First quantized electron and photon model of QED and radiative processes

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Abstract

In this study we combine the classical models of the massive and massless spinning particles, derive the current-current interaction Lagrangian of the particles from the gauge transformations of the classical spinors, and discuss radiative processes in electrodynamics by using the solutions of the Dirac equation and the quantum wave equations of the photon. The longitudinal polarized photon states give a new idea about the vacuum concept in electrodynamics.

1 Introduction

The perturbative formulation of quantum electrodynamics is based on at least four different approaches: (1) The standard operator expansion in the quantum field theory [1]; (2) Green’s function expansion [2]; (3) S-matrix unitary expansion [3] and (4) path integral quantization of classical particle trajectories [4]. There are also various nonperturbative approaches [5, 6]. In some of these approaches electrons are represented by the solution of the Dirac equation instead of the second quantized Dirac fields. But photons are represented as second quantized Maxwell fields, which have infinite degrees of freedom. In classical electrodynamics the radiative processes and the radiation reaction are described by the self-interactions of the electrons with themselves and also in this formalism the electromagnetic field is represented by Green’s function.

Up to now there have been a lot of attempts to understand the dynamical structure of the electron. The Heisenberg equations of the Dirac electron give a helical trajectory as its natural free motion (the zitterbewegung). Barut and Zanghi derived a classical analogue of the zitterbewegung oscillations [7], the quantization of this model gives the Dirac equation as well as higher spin wave equations and the first quantized formulation of the QED can be derived by the quantization of the classical action of this model [8].

Recently, one of us proposed a classical model for the massless spinning particles and the quantization of this model gives a new wave equation for the massless, spin-1 particles as well as the wave equation for Majorana neutrinos [9].

The aim of this study is to derive a completely first quantized version of the QED in which the electrons and photons are described classically as spinning
particles with a finite degree of freedom. In Section II, we discuss the free particle Lagrangian of the massive and massless spinning particles and try to derive the interaction term by using a new global gauge principle for the classical spinors of these particles. The local form of the gauge principle is used to derive the Dirac equation and the wave equation for photons in curved space-times separately [10]. In Section III, we discuss the quantization of the composite spinning two-particle system. In Section IV, we discuss the solutions of the wave equation for the photon, which is the analog of the Dirac equation for the electron. Since this new wave equation is similar to the Dirac equation and has first order derivatives with respect to the space-time coordinates, it has a probability interpretation. It has three spin states and describes the probability amplitudes for the photon. The solutions of this new wave equation are in the same form as the solutions of the Maxwell equations in the free space and curved space-time, but the physical meaning of these functions is different. The solutions to the Maxwell equations and this new equation represent the amplitude of the energy waves and the probability waves of the photons, respectively. In Section V, we discuss the radiative processes and evaluate the scattering amplitudes by using this new interaction Lagrangian of the photon current and the electron current. Section VI is the conclusion.

2 The classical system

In this section, we develop a formalism to describe the electron and the photon as the classical spinning particles. Then the phase space of the classical system is characterized by the pair of conjugate dynamical variables of the electron and the photon. These are $[x_1^\mu (\tau_1), p_1^\mu (\tau_1)]$ and $[\vec{z}(\tau_1), -iz (\tau_1)]$ for the external and internal dynamical variables of the electron, respectively, and $[x_2^\mu (\tau_2), p_2^\mu (\tau_2)]$ and $[\eta^+ (\tau_2) - i\eta (\tau_2)]$ for the external and internal dynamical variables of the photon. $x^\mu$ and $p^\mu$ are four vectors of the coordinate and momentum, respectively. Here, $\tau_1$ and $\tau_2$ are the invariant time parameters of the electron and photon, respectively. $\vec{z}$ and $z$ are four component complex spinors, which represent internal degrees of freedom for a particle (the electron) with negative energy. $\eta^+$ and $\eta$ are two component complex spinors which represent internal degrees of freedom for a particle (the photon) without negative energy. The units are $c = \hbar = 1$.

The action of the free electron and photon system is given in the Cartan form:

$$A = \int \left[ p_1^\mu (dx_1^\mu - \vec{z}\gamma^\mu z d\tau_1) - izd\vec{z} + p_2^\mu (dx_2^\mu - \eta^+ \sigma^\mu \eta d\tau_2) - i\eta d\eta^+ \right]$$

(1)

where $\gamma_\mu$ and $\sigma_\mu = (1, \vec{\sigma})$ are the Dirac and Pauli matrices, respectively. The Hamiltonian of the system ($H$) is in the covariant form and is the function of the internal and external dynamical variables of the particles.

The interaction of the electron and photon can be derived by using the
following transformation for $z$ and $\eta$:

$$z \rightarrow e^{i\alpha(\tau_1)} z$$

$$\bar{z} \rightarrow \bar{z} e^{-i\alpha(\tau_1)}$$

$$\eta \rightarrow e^{i\beta(\tau_2)} \eta$$

$$\eta^\dagger \rightarrow \eta^\dagger e^{-i\beta(\tau_2)} \quad (2)$$

We substitute them into Eq. (1). The result is

$$A = \int d\tau_1 \left[ p_1\mu \left( \frac{dx_1^\mu}{d\tau_1} - \bar{z}\gamma^\mu z \right) - \frac{d\alpha}{d\tau_1} - iz \frac{d\bar{z}}{d\tau_1} \right]$$

$$+ \int d\tau_2 \left[ p_2\mu \left( \frac{dx_2^\mu}{d\tau_2} - \eta^\dagger \sigma^\mu \eta \right) - \frac{d\beta}{d\tau_2} - i\eta \frac{d\eta^\dagger}{d\tau_2} \right] \quad (3)$$

We choose $\alpha$ and $\beta$ as

$$\frac{d\alpha}{d\tau_1} = \frac{e(\lambda)}{2} \bar{z}\gamma^\mu z \int d\tau_2 \delta [x_1(\tau_1) - x_2(\tau_2)] \eta^\dagger \sigma^\mu \eta$$

$$\frac{d\beta}{d\tau_2} = \frac{e(\lambda)}{2} \eta^\dagger \sigma^\mu \eta \int d\tau_1 \delta [x_1(\tau_1) - x_1(\tau_1)] \bar{z}\gamma^\mu z \quad (4)$$

Then the interaction term becomes

$$A_{\text{int}} = e(\lambda) \int d\tau_1 \bar{z}\gamma^\mu z \int d\tau_2 \int d\tau_2 \delta [x_1(\tau_1) - x_2(\tau_2)] \eta^\dagger \sigma^\mu \eta \quad (5)$$

Thus the $H_{\text{int}}$ is derived by the gauge symmetry of the classical spinors. It consists of the interaction between the electron and photon currents with the help of $\delta$ function by a new coupling constant $e(\lambda)$ which will be obtained in terms of the electron charge $e$.

We substitute the Eq. (5) into Eq. (3). Then the action becomes

$$A = \int d\tau_1 \left[ p_1\mu \left( \frac{dx_1^\mu}{d\tau_1} - \bar{z}\gamma^\mu z \right) - iz \frac{d\bar{z}}{d\tau_1} \right]$$

$$+ \int d\tau_2 \left[ p_2\mu \left( \frac{dx_2^\mu}{d\tau_2} - \eta^\dagger \sigma^\mu \eta \right) - i\eta \frac{d\eta^\dagger}{d\tau_2} \right]$$

$$- e(\lambda) \int d\tau_1 d\tau_2 \bar{z}(\tau_1) \gamma^\mu z(\tau_1) \int dx_2 \eta^\dagger(\tau_2) \sigma^\mu \eta(\tau_2) \delta [x_1(\tau_1) - x_2(\tau_2)] \quad (6)$$
2.1 Euler-Lagrange and Hamilton equations

The variations of the action in Eq. (5) are calculated and the Euler-Lagrange equations of the electron and photon system are obtained. For the electron, these equations are

\[
\frac{dz}{d\tau_1} = -i\gamma^\mu [p_{1\mu} - eA_\mu (x_1)] z
\]

\[
\frac{d\bar{\sigma}}{d\tau_1} = -i\bar{\sigma}^\mu [p_{1\mu} - eA_\mu (x_1)]
\]

\[
\frac{dx_1^\mu}{d\tau_1} = \bar{\sigma}^\mu z
\]

\[
\frac{dp_1^\mu}{d\tau_1} = -e\bar{\sigma}^\nu zA^\nu_\mu
\]

where \(A_\mu (x_1)\) is the four component vector potential defined as

\[
-eA_\mu (x_1) = e(\lambda) \int d\tau_2 \int dx_2 \eta^\dagger \sigma_\mu \eta \delta [x_1 (\tau_1) - x_2 (\tau_2)]
\]

(7)

which represents the field created by the photon current in the electron’s spacetime. The Eq. (7) is in the same form as the equations which were derived in the electron model in Ref. [7]. The Euler-Lagrange equations of the photon are

\[
\frac{d\eta}{d\tau_2} = -i\sigma^\mu [p_{2\mu} - e(\lambda) B_\mu (x_2)] \eta
\]

\[
\frac{d\eta^\dagger}{d\tau_2} = i\eta^\dagger \sigma^\mu [p_{2\mu} - e(\lambda) B_\mu (x_2)]
\]

\[
\frac{dx_2^\mu}{d\tau_2} = \eta^\dagger \sigma^\nu \eta
\]

\[
\frac{dp_2^\mu}{d\tau_1} = eB^\nu_\mu \eta^\dagger \sigma_\nu \eta
\]

(9)

where \(B_\mu (x_2)\) is the four component vector potential

\[
-eB_\mu (x_2) = e(\lambda) \int d\tau_1 \bar{\sigma} \gamma^\mu \delta [x_1 (\tau_1) - x_2 (\tau_2)]
\]

(10)

which represents the field created by the electron current in the photon’s spacetime. The Hamiltonian equations are the same as the Euler-Lagrange equations.
3 Quantumization

The configuration space of the two-particle system is $M^4 \otimes C^4 \otimes M^4 \otimes C^2$. The quantum states of the system are represented by the composite wave functions

$$\Psi = \Psi(x_1, z_1, x_2, \eta^\dagger; \tau_2)$$

The state function $\Psi$ satisfies the Schrödinger equation for the composite system in the center of mass system of the proper times. It is

$$i \left( \frac{\partial}{\partial \tau_1} + \frac{\partial}{\partial \tau_2} \right) \Psi(x_1, z, \tau_1, x_2, \eta^\dagger; \tau_2) = \hat{H} \Psi(x_1, z, \tau_1, x_2, \eta^\dagger; \tau_2)$$

(11)

where $p^\mu_1$ and $p^\mu_2$ are the momenta of the electron and photon respectively, the operator $\hat{H}$ is given as

$$\hat{H} = \bar{z} \gamma^\mu \hat{p}^\mu_1 + \hat{\eta}^\dagger \sigma^\mu \hat{p}^\mu_2 - e (\lambda) \bar{z} \gamma^\mu \hat{\eta}^\dagger \sigma^\mu \hat{\eta}$$

and the momentum operators representing the internal dynamics of the system are

$$\hat{z} = \frac{\partial}{\partial z}, \quad \hat{\eta} = \frac{\partial}{\partial \eta^\dagger}$$

In order to obtain the spin eigenstates of the composite system we expand the $\Psi$ into the power series of $z^\alpha$ and $\eta^\dagger_\beta$:

$$\Psi = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} (z^{\alpha_0} \ldots z^{\alpha_n}) (\eta^\dagger_\beta_0 \ldots \eta^\dagger_\beta_m) \psi_{\alpha_0 \ldots \alpha_n, \beta_0 \ldots \beta_m}(x_1, \tau_1, x_2, \tau_2)$$

(12)

In the Schrödinger equation, the magnitudes of the spin are conserved. Then we obtain the most general eigenvalue equation for the spin-$n/2$ and spin-$m/2$ particles separately by equalizing the coefficients of the $n^{th}$ power of $z$ and $m^{th}$ power of $\eta^\dagger$ on both sides of the Schrödinger equation:

$$\{ z^{\alpha_0} (\gamma^\mu \hat{p}_1)_\alpha^\dagger + \eta^\dagger_\beta (\sigma^\mu \hat{p}_2)_\beta^\dagger \} + e (\lambda) \int d\tau_1 \bar{z} \gamma^\mu (\gamma^\mu)_\alpha^\dagger \eta^\dagger_\beta (\sigma^\mu \eta)_{\beta^\dagger} \times \delta(x_1 - x_2) \}$$

$$\times \sum_{j=1}^{n} \delta^{\alpha_0}_j \ldots \delta^{\alpha_n}_j \times \sum_{k=1}^{m} \delta^{\beta_0}_k \ldots \delta^{\beta_m}_k = e (\bar{z} \gamma^\mu \ldots \gamma^\mu_n) (\eta^\dagger_\beta_1 \ldots \eta^\dagger_\beta_m) \psi_{\alpha_1 \ldots \alpha_n, \beta_1 \ldots \beta_m}$$

(13)
where $\epsilon$ is the total energy of the particles in their own rest frame. Eq. (13) gives the Schrödinger equation of the system of higher spin particles. Thus we derive the eigenvalue equation of the spin-$\frac{1}{2}$ and spin-1 particles as

$$
\sum_{\alpha} \left\{ (\gamma^\mu \hat{p}_{1\mu})_{\alpha\alpha'} \eta_{\beta_1}^\dagger \eta_{\beta_2}^\dagger + \delta_{\alpha\alpha'} \eta_{\beta_2}^\dagger (\sigma^\mu \hat{p}_{2\mu})_{\beta\beta'} \left( \delta_{\beta'\beta_1} \eta_{\beta_2}^\dagger + \eta_{\beta_1}^\dagger \delta_{\beta'\beta_2} \right) + \epsilon (\lambda) \int d\tau (\gamma^\mu)_{\alpha\alpha'} \eta_{\beta_2}^\dagger (\sigma^\mu)_{\beta\beta'} \left( \delta_{\beta'\beta_1} \eta_{\beta_2}^\dagger + \eta_{\beta_1}^\dagger \delta_{\beta'\beta_2} \right) \delta(x_1 - x_2) \right\} \psi_{\alpha', \beta_1, \beta_2}
$$

(14)

If we rewrite this equation in the variational form it becomes

$$
\int dx_1 dx_2 \psi_{\alpha_1, \beta_1, \beta_2} \left\{ (\gamma^\mu \hat{p}_{1\mu}(1 \otimes 1) + (\sigma^\mu \otimes 1 \otimes \sigma^\mu) \hat{p}_{2\mu}) d\tau_1 \right\} \times e (\lambda) \gamma^\mu (\sigma^\mu \otimes 1 \otimes \sigma^\mu) \delta(x_1 - x_2) d\tau_1 d\tau_2 \psi_{\gamma_1, \sigma_1, \sigma_2}
$$

(15)

$$
4 \text{ Free electron and photon wave functions}
$$

For the weakly interacting particles we separate the composite wave function $\psi$ in the following way:

$$
\psi(x_1; \tau_1, x_2; \tau_2) = \varphi(x_1; \tau_1) \otimes \phi(x_2; \tau_2)
$$

(16)

where $\varphi$ and $\phi$ are the electron and photon wave functions, respectively. We define $\Sigma^\mu$ as

$$
\Sigma^\mu = \sigma^\mu \otimes 1 + 1 \otimes \sigma^\mu
$$

Thus we obtain the following equations with the help of the variations of $\varphi$ and $\phi$:

$$
[\gamma^\mu (\hat{p}_{1\mu} - eA_\mu) - m] \varphi = 0
$$

(17)

$$
\Sigma^\mu (\hat{p}_{2\mu} - eB_\mu) \phi = 0
$$

(18)

Equations (17) and (18) are the wave equations for the electron and the massless spin-1 particle (photon) in the slowly varying field of the others. In this study we consider the space and time coordinates of the particles as operators and thus normalize the wave functions $\varphi$ and $\phi$ in the four dimensional space-time box with volume $VT$. 

6
4.1 Coupling constant $e(\lambda)$

$e(\lambda)$ is the parameter representing the coupling of the electron and photon in this new formulation of the electrodynamics. We compare it with the electron charge $e$. First we do the dimension analysis in the definition

$$[-eA_\mu(x) = e(\lambda) \overline{\psi}(x) \gamma_\mu \psi(x)]$$

where $e$ is the dimensionless electron charge. The dimension of the electromagnetic potential is $[\text{length}]^{-1}$. Since the wave functions are normalized in four-dimensions, the dimensions of the product is $[\text{length}]^{-4}$. Thus $e(\lambda)$ is found as

$$e(\lambda) = -e (VT)^\frac{3}{2}$$

(19)

4.2 Electron wave function

The expressions of the free electron wave functions, normalized in four-dimensions are

$$\varphi(x) = \frac{1}{\sqrt{VT}} \int \frac{d^4p}{(2\pi)^4} \sqrt{\frac{|p_0| + m}{2|p_0|}}$$

$$\times \left[ \left( \frac{1}{|p_0| + m} \right) \otimes \chi^{(s)} e^{ip \cdot \sigma \theta (|p_0|)} + \left( \frac{|p_0| + m}{1} \right) \otimes \chi^{(s)} e^{-ip \cdot \sigma \theta (-|p_0|)} \right]$$

(20)

where $\chi^{(s)}$ is the two component Pauli spinor. The first (second) term in the bracket corresponds to the positive (negative) energy solutions of the Dirac equations, which correspond to the forward (backward) moving electron in time.

4.3 Photon wave function

The eigenvalue equation of the free spin-1 particle (photon) is

$$\Sigma^\mu (p_\mu) \phi = 0$$

(21)

The solutions to Eq. (21) are

$$\phi^{(\pm)} (x_\parallel, t) = \frac{1}{\sqrt{VT}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-i(k(t-x_\parallel))} \text{ for } \omega = k$$

(22)

$$\phi^{(-)} (x_\parallel, t_2) = \frac{1}{\sqrt{VT}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{i(k(t+x_\parallel))} \text{ for } \omega = -k$$

(23)
The first two solutions correspond to the transverse polarized wave functions with positive and negative helicities and they correspond to the photon which carries energy and momentum. In the most general case, the transverse polarized wave functions are written as

$$\phi(x, t) = \frac{1}{\sqrt{2VT}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} e^{+ikx} \quad \text{for} \quad \omega = 0, k \neq 0$$  \hspace{1cm} (24)$$

$$\phi(x, t) = \frac{1}{\sqrt{2VT}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} e^{-ikx} \quad \text{for} \quad \omega = k = 0$$  \hspace{1cm} (25)$$

The equations (24) and (25) define the zero-energy longitudinal photon. If the energy of the photon is zero then the momentum of it is zero or not zero. The Maxwell fields represent the energy waves and for this reason they do not have zero energy solutions. We interpret these new solutions: The third solution (Eq. (24)), \( \omega = 0, \frac{|\vec{k}|}{k} \neq 0 \) case is considered as the longitudinal state of the photon which carries momentum without energy. The energy of the photon emitted

$$U_F(x_2 - x_1) = \frac{1}{\sqrt{VT}} \int \frac{d^4k}{(2\pi)^4} \frac{e^{-i\omega(t_2-t_1)+i\vec{k} \cdot (\vec{x}_2-\vec{x}_1)}}{2\omega}$$

$$\times \left( \frac{1}{\omega - |\vec{k}|} + \frac{1}{\omega + |\vec{k}|} \right)$$  \hspace{1cm} (27)$$

This is equivalent to the Green’s function of the plane electromagnetic waves in the Maxwell theory or in quantumelectrodynamics. The normalizations and dimensions are different. In the Maxwell theory, the Green’s function represents the transition amplitudes for the energy waves, but in here it represents the transition amplitudes for the probability waves.
from a heavy nucleus can approximately be zero. In this case, the emitting photon is longitudinally polarized. We derived the propagator for the photon in the self interaction of the charged particle or in the interaction of the two charged particles. The transition amplitudes between the longitudinal polarized states are

\[ U'_{F}(x_2 - x_1) = \]

\[ -\frac{1}{\sqrt{VT}} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x}_2 - \vec{x}_1)} \left( \frac{1}{\omega - \vec{k}} + \frac{1}{\omega + \vec{k}} \right) \delta(\omega) d\omega \]  

(28)

This is the expression of the Green’s function of the longitudinal polarized photon in the limit of zero-energy.

The last solution (Eq. (25)), \( \omega = k = 0 \) case is considered to be a vacuum state which carries neither energy nor momentum. Historically, there are many studies on the structure of the vacuum, which have not been finished completely. In this study we interpret the vacuum as a state function covering all space and time. In the flat space-time the photon wave-function is constant. This means that the photon has a constant probability in space-time even if its energy and momentum are zero. Thus the photon is neither created nor annihilated; only the energy, momentum and the polarization of the photon have changed. In QED this is formulated as the creation and annihilation of the photon. The probability amplitudes between the emission and absorption points \( x_1 \) and \( x_2 \) or the vacuum states are

\[ N_{F}(x_1 - x_2) = \frac{1}{\sqrt{VT}} \]  

(29)

The dimension of this propagator is \([L]^{-4}\)

5 Radiative processes

The S-matrix element, that will be covered in this study, is constructed upon the interaction of fields, which are produced by electron and photon currents. The free-incoming waves of the electron and photon are represented by \( \varphi_i(x) \) and \( \phi_i(x) \) respectively. \( \varphi_f(x') \) and \( \phi_f(x') \) show the outgoing free-waves of the electron and photon, respectively. The \( S \)–matrix elements are dimensionless. Finally, from the dimensional analysis we have obtained a factor that is represented by \( q(\theta) \). For each emitting and absorbing longitudinal polarized photon at vertices, we write

\[ q(\theta) = \sqrt{\frac{2\pi T}{\omega}} \]  

(30)
For each emitting and absorbing transverse polarized photon at vertices,

\[ q(\theta)_{i,f} = \sqrt{\frac{T}{\omega_{i,f}}} \]  

(31)

where the photon is real and, \( f \) and \( i \) subindexes represent the final and initial states respectively and \( \omega \) is the energy of transverse photon. We choose the dimension of Feynman propagator as \([L]^{-4}\). In this case, we obtain

\[ S_F(x' - x) = \frac{1}{(VT)^{\frac{1}{4}}} \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x' - x)}}{(p - m)} \]  

(32)

5.1 Compton scattering

The Feynman graph of the Compton scattering, which is one of the radiative processes, is shown in Fig. 1., where \( i \) and \( f \) subindexes represent the initial and the final states and \( p \) and \( k \) are the four momentum of the electron and photon, respectively. \( \epsilon \) represents the photon polarization, and \( s \) represents the electron spin. We can explain the Compton scattering as follows:

In the graph on the left, the electron and photon interact with each other in the space-time \( x \) and propagate to the space-time \( x' \). Since the photon does not have energy and momentum between the points \( x \) and \( x' \), it is longitudinal polarized. The electron and photon again interact with each other in the final position \( x' \) and then propagate to different space-time regions. In the graph on the right, the interaction positions are exchanged. The contributions of both cases are equal to each other. The electron and photon have been propagated into the future in the space-time. There is a second-order interaction in this process. The \( S \)-matrix element of the Compton scattering is

\[ S_{fi}^{CS} = i \int d^4x d^4x' f_i (x') (\gamma^\mu) i N_F (x' - x) (\gamma^\nu) \phi_i (x) \]

\[ \times \left[ \phi_f (x') \gamma^\mu_i N_F (x' - x) (\gamma^\nu) \phi_i (x) \right. \]

\[ + \left. \phi_f (x) \gamma^\mu \phi_f (x) \right] \]

(33)

We carry out the integral upon the coordinates and momenta. Since the photon spinors are normal to each other, we obtain

\[ S_{fi}^{CS} = \frac{e^2}{VT^3} \sqrt{\frac{m^2}{E_i E_f \sqrt{2\omega_i}}} \frac{1}{(2\pi)^4} \delta^4(p_f + k_f - p_i - k_i) \]

\[ \times \pi(p_f, s_f) \left[ (-i\gamma_\mu) a_+^{\mu} \frac{i}{\gamma_i + \gamma_f - m} (-i\gamma_\nu) \Sigma_+ \right] \]
\[ (+ i \gamma_\mu) \Sigma^\mu a^\dagger \frac{i}{p_f - k_f - m} \]  
\[ (- i \gamma_\nu) a^\dagger \Sigma^\nu] u(p_i, s_i) \]  
\quad (34)

Eq. (34) corresponds to the Feynman graphs in Fig. 1, where the conservation of energy and momentum \( p_f + k_f = p_i + k_i \) have been satisfied. This condition corresponds to the process of an incident free electron and photon transforming to a final state of a free electron and photon. We define the transverse polarization conditions of the photon as shown below:

\[ \gamma_\mu a^\dagger \Sigma^\mu \equiv \gamma_\mu \xi_{f+} = \xi_{i+}, \quad \gamma_\nu \Sigma^\nu a_+ \equiv \gamma_\nu \xi_{i+} = \xi_{i+} \]  
\[ \gamma_\nu a^\dagger \Sigma^\nu \equiv \gamma_\nu \xi_{f-} = \xi_{f-}, \quad \gamma_\mu \Sigma^\mu a_- \equiv \gamma_\mu \xi_{i-} = \xi_{i-} \]  
\quad (35)

Finally, we obtain

\[ S_{CFS}^{CS} = \frac{e^2}{VT^3} \sqrt{\frac{m^2}{E_f E_i}} \frac{1}{\sqrt{2 \omega_f 2 \omega_i}} (2\pi)^4 \delta^4 (p_f + k_f - p_i - k_i) \pi(p_f, s_f) \]
\[ \times \{ (-i \xi_{f+}) \frac{i}{p_f + k_f - m} (-i \xi_{i+}) + (-i \xi_{i-}) \frac{i}{p_i - k_f - m} (-i \xi_{f-}) \} u(p_i, s_i) \]  
\quad (36)

### 5.2 Bremsstrahlung

In this process radiation is emitted by an electron bound to a heavy nucleus with the charge \( +Z \alpha \). The process is shown in Fig. 2. Since the energy-momentum of the photon incoming to the heavy nucleus is zero and the momentum of the photon, outgoing from the nucleus, is not zero \( (\vec{k} \neq 0) \), we consider the Coulomb interaction between the electron and nucleus as the current of the longitudinal polarized photon. The propagator of the longitudinal polarized photon is given in Eq. (28). The scattering photon is transverse polarized and the \( S \)-matrix element of this process is:

\[ S_{fi}^B = i \int d^4x' d^4x d^4x_0 \varphi_{f-}(x') (-i) e(\lambda) \gamma_\mu iS_F(x' - x) (-i) e(\lambda) \gamma_0 \varphi_{i-}(x) \]
\[ \times \{ \varphi^\dagger_f(x') (-i) q(\theta)_f \Sigma^\mu iN_F(x' - x) (-i) q(\theta) \Sigma^0 U'_{F'}(x - x_0) \]
\[ + \varphi^\dagger_f(x) (-i) q(\theta)_f \Sigma^0 iN_F(x - x') (-i) q(\theta) \Sigma^\mu U'_{F'}(x' - x_0) \]  
\[ \times (-i) q(\theta) \Sigma_0 iN_F \varphi'(x_0) (-i) Ze(\lambda) \gamma_0 \varphi'(x_0) \]  
\quad (37)
The current of the nucleus is in the form of \( \Psi(x_0) \gamma_0 \Psi(x_0) \) and its wave function is a positive energy free-plane wave:

\[
\Psi(x_0) = \frac{1}{\sqrt{VT}} e^{-ip_0 \cdot x_0} b(p_n, s_n)
\]  

(38)

Thus we obtain S-matrix

\[
S_{fi}^B = \frac{-Ze^3}{VT^{3/2}} (2\pi) \delta(E_{f-} + \omega_f - E_{i-}) \sqrt{\frac{m^2}{2\omega_f}} \frac{1}{2\omega_f} \frac{1}{|\rho f - \rho f^- - \rho f^-|} 
\]

\[
\times \prod(p_{f-}, s_{f-}) \left[ \frac{i}{\gamma_{f-} m} (-i\gamma_0) + \frac{i}{\gamma_{i-} m} (-i\gamma_0) \right] u(p_{i-}, s_{i-})
\]

(39)

where \( E_{i-} = E_{f-} + \omega_f \) represents the conservation of the energy.

5.3 Annihilation of electron-positron pair into gamma rays

The Feynman graph of this process is the same as Compton scattering. But here, the electron-positron pair decays into two photons. The interaction is in the order of \( e^2 \). The initial and final photons are transverse polarized and the photon between the interaction points is longitudinal polarized. By using the similarity between Compton scattering and pair annihilation we obtain the following substitutions:

\[
CS \leftrightarrow PA \\
\epsilon_f, k_f \leftrightarrow \epsilon_f, k_f \\
\epsilon_i, k_i \leftrightarrow \epsilon_i, -k_i \\
p_{f+}, s_{f+} \leftrightarrow -p_+, s_+ \\
p_{i-}, s_{i-} \leftrightarrow p_-, s_-
\]

Then the matrix elements are

\[
S_{fi}^{PA} = i \int d^4x' d^4x \overline{\phi}_f(x') (-i) e(\lambda) \gamma_\mu iS_F(x' - x) (-i) e(\lambda) \gamma_\nu \phi_-(x) \\
\times \{ \phi_f^\dagger(x') (-i) q(\theta)_f \Sigma^\mu iN_F(x' - x) (-i) q(\theta)_i \Sigma^\nu \phi_i(x) \\
+ \phi_f^\dagger(x) (-i) q(\theta)_f \Sigma^\nu iN_F(x - x') (-i) q(\theta)_i \Sigma^\mu \phi_i(x') \} 
\]  

(40)
and it is written finally as

\[ S_{f_1}^{PA} = \frac{e^2}{VT^3} \sqrt{\frac{m^2}{E_+E_-}} \frac{1}{\sqrt{2\omega f 2\omega_i}} (2\pi)^4 \delta^4 \left( -p_+ + k_f - p_- + k_i \right) \nabla(p_+, s_+) \]

\times \left\{ \begin{array}{l}
  \frac{i}{\not{\gamma} - k_i - m} (-i \not{\gamma} + i) + \frac{i}{\not{\gamma} - k_f - m} (-i \not{\gamma}) \\
  \frac{i}{\not{\gamma} - k_i - m} (-i \not{\gamma} + i) + \frac{i}{\not{\gamma} - k_f - m} (-i \not{\gamma}) \end{array} \right\} u(p_-, s_-)

(41)

Here we also have the exchange symmetry of the two photons according to Bose-Einstein statistics. \( \delta \) function in Eq. (41) shows the conservations of energy and momentum.

### 5.4 Pair production

When an energetic photon passes from a region around a heavy nucleus it decays into an electron-positron pair. We explain this process as follows: The charge of the heavy nucleus is \(-Ze\) and its Coulomb field is described in a similar way to that of the Bremsstrahlung. When the transverse polarized initial photon with momentum \( k_i \) enters into this field, it gives its energy-momentum to the pair of particles and it becomes longitudinal polarized.

The Feynman graph of this process is similar to that of the Bremsstrahlung. By comparing these two processes we obtain the following substitution rules:

\[ B \leftrightarrow PP \]

\[ \in_f, k_f \leftrightarrow \in_i, -k_i \]

\[ p_{f-}, s_{f-} \leftrightarrow p_+, s_+ \]

\[ p_{i-}, s_{i-} \leftrightarrow -p_-, s_- \]

In the momentum space we finally obtain \( S \)-matrix as

\[ S_{f_1}^{PP} = -\frac{Ze^3}{VT^3 \sqrt{2}} \frac{1}{(2\pi)} \frac{1}{\sqrt{2\omega_i}} \left| \begin{array}{l}
  \frac{m^2}{E_+E_-} \frac{1}{\sqrt{2\omega}} \\
  -k_i + p_+ + \not{p} \\
  \end{array} \right|^2 \\

\times \nabla(p_+, s_+) \left\{ \begin{array}{l}
  \frac{i}{\not{\gamma} - k_i - m} (-i \not{\gamma} + i) + \frac{i}{\not{\gamma} - k_f - m} (-i \not{\gamma}) \\
  \frac{i}{\not{\gamma} - k_i - m} (-i \not{\gamma} + i) + \frac{i}{\not{\gamma} - k_f - m} (-i \not{\gamma}) \end{array} \right\} u(p_-, s_-)

(42)

where \( \delta \) function shows that the photon energy has been transformed to electron-positron pairs.
5.5 Positive energy electron-electron scattering

The process represents the scattering of two identical particles and the Feynman graph is shown in Fig. 3. The interacting electrons are labeled with subindexes 1 and 2, and their initial and final states are in the form of free-plane waves. In the first Feynman graph, the incoming positive energy electron-1 is interacted with the incoming zero energy-momentum photon at interaction point \( x \). As a result of this interaction, a transverse polarized photon is emitted, this photon interacts with electron-2 at interaction point \( x' \) and loses its energy-momentum. In the second Feynman graph, electron-1 and electron-2 have been exchanged in the final state according to Fermi-Dirac statistics. The corresponding \( S \)-matrix element of the process of positive energy electron-electron scattering is

\[
S_{EES}^{f_1} = i \int d^4x' d^4x' [\overline{\psi}_{f_2} (x') (-i\gamma_\mu) e (\lambda) \varphi_{i_2} (x') iN_F (-i\Sigma_0) q (\theta) iU_F (x' - x) \\
\times (-i\Sigma_0) q (\theta) N_F \overline{\psi}_{f_1} (x) (-i\gamma_\mu) e (\lambda) \phi_{i_1} (x) - \overline{\psi}_{f_1} (x') (-i\gamma_\mu) e (\lambda) \phi_{i_2} (x') \\
\times iN_F (-i\Sigma_0) q (\theta) iU_F (x' - x) (-i\Sigma_0) q (\theta) N_F \overline{\psi}_{f_2} (x) (-i\gamma_\mu) e (\lambda) \varphi_{i_1} (x)]
\]

(43)

and we finally obtain

\[
S_{EES}^{f_1} = \frac{-e^2m^2}{V^{3/2}T^{3/2}} \sqrt{E_{f_2}E_{i_2}E_{f_1}E_{i_1}} (2\pi)^4 \delta^4 (p_{f_2} + p_{f_1} - p_{i_2} - p_{i_1}) \\
\times \left[ \overline{\pi} (p_{f_2}, s_{f_2}) (-i\gamma_\mu) u (p_{i_2}, s_{i_2}) \frac{i}{(p_{i_1} - p_{f_1})^2} \pi (p_{f_1}, s_{f_1}) (-i\gamma_\mu) u (p_{i_1}, s_{i_1}) \\
- \overline{\pi} (p_{f_1}, s_{f_1}) (-i\gamma_\mu) u (p_{i_2}, s_{i_2}) \frac{i}{(p_{i_1} - p_{f_2})^2} \pi (p_{f_2}, s_{f_2}) (-i\gamma_\mu) u (p_{i_1}, s_{i_1}) \right]
\]

(44)

5.6 Positive energy (forward moving in time) and negative energy (backward moving in time) electron scattering

The Feynman graph for this process is similar to positive energy electron-electron scattering. We obtain the following substitution rules:

\[
\begin{align*}
\text{EES} & \leftrightarrow \text{EPS} \\
p_{i_1} & \leftrightarrow p_{i-} \\
p_{f_1} & \leftrightarrow p_{f-} \\
p_{i_2} & \leftrightarrow -p_{f+} \\
p_{f_2} & \leftrightarrow -p_{i+}
\end{align*}
\]
We thus write the S-matrix element as

\[
S_{EP}^{\phi \phi} = -i \int d^4x' d^4x \overline{\psi}_{f+} (x') (-i\gamma_\mu) e (\lambda) \phi_{i+} (x') iN_F (-i\Sigma_0) q (\theta) iU_F (x' - x)
\]

\[
\times (-i\Sigma_0) q (\theta) N_F \overline{\psi}_{f-} (x) (-i\gamma^\mu) e (\lambda) \phi_{i-} (x) - \overline{\psi}_{f-} (x') (-i\gamma_\mu) e (\lambda) \phi_{f+} (x')
\]

\[
\times iN_F (-i\Sigma_0) q (\theta) iU_F (x' - x) (-i\Sigma_0) q (\theta) N_F \overline{\psi}_{i+} (x) (-i\gamma^\mu) e (\lambda) \phi_{i-} (x)
\]

and it becomes

\[
S_{EP}^{\phi \phi} = \frac{e^2 m^2}{2T^4} \frac{1}{\sqrt{E_f + E_i + E_f - E_i}} (2\pi)^4 \delta^4 (p_{f+} - p_{i+} + p_{f-} - p_{i-})
\]

\[
\times \frac{i}{(p_{i-} - p_{f-})^2} \overline{\psi} (p_{i+}, s_{i+}) (-i\gamma_\mu) \nu (p_{f+}, s_{f+}) \frac{i}{(p_{i-} + p_{f+})^2} \overline{\psi} (p_{f-}, s_{f-}) (-i\gamma^\mu) u (p_{i-}, s_{i-})
\]

\[
- \frac{i}{(p_{i-} - p_{f-})^2} \overline{\psi} (p_{f-}, s_{f-}) (-i\gamma_\mu) \nu (p_{f+}, s_{f+}) \frac{i}{(p_{i-} + p_{f+})^2} \overline{\psi} (p_{i+}, s_{i+}) (-i\gamma^\mu) u (p_{i-}, s_{i-})
\]

The first term in the bracket represents the scattering of the positive energy electron with the negative energy electron. We can interpret the second term as the exchange of the positive energy electrons and negative energy electrons according to Fermi-Dirac statistics. By using this interpretation we can also explain the annihilation of the positronium [12, 13]. Although the electron and positron do not have the same charge, they are solutions to the same equation.

**5.7 Vacuum polarization**

In vacuum polarization a free transverse polarized photon is transformed at point \( x \) into a longitudinal polarized photon with zero energy-momentum and an electron-positron pair, and they are annihilated at \( x' \). The Feynman graph is shown in Fig. 4 and the S-matrix for vacuum polarization is

\[
S_{V}^{\phi \phi} = - \int d^4x' d^4x (-i) e (\lambda) \gamma_\alpha^{\mu\beta} iS_F (x' - x) \beta (\lambda) \nu (p_{f+}, s_{f+})
\]

\[
\times \partial_\mu (x') (-i\Sigma_\mu) q (\theta) iN_F (x' - x) (-i\Sigma_\nu) q (\theta) \nu (p_{i-}, s_{i-})
\]

\[
(46)
\]
By performing the $x$ and $x'$ integrations we obtain the S-matrix element of the vacuum polarization as

$$
S_{fi}^{VP} = -ie^2 \frac{1}{T^3 \sqrt{2\omega_f 2\omega_i}} (2\pi)^4 \delta^4 (k_i - k_f)
$$

$$
\times \int \frac{d^4p}{(2\pi)^4} \frac{(-i) \gamma_a^\mu \in_{\mu f} \frac{i}{(p' - m)_{\beta \lambda}} (-i) \gamma_{\lambda \beta} \in_{\nu i} \frac{i}{(p' - k_i' - m)_{\theta \alpha}}} \quad (47)
$$

The closed fermion loop gives the trace of the matrices. As a result, we obtain

$$
S_{fi}^{VP} = -i \frac{1}{T^3 \sqrt{2\omega_f 2\omega_i}} (2\pi)^4 \delta^4 (k_i - k_f)
$$

$$
\times \int (\frac{-i}{2}) e^2 \frac{d^4p}{(2\pi)^4} \frac{Tr \{ \xi_f (p' + m) \xi_i (p' - k'_i + m) \}}{(p - k_i)^2 - m^2 - i\varepsilon} \quad (48)
$$

This integral has a logarithmic singularity at large $p$, hence it is divergent. The general procedure used to evaluate them is dimensional regularization and renormalization. A convenient way of the regularization is to evaluate the divergent terms in the $d$-dimension. We show the divergent term as $\Pi_d^{\mu\nu} (k_i)$, which can be written as

$$
\Pi_d^{\mu\nu} (k_i) = (g^{\mu\nu} k_i^2 - k_i^\mu k_i^\nu) \Pi_d (k_i^2) \quad (49)
$$

where $\Pi_d (k_i^2)$ is the scalar part of the equation:

$$
\Pi_d (k_i^2) = \frac{4e^2}{(4\pi)^{d/2}} \Gamma \frac{(3 - d/2)}{(2 - d/2) \Gamma (2)} \int_0^1 dx \frac{2x (1 - x)}{[m^2 - k_i^2 x (1 - x)]^{2-d/2}} \quad (50)
$$

Here the singularity for $d = 4$ dimensions is separated. Vacuum polarization is convergent for $d < 4$. The physical result can be obtained from the limit $d \to 4$. To obtain this expression we write the scalar vacuum polarization in Eq. (50) as the sum of two terms, a finite and an infinite term. The infinite term is a constant corresponding to the value of $\Pi_d (k_i^2)$ at $k_i^2 = 0$ and the finite term is obtained by subtracting the integral in Eq. (50) at $k_i^2 = 0$. This gives

$$
\Pi_d (k_i^2) = \Pi_d (0) + [\Pi_d (k_i^2) - \Pi_d (0)]
$$

$$
= \Pi_d (0) + \Pi (k_i^2) \quad (51)
$$

We obtain the singular part (the infrared divergent)

$$
\Pi_d (0) = \frac{\alpha}{\pi} \frac{2\Gamma (1 + \varepsilon/2)}{\varepsilon (4\pi)^{-\varepsilon/2}} \int_0^1 dx \frac{2x (1 - x)}{m^2} \quad (52)
$$
where $\varepsilon = 4 - d$. The finite part is obtained by taking the limit $\varepsilon \to 0$:

$$\Pi_d (k_i^2) \to \Pi (k_i^2) = \frac{2\alpha}{\pi} \int_0^1 dx \frac{1}{(1-x)x \log \left[ 1 - \frac{k_i^2}{m^2} (1-x) \right]}$$

(53)

Eq. (53) is zero for $k_i^2 = 0$ and it gives

$$\Pi (k_i^2) \bigg|_{k_i^2 \ll m^2} \to \frac{\alpha}{15\pi m^2} \frac{k_i^2}{m^2}$$

(54)

for $|k_i^2| \ll m^2$. Eq. (53) is complex for $k_i^2 \geq m^2$. We evaluate the contribution of vacuum polarization for the longitudinal photon with $k_i = 0$. As shown in the Feynman graph in Fig. 5, this corresponds to the production and annihilation of the positronium. In this process, the initial and the final photons have zero energy and momentum and are longitudinal polarized.

The S-matrix element is

$$S_{ij}^{\text{VP}} = -\frac{1}{2} i \int d^4 x' d^4 x (-i) \gamma^\mu S_F (x' - x) g_{\mu\nu} e \gamma^\nu \phi_i (x)$$

$$\times (i) \gamma^\rho \phi_f (x) - i\Sigma_0 q (\theta) i N_F (x') \phi (x')$$

(55)

This integral gives zero:

$$S_{ij}^{\text{VP}} = 0$$

(56)

This shows that $k_i = 0$ is the vacuum case for the photon, and positronium can be produced and annihilated in a vacuum, but this term gives no contribution to vacuum polarization.

5.8 Electron self-energy

Electron self energy is explained here as an electron that interacts the longitudinal photon at $x$ by transforming it to the transverse state with the energy and momentum photon. At $x'$ the process is reversed. The Feynman graph of the process is shown in Fig. 6.

We write the S-matrix element of the process as

$$S_{ij}^{\text{ESE}} = i \int d^4 x' d^4 x e \gamma^\mu (x') (i \gamma^\rho) e (\lambda) S_F (x' - x) g_{\mu\nu} e (\lambda) \phi_i (x)$$

$$\times i N_F (i\Sigma_0) q (\theta) i U_F (x' - x) (i\Sigma_0) q (\theta) N_F$$

(57)
This integral can be evaluated by the method used in vacuum polarization. As a result, we obtain

\[ S_{fi}^{ESE} = -\frac{i}{\sqrt{1/2T_{i3}}} \sqrt{\frac{m}{E_f E_i}} (2\pi)^4 \delta^4 (p_f - p_i) \Omega (p_i) u (p_i, s_i) \]  

(58)

where \( \Omega (p_i) \) gives the self energy of the electron:

\[ \Omega (p_i) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu (m + \not{k} - \not{p_i}) \gamma^\mu}{m^2 - (p_i - k)^2 - i\epsilon} \]  

(59)

This integral is divergent. We regularize it using dimensional regularization and perform the integrals in \( d \) dimensions. Then we substitute \( 4 + \varepsilon \) for \( d \) and evaluate \( \Omega (p_i) \) when \( \varepsilon \to 0 \):

\[ \Omega_\varepsilon (p_i) = \frac{\alpha}{(4\pi)^{1-\varepsilon/2} m^2} \frac{1}{\Gamma (2)} \int_0^1 \frac{dz}{z^{\varepsilon/2}} \left( \frac{1}{\varepsilon} [2m - (1 - z) p_i^2] + (1 - z) p_i^2 \left[ 1 + \left( 1 - \frac{\varepsilon}{2} \right) \log \left( 1 - \frac{p_i^2 (1 - z)}{m^2} \right) \right] - m \left[ 1 + \left( 2 - \frac{\varepsilon}{2} \right) \log \left( 1 - \frac{p_i^2 (1 - z)}{m^2} \right) \right] \right) \]  

(60)

In Eq. (60) the first term in the curly bracket goes to infinity in the limiting case \( \varepsilon \to 0 \) and the second and third terms are finite in this limit. If \( p_i^2 = m^2 \), the electron is on the mass shell. Near the mass shell, that is, when \( p_i^2 \approx m^2 \), we obtain

\[ \Omega_\varepsilon (p_i) = \frac{\alpha}{(4\pi)^{1-\varepsilon/2} m^2} \left( \frac{1}{\varepsilon} [3m - (\not{p_i} - m)] - (\not{p_i} - m) \log \left( \frac{m^2 - p_i^2}{m^2} \right) \right) \]  

(61)

where the last term has logarithmic singularity. \( \Omega (p_i) \) becomes complex for \( p_i^2 > m^2 \). This case corresponds to the existence of the process of virtual electron decaying into electron and photon.

### 5.9 Energy shifts

In this subsection we evaluate the energy shifts for the bound electron by using the scattering matrix \( S_{fi} \). If we consider Fig. 7, the \( S \)-matrix element is

\[ S_{fi} = i \int d^4y d^4x \bar{\varphi}_d (y) (-i\gamma^\mu) \epsilon (\lambda) \varphi_i (y) \bar{\varphi}_b (x) (-i\gamma^\nu) g_{\mu\nu} \epsilon (\lambda) \varphi_o (x) \]

\[ \times i N_F (-i\Sigma_0) q (\theta) \phi_\alpha (y) \phi_\beta (y) (-i\Sigma_0) q (\theta) N_F \]  

(62)
where \( x = (t, \vec{x}) \) and \( y = (t', \vec{y}) \) are the coordinates of the interaction points. The propagator of the electron and photon are written in terms of the wave functions:

\[
U_F (y - x) = \phi_\beta^\dagger (y) \phi_\alpha (x)
\]

\[
S_F (y - x) = \phi^c (y) \phi^b (x)
\]

Generally, \( \varphi (z) = \varphi (t, \vec{z}) \) is the first quantized wave function of the electron. The Fourier expansion of this wave function in terms of the time variable gives

\[
\varphi (z) = \frac{1}{\sqrt{T}} \sum_n \varphi_n (\vec{z}) e^{-iE_n t}
\]  

where \( 1/\sqrt{T} \) is obtained from the normalization of the function. Eq. (63) is called Coulomb series expansion, and is different from those used in standard QED and quantum-optics. As a result, the \( S \)-matrix element is obtained:

\[
S_{fi} = -e^2 \sum_{d,c,b,a} 2\pi \delta (E_d - E_c + E_b - E_a) \int d^3 y \varphi^d (\vec{y}) \gamma^\mu \varphi^c (\vec{y})
\]

\[
\times \int d^3 x \varphi^b (\vec{x}) \gamma_\mu \varphi^a (\vec{x}) \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{y} - \vec{x})} \frac{1}{2k}
\]

\[
\times \{ P \left( \frac{1}{E_d - E_c - k} - \frac{1}{E_d - E_c + k} \right) - i\pi \left[ \delta (E_d - E_c - k) - \delta (E_d - E_c + k) \right] \}
\]

(64)

where \( P \) is the principal value of the integral and it is in the following form:

\[
\frac{1}{E_d - E_c - k} - \frac{1}{E_d - E_c + k} - i\pi \delta (E_d - E_c - k)
\]

The total energy of the system and the \( S \)-matrix element depend on each other with the \( \delta \)- function. If we consider \( \delta (E_d - E_c + E_b - E_a) \), the energy is conserved according to the relation of

\[
E_d - E_c = 0 ; \quad E_b - E_a = 0
\]

or

\[
E_d - E_c = E_a - E_b
\]
To obtain the energy shift of a certain state \( d \), we eliminate \( \delta \) function from the \( S \)-matrix element in Eq. (64), and we obtain the sum of all \( d \) levels. Thus

\[
\Delta E_d = -2\pi e^2 \sum_b \int d^3x \int d^3y \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{y} - \vec{x})} \\
\times \left\{ \left( -\frac{P}{k^2} \right) \frac{\gamma^\mu \varphi_d(\vec{y}) \gamma^\nu \varphi_d(\vec{x}) + \gamma^\mu \varphi_b(\vec{y}) \gamma^\nu \varphi_b(\vec{x})}{(E_d - E_b + k)^2 - (E_d - E_b - k)^2} \right\}
\]

As seen in from Eq. (65), the energy shifts are complex. The first term corresponds to the contribution of vacuum polarization and the third term gives Lamb-shift. These terms are real. The second term is imaginary and it gives the spontaneous emission and absorption. For \( \delta (E_d - E_b - k) \) function represents the conservation of energy for the transformation of the photon at level \( d \) and this corresponds to the spontaneous emission. \( \delta (E_d - E_b + k) \) represents the energy conservation for the inverse transformation of the photon at level \( b \) and this corresponds to the spontaneous absorption.

6 Conclusion

In this study we discussed the possibility of the representation of the photons as spinning particles. First we derived the Euler-Lagrange equations of the photon as a classical spinning particle and then studied the current-current interaction of it with the electron. In this formulation the electron and the photon have equal status. In the Dirac equation the electromagnetic potential is replaced by the velocity field created by the photon current in the space-time of the electron. Meanwhile, in the photon wave equation there is a similar term, which represents the field created by the electron current in the space-time of the photon. These kind of effects are observed when the motion of light in intense electron fields is investigated [11].

We obtained the transverse and longitudinal states for the photons and showed that the transverse photon states correspond to the plane wave solutions of the Maxwell equations. We also discussed the longitudinal photon states. The longitudinal, zero energy and non zero momentum states correspond to the static solutions of the Maxwell equations. We showed the existence of the zero energy and momentum states of the photons which can be interpreted as the
vacuum states of the photon. In this formulation the vacuum state has zero energy and momentum, instead of the $\hbar\omega/2$ vacuum energy, but it has a nonzero probability amplitude.

Finally we studied the radiative processes of the QED by using the solutions of the Dirac electron and the photon wave equation and showed that these processes can be described by representing the photon as a particle with conserved probability amplitudes, without being created and annihilated and their $S$-matrix elements give the same result.

We also discussed the generalization of the Pauli principle and indistinguishability of the electron and its antiparticle (positron) which are the forward and backward moving solutions of the Dirac equation in time. We have shown that the particles represented by the solutions of the same equation can be thought of as identical.

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