Confidence Dimension for Deep Learning based on Hoeffding Inequality and Relative Evaluation

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Abstract

Research on the generalization ability of deep neural networks (DNNs) has recently attracted a great deal of attention. However, due to their complex architectures and large numbers of parameters, measuring the generalization ability of specific DNN models remains an open challenge. In this paper, we propose to use multiple factors to measure and rank the relative generalization of DNNs based on a new concept of confidence dimension (CD). Furthermore, we provide a feasible framework in our CD to theoretically calculate the upper bound of generalization based on the conventional Vapnik-Chervonenkis dimension (VC-dimension) and Hoeffding's inequality. Experimental results on image classification and object detection demonstrate that our CD can reflect the relative generalization ability for different DNNs. In addition to full-precision DNNs, we also analyze the generalization ability of binary neural networks (BNNs), whose generalization ability remains an unsolved problem. Our CD yields a consistent and reliable measure and ranking for both full-precision DNNs and BNNs on all the tasks.

1 Introduction

Deep neural networks (DNNs) contribute significantly to many computer vision tasks over the past decade. However, most DNNs are heavily over-parameterized, which means that the models can achieve near-perfect performance on the training data but have a performance gap on the test data. Measuring the generalization ability is complex. This raises a fundamental question: can we fairly and objectively evaluate the generalization ability of a deep learning model?

Theoretical research on the image classification capacity of shallow models starts with Vapnik-Chervonenkis Dimension (VC-dimension) [Vapnik and Chervonenkis(1971)]. Taking the data distribution into account, Bartlett et al. [Bartlett and Mendelson(2002)] further proposes the Rademacher complexity, a stricter generalization error bound.

The training of DNNs is more complicated and involves solving nonconvex optimization problems in a very high-dimensional space. Moreover, it is difficult to find the global minimum of DNNs analytically. Previous works [Dziugaite and Roy(2016)] have shown that there is a degree of correlation between the optimizer and the generalization ability of DNNs. Inspired by this, Fort et al. [Fort et al.(2019)Fort, Nowak, Jastrzebski, and Narayanan] proposes a gradient alignment measure to understand the generalization. However, the optimizer is not the only factor that affects the generalization ability. For example, training different network architectures with the same optimizer may lead to other generalization errors. Furthermore, batch size and regularization techniques such as dropout [Srivastava et al.(2014)Srivastava, Hinton, Krizhevsky, Sutskever, and Salakhutdinov], weight decay, and early stopping also influence the generalization ability. Those regularization techniques lower the model's complexity and therefore affect the generalization.

Based on these observations, the generalization of DNNs should consider both the network architecture and the optimization. Due to finite sampling set, traditional VC-dimension can only produce quite loose generalization bounds, which are unsuitable to describe DNNs with more parameters than training samples [Valle-Pérez and Louis(2020); Zhang et al.(2017)Zhang, Bengio, Hardt, Recht, and Vinyals]. Tighter generalization bounds [Neyshabur et al.(2019)Neyshabur, Li, Bhojanapalli, LeCun, and Srebro; Valle-Pérez and Louis(2020)] have been proposed to understand the generalization ability of deep models better. However, these generalization bounds are still impractical for real applications because of their restrictive assumptions. For example, Valle et al. [Valle-Perez et al.(2018)Valle-Perez, Camargo, and Louis] ignores the effects of different training “tricks” on the generalization. Furthermore, Neyshabur et al. [Neyshabur et al.(2019)Neyshabur, Li, Bhojanapalli, LeCun, and Srebro] limits the architecture to two-layer networks, and Valle et al. [Valle-Pérez and Louis(2020)] restricts the studied image classification tasks to binary problems. Moreover, previous methods based on VC-dimension often offer an unstable explanation and occasionally show trends opposite the actual error. Previous bound predictions increase with the increasing training set size, while the measured error decreases [Nagarajan and Kolter(2019)]. As a result, for real applications only when a finite training sample set is available for the VC-dimension calculation, a theoretical upper bound for generalization will provide a guidance to analyze DNNs from a pragmatic perspective.

So far, there has not been any effective method to mea-
measure generalization. In this paper, we propose a new concept of confidence dimension (CD) based on VC-dimension and Hoeffding’s inequality to fairly evaluate the generalization ability of DNNs, by considering both the network architecture and the optimization process in a unified framework, as shown in Fig. 1. Many factors are not taken into account in the VC-dimension [Zhang et al.(2017)Zhang, Bengio, Hardt, Recht, and Vinyals]. Now we consider inputs and process of optimization as a correction term to obtain the upper bound of generalization, so as to ensure that the generalization of models are compared within a unified and relatively completed framework. It also can be used to measure the generalization of models on a unified scale under different experimental conditions. We investigate the generalization evaluation based on a relative assessment of different DNN models, i.e., focus on the CD ranking consistency of DNN models across different situations, which is more practical than existing methods [Bartlett and Mendelson(2002); Vapnik and Chervonenkis(1971)]. We take two steps to establish an upper bound for generalization, leading to our confidence dimension (CD), and provide explanatory power for DNNs. First, DNN models are trained on randomly and accurately labeled data to calculate the VC-dimension. Then, based on Hoeffding’s inequality, we calculate the corrected term of generalization by considering the optimization process [Dziugaite et al.(2020)Dziugaite, Drouin, Neal, Rajkumar, Caballero, Wang, Mitliagkas, and Roy] into calculating the CD. According to the law of large numbers, the CD will become stable by performing large numbers of experiments.

We demonstrate the rationale of our method theoretically and experimentally. We first provide a theoretical derivation of the upper bound of generalization based on the VC-dimension and Hoeffding’s inequality. Then, we further extensively validate the generalization bound CD by cross-tasks, including image classification and object detection. Moreover, different from existing works analyzing the generalization ability only for full-precision models, we also evaluate the generalization ability for binary neural networks (BNNs). BNNs [Gu et al.(2019)Gu, Li, Zhang, Han, Cao, Liu, and Doermann; Hubara et al.(2016)Hubara, Courbariaux, Soudry, El-Yaniv, and Bengio; Liu et al.(2020)Liu, Shen, Savvides, and Cheng] are among the most compressed deep models and are worth exploring their generalization performance, which quantize weights and features to single bits and have attracted intense interest for their excellent computation acceleration and model compression. The main contributions of this paper are as follows:

- We propose a confidence dimension (CD) to measure the relative generalization ability of deep models and provide a new approach for generalization prediction.
- We show that theoretically, our CD is the upper bound of generalization based Hoeffding’s inequality, which is more feasible than conventional measures when only a data-driven method is available to measure the generalization ability of DNNs.
- We extensively validate the performance of our CD by cross-tasks, including image classification and object detection, BNNs and full-precision DNNs, which are consistent with the theoretical results. The CD measure of a DNN is stable for various tasks and is potentially a useful tool to evaluate the generation for DNNs.

2 Related Work

The VC-dimension [Vapnik and Chervonenkis(1971)] provides a general measure of the complexity of traditional models and reflects their learning ability. It is closely related to the generalization performance of the underlying models. To guarantee good generalization performance, we expect a novelty dimension to give a tight bound on the expected error. Unfortunately, the VC-dimension has practical limitations when applied to the generalization of DNNs. Contrary to VC-dimension theory, decreasing the number of parameters in DNNs may achieve a better generalization performance. This is primarily because having more parameters than the number of training examples usually results in a loose generalization bound based on the VC-dimension [Zhang et al.(2017)Zhang, Bengio, Hardt, Recht, and Vinyals]. Furthermore, different optimizers or regularization techniques have an influence on the optimization process and lower the model complexity, affecting the generalization [Srivastava et al.(2014)Srivastava, Hinton, Krizhevsky, Sutskever, and Salakhutdinov]. Gintare et al. [Dziugaite et al.(2020)Dziugaite, Drouin, Neal, Rajkumar, Caballero, Wang, Mitliagkas, and Roy] argues that generalization measures should instead be evaluated within the framework of distributional robustness. Bengio et al. [Jiang et al.(2019)Jiang, Neyshabur, Mobahi, Krishnan, and Bengio] confirms that implicit regularization and optimizer influence the model in the same trend, i.e., the process of optimization is a comprehensive measurement of optimizer and regularization.
Existing works mainly focus on estimating a tighter generalization bound to understand the generalization ability of DNN models better. Sun et al. [Sun et al.(2016)]Sun, Chen, Wang, Liu, and Liu] expects the generalization error to be bounded by an empirical margin error plus the Rademacher Average term. Instead, works [Valle-Pérez and Louis(2020)] prefer the PAC-Bayes bounds. Valle et al. [Valle-Perez et al.(2018)]Valle-Perez, Camargo, and Louis] computes the PAC-Bayes generalization error bounds using the Gaussian process approximation of the prior over functions. Furthermore, Valle et al. [Valle-Perez and Louis(2020)] proposes a marginal-likelihood PAC-Bayes bound under the assumption of a power-law asymptotic behavior with training set size.

Yet strong assumptions are still key bottlenecks preventing those generalization bounds from practical use. To evaluate the generalization of DNNs more flexibly, we propose a new measurement method that comprehensively considers the model’s structure, optimization, and other training details like the training set size. In this case, we can compare the upper bound of the generalization ability between models in practical applications.

3 Measuring Relative Generalization Ability

Our goal is to present a flexible framework to compare the generalization ability of different networks. Based on randomization tests also used in VC-dimension, we calculate the confidence dimension (CD) to measure the models’ generalization performance. In the following, we will introduce VC-dimension, CD, and then analyze CD theoretically.

3.1 VC-dimension Estimation

Let \( \mathcal{H} \) be a hypothetical space, and \( \mathcal{X} = \{ x_1, x_2, ..., x_m \} \) be a sampling set with size \( m \). Each hypothesis \( h \) in \( \mathcal{H} \) marks a sample in \( \mathcal{X} \), and the result is expressed as:

\[
\text{h}\mid\mathcal{X} = \{ h(x_1), h(x_2), ..., h(x_m) \}.
\]

(1)

As the size of sample \( m \) increases, the number of corresponding examples in \( \mathcal{X} \) may also increase. For \( m \in N \), the growth function is defined as:

\[
\Pi_\mathcal{H}(m) = \max_{x_1, ..., x_m \in \mathcal{X}} |\{ h(x_1), h(x_2), ..., h(x_m) | h \in \mathcal{H} \} |.
\]

(2)

The growth function \( \Pi_\mathcal{H}(m) \) represents the maximum number of possible results that can be labeled with the hypothesis space \( \mathcal{H} \) for \( m \) examples. The greater the number of possible results that \( \mathcal{H} \) can label for these examples, the stronger the expression ability of \( \mathcal{H} \). In deep learning, the DNN is the hypothesis space \( \mathcal{H} \).

Now let us double the number of sample sets \( \mathcal{X} = \{ x_1, ..., x_m, x_{m+1}, ..., x_{2m} \} \) and generate subsets \( \mathcal{X}_1 = \{ x_1, ..., x_m \} \) and \( \mathcal{X}_2 = \{ x_{m+1}, ..., x_{2m} \} \). The risk of \( \mathcal{X} \) in the deep learning model is defined as [Vapnik and Chervonenkis(1971)]:

\[
v(\mathcal{X}) = \frac{1}{2m} \sum_{i=0}^{2m} |y_i - f(x_i)|,
\]

(3)

where \( y_i \) is the label of the dataset and \( f(x_i) \) is the classification results of the model. Therefore, the risks of the two subsets are:

\[
v(\mathcal{X}_1) = \frac{1}{m} \sum_{i=0}^{m} |y_i - f(x_i)|,
\]

\[
v(\mathcal{X}_2) = \frac{1}{m} \sum_{i=m+1}^{2m} |y_i - f(x_i)|.
\]

(4)

According to previous work [Vapnik and Chervonenkis(1971)], the difference between the two risks \( v(\mathcal{X}_1) \) and \( v(\mathcal{X}_2) \) is positively correlated with the sample set size \( m \).

\[
p = \sup \{ v(\mathcal{X}_1) - v(\mathcal{X}_2) \} \propto m,
\]

(5)

\( \sup \) denotes the upper limit, which can be obtained by maximizing:

\[
\frac{1}{m} \sum_{i=1}^{m} |y_i - f(x_i)| - \frac{1}{m} \sum_{i=m+1}^{2m} |y_i - f(x_i)|
\]

\[
= (1 - \frac{1}{m} \sum_{i=1}^{m} |\hat{y}_i - f(x_i)|) - \frac{1}{m} \sum_{i=m+1}^{2m} |y_i - f(x_i)|,
\]

(6)

which is equivalent to:

\[
p = \inf \{ (\frac{1}{m} \sum_{i=1}^{m} |\hat{y}_i - f(x_i)| + \frac{1}{m} \sum_{i=m+1}^{2m} |y_i - f(x_i)|) \},
\]

(7)

where \( \hat{y}_i \) is the incorrect label of the dataset, \( \inf \) denotes lower limit. When the sample set size \( m \) is the same, VC-dimension is proportional to \( p \). The result of \( p \) will be normalized. There exist \( \zeta, \varepsilon \in (0, \infty) \), such that the form of VC-dimension is shown below [Vapnik and Chervonenkis(1971)]:

\[
VC \propto \frac{\zeta}{\varepsilon\ln(\frac{m^2}{\alpha})}.
\]

(8)

We can know that \( p \) is a measure of VC-dimension, which can be used to rank VC-dimension of different models. The minimum value in Eq. 7 cannot be easily estimated because of the huge complexity of deep learning models. We can only approximate the global minimum with a local minimum, which is actually related to the used optimizer. Furthermore, finding the maximum number of shattered samples still remains an open problem. As a result, we can only obtain an inaccurate generalization measure for DNNs by the method of VC-dimension.

3.2 Confidence Dimension

We consider multiple elements to define a new measure CD, which can provide a much more stable measure than VC-dimension. To this end, we calculate the correction term \( \delta \) of generalization, which can be used to bound the generalization ability. \( \delta \) is estimated as:

\[
\delta = \alpha \sqrt{\frac{\ln(2 + E_{\text{err}})}{m}}
\]

(9)

where \( E_{\text{err}} \) denotes the training error for the sample set \( \mathcal{X} \), \( \alpha \) denotes the coefficient of correction terms, and \( m \) is the size of \( \mathcal{X} \). We then define the confidence dimension (CD) as:

\[
CD = \sup \{ v(\mathcal{X}_1) - v(\mathcal{X}_2) \} + \delta.
\]

(10)
in order to ensure $CD \in [0, 1]$, $CD$ will be normalized. The advantages of our measure lie in that: 1) Our $CD$ can be more consistent than VC-dimension. Based on our theoretical investigation, we show that our measure is the upper bound of generalization based on the VC-dimension, which might have a tighter constraint; 2) Our $CD$ can effectively measure the relative generalization ability of different DNNs based on a finite sample size.

**Theorem 1.** The Confidence Dimension is an upper bound of the generalization ability of any model with a probability of

$$1 - \frac{1}{(2 + Err)^4}$$

where $Err$ is training error.

**Proof.** We denote $\hat{p}_i$ as the normalized optimal value for the $i^{th}$ batch of independent and identically distributed samples, which is the infimum calculated by Eq. 7. We also define:

$$\hat{p} = \frac{\sum \hat{p}_i}{m},$$

where $m$ is the number of input samples. For easy proof our result, we reasonably assume $\hat{p}$ as the probability measure of obtaining the upper bound of generalization. Due to the independence of $CD_i$ (in the $i^{th}$ batch), we can get Eq. 11 based on the Hoeffding’s inequality [Dubhashi and Panconesi(2009)]:

$$E[e^{\lambda CD}] = E[e^{\hat{\lambda} m \sum_i (CD_i - \hat{p}_i)}] = \prod_i E[e^{\hat{\lambda} CD_i}] = \prod_i (\hat{p}_i e^{\hat{\lambda} m} + \hat{q}_i) \leq \left( \sum_i (\hat{p}_i e^{\hat{\lambda} m} + \hat{q}_i) \right)^m = (\hat{p} e^{\hat{\lambda} m} + \hat{q})^m,$$

where $\hat{q}_i = 1 - \hat{p}_i$ and $\hat{q} = 1 - \hat{p}$. Given Markov’s inequality, we have:

$$P(CD \geq \hat{p} + \delta) = P\left(\frac{1}{m} \sum_i (CD_i - \hat{p}_i) \geq \delta\right) = P\left[e^{\hat{\lambda} m \sum_i (CD_i - \hat{p}_i)} \geq e^{\lambda \delta}\right] \leq \frac{E[e^{\hat{\lambda} m \sum_i (CD_i - \hat{p}_i)}]}{e^{\lambda \delta}}.$$

We also have $1 + x \leq e^x \leq 1 + x + x^2$, when $0 \leq |x| \leq 1$. We thus have $E[e^{\hat{\lambda} m \sum_i (CD_i - \hat{p}_i)}]$ in Eq. 12, which can be further approximated as:

$$E[e^{\hat{\lambda} m \sum_i (CD_i - \hat{p}_i)}] = \prod_i E[e^{\hat{\lambda} CD_i}],$$

$$\leq \prod_i E[1 + \frac{\lambda}{m} (CD_i - \hat{p}_i) + (\frac{\lambda}{m})^2 (CD_i - \hat{p}_i)^2] \leq \prod_i (1 + (\frac{\lambda}{m})^2 \delta^2) \leq e^{(\frac{\lambda}{m})^2 \delta^2},$$

where $\delta$ denotes the variance of $CD_i$, $\nu$ denotes the variance of $CD$. Combining Eq. 12 and Eq. 13 gives $P(CD \geq \hat{p} + \nu \delta)$.

Since $\lambda$ is a non-negative constant and $CD \in [0, 1]$, according to the root formula, it can be obtained by transforming values such that $P(CD \geq \hat{p} + \delta) \leq e^{-2m\delta^2}$ when $m \geq 8$. From the symmetry of the distribution, we have $P(CD < \hat{p} - \delta) \leq e^{-2m\delta^2}$. Finally, we get the inequality:

$$P(CD - \hat{p} \leq \delta) \geq 1 - 2e^{-2m\delta^2},$$

where $\hat{p}$ is the normalized theoretical VC-dimension value. We guarantee that $\delta$ decrease as the size of the input samples increases. Therefore, we choose $\alpha\sqrt{\ln(2 + Err)}$ to be $\delta$ and have:

$$\hat{p} - \alpha\sqrt{\ln(2 + Err)} \leq CD \leq \hat{p} + \alpha\sqrt{\ln(2 + Err)},$$

which shows that the value of $CD$ as defined in Eq. 10 is the upper bound that the model can reach with at least a probability of $1 - \frac{1}{(2 + Err)^4}$ [Dubhashi and Panconesi(2009)].

When $m$ is infinite, the value of the correction term will gradually decrease, and $\hat{p}$ will finally approach the $CD$. Eq. 16 shows that we can achieve a 0.988 probability when the training error gets to 1.

$$1 - \frac{1}{(2 + Err)^4} = \begin{cases} 0.938 & Err = 0 \\ 0.988 & Err = 1. \end{cases}$$

By introducing a correction term into VC-dimension, we obtain a better estimation of the generalization ability. Experimentally, we also show that it can provide a relatively stable measure of generalization ability for different DNNs.

### 4 Experiments

We analyze the generalization ability for a variety of DNNs including BNNs based on $CD$. We also validate the performance of different object detectors.

#### 4.1 Implementation Details

**Cross-tasks:** Image classification and object detection are two fundamental computer vision tasks. We term traditional image classification as standard image classification, which involves predicting the category of one object in an image. As two different tasks (cross-tasks) are based on the same architecture, we use the performance degradation to validate the generalization ability of DNNs. Specifically, we calculate $CD$ of DNNs via image classification and validate it via object detection as shown in Fig. 2. We demonstrate the consistency of $CD$ across two different tasks which better than VC-dimension.

**Randomization tests:** Our methodology is based on a randomization test [Zhang et al. 2017]. A randomization test is a permutation test based on randomization from non-parametric statistics. We train several candidate architectures on a copy of the data where half of the correct labels are replaced with incorrect labels as described in Eq. 7. For brevity, we define two kinds of labels in our experiments: 1) **Correct labels:** the original labels without modification, 2) **Incorrect labels:** correct labels replaced with incorrect ones.
Figure 2: The obtainment and validation of CD. Based on the measure of VC-dimension ($p$), training set size, and training error, we calculate the $CD$ of DNNs via a image classification task (Randomization test). Since our proposed $CD$ is a general performance metric regardless of the task, we further validate $CD$ via the cross-tasks (standard image classification and object detection).

**Dataset:** We modify CIFAR-10 [Krizhevsky and Hinton(2009)] and ImageNet ILSVRC 2012 [Deng et al.(2009)] for a more straightforward calculation of VC-dimension based on [Vapnik and Chervonenkis(1971)], which is a fundamental step to calculate the $CD$ of DNNs, then we validate the $CD$ on the PASCAL VOC dataset [Everingham et al.(2015)] and MS COCO dataset [Lin et al.(2014)]. In order to test the effect of training set size on $CD$, we extract 2, 5, 7, 10 classes from the CIFAR-10 dataset and 2, 10, 50 classes from the ImageNet dataset. We re-label these datasets with random half incorrect and correct labels as required by the randomization test method.

**BNNs:** BNNs aim to compress the CNN models by quantizing the weights in the trained CNNs. We are the first to study the generalization ability of BNNs and estimate the performance bound of BNNs. We have selected two commonly used BNNs, PCNN [Gu et al.(2019)] and ReActNet [Liu et al.(2020)], in our experiments. We choose PCNN in our experiments because it employs a new BP algorithm in the training process, which can be used to evaluate the generalization ability of different BNNs considering the optimization. ReActNet proposes using RSign and RPReLU to enable explicit learning of the distribution reshape and shift at near-zero extra cost.

### 4.2 Obtainment: Image Classification

**Experimental Setting:** We calculate the $CD$ for several commonly used neural networks, including ResNet-18 [He et al.(2016)] and MobileNetV2 [Sandler et al.(2018)]. The $CD$ of DNNs (ReActNet and PCNN) is obtained in the image classification task with a SGD optimizer on the CIFAR-10 and ImageNet datasets. In the experiment, we set the learning rate to 0.01, $\alpha = 1$ in the Eq. 9, and train the network for 256 epochs. We also show the effect of training set size ($10^3$) and training error. We use different numbers of classes in the experiments to explore the effect of various elements on VC-dimension and $CD$. In the section, we will explore the relative generalizations of the different models. The goal is to keep the generalization ranking of different models consistent in different situations. $p$ is calculated based on Eq. 7. As shown in Eq. 8, $p$ is a measure of VC-dimension. We will use the rank of $p$ to represent the rank of VC-dimension.

To evaluate the relationship between the generalization ability of BNNs and their full-precision counterparts. We use ResNet-18 as the backbone. To ensure a fair and accurate comparison, we adopt the same experimental setting for different models.

| #Class | Models       | Training set size ($10^3$) | Training error | $p$/Rank   | $CD$/Rank |
|--------|--------------|-----------------------------|----------------|------------|-----------|
| 2      | ReActNet     | 10                          | 0.512          | 0.385/4    | 0.598/4   |
|        | PCNN         | 10                          | 0.371          | 0.362/3    | 0.582/3   |
|        | ResNet-18    | 10                          | 0.034          | 0.102/2    | 0.413/2   |
|        | MobileNetV2  | 10                          | 0.004          | 0.022/1    | 0.355/1   |
| 5      | ReActNet     | 25                          | 0.801          | 0.573/3    | 0.580/4   |
|        | PCNN         | 25                          | 0.734          | 0.583/4    | 0.564/3   |
|        | ResNet-18    | 25                          | 0.085          | 0.163/2    | 0.275/2   |
|        | MobileNetV2  | 25                          | 0.001          | 0.011/1    | 0.229/1   |
| 7      | ReActNet     | 35                          | 0.785          | 0.638/4    | 0.698/4   |
|        | PCNN         | 35                          | 0.876          | 0.634/3    | 0.696/3   |
|        | ResNet-18    | 35                          | 0.124          | 0.291/2    | 0.437/2   |
|        | MobileNetV2  | 35                          | 0.001          | 0.010/1    | 0.203/1   |
| 10     | ReActNet     | 50                          | 0.900          | 0.669/4    | 0.714/4   |
|        | PCNN         | 50                          | 0.792          | 0.669/3    | 0.713/3   |
|        | ResNet-18    | 50                          | 0.133          | 0.300/2    | 0.423/2   |
|        | MobileNetV2  | 50                          | 0.001          | 0.006/1    | 0.272/1   |
Table 2: The CD of different models on variants of ImageNet with batch size of 128.

| #Class | Models     | Training set size (10^3) | Training error | p/Rank | CD/Rank |
|--------|------------|--------------------------|----------------|--------|---------|
| 2      | ReActNet   | 2                        | 0.490          | 0.349/3 | 0.579/4 |
|        | PCNN       | 2                        | 0.322          | 0.354/4 | 0.576/3 |
|        | ResNet-18  | 2                        | 0.281          | 0.342/2 | 0.567/2 |
|        | MobileNetV2| 2                        | 0.378          | 0.278/1 | 0.538/1 |
| 10     | ReActNet   | 10                       | 0.117          | 0.645/3 | 0.688/4 |
|        | PCNN       | 10                       | 0.195          | 0.642/3 | 0.687/3 |
|        | ResNet-18  | 10                       | 0.082          | 0.601/2 | 0.654/2 |
|        | MobileNetV2| 10                       | 0.180          | 0.574/1 | 0.634/1 |
| 50     | ReActNet   | 50                       | 0.133          | 0.740/4 | 0.752/4 |
|        | PCNN       | 50                       | 0.087          | 0.701/3 | 0.716/3 |
|        | ResNet-18  | 50                       | 0.0347         | 0.674/2 | 0.691/2 |
|        | MobileNetV2| 50                       | 0.333          | 0.645/1 | 0.667/1 |

Table 3: ReActNet with different optimizers on variants of ImageNet with batch size of 128.

| #Class | Models | Training set size (10^3) | Training error | p/Rank | CD/Rank |
|--------|--------|--------------------------|----------------|--------|---------|
| 2      | SGD    | 2                        | 0.489          | 0.349/3 | 0.576/3 |
|        | Adam   | 2                        | 0.190          | 0.212/1 | 0.491/2 |
|        | AdamW  | 2                        | 0.013          | 0.228/2 | 0.489/1 |
| 10     | SGD    | 10                       | 0.117          | 0.645/3 | 0.688/3 |
|        | Adam   | 10                       | 0.008          | 0.328/2 | 0.441/2 |
|        | AdamW  | 10                       | 0.004          | 0.299/1 | 0.418/1 |
| 50     | SGD    | 50                       | 0.134          | 0.740/3 | 0.752/3 |
|        | Adam   | 50                       | 0.027          | 0.658/2 | 0.677/2 |
|        | AdamW  | 50                       | 0.019          | 0.576/1 | 0.603/1 |

The results of ReActNet and PCNN show that even PCNN achieves a much worse performance on the image classification task than ReActNet, it still can achieve close performance.

We can find that the volatility with different optimizers is limited to 44% in CD versus 71% in VC-dimension. Our CD reserves the same ranking for different parameter settings while VC-dimension is inconsistent, which strongly validates that CD can reliably and stably measure the generalization ability of DNNs. In addition, we further validate CD on object detection as a stable metric for the generalization ability measure in Section 4.3.

4.3 Validation: Object Detection

Experimental setting: Our experiments are based on Faster R-CNN [Ren et al.(2015)Ren, He, Girshick, and Sun] for object detection. We test the performance of Faster R-CNN with PCNN and ReActNet (backbone is ResNet-18 or MobileNet V2) in Tab. 4, to validate whether the generalization of them are consistent with the rank of the CD as shown in Tab. 1 and Tab. 2. We first initialize the backbone of Faster R-CNN from the pre-trained models, i.e., the models from the image classification task. Note that the convolutions in neck and head of Faster R-CNN are also binarized by corresponding method. Then, we use the SGD optimizer with a batch size of 8 and a learning rate of 0.0015, to finetune the networks for 36 epochs.

We evaluate the generalization ability of deep detectors based on the rate of performance change on image classification versus object detection. Since deep detectors are significantly affected by the initially pre-trained models based on image classification, we believe the model with lower rate of performance degradation on object detection than that on image classification achieves better generalization. For example, as shown in Table 4, the model, of larger performance degradation on object classification (13.2%) but becoming smaller (3.9%) on object detection, gains a better generalization. Through object detection, we reach a safe conclusion that the generalization ranking of the models is consistent with CD in Section 4.2.

Table 4: Test results of BNNs on object detection. The used detection datasets are Pascal VOC and MS COCO. The used classification dataset is ImageNet. ‘W’ and ‘A’ refer to the weight and activation bitwidth, respectively.

| Backbone | Binary method | W | A | Accuracy of classification detection | Better generalized model |
|----------|---------------|---|---|-------------------------------------|-------------------------|
| VOC2007+ImageNet |               |   |   |                                     |                         |
| ResNet-18 | ReAct        | 1 | 1 | 65.9                                | 69.3                    |
| ResNet-18 | PCNN         | 1 | 1 | 57.2                                | 72.0                    |
| Rate of performance change | 13.2% ↓ | 3.9% ↑ | PCNN |
| VOC2007+ImageNet |               |   |   |                                     |                         |
| ResNet-18 | ReActV2      | 1 | 1 | 65.9                                | 69.3                    |
| MobileNetV2 | ReAct      | 1 | 1 | 69.5                                | 74.1                    |
| Rate of performance change | 5.5% ↑ | 6.9% ↑ | MobileNet V2 |
| COCO2017+ImageNet |               |   |   |                                     |                         |
| ResNet-18 | ReAct        | 1 | 1 | 65.9                                | 27.5                    |
| ResNet-18 | PCNN         | 1 | 1 | 57.2                                | 26.9                    |
| Rate of performance change | 13.2% ↓ | 2.2% ↓ | PCNN |

The different datasets. The CD of ReActNet and PCNN are shown in Tab. 1. We can observe that the CD metric yields a much more stable ranking for these models on various experimental settings, which proves that the CD metric is more stable and reliable than VC-dimension even when the training set size (10^3) and training error vary. The CD of PCNN is lower than that of ReActNet, indicating that PCNN has better generalization ability. This explains why the performance of the binary network PCNN is better than ReActNet in object detection, even though the pre-trained model of ReActNet is much better performance than PCNN.

To further compare the generalization of the CD metric, we also calculate the CD on ImageNet, and the experimental settings are the same with CIFAR-10. The result is shown in Tab. 2, which reveals full-precision networks always have higher generalization than BNNs. Besides, it can be found that the span of CD is much narrower than that of VC-dimension when using different numbers of classes and training set sizes in Tab. 1 and Tab. 2, which might be a good property of CD metric for DNNs. In the conditions of different numbers of classes and different datasets, the ranking of CD is always consistent though the ranking of VC-dimension is disorder. Even the relationship between errors and CD is not monotonic, CD actually provides a complementary measure based on the training error and VC-dimension. As a result, CD is more stable than VC-dimension.

We validate the effect of the optimizer for CD in Tab. 3.
(on COCO) or better performance (on VOC) than ReActNet. These results indicate that PCNN trained on the image classification task has a better generalization than ReActNet. We also compare ResNet-18, MobileNetV2 with ReAct. We can see that MobileNetV2 achieves a 5.5% improvement on the image classification task and a 6.9% improvement on the object detection task, which indicates MobileNetV2 performed better on cross-tasks. Conclusively, the generalization of the MobileNetV2 backbone is slightly better than that of the ResNet-18 backbone, which is also in line with the CD ranking. All results demonstrate that the CD is a reliable metric to measure the model’s generalization ability.

5 Conclusion
In this work, we present a flexible framework to understand the capacity and generalization of DNNs. We introduce confidence dimension (CD) to measure deep models’ relative generalization ability together with a theoretical analysis, which proves to be the upper bound of the generalization. We validate our CD on cross tasks of object recognition and detection over BNNs and their full-precision counterparts. In future, we will try more applications to validate the effectiveness of our method.

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