Pinning Transition of Bose-Einstein Condensates in Optical Ring Resonators

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We experimentally explore the stability of Bose-Einstein condensates inside an optical ring resonator with additional periodic pinning potential. Beyond a critical resonator detuning the system undergoes a transition to a new stable phase. Phase diagrams and quench curves are recorded and described by numerical simulations. We also discuss a physical explanation based on a geometrical interpretation of the underlying nonlinear equations of motion.

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Cold atoms in optical resonators is a classic in quantum optics. Experimentally and theoretically equally accessible, these model systems allow to explore a broad spectrum of fundamental topics [1]. Current research concentrates on cavity mediated long range atomic interactions, quantum phase transitions, quantum many body physics, and solid state modelling [2], [3], [4], [5]. While the majority of experiments are carried out with standing wave resonators, the specific properties of ring resonators are far less explored. Different to standing wave resonators, the position of an optical standing wave in an ideal ring cavity is not fixed by the mirrors. The emergence of atomic density patterns which breaks this continuous symmetry was predicted in 1998 and interpreted as cold atom analogy to the free electron laser [6]. Early continuous symmetry was predicted in 1998 and interpreted as cold atom analogy to the free electron laser [6]. Early experiments have observed this effect already more than a decade ago [7],[8] but only recently the related phase diagram has been recorded with Bose-Einstein condensates which has been investigated in a series of pioneering experiments [9].

While the physics in ring resonators may be interpreted as an extension of the paradigmatic Dicke phase transition which has been investigated in a series of pioneering experiments with standing wave resonators [12], [2]. Ring resonators are also discussed as promising candidates for the realization of super solid states in atomic quantum gases. [13]

The here described experiment is based on the standard setup of a $^{87}$Rb-Bose-Einstein condensate in a ring resonator that is longitudinally pumped in one direction (pump mode) [2]. In addition, some light is injected also into the reverse propagating mode (probe mode). The interference between the pump mode and the probe mode generates a very weak periodic pinning potential that competes with the optical lattice generated by the atoms. We observe a new stable phase that appears if the resonator is detuned beyond a critical value. Below this value the self generated optical lattice of the atoms overrides the pinning potential and the system becomes unstable. The observations are well described by numerical simulations; however, a more physical explanation is possible by means of a geometric interpretation of the nonlinear equations of motion in the vicinity of the phase boundary.

The experimental setup is similar as described in [2]; however we now use a much larger resonator with a round trip length of 39 cm a beam waist at the position of the condensate of $w_0 = 170 \mu m$ and a mode volume of $V = 18.2 \text{mm}^3$. For $s$-polarized light the decay rate for the electric field amplitude in the resonator amounts to $\kappa = 2\pi \cdot 5 \text{kHz}$ which is about 3 times smaller than the recoil shift $\omega_r = 2\hbar k^2/m = 2\pi \cdot 14.5 \text{kHz}$ due to momentum absorption of an initially non moving atom that scatters a photon from a pump beam into the probe beam. Here, $k$ and $m$ are the wave vector of the pump light and the mass of the atom. For $p$-polarized light we observe a 3 times larger decay rate. The $s$-polarized forward propagating TEM$_{00}$ mode ("pump mode") is longitudinally pumped from one side with up to 6 mW from an amplified diode laser system at a frequency $\omega$ detuned by $\Delta_0 = \omega - \omega_0 = -60 \text{GHz}$ relative to the atomic transition frequency $\omega_0$ (D1 Line: 5s1/2, F = 2 to 5p1/2, F = 2).

Part of the laser output is used to electronically stabilize the laser to the reverse propagating $p$-polarized TEM$_{10}$ mode with a precision of about $2\pi \cdot 200 \text{Hz}$. Frequency and amplitude of the injected pump light is controlled by an acousto optical modulator. The pump frequency $\omega$ can be tuned relative to the resonance frequency $\omega_c$ of the TEM$_{00}$ mode over a range of $\Delta_c = \omega_c - \omega = \pm 10 \omega_r$. The power in the TEM$_{00}$ mode and in the counter-propagating TEM$_{00}$-mode (probe mode) is monitored by recording the light leaking out of the resonator mirrors with sensitive avalanche diodes.

The pinning potential required to generate a stable phase is very small such that we don’t have to inject the probe mode externally but rather exploit coherent scattering of pump light into the probe mode due to inhomogeneities in the mirror coatings. The scattered light from the three mirrors interferes according to their relative positions and to the wavelength of the light [13]. Different longitudinal modes may thus exhibit very different total mirror scattering. Furthermore, the total mirror scattering can be varied up to a factor of three by controlling the position of one of the mirrors with a piezo element. In the experiment, the total mirror scattering rate $\kappa = \kappa \cdot \sqrt{\varepsilon}$ is determined for each cycle by recording the resonant power ratio $\varepsilon$ of the pump and the probe mode right before the atoms are loaded into the cavity. The magnetically trapped Bose-Einstein conden-
sate of $^{87}$Rb-atoms is placed at the intensity minimum at the center of the TEM$_{00}$ mode where the atoms are least affected by the locking light. During preparation of the condensate, the laser beams are switched off and held at one fixed frequency for about 20s. Once the condensate is in place, the locking is reactivated within 300 ms and the pump light is then ramped up to a final value within 50 μs, slow enough to avoid ringing of the high finesse resonator. After a holding time of 1.5 ms the atoms are released from the trap and the population of the momentum states are derived from absorption images after 35 ms of ballistic expansion.

Data are taken from 20000 experimental cycles for various cavity pump detuning $\Delta_c$ and photon numbers $|a_P|^2$ in the pump mode. The data are post selected according to the value of the Ratio $R := \kappa_s/(2U_0 N)$ for the specific cycle. The denominator contains the total number of atoms $N$ and the single photon light shift $U_0 = g_c^{\text{eff}}/\Delta_a$ with the coupling constant $g_c^{\text{eff}} = \frac{d^2}{6\pi \varepsilon_0 a} = 2\pi \cdot 19 \text{kHz}$, the dipole moment of the atomic transition $d$ and the permittivity of free space $\varepsilon_0$. The ratio $R$ turns out to be the relevant parameter to specify the strength of the pinning potential (see theory part below). Figure 1 shows the observed population $|c_0|^2$ of the zero momentum state in the case of large mirror scattering ($R = 0.15$). The detuning $\Delta_c = \Delta_c + U_0 N$ plotted along the vertical axis is corrected for the index of refraction due to the atoms. The blue area where the system is unstable and almost all atoms are excited into higher momentum states is clearly separated from the stable regime where at least half of the population $|c_0|^2$ persists. For $\Delta_c = -\omega_r$ the critical pump photon number for entering the unstable regime is smallest since light scattered from the initial condensate is recoil shifted by one $\omega_r$. For $\Delta_c < -\omega_r$ the phase boundary between the stable and the unstable regime follows the prediction of a numerical simulation which ignores the pinning potential (solid line in the left and right subplot). Obviously, the pinning potential has only little effect in this regime. This is because at threshold the system jumps from a homogeneous superfluid state directly into a state where the atoms form a density grating that moves with a finite start velocity $\epsilon$. In the reference frame of the moving atoms the pinning potential averages out and has no effect. On the contrary, for positive detuning the atoms form a stationary density grating which can be seeded efficiently by the pinning potential. In fact, the observed phase boundary steeply increases in this regime and asymptotically approaches a vertical line positioned at a critical detuning of $\Delta_0 \approx 0.7 \omega_r$ (dashed line in the left and right subplot). A numerical simulation which includes pinning reproduces this behavior (right subplot).

The theoretical analysis of the experiment describes the light in the pump mode and the probe mode by the field operators $A_p = a_p e^{i k z}$ and $A = a e^{-i k z}$. The atomic matter field $\psi = \sum c_n e^{i n k z}$ is expanded into momentum eigenstates separated by $2\hbar$ which is the momentum transferred to the atoms by scattering a single photon from the pump mode into the probe mode. The atoms and the light interact via the optical dipole potential $H_{\text{int}} = \hbar U_0 \int \psi^\dag \psi (\hat{A}_p + \hat{A}) (\hat{A}_p^\dag + \hat{A}^\dag) \, dV$. Mirror scattering couples the pump mode and the probe mode and forms the pinning potential, $H_p = -\hbar \kappa_s (\hat{A}_p \hat{A} + \hat{A}_p^\dag \hat{A}^\dag)$. The equations of motion are derived from the Hamiltonian $H = H_0 + H_{\text{int}} + H_p$ with $H_0 = \int (\psi^\dag (-\hbar^2 \nabla^2 / (2m)) \psi + h \Delta_c (\hat{A}^\dag A + A^\dag A_p)) \, dV$. Atomic collisions are neglected. In mean field approximation, operators are replaced by their expectation values $a_p, a$, and $c_n$. Since the power of the pump mode is electronically stabilized we set $a_p$ to be constant. Since only the relative phase between $a$ and $a_p$ is physically relevant we also set $a = |a_p|$. For the equations of motion one then gets

$$
\dot{c}_n = -i n^2 \omega_r c_n - i \sigma (c_{n-1} a^* + c_{n+1} a) \tag{1}
$$
$$
\dot{a} = - (\kappa + i \Delta_c) a - i \sigma \sum_n c_n^* c_{n-1} - i \kappa_s |a_p|
$$

with the coupling constant $\sigma := U_0 |a_p|$ and the total number of atoms $N = \sum c_n^* c_n$. The finite cavity line width is taken into account by adding the decay term $-\kappa a$. The simulations in FIG. 1 are based on equ. (1) with the sum ranging from $n = -5$ to $n = 5$.

To gain further physical insight we interpret equations (1) in the vicinity of the threshold. Higher momentum

![FIG. 1: (color online). Phase diagram. Population of the zero momentum state after 1.5 ms of interaction with the light in the resonator for various photon numbers in the pump mode and cavity pump detuning. For negative detuning the experimental observations (left subplot) are well described by the phase boundary derived from a numerical simulation that does not include the pinning potential (solid line). The simulation shown in the right subplot includes the pinning potential. The dashed line indicates the critical detuning in the limit of strong pumping according to equ. (5).](image-url)
states with $|n| > 1$ can then be neglected yielding
\begin{align}
\dot{c}_{-1} &= -i\omega_c c_{-1} - ia c_0 \\
\dot{c}_0 &= -i\sigma (ac_1 + a^* c_{-1}) \\
\dot{c}_1 &= -i\omega_c c_1 - i\sigma a^* c_0 \\
\dot{a} &= -i\Delta a - i\sigma b - i\kappa_s |a_p|.
\end{align}

Here, we introduce the complex detuning $\Delta = |\Delta| e^{i\delta} := \Delta_c - i\kappa$ and the complex structure factor $b = |b| e^{i\varphi_b} := c^*_1 c_0 + c^*_0 c_{-1}$. Without mirror scattering, the population of the zero momentum component $|c_0|^2$ remains undepleted until close to threshold $a$ and $c_{\pm 1}$ become nonzero. In previous work, $c_0$ was thus approximated as constant near threshold. The equations then become linear and can be solved analytically \cite{10,17}. If, in contrary, mirrors scatter pump light into the probe mode, a standing wave is formed and the resulting optical lattice potential depletes the zero momentum component even below threshold. Thus $c_0$ has to be kept as variable and the equations resume their nonlinear character. Treating equ. (2a)-(2d) by linearization around the steady state so-determine how, vice versa, a given structure factor leads to 

\begin{align}
B &= \frac{A/A_s}{\sqrt{1 + |A|^2 / A_s^2}},
\end{align}

introducing the normalized strength of the structure factor $B := |b|/(2N\sigma)$, the normalized amplitude of the light mode $A := |a| |\Delta|/(2N\sigma)$ and the saturation parameter $A_s := |\Delta|/2\sqrt{8N\sigma^2})$. For both states the structure factor is time independent and saturates at a maximum $B_m := 1/\sqrt{8}$ as $A_s$ approaches zero for strong pumping. The two states differ in the limit of vanishing $a$ where the population $|c_0|^2$ approaches either zero or $N$. We thus disregard the first case since in the experiment all atoms are initially in the condensate. The calculation shows that in this case the phases of the structure factor and the light field are equal, $\varphi_b = \varphi_a$. In a second step we determine how, vice versa, a given structure factor leads to a stable light field. Setting $\dot{a} = 0$ in equ. (2d) yields
\begin{align}
A^2 + B^2 + 2AB \cos \delta = R^2.
\end{align}

In figure 2 equ. 2 and 4 are plotted. Equ. 4 forms an ellipse tilted by $45^\circ$. Its long axis varies between $2R$ (circle) for $\Delta_c = 0$ and infinity for $\Delta_c \gg \kappa$. Equilibrium states exist at the intersection points of both curves. The stability of equilibrium points is determined by reading from the diagram how a given field $A_1$ results in a structure factor $B$ (vertical arrows) and how the so-determined $B$ generates a new light field $A_{s+1}$ (horizontal arrows). By repeating this sequence, the resulting series $A_1$ converges for stable equilibrium and diverges otherwise. For small detuning (left subplot) and weak pumping (saturation curve 1) one finds a single point of stable equilibrium (indicated by "s"). For stronger pumping the point moves to smaller $A$ and eventually becomes unstable (saturation curve 2, "u"). Without condensate depletion (neglecting the second term in the square root of equ.4) the system becomes unstable for $A_s = 1/\sqrt{8}$ which reproduces the threshold behavior found in previous models \cite{10,17,16}. For large detuning, the stable point remains stable even for large pumping. This is true for arbitrary pump strength only if the maximum of the ellipse $R/\sin(\delta)$, exceeds the maximum value of the structure factor $B_m$ (dashed line). This condition determines the critical detuning $\sin(\delta_0) = R/\sqrt{8}$. After replacing the above definitions the critical detuning that defines the vertical phase boundary in the limit of strong pumping reads
\begin{align}
\frac{\Delta_0}{\kappa} = \sqrt{\frac{1}{8} \left( \frac{2U_0 N}{\kappa_s} \right)^2 - 1}.
\end{align}

It depends on the strength of the pinning potential $\kappa_s$ via the ratio $1/R = 2U_0 N/\kappa_s$.

We tested this relation by recording the phase boundary for various mirror scattering $\kappa_s$ and atom number $N$. The phase boundary is detected by sweeping the detuning $\Delta_c$ from large to small values within 1ms while the photon number in the pump mode is electronically stabilized to a constant value of $|a_p|^2 = 4 \cdot 10^6$ (inset in FIG. 3).

While sweeping, the power in the probe mode increases until eventually the threshold is reached. We identify
By repeating the experiment for various values the probe mode drops quickly and the system becomes unstable. By seeding the probe mode, the transition from a ring geometry to a standing wave geometry can be explored similar as in recent work with a condensate replaced by a nano membrane. More work is required to understand the role of the two additional points of equilibrium which appear above the critical detuning. Also unclear is the classification of the phase transition including noise properties near threshold and possible metastability.

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In summary, we have investigated an atomic Bose Einstein condensate in an optical ring resonator with additional pinning potential. A new stable phase was identified above a critical value of the cavity pump detuning. The phase boundary is defined by the competition of the pinning potential and the optical potential generated by the atoms. The observations are quantitatively described by simulating the nonlinear equations of motion including depletion of the condensate. A geometric interpretation is introduced to determine equilibrium and stability of the system and an analytic expression for the phase boundary is derived in the limit of strong pumping.

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