Metallic Glass in Two Dimensional Disordered Bose System; A Renormalization Group Approach.

M. Crisan and I. Grosu

Department of Theoretical Physics, University of Cluj, 3400 Cluj-Napoca, Romania

I. Tifrea

Department of Theoretical Physics, University of Cluj, 3400 Cluj-Napoca, Romania and Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242, USA

We consider the two dimensional disordered Bose gas which present a metallic state at low temperatures. A simple model of an interacting Bose system in a random field is propose to consider the interaction effect on the transition in the metallic state.

I. INTRODUCTION

The recent experimental data obtained on the two-dimensional (2d) systems showed the possibility of the occurrence for the metallic phase in the insulator-superconductor transition. The model proposed by Das and Doniach is a uniform phase lacking both phase and charge order. This phase presents translationally and rotationally invariance. It has to appear at finite temperature, because at \( T = 0 \) there is no phase transition which is lacking both charge and phase order. The occurrence of the metallic phase in a disordered two dimensional (2d) bosonic system has been predicted by Wagenblast et al. in the insulator-superconductor transition, but only at the separatrix line. The important result has been obtained by Daidovich and Phillips, who showed that this state can appear in disordered two dimensional (2d) Bose systems. This model is based on the quantum version of the spin glass state which can be described using the replica trick method. The initial free energy has the universality class \( z = 1 \) (\( z \) is the dynamical critical exponent) with a local breaking spin rotation invariance. The effective order parameter \( \phi \), is given in terms of the phase \( \varphi(r, \tau) \) (\( \tau \) is the imaginary time) as

\[
Q(\tau) = \langle \exp(i\varphi(r_1, \tau) - i\varphi(r_2, \tau)) \rangle \exp(2i\varphi(r_1, \tau))
\]

and this gives rise to excitations which scales as \( |\omega| \) and changes the universality class to \( z = 2 \). Such a dissipative model has been treated in the Gaussian approximation and the conductivity \( \sigma(T = 0, \omega = 0) \) has been obtained as finite, which proves that the state is metallic.

In this paper we present an equivalent model for the 2d disordered Bose system which can be treated with the Renormalization Group (RG) method. The model was used to describe the spin glass state where disorder was introduced by a random field which can be connected to the effective order parameter of the spin glass state. The quantum version of this method was given in where was applied for the study of the critical behavior of the interacting Bose system. However, we mention that the theory of the critical behavior of the transport is not trivial because at finite temperature the limits \( \sigma(T = 0, \omega) \) and \( \sigma(\omega = 0, T) \) are not equal, but we will be interested in the calculation of the \( \sigma(0, 0) \) near the critical region. The model of our disordered Bose system will be presented in Sec.II, where we will discuss the equivalence with the model from Ref. 8. In Sec.III we solve the flow equations of the RG equations of this model. The strategy of our calculations is different to this from [11, 12], because we are interested in the case of \( d = 2 \) and \( z = 2 \). In this case we can solve the differential equations exactly, studying the influence of the temperature on the transition. In fact this procedure give us the possibility to analyze the relevance of the parameters \( T \) and the interaction \( u_0 \) near the fixed point \( T = 0 \). We expect that the new result for conductivity to contain a correction given by the interactions between bosons which is in fact small and are in fact corrections to the Gaussian behavior. The conductivity near the critical region will be calculated following the method given in [6], and applied recently [15] for the study of the fluctuations of the conductivity in the d-wave superconductors near the critical disorder.

The last section, Sec.IV, will be devoted to the calculation of the conductivity and the result will be compared with the simple Gaussian model. The values given by the experimental are different from the standard result obtained for the simple model which gives \( \sigma(0) = 4e^2/h \) (2e is the pair charge and \( h \) is the Planck constant) the well known result of Fisher and Grinstein. This difference is not yet clear, but the accuracy of the measurements, as well as, the interactions from the systems can generate this deviation. The present results confirms the exciting idea of the existence of a metallic state in the insulator-superconductor transition which is driven by disorder, which in fact is not favorable for the superconducting state. We expect that this intermediate metallic state to appear in the the d-wave superconductors containing nonmagnetic impurities. The model from Ref. 15 contains also the influence of the temperature on the metallic conductivity near the critical concentration which destroys d-waves superconducting state.
II. MODEL.

Before the presentation of the model of an interacting Bose system we start with a short presentation of the model proposed by Dalidovich and Phillips [7, 8], which contains the physics of the problem. This discussion is also relevant to make clear the validity and advantage of the model proposed in this section.

A. Dalidovich-Phillips model

The Landau action for this problem can be obtained using the replicas to perform the average on the disordered state. The quadratic and quartic terms describe, in the spin glass theory the interaction between spins that appears by the average procedure on the disordered state, and can be decoupled by introducing the auxiliary fields \( Q_{\mu \nu}(k, \tau, \tau') \) and \( \Psi^a_\mu(\tau) \), where the superscriptions \( a \) and \( b \) represent the replica indices. A finite value of \( \Psi^a_\mu \) is equivalent with the phase ordering in the charge of 2e condensate, and \( \langle \Psi^a_\mu \rangle = 0 \) describe the disordered phase.

As we mentioned in introduction for the quasi-order in the charge of 2e condensate, and \( \langle \Psi^a_\mu \rangle \) are the effective order parameter. The behavior of this parameter leads to the change of the critical dynamic exponent \( z = 1 \) to \( z = 2 \). The free energy given in [7] contain a Gaussian and a quartic term as in the standard \( \Phi^4 \)-theory, and the contribution which couples the charge and the glassy degrees of freedom, and is considered by the authors as dominant in the bicritical point. However, using such a form for action the application of the RG method seems to be very difficult, and even the authors considered only the Gaussian approximation.

We will show that this state, studied in [7] can be modelled by an action which describes the effect of a random field on the interacting Bose system, using the RG in the quantum limit. In the following we will present the model which gives a similar result with the Gaussian model containing the corrections given by the interaction between bosons. The effect of disorder is contained only in the gaussian contribution, and we will show that in the lowest order the gaussian term is identical with the expression from [7].

B. Random field model

We consider a \( d \)-dimensional Bose model in a random field \( h(x) \) described by the Hamiltonian:

\[
H = \int d^d x \Phi^\dagger(x) \left[ - \nabla^2 + r_0 \right] \Phi(x) + \frac{u_0}{4} \int d^d x |\Phi(x)|^4 + \int d^d x |h(x)\Phi^\dagger(x) + h^\dagger(x)\Phi(x)|. \tag{1}
\]

In this Hamiltonian \( (h^2 = 2m = 1) \), \( \Phi(x) \) is the bosonic field, \( r_0 \) the control parameter (for the standard case it is the chemical potential), and \( u_0 > 0 \) is the bare coupling constant. The random field \( h(x) \) is a Gaussian random variable with a Fourier transform \( h(k) \) which satisfies:

\[
< h(k) >= \delta_{k,0}, \quad < h^\dagger(k)h(k^\prime) >= \delta_{k,0} \delta_{k^\prime,0} \tag{2}
\]

where \( <> \) indicates an average over the possible configurations of the random field and will serve to introduce a an effective Edwards-Anderson [17] spin-glass parameter denoted by \( q \). This Hamiltonian has been used to develop a functional theory [11, 12] described by the action:

\[
S[\Phi] = S^{(0)}[\Phi] + S^{(in)}[\Phi] \tag{3}
\]

where:

\[
S^{(0)}[\Phi] = \frac{u_0}{4} \sum_{a,b=1}^{m} \sum_{k} [(r_0+k^2-i\omega_n)\delta_{ab}-(q/T)\delta_{\omega_n0}] \Phi_a(k)\Phi_b(k) \tag{4}
\]

In this equation \( k \equiv (k, \omega_n) \) the indices \( a, b \) and \( m \) are the replicas indices from the standard spin-glass theory [17], method used in [11, 12] to calculate the free energy of this disordered system. In the replica-trick theory the calculations are performed taking \( m \rightarrow 0 \). The interaction contribution has the form:

\[
S^{(in)}[\Phi] = \frac{u_0}{4} \sum_{a=1}^{m} \sum_{k_1} \cdots \sum_{k_4} \Phi_a(k_1)\Phi_a(k_2)\Phi_a(k_3)\Phi_a(k_4)\delta(k_1+\cdots+k_4) \tag{5}
\]

The Gaussian propagator in the limit \( m \rightarrow 0 \) has the form [8, 11]:

\[
G_{a,b}(k) = G_{0}(k)\delta_{a,b} + \beta G_{0}^{2}(k)q\delta_{\omega_n0} \tag{6}
\]

where \( \beta = 1/T \) and \( G_{0}(k, \omega_n) = (r_0+k^2+\eta||\omega_n||)^{-1} \). We mention that the damping term linear in energy has been introduced in the model following the physical considerations presented in [8, 13], and it has a importance, because it keeps the universality class \( z = 2 \) for the model.

In the next section we will use the results from [11] to write the flow equations for the model in the limit \( m \rightarrow 0 \). We also considered the replica symmetric case, as in Ref. [13], but we do not have a term which couples the bosonic field \( \Phi \) and the order parameter \( q \), which in this model has been considered only spatial dependent. This
may be considered as a very poor approximation, because we used the an free propagator with the Ohmic dissipative term, which implies a time dependence of the effective parameter $q$. Anyway, even this simple model, which is tractable by RG-theory, contains more than the Gaussian model, and can give us an idea of the simple approximation performed in [13].

III. RENORMALIZATION GROUP EQUATIONS

The flow equations for the RG. can be obtained using the recursion relations [1] (where we take $b = e^t$) as:

$$\frac{dT}{dl} = 2T.$$  

(7)

$$\frac{dq}{dl} = 4q.$$  

(8)

$$\frac{dr}{dl} = 2r + uF_1 + uqF_2.$$  

(9)

$$\frac{du}{dl} = [4 - (d + z)]u - \frac{u^2}{4}[8F_3 + 2F_4] + 5u^2q.$$  

(10)

In these equations we used for the number of components of the bosonic field $n = 2$, and we will take $d = 2$ and $z = 2$. The functions $F_i,(i = 2...5)$ can be calculated [1], (in this case there is a difference because of the damping $\eta$) and only the function $F_1(T(l) = K_2[\exp(1/T(l))−1]^{-1}$ ($K_2 = 1/(2\pi)$) is important because contains the relevant parameter $T(l)$. The Eqs.(9-10) will be solved in the low temperature limit and when we can take: $8F_3 + 2F_4 \simeq K_2$, and $F_5 \simeq K_2$.

The solutions of the Eqs.(7-8) are $T(l) = Te^{2l}$ and $q(l) = ge^{4l}$. Using these results we solved exactly the Eq.(9) and the solution is:

$$u(l) = \frac{4}{K_2l + l_0 + (e^{4l} - 1)},$$  

(11)

where $l_0 = 4/K_2u_0$.

The general solution of the Eq.(9) has the form:

$$r(l) = e^{2l}[r_0 + I_1(T) + 4q \int_0^l dl' \frac{e^{2l'}}{l' + l_0 + 5q(e^{4l'} - 1)}],$$  

(12)

where $I_1(T)$ is given by the expression:

$$I_1(T) = 4 \int_0^l dl' \frac{\exp(-2l')}{\exp(1/T(l')) - 1}.$$  

(13)

We will consider the case of low temperatures when $I_1$ can be neglected, and $q$ is small, so we can approximate Eq.(12) by:

$$r(l) = e^{2l}[r_0 + 4q \int_0^l dl' \frac{e^{l'}}{l' + l_0}].$$  

(14)

This integral can be given by the function $Ei(x)$ defined by

$$Ei(xy) = e^{xy} \int_0^1 dt \frac{t^{y-1}}{x + \ln t}.$$  

(15)

For $y = 1$ and big argument $Ei(x) \simeq e^x/x$ and we get for Eq.(14) the expression:

$$r(l) \simeq e^{2l} \left[r_0 + 2q (\frac{e^{2l}}{l + l_0} - \frac{1}{l_0})\right].$$  

(16)

Next important step is to calculate the stop scaling parameter $l^*$ from the condition $r(l^*) = 1$. This procedure applied first in [7] gives the possibility to study the influence of the temperature on the critical behavior of the transport properties in the critical region of the $z = 2$ insulator-superconductor transition. In our case the Gaussian fixed point $T = 0$ is stable, and we will expect a correction given by the interaction between fluctuations on this behavior. Indeed from Eq.(16) we obtain the important result:

$$e^{4l^*} \simeq (1 + u_qq^2)$$  

(17)

a result which will be used to calculate the conductivity of the system considered, at $T = 0$. The conductivity will be calculated using the Kubo formula generalized for the replicated action (1) which contains the influence of disorder. In the next section we will calculate $\sigma(\omega_n)$ following the approximations from [13] but using the renormalized value for $q(l)$ which is a relevant parameter. This parameter is strongly dependent of the variable $l$ and in this way we can fix it near the critical point studding the competition between disorder and the interaction between fluctuations in the occurrence of the metallic phase. Before starting this calculation, which will follow the approximations from [7], we mention that even if our model is slightly different from that developed by Dalidovich and Phillips [8], their approximations remain valid for our model because are related to the replica-trick method, and of the propagators of the replicated systems, which are the same for both models. On the other hand, the existence of the dissipation is essential for the occurrence of the metallic state as was first pointed out in [8]. However, if our model appear to be more tractable analytically by RG method, the main approximation of our model remain the dissipative form of the free propagator $G_0(k,\omega_n)$. Such a form can be easy obtained if we adopt a model similar with this from [12] where the free propagator contains the scattering effects on the non-magnetic impurities, or the coupling to an Ohmic heat bath [18]. This approximation is well justified by the results, and a RG-treatment containing more complicated form of the action is difficult to be controlled. The higher order corrections mentioned in [8] leads the terms proportionally to $u_qq^2$ which appear also in our calculations, but we cannot consider these corrections as having the same origin. In fact the vertex corrections presented later by the authors in Ref. [8]...
leads to the same qualitative behavior for the $T = 0$ case, the difference being the power of $r_0$ from the denominator, a result similar with the calculations from [12], performed for the case of layered quasi-two dimensional superconductor. In fact the generalization of our model for this case is very simple problem and we do not expect qualitatively new results. The origin of the driving parameter $r_0$ is another problem which can be discussed because this transition is a $T = 0$ transition, and in fact is a Quantum Phase Transition (QPT). In the next section we used the results from [7, 18] for the calculation of conductivity, without giving many technical aspects, and using the same approximations. However, we mention that the calculations will be performed taking the conductivity to one-loop order per replica as in the Gaussian approximation. The new point is that we consider the influence of the fluctuations near the critical region using $q(l^*)$ where $l^*$ is given by Eq.(17).

IV. CONDUCTIVITY

The Kubo formula for a disordered system has the general form [7]:

\[
\sigma(i\omega_n) = \frac{8e^2}{\pi \omega_n} \sum_{a,b,\omega} \int \frac{d^2k}{(2\pi)^2} \left[ G_{a,b}^0(k, \omega_i) \delta_{a,b} - 2k_x^2 G_{a,b}^0(k, \omega) G_{a,b}^0(k, \omega_1 + \omega_n) \right] (18)
\]

Following the way from [7] we transform this equation in the simple form:

\[
\sigma(\omega_n) = \frac{8q(l^*)e^2}{\omega_n} \int \frac{d^2k}{(2\pi)^2} k_x^2 G_0^2(k, 0) [G_0(k, 0) - G_0(k, \omega_n)] (19)
\]

In this point we mention that, differently from [7] we considered the $l$-dependence of the effective order parameter $q$ for the disordered state in order to take into consideration the influence of the interaction. This is not necessary in the case of the Gaussian approximation mention above. Using now the simple expression for $G_0$ and the Eq.(17) for $l^*$ from Eq.(19) and $q(l^*) = qe^{il^*}$ we get:

\[
\sigma(\omega = 0, T \rightarrow 0) = \text{const} \frac{q^2(1 + u_0q)^2}{r_0^2} (20)
\]

This result shows that for $u_0 = 0$, which is Gaussian approximation we get the same result as in Ref [7], and the metallic state is robust to the interaction between bosons.

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[1] H. M. Jager, D. B. Haviland, B. G. Orr, and A. M. Goldman, Phys. Rev. B40, 182 (1989).
[2] D. Ephron, A. Yazdani, A. Kapitulnik and, M. R. Beasley, Phys. Rev. Lett. 76, 1529 (1996).
[3] N. Manson and A. Kapitulnik, Phys. Rev. Lett 82,5341 (1999).
[4] D. Das and S. Doniach, Phys. Rev. B 60, 1261 (1999).
[5] R. Fazio and G. Schön, Phys. Rev. B 43, 5307 (1991)
[6] K. Wagenblast, A. van Otterlo, G. Schön, and G. Zimanyi Phys. Rev. Lett. 78 , 1779 (1997)
[7] D. Dalidovich and P. Phillips, Phys. Rev. Lett. 89, 27001 (2002).
[8] P. Phillips and D. Dalidovich, Science 302, 243 (2003).
[9] A. A. Abrikosov and S. I. Moukhin, J. Low Temp. Phys.33, 207 (1978).
[10] M. V. Medvedev, and A. V. Zaborov, Pys. Stat. Sol. (b)79, 379 (1977).
[11] G. Busiello, L. De Cesare, and I. Rabuffo, Phys. Rev. B 28, 6463 (1983).
[12] G. Busiello, L. De Cesare, and I. Rabuffo, Phys. Rev.B 29, 4189 (1984).
[13] K. Damle and S. Sachdev, Phys. Rev. B 56, 8717 (1997); Phys. Rev. B 57, 8307 (1998).
[14] S. Sachdev, Quantum Phase Transitions (Cambridge University Press, Cambridge, 1999).
[15] I. Tifrea, D. Bodea, I. Grosu, and M. Crisan, Eur. Phys. J. B 36, 377 (2003).
[16] M. P. A. Fisher and G. Grinstein, Phys. Rev. Lett.60, 208 (1988).
[17] S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).
[18] D. Dalidovich and P. Phillips, Phys. Rev. B 63, 224503 (2001).