Testing the stationarity of white dwarf light-curves

L Molnár, Z Kolláth, E Plachy and M Paparó
Konkoly Observatory, P.O. Box 67. H-1525 Budapest, Hungary
E-mail: lmolnar@konkoly.hu

Abstract. Long period white dwarfs show changes in their frequency spectra from one observing season to another, i.e. their light-curves cannot be considered as stationary multiperiodic variations on long timescales. However, due to the complex frequency spectra of these stars and the narrow frequency spacing, it is still unknown, what the shortest time scale is, where real physical modulation exists. We present tests on artificial data, resembling the observations, using time-frequency distributions (TFDs), Fourier-analysis and the analytical signal method.

1. Introduction
Temporal modulations of light-curves provide a great deal of information on the nature of dynamics behind stellar pulsation. However, from a finite dataset it is usually very hard to conclude whether the signal is multiperiodic or a time varying one, with real amplitude and/or phase (frequency) variations. Furthermore, the decision whether a non-stationary signal is modulated due to nonlinear dynamics or some stochastic process, is even more challenging. Time-frequency representation (Cohen 1995) of a non-stationary signal yields information about characteristics of the dataset in the time-frequency plane. Then transient events or frequency/amplitude changes can be monitored easily with these tools. Time-frequency distributions are proved to be useful for analyzing non-stationary light-curves, e.g. semiregular variables (Buchler et al. 2004) or stellar activity (Kolláth and Oláh 2008). However, these methods can lead to false results when applied to multiperiodic variations with narrow frequency spacing, as the beating produces structures very similar to the modulationary one on the time-frequency plots.

Long term multisite observations provide very good frequency resolution, resulting in a good overall fit of the observations. But in some cases, single night observations deviate significantly from the multiperiodic curve fitted to the whole data. Whether it is a sign of ill frequency resolution or nonstationarity of the process, is another question hard to answer. The compatibility of single night data to a general model cannot be tested by Fourier transformation itself, if the length of the night is short compared to the timescale related to the frequency spacing. But it could be useful to investigate other methods, whether any fingerprints of nonstationarity can be obtained by them.

We have performed our tests based on published frequencies of pulsating white dwarf stars. We generated artificial light-curves using fixed frequencies and amplitudes. The frequencies have been derived from the WET (Whole Earth Telescope) observations of GD154 (Pfeiffer et al. 1996). For the sake of simplicity we concentrated on the frequency range 60-100 c/d, mainly on the structures around 73 c/d. We have to note that the narrowly spaced frequencies (e.g.
a triplet at 73 c/d) practically act as a single frequency, as the time base of our tests are very small compared the characteristic time of the beating.

2. Time-Frequency Distributions

We calculated time-frequency distributions (TFDs) of different synthetic signals to check the characteristics of modulation and beats on single night observations. To construct TFDs, we used the TiFrAn (Time Frequency Analyzer) package, which has been developed at the Konkoly Observatory (see http://konkoly.hu/tifran). Different types of distributions have been integrated to this tool, for example Short-Term Fourier Transform (STFT), Choi-Williams- and Zhao-Atlas-Marks-Distributions (Cohen 1995).

A typical sample of synthetic variation calculated from the frequencies of GD154 is exhibited on panel (a) of Figure 1. With different randomized values of the phases the signal changes greatly, as light-curves do on different nights of observations. The Fourier-spectrum, right to the time-frequency distribution, clearly shows the three (groups of) frequencies. The TFD plot displays the traces of these peaks, but they are rather distorted by the modulation due to the close frequency spacing. TFDs have a very good noise rejection ratio, it is demonstrated on panel (b) of Fig. 1. Here we added a large Gaussian noise with a standard deviation of 0.02 to the synthetic data. Even with this amount of noise, the structure on the time-frequency plot remains unaltered, indicating that TFDs provide a good tool to compare noisy data with synthetic signals.

For comparison in panel (c) the same analysis of a single amplitude modulated (AM) signal is exhibited. For this tests we replaced the main triplet with an AM signal at 73 c/d. This simple test signal can be easily distinguished from the multiperiodic variation shown on panel
Figure 2. Probability distributions: main peak amplitudes (a), frequencies of the two largest peaks (b). Panel (c) and (d) are similar, with the 68 c/d frequency added. Open squares indicate the WET observation values (1991, partial), while solid squares show our observations (2006).

(a). However, if any component is included in the test signal in addition to the AM part, the resulting TFD looses the significant characteristic shown in panel (c). This effect is clearly demonstrated on panel (d) of Fig. 1, where a triplet at 79 c/d is included in the synthetic variation. The datasets are 0.4 day long to be comparable to a single-night observation which is far too short to resolve the modulation of the triplet itself.

Another useful tool is to calculate the instantaneous frequencies and amplitudes by band filtered analytical signal (Kolláth and Buchler 2001). This method provides an efficient way to reconstruct a signal by a single $A(t) \exp(i\varphi(t))$ term, where $A(t)$ and $\varphi(t)$ are the time dependent instantaneous amplitude and phase. It has similar properties (e.g. in noise rejection) to time-frequency distributions, and the band filtered analytical signal method can be used together with TFDs to get quantitative information on modulations.

3. Monte-Carlo Simulations

Based on the hypothesis of stationary multiperiodic data, probability distribution functions of the parameters of single night observations can be constructed by Monte-Carlo simulation. We fixed the periods and amplitudes while phases were randomized. The simulation includes more than 100,000 artificial light-curves. This method allows us to define the probable parameter range of single-night observations, like the central frequency and amplitude of the largest Fourier-
peak. Then the parameters of real observations can be compared to the original hypothesis. If the observed frequency and amplitude values fall outside the possible range, then that night is not compatible with the original assumption. Similar test can be performed using only the frequencies of the two largest peaks. In that case the results are independent from the amplitude scale. To minimize the effect of the side-lobes of the peaks in the Fourier-transform, we used a Gaussian windowing of the data. This preprocessing results in a better defined probability distribution of the Fourier peak parameters.

The published frequencies and amplitudes from WET observations of GD154 (Pfeiffer et al. 1996) have been used to construct the upper panels of Fig. 2, while the lower panels include an additional frequency at 68 c/d based on recent observations (Hürkal et al. 2005). The probability distribution functions of the largest Fourier-peak are shown in the left panels. The right panels display the possible ranges of the frequencies of the two largest peaks.

The WET observations are displayed with open squares on the figures. While the majority of the nights coincides with the most probable ranges, some observations are clearly show deviations from the assumed values. The additional frequency at 68 c/d helps in resolving the discrepancies. Preliminary results of recent observations of GD154 are also plotted by solid squares – these data suggest further departure from the original hypothesis.

4. Conclusion

It is an important question, whether white-dwarf light-curves are considered to be stable (stationary) on short timescales. We investigated several nonstandard methods to test if they are capable on tracing deviations from a multiperiodic variation in single night observations.

(i) Time-frequency distributions itself cannot show differences between multiperiodic and nonstationary time varying signals, if the timebase of the data is comparable to the modulation periods due to beats. However, due to the good signal-to-noise ratio in the TFD plots, it can be used to compare single night observations to the best fit of the whole dataset.

(ii) For continuous long timebase observations expected in the future from space-born observatories, these methods will be substantial tools to test the nonstationarity of the light variations.

(iii) With Monte-Carlo simulations based on the mathematical model of multisite observations, the probability distribution of the largest amplitudes and the corresponding frequencies in a single night can be calculated. Then the observed quantities, as compared to the probability distribution, provides information on the quality of the fit or on the validity of the stationarity of the data.

References

Buchler J R Kolláth Z and Cadmus R R 2004 ApJ 613 532
Cohen L 1995 Time-Frequency Analysis (Englewood Cliffs, NJ: Prentice-Hall PTR)
Hürkal D O, Handler G, Steininge B A and Reed M D 2005 ASPC 334 577
Kolláth Z and Oláh K 2008 A&A submitted
Kolláth Z and Buchler J R 2001 Stellar Pulsation – Nonlinear Studies (Astrophysics and Space Science Library Series vol 257) ed M Takeuti and D D Sasselov (Dordrecht: Kluwer Academic Publishers) pp 29-60
Pfeiffer B et al. 1996 A&A 314 182-90