Revivals and oscillations of the momentum of light in a planar multimode waveguide

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Abstract

The evolution of the transverse momentum of monochromatic light entering a multimode planar waveguide at large angle is investigated. We report on oscillations of the momentum caused by the beatings between the adjacent populated modes of the waveguide and their periodic collapses and revivals. A new type of an interferometer based on this effect with fringe spacing as small as $\lambda/9$ is demonstrated experimentally and periods as small as $\lambda/1000$ seem to be feasible.
The propagation of light along a planar multimode waveguide is one of the fundamental questions of wave optics. This problem is closely related to the evolution of atomic wavepackets in square potential wells [1]. As it was shown in [2], [3] it is also related to the propagation of light in a periodic grating. The situation, when the monochromatic light enters the waveguide with no average transverse momentum, was studied in [3], [4].

This letter considers the case, when the monochromatic light beam enters the planar metal multimode waveguide at some large angle. In this case the light experiences not only full and fractional revivals [3] of its initial phase distribution along the waveguide, but also additional oscillations of the transverse momentum. This is related to the beatings between the adjacent populated transverse modes of the waveguide. These oscillations are similar to the Pendellösung oscillations [5] for the Bragg reflection of waves at diffraction gratings, which were observed in many different systems [6], [7].

Based on this effect, a new type of interferometer with a fringe spacing much smaller than the wavelength of light has been demonstrated.

The basic setup is shown in Fig. 1. The waveguide is realized by two metal mirrors placed parallel to each other at distance $d$. The plane monochromatic light wave enters the waveguide at an angle $\beta$. The amplitude of the transverse distribution of the light field at the front of the waveguide is $E_0(x, y < 0) = E_0 \exp (-ik \sin (\beta) x) \exp (-ik \cos (\beta) y)$, where $k = 2\pi/\lambda$ and $\lambda$ is the wavelength of the light. Therefore, the initial distribution has a constant amplitude and its phase changes linearly in $x$ direction. The modes propagating along the waveguide have to satisfy the condition of quantization of the transverse component of their wavevector

$$kd \sin \alpha_n = n\pi,$$

where $n=1,2,3...$ and $\alpha_n$ is the angle of the wavevector in the $n$-th mode with respect to the boundaries of the guide. The field amplitude inside the waveguide is given by

$$E(x, y) = \sum_{n=1}^{\infty} c_n \sin \left( \frac{n\pi}{d} x \right) \exp \left( -ik \sqrt{1 - \left( \frac{n\pi}{kd} \right)^2} y \right) \exp (-\gamma_n y).$$

The exponents give the phase and the loss of the $n$-th mode along the waveguide and $\sin (n\pi x/d)$ - is the distribution of the light amplitude in the $n$-th mode. The coefficients $c_n$ are determined by the projection of the initial distribution $E_0(x, y < 0)$ onto the modes of the waveguide

$$c_n = \frac{2}{d} \int_0^d \sin \left( \frac{n\pi}{d} x \right) E_0 (x, y < 0) \, dx,$$

which can be integrated analytically. The transverse momentum distribution of the light emerging from the waveguide can be found by the Fourier transform

$$E(k_x) \propto \frac{1}{d} \int_0^d E(x, y = L) \exp (-ik_x x) dx,$$

of the spatial distribution of the light amplitude $E(x, L)$ at the output of the waveguide, where $L$ is the length of the waveguide and $k_x$ is the transverse component of the light wavevector.
The numerical calculations of the light momentum distribution at the output of the waveguide based on equations (2-4) show quasi-periodic oscillations between the two main directions $\beta$ and $-\beta$ (see Fig. 1) as a function of the length $L$ of the waveguide. We shall label these two output beams respectively as transmitted and reflected beams. For incidence angles $\beta > \arcsin(\lambda/d)$ the two output beams are well separated from each other in momentum. The solid curve in Fig. 2 shows the calculated intensity of the transmitted light beam as a function of the waveguide length $L$. In these calculations we assumed large width of the waveguide $d >> \lambda$ and small angles of incidence $\sin(\beta) << 1$. We used a dimensionless relative length $s = L/L_r$, corresponding to the number of revival, or Talbot, or self imaging periods

$$L_r = \frac{8d^2}{\lambda}$$

(5)

and a constant propagation parameter $f = 48$,

$$f = \frac{L_r}{L_b} = \frac{4d \sin(\beta)}{\lambda},$$

(6)

which is equal to the ratio of the revival period $L_r$ to the period of light momentum beatings

$$L_b \simeq \frac{2d}{\sin(\beta)}.$$  

(7)

This is related to the situation when the waveguide width $d$ is fixed and its length $L$ is varied. For this setting there are only a few transverse modes of the waveguide significantly populated, which satisfy the condition $\alpha_e \simeq \beta$. The length $L_b$ corresponds to the period of beatings between these modes. The expression (7) is valid only for $d >> \lambda$ and $\sin(\beta) << 1$, which is the case for our experiment. Without this assumption the period of the beatings between two adjacent modes is

$$L_b = \lambda / \left( \sqrt{1 - \sin^2 \alpha_e^n} - \sqrt{1 - \sin^2 \alpha_e^{n+1}} \right).$$

(8)

Note that the period (7) can be found also from a zigzag geometrical ray propagation of the ingoing light beam through the waveguide. The corresponding classical oscillations of the intensity in the transmitted beam are shown in Fig. 2 by the dashed sinusoidal trace possessing a period $L_b$. However these classical oscillations do not depend on the light wavelength.

The real wave behavior (solid curve) is quite different and can be characterized by two main periods $L_r$ and $L_b$. Despite the fact, that the period of the momentum beatings is approximately the same as in the classical case, the location of the peaks is determined by the position of revival resonances and therefore depends on the wavelength of light. The amplitude of the beatings is modulated in such a way that they are maximal around the relative lengths of the waveguide $s_{\text{max}}^m = m/2$ and minimal around $s_{\text{min}}^m = 1/4 + m/2$, with $m = 0, 1, 2, ...$. It means the revivals and collapses of momentum state oscillations take place. For full revival lengths, when $m$ is even and $s_{\text{max}}^m = 0, 1, 2, ...$, the revivals of the initial light momentum take place and all light is concentrated in the transmitted output beam.
For half-revival lengths, when \( m \) is even and \( s_m^{\text{max}} = 1/2, 3/2, 5/2, \ldots \), the phase of the initial momentum distribution is inverted and all light is concentrated in the reflected beam. It can be seen from Fig. 2 that the phase of momentum oscillations at \( s = 1/2 \) is inverted with respect to their phase at \( s = 0 \), which is not the case for classical oscillations. From equations (5-7) it follows that the tuning of the waveguide from one transmission fringe to the next can be done by changing either the length or the width of the waveguide, or the light wavelength according to

\[
\frac{\delta L}{L} = \frac{2\delta d}{d} = \frac{\delta \lambda}{\lambda} = \frac{L}{s_f} \sim \frac{1}{s f} = \frac{2d}{L \sin (\beta)}.
\] (9)

One way to change the relative length \( s \) of the waveguide is to vary the width \( d \), while the length of the waveguide \( L \) fixed. As \( s = L\lambda/8d^2 \) is nonlinear in \( d \), the fringe spacing \( \delta d \) also changes nonlinearly in \( d \) as

\[
\delta d = \frac{d^2}{L \sin (\beta)}.
\] (10)

The maximum amplitudes of the momentum oscillations are located at half and full revival distances, which are characterized by the following widths of the waveguide

\[
d_m = \sqrt{\frac{L\lambda}{8s_m^{\text{max}}}} = \sqrt{\frac{L\lambda}{4m}},
\] (11)

where \( m \) is an integer.

Compared to all other types of interferometers, where the minimal fringe spacing is equal to \( \lambda/2 \), the distance \( \delta d \) can be much smaller than the light wavelength. This can be explained by the fact, that in a waveguide the interfering modes experience multiple reflections between the mirrors before their phase difference is observed.

Experimentally the waveguide was formed by two parallel flat mirrors of equal length \((L = 5 \text{ cm})\), coated with bare gold. The substrates were made of fused silica and polished to a surface figure \( \sim \lambda/20 \). All degrees of freedom of one of the mirrors were adjustable. This mirror was attached to a precise mechanical translational stage driven by a “COHERENT” Encoder Driver System 37-0486 which provided a resolution in translation of \( \sim 10 \text{ nm} \) and a total range of 13 mm in the \( x \) direction.

We used a He-Ne laser with wavelength \( \lambda = 633 \text{ nm} \), divergence about 1 mrad, linear polarisation, directed parallel to the mirrors, and a beam diameter of \( \sim 2 \text{ mm} \). In our first experiment the entrance edge of the waveguide was illuminated at an angle of \( \beta = 0.253 \text{ rad} \).

To detect the oscillations we used a photodiode, which was placed in the reflected output light beam (Fig. 1) at a distance \( D \sim 10 \text{ cm} \) behind the waveguide. Each measurement of the light intensity in the reflected beam as a function of the waveguide width \( d \) was done in a single sweep of the driven mirror. The corresponding dependencies are shown in Fig. 3. We have observed quasi-periodic oscillations of the light intensity in the reflected and the transmitted output light beam, shifted by \( \pi \) with respect to each other. This leads to the conclusion, that the output light momentum experiences quasi-periodic oscillations between
the two directions $\beta$ and $-\beta$. Fig. 3 also shows that these oscillations experience quasi-periodic collapses and revivals as a function of distance $d$. In full agreement with equation (10) the maximum amplitudes of the oscillations were observed at $d_1 \simeq 89\,\mu m$, $d_2 \simeq 63\,\mu m$, $d_3 \simeq 51.5\,\mu m$, $d_4 \simeq 44.5\,\mu m$, $d_5 \simeq 40\,\mu m$, $d_6 \simeq 36\,\mu m$, $d_7 \simeq 33.5\,\mu m$, $d_8 \simeq 31.5\,\mu m$ and $d_9 \simeq 29.7\,\mu m$, where half or full revivals take place.

The amplitude of the fringes become smaller with decreasing the width $d$ of the waveguide, which has several reasons. First, by extinction of the total light power coupled to the waveguide; second, by increasing the number of reflections inside the waveguide and the corresponding losses; third, for smaller width of the waveguide the divergence of the outgoing light beams becomes larger and the intensity decreases. The period of fringes is decreasing with reducing the width $d$ of the waveguide in full accordance with equation (9). For $m = 9$ ($d \simeq 30\,\mu m$) we have observed fringes with a period $\delta d \simeq 70\,\text{nm}$. These fringes are shown in the inset of Fig. 3c. For such a small fringe spacing the single steps of the Encoder Driver, which are of the order of 10 nm, were resolved.

To observe directly the oscillations of the output light between the two directions $\beta$ and $-\beta$ we used a CCD camera placed at distance $D \simeq 12\,\text{cm}$ behind the waveguide, which detected the transmitted and reflected output beams simultaneously. The detected transverse spatial distribution of light around the $m = 1$ revival resonance are presented in the left column of Fig. 4. The right column of the Fig. 4 shows corresponding momentum distributions calculated numerically from equations (2), (3) and (4).

According to the equation (10) the position of fringes depends on the light wavelength. To demonstrate this fact experimentally, we have illuminated the waveguide with two copropagating and well overlapped laser beams of different frequency ($\lambda_1 = 633\,\text{nm}$ and $\lambda_2 = 532\,\text{nm}$). For the latter wavelength we used a frequency-doubled Nd:Vanadate single-frequency laser. At large widths $d$ of the waveguide the outgoing light distributions are identical for both frequencies, but for small $d$, the fringes are shifted with respect to each other. As an example Fig. 4c shows spatial distributions of the output light at width $d = 86\,\mu m$. Under these conditions the red light intensity ($\lambda_1 = 633\,\text{nm}$) is almost completely contained in the transmitted beam and the green light ($\lambda_2 = 532\,\text{nm}$) in the reflected one. Therefore the two different frequency components are well separated in momentum and space.

From all these observations we come to the conclusion, that the multimode waveguide at large incidence angles of the input light beam can be considered as a new kind of an interferometer. The definite advantage of such an interferometer, compared to other known types, like Fabry-Perot or Michelson interferometers, is that its fringe spacing (5) can be much smaller than $\lambda/2$. The observed period of $\lambda/9$ was soley limited by technical factors in our first experiment. For a very narrow waveguide ($d \sim \lambda$) the period of revivals and mode beatings become smaller than in the multimode waveguide considered above ($d >> \lambda$). Especially interesting is the limit, when the waveguide contains only two propagating modes. The fringes of such a waveguide becomes uniform and periodic, and their period can be calculated from the formula (8), where $n = 1$ should be used. The fringe spacing of such an interferometer is expected to be very small. For example in a case of a waveguide with $d = 0.7\,\mu m$, $L = 100\,\mu m$, $\lambda = 633\,\text{nm}$ and $\beta = 33^\circ$, the fringe spacing is expected to be $\delta d = 0.56\,\text{nm}$.

Such a new kind of interferometer, especially in dielectric multimode waveguides, can find
many applications in precision optical measurements and also in switching and modulation of optical signals.

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FIG. 1. Experimental setup of the planar metal waveguide for observing the oscillations and revivals of the transverse momentum of light.

FIG. 2. Light intensity of the transmitted output beam of the waveguide as a function of its relative length, measured in revival periods $L_r = 8d^2/\lambda$. In this calculation the propagation parameter $f = 48$ was used. The influence of several waveguide modes leads to a wavelength dependent deviation from the geometric optics case for $L > L_r/8$. 

$L \left[ \frac{8 \ d^2}{\lambda} \right]$
FIG. 3. Light intensity in the reflected beam of the light past the waveguide as a function of the waveguide width $d$. 
FIG. 4. a), b), c) - experimental profiles of the light distributions in a far zone behind the waveguide; d), e), f) - numerically calculated transverse momentum distribution of light behind the waveguide.