Cyclostationary-based BSS method for time-frequency overlapped BPSK signals in electromagnetic surveillance

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Abstract. A cyclostationary-based blind signal separation (BSS) algorithm is proposed in this paper. The proposed separation algorithm is used for simultaneously received multiple spectrum overlapped BPSK signals in single channel electromagnetic surveillance system. The mixed signal is separated by means of blind adaptive frequency shift (BA-FRESH) filter based on the independent signals’ cyclostationary. This BA-FRESH filtering technique does not require training signals. It can separate the independent signals only by knowing some of their cycle frequencies, which is gained by the second order cycle cumulants in this work. Numerical simulation results proved the effectiveness of this separation method.

1. Introduction

In modern information age, the communication surveillance system becomes even more complex and varied electromagnetic environment. The probability that receiver receives the mixture of multiple independent signal has increased greatly. How to intercept and capture useful components from the mixture develops into an important research topic, especially for single channel surveillance system. In electromagnetic surveillance, single-channel BSS (SCBSS) has great significance.

In the uncooperative communication, the signals received by single-channel receiver are usually not only overlapped in time domain but also in frequency domain (called time-frequency overlapped signal), so it can not separate each signal contained in the mixture by time filter or frequency filter. Besides, the conventional separation methods become ineffective for the lack of prior information. Therefore, SCBSS of time-frequency overlapped signal is an important research project full of challenges.

Independent component analysis (ICA), firstly proposed by Jutten and Herault [1,2], improved by Common [3,4], is the most widely used separation method in BSS. However, most of ICA algorithms rely on spatial (i.e., multichannel) analysis. In [5], a method for extracting information from single channel recordings of electroencephalogram (EEG) signals is introduced through ICA in a dynamical system framework. For the base-band communication signal separation, several previous papers have been published [6, 7, 8, 9]. For the mixture of signals with distinct powers, the reference [6] uses maximum likelihood and maximum a posteriori criteria to derive co-channel symbol detectors. In [7], the differences between shaping filter are used by Warner to handle single-channel separation. The limited character set feature of digital communication signal is employed by Szu, and proposed an effective single-channel BSS method. A particle filtering based algorithm for SCBS of two convolutionally coded signals is proposed in [9]. However, as above said, those four algorithms are only suit to base-band signal. Comparing with base-band communication signal separation, there are less papers about the intermediate frequency signal separation. By joining the singular spectrum
analysis and blind source separation techniques, [10] introduced an effective separation approach, however, it can hardly handle the frequency overlapped signals. [14] explores the capabilities of cyclostationary for interference rejection when only one sensor is available, however, it needs known some of the interfaces’ parameters.

Based on cyclostationary, a BSS algorithm for simultaneously received multiple spectrum overlapped BPSK signals in single channel electromagnetic surveillance system is proposed in this paper. In section II, the BA-FRESH filter separation system, containing FRESH filter system and blind adaptive separation system, is discussed in details. The separation procedure of this algorithm is presented in section III. Numerical simulation results are given in section IV to verify this method. Section V is the conclusion.

2. Ba-Fresh Filter Separation System

Signals can be obtained from different domains, where they are not overlapped, such as spectrdomains, temporaldomains or spatial domains. However, they are all not suit to the separation problem here discussing for the single channel surveillance system and signals overlapping in frequency domain. Herein, cycle spectral domain is used to separate time-frequency overlapped signals using BA-FRESH filter. cyclostationary is an inherent feature of BPSK signal [11,12]. For a real input signal \( x(n) \), the output of a FRESH [12] is given by

\[
y(n) = \sum_m h_m(n) \ast \left[ s(n) e^{j2\pi a_m n} \right]
\]  

(1)

whose Fourier transform as

\[
Y(f) = \sum_m H_m f \times f - a_m
\]  

(2)

Where, \( h_m(n) \) is the impulse response of the complex time-invariant filter, \( a_m \) is cycle frequency and \( \ast \) means convolution. Designing an optimum FRESH filter is to extract the useful signals’ information of the auto-spectral density matrix of input signal and the cross spectral density vector between input signal and desired output signal. In [13], the standard least-mean-square and recursive least-square adaptive filter algorithm are selected to yield the optimum FRESH filter. However, a training signal is needed in this algorithm, so it cannot be used here. Now, considering the input of FRESH filter is an instantaneous linear mixture of several independent communication signals recorded by electromagnetic surveillance system with one antenna, as equation (3) depicts.

\[
x(n) = \sum_i a_i y_i(n) + v(n)
\]  

(3)

Where, \( s_i(n) \) is the \( i \)th original signal, \( v(n) \) is additive white Gaussian noise with variance \( \sigma^2 \), \( a_i \) is a positive constant determining the independent signals’ participation degree in the mixture, \( D \) is the original sources number. In this paper, the cyclic spectrum of each independent signal must be exclusively different, and our purpose is to extract them from the mixture one by one based cyclostationary through BA-FRESH filter. Taking extract the \( j \)th signal \( s_j(n) \) for example, we can regard \( s_j(n) \) as the desired signal and \( \sum_{i \neq j} \sum_i s_i(n) \) as interference, for convenient using \( s(n) \) and \( u(n) \) to express them respectively. Then, (3) can be written into (4).

\[
x(n) = s(n) + u(n) + v(n)
\]  

(4)

For the lack reference signal, a suitable reference is yield by adding a secondary branch as shown in Figure 1. In Figure 1, the output of the second branch \( r(n) \) is an \( a' \) shift version of the input \( x(n) \), and one cycle frequency of \( s(n) \) is \( a' \). The rationale is that if the output \( y(n) \) is a close approximation of \( s(n) \) and is relatively free from containing \( u(n) \) and \( v(n) \), then it must have high correlation with the \( r(n) \). Thus, the correlation of the two signals may provide a measure of the suppression of \( u(n) \) and \( v(n) \).
2.1 FRESH filter separation system

In this section, only the FRESH filter is considered, as the gray shaded part shown in Figure 1. Here, using $\phi_j$ stands for the cycle frequency set of the $j$th independent signal, as

$$\phi_j = \left\{ \phi_{j1}, \phi_{j2}, \ldots, \phi_{jM} \right\}$$

Then, the cycle frequency set of the equation (4) can be expressed as

$$\phi_i = \phi_j^*, \phi_k = \phi_i^*, i = 1, 2, \ldots, M, i \neq j$$

Figure 1. BA-FRESH filter structure.

And using $\phi_i$ expresses the cycle frequency of white Gaussian noise $v(n)$, then $\phi_i \cap \phi_k \cap \phi_i = \phi$. $\phi$ means empty set. The strategy used in this paper is to take cycle frequencies of $\phi_i$, exclusively related to $s(n)$, which show the highest spectral correlation level. Then, if given $\phi_i$, the optimum FRESH filter (as the gray shaded part in Figure 1) can be defined by the multi-dimensional Wiener filter theory, the frequency domain results equation (7) shows [14].

$$\mathbf{W_f} = \mathbf{S_{ss}^{-1}} f = \mathbf{S_{ss}^{-1}} f \left[ \mathbf{S_{ss} f + S_{ssu} f + S_{ssM} f} \right]$$

Where, the single-overline represents vector notation and a double-overline represents a matrix. $\xi$ is the noise power spectral density and $\mathbf{V_M}$ is the identity matrix of size $M \times M$. $\mathbf{W_f}^T$ is the column vector of the FRESH filter as

$$s^{\phi_i}_f$$

is the spectral correlation function [15] of the $s(n)$, and $s^{\phi_i}_f$ is the row vector of spectral correlation of $s(n)$, as

$$s^{\phi_i}_f = \left[ s_{\phi_i}^{\phi_1} f, s_{\phi_i}^{\phi_2} f, \ldots, s_{\phi_i}^{\phi_M} f \right]$$

$$s^{\phi_i}_f = F.T. \left[ s_{n+1m} \phi_i \right], m = 1, 2, \ldots, M$$

Where, $\langle \rangle$ and $n$ mean the time-average with respect to $n$ and the conjugate transpose respectively. Additionally, the definition of matrix $\mathbf{S_{ss} f}$ is shown in equation (11).

$$\mathbf{S_{ss} f} = F.T. \left[ s_{n+1m} \phi_i \right]$$

2.2 Blind adaptive separation system

The optimum FRESH system defined in equation (7) depends on the spectral correlation function of the interest signal, however, for blind signal separation, which is unknown. Therefore, an adaptive algorithm is employed to adjust the FRESH filter constituting $\mathbf{W_f}^T$ . Adaptive algorithms need a reference signal to compute the estimation error [13]. The optimum reference is a sample of the original signals, which is not available. Consequently, the second branch is used to yield as reference...
signal. In this work, it must ensure that the FRESH filter in the primary branch has no common frequency shift with the secondary branch [13]. The following is how to adjust the coefficient of the set of the FRESH filter so as to minimize the estimation error.

As shown in Figure 1, the coefficient vector of the primary FRESH filter is

\[ h_m = [h_m^0, h_m^1, \ldots, h_m^{L-1}]^T, m = 1, \ldots, M \]  

(12)

The output \( y(n) \) of the BA-FRESH filter can be written as

\[ y(n) = \sum_{m=1}^{M} h_m^T \tau_m(n) = h^T \tau(n) \]  

(14)

Where, \( \tau(n) = [\tau^0(n), \tau^1(n), \ldots, \tau^L(n)]^T \), with \( \alpha_m \neq \alpha_n, m = 1, \ldots, M \). The BA-FRESH filter seeks to maximize the normalized correlation between \( y(n) \) and \( r(n) \) by adjusting the coefficients \( h \), where \( r(n) = x(n) e^{j2\pi \alpha_n} \).

\[ \max J(h) = \max \left( \frac{|R_{yy}|}{|R_{yr}||R_{yr}|} \right) \]  

(15)

Where \( R_{yy} = \langle \tau^T(n) \tau(n) \rangle, R_{yr} = \langle \tau^T(n) r(n) \rangle \). And \( \kappa_{xy} = \langle \tau^T(n) \tau(n) \rangle, \rho_{xy} = \langle \tau^T(n) r(n) \rangle \), then according to Cauchy-Schwarz inequality, we can got

\[ J(h) = \frac{|h^T R_{yy}^{1/2} \rho_{xy}^{1/2}|^2}{|h^T R_{yy}| |h^T R_{yr}|} \leq \frac{|h^T R_{yy}^{1/2} \rho_{xy}^{1/2}|^2}{|h^T R_{yy}| |h^T R_{yr}|} \]  

(16)

For equality, we have

\[ h_{BF} = R_{xx}^{-1} \rho_{xy} \]  

(17)

It is the Wiener-Hopf equation for BA-FRESH filter. As \( \kappa_{xy} \) and \( \rho_{xy} \) are not available. So a recursive method for updating the tap-weights of the FRESH filter in the primary branch can be obtained, following the Widrow-HoffLMS algorithm [13] by using the output \( r(n) \) of the secondary branch as the reference and choosing an appropriate step-size. Alternatively, we can window the frequency shifted data vectors \( \tau(n) \) to obtain the cross-correlation vector \( \rho_{xy} \) and finite-sample time-average estimates of the data correlation matrix \( \kappa_{xy} \). Using \( \kappa_{xy} \), \( \rho_{xy} \) and \( h_{BF} \), denote \( \hat{\kappa}_{xy} \), \( \hat{\rho}_{xy} \) and \( \hat{h}_{BF} \) respectively, we can obtain

\[ \hat{h}_{BF} = R_{xx}^{-1} \hat{\rho}_{xy} \]  

(18)

\[ \hat{\kappa}_{xx} n = \left( 1 - \frac{1}{n} \right) \hat{\kappa}_{xx} n-1 + \frac{1}{n} \tau(n) \tau^T(n), \hat{\rho}_{xy} n = \left( 1 - \frac{1}{n} \right) \hat{\rho}_{xy} n-1 + \frac{1}{n} \tau(n) r^T(n) \]  

(19)

Using the matrix inversion lemma, we have

\[ \hat{R}_{xx}^{-1} n = \frac{\hat{R}_{xx}^{-1} n-1 - \frac{n-1}{n} \hat{R}_{xx}^{-1} n-1} {\frac{n-1}{n} \hat{R}_{xx}^{-1} n-1} \]  

(20)

Then, arriving at the recursive formula of the update desired filter response as

\[ h_{BF}(n) = (I - \frac{1}{n-1} \hat{R}_{xx}^{-1} (n-1) \hat{\kappa}(n) \hat{\rho}(n)) \times (b(n-1) + \frac{1}{n} \hat{R}_{xx}^{-1} (n-1) \hat{\kappa}(n) \hat{\rho}(n)) \]  

(21)

3. Separation procedure
From derivation in section II, we know that some of the cycle frequencies of the original signals must be known before using the BA-FRESH filter separation method. Reference [11] proved the BPSK has the cyclostationary feature. In [12], Gardner pointed out the cycle frequency set of BPSK is 
\[ \alpha = \{ 2f_c \pm k/T_b \} \] , where \( f_c \) and \( T_b \) are the carrier frequency and the bit rate of the signal. As the estimation of the carrier frequency and bit rate are all bring in estimation error. In order to reduce those influence, the cycle frequency \( 2f_c \), when \( k = 0 \), is used here. That is to say, the primary branch of FRESH has one filter \( ( M = 1 ) \) and its cycle frequency is set by \( 2f_c \). Additionally, the second -order cyclic cumulants are used to estimate the frequencies of each original signals from the the single channel recording [16] [17]. Alternatively, the received signal \( x(n) \) is used as a reference. To be exactly, \( r(n) \) is replaced by \( x(n) \). Numerical simulation results in section 4 demonstrate that this separation system can capture the necessary information of the original signal.

The whole separation procedure can be summarized into two main steps. Step 1: use the second-order cyclic cumulants to estimate the frequencies of each original signals from the mixture. Step 2: use the BA-FRESH filter separation system to extract the independent signals from the single channel mixture one by one based on the cyclostationary. Therefore, the cyclic frequencies of each independent signal must satisfy equation (6). Here, it means the carry frequency of each independent component must be different.

In all, this section introduced the BA-FRESH filter separation system and the main steps of the separation process. The effectiveness of this separation approach will be presented in section 4.

4. Numerical simulation

Numerical simulation results of the proposed method is presented applied in electromagnetic surveillance field. First of all, the definition of frequency overlapped level (FOL), correlation coefficient and normalization mean square error (NMSE) are given, as

\[ \eta_i = \frac{\text{Bandwidth}_i}{\text{Bandwidth}_i (s_i)} \]
\[ \rho_{\eta_i} = \frac{\text{cov}(s_i, \delta_i)}{\sqrt{\text{Bandwidth}_i (s_i) \text{Bandwidth}_i (\delta_i)}} \]
\[ \text{NMSE}_i = \frac{\sum_{n=1}^{N_i} |s_i(n) - \delta_i(n)|^2}{\text{Bandwidth}_i (s_i) \text{SampleTime}} \]

Where \( i = 1, 2 \cdots D \). \( \text{Bandwidth}_i \) and \( \text{Bandwidth}_i (s_i) \)are the bandwidth of the \( i \)th signal \( s_i \) and its frequency overlapped bandwidth with the other signal respectively. \( \eta_i \) and \( \rho \) are the FOL of each original source and the FOL of mixed signal respectively. \( s_i \) and \( \delta_i \) are original components and corresponding estimated one.

Table 1. Simulation Conditions.

| Source number | Signaltypes | \( f_r \) (MHz) | \( \alpha \) | Bit rate \( s_i \) (Mb/s) | SampleTime (\( \mu s \)) | \( f_r \) (MHz) | SNR (dB) |
|---------------|-------------|---------------|-------------|-----------------|----------------|---------------|-----------|
| 3             | BPSK        | 12            | 1           | 1               | 200            | 100           | 10        |
|               |             | 13            | 1           |                 |                |               |           |
|               |             | 14            | 1           |                 |                |               |           |

Table 1 shows the simulation conditions. The signal-noise ratio (SNR) is defined by
\[ \text{SNR} = 10 \log \left( \frac{\alpha^2}{\sigma^2} \right) \], and \( \sigma^2 \) stands for the total power of original signals. The FOL calculated by (13) is 100%, and the FOL of each components are 50%, 100% and 50% respectively.

Figure2 shows frequency estimation results by the second-order cyclic cumulants estimation method. Figure 3 shows how noise influences the absolute error of frequency estimation results by 100 Monte Carlo experiments. It demonstrates that the algorithm can provide high estimation results even
in low SNR environment. When \( SNR \geq 5dB \), the algorithm will not be influenced by SNR, then the sample number and FOL become the main factors that influence the estimation accuracy.

![Figure 2. Frequency estimation results](image)

![Figure 3. Frequency estimation error changing trend with SNR.](image)

The simulation conditions as shown in Table 1, only change SNR, Figure 4 (a) and (b) exhibit the comparison of estimated signal and its corresponding original source in form of NMSE and bit error rate (BER) changing trend with SNR. When \( SNR \geq 5dB \), the \( NMSE_{\hat{s}_i} \) and \( BER_{\hat{s}_i} \) will reach a relatively stable state (\( NMSE_{\hat{s}_i} < -5dB \) and \( BER_{\hat{s}_i} < 10^{-2} \)). The separation accuracy of signal one and three are similar to each other and are much better than that of signal two, which is because the FOL of signal two is higher than that of the other two signals.

![Figure 4. The NMSE and BER variation trend with SNR](image)

Frequency band of digital communication signal is controlled by its bit rate and carrier frequency. So the FOL of each original signal can be revised by renew \( f_c \), or \( r_i (i = 1, 2, 3) \), or both of them. Here, revision of \( \eta_i (i = 1, 2, 3) \) is adopted. Still use simulation conditions shown in Table 1, except for \( r_b \) and SNR. Table 2 gives the value of \( \eta_i \) and the corresponding \( \eta \) as well as \( \eta_i \) of four different FOL environment. Figure 5 gives its numerical simulation results.

| \( r_b \) (Mb/s) | \( r_{s_i} \) (Mb/s) | \( r_{s_i} \) (Mb/s) | \( \eta_i \) | \( \eta \) | \( \eta_i \) | \( \eta_i \) |
|-----------------|------------------|------------------|--------|--------|--------|--------|
| 0.5             | 1.5              | 2.5              |        |        |        |        |
| 1               | 2                | 3                |        |        |        |        |

![Table 2. The Bit Rate For Four Different FOL](image)
Here, NMSE is calculated by (23) (b), the BER is calculated by $BER = \frac{1}{D} \sum_{i=1}^{D} BRE_{S_i,b_i}$.

Form Figure 5, we can see lower FOL yields better separation results. However, even when FOL reaches 100%, the proposed approach still can provide high accuracy separation results. As shown in Figure 5, when $SNR \geq 10dB$, the method will reach a stable stage with $NMSE < -8dB$ and $BER < 10^{-2}$. The fourth group is significantly better than that of the other three groups, that is not only because $\eta$ is smaller but due to all $\eta_i (i=1,2,3)$ are much smaller. That also illustrates FOL is an important factor influenced separation accuracy.

5. Conclusion
To deal with the varied and complex electromagnetic surveillance, a SCBSS is proposed in this work based on independent components’ cyclostationary. The separation system and procedure have been introduced in details in section 2 and 3 respectively. Its effectiveness and high separation accuracy is proved by numerical simulation from all aspects. It has lower NMSE and BER, no more than -8dB and $10^{-2}$ respectively, when $SNR \geq 10dB$, which can absolutely guarantee the surveillance system captures all information sent by others from its single-channel recording. Additionally, the BA-FRESH filter separation system does not require training signals which greatly reduce computation complexity.

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