Analysis of prospective teacher’s mathematical problem solving based on taxonomy of reflective thinking

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Abstract. Reflective thinking is a mental activity that produces changes in viewpoint through the process of re-interpretation which involves the process of analysis and decision-making about what has been done. Reflective thinking in solving problems plays an important role in the next problem solving process. It is because the students are able to realize and think about what they have done and they can use it for solving the next problems. One way to see how far reflective thinking has been used in problem solving is taxonomy of reflective thinking. This study aims to describe taxonomy of reflective thinking of teachers to be in problem solving. The subject of research is a student majoring in mathematics at Universitas Muhammadiyah Makassar. The results showed that subject met the six levels of reflective thinking starting from level (1) remembering, (2) understanding (3) applying, (4) analyzing, (5) evaluating and (6) creating. The conclusion provides ideas about the taxonomy of reflective thinking that describes the depth of reflective thinking in solving mathematical problems. Each level illustrates the skill characteristic to use mathematical knowledge. Teachers are expected to have awareness to teach students to think reflectively up to the highest level.

1. Introduction
Mathematics can be used as a means to develop the students’ thinking skills, especially high-order thinking. To achieve this objective, handling of the learning process that can train students’ thinking skills is required. One of the skills that can be taught and is needed much in mathematics learning is reflective thinking. This is consistent with what proposed by Odafe reflective thinking in mathematics is important because it is one of the high-order thinking skills that can motivate the curiosity of students [1,2]. Reflective thinking helps a person in solving complex problems, due to the reflective thinking helps students to identify concepts, facts, formulas and relevant theories to the solution of the problem solved so that the answers obtained tend to be true and correct [3]. In addition, reflective thinking also involves the process of analyzing, comparing, synthesizing, clarifying and choosing what someone will do [4,5]. It indicates that the reflective thinking can enhance students’ problem solving abilities that are the essential competencies and should be owned by the students in mathematics [6].

In the last few years, reflective thinking becomes very popular term in the world of education, especially in professional education of teachers [7,8]. For many reasons, educators are more interested in teaching about thinking skills in various ways. Reflective thinking is active, persistent, and careful consideration of any belief or suppose from of knowledge in the light of the grounds that support it and the conclusion to which it tends [9]. Schelke and Steinbring defined reflective thinking as a cognitive
activity which results a change in understanding or perspectives, through process of re-interpretation [10]. While Atkins and Murphy suggested that reflective thinking is a thinking process to realize something based on the experience and then how to interpret it [5].

Pappas had developed a model to investigate the extent of reflective thinking used in solving problems known as taxonomy of reflective thinking [11]. Taxonomy is a special classification that is based on scientific research data related to the matters that are classified in a particular systematic [12]. Taxonomy is structured to see how far the students’ reflective thinking ability to problem solving that has been carried out, starting from the most basic level that is remembering to the highest level that is creating. Here is a picture of a reflective thinking taxonomy model that is described by Pappas with the questions that can be asked to determine reflective thinking in every stage [11].

| A Taxonomy of Reflective Thinking |
|----------------------------------|
| Creating: What should I do next? |
| Evaluating: How well did I do?   |
| Analyzing: Do I see any patterns in what I did? |
| Applying: Where could I use this again? |
| Understanding: What was important about it? |
| Remembering: What did I do?      |

**Figure 1. Model of reflective thinking.**

Based on the figure 1, it can be explained that the reflective thinking taxonomy consists of six levels aligned with Bloom's taxonomy. The six levels are (1) **Remembering**, which corresponds to the question What did I do?, (2) **Understanding** with the question: What was important about it? (3) **Applying** with the question: Where could I use this again? (4) **Analyzing** with the question: Do I see any patterns in what I did? (5) **Evaluating**: with the question How well did I do?, and (6) **Creating** with the question: What should I do next?. Furthermore, Pappas states that the objective to develop the model of this taxonomy is to see reflective thinking skills by checking, evaluating or testing the correctness and accuracy of the problem-solving tasks that have been completed or in learning that has been done in the classroom [11]. By knowing one's taxonomy of reflective thinking in solving the problem, it will be known how far and how someone use reflective thinking in solving problems. If the reflective thinking in solving problem of students is only at the level of understanding, it means that students remember what they have done in solving problems and understanding what they have done and find out the important points of solving the problem, but they do not think about how the solution that has been done is used in solving other problems, and they do not think the patterns that exist in the solution of the problem and, they do not evaluate the solution.

The learning process should be designed to facilitate and help students acquire the knowledge, skill and expertise. Based on the statement, to learn, master, process information, or solve the problem, students will do it in various ways [13]. It implies that when students finish solving mathematical problems, their reflective thinking is different towards solving the problem that has been done. To teach the reflective thinking in problem solving up to the highest level, it needs a teacher who also applies reflective thinking in solving the problem. Therefore, by knowing how reflective thinking of preservice teachers to be in problem solving, it will have an impact on the reflective thinking learning of students.
2. Experimental method
This research was a descriptive analysis of qualitative approach that aimed to describe deeply about reflective thinking taxonomy of mathematical problem solving of the preservice teachers. To achieve the objective, the subjects were given the task of a mathematical problem solving to be solved. This research was conducted at Muhammadiyah University of Makassar, Indonesia, involving 83 students of mathematics education program to be in semester 5. After solving problems, 83 students were given a questionnaire related to their reflective thinking to see how their reflective thinking taxonomy was. Based on the results of the questionnaire, one student who used reflective thinking to the highest level namely creating was chosen, to become the subject of the research and was interviewed deeply to explore the data of his reflective thinking at each level. Data from the solving problem and the interview were analyzed. To obtain the valid data, the researchers used a time triangulation. The mathematics problem given to the subject of this research was as follows:
Mr. Rudding is a farmer and he has a rice field located on the highlands within 20 km from his house. One day, Patimasang, Rudding’s wife, asked Ramli, her son to bring drinking water to the field. The drinking water that would be taken there was two boxes in which each box contained 20 cups. Ramli would bring a basket to carry water that can fit maximum of 20 cups. Because the way to the field was on the highlands, then every 1 km to the field, Ramli drank 1 cup of water. Try to explain, how you can help Mrs. Patimasang or Ramli to make water can be in Mr. Rudding’s rice field as much as possible.

3. Result and discussion
In this section, the process of subject selection, the results obtained in this study and related discussion of results will be discussed. The results of the questionnaire, related to the level of reflective thinking in solving problems given to 83 prospective teachers to be to solve the problem, described that there was no student at a level below the applying. Clearly, it is described in the following table 1.

| Level of reflective thinking | Sum of the students | Percentage (%) |
|-----------------------------|---------------------|----------------|
| Applying                    | 44                  | 53             |
| Analyzing                   | 21                  | 25             |
| Evaluating                  | 13                  | 16             |
| Creating                    | 5                   | 6              |

Table 1 shows that there were 44 students (53%) at the applying level, 21 students (25%) at the level of analyzing, 13 students (16%) at the evaluating level and 5 students (6%) at the level of creating. One of the five students on a creating level was selected, which was communicative and willing to be the subject of the research.

The reflective thinking taxonomy in solving mathematical problems of the subject will be described as follows. The first level, remembering, is the ability to mention the information or knowledge stored in the memory associated with a problem that has been solved, what problems that have been solved and how to handle the problem related to the content or the stages that have been made in solving the problem. At this level, subject restated all information using his own words in good order. Subject could mention the main point of the problem. In addition, to ensure the answer obtained was correct, the subject crossed out the answers that were less precise. For more details, it can be described based on the interview of the research subject below:

Researcher:  Would you please explain, what do you remember related to the mathematical problem solving that you have done?

Subject:  I remember that Rudding is a farmer. The distance between the house and the field is 20 km. ... Each box contains 20 cups. ... When I solved this problem, I always circled the...
answer that I thought it was right, and sometimes I crossed out the wrong answer and replaced it with the correct answer.

Then the subject explained the steps he did to solve the problem. Based on the interview above, it could be stated that at remembering level, subject understood the problem by reading and mentioning all the information using his own words and he explained in good order without looking at the text. Moreover, the subject remembered when he solved problems he circled the answer that he considered right and crossed out the answer that was less precise, and he remembered the steps of solving that had been done. The second level, understanding, associates with ability to use reflective thinking in understanding what has been done and to think about the important points of the problems that have been solved. At this level, the subject explained the solution steps which he did by giving reasons, and said that the mathematical problem solved demanded to determine the strategy in solving problems and he also explained the strategy he used. The interview result related to the subject:

Researcher: Please explain what you understand relating to your solution on solving the problem that you have done.
Subjects: In this problem, I was asked to help Patimasang. Indirectly I have to find my solution strategy. ... To solve this problem, I use the method of drawing because it makes me easier to solve the problem ...

Based on the interview above, it can be explained that based on the understanding of the subject, the problem solved was a problem associated with the determination of a strategy in finding a solution, and he thought that the right strategy in solving the problem was by drawing because it was easier to find a solution through drawing. In addition, by drawing pictures, the subject was more confident and believed that the answers obtained were right. Subjects also compared all the answers to decide and choose the most correct answer. The third level, implementing or applying, associates with the ability to apply the concept in certain situations, namely how to apply knowledge of content, process, or product or result of problems solved before. In addition, this level requires the ability to adapt and modify the approaches previously used towards solving other problems. At this level, the subject explained that the mathematical problems could be applied in daily life, for example in the field of economics or education. This can be explained based on the interview with the subject below:

Researcher: After completing this problem, where do you think this mathematical problem can be applied?
Subject: In my opinion, the problem of mathematics can be applied not only in the economic field but it can also be implemented in education field. For example, in economy, in a soy sauce company ... The company should think about the delivery strategy ... so that the company does not lose. In the field of education, it can be applied in learning process ... so that I can use this mathematical problem ... it can also train the students to follow Mathematical Olympiad because this problem is very complex.

Based on the above explanation, it can be explained that the subject described the problem that he had solved could be applied in the fields of economy and education. In details, in the economic field, it associated with the ability to make profit as much as possible with minimum production cost. In the field of education, the subject mentioned that this issue could be applied in teaching mathematics; in particular, it developed the questions to prepare students following mathematics Olympiad.

The fourth level, analyzing, is the ability to separate a concept into several components and relate each other to gain an understanding of the concept as a whole. It is related to the effectiveness of the strategies, skills and procedures used in problem solving, the patterns used as an approach in making a decision. At this level, the subject explained that to analyze the problem, the subject looked for the correlation of some answers that had been obtained. For the subject, with this way he could make the link for the answers. The details can be described based on interviews with the following subjects:
Researcher: Tell me, how do you analyze your problem solving?
Subject: I analyzed the correlation among all my answers. For example, I tried to find a relationship between my first answer with another answer. My first answer was 10. The distance was 20 km. In my opinion, to get 10, first, I divided two parts of street Sir. Then I divided the 20 km with two parts of the road. The result was 10. Then I subtracted 20 km with 10, so I got the final answer 10. .... The second way, the third way .... So, I can conclude that I split the distance and the number of stops, then I subtract the distance and the result of the previous division. However, it cannot be used generally because it does not happen because I get 0.3 answers and 16 others.

Based on the interview above, it can be concluded that in analyzing level, the subject gained the answers patterns by linking the answers in order to obtain a new pattern conclusion that can solve the problem with specific strategies used by the subject.

Fifth level, evaluating, is the ability to determine the degree of something based on certain criteria or benchmarks toward solving the problem that has been done. To support the decision taken, the important aspects related to the ability to measure, to correct measurements are required, so the decision is based on the knowledge and skill that have been achieved. At this level, the subject stated that what he had done was good enough, and there were several ways to evaluate mathematical problem solving that he had done. Furthermore, the subject explained that the subject had corrected and re-checked the answers obtained. The correction was made related to the use of formula, operation, or a plan for accomplishing, so according to subject, the problem solving has been checked. The details can be explained in the interview with the subject below:

Researcher: How do you evaluate your completion or solution?
Subject: .... to make sure that I get the correct answer, I rechecked my answer from the beginning to the final result obtained namely the formula that I used, correct operation and procedure. .... Finally, I compared some of the possibilities and decided to choose the most correct answer that was 16 because there were two same answers, namely 16 ...

From the interview excerpts above, it can be explained that the subject judged the problem-solving process that he did was quite well because he had done the evaluation process by improving and re-checking the answers related to the use of a formula, operation and procedure. Furthermore, subject compared all of the answers he got and decided to choose the correct answer.

The sixth level, creating, is the ability to combine the elements into a new shape which is intact and coherent, or to create something original, how to customize the content knowledge or skill to make a difference in solving the problem. At this level, the subject stated that the solution of problems helped create an idea, including tricks to avoid a loss because he thought that this problem could be applied to a company. Additionally, the subject could test the ability and competence in solving mathematical problems. In details, it is described based on the interview with the subject:

Researcher: After solving this problem, what idea can you get?
Subject: The idea is a trick to help companies avoid losses. By solving this problem, I think that there are many problems that are quite interesting and challenging to solve. To test my ability, I will create a problem that has the same principle to this problem that is to look for the completion strategy and try to solve it. Thus, if one day I becomes a teacher, I do not have to suffer anymore to make questions because previously I've trained myself ... I am also looking for problems from the internet and modifying them ... I will teach students how to solve the problem well. ...

From the interview excerpts above, it can be explained that the subject had a new idea to solve the problems of everyday life. Additionally, the subject also stated that the subject tried to look for other problems and tried to modify them. Furthermore, the subject revealed that when he became a teacher, he will teach students how to solve the problem well.
4. Conclusion
Based on these results of this research, it can be concluded that it is the possible for the preservice teacher to be use reflective thinking to the highest level that is creating in solving a problem. At the level of remembering, the subject used the reflective thinking in formulating the problem with their own words in sequence as well as remembering the completion steps that had been made. At the level of understanding, subject understood what they had done when they solved the problem by giving the reasons why the strategy undertaken was appropriate and facilitated to obtain solutions and determined the essential points of the problem related to define strategies and to compare some answers to make a decision about the most correct answer. At the third level, namely applying, subject used the thinking reflective in thinking about the application of new issues resolved into everyday life, namely in the economic field, avoiding the loss of a company related to the delivery of goods, and in the field of education, in learning or assistance training of Olympiad for students. The fourth level is analyzing. At this level, the findings obtained relate to the ability to get a pattern that describes the relationship between the answers obtained. It is related to the effectiveness of a strategy, skills and procedures used in problem solving. Subject used reflective thinking to get the pattern of the various solutions found and determined the relationship to make optimal answers. The fifth level is evaluating. At this level, the subject assessed what they had been done when the problem solving was good enough because the evaluating process had been carried out by the subject by rechecking the answers related to the use of a formula, operations and completion procedures. The last level is creating. At this level the finding obtained from the subject is the ability to create ideas that can be used to solve everyday problems. Subject tried to modify and created new problems to improve problem solving skills that would have an impact when the subjects became teachers to teach math problem solving.

The conclusion is this study provides ideas about the taxonomy of reflective thinking that describes the depth of reflective thinking in solving mathematical problems. Each level illustrates the skill characteristic to use mathematical knowledge. Teachers are expected to have an awareness of reflective thinking and willing to teach or train students to think reflectively up to the highest level.

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