Inflation is well known to be difficult in the context of supergravity, if the potential is dominated by the $F$ term. Non-renormalizable terms generically give $|V''| \sim V/M^2$, where $V(\phi)$ is the inflaton potential and $M$ is the scale above which the effective field theory under consideration is supposed to break down. This is equivalent to $|\eta| \sim (M_{Pl}/M)^2 > 1$ where $M_{Pl} = (8\pi G)^{-1/2}$, but inflation requires $|\eta| < 0.1$. I here point out that all of the above applies also if the $D$ term dominates, with the crucial difference that the generic result is now easily avoided by imposing a discrete symmetry. I also point out that if extra spacetime dimensions appear well below the Planck scale, as in a recent M-theory model, one expects $M \ll M_{Pl}$, which makes the problem worse than if $M \sim M_{Pl}$. 
1. To achieve slow-roll inflation, the potential \( V(\phi) \) must satisfy the flatness conditions \( \epsilon \ll 1 \) and \( |\eta| \ll 1 \), where

\[
\epsilon \equiv \frac{1}{2} M_{Pl}^2 (V'/V)^2 \\
\eta \equiv M_{Pl}^2 V''/V
\]  

and \( M_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck mass. When these are satisfied, the time dependence of the inflaton \( \phi \) is generally given by the slow-roll expression \( 3H\dot{\phi} = -V' \), where \( H \simeq \sqrt{1/3} M_{Pl}^{-2} V \) is the Hubble parameter during inflation. On a given scale, the spectrum of the primordial curvature perturbation, thought to be the origin of structure in the Universe, is given by

\[
\delta_H^2(k) = \frac{1}{150 \pi^2 M_{Pl}^4} V \epsilon
\]  

The right hand side is evaluated when the relevant scale \( k \) leaves the horizon. On large scales, the COBE observation of the cmb anisotropy corresponds to

\[
V^{1/4}/\epsilon^{1/4} = .027 M_{Pl} = 6.7 \times 10^{16} \text{ GeV}
\]  

The spectral index of the primordial curvature perturbation is given by

\[
n - 1 = 2\eta - 6\epsilon
\]  

A perfectly scale-independent spectrum would correspond to \( n = 1 \), and observation already demands \( |n - 1| < 0.2 \). Thus \( \epsilon \) and \( \eta \) have to be \( \lesssim 0.1 \) (barring a cancellation) and this constraint will get tighter if future observations move \( n \) closer to 1. Many models of inflation predict that this will be the case, some giving a value of \( n \) completely indistinguishable from 1.

Usually, \( \phi \) is supposed to be charged under at least a \( Z_2 \) symmetry \( \phi \rightarrow -\phi \), which is unbroken during inflation. Then \( V' = 0 \) at the origin, and inflation typically takes place near the origin. As a result \( \epsilon \) negligible compared with \( \eta \), and \( n - 1 = 2\eta \equiv 2M_{Pl}^2 V''/V \). We assume that this is the case in what follows. If it is not, the nonrenormalizable terms generically give both \( |\eta| \sim 1 \) and \( \epsilon \sim 1 \) at a generic point in field space, making model-building even more tricky.

2. In supergravity, the tree-level potential is the sum of an \( F \)-term and a \( D \)-term \[2\],

\[
V = V_F + V_D
\]  

Supergravity is a non-renormalizable field theory, whose lagrangian presumably contains an infinite number of non-renormalizable terms. Taking the theory to be an effective one, holding up to some scale \( M \), the coefficients of the non-renormalizable terms are expected to be generically of order 1, in units of \( M \). Hopefully, they can be calculated if one understands what goes on at scales above \( M \). The most optimistic view is to take \( M \) to be the scale above
which field theory itself breaks down, due to gravitational effects such as the quantum fluctuation in the spacetime metric. It is not unreasonable to take this view in the context of inflation, if the inflaton is a gauge singlet, and we adopt it here. With four spacetime dimensions, this means that $M = M_{\text{Pl}}$. If more dimensions open up well below the Planck scale, the answer is less obvious (see below).

The non-renormalizable terms respect the gauge symmetries possessed by the renormalizable theory, but they need not respect its global symmetries. On the contrary, it is generally felt that the usual continuous global symmetries (built out of $U(1)$’s acting on the phases of the complex fields) will not generically be respected, at least if $M$ is the scale at which gravitational effects spoil field theory. This viewpoint derives in part from superstring theory, but that theory also suggests that discrete subgroups of the global symmetries (built out of $Z_N$’s) will occur. By imposing suitable discrete symmetries, one can ensure that a global continuous symmetry is approximately preserved at field values $\ll M$ [3]. This is important, because several cases are known where an approximate global $U(1)$ is phenomenological desirable. The best known case is Peccei-Quinn symmetry, which must be preserved to high accuracy in order to keep the axion sufficiently light.

Some time ago [4], it was emphasized that non-renormalizable terms will contribute to $V_F$, generically giving $|V_F''| \sim V_F/M^2$ [3]. Most models of inflation have the $F$-term dominating ($F$-term inflation) and then the non-renormalizable terms in $V_F$ give

$$|\eta| \sim M_{\text{Pl}}^2/M^2$$

This generic result is too big, since slow-roll inflation per se requires $|\eta| \ll 1$, and the observational bound on $n$ requires $|\eta| < 0.1$.

A little later [7] it was pointed out that the problem may disappear if instead $V_D$ dominates ($D$-term inflation). It was also pointed out that $D$-term hybrid inflation occurs quite naturally, if the lagrangian contains a Fayet-Illiopoulos term. (Within the single-field paradigm, $D$-term inflation is essentially impossible, because the inflaton then has to be charged under the relevant $U(1)$ and the gauge couplings spoil inflation unless they are unreasonably small [S].) Several authors [8–15] have since studied models of $D$-term inflation, always under the same assumption that non-renormalizable terms can be ignored in that case.

Here I point out that non-renormalizable terms also contribute to $V_D$, through the gauge kinetic function. They generically give $|\eta| \sim (M_{\text{Pl}}/M)^2$ in that case also, but in contrast with the case of the $F$ term this generic result is easy to evade by imposing a discrete symmetry. The reason is that the gauge kinetic function appearing in the $D$ term is holomorphic, in contrast with the Kahler potential which appears in the $F$ term.

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1 For fields charged under the Standard Model gauge interactions one should use $M = M_{\text{GUT}} \simeq 10^{16}$ GeV in the low energy theory, and for hidden/secluded sector fields below the condensation scale $\Lambda$ one should use $M = \Lambda$.

2 The result holds actually for the second derivative in any field direction, as was noted a long time ago [3].
I consider the usual model of $D$-term hybrid inflation \[7,9–12,15\]. There is just one non-vanishing $D$ field, which contains a Fayet-Illiopoulos term with coefficient $\xi$. The fields charged under the relevant local $U(1)$ have been driven to zero (or anyhow sufficiently small values) along with $V_F$. Then

\[ V \simeq V_D \simeq \frac{\xi^2 g^2}{2} \text{Re} f^{-1} \]  

(8)

where $g$ is the gauge coupling of the relevant $U(1)$, and the gauge kinetic function $f$ is a holomorphic function of all of the complex scalar fields $\phi_n$. If we regard $g^2$ as fixed, $f$ must be invariant under all internal symmetries.

At a given point in field space, one can choose $f = 1$ corresponding to canonical normalization of the gauge field of the relevant $U(1)$. This point is conveniently taken to be the origin, defined as the fixed point of the usual internal symmetries. Then, along say the $\phi_1$ direction,

\[ \frac{1}{f} = 1 + \lambda M^{-2} \phi_1^2 + \cdots \]  

(9)

There is no linear term, unless $\phi_1$ is a singlet under all of the internal symmetries that are unbroken during inflation. The quadratic term is allowed if the only symmetry is $\phi_1 \to -\phi_1$. Then, if inflaton is $\phi = \sqrt{2} \text{Re} \phi_1$, it gives a contribution $\eta = \lambda (M_{\text{Pl}}/M)^2$, with $\lambda$ generically of order 1.

The offending term is forbidden if there is a $Z_N$ symmetry ($\phi_1 \to \exp(i\alpha)\phi_1$ with $\alpha = 2\pi/N$) with $N \geq 3$. It is also forbidden if there is a global $U(1)$ symmetry, corresponding to arbitrary $\alpha$. In all of the models of $D$-term inflation proposed so far, such a global $U(1)$ is present as an $R$ symmetry, with $W \propto \phi_1$. In the simplest case,

\[ W = c\phi_1\phi_2\phi_3 \]  

(10)

where $\phi_2$ and $\phi_3$ oppositely charged under the relevant gauge $U(1)$. One indeed needs to forbid terms in $W$ of the form $\phi_1^n (n \neq 1)$ because they would generate a quartic term in the inflaton potential and spoil inflation. But to achieve this it is enough to have the symmetry $\phi_1 \to -\phi_1$ (acting on $W$ as an $R$ symmetry). I am here pointing out that this symmetry is not enough to forbid the disastrous quadratic term in $f$; it has to be promoted to $Z_{2N}$ with $N > 1$, or to the full $U(1)$. One hopes that the superstring will ultimately determine which discrete symmetry actually holds, if any.

In one model of $D$-term inflation [15], the global $U(1)$ is the Peccei-Quinn symmetry, whose non-perturbative breaking ensures the CP invariance of the strong interaction. It has to be preserved to high accuracy in order to keep the axion sufficiently light, and it appears that discrete symmetries of the model ensure this\[3\]. Of course, this means that the quadratic term in $f$ is killed. This model has the virtue of making contact with both Peccei-Quinn symmetry and the Standard Model. On the negative side, it so far lacks a definite origin for the Fayet-Iliopoulos

\[3\]In an earlier version of this paper I stated that they will not do so.
term: in contrast with the other models of $D$-term inflation, a direct origin in the superstring seems impossible since $V^{1/4}$ is extremely low.

3 Let us briefly recall the situation for the $F$ term [4]. At least at tree level,

$$V_F = e^{K/M_{Pl}^2} \left[ \sum_{nm} (W_n + M_{Pl}^{-2} W K_n) K^{nm} \left( \bar{W}_m + M_{Pl}^{-2} \bar{W} K_m \right) - 3M_{Pl}^{-2}|W|^2 \right]$$  \hspace{1cm} \text{vf} \tag{11}

Here, $W$ is the superpotential (holomorphic in the fields) and $K$ is the Kahler potential (a non-singular function of the fields and their complex conjugates). A subscript $n$ denotes $\partial/\partial \phi_n$, and a subscript $\bar{m}$ denotes $\partial/\partial \bar{\phi}_m$. Also, $K^{nm}$ is the matrix inverse of $K_{nm}$. Only the combination $G \equiv K + \ln |W|^2$ is physically significant.

Canonically normalizing the fields at (say) the origin corresponds to

$$K = \sum_n |\phi_n|^2 + O(\phi_n^3)$$  \hspace{1cm} \text{(12)}

(Any linear term can be absorbed into $W$.) Then,

$$K^{nm} = \delta_{nm} + M^{-2} \sum_n \lambda_n |\phi_n|^2 + \cdots$$  \hspace{1cm} \text{(13)}

In the last expression, the terms not displayed are linear and higher; the quadratic term displayed is a particularly simple one, which cannot be forbidden by any of the usual symmetries.

Now make again the assumption that $\phi = \text{Re} \phi_1/\sqrt{2}$; as we remark in a moment, the opposite assumption that $\phi$ corresponds instead to the phase of $\phi_1$ would give something dramatically different. For simplicity, also set $e^K = 1$ corresponding to small field values. Then, one can identify some contributions to $V''$,

$$V'' = M_{Pl}^2 \left[ V - |W_1|^2 \right] + M^{-2} \left[ \sum_n \lambda_n |W_n|^2 + \cdots \right]$$  \hspace{1cm} \text{(14)}

In the first bracket, $V$ comes from differentiating $e^K$, and the coefficient of $-|W_1|^2$ is the sum of +2 coming from the first term in the bracket of Eq. (11), and $-3$ coming from the $-3M_{Pl}^{-2}|W|^2$. The three terms displayed will give a contribution

$$\eta = 1 - a + b (M_{Pl}/M)^2$$  \hspace{1cm} \text{(15)}

with generically $|b| \sim 1$.

It was pointed out in [4] if $W = \Lambda^2 \phi_1$ during inflation, then $a = 1$. (It is easy to construct models of inflation where this is exactly [4,16] or approximately [17] true.) In that case, one need only require $|b| \ll 1$, corresponding to an accidental suppression of the other non-renormalizable contributions. Some authors [17,18] have taken the view that this is an improvement on the generic situation, where one needs in addition the accident $|a - 1| \ll 1$.

Notice that Eq. (11) contains $M_{Pl}$, as distinct from the scale $M$ above which field theory is supposed to break down. Taking $M_{Pl}$ to infinity with $M$ fixed converts supergravity into a non-renormalizable globally supersymmetric theory. More usually, one considers the limit where $M_{Pl}$ and $M$ go to infinity together, corresponding to a renormalizable...
globally supersymmetric theory. It is clear from the form of Eqs. (4) and (15) that, during inflation, neither of these prescriptions should be used without justification. As we have just seen though, the use of non-renormalizable global supersymmetry can be justified if the superpotential is linear.

If \( b \sim 1 \) (the generic case) the only way of evading the result \(|\eta| \sim (M_{Pl}/M)^2\) is to suppose that \( \phi \) is the pseudo-Goldstone boson of a global symmetry \( [20] \) acting on (say) \( \phi_1 \). Then all \(|\phi_n|\) are fixed during inflation, and if the symmetry is broken only by \( W \) and not by \( K \), the non-renormalizable terms in the latter need cause no problem \( [4] \), though one still has the problem of understanding why a global continuous symmetry is respected by non-renormalizable terms. A more promising avenue is to suppose that \( K \) and \( W \) have very special forms \( [7,21,22] \), corresponding roughly to versions of no-scale supergravity. In this case, it may be justified to use the limit of renormalizable global supersymmetry.

4. Finally, I note that the scale \( M \) might not be as big as \( M_{Pl} \) if additional space dimensions open up below \( M_{Pl} \). I have in mind a specific example, where the underlying theory is an M theory \( [23] \). A field theory is valid below some scale \( Q < M_{Pl} \), and it is attractive to choose \( Q \sim 10^{-2}M_{Pl} \sim 2 \times 10^{16} \text{ GeV} \) to account for the apparent unification of the gauge couplings. This field theory lives in effectively five spacetime dimensions, until we get down to some still lower scale. If the fifth dimension makes no significant difference, the scale \( M \) of the non-renormalizable terms will be \( M = Q \sim 10^{-2}M_{Pl} \). Otherwise \( M \) will presumably be lower, though with two mass scales in the underlying theory the concept of a single scale \( M \) may not even be useful. As yet nothing has been worked regarding these questions, even in the specific example mentioned.

If \( M \) is indeed of order \( 10^{16} \text{ GeV} \), a field-theory model of inflation will not make sense \( [24] \) if \( V^{1/4} \sim 10^{16.5} \text{ GeV} \), the maximum allowed by the COBE normalization Eq. (4). A four-dimensional field theory model will not make sense unless \( V^{1/4} \) is even lower. However, many models have been proposed with low \( V^{1/4} \).

Of these, the simplest is the \( D \)-term model, with the slope of the potential coming from the 1-loop correction \( [9,10] \). In that case COBE normalization requires \( [11,12] \)

\[
\frac{V^{1/4}}{(g^2/2)^{1/4}} = \sqrt{\xi} = 3 \times 10^{15} \text{ GeV}
\]

This might be low enough to justify the use of four-dimensional field theory. Also, if \( \xi \) comes the superstring, it will be a loop suppression factor times \( M^2 \), which might be compatible with the M theory model just mentioned. Again, this has yet to be investigated.

The most promising \( F \)-term model is probably that of \( [25] \). Starting at the scale \( \phi = M \) with the generic \( V''(\phi) \sim (M_{Pl}/M)^2V \), renormalization group equations are invoked to run this quantity to zero at a scale \( \phi \ll M \), where inflation occurs. In this case one can have \( V^{1/4} \) as low as \( \sqrt{Mm_s} \) where \( m_s = 100 \text{ GeV} \), amply justifying the use of four-dimensional field theory. As given, this model takes \( M = M_{Pl} \), and it is unclear whether the change \( M \sim M_{Pl}/100 \) would make an important difference.
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