Flow simulation and topological rearrangements of ordered foams

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Abstract. Flow through a narrow bent channel may induce topological rearrangements in a two-dimensional monodispersed dry liquid foam. We use the Cellular Potts Model to simulate a foam under a variable driving force in order to investigate the strain-rate response from these rearrangements. We observe a set of foams' behaviors ranging from elastic, viscoelastic to fluid regime. Bubble’s topological rearrangements are localized and their cumulative rearrangements change linearly with time, thus non avalanches critical behavior is found. The strain-rate affects the rate of topological rearrangements, its dependence on the drag force is nonlinear, obeying a Herschel-Bulkley like relationship below the foam’s flow point.

1. Introduction
Foams consist of two continuous liquid fluid phases walls enclosing gas cells where a surfactant stabilizes the liquid walls against rupture by reducing the liquid-gas surface energy. Dry foams contain less than 5% liquid per volume, the liquid forms channels of finite width (Plateau borders) rounding the sharp edges and corners of polyhedral cells. In two-dimensions (2D) dry foams consist of polygonal cells, while bubbles in wet foams are nearly circular [1]. Bubbles in foams have an unusual set of physical properties [2]: They deform easily with bubbles adjusting their positions and shapes in order to reach a local minimum of total interfacial area [3]. Bubbles evolve slowly by diffusion of gas from smaller few-sided bubbles to larger many-sided bubbles, leading to coarsening. Generally a foam’s configuration is very close to a true equilibrium configuration except when a local topological change involving bubble disappearance or irreversible bubbles neighbor switching sides (T₁) forces the foam system to evolve from a metastable configuration to a new one[4].

Foam rheological properties depend on the liquid’s fraction, bulk viscosity, surface viscosity, surface tension and geometry. Foams behave like solids under small applied shear stress, its elastic modulus is very small because elasticity originates from the small surface tension of the soap films. Foam rheological properties depend on the liquid’s fraction, bulk viscosity, surface viscosity, surface tension and geometry. Foams behave like solids under small applied shear stress, its elastic modulus is very small because elasticity originates from the small surface tension of the soap films. For intermediate stress (beyond a yield stress), foams behave like viscoelastic fluids, while for large deformations the energy barrier to T₁ is relatively negligible and the foam flows like a liquid[2]. In order to understand the mechanical response of foams under shear it may be necessary to consider different aspects of foam physics, especially the role of T₁ in shear and flow, as well as the relation between the dynamics of individual bubbles and the bulk. The spatial and temporal distributions of T₁ under increasing applied stress will determine the mechanical (inelastic) properties of the foam and how it releases energy during flow. We use the
Figure 1. The bent channel with bubbles moving counterclockwise dragged by a force. The spot in the middle shows a typical T<sub>1</sub> event.

Figure 2. The dependence of the driving force on the strain-rate for the elastic (1), viscoelastic (2) and fluid (3) regimes. The inset shows a log-log plot of the same order parameters showing a power law dependence for regimes (1) and (2).

Cellular Potts Model (CPM) to investigate the strain-rate response of 2D dry ordered foams subject to drag forces in a bent channel. We present the CPM algorithm and the numerical simulation in section 2. Next, in section 3 are presented our numerical results and discussions. Conclusions are finally presented in section 4.

2. The Cellular Potts model and the Numerical Simulation

The CPM represents bubbles on a lattice by a spin variable σ<sub>i</sub> within bubbles, while film boundaries are domain walls between regions of different spins. The basic foam Free Energy (<i>E</i>) consists of tree terms: The first one is the exchange energy between neighboring spins at bubbles boundaries, representing the surface tension. The next term describe the gas compressibility, an energy penalty for changes in the areas of bubbles, where <i>i</i> represents a site with coordinates (x<sub>i</sub>, y<sub>i</sub>), <i>J</i><sub>i,j</sub> is the energy per unit contact length between neighboring bubbles σ<sub>i</sub> and σ<sub>j</sub>, λ is the inverse of the gas compressibility, <i>a</i><sub>n</sub> is the area of the <i>n</i>th bubble, <i>A</i><sub>n</sub> its corresponding area under no compression. The third term considers a driving force that biases the probability of a spin reassignment in some given direction, where <i>f</i> is the driving field and <i>δl</i> is a unit vector pointing along the direction of the channel [5],

\[
\mathcal{E} = \sum_{i,j}^{\text{neighbors}(i,j)} J_{i,j} \left( 1 - \delta_{\sigma_i, \sigma_j} \right) + \lambda \sum_n \left( a_n - A_n \right)^2 + \sum_i f(x_i, y_i) \cdot \delta l .
\]

The foam evolves via Monte Carlo Metropolis dynamics at zero temperature. A site is chosen at random and a trial reassignment of its value to one of its neighboring sites. This step is is accepted if the energy change is negative. No change in the energy is accepted with probability 1/2. Time is measured in units of Monte Carlo Steps (MCS), consisting of as many spin attempts as the number of lattice sites. We use fourth-nearest neighbors interactions on a square lattice to minimize anisotropy.

We consider a narrow bent channel to drive bubbles through the curve region. Here we restrict ourselves to dry ordered foam at zero temperature, although the CPM is readily amenable for general cases such as wet and disordered foams at non-zero temperature [4, 6, 7]. We work in a lattice of 378 × 90 pixels<sup>2</sup> with a racetrack channel of 23 pixels width with semicircular caps. The radius of the outer channel boundary is 45 pixels while the inner channel boundary radius is 22 pixels. A linear channel of length 288 pixels separates the curved regions. We arrange the initial bubbles consistently with the boundary conditions. The pattern is generated by partitioning the network into equal size domains, each containing 22 × 20 pixels<sup>2</sup>. The number of bubbles...
determines each bubble’s area, we fix all target areas to $A_n = \text{total-area/number-of-bubbles}$ and choose the area constraint $\lambda$ to guarantee that the bubble area dispersion is smaller than 0.075. We use the following parameter values: $n = 56$, the coupling parameter $J_{ij} = 2.0$, the area constraint $\lambda = 2 \times 10^{-4}$ and the target area $A_n = 485 \text{ pixel}^2$. The initial pattern is obtained by turning off the driving term and relaxing the foam for around 100 $MCS$ to produce 31 bubbles in the outer layer and 25 in the inner one. Simulated foams are now driving by a constant applied force, under these conditions the relaxation time is no longer than 30 $MCS$, provided the bubble area dispersion is as specified above. We consider the driving force values (in $arb.\text{unit}$) into a range of $[0, 1.4]$ , and for an specific $f$ value we compute the $T_{18}$ events by comparing bubbles neighbors side after each $MCS$ and relaxation.

3. Results and Discussions

We found that $T_{18}$ events take place at specific locations in the bent channel, with a systematic offset from the center of the bend, independent of the driving force value. Figure 1 shows a typical bubble rearrangement at $2.39 \text{rad}$, in the upper left-side channel, for a driving force equal to $0.5 (arb.\text{unit}/\text{pixel})$, occurred precisely at $2025\ MCS$. We discuss foams macroscopic physics, by computing bubbles’ mean velocities by tracking them during many circuits around the channel, for a time interval of around $10^5\ MCS$. The velocity fluctuations normal to the direction of motion are negligibly small. The strain-rate $\dot{\gamma}$ is computed as in Couette’s experiment[8] , it is proportional to the mean velocity, $\dot{\gamma} \propto \Delta \omega = V_{\overrightarrow{\delta l}} / D$, where $\Delta \omega$ is the relative angular velocity of the bubbles in the two layers, $V_{\overrightarrow{\delta l}}$ is the tangential mean velocity and $D$ is the bubble diameter. Figure 2 shows a rheogram-plot of the driving force as a function of the strain-rate. Here, we can distinguish three different regimes: (1) $\dot{\gamma} \leq 0.0105 \ (f \leq 0.1946)$, (2) $0.0105 < \dot{\gamma} < 0.027$ and (3) $\dot{\gamma} \geq 0.027 (f \geq 0.40)$. There are many constitutive models availables to describe rheological fluid behavior, Herschel-Bulkley like foams behavior has been reported in foams experiments [9, 10, 11]. The model employs three rheological parameters: the yield stress, the consistency index and the power-law index. We use a similar power law relationship to describe the bubbles regimes $(1-2)$ as shown in the inset of Figure 2, i.e.: $\dot{f}_i = f_1^* + K_i (\dot{\gamma} - \dot{\gamma}_1^*)^n$, the parameters are computed by extrapolation using the Lavenberg-Marquadt method [12]. We found for regime $(1): f_1^* \approx 0.0125, \dot{\gamma}_1^* \approx 0.00012, K = 0.286$ and $\psi_1 = 0.08$. It is called elastic, because it also shows a jamming behavior. The regime $(2)$ is called viscoelastic with parameters: $f_2^* \approx 0.1946, \dot{\gamma}_2^* \approx 0.0105, K_2 = 4.351$ and $\psi_2 = 0.73$. The third regime is called a fluid regime and obeys a more complex rational function of the form $h(\dot{\gamma}) / g(\dot{\gamma})$, where both functions are

![Figure 3](image1.png)  
**Figure 3.** The cumulative $T_{18}$ linear behavior versus time in $MCS$, for different $f$ values decreasing from the top to the bottom.

![Figure 4](image2.png)  
**Figure 4.** Comparison between the strain-rate and the rate of cumulative topological events, a linear dependence is described by the slope approximately equal to 1.0675.
polynomials of fourth order.

Let us come back to the microscopic rearrangements. We found that $T_{1s}$ events are roughly periodic with approximately 100 per round trip. There are no $T_{1s}$ avalanche events so that its cumulative amount grow linearly with MCS time, irrespective of the value of the driving force applied, as shown in Figure 3. Following we compute the slope for these curves, which represents the rate of number events occurring at specific positions in the bended channel. They are related to the viscous gradient of $T_{1s}$ spatial distribution as reported in [13]: it relates the advective charge induced by the imposed shear at strain-rate $\dot{\gamma}$ to the cumulative relaxation of stress associated with $T_{1s}$. We compare these two different time scale processes in Figure 4 by plotting the strain-rate as a function of the rate of topological events. A linear behavior predominates in regions $(1-2)$ and even in a considerable part of region $(3)$, with the slope approximately equal to 1.0675, in agreement with [13].

4. Conclusions
Our CPM results agree qualitatively with observed experimental foam behavior, since we consistently find elastic, viscoelastic and fluid type responses, depending on the applied dragging force. Perhaps our more important result is to show that the dragging force is naturally related to the strain-rate by a power law relationship with a given exponent. We used a similar Herschel-Bulkley expression in order to describe the elastic and viscoelastic foam regimes. For the later we found the exponent 0.73, closer to the $2/3$ as computed from analytic approximations and it has indeed been measured [14, 15, 16]. But some experiments suggest this value is in the range $(0.25, 1.0)$ [9, 17]. The strain-rate response arises from topological rearrangements which occurred at specific locations at the channel bend regions and no avalanche phenomena are found. This lack of self-organized criticality has also been reported in both monolayer and bulk foams experiments on 2 and 3D, including some simulations [18, 19]. We found that the rate of localized rearrangements events is proportional to the (low) shear-rate, in agreement with kinematic expression reported in [13].

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