Extended Nodal Admittance Matrix Based Stability Analysis of HVDC Connected AC Grids

Yonggang Zhang¹, Daniel Duckwitz¹,², Nils Wiese¹, Martin Braun¹,²

¹Department of Energy Management and Power System Operation, University of Kassel, Kassel, Germany
²Fraunhofer Institute for Energy Economics and Energy System Technology (IEE), Kassel, Germany

Corresponding author: Yonggang Zhang (e-mail: yonggang.zhang@uni-kassel.de).

The work was supported by the German Federal Ministry for Economic Affairs and Climate Action and the Projekträger Jülich within the project "Netzregelung 2.0" (FKZ 0350023C). Only the authors are responsible for the content of this publication. This paper does not necessarily reflect the consolidated opinion of the project consortium "Netzregelung 2.0".

ABSTRACT Motivated by emerging oscillation issues of high-voltage direct-current (HVDC) transmission system resulted from the interaction with AC grids, impedance-based stability analysis becomes an attractive approach in the de-risking studies due to the capability of black-box modelling and clear physical meaning. However, impedance-based stability assessment of HVDC is typically conducted either from the AC side or from the DC side, without fully considering both AC and DC grid dynamics, thus impairs evaluation accuracy. In this paper, the nodal admittance matrix based resonance mode analysis method is extended to include both AC and DC grid dynamics through formulating a hybrid AC-DC admittance matrix. To achieve this goal, HVDC interfacing-converters are modelled as three-port admittance networks with one DC port and two AC ports (in dq-frame) integrating both the AC side frequency coupling and the AC-DC dynamic coupling. In contrast to the widely adopted Nyquist-based methods, the proposed method does not requires knowing the number of right half-plane poles of system minor-loop gain, thus simplifies stability analysis, and moreover, the participation factors of both AC and DC grid nodes in critical resonance modes can be identified through eigenvalue-decomposition analysis, which is in favor of the design of mitigation measures. The effectiveness of the proposed method is validated by EMT-simulations in MATLAB/Simulink.

INDEX TERMS AC-DC dynamic coupling, dynamic mode, damping ratio, HVDC, stability assessment.

I. INTRODUCTION

In order to achieve massive grid integration of renewable energy sources (RES) and long distance power transmission, multiple HVDC links have been constructed and integrated into German grids [1], [2]. Since both HVDC and RES are converter-interfaced components, the fast growth of such power electronics devices in modern power systems have resulted new dynamics and stability issues, and the most commonly encountered one is the wideband resonance stability [3]-[5]. Over the last decade, a series of oscillation incidents involving HVDC have been reported [6], [7]. The recorded oscillation frequencies range from several Hz to above 1 kHz. Typically they are triggered by the change of operating conditions in involved systems [3], [4], [8], such as the grid topology change induced by grid disturbances.

To reveal the mechanisms of such issues and prevent them happening again, e.g. through adopting proper measures in power system planning or operation stages, quite a few studies have been conducted, either through electromagnetic transient (EMT) analysis or analytical stability assessment [4], [7], [8]. Although EMT analysis can usually give plausible stability evaluation results through applying detailed nonlinear grid models, EMT simulation is in most cases time-consuming and incapable of capturing the underlying mechanism of oscillation phenomenon, thus will mainly be used for validation in the paper [9]. As for the analytical stability assessment, several widely used methods are: state-space based method, Nyquist based method and nodal-based Resonance Mode Analysis (RMA) method [5], [10]. For easy understanding, the nodal-based RMA method will be named as Ybus based method. Both Nyquist and Ybus based methods are impedance methods, which are more popular than state-space based method due to the strengths in natural association with physical circuits and the capability of black-box modelling. However, their application in the stability assessment of
HVDC and the resulted hybrid AC/DC grids is typically conducted either in AC side or DC side while partly neglecting or simplifying the dynamics of AC and DC side grids [11]-[14], which may fail to identify the potential arising stability problems when the AC-DC dynamic coupling is strong.

In order to get more complete model for hybrid AC/DC grids including detailed AC and DC grid dynamics that have been partly neglected or simplified in the past, multiport admittance model of HVDC converter that reserves the AC-DC dynamic coupling has been developed [15], [16] and furtherly applied in enhanced Nyquist based stability analysis [17], [18]. However, these methods fail to identify the critical dynamic modes and the participation factors (PFs) of grid components, which hinders the identification of risk sources. This issue was properly treated in the Ybus based method, which uses nodal admittance matrix for stability assessment [19]. But Ybus based method is typically applied to single sequence component system, which neglects the coupling between positive- and negative-sequence impedances [20]. Moreover, the integration of AC-DC dynamic coupling into Ybus based method has not been found in literatures so far. To bridge the above gaps, this paper extends the traditional Ybus based method to integrate full dynamic couplings and analyzes various stability impacts in hybrid AC/DC grids, simultaneously contributing to accurate stability assessment and better support in mitigation design.

The rest of this paper is organized as follows: Section II compares existing stability analysis methods and introduces the proposed enhancement. Section III presents the test grid including the detailed nonlinear model and the small-signal modelling. Section IV shows the effectiveness of the proposed method through case studies. Section V concludes the work.

II. LIMITATIONS IN EXISTING STABILITY ANALYSIS METHODS AND THE PROPOSED ENHANCEMENT

Figure 1. (a) shows a simple hybrid AC-DC grid with embedded HVDC, synchronous machine (SM) and renewable energy source (RES). Two-level voltage-source converter (VSC) based HVDC is adopted in the study. The HVDC is parallel with a high-voltage AC (HVAC) branch. Figure 1. (b) shows the RES and the feeding route, e.g. for representing the application of wind park integration. An aggregated single-converter model was adopted.

Multiple converter-interfaced components and controllers are involved in the system, with the potential for stability-critical interactions. For the stability check of such a test grid, the most widely used analytical methods including the state-space based method, Nyquist based method and Ybus based method are compared in TABLE I. Limitations of the compared methods regarding the state-of-the-art are marked with red letters. Some disadvantages are marked with orange letters.

For the state-space based method, it requires detailed information on the circuit, control and parameters of a system to build system state-space model, which is naturally incapable of black-box modelling and lack of scalability [5], [21]. While for the Nyquist based methods, it is shown in [10] that bode plots can not predict system stability accurately under certain conditions and Nyquist diagrams are not straightforward in checking the right-half plane (RHP) poles and the encirclements of (-1, 0). Moreover, when checking the stability of strongly meshed AC/DC grids using Nyquist based methods, it is difficult to partition the system into source-load subsystems at any point and it may require the stability check at multiple partitioning points [14], which weakens the suitability of the method for complex systems. As for the Ybus based method, it has the best scalability but lacks the integration of asymmetrical control coupling and AC-DC dynamic coupling regarding the state-of-the-art. To simultaneously achieve accurate stability check and stability risk localization for a hybrid AC/DC grid, the Ybus based method will be enhanced to eliminate its limitations as listed in TABLE I.

As the basis for enhancement, the nodal admittance matrix of the test system should be formulated, and before that, all components should be modelled as parallel or series admittance networks. To integrate the dynamic coupling induced by asymmetrical converter control in admittance...
modelling, the AC side of grid components should include positive- and negative-sequence ports when modelled in the natural reference frame, or d- and q-axis ports when modelled under dq-frame [7]. The sequence ports and dq ports can be equivalently transformed to each other through linear transformation and frequency shifting [7]. For the convenience of determining the PFs of d- and q-control loops in the critical dynamic modes, the dq-frame modelling is selected. With the objective of integrating AC-DC dynamic coupling in admittance modelling, each HVDC converter is modelled as a three port hybrid AC/DC admittance network, with two ports at AC side and one port at DC side [17]. Then the SM, RES, AC filters and external grid (as parallel AC grid components) are modelled as two port admittance network. The AC lines and transformers (as series AC grid components) are modelled as four port admittance network. The HVDC cable (as a series DC grid component) is modelled as a two port admittance network.

Since the admittance modelling of active components including SM, RES and HVDC converter is locally linearized, which depends on the angle of the local reference frame, the models developed under local reference frames will be rotated to a defined global reference frame [14]. Note that the modelling of DC components and DC ports of hybrid AC/DC components does not involve dq-frame rotations. In addition, the stationary frame in DC side can be mapped to the dq-frame of AC side without frequency shifting. On this basis, all components can be directly connected in circuits for system level analysis. Nodal admittance matrix of the studied system is formulated in (1), where \( i_{AC1} \) and \( I_{DCS} \) denote the current source injections to the nodes AC1 and DC5, \( u_{AC1} \) and \( U_{DCS} \) denote the voltages of AC1 and DC5, \( Y_{11} \) and \( Y_{55} \) denote the nodal admittances of AC1 and DC5, and other terms follow the same notation.

\[
\begin{bmatrix}
  i_{AC1} \\
  i_{AC2} \\
  i_{AC3} \\
  i_{AC4} \\
  i_{DC5} \\
  i_{DC6} \\
  i_{DC7} \\
  i_{DC8}
\end{bmatrix}
= \begin{bmatrix}
  Y_{11} & 0_{22} & 0_{33} & 0_{23} & 0_{24} & 0_{25} & 0_{26} & 0_{27} \\
  0_{32} & Y_{22} & 0_{33} & 0_{23} & 0_{24} & 0_{25} & 0_{26} & 0_{27} \\
  0_{43} & 0_{34} & Y_{33} & 0_{34} & 0_{35} & 0_{36} & 0_{37} & 0_{38} \\
  0_{45} & 0_{54} & 0_{45} & Y_{44} & 0_{45} & 0_{46} & 0_{47} & 0_{48} \\
  0_{56} & 0_{65} & 0_{56} & 0_{57} & 0_{58} & 0_{66} & 0_{67} & 0_{68} \\
  0_{67} & 0_{76} & 0_{67} & 0_{68} & 0_{69} & 0_{77} & 0_{78} & 0_{79} \\
  0_{78} & 0_{87} & 0_{78} & 0_{79} & 0_{78} & 0_{88} & 0_{89} & 0_{99} \\
  0_{89} & 0_{98} & 0_{89} & 0_{89} & 0_{89} & 0_{99} & 0_{99} & 0_{99}
\end{bmatrix}
\begin{bmatrix}
  u_{AC1} \\
  u_{AC2} \\
  u_{AC3} \\
  u_{AC4} \\
  U_{DC5} \\
  U_{DC6} \\
  U_{DC7} \\
  u_{DC8}
\end{bmatrix}
\]

(1)

In comparison to the commonly used nodal admittance matrix that is formulated using single phase or sequence component variables, the nodal current of each AC node in (1), e.g. \( i_{AC1} \), is a column vector comprised of d- and q-axis currents under global reference frame, e.g. as denoted by \( I_{d1} \) and \( I_{q1} \). Each AC admittance element in the nodal admittance matrix is a 2-order matrix instead of a single quantity. The integration of DC grid nodes in the node-voltage equation gives a hybrid AC-DC nodal admittance matrix.

Having \( L_K \) and \( T_K \) as the left and right eigenvector matrices of \( Y_{kk} \) obtained through matrix decomposition [22], the eigenvalue admittance matrix \( Y_m \) of \( Y_{kk} \) is also obtained, as illustrated in Figure 2. Through setting \( i_m = T_K^*i_K \) and \( u_m = T_K^*u_K \), the system \( i_m = Y_{kk} u_m \) in nodal coordinate system can be transformed to \( i_K = Y_{kk} u_K \) in modal coordinate system. Then inverting the modal admittance matrix \( Y_m \) yields the modal impedance matrix

\[
Z_m = Y_m^{-1} = L_K Y_{kk}^{-1} T_K
\]

(2)

where \( L_K = T_K^{-1} \). Performing frequency scanning to the frequency-dependent modal impedances \( Z_{m1}(s) \), \( Z_{m2}(s) \), ... \( Z_{mn}(s) \), i.e. the diagonal terms of \( Z_m \), the critical dynamic modes can be determined by the peaks of the impedance amplitude versus frequency curves, which is the basis concept of resonance mode analysis (RMA) [19]. The frequencies at curve peaks indicate the frequencies of dynamic modes. The stability of the \( k^{th} \) dynamic mode can firstly be checked by the real part value of the corresponding complex-valued modal impedance at the oscillation frequency \( f_k \), i.e. the \( R(f_k) \) of

\[
Z_{m1}(f_k) = R(f_k) + jX(f_k)
\]

(3)

if \( R(f_k) < 0 \), the dynamic mode is unstable due to negative damping while the dynamic mode is stable if \( R(f_k) > 0 \) due to positive damping, and the damping ratio \( \xi_k \) can be defined as

\[
\xi_k = \frac{1}{2Q_k} = \frac{\Delta f_k}{2f_k}
\]

(4)

where \( Q_k \) denotes the quality factor of \( k^{th} \) resonance circuit, as defined by the frequency-to-bandwidth ratio of the resonance circuit, and \( \Delta f_k \) denotes the resonance bandwidth at half maximum, which is the difference between the two neighbored frequencies (of \( f_k \)) at which the impedance amplitudes are equal to half of the peak amplitude value (at \( f_k \)) [23]. The other common nearly equivalent definition for \( Q \) is the ratio of the energy stored in the oscillating resonance circuit to the energy dissipated per cycle by damping processes [23].

After identifying critical dynamic modes, the system can be transformed from modal coordinate system back to nodal coordinate system, as illustrated in Figure 2. The eigenvector matrix \( T_K \) or \( L_K \) can be used to determine the PFs of both AC and DC grid nodes for the dynamic modes.

An alternative way of dynamic mode identification and stability assessment is to evaluate the poles of the closed loop system [24], [25]

\[
u_K = Y_{kk} (s) \ i_K
\]

(5)

taking the nodal current vector \( i_K \) as input and the bus voltage vector \( u_K \) as output, i.e. solving...
In nodal coordinate system

\[ i_k = Y_{kk} u_k \]

Formulate hybrid AC/DC node voltage equation

\[ Y_{kk} = L_R Y_{TT} \]
\[ i_k = T_k \]

In modal coordinate system

\[ T_k u_k = Y_{kk}^{-1} T_k \]

Identify dynamic modes using \( Z_M \)

\[ Z_{kk} = L_k Z u_k T_k \]

Identify participation factors (PFs) of AC and DC nodes using \( T_k \)

\[ u_k = Z_{kk} i_k \]

FIGURE 2. Schematic diagram of resonance mode analysis (RMA)

| \( \text{det}[Y_{kk}(s)] = 0 \Rightarrow s_j = \sigma_j + j \omega_j \) (k = 1, 2, ..., n) \]

(6)

where \( \sigma_j \) is the attenuation factor of the mode and \( \omega_j = 2\pi f_j \) is the oscillation angular frequency. The damping ratio \( \xi_j \) can be defined as

\[ \xi_j = \frac{-\sigma_j}{\sqrt{\sigma_j^2 + \omega_j^2}} \]

(7)

if \( \sigma_j > 0 \), the dynamic mode is unstable.

Note that the real part \( R(f_j) \) of modal impedance in (3), in which the Laplace operator \( s \) is an imaginary value, and the attenuation factor \( \sigma_j \) of the closed system pole in (6), in which \( s \) is a complex value, are complete different quantities, which should not be mixed up in the formulation of damping ratio.

Moreover, finding the poles of closed-loop system transfer function through solving the roots of \( \text{det}[Y_{kk}(s)] = 0 \) requires that each element of the matrix \( Y_{kk}(s) \) is given in the form of continuous function of frequency [26], which is not the case as illustrated in Appendixes A-C, thus the point-by-point numerical manipulation over frequency and the approximation, e.g. by vector fitting [27], on each set of frequency response by a continuous function of frequency is needed, which is a major disadvantage of this approach. Furthermore, due to the better visualization and physical meaning of impedance versus frequency curves in dynamic mode identification than the poles of closed-loop transfer function, only the (modal impedance) frequency scanning method as illustrated in Figure 2 and Eq. (2)-(4) will be considered for further analysis.

III. GRID MODEL

The 8-bus grid in Figure 1 is adopted for analysis. The component models and their main parameters are given in TABLE II. For the SM, the detailed round rotor synchronous machine model from MATLAB / Simscape library is adopted [28]. For the RES, the virtual synchronous machine (VSM) control, as one of the grid-forming (GFM) control methods, is adopted, as shown in Figure 3. The readers are referred to [29] for the details on the design of the VSM. For the HVDC, the generally used grid-following (GFL) control in real applications is adopted. Detailed circuit and control diagrams of the HVDC converters are shown in Figure 4.

GFL converters use phase-locked loop (PLL) to synchronize with grid, while the GFM converter automatically synchronize with grid through the swing blocks in VSM core, as shown in Figure 3 and Figure 4 individually. For the transformers, the typically used T model is considered, the series branch is described by the short-circuit voltage expressed in percentage (\( u_e \) as resistive part and \( u_x \) as inductive part), and the shunt magnetizing branch is described by paralleled resistance and inductance, both of which are set as 500 pu [7].

With the above configuration, the interaction of GFL converter, GFM converter and SM is involved in the test system. To prepare for the stability analysis, the dq-frame admittance models of SM and RES are derived referring to [30], as presented in Appendix A and Appendix B. Similarly, the three-port admittance modelling of HVDC converters through combining the control and power stage equations (including the AC-DC power balance) is conducted and validated through EMT simulations, as presented in Appendix C. Note that two DC voltage compensation modes for the modulation of HVDC converters are commonly seen in literatures [18], [31], as distinguished by constant DC voltage \( U_{dC} \) compensation and instantaneous DC voltage \( u_{dC} \) compensation, as shown in in Figure 4. Both modes will be considered to show their impact on stability assessment.

| TABLE II. Brief description and main parameters of the grid model |
|---------------------------------------------------------------|
| **Generations** | RES: 650 V, 1.4 GVA, SM: 27 kV, 1.4 GVA (see Appendix A and B for detailed models and parameters) |
| **HVDC converter** | 400 kV (AC) ±350 kV (DC), 1 GW (see Appendix C for detailed models and parameters) |
| **HVDC line** | 100 km DC cable (π model): \( R = 0.0139 \Omega/km, L = 0.159 \text{ mH/km, } C = 231 \text{ nF/km} \) |
| **HVAC line** | 80 km AC lines, 100 km, π model: \( R = 0.025 \Omega/km, L = 0.796 \text{ mH/km, } C = 14 \text{ nF/km} \) |
| **Transformer T1, T2** | 400 / 400 kV, 1 GVA, \( u_e = 0.18\% \), \( u_x = 16.4\% \) |
| **Transformer T3** | 20 / 0.65 kV, 1.5 GVA, \( u_e = 1\% \), \( u_x = 4\% \) |
| **Transformer T4** | 110 / 20 kV, 1.5 GVA, \( u_e = 0.6\% \), \( u_x = 12\% \) |
| **Transformer T5** | 400 / 110 kV, 1.5 GVA, \( u_e = 0.18\% \), \( u_x = 16.4\% \) |
| **HV line - 110 kV** | \( R = 0.04 \Omega, L = 0.414 \text{ mH (lumped RL model)} \) |
| **MV line - 20 kV** | \( R = 0.001 \Omega, L = 3.82 \mu H (lumped RL model) \) |
| **AC Grid-B** | Thevenin circuit with \( X/R = 50 \) and variable capacity |

IV. CASE STUDIES

After validating the derived admittance models of RES, SM and HVDC converters, the nodal admittance matrix of the test grid can be formulated through connecting the grid components into circuits and used for verifying the effectiveness of the proposed method. An overview of the case studies is given in TABLE III.
TABLE III. An overview of the case studies

| Case | Stability impact of weak grid connections for HVDC | Stability impact of different DC voltage compensation modes for the modulation of HVDC converters | Stability impact of the switching states of (single or double) parallel AC line(s) for HVDC | Stability impact of SM/RES in contrast to a simplified Thevenin circuit with same short circuit capacity |
|------|---------------------------------------------------|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Case A | Stability impact of weak grid connections for HVDC | Stability impact of different DC voltage compensation modes for the modulation of HVDC converters | Stability impact of the switching states of (single or double) parallel AC line(s) for HVDC | Stability impact of SM/RES in contrast to a simplified Thevenin circuit with same short circuit capacity |

A. STABILITY IMPACT OF WEAK GRID CONNECTION

Firstly, the simplified grid as shown in Figure 5 is adopted for analysis. The AC Grid-A and AC Grid-B are both modelled as ideal voltage sources with series RL impedances, and they are characterized by the short circuit ratios of SCR-A and SCR-B. The X/R ratio of their RL impedances is fixed to 50. The left side HVDC converter is operated in the active power control mode, and its setpoint is set as $P=0.6$ pu. The right side HVDC converter is operated in the DC voltage control mode. Both converters adopt the instantaneous DC voltage ($u_{dc}$) compensation. Initially, both SCR-A and SCR-B are set to 4, i.e. the short circuit capacities of AC Grid-A and AC Grid-B are 4 GVA, then they are reduced to 2 to show the stability impact of weak grid.

Frequency scanning of the resulted modal impedances $Z_{pq}(s), Z_{qq}(s)$ ... $Z_{mm}(s)$ for SCR-A=2 and SCR-B=2 are presented in Figure 6. Curve peaks in the amplitude plot indicate dynamic/resonance modes, and the corresponding values in the real part plot indicate if the damping is positive or negative. For simple illustration, only two critical dynamic modes at around 7 Hz and 18 Hz are shown, as indicated by the orange and green solid lines for the instantaneous DC voltage ($u_{dc}$) compensation mode. Note that the green solid line partly overlap with the blue dashed line. The positive damping of the 7 Hz resonance mode indicates a damped oscillation and the damping ratio is around $\zeta = 0.7 \text{ Hz} / (2 \times 7 \text{ Hz}) = 0.05$ according to (4). The negative damping of the 18 Hz resonance mode indicates an undamped oscillation, as validated in Figure 7 (a). Note that the unstable resonance mode from analytical results matches the sustaining oscillation waveform due to the controller saturation as oscillation grows.

Node participation factors for each resonance mode (total value is 100 in %) are presented in TABLE IV, from which the domination of the 7 Hz mode by the q-ports of AC3 and AC4, and the domination of the 18 Hz mode by the d-ports of AC7 and AC8 are identified. In other words, the 7 Hz mode is induced by the interaction of the left side HVDC converter (VSC-A) with the weak grid connection (AC Grid-A), mainly involving the q-control loop (including PLL) of VSC-A, while the 18 Hz mode is induced by the interaction of the right side HVDC converter (VSC-B) and the weak grid connection, mainly involving the d-control loop of VSC-B. The contributions of DC nodes to the 18 Hz mode are also indicated by the PFs of the nodes DC5 and DC6. However, the instantaneous DC voltage ($u_{dc}$)
compensation in VSC-A blocks the propagation of the 18 Hz oscillation from AC Grid-A to AC Grid-B.

**B. STABILITY IMPACT OF DC VOLTAGE COMPENSATION MODE**

DC voltage compensation using either the constant value $U_{\text{dc0}}$ or the instantaneous value $u_{\text{dc}}$ is widely used in converter modulation [18], [31]. For the PQ-controlled HVDC converter (VSC-A), replacing instantaneous $u_{\text{dc}}$ compensation with constant $U_{\text{dc0}}$ compensation mainly influences the 7 Hz resonance mode, as indicated by the modal impedances shown in Figure 6. For the $U_{\text{dc0}}$ compensation mode, the damping of the 7 Hz resonance (red dashed line) is positive and the damping ratio is around $\varepsilon = 1\text{ Hz}/(2\times7\text{ Hz}) \approx 0.07$ according to (4), which is slightly larger than the damping ratio of the $u_{\text{dc}}$ compensation mode around 0.05, as calculated in Section IV.C. For simplicity, this statement can be qualitatively obtained by looking at the sharpness of the amplitude curve peaks. A less sharpened amplitude peak indicates larger damping ratio, which can be validated by comparing the EMT simulation results in Figure 7 (a) and (b). Another observation from the EMT simulation in Figure 7 (b) is, the sending end 7 Hz waveform (blue line) in the time range of 3-4 s is superimposed by the 18 Hz oscillation component in the time range of 4-5 s. And this phenomenon can be furtherly explained by the node PF analysis as shown in TABLE IV. When VSC-A adopts $U_{\text{dc0}}$ compensation, the AC Grid-A side nodes also participate the 18 Hz resonance mode, besides, the AC Grid-B side nodes have slight participation in the 7 Hz resonance mode. The observed phenomenon clearly reflects the AC-DC dynamic coupling.

**C. STABILITY IMPACT OF THE PARALLEL AC LINE FOR HVDC**

As already shown in above subsections, the simplified test grid in Figure 5 is stable at SCR-A=4 and SCR-B=4, and becomes unstable when SCR-B is reduced to 2. If increasing SCR-A to 6 and keeping SCR-B as 2, the overall SCR of the HVDC connections is still 8. Under this condition, it will be checked if the 18 Hz resonance mode can be stabilized by connecting the parallel AC line for HVDC. Then the new test grid is illustrated by Figure 8.

Figure 9 shows the modal impedances considering the following Scenarios: (a) no AC line connection; (b) single AC line connection; (c) double AC line connection. All the scenarios have SCR-A=6, SCR-B=2 and $U_{\text{dc0}}$ compensation mode for VSC-A. Note that all the SCR values in the paper scenarios have SCR-A=6, SCR-B=2 and $U_{\text{dc0}}$ compensation mode around 0.05, as calculated in Section IV.C. For simplicity, this statement can be qualitatively obtained by looking at the sharpness of the amplitude curve peaks. A less sharpened amplitude peak indicates larger damping ratio, which can be validated by comparing the EMT simulation results in Figure 7 (a) and (b). Another observation from the EMT simulation in Figure 7 (b) is, the sending end 7 Hz waveform (blue line) in the time range of 3-4 s is superimposed by the 18 Hz oscillation component in the time range of 4-5 s. And this phenomenon can be furtherly explained by the node PF analysis as shown in TABLE IV. When VSC-A adopts $U_{\text{dc0}}$ compensation, the AC Grid-A side nodes also participate the 18 Hz resonance mode, besides, the AC Grid-B side nodes have slight participation in the 7 Hz resonance mode. The observed phenomenon clearly reflects the AC-DC dynamic coupling.

**TABLE IV. Node PFs (%) for different DC voltage compensation modes**

| PF in % | AC3-d/q | AC4-q | DC5 | DC6 | AC7-d/q | AC8-d/q |
|---------|---------|-------|-----|-----|---------|---------|
| SCR-A with $u_{\text{dc}}$ compensation | 14/23 | 23/40 | 0 | 0 | 0 | 0 |
| 7 Hz Mode | 14/23 | 23/40 | 0.1 | 0.1 | 0.3 | 0.2 | 0.2 | 0.1 |
| SCR-A with $U_{\text{dc0}}$ compensation | 3/0 | 4/0 | 7 | 7 | 23/28 | 13/15 |
| 18 Hz Mode | 3/0 | 4/0 | 7 | 7 | 23/28 | 13/15 |

compensation in VSC-A blocks the propagation of the 18 Hz oscillation from AC Grid-A to AC Grid-B.

**B. STABILITY IMPACT OF DC VOLTAGE COMPENSATION MODE**

DC voltage compensation using either the constant value $U_{\text{dc0}}$ or the instantaneous value $u_{\text{dc}}$ is widely used in converter modulation [18], [31]. For the PQ-controlled HVDC converter (VSC-A), replacing instantaneous $u_{\text{dc}}$ compensation with constant $U_{\text{dc0}}$ compensation mainly influences the 7 Hz resonance mode, as indicated by the modal impedances shown in Figure 6. For the $U_{\text{dc0}}$ compensation mode, the damping of the 7 Hz resonance (red dashed line) is positive and the damping ratio is around $\varepsilon = 1\text{ Hz}/(2\times7\text{ Hz}) \approx 0.07$ according to (4), which is slightly larger than the damping ratio of the $u_{\text{dc}}$ compensation mode around 0.05, as calculated in Section IV.C. For simplicity, this statement can be qualitatively obtained by looking at the sharpness of the amplitude curve peaks. A less sharpened amplitude peak indicates larger damping ratio, which can be validated by comparing the EMT simulation results in Figure 7 (a) and (b). Another observation form the EMT- simulation in Figure 7 (b) is, the sending end 7 Hz waveform (blue line) in the time range of 3-4 s is superimposed by the 18 Hz oscillation component in the time range of 4-5 s. And this phenomenon can be furtherly explained by the node PF analysis as shown in TABLE IV. When VSC-A adopts $U_{\text{dc0}}$ compensation, the AC Grid-A side nodes also participate the 18 Hz resonance mode, besides, the AC Grid-B side nodes have slight participation in the 7 Hz resonance mode. The observed phenomenon clearly reflects the AC-DC dynamic coupling.
FIGURE 9. Modal impedances of the system by: no parallel AC line (solid line), single AC line (dashed line), double AC line (dotted line)

FIGURE 11. Modal impedances when AC Grid-A is modelled as Thevenin circuit with SCR-A=2.8 or as SM and RES

FIGURE 12. EMT simulations with different models of AC Grid-A

TABLE V. PFs (%) of the 18 Hz mode for different AC line cases

| PF in % | AC3-d/q | AC4-d/q | DC5 | DC6 | AC7-d/q | AC8-d/q |
|---------|---------|---------|-----|-----|---------|---------|
| No      | 0.4 / 0.1 | 1.2 / 0.34 | 5.3 | 5   | 32.7 / 23.3 | 18.5 / 13.2 |
| Single  | 5.3 / 7.6 | 5.4 / 14.3 | 2.5 | 2.2 | 16.9 / 21.1 | 11.4 / 13.4 |
| Double  | 6.7 / 8.9 | 7.2 / 16.6 | 2.6 | 2.3 | 15.6 / 18.4 | 10 / 11.7 |

TABLE VI. PFs (%) of the 18 Hz mode for SCR-B=2 and different AC Grid-A models

| SCR-B | AC1-d/q | AC2-d/q | AC3-d/q | AC4-d/q | DC5 | DC6 | AC7-d/q | AC8-d/q |
|-------|---------|---------|---------|---------|-----|-----|---------|---------|
| Thevenin circuit | 7.6 / 7.4 | 7.5 / 12 | 6.4 | 5.4 | 24 / 11.5 | 10.5 / 7.8 |
| SM / RES | 1.1 / 1.4 | 4.4 / 4.5 | 7.1 / 9 | 6.5 / 14.9 | 2.2 | 1.9 | 12.7 / 14.8 | 9.1 / 10.5 |

The variation of SCR-B from 4 to 2 does not destabilize the system. This stability assessment result is furtherly validated by the EMT simulations as shown in Figure 12. Note that the 7 Hz resonance mode observed in Section IV.A and IV.B has not been observed again as the strength of AC Grid-A has not been decreased down to SCR-A=2 again and the parallel AC lines of HVDC are connected in the system.

For the scenarios with SCR-B=2, the PFs of grid nodes are presented in TABLE VI. After replacing the AC Grid-A (Thevenin circuit) with SM and RES, the left side AC grid has more participation in the 18 Hz dynamic mode as the summed PF of AC1-AC4 becomes larger, while the AC-DC dynamic coupling becomes weaker as indicated by the smaller PFs of the DC grid nodes. Another finding from the PF results is, the SM branch participates several times more than the RES branch in the 18 Hz dynamic mode.

D. STABILITY IMPACT OF SM AND RES

In above analysis, the left and right side external grids are both modelled as Thevenin circuits. To create a more practical impedance profile, the AC Grid-A is replaced by SM and RES (see Figure 1). And the influence of replacing the Thevenin circuit (with SCR-A=2.8) with SM and RES (in total of 2.8 GVA capacity) is analyzed for the scenarios of varying SCR-B from 4 to 2. Note that the parallel double AC lines of the HVDC link are kept connected in the system. Figure 11 shows the analytical results. When AC Grid-A is modelled as Thevenin circuit, varying SCR-B from 4 to 2 distabilize the system at the 18 Hz dynamic mode. In comparison to Section IV.C, the strength of AC Grid-A is set as SCR-A=2.8 (unstable) instead of SCR-A=6 (stable). After replacing Thevenin circuit with SM / RES, coupling in HVDC is weakened by the parallel AC line in this case.
furtherly increasing the RES penetration, the stabilizing effect from SM and RES in this case may become weakened. Further analysis on this effect will be presented in a separate paper.

E. DISCUSSION OF OSCILLATION MITIGATION DESIGN
In above subsections, participation factor analysis for the dynamic modes of a simple AC/DC grid was conducted considering different stability impacts. According to the results in TABLE IV - TABLE VI, the grid nodes AC3 and AC4 have the dominating PFs for the around 7 Hz dynamic mode and the grid nodes AC7 and AC8 have the dominating PFs for the around 18 Hz dynamic mode in most grid conditions as shown in Figure 1, Figure 5 and Figure 8, thus an oscillation mitigation scheme might be preferably configured at these grid nodes [9]. A very effective way is to add active damping control in the HVDC converters at AC3 and AC7, so that no additional passive or active damping devices should be installed there [7], [20]. Moreover, proper damping design in HVDC converters can de-risk the potential oscillation stability for a wide frequency range until the limits of their PWM modulation capability are reached [8], [32], [33].

Apart from the mitigation design in HVDC converters, the damping capability of the RES at AC3, i.e. the active damping in the VSM control scheme here, could also be fully deployed to furtherly increase the overall damping level of the system [29]. However, careful design should be conducted to prevent potential control interactions.

V. CONCLUSION
A straightforward nodal admittance matrix based method, that integrates the detailed dynamics of AC and DC grids as well as AC-DC dynamic coupling, is proposed to assess the stability of converter-interfaced hybrid AC/DC grids. The proposed method not only prevents knowing the number of RHP poles of system minor loop gain as required by Nyquist based method, but also helps identify dynamic modes and PFs of grid components. It is especially advantageous compared to state-space based method, because it does not strictly rely on white-box models, the partial admittance models can also be measured or obtained from product vendors.

The effectiveness of the proposed method is illustrated by case studies with a simple hybrid AC/DC grid consisting embedded HVDC, SM and RES. The stability impacts of varying grid strengths, converter DC voltage compensation mode and switching states of parallel AC line as well as the model variation of AC Grid-A are analyzed. Influences on the AC-DC dynamic coupling are analyzed through grid node PF analysis. It is shown, the GFL controlled HVDC faces potential resonance risk from weak grid connection, and the observed AC-DC dynamic coupling from HVDC converters can introduce the interaction of the connected AC grids. Besides, modelling the connected AC grid of an HVDC converter as simplified Thevenin circuit greatly changes the stability assessment result compared to detailed AC grid model. Moreover, the GFL and GFM converters and SM can operate stably in the considered conditions, and the SM branch participates more than the VSM-controlled RES branch in stabilizing the system in the case study.

In the future, highly meshed AC/DC grids are expected, and the AC-DC dynamic coupling effect is expected to get stronger when HVDC is configured with GFM control (due to the conflict between AC side power availability and DC voltage control) or in a multi-terminal HVDC system with drooped DC voltage control. Moreover, the ever-growing complexity of DC grid may introduce new dynamic modes and/or potentially have larger impact on the dynamic modes which are presently dominated by AC grid, i.e. some dynamic mode could be simultaneously dominated by both AC and DC grid nodes. The proposed method could be used to analyze the stability risks from such configurations. Furthermore, as the study case is a small system, whether there is any limitation of the proposed method from system size or not would be addressed in future works.

APPENDIX
A. SM MODEL
A round rotor SM is considered. The machine has one damping winding in the direct-axis (subscript D) and two damping windings in the quadrature-axis (subscript Q1 and Q2). The field winding is oriented in the direct-axis. When parameterized with fundamental parameters and written in per-unit form, the flux equations of the SM are formulated in (8), where $\omega_b$ and $\omega$ denote the base and rotor angular frequency, the subscripts d, q represent stator d-axis and q-axis under the rotating rotor dq-frame, D, Q denote rotor direct- and quadrature-axis, f denotes the field winding in direct-axis, $L_d$ and $L_q$ are synchronous inductances, $L_{q1}$, $L_{q2}$ and $L_{q3}$ are the self-inductances of the field winding and damper windings in direct and quadrature axes, $L_{sd}$ and $L_{sq}$ are d and q axis mutual inductances, which satisfy $L_{sd} = L_{s} + L_{ad}$, $L_{sq} = L_{s} + L_{aq}$ and $L_{dq} = L_{s} + L_{ad}$, with $L_{s}$ denoting stator leakage inductance and $L_{qf}$ denoting rotor field circuit inductance.

\[
\begin{align*}
\psi_d &= -R_i i_d + \psi_d / \omega_b - \omega \psi_q / \omega_b \\
\psi_q &= -R_i i_q + \psi_q / \omega_b - \omega \psi_d / \omega_b \\
\psi_f &= R_f i_f + \psi_f / \omega_b \\
\psi_q1 &= -L_{sd} i_q + L_{q1} i_d + L_{qf} i_d \\
\psi_q2 &= -L_{sq} i_q + L_{q2} i_d + L_{qf} i_d \\
\psi_d &= -L_{ad} i_d + L_{qf} i_d + L_{qf} i_d
\end{align*}
\]

(8)

Swing and electrical torque equations in per-unit form are:

\[
J \ddot{\theta} = T_m - T_e - D \omega, \quad \dot{\theta} = \omega - \omega_b, \quad T_e = \psi_d i_d - \psi_q i_q
\]

(9)

where $T_e$ and $T_m$ are the electrical and mechanical torques, $\theta$ is the rotor angle, and $\omega_b$ is the per-unit steady-state.
frequency. On the basis that the prime-mover’s speed governor is slow compared to the fast transients under consideration, $T_{in}$ is assumed to be constant. Moreover, the exciter and power system stabilizer (PSS) dynamics are disregarded, resulting a constant field voltage $u_c$, if desired, these dynamics can be included in the model [34]. Then performing small-signal analysis to (8) and (9), rewriting them in Laplace-form and solving rotor speed dynamic from the swing and electrical torque equations yield

$$\Delta u_s = -Z_{ss} \Delta i_s - \Delta \omega L_{i} i_s + \Delta \omega L_{ss} i_{10}$$

$$0 = -Z_{ss} \Delta i_s + Z_{ss} \Delta i_s$$

(10)

where

$$Z_{ss} = \begin{bmatrix} R_t + L_s s & \omega_b \omega_{Lq} & 0 & 0 & -L_{dq} & -L_{di} \\ \omega_b \omega_{Lq} & R_t + L_s s & 0 & 0 & -L_{di} & -L_{dq} \\ 0 & 0 & L_{di} & L_{dq} & 0 & 0 \\ 0 & 0 & -L_{dq} & -L_{di} & L_{di} & L_{dq} \\ 0 & 0 & 0 & 0 & R_{q1} + L_s s & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{q1} + L_s s \\ L_{sR} \omega_b & L_{sR} \omega_b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b \\ 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b \end{bmatrix}$$

$$L_{mv} = \begin{bmatrix} \omega_b L_{mq} & \omega_b L_{mq} & 0 & 0 & -L_{dq} & -L_{di} \\ \omega_b L_{mq} & \omega_b L_{mq} & 0 & 0 & -L_{dq} & -L_{di} \\ 0 & 0 & L_{di} & L_{dq} & 0 & 0 \\ 0 & 0 & -L_{dq} & -L_{di} & L_{di} & L_{dq} \\ 0 & 0 & 0 & 0 & R_{q1} + L_s s & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{q1} + L_s s \\ 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b \\ 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{sR} \omega_b & L_{sR} \omega_b \end{bmatrix}$$

The symbol $\Delta$ denotes small-signal value, $\Delta u_s = \begin{bmatrix} \Delta u_{sR} \\ \Delta u_{sQ} \\ \Delta u_{ss} \end{bmatrix}$ and $\Delta i_s = \begin{bmatrix} \Delta i_{sR} \\ \Delta i_{sQ} \\ \Delta i_{ss} \end{bmatrix}$ denote the column vectors of stator small-signal voltages and currents, $\Delta i_s = \begin{bmatrix} \Delta i_{sR} \\ \Delta i_{sQ} \end{bmatrix}$ denotes the column vector of rotor small-signal currents, $i_{sR}$ and $i_{sQ}$ are the steady-state values of $i_s$. Jointly solving the small-signal equations in (10) gives the SM impedance under local rotor rotating dq-frame

$$\Delta u_{sdq} = -Z_{dq} \Delta i_{dq} = Z_{dq}^* \Delta i_{dq}$$

$$Z_{dq} = Z_{ss} - s \omega_b \omega_{Lq} + \frac{G_m}{s} \omega_b \omega_{Lq} \begin{bmatrix} C_2 + 2L_{逃避} \frac{Z_{逃避}}{Z_{逃避}} \end{bmatrix} \begin{bmatrix} L_{逃避} \frac{i_{逃避}}{L_{逃避}} - L_{逃避} \frac{i_{逃避}}{L_{逃避}} \end{bmatrix}$$

Then $Z_{dq}$ is furtherly transformed to the local steady-state dq-frame through rotating back the angle deviation $\delta_0$ between rotor and stator, as formulated in

$$Z_{dq} = T_{dq}^{-1} Z_{dq}^* T_{dq} \text{ with } T_{dq} = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \delta_0 & \sin \delta_0 \\ -\sin \delta_0 & \cos \delta_0 \end{bmatrix}$$

(12)

where $Z_{dq}$ is the SM output impedance under system dq-frame. The corresponding admittance can be simply obtained by matrix inversion. The linearized impedance model is then validated using detailed time-domain model from MATLAB / Simscape, the validation results are omitted for saving space. The adopted standard parameters, as given in TABLE VII, can be transformed to the fundamental parameters as used in the derived model according to [35].

### Table VII. SM standard parameters in per-unit values

| Voltage / Capacity: 27 kV / 1.4 GVA (base system), $J = 10$ s, $D = 0.01$ |
|---|---|
| $X_{r1} = 2.63, X_{r2} = 2.57, X_{r0} = 0.34, X_{r0}'' = 0.54, X_{r0}'''' = 0.26, X_{r0}''' = 0.27, X_{r0}'' = 0.228$ |
| $R = 0.0021, T_y = 0.778, T_y'' = 0.165, T_y''' = 0.016, T_y'''' = 0.0016$ |

### B. RES MODEL

The VSM model in [29] is applied for the RES generation, as shown in Figure 3. The VSM is composed of a VSM core, a virtual circuit and an active damping. The VSM core is used to provide a reference for the inner source voltage. The virtual circuit introduces a virtual impedance in series to the RES output filter, which increases the damping of the dynamic modes around nominal frequency. The active damping employs a lead-lag filter to counteract the high-frequency LC oscillation from the AC filter.

The instantaneous power measured with phase (abc) quantities is equivalent to

$$p = i_d u_d + i_q u_q$$

(13)

where $u_{d0} = \begin{bmatrix} u_d \\ u_q \end{bmatrix}$ and $i_{d0} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$ are the column vectors of the system dq-frame current and voltages. The Q & V droop control in VSM core is deactivated by setting $k_v = 0$. Linearizing the active power equation for the operating point $u_0 = \begin{bmatrix} U_{d0} \\ U_{q0} \end{bmatrix}$ and $i_0 = \begin{bmatrix} I_{d0} \\ I_{q0} \end{bmatrix}$ yields

$$\Delta p = i_{d0}^T \Delta u_{d0} + i_{q0}^T \Delta u_{q0}$$

(14)

Then linearizing the swing equation

$$\omega = \frac{(P_{ref} - p - p_{pr})}{(T_{pr}s)} = k_{pr}(\omega - \omega_{ref})$$

(15)

and substituting $\Delta p$ into it leads to

$$\Delta \theta = G_{sync}(i_{d0}^T \Delta u_{d0} + i_{q0}^T \Delta u_{q0})$$

(16)

Note that the $P / f$ droop control is configured to work under the limited frequency sensitive mode at over frequency (LFSM-O). When the frequency is within 50.2 Hz, this control function is not activated, thus can be considered as $k_{pr} = 0$. Then performing small-signal analysis to the inner voltage control, virtual circuit and active damping branches yields

$$\Delta m_{dq} = \Delta e_{dq} - G_{vc} \Delta v_{c} + G_{ad} \Delta u_{dq}$$

(17)

where the superscript c indicates the variables under control dq-frame. The transformation between control dq-frame and system dq-frame can be illustrated by

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos(\Delta \theta) & \sin(\Delta \theta) \\ -\sin(\Delta \theta) & \cos(\Delta \theta) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} = T_{dq} \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

(18)
where the angle deviation $\Delta \theta$ equals to the VSM core angle $\theta$ minus the grid voltage angle $\theta_0$. Applying this transformation to current, voltage and modulation index yields

$$\begin{align*}
\Delta m_{dq} &= -C_2 m_0 \Delta \theta + \Delta m_{d_0}^c \\
\Delta \theta_{d_0} &= C_2 u_{dq} \Delta \theta + \Delta u_{d_0}^c \\
\Delta \theta_{d_0} &= C_2 i_{dq} \Delta \theta + \Delta i_{d_0}^c
\end{align*}$$

$\Delta m_{dq}$, $\Delta \theta_{d_0}$, and $\Delta i_{d_0}^c$ follow the same notations. Then the power-stage small-signal equation can be written as

$$\Delta \Delta m_{d} = -C_2 m_0 \Delta \theta + \Delta m_{d_0}^c$$

$\Delta \Delta \theta_{d_0} = C_2 u_{dq} \Delta \theta + \Delta u_{d_0}^c$

$\Delta \Delta i_{d_0}^c = C_2 i_{dq} \Delta \theta + \Delta i_{d_0}^c$

where $\Delta m_0$ denotes the steady state values of $m_{d_0}$, and $u_0$ and $i_0$ follow the same notations. Then the power-stage small-signal equation can be written as

$$e^{-sT_{in}}(1-e^{-sT_{in}}) \Delta m_{dq} = -R_s L_c / \omega_b - L_v + R_s L_c / \omega_b \Delta \Delta m_{dq}$$

where $G_D$ models the control delay and sampling process. Jointly solving the above linearized equations yields

$$\Delta \Delta m_{dq} = \left(1 - G_D G_s \right) Z_{dq} - G_S V_{VSM} \frac{z_{dq}}{z_m}$$

The derived output impedance of VSM under system dq-frame $Z_{dq}$ is validated using detailed nonlinear EMT model with the parameters as given in TABLE VIII. Due to space reason, the validation results are not presented here.

### TABLE VIII. Parameters of the aggregated VSM-controlled RES

| Voltage / Capacity | 650 V(AC)/1800 V(DC) / 1.4 GVA (base system for AC circuits) |
|--------------------|---------------------------------------------------------------|
| $R_s$              | 8.5 $\mu$Î2, $L_c = 0.085 \mu$Î2, $R_s = 8.5 $\mu$Î2, $C_s = 0.2769$ F |
| VSM core           | $k_0 = 20$, $k_g = 0.125$, $T_s = 10$, $k_0 = 10$, $T_s = 1$ |
| Virtual circuit    | $r_s = 0.13$, $x_s = 0.11$, $x_s = 0.008$ |
| Active damping     | $k_{act} = 1$, $k_{act} = 0$, $k_{act} = 1500 - 2t$ |

### C. HVDC Converter Model

Circuit and control diagrams of HVDC converters are shown in Figure 4. For both converters, the transfer function of the standard SRF-PLL is

$$\theta_{PLL} = \omega_{PLL} / s = u_q^c H_{PLL} (s) / s$$

where $H_{PLL}(s)$ denotes the transfer function of the PI controller in PLL. Assuming a small-signal perturbation $\Delta \theta_{PLL}$ on the PLL angle, the variables in system dq-frame can be rotated to the control dq-frame by multiplying

$$T_{dq} = \begin{bmatrix} \cos(\Delta \theta_{PLL}) & \sin(\Delta \theta_{PLL}) \\ -\sin(\Delta \theta_{PLL}) & \cos(\Delta \theta_{PLL}) \end{bmatrix} = \begin{bmatrix} 1 & \Delta \theta_{PLL} \\ -\Delta \theta_{PLL} & 1 \end{bmatrix}$$

Then we will have

$$\begin{align*}
\Delta \theta_{dq} &= T_{dq} \Delta \theta_{PLL} \\
i_{dq} &= T_{dq} i_{PLL} \\
m_{dq} &= T_{dq} m_{PLL}^c
\end{align*}$$

Performing small-signal analysis to above equations and jointly solving them yields

$$\Delta \theta = G_{PLL} \Delta u_{dq}, G_{PLL} = H_{PLL} / (s + U_{grid} H_{PLL}), \Delta u_{dq}^c = G_{PLL}^c \Delta m_{dq}$$

$$G_{PLL} = \begin{bmatrix} G_{PLL} & \Delta \theta_{dq} \\ \Delta \theta_{dq}^c & \Delta \theta_{dq} \end{bmatrix}$$

$\Delta \theta_{dq}$, $\Delta \theta_{dq}^c$, $\Delta \theta_{dq}$ denote the steady state voltages, currents and modulation indexes and can be written in vector form as

$$\Delta \theta_{dq} = G_{PLL} \Delta \theta_{dq}^c + \Delta \theta_{dq}$$

The power-stage and control equations can be written in small-signal form as

$$\Delta \theta_{dq} = \Delta \theta_{dq} = \Delta \theta_{dq}$$

where $H_{PLL}$ describes the current PI controller and $u_{dq}$ describes the modulated converter voltage as represented by

$$\begin{align*}
u_{dq} &= G_{PLL} u_{dq} + \Delta \theta_{dq} \\
u_{dq}^c &= G_{PLL} u_{dq} + \Delta \theta_{dq}^c
\end{align*}$$

for the $u_{dq}$ and $U_{dq}$ compensation mode respectively. Linearizing them leads to

$$\Delta \theta_{dq} = G_{PLL} \Delta \theta_{dq}^c, \Delta \theta_{dq} = \Delta \theta_{dq}^c + \Delta \theta_{dq}$$

Then assuming an AC-DC power lossless conversion, we have

$$p = i_q u_{dq} + i_d u_{dq}^c = i_q u_{dq} - 0.5 C_{dc} u_{dq}^2 / \omega_b$$

which can be linearized to

$$i_q^0 \Delta u_{dq} + u_{dq}^c \Delta \theta_{dq} = i_q^0 \Delta u_{dq} + i_d^0 \Delta u_{dq} - C_{dc} \Delta u_{dq}^2 / \omega_b$$

Fort the power control mode converter, it is set $\Delta \theta_{dq}^c = 0$. For the voltage control mode converter, the current reference dynamics can be written as

$$\begin{align*}
\Delta i_{dq} &= \begin{bmatrix} 0_{4 \times 2} \\ -H_{Vdq} \eta_i \delta_{G_{PLL}} \end{bmatrix} \delta_{i_{dq}} + \Delta i_{dq} \\
\Delta i_{dq} &= \begin{bmatrix} Y_{dq} & Y_{dq} \delta_{G_{PLL}} \\ Y_{dq} \delta_{G_{PLL}} & Y_{dq} \delta_{G_{PLL}} \end{bmatrix} \begin{bmatrix} Y_{dq} \delta_{G_{PLL}} \\ Y_{dq} \delta_{G_{PLL}} \delta_{G_{PLL}} \end{bmatrix} \delta_{u_{dq}}
\end{align*}$$

where $I_0$ denotes the second-order identity matrix and

$$\begin{align*}
\eta_i &= (Z_{VSM} + G_{PLL} C_{dc}) \begin{bmatrix} 1 \end{bmatrix} \\
\eta_i &= \begin{bmatrix} 0 \ end{bmatrix} \\
y_{dq} &= \begin{bmatrix} i_q^0 \ i_d^0 \end{bmatrix} \quad Z_{VSM} + G_{PLL} C_{dc} \delta_{u_{dq}} / \omega_b
\end{align*}$$

for the power control mode converter while

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109ACCESS.2023.3177232, IEEE Access
\[
 Y_{dqc} = (Z_{RL} + G_{CC}G_{iCC})^{-1} \left( I_{2} - G_{0}(G_{CC}^{*} + H_{CC}A_{2} - G_{CC}^{*}G_{PLL}^{*}) \right)
\]

\[
y_{dqc} = -Z_{RL} + G_{0}G_{CC}G_{iCC} \frac{G_{PLL}}{\omega_{b}}
\]

\[
y_{dqc1} = (i_{d}^{T} Z_{RL} + m_{i}^{T} G_{0})Y_{dqc} + I_{d0} / U_{d0} + C_{dc} / \omega_{b}
\]

\[
y_{dqc2} = -(i_{q}^{T} Z_{RL} + m_{q}^{T} G_{0})Y_{dqc} + I_{q0} / U_{d0} + C_{dc} / \omega_{b}
\]

(36)

for the voltage control mode converter. Comparing (35) and (36), the power control mode has weaker AC-DC coupling than the voltage control mode converter. However, if replacing \( u_{dc} \) compensation with constant DC voltage \( (U_{d0}) \) compensation in the power control mode converter, we have

\[
y_{dqc} = -(Z_{RL} + G_{0}G_{CC}G_{iCC})^{-1} m_{0} / U_{d0}
\]

\[
y_{dqc1} = -(i_{d}^{T} Z_{RL} + m_{d}^{T} G_{0})Y_{dqc} + I_{d0} / U_{d0} + C_{dc} / \omega_{b}
\]

(37)

which strengthens the AC-DC coupling. Similarly, the admittance model of the voltage control mode converter can be adapted for \( U_{d0} \) compensation. Note that the per-unit base system for the AC and DC pu quantities of above models are chosen in the way to achieve invariant AC-DC power transformation [7].

Using the parameters as given in TABLE IX, the three-port admittance model in (34) is validated through EMT simulation. As an example, the validation of the power control model converter with \( U_{d0} \) compensation is presented in Figure 13. The impedance amplitudes are shown in dB of Ohms and the impedance angles are shown in degrees, as denoted by \(^{\circ}\). The observed error around 2 Hz may come from the inaccurate frequency extraction. As the low frequency near 0 Hz in dq-frame corresponds to a frequency around the nominal grid frequency 50 Hz in the phase or sequence domain. It is difficult to accurately extract a small-signal frequency component near 50 Hz from a measured signal dominated by the fundamental frequency component.

**TABLE IX. Parameters of HVDC converters**

| HVDC converter VSCA | AC circuits | \( R_{L} = 0.45 \Omega, L_{c} = 0.143 \text{ mH} \) |
|---------------------|------------|--------------------------------------------------|
| Current control     | \( H_{cc} = 0.422 + 39.38 / s, K_{c} = L_{c} \) |
| PLL PI controller   | \( H_{cc} = 50 + 625 / s \) (same for VSC-B) |

| HVDC converter VSC-B | AC circuits | \( R_{L} = 0.45 \Omega, L_{c} = 0.143 \text{ mH} \) |
|----------------------|------------|--------------------------------------------------|
| Current control      | \( H_{cc} = 0.422 + 39.38 / s, K_{c} = L_{c} \) |
| Voltage control      | \( H_{cc} = 15 + 1050 / s, H_{cc} = -5 / (1 + 0.01 s) \) |

**ACKNOWLEDGMENT**

The authors would like to acknowledge the financial support received from the University of Kassel, funds for open access publications, Kassel, Germany. The authors are fully responsible for the content of this publication.

**REFERENCES**

[1] X. Liu, “Control of Voltage Source Converter Based High Voltage Direct Current Transmission Systems for Grid Code Compliance,” Diss., Institute of Electric Power Systems, Otto-von-Guericke-University Magdeburg, 2016.

[2] S. Rath, D. Beck, T. Bongers and I. Sander, “ALEGrO – Market Integration and System Operation Aspects of the new HVDC link between Germany and Belgium,” ETG Congress 2021, 2021, pp. 1-6.

[3] C. Buchhagen, C. Rauscher, A. Menze, and J. Jung, “BorWin1 – first experiences with harmonic interactions in converter dominated grids,” in Proceedings of International ETG Congress 2015, 2015, pp. 1-7.

[4] C. Buchhagen, M. Greve, A. Menze, and J. Jung, “Harmonic stability - practical experience of a TSO,” in Proceedings of the 15th Wind Integration Workshop, 2016, pp. 1-6.

[5] X. Wang and F. Blaabjerg, “Harmonic Stability in Power Electronics-Based Power Systems: Concept, Modeling, and Analysis,” in IEEE Transactions on Smart Grid, vol. 10, no. 3, pp. 2858-2870, May 2019.

[6] C. Yin, X. Xie, S. Xu and C. Zou, “Review of oscillations in VSC-HVDC systems caused by control interactions”, The Journal of Engineering, vol. 2019, issue 16, pp. 1204-1207.

[7] Y. Zhang, “Analysis and Control of Resonances in HVDC Connected DFIG-based Offshore Wind Farm,” Dissertation, Institute of Electric Power Systems, Otto-von-Guericke-University Magdeburg, 2021.

[8] C. Zou et al., “Analysis of Resonance Between a VSC-HVDC Converter and the AC Grid,” in IEEE Transactions on Power Electronics, vol. 33, no. 12, pp. 10157-10168, Dec. 2018.

[9] Y. Zhang, C. Klabunde and M. Wolter, “Harmonic Filtering in DFIG-based Offshore Wind Farm through Resonance Damping,” 2019 IEEE PES Innovative Smart Grid Technologies Europe (ISGT-Europe), 2019.

[10] L. Fan and Z. Miao, “Admittance-Based Stability Analysis: Bode Plots, Nyquist Diagrams or Eigenvalue Analysis?,” in IEEE Transactions on Power Systems, vol. 35, no. 4, pp. 3312-3315, July 2020.

[11] Ö. C. Sakinci et al, “Generalized Dynamic Phasor Modeling of the MMC for Small-Signal Stability Analysis,” in IEEE Transactions on Power Delivery, vol. 34, no. 3, pp. 991-1000, June 2019.

[12] K. Ji, G. Tang et al, “Harmonic Stability Analysis ofMMC-Based DC System Using DC Impedance Model,” in IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 8, no. 2, June 2020.

[13] Y. Chen, L. Xu, A. Egea-Álvarez et al., “MMC Impedance Modelling and Interaction of Converters in Close Proximity,” in
IEEE Journal of Emerging and Selected Topics in Power Electronics, Oct. 2020.

[14] C. Zhang et al., “Impedance-Based Analysis of Interconnected Power Electronics Systems: Impedance Network Modeling and Comparative Studies of Stability Criteria,” in IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 8, no. 3, Sept. 2020.

[15] S. Shah and L. Parsa, “Impedance Modeling of Three-Phase Voltage Source Converters in DQ, Sequence, and Phasor Domains,” in IEEE Transactions on Energy Conversion, vol. 32, no. 3, pp. 1139-1150, Sept. 2017.

[16] J. Sun, “Two-Port Characterization and Transfer Impedances of AC-DC Converters Part I: Modelling,” in IEEE Open Journal of Power Electronics, 2021.

[17] J. Pedra et al., “Three-Port Small Signal Admittance-Based Model of VSCs for Studies of Multi-Terminal HVDC Hybrid AC/DC Transmission Grids,” in IEEE Transactions on Power Systems, vol. 36, no. 1, pp. 732-743, Jan. 2021, doi: 10.1109/TPWRS.2020.3003835.

[18] H. Zhang et al., “Impedance Analysis and Stabilization of Point-to-Point HVDC Systems Based on a Hybrid AC-DC Impedance Model,” in IEEE Transactions on Industrial Electronics, vol. 68, no. 4, pp. 3224-3238, April 2021, doi: 10.1109/TIE.2020.2978706.

[19] W. Xu et al., “Harmonic resonance mode analysis,” in IEEE Transactions on Power Delivery, vol. 20, no. 2, pp. 1182-1190, April 2005.

[20] Y. Zhang, C. Klabunde and M. Wolter, “Study of Resonance Issues between DFIG-based OWF and HVDC Transmission,” in Electric Power Systems Research, vol. 190, article 106767, Jan. 2021.

[21] F. Thams, R. Eriksson and M. Molinas, “Interaction of Droop Control Structures and Its Inherent Effect on the Power Transfer Limits in Multiterminal VSC-HVDC,” in IEEE Transactions on Power Delivery, vol. 32, no. 1, pp. 182-192, Feb. 2017, doi: 10.1109/TPOWER.2016.2600028.

[22] R. Bellman, Introduction to Matrix Analysis, 2nd ed. New York: McGraw-Hill, 1970.

[23] Green, Estill I. (October 1955). "The Story of Q" (PDF). American Scientist. 43: 584-594. Archived (PDF) from the original on 2012-12-03. Retrieved 2012-11-21.

[24] E. Ebrahizadeh, F. Blaabjerg, X. Wang and C. L. Bak, "Bus Participation Factor Analysis for Harmonic Instability in Power Electronics Based Power Systems," in IEEE Transactions on Power Electronics, vol. 33, no. 12, pp. 10341-10351, Dec. 2018, doi: 10.1109/TPWEL.2018.2803846.

[25] G. He, W. Wang and H. Wang, "Coordination control method for multi-wind power systems to prevent sub/super-synchronous oscillations," in CSEE Journal of Power and Energy Systems, doi: 10.17775/CSEEJPS.2020.06550.

[26] J. Sun, “Frequency-Domain Stability Criteria for Converter-Based Power Systems,” in IEEE Open Journal of Power Electronics, doi: 10.1109/JOPEL.2022.3155568.

[27] B. Grøtnes and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp. 1052-1061, July 1999.

[28] P. Kundur, Power system stability and control, New York: McGraw-Hill, 1994.

[29] D. Duckwitz, “Power System Inertia: Derivation of Requirements and Comparison of Inertia Emulation Methods for Converter-based Power Plants”, Dissertation, Universität Kassel and Fraunhofer IEE, 2019.

[30] Y. Zhang, C. Klabunde and M. Wolter, "Frequency-Coupled Impedance Modeling and Resonance Analysis of DFIG-Based Offshore Wind Farm With HVDC Connection," in IEEE Access, vol. 8, 2020.

[31] M. Amin and M. Molinas, "Understanding the Origin of Oscillatory Phenomena Observed Between Wind Farms and HVdc Systems," in IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 5, no. 1, pp. 378-392, March 2017.

[32] J. Lyu, X. Cai and M. Molinas, "Optimal Design of Controller Parameters for Improving the Stability of MMC-HVDC for Wind Farm Integration," in IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 6, no. 1, pp. 40-53, March 2018, doi: 10.1109/JESTPE.2017.2759096.

[33] B. Pang, H. Nian and Y. Xu, "Mechanism Analysis and Damping Method for High Frequency Resonance Between VSC-HVDC and the Wind Farm," in IEEE Transactions on Energy Conversion, vol. 36, no. 2, pp. 984-994, June 2021, doi: 10.1109/TEC.2020.3023428.

[34] L. Harnefors, "Analysis of Subsynchronous Torsional Interaction With Power Electronic Converters," in IEEE Transactions on Power Systems, vol. 22, no. 1, pp. 305-313, Feb. 2007.

[35] J. A. Martinez et al, "Parameter determination for modeling system transients - Part IV: rotating machines," in IEEE Transactions on Power Delivery, vol. 20, no. 3, pp. 2063-2072, July 2005.

YONGGANG ZHANG (M’21) received the B.S. degree from Chang’an University, Xi’an, China, in 2007 and the M.S. degree from Duisburg-Essen University, Germany, in 2010, and the Ph.D. degree from the Otto-von-Guericke University Magdeburg, Germany, in 2021, all in electrical engineering. He is currently a postdoctoral researcher in the Department of Energy Management and Power System Operation at the University of Kassel.

DANIEL DUCKWITZ received the Dipl.-Ing. degree in mechatronical engineering from the University of Erlangen–Nuremberg, Erlangen, Germany, in 2010, and the Ph.D. degree in electrical engineering from the University of Kassel, Germany, in 2019. From 2019 to 2021, he has been a team leader in the Department of Energy Management and Power System Operation at the University of Kassel. He is currently a product manager of grid services at SMA Solar Technology AG.

NILS WIESE received the bachelor’s and master’s degrees in Renewable Energy Technologies und Systems Engineering from Flensburg University of Applied Sciences, Flensburg, Germany, in 2017 and 2019, respectively. He is currently pursuing the Ph.D. degree with the Department of Energy Management and Power System Operation, University of Kassel, Kassel, Germany.

MARTIN BRAUN (Senior Member, IEEE) received the Diploma degree in electrical engineering and in technically oriented business administration from the University of Stuttgart, Stuttgart, Germany, in 2005, and the Ph.D. degree in engineering from the University of Kassel, Kassel, Germany, in 2008. He is currently a Professor with the Department of Energy Management and Power System Operation, University of Kassel, and the Director of the Grid Planning and Grid Operation Division, Fraunhofer Institute for Energy Economics and Energy System Technology, Kassel.