Heavy meson physics: what have we learned in twenty years?

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Abstract.
I give a personal account of the development of the field of heavy quarks. After reviewing the experimental discovery of charm and bottom quarks, I describe how the field’s focus shifted towards determination of CKM elements and how this has matured into a precision science.

1. Introduction

Let me start with a warning: this is not a review of heavy quark physics. I have been honored by the División de Partículas y Campos of the Sociedad Mexicana de Física with the Medalla 2003, for which I am grateful and humbled. This talk was given on occasion of the medal being conferred. I tried to describe, from my personal viewpoint, how the field of $B$ physics evolved since its inception to the present, with a shameful bias towards my own contributions.

2. Ancient history

2.1. While I was in High School . . .

In November of 1994 two experimental collaborations announced the discovery of a new very narrow resonance with mass 3.1 GeV. They had unearthed evidence for the charm quark, and for the validity of an asymptotically free theory, like QCD, for strong interactions. These events had extraordinary consequences, affecting the way we think today about particle physics. They are often referred to as the “November Revolution”.

A MIT–Brookhaven collaboration, led by S. Ting, found evidence for the new resonance by measuring the $e^+e^−$ mass spectrum in $p + Be → e^+ + e^− + X$ with a precise pair spectrometer at Brookhaven Natl. Lab.’s 30 GeV AGS[1].

The Mark I collaboration, from SLAC and LBL, led by B. Richter was conducting experiments at the newly constructed $e^+e^−$ ring, SPEAR, at SLAC. Their detector consisted of a spark chamber embedded in a solenoidal magnetic field, and surrounded by time-of-flight counters, shower counters and proportional counters embedded in slabs of iron for muon identification. They[2] “observed a very sharp peak in the cross sections for $e^+e^− →$ hadrons, $e^+e^−$, and $μ^+μ^−$ at a center of mass energy of $3.105 ± 0.003$ GeV” and found an upper bound on the width of 1.3 MeV.
The MIT-BNL collaboration called the new resonance “J”, Mark I called it “ψ”, so it’s now known as the J/ψ. The Mark I collaboration soon found a second narrow resonance[3], the ψ′, at a mass of 3.695±0.004 GeV. No other narrow resonances were found in the total e+e− cross sections at SPEAR, but broader structures did appear at energies above the ψ′[4]. Other narrow structures that could not be directly produced in e+e− collisions were found through cascade decays of the ψ′. The DASP collaboration working at DESY’s e+e− storage ring DORIS found[5] the first χ state in ψ′ → χ + γ → ψ + γ + γ. The Crystal Ball collaboration[6] detector provided the high spatial and energy resolution needed to finally unravel the spectroscopic levels of charmonium.

The interpretation of these resonance soon became clear: they are atom-like bound states of a charm quark-antiquark pair. The interaction between the rather heavy quarks is coulomb like, since at short distances QCD becomes weak and a single gluon exchange gives an attractive coulomb potential between the quarks. The potential is not really coulomb: for one thing, it must be confining so it must grow without bound at long distances. But the physics of the spectrum of bound states is dominated by the short distance interaction and is not dissimilar from the physics of the hydrogen atom. The ηc and ψ families are in a quark-spin singlet and triplet state, respectively, with orbital angular momentum 0, giving JPC = 0−+ and 1−−, respectively. The χc0, χc1 and χc2 are in a spin triplet, with JPC = 0++, 1++ and 2++, respectively.

Charmonium states have zero charm number, C. States with |C| = 1, with so called “naked charm”, were first convincingly observed by Mark I at SPEAR[7]. They observed narrow peaks in the invariant mass spectra for neutral combinations of charged particles in Kπ and K3π. They inferred the existence of an object of mass 1865 ± 15 MeV and put an upper limit on its width of 40 MeV. The new state, with C = 1 (−1), was the D( ¯D) pseudoscalar meson. They found “it significant that the threshold energy for pair-producing this state lies in the small interval between the very narrow ψ′ and the broader . . .” ψ″. That is, the ψ″ is much broader because it decays strongly into a D– ¯D pair.

2.2. ...and then in college

The discovery of “naked bottom” (or “naked beauty”, outside the Americas) paralleled in many ways that of charm. Although a new sequential heavy lepton, the τ, had been discovered, and therefore the existence of beauty and top expected, the masses of these quarks were unknown.

L. Lederman led a collaboration at Fermilab that used a two arm spectrometer to search for muon pairs in 400 GeV proton-nucleus collisions. They had some experience. Years earlier the group conducted a similar experiment at BNL’s AGS. Because their apparatus had smaller resolution than that of the MIT-BNL group, they did not report any evidence for a resonance. They had seen a cross section that, except for a small plateau in the 3 GeV region, fell with invariant mass as expected. After missing the J/ψ, they were ready for the discovery of bottomonium. They observed[8] a similar effect in the new experiment, and correctly interpreted it as a dimuon resonance at about 9.5 GeV. A refined analysis of the experiment revealed actually two peaks, at 9.44 and 10.17 GeV. The states were named “Υ” and “Υ′”.

An upgrade of the energy of DORIS made it possible for the PLUTO and DASP II collaborations to observe the Υ in e+e− annihilation[9, 10]. A further energy upgrade made the Υ′ accessible too[11, 12].

After the Cornell Electron Storage Ring (CESR) was commissioned, the CUSB and CLEO collaborations successfully observed the Υ in e+e− annihilation[11, 10]. All three resonances, with masses 9.460, 10.023 and 10.355 GeV are narrow. Shortly afterwards the two collaborations established the existence of a broader resonance, the Υ″, at a mass of ∼ 10.55 GeV and a width of about 12.6 MeV. This is significant because, following the charm experience, it suggests looking for naked beauty in the decay of Υ″. B-mesons were first found and reported by the CLEO
collaboration in a paper which for once is straight and to the point in its title (“Observation of Exclusive Decay Modes of $b$-Flavored Mesons”) and in its abstract (see Ref. [13]). To be sure, $B$-mesons had been inferred from the observation of high momentum leptons in $\Upsilon''$ decays, but it was the reconstruction of a few exclusive decays that demonstrated their existence conclusively. Today $D$ and $B$ mesons are universally accepted established resonances. They are the closest we can get to having naked charm and beauty. Their masses have been measured to high accuracy[14]:

\[
\begin{align*}
    m_{D^\pm} &= 869.4 \pm 0.5 \text{ MeV} \\
    m_{D^0} &= 864.6 \pm 0.5 \text{ MeV} \\
    m_{B^\pm} &= 279.0 \pm 0.5 \text{ MeV} \\
    m_{B^0} &= 5279.4 \pm 0.5 \text{ MeV}
\end{align*}
\]

3. Lifetimes, CKM texture, Semileptonic and ISGW

One of the first surprises encountered in the early 80’s was the long lifetime of naked $B$. The 1984 PDG Review of Particle Properties[15] gives:

\[
\begin{align*}
    \tau_{D^\pm} &= 0.92 \pm 0.15 \text{ ps} \\
    \tau_{D^0} &= 0.44 \pm 0.07 \text{ ps} \\
    \tau_B &= 1.4 \pm 4 \text{ ps}
\end{align*}
\]

The $B^\pm$ and $B^0$, $\bar{B}^0$ lifetimes are not separated. The $B$ lifetime is longer than the $D$ lifetime. Since naively $\tau_B = (m_b/m_c)^5 (x|V_{cb}|^2 + |V_{ub}|^2)/|V_{cs}|^2$, where $x \approx 0.5$ is a phase space suppression factor, one has to conclude that both $V_{cb}$ and $V_{ub}$ are small:

\[
\sqrt{x|V_{cb}|^2 + |V_{ub}|^2} \approx \left( \frac{m_c}{m_b} \right)^5 |V_{cs}| \sqrt{\frac{\tau_D}{\tau_B}} \approx 0.03 - 0.05. \tag{1}
\]

Moreover, already the 1984 PDG Review listed the decay branching fraction into charmed final states as $80 \pm 28\%$, implying that $|V_{ub}| \ll |V_{cb}|$ so that the above estimate gives $|V_{cb}| \approx 0.05 - 0.07$. Unitarity of the CKM matrix requires $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$, so we learned, in addition, that $|V_{tb}| \approx 1$. There was a quantum leap in understanding of the texture of the CKM matrix: it became evident that it has 1’s along the diagonal, numbers of order 0.1 off the diagonal, and very likely much smaller than 0.1 two steps removed from the diagonal. The rule of thumb was (and still is) $V_{ij} \sim (0.1)^{|i-j|}$. In what used to be the standard parametrization of the CKM matrix in terms of angles $\theta_i$ and $\delta$, this meant that all angles are of the order of 0.1. The origin of this texture remains a mystery, and a challenge to model builders.

How can we progress from an estimate of the CKM angles to a precision measurement? This is an example of a story that repeats itself over and over in the study of heavy quarks: experiment and theory have to work together to find a route to the answer of this question. Experimentally it would be easiest to measure with great precision the lifetime of the $B$ meson, but theorists did not have (nor do they have today) a complete theory of lifetimes. Theory prefers inclusive widths of semileptonic decays, but experimental backgrounds make this a tough, if not impossible measurement, particularly when restricting to non-charm final states as is necessary for the determination of $|V_{ub}|$. The difficulty with the charm background is easily grasped from Fig. 1a, which shows data from CLEO[16] and ARGUS[17] superimposed on model calculations (see below) of the semileptonic decay spectrum into $c$- and $u$-quarks, assuming $V_{ub} = V_{cb}$.

Not knowing how to calculate the decay rate from first principles, theorists resorted to reasonable guesses, or “hadron models.” The GSW model[18], and later incarnations as the
Figure 1. (a) Data from CLEO[16] and ARGUS[17] superimposed on quark-model calculations of the $B$-meson semileptonic decay spectrum into $c$- and $u$-flavored mesons, assuming $V_{ub} = V_{cb}$. (b) Individual charmed resonances contributions to the total $B$ meson decay spectrum. The sum, indicated as “total,” is in remarkable agreement with the decay rate of a free $b$-quark, indicated as “free.”

Figure 2. Quark model calculation of non-charm semileptonic $B$-meson decay spectrum showing how contributions of several resonances add up. The deficit (relative to the free quark rate) at large energy cannot be supplied by massive resonances, even if they may for lower energies.

ISGW[19] and ISGW-II[20] models, was simply an application of the quark-potential model of hadrons to the computation of matrix elements of $V - A$ charged currents between an initial $B$ meson state and a final state consisting of a single meson with either a charm quark (for $V_{cb}$) or a $u$-quark (for $V_{ub}$). It was reassuring that adding over a few final state charmed mesons, the semi-inclusive decay rate into charm gives approximately the same answer as the free quark decay rate. Fig. 1b shows how the individual charmed resonances add up.

In order to determine $V_{ub}$ the charm background has to be controlled. A favorite experimental method was to measure the inclusive semileptonic decay spectrum as a function of the readily measurable electron (or muon) energy, and to focus on energies large enough that decay into charm is forbidden. The calculation of GSW, Fig. 2, showed that the rate in the restricted region is dominated by a few final state resonances which are only a fraction of the total semi-inclusive rate as calculated directly at the parton level ($b \to ue\bar{\nu}$). This suggested that the parton level
calculation is highly unreliable in the end-point region. Moreover, the result of summing over a few resonances had a different shape and end-point than the partonic result. And, as if not enough, the model computation was fairly sensitive to the choice of model parameters. Ouch!

4. Mixing, Heavy Top, Rare Decays

The standard model predicts $B^0 - \bar{B}^0$ oscillations, much in the same way as for $K^0 - \bar{K}^0$. The underlying weak process is well known. Since mixing requires a change of $b$-number by two units, there must be two $W^\pm$ exchanged, and the process is doubly-weak. The amplitude is given by the “box” Feynman diagram of Fig. 3a. Keeping track of the CKM factors from both fermion lines in the box diagram, one has

$$Amp \propto G_F^2 \sum_{i,j=u,c,t} (V_{ib}V_{id}^*) (V_{jb}V_{jd}^*) F(m_i^2/M_W^2, m_j^2/M_W^2), \quad (2)$$

where $F$ is a function that arises from computing the one loop graph. If the three intermediate quarks were degenerate the amplitude would vanish identically since the CKM matrix is unitary: $\sum_{i=u,c,t} V_{ib}V_{id}^* = 0$. Expanding $F$ in the quark masses we have,

$$Amp \propto G_F^2 \left( \sum_{i=u,c,t} V_{ib}V_{id}^* \frac{m_i^2}{M_W^2} \right)^2, \quad (3)$$

Since each of the factors $V_{ib}V_{id}^*$ involves a jump of two generations, the rule of thumb of the previous sections says that all these are similar (and of order $0.1^2$),

$$|V_{ub}V_{ud}^*| \sim |V_{cb}V_{cd}^*| \sim |V_{tb}V_{td}^*|. \quad (4)$$

It follows immediately that the top quark gives the dominant contribution to the mixing amplitude in Eq. (3).

In the absence of mixing, a $B^0 - \bar{B}^0$ pair produced in an $e^+e^-$ collision will produce opposite charged leptons when both $B$’s decay semileptonically. Mixing implies that some fraction of the time the two semileptonic decays produce the same charge leptons. The ARGUS collaboration discovered this phenomenon and reported\cite{21}

$$r \equiv \frac{N(B^0\bar{B}^0) + N(\bar{B}^0B^0)}{N(B^0\bar{B}^0)} = 0.21 \pm 0.08. \quad (5)$$

The rate of mixing is given by the amplitude above times $f_B^2 B_B$, which characterizes the matrix element, times some fixed numbers (including short distance QCD corrections). To explain this
rather large mixing within the standard model of electroweak interactions we could assume $f_B^2B_B$ was larger than estimated. But even taking rather extreme values for $f_B^2B_B$ (corresponding to more than four times the modern accepted value!), and taking $|V_{tb}V_{ts}^*|$ as large as possible, we were forced to require a large top quark mass, in excess of 50 GeV. In 1987 the direct bound on the mass of the top quark was 15 GeV. It was expected that the $t$-quark would be much lighter than the $W$, a reasonable guess given the masses of all other quarks. The evidence from $B^0 - \bar{B}^0$ mixing was that the top quark was much heavier, possibly heavier than the $W$. This had immediate, surprising implications.

For one, the GIM mechanism is a bit of a fluke. Take for example the prediction of the mass of the charm quark. GIM cancellations bring in a suppression factor of $m_c^2/M_W^2$, as can be seen by adapting the result in Eq. (3) to the case of $K^0 - \bar{K}^0$ mixing. The top quark contribution is not suppressed by $m_t^2/M_W^2$ simply because this factor exceeds unity. In fact, for the top quark it makes no sense to approximate the function $F$ in (2) by the expansion in (3). The top contribution to $K^0 - \bar{K}^0$ mixing involves $V_{td}V_{ts}^*$. This is a three generation jump, as compared to the single jump for the charm contribution, so we get a suppression factor of $(0.1)^2$. But, by comparison, the charm contribution is suppressed by $m_c^2/M_W^2 \sim 0.0004$.

A second implication is that processes mediated by virtual top-quarks may have larger rates than we had thought by 1987. The first such process you would think of is the radiative decay, $b \rightarrow s\gamma$. The lowest order Feynman diagram is the one loop graph shown in Fig. 3b. This gives an effective Hamiltonian for the radiative decay

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e m_b}{16\pi^2} A(\bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} b_R).$$  \hspace{0.5cm} (6)$$

Here we have neglected the contribution of $u$ and $c$ quarks, and $A$ is a function of the top quark mass plotted in Fig. 4a as a function of $x = m_t^2/M_W^2$.

Short distance QCD corrections enhance the amplitude by about 70%, so the rate is enhanced by a full factor of three; see Fig. 4b. The reason the QCD corrections are so large is that the function $A$ in (6) is accidentally small. The QCD corrections are suppressed by the strong coupling constant but enhanced by large logarithms. Resumming large logarithms of the ratio of $m_b$ to $M_W$ or $m_t$ (assuming $m_t$ is the same scale as $M_W$) has the effect of replacing $A$
Figure 5. (a) Rate for radiative $B$ decay in a model with two Higgs doublets (type II) with charged Higgs mass of 100 GeV and $\tan \beta = 1$. (b) Lepton mass spectrum in $B \to X_s \ell\ell$ for $m_t = 125$ GeV (bottom curve) and 150 GeV (top curve).

in Eq. (6)[22]:

$$A \to \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^\frac{16}{23} \left\{ A + \frac{3a}{10} \left[ \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^{\frac{10}{23}} - 1 \right] + \frac{3a}{28} \left[ \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^{\frac{28}{23}} - 1 \right] \right\}.$$  

Here $a = 232/81$ ($-140/81$ for the case of $c \to u\gamma$).

This is another interesting GIM fluke. When we first computed these QCD corrections we thought they would dominate because GIM gives an amplitude with a suppression of $m_t^2/M_W^2$, while our QCD correction would involve this ratio logarithmically. With $m_t \approx 2M_W$ this argument does not hold water, but remarkably the QCD logarithmic “corrections” are still the dominant contribution, because the function $A$ is small. Remarkably, the prediction we made in 1987 has changed little by theory refinements and higher order calculations[23]. The reason is that $\alpha_s(m_b)$ is sufficiently small that perturbation theory works rather well.

Radiative $B$ decays are interesting because they proceed only at one loop in the standard model. Therefore, new physics at short distances may in principle compete favorably with the standard model contribution to the process. To get an idea of how sensitive to new physics the process is we[24] chose to explore one of the simplest extensions of the standard model, namely, a multi-Higgs model. Consider an extension of the standard model with two Higgs doublets. In order to naturally suppress flavor changing neutral currents[25] it is customary to impose a symmetry so that all quarks receive their mass from Yukawa couplings to only one of the two Higgs doublets (“model I”), or so that all charge $+2/3$ quarks get their masses from one Higgs doublet and the charge $-1/3$ quarks get their masses from the other (“model II”). Model II is similar to the minimal supersymmetric extension of the standard model, so it has received more attention. It is easy to see that the rate $\Gamma(B \to X_s\gamma)$ is strictly larger in model II than in the standard model. Fig. 5a show the rate in model II as a function of $m_t$ for a charged Higgs mass of 100 GeV and with equal expectation values of the two Higgs doubles ($\tan \beta = 1$). Comparing with Fig. 4b we see that the standard model rate is several times smaller. Since experiment is consistent with the standard model to within 20%, the mass of the charged Higgs in model II has to be much larger than 100 GeV. The same is true in minimal supersymmetry, with the caveat that a light Higgs is allowed if one fine tunes additional contributions (like higgsinos running in the loop) to cancel the charged Higgs graph.

Many other processes are enhanced by a large top quark mass. This is demonstrated in
Fig. 5b[26], which compares the predicted lepton mass spectrum in the decay $B \rightarrow X_{s}e^{+}e^{-}$, in units of the semileptonic decay rate, for $m_{t} = 125$ GeV (lower curve) and 150 GeV (higher curve).

5. Precision CKM, HQET

5.1. Introduction

It did not escape the attention of many that the quark model prediction of the inclusive decay rate, shown in Fig. 1b, is dominated by the lowest two charmed states: the $D$ and $D^{*}$ mesons. Nussinov and Wetzel[27] emphasized that this calculation has nothing to do with the details of the quark model used. They argued that the $D$, $D^{*}$ and $B$ wavefunctions are the same, so that when the $D$ or $D^{*}$ mesons are not recoiling in the decay the amplitude is given by the overlap of identical wave-functions which is fixed to unity for normalized wave-functions. Moreover, the higher resonances have wavefunctions that are orthogonal to these, so clearly their contributions are small. The Nussinov-Wetzel argument relied on the quark-model picture that the meson is a two body system with a potential binding. Since the $b$ and $c$ are heavy, the reduced mass (which governs all the dynamics) is the same for both $D$ and $B$ mesons. Moreover, the heavy spin decouples, since the coupling arises from the magnetic moment which scales inversely with the large mass. So the wavefunction for the $D^{*}$ agrees with that of the $D$.

If this were true in QCD then one could begin a program of precision CKM determination. Enter HQET. Heavy Quark Effective Theory (HQET) is a calculational method that exploits enhanced symmetries of QCD that appear when one restricts attention to a very specific sector of the theory. I stress that HQET is QCD. This is in contrast with hadronic models, like the quark potential model, which had been used to calculate form factors for heavy meson decays.

The successes of the constituent quark model is indicative of the fact that, inside hadrons, strongly bound quarks exchange momentum of magnitude a few hundred MeV. We can think of the typical amount $\Lambda$ by which the quarks are off–shell in the nucleon as $\Lambda \approx m_{p}/3 \approx 330$ MeV. In a heavy hadron the same intuition can be imported, and again the light quark(s) is(are) very far off–shell, by an amount of order $\Lambda$. But, if the mass $M_{Q}$ of the heavy quark(s) is large, $M_{Q} \gg \Lambda$, then, in fact, this quark is almost on–shell. Moreover, interactions with the light quark(s) typically change the momentum of $Q$ by $\Lambda$, but change the velocity of $Q$ by a negligible amount, of the order of $\Lambda/M_{Q} \ll 1$. It therefore makes sense to think of $Q$ as moving with constant velocity, and this velocity is, of course, the velocity of the heavy hadron.

In the rest frame of the heavy hadron, the heavy quark is practically at rest. The heavy quark effectively acts as a static source of gluons. It is characterized by its flavor and color--$SU(3)$ quantum numbers, but not by its mass. In fact, since spin–flip interactions with $Q$ are of the type of magnetic moment transitions, and these involve an explicit factor of $g_{s}/M_{Q}$, where $g_{s}$ is the strong interactions coupling constant, the spin quantum number itself decouples in the large $M_{Q}$ case. Therefore, the properties of heavy hadrons are independent of the spin and mass of the heavy source of color.

The HQET is nothing more than a method for giving these observations a formal basis. It is useful because it gives a procedure for making explicit calculations. But more importantly, it turns the statement ‘$M_{Q}$ is large’ into a systematic perturbative expansion in powers of $\Lambda/M_{Q}$. Each order in this expansion involves QCD to all orders in the strong coupling, $g_{s}$. Also, the statement of mass and spin independence of properties of heavy hadrons appears in the HQET as approximate internal symmetries of the Lagrangian.
5.2. Effective Lagrangian and New symmetries

We shall focus our attention on the calculation of Green functions in QCD, with a heavy quark line, its external momentum almost on-shell. The external momentum of gluons or light quarks can be far off-shell, but not much larger than the hadronic scale $\Lambda$. This region of momentum space is interesting because physical quantities — $S$–matrix elements— live there. And, as stated in the introduction, we expect to see approximate symmetries of Green functions in that region which are not symmetries away from it. That is, these are approximate symmetries of a sector of the $S$–matrix, but not of the full QCD Lagrangian.

The effective Lagrangian $L_{\text{eff}}$ is constructed so that it will reproduce these Green functions, to leading order in $\Lambda/M_Q$. It is given, for a heavy quark of velocity $v$, by[28],

$$L^{(v)}_{\text{eff}} = \bar{Q}_v i v \cdot D Q_v ,$$  

(7)

where the covariant derivative is

$$D_\mu = \partial_\mu + i g_s A_\mu^a T^a ,$$  

(8)

and the heavy quark field $Q_v$ is a Dirac spinor that satisfies the constraint

$$\left( 1 + \frac{\not{v}}{2} \right) Q_v = Q_v .$$  

(9)

In addition, it is understood that the usual Lagrangian $L_{\text{light}}$ for gluons and light quarks is added to $L^{(v)}_{\text{eff}}$.

We have introduced an effective Lagrangian $L^{(v)}_{\text{eff}}$ such that Green functions $\tilde{G}_v(k;q)$ calculated from it agree, at tree level, with corresponding Green functions $G(p;q)$ in QCD to leading order in the large mass

$$G(p;q) = \tilde{G}_v(k;q) + \mathcal{O}(\Lambda/M_Q) \quad \text{(tree level)} .$$  

(10)

Here, $\Lambda$ stands for any component of $k_\mu$ or of the $q$’s, or for a light quark mass, and $p = M_Q v + k$. It is straightforward to verify Eq. (10).

Beyond tree level the corrected version is still close in form to this[29],

$$G(p;q;\mu) = C(M_Q/\mu, g_s) \tilde{G}_v(k;q;\mu) + \mathcal{O}(\Lambda/M_Q) .$$  

(11)

The Green functions $G$ and $\tilde{G}_v$ are renormalized, so they depend on a renormalization point $\mu$. The function $C$ is independent of momenta or light quark masses: it is independent of the dynamics of the light degrees of freedom. It is there because the left hand side has some terms which grow logarithmically with the heavy mass, $\ln(M_Q/\mu)$. The beauty of Eq. (11) is that all of the logarithmic dependence on the heavy mass factors out. Better yet, since $C$ is dimensionless, it is a function of the ratio $M_Q/\mu$ only, and not of $M_Q$ and $\mu$ separately. (Actually, additional $\mu$ dependence is implicit in the definition of the renormalized coupling constant $g_s$. This reflects itself in the explicit form of $C$.) To find the dependence on $M_Q$ it suffices to find the dependence on $\mu$. This in turn is dictated by the renormalization group equation.

It is appropriate to think of the HQET as a factorization theorem, stating that, in the large $M_Q$ limit, the QCD Green functions factorize into a universal function of $M_Q$, $C(M_Q/\mu, g_s)$, which depends on the short distance physics only, times a function that contains all of the information about long distance physics and is independent of $M_Q$, and can be computed as a Green function of the HQET lagrangian.
The Lagrangian for $N$ species of heavy quarks, all with velocity $v$, is

$$\mathcal{L}_{\text{eff}}^{(v)} = \sum_{j=1}^{N} \bar{Q}^{(j)}_{\nu} iv \cdot DQ^{(j)}_{\nu}. \quad (12)$$

This Lagrangian has a $U(N)$ symmetry\cite{30, 31}. The subgroup $U(N)$ corresponds to flavor conservation of the strong interactions, and was a good symmetry in the original theory. The novelty in the HQET is then the non-Abelian nature of the symmetry group. This leads to relations between properties of heavy hadrons with different quantum numbers. Please note that these will be relations between hadrons of a given velocity, even if of different momentum (since typically $M_{Q_i} \neq M_{Q_j}$ for $i \neq j$). Including the $b$ and $c$ quarks in the HQET, so that $N = 2$, we see that the $B$ and $D$ mesons form a doublet under flavor–$SU(2)$.

This flavor–$SU(2)$ is an approximate symmetry of QCD. It is a good symmetry to the extent that $m_c \gg \Lambda$ and $m_b \gg \Lambda$. (13)

These conditions can be met even if $m_b - m_c \gg \Lambda$. This is in contrast to isospin symmetry, which holds because $m_d - m_u \ll \Lambda$.

In atomic physics this symmetry implies the equality of chemical properties of different isotopes of an element.

**Spin – $SU(2)$:** The HQET Lagrangian involves only two components of the spinor $Q_{\nu}$. Recall that

$$\left(1 - \frac{\not{v} \cdot \not{v}'}{2}\right) Q_{\nu} = 0. \quad (14)$$

The two surviving components enter the Lagrangian diagonally, i.e., there are no Dirac matrices in $\mathcal{L}_{\text{eff}}$ in Eq. (7). Therefore, there is an $SU(2)$ symmetry of this Lagrangian which rotates the two components of $Q_{\nu}$ among themselves\cite{30, 32}.

Please note that this “spin”–symmetry is actually an *internal* symmetry. That is, for the symmetry to hold no transformation on the coordinates is needed, when a rotation among components of $Q_{\nu}$ is made. On the other hand, to recover Lorentz covariance, one does the usual transformation on the light–sector, including a Lorentz transformation of coordinates and in addition a Lorentz transformation on the velocity $v_{\mu}$. A spin–$SU(2)$ transformation can be added to this procedure, to mimic the original action of Lorentz transformations.

### 5.3. Exclusive Semileptonic Decays

The symmetries in HQET are sufficient to give us the matrix elements for semileptonic $B$ decay in terms of one undetermined “Isgur-Wise” function $\xi(v \cdot v')$:

$$\langle D(v')|V_{\mu}|\bar{B}(v)\rangle = \xi(v \cdot v')(v_{\mu} + v'_{\mu}), \quad (15)$$

$$\langle D^*(v')\epsilon|V_{\mu} - A_{\mu}|\bar{B}(v)\rangle = \xi(v \cdot v')[i\epsilon_{\mu\nu\lambda\sigma}v^\nu v'^\lambda v^\sigma + \epsilon^*_{\mu}(1 + v \cdot v') - v'_{\mu} \epsilon \cdot v]. \quad (16)$$

Moreover, the Isgur-Wise function satisfies a normalization condition, $\xi(1) = 1$. It is easy to see where this comes from. The forward scattering amplitude of the $B$ meson by the $b$-current, $b\gamma^\mu b$, is normalized by charge conservation (just like electromagnetic charge form factors). An HQ-flavor $SU(2)$ transformation relates this to the first matrix element in (15) and a further HQ-spin $SU(2)$ transformation relates it to the second line in (15). This is remarkable: six unknown form factors are given in terms of one, which, in addition, is known at one kinematic point!

A remarkable theorem by Luke\cite{33} states that even after including corrections of order $1/M_Q$ some form factors are still normalized. The remaining irreducible uncertainty in the
determination of $|V_{cb}|$ is of order $(\Lambda/2m_c)^2 \sim 0.01$, so this should give a determination with precision of a few per-cent. The measurement is complicated by the fact that the decay rate vanishes at $v \cdot v' = 1$, so the measurement needs to be extrapolated. Fortunately, QCD restricts significantly the extrapolation[34] so little uncertainty is introduced.

5.4. Inclusive Semileptonic Decays and Duality
Quark-hadron duality, the imprecise statement that a quantity can be computed directly at the parton level if it is inclusive enough and the energy involved is large enough, had long been thought to hold for decay rates of heavy mesons. However, in fact, the statement

$$\sum_{X_q} \frac{d\Gamma(B \to X_qe\bar{\nu})}{dE dm^2_{e\bar{\nu}}} = \frac{d\Gamma(b \to qe\bar{\nu})}{dE dm^2_{e\bar{\nu}}},$$

with $q = u$ or $c$, is generally not correct. But if we smear over the electron energy things work out[35, 36]:

$$\sum_{X_q} \langle \frac{d\Gamma(B \to X_qe\bar{\nu})}{dE dm^2_{e\bar{\nu}}} \rangle_f = \langle \frac{d\Gamma(b \to qe\bar{\nu})}{dE dm^2_{e\bar{\nu}}} \rangle_f + O\left(\frac{1}{m_b^2}\right),$$

where the smearing is defined by

$$\langle g \rangle_f \equiv \int dEf(E)g(E)$$

with $f$ a smooth function.

The result is obtained by simultaneous short distance (OPE) and heavy quark mass expansions of a Green’s function, the physical rate given by its imaginary part (discontinuity across the cut). One therefore gets, in addition, a means for systematically improving the expansion, order by order in $1/m_b$. We have indicated this in Eq. (18), which shows the remarkable result[36] that there are no corrections at first order in $1/m_b$. Moreover, by choosing the function $f$ appropriately one can find new sum rules. These allow, for example, experimental determination of the unknown parameters that appear at order $1/m_b^2[38]$ by computing moments of the spectrum[37].

5.5. Nailing down $|V_{cb}|$ and $|V_{ub}|$
For semileptonic decays to charm we have seen that our theoretical understanding of both exclusive and inclusive widths is solid. At the time of the Colima conference HFAG was quoting remarkable agreement in the determination of $|V_{cb}|$ by both means:

$|V_{cb}| = \begin{cases} (41.9 \pm 1.1_{\text{exp}} \pm 1.8_{\xi(1)}) \times 10^{-3} \text{ exclusive,} \\ (41.2 \pm 0.7_{\text{exp}} \pm 0.6_{\text{th}}) \times 10^{-3} \text{ inclusive.} \end{cases}$

The updated results can be found in HFAG’s website[39].

The prospect of making a precise determination of $|V_{ub}|$ is not as bright. There is no simple way of determining the form factors. One can either rely on the prospect of future precise lattice computations, or on using symmetry to fix the form factors indirectly. It may be possible to determine $|V_{ub}|$ to about 10% accuracy by comparing the rates for $B \to \rho \ell \nu$, $B \to K^*\ell\bar{\ell}$, $D \to \rho \ell\nu$ and $D \to K^*\ell\bar{\nu}[40]$. The endpoint of the electron/muon energy spectrum in $b \to u\ell\nu$ cannot be reliably described theoretically: the OPE/HQET expansion breaks down (not enough smearing in Eq. (18). One may limit the charm contamination by restricting other kinematic variables. Decay to charm is not allowed for final state hadronic invariant mass $m_X < m_D$, or for final state lepton pair invariant mass square $q^2 > (m_B - m_D)^2$. The best method will most likely involve carving out a region of $m_X$ vs $q^2$ space that minimizes theoretical errors while keeping the charm background under control[41].
6. Factorization, LEET and SCET

Roughly speaking, factorization in $B \to D\pi$ means that the following holds,

$$\langle D\pi|\langle ud\rangle_{V-A} \langle \bar{b}c\rangle_{V-A}|B\rangle \approx \langle D|\langle \bar{b}c\rangle_{V-A}|B\rangle \langle \pi|\langle ud\rangle_{V-A}|0\rangle,$$

\hspace{1cm} \text{(21)}

and can be tested experimentally by comparing $\Gamma(B \to D\pi)$ and $f_{\pi}^2 d\Gamma(B \to D\ell\nu)/dM_{\ell\nu}^2$ at $M_{\ell\nu}^2 = m_{\pi}^2$. Factorization holds in the large $N_c$ expansion of QCD to leading order in $1/N_c$, but this explanation of factorization is a bit too democratic: it gives that factorization holds with the same accuracy for light meson decays, where it is known to fail badly. Bjorken suggested that factorization may hold by color transparency. One can quantify and systematize Bjorken’s ideas by showing that factorization holds to all orders in QCD perturbation theory. One needs to find a proper expansion parameter such that in leading order gluons originating from the $B/D$ system do not couple to the $\pi$ system.

One can easily accomplish this[43]. The idea is to expand in the large energy, $E = (m_B^2 - m_D^2)/2m_B$, released to the light meson. We treat $b$ and $c$ as heavy quarks, expanding in $1/m_b$ while keeping $m_c$ fixed. Then the energy $E$ scales with $m_b$ while the relativistic $\gamma$-factor of the recoiling $D$ meson is fixed and small, $\gamma = v\cdot v' = 1/2[(m_D/m_B) + (m_B/m_D)] \approx 1.6$. Since the velocity of the recoiling $D$ is small, the spectator quark (the light quark bound in the $B$ meson) only requires a soft kick to be incorporated into the final $D$ meson. The two light quarks from the $W$-vector-boson of the weak interaction must produce a single light meson. So the $\bar{u}d$ quark pair is produced with large energy and low invariant mass. This mean they travel in the same light-like direction, and therefore their color charge cancels. This can be made quantitatively precise by introducing an effective theory that systematizes an expansion in $1/E$. The LEET[43] is an effective theory that includes the interactions of soft gluons with the relativistic low mass quark-antiquark pair. Decoupling of the pair from soft gluons is trivial to show in the LEET. Hard gluons do not decouple, but their effects are suppressed by $\alpha_s(m_b)$, so corrections to factorization come in both at order $1/E$ and $\alpha_s(m_b)$. This argument misses the possible effects of collinear gluons, but these are easily incorporated in SCET, an effective theory with both soft and collinear gluons[44].

The LEET/SCET justification for factorization in $B \to D\pi$ is interesting in several respects. First, it is patently different from large $N_c$ arguments. It applies only to decays of a heavy meson to a heavy meson plus a light one. This is good. Empirically, factorization does not hold in $D$ decays. It does not hold in decays of $B$ to two light mesons, but this case is complicated by the presence of penguins. In LEET/SCET factorization the spectator quark needs only a soft kick to join the final state. When the final state has two light mesons the spectator quark needs a hard kick, but hard kicks break factorization (in the sense of Eq. (21) above. To be sure, there is a factorizable contribution to the amplitude, but it is by no means dominant. Second, although the argument was inspired by Bjorken’s color transparency ideas, it is by no means equivalent. Not only does the effective theory provide a systematic approach to the study of factorization (or rather, to violations to factorization) but also it does not justify factorization in some processes for which color transparency applies, like $B \to J/\psi K$. And third, it is predictive and therefore testable. For example, it predicts that the whole amplitude for $B^0 \to D^0\pi^0$ is suppressed by the amount by which factorization is violated,

$$\frac{\mathcal{M}(B^0 \to D^0\pi^0)}{\mathcal{M}(B^0 \to D^-\pi^+)} \sim \frac{1\text{GeV}}{m_B}.$$  \hspace{1cm} \text{(22)}

Experimentally, $\text{Br}(B^0 \to D^0\pi^0)/\text{Br}(B^0 \to D^-\pi^+) = (1.0 \pm 0.3) \times 10^{-1}$ in agreement with expectations.

LEET/SCET factorization in $B \to DX$ holds provided the hadronic system $X$ has small invariant mass. This is verified experimentally in the case when $X$ is a single resonance,
\( X = \pi, \rho, a_1 \). As the invariant mass of \( X \) increases and eventually scales with \( m_b \) the LEET/SCET argument fails. This could be tested with \( X \) a multi-pion final state, by measuring the rate as a function of its invariant mass, \( m_X \)[43]. There is evidence that factorization holds even as a function of \( m_X \)[45]. There is no convincing explanation for this.

7. Duality, CP violation, Conclusions

Time and space limitations preclude me from discussing other subjects which are very interesting. I would like to at least mention two of them:

**Quark-Hadron Duality:** I explained above how quark-hadron duality works in semileptonic decays. There is no analogous treatment for purely hadronic decays and hence for lifetimes. The problem is that there is no external kinematic variable that allows us to consider a Green function, rather than a decay rate. The Green function can be studied at non-physical momenta and is amenable to an OPE. In the absence of an OPE we are left with educated guesses. We can study the question in soluble models. In \( 1 + 1 \) QCD in the large \( N_c \) limit one can compute exactly the total width of a heavy meson. It is found that generally the width (and hence the lifetime) differs from the local quark-hadron duality prediction at order \( 1/m_b \)[46], to be contrasted with the \( 1/m_b^2 \) corrections in the semileptonic case. Moreover and also in contrast with the semileptonic case, there is no systematic theory of these corrections.

**CP violation:** The effort to precisely determine the elements of the CKM matrix complements the measurement of the angles of the unitarity triangle through CP violating asymmetries at \( B \)-factories. The idea is to over-constrain the triangle in an effort to ferret out any hint of new physics hiding under the surface. Bigi and Sanda observed that CP asymmetries through the interference of mixing and decay of neutral \( B \) mesons to states that are CP eigenstates determine unitarity angles cleanly, without contamination from unknown hadronic matrix elements[47]. This works extremely well for determinations of \( \sin(2\beta) \) through, for example, \( B \to J/\psi K_S \). One of the challenges of modern and future \( B \) factories is to determine \( \alpha \) and \( \gamma \) as well. A very unwelcome surprise came several year after Bigi and Sanda’s ground-breaking paper when it was first realized that the determination of \( \sin(2\alpha) \) in, say, \( B \to \pi \pi \), could be compromised by unknown hadronic matrix elements because the effects of penguin diagrams could be significant[48].

**Conclusions** I learned about the discovery of charm, the third generation of leptons and then of \( b \) quarks from Scientific American articles when I was in High School and then in College. It has been a fabulous opportunity to contribute to the understanding of these beasts throughout the time over which this field has evolved. The story is similar to that of electroweak-physics, and one could say that the \( B \) meson is to CKM as the \( Z \) vector-boson is to the SM. With the big caveat that \( B \) physics is still furiously evolving and may still hold some surprises.

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