The Problem of Axion Quality: A Low Energy Effective Action Perspective

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ABSTRACT: Any would-be Peccei-Quinn (PQ) symmetry is vulnerable to various types of explicit breaking. It has long been recognized that these can disrupt the axion solution to the strong CP problem. There have also been suggestions that, under certain circumstances, these can lead to a surprisingly large mass for the axion. Two types of corrections to this computation have been widely considered: higher dimension symmetry breaking operators, in theories where there is a complex field responsible for the spontaneous breaking of the PQ symmetry, and small instantons. Motivated by situations where small instantons dominate the $\theta$ potential of non-abelian gauge theories, we formulate the question of axion quality in terms of constraints on a Wilsonian effective action at scales somewhat above the scale of QCD. In this language, the standard axion mass computation assumes a nearly exact PQ symmetry in this action. The higher dimension operators and/or small instantons represent symmetry breaking terms in the Wilsonian action. This picture permits a uniform treatment of the problem of axion quality. If one assumes order one CP violation at high energies, then solving the strong CP problem constrains the axion potential terms in this Wilsonian action; in particular, the axion mass must be extremely close to its “standard” value.
1 Introduction and Overview

The axion solution to the strong CP problem relies on the existence of a global, Peccei-Quinn (PQ) symmetry of high quality. As we don’t expect the underlying theory of nature to exhibit exact global symmetries, this requires some explanation. In the literature, at least two sources of symmetry violation have been considered:

1. In situations where the effective theory, at the Peccei-Quinn scale, contains a field, $\Phi$, responsible for spontaneous breaking of the PQ symmetry, high dimension operators involving $\Phi$ can explicitly break the symmetry[1]. Typically, operators of very high dimension must be suppressed. This can be achieved, for example, with intricate discrete symmetries[2].

2. QCD itself violates the symmetry. In situations where the QCD $\beta$ function is small at high energies, small scale instantons can make substantial contributions to the axion potential, possibly dwarfing the standard result. This was pointed out some time ago in [3], and reiterated more recently in [4–8].

But as pointed out in [9], such small instanton contributions are sensitive to unknown short distance CP violating effects, raising the question of whether unknown, and possibly uncontrolled, non-perturbative string theory effects might spoil the PQ solution. String theory, which motivated that study, is remarkable, in fact, in providing approximate PQ
symmetries at weak coupling. But how large the breaking of the symmetry might be is an open question, for which the instanton analysis, at best, suggests lower bounds.

It is worth articulating what distinguishes small and large instantons. A standard way to compute the axion potential in QCD is to study an effective theory at scales below the scale of chiral symmetry breaking, which includes eight Goldstone bosons and the axion. In addition to the eight axial $SU(3)$ currents and the corresponding Goldstone bosons, one can then define a ninth current which is free of anomalies. This current is not conserved already classically, with divergence proportional to light quark masses. Matrix elements of this current are studied in the non-linear sigma model. Thinking of this as a Wilsonian theory with an upper cutoff $M \sim 1$ GeV, this procedure accounts for non-perturbative effects such as instantons at scales below $M$. Small instantons and other short distance non-perturbative effects would yield PQ symmetry violating terms in the Wilsonian action, and these affects must be accounted for separately.

In this note, in addition to providing a unified view of these problems, we ask the extent to which these types of contributions to the Wilsonian action endanger the PQ solution of the strong CP problem. If one assumes order one CP violation at the relevant energy scales, we will see that the axion mass must be extremely close to its value in the standard computation. The question of higher dimension operators is well studied[1] and the results can be understood in this language. For small instantons, this translates into constraints on the $\beta$ function. Interesting models, such as minimal gauge mediation (and certainly non-minimal gauge mediation) are susceptible to large corrections. Indeed, in such cases, small instantons are potentially disruptive of supersymmetry breaking itself.

In the next section, we discuss the Wilsonian effective action. In section 3, we review the issues associated with higher dimension operators in theories in which PQ symmetry breaking is driven by a single (or small number of) complex fields. In section 4, we review the potential problems associated with small instantons, noting that, quite generally, in non-supersymmetric theories, if the leading term in the QCD $\beta$ function is less than four, then generically small instantons dominate, and high energy CP violation endangers the PQ solution of the strong CP problem. In section 5, we consider the question of small instantons in supersymmetric theories. Here, on the one hand, the $\beta$ function, for a given number of generations, is smaller; on the other hand, the need for supersymmetry-breaking insertions softens the high energy behavior. We review, as noted in [9], the argument that the axion potential is first order in the gluino mass. We explain why, in cases where they are the dominant contribution to the axion potential, small instantons are potentially the principle source of supersymmetry breaking. Noting, as discussed in [9], the way in which small instanton effects may be cut off in grand unified theories above the unification scale, in section 6, we speculate on possible behaviors of instanton computations in “ultraviolet complete” theories such as string theory. In section 7, we consider the implications of these results for particular cases: non-supersymmetric theories with more than four generations of quarks and leptons, supersymmetric theories with additional quarks and leptons, such as required in gauge mediated models, and other situations. We stress that, quite generally, corrections to the standard formula for the axion mass must be extremely small if the axion is to solve the strong CP problem without strong assumptions about CP violation at short
We consider, here, a Wilsonian effective action for QCD with an axion, defined at a scale above the scale of QCD, but well below the weak scale. We assume an approximate PQ symmetry, and write an effective action for the axion field, analogous to the non-linear effective action for the pseudoscalar mesons. The action is built out of an object with nice transformation properties under the Peccei-Quinn symmetry:

$$\Phi = f_a e^{i a e}.$$  \hspace{1cm} (2.1)

The action includes derivative terms, which respect the PQ symmetry. It includes also PQ violating potential terms, as well as derivative terms which violate the symmetry. If these terms are small, the computation of the axion mass is the standard one, plus perturbations.

More precisely, the effective action has the structure:

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu \nu}^2 + \frac{a(x)}{16f_a \pi^2} \tilde{F} F + \text{quark kinetic and mass terms}$$

$$+ \frac{1}{2} (\partial_\mu \Phi) \Phi + \text{symmetry violating derivative terms} + \sum_i \Lambda_i^{1-n_i} \Phi^n_i + \text{c.c.} + \ldots$$  \hspace{1cm} (2.2)

Here the $\Lambda_i$ terms represent symmetry breaking effects, and the parameters $\Lambda_i$ are themselves, in general, complex. We have defined $a$ through its coupling to $\tilde{F} F$.

If the potential terms are small, in the standard way, we can calculate the axion potential from this action. Recall that this “Standard Computation” proceeds by first modifying the axion current defining a non-anomalous, non-conserved current; the corresponding symmetry is classically violated by light quark mass terms. Then the quark mass terms are proportional to $e^{i A_a \pi / \pi} \frac{e F}{2 \pi}$. The leading axion mass may be calculated in a straightforward way.

We can then treat the $\Lambda_i$ terms as perturbations. This Wilsonian setup allows a uniform treatment of:

1. Higher dimension, PQ symmetry violating operators originating from an effective action at scales well above QCD scales[1].

2. Small instantons

3. Unknown non-perturbative effects, e.g. in string theory.

Let’s consider each of these in turn.

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1 This can be seen by rewriting these terms in terms of the degrees of freedom of the non-linear sigma model, and requiring that, in the ground state, there be no tadpole for the pions or the axion.
3 Higher Dimension Symmetry Violating Operators

Suppose we have a theory with a complex field, $\phi$, $\langle \phi \rangle = f_a$, transforming under the PQ symmetry, which acquires an expectation value of order $f_a$, and which creates a light axion and a massive field. Then it has long been known\cite{1} that symmetry violating operators of the form

$$L_{\text{symmetry-breaking}} = \frac{1}{\Lambda^{n-4}} \phi^n + c.c. \quad (3.1)$$

endanger the PQ solution to the strong CP problem, even if $\Lambda \sim M_P$. Indeed, below the mass of Re $\phi$, these operators have precisely the form of our symmetry violating operators in the Wilsonian effective action of equation 2.2. Achieving the required quality of the Peccei-Quinn symmetry requires suppressing operators up to $n \sim 12$. If this is to be achieved through, say, discrete symmetries, these must be rather intricate\cite{2}.

4 Small Instantons in Non-Supersymmetric Theories

The possibility that small instantons might enhance the axion mass was raised in \cite{3}. In \cite{9}, motivated by then popular string theory constructions, theories were considered where, at high energy scales, the QCD beta function is, at leading order, small. If one assumes that it is possible to tie together all of the fermion zero modes, with no additional chiral suppression, then, writing the leading order term in the $\beta$ function, $b_0$:

$$\beta(g) = -b_0 \frac{g^3}{16\pi^2} \quad (4.1)$$

one has, for the instanton contribution to the axion potential at small $\rho$:

$$K e^{i \frac{\pi}{7} + i \delta} \int \frac{d\rho}{\rho^5} e^{-\frac{8\pi^2}{\pi^2} (\Lambda \rho)^4} + c.c. = e^{i \frac{\pi}{7} + i \delta} \int \frac{d\rho}{\rho^5} (\Lambda \rho)^{b_0} \quad (4.2)$$

for some constant $K$. For small enough $b_0$,

$$b_0 \leq 4 \quad (4.3)$$

small instantons dominate. In general, the phase, $\delta$, depends on unknown high energy physics. To make sense of this expression, we must assume that the scale size integral is cut off by some additional dynamics at some small $\rho = \rho_0$. In this case, again, the action is of the general form we have described above.

To be specific, following \cite{9}, we can consider an $SU(5)$ grand unified theory. With three generations and without additional colored scalars or other fields, the $\beta$ functions for $SU(3)$ and $SU(2)$, below the unification scale, are:

$$b_0^{(3)} = 7; b_0^{(2)} = 10 \quad (4.4)$$

For $SU(3)$, the $\beta$-function is not small enough that small instantons dominate. For $SU(2)$ they do, but assuming coupling unification, $SU(2)$ instantons are still highly suppressed.
at the unification scale. Above that scale, the $b_0$ of the full $SU(5)$ theory is $42/3$, and instanton effects die out rapidly.

Things are different if there are thresholds for additional fields, say scalars transforming under $SU(3) \times SU(2) \times U(1)$, slightly above the weak scale. For example, if there are several additional scalar $SU(3)$ triplets, such that

$$b_{0}^{(3)} = 2,$$

(4.5)

small instantons dominate up to the unification scale. One can close up the instanton with insertions of the Higgs field; there might be additional possibilities to obtain a non-zero axion potential involving new physics at very short distances. From the Higgs field, the result is proportional to

$$M_{GUT}^4 e^{-\frac{16 \pi^2}{3 \alpha} \det(y_U) \det(y_D)} (4.6)$$

Taking $M_{GUT} = 10^{16}$GeV, this is orders of magnitude larger than typical QCD energy scales, $m_\pi^2 f_\pi^2$, even allowing possible substantial numerical suppression. As a result, the QCD vacuum angle is at least as large as the phase of the determinant of the CKM matrix, of order $10^{-5}$. It might be larger if there are additional CP violating effects at scales near the GUT scale.

It is perhaps worth recalling how one computes the effective $\theta$, $\langle a \rangle_{f_a}$ assuming small instantons are suppressed. At low energies, including only dimension four terms in the QCD lagrangian, the theory has a symmetry of CP + $\theta \rightarrow -\theta$. So before including higher dimension operators, the minimum of the potential for $\theta$ is at $\theta = 0$. Including higher dimension, CP violating operators, the minimum shifts, by amount of order:

$$\theta_{min} = \frac{A_{QCD}^2}{M^2} \alpha.$$ (4.7)

Here we have assumed a dimension six operator, scaling with $1/M^2$; $\alpha$ is a CP violating phase. In the Standard Model, we would expect

$$\alpha \sim \text{Im} \det(Y_U) \det(Y_D).$$ (4.8)

The CP violating phase, in this case, is of order the phase of the determinant of the KM matrix, roughly $10^{-5}$. So we see that, when small instantons are important, they dominate and lead to a far larger phase, even though the underlying CP violating phases have a similar origin. This suggests that aligning the minimum of the $\theta$ potential associated with small instantons with that from low energy QCD is difficult.

5 Small Instantons in Supersymmetric Theories

In the supersymmetric case, there are new features. In particular, at energy scales much above the supersymmetry breaking scale (instanton scale sizes correspondingly smaller) the axion ($\theta$) potential will be suppressed by powers of some supersymmetry breaking scale. In [9] it was argued that one should be able to tie together all of the instanton zero modes
by including a single insertion of $m_\lambda$ (as well as suitable Yukawa couplings). So now the 
axion ($\theta$) potential behaves as:

$$K' e^{i \frac{2\pi}{f_\theta} + i \delta} \int m_\lambda \frac{d\rho}{\rho^2} e^{-\frac{8\pi^2}{g_2(\rho)}} + c.c. = K' e^{i \frac{2\pi}{f_\theta} + i \delta} m_\lambda \int \frac{d\rho}{\rho^4} (\Lambda \rho)^{b_0}$$

(5.1)

for some constant $K'$. In this expression, small instantons dominate if $b_0 \leq 3$. In the 
simplest $SU(5)$ model with three generations, $b_0 = 3$, so we are precisely at the boundary. 
If there are additional chiral multiplets in the 3 and $\bar{3}$, as in the simplest gauge mediated 
models, for example, we will be in a regime of small instanton domination, and the PQ 
solution of the strong CP problem is vulnerable to unknown high energy CP-violating 
effects. If, instead, instanton effects are second order in $m_\lambda$, things are slightly better. 

It is interesting to consider the case that instanton effects are linear in $m_\lambda$ from another 
point of view. Thinking of $m_\lambda$ as linear in a supersymmetry breaking spurion, the potential 
due to instantons is \textit{linear} in the spurion, as would be the case for the leading term in 
an O’Raifeartaigh-like model. If small instantons dominate, there is the potential for 
this linear term to be the dominant source of supersymmetry breaking. To gain some 
feeling for the potential problems, consider the possibility that there is an underlying gauge 
mediated theory with a single generation of vectorlike messenger chiral fields. In that case, 
$b_0^{(3)} = -1$, and, assuming small instantons are cut off at $M_{GUT}$, the linear term in $m_\lambda$ is 
of order $M_{GUT}^3 \left( \frac{M_{GUT}}{\Lambda_{QCD}} \right)$. This is potentially a huge enhancement, though one also expects 
significant suppression by powers of Yukawa couplings and possible numerical factors.

6 Unknown Non-Perturbative Effects

If we consider the sorts of axions which arise in string theory, we might expect that PQ 
violating effects could arise from sources beyond those so far considered. Non-perturbative 
effects in these theories are not well understood, and one might expect effects at high 
energies larger than those anticipated from instantons. The instanton effects we have 
described so far are presumably just a lower bound. We have seen, for example, how 
the unification scale can act as a cutoff on small scale QCD instantons. The cutoff in 
a string theory might operate differently. Factors of a few change in the effective cutoff 
could significantly change the size of the result. Lacking such an understanding, we have 
to acknowledge that the successful implementation of the PQ solution to the strong CP 
problem relies on strong assumptions about very short distance physics.

7 Conclusion: Possible Implications of Small Instantons for Interesting 
Models

Simply allowing additional colored fermion or boson fields in the Standard Model, we have 
seen, could substantially enhance the role of small instantons. Supersymmetric theories are 
even more susceptible to such enhancements. We have noted at least one well motivated 
model where small instantons would play a role: theories of gauge mediated supersymmetry 
breaking.
7.1 Heavy Axions

More generally, over the years and particularly quite recently, there has been some interest in solutions to the strong CP problem where the axion might be heavy; a partial list includes [3–8]. From the perspective of the effective action for the axion which we have advocated in this paper, such heavy axions are problematic. Again, by assumption, the action has an approximate $U(1)$ symmetry, and the axion is a compact field. So the effective action is a sum of terms of the form:

$$V(a) = m_a^2 f^2 \pi \cos\left(\frac{a}{f_a}\right) + \sum \Lambda_i^4 \cos\left(\frac{n_i a}{f_a} + \alpha_i\right)$$

(7.1)

Here we have used an old argument of Weinberg’s to write the leading QCD effects in terms of an even function of the axion field ($\theta$): the low energy theory has a symmetry which is conventional CP accompanied by a reversal of the sign of $\theta \left(\frac{a}{f_a}\right)$.

If $\Lambda_i$ for a particular $i$ is the dominant term here, then we have

$$\frac{\delta m_a^2}{m_a^2} = \frac{\Lambda_i^4}{m_a^2 f_a^2}$$

(7.2)

The effective $\theta$ is

$$\langle \frac{a}{f_a} \rangle = \alpha_i \frac{\delta m_a^2}{m_a^2}$$

(7.3)

If the phase $\alpha_i$ is of order one, then we require:

$$\frac{\delta m_a^2}{m_a^2} < 10^{-10}.$$  \hfill (7.4)

In other words, we can only have an enhanced axion mass if new sources of CP violation, connected with this mass, are small. We have seen this in various contexts in this note: higher dimension operators, small instantons.

7.2 Gauge Mediated Supersymmetry Breaking

We have noted that gauge mediated supersymmetry breaking, where $b_0^{(3)} \leq 2$ is particularly prone to domination by small instantons. There is the possibility that such instantons in fact are the dominant source of SUSY breaking. This may pose an obstacle to the implementation of the PQ solution of the strong CP problem in such models, and possibly even an obstruction in some cases to the construction of such models. It is possible that in some models one can’t close up the zero modes without multiple supersymmetry breaking insertions, thus suppressing small instantons. This is a subject worthy of further investigation.

7.3 Pitfalls for Model Builders – and Nature – To Avoid

We can summarize by saying that we require of nature, if we are to successfully implement the PQ solution of the strong CP problem

1. Before considering non-perturbative QCD effects, a high quality PQ symmetry.
2. Non-perturbative QCD effects must not be larger than $10^{-10}$ of the axion mass-squared.

By definition, we don’t have direct experimental knowledge, at present, of these high energy effects. So the PQ mechanism to solve the strong CP problem is contingent on features of high energy physics which are currently unknown. Said another way, discovery of a Weinberg-Wilczek axion would set strong limits on possible ultraviolet physics.

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