Resistive state of superconducting structures with fractal clusters of a normal phase

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The effect of morphologic factors on magnetic flux dynamics and critical currents in percolative superconducting structures is considered. The superconductor contains the fractal clusters of a normal phase, which act as pinning centers. The properties of these clusters are analyzed in the general case of gamma-distribution of their areas. The statistical characteristics of the normal phase clusters are studied, the critical current distribution is derived, and the dependencies of the main statistical parameters on the fractal dimension are found. The effect of fractal clusters of a normal phase on the electric field induced by the motion of the magnetic flux after the vortices have been broken away from pinning centers is considered. The voltage-current characteristics of fractal superconducting structures in a resistive state for an arbitrary fractal dimension are obtained. It is found that the fractality of the boundaries of normal phase clusters intensifies magnetic flux trapping and thereby increases the current-carrying capability of the superconductor.

I. INTRODUCTION

A significant property of clusters of a normal phase embedded in a superconducting medium consists in their capability to trap a magnetic flux and hold the vortices from moving under the action of the Lorentz force. As a result, these clusters can act as effective pinning centers [1] - [4]. This property is widely used in creating composite superconducting materials with high critical current values [5], [6]. The morphologic characteristics of the normal phase clusters essentially affects the dynamics of trapped magnetic flux, especially if the clusters have fractal boundaries [7] - [9]. In the present work we consider in detail the geometric probability properties of these fractal clusters and their influence on the critical current and dynamics of trapped magnetic flux near the transition of the superconductor into a resistive state.

II. FRACTAL GEOMETRY OF NORMAL PHASE CLUSTERS AND DISTRIBUTION OF CRITICAL CURRENTS

Let us consider a superconductor containing fragments of a normal phase. Suppose that dimension of these fragments along one direction significantly exceed the other sizes. Similar columnar defects are of great interest in creating artificial pinning centers [1], [10] - [12]. If such a superconducting structure is cooled in a magnetic field directed along the axis of alignment of these defects below the critical temperature, then the distribution of magnetic flux trapped in the clusters of a normal phase will be two-dimensional. This can be done especially easily with a superconducting film where such clusters are formed near defects at the boundary with the substrate during the growth process and are directed transversely to the film plane [13], [14]. Let us suppose that the film surface fraction covered by the normal phase is below the percolation threshold for the transfer of magnetic flux (i.e., less than 50% for 2D-percolation [15]). In this case the fraction of the superconducting phase exceeds the percolation threshold, so there is a superconducting percolation cluster that can carry a transport current in the film plane. This kind of structure provides for the effective pinning and thereby raises the critical current, for the magnetic flux is trapped in isolated clusters of the normal phase and the vortices cannot leave them without crossing the surrounding superconducting space. As the current increases, the trapped flux will remain unchanged until the vortices begin to break away from those clusters for which the pinning force is less than the Lorentz force caused by the transport current. When the magnetic flux is breaking away from the pinning centers, each vortex must cross the infinite superconducting cluster. In this case the vortices will move primarily along the weak links that connect the clusters of a normal phase to each other [3], [14] - [17]. These weak links form readily in high-temperature superconductors (HTS) characterized by a small coherence length [16]. Structural defects, which could act simply as scattering centers at a large coherence length, create weak links in HTS. There is a large variety of weak links in a wide range of spatial scales in HTS [3], [14] - [18]. At an atomic level
weak links are created by structural point defects, primarily by oxygen vacancies [16], [19]. On a mesoscopic scale weak links are efficiently formed on twin boundaries [13], [16], [18], [20]. Twins can be located at a distance of several tens of nanometers from each other, therefore even a single crystal can have a fine substructure created by twins. Finally, on a macroscopic scale weak links are created by various structural defects, such as boundaries of grains and crystallites or barriers arising from the secondary degrading of a non-stoichiometric crystal into the domains with a high and low oxygen content [17], [19]. Moreover, a magnetic field favors the coherence length to further decrease [21], so weak link formation is facilitated still more. In conventional low-temperature superconductors, which are of large coherence length, weak links are formed due to the proximity effect at sites where the distance between neighboring clusters of a normal phase is minimum.

Thus, regardless of their origin, weak links form channels for vortex transport where the pinning force is less than the Lorentz force caused by the transport current. According to their configuration each cluster of the normal phase has its own value of the depinning current that makes the contribution to the overall statistical distribution of the critical currents. When a transport current is passed through the sample, vortices will be broken away first from clusters of a smaller pinning force and, accordingly, a smaller critical current of depinning. So the change in the trapped magnetic flux will be proportional to the number of the normal phase clusters which have the critical current less than the preset value $I$. Therefore, the relative decrease in the flux can be expressed through the function of cumulative probability $F = F(I)$ for the distribution of critical currents of the clusters:

$$\frac{\Delta \Phi}{\Phi} = F(I)$$

where

$$F(I) = \Pr \{ \forall I_j < I \}$$

The right-hand side of Eq. (1) is the probability that any $j$-th cluster has a critical current $I_j$ less than the given upper bound $I$. On the other hand, the magnetic flux trapped in a single cluster is proportional to its area $A$. Therefore, the decrease in the trapped flux can be expressed through the function of cumulative probability $W = W(A)$ for the distribution of the areas of normal phase clusters, which is proportional to the number of clusters of area smaller than a given value of $A$

$$\frac{\Delta \Phi}{\Phi} = 1 - W(A)$$

where

$$W(A) = \Pr \{ \forall A_j < A \}$$

Generally, the distribution of cluster areas can be described by the gamma-distribution, for which the function of cumulative probability has the form

$$W(A) = \frac{\gamma (g + 1, A/A_0)}{\Gamma (g + 1)}$$

where $\gamma (\nu, z)$ is the incomplete gamma function, $\Gamma (\nu)$ is Euler gamma function, and $A_0$ and $g$ are the parameters of the gamma-distribution that determine the mean area of the normal phase cluster $\overline{A} = (g + 1)A_0$ and its standard deviation $\sigma_A = A_0\sqrt{g + 1}$. The distribution function of cluster areas can be found from the geometric probability analysis of electron photomicrographs of superconducting films [3], [11], [12]. For example, in the practically important case of YBCO films with columnar defects [12] the exponential distribution is realized, which is a special case of the gamma-distribution of Eq. (3) with $g = 0$.

To clarify how the transport current affects the trapped magnetic flux, it is necessary to find the relationship between the distribution of critical currents of clusters of Eq. (1) and the distribution of their areas of Eq. (2). The larger the normal phase cluster size, the more weak links rise from its perimeter bordering with the surrounding superconducting space, and therefore, the smaller is the critical current at which the magnetic flux breaks away from this cluster. Let us suppose that the weak link concentration per unit length of the perimeter is the same for all clusters and all the clusters of equal perimeters have the same pinning force. Then the critical current $I$ is inversely proportional to the perimeter $P$ of the normal phase cluster: $I \propto 1/P$, because the probability to find a weak link over the larger perimeter is higher. It is suggested here that the vortex, driven by the Lorentz force, passes through the weak link from one normal phase cluster to another with the probability of 100%. In this case magnetic flux is transferred through the superconducting medium by Josephson vortices. The probability to trap the vortex when it moves through the weak link under the action of the Lorentz force is negligible, inasmuch as the Josephson penetration depth in the considered materials significantly exceeds the sizes of possible irregularities along the weak
I current of the cluster: proportionality of the critical current of a cluster to its perimeter, we arrive at the following expression for critical current:

\[ I = \alpha A^{D/2} \]

where \( D \) is the fractal dimension of the cluster boundary.

Relation of Eq. (4) agrees with the generalized Euclid theorem [23], according to that the ratios of the corresponding measures are equal when they are reduced to the same dimension. So \( P^{1/D} \propto A^{1/2} \), and this relation holds true both for Euclidean clusters (for which the Hausdorff–Bezikovich dimension of the perimeter is equal to the topological dimension of a line, \( D = 1 \)), and for fractal clusters (for which the Hausdorff–Bezikovich dimension of the boundary strictly exceeds the topological one, \( D > 1 \)).

Note that it is just the statistical distribution of cluster areas, rather than their perimeters, is fundamental for deriving the critical current distribution. Inasmuch as the Hausdorff–Bezikovich dimension of a fractal line exceeds unity, the perimeter of a fractal cluster is not well defined: it diverges when the precision of its measurement increases indefinitely [22]. At the same time, the topological dimension of the cluster area coincides with the Hausdorff–Bezikovich dimension (both are equal to 2). Therefore, the area of a surface confined by the fractal curve is a finite, well-defined quantity.

Analyzing the geometric characteristics of normal phase clusters, we are considering the cross-section of extended columnar defects by the plane the transport current flows through. Therefore, though a normal phase cluster is a self-affine fractal [24], we can consider its geometric probability properties in the planar section only, where the boundary of cluster is statistically self-similar.

Now, using relation of Eq. (4) between the fractal perimeter and area, together with the assumption of inverse proportionality of the critical current of a cluster to its perimeter, we arrive at the following expression for critical current of the cluster: \( I = \alpha A^{D/2} \), where \( \alpha \) is a form factor. Taking into account our initial relations of Eqs. (1) and (2), we can find the distribution of critical currents in the general case of the gamma-distributed cluster areas:

\[ F(i) = \frac{\Gamma(g + 1, Gi^{-2/D})}{\Gamma(g + 1)} \]

where

\[ G = \left( \frac{\theta^D}{\theta^D - (D/2)} \right)^{\theta} \exp \left( \frac{\theta}{\theta^D} \right) \Gamma(\theta + 1, \theta) \]

\( \Gamma(\nu, z) \) is the complementary incomplete gamma function, \( i \equiv I/I_c \) is the dimensionless electric current, and \( I_c = \alpha (A_0 G)^{-D/2} \) is the critical current of the transition into a resistive state. The found function of cumulative probability of Eq. (5) provides a complete description of the effect of the transport current on the trapped magnetic flux. Using this function, we can easily derive the probability density \( f(i) = dF/di \) for the distribution of depinning currents, which has the form

\[ f(i) = \frac{2G^{g+1}}{\Gamma(g + 1)} i^{-\frac{g+1}{2}} \exp \left( -G i^{-2} \right) \]

The probability density is normalized to unity over all possible positive values of the critical currents. The relative change in the trapped flux \( \Delta \Phi/\Phi \), which can be directly found from Equation (5), also determines the density of vortices \( n \) broken away from pinning centers by the current of a given magnitude \( i \)

\[ n(i) = B \Phi_0 \int_0^i f(\tilde{i}') d\tilde{i}' = \frac{B}{\Phi_0} \frac{\Delta \Phi}{\Phi} \]

where \( B \) is the magnetic field, and \( \Phi_0 \equiv hc/(2e) \) is the magnetic flux quantum (\( h \) is Planck constant, \( c \) is the velocity of light, and \( e \) is the electron charge).

Figure 1 shows how \( g \)-parameter of the gamma-distribution affects the distribution of the critical currents of the clusters. As an example, we have taken a value of the fractal dimension \( D = 1.5 \), which is near to the value \( D = 1.44 \pm 0.02 \) found earlier [8] for the normal phase clusters in YBCO film structures. On the other hand, the value \( D = 1.5 \) is intermediate between two limiting cases: \( D = 1 \) for the Euclidean clusters and \( D = 2 \) for the clusters of the most fractality. When the fractal dimension differs from unity so much, the fractal properties of the cluster structure of a superconductor are of great importance. In Fig. 2 the cluster area distribution is presented. The corresponding probability density has the following form.
\[ w(a) = \frac{(g+1)^{g+1}}{\Gamma(g+1)} a^2 e^{-(g+1)a} \]  

(8)

where \( a \equiv A/\mathcal{A} \) is the dimensionless cluster area, with \( \pi = 1 \) and \( \sigma_a = 1/\sqrt{g+1} \). The function of cumulative probability for the dimensionless area is related with function of Eq. (8) by the formula \( W(a) = \int_0^a w(a') da' \) and can be written as

\[ W(a) = \frac{\gamma((g+1),(g+1)a)}{\Gamma(g+1)} \]

As is seen from Fig. 3 when \( g \)-parameter decreases, the distribution \( f = f(i) \) spreads to the right, spanning higher and higher range of critical currents. It might be expected that the highest current-carrying capability of the superconductor can be achieved in the limiting case of \( g = 0 \), when the gamma-distribution of Eq. (8) is reduced to the exponential one \( w(a) = \exp(-a) \). In this case the small clusters give the most contribution into the overall distribution (curve 1 in Fig. 3). For \( g = 0 \) expressions of Eqs. (9) and (10) can be simplified

\[ F(i) = \exp \left( -\left( \frac{2+D}{2} \right)^{\frac{1}{4}} i^{-\frac{1}{4}} \right) \]

(9)

\[ f(i) = \frac{2}{D} \left( \frac{2+D}{2} \right)^{\frac{1}{4}} i^{-1} \exp \left( -\left( \frac{2+D}{2} \right)^{\frac{1}{4}} i^{-\frac{1}{4}} \right) \]

(10)

where, as above, \( i \equiv I/I_c \), and the critical current of the resistive transition can be calculated using a simpler formula:

\[ I_c = (2/(2+D))^{(2+D)/2} \alpha A^{-D/2} \]

Figure 3 shows how the fractal dimension of the cluster boundaries affects the distribution of the critical currents of Eq. (10). As is clearly seen from the figure, with increasing fractal dimension the critical current distribution spreads and shifts to the higher values of current. This shift can be characterized by the dependence of the mean critical current on the fractal dimension, as is shown in Fig. 4. Although the mode of the distribution \( \text{mode} (i) = (2+D)/2 \) is linear with respect to the fractal dimension, the mathematical expectation obeys a much stronger superlinear law controlled by Euler gamma function

\[ \tau = \left( \frac{2+D}{2} \right)^{2+D} \Gamma \left( 1 - \frac{D}{2} \right) \]

(11)

Thus, the geometric probability properties of the normal phase clusters determine the statistical distribution of depinning currents. Using this distribution along with Equation (1), the relative change in the trapped flux under the action of the transport current can be found.

### III. DYNAMICS OF MAGNETIC FLUX TRAPPED IN NORMAL PHASE CLUSTERS WITH FRAC TAL BOUNDARIES

The effect of the transport current on the trapped magnetic flux is demonstrated in Fig. 3. The change in the trapped flux was calculated using formula of Eq. (4) for the exponential-hyperbolic critical current distribution of Eq. (1). Curves 1 and 5 in Fig. 3 correspond to the limiting cases of the Euclidean clusters (\( D = 1 \)) and the clusters of maximum fractality (\( D = 2 \)), respectively; they confine the region of changes in the trapped flux for all possible values of the fractal dimension. The function \( F = F(i) \) reveals one important property; namely, it is very flat near the coordinate origin. It can readily be shown that all its derivatives become zero at the origin point: \( d^kF(0)/di^k = 0 \) for any values of \( k \). Therefore even the Taylor series expansion of the function in the vicinity of the origin converges to zero value but not to the quantity \( F \) itself. This mathematical feature has a clear physical sense: such a small transport current does not affect the trapped magnetic flux, because there are no pinning centers with such small critical currents in the total statistical distribution. The decrease in the magnetic flux becomes noticeable only near the point of a resistive transition (\( i = 1 \)). A use of the exponential-hyperbolic distribution of critical currents of Eq. (9) excludes the uncertainty caused by truncation of non-physical negative values of the depinning currents, which may happen, e.g., in the case of the normal distribution [25], [27].

As is seen from Fig. 3, the breaking of the vortices away is observed mainly at \( i > 1 \), when the sample undergoes a transition into a resistive state. Figure 3 demonstrates the practically important property of a superconducting...
structure with fractal normal phase clusters: the fractality favors magnetic flux trapping, and thereby increases the critical current magnitude below which the sample remains in a superconducting state. Indeed, the transport current of the value \( i = 2 \) causes the 43% of the total trapped magnetic flux to break away from the usual Euclidean clusters \((D = 1, \text{curve 1})\), whereas this value equals only to 13.5% for the fractal normal phase clusters of the greatest possible fractal dimension \((D = 2, \text{curve 5})\). This is equivalent to the pinning increase of 218% in the latter case. The enhancement of pinning due to fractality can be characterized by the pinning gain factor

\[
k_\Phi = 20 \log \frac{\Delta \Phi (D = 1)}{\Delta \Phi (D)}, \text{ dB}
\]

which is equal to the relative decrease in the fraction of the magnetic flux broken away from the fractal clusters of dimension \( D \) in comparison with the Euclidean ones \((D = 1)\). In Fig. 6 the dependencies of the pinning gain factor on the transport current and on the fractal dimension are presented. The maximum pinning gain is achieved when the boundaries of clusters have the greatest possible fractal dimension \((D = 2)\). Note that the pinning gain factor characterizes the properties of the superconductor in the range of transport currents corresponding to a resistive state \((i > 1)\). With smaller currents, the trapped magnetic flux virtually does not change, because there are no pinning centers with such small critical currents (Figs. 1, 3) and the breaking of the vortices away has not started yet. In the presence of a finite resistance any current flow is accompanied by energy dissipation. As for any hard superconductor (type-II, with pinning centers) the energy dissipation in a resistive state does not mean the phase coherence destruction yet. Some dissipation accompanies any motion of the magnetic flux; this effect can be observed in a hard superconductor even for small transport currents. Therefore, the critical current in such materials cannot be determined as the greatest dissipationless current. The superconducting state collapses only when the dissipation increases in an avalanche-like manner as a result of the thermo-magnetic instability.

The reason for the pinning gain caused by the fractality of the cluster boundaries lies in the fundamental properties of the statistical distribution of critical currents (see Fig. 6). In the case of Euclidean clusters the mean value of the critical current calculated in accordance with Eq. (11) is equal to \( i (D = 1) = (3/2)^{1/2} \sqrt{\pi} = 3.2562 \), whereas for the clusters of the most fractality \((D = 2)\) it is infinitely high. As is seen from Fig. 6, when the fractal dimension increases, the contribution of the clusters with high depinning currents to the overall statistical distribution increases, too, which results in the enhancement of magnetic flux trapping.

In a resistive state a hard superconductor can be adequately described by its voltage-current characteristics. Using the fractal distribution of the critical currents of Eq. (6), we can find the electric field caused by magnetic flux motion after the vortices have been broken away from pinning centers. Inasmuch as each cluster of the normal phase contributes to the overall distribution of the critical currents, the voltage across the superconductor \( V = V (i) \) is a response to the sum of effects made by contribution from each cluster. This response can be expressed as a convolution integral

\[
V = R_f \int_0^i (i - i') f(i')di'
\]

(12)

where \( R_f \) is the flux flow resistance. This representation for the voltage across the sample is often used to consider the pinning of bunches of vortex lines in a superconductor [28], as well as to analyze the critical scaling of voltage-current characteristics [22], i.e., in all the cases when the distribution of depinning critical currents occurs. In the following analysis we will focus on the results of the properties of the exponential-hyperbolic distribution in Eq. (11); we will not consider questions associated with the possible dependence of the flux flow resistance \( R_f \) on the transport current.

In the simplest case, when all pinning centers have the same critical current \( i_c \), all vortices are released simultaneously at \( i = i_c \), and their density, in accordance with Eq. (5), has the form

\[
n = \frac{B}{\Phi_0} \int_0^i \delta (i' - i_c) di' = \frac{B}{\Phi_0} h (i - i_c)
\]

where \( \delta (i) \) is the Dirac delta function, and \( h (i) = \begin{cases} 1 & \text{for } i \geq 0 \\ 0 & \text{for } i < 0 \end{cases} \) is the Heaviside function. The trapped flux would change at once by 100% in this case: \( \Delta \Phi / \Phi = h (i - i_c) \).

Thus, accordingly with Equation (12), for the \( \delta \)-like distribution of the critical currents in the regime of viscous flow of the magnetic flux the voltage across the superconductor is controlled by a simple linear dependence \( V = R_f (i - i_c) h (i - i_c) \). The corresponding voltage-current characteristic is shown in Fig. 7 by the dashed line \( a \). In the same figure the voltage-current characteristics of the superconductor in a still simpler approximation of the critical state model is represented, too (the dashed curve \( b \)). In this case the response on any external action, which results in the appearance of an electric field in a hard superconductor, is the flow of the current that is equal to the critical
one $i = i_c$, regardless of the value of the voltage across the sample. (The dimensionless critical current is equal to unity due to the normalization chosen above, $i \equiv I/I_c$.)

For the fractal distribution of the critical currents the situation changes radically, because now the vortices are being broken away in a wide range of transport currents. We shall consider the case of the exponential distribution of the cluster areas ($g = 0$), where the current-carrying capability of the superconductor is maximum. After substitution of the distribution of Eq. (10) into Eq. (12) followed by integration, the voltage across the sample can be expressed through the function of cumulative probability of Eq. (4)

$$V = R_f \int_0^i F(i') \, di'$$

Upon integration we get

$$V = R_f \left[ i \exp \left( -\left( \frac{2 + D}{2} \right)^{D+1} i^{1-D} \right) - \left( \frac{2 + D}{2} \right)^{D+1} \Gamma \left( 1 - \frac{D}{2}, \left( \frac{2 + D}{2} \right)^{D+1} i^{1-D} \right) \right]$$

(13)

In the limiting cases of $D = 1$ and $D = 2$, Equation (13) can be simplified.

For Euclidean clusters ($D = 1$) the voltage across the superconductor has the form

$$V = R_f \left[ i \exp \left( -\frac{3.375}{i^2} \right) - \sqrt{3.375 \pi} \text{erfc} \left( \sqrt{\frac{3.375}{i}} \right) \right]$$

(14)

where the complementary incomplete gamma function is expressed through the complementary error function $\Gamma(1/2, z) = \sqrt{\pi} \text{erfc}(\sqrt{z})$.

For clusters of maximum fractality ($D = 2$), after substitution of the representation for the complementary incomplete gamma function $\Gamma(0, z) = -\text{Ei}(-z)$, Equation (13) for the voltage takes its final form

$$V = R_f \left[ i \exp \left( -\frac{4}{i} \right) + 4 \text{Ei} \left( -\frac{4}{i} \right) \right]$$

(15)

where $\text{Ei}(-z)$ is the exponential integral function.

In Fig. 5 there are shown the voltage-current characteristics of a superconductor containing fractal clusters of a normal phase. For all values of the fractal dimension there is a noticeable voltage drop starting from the transport current of $i = 1$ that coincides with the value found earlier from the critical current distribution of Eqs. (8) and (9) for the resistive transition current. Equations (14) and (15) describe the dependencies of the voltage on the transport current for the limiting values of the fractal dimension. Whatever the geometric probability properties of the normal phase clusters may be, the voltage-current characteristics of a superconductor with these clusters will lie in the region confined by these dependencies (curves 1 and 5 in Fig. 5). As is seen from the figure, the fractality significantly reduces the electric field induced by magnetic flux motion. In Fig. 6 the dependencies of the attenuation factor of dissipation

$$k_V \equiv 20 \log \frac{V(D = 1)}{V(D)} \text{ dB}$$

on the transport current at various values of the fractal dimension are presented. The decrease in the electric field with increasing fractal dimension is especially appreciable in the range of currents $1 < i < 3$, where the pinning gain also has a maximum (Fig. 6). Both of these effects have the same nature, since their reason lies in the peculiarities of the fractal distribution of the depinning currents. As is seen from Figs. 5 and 6 an increase in the fractal dimension leads to considerable spreading of the tail of the distribution $f = f(i)$. This means that more and more clusters of small sizes, which can best trap the magnetic flux, are being involved in the game. As a result, the density of vortices broken away from pinning centers by the Lorentz force decreases, so the smaller part of magnetic flux can flow creating a lower electric field. In turn, the smaller the electric field, the smaller energy is dissipated when the transport current passes through the sample. So the decrease in heat evolution, which could cause a transition into a normal state, leads to an increase in the current-carrying capability of the superconductor containing such fractal clusters.

Thus, Figs. 5-8 clearly demonstrate the most important result: the fractality of the normal phase clusters prevents destruction of the superconductivity by the transport current, and thereby enhances the current-carrying capability of the superconductor. The fractal clusters essentially affect the dynamics of the magnetic flux trapped in the superconductor. The crucial change of the depinning current distribution caused by increasing the fractal dimension of the
clusters is the reason of this effect. The fractality of the boundaries of the normal phase clusters intensifies pinning and slows down destruction of the superconductivity by the transport current. This point provides principally new possibilities for increasing the critical currents of composite superconductors by optimizing their geometric morphological properties.

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FIG. 1. Distribution of the critical currents for various values of $g$-parameter of the gamma-distribution at the fractal dimension of $D = 1.5$. Curve (1) corresponds to the case of $g = 0$, (2) - $g = 1$, (3) - $g = 2$, (4) - $g = 5$, and (5) - $g = 10$. 
FIG. 2. Gamma-distribution of the areas of normal phase clusters. Curve (1) corresponds to $g = 0$, (2) - $g = 1$, (3) - $g = 2$, (4) - $g = 5$, and (5) - $g = 10$. 
FIG. 3. Effect of the fractal dimension of the cluster boundary on the critical current distribution at $g = 0$. Curve (1) corresponds to the case of Euclidean clusters ($D = 1$), curve (2) - to the clusters of fractal dimension $D = 1.25$, (3) - $D = 1.5$, (4) - $D = 1.75$, and (5) - to the clusters of boundaries with maximum fractality ($D = 2$).
FIG. 4. Dependence of the mean critical current on the fractal dimension.
FIG. 5. Effect of a transport current on the magnetic flux trapped in fractal clusters of a normal phase at $g = 0$. Curve (1) corresponds to $D = 1$, (2) - $D = 1.25$, (3) - $D = 1.5$, (4) - $D = 1.75$, (5) - $D = 2$. 
FIG. 6. The pinning gain factor at different values of the fractal dimension. Curve (2) corresponds to $D = 1.25$, (3) - $D = 1.5$, (4) - $D = 1.75$, (5) - $D = 2$. 
FIG. 7. Current-voltage characteristics of fractal superconducting structures at $g = 0$. Curve (1) corresponds to $D = 1$, (2) - $D = 1.25$, (3) - $D = 1.5$, (4) - $D = 1.75$, (5) - $D = 2$. Dashed line $a$ corresponds to viscous flow of a magnetic flux for the case of $\delta$-like distribution of the critical currents, and dashed line $b$ describes the superconductor in a critical state.
FIG. 8. Attenuation factor of dissipation at different values of the fractal dimension. Curve (2) corresponds to $D = 1.25$, (3) - $D = 1.5$, (4) - $D = 1.75$, (5) - $D = 2$. 