On $SU(3)_{F}$ Breaking through Final State Interactions and CP Asymmetries in $D \rightarrow PP$ Decays

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ABSTRACT: We analyse $D$ decays to two pseudoscalars ($\pi, K$) assuming the dominant source of $SU(3)_F$ breaking lies in final state interactions. We obtain an excellent agreement with experimental data and are able to predict CP violation in several channels based on current data on branching ratios and $\Delta A_{CP}$. We also make predictions for $\delta_{K\pi}$ and the branching fraction for the decay $D^{+}_{s} \rightarrow K^{+}K_{L}$. 

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1 Introduction

The search for physics beyond the Standard Model (SM) has brought about a search for CP violation beyond what is generated by the phase in the CKM matrix [1–3]. The challenging measurement and conclusive evidence of a non-vanishing value for the ratio $\epsilon'/\epsilon$ [4–6] has excluded the superweak hypothesis of Wolfenstein [7]. However, $\epsilon'/\epsilon$ has not yet facilitated a severe test of the SM. This is primarily due to the poor predictive power plaguing this ratio in the presence of cancellations between QCD and electroweak penguins. The possibility of a quantitative evaluation of these matrix elements often transforms into a theological debate, in spite of the encouraging and pioneering lattice results by the RBC and UKQCD collaborations [8]. On the other hand, the triumphant measurement of $O(1)$ CP violation in the golden decay channel $J/\psi K_S$ [9–12] of the neutral $B$ meson, where the measured time-dependent asymmetry depends, to an excellent approximation, only on the CP violating phase of the CKM matrix [13, 14], has been a striking confirmation of the SM. More recently, LHCb has also confirmed the validity of the SM through a measurement of CP violation in $B_s$ physics [15]. LHCb has also played a key role in bringing the pioneering first results on neutral $D^0$ meson mixing by previous experiments [16–41] to a mature stage, along with an impressive progress in the measurement of CP violation [42–49].

The controversial measurements [40–44, 50–53] of the CP violating asymmetry found in the decay of the neutral $D^0$ meson to pairs of charged kaons and pions, had effectively
stirred the question of whether such rather high values found in the first experimental results could be accommodated within the Standard Model. While many arguments were placed in favour of contributions coming from beyond the SM [54–67], concrete arguments were also made for the presence of large phases coming from Final State Interactions (FSI) allowing for the accommodation of the asymmetry within the SM [68–75]. In fact, both the isospin relations for the Cabibbo allowed (CA) decays into $K\pi$ and singly Cabibbo suppressed (SCS) decays into $\pi\pi$ of $D$ mesons are characterized by large angles in the complex plane for the corresponding triangles [76, 77]. In the case of the $\pi\pi$ final states the phase difference between the $I = 2$ and $I = 0$ amplitudes is about $\pi/2$. These large phases have been, for a long time, advocated as the main cause for the large $SU(3)_F$ violations in exclusive $D$ decays [78, 79]. Indeed, in the $D$ mass region there is a nonet of scalar resonances and their mass splittings imply large $SU(3)_F$ violations generated by FSI.

Identifying the dominant source of $SU(3)_F$ violation is of crucial phenomenological importance in $D$ decays, since on one hand the imposition of exact $SU(3)_F$ completely fails in reproducing experimental data, while on the other hand introducing $SU(3)_F$ breaking in a general manner leads to a complete loss of predictivity due to the proliferation of independent parameters (see e.g. ref. [75]). Several interesting attempts at reducing the number of parameters have been made. The authors of refs. [80–82] advocated the use of $1/N_c$ counting to reduce the size of the parameter set to a tractable number. However, relying on the $1/N_c^2$ suppression of formally divergent corrections seems questionable. In ref. [69], the dominance of lower rank representations was argued for, and only $SU(3)_F$ triplets were considered as additional operators in the effective Hamiltonian. However, there is no compelling reason to truncate the effective Hamiltonian in such a drastic manner.

In contrast to the above-mentioned approaches, the assumption that FSI is the dominant source of $SU(3)_F$ breaking rests solidly on the large observed strong phases, it provides a very good description of available experimental data, it allows to predict several CP asymmetries which are currently poorly measured and it can be tested against independent determinations of the relevant rescattering matrices. It also allows us to predict the relative strong phase between the doubly Cabibbo suppressed (DCS) and CA charged $K\pi$ decays, $\delta_{K\pi}$.

The paper is organized as follows. In the next section we write the amplitudes for all the decays considered. In section 3 we discuss the parameterization of the $\Delta U = 0$ part of the amplitudes proportional to $V_{cb}^* V_{ub}$. In section 4 we give a brief overview of the current status of experimental measurements of CP asymmetries in SCS decays. Using the experimental branching ratios of $D^0$, $D^+$ and $D_s^+$ into final states with kaons and/or pions and measurements of CP asymmetries we fit the values of the parameters in section 5. In this section we also take a critical look at our parameterization and its consequences on correlations between CP asymmetries followed by an estimate of how future measurements will improve the constraints on penguin amplitudes. We finally summarize our results in section 6. The details of the fit results are given in the appendix.
2 Amplitudes and Parametrization

It is customary to describe weak decay amplitudes in terms of topologies of Wick contractions (or renormalization-group-invariant combinations thereof). Notice that any Wick contraction, as defined in refs. [83, 84], can be seen as an emission followed by rescattering [83, 85]. Thus, rescattering establishes a link between emissions and long-distance contributions to other subleading topologies such as penguins, annihilations, weak exchange, etc. The large phases observed in two-body nonleptonic $D$ decays imply the importance of FSI, leading to an effective description of decay amplitudes in terms of emissions followed by rescattering. This description was developed in refs. [78, 79], where FSI effects were parameterized in terms of resonances.

In a previous study [86] of the SCS decays of the $D^0$ into a pair of pseudoscalars, exact SU(3)$_F$ symmetry was assumed amongst the emission matrix elements of the nonleptonic Hamiltonian. The necessary SU(3)$_F$ breaking was determined by FSI, described as the effect of resonances in the scattering of the final particles. Assuming no exotic resonances belonging to the 27 representation, the possible resonances have SU(3)$_F$ and isospin quantum numbers $(8, I = 1)$, $(8, I = 0)$ and $(1, I = 0)$. Moreover, the two states with $I = 0$ can mix, yielding two resonances:

$$|f_0⟩ = \sin φ |8, I = 0⟩ + \cos φ |1, I = 0⟩,$$

$$|f'_0⟩ = -\cos φ |8, I = 0⟩ + \sin φ |1, I = 0⟩. \tag{2.1}$$

The main contribution from the Hamiltonian, $H(|[ΔC] = 1, ΔS = 0)$, transforms as a $U$–spin triplet and therefore relates the $D^0$, which is a $U$–spin singlet, to the $U$–spin triplets of the 8 and 27 representation of SU(3)$_F$. So in ref. [86] two parameters were introduced for the matrix elements of the weak Hamiltonian, namely $T$ and $C$. The phase of the $I = 1$ octet amplitude, $δ_1$, and the two phases and mixing angle between the $I = 0$ singlet and octet amplitude, $δ_0$, $δ'_0$ and $φ$, were taken as free parameters. The strong phases should be related to the mass and width of the resonances. However, the lack of complete experimental information on the scalar resonances do not allow for the determination of the strong phases and so we determine them from the fit. It should also be noted that notwithstanding the lack of exotic resonances belonging to the 27 representation, there is a small phase associated with this amplitude which is compatible with 0 [77]. We set this phase to 0 and hence all the other phases should be interpreted as a difference with respect to this phase.

Other attempts have been made previously to study $D \rightarrow PP$ decays in the SU(3)$_F$ framework with perturbative breaking of the symmetry [72, 75, 80–82, 87]. The spotlight has always been on prescriptions for estimating the penguin amplitudes by formulating a reasonable parametrization in the SU(3)$_F$ framework and then using available data on branching fractions and CP asymmetries. In this work we extend the formalism that was developed in ref. [86] by including more decay modes of the $D$ meson system and introducing new parameters to aptly parametrize the additional decay amplitudes.

The $D^+$ and $D^+_s$ form a $U$–spin doublet and the matrix elements of the weak Hamiltonian, which relate $D^0$ to the $Q = 0, U = 1$ and $D^+$ to the $Q = 1, U = \frac{1}{2}$ of the octet,
are independent. This requires the introduction of a SU(3)$_F$ invariant parameter $\Delta$ in the $\Delta U = 1$ part of the amplitude. The terms proportional to $\Delta$ vanish in the factorization ansatz and appears only in the $D_s^+$ decay amplitudes. As we will explain in section 3, $\Delta$ is related by SU(3)$_F$ to a vanishingly small contribution in the $\Delta U = 0$ part of the amplitude suppressed by an approximate selection rule.

To expand the FSI description we need a phase, $\delta_{1\frac{7}{2}}$, for the FSI of the $I = \frac{1}{2}$ member of the octet. The phases $\delta_0$, $\delta_0'$, $\delta_1$ and $\delta_{1\frac{7}{2}}$ and the mixing angle $\phi$ are defined such that in the the limit of SU(3)$_F$ conservation $\delta_0 = \delta_1 = \delta_{1\frac{7}{2}}$, the amplitudes are independent of $\delta_0'$ and $\phi = \pi/2$. The phase for the decay modes with $D_s^+$ in the initial state is expected to be different from those in the $D^0$ and $D^+$ decay modes as an effect of SU(3)$_F$ breaking and the consequent shift in the mass of the $D_s^+$. Keeping in mind that both the phases shift in the same direction, we parameterize the phases with $\epsilon_\delta$ and $\delta'_{1\frac{7}{2}}$ as

$$
\delta'_1 = \delta_1 (1 - \epsilon_\delta) \quad \text{and} \quad \delta'_{1\frac{7}{2}} = \delta_{1\frac{7}{2}} (1 - \epsilon_\delta).
$$

(2.3)

The extension to the CA and DCS final states requires the introduction of additional sources of SU(3)$_F$ violation in addition to $\delta_{1\frac{7}{2}}$ in the $I = \frac{1}{2}$ octet channel. To understand this better one must note that the SU(3)$_F$ relationship for $D^+$ decays:

$$
\tan \theta_C A(D^+ \rightarrow \bar{K}^0\pi^+) = \sqrt{2} A(D^+ \rightarrow \pi^0\pi^+),
$$

(2.4)

which implies (neglecting the interference with the DCS final state) the ratio of the decay amplitudes into two pions and into $K_S\pi^+$ being equal to tan $\theta_C$ is in disagreement with data. To correct for this discrepancy we allow for a breaking of the 27 amplitudes through the introductions of a parameters $\kappa$ and $\kappa'$ which, respectively, split the 27 matrix element in the CA and DCS channels from the 27 matrix element in the SCS channel.

Next, we observe that the ratio of the branching fractions of the DCS to the CA decays of $D^0$ into a kaon and a pion with opposite electric charge given by

$$
\frac{\text{BR}(D^0 \rightarrow K^+\pi^-)}{\text{BR}(D^0 \rightarrow K^-\pi^+)} = \tan^4 \theta_C.
$$

(2.5)

is violated and the ratio is actually larger than the value $\tan^4 \theta_C$. To accommodate for this we allow for the SU(3)$_F$ breaking parameter $K$ which contributes with opposite signs to the octet part for the CA and DCS channels to correct the prediction in eq. 2.5. This parameter represents the non-conservation of the strangeness changing current which, in the factorization ansatz, corresponds to the axial current that destroys the initial $D$ meson state and the divergence in the vector current proportional to the mass difference of the strange quark and that of the lighter quarks $u$ and $d$. This also generates a term proportional to $K'$ in the DCS decays of the $D^+$ and the CA decays of the $D_s^+$ meson which comes from the fact that for annihilating the charged mesons a charged current is necessary.

With this parametrization we arrive at the following amplitudes for the SCS, CA and DCS amplitudes. Although we present the amplitudes with $\eta_8$ in the final state, we do not make any attempt to include $\eta - \eta'$ mixing in this work and hence do not use the experimental measurements of these channels for the fits. While considering the singlet
state, $\eta_1$, would increase the number of measurements that we could fit the parameters to, including the singlet state would also require additional parameters since it is has a significant gluonic content [88, 89]. A discussion of the complexities of addressing $\eta - \eta'$ mixing can be found in [90] and references therein. Hence, we shall postpone this exercise to a future work.

The CKM factors are to be kept explicit and hence the following amplitudes given in equations (2.7)–(2.9) should be multiplied by $\frac{1}{2}(V_{us}V_{us}^* - V_{ud}V_{ud}^*)$, $V_{ud}V_{cs}^*$ and $-V_{us}V_{cd}^*$ for SCS, CA and DCS modes respectively. The branching fraction is then defined as

$$\text{BR}(D \to P_1 P_2) = \frac{\tau_D G_F^2}{16\pi m_D^3} \frac{\sqrt{(m_D^2 - (m_{P_1} + m_{P_2})^2)(m_D^2 - (m_{P_1} - m_{P_2})^2)}}{2m_D} |A(D \to P_1 P_2)|^2,$$

(2.6)

where $m_D$, $m_{P_1}$ and $m_{P_2}$ are the masses of the $D$ meson in the initial state and the pseudoscalars in the final state respectively, $G_F$ is the Fermi constant and $\tau_D$ is the relevant $D$ meson lifetime.

**SCS modes** (to be multiplied by $\frac{1}{2}(V_{us}V_{us}^* - V_{ud}V_{ud}^*)$)

$$A(D^0 \to \pi^+\pi^-) = (T - \frac{2}{3} C) \left[ -\frac{3}{10} \left( e^{i\delta_0} + e^{i\delta_0'} \right) + \left( -\frac{3}{10} \cos(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_0'} - e^{i\delta_0} \right) \right] - \frac{2}{5} (T + C)$$

$$A(D^0 \to \pi^0\pi^0) = (T - \frac{2}{3} C) \left[ -\frac{3}{10} \left( e^{i\delta_0} + e^{i\delta_0'} \right) + \left( -\frac{3}{10} \cos(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_0'} - e^{i\delta_0} \right) \right] + \frac{3}{5} (T + C)$$

$$A(D^0 \to K^+K^-) = (T - \frac{2}{3} C) \left[ \frac{3}{20} \left( e^{i\delta_0} + e^{i\delta_0'} \right) + \left( \frac{3}{20} \cos(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_0'} - e^{i\delta_0} \right) \right] + \frac{3}{10} e^{i\delta_1} + \frac{2}{5} (T + C)$$

$$A(D^0 \to K^0\bar{K}^0) = (T - \frac{2}{3} C) \left[ \frac{3}{20} \left( e^{i\delta_0} + e^{i\delta_0'} \right) + \left( \frac{3}{20} \cos(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_0'} - e^{i\delta_0} \right) \right] - \frac{3}{10} e^{i\delta_1}$$

$$A(D^0 \to \eta\eta_h) = (T - \frac{2}{3} C) \left[ \frac{3}{10} \left( e^{i\delta_0} + e^{i\delta_0'} \right) + \left( \frac{3}{10} \cos(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_0'} - e^{i\delta_0} \right) \right] - \frac{3}{5} (T + C)$$

$$A(D^0 \to \pi^0\eta_h) = \frac{\sqrt{3}}{5} \left[ \left( T - \frac{2}{3} C \right) e^{i\delta_1} - (T + C) \right]$$

$$A(D^+ \to K^+\bar{K}^0) = \frac{1}{5}(2T - 3C + \Delta)e^{i\delta_1} + \frac{3}{5} (T + C)$$

$$A(D^+ \to \pi^+\pi^0) = \frac{1}{2\sqrt{2}} (T + C)$$

$$A(D^+ \to \pi^+\eta_h) = \frac{\sqrt{3}}{5\sqrt{3}} (2T - 3C + \Delta)e^{i\delta_1} - \frac{3\sqrt{3}}{5\sqrt{2}} (T + C)$$

$$A(D^+ \to \pi^+K^0) = -\frac{1}{5}(2T - 3C + \Delta - K')e^{i\delta_1} - \frac{3}{5} (T + C)$$

$$A(D^+ \to \pi^0K^+) = -\frac{1}{2\sqrt{2}} (2T - 3C + \Delta - K')e^{i\delta_1} + \frac{2}{5\sqrt{2}} (T + C)$$

$$A(D^+ \to K^+\eta_h) = \frac{1}{5\sqrt{6}} (2T - 3C + \Delta - K')e^{i\delta_1}' - \frac{2\sqrt{6}}{5} (T + C)$$

(2.7)
CA modes (to be multiplied by $V_{cs}V_{us}^\ast$)

\[
A(D^0 \rightarrow \pi^+ K^-) = \frac{1}{5}(3T - 2C - K)e^{i\frac{\phi}{2}} + \frac{2}{5}(T + C + \kappa)
\]
\[
A(D^0 \rightarrow \pi^0 K^0) = \frac{1}{5\sqrt{2}}(3T - 2C - K)e^{i\frac{\phi}{2}} + \frac{3}{5\sqrt{2}}(T + C + \kappa)
\]
\[
A(D^0 \rightarrow \bar{K}^0\eta_b) = -\frac{1}{5\sqrt{6}}(3T - 2C - K)e^{i\frac{\phi}{2}} + \frac{3}{5\sqrt{6}}(T + C + \kappa)
\]
\[
A(D^+ \rightarrow \pi^+ K^0) = (T + C + \kappa)
\]
\[
A(D^+_s \rightarrow K^+ K^0) = -\frac{1}{5}(2T - 3C + \Delta)e^{i\frac{\phi'}{2}} + \frac{2}{5}(T + C + \kappa)
\]
\[
A(D^+_s \rightarrow \pi^+ \eta_b) = -\sqrt{\frac{3}{5\sqrt{3}}} (2T - 3C + \Delta)e^{i\frac{\phi'}{2}} - \sqrt{\frac{3}{5}}(T + C + \kappa)
\]  

DCS modes (to be multiplied by $-V_{cd}V_{us}^\ast$)

\[
A(D^0 \rightarrow \pi^0 K^0) = \frac{1}{5\sqrt{2}}(3T - 2C + K)e^{i\frac{\phi}{2}} - \frac{3}{5\sqrt{2}}(T + C + \kappa')
\]
\[
A(D^0 \rightarrow \bar{K}^0\eta_b) = \frac{1}{5\sqrt{6}}(3T - 2C + K)e^{i\frac{\phi}{2}} - \frac{3}{5\sqrt{6}}(T + C + \kappa')
\]
\[
A(D^+ \rightarrow \pi^+ K^0) = \frac{2}{5}(2T - 3C + \Delta - K')e^{i\frac{\phi}{2}} - \frac{2}{5}(T + C + \kappa')
\]
\[
A(D^0 \rightarrow \pi^- K^+) = -\frac{1}{5}(3T - 2C + K)e^{i\frac{\phi}{2}} + \frac{2}{5}(T + C + \kappa')
\]
\[
A(D^+ \rightarrow \pi^0 K^+) = \frac{1}{5\sqrt{2}}(2T - 3C + \Delta - K')e^{i\frac{\phi}{2}} + \frac{3}{5\sqrt{2}}(T + C + \kappa')
\]
\[
A(D^+ \rightarrow K^+ \eta_b) = -\frac{1}{5\sqrt{6}}(2T - 3C + \Delta - K')e^{i\frac{\phi}{2}} - \frac{3}{5\sqrt{6}}(T + C + \kappa')
\]
\[
A(D^+_s \rightarrow K^+ K^0) = -(T + C + \kappa')
\]  

3 The $\Delta U = 0$ amplitudes

The $\Delta U = 0$ contributions to the SCS decays proportional to $V_{cb}V_{ub}^\ast$ need to be considered both for the amplitudes related to the penguin operator:

\[
\bar{u}_L(x)\gamma^\mu \lambda_a c_L(x)[(\bar{u}(x)\gamma_\mu \lambda_a u(x) + \bar{d}(x)\gamma^\mu \lambda_a d(x) + \bar{s}(x)\gamma_\mu \lambda_a s(x)]
\]  

and to the operator:

\[
\bar{u}_L\gamma_\mu s_L \bar{s}_L \gamma^\mu c_L(x) + \bar{u}_L\gamma_\mu d_L \bar{d}_L \gamma^\mu c_L(x).
\]

The latter have to be considered as a consequence of the unitarity of the CKM matrix and are referred to as the pseudo-penguin operators. Parameterizing this part of the amplitude requires the introduction of three additional real parameters $P$, $\Delta_3$ and $\Delta_4$. The penguin contributions are encapsulated in $P$. The matrix elements of the operator defined in equation (3.2) depend on four reduced matrix elements, $\langle 27|15|3 \rangle$, $\langle 8|15|3 \rangle$, $\langle 8|3|3 \rangle$ and $\langle 1|3|3 \rangle$. The first two are related to the ones for the $\Delta U = 1$ part. We introduce two parameters, $\Delta_3$ and $\Delta_4$, which are combinations of the four reduced matrix
elements which are defined in such a way that, by neglecting final state interactions, one has:

\[ B(D^0 \to K^+ K^-) = P + T + \Delta_3, \]
\[ B(D^0 \to K^0 \bar{K}^0) = \Delta_4. \]

(3.3)

The asymmetries consist of three contributions. The first contribution comes from the terms proportional to \( P = (P + T + \Delta_4) \). While \( T \) can be extracted from the branching fraction data neither \( P \) nor \( \Delta_3 \) can be estimated from first principles. The second contribution is proportional to \( T + C \) and it can be completely determined from the branching fraction data. The third contribution is proportional to \( \Delta_4 \). This contribution, which is a sum of penguin and pseudo-penguin contributions, is vanishingly small due to an approximate selections rule which disfavors the simultaneous creation of \( d \bar{d} \) and \( s \bar{s} \) pairs\(^1\).

Moreover, \( SU(3)_F \) relates the \( \Delta \) in the \( \Delta U = 1 \) part to the \( \Delta_4 \) in the \( \Delta U = 0 \) part. It is interesting to note here that indeed the \( \Delta U = 0 \) contributions of the 15 would allow a contribution to \( D^0 \to K^0 \bar{K}^0 \). To forbid it, according to the selection rule, one should put \( \Delta \sim 0 \). The fact that the fit to the branching ratios implies for \( \Delta \) a small value consistent with 0 lies in favor of the selection rule. However \( \Delta_3 \) is not affected by this approximate selection rule and hence does not need to be vanishingly small. There is a contribution proportional to \( \Delta \) in the \( \Delta U = 0 \) part of the amplitudes of the \( D^+ \) modes. However, as we shall see, \( \Delta \) is very small and hence this contribution turns out to be insignificant. The explicit form of the \( \Delta U = 0 \) part of the amplitude are as follows:

\[
B(D^0 \to \pi^+ \pi^-) = \mathcal{P} \left( \frac{1}{2} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \left( e^{i\delta_d} - e^{i\delta_s} \right) \left( \frac{1}{6} \cos(2\phi) - \frac{7}{4\sqrt{10}} \sin(2\phi) \right) \right) + (T + C) \left( -\frac{3}{20} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \frac{3}{10} + \frac{1}{60} \cos(2\phi) + \frac{1}{2\sqrt{10}} \sin(2\phi) \right) \left( e^{i\delta_d} - e^{i\delta_s} \right),
\]
\[
B(D^0 \to \pi^0 \pi^0) = B(D^0 \to \pi^+ \pi^-) - (T + C),
\]
\[
B(D^0 \to K^+ K^-) = \mathcal{P} \left( \frac{1}{4} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \left( e^{i\delta_d} - e^{i\delta_s} \right) \left( -\frac{5}{12} \cos(2\phi) + \frac{1}{4\sqrt{10}} \sin(2\phi) \right) + \frac{1}{2} e^{i\delta_k} \right) + (T + C) \left( -\frac{1}{20} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \frac{3}{10} + \frac{7}{60} \cos(2\phi) \left( e^{i\delta_d} - e^{i\delta_s} \right) - \frac{1}{5} e^{i\delta_k} \right) + \Delta_4 \left( \frac{1}{4} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \left( e^{i\delta_d} - e^{i\delta_s} \right) \left( -\frac{1}{12} \sin(2\phi) + \frac{3}{4\sqrt{10}} \sin(2\phi) \right) - \frac{1}{2} e^{i\delta_k} \right),
\]
\[
B(D^0 \to K^0 \bar{K}^0) = \mathcal{P} \left( \frac{1}{4} \left( e^{i\delta_d} + e^{i\delta_s} \right) + \left( e^{i\delta_d} - e^{i\delta_s} \right) \left( -\frac{5}{12} \cos(2\phi) + \frac{1}{4\sqrt{10}} \sin(2\phi) \right) - \frac{1}{2} e^{i\delta_k} \right).
\]

\(^1\)The approximate selection rule that leads to the suppression of the simultaneous creation of \( d \bar{d} \) and \( s \bar{s} \) pairs in our framework is analogous to a Zweig suppression (or OZI rule) [91–93]. We refrain from calling it the Zweig suppression since the process \( \phi \to \rho \tau \), forbidden by the Zweig rule, might occur by allowing the conversion of the initial \( s \bar{s} \) into a pair of light quarks accompanied by the creation of another pair of light quarks. To exclude the formation of a \( K^0 \bar{K}^0 \) pair through the \( \Delta U = 0 \) decay amplitude we do not allow the conversion of the \( u \bar{u} \) pair into a \( d \bar{d} \) or \( s \bar{s} \) pair and assume that the constituents of the final mesons are just the three quarks produced in the \( c \) decay and the spectator \( \bar{q} \). This has a consequence also for the \( \Delta U = 1 \) part related by \( SU(3)_F \) to the \( \Delta U = 0 \). Our selection rule is better motivated, since asymptotic freedom implies that the strong coupling constant is a decreasing function of the scale.
written as CP asymmetry in this channel \[82, 94\] due to the Zweig suppression of the \[\Delta U\] topology. This contribution can be potentially large and lead to the enhancement of the measurement of \[\Delta A\] is measured. The most notable of the CP asymmetry measurements is the very precise \[D\] CP asymmetries only. In the decay of the charged \[D\] mesons the direct and indirect CP asymmetries (time integrated), while the HFLAV averages are direct and hence related to the parameters in the \[\Delta U\] contribute. In our framework, this weak exchange topology is generated by rescattering in this channel from the other SCS channels where the weak exchange topology does not contribute and the suppression of the branching fraction decorrelates the CP asymmetry of the amplitude. It has been pointed out in \[82\] that the possibly large weak exchange with weak exchange topology leads to the parametric correlation between all the SCS \[\Delta U\] exchange topology by rescattering was also discussed in \[95\]. This characterization of the weak exchange topology can be found on the HFLAV \[96\] website. It is important to note here that much progress has been made in the measurement of CP asymmetries, a compendium of which can be found on the HFLAV \[96\] website. It is important to note that the CP asymmetries measured by the experiments in the neutral \[D^0\] channel is the sum of direct and indirect CP asymmetries (time integrated), while the HFLAV averages are direct CP asymmetries only. In the decay of the charged \[D\] mesons the direct CP asymmetry is measured. The \[\Delta U = 0\] part of the \[D^0 \rightarrow K_S K_S\] decay amplitude includes a weak exchange topology. This contribution can be potentially large and lead to the enhancement of the CP asymmetry in this channel \[82, 94\] due to the Zweig suppression of the \[\Delta U = 1\] part of the amplitude. It has been pointed out in \[82\] that the possibly large weak exchange contribution and the suppression of the branching fraction decorrelates the CP asymmetry in this channel from the other SCS channels where the weak exchange topology does not contribute. In our framework, this weak exchange topology is generated by rescattering and hence related to the parameters in the \[\Delta U = 1\] amplitude. The generation of the weak exchange topology by rescattering was also discussed in \[95\]. This characterization of the weak exchange topology leads to the parametric correlation between all the SCS \[\Delta U = 0\] amplitudes leading to a correlation amongst the CP violation in these channels.

4 Measurements of CP asymmetries

Much progress has been made in the measurement of CP asymmetries, a compendium of which can be found on the HFLAV \[96\] website. It is important to note here that the CP asymmetries measured by the experiments in the neutral \[D^0\] channel is the sum of direct and indirect CP asymmetries (time integrated), while the HFLAV averages are direct CP asymmetries only. In the decay of the charged \[D\] mesons the direct CP asymmetry is measured. The most notable of the CP asymmetry measurements is the very precise measurement of \[\Delta A_{\text{CP}} = A_{\text{CP}}(D^0 \rightarrow K^+ K^-) - A_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^-)\] by LHCb \[44\] with their 7 TeV and 8 TeV data. Combining this result with the previous LHCb measurement \[43\] and the LHCb measurement of indirect asymmetry \[42, 105\] using \[A_\Gamma \sim -\Delta A_{\text{CP}}^{\text{ind}}\] and the
measurement of $y_{\text{CP}}$ [47] they extracted the difference of the direct CP asymmetry as

$$\Delta A_{\text{CP}}^{\text{dir}} = (-0.061 \pm 0.076)\%,$$

while the HFLAV world average stands at [96]:

$$\Delta A_{\text{CP}}^{\text{dir}} = (-0.137 \pm 0.070)\%.$$

We do not use the measurement of the individual asymmetries $A_{\text{CP}}(D^0 \to \pi^+\pi^-)$ and $A_{\text{CP}}(D^0 \to K^+K^-)$ since the LHCb results for these [43] are used in the estimation of $\Delta A_{\text{CP}}^{\text{dir}}$. The results from the other experiments on these individual asymmetries do not improve the fit in any manner since they are much less precise. We have numerically checked the validity of this statement. For the sake of completeness we list some of the relevant CP asymmetries in table 1 that have been measured till date. We do not use these measurements in the fit but predict them from a fit to the branching fractions and the HFLAV average of $\Delta A_{\text{CP}}^{\text{dir}}$.

In the recent past some theoretical effort has been put on estimating CP asymmetry in $D^0 \to K_SK_S$ [72, 75, 82] along with experimental measurements being performed at LHCb [103] and Belle [104] as listed in table 1. There is an older measurement by CLEO [97] which we do not quote here since it is much less precise. In [72], $A_{\text{CP}}(D^0 \to K_SK_S)$ was estimated to be about 0.6% in magnitude. In [75] the CP asymmetry in $D^0 \to K_SK_S$ was related to $\Delta A_{\text{CP}}^{\text{dir}}$ and an estimation of about 0.4% was made for the former. In [82] it was shown that this asymmetry can be of $O(1\%)$ due to possibly large contributions from the weak exchange diagrams to the $\Delta U = 0$ part of the amplitude. In the following section we present our results for $A_{\text{CP}}(D^0 \to K_SK_S)$ using data on both branching fractions and $\Delta A_{\text{CP}}^{\text{dir}}$ to constrain the parameters along with the prediction of CP asymmetries of several other SCS modes.

The only SCS channel for which the CP asymmetry is predictably 0 in the SM is that in $D^+ \to \pi^+\pi^0$ [106, 107] since it is driven by a single isospin amplitude and hence lacks the two separate strong and weak phases necessary for a non-zero CP asymmetry. Recently Belle has measured a CP asymmetry in this channel consistent with the null SM

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
channel & mean $\pm$ rms (%) & reference \\
\hline
$D^0 \to K^+K^-$ & $-0.16 \pm 0.12$ & HFLAV [96] \\
$D^0 \to \pi^+\pi^-$ & $0.00 \pm 0.15$ & HFLAV [96] \\
$D^0 \to \pi^0\pi^0$ & $-0.03 \pm 0.64$ & HFLAV [97, 98] \\
$D^+ \to K^+K_S$ & $-0.11 \pm 0.25$ & HFLAV [99–102] \\
$D_s^+ \to K_S\pi^+$ & $0.38 \pm 0.48$ & HFLAV [96] \\
$D_s^+ \to K^+\pi^0$ & $-0.266 \pm 0.238 \pm 0.009$ & CLEO [100] \\
$D^0 \to K_SK_S$ & $-2.9 \pm 5.2 \pm 2.2$ & LHCb [103] \\
$D^0 \to K_SK_S$ & $-0.2 \pm 1.53 \pm 0.17$ & Belle [104] \\
\hline
\end{tabular}
\caption{Measurements of CP asymmetries in various channels.}
\end{table}
value \[108\]:

$$A_{CP}(D^+ \to \pi^+\pi^0) = (2.31 \pm 1.24 \pm 0.23)\%$$ \hspace{1cm} (4.3)

which has a much better precision than the previous CLEO measurement \[100\] which has an error of 2.9\% and is consistent with the null SM value.

The BESIII Collaboration has performed the first measurements of CP asymmetry in $D^+ \to K^+K_S$ and $D^+ \to K^+K_L$ \[109\] which should be exactly equal since both are driven by $D^+ \to K^+K^0$ only. The two measurements are in good agreement with each other and consistent with 0:

$$A_{CP}(D^+ \to K^+K_S) = (-1.8 \pm 2.7 \pm 1.6)\%$$
$$A_{CP}(D^+ \to K^+K_L) = (-4.2 \pm 3.2 \pm 1.2)\%$$ \hspace{1cm} (4.4)

5 Results and their consequences

We use HEPfit \[110\] to perform a fit in the Bayesian framework. The 7 amplitudes ($T, C, \Delta, K, K', \kappa$ and $\kappa'$), the SU(3)$_F$ breaking parameter quantifying the shift in the $D^+_s$ phase ($\epsilon_3$), 4 phases ($\delta_0, \delta'_0, \delta_2, \delta_1$) and a mixing angle $\phi$ are constrained using 17 branching fractions. The $\Delta U = 0$ part of the amplitudes require three additional parameters $P, \Delta_3$ and $\Delta_4$. The first two, $P$ and $\Delta_3$ always appear as a sum in the $\Delta U = 0$ part of all the decay amplitudes and hence it is not possible to disentangle them individually from data. Moreover, it is not possible to estimate the sizes of these parameters from first principles and hence we work with the ratio $(P + \Delta_3)/T$ in our fit and use $\Delta A_{CP}^{dir}$ to constrain this combination. As discussed before, $\Delta_4$ is expected to be tiny due to the approximate selection rule and so we set it to 0 in our fit. The experimental numbers used for the fit are listed in table 3 and in equation (4.1). For the branching fractions we use the $D^0$ and the $D^+_s$ decays with only $\pi$ and $K$ in the final state. In addition to the PDG averages listed in table 3 we also use the recent measurements made by the BESIII Collaboration \[109, 111\] which are comparable or better than the PDG averages.

5.1 Fit to branching fractions and $\Delta A_{CP}^{dir}$

The fit results are presented in table 2 along with the correlation matrix for the fitted parameters. The parameter $(P + \Delta_3)/T$ is excluded from the correlation matrix because it is essentially uncorrelated with the other parameters being determined by $\Delta A_{CP}^{dir}$ while the other parameters are determined by the branching ratio data. As a cross-check we also performed a fit using MINUIT routines and verified that the results are exactly the same. The error analysis was done in HEPfit and is taken as the RMS of the posterior distributions of the parameters and observables.

We find two equivalent solutions for the parameters in the $\Delta U = 1$ part of the amplitude from the branching fractions alone. The solutions are distinct only for the posterior distributions of the 4 phases ($\delta_0, \delta'_0, \delta_2, \delta_1$) with $\delta_i \rightarrow -\delta_i$ relating these two solutions. The rest of the parameters have identical solutions. However, these two solutions lead to very different fits for $(P + \Delta_3)/T$ from the $A_{CP}^{dir}$ data and hence we present both the solutions.
I can be seen that the mass of the resonance corresponding to the inferred upon by using the Gell-Mann-Okubo mass formula \([114-117]\). From the latter it generated by rescattering due to the presence of resonances, the phases should follow a results with the negative solution keeping in mind that it is better motivated. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution. In the following discussion we shall focus more on the favourable solution.

| \((\mu \pm \sigma)\) | \((\mu \pm \sigma)\) |
|-------------------|-------------------|
| \(T\)             | 0.424 ± 0.003     | \(\delta_0\)     | −2.370 ± 0.062 |
| \(C\)             | −0.211 ± 0.003    | \(\delta_0'\)    | −0.840 ± 0.046 |
| \(\kappa\)        | −0.036 ± 0.004    | \(\delta_2\)     | −1.630 ± 0.020 |
| \(\kappa'\)       | −0.062 ± 0.088    | \(\delta_1\)     | −1.080 ± 0.039 |
| \(K\)             | 0.100 ± 0.012     | \(\epsilon_3\)   | 0.067 ± 0.061 |
| \(K'\)            | −0.153 ± 0.072    | \(\phi\)         | 0.435 ± 0.025  |
| \(\Delta\)        | −0.026 ± 0.019    | \((P + \Delta_3)/T\) | −1.700 ± 0.508 |
| \(\epsilon_3\)    | 0.067 ± 0.061     | \(\phi\)         | 0.435 ± 0.025  |

(a) Fit values of the parameters. \(T, C, \Delta, \kappa^{(r)}\) and \(K^{(r)}\) are in units of \(\text{GeV}^3\). The angle \(\phi\) and the phases \(\delta_0, \delta_0', \delta_1\) and \(\delta_1\) are in radians. The parameter \(\epsilon_3\) is dimensionless. The phases \(\delta_i\) have two solutions, positive and negative. The negative solution is better motivated as explained in the text. The solution for \((P + \Delta_3)/T\) changes accordingly while the solutions for all other parameters remain the same.

| \(T\) | 1 | −0.39 | −0.37 | −0.20 | −0.20 | 0.70 | 0.14 | 0.28 | −0.40 | −0.36 | −0.24 | −0.40 | −0.32 | −0.56 |
| \(C\) | 1.00 | −0.58 | 0.25 | 0.26 | −0.57 | −0.09 | −0.20 | 0.19 | 0.18 | 0.18 | 0.27 | 0.14 | 0.12 | 0.22 |
| \(\kappa\) | 1.00 | −0.33 | −0.10 | 0.03 | −0.05 | −0.04 | 0.11 | 0.09 | −0.04 | 0.15 | 0.12 | 0.26 |
| \(\kappa'\) | 1.00 | 0.19 | −0.17 | 0.11 | −0.03 | 0.12 | 0.10 | 0.26 | 0.09 | 0.08 | −0.08 |
| \(\Delta\) | 1.00 | −0.25 | 0.62 | 0.54 | −0.19 | −0.21 | −0.00 | −0.30 | −0.21 | 0.11 |
| \(K\) | 1.00 | 0.07 | 0.14 | −0.20 | −0.17 | 0.11 | −0.17 | −0.14 | −0.54 |
| \(K'\) | 1.00 | 0.81 | −0.40 | −0.42 | −0.06 | −0.52 | −0.37 | 0.02 |
| \(\epsilon_3\) | 1.00 | 0.94 | 0.24 | 0.81 | 0.85 | −0.11 |
| \(\delta_0\) | 1.00 | 0.22 | 0.81 | 0.86 | −0.11 |
| \(\delta_0'\) | 1.00 | 0.25 | 0.19 | −0.04 |
| \(\delta_1\) | 1.00 | 0.74 | −0.14 |
| \(\phi\) | 1.00 | −0.10 |
| \(\lambda\) | 1.00 |

(b) Correlation matrix of the parameters, \(\lambda\) being the Wolfenstein parameter in the CKM matrix.

**Table 2: The fit value of the parameters and their correlations**

Since the phases coming from final state interactions should be interpreted as being generated by rescattering due to the presence of resonances, the phases should follow a distinct pattern determined by the masses of the resonances corresponding to particular isospin quantum numbers. The spectrum of the masses of these scalar resonances can be inferred upon by using the Gell-Mann-Okubo mass formula \([114-117]\). From the latter it can be seen that the mass of the resonance corresponding to the \(I = 1\) channel is smaller than the mass of the resonance corresponding to the \(I = 1/2\) channel. This implies that the strong phase shift due to resonance rescattering in the \(I = 1\) channel should be larger than that in the \(I = 1/2\) channel. As a consequence, the solution with negative phases seem to be the favourable solution. In the following discussion we shall focus more on the results with the negative solution keeping in mind that it is better motivated.
Table 3: The branching fractions that were used in the fit [109, 111, 112]. We also use BR(D_s^+ \rightarrow K^+ K_{SL}) = (29.5 \pm 1.4) \times 10^{-3} [113] measured by Belle. The fit results are the same for both the negative and the positive solutions for the phases, \delta_i.

| Channel                | Fit (x10^{-3}) | PDG (x10^{-3}) | BESIII (x10^{-3}) |
|------------------------|---------------|---------------|-------------------|
|                         | SCS           |               |                   |
| D_s^0 \rightarrow \pi^+ \pi^- | 1.448 \pm 0.019 | 1.407 \pm 0.025 | 1.508 \pm 0.028   |
| D_s^0 \rightarrow \pi^0 \pi^0 | 0.816 \pm 0.025 | 0.822 \pm 0.025 | -                 |
| D_s^0 \rightarrow \pi^+ \pi^0 | 1.235 \pm 0.033 | 1.17 \pm 0.06  | 1.259 \pm 0.040   |
| D_s^0 \rightarrow K^+ K^- | 4.064 \pm 0.044 | 3.97 \pm 0.07  | 4.233 \pm 0.067   |
| D_s^0 \rightarrow K_SK_S   | 0.168 \pm 0.012 | 0.17 \pm 0.012 | -                 |
| D_s^+ \rightarrow K^+ K_S   | 3.164 \pm 0.056 | 2.83 \pm 0.16  | 3.183 \pm 0.067   |
| D_s^+ \rightarrow \pi^0 K^+  | 1.41 \pm 0.15  | 0.63 \pm 0.21  | -                 |
| D_s^+ \rightarrow \pi^+ K_S   | 1.24 \pm 0.06  | 1.22 \pm 0.06  | -                 |
|                         | CA & DCS      |               |                   |
| D_s^+ \rightarrow \pi^+ K_S   | 15.80 \pm 0.29 | 14.7 \pm 0.8   | 15.91 \pm 0.31    |
| D_s^+ \rightarrow \pi^+ K_L   | 14.37 \pm 0.52 | 14.6 \pm 0.5   | -                 |
| D_s^0 \rightarrow \pi^+ K^-   | 38.96 \pm 0.32 | 38.9 \pm 0.4   | -                 |
| D_s^0 \rightarrow \pi^0 K_S   | 12.29 \pm 0.21 | 11.9 \pm 0.4   | 12.39 \pm 0.28    |
| D_s^0 \rightarrow \pi^0 K_L   | 9.73 \pm 0.21  | 10.0 \pm 0.7   | -                 |
| D_s^+ \rightarrow K^+ K_S     | 14.67 \pm 0.41 | 15.0 \pm 0.5   | -                 |
| D_s^+ \rightarrow \pi^0 K^+   | 0.151 \pm 0.013 | 0.181 \pm 0.027 | 0.231 \pm 0.022  |
| D_s^0 \rightarrow \pi^- K^+   | 0.141 \pm 0.003 | 0.1385 \pm 0.0027 | -               |
| D_s^0 \rightarrow \pi^+ K^-   | 39.1 \pm 0.32  | -               | 38.98 \pm 0.52    |

Table 4: Predictions of CP asymmetries using the branching fraction data and the HFLAV average of \Delta A_{CP}^{dir}.

| A_{CP} (D_s^0) | (\mu \pm \sigma) (%) | \delta_i \rightarrow \pm ve | A_{CP} (D_s^+ \rightarrow \pi^+ K_S) | (\mu \pm \sigma) (%) | \delta_i \rightarrow \pm ve |
|---------------|----------------------|------------------|--------------------------------|----------------------|------------------|
| D_s^0 \rightarrow \pi^+ \pi^- | 0.098 \pm 0.050 | 0.099 \pm 0.050 | D_s^+ \rightarrow K^+ K_S | -0.023 \pm 0.011 | -0.022 \pm 0.011 |
| D_s^0 \rightarrow \pi^0 \pi^0 | 0.003 \pm 0.018 | 0.072 \pm 0.019 | D_s^+ \rightarrow \pi^0 K^- | -0.034 \pm 0.016 | -0.030 \pm 0.016 |
| D_s^0 \rightarrow K^+ K^- | -0.030 \pm 0.020 | -0.039 \pm 0.020 | D_s^+ \rightarrow \pi^0 K^+ | 0.044 \pm 0.011 | -0.006 \pm 0.010 |
| D_s^0 \rightarrow K_S K_S | 0.036 \pm 0.017 | 0.031 \pm 0.017 |

The strongest constraint, by far, on (P + \Delta_3)/T comes from \Delta A_{CP}^{dir}. In figure 1 we show the posterior distribution of (P + \Delta_3)/T. It was pointed out earlier that the CP asymmetries in the different channels are parametrically correlated. Hence the constraints from \Delta A_{CP}^{dir} also put constraints on CP asymmetries in the other decay modes. We use this to make predictions for the CP asymmetries in the other SCS channels which we present in table 4 including the single channels A_{CP}(D_0 \rightarrow \pi^+ \pi^-) and A_{CP}(D_0 \rightarrow K^+ K^-). The errors in the prediction of the asymmetries vindicate our deduction that amongst the CP asymmetries \Delta A_{CP}^{dir} puts the strongest constraint on (P + \Delta_3)/T by far.

It is important to note here that while \Delta A_{CP}^{dir} is compatible with 0, the fit to (P + \Delta_3) is
comparable in size to the tree amplitudes parametrized by $T$. This implies that the penguin amplitudes, which appear in the terms proportional to $(P + T + \Delta_3)$, can be much smaller than – or, at best, comparable in size to – the tree amplitudes. As a result, the penguin amplitudes can no longer be considered the dominant contribution in CP asymmetries of these channels. While it is still sizable compared to the contribution proportional to $T + C$, it can only bring about a factor of few, and not an order of magnitude, enhancement contrary to what was previously expected. This can be clearly gauged from figure 1. Of course, on the contrary, it can effect a cancellation with the part proportional to $T + C$ and hence make the CP asymmetry vanishingly small, but one is allowed to hope that this scenario will not be manifest. What is quite important to note is that the predictions of CP asymmetries in these SCS modes using only the $\Delta A_{\text{CP}}$ data has errors bars that are almost the same, or significantly better than, what is projected as the sensitivity at Belle II with 50 ab$^{-1}$ of data as shown in table 6. We will see in section 5.5 what this implies for constraints on the penguin amplitudes in the future.

Finally, we make some predictions from our fit. The branching fraction of the decay mode $D_s^+ \to K^+ K_L$ is yet unmeasured. However, the sum of the branching fractions for $D_s^+ \to K^+ K_S$ and $D_s^+ \to K^+ K_L$ has been measured by Belle yielding [113]:

$$\text{BR}(D_s^+ \to K^+ K_S) + \text{BR}(D_s^+ \to K^+ K_L) = (29.5 \pm 1.1 \pm 0.9) \times 10^{-3} \quad (5.1)$$

while the branching fraction $\text{BR}(D_s^+ \to K^+ K_S) = (15.0 \pm 0.5) \times 10^{-3}$. Several predictions
have been made in the past for the rate asymmetry between $D_s^+ \rightarrow K^+ K_L$ and $D_s^+ \rightarrow K^+ K_S$ which is tantamount to predicting the branching fraction of the former since the branching fraction of the latter mode is measured to a very good precision. In $[80, 118]$ the branching fraction of $D_s^+ \rightarrow K^+ K_L$ is predicted to be smaller than the branching fraction of $D_s^+ \rightarrow K^+ K_S$. In contrast, we predict:

$$\text{BR}(D_s^+ \rightarrow K^+ K_L) = (14.98 \pm 0.39) \times 10^{-3},$$

which is almost equal to the branching fraction of $D_s^+ \rightarrow K^+ K_S$ with the central value of the former being greater than the latter. The discrepancy is discussed in the next section where we also discuss the rate asymmetry and compare with results in the literature. If we do not use the result in eq. (5.1) we get

$$\text{BR}(D_s^+ \rightarrow K^+ K_L) = (15.01 \pm 0.47) \times 10^{-3}.$$

We also predict the relative strong phase between the amplitudes of the modes $D^0 \rightarrow K^+ \pi^-$ and $D^0 \rightarrow K^- \pi^+$. The world average of the measured value of this phase is $(8.8^{+9.1}_{-9.7})^\circ [96]$ when one assumes that there is no CP violation in the DCS decays. From our fit we get:

$$\delta_{K\pi} = \delta_{K^- \pi^+} - \delta_{K^+ \pi^-} = 3.14^\circ \pm 5.69^\circ$$

which is compatible with the measured value. Various other estimates of $\delta_{K\pi}$ can be found in $[78, 119-122]$. In the exact SU(3)$_F$ limit $\delta_{K\pi}$ should be $0$ $[123-125]$, the deviation from which, as indicated by the fit result, underscores the significance of SU(3)$_F$ breaking through strong phases in the framework that we use.

### 5.2 Rate asymmetries

One can also define rate asymmetries involving interference of CA and DCS decays of the neutral $D^0$ meson to the neutral $K\pi$ final state. A method for measuring this was first proposed in $[126]$. The rate asymmetry for the neutral $D^0$ initial state is defined as

$$R(D^0, \pi^0) \equiv \frac{\Gamma (D^0 \rightarrow K_S \pi^0) - \Gamma (D^0 \rightarrow K_L \pi^0)}{\Gamma (D^0 \rightarrow K_S \pi^0) + \Gamma (D^0 \rightarrow K_L \pi^0)}.$$  \hspace{1cm} (5.5)

For the charged $D^+$ in the initial state, the rate asymmetry is defined as $R(D^+, \pi^+)$ with the substitutions $D^0 \rightarrow D^+$ and $\pi^0 \rightarrow \pi^+$. For $D_s^+$ the rate asymmetry is defined as $R(D_s^+, K^+)$ with the substitution $D^0 \rightarrow D_s^+$ and $\pi^0 \rightarrow K^+$ in the above relation. The rate asymmetry in eq. (5.5) leads us to another U-spin breaking parameter $\epsilon_0$, the real part of which is can be shown to be given by

$$\text{Re} (\epsilon_0) = \frac{R(D^0, \pi^0)}{4 \tan^2 \theta_C} - \frac{1}{2}. \hspace{1cm} (5.6)$$

From a CLEO Collaboration measurement of $K_S - K_L$ asymmetry $[127]$, the measured values for $R(D^0, \pi^0)$, $R(D^+, \pi^+)$ are

$$R(D^0, \pi^0)_{\text{CLEO}} = 0.108 \pm 0.025 \pm 0.024, \hspace{0.5cm} R(D^+, \pi^+)_{\text{CLEO}} = 0.022 \pm 0.016 \pm 0.018 \hspace{1cm} (5.7)$$

$^2$The predicted value has the opposite sign for the positive solutions of the strong phases.
leading to a value of $\text{Re}(\epsilon') = 0.00 \pm 0.16$ [128]. We compute $R(D^0, \pi^0)$, $R(D^+, \pi^+)$ and $\text{Re}(\epsilon')$ from our fit to the branching fractions and CP asymmetries and get (c.f. figure 2)\(^3\)

$$R(D^0, \pi^0) = 0.1166 \pm 0.0061, \quad R(D^+, \pi^+) = 0.048 \pm 0.020, \quad \text{Re}(\epsilon') = 0.045 \pm 0.029. \quad (5.8)$$

which is in fair agreement with the CLEO measurements. The various predictions for $R(D^0, \pi^0)$ and $R(D^+, \pi^+)$ that have been made previously are listed in table 5.

We present a prediction of $R(D_s^+, K^+)$ (c.f. figure 2)\(^4\):

$$R(D_s^+, K^+) = -0.0103 \pm 0.0074. \quad (5.9)$$

which can be compared with other predictions made in the past as listed in table 5. All the results are reasonably compatible. However, these imply that $\text{BR}(D_s^+ \to K^+ K_S) > \text{BR}(D_s^+ \to K^+ K_L)$ in [80, 118], whereas for the rest the contrary is true if one considers the central values of the ratio. Since this rate asymmetry depends on the estimate of the strong phases, a measurement of the latter can be used to test our predictions of the strong phases.

---

\(^3\)The posterior distributions of all three observables are non-Gaussian and hence, the error bars have been interpreted as the RMS of the distributions.

\(^4\)The predicted value is the same for both the solutions of the strong phases, negative and positive.
Figure 3: Fit results for $\Delta R$, $\epsilon_1$ and $\epsilon_2$ using the branching fraction data in table 3 and the HFLAV world average of $\Delta \Lambda_{CP}$. The green, red and orange regions are the 68%, 95% and 99% probability regions respectively.

5.3 Amplitude relations and SU(3)$_F$ breaking

While our parametrization is well motivated by SU(3)$_F$ arguments, it is also good to check if there are some ways of validating it. Here we follow a more general theoretical construction of SU(3)$_F$ arguments put forward by Gronau in [128] which also allows for a measure of the degree at which SU(3)$_F$ is broken by applying a higher order perturbation expansion in SU(3)$_F$ breaking. The amplitude relations for $D^0$ decays to pairs of neutral pseudoscalar mesons can be written as:

$$
R_1 \equiv \frac{|A(D^0 \to K^+\pi^-)|}{|A(D^0 \to \pi^+K^-)|\tan^2\theta_C} ,
$$

$$
R_2 \equiv \frac{|A(D^0 \to K^+K^-)|}{|A(D^0 \to \pi^+\pi^-)|} ,
$$

$$
R_3 \equiv \frac{|A(D^0 \to K^+\pi^-)| + |A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)|\tan\theta_C + |A(D^0 \to K^+\pi^-)|\tan^{-1}\theta_C} ,
$$

$$
R_4 \equiv \frac{\sqrt{|A(D^0 \to K^+K^-)||A(D^0 \to \pi^+\pi^-)|}}{|A(D^0 \to \pi^+K^-)||A(D^0 \to K^+\pi^-)|} .
$$

These four ratios are not mutually independent. They obey a trivial identity

$$
R_4 = R_3 \sqrt{\frac{1 - [(R_2 - 1)/(R_2 + 1)]^2}{1 - [(R_1 - 1)/(R_1 + 1)]^2}} .
$$

It can be shown that $R_i = 1$ in the limit of SU(3)$_F$ and the relation

$$
\Delta R \equiv R_3 - R_4 + \frac{1}{8} \left[ (\sqrt{2R_1 - 1} - 1)^2 - (\sqrt{2R_2 - 1} - 1)^2 \right] = \mathcal{O}(\epsilon_1^4, \epsilon_2^4) + \mathcal{O}(\hat{\delta}_1\epsilon_1^2, \hat{\delta}_2\epsilon_2^2) .
$$

(5.12)

differs from zero by terms of the order $\mathcal{O}(\epsilon_1^4, \epsilon_2^4) + \mathcal{O}(\hat{\delta}_1\epsilon_1^2, \hat{\delta}_2\epsilon_2^2)$, where $\epsilon_i$ and $\hat{\delta}_i$ are U-spin and Isospin breaking terms, respectively. One can then write the real parts of the SU(3)$_F$
breaking parameters $\epsilon_1$ and $\epsilon_2$ as

$$\text{Re}(\epsilon_i) = \frac{1}{2} \left( \sqrt{2R_i} - 1 - 1 \right) - \text{Re}(\delta_i) - 2\text{Re}(\delta_i)\text{Re}(\epsilon_i) + \mathcal{O}(\delta_i\epsilon_i) + \mathcal{O}(\epsilon_i^2)$$

(5.13)

with $i = 1, 2$. The U-spin breaking in $D^0 \rightarrow K^+\pi^-$ is denoted by $\epsilon_1$ and that in $D^0 \rightarrow K^+K^-$ is denoted by $\epsilon_2$. It is expected [128] that $\epsilon_2$ quantifies breaking in both the tree and penguin amplitudes while $\epsilon_1$ quantifies the breaking in only tree amplitudes. Hence, the former is expected to be somewhat larger than the latter. In our work we do not consider isospin breaking and hence $\delta_i = 0$. We test these relations in our parameterization of the amplitudes and use the parameters extracted from the branching fractions as inputs. We find a fair agreement with the results quoted in [128] for $\Delta R$, $\text{Re}(\epsilon_1)$ and $\text{Re}(\epsilon_2)$ as is evident from figure 3.

5.4 Correlations between CP asymmetries

![Correlations between CP asymmetries](Figure 4)

**Figure 4:** Correlations between asymmetries (in %) as given in equation (5.16) using the branching fraction data in table 3 and the HFLAV world average of $\Delta \Lambda_{\text{CP}}$ quoted in section 3. The orange, red and green regions are the 68%, 95% and 99% probability regions respectively.

As a second test of our parameterization we propose the correlation between the CP asymmetries that we have earlier explained are parametrically correlated. Since the asymmetries are correlated through the combination of parameters, $(P + \Delta_3)/T$, it is possible
to combine the expression for the asymmetries to obtain relations between them. By considering the $\pi\pi$ and $KK$ final states we have, symbolically,

$$A_{CP}(D \rightarrow KK) = f_{KK} \langle \vec{p} \rangle A_{CP}(D^0 \rightarrow \pi^+\pi^-) + g_{KK} \langle \vec{p} \rangle A_{CP}(D^0 \rightarrow \pi^0\pi^0) + h_{KK} \langle \vec{p} \rangle,$$

(5.14)

where $f_{KK} \langle \vec{p} \rangle$, $g_{KK} \langle \vec{p} \rangle$ and $h_{KK} \langle \vec{p} \rangle$ are functions of $\vec{p} = \{T, C, \kappa, \kappa', K, K', \Delta, \phi, \epsilon, \delta_0, \delta_0', \delta_1, \delta_1' \}$ and depend on the final $KK$ pair. With the central values for the parameters from our fits in table 2 we get for negative phases

$$A_{CP}(D^0 \rightarrow K^+K^-) = -0.657A_{CP}(D^0 \rightarrow \pi^+\pi^-) + 0.750A_{CP}(D^0 \rightarrow \pi^0\pi^0) + 2.78 \times 10^{-4},$$

$$A_{CP}(D^0 \rightarrow K_SK_S) = +3.47A_{CP}(D^0 \rightarrow \pi^+\pi^-) - 8.88A_{CP}(D^0 \rightarrow \pi^0\pi^0) - 3.28 \times 10^{-3}.$$

(5.15)

when we consider the fit with positive phases the constant terms change their signs. In the case in which we consider the limit $\Delta_4 \rightarrow 0$ we have,

$$A_{CP}(D^0 \rightarrow K^+K^-) = -0.394A_{CP}(D^0 \rightarrow \pi^+\pi^-) - 1.05 \times 10^{-6},$$

$$A_{CP}(D^0 \rightarrow K_SK_S) = +0.342A_{CP}(D^0 \rightarrow \pi^+\pi^-) + 2.75 \times 10^{-5},$$

$$A_{CP}(D^0 \rightarrow \pi^0\pi^0) = +0.352A_{CP}(D^0 \rightarrow \pi^+\pi^-) - 3.72 \times 10^{-4}.$$  

(5.16)

Likewise, the CP asymmetries in the other channels can also be correlated. These correlations between the predicted asymmetries are plotted in figure 4 and includes several SCS decay modes in which CP violation is possible. A deviation from these correlations would indicate a breakdown of our parameterization. The correlations have been derived by using only the branching fraction data and the measurement of $\Delta A_{CP}$ from LHCb. Most notably, the formalism we use renders the CP asymmetry in $D^0 \rightarrow K_SK_S$ completely correlated to $\Delta A_{CP}$ since the weak exchange diagram present in the $\Delta U = 0$ part of the amplitude of the former decay mode, and absent in the latter, is generated by rescattering and is not an independent contribution.

### 5.5 Constraints on penguin amplitudes from future measurements

With Belle II starting up and LHCb building a very strong charm program over the next few years, it is instructive to see what these measurements will mean in terms of constraining the penguin amplitudes. While the measurements of the branching fractions are expected to improve significantly too, this will not additionally constrain the penguin amplitudes directly. However, the ratio $(P + \Delta_3)/T$ would certainly benefit from an improved determination of $T$. Considering $T$ is already extracted at precision less than $O(1\%)$, improvements in this parameter will leave a negligible effect. On the other hand, not all the phases appearing in the SCS decays are very well constrained. An improvement in these would certainly improve the constraints on the penguin amplitudes. In particular, an improved measurement of $D^0 \rightarrow K_SK_S$, which is non-vanishing only when $SU(3)_F$ is broken, is quite important for further constraining the parameters that arise from this breaking specially because the branching ratio of this channel is not well measured currently.
Figure 5: Fit results for $P/T$ using the branching fraction data in table 3 and CP asymmetries listed in table 6. The green, red and orange regions are the 68%, 95% and 99% probability regions respectively. (BII-50: Belle II 50 ab$^{-1}$, LHCb-5: LHCb 5 fb$^{-1}$, LHCb-50: LHCb 50 fb$^{-1}$)

| $A_{CP}$ (channel) | mode (%) | RMS (%) |
|--------------------|----------|---------|
|                    | Current Fit | Belle II 50 ab$^{-1}$ | LHCb 5 fb$^{-1}$ | LHCb 50 fb$^{-1}$ |
| $D^0 \rightarrow \pi^+\pi^-$ | 0.0977 | 0.050 | 0.05 | – | – |
| $D^0 \rightarrow \pi^0\pi^0$ | -0.0035 | 0.018 | 0.09 | – | – |
| $D^0 \rightarrow K^+K^-$ | -0.0387 | 0.020 | 0.03 | – | – |
| $D^0 \rightarrow K_SK_S$ | 0.0362 | 0.017 | 0.17 | – | – |
| $D^+ \rightarrow K^+K_S$ | -0.0399 | 0.011 | 0.05 | – | – |
| $D^+_s \rightarrow \pi^+K_S$ | -0.0231 | 0.016 | 0.29 | – | – |
| $\Delta A_{CP}$ | -0.137 | – | – | 0.05 | 0.01 |

Table 6: Numbers used to generate the constraints on $P/T$ from future experiments. The column marked “Current Fit” shows the RMS from our prediction of the asymmetries using the branching fraction data and the LHCb measurement of $\Delta A_{CP}$ only and for the negative solution for the phases.

To keep the analysis simple and on the more conservative side we do not take into account any improvement in the measurement of the branching fractions. We project the central values of the CP asymmetries using their value at the global mode of the current fit and use the errors projected by the experiments. We use projections for Belle II at 50 ab$^{-1}$ for various asymmetries. We also use the projected measurement of $\Delta A_{CP}$ at LHCb with 5 fb$^{-1}$ and 50 fb$^{-1}$ data. Finally, we also combine all these projected measurements. These are tabulated in table 6.

In figure 5 we show how the constraints on $P/T$ will change with additional data. As is evident, the constraints are not much better than what we see in figure 1 with only the full Belle II data. The reason for this is that the projected precision of measurement of these asymmetries from the full Belle II data of 50 fb$^{-1}$ is comparable or worse than the precision of the prediction of the asymmetries from the current measurement of $\Delta A_{CP}$ as can be seen from table 6. Once the measurement of $\Delta A_{CP}$ improves, the constraint on
\( \psi/T \) gets much better, but significantly so only after 50 fb\(^{-1} \) of data from LHCb which is projected to be available by 2025.

6 Summary

In this work we extend the formalism presented in [86] with a larger menu of branching fractions for \( D \to PP \) with \( P = K, \pi \) but excluding the branching fractions which have \( \eta/\eta' \) in the final state. We extend the old parameterization with the parameters \( K^{(t)} \) and \( \kappa^{(t)} \) to address SU(3)\(_F\) breaking effects both in the tree and colour suppressed amplitudes. We introduce \( \epsilon_5 \) to address the splitting of the phases due to mass splitting between the \( D_0^- \) and the \( D_s^+ \). Another parameter \( \Delta \) is included to address the decays of \( D_{(s)}^+ \) mesons. To accommodate for CP asymmetry in the SCS decays we introduce three parameters \( P \), \( \Delta_3 \) and \( \Delta_4 \). The latter is parametrically suppressed due to an approximate selection rule. The former two cannot be resolved from CP asymmetries of the SCS decays we consider and hence only the sum can be extracted from data and we deem its ratio with \( T \) as the parameter relevant for the fit.

We perform a fit of the parameters to the branching fractions and \( \Delta A_{CP} \) using HEPfit and predict several CP asymmetries using our parametrization. In our framework, ignoring very small effects, the CP asymmetries show distinct correlations which can serve as a test of our framework. We also explore SU(3)\(_F\) breaking effects as advocated by Gronau [128] and find a good agreement with the results from that work. The rate asymmetries extracted from the branching fraction data agrees well with the CLEO collaboration data. As a future extension of this work, we will extend the parameterization to final states with \( \eta/\eta' \).

Within the ambit of our work we find reasonable success in trying to parameterize \( D \to PP \) decays within a SU(3)\(_F\) framework. The important conclusions of our work are:

- We succeed in describing the measured branching fractions by invoking SU(3)\(_F\) breaking using large phases from FSI, non-conservation of the strangeness changing vector current and slight shifts in the reduced matrix elements for CA and the DCS vs. the SCS decay amplitudes. This does not require the introduction of the parameter \( P \), \( \Delta_3 \) or \( \Delta_4 \).

- The values of the FSI phases, when considering the negative solutions, fall nicely along the pattern of the expected mass ordering of the resonance from the presence of which these FSI phases are generated. This also fixes the imaginary parts, which are relevant for the CP violating asymmetries. The negative solution is motivated by considering the masses of the resonances to be arranged according to the Gell-Mann-Ne’eman-Okubo mass formula which requires the strong phase in the \( I = 1 \) channel to be larger than that in the \( I = 1/2 \) channel.

- Once we relate the 15 in the \( \Delta U = 0 \) part of the amplitude to that in the \( \Delta U = 1 \) part of the amplitude the asymmetries depend on three new parameters. Of these, the combination \( P + T + \Delta_3 \) incorporates the uncertain strength of the penguin contributions. Then we apply an approximate selection rule that forbids the simultaneous
creation of a $d\bar{d}$ and a $s\bar{s}$ pair, similar to the OZI rule, to the penguin annihilation contribution coming from the 3. Hence, the third parameter, $\Delta_4$, is expected to be small by this approximate selection rule and contributes mainly to the asymmetry in $D^0 \to K_SK_S$ as can be seen from the SU(3)$_F$ limit. Moreover, the terms proportional to $T + C$ are constrained by the branching fraction data. The combination $P + \Delta_3$ cannot be disentangled from measurements. Hence all the CP asymmetries depend on this combination of parameter and are thus correlated.

- We show that amongst the current measurement of CP asymmetries, $\Delta A_{\text{dir}}^{\text{CP}}$ is by far the strongest constraint on the combination $(P + \Delta_3)/T$. We use this fact and the parametric correlation between the $\Delta U = 0$ part of the amplitudes to predict several asymmetries which are listed in table 4. Since $\Delta A_{\text{dir}}^{\text{CP}}$ constrains the penguin amplitude to $P \sim \mathcal{O}(T)$, the part proportional to it no longer dominates the CP asymmetries. Indeed the part proportional to $(T+C)$ becomes sizable in comparison. Hence the penguin amplitudes can no longer be expected to bring about an order of magnitude enhancement beyond the 1% level in the CP asymmetries of several channel and can enhance them by only a factor of few. This lies in contrast with what was previously expected as the effect of penguin amplitudes in the CP asymmetries of SCS $D \to PP$ decays.

- With the correlations between the asymmetries and the direction pointed at by the data we can propose methods for validating our SU(3)$_F$ framework by looking at rate asymmetries between several $K_S - K_L$ final states and the correlation between CP asymmetries in different channels. In particular, as a consequence of the strong phases determined by the fit, we predict the yet unmeasured rate asymmetry:

$$R(D_s^+, K^+) = -0.0103 \pm 0.0074.$$ 

- When we choose the negative solution for the phases, we also predict

$$\delta_{K\pi} = \delta_{K^+\pi^-} - \delta_{K^+\pi^-} = 3.14^\circ \pm 5.69^\circ.$$ 

In this framework of SU(3)$_F$ breaking that is driven by large phase from FSI due to rescattering through scalar resonances, it can be shown that CP asymmetries in all SCS modes are constrained to the per mille level by the current measurement of $\Delta A_{\text{dir}}^{\text{CP}}$.

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Figure 6: The marginalized posterior distributions of the parameters from the fit as given in table 2. The green, red and orange regions are the 68%, 95% and 99% probability regions respectively. The bottom three 2D marginalized plots show the correlations between the parameters $\delta_0$, $\delta'_0$ and $\phi$. The orange, red and green regions are the 68%, 95% and 99% probability regions respectively.

A Posterior distributions of the parameters for the full fit

The fit of the parameters to the branching fractions and $\Delta A_{\text{CP}}^{\text{dir}}$ and the predictions for the CP asymmetries was done with HEPfit. A model was built specifically for this purpose. The code necessary for replicating this analysis can be made available on request. In figure 6 we show the posterior distributions of the parameters from the fit with the mean and RMS listed in table 3 and $\Delta A_{\text{CP}}^{\text{dir}}$ quoted in section 3. Only the posteriors for $\delta_0$ and
show some deviation from being Gaussian distributions. The two phases $\delta_0$ and $\delta'_0$ and the angle $\phi$ that appear in the SCS decays are highly correlated. We show the correlation plots for these parameters in the bottom three plots of figure 6. The values of the CKM parameters used in these fits are from the UTfit average \cite{133}:

$$\lambda = 0.22534 \pm 0.00089 \quad A = 0.833 \pm 0.012$$  \hspace{1cm} (A.1)

$$\bar{\rho} = 0.153 \pm 0.013 \quad \bar{\eta} = 0.343 \pm 0.011$$  \hspace{1cm} (A.2)

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