Leptonic Decay of $\Upsilon$, a Possible Signature of New Physics

Yu-Jie Zhang \textsuperscript{(a)}, Hua-Sheng Shao \textsuperscript{(b)}

\textsuperscript{(a) Key Laboratory of Micro-nano Measurement-Manipulation and Physics (Ministry of Education) and School of Physics, Beihang University, Beijing 100191, China}

\textsuperscript{(b) Department of Physics and National Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China}

Abstract

We calculate the inclusive decay width of $\Upsilon \rightarrow l^+l^-$. Then we get the ratio $R_{\tau\mu} = \Gamma[\Upsilon \rightarrow \tau^+\tau^-]/\Gamma[\Upsilon \rightarrow \mu^+\mu^-]$ to $O(\alpha)$ and $O(\alpha_s^2)$ within the Standard Model(SM). Comparing with the recent Babar’s data $R_{\tau\mu} = 1.005 \pm 0.013 \pm 0.022$, we find that SM prediction $R_{\tau\mu}$ is not consistent with the experimental data in the error bar. The discrepancy is about 1.25$\sigma$. So leptonic decay of $\Upsilon$ may be a possible signature of New Physics(NP). We present a better approach to test the Standard Model, $R_{\tau\mu}(E_{soft}) = \Gamma[\Upsilon \rightarrow \tau^+\tau^- + X]/\Gamma[\Upsilon \rightarrow \mu^+\mu^- + X]|_{E_X < E_{soft}}$. After resumming the large logarithms, we get $R_{\tau\mu}(E_{soft})$ at the precision level of 0.1%. It can be compared with experimental data more precise. We also consider the impact of $R_{\tau\mu}(E_{soft})$ and $R_{\tau\mu}$ from light Higgs $h$ and pseudoscalar Higgs $A_0$. 


I. INTRODUCTION

Although the Standard Model (SM) of particle physics describes the interactions of elementary particles very successfully, it is believed that SM is not the final theory and there should be New Physics (NP) beyond SM. So the hunting of NP is one of the hottest topics for theorist and experimentalist. The B factories gave a very clear channel to test SM, just as $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon \rightarrow l^+l^-$ ($l = \tau, \mu$). Recent Babar measured the ratio $R_{\tau\mu} = \frac{Br[\Upsilon \rightarrow \tau^+\tau^-]}{Br[\Upsilon \rightarrow \mu^+\mu^-]} = 1.005 \pm 0.013 \pm 0.022$, \hfill (1)

where branch ratio $Br[\Upsilon \rightarrow \tau^+\tau^- (\mu^+\mu^-)]$ is corresponded to inclusive decay width. The final states radiations (FSR) effects due to photon(s) and gluon(s) are taken into account in MC generator. The Leading Order (LO) SM prediction of $R_{\tau\mu}$ is 0.992 \hfill [3, 4]. It is consistent with experimental data. Then Babar claimed “No significant deviation of the ratio $R_{\tau\mu}$ from the SM expectation is observed”.

Theoretically, the high order corrections of the ratio $R_{\tau\mu}$ should be taken into account. The SM predictions should be compared with experimental data beyond tree level. At the same time, $R_{\tau\mu}$ is sensitively on the coupling of $h(A_0)bb$ and $h(A_0)l^+l^-$ within NP. It is an excellent probe for the new Higgs interactions in some NP Model, where the coupling of Higgs $bb$ and Higgs $l^+l^-$ is enhanced \hfill [5]. Then we should calculate the ratio $R_{\tau\mu}$ and compare with the experimental data to test SM or hunt NP.

There are some theoretical and experimental works related with leptonic decay of $\Upsilon$. The Quantum Chromodynamics (QCD) corrections of $\Upsilon \rightarrow l^+l^-$ have been calculated to two-loop \hfill [6]. We have calculated $\Upsilon$ decay to charm jet \hfill [7]. The leptonic decay of vector bosons has been calculated to Next-to-Leading Order (NLO) in Quantum Electrodynamics (QED) \hfill [8]. The CLEO got the ratio $R_{\tau\mu} = 1.02\pm0.02\pm0.05$ in 2006 \hfill [9]. The MC simulation of $\Upsilon \rightarrow l^+l^-$ has been studied, where large logarithms have been resummed \hfill [10]. The pseudoscalar Higgs $A_0$ is also introduced in those processes \hfill [3, 4, 11, 12]. Babar has searched for a light Higgs boson $A_0$ in the radiative decay of $\Upsilon(nS) \rightarrow \gamma A_0$, $A_0 \rightarrow l^+l^-$ for $n = 1, 2, 3$. They found no evidence for such processes in the mass range $0.212GeV \leq M_{A_0} \leq 9.3GeV$ and no narrow structure with $4.03GeV \leq M_{\tau^+\tau^-} \leq 10.10GeV$ \hfill [13, 14].

In this paper, we calculate the inclusive decay width of $\Upsilon \rightarrow l^+l^-$. Then we get the precise prediction within SM. We also consider the impact from light Higgs $h$ and pseudoscalar Higgs $A_0$. 


II. SM PREDICTION

The LO QED Feynman diagrams of $\Upsilon \rightarrow l^+ l^-$ are shown in Fig. 1. Followed the process of $\Upsilon \rightarrow c \bar{c}$ in Ref. [7], we can get the LO amplitude and decay width of $\Upsilon \rightarrow l^+ l^-$,

$$M_{LO}[\Upsilon \rightarrow l^+ l^-] = \sqrt{\frac{16\pi}{3M^2}} \alpha |R(0)| \bar{l} \neq l, \tag{2}$$

$$\Gamma_{LO}[\Upsilon \rightarrow l^+ l^-] = \frac{4|R(0)|^2\alpha^2\sqrt{1 - 4r_l(1 + 2r_l)}}{9M^2}, \tag{3}$$

where $r_l = M_l^2/M^2$, $|R(0)|$ is the radial wave function of $\Upsilon$ at origin, $\epsilon$ is the polarization vector of $\Upsilon$. If expanded with $r_l$, we can get

$$\Gamma_{LO}[\Upsilon \rightarrow l^+ l^-] = \frac{4|R(0)|^2\alpha^2}{9M^2} \left(1 - 6r_l^2 + O(r_l^3)\right). \tag{3}$$

We take into account the NLO QED correction here. The renormalization of lepton and $b$ quark wave function, and electron charge should appear. We use $D = 4 - 2\epsilon$ space-time dimension to regularize the divergence. On-mass-shell (OS) scheme is selected for $Z_{2b(l)}$ and modified minimal-subtraction ($\overline{MS}$) scheme for $Z_e$:

$$\delta Z_{2f}^{OS} = -\frac{Q_f^2\alpha}{4\pi} \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3\ln\frac{4\pi\mu^2}{M^2} + 4\right],$$

$$\delta Z_{e}^{\overline{MS}} = \frac{\alpha}{6\pi} \left(3 + \frac{10}{3}\right) \left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi)\right), \tag{4}$$

where $\mu$ is the renormalization scale, $\gamma_E$ is the Euler’s constant, $f = b, l$, and $Q_f$ is the charge of fermion $f$ in unit of electron charge. The factor $3 + \frac{10}{3}$ is from the charge and color factor of three flavor lepton $e, \mu, \tau$ $(3 \times 1)$ and four flavor quark $u, d, s, c$ $(2 \times 3 \times (1/9 + 4/9))$. If we ignore the self energy of photon and the renormalization of $\alpha$, the NLO QED correction is just replaced $4\alpha_s/3$ with $\alpha$ from $\Upsilon \rightarrow c\bar{c}$ [7].
TABLE I. The numerical decay width of inclusive processes $\Upsilon \rightarrow l^+l^-(l = \tau, \mu)$ in unit of $|\Gamma(0)|^2 / 10^7 \text{GeV}^2$ and $R_{\tau\mu}$ within SM.

|       | $\Gamma[\tau]$ | $\Gamma[\mu]$ | $R_{\tau\mu}$  |
|-------|-----------------|----------------|----------------|
| LO    | 2.822           | 2.844          | 0.992          |
| NLO QED | 2.777          | 2.798          | 0.993          |
| NLO QED, $l^+l^-gg$ | 2.780      | 2.836          | 0.980          |
| NLO QED\&QCD, $l^+l^-gg$ | 1.743   | 1.791          | 0.973 ± 0.001  |
| Babar | -               | -              | 1.005 ± 0.026  |

For the corrections due to gluons in the final state are considered in experimental Monte Carlo, we should consider the QCD processes $\Upsilon \rightarrow l^+l^- + gg$. We also consider NLO QCD corrections to the decay width of $\Upsilon \rightarrow l^+l^-$, which give a factor of $1 - 4C_f\alpha_s/\pi$ to suppress the LO decay width [6].

In numerical calculation, the parameters are selected as:

$$M_e = 0.5110 \text{MeV}, \quad M_d = 0.00 \text{MeV}, \quad M_u = 0.00 \text{MeV},$$

$$M_\mu = 0.1057 \text{GeV}, \quad M_s = 0.10 \text{GeV}, \quad M_c = 1.30 \text{GeV},$$

$$M_\tau = 1.7768 \text{GeV}, \quad M_b = 4.73 \text{GeV}, \quad \alpha = 1/132.33.$$  \hspace{1cm} (5)

Here $M_b = M_\Upsilon / 2$. The renormalization scale $\mu$ is selected as $\mu = M_\Upsilon$, and the fine structure constant $\alpha$ is calculated with the program alphaQED.f [15]. The numerical $\Gamma[\tau(\mu)]$ and $R_{\tau\mu}$ are listed in Table I. The LO prediction of $R_{\tau\mu}$ is 0.992. It is used in Ref. [1, 2], where claimed “No significant deviation of the ratio $R_{\tau\mu}$ from the SM expectation is observed”. But the QCD corrections should suppress the SM prediction and drive $R_{\tau\mu}$ away from the experiment data.

We should calculate the uncertainty for the theoretical prediction. As an order estimate, one can get $R_{\tau\mu}^{\text{LO}} \sim \mathcal{O}((\alpha/\pi)^0)$. For the NLO QED corrections have been taken into account, the uncertainty from higher order QED contributions is $\mathcal{O}(\alpha^2/\pi^2) \sim 6 \times 10^{-6}$. In the same way as QED, the uncertainty from higher order QCD contributions is $\mathcal{O}(\alpha_s^3/\pi^3) \sim 3 \times 10^{-4}$. $Z$ can contribute to $\Upsilon \rightarrow l^+l^-$ at tree level through replacing photon with $Z$. We can get

$$\frac{\mathcal{M}^Z_{LO}[\Upsilon \rightarrow l^+l^-]}{\mathcal{M}^\Upsilon_{LO}[\Upsilon \rightarrow l^+l^-]} = f_Z \frac{[4 \sin^2 \theta_W - 1][\epsilon^+ \epsilon^+^5]}{\frac{1}{l} \frac{1}{l}}.$$  \hspace{1cm} (6)
\[ f_z = \frac{M_T^2 (3 - 4 \sin^2 \theta_W)}{16 (M_T^2 - M_Z^2) (1 - \sin^2 \theta_W) \sin^2 \theta_W}. \] (7)

Here \( f_z \sim -M_Z^2/M_T^2 \sim -10^{-2} \). The vector current term should change the LO amplitude in Eq.(2) by a factor \( f_z (-1 + 4 \sin^2 \theta_W) \sim 10^{-3} \). It is just like replacing \( \alpha \) with \( \alpha (1 - f_z (1 - 4 \sin^2 \theta_W)) \) in Eq.(2) but it is a global factor for three lepton at LO. Then the uncertainty from \( Z \) of \( R_{\tau \mu} \) should be \( \mathcal{O}(f_z (1 - 4 \sin^2 \theta_W) (R_{\tau \mu}^{QED} - R_{\tau \mu}^{LO})) \sim \mathcal{O}(10^{-6}) \). Here superscript \( QED \) means NLO QED has been taken into account. The axial vector current is not coherent with the vector current in Eq.(2). It change the width with a factor \( O(\tau \mu) \) means NLO QED has been taken into account. The axial vector current is not coherent with the vector current in Eq.(2). It change the width with a factor \( O(\tau \mu) \)

Then the weak contributions from \( W^\pm, Z, H \) can be ignored safely. Within SM, it should be considered that \( \Upsilon \to \gamma \eta_b \), where \( \eta_b \to l^+l^- \) is followed [12]. The energy of \( \gamma \) is about 70 MeV in \( \Upsilon \to \gamma \eta_b \) and \( Br[\eta_b \to l^+l^- (+\gamma_{soft})] \sim 10^{-8} \) [10, 17]. For \( \Upsilon \to \gamma \eta_b \) is a P wave process, we can estimate \( Br[\Upsilon \to \gamma \eta_b] \) through

\[ \frac{\Gamma[\Upsilon \to \gamma \eta_b]}{\Gamma[J/\psi \to \gamma \eta_c]} \sim \left( \frac{e_b}{e_c} \right)^2 \left( \frac{M_{J/\psi}(M_\Upsilon - M_{\eta_b})}{M_\Upsilon(M_{J/\psi} - M_{\eta_c})} \right)^3. \] (8)

Then \( Br[\Upsilon \to \gamma \eta_b] \sim 10^{-5} \). So \( Br[\Upsilon \to \gamma \eta_b] \times Br[\eta_b \to l^+l^- (+\gamma_{soft})] \sim 10^{-12} \). This can be ignored safely. The uncertainties of \( R_{\tau \mu} \) within SM are listed in Tab.II. Then SM prediction is

\[ R_{\tau \mu} = 0.973 \pm 0.001. \] (9)

Compared with Eq.(11), it is not consistent with the experimental data in the error bar. The discrepancy is about 1.25\( \sigma \).

The QCD contributions have been taken into account in Eq.(9). It is difficult to measure. So we present a better approach to test the Standard Model, \( R_{\tau \mu}(E_{soft}) = \Gamma[\Upsilon \to \tau^+ \tau^- + X]/\Gamma[\Upsilon \to \mu^+ \mu^- + X]|_{M_X < E_{soft}} \). If we select \( E_{soft} \sim 5 GeV \), \( \Gamma[\Upsilon \to l^+l^- + gg]|_{M_X < E_{soft}} \) is less than \( \Gamma[\Upsilon \to l^+l^-]/1000 \), then the impact on \( R_{\tau \mu}(E_{soft}) \) is less than \( 2 \times 10^{-5} \), but the large logarithms appear

\[ L = \ln \frac{4E^2_s}{M_T^2} \ln \frac{4M_T^2}{M_T^2}. \] (10)

We resum the large logarithms with YFS resummation scheme[10, 18],

\[ Y = -\frac{\alpha}{\pi} \left( 2 (\ln r_l + 1) \ln \frac{2E_s}{M_T} + \ln r_l \frac{\pi^2}{3} + 1 \right). \] (11)
TABLE II. The uncertainties of $R_{\tau\mu}$ within SM.

| Order      | Numerical |
|------------|-----------|
| QED        | $\alpha^2/\pi^2$ | $6 \times 10^{-6}$ |
| QCD        | $\alpha_s^3/\pi^3$ | $3 \times 10^{-4}$ |
| $Z(W^\pm, H)$ | $M^2_{W}/M^2_\tau$ or $\alpha M^2_\tau/(M^2_\tau \pi)$ | $4 \times 10^{-6}$ |
| $\eta_b$  | $Br[\Upsilon \rightarrow \gamma \eta_b] \times Br[\eta_b \rightarrow l^+l^-]$ | $1 \times 10^{-12}$ |

The resumed results are

$$\Gamma^{res}_{LO} = e^Y \Gamma_{LO},$$

$$\Gamma^{res}_{NLO} = (e^Y - 1 - Y) \Gamma_{LO} + \Gamma_{QED}. \quad (12)$$

After the large logarithms are resummed, we get $R_{\tau\mu}(E_{soft})$ with a soft cut at the precision level of 0.1%. The numerical $\Gamma_{\tau\mu}(E_{soft})$ in unit of $R(0)_{10^6 GeV^2}$ and $R_{\tau\mu}(E_{soft})$ with different energy cut $E_s$ are listed in Table. The dependence of $R_{\tau\mu}(E_{soft})$ on the soft cut $E_s$ is shown in Fig.2. If we select $E_s = 0.2 GeV$. Including the uncertainty, the ratio is

$$R_{\tau\mu}(0.2 GeV) = 1.0628 \pm 0.0011. \quad (13)$$

The effect of QCD is very weak in this channel. $R_{\tau\mu}(E_{soft})$ can be compared with experimental data more precise.

III. IMPACT FROM NP

NP may play a role in the discrepancy between theoretical prediction and experimental data of $R_{\tau\mu}$ in Eq.(9) and Eq.(11). We only consider the scheme of light Higgs $h$ and pseudoscalar Higgs $A_0$ here.

The Feynman rules are $ieM_f C_h/(2M_W \sin \theta_W) \bar{f} f$ and $-eM_f C_{A0}/(2M_W \sin \theta_W) \bar{f} f$ for $hf \bar{f}$ and $A_0 f \bar{f}$ vertex respectively, here $f = l, b$. $C_{A0(h)}$ are different in the special model, we consider them as parameters. For it is IR finite which $A_0(h)$ involved in $\Upsilon \rightarrow \gamma_{soft} l^+ l^-$, so its contributions are suppressed by $E_s/M_b \sim 4 \times 10^{-2}$ when compared with virtual processes. So
FIG. 2. The dependence of \( R_{\tau\mu}(E_{\text{soft}}) \) on the soft cut \( E_s \) within SM.

| \( E_s \) (GeV) | \( \Gamma[\tau] \) | \( \Gamma[\mu] \) | \( R_{\tau\mu}(E_{\text{soft}}) \) |
|----------------|----------------|----------------|-------------------|
| LO            | 2.8221         | 2.8444         | 0.9922            |
| LOYFS\( E_s=0.10 \) | 2.7277         | 2.4925         | 1.0944            |
| NLO\( E_s=0.05 \) | 2.6744         | 2.3932         | 1.1174            |
| NLOYFS\( E_s=0.05 \) | 2.6768         | 2.4272         | 1.1028            |
| NLO\( E_s=0.10 \) | 2.6954         | 2.4678         | 1.0922            |
| NLOYFS\( E_s=0.10 \) | 2.6970         | 2.4916         | 1.0824            |
| NLO\( E_s=0.20 \) | 2.7158         | 2.5411         | 1.0688            |
| NLOYFS\( E_s=0.20 \) | 2.7168         | 2.5564         | 1.0628            |
| NLO\( E_s=0.45 \) | 2.7385         | 2.6236         | 1.0438            |
| NLOYFS\( E_s=0.45 \) | 2.7389         | 2.6312         | 1.0409            |

we ignored the real processes and included the virtual processes only when we considered the impact of \( A_0(h) \) to \( R_{\tau\mu}(E_{\text{soft}}) \). The Feynman diagrams are shown in Fig.3. The Feynman diagrams which exchange \( A_0(h) \) between \( b\bar{b} \) are ignored for it should not change the ratio \( R_{\tau\mu} \). Compared with \( \Gamma[\Upsilon \to \tau^+\tau^-] \), the impact of \( A_0(h) \) to \( \Gamma[\Upsilon \to \mu^+\mu^-] \) is suppressed by \( M_\mu^2/M_\tau^2 \), for the \( A_0(h)l^+l^- \) coupling and the spin flip between \( \bar{l}\gamma\nu l^- \) and \( \bar{l}\gamma^5l^- \) (\( l\bar{l} \)) are both
FIG. 3. Part of the Feynman diagrams of $\Upsilon \to l^+l^-$ which $A_0(h)$ involved. The Feynman diagrams which exchange $A_0(h)$ between $b\bar{b}$ are ignored for it should not change the ratio $R_{\tau\mu}$.

FIG. 4. The $A_0(h)$ impact on $\Upsilon \to \tau^+\tau^-$ as a function of $M_{A_0(h)}$. The $A_0(h)$ impact on real contributions ignored for it is suppressed by $E_s/M_b$ and $\Upsilon \to \mu^+\mu^-$ is ignored for it is suppressed by $M_\mu^2/M_\tau^2$. The Feynman diagrams which exchange $A_0(h)$ between $b\bar{b}$ are ignored for it should not change the ratio $R_{\tau\mu}$.

proportional to $M_l$. So we ignore the contributions for $\Upsilon \to \mu^+\mu^-$. Only $\Upsilon \to A'^*_0\gamma^* \to \tau^+\tau^-$ and $\Upsilon \to h^*\gamma^* \to \tau^+\tau^-$ are taken into account. The contributions with $A'^*_0A'^*_0$, $h^*h^*$, or $A'^*_0h^*$ in the loop are zero for $J^{PC}$. The numerical result of the $A_0(h)$ impact as a function of $M_{A_0(h)}$ from the loop Feynman diagram is shown in Fig. 4. The $A_0(h)$ impact on $R_{\tau\mu}$ is $R_{\tau\mu}^{LO}\Gamma^{A_0(h)}[\tau]/\Gamma^{LO}[\tau]$.

If we consider the $R_{\tau\mu}$, we should include the real correction too. If we select $10.3 GeV < M_{A_0(h)} < 10.6 GeV$, $\Gamma^{A_0}[\tau]/\Gamma^{LO}[\tau] \sim -4 \times 10^{-6}C_{A_0}^2 + 5 \times 10^{-10}C_{A_0}^4$, and $\Gamma^h[\tau]/\Gamma^{LO}[\tau] \sim 3 \times 10^{-6}C_h^2 + 8 \times 10^{-10}C_h^4$. 

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IV. SUMMARY

In summary, we calculate the inclusive decay width of $\Upsilon \to l^+ l^- \ (l = \tau, \mu)$. then we get the ratio $R_{\tau\mu} = \frac{\Gamma[\Upsilon \to \tau^+ \tau^-]}{\Gamma[\Upsilon \to \mu^+ \mu^-]}$ to $O(\alpha)$ and $O(\alpha_s^2)$ within SM. Compared with the recent Babar’s data $R_{\tau\mu} = 1.005 \pm 0.013 \pm 0.022$, we find that SM prediction $R_{\tau\mu} = 0.973 \pm 0.001$ is not consistent with the experimental data. The discrepancy is about $1.25 \sigma$. So leptonic decay of $\Upsilon$ may be a possible signature of NP. We present a better approach to test the Standard Model, $R_{\tau\mu}(E_{soft}) = \frac{\Gamma[\Upsilon \to \tau^+ \tau^- + X]}{\Gamma[\Upsilon \to \mu^+ \mu^- + X]}|_{E_X < E_{soft}}$. After resumming the large logarithms, we get $R_{\tau\mu}(E_{soft})$ with a soft cut at the precision level of $0.1\%$. The effect of QCD is very weak in this channel. It can be compared with experimental data more precise. We also consider the possible solution, light Higgs $h$ and pseudo scalar Higgs $A_0$. To clarify the discrepancy, more work should be done by theorist and experimentalist.

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