Top-heavy integrated galactic stellar initial mass functions (IGIMFs) in starbursts

C. Weidner\textsuperscript{1,2⋆}, P. Kroupa\textsuperscript{3†} and J. Pflamm-Altenburg\textsuperscript{3‡}

\textsuperscript{1}Scottish Universities Physics Alliance (SUPA), School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, UK
\textsuperscript{2}Departamento de Astronomía y Astrofísica, Pontificia Universidad Católica de Chile, Av. Vicuña MacKenna 4860, Macul, Santiago, Chile
\textsuperscript{3}Argelander-Institut für Astronomie (Sternwarte), Auf dem Hügel 71, D-53121 Bonn, Germany

\textsuperscript{⋆}E-mail: cw60@st-andrews.ac.uk
\textsuperscript{†}E-mail: pavel@astro.uni-bonn.de
\textsuperscript{‡}E-mail: jpflamm@astro.uni-bonn.de

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ABSTRACT
Star formation rates (SFR) larger than 1000 \(M_\odot\) yr\(^{-1}\) are observed in extreme starbursts. This leads to the formation of star clusters with masses > \(10^6 M_\odot\) in which crowding of the pre-stellar cores may lead to a change of the stellar initial mass function (IMF). Indeed, the large mass-to-light ratios of ultra-compact dwarf galaxies and recent results on globular clusters suggest the IMF to become top-heavy with increasing star-forming density. We explore the implications of top-heavy IMFs in these very massive and compact systems for the integrated galactic initial mass function (IGIMF), which is the galaxy-wide IMF, in dependence of the star-formation rate of galaxies. The resulting IGIMFs can have slopes, \(\alpha_3\), for stars more massive than about \(1 M_\odot\) between 1.5 and the Salpeter slope of 2.3 for an embedded cluster mass function (ECMF) slope (\(\beta\)) of 2.0, but only if the ECMF has no low-mass clusters in galaxies with major starbursts. Alternatively, \(\beta\) would have to decrease with increasing SFR > \(10 M_\odot\) yr\(^{-1}\) such that galaxies with major starbursts have a top-heavy ECMF. The resulting IGIMFs are within the range of observationally deduced IMF variations with redshift.

Key words: stars: formation – stars: luminosity function, mass function – galaxies: evolution – galaxies: starburst – galaxies: star clusters – galaxies: stellar content

1 INTRODUCTION: CLUSTERED STAR-FORMATION AND THE IMF

The stellar initial mass function (IMF) is one of the fundamental astrophysical distribution functions. It defines the ratio of low-mass stars, which do not contribute to the chemical evolution over a Hubble time but lock-up baryonic matter, to high-mass stars, which power the interstellar medium and enrich it with metals through AGB-winds and supernovae. It further determines the mass-to-light ratios of stellar populations and influences the dynamical evolution of star clusters and whole galaxies.

The low-mass end of the IMF is found to be independent of environmental influences like metallicity and density, which may be understandable theoretically by a nearly constant Jeans-mass due to the way how molecular cooling rates scale with density and temperature (Elmegreen et al. 2008). In the regime around and below the hydrogen burning mass-limit it now seems that brown dwarfs form an individual distribution disjoint but related to the low-mass IMF (Thies & Kroupa 2007, 2008). The formation and theoretical basis of the IMF of massive stars (> \(10 M_\odot\)) is less well understood with at least two competing theories (competitive accretion vs. single star accretion) having been developed (Bonnell et al. 1998, 2003; Bonnell & Bate 2006; Tan et al. 2006; Krumholz et al. 2009). Observationally, though, the slope of the high-mass IMF within star clusters seems to be as independent from the environment as the low-mass slope. Unresolved multiple systems have been shown to have no effect on the \(m > 10 M_\odot\) IMF (Maíz Apellániz 2008; Weidner et al. 2009) but for \(m \leq 1 M_\odot\) unresolved multiple stars effect the observed slope of the IMF substantially (Kroupa et al. 1991).

Over the last years it has become clear that star formation takes place mostly in embedded clusters (Lada & Lada 2003), each cluster containing a dozen to many million of stars (Kroupa 2005). Within these clusters stars appear to form following the canonical IMF, \(\xi(m) \propto m^{-\alpha}\), with a slope of 1.3 for low mass stars and the Salpeter/Massey-slope of...
2.35 for massive stars (for more details on the canonical IMF see Appendix A).

Not only stars follow a mass function but also (young, embedded) star clusters. The embedded cluster mass function (ECMF) has been found to be a power-law, $\xi_{\text{ecl}} \propto M_{\text{ecl}}^{-\alpha}$, with a rather constant slope of $\approx 2$ for differently formed environments from the quiescent solar neighbourhood to the vigorously star-forming Antennae galaxies (Lada & Lada 2003; Hunter et al. 2003; Zhang & Fall 1999).

The upper mass end of the ECMF (the most-massive young cluster within a star-forming galaxy) has been found to depend on the SFR of the galaxy (Weidner et al. 2004; Bastian 2008). Furthermore, it appears that the mass of the most-massive star in a star cluster is related to the mass of the cluster in a non-trivial way (Weidner et al. 2010). Low-mass clusters seem to be unable to form very massive stars. The physical reason for this empirical relation has not yet been found. But it can be presumed that the interplay between stellar feedback (ionising radiation and winds) from the massive stars and the gravitational potential of the star-forming cloud might play a role. In this picture, the feedback of the massive stars overcomes the binding energy of the cloud and star-formation is terminated. Such a process would directly couple the mass of the cloud with the feedback of the stars which is directly correlated with their mass (Elmegreen 1984). The majority of these embedded star clusters then dissolve quickly due to gas-expulsion (Kroupa et al. 2001; Adams & Myers 2001; Kroupa & Boily 2002; Lada & Lada 2003; de la Fuente Marcos & de la Fuente Marcos 2004; Parmentier et al. 2009).

A direct consequence from clustered star-formation is that the composite stellar population in a galaxy, which results from many star-forming events, is the sum of the dissolving star clusters. Thus the integrated galactic initial mass function (IGIMF) is the sum of all the IMFs of all the star clusters (Kroupa & Weidner 2003; Weidner & Kroupa 2004; see also Vanbeveren 1982).

$$
\xi_{\text{IGIMF}}(m,t) = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}(SFR(t))} \xi(m \leq m_{\text{max}}(M_{\text{ecl}})) \cdot \xi_{\text{ecl}}(M_{\text{ecl}}) \, dM_{\text{ecl}},
$$

(1)

where $M_{\text{ecl,min}}$ is the minimal embedded cluster mass, $M_{\text{ecl,max}}(SFR(t))$ is the maximum embedded cluster mass which is dependent on the SFR of the galaxy and is given by the following equation (Weidner et al. 2004).

$$
M_{\text{ecl,max}} = 8.510^4 \times SFR^{0.75} M_\odot.
$$

(2)

And $\xi_{\text{ecl}}(M_{\text{ecl}})$ is the ECMF with $dN_{\text{ecl}} = \xi_{\text{ecl}}(M_{\text{ecl}}) \, dM_{\text{ecl}} \propto M_{\text{ecl}}^{-\alpha} \, dM_{\text{ecl}}$ being the number of just formed embedded clusters with stellar mass in the interval $M_{\text{ecl}} \leq M_{\text{max}}(M_{\text{ecl}})$.

Note that, strictly, $\xi_{\text{ecl}}$ describes the distribution of star-forming molecular cloud cores containing only the stellar mass formed. The “embedded clusters” in eq. (1) do not, under any circumstances, mean bound or radially well-defined stellar ensembles. Rather, eq. (1) is an integral over all locally correlated star forming events, a small fraction of which will hatch from the clouds as bound clusters, while the majority disperse within about 10 Myr.

The IGIMF is steeper than the individual canonical IMFs in the actual clusters, hereby immediately explaining why $\alpha_{\text{field}} = 2.7 > \alpha = 2.35$, where $\alpha_{\text{field}}$ is the slope of the IMF derived by Scalo (1986) and Reid et al. (2002) from OB star counts in the Milky Way field, and $\alpha = 2.35$ is the Salpeter (1955) index. This is due to the fact that low-mass star clusters are numerous but can not have massive stars.

The high mass part of the resulting IGIMF is strongly dependent on the SFR of a galaxy (Haas & Anders 2010). This result has been found to be naturally (without parameter adjustments) able to explain the mass-metallicity relation of galaxies (Köppen et al. 2007), the alpha-element abundances as a function of galaxy mass (Recchi et al. 2009) and the Hα cut-off in star-forming galaxies (Pflamm-Altenburg & Kroupa 2008). It also predicted a discrepancy between SFRs derived from UV- and Hα-fluxes (Pflamm-Altenburg et al. 2007, 2009), a result which has recently been confirmed qualitatively (Meurer et al. 2009) and quantitatively (Lee et al. 2009) by observations. Furthermore, the Hα-SFR relation calculated in the IGIMF theory leads to higher SFRs of Hα faint star-forming dwarf galaxies (Pflamm-Altenburg et al. 2007) and reveals a simple linear relation between the total neutral gas mass and SFRs of galaxies (Pflamm-Altenburg & Kroupa 2009).

The IGIMF theory (eq. 1) is based solely on observed correlations ($m_{\text{max}}$ vs $M_{\text{ecl}}$, $M_{\text{ecl,max}}$ vs $SFR$) and distribution functions (IMF in the correlated star formation events, or “embedded clusters”, and the mass function of stellar masses formed in these events, the ECMF). It is these observed correlations and distribution functions which contain the relevant star-formation physics. It is this basis which allowed the IGIMF theory to be so successful since its first formulation (Kroupa & Weidner 2003). The empirical basis of the IGIMF theory is important because star-formation theory is not advanced enough to make reliable statements on the galaxy-wide IMF, and would in any case lead directly to the IGIMF theory since star-formation theory will have to account for the observed correlations and distribution functions.

In the case of very high SFRs (> 30 $M_\odot$/yr) the $M_{\text{ecl,max}} = SFR$-relation results in very massive star clusters ($M_{\text{ecl}} > 10^5 M_\odot$) which may be the progenitors of the Ultra Compact Dwarf galaxies (UCD), as these most-likely formed as compact objects of a few pc size (Dabringhausen et al. 2009). And indeed recent HST/Spitzer observations found very young massive ($10^4 M_\odot$) objects with radii below 100 pc in low-redshift Lyman-Break-Analog galaxies with high star-formation rates (Overzier et al. 2009).

In this contribution we explore the consequences of very high galaxy-wide SFRs on the IGIMF by taking into account recently acquired evidence for top-heavy IMFs when UCDs...
and possibly also globular cluster are formed. This advances the IGIMF theory into a physical regime of cosmological significance since very high SFRs are typically observed at high redshifts (Tresse et al. 2007).

The observational situation regarding top-heavy IMFs and crowding in massive proto-clusters is discussed in Section 2 while the top-heavy IGIMF model is presented in Section 3. The following Section 4 then shows the results of the model calculations which are discussed in Section 5. The canonical IMF within star clusters used throughout this work is presented in Appendix A.

2 OBSERVATIONAL SITUATION

2.1 Top-heavy IMF

Despite all the evidence for a universal IMF, recently several indications have begun emerging for a possible dependence of the shape of the IMF on environment. Several observational and theoretical indications suggest the IMF to become top-heavy under extreme starburst conditions. For example, Baugh et al. (2005) find that a flatter high-mass slope of the IMF at higher redshifts is needed to explain the observed numbers of sub-mm galaxies. Stanway et al. (2003) observe rather steep slopes in the spectral energy distribution of high-redshift objects in the Hubble Ultra Deep Field. They conclude this can be either explained by a 50% higher star formation rate than the current calibrations yield or a top-heavy IMF at very high-redshifts (e.g. $z > 10-15$). This is actually predicted by Clarke & Bromm (2003).

A similar evolution of the IMF with redshift such that at high-z star-formation is biased towards more massive stars is found by studying the amplitude of the galaxy-stellar-mass-star-formation-rate relationship (Dave 2008) and by comparing the rate of the luminosity evolution of massive early-type galaxies in clusters to the rate of their colour evolution (van Dokkum 2008). As star-formation rates are also higher at high-z this can be seen as an indication for a top-heavy IMF in starbursts. When integrating the redshift evolution of the SFR over cosmological time-scales within the A cold dark matter cosmology Wilkins et al. (2008) find that they need a top-heavy IMF with $\alpha_3 = 2.15$ to reproduce the present-day stellar mass density. However, in order to explain the stellar mass density at higher redshifts an even flatter slope is needed. Other constraints also come from the element abundances observed in galaxies. Finoguenov et al. (2003) study the element abundances in groups and clusters of galaxies with the use of XMM-Newton and ASCA. They show “that while the metal production in groups could be described by a stellar population with a standard local initial mass function, clusters of galaxies require a more top-heavy IMF.” From modelling the iron evolution in galaxy clusters and the field and comparing the results with observations, Loewenstein (2006) concludes that a top-heavy IMF is indeed needed in galaxy clusters but not for galaxies in the field. Tomo et al. (2004) find a similar result in their N-body-Tree + SPH simulations of a galaxy cluster. A Salpeter IMF produces subsolar $[\text{O/Fe}]$ abundances. They can explain solar $[\text{O/Fe}]$ if the IMF is top-heavy for $z > 2$ and Salpeter afterwards. To account for the $[\alpha$/Fe]-relation observed in local elliptical galaxies Calura & Menci (2007) also require a top-heavy IMF. Ballero et al. (2007) infer the need of a top-heavy IMF in the Galactic bulge in order to explain the difference in the $[\text{O/Mg}]$ to $[\text{Mg/H}]$ ratios in bulge Giants compared to the Galactic disk. But McWilliam et al. (2008) find that metallicity-dependent stellar yields for massive stars equally explain the ratios. Hints for a top-heavy IMF are also found in the centre of the Milky Way. By examining the evolution of disk stars orbiting a central black hole Alexander et al. (2007) found strong evidence for a top-heavy IMF in order to explain the observed ring of massive stars orbiting about 0.1pc around the Galactic centre. Using Chandra X-ray data of the Galactic centre Navakshin & Sunyaev (2005) found 10 times less X-ray emission from low-mass stars than expected if the observed massive stars formed from the canonical IMF. Elmegreen (2002) did an extensive literature study of Galactic and extragalactic observations and concluded that dense star-forming regions like starbursts might have a slightly shallower IMF, a view shared by Eisenhauer (2001). Furthermore, Brown et al. (2007) find that the majority of observed hypervelocity stars ejected by the super-massive black hole in the centre of the Galaxy have a mass around 3 or 4 $M_\odot$ whilst lower mass stars are lacking from observations. The high number of carbon-enhanced, s-process-enriched unevolved stars among extremely metal-poor stars in the halo of the MW has been interpreted by Lucatello et al. (2003) as evidence for a top-heavy IMF during the formation of the halo. A recent study found that the central regions of disturbed galaxies host far more supernovae Ib and Ic (originating from very massive stars) while in all other environments supernova typ II dominate (Habergham et al. 2010). This can be attributed to a top-heavy IMF in starbursts after galaxy interactions and mergers.

Marks, Kroupa & Dabringhausen (in preparation) analyse the stellar content of globular clusters and the dynamical M/L ratios of UCDs using $N$-body models and deduce that the IMF appears to become top-heavy with increasing density of the forming object. Given the mass-radius relation inferred from the proto-globular clusters and proto-UCDs this translates into a dependence of the IMF slope $\alpha_3$ on the object’s mass. The application of the IGIMF theory to high-galaxy-wide SFRs is based on this result. Papadopoulos (2010) shows that the molecular gas is likely to have a higher temperature in massive star bursts as a result of heating through supernova generated cosmic rays. This may be a theoretical reason for expecting IMFs that are top-heavy in such systems.

2.2 Crowding in very massive pre-cluster cloud cores

A substantial fraction of stars seems to form in star cluster-like correlated events (Adams & Myers 2001; Lada & Lada 2003) and these are observed to be very compact during their final stages of collapse ($r_{\text{core}} \lesssim 1$ pc; Testi et al. 1998; Kroupa 2003; Gutermuth et al. 2005; Testor et al. 2005; Rathborne et al. 2006; Scheepmaker et al. 2009) and are therefore in very dense configurations. Also, strikingly similar values are found for the size of pre-stellar cloud cores, despite large differences in mass. Furuva et al. (2006) find radii of about 5000 AU for a low-mass star-forming environment. Baczyn et al. (2001) find
5000 to 10000 AU and van der Tak (2000) find 20000 AU for massive star progenitors.

Furthermore, within starbursts extreme star formation rates (> 100 M⊙/yr−1, Yan et al. 2004; Daddi et al. 2004; Choi et al. 2006) are reached, e.g. in the case of Arp 220 (Wilson et al. 2006). The recent Herschel study by Magnelli et al. (2010) of submillimetre galaxies (SMG) found that for SMGs with fluxes S(850µm) > 5 mJy the median SFR is 960 M⊙ yr−1. The highest S(850µm) flux in their sample is about 10 mJy, corresponding to a SFR of several 103 M⊙ yr−1.

This leads to the formation of very massive star clusters according to eq. 2. For the characteristic time-scale in stars, ξ, of about 10 Myr (Fgas et al. 2004, 2006), the total mass in stars is Mtot = SFR × ξ. But as also Mtot = \int_{M_{\text{cl,min}}}^{M_{\text{cl,max}}} \xi_{\text{cl}}(M_{\text{cl}}) dM_{\text{cl}}, M_{\text{cl,max}} increases as well when the SFR increases. The normalisation of ξ_{\text{cl}}(M_{\text{cl}}) is obtained by stating there is one most-massive cluster, 1 = \int_{M_{\text{cl,min}}}^{M_{\text{cl,max}}} \xi_{\text{cl}}(M_{\text{cl}}) dM_{\text{cl}}, forming during each 10 Myr interval.

Because young pre-star clusters are compact (r ≈ 1 pc) and since the contracting pre-stellar cores within them have dimensions of a few thousand AU the pre-stellar cores are likely to begin interacting and coalescing above a certain mass limit. Bonnell et al. (1998) predict an influence on the IMF of the forming star cluster, and so do Elmegreen & Shadmehri (2003) who investigated this question of crowding further. They concluded that crowding should be relevant in the most massive clusters (e.g. progenitors of globular clusters) leading to a top-heavy IMF, because the crowded medium would have a higher low-mass limit for star formation and therefore more massive stars need to be build to result in the same total mass. Elmegreen (2004) constructed a three-component IMF model from these results. Shadmehri (2004) further showed that the effect of crowding must have an influence on the IMF in starburst clusters. While Elmegreen (2004) and Shadmehri (2004) present models for the IMF in a starburst they do not discuss the overall integrated IMF for a whole galaxy with a starburst. Certainly, in massive clusters, or UCDs, the IMF will be effected by crowding (higher M/L ratio, Dabringhausen et al. 2004), but the question remains if this change is large enough to influence the IGIMF significantly.

In Fig. 4 the crowding is illustrated. It shows how many proto-stars of a fixed pre-stellar cloud core size can be fitted into a spherical proto-cluster with a radius of 1 pc. This is done by taking a range of proto-cluster masses and dividing these masses by the mean mass of the canonical IMF (m_{\text{mean}} ≈ 0.36 M_\odot) in order to get the expected number of stars, N_{\text{exp}}, in the cluster. N_{\text{exp}} is then multiplied by the volume of three different pre-stellar cloud core sizes - 1000 AU (thick solid line), 5000 AU (thick short-dashed line), and 10000 AU (thick long-dashed line). Fig. 4 shows the volume of the proto-cluster divided by the total volume of the pre-stellar cloud cores. For clusters with more than ≈ 10^5 M_\odot crowding will be a problem. It is therefore appropriate to assume that such high-density environments may have an influence on the IMF within such star clusters.

An alternative or additional mechanism which may induce star bursts leading to top-heavy IMFs may be through the heating of the molecular gas by supernova generated cosmic rays (Papadopoulos 2010).

### 3 THE MODEL

While the above discussion suggests that the IMF may become top-heavy, it remains, unfortunately, unclear as to how a systematic variation of the IMF with increasing M_{\text{cl}} ought to be realised. Elmegreen (2004) and Shadmehri (2004) already presented descriptions of the change of an IMF in starbursts. They based their models on a log-normal IMF (Scalo 1986) but we use the multi-power law formulation of the stellar IMF.

The modelling of the IGIMF starts from a SFR which determines the upper mass limit of the ECMF according to eq. 2. In order to calculate the total amount of stars formed in a star-forming period the SFR is multiplied by a time-step which is assumed to be 10 Myr. With the use of the total mass the ECMF can be normalised for the time-step. The ECMF is then divided in 1000 logarithmic bins. As the embedded cluster mass limits the most-massive star in each cluster, the upper limit of the IMF in each ECMF-bin can be calculated. As the number of clusters per bin is given by the normalised ECMF, the IMFs of each ECMF-bin can be multiplied by the number of clusters in order to give the integrated galactic IMF of the time-step. The slope
of this IGIMF above $1.3 \, M_\odot$ can then be computed by a least-squares fit to the calculated IGIMF. The value of the so-derived $\alpha_3$ gives a rough indication of the top-heaviness, while detailed astrophysical parameters would be derived from using the actually computed IGIMF.

In order to include the possible effect of crowding into the framework of the IGIMF theory (§4.2), the results of Marks, Kroupa & Dabringhausen (in preparation) on the high-mass IMF slope in globular clusters and UCDs are used. They derive the following dependence of the IMF slope for stars more massive than $1 \, M_\odot$ and for clusters with initial masses $M_{ecl} \geq 2 \times 10^5 \, M_\odot$,

$$\alpha_3(M_{ecl}) = -1.67 \times \log_{10} \left( \frac{M_{ecl}}{10^5 \, M_\odot} \right) + 1.05.$$  \hspace{1cm} (3)

The limit of $2 \times 10^5 \, M_\odot$ is chosen because for clusters with masses below this limit Marks, Kroupa & Dabringhausen (in preparation) results infer a Salpeter slope. Note that while the parametrisation in eq. (3) rests on empirical globular cluster data which have been corrected for stellar and dynamical evolution, the parametrisation we adopt in eq. (3) is not to be seen as an established dependence of $\alpha_3$ on $M_{ecl}$. Rather, while giving a good clue as to how the IGIMF may change with increasing SFR, the calculations performed here are independent of the parametrisation and may thus be easily adopted for different parametrisations.

### 3.1 Model parameters

Besides the change in the IMF for very massive clusters, the most important parameters in our model are the SFRs of the models (which give the upper mass limit of the forming clusters, $M_{ecl,max}$), the slope ($\beta$) of the ECMF, and the lower mass limit of the ECMF, $M_{ecl,min}$. For this study of starbursts four SFRs (10, 100, 1000 and 10000 $M_\odot$ yr$^{-1}$) are chosen. Below 10 $M_\odot$ yr$^{-1}$ little impact on the IGIMF from the top-heavy IMFs in very-massive clusters is to be expected as only very few or none of them are formed according to eq. (2). As already discussed in the introduction, SFRs of more than $1000 \, M_\odot$ yr$^{-1}$ have recently been found (Magnelli et al. 2010). The upper limit for the SFRs in the models is therefore chosen to be $10000 \, M_\odot$ yr$^{-1}$. The value of the slope of the ECMF is usually given with a $\beta$ around 2 with errors between 0.2 and 0.5 (Larsen 2009), though some studies find systematically flatter slopes like 1.8 (Dowell et all 2008). And the mass spectrum of giant molecular clouds, the precursors of star clusters, shows a slope of 1.7 (Rosolowsky 2003). As the identification of embedded clusters in extra-galactic objects is very challenging, we allow a greater range for this parameter and study $\beta = 0.5$ to 2.35. The lower mass limit of the ECMF, $M_{ecl,min}$, is even worse constrained than the slope of the ECMF. We therefore study a broad range of possible $M_{ecl,min}$ from 5 to $10^5 \, M_\odot$.

The lower mass limit of the canonical IMF in each cluster is fixed to 0.01 $M_\odot$ and the upper mass limit is determined by the cluster mass (Weidner & Kroupa 2004).

### 4 RESULTS

Tab. 1 shows the IGIMF slopes for stars more massive than $1.3 \, M_\odot$ for different star-formation rates, different values of the ECMF slope, $\beta$, and for several lower limits of the ECMF, $M_{ecl,min}$. For clusters above $2 \times 10^5 \, M_\odot$ the IMF slope for stars above $1 \, M_\odot$, $\alpha_3$, is changed according to eq. (3) while below $2 \times 10^5 \, M_\odot$ $\alpha_3$ is kept constant at the canonical (Salpeter) index 2.35. For $\alpha_3$ a lower limit of 1 is used because clusters with an $\alpha_3$ of 1 or less contain more than 90% of the mass in stars more massive than 8 $M_\odot$. When these stars explode as supernovae the resulting extreme mass-loss would completely disperse the star clusters. In such a case no globular clusters would survive until today. For clusters more massive than $\approx 1.1 \times 10^6 \, M_\odot$ this limit is reached and $\alpha_3$ is fixed to 1.0 for these objects.

In Fig. 2 three IGIMF examples from Tab. 1 are shown together with the input canonical IMF. Depending on the parameters a strong top-heaviness can be seen.

In Fig. 3 the results of Tab. 1 are summarised for $\beta = 2.35, 2.0$ and 1.6. Generally, the IGIMF slope decreases with increasing SFR for a given $\beta$. This is due to the relation between the maximal cluster mass and the SFR (eq. (2)). The higher the SFR the more-massive is the upper limit of the ECMF. As massive clusters develop a top-heavy IMF according to eq. (3) a larger fraction of massive stars are formed with increasing SFRs. Fig. 3 shows the dependence of $\alpha_3$ on the lower limit of the ECMF ($M_{ecl,min}$). For very flat ECMFs (dashed lines) this dependency is nearly

![Figure 2. Three different IGIMF's from Tab. 1 together with the input canonical IMF (solid line, eq. (2)). All three models have $\beta = 2.0$ and for the dotted line $SFR = 10 \, M_\odot$ yr$^{-1}$ and $M_{ecl,min} = 1000 \, M_\odot$, for the short-dashed line $SFR = 100 \, M_\odot$ yr$^{-1}$, $M_{ecl,min} = 10000 \, M_\odot$ and for the long-dashed line $SFR = 10000 \, M_\odot$ yr$^{-1}$, $M_{ecl,min} = 100000 \, M_\odot$. All IMFs are normalised to the same total mass in stars ($10^{11} \, M_\odot$).](image-url)
negligible. This is because for slopes of $\beta$ less than 2.0 the ECMF is dominated by massive clusters and reducing the number of low-mass cluster by increasing $M_{\text{min}}$ does not affect the IGIMF significantly. For $\beta = 2.35$ the opposite is the case. For steep slopes, low-mass clusters dominate the ECMF and therefore have a strong influence on the IGIMF. Here, increasing $M_{\text{min}}$ changes the IGIMF strongly by eliminating the low-mass clusters.

5 DISCUSSION

With this contribution the SFR dependent IGIMF theory is extended into the starburst regime. The SFR dependent IGIMF, which produces results agreeing with observations for massive and dwarf galaxies (Kroupa & Weidner 2003, Weidner & Kroupa 2005, Köppen et al. 2007, Recchi et al. 2009), leads to top-heavy galaxy-wide IMFs in massively starbursting galaxies when combined with the cluster-mass dependent high-mass IMF slope for globular clusters and UCDs (eq. 3) by Marks, Kroupa

| $M_{\text{cl.min}}$ [M$_\odot$] | SFR: 10 | 100 | 1000 | 10000 | 10 | 100 | 1000 | 10000 | 10 | 100 | 1000 | 10000 |
|--------------------------------|----------|-----|------|-------|---|----|------|-------|---|----|------|-------|
| $\beta = 0.5$ $\beta = 0.5$ $\beta = 0.5$ | $\beta = 1.0$ $\beta = 1.0$ $\beta = 1.0$ | $\beta = 1.0$ $\beta = 1.0$ $\beta = 1.0$ |
| 5 | 1.96 | 1.22 | 1.03 | 1.00 | 1.00 | 0.93 | 0.99 | 1.01 | 0.016 | 0.019 | 0.019 | 0.019 |
| 100 | 1.96 | 1.22 | 1.03 | 1.00 | 1.00 | 0.93 | 0.99 | 1.01 | 0.016 | 0.019 | 0.019 | 0.019 |
| 1000 | 1.96 | 1.22 | 1.03 | 1.00 | 1.00 | 0.93 | 0.99 | 1.01 | 0.016 | 0.019 | 0.019 | 0.019 |
| 10000 | 1.96 | 1.22 | 1.03 | 1.00 | 1.00 | 0.93 | 0.99 | 1.01 | 0.016 | 0.019 | 0.019 | 0.019 |
| 100000 | 1.92 | 1.21 | 1.02 | 1.00 | 1.00 | 0.94 | 1.00 | 1.01 | 0.017 | 0.019 | 0.019 | 0.019 |

| $\beta = 1.0$ $\beta = 1.0$ $\beta = 1.0$ | $\beta = 1.6$ $\beta = 1.6$ $\beta = 1.6$ | $\beta = 2.0$ $\beta = 2.0$ $\beta = 2.0$ |
| 5 | 2.04 | 1.39 | 1.12 | 1.03 | 1.00 | 0.88 | 0.96 | 0.99 | 0.015 | 0.019 | 0.019 | 0.019 |
| 100 | 2.04 | 1.39 | 1.12 | 1.03 | 1.00 | 0.88 | 0.96 | 0.99 | 0.015 | 0.019 | 0.019 | 0.019 |
| 1000 | 2.04 | 1.39 | 1.12 | 1.03 | 1.00 | 0.88 | 0.96 | 0.99 | 0.015 | 0.019 | 0.019 | 0.019 |
| 10000 | 2.03 | 1.39 | 1.11 | 1.03 | 1.00 | 0.88 | 0.96 | 0.99 | 0.015 | 0.019 | 0.019 | 0.019 |
| 100000 | 1.97 | 1.34 | 1.09 | 1.02 | 1.00 | 0.91 | 0.97 | 1.00 | 0.016 | 0.019 | 0.019 | 0.019 |

| $\beta = 2.0$ $\beta = 2.0$ $\beta = 2.0$ | $\beta = 2.35$ $\beta = 2.35$ $\beta = 2.35$ |
| 5 | 2.52 | 2.32 | 2.16 | 2.05 | 1.00 | 0.88 | 0.78 | 0.74 | 0.007 | 0.009 | 0.010 | 0.011 |
| 100 | 2.36 | 2.15 | 2.01 | 1.91 | 1.11 | 0.88 | 0.81 | 0.77 | 0.010 | 0.011 | 0.012 | 0.013 |
| 1000 | 2.29 | 2.06 | 1.91 | 1.81 | 1.00 | 0.86 | 0.80 | 0.78 | 0.011 | 0.013 | 0.014 | 0.015 |
| 10000 | 2.24 | 1.93 | 1.78 | 1.68 | 1.00 | 0.83 | 0.80 | 0.79 | 0.011 | 0.014 | 0.015 | 0.016 |
| 100000 | 2.08 | 1.69 | 1.53 | 1.45 | 1.00 | 0.85 | 0.84 | 0.85 | 0.014 | 0.017 | 0.018 | 0.018 |

| mean SFR: | $7.1 \times 10^{-3}$ | $7.1 \times 10^{-2}$ | $7.1 \times 10^{-1}$ | 7.1 |
|-----------|---------------------|---------------------|---------------------|-----|
| $M_{\text{cl.max}}$ [M$_\odot$] | $4.75 \times 10^5$ | $2.65 \times 10^6$ | $1.48 \times 10^7$ | $8.25 \times 10^7$ |
| 0.0064 | 0.0068 | 0.0071 | 0.0073 |
values of IMFs are difficult to reconcile with the here presented model even for rather flat IMFs already for relatively low SFRs and are different with the constraints for the Galactic and M31 bulge also be seen in Fig. 4 the IGIMF results are in good agreement with the observations do not measure IMF slopes and SFRs for individual galaxies but study indirect evidence (luminosities, chemical enrichment, etc.) for whole populations and average the results over the galaxy luminosity function. The most-massive star cluster for which the IMF has been determined by star counts, R136 in the LMC, shows no evidence of a non-Salpeter slope of the high-mass stars (Selman et al. 1999). However, this cluster falls below the mass limit above which the parametrisation chosen here (eq. 1) implies a top-heavy IMF. Additionally, no common description is used in the literature to characterize the top-heaviness. Some authors vary the peak of the IMF while others vary the slope or the range of model results for an ECMF slope $\beta = 2.0$ while the dashed lines show the full envelope constrained by all models. The three asterisks are from Davé (2008), the one triangle around $\alpha_3 \approx 0.7$ is from van Dokkum (2008), the filled circle with error bars is from Ballero et al. (2007a) while the open circle with error bars corresponds to the Baugh et al. (2005) result. The dashed horizontal line marks the Wilkins et al. (2008) constrain with the light dashed lines 0.15 dex above and below being their uncertainty range.

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Figure 3. Dependence of the IGIMF slope above $1.3 M_\odot (\alpha_{\text{IGIMF}})$ on the lower mass limit of the ECMF ($M_{\text{ecl,min}}$). Solid lines show the results for $\beta = 2.35$ for SFRs from 10 (upper-most line) over 100 and 1000 to 10000 $M_\odot$ yr$^{-1}$ (lowest line). The dotted lines are like the solid ones, but for $\beta = 2.0$ and the dashed lines for $\beta = 1.6$.

Figure 4. IGIMF slopes above $1.3 M_\odot (\alpha_{\text{IGIMF}})$ for the SFR values of Tab 1 and some observational constrains in dependence of the total SFR. The shaded region between the solid lines marks the range of model results for an ECMF slope $\beta = 2.0$ while the dotted lines show the full envelope constrained by all models. The three asterisks are from Davé (2008), the one triangle around $\alpha_3 \approx 0.7$ is from van Dokkum (2008), the filled circle with error bars is from Ballero et al. (2007a), as well as the Wilkins et al. (2008) constrains for the present-day mass density from the cosmological SFH (dashed lines). The GAMA-team finds a very similar trend of decreasing slope with SFR as the models in their sample of $\sim 40000$ galaxies (Gunawardhana et al. 2010). Likewise, the Davé (2008) study of the amplitude of the galaxy stellar mass-star formation rate relationship seems to be in reasonable agreement with the model. Though for this agreement it is necessary that for galaxies with SFRs above $10 M_\odot$ yr$^{-1}$ the ECMF has either a lower limit larger then $1000 M_\odot$ or a slope flatter then 1.5. Generally, it should be possible to test these scenarios observationally by observing the ECMF.

Summarising, the main physical reason why the IGIMF becomes top-heavy at galaxy-wide SFRs $> 10 M_\odot$ yr$^{-1}$ is the formation of very massive star clusters with masses $M_{\text{cl}} > 10^6 M_\odot$. To achieve such high SFRs a galaxy with neutral gas needs to either globally be unstable (e.g. at early cosmological times when the gas fraction was very high) or be compressed globally due to an external tidal force field. Under these conditions the pressurised interstellar medium will collapse to massively giant molecular cloud complexes with masses of $10^8 M_\odot$ or larger. In these complexes, ultra-compact-dwarf galaxy type star clusters may form with in-
 individul masses \(>10^6 M_\odot\), and these would be having top-heavy star IMFs \cite{Dabringhausen et al. 2009}, as introduced into the IGIMF theory here. It may also be possible that at the same time the formed star-cluster mass function becomes bottom light under extreme SFRs, which may be due to the conditions for forming low-mass \((<100-1000 M_\odot)\) clusters not being available, perhaps due to the intense stellar feedback which may suppress the formation of such low-mass molecular cloud cores.

It should be noted here that the two rather extreme top-heavy IMF suggestions \cite{Baugh et al. 2005, van Dokkum 2008} with \(\alpha_{\text{empirical}} \leq 1\) are difficult to reconcile with the existence of globular clusters today. To obtain such flat IGIMFs the massive star clusters would need to have such flat IMFs that more than 90\% of their mass would be in stars which explode as supernovae. Such extreme mass-loss would quickly fully disperse these clusters, leaving no globular clusters behind.

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APPENDIX A: THE CANONICAL IMF

The following multi-component power-law IMF is used throughout the paper:

\[
\xi(m) = k \left\{ \begin{array}{ll}
\left( \frac{m}{m_{\text{H}}} \right)^{-\alpha_0}, & m_{\text{low}} \leq m < m_{\text{H}}, \\
\left( \frac{m}{m_{\text{H}}} \right)^{-\alpha_1}, & m_{\text{H}} \leq m < m_0, \\
\left( \frac{m}{m_{\text{H}}} \right)^{-\alpha_2}, & m_0 \leq m < m_1, \\
\left( \frac{m}{m_{\text{H}}} \right)^{-\alpha_3}, & m_1 \leq m < m_{\text{max}},
\end{array} \right.
\]

(A1)

with exponents

\[
\begin{align*}
\alpha_0 &= +0.30, & 0.01 \leq m/M_\odot < 0.08, \\
\alpha_1 &= +1.30, & 0.08 \leq m/M_\odot < 0.50, \\
\alpha_2 &= +2.35, & 0.50 \leq m/M_\odot < 1.00, \\
\alpha_3 &= +3.35, & 1.00 \leq m/M_\odot \leq m_{\text{max}}.
\end{align*}
\]

where \(dN = \xi(m) dm\) is the number of stars in the mass interval \(m\) to \(m + dm\). The exponents \(\alpha_i\) represent the standard or canonical IMF \cite{Kroupa 2001, 2002}. Though, \(\alpha_3\) is kept constant at 2.35 only for star clusters with \(M_{\text{cl}}\) less than \(2 \times 10^7 M_\odot\). For more massive clusters \(\alpha_3\) is changed with cluster mass according to eq. \[3\].

The advantage of such a multi-part power-law description are the easy integrability and, more importantly, that different parts of the IMF can be changed readily without affecting other parts. Note that this form is a two-part power-law in the stellar regime, and that brown dwarfs contribute about 4 per cent by mass only and that a discontinuity near \(m_{\text{H}}\) implies brown dwarfs to be a separate population \(k' \sim 0.5\) \cite{Thies & Kroupa 2007, 2008}. A log-normal form in the stellar regime but below \(1 M_\odot\) with a power-law extension to high masses was suggested by \cite{Chabrier 2003} but is virtually identical to the canonical IMF (fig. 8 in \cite{Dabringhausen et al. 2008}).

The observed IMF is today understood to be an invariant Salpeter/Massey power-law slope \cite{Salpeter 1955, Massey 2003} above \(0.5 M_\odot\), being independent of the cluster density and metallicity for metallicities \(Z \gtrsim 0.002\) \cite{Massey & Hunter 1998, Sirianni et al. 2000, 2002, Parker et al. 2001, Massey 1998, 2003, Wyse et al. 2002, Bell et al. 2003, Piskunov et al. 2004, Pflamm-Altenburg & Kroupa 2006}. Furthermore, unresolved multiple stars in the young star clusters are not able to mask a significantly different slope for massive stars \cite{Maiz Apellanz 2008, Weidner et al 2009, Kroupa 2002} has shown that there are no trends with physical conditions and that measured high-mass slopes, \(\alpha_3\), are a Gaussian distribution about the Salpeter value thus allowing us to assume for now that the stellar IMF is invariant and universal in each cluster. There is evidence of a maximal mass for stars \((m_{\text{max}} \approx 150 M_\odot)\) \cite{Weidner & Kroupa 2004}, a result later confirmed by several independent studies \cite{Oev & Clarke 2003, Figer 2005, Koen 2006}. However, according to \cite{Crowther et al. 2010} \(m_{\text{max}}\) may also be as high as \(300 M_\odot\).

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