The principle of materiality of space and the theory of fundamental fields

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Abstract. The work formulates the principle of materiality of space and on its basis a brief critical analysis of the general ideology of the Special and General Theories of Relativity is carried out. The connection of the new principle with the previously developed Topological Theory of Fundamental Fields (TTFF) is considered. A method of constructive implementation of the principle of materiality in the framework of the physical theory of fundamental fields is considered. General equations of the dynamics of markers of material points of physical space are derived and their physical meaning is established.

1. Introduction

In papers [1, 2, 3, 4, 5, 6, 7, 8] there is a new approach for describing the dynamic of matter and its structures, including electromagnetic and gravitational fields, which will be further called topological theory of fundamental fields or short TTFF. One of the features of this approach is the possibility, based on geometric and topological ideas about the structure of space, to obtain a completely adequate unified description of not only gravity and electromagnetism, but also quantum phenomena, including elementary particles. This geometric approach is an alternative to the geometric approach of General Relativity (GR) [9]. In contrast to GR, topological and geometric ideas in TTFF were initially applied not to the gravitational field, but to the electromagnetic field, and first of all, to the concept of an electric charge. But on the basis of a developed ideology, this theory served as the basis for a new description of the gravitational field. Although the general construction of the developed theory is not yet closed ([5]), nevertheless, a number of important problems of modern physics find a fairly clear explanation within the framework of this theory. These include, for example, the topological interpretation of electric and baryon charges, which explains their discreteness. The geometric interpretation is given to the concept of mass and the concept of the wave function of particles, which reduces Born’s probabilistic postulate to the principle of geometric averaging.

The main problem of TTFF, to which special attention was paid in papers [3, 8], is the absence of a physical ideology in it, with the help of which one can describe geometrodynamics of a physical material hypersurface $V^3$ (PMH), embedded in the ambient space $W^4$ per unit of a larger number of dimensions. The geometry of space in TPFF is simpler than in GR, and is determined by the embedding of a three-dimensional physical hypersurface into a
Euclidean space of dimension 4. Therefore, all the properties of such a hypersurface at any given moment of time $t$ are determined by only one function of height $\mathcal{F}(\mathbf{x},t)$ But until the physical nature of the hypersurface itself and its environment is known, it is not possible to fully formulate the principles of describing its dynamics. It should be noted that a similar problem stood at the first stages of the creation of general relativity. This was due to the fact that there was no direct data on what determines the properties of the curvature of space-time. The solution was found in the form of postulating the principle of least action in the form of the Hilbert-Einstein variational principle. This principle was involved in general relativity on the basis of some indirect ideas concerning the invariant description of the dynamics of matter and material fields in pseudo-Riemannian space-time, provided that they should transform into the equations of Newton’s theory of gravitation in the limit to flat space-time. The last requirement is a particular consequence of the correspondence principle, which plays an important role in modern physics.

In TTFF, the solution to the formulated problem also has to be sought, relying on some indirect data. For example, the passage to the limit to the properties of electromagnetic fields in vacuum can serve as such indirect information. In the case of weak fields, electromagnetic waves in empty space propagate at a fixed speed - the speed of light and are described by the d’Alembert equations. In TTFF, such a transition should also take place. By analogy with general relativity, there should also be a passage to the limit to Newton’s theory of gravitation. Since the TTFF claims to describe quantum phenomena as well, then there must be a passage to the limit in it and to quantum mechanics. All these passage to the limit were found in the above papers. All of them are a consequence of the mathematical properties of describing the dynamics of PMH within the framework of the proposed interpretation of electric charge, mass, etc. Due to this interpretation, all the attributes of matter necessary for modern physics appear in theory in the form of mathematical connections between the introduced fields and objects of geometry and topology.

At the same time, in order to substantiate the TTFF, it is required to carry out a fundamental analysis of the modern geometric theory of space-time - GR. This analysis is necessary for several reasons. The first of these reasons concerns the need to explain why the method chosen in GR to describe the geometry of space-time and its relationship with the material objects in it cannot be considered physically adequate to the general concepts of matter as such. With the mathematical flawlessness of general relativity, it contradicts the basic concepts of the properties of matter. The second reason is that such an analysis can provide the missing elements for describing the dynamics of space, which are necessary to complete the whole concept of TTFF.

Partially a preliminary analysis of the justification for the need to revise the entire concept of general relativity was carried out in the paper [7]. In this work, we will conduct a more detailed study of the difficulties of SR and GR, which will give us the opportunity to formulate a special principle for constructing theories of matter and fields, including the concept of space and time. Formalization of this principle, in turn, will make it possible to obtain general information about how it is necessary to formulate the equations of the dynamics of space as a hypersurface embedded in the ambient space of four dimensions. The need for such an approach is due to the fact that the entire concept of TTFFT is based on a set of mathematical identities that allow one to construct from a unified standpoint the entire set of equations for the dynamics of particles with mass, electric and baryon charges, which are involved in electromagnetic and gravitational interactions. However, the theory lacks a dynamic principle that would single out, among all the possible variants of the geometric dynamics of a three-dimensional hypersurface in the ambient four-dimensional space, the only one that corresponds to the dynamics being realized in reality. From a physical point of view, this problem can be formulated as the problem of the absence of any knowledge about what physical properties the physical hypersurface possesses as a material object and what properties the "physical environment" that surrounds it in the ambient space.
of four dimensions possesses. This formulation becomes possible only after the analysis of the admissible general description of the dynamics of material objects from the point of view of markers is carried out.

At the first stage in the proposed work, after discussing the main difficulties of SRT and GRT, the principle of materiality of PMH is formulated, which is absent in SR and GR. This principle is a revision of a number of remarks concerning problems with the materiality of space-time in general relativity and special relativity, which was given in the papers [15, 13, 14, 16], equations for the fundamental potential are derived in general form - functions of the FMH height, written in terms of the marker transfer equation. These equations are the main goal of this work, which represent a tool that allows one to formulate specific models of FMG dynamics also in terms of marker dynamics.

2. Fundamental disadvantages of SRT and GRT

One of fundamental disadvantages of the Special relativity (SR) and General relativity (GR) ([7]) is the immateriality of physical space. Formally, at least within the framework of GR, sometimes space-time is thought of as a material object, on the natural basis that it is endowed with physical properties of curvature. The problem is that no direct measurements for fixing any elements of this space, as material objects, are provided in SR and GR. This fact can be formulated approximately as follows: points of the space-time of GR cannot be marked with any physical marker. However, in the absence of such a procedure, space-time cannot be interpreted as a material object. Thus, in SR and GR there is matter and an intangible object - space-time, which nevertheless in some way interact with each other.

The absolute space of classical mechanics is also not material, but this space has no other properties, except for the obvious property of extension. This means that in reality, distances in space can be measured exclusively between material bodies or parts of the same material body. In the latter case, one speaks of length. However, it is not very convenient to remember the set of individual distances and lengths between the gigantic set of individual material objects and their component parts - points. It is convenient to combine all these distances into a common consistent scheme - a space that has three dimensions. According to the observations of classical mechanics, a consistent scheme is a mathematical construction - a Euclidean three-dimensional space. Therefore, the absolute Euclidean space of classical mechanics is not a material object, but a convenient way to describe the set of distances between material bodies. It is important to state that the absolute space of classical mechanics itself, being a mathematical construction, does not have any effect on material bodies, which is just postulated in Newton's first law and Galileo's principle of relativity.

Newton's corpuscular theory of light did not contradict the idea of the immateriality of space and time. Individual light particles were some localized material objects, and in this sense, the distances to them were included in the general scheme of classical mechanics. However, the contradiction to this fact of the immateriality of space and time was the wave theory of light, which was first formulated clearly by Huygens. If light is waves, then some medium in which these waves propagate must fill the entire space as a whole. In this case, it is possible to check the properties of the model of space as a Euclidean three-dimensional space, considering changes in the characteristics of the waves that have passed through it. However, in this case, there is a problem with the need to distinguish the space itself from the environment in it. This problem became acute after Maxwell created the general theory of electromagnetism. This medium, which was called ether, had to be discovered and its properties established.

As you know, in the Michelson-Morley experiments, the ether could not be detected. Actually SR appeared as a theory that tries to build mechanics and electrodynamics without the presence of ether - a luminiferous medium. In SR, with the help of Lorentz transformations, it is possible to eliminate from the equations of electrodynamics the terms arising from the transition from
one inertial frame of reference to another (as in acoustics), moving uniformly relative to the first frame. However, the environmental problem remained unchanged. If electromagnetic waves are a material object that carries recorded energy, then the question of what oscillates at the observation point when an electromagnetic wave passes through it remains unanswered. As a result, we have to admit that the electromagnetic field itself is a specific form of matter, which does not have mass and other natural properties of matter.

A completely analogous situation exists with any other field, in particular, the gravitational field. As a result, the fields are singled out into a separate category of matter, which does not have any material embodiment, but has a number of measurable and observable characteristics, i.e. fields have a measurable effect on matter.

But if the properties of the electromagnetic field are to some extent similar to matter in the sense that, according to the quantum theory, it has corpuscular properties, then the situation with the gravitational field turns out to be much more complicated. Gravitation does not show any corpuscular properties. The forces of interaction of this field with matter are many times less than the electromagnetic one. But nevertheless, it has a significant effect on bodies of large mass and determines the entire dynamics of celestial bodies. However, the gravitational field in the area between the bodies does not manifest itself in any way. To explain this behavior of the gravitational field at the end of the 19th century, the general idea was proposed that the gravitational field is a manifestation of the properties of space itself. If there is no environment, then only the space itself can be endowed with the necessary properties. It was this idea that formed the basis of Einstein's theory of gravitation in the form of general relativity, although a more radical idea was put forward by Clifford [11].

How does SR solve the problem of the absence of an environment as a material object that would be a carrier of electromagnetic waves and a repository of other fields? For this, in SR space-time is endowed with special properties, consisting in the fact that it affects the movements of material bodies, although in itself, as noted above, it is not a material object. This influence consists in the fact that, without any physical reason, the transition from one inertial frame of reference to another leads to a change in the scales of length and time. This change is detected by comparing the standards of length and time in close contact with the help of light signals, i.e. is a physically detectable phenomenon. It is important to emphasize that the reason for the change in the standards of length and time under these conditions cannot be any physical mechanism, since, according to the first postulate of the SR, all physical laws are the same in all inertial reference frames. Thus, a fundamental contradiction of SR appears - an intangible object - space-time, has quite measurable physical properties.

In general relativity, the non-material space-time is endowed with an even greater number of physical properties - the components of the curvature tensor of the four-dimensional pseudo-Riemannian space-time. The curvature of non-material space-time in general relativity affects the motion of material bodies and, moreover, determines the observed properties of material bodies - their interaction with each other using the forces of gravity. The result of including both material and non-material objects in the theory is the presence of a number of paradoxes in SR and GR. An example is the paradox of the SR twins. In general relativity, this is, for example, the Unruh effect, as well as an intractable problem with the energy of the gravitational field [12].

2.1. Kinematic paradoxes of SR and GR
All explanations of paradoxes in SR and GR are usually based on proving the mathematical consistency of the theory itself, and the irrationality of some of the conclusions of these theories usually refers to the inability to understand reality from the point of view of the practical experience of a person living in the macrocosm of classical mechanics. For example, the paradox of twins requires for its explanation the fact that the frames of reference in which the twins
are located are not "symmetric". If we compare the frames of reference moving uniformly and rectilinearly relative to each other, then all the physical laws in them are the same and there is no way to indicate the mechanism of occurrence of the discrepancy in the clock speed and the change in the length of the rulers in them. Actually, this is the twins paradox. For twins, the problem is that aging is a physical process that takes place in the cells of a living organism. Since all physical processes in both frames of reference are exactly the same, the faster aging of the twin remaining on Earth is not associated with any physical mechanism. From a formal point of view, the asymmetry between twins can be found if we take into account the presence of a mandatory segment of the path of the departing twin astronaut, on which the spacecraft accelerates. In the frame of reference associated with the departing twin, inertial forces arise, and in the frame of reference associated with the Earth, there are no inertial forces. Therefore, it is logical to assume that the discrepancy in the readings of the clocks and the length of the rulers arises precisely because of the presence of inertial forces in the frame of reference of the departing twin. However, inertial forces, generally speaking, are the kinematic effect of the formal transformation of coordinates from one frame of reference to another. Therefore, this asymmetry is irrational from a physical point of view. The forces of inertia in the accelerated frame of reference are manifested only in the emergence of reaction forces of the support acting on the astronaut, which ensure its acceleration. Literally, the departing cosmonaut is "pressed" against the wall of the spacecraft, which creates asymmetry from the point of view of material forces. However, it is difficult to imagine that it is the impact of the spacecraft wall on the astronaut that causes changes in the course of his biological clock. With such an explanation, it can be assumed that any action of the reaction force of the support should lead to a change in the lengths of the standards and hours. But even in this case, it remains unclear why the Lorentz transformations do not contain any references to the presence of inertial forces. With a "mathematical" explanation of this paradox, they usually stop at the very fact of the presence of asymmetry in frames of reference, considering that the intangible space-time itself "somehow" manages to influence all clocks and rulers, regardless of their device. A number of problems of this kind were discussed in the paper of Brillouin [10].

The "muon effect" is often cited as evidence that scale contractions in a moving frame of reference take place in reality. This effect consists in the fact that decaying muons, being born in the upper layers of the atmosphere, have time to fly to the surface of the Earth. Since their lifetime in a laboratory experiment in a stationary frame of reference is approximately $2 \times 10^{-6}$ ns, even at the speed of light, these particles could not travel to decay a distance greater than 600 m. The distance from the region where these particles are born to the Earth’s surface is tens of kilometers. However, it should be borne in mind that today there is no real theory of the muon decay process. Therefore, it can be assumed that there is a real physical mechanism that is not associated with the Lorentz transformations, which explains the fact of slowing down of muon decay. This is all the more important, since, as already noted, the physical processes in the own frame of reference and the frame of reference associated with the Earth should be the same for a muon.

3. The principle of materiality

Considering the above analysis as a program for the formation of the concept of three-dimensional space as a material object, we introduce a special principle of materiality, which will be the starting point in the creation of a consistent physical theory of fields and particles. The essence of this principle can be summarized as the following statement: Any object that has a physically measurable (detectable with the help of instruments) effect on material bodies must itself be a material object.

For the practice of physical research of observed material objects, this principle does not provide anything particularly new. However, in relation to the concepts of field, space and time,
this principle leads to important consequences in relation to their physical properties. For space and time, this principle has at least two ways of consistent implementation.

The first method is demonstrated by classical mechanics, according to which space and time do not possess any properties of materiality and serve as a formal mathematical model for describing lengths and distances on the one hand, and periods and durations of phenomena on the other. It is important to emphasize here once again that distances in the absolute space of classical mechanics can be measured and compared only to certain material objects. At the same time, it is postulated that material objects are something completely different from immaterial space and time. Only the distance from one material body to another has physical meaning, although formally the entire space is covered by a coordinate grid, which reflects a consistent way of conveniently generalizing the set of measurements of distances between material objects. Therefore, in classical mechanics, the methods for measuring length and time are the same in all space and do not depend on the choice of an inertial frame of reference. At the same time, the limitation of the area of operation of this principle only by inertial reference frames is explained by the possibility of the appearance in non-inertial reference frames of forces that change the length of the rulers and the speed of the clock due to obvious physical mechanisms. In a non-inertial frame of reference, for example, support forces must always arise, forcing the standard to accelerate. Support reaction forces acting on elastic bodies cause their relative lengthening or contraction. These facts were discussed by Brillouin in paper [10]. In contrast to this, in SR, the change in the length of the standards occurs during the transition from one inertial frame of reference to another, although the first postulate of this theory is the sameness of all the laws of physics in these frames of reference. Therefore, the change in the scales of length and time in SRT cannot be associated with any real physical mechanism of interaction of material bodies.

Another way of interpreting space, corresponding to the outlined new theory, is to recognize space as a material object with measurable physical properties. In this case, the points of this space are recognized as material objects, and any change in the distances between these points must be caused and explained by specific physical reasons. As a way of describing the geometric properties of such a material space, a model of embedding of a three-dimensional smooth hypersurface into an enclosing four-dimensional space is proposed $W^4$. This model means that the properties of a three-dimensional material hypersurface differ in a measurable way from the properties of a four-dimensional enclosing space. Since this model includes a new mathematical object - four-dimensional ambient space and separately time, then the same initial dilemma arises with a single description of them. It is necessary to make a choice, either we believe that this space is only a formal way of calculating the lengths and distances in it, or we believe that this space is also material. In the first case, we can rely on our experience in measuring distances in classical mechanics, since the absolute space of this theory is part of the ambient space. In this case, the most natural model for calculating the lengths in this four-dimensional space is the four-dimensional Euclidean space $W^4$, at all points of which the length of the standards remains unchanged. Moreover, in order for such a procedure to be implemented, the presence of material objects in $W^4$ is required. As in classical mechanics, in TTFF distances can be measured only between material objects. Such material objects in TTFF are points of a three-dimensional physical material hypersurface (PMH).

In the second case, it is necessary to indicate the model of materialization of this space, which will determine its geometry as a material object. However, in the absence of any precise information about this space, it is rather difficult to do this. Due to this, the most logical choice is the assumption that the ambient four-dimensional space $W^4$ is an analogue of the absolute space of classical mechanics, but having one more dimensions. This approach was adopted earlier in the discussed new theory of TTFF.

However, in contrast to classical mechanics, in which the very practice of a physical experiment was based on the possibility of decomposing matter into separate material points, in TTFF it
is necessary to restrict ourselves, at least for now, to distances only between the points of the physical material hypersurface itself. This creates certain difficulties in attempts to formulate the theory of ambient space, relying on standard classical techniques. Since we have no idea of how a material hypersurface is arranged in a small one, we can still formally rely on mathematical axioms, according to which a hypersurface in Euclidean space can be represented as a set of mathematical points, which can be formally associated with some physical parameters, for example, density masses, energy density, speed, etc. In this case, the PMH is represented as a continuous and even smooth hypersurface $V^3 \in W^4$ with physical quantities distributed on it. As a result, we get an object consisting of many points, the physical meaning of which is not fully defined, but which can serve as a good model for describing the dynamics of matter. It should be emphasized that these points are not directly related to the material points of classical or quantum mechanics. The objects of these theories are particles, which are treated in the TTFF as extended FMG regions, distinguished in a certain way (6, 2, 3, 4, 5, 8).

Suppose $W^4$ as Euclidean space, a Cartesian coordinate system of the following form can be introduced: $X = (x_1, x_2, x_3, x_4) = (x, u)$, where one of the coordinates is highlighted, in this case $u = x^4$, and orthogonal hyperplane $P^3$ with Cartesian coordinates $x = (x_1, x_2, x_3)$. Then any hypersurface $V^3 \in W^4$ can be mathematically singled out unambiguously using one height function $F(x, t)$ using the equation:

$$u = F(x, t). \quad (1)$$

It is in this form that the physical material hypersurface (PMH) was described in the papers [5, 8]. However, this selection implies that in $W^4$ it is possible not only formally, but also physically, to single out the hyperplane $P^3$. As pointed out in [5, 8], such hyperplane in $W^4$ stands out by us in the form of an intuitive image based primarily on the rectilinear propagation of light in a vacuum. Since light does not pass through dense matter in a straight line or does not pass through individual material objects at all, the idea of the flatness of our world intuitively continues into the area of space occupied by these objects. Thus, the hyperplane $P^3$ - is a set of our intuitive ideas in relation to the PMH about the location of material objects relative to each other, which can be interpreted as the choice of a physical frame of reference.

Following this analysis and the general idea underlying the hypothesis of Clifford and Einstein that matter and material fields, gravitational and electromagnetic, should be explained by the properties of the geometry of some space and time, it is logical to relate them to the geometric and topological properties of the physical hypersurface $V^3$. But now this method should be based not on formal considerations such as choosing the Ricci curvature tensor as the density of the Lagrangian function of the gravitational field of the scalar, but on the very ideology of materiality of the physical hypersurface. Such an ideology, first of all, should reflect the very meaning of defining the materiality of an object as an object that can be observed in a physical experiment.

4. A method for describing the properties of a material hypersurface
The most obvious property of material objects is their observability, i.e. the ability to track them using one or another direct physical measurements. This means that each point of a material object can be associated with a set of physical markers that reflect measurable properties that distinguish one point from others. Such markers in hydrodynamics are called Lagrangian markers and represent the coordinates of each material point at a certain fixed moment in time. It is these Lagrangian markers that are the most general way to describe any moving material medium at subsequent times. In this regard, to describe the distribution of matter in the ambient four-dimensional space $W^4$ it is also necessary to introduce a set of markers $E^a(x, t)$, that uniquely highlight each point of matter located in it at a point with a coordinate $X = (X_1, X_2, X_3, X_4)$ in time moment $t$. As noted, we will assume that $W^4$ is a Euclidean space and therefore the coordinates $X$ can be chosen as Cartesian without loss of generality.
Suppose that a material object has the form of a hypersurface \( V^d \) of dimension \( d \), embedded in the ambient space \( V^D \) of dimension \( D > d \). The physical material hypersurface distinguishes the object under study, and the enveloping space is necessary for introducing the extension property into the theory of the standard, comparing with which we can get an idea of the change in dimensions and distances on the physical hypersurface. Geometric markers \( e^a(\mathbf{x}, t), \ a = 1, \ldots \) will further mean the functions of the coordinates of the ambient space and time, which are associated with each point of the hypersurface \( V^d \subset V^D \) and allow one point to be uniquely distinguished from other points of this hypersurface. By analogy with hydrodynamics, the value of the marker function can be the value of the coordinate of a material point in \( V^D \) at one selected moment in time \( t = 0 \) conduct the measurement \( e^a_0(\mathbf{x}) \). Based on this, the number of geometric markers \( e^a(x_1, x_2, \ldots, x_D), \ a = 1, \ldots, D \) should be equal to \( D \) independent functions. In this case, all markers of points of a material object should be divided into two classes. The first class is global markers, highlighting the physical material hypersurface in the ambient space as a whole. The number of such markers must be equal to \( M = D - d \). Indeed, the material hypersurface of dimension \( d \) in the Euclidean space \( V^D \) at time moment \( t = 0 \) can be specified using a system of \( M \) algebraic equations:

\[
e^a(\mathbf{x}, t) = C_a = \text{const}, \ a = d + 1, \ldots, D.
\]

\( C^a \) is a collection of a certain set of real constants that distinguishes a particular hypersurface among all possible hypersurfaces with other values of global markers. A collection of functions \( e^a(\mathbf{x}, t) \) can be considered as \( M \) markers, which will be called global markers. Their values are determined at the moment \( t = 0 \) using one or another measuring procedure and remain unchanged at any point of the PMH for any changes in its geometry.

The second class of markers should highlight the points of the FMG itself. These markers reflect the movement of points of the FMH with specified physical properties along the PMH itself when its geometry changes. These markers will be called local or geometric markers. This means that local markers must be associated with such transformations of the functional dependence of global markers on \( \mathbf{X} \rightarrow e^a(\mathbf{X}, t) \), that do not change the geometry of the PMH, but only reduce to redesignating its points.

To describe the motion of the hypersurface \( V^d \) with time, it is now sufficient to introduce into the theory the equation of transfer of markers:

\[
\frac{\partial e^a}{\partial t} + U^i(\mathbf{x}, t) \frac{\partial e^a}{\partial x^i} = 0, \ a = 1, \ldots, D; \ i = 1, \ldots, D. \tag{2}
\]

The field \( U(\mathbf{x}, t) \) with components \( U^i(\mathbf{x}, t) \) will be called the marker transfer rate field. Along the integral curves of this field, i.e. integral curves of the system of equations:

\[
\frac{d\mathbf{x}^i}{dt} = U^i(\mathbf{x}, t), \ \frac{de^a}{dt} = 0,
\]

Geometric marker values remain constant. Therefore, the field \( U(\mathbf{x}, t) \) of the marker transfer rate should play a fundamental role in the theory of PMH dynamics, since it is associated with all the main elements of the marker movement and, as a consequence, the entire material hypersurface. In this regard, the equation (2) is a mathematical expression of the second postulate of materiality. Namely: The main mathematical method for describing the dynamics of a material hypersurface is the equation (2), and the field \( U(\mathbf{x}, t) \) of the transfer of markers is the fundamental field for describing the dynamics of a material object \( V^d \).
5. Three-dimensional material hypersurface in $W^4$

In accordance with the main provisions of the TTFF, the physical material hypersurface $V^3$ has dimension $d = 3$ and is embedded in the ambient Euclidean space $W^4$ of dimension $d = 4$: $V^3 \in W^4$. In this case, among the four markers, one of the markers $E^4$, which we will further denote as the function $\phi$, is a global marker, and the other three are local.

Since the function $\phi(X, t)$ is a global marker, the FMG can be distinguished in $W^4$ using one algebraic equation:

$$\phi(X, t) = \phi_0 = \text{const.} \quad (3)$$

However, in TTFF, to describe the geometry of the FMH and the physical properties of material objects that we identify as particles of matter, not the $\phi$ function, but the height function $F(x, t)$, is used, with the help of which the PMH is distinguished in accordance with equation:

$$F(x, t) = u = x^4, \quad x = (x_1, x_2, x_3). \quad (4)$$

here $u$ - the coordinate of $W^4$, corresponding to the orthogonal direction to some distinguished hyperplane $P^3 \subset W^4$, on which the Cartesian coordinates are given $x = (x_1, x_2, x_3)$. It is this function, called the fundamental potential in the TTFF, that plays the most important role in the description of the fundamental fields - electromagnetic and gravitational. In particular, the solution of the problem of a topological-geometric description of the properties of electric charges, electromagnetic and gravitational fields in the TTFF is based on the fact that the function of the height of the PMH $F$ is associated with local markers $e$ on each simple topological cell (2, 3, 4, 5 Appendix) using the equation:

$$F = F_0 + \frac{\varepsilon}{2} |e(x, t)|^2. \quad (5)$$

Here $|e|^2 = R^2(x, t) = (e^1)^2 + (e^2)^2 + (e^3)^2$, and $F_0$ - the function value $F$ at its extremum belonging to a given cell, and $\varepsilon = \pm 1$ depending on whether the extremum is a maximum or a minimum.

Marker transfer equation (2) should, by definition, look like:

$$\frac{\partial e^a}{\partial t} + U^i(X, t) \frac{\partial e^a}{\partial x^i} = 0, \quad a = 1, \ldots, 4; \quad i = 1, \ldots, 4. \quad (6)$$

However, in papers [6, 2, 3, 4, 5, 8] only local markers were considered, the transfer equation of which has the following form:

$$\frac{\partial e^a}{\partial t} + V^\alpha(x, t) \frac{\partial e^a}{\partial e^{\alpha}} = 0, \quad a = 1, \ldots, 3; \quad \alpha = 1, \ldots, 3, \quad (7)$$

where the field $V^\alpha(x, t)$ was specified on the hyperplane $P^3$ and associated with an electromagnetic field. In particular, it was shown that by introducing geometric averaging with density $|J|$, the role of which is played by the Jacobian of the transformation $x = (x^1, x^2, x^3) \rightarrow e = (e^1, e^2, e^3)$, it is possible to construct all the basic equations of classical and quantum physics, as well as the equations of the classical theory of gravity in an extended form. It was also shown [3, 8], that a simple modification of the theory of gravitation in the classical form makes it easy to explain the phenomenon called dark matter in modern physics. In fact, this means that the concept of markers is a very effective tool for solving a number of problems in modern physics. Therefore, we can assume that the equations (6) for local markers should be reduced to the form of equations (7).
The equation for the global marker can now be constructed from the definition of the height function. Since on FMG the global marker takes some constant value, then based on this we can assume that the global marker \(\phi(x, t, u)\) is some function of the following form:

\[
\phi = \Phi(u - F(x, t)).
\]  

(8)

Differentiating this equation with respect to \(x\), \(t\) and \(u\), and then, excluding the derivative of the function \(\Phi'(\xi)\), from the obtained relations, we arrive at the following system of equations for the function \(\phi(x, t, u)\):

\[
\frac{\partial \phi}{\partial x_i} = -\frac{\partial F}{\partial x_i} \frac{\partial \phi}{\partial u}, \quad i = 0, 1, 2, 3. 
\]  

(9)

This system of equations was considered in [14, 7] and describes a special class of waves - rivertos. Convolving this system with respect to the index \(i\) with an arbitrary continuous vector field \(u_i(x, t, u)\), we arrive at the following equation for \(\phi(x, t, u)\):

\[
u^0 \frac{\partial \phi}{\partial t} + u^\alpha \frac{\partial \phi}{\partial x^\alpha} + (u, \nabla F) \frac{\partial \phi}{\partial u} = 0, \quad (u, \nabla F) = u^i F_{i}.\]  

(10)

This equation can be given the following form:

\[
\frac{\partial \phi}{\partial t} + U^\alpha(x, t) \frac{\partial \phi}{\partial x^\alpha} + U^4(x, t) \frac{\partial \phi}{\partial u} = 0. 
\]  

(11)

Here:

\[
U^\alpha = \frac{u^\alpha}{u^0}, \quad U^4 = \frac{(u, \nabla F)}{u^0}. 
\]  

(12)

The field \(U\), defined on \(W^4\), with the components \(U^i\) (12) is the field for transferring the global marker to \(W^4\). It follows from the last relation that the component \(U^4\) of the field \(U\) can be represented as follows:

\[
U^4 = U^\alpha \frac{\partial F}{\partial x^\alpha} + \frac{\partial F}{\partial t}. 
\]

Therefore, this component \(U^4\) of the field \(U\) is determined by the derivatives of the height function \(F\) and the components of the field \(V = (U^1, U^2, U^3, 0)\), tangent to the hyperplanes \(P^3\). The case \(U^4 = 0\), equivalent to the execution of the equation:

\[
\frac{\partial F}{\partial t} + V^\alpha \frac{\partial F}{\partial x^\alpha} = 0, 
\]

defines the transfer equation of the height function and, as a consequence, local markers.

Note that the relations (12) can be slightly changed, proceeding from the fact that the relation (8) can be generalized, assuming:

\[
\phi = \Phi(u - F(x, t, \phi)).
\]  

(13)

This generalization is connected with the fact that, solving the equation (3) with respect to \(u\), we have the relation:

\[
u = F(x, t, \phi),
\]

which is equivalent to (4) for some specific constant value \(\phi(x, t, u) = C = \text{const}\), which is what the PMH selects. With this generalization, the global marker can be a multivalued function of coordinates in \(W^4\), which extends the admissible structure of the PMH.
Differentiating (13) with respect to $x$, $t$ and $u$ and excluding the derivative $\Phi'(\xi)$, we have the same equation (9):

$$\frac{\partial \phi}{\partial x^i} = -A_i \frac{\partial \phi}{\partial u}, \quad A_i = \frac{\partial F(x, t, \phi)}{\partial x^i} \bigg|_{\phi=\text{const}}, \quad i = 0, 1, 2, 3,$$

with the difference that the function $F(x, t, \phi)$ contains a dependence on $\phi$, which is not affected by differentiation with respect to coordinates and time. Thus, all the relations (10),(11) remain valid in this generalized case, and in the equation (12) it is necessary to take into account that the derivatives with respect to $x^i$ on the right-hand side are calculated under the condition $\phi = \text{const}.$

### 6. Equation of geometrodynamics of the second order

The resulting transfer equation for the global marker contains the vector $U$ with its arbitrary distribution on the distinguished hyperplane $\mathcal{P}^3$. This means that the introduction of a global marker does not yet solve the problem of distinguishing among all possible types of FMG dynamics in $\mathcal{W}^4$ the one that corresponds to the real state of affairs. For such a possibility to arise, it is necessary to obtain some consequences of the equations derived in the previous section, which can be related to some inherently clear dynamic FMG models. Such consequences can be second order equations for $\phi$, which can be obtained from equations (9) or (14), and which can be interpreted by comparing with some form of dynamics of classical or mathematical physics.

The calculations will be carried out for the generalized system (14). Instead of the time $t$ we introduce the variable $x^0 = ct$, where $c$ - is a constant that further plays the role of the speed of light. This is useful for comparison with existing classical theories. Let us reduce the equations of the system (14) with the components $g^{ij}$ of the diagonal matrix $\hat{g}$, which has the form:

$$\hat{g} = \text{diag}(1, -1, -1, -1),$$

which imitates the Minkowski space-time metric. As a result, we get the following equation:

$$g^{ij} \frac{\partial \phi}{\partial u^j} = g^{ij} A_j(\phi, x) \frac{\partial \phi}{\partial u^i}, \quad i = 0, 1, 2, 3.$$

Calculating now the divergence from the right and left sides of the last equation, we arrive at the following second-order equation:

$$\nabla \phi = \frac{\partial}{\partial u} \left( |A(\phi, x)|^2 \frac{\partial \phi}{\partial u} \right) + \nabla F \frac{\partial \phi}{\partial u},$$

where

$$\nabla = g^{ij} \frac{\partial^2}{\partial x^i \partial x^j} = \frac{\partial^2}{\partial x^0 \partial x^i} - \sum_{\alpha=1}^{3} \frac{\partial^2}{\partial x^\alpha \partial x^i}$$

- is the d’Alembert operator in the coordinate space-time, the action of which on the function $F(x, t, \phi)$ is carried out under the condition $\phi = \text{const}.$ In addition, the designation has been introduced:

$$|A(x, t, \phi)|^2 = g^{ij} A_i A_j = \left( A_0(\phi, x) \right)^2 - \sum_{\alpha=1}^{3} \left( A_\alpha(\phi, x) \right)^2.$$

Note that from the formal point of view, the choice of the constant nondegenerate matrix $\hat{g}$, over which the convolution is performed in the equation (16), is arbitrary. For example, as a matrix $\hat{g}$,
one could choose any non-degenerate matrix with signature \(((+, +, +, +))\) or \(((+, +, -, -))\). In this case, instead of the standard d’Alembert operator, the equation (16) would have an operator with the corresponding signature of the second derivatives. In particular, for the signature \(((+, +, +, +))\) it would be the Laplace operator in four-dimensional space. This choice obviously corresponds to a different version of the FMG dynamics, which could be realized in some other situation than the one that is realized for our Universe.

Regardless of the choice of the matrix \(g_{ij}\), the equation (16) is the equation of the dynamics of the global marker, which can be compared to some equations of classical physics in order to understand its interpretation from the experimental point of view. First of all, it should be noted that the reduced general form of the equation (16) is a general and natural consequence of the original postulate that the FMG is a three-dimensional hypersurface in \(W^4\). Because of this, the equation contains two functional parameters \(\Lambda = |A|^2\) and \(P = \nabla F\), connected exclusively with the height function \(F\):

\[
\Lambda = |A|^2 = \left(\left(\frac{\partial F(x, t, \phi)}{\partial x^0}\right)^2 - \sum_{\alpha=1}^{3} \left(\frac{\partial F(x, t, \phi)}{\partial x^\alpha}\right)^2\right)|_{\phi=\text{const}},
\]

\[
P = \nabla F = \frac{\partial^2 F}{\partial x^0^2} - \sum_{\alpha=1}^{3} \frac{\partial^2 F}{\partial x^\alpha^2}.
\]

Hence it follows that the geometrodynamics of the FMG is determined by two functions \(\Lambda\) and \(P\), which must be related to the physical quantities that are measured in the experiment. In fact, if \(\Lambda\) and \(P\) are represented in the form of measurable distributions of some physical parameters of matter, then the relations (17) can be considered as equations for the fundamental potential - the height function \(F\). From this point of view, the choice of the matrix \(\hat{g}\) in the form (15) is only a way to bring such equations for \(F\) to a form close to the form of the wave equations of classical physics. The meaning of this choice lies in the assumption that in the limit when the functions \(\Lambda\) and \(P\) are small, which can apparently be interpreted as the absence of matter in space, the processes of change in \(F\) and \(\phi\) should be are similar to the propagation of waves with a certain fixed speed \(c\), which coincides with the speed of light.

7. Interpretation of the equations of geometrodynamics

Note that from the point of view of the measurements that we can make as an element of the FMG, the equations (16) taking into account (17) are not very useful, since we do not have, at least not yet, information about the ambient space \(W^4\). Because of this, the equations (17), which relate to the height function and do not contain dependence on the variable \(u\), are of particular interest. The function \(F(x, t, \phi)\) itself formally depends on the value of \(\phi\) on the PMH. But this dependence is purely parametric and does not actually affect the functional dependence \(F(x, t, \phi)\) with respect to \(x\) and \(t\). Moreover, if the dependence is known \(F(x, t, \phi)\) with respect to \(x\) and \(t\), then the dependence \(\phi(x, t, u)\) will be determined directly from (8), or from the solution of the equation (16) or (17) with additional boundary or initial conditions. Thus, to clarify the dynamics of the PMH, it is necessary to focus on the interpretation of the (17) equations from the point of view of a physical experiment.

The first thing to notice is that the equation:

\[
\nabla P = \nabla (\partial F) = P
\]

has an obvious similarity with the equation of vibrations of a thin three-dimensional membrane in four-dimensional Euclidean space under the action of the "force" \(P\), referred to the unit volume of the PMH, and applied to the membrane along the \(u\) axis, orthogonal to the hyperplane \(P^3 \in W^4\). Indeed, the equation of vibrations of a thin two-dimensional membrane has a similar form, but
with the dimension of the coordinate space less than one. For a two-dimensional membrane, the $P$ function is the ratio of the external pressure applied to the membrane to the density of the membrane material. As a result, we can interpret the equation (18) as the equation of vibrations of an elastic three-dimensional membrane $V^3$ in $W^4$, under the action of some "force" $P$.

Since to calculate the function $F(x, t, \phi)$, one equation (18) is sufficient in the presence of boundary and initial conditions, the second equation of the system (17):

$$\Lambda = \left( \left( \frac{\partial F(x, t, \phi)}{\partial x^0} \right)^2 - \sum_{\alpha=1}^{3} \left( \frac{\partial F(x, t, \phi)}{\partial x^\alpha} \right)^2 \right)_{|\phi=\text{const}}.$$  \hspace{1cm} (19)

must be a consequence of (18). However, there is no simple link between (18) and (19), since there is no simple link between the functions $P$ and $\Lambda$. Such a connection can appear if we use a formal factorization of these equations, similar to the relations (14) for the function $\phi$. This possibility will be discussed later. For the function $\Lambda$ there is a formal interpretation as the Lagrangian of the linear part of the equation (18). However, this interpretation does not make much physical sense yet.

8. **General principles of the dynamics of a physical hypersurface**

Interpreting the equation (18) as the equation of vibrations of a three-dimensional membrane in four-dimensional Euclidean space under the action of the "forces" described by the function $P$ allows us to analyze the meaning and properties of $P$ from the point of view of known facts. The first important fact that must be taken into account is the fulfillment of the law of conservation of energy in our Universe on all scales observed today. In particular, this means that the equation (18) itself should most likely be autonomous. The condition of autonomy guarantees that no external factors interfere with the dynamics of the system, which, as a rule, lead to a violation of the laws of conservation of momentum and energy. The latter means that the force acting on the PMH should depend only on the parameters of the PMH itself and its derivatives:

$$P = P(F, F_{,\alpha}, F_{,\alpha,\beta}, \ldots).$$

In the simplest case, we can assume that the function $P(F, F_{,\alpha}, F_{,\alpha,\beta}, \ldots)$ explicitly depends only on the function $F$ itself. In this case, the equation (18) is the nonlinear Klein-Gordon equation, which occurs in various physical applications. In particular, in the work [16], exact solutions of the nonlinear Klein-Gordon equations in the class of rivertons were found. The solutions obtained in this work are multivalued, which seems to be quite useful for using in the TTFF, in which the multivalued structures of the fundamental hypersurface are associated with baryons [5].

Similar reasoning can be applied to the function $\Lambda$, assuming that the equation (19) must also be autonomous. In this case, the function $\Lambda$ must be a function of only $F$ and its derivatives:

$$\Lambda = \Lambda(F, F_{,\alpha}, F_{,\alpha,\beta}, \ldots).$$

Unfortunately, until the direct connections of the equations (18) and (19) with experimentally measurable quantities have not been established, it is difficult to give any instructions for the functional choice of $P$ as a function of $F$ and its derivatives. However, we can establish some useful properties of this function by considering some limiting situations that should be realized in reality.

As an example of the "simplest" state of pmh, consider the equations for the fundamental potential corresponding to the situation: $P = 0$. In this case, the complete equation (16) turns into an equation of the following form:

$$\Box \phi = \frac{\partial}{\partial u} \left( |A(\phi, x)|^2 \frac{\partial \phi}{\partial u} \right).$$
Accordingly, the equations (18) and (19) take the following form:

\[ \Box F = 0, \]  
\[ \left( \frac{\partial F(x,t,\phi)}{\partial x^0} \right)^2 - \sum_{\alpha=1}^{3} \left( \frac{\partial F(x,t,\phi)}{\partial x^\alpha} \right)^2 \bigg|_{\phi=\text{const}} = \Lambda. \]  

The first equation of this system is the equation of free vibrations of a three-dimensional thin membrane in four-dimensional space. The phase velocity of waves arising due to edge effects is determined by the number \( c \), which is related to the relations (15). This speed in the experiment is obviously equivalent to the speed of light. The function \( \Lambda \) in this case is some equivalent of the energy density of such waves, corresponding to each specific solution for \( F \). In this case, such a situation can be correlated with PMH oscillations in the absence of complex topological structures, which are compared in the TTFF with elementary particles [5].

However, the equality to zero of \( P \) is obviously a very special case of possible situations. Although the set of solutions to the D'Alembert equation (20) contains an extensive subset of multivalued solutions ([13, 14]) - rivertones, nevertheless, solutions from this set alone can hardly give something similar Wheeler's handles [5]. However, in order for such a possibility to appear, there is an extended version of using the results of [13, 14], described in the work [16]. In this last work, using the theory of rivertons, multivalued solutions of first-order quasilinear equations, an extensive set of multivalued solutions of multidimensional nonlinear Klein-Gordon equations of general form was constructed. Those equations of the form:

\[ \Box F = F(F), \]

where \( F(F) \) - the quite function general.

9. Rivertons and multivalued solutions of equations of geometrodynamics

The idea of constructing solutions of multidimensional nonlinear Klein-Gordon equations using rivertons is reduced to the following. By rivertons we mean solutions of a system of quasilinear equations of the following form:

\[ \frac{\partial \psi}{\partial x^\alpha} = B_\alpha(\psi) \frac{\partial \psi}{\partial t}, \quad \alpha = 1, \ldots, n. \]  

The general solution to this system [13, 14] can be written as follows:

\[ H(\psi, t + U(B, x)) = 0, \]  

where:

\[ U(B, x) = \sum_{\alpha=1}^{n} B_\alpha(\psi)x^\alpha. \]

Simple differential consequence (22) is the equation:

\[ \Delta \psi = \frac{\partial}{\partial t} \left( |B|^2 \psi_t \right). \]  

Since the components of the vector field \( B(\psi) \) are arbitrary functions of \( \psi \), they can be chosen in such a way that the condition is satisfied:

\[ |B|^2 = c^{-2}, \]  

where \( c \) is the speed of light.
where $c$ - constant from relations (16). As a result, the equation (24) becomes the equation (20). Note that in this case the equation (19) with $\Lambda = 0$ will be automatically satisfied for the functions $\psi$. This follows directly from the equations (22) and the choice of the functions $B(\psi)$ in accordance with (25).

In order to obtain in a similar way the solutions of equations of the (18) type, it is necessary to consider functions of the following general form as $\mathcal{F}$:

$$
\mathcal{F} = W(\psi, \psi_t, \psi_{tt}, \ldots),
$$

(26)

where $\psi$ - riverton, and $W$ - some differentiable function of its arguments, the form of which will be determined by the functional form of the function $P$ as a function of $\mathcal{F}$ and its derivatives.

For simplicity, consider the case $\mathcal{F} = W(\psi, \psi_t)$. Let us introduce additional notation $u = \psi_t$, $v = \psi_{tt}$. Differentiate (26) with respect to independent variables $x^\alpha$ and $t$. As a result, we find:

$$
\mathcal{F}_{,\alpha} = W\psi\psi_{,\alpha} + W_u\psi_t\psi_{,\alpha} = W\psi B_\alpha u + W_u \frac{\partial}{\partial t}(B_\alpha u), \\
\mathcal{F}_t = W\psi u + W_u u_t.
$$

using the last relation, we find:

$$
\mathcal{F}_{,\alpha} = B'_\alpha F(\psi, u) + B_\alpha \mathcal{F}_t.
$$

Where:

$$
F(\psi, u) = u^2 \frac{\partial W}{\partial u};
$$

From the last relation, after some transformations ([16]) follows:

$$
\Delta \mathcal{F} - R\mathcal{F}_{tt} = R'' u F + \Omega^2 (u^2 F_{,u} - 2u F) + R' \left( u F_{,\psi} + F_{,u} u_t + u \mathcal{F}_t \right).
$$

(27)

The notation is introduced here:

$$
R = \sum_{\alpha=1}^{n} B^2_\alpha(\psi), \quad \Omega^2 = \sum_{\alpha=1}^{n} B'^2_\alpha(\psi), \quad B'_\alpha = \frac{d}{d\psi} B_\alpha.
$$

In the case of a special choice $R = \text{const} = c^{-2}$ the equation (27) turns into the equation:

$$
\Delta \mathcal{F} - R\mathcal{F}_{tt} = \Omega^2 \left( u^2 F_{,u} - 2u F \right),
$$

which turns into the Klein-Gordon equation if the function $W(\psi, u)$ satisfies the equation:

$$
u^2 F_{,u} - 2u F = u^2 \frac{\partial}{\partial u} \left( u^2 \frac{\partial W}{\partial u} \right) - 2u^2 \frac{\partial W}{\partial u} = u^3 \frac{\partial^2 W}{\partial u^2} = \frac{1}{\Omega^2(\psi)} G(W).
$$

(28)

If $W(\psi, u)$ is the solution of this simple differential equation, then the function $\mathcal{F}(x, t) = W(\psi(x, t), \psi_t(x, t))$, where $\psi(x, t)$ - is one of rivertons, is a solution to the nonlinear Klein-Gordon equation:

$$
\Delta \mathcal{F} - c^{-2} \mathcal{F}_{tt} = G(\mathcal{F})
$$

for an arbitrary choice of the function $G(W)$. This equation is standalone and can serve as a variant of the (18) equation. Since the choice of the function $G(W)$ is not predetermined in advance, it is possible to select the properties of feasible solutions based on the choice of the
function $G(W)$. Note that in [16] more general options for constructing solutions of equations of the Klein-Gordon type and nonlinear telegraph equations based on the general dependence (26) were described. Although this approach has not been studied well enough from the point of view of constructing a general model of FMG dynamics based on the equation (18), nevertheless, in the absence of direct connections between the FMG geometry parameters and ordinary experimental data, this approach seems to be quite natural for solving the problems posed. However, solving such problems is beyond the scope of this article.

10. Equations of the dynamics of fundamental fields and local markers

The previous constructions are a natural consequence of the principle of materiality and its embodiment in the form of dynamics of PMH point markers. As shown in the papers [2, 3, 4, 5], all the basic equations of the dynamics of particles and physical fields, including electrodynamics, the theory of gravity and mechanics in the form of a modified Newtonian theory and quantum mechanics, follow from the equations of the dynamics of local markers. To clarify the role played by the global marker equation in the TTFF, it is useful to consider the basic principles of deriving the equations of modern physical theories from the set of marker transfer equations (7). Using the conclusions [2, 3, 4, 5], we will show that all these equations are, in fact, a consequence of only the equations (7) in some of their natural interpretation.

The first step is to derive the differential density conservation equation

$$J = \frac{1}{\mu} \int f(x) J dV, \quad \mu = \int J I(|e|) dV,$$

where $\mu$ is a certain dimensional factor. The function $I(|e|)$ is an invariant associated with the metric of the marker space, which determines the deviations from Newton’s law of gravitation of the "dark matter" type. See [2, 3, 4, 5, 8]. The presence of the conserved density $J$ allows one to introduce into the theory geometric averaging for any function $f(x)$ according to the rule:

$$\bar{f}(t) = \frac{1}{\mu} \int f(x) J dV, \quad \mu = \int J I(|e|) dV.$$

The velocities and accelerations associated with the average coordinates are as follows:

$$U^\alpha = \frac{dX^\alpha}{dt} = \int V^\alpha J dV,$$

$$A^\alpha = \frac{d^2 X^\alpha}{dt^2} = \int \left( \frac{\partial}{\partial t} V^\alpha + V^\beta \frac{\partial}{\partial x^\beta} V^\alpha \right) J dV.$$

where $\mu$ is a certain dimensional factor.
If we formally represent the transfer field of markers \( V \) as a set of vortex \( \gamma_0A \) and potential \( \nabla \chi \) parts:
\[
V = -\gamma_0A + \nabla \chi,
\]
where \( \gamma_0 \) is a dimensional constant, then the last equation in (33) can be considered as Newton’s equation of the averaged motion of a charged particle in an averaged electromagnetic field with a vector potential \( A \) [2]:
\[
d^2X^\alpha dt^2 = \gamma_0E - \gamma_0(V \times B) - \nabla X^\alpha U + F_q. \tag{34}
\]
Here \( E \) and \( B \) is average strength of electric and induction of magnetic fields:
\[
E = -\frac{1}{c} \frac{\partial A^\alpha}{\partial t} - \frac{\partial \Phi}{\partial X^\alpha}, \quad B = \text{rot}_X A,
\]
\( F_q \) is a quantum addition to the forces acting on a particle and \( U(X, t) \) is an additional potential:
\[
U(X, t) = -\frac{1}{2} |V|^2 - \chi_t - \gamma_0c\Phi, \tag{35}
\]

\( \Psi \) presumably playing the role of averaged gravitational forces ( [2]). For a given potential \( U(x, t) \), the ratio (35) can be considered as the Jacobi equations:
\[
\chi_t + \frac{1}{2} \left( |\nabla \chi|^2 + 2\gamma_0(A, \nabla \chi) + \gamma_0|A|^2 \right) + \gamma_0c\Phi + U = 0. \tag{36}
\]

for function \( \chi \), which in this case is the action function for a particle in an electromagnetic field with potentials \( \Phi \) and \( A \) and in a scalar field with potential \( U \). This also implies that the function
\[
\Psi = \sqrt{|J|} e^{i\chi/\hbar}, \tag{37}
\]
where \( \hbar \) - Planck’s constant, satisfies the Schrödinger equation
\[
\imath\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2} \left( -\imath\hbar \nabla - \gamma_0A \right)^2 \Psi + \left( \gamma_0\Phi - U_G \right) \Psi, \tag{38}
\]
where \( \Phi \) is the potential of the electric field and
\[
U_G = U - \frac{\hbar^2}{2} \frac{\Delta |J|}{|J|}. \tag{39}
\]

Thus, averaging (31) automatically leads to the equations of quantum theory and classical mechanics in the form of averaged equations of motion with a geometric interpretation of the wave function (37).

The field part of the theory is based on identities connected by local markers \( e^a(x) \). The basic principle, according to which equations of fundamental fields arise in TTFF, is that all fundamental fields are functions of the markers themselves and their derivatives. Let \( e^a(x) \) be markers as functions of coordinates \( x^\alpha, \alpha = 1, 2, 3 \) on the hyperplane \( P^3 \in \mathcal{M}^4 \) distinguished in the ambient space and time \( t \). The set of values \( \psi \) of the functions \( e^a(x) \) can be considered as some coordinate space \( E^3 \) with the metric induced by the mapping: \( P^3 \rightarrow E^3 \). This space \( E^3 \) will be called the marker space. Two obvious differential identities hold in the marker space:
\[
\frac{\partial e^a}{\partial e^a} = 3, \quad \frac{\partial}{\partial e^a} \left( \frac{e^a}{|e|^3} \right) = 4\pi \delta(e), \tag{40}
\]
where \( e = (e_1, e_2, e_3) \) - the radius vector on a Cartesian marker space map, \(|e|^2 = (e_1)^2 + (e_2)^2 + (e_3)^2 \) and \( \delta(e) \) - Dirac delta function with a carrier at the origin of the Cartesian map \( e^3 \). After converting to coordinates on the identities (40) turn in to the following equations:

\[
\frac{\partial}{\partial x^\alpha} g^\alpha = 4\pi G \zeta(|e|) \rho_m, \quad \frac{\partial}{\partial x^\alpha} D^\alpha = 4\pi \rho_e, \tag{41}
\]

where the following notation is introduced:

\[
g^\alpha = \frac{4\pi m_0 G}{3} I(|e|) |e| e^a \frac{\partial x^\alpha}{\partial e^a}, \quad D^\alpha = \frac{|J|}{|e|^3} e^a \frac{\partial x^\alpha}{\partial e^a},
\]

\[
\rho_e = \sum_{k=1}^N \varepsilon_k \delta(x - x_k), \quad \zeta(R) = 1 + \frac{e^a}{3} \frac{\partial \ln I(e)}{\partial e^a}.
\]

The factor \( G \) - Newton’s constant of gravity, function \( \rho_m \) is mass density given by the ratio (30), \( \rho_e \) - electric charge density, which is the density of point charges associated with critical points of the fundamental potential \( F \). See papers [6, 2, 3, 4, 5, 8]. As a result, the identities (41) take the standard form of the Poisson equations for the gravitational field strength \( g \) (with components \( g^\alpha \)) and Maxwell’s first equation for the electric induction field \( D \) (with components \( D^\alpha \)). Another pair of equations is obtained directly from (7) using some transformations, the details of which can be found in [4, 8]. This pair of equations can be represented in this form:

\[
\frac{\partial D}{\partial t} = -\text{rot} \left( D \times V \right) - 4\pi \rho_e V, \tag{42}
\]

\[
\frac{\partial g}{\partial t} = -\text{rot} \left( g \times V \right) - 4\pi G \zeta(R) \rho_m V. \tag{43}
\]

Here \( V \) - markers transfer field. The last pair of equations are analogs of the equations of induction of electromagnetic and gravitational fields. The other two Maxwell equations actually relate the magnetic and electric fields and require the absence of magnetic charges. In this theory, the magnetic field strength looks like this:

\[
H = -\frac{1}{c} \left( D \times V \right) + \nabla \Phi_H, \tag{44}
\]

where \( \Phi_H \) - an initially arbitrary potential that leaves the equation (42) unchanged, calculated from the equation of the absence of magnetic charges:

\[
\text{div} H = 0.
\]

The equation (43) is the equation of induction of the gravitational field, which is similar to the equation of induction of electromagnetic fields, which is an extension of Newton’s theory of gravitation, which consists in introducing the gravitational field strength and, as a consequence, the gravimagnetic field into the theory of vortex fields \( Z = [g \times V] + \nabla \Phi_g \) with a potential that also, by analogy with (44), find from the equation of the absence of gravimagnetic charges:

\[
\text{div} Z = 0.
\]

The entire set of the presented equations forms a complete system of equations for describing the fundamental electromagnetic and gravitational fields, which in TTFF turn out to be closely related, since the field strengths \( D \) and \( g \) differ only in scalar factors from each other and contain in their notation the fundamental field \( K \):

\[
K = e^a \frac{\partial x^\alpha}{\partial e^a}, \quad D = \frac{|J|}{|e|^3} K, \quad g = \frac{4\pi m_0 G}{3} I(|e|) |J| K. \tag{45}
\]
Thus, from the system (7) all the basic equations of the modern physical picture of the world follow. In TTFF, this picture is supplemented by a topological interpretation of the electric charge, which provides an explanation of the charge properties of elementary particles, including electric and baryon charges (2, 5). There is one flaw in this scheme, which is that for the function $R = |e|$, which plays an important role in the theory, there is no equation describing the variation of this function with time. However, since $R$, according to (5), is associated with the fundamental potential $F$, the missing equation is the equation for $F$, the role of which is played by the equation (18) with some interpretation of the function "external pressure".

11. Conclusion

The General Theory of Relativity paved the way for a completely new approach in physics - the description of the properties of matter and fields, based on the properties of non-Euclidean space-time geometry. In fact, thanks to general relativity, concepts such as the theory of the Grand Unification or the theory of "everything" appeared in physics. However, in the process of implementing such unifying ideas, a number of problems of the general concept of general relativity emerged, which served as the basis for many attempts to construct its generalizations that would solve at least some of these problems. From a mathematical point of view, the whole concept of general relativity looks quite adequate. Nevertheless, problems, for example, the energy of the gravitational field, deprive general relativity of completeness and do not provide an opportunity to adequately relate to quantum theory. In this work, it was demonstrated that the main disadvantage of the path chosen in SRT and GRT to the geometrization of physics is the endowment of non-material objects with physical properties, in particular, Einstein’s space-time itself. Proceeding from this, the problem of energy seems to be natural and requires a solution for at least some real theory of matter.

The main conclusion of this work can be an indication that in order to construct a theory that would correctly include electromagnetic and gravitational fields, quantum particles with electric and baryon charges, and even "dark matter", it is necessary to initially involve the principle of materiality of space. This principle excludes from the theory non-material objects that can be implicitly endowed with physical properties, which leads to various paradoxes with the outwardly consistent of the theory from a mathematical point of view.

In addition to formulating the principle of materiality of space, the paper proposes a method for its general implementation, which would make it possible to initially formulate the geometric theory in terms of material particles and objects. This approach is closely related to the TTFF [6, 2, 3, 4, 5, 8], which provides a deeper justification for this theory. In this work, the TTFF problem of deriving the FMG dynamics equation was formally solved, relying on general ideas about fundamental and geometric markers. Nevertheless, the theory cannot be considered complete, since it lacks ideas for a general description of the $P$ function in the geometrodynamics equation (18) based on experimental data. The problem is that there are no such experimental data at this time. The paper proposes a way to derive the general form $P$, relying on the selection of this function, proceeding from the analysis of solutions to this equation in the class of rivertones. However, such an analysis is beyond the scope of this work.

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