 | 4.1340 | 14.3954 | 10.5918 | 3.8794 | 14.4712 | 11.0869 | 3.5373 | 14.6233 | 11.4874 | 3.2887 | 14.7761 |
| 60 | 120 | 9.9940 | 4.4564 | 14.3458 | 10.1425 | 4.2293 | 14.3718 | 10.4285 | 3.9969 | 14.4254 | 10.6607 | 3.8196 | 14.4803 |
| 90 | 90 | 9.5994 | 4.7004 | 14.3198 | 9.5427 | 4.7771 | 14.3195 | 9.5591 | 4.7604 | 14.3419 | 9.5918 | 4.7271 | 14.3199 | 9.6246 | 4.6938 | 14.3184 |
| 120 | 60 | 9.1882 | 5.1511 | 14.3393 | 9.1508 | 5.1832 | 14.3440 | 9.0092 | 5.3582 | 14.3674 | 8.8223 | 5.5907 | 14.4130 | 8.6892 | 5.7681 | 14.4573 |
| 150 | 30 | 8.9393 | 5.4403 | 14.3796 | 8.8742 | 5.5202 | 14.3944 | 8.6270 | 5.8416 | 14.4686 | 8.2990 | 6.3171 | 14.6161 | 8.0639 | 6.6992 | 14.7631 |
| 180 | 0 | 8.8560 | 5.5500 | 14.4000 | 8.7752 | 5.6448 | 14.4200 | 8.4913 | 6.0287 | 14.5200 | 8.1156 | 6.6044 | 14.7200 | 7.5469 | 7.0731 | 14.9200 |

4. Conclusion and discussion
The higher effective fusion cross section, the higher the probability of fusion. In general, the probability of fusion increases with the kinetic energy of the nuclei. Kinetic energies of nuclei are proportional to their temperatures in T, so the T-$^3$He fusion has the highest cross section nearly 1000 KeV temperature.
The energy of a reaction product depends on the masses of the particles involved in the reaction (it is fortunate that substantial reductions occur in fusion collisions between the light nuclei) and on the velocities of reactants, the reactants have Maxwellian velocity distributions. The maximum fusion energies for the heaviest product obtained at zero reaction angle $\phi$, the minimum than at $180^\circ$. The most common collisions are those that lead to the deviation of the fusion product by angle $90^\circ$.

It is necessary to see that we arrive or bring excellent facts which reflect the rule or the basic phenomena of resulting the fusion triton with energies corresponding to the reaction angles and this results completely described in (table 2). Our conclusion concerned that we arrive a suitable agreement between the calculated results and the corresponding experimental results and in terms one can be used our theoretical model simply to calculate the interested fusion parameters and other related parameters and this fact can be related by observing the Alpha particles, neutrons, and proton fusion products energy spectrums for the different types of the interested thermonuclear fusion reactions.

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