Generalized States for Multipion Production and Disoriented Chiral Condensates

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Abstract

We show that the sudden quenching mechanism responsible for the production of the disoriented chiral condensate gives rise automatically to squeezed states. We compare the distribution of charged and neutral pions in the two extreme limits of a sudden quench and a slow adiabatic relaxation and show that in the former case the difference in the distributions is much more dramatic than in the latter. We also examine the isospin structure of the resulting squeezed pion distributions.

1 Introduction

In high energy collisions, besides the quark-hadron phase transition, the particular phase transition exciting great interest these days is chiral phase transition. Following the pioneering work of Bjorken [1], Kowalski, Taylor [2], Wilczek and Rajagopal [3], there exists the exciting possibility that in the upcoming high luminosity hadron and heavy ion collisions, after a chiral phase transition, a disoriented chiral condensate (DCC) may be formed. The physics of the formation mechanism in various low energy models of QCD has been presented in the talk of Dr. A. M. Srivastava in these proceedings. Theoretical treatments of the DCC have been based on coherent pion states and recently Amado and Kogan [4] have shown that a mean field quantization of the linear $\sigma$ model (which is one of the canonical effective models of QCD used in the description of the DCC) leads to a squeezed state description of the DCC at the quantum level. In this paper we examine this description in more detail and show that the amount of squeezing which leads to an amplification of the low-momentum pion modes is directly related to the mechanism by which the system goes through the phase transition. We show that if the transition is a sudden quench the squeezing and amplification is much more dramatic than if the transition is a slow adiabatic one.
2 Quenching and Squeezing

In the quantum treatment of sigma model in mean field approximation the mechanism for amplification of low momentum modes is done by considering the equation of motion for the pion field with \( \phi^a \phi_a = \langle \phi_a \phi^a \rangle (t) \).

\[
\frac{\partial^2 \pi(k, t)}{dt^2} + [k^2 + \lambda(\langle \phi^2 \rangle(t) - v^2)] \pi(k, t) = 0
\]

(1)

For initial condition \( \langle \phi^2 \rangle < v^2 \) long wavelength modes grow exponentially and \( \langle \phi \rangle \) oscillates around \( v^2 \) damps down to stability. Each long wavelength mode becomes parametrically excited oscillator and amplification of zero point oscillations gives DCC. Hence coherent state descriptions are often used.

In Mean Field approximation with pion wave function \( |\Psi > = \Pi_{i,k} |\psi >_{i,k} \) the equation of motion for the pion field can be viewed as a time dependent oscillator equation with a time dependent frequency \( \Omega(k, t) = \bar{k}^2 + \lambda(\langle \phi^2 \rangle(t) - v^2) \) evolving by the Hamiltonian

\[
H_k(t) = \left[ \frac{1}{2}(P^2_{\pi} + \Omega^2(k, t)\pi^2(k)) \right]
\]

(2)

Creation and annihilation operators of the observed pions in the decay of DCC are given by \( a(k) = \frac{w(k)\pi(k) + iP_n}{\sqrt{2w(k)}} \) and \( a^\dagger(k) = \frac{w(k)\pi(k) - iP_n}{\sqrt{2w(k)}} \) with \( w(k) = \Omega(k, \infty) = \sqrt{k^2 + m^2_\pi} \). The Hamiltonian \( H_k(t) \) can be written as:

\[
H_k(t) = \Omega_{k,t} A^\dagger(k, t) A(k, t)
\]

(3)

Where \( A, A^\dagger \) are related to \( a, a^\dagger \) by the Unitary evolution operator corresponding to \( H_k(t) \) i.e. \[3\]

\[
A(k, t) = U(t, t_0) a(k) U^\dagger(t, t_0)
\]

(4)

it can be shown using SU(1,1) algebra that this reduces to a Bogolubov (squeezing) transformation \( A = \mu(t)a(k) + \nu(t)a^\dagger(k) \) with \( \mu = \text{Sinh}(r) \) and \( \nu = \text{Cosh}(r) \), where \( r \) is the squeezing parameter and \( \mu \) and \( \nu \) are functions of the sums and differences of \( w(k) \) and \( \Omega(k, t) \). In the case of the DCC an initial vacuum \( |0> \) is transformed to:

\[
S(z) = e^{\frac{1}{2}(z^2a^2 - za^2)} |0>
\]

(5)

\[z = re^{i\theta}\] \[3\].

The wave function \( |\psi_k > \) evolves as

\[
i \frac{\partial|\psi_k >}{\partial t} = H_k(t)|\psi_k >
\]

(6)

In the co-ordinate representation the equation of motion for \( \pi \) can be viewed as a Shrodinger Equation for \( \psi \) with a time dependent potential \( V(t) = -\lambda(\langle \phi^2 > (t) - v^2) \) so that this reduces to a Bogolubov (squeezing) transformation

\[
-\psi'' + V(t)\psi = \omega^2(k)\psi
\]

(7)

For a general time dependent oscillator with variable frequency where \( V(t) \) takes constant values at \( t \rightarrow \pm \infty \) the problem becomes one of a potential barrier reflection and it can be shown that the transmission coefficient of a wave passing through the barrier can be related to the squeezing parameter in a particle creation problem by the relation

\[
\text{Sinh}^2 r = \frac{T - 1}{T} = < n >
\]

(8)

Now we are in a position to examine the problem of quenching versus adiabaticity for squeezing. Let us take \( < \phi^2 > (t) \) given by:

\[
< \phi > (t) = \sqrt{f_\pi[Tanh[(t - \tau)/a] - Tanh[-\tau/a]}
\]

(9)
Then the limit \( a^- > 0 \) corresponds to the quenching limit and \( a^- > \infty \) corresponds to the adiabatic limit as shown in fig.1. Here \( \tau \) represents the damping time of \( < \phi(t) > \) and \( a \) is given in units of \( \tau \). Typically \( \tau \) is taken to be \( 3 - 6m^\sigma_1 \). The corresponding potential barrier is of the modified Poschl Teller Potential given in fig.2 which in the quenched limit corresponds to the rectangular barrier. The potentials in both limits are exactly soluble and the value of the transmission coefficients as a function of \( k \) can be calculated. Using these and equation 8 fig. 3 shows the variation of \( < n > \) with \( k \) for various values of \( a \). In the quenching limit, the long wavelength modes are clearly amplified and the squeezing is very pronounced. In the adiabatic limit no substantial squeezing is present. This clearly illustrates the connection between quenching and squeezing. A detailed calculation tabulating various forms of \( < \phi(t) > \) and the resulting amplification will be published in a later communication.

An alternate way to vary the frequency of the time dependent oscillator would be to take the pion mass as an exponential function of temperature and examine the evolution in terms of thermal state in terms of a time dependent temperature. The squeezed state formalism is particularly useful for this and the methods of thermofield dynamics (TFD) can be used effectively. This work is in progress.

3 Pion Radiation from DCC

Having shown that the mean Field treatment of DCC suggests Squeezed State

\[ < \phi(t) > \]

\[ V(t) \]

\[ N(t) \]

Figure 1: Shows the variation of \( < \phi(t) > \) with \( t \) for various values of \( a \).

Figure 2: Shows potential \( V(t) \) for \( a=10^{-4}(\text{rectangular barrier}),.1 (\text{gradual quench}),1 (\text{Poschl-Teller}) \).

Figure 3: Shows the variation of \( < n > \) with wave number \( k \) for values of \( a = .0001 , .1 ,1 ,10 . \).

3
Treatment of emerging pion waves and that this effect is more pronounced in quenching we now proceed to construct most general class of isospin squeezed states which can produce the phenomenology of DCC decay based on first principles. The most general eigenstate of $I$ and $I_3$ and having properties of squeezing can be constructed by

$$F(A, A^\dagger, N)Y_m^i(a^\dagger)|0 >.$$  \hspace{1cm} (10)

where $F = e^{\frac{i}{2}(fA^\dagger - f^*A)}$.

The state $|\psi, f >$ is the correct displacement operator coherent state of definite isospin $|\frac{1}{2}, \frac{1}{2} \rangle$. Where bilinear operators $A, A^\dagger$ and $N$ which commute with $I$ and $I_3$, in terms of pion creation and annihilation operators are:

$$A = \vec{a} \cdot \vec{a} = a_0 a_0 - 2a_+ a_-$$

$$A^\dagger = \vec{a}^\dagger \cdot \vec{a}^\dagger = a_0^\dagger a_0^\dagger - 2a_+^\dagger a_-^\dagger$$

$$N = a_0^\dagger a_0 + a_1^\dagger a_1 + a_2^\dagger a_2$$  \hspace{1cm} (12)

The multiplicity distributions for $\pi_0, \pi^+$ and $\pi^-$ can be written as:

$$| < n_0, n_+, n_- | \psi, f, l, m > |^2 = (\frac{1}{2})^{2m} [(2l + 1)(l - m)!(l + m)!]$$

$$\sum_{k=0}^{(l-m)/2} \frac{2^{m-k}}{(l - m - 2k)!k!(m + k)!} S_{n_0, l-m-2k}S^\dagger_{n_+, l-m+k,k} |^2$$  \hspace{1cm} (13)

where $S(f)$ is the one mode squeezing operator

$$S(f) = < n_0 | e^{i\frac{f}{2}(a_0^\dagger + f^*a_0^\dagger)} | l - m - 2k > = S_{n_0, l-m-2k}.$$  \hspace{1cm} (14)

$S'(f)$ is then the two mode squeezing operator

$$< n_+, n_- | e^{i(fa_+^\dagger + f^*a_+^\dagger - fa_-^\dagger - f^*a_-^\dagger)} | m + k, k > = S_{n_+, n_-, m+k,k}.$$  \hspace{1cm} (15)

For $p\overline{p}$ collisions, isospin conservation requires $I = 0, 1$. For $pp$ collisions $I = 0, 1, 2$  
$\pi p$ collisions can have up to $I = 3$. Charge conservation implies that $n_{\pi^+} = n_{\pi^-}$ so that we have predominantly $I_3 = 0$ states. For $I = 0, I_3 = 0 \rightarrow n_+ = n_-$, the distribution reduces to the product of squeezed distributions for charged and neutral pions and only even number of pions emerge. The distribution of charged particles is

$$P_n = \frac{(\tanh(f))^{2n_c}}{(\cosh(f))^2}$$  \hspace{1cm} (16)

and for neutral particles:

$$P_n = \frac{n_0!(\tan(f))^{n_0}}{((\frac{n_0}{2})!)^2\cosh(f)^{2n_0}}$$  \hspace{1cm} (17)

and corresponds to the product of distributions of a one-mode and two-mode squeezed state. Case $I = 1; m = 0$: Again, $n_+ = n_-$ and

$$| < n_0, n_c | \psi, f, 1, 0 > |^2 = | S_{n_0,1}S^\dagger_{n_+, n_-, 0,0} |^2$$  \hspace{1cm} (18)

so the neutral pion distribution is affected by isospin and not the charged pion distribution. $n_0$ must be odd $= 2m + 1$. 

$$| 2m + 1, 2n_c | \psi, f, 1, 0 > |^2 = \frac{(2m + 1)!(\tan(f))^{2m+2n_c}}{((\frac{n_c}{2})!)^2\cosh^2(f)}$$  \hspace{1cm} (19)
The generalized squeezed Isospin eigenstate leads to products of two types of squeezed states of pions, the neutral pions being in a one mode squeezed state and the charged pions being in an SU(1,1) coherent or two-mode squeezed state. Thus the neutral and charged pion distributions are significantly different as the two types of states have different properties. We now illustrate the effect of quenching versus adiabaticity on these two distributions. Figs. 4 and 5 show the difference in the charged and neutral pion distribution for the adiabatic limit where the difference is negligible and the quenched limit where the difference is pronounced. In conclusion, we have shown that the sudden quench approximation in the evolution of the disoriented chiral condensate leads to a substantial amount of squeezing which manifests itself in the dramatic difference between charged and neutral distributions. For an adiabatic quench the difference is much less, so that the characteristic signal examined in literature for the DCC is related directly to the way in which the DCC relaxes to the vacuum in a chiral phase transition.

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