Effect of in-medium $\pi$ and $\eta$ propagators to charge symmetry breaking interaction

Subhrajyoti Biswas

$^1$Department of Physics, Rishi Bankim Chandra College, Naihati - 743165, West Bengal, India

We revisit the charge symmetry breaking (CSB) in nucleon-nucleon (NN) interaction caused by $\pi$-$\eta$ mixing in nuclear matter (NM) employing one boson exchange (OBE) model. The medium effect on CSB is incorporated through the in-medium $\pi$ and $\eta$ propagators. In addition, the medium modification to the nucleon mass is also taken into account to construct the CSB class III potential considering off-shell $\pi$-$\eta$ mixing amplitude and large contribution of in-medium meson propagators to CSB potential is found for the pseudoscalar (PS) interaction compared to that of pseudovector (PV) interaction.

PACS numbers: 21.65.Cd, 13.75.Cs, 13.75.Gx, 21.30.Fe

I. INTRODUCTION

The charge symmetry (CS) is broken inherently by small amount in the nucleon-nucleon ($NN$) interaction. This symmetry breaking effect is observed trivially in the neutron-neutron ($nn$) and proton-proton ($pp$) systems through the presence of the Coulomb interaction. However, it is difficult to separate the strong interaction part model independently from the Coulomb interaction.

At the fundamental level CS of the nuclear force is broken due to the down ($d$) and up ($u$) quark mass difference i.e. $m_d \neq m_u$ and the electromagnetic interactions among the quarks. The $d$ and $u$ quark mass difference along with the electromagnetic effects is responsible for the observed mass differences between hadrons of the same isospin multiplets. Such mass splitting causes CSB at the hadronic level.

Many experiments have been designed to detect and measure CSB effects in various observables. It is seen clearly in the small difference between the $^1S_0$ $nn$ and $pp$ scattering lengths. The latest data of scattering experiment show that the amount of CSB is $\Delta a_{CSB} = a_{pp}^N - a_{nn}^N = 1.6 \pm 0.6$ fm where the superscript $N$ indicates the nuclear effect only.

Another convincing observation of CSB is found in the difference of ground state binding energies between the mirror nuclei $^3He$ and $^3H$. After excluding the corrections due to the static Coulomb interaction (648±4 KeV) electromagnetic effect (35±3 KeV) and $n$-$p$ mass difference in the kinetic energy (14±2 KeV), the remaining 67±9 KeV is believed to be accounted for the CSB interaction.

Similar phenomena have been investigated for other mirror nuclei. The Coulomb displacement energies of mirror nuclei are found different. This is known as the Okamoto-Nolen-Schiffer (ONS) anomaly. Many efforts, considering electromagnetic corrections, many-body correlations etc. have been made to explain this anomaly. In addition other manifestation of CSB interaction are the difference of $n$-$p$ form factors, correction to $g - 2$ etc.

The well known mechanism that generates CSB nucleon-nucleon ($NN$) interaction is the mixing of neutral mesons with different isospins but same spin and parity like $\pi$-$\eta$, $\pi$-$\eta'$, $\rho$-$\omega$ etc. mixing. Such mixing of isospin pure resonance states is caused by the $d$-$u$ quark mass difference and electromagnetic interactions.

In a quark model calculation, Goldman, Henderson, and Thomas showed that $\rho$-$\omega$ mixing amplitude has a substantial momentum dependence employing free constituent quark propagators and phenomenological meson-quark-antiquark vertex form factors. A QCD sum rule calculation and other investigations also reported strong momentum dependence of $\rho$-$\omega$ mixing.

Using chiral perturbation theory Maltman shows significant change of $\pi$-$\eta$ mixing amplitude in going from timelike to spacelike $q^2$. Similar $q^2$ dependence is found at the leading order contribution of $\pi$-$\eta$ mixing obtained from chiral effective Lagrangian model, calculations limited to the one loop order. In the $\pi$-$\eta$ mixing matrix element is calculated from the decays of $\eta$ and $\eta'$ considering $q^2$-independent $\pi$-$\eta$ vertex.

At the hadronic level, $n$-$p$ mass difference ($M_n \neq M_p$) causes to mix various isospin states in vacuum and meson fields in vacuum. The mixing amplitudes then used to construct the CSB two body potential. In 22, 23 and 26, $\pi$-$\omega$ potentials have been constructed using on-shell and constant $\rho$-$\omega$ mixing amplitudes. Various CSB observables have been calculated considering either constant or on-shell mixing amplitude and claimed successful for explaining the CSB observables. Though the mixing amplitude as shown in 29, 30, 32, 40, 49 has strong momentum dependence. On the basis of it the success of 26, 50 has been put into question.

There is another class of mixing mechanism which is completely different in origin. The mixing of different mesons in NM stems from the absorption and emission of intermediate mesons by neutron and proton Fermi spheres. Such mixing takes place in matter if the ground state contains unequal number of $n$ and $p$, i.e. $N \neq Z$ where $N$ and $Z$ represent the neutron and proton num-

*Electronic address: anjansubhra@gmail.com
bers respectively, called the asymmetric nuclear matter (ANM). The asymmetry parameter $\alpha$ is defined as $\alpha = (\rho_n - \rho_p) / (\rho_n + \rho_p)$ where $\rho_n$ and $\rho_p$ correspond to the neutron and proton densities, (and $k_n$ and $k_p$ Fermi momenta).

In ANM the emission and absorption of different mesons by the neutron and proton Fermi spheres are such that their contributions do not cancel and it gives rise to a non vanishing mixing amplitude. If $N = Z$ i.e. symmetric nuclear matter (SNM), the contributions of neutron and proton Fermi spheres will cancel only if the ground state respects the symmetry. Otherwise, such cancellation does not take place even in SNM. Therefore, the matter induced mixing is an additional source of CSB in $NN$ interaction.

The possibility of such matter induced mixing was first investigated by Dutt-Mazumder, Dutt-Roy and Kundu in ANM in the Walecka model and subsequently in similar investigations have been made. In matter induced mixing was studied using phenomenological parametrization to incorporate the results of all models of meson properties in medium. Most of the above works investigated the role of matter induced mixing on dilepton spectrum, pion form factors, etc. Mori and Saito studied the properties of CSB meson mixing in ANM within the framework of quantum hadrodynamics and constructed CSB potentials in spacelike region. Similarly matter induced $\rho$- and $\pi$-$\eta$ mixing in ANM have been investigated in [54] and also constructed CSB two body $NN$ potentials. In [61, 62] the effect of asymmetry and medium to the CSB potentials are incorporated through the in-medium mixing amplitudes. The mixing amplitudes are calculated using in-medium nucleon propagators [54] which contains the medium modified nucleon mass and density of the NM.

The main motivation of the present work is to study the medium effect, particularly the role of the in-medium $\pi$ and $\eta$ meson propagators, and effective nucleon mass to the CSB potential. Inclusion of such in-medium meson propagators and medium modified nucleon mass in the nucleon spinor of the external nucleon legs are consistent for the construction of CSB potential in the nuclear medium which, in the previous works were not considered. In addition, we also studied the role of effective masses of $\pi$ and $\eta$ mesons to the CSB potential simply replacing their bare masses of the respective propagators.

The paper is organized as follows. We calculate $\pi$ and $\eta$ meson self-energies in Sec. II considering both $PS$ and $PV$ interactions. These self-energies are then used to calculate the in-medium $\pi$ and $\eta$ meson propagators in sub section II B. In Sec. III we calculate $\pi$-$\eta$ mixing amplitudes and construct the CSB two body $NN$ class III potentials in Sec. IV. The Sec. V is devoted for presenting numerical results and discussion. And finally we summarize in Sec. VI.

II. $\pi$ AND $\eta$ MESON SELF-ENERGIES AND IN-MEDIUM PROPAGATORS

The meson self-energies and propagators in the medium are essential ingredients of the present work. To calculate the meson self-energies in medium one uses the in-medium nucleon propagator $\tilde{G}_N(k)$ which consists of the Dirac sea and Fermi sea contributions known as the usual vacuum part $G_v(k)$, and the density dependent part $G_m(k)$, respectively [53]:

$$\tilde{G}_N(k) = G_v(k) + G_m(k) ,$$

where

$$G_v(k) = \frac{k + \tilde{M}_N}{k^2 - M_N^2 + i\xi} ,$$

$$G_m(k) = \frac{i\pi}{E_N} \delta(k_0 - E_N)\delta(k_N - |k|) .$$

In the above equations $N$ represents the nucleon index $p$ for proton and $n$ for neutron and $k = (k_0, k)$ denotes the four momentum of the loop nucleon. The nucleon energy is denoted by $E_N = \sqrt{M_N^2 + k^2}$, where $M_N$ is the in-medium nucleon mass.

Note that the $\delta$-function in Eq. (2b) indicates the nucleons are on-shell and the $\theta$-function, called Pauli blocking, ensures that the momentum of the nucleon propagating in the medium must be less than the Fermi momentum $k_N$.

In quantum hadrodynamics ($QHD - I$) the nucleon is assumed to move in the mean field produced by the neutral scalar and vector mesons. The scalar field modifies the nucleon mass as,

$$\tilde{M}_N = M_N - \frac{g^2_{\sigma\sigma}}{m_{\sigma}}(\rho^*_p + \rho^*_n) .$$

while the vector field causes to shift the energy which we do not consider in this work.

In the above equation $m_{\sigma}$ and $g_{\sigma}$ represent the mass and coupling constant of the scalar meson $\sigma$, respectively. The bare nucleon mass is denoted by, $M_N$ and $\rho^*_N$ denotes the scalar density:

$$\rho^*_N = \frac{\tilde{M}_N}{2\pi^2} \left[ \frac{\tilde{E}_N k_N - \tilde{M}_N^2}{\tilde{M}_N} \ln \left( \frac{\tilde{E}_N + k_N}{\tilde{M}_N} \right) \right] .$$

The in-medium nucleon mass $\tilde{M}_N$ can be determined from Eq. (3) solving it self consistently.

A. The Meson Self-Energies

The calculations of the in-medium meson self-energies have been restricted up to one loop order (see Fig. 1).
which have been calculated using the formula [70]:

$$\Pi_{ab}^{(N)}(q^2) = -\int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \Gamma_a(q) \tilde{G}_N(k) \Gamma_b(-q) \tilde{G}_N(k+q) \right],$$

(5)

where $a$ (or $b$) denotes $\pi$ or $\eta$ and $\Gamma_{a,b}(q)$ represent the meson-nucleon-nucleon vertex factor. The dashed lines in Fig. 1 represent the meson propagators and the solid lines denote the in-medium nucleon propagators.

$$\Pi_{ab}^{(N)} = \frac{1}{k}$$

FIG. 1: The generic diagram of meson self-energy at the one loop order.

Substituting the in-medium nucleon propagator, $G_N(k)$ in the expression of Eq. (5) one may distinguish four terms with the combinations like $\Gamma_a G_v \Gamma_b G_v$, $\Gamma_a G_v \Gamma_b G_n$, $\Gamma_a G_n \Gamma_b G_v$, and $\Gamma_a G_n \Gamma_b G_n$. Out of which first three terms have been considered in this paper for calculating the total meson self-energy in the medium. The fourth term contains two $\delta$-functions which means both the loop nucleons, in Fig. 1 are on-shell which means that the mesons will decay into nucleon - antinucleon pair and it happens if the meson momentum is larger than two times the nucleon momentum, i.e. $q > 2k_N$ (also $q_0 > 2E_N$). Since the present work is restricted only for the low energy collective excitation of mesons in the medium, the fourth term has been neglected [60].

The meson self-energy is the sum of the contributions of $p$ and $n$ loops:

$$\Pi_{aa}(q^2) = \Pi_{aa}^{(p)}(q^2) + \Pi_{aa}^{(n)}(q^2).$$

(6)

Note that the part of the self-energy containing the term $\Gamma_a G_v \Gamma_b G_v$ is called vacuum part while the other part, i.e. the sum of the terms $\Gamma_a G_v \Gamma_b G_n$ and $\Gamma_a G_n \Gamma_b G_v$ is called the medium part of the self-energy.

A1. Pseudoscalar interaction

We shall first calculate the self-energies of $\pi^0$ and $\eta$ mesons considering the pseudoscalar interactions described by the Lagrangians

$$\mathcal{L}_{\pi}^{PS} = -ig_\pi \left[ \bar{\psi}_p \gamma_5 \Phi \psi_p - \bar{\psi}_n \gamma_5 \Phi \psi_n \right],$$

(7a)

$$\mathcal{L}_\eta^{PS} = -ig_\eta \left[ \bar{\psi}_p \gamma_5 \Phi \psi_p + \bar{\psi}_n \gamma_5 \Phi \psi_n \right].$$

(7b)

In the above Lagrangians, $\psi$ and $\phi$ represent the nucleon spinor and meson fields, respectively. $g_\pi$ ($g_\eta$) is the meson-nucleon coupling constants. The vertex factor $\Gamma_a(q) = -ig_a \gamma_5$. The contribution of nucleon loop (i.e. either proton or neutron loop) at one loop order to the vacuum part of the meson self-energy can be written from Eq. (6) as

$$\Pi_{ab,v}^{(N)}(q^2) = -4\int \frac{d^4k}{(2\pi)^4} \text{Tr}[\tilde{G}_N(k) \Gamma_b(q) \tilde{G}_N(k+q)]$$

$$= 4ig_a g_b \int \frac{d^4k}{(2\pi)^4} \left[ \frac{\tilde{M}_N^2 - k \cdot (k+q)}{(k^2 - M_N^2)((k+q)^2 - M_N^2)} \right].$$

(8)

The above self-energy integral is found to be quadratically divergent at the one loop order and needs to be regularized. The divergent terms can be isolated following the method of dimensional regularization [74–76]. The $PS$ interaction is renormalizable by adding appropriate counter terms to the Lagrangian [53, 72]. Renormalization means the self-energy integral becomes divergence free for all orders [74]. The renormalized vacuum contribution to the self-energy of nucleon loop has been borrowed from Ref. [75]:

$$\Pi_{aa,v}^{(N)}(q^2) = \frac{(g_a)^2}{2m_N^2} \left[ 3(M_N^2 - M_v^2) + (q^2 - m_a^2) \left( \frac{1}{3} + \frac{M_N^2}{m_a^2} \right) \right.$$ 

$$- 2M_N^2 \ln \left( \frac{M_N^2}{M_N^2 - M_v^2} \right) + \frac{8M_N^2 (M_N^2 - M_v^2)^2}{(4M_N^2 - m_a^2)^2} \right.$$ 

$$- 2M_N^2 \sqrt{4M_N^2 - q^2} \tan^{-1} \left( \frac{q}{\sqrt{4M_N^2 - q^2}} \right)$$

$$+ 2M_N^2 \sqrt{4M_N^2 - m_a^2} \tan^{-1} \left( \frac{m_a}{\sqrt{4M_N^2 - m_a^2}} \right)$$

$$+ \left( M_N^2 - M_v^2 \right) + \frac{m_a^2 (M_N^2 - M_v^2)^2}{(4M_N^2 - m_a^2)^2} + \frac{M_N^2 (q^2 - m_a^2)}{m_a^2} \right)$$

$$\times \frac{8M_N^2}{m_a \sqrt{4M_N^2 - m_a^2}} \tan^{-1} \left( \frac{m_a}{\sqrt{4M_N^2 - m_a^2}} \right)$$

$$+ \int_0^1 dx 3x(1-x)q^2 \ln \left( \frac{M_N^2 - q^2x(1-x)}{M_N^2 - m_a^2x(1-x)} \right) \right].$$

(9)

The vacuum part of the meson self-energy may be written after suitable approximation as

$$\Pi_{aa}^{(N)}(q^2) \approx A_{0a} + A_{1a} q^2,$$

(10)

keeping terms up to orders $q^2/M_N^2$ and neglecting its higher orders, where

$$A_{0a} = \frac{(g_a)^2}{2m_N^2} \left[ 3(M_N^2 - M_v^2) + 2M_v^2 \ln \left( \frac{M_N}{M_v} \right) \right.$$ 

$$+ 3(M_N^2 - M_v^2) + 2M_v^2 \ln \left( \frac{M_N}{M_v} \right) \right],$$

(11a)

$$A_{1a} = 3 \frac{(g_a)^2}{2m_N^2} (M_N^2 + M_v^2)/m_a^2.$$
The contribution of the medium part of the self-energy reads
\[ \Pi^{(N)}_{ab,m}(q^2) = -i \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ (-ig_a \gamma_3) G_v(k)(-ig_b \gamma_3) G_m(k+q) + (-ig_a \gamma_3) G_m(k)(-ig_b \gamma_3) G_v(k+q) \right], \]
which after calculating the trace and performing the integration of \( k_0 \), can be written as
\[ \Pi^{(N)}_{ab,m}(q^2) = -2g_ag_b \int \frac{d^3k}{(2\pi)^3 E_N} \left[ \frac{4(k \cdot q)^2}{q^4 - 4(k \cdot q)^2} \theta(k_N - |\mathbf{k}|) \right]. \]
(13)
The calculation is restricted to the low momentum excitation as mentioned earlier. That means \( q^2 < k^2 \). This allows one to write Eq. (13) as
\[ \Pi^{(N)}_{ab,m}(q^2) \approx 2g_ag_b \int \frac{d^3k}{(2\pi)^3 E_N} \left[ 1 + \frac{q^2}{4(k \cdot q)^2} \right] \theta(k_N - |\mathbf{k}|), \]
(14)
which after integration and algebraic manipulation (see Appendix A) reduces to
\[ \Pi^{(N)}_{ab,m}(q^2) \approx 2 \left( \frac{g_a}{2\pi} \right) \left( \frac{g_b}{2\pi} \right) \times \left\{ \frac{k_N E_N}{2} - \frac{\sqrt{M^2_k}}{\pi} \ln \left( \frac{E_N + k_N}{E_N - k_N} \right) \right\}. \]
(15)

Similar to the vacuum part one may approximate the medium part of the self-energy.
\[ \Pi_{aa,m}(q^2) \approx B_{0a} + B_{1a}(q_0^2 - q^2), \]
(16)
where
\[ B_{0a} = 2 \left( \frac{g_a}{2\pi} \right)^2 \left\{ \frac{k_p E_p + k_n E_n}{M_p^2} - \frac{\sqrt{M^2_k}}{\pi} \ln \left( \frac{E_p + k_p}{E_p - k_p} \right) + \frac{\sqrt{M^2_k}}{\pi} \ln \left( \frac{E_n + k_n}{E_n - k_n} \right) \right\}, \]
(17a)
\[ B_{1a} = \left( \frac{g_a}{2\pi} \right)^2 \left\{ \frac{k_p E_p + k_n E_n}{M_p^2} - \frac{\sqrt{M^2_k}}{\pi} \ln \left( \frac{E_p + k_p}{E_p - k_p} \right) + \frac{\sqrt{M^2_k}}{\pi} \ln \left( \frac{E_n + k_n}{E_n - k_n} \right) \right\}. \]
(17b)
The total self-energy is the sum of vacuum and medium contributions:
\[ \Pi_{aa}(q^2) = \Pi_{aa,v}(q^2) + \Pi_{aa,m}(q^2). \]
(18)
The space like self-energy is required for construction of the CSB potential in momentum space which is obtained by substituting \( q_0 = 0 \) in the expression of \( \Pi_{aa,v} \).
\[ \Pi_{aa}^{PS}(q^2) = (A_{0a} + B_{0a}) - (A_{1a} + 2B_{1a})q^2. \]
(19)

### A2. Pseudovector interaction

In this section we calculate the meson self-energies considering pseudovector meson-nucleon interactions:
\[ \mathcal{L}^{PV}_v = -\frac{g_v}{2M_N} \bar{\Psi}_n \gamma_5 \Phi \gamma_5 \Psi_n - \bar{\Psi}_n \gamma_5 \Phi \gamma_5 \Psi_n, \]
(20a)
\[ \mathcal{L}^{PV}_q = -\frac{g_q}{2M_N} \bar{\Psi}_n \gamma_5 \Phi \gamma_5 \Psi_n. \]
(20b)

Following Eq. (15), one may write the contribution of nucleon loop to the vacuum part of the self-energy
\[ \Pi^{(N)}_{ab,v}(q^2) \approx -i \int \frac{d^4k}{(2\pi)^4} \times \text{Tr} \left[ \left( -\frac{g_a}{2M_N} \gamma_5 \Phi \gamma_5 \Psi_n - \bar{\Psi}_n \gamma_5 \Phi \gamma_5 \Psi_n \right) \right]. \]
(21)

The above integral is also divergent. Similar to the \( PS \) interaction one may invoke dimensional regularization to extract the diverging parts of this integral. After dimensional regularization the integral of Eq. (21) reduces to
\[ \Pi^{(N)}_{ab,v}(q^2) = \left( \frac{g_a}{2\pi} \right)^2 \left\{ \frac{\sqrt{M^2_N}}{\pi} \ln \left( \frac{\sqrt{4M^2_N - q^2}}{q} \right) \right\} \tan^{-1} \left( \frac{q}{\sqrt{4M^2_N - q^2}} \right). \]
(22)

Note that \( \epsilon = (4 - D)/2 \) and \( \mu \) is an arbitrary scaling parameter. \( D \) is the dimension of integration. \( \gamma_E \) is the Euler-Mascheroni constant. The above integral diverges for \( D = 4 \).

The pseudovector interaction is non-renormalizable because of the derivative term. That means one can not eliminate the divergences for all orders by adding appropriate counterterms in the Lagrangian \[74\]. Various renormalization methods have been discussed in \[74\]. It is to be mentioned that the result depends on a particular method \[74\]. Here we adopt subtraction scheme \[75\] to eliminate the divergences (see Appendix A) of Eq. (22):
\[ \Pi^{(N)}_{aa,v}(q^2) = \left( \frac{g_a}{2\pi} \right)^2 q^2 \times \left\{ \frac{\sqrt{4M^2_N - q^2}}{q} \tan^{-1} \left( \frac{q}{\sqrt{4M^2_N - q^2}} \right) \right\} \left( \frac{4M^2_N - m^2_a}{m_a} \right)^{1/2} \tan^{-1} \left( \frac{m_a}{\sqrt{4M^2_N - m^2_a}} \right). \]
(23)
Now one may approximate vacuum part of the self-energy similar to the PS interaction:

$$\Pi_{aa,v}(q^2) \approx A'_1 a q^2,$$  \hspace{1cm} (24)

where

$$A'_1 a = \frac{1}{12} \left( \frac{g_a}{2\pi} \right)^2 \left( \frac{m_a^2}{M_p^2} + \frac{m_a^2}{M_N^2} \right).$$  \hspace{1cm} (25)

The contribution of nucleon loop to the medium part of the meson self-energy reads

$$\Pi^{(N)}_{ab,m}(q^2) = -i \int \frac{d^4k}{(2\pi)^4} \times \text{Tr} \left[ \left( -\frac{g_a}{2M_N} \gamma_5 \right) G_v(k) \left( \frac{g_b}{2M_N} \gamma_5 \right) G_m(k+q) \right. \right.$$  

$$\left. + \left( -\frac{g_b}{2M_N} \gamma_5 \right) G_m(k) \left( \frac{g_a}{2M_N} \gamma_5 \right) G_v(k+q) \right],$$  \hspace{1cm} (26)

which after calculating the trace and performing the integration of $k_0$ similar to that of the medium part of $PS$ interaction, reduces to

$$\Pi^{(N)}_{ab,m}(q^2) = -8 \left( \frac{g_a}{2M_N} \right) \left( \frac{g_b}{2M_N} \right) \tilde{M}_N^2 q^4 \times \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{4(kq)^2} \right] \theta(k_N - |k|).$$  \hspace{1cm} (27)

Now the Eq. (27) may be evaluated and approximated as discussed in Appendix A

$$\Pi^{(N)}_{ab,m}(q^2) \approx (q_0^2 - 2q^2) \left( \frac{g_a}{2\pi} \right) \left( \frac{g_b}{2\pi} \right)$$  

$$\times \left[ \left( \frac{M_N}{M_N} \right)^2 - \frac{ \tilde{M}_N^2}{M_N^2} \frac{1}{2} \right] \ln \left( \frac{E_N + k_N}{E_N - k_N} \right).$$  \hspace{1cm} (28)

The medium part of the self-energy can be written as

$$\Pi^{(N)}_{aa,m}(q^2) \approx B'_{1a}(q_0^2 - 2q^2)$$  \hspace{1cm} (29)

where,

$$B'_{1a} = \left( \frac{g_a}{2\pi} \right)^2 \frac{1}{2} \left[ \left( \frac{M_p}{M_p} \right)^2 + \frac{M_a}{M_n} \right]$$

$$- \frac{1}{2} \left( \frac{\tilde{M}_n}{M_N} \right)^2 \ln \left( \frac{\tilde{E}_n + k_n}{\tilde{E}_n - k_n} \right)$$

$$+ \left( \frac{\tilde{M}_n}{M_N} \right)^2 \ln \left( \frac{\tilde{E}_n + k_n}{\tilde{E}_n - k_n} \right) \right].$$  \hspace{1cm} (30)

The total space like self-energy is given by

$$\Pi^{PV}_{aa} = -(A'_{1a} + 2B'_{1a})q^2.$$  \hspace{1cm} (31)

B. Meson Propagators in medium

While mesons propagate through the nuclear medium they are scattered by the Fermi spheres and receive corrections to their masses and energies. This effect may be incorporated through the in-medium meson propagators. Such in-medium propagator may be derived from standard covariant perturbation theory. Here we will solve the Swinger - Dyson equation to find the in-medium meson propagator:

$$\tilde{D}_a(q^2) = D_a(q^2) + D_a(q^2)\Pi_{aa}(q^2)D_a(q^2) \cdots + \cdots,$$  \hspace{1cm} (32)

where $D_a(q^2) = (q^2 - m_a^2 + i\epsilon)^{-1}$ is representing the bare meson propagator while $\tilde{D}_a(q^2)$ is the in-medium meson propagator which after solving Eq. (32) can be written as

$$\tilde{D}_a(q^2)^{-1} = q^2 - m_a^2 - \Pi_{aa}(q^2).$$  \hspace{1cm} (33)

FIG. 2: Diagrammatic presentation of Swinger - Dyson equation.

In Eq. (33) the imaginary part, $i\epsilon$ has been dropped as it is not important in the present context.

For the construction of CSB potential one needs space like in-medium meson propagators which is obtained from Eq. (33) by substituting $q_0 = 0$ in this equation. The total meson self-energies from Eq. (19) and Eq. (31) in Eq. (33) one can write the space like meson propagators $D_a(q^2)$ and $\tilde{D}_a(q^2)$ in medium for $PS$ and $PV$ interactions respectively,

$$\tilde{D}_a(q^2) = \frac{1}{(1 - A_{1a} - 2B_{1a})(q^2 + M_a^2)}$$  \hspace{1cm} (34a)

$$\tilde{D}'_a(q^2) = \frac{1}{(1 - A'_{1a} - 2B'_{1a})(q^2 + M_a'^2)},$$  \hspace{1cm} (34b)

where

$$M_a^2 = \frac{m_a^2 + A_{1a} + B_{0a}}{1 - A_{1a} - 2B_{1a}}$$  \hspace{1cm} (35a)

$$M_a'^2 = \frac{m_a'^2}{1 - A'_{1a} - 2B'_{1a}}$$  \hspace{1cm} (35b)

Note that $M_a$ (or $M_a'$) is not the effective mass of meson. The effective mass of meson can be obtained from Eq. (33) by solving $D_a(q^2 = 0)^{-1} = 0$.

III. π-η MIXING AMPLITUDE

In this section π-η mixing amplitudes have been calculated at the one loop order for both $PS$ and $PV$ interactions. The mixing amplitude, $\Pi_{ab}(q^2)$ is generated
by the $p$-loop contribution, $\Pi_{ab}^{(p)}(q^2)$ minus the $n$-loop contribution, $\Pi_{ab}^{(n)}(q^2)$ as shown in Fig. 3 where, the continuous line represents the loop nucleon and mesons by the dashed lines.

$$\Pi_{ab} = \cdots a \cdots p - \text{loop} \quad b \quad \cdots \quad a \cdots n - \text{loop}$$

**FIG. 3:** Generic diagram for mixing amplitude.

The origin of negative sign between the $p$-loop and $n$-loop contributions may be understood from the interaction Lagrangians given in Eq. (4) or Eq. (20). Note that $\pi^0$ and $\eta$ couple to proton with the same sign while they couple to the neutron with opposite sign. This brings the negative sign between $p$ and $n$ loop contributions. Therefore the mixing amplitude,

$$\Pi_{ab}(q^2) = \Pi_{ab}^{(p)}(q^2) - \Pi_{ab}^{(n)}(q^2). \quad (36)$$

First we proceed to calculate the mixing amplitude for $PS$ interaction.

### A. Pseudoscalar interaction

The vacuum contribution of the nucleon loop to the $\pi-\eta$ mixing amplitude can be obtained from Eq. (3). As discussed before, this part contains divergent terms, one of which is proportional to $M_N$. This divergent term may be eliminated by subtracting $\Pi_{\pi\eta,v}(q^2 = 0)$ from $\Pi_{\pi\eta,v}(q^2)$. After this subtraction the vacuum contribution reads

$$\Pi_{\pi\eta,v}^{(N)}(q^2) = \left(\frac{g_{\pi\eta}}{2\pi}\right)^2 \left\{7 \left[\frac{1}{3} - \frac{1}{\epsilon} + \gamma_E - \ln(4\pi\mu^2)\right] + \ln(M_N^2 - q^2) - \frac{q}{4M_N^2 - q^2}\right\}. \quad (37)$$

Note that this simple subtraction, however, removes the divergence proportional to $M_N^2$ but can not remove the divergence completely. There is still one divergent term proportional to $q^2$ as seen in Eq. (37). The subtraction of $n$-loop contribution from the $p$-loop contribution completely removes this divergent term yielding the vacuum part of the mixing amplitude finite. Thus the vacuum part of the mixing amplitude can be approximated as

$$\Pi_{\pi\eta,v}^{PS}(q^2) \approx A_{1\pi\eta}q^2, \quad (38)$$

where the constant

$$A_{1\pi\eta} = \left(\frac{g_{\pi\eta}}{2\pi}\right)^2 \ln(M_p/M_n). \quad (39)$$

It is important to note that in case of mixing in vacuum, the mixing amplitude can be obtained by replacing $M_p$ (or $M_n$) with $M_p (or M_n)$ in Eq. (39). The mixing amplitude vanishes if $M_p = M_n$ and CSB interaction in vacuum vanishes.

The medium contribution to the mixing amplitude can be obtained from the Eq. (15). Therefore the medium part of the mixing amplitude may be written as

$$\Pi_{\pi\eta,m}^{PS}(q^2) \approx B_{0\pi\eta} + B_{1\pi\eta}(q_0^2 - 2q^2). \quad (40)$$

The constants

$$B_{0\pi\eta} = 2 \left(\frac{g_{\pi\eta}}{2\pi}\right)^2 \left[\left(k_p E_p - k_n E_n\right) \ln\left(E_p + k_p\right) - \ln\left(E_n + k_n\right)\right] \quad (41a)$$

$$B_{1\pi\eta} = \left(\frac{g_{\pi\eta}}{2\pi}\right)^2 \left[\left(k_p E_p - k_n E_n\right) \ln\left(E_p + k_p\right) - \ln\left(E_n + k_n\right)\right] \quad (41b)$$

The in-medium nucleon mass $\tilde{M}_N$ and nucleon energy $\tilde{E}_N$ depend on the Fermi momentum $k_F$ of the nucleon, which is a function of baryon density $\rho_B$ and the asymmetry parameter $\alpha$ as discussed in section II. Therefore the constants, $A_{1\pi\eta}, B_{0\pi\eta}$ and $B_{1\pi\eta}$ depend on $\rho_B$ and $\alpha$. Thus for the mixing amplitudes in this case, both the vacuum and medium parts are driven by the asymmetry parameter $\alpha$.

To construct CSB potential in momentum space one needs space like mixing amplitude which reads as

$$\Pi_{\pi\eta,v}^{PS}(q^2) \approx -A_{1\pi\eta}q^2, \quad (42a)$$

$$\Pi_{\pi\eta,m}^{PS}(q^2) \approx B_{0\pi\eta} - 2B_{1\pi\eta}q_0^2. \quad (42b)$$

The mixing amplitude contains both the vacuum and medium parts,

$$\Pi_{\pi\eta}(q^2) = \Pi_{\pi\eta,v}^{PS}(q^2) + \Pi_{\pi\eta,m}^{PS}(q^2). \quad (43)$$

### B. Pseudovector interaction

The vacuum contribution of $\pi-\eta$ mixing amplitude in case of $PV$ interaction can be obtained from Eq. (23) following the method discussed in appendix B. The vacuum contribution of the mixing amplitude therefore reads

$$\Pi_{\pi\eta,v}^{PV}(q^2) \approx A_{1\pi\eta}q^2. \quad (44)$$

Note that the value of the constant $A_{1\pi\eta}$ depends on the condition (32). If one consider $m_{\pi}^2 = m_{\pi}^2$ in Eq. (32) then

$$A_{1\pi\eta} = \frac{1}{12} \left(\frac{g_{\pi\eta}}{2\pi}\right)^2 \left(M_p^2 - m_{\pi}^2\right) \quad (45)$$
and if $m_a^2 = m_n^2$ is chosen in Eq. (42) then

$$A'_1 \pi \eta - \frac{1}{12} \left( \frac{g_a}{2 \pi} \right)^2 \left( \frac{m_\eta^2}{M_p^2} - \frac{m_n^2}{M_n^2} \right).$$

(46)

The vacuum contribution to the mixing amplitudes as obtained above also vanish in the limit $M_p = M_n$, but unlike the $PS$ interaction, independent of medium effect because of the method adopted to remove the divergence. The density dependent part of the mixing amplitude may be obtained from Eq. (28):

$$\Pi'_{\pi \eta, n}(q^2) \approx B'_{1 \pi \eta}(q_0^2 - 2q^2),$$

(47)

where the constant $B'_{1 \pi \eta}$ is given by

$$B'_{1 \pi \eta} = \left( \frac{g_a}{2 \pi} \right)^2 \frac{1}{2} \left( \frac{k_\eta E_\eta}{M_p^2} - \frac{k_n E_n}{M_n^2} \right) - \frac{1}{2} \left( \frac{M_p}{M_n} \right)^2 \ln \left( \frac{E_\eta + k_\eta}{E_\eta - k_\eta} \right) - \left( \frac{M_n}{M_p} \right)^2 \ln \left( \frac{E_n + k_n}{E_n - k_n} \right).$$

(48)

The space like mixing amplitudes are given by

$$\Pi'_{\pi \eta, n}(q^2) \approx -A'_{1 \pi \eta} q^2,$$

(49a)

$$\Pi'_{\pi \eta, n}(q^2) \approx -2B'_{1 \pi \eta} q^2.$$  

(49b)

IV. CHARGE SYMMETRY BREAKING POTENTIAL

In this section we construct $CSB$ potential employing the one-boson exchange ($OBE$) model considering the in-medium meson propagators and the in-medium spinors for external nucleon legs as shown in the relevant Feynman diagrams in Fig. 4. To construct the $CSB$ potential one should first calculate the nucleon-nucleon scattering amplitude $M_{\pi \eta}^{(NN)}(q^2)$ using the Fig. 4

$$M_{\pi \eta}^{(NN)}(q^2) = \{ U_3(P_3, s_3) \tau_3(1) \Gamma_\pi (q^2) U_1(P_1, s_1) \}
\times \{ \bar{D}_\eta(q^2) \Pi_\eta(q^2) \bar{D}_\eta(q^2) \}
\times \{ U_4(P_4, s_4) \Gamma_\eta(-q^2) U_2(P_2, s_2) \}
\times \{ \bar{U}_3(P_3, s_3) \Gamma_\eta(q^2) U_1(P_1, s_1) \}
\times \{ \bar{D}_\eta(q^2) \Pi_\eta(q^2) \bar{D}_\eta(q^2) \}
\times \{ \bar{U}_4(P_4, s_4) \tau_3(2) \Gamma_\eta(-q^2) U_2(P_2, s_2) \}. \quad (50)$$

In the above expression $U_l(P_l, s_l)$ where $l = 1, \ldots, 4$ represents in-medium spinors for the external nucleon legs as represented by solid straight lines in Fig. 4 with four momentum $P_l$ and nucleon spin $s_l$. The meson-nucleon-nucleon interaction vertices are labeled by 1 and 2. The isospin operator $\tau_3$ takes care of the fact that only the neutral pion couple with the nucleon.

The potential in the momentum space is obtained from the scattering amplitude given in Eq. (50) by substituting $q_0^2 = 0$:

$$V_{CSB}^{(NN)}(q^2) = M_{\pi \eta}^{(NN)}(0, q^2),$$

(51)

along with the non-relativistic form of the spinors obtained by suitable expansion of the nucleon energy $E_N = \sqrt{M_N^2 + p_l^2}$ as

$$U(p_l, s_l) \sim \left( 1 - \frac{p_l^2}{8M_N^2} - \frac{q^2}{32M_N^2} \right) \left( g \cdot (p_l + q/2) \right). \quad (52)$$

in terms of the average nucleon momentum defined by $p = (p_1 + p_3)/2 = (p_2 + p_4)/2$ and the three momentum of meson, $q = p_1 - p_3 = p_2 - p_4$. Such expansion of the nucleon energy in the spinors helps one to simplify the spin structure of the $NN$ potential. $g$ represents the Pauli spin matrices.

Since the mixing amplitude contains both the vacuum and medium contributions, the $CSB$ potential also contains two parts, namely a vacuum part $V_{CSB,v}^{(NN)}(q^2)$ and a medium part $V_{CSB,m}^{(NN)}(q^2)$:

$$V_{CSB}^{(NN)}(q^2) = V_{CSB,v}^{(NN)}(q^2) + V_{CSB,m}^{(NN)}(q^2). \quad (53)$$

It is important to note that mesons and nucleons are not point like particles. They have quark structures. One must include the form factors at each nucleon-nucleon-meson interaction vertex while derive the potential within the framework of $OBE$ model [53, 76]. In the most of the calculations phenomenological form factors either monopole type [54, 61, 77] or dipole type [50] are used at each vertex. Such form factors are obtained from the phenomenological fit of the two nucleon data. However, the vertex factor ought to be calculated from the same theory that provides the propagator [53]. The form factor also cures the problem of contact term in $OBE$ potential [77].

In the present calculation we use phenomenological form factors of dipole type in space like region at each vertex. At each vertex the coupling constant $g_a$ in Eq. (51) is to be replaced as follows:

$$g_a \rightarrow g_a \left( \frac{A_a^2 - m_a^2}{A_a^2 + m_a^2} \right). \quad (54)$$
The cut-off parameter \( \Lambda_a \) in the OBE model separates the short ranged effects of the nuclear force while the long ranged parts are handeled by the meson propagators \(^{53, 76}\).

Once we have the potential in momentum space we can easily obtain it in the coordinate space by Fourier transformation. Thus Eq. (55) reduces to

\[
V^{(NN)}_{CSB}(r) = \int \frac{d^3q}{(2\pi)^3} V^{(NN)}_{CSB}(q^2) \exp(-i\mathbf{q} \cdot \mathbf{r}). \tag{55}
\]

### A. Pseudoscalar interaction

Substituting the in-medium meson propagators, phenomenological form factors, non-relativistic spinors, mixing amplitude and keeping the terms of the order \( \mathcal{O}(q^2/M_N^2) \) with some algebraic calculation one may write the CSB potential in momentum space as

\[
V^{(NN)}_{CSB,v}(q^2) = T^+_3 C_{\pi\eta}(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{4M_N^2}
\times \left[ \frac{1}{M^2_\pi - M^2_\eta} \left\{ \frac{M^2_\pi \delta_{\pi\eta}}{q^2 + M^2_\eta} - \frac{M^2_\eta \delta_{\eta\pi}}{q^2 + M^2_\pi} \right\} \right.
\times \frac{1}{\Lambda^2_\eta - \Lambda^2_\pi}
\left\{ \frac{\Lambda^2_\eta \delta_{\pi\eta}}{q^2 + M^2_\eta} - \frac{\Lambda^2_\pi \delta_{\eta\pi}}{q^2 + M^2_\pi} \right\}, \tag{56}
\]

and

\[
V^{(NN)}_{CSB,m}(q^2) = T^+_3 C_{\pi\eta}(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{4M_N^2}
\times \left[ \frac{B_{\pi\eta}}{M^2_\eta - M^2_\pi} \left\{ \frac{\delta_{\pi\eta}}{q^2 + M^2_\eta} - \frac{\delta_{\eta\pi}}{q^2 + M^2_\pi} \right\} \right.
\times \frac{1}{\Lambda^2_\eta - \Lambda^2_\pi}
\left\{ \frac{\Lambda^2_\eta \delta_{\pi\eta}}{q^2 + M^2_\eta} - \frac{\Lambda^2_\pi \delta_{\eta\pi}}{q^2 + M^2_\pi} \right\}
\times \frac{1}{\Lambda^2_\eta - \Lambda^2_\pi}
\left\{ \frac{\Lambda^2_\eta \delta_{\pi\eta}}{q^2 + M^2_\eta} - \frac{\Lambda^2_\pi \delta_{\eta\pi}}{q^2 + M^2_\pi} \right\}\] (57).

In the above equations \( T^+_3 = \tau_3(1) + \tau_3(2) \),

\[
C_{\pi\eta} = \frac{1}{(1 - A_{1\pi} - 2B_{1\pi})(1 - A_{1\eta} - 2B_{1\eta})}, \tag{58}
\]

and

\[
\delta_{ab} = \left( \frac{\Lambda^2_a - m^2_a}{\Lambda^2_\pi - M^2_\pi} \right) \left( \frac{\Lambda^2_b - m^2_b}{\Lambda^2_\eta - M^2_\eta} \right), \tag{59a}
\]

\[
\tilde{\delta}_{ab} = \left( \frac{\Lambda^2_a - m^2_a}{\Lambda^2_\pi - M^2_\pi} \right) \left( \frac{\Lambda^2_b - m^2_b}{\Lambda^2_\eta - M^2_\eta} \right). \tag{59b}
\]

One may easily obtain the coordinate space potential by Fourier transform of the momentum space potential once constructed. Thus from Eq. (56) and Eq. (57), the vacuum and medium parts of the potentials in coordinate space, respectively are as follows:

\[
V^{(NN)}_{CSB,v}(r) = -T^+_3 C_{\pi\eta}(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{48\pi M^2_N}
\times \left[ \frac{M^5_\pi \delta_{\pi\eta} U(x_\pi) - M^5_\eta \delta_{\eta\pi} U(x_\eta)}{M^2_\pi - M^2_\eta}
\right.
\times \frac{\Lambda^5_\pi \delta_{\pi\eta} U(x_\pi) - \Lambda^5_\eta \delta_{\eta\pi} U(x_\eta)}{\Lambda^2_\pi - \Lambda^2_\eta}, \tag{60}
\]

and

\[
V^{(NN)}_{CSB,m}(r) = -T^+_3 C_{\pi\eta}(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{48\pi M^2_N}
\times \left[ \frac{B_{\pi\eta}}{8M^2_N} \left\{ \frac{M^5_\pi \delta_{\pi\eta} U(x_\pi) - M^5_\eta \delta_{\eta\pi} U(x_\eta)}{M^2_\pi - M^2_\eta}
\right. \right.
\times \frac{\Lambda^5_\pi \delta_{\pi\eta} U(x_\pi) - \Lambda^5_\eta \delta_{\eta\pi} U(x_\eta)}{\Lambda^2_\pi - \Lambda^2_\eta}
\left. \right. \right)
\times \left[ \frac{B_{\pi\eta}}{8M^2_N} + 2B_{1\pi\eta} \right]
\left\{ \frac{M^5_\pi \delta_{\pi\eta} U(x_\pi) - M^5_\eta \delta_{\eta\pi} U(x_\eta)}{M^2_\pi - M^2_\eta}
\right. \right.
\times \frac{\Lambda^5_\pi \delta_{\pi\eta} U(x_\pi) - \Lambda^5_\eta \delta_{\eta\pi} U(x_\eta)}{\Lambda^2_\pi - \Lambda^2_\eta}
\left. \right. \right).
\tag{61}
\]

Note that \( x_\pi = M_a r \) and \( x_\eta = \Lambda_a r \). We denote

\[
U(x) = \left[ (\sigma_1 \cdot \sigma_2) + S_{12}(\hat{r})(1 + \frac{3}{x} + \frac{3}{x^2}) \right] e^{-x}, \tag{62}
\]

\[
S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - (\sigma_1 \cdot \sigma_2). \tag{63}
\]

### B. Pseudovector interaction

Similar to the pseudoscalar interaction, the CSB potential in momentum space for vacuum and medium part in pseudovector interaction can be written as

\[
V^{(NN)}_{CSB,v}(q^2) = T^+_3 C_{\pi\eta}(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{4M_N^2}
\times \left[ \frac{1}{M^2_\pi - M^2_\eta} \left\{ \frac{M^2_\eta \delta_{\eta\pi} U(x_\pi) - M^2_\eta \delta_{\eta\pi} U(x_\eta)}{q^2 + M^2_\eta}
\right. \right.
\times \frac{1}{\Lambda^2_\eta - \Lambda^2_\pi}
\left. \right. \right)
\times \left[ \frac{1}{\Lambda^2_\pi - \Lambda^2_\eta}
\left. \right. \right] \tag{64}
\]

and

\[
V^{(NN)}_{CSB,m}(q^2) = T^+_3 C_{\pi\eta}(2B_{1\pi\eta})(\sigma_1 \cdot \mathbf{q})(\sigma_1 \cdot \mathbf{q}) \frac{g_{\pi\pi\eta}}{4M^2_N}
\times \left[ \frac{1}{M^2_\pi - M^2_\eta} \left\{ \frac{M^2_\eta \delta_{\eta\pi} U(x_\pi) - M^2_\eta \delta_{\eta\pi} U(x_\eta)}{q^2 + M^2_\eta}
\right. \right.
\times \frac{1}{\Lambda^2_\eta - \Lambda^2_\pi}
\left. \right. \right)
\times \left[ \frac{1}{\Lambda^2_\pi - \Lambda^2_\eta}
\left. \right. \right], \tag{65}
\]
where,

\[ C'_{\pi\eta} = \frac{1}{(1 - A'_{\pi} - 2B'_{\pi})(1 - A'_{\eta} - 2B'_{\eta})}. \]  

(66)

Note that \( \delta'_{ab} \) and \( \delta'_{\eta} \) can be obtained from Eq. (59) by replacing \( M_a \rightarrow M'_a \) and \( M_b \rightarrow M'_b \). The Fourier transformation of Eq. (64) and Eq. (65) give us the potentials in coordinate space.

\[
V^{(NN)}_{CSB,v}(r) = -\frac{T^+ C'_{\pi\eta}A'_{\pi\eta}}{48\pi M_N^2} \times \left[ M^0_a \delta'_{\pi\eta} U(x'_a) - M^0_b \delta'_{\pi\eta} U(x'_b) \right] + \frac{\Lambda^2_{\pi} \delta'_{\pi\eta} U(x_a) - \Lambda^2_{\pi} \delta'_{\pi\eta} U(x_b)}{\Lambda^2_{\pi} - \Lambda^2_{\eta}} \right],
\]

(67)

and

\[
V^{(NN)}_{CSB,m}(r) = -\frac{T^+ C'_{\pi\eta}(2B'_{\pi\eta})}{48\pi M_N^2} \times \left[ M^0_a \delta'_{\pi\eta} U(x'_a) - M^0_b \delta'_{\pi\eta} U(x'_b) \right] + \frac{\Lambda^2_{\pi} \delta'_{\pi\eta} U(x_a) - \Lambda^2_{\pi} \delta'_{\pi\eta} U(x_b)}{\Lambda^2_{\pi} - \Lambda^2_{\eta}} \right],
\]

(68)

where, \( x'_a = M'_a r \).

V. RESULT AND DISCUSSION

In this section we discuss our numerical results. To generate numerical data following meson parameters listed in table have been used. The nucleon mass in nuclear medium is estimated by solving Eq. (31) self-consistently. We borrowed the values of coupling constant \( g_{\sigma} \) and mass \( m_{\sigma} \) of the scalar meson \( \sigma \) from Ref. [5] to calculate the effective nucleon mass \( M_N \) in nuclear matter.

| Mesons | \( m_{\sigma} \) (MeV) | \( \frac{\Lambda^2_{\pi}}{M_N^2} \) | \( \Lambda_{\pi} \) (MeV) |
|--------|-----------------|-----------------|-----------------|
| \( \pi \) | 138.6 | 14.6 | 1300 |
| \( \eta \) | 548.0 | 5.0 | 1500 |

All the Figures in this section represent the difference of CSB potentials between \( mn \) and \( pp \) systems in the coordinate space in \( ^1S_0 \) state. We denote \( \Delta V = V^{(CSB)}_{nn} - V^{(pp)} \), where \( V^{(CN)} = V^{(CSB)}_{\pi\eta} = V^{(CSB,m)} \). We consider the nuclear matter density \( \rho_0 = 0.148 \text{ fm}^{-3} \).

At first we present the vacuum contribution, \( \Delta V_v \) and medium contribution, \( \Delta V_m \) for PS interaction in Fig. 4 and the same for PV interaction in Fig. 5 respectively.

These figures show the variation of vacuum and medium parts with distance \( r \) at density \( \rho_B = 1.2\rho_0 \) and asymmetry \( \alpha = 0.2 \). The vacuum contribution is found to dominate over the medium contribution below \( r \sim 0.25 \text{ fm} \) for PS interaction while for PV interaction the same effect is observed below \( r \sim 0.5 \text{ fm} \). In addition, the medium contribution of PS interaction is found \( \sim 10^3 \) times larger than that of PV interaction below \( r \sim 0.5 \text{ fm} \), however the vacuum contributions for both cases are comparable.

\[ \Delta V_v \] and \( \Delta V_m \) show the difference of CSB potentials i.e., \( \Delta V \) for PS and PV interactions at different densities but the same asymmetry \( \alpha = 0.2 \). The dotted, solid and dashed curves represent \( \Delta V \) at densities \( \rho_B = \rho_0, 1.2\rho_0 \) and \( 1.4\rho_0 \) respectively.

It is observed that below \( r \sim 0.5 \text{ fm} \), \( \Delta V \) for PS interaction is \( \sim 10^3 \) times larger than that of PV interaction as shown in Fig. 4 and Fig. 5. The origin of such large value possibly arises from the in-medium meson propagators. For PS interaction \( C^{\pi\eta} \) is found \( \sim 10^{-5} \) times of \( C^{\pi\eta} \) for PV interaction. And this causes large
FIG. 7: The plot shows variation of $\Delta V$ with $r$ at different densities for $PS$ interaction considering the in-medium $\pi$ and $\eta$ meson propagators.

FIG. 8: The plot shows variation of $\Delta V$ with $r$ at different densities for $PV$ interaction considering the in-medium $\pi$ and $\eta$ meson propagators.

FIG. 9: The variation of $\Delta V$ with $r$ at $\alpha = 0.2$ and $\rho_B = 1.2\rho_0$ for $PS$ (dashed curve) and $PV$ (dotted curve) interactions is shown. The effective $\pi$ and $\eta$ meson masses are used to generate this graph, instead of propagators.

FIG. 10: The plot shows the variation of $\Delta V$ with $r$ both for $PS$ and $PV$ interactions considering the effective masses of $\pi$ and $\eta$ mesons instead of in-medium propagators. To study the role of effective mass of meson we simply replaced the bare masses with their effective masses to the meson propagators. It is clear from Fig. 10 that such replacement of bare mass with in-medium mass of meson makes $\Delta V$ comparable for both $PS$ and $PV$ interactions. At $r \sim 0.2$ fm, $\Delta V$s are found to be equal for both the cases and $\Delta V$ is positive between $r \sim 0.25 - 0.75$ fm for $PS$ interaction.

VI. SUMMARY

In the present work we have revisited $CSB$ due to $\pi$-$\eta$ mixing in nuclear matter employing the $OBE$ model and constructed the two-body $CSB$ potential which is class $III$ type. The in-medium nucleon propagator is used to calculate the meson self-energies and $\pi$-$\eta$ mixing amplitude and both calculations are restricted to the one loop order. We used these meson self-energies to obtain the in-medium meson propagators by solving the Swinger-Dyson equation. The bare propagators in the $CSB$ potential in momentum space is replaced by the in-medium $\pi$ and $\eta$ propagators.

Furthermore, we also used the mixing amplitude in space like region to construct the two-body $CSB$ potential instead of constant or on-shell mixing amplitude and the effect of external nucleon legs is also taken into account.

We noticed that the difference of $CSB$ $nn$ and $pp$ potentials, $\Delta V$ for $PV$ interaction is insignificant compared to that for $PS$ interaction while the effective masses of $\pi$ and $\eta$ instead of their in-medium propagators shows comparable contributions for both the cases.

ACKNOWLEDGMENT

The author thanks Prof. Pradip K. Roy, Saha Institute of Nuclear Physics, and Mahatsab Mandal, Government General Degree College of Kalna, East Bardwan, India for their valuable comments and suggestions.
Appendix A

Integrating over the azimuthal angle $\phi$, Eq. (14) reads

$$\Pi_{\text{ab,m}}^{(N)}(q^2) \approx 2 \left( \frac{g_A}{2\pi} \right) \int_0^{k_N} \frac{k^2dk}{\sqrt{M_N^2 + k^2}} \left[ \frac{1 + q^4}{4(k^2q^2)} \right] \left( I_1 + I_2 \right), \quad (A1)$$

where $x = \cos \theta$,

$$I_1 = 2 \int_0^{k_N} \frac{k^2dk}{\sqrt{M_N^2 + k^2}} \left( \frac{\bar{E}_N}{E_N - k_N} - \frac{1}{2} \right) \ln \left( \frac{\bar{E}_N + k_N}{E_N - k_N} \right), \quad (A2)$$

and

$$I_2 = \frac{q^4}{4g_0^2} \int_0^{k_N} \frac{k^2dk}{\sqrt{M_N^2 + k^2}} \left[ \frac{2}{(q_0E_N - |k||q|x)^2} \right] \left( \frac{\bar{E}_N}{E_N - k_N} - \frac{1}{2} \right) \ln \left( \frac{\bar{E}_N + k_N}{E_N - k_N} \right) \left( \frac{\bar{E}_N}{E_N - k_N} - \frac{1}{2} \right), \quad (A3)$$

where, $z = 1 - q^2/q_0^2$. Note that $z < 1$ and $2k^2 << \bar{M}_N^2$. Thus we neglected the term proportional to $z$ in Eq. (A3). Thus Eq. (A4) can be written as

$$I_2 \approx (q_0^2 - 2q^2) \frac{1}{4} \ln \left( \frac{E_N + k_N - 2q^2}{E_N - k_N} \right). \quad (A5)$$

Substituting $I_1$ and $I_2$ in Eq. (A1) we obtain the contribution of nucleon loop to medium part Eq. (A5).

Appendix B

To remove the diverging part from Eq. (22) we use simple subtraction method [73]. Let us denote

$$\Pi_v^{(N)}(q^2) = \left( \frac{M_N}{M_N} \right)^2 \left[ -2 - \frac{1}{\epsilon} + \gamma_E + \ln(4\pi\mu^2) \right] + 2\ln(\bar{M}_N) \quad (B1)$$

and

$$\tilde{\Pi}_v^{(N)}(q^2) = \Pi_v^{(N)}(q^2) - \Pi_v^{(N)}(q^2 = m^2). \quad (B2)$$

This will remove the divergence yielding the finite vacuum part of the self-energy:

$$\Pi_{\text{aa,m}}^{(N)}(q^2) = \left( \frac{g_A}{2\pi} \right)^2 \tilde{\Pi}_v^{(N)}(q^2) q^2. \quad (B3)$$

References:

[1] E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 17.
[2] E. M. Henley and G. A. Miller, in *Mesons in Nuclei*, edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), p. 405.
[3] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).
[4] G. A. Miller, M. K. Nefkens, and I. Slaus, Phys. Rep. 194, 1 (1990).
[5] G. A. Miller and W. H. T. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W. C. Haxton and E. M. Henley (World Scientific, Singapore, 1995), p. 127.
[6] G. A. Miller, A. W. Thomas, and A. G. Williams, Phys. Rev. Lett. 56, 2567 (1986).
[7] A. W. Thomas, and A. G. Williams, and G. A. Miller, Phys. Rev. C 36, 156 (1986).
[8] L. Ge and J. P. Svenne, Phys. Rev. C 33, 417 (1986).
[9] A. Gersten et al., Few Body Sys. 3, 171 (1988).
[10] B. Holzenkamp, K. Holinde, and A. W. Thomas, Phys. Lett. B 195, 121 (1987).
[11] J. A. Niskanen and A. W. Thomas, Aust. J. Phys. 41, 31 (1988).
[12] M. J. Iqbal, J. Thaler, and R. M. Woloshyn, Phys. Rev. C 36, 2442 (1987).
[13] M. J. Iqbal, and J. A. Niskanen, Phys. Rev. C 38, 2259 (1988).
[14] C. R. Howell et al., Phys. Lett. B 444, 252 (1998).
[15] D. E. Gonzalez Trotter et al., Phys. Rev. Lett. 83, 3788 (1999).
[16] S. A. Coon and R. C. Barret, Phys. Rev. C 36, 2189
