Static mode of the mathematical model of an electric multipole with memresistive branches in conditions of interval uncertainty

A V Bondarev¹ and V N Efano²
¹FSBEI HE "Orenburg State University", Kumertau Branch OSU, 3b, 2 Soviet lane, Kumertau, 453300, Russia
²FSBEI HE "Ufa State Aviation Technical University", 12, C. Marx str., Ufa, 450000, Russia
E-mail: bondarevav@kfosu.edu.ru

Abstract. This article discusses a method for obtaining a mathematical model of an electric multipole with memresistive branches in finite parameter increments for a large signal mode when calculating a direct current based on a generalized model obtained earlier. A computational algorithm for calculating nanoelectronic circuits based on methods of interval arithmetic is also given.

1. Introduction
Recently, a wide range of nanoelectronic components have emerged that are used for the design of information and computing systems. This relatively new direction in the development of computer technology still has many unsolvable problems of practical use. One of these problems was the use of such elements in conditions other than laboratory ones. In this case, interval uncertainties arise when determining the electrical parameters of these devices, which is superimposed on the uncertainty of the entire system as a whole and complicates the circuit analysis, as well as the entire design process as a whole.

Various authors, of course, have long studied the issues of improving the operational characteristics of computer technology in the design of the basic elements of digital technology. Now there is quite a lot of both new research and academic works of the 20th century on this score. However, a new element base for nanoelectronic technology requires new studies of the robustness of the electromagnetic parameters of the components used.

One of the newest elements of nanoelectronics that appeared not so long ago is the memristor, a passive element in microelectronics capable of changing its resistance depending on the charge flowing through it. For a long time, the memristor was considered a theoretical model [1-7], which could not be realized in practice until the first sample of an element demonstrating the properties of a memristor was created in 2008 by a team of scientists led by R. S. Williams at the Hewlett-Packard research laboratory. The device does not accumulate charge like a capacitor, does not support magnetic flux like an inductor. The change in the properties of the device is provided by chemical reactions in a thin two-layer film of titanium dioxide (5 nm). One layer of the device's film is slightly oxygen-depleted and oxygen vacancies migrate between the layers as the voltage changes. This implementation of the memristor belongs to the
class of nanoionic devices. The observed phenomenon of hysteresis in the memristor makes it possible to use it, among other things, as a memory cell [8-16].

The well-known properties of memristors make it possible to create computers of a fundamentally new architecture, significantly exceeding semiconductor ones in performance. The projected memristor information-computing systems assume the parallel and independent operation of many modules, and the ability to memorize and operate an unlimited set of values from 0 to 1 means that the executable programs are not limited to binary code [5, 8, 17-20].

In connection with the above, it is necessary to take a fresh look at the mathematical model of an electric multipole using a memristor as one of the elements of the basic set. The introduction of such an addition to the basic set of elements also determines the appearance of new memristive branches of the electric multipole.

According to the well-known classification [22], it is customary to distinguish the following typical operating modes of the investigated electronic devices, which is quite true for the elements of nanoelectronics:

- Quasi-linear mode of a small signal when calculating with an alternating current;
- Quasi-linear small signal mode when calculating in the time domain;
- Large signal mode when calculating for direct current (static mode);
- Large signal mode when calculating in the time domain (dynamic mode).

Depending on the selected mode, the initial model of the investigated device will be an electrical multipole of one of the following types: linear resistive, linear reactive, nonlinear resistive and nonlinear reactive. At the same time, the general methodological basis for studying all the listed types of models is their description in the basis of finite deviations of characteristics within specified limits in the presence of disturbing influences with undefined properties. This article analyzes the third large signal mode in the specified classification when calculating by direct current (static mode) for an electric multipole with memristive branches in finite increments.

2. Materials and methods

Mathematical models of an electric multipole with memristive branches in the nominal form and using finite increments were built earlier [1-3] using the method of decomposition of branches of the directed graph of the circuit described in [16; 19-22].

A full-size mathematical model in a full hybrid basis in finite increments of currents and voltages will look like this [19]:

\[
\begin{align*}
\begin{bmatrix} 0 & \hat{L} \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \Delta U_C^P \\ \Delta I_L^X \end{bmatrix} &= Q_1 \begin{bmatrix} \Delta U_C^P \\ \Delta I_L^X \end{bmatrix} + Q_2 \begin{bmatrix} \Delta U_H^P \\ \Delta I_H^X \end{bmatrix} + Q_3 \begin{bmatrix} E_N^R \\ J_N^R \end{bmatrix} + Q_4 \begin{bmatrix} \Delta U_M^P \\ \Delta I_M^X \end{bmatrix} - \begin{bmatrix} E_E^R \\ J_E^R \end{bmatrix} \\
\begin{bmatrix} \Delta U_H^P \\ \Delta I_H^X \end{bmatrix} &= A_1 \begin{bmatrix} \Delta U_C^P \\ \Delta I_L^X \end{bmatrix} + A_2 \begin{bmatrix} \Delta U_R^P \\ \Delta I_R^X \end{bmatrix} + A_3 \begin{bmatrix} \Delta U_M^P \\ \Delta I_M^X \end{bmatrix} - H \cdot F^I \\
\begin{bmatrix} \Delta U_R^P \\ \Delta I_R^X \end{bmatrix} &= G_1 \begin{bmatrix} \Delta U_C^P \\ \Delta I_L^X \end{bmatrix} + G_2 \begin{bmatrix} \Delta U_H^P \\ \Delta I_H^X \end{bmatrix} + G_3 \begin{bmatrix} \Delta U_M^P \\ \Delta I_M^X \end{bmatrix} - W \begin{bmatrix} E_N^R \\ J_N^R \end{bmatrix} \\
\begin{bmatrix} \Delta U_M^P \\ \Delta I_M^X \end{bmatrix} &= S_1 \begin{bmatrix} \Delta U_C^P \\ \Delta I_L^X \end{bmatrix} + S_2 \begin{bmatrix} \Delta U_H^P \\ \Delta I_H^X \end{bmatrix} + S_3 \begin{bmatrix} \Delta U_R^P \\ \Delta I_R^X \end{bmatrix} - K \begin{bmatrix} E_N^R \\ J_N^R \end{bmatrix}
\end{align*}
\]

where \( \hat{L} = diag(L_k + \Delta L_k) \), \( \hat{C} = diag(C_k + \Delta C_k) \) is the matrix of inductances \((k = 1, n_{HL}, l = 1, n_{HL}, n_{HL} + n_{HL} = dim \Delta U_L)\); \( \hat{C} = diag(C_k + \Delta C_k) \) is the capacity matrix...
Actual problems of the energy complex 2020

IOP Conf. Series: Materials Science and Engineering 976 (2020) 012012  doi:10.1088/1757-899X/976/1/012012

\( k = 1, n^{Xc}_{\mathcal{H}c}, l = 1, n^{Xc}_{\mathcal{H}c}, n^{Xc}_{\mathcal{H}c} + n^{Yc}_{\mathcal{H}c} = \text{dim} \Delta U^P; \Delta U^X_{\mathcal{H}c}, \Delta I^X_{\mathcal{H}c} \) - increments of volatges and currents on inductive chords; \( \Delta U^P_{\mathcal{H}c}, \Delta I^P_{\mathcal{H}c} \) - voltage and current increments on the capacitive edges; \( \Delta U^P_{\mathcal{H}c}, \Delta I^P_{\mathcal{H}c}, \Delta I^X_{\mathcal{H}c} \) - voltage and current increments on resistive edges and chords; \( \Delta U^P_{\mathcal{H}c}, \Delta U^P_{\mathcal{H}c}, \Delta I^P_{\mathcal{H}c}, \Delta I^X_{\mathcal{H}c} \) - voltage and current increments on nonlinear edges and chords; \( \Delta U^P_{\mathcal{H}c}, \Delta U^P_{\mathcal{H}c}, \Delta I^P_{\mathcal{H}c}, \Delta I^X_{\mathcal{H}c} \) - voltage and current increments on memresistive edges and chords; \( Q_1 = \left[ -B_{II} \; Z_E \right] \left[ V_E \; -D_{IV} \right] \) and \( Q_2 = \left[ -B_{IV} \; 0 \; 0 \; -D_{IV} \right] \) \( G_1, \; Q_2 = \left[ -B_{II} \; 0 \; -D_{II} \right] + \left[ -B_{IV} \; 0 \; -D_{IV} \right] \cdot G_2, \; Q_3 = \left[ -B_{II} \; 0 \; -D_{II} \right] \cdot W \) and \( Q_4 = \left[ -B_{III} \; 0 \; -D_{III} \right] \) + \( \left[ -B_{IV} \; 0 \; -D_{IV} \right] \); \( Z_E = [0]_{n^{Xc}_{\mathcal{H}c} \times n^{Xc}_{\mathcal{H}c}} \text{diag}\{ (Z_{E_1}^I + Z_{E_1}^R) \} \) - matrix of equivalent resistances and \( V_E = [0]_{n^{Xc}_{\mathcal{H}c} \times n^{Xc}_{\mathcal{H}c}} \text{diag}\{ (G_{E_1}^C + G_{E_1}^N) \} \) - equivalent conductivities of nonlinear inductances and capacities; \( D \) is the matrix of the main contours of the directed graph of the circuit; \( n \) is the number of branches and \( k \) is the number of nodes; \( E_{E}^1 = \left[ (E_N)_k \right]_{n^{Xc}_{\mathcal{H}c} \times 1} \left[ (E_E^1 + E_E^2 (\Delta t))_k \right]_{n^{Xc}_{\mathcal{H}c} \times 1}^T \) and \( f_{E}^1 = \left[ (U_N)_k \right]_{n^{Xc}_{\mathcal{H}c} \times 1} \left[ (U_E^1 + U_E^2 (\Delta t))_k \right]_{n^{Xc}_{\mathcal{H}c} \times 1}^T \) \( A_1 = \left[ \Delta M^{-1} \; Z_X (\Delta I^X_{\mathcal{H}c}) \right] \left[ G_P (\Delta U^P_{\mathcal{H}c}) \Delta M \right] \), \( A_2 = \left[ \Delta M^{-1} \; Z_X (\Delta I^X_{\mathcal{H}c}) \right] \left[ G_P (\Delta U^P_{\mathcal{H}c}) \Delta M \right] \), \( A_3 = \left[ \Delta M^{-1} \; Z_X (\Delta I^X_{\mathcal{H}c}) \right] \left[ G_P (\Delta U^P_{\mathcal{H}c}) \Delta M \right] \); \( E^I = \left[ E \; 0 \; 0 \; E \; 0 \; 0 \; E \; 0 \; E \; 0 \; 0 \; E \; 0 \; 0 \; E \; 0 \; 0 \; E \right] \).

and \( \Delta M \) are diagonal matrices of equivalent inverse and forward memresistivities; \( E \) is the identity matrix; \( E_{E}^1, \; E_{E}^2, \; E_{\mathcal{H}c}, \; f_{E}^1, \; f_{E}^2, \; f_{\mathcal{H}}^1, \; f_{\mathcal{H}}^2 \) and \( f_{\mathcal{H}}^3 \) - equivalent sources of EMF and current on nonlinear chords and edges; \( G_1 = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \), \( G_2 = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \), \( G_3 = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \) and \( W = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \); \( S_1 = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \), \( S_2 = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \) and \( K = \left[ B_{X_1} \; \hat{R} \; \hat{X} \; D_{X_1} \right]^{-1} \cdot \left[ -B_{X_1} \; 0 \; -D_{X_1} \right] \); \( \hat{M} = M + \Delta M \) is the matrix of the branch memresistivities, \( \hat{M}^{-1} = M^{-1} + \Delta M^{-1} \) is the matrix of inverse memresistivities of the branch; \( E_{E}^1, \; f_{E}^1 \) - equivalent vectors of independent voltage and current sources on memresistive branches.

Mathematical model of a multipole with memresistive branches in a static mode. Multipole calculation in this mode, DC calculation is carried out to determine the parameters of the operating point of nonlinear electronic devices, as well as static corresponding electronic devices, for example, DC transmission ratios, input, and output resistances, etc.
The equivalent circuit of the investigated device in this mode does not contain reactive elements - capacitors or inductors, therefore it is a nonlinear resistive-memresistive multipole. When compiling mathematical models in a reduced hybrid basis, we will assume that the multipole topology satisfies the following two constraints:

- Non-monotonic voltage-controlled resistances do not form a loop;
- Non-monotonic current-controlled memresistivities do not form cross sections.

Non-monotonic voltage-controlled resistances and non-monotonic current-controlled resistances belong to special branches of an electrical multipole, with the first of the mentioned resistances being assigned to the edges, and the second to chords. To ensure the formulated requirement, the numbering of the multipole branches starts with nonmonotonic voltage-controlled resistances and ends with nonmonotonic current-controlled memresistivities.

Earlier in [21-22] it was shown that when composing equivalent circuits in nominal and perturbed operating modes, the circuit topology does not change. Consequently, using the already known expressions of the generalized model of an electric multipole, we will immediately write down expressions for currents and voltages of the branches in finite increments.

Considering the remarks made, the vectors of currents and voltages increments for all branches of the investigated electric multipole are divided into the following characteristic groups: nonlinear edges, linear edges, linear chords. Based on the above, the mathematical model of a multipole with memresistive branches (1) in a static mode will take the form:

\[
\begin{align*}
\begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} &= A_2 \begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} + A_3 \begin{bmatrix}
\Delta U_R^m \\
\Delta I_R^m
\end{bmatrix} - H \cdot F^l \\
\begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} &= G_2 \begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} + G_3 \begin{bmatrix}
\Delta U_R^m \\
\Delta I_R^m
\end{bmatrix} - W \begin{bmatrix}
E_N^p \\
J_N^p
\end{bmatrix} \\
\begin{bmatrix}
\Delta U_R^m \\
\Delta I_R^m
\end{bmatrix} &= S_2 \begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} + S_3 \begin{bmatrix}
\Delta U_R^m \\
\Delta I_R^m
\end{bmatrix} - K \begin{bmatrix}
E_N^m \\
J_N^m
\end{bmatrix}
\end{align*}
\]

Expression (2) belongs to the class of interval systems of linear algebraic equations, for the solution of which it is permissible to use Kahan's interval arithmetic and the interval methods of Newton and Krawczyk.

The essence of Kahan's interval arithmetic is that operations with intervals containing zero have the same result as in the case of other intervals and allows you to keep the interval expansion of functions unchanged, and also guarantees, under certain conditions, not only the distributiveness of operations, but also monotonicity with respect to inclusion [20-23].

3. Results

Computational algorithm for external estimation of the solution sets of the mathematical model of a multipole with memresistive branches in a static mode. To solve such interval systems of nonlinear algebraic equations, one can use Newton's method [23]. For this, we bring system (2) to the form:

\[ f_p(x, y) = 0 \] (3)

Let us denote \( IGA (A, b) \) - the result of applying the Gaussian method to the interval system \( Ax = b \). Then for the first equation of system (2), we get:

\[ A = A_2; x = \begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix}; b = A_3 \begin{bmatrix}
\Delta U_R^m \\
\Delta I_R^m
\end{bmatrix} - \begin{bmatrix}
\Delta U_R^p \\
\Delta I_R^p
\end{bmatrix} - H \cdot F^l \] (4)

\[ (\Delta U)_{k+1} = \cap((\Delta U)_k) \cap (\Delta U)_k \] (5)
where $\mathcal{N}(\Delta U)$ is the Newton operator of the following form:

$$\mathcal{N}(\Delta U) = \text{mid}(\Delta U) - IGA(F'(\Delta U), f(\text{mid}(\Delta U)))$$  \hspace{1cm} (6)

On the direct course of calculations, we find the Jacobi matrix:

$$\hat{A} = 
\begin{bmatrix}
\frac{\partial F_1^{(1)}(\Delta U)}{\partial U^H_{11}} & \frac{\partial F_1^{(1)}(\Delta U)}{\partial I^H_{11}} \\
\frac{\partial F_2^{(1)}(\Delta U)}{\partial U^H_{11}} & \frac{\partial F_2^{(1)}(\Delta U)}{\partial I^H_{11}}
\end{bmatrix}$$  \hspace{1cm} (7)

We calculate:

$$r_{ij} \leftarrow \frac{a_{ij}}{a_{jj}}$$  \hspace{1cm} (8)

For all $j = 1$ to $n-1$ and for all $i = j + 1$ to $n$. Here $a$ are the elements of the Jacobi matrix.

On the reverse, using the elements of the matrix $A$ and the vector $b$ according to (4), we find that:

$$y_2 = \frac{b_{2}^{**}}{a_{2}^{**}}$$  \hspace{1cm} (9)

Then

$$y_1 = b_{1}^{**} - a_{21}^{**}y_2^{**}$$  \hspace{1cm} (10)

Newton's interval method has a drawback, the essence of which is that even if the matrix of the natural interval expansion of the function is non-singular, in the general case it can be calculated $IGA[F'(\Delta U), f(\text{mid}(\Delta U))]$ only if the width $\text{mid}(\Delta U)$ is small enough.

This drawback is eliminated in the method for solving systems of nonlinear interval equations, called the Krawczyk method, by introducing the mapping of the interval function (the Krawczyk operator) [24].

The solution of system (3) by the Krawczyk method in the general case will have the following form:

$$\Delta U^{K+1} = \mathfrak{R}(\Delta U_K) \cap \Delta U_K$$  \hspace{1cm} (11)

where $\mathfrak{R}(\Delta U)$ is the Krawczyk operator of the following form:

$$\mathfrak{R}(\Delta U_K) = \text{mid}(\Delta U_K) - C \cdot f(\text{mid}(\Delta U_K)) + (E - C \cdot f'(\Delta U_K)) \cdot (\Delta U_K - \text{mid}(\Delta U_K))$$  \hspace{1cm} (12)

where $C$ is a non-singular real matrix of weight coefficients, the elements of which are selected empirically in such a way as to improve the convergence of the method; $f$ is function (3).

For a fixed matrix $C$, the Krawczyk method is defined by the expression:

$$\Delta U^{K+1} = \mathfrak{R}(\Delta U^K) \cap \Delta U^K$$  \hspace{1cm} (13)

The algorithm for solving the second and third equations of system (2) is implemented in a similar way.

4. Discussion

The static regime considered in this article for an electric multipole with memresistive branches confirms the applicability of the chosen approach to the analysis of nanoelectronic circuits of both memory elements and the architecture of information-measuring systems.

5. Conclusion

The form of presentation of the mathematical model for each mode of operation is quite consistent with the forms required for using the methods of interval analysis. Of course, it is necessary to check the
adequacy of the presented model of an electric multipole with memresistive branches both in general form and in dynamic and static modes for real nanoelectronic circuits, which is the subject of further research.

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