Generation of large scale magnetic fields at recombination epoch

Z. Berezhiani\textsuperscript{a} and A.D. Dolgov\textsuperscript{b,c}

\textsuperscript{a} Dipartimento di Fisica, Università di L’Aquila, 67010 Coppito, L’Aquila, and INFN, Laboratori Nazionali del Gran Sasso, 67010 Assergi, L’Aquila, Italy; e-mail: berezhia@fe.infn.it

\textsuperscript{b} INFN, sezione di Ferrara, Via Paradiso, 12 - 44100 Ferrara, Italy

\textsuperscript{c} ITEP, Bol. Cheremushkinskaya 25, Moscow 113259, Russia; e-mail: dolgov@fe.infn.it

Abstract

It is argued that large scale cosmic magnetic field could be generated in the primeval plasma slightly before hydrogen recombination. Non-zero vorticity, necessary for that, might be created by the photon diffusion in the second order in the temperature fluctuations. The resulting seed fields at galactic scale would be only 4 orders of magnitude smaller than the observed ones and with a mild galactic dynamo amplifying the seed fields by the factor $\sim 10^4$ an existence of coherent magnetic fields in galaxies may be explained.

\textit{PACS: 95.30.Qd,98.62.En}

Keywords: galactic and intergalactic magnetic fields, density perturbation.

1 Introduction

Existence of magnetic fields on astronomically large scales remains one of unsolved cosmological mysteries. It is known from observations that there are magnetic fields in galaxies with the strength about micro-gauss which are homogeneous over galactic size $l_{gal} \sim (a few)$ kpc, and maybe, even more puzzling, intergalactic magnetic fields
with the strength approximately three orders of magnitude weaker but homogeneous on much larger intergalactic scale of hundreds kpc (for review see e.g. refs. [1]). Though in stellar processes much stronger magnetic fields can be generated, their coherence scale is negligible in comparison with $l_{gal}$. There are two competing ideas of explanation of the phenomenon [1, 2]. The first one is based on traditional astrophysics and pursues the possibility that strong magnetic fields generated in multiple stellar catastrophes could be ejected into interstellar space and by field line reconnection might create magnetic fields coherent on galactic size (for a recent review see ref. [3]). However there are no compelling quantitative arguments in favor of this hypothesis. Moreover, the energy density of galactic magnetic fields are of the same magnitude as the energy density of cosmic microwave background radiation (CMBR) so its magnitude is about $10^{-10}$ of the total galactic mass/energy. Such a huge contribution is difficult to explain by conventional mechanisms. It is even more difficult to explain in this way intergalactic magnetic fields, if they exist.

Other possible mechanisms of generation of galactic magnetic fields are based on physical processes in the early universe - for the review see e.g. refs. [2, 4]. One can also find an extensive list of literature in ref. [5]. Basically there are three different mechanisms discussed:

1. Breaking of conformal invariance of electromagnetic interaction at inflationary stage. The latter could be realized either through new non-minimal (and possibly non gauge invariant) coupling of electromagnetic field to curvature [6], or in dilaton electrodynamics [7], or by conformal anomaly in the trace of the stress tensor induced by quantum corrections to Maxwell electrodynamics [8].

2. First order phase transitions in the early universe [9] producing bubbles of new phase inside the old one. A different mechanism but also related to phase transitions is connected with topological defects, in particular, cosmic strings [10]. A recent discussion and a model of generation of large scale magnetic field can be found in
3. Creation of stochastic inhomogeneities in cosmological charge asymmetry, either electric [12], or e.g. leptonic [13] at large scales which produce turbulent electric currents and, in turn, magnetic fields.

In this work we will consider a new mechanism somewhat similar to those mentioned in point 3 above but in contrast to them this mechanism does not demand any new physics and could be realized at relatively late stage of the universe evolution, namely, at red-shifts $z \sim 10^3 - 10^4$, near hydrogen recombination. The mechanism suggested here is intermediate in time between the early universe ones and the astrophysical mechanism which took place practically in contemporary universe after galaxies were formed and stellar explosions took place.

The basic features of the suggested model of magnetic field generation are the following. We show that, despite low Reynolds number (3), non-zero, though small, vorticity can be generated in the cosmic electron-photon fluid due to inhomogeneity of the latter and especially due to different spectrum of inhomogeneities of electronic and photonic components. Such difference could be created at relatively late stage of cosmological evolution (near hydrogen recombination) even from initially adiabatic density perturbations. The motion of cosmic plasma would create electric currents because of different velocities of electrons and protons and since the vorticity of the motion would be non-vanishing the same would be true for the currents, $\nabla \times \mathbf{J} \neq 0$. Such currents are known to generate magnetic fields. An attractive feature of the proposed mechanism is that no new physics has to be invoked. The mechanism operates in the standard cosmological model in the frameworks of the usual Maxwell electrodynamics. In what follows we will estimate the magnitude of the generated magnetic field and show that it can be strong enough and have sufficiently large coherence scale so that after dynamo amplification (for the review of the latter see refs. [14]) it can explain the observed fields in the galaxies and, possibly, even intergalactic ones.
2 Hydrodynamics of cosmic plasma at recombination

Fluid motion under pressure forces is governed by the hydrodynamical equation (see e.g. the book [15]):

$$\rho (\partial_t v_i + v_k \partial_k v_i) = -\partial_i p + \partial_k \left[ \eta \left( \partial_k v_i + \partial_i v_k - \frac{2}{3} \delta_{ik} \partial_j v_j \right) + \partial_i (\zeta \partial_j v_j) \right]$$ (1)

where $v$ is the velocity of the fluid element, $\rho$ and $p$ respectively are the energy and pressure densities of the fluid, and $\eta$ and $\zeta$ are the first and second viscosity coefficients. In the case of constant viscosity coefficients this equation is reduced to the well known Navier-Stokes equation. The coefficient $\eta$ is related to the mean free path of particles in fluid as

$$\eta/\rho \equiv \nu = l_f$$ (2)

In what follows we disregard second viscosity $\zeta$.

The character of the solution to eq. (1) crucially depends upon the value of the Reynolds number

$$R_\lambda = \frac{v \lambda}{\nu}$$ (3)

where $\lambda$ is the wavelength of the velocity perturbation. If $R \gg 1$, then the fluid motion would become turbulent and non-zero vorticity would be created by spontaneously generated turbulent eddies. In the opposite case of low $R$ the motion is smooth and in the case of incompressible fluid with homogeneous $\rho$ and viscosity coefficients $\eta$ and $\zeta$ the fluid velocity would have potential character with vanishing vorticity, $\nabla \times v = 0$. However, the above assumptions of homogeneity is not precise and some vorticity can be generated even with low $R$.

Let us consider now the cosmological epoch before the hydrogen recombination, when the plasma consists of three components: photons, electrons, and baryons
mainly constituted by protons (for simplicity, we shall neglect small amount of $^4\text{He}$ nuclei). For temperatures $T \geq 1$ eV the energy density of ($e, p, \gamma$)-plasma is dominated by photons. Indeed, at the present day the CMBR energy density is $\rho_\gamma \approx 0.26$ eV/cm$^3$, i.e. approximately $10^3$ smaller than the energy density of baryonic matter $\rho_b$, and thus $\rho_b$ and $\rho_\gamma$ become comparable at red-shift $z \approx 10^3$ or at $T \approx 0.23$ eV.\(^1\) The contribution of dark matter (DM) into energy density is not important for plasma hydrodynamics as far as the DM does not interact with plasma. For the moment, we also neglect cosmological expansion considering characteristic times smaller than the Hubble time $H^{-1}$ at the appropriate epoch:

$$H^{-1} = \frac{27 \text{ kpc}}{T_{\text{eV}}^{3/2} [T_{\text{eV}} + 0.76]^{1/2}},$$  \hspace{1cm} (4)

where $T_{\text{eV}} = (T/1 \text{ eV})$. At the present scale, it corresponds to $(1 + z)H^{-1} = 110 T_{\text{eV}}^{-1/2} [T_{\text{eV}} + 0.76]^{-1/2}$ Mpc. This result is obtained for cosmological relativistic matter consisting from photons and neutrinos (contributing 68% with respect to photons); the cosmological constant is not essential. For labeling the cosmological epoch we will interchangeably use the temperature $T$ or red-shift $1 + z = 4260 T_{\text{eV}}$.

As far as the cosmological plasma was dominated by photons, the viscosity coefficient $\nu$ is determined by the photon mean free path:

$$\nu = l_\gamma = \frac{1}{\sigma_T n_e X_e} \approx \frac{30 \text{ pc}}{X_e(T) T_{\text{eV}}^3},$$  \hspace{1cm} (5)

where $\sigma_T = 8\pi\alpha^2/3m_e^2 = 6.65 \times 10^{-25}$ cm$^2$ is the Thomson cross-section, $n_e = \beta n_\gamma = 6 \cdot 10^{-10} \cdot 0.24T^3$ is the electron number density, and $X_e$ is a fraction of the free electrons: $X_e(z)$ is practically 1 for $z > 1500$, and sharply decreases for smaller $z$’s, reaching values $\sim 10^{-5}$ at $z < 1000$. For $T \geq 1$ eV the mean free path (5) is much

\(^1\)Throughout this paper, we use the following values of the cosmological parameters: the CMBR temperature $T^{(0)}_\gamma = 2.725$ K, the baryon-to-photon ratio $\beta = n_B/n_\gamma = 6 \times 10^{-10}$, the Hubble constant $h = 0.71$, the matter density of the universe $\Omega_m h^2 = 0.135$, and the baryon density $\Omega_B h^2 = 0.0224$. 

5
smaller than the Hubble horizon:

\[
\frac{l_\gamma}{H^{-1}} \approx \frac{1.1 \times 10^{-3}}{X_e(T)T_{eV}} \left[1 + 0.76 T_{eV}^{-1}\right]^{1/2}
\]  \(6\)

This ratio becomes comparable to unity only at temperatures \(T \sim 0.3 \text{ eV}\), when \(X_e \ll 1\). At the present day scale the photon mean free path (5) would be \(l_\gamma^{(0)} \approx 130 \text{ kpc} X_e^{-1}T_{eV}^{-2}\).

Let us first estimate the Reynolds number of the fluid motion created by the pressure gradient in eq. (1). To this end we assume that the liquid is quasi incompressible and homogeneous, so that the second term in the r.h.s. of this equation can be neglected. This is approximately correct and the obtained magnitude of fluid velocity is sufficiently accurate. In this approximation eq. (1) reduces to a much simpler one:

\[
\partial_t \mathbf{v} + (\mathbf{v} \nabla) \mathbf{v} - \nu \Delta \mathbf{v} = -\nabla p \rho
\]  \(7\)

A comment worth making at this stage. The complete system of equations includes also continuity equation which connects the time variation of energy density with the hydrodynamical flux (see below eq. (19) and the Poisson equation for gravitational potential induced by density inhomogeneities. We will however neglect the gravitational force and the back reaction of the fluid motion on the density perturbation. This approximation would give a reasonable estimate of the fluid velocity for the time intervals when acoustic oscillations are not yet developed, i.e. for \(t < \lambda/v_s\), where \(\lambda\) is the wave length of the perturbation and \(v_s\) is the speed of sound (in the case under consideration \(v_s^2 = 1/3\)). In fact the wave length should be larger than the photon mean free path, to avoid diffusion damping, and the characteristic time interval, as we see below, should be somewhat larger than \(\lambda\). So we may hope that our estimates of the velocity are reasonable enough. Neglecting gravitational forces, especially those induced by dark matter would result in a smaller magnitude of the fluid velocity, so the real effect should be somewhat larger.
For small velocities (or sufficiently small wavelengths) we may neglect the second term in the l.h.s. with respect to the third one. In this approximation the equation becomes linear and can be easily solved for the Fourier transformed quantities. Assuming that the parameters are time-independent (though it is not necessary) we obtain:

\[ v_k = -\frac{ik}{3k^2\nu} \delta_k \left[ 1 - \exp(-\nu k^2 t) \right] \]  

(8)

where \( \delta_k = (\delta\rho/\rho)_k \) is the Fourier transform of relative density perturbations, \( \delta\rho/\rho \); its natural value is \( \sim 10^{-4} \), though it might be much larger at small scales. The coefficient \( 1/3 \) comes from equation of state of relativistic fluid, \( p = \rho/3 \).

Therefore, for the Reynolds number we obtain:

\[ R_k = \frac{\delta_k}{3(k\nu)^2} \left[ 1 - \exp\left(-\nu k^2 t\right) \right]. \]  

(9)

If \( \delta_k \) is weakly dependent on \( k \), then \( R_k \) is a monotonically rising function of the wavelength \( \lambda = 2\pi/k \). For \( t \ll \lambda^2/\nu \) it takes the value

\[ R_k = \frac{t}{3\nu} \delta_k \]  

(10)

Therefore, considering the perturbation with the wavelength \( \lambda = 2\pi/k \) which enters the horizon at the time \( t \sim H^{-1} \), we find that the maximum value of the Reynolds number is given by \( R_k^{\max} \approx (H^{-1}/3l_\gamma)\delta_k \). For later times, the Reynolds number for the comoving wavelength decreases approximately as \( \propto T \) until the density perturbation is completely damped by the photon diffusion. Thus, for \( T \gg 1 \) eV, using eq. (6), we obtain \( R_k^{\max} \approx 300\delta_k T_{eV} \), and so for the development of turbulence one needs \( \delta_k T_{eV} > 0.1 \). This condition can be satisfied either at high temperatures, \( T \geq 100 \) eV, or for large density perturbations, \( \delta_k \gg 10^{-4} \). This is not the case for the adiabatic fluctuations with a nearly flat spectrum, as is generically predicted in inflationary scenarios. However, one cannot exclude a situation that at smaller
wavelengths, corresponding to the present day scales \( \leq 100 \text{kpc} \), there are substantial isocurvature fluctuations, e.g. the baryon density perturbations \((\delta \rho/\rho)_b\) are larger than the dark matter fluctuations \((\delta \rho/\rho)_{DM}\).

The baryon density perturbation of the wavelength \(\lambda\), after many oscillations, will be damped by the Silk effect \cite{16} at the effective timescale \(t_{\text{eff}} \sim \lambda^2/\nu\). However, because of a large Reynolds number, it could happen that during this time the photon diffusion dragging electrons would also produce non-zero vorticity.\(^2\) In this case the generation of magnetic fields might be strongly amplified. We shall keep in mind an interesting possibility of large \(\delta_k\) at smaller scales, however now we shall mostly concentrate on a more plausible possibility of small Reynolds numbers and non-turbulent flow of cosmic fluid in \(T \sim 1 \text{ eV}\) range of temperatures.

At first sight vorticity in laminar fluid motion is not generated in the approximation given by eq. (7). However, this is not quite so. If both \(p\) and \(\rho\) are different functions of space points, pressure gradient may create motion with non-vanishing \(\nabla \times \mathbf{v}\). Indeed, from eq. (7) follows

\[
\frac{\partial}{\partial t} \Omega - \nu \Delta \Omega = -\nabla \times \left( \frac{\nabla p}{\rho} \right)
\]  
(11)

where \(\Omega = \nabla \times \mathbf{v}\) and we assume that velocity is small so that the term quadratic in \(v\) was neglected. If the r.h.s. is non-vanishing, then \(\Omega\) would be non-zero too. However usually pressure density is proportional to the energy density, \(p = w \rho\), with a constant coefficient \(w\) and hence \(\nabla \times (\nabla p/\rho) = 0\).

In the case under consideration, electrons are strongly coupled to photons. Their effective mean free path is

\[
l_e = \frac{\sqrt{3m_e/T}}{\sigma_{\text{Th}} n_\gamma}
\]  
(12)

\(^2\)The case of large Reynolds numbers, that might be created at earlier epoch, \(T \sim 1 \text{ MeV}\), due to large leptonic asymmetries, was considered in ref. \cite{13}.
The factor in the numerator takes into account large thermal momentum of electrons, $\langle p \rangle \sim \sqrt{3m_eT}$, so that they need many collisions, with momentum transfer $\Delta p = T$, for a significant change of their momentum. Still despite the presence of this factor, the mean free path of electrons is much smaller than that of photons simply because there are more than billion photons per a single electron in the plasma. So electrons are practically frozen in the plasma, while photons may diffuse to much larger distance. One should keep in mind that the plasma must be locally electrically neutral and thus redistribution of electron inhomogeneities is accompanied by the same redistribution of baryonic ones. This makes the mean free path of charged species of matter even smaller. On the other hand, a motion of homogeneous component of electron number density does not require dragging of baryons.

For the wavelengths larger than mean free path of charged particles we may consider plasma as a fluid where charged particles are strongly coupled to photons. So for the estimate of the rotational velocity we will use the hydrodynamic equation (11). As we have already mentioned, vorticity, $\nabla \times \mathbf{v}$ could be non-zero only if the fluid is non-homogeneous and strictly speaking we should use eq. (1) with all parameters depending upon space points. However, we believe that for a simple estimate eq. (11) may be sufficient. Since before recombination the interaction rate between radiation and charged particles is very high, plasma/liquid should be in local thermal equilibrium with all constituents having the same temperature $T(x)$. If $T$ would be the only parameter which determines the state of the medium, then vorticity would not be generated because we would have in our disposal only $\nabla T$ and it is impossible to construct non-vanishing $\nabla \times \mathbf{v}$ from the gradient of only one scalar function. However, distributions of charged particles depend upon one more function, their chemical potential:

$$f = \exp \left[ -\frac{E}{T(x)} + \xi(x) \right]$$

(13)
where the dimensionless chemical potential \( \xi \) can be readily expressed through particle number density \( n_e \approx n_B = \beta(x)n_\gamma \) with \( \beta(x) = 6 \cdot 10^{-10} + \delta\beta(x) \):

\[
\xi(x) = \ln(\beta(x)) + \text{const}
\]  

(14)

Hence we will find that the source term in the vorticity equation (11) is equal to:

\[
S_k \equiv -\epsilon_{ijk} \partial_j \left( \frac{\partial_i p}{\rho} \right) = \epsilon_{ijk} \frac{\partial_i \rho_\gamma}{3\rho_{\text{tot}}} \frac{\partial_j \beta}{\beta} \frac{\rho_b}{\rho_{\text{tot}}}
\]  

(15)

An essential feature here is that the spatial distribution of charged particles does not repeat the distribution of photons and hence the vectors \( \nabla \rho_\gamma \) and \( \nabla \beta \) are not collinear. This could occur if for lambda corresponding to subgalactic scales, there exist baryon isocurvature fluctuations and thus \( \rho(x) \) and \( \beta(x) \) have different profiles. As we have mentioned above, different mean free paths of photons and charged particles would maintain such non-collinearity of the order of unity at the scales \( \lambda \sim l_\gamma \). Moreover, even in the case of adiabatic perturbations a shift in the distribution of photons and charged particles could also be created because of acoustic oscillations that proceeded with different phases of radiation and matter densities. At the scales \( \lambda \leq l_\gamma \) perturbations in the plasma temperature would be erased by the diffusion damping [16], while for \( \lambda \gg l_\gamma \) diffusion processes are not efficient and one would expect self-similar perturbation leading to collinearity of \( \nabla \rho_\gamma \) and \( \nabla \beta \). On the other hand, when \( \lambda \) entered under horizon acoustic oscillations begun which destroyed the self-similarity. Thus the expected wavelengths of vorticity perturbations should be between \( l_\gamma < \lambda < H^{-1} \).

We assume that there is no additional suppression of the source term (15) and by the order of magnitude its amplitude corresponding to the wavelength \( \lambda \) can be evaluated as

\[
S \sim \frac{16\pi^2}{3\lambda^2} \left( \frac{\delta T}{T} \right)_\lambda \left( \frac{\delta \beta}{\beta} \right)_\lambda \left( \frac{\rho_b}{\rho_{\text{tot}}} \right) \sim 10 \left( \frac{\delta T}{T} \right)_\lambda \left( \frac{\delta \beta}{\beta} \right)_\lambda T_{eV}^{-1}
\]  

(16)
Taking $\delta T/T \sim \delta \beta/\beta \sim 3 \cdot 10^{-5}$ we obtain $S \sim 10^{-8} T_{eV}^{-1}$.

Now equation (11) can be solved in the same way as eq. (7) and we find:

$$|\Omega| = \frac{0.27}{l_\gamma T_{eV}} \left( \frac{\delta T}{T} \right) \lambda \left( \frac{\delta \beta}{\beta} \right) \lambda \left[ 1 - \exp \left( -\frac{4\pi^2 l_\gamma t}{\lambda^2} \right) \right]$$

with $l_\gamma$ given by eq. (5).

Vorticity could also be generated even if perturbations in plasma are determined by a single scalar function, for example, by $T(t, x)$ because it might be proportional to the product $\partial_i T(t, x) \partial_j T(t', x)$. These two gradients generally are not collinear if taken at different time moments $t$ and $t'$. To see that, let us start from the Boltzmann equation for the distribution function $f(t, x, E, p)$ of photons:

$$\left( \frac{\partial}{\partial t} + V \cdot \nabla - H \frac{\partial}{\partial p} + F \frac{\partial}{\partial p} \right) f(t, x, E, p) = I_{\text{coll}} [f_a] \ ,$$

where $V = p/E$ is the particle velocity (not to be confused with the velocity $v$ of macroscopic motion of the medium), for photons $V = 1$, while $v \ll 1$, $E$ and $p$ are respectively the particle energy and spatial momentum, $H$ is the universe expansion rate, $F$ is an external force acting on particles in question (the latter is assumed to be absent), and $I[f_a, f_b, ...]$ is the collision integral depending on the distributions $f_a$ of all participating particles (for the definition of the $I_{\text{coll}}$ see e.g. eq. (43) of review [17]).

At temperatures in eV-range only the Thomson scattering of photons on electrons is essential, so the collision integral is dominated by the elastic term. Integrating both parts of eq. (18) over $d^3p/(2\pi)^3$ we arrive to the continuity equation:

$$\dot{n}(x) + \nabla J = 0$$

where $J$ is the photon flux given by

$$J \equiv \mathbf{vn} = \int \frac{d^3p}{(2\pi)^3} \frac{P}{E} f$$
and $v$ is the average macroscopic velocity of the photon plasma. Using the standard arguments one can derive from eq. (19) the diffusion equation:

$$\dot{n} = D \Delta n$$

(21)

where $D \approx l_\gamma / 3$ is the diffusion coefficient. We will use this equation below to determine time evolution of the photon temperature $T$.

If the elastic reaction rate $\Gamma_{el} = \sigma_{Th} n_e X_e = 1/l_\gamma$ is sufficiently large, local thermal equilibrium would be established and the photon distribution would be approximately given by

$$f \approx f_0 = \exp (-E/T + \xi)$$

(22)

where the temperature and effective chemical potential could be functions of time and space coordinates: $T = T(t, x)$ and $\xi = \xi(t, x)$, and the photon mean free path is given by eqs. (5,6). Evidently $f_0$ annihilates the collision integral. We can find correction to this distribution, $f = f_0 + f_1$, substituting this expression into kinetic equation (18) and approximating the collision integral in the usual way as $-\Gamma_{el} f_1$:

$$(K + \Gamma_{el}) f_1 = -K f_0$$

(23)

where $K$ is the differential operator, $K = \partial_t + (V \nabla)$. The solution of this equation is straightforward:

$$f_1(t, x, E, V) = - \int_0^t d\tau_1 \exp \left[ - \int_0^{\tau_1} d\tau_2 \Gamma_{el} (t - \tau_2, x - V\tau_2) \right] K f_0(t - \tau_1, x - V\tau_1)$$

(24)

Using this result we can calculate the average macroscopic velocity of the plasma. The calculations are especially simple if elastic scattering rate is high and the integrals are dominated by small values of $\tau_1$. In this case we obtain:

$$v_j(t, x) = \frac{\int d^3p V_j f_1(t, x, E, V)}{\int d^3p f_0(t, x, E)}$$

(25)
and its vorticity, $\Omega = \epsilon_{ijk} \partial_j v_k$:

$$\Omega_i \approx 6 \epsilon_{i j l} l^2 \frac{T}{\partial_j T} \left( \frac{\partial_i T}{T} \right)$$  \hspace{1cm} (26)

To estimate time derivatives of the temperature we will use the diffusion equation (21), from which we find $\partial_t T = D \Delta T$ and finally obtain for vorticity at the scale $\lambda$:

$$|\Omega|_\lambda \approx 2 \left( \delta T \right)^2 \frac{l^2}{\lambda} k^4 \approx 3 \cdot 10^3 \left( \delta T \right)^2 \frac{l^2}{\lambda^3}$$  \hspace{1cm} (27)

Since the photon diffusion erases temperature fluctuations at the scales $\lambda < l_\gamma$, the vorticity has the maximum near $\lambda \sim l_\gamma$. This magnitude of vorticity is considerably larger than found previously (17) and we will rely on it in the estimates of magnetic field presented in the following section.

To avoid confusion let us mention that at the moment when perturbation with a given wavelength enters the horizon, $\lambda \gg l_\gamma$. Later $(\lambda/l_\gamma)$ scales as $\sim a(t)^{-2}$ and till $\lambda$ remains larger than $l_\gamma$ the amplitude of perturbations does not decrease significantly and only when $\lambda \leq l_\gamma$ the photon diffusion damps density perturbations.

### 3 Electrodynamics of cosmic plasma with helical flows and magnetic field generation

Generation and evolution of magnetic field strongly depends upon the electric conductivity of plasma which can be estimated as follows. Equation of motion of a charged particle in external electric field $E$ has the usual form

$$m \dot{V}_d = eE$$  \hspace{1cm} (28)

So the drift velocity gained during the time $\Delta t$ is equal to $V_d = eE \Delta t/m$. The charged particles (electrons) keep on to be accelerated approximately during time between
collisions, $\Delta t = l_e / V_T$ where $V_T = \sqrt{3T/m}$ is the thermal velocity and the electron mean free path $l_e$ is given by eq. (12). After collision the particle loses (forgets) its previous velocity and the process repeats. This is true for sufficiently weak fields when the drift velocity is small in comparison with thermal velocity, otherwise run-away charge carriers would be produced and the conductivity would be much larger.

The induced electric current is $J = en_e V$, where $n_e$ is the number density of charge carriers. Comparing this with the definition of current conductivity $\kappa = J/E$ we find:

$$\kappa = \frac{3}{2\alpha} \frac{n_e m_e^2}{n_e n_{\gamma} T}$$

(29)

The conductivity is very high, so the generation of magnetic field by the source currents, created by the cosmological inhomogeneities, is governed by the well know equation of magnetic hydrodynamics:

$$\partial_t B = \nabla \times (v \times B) + \frac{1}{\kappa} \nabla \times J$$

(30)

The electric current $J$ induced by the relaxation of the density inhomogeneities would contain two components: electronic and protonic. However the first one is surely dominant because it is much easier to drift electrons than heavier protons. This is why a non-zero current can be induced. Of course motion of electrons should not produce any excess of electric charge but this can be realized because the current is created in the dominant homogeneous part of the charge particle distribution.

The solution of eq. (30) can be roughly written as

$$B \sim \int_0^t dt_1 \left( \frac{2 \pi J}{\lambda \kappa} \right) e^{2\pi v t_1 / \lambda}$$

(31)

The exponential factor under the integral, which presents pregalactic dynamo effect is normally rather weak, the exponent is about

$$\frac{2\pi vt}{\lambda} \approx 500 T_{eV} \left( \frac{\delta T}{T} \right)$$

(32)
where eqs. (8) and (6) have been used and the density fluctuations were taken as
\[ \delta_\lambda = 4 \left( \frac{\delta T}{T} \right)_\lambda. \]
With \( (\delta T/T)_\lambda \sim 3 \cdot 10^{-5} \) and \( T \sim \text{eV} \) the exponent is about
\[ 0.015 T_{\text{eV}} < 1 \]
and pregalactic dynamo is not important. It may be significant if the
density perturbations at relatively small scales are much larger than their accepted
canonical values. Exponential enhancement can be large at higher temperatures, \( T > 10^3 \text{ eV} \), which correspond to the present day scales below kpc. Equipartition between
magnetic field and CMBR can be expected on such scales and after 'Brownian' line
reconnection and relatively mild galactic dynamo the observed galactic magnetic fields
may be created.

An estimate of magnetic field without pregalactic dynamo enhancement can be
easily done if the helical source current is known, \( \nabla \times \mathbf{J} = e n_e \mathbf{\Omega} \). With \( \mathbf{\Omega} \) given by
eq. (27) and \( B \) by eq. (31) we obtain:
\[ \frac{B_0}{T^2} = 0.24 \cdot 10^3 (4 \pi \alpha)^{3/2} \left( \frac{l}{\lambda} \right)^3 \left( \frac{T}{m_e} \right)^2 \approx 10^{-8} T_{\text{eV}}^3 \]
where we took the wavelength equal to the photon mean free path, \( \lambda = l_\gamma. \)

If we take into account that linear compression of pregalactic medium in the
process of galaxy formation is approximately \( r \sim 10^2 \), the seed field in a galaxy
after its formation would be \( r^2 B_0 \), i.e. 4 orders of magnitude larger than that given
by eq. (33) and, for \( T = 1 \text{ eV} \), a relatively mild galactic dynamo, about \( 10^4 \), is
necessary to obtain the observed galactic magnetic field of a few micro-Gauss at the
scale \( l_B \sim (100/r) \text{ kpc} = 1 \text{ kpc} \). The seed magnetic fields formed earlier (at higher \( T \))
would have larger magnitude (\( \sim T^3 \)) but their characteristic scale would be smaller by
factor \( 1/T^2 \). Chaotic line reconnection could create magnetic field at larger, galactic
scale \( l_{gal} \), but the magnitude of this field would be suppressed by Brownian motion
law - it would drop by the factor \( (l_B/l_{gal})^{3/2} \). It is interesting that according to our
results all scales give comparable contributions at \( l_{gal} \). This effect may lead to an
enhancement of the field but it is difficult to evaluate the latter. Let us also note that
magnetic fields generated by the discussed mechanism at the cluster scale, 10 Mpc, should be not larger than $10^{-8} \mu G$ if no additional amplification took place.

4 Discussion and conclusion

We have shown in this work that photon diffusion produced by temperature fluctuations of CMBR could create vortical flows in the primeval plasma in the second order, i.e. proportional to $(\delta T/T)^2$. Because of different mobility of electrons and protons (or helium ions) in the plasma the helical macroscopic motion of the latter would create helical electric currents which would in turn generate magnetic fields. The amplitude of such fields generated at the moment when plasma temperature was equal to $T$ is given by eq. (33). The characteristic scale of the field should be equal to the photon mean free path in the plasma at temperature $T$ which at the present day is about 100 kpc $(eV/T)^2$. This scale is quite close to the galactic size. Adiabatic compression in the process of galaxy formation by the factor $r \approx 100$ would lead to an enhancement of magnetic field by $r^2 = 10^4$. Thus to meet the observational data this seed field should be amplified by the galactic dynamo only by 4-5 orders of magnitude. Even if the magnitude of the seed field presented above is somewhat overestimated there is a huge reserve in the amplification by the galactic dynamo which could be as large as 15 orders of magnitude [14].

A nice feature of this mechanism is that it does not demand any new physics for the realization. Of course, density/temperature perturbations at large scales were, most probably, created during inflationary epoch and in this sense new physics is invoked but independently of the mechanism we know from observations that $\delta T/T \neq 0$ and, based on these data, we may estimate the seed magnetic fields using good old physics.

It is interesting that, according to the existing indications to intergalactic magnetic fields, their strength is 3-4 orders of magnitude weaker than the strength of galactic
fields. This fact (if it is a fact) hints that intergalactic and galactic magnetic fields might have common origin, but galactic fields are larger due to mentioned above adiabatic compression which enhanced the field by 4 orders of magnitude. If it is so, the mechanism discussed in this work may be irrelevant because some galactic dynamo is needed to amplify the galactic seed field up to the observed magnitude. On the other hand, such dynamo seemingly does not operate on intergalactic scales.

Larger density perturbations would be helpful for generation of larger magnetic field for which dynamo might be unnecessary. Though much bigger $\delta T$ is not formally excluded at the scale about 100 kpc, but to have them at the level $(\delta T/T)^2 \sim 10^{-4}$ seems to be too much. A natural idea is to turn to a later stage, to onset of structure formation when $\delta \rho/\rho$ becomes larger than $10^{-2}$. With such density perturbations strong enough magnetic fields may be generated without dynamo amplification. However after recombination the number density of charge carriers drops roughly by 5 orders of magnitude. Correspondingly $l_\gamma$ rises by the same amount and the strength of the seed field would be 5 orders of magnitude smaller if density perturbations and the temperature of formation remained the same. However both became very much different. Density perturbations rose as scale-factor, $(\delta \rho/\rho)^2 \sim (T_{eq}/T)^2$, where $T_{eq} \sim 1$ eV is the temperature when radiation domination changed into matter domination and density perturbations started to rise. Since, $B/T^2 \sim T^3$, according to eq. (33), the net effect of going to smaller $T$ is a decrease of $B/T^2$ which would be difficult to cure even by later reionization. Still, as argued in ref. [18], magnetic field generation, driven by anisotropic and inhomogeneous radiation pressure (and in this sense similar to our mechanism) at the epoch of reionization, could end up with the field of about $8 \cdot 10^{-6} \mu G$. This result is 8 orders of magnitude larger than that found in the earlier papers [19] and quite close to ours (33), though these two mechanisms operated during very different cosmological epochs and were based on different physical phenomena.

After this paper was sent to astro-ph we became aware of the work [20] "Gener-
ation of Cosmic Magnetic Fields at Recombination” where a much weaker effect was found. We think that the difference can be attributed to the following effects. We considered earlier period when the photon mean free path was much smaller than the horizon. It gives a factor about $10^3$ in fluid velocity, eq.(8). Moreover, since in our case the electrons are tightly bound to photons the electron-photon fluid moves as a whole (while protons and ions are at rest) and the electric current induced by macroscopic motion/oscillations of plasma is noticeably larger.

**Acknowledgments**

We thank C. Hogan and P. Naselsky for critical comments. This work was partially supported by the MIUR research grant ”Astroparticle Physics”.

**References**

[1] P.P. Kronberg, Rep. Prog. Phys., 57 (1994) 325;  
R. Beck et al, Ann. Rev. Astron. Astrophys., 34 (1996) 155.

[2] D. Grasso, H.R. Rubinstein, Phys. Repts., 348 (2001) 161.

[3] P.L. Biermann, C.F. Galea, to be published in Proc. Palermo Meeting, Sept. 2002, Eds. N. G. Sanchez et al., astro-ph/0302168.

[4] A.D. Dolgov, hep-ph/0110293; to be published in Collection of papers dedicated to 70th anniversary of S.G. Matinyan.

[5] K. Dimopoulos, T. Prokopec, O. Tornkvist, A. C. Davis, Phys. Rev. D65 (2002) 063505.

[6] M.S. Turner, L.M. Widrow, Phys. Rev., D37 (1988) 2743.
[7] B. Ratra, Astrophys. J., 391 (1992) L1;
    D. Lemoine, M. Lemoine, Phys. Rev., D52 (1995) 1955;
    M. Gasperini, M. Giovannini, G. Veneziano, Phys. Rev. Lett., 75 (1995) 3796;
    M. Gasperini, Class. Quant. Grav., 17 (2000) R1;
    M. Giovannini, hep-ph/0104214.

[8] A.D. Dolgov, Phys. Rev., D48 (1993) 2499;
    A.D. Dolgov, ZhETF, 81 (1981) 417 [Sov. Phys. JETP, 54 (1981) 2223];
    A.-C. Davis, K. Dimopoulos, T. Prokopec, O. Tornkvist Phys. Lett. B501 (2001)
       165;
    T. Prokopec, astro-ph/0106247.

[9] C.H. Hogan, Phys. Rev. Lett., 51 (1983) 1488.

[10] T. Vachaspati, A. Vilenkin, Phys. Rev. Lett., 67 (1991) 1057.

[11] D. Boyanovsky, H.J. de Vega, M. Simionato, Phys.Rev. D67 (2003) 023502; hep-
    ph/0211022 (to be published in Phys.Rev. D).

[12] A.D. Dolgov, J. Silk, Phys.Rev., D47 (1993) 3144.

[13] A.D. Dolgov, D. Grasso, Phys. Rev. Lett. 88 (2002) 011301.

[14] A.A. Ruzmaikin, A.M. Shukurov, D.D. Sokolov, “Magnetic Fields in Galaxies”,
    Kluwer Academic Publishers, Dordrecht, 1988;
    R.M. Kulsrud, Ann. Rev. Astron. Astrophys. 37 (1999) 37;
    L. Malyshkin, R. Kulsrud, astro-ph/0202284.

[15] L.D. Landau, E.M. Lifshits, “Gidrodinamika” (Hydrodynamics) (Moscow:
    Nauka, 1986) [Translated into English “Fluid Mechanics” (Oxford: Pergamon
    Press, 1987)].
[16] J. Silk, Astrophys. J. 151 (1968) 459.

[17] A.D. Dolgov, Phys. Rept. 370 (2002) 333.

[18] M. Langer, J.-L. Puget, N. Aghanim, Phys. Rev. D67 (2003) 043505.

[19] K. Subramanian, D. Narasimha, S.M. Chitre, Mon. Not. Roy. Astr. Soc. 271 (1994) L15;
    N.Y. Gnedin, A. Ferrara, E.G. Zweibel, Astrophys. J. 539 (2000) 505.

[20] C. Hogan, astro-ph/0005380.