Lightweight Mediated Semi-Quantum Secret Sharing Protocol

Chia-Wei Tsai\(^1\), Zong-Liang Zhang \(^2\), Bo-Cheng Jian \(^3\), and Yao-Chung Chang\(^4\)

Department of Computer Science and Information Engineering, National Taitung University, Taitung 95092, Taiwan

\(^1\)cwtsai@nttu.edu.tw
\(^2\)study00000001@gmail.com
\(^3\)jejoru03@yahoo.com
\(^4\)ycc@nttu.edu.tw

Abstract

Due to the exiting semi-quantum secret sharing protocol have two challenges including (1) the dealer must be the quantum user, and (2) the classical users must equip with the Trojan Horse detectors, this study wants to propose a novel mediate semi-quantum secret sharing (MSQSS) protocol to let a classical dealer can share his/her secrets to the classical agents with the help of a dishonest third-party (TP). The proposed MSQSS protocol adopts the one-way quantum communication and thus it is free from the Trojan Horse attacks. Furthermore, the security analysis is given for proving that the proposed protocol can be against the collective attack. Comparing to the exiting SQSS protocols, the proposed MSQSS protocol is more lightweight and more practical.

Keywords: semi-quantum; secret sharing protocol; dishonest third party; Trojan Horse attack.

1. Introduction

How to let a dealer share his/her secrets with the agents and the dealer’s secret can be recover when enough number of agents cooperate is an important research issue. For this issue, Shamir [1] proposed the first solution - the secret sharing protocol, in which a master can split their secret to several secret shadows, transmit these shadows to their agents, and then allow the agents to recover the master’s secret. The security of classical secret sharing (CSS) protocol is depending on high computational complexity (e.g., factorization of large numbers) mathematical problems. Therefore, the security of these CSS protocols is conditional. That is to say, they can be insecure if some technologies can solve these computational complexity mathematical problems (e.g., quantum algorithms). To overcome this issue, Hillery et al. [2] proposes the quantum secret sharing (QSS) protocol by using Greenberger–Horne–Zeilinger (GHZ) state in 1999. The QSS protocol used quantum mechanics to complete the same goal of CSS protocols. After Hillery et al.’s QSS protocol, various QSS protocols [3–44] have been since proposed. However, the aforementioned QSS protocols always assume that both the master and agents have the complete quantum devices (e.g., quantum memory, quantum measurement device, and generator for various quantum states). Some quantum devices remain expensive and difficult to implement. Therefore, whether all participants should use complete quantum capabilities in quantum protocols is an interesting research challenge. For this challenge, Boyer et al. [45] given a positive answer, and a novel environment, semi-quantum environment, is proposed for improving the practicality of quantum protocol. In the semi-quantum environment, there are two kinds of users, the quantum users who have the complete quantum capabilities, and the classical users who only equip with the limited quantum devices. Table 1 summarizes the quantum capabilities of the classical users depending on the various semi-quantum environment.

| Environment               | Capabilities of classical user                                      |
|---------------------------|---------------------------------------------------------------------|
| Measure–Resend            | (1) generating Z-basis qubits                                       |
|                           | (2) Z-basis measurement                                             |
|                           | (3) reflecting photons without disturbance                          |
| Randomization-Based       | (1) Z-basis measurement                                             |
|                           | (2) reordering photons by using different delay lines               |
|                           | (3) reflecting photons without disturbance                           |
| Measurement-Free          | (1) generating Z-basis qubits                                       |
|                           | (2) reordering photons by using different delay lines               |
|                           | (3) reflecting photons without disturbance                           |

Table 1. Limited quantum capabilities of a classical user in various environments
After Boyer et al. [45] proposed the first semi-quantum key distribution (SQKD) protocol, various quantum protocols were proposed, in which the semi-quantum secret sharing (SQSS) protocol is also an important research issue. There are literatures [46-60] proposed for this issue. In 2011, Li et al. [46] proposed the first SQSS protocol to let the dealer Alice (the quantum user) share her secrets with two agents, Bob and Charlie (the classical users) by using GHZ-type state. Wang et al. [47] used the two-particle entangled states to design SQSS protocol. Li et al. [48] proposed the first n-party SQSS protocol using the product states, and then this design was improved by Yang and Hwang [49]. [50] proposed a SQSS protocol let Alice can share her secrets with the agents immediately without needing an additional transmission of $s \bigotimes k_{Alice}$. In 2016, Gao et al. [48] proposed a multiparty SQSS scheme that used Bell states. In 2018, Yi and Tong [50] proposed the multiparty SQSS protocol, which used a unique two-particle entangled state. In 2019, Tsai et al. [60] also used W state to propose SQSS protocol. However, the existing SQSS protocols have the two challenges which including (1) whether let a classical user can be the dealer to share his/her secret or not, (2) the classical users must equip with the additional detectors for avoiding the Trojan Horse attacks (that violate the original intention of the semi-quantum environment obviously).

In view of this, this study wants to adopt the concept of mediated semi-quantum key distribution protocol [61] to overcome the 1st challenge; that is, a classical dealer can share his/her secret with the classical agent with the help of the dishonest three-party (TP). For the 2nd challenge, the one-way quantum communication manner will be used to let the proposed protocol can be free from the Trojan Horse attacks, and thus the classical users do not equip with any additional detector. Furthermore, the classical users only the two quantum capabilities including (1) performing the single qubit unitary operations, and (2) measuring qubits with $Z$-basis $\{0, 1\}$. Therefore, the proposed protocol still is lightweight and practical.

The rest of this paper is organized as follows: Section 2 describes the entanglement characteristic between the GHZ state and Hadamard operation, and then the processes of MSQSS are presented in Section 3. The security of the proposed MSQSS protocol is analyzed in Section 3. A conclusion and the directions for future studies are provided in Section 5.

2. Entanglement characteristic between GHZ state and Hadamard operation

Before describing the processes of the proposed MSQSS protocol, this study firstly indicates the property between GHZ state and Hadamard operation, where the general form of GHZ state and the Hadamard operation are shown respectively as follows:

$$|\Psi\rangle_{12...n} = \frac{1}{\sqrt{2}}(|x_1x_2 \ldots x_n \rangle \pm |x_1\bar{x}_2 \ldots \bar{x}_n \rangle)_{12...n},$$

where $x_i \in \{0, 1\}$.

$$H = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle + (|0\rangle - |1\rangle)|1\rangle.$$

Here, we take the two kinds of 3-particles GHZ state (i.e., $|\Psi^\pm\rangle_{123} = \frac{1}{\sqrt{2}}(|x_1x_2x_3 \rangle \pm |x_1\bar{x}_2\bar{x}_3 \rangle)_{123}$) as the example to explain the properties between GHZ state and Hadamard operation. There are three situations to show these properties including (1) performing Hadamard operation one particle of GHZ state, (2) performing Hadamard operation on any two particles of GHZ state, and (3) performing Hadamard operation on each particle of GHZ state.

**Situation 1:** when performing Hadamard operation one particle of GHZ state, the GZH state will become as follows:

$$H \otimes I \otimes I |\Psi^\pm\rangle_{123} = \frac{1}{2}(|0\rangle_1(|x_2x_3 \rangle \pm |x_2\bar{x}_3 \rangle)_{23}$$

$$+ |1\rangle_1((-1)^{x_1}|x_2x_3 \rangle \pm (-1)^{\bar{x}_1}|x_2\bar{x}_3 \rangle)_{23}$$

$$I \otimes H \otimes I |\Psi^\pm\rangle_{123} = \frac{1}{2}(|0\rangle_2(|x_1x_3 \rangle \pm |x_1\bar{x}_3 \rangle)_{13}$$

$$+ |1\rangle_2((-1)^{x_2}|x_1x_3 \rangle \pm (-1)^{\bar{x}_2}|x_1\bar{x}_3 \rangle)_{13}$$

$$I \otimes I \otimes H |\Psi^\pm\rangle_{123} = \frac{1}{2}(|0\rangle_3(|x_1x_2 \rangle \pm |x_1\bar{x}_2 \rangle)_{12}$$

$$+ |1\rangle_3((-1)^{x_3}|x_1x_2 \rangle \pm (-1)^{\bar{x}_3}|x_1\bar{x}_2 \rangle)_{12}$$

According to the above-mentioned equations, we can find out which the remaining qubits still are entangled if the qubit performed by Hadamard is measured using $Z$-basis $\{0, 1\}$. Therefore, the dealer and agents can use the qubits to check eavesdroppers in this situation.

**Situation 2:** performing Hadamard operation on any two particles of GHZ state, the GHZ state will become as follows:
\[ H \otimes H \otimes I |\Psi^+\rangle_{123} = \frac{1}{2\sqrt{2}} (|100\rangle_{123}(|x_3\rangle \pm |\overline{x_3}\rangle)_3 \\
+ |011\rangle_{123}((-1)^{x_2}|x_1\rangle \pm (-1)^{x_2}|\overline{x_1}\rangle)_1 \\
+ |100\rangle_{123}((-1)^{x_1}|x_2\rangle \pm (-1)^{x_1}|\overline{x_2}\rangle)_1 \\
+ |111\rangle_{123}((-1)^{x_1+x_2}|x_3\rangle \pm (-1)^{x_1+x_2}|\overline{x_3}\rangle)_3 \\
\] 
\[ = \frac{1}{2\sqrt{2}} (|100\rangle_{123}(|x_3\rangle \pm |\overline{x_3}\rangle)_3 \\
+ (-1)^{x_2}|011\rangle_{123}(|x_1\rangle \mp |\overline{x_1}\rangle)_1 \\
+ (-1)^{x_1}|100\rangle_{123}(|x_2\rangle \mp |\overline{x_2}\rangle)_1 \\
+ (-1)^{x_1+x_2}|111\rangle_{123}(|x_3\rangle \pm |\overline{x_3}\rangle)_3) \]

\[ I \otimes H \otimes H |\Psi^\pm\rangle_{123} = \frac{1}{2\sqrt{2}} (|000\rangle_{123}(|x_1\rangle \pm |\overline{x_1}\rangle)_1 \\
+ |011\rangle_{123}((-1)^{x_3}|x_1\rangle \pm (-1)^{x_3}|\overline{x_1}\rangle)_1 \\
+ |100\rangle_{123}((-1)^{x_2}|x_1\rangle \pm (-1)^{x_2}|\overline{x_1}\rangle)_1 \\
+ |111\rangle_{123}((-1)^{x_2+x_3}|x_1\rangle \pm (-1)^{x_2+x_3}|\overline{x_1}\rangle)_1 \\
\] 
\[ = \frac{1}{2\sqrt{2}} (|000\rangle_{123}(|x_1\rangle \pm |\overline{x_1}\rangle)_1 \\
+ (-1)^{x_3}|011\rangle_{123}(|x_1\rangle \mp |\overline{x_1}\rangle)_1 \\
+ (-1)^{x_2}|100\rangle_{123}(|x_1\rangle \mp |\overline{x_1}\rangle)_1 \\
+ (-1)^{x_2+x_3}|111\rangle_{123}(|x_1\rangle \pm |\overline{x_1}\rangle)_1) \]

\[ H \otimes I \otimes H |\Psi^\pm\rangle_{123} = \frac{1}{2\sqrt{2}} (|000\rangle_{123}(|x_2\rangle \pm |\overline{x_2}\rangle)_2 \\
+ (-1)^{x_3}|011\rangle_{123}(|x_2\rangle \mp |\overline{x_2}\rangle)_2 \\
+ (-1)^{x_1}|100\rangle_{123}(|x_2\rangle \mp |\overline{x_2}\rangle)_2 \\
+ (-1)^{x_1+x_3}|111\rangle_{123}(|x_2\rangle \pm |\overline{x_2}\rangle)_2) \]

\[ H \otimes H \otimes H |\Psi^+\rangle_{123} = \frac{1}{4} (|200\rangle + |000\rangle + |011\rangle + |010\rangle) \\
+ (-1)^{x_3} |100\rangle + (-1)^{x_3} |111\rangle + (-1)^{x_2} |010\rangle + (-1)^{x_2} |110\rangle + (-1)^{x_1} |100\rangle + (-1)^{x_1} |010\rangle \]

By observing the above-mentioned equations, we can know that the entanglement of GHZ state is broke, but the measurement results of the qubits performed Hadamard operations can infer the measurement result of the remaining qubit in X-basis \([|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\); that is to say, if the initial state is \(|\Psi^+\rangle_{123}\) and the measurement results of the qubits performed Hadamard operations is \((0,0)\) or \((1,1)\) (\((0,1)\) or \((1,0)\)), the measurement result of the remaining qubit in X-basis will be \(|+\rangle\) (\(|-\rangle\)). However, the dealer and the agents only perform Z-basis measurement, and thus the dealer and agent cannot use the measurement results of the qubits to share secrets or check eavesdroppers.

**Situation 3:** performing Hadamard operation on each particle of GHZ state, the GHZ state will become as follows:

H, there, we can find out that XOR value of all measurement results with Z-basis will be 0 if the initial state is \(|\Psi^+\rangle_{123}\); otherwise, the XOR value will be 1. Therefore, the dealer and agent can use the measurement results of the qubits to share secrets or check eavesdroppers in this situation.

3. Proposed Lightweight MSQSS Protocol

Before describing the processes of the proposed protocol, this study explains the environment assumptions and the quantum capabilities of the dealer and the agents. In the proposed protocol, there are four participants including the three-party TP, the dealer, Alice, and the two agents, Bob and Charlie, in which all participants are the classical users except TP. Alice has the two quantum capabilities: (1) performing single qubit operations including Hadamard operation and \(\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|\) operation, and (2) measuring qubits in Z-basis. Bob and Charlie both have the two quantum capabilities: (1) performing Hadamard operation, and (2) measuring qubits in Z-basis. TP is a quantum user which can generate GHZ states and store these states. Here, for getting closer to reality, we take the trustworthiness level of TP as be dishonest; that is, the TP may perform any possible attacks to obtain the information about the secrets of Alice. Furthermore, there are quantum channels between TP and the other participants with the noise rate \(\varepsilon\). Alice, Bob and Charlie have the authenticated classical channels (i.e.,
the messages in the authenticated classical channel can be eavesdropped but cannot be modified).

This study assumes that Alice wants to share n-bit secret to Bob and Charlie with the help of TP. The processes of the proposed MSQSS protocol are described as follows (also shown in Figure 1).

**Step 1.** TP generates the GHZ state $|\Psi^+\rangle_{123}$, sends the 1st particle to Alice and caches the remaining particles.

**Step 2.** After receiving the qubit, Alice performs the identity operation $I$ or $\sigma_z$ operation on the qubit depending on her secret bit $s_i$, performs Hadamard operation $H$ on the qubit with the probability of 50%, then she measures the qubit in Z-basis and stores the measurement result $m_{r_i}^A$, where $i$ indicates the $i$-th transmission. After achieving the above-mentioned task, she will return “ACK” to TP.

**Step 3.** After receiving “ACK” sent from Alice, TP send the 2nd and 3rd particles to Bob and Charlie, respectively.

**Step 4.** Bob (Charlie) performs Hadamard operation on the received qubit with the probability of 50%, measures the qubit in Z-basis and then stores the measurement result $m_{r_i}^B$ ($m_{r_i}^C$), where $i$ also indicates the $i$-th transmission.

TP, Alice, Bob and Charlie repeat the above-mentioned steps (i.e., Step 1 ~ 4) 16n times.

**Step 5.** According to Alice’s, Bob’s and Charlie’s operations, there are 16 possible cases summarized in Table 2.

### Table 2. 16 possible cases among Alice’s, Bob’s, and Charlie’s operations

| Case | Alice’s, Bob’s, and Charlie’s Operation | Purpose |
|------|----------------------------------------|---------|
| 1    | $I \cdot I \otimes I \otimes I$        | All measurement results are used to check TP or eavesdroppers |
| 2    | $I \cdot I \otimes I \otimes H$        | Alice’s and Bob’s measurement results are used to check TP or eavesdroppers |
| 3    | $I \cdot I \otimes H \otimes I$        | Alice’s and Charlie’s measurement results are used to check TP or eavesdroppers |
| 4    | $H \cdot I \otimes I \otimes I$        | Bob’s and Charlie’s measurement results are used to check TP or eavesdroppers |
| 5    | $I \cdot I \otimes H \otimes H$        | Discarding the measurement results |
| 6    | $H \cdot I \otimes I \otimes H$        | Discarding the measurement results |
| 7    | $H \cdot I \otimes H \otimes I$        | Discarding the measurement results |
| 8    | $H \cdot I \otimes H \otimes H$        | Sharing a raw secret bit 0 |
| 9    | $I \cdot \sigma_z \otimes I \otimes I$ | All measurement results are used to check TP or eavesdroppers |
| 10   | $I \cdot \sigma_z \otimes I \otimes H$ | Alice’s and Bob’s measurement results are used to check TP or eavesdroppers |
| 11   | $I \cdot \sigma_z \otimes H \otimes I$ | Alice’s and Charlie’s measurement results are used to check TP or eavesdroppers |
| 12   | $H \cdot \sigma_z \otimes I \otimes I$ | Bob’s and Charlie’s measurement results are used to check TP or eavesdroppers |
| 13   | $I \cdot \sigma_z \otimes H \otimes H$ | Discarding the measurement results |
| 14   | $H \cdot \sigma_z \otimes I \otimes H$ | Discarding the measurement results |
| 15   | $H \cdot \sigma_z \otimes H \otimes I$ | Discarding the measurement results |
| 16   | $H \cdot \sigma_z \otimes H \otimes H$ | Sharing a raw secret bit 1 |

Alice, Bob, and Charlie announce the information about whether they perform Hadamard operations or not in each round, and then they use the measurement results to check TP or eavesdroppers by using the authenticated classical channels in the appropriate cases (i.e., Case 1, 2, 3, 4, 9, 10, 11 and 12). If the error rate is more than the preset threshold (we can use the noise rate of the quantum channel as the preset threshold), the participants will abort this session and restart the protocol. Otherwise, the participants can share the raw shadow bits in Case 8 and 16.
Step 6. Alice chooses the part of the raw shadow bits as the checking bits, and requests Bob and Charlie to announce the values in the corresponding positions. Then, Alice check the values depending on her operations. If the error rate is more than the preset threshold, this session will be aborted, and a new session will be launched. Otherwise, Alice, Bob, and Charlie will take the remaining raw shadow bits as the secret shadow bits \( \{s_A^1, s_A^2, \cdots, s_A^n\} \), \( \{s_B^1, s_B^2, \cdots, s_B^n\} \) and \( \{s_C^1, s_C^2, \cdots, s_C^n\} \) , respectively. Then, Alice will announce \( \{s_A^1, s_A^2, \cdots, s_A^n\} \). When Bob and Charlie cooperate each other, they can recovery Alice’s secret bit by \( s^j = s_A^j \oplus s_B^j \oplus s_C^j \).

![Figure 1. The processes of the proposed MSQSS protocol](image)

4. Security Analysis

In this section, we analyze the security of the proposed MSQSS protocol. In terms of security analysis, the collective attack is an especially important class of attacks, and the assumption of attacker’s power in the collective attack is more powerful than the individual attack (e.g., the intercept-and-resend attack). Thus, a complete analysis of the collective attack is given for proving the security of the proposed MSQSS protocol firstly, and then we also describe that the proposed MSQSS protocol is free from the Trojan Horse attacks.

4-1 Collective Attack

In the proposed MSQSS protocol, because the TP has many advantages over the other insider or outsider attackers, the study wants to prove that the proposed protocol is robustness under TP execute the collective attack; that is, the TP’s collective attack must be detected by the participants if the TP wants to obtain some information about the Alice’s secret or the agents’ secret shadows.

Theorem 1: the TP perform the collective attack (i.e., she/he performs a unitary operation \( U_e \) which must comply with the quantum mechanical theorems) to attack the qubit that sent to Alice, Bob and Charlie. However, there is no strategy of the collective attack (i.e., no unitary operation) that allows Eve to obtain information about the participants’ secret key without being detected.

Proof: Before the TP send the particles of GHZ states to Alice, Bob and Charlie, the TP perform \( U_e \) on each GHZ state to insert her/his probe qubits \( E = \{ |E_1 \rangle, |E_2 \rangle, \cdots, |E_{16n} \rangle \} \). the TP will keep the probe qubits in her/his hand, and then the TP can measure these qubits to extract the information about Alice’s secret or the agents’ secret shadows when the participants accomplish the protocol. To clear elaborate the proving processes, we take one kind of GZZ state, \( |\Psi\rangle_{123} = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{123} \) as example. According to quantum mechanical theorems, \( U_e \) can be defined as follows:

\[
U_e |\Psi\rangle_{123} \otimes |E_i \rangle = a_1 |000\rangle |e_1 \rangle + a_2 |100\rangle |e_2 \rangle + a_3 |011\rangle |e_3 \rangle + a_4 |111\rangle |e_4 \rangle
\]

, where \( i \) also indicates the \( i \)-th transmission, \( |e_1 \rangle, |e_2 \rangle, |e_3 \rangle \) and \( |e_4 \rangle \) are four states that can be distinguished by Eve and \( |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1 \). Then, Alice performs the corresponding operations. Here, the qubit system become the four types depending on Alice’s operations. If Alice perform \( I \cdot I, I \cdot a_x, H \cdot I, \text{ or } H \cdot a_x \), the qubit system will become the following systems, respectively.

\[
|00 \rangle \otimes (a_1 |00 \rangle |e_1 \rangle + a_3 |11 \rangle |e_3 \rangle) + |1 \rangle \otimes (a_2 |00 \rangle |e_2 \rangle + a_4 |11 \rangle |e_4 \rangle)
\]

\[
|0 \rangle \otimes (a_1 |00 \rangle |e_1 \rangle + a_3 |11 \rangle |e_3 \rangle) - |1 \rangle \otimes (a_2 |00 \rangle |e_2 \rangle + a_4 |11 \rangle |e_4 \rangle)
\]

\[
\frac{1}{\sqrt{2}} \begin{pmatrix}
|00 \rangle \otimes (a_1 |e_1 \rangle + a_2 |e_2 \rangle) + \\
|1 \rangle \otimes (a_3 |e_3 \rangle + a_4 |e_4 \rangle)
\end{pmatrix}
\]

or

\[
\frac{1}{\sqrt{2}} \begin{pmatrix}
|00 \rangle \otimes (a_1 |e_1 \rangle - a_2 |e_2 \rangle) + \\
|1 \rangle \otimes (a_3 |e_3 \rangle - a_4 |e_4 \rangle)
\end{pmatrix}
\]

Then, this study also summaries whole qubit systems after Bob and Charlie performed the corresponding operations in Table 3. Notice the quantum systems that will be discarded by the participants (i.e., Case 5–7 and Case 13–15 in Table 2) are not been discussed.
Table 3. the corresponding quantum systems with the participants’ operations

| Situation | Alice’s, Bob’s and Charlie’s Operation | Quantum System |
|-----------|----------------------------------------|----------------|
| 1 | \( I \otimes I \otimes I \otimes I \) | - Alice’s measurement result = \([0]\):
\[ \alpha_1|00\rangle|e_1\rangle + \alpha_3|11\rangle|e_3\rangle \]
- Alice’s measurement result = \([1]\):
\[ \alpha_2|00\rangle|e_2\rangle + \alpha_4|11\rangle|e_4\rangle \]
| 2 | \( I \otimes I \otimes I \otimes H \) | - Alice’s measurement result = \([0]\):
\[ \frac{1}{\sqrt{2}}(\alpha_1|00\rangle|e_1\rangle + \alpha_2|01\rangle|e_1\rangle + \alpha_3|10\rangle|e_3\rangle - \alpha_3|11\rangle|e_3\rangle) \]
- Alice’s measurement result = \([1]\):
\[ \frac{1}{\sqrt{2}}(\alpha_2|00\rangle|e_2\rangle + \alpha_2|01\rangle|e_2\rangle + \alpha_4|10\rangle|e_4\rangle - \alpha_4|11\rangle|e_4\rangle) \]
| 3 | \( I \otimes H \otimes I \) | - Alice’s measurement result = \([0]\):
\[ \frac{1}{\sqrt{2}}(\alpha_1|00\rangle|e_1\rangle + \alpha_1|01\rangle|e_1\rangle + \alpha_3|10\rangle|e_3\rangle - \alpha_3|11\rangle|e_3\rangle) \]
- Alice’s measurement result = \([1]\):
\[ \frac{1}{\sqrt{2}}(\alpha_2|00\rangle|e_2\rangle + \alpha_2|01\rangle|e_2\rangle + \alpha_4|10\rangle|e_4\rangle - \alpha_4|11\rangle|e_4\rangle) \]
| 4 | \( H \otimes I \otimes I \) | - Alice’s measurement result = \([0]\):
\[ |00\rangle\otimes(\alpha_1|e_1\rangle + \alpha_2|e_2\rangle) + |11\rangle\otimes(\alpha_3|e_3\rangle + \alpha_4|e_4\rangle) \]
- Alice’s measurement result = \([1]\):
\[ |00\rangle\otimes(\alpha_1|e_1\rangle - \alpha_2|e_2\rangle) + |11\rangle\otimes(\alpha_3|e_3\rangle - \alpha_4|e_4\rangle) \]
| 5 | \( H \otimes I \otimes H \) | - Alice’s measurement result = \([0]\):
\[ \frac{1}{\sqrt{2}}(|00\rangle\otimes(\alpha_1|e_1\rangle + \alpha_2|e_2\rangle + \alpha_3|e_3\rangle + \alpha_4|e_4\rangle) \]
\[ + |01\rangle\otimes(\alpha_1|e_1\rangle + \alpha_2|e_2\rangle - \alpha_3|e_3\rangle - \alpha_4|e_4\rangle) \]
\[ + |10\rangle\otimes(\alpha_1|e_1\rangle + \alpha_2|e_2\rangle - \alpha_3|e_3\rangle - \alpha_4|e_4\rangle) \]
\[ + |11\rangle\otimes(\alpha_1|e_1\rangle + \alpha_2|e_2\rangle + \alpha_3|e_3\rangle + \alpha_4|e_4\rangle) \]
- Alice’s measurement result = \([1]\):
\[ \frac{1}{\sqrt{2}}(|00\rangle\otimes(\alpha_1|e_1\rangle - \alpha_2|e_2\rangle + \alpha_3|e_3\rangle - \alpha_4|e_4\rangle) \]
\[ + |01\rangle\otimes(\alpha_1|e_1\rangle - \alpha_2|e_2\rangle - \alpha_3|e_3\rangle + \alpha_4|e_4\rangle) \]
\[ + |10\rangle\otimes(\alpha_1|e_1\rangle - \alpha_2|e_2\rangle - \alpha_3|e_3\rangle + \alpha_4|e_4\rangle) \]
\[ + |11\rangle\otimes(\alpha_1|e_1\rangle - \alpha_2|e_2\rangle + \alpha_3|e_3\rangle - \alpha_4|e_4\rangle) \]
| 6 | \( I \otimes \sigma_z \otimes I \otimes I \) | - Alice’s measurement result = \([0]\):
\[ \alpha_1|00\rangle|e_1\rangle + \alpha_3|11\rangle|e_3\rangle \]
- Alice’s measurement result = \([1]\):
\[ \alpha_2|00\rangle|e_2\rangle + \alpha_4|11\rangle|e_4\rangle \]
| 7 | \( I \otimes \sigma_z \otimes I \otimes H \) | - Alice’s measurement result = \([0]\):
\[ \frac{1}{\sqrt{2}}(\alpha_1|00\rangle|e_1\rangle + \alpha_4|01\rangle|e_1\rangle + \alpha_3|10\rangle|e_3\rangle - \alpha_3|11\rangle|e_3\rangle) \]
- Alice’s measurement result = \([1]\):
\[ \frac{1}{\sqrt{2}}(\alpha_2|00\rangle|e_2\rangle + \alpha_2|01\rangle|e_2\rangle + \alpha_4|10\rangle|e_4\rangle - \alpha_4|11\rangle|e_4\rangle) \]
| 8 | \( I \otimes \sigma_z \otimes H \otimes I \) | - Alice’s measurement result = \([0]\):
\[ \frac{1}{\sqrt{2}}(\alpha_1|00\rangle|e_1\rangle + \alpha_4|01\rangle|e_1\rangle + \alpha_3|10\rangle|e_3\rangle - \alpha_3|11\rangle|e_3\rangle) \]
- Alice’s measurement result = \([1]\):
\[ \frac{1}{\sqrt{2}}(\alpha_2|00\rangle|e_2\rangle + \alpha_2|01\rangle|e_2\rangle + \alpha_4|10\rangle|e_4\rangle - \alpha_4|11\rangle|e_4\rangle) \]
Here, in the light of Table 4, the TP will adjust the parameters of $U_e$ to pass the participants’ detection in Step 5 and Step 6 of the proposed protocol. According to the proposed protocol, the participants use the measurement results (the part of raw shadow bits) to public discussion in Case 1–4 and Case 9–12 (Case 8 and 16). Firstly, the TP will set $α_2 = α_3 = 0$ in the Situation 1 and Situation 2 for passing the detections in Case 1 and Case 9, and then the TP can set $α_1|e_1⟩ + α_3|e_3⟩ = α_2|e_2⟩ + α_4|e_4⟩ = 0$ in Situation 2–4 and 7–9 for the detections in Case 2–4 and Case 10–12. Finally, to pass the detections in Case 8 and 16, the TP can set $α_1|e_1⟩ + α_2|e_2⟩ - α_3|e_3⟩ - α_4|e_4⟩ = α_1|e_1⟩ - α_2|e_2⟩ + α_3|e_3⟩ - α_4|e_4⟩ = 0$ in Situation 5 and 10. Because the TP cannot know the participants’ operations in advance, he/she must adjust $U_e$ to pass all situations. According to $α_2 = α_3 = 0$ and $α_1|e_1⟩ + α_2|e_2⟩ - α_3|e_3⟩ - α_4|e_4⟩ = α_1|e_1⟩ - α_2|e_2⟩ + α_3|e_3⟩ - α_4|e_4⟩ = 0$, we can infer that $α_1|e_1⟩ = α_4|e_4⟩$, and then the result $α_1|e_1⟩ = α_4|e_4⟩ = 0$ is also obtained from $α_1|e_1⟩ + α_3|e_3⟩ = α_2|e_2⟩ + α_4|e_4⟩ = 0$. Therefore, if the TP wants to obtain any information about the Alice’s secret or the agents’ shadows without being detected, he/she will set each probe qubit as be 0; that is, he/she cannot measure the probe qubit to obtain any information.

4-2 Trojan horse attack

In terms of implementation-dependent attacks, Trojan horse attack [62,63] is common. In Trojan horse attack, the attacker can insert the probing photons into the qubits sent from the TP. Then, the attacker attempts to obtain Alice’s, Bob’s, Charlie’s secret shadow bits using these probing photons. However, in the proposed protocol, the quantum transmission strategy of qubits works one way, implying that the qubits are only sent from the TP to the classical participants. Although the attacker can insert probing photons into each qubit, on information about the participants’ secret shadow bits cannot be extracted because the probing photons cannot be retrieved; that is, the attacker cannot recycle the probing photons. Therefore, the proposed protocol is free from the Trojan horse attacks. Hence, the classical participants do not need to be equipped with expensive Trojan horse detectors (such as the photon number splitter and optical wavelength filter devices) to avoid Trojan horse attacks.

5. Conclusions

This study proposes the first mediated SQSS protocol that let the classical dealer can share his/her secret with the classical agents with the help of the dishonest TP, in which the classical dealer and agents only equip the single-qubit unitary operations and Z basis measurement without any Trojan Horse detector. Comparing to the existing SQSS protocols, the proposed protocol is more lightweight than the existing SQSS protocols. Therefore, the proposed protocol solves the two challenges ((1) the dealer must be the quantum user, and (2) the classical users must equip with the Trojan Horse detectors) in the existing SQSS protocols. However, how to extend the proposed protocol to n-party agents is an important issue. We will take this issue as future work.

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