Probing the muon $g_\mu - 2$ anomaly, $L_\mu - L_\tau$ gauge boson and Dark Matter in dark photon experiments

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Abstract

In the $L_\mu - L_\tau$ model the 3.6 $\sigma$ discrepancy between the predicted and measured values of the anomalous magnetic moment of positive muons can be explained by the existence of a new dark boson $Z'$ with a mass in the sub-GeV range, which is coupled at tree level predominantly to the second and third lepton generations. However, at the one-loop level the $Z'$ coupling to electrons or quarks can be induced via the $\gamma - Z'$ kinetic mixing, which is generated through the loop involving the muon and tau lepton. This loophole has important experimental consequences since it opens up new possibilities, in particular for the complementary searches of the $Z'$ in the ongoing NA64 and incoming dark photon experiments with high-energy electrons. An extension of the model able to explain relic Dark Matter density is also discussed.
At present there are several signals that new physics beyond the standard model (SM) exists. The most striking is the observation of Dark Matter (DM). Among the many models of DM, for a review, see e.g. \[1 - 4\], those that motivate the existence of light vector(scalar) bosons with a mass $m_d \leq O(1) \text{ GeV}$ are rather popular now \[5, 6\]. The main idea is that in addition to gravity a new interaction between visible and dark sector exists which is mediated by this gauge boson \[6\].

Another possible hint in favour of new physics is the muon $g - 2$ anomaly, which is the 3.6 $\sigma$ discrepancy between the experimental values \[7, 8\] and the SM predictions \[9, 10, 11, 12\] for the anomalous magnetic moment of the muon.

Among several extensions of the SM explaining the anomaly, the models predicting the existence of a weak leptonic force mediated by a sub-GeV gauge boson $Z'$ that couples predominantly to the difference between the muon and tau lepton numbers, $L_\mu - L_\tau$, are of general interest. The abelian symmetry $L_\mu - L_\tau$ is an anomaly-free global symmetry within the SM \[13, 14, 15\]. Breaking $L_\mu - L_\tau$ is crucial for the appearance of a new relatively light, with a mass $m_{Z'} \leq 1 \text{ GeV}$, vector boson ($Z'$) which couples very weakly to muon and tau-lepton with the coupling constant $\alpha_\mu \sim O(10^{-8})$ \[16 - 29\]. The $Z'$ interaction with $L_\mu - L_\tau$ vector current given by

$$L_{Z'} = e_\mu Z'_\nu [\bar{\mu} \gamma^\nu \mu - \bar{\tau} \gamma^\nu \tau + \bar{\nu}_\mu \gamma^\nu \nu_\mu - \bar{\nu}_\tau \gamma^\nu \nu_\tau]$$

leads to additional contribution to the muon anomalous magnetic moment \[30\]

$$\delta a = \frac{\alpha_\mu}{2\pi} F\left(\frac{m_{Z'}}{m_\mu}\right),$$

where

$$F(x) = \int_0^1 dz \frac{[2z(1-z)^2]}{[(1-z)^2 + x^2 z]}$$

and $\alpha_\mu = \frac{e_\mu^2}{4\pi}$. The use of the formulae (2,3) allows to determine the coupling constant $\alpha_\mu$ which explains the value of the $g - 2$ anomaly \[16 - 29\] and it does not contradict to existing experimental bounds for $m_{Z'} \leq 2m_\mu$ \[29\]. Namely, for $m_{Z'} \ll m_\mu$ \[30\]

$$\alpha_\mu = (1.8 \pm 0.5) \times 10^{-8}.$$
For another limiting case $m_{Z'} \gg m_\mu$ the $\alpha_\mu$ is

$$\alpha_\mu = (2.7 \pm 0.7) \times 10^{-8} \times \frac{m_{Z'}^2}{m_\mu^2}. \quad (5)$$

In addition to the case of the $g_\mu - 2$ anomaly, there are also other implications of $Z'$ \cite{16-29}. For example, in neutrino sector, the $L_\mu - L_\tau$ model can provide a natural explanation of a zeroth-order approximation for neutrino mixing with a quasi-degenerate mass spectrum predicting a maximal atmospheric and vanishing reactor neutrino mixing angle \cite{31, 32, 33}, small masses of neutrinos and its oscillations by extending the model with the left-right gauge symmetry \cite{34}, the $R_K$ puzzle in LHCb data and the $g_\mu - 2$ anomaly can be simultaneously explained with the $\simeq 10$ MeV $Z'$ which also induces the nonstandard matter interactions (NSI) of neutrinos \cite{35}. The later could also provide LMA-Dark solution to solar anomaly, which also requires NSI \cite{36}. Recently, it has been pointed out that specific features of cosmic neutrino spectrum reported by the IceCube Collaboration can be explained by a mass of the MeV scale \cite{37, 38}, which can be of interest for the search at Belle II \cite{39}. Moreover, below we show that the $L_\mu - L_\tau$ model with a $\simeq 10$ MeV $Z'$ boson interacting with a light DM can also explain the observed relic DM density. All these solutions employ a SM extension with a gauge $L_\mu - L_\tau$ model.

It is generally assumed that searches for the $L_\mu - L_\tau$ gauge boson are difficult as it couples only to the muon and tau lepton family. The relevant bounds can be extracted from the measurements of the neutrino trident production $\nu_\mu N \rightarrow \nu_\mu \mu^+ \mu^- N$ \cite{20, 21}, from the search for a muonic dark force at BABAR \cite{40}, and from the data of the Borexino experiment \cite{23}. Currently, the allowed $Z'$ mass region for the explanation of the $g_\mu - 2$ anomaly is constrained to $m_{Z'} \lesssim 400$ MeV from by the neutrino trident production \cite{41, 42}, while the BABAR search excluded masses $m_{Z'} \gtrsim 200$ MeV. Besides this, if the $Z'$ is light it would increase the number of relativistic degrees of freedom that spoils the success of the standard Big Bang nucleosynthesis (BBN) predictions. This leads to the lower bound $m_{Z'} \geq 1$ MeV \cite{43}. Moreover there is more restrictive bound $m_{Z'} \geq (3 - 5) \, MeV$ \cite{44} based on the fact that relatively light $Z'$ may indirectly contribute to the number of effective relativistic degrees of freedom $N_{\text{eff}}$ through the raise of the
temperature $\nu_\mu$ and $\nu_\tau$. From the requirement $\Delta N_{\text{eff}} < 0.7$ more restrictive bound $m_{Z'} \geq 5 \text{ MeV}$ arises \[44\]. To search for the $Z'$ in the still unconstrained mass region $5 \lesssim m_{Z'} \lesssim 200 \text{ MeV}$ is challenging as the $Z'$ dominant decay $Z' \to \nu \bar{\nu}$ is invisible. The direct search for such $Z'$ by using the reaction $\mu Z \to \mu ZZ'; Z' \to \text{invisible}$ of the $Z'$ production in high-energy muon scattering off heavy nuclei at the CERN SPS was proposed in Ref. \[49\]. The experiment is expected to improve the sensitivity to the coupling $\alpha_\mu$ by a few orders of magnitude and fully cover the parameter region referred with Eqs. (4) and (5).

Let us now discuss the mixing between the $Z'$ and ordinary photons. An account of one-loop diagrams, which is in our case propagator diagrams with virtual $\mu$- and $\tau$-leptons in the loop, leads to nonzero $\gamma - Z'$ kinetic mixing $\frac{\epsilon}{2} F^{\mu\nu}_{Z}\bar{Z}^{\mu\nu}$ where $\epsilon$ is the finite mixing strength given by \[19\]

$$\epsilon = \frac{8}{3} \frac{e e_\mu}{16 \pi^2} \ln \left( \frac{m_\tau}{m_\mu} \right) = 1.43 \cdot 10^{-2} \cdot e_\mu.$$  \(6\)

Here $e$ is the electron charge and $m_\mu$, $m_\tau$ are the muon and tau-lepton masses respectively. It should be stressed that we assume that possible tree level mixing $\frac{\epsilon_{\text{tree}}}{2} F^{\mu\nu}_{Z}\bar{Z}^{\mu\nu}$ is absent or much smaller than one-loop mixing $\frac{\epsilon}{2} F^{\mu\nu}_{Z}\bar{Z}^{\mu\nu}$. To be precise, we assume that there is no essential cancellation between tree level and one loop mixing terms $|\epsilon_{\text{tree}} + \epsilon| \geq |\epsilon|$ . For $m_{Z'} \ll m_\mu$ the value $e_\mu = (4.75 \pm 0.8) \cdot 10^{-4}$ from Eq. \(4\) leads to the prediction of the corresponding mixing value

$$\epsilon = (6.8 \pm 1.1) \cdot 10^{-6}$$  \(7\)

Thus, one can see that the $Z'$ interaction with the $L_\mu - L_\tau$ current induces at one-loop level the $\gamma - Z'$ mixing of $Z'$ with ordinary photon which allows to probe $Z'$ not only in muon or tau induced reactions but also with intense electron beams. In particular, this loophole opens up the possibility of searching the new weak leptonic force mediated by the $Z'$ in experiments looking for dark photons ($A'$).

The fact that the $\gamma - Z'$ mixing of Eq.\(7\) is at an experimentally interesting level is very exciting. We point out further that a new intriguing possibilities for the complementary
searches of the $Z'$ in the currently ongoing experiment NA64 \[45, 46\] exists. Indeed, the NA64 aimed at the direct search for invisible decay of sub-GeV dark photons in the reaction $e^- + Z \rightarrow e^- + Z + A'$; $A' \rightarrow invisible$ of high energy electron scattering off heavy nuclei \[47, 48\]. The experimental signature of the invisible decay of $Z'$ produced in the reaction $e^- + Z \rightarrow e^- + Z + Z'$; $Z' \rightarrow invisible$ due to mixing of Eq.(6) is the same -- it is an event with a large missing energy carried away by the $Z'$. Thus, by using Eq.(6) and bounds on the $\gamma - A'$ mixing the NA64 can also set constraints on coupling $e_\mu$.

The current NA64 bounds on $\epsilon$ parameter for the dark photon mass region $1 \lesssim m_{Z'} \lesssim 10$ MeV are in the range $10^{-5} \lesssim \epsilon \lesssim 10^{-4}$ for the number of accumulated electrons on target (EOT) $n_{EOT} \simeq 4.3 \cdot 10^{10}$ \[46\]. Taking into account that the sensitivity of the experiment scales as $\epsilon \sim 1/\sqrt{n_{EOT}}$, results in required increase of statistics by a factor $\simeq 100$ in order to improve sensitivity up to the mixing value of Eq.(7) for this $Z'$ mass region. This would allow either to discover the $Z'$ or exclude it as an explanation of the $g_\mu - 2$ anomaly for the substantial part of the mass range $m_{Z'} \ll m_\mu$ by using the electron beam. The direct search for the $Z'$ in missing-energy events in the reaction $\mu Z \rightarrow \mu ZZ'; Z' \rightarrow invisible$ in the dedicated experiment of Ref.\[49\] with the muon beam at CERN would then be an important cross check of results obtained with the electron beam. Let us note that the mixing given by the Eq.(6) would also lead to an extra contribution to the elastic $\nu_e \rightarrow \nu_e$ scattering signal in the solar neutrino measurement at the Borexino experiment \[38\]. The BOREXINO data on the elastic $\nu_\mu e$ scattering \[23\] lead to lower bound on $m_{Z'} \geq (5 - 10)$ MeV by assuming that muon anomaly is explained due to existence of light $Z'$ boson interacting with $L_\mu - L_\tau$ current and there is no tree level mixing between photon and $Z'$, i.e. $\epsilon_{tree} = 0$. The measurement of $\nu - e$ elastic scattering in the LSND experiment \[24\] set a similar bound to the $e_\mu$ coupling for $m_{Z'} \lesssim 10$ MeV \[38\]. The expected 90% C.L. NA64 exclusion regions in the $(m_{Z'}, e_\mu)$ plane (dashed curves) from the measurements with the election beam for $\simeq 4 \times 10^{12}$ and $\simeq 4 \times 10^{13}$ EOT \[45, 46, 48\] and muon beams for $\simeq 10^{12}$ muons on target (MOT) \[49\] are shown in Fig.1. Constraints from the BOREXINO \[38\], CCFR \[42\], and BABAR \[40\] experiments, as well as the BBN excluded area \[38, 50\] are also shown. The parameter
space shown in Fig.1 could also be probed by other electron experiments such as Belle II [39], BDX [53], and LDMX [54], which would provide important complementary results.

Figure 1: The NA64 90% C.L. expected exclusion regions in the $(m_{Z'}, e_{\mu})$ plane (dashed curves) from the measurements with the electron (NA64, $\simeq 4 \times 10^{12}$ EOT and $\simeq 4 \times 10^{13}$ EOT) [45, 46, 48] and muon (NA64-µ, $\simeq 10^{12}$ MOT) [49] beams. Constraints from the BOREXINO [38], CCFR [42], and BABAR [40] experiments, as well as the BBN excluded area [38, 50] are also shown. Two triangles indicate reference points corresponding to the mass $m_{Z'} = 9$ and 11 MeV, and coupling $e_{\mu} = 4 \times 10^{-4}$ and $5 \times 10^{-4}$, respectively, which are used to explain the IceCube results, see Ref. [38] for details.

Another possible way to search for the $Z'$ is based on production and detection of its visible decay mode, $Z' \rightarrow e^+ e^-$, which can also occur at the one-loop level. The flux of $Z'$s would be generated in a high intensity beam dump experiment through the mixing with photon produced either directly in the dump [51] or, e.g., in the $\pi^0$, $\eta$, $\eta'$ decays [52]. The $Z'$s would then penetrate the dump without significant interaction and decay in flight into $e^+ e^-$ pairs which can be observed in a far detector. For a given flux
\[ \frac{d\Phi(m_{Z'}, E_{Z'}, N_{POT})}{dE_{Z'}} \] of \( Z' \)'s from the dump the expected number of \( Z' \rightarrow e^+e^- \) decays occurring within the fiducial length \( L \) of a far detector located at a distance \( L' \) from the beam dump is given by

\[
N_{Z'\rightarrow e^+e^-} = Br(Z' \rightarrow e^+e^-) \int \frac{d\Phi}{dE_{Z'}} exp\left( -\frac{L'm_{Z'}}{P_{Z'}\tau_{Z'}} \right) \\
\cdot \left[ 1 - exp\left( -\frac{Lm_{Z'}}{P_{Z'}\tau_{Z'}} \right) \right] \epsilon_{eff} AdE_{Z'}
\]

where \( E_{Z'}, P_{Z'}, \) and \( \tau_{Z'} \) are the \( Z' \) energy, momentum and the lifetime at rest, respectively, \( \epsilon_{eff}, A \) are the \( e^+e^- \) pair reconstruction efficiency and acceptance, \( N_{POT} \) is the number of primary particles on target (dump). For the mass region \( 1 \lesssim m_{Z'} \lesssim 200 \text{ MeV} \) the branching fraction is given by

\[
Br(Z' \rightarrow e^+e^-) = \frac{\Gamma(Z' \rightarrow e^+e^-)}{\Gamma(Z' \rightarrow e^+e^-) + \Gamma(Z' \rightarrow \nu\nu)}
\]

where the decay rate of the \( Z' \) into neutrino, \( \Gamma(Z' \rightarrow \nu\nu) \) (\( \nu = \nu_\mu, \nu_\tau \)) and \( e^+e^- \) pairs, \( \Gamma(Z' \rightarrow e^+e^-) \) is given by

\[
\Gamma(Z' \rightarrow e^+e^-) = \frac{e_\mu^2}{12\pi} m_{Z'}
\]

and

\[
\Gamma(Z' \rightarrow e^+e^-) = \frac{\alpha}{3} e^2 m_{Z'} \sqrt{1 - \frac{4m_e^2}{m_{Z'}^2}} \left( 1 + \frac{2m_e^2}{m_{Z'}^2} \right)
\]

respectively. Using Eqs.(9 - 11) we found the \( Z' \) lifetime and branching fraction to be in the range \( 10^{-15} \lesssim \tau_{Z'} \lesssim 10^{-13} \) s. This results in a very short \( Z' \) decay length \( c\tau_{Z'\gamma} \approx 150 \) cm even for the \( m_{Z'} \approx 1 \) MeV and \( E_{Z'} \approx 50 \) GeV. Thus, the attenuation of the \( Z' \) flux due to \( Z' \) decays in flight which is given by the term \( exp\left( -\frac{L'm_{Z'}}{P_{Z'}\tau_{Z'}} \right) \) in Eq.(8), give a suppression factor \( \ll 10^{-15} \) for any beam dump experiment searching for an excess of \( e^+e^- \) pairs from dark photon decays \[8\], as they typically used \( L' \gtrsim 100 \) m and \( E_{Z'} \lesssim 50 \) GeV. Because the effective coupling of \( Z_\mu \) to electrons (or quarks) due to the mixing of Eq.(6) is suppressed by a factor \( \approx 10^{-2} \), the branching fraction \( Br(Z' \rightarrow e^+e^-) \) is estimated to be \( \approx O(10^{-4}) \). Taking all these into account makes current constraints \( 10^{-8} \lesssim \epsilon \lesssim 10^{-4} [8] \) from the beam dump experiments searching for visible \( A' \rightarrow e^+e^- \) decays of dark photons in the mass range \( 1 \lesssim m_{A'} \lesssim 200 \) MeV inapplicable to the \( Z' \)
case and much more weaker than the value of Eq.(7) as they were obtained under the assumption that this decay mode is dominant.

Thus, the advantage of searching for $Z'$ in a missing-energy type experiment, e.g. such as NA64, is that its sensitivity is roughly proportional to the mixing squared, $\epsilon^2$ associated with the $Z'$ production in the primary reaction and its subsequent invisible decay, while for the visible case it is proportional to $\epsilon^2 \times Br(Z' \to e^+e^-)$. The factor $\epsilon^2$ is coming from the $Z'$ production process and another suppression factor $Br(Z' \to e^+e^-) = O(10^{-4})$ from the $Z' \to e^+e^-$ decay in the detector. Similar arguments are also valid for the experiments that searched for the $A'$ in particle decays, because their exclusion area is $\epsilon \gtrsim 10^{-4} - 10^{-3}$ for the mass range $1 \lesssim m_{A'} \lesssim 200$ MeV [8]. As a consequence, taking into account the previous discussions, in any beam dump or decay experiment using electrons or quarks as a source of $Z'$s through the mixing of (6), the number of visible $Z' \to e^+e^-$ signal events would be highly suppressed resulting in a weak bound on $\alpha_\mu$.

Similar considerations results in rather modest constraints on invisible decays of $Z'$ which one can extract from the present results of dark-photon experiments searching for the invisible $A'$ decays [8]. For example, the bound on the coupling $\alpha_\mu$ from the $K^+ \to \pi^+ + \text{missing energy}$ decay is at the level $\alpha_\mu \leq O(10^{-3})$, which is several orders of magnitude below the value from Eq.(4).

Finally, note that in order to cover the range $\epsilon \lesssim 10^{-5}$ for the $Z' \to e^+e^-$ decays the trick would be to try to run a corresponding experiment in a very short-length beam dump mode. A good example of such approach is the AWAKE experiment, which plan to search for dark photon decays $A' \to e^+e^-$ with a $\simeq 50$ GeV electron beam by using short W-dump and a detector located at a distance $L' \simeq a$ few meter [55]. This experiment would be very complementary to the $Z'$ searches in invisible decay mode provided the accumulation of $\gtrsim 10^{16}$ EOT is feasible. Another experiment, which potentially might be sensitive to the values around of those from of Eqs.(6),(5) for the masses $m_{Z'} \simeq 100$ MeV, is the HPS [56], which currently aims at reaching the sensitivity $\epsilon \lesssim 10^{-5}$ for the $A' \to e^+e^-$ decays.

Let us show now that an extension of the $L_\mu - L_\tau$ model is able to explain today DM
density in the Universe. Consider the simplest SM extension with an additional complex scalar field $\phi$. The charged dark matter field $\phi_d$ interaction with the $Z'$ field is

$$L_{\phi Z'} = (\partial^\mu \phi - ie_d Z^\mu \phi)^* (\partial_\mu \phi - ie_d Z'_\mu \phi) - m_{DM}^2 \phi^* \phi - \lambda \phi^*(\phi^* \phi)^2$$  \hspace{1cm} (12)$$

The annihilation cross section $\phi_d \bar{\phi}_d \rightarrow \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau$ for $s \approx 4m_{DM}^2$ has the form\footnote{The annihilation cross-section for scalar DM has $p$-wave suppression that allows to escape CMB bound \cite{57}.}

$$\sigma v_{rel} = \frac{8\pi}{3} \frac{\epsilon^2 \alpha_d m_{DM}^2 v_{rel}^2}{(m_{Z'}^2 - 4m_{DM}^2)^2},$$ \hspace{1cm} (13)$$

We use standard assumption that in the hot early Universe DM is in equilibrium with ordinary matter. During the Universe expansion the temperature decreases and at some point the thermal decoupling of the Dark Matter starts to work. Namely, at some freeze-out temperature the cross-section of annihilation $DM$ particles $\rightarrow SM$ particles becomes too small to obey the equilibrium of DM particles with the SM particles and DM decouples. The experimental data are in favour of scenario with cold relic for which the freeze-out temperature is much lower than the mass of the particle. In other words DM particles decouple in the non-relativistic regime. The value of the DM annihilation cross-section at the decoupling epoch determines the value of the current DM density in the Universe. Too big annihilation cross-section leads to small DM density and vise versa too small annihilation cross section leads to DM overproduction. The observed value of the DM density $\frac{\rho_{DM}}{\rho_c} \approx 0.23$ allows to estimate the DM annihilation cross-section into the SM particles and hence to estimate the discovery potential of light dark matter both in direct underground and accelerator experiments.

The dark matter relic density can be numerically estimated as \footnote{Here we consider the case $m_{Z'} > 2m_{DM}$.}

$$\Omega_{DM} h^2 = 0.1 \left[ \frac{(n + 1)x_f^{n+1}}{(g^{*s} / g^{*1/2})} \right] \frac{0.856 \cdot 10^{-9} GeV^{-2}}{\sigma_0},$$ \hspace{1cm} (14)$$

where $<\sigma v_{rel}> = \sigma_0 x_f^{-n}$, $x_f = \frac{m_{DM}}{T_{dec}}$ and

$$x_f = c - (n + \frac{1}{2})\ln(c),$$ \hspace{1cm} (15)$$
\[ c = \ln \left[ 0.038(n + 1) \frac{g}{\sqrt{g_*}} M_P m_{DM} \sigma_0 \right]. \quad (16) \]

For the case where dark matter consists of dark matter particles and dark matter antiparticles the \( DM \rightarrow SM \) particles annihilation cross section \( \sigma = \frac{a_{an}}{2} \). Numerically we find that

\[ k(m_{DM}) \cdot 10^{-6} \cdot \left( \frac{m_{DM}}{\text{GeV}} \right)^2 \cdot \left[ \frac{m_{A'}^2}{m_{DM}^2} - 4 \right] = \epsilon^2 \alpha_d. \quad (17) \]

Here the coefficient \( k(m_{DM}) \) depends logarithmically on the dark matter mass \( m_{DM} \) and \( k_{DM} \approx 0.5(0.9) \) for \( m_{DM} = 1(100) \) MeV. For instance, for \( m_{A'} = 2.2 \ m_{DM} \) we have

\[ 0.71k(m_{DM}) \cdot 10^{-6} \cdot \left[ \frac{m_{DM}}{1 \text{ GeV}} \right]^2 = \epsilon^2 \alpha_d. \quad (18) \]

As a consequence of (14) we find that for \( m_{Z'} \ll m_{\mu} \) the values \( \epsilon^2 = (2.5 \pm 0.7) \cdot 10^{-6} \) and \( \alpha_d = (0.28 \pm 0.08)k(m_{DM}) \cdot \left[ \frac{m_{DM}}{1 \text{ GeV}} \right]^2 \) explain both the \( g_{\mu} - 2 \) muon anomaly and today DM density. We can rewrite the equation (15) in the form

\[ \frac{e_d^2}{e_{\mu}^2} = (16 \pm 9)k(m_{DM}) \cdot \left[ \frac{m_{DM}}{1 \text{ MeV}} \right]^2. \quad (20) \]

So we see that for \( m_{DM} \geq 1 \) MeV we have \( e_d \gg e_{\mu} \), i.e. the \( Z' \) must interact much more strongly with light DM than with the SM matter.

In summary, the \( L_\mu - L_{\tau} \) model with the light vector boson \( Z' \) interacting with \( L_\mu - L_{\tau} \) current is a well-motivated SM extension, with impressive indirect support from the possible explanation of the muon \( g_{\mu} - 2 \) anomaly and several observations in neutrino sector and astrophysics. While the model can be effectively tested with the direct high-energy muon experiment at the CERN SPS [49], we show that nonzero \( \gamma - Z' \) mixing generated in the model at the one-loop level strongly motivates the complementary searches of the light \( Z' \) with high-energy electron beams. This open up an intriguing possibility for probing the \( L_\mu - L_{\tau} \) gauge boson \( Z' \) in the near future with ongoing NA64 experiment with the statistics increased by a factor \( \simeq 10 - 100 \). The \( Z' \) searches can be as well performed in the incoming dark photon experiments, e.g such as AWAKE, Belle-II, BDX, and LDMX. Moreover an extension of the \( L_\mu - L_{\tau} \) model allows to explain relic Dark Matter density
for $m_{Z'} \simeq O(10)$ MeV, which strengthen motivation for the experimental search of the $L_{\mu} - L_{\tau}$ mediator of the DM production in invisible decay mode. Finally, we note that if the $Z'$ couples to light DM, then an additional contribution from the invisible decay mode $Z' \rightarrow dark\ matter$ increases the $Z' \rightarrow invisible$ decay rate as a consequence for $m_{Z'} > 2m_{\mu}$ visible decay $Z' \rightarrow \mu^+\mu^-$ is suppressed.

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References

[1] D.S. Gorbunov and V.A. Rubakov, Introduction to the theory of the early Universe, World Scientific Publishing Co. Pt. Ltd., 2011.

[2] S. Profumo, TASI 2012 lectures on astrophysical ptobes of dark matter, arXiv:1301.0952.

[3] S. Dodelson, “Modern cosmology”, Amsterdam, Netherlands: Academic Press (2003) 440 p.

[4] G. Arcadi et al., arXiv:1703.07364 (2017).

[5] C. Boehm, T. Ensslin and J. Silk, J. Phys. G30 (2004) 279;
   G. Boehm and P. Fayet, Nucl. Phys. B683 (2004) 219.

[6] As a review of current and future efforts to discover light Dark Matter see:
   J. Alexander et al., arXiv:1608.08632 (2016).

[7] G.W. Bennett et al. (Muon g-2Collaboration), Phys. Rev. D 73, 072003 (2006).
[8] C. Patrignani et al. (Particle Data Group Collaboration), Chin. Phys. C 40, 100001 (2016).

[9] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, Eur. Phys. J. C 71, 1515 (2011); 72, 1874 (2012) [erratum].

[10] F. Jegerlehner and R. Szafron, Eur. Phys. J. C 71, 1632 (2011).

[11] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, J. Phys. G 38, 085003 (2011).

[12] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. 109, 111808 (2012).

[13] X. He, G. C. Joshi, H. Lew, and R. Volkas, Phys. Rev. D 43, 22 (1991).

[14] R. Foot, Mod. Phys. Lett. A6, 527 (1991).

[15] X.-G. He, G. C. Joshi, H. Lew, and R. Volkas, Phys. Rev. D 44, 2118 (1991).

[16] S.N. Gninenko and N.V. Krasnikov, Phys. Lett. B513, 119 (2001).

[17] S. Baek et al., Phys.Rev. D64, 055006 (2001).

[18] E. Ma, D. Roy and S. Roy, Phys.Lett. B525, 101 (2002).

[19] See, for example, B. Holdom, Phys. Lett. B 166, 196 (1986).

[20] D. Geiregat et al. (CHARM-II Collaboration), Phys. Lett. B 245, 271 (1990).

[21] S.R. Mishra et al. (CCFR Collaboration), Phys. Rev. Lett. 66, 3117 (1991).

[22] J.P. Lees et al. (BaBar Collaboration), Phys.Rev. D 94, 011102 (2016).

[23] G. Bellini et al., Phys.Rev.Lett. 107, 141302 (2011).

[24] L. B. Auerbach et al. (LSND Collaboration), Phys. Rev. D 63, 112001 (2001).

[25] P. Fayet, Phys. Rev. D 75, 115017 (2007).
[26] M. Pospelov, Phys. Rev. D 80, 095002 (2009).

[27] J. Heeck, and W. Rodejohann, Phys. Rev. D 84, 075007 (2011).

[28] W. Altmannshofer, M. Carena and A. Crivellin, Phys. Rev. D 94, 095026 (2016).

[29] As a recent review, see for example: N.V.Krasnikov, arXiv:1702.04596 (2017).

[30] As a review, see for example:
   F. Jegelehner and A. Nyffeler, Phys. Rep. 477, 1 (2009);
   F.S. Queiroz and W. Shepherd, Phys. Rev. D 89, 095024 (2014).

[31] P. Binetruy, S. Lavignac, S. T. Petcov, and P. Ramond, Nucl. Phys. B 496, 3 (1997).

[32] N. F. Bell and R. R. Volkas, Phys. Rev. D 63, 013006 (2001).

[33] S. Choubey and W. Rodejohann, Eur. Phys. J. C 40, 259 (2005).

[34] A. Dev, arXiv:1710.02878.

[35] A. Datta, J. Liao, D. Marfatia, Phys. Lett. B 768, 265 (2017).

[36] Y. Farzan, Phys. Lett. B 748, 311 (2015).

[37] T. Araki, F. Kaneko, Y. Konishi, T. Ota, Joe Sato, T. Shimomura, Phys. Rev. D 91, 037301 (2015).

[38] T. Araki, F. Kaneko, T. Ota, J. Sato, T. Shimomura, Phys. Rev. D 93, 013014 (2016).

[39] T. Araki, S. Hoshino, T. Ota, J. Sato, T. Shimomura, Phys. Rev. D 95, 055006 (2017).

[40] J.P. Lees et al. (BaBar Collaboration), Phys. Rev. D 94, 011102 (2016).

[41] W. Altmannshofer, S. Gori, M. Pospelov, I. Yavin, Phys. Rev. D 89, 095033 (2014).

[42] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, Phys. Rev. Lett. 113, 091801 (2014).
[43] B. Ahlgren, T. Ohlsson and S. Zhou, Phys. Rev. Lett. 111, 199001 (2013).

[44] A. Kamada and H.-B. Yu, Phys. Rev. D 92, 113004 (2015).

[45] D. Banerjee et al. (NA64 Collaboration), Phys. Rev. Lett. 118, 011802 (2017).

[46] D. Banerjee et al. (NA64 Collaboration), arXiv:1710.00971 [hep-ex].

[47] S.N. Gninenko, Phys. Rev. D 89, 075008 (2014).

[48] S. Andreas et al., arXiv:hep-ph/1312.3309(2013).

[49] S.N. Gninenko, N.V. Krasnikov and V.A. Matveev, Phys. Rev. D 91, 095015 (2015).

[50] A. Kamada and H.-B. Yu, Phys. Rev. D 92, 113004 (2015).

[51] J. Blümlein and J. Brunner, Phys. Lett. B 701, 155 (2011);
    J. Blümlein and J. Brunner, Phys. Lett. B 731, 320 (2014).

[52] S.N. Gninenko, Phys. Rev. D 85, 055027 (2012);
    S.N. Gninenko, Phys. Lett. B 713, 244 (2012).

[53] M. Battaglieri et al. (BDX Collaboration), arXiv:1712.01518

[54] J. Mans (LDMX Collaboration), EPJ Web Conf. 142 (2017) 01020.

[55] See, for example, Edda Gschwendtner, ”PBC Exploratory Study for AWAKE Applications”, Talk given at the Physics Beyond Colliders Annual workshop, CERN, 21-22 November, 2017.

[56] See, for example, I. Balossino et al., arXiv:1610.04319v3 [physics.ins-det], and references therein.

[57] P.A.R. Ade et al.(Planck), arXiv:1502.01589 (2015).

[58] E.W. Kolb and M.S. Turner, The early Universe, Front. Phys. 69 (1990) 1-547.