Nonlinear frequency conversion and manipulation of vector beams in a Sagnac loop
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ABSTRACT

We report an experimental realization of nonlinear frequency conversion (NFC) and manipulation of vector beams (VBs) that can be used to expand the available frequency band. The main idea of our scheme is introduction of a Sagnac loop to solve the polarization dependence problem of NFC in nonlinear crystals. The experimental results agree well with those of our theoretical model.

Keywords: Vector beam, nonlinear frequency conversion

1. INTRODUCTION

Recently, laser beams with spatially inhomogeneous SOPs such as vector-beam (VB) solutions of Maxwell’s equations have been attracting increasing attention because of the unique intensity and polarization distributions in their transverse sections\textsuperscript{1}. One particular example with cylindrical symmetry of polarization is cylindrical vector beams (CVBs)\textsuperscript{2}, which include radially and azimuthally polarized VBs, and can be applied in high-numerical-aperture focusing\textsuperscript{3}, optical trapping\textsuperscript{4}, laser machining\textsuperscript{5}, optical cages\textsuperscript{6}, super-resolution imaging\textsuperscript{7}, high-capacity communications\textsuperscript{8}, and quantum information science\textsuperscript{9-11}. Given their broad application prospects, researchers have presented several schemes to generate CVBs, which can be classified into active\textsuperscript{12,13} and passive\textsuperscript{14-18} schemes.

As a highly important laser technology, nonlinear frequency conversion (NFC) provides an important way to expand the available frequency range of CVBs when they are difficult to generate directly, such as ultraviolet CVBs, which are more useful in lithography techniques because their shorter wavelengths provide a tighter focus. NFC of structured light beams has attracted growing research interest in recent years\textsuperscript{19-27}. Most work on NFC of structured light beams to date has focused on optical vortex beams with uniform polarization distributions\textsuperscript{19-21}. The NFC of structured light beams with nonuniform polarization distributions have only recently been studied theoretically\textsuperscript{22,23} and experimentally\textsuperscript{25-27}. To date, realization of NFC of CVBs remains challenging because of the polarization-sensitive phase matching condition in NFC. In this letter, we use a passive scheme to generate CVBs and demonstrate a scheme to realize second harmonic generation (SHG) of a CVB. The main idea is introduction a Sagnac loop to resolve the polarization sensitivity problem in NFC. Fig. 1 shows a schematic of our experiment where (a) and (b) correspond to the generation and SHG of CVBs respectively.
2. GENERATION OF CVB

The pump CVB at the fundamental wavelength of 1550 nm is generated using a Sagnac loop with a vortex phase plate (VPP). The mathematical expression for the target linearly polarized CVB is

\[\mathbf{E}_2 = S(l)Q \left(\frac{-\pi}{4}\right)H(\alpha)E_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i(\theta_0+2\alpha)} \\ ie^{-i(\theta_0+2\alpha+\Delta)} \end{pmatrix},\]

(1)

where \(S(l)=\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i(\theta+\Delta)} \end{pmatrix}\) represents the Jones matrix of the Sagnac loop, \(H(\alpha)=\begin{pmatrix} \cos2\alpha & \sin2\alpha \\ \sin2\alpha & -\cos2\alpha \end{pmatrix}\) and \(Q(-\frac{\pi}{4})=\begin{pmatrix} 1 \\ i \end{pmatrix}\) represent the Jones matrices of the HWP and the QWP respectively, from which the global phase has been omitted. This beam is a hybrid-polarized CVB\(^{28}\), which can be transformed into a linearly polarized VB using QWP2 with its fast axis oriented at \(\pi/4\). The linearly polarized VB can
be described as $Q \left( \frac{\pi}{4} \right) E_2 = e^{-i\Delta/2} \left( \cos(\theta + \varphi) \right)$, where $Q \left( \frac{\pi}{4} \right) = \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$ and $\varphi = 2(\alpha + \Delta/4)$ is the initial phase of the CVB, which can be controlled by adjusting the optical axis angle $\alpha$ of HWP. According to the calculation, a linearly polarized CVB will be generated after QWP2, and we record its intensity distribution using an infrared charge-coupled device (CCD). The results are shown in the top row of Fig. 2. The arrows represent the polarization direction. The hole at the center of the field arises from the phase singularity. To check the polarization distribution, we used a polarizer to perform projection measurements. The second row of Fig. 2 shows the results of measurements using a horizontal direction polarizer. The direction is shown at the top right corner. Only the polarization in the same direction as that of the polarizer can be recorded, while the orthogonal polarization is extinguished.

Fig. 2. Fundamental (top and second rows) and SHG CVBs (third and bottom rows) with $l = 1$; arrows indicate the SOP distribution in each case, and the second and the bottom rows show the results measured using a horizontal polarizer. The different SOP distribution is controlled by the axis angle of HWP; From left to right, the initial phase $\varphi = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}$.

3. FREQUENCY CONVERSION OF CVB

In the second Sagnac loop, a dichroic PBS (DPBS) is used to separate the horizontal and vertical polarizations; one of these polarizations is then transformed to the orthogonal polarization using a dichroic HWP (DHWP) before the crystal. The fundamental beam thus pumps the nonlinear crystal from opposite directions with the same polarization. Two SHG beams with the same polarization are then generated; before the DPBS, one of the SHG beams is rotated to its orthogonal polarization using the DHWP and thus the two SHG beams are combined at the DPBS with orthogonal polarizations. In our experiment, we use a type-0 (zzz) crystal, which only responds to the vertical polarization. Therefore, the DHWP is set in the transmission side, as shown in Fig. 1 (b), and the counter propagating beams will return along the original path after they are combined. The pump beam passes through the dichroic mirror (DM) and the SHG beam is then separated by reflection.

The transformation of the polarization states in the Sagnac loop can be described using the operator $T = \begin{pmatrix} 0 & 1 \\ e^{i\Delta} & 0 \end{pmatrix}$, where $\Delta$ is the phase difference in the second Sagnac loop that is mainly caused by the asymmetric positioning of the crystal. Unlike the linear process, the nonlinear process cannot be represented using Jones matrices. Consideration of the relationship $E^{2\omega} \propto (E^n)^2$ between the SHG beam and the fundamental beam under a paraxial and undepleted pump
approximation indicates that the complete process in the Sagnac loop could be represented by $E^{2\omega} \propto T(E^{\omega})^2$. After frequency doubling of the vector beam $\vec{E}_2$, we use QWP3 with its fast axis oriented at $\pi/4$ to obtain the linearly polarized VB. This process can be described as follows:

$$\bar{E}_2^{2\omega} \propto Q \left(-\frac{\pi}{4}\right) T(\bar{E}_2)^2 \propto \left( \cos(2\theta + \psi) \right),$$

(2)

where $\Delta_2$ is the phase difference in the second Sagnac loop and $\psi = 4\alpha + \Delta_1 + \Delta_2/2 - \pi/4$ is the initial phase of the CVB, which is still controlled by HWP.

The SHG beam separated by a DM is measured using a polarizer and finally recorded using a visible band CCD. The third and bottom row of Fig. 2 show the results of the SHG experiments, with their fundamental beams corresponding to the same columns in the top row. As shown in equation (2), the topological charge of the second harmonic CVB has been doubled because of the conservation of angular momentum. The results for four petals in Fig. 2 are thus consistent with the theory. We then change the topological charge of the VPP to arrive at the higher-order CVBs. Fig. 3 shows the experimental results for fundamental CVBs and their second-harmonic beams with $l = 2, 3, 4$ and $\theta = 0$, which are also consistent with the theory.

![Fig 3. Fundamental (first and second columns) and SH CVBs (third and fourth columns) with $l = 2, 3, 4$ (from the top row to the bottom row). The arrows indicate the SOP distribution, and the second and fourth columns show the results measured using the horizontal polarizer.](image)

**4. CONCLUSION AND DISCUSSION**

In conclusion, we have realized nonlinear frequency conversion of CVBs using a Sagnac loop. The method and the theoretical analysis that were presented here are also applicable to other wavebands and second-order nonlinear processes. In particular, sum-frequency generation will allow us to select target frequency bands and topological charges with greater flexibility. In our experiment, the frequency conversion efficiency is approximately 0.01% when the pump power is 500 mW. The efficiency can be improved through use of a strong pulse pump or insertion of a symmetric confocal resonator in the Sagnac loop in future experiments.

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