On the Modeling of Thin Bodies of Revolution

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Abstract. The new parametrization of the three-dimensional thin domain of an arbitrary body of rotation is considered, consisting in using several base surfaces in contrast to the classical approaches. The vector parametric equation is given. The geometric characteristics inherent to the new parameterization are determined. Expressions for the translation components of the unit tensor of the second rank and also relations connecting different bases and geometric characteristic generated by them are written. The determination of the moment of an arbitrary function is given. The motion equations and CR of the micropolar theory of elastic rotation bodies without a symmetry center of variable thickness are obtained under the new parametrization, from which equations are obtained for very thin and shallow rotation bodies, both variable and constant thickness. The equations of motion and CR of any approximation in the moments of unknown functions with respect to an arbitrary system of orthogonal polynomials and, in particular, for the system of Legendre polynomials are derived. The questions of setting initial-boundary problems are discussed.

1. Introduction
The use of composite materials of layered and fibrous structures in various branches of technology has led to the need to develop new methods for calculating and designing thin bodies. In connection with the use of thin bodies (one-, two-, three- and multilayer structures) there is a need to create new refined theories of thin bodies and improved methods for their calculation. One of the methods for constructing the theory of thin bodies is the analytical method. In particular, it is used the expansion in series of the system of orthogonal Legendre polynomials [1–20] (these decompositions are used equally to construct any theory of thin bodies [17]). In this work, we use the Legendre polynomial expansion for modeling micropolar bodies of revolution.

2. To new parametrization of the domain of a rotation body with one small size
We consider a new parametrization [21, 22] of the domain of a 3D Euclidean space that is bounded by two face surfaces \((-S)\) and \((+S)\) and a lateral surface \(\Sigma\) (Fig. 1). The face surface \((-S)\) is called inner base surface, and the face surface \((+S)\) is called outer base surface. The radius-vector of an arbitrary point of the domain of a thin body is introduced in the following form (Fig. 1)

\[
\mathbf{r}(x', x^3) = \mathbf{r}(-) (x') + x^3 \mathbf{h}(x') = (1 - x^3)(-r)(x') + x^3 (+r)(x'), \quad x' = (x^1, x^2), \quad \forall x^3 \in [0, 1],
\]

where relations \(\mathbf{r}(-) = \mathbf{r}(-)(x')\), \(\mathbf{r}(+) = \mathbf{r}(+) (x')\) are the vector parametric equations of the base surfaces \((-S)\) and \((+S)\), respectively, \(x' = (x^1, x^2)\) is an arbitrary point on \((-S)\), \(x^1\) and \(x^2\) is an curvilinear
coordinates on inner base surface \( S \). Vector \( h(x') = r' - x' \) performing topological mapping of the inner base surface \( S \) on the outer surface \( S' \), in general is not perpendicular to the base surfaces. The relation (1) is exactly the vector parametric equation of the domain of a thin body. When presenting the material, the usual rules for tensor calculus are applied [21–23]. Uppercase and lowercase latin indices take values of 1, 2 and 1, 2, 3 respectively. In addition, short entries are used later on, e.g. \( M \in S; \) \( * \in \{ -, \emptyset, + \} \) or \( r_p = g_{p\bar{q}} r_{\bar{q}} \), \( \sim , \emptyset \in \{ -, \emptyset, + \} \) where \( \emptyset \) denotes the empty set. The first entry means that if \( * = - \) then \( M \in S; \) if \( * = \emptyset \), then \( M \in S; \) if \( * = + \), then \( M \in S' \). The second entry means that if \( \sim = \emptyset \) and \( - = \emptyset \), then \( r_p = g_{p\bar{q}} r_{\bar{q}} \), etc. Going through all the values, one can get all the relations. Figure 1 shows the body of rotation with \( x_1 = \theta \) (the angle between the rotation axis and the normal to \( S \) and \( x_2 = \varphi \) (azimuth angle). Further, we introduce the following notation for the derivatives of the \( r \) and \( r' \) with respect to \( x^p \) at points \( M \in S; \) \( * \in \{ -, \emptyset, + \} \)
\[
  r_p = \partial_p r = \partial r/\partial x^p, \quad r'_p = \partial r'/\partial x^p, \quad * \in \{ -, + \}.
\] (2)

The pair of vectors \( r_1, r_2, * \in \{ -, \emptyset, + \} \), defined at points \( M \in S; \) \( * \in \{ -, \emptyset, + \} \) form two-dimensional covariant surface bases. It is well known that based on these bases [21–23], we can construct the corresponding contravariant bases \( r^1, r^2, * \in \{ -, \emptyset, + \} \). Taking into account the expression of the radius vector \( r \) (1) in the first relation of (2) and introducing denotation \( h_p = \partial h/\partial x^p = \partial_p h \) one can get
\[
r_p = r'_p + x^3 h_p = (1 - x^3) r'_p + x^3 r^3_p, \quad r_3 = \partial_3 r = \partial r/\partial x^3 = h(x'), \quad \forall x^3 \in [0, 1].
\] (3)

Based on the second relation (3) we can assume
\[
r_3 = r_3 = r' = h(x'), \quad \forall x^3 \in [0, 1].
\] (4)

Relation (4) defines the spatial covariant bases \( r'_p, * \in \{ -, + \} \) at points \( M \in S; \) \( * \in \{ -, + \} \). Thus, the third basis vector of the spatial covariant bases at points \( M \in S; \) \( * \in \{ -, \emptyset, + \} \) is the same vector \( h(x') \). Given (4), relations (3) can be combined in the following form
\[
r_p(x', x^3) = r'_p(x') + x^3 h_p(x') = (1 - x^3) r'_p(x') + x^3 r^3_p(x').
\] (5)
Three vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ast \in \{-, \emptyset, +\}$ defined in the considered points $M \in S^1$; $\ast \in \{-, \emptyset, +\}$ form three-dimensional (spatial) covariant bases. It is well known that based on these bases [21–23], one can construct the corresponding contravariant bases $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ast \in \{-, \emptyset, +\}$. Further, the geometrical characteristics are written down as in [17,21,22] and in the important special case, when $\mathbf{h} \perp S^1$ (h = h ($\mathbf{n}$)), for the rotation bodies we find

$$
\mathbf{r}_1^\ast = a \mathbf{a} + f(a) \mathbf{k}_3, \quad a = \cos \varphi \mathbf{k}_1 + \sin \varphi \mathbf{k}_2 \quad a = R_2 \sin \theta, \\
\mathbf{r}_2^\ast = \mathbf{r}_1^\ast (\theta, \varphi) + \mathbf{h}, \quad \mathbf{r}_3^\ast = \mathbf{r}_1^\ast (\theta, \varphi) + x^3 \mathbf{h}(\theta, \varphi), \quad (6)
$$

where to reduce the letter, instead of $\mathbf{R}_2$ and $\theta$ the notations $R_2$ and $\theta$ are introduced, respectively. Further, introducing the notations $x^1 = \theta$ and $x^2 = \varphi$, it is easy to find the geometric characteristics (covariant and contravariant bases, Christoffel symbols, components of the unit tensor of the second rank, etc.) for the body of rotation under the new parameterization. To make shorter the work we will not go into detail, but we give some necessary relations without proof. Introducing $c = \partial_\theta \mathbf{a} = -\sin \varphi \mathbf{k}_1 + \cos \varphi \mathbf{k}_2$ ($\mathbf{a} \cdot \mathbf{c} = 0$), according to (6) and easily proved formulas (the proof of the first formula can be found in [24])

$$
\frac{da}{d\theta} = R_1 \cos \theta, \quad \partial_1 f(a) = -R_1 \cos \theta \delta_{11}, \quad \partial_1 \mathbf{h} = \frac{g_3^\ast (\mathbf{n})}{(\mathbf{n})} - h \frac{\partial h}{\partial \theta}, \quad h' = \frac{dh}{d\theta}, \quad g_3^\ast = h^{-1} h' \delta_{11}
$$

for the covariant and contravariant bases, and also for the orthonormal basis at the point $M$ we have expressions

$$
\mathbf{r}_1 = R_1 \cos \theta \mathbf{a} - R_1 \sin \theta \mathbf{k}_3, \quad \mathbf{r}_2 = R_2 \sin \theta \mathbf{c}, \quad \mathbf{r}_3 = \mathbf{h} = h \mathbf{n} = h (\sin \theta \mathbf{a} + \cos \theta \mathbf{k}_3), \\
\mathbf{r}_1^\ast = \frac{1}{R_1} (\cos \theta \mathbf{a} - \sin \theta \mathbf{k}_3), \quad \mathbf{r}_2^\ast = \frac{1}{R_2 \sin \theta} \mathbf{c}, \quad \mathbf{r}_3^\ast = h^{-2} \mathbf{h} = h^{-1} (\mathbf{n}), \quad \sqrt{g} = h R_1 R_2 \sin \theta, \quad (7)
$$

$$
\mathbf{e}_1 = \cos \theta \mathbf{a} - \sin \theta \mathbf{k}_3, \quad \mathbf{e}_2 = \mathbf{c}, \quad \mathbf{e}_3 = \mathbf{n} = \sin \theta \mathbf{a} + \cos \theta \mathbf{k}_3, \quad \mathbf{e}_p \cdot \mathbf{e}_q = \delta^p_{pq},
$$

where $\delta^p_{pq}$ is the Kronecker delta, $\mathbf{e}_+, \mathbf{e}_-, \mathbf{e}_1$ is the orthonormal basis at the point $M$, $h, \mathbf{h}$ are the components of the second tensor of the inner base surface.

Taking into account $\mathbf{h}_1 = \partial_1 \mathbf{h} = (h \cos \theta + h' \sin \theta) \mathbf{a} + (h' \cos \theta - h \sin \theta) \mathbf{k}_3$, $\mathbf{h}_2 = \partial_2 \mathbf{h} = h \sin \theta \mathbf{c}$, similarly to (7) for the covariant bases at the points $M$ and $M^\ast$, we obtain

$$
\mathbf{r}_1^\ast = [(R_1 + h \cos \theta + h' \sin \theta) \mathbf{a} + [h' \cos \theta - (R_1 + h) \sin \theta] \mathbf{k}_3, \\
\mathbf{r}_2^\ast = (R_2 + h) \sin \theta \mathbf{c}, \quad \mathbf{r}_3^\ast = R_3 = h \mathbf{n} = h (\sin \theta \mathbf{a} + \cos \theta \mathbf{k}_3), \\
\mathbf{r}_1 = [(R_1 + x^3 h) \cos \theta + x^3 h' \sin \theta] \mathbf{a} + [x^3 h' \cos \theta - (R_1 + x^3 h) \sin \theta] \mathbf{k}_3, \\
\mathbf{r}_2 = (R_2 + x^3 h) \sin \theta \mathbf{c}, \quad \sqrt{g} = h (R_1 + x^3 h) (R_2 + x^3 h), \quad \sqrt{g}^\ast = h (R_1 + h) (R_2 + h), \quad (8)
$$

Further according to (7) and (8) it is easy to find the components of the unit tensor of the second rank $g_{pq}, g^p_{pq}, g_{pq}^q$ and $g^p_{q}$. In fact, after simple calculations, for example, for the components
Here we have found expressions for \( g_{pq}^{-}, g_{q}^{+} \), and \( g_{q}^{p} \), we find the next expressions:

\[
\begin{align*}
g_{1}^{1} &= 1 + hk_{1}x^{3}, \quad g_{1}^{2} = 0, \quad g_{1}^{3} = x^{3}g_{1}^{1}; \\
g_{2}^{1} &= 0, \quad g_{2}^{2} = 1 + hk_{2}x^{3}, \quad g_{2}^{3} = 0; \\
g_{3}^{1} &= 0, \quad g_{3}^{2} = 0, \quad g_{3}^{3} = 1; \\
g_{1}^{1} &= (1 + hk_{1}x^{3})^{-1}, \quad g_{2}^{1} = 0, \quad g_{3}^{1} = 0, \\
g_{2}^{2} &= 0, \quad g_{2}^{2} = (1 + hk_{2}x^{3})^{-1}, \quad g_{2}^{2} = 0, \\
g_{3}^{3} &= -h^{-1}h'x^{3}, \quad g_{3}^{3} = 0, \quad g_{3}^{3} = 1.
\end{align*}
\]

Note that based on (8) it is easy to find the expressions for \( g_{p}^{-}, g_{q}^{+}, g_{q}^{p} \), and from it we find the expressions for \( r^{i} \). In fact, we have

\[
r^{i} = \theta^{-1} \epsilon^{ij} \epsilon_{LJK} g_{j}^{L} r^{K}, \quad r^{3} = r_{3}^{i} - g_{j}^{i} g_{j}^{r} r^{r} L, \quad (\theta^{-1} = (1 + hk_{1}x^{3})(1 + hk_{2}x^{3}), \quad k_{l} = R_{l}^{i}.
\]

Taking into account the appropriate relations (7) and (9), we obtain the expressions for \( r^{3} \) from the second formula (10), and from it we find the expression for \( r_{3}^{3} \). We will get

\[
\begin{align*}
r^{3} &= \frac{1}{h(R_{1} + h x^{3})} \left\{ \left[R_{1} + h x^{3} \right] \sin \theta - x^{3} h' \cos \theta \right| \right| a + \left\{ \left[R_{1} + h x^{3} \right] \cos \theta + x^{3} h' \sin \theta \right| \right| k_{3}, \\
r_{3}^{3} &= \frac{1}{h(R_{1} + h x^{3})} \left\{ \left[R_{1} + h x^{3} \right] \sin \theta - h' \cos \theta \right| \right| a + \left\{ \left[R_{1} + h x^{3} \right] \cos \theta + h' \sin \theta \right| \right| k_{3}, \\
g_{3}^{33} &= \frac{1}{h^{2}(R_{1} + h x^{3})^{2}} [(R_{1} + h x^{3})^{2} + (x^{3} h')^{2}], \quad g_{3}^{33} = \frac{1}{h^{2}(R_{1} + h x^{3})^{2}} [(R_{1} + h x^{3})^{2} + (h')^{2}].
\end{align*}
\]

Here we have found expressions for \( g_{3}^{33}, g_{3}^{33} \). Let us find the expression for the unit normal to the surface \( S \) at the point \( M \). Denoting it by \( \mathbf{n} \), from the first formula (11) we get

\[
\mathbf{n} = \mathbf{n} = \frac{r^{3}}{\sqrt{g_{3}^{3}}} = \frac{1}{\sqrt{h(R_{1} + h x^{3})^{2} + (x^{3} h')^{2}}} \left\{ \left[R_{1} + h x^{3} \right] \sin \theta - x^{3} h' \cos \theta \right| \right| a + \left\{ \left[R_{1} + h x^{3} \right] \cos \theta + x^{3} h' \sin \theta \right| \right| k_{3}, \quad (\theta = \mathbf{n} | x^{3} = 1.
\]

Note that the relations (11) – (12) are used when writing boundary conditions.

3. Equations and constitutive relations of the theory of thin elastic bodies

Motion equations in terms of stress and moment stress tensors [17]

\[
g_{p}^{P} N_{p}^{M} P_{M}^{M} + \partial_{p} P_{M}^{M} + \rho F = \rho \partial_{p}^{2} \mathbf{u}, \quad g_{p}^{P} N_{p}^{M} P_{M}^{M} + \partial_{p} P_{M}^{M} + C_{p}^{2} P_{M} + \rho \mathbf{m} = \mathbf{J} \cdot \partial_{p}^{2} \mathbf{u};
\]

\[
N_{p} = \partial_{p} - g_{p}^{3} \partial_{3}, \quad \partial_{i} = \partial / \partial x^{i}, \quad g_{p}^{3} = x^{3} g_{3}^{3}, \quad P_{M}^{M} = \mathbf{r} \cdot \mathbf{P}, \quad \mathbf{m} = \mathbf{r} \cdot \mathbf{m} \mathbf{u},
\]

\[
g_{p}^{P} = \sum_{s=0}^{\infty} A_{p}^{P} (x^{3})^{s}, \quad A_{p}^{P} = g_{p}^{P}, \quad A_{p}^{P} = g_{p}^{P} - g_{p}^{P}, \ldots, \quad A_{p}^{P} = g_{p}^{P} - g_{p}^{P} \ldots, \quad (g_{n}^{P} - g_{n}^{P}) \ldots (g_{n}^{P} - g_{n}^{P})
\]

Similarly to (13), from constitutive relations (CR) of micropolar linear elasticity theory one has [17]
\[
\begin{align*}
P &= A^\gamma \cdot g^P \cdot N_P \cdot u + A^\gamma \cdot \partial_3 \cdot u + B_\gamma \cdot g^P \cdot N_P \cdot \varphi + B_\gamma \cdot \partial_3 \cdot \varphi - A^\gamma \otimes C \cdot \varphi, \\
\mu &= C^\gamma \cdot g^P \cdot N_P \cdot u + C^\gamma \cdot \partial_3 \cdot u + D^\gamma \cdot g^P \cdot N_P \cdot \varphi + D^\gamma \cdot \partial_3 \cdot \varphi - C^\gamma \otimes C \cdot \varphi, \\
\bar{A}^m &= A^{ijm} \cdot r_i \cdot r_j \cdot r_l, \\
\bar{B}^m &= B^{ijm} \cdot r_i \cdot r_j \cdot r_l, \\
\bar{C}^m &= C^{ijm} \cdot r_i \cdot r_j \cdot r_l, \\
\bar{D}^m &= D^{ijm} \cdot r_i \cdot r_j \cdot r_l.
\end{align*}
\]  

where \( P \) and \( \mu \) are the stress and moment stress tensors, \( C \) is the discriminant tensor [23], \( u \) and \( \varphi \) are the displacement and rotation vectors, \( \rho \) is the material density, \( F \) and \( m \) are the mass force and mass momentum densities, \( J \) is a tensor of inertia moment (inner property of the medium), \( \nabla \) is the Hamilton nabla-operator, \( \partial_3 = \partial / \partial t \), \( t \) is a time, \( \otimes \) is an inner 2-product [22, 23].

In this case, by virtue of the corresponding relations (9), we have relations

\[
N_1 = \nabla_1 - g_1^3 \partial_3, \quad N_2 = \nabla_2, \quad g^P \cdot N_P \cdot P^M = g_1^1 (\nabla_1 - g_1^3 \partial_3) \cdot P^1 + g_2^2 \nabla_2 \cdot P^2,
\]

in virtue of which the equations (13) and CR (14) for rotation bodies of variable thickness are represented in the form

\[
\begin{align*}
g_1^1 (\nabla_1 - g_1^3 \partial_3) \cdot P^1 + g_2^2 \nabla_2 \cdot P^2 + \partial_3 \cdot \rho F &= \rho \partial_3^2 \cdot u, \\
g_1^1 (\nabla_1 - g_1^3 \partial_3) \cdot \mu^1 + g_2^2 \nabla_2 \cdot \mu^2 + \partial_3 \cdot \mu^3 + C \otimes P + \rho m &= J \cdot \partial_3^2 \cdot \varphi;
\end{align*}
\]

\[
\begin{align*}
P = [g_1^1 A^{-1} (\nabla_1 - g_1^3 \partial_3)] + g_2^2 A^2 \cdot \nabla_2 \cdot u + A^3 \cdot \partial_3^3 \cdot u + \\
+ [g_1^1 B^{-1} (\nabla_1 - g_1^3 \partial_3)] + g_2^2 B^2 \cdot \nabla_2 \cdot \varphi + B^3 \cdot \partial_3^3 \cdot \varphi - A^2 \otimes C \cdot \varphi, \\
\mu = [g_1^1 C^{-1} (\nabla_1 - g_1^3 \partial_3)] + g_2^2 C^2 \cdot \nabla_2 \cdot u + C^3 \cdot \partial_3^3 \cdot u + \\
+ [g_1^1 D^{-1} (\nabla_1 - g_1^3 \partial_3)] + g_2^2 D^2 \cdot \nabla_2 \cdot \varphi + D^3 \cdot \partial_3^3 \cdot \varphi - C^2 \otimes C \cdot \varphi.
\end{align*}
\]

It should be noted that (16) and (17) represent the equations of motion and CR of the micropolar linear theory of elastic rotation bodies without a center of symmetry of variable thickness. For very thin and shallow rotation bodies \( g_1^1 \approx 1 \) and \( g_2^2 \approx 1 \), and \( g_1^3 = 0 \) for a constant thickness, therefore in these cases the equations of motion and the CR are simplified. It is not difficult to write them out, so we will not write them out to shorten the letter. However, we shall write out them below for very thin and shallow rotation bodies of constant thickness. Taking into account the above, we shall have

\[
\begin{align*}
\nabla_1 \cdot P^1 + \partial_3 \cdot P^3 + \rho F &= \rho \partial_3^2 \cdot u, \\
\nabla_1 \cdot \mu^1 + \partial_3 \cdot \mu^3 + C \otimes P + \rho m &= J \cdot \partial_3^2 \cdot \varphi;
\end{align*}
\]

\[
\begin{align*}
P &= A^{-1} \cdot \partial_1 \cdot u + A^3 \cdot \partial_3 \cdot u + B^{-1} \cdot \partial_1 \cdot \varphi + B^3 \cdot \partial_3 \cdot \varphi - A^2 \otimes C \cdot \varphi, \\
\mu &= C^{-1} \cdot \partial_1 \cdot u + C^3 \cdot \partial_3 \cdot u + D^{-1} \cdot \partial_1 \cdot \varphi + D^3 \cdot \partial_3 \cdot \varphi - C^2 \otimes C \cdot \varphi.
\end{align*}
\]

Applying the \( k \)-th order operator [17, 18] to (18) and (19), we obtain the equations of motion and the CR of the micropolar linear theory of very thin and shallow elastic bodies of constant thickness without a center of symmetry in moments of unknown functions with respect to any system of orthogonal polynomials

\[
\begin{align*}
\nabla_1 \cdot P^{(k)} + P^{(k)} \cdot \partial_3^{(k)} + \rho \cdot F &= \rho \partial_3^{(k)} \cdot u, \\
\nabla_1 \cdot \mu^{(k)} + \mu^{(k)} \cdot \partial_3^{(k)} + C^{(k)} \otimes P + \rho \cdot m &= J \cdot \partial_3^{(k)} \cdot \varphi, \quad k \geq 0;
\end{align*}
\]
\[
\begin{align*}
\Pi = & \mathbf{A} \cdot \partial_1 (u) + \mathbf{A} \cdot \partial_1 (\varphi) + \mathbf{B} \cdot \partial_1 (\varphi) + \mathbf{B} \cdot \partial_1 (\varphi) - \mathbf{A} \otimes \mathbf{C} (\varphi), \\
\mu = & \mathbf{C} \cdot \partial_1 (u) + \mathbf{C} \cdot \partial_1 (\varphi) + \mathbf{D} \cdot \partial_1 (\varphi) + \mathbf{D} \cdot \partial_1 (\varphi) - \mathbf{C} \otimes \mathbf{C} (\varphi),
\end{align*}
\]

where the definition of the moment of the kth order \( F \) and the expression for the operator "prime" \( F' \) of any function \( F \), for example, with respect to the system of Legendre polynomials are represented in the form

\[
M_F(k) = (2k + 1) \int_0^1 F(x', x^3) P_k(x^3) dx^3, \quad F'(x') = 2(2k + 1) \sum_{m=0}^{(k+2m+1)} F =
\]

\[
= (2k+1) \{ [F - (-1)^k F] - \sum_{p=0}^{k} [1 - (-1)^{k+p}] F(x') \}, \quad k \geq 0, \quad F = F|_{x^3 = 0}, \quad F = F|_{x^3 = 1}.
\]

Note that from (20) and (21) one can get the equations of motion and CR of any approximation. Of course, in order to correctly formulate initial boundary value problems for (20) and (21), depending on the type of initial boundary value problems, it is necessary to add the corresponding initial and boundary conditions in the moments that are obtained similarly to (20) and (21). We do not dwell on them in order to shorten the letter, but we refer the interested reader to [17, 25], in which the analogous questions for thin bodies of other configurations are described in detail.

4. Conclusion

Some questions of the new parametrization of the three-dimensional region of the body of revolution are considered. In particular, the vector parametric equation of the body of revolution is written out. Covariant and contravariant bases are constructed at the points of the inner and outer base surfaces, as well as at arbitrary points of the body. By virtue of these bases, expressions of some geometric characteristics necessary for the presentation of the material are found. The motion equations and CR of the micropolar theory of elastic rotation bodies without a symmetry center of variable thickness are obtained under the new parametrization, from which equations are obtained for very thin and shallow rotation bodies, both variable and constant thickness. Finally, equations of motion and CR of any approximation in the moments of unknown functions with respect to an arbitrary system of orthogonal polynomials and, in particular, for the system of Legendre polynomials are derived. The questions of setting initial-boundary problems are discussed.

Note also that of great interest are works on the thermomechanics of composite structures under high temperature [26-30], which can be used when considering problems of thermomechanics for thin structures.

Acknowledgement: The work was supported by the Russian Foundation for Basic Research (project no.18-29-10085-mk).

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