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Citation
Ferrara, Andrea, and Abraham Loeb. 2013. “Escape Fraction of the Ionizing Radiation from Starburst Galaxies at High Redshifts.” Monthly Notices of the Royal Astronomical Society 431 (3): 2826–33. https://doi.org/10.1093/mnras/stt381.

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Escape fraction of the ionizing radiation from starburst galaxies at high redshifts

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Accepted 2013 February 26. Received 2013 February 5; in original form 2012 September 11

ABSTRACT

Recent data indicate that the cosmic ultraviolet emissivity decreased with decreasing redshift $z$ near the end of reionization. Lacking evidence for very massive early stars, this could signal a decline with time in the mass-averaged escape fraction of ionizing radiation from galaxies $\langle f_{\text{esc}} \rangle$ at $z \gtrsim 6$. We calculate the evolution of ionization fronts in dark matter haloes which host gas in hydrostatic equilibrium at its cooling temperature floor ($T \approx 10^4$ K for atomic hydrogen). We find a high escape fraction only for the lowest mass haloes [with $M < 10^{9.7}$ M$_\odot$ at $(1 + z) = 10$], provided their star formation efficiency $f_s \gtrsim 10^{-3}$. Since the low-mass galaxy population is depleted by radiative feedback, we find that indeed $\langle f_{\text{esc}} \rangle$ decreases with time during reionization.

Key words: galaxies: star formation – dark ages, reionization, first stars.

1 INTRODUCTION

The first generation of galaxies are expected to have reionized cosmic hydrogen by a redshift $z \approx 7$ (Loeb & Furlanetto 2013). One of the most important unknown parameters regulating the way in which reionization proceeds is the fraction of ionizing photons, $f_{\text{esc}}$, that escape outside the virial radius of the galaxies in which they were produced and into the surrounding intergalactic medium (IGM). The growth of ionized regions with cosmic time depends on the average value of this parameter over the viewing angle per galaxy and over the galaxy population (Wyithe & Loeb 2003; Trac & Cen 2007; Bouwens et al. 2012; Finkelstein et al. 2012).

Observations of galaxies at $z = 1$–3 indicate a broad set of escape fraction values ranging from a few per cent to tens of per cent (Steidel, Pettini & Adelberger 2001; Fernández-Soto, Lanzetta & Chen 2003; Inoue, Iwata & Deharveng 2006; Shapley et al. 2006; Siana et al. 2007; Giallongo et al. 2008; Iwata et al. 2009; Boutsia et al. 2011; Grazian et al. 2011; Vanzella et al. 2012a,b). This potentially reflects strong variations of the escape fraction with viewing angle and evolutionary time.

A calculation of the escape fraction of ionizing radiation from first principles is difficult, as it depends on the spatial distribution of ionizing sources relative to the neutral hydrogen in galaxies and is therefore sensitive to the small-scale clustering of young stars and the interstellar medium. Due to their higher density, high-redshift galactic discs are expected to have small escape fractions during reionization (Wood & Loeb 2000; Razoumov & Sommer-Larsen 2006; Grazian et al. 2008; Wise & Cen 2009). Even for local disc galaxies, Dove, Shull & Ferrara (2000) concluded that $f_{\text{esc}}$ should not exceed a few per cent in most cases, as a result of efficient radiation trapping by the shells of the expanding superbubbles around OB star associations.

Since the mean density of the Universe (and hence galaxies) scales as $(1 + z)^3$, theoretical calculations tend to conclude that $f_{\text{esc}}$ should decrease with increasing redshift. This expectation appears to be in conflict with recent reionization models and data (Bolton & Haehnelt 2007; Salvaterra, Ferrara & Dayal 2011; Finkelstein et al. 2012; Kuhlen & Faucher-Giguère 2012; Mitra, Choudhury & Ferrara 2012; Mitra, Ferrara & Choudhury 2013) that point towards the need for an increasing ultraviolet (UV) emissivity towards high redshifts. This can be achieved in two ways. The first is to postulate an increasingly top-heavy initial mass function (IMF) of stars that would increase the number of photons emitted per baryon incorporated in stars. Although possible, this explanation seems to be disfavoured by the lack of observational evidence for an early population of substantially more massive stars (Cayrel et al. 2004; Caffau et al. 2011) and by recent numerical simulations (Greif et al. 2012). A more appealing solution is represented by a possible increase in the average escape fraction $\langle f_{\text{esc}} \rangle$ during reionization, recently suggested by Mitra et al. (2013). Here we explore a novel physical explanation for this unexpected trend.

We show that the lowest mass galaxies near or below (so-called minihaloes) the hydrogen cooling threshold have high escape fractions but their contribution to the UV production rate decreases with cosmic time due to large-scale radiative feedback processes during reionization that either photoheat or sterilize them by dissociating their H$_2$. However, internal feedback is of foremost importance as well: as these small systems achieved large $f_{\text{esc}}$ by rapidly (within $\sim 10^8$ yr) ionizing their entire interstellar medium,
this gas will become loosely bound to the host galaxy. Under these conditions it is difficult to sustain a continuous mode of star formation (SF), particularly because following the death of massive stars powerful supernova (SN) explosions will clear the gas out of the potential well before a stable disc structure is established. More massive galaxies will form a disc with the net result of trapping most of their ionizing photons.

If high-redshift dwarf galaxies form stars predominantly in episodic bursts, as suggested by the high incidence of galaxy mergers at increasing redshift (Muñoz & Loeb 2011), the conventional argument that their SF efficiency is suppressed by SN feedback may not be valid. In particular, SN feedback is inhibited if the duration of the starburst is shorter than ~10 Myr, the lifetime of SN progenitors.

In this regime, the global radiative feedback sets the amount of gas initially available for making stars and hence the overall efficiency of SF, $f_s$. In our calculations, we will calibrate the value of $f_s$ based on recent measurements of the cosmic stellar mass density at high redshifts (González et al. 2011).

For an instantaneous starburst, $f_{\text{esc}}$ can be large only if an escape route for ionizing photons is opened within a few Myr, prior to the death of the massive stars that produce these photons. This process cannot be mediated by SN explosions, which occur after the emission of ionizing photons has already started to decline. The key question is therefore whether the surrounding blanket of absorbing hydrogen atoms can be ionized before the massive stars in the starburst end their life. We address this question within the context of a simple model in which we populate dark matter haloes with primordial gas that cooled to the temperature floor of atomic hydrogen, $\sim 10^4$ K.

Our basic point is simple: massive galaxies exist at all redshifts and the escape fraction of ionization from them is calculated and observed to be small; however, the lowest mass galaxies ($T_{\text{vir}} \lesssim 10^4$ K) exist only before and during reionization and have a high $f_{\text{esc}}$. Our assumption of a short-lived SF phase without SN feedback is self-consistent, as we will show that photoionization heating of the gas suppresses SF in dwarf galaxies after a period of time much shorter than the lifetime of massive stars.

In Section 2, we describe the details of our calculation and in Section 3 we present our numerical results. Finally, we discuss our main conclusions in Section 4. Throughout the paper, we adopt the WMAP7 set of cosmological parameters with $\Omega_m = 0.27$, $\Omega_{\Lambda} = 1 - \Omega_m$, $\Omega_b h^2 = 0.023$, $h = 0.71$, $\sigma_8 = 0.81$, $n_s = 0.97$ and $dn_k/dn_k = 0$ (Larson et al. 2011).

2 METHOD OF CALCULATION

We derive the escape faction of the ionizing photons, $f_{\text{esc}}$, based on a set of simplifying assumptions. We consider a spherical dark matter halo of mass, $M$, virializing at redshift $z$ and characterized by an internal density (spherically averaged) profile following the Navarro–Frenk–White (Navarro, Frenk & White 1997, NFW) form

$$\rho(r) = \frac{\rho_s \delta_c}{c x (1 + cx)^2},$$

(1)

where $z \equiv r/r_{\text{vir}}$, $r_{\text{vir}}$ is the virial radius of the system, $c$ is the halo concentration parameter, $\delta_c = (18\pi^2/3)c^3/F(c)$ is a characteristic overdensity and

$$F(t) = \ln(1 + t) - \frac{t}{1 + t}.$$  

(2)

For the concentration parameter we use the results of Prada et al. (2012), extrapolating them towards lower masses and higher redshifts when necessary. The circular velocity, $v_c^2(r) = GM(r)/r$, can be expressed as

$$v_c^2 = \frac{V^2 F(cx)}{x F(c)},$$

(3)

with $V_c^2 = GM/r_{\text{vir}}$. Baryons that fall into the dark matter halo potential well are shock-heated to the virial temperature $T_{\text{vir}} = (\mu m_p/2k_B)V_c^2$, and then relax to a hydrostatic configuration (Makino, Sasaki & Suto 1998),

$$\ln \rho(r) = \ln \rho_0 - \frac{V_c^2}{V_c^2} [v_c^2(r = 0) - v_c^2(r)].$$

(4)

Here, $v_c$ is the halo escape velocity from the halo,

$$v_c^2(r) = 2V_c^2 \frac{F(cx) + cx/(1 + cx)}{x F(c)},$$

(5)

$\mu = 1.22$ is the mean molecular weight of a neutral primordial H–He gas; $k_B$ is the Boltzmann constant and $m_p$ the proton mass. We have also inserted in equation (4) the extra factor $V_c^2/V_c^2 = T_{\text{vir}}/T > 1$, where $c^2 = 2k_B T/\mu m_p$ is the effective sound speed at a generic temperature $T \neq T_{\text{vir}}$. The standard case in which $T = T_{\text{vir}}$ is obtained by setting $V = 1$. By manipulating equation (4) we then get the final expression for the gas density profile as a function of radius,

$$\rho(x) = \rho_0 e^{-x^2/2} (1 + cx)^{-5/2}. \approx \mu m_p n_0 \gamma(x),$$

(6)

where we have defined $\gamma = 2c^2 / F(c)$, and the ancillary function $\gamma(x) = e^{-x^2/2} (1 + cx)^{-5/2}$. The central density, $\rho_0$, is obtained by requiring that the total gas mass in the halo is equal to the cosmological value, i.e. $M_b = (\Omega_b/\Omega_m)M = f_s M$. This procedure yields

$$\rho_0 = (18\pi^2/3)f_s c^2 e^2 \left[ \int_0^\infty (1 + t)^{-5/2} t^2 \right]^{-1}.$$  

(7)

Table 1 provides the values of the central number density, $n_0$, for different halo masses and $V$ values at a fixed redshift, $z = 9$. As expected, $n_0$ increases in larger masses even in the case of $V = 1$ in which the gas is at the virial temperature of the halo. This reflects the strong dependence of $n_0$ on the concentration parameter, which increases from $c \approx 4$ for $M = 10^{10} M_\odot$ to $c \approx 6.5$ for $M = 10^{11} M_\odot$. This increasing trend is amplified if the gas is cooler than the virial temperature ($V > 1$) or if it is thermostated at a fixed temperature (such as $T = 2 \times 10^4$ K in Table 1), leading to unrealistic densities for the most massive haloes with $M = 10^{11} M_\odot$ (which represent rare $3\sigma$ fluctuations of the density field at $z = 9$). For massive haloes, we obtain artificially high densities because we ignore angular momentum. Once the gas condenses to a sufficiently small scale, its rotation will halt contraction and it will settle to a disc. Discs were considered by Wood & Loeb (2000) and Dove et al. (2000) who independently concluded that the escape fraction

| $\log_{10}T_{\text{vir}}$ (K) | 4.31 | 4.98 | 5.65 | 6.32 |
|-----------------------------|------|------|------|------|
| $M$ ($M_\odot$)             | $10^8$ | $10^9$ | $10^{10}$ | $10^{11}$ |
| $V^2 = 1.0$                 | 20.6 | 23.9 | 36.0 | 167.8 |
| $T = 2 \times 10^4$ K       | 24.3 | $1.9 \times 10^4$ | 4.3 | $10^6$ |
was negligible for a smooth gas distribution in high-redshift discs and also local spirals. We therefore argue that once the gas contracts enough to make a disc, its density will not increase by as much as our spherical model predicts but UV photons will not be able to escape from it anyway, based on earlier studies.

2.1 Ionization front evolution

Having specified the gas density distribution inside haloes, we may now derive the evolution of an ionization front (IF) driven by the emission of photons with energy > 13.6 eV from a burst of SF at the halo centre. We assume that a fraction \( f_s \) of the available gas, \( f_s M \), of a given halo is instantaneously converted into stars distributed in mass according to a Salpeter IMF in the range \( m_{\text{min}}, m_{\text{max}} = (1 \, M_\odot \text{, } 100 \, M_\odot) \) and with absolute metallicity \( Z = 10^{-3} \).

The ionizing photon production rate, \( Q(t) \), by the stellar cluster can be computed exactly from population synthesis models: we use here \texttt{STARBURST99}\(^1\) (Leitherer et al. 1999). The time dependence of the production rate of ionizing photons under these conditions is

\[
Q(t) = \frac{Q_0}{1 + (t/t_0)^4},
\]

with \( (Q_0, t_0) = (10^{47} \text{s}^{-1} \, M_\odot^{-1}, 10^6 \text{ yr}) \). Equation (8) illustrates the important point that after \( \sim 4 \) Myr, the production rate of ionizing photons rapidly drops as a result of the death of short-lived massive stars. This has important implications for \( f_{\text{esc}} \) as discussed hereafter. Note that equation (8) implies that the number of ionizing photons emitted per baryon incorporated into stars is \( N_\gamma \approx 0.5Q_0 m/\dot{M}_\odot = 5 \times 10^3 \).

The time evolution of the IF radius, \( r_I \), is described by an ordinary differential equation that expresses a detailed balance between the ionization and recombination rates within the volume enclosed by the IF,

\[
\frac{dr_I}{dt} = \frac{1}{4 \pi n_H r_I^2} \left[ Q(t) - \frac{4\pi}{3} N_\gamma n_H^2 \alpha_{\text{rec}}^2 \right],
\]

where \( n_H = 0.92n_B \) is the hydrogen density for a primordial gas and \( \alpha_{\text{rec}}^2 = 2.6 \times 10^{-13} / (T/10^4 \text{ K})^{-1/2} \) is the Case B recombination rate of hydrogen (Maselli, Ferrara & Ciardi 2003). Normalizing by an effective recombination rate in the halo, \( N_\gamma \approx (4\pi/3) \alpha_{\text{rec}}^2 n_H^2 r_I^3 \), and adopting a dimensionless time variable \( \tau = t/t_{\text{esc}} = \alpha_{\text{rec}}^2 n_H t \), equation (9) can be written in a dimensionless form,

\[
\frac{dr_I}{d\tau} = \frac{Q(\tau)}{3N_\gamma \eta(x)} x^2 - \frac{1}{3} x \eta(x).
\]

Equation (10) describes the expansion of the H \( \equiv \) region into the stratified gas distribution within the halo. As the density decreases outwards, the IF accelerates and eventually exits the virial radius at a time \( t_{\text{out}} \equiv t(x = 1) \). Until \( t_{\text{out}} \) all the ionizing photons are absorbed inside the halo\(^2\); hence, \( f_{\text{esc}} \approx 0 \). However, for \( t > t_{\text{esc}} \), ionizing photons will be only used to keep the halo ionized by balancing recombinations within it, but a large fraction of them will be able to escape into the IGM, thus making \( f_{\text{esc}} > 0 \). We then express the net escape fraction as follows:

\[
f_{\text{esc}} = \frac{\int_{t(\tau = 1)}^\infty dt Q(t)}{\int_0^\infty dt Q(t)}.
\]

As we will see shortly, though, either the IF is efficiently confined by the density, \( dx/dr \rightarrow 0 \), or the blowout will occur on a very short time-scale, thus making \( f_{\text{esc}} \approx 1 \). We will refer to these two different situations as a ‘confined’ or ‘unconfined’ IF in the rest of the paper.

3 RESULTS

We solved numerically equation (10) for a number of halo masses with \( V = 1 \) in the range \( M = 10^{8–10} M_\odot \) at two selected redshifts, \( z = 9 \) and 14. Figs 1 and 2 show the results for \( f_s = 0.2 \) per cent, a reasonable value for nearly primordial star-forming haloes (Barkana & Loeb 2001; Ciardi & Ferrara 2005; Pawlik, Milosavljevic & Bromm 2012); note that Wise & Cen (2009) found that \( f_{\text{esc}} f_s \approx 0.02 \) averaged over all atomic cooling \( (T_{\text{vir}} \geq 8000 \text{ K}) \) galaxies assuming a Salpeter IMF.

IFs in these early haloes evolve rapidly to the strong radiative input by the stars and reach a radius \( r_I = r_{\text{vir}} \) within \( \sim 1 \) Myr, yielding \( f_{\text{esc}} \approx 0.98–0.90 \) for halo masses up to \( M \approx 2 \times 10^{10} M_\odot \), beyond which the IF is confined within the halo, and hence \( f_{\text{esc}} \approx 0 \).

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\(^1\) http://www.stsci.edu/science/starburst99/

\(^2\) Here we ignore the possibility of a highly inhomogeneous medium in which low-density channels for escape exist.

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Figure 1. Time evolution of the IF radius, \( r_I \), at \( z = 9 \) within haloes of different total mass \( M = 10^{8–10} M_\odot \), with \( j = 0, \ldots, 9 \) from the top curve \((M = 10^8 M_\odot)\) to the bottom one \((M = 10^{10} M_\odot)\). The assumed SF efficiency is \( f_s = 0.002 \) and the gas temperature factor \( V = (T_{\text{vir}}/T)^{1/2} \approx 1 \).

Figure 2. Same as Fig. 1, but for \( z = 14 \).
It requires fine-tuning for a single object to achieve intermediate $f_{\text{esc}}$ values; rather, the process favours on–off states implying that the escape fraction of photons from galaxies can be intermittent: depending on variations of $f_{\text{esc}}$ in different bursts (say, due to the increasing gas metallicity or decreasing gas fraction due to mass-loss from SN-driven winds), the same halo might switch from being dark to bright above the Lyman limit. This is consistent with the results of Shapley et al. (2006) who showed that the probability distribution function of the escape fraction from $z = 3$ star-forming galaxies is bimodal (see their fig. 5). Note also that for the largest IF-confined haloes, the H$\alpha$ region starts to shrink at $t \gtrsim 3$ Myr as a result of recombinations and the decline in the supply rate of ionizing photons.

The situation is dramatically different at $z = 14$ (for the same $f_{\text{esc}}$): at that epoch all haloes confine their IF and have $f_{\text{esc}} = 0$. This is due to the increase in the gas density and the larger concentration factor at a fixed mass. A more global view of the dependence of $f_{\text{esc}}$ on halo mass and SF efficiency at $z = 9$ is given by Fig. 3. The abrupt transition from $f_{\text{esc}} \approx 1$ to 0 is very weakly dependent on halo mass, except for the narrow range of $-2.8 < \log f_{\text{esc}} < -2.2$ in which larger haloes tend to be more opaque to ionizing photons. Above this range, $f_{\text{esc}} = 1$ independently of halo mass and SF efficiency, and below this range, $f_{\text{esc}} = 0$.

Fig. 4 shows the examples with $V > 1$ introduced in Table 1, for which $T = 2 \times 10^4$ K, corresponding to $V^2 = (1.04, 4.82, 22.39, 103.96)$ for haloes of mass $M/M_\odot = (10^8, 10^9, 10^{10}, 10^{11})$ and hence resulting in considerably denser and more compact gas configurations. The IF is now largely confined within all haloes with $M > 10^9 M_\odot$ even if the SF efficiency has been increased by a factor of 50 with respect to the case with $V = 1$ shown in Fig. 1. In addition, the IF escapes the virial radius in a very short time, $t_{\text{esc}} \sim 10^4$ yr.

The different behaviour relative to the case of $V = 1$ is caused by the associated high gas density (see Table 1). The small scaleheight of the gas distribution, correspondingly reduced by an analogous factor $\alpha V^2$, forces the steep acceleration seen just prior to blowout.

3 We have checked that the IF speed is subluminal, $dr_1/dt < c$.
seen only for values of $x_{\text{esc}} \gtrsim 0.1$. Stated differently, it would be necessary to completely remove all the gas within 10 per cent of the virial radius to produce appreciable deviations from what we have obtained with our density profile. Note that with the procedure used for the two tests, we are removing partly or completely the mass within $r_{\text{core}}$ without redistributing it outside the core, i.e. the actual halo baryonic content is slightly decreased. This further enhances the chances for the IF to escape to infinity. Hence, we conclude that our results should be robust with respect to the details of the density distribution.

The existence of a minimum value of $f_e \approx 10^{-3}$ to allow the escape of photons can be understood based on a simple argument. This minimum number of ionizing photons provided to each hydrogen atom in the halo must balance the number of recombinations it undergoes during the lifetime of the starburst, approximately equal to $t_0 = 10^{6.6}$ yr (see equation 8). This can be expressed by the following inequality:

$$f_e \geq \frac{C}{\mathcal{N}_\nu} \frac{\alpha_{20}^2 t_0}{\mu m_p} (18 \pi^2) \Omega_b \rho_b(z)$$

$$= 0.7 \times 10^{-3} \left( \frac{C}{3} \right) \left( \frac{5 \times 10^{3}}{\mathcal{N}_\nu} \right) \left( \frac{1 + z}{10} \right)^3,$$

where $C = (n_i^2)/\langle n_H \rangle^2 \approx 3$ is the clumping factor accounting for the gas density structure inside a NFW halo.

The results for haloes with $M \lesssim 10^8 M_{\odot}$ in which $T_{\text{vir}} \lesssim 2 \times 10^4$ K at $z = 9$ need some extra attention. In fact, by fixing the gas temperature to $2 \times 10^4$ K, the gas might have never been accreted in the first place; this is the case if, for example, the halo is located within an already ionized region. To correct this problem, we have set $V = 1$ in these minihaloes; of course, this requires that some cooling agent, such as H$_2$, HD or H$_2$O, would cool the gas in these systems to make stars.

### 3.1 Global redshift evolution

Next we would like to constrain the redshift evolution of $f_{\text{esc}}(z)$ when averaged over the entire galaxy population.

The dark matter halo mass function, $n(M, z)$, is well described by the Sheth & Tormen (2002) form

$$n(M, z) dM = A \left[ 1 + \left( \frac{1}{\sqrt{2}} \right)^{1/\eta} \right] \frac{M}{\pi M_0} \frac{dM}{\sqrt{M_0}} \exp \left[ -\frac{v^2}{2} \right] dM,$$

where $A = 0.322$, $q = 0.3$, $v = \sqrt{a v}$, $a = 0.707$, $v = \delta_e/D(z) \sigma(M)$, with $\delta_e = 1.686$, $D(z)$ being the linear growth factor and $\sigma(M)$ being the rms amplitude of density fluctuations on a mass scale $M$.

We define the mass-averaged escape fraction, $\langle f_{\text{esc}} \rangle$ at a given $f_e$ and redshift $z$ as follows:

$$\langle f_{\text{esc}} \rangle(z, f_e) = 1$$

$$= F(\nu M_0(z_i) f_e)$$

$$= M_d(z),$$

where $F(\nu M_0, z) = \int_{M_0}^{\nu M_0} n M dM / \int_{M_0}^{\nu M_0} n M dM$ is the collapse fraction of all matter in haloes with mass exceeding $M$ at redshift $z$.

Equation (15) contains two important characteristic masses. The first, $M_0(z, f_e)$, is defined as the halo mass above which, for a given redshift $z$ and SF efficiency $f_e$, no ionizing photons can escape, i.e. $f_{\text{esc}} = 0$. The results presented above allow us to precisely determine $M_0(z, f_e)$.

The second mass scale, $M_d(z)$, denotes the redshift-dependent minimum mass of star-forming haloes which is set by radiative feedback. The evolution of $M_d$ is determined by two distinct radiative feedback processes. The first one is related to the increase of the cosmological Jeans mass in progressively ionized cosmic regions; as a result, the infall of gas in haloes below a given virial temperature is quenched. The evolution of $M_d(z)$ depends on the details of the reionization history (Gnedin 2000; Okamoto, Gao & Theuns 2008; Schneider et al. 2008). Guided by these findings, we adopt the value $M_d = M(T_{\text{vir}} = T_z) / T_z = 3 \times 10^4$ K after the end of reionization, here assumed to occur at $z_{\text{reion}} = 6$.

Before reionization (at least in the mostly neutral regions) a second type of feedback, involving the photodissociation of H$_2$ molecules by the Lyman–Werner (LW) background photons, becomes important. As H$_2$ is the primary coolant for minihaloes ($T_{\text{vir}} < 10^4$ K), fragmentation of the gas to stars could be suppressed in these objects through H$_2$ dissociation by the UV background (Haiman, Rees & Loeb 1996; Ciardi, Ferrara & Abel 2000; Kitayama et al. 2000; Machacek, Bryan & Abel 2001). According to Dijkstra et al. (2004), $z \sim 10$ minihaloes with $T_{\text{vir}} = T_z \approx 2 \times 10^4$ K can self-shield H$_2$ and cool; hence, we use this value as the lower threshold for SF. During reionization the interplay between the photoionization heating and LW feedback is complicated [see the discussion in Mesinger & Dijkstra (2008) and fig. 25 of Ciardi & Ferrara (2005)] and the evolution of $M_d$ during this epoch is uncertain. To circumvent this uncertainty, we follow the phenomenological approach of Salvadori & Ferrara (2009) and Salvadori & Ferrara (2012) who suggested that the metallicity distribution and iron–luminosity relation of the ultraviolet dwarf spheroidals in the Milky Way halo can be explained if these objects are relic minihaloes that formed their stars prior to reionization. With this perspective, one can reconstruct $M_d(z)$ by fitting local data. This approach yields the functional form

$$M_d(z) = M[T_{\text{vir}} = T_z] e^{(z-z_{\text{reion}})/\Delta z + T_z},$$

with $\Delta z = 4$, which we adopt here.

In general $M_0(z, f_e) > M_d(z)$. However, for sufficiently low values of $f_e$, $M_0(z, f_e) = M_d(z)$ and $\langle f_{\text{esc}} \rangle = 0$. To illustrate this point (see Table 2), we examine the results at $z = 9$ in Fig. 5. For $f_e = 0.03$, stars are formed only in haloes more massive than $M_d = 1.17 \times 10^7 M_{\odot}$. Ionizing photons can only escape from haloes less massive than $M_0 = 2.19 \times 10^6 M_{\odot}$. As a result, there is a narrow range of objects that contribute to the ionizing photon budget in the IGM. Below that range stars cannot form and above it the gas is sufficiently dense to confine the H ii region. When weighted with the halo mass function, though, the collapsed mass fraction above $M_d$ is $(0.113, 0.059)$, yielding the large value $\langle f_{\text{esc}} \rangle(z = 9, f_e = 0.01) = 0.847$. Since H ii regions in abundant low-mass haloes are unbound, the average escape fraction is high. The width of the range progressively shrinks as SF becomes less efficient. This is evident from Table 2, where for $z = 9$ and $f_e = 10^{-4}$, $M_0(z, f_e) = M_d(z)$ and the escape fraction drops to zero. These results underline the importance of the possible evolution in the SF efficiency, which we address below.

The redshift evolution of the results can be understood in terms of a balance between two opposite trends. On the one hand, the halo central densities tend to become larger with increasing redshift, providing a more efficient confinement of the IF and acting to decrease $\langle f_{\text{esc}} \rangle$. On the other hand, more dwarf galaxies and minihaloes are able to produce ionizing photons as the critical mass $M_d$ decreases with increasing $z$. The overall evolution for the two specific values $f_e = (0.03, 0.003)$ can be seen in Fig. 6 (solid curves). For the high-$f_e$ case, the escape fraction continues to increase with redshift, reaching $\langle f_{\text{esc}} \rangle = 0.83$ at $z = 20$. If instead $f_e = 0.003$, the escape
Table 2. Values of various quantities related to the global evolution of the mass-averaged escape fraction. See text for more details.

| \(M_\text{d} \) (\(M_\odot\)) | \(M_\text{f} \) (\(M_\odot\)) | \(M_\text{f0} \) (\(M_\odot\)) | \(f_{\text{coll}} (> M_\text{d}) \) | \(f_{\text{coll}} (> M_\text{f0}) \) | \(f_{\text{esc}} \) |
|-----------------|------------------|-----------------|----------------|----------------|----------------|
| \(z = 6 \) | 2.977e+08 | 2.977e+08 | 4.852e+08 | 1.379e−01 | 1.262e−01 | 8.478e−02 | 3.000e−02 |
| \(z = 9 \) | 1.169e+07 | 1.169e+07 | 2.190e+08 | 1.130e−01 | 5.936e−02 | 4.747e−01 | 3.000e−02 |
| \(z = 9 \) | 1.169e+07 | 1.169e+07 | 1.253e+08 | 1.130e−01 | 6.897e−02 | 3.897e−01 | 3.000e−03 |
| \(z = 9 \) | 1.169e+07 | 1.169e+07 | 5.818e+08 | 1.130e−01 | 4.374e−02 | 6.130e−01 | 1.000e+00 |
| \(z = 9 \) | 1.169e+07 | 1.169e+07 | 1.169e+07 | 1.130e−01 | 1.130e−01 | 0.000e+00 | 1.000e−04 |
| \(z = 14 \) | 1.634e+06 | 1.634e+06 | 8.131e+07 | 5.373e−02 | 1.471e−02 | 7.263e−01 | 3.000e−02 |

Figure 6. Redshift evolution of the mass-averaged escape fraction for different assumptions concerning the SF efficiency. The solid lines refer to the redshift-independent values \(f_\text{c} = 0.03, 0.003\) as indicated by labels; the dotted lines are the same cases in which a fixed \(f_{\text{esc}} = 0.05\) has been assumed for large haloes with \(M > M_\text{f0}\) (see Section 4); the shaded area refers to \(f_\text{c}(z)\) (shown by the shaded cyan area and multiplied by a factor of 100) derived from the stellar mass density evolution by González et al. (2011) based on equation (17).

The escape fraction reaches a peak value of 0.59 at \(z = 11.5\) where it starts to decrease until it reaches zero at \(z = 17.5\) when the density effect dominates over the enhanced photon production rate. Interestingly, a nearly-redshift-independent \(f_\text{c}\), above a threshold of \(\approx 0.1\) per cent (see Fig. 5) always leads to \(\text{d}(f_{\text{esc}})/\text{d}z > 0\) during the final phase of reionization.

Although a rather weak mass dependence of \(f_\text{c}\) for galaxies of stellar mass \(M_\text{s} > 3 \times 10^9 M_\odot\) is inferred from local SDSS data (Dekel & Woo 2003), there are indications of a decreasing SF efficiency for high-\(z\) galaxies. This is in line also with the conclusion (based on numerical simulations) by Munshi et al. (2013) that low-mass \(M < 10^9 M_\odot\) haloes have generally lower (0.1 per cent) values of \(f_\text{c}\). Recently, different groups (Stark et al. 2009; Labbé et al. 2010; González et al. 2011) have obtained stellar masses of galaxies in the redshift range \(4 < z < 7\) from spectral energy distribution fitting to rest-frame optical and UV fluxes from HST-WFC3/IR camera observations of the Early Release Science field combined with the deep GOODS-S Spitzer/IRAC data. From these data it has been possible to reconstruct (see fig. 4 of González et al. 2011) the evolution of the comoving stellar mass density (SMD), which we define here as \(\rho_\text{s}(z)\). The main result is that \(\rho_\text{s}(z = 6) = 10^3 M_\odot \text{Mpc}^{-3}\); in addition, the SMD grows with decreasing redshift as \(\alpha(1+z)^{-3.4 \pm 0.8}\). We can exploit this observational result to obtain a robust estimate for \(f_\text{c}(z)\) based on the relation

\[
f_\text{c}(z) = \frac{\rho_\text{s}(z)}{F[M_{\text{d}}(z)] \Omega_0 \rho_c}.
\]

In other words, we tune the SF efficiency so that baryons in collapsed haloes account for the abundance of stars observed at \(z < 7\). Note that the González et al. (2011) SMD has also been corrected for incompleteness at low masses. The range of allowed values of \(f_\text{c}\) implied by the data and obtained through equation (17) are plotted as a shaded area in Fig. 6. The data suggest \(f_\text{c} = 0.003\) at \(z = 6\) (for display reasons we have multiplied the values by a factor of 100 in the figure) decreasing thereafter by a factor of about 3 above \(z = 10\).

The main consequence of a suppressed SF efficiency in low-mass objects is to introduce a new intermediate mass scale \(M_\text{f0}\). Because of their lower SF efficiency, haloes above \(M_\text{d}\) might not produce enough stars to ensure \(f_{\text{esc}} = 1\). IfFs become unconfined only for somewhat larger haloes of characteristic mass \(M_\text{f0}\), such that \(M_\text{d} < M_\text{f0} < M_\text{f}\). Thus, \(f_{\text{esc}} = 0\) only in the interval \([M_\text{f0}, M_\text{f}]\). Stated differently, even objects that are above \(M_\text{d}\) and can form stars are now more efficient at confining their IFs; therefore, their escape fraction becomes zero. Table 2 shows how \(M_\text{f0}\) gradually approaches \(M_\text{d}\) towards high redshifts, closing the non-vanishing escape fraction window.

Another interesting question is how the global production of ionizing photons is distributed among haloes of different masses. We quantify this through the cumulative fraction of ionizing photons produced by haloes below a given mass,

\[
f_\gamma(<M_\text{d}, z) \propto \int_{M_\text{d}}^{M} f_{\text{esc}}(M', z; f_\text{c}) f_\text{s}(M_\text{f}, z; M') \text{d}M',
\]

as shown in Fig. 7. Due to their large escape fraction, haloes with masses \(M \lesssim 10^9 M_\odot\) are the dominant contributors to the cosmic ionizing photon budget. This holds for the more realistic redshift-dependent \(f_\text{c}\), but also for the constant-\(f_\text{c}\) case. Thus, the low-mass galaxies and minihaloes might have dominated reionization. In particular, at \(z = 10\) the mass corresponding to \(T = 10^4\) K limiting the mini-halo mass range from the above is \(\sim 3 \times 10^7 M_\odot\). Fig. 7 shows...
that independently of the SF efficiency prescription, we predict that minihaloes contribute $\sim 40$ per cent of the total emissivity.

Reionization models suggest an increasing emissivity towards high redshifts (Bolton & Haehnelt 2007; Kuhlen & Faucher-Giguère 2012; Mitra et al. 2012, 2013), whose most natural explanation is in terms of an $f_{\text{esc}}$ increasing with redshift as we find here. For example, Mitra et al. (2013) find that: (i) the escape fraction increases from $(f_{\text{esc}}) = 0.068^{+0.054}_{-0.047}$ at $z = 6$ to $0.179^{+0.331}_{-0.132}$ at $z = 8$; and (ii) at $z = 10$, $(f_{\text{esc}}) > 0.146$.

4 DISCUSSION

We have calculated the escape fraction of ionizing photons from starburst galaxies during reionization, assuming that galactic gas cools down to a temperature floor ($\sim 10^4$ K for atomic hydrogen and $\sim 2 \times 10^4$ K for self-shielded molecular hydrogen). We have found that most of the escaping ionizing photons originates in the lowest mass galaxies with $T_{\text{vir}} \lesssim 10^4$ K. Due to the shallowness of the gravitational potential well in these galactic haloes, the gas maintains a low density and its recombination rate is low. This allows the IF to break through the virial radius rapidly, resulting in a high escape fraction of the emitted ionizing radiation. As reionization progresses, the IGM is photoheated and the abundance of galaxies with $T_{\text{vir}} \sim 10^4$ K declines, leading to a decrease in $(f_{\text{esc}})$. This physical mechanism may account for the increase of $(f_{\text{esc}})$ with redshift inferred by Mitra et al. (2013).

Our study is consistent with previous studies of low-mass galaxies by Wise & Cen (2009), who found a high escape fraction ($\sim 0.1$). Low-mass galaxies do not form a disc (Pawlik, Milosavljević & Bromm 2011), which could trap effectively ionizing photons. The relative importance of minihaloes for the global production rate of ionizing photons is determined by the product $f_{\text{esc}} \times f_{\text{i}}$. Here we have used a phenomenological calibration of $f_{\text{i}}$. In order to produce the observed stellar density at $z = 6-7$ by minihaloes, $f_{\text{i}}$ has to be $\gtrsim 0.001$. This value lies in the middle range resulting from Wise & Cen (2009) idealized halo simulations, but they find the SF efficiency in haloes with $M < 10^7$ $M_\odot$ to be smaller. We find that the required increase of $f_{\text{esc}}$ with redshift cannot be achieved if minihaloes are sterile or form stars inefficiently.

Our simplified calculation ignored angular momentum which drives cold gas to a rotating disc configuration. Our model allowed cold gas to condense to arbitrarily small scales without making a disc, and so one might worry that it underestimates $f_{\text{esc}}$. However, previous calculations have demonstrated that on the scale of galactic discs the escape fraction is very small (Wood & Loeb 2000; Razoumov & Sommer-Larsen 2006; Gnedin et al. 2008; Wise & Cen 2009), and so our inclusion of angular momentum could not increase $f_{\text{esc}}$ significantly. In particular, the conclusion that $f_{\text{esc}}$ is high in low-mass haloes must be robust. Although our simplified model predicts zero escape fractions from brief starbursts in massive haloes, the actual value of $f_{\text{esc}}$ would be finite in reality due to persistent SF histories and winds driven by SN feedback or Ly$\alpha$ radiation pressure (Dijkstra & Loeb 2008). Our results are in qualitative agreement with Gnedin et al. (2008) and Razoumov & Sommer-Larsen (2006) who simulated more massive galaxies ($M \gtrsim 10^{10}$ $M_\odot$) and inferred extremely small escape fraction values $f_{\text{esc}} \lesssim 1$ per cent, even in situations where the SF rate is high ($\sim 10$ M$\odot$ yr$^{-1}$). This results from the high gas densities in massive galaxies due the presence of a rotationally supported, geometrically-thin disc.

To get a simple estimate of the impact of these larger galaxies, we have also run cases allowing a certain ionizing photon fraction (here assumed to be 5 per cent, guided by local observations) to escape from haloes of mass $M > M_{\text{f}}$. The results are shown for two cases in Fig. 6 (dotted lines). The reported trend of an increasing $f_{\text{esc}}$ for $z < 11$ remains clearly visible.

Finally, we note that the trend we find for the increase of the escape fraction with redshift should be enhanced if the IMF of stars were tilted towards more massive stars. This is because a higher production rate of ionizing photons would allow the IF to ionize the surrounding hydrogen blanket even faster than we calculated.

ACKNOWLEDGEMENTS

Our computations were based on the SciPy open source scientific tools for Python: http://www.scipy.org/. This work was supported in part by NSF grant AST-0907890 and NASA grants NNX08AL43G and NNA09DB30A.

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