CHEMICAL EQUILIBRATION OF ANTIHYPERONS

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Abstract

Rapid chemical equilibration of antihyperons by means of the interplay between strong annihilation on baryons and the corresponding back-reactions of multi-mesonic (fusion-type) processes in the later, hadronic stage of an ultrarelativistic heavy ion collision will be discussed. Explicit rate calculations for a dynamical setup are presented. At maximum SPS energies yields of each antihyperon specie are obtained which are consistent with chemical saturated populations of $T \approx 150 – 160$ MeV. The proposed picture supports dynamically the popular chemical freeze-out parameters extracted within thermal models.

1 Physics of antihyperon production

Strangeness enhancement has been predicted a long time ago as a diagnostic probe to prove for the short-time existence of a QGP in the course of a relativistic heavy ion collision. With respect to the high production thresholds in the various binary hadronic reaction channels, especially the antihyperons were then advocated as the appropriate candidates [1]. Assuming the existence of a temporarily present phase of QGP and following statistical coalescence estimates, the abundant (anti-)strange quarks can easily be redistributed to combine to strange (anti-)baryons [1], which do then, in return, come close to their chemical equilibrium values at the onset of the hadronic phase.

Indeed, such a behaviour of nearly chemically saturated populations of the antihyperons has been experimentally demonstrated with the Pb+Pb experiments NA49 and WA97 at CERN-SPS. For this statement, of course, a quantitative, theoretical analysis within a thermal model has to be invoked by fitting the thermodynamical parameters to the set of individual (strange) hadronic abundancies [2]. Some phenomenological attempts to explain a more abundant production of antihyperons within a hadronic transport description [3] do exist like the color rope or the high-dense cluster formation. The underlying mechanisms, however, have to be considered as exotic. In particular, a dramatic role of antibaryon annihilation is observed, which, in return, has
to be more than counterbalanced in a rather ad hoc way by these more exotic mechanisms.

Be it as it is, a correct incorporation of the strong baryonic annihilation channels had actually not been achieved within the various microscopic implementations. Recently we have conjectured that a sufficiently fast redistributions of strange and light quarks into (strange) baryon-antibaryon pairs should be achieved by multi-mesonic fusion-type reactions of the type

$$n_1\pi + n_2K \leftrightarrow \bar{\Lambda} + p$$

(1)

for a moderately dense hadronic system \[4\]. The beauty of this argument lies in the fact that (at least) these special kind of multi-hadronic reactions have to be present because of the fundamental principle of detailed balance. As the annihilation of antihyperons on baryons is of dramatic relevance, the multi-mesonic (fusion-like) ‘back-reactions’ involving $n_1$ pions and $n_2$ kaons, where $n_2$ counts the number of anti-strange quarks within the antihyperon $\bar{\Lambda}$, must, in principle, be taken care of in a dynamical simulation. This reasoning has first been raised by Rapp and Shuryak concerning the production of anti-protons \[5\], which then triggered our work.

It is plausible to assume that the annihilation crosssections are approximately the same as for $N\bar{p}$ at the same relative momenta, i.e. $\sigma_{\bar{Y}N \rightarrow n\pi + n\Lambda K} \equiv \sigma_0 \approx 50\text{ mb}$. The equilibration timescale $(\Gamma_{\bar{Y}})^{(-1)} \sim 1/(\sigma_{\bar{Y}N}v_\Lambda \rho_B)$ is to a good approximation proportional to the inverse of the density of baryons and their resonances. Adopting an initial density of 1–2 times normal nuclear matter density $\rho_0$ for the initial and thermalized hadronic fireball, the antihyperons will equilibrate on a timescale of 1–3 fm/c! This timescale competes with the expansion timescale of the late hadronic fireball. To be more quantitative rate calculations for a dynamical setup will be presented in the next section.

2 Rate equation results and their implications

Following the concepts of kinetic theory, the microscopic starting point would be a Boltzmann-type equation of the form

$$\partial_t f_{\bar{Y}} + \frac{p}{E_{\bar{Y}}} \nabla f_{\bar{Y}} = \sum_{\{n_1, n_2\}; B} \frac{1}{2E_{\bar{Y}}} \int \frac{d^3p_B}{(2\pi)^3} \frac{\prod_{\{n_1, n_2\}} \int \frac{d^3p_i}{(2\pi)^3} \delta^4(p_{\bar{Y}} + p_B - \sum_{\{n_1, n_2\}} p_i)}{(2\pi)^4}$$

$$\langle \langle n_1, n_2|T|\bar{Y}B\rangle \rangle^2 \left\{ (-) f_\Lambda f_B \prod_{\{n_1, n_2\}} (1 + f_i) + \prod_{\{n_1, n_2\}} f_i(1 - f_\Lambda)(1 - f_B) \right\},$$
where
\[
\sigma_{YB}^{\{n_1\}} = \frac{1}{\text{Flux}^{\{n_1, n_2\}}} \prod d^3p_i \frac{(2\pi)^4 2E_i}{2} (2\pi)^4 \delta^4 \left(p_Y + p_B - \sum_{\{n_1, n_2\}} p_i\right) |\langle\langle n_1, n_2 | T | YB\rangle\rangle|^2
\]

corresponds to the annihilation cross section. Assuming \(v_{\text{rel}} \sigma_{YB}(\sqrt{s})\) to be roughly constant, which is actually a good approximation for the \(p\bar{p}\)-annihilation, or, invoking a standard coarse grained description of thermally averaged cross sections and distributions, and furthermore taking the distributions in the Boltzmann approximant, the following master equation for the respectively considered antihyperon density is obtained
\[
\frac{d}{dt} \rho_{\bar{Y}} = -\langle\langle \sigma_{YB} v_{YB} \rangle\rangle \left\{ \rho_{\bar{Y}} \rho_B - \sum_{\{n_1\}} \mathcal{R}_{(n_1, n_2)}(T, \mu_B, \mu_s)(\rho_{\pi})^{n_1}(\rho_K)^{n_2} \right\}, \quad (2)
\]

where the ‘back-reactions’ of several effectively coalescing pions and kaons are incorporated in the ‘mass-law’ factor
\[
\mathcal{R}_{(n_1, n_Y)}(T, \mu_B, \mu_s) = \frac{\rho_{\bar{Y}}^{eq} \rho_B^{eq}}{(\rho_{\pi}^{eq})^{n_1}(\rho_K^{eq})^{n_2}} p_{n_1}.
\]

Here \(p_{n_1}\) states the relative probability of the reaction (1) to decay into a specific number \(n_1\) of pions and \(\rho_B\) denotes the total number density of baryonic particles. \(\mathcal{R}\) depends only on the temperature and the baryon and strange quark chemical potentials. \(\Gamma_{\bar{Y}} \equiv \langle\langle \sigma_{YN} v_{YN} \rangle\rangle \rho_B\) gives the effective annihilation rate of the respective antihyperon specie on a baryon.

Nonequilibrium inelastic hadronic reactions can explain to a good extent the overall strangeness production seen experimentally: The major amount of the produced kaons at SPS-energies can be understood in terms of still early and energetic non-equilibrium interactions [6, 7]. Refering to the master equation (2), one can then take the pions, baryons and kaons to stay approximately in thermal equilibrium throughout the later hadronic evolution of the collision, the later being modelled to be an isentropic expansion with fixed total entropy content being specified via the entropy per baryon ratio \(S/A\). (2) becomes
\[
\frac{d}{dt} \rho_{\bar{Y}} = -\Gamma_{\bar{Y}} \left\{ \rho_{\bar{Y}} - \rho_{\bar{Y}}^{eq} \right\}. \quad (3)
\]

The ‘effective’ (global or at midrapidity) volume \(V(t)\) is parametrized as function of time by longitudinal Bjorken expansion and including an accelerating radial flow, e.g.
\[
V_{\text{eff}}(t \geq t_0) = \pi (ct) \left( R_0 + v_0(t - t_0) + 0.5a_0(t - t_0)^2 \right)^2 \quad (4)
\]
with $R_0 = 6.5 \text{ fm}$, $v_0 = 0.15 \text{ c}$ and $a_0 = 0.05 \text{ c}^2/\text{fm}$. At starting time $t_0$ an initial temperature $T_0$ is chosen. ($T_0$ is set to 190 MeV for the SPS and 150 MeV for the AGS situation, while the initial energy densities are then about 1 GeV/fm$^3$.) From (3) together with the constraint of conserved entropy the temperature and the chemical potentials do follow as function of time. As a minimal assumption the initial abundance of antihyperons is set to zero. Equation (3) is solved for each specie.

$$
0.000 \quad 0.002 \quad 0.004 \quad 0.006 \quad 0.008 \quad 0.010
\begin{array}{cc}
0.4 & 0.5 \quad 0.6 \quad 0.8 \quad 1.0
\end{array}
$$

**Fig. 1:** The anti-$\Lambda$ to baryon number ratio $N_{\bar{\Lambda}}/N_B(t)$ as a function of time for various implemented annihilation cross section $\sigma_{eff} \equiv \lambda \sigma_0$. The entropy per baryon is taken as $S/A = 30$, $t_0 = 3 $ fm/c and $T_0 = 190 $ MeV.

In Fig. 1 the number of $\bar{\Lambda}$s (normalized to the conserved net baryon number) as a function of time is depicted. The entropy per baryon is chosen as $S/A = 30$ being characteristic to global SPS results. In addition the cross section employed is varied by a constant factor, i.e. $\sigma_{eff} \equiv \lambda \sigma_0$. The characteristics is that first the antihyperons are dramatically being populated, and then in the very late expansion some more are still being annihilated. The results are rather robust against a variation by a factor of 2 in the cross section.

In Fig. 2 the number of antihyperons of each specie are now shown as a function of the decreasing temperature $T(t)$ of the hadronic system. For a direct comparison the instantaneous equilibrium abundance $N_{\bar{Y}}^{eq}(T(t), \mu_B(t), \mu_s(t))/N_B$ is also given. As noted above, after a fast initial population, the individual yields of the antihyperons do overshoot their respective equilibrium number and then do finally saturate at some slightly smaller value. Moreover, one notices that the yields effectively do saturate at a number which can be compared to an equivalent equilibrium number at a temperature parameter around
$T_{\text{eff}} \approx 150 - 160 \text{ MeV}$, being strikingly close to the ones obtained within the various thermal analyses.

In Fig. 2 the antihyperon to baryon number ratio $N_{\bar{Y}}/N_B(T)$ and $N_{\bar{Y}}^{eq.}/N_B(T)$ (dotted line) as a function of the decreasing temperature. Parameters are the same as in Fig. 1.

In Fig. 3 the number of anti-$\bar{\Lambda}$s as a function of time is given for various entropy per baryon ratios. One notices that the final value in the yield significantly depends on the entropy content, or, in other words, on the baryochemical potential. We note that the results at midrapidity from WA 97 can best be reproduced by employing an entropy to baryon ratio $S/A = 40$, where one qualitatively expects a larger entropy content.

There is also a clear hint at AGS energies of enhanced anti-$\Lambda$ production. For most central collisions more anti-$\Lambda$s are found compared to anti-protons, which is quite puzzling as within a thermal model analysis this ratio is found not to be larger than 1. This enhanced ratio of anti-$\Lambda$s compared to anti-protons at AGS energies one can understand in a way that one assumes that their annihilation cross section on baryons is just slightly smaller than for the antiprotons. In Fig. 4 a similar study like that of Fig. 1 is shown for a characteristic situation at AGS. For smaller, yet not too small effective cross sections the final yield can here be enlarged by a factor of 2 compared to the case with a ‘full’ crosssection, as the final reabsorption is not as effective. But, of course, this idea is speculation at present. Also, we remark that the $\bar{\Lambda}$s effectively do saturate at an equivalent equilibrium number at a temperature parameter around $T_{\text{eff}} \approx 120 - 130 \text{ MeV}$. Unfortunately, there are no data for
Ξ at AGS. A detailed measurement of all antihyperons represents an excellent opportunity for future heavy ion facilities at an energy upgraded GSI.

Fig. 3: $N_\bar{\Lambda}/N_B(t)$ as a function of time for various entropy content described via the entropy per baryon ratio ($S/A = 20 - 40$). Other parameters are as in Fig. 1.

Fig. 4: $N_\bar{\Lambda}/N_B(T)$ and $N_{\bar{\Lambda}eq}/N_B(T)$ as a function of decreasing temperature for a characteristic AGS situation with an entropy content of $S/A = 12$ for various implemented annihilation cross section $\sigma_{eff} \equiv \lambda \sigma_0$. $t_0 = 5$ fm/c and $T_0 = 150$ MeV.

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