DOUBLE BETA DECAY

S. P. Rosen

College of Science
The University of Texas at Arlington
Arlington, Texas 76019

ABSTRACT

I review the subject of double beta decay with emphasis upon its implications for the neutrino. It is shown that even if right-handed currents provide the phenomenological mechanism for no-neutrino decay, the fundamental mechanism underlying the process must be neutrino mass. Estimates of the mass required suggest that this mechanism is less likely than the direct mechanism of Majorana mass terms in the mass matrix.

Invited talk at the Franklin Symposium honoring Frederick Reines and the discovery of the neutrino held at the Franklin Institute, Philadelphia
April 30, 1992.

This research is supported in part by the U. S. Department of Energy under grant DE-FG05-92ER40691.
DOUBLE BETA DECAY

S. P. Rosen

College of Science
The University of Texas at Arlington
Arlington, Texas 76019

1 Introduction

Nature has a way of teasing us with her most intimate secrets: she wants us to know that they are there, but she is reluctant to let us find out exactly what they are. So she throws out a clue to catch our attention and, once we have found it, she puts all kinds of obstacles in our way and draws false leads across our path. It is only with great persistence and painstaking attention to every detail that we have any chance of wresting some of these secrets from her.

Fred Reines is a physicist who loves to take up such challenges. Just as he set out to observe the neutrino, which had eluded us for twenty five years, and was prepared to explode a bomb to do it,\(^1\) so he has gone seeking for proton decay, for supernovae, for neutrino-electron interactions, and for double beta decay. All are either rare, or extremely rare phenomena, but all are associated with some fundamental principle of physics; and it is this association that adds so much spice to the pursuit.

Inevitably the pursuit takes a long time. In the case of double beta decay, it took thirty years to show that the phenomenon does indeed occur in ancient ores\(^2\) and another twenty to observe an actual double beta event in the laboratory.\(^3\) In the mid fifties, Fred and his colleagues Cowan, Harrison, and Langer\(^4\) undertook a pioneering search for \(^{150}\text{Nd} \rightarrow ^{150}\text{Sm}\), which has a relatively large \(Q\)-value of 3.5 MeV and which Mike Moe has only recently started to re-investigate. Today we are still searching for one form of double beta decay, which, should it occur, will have profound implications for the question of neutrino mass and for physics beyond the standard electroweak model. The problem is, and always has been that double beta decay is so slow that minute traces of intrinsically faster radioactivities, such as those associated with the Uranium and Thorium decay chains, become significant backgrounds mimicking the signal for double beta decay.

Like its sister phenomenon of single beta decay, double beta decay is a process in which a nucleus undergoes a transmutation from one element to another. Whereas in single beta decay, the atomic number usually increases by one unit, from \(Z\) to \(Z + 1\), and an electron, or negative beta ray, is emitted in order to conserve electric charge, so in double beta decay \(Z\) increases by two units and two electrons are emitted. Lifetimes of single beta decay vary from fractions of a second to hundreds of thousands of years depending upon the nature of the nuclear transmutation and the total amount of
energy released in the process. Lifetimes of double beta decay are tens of billions of billions of years and longer.

To ensure that energy, momentum, and angular momentum are conserved in single beta decay, each beta ray is always accompanied by a neutral particle of spin 1/2; by convention the particle accompanying a negative beta ray is called an anti-neutrino and the one accompanying a positive beta ray is called a neutrino. Since negative and positive beta rays are themselves particle and anti-particle respectively, namely electrons and positrons, the nomenclature is clearly designed to suggest another conservation law. Assigning a leptonic ‘charge’ of +1 unit to ‘leptons’, that is, electrons and neutrinos, and −1 to anti-leptons, namely positrons and anti-neutrinos, we might be tempted to think that lepton charge, or lepton number is conserved in beta decay. But is that really the case?

According to the standard model of Glashow, Weinberg, and Salam, the neutrino is distinguished from its anti-particle by virtue of helicity: the neutrino is perfectly left-handed and the anti-neutrino perfectly right-handed. As long as the neutrino has precisely zero mass, these two particle states are orthogonal to one another and cannot ‘communicate’; but if the neutrino acquires a mass, no matter how small, then the possibility arises that neutrino and anti-neutrino will no longer be orthogonal. Such a neutrino is called a Majorana particle and there are numerous theories beyond the standard model, for example those unifying all the known interactions, which predict that the neutrino is indeed a Majorana particle.

The key test for a Majorana neutrino, originally proposed by G. Racah in 1937, is whether an anti-neutrino emitted in the beta decay of one neutron can interact with another neutron and cause it to transform into a proton and an electron. Such a combination of events would violate the conservation of lepton number because there would be two neutrons and no leptons in the initial state and two protons, two electrons and no anti-neutrinos in the final state; lepton number would have changed from zero to 2 units. With the discovery of parity nonconservation and the two-component neutrino in 1957, it was recognised that the two-step process of Racah is inhibited by helicity: the right-handed anti-neutrino emitted by the first neutron is in the wrong helicity state to be re-absorbed by another neutron. In order to complete the second step of the Racah process, the anti-neutrino must be able to flip its helicity and turn itself into a neutrino. We show below that such a double flip may be induced by a mass term unique to electrically neutral fermions and known as a Majorana mass term.

One way in which to study the Racah process is to use real anti-neutrinos. This was, in fact the first experiment Ray Davis performed with the famous $^{37}$Cl → $^{37}$Ar reaction, using anti-neutrinos from an atomic reactor in place of neutrinos from the sun. (Rumors of a positive result reached Bruno Pontecorvo in Moscow in 1957 and caused him to invent neutrino oscillations in direct analogy with the Gell-Mann–Pais analysis of neutral Kaon decay. The rumors eventually died out but the idea of oscillations is still alive and kicking.) A much more sensitive method is to study
Double beta decay.

Double beta decay, because of its extremely long lifetime, is regarded as a second-order effect of the same interaction as gives rise in first order to single beta decay. Neutrinos are not needed to conserve energy and momentum in double beta decay and so we may ask whether they must always accompany the emitted electrons. If the neutrino and anti-neutrino are Majorana particles and hence not orthogonal to one another, then it is possible to reproduce the Racah process inside the nucleus: the anti-neutrino emitted by one neutron may be reabsorbed by a second to give a form of decay in which no neutrinos materialise. W. H. Furry,\textsuperscript{10} in 1939, was the first to realise this possibility, and he showed that, absent the helicity suppression, the virtual neutrino was much better at stimulating the Racah process because its effective energy would be much greater than that of a real neutrino. The enhancement of the rate could be as large as six orders of magnitude and this makes double beta decay sensitive to very small lepton number nonconserving effects.

We see from this analysis that there are two forms of double beta decay, the two-neutrino decay in which two neutrinos are emitted together with the two electrons and the no-neutrino decay in which no neutrinos are emitted. Two-neutrino decay will occur irrespective of the nature of the neutrino, but no-neutrino decay can occur only if the neutrino is a Majorana particle. It is also worth mentioning that certain theories of neutrino mass predict other forms of decay in which no neutrinos are emitted, but the two electrons are accompanied by one or more scalar particles called Majorons.\textsuperscript{11} We shall argue below that the occurrence of no-neutrino decay implies that the neutrino must have a non-zero mass and sets lower bounds on this mass.

Before delving more deeply into the theory of the neutrino, let us consider the nuclear setting in which double beta decay takes place and the original clue that Nature put in our way. The story began in the 1930’s with the question of the stability over geological time of certain nuclei which, on the basis of energetics alone, could not be absolutely stable. These nuclei are all composed of an even number of neutrons $N$ and an even number of protons $Z$ and, because the nuclear force tends to bind pairs of like particles more tightly than pairs of different particles, they are lighter than neighboring odd-odd nuclei which contain the same total number of nucleons $A = N + Z$, but odd numbers of neutrons and of protons. It can, and does happen that a particular even-even nucleus ($A, N, Z$) is lighter than its nearest neighbor ($A, N - 1, Z + 1$) but heavier than the next nearest neighbor ($A, N - 2, Z + 2$). Therefore, while decay from $Z$ to $Z + 1$ is forbidden, decay from $Z$ to $Z + 2$ is allowed energetically and it should indeed take place.

The problem is to find a decay mechanism sufficiently slow that known even-even nuclei, for example $^{130}\text{Te}$ and $^{82}\text{Se}$ could survive in significant quantities over periods of the order of several billion years. In 1935 Maria Mayer\textsuperscript{12} realised that second-order beta decay could provide the needed mechanism. She used the then new Fermi theory of beta decay to estimate the lifetime for the two-neutrino mode, and found it well in excess of $10^{17}$ years, which is exceedingly slow even on geological time scales.
Four years later Wendell Furry\textsuperscript{10} estimated the lifetime of no-neutrino decay to be a million times shorter than the two-neutrino one; nevertheless it was still long enough to account for the stability of even-even nuclei.

Today we know that the lifetimes of both modes are much longer than these early estimates. Two-neutrino decay has been observed in several nuclei with lifetimes ranging from $10^{19}$ years in $^{100}$Mo to $10^{21}$ years in $^{130}$Te. Prolonged searches for no-neutrino decay have been made, so far without success, and the best lower bound on its lifetime is of order $10^{24}$ years in the decay of $^{76}$Ge. In this talk I shall discuss the implications of these results and show how they may be used to set lower bounds on the mass of neutrinos.

2 The neutrino mass matrix

I now turn to neutrino properties. In the context of the Dirac equation for spin 1/2 particles, the mass term serves to change helicity from left to right and vice versa. Consider for the moment an electron: its four states can be described as an electron with left-handed helicity (spin anti-parallel to momentum) $e^{-}_L$, an electron with right-handed helicity (spin parallel to momentum) $e^{+}_L$, a left-handed positron $e^{-}_L$, and a right-handed positron $e^{+}_R$. The mass term transforms the two particle states into each other

$$e^{-}_L \leftrightarrow e^{+}_R ,$$

and the two anti-particle states into each other

$$e^{+}_L \leftrightarrow e^{-}_R ,$$

but the transformation of a left-handed particle into a right-handed anti-particle

$$e^{-}_L \leftrightarrow e^{+}_R$$

is forbidden by the conservation of electric charge.

Next consider the four corresponding electron-neutrino states: a left-handed neutrino $\nu_{eL}$, a right-handed neutrino $\nu_{eR}$, a left-handed anti-neutrino $\bar{\nu}_{eL}$, and a right-handed anti-neutrino $\bar{\nu}_{eR}$. In the standard model only $\nu_{eL}$ and $\bar{\nu}_{eR}$ take part in weak interactions. Exactly in parallel with the electron mass term, a neutrino mass term could induce left-right transitions between particles and between anti-particles

$$\nu_{eL} \leftrightarrow \nu_{eR}$$

$$\bar{\nu}_{eL} \leftrightarrow \bar{\nu}_{eR} ,$$

but in addition electric charge conservation no longer forbids transitions between particle and anti-particle

$$\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$$

$$\bar{\nu}_{eL} \leftrightarrow \nu_{eR} .$$
These latter transitions are exactly what we need to make the Racah process work and the presence of such terms in the neutrino mass-matrix gives rise to Majorana mass eigenstates.

To see how this comes about in a formal way, let us consider the electron-neutrino field $\psi$ and its anti-neutrino field $\psi^c = C\bar{\psi}^T \equiv \gamma_2\psi^*$. The helicity projections of the field are

$$\psi_L = \frac{1}{2}(1 + \gamma_5)\psi; \quad \psi_R = \frac{1}{2}(1 - \gamma_5)\psi,$$  \hspace{1cm} (6)

and those of the charge conjugate are

$$(\psi_L)^c = \frac{1}{2}(1 - \gamma_5)\psi^c = (\psi^c)_R$$

and

$$(\psi_R)^c = \frac{1}{2}(1 + \gamma_5)\psi^c = (\psi^c)_L.$$  \hspace{1cm} (7)

The most general form of the mass matrix can be written in terms of all four components of the fermion field as\textsuperscript{13}

$$(\bar{\psi}_L, (\bar{\psi}_R)^c, (\bar{\psi}_L)^c, \bar{\psi}_R) \begin{pmatrix} O & (m_L & m_D) \\ (m_D & m_R) & O \end{pmatrix} \begin{pmatrix} \psi_L \\ (\psi_R)^c \\ (\psi_L)^c \\ \psi_R \end{pmatrix}$$  \hspace{1cm} (8)

The matrix element $m_D$ gives rise to the transitions of eq. (4) and is called the Dirac mass term, while the matrix elements $m_L$ and $m_R$ give rise to the transitions of eq. (5) and are known as the Majorana mass terms.

We use the phrase ‘Majorana mass terms’ because as long as one of $m_L$ and $m_R$ is non-zero, the eigenvectors of the mass matrix are linear combinations of $\psi$ and $\psi^c$ and transform into themselves under the charge conjugation operation. For example, if $CP$ is conserved and $m_L = m_R = m_M$, then the mass eigenvalues are

$$M_\pm = \frac{1}{2}(m_M \pm m_D)$$  \hspace{1cm} (9)

and the corresponding eigenvectors are

$$\Psi_\pm = \frac{1}{\sqrt{2}} (\psi \pm \psi^c)$$  \hspace{1cm} (10)

More generally, if we define

$$L = \begin{pmatrix} \psi_L \\ (\psi_R)^c \end{pmatrix}$$

and

$$L^c = \begin{pmatrix} (\psi_L)^c \\ \psi_R \end{pmatrix}$$  \hspace{1cm} (11)
and $m_L \neq m_R$, then the eigenvectors of the mass matrix are

$$L_\pm = \frac{1}{\sqrt{2}}(L \pm L^c)$$ (12)

Although these fields have the formal appearance of eigenvectors of the charge-conjugation operation $\psi \to \psi^c$ with eigenvalues $\pm 1$, the physical helicity states upon which they act are eigenstates of $CP$. This is as it should be since the mass matrix conserves $CP$, but not $C$. We refer to this eigenvalue as the $CP$ eigenvalue, noting that in the most general case it is a complex phase and that the parity of a Majorana field is actually imaginary. For our purposes, however, it is the opposite algebraic signs that carry the important physical information. We also note that the original (Dirac) field $\psi$ can be expressed as a linear combination of two Majorana fields with opposite $CP$ eigenvalues, for example

$$\psi_{\text{Dirac}} = \frac{1}{\sqrt{2}}(\Psi_+ + \Psi_-)$$ (13)

3 Physical processes

In the standard model, the effective beta decay Hamiltonian consists of products of left-handed currents for nucleons and leptons with the appropriate Hermitian adjoint currents. It gives rise to a set of basic physical processes involving transitions between neutrons and protons and the corresponding emission or absorption of leptons and anti-leptons. We summarise these processes in the following Table.

| Process       | Nucleon Transition | Beta Ray | Neutrino |
|---------------|--------------------|----------|----------|
| Emission      | $n \to p$          | $e^-_L$  | $\bar{\nu}_{eR}$ |
| Emission      | $p \to n$          | $e^+_R$  | $\nu_{eL}$   |
| Absorption    | $n \to p$          | $e^-_L$  | $\nu_{eL}$   |
| Absorption    | $p \to n$          | $e^+_R$  | $\bar{\nu}_{eR}$ |

How does this affect no-neutrino double beta decay? The first and third lines of Table 1 illustrate the point made in the Introduction, namely that in order for the neutrino emitted in the first step of the process to be reabsorbed in the second step, a $\bar{\nu}_{eR}$ must be able to transform itself into a $\nu_{eL}$. One way to accomplish this
transformation is through the Majorana mass terms of eqs. (5, 8) above; another way is to include small admixtures of right-handed currents in the predominantly left-handed currents of the effective Hamiltonian, treating the electron-neutrino as a Majorana particle identical with its anti-particle. We can represent these admixtures insofar as neutrinos are concerned by modifying the first and third lines of Table 1:

| Process   | Nucleon | Beta Ray | Neutrino                  |
|-----------|---------|----------|---------------------------|
| Emission  | $n \rightarrow p$ | $e^-$     | $\bar{\nu}_{eR} + \eta \nu_{eL}$ |
| Absorption| $n \rightarrow p$ | $e^-$     | $\nu_{eL} + \eta \nu_{eR}$  |

Table 2. Emission and Absorption of Neutrinos in a Modified Standard Model.

The parameter $\eta$ denotes small admixtures of opposite helicities and the assumption of a Majorana neutrino means that $\bar{\nu} \equiv \lambda \nu$ where $\lambda = \pm 1$.

The leptonic part of the no-neutrino double beta decay is thus proportional to:

$$\mathcal{L} = \frac{1}{\sqrt{2}}(1 - P(e_1, e_2))\langle L_\mu L_\alpha \rangle$$

$$\langle L_\mu L_\alpha \rangle = \left\langle [e_1 \gamma_\mu (\nu_L + \eta \nu_R)] [(\nu_L + \eta \nu_R)^T \gamma_\alpha (e_2)^T] \right\rangle,$$

where the overbrace indicates that we must contract the neutrino fields using the propagator

$$\frac{i q + m}{q^2 + m^2}.$$ (15)

Fields with the same helicity are linked by the mass term and those with opposite helicity by the $q$ term. Thus the two mechanisms have amplitudes proportional to:

$$\text{Mass-mechanism} \sim \frac{\lambda m}{q^2 + m^2}$$

$$\text{RHC-mechanism} \sim \frac{\lambda \eta \cdot q}{q^2 + m^2}.$$ (16)

where $\lambda$ is the $CP$ eigenvalue of the neutrino.

Although we are treating the admixture of right-handed currents (RHC-mechanism) as a separate phenomenological mechanism for no neutrino double beta decay, we intend to show later that it is not a separate fundamental mechanism. We shall argue that in gauge theories, the mass is the fundamental mechanism for lepton number nonconserving processes. In other words the RHC-mechanism will not work unless there are mass terms present in the neutrino mass matrix.
4 Neutrino mixing

Although the preceding discussion has treated the electron-neutrino as if it were a single particle, we must recognize that, like its companion muon- and tau-neutrinos, it is really a superposition of the mass eigenstates of the neutrino mass matrix. To allow for the three families, each of the entries in the row and column of fields in eq. (8) becomes a 3-fold object and each entry in the matrix becomes a 3-by-3 matrix; in addition, the sub-matrix in the lower left quarter becomes the Hermitian adjoint of the one in the upper right quarter. As long as the Majorana matrices corresponding to $m_L$ and $m_R$ do not vanish, the eigenvectors will be CP eigenstates analogous to those in eqs. (10, 11, 12) and with eigenvalues $\pm 1$. We denote them as $\nu_i$ with mass $m_i$ and CP eigenvalue $\lambda_i$:

$$ (\nu_i)^c = \lambda_i \nu_i , \quad \lambda_i = \pm 1 . $$

(17)

The flavor eigenstates $\nu_{eL}$ and $\nu_{eR}$ can now be written as linear superpositions of $\nu_{iL}$ and $\nu_{iR}$:

$$ \nu_{eL} = \sum_i U_{ei} \nu_{iL} $$
$$ \nu_{eR} = \sum_i V_{ei} \nu_{iR} , $$

and similarly for $\nu_\mu$ and $\nu_\tau$. The mixing matrices $U_{ei}$ and $V_{ei}$ are both unitary,

$$ \sum_i U_{ei}^* U_{ei} = \sum_i V_{ei}^* V_{ei} = 1 $$

(19)

but not necessarily orthogonal to one another.

The leptonic factors for the two mechanisms for no-neutrino double beta decay described in eq. (16) must now be modified to take neutrino mixing into account. For the mass mechanism, we replace the expression in eq. (16) by an effective mass $m_{\beta\beta}$, where

$$ m_{\beta\beta} = \sum_i m_i \lambda_i (U_{ei})^2 ; $$

(20)

and for the RHC-mechanism we use

$$ \eta_{LR} = \sum_i \lambda_i U_{ei} V_{ei} \frac{d}{q^2 + (m_i)^2} . $$

(21)

The effective mass $m_{\beta\beta}$ obviously vanishes when all the mass eigenvalues $m_i$ vanish, but $\eta_{LR}$ would appear not to do so. In other words, it would seem that the RHC-mechanism could give rise to no-neutrino double beta decay even in the absence of neutrino mass. We shall now argue that this cannot happen in gauge theories because of their renormalizability.
5 Limitation from high energy behavior

The renormalizability properties of gauge theories implies that the amplitudes for physical processes cannot diverge at high energies: the amplitudes for specific diagrams contributing to such processes will either be finite themselves at high energy or their divergent parts will cancel amongst one another. For example, the creation of a W-boson pair in $e^+e^-$ annihilation at high energies can take place through two diagrams:

![Figure 1. Feynman diagrams for $e^+e^- \rightarrow W^+W^-$ via neutrino and $Z^0$ exchange.](image)

one involving neutrino exchange and the other a virtual $Z^0$-boson. By themselves, these diagrams are quadratically divergent at high energies; but, because of the relationships amongst coupling constants imposed by the standard Glashow-Weinberg-Salam model, the divergent parts of the two diagrams cancel each other.

If we look carefully at the diagram for no-neutrino decay, we find a sub-diagram which corresponds to the process (Fig. 2)

$$W^-W^- \rightarrow e^-e^-.$$  \hspace{1cm} (22)

The amplitude via the mass mechanism does not diverge at high energies, but the amplitude via the RHC-mechanism does. Gauge theories must therefore produce a way of canceling this divergence. One way would be to have a gauge group with doubly-charged gauge bosons to cancel the neutrino exchange of Fig. 2. There is, however, no evidence for the existence of such bosons, and so we must confine ourselves to models without them. In this case the only way to eliminate the divergent part of the amplitude is to require its coefficient to be zero.

![Figure 2. Feynman diagrams for no-neutrino double beta decay. Note the subdiagram corresponding to $W^-W^- \rightarrow e^-e^-$.](image)
It is not difficult to show that this coefficient is:

\[ A_{LR} \equiv \sum_i \lambda_i U_{ei} V_{ei}. \]  

(23)

Therefore the gauge theory requirement of good high energy behavior for \( W^- W^- \rightarrow e^- e^- \) is that

\[ A_{LR} = 0 \]  

(24)

This is a completely general requirement and, as shown by Kayser, Petcov, and Rosen, it holds in a large class of gauge theories. Comparing eqs. (23, 24) with eq. (21) above, we see that the vanishing of \( A_{LR} \) is sufficient to forbid no-neutrino double beta decay when all neutrino masses vanish, or when they are all completely degenerate. In the case when all the neutrino mass eigenvalues are small compared with \( q^2 \), we can expand eq. (21) in powers of \((m_i/q)^2\) to obtain

\[ \eta_{LR} = \sum_i \lambda_i U_{ei} V_{ei} \left( \frac{q}{q^2} \right)^{-1} \frac{-(m_i)^2}{q^2}. \]  

(25)

Note that the typical value of \( q^2 \) corresponds to a mean inter-nucleon separation of a few fermi and has a value of order \((50 \text{ MeV})^2\).

6 Nuclear physics

Most double beta decay transitions take place between the groundstates of even-even nuclei which always have zero spin and positive parity: \( 0^+ \rightarrow 0^+ \). Many daughter nuclei have excited states with spin and parity \( 2^+ \) about 500 keV above the groundstate and transitions from the parent to these excited states are also possible. In the case of no-neutrino decay, such \( 0^+ \rightarrow 2^+ \) transitions can only arise through the RHC-mechanism and so the observation of them would provide important information about the phenomenological mechanism for double beta decay.

Typical energy releases \( (Q) \) in double beta decay are in the range of 2–3 MeV, but there are examples like \( ^{128}\text{Te} \rightarrow ^{128}\text{Xe} \) and \( ^{238}\text{U} \rightarrow ^{238}\text{Pu} \) with \( Q \)-values closer to 1 MeV. Two-neutrino decay has a four-body phase space corresponding to the two electrons and two neutrinos emitted and it is usually a polynomial of degree 10-11 in \( Q \):

\[ F_{2\nu} = \text{Phase Space} \sim Q^{10-11} \]  

(26)

No-neutrino decay has a two-body phase space together with an integration over the energy of the virtual neutrino; the phase space is a polynomial of degree 5 in \( Q \), and the integration is proportional to the fifth power of the energy \( E_\nu \) of the virtual

\[ \]
neutrino. Thus the no-neutrino factor corresponding to the four-body phase space of two-neutrino decay is:

\[ \Phi_{0\nu} = (E_\nu)^5 F_{0\nu} \sim (E_\nu Q)^5 \]  

(27)

For a typical separation between neutrons inside the nucleus of 4–5 fermi, the mean energy of the virtual neutrino is about 50 MeV and so the ratio of phase space factors favors the no-neutrino decay over the two-neutrino mode by a factor

\[ R \left( \frac{0\nu}{2\nu} \right) \sim \left( \frac{E_\nu}{Q} \right)^5 \sim 10^6. \]  

(28)

It is this factor that enables us to set sensitive limits on lepton number nonconserving parameters.

7 Nuclear matrix elements and two-neutrino half-lives

In second order perturbation theory, the matrix element for double beta decay takes the general form

\[ M(i \to f) = \sum_k \langle f | H_\beta | k \rangle \frac{1}{E_k - E_i} \langle k | H_\beta | i \rangle \]  

(29)

where \( i \) denotes the initial state of the parent nucleus, \( k \) the intermediate state of the nucleus plus one electron and one neutrino, and \( f \) the final state of the nucleus plus two electrons and either two neutrinos or no neutrinos. The energies of these states are denoted by \( E_k \) and \( E_i \). To carry out the sum over intermediate states, we replace the energy denominator by an average value and use closure over the states of the intermediate nucleus. While closure is a good approximation for the no-neutrino mode, its validity for two-neutrino decay is a matter of some debate.\(^{16}\)

For \( 0^+ \to 0^+ \) nuclear transitions the dominant contribution to the nuclear matrix element comes from the axial vector part of the effective weak Hamiltonian \( H_\beta \). The matrix elements for two-neutrino and no-neutrino decay are respectively:

\[ M^{2\nu} = \langle f | \sum_{jk} \tau_j \tau_k \sigma_j \cdot \sigma_k | i \rangle \]  

(30)

\[ M^{0\nu}(m_n) = \langle f | \sum_{jk} \tau_j \tau_k \sigma_j \cdot \sigma_k \frac{I(r_{jk}, m_n)}{r_{jk}} | i \rangle, \]

where each operator \( \tau_l \) transforms a neutron in the initial state into a proton in the final state and the function \( I(r_{jk}, m_n)/r_{jk} \) in \( M_{0\nu} \) represents the propagator for a neutrino of mass \( m_n \). When we multiply the no-neutrino matrix element by the
lepton number nonconserving parameters of eqs. (20, 21) we must carry out a sum over $n$.

The integration over the neutrino momentum in the function $I(r_{jk}, m_n)$ leads to standard functions in the case of zero neutrino mass, but it is much harder to carry out when the mass is nonzero. Haxton has found an empirical representation which works well for small masses. Expanding this representation to lowest order in $m_n$, we find that

$$\frac{I(r, m)}{r} \approx \frac{I(r, 0)}{r} - \frac{m^2}{a\langle E_{ki} \rangle}, \quad (31)$$

where $a$ is a slowly varying parameter roughly equal to 0.4 in the range of interest, and $\langle E_{ki} \rangle$ is an average value of the energy denominator of eq. (29).

That the lowest term in eq. (31) is quadratic in $m$ is easily understood from the propagator in eq. (16). It is also noteworthy that this term is independent of the nucleon separation variable $r$, just like the two-neutrino matrix element $M_{2\nu}$ of eq. (30). This means that the corresponding contribution to the no-neutrino lifetime can be expressed in terms of the two-neutrino lifetime, a result which will be very useful when the term independent of $m$ vanishes.

We can now write the half-life for two-neutrino decay in the form

$$\frac{1}{\tau_{2\nu}} = F_{2\nu}(Q)|M_{2\nu}|^2\langle E_{ki} \rangle^{-2}, \quad (32)$$

where $\langle E_{ki} \rangle$, the average energy denominator in eq. (29), is roughly one-half the energy release $Q$. The matrix elements and resulting half-lives for several nuclei have been calculated by various authors using the shell model and using the quasi-particle random phase approximation (QRPA); the results and the comparison with experiment are given in the following Table 3.

| Parent Nucleus | Q-Value (MeV) | $F_{2\nu}$ (MeV²/10²¹yr) | $\tau_{2\nu}$ (10²⁰yr) |
|----------------|--------------|--------------------------|------------------------|
| ⁷⁶Ge           | 2.041        | 34.70                    | 10.1                   |
|                |              |                          | 16–63                  |
|                |              |                          | 9 ± 1 Ref. 20          |
| ⁸²Se           | 2.995        | 1151                     | 0.64                   |
|                |              |                          | 1–6                    |
|                |              |                          | 1.1⁺⁻⁰.⁸⁻⁻⁰.³ Ref. 3   |
| ¹⁰⁰Mo          | 3.034        | 2502                     | —                      |
|                |              |                          | 0.4–0.04               |
|                |              |                          | 0.12⁺⁻⁰.³⁴⁻⁻⁰.₀¹ Ref. 21|

The comparison between theory and experiments works quite well.
8  No-neutrino half-lives and neutrino mass bounds

In this section we give a somewhat simplified version of the no-neutrino half-life, keeping the leading matrix elements and dropping smaller ones. For a more complete account the reader is referred to the recent review article by Tomoda.\(^{17}\) The matrix elements of eqs. (30, 31) are functions of the mass of the exchanged neutrinos and so we must sum them over the spectrum of mass eigenstates using weighting factors appropriate to the mass- and RHC-mechanism. The half-life can then be written as

\[
\frac{1}{\tau_{0\nu}} = A_{mm} + 2B_{m\eta} + C_{\eta\eta},
\]

where \(A\) arises from the mass-mechanism, \(C\) from the RHC-mechanism and \(B\) from the interference between them:

\[
A_{mm} = \left[ \sum_n m_n \lambda_n (U_{en})^2 M^{0\nu}(m_n) \right]^2 F_{11}^0
\]

\[
B_{m\eta} = \left[ \sum_n m_n \lambda_n (U_{en})^2 M^{0\nu}(m_n) \right] \left[ \sum_n \lambda_n U_{en} V_{en} M^{0\nu}(m_n) \right] 2F_{13}^0
\]

\[
C_{\eta\eta} = \left[ \sum_n \lambda_n U_{en} V_{en} M^{0\nu}(m_n) \right]^2 4F_{33}^0.
\]

The \(F_{kl}^0\) are phase space and Coulomb factors defined and tabulated by Tomoda.\(^{17}\) In the RHC-mechanism part of these expressions we have kept only those terms which will give a significant contribution when the gauge theory condition \(A_{LR} = 0\) of eqs. (23, 24) is satisfied.

Using the gauge theory condition and the expansion of eq. (31) for the neutrino propagator, we can rewrite the \(A, B, C\) expressions of eq. (34) as

\[
A_{mm} = \left[ m_{\beta\beta} M^{0\nu}(0) \right]^2 F_{11}^0
\]

\[
B_{m\eta} = \left[ m_{\beta\beta} M^{0\nu}(0) \right] \left[ \sum_n \lambda_n U_{en} V_{en} \frac{(m_n)^2}{a(E_{Ni})} M^{2\nu} \right] 2F_{13}^0
\]

\[
C_{\eta\eta} = \left[ \sum_n \lambda_n U_{en} V_{en} \frac{(m_n)^2}{a(E_{Ni})} M^{2\nu} \right]^2 4F_{33}^0.
\]

Notice that the two-neutrino matrix element now appears in the coefficients \(B\) and \(C\) because of the absence of the nucleon separation variable in the second term of eq. (31).

At the present time, the best limit on no-neutrino decay comes from studies of \(^{76}\)Ge \(\rightarrow^{76}\)Se which show that the half-life must be longer than about \(2 \times 10^{24}\) yrs.\(^{22}\) In terms of an effective mass for double beta decay this gives a limit of \(1\) eV:\(^{17}\)

\[
m_{\beta\beta} \equiv \sum_n m_n \lambda_n (U_{en})^2 \leq 1\ \text{eV}.
\]
For the RHC-mechanism, we use the presence of the two-neutrino matrix element in
the expression for $C_{\eta\eta}$ together with eq. (32) to write

$$\frac{1}{\tau_{0\nu}} = a^{-2} \left[ \sum_n \lambda_n U_{en} V_{en} (m_n)^2 \right]^2 \times \frac{4}{F_{33}^2} \times \frac{1}{\tau_{2\nu}}. \tag{37}$$

The measured lifetime for two-neutrino decay in $^{76}$Ge is very close to $10^{21}$ yrs and
the ratio of phase space factors can be estimated from Table A1 of Tomoda to be
$3.3 \times 10^3/\text{MeV}^4$. Taking the parameter $a$ to be 0.5, we then obtain the bound

$$\sum_n \lambda_n U_{en} V_{en} (m_n)^2 \leq 200 \text{ keV}^2 \tag{38}$$

If we include the interference terms between the two mechanisms, then we obtain the
usual quadratic forms in the two-dimensional lepton number nonconserving parameter
space.

9  Implications of seeing no-neutrino decay

Suppose now that at some future date, the no-neutrino mode will actually be seen in
$^{76}$Ge decay with a half-life of $2N^2 \times 10^{24}$ yrs. What will this mean for neutrino mass
aside from the obvious implication that the mass must be nonzero\(^23\)?

The first implication is that the above inequalities will be replaced by one of the
equalities

$$\sum_n m_n \lambda_n (U_{en})^2 = \frac{1}{N} \text{ eV}$$

$$\sum_n \lambda_n U_{en} V_{en} (m_n)^2 = \frac{200}{N} \text{ keV}^2, \tag{39}$$

depending upon which mechanism is at work. If we assume that all neutrino mass
eigenstates are much lighter than $q$, the momentum of the exchanged neutrino (roughly
50 MeV), then we can use these equations to set lower limits on the highest eigenvalue
$m_{\max}$.

In the case of the mass mechanism we find that

$$m_{\max} \sum_n \lambda_n (U_{en})^2 \geq \frac{1}{N} \text{ eV} ; \tag{40}$$

and, since the sum of squares of mixing coefficients $U_{en}$ can never exceed unity, it
follows that

$$m_{\max} \geq \frac{1}{N} \text{ eV}. \tag{41}$$
In the case of the RHC-mechanism, we find by a similar argument\(^{14}\) that

\[
m_{\text{max}} \geq \frac{14}{\sqrt{N}} \text{ keV}.
\] (42)

The physical implications of these bounds are not insignificant, especially in the case of the RHC-mechanism. Equation (42) implies that for no-neutrino double beta decay to be observed, there must exist at least one neutrino with a mass of several keV. Given the apparent demise of the 17 keV neutrino,\(^{24}\) this is unlikely; and thus RHC-induced double beta decay is also unlikely. The mass-mechanism requires a neutrino of mass in the eV range or less, a more plausible possibility.

10 Majoron emission

One model of neutrino mass is based upon the coupling of neutrinos to a light pseudo-Goldstone boson associated with the spontaneous breakdown of lepton conservation.\(^{11}\) This can give rise to double beta decay in which two electrons and a spinless boson, the Majoron, are emitted, but they are not accompanied by neutrinos. The half-life for this process can be expressed in terms of the same matrix element as occurs in no-neutrino decay, the coupling \(g_M\) of the neutrino to the Majoron and a three-body phase space factor \(F_M\):

\[
\frac{1}{\tau_{0\nu M}} = [(g_M)M^{0\nu}(0)]^2 F_M,
\] (43)

where \(F_M\) is tabulated by Tomoda\(^{17}\) and has a value of \(1.3 \times 10^{-14} \text{ fm}^{-2} \text{ yr}^{-1}\) for \(^{76}\text{Ge}\).

We can search for the Majoron mode by studying the double beta decay spectrum as a function of the sum \(E\) of the electron energies. For the no-neutrino mode \(E\) is always equal to the total energy released, \(Q\), because there are no other leptons to carry off the available energy; for the two-neutrino mode, the energy is shared between the electrons and neutrinos and so the spectrum is continuous. It peaks below the mid-point \(Q/2\), approaches the end-point like \((Q - E)^5\) and is virtually zero for the last 20% of the spectrum.

By contrast, the two electrons in the Majoron mode carry off most of the available energy. Since the Majoron is spinless and the electrons have the same helicity, they have to emerge predominantly in a back-to-back configuration in order to conserve total angular momentum in the \(0^+ \rightarrow 0^+\) nuclear transition. Hence the Majoron tends to be emitted softly. As a result the spectrum for the Majoron mode peaks in the region near the end-point where the two-neutrino spectrum vanishes.

As we shall hear from Prof. Moe,\(^{25}\) there appear to be anomalous numbers of events in this region in the observed spectra for \(^{82}\text{Se}, \, ^{100}\text{Mo}\) and \(^{150}\text{Nd}\). The fact that these nuclei have different \(Q\) values tends to suggest that the phenomenon is real, but it is too early to rule out some as yet unanticipated background as the source of
the anomaly. Recent experiments with enriched sources of $^{76}\text{Ge}$, which has a lower end-point than the other nuclei, find no evidence for the anomaly and argue against an interpretation as the Majoron mode. The effect Moe appears to observe could still be real, but with a completely different interpretation of a nuclear, rather than particle nature.

### 11 Limits on heavy neutrinos

In addition to the light neutrinos that we have considered so far, we could also discuss heavy neutrinos, with masses much greater than the mean neutrino momentum of $q \approx 50\text{MeV}$. For mass-induced no-neutrino decay, the crucial factor in the leptonic matrix element is the product

$$P = \frac{q m_\nu}{q^2 + m_\nu^2}$$

For light neutrinos, $m_\nu \ll q$ and

$$P \approx \frac{m_\nu}{q} ,$$

while for heavy ones, $M_\nu \gg q$ and

$$P = \frac{q}{M_\nu} .$$

From the equivalence between these two forms for $P$, we obtain a ‘see-saw’ for light and heavy neutrinos:

$$m_\nu M_\nu \approx q^2 \left(\frac{M_W}{M_{W'}}\right)^4$$

where we have allowed for different gauge boson masses $M_W$ and $M_{W'}$ associated with the light and heavy neutrinos respectively. Taking $q$ to be $50\text{MeV}$ and the ratio of boson masses to be about $1/10$, we obtain

$$m_\nu M_\nu \approx (m_e)^2 .$$

If the light mass is of order $1 \text{ eV}$, then the heavy one must be of order $100 \text{ GeV}$.

### 12 Conclusion

In the years since Cowan and Reines opened the era of *Experimental Neutrino Physics*, great progress has been made in the study of double beta decay. The two-neutrino mode has been observed in the laboratory with half-lives as long as $10^{21} \text{ yrs}$ and the
so-called ‘geochemical method’ has been used to detect double beta decay with half-lives of order $10^{23}$ yrs. Bounds on the no-neutrino mode have been extended beyond $10^{24}$ yrs and, with the advent of enriched sources, it may be possible to push the range of sensitivity to $10^{26}$ yrs, corresponding to an effective mass of 0.1 eV. We are still in search of the holy grail of double beta decay without neutrinos, and thus we are intrigued by the apparent anomaly found by Moe. If it cannot be shown to be a background, then it should be attacked experimentally on the same scale as the solar neutrino problem.

Our theoretical understanding of double beta decay has also advanced. We are now able to calculate the nuclear matrix elements much more accurately than in the early days, and we have a much clearer understanding of the fundamental role of neutrino mass in relation to the no-neutrino mode. If this mode is eventually observed, then not only will we be able to conclude that the neutrino has a mass, but we will also be able set a lower bound on the mass of the heaviest mass eigenvalue. Such a bound would be important for other phenomena that are associated with mass, for example neutrino oscillations.

I hope that Fred will take much pleasure in the progress that has been and that he will take paternal pride in Neutrino Physics and its practitioners.

References

1. F. Reines in *Neutrino Physics and Astrophysics* (Proceedings of Neutrino ’80) edited by E. Fiorini (Plenum Press, New York, 1982) p.11–28.

2. T. Kirsten, O. Schaeffer, E. Norton, and R.W. Stoenner, Phys. Rev. Lett. 20, 1300 (1968).

3. S.R. Elliott, A.A. Hahn, and M.K. Moe, Phys. Rev. Letters 59, 2020 (1987).

4. C. L. Cowan, F. B. Harrison, L. M. Langer, and F. Reines, Nuovo Cim. 3, 649 (1956).

5. S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in *Elementary Particle Theory, Relativistic Groups, and Analyticity* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968) p. 367.

6. E. Majorana, Nuovo Cimento 14, 171 (1937).

7. G. Racah, Nuovo Cimento 14, 322 (1937). The notion of neutrino-induced reactions was first discussed by H. Bethe and R. Peierls, Nature 133, 532 and 689 (1934).

8. R. Davis, Jr., Phys. Rev. 97, 766 (1955).
9. B. Pontecorvo, Sov. Phys. JETP 6, 429 and 7, 1972 (1957). See also B. Pontecorvo in Neutrino Physics and Astrophysics (Proceedings of Neutrino ’80) edited by E. Fiorini (Plenum Press, New York, 1982) p.52.

10. W. H. Furry, Phys. Rev. 56, 1184 (1939).

11. Y. Chikashige, R.N. Mohapatra, and R.D. Peccei, Phys. Lett. 98B, 265 (1981); G.B. Gelmini and M. Roncadelli, ibid 99B, 411 (1981); and H. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

12. M. Goeppert-Mayer, Phys. Rev. 48, 512 (1935).

13. S. P. Rosen, Phys. Rev. D29, 2535 (1984) and D30, 1995 (E).

14. B. Kayser, S. Petcov, and S. P. Rosen (in preparation).

15. S. P. Rosen, in Gauge Theories, Massive Neutrinos, and Proton Decay edited by A. Perlmutter (Plenum Press NY 1981) p. 333.

16. J. Engel, W. C. Haxton, and P. Vogel, Preprint INT92-07-12 (submitted to Phys. Rev. C).

17. T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

18. W. C. Haxton, Phys. Rev. Lett. 67, 2431 (1991).

19. For a review, see talk by W. C. Haxton in Proceedings of Neutrino ’92, preprint No.40427-21-N92, Institute for Nuclear Theory.

20. F. T. Avignone III, et al., Phys. Lett. B256, 559 (1991).

21. M. Moe, M. Nelson, M. Vient and S. Elliott, preprint UCI-NEUTRINO 92-1 (1992).

22. A. Piepke, talk at XXVI International Conference on High Energy Physics, Dallas, Texas 5–12 August, 1992; D. Caldwell et al., Nucl. Phys. (Proc. Suppl.) B13, 547 (1990).

23. J. Schechter and J. W. F. Valle, Phys. Rev. D25, 2951 (1982).

24. R. G. H. Robertson, Plenary talk at XXVI International Conference on High Energy Physics, Dallas, Texas 5–12 August, 1992.

25. M. Moe, following talk.

26. See A. Piepke, reference 22.

27. T. Bernatowicz et al., Phys. Rev. Lett. (to be published) (1992).