Information system model for implementation and maintenance of stochastic control over a nonlinear object on manifolds

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Abstract. The paper presents a problem statement for a robust stochastic regulator synthesis based on the principles of control on manifolds, a solution algorithm of a new information system and its structure maintaining the synthesis algorithm for a stochastic discrete object. The object under study is presented as a system of stochastic, difference nonlinear equations. The mathematical tools of the classical method of analytical design of aggregated regulators are used, which were developed earlier for a deterministic nonlinear object with a complete description. The robust stochastic nonlinear regulator ensures the following characteristics of the control system: a) minimum deviation of the output variable; b) minimum dispersion of the target invariant; c) minimum average value of the control quality functional.

1. Introduction

The purpose of the paper is, on the one hand, to attract attention to the recent and, in its essence, versatile technique [1-7], which allows the following goal-directed ‘manipulations’ with nonlinear (including stochastic) objects (not specifically of engineering applications), for which, as it is well known, the construction of a general theory is currently impossible (according to Neyman):

- identification of new features of behavior of nonlinear objects, different from those in a free state;
- knowledge acquisition on a poorly formalized object relying on an analog of the least-action principle historically related with mechanics (see examples in [5]);
- presentation of a nonlinear multidimensional object in an analytically pre-described target manifold (a variety of an attractive set of states);
- control over a multi-dimensional, multi-loop nonlinear object under the conditions of deterministic and stochastic additive disturbances;
- control over a sophisticated object under the conditions of an integral-type parametric uncertainty.

On the other hand, our aim is to propose a straightforward generalization of this, in many respects attractive, deterministic technique for analysis of nonlinear objects (and synthesis of objects with new properties) for the objects functioning under the conditions of random additive noise, with the output macro variable being minimized.

Here we are not strictly using the terminology of the author of the method for analytical design of aggregated regulators (ADAR) (see, e.g., [5]).
A control system invariant is understood to be a certain analytically formulated law, which the motion of the image point (IP) of the system would obey in a steady-state mode.

Historically the formalism of invariants was used earlier to construct a regulator for a linear system with a full compensation of disturbances [8], for which the author of this idea was accused for the non-scientific approach and was deprived of all degrees, awards and positions, which were re-established only 17 years later.

Set (of object’s states) \( V \) is referred to as attractive, if it is a closed and invariant set.

The macro variables are understood to be certain defined functions \( \psi(x) \) of the object’s coordinates; equality \( \psi(x) = 0, \psi \in \mathbb{R}^m, m \leq n \) determines the target set of states \( x \in \mathbb{R}^n \), which would possess the property of attractiveness (in other words, if the IP gets into set \( V \), it remains in this set with the time tending to infinity).

The mathematical tools of the deterministic method of analytical design of aggregated regulators relies on the results of theoretical mechanics (e.g., [9-11]), in particular on the least action principle.

2. New Stochastic Formulation of the Problem of Discrete Control on Manifolds

The formalization of this problem, based on a combined use the deterministic ADAR-method [5] and the strategies of control minimizing the dispersion of the input variable [12], is similar to the deterministic discrete ADAR-problem statement and involves the following [13-15]:

a) object of control, prescribed by a system of difference (nonlinear) stochastic equations

\[
X[t+1] = F[x] + u[t] + \xi[t+1] + c\xi[t],
\]

where \( X[t] = (X_1[t], ..., X_n[t]) \) is the vector variable of state; \( F[x] : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is the nonlinear function; \( u \in \mathbb{R}^m, m \leq n \) is the control; \( \xi[t] \in \mathbb{R}^i, i \leq n \) is the random function with the properties \( \mathbb{E} \{ \xi[t] \} = 0, \mathbb{D} \{ \xi[t] \} = \sigma^2_i, i = 1, I; \) \( 0 < c < 1 \) is a certain constant interpreted as a noise attenuation index of the previous measurement; \( t \in \{0,1,...\} \); a pair wise independence of random quantities \( \{\xi_i[t], \xi_j[t], X[t] \}, i = \overline{1,n}, k \neq d \) from the following system is assumed as well as disconnection of the random variables \( \xi_i[t], i = \overline{1,I} \):

b) target of control in the form of an analytically prescribed equation

\[
\mathbb{E} \{ \psi(X) \} = 0, \psi \in \mathbb{R}^m, X \in \mathbb{R}^n, m \leq n; \]

c) fulfilment of the conditions: a set of states, obeying the description \( \psi(X) = 0 \), is an invariant manifold; the solutions of the initial system of equations are limited; there is an object stabilization mode in the neighbourhood \( \psi(X) = 0, \psi \in \mathbb{R}^m, X \in \mathbb{R}^n \).

d) requirements on the sought-for law of control \( u \in \mathbb{R}^n \) (\( t \rightarrow \infty \)):

\[
\begin{align*}
\mathbb{D} \{ \psi_j[t+1] + \omega_j \psi_j[t] \} & \rightarrow \text{min}, j = \overline{1,m}, \\
\mathbb{E} \{ \psi_j[t+1] + \omega_j \psi_j[t] \} & = 0, 0 < \omega_j = \text{const} < 1, \\
\mathbb{E} \{ \Phi \} & = \mathbb{E} \left\{ \sum_{j=1}^{m} \sum_{i=1}^{n} \left( \alpha_j^2 \left( \psi_j[t] \right)^2 + \left( \Delta \psi_j[t] \right)^2 \right) \right\} \\ & \rightarrow \text{min}.
\end{align*}
\]

Coefficients \( \alpha_j, \omega_j, j = \overline{1,m} \) are the parameters of the synthesized control system; they have a conceptual meaning and are related via a dependence given by \( \omega_j - (2 + \alpha_j^2) \omega_j + 1 = 0, j = \overline{1,m} \).

In a simplified form the process of formulation of the stochastic control problem can be interpreted by a scheme given in Fig.1.
2.1. Comments to a stochastic statement of the problem of discrete control on a manifold

This is an attractive synergetic control theory in many respects, specifically:

- its principal method of analytical design of aggregated regulators (developed by the scientific school of A.A. Kolesnikov [5]) is straightforward and physically reasonable and so is the algorithm of control synthesis for sophisticated objects due to their nonlinearity, multi-loop nature, and multidimensionality;

- the resulting control system is robust;

- it guarantees reaching the vicinity of the target manifold and ensures an asymptotically stable maintenance of the object in its vicinity (given the availability and boundedness of the solution of the initial system of equations and potential attractiveness of the target manifold); and

- meets the requirements of the physical theory of control [1], and, in essence, implements the following statement: ‘there are available answers to all questions; all that is necessary to do is to ask correctly in order to get them’.

It is evident that an application of the above discussed technique of designing control on manifolds to a non-engineering task faces a problem of “what is control as a process of an external material action on an object of control?”

Positive solutions of the above two questions (the formulated target of control and a meaningful interpretation of the control variable) ensure a well-developed, logical and transparent algorithm for synthesizing an energy-saving control leading to the sought-for target.

3. Principal Steps of a New Algorithm for Analytical Design of Aggregated Regulator for a Stochastic Object (ADAR(S))

For the sake of a convenient reference to the proposed algorithm of analytical design of a stochastic discrete regulator, we denote it as ADAR(S) (Analytical Design of Aggregated Regulator for a Stochastic discrete object).

1. Using ADAR, synthesize \( \hat{u}^k[k] \) at fixed random functions.
2. Perform an operation of conditional mathematical expectation

\[
\hat{u}[k] = E\{\hat{u}^k[k] | \xi^k \} = (\xi[k], \xi[k - 1], ..., \xi[0])
\]

3. Decompose the initial description based on \( \hat{u}[k] \).
4. Determine the dependence of $\xi^4$ on the observation of states.

5. Substitute the dependence obtained in Step 4 into $\hat{u}[k] \cdot u[k]$ - the sought-for control.

4. Applied Problems of Stochastic Control Based on ADAR(S)

Let us enumerate some of the problems, stochastic in their essence, whose nonlinear solutions were obtained using the ADAR (S) algorithm [13-15]. These problems and their applications can be extended to practically any area of knowledge or activity, where an object’s formalization is admissible [16-21].

An unformalized problem statement for applying a synergetic theory of control and its statistical generalization can be formulated as follows: a nonlinear multidimensional object is given, the initial description of which can be presented by a system of nonlinear, stochastic difference equations or ordinary differential equations; construct a control system for reaching the prescribed invariant with the minimum dispersion of the output variable.

4.1. Problem of Environmental Control

Object of control and problem statement

A third-order nonlinear continuous system based on the predator-prey model is given, extended by an equation simulating control of the victim’s nutrition dynamics:

$$\dot{x}_1 = \alpha_1(t)x_1 - \beta_1x_1x_2,$$
$$\dot{x}_2 = -\alpha_2x_2 + \beta_2x_1x_2,$$
$$\alpha_1 = u(t).$$

Here the following interpretation of variables $\alpha_1(t), x_1(t), x_2(t)$ is possible: victim ($x_1$), predator ($x_2$) and nutrition control $\alpha_1(t)$ are phytoplankton, zooplankton and a dynamics of the victim’s nutrition in the form of nitrogen or phosphorus, respectively.

It is necessary to determine the control law $u(t)$, ensuring the target

$$\nu(t) = x_i(t) - x^* \xrightarrow{t \to \infty} 0, x^*_i = \text{const}$$

is fulfilled in noise condition (in off-design conditions).

4.2. Problem of Immune Response Control Design

The basic model for any infectious diseases is well known among specialists [16], which is a system of ordinary differential equations with delay $\tau_0 > 0$.

To apply the new discrete technique of the ADAR (S) method, we consider the following discrete extended analogue of this system:

$$V[k+1] = V[k] + \tau_0(a_iV[k] - a_2F[k]V[k]),$$
$$S[k+1] = S[k] + \tau_0(\alpha_1(\zeta(m)Y_i[k]Y_i[k] - a_i(S[k] - 1))),$$
$$F[k+1] = F[k] + \tau_0(a_i(S[k] - F[k]) - a_iF[k]V[k] + u[k]),$$
$$m[k+1] = m[k] + \tau_0(a_iV[k] - a_2m[k]),$$
$$Y_i[k+1] = F[k]Y_i[k] + 1, 2, ..., 8$$

where the value $\tau_0$ is a discretization parameter; variables $Y_i[k], Y_i[k+1]$ are fictitious variables to explain the lag.

Here $V[k], S[k], F[k], m[k]$ are relative concentrations of antigens, plasma cells and antibodies, the proportion of target organ cells destroyed by antigens, respectively; $a_i, i = 1, 8; \zeta(m)$ are model parameters [17].
It is necessary to determine the control law $u[k]$, ensuring that $V[k] = V' - k \to 0, k \to \infty$, and the (2) properties of the control system are fulfilled (here $V'$ is the target value, for example, $V' = 0$).

### 4.3. Control of an Economic Object

The description of the main model of stock theory, as a rule, has the form of a system of stochastic difference equations

\[
\begin{align*}
x[k+1] &= f(x[k]) + u[k] - s[k], \\
s[k+1] &= as[k] + b + c\xi[k] + \xi[k+1],
\end{align*}
\]

where $x[k]$ is the storage at time $k$, $u[k]$ is the scope of delivery at time $k$, $s[k]$ is the current demand; $a, b, c$ are the object’s parameters; $f(x[k])$ is a nonlinear function.

It is necessary to determine the control law $u[k]$, ensuring that the (2) properties of the control system are fulfilled.

Let us briefly present some arguments in favour of the above-described approach to control over nonlinear stochastic systems for non-engineering applications, based on the prescription of target manifolds.

1. Social dynamic systems contain all attributes of ‘sophisticated’ systems (according to L.A. Rastrigin [18]); their behaviour is similar to the models giving rise to chaotic behaviour in physics and biology [19-22].

2. ..., “an economy cannot be regulated from the top, least of all it can be controlled from the top” [23, 24], which points to a well-founded reason for triggering the mechanism of target-oriented self-organization of an economic object (if the target-setting is consistent with the physical properties of the initial system).

3. Limited number of situations, where an application of statistical models as a method for examination of the empirical data is correct.

4. Consistency of the methods for designing control over nonlinear multidimensional objects on the basis of dependency of the partial controls with respect to each of the independent control subsystems [25].

5. Knowledge of the target manifolds and their analytical description is the ‘price’ for a successful design of control, which guarantees not only achieving the goal, but also implies an asymptotic maintenance of a sophisticated object in the neighbourhood of the target manifold, which is in agreement with a well-known idea that ‘scientists in a hundred and one way express their astonishment that, given a correct formulation of the problem, they would be able to unravel any puzzle set by the nature’ [20].

6. In accordance with the principle of an English cybernetician Anthony Stafford Beer; the degrees of sophistication and uncertainty of an object should correspond to those of the regulator.

7. “There are no optimal solutions in the nature” and it makes sense to treat the progress of any process as “a certain extreme, for which there is always its own functional that it minimizes” (according to N.N. Moiseev).

### 5. Structure of an information system for maintaining control over a sophisticated object

A set of interrelated elements, functioning consistently and in a target-oriented manner (Fig. 2), form a structural mechanism of a stochastic control system lending itself to algorithmization under the conditions of its software implementation.
The availability of a software support of such a technology for every applied problem (real process model) could become a specialized decision support system, whose major function would be to provide recommendations on the choice of appropriate macro variables $\psi(X) \in \mathbb{R}^m$, $X \in \mathbb{R}^n$, $m \leq n$, whose equality to zero determines the quality properties of the target control system (including an evaluation of the consequences of the available behavioural versions) and the target state as an implementation of a well-known statement that ‘there is already an answer to any question, all that is needed is to formulate the latter correctly’.

6. Sample Solutions of the Applied Problems on the Basis of the ADAR(S) Algorithm
Considering two non-linear control objects, for which, on the one hand, it is possible to design control systems based on the above algorithms, on the other hand, for the application of the developed control systems it is required that their parameters (in a real application) be adjusted due to an unknown noise character when changing coordinates of the state of the control object.

6.1. Case study 1. Stochastic control problem
We consider first the continuous description of an object of the predator – prey type in a scalar form

$$\begin{align*}
\dot{x}(t) &= ax(t) - bx(t)y(t) + u(t), \\
\dot{y}(t) &= -cy(t) + mx(t)y(t), \\
x(0) &= x_0 > 0, \ y(0) = y_0 > 0,
\end{align*}$$

where $x$ is the number of antigens (preys); $y$ is the number of antigens (predators); $a, b, c, m$ – positive coefficients characterizing (interspecies) interactions between the system’s variables.

Here the control variable $u(t) = u(x(t), y(t))$ means a possibility of influencing the system (6) via establishing a target-oriented character of the antigen’s dynamics (e.g., biostimulator injections).
The goal of control consists in ensuring stabilization of the function $y(t)$ in the vicinity of the set value, in other words, according to the principal method synergetic control theory the method of analytical design of aggregated regulators (ADAR) [5], we have to set the goal of control in the following form:

$$
(\psi(t) - y) = 0, t \geq t^*,
$$

where $y^*$ is the target value of antigens ($t^*$ - the point of reaching the vicinity of the target value with a satisfactory deviation from the exact value $y^*$).

Comment 1. The basic concepts of the synergetic control theory are implemented by the method of designing a control ‘pushing’ self-organization of the initial system in the ‘preferred’ direction, which does not contradict the evolution of biological systems if the set ‘goal-setting’ does not contradict its natural properties. Given a positive result, this technique could be well justified in specialized decision support systems for generating the “if…, then…” rules.

An application of the synergetic control theory to a biological object requires certain necessary conditions. Firstly, such systems are open (an evident interrelation with the medium); secondly, an obvious self-organization (appearance of stable structures in the form of disease – recovery and their gradations); thirdly, the components of such objects form a multi-loop, structurally sophisticated system (not liable to a strict and full formal description) functioning consistently and in a target-oriented manner.

The synthesis of the control law $u(t) = u(x(t), y(t))$ based on the ADAR method consists in performing the steps determined below, whose theoretical validation consists in the main concepts of synergetic control theory and the results of theoretical mechanics [6].

Step 1. Formation of control structure. Introduce an auxiliary macro variable

$$
\psi^{(1)}(x(t), y(t)) = x(t) - \phi(y(t)),
$$

where function $\phi(y(t))$ is subject to being defined in the subsequent steps.

According to ADAR, function $\phi(y(t))$ is referred as the ‘internal’ control, and the sought-for variable $u(t):=u(x(t), y(t))$ – as ‘external’ control. In this stage, the goal of control is the set $\{(x, y):\psi^{(1)}(t):=\psi^{(1)}(x(t), y(t)) = 0, t > 0\}$.

The variation problem, relying on which the control variable is derived, is given by

$$
\Phi_1(\psi^{(1)}) = \int_0^\infty \left(\left(\psi^{(1)}\right)^2(t) + T_1^2 \left(\psi^{(1)}\right)^2\right) dt \to \min.
$$

The external control structure (according to ADAR) is determined on the basis of a functional equation

$$
T_1\psi^{(1)}(t) + \psi^{(1)}(t) = 0
$$

where $\psi^{(1)}(t) = \psi^{(1)}(x(t), y(t)), T_1 = \text{const} > 0$. It follows from this equation that

$$
u(t) = -T_1^{-1}\psi^{(1)}(t) - ax(t) + bx(t) y(t) + \dot{\phi}(y(t))
$$

Step 2. Control structure refinement. Reduce the system on the achieved manifold $\psi^{(1)}(x(t), y(t)) = 0$, where $x(t) = \phi(y(t))$ is valid, and the initial description will acquire the following form:

$$
\dot{y}(t) = -cy(t) + m\psi(y(t)).
$$

Formulate the next variation problem

$$
\Phi_2(\psi) = \int_0^\infty \left(\psi^2(t) + T_2^2 \psi^2(t)\right) dt \to \min.
$$

$$
\psi(t) = y(t) - y^* = 0, t \to \infty
$$
From the respective functional equation (according to ADAR) $T_1\psi(t) + \psi(t) = 0$, $T_2 = \text{const} > 0$, relying on the already decomposed description (5), find the expressions for $\phi(y(t))$ and its derivative

$$\phi(y(t)) = m^{-1}(c - T_2) + \left(mT_2\right)^{-1} y^*(t)$$

$$\phi(y) = T_2^{-1} y^*(t)(cm^{-1} - x(t)).$$

Equations (3), (4), (6) form the final control system.

**Comment 2.** Positive constants $T_1$, $T_2$ are the parameters of regulator $u(t)$; their conceptual meaning is that of the durations of reaching the manifolds $y^{(i)}(t) = 0$, $\psi(t) = 0$, respectively (see [5]).

The control system (3), (4), (6) was modelled at the following parameter values and initial conditions:

$$a = 3, b = 2.7, c = 2, m = 1, T_1 = T_2 = 1,$$

$$y^* = 7, x(0) = 5, y(0) = 3.$$  

The graph of variation in the number of predators and prey is shown in Fig. 3.

6.2. **Case study 2. Stochastic control problem**

Control problem over an unstable open-loop system of the predator-prey type, whose application is not limited to its biological component.

In accordance with the Euler difference scheme (with the error $O(\tau_0)$) with regard to the initial description taking into account we obtain a system of difference equations for the predator–prey model ($\tau_0$ - sample spacing, $\alpha_i, \beta_i, i = 1, 2$ - object’s parameters):

$$X_1[k+1] = X_1[k] + \tau_0 f_1[k] + \tau_0 u[k] + \xi_1[k+1] + c \xi_1[k],$$

$$X_2[k+1] = X_2[k] + \tau_0 f_2[k],$$

$$f_1[k] = \alpha_1 X_1[k] - \beta_1 X_1[k] X_2[k],$$

$$f_2[k] = -\alpha_2 X_2[k] + \beta_2 X_1[k] X_2[k], \quad k = 0, 1, 2, ...$$

Let the control goal be:

$$\psi[k] = X_2[k] - X_2^*, X_2^* = \text{const}, k \to \infty$$

**Solution of the problem** on the basis of the scheme in Item 3 is implemented using the following algorithm for synthesis of a stochastic regulator.
1. At fixed values of noise $\xi_i[k], i=1,2$, in accordance with the deterministic case, obtain the regulator structure $\hat{u}[k]$ given by the following (see ADAR-method [5]):

$$\tau_0\hat{u}[k] = -\lambda_1\psi_1[k] - \left(\xi_i[k+1] + c\xi_i[k]\right) - X_1[k] - \tau_0f_1[k] + \varphi\left(X_2[k] + \tau_0f_2[k]\right),$$

$$\psi_1[k] = X_1[k] - \varphi[k],$$

$$\varphi[k] = -\left(\tau_0k\beta_1X_2[k]\right)^{-1}\left(\lambda_2 + 1\right)\psi[k] + \alpha_2\left(k\beta_1\right)^{-1}.$$

2. Find the conditional mathematical expectation $\tau_0u[k] = \mathbb{E}\left\{\tau_0\hat{u}[k] | \xi^k\right\}$ (according to condition $\mathbb{E}\left\{\xi[k]\right\} = 0, i=1,2$; the random variables $\xi_i[k], \xi_i[k+1], X_i[k], i=1,2$ are pair wise independent):

$$\tau_0u[k] = \mathbb{E}\left\{\tau_0\hat{u}[k] | \xi^k\right\} =$$

$$= \mathbb{E}\left\{-\lambda_1\psi_1[k] - \left(\xi_i[k+1] + c\xi_i[k]\right) - X_1[k] - \tau_0f_1[k] + \varphi\left(X_2[k] + \tau_0f_2[k]\right) | \xi^k\right\} =$$

$$= -\lambda_1\psi_1[k] - c\xi_i[k] - X_i[k] - \tau_0f_1[k] + \varphi\left(X_2[k] + \tau_0f_2[k]\right).$$

3. Eliminate the summand contained in $c\xi_i[k]$. To this aim, substitute expression (9) into the first equation in (7) to obtain

$$\xi_i[k] = \psi_1[k] + \lambda_1\psi_1[k-1].$$

Taking this relation and the form of the target variable into account (8), we have the final control system acquires the following form:

$$X_1[k+1] = X_1[k] + \tau_0f_1\left(X_1[k], X_2[k]\right) + \tau_0u\left(X_1[k], X_2[k]\right) + \xi_i[k+1] + c\xi_i[k],$$

$$X_2[k+1] = X_2[k] + \tau_0f_2\left(X_1[k], X_2[k]\right).$$

$$\tau_0\hat{u}[k] = -\left(\lambda_1 + c\right)\psi_1[k] - c\lambda_1\psi_1[k-1] - X_1[k] - \tau_0f_1[k] + \varphi\left(X_2[k] + \tau_0f_2[k]\right)$$

$$\psi_1[k] = X_1[k] - \varphi[k],$$

$$\varphi[k] = -\left(\tau_0k\beta_1X_2[k]\right)^{-1}\left(\lambda_2 + 1\right)\psi[k] + \alpha_2\left(k\beta_1\right)^{-1}.$$

The results of numerical modelling of the control system (10) discretized by the system of the predator – prey type are presented in Fig. 4 with the object’s

$$\alpha_1 = 1.1; \alpha_2 = 0.4; \beta_1 = 0.1; \beta_2 = 0.3;$$

$$B = 18; c = 0.05; X_1(0) = 20; X_2(0) = 10; X_2^* = 40.$$

and regulator’s $\Delta t = 0.1; \lambda_1 = \lambda_2 = 0.1$ parameters.
Figure 4. Transient process: for the target macro variable $\psi = X_2[k] - X_2^*.$

7. Summary
In this paper:
- a number of arguments have been presented in favour of application of the methods of designing control on manifolds not only to engineering control problems;
- a stochastic problem statement has been reported and an ADAR(S) algorithm for its solution as a generalized classical method of analytical nonlinear design of aggregated regulators for a random object;
- a need in the information system maintaining the synthesis of stochastic regulation has been demonstrated for every problem of nonlinear stochastic control, and a rough scheme of such an information system has been given;
- sample formulations of applied non-engineering problems have been presented, which are solved on the basis of the ADAR(S) algorithm generalizing the principal method of synergetic ADAR-control in the class of discrete systems.

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