Dark Matter and Structure Formation
with Late Decaying Particles

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Abstract

Since it became evident that the CDM model for cosmic structure formation predicts smaller power on large scales than observed, many alternatives have been suggested. Among them, the existence of late decaying particle can cure it by delaying the beginning of the matter domination and increasing the horizon length at that time. We discuss the realization of this scenario and present the light neutrino and the light axino as possible examples of working particle physics model. We point out that the increased power at sub-galaxy scale predicted by this scenario could lead to rich sub-galaxy structures.
I. INTRODUCTION

In the past decade, the cold dark matter (CDM) model has attracted a great attention as a possible theory of the cosmic structure formation. It is characterized by a spatially flat universe with $\sim 90\%$ of the mass density formed by cold dark matter and the scale invariant primordial fluctuation spectrum. It was supported by its successful features, the simplicity of its underlying assumptions and its link to the physics of the early universe. The basic idea that the large scale structure seen today evolved from very small primordial density fluctuations was strengthened by the recent detection of large scale anisotropies in the cosmic microwave background \[1\]. The candidates for cold dark matter were provided by particle physics models. The scale invariant fluctuation spectrum was naturally obtained from inflation which is an indispensable ingredient of current cosmology.

Recently, however, it became evident that this model is in conflict with observations \[2\]. The main difficulty was that the predicted power spectrum could not fit well with all the observational data simultaneously. The reason might be attributed to the relative smallness of the horizon length when the universe passes from relativistic matter domination to nonrelativistic matter domination which is the unique length scale characterizing the simple CDM model. Therefore, a more complicated model may be needed.

There are two main variables in the structure formation theory, the content of matter in the universe and the shape of the primordial density fluctuations. A variation of two ingredients leads to a different prediction on the power spectrum. Actually several variants yield better agreements with observations than the cold dark matter model \[3\]. The cold plus hot dark matter model attracted broad interest recently in this regard \[4\]. The warm dark matter model is another appealing possibility which draws new attention \[5\]. Altering the primordial fluctuation spectrum is another alternative \[6\]. In addition, there is a way to maintain the matter content as cold dark matter by increasing the horizon length at the time of radiation matter equality by appropriate amount. Introduction of cosmological constant is one way to achieve this scenario \[7\]. The existence of late decaying particles which shift
the beginning of matter domination era is another possibility [3]. At present, the large scale structure phenomenology does not favor any specific one among these variants.

From the point of view of particle physicists, we have a similar situation because most dark matter candidates have no strong support for their existence from the particle physics phenomenology and variant models require unconfirmed new physics. Therefore, the present criterion in particle physics is the necessity and plausibility of the models accommodating these variants. In view of this criterion, the problem in the hot plus cold dark matter model is the one of explaining the comparable amount of hot and cold dark matter without attributing it to a mere chance. At first glance, the massive neutrino and the lightest supersymmetric particle as the hot and cold dark matter candidates seem to have no relation between them. Recently, some suggestions were made, connecting the amount of cold dark matter to that of hot dark matter which is usually assumed to be neutrinos [3]. For the cold plus hot dark matter models, one usually introduces additional parameters which need explanation. In a similar manner, introduction of late decaying particles faces the problem of explaining the simultaneous existence of cold dark matter with critical density and a late decaying particle with appropriate lifetime. It would face the similar problem of plausibility as the hot plus cold dark matter model does. At present, however, it is welcome to have a working model for it. This is our philosophy in this paper. We pursue for the possibility of late decaying particles dominating the energy density of the universe for some time after nucleosynthesis and decaying to extremely relativistic, weakly interacting particles. They shift the time of radiation matter equality, which leads to the elongation of the horizon length at that time and the suppression of the power on large scales compared to the value predicted in the simple CDM model. This idea was first pointed out in Ref. [3]. Later, the 17 keV neutrino was used to implement the idea [3], but its existence is questioned now [10]. Recently, this idea has been rediscovered in the light axino decay [11], through the naive expectation that the late decaying particles producing relativistic particles can mimick the cold plus hot dark matter scenario.

In this paper, we emphasize the role of late decaying particles in cosmic structure for-
mation and obtain the conditions on the relic amount and lifetime applicable to the generic case. Then the explicit models for late decaying particles are presented. The plan of the paper is as follows. In section II, we give a brief review on the structure formation theories and compare the several alternatives to the simple CDM model. In section III, the role of late decaying particles in structure formation is explained and the condition for better agreement with the observational data is presented. In section IV, we present the scenarios for the light neutrino and the light axino as explicit models for the cosmology of late decaying particles. Section V is a discussion and a conclusion.

II. THE STRUCTURE FORMATION THEORIES AND DARK MATTER

Structure formation and dark matter are closely related. Any theory for the formation of cosmic structure must specify two chief ingredients: the amount and nature of the material that fills the universe, and the properties of the seed density fluctuations from which galaxies and clusters developed.

The amount of each component of materials in the universe is represented by the ratio of its mass density to the critical density: \( \Omega_i = \frac{\rho_i}{\rho_{\text{crit}}} \). There are two widely accepted beliefs concerning the amount of matter \(^{[12]}\). One is that the total amount of matter is nearly critical (\( \Omega \equiv \sum_i \Omega_i = 1 \)). The other is that the amount of baryons is less than 10% of the critical density (\( \Omega_B \lesssim 0.1 \)). The former follows from most inflationary scenarios. The strongest argument for the latter is the light element production in nucleosynthesis. The gap between \( \Omega \) and \( \Omega_B \) can be filled by the existence of nonbaryonic dark matter or/and the cosmological constant. Dark matters are classified into hot, warm and cold dark matters by the characteristic scales they develop during the evolution of seed density fluctuations.

At present, two types of origins for the seed density fluctuations are widely discussed: inflation and topological defects. In the inflationary scenario the quantum fluctuations of the inflaton field are changed into the density fluctuations, which is featured by the scale invariant fluctuation spectrum \(^{[13]}\). The topological defects are created during the cosmic
phase transitions in some models with a spontaneously broken global symmetry. Cosmic strings and textures have been discussed in this context.

The seed density fluctuations evolve under the act of gravity. For small fluctuations, the evolution is studied by the numerical integration of coupled perturbed-Einstein-Boltzmann equations. With the seed density fluctuation spectrum as initial condition, we obtain the evolved linear fluctuation spectrum. For large fluctuations, numerical simulation is used to show the nonlinear dynamics of clustering. Much work has been done with various dark matter contents and seed density fluctuation spectra. In the literature, one can find the numerical fit formulae for the evolved linear fluctuation spectra and the results of numerical simulations for many cases [14]. We can understand the qualitative features of these spectra by the characteristic scales inherent to the seed density fluctuations or developed by the matter content during the evolution. Important ones are the following. The Jeans length $\lambda_J$ separates the gravitationally stable and unstable modes. The free streaming length $\lambda_{FS}$ is a scale below which fluctuations in a nearly collisionless component are damped due to free streaming. Another significant length scale is the horizon length at the time of the radiation matter equality $\lambda_{EQ}$. This scale appears because the growth of the density fluctuation of nonrelativistic matter within the horizon is suppressed during the radiation dominated era while begins as the matter domination era starts.

Among the various models, the CDM model was most successful in the past decade. The CDM model assumes a flat ($\Omega = 1$) universe in which $5 \sim 10\%$ of the critical density is provided by ordinary matter and the rest by the weakly interacting, yet unknown, cold dark matter. In addition, the seed fluctuations are assumed to be of the scale invariant form. The key assumptions of this model fit nicely with ideas on inflation. The evolved fluctuation spectrum is given by [14]

$$|\delta_k|^2 = \frac{A_k}{(1 + \alpha k + \beta k^{3/2} + \gamma k^2)^2}$$  \hspace{1cm} (1)

where $A$ is a normalization constant and $\alpha = 1.7 \, l$, $\beta = 9.0 \, l^{3/2}$, $\gamma = 1.0 \, l^2$, and $l = (\Omega h^2)^{-1} \theta^{1/2}\text{Mpc}$. The scaling of $l$ with $(\Omega h^2)^{-1} \theta^{1/2}$ simply reflects the horizon length at the
time of radiation matter equality which is given by

\[ \lambda_{EQ} \simeq 30(\Omega h^2)^{-1}\theta^{1/2} \text{ Mpc}, \]  

where \( \theta = \rho_{\text{rel}}/1.68\rho_\gamma \) measures the present energy density of extremely relativistic particles and is 1 for photon and three massless neutrino flavors. It is independent of the nature of CDM and \( \lambda_{EQ} \) is a single physical length scale characterizing the above spectrum. The power spectrum of the model is compared to that extracted from the observational data \[15\] in Fig. 1. It shows that with the standard choice \( \Omega h = 0.5 \) the model does not fit the data. With the normalization fixed at COBE data, we can say that the standard CDM model predicts more power at small scales than observed. From the figure, we intuitively reason that the problem in the CDM model is that it has a relatively small \( \lambda_{EQ} \).

Several cures for the excessive power of the simple CDM model at small scales have been considered. We put them into four categories: (i) assuming large biasing or antibiasing, (ii) the initial fluctuation spectrum with less power at small scales, (iii) the mixed dark matter contents, and (iv) increasing \( \lambda_{EQ} \). Biasing is the modulation of the galaxy distribution by processes associated with galaxy formation. Though a complicated version in this category met some successes, we don’t know its detailed physical mechanism. The initial fluctuation spectrum with less power at small scales can be obtained in fine tuned inflationary models \[6\] or in cosmic string and texture models. But they require peculiar models with fine tuning of parameters. The mixed dark matter content will change the evolution of the initial power spectrum. Cold plus hot dark matter model was shown to be successful in this regard. With the massive neutrino as hot dark matter, the best fit was obtained for \( \Omega_{\text{CDM}} = 0.6 \), \( \Omega_\nu = 0.3 \) and \( \Omega_B = 0.1 \) \[16\]. While this model met many cosmological successes, it also has a plausibility problem: how to explain the comparable amount of hot and cold dark matter without attributing it to a mere chance. It might lead to fine tuning again. Warm dark matter model may give a promising result. In increasing \( \lambda_{EQ} \), the simplest idea is to lower \( \Omega \) or \( h \) to make \( \Omega h \simeq 0.2 - 0.3 \). (See Fig. 1. We allow small biasing.) But this conflicts with other observational data \[12\]. Introduction of small cosmological constant
allows $\Omega_{CDM} = 0.2$ in the flat universe, resulting in the increase of $\lambda_{EQ}$. But this requires severe fine tuning of the cosmological constant, which is not solved yet in particle physics. Late decaying particles can lead to the retardation of matter domination with the desired increase in $\lambda_{EQ}$. We will discuss this scenario in detail in the following section.

III. THE ROLE OF LATE DECAYING RELIC PARTICLES IN STRUCTURE FORMATION

In the CDM model, the power on small scales can be reduced by delaying the beginning of matter domination era. Since the density fluctuations of cold dark matter grow significantly during the matter domination era, the growing time for small scales is reduced as much as one delays the time for the small scales to enter the horizon before the radiation matter equality. The $\Omega h = 0.2$ CDM model is very successful in this way. But the problem is that one wants to get the same success while maintaining $\Omega = 1$. One way to achieve this is by postulating a late decaying relic particle which dominates the energy density of the universe for some time and decays into extremely relativistic particles. By producing relativistic particles, the universe enters into a radiation dominated era around the time of its decay.

Consider a late decaying relic particle $X$ with the mass $m$, the lifetime $\tau \gg 1$ sec (the time of neutrino decoupling) and the ratio of the number density to entropy density $Y$ which is kept constant when the processes creating or destroying $X$ are frozen. The value of $Y$ varies from $\sim 10^{-2}$ to $10^{-9}$. The number density of $X$ at temperature $T$ is given by $n_X(T) = Ys(T)$, where $s(T) = \frac{2\pi^2}{45}g_{s}(T)T^3$ is the entropy density. For $T \ll m$, $X$ is nonrelativistic and the energy density is given by $\rho_X(T) = mn_X(T)$.

Let us consider the relevant range of the values of $mY$ and $\tau$. There are two obvious bounds on $mY$ and $\tau$. The first concerns nucleosynthesis. At the time of neutrino decoupling, one customarily requires that the energy density of $X$ should be less than that of one neutrino species. This imposes an upper bound on $mY$ which applies to the case that $m > 1$ MeV,

$$\left(\frac{mY}{\text{MeV}}\right) < 0.107. \quad (3)$$
The second comes from the condition that the energy density of the decay products of $X$ should not overclose the universe:

$$\left(\frac{\tau}{\text{sec}}\right)\left(\frac{m_Y}{\text{MeV}}\right)^2 < 2 \times 10^6 h^3. \quad (4)$$

If $X$ can decay into photons or ordinary particles, a severe restriction can be imposed by the present observation of microwave background radiation [17]. Therefore, we consider cases in which the branching ratio of $X$ into photons is negligible and decay products of $X$ are very weakly interacting so as not to disturb the spectra of observable particles.

In this scenario, there exist two eras of matter domination. The first era begins when the energy density of the late decaying particles dominates the energy density of the universe. The first matter dominated era ends when the late decaying particle decays into extremely relativistic particles. The second era begins when the energy density of cold dark matter dominates over the energy density of the decay products. The evolution of the energy densities is shown in Fig. 2. Four interesting scale factors appear in this scenario: $R_{EQ1}$ (the scale factor at which the energy density of $X$'s and the energy density of photons and neutrinos become equal), $R_D$ (the scale factor at which $X$ decays), $R_{EQ}$ (the scale factor at which the energy density of cold dark matter and the energy density of photons and neutrinos become equal), and $R_{EQ2}$ (the scale factor at which energy densities of cold dark matter and extremely relativistic decay products become equal). $R_{EQ}$ is the scale factor at the radiation–matter equality epoch in the simple CDM model. In our scenario, we make $R_{EQ2} > R_{EQ}$.

The temperature and the time at which the first matter domination begins are given by

$$T_{EQ1} = 1.55 \ m_Y,$$

$$t_{EQ1} = 0.55 \left(\frac{\text{MeV}}{m_Y}\right)^2 \text{sec}, \quad (5)$$

if the lifetime $\tau$ is larger than $t_{EQ1}$. Actually, the conditions for which $X$-domination really occurs are $\tau > t_{EQ1}$ and $t_{EQ1} < t_{EQ}$, which are converted to

$$\left(\frac{\tau}{\text{sec}}\right)\left(\frac{m_Y}{\text{MeV}}\right)^2 > 0.55, \quad (7)$$
\[
\left( \frac{m_Y}{\text{MeV}} \right) > 3.6 \times 10^{-6} h^2.
\] (8)

To find out the scale factor \( R_{EQ2} \), we use the rough approximation of simultaneous decay and sudden change of radiation and matter domination. The condition for \( R_{EQ} \) and \( R_{EQ2} \) read

\[
\rho_0 \left( \frac{R_0}{R_{EQ}} \right)^3 = \rho_{EQ1} \left( \frac{R_{EQ1}}{R_{EQ}} \right)^4 \]

(9)

\[
\rho_0 \left( \frac{R_0}{R_{EQ2}} \right)^3 = \rho_{EQ1} \left[ \left( \frac{R_{EQ1}}{R_D} \right)^3 \left( \frac{R_D}{R_{EQ2}} \right)^4 + \left( \frac{R_{EQ1}}{R_{EQ2}} \right)^4 \right]
\]

(10)

where \( \rho_{EQ1} \) is the energy densities of \( X \) at the first radiation matter equality and \( \rho_0 \) is the present matter energy density. From these, it follows that the second matter domination in our scenario occurs later than the well-known matter domination of the simple CDM model by a factor

\[
\frac{R_{EQ2}}{R_{EQ}} \simeq \frac{R_D}{R_{EQ1}} + 1 \simeq \left( \frac{\tau}{t_{EQ1}} \right)^{2/3} + 1,
\]

(11)

where the last equality comes from the relation \( R \propto t^{2/3} \) during the matter dominated era.

The evolution of the fluctuation spectrum is obtained by the linear perturbation theory as done in Ref. [9] for the 17 keV neutrino case. But here, we present a rough behavior and a simple estimate. The fluctuation spectrum is characterized by two length scales, associated with the horizon lengths at two instances of radiation matter equality,

\[
\lambda_{EQ1} \simeq 8 \times 10^{-2} \left( \frac{\text{MeV}}{m_Y} \right) \text{kpc},
\]

(12)

\[
\lambda_{EQ2} \simeq 30(\Omega h^2)^{-1} \left[ \left\{ \frac{1}{0.55} \left( \frac{\tau}{\text{sec}} \right) \left( \frac{m_Y}{\text{MeV}} \right)^2 \right\}^{2/3} + 1 \right]^{1/2} \text{Mpc}.
\]

(13)

The existence of the scale \( \lambda_{EQ1} \) distinguishes this scenario from the standard CDM model. Objects on this and smaller scales would collapse at high red shifts and more structures exist on these scales than the standard CDM model. We will discuss about it later.

\( \lambda_{EQ2} \) is larger than \( \lambda_{EQ} \) when the X-domination condition, eq. (8) holds. In the previous section, we mentioned that the \( \Omega h = 0.2 \) CDM model have a good fit with possible small
biasing taken into account. Lowering $\Omega h$ has an effect of making $\lambda_{\text{EQ}}$ large. But this model is in conflict with the inflationary paradigm $\Omega = 1$ and the observed Hubble constant $h \simeq 0.4 - 1$. In the late decaying particle scenario, we have the same effect of making $\lambda_{\text{EQ}}$ large but still can keep $\Omega = 1$. The value $\Omega h = 0.2$ gives a rough estimation of the desired value for $\lambda_{\text{EQ}}$: $\lambda_{\text{EQ}}/h^{-1}$ Mpc $\simeq 150$. It can be rewritten as a condition on $\tau$ and $m_Y$ for $\Omega = 1$,

$$\left( \frac{\tau}{\text{sec}} \right) \left( \frac{m_Y}{\text{MeV}} \right)^2 \simeq 0.55 \left[ (h/0.2)^2 - 1 \right]^{3/2}.$$  \hspace{1cm} (14)

This together with the eqs. (3) and (8) forms a main condition of the late decaying particle scenario with $\Omega = 1$.

**IV. MODELS OF A LATE DECAYING PARTICLE**

The long lifetime and the small radiative branching ratio of a late decaying particle make it difficult to construct particle physics models for it. But it is certainly not impossible. It is possible to use symmetry and/or coupling suppressed by a large mass scale to obtain the long lifetime and to suppress the radiative decay. In this section, we present two models: the massive neutrino and the light axino.

**A. A Late Decaying Neutrino**

Cosmological implications of the massive neutrino vary depending on the size of mass and lifetime. At present, the stable $\sim 30$ eV mass neutrino is a good candidate for hot dark matter. Here we consider a different region in mass and lifetime. The relic abundance of a light neutrino species is $Y = 3.9 \times 10^{-2}$. The late decaying particle scenario works for the neutrino mass in the range $100$ eV $\lesssim m_\nu \lesssim 1$ MeV. Hereafter, we use $m_\nu = 17$ keV as the representative value and adopt the notation $m_{17} = m_\nu/17$ keV. Then $m_Y = 0.66m_{17}$ keV and the eq. (14) demands the lifetime to be

$$\tau \simeq 0.5m_{17}^2 \text{ yr},$$  \hspace{1cm} (15)
for the standard value $h = 0.5$. (Actually, the use of the formulae of the previous section needs a slight modification due to the change in the number of relativistic neutrino species. But we still use them because the estimations are not affected very much by this modification.) The length scale of the first radiation matter equality is

$$\lambda_{EQ1} \simeq 120 m_{\nu}^{-1/2} \text{ kpc.} \qquad (16)$$

The well-known way to obtain the neutrino mass in the above mentioned range is to use the seesaw mechanism. The long lifetime required in the eq. (15) can be obtained in several models [18]. They considered schemes in which the massive neutrino decays into exotic particles — majoron, familon or techniphoton. In either case, we have to make a judicious choice of unknown parameters to give such a long lifetime.

**B. A Light Axino and A Lighter Gravitino**

The importance of the cosmological effects of axinos and gravitinos arises from the weaknesses of their interactions due to the suppression factors of the axion decay constant for the axino and the Planck mass for the gravitino. Here we focus on the scenario in which the gravitino is the lightest supersymmetric particle (LSP) and the axino is the second lightest supersymmetric particle (SLSP).

For this, we need the simultaneous implementation of local supersymmetry and the axion solution of the strong $CP$ problem. Supersymmetry has been introduced to solve the gauge hierarchy problem. Realistic models incorporating low energy supersymmetry consider local supersymmetry (supergravity), spontaneously broken by the hidden sector [19]. Most of them have an $R$ symmetry to suppress the unwanted large $B$ and $L$ violations. The $R$ symmetry dictates the existence of a stable particle commonly called the LSP which is a good candidate for dark matter. The LSP is model dependent and a popular candidate is neutralino, a linear combination of neutral gauginos and neutral higgsinos. If we adopt local supersymmetry, the gravitino is another natural candidate for the LSP. Peccei-Quinn
symmetry is needed for a solution of the strong $CP$ problem, and no idea seems to be compelling in this regard [20]. It predicts the existence of the axion [21] and the coherent classical axion oscillation created at $T \simeq \Lambda_{\text{QCD}}$ during the expansion of the universe is a good candidate for dark matter [22]. The simultaneous implementation of these two ideas can affect the status of cold dark matter candidates [23]. The axino (the superpartner of axion) can and generally do destabilize the lightest ordinary supersymmetric particle (LOSP) which is a popular dark matter candidate. Then the viable candidates for dark matter are gravitinos, axinos and axions. For the gravitino and the axino to be LSP and SLSP, we need gravitino and axino lighter than the LOSP.

Currently favored gravitino mass is identified with the electroweak scale of $10^{2} - 10^{3}$ GeV and the gravitino with such mass cannot be the LSP because there is usually a lighter supersymmetric particle to which the gravitino can decay. Recently, however, the old idea of supercolor at multi TeV region has been reinvestigated without phenomenological difficulties [24]. In this case the gravitino mass falls in the $0.4 \, \text{meV} - 0.4 \, \text{keV}$ region for $\Lambda_{\text{supercolor}} = 1 - 10^{3} \, \text{TeV}$. Therefore, the phenomenologically favored gravitino mass is still open in these two regions and it is worthwhile to consider the cosmological consequence of this light gravitino scenario.

In the case that $R$ symmetry is exact and the axino is lighter than the LOSP, the LSP lighter than the axino must exist for the axino to decay. The presently known candidate is the gravitino. The possibility and cosmological constraints of the gravitino as the LSP were previously considered [25]. Here, we summarize the results. Gravitinos typically decouple just after the Planck time. From this one obtains a gravitino mass bound $m_{3/2} \leq 2 \, \text{keV}$ in the standard big bang cosmology. But the inflation after the gravitino decoupling might have washed out relic gravitinos almost completely and we have no such bound. Gravitinos can be regenerated through the reheating process subsequent to inflation. The requirement that the regenerated gravitinos should not overclose the universe gives a constraint between the reheating temperature and the gravitino mass [26]. If we assume that the reheating temperature is larger than the axino decoupling temperature $10^{11}$ GeV, a region of gravitino
mass from a few keV to a few × 10 GeV is excluded. There is also a bound in the extremely light gravitino. For the very light gravitino whose mass $m_{3/2}$ is small compared to the typical interaction energy, the helicity $\pm \frac{1}{2}$ component of the gravitino dominate the interaction and the strength of the interaction becomes larger as $m_{3/2}$ becomes smaller. Therefore, the extremely light gravitino ($m_{3/2} \ll 1$ keV) interacts strongly and decouples well below the weak scale. The standard nucleosynthesis scenario constrains the number of the species of the neutrino-like particles to be smaller than 3.3 at the time of nucleosynthesis. Then, gravitinos must decouple before $T \simeq 200$ MeV to get a sufficient dilution factor. This yields a gravitino mass bound

$$m_{3/2} \gtrsim 10^{-4} \left( \frac{m_{\tilde{t}}}{100 \text{GeV}} \right) \text{ eV}$$

where $m_{\tilde{t}}$ is the slepton mass.

The axino mass is predicted to be in a region between 10 GeV and keV in supergravity models with supersymmetry broken in the hidden sector. Usually one adopts the minimal kinetic energy term. In general, there exist other loop contributions and the kinetic energy term can be of nonminimal form. Furthermore, we can adopt the supersymmetry breaking by supercolor type interactions. Therefore, the axino mass heavier than keV is a theoretical possibility.

Now, we calculate the axino lifetime. The dominant coupling of axion to gravitino is

$$\mathcal{L}_{a\tilde{a}G} = \frac{1}{M} \bar{\psi}_\mu \gamma^\nu \partial_\nu z^* \gamma^\mu \tilde{a}_L + \text{h.c.}$$

where $z = (s + ia)/\sqrt{2}$ in terms of saxino $s$ and axion $a$. There exists another interaction term proportional to $m_{3/2}$ which is negligible compared to the above interaction. In the case that the axino is heavier than the gravitino, the axino decays into the gravitino and the axion. In the limit of vanishing axion mass, the axino lifetime is

$$\tau_a = \frac{96\pi M^2 m_{3/2}^2}{m_{\tilde{a}}^5} = 1.2 \times 10^{12} \left( \frac{\text{MeV}}{m_{\tilde{a}}} \right)^5 \left( \frac{m_{3/2}}{\text{eV}} \right)^2 \text{ sec.}$$

Axinos are kept in thermal equilibrium at high temperature through the process $q\bar{q} \leftrightarrow \tilde{a}g$ with an intermediate gluon. From the interaction lagrangian
\[ \mathcal{L} = \frac{\alpha_c}{8\pi F_a} \left( aG^a_{\mu\nu} \bar{G}^{a\mu\nu} + sG^a_{\mu\nu} \bar{G}^{a\mu\nu} + \bar{a}\gamma_5\sigma_{\mu\nu} \tilde{g}^a G^{a\mu\nu} + \cdots \right), \]  

(20)

the reaction rate can be calculated to give \( \Gamma \sim \alpha_c^3 T^3 / 16\pi F_a^2 \) for \( T < F_a \). On the other hand, the expansion rate is \( H = 1.66g_*^{1/2}T^2/M_{pl} \) where \( g_* = 915/4 \) for the particle content in the minimal supersymmetric standard model. Therefore, the decoupling temperature is

\[ T_{\tilde{a}} = 10^{11} \left( \frac{F_a}{10^{12}\text{GeV}} \right)^2 \left( \frac{0.1}{\alpha_c} \right)^3 \text{GeV}. \]  

(21)

The amount of relic axinos depend on the decoupling temperature. (There are also axinos from decays of relic LOSPs. But the number density of relic LOSPs is usually much smaller than that of hot thermal relic and hence axinos from decays of relic LOSPs can be neglected.) The axino decoupling temperature \( T_{\tilde{a}} \) is so high that it can be comparable to or larger than the reheating temperature \( T_R \). For simplicity, we consider two extreme cases which differ from each other significantly.

(i) \( T_R \gg T_{\tilde{a}} \): The relic axinos are hot thermal relic so that \( Y \simeq 1.8 \times 10^{-3} \). The desired value of \( \lambda_{EQ2} \) is obtained for

\[ \left( \frac{\text{MeV}}{m_{\tilde{a}}} \right)^3 \left( \frac{m_{3/2}}{\text{eV}} \right)^2 \simeq 1.4 \times 10^{-7} (25h^2 - 1)^{3/2}, \]  

(22)

within the mass range

\[ 2 \text{ keV} \lesssim m_{\tilde{a}} \lesssim 60 \text{ MeV}, \]  

(23)

\[ \left( \frac{\text{MeV}}{m_{\tilde{a}}} \right)^3 \left( \frac{m_{3/2}}{\text{eV}} \right)^2 < 0.5h^3, \]  

(24)

which come from the eqs. (3), (4) and (8). This requires the MeV mass axino and the very light, sub-eV mass gravitino. The cosmologically allowed mass ranges and the region corresponding to the eq. (22) are shown in Fig. 3. The smaller characteristic length scale is given by

\[ \lambda_{EQ1} = 44 \left( \frac{\text{MeV}}{m_{\tilde{a}}} \right) \text{kpc}. \]  

(25)

With the MeV mass axino, it is about the globular cluster scale.
(ii) \( T_R \ll T_{\tilde{a}} \): The hot thermal relic axinos are diluted away by inflation and relic axinos are those regenerated through the nonequilibrium processes, which yields

\[
Y \simeq 7 \times 10^{-6} \tilde{F}_a^{-2} \tilde{T}_R
\]

where \( \tilde{F}_a = F_a/10^{12}\text{GeV} \) and \( \tilde{T}_R = T_R/10^6\text{GeV} \). Now the corresponding equations of the eqs. (22)–(25) contain \( F_a, T_R \) as well as \( m_{\tilde{a}}, m_{3/2} \) and are given by

\[
\tilde{F}_a^{-4} \tilde{T}_R^2 \left( \frac{\text{GeV}}{m_{\tilde{a}}} \right)^3 \left( \frac{m_{3/2}}{\text{keV}} \right)^2 \simeq 8.8 \left( 25h^2 - 1 \right)^{3/2},
\]

(26)

\[
5 \times 10^{-4} \lesssim \tilde{F}_a^{-2} \tilde{T}_R \left( \frac{m_{\tilde{a}}}{\text{GeV}} \right) \lesssim 15,
\]

(27)

\[
\tilde{F}_a^{-4} \tilde{T}_R^2 \left( \frac{\text{GeV}}{m_{\tilde{a}}} \right)^3 \left( \frac{m_{3/2}}{\text{keV}} \right)^2 < 3 \times 10^7 h^3,
\]

(28)

and

\[
\lambda_{EQ1} = 11 \tilde{F}_a^2 \tilde{T}_R^{-1} \left( \frac{\text{GeV}}{m_{\tilde{a}}} \right)^2 \text{kpc}.
\]

(29)

This case opens a possibility of the heavier axino with GeV mass depending on the size of the reheating temperature.

V. DISCUSSION AND CONCLUSION

In our scenario of late decaying particles, there exists a brief era of matter domination between \( \lambda_{EQ1} \) and \( \lambda_D \). During this epoch, there exists a possibility that some scales can grow and form structures. Because there exists a characteristic scale of globular clusters, it is worthwhile to see if the structure formation is triggered during this brief period and to check whether it coincides with the scale of globular clusters. If the conclusion is affirmative, then the scenario of the late decaying particle provides an explanation for the size of globular clusters. Furthermore, such a large power at scale smaller than the galaxy scale is advantageous on explaining the Lyman-line-absorbing clouds seen in quasar spectra [27]. The problem is that it leads to large anisotropies in microwave background radiation at small angles. But it is known that this early structure formation, e.g. the early population of quasi-stellar objects, could reionize the universe shortly after the normal epoch.
of photon–baryon decoupling at $z \sim 1000$, thereby reducing small angle anisotropies in the microwave background radiation.

There are a few merits of the axino-gravitino scenario. First, it has the classical axion oscillation as a natural candidate for cold dark matter. Second, the interactions of the axino and the gravitino are well known so that the unknown parameters are mainly their masses and no unknown coupling is involved. Third, the explanation of the globular cluster scale or other sub-galaxy structures may fix their masses.

In summary, we considered the late decaying particle scenario in the $\Omega = 1$ CDM model as a viable alternative to the simple CDM model which failed in fitting all the observational data simultaneously. The late decaying particles delay the beginning of the matter domination, thereby reducing the excessive power at small scales which was the problem of the simple CDM model. This scenario has another peak in the power spectrum developed during the earlier matter domination. Its length scale is smaller than the galaxy scale, thereby having implications for sub-galaxy structures, which need further investigations. The particle physics model realizing this scenario is difficult to construct due to the required long lifetime and small radiative branching ratio of a late decaying particle, but certainly not impossible. We presented two examples. The keV mass neutrino with lifetime of order 1 yr, which has been considered by many authors, is a possible candidate of the late decaying particle. The axino-gravitino scenario requires the MeV mass axino and the sub-eV mass gravitino (the sufficiently low reheating temperature can raise the required masses). It possesses all the necessary ingredients of the late decaying particle scenario with the simultaneous implementation of local supersymmetry and Peccei-Quinn symmetry.

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FIGURES

FIG. 1. Comparison of the CDM model power spectrum with the observational data. The solid line corresponds to the $\Omega_h = 0.3$ CDM model, while the dashed and dotted lines correspond to the $\Omega_h = 0.5$ and 1 CDM models respectively.

FIG. 2. The evolution of energy densities of radiation ($\gamma, \nu$), late decaying particles ($X$), their decay products and the cold dark matter.

FIG. 3. Cosmologically allowed region in the axino mass – gravitino mass plane. Dotted regions are excluded by the closure bound and the nucleosynthesis bound (the equation numbers represent the relevant equations in the text). The heavy dotted line corresponds to the eq. (22) for $h = 0.5$. The white island below the dashed line is the mass range allowed by the eqs. (23) and (24).
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