Identify student mathematical understanding ability through direct learning model

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Abstract. The ability of mathematical understanding is an important basic skill possessed by students. The purpose of this study is to identify the ability of the students mathematical understanding through direct learning model. The type of research is descriptive qualitative research. The subjects were 6 students divided into 3 criteria: students who have good, medium, and low mathematics skills. The learning process is done by using direct learning model. All students were given diagnostic questions and the results were corrected for analysis. The classification of misconceptions made by students uses four errors, namely: (1) Basic error that is the basic concept mistake or misunderstanding of the problem; (2) Appropriate error of understanding some concepts; (3) Missing information that is not able to answer completely the problem given; (4) Partial insight error caused by miscalculation or carelessness. Based on the result of the research, it is found that in solving basic mathematics problems, students make mistakes of mathematical understanding on basic error type, missing information, and partial insight. After being given direct learning, misunderstanding of students decreased.

1. Introduction

Mathematics is part of education that has a goal to grow maturity of thinking. The patterns of thinking start from concrete thinking to abstract thinking. The ability to think mathematically becomes a clue for all learners to be able to use to solve problems in life. Solving the problem cannot be felt directly can be seen from the ability to analyze the problem can be obtained the solution to solve it. This can be learned in a person's math exercise-trained to have various mathematical abilities ranging from the ability of understanding to high-order thinking skills.

Mathematics is not only studied in their field but integrated with other fields. For example, mathematics also studies in economics, pharmacy, chemistry, physics, and others. Therefore mathematics can be said to be a minister of knowledge because mathematics can help to understand concepts in other fields. A student who uses mathematics as a tool to understand the concept he is learning needs to first understand the concept of mathematics itself. In this case that needs to be possessed is the ability to understand mathematically because it becomes the basis for the concept of mathematics can be mastered.

The ability to understand mathematically is an important foundation for thinking in solving mathematical problems. [1] suggests that mathematical understanding is an important aspect of the principle of mathematics learning. Therefore, in terms of basic mathematics courses, students must have the ability to understand mathematically. [2] distinguishes two types of understanding, namely
instrumental understanding and relational understanding. The instrumental understanding of a concept is the notion of a separate concept and only by reason. Relational understanding is a complex and interconnected structure of knowledge to be used in solving mathematical problems.

[3] saw understanding as the ability of students to explain concepts, using concepts, and develop mathematical concepts. Explaining the meaning of mathematical concepts students' ability to express again what has been gained. Using the mathematical concept means the ability of students to use mathematical concepts in a variety of different situations. Developing mathematical concept means the ability of students to develop some of the consequences of a concept in solving mathematical problems using mathematical concepts.

Knowledge of concepts is called conceptual knowledge [4][5][6]. Conceptual knowledge is knowledge of the mastery of certain concepts of matter. Conceptual knowledge is not just to know about a particular concept but also know where these concepts appear. While knowledge of the procedure is often called procedural knowledge [5][6]. Procedural knowledge is knowledge about the steps needed to achieve various objectives. In solving math problems, requires a procedure that is prepared with the right concept. In this case, the student needs to have a strong knowledge of understanding basic math concepts so that the problem-solving process, students can make the correct procedure.

Cultivate the ability of mathematical understanding is not easy. However, the need for an effective way of learning is one of them with direct learning model. [7] states that the direct learning model is an effective way of developing procedural skills. Direct learning consists of orientation activities of learning goals, presentations, structured exercises, guided training, and self-training. The ability of mathematical understanding is closely related to procedural skills. In the process of mathematical problem solving, the ability of understanding serves as the foundation of every procedure.

The ability of mathematical understanding is a low-level ability in one's thinking process. However, this understanding determines the success of other abilities. Therefore, the learning process needs to be designed in such a way that all students can have mathematical skills before leading to high-level thinking skills. [8] mentioned that the type of activity performed through direct learning is more suitable for low-level thinking skills.

This study intends to identify the ability of the students mathematical understanding through hands-on learning. The identification is done by analyzing the mistakes made by the students as a measure of the difficulties experienced by students. [9] stated that the errors of students in mathematics are the primary source to determine the difficulties experienced in the learning of mathematics. Difficulty undertook by students as part of the lack of ability of the students mathematical understanding.

[10] revealed the cause of the errors made by the students, namely: Basic error, Appropriate errors, missing information, and partial insight. Basic Error is a mistake or error to understand the basic concepts of matter. The appropriate error is part of the concept is not understood yet already understand the basic concepts. Missing information which cannot fully answer the answer to such problems. Partial insight that bit errors caused by miscalculations or errors due to carelessness.

2. Methodology
The research method used in this research is descriptive qualitative. Before conducting the selection of research subjects, the researchers conducted a descriptive data analysis of pretest, pretest, and normalized gain data to determine the improvement of students' mathematical understanding ability. After the categorization of student groups is known based on the level of achievement of the ability of understanding after the direct learning, the researcher took the subject of research as many as 6 students selected from 51 students from Universitas Garut. The six students are divided into three groups of students who have high, medium, and low mathematical skills with each group consisting of two students. The researchers used a diagnostic test that contained problems with mathematical understanding. The diagnostic test is given to six students and then the results are analyzed for known bugs mathematical understanding what is done by the students.
The qualitative data analysis technique used in this research is to make the student answer record correct or wrong, to identify the factors that become the thinking of the mistake in doing the problem, to classify the students' difficulties based on the four mistakes developed by [10] is basic error, appropriate error, missing information, and partial insight. As a matter of diagnostics given are:

### Table 1. Problem Diagnostic test

| No | Question |
|----|----------|
| 1  | If $f(x) = -4x^2 + 2$, then calculate the value of the $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ |
| 2  | Find the derivative of  
|    | a. $f(x) = \cos^3(x^2 + 5)^2$  
|    | b. $2x^2y + 4y^2x = x + 3y^3$ |
| 3  | Determine the value of:  
|    | $\int \frac{5}{2 - 4x} \, dx$ |

### 3. Results

The objective of this research is to know how to increase the ability of students mathematical understanding and identifying errors which students in completing test regarding the ability of mathematical understanding. Before given the direct learning, students must first be given about the pretest to determine how the initial ability to understanding the mathematical ability of students. At the end of direct learning, students are given about the post-test to see how the achievement of mathematical understanding after learning abilities. To determine the increase mathematical ability, researchers used data for the pretest and posttest were analyzed using normalized gain formula.

The results of the analysis of data descriptive of the pretest, posttest, and the gain normalization is as follows:

### Table 2. Descriptive Analysis of Student Mathematical Understanding

| Description       | Pretest | Posttest | N-gain |
|-------------------|---------|----------|--------|
| Average           | 68,82   | 80,61    | 0,40   |
| Standard deviation| 17,65   | 12,27    | 0,21   |

According to the table above shows that the average post-test results of mathematical ability of students is greater understanding of the results of the pretest. In addition, there is increased understanding of mathematical ability of students after direct learning of 0.40 with medium improvement category.

In order to identify errors ability mathematical understanding of students, researchers took the data postest as a reference to determine the sample as many as 6 students, with each divided into three groups with high achievement category, medium, and low. The overall presentation of categories of achievement can be seen in the following table:

### Table 3. Percentage of Student Mathematical Understanding Achievement

| Category | Amount | Percentage |
|----------|--------|------------|
| High     | 28     | 54.90%     |
| Medium   | 21     | 41.18%     |
| Low      | 2      | 3.92%      |
Based on the categorization, taken from each category as much as two students to be identified the ability of mathematical understanding through the mistake he made.

a. High Subject Mathematical Understanding Student

Mathematical misunderstanding by students in a high category is as follows:

In question 1, the first student has used the concept correctly but carelessly in the calculation is
\[-4(x + h)^2 = -4x^2 + 8xh + 4x^2\]
the answer should be 
\[-4(x + h)^2 = -4x^2 - 8xh + 4x^2.\]
This is due to the student's haste in doing the calculations so that less careful in writing positive signs that should be negative. Student answers to problem 1 can be seen in the following figure:

![Figure 1. First Student Error in High Category](image)

Furthermore, the second student's misconception was made on problem 2a as a result of the student's carelessness on the matter of trigonometric function derivatives i.e. error decreasing \(x^2\) to 2. But overall the process is correct. Results from the second student's answer can be seen in the following figure:

![Figure 2. Second Student Error in High Category](image)

Based on the description of both students' mathematical understanding errors can be concluded that students in the high category already have an instrumental understanding and relational understanding well. This means that both students have understood the concept and apply it in solving mathematical problems. But that understanding is not accompanied by prudence in the work of the problem so that the completion of the problem is not perfect (partial insight).

b. Student of Medium Mathematical Understanding Category

Mathematical misunderstanding by students in the medium category is as follows:
In Problem 2a, the first student makes a mistake in deriving \( \cos(x^2 + 5)^2 \) into \( \sin(x^2 + 5)^2 \cdot 2(x^2 + 5) \cdot 2 \) the answer should be \( -\sin(x^2 + 5)^2 \cdot 2(x^2 + 5) \cdot 2x \). The student has understood the procedure of decreasing trigonometric function, but still careless in writing negative signs and multiplication of number. The student's reply can be seen in the following figure:

![Figure 3. First Student Error in Medium Category](image)

Furthermore, the second student's misconception is made on a problem no 3 i.e. only doing part of the settlement that is not to change the return of the form of u to its original form. This is because only understand some integral concepts. Results from the second student's answer can be seen in the following figure:

![Figure 4. Second Student Error in Medium Category](image)

Based on the description of both students' mathematical understanding errors, it can be concluded that students in the medium category already have an instrumental understanding but do not yet have a good relational understanding. This means that the student has understood the concept but has not been able to apply it in solving the mathematical problem so as to make mistakes due to carelessness (partial insight) and working on some concepts (missing information).

c. **Student of Low Mathematical Understanding Category**
Mathematical misunderstanding by students in the low category is as follows:
In question no 1, the first student does not understand the concept well. This can be seen from the many errors made in solving algebraic derivatives by using the definition of limits. The mistake made is that the student changed the value \( -4(x + h)^2 \) to \( -4x^2 - 4h^2 \). This is because the
student does not understand the concept of quadratic equations so that the derivative work is wrong. The student's reply can be seen in the following figure:

![Figure 5. First Student Error in Low Category](image1.jpg)

Furthermore, the second student in the low category made a mistake in answering the problem of no 3 i.e. decreasing \(2 - 4x\) to 4 and error in applying the derived concept in integral. This is because the student has not understood the integral concept well. The answer to question no 3 can be seen in the following figure:

![Figure 6. Second Student Error in Low Category](image2.jpg)

Based on the description of both students' mathematical understanding errors can be concluded that the students in the low category already have an instrumental understanding but not yet have a good relational understanding. This means that the two students know about the settlement procedure but have not understood the basic concept in solving derived problems and integral (basic error).

4. Discussion
The ability of mathematical understanding is an important part of mathematics learning. The ability of mathematical understanding is the basis of thinking in matters of mathematics as well as in issues of everyday life. The direct learning model is one alternative that can be used to improve the ability of mathematical understanding. Based on the research results obtained a moderate increase in the ability of students' mathematical understanding of learning using direct learning model. [8] mentioned that the type of activity performed through direct learning is more suitable for low-level thinking skills.
Direct learning involves various exercises in the development of concepts to improve students' mathematical understanding. Concept development can be done by emphasizing student-centered learning. Interventions given to each exercise have an effect on improving student motivation and achievement in mathematics [11]. Optimal efforts can be made on direct learning by providing structured interventions in each implementation of the exercise so that increased understanding of students can achieve high categories.

Direct learning allows for discussion. This activity creates a social environment that will foster students' mathematical understanding through the question and answer process in the discussion of the practice. Collaborative activities help to improve students' mathematical understanding [12].

Based on the identification of students' mathematical misunderstanding after the direct learning, the results obtained that students make mistakes on the type of basic error, missing information, and partial insight. Basic Error is a student error in understanding the basic concepts to be used in solving math problems. This basic concept is a prerequisite concept that must be possessed by students to be applied in other concepts. Just as in working on a derivative using the limit definition requires the concept of quadratic equations. So also in doing integral by substitution requires a prerequisite that is a derivative concept. Looking at the answers from students, it is known that they know the procedure in doing the problem but wrong in applying the basic concept. Procedural knowledge is the knowledge of the sequence of steps to be taken in solving the problem. This means that procedural knowledge is still related to instrumental understanding. [13] suggests that procedural knowledge is identical to instrumental understanding.

Missing information is a student error caused by understanding some concepts so as not to work out the answer optimally. Understanding of a concept needs to be done thoroughly in order to avoid mistakes in interpretation in answering a problem. Conceptual understanding positively affects procedural understanding [14]. Therefore, in order to avoid missing information, students need to be trained to have conceptual understanding in order to lead to procedural understanding.

Basic Error is a student error caused by doing carelessness in doing math problems. Carelessness is part of the lack of conceptual understanding that students have. A person who has conceptual understanding will be able to evaluate the mistakes he or she makes [15].

It is relatively easy to judge procedural understanding compared to conceptual understanding [16]. This is seen in missing information and basic error. The mistakes of students who only work on some concepts and because of carelessness in reply can be interpreted that the student has conceptual understanding but has no procedural understanding or otherwise.

These three errors are caused by a lack of understanding of students in mathematics. Therefore it is necessary to develop the concept of learning process in mathematics. Conceptual development can be done by rediscovering concepts, proposing concepts, and using examples and comparisons for analogical reasoning [17].

5. Conclusion

Based on the results and discussion, it can be concluded that direct learning can improve students' mathematical understanding ability. In the identification of students' mathematical understanding errors, it can be concluded that the students made mistakes of mathematical understanding on a basic error, missing information, and partial insight types. This shows there are still some students who have an instrumental understanding but are still not good in relational understanding.

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