Efficient CNN with uncorrelated Bag of Features pooling

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Abstract—Despite the superior performance of CNN, deploying them on low computational power devices is still limited as they are typically computationally expensive. One key cause of the high complexity is the connection between the convolution layers and the fully connected layers, which typically requires a high number of parameters. To alleviate this issue, Bag of Features (BoF) pooling has been recently proposed. BoF learns a dictionary, that is used to compile a histogram representation of the input. In this paper, we propose an approach that builds on top of BoF pooling to boost its efficiency by ensuring that the items of the learned dictionary are non-redundant. We propose an additional loss term, based on the pair-wise correlation of the items of the dictionary, which complements the standard loss to explicitly regularize the model to learn a more diverse and rich dictionary. The proposed strategy yields an efficient variant of BoF and further boosts its performance, without any additional parameters.

Index Terms—deep learning, CNN, diversity, bag of features pooling

I. INTRODUCTION

In recent years, Convolutional Neural Networks (CNNs) have significantly advanced many tasks in the computer vision field due to their ability to learn ‘good’ feature representation in an end-to-end manner [1], [2]. However, despite their superior performance across multiple tasks, e.g., image classification [3]–[5], object detection [6]–[8], anomaly detection [9]–[12], deploying CNN-based solutions on low computational power devices, such as mobile phones, is still limited as most of the high-accuracy models are typically computationally expensive [1], [13], [14]. Thus, they are inefficient in terms of time and energy consumption [15]. To alleviate this issue, several approaches have been proposed to reduce the number of parameters required by a CNN model [16]–[20].

The standard CNN model is usually composed of two parts: The first part is formed of convolutional layers typically coupled with max-pooling operations. Then, in the second part, fully connected layers are connected directly to a flattened version of the last convolutional layer output. This connection dramatically increases the total number of parameters, as convolutional layer outputs usually have high dimensionality. Recent approaches mitigate this problem by developing better mechanisms for connecting both parts, e.g., global average pooling and Bag of Features (BoF) pooling.

BoF pooling is a neural extension [18], [19] of the famous Bag-of-Visual Words [21]–[26]. An illustration of a simple BoF-based CNN model is presented in Figure 1. Based on the convolutional output, BoF pooling learns a codebook (dictionary) and outputs a shallow histogram representation of the input. The items of the dictionary are optimized in an end-to-end manner during the standard back-propagation. This yields powerful and efficient models, that achieve a high performance with a low computational footprint. Recently, CNN models, based on BoF pooling, have been used to solve multiple tasks [27]–[32], such as action recognition [30], information retrieval [33], and illumination estimation [34].

In this paper, we propose an approach that builds on top of BoF pooling to boost its efficiency by ensuring that the items of the learned dictionary are non-redundant. Forcing an
uncorrelated structure on the codebook yields a more powerful model which can achieve a high performance with minimal dictionary size. Diversity in deep learning context has been shown to lead to better results [35]–[48]. To this end, we propose to augment the loss of the model to penalize pairwise correlations between the items of the codebook. The proposed technique requires no additional parameters and can be incorporated in any BoF-based CNN model to boost the performance of the CNN model.

The contributions of this paper can be summarized as follows:

- We propose a scheme to avoid redundant items in the dictionary learned by the BoF.
- We propose to augment the CNN-loss to explicitly penalize the pairwise correlations between codebook items and learn rich compressed dictionary.
- The proposed regularizer acts as an unsupervised regularizer on top of the BoF pooling layer and can be integrated into any BoF-based CNN model in a plug-and-play manner.
- The proposed approach is evaluated with three datasets.
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The rest of this paper is organized as follows. First, provide a brief overview of BoF in Section II. In Section III, we present our approach. In Section IV, we empirically evaluate the performance of our method on three different datasets. We conclude the paper in Section V.

II. BAG-OF-FEATURES POOLING

In this section, we briefly describe the BoF pooling mechanism. BoF [18], [19] has been incorporated in a variety of applications and often led to superior results [31], [33], [34]. The BoF pooling is parameterized with a dictionary. Given an input, i.e., the output maps of the last convolutional layer, a histogram representation is compiled based on the dictionary. In the training phase, the items of the dictionary are optimized with the traditional back-propagation. The size of the dictionary is a hyper-parameter that can be adjusted with a validation set to avoid over-fitting.

BoF pooling is formed using two inner layers: a Radial Basis Function (RBF) layer that measures the similarity of the input features to the RBF centers and an accumulation layer that builds a histogram of the quantized feature vectors. Formally, let \( X \) be the input image and \( \rho(X) \in \mathbb{R}^{D \times P} \) the output of the convolutional layer, the RBF layer outputs a sequence of quantized representations:

\[
\Psi = [\psi_1, \psi_2, \cdots, \psi_P] \in \mathbb{R}^{K \times P},
\]

where \( \psi_i \) is the representation corresponding to the \( i^{th} \) feature, i.e.,

\[
\psi_i = [\psi_{i,1}, \cdots, \psi_{i,K}].
\]

The output of the \( i^{th} \) RBF unit is as follows:

\[
\psi_{n,i} = \frac{\exp(-||\rho(X)_n - c_i||/m_i)}{\sum_j \exp(-||\rho(X)_n - c_j||/m_j)},
\]

where \( c_i \) is the center of the \( i^{th} \) RBF neuron, and \( m_i \) is a scaling factor. The outputs of the \( P \) RBF neurons are accumulated in the next layer in order to obtain the final representation \( \Phi \) of each image:

\[
\Phi = \frac{1}{P} \sum_j \psi_j.
\]

To summarize, BoF receives as input a feature representation, usually in high dimension, and quantizes it into a fixed-size shallow histogram representation. The quantization is based on the inner dictionary, \( \{c_1, \cdots, c_K\} \), which can be learned jointly with the rest of the parameters in an end-to-end manner.

III. OUR APPROACH

BoF pooling layer is a key technique that can be used in CNNs to construct powerful models with a low computational cost. The BoF relies on a dictionary, learned during the training, to compute its shallow output. In this paper, we propose an approach that builds on top of BoF pooling to boost its efficiency by explicitly forcing the items of the learned dictionary to be distinct and non-redundant. We propose a simple additional regularizer that penalizes the similarities between the codebook items. This can further boost the performance of the model, without any additional parameters.

The dictionary learned the BoF layer plays a critical role in the global performance of the model. Intuitively, Learning a diverse and rich dictionary yields in a robust codebook and increases the efficiency of the global model. Given a CNN model containing a BoF pooling layer with an inner dictionary \( \{c_1, \cdots, c_K\} \) of size \( K \), the similarity \( SIM \) between two elements \( c_i \) and \( c_j \) of this dictionary can be measured with the squared correlation:

\[
SIM(c_i, c_j) = (corr(c_i, c_j))^2,
\]

where \( corr(\cdot, \cdot) \) is the correlation operator.

Intuitively, \( SIM \) measures how similar two items are. Our goal is to regularize the similarities between the elements of the dictionary. So, the global similarity regularizer can be computed as the sum of the pairwise similarities, i.e.,

\[
\sum_{i \neq j} SIM(c_i, c_j).
\]

Given the original loss \( L \), e.g., least squares or cross entropy, we propose to regularize it as follows:

\[
L_{\text{new}} \triangleq L + \beta \sum_{i \neq j} SIM(c_i, c_j),
\]

where \( L_{\text{new}} \) is the augmented loss and \( \beta \) is a hyper-parameter employed to control the contribution of the supplementary regularizer in the global loss of the model. The computation of the total loss is illustrated in Figure 2.

Setting \( \beta = 0 \) corresponds to the standard BoF case, while a higher \( \beta \) yields a loss dominated by the regularizer. In the training phase, at each step in the back-propagation, the
of 50,000 and 10,000 samples for training and testing, respectively.

2) Training & Testing: In all our experiments, we hold 20% of the training data for validation and hyper-parameter selection. We also experiment with different values for the number of filters in the last convolutional layer. The full topology of the CNN models used in MNIST/fashionMNIST and CIFAR10 experiments are reported in Table I and Table II, respectively.

For MNIST and fashionMNIST experiments, all the models are trained for 50 epochs using Adam [52] regularizer with a 0.001 learning rate and a batch-size of 128. For CIFAR10 experiments, all the models are trained for 200 epochs with standard data augmentation [4] using Adam regularizer with a 0.0001 learning rate and a batch-size of 128.

We report the competitive results of the different pooling strategies, namely global max pooling (GMP) [15], global average pooling (GAP) [15], BoF [18], [19], and our approach. The size of the codebook is a hyperparameter for both BoF and our approach. It is optimized with the validation set from {8, 16, 32, 64, 128} for MNIST and fashionMNIST and from {32, 64, 128, 256} for CIFAR10. The hyper-parameter $\beta$ in Eq. (5), used for controlling the contribution of the proposed regularizer in the global loss, is selected from {0.1, 0.01, 0.001, 0.0001} using the validation set in all experiments.

B. Empirical Results

In Table III and Table IV, we report the average error rates and standard deviations for the different filter sizes, i.e., $C$ in Table I, on MNIST and fashionMNIST datasets, respectively. Compared to standard pooling approaches, i.e., GMP and GAP, we note that both variants of BoF consistently yield a better performance. For the 16 filter case, for example, GMP and GAP reach 3.63% and 4.67% errors on MNIST, respectively, whereas standard BoF and our variant of BoF reach 1.03 and 1.00% for the same case, respectively.
best results achieved by GMP, GAP, and the standard BoF, respectively.

V. CONCLUSION AND FUTURE WORK

In this paper, we proposed a scheme that builds on top of BoF pooling to improve its performance. We proposed a regularizer, based on the pair-wise correlation of the items of the dictionary, which ensures the diversity and the richness of the learned dictionary within the BoF layer. It led to an efficient variant of BoF and further improved its capability, without any additional parameters. The proposed approach can be incorporated in any BoF-based model in a plug-and-play manner. Empirical results over three different dataset showed that the proposed regularizer boosts the performance of the model and led to lower error rates.

Future directions include more extensive experimental evaluation of the proposed approach over larger datasets and proposing more advanced techniques for quantifying the similarities between the codebook elements.

REFERENCES

[1] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton, “Deep learning,” nature, vol. 521, no. 7553, pp. 436–444, 2015.
[2] Ross Girshick, “Fast r-cnn,” in Proceedings of the IEEE international conference on computer vision, 2015, pp. 1440–1448.
[3] Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, “Imagenet classification with deep convolutional neural networks,” Advances in neural information processing systems, vol. 25, 2012.
[4] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun, “Deep residual learning for image recognition,” in Proceedings of the IEEE conference on computer vision and pattern recognition, 2016, pp. 770–778.
[5] Karen Simonyan and Andrew Zisserman, “Very deep convolutional networks for large-scale image recognition,” arXiv preprint arXiv:1409.1556, 2014.
[6] Shaoqing Ren, Kaiming He, Ross Girshick, and Jian Sun, “Faster r-cnn: Towards real-time object detection with region proposal networks,” Advances in neural information processing systems, vol. 28, 2015.
[7] Zhong-Qui Zhao, Peng Zheng, Shou-tao Xu, and Xindong Wu, “Object detection with deep learning: A review,” IEEE transactions on neural networks and learning systems, vol. 30, no. 11, pp. 3212–3232, 2019.
[8] Mingxing Tan, Ruoming Pang, and Quoc V Le, “Efficientdet: Scalable and efficient object detection,” in Proceedings of the IEEE/CVF conference on computer vision and pattern recognition, 2020, pp. 10781–10790.
[9] Guansong Pang, Longbing Cao, and Charu Aggarwal, “Deep learning for anomaly detection: Challenges, methods, and opportunities,” in Proceedings of the 14th ACM International Conference on Web Search and Data Mining, 2021, pp. 1127–1130.
[10] Lukas Ruff, Robert Vandermeulen, Nico Goernitz, Lucas Deecke, Shoaib Ahmed Siddiqui, Alexander Binder, Emmanual Müller, and Marius Kloft, “Deep one-class classification,” in International conference on machine learning. PMLR, 2018, pp. 4393–4402.
[11] Hyunjong Park, Jongyoun Noh, and Bumsub Ham, “Learning memory-guided normality for anomaly detection,” in Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, 2020, pp. 14372–14381.
[12] Taejoon Kim, Sang C Suh, Hyunjoo Kim, Jonghyun Kim, and Jimoh Kim, “An encoding technique for cnn-based network anomaly detection,” in 2018 IEEE International Conference on Big Data (Big Data). IEEE, 2018, pp. 2960–2965.
[13] Jisu Chen and Xukan Ran, “Deep learning with edge computing: A review,” Proceedings of the IEEE, vol. 107, no. 8, pp. 1655–1674, 2019.
[14] Firas Laakom, Jenni Raitoharju, Alexandros Iosifidis, Jarno Nikkanen, and Moncef Gabbouj, “Color constancy convolutional autoencoder,” in 2019 IEEE Symposium Series on Computational Intelligence (SSCI). IEEE, 2019, pp. 1085–1090.
| method   | 16 filters | 32 filters | 64 filters | 128 filters |
|----------|------------|------------|------------|-------------|
| GMP      | 3.63 ± 0.31| 1.97 ± 0.20| 1.39 ± 0.07| 1.09 ± 0.08 |
| GAP      | 4.67 ± 1.17| 2.01 ± 0.09| 1.31 ± 0.05| 1.06 ± 0.02 |
| BoF      | 1.03 ± 0.08| **0.97 ± 0.11**| 1.00 ± 0.08| 1.03 ± 0.06 |
| BoF (ours)| 1.00 ± 0.06| 0.98 ± 0.06| **0.87 ± 0.10**| 0.98 ± 0.08 |

**TABLE III**

AVERAGE ERROR RATES AND STANDARD DEVIATION OF DIFFERENT APPROACHES FOR DIFFERENT NUMBER OF FILTERS IN THE LAST CONVOLUTIONAL LAYER ON THE MNIST DATASET. RESULTS ARE AVERAGED OVER 5 RANDOM SEEDS. TOP RESULTS FOR EACH APPROACH ARE IN BOLD AND BEST GLOBAL RESULT IS UNDERLINED.

| method   | 16 filters | 32 filters | 64 filters | 128 filters |
|----------|------------|------------|------------|-------------|
| GMP      | 14.94 ± 0.70| 12.13 ± 0.30| 10.46 ± 0.18| 9.48 ± 0.12 |
| GAP      | 15.09 ± 0.19| 12.30 ± 0.20| 10.91 ± 0.21| 9.97 ± 0.06 |
| BoF      | 9.55 ± 0.29| 9.44 ± 0.25| 9.04 ± 0.22| **9.02 ± 0.15**|
| BoF (ours)| 9.52 ± 0.29| 9.14 ± 0.12| 8.98 ± 0.18| **8.77 ± 0.22**|

**TABLE IV**

AVERAGE ERROR RATES AND STANDARD DEVIATION OF DIFFERENT APPROACHES FOR DIFFERENT NUMBER OF FILTERS IN THE LAST CONVOLUTIONAL LAYER ON THE FASHION-MNIST DATASET. RESULTS ARE AVERAGED OVER 5 RANDOM SEEDS. TOP RESULTS FOR EACH APPROACH ARE IN BOLD AND BEST GLOBAL RESULT IS UNDERLINED.

| method   | 32 filters | 64 filters | 128 filters |
|----------|------------|------------|-------------|
| GMP      | 22.31 ± 0.48| 20.33 ± 0.67| **18.80 ± 1.03**|
| GAP      | 20.94 ± 0.53| 20.08 ± 0.99| **17.61 ± 0.33**|
| BoF      | 17.15 ± 0.58| 17.05 ± 0.11| **16.10 ± 0.20**|
| BoF (ours)| 17.21 ± 0.71| 16.57 ± 0.25| **15.93 ± 0.14**|

**TABLE V**

AVERAGE ERROR RATES AND STANDARD DEVIATION OF DIFFERENT APPROACHES FOR DIFFERENT NUMBER OF FILTERS IN THE LAST CONVOLUTIONAL LAYER ON THE CIFAR10 DATASET. RESULTS ARE AVERAGED OVER THREE RANDOM SEEDS.

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[15] Ian Goodfellow, Yoshua Bengio, and Aaron Courville, *Deep learning*, MIT press, 2016.

[16] Yuchao Li, Shaohui Lin, Baochang Zhang, Jianzhuan Liu, David Doermann, Yongqian Wu, Feiyue Huang, and Rongrong Ji, “Exploiting kernel sparsity and entropy for interpretable cnn compression,” in *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2019.

[17] Shaohui Lin, Rongrong Ji, Xiaowei Guo, Xue Long Li, et al., “Towards convolutional neural networks compression via global error reconstruction,” in *IJCAI*, 2016, pp. 1753–1759.

[18] Nikolaos Passalis and Anastasios Tefas, “Neural bag-of-features learning,” *Pattern Recognition*, pp. 277–294, 2017.

[19] Nikolaos Passalis and Anastasios Tefas, “Learning bag-of-features pooling for deep convolutional neural networks,” in *IEEE International Conference on Computer Vision*, 2017.

[20] Gaurav Menghani, “Efficient deep learning: A survey on making deep learning models smaller, faster, and better,” *arXiv preprint arXiv:2106.08962*, 2021.

[21] Fei-Fei Li and Pietro Perona, “A bayesian hierarchical model for learning natural scene categories,” in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2005, pp. 36–43.

[22] Gu. Qiu, “Indexing chromatic and achromatic patterns for content-based colour image retrieval,” *Pattern Recognition*, pp. 1675–1686, 2002.

[23] Jun Yang, Yu-Gang Jiang, Alexender G Hauptmann, and Chong-Wah Ngo, “Evaluating bag-of-visual-words representations in scene classification,” in *Proceedings of the international workshop on Workshop on multimedia information retrieval*, 2007, pp. 197–206.

[24] Gabriela Csurka and Florent Perronnin, “Fisher vectors: Beyond bag-of-visual-words image representations,” in *International Conference on Computer Vision, Imaging and Computer Graphics*. Springer, 2010, pp. 28–42.

[25] Jyoti S Shukla, Kriti Rastogi, Hebal Patel, Gaurav Jain, and Shishakt Sharma, “Bag of visual words methodology in remote sensing—a review,” in *Proceedings of the International e-Conference on Intelligent Systems and Signal processing*. Springer, 2022, pp. 475–486.

[26] Yen Zhang, Rong Jin, and Zh-Hua Zhou, “Understanding bag-of-words model: a statistical framework,” *International journal of machine learning and cybernetics*, vol. 1, no. 1, pp. 43–52, 2010.

[27] Nikolaos Passalis and Anastasios Tefas, “Training lightweight deep convolutional neural networks using bag-of-features pooling,” *IEEE transactions on neural networks and learning systems*, vol. 30, no. 6, pp. 1705–1715, 2018.

[28] Kateryna Chumachenko, Alexandros Iosifidis, and Moncef Gabboj, “Self-attention neural bag-of-features,” *arXiv preprint arXiv:2201.11092*, 2022.

[29] Nikolaos Passalis, Anastasios Tefas, Juho Kanniainen, Moncef Gabboj, and Alexandros Iosifidis, “Temporal bag-of-features learning for predicting mid price movements using high frequency limit order book data,” *IEEE Transactions on Emerging Topics in Computational Intelligence*, vol. 4, no. 6, pp. 774–785, 2020.

[30] Alexlndros Iosifidis, Anastasios Tefas, and Ioannis Pitas, “Discriminant bag of words based representation for human action recognition,” *Pattern Recognition Letters*, pp. 185 – 192, 2014.

[31] Nikolaos Passalis, Jenni Raitoharju, Anastasios Tefas, and Moncef Gabboj, “Efficient adaptive inference for deep convolutional neural networks using hierarchical early exits,” *Pattern Recognition*, vol. 105, pp. 107346, 2020.

[32] Dat Thanh Tran, Nikolaos Passalis, Anastasios Tefas, Moncef Gabboj, and Alexandros Iosifidis, “Attention-based neural bag-of-features learning for sequence data,” *arXiv preprint arXiv:2012.12250*, 2020.

[33] Yu-Gang Jiang, Chong-Wah Ngo, and Jun Yang, “Towards optimal bag-of-features for object categorization and semantic video retrieval,” in *ACM International Conference on Image and Video Retrieval*, 2007, pp. 494–501.

[34] Firas Laakom, Nikolaos Passalis, Jenni Raitoharju, Jarno Nikkanen, Anastasios Tefas, Alexandros Iosifidis, and Moncef Gabboj, “Bag of color features for color constancy,” *IEEE Transactions on Image Processing*, vol. 29, pp. 7722–7734, 2020.

[35] Huanhuan Chen, Diversity and regularization in neural network ensembles, Ph.D. thesis, University of Birmingham, 2008.

[36] Mehrdad J Gangeh, Ahmed K Farahat, Ali Ghodsi, and Mohamed S Kamel, “Supervised dictionary learning and sparse representation—a review,” *arXiv preprint arXiv:1502.05928*, 2015.

[37] R Rajesh and Atul Negi, “Heuristic based learning of parameters for dictionaries in sparse representations,” in *2018 IEEE Symposium Series on Computational Intelligence (SSCI)*. IEEE, 2018, pp. 1013–1019.

[38] Babajide O Ayinde, Tamer Iancu, and Jacek M Zurada, “Regularizing deep neural networks by enhancing diversity in feature extraction,” *IEEE transactions on neural networks and learning systems*, vol. 30, no. 9, pp. 2650–2661, 2019.
Yashar Naderhassan, Soosan Beheshti, and Mohammad Ali Tinati, “Correlation based online dictionary learning algorithm,” *IEEE Transactions on Signal Processing*, vol. 64, no. 3, pp. 592–602, 2015.

Hien Van Nguyen, Vishal M Patel, Nasser M Nasrabadi, and Rama Chellappa, “Kernel dictionary learning,” in *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2012, pp. 2021–2024.

Xiao-Yuan Jing, Rui-Min Hu, Fei Wu, Xi-Lin Chen, Qian Liu, and Yong-Fang Yao, “Uncorrelated multi-view discrimination dictionary learning for recognition,” in *Proceedings of the AAAI Conference on Artificial Intelligence*, 2014, vol. 28.

Jiayun Wang, Yubei Chen, Rudrasis Chakraborty, and Stella X Yu, “Orthogonal convolutional neural networks,” in *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, 2020, pp. 11505–11515.

Jure Zbontar, Li Jing, Ishan Misra, Yann LeCun, and Stéphane Deny, “Barlow twins: Self-supervised learning via redundancy reduction,” in *International Conference on Machine Learning*. PMLR, 2021, pp. 12310–12320.

Pengtao Xie, Aarti Singh, and Eric P. Xing, “Uncorrelation and evenness: a new diversity-promoting regularizer,” in *Proceedings of the 34th International Conference on Machine Learning*, 2017, pp. 3811–3820.

Michael Cogswell, Faruk Ahmed, Ross Girshick, Larry Zitnick, and Dhruv Batra, “Reducing overfitting in deep networks by decorrelating representations,” *arXiv preprint arXiv:1511.06068*, 2015.

Firas Laakom, Jenni Raitoharju, Alexandros Iosifidis, and Moncef Gabbouj, “On feature diversity in energy-based models,” in *Energy Based Models Workshop-ICLR 2021*, 2021.

Firas Laakom, Jenni Raitoharju, Alexandros Iosifidis, and Moncef Gabbouj, “Reducing redundancy in the bottleneck representation of the autoencoders,” *arXiv preprint arXiv:2202.04629*, 2022.

Firas Laakom, Jenni Raitoharju, Alexandros Iosifidis, and Moncef Gabbouj, “Within-layer diversity reduces generalization gap,” *arXiv preprint arXiv:2106.06012*, 2021.

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner, “Gradient-based learning applied to document recognition,” *Proceedings of the IEEE*, vol. 86, no. 11, pp. 2278–2324, 1998.

Han Xiao, Kashif Rasul, and Roland Vollgraf, “Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms,” *arXiv preprint arXiv:1708.07747*, 2017.

Alex Krizhevsky, Geoffrey Hinton, et al., “Learning multiple layers of features from tiny images,” 2009.

Diederik P Kingma and Jimmy Ba, “Adam: A method for stochastic optimization,” *arXiv preprint arXiv:1412.6980*, 2014.