Geometrical product specifications: A structure of linear dimensions tolerances

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Abstract. The paper reveals the relationship of linear dimensions tolerances with other geometrical specifications: coordinates, deviations of position and deviations of surface configuration of component geometric features. The dependence of the precision structure of the linear dimensions on the functional purpose of features. The basic concept of the paper is informativeness of the feature, which represents the total number of linear and angular degrees of freedom, constrained by the feature in the component at operation. The very informativeness of the feature determines the two-dimensional linear-angular composition of the structure of the linear size, its tolerance and standard grade. This paper discusses only the linear dimensions of the features involved in couplings and forming fits.

1. Introduction
Two indicators characterize the precision of the component feature: a dimensional precision and a geometric one. The dimensional precision is a precision of dimensions of the feature, whereas the geometric precision is a total precision of deviations of position and surface configuration of the feature or geometric deviations. Dimension tolerance is an overall characteristic of dimensional and geometric precision.

Geometric precision of the component feature is determined by the deviations of the surface configuration and the deviations of position, which are included into the tolerance structure of the feature dimension. In Russian national standardization, the basic norms of interchangeability of components by geometric precision are divided into three levels A, B, and C. Level A of normal geometric precision means that 60% of the tolerance zone of the dimension is occupied by the total tolerance zone of geometric deviations in the diametric expression. Level B of the extended geometric precision occupies 40% and level C of high geometric precision occupies 25% of the tolerance of the feature dimension. Since the geometric precision determines the difference of dimensions of each feature, two dimensions of one feature different from each other by 25…60% of the dimension tolerance is an objective reality, which must be considered at normalization of precision [1]. Consequently, the share of dimensional accuracy of the feature accounts for only 40…75% of the dimension tolerance [2].

2. Setting goals
In order to optimize the normalization of precision during designing of technical products based on registration of informativeness of geometric features of components the authors of the paper have set and solved the following tasks:
• the informativeness of prismatic and cylindrical features of different functional purpose has been determined;
• a precision structure of dimensions and tolerances for two-, four-, six-sided orthogonal prismatic features has been discovered;
• a precision structure of sizes and tolerances of diameters of cylindrical features of different informativeness has been revealed.

3. Features informativeness
A geometric feature of a component involved in coupling and fitting, serves as a constructional datum depriving the attached component of several degrees of freedom (from one to five). Only three of them prevent from linear movement along three mutually perpendicular axes or prevent from three angular rotations around these axes [1]. The number of degrees of freedom constrained by the feature in datum settings can serve as a quantitative characteristic of its informativeness. If a component feature functions as an actuating feature then, in case of contact loss with a coupling component, its informativeness drops to zero (0). Prismatic features with pairwise parallel flat planes can have informativeness equal to 3, 2, 1, 0, while cylindrical ones – 4, 2 or 0. This difference in feature informativeness impacts the structure of dimension tolerance, and consequently, the feature precision, tolerance numeric value and precision grade.

4. The sizes and tolerances structure for prismatic features
In prismatic features the dimensions form two nominally parallel flat planes, one of which represents the datum depriving the component of 3, 2 or 1 degrees of freedom and determining informativeness of the prismatic feature on the whole, and the second one is the actuator surface with zero informativeness. This prismatic feature peculiarity specifies the structure of the tolerance zone, which consists of two pieces: $T'$ depends on datum deviations and $T''$ – on the executive surface deviations (Figure 1-4).

If datum informativeness equals 3, the height of prismatic feature is $h_3$ (for the shaft) or $H_3$ (for the hole) (Fig.1). The structure of the first piece of tolerance includes only datum flatness deviation $EFE_1$, while the structure of the second one encompasses the deviation of the centre coordinate of actuator surface $Ezc$ ($Ezc'$), the deviation from parallelism $EPA$ of the executive surface relatively the assembly base and the deviation from flatness $EFE_2$ of the executive surface:

\[
\begin{align*}
T'h_3 &= T''h_3 + T''H_3, & T'H_3 &= T''H_3 + T''EFE_1, \\
T''H_3 &= Ezc + EPA + EFE_2; & T''H_3 &= Ezc' + EPA + EFE_2
\end{align*}
\]
In the course of base informativeness reduction, the structure of the tolerance of prismatic feature dimension becomes more complex. Width \( w_2(W_2) \) of the feature results from the base with informativeness 2 (Figure 2). The structure of the first piece of tolerance \( T'w_2(T'W_2) \), along with the deviation from flatness of base \( EFE1 \), includes the deviation from perpendicularity \( EPE1 \) of base \( B2 \) with respect to base \( A3 \) with informativeness 3. In the structure of second piece \( T''w_2(T''W_2) \), as compared to the structure of the zone of height tolerance \( T''h_3(T''H3) \) the deviations from parallelism \( EPA \) is replaced by the deviation from the perpendicularity of executive surface \( EPE2 \) with respect to base \( A3 \) and the deviation of coordinate \( Ezc(Ez'c) \) is replaced by deviation \( Exc(Ex'c) \):

\[
\begin{align*}
T'w_2 &= T'W_2 + T''w_2, \\
T''w_2 &= EPE1 + EFE1;
\end{align*}
\]

for the shaft \( Tw_2 = T'w_2 + T''w_2 \),

\[
\begin{align*}
T'w_2 &= Exc + EPE2 + EFE2; \\
T''w_2 &= EPE2 + EFE1,
\end{align*}
\]

for the hole \( Tw_2 = T'W_2 + T''W_2, \)

\[
\begin{align*}
T'W_2 &= T'W_2 + T''W_2; \\
T''W_2 &= EPE1 + EFE1; \\
T''W_2 &= Ex'c + EPE2 + EFE2.
\end{align*}
\]

\( (2) \)

\( (3) \)

Figure 2. The structure of width dimensions of prismatic features with informativeness 2 for the shaft (a) and for the hole (b).

The makeup of the tolerance of length dimension \( l_1(L_1) \) calculated from datum \( C1 \) of the prismatic feature with informativeness 1 (Figure 3) is supplemented with deviations from perpendicularity \( EPE3 \) of datum \( C1 \) and from perpendicularity \( EPE4 \) of executive surface \( K\Theta \) relatively the datum of component \( B2 \) with informativeness 2:

\[
\begin{align*}
l_1 &= T' l_1 + T'' l_1; \\
l_1 &= EPE1 + EPE3 + EFE1; \\
l'' l_1 &= Exc + EPE2 + EPE4 + EFE2;
\end{align*}
\]

for the shaft \( Tl_1 = T' l_1 + T'' l_1; \)

\[
\begin{align*}
l_1 &= EPE1 + EPE3 + EFE1; \\
l'' l_1 &= EPE1 + EPE3 + EFE1; \\
l'' l_1 &= Ex'c + EPE2 + EPE4 + EFE2.
\end{align*}
\]

\( (3) \)

Figure 3. The structure of length dimensions of prismatic features with informativeness 1 to shaft (a) and hole (b).
Finally, in case both planes of prismatic feature function as executive surfaces the feature becomes functionally symmetrical (Figure 4). It forms an overall dimension of height $g(G)$, the structure of which tolerance $T_g$ ($T_G$) include: four deviations $Ezc$ ($E_zc'$) from nominal coordinate $zc$ of centre $C$ of the plane of feature symmetry, doubled deviation from perpendicularity $EPE$ of the plane of the feature symmetry relatively axis $Z$ of generalized coordinate system of the component, represented base $A_4$, deviation of proper height $Egc$ ($EGc$) from the nominal value in the middle cross-section of the feature and two deviations from the plane of executive surfaces $EFE$:

for the shaft $Tg0=4Ezc +4EPE+ Egc+2EPA+2EFE$;

for the hole $TG0=4Ezc'+4EPE+ EGc+2EPA+2EFE$.  \hspace{1cm} (4)

It stands to reason that such complex tolerance framework does not allow a precise coupling for the executive surfaces of connecting overall dimensions of prismatic features of the components, for instance, in an automobile door both at height and in width.

![Figure 4. The structure of dimensions of the length of prismatic features with informativeness $\Theta$ (zero) for shaft (a) and for hole (b).](image)

5. The dimension and tolerance structure for cylindrical features

The tolerance zone with diameter size $T_d4$ ($TD4$) of cylindrical element serving as a component of main or auxiliary datum with informativeness 4 (Figure 5) represents a circular area in the component material included between two cylinders. These cylinders are coaxial with adjoining cylinder $ZE$, the axis of which represents an axis of the generalized or auxiliary coordinate system of the component. The dimensions of two cylinders of the tolerance zone are the limits of maximum $dl_{\text{max}}$ ($Dl_{\text{mm}}$) and minimum $dl_{\text{min}}$ ($Dl_{\text{mm}}$) of the material. The thickness of the circular tolerance zone is equal to half of the tolerance for diameter $0.5Td$ ($0.5TD$). The tolerance structure of the feature of geometric deviations includes only the deviations in configuration of cylinder surface $E_{FZ}$ (Figure 6). They include conicity $EFC$, because of the angular distortion of the cylinder feature in the linear expression, which in essence is a deviation from parallelity of the features, that is, angular deviation of position, and deviation of longitudinal profile $EFP$ (barrel or bow), and deviation from the straightness of axis $EFL$, and deviation from circularity of ovality-type $EFV$ and faceting $EFH$. The dimensional constituent of the tolerance is a doubled deviation of radius $Er(ER)$ of the cylinder in the midsection from nominal value.

Finally, at informativeness 4 of the cylinder feature we have:

for the shaft $Td4=2Er+2EFZ$;

for the hole $TD4=2ER+2EFZ$;

where $EFZ= EFC+ EFP+ EFL +EFV+ EFH$.   \hspace{1cm} (5)
The overall diameter tolerance zone cannot be exclusively represented by form of deviations because otherwise it would be necessary to make the cylinder radius equal to the nominal one, that is, with zero deviation, which is impractical. Moreover, safe deviations of the radius dimension generally determine the numerical value of the tolerance necessary for compensation of all constituents of machining errors, namely locating, static and dynamic setting, power and temperature-induced deformations.

In the conventional pictorial symbols of tolerance zones of dimensions accepted by the Unified standard system of tolerances and fits in the form of rectangles located relatively a zero line corresponding to nominal diameter \( d_0 (D_0) \) (Figure 5b), a part of the tolerance zone occupied by shape deviations should be shown in diametrical expression \( TFd \) (\( TFD \)) and be equal to a doubled standardized tolerance of the shape in radius expression \( 2TFZ \).

Diametrical tolerance zone of the shape is adjacent to minimum limit of the material as it determines the structure of the dimension of the minimum of the material. However it in essence is movable within the tolerance zone of dimension as it limits the allowable difference of the maximum and minimum dimensions of one and the same feature.

![Figure 5. The sizes and tolerance structure diameter of cylindrical features with informativeness 4 in cross-section (a) and size distribution laws (b).](image)

The material dimensions of maximum \( d_{mm} \) and \( D_{mm} \) (maximum is for the shaft and minimum for the hole) and minimum \( d_{mm} \) and \( D_{mm} \) (minimum is for the shaft and maximum is for the hole) serve different purposes. Maximum material dimensions play a key role since they determine the nature of fittings between coupled components and materialize the axis of the coordinate system of the component by the axis of the adjacent feature cylinder. Thus, the maximum dimension deviations of the feature material relatively nominal dimension are in fact critical, and therefore they should serve as a basis for the whole system of tolerances and fits. Nevertheless, in the standardized system the primary deviations of tolerance zones are the deviations that are closest to a zero line, which is not scientifically substantiated.

Maximum and minimum feature dimensions of the material comply with their own laws of distribution (Figure 5b) that are displaced relatively each other by the mean value of shape deviations \( EFZ \) taken for a component lot. The structure of shape deviation depends on coupling length \( lm \), on which the feature serves its purpose (Figure 6).
As cylindrical element informativeness decreases from 4 to 2 (in case it serves as an assembly base with informativeness 2) (Figure 7) the structure of the tolerance zone of the diameter dimension expands only due to geometrical angular deviations $\Delta E$ of the axes of two coaxial bases relatively common axis $Z_4$, which is the axis of the generalized coordinate system of the component (Figure 7a) or axis $Z'_4$ of the auxiliary coordinate system (Figure 7b). Linearly expressed angular displacements are limited to deviations from coaxiality relatively the common axis in diametrical expression $E_{PC}$. These deviations are included in the structure of tolerance zone of diameter dimension as doubled value $2E_{PC}$, which is typical only for symmetrical features:

- for the shaft: $T_d=2E_r+2E_{PC}+2E_{FZ}$;
- for the hole: $T_D=2E_R+2E_{PC}+2E_{FZ}$.

Despite the angular misalignment of axes, tolerance zones $T_D$ of features diameters are arranged in circular zones nominally relatively coordinate axes $Z_4$ and $Z'_4$ represented by two common axes of two bases. No matter to what extent the generators of the features are distorted, but if they fit in their tolerance zones along with shape deviations, the diameters of such features – maximum and minimum – will not go beyond the tolerance zones limits. Hence, the component will be fit and will serve its purpose. In case of a similar length of coupling cylinder features with informativeness 2 are less accurate than those with informativeness 4 due to more complex tolerance structure.

![Figure 6](image)

**Figure 6.** The tolerance structure of diameter dimensions of a cylindrical feature with informativeness 4 in the longitudinal section, depending on the length of the coupling (a) and (b).

![Figure 7](image)

**Figure 7.** The tolerance structure of diameter dimensions of a cylindrical feature with informativeness 2 for the shaft (a) and for the hole (b).

As the informativeness of the cylindrical feature drops to zero (which is true, when it operates in assembly in the coupling without direct contact with the coupling component (Figure 8)), tolerance structure $T_d$ (TD) additionally increases by four eccentricities $EE$ of the axis of the executive surface with respect to the axis of the coaxial cylindrical datum with informativeness 4. However, there is no provision for standardization of eccentricity by the International Standard [3]. That is why the eccentricity of the axis of the executive surface is subjected to limitation in tandem with linearly expressed angular misalignment $\Delta E$ in terms of standardized deviation from coaxiality relatively the reference axis in diametrical expression $E_{PC}$. The part of tolerance zone of diameter, which will be
occupied by eccentricity and misalignment, is equal to doubled deviation form coaxiality in diametrical expression 2EPC:

\[
\begin{align*}
\text{for the shaft: } Td\theta &= 2Er + 2EPC + 2EFZ; \\
\text{for the hole: } 2D\theta &= 2ER + 2EPC + 2EFZ.
\end{align*}
\]

Figure 8. The tolerance structure of the diameter dimensions of the cylindrical feature with informativeness \( \theta \) for the shaft (a) and for the hole (b).

Taking into consideration that such significant tolerance expansion occurs both in the encompassed and in the encompassing components, the precision of executive surfaces fitting cannot be high. If a cylindrical executive surface of the hole with informativity \( \Theta \) (zero) is placed in the generalized coordinate system, the component represented by a set of three flat datum (Figure 9) the structure of tolerance zone \( T_D0 \) of its complex diameter will include the doubled tolerances of position deviation in diametric expression \( 2TP0 \) of executive surface axis relatively three flat data. These tolerances encompass both sides of tolerance zone \( T_Dc \) of proper diameter \( Dc \), of the executive surface and thereby constituting the tolerance zone of complex diameter \( TD0 \):

\[
TD0 = T_Dc + 2TP0.
\]

Figure 9. The structure of tolerance of diameter dimensions of the cylindrical hole with informativeness \( \Theta \) for the model (a) and for the tolerance zone (b).
6. Discussion of scientific results

The following scientific results are submitted for a discussion:

The precision of linear dimensions is characterized by tolerance, which consists of two parts - dimensional precision and geometric precision. Dimensional precision determines the precision of the maximum material dimension or envelope size. Dimensional precision is 40…70 % of the dimensional tolerance. Geometric precision is defined by deviations from the shape and location of feature surfaces. Geometric accuracy amounts to 25…60 % of the dimensional tolerance.

The precision structure of the linear dimensions of the features includes two kinds of components: linear and angular, that is, it is two-dimensional. The composition of constituents in the structure of a linear dimension depends on the informativeness of the feature.

A valid feature dimension is the maximum material dimension, serving for defining the nature of the fitting in coupling. Therefore, the maximum material dimension should determine the position of the underlying deviation in the system of tolerances on the linear dimensions.

The geometric deviations of a single feature are systematic functional distortions that create the variability of feature dimension and form two boundary values: maximum material dimension and minimum material dimension.

The maximum and minimum material dimensions in the kit of parts have different distribution laws, shifted in relation to each other on the average value of the total geometrical deviations in the diametric expression.

References

[1] Glukhov V I 2014 International Scientific and technical Conference on in Dynamics of Systems, Mechanisms and Machines 1–9
[2] ISO 286-1-2010 Geometrical product specifications (GPS) – ISO code system for tolerances on linear sizes – Part 1: Basis of tolerances, deviations and fits
[3] ISO 1101-2011 Geometrical product specifications (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out
[4] ISO 8015-2011 Geometrical product specifications (GPS) – Fundamentals – Concepts, principles and rules