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A prediction of $\delta_{CP}$ for a normal neutrino mass hierarchy in an extended standard model with an A4 flavour symmetry

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Abstract. A method of diagonalization of a general neutrino mass matrix in an extended standard model with an A4 flavour symmetry is used. The method allows us to determine a relation between the mixing angles ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$) and the CP-violation Dirac phase ($\delta_{CP}$). The current prediction of $\delta_{CP}$ is quite good, near the $1\sigma$ region of the best fit of the experimental data at a normal neutrino mass hierarchy.

1. Introduction

The neutrino masses and mixings are among the phenomena calling for extending the standard model (SM) [1, 2]. One of the extensions of the SM is to add a flavour symmetry. A popular flavor symmetry intensively investigated in literature is A4 (see, for instance, [3, 4]) which allows to obtain a tribi-maximal neutrino mixing (TBM). The recent experimental data [5] showing, however, a non-zero mixing angle $\theta_{13}$ and a possible CP-violation Dirac phase $\delta_{CP}$, rejects the TBM. There have been many attempts to explain these new phenomena. Here, using a perturbation method we can diagonalize a general neutrino mass matrix and derive a relation between $\delta_{CP}$ and the mixing angles $\theta_{ij}$. The latter allows us to calculate $\delta_{CP}$ numerically using experimental data on the mixing angles.

We will make in the next section a quick introduction to the representations of A4 and their application to a simple model. In Sect. 3 we will deal with the diagonalization of the neutrino mass matrix obtained for this model from which we can get a mass spectrum and a relation between $\delta_{CP}$ and $\theta_{ij}$. 

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2. Extended standard model with A4 flavour symmetry

2.1. Representations of A4 in brief

Here we give a brief information on representations of A4 [6, 7]. A4 group has 12 elements and it can be generated by two basic permutations S and T

\[ S^2 = T^3 = (ST)^3 = 1. \]  (1)

A4 has three one-dimensional unitary representations 1, 1′ and 1′′ generated by

1 : \[ S = 1 \quad T = 1, \]  (2)

1′ : \[ S = 1 \quad T = e^{i \frac{2 \pi}{3}} = \omega, \]  (2a)

1′′ : \[ S = 1 \quad T = e^{i \frac{4 \pi}{3}} = \omega^2. \]  (2b)

and a three-dimensional unitary representation with the generators

\[
S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \]  (3)

In applications of a group we often need to know the multiplication rule of its (irreducible) representations which in the case of A4 are

\[ 1 \times 1 = 1, \]  (4)

1′ × 1′′ = 1,

3 × 3 = 1 + 1′ + 1′′ + 3S + 3AS.

If we have two triplets \( 3_a \sim (a_1, a_2, a_3) \) and \( 3_b \sim (b_1, b_2, b_3) \), their direct product can be decomposed into irreducible representations as follows

\[ 1 = a_1 b_1 + a_2 b_2 + a_3 b_3, \]  (5a)

\[ 1′ = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \]  (5b)

\[ 1′′ = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \]  (5c)

\[ 3_1 \sim (a_2 b_3, a_3 b_1, a_1 b_2), \]  (5d)

\[ 3_2 \sim (a_3 b_2, a_1 b_3, a_2 b_1). \]  (5e)

The above given information is used in constructing the Lagrangian of a considered model with an A4 symmetry, for example, the Lagrangian (6).

2.2. A simple model

In any model with three active Majorana neutrinos, the most general neutrino mass matrix is symmetric and parametrized by 6 independent parameters. As an example, we consider here a model studied in [8, 9, 10]. The lepton sector (the quark sector is not considered here) of this model includes an A4 triplet \( N \) which is an SU(2)_L singlet (called also right-handed neutrino), and the SM leptons among which the left-handed leptons \( l_L, l = e, \mu, \tau \), transform as A4 triplets, while the right-handed ones \( e_R, \mu_R \) and \( \tau_R \) transform as A4 singlets 1, 1′ and 1′′, resp. Besides the SM Higgs \( \phi_h \) which is a singlet under A4, the scalar sector of the model has five additional
For the neutrino sector, we get a Majorana mass matrix

\[ M_N = \begin{pmatrix} c_a + c_b + c_d & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & c_a + \omega c_b + \omega^2 c_d & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & c_a + \omega^2 c_b + \omega c_d \end{pmatrix} = \begin{pmatrix} a & b & c \\ b & e & d \\ c & d & f \end{pmatrix}, \]

where \( c_a = c_\xi u_a, \) \( c_b = c_{\xi'} u_b, \) \( c_d = c_{\xi''} u_c, \) \( c_1 = c_N v_1, \) \( c_2 = c_N v_2, \) \( c_3 = c_N v_3. \) We also get a Dirac mass matrix

\[ M_D = \begin{pmatrix} \lambda_{N_1} v & 0 & 0 \\ 0 & \lambda_{N_2} v & 0 \\ 0 & 0 & \lambda_{N_3} v \end{pmatrix} = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}. \]

Following the seesaw mechanism we get a neutrino mass matrix,

\[ M_\nu = -M_D^T M_N^{-1} M_D, \]
which, with (12) and (13) taken into account, has the form

\[
M_\nu = -\frac{1}{\text{det}(M)} \begin{pmatrix}
(d^2 - ef) x^2 & -(cd + bf) xy & -(bd + ce) xz \\
-(cd + bf) xy & (c^2 - af) y^2 & -(bc + ad) yz \\
-(bd + ce) xz & -(bc + ad) yz & (b^2 - ae) z^2 \\
\end{pmatrix} = \begin{pmatrix} A & B & C \\
B & E & D \\
C & D & F \end{pmatrix},
\]

(15)

where \(\text{det}(M) = -2bcd + c^2e + b^2f + a(d^2 - ef)\) is the determinant of \(M_N\).

One of the key problems of a neutrino mass model is to diagonalize the corresponding neutrino mass matrix, in this case, the matrix (15). To solve this problem, different methods and tricks have been used. Below we will follow a perturbation approach.

3. Diagonalization of the neutrino mass matrix

In the basis of the diagonalized charged lepton mass matrix (i.e., in the basis where this matrix has a diagonal form), where \(M_\nu = U_0^T M_0^0 U_0\), and, as shown by the current neutrino oscillation experimental data, \(U_{PMNS}\) is a small deviation from the tribi-maximal form, one always has (see also [11])

\[
M_\nu = M_0 + \lambda V,
\]

(16)

with

\[
M_0 = \begin{pmatrix}
A & B \\
B & E \\
-(A - E + B) & E \\
\end{pmatrix},
\]

\[
\lambda V = \begin{pmatrix} 0 & 0 & e_1 \\
0 & 0 & e_3 \\
e_1 & e_3 & e_2 \end{pmatrix},
\]

(17)

where \(\lambda\) is a small (perturbation) parameter and \(M_0\) can be diagonalized

\[
\text{diag}(m_1^0, m_2^0, m_3^0) = U_{TBM}^\dagger M_0 U_{TBM},
\]

(18)

by

\[
U_{TBM} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\end{pmatrix} \sim (|1^0\rangle, |2^0\rangle, |3^0\rangle).
\]

(19)

Using the perturbation expression

\[
|n\rangle = |n^0\rangle + \lambda \sum_{k \neq n} |k^0\rangle \frac{V_{kn}}{m_k^0 - m_n^0}; \quad n, k = 1, 2, 3,
\]

(20)

with

\[
V_{kn} = \langle k|V|n\rangle,
\]

(21)

one can diagonalize the matrix \(M_\nu\) by

\[
U_{PMNS} = \begin{pmatrix}
\sqrt{\frac{2}{3}} + \sqrt{\frac{1}{3}} x^* & \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}} x & -\sqrt{\frac{2}{3}} y - \sqrt{\frac{1}{3}} z \\
-\sqrt{\frac{1}{6}} + \sqrt{\frac{1}{3}} x^* + \sqrt{\frac{1}{2}} y^* & \sqrt{\frac{1}{3}} + \sqrt{\frac{1}{6}} x + \sqrt{\frac{1}{2}} z^* & \sqrt{\frac{1}{6}} + \sqrt{\frac{1}{2}} y - \sqrt{\frac{1}{3}} z \\
\sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}} x^* + \sqrt{\frac{1}{2}} y^* & -\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}} x + \sqrt{\frac{1}{2}} z^* & \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{2}} y + \sqrt{\frac{1}{3}} z \\
\end{pmatrix},
\]

(22)

where

\[
x = \frac{\sqrt{2}}{6} \left(\frac{2e_3 - e_1 - e_2}{m_1^0 - m_2^0}\right), \quad y = \frac{\sqrt{3}}{6} \left(\frac{2e_1 + e_2}{m_1^0 - m_3^0}\right), \quad z = \frac{1}{\sqrt{6}} \left(\frac{e_1 + e_2}{m_2^0 - m_3^0}\right).
\]

(23)
4. Relation between mixing angles and $\delta_{CP}$

Denoting with $U_{ij}$, $i, j = 1, 2, 3$, a matrix element of (22), we obtain the relation

$$U_{21} + \sqrt{2}U_{22} - U_{31} - \sqrt{2}U_{32} = 2U_{11} - \sqrt{2}U_{12}. \quad (24)$$

The latter compared with the elements of the matrix $U_{PMNS}$ of a general form

$$U_{PMNS} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}e^{i\delta} \\
    s_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & -c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}, \quad (25)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$; $i, j = 1, 2, 3$, leads to a relation between $\delta_{CP} \equiv \delta$ and the mixing angles $\theta_{ij}$:

$$\cos \delta \tan \theta_{13} = \frac{(\sqrt{2} - \tan \theta_{12})}{(1 + \sqrt{2} \tan \theta_{12})(1 - \tan \theta_{23})} \left( \frac{\sqrt{2}}{c_{23}} - \frac{1 + \tan \theta_{23}}{c_{13}} \right). \quad (26)$$

In general, it is not easy to determine $\delta_{CP}$ both theoretically and experimentally. Here, we obtain an explicit $\delta_{CP}$ as a function of the mixing angles which, thus, $\delta_{CP}$, could be determined experimentally. Based on the relation (1) we can calculate $\delta_{CP}$ numerically using experimental inputs. The distributions of $\delta_{CP}$ are plotted in Fig. 1 and Fig. 2.

![Figure 1: Distribution of $\delta_{CP}$](image1.png)

![Figure 2: $\delta_{CP}$ versus $\sin^2 \theta_{13}$](image2.png)

From these figures we see that $\delta_{CP}$ distributes in the region $1.78 < \delta_{CP} < 3.2$. This distribution has a mean value at $\delta_{CP} = 2.064$ which is on the edge of the $1\sigma$ region from the best fit value (BFV) to the experimental data and reaches a maximum value around $\delta_{CP} = 1.9$ which is between $1\sigma$ from the BFV [5].
Conclusions

A general neutrino mass matrix in an extended SM with an A4 flavour symmetry can be diagonalized by a perturbation method. Then, an explicit dependence between the CP-violation Dirac phase and the mixing angles leads to a quite good fit (near the 1σ region of the best fit) with recent experimental data at the normal hierarchy [5]. This result provides a more explicit form of neutrino mass and mixing matrices. The case with the inverse hierarchy is being investigated.

We are also considering perturbation of higher orders which may give a better fit (e.g., between the 1σ region) with the experimental data.

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