Abstract

Using an effective theory approach, we calculate the neutron electric dipole moment (nEDM) in the minimal left-right symmetric model with both explicit and spontaneous CP violations. We integrate out heavy particles to obtain flavor-neutral CP-violating effective Lagrangian. We run the Wilson coefficients from the electroweak scale to the hadronic scale using one-loop renormalization group equations. Using the state-of-the-art hadronic matrix elements, we obtain the nEDM as a function of right-handed W-boson mass and CP-violating parameters. We use the current limit on nEDM combined with the kaon-decay parameter $\epsilon$ to provide the most stringent constraint yet on the left-right symmetric scale $M_{W_R} > (10 \pm 3)$ TeV.
I. INTRODUCTION

The electric dipole moment (EDM) of an elementary particle has been a subject of strong interest from both experimentalists and theorists for over half a century [1, 2]. A non-vanishing intrinsic EDM indicates violation of time-reversal (T), parity (P) and charge-conjugation-parity (CP) invariance at fundamental level. Although the standard model (SM) of particle physics predicts non-vanishing EDM for leptons and quarks from both the CP phase in the Cabbibo-Kobayashi-Moskawa (CKM) matrix and the quantum chromodynamic (QCD) $\theta$-angle, it is widely believed that new physics must exist, allowing EDMs at least competitive with or even dominate the SM predictions [2, 3]. New CP-violating physics is needed to explain, for example, the baryon number asymmetry observed in the universe today [4]. In this article, we are interested in the EDM of a strongly-interacting bound state—the free neutron. Because of its charge-neutrality, the neutron EDM ($n$EDM) is relatively “easier” to measure. The current bound is $2.9 \times 10^{-26}$ e cm [5], which is already very constraining for new physics. The upcoming experiments will enhance the current sensitivity by one to two orders of magnitude [6], which either will rule out many of the new physics models under investigation, or will provide the first opportunity to reveal an intrinsic nEDM.

Critical to understanding the experimental data is a theoretical nEDM calculation with controlled precision of non-perturbative strong-interaction physics. The result is indispensable for extracting or constraining the new interaction parameters. In the past, many calculations have been made in the literature, and most of which are done in the context of ad hoc models [7, 8, 9, 10, 11, 12, 13]. In one class of models, the quark EDM is first obtained, and the neutron EDM is calculated through constituent quark models. In another class of models, the T-odd pion-nucleon interaction vertices are first derived and then hadron physics effect is calculated through pion loops. The relationship among different contributions is often unclear and confusing. Depending on different modelings, there are often large uncertainties in the final result.

In this paper, we follow an effective theory approach to calculate the nEDM in the left-right symmetric model (LRSM) [14]. The model was motivated by the hypothesis that parity is a perfect symmetry at high-energy, and is broken spontaneously at low-energy due to the asymmetric vacuum. This model has a number of attractive features, including a natural explanation of weak hyper-change in terms of baryon and lepton numbers, existence of right-handed neutrinos and the seesaw mechanism for neutrino masses, and possibility of spontaneous CP violation. In a recent paper, we found a complete solution of the CP violation structure of the minimal left-right symmetric model (mLRSM) [15]. Our goal here is to derive a factorization formula for nEDM in this model, with QCD and other short-distance physics in the Wilson coefficients, and with long-distance physics in hadronic matrix elements ready for, for example, lattice QCD calculations. Using the state-of-the-art hadronic matrix elements, we derive the best constraints on the model parameters. In particular, we find the most stringent bound yet on the left-right symmetric scale $10 \pm 3$ TeV, which is beyond the detection capability of the Large Hadron Collider (LHC) [16].

Before starting, let us make a number of relevant comments. First of all, it is possible that the entire nEDM to be measured can be explained by the so-called QCD $\theta$-term, a term in the QCD Lagrangian which will contribute to nEDM due to instanton effect. Its contribution to nEDM has been calculated in several different ways [17]. The nEDM constraint on $\theta$-term is so strong $\theta < 10^{-10}$ [17] that there should be some mechanism, for example the Peccei-Quinn
symmetry [18], to cure this so-called strong CP problem. We will not consider the induced \( \theta \)-contribution to nEDM in mLRSM. However, this term seems unlikely to be the only or the most important source for nEDM. A typical beyond-SM physics model allow natural sizes of the nEDM on the order currently been probed by experiments. Second, there has been a number of papers in the literature about the perturbative QCD effects for nEDM in various versions of LRSM without general CP structure [10, 19, 20]. We will use some of these results to make a coherent formulation, taking into account various effects consistently. Finally, the EDM for spin \( \vec{s} \) has an interaction term in the hamiltonian \( H = -d \vec{s} \cdot \vec{E} / |\vec{s}| \), which corresponds to the following term in the effective lagrangian density

\[
\mathcal{L} = -\frac{1}{2} d \bar{\psi} \sigma_{\mu\nu} i \gamma_5 \psi F^{\mu\nu},
\]

where \( \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \) and \( F^{\mu\nu} \) is the electromagnetic field strength and \( \psi \) is the spin-1/2 Dirac field.

The presentation of the paper is organized as follows: In Sec. II, we collect all the P-odd and CP-odd operators up to dimension-six and discuss their short-distance QCD effects. In Sec. III, we calculate the Wilson coefficients of quark EDM, CDM operators, four-quark operators and the Weinberg operator in the framework of mLRSM using effective theory by integrating out heavy particles. In Sec. IV, we study nEDM in mLRSM numerically and get the lower bound of the mass of the righthanded W-boson (\( W_R \)). It turns out that the biggest contribution comes from four-quark operators. This property is true not only in mLRSM, but for a large category of models, for example two-Higgs-doublet Models (see Ref. [21] for a good review). Recently, we have made a comprehensive study of the matrix elements of the CP-odd and P-odd four-quark operators in the neutron state [22], which makes it possible to calculate the nEDM more reliably. We conclude the paper in Sec. V.

II. GENERAL CP-VIOLATING EFFECTIVE LAGRANGIAN

In this section, we lay out a general approach to calculating the neutron EDM using the effective Lagrangian method, independent of new physics. In this approach, one integrates out all heavy particles including SM gauge bosons and heavy-quarks. The resulting flavor neutral CP-violating effective Lagrangian has an expansion in terms of operators consisting of light-quark fields, \( u, d, \) and \( s \) and the gluon field \( G^{\mu\nu} \), with increasing dimensions,

\[
\mathcal{L}_{\text{CP-odd}} = \sum_{i=4,...} \mathcal{L}_{i}^{\text{CP-odd}},
\]

where \( \mathcal{L}_i \) contains \( i \)-dimensional operator. The Wilson coefficient of each operator depends on the renormalization scale \( \mu \) which in the end will be chosen as hadronic physics scale, about 1 GeV or lattice cut-off \( 1/\alpha \), where \( \alpha \) is lattice spacing. The QCD operators also depend on the renormalization scale, but physical results do not.

At the lowest dimension, there are two CP-odd operators,

\[
\mathcal{L}_4 = -\frac{g^2 \theta}{32\pi^2} G^{\mu\nu} \bar{G}_{\mu\nu} + \sum_q \bar{m}_q q \gamma_5 q,
\]

Through \( SU(3) \) chiral rotations, the CP-odd quark-mass term can be rotated into a chiral singlet. Furthermore, one can eliminate either \( G \bar{G} \) or the singlet quark-mass term through \( U_A(1) \) chiral rotation \( q \rightarrow e^{i\gamma_5 \alpha} q \).
At dimension-five level, there are two kinds of flavor neutral P-odd and CP-odd operators, namely, the quark EDM operators and the chromo electric dipole moment (CDM) operators,

\[ L_5 = \sum_q d_q^E(\mu)O_q^E(\mu) + \sum_q d_q^C(\mu)O_q^C(\mu), \quad (4) \]

where \( O_q^E = -\frac{1}{2}\bar{q}\sigma^{\mu\nu}i\gamma_5 q F_{\mu\nu} \) and \( O_q^C = -\frac{1}{2}\bar{q}\sigma^{\mu\nu}i\gamma_5 t^a q G_{\mu\nu}^a \), and \( F_{\mu\nu} \) and \( G_{\mu\nu}^a \) are the electromagnetic and gluon field strengths, respectively, and \( t^a \) are generators of the SU(3) gauge group. The one-loop evolution equations are \[ \frac{d}{d\mu^2}O_q^C(\mu) = -\left( \frac{2}{3} - \frac{b_f}{2} \right) \frac{\alpha_s(\mu)}{4\pi} O_q^C(\mu), \quad (5) \]

\[ \frac{d}{d\mu^2}O_q^E(\mu) = -\frac{4}{3} \frac{\alpha_s(\mu)}{4\pi} O_q^E(\mu), \quad (6) \]

where \( b_f = 11 - 2n_f/3 \), \( n_f \) is the number of quark flavors. It is easy to see that the dependence of the evolution of the quark CDM on \( n_f \) is the same as that of the strong coupling, since they are both derived from wave function renormalization of the gluon field.

At dimension-six, there are a number of four-quark flavor-neutral CP-odd operators and the Weinberg’s three-gluon operator \[ \mathcal{L}_6 = \sum_i C_i(\mu)O_{4i}(\mu) + C_g(\mu)O_g(\mu), \quad (7) \]

where the four-quark CP-odd operators can be divided into two groups. The first group includes operators with two different light flavors \[ O_{11} = (\bar{q}i\gamma_5 q)^{(\bar{q}' q')}, \]

\[ O_{12} = (\bar{q}q)^{(\bar{q}' i\gamma_5 q')}, \]

\[ O_{21} = (\bar{q}i\gamma_5 t^a q)^{(\bar{q}' i\gamma_5 t^a q')}, \]

\[ O_{22} = (\bar{q}t^a q)^{(\bar{q}' i\gamma_5 t^a q')}, \]

\[ O_{3} = (\bar{q}i\gamma_5 \sigma^{\mu\nu} q)^{(\bar{q}' \sigma_{\mu\nu} q')}, \]

\[ O_{4} = (\bar{q}i\gamma_5 t^a \sigma^{\mu\nu} t^a q)^{(\bar{q}' \sigma_{\mu\nu} t^a q')}, \quad (8) \]

where \( q, q' = u, d, s \) and \( q \neq q' \). The second group includes operators with one quark flavor

\[ O'_{1} = (\bar{q}i\gamma_5 q)^{(\bar{q} q)}, \]

\[ O'_{2} = (\bar{q}i\gamma_5 t^a q)^{(\bar{q} t^a q)}. \quad (9) \]

The Weinberg operator is defined as

\[ O_g = -\frac{1}{6} f^{abc} \epsilon^{\mu\nu\alpha\beta} G_{\mu\rho}^a G_{\nu\sigma}^b G_{\alpha\beta}^c, \quad (10) \]

where \( \epsilon^{0123} = 1 \).

It is not difficult to see that all the dimension-six operators listed above are CP-odd. In the first group there are two different flavors in each operator. These operators are constructed by a pseudoscalar current coupled to a scalar one, a pseudo-tensor current coupled to a tensor one. No operator is constructed from an axial-vector current coupled to a vector current,
since CP-odd operators cannot be generated in this way. And since \( \gamma_5 \sigma^{\mu\nu} = \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta} \), the two operators \( O_3 \) and \( O_4 \) are enough to describe the CP-odd pseudo-tensor and tensor coupling. Therefore, the first group includes all the P and CP-odd four-quark operators constructed by two different quark flavors. For the one-flavor case, the first four operators in the first group automatically become \( O'_1 \) and \( O'_2 \) in the second group, and using the Fierz transformation one can easily see that the operators described the tensor-pseudotensor coupling are not independent of \( O'_1 \) and \( O'_2 \). Therefore, the second group includes all the flavor-neutral CP-odd four-quark operators with single quark flavor.

The leading-order QCD evolution equations for dimension-six operator are as follows

\[
\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O_{11} \\ O_{12} \\ O_{21} \\ O_{22} \\ O_3 \\ O_4 \end{pmatrix} = \frac{\alpha_s(\mu)}{4\pi} \begin{pmatrix} 8 & 0 & 0 & 0 & 0 & 1 \\ 0 & 8 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & \frac{24}{5} & \frac{12}{5} \\ 0 & 0 & 0 & -1 & \frac{5}{9} & \frac{5}{9} \\ 16 & 16 & 10 & 10 & 0 & \frac{10}{3} \end{pmatrix} \begin{pmatrix} O_{11} \\ O_{12} \\ O_{21} \\ O_{22} \\ O_3 \\ O_4 \end{pmatrix},
\]

(11)

\[
\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O'_1 \\ O'_2 \end{pmatrix} = \frac{\alpha_s}{4\pi} \begin{pmatrix} \frac{40}{9} & 0 \\ \frac{80}{27} & -\frac{10}{9} \end{pmatrix} \begin{pmatrix} O'_1 \\ O'_2 \end{pmatrix},
\]

(12)

\[
\mu^2 \frac{d}{d\mu^2} O_g = \frac{\alpha_s}{4\pi} \gamma_{gg} O_g.
\]

(13)

The anomalous dimension of the Weinberg, \( \gamma_{gg} \), has been calculated in the literature \[25\], \( \gamma_{gg} = -\frac{C_A}{2} - n_f \), where \( C_A = 3 \). The dimension-six operators mix with the dimension-five operators when scale evolves, however at the energy scale where only the light quarks exist, the mixing can be neglected because the dimension-five quark EDM and CDM are chirality flipping and thus proportional to the quark mass. At higher energies, the mixing is important and we will discuss it in the following sections.

There is no mixing between the Weinberg operator and the four-quark operators listed in Eqs. (8) and (9). To see this, we can decompose the four-quark operators into irreducible representations of the \( SU(3)_L \times SU(3)_R \) chiral group and only \((3,3), (6,6), (8,8)\) and their conjugate representations are found \[22\]. On the other hand, the three-gluon operator is a chiral singlet. QCD evolution maintains the chiral structure of operators.

When scale changes, the pure quark-gluon CP-odd operators generate perturbative contributions to quark EDM through the following \( T \)-product

\[
\int d^4 x \ T \left( e A_\mu(x) j^{\mu}_{em}(x) \sum_i O_i(0) \right),
\]

(14)

where the summation neglects the quark EDM operator itself. The contributions are divergent so they induce additional running of the CP-odd operators. The contributions from the dimension-six operators are proportional to the mass of light quarks and can be neglected. The only large contribution is from the quark CDM operator, whose running has an effective inhomogenous term, \[19\]

\[
\mu^2 \frac{d}{d\mu^2} O_q^C = \frac{\alpha_s(\mu)}{4\pi} \left( -\left( \frac{2}{3} - \frac{b_f}{2} \right) O_q^C - \frac{16}{3} g_s(\mu) Q_q O_q^E \right),
\]

(15)
where $Q_q$ is the electric charge of the quarks and $g_S$ is the coupling of strong interaction. Inversely, the quark EDM operators can also generate quark CDM operators through the electromagnetic interaction which is, however, proportional to the electromagnetic fine-structure constant.

Therefore, omitting the $\theta$-contribution, one can define the following electric dipole form factor

$$-F_n^E(q^2)\bar{U}_n(\vec{k}_2)\sigma^{\mu\nu}\gamma_5 q_\mu U_n(\vec{k}_1)\epsilon_\nu(q) = \langle N(\vec{k}_2)| \sum_q d_q^E(\mu)O_q^E(0;\mu)$$

$$+i \int d^4x T \left[ eA_\mu(x)j_{em}^\mu(x) \left( \sum_q d_q^C(\mu)O_q^C(0;\mu) + \sum \right) C_i(\mu)O_{4i}(0;\mu) \right] |\gamma(q)N(\vec{k}_1)\rangle,$$

where $q^\mu = k_2^\mu - k_1^\mu$ and $U_n$ is the wavefunction of neutron and $\epsilon^\nu$ is the polarization of the incoming photon. The static nEDM is just the zero-momentum limit of the form factor $d_n^E = F_n^E(0)$.

### III. WILSON COEFFICIENTS IN LRSM

Following the previous section, we make calculation of nEDM in the mLRSM by first evaluating the Wilson coefficients of the effective quark-gluon operators at the electroweak scale, and subsequently running them to hadronic scale. The detail of the model can be found in Ref. [15], in which the spontaneous CP-violation is controlled by a phase angle $\alpha$ in the Higgs sector, and additional parameters of the model include, among others, the masses of the right-handed gauge boson and the new Higgs bosons. In the following subsections, we study the Wilson coefficients of various CP-violating operators separately. We will ignore the contribution of the $\theta$-term as it will usually generate a much too large nEDM: We assume certain mechanisms such as Peccei-Quinn symmetry [18] is in operation to suppress it.

#### A. CP-Odd Four-Quark Operators

To leading order, diagrams in Fig. 1 generate the CP-odd four-quark operators induced by the exchange of gauge bosons and Higgs bosons. The operators are listed in Eq. (8) and
The corresponding Wilson coefficients can be easily read through the diagrams,

\[
C_{11}^{ab} = \frac{\sqrt{8}G_F}{6} \sin 2\zeta \text{Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) + \frac{\sqrt{8}G_F}{M_{H_2}^2} \text{Im}(C^{aa}D^{bb}) \\
+ \frac{\sqrt{8}G_F}{6M_{H_2}^2}(m_a^2 - m_b^2)\xi \text{ Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) ,
\]

\[
C_{12}^{ab} = -\frac{\sqrt{8}G_F}{6} \sin 2\zeta \text{Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) + \frac{\sqrt{8}G_F}{M_{H_2}^2} \text{Im}(C^{aa}D^{bb}) \\
+ \frac{\sqrt{8}G_F}{6M_{H_2}^2}(m_a^2 - m_b^2)\xi \text{ Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) ,
\]

\[
C_{21}^{ab} = \sqrt{8}G_F \sin 2\zeta \text{Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) \\
+ \frac{\sqrt{8}G_F}{M_{H_2}^2}(m_a^2 - m_b^2)\xi \text{ Im}(e^{-i\alpha}V_L^{\alpha\beta}V_R^{\alpha\beta*}) ,
\]

\[
C_{22}^{ab} = -\sqrt{8}G_F \sin 2\zeta \text{Im}(e^{-i\alpha}V_L^{ab}V_R^{ab*}) \\
+ \frac{\sqrt{8}G_F}{M_{H_2}^2}(m_a^2 - m_b^2)\xi \text{ Im}(e^{-i\alpha}V_L^{\alpha\beta}V_R^{\alpha\beta*}) ,
\]

(17)

\[\begin{array}{c}
\begin{array}{c}
\text{(a)}\\
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
\text{(b)}
\end{array}
\end{array}\]

FIG. 1: Effective four-quark operators generated by integrating out $W_1$-boson: (a) the diagrams in the full theory and (b) the effective operator.
\[ C_{3}^{ab} = \frac{\sqrt{8}G_{F}}{6M_{H_{2}}^{2}}(m_{a}^{2} - m_{b}^{2})\xi \text{Im}(e^{-i\alpha}V_{L}^{\alpha\beta}V_{R}^{\alpha\beta^{*}}), \]
\[ C_{4}^{ab} = \frac{\sqrt{8}G_{F}}{M_{H_{2}}^{2}}(m_{a}^{2} - m_{b}^{2})\xi \text{Im}(e^{-i\alpha}V_{L}^{\alpha\beta}V_{R}^{\alpha\beta^{*}}), \]
\[ C_{11}^{aa'} = \frac{2\sqrt{8}G_{F}}{M_{H_{0}}^{2}}\text{Im}(C_{aa}^{a'a^{*}}), \]
\[ C_{12}^{aa'} = -\frac{2\sqrt{8}G_{F}}{M_{H_{0}}^{2}}\text{Im}(C_{aa}^{a'a^{*}}), \]
\[ C_{11}^{bb'} = \frac{2\sqrt{8}G_{F}}{M_{H_{0}}^{2}}\text{Im}(D_{aa}^{a'a^{*}}), \]
\[ C_{12}^{bb'} = -\frac{2\sqrt{8}G_{F}}{M_{H_{0}}^{2}}\text{Im}(D_{aa}^{a'a^{*}}), \]

where \( a, a' \in u, c, t, a \neq a' \) and \( b, b' \in d, s, b, b \neq b' \), \( C = V_{L}^{\dagger}M_{D}V_{R}^{\dagger} - 2\xi e^{i\alpha}M_{U}, D = V_{L}^{\dagger}M_{D}V_{R}^{\dagger} - 2\xi e^{-i\alpha}M_{D}, M_{H_{0}} \) is the mass of the flavor changing neutral Higgs (FCNH) and \( M_{H_{2}} \) is the mass of \( H_{2}^{+} \) which is a charged Higgs in mLRSM \[15\]. \( \hat{M}_{U} \) and \( \hat{M}_{D} \) are diagonalized quark mass matrices. \( \zeta \) is the mixing angle between the left-handed and right-handed W-bosons that
\[
\sin 2\zeta \approx -r \frac{4m_{b}}{m_{t}} \left( \frac{M_{1}}{M_{2}} \right)^{2},
\]
where \( r \equiv (m_{t}/m_{b})\xi \) and \( \xi \) is the ratio between the two vevs of the Higgs bidoublet in mLRSM \[15\]. The contributions due to the Higgs exchanges are always proportional to quark masses. Since we are only interested in operators with at least two of the quarks being light, the Wilson coefficients are always proportional to at least one light quark mass, or they are proportional to heavy quark masses but must be suppressed by the non-diagonal CKM matrix elements. Furthermore, the mass of FCNH is strongly constrained to very large value by the mass differences and the CP-violating decay properties of the neutral K-bosons and B-bosons \[15, 26\], and detailed calculation shows \( H_{2}^{+} \) is as heavy as FCNH. If we are interested in the case of a few TeV right-handed W-boson mass, we can safely neglect the Higgs exchange contributions. Then at the electroweak scale the Wilson coefficients of the CP-odd four-quark operators can be simplified to
\[
C_{11}^{ab} = -C_{12}^{ab} = \frac{\sqrt{8}G_{F}}{6} \sin 2\zeta \text{Im}(e^{-i\alpha}V_{L}^{ab}V_{R}^{ab^{*}}),
\]
\[
C_{21}^{ab} = -C_{22}^{ab} = \sqrt{8}G_{F} \sin 2\zeta \text{Im}(e^{-i\alpha}V_{L}^{ab}V_{R}^{ab^{*}}).
\]

We will take this simple limit in the following discussion.

### B. Quark EDM and CDM Operators

The one-loop contributions to the quark EDM from the gauge interactions are shown in Fig. 2, where the internal wavy lines represent the light charged gauge-boson \( W_{1} \) which is dominated by \( W_{L} \), but has a small admixture of \( W_{R} \). The dashed lines represent the charged-Goldstone boson present in Feynman gauge, and the external wavy line is the static electric...
field or photon. Diagrams a) and b) have the photon interacting with the quarks directly, and these from c) to f) have the photon interacting with charged bosons. For the quark CDM case we have the first two diagrams only with the external wavy line representing a gluon.

These diagrams have been calculated in the literature long ago [8], our result is somewhat different from theirs in the infrared part. The CP-odd part of the diagrams in Fig. 2 can be expressed in terms of the coefficients of the EDM and CDM operators. For the up quark, we have $d_u^E O_u^E + d_u^C O_u^C$ with,

\[
d_u^E = \frac{1}{16\pi^2} \sum_{i=d,s,b} m_{di} e \sqrt{8} G_F \sin 2\zeta \text{Im}(e^{-ia} V_{Li}^* V_{Ri}^L) \\
\times \frac{1}{(1-r_i)^3} \left( \frac{4}{3} - 4r_i + 3r_i^2 - \frac{1}{3}r_i^3 + \frac{1}{2} r_i \ln r_i - \frac{3}{2} r_i^2 \ln r_i \right),
\]

\[
d_u^C = \frac{1}{16\pi^2} \sum_{i=d,s,b} m_{di} g_s \sqrt{8} G_F \sin 2\zeta \text{Im}(e^{-ia} V_{Li}^* V_{Ri}^L) \\
\times \frac{1}{(1-r_i)^3} \left( 1 - \frac{3}{4} r_i - \frac{1}{4} r_i^3 + \frac{3}{2} r_i \ln r_i \right).
\]

(21)
And for the down quark, the contribution is $d_d^E O_d^E + d_d^C O_d^C$ with
\[
d_d^E = \frac{1}{16\pi^2} \sum_{i=u,c,t} m_{ui} e \sqrt{8G_F} \sin 2\zeta \text{Im}(e^{-i\alpha} V_L^{1l} V_R^{1\ast l}) \\
\times \frac{1}{(1 - r'_i)^3} \left( \frac{5}{3} - \frac{17}{4} r'_i + 3 r'_i^2 - \frac{5}{12} r'_i^3 + r'_i \ln r'_i - \frac{3}{2} r'_i^2 \ln r'_i \right),
\]
\[
d_d^C = \frac{1}{16\pi^2} \sum_{i=u,c,t} m_{ui} g_s \sqrt{8G_F} \sin 2\zeta \text{Im}(e^{-i\alpha} V_L^{1l} V_R^{1\ast l}) \\
\times \frac{1}{(1 - r'_i)^3} \left( 1 - \frac{3}{4} r'_i - \frac{1}{4} r'_i^3 + \frac{3}{2} r'_i \ln r'_i \right).\tag{22}
\]

It is easy to see that this mixing angle is suppressed by the mass ratio of bottom and top quarks and by the ratio of the left and right handed $W$-boson masses. $m_{ui}$ are the masses of up-type intermediate quarks, $r_i = m_{di}/M_W$, $r'_i = m_{ui}/M_W$, $V_L$ and $V_R$ are the left and right-handed CKM mixing matrices, respectively, $\alpha$ is the spontaneous CP phase mentioned earlier.

![Diagram](image)

FIG. 3: Higgs-induced quark EDM. The dashed lines here represents the Higgs bosons.

In mLRSM, $H_2^+$ also gives contribution to the quark EDM and CDM. The relevant diagrams are shown in Fig. 3 and the result is
\[
d_u^E = \sum_{a \in \{d,s,b\}} \frac{1}{16\pi^2} \sqrt{8G_F} \frac{2m_a(m_a^2 - m_g^2)}{M_{H_2}^2} \xi \text{Im}(e^{-i\alpha} V_L^{1\ast a} V_R^{1l}) \\
\times \left[ e_d \frac{3 - 4r_j + r_j^2 + 2 \ln r_j}{2(-1 + r_j)^3} - e^{-1 + r_j^2 - 2r_j \ln r_j} \right],
\]
\[
d_u^C = \sum_{a \in \{d,s,b\}} \frac{g_s}{16\pi^2} \sqrt{8G_F} \frac{2m_a(m_a^2 - m_g^2)}{M_{H_2}^2} \text{Im}(e^{-i\alpha} V_L^{1\ast a} V_R^{1\ast l}) \frac{3 - 4r_j + r_j^2 + 2 \ln r_j}{2(-1 + r_j)^3},\tag{23}
\]
\[
d_d^E = \sum_{a \in \{u,c,t\}} \frac{1}{16\pi^2} \sqrt{8G_F} \frac{2m_a(m_a^2 - m_g^2)}{M_{H_2}^2} \xi \text{Im}(e^{-i\alpha} V_L^{a1} V_R^{1\ast l}) \\
\times \left[ e_u \frac{3 - 4r_j + r_j^2 + 2 \ln r_j}{2(-1 + r_j)^3} + e^{-1 + r_j^2 - 2r_j \ln r_j} \right],
\]
\[
d_d^C = \sum_{a \in \{u,c,t\}} \frac{g_s}{16\pi^2} \sqrt{8G_F} \frac{2m_a(m_a^2 - m_g^2)}{M_{H_2}^2} \text{Im}(e^{-i\alpha} V_L^{a1} V_R^{1\ast l}) \frac{3 - 4r_j + r_j^2 + 2 \ln r_j}{2(-1 + r_j)^3},\tag{24}
\]

10
in which

\[ r_j = \frac{M^2_{D_j}}{M^2_{H_2}}, \quad r'_j = \frac{M^2_{U_j}}{M^2_{H_2}}, \]

Therefore, if the right-handed \( W \)-boson has a moderate mass, say, a few TeV, the contribution from \( H^+_2 \) to the quark EDM and CDM can be neglected in comparison to that from the right-handed gauge boson.

Actually, there are both long-distance and short-distance contributions from the one-loop diagrams in Fig. 2 and Fig. 3. The short-distance contributions come from the integration region where the internal momentum is around \( M_W \); and the long-distance one from the loop momentum around the internal light quark masses. Due to asymptotic freedom of the strong interaction, the short-distance contributions can be calculated accurately using perturbation theory. The long-distance contributions, however, suffer from non-perturbative QCD effects, and the only known way to calculate it correctly is by Lattice QCD. In the matching calculation, the long distance contribution has to be subtracted to obtain the Wilson coefficients, which is shown in Fig. 4. This contribution can be calculated using a certain UV regulator, such as dimensional regulation or momentum cut-off or lattice regularization. Any regularization preserving a certain Fierz identity will give a zero answer as the loop integral involves only the photon or gluon external momentum. Other regularizations, such as naive dimensional regularization, will find a finite contribution. One must be careful though that the vanishing of long-distance contribution is only true at one-loop level: as soon as the QCD corrections are taken into account, the result becomes non-zero. Therefore, to the leading order, we can directly read off the Wilson coefficients of quark EDM and CDM operators from Eqs.(21), (22), and (23).

C. Weinberg Operator

In mLRSM, the Weinberg operator can be induced from diagrams in Fig. 5. Since the result is proportional to the quark masses, the leading contribution comes from the third generation of the quarks running in the loop. These are two-loop diagrams, the Weinberg operator comes out after one integrates out the internal quarks and bosons entirely. If one follows the effective theory approach, in which the top quark and the W-boson are first...
FIG. 5: Diagrams contributing to Weinberg operator in mLRSM. The first diagram is induced by the W-boson exchange, the second by Goldstone exchange and the third by the charged Higgs boson.

FIG. 6: Contribution to the three-gluon vertex after integrating out the top quark, the Higgs boson and the W-bosons. The black dot labels the bottom quark CDM operator.

integrated out, the CDM operator of the bottom quark emerges and one can get its wilson coefficient from Eq. (22).

Then from Fig. 6 one gets the major contribution to the Weinberg three gluon vertex. Because this diagram would diverge quadratically in the infrared if the mass of the bottom quark was zero, this diagram should be proportional to $1/m_b^2$. However, chirality flipping is needed or otherwise the fermion loop will vanish, so the numerator of the diagram must be proportional to $m_b$. Combining the two effects together, this diagram is proportional to $d^C_b/m_b$, where $d^C_b$ is the bottom quark CDM which is proportional to $m_t$. Therefore this diagram has an enhancement of a factor of $m_t/m_b$, about 40, which was first found in Ref. [27]. Detailed calculation gives the Wilson coefficient

$$C_g(m_b) = \frac{g_s^2(m_b) d^C_b(m_b)}{16 \pi^2 m_b}.$$  \hspace{1cm} (26)

This contribution is seemingly large, however, it is suppressed by a numerical factor, $1/(1 - m_t^2/M^2_1)^3 \approx -0.02$ in Eq. (22). Therefore, the effect of the enhancement is totally canceled. Furthermore, the evolution also makes the contribution of this operator to be smaller at the low energy region [25]. Therefore, we safely neglect its contribution to nEDM in the following calculations.
D. Wilson Coefficients at Hadronic Scale Through Leading-Order QCD Evolution

The Wilson coefficients of the dimension-six operators can be written as

\[ O_{1}^{ab} = O_{11}^{ab} - O_{12}^{ab}, \]
\[ O_{2}^{ab} = O_{21}^{ab} - O_{22}^{ab}, \]

with the Wilson coefficients \( C_{1}^{ab} = C_{11}^{ab} \) and \( C_{2}^{ab} = C_{21}^{ab} \), respectively. Therefore, the RGEs of the Wilson coefficients of the dimension-six operators can be written as

\[ \mu^2 \frac{d}{d\mu^2} C_{1}^{ab}(\mu) = -8\frac{\alpha_s(\mu)}{4\pi} C_{1}^{ab}(\mu); \]
\[ \mu^2 \frac{d}{d\mu^2} C_{2}^{ab}(\mu) = \frac{\alpha_s(\mu)}{4\pi} C_{2}^{ab}(\mu), \]

which shows that \( C_1 \) grows as the scale goes down, whereas \( C_2 \) does the opposite.

The RGE of the quark CDM operators are a little bit complicated. For \( d \) quark and \( s \) quark CDM operators, as we discussed before, the \( c \) quark internal line gives a large contribution. Therefore, the RGEs of \( d \) and \( s \) quark CDM operators can be written as

\[ \mu^2 \frac{d}{d\mu^2} d_{d,s}^{C}(\mu) = -\frac{g^3_s(\mu)}{(16\pi)^2} m_c(\mu) \left( \frac{2}{3} \gamma_{31} C_{2}^{c(d,s)}(\mu) - 4 \gamma_{32} C_{1}^{c(d,s)}(\mu) \right) \]
\[ -\frac{g^2_s(\mu)}{16\pi^2} (\gamma_{33} + b_f/2 - \delta) d_{d,s}^{C}(\mu). \]

The Wilson coefficient of the up quark CDM operator is one order of magnitude smaller than that of the of \( d \) quark due to that \( m_s/m_c \sim 1/10 \). In the above formula, \( \gamma_{31} = 5/2, \gamma_{32} = -1, \gamma_{33} = -14/3, \) and \( \delta = -4 \) is the anomalous dimension of the quark mass. Detailed calculation gives, at \( m_c \), the relevant Wilson coefficients are

\[ C_{1}^{u(d,s)}(m_c) = 3.0 C_{1}^{u(d,s)}(M_L), \]
\[ C_{2}^{u(d,s)}(m_c) = 0.87 C_{2}^{u(d,s)}(M_L), \]
\[ d_{d,s}^{C}(m_c) = 1.7 \frac{m_c}{16\pi^2} C_{1}^{c(d,s)}(M_L) + 0.34 \frac{m_c}{16\pi^2} C_{2}^{c(d,s)}(M_L) + 1.6 d_{d,s}^{C}(M_L). \]

The CP-odd operators generate additional running of the quark EDM operators through the electromagnetic interaction. The RGE of the down quark EDM operator can be written as

\[ \mu^2 \frac{d}{d\mu^2} d_{d}^{E}(\mu) = -\frac{2 e m_c(\mu) g^2_s(\mu)}{3 (16\pi^2)^2} \gamma_{41} C_{2}^{c(d)}(\mu) - \frac{e g_s(\mu)}{16\pi^2} \gamma_{43} d_{d}^{C}(\mu) - \frac{g^2_s(\mu)}{16\pi^2} (\gamma_{44} - \delta) d_{d}^{E}(\mu). \]
where $\gamma_{41} = 16/3$, $\gamma_{43} = 16/9$, $\gamma_{44} = -16/3$, and similarly for the strange quark. The RGE of the electromagnetic coupling $\epsilon$ does not depend on the strong coupling constant $g_s$ up to one-loop, therefore, can be treated as a constant. At the charm quark mass scale, one can get

$$d_{d,s}^E(m_c) = \frac{em_c}{16\pi^2}(0.07C_1^{c(d,s)}(M_L) + 0.34C_2^{c(d,s)}(M_L)) + 0.17ed_{d,s}^C(M_L) + 0.83d_{d,s}^E(M_L).$$

which shows the explicit contributions from the running of the four-quark operators as well as CDM operators.

IV. NEDM IN MLRSM AND CONSTRAINT ON LEFT-RIGHT SYMMETRY SCALE

In this section, we carry out the last step of the nEDM calculation in mLRSM by incorporating the neutron matrix elements of hadronic operators. We collect the state-of-art results in the literature and use them to constrain the parameters in mLRSM. We find that in order to satisfy the current experimental bound on nEDM and the data on kaon-decay parameter $\epsilon$, the right-handed gauge boson $W_R$ might be as heavy as $10 \pm 3$ TeV. This bound is far higher than the bound obtained previously from the kaon mass difference, making it difficult to discover left-right symmetry at LHC.

A. Hadronic Matrix Elements

The most difficult part in calculating nEDM is to estimate the hadronic matrix elements. In the literature, many different approaches, such as the $SU(6)$ quark model, bag models, QCD sum rules, and chiral perturbation theory have been used to make estimations. In this subsection, we summarize the results and get some idea about their uncertainties.

1. Contribution from Quark EDM

   In the $SU(6)$ constituent quark model, the matrix elements of the quark tensor operators are simple and scale-independent [7, 8], leading to

   $$d_{N}^{(1)} = -\frac{1}{3}d_u^E + \frac{4}{3}d_d^E.$$

   Although it has been suggested that one should use the constituent quark masses in the formulas of quark EDM [8], this is incorrect from the point of view of factorization.

   In the parton quark model discussed in [9], it was found,

   $$d_{N}^{(1)} = -0.508d_u^E + 0.746d_d^E - 0.226d_s^E.$$

   From the QCD sum rules, one gets [28]

   $$d_{N}^{(1)} = (1 \pm 0.5) \times 0.7(-0.25d_u^E + d_d^E).$$

   Different approximations are largely consistent.
2. Contribution from Quark CDM

The contribution to nEDM from the quark CDM in the constituent quark model is [7]

\[ d_N^{(2)} = \frac{4}{9} e g_s d_C^u + \frac{8}{9} e g_s d_C^d, \]  

(36)

where \( g_s \) is the coupling of strong interaction at the energy scale where the model is applicable. In this calculation, the authors assumed first that the neutron is composed of constituent quarks, and then treated the gluon field inside the neutron as a background, neglecting its kinetic energy. Therefore, Eq. (36) can only be seen as an order-of-magnitude estimate.

Weinberg’s naive dimensional analysis has also been used to estimate this contribution [11, 12, 23],

\[ d_N^{(2)} \sim \frac{e}{4\pi} (O(1)d_C^u + O(1)d_C^d). \]  

(37)

In Ref. [13], the authors used the chiral perturbation theory to calculate the singular part of the long distance contribution,

\[ d_N \simeq \frac{0.7e}{g_s} (d_C^u + d_C^d). \]  

(38)

And finally, QCD sum rules analysis in Ref. [28] gives

\[ d_N^{(2)} = (1 \pm 0.5) \times \frac{0.55e}{g_s} (0.5d_C^u + d_C^d), \]  

(39)

where \( g_s \) is the strong coupling constant at 1 GeV, about 2.5.

3. Contribution from Weinberg Operator

The contribution from the Weinberg’s operator \( O_W \) can be estimated by Weinberg’s naive dimensional analysis [23], which is an order-of-magnitude estimate

\[ d_N^{(3)} \simeq eMC_g(\mu)/4\pi \approx 100 \text{ MeV} e C_g(1\text{GeV}), \]  

(40)

where \( M = 4\pi F_\pi \approx 1190 \text{ MeV} \) and \( \mu \) is the hadronic scale taking as 1 GeV.

On the other hand, the estimate based on QCD sum rules gives [29]

\[ d_N^{(3)} \simeq (10 - 30)\text{MeV} e C_g(1 \text{GeV}), \]  

(41)

which is considerably smaller. In any case, because of the small coefficient function, the Weinberg operator contribution can essentially be neglected.

4. Contribution from Four-Quark Operators

The hadronic matrix elements of the four-quark operators have been studied and reviewed in Ref. [22]. In this work we will take the results from that paper.
B. Numerical Results

As discussed in Ref. [15], combining with the kaon indirect CP-violation $\epsilon$ parameter, one can use nEDM to get the most stringent lower bound on the mass of the right-handed $W$ boson in the context of the mLRSM. In Ref. [15], the authors used naive factorization [10] to estimate the contribution of four-quark operators. However, this method for baryons may not be valid even in the large-$N_C$ limit, and the uncertainty is unknown. Therefore, we have assumed a very large error on their matrix elements and the resulting constraint on the left-right symmetry scale is not very strong. In a dedicated study of these matrix elements [22], we have gotten a much better understanding on their contribution. In Ref. [22], the contribution of four-quark operators to nEDM was separated into two parts, the direct contribution and the meson-condensate contribution. For the direct contribution, quark models were employed to calculate the hadronic matrix elements, which is only an order-of-magnitude estimate. However, for the meson-condensate contribution, the factorization method was used to calculate the meson matrix elements, which can be justified in the large-$N_C$ limit. Since the meson-condensate contribution dominates over the direct one, we believe that we reached a factor-of-two accuracy in the matrix elements of four-quark operators.

![Graph](image)

**FIG. 7:** nEDM contributed from operators, $\bar{u}i\gamma_5u\bar{d}d$ (short dashed red line), $\bar{u}i\gamma_5u\bar{s}s$ (long dashed green line), down quark EDM and CDM operators (solid blue line).

In mLRSM, after neglecting the contributions from FCNH and the charged higgs boson exchange, nEDM depends only on three parameters, $r$, $\alpha$, and $M_{W_R}$, where $\alpha$ is the new source of CP-violation. Therefore, if $\alpha = 0$, nEDM predicted by the mLRSM will be the same as that predicted by SM, about five orders of magnitude smaller than the upper bound given by the current experiment [30]. Whereas for $\epsilon$, there are two new contributions in mLRSM [15], the Dirac phase in the righthanded CKM matrix inherited from the lefthanded CKM matrix, and the spontaneous phase $\alpha$. The new contribution from the Dirac phase is enhanced compared to the similar contribution in SM due to the chiral enhancement in the hadronic matrix element (see Ref. [21] for a good review). The contribution of the spontaneous CP-phase $\alpha$ must be adjusted to cancel the contribution of the Dirac phase. Therefore, in mLRSM there is a tension between nEDM and $\epsilon$ that one cannot only adjust $\alpha$ to suppress all the new CP-violation sources, and a large $M_{W_R}$ is needed. As a result, nEDM and $\epsilon$ together give a lower bound on $M_{W_R}$. 
FIG. 8: Constraints on the mass of $W_R$ and the spontaneous CP-violating parameter $\alpha$ from the kaon decay parameter $\epsilon$ ($M_{H_0} = \infty$, red dots; $M_{H_0} = 50$ TeV, blue dots) and nEDM (green dots). For nEDM, we use the current experimental upper bound as the constraint and for $\epsilon$ we use the criteria that the beyond-SM-physics contribution should not exceed $1/4$ of the experimental value.

In this new study, we use the QCD sum rules to estimate the contribution of the quark EDM and CDM operators, and use the results in Ref. [22] for the contribution of the four-quark operators. Fig. [7] shows the contributions to nEDM from different operators at fixed $M_{W_R}$ and $r$. The result from the Weinberg operator is too small to be included in the figure. It is clear that the contributions from four-quark operators are much larger than from quark EDM and CDM operators. One way to understand this is that in mLRSM the quark EDM and CDM operators are generated in the same way as the four-quark operators. The quark EDM and CDM operators are generated through diagrams in Fig. [1] and the four-quark operators are generated through diagrams in Fig. [2]. The Wilson coefficients roughly have the following relations

$$d^E_q \simeq \frac{e m_q A}{16\pi^2} C_4 ; \quad d^C_q \simeq \frac{g_s m_q A'}{16\pi^2} C_4 ,$$

(42)

where $A$ and $A'$ are two proportionality coefficients, $C_4$ is the Wilson coefficient of certain four-quark operators. Take the down quark EDM as an example, $A$ can be written as $\sin^2 \theta_C m_c/m_u \simeq 15$, where $\theta_C$ is the Cabibbo angle. From QCD sum rules, nEDM contributed by the down-quark EDM operator is approximately the down-quark EDM itself, whereas the nEDM contributed directly from the four-quark operator can be written as

$$d^\text{four-quark}_N \simeq \frac{e}{16\pi^2} B_0 C_4 ,$$

(43)

where $B_0 \simeq 2.2$ GeV is related to SSB of the chiral symmetry. Since $B_0 \gg A m_d$, nEDM directly from the four-quark operator $\bar{u}i\gamma_5 u\bar{d}d$ is much larger than the contribution from the down quark EDM operator. Indeed, this is a common phenomenon in left-right models and two-Higgs-doublet models, where the quark EDM and CDM operators are always generated by the triangle diagrams in Fig. [2] and the internal lines are always quarks. In other types of new physics models, the internal lines can be other kind of fermions. For example, in supersymmetric models, they can be gauginos, and in extra dimension models, they can be KK-fermions, where the above relation between quark EDM operators and four-quark
operators is no longer hold. In these models, quark EDM and CDM operators might be more important that four-quark operators.

Using the matrix elements in Ref. [22], we calculate the constraint from the nEDM and kaon-decay parameter $\epsilon$ on the allowed parameter space of mLRS. The result is shown in Fig. 8. The allowed parameter region by the experimental upper bound on nEDM is shown as green dots. The constraints from $\epsilon$-parameter depends strongly on the mass of the FCNH in the theory. We have shown two possible values of $M_{H_0}$, 50 TeV and $\infty$ for simplicity. We assume for $\epsilon$ the new contribution should not exceed 1/4 of the experimental value. From Fig. 8 one can see that the lower bound for the $M_{W_R}$ from nEDM and $\epsilon$ is around 10 TeV. If we assume a factor of 2 uncertainty on the hadronic matrix elements, the actual bound is $10 \pm 3$ TeV. This will make a direct detection of the right-handed gauge boson very difficult at LHC if it exits.

V. CONCLUSION

In this paper, we have studied nEDM in mLRS systematically by using effective field theory approach. The formula for calculating nEDM is given in Eq. (16). The contribution of four-quark operators is found to be the most important. The contribution of Weinberg operator to nEDM has been discussed systematically. A numerical suppression is found which counteracts the infrared enhancement and makes the contribution of this operator negligible. We have found a lower bound on the mass of $W_R$ which is about $(10 \pm 3)$ TeV. This is higher than what have been found before [15, 31] and certainly cannot be detected at LHC.

In a more complicated non-supersymmetric scenario of LRSM, although the CP-violation pattern in the Higgs sector might be change, the tension between $\epsilon$ and nEDM discussed in Sec. IV still exists. Therefore, one can also use this analysis to set a lower bound on the righthanded scale. In the supersymmetric LRSM, there are new CP-violation sources from the soft terms, which can contribute to both nEDM and $\epsilon$. Furthermore, in supersymmetric LRSM [32], the lefthanded and righthanded CKM matrices must be equal to each other up to a sign, therefore, if one assumes certain scenarios of the breaking mechanism of supersymmetry, $\epsilon$ itself can give a constraint on the righthanded scale [33].

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