Comparison of Two Independent Sr Optical Clocks with $1 \times 10^{-17}$ Stability at $10^3$ s

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(Dated: May 10, 2014)

Many-particle optical lattice clocks have the potential for unprecedented measurement precision and stability due to their low quantum projection noise. However, this potential has so far never been realized because clock stability has been limited by frequency noise of optical local oscillators. By synchronously probing two $^{87}$Sr lattice systems using a laser with a thermal noise floor of $1 \times 10^{-15}$, we remove classically correlated laser noise from the intercomparison, but this does not demonstrate independent clock performance. With an improved optical oscillator that has a $1 \times 10^{-16}$ thermal noise floor, we demonstrate an order of magnitude improvement over the best reported stability of any independent clock, achieving a fractional instability of $1 \times 10^{-17}$ in 1000 s of averaging time for synchronous or asynchronous comparisons. This result is within a factor of 2 of the combined quantum projection noise limit for a 160 ms probe time with $\sim 10^8$ atoms in each clock. We further demonstrate that even at this high precision, the overall systematic uncertainty of our clock is not limited by atomic interactions. For the second Sr clock, which has a cavity-enhanced lattice, the atomic-density-dependent frequency shift is evaluated to be $-3.11 \times 10^{-17}$ with an uncertainty of $8.2 \times 10^{-19}$.

Precise time keeping is foundational to technologies such as high-speed data transmission and communication, GPS and space navigation, and new measurement approaches for fundamental science. Given the increasing demand for better synchronization, more precise and accurate clocks are needed, motivating the active development of atomic clocks based on optical transitions. Several optical clocks have surpassed the systematic uncertainty of the primary Cs standard $^{1,2}$. Two examples are the NIST trapped Al$^+$ single ion clock, with a systematic uncertainty of $8.6 \times 10^{-18}$ $^{3}$, and the JILA $^{87}$Sr neutral atom lattice clock, at the $1.4 \times 10^{-16}$ level $^{4,5}$. The field of optical atomic clocks has been very active in recent years, with many breakthrough results coming from both the ion clock $^{6–9}$ and lattice clock $^{10–15}$ communities.

In principle, the stability of an optical lattice clock can surpass that of a single-ion standard because the simultaneous interrogation of many neutral atoms reduces the quantum projection noise (QPN) of the lattice clock $^{14,17}$. QPN determines the standard quantum limit to the clock stability, and it can be expressed as

$$\sigma_{\text{QPN}}(\tau) = \frac{X}{\pi Q} \sqrt{\frac{T_c}{N\tau}} \tag{1}$$

Here, $\sigma_{\text{QPN}}(\tau)$ is the QPN-limited fractional instability of a clock, $Q$ is the quality factor of the clock transition, $N$ is the number of atoms, $T_c$ is the clock cycle time, $\tau$ is the averaging time (in seconds), and $\chi$ is a numerical factor near unity that is determined by the line shape of the clock transition spectroscopy. In a typical lattice clock, $N$ is on the order of $10^3$. In the case of the Al$^+$ ion clock, $N = 1$, and a fractional instability of $2.8 \times 10^{-15}/\sqrt{\tau}$ for a two-clock comparison has been demonstrated $^{3}$. For typical values of $T_c$ and $N$, a QPN-limited $^{87}$Sr lattice clock could potentially reach a given stability 500 times faster than the Al$^+$ clock.

Despite this promise, thus far the instability of lattice clocks has been far worse than the QPN limit. Instead, demonstrated lattice clock instability has been dominated by downsampled broadband laser noise (the Dick effect $^{18}$) at a few times $10^{-15}/\sqrt{\tau}$, similar to that of the best ion systems $^{4,11,12,19}$. To improve the precision of lattice clock systematic evaluations while avoiding the challenge of building more stable clock lasers, a synchronous interrogation method can be implemented $^{12,20}$. Synchronous interrogation facilitates laser-noise-free differential measurements between two atomic systems; however, in this approach, these systems are not independent clocks.

In this work, we achieve instability at the $10^{-16}/\sqrt{\tau}$ level for two independent $^{87}$Sr optical lattice clocks. Using a new clock laser stabilized to a 40 cm optical reference cavity $^{2}$ with a thermal noise floor $^{21}$ of $1 \times 10^{-16}$, we directly compare two independently operated $^{87}$Sr clocks. The combined stability of these clocks is within a factor of 2 of the QPN limit, reaching $1 \times 10^{-17}$ stability in only 1000 s. We also use synchronous interrogation to study the effect of laser noise on clock stability, demonstrating its effectiveness in removing correlated noise arising from a 7 cm cavity with a $1 \times 10^{-15}$ thermal noise floor. Operating with the 40 cm cavity, on the other hand, synchronous and asynchronous interrogations (the latter of which demonstrates independent clock performance) yield nearly the same measurement precision for a given averaging time.

This high measurement precision will permit much shorter averaging times for a range of applications, including investigations of systematic uncertainties in lattice clocks. In particular, we are able to characterize one of the most challenging systematics in a many-particle
clock—the density-related frequency shift $10^{-22} 24$—at an uncertainty below $1 \times 10^{-18}$ for our second Sr clock. The only remaining major systematic uncertainty for lattice clocks is the blackbody-radiation-induced Stark shift $13, 25, 27$. One can mitigate this effect by trapping atoms in a well-characterized blackbody environment or cold enclosure [28].

Our previous clock comparisons involved referencing our first generation $^{87}$Sr clock (Sr1) to various clocks at NIST using a 3.5 km underground fiber optic link [1, 26]. To evaluate the stability of the $^{87}$Sr clock at the highest possible level, we constructed a second Sr clock (Sr2) for a direct comparison between two systems with similar performance. In both systems, $^{87}$Sr atoms are first cooled with a Zeeman slower and a magneto-optical trap (MOT) on a strong 30 MHz transition at 461 nm. Then a second MOT stage, operating on a 7.5 kHz intercombination transition at 689 nm, cools the atoms to a few $\mu$K. Atoms are then loaded from their 689 nm MOTs into 1D optical lattices and are nuclear spin polarized. The lattices operate near the “magic” wavelength at 813 nm where the differential AC Stark shift for the $^{1}S_0$ and $^{3}P_0$ clock states is identically zero [29].

The lattice for Sr1 is made from the standing wave component of a retroreflected optical beam focused to a 32 $\mu$m radius. The power in one direction of this beam is 140 mW, corresponding to measured trap frequencies of 80 kHz along the lattice axis and 450 Hz in the radial direction. From this trap frequency we estimate a 22 $\mu$K trap depth. The Sr2 lattice utilizes an optical buildup cavity so that laser power in one direction of this lattice is 6 W. The cavity has a finesse of 120 and is mounted outside our vacuum chamber. The intracavity beam radius for this lattice is 160 $\mu$m, which yields a much greater trap volume. Trap frequencies in this lattice are 100 kHz and 120 Hz in the axial and radial directions, respectively. We estimate a 35 $\mu$K trap depth for the cavity-enhanced lattice.

The optical local oscillator for the Sr1 and Sr2 systems is derived from a common cavity-stabilized diode laser at 698 nm, but two different acousto-optic modulators (AOMs) provide independent optical frequency control for each system [Fig. 1(a)]. For all measurements presented in this Letter, we use Rabi spectroscopy with a 160 ms probe time, corresponding to a Fourier-limited linewidth of 5 Hz. For the stability measurements, we use 1000 atoms for Sr1 and 2000 atoms for Sr2. The optical frequency is locked to the clock transition using a digital servo that provides a correction to the AOM frequency for the corresponding clock.

To provide a quantitative understanding of the role of laser noise in our clock operations, we use two different clock lasers in our experiment. The first clock laser is frequency stabilized to a vertically oriented 7 cm long cavity with a thermal noise floor of $1 \times 10^{-15}$. This 7 cm reference cavity was used in much of our previous clock work and represented the state-of-the-art in stable lasers until recently. The second laser is stabilized to a horizontal 40 cm long cavity with a thermal noise floor of $1 \times 10^{-16}$ [Fig. 2(a)], which is similar to the record performance achieved with a silicon-crystal cavity [31]. The greater cavity length and use of fused silica mirror substrates both reduce the thermal noise floor of this laser [3]. Other significant improvements for the 40 cm system include a better vacuum, active vibration damping, enhanced thermal isolation and temperature control, and an improved acoustic shield.

When comparing the two $^{87}$Sr systems using the 7 cm reference cavity, the probe pulses for the Sr1 and Sr2 clock transitions are precisely synchronized [Fig. 1(b)]. The responses of both digital atomic servos are also matched. This synchronous interrogation allows each clock to sample the same laser noise; therefore, the difference between the measured clock transition frequencies for Sr1 and Sr2 benefits from a common-mode rejection of the laser noise. Because of this common-mode laser noise, simultaneous measurements of the excitation fraction for the Sr1 and Sr2 atomic servos show classical correlations [Fig. 2(b)], as evidenced by the distribution of these measurements in the shape of an ellipse stretched along the correlated (diagonal) direction. The minor axis of this distribution indicates uncorrelated noise such as QPN.

The 40 cm cavity supports a tenfold improvement in laser stability, and we estimate that the Dick effect contribution is close to that of QPN for clock operation.
involves comparing two independent atomic servos on the full stability of our clocks [Fig. 3(a)]. A self-comparison
measures the short- and long-term stability in two ways.

By drifting systematic shifts. Using the 40 cm cavity, we
term (\(\nu_1 - \nu_2\)) by \(\nu_1\) and \(\nu_2\) are the frequencies to which the two servos are
locked. Dividing (\(\nu_1 - \nu_2\)) by \(\sqrt{2}\) extrapolates the self-comparison stability to the expected performance of a
comparison between the Sr2 system [10]. Updates for these two digital servos alter-
ternate for each experimental cycle. Thus the difference
between these servo frequencies is sensitive to the Dick
effect and QPN and therefore represents the short-term
stability of an independent clock [32, 33]; however, it does not measure the clock’s long-term stability as it is insen-
tive to all drifts at time scales greater than 5 s. The
other component of this approach, the synchronous com-
parison, is sensitive to long-term drifts on either system,
but in the short term it is free of correlated laser noise.
Together these two data sets provide a complete picture
of our clock’s short- and long-term stability, and the small
difference between them after about 10 s implies that our
clocks are only minimally affected by correlated noise.

In the second approach, we measured the full stability
of our clock with an asynchronous comparison, which is

To test this, we operate the two clocks asynchronously,
where the clock probes are timed such that the falling
edge of the Sr1 pulse and the rising edge of the Sr2 pulse are always separated by 10 ms [Fig. 1(b)]. During
this asynchronous comparison, the two clocks sample
different laser noise, preventing common-mode laser noise
rejection. The Sr1–Sr2 excitation fraction scatter plot
[Fig. 2(b)] resembles a 2D Gaussian distribution, which is
consistent with both clocks being dominated by un-
correlated white noise. Synchronous comparisons with
the 40 cm cavity were also performed, indicating a similar
distribution for the scatter plot of the Sr1 vs. Sr2 excitation [Fig. 2(b) inset].

With this understanding of laser noise effects in our
clocks, we now evaluate the clock stability. In the short
term (<100 s) the clock stability is limited by laser noise
and QPN, and in the long term (\(\sim 1000\) s) it is limited by
drifting systematic shifts. Using the 40 cm cavity, we
measure the short- and long-term stability in two ways.
The first approach combines information from both a self-
comparison and a synchronous comparison to infer the
full stability of our clocks [Fig. 3(a)]. A self-comparison
involves comparing two independent atomic servos on the
Sr2 system [10]. Updates for these two digital servos alter-
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In the second approach, we measured the full stability
of our clock with an asynchronous comparison, which is
sensitive to both short- and long-term instability. Beyond the atomic servo response time (\(>20\) s), an asynchronous comparison reflects the performance of two independent clocks. Analysis of the Dick effect for our asynchronous pulse sequence (and a thermal-noise-limited local oscillator) shows that our asynchronous comparison reproduces independent clock performance within 6%. The Allan deviation of the comparison signal is shown in Fig. 3(b). These results demonstrate that one or both of our \(^{87}\text{Sr}\) clocks reaches the \(1 \times 10^{-17}\) level in 1000 s, representing the highest stability for an individual clock and marking the first demonstration of a comparison between independent neutral-atom optical clocks with a stability well beyond that of ion systems.

The enhanced stability of many-particle clocks can come at the price of higher systematic uncertainty due to density-dependent frequency shifts, which arise from atomic interactions. This shift has received a great deal of attention in recent years, with experiments and theory centered around schemes for explaining and minimizing this effect for optical lattice clocks \([10, 23, 34–36]\). These shifts are sensitive to both short- and long-term instability. Beyond the highest stability for an individual clock and marking the first demonstration of a comparison between independent neutral-atom optical clocks with a stability well beyond that of ion systems.

To verify that the shift is linear in atom number, we vary \(\Delta N\) by changing \(N_{\text{high}}\) while setting \(N_{\text{low}}\) to 2000–3000 atoms [Fig. 4]. We analyze the density shift data using the statistical analysis from our previous work \([10]\).

**FIG. 4:** The measured \(^{87}\text{Sr}\) density shift as a function of \(\Delta N\). Each point on this plot represents an average over a bin of 30 measurements of \((\nu_{\text{high}} - \nu_{\text{low}})/(N_{\text{high}} - N_{\text{low}})\). Our statistics show that the density shift for our trapping conditions is linear within our quoted uncertainty of \(8.2 \times 10^{-19}\). **Inset:** A single 2000 s long density shift measurement with \(\Delta N \approx 41000\). The shift per atom was measured and then scaled up to 2000 atoms for a typical running condition. This measurement shows that a single density shift evaluation for 1000 s using a large atom number modulation is sufficient for a \(1 \times 10^{-18}\) clock.

To determine the \(^{87}\text{Sr}\) density shift of \((-3.11 \pm 0.08) \times 10^{-17}\) at 2000 atoms. At this atom number, the total shift is sufficiently small such that our clock is stable at the \(1 \times 10^{-18}\) level in the presence of typical atom number drifts.

In summary, we have demonstrated comparisons between two independent optical lattice clocks with a combined instability of \(4.4 \times 10^{-16}/\sqrt{T}\), with a single clock demonstrating \(1 \times 10^{-17}\) level instability at 1000 s. We have also determined the density-dependent frequency shift uncertainty in our cavity-enhanced lattice at \(8.2 \times 10^{-19}\), with single measurements averaging down to the \(1 \times 10^{-18}\) level in 1000 s.

We acknowledge funding support from NIST, NSF, and DARPA. JRW is supported by NRC RAP. MB acknowledges support from NDSEG. SLC acknowledges support from NSF. We thank X. Zhang and W. Zhang for technical contributions.

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