We show that if global lepton number symmetry is spontaneously broken in a post inflation epoch, then it can lead to the formation of cosmological domain walls. This happens in the well-known “majoron paradigm” for neutrino mass generation. We propose some realistic examples which allow spontaneous lepton number breaking to be safe from such domain walls.

I. INTRODUCTION

Topological defects such as monopoles, strings and domain walls can arise in many gauge theories including grand unification. In addition, there can appear (hybrid) configurations such as monopoles connected via strings or walls bounded by strings. Two well known examples of the latter arise in SO(10) and axion models. Stable or sufficiently long lived domain walls, associated with symmetry breaking scales comparable to or larger than in the Standard Model (SM) will sooner or later become the dominant energy component of the early universe. As a consequence such domain walls pose a serious challenge in cosmology and should therefore be avoided in realistic model building (some possibilities were recently discussed in [5]).

Domain walls are well known to appear associated with the spontaneous breaking of the Peccei-Quinn symmetry. Here we note that also the “weak” SU(2) may be associated to the presence of domain walls. This may happen in the context of spontaneous violation of lepton number symmetry. Indeed, such models in which lepton number is violated by a gauge singlet Higgs vacuum expectation value (vev) provides an attractive way to generate Majorana masses for neutrinos, as needed to account for current neutrino data. In addition, it implies the existence of a physical Nambu-Goldstone boson, called majoron. The latter may pick up a mass from explicit symmetry breaking by gravity effects. Under such circumstances the majoron may provide a good dark matter candidate.

The origin of the domain wall problem in this case stems from the existence of an unbroken residual subgroup Z2 arising from the spontaneous lepton number violation, which clashes with the unbroken Z3 from the non-perturbative instanton effects associated with the weak SU(2). This implies that the domain wall problem associated to the weak SU(2) exists in a broad class of majoron models of neutrino mass generation. A standard mechanism for evading the domain wall problem is to invoke a suitable inflationary phase during their formation such that the walls are inflated away. In this letter we propose a more direct resolution of the domain wall problem which does not rely on inflation. We present various possible mechanisms for having realistic majoron models, with and without supersymmetry, which allow spontaneous lepton number violation to occur without encountering a domain wall problem.
II. GLOBAL LEPTON NUMBER AND DOMAIN WALL PROBLEM

Apart from the gauge symmetries it is well known that in the Standard Model there are two “accidental” global $U(1)$ symmetries namely the Baryon number $U(1)_B$ and the Lepton number $U(1)_L$ symmetries. Although, accidental within Standard Model, these symmetries nonetheless play a very important role. The baryon number symmetry $U(1)_B$ is responsible for the stability of the proton and the lepton number symmetry plays a key role in neutrino mass generation and in determining the Dirac or Majorana nature of neutrinos. Lepton number in the Standard Model is conserved at the Lagrangian level to all orders in perturbation theory. However, lepton number is an anomalous symmetry, hence it is explicitly broken by non-perturbative effects \[^2\]. In particular, owing to the $[SU(2)_L]^2 \times U(1)_L$ anomaly, the non-perturbative instantons will explicitly break the initial lepton number symmetry $U(1)_L$ down to the discrete $Z_N$ subgroup, with

$$N = \sum_R N(R) \times L(R) \times T(R) = 3 \times 1 \times 1 = 3,$$

where $N(R)$ is the number of copies of a given fermion in representation $R$, $L(R)$ is the lepton number of the fermion and $T(R)$ is the $SU(2)_L$ Dynkin multiplicity index. For the $SU(2)_L$ group, the index $T(R)$ for the lowest representations, singlets, doublets and triplets are: for a singlet $T(1) = 0$, for a doublet $T(2) = 1$ and for a triplet $T(3) = 4$.

It is clear, from \[^1\], that the non-perturbative instantons associated to the weak SU(2) break $U(1)_L \rightarrow Z_3$. Notice also that the threefold family replication in the Standard Model plays a crucial role in dictating the breaking $U(1)_L \rightarrow Z_3$. The residual $Z_3$ symmetry is exact at the classical and quantum level, implying the existence of degenerate vacua in our theory. Notice also that, in contrast to the case of axions, where the anomaly is related to the $U(1)$ Peccei-Quinn assignments, in the case of lepton number there is an anomaly intrinsically associated to the chiral nature of weak SU(2).

Although $SU(2)_L$ instanton effects are extremely small at zero temperature, they become relevant at high temperatures \[^21, 22\]. In fact, for temperatures above the sphaleron mass $E_{\text{sph}} \sim m_W(T)/\alpha_W$, the typical exponential suppression of instantons is absent \[^4\]. This remains true even above the critical temperature for the electroweak phase transition. Thus, the $B$ and $L$ violating reactions at high temperatures are fast, so that the $U(1)_L$, $U(1)_B$ are explicitly broken by non-perturbative effects, down to discrete $Z_3$ symmetries.

If the Standard Model is the final gauge theory, the non-perturbative breaking of lepton number won’t be a serious issue. However, a dynamical understanding of the smallness of neutrinos mass often requires that lepton number is further broken down either explicitly or spontaneously by the new physics associated to neutrino mass generation. A popular and well studied scenario is the case of spontaneous breaking of lepton number \[^8\]. This is a specially attractive scenario that not only leads to Majorana masses, but also implies the existence of a Nambu-Goldstone boson, called majoron. It breaks the global $U(1)_L$ lepton number symmetry down to a $Z_2$ subgroup through the vev of a $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ singlet scalar carrying two units of lepton number. However, we notice the mismatch between the unbroken residual subgroup $Z_2$ arising from the spontaneous lepton number violation and that obtained by the non-perturbative effects $Z_3$. Owing to this mismatch the domain walls will appear, as already explained. This reasoning is general and applies to all majoron models with spontaneously broken lepton number: from the seesaw to radiative models. It is striking how this apparently harmless SM extension can suffer from the cosmological domain wall problem.

One may achieve the above spontaneous breaking of $U(1)_L \rightarrow Z_2$ by the vev of a field carrying two units of lepton number. Its connection to neutrino masses and the resulting majoron can be described in full generality at the

\[^2\] Note that both baryon and lepton numbers are anomalous symmetries however a particular combination $U(1)_{B-L}$ is anomaly free. The other orthogonal combination $U(1)_{B+L}$ remains anomalous and hence it is explicitly broken by non-perturbative effects \[^20\].

\[^3\] Note that the tunneling amplitude from one vacuum to another due to instantons is proportional to $\exp(-2\pi/\alpha_W)$. 

operator level. Consider the \( U(1)_L \) invariant effective operator

\[
\frac{1}{\Lambda^2} L H H \sigma L .
\]  

(2)

In (2) the field \( L \) is the \( SU(2)_L \) lepton doublet, \( H \) is the Higgs doublet and \( \sigma \) is a Standard Model gauge singlet scalar field charged under the \( U(1)_L \) symmetry. Also, \( \Lambda \) is the cutoff scale for the effective operator above which the full Ultra-Violet complete theory should be specified. This operator is \( U(1)_L \) invariant if \( \sigma \) has charge \(-2\) under the \( U(1)_L \) symmetry. After \( \sigma \) develops a non-zero vev, \( \langle \sigma \rangle \), \( U(1)_L \) is broken down to \( Z_2 \) and the expression in Eq. 2 reduces to the famous Weinberg operator \[23\]. Again the CP odd part of \( \sigma \) will be a Nambu-Goldstone boson, the majoron.

III. SOLUTIONS TO THE DOMAIN WALL PROBLEM

In this section we consider alternative solutions to the domain wall problem which arises from the spontaneous breaking of \( U(1)_L \) by the vev of a lepton-number-carrying scalar field. The examples in Sec III A, III B and III D involve only the Standard Model gauge structure. On the other hand the model considered in Sec III C requires an extension of the Standard Model with a gauge family symmetry.

A. Majoron with Singlet-Triplet Seesaw

The simplest solution of the domain wall problem in the majoron model uses only the usual Standard Model gauge framework. It requires, in addition to the Standard Model fields, the following new ones with their \( SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y} \) quantum numbers indicated in parenthesis and subscripts denoting their charges under \( U(1)_L \):

\[
\nu_R = (1, 1, 0)_+ , \Sigma_R = (1, 3, 0)_+ , \sigma = (1, 1, 0)_{-2} ,
\]  

(3)

where the first field \( (\nu_R) \) is a gauge singlet right-handed neutrino present in seesaw schemes \[24\] \[28\] (with arbitrary multiplicity, which we take equal to one for simplicity, given that this is sufficient to account for the current neutrino data). The second field \( (\Sigma_R) \) is a \( SU(2)_L \) triplet right-handed fermion and, the last field is the complex scalar whose vev \( \langle \sigma \rangle \) is responsible for the spontaneous lepton number breaking. The Lagrangian will now contain the following new couplings:

\[
\mathcal{L}_{\text{new}} = y_{\nu_R}^D \bar{L}^i H \nu_R + y_{\Sigma}^D \bar{L}^i H \Sigma_R + y_{\Sigma}^M \sigma \Sigma_R \Sigma_R + y_{\nu_R}^M \sigma \bar{\nu}_R \nu_R .
\]  

(4)

After electroweak symmetry breaking the Higgs field will get a vev \( \langle H \rangle = v \) and we will have a seesaw-like mechanism for light neutrinos with mass matrix \( m_\nu = M_D^T M_R^{-1} M_D \) where

\[
M_D = \begin{pmatrix}
    v y_{\nu}^{D1} & v y_{\nu}^{D2} & v y_{\nu}^{D3} \\
    v y_{\Sigma}^{D1} & v y_{\Sigma}^{D2} & v y_{\Sigma}^{D3}
\end{pmatrix},
\] 

\[
M_R = \begin{pmatrix}
    y_{\nu}^M \langle \sigma \rangle & 0 \\
    0 & y_{\Sigma}^M \langle \sigma \rangle
\end{pmatrix}.
\]  

(5)

The resulting matrix, \( m_\nu \), has rank 2 leaving one light neutrino massless. Note that, since \( \Sigma_R \) has non-trivial \( SU(2)_L \) quantum numbers it produces a significant change in the \( [SU(2)_L]^2 \times U(1)_L \) anomaly, which is now given by

\[
N = \sum_R N(R) \times L(R) \times T(R) = 3 \times 1 \times 1 - 1 \times 1 \times 4 = -1 ,
\]  

(6)

By computing the anomaly factor one sees that the domain wall problem is absent in this extension. Therefore, the heavy triplet \( \Sigma_R \) acts as an auxiliary Majorana field to address the domain wall issue. Moreover, it also acts as heavy messenger for small neutrino mass generation through the seesaw mechanism.
B. Majoron seesaw within Supersymmetry

The simple solution illustrated in the previous section can be generalized within a Supersymmetric (SUSY) context. We present here a simple supersymmetric model which also addresses the domain wall problem. The particle content and charges of the superfields are as shown in Table I.

| Superfields | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_R$ | $U(1)_L$ | $U(1)_R$ |
|-------------|-----------|-----------|----------|----------|----------|----------|
| $Q_i$       | 3         | 2         | $1/6$    | $1/3$    | 0        | 1        |
| $u_i^c$     | 3         | 1         | $-2/3$   | $-1/3$   | 0        | 1        |
| $d_i^c$     | 3         | 1         | $1/3$    | $-1/3$   | 0        | 1        |
| $L_i$       | 1         | 2         | $-1/2$   | 0        | 1        | 1        |
| $e_i^c$     | 1         | 1         | 1        | 0        | -1       | 1        |
| $\nu_i^c$   | 1         | 1         | 0        | 0        | -1       | 1        |
| $T$         | 1         | 3         | 1        | 0        | -1       | 1        |
| $\bar{T}$  | 1         | 3         | $-1$     | 0        | 0        | 1        |
| $H_u$       | 1         | 2         | $1/2$    | 0        | 0        | 0        |
| $H_d$       | 1         | 2         | $-1/2$   | 0        | 0        | 0        |
| $S$         | 1         | 1         | 0        | 0        | 0        | 2        |
| $\phi$      | 1         | 1         | 0        | 0        | $-1$     | 0        |
| $\bar{\phi}$| 1         | 1         | 0        | 0        | 1        | 0        |

Table I. Particle content and charges. $U(1)_R$ is an R-Symmetry under which the superpotential $W$ has R-charge of 2 units.

In addition to the usual MSSM superfields and the right-handed neutrinos ($\nu^c$), one adds the $SU(2)_L$ triplet superfields $T, \bar{T}$ and the gauge singlet superfields $S, \phi, \bar{\phi}$ with charges as listed in Table I. The superpotential of our model is given by

$$W = \kappa S(\bar{\phi}\phi - M^2) + y_i^u H_u Q_i u_i^c + y_i^d H_d Q_i d_i^c + y_i^\nu H_u L_i \nu_i^c + y_i^\nu H_d L_i \nu_i^c + \lambda S H_u H_d + y^{T}_{i,j} TL_i H_d + y^{T}_{i,j} \bar{T} \bar{L} + y^{\phi}_{i,j} \frac{\bar{\phi}^2 \nu^c \nu^c}{m_P},$$

(7)

where $i, j = 1, 2, 3$ are generation index and $m_P$ is the reduced Planck scale.

Owing to the presence of the triplet superfield $T$, the $[SU(2)_L]^2 \times U(1)_L$ anomaly is again found to be

$$N = \sum_R N(R) \times L(R) \times T(R) = 3 \times 1 \times 1 - 1 \times 1 \times 4 = -1.$$  

(8)

Thus, unlike the usual majoron models, here the instanton effects will break $U(1)_L \to Z_1$, avoiding the domain wall problem. As in the previous case, this holds irrespective of the number of right-handed neutrino superfields, the minimal realistic model has just one.

Notice that this solution differs from the standard seesaw mechanism in that the majoron coming from the imaginary parts of the $\phi, \bar{\phi}$ scalars carry one unit of lepton number, instead of two. Moreover, our model has other attractive features which make it quite appealing. Apart from solving the domain wall problem, it automatically addresses the so-called “$\mu$-problem” of the MSSM [29]. In addition we also have a R-symmetry which contains the usual R-parity of the MSSM, forbidding all the potentially dangerous terms in the superpotential, [30]. Finally, right-handed neutrino masses arise through the non-renormalizable term $\bar{\phi}^2 \nu^c \nu^c / m_P$, where we take the high scale as $m_P$. 


C. SU(3) family symmetry for leptons

Consider now a $SU(3)_{\text{lep}}$ gauge extension of the Standard Model scenario. Let quarks be singlets under this group, while leptons transform under it in a vector-like way,

\begin{align*}
L &= (1, 2, -1/2, 3), \\
e_R &= (1, 1, -1, 3), \\
\nu_R &= (1, 1, 0, 3),
\end{align*}

with the first three entries in parenthesis indicating the standard model properties and the last entry the $SU(3)_{\text{lep}}$ representation. This extension has several consequences. First of all, right-handed neutrinos cannot have a bare mass term. Their masses must be generated through the spontaneous violation of $U(1)_L$. This is related with the breaking of $SU(3)_{\text{lep}}$ and is achieved by the vev of a flavour sextet scalar field $\sigma$ with lepton number $-2$ via the coupling

$$\sigma \bar{\nu}_R \nu_R.$$

The second and more important implication is that this extension automatically solves the domain wall problem. The reason is that the center of $SU(3)_{\text{lep}}$ which is a $Z_3$, exactly coincides with the discrete $Z_3$ subgroup of $U(1)_L$ left unbroken by the anomaly. Since this accidental subgroup can be embedded in the continuous gauge group $SU(3)_{\text{lep}}$, the degenerate minima are now connected by a gauge transformation, so that any difference among them becomes unphysical. In this way, the domain wall problem is solved. This is a majoron variant of the domain wall axion solution given in the GUT context in Ref. [30, 31].

D. Dirac Solution

Another possible solution to the domain wall problem is obtained by enforcing that the spontaneous lepton number breaking is such that $U(1)_L \rightarrow Z_3$ instead of $Z_2$. In this case there is no mismatch between the residual subgroup preserved by the anomaly and that preserved by the spontaneous lepton number violation due to $\langle \sigma \rangle$, so the domain wall problem will be automatically solved. Clearly, the $U(1)_L \rightarrow Z_3$ spontaneous breaking cannot be accomplished within the framework of the canonical majoron model. In fact, if $Z_3$ is the residual unbroken symmetry then neutrinos cannot be Majorana particles. However, we note that for Dirac neutrinos the $U(1)_L \rightarrow Z_3$ breaking is viable, and will lead to a solution of the domain wall problem within a variant of the “Diracon models” [32, 33].

To see this Diracon solution, the first thing is to realize that the lepton number of right handed neutrinos $\nu_R$ need not be the same as that of the left handed neutrinos $\nu_L$. In fact, a non-conventional lepton number assignment of $(4, 4, -5)$ for the three generations of $\nu_i, R; i = 1, 2, 3$, proposed in [36, 37] is equally acceptable.

If the $\nu_{i, R}$ transform with such non-conventional charges under $U(1)_L$ then one cannot write down the tree level Dirac term $\bar{L}H \nu_{i, R}$ nor the majoron Weinberg operator of $\bar{\nu}_R$. However, one can still write down the following $U(1)_L$ invariant operators

$$\frac{1}{\Lambda} \bar{L} H \chi \nu_{i, R}, \quad \frac{1}{\Lambda^2} \bar{L} H \chi^* \chi^* \nu_{3, R},$$

where $\nu_{i, R}; i = 1, 2$ are the two right handed neutrinos carrying charge 4 units under $U(1)_L$, and $\nu_{3, R}$ has $U(1)_L$ charge of $-5$. Also, the field $\chi$ has charge of $-3$ under $U(1)_L$. It can be easily seen that the vev of the $\chi$ field will spontaneously break $U(1)_L \rightarrow Z_3$ with the resulting neutrinos being Dirac in nature. Furthermore, the CP odd part of $\chi$ will be a Nambu-Goldstone boson which we call Diracon and is associated with the Dirac mass generation of the neutrinos. Now, since the $U(1)_L$ in this case is spontaneously broken to the same residual subgroup $Z_3$ as that preserved by the non-perturbative $SU(2)_L$ instantons, there is no mismatch and hence the problem of domain walls is automatically avoided.
IV. CONCLUSIONS

We have shown that if the global total lepton number symmetry is broken spontaneously in a post inflationary epoch, then it can lead to the formation of cosmological domain walls. Since the presence of these domain walls may spoil the standard picture of cosmological evolution, we have studied the conditions to prevent their formation as a result of spontaneous symmetry breaking. We have shown that the simplest seesaw majoron models of neutrino masses have, in principle, a domain wall problem associated with the chiral SU(2) gauge group describing the weak interaction. We have also provided some explicit and realistic solutions which allow a safe spontaneous breaking of lepton number free of domain walls. Some of these models involve new particles that could potentially lead to phenomenological implications.

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