Color Glass Condensate and Glasma

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1. **Color Glass Condensate**
   - Why small-x gluons matter
   - Color Glass Condensate

2. **Just before the collision: Factorization**
   - Stages of AA collisions
   - Leading Order
   - Leading Log resummation

3. **Just after the collision: Glasma fields**
   - Initial color fields
   - Link to the Lund model
   - The ridge in Au-Au collisions

4. **Matching to hydrodynamics**
   - Glasma instabilities
   - Hydro in a toy model
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   Glasma instabilities
   Hydro in a toy model
• 99% of the multiplicity below $p_\perp \sim 2$ GeV
• $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
• $x \sim 4 \times 10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

▷ partons at small $x$ are the most important
Saturation domain

\[
\log(x^{-1}) \quad \Lambda_{\text{QCD}} \quad \log(Q^2)
\]
Implications for a QCD approach

- Main difficulty: How to treat collisions involving a large number of partons?
Implications for a QCD approach

- **Main difficulty**: How to treat collisions involving a large number of partons?

- **Dilute regime**: one parton in each projectile interact (what the standard perturbative techniques are made for)
Main difficulty: How to treat collisions involving a large number of partons?

Dense regime: multiparton processes become crucial
- new techniques are required
- multi-parton distributions are needed
CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The **fast partons** \((k^+ > \Lambda^+)\) are frozen by time dilation
  ➤ described as **static color sources** on the light-cone:
  
  \[
  J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (0 < x^- < 1/\Lambda^+)
  \]

- **Slow partons** \((k^+ < \Lambda^+)\) cannot be considered static over the time-scales of the collision process
  ➤ must be treated as standard gauge fields
  ➤ eikonal coupling to the current \(J^\mu : A_\mu J^\mu\)

- The color sources \(\rho\) are **random**, and described by a distribution \(W_{\Lambda^+}[\rho]\), with \(\Lambda^+\) the longitudinal momentum that separates “soft” and “hard”
Independence w.r.t $\Lambda^+$ → evolution equation (JIMWLK):

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H} \ W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int \frac{\delta}{\delta \alpha(\vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \alpha(\vec{x}_\perp)}$$

where $-\partial_\perp^2 \alpha(\vec{x}_\perp) = \rho(1/\Lambda^+, \vec{x}_\perp)$

• $\eta(\vec{x}_\perp, \vec{y}_\perp)$ is a non-linear functional of $\rho$

• Resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/\rho_\perp$ that arise in loop corrections

• Simplifies into the BFKL equation when the source $\rho$ is small (expand $\eta$ in powers of $\rho$)
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**The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time \( \tau \sim Q_s^{-1} \)**

**Subsequent stages are described as fluid dynamics**
Reminder on hydrodynamics

Equations of hydrodynamics:

\[ \partial_\mu T^{\mu \nu} = 0 \]

Additional inputs:

EoS: \( p = f(\epsilon) \), Transport coefficients: \( \eta, \zeta, \cdots \)

- Required initial conditions: \( T^{\mu \nu}(\tau = \tau_0, \eta, \vec{x}_\perp) \)
Initial conditions from CGC: power counting

- CGC effective theory with cutoff at the scale $\Lambda_0^+$:

  \[
  \mathcal{L} = \frac{1}{2} \text{tr} \, F_{\mu\nu} F^{\mu\nu} + \left( J^\mu_1 + J^\mu_2 \right) A_\mu
  \]

  \[
  T^{\mu\nu} = \frac{Q^4_s}{g^2} \left[ c_0 + c_1 \, g^2 + c_2 \, g^4 + \cdots \right]
  \]

- Expansion in $g^2$ in the saturated regime:

  \[
  T^{\mu\nu} = \frac{Q^4_s}{g^2} \left[ c_0 + c_1 \, g^2 + c_2 \, g^4 + \cdots \right]
  \]
The Leading Order contribution is given by classical fields:

\[ T_{LO}^{\mu\nu} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F_{\nu\lambda} \]

with \[ [D_\mu, F^{\mu\nu}] = J^\nu \]

Yang–Mills equation

The Yang-Mills equations have been solved numerically
Krasnitz, Venugopalan (1998-2000)
Lappi (2003)
Krasnitz, Nara, Venugopalan (2001-2003)
Initial condition from CGC: Leading Logs

• Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^+ < k^+ < \Lambda_0^+$:

\[
\delta T^\mu_\nu_{\text{NLO}} = \left[ \ln \left( \frac{\Lambda_0^+}{\Lambda_1^+} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H}_2 \right] T^\mu_\nu_{\text{LO}}
\]

(FG, Lappi, Venugopalan (2008))

• At leading log accuracy, the contribution of the quantum modes in that strip can be written as:

\[
\delta T^\mu_\nu_{\text{NLO}} = \left[ \ln \left( \frac{\Lambda_0^+}{\Lambda_1^+} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H}_2 \right] T^\mu_\nu_{\text{LO}}
\]
Initial condition from CGC: Leading Logs

- These corrections can be absorbed in the LO result,

\[
\langle T_{\text{LO}} + \delta T_{\text{NLO}} \rangle_{\Lambda_0} = \langle T_{\text{LO}} \rangle_{\Lambda_1}
\]

provided one defines a new effective theory with a lower cutoff \(\Lambda^{\pm}_{1}\) and an extended distribution of sources \(W^{\pm}_{\Lambda_{1}}[\rho] :\)

\[
W^{\pm}_{\Lambda_{1}} \equiv \left[ 1 + \ln \left( \frac{\Lambda^{\pm}_{0}}{\Lambda^{\pm}_{1}} \right) * H_{1,2} \right] W^{\pm}_{\Lambda^{\pm}_{0}}
\]

(JIMWLK equation for a small change in the cutoff)
Initial condition from CGC: Leading Logs

- Iterate this step to integrate out all the slow field modes at leading log accuracy:

\[
\langle T_{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle_{\text{LLog}} = \int \left[ D\rho_1 \ D\rho_2 \right] W_1[\rho_1] \ W_2[\rho_2] \ T_{\mu\nu}^{\text{LO}}(\tau, \vec{x}_\perp) \tag{1}
\]

for fixed \( \rho_{1,2} \)

- At leading log accuracy, the rapidity dependence comes entirely from the wavefunctions of the projectiles

- This factorization establishes a link to other reactions (such as DIS on a nuclear target) in the saturated regime

- Works for all sufficiently inclusive observables

Energy-Momentum tensor at Leading Log accuracy

\[
\langle T_{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle_{\text{LLog}} = \int \left[ D\rho_1 \ D\rho_2 \right] W_1[\rho_1] \ W_2[\rho_2] \ T_{\mu\nu}^{\text{LO}}(\tau, \vec{x}_\perp) \tag{1}
\]

for fixed \( \rho_{1,2} \)
Correlations in $\eta$ and $\vec{x}_\perp$ at Leading Log

- The factorization valid for $\langle T^\mu_\nu \rangle$ can be extended to multi-point correlations:

$$\langle T^{\mu_1\nu_1}(\tau, \eta_1, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}(\tau, \eta_n, \vec{x}_{n\perp}) \rangle_{\text{LLog}} = $$

$$= \int \left[ D\rho_1 \, D\rho_2 \right] W_1[\rho_1] \, W_2[\rho_2] \times T^{\mu_1\nu_1}_{\text{LO}}(\tau, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}_{\text{LO}}(\tau, \vec{x}_{n\perp})$$

- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$ they are a property of the pre-collision initial state.

- This formula predicts long range ($\Delta \eta \sim \alpha_s^{-1}$) rapidity correlations for points located at the same impact parameter.
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Initial classical fields, Glasma
Lappi, McLerran (2006)

- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant:

\[ \frac{[g^2 \mu^4]}{g^2} \]

- Glasma = intermediate stage between the CGC and the quark-gluon plasma
Glasma flux tubes

- The initial chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal “flux tubes” extending between the projectiles:

  ![Diagram of flux tubes](image)

- Correlation length in the transverse plane: $\Delta r_\perp \sim Q_s^{-1}$
- Correlation length in rapidity: $\Delta \eta \sim \alpha_s^{-1}$
- The flux tubes fill up the entire volume
Glasma flux tubes

- A classical field configuration where $B^i_a = \lambda E^i_a$ has an energy-momentum tensor of the form:

$$
\langle T^{\mu\nu}(0^+, \eta, \vec{x}) \rangle = \begin{pmatrix}
\epsilon & \epsilon \\
\epsilon & -\epsilon
\end{pmatrix}
$$

- Multiplicity distribution: Negative Binomial (T. Lappi’s talk)
- Long range correlations in rapidity survive in the final state
- $E$ parallel to $B \triangleright$ non-zero $\tilde{FF}$
Link to the Lund string model

- Tanji (2008), Fukushima, FG, Lappi (2009): the yield from the Schwinger mechanism has exactly the same form as the NLO correction in the CGC

- Analogies between the **Glasma** and the **Lund strings**:
  - Glasma tubes $\longleftrightarrow$ strings
  - Negative $P_z$ $\longleftrightarrow$ string tension
  - Glasma instability $\longleftrightarrow$ string breaking

- Differences:
  - $B$ field in the Glasma
  - The “string size” is set dynamically in the Glasma
  - The Glasma is more closely related to QCD
Importance of initial rapidity correlations

Early physics can survive in long range rapidity correlations

\[ t_{\text{correlation}} \leq t_{\text{freeze out}} \ e^{-\frac{1}{2} |\eta_A - \eta_B|} \]

A \hspace{2cm} B

Detection

Freeze out

Latest correlation
2-hadron correlations at RHIC

- Long range correlation in $\Delta \eta$ (rapidity)
- Narrow correlation in $\Delta \varphi$ (azimuthal angle)
2-hadron correlations at RHIC

Dumitru, FG, McLerran, Venugopalan (2008)
Dusling, Fernandez-Fraile, Venugopalan (2009)
Dusling, FG, Lappi, Venugopalan (2009)

- $\eta$-independent fields lead to long range correlations:
2-hadron correlations at RHIC

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• $\eta$-independent fields lead to long range correlations:

• Particles emitted by different flux tubes are not correlated
  \[ (RQ_s)^{-2} \] sets the strength of the correlation
2-hadron correlations at RHIC

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- $\eta$-independent fields lead to long range correlations:

- Particles emitted by different flux tubes are not correlated
  \[ (RQ_s)^{-2} \] sets the strength of the correlation

- At early times, the correlation is flat in $\Delta \varphi$
2-hadron correlations at RHIC

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- $\eta$-independent fields lead to long range correlations:

Particles emitted by different flux tubes are not correlated:
- $(RQ_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta \varphi$
  A collimation in $\Delta \varphi$ is produced later by radial flow
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\( T^{\mu \nu} \) of the Glasma

- Glasma:

\[ \langle T^{\mu \nu}(0^+, \eta, \vec{x}) \rangle = \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & -\epsilon \end{pmatrix} \]

- Ideal hydro:

\[ T^{\mu \nu}_{\text{ideal}}(0^+, \eta, \vec{x}) = \begin{pmatrix} \epsilon & p \\ p & p \end{pmatrix} \]

- If a smooth matching from the Glasma to Hydro is possible, one should be able to recover the fluid behavior from classical fields
Unstable fluctuations

Romatschke, Venugopalan (2005)

- Perturbations to the classical fields grow like $\exp(\sqrt{Q_s \tau})$ until the non-linearities become important:

\[ \tau \sim \tau_{\text{max}} \sim Q_s^{-1} \ln^2(1/\alpha_s) \]
Resummation of the unstable terms

- To go beyond the time $\tau_{\text{max}}$, one must resum all the fastest growing terms $\sim [g^2 e^{\sqrt{Q_s \tau}}]^n$.

- This amounts to superimposing fluctuations to the initial classical field:

$$\langle T^{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle \overset{\text{resummed}}{=} \int [D\rho_1 D\rho_2] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2]$$

$$\times \int [Da] F[a] T^{\mu\nu}_{\text{LO}}[\mathcal{A} + a] \text{ initial field}$$

- Fukushima, FG, McLerran (2006): this result can be obtained in a semi-classical approach (with Gaussian $F[a]$).

- FG, Lappi, Venugopalan (2008): can be obtained by a resummation of the NLO result in the CGC.
Hydro behavior in a toy model

- Including the fluctuations in the calculation of $T^{\mu \nu}$ is hard:
  - Space-dependent fluctuations need to be renormalized (because of UV divergences)
  - The QCD classical equations are difficult to solve

Toy model: scalar field, uniform in $\eta$

Equation of motion: $\ddot{\phi} + \frac{1}{\tau} \dot{\phi} - \nabla_\perp^2 \phi + V'(\phi) = 0$

Interaction potential: $V(\phi) \sim g^2 \phi^4$

Initial conditions at $\tau = \tau_0$: $\phi = \varphi_0$, $\dot{\phi} = \dot{\varphi}_0$

Gaussian fluctuations of $\varphi_0$ and $\dot{\varphi}_0$
Without fluctuations, $p$ oscillates forever
Hydro behavior in a toy model (1+0 dim)

**Classical field evolution with fluctuations**

- Without fluctuations, $p$ oscillates forever
- With fluctuations, $p$ relaxes quickly to $\epsilon/3$
Hydro behavior in a toy model (1+0 dim)

Classical field evolution with fluctuations

- Without fluctuations, $p$ oscillates forever
- With fluctuations, $p$ relaxes quickly to $\epsilon/3$
- $\epsilon$ and $p$ decrease as $1/\tau^{4/3}$
  - same behavior as in ideal hydro with EoS $\epsilon = 3p$...
Hydro behavior in a toy model (1+2 dim)

- Left: $\log |(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log |(\epsilon - 3p)/\epsilon|$ with fluctuations

**Summary**

- Hydro behavior in a toy model (1+2 dim)
- Initial color fields
- Link to the Lund model
- Rapidity correlations
- Matching to hydro
- Glasma instabilities
- Hydro in a toy model

**Extra bits**

- Hydro behavior in a toy model (1+2 dim)
  - Left: $\log |(\epsilon - 3p)/\epsilon|$ without fluctuations
  - Right: $\log |(\epsilon - 3p)/\epsilon|$ with fluctuations

**Factorization**

- Stages of AA collisions
- Leading Order
- Leading Logs
- Glasma fields
  - Initial color fields
  - Link to the Lund model
  - Rapidity correlations
Hydro behavior in a toy model (1+2 dim)

- Left: $\log \left| \left( \epsilon - 3p \right) / \epsilon \right|$ without fluctuations
- Right: $\log \left| \left( \epsilon - 3p \right) / \epsilon \right|$ with fluctuations

**time = 0.35**
Hydro behavior in a toy model (1+2 dim)

- Left: $\log \left| \frac{\epsilon - 3p}{\epsilon} \right|$ without fluctuations
- Right: $\log \left| \frac{\epsilon - 3p}{\epsilon} \right|$ with fluctuations

**time = 1.20**

![Graphs showing hydro behavior in a toy model](image-url)
Hydro behavior in a toy model (1+2 dim)

- Left: \( \log |(\epsilon - 3p)/\epsilon| \) without fluctuations
- Right: \( \log |(\epsilon - 3p)/\epsilon| \) with fluctuations

\[ \text{time} = 4.16 \]
Hydro behavior in a toy model (1+2 dim)

- Left: \( \log |(\epsilon - 3p)/\epsilon| \) without fluctuations
- Right: \( \log |(\epsilon - 3p)/\epsilon| \) with fluctuations

**time = 14.43**
Hydro behavior in a toy model (1+2 dim)

- Left: magnitude of the viscous tensor $\log(\Pi_{\mu\nu}/\epsilon)$
- Right: velocity field
Summary

**Initial state, up to $\tau = 0^+$**

- Consistent framework to include the non-linear saturation effects in heavy ion collisions
- Factorization of the large logs of $x_{1,2}$ into universal distributions $W[\rho]$
- Implies long range rapidity correlations, in good agreement with RHIC data

**Final state evolution**

- Unstable fluctuations need to be resummed, but the machinery for doing that is not fully developed
- An equation of state may be obtained by superimposing quantum fluctuations to the classical fields, without complete thermalization of the system
2-field model

\[ \mathcal{L} = \frac{1}{2} \left[ \dot{\phi}_1^2 + \dot{\phi}_2^2 \right] - \frac{g^2}{4!} \left[ \phi_1^2 + \phi_2^2 \right]^2 \]