Quantum Lenoir Engine with a Single Particle System in a One Dimensional Infinite Potential Well

Yohanes Dwi Saputra

Physics Department, Institut Teknologi Kalimantan, Jln. Soekarno-Hatta Km 15, Balikpapan, Indonesia

Email: yohanesngawi@lecturer.itk.ac.id

(Accepted 21 Agustus 2019; Approved 1 Oktober 2019; Published 30 November 2019)

Abstract

Lenoir engine based on the quantum system has been studied theoretically to increase the thermal efficiency of the ideal gas. The quantum system used is a single particle (as a working fluid instead of gas in a piston tube) in a one-dimensional infinite potential well with a wall that is free to move. The analogy of the appropriate variables between classical and quantum systems makes the three processes for the classical Lenoir engine applicable to the quantum system. The thermal efficiency of the quantum Lenoir engine is found to have the same formulation as the classical one. The higher heat capacity ratio in the quantum system increases the thermal efficiency of the quantum Lenoir engine by 56.29% over the classical version at the same compression ratio of 4.41.

Keywords: compression ratio, potential well, thermal efficiency, quantum Lenoir engine

1. Introduction

The heat engine is known as a device that acts to convert heat energy into work with a certain thermal efficiency. This engine is inseparable from the second law of thermodynamics, where the engine can not entirely convert the heat energy entering the system into work, but there is residual heat wasted into the environment [1, 2]. This opens up opportunities for much research in efforts to obtain alternative systems for heat engines other than gas in the piston tube and efforts to improve the efficiency of heat engines [3].

Quantum physics offers a quantum system as a substitute for the classical system to improve the thermal efficiency of a classical system engine. Quantum systems used are potential wells [4, 5, 6, 7, 8, 9, 10, 11, 12, 13], quantum harmonic oscillators [14, 15], and others.

Lenoir engine is an internal combustion engine which was first thought to be mass-produced. This engine works based on three thermodynamic processes, namely isochoric, adiabatic expansion, and isobar compression in one cycle [16]. The existing classical Lenoir engine needs to be improved in its efficiency. This research offers a quantum system in the form of a single particle trapped in an infinite one-dimensional potential well instead of gas in a piston tube. A particle in this potential well is an interpretation of copies of an infinite number of particles where each has its potential well [17]. This study opens the opportunity for other studies on the application of quantum systems, especially in the Lenoir engine, to achieve increased thermal efficiency.

2. Theoretical Model

The theoretical model used as a substitute for the classical system, gas in a piston tube, is a single particle of mass m confined in a one-dimensional infinite potential well of width L, as shown in Figure 1. The behavior of a single particle at $0 < x < L$ is represented by one-dimensional independent time Schrödinger equation,

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}. \quad (1)$$

With the given boundary conditions at both ends of the potential wall, $\psi(0) = \psi(L) = 0$, the solution of

![Figure 1. A particle in a one-dimensional infinite potential well as a quantum system.](image)
equation (1) describes the behavior of particles in the walls,

\[ \phi_n(x) = \frac{2}{L} \sin \frac{n\pi x}{L} \]  

with quantum number \( n = 1, 2, 3, \ldots \) and \( L \) is the width of the potential well from 0 to \( \infty \). The total energy of each state of the \( n \)-th is

\[ E_n(L) = \frac{n^2\pi^2\hbar^2}{2mL^2}. \]  

The wave function of a particle as a superposition of states is

\[ \psi(x) = \sum_n a_n \phi_n(x) \]  

with an average total energy

\[ E(L) = \langle \psi(x) | \hat{H} | \psi(x) \rangle = \sum_n |a_n|^2 \frac{n^2\pi^2\hbar^2}{2mL^2}. \]  

The potential well is driven by mechanical force according to the equation

\[ F(L) = -\frac{dE(L)}{dL} = -\sum_n |a_n|^2 \frac{n^2\pi^2\hbar^2}{mL^3}. \]  

Equation (6) means that the potential well driving force depends on the quantum number \( n \) achieved by the particle and the width of the potential well. The higher the value of \( n \) causes \( F(L) \) to increase, the wider the potential well will reduce \( F(L) \).

The relationship between classical and quantum systems is based on the analogy of the variables originally proposed by Bender et al. [4], according to Table 1.

**Table 1.** Correspondence of variables from the classical system to the quantum system.

| Num. | Classical System | Quantum System |
|------|------------------|----------------|
| 1    | Pressure (P)     | Mechanical Force of Potential Well (\( F \)) |
| 2    | Volume (V)       | Width of Potential Well (\( L \)) |
| 3    | Temperature (T)  | Total Energy Expectation Value (\( E \)) |

Given the analogy of these variables, the curve of gas pressure as a function of the tube volume is analogous to the graph of the potential well mechanical force as a function of the potential well width, according to Figure 2. However, in ideal conditions that are reversible, every thermodynamic process is assumed to take place in a quasi-static state which occurs in a very long time [3,17], so that each point or intermediate states that make up a process is in a state of thermodynamic equilibrium (mechanical equilibrium, thermal equilibrium, and chemical equilibrium) all the time. This leads to equations (2) and (3) still considered satisfying [3].

Point A is chosen as the starting point. The particle is assumed to be in a ground state (\( n = 1 \)) with a wave function \( \phi_1 \) and expectation value of total energy \( E_1 \).

The first step (A→B) is an isochoric process where the particle condition changes from a ground state (\( n = 1 \)) to an excited state with the quantum number \( n \). Superposition of two-particle states occurs to form a wave function

\[ \psi_{AB}(x) = a_1 \phi_1(x) + a_n \phi_n(x) \]  

with the total energy expectation value

\[ E_{AB}(L) = |a_1|^2 E_1(L) + |a_n|^2 E_n(L). \]  

The heat energy that enters the system from a heat bath is equal to the total energy difference between the two states of the particle from points A and B,

\[ Q_{in} = Q_{AB} = (n^2 - 1) \frac{\hbar^2}{2mL_c^2}. \]  

The width of the potential well does not change, so there is no work at this step. All incoming heat energy is converted to increase the energy level of a single particle from \( n = 1 \) to \( n = n \).

The second step (B→C) is an adiabatic expansion process that does not change the probability of each state so that the particle remains in the excited state with \( n = n \),

\[ \psi_{BC}(x) = \phi_n(x), \]  

with the total energy expectation

![Figure 2. The curve of F(L) on the quantum system.](image)
Based on equation (6, a potential wall mechanical force reads

\[ F_{BC}(L) = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \] (12)

so the work done by the quantum system to the surrounding is

\[ W_{BC} = \int_{L_B}^{L_A} F_{BC} dL = \frac{n^2 \pi^2 \hbar^2}{2m} \left( \frac{1}{L_A^2} - \frac{1}{L_C^2} \right) \] (13)

The third step (C→A) is an isobar compression process in which a single particle from an excited state with the quantum number \( n \) returns to the ground state (\( n = 1 \)) according to the wave function

\[ \psi_{CA}(x) = a_1 \phi_1(x) + a_n \phi_n(x). \] (14)

The normalized condition of equation (14 results in a relationship of two complex constants

\[ |a_n|^2 = 1 - |a_1|^2 \] (15)

so the expectation value of the total energy in this process can be expressed as

\[ E_{CA}(L) = [n^2 + |a_1|^2(1 - n^2)] \frac{\pi^2 \hbar^2}{2mL^2}. \] (16)

The mechanical force acting on the wall potential is obtained for

\[ F_{CA}(L) = [n^2 + |a_1|^2(1 - n^2)] \frac{\pi^2 \hbar^2}{mL^3}. \] (17)

During this isobar process, mechanical force is always constant \((F_C = F_{CA})\) to give a relation

\[ L = \left[ \frac{n^2 + |a_1|^2(1 - n^2)}{n^2} \right]^3 L_C. \] (18)

According to equation (18), the width of the potential well is valued from \( L_C \) to \( L_A = (1/n)^{2/3} L_C \) in this step. Thus the surrounding work to compress the width of the well is calculated

\[ W_{CA} = \int_{L_C}^{L_A} F_{CA} dL = \frac{n^2 \pi^2 \hbar^2}{mL_C^2} \left( \frac{1}{n} \right)^{2/3} - 1 \right. \] (19)

The decrease in the total energy level of a single particle from a state with a quantum number \( n \) to the ground state indicates that the energy released into the cold bath. According to the first thermodynamics law, we have

\[ |Q_{out}| = |Q_{CA}| = 3 \left( \frac{2}{n^3} - 1 \right) \frac{\pi^2 \hbar^2}{2mL_C^2} \] (20)

The total work that arises in one Lenoir cycle is the sum of work for the three steps. We find

\[ W_t = \left[(n^2 - 1) n^4 - 3 \left( n^2 - n^3 \right) \right] \frac{\pi^2 \hbar^2}{2mL_C^2}. \] (21)

The thermal efficiency of the quantum Lenoir engine is raised by dividing equation (21) by equation (9),

\[ \eta = 1 - 3 \frac{n^2 - 1}{n^2 - 1} = 1 - \frac{\gamma r - 1}{\gamma - 1} \] (22)

Where \( r \) is the compression ratio between \( L_C \) and \( L_A \)

\[ r \equiv \frac{L_C}{L_A} = n^\frac{2}{3}, \] (23)

while \( \gamma \) is the heat capacity ratio to a quantum system, a particle in an infinite potential well, which has a value of 3.

3. Result and Discuss

Making use of equation (22), we get a graph of the thermal efficiency as a function of the excited state with \( n = n \) (Figure 3). As is seen, the higher the \( n \)-th state the particle can achieve, the greater the thermal efficiency value.

If equation (22) is compared with the classical version of the Lenoir engine, the same thermal efficiency formulations are observed. The difference is in the value of the heat capacity ratio wherein the quantum version is 3 while in the classical version it is 1.4 with air as working fluid. Figure 4 sdmas a graph of the thermal efficiency of the two versions at five different compression ratio values. It can be observed that the quantum version of the thermal efficiency is higher than the thermal efficiency of its classical counterpart on the same compression ratio. This is due to the higher value of

![Figure 3. Curve \( \eta(n) \) of quantum Lenoir engine.](image-url)
the heat capacity ratio of the quantum version than the classical version. Increasing the value of the compression ratio successively 1.4, 2.3, 4, 4.41 to 5 is also followed by an increase in the value of thermal efficiency in the quantum version. This is closely related to equation (23) which indicates that the higher the engine compression ratio, the higher the excited state that can be achieved by a single particle.

Based on the equation (22, the thermal efficiency difference of the quantum Lenoir engine \( \gamma_0 = 3 \) and the classical version \( \gamma_c = 1.4 \) can be defined as

\[
\Delta \eta \equiv \eta_Q - \eta_C = \gamma_C \frac{r - 1}{\gamma_C r - 1} - \gamma_Q \frac{r - 1}{\gamma_Q r - 1}
\]

(24)

illustrated in Figure 5. The efficiency difference of both the engine reaches a maximum of 56.29% at the same compression ratio \( r = 4.41 \).

4. Conclusion

From the results and discussion presented in the previous section, it implies that the higher the excited state that can be achieved by a particle, the higher the thermal efficiency of the quantum Lenoir engine. This can be realized by increasing the compression ratio of the quantum Lenoir engine. At the same compression ratio, the quantum Lenoir engine is more efficient than the classical version due to the higher heat capacity ratio in the quantum system compared to the classical system. The quantum system used can increase the efficiency of the Lenoir engine up to 56% at a compression ratio of 4.41.

5. Acknowledgment

The author acknowledges LPPM ITK for funding this research by contract no. 610/IT10.III/PPM.01/2019.

References

[1] Zemansky, M. W. and Dittman, R. D., Heat and Termodynamics,McGraw-Hill Companies, Inc, 1997.

[2] Cengel, Y. A. and Boles, M. A.,Thermodynamics: an Engineering Approach, McGraw-Hill Education, 2015.

[3] Quan, H. T. , Liu, Y. , Sun, C. P., and Nori, F. , Quantum thermodynamic cycles and quantum heat engines, Physical Review E, 76(3), pp.1–19, 2007.

[4] Bender, C. M., Brody, D. C., and Meister, B. K., Quantum mechanical Carnot engine, Journal of Physics A: Mathematical and General, 33(24), pp.4427–4436, 2000.

[5] Purwanto, A., Sukamto, H., Subagyo, B. A. , and Taufiqi, M. , Two Scenarios on the Relativistic Quantum Heat Engine, Journal of Applied Mathematics and Physics, 04(07), pp.1344–1353, 2016.

[6] Sukamto, H., Purwanto, A, and Subagyo, B. A., Mesin Panas Kuantum Partikel Relativistik pada Sumur Potensial 2 Dimensi, Jurnal Fisika dan Aplikasinya, 10(2), pp.103, 2014.

[7] Latifah, E. and Purwanto, A, Multiple-State Quantum Carnot Engine, Journal of Modern Physics, 02(11), pp.1366–1372, 2011.

[8] Latifah, E. and Purwanto, A, Quantum Heat Engines; Multiple-State 1D Box System, Journal of Modern Physics, 04(08), pp.1091–1098, 2013.

[9] Akbar, M. S., Latifah, E., Qomariyah,S. N., Setyo, D. P., Wisodo, H. , and Hidayat, A., Proses Adiabatis dan Isovolume Kuantum Sistem Dua Partikel Simetri, JPSE (Journal of Physical Science and Engineering), 2(2), pp.55–65, 2018.
[10] Akbar, M. S., Latifah, E., and Wisodo, H., Limit of Relativistic Quantum Brayton Engine of Massless Boson Trapped 1 Dimensional Potential Well, Journal of Physics: Conference Series, 1093(1), 2018.

[11] Setyo, D. P., Latifah, E., Hidayat, A., and Wisodo, H., Quantum Relativistic Diesel Engine with Single Massless Fermion in 1 Dimensional Box System, Jurnal Penelitian Fisika dan Aplikasinya (JPFA), 8(1), pp.25, 2018.

[12] Setyo, D. P. and Latifah, E., Quantum Otto Engine based on Multiple-State Single Fermion in 1D Box System, Journal of Physics: Conference Series, 1093(1), 2018.

[13] Singh, S., Quantum Braton Engine of Non-Interacting Fermions in One-Dimensional Box(August), 2019.

[14] Kosloff, R. and Rezek, Y., The quantum harmonic otto cycle, Entropy, 19(4), pp.1–36, 2017.

[15] Jaramillo, J., Beau, M., and Campo, A. Del, Quantum supremacy of many-particle thermal machines, New Journal of Physics, 18(7), 2016.

[16] Balmer, R. T., Modern Engineering Thermodynamics, Academic Press, 2011.

[17] Bender, B. C. M., Brody, D. C., and Meister, B. K., Entropy and Temperature of a Quantum Carnot Engine, Royal Society, 2002.