New Physics, precision electroweak data and an upper bound on higgs mass

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Abstract

In this paper we express the effect of new physics on gauge boson self energy corrections through non-renormalizable dimension six operators. Using the precision electroweak data we then determine a lower bound on the scale \( \Lambda \) associated with the underlying new physics. The lower bound on \( \Lambda \) is then used to derive an upper bound on \( m_h \) through the triviality relation.
Introduction

The Standard Model (SM) of strong and electroweak interactions based on the gauge group $SU(3)_c \times SU(2)_l \times U(1)_y$ has been extremely successful in explaining all the experimental data so far. No significant deviations from the SM or evidence for new physics has been found so far. However, despite this extraordinary phenomenological success many theorists consider the SM as incomplete and at best a low energy description of some underlying high energy theory. The reasons behind this attitude are many. The SM does not provide any answer to the hierarchy problem, to the underlying physics that gives rise to the complex pattern of fermion masses or CP violation. In this paper we shall also take this attitude and assume that some unknown new physics will manifest itself above some high energy scale $\Lambda$ ($\Lambda \gg v$) which will provide answers to some of these problems. The effect of new heavy physics at scales much smaller compared to $\Lambda$ say near the EW scale can be described by non-renormalizable operators constructed out of the relevant degrees of freedom of low energy theory i.e. the SM fields. These non-renormalizable operators can be systematically constructed by starting from the high energy theory and integrating out the heavy fields. The effective Lagrangian for describing physics near the $Z$ pole can therefore be written as [1]

$$L_{\text{eff}} = L_{\text{sm}} + L_\Lambda = L_{\text{sm}} + \sum_i c_i \frac{O_i}{\Lambda^{d_i-4}}. \quad (1)$$

Here $L_\Lambda$ represents the effects of new heavy physics at scales much smaller than $\Lambda$. $O_i$ is a non-renormalizable operator of canonical dimension $d_i$. $c_i$ are dimensionless coefficients of order one if the new physics above $\Lambda$ is strongly coupled which we assume to be the case. The operator $O_i$ like the renormalizable SM Lagrangian $L_{\text{sm}}$ must be invariant under the $SU(3)_c \times SU(2)_l \times U(1)_y$ gauge group and the relevant global symmetries. In the following we shall assume that the EW gauge symmetry is linearly realized on the field content of the theory.

It is clear that in order to construct non-renormalizable operators representing the
effects of new physics that contribute to the gauge boson self energy corrections we must concentrate on the gauge-higgs system. We shall show that the following dimension six operators contribute to the S, T and U parameters.

\[ O_1 = \frac{1}{\Lambda^2} (D_\mu \phi)^+ \frac{\tau_a}{2} (D^\mu \phi) (\phi^+ \frac{\tau_a}{2} \phi). \]  \hspace{1cm} (2)

\[ O_2 = \frac{1}{\Lambda^2} (D_\nu D_\mu \phi)^+ (D^{\nu} D^{\mu} \phi). \]  \hspace{1cm} (3)

\[ O_3 = -\frac{1}{\Lambda^2} W^a_{\mu\nu} B^{\mu\nu} \phi^+ \frac{\tau_a}{2} \phi. \]  \hspace{1cm} (4)

Note that the negative sign associated with \(O_3\) is in accordance with usual gauge kinetic energy term.

**S, T and U parameters**

The S, T and U parameters introduced by Peskin and Takeuchi [2] describe the effects of new heavy physics, that do not have direct couplings to ordinary fermions, on the gauge boson self energies. In the \(\overline{MS}\) scheme they are defined as [3,4]

\[ \alpha(M_z) T \equiv \frac{\Pi_{ww}^{new}(0)}{M_w^2} - \frac{\Pi_{zz}^{new}(0)}{M_z^2}. \]  \hspace{1cm} (5)

\[ \alpha(M_z) S \equiv \frac{\Pi_{ww}^{new}(M_z^2) - \Pi_{zz}^{new}(0)}{M_z^2}. \]  \hspace{1cm} (6)

\[ \alpha(M_z) (S + U) \equiv \frac{\Pi_{ww}^{new}(M_z^2) - \Pi_{zz}^{new}(0)}{M_w^2}. \]  \hspace{1cm} (7)

Here \(\Pi_{ww}^{new}\) and \(\Pi_{zz}^{new}\) are the contributions of new heavy physics to W and Z self energies. \(s_z^2 = \sin^2 \hat{\theta}_w(M_z) = .2311 \pm .0003\) in the \(\overline{MS}\) scheme. Note that T is proportional to the W and Z self energies at \(q^2 = 0\) and hence it measures the strength of vector SU(2) breaking. On the other hand \(S(S + U)\) are proportional to the difference between Z(W)
self energies at $q^2 = M_Z^2(M_W^2)$ and $q^2 = 0$. Hence they measure the strength of axial SU(2) breaking. The values of $S$, $T$ and $U$ as determined from the EW data are given by

$$S = -0.28 \pm 0.19^{-0.08}_{+0.17}$$

$$T = -0.20 \pm 0.26^{+0.17}_{-0.12}$$

$$U = -0.31 \pm 0.54$$

The first uncertainties are from the inputs. The central values assume $m_h = 300$ GeV and the second uncertainty is the change for $m_h = 1000$ GeV (upper) and 60 GeV (lower).

**Bound on $\Lambda$ from T parameter**

The operator $O_1$ given above contributes to the vector SU(2) breaking $T$ parameter. In order to see that we write

$$\phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

We then get

$$O_1 = \frac{-v^2}{8\Lambda^2}(D_\mu \phi)^+ \frac{\tau_3}{2}(D^{\mu} \phi) \phi + \frac{\tau_3}{2} \phi$$

$$= \frac{iv^4}{64\Lambda^2}(0, 1)[-ig^2 \tau_3 (W_{1\mu}W_{1}^{\mu} + W_{2\mu}W_{2}^{\mu}) + ig^2 \tau_3 W_{3\mu}W_{3}^{\mu}$$

$$+ 2gg'W_{3\mu}B^{\mu} + g'^2 \tau_3 B_{\mu}B^{\mu}] \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{v^4}{64\Lambda^2}[(g^2 + g'^2)Z_\mu Z^{\mu} - 2g^2 W_{\mu}^{+}W^{-\mu}]. \tag{8}$$

It follows from the above expression that $\Pi_{ww}^{new}(0) = -\frac{\pi\alpha v^4}{8s_w^2\Lambda^2}$ and $\Pi_{zz}^{new}(0) = \frac{\pi\alpha v^4}{8s_w^2c_w^2\Lambda^2}$ and hence $T = -\frac{\pi\alpha v^4}{8s_w^2\Lambda^2}(\frac{1}{M_Z^2} + \frac{1}{c_w^2M_Z^2})$. Note that due to custodial SU(2) symmetry breaking by $O_1$, $\Pi_{ww}^{new}(0)$ and $\Pi_{zz}^{new}(0)$ turn out to be of opposite sign. Using the central value of $T$ as determined from electroweak data we obtain a lower bound of 3.12 TeV on $\Lambda$. 

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Bound on $\Lambda$ from S parameter

The operator $O_2$ can contribute to the S parameter. To see that we replace $\phi$ by its vev. Remembering that for a non-vanishing contribution to S we need two powers of $\partial_\mu$ we get

$$O_2 = \frac{1}{4}[g^2 v^2 \partial_\nu W^+ W^- + \frac{v^2}{2} (g^2 + g'^2) \partial_\nu Z^\mu \partial^\nu Z_\mu + \ldots]. \quad (9)$$

In the above expression the terms that do not contribute to S has been represented by dots. It follows from the above that $\Pi_{ww}^{new}(q^2) = -\frac{1}{4} \frac{g^2 v^2}{\Lambda^2} q^2$, $\Pi_{zz}^{new}(q^2) = -\frac{1}{4} \frac{g^2 + g'^2 v^2}{\Lambda^2} q^2$ and hence $S = -4\pi \frac{v^2}{\Lambda^2}$. Using the central value of the S parameter we get $\Lambda \geq 1.65$ TeV.

Note that the operator $O_2$ does not contribute to the U parameter which measures the difference between W and Z wavefunction renormalizations.

Bound on $\Lambda$ from U parameter

The operator $O_3$ contributes to both S and U. To see that we again replace $\phi$ by its vev to obtain

$$O_3 = \frac{v^2}{4\Lambda^2} W^\mu_3 W^\nu_3 B_{\mu\nu}$$

$$= \frac{v^2}{4\Lambda^2} (\hat{c}_z F_{\mu\nu} + \hat{s}_z Z_{\mu\nu})(\hat{c}_z F^{\mu\nu} - \hat{s}_z Z^{\mu\nu}) + \ldots$$

$$= -\frac{v^2}{2\Lambda^2} \hat{c}_z \hat{s}_z \partial_\mu Z_\nu \partial^\mu Z^{\nu} + \ldots \quad (10)$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. It follows from the above expression that $\Pi_{ww}^{new}(q^2) = 0$ and $\Pi_{zz}^{new}(q^2) = \frac{v^2}{4\Lambda^2} \hat{c}_z \hat{s}_z q^2$ and hence $U = -S = -4\frac{\hat{c}_z \hat{s}_z^3}{\alpha(M_Z)} \frac{v^2}{\Lambda^2}$. Using the central value of the U parameter we get a lower bound of 2.75 TeV on $\Lambda$. Note that the operator $O_3$ also makes a positive contribution to the S parameter. However since the present data favours a slightly negative value of S we do not use it to derive a bound on $\Lambda$. It is interesting to note that the lower bounds on $\Lambda$ determined from the experimental values of S, T and U are in close agreement with each other.
Triviality bound on the higgs mass

The lower bounds on the Λ derived in the previous sections can be used to derive an upper bound $m_h(Λ)$ on the higgs mass using the triviality relation [5]. An approximate perturbative estimate of $m_h(Λ)$ can be obtained by using the one loop RG equation of the higgs self coupling $λ$ and assuming that $λ$ is much greater than the other couplings that enter the equation i.e. $λ ≫ g_t, g, g′$. Such an approximation can be justified for a very heavy higgs boson. In this limit the scalar sector of the EW theory can be considered in isolation from the gauge bosons and the fermions. Let the scalar potential of the SM be given by

$$V(φ) = μ^2(φ^+φ) + λ(φ^+φ)^2. \quad (11)$$

where $μ^2 < 0$ and $λ = \frac{m_h^2}{2v^2}$ is the higgs self coupling. The one loop RG equation for $λ$ in a pure scalar theory is given by

$$\frac{dλ}{dt} = \frac{3λ^2}{4π^2}. \quad (12)$$

where $t = \ln \frac{Q^2}{Q_0^2}$ and $Q_0$ is some reference scale which we shall take to be equal to $v$. Integrating the above differential equation we get

$$\frac{1}{λ(Q)} = \frac{1}{λ(Q_0)} - \frac{3}{4π^2} \ln \frac{Q^2}{Q_0^2}. \quad (13)$$

We find that irrespective of how small $λ(Q_0)$ is $λ(Q)$ will ultimately become infinite at some very large energy scale $Q$. A bound on $m_h$ can be obtained by requiring that $λ$ is large but finite at some large energy scale where new physics appears. This consideration leads us to an approximate upper bound on the higgs mass

$$m_h^2 < \frac{8π^2v^2}{3 \ln(\frac{Λ^2}{v^2})}. \quad (14)$$
The perturbative bound on the higgs mass derived from triviality is cut off dependent in contrast to the lattice results. The average value of $\Lambda$ determined from S, T and U parameters in this paper is equal to 2.5 TeV. The triviality bound on the higgs mass for this value of $\Lambda$ is 586 GeV. However our perturbative estimate will be valid only if the one loop RG equation used by us provides an accurate description of the theory at large $\Lambda$. For large $\Lambda$, however higher order corrections and non-perturbative effects must be included to get a more reliable estimate of $m_h(\Lambda)$. Lattice gauge theory calculations based on a pure scalar theory gives a limit $m_h(\text{lattice}) < 640$ GeV [6].

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