Characterization of Programmable Integrated Quantum Voltage Noise Source with Variable Power Spectral Density

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Abstract A quantum voltage noise source (QVNS) based on the Josephson effect as a reference signal is indispensable in precise Johnson noise thermometry (JNT) because it provides quantum accurate power spectral density (PSD) represented by some physical constants and design parameters. An integrated QVNS (IQVNS) device that integrates the circuit elements required for generation of a QVNS signal into one chip has been developed at AIEST. The PSD of IQVNS should be variable for precise JNT measurement over a wide temperature range. Here, a new IQVNS device with variable PSD—programmable IQVNS—was developed and the values of the PSD were experimentally confirmed to be consistent with the theoretical values for various design parameters. Thus, JNT measurement over a wide temperature range can be realized with the IQVNS.

key words: Johnson noise thermometry, Josephson junction, rapid single flux quantum circuit, pseudo-random number, integrated quantum voltage noise source, thermodynamic temperature.

Classification: Superconducting electronics

1. Introduction

Johnson noise thermometry (JNT) [1, 2, 3, 4] is a primary thermometry that measures the thermodynamic temperature based on the Nyquist equation: the power spectral density (PSD) $S_R$ of a resistor $R$ at a temperature $T$ is represented as

$$S_R = 4kRT$$

(1)

in a low-frequency and high-temperature limit [5, 6], where $k$ is the Boltzmann constant.

A quantum voltage noise source (QVNS) that generates pseudo-random voltage noise is used as a reference signal to measure $S_R$ precisely [7, 8, 9, 10]. In these previous works, the quantum voltage noise was generated by driving the Josephson junction (JJ) arrays at a cryogenic temperature with a pulse-pattern generator or arbitrary waveform generator at room temperature [9, 10].

In contrast, the National Institute of Advanced Industrial Science and Technology (AIEST) has developed an integrated QVNS (IQVNS) device that includes the rapid single-flux quantum (RSFQ) circuit [11] elements required to generate a QVNS signal in a single chip. The IQVNS allows for a simple experimental apparatus without the digitally controlled high-frequency electronics at room temperature needed for the QVNS. This reduces costs and eliminates electromagnetic noise from the electronics. Moreover, the tone spacing of the PSD of the IQVNS is much smaller than that of the QVNS because the period of the IQVNS signal is much longer than that of the QVNS.

The PSD of the IQVNS output $S_{IQ}$ is expressed as

$$S_{IQ} = \frac{2(M + 1)}{M} \left( \frac{h}{2e} \right)^2 f_L N_1^2 N_2^2$$

(2)

where $h$ and $e$ are the Planck constant and the elementary charge, respectively [4, 12, 13, 14, 15, 16, 17, 18]. The meanings of the design parameters ($M, f_L, N_1$, and $N_2$) are explained later.

JNT based on the IQVNS (IQVNS-JNT) had been developed for determination of the Boltzmann constant and the elementary charge, respectively [4, 12, 13, 14, 15, 16, 17, 18]. The meanings of the design parameters ($M, f_L, N_1$, and $N_2$) are explained later.

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of the $T - T_{90}$ values.

To measure the thermodynamic temperature precisely with IQVNS-JNT over a wide temperature range, the PSD of the IQVNS should be variable to match the variation of $S_R$ because the ratio of $S_{1Q}/S_R$ should be close to 1 to avoid a detector nonlinearity effect [3, 7]. Therefore, a programmable IQVNS (pIQVNS) device that has variable $S_{1Q}$ by changing a design parameter from a computer was developed, whereas the design parameter was fixed in the previous version of the IQVNS. In this article, the design and characterization of the pIQVNS device with variable PSD are presented.

2. Design of the Programmable IQVNS Device and Experimental Setup

2.1 Programmable IQVNS design

Fig. 1 shows a block diagram of an entire pIQVNS system, which includes the pIQVNS (dashed square) and room-temperature electronics. The pIQVNS contains seven components: a prescaler (PRSCL), an $m$-bit pseudo-random noise generator (PRNG), a variable pulse number multiplier (VPN M) with a multiplication factor $N_1$, a deserializer (DS), a buffer, a demultiplexer (DMUX), and a bipolar voltage multiplier (BPVM) with a multiplication factor $N_2$. The PRSCL is a device that divides the input clock frequency $f_{\text{clk}}$ of 8.192 GHz into $f_{\text{LF}} = f_{\text{clk}}/4096$. The PRNG generates pseudo white noise based on a maximum length sequence algorithm. The PRNG is configured by an $m = 21$ stage feedback shift register. The total length of the pseudo-random signal $M$ is expressed as $M = 2^m - 1 = 2097151$. The VPNM produces $N_1$ pulses with a pulse repetition frequency of $f_{\text{clk}}$. To enable JNT measurement over a wide temperature range, the design was revised so that the factor $N_1$ is variable in the range of 16 to 271, based on the previous design with a fixed factor of $N_1 = 74$ specialized for measurement at $T_{\text{TPW}}$ [15]. The BPVM generates a quantum accurate bipolar voltage waveform $v_{1Q}(t)$ by multiplying the input SFQ pulses by the factor $N_2$. The BPVM is a pair of series-connected arrays, where each array contains $N_2 (= 4)$ dc superconducting quantum interference devices (SQUIDs) [28] connected in series. The design and operation of these components, except the DS and buffer, are basically the same as those previously explained in Ref. [15]. On the other hand, the DS and buffer are newly introduced to control the VPNM. Details of the VPNM, DS, and buffer are described later.

The VPNM was revised according to the design shown in Ref. [15]. The VPNM contains an 8-bit resettable toggle flip-flop (RTFF)-based counter with a 4-stage delay flip-flop (DFF)-based timing buffer. Connection of the 8-bit control reset signals $r_i = 0, 1 (0 \leq i \leq 7)$ to 8 RTFFs enables initialization of the counter to an arbitrary state. The factor $N_1$ depends on the counter state; therefore, the relationship between the values $r_0-r_7$ and $N_1$ is as follows.

$$N_1 = 2^8 - (r_7 \times 2^7 + r_6 \times 2^6 + r_5 \times 2^5 + r_4 \times 2^4)$$
$$+ (r_3 \times 2^3 + r_2 \times 2^2 + r_1 \times 2 + r_0 \times 2^0),$$  \hspace{1cm} (3)

where the factor $N_1$ ranges from 16 to 271, which corresponds to a sufficiently wide range for JNT measurement from 12.5 K to 3640 K when using a 100 Ohm sensing resistor.

The 1:8 DS based on shift-and-dump architecture [29] was adopted to introduce the 8-bit control signals $r_0-r_7$, which reduces the number of input channels from 8 to 3. A non-destructive readout (NDRO)-based 8-bit buffer was also used. A serialized control signal is introduced from $N_1$ cont to the DS with a low frequency clock $(ds\_clk)$. The internal state of the DS is then dumped and stored in the NDRO-based buffer as control signals $r_0-r_7$ for the RTFFs. Initialization of the counter in the VPNM is then repeatedly possible at high speed by readout from the buffer, which enables repeated operation of pulse number multiplication. Robust timing design was applied to all elemental circuits, including the VPNM of the RTFF-based counter, to ensure stable operation up to $f_{\text{clk}}$ of approximately 10 GHz.

An RSFQ cell library and tools for circuit design [30] were used. The circuit is composed of a total of 1529 JJs. The DC bias currents are 131 mA for the digital components, and 173 mA for entire pIQVNS including auxiliary circuits. The chip was fabricated using the AIST Nb standard II process [31], in which Nb/AIOx/Nb JJs have a critical current density of $J_c = 2.5 \text{ kA/cm}^2$.

2.2 Experimental setup and evaluation
Fig. 1 also shows the room temperature electronics, such as a digital-to-analog converter (DAC), a signal generator (SG) and a dc bias current supply. To control the factor $N_1$ for the pIQVNS, a control signal is introduced using $N_1_{cont}$, $ds_{clk}$, and $dump$ with the DAC controlled by a computer. The SG applies a clock signal at a frequency of $f_{clk} = 8.192$ GHz. A pseudo-random noise waveform $v_{IQ}(t)$, with a PSD determined using Eq. (2), is then obtained. The PRNG produces a pseudo-random signal with a length $M = 2^{20}$ at clock frequency $f_{LF} = f_{clk}/4096 = 2$ MHz; therefore, the waveform is periodic with a repetition period of $T = M/f_{LF}$ and the PSD is discrete with a tone spacing of $\Delta f_{IQ} = 1/T = f_{LF}/M = 0.954$ Hz.

The pIQVNS device in a liquid helium cryostat was characterized using the noise thermometer developed for measurements of the Boltzmann constant [18]. The cross power spectrum of the pIQVNS output signals was compared with that for a 100 $\Omega$ resistor at $T_{TPW}$. The details of equipment and methods used to obtain the cross power spectrum are described in our previous report [18].

Fig. 2(a) shows the cross power spectrum of a 100 $\Omega$ resistor at $T_{TPW}$ (273.16 K) and (b) cross power spectra of the pIQVNS for $N_1 = 32, 64, 74, 128$, and 256.

### Fig. 2
(a) Cross power spectrum of a 100 $\Omega$ resistor at $T_{TPW}$ (273.16 K) and (b) cross power spectra of the pIQVNS for $N_1 = 32, 64, 74, 128$, and 256.

### Fig. 3
(a) Spectral ratio of output signal of the pIQVNS to thermal noise of a 100 $\Omega$ resistor at $T_{TPW}$. The horizontal and vertical axes correspond to theoretical values ($r_{calc}$) and experimental values ($r_{exp}$), respectively. (b) Relative difference between $r_{exp}$ and $r_{calc}$ with respect to $r_{calc}$. (c) Difference between experimental and theoretical value in terms of $N_1$. The circles and the crosses are the results for the calculation without the removal of unwanted tones and for with the removal of unwanted tones, respectively.

### 3. Results and Discussion

Functional tests were performed for the pIQVNS digital components for several typical factors, $N_1 = 16, 74, 78, 235$ and 271, at a clock frequency of 10 kHz. Correct operation was confirmed with operation from 105 mA to 145 mA based on the dc bias current. The current-voltage characteristics for the BPVM were also evaluated. The widths of the constant voltage steps were from 209 $\mu$A to 301 $\mu$A for both positive and negative arrays, which means that stable operation is possible for JNT measurement.

Fig. 2(a) shows the cross power spectrum of a 100 $\Omega$ resistor ($S_N$) placed at $T_{TPW}$. The cross power exhibits a
flat behavior over a wide frequency range from approximately 1 kHz to several hundred kilohertz. This flat part corresponds to the right side of the Nyquist Eq. (1). No obvious unwanted tones were observed. This result is consistent with the previous results reported to date [16, 17, 18].

Fig. 2(b) shows the cross power spectra of the pIQVNS output signal \( S_{IQ} \) for \( N_1 = 32, 64, 74, 128, \) and 256. The signals generated by the pIQVNS are periodic and therefore their spectra are discrete; however, the spectra appear continuous like that of the thermal noise of a resistor because the frequency resolution of the spectrum measurement is adjusted to be the same as the tone spacing of 0.954 Hz. The values of \( S_{IQ} \) in the flat part increase with \( N_1 \) in Fig. 2(b). As expected from Eq. (2), the spectra for \( N_1 = 32, 64, 128 \) and 256 seem to be arranged in equal intervals on a logarithmic scale, which implies that the pIQVNS basically operates as designed.

However, on the other hand, many unwanted tones were evident, especially in the frequency range above 50 kHz, whereas these tones were not observed with the previous IQVNS using a fixed \( N_1 \). The pIQVNS device was directly connected to the computer with metallic wires for control of the DS and buffer whereas there was no direct connection with metallic wires in the previous IQVNS. Therefore, the unwanted tones were probably due to the electrical noise from the computer through the metallic wires. It is expected that these unwanted tones could be eliminated, in principle, by isolating the pIQVNS device from the computer, such as via the introduction of an optical fiber connection instead of the metallic wires.

To verify that the PSD of the pIQVNS matched the value calculated by Eq. (2), spectral ratios \( S_{IQ}/S_R \) were evaluated for various \( N_1 \); an experimental spectral ratio \( r_{exp} \) obtained by the measured \( S_{IQ} \) and \( S_R \) was compared with a theoretical spectral ratio \( r_{calc} \) calculated from Eqs. (1) and (2). The experimental spectral ratio \( r_{exp} \) is the value in the low-frequency limit when a ratio spectrum \( S_{IQ}/S_R \) (calculated from measured \( S_{IQ} \) and \( S_R \)) is fitted using a model function of \( S_{IQ}/S_R = a_0 + a_2 f^2 \), i.e., \( r_{exp} = a_0 \). The circle data points in Fig. 3(a) show \( r_{exp} \) for various \( N_1 \) as a function of \( r_{calc} \). The points are expected to be plotted on the dashed line in Fig. 3(a), which represents \( r_{exp} = r_{calc} \). The results for \( N_1 = 78 \) are also shown in Fig. 3 and as not shown in Fig. 2. When \( N_1 = 74, S_{IQ} \) is almost the same as \( S_R \) for a 100 Ω resistor at the triple point of gallium temperature (302.91 K), and \( r_{calc} \) is 1.105.

The circle data in Fig. 3(b) show the relative difference of \( r_{exp} \) and \( r_{calc} \) with respect to \( r_{calc} \). The difference is small around \( r_{calc} = 1 \) (\( N_1 = 74 \)) and becomes larger as \( r_{calc} \) deviates from 1. This difference in spectral ratio can be converted to a difference for \( S_{IQ} \) as shown in Fig. 3(c). The difference in terms of \( N_1 \) is estimated as the difference between the set values \( N_1\text{(set)} \) and the experimental values \( N_1\text{(exp)} \) calculated using Eqs. (1) and (2) from \( r_{exp} \). Although the maximum difference in \( N_1 \) of 2.4 corresponds to the error in the least-significant bit (LSB) of the buffer, it is unlikely to occur according to the results of functional tests for the pIQVNS digital components described at the beginning of this section. Similarly, external random noise would affect not only the LSB but other bits, which would result in a much larger error in \( N_1 \) than 2.4. For these reasons, it is not likely that the difference in \( N_1 \) is attributed to a malfunction of the digital electronics of the pIQVNS device, but rather can be attributed to analog error, especially unwanted tones.

To estimate the effect of unwanted tones, we have tested another calculation for \( r_{exp} \) using \( S_{IQ} \) without unwanted tones by removing the region around them and \( S_R \) without the same region as \( S_{IQ} \). Examples of the ratio spectra \( S_{IQ}/S_R \) with and without unwanted tones for \( N_1 = 32 \) are shown in Fig. 4. The spectrum with removal of the unwanted tones is multiplied by 1/10 for clarity, and the shaded areas indicate the removed regions. As a result, the deviation from the theoretical value was decreased, as shown by the crosses in Figs. 3(a), (b), and (c). Qualitatively, the deviation from the theoretical value in the spectral ratio \( \delta r \) is \( 3.4 \times 10^{-2} \) with the removal of unwanted tones, while \( \delta r \) is \( 5.4 \times 10^{-2} \) without removal of the unwanted tones; here, \( \delta r \) is defined as the square root of sum of squares of the relative difference in the spectral ratio:

\[
\delta r = \sqrt{\sum N_1 \left( \frac{r_{exp,N_1} - r_{calc,N_1}}{r_{calc,N_1}} \right)^2}
\]

for \( N_1 = 32, 64, 74, 78, 128 \) and 256. Similarly, assuming the
deviation in $N_1$ as \[ \delta N_1 = \sqrt{\sum_{i=1}^{N_1} (N_{1\text{exp}} - N_{1\text{set}})^2}, \]
\[ \delta N_1 = 1.0 \text{ with removal of the unwanted tones, while} \]
\[ \delta N_1 = 2.6 \text{ without removal of the unwanted tones. This} \]
implies that the PSD of the pIQVNS would be closer to the
theoretical value, if the unwanted tones were eliminated experimentally. Note that the removed
unwanted tones are limited to noticeable tones, as shown in
Fig. 4. Therefore, it is difficult to fully calculate the
effect of unwanted tones because there may be many
invisible unwanted tones left in $S_{IQ}$. The cause of $\delta r$ or
$\delta N_1$ with removal of the unwanted tones is considered
to be the effect of remnant unwanted tones and the
nonlinearity due to the difference in the PSD level. In
conclusion, it was demonstrated that the PSD of the
pIQVNS device is controlled according to the input
value of $N_1$, so that the pIQVNS device operated as
designed.

4. Summary

In this work, a pIQVNS device was developed so that
PSD could be varied according to the thermal noise of a
resistor over a wide temperature range. To make the PSD
of the IQVNS variable, the design parameter $N_1$ in the
VPNM was modified to be programmable from a
computer by the introduction of a DS and 8-bit buffer.
For characterization of the pIQVNS device, the
experimentally obtained ratio values of the PSD of the
pIQVNS to that of a 100 $\Omega$ resistor at $T_{TPW}$ were
compared to the theoretical values for various $N_1$. As a
result, the experimental values were in good agreement
with the theoretical values over a wide $N_1$ range, which
verified that the pIQVNS device was operated as
designed. By the elimination of unwanted tones, the
pIQVNS device is expected to be applicable as a reference signal for JNT measurements over a wide
temperature range.

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