Research Article

Efficient Auxiliary Information–Based Control Charting Schemes for the Process Dispersion with Application of Glass Manufacturing Industry

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The hybrid exponentially weighted moving average (HEWMA) control chart is an enhanced version of the EWMA control chart that monitors the process parameters effectively. Similarly, the auxiliary information-based (AIB) EWMA control charts are very efficient for monitoring process parameters. The purpose of this paper is to propose two new control charts for the improved monitoring of process dispersion referred to as HEWMA\(^{(1)}\)AIB and HEWMA\(^{(2)}\)AIB control charts. A simulation study is carried out to assess the performance of the proposed HEWMA\(^{(1)}\)AIB and HEWMA\(^{(2)}\)AIB control charts. Average run length, extra quadratic loss, relative average run length, and performance comparison index are used to compare the performance of the proposed control charts against the existing counterparts. The comparisons reveal the superiority of the proposed control charts against other competing control charts, particularly for small to moderate shifts in the process dispersion. Finally, a real-life data set from the glass industry is used to demonstrate the practical implementation of the proposed control charts.

1. Introduction

There are two types of variations in manufacturing and service processes; common (random) cause variations and special (assignable) cause variations. The common cause variations are an inherent part of every process and cannot be removed entirely. However, the special cause variations are harmful and may distract the processes from their target which results in shifts in the process parameter(s) (location and/or scale). As a result, the practitioner needs to identify and eliminate the assignable cause variations in the process. The statistical control chart is a primary tool in the statistical process control (SPC) toolkit that identifies and rectifies the special cause variations in the process. Memory-less control charts introduced by Shewhart [1] are used to monitor large shifts in the process. On the other hand, the classical memory control charts like cumulative sum (CUSUM) control chart designed by Page [2] and exponentially weighted moving average (EWMA) control chart offered by Roberts [3] are used to monitor small to moderate shifts in the process.

Generally, the classical EWMA control chart has been used to detect small shifts in the process mean. However, in many practical situations, the shifts may also occur in the process variance (or standard deviation); when the process variance increases, the productivity and capability of the process may be damaged. If the process variance decreases, more units will be closer to their target value, resulting in
improved process functionality. If these changes in the process dispersion are not rectified quickly, unnecessary losses may occur. Numerous authors have constructed different EWMA control charts for the process variance. For example, Crowder and Hamilton [4] used a suitable log transformation to $S^2 / \sigma^2_0$, and designed the EWMA control chart for monitoring the process standard deviation, where $\sigma^2_0$ is the in-control (IC) process variance. Following the same lines, Shu and Jiang [5] suggested the new EWMA control chart in which $\ln(S^2 / \sigma^2_0)$ truncated to its IC mean whenever it is less than zero. Similarly, Chang and Gan [6] constructed a one-sided optimal EWMA for monitoring the process variance. Likewise, Khoo [7] introduced AIB EWMA for the process mean, named as EWMA_AIB control chart. Likewise, Haq [27] recommended two new AIB EWMA control charts named as EWMA-1_AIB and EWMA-2_AIB control charts that efficiently monitor the process dispersion. Similarly, Haq [28] provided AIB maximum EWMA control chart for process location and dispersion. Hussain, et al. [29] suggested EWMA control chart based on dual auxiliary information-based estimator for the monitoring of process location. Similarly, Abbasi and Riaz [30] provides the control chart using dual auxiliary information under different ranked set sampling schemes. On the same lines, Riaz, et al. [31] suggested variability control chart using dual auxiliary information-based estimators under different ranked set sampling techniques and different runs rules. Besides, Noor-ul-Amin, et al. [32] suggested the MaxEWMA_AIB control chart for the simultaneous monitoring of the process mean and coefficient of variation. Recently, Anwar, et al. [33] introduced an AIB combined mixed EWMA-CUSUM control chart for joint monitoring of process parameters.

As mentioned before, Ali and Haq [16] proposed the HEWMA control chart for the process dispersion which is more sensitive than the classical EWMA control chart. Sometimes, researchers, engineers, and practitioners are interested in utilizing the features of HEWMA control charts when the original variable carries other information, such as an auxiliary variable, to improve the process’s effectiveness. In this case, the HEWMA control chart will remain inefficient. So, to address this deficiency, this study introduces two auxiliary information-based HEWMA, symbolized as HEWMA_{1_AIB} and HEWMA_{2_AIB} control charts to monitor the small shifts in the process dispersion. To evaluate the performance of the proposed HEWMA_{1_AIB} and HEWMA_{2_AIB} control charts against other control charts, specific performance evaluation measures such as average run length (ARL), extra quadratic loss (EQL), performance comparison index (PCI), and relative ARL (RARL) measures are considered. Besides, an algorithm is designed in R using the Monte Carlo simulations method to calculate the performance evaluation measures. Existing control charts such as HEWMA, adaptive EWMA (AEWMA), HHW1, HHW2, EWMA-1_AIB, and EWMA-2_AIB control charts are considered for comparison. Moreover, the proposed control charts are also implemented with real-life applications to show the significance for practical importance.

The article’s remainder is organized as follows: variable of interest, auxiliary information, transformation based on auxiliary information, and the existing HEWMA control chart are highlighted in Section 2. Section 3 presents the design structure of the proposed HEWMA_{1_AIB} and HEWMA_{2_AIB} control charts. Besides, Section 4 highlights the performance evaluation measures. Furthermore, Section 5 consists of the performance comparison of the proposed HEWMA_{1_AIB} and HEWMA_{2_AIB} control charts against the existing control charts. Similarly, the real-life application of the proposed control charts is provided in Section 6. The last section presents an overall summary and conclusions.
2. Existing Method

This section provides insight into the variable of interest and transformation in Subsection 2.1. Likewise, the methodology of the HEWMA control charts are presented in Subsection 2.2.

2.1. Variable of Interest and Transformation. Suppose $Y$ be normally distributed process variable, that is, $Y \sim N(\mu_Y, \sigma_Y^2)$. It is assumed that over a certain period, the underlying process remains IC with variance $\sigma_Y^2$, but afterwards, it becomes out-of-control (OOC) with variance $\sigma_{Y,t}^2$. Let $\tau = \sigma_{Y,t}/\sigma_Y$ be the amount of shift in process standard deviation $\sigma_Y$. In the case of the IC process, $\tau = 1$ and OOC process, $\tau \neq 1$. Also, $\overline{Y}$ represents the the sample mean and $S_Y^2$ denotes sample variance of the process variable $Y$.

2.2. Transformation. Suppose, $X$ be an auxiliary information variable of $Y$ variable, then $X$ and $Y$ follow a bivariate normal distribution. Suppose $(Y_i, X_i)$, for $i = 1, 2, \ldots, n$ be a random sample of size $n$. Let $X$ and $S_X^2$ be the sample mean and the sample variance of $X$, respectively. According to Garcia and Cebrian [34], the unbiased regression estimator of $\sigma_Y^2$ say $S_Y^2*$, is given by

$$S_Y^2* = S_Y^2 + \rho^2 \left( \frac{\sigma_Y^2}{\sigma_X^2} \right) (\sigma_X^2 - S_X^2). \quad (1)$$

Where $\rho$ is the correlation coefficient. The mean and variance of $S_Y^2*$ are given as $E(S_Y^2*) = \sigma_Y^2$, $V(S_Y^2*) = (2\sigma_Y^2/n - 1)(1 - \rho^2)$.

Similarly, Haq [27] suggested the difference estimator for process dispersion given as

$$D_t = M_{Y,t} - \rho^* M_{X,t}, \quad (2)$$

where $M_Y = \Phi^{-1}(G((n-1)S_Y^2(\sigma_Y^2)))$, $M_X = \Phi^{-1}(G((n-1)S_X^2(\sigma_X^2)))$ and $t$ represents the sample number. Also, $G(\cdot : n-1)$ is the cumulative distribution function (CDF) of chi-square distribution at $n - 1$ degrees of freedom, and $\Phi^{-1}(\cdot)$ denotes the inverse CDF of the standard normal distribution. The $\rho^*$ is the correlation between $M_Y$ and $M_X$. The mean and variance of $D_t$ given by $E(D_t) = 0$ and $V(D_t) = 1 - \rho^*$.

2.3. HEWMA Control Chart for Process Dispersion. Ali and Haq [16] proposed the HEWMA control chart for the monitoring of process dispersion. Let $\{HE_t\}$ for $t \geq 1$ be the sequence of independently and identically distributed (IID) observations based on the other sequence $\{M_{Y,t}\}$, then the plotting statistic $HE_t$, for the HEWMA control chart is defined as:

$$E_t = (1 - \lambda_1)E_{t-1} + \lambda_1 M_{Y,t}, \quad 0 < \lambda_1 \leq 1, \quad (3)$$

$$HE_t = (1 - \lambda_2)HE_{t-1} + \lambda_2 E_t, \quad 0 < \lambda_2 \leq 1, \quad (3)$$

where $HE_0 = E_0 = 0$, and $\lambda_1$ and $\lambda_2$ are smoothing constants. The mean and variance of $HE_t$ are, respectively, given as $E(HE_t) = 0$, and $V(HE_t) = \lambda_1(1-\lambda_1)^2/\lambda_2, \quad \sum_{k=1}^{\infty} ((1-\lambda_2)^k + (1-\lambda_2)^{k+1})/1 - (1 - \lambda_1)^2 (1 - \lambda_2)^2)$. The lower and upper control limits of the HEWMA control chart at the time $t$, are presented as

$$\left\{ \begin{array}{c} UCL_{(HEWMA)} = LCL_{(HEWMA)} = \pm L_{HEWMA} \sqrt{V(HE_t)} \end{array} \right., \quad (4)$$

where $L_{HEWMA}$ control chart coefficient is used to adjust the IC ARL of the HEWMA control chart at a pre-specified desired level. The HEWMA control chart triggers an OOC signal whenever $HE_t$ fall outside of the control limits $\left( UCL_{(HEWMA)}, LCL_{(HEWMA)} \right)$.

3. Proposed Methods

This section contains the methodology of the proposed HEWMA$^{(1)}_{\text{AIB}}$ and HEWMA$^{(2)}_{\text{AIB}}$ control charts for monitoring the process dispersion. Subsection 3.1 covers the design structure of the proposed HEWMA$^{(1)}_{\text{AIB}}$ control chart, whereas, the HEWMA$^{(2)}_{\text{AIB}}$ control chart are given in Subsection 3.2.

3.1. HEWMA$^{(1)}_{\text{AIB}}$ Control Chart. Let $\{S_{Y,t}^{2*}\}$ for $t \geq 1$ be the sequence of IID random variables, then the plotting statistic $H_{1,t}$, for HEWMA$^{(1)}_{\text{AIB}}$ control chart using the recurrence formula, given by

$$H_{1,t} = (1 - \lambda_2)H_{1,t-1} + \lambda_2 E_{1,t}, \quad H_{1,0} = \sigma_Y^2. \quad (5)$$

Here $\lambda_1$ and $\lambda_2 \in (0, 1)$ are smoothing constants. The mean and variance of the $H_{1,t}$ can be given by the expression as

$$E(H_{1,t}) = \sigma_Y^2 \quad \text{and} \quad V(H_{1,t}) = \left( (1 - \rho^2)^2 \right) \left( \sum_{k=1}^{\infty} \left( \sum_{k=1}^{\infty} ((1 - \lambda_2)^k + (1 - \lambda_2)^{k+1})/1 - (1 - \lambda_1)^2 (1 - \lambda_2)^2 \right) \right) \quad (6)$$

where $L_{HEWMA}^{(1)_{\text{AIB}}}$ is the coefficient for the HEWMA$^{(1)}_{\text{AIB}}$ control chart at a pre-specified false alarm rate. The $H_{1,t}$ statistic is plotted against the $UCL_{(HEWMA)_{\text{AIB}}}^{(1)}$ and $LCL_{(HEWMA)_{\text{AIB}}}^{(1)}$. The process is considered to be OOC when $H_{1,t} > UCL_{(HEWMA)_{\text{AIB}}}^{(1)}$ or $H_{1,t} < LCL_{(HEWMA)_{\text{AIB}}}^{(1)}$; otherwise, it is IC.

3.2. HEWMA$^{(2)}_{\text{AIB}}$ Control Chart. Let $\{D_t\}$ for $t \geq 1$ be the sequence of IID random variables, based on $\{D_t\}$, we defined a new sequence $H_{2,*}$, using the recurrence formula, given by
\[ E_{2,t} = \lambda_1 D_t + (1 - \lambda_1) E_{2,t-1}, \quad E_{2,0} = 0, \]
\[ H_{2,t} = (1 - \lambda_2) H_{2,t-1} + \lambda_2 E_{2,t}, \quad H_{2,0} = 0. \]

Here \( \lambda_1 \) and \( \lambda_2 \in (0,1) \) are smoothing constants. The mean and variance of \( H_{2,t} \) can be given by the expression as 
\[ E(H_{2,t}) = 0 \quad \text{and} \quad V(H_{2,t}) = (1 - \rho^*)(\lambda_1 \lambda_2/\lambda_1 - \lambda_2)^2 \left\{ \frac{1}{1 - \lambda_2^2} \left[ (1 - \lambda_2)^2 - (1 - \lambda_2^2) \right] / (1 - \lambda_2) - (2 - 1 - \lambda_2) \left[ (1 - 1 - \lambda_2)^2 / (1 - \lambda_2) - (1 - \lambda_2) \right] \right\}. \]

The control limits for the HEWMA\(_2\) AIB control chart are given by
\[ \text{LCL}_{\text{HEWMA}(2)\text{AIB}} = \pm L_{\text{HEWMA}(2)\text{AIB}} \sqrt{V(H_{2,t})}, \]
\[ \text{UCL}_{\text{HEWMA}(2)\text{AIB}} = \text{LCL}_{\text{HEWMA}(2)\text{AIB}}. \]

The parameters of the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts are \( \lambda_1, \lambda_2, \) and \( \rho \). Several settings of these parameters are used, and hence corresponding ARL, and standard deviation of RL (SDRL) are computed. The values of \( (\lambda_1, \lambda_2) \) are set as \((0.05, 0.05), (0.05, 0.05), (0.05, 0.05), (0.1, 0.05), (0.05, 0.2), (0.1, 0.2), (0.2, 0.2), (0.2, 0.1), (0.05, 0.2), (0.2, 0.2), (0.3, 0.05), (0.3, 0.1), \) and \((0.3, 0.2)\) and the values of \( L_{\text{HEWMA}(1)\text{AIB}} \) and \( L_{\text{HEWMA}(2)\text{AIB}} \) are determined to obtain ARL\(_0 = 200\). The different settings of \( \rho \) and \( \rho^* \) are taken from the Haq [28] given as \((\rho, \rho^*) = (0.25, 0.0563898), (0.50, 0.2293317), (0.75, 0.5313626), (0.90, 0.7870992), (0.95, 0.8880799). \) The numerical results of the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts are presented in Tables 1–8.

4. Performance Evaluation Measures

Quality experts use different performance evaluation measures to evaluate the control charts’ performance. ARL is mostly used for a single shift, while EQL, RARL, and PCI are used to assess the overall performance of control charts. An algorithm is developed in R software, and the Monte Carlo simulation technique is used to compute the numerical results. Monte Carlo simulation with 20000 iterations is performed at each shift \( \tau \), where \( \tau = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, \) and 2.0. The details of these performance measures are given in the following subsections.

4.1. ARL Measure.

The ARL is defined as the average number of sample points plotted until the OOC signal is detected. The ARL is categorized as IC ARL (ARL\(_0\)) and OOC ARL (ARL\(_1\)). If the process is IC state, the ARL\(_0\) needed to be large enough to avoid frequent false alarms. However, the ARL\(_1\) should be small enough that it quickly detects the shift(s) in the process parameters. It is necessary for the better performance of the control chart that it should have a smaller ARL\(_1\) with fixed ARL\(_0\) at the desired level.

4.2. Overall Performance Measures.

The EQL, RARL, and PCI performance evaluation measures evaluate a control chart’s overall effectiveness by comparison method. The EQL evaluates the overall performance of control charts over a specific range of shifts (Raza, et al. [35]). It is based on the loss function and is defined as:
\[ \text{EQL} = (\tau_{\text{max}} - \tau_{\text{min}})^{-1} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \tau^2 \text{ARL}(\tau) \, d\tau, \]
where ARL\(_\tau\) is the ARL of a particular control chart at shift \( \tau \). The EQL is a weighted average ARL over the entire shift domain \( \tau_{\text{min}} < \tau < \tau_{\text{max}} \) using the square of shift \( (\tau^2) \) as weight. A control chart with a minimum EQL value is preferred over other control charts (Anwar, et al. [36]).

The RARL is the average of the ratios among the ARL of a particular control chart with the ARL of a benchmark control chart for all desired shifts.
\[ \text{RARL} = (\tau_{\text{max}} - \tau_{\text{min}})^{-1} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \frac{\text{ARL}(\tau)}{\text{ARL}_{\text{BM}}(\tau)} \, d\tau, \]
where ARL\(_\tau\) and ARL\(_{\text{BM}}(\tau)\) symbolize the ARL of a particular control chart and a benchmark control chart for the desired shift, respectively. The benchmark control chart is the control chart with the least EQL. The RARL value for the benchmark control chart is one, and for the other control charts, it is greater than 1.

The PCI evaluates the performance of the best control chart. It is defined as the ratio between the EQL of a control chart and the EQL of the benchmark control chart.
\[ \text{PCI} = \frac{\text{EQL}_{\text{BM}}}{\text{EQL}}. \]

where EQL\(_{\text{BM}}\) is the EQL of the best-performing control chart. The PCI for the benchmark control chart is 1, while the other control chart’s PCI is greater than 1.

4.3. Choices of Parameters.

The parameters of the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts are \( \lambda_1, \lambda_2, \) and \( \rho \). Several settings of these parameters are used, and hence corresponding ARL, and standard deviation of RL (SDRL) are computed. The values of \( (\lambda_1, \lambda_2) \) are set as \((0.05, 0.05), (0.05, 0.05), (0.05, 0.05), (0.1, 0.05), (0.05, 0.2), (0.1, 0.2), (0.2, 0.2), (0.2, 0.1), (0.2, 0.2), (0.3, 0.05), (0.3, 0.1), \) and \((0.3, 0.2)\) and the values of \( L_{\text{HEWMA}(1)\text{AIB}} \) and \( L_{\text{HEWMA}(2)\text{AIB}} \) are determined to obtain ARL\(_0 = 200\). The different settings of \( \rho \) and \( \rho^* \) are taken from the Haq [28] given as \((\rho, \rho^*) = (0.25, 0.0563898), (0.50, 0.2293317), (0.75, 0.5313626), (0.90, 0.7870992), (0.95, 0.8880799). \) The numerical results of the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts are presented in Tables 1–8.

5. Evaluation and Performance Comparison

This section provides extensive comparisons of the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts with the HEWMA (Ali and Haq [16]), AEWMA (Haq [37]), HHW1 and HHW2 (Huwang, et al. [11]), and EWMA-I\(_{AIB}\) and EWMA-II\(_{AIB}\) (Haq [27]) control charts.

5.1. Proposed versus HEWMA Control Chart.

The proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts are compared with the HEWMA control chart. The proposed control charts outperform the HEWMA control chart. For instance, at ARL\(_0 = 200, \lambda_2 = 0.1, \lambda_1 = 0.5, \) and \( \tau = 1.1, 1.2, \) the proposed HEWMA\(_1\) and HEWMA\(_2\) AIB control charts
Table 1: Run-length profile of two-sided HEWMA\(^{(1)}\) control chart at ARL\(_0 \approx 200\).

| \(\rho\) | \(\lambda_1 = 0.05\), \(\lambda_2 = 0.05\) | \(\lambda_1 = 0.05\), \(\lambda_2 = 0.10\) | \(\lambda_1 = 0.05\), \(\lambda_2 = 0.20\) |
|---|---|---|---|
| | \(L_{\text{HEWMA}}^{(1)}\) AIB | \(L_{\text{HEWMA}}^{(1)}\) AIB | \(L_{\text{HEWMA}}^{(1)}\) AIB |
| \(\tau\) | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
| 0.5 | 3.41 | 1.02 | 3.28 | 1.14 | 2.71 | 1.29 | 1.93 | 1.08 | 1.54 | 0.82 |
| 0.6 | 4.69 | 1.90 | 4.48 | 1.93 | 3.58 | 1.87 | 2.38 | 1.43 | 1.79 | 1.05 |
| 0.7 | 7.28 | 3.80 | 6.91 | 3.69 | 5.41 | 3.14 | 3.32 | 2.15 | 2.31 | 1.51 |
| 0.8 | 13.67 | 8.88 | 13.02 | 8.58 | 10.12 | 6.82 | 5.90 | 4.16 | 3.73 | 2.69 |
| 0.9 | 38.48 | 32.49 | 36.60 | 30.96 | 28.83 | 24.02 | 17.10 | 13.80 | 10.54 | 8.47 |
| 1.0 | 200.63 | 237.56 | 200.24 | 238.26 | 200.61 | 239.81 | 200.47 | 243.01 | 200.86 | 244.70 |
| 1.1 | 33.98 | 35.79 | 32.46 | 34.08 | 25.89 | 26.97 | 15.70 | 15.74 | 9.85 | 9.63 |
| 1.2 | 12.39 | 12.68 | 11.66 | 11.94 | 9.02 | 9.24 | 5.62 | 5.51 | 3.62 | 3.31 |
| 1.3 | 6.73 | 6.86 | 6.47 | 6.57 | 5.02 | 5.01 | 3.18 | 2.90 | 1.15 | 1.77 |
| 1.4 | 4.39 | 4.39 | 4.19 | 4.15 | 3.35 | 3.16 | 2.22 | 1.85 | 1.67 | 1.15 |
| 1.5 | 3.20 | 3.05 | 3.10 | 2.93 | 2.54 | 2.26 | 1.77 | 1.29 | 1.41 | 0.82 |
| 1.6 | 2.54 | 2.29 | 2.45 | 2.20 | 2.06 | 1.70 | 1.52 | 0.99 | 1.29 | 0.64 |
| 1.7 | 2.10 | 1.75 | 2.03 | 1.68 | 1.78 | 1.35 | 1.37 | 0.78 | 1.19 | 0.50 |
| 1.8 | 1.85 | 1.43 | 1.78 | 1.35 | 1.57 | 1.07 | 1.28 | 0.64 | 1.15 | 0.43 |
| 1.9 | 1.65 | 1.19 | 1.61 | 1.13 | 1.44 | 0.90 | 1.21 | 0.54 | 1.12 | 0.37 |
| 2.0 | 1.52 | 1.01 | 1.49 | 0.95 | 1.36 | 0.79 | 1.17 | 0.47 | 1.09 | 0.31 |

\(\lambda_1 \approx 0.05\), \(\lambda_2 \approx 0.10\), \(\lambda_1 \approx 0.10\)
Table 1: Continued.

| \( \rho \) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|-------|-----|-----|-----|-----|-----|

| \( \lambda_1 = 0.10, \lambda_2 = 0.05 \) | \( I_{1.8211}^{\text{HEWMA}}(\lambda) \) | \( I_{1.8255}^{\text{HEWMA}}(\lambda) \) | \( I_{1.835}^{\text{HEWMA}}(\lambda) \) | \( I_{1.8406}^{\text{HEWMA}}(\lambda) \) | \( I_{1.8405}^{\text{HEWMA}}(\lambda) \) |
|-----|-----|-----|-----|-----|-----|

| \( \tau \) | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.5 | 3.89 | 1.03 | 3.72 | 1.18 | 3.04 | 1.38 | 2.10 | 1.17 | 1.63 | 0.89 |
| 0.6 | 5.27 | 1.92 | 5.05 | 1.98 | 4.03 | 1.96 | 2.60 | 1.52 | 1.91 | 1.13 |
| 0.7 | 8.09 | 3.82 | 7.68 | 3.75 | 6.02 | 3.24 | 3.65 | 2.26 | 2.51 | 1.60 |
| 0.8 | 15.03 | 8.98 | 14.40 | 8.63 | 11.08 | 6.90 | 6.48 | 4.26 | 4.08 | 2.80 |
| 0.9 | 42.62 | 34.84 | 40.34 | 32.99 | 31.97 | 25.48 | 18.54 | 14.18 | 11.32 | 8.62 |
| 1.0 | 200.88 | 227.13 | 200.82 | 227.99 | 200.73 | 228.04 | 200.16 | 228.14 | 200.20 | 230.48 |
| 1.1 | 35.34 | 36.33 | 33.68 | 34.69 | 26.92 | 19.19 | 5.89 | 5.52 | 3.84 | 3.39 |
| 1.2 | 12.82 | 12.65 | 12.28 | 12.08 | 9.46 | 9.19 | 5.89 | 5.52 | 3.84 | 3.39 |
| 1.3 | 6.99 | 6.86 | 6.63 | 6.50 | 5.27 | 5.06 | 3.36 | 3.01 | 2.29 | 1.83 |
| 1.4 | 4.58 | 4.41 | 4.40 | 4.21 | 3.53 | 3.25 | 2.34 | 1.90 | 1.72 | 1.19 |
| 1.5 | 3.33 | 3.12 | 3.21 | 2.96 | 2.65 | 2.31 | 1.83 | 1.34 | 1.46 | 0.86 |
| 1.6 | 2.60 | 2.27 | 2.53 | 2.21 | 2.15 | 1.76 | 1.57 | 1.04 | 1.32 | 0.68 |
| 1.7 | 2.19 | 1.82 | 2.12 | 1.72 | 1.82 | 1.37 | 1.41 | 0.81 | 1.21 | 0.53 |
| 1.8 | 1.91 | 1.48 | 1.85 | 1.39 | 1.62 | 1.12 | 1.30 | 0.68 | 1.17 | 0.46 |
| 1.9 | 1.70 | 1.23 | 1.65 | 1.17 | 1.49 | 0.95 | 1.23 | 0.58 | 1.13 | 0.39 |
| 2.0 | 1.56 | 1.06 | 1.53 | 1.01 | 1.39 | 0.80 | 1.18 | 0.49 | 1.10 | 0.33 |

\[ \lambda_1 = 0.10, \lambda_2 = 0.10 \]

| \( \lambda_1 = 0.10, \lambda_2 = 0.20 \) | \( I_{1.9664}^{\text{HEWMA}}(\lambda) \) | \( I_{1.970}^{\text{HEWMA}}(\lambda) \) | \( I_{1.983}^{\text{HEWMA}}(\lambda) \) | \( I_{1.991}^{\text{HEWMA}}(\lambda) \) | \( I_{1.993}^{\text{HEWMA}}(\lambda) \) |
|-----|-----|-----|-----|-----|-----|
| \( \tau \) | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.5 | 4.39 | 1.07 | 4.21 | 1.22 | 3.40 | 1.49 | 2.29 | 1.27 | 1.74 | 0.97 |
| 0.6 | 5.92 | 1.95 | 5.66 | 2.03 | 4.50 | 2.07 | 2.85 | 1.64 | 2.05 | 1.22 |
| 0.7 | 8.99 | 3.90 | 8.54 | 3.81 | 6.68 | 3.35 | 4.00 | 2.37 | 2.71 | 1.71 |
| 0.8 | 16.61 | 9.46 | 15.82 | 9.12 | 12.30 | 7.20 | 7.05 | 4.42 | 4.46 | 2.94 |
| 0.9 | 49.77 | 41.40 | 47.20 | 38.75 | 36.24 | 29.01 | 20.42 | 15.29 | 12.32 | 8.91 |
| 1.0 | 200.09 | 216.15 | 200.79 | 217.67 | 200.16 | 219.18 | 200.52 | 222.13 | 200.92 | 222.16 |
| 1.1 | 36.67 | 37.22 | 35.09 | 35.41 | 28.48 | 28.08 | 17.53 | 16.34 | 11.09 | 9.93 |
| 1.2 | 13.39 | 12.79 | 12.77 | 12.15 | 9.96 | 9.28 | 6.29 | 5.65 | 4.05 | 3.44 |
| 1.3 | 7.26 | 6.89 | 6.94 | 6.58 | 5.54 | 5.12 | 3.52 | 3.06 | 2.41 | 1.92 |
| 1.4 | 4.77 | 4.45 | 4.60 | 4.25 | 3.74 | 3.37 | 2.41 | 1.95 | 1.80 | 1.26 |
| 1.5 | 3.44 | 3.11 | 3.33 | 3.02 | 2.76 | 2.38 | 1.91 | 1.41 | 1.51 | 0.91 |
| 1.6 | 2.74 | 2.38 | 2.62 | 2.25 | 2.22 | 1.80 | 1.62 | 1.07 | 1.34 | 0.71 |
| 1.7 | 2.27 | 1.85 | 2.21 | 1.79 | 1.89 | 1.42 | 1.44 | 0.86 | 1.25 | 0.57 |
| 1.8 | 1.97 | 1.54 | 1.90 | 1.45 | 1.68 | 1.16 | 1.33 | 0.70 | 1.18 | 0.48 |
| 1.9 | 1.75 | 1.27 | 1.70 | 1.21 | 1.52 | 0.97 | 1.25 | 0.60 | 1.14 | 0.40 |
| 2.0 | 1.60 | 1.08 | 1.57 | 1.05 | 1.42 | 0.84 | 1.21 | 0.52 | 1.10 | 0.35 |

\[ \lambda_1 = 0.10, \lambda_2 = 0.20 \]
Table 2: Run-length profile of two-sided HEWMA\((1)\) control chart at ARL\(_n = 200\).

| \(\rho\) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---------|------|------|------|------|------|
| \(\lambda_1 = 0.20\), \(\lambda_2 = 0.05\) |
| \(\lambda_1\) | 1.952 | 1.957 | 1.9672 | 1.9751 | 1.9755 |
| \(\tau\) | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
| 0.5 | 4.28 | 1.04 | 4.10 | 1.20 | 3.32 | 1.45 | 2.26 | 1.24 | 1.72 | 0.95 |
| 0.6 | 5.76 | 1.91 | 5.51 | 1.98 | 4.37 | 2.01 | 2.79 | 1.60 | 2.02 | 1.19 |
| 0.7 | 8.71 | 3.78 | 8.32 | 3.73 | 6.48 | 3.26 | 3.91 | 2.31 | 2.65 | 1.67 |
| 0.8 | 16.07 | 9.21 | 15.34 | 8.84 | 11.86 | 7.00 | 8.66 | 4.33 | 4.34 | 2.85 |
| 0.9 | 46.50 | 37.50 | 44.09 | 35.31 | 34.43 | 26.74 | 19.68 | 14.60 | 11.94 | 8.66 |
| 1.0 | 200.14 | 219.48 | 200.23 | 221.63 | 200.88 | 222.78 | 201.02 | 224.93 | 200.15 | 224.44 |
| 1.1 | 35.45 | 35.95 | 34.01 | 34.50 | 27.48 | 27.19 | 16.83 | 15.82 | 10.71 | 9.66 |
| 1.2 | 12.89 | 12.42 | 11.88 | 9.61 | 9.02 | 6.11 | 3.45 | 3.00 | 2.38 | 1.87 |
| 1.3 | 7.08 | 6.73 | 6.74 | 6.40 | 5.39 | 4.98 | 3.45 | 3.00 | 2.38 | 1.87 |
| \(\lambda_1 = 0.20\), \(\lambda_2 = 0.10\) |
| \(\lambda_1\) | 2.112 | 2.114 | 2.1312 | 2.143 | 2.145 |
| \(\tau\) | ARL | SDRL | ARL | SDRL | ARL | SDRL |
| 0.5 | 4.85 | 1.07 | 4.63 | 1.24 | 3.73 | 1.54 | 2.47 | 1.35 | 1.84 | 1.03 |
| 0.6 | 6.48 | 1.96 | 6.18 | 2.04 | 4.89 | 2.12 | 3.09 | 2.18 | 2.18 | 1.28 |
| 0.7 | 9.78 | 4.00 | 9.30 | 3.91 | 7.21 | 3.44 | 4.32 | 2.44 | 2.90 | 1.78 |
| 0.8 | 18.53 | 17.56 | 17.36 | 10.16 | 13.44 | 7.79 | 7.61 | 4.57 | 4.77 | 3.02 |
| 0.9 | 60.10 | 51.47 | 56.51 | 48.34 | 42.42 | 35.00 | 22.70 | 17.21 | 13.31 | 9.49 |
| 1.0 | 200.90 | 211.43 | 200.89 | 213.66 | 200.76 | 214.83 | 200.78 | 216.18 | 200.09 | 215.64 |
| 1.1 | 36.95 | 37.48 | 35.65 | 35.57 | 29.37 | 28.82 | 18.26 | 16.79 | 11.59 | 10.11 |
| 1.2 | 13.59 | 12.85 | 13.24 | 12.24 | 10.25 | 9.36 | 6.51 | 5.65 | 4.22 | 3.46 |
| 1.3 | 7.44 | 6.84 | 7.05 | 6.52 | 5.69 | 5.06 | 3.64 | 3.04 | 2.50 | 1.94 |
| 1.4 | 4.86 | 4.38 | 4.67 | 4.18 | 3.82 | 3.35 | 2.50 | 1.98 | 1.86 | 1.28 |
| \(\lambda_1 = 0.20\), \(\lambda_2 = 0.20\) |
| \(\lambda_1\) | 2.264 | 2.269 | 2.289 | 2.3029 | 2.308 |
| \(\tau\) | ARL | SDRL | ARL | SDRL |
| 0.5 | 5.33 | 1.09 | 5.11 | 1.27 | 4.08 | 1.61 | 2.66 | 1.42 | 1.96 | 1.09 |
| 0.6 | 7.11 | 2.07 | 6.79 | 2.13 | 5.36 | 2.20 | 3.34 | 2.34 | 3.34 | 1.36 |
| 0.7 | 10.98 | 4.66 | 10.44 | 4.51 | 8.01 | 3.76 | 4.66 | 2.56 | 3.11 | 1.86 |
| 0.8 | 23.27 | 15.49 | 21.79 | 14.25 | 15.73 | 9.92 | 8.35 | 5.06 | 5.13 | 3.15 |
| 0.9 | 91.32 | 84.13 | 84.04 | 77.20 | 58.15 | 51.79 | 27.54 | 22.55 | 15.11 | 11.32 |
| 1.0 | 200.51 | 205.99 | 200.00 | 205.80 | 200.79 | 207.33 | 200.97 | 209.96 | 200.88 | 209.26 |

| \(\lambda_1\) | 1.952 | 1.957 | 1.9672 | 1.9751 | 1.9755 |
| \(\lambda_1\) | 2.112 | 2.114 | 2.1312 | 2.143 | 2.145 |
| \(\lambda_1\) | 2.264 | 2.269 | 2.289 | 2.3029 | 2.308 |

\(\rho\) values are the proportion of false alarms expected when the process is in control.

\(\lambda\) values are the in-control ARLs.
| $\rho$ | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|-------|------|------|------|------|------|
| $\lambda_1 = 0.30$, $\lambda_2 = 0.05$ |      |      |      |      |      |
| **^{1}_{HEWMA}(10)** | 2.029 | 2.030 | 2.042 | 2.051 | 2.053 |
| $\tau$ | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
| 0.5 | 4.47 | 1.03 | 4.28 | 1.19 | 3.45 | 1.46 | 2.33 | 1.26 | 1.77 | 0.97 |
| 0.6 | 5.99 | 1.89 | 5.72 | 1.96 | 4.54 | 2.01 | 2.88 | 1.61 | 2.07 | 1.21 |
| 0.7 | 9.03 | 3.83 | 8.57 | 3.73 | 6.68 | 3.27 | 4.02 | 2.32 | 2.73 | 1.68 |
| 0.8 | 16.68 | 9.35 | 15.84 | 8.98 | 12.26 | 7.13 | 7.01 | 4.34 | 4.44 | 2.86 |
| 0.9 | 48.90 | 39.23 | 46.20 | 36.73 | 35.58 | 27.54 | 20.22 | 14.89 | 12.21 | 8.78 |
| 1.0 | 200.92 | 219.58 | 200.11 | 219.30 | 200.28 | 219.35 | 200.96 | 224.86 | 200.48 | 222.50 |
| 1.1 | 35.24 | 35.77 | 33.84 | 34.31 | 27.53 | 27.03 | 16.87 | 15.73 | 10.80 | 9.57 |
| 1.2 | 12.83 | 12.36 | 12.28 | 11.72 | 9.62 | 8.94 | 6.13 | 5.42 | 3.99 | 3.32 |
| 1.3 | 7.03 | 6.61 | 6.73 | 6.31 | 5.36 | 4.89 | 3.47 | 2.95 | 2.40 | 1.85 |
| 1.4 | 4.67 | 4.28 | 4.45 | 4.04 | 3.66 | 3.18 | 2.40 | 1.87 | 1.79 | 1.22 |
| 1.5 | 3.41 | 3.01 | 3.30 | 2.89 | 2.73 | 2.28 | 1.91 | 1.38 | 1.52 | 0.91 |
| 1.6 | 2.68 | 2.23 | 2.61 | 2.17 | 2.21 | 1.74 | 1.63 | 1.05 | 1.34 | 0.70 |
| 1.7 | 2.26 | 1.81 | 2.18 | 1.71 | 1.89 | 1.38 | 1.45 | 0.85 | 1.25 | 0.58 |
| 1.8 | 1.96 | 1.47 | 1.90 | 1.39 | 1.68 | 1.15 | 1.34 | 0.70 | 1.19 | 0.49 |
| 1.9 | 1.76 | 1.26 | 1.70 | 1.18 | 1.53 | 0.95 | 1.26 | 0.60 | 1.14 | 0.41 |
| 2.0 | 1.59 | 1.05 | 1.57 | 1.02 | 1.41 | 0.82 | 1.20 | 0.51 | 1.11 | 0.35 |
| **^{1}_{HEWMA}(10)** | 2.1939 | 2.198 | 2.217 | 2.2291 | 2.2348 |
| $\lambda_1 = 0.30$, $\lambda_2 = 0.10$ |      |      |      |      |      |
| $\lambda_1 = 0.30$, $\lambda_2 = 0.20$ |      |      |      |      |      |
| **^{1}_{HEWMA}(10)** | 2.3616 | 2.369 | 2.3865 | 2.401 | 2.404 |
| $\lambda_1 = 0.30$, $\lambda_2 = 0.20$ |      |      |      |      |      |
| $\lambda$ | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|-----------|------|------|------|------|------|
| $\rho = 0.25$ | $\tau$ | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL |
| 1.0 | 200.38 | 248.99 | 200.08 | 248.42 | 200.16 | 248.03 | 200.74 | 250.04 | 200.19 | 250.51 |
| 1.1 | 22.77 | 26.39 | 21.87 | 25.34 | 17.47 | 20.05 | 10.75 | 12.14 | 6.89 | 12.05 |
| 1.2 | 8.81 | 10.05 | 8.48 | 9.67 | 6.73 | 7.54 | 4.90 | 4.25 | 2.72 | 2.54 |
| 1.3 | 4.90 | 5.36 | 4.66 | 5.07 | 3.84 | 4.05 | 2.46 | 2.28 | 1.78 | 1.36 |
| 1.4 | 3.40 | 3.54 | 3.20 | 3.29 | 2.61 | 2.51 | 1.83 | 1.45 | 1.44 | 0.89 |
| 1.5 | 2.54 | 2.42 | 2.47 | 2.37 | 2.08 | 1.79 | 1.55 | 1.05 | 1.28 | 0.64 |
| 1.6 | 2.10 | 1.83 | 2.03 | 1.75 | 1.72 | 1.35 | 1.36 | 0.78 | 1.18 | 0.49 |
| 1.7 | 1.82 | 1.48 | 1.76 | 1.36 | 1.54 | 1.07 | 1.27 | 0.64 | 1.13 | 0.40 |
| 1.8 | 1.61 | 1.20 | 1.58 | 1.13 | 1.40 | 0.86 | 1.20 | 0.53 | 1.10 | 0.34 |
| 1.9 | 1.49 | 1.00 | 1.46 | 0.96 | 1.31 | 0.72 | 1.15 | 0.44 | 1.07 | 0.28 |
| 2.0 | 1.37 | 0.83 | 1.36 | 0.80 | 1.25 | 0.63 | 1.11 | 0.38 | 1.06 | 0.24 |

| $\lambda$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|-----------|------|------|------|------|------|
| $\rho = 0.10$ | $\tau$ | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL |
| 1.0 | 200.84 | 237.00 | 200.61 | 236.04 | 200.10 | 236.36 |
| 1.1 | 24.80 | 27.24 | 23.69 | 25.98 | 19.00 | 20.38 | 11.80 | 12.41 | 7.57 | 12.05 |
| 1.2 | 9.51 | 10.17 | 9.05 | 9.75 | 5.39 | 5.94 | 3.84 | 3.31 | 2.72 | 2.54 |
| 1.3 | 5.39 | 6.64 | 5.09 | 5.29 | 4.13 | 4.16 | 2.66 | 2.44 | 1.91 | 1.46 |
| 1.4 | 3.64 | 3.62 | 3.51 | 3.50 | 2.84 | 2.68 | 1.97 | 1.59 | 1.51 | 0.96 |
| 1.5 | 2.79 | 2.64 | 2.65 | 2.42 | 2.18 | 1.89 | 1.61 | 1.10 | 1.32 | 0.70 |
| 1.6 | 2.24 | 1.99 | 2.19 | 1.92 | 1.83 | 1.44 | 1.43 | 0.87 | 1.22 | 0.55 |
| 1.7 | 1.90 | 1.59 | 1.81 | 1.48 | 1.62 | 1.16 | 1.30 | 0.69 | 1.15 | 0.44 |
| 1.8 | 1.70 | 1.27 | 1.66 | 1.23 | 1.46 | 0.93 | 1.22 | 0.56 | 1.11 | 0.36 |
| 1.9 | 1.54 | 1.06 | 1.49 | 0.99 | 1.36 | 0.80 | 1.17 | 0.48 | 1.09 | 0.32 |
| 2.0 | 1.44 | 0.90 | 1.40 | 0.86 | 1.27 | 0.64 | 1.13 | 0.41 | 1.07 | 0.28 |

| $\lambda$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|-----------|------|------|------|------|------|
| $\rho = 0.05$ | $\tau$ | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL | AIB | ARL | SDRL |
| 1.0 | 200.79 | 225.52 | 200.76 | 225.46 | 200.00 | 222.81 | 200.03 | 223.34 | 200.70 | 225.34 |
| 1.1 | 27.45 | 28.95 | 26.27 | 27.44 | 21.21 | 21.79 | 13.10 | 13.08 | 8.50 | 8.15 |
| 1.2 | 10.49 | 10.56 | 9.94 | 10.05 | 8.11 | 8.09 | 4.95 | 4.67 | 3.26 | 2.88 |
| 1.3 | 5.96 | 5.90 | 5.69 | 5.65 | 4.51 | 4.35 | 2.94 | 2.63 | 2.06 | 1.60 |
| 1.4 | 4.06 | 3.94 | 3.89 | 3.70 | 3.07 | 2.82 | 2.11 | 1.68 | 1.58 | 1.04 |
| 1.5 | 3.01 | 2.79 | 2.90 | 2.66 | 2.39 | 2.07 | 1.72 | 1.22 | 1.38 | 0.78 |
| 1.6 | 2.41 | 2.10 | 2.32 | 2.01 | 1.97 | 1.56 | 1.48 | 0.91 | 1.25 | 0.59 |
| 1.7 | 2.06 | 1.69 | 1.97 | 1.59 | 1.70 | 1.22 | 1.35 | 0.74 | 1.18 | 0.48 |
| 1.8 | 1.79 | 1.36 | 1.75 | 1.32 | 1.53 | 0.99 | 1.26 | 0.61 | 1.14 | 0.41 |
| 1.9 | 1.63 | 1.15 | 1.59 | 1.08 | 1.42 | 0.86 | 1.20 | 0.52 | 1.10 | 0.34 |
| 2.0 | 1.49 | 0.97 | 1.46 | 0.92 | 1.33 | 0.73 | 1.16 | 0.45 | 1.08 | 0.29 |
provides the ARL1 values are (23.69, 9.05) and (25.05, 9.75), respectively, whereas the HEWMA control chart has the ARL1 values equal to (25.79, 10.15) (see Tables 3, 7 versus 9). Similarly, the proposed control charts’ superiority over the HEWMA control chart can also be visualized in Figure 1. Besides, the overall performance
Table 5: Run-length profile of two-sided HEWMA\(^{(2)}\) control chart at ARL\(_0 = 200.

| \(\rho\) | \(\tau\) | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL | ARL | SDRL |
|-------|------|-----|------|-----|------|-----|------|-----|------|-----|------|
| 0.5   | 0.50 | 1.722 | 1.718 |
| 0.6   | 0.75 | 1.715 | 1.710 |
| 0.7   | 0.90 | 1.710 | 1.710 |
| 0.8   | 0.95 | 1.710 | 1.710 |
| 0.9   | 1.00 | 1.722 | 1.718 |
| 1.0   | 1.05 | 1.715 | 1.710 |
| 1.1   | 1.10 | 1.710 | 1.710 |
| 1.2   | 1.15 | 1.710 | 1.710 |
| 1.3   | 1.20 | 1.710 | 1.710 |
| 1.4   | 1.25 | 1.710 | 1.710 |
| 1.5   | 1.30 | 1.710 | 1.710 |
| 1.6   | 1.35 | 1.710 | 1.710 |
| 1.7   | 1.40 | 1.710 | 1.710 |
| 1.8   | 1.45 | 1.710 | 1.710 |
| 1.9   | 1.50 | 1.710 | 1.710 |
| 2.0   | 1.55 | 1.710 | 1.710 |

\(\lambda_1 = 0.05, \lambda_2 = 0.05\)

\(\lambda_1 = 0.05, \lambda_2 = 0.10\)

\(\lambda_1 = 0.05, \lambda_2 = 0.20\)
| \( \rho \) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---|---|---|---|---|---|
| \( \lambda_1 \) = 0.10, \( \lambda_2 \) = 0.05 | 1.851 | 1.849 | 1.846 | 1.846 | 1.845 |
| \( \lambda_1 \) = 0.10, \( \lambda_2 \) = 0.10 | 1.997 | 1.997 | 1.992 | 1.995 | 1.990 |
| \( \lambda_1 \) = 0.10, \( \lambda_2 \) = 0.20 | 2.1406 | 2.1404 | 2.1402 | 2.1399 | 2.1397 |
Table 6: Run-length profile of two-sided HEWMA\(^{(2)}\) control chart at ARL\(_{0}\) = 200.

| \(\rho\) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|-------|------|------|------|------|------|
| \(\lambda_1 = 0.20, \lambda_2 = 0.05\) |
| \(\lambda_1 = 0.20, \lambda_2 = 0.10\) |
| \(\lambda_1 = 0.20, \lambda_2 = 0.20\) |

\(\lambda_1, \lambda_2\) denote the parameters of the HEWMA control chart.
| Table 6: Continued. |
|---------------------|
| \( \rho \)   | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
| \( \lambda_1 = 0.30, \lambda_2 = 0.05 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.10 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.20 \) |

```
| \( \rho \) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---------------------|
| \( \lambda_1 = 0.30, \lambda_2 = 0.05 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.10 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.20 \) |
```

Table 6 (continued):

| \( \rho \) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---------------------|
| \( \lambda_1 = 0.30, \lambda_2 = 0.05 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.10 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.20 \) |

```
| \( \rho \) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---------------------|
| \( \lambda_1 = 0.30, \lambda_2 = 0.05 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.10 \) |
| \( \lambda_1 = 0.30, \lambda_2 = 0.20 \) |
```
| $\tau$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| ARL  | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| SDRL | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 | 235.98 |
| ARL  | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 | 200.01 |
| SDRL | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 | 236.51 |
| ARL  | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 |
| SDRL | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 | 222.30 |
| ARL  | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 | 200.15 |
| SDRL | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 | 222.82 |
| ARL  | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 | 200.34 |
| SDRL | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 | 222.72 |
| ARL  | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 | 200.11 |
| SDRL | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 | 221.85 |
| ARL  | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| SDRL | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 | 221.71 |

$\rho = 0.25, 0.50, 0.75, 0.90, 0.95$
Table 8: Run-length profile of one-sided HEWMA\(^{(2)}\) control chart at ARL\(_0\) = 200.

| \(\rho\) | 0.25 | 0.50 | 0.75 | 0.90 | 0.95 |
|---------|------|------|------|------|------|
| \(\lambda_1 = 0.20\), \(\lambda_2 = 0.05\) |
| \(\lambda_1 = 0.20\), \(\lambda_2 = 0.05\) |
| \(\lambda_1 = 0.30\), \(\lambda_2 = 0.05\) |
| \(\lambda_1 = 0.30\), \(\lambda_2 = 0.10\) |

\[\text{ARL SDRL ARL SDRL ARL SDRL ARL SDRL ARL SDRL ARL SDRL ARL SDRL ARL SDRL}\]
5.2. Proposed versus AEWMA Control Chart. The proposed HEWMA(1) AIB and HEWMA(2) AIB control charts are more efficient than the AEWMA control chart. For example, with $(\lambda, \lambda_1, \lambda_2) = (0.1, \lambda_1 = 0.05, \delta = 1.1, \rho = 0.5)$, the proposed HEWMA(1) AIB and HEWMA(2) AIB control charts attain the ARL1 values as (23.69, 9.05) and (25.05, 9.75), respectively, while the HHW1 and HHW2 control charts have the ARL1 values as (34.52, 14.09) and (32.11, 12.72), respectively (see Tables 3, 7 versus 9). Furthermore, Figure 2 also shows the superiority of the proposed control charts over the HHW1 and HHW2 control charts. In terms of overall effectiveness (see Table 10), the HEWMA(1) AIB and HEWMA(2) AIB control charts are superior to the HHW1 and HHW2 charts. For instance, the proposed HEWMA(1) AIB and HEWMA(2) AIB control charts attain the EQL, PCI, and RARL values as 13.905, 13.918, and 19.752, 19.609, respectively (see Table 10).

5.3. Proposed versus HHW1 and HHW2 Control Charts. The proposed HEWMA(1) AIB and HEWMA(2) AIB control charts provide better performance against the HHW1 and HHW2 control charts. For example, at $(\rho, \lambda_1, \lambda_2) = (0.5, 0.05, 0.95)$ and $(\lambda_1, \lambda_2) = (1.0, 0.10, 0.10)$ and $(\rho, \lambda_1, \lambda_2) = (1.0, 0.10, 0.10)$ and $(\rho, \lambda_1, \lambda_2) = (1.0, 0.10, 0.10)$, the proposed HEWMA(1) AIB and HEWMA(2) AIB control charts attain the ARL1 values as (23.69, 9.05) and (25.05, 9.75), respectively, while the HHW1 and HHW2 control charts have the ARL1 values as (34.52, 14.09) and (32.11, 12.72), respectively (see Tables 3, 7 versus 9). Furthermore, Figure 2 also shows the superiority of the proposed control charts over the HHW1 and HHW2 control charts. In terms of overall effectiveness (see Table 10), the HEWMA(1) AIB and HEWMA(2) AIB control charts are superior to the HHW1 and HHW2 charts. For instance, the proposed HEWMA(1) AIB and HEWMA(2) AIB control charts attain the EQL, PCI, and RARL values as 13.905, 13.918, and 19.752, 19.609, respectively (see Table 10).

5.4. Proposed versus EWMA-I AIB- and EWMA-II AIB-Control Charts. The proposed HEWMA(1) AIB and HEWMA(2) AIB control charts attain outstanding performance against the EWMA-I AIB and EWMA-II AIB control charts. For example, when $(\lambda_1, \lambda_2) = (0.1, \lambda_2 = 0.05, \rho = 0.95)$, and $\tau = 1.0$, the ARL1 values of proposed HEWMA(1) AIB and HEWMA(2) AIB control charts are 7.57 and 8.52, whereas, the ARL1 values of EWMA-I AIB and EWMA-II AIB control charts are 34.52 and 14.09, respectively (see Tables 4, 7 versus 9). Additionally, the dominance of the proposed HEWMA(1) AIB and HEWMA(2) AIB control charts over the EWMA-I AIB and EWMA-II AIB control charts can be seen in Figure 3. For overall performance, the proposed HEWMA(1) AIB and

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### Table 9: Run-length profile of existing charts at ARL0 = 200.

| HEWMA | AEWMA | HHW1 | HHW2 | EWMA-I AIB | EWMA-II AIB | HEWMA(1) AIB | HEWMA(2) AIB |
|-------|-------|------|------|------------|------------|-------------|-------------|
| $\tau$ | $\lambda_1 = 0.05, \lambda_2 = 0.10$ | $\lambda = 0.10$ | $\lambda = 0.10$ | $\lambda = 0.10$ | $\lambda = 0.10$ | $\lambda_1 = 0.10, \lambda_2 = 0.05$ | $\lambda_1 = 0.10, \lambda_2 = 0.05$ |
| 1.0  | 201.71 | 200.76 | 199.69 | 200.12 | 200.42 | 200.3  | 200.3  |
| 1.1  | 25.79  | 26.04  | 34.52  | 32.11  | 9.73   | 10.96  | 10.96  |
| 1.2  | 10.15  | 10.35  | 14.09  | 12.72  | 3.72   | 4.03   | 4.03   |
| 1.3  | 5.79   | 5.71   | 8.2    | 7.22   | 2.34   | 2.37   | 2.37   |
| 1.4  | 3.99   | 3.78   | 5.64   | 4.6    | 1.79   | 1.76   | 1.76   |
| 1.5  | 2.96   | 2.83   | 4.28   | 3.67   | 1.51   | 1.45   | 1.45   |
| 1.6  | 2.39   | 2.29   | 3.46   | 2.96   | 1.36   | 1.28   | 1.28   |
| 1.7  | 2.03   | 1.97   | 2.91   | 2.48   | 1.26   | 1.19   | 1.19   |
| 1.8  | 1.78   | 1.75   | 2.53   | 2.16   | 1.2    | 1.13   | 1.13   |
| 1.9  | 1.62   | 1.58   | 2.25   | 1.93   | 1.15   | 1.09   | 1.09   |
| 2.0  | 1.49   | 1.47   | 2.05   | 1.77   | 1.12   | 1.07   | 1.07   |

### Table 10: Overall performance measures of proposed vs existing charts.

| HEWMA | AEWMA | HHW1 | HHW2 | EWMA-I AIB | EWMA-II AIB | HEWMA(1) AIB | HEWMA(2) AIB |
|-------|-------|------|------|------------|------------|-------------|-------------|
| $\lambda_1 = 0.10, \lambda_2 = 0.05$ | $\lambda = 0.10$ | $\lambda = 0.10$ | $\lambda = 0.10$ | $\rho = 0.95$ | $\rho = 0.95$ | $\lambda_1 = 0.10, \lambda_2 = 0.05$ | $\lambda_1 = 0.10, \lambda_2 = 0.05$ |
| EQL   | 19.752| 19.609| 23.413| 21.954 | 14.560 | 14.638 | 13.905 | 13.918 |
| PCI   | 1.421 | 1.410 | 1.684 | 1.579  | 1.047 | 1.053 | 1.000 | 1.001 |
| RARL  | 2.273 | 2.234 | 3.171 | 2.793  | 1.145 | 1.140 | 1.000 | 0.989 |
AIB control charts have smaller EQL, PCI, and RARL values than the EQL, PCI, and RARL values of the EWMA-IAIB and EWMA-IIAIB control charts, respectively (see Table 10).

5.5. Main Outcomes of the Study. Some interesting outcomes of the proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts are listed as follows:

(i) The use of Hybrid EWMA statistic certainly boosts the detection ability of the proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts.

(ii) The performance of the proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts improve with the induction of suitable auxiliary information in the model.

(iii) The ARL\textsubscript{1} values of the proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts are smaller than HEWMA, AEWMA, HHW1, HHW2, EWMA-IAIB, and EWMA-IIAIB control charts.

(iv) The overall performance evaluation measures show the dominance of the HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts against other control charts (see Subsections 5.1–5.4).

(v) The proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts provide the best performance for larger values of $\rho$ (see Figures 4 and 5).

(vi) The ARL\textsubscript{1} performance of the proposed HEWMA\textsubscript{AIB} and HEWMA\textsubscript{AIB} control charts are increased for smaller values of $\lambda_1$ and $\lambda_2$ (see Tables 1–8).
6. Real-Life Application

To demonstrate the practical implementation of the proposed HEWMA\(_{\text{AIB}}^{(1)}\) and HEWMA\(_{\text{AIB}}^{(2)}\) control charts, a real-life data set of glass thickness (\(X\)), and its impact on the stress strength (\(Y\)) of glass bottles is considered from Asadzadeh and Kiadaliry [38]. This data set contains 40 samples, each of size 5, of stress strength (kg/cm\(^2\)), thickness (cm). The proposed control charts are constructed under the assumption of known parameters. However, in real-life data application of the proposed control chart, the population parameters are not available. Therefore, for the practical implementation of the control charts, the estimated parameters are used for the empirical quantification of the quantities required to show the proposed control charts’ implementation. The estimates of process parameters are given as: \(\hat{\mu}_Y = 6.36, \hat{\sigma}_Y = 8.92, \hat{\mu}_X = 1.38, \hat{\sigma}_X = 0.62\), and \(\hat{\rho} = 0.905\). In the data set, the first 20 samples are treated as IC, while the rest of the 20 samples are considered OOC. Following Anwar, et al. [39], the \(Y\) is multiplied by 1.3 for the OOC scenario.

The parameters of proposed HEWMA\(_{\text{AIB}}^{(1)}\) and HEWMA\(_{\text{AIB}}^{(2)}\) control charts are set on \(\lambda_1 = 0.2\), \(\lambda_2 = 0.2\), \(\lambda_{\text{HEWMA}}^{(1)} = 2.3029\), \(\lambda_{\text{HEWMA}}^{(2)} = 2.299\) and \(\rho = 0.905\) with ARL\(_0 = 200\). Similarly, the control chart parameters of the existing EWMA-I\(_{\text{AIB}}\) and EWMA-II\(_{\text{AIB}}\) control charts are \((\lambda, \lambda_{\text{EWMA-I}}^{\text{AIB}}, \lambda_{\text{EWMA-II}}^{\text{AIB}}) = (0.2, 2.785, 0.2, 2.677)\) at \(\rho = 0.905\) and ARL\(_0 = 200\).

The EWMA-I\(_{\text{AIB}}\), EWMA-II\(_{\text{AIB}}\), proposed HEWMA\(_{\text{AIB}}^{(1)}\), and proposed HEWMA\(_{\text{AIB}}^{(2)}\) control charts detect the first OOC signal at sample number 29 (see Figures 6–9). Overall, the existing EWMA-I\(_{\text{AIB}}\) control chart detects 2 OOC signals while the HEWMA\(_{\text{AIB}}^{(1)}\) control chart detects 8 OOC signals. In the same manner, the EWMA-II\(_{\text{AIB}}\) control chart detects a total of 2 OOC signals, and the HEWMA\(_{\text{AIB}}^{(2)}\) control chart detects 5 OOC signals. This indicates that the proposed HEWMA\(_{\text{AIB}}^{(1)}\) and HEWMA\(_{\text{AIB}}^{(2)}\) control charts are more efficient than the existing EWMA-I\(_{\text{AIB}}\) and EWMA-II\(_{\text{AIB}}\) control charts.
HEWMA AIB
Control Limits
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8

Figure 9: The proposed HEWMA\textsuperscript{(1)} AIB control chart for real-life data using $\rho = 0.905$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $L_{\text{HEWMA}_{AIB}} = 2.299$ and $\text{ARL}_0 = 200$.

7. Concluding remarks

This study presented the two new auxiliary information-based hybrid EWMA control charts, named HEWMA\textsuperscript{(1)} AIB and HEWMA\textsuperscript{(2)} AIB control charts for process dispersion. The HEWMA\textsuperscript{(1)} AIB control chart used the auxiliary information through the regression estimator for the population variance, whereas the HEWMA\textsuperscript{(2)} AIB control chart used the auxiliary information through the difference estimator. The HEWMA\textsuperscript{(1)} AIB and HEWMA\textsuperscript{(2)} AIB control charts are constructed by combining the features of the AIB dispersion estimators with the HEWMA control chart. The proposed control charts’ performance based on average run length, extra quadratic loss, relative average run length, and performance comparison index measures reveal the superiority over the competitive control charts. It is worth mentioning that the proposed HEWMA\textsuperscript{(1)} AIB and HEWMA\textsuperscript{(2)} AIB control charts performed very well to monitor small to moderate shifts in process dispersion, especially for large correlation coefficient values. Besides, a real-life application is also provided for users and practitioners to demonstrate the implementation of the proposed study from a practical perspective. This work can be extended to a non-normal process(s) and a multi-variate case.

Data Availability

The data is available in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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