Statistical modelling of tropical cyclone tracks: modelling cyclone lysis

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February 7, 2022

Abstract

We describe results from the fifth stage of a project to build a statistical model of tropical cyclone tracks. The previous stages considered genesis and the shape of tracks. We now consider in more detail how to represent the lysis (death) of tropical cyclones. Improving the lysis model turns out to bring a significant improvement to the track model overall.

1 Introduction

The impact of tropical cyclones on communities in various parts of the world motivates the development of accurate methods for estimating tropical cyclone risks. One set of methods that have been used for estimating these risks are the so-called basin-wide tropical cyclone models, examples of which have been described in Fujii and Mitsuta (1975), Drayton (2000), Vickery et al. (2000) and Emanuel et al. (2005). These models attempt to represent the entire life-cycle of tropical cyclones statistically, and thus allow the generation of an unlimited number of random but realistic cyclones that can be used for risk estimation. This article describes the latest results from a research effort aimed at developing new and more accurate basin-wide tropical cyclone models. Our previous articles on this subject have described models for mean tropical cyclone tracks (Hall and Jewson, 2005a), for the variance of fluctuations around the mean (Hall and Jewson, 2005b), for the autocorrelation of the fluctuations around the mean (Hall and Jewson, 2005c), and for genesis (Hall and Jewson, 2005d). Hall and Jewson (2005c) also included a very simple representation of the death, or lysis, of tropical cyclones. We now describe a model that gives a more realistic representation of lysis, by modelling the probability of lysis as a function of location. We find that this significantly improves the performance of the track simulation overall.

2 Data

We continue with our focus on tropical cyclones in the Atlantic basin, and the data we use is the ‘official’ National Hurricane Center track data set, known as HURDAT (Jarvinen et al., 1984). We only use data from 1950, since this is the only data that we consider to be sufficiently reliable. HURDAT data from 1950 to 2003 contains 524 tropical cyclones, and these are the data that we use.

3 Methods

In Hall and Jewson (2005d) we show simulations of Atlantic hurricane tracks, and make a number of comparisons between the simulated tracks and the historical observations. The simulated tracks are somewhat realistic, but a detailed comparison with observations indicates various systematic errors. We are gradually testing various possible improvements to the model in order to reduce these errors to a more acceptable level. One process that was only represented in a very rudimentary fashion in Hall and Jewson (2005d) is the lysis of hurricanes: simulated hurricanes originate from historical genesis points, and each is given a lifetime equal to the lifetime of the historical hurricane that originated from the same point. This gives a realistic distribution of hurricane lifetimes overall (by definition) but also leads to some physical

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inconsistencies. For instance, some simulated hurricane tracks wander into regions where real hurricanes would die, and too many hurricanes die in regions where real hurricanes only die infrequently. This, in turn, affects the distribution of the density of hurricane tracks in different regions, and would ultimately affect estimates of risk.

To remedy these problems we now describe a more complex lysis model. The model is very similar to the models we use for genesis and track dynamics in the papers cited above in that it is a non-parametric model based on spatial averaging of the behaviour of the observations. The optimum averaging length-scale is determined using cross-validation, with ‘optimum’ defined by maximising the out-of-sample likelihood. At each time step, every simulated hurricane has a probability $p$ of lysis. This probability varies in space, and is determined as follows:

- A two dimensional Gaussian window, with lengthscale $L$, is constructed around the current location of a simulated hurricane. We then note that every historical storm point in the basin is either the last point of a hurricane track, or not.
- $p$ is estimated as the sum of the Gaussian weights over the last points divided by the sum of the Gaussian weights over all points.

The lengthscale $L$ is determined from historical data using a maximum out-of-sample likelihood jack-knife cross-validation scheme as follows:

- For a given value of $L$, for each year, and for each observed hurricane point in that year, we calculate the value of $p$ as above, but only using data from all other years.
- The likelihood for the actual outcome for that hurricane point is then calculated. This likelihood is either $p$ if the hurricane point is the last for that track, or $1 - p$ otherwise.
- The likelihood for each year is calculated as the sum of the likelihoods for all points in that year.
- The lengthscale $L$ is varied to find the maximum of the likelihood.

There are various benefits from using a non-parametric scheme based on the methods of cross-validation and out-of-sample likelihood, such as the avoidance of overfitting and the optimisation of out-of-sample performance of the model. These benefits are discussed in more detail in our previous articles.

4 Results

We now show some results from the model described above.

First, in figure 1 we show the out-of-sample likelihood as a function of the lengthscale $L$. There is a very clear maximum at 360km (accurate to within a tolerance of 10km). The maximum represents a balance between two competing effects: (1) decreasing $L$ increases random sampling errors, as fewer lysis events receive significant weight; and (2) increasing $L$ results in averaging of real spatial structure in the lysis distribution.

In figure 2 we show the locations of historical hurricane lysis (panel (a)) and three realisations of simulations of lysis from the track model of Hall and Jewson (2005c), but with the new lysis scheme. The new lysis model is reasonable, in that the overall distribution of locations of observed hurricane lysis is captured, but it is not perfect. There are too few deaths off the coast of Newfoundland and too many in the region North of the Canary Islands. We think that these discrepancies are caused by the track model, rather than lysis directly. A good lysis model is necessary, but not sufficient, for a realistic lysis distribution, because the lysis distribution and the tracks are coupled. For reasons we have yet to determine there is a weak tendency for our simulated tracks to curve eastward too far south. The new lysis model has greatly improved this problem compared to Hall and Jewson (2005c), but a small discrepancy remains. A few too many simulated storms move into the eastern subtropical Atlantic where historical storms, and, thus, historical lysis events, are rare. The lysis model must rely on remote lysis events to ultimately kill these storms. This is an advantage, as it prevents the simulated storms from wandering even further a field. However, it also generates lysis events where none occur historically.

Figure 3 shows tracks simulated from the track model. This figure should be compared with figure 14 in Hall and Jewson (2005c). In comparison with that figure, we see that (a) the spurious point of track convergence in the East Atlantic has disappeared and (b) the high density of tracks off the East
Coast of the US has been simulated more realistically. However, as noted above, there is still a weak tendency for storms to bend too far eastward in the subtropical Atlantic.

We now consider a more quantitative evaluation of the performance of the track model with the new lysis process. Figure 4 is a new version of figure 17 from Hall and Jewson (2005d), and figure 5 is a new version of figure 18. They are created by simulating 20 realisations from the track model, each with 524 hurricanes. These 20 realisations define a distribution for the numbers of tracks crossing lines of latitude and longitude in a 54 year track set, and these distributions can be compared with the observed numbers of tracks crossing these lines. One would hope that the observations would lie within the distribution created from the statistical model. In figure 4 which shows numbers of tracks crossing lines of latitude, we see close consistency between the observations (dotted line) and the distribution of simulated tracks (illustrated using solid lines at plus and minus one standard deviation). The results are much better than those from the previous model, and the systematic error of having too few storms at certain latitudes has been fixed (it seems that the problem was that too many storms were dying too early in their life, and were not reaching the region of high storm density off the east coast of the US). In figure 5 which shows numbers of tracks crossing lines of longitude, the model results at low latitudes are consistent with the observations. The results at higher latitudes (in the westerlies) are good, but there are still some slight discrepancies in the far east of the domain. Again, however, the major systematic errors in the previous model have disappeared.

Figure 6 is a repeat of figure 20 from the earlier paper, but now for the new track model. The first panel shows the density of hurricane points from the simulations (based on the ensemble of 20 realisations used in the previous figures), while panel (b) shows the same for observations. Panel (c) shows the difference between the two, and panel (d) shows the standard deviation of the ensemble. The track model now simulates the density of tracks consistently with the observations almost everywhere, except in a very small region off the coast of Georgia and the Carolinas. In this region the observations show a very high density of tracks. The model also simulates a high density of tracks in this region, but the peak density is lower and the location of the maximum is shifted slightly to the south. These differences appear to be significant.

4.1 Landfalling rates

In figures 7 and 8 we show various aspects of a diagnostic designed to analyse the extent to which our model simulates the correct number of tracks making landfall in different regions. We use a simple representation of the North and Central American eastern coastline, as shown in the map in the left panel in each figure. This modelled coastline consists of 39 straight line segments of different lengths (using different length segments gives a better fit than using identical length segments). In the right hand panels of both figures we straighten this coastline out, with the horizontal axis running from North to South, and with 10 reference points marked on the map and the graph to help understand the correspondence between the two.

In the top right hand panel of figure 7 the black line shows the observed rate at which hurricanes crossed each of the 39 straight line segments, in numbers of hurricanes per 100km per year, based on the 54 years from 1950 to 2003. There are very large fluctuations in the crossing rate depending on location on the coast. The highest rates from this analysis are at Cape Hatteras, followed by some parts of Florida. It is clear from the high level of variability in the rates from segment to segment that the crossing rate at each location depends in a sensitive way on the geometry of the coastline and the geometry and variability of the hurricane tracks. This has implications for the extent to which these results can be considered general: if we repeated this analysis with a slightly different approximation for the coastline then they might look very different. It is therefore only reasonable to compare crossing rates between model and observations when both are calculated for exactly the same coastline (as we will do below). Our crossing rates should not, however, be compared with results from the same analysis that use different models for the coastline. For instance, the results that are shown on page 172 of Diaz and Pulwarty (1997) were generated using a different approximation to the coastline. As a result, although the pattern of landfall rates displays similarly located minima and maxima the relative magnitudes of these extrema are different, and the maximum landfall rates in that analysis are in Florida rather than near Cape Hatteras.

The red lines in the top right panel of figure 7 show the crossing rates derived from simulations from the model. Each line corresponds to one of the 54 year realisations described above, and there are 20 lines corresponding to the 20 realisations. We see considerable variability between the different realisations: this suggests that the historical observed crossing rates themselves could have been rather different, and are very affected by randomness. For instance, some of the realisations have the highest crossing rates in Florida rather than Cape Hatteras, and there are many locations where the crossing
rates vary by a factor of 2 or more between different 54 year realisations. We will investigate this latter point further below.

The middle panel of figure 7 shows the mean crossing rates for the simulations from all 20 realisations together. The agreement between the model and the observations in this and the top panel suggests that various features in the observations are real, rather than statistical artefacts. These include the deep minimum of rates between points C and D, on the North East coast of Florida.

The lower right panel of figure 7 shows the standard deviation across the 20 realisations. There are considerable variations in this standard deviation with location, with a pattern that resembles the mean. These variations show that not only is the mean crossing rate affected by the geometry of the coastline and the tracks, but also the variability among different 54 year samples.

Figure 8 shows three more ways in which we can compare the observed and simulated landfalling rates. First, the top right hand panel shows the observed landfall rates, with the plus and minus one standard deviation range for the model, derived from the 20 realisations. Second, the middle panel shows the difference between the mean model rate and the observations, divided by the ensemble standard deviation. Considering these two diagnostics we see that there are two places in particular where the observations may be inconsistent with the simulations: between B and C, and between G and H. Does this indicate flaws in the model? In fact it seems likely that the difference between B and C is related to the errors in the simulation of the high density of tracks off the coast of the Carolinas, as illustrated in figure 6 and discussed above. This is because the location of the difference in coastal rates is directly ‘downstream’, in a track sense, from the high density region. The difference between G and H is smaller, although more significant, is less easy to understand.

The lower right hand panel in figure 8 shows the ratio of the observed crossing rates to the model ensemble standard deviation. This gives an indication of where the observed rates are most affected by randomness: high values indicate that the role of randomness is small and that the observed rates are presumably a good estimator of the (unknown) real rates, while smaller values indicate that there is a large effect of randomness. In general, this ratio follows the observed rates. In other words, it is more or less the case that in regions where there are more hurricanes the landfalling rates can be estimated more accurately than in regions where there are fewer hurricanes (as one might expect). This is of some relevance to the insurance industry, as follows. One would expect property insurance rates to be lower in regions where landfalling rates are observed to be lower on average. However, one would also expect insurance rates to depend, in part, on the uncertainty in the estimation of the hurricane rates. Some of the regions where the observed rates are low, but the uncertainty is high (i.e. there is the possibility that the observed rates are only low by good fortune over the last 54 years), should, perhaps, have higher insurance rates than one might otherwise expect.

5 Conclusions

We have described the latest version of our hurricane track model. Relative to the previous version, we have included a more realistic lysis process. Ultimately, we hope to model fluctuations in the pressure of hurricanes as well as the tracks, and a good pressure model would take care of lysis automatically. However, pressure modelling is not simple, and at this point we have created a purely empirical model for lysis based on the observed distribution.

The new lysis model has significantly improved the simulation of hurricane tracks, has removed the major systematic errors in our model, and has partly solved the problem of how to simulate the high density of tracks off the US East Coast. A few smaller systematic errors remain, such as the precise location and magnitude of the peak density of tracks in this region.

There is still more work to do, however. We have not addressed the question of the non-normality of the residuals in the track model, as shown in figure 11 in Hall and Jewson (2005c). Presumably better modelling of the residuals would lead to further improvements in the large-scale behaviour of the tracks. And our model is also entirely non-seasonal at this point, even though the genesis of hurricanes, and possibly also the track dynamics and lysis, are definitely not constant across the hurricane season. If one is interested in the seasonal variation of risk then modelling these seasonal variations is imperative. And modelling seasonal variations correctly may also help in the modelling of overall annual risk.
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Figure 1: The variation of the out-of-sample likelihood with the averaging length-scale for the lysis model described in the text.
Figure 2: The observed locations of hurricane lysis for the 524 storms from 1950 to 2003 (panel (a)), and three simulations of hurricane lysis, each for 524 storms, for the model described in the text.
Figure 3: Observed (panel (a)) and three realisations of simulated hurricane tracks from the model described in the text.
Figure 4: The number of tracks from observations and from simulations that cross certain lines of latitude (equally spaced from 10N to 50N, from bottom to top), in a northward direction (left panel) and in a southward direction (right panel).
Figure 5: The number of tracks from observations and from simulations that cross certain lines of longitude (equally spaced from 80W to 20W, from left to right), in an eastward direction (panel (a)) and in a westward direction (panel (b)).
Figure 6: Track densities for model and observations. Panel (a) shows the track density for the model, averaged over 20 realisations of 524 storms. Panel (b) shows the track density for observations, for 524 storms. Panel (c) shows the difference of these densities, and panel (d) shows the standard deviation of the density from the model (across the 20 simulations of 524 storms).
Figure 7: The left hand panel shows a model for the coastline of North and Central America, consisting of 39 segments. The right hand panels show various diagnostics for the number of hurricanes crossing each of these segments in the observations and in the model. The top panel shows 54 years of observations (black line) and 20 realisations of 54 years from the model (red lines). The middle panel shows the observations and the mean of the model realisations. The lower panel shows the standard deviation of the model realisations.
Figure 8: As for figure 8, but now panel (a) shows the observations, with lines for plus and minus one standard deviation from the model, panel (b) shows the difference between model and observations normalised using the standard deviation from the model and panel (c) shows the observations divided by the standard deviation from the model.