Jet broadening in deeply inelastic scattering *

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Abstract

In deeply inelastic lepton-nucleus scattering (DIS), the average jet transverse momentum is broadened because of multiple scattering in the nuclear medium. The size of jet broadening is proportional to the multi-parton correlation functions inside nuclei. We show that at the leading order, jet broadening in DIS and nuclear enhancement in di-jet momentum imbalance and Drell-Yan $\langle q_T^2 \rangle$ share the same four-parton correlation functions. We argue that jet broadening in DIS provide an independent measurement of the four-parton correlation functions and a test of QCD treatment of multiple scattering.

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I. INTRODUCTION

When a parton propagates through a nuclear matter, the average transverse momentum may be broadened because of multiple scattering of the parton. Nuclear dependence of such broadening provides an excellent probe to study QCD dynamics beyond the single hard-scattering picture. Reliable calculation of multiple scattering in QCD perturbation theory requires to extend the factorization theorem beyond the leading power. Qiu and Sterman showed that the factorization theorem for hadron-hadron scattering holds at the first-nonleading power in momentum transfer, which is enough to study double scattering processes in hadronic collisions. According to this generalized factorization theorem, the double scattering contribution can be expressed in terms of universal four-parton (twist-four) correlation functions. The predictions of the multiple scattering effects rely on the accurate information of the four-parton correlation functions. Various estimate of the size of these four-parton correlation functions were derived recently. It is the purpose of this paper to show that jet broadening in deeply inelastic lepton-nucleus scattering (DIS) can be used as an independent process to test the previous estimate of the four-parton correlation functions.

The four-parton (twist-4) correlation functions are as fundamental as the normal twist-2 parton distributions. While twist-2 parton distributions have the interpretation of the probability distributions to find a parton within a hadron, four-parton correlation functions provide information on quantum correlations of multi-partons inside a hadron. Just like normal twist-2 parton distributions, multi-parton correlation functions are non-perturbative. QCD perturbation theory cannot provide the absolute prediction of these correlation functions. But, due to factorization theorems, these correlation functions are universal; and can be measured in some processes and be tested in other processes.

Luo, Qiu and Sterman (LQS) calculated the nuclear dependence of di-jet momentum imbalance in photon-nucleus collision in terms of nuclear four-parton correlation functions.
\[
T_{q/A}(x) = \int \frac{dy^-}{2\pi} e^{ix p^+ y^-} \frac{dy_1^- dy_2^-}{2\pi} \theta(y_1^- - y^-) \theta(y_2^-) \\
\times \frac{1}{2} \langle p_A | \bar{\psi}_q(0) \gamma^+ F^+_\sigma(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | p_A \rangle ; \quad (1a)
\]
and
\[
T_{g/A}(x) = \int \frac{dy^-}{2\pi} e^{ix p^+ y^-} \frac{dy_1^- dy_2^-}{2\pi} \theta(y_1^- - y^-) \theta(y_2^-) \\
\times \frac{1}{x p^+} \langle p_A | F^+\alpha(0) F^+_\sigma(y_2^-) F^{+\sigma}(y_1^-) F^{+\alpha}(y^-) | p_A \rangle . \quad (1b)
\]

By comparing the operator definitions of these four-parton correlation functions and the definitions of the normal twist-2 parton distributions, LQS proposed the following model [3,6]:

\[
T_{f/A}(x) = \lambda^2 A^{4/3} \phi_{f/N}(x) , \quad (2)
\]
where \( \phi_{f/N}(x) \) with \( f = q, \bar{q}, g \) are the normal twist-2 parton distribution of a nucleon, and \( \lambda \) is a free parameter to be fixed by experimental data. Using the Fermilab E683 data on di-jet momentum imbalance, LQS estimated the size of the relevant four-parton correlation functions to be of the order \( \lambda^2 \approx 0.05 \sim 0.1 \text{ GeV}^2 \) [3].

The nuclear enhancement of the average Drell-Yan transverse momentum, \( \Delta \langle q_T^2 \rangle \), also depends on the similar four-parton correlation functions [4]. However, Fermilab and CERN data on nuclear dependence of the Drell-Yan \( \Delta \langle q_T^2 \rangle \) prefer a much smaller size of the four-parton correlation functions [4,7,8]. Actually, as we will show below, the Drell-Yan data favor the four-parton correlation functions about five times smaller than what was extracted from the di-jet data. This discrepancy may result from different higher order contribution to the Drell-Yan \( \Delta \langle q_T^2 \rangle \) and the di-jet momentum imbalance. We have pure initial-state multiple scattering for the Drell-Yan process, while pure final state multiple scattering for a di-jet system. It is necessary to study the high order corrections to these two observables in order to test QCD treatment of multiple scattering. It was argued in Ref. [9,10] that the high order contribution to the Drell-Yan nuclear enhancement is important. Meanwhile, it is also important and necessary to find different observables that depend on the same four-parton correlation functions.

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The jet broadening in deeply inelastic lepton-nucleus scattering provides an independent test of the size of the four-parton correlation functions. At the leading order in deeply inelastic scattering, a quark scattered of the virtual photon forms a jet (known as the current jet) in the final state. Because of the multiple interaction when the scattered parton propagates through the nuclear matter, the transverse momentum of the final state jet is broadened in deeply inelastic lepton-nucleus scattering \[1\]. We show below that the size of the jet broadening is directly proportional to the four-parton correlation functions. These four-parton correlation functions are the same as those appeared in the nuclear enhancement of the di-jet momentum imbalance. Current data for the di-jet momentum imbalance and the Drell-Yan $\Delta \langle q_T^2 \rangle$ provide two different sizes of the four-parton correlation functions. Using these data, without additional parameters, we predict a range of the jet broadening in DIS. Existing data from Fermilab E665 and future HERA nuclear beam data can provide a direct test of QCD treatment of multiple scattering.

The rest of this paper is organized as follows. In Sec. II, we derive the nuclear enhancement of the Drell-Yan $\langle q_T^2 \rangle$ by using the same method used by LQS to derive the di-jet momentum imbalance, and show that our result is consistent with that presented in Ref. \[4\]. In Sec. III, using the same method, we derive jet broadening in DIS in terms of the same four-parton correlation functions. Finally, in Sec. IV, we use the values of $\lambda^2$ extracted from data on di-jet momentum imbalance and Drell-Yan $\Delta \langle q_T^2 \rangle$ to estimate jet broadening in DIS. We also present our discussions and conclusions in this section.

**II. NUCLEAR ENHANCEMENT FOR DRELL-YAN $\langle Q_T^2 \rangle$**

Consider the Drell-Yan process in hadron-nucleus collisions, $h(p') + A(p) \rightarrow \ell^+ \ell^-(q) + X$, where $q$ is the four-momentum for the virtual photon $\gamma^*$ which decays into the lepton pair. $p'$ is the momentum for the incoming beam hadron and $p$ is the momentum per nucleon for the nucleus with the atomic number $A$.

Let $q_T$ be the transverse momentum of the Drell-Yan pair, we define the averaged trans-
verse momentum square as

$$\langle q_T^2 \rangle^{hA} = \int dq_T^2 \cdot q_T^2 \cdot \frac{d\sigma^{hA}}{dQ^2 dq_T^2} \left/ \frac{d\sigma^{hA}}{dQ^2} \right..$$  \hspace{1cm} (3)$$

In Eq. (3), $Q$ is the total invariant mass of the lepton pair with $Q^2 = q^2$. The transverse momentum spectrum, $d\sigma/dQ^2dq_T^2$, is sensitive to the multiple scattering, and has the $A^{1/3}$-type nuclear size effect. If we write the cross section as

$$\sigma_{hA} = \sigma_{hA}^S + \sigma_{hA}^D + \ldots,$$  \hspace{1cm} (4)$$

where $\sigma_{hA}^S$, $\sigma_{hA}^D$, and “…” represent the single, double and higher multiple scattering, respectively. The single scattering is localized, hence, $\sigma_{hA}^S$ does not have a large dependence on the nuclear size. Therefore, $\sigma_{hA}^S \approx A\sigma_{hN}$ with $N$ represents a nucleon. As we will demonstrate explicitly, the inclusive cross section $d\sigma/dQ^2$ has much weaker nuclear dependence after integration over the whole momentum spectrum. Hence,

$$\int dq_T^2 \cdot q_T^2 \cdot \frac{d\sigma_{hA}^S}{dQ^2 dq_T^2} \left/ \frac{d\sigma_{hA}}{dQ^2} \right. \approx \langle q_T^2 \rangle^{hN}. \hspace{1cm} (5)$$

To extract effect due to multiple scattering we introduce the nuclear enhancement of the Drell-Yan $\langle q_T^2 \rangle$ as

$$\Delta\langle q_T^2 \rangle \equiv \langle q_T^2 \rangle^{hA} - \langle q_T^2 \rangle^{hN}. \hspace{1cm} (6)$$

If we keep only double scattering and neglect contribution from the higher multiple scattering, we have

$$\Delta\langle q_T^2 \rangle \approx \int dq_T^2 \cdot q_T^2 \cdot \frac{d\sigma_{hA}^D}{dQ^2 dq_T^2} \left/ \frac{d\sigma_{hA}}{dQ^2} \right.. \hspace{1cm} (7)$$

From our definition, $\Delta\langle q_T^2 \rangle$ represents a measurement of QCD dynamics beyond the traditional single-hard scattering picture. The nuclear enhancement of Drell-Yan $\langle q_T^2 \rangle$, defined in Eq. (3), is a result of multiple scattering between the incoming beam parton and the nuclear matter before the Drell-Yan pair is produced.

Since we are interested in the averaged transverse momentum square, we do not need to know the angular information of the Drell-Yan lepton pair. After integration over the lepton’s angular information, the Drell-Yan cross section can be written as
\[ d\sigma_{hA\to\ell^+\ell^-} = \left(\frac{2\alpha_{em}}{3Q^2}\right) (-g_{\mu\nu} W_{hA\to\gamma^*}(q)) , \quad (8) \]

where \( W^{\mu\nu} \) is the hadronic tensor \[12\]. To simplify the notation of following derivation, we introduce

\[ W_{hA\to\gamma^*}(q) \equiv -g_{\mu\nu} W_{hA\to\gamma^*}(q) . \quad (9) \]

At the leading order, the single scattering contribution to \( W_{hA\to\gamma^*}(q) \) is given by

\[ W^S_{hA\to\gamma^*}(q) = \sum_{q} \int dx' \phi_{q/h}(x') \int dx \phi_{q/A}(x) \left[ e_q^2 \frac{2\pi\alpha_{em}}{3} \right] , \quad (10) \]

where superscript “S” signals the single scattering; and the corresponding inclusive single scattering cross section is \[12\]

\[ \frac{d\sigma_{hA\to\ell^+\ell^-}}{dQ^2} = \sigma_0 \sum_{q} e_q^2 \int dx' \phi_{q/h}(x') \int dx \phi_{q/A}(x) \delta(Q^2 - xx's) , \quad (11) \]

with \( s = (p + p')^2 \) and the Born cross section

\[ \sigma_0 = \frac{4\pi\alpha_{em}^2}{9Q^2} . \quad (12) \]

Consider only double scattering inside the nuclear target, we can factorize the double scattering contribution to \( W(q) \) as

\[ W^D_{hA\to\gamma^*}(q) = \sum_{f} \int dx' \phi_{f/h}(x') \cdot \hat{W}^D_{fA\to\gamma^*}(x', q) , \quad (13) \]

where superscript “D” indicates the double scattering contribution, and \( f \) sums over all parton flavors. In Eq. \[13\], \( \hat{W}^D_{fA\to\gamma^*}(x', D) \) is the hadronic contribution from double scattering between a parton \( f \) and the nucleus. At the leading order in \( \alpha_s \), \( \hat{W}^D_{fA\to\gamma^*}(x', D) \) is given by the Feynman diagrams in Fig. \[4\]. According to the generalized factorization theorem \[2\], \( \hat{W}^D_{fA\to\gamma^*}(x', D) \) can be factorized as

\[ \hat{W}^D_{hA\to\gamma^*}(x', D) = \frac{1}{2x's} \int dx \int dx_1 \int dx_2 \int dk_T^2 T^{(f)}(x, x_1, x_2, k_T) \tilde{H}(x, x_1, x_2, k_T, x', p, q) \Gamma^D , \quad (14) \]
where $\Gamma^D$ is the phase space factor for the double scattering. For diagram shown in Fig. 1a, for example, the phase space factor is given by

$$
\Gamma^D_a = \frac{d^4 q}{(2\pi)^4} \cdot (2\pi)^4 \delta^4 (xp + x_1 p + x' p' + k_T - q) \\
= \frac{1}{x's} \delta(x + x_1 - \frac{k^2_T}{x's} - \frac{Q^2}{x's}) \delta(\tilde{q}^2_T - k^2_T) \, dq^2 \, dq^2.
$$

(15)

At this stage of the derivation, $\hat{T}$ and $\hat{H}$ in Eq. (14) both depend on gauge choice, even though $W(q)^D$ is gauge invariant. In this paper, Feynman gauge is used in our calculation.

In Eq. (14), the hadronic matrix element

$$
\hat{T}^{(f)}(x, x_1, x_2, k_T) = \int \frac{dy^-_1}{2\pi} \frac{dy^-_2}{2\pi} \frac{d^2 y_T}{(2\pi)^2} \\
\times e^{ixp^+} e^{ix_1 p^+} e^{ix_2 p^+} e^{ik T \cdot y_T} \\
\times \frac{1}{2} \langle p_A | A^+(y_2, 0_T) \psi_\uparrow(0) \gamma^+ \psi_\uparrow(y^-) A^+(y_1, y_1 T) | p_A \rangle.
$$

(16)

Because of the exponential factors in Eq. (16), the position space integration, $dy^-$’s cannot give large dependence on nuclear size unless the parton momentum fraction in one of the exponentials vanishes. If the exponential vanishes, the corresponding position space integration can be extended to the size of whole nucleus. Therefore, in order to get large nuclear enhancement or jet broadening, we need to consider only Feynman diagrams that can provide poles which set parton momentum fractions on the exponentials to be zero [3]. At the leading order, only diagrams shown in Fig. 1 have the necessary poles. These diagrams contribute to the double scattering partonic part $\hat{H}(x, x_1, x_2, k_T, x' p', p, q)$ in Eq. (14).

For the leading order diagrams shown in Fig. 1, the corresponding partonic parts have two possible poles. For example, for diagram shown in Fig. 1, the partonic part has following general structure

$$
\hat{H}^a \sim \frac{1}{(x' p' + x_1 p + k_T)^2 + i\epsilon} \cdot \frac{1}{(x' p' + (x_1 - x_2)p + k_T)^2 - i\epsilon} \cdot (-g_{\mu\nu}) \hat{H}^{\mu\nu}
$$

(17)

where $\hat{H}^{\mu\nu}$ is the numerator which is proportional to a quark spinor trace.
The first step to evaluate the Eq. (14) is to carry out the integration over parton momentum fractions \( dx_1 dx_2 \). Using one of the \( \delta \)-functions in phase space factor in Eq. (15) to fix \( dx \) integration, and the two poles in the partonic part in Eq. (17) to perform the contour integration for \( dx_1 dx_2 \), we obtain

\[
I_a \equiv \int dx_1 dx_2 \ e^{ix_1 p^+ y^-} e^{ix_2 p^+ (y_1^- - y_2^-)} e^{ix_2 p^+ y_2^-} \\
\times \frac{1}{x_1 - \frac{k_T^2}{x's} + i\epsilon} \cdot \frac{1}{x_1 - x_2 - \frac{k_T^2}{x's} - i\epsilon} \cdot \delta(x + x_1 - \frac{k_T^2}{x's} - \frac{Q^2}{x's}) \\
= (4\pi^2) \theta(y^- - y_1^-) \theta(-y_2^-) e^{i(\tau/x') p^+ y^-} e^{i(k_T^2/(x's)) p^+ (y_1^- - y_2^-)},
\]

with \( \tau = Q^2/s \). For the diagram shown in Fig. 1b, we have a slightly different phase space factor

\[
\Gamma_b = \frac{1}{x's} \delta(x + x_2 - \frac{Q^2}{x's}) \delta(q_T^2) dQ^2 dq_T^2.
\]

The corresponding partonic part \( \bar{H}^b \) has slightly different poles

\[
\bar{H}^b \propto \frac{1}{x_1 - \frac{k_T^2}{x's} + i\epsilon} \cdot \frac{1}{x_2 + i\epsilon}.
\]

Integrating over parton momentum fractions, \( dx_1 dx_2 \), we have

\[
I_b \equiv \int dx_1 dx_2 e^{ix_1 p^+ y^-} e^{ix_2 p^+ (y_1^- - y_2^-)} e^{ix_2 p^+ y_2^-} \\
\times \frac{1}{x_1 - \frac{k_T^2}{x's} + i\epsilon} \cdot \frac{1}{x_2 + i\epsilon} \cdot \delta(x + x_2 - \frac{Q^2}{x's}) \\
= (-4\pi^2) \theta(y_2^- - y_1^-) \theta(y^- - y_2^-) e^{i(\tau/x') p^+ y^-} e^{i(k_T^2/(x's)) p^+ (y_1^- - y_2^-)}. 
\]

Similarly, for diagram shown in Fig 1c, we have

\[
I_c = (-4\pi^2) \theta(y_1^- - y_2^-) \theta(-y_1^-) e^{i(\tau/x') p^+ y^-} e^{i(k_T^2/(x's)) p^+ (y_1^- - y_2^-)}. 
\]

It is easy to show that the numerator spinor trace gives the same factor for all three diagrams. Therefore, total contribution to jet broadening is proportional to \( I_a + I_b + I_c \), and this sum has the following feature
\[
\frac{d\sigma_{hA}}{dQ^2 dq_T^2} \propto I_a + I_b + I_c \\
\propto \theta(y^- - y_{\bar{1}}^-) \theta(-y_2^-) \left[ \delta(q_T^2 - k_T^2) - \delta(q_T^2) \right] \\
+ \left[ \theta(y^- - y_{\bar{1}}^-) \theta(-y_2^-) - \theta(y_2^- - y_{\bar{1}}^-) \theta(y^- - y_2^-) - \theta(y_{\bar{1}}^- - y_2^-) \theta(-y_{\bar{1}}^-) \right] \delta(q_T^2) .
\]

(23)

It is clear from Eq. (24) that for the inclusive Drell-Yan cross section, \(d\sigma/dQ^2\), the double scattering contribution, \(d\sigma_{hA}^D/dQ^2\), does not have large dependence on the nuclear size. The integration over \(dq_T^2\) eliminates the first term in Eq. (24), while the second term is localized in space if \(\tau/x'\) is not too small. When the \(\tau/x'\) is finite, and \(p^+\) is large, the \(\exp[i(\tau/x')p^+y^-]\) effectively restricts \(y^- \sim 1/((\tau/x')p^+) \to 0\). When \(y^- \to 0\), the combination of three pairs of \(\theta\)-functions in Eq. (24) vanishes,

\[
\frac{d\sigma_{hA}^D}{dQ^2} = \int dq_T^2 \left( \frac{d\sigma_{hA}^D}{dQ^2 dq_T^2} \right) \\
\propto \left[ \theta(y^- - y_{\bar{1}}^-) \theta(-y_2^-) - \theta(y_2^- - y_{\bar{1}}^-) \theta(y^- - y_2^-) - \theta(y_{\bar{1}}^- - y_2^-) \theta(-y_{\bar{1}}^-) \right] \\
\to 0 \quad \text{as} \quad y \to 0 .
\]

(25)

Physically, Eq. (25) says that all integrations of \(y^-\)'s are localized. Actually, at the leading order, the term proportional to \(\theta(y^- - y_{\bar{1}}^-) \theta(-y_2^-) - \theta(y_2^- - y_{\bar{1}}^-) \theta(y^- - y_2^-) - \theta(y_{\bar{1}}^- - y_2^-) \theta(-y_{\bar{1}}^-)\) is the eikonal contribution in Feynman gauge to make the normal twist-2 quark distribution \(\phi_{q/A}(x)\) in Eq. (11) gauge invariant. Eq. (25) is a good example to demonstrate that the double scattering does not give large nuclear size effect to the total inclusive cross section.

On the other hand, from Eq. (24), the double scattering contribution to the averaged transverse momentum square can gain large size effect

\[
\Delta\langle q_T^2 \rangle \sim \int dq_T^2 q_T^2 \left( \frac{d\sigma_{hA}^D}{dQ^2 dq_T^2} \right) \\
\propto \int dq_T^2 q_T^2 \left[ \delta(q_T^2 - k_T^2) - \delta(q_T^2) \right] \\
\sim k_T^2 .
\]

(26)

Actually, \(k_T^2\) in Eq. (26) needs to be integrated first. But Eq. (26) already demonstrates that \(\Delta\langle q_T^2 \rangle\) is proportional \(k_T^2\), which is the kick of the transverse momentum the parton
received from the additional scattering. The bigger the nuclear size, the bigger effective \( k_T^2 \).

As shown below, \( \Delta \langle q_T^2 \rangle \) is proportional to the nuclear size.

After demonstrating the physical picture of transverse momentum broadening in the Drell-Yan process, we are now ready to carry out the algebra. Working out the spinor trace for all three diagrams in Fig. 1 and combining three diagrams and drop the term localized in space, we obtain

\[
\Delta \langle q_T^2 \rangle \frac{d\sigma_{hA}}{dQ^2} = \int dq_T^2 q_T^2 \sigma_0 \sum_q \int dx' \phi_{q/h}(x') \int dx \delta(Q^2 - xx's) \\
\times \int \frac{dy_2}{2\pi} \frac{dy_1}{2\pi} \theta(y^- - y_1^-) \theta(-y_2^-) e^{ixp^+y^-} \\
\times \left[ \int dq_T^2 e^{ik_T^+ y_T^+} e^{i(k_T^0/(x's))y^+(y^-_1 - y^-_2)} \right] \\
\times \left[ \sum_q \int dx \phi_{q/h}(x') \int dx T^{(I)}_{q/A}(x) \delta(Q^2 - xx's) \right],
\]

where \( \sigma_0 \) is defined in Eq. (12) and the four-parton correlation function is given by

\[
T^{(I)}_{q/A}(x) = \int \frac{dy^-_2}{2\pi} e^{ixp^+y^-} \int \frac{dy^-_1}{2\pi} \theta(y^- - y_1^-) \theta(-y_2^-) \\
\times \left[ \sum_q \int dx' \phi_{q/h}(x') \int dx T^{(I)}_{q/A}(x) \delta(Q^2 - xx's) \right],
\]

In deriving Eq. (28), we expand \( \delta(q_T^2 - k_T^2) \) at \( k_T = 0 \), known as the collinear expansion, and keep only the first non-vanishing term which corresponds to the second order derivative term,

\[
\delta(q_T^2 - k_T^2) - \delta(q_T^2) \approx -\delta'(q_T^2)(-g_{\alpha\beta}^0) k_T^\alpha k_T^\beta.
\]

We use the factor \( k_T^\alpha k_T^\beta \) in Eq. (30) to convert the \( k_T^\alpha A^+ \) into field strength \( F^\alpha + F^\beta \) by partial integration. Here, we work in Feynman gauge. The terms associated with other components of \( A^\rho \) are suppressed by \( 1/p^+ \) compared to those with \( A^+ \), because of the requirement of the Lorentz boost invariance for the matrix elements 3. \( T^{(I)}_{q/A}(x) \) defined in
Eq. (29) is the four-parton correlation function in a nucleus. The superscript (I) represents the initial state interaction, in order to distinguish from the similar four-parton correlation function defined in Eq. (14). More discussion on the relation between \( T_{q/A}(x) \) and \( T_{q/A}^{(I)}(x) \) will be given in Sec. IV.

Combining Eqs. (6), (11), and (28), we obtain the nuclear enhancement of Drell-Yan \( \langle q_T^2 \rangle \) at the leading order in \( \alpha_s \)

\[
\Delta \langle q_T^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \frac{\sum q e_q^2 \int dx' \phi_{q/h}(x') T_{q/A}^{(I)}(\tau/x')/x'}{\sum q e_q^2 \int dx' \phi_{q/h}(x') \phi_{q/A}(\tau/x')/x'} \cdot (31)
\]

III. JET BROADENING IN DEEPLY INELASTIC SCATTERING

Consider the jet production in the deeply inelastic lepton-nucleus scattering, \( e(k_1) + A(p) \rightarrow e(k_2) + jet(l) + X \). \( k_1 \) and \( k_2 \) are the four momentum of the incoming and outgoing lepton respectively, and \( p \) is the momentum per nucleon for the nucleus with the atomic number \( A \). With \( l \) being the observed jet momentum, we define the averaged jet transverse momentum square as

\[
\langle l_T^2 \rangle^{eA} = \int d\tau_T d\phi_T \frac{d\sigma_{eA}}{dx_B dQ^2 d\tau_T^2} \frac{d\sigma_{eA}}{dx_B dQ^2} \cdot (32)
\]

where \( x_B = Q^2/(2p \cdot q) \), \( q = k_1 - k_2 \) is the momentum of the virtual photon, and \( Q^2 = -q^2 \).

The jet transverse momentum \( l_T \) depends on our choice of the frame. We choose the Breit frame in the following calculation. Similar to Drell-Yan \( d\sigma/dQ^2 dq_T^2 \), the jet transverse momentum spectrum, \( d\sigma/dx_B dQ^2 dl_T^2 \), is sensitive to the \( A^{1/3} \) type nuclear size effect due to multiple scattering. On the other hand, the inclusive DIS cross section \( d\sigma/dx_B dQ^2 = \int d\tau_T d\phi_T d\sigma/dx_B dQ^2 d\tau_T^2 \) does not have the \( A^{1/3} \) power enhancement. Instead, it has the much weaker \( A \)-dependence, such as the EMC effect and nuclear shadowing. To separate the multiple scattering contribution from the single scattering, we define the jet broadening as

\[
\Delta \langle l_T^2 \rangle \equiv \langle l_T^2 \rangle^{eA} - \langle l_T^2 \rangle^{eN} . \quad (33)
\]

Keeping only contribution from the double scattering, similar to Eq. (3), we have
\[ \Delta \langle l_T^2 \rangle \approx \int d\ell_T^2 \cdot \left( \frac{d\sigma_{vA}}{dx_B dQ^2 d\ell_T^2} \right) / \left( \frac{d\sigma_{vA}}{dx_B dQ^2} \right). \]  

(34)

In the rest of this section, we derive the leading contribution to \( \Delta \langle l_T^2 \rangle \) by using the same technique used to derive the Drell-Yan \( \Delta \langle q_T^2 \rangle \) in last section.

The general expression for the cross section in DIS is

\[ d\sigma = \frac{1}{2s} \frac{e^2}{Q^4} L_{\mu\nu} W^{\mu\nu} \frac{d^3 k_2}{(2\pi)^3 2E_2}, \]  

(35)

with \( s = (p + k_1)^2 \). In Eq. (35), the leptonic tensor \( L_{\mu\nu} \) is given by the diagram in Fig. 2a,

\[ L_{\mu\nu} = \frac{1}{2} \text{Tr}(\gamma \cdot k_1 \gamma_{\mu} \gamma \cdot k_2 \gamma_{\nu}), \]  

(36)

and \( W^{\mu\nu} \) is the hadronic tensor given by the diagram shown in Fig. 2b. The leading order double scattering diagrams contributing to jet broadening are given in Fig. 3.

Feynman diagrams in Fig. 3 give the double scattering contribution to the hadronic tensor \( W_{\mu\nu} \) as,

\[ W^{D}_{\mu\nu} = \sum_a \int dx dx_1 dx_2 \int dk_T^2 \mathcal{T}^{(F)}(x, x_1, x_2, k_T) \tilde{H}_{\mu\nu}(x, x_1, x_2, k_T, p, q, l), \]  

(37)

with the matrix element

\[ \mathcal{T}^{(F)}(x, x_1, x_2, k_T) = \int \frac{dy^-}{2\pi} \frac{dy_1^-}{2\pi} \frac{dy_2^-}{2\pi} \frac{d^2 y_T}{(2\pi)^2} e^{ixp} e^{ix_1p} e^{ix_2p} e^{ik_T y_T} \times e^{ixp} e^{iy_2} \left( -y_1 - y_2 \right) e^{ix_1p} e^{ix_2p} e^{ik_T y_T} \times \frac{1}{2} \langle pA|\bar{\psi}_q(0) \gamma^+ A^+(y_2, 0_T) A^+(y_1, y_T) \psi_q(y^-)|pA \rangle, \]  

(38)

where superscript \( (F) \) indicates the final-state double scattering. The matrix element \( \mathcal{T}^{(F)} \) is equal to the \( \mathcal{T}^{(l)} \) in Eq. (16) if we commute the gluon fields with the quark fields [2,13]. In Eq. (37), \( \tilde{H}_{\mu\nu}(x, x_1, x_2, k_T, p, q, l) \) is the corresponding partonic part.

For the diagram shown in Fig. 3a, the partonic part is given by

\[ \tilde{H}^a_{\mu\nu} = C' \cdot \tilde{H}_{\mu\nu} \cdot \frac{1}{(xp + q)^2 + i\epsilon} \cdot \frac{1}{(xp + x_2p + q)^2 - i\epsilon} \times (2\pi) \delta((xp + x_1p + k_T + q)^2) \delta(l_T^2 - k_T^2) d\ell_T^2 \]

\[ \times \delta(x + x_1 - x_B - \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q}) \cdot \frac{1}{x - x_B + i\epsilon} \cdot \frac{1}{x + x_2 - x_B - i\epsilon}, \]  

(39)
where $C$ is the color factor for the diagrams in Fig. 3, and given by $C = 1/6$. In Eq. (39), the $\delta$-function is from the phase space, and $\hat{H}_{\mu\nu}$ represents the spinor trace and is given by

$$\hat{H}_{\mu\nu} = (4\pi)^2 \alpha_s \alpha_m e^2 \text{Tr} [\gamma \cdot p \gamma \cdot (xp + x_2 p + q) \gamma^\sigma \gamma \cdot (xp + q) \gamma^\nu] p_\mu p_\nu, \quad (40)$$

where $p_\mu, p_\nu$ is a result of our definition for the hadronic matrix element in Eq. (38).

Following the derivation of the Drell-Yan $\langle q_1^2 \rangle$, first, we carry out the integrations of the parton momentum fractions by using the $\delta$-function and two poles in Eq. (39),

$$H^a \equiv \int dx \, dx_1 \, dx_2 \, e^{ixp^+ y^-} e^{ix_1 p^+(y_1^- - y_2^-)} e^{ix_2 p^+ y_2^-} \delta(l_T^2 - k_T^2)$$

\[ \times \delta(x + x_1 - x_B - \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q}) \cdot \frac{1}{x - x_B + i\epsilon} \cdot \frac{1}{x + x_2 - x_B - i\epsilon} \]

$$= (2\pi)^2 \theta(y_1^- - y^-) \theta(y_2^-) e^{ix_B p^+ y^-} \delta(l_T^2 - k_T^2) e^{i \left( \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q} \right) p^+(y_1^- - y_2^-)}. \quad (41)$$

Similarly, we have the corresponding integrations for the interference diagram shown in Fig. 3:

$$H^b \equiv \int dx \, dx_1 \, dx_2 \, e^{ixp^+ y^-} e^{ix_1 p^+(y_1^- - y_2^-)} e^{ix_2 p^+ y_2^-} \delta(l_T^2)$$

\[ \times \delta(x + x_1 - x_B) \cdot \frac{1}{x - x_B + i\epsilon} \cdot \frac{1}{x + x_1 - x_B - \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q} + i\epsilon} \]

$$= -(2\pi)^2 \theta(y^- - y_1^-) \theta(y_1^- - y^-) e^{ix_B p^+ y^-} \delta(l_T^2) e^{i \left( \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q} \right) p^+(y_1^- - y_2^-)}, \quad (42)$$

and for the diagram in Fig. 3:

$$H^c \equiv \int dx \, dx_1 \, dx_2 \, e^{ixp^+ y^-} e^{ix_1 p^+(y_1^- - y_2^-)} e^{ix_2 p^+ y_2^-} \delta(l_T^2)$$

\[ \times \delta(x - x_B) \cdot \frac{1}{x + x_1 - x_B - \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q} - i\epsilon} \cdot \frac{1}{x + x_2 - x_B - i\epsilon} \]

$$= -(2\pi)^2 \theta(y^- - y_2^-) \theta(y_1^- - y_2^-) e^{ix_B p^+ y^-} \delta(l_T^2) e^{i \left( \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q} \right) p^+(y_1^- - y_2^-)}. \quad (43)$$

Combining Eqs. (11), (12) and (13), we again have the same structure as that in Eq. (24). Therefore, like the Drell-Yan case, we conclude that the double scattering does not result into any large nuclear size dependence to the inclusive DIS cross section. For the jet broadening
defined in Eq. (34), we drop the term proportional to \( \theta(y_1^- - y^-) \theta(y_2^- - y^-) \theta(y_1^- - y^-) - \theta(y_1^- - y^-) \theta(y_2^-) \), which is localized as the single scattering. After carrying out the algebra similar to those following Eq. (26), we derive the jet broadening in DIS as

\[
\Delta \langle l_T^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \frac{\sum_q e_q^2 T_{q/A}(x_B)}{\sum_q e_q^2 \phi_{q/A}(x_B)},
\]

where \( \sum_q \) sums over all quark and antiquark flavors. In Eq. (44), \( \phi_{q/A}(x) \) is the normal twist-2 quark distribution inside a nucleus, and the four-parton correlation function, \( T_{q/A}(x_B) \) is defined in Eq. (1a).

From Eq. (44), we conclude that based on the leading order calculation, we can uniquely predict the jet broadening in DIS, without any free parameter, if the four-parton correlation function, \( T_{q/A}(x_B) \), is measured in another experiment. Therefore, jet broadening in DIS can be used to test the QCD treatment of multiple scattering in nuclear environment. In addition, the jet broadening given in Eq. (44) is directly proportional to \( T_{q/A}(x_B) \), \( x_B \)-dependence of \( \Delta \langle l_T^2 \rangle \) can provide immediate information on the functional form of the four-parton correlation function.

**IV. DISCUSSION AND CONCLUSIONS**

From Eqs. (31) and (44), both jet broadening in DIS and the nuclear enhancement of the Drell-Yan \( \langle q_T^2 \rangle \) are directly proportional to the four-parton (twist-4) correlation functions, which are defined in Eqs. (1a) and (29), respectively. Since field operators are commuted on the light-cone \[2,13\], the four-parton correlation functions, \( T_{q/A}(x) \) and \( T_{Iq/A}(x) \), are the same if the phase space integral are symmetric. With the model given in Eq. (2), we predict the jet broadening in DIS as

\[
\Delta \langle l_T^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \frac{\lambda^2}{A^{1/3}}; \quad (45)
\]

and predict the nuclear enhancement of Drell-Yan transverse momentum square as

\[
\Delta \langle q_T^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \lambda^2 A^{1/3}. \quad (46)
\]
which is the same as that obtained in Ref. [4]. It is a direct result of the model proposed by LQS [3] that both jet broadening in DIS and nuclear enhancement in Drell-Yan $\Delta \langle q_T^2 \rangle$ have the very simple expressions, shown in Eqs. (45) and (46). In addition, Eqs. (45) and (46) tell us that at the leading order, jet broadening in DIS and nuclear enhancement in Drell-Yan $\Delta \langle q_T^2 \rangle$ are the same, if the averaged initial-state gluon interactions is equal to the corresponding final-state gluon interactions, i.e., $T_{q/A}(x) = T_{q/A}^I(x)$.

From the simple expression in Eq. (45), we conclude that at the leading order, the jet broadening in DIS has a strong scaling property, and it does not depend on beam energy, $Q^2$ and $x_B$. However, the $x_B$-dependence needs to be modified if $x_B$ is smaller than 0.1. When $x_B \leq 0.1$, the localized term, $	heta(y_1^- - y^-) \theta(y_2^-) - \theta(y_2^- - y_1^-) \theta(y_1^- - y^-) - \theta(y_1^-- y_2^-) \theta(y_2^-)$, is no longer localized [11,14]. Therefore, we have to keep this term for jet broadening calculation if $x_B$ is small. In addition, $Q^2$-dependence may be modified because the four-parton correlation function $T_{q/A}(x)$ and the normal quark distribution $\phi_{q/A}(x)$ can have different scaling violation. Of course, all dependence or whole conclusion could be modified due to possible different high order corrections. Nevertheless, we believe that experimental measurements of the jet broadening in DIS can provide valuable information on strength of multi-parton correlations and dynamics of multiple scattering.

Similarly, from Eq. (46), we can also conclude that nuclear enhancement in the Drell-Yan $\langle q_T^2 \rangle$ has small dependence on beam energy and $Q^2$ of the lepton pair. However, data from Fermilab E772 and CERN NA10 [7,8] demonstrate some energy dependence. It signals that high order corrections for Drell-Yan $\langle q_T^2 \rangle$ is important [9,10], or the simple model of LQS for four-parton correlation functions is too simple. On the other hand, the weak energy dependence from the data shows that the leading order calculation given here can be useful, and QCD treatment of multiple scattering can be eventually tested and understood.

Use Eq. (46) and data from E772 and NA10 on nuclear enhancement of Drell-Yan $\langle q_T^2 \rangle$, we estimate the value of $\lambda^2$ as

$$\lambda_{DY}^2 \approx 0.01\text{GeV}^2,$$  

(47)
which is at least a factor of five smaller than $\lambda_{\text{di-jet}}^2 \approx 0.05 \sim 0.1 \text{ GeV}^2$, estimated by LQS from momentum imbalance of di-jet data [3].

Since jet broadening in DIS and the momentum imbalance in di-jet are both due to the final-state multiple scattering, we will use the value of $\lambda_{\text{jk-jet}}^2$ to predict the jet broadening as

$$\Delta \langle l_T^2 \rangle \approx (0.66 \sim 1.31) \alpha_s A^{1/3} \ , \quad (48)$$

On the other hand, with a smaller value of $\lambda_{\text{DY}}^2$ in Eq. (47), we will predict a much smaller jet broadening $\sim 0.13 \alpha_s A^{1/3}$. Direct experimental measurement on jet broadening can certainly provide an independent test of our QCD treatment of multiple scattering. Early data on jet production from Fermilab E665 can be used to calculate the jet broadening. Future experiments at HERA with a heavy ion beam [11] should be able to provide much more information on dynamics of parton correlations.

In summary, the jet broadening in DIS provides an independent measurement of four-parton correlation functions, and tests of QCD dynamics beyond simple parton model. At the leading order, jet broadening is directly proportional to the four-parton correlation functions. The size of jet broadening provides a direct measurement of the size of the four-parton correlation functions. In addition, by varying the $x_B$, we can gain valuable information on $x$-dependence of the four-parton correlation functions. Information on $x_B$-dependence of the jet broadening, $\Delta \langle l_T^2 \rangle$ in Eq. (44), can provide a direct measurement of the functional form of $T_{q/A}(x_B, A)$ defined in Eq. (14).
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FIGURES

FIG. 1. Lowest order double scattering contribution to the nuclear enhancement of Drell-Yan \( \langle q_T^2 \rangle \); (a) symmetric diagram; (b) and (c): interference diagrams.

FIG. 2. Diagrams for DIS: (a) Diagram representing \( L_{\mu \nu} \); (b) Diagram representing \( W^{\mu \nu} \).

FIG. 3. Lowest order double scattering contribution to jet broadening: (a) symmetric diagram; (b) and (c): interference diagrams.
Fig. 1
Fig. 2
Fig. 3
Fig. 3