Information Retrieval via Truncated Hilbert-Space Expansions

Patricio Galeas\textsuperscript{1}, Ralph Kretschmer\textsuperscript{2}, and Bernd Freisleben\textsuperscript{1}

\textsuperscript{1} Dept. of Mathematics and Computer Science, University of Marburg, Hans-Meerwein-Str. 3, D-35032 Marburg, Germany
\{galeas, freisleb\}@informatik.uni-marburg.de
\textsuperscript{2} Raytion GmbH, Kaiser-Friedrich-Ring 74, D-40547 Düsseldorf, Germany
ralph.kretschmer@raytion.com

Abstract. In addition to the frequency of terms in a document collection, the distribution of terms plays an important role in determining the relevance of documents. In this paper, a new approach for representing term positions in documents is presented. The approach allows an efficient evaluation of term-positional information at query evaluation time. Three applications are investigated: a function-based ranking optimization representing a user-defined document region, a query expansion technique based on overlapping the term distributions in the top-ranked documents, and cluster analysis of terms in documents. Experimental results demonstrate the effectiveness of the proposed approach.

1 Introduction

The information retrieval (IR) process has two main stages. The first stage is the indexing stage in which the documents of a collection are processed to generate a database (index) containing the information about the terms of all documents in the collection. The index generally stores only term frequency information, but in some cases positional information of terms is also included, substantially increasing the memory requirements of the system.

In the second stage of the IR process (query evaluation), the user sends a query to the system, and the system responds with a ranked list of relevant documents. The implemented retrieval model determines how the relevant documents are calculated. Standard IR models (e.g. TFIDF, BM25) use the frequency of terms as the main document relevance criterion, producing adequate quality in the ranking and query processing time. Other approaches, such as proximity queries or passage retrieval, complement the document relevance evaluation using term positional information. This additional process, normally performed at query time, generally improves the quality of the results but also slows down the response time of the system. Since the response time is a critical issue for the acceptance of an IR system by its users, the use of time-consuming algorithms to evaluate term-positional information at query time is generally inappropriate.

The IR model proposed in this paper shifts the complexity of processing the positional data to the indexing phase, using an abstract representation of the term positions...
and implementing a simple mathematical tool to operate with this compressed representation at query evaluation time. Thus, although query processing remains simple, the use of term-positional information provides new ways to optimize the IR process. Three applications are investigated: a function-based ranking optimization representing a user-defined document region, a query-expansion technique based on overlapping the term distributions in the top-ranked documents, and cluster analysis of terms in documents. Experimental results demonstrate the effectiveness of the proposed approach for optimizing the retrieval process.

The paper is organized as follows. Section 2 discusses related work. Section 3 presents the proposed approach for representing term positions based on truncated Hilbert space expansions. In Section 4, applications of the approach are described. Section 5 concludes the paper and outlines areas for future work.

2 Related Work

An early approach to apply term-positional data in IR is the work of Attar and Fraenkel [2]. The authors propose different models to generate clusters of terms related to a query (searchonyms) and use these clusters in a local feedback process. In their experiments they confirm that metrical methods based on functions of the distance between terms are superior to methods based merely on weighted co-occurrences of terms. There are several other approaches that use metrical information [3, 7].

One of the first approaches using abstract representations of term distributions in documents is Fourier Domain Scoring (FDS), proposed by Park et al. [6]. FDS performs a separate magnitude and phase analysis of term position signals to produce an optimized ranking. It creates an index based on page segmentation, storing term frequency and approximated positions in the document. FDS processes the indexed data using the Discrete Fourier Transform to perform the corresponding spectral analysis.

A recent approach based on an abstract representation of term position is Fourier Vector Scoring (FVS) [4]. It represents the term information (Fourier coefficients) directly as an $n$-dimensional vector using the analytic Fourier transform, permitting an immediate and simple term comparison process.

3 Analyzing Term Positions

In this section, a general mathematical model to analyze term positions in documents is presented, making it possible to effectively use the term-positional information at query evaluation time.

Consider a document $D$ of length $L$ and a term $t$ that appears in $D$. The distribution of the term $t$ within the document is given by the set $P_t$ that contains all positions of $t$, where all terms are enumerated starting with 1 for the first term and so on. For example, a set $P_t = \{2, 6\}$ represents a term that is located at the second and sixth position of the document body. A characteristic function

$$f^{(t)}(x) = \begin{cases} 1 & \text{for } x \in [p - 1, p] \text{ if } p \in P_t, \\ 0 & \text{otherwise} \end{cases},$$

(1)
defined for \( x \in [0, L] \), is assigned to \( P_t \).

The proposed method consists of approximating this characteristic function by an expansion in terms of certain sets of functions. In order to do so, some concepts of functional analysis are introduced. Details can be found in the book of Yosida [9].

3.1 Expansions in Hilbert Spaces

A Hilbert space \( \mathcal{H} \) is a (possibly infinite-dimensional) vector space that is equipped with a scalar product \( \langle , \rangle \), i.e. two elements \( f, g \in \mathcal{H} \) are mapped to a real or complex number \( \langle f, g \rangle \). We only consider real scalar products here.

An example of a Hilbert space is the space \( L^2([0, L]) \) defined as the set of all functions \( f \) that are square-integrable in the interval \([0, L] \), i.e. functions for which \( \int_0^L (f(x))^2 \, dx < \infty \). In this vector space, the addition of two functions \( f \) and \( g \), and the multiplication of a function \( f \) by a scalar \( \alpha \in \mathbb{R} \) are defined point-wise:

\[
(f + g)(x) = f(x) + g(x), \quad (\alpha f)(x) = \alpha f(x).
\]

The scalar product in \( L^2([0, L]) \) is defined by

\[
\langle f, g \rangle = \int_0^L f(x)g(x) \, dx.
\]

Two vectors with vanishing scalar product are called orthogonal.

The scalar product induces a norm (an abstract measure of length)

\[
\|f\| = \sqrt{\langle f, f \rangle} \geq 0.
\]

With the help of this norm, the notion of convergence in \( \mathcal{H} \) can be defined: A sequence \( f_0, f_1, \ldots \) of vectors of \( \mathcal{H} \) is said to converge to a vector \( f \), symbolically \( \lim_{n \to \infty} f_n = f \), if \( \lim_{n \to \infty} \| f_n - f \| = 0 \). This allows to define an expansion of a vector \( f \) in terms of a set of vectors \( \{ \varphi_0, \varphi_1, \ldots \} \). One writes

\[
f = \sum_{k=0}^{\infty} \gamma_k \varphi_k,
\]

where the \( \gamma_k \) are real numbers, if the sequence \( f_n = \sum_{k=0}^{n} \gamma_k \varphi_k \) of finite sums converges to \( f \). This kind of convergence is called norm convergence.

Of particular importance are so-called complete, orthonormal sets \( \{ \varphi_0, \varphi_1, \ldots \} \) of functions in \( \mathcal{H} \). They have the following properties: (a) The \( \varphi_i \) are mutually orthogonal and normalized to unity:

\[
\langle \varphi_n, \varphi_m \rangle = \delta_{nm} = \begin{cases} 1 & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases}
\]

(b) The \( \varphi_i \) are complete, which means that every vector of the Hilbert space can be expanded into a convergent sum of them.

Important properties of expansions in terms of complete orthonormal sets are: (a) The expansion coefficients \( \gamma_k \) are given by

\[
\gamma_k = \langle \varphi_k, f \rangle.
\]
(b) They fulfill
\[ \sum_{k=0}^{n} \gamma_k^2 \leq \| f \|^2 \text{ for all } n, \text{ and } \sum_{k=0}^{\infty} \gamma_k^2 = \| f \|^2 \] (Bessel’s inequality and Parseval’s equation).

Given two expansions \( f = \sum_{k=0}^{\infty} \gamma_k \varphi_k, \ g = \sum_{k=0}^{\infty} \gamma'_k \varphi_k \), the scalar product can be expressed as
\[ \langle f, g \rangle = \sum_{k=0}^{\infty} \gamma_k \gamma'_k. \] (8)

If the expansion coefficients are combined into coefficient vectors \( c = (\gamma_0, \gamma_1, \ldots), \ c' = (\gamma'_0, \gamma'_1, \ldots) \), the preceding equation takes the form \( \langle f, g \rangle = c \cdot c' \).

The Fourier expansions considered by Galeas et al. [4] are an example of such an expansion. The functions
\[ \varphi^{\text{Fo}}_0(x) = \frac{1}{\sqrt{L}}, \varphi^{\text{Fo}}_{2k-1}(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{2\pi k}{L} x \right), \varphi^{\text{Fo}}_{2k}(x) = \sqrt{\frac{2}{L}} \cos \left( \frac{2\pi k}{L} x \right) \] (9)

\((k > 0)\) form a complete orthonormal set in \( L_2([0, L]) \), leading to an expansion
\[ f(x) = \frac{a_0}{\sqrt{L}} + \sqrt{\frac{2}{L}} \sum_{k=1}^{\infty} \left[ a_k \cos \left( \frac{2\pi k}{L} x \right) + b_k \sin \left( \frac{2\pi k}{L} x \right) \right], \] (10)

where \( a_0 = \gamma_0 \) and \( a_k = \gamma_{2k}, b_k = \gamma_{2k-1} \) for \( k > 0 \).

Another complete set of orthonormal functions of \( L_2([0, L]) \) is given by
\[ \varphi^{\text{Le}}_k(x) = \sqrt{\frac{2k+1}{L}} P_k^*(x/L) \quad k \geq 0, \] (11)

where the \( P_k^*(x) \) are so-called shifted Legendre polynomials [1]. These polynomials are of order \( k \). The first few of them are \( P_0^*(x) = 1, P_1^*(x) = 2x - 1, P_2^*(x) = 6x^2 - 6x + 1, P_3^*(x) = 20x^3 - 30x^2 + 12x - 1 \). Fig. 1 (left) shows \( \varphi^{\text{Le}}_k(x) \) for \( 0 \leq k \leq 4 \) in the range \( x \in [0, L] \) for \( L = 1 \).

Another example that will be used later is a complete set for the space \( L_2([R_+]) \) (the space of square-integrable functions for \( 0 \leq x < \infty \)):
\[ \varphi^{\text{L}}_k(x) = \frac{e^{-x/(2\lambda)}}{\sqrt{\lambda}} L_k(x/\lambda), \quad k \geq 0, \] (12)

Here, \( \lambda \) is a positive scale parameter and the \( L_k(x) \) are Laguerre polynomials [1], the first few of which are \( L_0(x) = 1, L_1(x) = -x + 1, L_2(x) = x^2/2 - 2x + 1, L_3(x) = -x^3/6 + 3x^2/2 - 3x + 1 \), see Fig. 1 (right).

### 3.2 Truncated Expansions of Term Distributions

As explained above, the finite sums \( f_n = \sum_{k=0}^{n} \gamma_k \varphi_k \) converge to the function \( f \) in the sense of norm convergence. As a consequence of Bessel’s inequality (7) they approximate \( f \) increasingly better for increasing \( n \). An essential ingredient for the following
discussion is to consider a truncated expansion, i.e., the mapping

\[ P_n : f^{(t)} \mapsto f_n^{(t)}, \]  

which associates to a term distribution \( f^{(t)} \) of the form (1) its finite-order approximation \( f_n^{(t)} \) in terms of some complete orthonormal set for some order \( n \).

Figure 2 shows an example for the Fourier expansion. One can observe the characteristic broadening effect generated by the reduction of the expansion order (truncation).

The \( L_2 \) scalar product of two truncated term distributions \( f_n \) and \( g_n \),

\[ \langle f_n, g_n \rangle = \int f_n(x) g_n(x) \, dx \]  

has the meaning of an overlap integral: The integrand is only large in regions in which both functions \( f_n(x) \) and \( g_n(x) \) are large, so that \( \langle f_n, g_n \rangle \) measures how well both functions overlap in the whole integration range.

Given \( f_n \) and \( g_n \), two truncated term distributions describing the term positions and their neighborhood in a certain document, we introduce the concept of semantic interaction range: Two terms that are close to each other present a stronger interaction.
because their truncated distributions have a considerable overlap. This semantic interaction range motivates the following definition of the similarity of two term distributions $f$ and $g$: For some fixed order $n$, one sets

$$\text{sim}(f, g) = \langle f, g_n \rangle = \langle P_n f, P_n g \rangle.$$  

(15)

In this definition, the truncation $P_n : f \mapsto f_n$ is essential, because the original term distributions $f$ and $g$ are always orthogonal if they describe two different terms. This is so because different terms are always at different positions within a document, so that their overlap always vanishes.

Definition (15) is only one possibility. In fact, any definition based on the scalar product $\langle f_n, g_n \rangle$ can be utilized. For example, in Galeas et al. [4] a cosine definition

$$\cos \vartheta = \frac{\langle f_n, g_n \rangle}{\|f_n\| \|g_n\|}$$

has been used. Another choice is the norm difference

$$\|f_n - g_n\| = \left( \int (f_n(x) - g_n(x))^2 \, dx \right)^{1/2} = \sqrt{\|f_n\|^2 + \|g_n\|^2 - 2\langle f_n, g_n \rangle}.$$  

(16)

Using different measures based on $\langle f_n, g_n \rangle$, we have found no significant differences in the final retrieval results in several experiments.

The scalar product of the truncated distributions can be easily calculated using the coefficient vectors: If the original distributions $f$ and $g$ have the infinite-dimensional coefficient vectors $c = (\gamma_0, \gamma_1, \ldots)$ and $c' = (\gamma'_0, \gamma'_1, \ldots)$, respectively, then the truncated distributions $f_n$ and $g_n$ have the $(n+1)$-dimensional coefficient vectors $c_n = (\gamma_0, \gamma_1, \ldots, \gamma_n)$ and $c'_n = (\gamma'_0, \gamma'_1, \ldots, \gamma_n)$, resp., and their scalar product is the finite sum

$$\langle f_n, g_n \rangle = c_n \cdot c'_n = \sum_{k=0}^n \gamma_k \gamma'_k.$$  

(17)

### 3.3 The Semantic Interaction Range

In this section, a precise definition of the semantic interaction range is given.

In abstract terms, the truncation $P_n : f \mapsto f_n$ is a filtering or a projection: In the expansion $f(x) = \sum_{k=0}^\infty \gamma_k \varphi_k(x)$ the components $\varphi_k$ for $k > n$ are filtered out, which amounts to a projection of $f$ onto the components $\varphi_0, \ldots, \varphi_n$. Thus, $P_n$ is a projection operator in the Hilbert space. To derive a closed expression for the operator $P_n$, one combines $(P_n f)(x) = f_n(x) = \sum_{k=0}^n \gamma_k \varphi_k(x)$, with (6) to obtain

$$(P_n f)(x) = \sum_{k=0}^n \left( \int \varphi_k(y) f(y) \, dy \right) \varphi_k(x) = \int \left( \sum_{k=0}^n \varphi_k(y) \varphi_k(x) \right) f(y) \, dy.$$  

(18)

One can write the last expression as $\int p_n(y, x) f(y) \, dy$ with the projection kernel

$$p_n(y, x) = \sum_{k=0}^n \varphi_k(y) \varphi_k(x).$$  

(19)
as an integral representation of $P_n$ in the sense of a convolution. It has the advantage that one can study the properties of the truncation independently of the function $f$.

The width of $p_n(y, x)$ as a function of $x$ is a lower bound for the width of a truncated expansion of a term located at $y$. Therefore, this width will be used as the semantic interaction range for a term at position $y$.

For the Fourier expansion, $p_{2k}$ is given by

$$p_{2k}^{Fo}(y, x) = \frac{\cos(4\pi k(y - x)/L) - \cos(2\pi(2k + 1)(y - x)/L)}{L(1 - \cos(2\pi(y - x)/L))}.$$  \hspace{1cm} (20)

(We consider only even orders $n = 2k$, because for these orders the expansion consists of an equal number of sine and cosine terms, see (9).) The maximum of $p_{2k}^{Fo}(y, x)$ is at $x = y$ and the two zeros closest to the maximum are at $x = y \pm L/(2n + 1)$. Thus, the semantic interaction range for a Fourier expansion of order $n$ may be defined to be

$$\varrho_n^{Fo} = \frac{2L}{2n + 1}.$$  \hspace{1cm} (21)

Fig. 3 (left) shows $p_{6}^{Fo}(20, x)$ and $p_{6}^{Fo}(100, x)$ for $L = 200$.

For the expansions in terms of Legendre and Laguerre polynomials, the projection kernels can be calculated with the Christoffel-Darboux equation [1]. The results are

$$p_{n}^{i}(y, x) = \alpha_{n}^{i} \frac{\varphi_{n+1}^{i}(y)\varphi_{n}^{i}(x) - \varphi_{n}^{i}(y)\varphi_{n+1}^{i}(x)}{y - x},$$  \hspace{1cm} (22)

$i = Le, La$, with $\alpha_{n}^{Le} = (L/2)(n + 1)/(2n + 1)$ and $\alpha_{n}^{La} = -\lambda(n + 1)$. These kernels are no longer functions of $y - x$, meaning that the broadening of a term distribution depends on the position $y$ of the term distribution within the document.

Fig. 3 (right) shows the projection kernel $p_{6}^{La}(y, x)$ for $y = 20$ and $y = 100$. One can see that the spatial resolution of the truncated expansion decreases for terms that are far away from the beginning of the document.
4 Applications

The goal of our approach is to shift the complexity of processing the positional data from the query evaluation phase to the (not time critical) indexing phase, reducing the ranking optimization via term positions to a simple mathematical operation.

Hence, we propose to calculate the expansion coefficients $\gamma_k$ of the term distributions in the indexing phase and to store this abstract term positional information in the index. This permits a considerably faster query evaluation, compared with methods that use the raw term-positional information.

Thus, the index contains an $(n+1)$-dimensional coefficient vector $c_n = (\gamma_0, \gamma_1, \ldots, \gamma_n)$ for each term and each document in the collection. The $\gamma_k$ are calculated analytically via (6). To give an example of the complexity involved,

$$\gamma_k = \sum_{p \in P} \sum_{j=0}^{k} \alpha_j \left[ \left( \frac{p}{L} \right)^{j+1} - \left( \frac{p-1}{L} \right)^{j+1} \right]$$

with $\alpha_j = \sqrt{(2k+1)L a_j} / (j+1)$ is the expression for the expansion coefficients in the case of the expansion in terms of Legendre polynomials, cf. (1). (The $a_j$ are the polynomial coefficients of the shifted Legendre polynomial of order $k$.) Calculations of this kind can be easily performed in the indexing stage.

The retrieval scenarios that we have investigated are: (a) ranking optimization based on user-defined objective functions and (b) query expansion based on term-positional information [4], and (c) cluster analysis of terms in documents. They all involve a calculation of the similarity of term distributions.

4.1 Ranking Optimization

The first scenario states document ranking as an optimization problem that is based on the query term distribution function $f_{q,d}$ and a user-defined objective function $f_o$ representing the optimal query term distribution in the document body:

$$\text{Maximize } \{ \text{sim}(f_{q,d}, f_o) \} \quad \forall f_{q,d} \in A$$

where $A$ represents the query term distributions in a document set, $f_{q,d}$ is the query term distribution function for query $q$ in document $d$, and $f_o$ is a user-defined objective function, representing the optimal query term distributions for the documents in the document ranking. Experiments based on the TREC-8 collection and the software Terrier [5], carried out to order $n = 6$, show the accuracy of the term distributions in a ranking based on user-defined objective functions. As depicted in Figure 4, the Fourier and Legendre models present a high accuracy for the distribution of query terms in the top-20 ranked documents, based on two different objective functions: The first function (denoted $f_o = 1|3$) selects terms located in the first third of the document, and the second ($f_o = 3|3$) selects terms located in the last third of the document [4].
4.2 Query Expansion

The second scenario considers the top-$r$ documents $D = \{d_1, d_2, \ldots, d_r\}$ of an initial ranking process and the functions $f_{q,d}$ with $d \in D$. The set of terms $T_q$ whose elements $t$ maximize the expression $\text{sim}(f_{q,d}, f_{t,d})$ is computed. It contains the terms for all documents in $D$ that have a similar distribution as the query, i.e. terms positioned near the query in the top ranked documents. This set $T_q$ is used to expand $q$.

As depicted in Figure 5, experiments executed on the TREC-8 collection demonstrate that query expansion based on the proposed orthogonal functions (Fourier and Laguerre) outperform state-of-the-art query expansion models, such as Rocchio and Kullback-Leibler [5]. The term position models (left) differ from the other models (right) because the former tend to increase the retrieval performance by increasing the number of expansion documents and expansion terms, while for the other models, the performance drops beyond roughly the 15th expansion document.

Figure 6 (left) shows a fixed query expansion configuration in which the other models show their best performance. Nevertheless, the term distribution models perform better. Any increase in the number of expansion documents or expansion terms makes the superiority of the term distribution models even clearer.

4.3 Cluster Analysis of Terms in Documents

Given a document, one may ask whether there are groups (clusters) of terms whose elements all have similar distributions. One may then infer that all terms inside a cluster
describe related concepts [2]. In this section, some properties of the proposed method will be explained that may be useful for the analysis of term clusters.

Consider a document of length $L$. Since at every position within the document a particular term may either be present or not, there are in total $N = 2^L$ possible term distributions. Each of these distributions is mapped to a point in an $(n + 1)$-dimensional Hilbert space. If the norm difference (16) is used as the similarity criterion, then clusters of similar term distributions are just Euclidean point clusters in the Hilbert space.

We will now investigate the geometrical structure of the set of all possible term distributions. Let us first calculate the center $\tilde{f}(x) = (1/N) \sum_{\nu=1}^{N} f^{(\nu)}(x)$ of all term distributions (here $f^{(\nu)}(x), \nu = 1, \ldots, N$, is an enumeration of distributions of the form (1)). At any position $x$, half of all $N$ distributions have a term present ($f^{(\nu)}(x) = 1$) and the other half does not ($f^{(\nu)}(x) = 0$), so that $f(x) = 1/2 = \text{const}$ for all $x \in [0, L]$. This average distribution is mapped to a non-truncated, in general infinite-dimensional coefficient vector $\tilde{c}$, whose length $|\tilde{c}|$ is given by the norm $||\tilde{f}|| = \left[ \int_0^L dx / 4 \right]^{1/2} = \sqrt{L}/2$. The squared distance between the center point and the coefficient vector $c^{(\nu)}$ of a distribution $f^{(\nu)}$ is $|\tilde{c} - c^{(\nu)}|^2 = ||f - f^{(\nu)}||^2 = \int_0^L (1/2 - f^{(\nu)}(x))^2 dx$. Since $f^{(\nu)}(x)$ is either 0 or 1, it follows that $(1/2 - f^{(\nu)}(x))^2 = 1/4 = \text{const}$ for all $x \in [0, L]$, giving
Fig. 6. Left: Query Expansion performance for the term distribution models (Fourier, Legendre and Laguerre) and the other models, using a configuration of 15 expanded documents and 40 expanded terms. Right: Three dimensional sphere of all 512 possible term distributions in a document of length $L = 9$ for the expansion in terms of Legendre polynomials.

$|c^{(\nu)} - \bar{c}| = \sqrt{L}/2$ for all $\nu$. This means that the non-truncated coefficient vectors of all term distributions lie on the surface of a sphere with radius $\sqrt{L}/2$ whose center is at $\bar{c}$. Because $|\bar{c}| = \sqrt{L}/2$, this sphere touches the origin of the Hilbert space.

Bessel’s inequality (7) leads to $|c^{(\nu)}_n - \bar{c}_n| \leq \sqrt{L}/2$ for all $\nu$ for the coefficient vectors truncated to order $n$. Thus, the truncated vectors all lie within a sphere of radius

$$R_0 = \sqrt{L}/2$$

in the $(n+1)$-dimensional Hilbert space. The center of this sphere is at $\bar{c}_n$. If—as in the Fourier and Legendre cases—one of the expansion functions, say $\varphi_0(x)$, is constant, the vector $\bar{c}$ describing itself a constant function has only a non-vanishing zero component: $\bar{c} = \bar{c}_n = (\sqrt{L}/2, 0, 0, \ldots)$. Fig. 6 (right) shows this term sphere in $n + 1 = 3$ dimensions for a document of length $L = 9$ and the expansion in terms of Legendre polynomials.

The fact that all possible truncated coefficient vectors $c^{(\nu)}_n$ lie within a sphere whose radius and center are known is very useful for clustering analysis. First of all, it shows where in the Hilbert space to look for clusters. Secondly, assume one has found a cluster $K = \{k_1, \ldots, k_q\}$ of term distributions by some clustering algorithm (for an $n$th order truncation). The volume of this cluster can be estimated by calculating the standard deviation $R_K = [(1/q) \sum_{i=1}^q (k_i - \bar{k})^2]^{1/2} = [(1/(2q^2)) \sum_{i,j=1}^q (k_i - k_j)^2]^{1/2}$ (here $\bar{k}$ is the center of the cluster) and approximating the cluster by a sphere of radius $R_K$. Since the volume of a sphere of radius $R_K$ in $n+1$ dimensions is proportional to $R_K^{n+1}$, the cluster occupies approximately a part $\xi = (R_K/R_0)^{n+1} = (2R_K/\sqrt{L})^{n+1}$ of the theoretically available space. A cluster would then be considered as significant only if $\xi \ll 1$. An analysis of this kind may be useful to generate an ontology of terms based on individual documents.
It has been conjectured that the use of quantum mechanical methods, in particular infinite-dimensional Hilbert spaces and projection operators, may be advantageous in IR [8]. The approach presented here goes into this direction, because constructing appropriate sets of orthogonal functions is a standard technique in quantum mechanics. Still, we emphasize that our approach is essentially classical, not quantum mechanical, since it does not use any of the interpretational subtleties of quantum mechanics.

5 Conclusions

In this paper, a new approach to improve document relevance evaluation using truncated Hilbert space expansions has been presented. The proposed approach is based on an abstract representation of term positions in a document collection which induces a measure of proximity between terms (semantic interaction range) and permits their direct and simple comparison. Based on this abstract representation, it is possible to shift the complexity of processing term-positional data to the indexing phase, permitting the use of term-positional information at query time without significantly affecting the response time of the system. Three applications for IR were discussed: (a) ranking optimization based on a user-defined term distribution function, (b) query expansion based on term-positional information, and (c) a cluster analysis approach for terms within documents.

There are several areas of future work. For example, (a) quantifying the effect of the abstract term positions representation in the index size, (b) measuring the effectiveness of the proposed clustering approach, and (c) studying objective functions in documents having homogeneous structures (forms) are some of the topics that should be investigated.

References

1. M. Abramowitz, I. Stegun, M. Danos, and J. Rafelski. Pocketbook of Mathematical Functions. H. Deutsch, 1984.
2. R. Attar and A. S. Fraenkel. Local feedback in full-text retrieval systems. Journal of the ACM, 24(3):397–417, 1977.
3. M. Beigbeder and A. Mercier. An information retrieval model using the fuzzy proximity degree of term occurrences. In SAC ’05: Proceedings of the 2005 ACM Symposium on Applied Computing, pages 1018–1022, New York, NY, USA, 2005. ACM.
4. P. Galeas, R. Kretschmer, and B. Freisleben. Document relevance assessment via term distribution analysis using Fourier series expansion. In JCDL ’09: Proceedings of the 2009 Joint International Conference on Digital Libraries, pages 277–284, New York, NY, USA, 2009. ACM.
5. I. Ounis, G. Amati, V. Plachouras, B. He, C. Macdonald, and C. Lioma. Terrier: A high performance and scalable information retrieval platform. In Proceedings of ACM SIGIR’06 Workshop on Open Source Information Retrieval (OSIR 2006), 2006.
6. L. A. Park, K. Ramamohanarao, and M. Palaniswami. Fourier domain scoring: A novel document ranking method. Transactions on Knowledge and Data Engineering, 16(5):529–539, May 2004.
7. T. Tao and C. Zhai. An exploration of proximity measures in information retrieval. In *SIGIR ’07: Proceedings of the 30th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 295–302, New York, NY, USA, 2007. ACM.
8. C. J. van Rijsbergen. *The Geometry of Information Retrieval*. Cambridge University Press, New York, NY, USA, 2004.
9. K. Yosida. *Functional Analysis*. Springer, 1980.