Chiral Transition and the Scalar and Vector Correlations

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Abstract

The properties of the scalar and vector correlations in the hot and/or dense hadronic matter close to chiral transition are discussed. Presuming that the linear realization of chiral symmetry will become appropriate at least near the critical point, we argue that the strength function in the $I = J = 0$ channel will soften near the critical point, and the sigma meson will accordingly become a clearer resonance in hot and/or dense medium than in the free space. It is shown that the steep rise of the baryon-number susceptibility $\chi_B$ around the critical point seen in the lattice simulations may suggest that the interactions between quarks in the vector channel become weak near the critical point; this implies that the peculiar behavior of the $\chi_B$ does not necessarily imply a proliferation of baryons and antibaryons (Skyrmions) which is though a natural scenario of chiral transition in the non-linear realization of chiral symmetry.

1 The sigma meson and the soft modes

Chiral transition is a phase transition of the QCD vacuum with $\langle \bar{q}q \rangle$ being the order parameter. When exploring a phase transition in any physical system, the study of fluctuations of physical quantities, especially ones related to the order parameter is as important as that of the phase diagram for the system in equilibrium. The fluctuations of observables are also related with dynamical phenomena such as the transport properties of the system.

If a phase transition is of 2nd order or weak 1st order, there exist soft modes, which are the fluctuations of the order parameter as is well known in condensed matter physics and nuclear physics. For the chiral transition, the fluctuation of the order parameter $\langle (\bar{q}q)^2 \rangle$ is a scalar-isoscalar meson which one calls the $\sigma$-meson. The $\sigma$ meson may become the soft mode of chiral transition at $T \neq 0$ and/or $\rho_B \neq 0$. In fact, the existence of the sigma meson as the quantum fluctuation of the order parameter of the chiral transition is still controversial, though the recent active studies on the phase-shift analysis of the $\pi-\pi$ scattering in the $I = J = 0$ channel, i.e., the sigma channel, have been confirming the existence of the pole deep in the second Riemann sheet with the real part ranging from 400 MeV to 800 MeV. If this pole is identified with the quantum fluctuation of the amplitude of the chiral condensate, it may imply that the linear realization of chiral symmetry is appropriate in the resonance energy region, although the chiral perturbation theory based on the non-linear chiral Lagrangians works well in the rather low-energy region.

The first half of this report is concerned with the sigma meson and the precursory soft modes for chiral transition in nuclear medium, and is essentially a recapitulation of the talk presented at Tohoku university, a week before of the present workshop; the manuscript for the proceedings is now on the net. Referring to the above proceedings for the detail of the content, here I only give a brief summary of the report and some comments:

(1) The existence of the $\sigma$ meson as the quantum fluctuation of the order parameter of the chiral transition accounts for various phenomena in hadron physics which otherwise remain mysterious.
There have been accumulation of experimental evidence of a low-mass pole in the $\sigma$ channel in the $\pi\pi$ scattering matrix\[^6\]. It should be emphasized that for obtaining this result, it is essential to respect chiral symmetry, analyticity and crossing symmetry even in an approximate way as in the $N/D$ method\[^12\].

Partial restoration of chiral symmetry in hot and dense medium leads to an enhancement in the spectral function in the $\sigma$ channel near the $2m_\pi$ threshold\[^13\]. Such an enhancement has been observed in the reaction

$$A(\pi^+, (\pi^+\pi^-)_{I=J=0})A'$$

by CHAOS collaboration\[^14\], which might be an experimental evidence of the partial restoration of chiral symmetry in heavy nuclei\[^16\][\[^17\)]. The conventional approach without incorporating a partial restoration of chiral symmetry in the nuclear medium\[^18\] failed to reproduce the CHAOS data.

The spectral enhancement near the $2m_\pi$ threshold in the $\sigma$ channel is predicted irrespective of the linear and nonlinear realization of chiral symmetry provided that the possible reduction of the quark condensate or $f_\pi$ is taken into account\[^19\].

One should confirm that the near $2m_\pi$-threshold enhancement observed in the $(\pi^+, \pi^+\pi^-)$ reactions by CHAOS collaboration is surely due to a partial restoration of chiral symmetry in nuclear medium by other means\[^10\]. For this purpose, the strength function in the $\sigma$ channel in the wider two-dimensional $(\omega, q)$ plane should be measured. To obtain such strength functions in the $(\omega, q)$ plane, various nuclear and electro-magnetic probes as well as heavy-ion collisions should can be utilized; for instance, photo- or electro-production\[^11\] of the $\sigma$ as well as the production by $(d, ^3\text{He})$ and $(d, t)$ reactions are interesting\[^16\][\[^20\).]

Such experiments with nuclear targets for exploring the possible restoration of chiral symmetry in nuclear medium will automatically give clearer confirmation of the existence of the $\sigma$ meson than is done in the free space.

It should be emphasized that in any hadronic medium where the baryon density is finite, there arises a scalar-vector mixing\[^21\][\[^11\][\[^22\)]

$$\sigma \leftrightarrow \gamma, \omega,$$

as is familiar in the $\sigma$-$\omega$ model\[^23\], which may cause a possible softening of the spectral functions at finite three momenta $q$, even in the vector channel, due to the softening of that in the sigma channel associated with the partial restoration of chiral symmetry in the hadronic medium. Such a softening may reflect in the spectral functions extracted in any experiment to try to see the spectral functions in the vector channel by seeing the lepton pairs from heavy-ion collisions such as CERES/NA45\[^24\], proton\[^25\], electro- or gamma-nucleus reactions and so on.

The formation of $\sigma$ mesic nuclei by $(d, ^3\text{He})$ and $(d, t)$ reactions are proposed as was done to produce the deeply-bound pionic atoms\[^21\]. To identify the $\sigma$ meson and the spectral function in that channel, detecting $2\pi^0$ and lepton pairs with $q \neq 0$ are interesting\[^11\].

2 Enhancement of the baryon-number susceptibility and the density fluctuations

The discussions in the preceding section put an emphasis on the sigma meson as a quantum fluctuation of the amplitude of the order parameter; this presumed that the linear realization of the chiral symmetry is appropriate at least near the critical point. I personally take it for granted that the linear realization is natural at least in the vicinity of the chiral critical point.

Nevertheless some stick to the nonlinear realization even in the vicinity of the critical point of the chiral transition. If the non-linear realization is appropriate even in the vicinity of the critical point, the chiral restoration might be associated with an anomalous proliferation of the baryons and the anti-baryons as Skyrmions and anti-Skyrmions near the critical point at finite temperature\[^26\].
which may account for a steep rise of the baryon-number susceptibility, $\chi_B$ obtained in the lattice simulations. It has also been suggested that the vector-realization, which is based on the non-linear sigma model with the vector mesons incorporated based on the ansatz of the hidden-local symmetry and similar to the Georgi’s vector limit, could be realized; then the decrease of the vector meson masses was conjectured.

In the second half of the present report, I showed that the baryon-number susceptibility give some information on the vector correlations at finite temperature, thereby the steep rise of $\chi_B$ seen in the lattice simulations may be accounted for without sticking to the non-linear realization of chiral symmetry.

The baryon-number susceptibility $\chi_B$ is the measure of the response of the baryon number density $\rho_B = \sum_{i=1}^{N_f} \rho_i$ to infinitesimal changes in the quark chemical potentials $\mu_i$:

$$\chi_B(T, \mu) = \left[ \sum_{i=1}^{N_f} \frac{\partial}{\partial \mu_i} \right] \left( \sum_{i=1}^{N_f} \rho_i \right) = \langle \langle N_B^2 \rangle \rangle / VT, \quad (1)$$

where $N_B$ is the baryon-number operator given by $N_B = \sum_{i=1}^{N_f} N_i$, with

$$\rho_i = \text{Tr} N_i \exp(-\beta (H - \sum_{i=u,d} \mu_i N_i)) / V \equiv \langle \langle N_i \rangle \rangle / V \quad (2)$$

the $i$-th quark-number density, $V$ the volume of the system and $\beta = 1/T$.

It is readily recognized that $\chi_B$ is the density-density correlation which is nothing but the 0-0 component of the vector-vector correlations:

$$\chi_B(T, \mu_q) = \beta \int d\mathbf{x} S_{00}(0, \mathbf{x}), \quad (3)$$

where

$$S_{\mu\nu}(t, \mathbf{x}) = \langle \langle j_\mu(t, \mathbf{x}) j_\nu(0, \mathbf{0}) \rangle \rangle,$$

with

$$j_\mu(t, \mathbf{x}) = \bar{q}(t, \mathbf{x}) \gamma_\mu q(t, \mathbf{x})$$

being the current operator. Using the fluctuation-dissipation theorem, one has

$$\chi_B(T, \mu_q) = -\lim_{k \to 0} L(0, \mathbf{k}), \quad (4)$$

where $L(\omega, \mathbf{k})$ is the longitudinal component of the retarded Green’s function or the response function in the vector channel;

$$R_{\mu\nu}(\omega, k) = \text{F.T.}(\lim_{t \to \infty} \langle \langle [j_\mu(t, \mathbf{x}), j_\nu(0, \mathbf{0})] \rangle \rangle).$$

These formula clearly show the relevance of the vector correlations to the baryon-number susceptibility.

I also discussed the density fluctuations around the critical point of the chiral transition at finite temperature $T$ and baryon density $\rho_B$. We notice that the baryon-number susceptibility at $\rho_B \neq 0$ is related with the (iso-thermal) compressibility of the system:

$$\kappa_T \equiv -N_B^{-1} (\partial V / \partial \mu)_{T,N_B} = \frac{\chi_B}{\rho^2}, \quad (5)$$

which tells that if $\chi_B$ is large and so is the density fluctuation, the system is easy to compress. One can then see that the stronger interaction in the vector channel suppress $\chi_B$. 
This part was largely based on a previous report by myself\cite{34}, so I only give a brief summary of this part here, referring to \cite{34} for the details:

1. The baryon-number susceptibility $\chi_B$ as an observable which reflects the confinement-deconfinement and the chiral phase transitions in hot and/or dense hadronic matter\cite{27}.

2. The suppression of $\chi_B$ at low temperatures and steep rise around the critical temperature as shown in the lattice QCD may be roughly attributed to the confinement-deconfinement transition\cite{29}. Nevertheless such a behavior of $\chi_B$ is also affected by the chiral transition\cite{22}.

3. Noting that $\chi_B$ is a measure of the rate of the density fluctuation in the system, one can see that the chiral transition at finite chemical potential especially leads to an interesting phenomenological consequence to $\chi_B$. When the vector coupling is small, the chiral transition at low temperatures is of first order in the density direction\cite{32, 36, 37}. This implies a divergent behavior of $\chi_B$, accordingly a huge density fluctuations\cite{34}.

4. A large enhancement of the fluctuation can also be expected for the scalar density fluctuations due to the scalar-vector mixing at finite density mentioned above\cite{22, 34}. Such a large enhancement may leads to an enhancement of the sigma-meson production due to the scalar-vector mixing at finite density. The above phenomena all have relevance to experiments to be done in RHIC and LHC\cite{38, 39}.

5. The nature of the chiral transition as to the first order or not etc is sensitively dependent on the strength of the vector coupling\cite{35}. An analysis of the lattice data suggests that the vector coupling is small in comparison with the scalar coupling at high temperature\cite{22, 40}.

6. The susceptibility $\chi_B$ is nothing but the generalized susceptibility $\chi(\omega, k)$ at $\omega = k = 0$. One should examine $\chi(\omega, k)$ in the whole region of $\omega$ and $k$ to get more information about the vector correlations and the density fluctuations theoretically and experimentally.

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