JT gravity from partial reduction and defect extremal surface

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ABSTRACT: We propose the three-dimensional counterpart for Jackiw-Teitelboim gravity coupled with CFT_2 bath based on partial reduction. The three-dimensional counterpart is classical AdS gravity with a defect brane which has small fluctuation in transverse direction. We derive full Jackiw-Teitelboim gravity action by considering the transverse fluctuation as a dilaton field. We demonstrate that the fine-grained entropy computed from island formula precisely agrees with that computed from defect extremal surface. Our construction provides a Lorentzian higher dimensional counterpart for Jackiw-Teitelboim gravity glued to a bath and therefore offers a framework to study problems such as black hole information paradox.

KEYWORDS: AdS-CFT Correspondence, Holography and Condensed Matter Physics (AdS/CMT)

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1 Introduction

Significant progress has been made in recent understanding of black hole information paradox [1]. In particular the island formula [2–4] for the von Neumann entropy of Hawking radiation [5] gives Page curve [6] and therefore maintains unitarity. The development relies on the quantum extremal surface formula [7] for the fine-grained entropy, which was based on the quantum corrected Ryu-Takayanagi formula in computing holographic entanglement entropy [8–10]. To justify the island formula of von Neumann entropy, one can employ replica trick and perform explicit gravitational path integral computation in lower dimensional systems. In particular the Jackiw-Teitelboim (JT) gravity coupled to a quantum bath provides a solvable model to explore the information transfer for black hole in two dimensions [2, 3]. It has been found that the island contribution corresponds to Euclidean replica wormholes [11, 12], which may cause the factorization puzzle [13, 14]. It is still quite interesting to explore the Lorentzian counterparts for the replica wormholes. There are many applications and generalizations of island formula, including the application in cosmology [15, 16] and the generalization to asymptotically flat space [17, 18]. For other related works, see [19–31, 31–45]. On the other hand, AdS gravity with a defect brane provides a natural holographic framework to study lower dimensional gravity coupled to a bath.
The defect brane model is based on AdS\(_{d+1}/BCFT_d\) [46], where the defect brane with a constant tension in the AdS bulk has AdS\(_d\) geometry. In [47] AdS/BCFT has been improved to include quantum degrees of freedom localized on the brane and it has been shown that the holographic counterpart of island formula is Defect Extremal Surface (DES) formula, which is the defect corrected Ryu-Takayanagi formula. By doing partial dimension reduction between zero tension brane and constant tension brane, one can obtain the brane world gravity, which is coupled to a CFT bath through boundary conditions. In three dimensional bulk, the simple dimension reduction leads to a two dimensional topological gravity on the brane [47]. It has been further demonstrated that entanglement entropy and reflected entropy computed from DES formula agree with island formula precisely [48]. See [49–52] for further works along this line.\(^1\) To fully reconcile the path integral approach and holographic approach, one may ask whether we can obtain JT gravity, which has a dilaton field, directly from dimension reduction in the defect brane model. If yes, what would be the holographic counterpart of the island formula for JT gravity coupled with CFT bath?

In this paper we answer these questions. We show that for the 3d AdS bulk with a defect brane, by considering the small transverse fluctuation, one can derive the full JT gravity action from partial reduction between zero tension brane and finite tension brane. In particular, the transverse fluctuation becomes the dilaton field on the brane world. For the remaining part of the bulk one can use AdS/CFT\(^2\) and obtain CFT\(_2\) bath on the asymptotic boundary. Eventually we obtain a 2d JT gravity coupled with CFT\(_2\) bath. The fact that the transverse fluctuation is small allows us to ignore higher order contributions in 2d action and obtain precisely the full 2d JT action including boundary term. We therefore obtain a higher dimensional counterpart for JT gravity coupled to a bath.

To support the 3d/2d relation, we compute the fine-grained entropy both from defect extremal surface formula in the bulk and from the boundary island formula. We find that the extremal conditions are consistent and the fined-grained entropy agree with each other precisely for small transverse fluctuation. We consider this agreement as strong evidence to support that JT gravity coupled to CFT bath can be described by semi-classical 3d gravity with a fluctuating defect brane.

Note added. After this work is finished, [54] appears in arXiv where the authors consider wedge holography with two finite tension branes.

2 Review of the model

We consider AdS\(_3/BCFT_2\) with the action given by

\[
I = \frac{1}{16\pi G_N} \int_N d^3x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G_N} \int_M d^2x \sqrt{-\gamma} K^{(\gamma)} + \frac{1}{8\pi G_N} \int_Q d^2x \sqrt{-h} K^{(h)} + I_Q + I_P, \tag{2.1}
\]

\(^1\)The defect brane model can also be generalized to higher dimensions [49, 53].

\(^2\)To be precise, we use \(Z_2\) quotient of AdS/CFT.
where $N$ denotes the bulk AdS spacetime, $M$ denotes the asymptotic boundary where the Dirichlet boundary condition is imposed and $Q$ the End of the World (EOW) brane where Neumann boundary condition is imposed. $I_Q$ is the action for matter fields constrained on $Q$ and $I_P$ is the counter term on the tip $P$. By varying this action, the Neumann boundary condition on $Q$ becomes
\[ K^{(h)}_{ab} - h_{ab} K^{(h)} = 8 \pi G_N T_{ab}, \]  
(2.2)
where $T_{ab} = -\frac{2}{\sqrt{-h}} \frac{\delta I_Q}{\delta h_{ab}}$ is the stress energy tensor coming from the variation of matter action.

Here we consider the bulk to be three dimensional and the brane $Q$ is thus two dimensional. There are two sets of useful coordinates: $(t, x, z)$ and $(t, \rho, y)$. Their relation is
\[ z = \frac{y}{\cosh \rho l}, \quad x = -y \tanh \frac{\rho}{l} \]  
(2.3)
and the bulk metric can be written in terms of either one
\[ ds_N^2 = \frac{l^2}{z^2} (-dt^2 + dz^2 + dx^2) \]
\[ = d\rho^2 + l^2 \cosh^2 \frac{\rho}{l} \cdot \frac{-dt^2 + dy^2}{y^2}, \]  
(2.4)
where $l$ is the AdS radius. It is also useful to introduce polar coordinate $\theta$ with $\frac{1}{\cos(\theta)} = \cosh (\frac{\rho}{l})$.

Consider the matter action on the brane $Q$ of the form\(^4\)
\[ I_Q = -\frac{1}{8 \pi G_N} \int_Q d^2 x \sqrt{-h} T, \]  
(2.5)
where $T$ is a constant tension. By solving (2.2), one can establish the relation between the tension and the location of the brane,
\[ T = \frac{\tanh \rho_0}{l}, \]  
(2.6)
where the brane locates at $\rho = \rho_0 = \arctanh \sin \theta_0$. See figure 1 for an illustration.

For an interval $I := [0, x_0]$ in BCFT, the entanglement entropy can be computed holographically using RT formula. As shown in figure 1, the minimal surface denoted by $\gamma_I$ terminates at a point on the brane which can be determined by minimization. The entanglement entropy is\(^5\)
\[ S_I = \frac{\text{Area}(\gamma_I)}{4 G_N} = \frac{c}{6} \log \frac{2x_0}{\epsilon} + \frac{c}{6} \rho_0 \]
\[ = \frac{c}{6} \log \frac{2x_0}{\epsilon} + \frac{c}{6} \arctanh(\sin \theta_0), \]  
(2.7)
\(^3\)Different boundary conditions are chosen because the metric on the AdS boundary is fixed while the metric on the EOW brane is not fixed.
\(^4\)This term represents a constant “vacuum energy” and becomes dominant in the low energy effective action on the brane, see [55, 56].
\(^5\)After minimization, $\gamma_I$ is a part of the circle ending on the brane in our case. See appendix D for more details.
Figure 1. The set up of AdS/BCFT where the brane tension is a constant.

where \( c \) is the CFT central charge and \( \epsilon \) is the UV cut off. The first term is precisely half of the CFT\(_2\) entanglement entropy for a single interval since \( \gamma_I \) is a quarter of a circle. The second term in eq. (2.7) is the so called boundary entropy. In particular the boundary entropy vanishes for \( \rho_0 = 0 \).

In [47], the authors improved AdS\(_3\)/BCFT\(_2\) by adding CFT matter localized on the brane. Notice that AdS\(_2\) is a maximally symmetric space, and the vacuum one point function of the CFT stress tensor takes the form

\[
\langle T_{ab} \rangle_{\text{AdS}_2} = \chi h_{ab},
\]

which contributes to Neumann boundary condition (2.2). Because of the entangled quantum matter on the defect brane, we should add the defect contribution to the ordinary holographic entanglement entropy. The improved holographic formula is therefore called Defect Extremal Surface (DES) formula [47],

\[
S_{\text{DES}} = \min_{\Gamma} \left\{ \text{ext}_{\Gamma, X} \left[ \frac{\text{Area}(\Gamma)}{4G_N} + S_{\text{defect}}[D] \right] \right\},
\]

where \( X = \Gamma \cap D \) with \( \Gamma \) the codimension two RT surface and \( D \) the defect. \( S_{\text{defect}}[D] \) is the entanglement entropy from quantum matter localized on the defect.

By using partial dimension reduction and AdS/CFT, one arrives at one lower dimensional effective theory which is gravity (coupled to brane CFT) glued with CFT bath. And this is precisely the effective theory in which island formula can be used. The procedure is shown in figure 2. The essential point here is that we only do partial dimensional reduction between zero tension brane and finite tension brane. From effective theory point of view, this corresponds to the fact that we glue brane CFT and bath CFT by a transparent interface which carries zero entropy. For an interval in the bath region, one can use either DES or island formula to calculate its fine-grained entropy and the results agree with each other. This provides a strong evidence that DES is the holographic counterpart of island formula [47].

However, the gravity theory obtained from partial reduction in [47] is topological while the gravity theory in the original paper [4] is JT gravity. Therefore it is natural to ask that if we can obtain JT gravity using partial dimension reduction. In particular whether DES
Figure 2. Effective description from partial dimension reduction plus AdS/CFT.

is still a promising holographic counterpart of island formula. These are the main questions we want to address in this paper. In the following sections, we will show that indeed we can obtain JT gravity from partial dimension reduction by considering EOW brane with small transverse fluctuation. Moreover, the fine grained entropy computed from DES and from island formula agree.

3 JT gravity from dimensional reduction

Now we derive JT gravity from dimension reduction of 3d AdS gravity action. Under the metric ansatz

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = d\rho^2 + l^2 \cosh^2 \frac{\rho}{l} \tilde{h}_{ab}(x^a)dx^a dx^b,$$

one can perform partial dimensional reduction for wedge $W_1 + \tilde{W}$ by integrating out the $\rho$ direction as shown in figure 3. The bulk action becomes

$$\frac{1}{16\pi G_N} \int_{W_1 + \tilde{W}} d^2x d\rho \sqrt{-g} (R - 2\Lambda) = \int d^2x \frac{\rho_0 + \tilde{\rho}}{16\pi G_N} \sqrt{-g^{(2)}} R^{(2)}$$

$$- \frac{1}{16\pi G_N} \int d^2x \frac{\sinh \left( \frac{2\rho}{T} \right)}{l \cosh^2 \frac{2\rho}{T}} \sqrt{-g^{(2)}},$$

where $\Lambda = -\frac{1}{l^2}$ and

$$g^{(2)}_{ab} = l^2 \cosh^2 \frac{\rho}{l} \tilde{h}_{ab}(x^a).$$

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The precise reduction of bulk Ricci scalar
\[ \sqrt{-g}R = \sqrt{-g^{(2)}} \left[ R^{(2)} - \frac{2}{l^2 \cosh^2 \frac{\rho}{l}} \right] \]
has been used in eq. (3.2). Notice that the brane $Q$ is located at
\[ \rho = \rho_0 + \hat{\rho}(x^a), \]
where $\hat{\rho}$ is a small fluctuation away from $\rho_0$, i.e. $\frac{\hat{\rho}}{\rho_0} \ll 1$. The fluctuation $\hat{\rho}$ is a function of the brane world coordinates and therefore should be treated as a field on the brane.

Next we consider Gibbons-Hawking term and the brane tension term. The extrinsic curvature of the brane is
\[ K = \nabla_\alpha n^\alpha = \frac{2 \tanh \left( \frac{\rho_0 + \hat{\rho}}{l} \right)}{l} - \frac{1}{l^2 \cosh^2 \frac{\rho}{l} \sqrt{-\hat{h}}} \partial_a \left( \sqrt{-\hat{h}} \hat{h}^{ab} \partial_b \hat{\rho} \right), \]
where $n^\alpha$ is the normal vector to the brane. Notice that the second term only contributes a total derivative in Gibbons-Hawking action. Also we note that the induced metric on the brane is \[ h_{ab} = l^2 \cosh^2 \frac{\rho}{l} \hat{h}_{ab}(x^a). \]

\[ ^6 \text{Here we neglect } O\left( \frac{\hat{\rho}^2}{\rho_0} \right) \text{ term in the induced metric, see appendix A.} \]
The tension of the brane remains a constant which equals to the tension (2.6) since the fluctuation of the $\rho$ coordinate will not affect the intrinsic brane tension. Up to a total derivative term, Gibbons-Hawking term plus the brane tension term is given by

$$
\frac{1}{8\pi G_N} \int_Q d^2x \sqrt{-h} (K - T) = \frac{1}{8\pi G_N} \int d^2x \frac{\sinh \left( \frac{2\rho_0 + 2\tilde{\rho}}{l} \right)}{l \cosh^2 \frac{\rho_0}{l}} \sqrt{-g^{(2)}}
$$

$$
- \frac{1}{8\pi G_N} \int d^2x \frac{\tanh \frac{\rho_0}{l} \cosh^2 \left( \frac{\rho_0 + \hat{\rho}}{l} \right)}{l \cosh^2 \frac{\rho_0}{l}} \sqrt{-g^{(2)}}.
$$

By adding (3.2) and (3.8) together and expanding with small $\tilde{\rho}/\rho_0$, we get the action of the 2d effective theory after partial dimension reduction

$$
I_{\text{tot}} = \frac{\rho_0}{16\pi G_N} \int d^2x \sqrt{-g^{(2)}} \frac{\hat{\rho}}{\rho_0} \left( R^{(2)} + \frac{2}{l^2 \cosh^2 \frac{\rho_0}{l}} \right)
$$

$$
+ \frac{\rho_0}{16\pi G_N} \int d^2x \sqrt{-g^{(2)}} R^{(2)} + O \left( \frac{\hat{\rho}^2}{\rho_0^2} \right).
$$

Neglecting $O \left( \frac{\hat{\rho}^2}{\rho_0^2} \right)$ terms, we see that the action is precisely the JT action, and $\frac{\hat{\rho}}{\rho_0}$ is identified as the dilaton field in JT gravity. We leave the discussion about $O \left( \frac{\hat{\rho}^2}{\rho_0^2} \right)$ terms in the appendix A.

To fully recover the JT action, we also need to reproduce the boundary term. Now we show that the JT boundary action can be obtained by doing dimension reduction from the Gibbons-Hawking term on the bulk cutoff surface.\(^7\) As shown in figure 4, the cutoff surface is denoted as $\Sigma$. Near the asymptotic boundary, the metric is taken to be

$$
ds^2 = d\rho^2 + l^2 \cosh^2 \frac{\rho}{l} \frac{-dt^2 + dy^2}{y^2}.
$$

Consider a generic cutoff surface $\Sigma$ parameterized by

$$
(y, t) = (y(u), t(u)),
$$

---

\(^7\)This part of calculation is closely related to the calculation in [36].
where $u$ is the time in the cutoff boundary. The induced metric at cutoff surface is

$$ds^2 = d\rho^2 + \ell^2 \cosh^2 \left( \frac{\rho}{\ell} \right) \left( \frac{y'^2 du^2 - t'^2 du^2}{y^2} \right),$$

where we denote $y' = \frac{dy}{du}$, $t' = \frac{dt}{du}$. One can compute the extrinsic curvature of $\Sigma$ by $K = g^{\mu\nu} \nabla_\mu n_\nu$ and the result is

$$K_{\Sigma} = \frac{1}{\ell \cosh \frac{\rho}{\ell}} \frac{t'^3 + yy' t'' - t'y'^2 - t'y y''}{(t'^2 - y'^2)^{3/2}} = \frac{1}{\ell \cosh \frac{\rho}{\ell}} K_{\Omega},$$

(3.13)

where $K_{\Omega} = \frac{t'^3 + yy' t'' - t'y'^2 - t'y y''}{(t'^2 - y'^2)^{3/2}}$ is the extrinsic curvature of the boundary $\Omega$ which is the intersection between $\Sigma$ and EOW brane. By doing dimension reduction of Gibbons-Hawking term on $\Sigma$, the action on $\Omega$ is obtained

$$I_{bJT} = \int_{\Sigma} d\rho du \sqrt{-\gamma} K_{\Sigma} = \frac{\rho_0}{8\pi G_N} \int du \sqrt{\frac{t'^2 - y'^2}{y^2}} \left( 1 + \frac{\bar{\rho}}{\rho_0} |_{\text{bdy}} \right) K_{\Omega},$$

(3.14)

where $\gamma_{\alpha\beta}$ is the induced metric on $\Sigma$. This is precisely the boundary term of JT gravity when $\frac{\bar{\rho}}{\rho_0}$ is identified as the dilaton field.

After reproducing full JT gravity action from dimension reduction, varying with respect to $\frac{\bar{\rho}}{\rho_0}$, we get the scalar curvature

$$R^{(2)} = -\frac{2}{\ell^2 \cosh^2 \frac{\rho}{\ell}}$$

(3.15)

and therefore the metric $\tilde{h}_{ab} dx^a dx^b = -dt^2 + \frac{dy^2}{y^2}$. Varying with respect to the metric, the fluctuation is

$$\frac{\bar{\rho}}{\rho_0} = \frac{\bar{\phi}_r}{y}.$$

(3.16)

At the intersection $\Omega$, employing the same trick in [57] one can fix the induced metric

$$g|_{\text{bdy}} = -\ell^2 \cosh^2 \left( \frac{\rho}{\ell} \right) \frac{\bar{\rho}}{\rho_0}.$$

(3.17)

By identifying the induced metric in (3.12) to (3.17), $y$ can be solved to the leading order in $\epsilon$, $y = \epsilon t' + O(\epsilon^2)$. By inserting the expression of $y$ into $K_{\Omega}$, we find

$$K_{\Omega} = 1 - \epsilon^2 \text{Sch}(t(u), u),$$

(3.18)

and the boundary JT action eq. (3.14) reads

$$I_{bJT} = \frac{\rho_0}{8\pi G_N} \int du \frac{K_{\Omega}}{\epsilon} + \frac{\rho_0}{8\pi G_N} \int du \frac{\epsilon^2}{\bar{\phi}_r(u)} K_{\Omega}.$$  

(3.19)

where we denote $\phi_r(u) = \frac{\bar{\phi}_r}{\rho_0}$. The first term is topological and the second term reads

$$\frac{\rho_0}{8\pi G_N} \int du \frac{\epsilon^2}{\bar{\phi}_r(u)} (1 - \epsilon^2 \text{Sch}(t(u), u)),$$

(3.20)

We choose the static profile.
where \( \phi_r(u) \) is the external coupling and \( t(u) \) is the field variable \([57]\). Notice that previously we neglect the total derivative term in eq. (3.6) when obtaining 2d JT action, however it may contribute to the JT boundary term. By calculating this term directly using Stokes theorem, we find that it takes the form \(-\int du \frac{\partial \phi_r(u)}{\partial u}\) and thus it precisely cancels the field independent term in eq. (3.20), leaving the boundary action at \(\Omega\) to be the Schwarzian action. We leave the detailed computation to appendix B and appendix C.

Thus after doing the partial dimension reduction for wedge \(W_1 + \tilde{W}\) and using standard AdS/CFT for wedge \(W_2\), which is the bulk region between zero tension brane and AdS boundary,\(^9\) we obtain the 2d effective theory to be a JT gravity together with the brane CFT, glued to a non-gravitational CFT bath through transparent boundary conditions, which is precisely the original set up to motivate the island formula. See figure 3 for an illustration of this procedure.

4 Entanglement entropy for an interval

4.1 Entanglement entropy from island formula

Since we have obtained the 2d effective description in terms of a JT gravity glued with a CFT bath, we can use island formula to calculate the von Neumann entropy for an interval \([0, L]\) on the asymptotic boundary

\[
S = \min_X \left\{ \text{ext} X \left[ \frac{\text{Area}(X)}{4G_N} + S_{\text{semi-cl}}(\Sigma_X) \right] \right\},
\]

where \(X = \partial I\) and \(I\) is the island, \(\Sigma_X\) is the associated region including the island and \(S_{\text{semi-cl}}(\Sigma_X)\) is the von-Neumann entropy of the quantum fields on \(\Sigma_X\) in the semi-classical description. The interval \([0, L]\) for which we compute the von Neumann entropy and the associated island are illustrated in figure 5. Introducing the 2d Newton constant

\[
G^{(2)}_N = \frac{G_N}{\rho_0},
\]

the action of 2d effective theory becomes

\[
I_{2d} = \frac{1}{16\pi G^{(2)}_N} \int d^2x \sqrt{-g^{(2)}} R^{(2)} + \frac{1}{16\pi G^{(2)}_N} \int d^2x \sqrt{-g^{(2)}} \frac{\hat{\rho}}{\rho_0} \left( R^{(2)} + \frac{2}{l^2 \cosh^2 \frac{a}{l}} \right) + I_{\text{CFT}}.
\]

Now we employ island formula to calculate the entropy of a single interval \([0, L]\). For simplicity we focus on static cases. The generalized entropy is given by

\[
S_{\text{gen}}(a) = S_{\text{area}} + S_{\text{matter}} = \frac{1}{4G^{(2)}_N} \left( 1 + \frac{\hat{\rho}}{\rho_0} \right) + \frac{c}{6} \log \frac{(L + a)^2}{a \cos \theta_0 e^a}.\]

\(^9\)It looks that we will introduce another boundary/brane in the bulk. However, since the brane has no tension, one can treat it as a transparent interface. The corresponding field theory which is dual to \(W_2\) region is thus a CFT with a transparent boundary.
Figure 5. The interval $[0, L]$ for which we compute the von Neumann entropy.

The extremization condition $\partial_a S_{\text{gen}}(a) = 0$ determines the boundary of island $a$ to be

$$a = \frac{L}{\mu} \left( \sqrt{36\mu^2 + 36\mu + 1} + 6\mu + 1 \right),$$

where $\mu$ is defined as $\mu = \frac{\rho_0 \bar{\phi}}{6L}$, which can also be expressed as $\frac{1}{6} \cdot \frac{\tilde{\rho}(a)}{L} \cdot \frac{a}{L}$, because of eq. (3.16). It characterizes the amplitude of the brane transverse fluctuation at location $a$, since $\frac{a}{L}$ measures the distance from the origin and $\tilde{\rho}$ measures the fluctuation in angular direction. In fact one can work out a simple relation between extremal point $a$ and $\tilde{\rho}(a)$

$$\frac{a}{L} = 1 + \frac{\tilde{\rho}(a)}{1 - \tilde{\rho}(a)}.$$ (4.6)

By plugging (4.5) into $S_{\text{gen}}(a)$ one can get the entropy $S_{\text{island}}$ computed from island formula.

### 4.2 Entanglement entropy from bulk DES

The defect extremal surface formula is the defect corrected Ryu-Takayanagi formula. For defect $D$ in the AdS bulk, the entanglement entropy is computed following

$$S_{\text{DES}} = \min_{\Gamma \subset X} \left\{ \text{ext}_{\Gamma \cap D} \left[ \frac{\text{Area}(\Gamma)}{4G_N} + S_{\text{defect}}[D] \right] \right\},$$

where $X = \Gamma \cap D$ and $\Gamma$ is the codimension two RT surface. We consider the interval $[0, L]$ on the asymptotic boundary and use DES formula to calculate its entropy. First we compute the RT surface $\frac{\text{Area}(\Gamma)}{4G_N}$ by using embedding coordinates. Considering $\tilde{\rho} / \rho_0 \ll 1$, the leading order result is

$$\frac{\text{Area}(\Gamma)}{4G_N} = \frac{l}{4G_N} \arccosh \left[ \frac{(L + a \sin \theta_0)^2 + a^2 \cos^2 \theta_0}{2a \cos \theta_0 \epsilon} \right] + \frac{\rho_0 \phi_r}{4G_N a}.$$ (4.8)

The computation detail is shown in appendix D. The brane CFT also contributes to the entanglement entropy

$$S_{\text{defect}}(D) = \frac{c'}{6} \log \frac{2l}{\epsilon_y \cos \theta_0}.$$ (4.9)
Then the total generalized entropy is

\[ S_{\text{gen}}(a) = \frac{c}{6} \log \left( \frac{(L + a \sin \theta_0)^2 + a^2 \cos^2 \theta_0}{a \cos \theta_0 c} \right) + \frac{\rho_0 \phi_r}{4 G_N a} + \frac{c'}{6} \log \frac{2l}{\epsilon_y \cos \theta_0}, \]  

(4.10)

where \( c = \frac{3l}{2G_N} \) is used and \( c' \) is the central charge of the brane CFT. To compare with the result obtained from boundary island formula, we take \( c' = c \).

From \( \partial_a S_{\text{gen}}(a) = 0 \), one obtains the extremal position of \( a \) to be

\[ a = \frac{(3)^{\frac{2}{3}} L (12 \mu^2 + 12 \mu \sin \theta_0 + 1)}{3 \nu} + 2 \mu L + \frac{L \nu}{3 \tau}, \]  

(4.11)

where

\[ \nu = \sqrt{72 \mu^3 + \frac{1}{6} \sqrt{\gamma} + 108 \mu^2 \sin \theta_0 + 36 \mu} \]  

(4.12)

\[ \gamma = 46656 \mu^2 \left( 2 \mu^2 + 3 \mu \sin \theta_0 + 1 \right)^2 - 108 \left( 12 \mu^2 + 12 \mu \sin \theta_0 + 1 \right)^3. \]

Plugging (4.11) into \( S_{\text{gen}}(a) \), we have the entropy result calculated from DES.

### 4.3 Comparison between DES and island formula

Now we compare the entropy result computed from DES and that from island formula.

We first consider \( \rho_0/l \gg 1 \) with the small fluctuation condition \( \tilde{\rho}/\rho_0 \ll 1 \) satisfied. This is the limit that the brane is nearly parallel to the asymptotic boundary. In this limit, the extremal point (4.5) and (4.11) coincide with each other and both DES and island formula give the same entropy. To see this, let us fix \( \mu \) and set \( \theta_0 = \frac{\pi}{2} - \omega \).

Expanding the extremal point and the entropy around \( \omega = 0 \), we get

\[ a_{\text{DES}} = L \left( \sqrt{36 \mu^2 + 36 \mu + 1} + 6 \mu + 1 \right) + \mathcal{O}(\omega^2) = a_{\text{island}} + \mathcal{O}(\omega^2), \]  

(4.13)

and

\[ S_{\text{DES}} = \frac{2 \mu}{6 \mu + 1 + \eta} + \frac{c}{6} \log \left( \frac{(6 \mu + 3 + \eta)^2 L}{(6 \mu + 1 + \eta) \epsilon} \right) + \frac{c}{6} \log \frac{l}{\omega^2 \epsilon_y} + \mathcal{O}(\omega^2), \]  

(4.14)

where

\[ \eta = \sqrt{36 \mu^2 + 36 \mu + 1}. \]  

(4.15)

Furthermore, we can consider a generic brane location, i.e. \( \rho_0/l \) is finite. By the small fluctuation condition \( \tilde{\rho}/\rho_0 \ll 1 \), we have that \( \tilde{\rho}/l \) is small, which implies that \( \mu = \frac{1}{6} \cdot \frac{l}{\epsilon_y} \cdot \frac{a}{L} \ll 1 \) provided that \( a/L \) is \( O(1) \). In this limit we can expand with small \( \mu \). The extremal point of bulk DES becomes

\[ a = L + 6(1 + \sin \theta_0)L \mu + \mathcal{O}(\mu^2), \]  

(4.16)

\[ ^{10} \text{Although } \rho_0 \text{ is very large, } \mu \text{ can be fixed.} \]
and the extremal point of island formula is

\[ a = L + 12L\mu + O(\mu^2). \]  

(4.17)

The entropy from bulk DES is

\[ S_{\text{DES}} = \frac{c}{6} \log 2 - \frac{L}{\epsilon} + \frac{c}{6} \log \frac{1 + \sin \theta_0}{\cos \theta_0} + \frac{c}{6} \log \frac{2l}{\epsilon_y \cos \theta_0} + c\mu + O(\mu^2), \]  

while the entropy from boundary island formula is

\[ S_{\text{island}} = \frac{c}{6} \log 2 - \frac{L}{\epsilon} + \frac{c}{6} \arctanh(\sin \theta_0) + \frac{c}{6} \log \frac{2l}{\epsilon_y \cos \theta_0} + c\mu + O(\mu^2). \]  

(4.19)

Comparing the above two results, we find that the entanglement entropy for single interval \([0, L]\) from DES and from island formula precisely match for small brane fluctuations.

### 4.4 Conclusion and discussion

In this paper we constructed the three-dimensional counterpart for Jackiw-Teitelboim gravity glued to CFT\(_2\) bath based on partial reduction. The bulk counterpart is classical AdS gravity with a defect brane which has small fluctuation in transverse direction. We obtain full Jackiw-Teitelboim gravity action by identifying the transverse fluctuation as a dilaton field on brane world. We further demonstrated that the fine grained entropy computed from island formula precisely agrees with that computed from defect extremal surface.

We discuss here some connections and/or difference between this work and various other works.

In [42], the authors also obtained the JT gravity from dimension reduction of the three dimensional bulk. The difference is that there is no brane in their set up. Furthermore, the dilaton field comes from a metric ansatz in [42] while we consider the dilaton as the transverse fluctuation of the brane.

Also note that our model is different from the “double holography” set up proposed in [4]. In our model, we do not replace the matter field on the brane by a higher dimensional bulk, instead we consider it as a defect theory, where there are quantum degrees of freedom localized on the brane.

In a very recent work [54] the authors also obtained JT gravity from wedge holography with two finite tension branes. To compare, we note that in that work, the two branes are both fluctuating while we only have a single fluctuating brane. A \(Z_2\) orbifolding will possibly connect their set up and the JT sector of our set up. Although the dimension reduction part looks similar, we treat the brane as a defect in the bulk which is different from [54]. Moreover, different from [54], the partial reduction indicates that JT is only a part of a larger quantum gravity, which probably relates to the ensemble nature of JT gravity.

There are a few interesting future questions: First, using our construction to understand JT gravity/ensemble relation. In our construction the JT gravity from partial reduction is dual to a defect theory in asymptotic boundary and it would be interesting to check the relation between this defect theory and the ensemble of quantum mechanics discussed
in [58]. Second, generalize our construction to higher dimensions. In higher dimensions, the dilaton field is known as radion field in the original Randall-Sundrum model [55]. It is quite interesting to work out the full brane world theory including the dilaton field from partial reduction. Last but not least, it is interesting to test our construction by other entanglement measures and to explore the physical implications of the dilaton in brane world cosmology [50].

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A Second order fluctuation

In this section, we will analyze $O\left(\tilde{\rho}^2\right)$ terms. The brane embedding is given by

$$\rho - \tilde{\rho} = \rho_0 = \text{const},$$

thus the normal vector is

$$n_\alpha = (n_\rho, n_t, n_y) = \left(1, -\frac{\partial \tilde{\rho}}{\partial t}, -\frac{\partial \tilde{\rho}}{\partial y}\right).$$

Then the extrinsic curvature can be computed,

$$K = \nabla_\alpha n^\alpha = \frac{2 \tanh \left(\frac{\rho_0}{l} + \tilde{\rho}\right)}{l} + \frac{y^2}{l^2 \cosh^2 \left(\frac{\rho_0}{l} + \tilde{\rho}\right)} \left(\frac{\partial^2 \tilde{\rho}}{\partial t^2} - \frac{\partial^2 \tilde{\rho}}{\partial y^2}\right),$$

where we neglect the normalization factor as it only contributes to higher order $\tilde{\rho}$ terms. For the hyper-surface parameterized by $(\rho_0 + \tilde{\rho}(x^\mu), t, y)$, we can construct the tangent vector

$$e^\alpha_t = \left(\frac{\partial \tilde{\rho}}{\partial t}, 1, 0\right),$$

$$e^\alpha_y = \left(\frac{\partial \tilde{\rho}}{\partial y}, 0, 1\right).$$

The induced metric $h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$ can be written up to the second order in $\tilde{\rho}$

$$h_{ab} = \begin{bmatrix}
\left(\frac{\partial \tilde{\rho}}{\partial t}\right)^2 - \frac{l^2 \cosh^2 \left(\frac{\rho_0}{l} + \tilde{\rho}\right)}{y^2} & \frac{\partial \tilde{\rho}}{\partial t} \frac{\partial \tilde{\rho}}{\partial y} \\
\frac{\partial \tilde{\rho}}{\partial t} \frac{\partial \tilde{\rho}}{\partial y} & l^2 \cosh^2 \left(\frac{\rho_0}{l} + \tilde{\rho}\right) \frac{\partial \tilde{\rho}}{\partial y}^2 + \left(\frac{\partial \tilde{\rho}}{\partial y}\right)^2
\end{bmatrix}. $$

The determinant of $h_{ab}$ is

$$\sqrt{-h} = \frac{l^2 \cosh^2 \left(\frac{\rho_0}{l} + \tilde{\rho}\right)}{y^2} - \frac{1}{2} \left[\left(\frac{\partial \tilde{\rho}}{\partial t}\right)^2 - \left(\frac{\partial \tilde{\rho}}{\partial y}\right)^2\right].$$
Combined with the bulk term, the action is given by up to second order

\[ S^{(2)} = \frac{1}{8\pi G_2} \int d^2x \sqrt{-g^{(2)}} \left[ \tanh \left( \frac{\phi}{\rho_0} \right) \frac{\rho_0}{2l} (\nabla_\mu \phi \nabla^\mu \phi) + \frac{\tanh \left( \frac{\phi}{\rho_0} \right)}{\cosh^2 \frac{\phi}{2l} l^2 \rho^2} \right], \tag{A.8} \]

where we identify \( \phi = \frac{\tilde{\phi}}{\rho_0} \). This is nothing but the free field action.

### B Schwarzian theory on the JT boundary

In this appendix we show further details of how to compute the JT boundary term and recognize that this is a Schwarzian theory.

We consider the UV cutoff surface \( \Sigma \). The tangent vector and normal vector are

\[ T^\nu = \frac{1}{l \cosh \left( \frac{\rho}{l} \right)} (t', y', 0), \tag{B.1} \]

\[ n_\mu = \left( \frac{y' \cosh \frac{\rho}{l}}{\sqrt{t^2 - y'^2}}, -\frac{t' \cosh \frac{\rho}{l}}{\sqrt{t^2 - y'^2}}, 0 \right). \tag{B.2} \]

The extrinsic curvature is given by

\[ K = g^{\mu\nu} \nabla_\mu n_\nu = -\frac{T^\nu}{T^2} \nabla_T n_\nu = -\frac{T^\nu}{T^2} (\partial_\mu n_\nu - \Gamma^\nu_{\mu\nu} n_\rho T^\rho) = \frac{1}{l \cosh \frac{\rho}{l}} \frac{t^3 + yy'y'' - t'y'^2 - t'yy''}{(t^2 - y'^2)^{3/2}}. \tag{B.3} \]

By using \( y = ct' \) and expanding \( K \) to \( O(\epsilon^2) \), we get

\[ K = \frac{1}{l \cosh \frac{\rho}{l}} (1 - \epsilon^2 \text{Sch}(t(u), u)), \tag{B.4} \]

where

\[ \text{Sch}(t(u), u) = \frac{2t't'' - 3t'^2}{2t'^2}. \tag{B.5} \]

### C Total derivative contribution to the JT boundary

The total derivative term does not affect the equation of motion but will contribute to the boundary term through Stokes theorem

\[ -\int dydt \partial_a (\sqrt{-h}h^{ab} \partial_b \tilde{\rho}) = -\int \sqrt{-\gamma} n_a V^a. \tag{C.1} \]

Here \( \gamma \) is the induced metric on the boundary and is given by (3.17), \( n_a \) is the outgoing normal vector

\[ n_a = \left( \frac{y' \cosh \frac{\rho}{l}}{\sqrt{t'^2 - y'^2}}, -\frac{t' \cosh \frac{\rho}{l}}{\sqrt{t'^2 - y'^2}} \right), \tag{C.2} \]

and \( V^a \) is

\[ V^a = h^{ab} \partial_b \tilde{\rho} = \frac{y^2}{l^2 \cosh \frac{\rho}{l}} (-\partial_t \tilde{\rho}, \partial_y \tilde{\rho}). \tag{C.3} \]
Thus the contribution to the boundary term becomes

$$- \frac{1}{8\pi G_N} \int \frac{du}{\epsilon} \ln \left( \frac{\rho}{\ell} \right) n_a V^a = - \frac{\rho_0}{8\pi G_N} \int \frac{y \tilde{\phi}_r}{\sqrt{t^2 - y^2}^2} \epsilon^2 \, du = - \frac{\rho_0}{8\pi G_N} \int \frac{d\rho_r(u)}{\epsilon^2}, \quad (C.4)$$

where the solution of $\tilde{\rho}$ (3.16) and the relation $\frac{t^2 - y^2}{y^2} = \frac{1}{\epsilon^2}$ are used. One can see that it precisely cancels the field independent term obtained in eq. (3.20).

### D Computation of bulk RT surface contribution

In this appendix, we compute the entropy contribution of RT surface by using embedding coordinates.

The embedding coordinates are

$$X^0 = z \left( \frac{1}{2} \ell^2 + x^2 - t^2 \right),$$
$$X^1 = \frac{\ell^2}{z} t, \quad X^2 = \frac{\ell^2}{z} x, \quad X^3 = \frac{z}{2} - \frac{1}{2z} \left( \ell^2 - x^2 + t^2 \right),$$

where $(X^0)^2 + (X^1)^2 - (X^2)^2 - (X^3)^2 = \ell^2$. Using these the geodesic distance $s$ between two points $(t_1, z_1, x_1)$ and $(t_2, z_2, x_2)$ is obtained as

$$s = l \arccosh \left[ \frac{-(t_2 - t_1)^2 + (x_2 - x_1)^2 + z_1^2 + z_2^2}{2z_1 z_2} \right]. \quad (D.2)$$

Plugging in two points $A = (t, a \cos \theta, -a \sin \theta)$ and $B = (t, \epsilon, L)$ where $A$ is the intersection point of DES and EOW brane and $B$ is the right boundary of the interval, one gets

$$\frac{\text{Area}(\Gamma)}{4G_N} = \frac{l}{4G_N} \arccosh \left[ \frac{(L + a \sin \theta)^2 + a^2 \cos^2 \theta}{2a \cos \theta \epsilon} \right]. \quad (D.3)$$

Considering $\frac{\rho}{\rho_0} \ll 1$, we can expand the RT result to the first order

$$\frac{\text{Area}(\Gamma)}{4G_N} = \frac{l}{4G_N} \arccosh \left[ \frac{(L + a \sin \theta_0)^2 + a^2 \cos^2 \theta_0}{2a \cos \theta_0 \epsilon} \right] + \frac{\rho_0 \hat{\phi}_r}{4G_N a}, \quad (D.4)$$

where $\frac{1}{\cos \theta} = \cosh \left( \frac{\rho_0 + \theta}{l} \right)$ and the solution (3.16) has been used.

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