Static chaos in spin glasses against quenched disorder perturbations

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Abstract

We study the chaotic nature of spin glasses against perturbations of the realization of the quenched disorder. This type of perturbation modifies the energy landscape of the system without adding extensive energy. We exactly solve the mean-field case, which displays a very similar chaos to that observed under magnetic field perturbations, and discuss the possible extension of these results to the case of short-ranged models. It appears that dimension four plays the role of a specific critical dimension where mean-field theory is valid. We present numerical simulation results which support our main conclusions.
1 Introduction

A long debated problem in spin glass theory concerns the correct description of the statics of the low temperature phase[1]. There is wide consensus on the fact that the mean-field theory is well understood in its essentials, while the nature of the equilibrium states for short-ranged models is still a controversial subject. Two competing pictures or approaches have been proposed: the mean-field picture and the droplet model. The mean-field theory has revealed enormously complex as a comprehensive approach to understand short range models. Consequently, the search for different approaches like droplet models [2] has been encouraged. These models, being phenomenological, try to capture the main aspects underlying the equilibrium and non-equilibrium properties of short-ranged models. Unfortunately, the mean-field way and these phenomenological approaches are far from being complementary and much effort has been devoted during the past years to discern what is the correct picture. Numerical simulations have played a prominent role in this task even though the main question still remains unsolved. The main problem relies on the large amount of computer time needed in order to reach the equilibrium.

Despite of the fact that both pictures are in fact contradictory in their essentials, there are however some common predictions in both approaches. Since it is very difficult to decide what is the correct picture, the strategy of searching for common features in both pictures can be useful to shed light on this controversy. Static chaos appears as a good starting point for this program. By static chaos we understand the sensitivity of the low temperature phase of spin glasses against static perturbations, like changes in the temperature or changes in the magnetic field. Mean-field theory [3] and droplet models [2] predict that spin-glasses, in the most general case, are chaotic. In mean-field theory, the mechanism of chaos is due to the small free energy differences between the different equilibrium states. These are of order $O(1/N)$ and a small perturbation completely reshuffles the Boltzmann weights $w_\alpha \sim \exp(-N\beta f_\alpha)$ of the different equilibrium states ($\alpha$ and $f_\alpha$ stand for equilibrium state and its free energy respectively). In droplet models, the application of a perturbation causes a reorganization of the spin-spin correlations at long distances. In both pictures the system is much sensitive to the applied perturbations.

A nice example of chaos concerns the sensitivity of spin-glasses against magnetic field perturbations [3, 4]. The chaos exponent (to be defined in the next section) for this type of perturbation has been computed in mean-field theory [3] and numerically measured in short-ranged models [4]. Surprisingly, this chaos exponent does not depend on the dimensionality of the system [4]. Even though we do not know a theoretical derivation of this result, it appears to be enough sound in order to be considered. Droplet models can give an explanation for this result under the assumption that 3 is the lower critical dimension in Ising spin glasses (also a long debated problem [5]). In the context of droplet models, the chaos exponent for magnetic field perturbations is related to the thermal exponent $\theta$ which measures the free energy cost of the droplet excitations. The result $\theta = \frac{d-3}{2}$ implies that the chaos exponent is 2/3 and does not depend on the dimension. This has to be compared to the known results, $\theta = -1$ (exact) in $d = 1$ [6], $\theta \simeq -0.48$ in $d = 2$[7] and the exact result for the chaos exponent (2/3) in the Gaussian approximation to mean-field theory [3] Apparently the simple expression previously reported for $\theta$ correctly matches the small $d$ regime to the infinite dimension result.

Regarding other type of perturbations, the situation is less clear. For instance in the case of temperature changes, it remains unclear how much chaotic is the system (see [8]
for recent results). If chaos exists then it is certainly small and the possibility that chaos is marginal [3] cannot be excluded. Numerical results in the case of short-range models [9] show that chaos in temperature is also very small, as in the mean-field case.

The perturbations previously commented share the common property that they add energy to the system. This work is devoted to the study of a perturbation which does not add extensive energy to the system. In particular we will study chaoticity against changes of the realization of the quenched disorder. Because of the self-averaging property we expect that changes in the realization of the disorder (keeping the form of the disorder distribution) should not add extensive energy to the system. Therefore chaoticity appears because of a complete reshuffling of the free energies of the configurations. We will show that the system displays chaos very similarly as for the case of magnetic field perturbations. Criticality of chaos against perturbations of the quenched disorder has been studied by other groups [10]. The perturbation we are interested in differs from others by the fact that we change the sample realization without moving the system to a new point in the phase diagram.

The paper is organized as follows. Section 2 is devoted to the study of chaos in mean-field models. In section 3 we discuss on the results for the short-ranged models. Section 4 presents the scaling approach we have used to obtain the chaos exponents and shows the numerical results. Finally we present our conclusions in section 5.

## 2 Chaos in mean-field theory

We consider the models described by the Hamiltonian

\[ H[\sigma] = - \sum_{(i,j)} J_{ij} \sigma_i \sigma_j - h \sum_i \sigma_i \]  \hspace{1cm} (1)

The couplings in (1) are Gaussian symmetrically distributed random variables with zero mean and $1/N$ variance, where $N$ is the number of spins. The perturbation we consider consists in changing randomly the sign of a fraction $r$ of the couplings, i.e. for each coupling we change its sign with probability $r$. On average, a total number $Nr$ of the couplings $J_{ij}$ are changed to $-J_{ij}$. In this way we keep the new configuration of $J'_s$ in the same ensemble of disorder realizations without moving the system in the phase diagram. This is different from other type of perturbations in which, for instance (see [10]), the $J_{ij}$ are changed by a small amount $\delta \cdot x_{ij}$ where $x_{ij}$ is a random number and $\delta$ is small. In this case the variance of the distribution is increased (it grows proportionally to $\delta^2$) and we add energy to the system. In what follows we will consider, for simplicity, the case of zero magnetic field.

Denoting by $R$ the set of couplings which change sign, then we can write the perturbed Hamiltonian as,

\[ H_r[\sigma] = - \sum_{(i,j)} J_{ij} \sigma_i \sigma_j + 2 \sum_{(i,j) \in R} J_{ij} \sigma_i \sigma_j \]  \hspace{1cm} (2)

The sum runs over nearest neighbors in a lattice of dimension $d$. The mean-field case can be obtained in several ways. In the infinite-range model or SK model, all the spins interact one to the other. Alternatively, one can consider the finite connectivity random lattices (with fixed or average number of neighbors [11]).
Once we have defined the perturbation we construct a full Hamiltonian $H_{12}[\sigma, \tau]$ defined in a space of two sets of variables $\{\sigma_i, \tau_i; i = 1, ..N\}$. The Hamiltonian $H_{12}$ is the sum of the unperturbed Hamiltonian $H[\sigma]$ plus the perturbed Hamiltonian $H_r[\tau]$,

$$H_{12}[\sigma, \tau] = H[\sigma] + H_r[\tau]$$  \hspace{1cm} (3)

We define the usual spin-glass correlation functions,

$$G(x) = \langle \sigma_0 \tau_0 \sigma_x \tau_x \rangle$$  \hspace{1cm} (4)

where $\langle .. \rangle$ means average over the quenched disorder and $\langle .. \rangle$ corresponds to the thermal average over the full Hamiltonian $H_{12}$. The degree of coherence of the two systems is measured by the overlap function,

$$P(q) = \langle \delta(q - \frac{1}{N} \sum_i \sigma_i \tau_i) \rangle$$  \hspace{1cm} (5)

At large distances $G(x)$ behaves like

$$G(x) \sim \exp(-x/\xi(r))$$  \hspace{1cm} (6)

where $\xi(r)$ is the chaos correlation length, which is finite for a finite perturbation $r$ (we identify the perturbation with the fraction $r$ of changed couplings). The chaos correlation length diverges when $r \to 0$ if the unperturbed system stays in the spin-glass phase, including the critical point. This is because in the limit $r \to 0$, $G(x)$ converges to the usual spin-glass correlation function which has infrared singularities due to the existence of zero modes. The chaos correlation length diverges like,

$$\xi(r) \sim r^{-\lambda}$$  \hspace{1cm} (7)

where $\lambda$ is the chaos exponent which can be exactly computed in some particular cases. From this definition it is clear that static chaos is absent when the system stays at a temperature above the spin-glass transition (the paramagnetic phase). But this assertion is true only if $\xi(r)$ smoothly converges to the finite correlation length at that temperature when $r \to 0$. This is the case in mean-field theory but should not be necessarily true in finite dimensions [12]. The exponent $\lambda$ can also depend in principle on the temperature. We will show in mean-field theory that $\lambda$ is constant in the low $T$ phase. Furthermore, at the critical point, we expect $\lambda$ to depend on the critical exponents, even though this is not always the case$^1$.

Now we face the problem of computing the exponent $\lambda$ in mean-field theory. We follow the standard procedures (see [4] for details) and we apply the replica method to the full Hamiltonian $H_{12}$ eq.(3),

$$\beta f = \lim_{n \to 0} \frac{\log(Z_f)}{nN}$$  \hspace{1cm} (8)

Introducing Lagrange multipliers for the different order parameters one gets a saddle point integral

$^1$For the problem of chaos in temperature there is a new chaos exponent independent of the usual critical exponents [13].
\[ Z_j = \int dP dQ dR \exp(-NA[PQR]) \] (9)

with

\[ A[PQR] = \frac{\beta^2}{2} \sum_{a<b} (P_{ab}^2 + Q_{ab}^2) + \frac{\beta^2}{2} \sum_{a,b} (R_{ab}^2) - \log(Tr_{\sigma\tau} \exp(L)) \] (10)

where \(a, b\) denote replica indices which run from 1 to \(n\), and

\[ L[\sigma, \tau] = \beta^2 \sum_{a<b} (Q_{ab}\sigma_a \sigma_b + P_{ab}\tau_a \tau_b) + \beta^2 \sqrt{1 - 2r} \sum_{a,b} (R_{ab}\sigma_a \tau_b) \] (11)

There is one stable solution to the equations of motion,

\[ Q_{ab} = P_{ab} = Q_{ab}^{SK} = 0 \] (12)

where \(Q_{ab}^{SK}\) is the solution for the unperturbed system. The order parameter \(R_{ab}\) measures the degree of correlation eq.(5) between the two systems via the relation,

\[ P(q) = \frac{1}{n^2} \sum_{a,b} \delta(q - R_{ab}) \] (13)

The stability of the solution \(R = 0\) means that there is chaos against coupling perturbations. This is indeed very similar to the case of chaos in a magnetic field. Now we can compute, in the Gaussian approximation, the correlation function \(G(x)\) of eq.(6). The computations can be easily done in Fourier space. We define,

\[ C(p) = \sum_x G(x)e^{ipx} \] (14)

In order to find \(C(p)\) we need to compute the spectrum of fluctuations in the direction \(R_{ab}\) around the stable solution (eq.(12)). The full expression has been reported in [3]. Its singular part is given by

\[ C(p) = \int_0^{q_{max}} dq \int_0^{q_{max}} dQ \frac{p^2 + 1 + \alpha(q)\alpha(Q)}{(p^2 + 1 - (1 - 2r)\alpha(q)\alpha(Q))^3} \] (15)

with

\[ \alpha(q) = \beta(1 - q_{max}^{\max} + \int_q^{q_{max}} dq x(q)) \] (16)

where \(\beta\) is the inverse temperature and \(q(x)\) is the order parameter function associated to the spin-glass. This expression yields the singular behavior of the correlation function in the spin-glass phase [4],

\[ C(p) \sim p^{-4}, \quad p \to 0 \] (17)

The chaos correlation length \(\xi(r)\) is given by the minimum eigenvalue of the stability matrix,

\[ \lambda_{\min} = 2\beta^2 r \] (18)

This yields,
\[ \xi(r) \sim \lambda_{\text{min}}^{-\frac{1}{2}} \sim r^{-\frac{1}{2}} \] (19)

This result is valid at and below the critical point. We expect it to be valid also in other mean-field models like, for instance, finite connectivity random lattices. In this case, where analytical calculations become much more involved, we expect to obtain the same results. This will be nicely corroborated by our numerical simulations in section 4.

3 Chaos in short-ranged systems

Now we face the problem of extending our results to finite dimensions. This is a non trivial task and we will present a derivation only for the one-dimensional case.

The chaos exponent can be exactly computed in one dimension since we know how to construct the ground state in this case. The exponent \( \lambda \) is a zero-temperature exponent because there is no phase transition at finite \( T \). The Hamiltonian reads,

\[ H = -\sum_i J_i \sigma_i \sigma_{i+1} \] (20)

The perturbation consists in changing the sign of a fraction \( r \) of the couplings in a random way, and we assume that the \( J' \)s are distributed around \( J = 0 \) with a finite weight at \( J = 0 \) (this is essential for the scaling arguments). When a fraction of the couplings is changed, the new ground state is constructed inverting domains close to the defects. The energy excess of these defects scales like \( rL \), where \( L \) is the length of the spins chain. On the other hand, domain excitations (in this case these excitations are inversions of compact domains) scale like \( L^\theta \) with \( \theta = -1 \), where \( \theta \) is the thermal exponent introduced in droplet models (see [6] for a derivation.). This gives the chaos correlation length,

\[ \xi \sim r^{-\frac{1}{2}} \] (21)

Domains of length \( L \) above this characteristic length \( \xi \) are destroyed by the bond defects. Below the characteristic length, the domains are nearly insensitive to the perturbation. This result is valid for a distribution of couplings with finite weight at \( J = 0 \). We have found therefore for the chaos correlation length the same result as in mean-field theory. Unfortunately we cannot do more in order to compute this exponent in other dimensions because we do not know the structure of the ground state. Anyway, we can try to estimate the energy the excitations after applying the perturbation. In the more general case, a perturbation of the type we are studying here will modify the ground state energy by a quantity proportional to the number \( r \) of created defects (because the total fraction of frustrated and unfrustrated bonds is finite). In addition, we can suppose that this energy will scale like the size of the system \( L^\alpha \) with \( \alpha \leq d \). In principle, the exponent \( \alpha \) is unknown and we do not know how to estimate it. If we assume \( \alpha = \frac{d+1}{2} \) (and using \( \theta = \frac{d-3}{2} \)), this gives the dimension independent result \( \xi \sim r^{-\frac{1}{2}} \). Unfortunately we are unable to estimate the exponent \( \alpha \) and a numerical computation of this exponent in 2 dimensions would be very interesting.

\(^2\)For a distribution of couplings with zero weight at \( J = 0 \) one finds \( \xi \sim r^{-1} \)
4 Numerical results

In this section we will discuss on our Monte Carlo simulations in order to test the results obtained in the previous sections for the chaos exponents. Furthermore, we will present simulations in four dimensions. Our results are compatible with the fact that the chaos exponents in finite dimensions are compatible with the mean-field ones.

We have simulated two types of mean-field models (the Sherrington Kirkpatrick -SK- model [14] and the random finite connectivity lattice model [11]) and a four-dimensional (4d) Ising spin glass for which the existence of a finite $T$ phase transition is well established [15]. Monte Carlo simulations implement the Metropolis algorithm (for the mean-field models) and the heat-bath algorithm (in the 4d case). Special attention has been payed in order to thermalize the samples.

4.1 The finite-size scaling approach

In order to measure the chaos exponents, we have performed a finite-size scaling analysis [4]. The idea is to compute the overlap between two copies of the system, one copy with an initial realization of the disorder, the other one with the perturbed realization. The overlap is defined as,

$$ q = \sum_{i=1}^{N} \sigma_i \tau_i $$

We define the chaos parameter $a(r)$,

$$ a(r) = \frac{\langle \sigma_i \tau_i \rangle_r^2}{\langle \sigma_i \tau_i \rangle_r^2_{r=0}} $$

i.e. we normalize the correlation between the unperturbed and the perturbed system to the autocorrelation of the unperturbed system. In this way $a(0) = 1$ by definition. The system is chaotic if the quantity $a(r)$ (in the thermodynamic limit) jumps to 0 as soon as $r$ is finite. This means that,

$$ \lim_{r \to 0} \lim_{N \to \infty} a(r) = 0 \quad \text{while} \quad a(0) = 1 $$

It is crucial to perform the limits in the order previously indicated. Since $a$ is an adimensional quantity, we expect it will scale like

$$ a \equiv f(L/\xi) $$

where $\xi$ is the chaos correlation length of eq.(6). In the mean-field case we find for the spin-glass phase (using eqs. (17) and (19))

$$ a \equiv f(Nr^2) $$

and at the critical point we get (using the singular behavior $C(p) \sim p^{-2}$ for $p \to 0$)

$$ a \equiv f(Nr^3) $$

In the case of short-range models we can derive the scaling behavior using equation (21)
\[ a \equiv f(rL^2) \]  

(28)

Since only one exponent (the chaos exponent) must be fitted, these scaling relations are highly predictive. Comparing equations (26) and (28) we observe that \( d_u = 4 \) plays the role of a specific critical dimension. The situation is the same as in the case of magnetic field perturbations [4], where the value of this dimension only depends on the behavior of the propagator \( C(p) \) in the limit \( p \to 0 \). In this case we expect the scaling functions \( f(x) \) in equations (26) and (28) to coincide except by the presence of some logarithmic corrections. A numerical test of this prediction is shown in the following subsections.

4.2 Numerical results in mean-field models

The SK model is defined by the following hamiltonian

\[ H = \sum_{i<j} J_{ij} \sigma_i \sigma_j \]  

(29)

with the \( J_{ij} \) distributed according to the function \( p(J_{ij}) \). In the thermodynamic limit, the only relevant feature of the \( p(J_{ij}) \) is its variance (we restrict to distributions with zero mean). To speed up the numerical computations we have taken a binary distribution of couplings, i.e. the \( J \)'s can take the values \( \pm \frac{1}{\sqrt{N}} \) with equal probability.

We have simulated the SK model at the critical temperature \( T = 1 \) and below the critical temperature. We have computed the chaos parameter \( a \) for different values of \( r \) (typically \( r \) runs from 0 to 0.5). Simulations were done for lattice sizes ranging from \( N = 32 \) to \( N = 1000 \). Figures 1 and 2 show the scaling laws eq.(26) and (27) at the critical point \( T = 1 \) and below the critical point \( T = 0.7 \) respectively. Data do nicely fit the predictions.

We have also simulated the random finite-connectivity lattice model (FC model). In this model each point of the lattice is connected (in average) to a finite number \( c \) of neighbors. In this case the FC model model is defined by,

\[ H = \sum_{i<j} J_{ij} \sigma_i \sigma_j \]  

(30)

where the \( J_{ij} \) are distributed according to,

\[ \mathcal{P}(J_{ij}) = \frac{c}{N} p(J_{ij}) + (1 - \frac{c}{N}) \delta(J_{ij}) \]  

(31)

and \( p(J_{ij}) \) is given by,

\[ p(J_{ij}) = \frac{1}{2} \delta(J_{ij} - 1) + \frac{1}{2} \delta(J_{ij} + 1) \]  

(32)

The parameter \( c \) is the average connectivity of the lattice. This model can be exactly solved, the only difference with respect to the SK model being that there appear an infinite set of order parameters, which can be absorbed in a global order parameter [11]. The model has a phase transition at a temperature \( \beta_c \) given by,

\[ 1 = (c - 1) \int_{-\infty}^{\infty} p(J) \tanh^2(\beta_c J) = (c - 1) \tanh^2(\beta_c) \]  

(33)

\(^3\)One can also consider the case in which the connectivity is fixed and equal to \( c \)
This expression implies that to have a phase transition, we need $c > 2$. To compare with the results of the 4d case, we have simulated the FC model with $c = 8$, in order to have the same number of nearest neighbors than the 4d model. The transition temperature is in this case $T_c \simeq 2.76$. We have simulated this model at the critical temperature and below that temperature, at $T = 2.0$. The results for the chaos parameter $a$ are shown in figures 3 and 4. The agreement with the scaling predictions (eqs.(26) and (27)) is also fairly good.

4.3 Numerical results in four dimensions

We have also done numerical simulations of the Ising spin glass model in four dimensions with the purpose of analysing the dimensionality effects on the chaos exponent. We have considered the Ising spin glass at $d = 4$ because it is widely accepted that there is a finite $T$ phase transition in this case.

We have simulated the model (eq.(1)) with a nearest neighbour interaction, periodic boundary conditions and using a discrete binary distribution of couplings as in eq.(32). We expect to obtain the same results as in the case of a continuous distribution of couplings with $p(J = 0)$ finite. The model has a transition at $T_c \sim 2.05$ [15]. We have done simulations at $T = T_c$ and $T = 1.7$. The results are shown in figures 5 and 6.

At the critical point we obtain a chaos exponent $\lambda \sim \frac{2}{3}$. It is not clear to us how to obtain this exponent in terms of the critical exponents and if it represents a new critical chaos exponent.

The results in figure 6 show that eq.(28) is in pretty good agreement with the data. Now we will show that four dimensions is well compatible with the upper critical dimension for the criticality of chaos. In order to get this result, we will compare the different values of the chaos parameter $a$ for different sizes with the corresponding values of the FC model with $c = 8$. We compare with the FC model, instead of the SK model, because we expect that logarithmic corrections, if present, should be smaller in the FC model than in the SK model. Both are mean-field models even though the FC model resembles the finite $d$ model much more than does the SK model. This fact should reflect in the nature of the corrections to the universal mean-field behavior. It is clear that in order to compare the FC model with the four dimensional model we have to put the system in equivalent points within the phase diagram. We expect the universal function $f(x)$ to depend on the temperature (which is an external parameter) in the following way

$$a \equiv f(A(T)(L/\xi)^d)$$

In four dimensions, the scaling function $f$ still depends on the temperature via the universal amplitude $A(T)$. It is reasonable to assume that the dependence of the amplitude $A(T)$ on the temperature enters through the spin-glass order parameter $q(T)$. More concretely, below but close to $T_c$ we expect,

$$A(T) \sim q^2(T)$$

because the argument of the scaling function $f$ of equation (34) scales like the singular part of the free energy which in mean field theory scales like $Q_{ab}^2$ (see eq.(10)). Consequently we have to normalize the adimensional ratio $(L/\xi)^d$ to the corresponding value of the Edwards-Anderson order parameter for that temperature. For $N = 256$ the FC model

\[4\]In the three dimensional case there is still much controversy on the existence of a finite $T$ transition[5]
gives $q(T = 2.0) \simeq 0.11$ and the 4d model at $L = 4$ gives $q(T = 1.5) \simeq 0.25$, the ratio of both numbers being 2.5. Simulation data for both models are shown in figure 7. If one considers the SK model then one observes that data fits well but not so nicely as in the case of the FC model.

5 Conclusions

We have investigated the sensitivity of spin glasses against the application of a particular static perturbation. In particular, we have studied the nature of the static chaos when a perturbation to the realization of the quenched disorder is applied to the system. This can be done in several ways. In our case we have considered a perturbation which, in average, does not add energy to the system. Due to the self-averaging property we expect that a change in the sign of a finite fraction of the total number of couplings in the system should not change its mean statistical properties (and in particular, its energy). This makes the new perturbed system to stay in the same point in the phase diagram. The existence of strong chaos for this type of perturbation proves that the reshuffling of the Boltzmann weights of the different states is complete. This differs from the case where the perturbation consists in applying a magnetic field to the system or where its temperature is changed. In these cases extra energy is supplied to the system.

We have solved the mean field theory and we have extracted the chaos exponent for this type of perturbation. The analytical solution of this problem is very similar to that of chaos against magnetic field perturbations where the chaos correlation length can be exactly computed [3]. This is in contrast to what happens when the temperature is changed. In the last case the system is much robust against the perturbation and a high degree of correlation between the configurations at both temperatures is preserved [8].

We have observed that the mean-field chaos exponent $1/2$ in the spin-glass phase is exact also at one dimension. A finite-size scaling approach to the criticality of chaos shows that $d = 4$ plays the role of an upper critical dimension for the chaos problem. Finite-size scaling studies are very powerful in order to get the chaos exponents. This is because we only need to determine one free parameter to make the data corresponding to different sizes to collapse in a unique scaling function. We have performed numerical simulations of mean-field models which are in agreement with the theory. Simulations in four dimensions are in very good agreement with the fact that 4 plays the role of an upper critical dimension for the criticality of chaos (see figure 7). Furthermore, the fact that the mean-field chaos exponent is also exact in one dimension suggests that mean-field theory is probably correct at any dimension. This is indeed very similar to what happens in the case of magnetic field perturbations.

Finally we would like to point out two possible extensions of this work. Firstly it would be interesting to make dynamical studies of the relaxation of the overlap function against this type of perturbation (as done for the remanent magnetization after application of a magnetic field). We expect to see aging effects as in the case of magnetic field perturbations. Secondly, it would be interesting to extend the study of chaos to the metastable states using the TAP formalism. Most probably, similar chaotic properties will be observed in the structure of the metastable states.
6 Acknowledgements

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References

[1] For general reviews on spin glasses see: M. Mézard, G. Parisi and M. A. Virasoro, Spin Glass Theory and Beyond (World Scientific, Singapore 1987); K. H. Fischer and J. A. Hertz, Spin Glasses (Cambridge University Press 1991); G. Parisi, Field Theory, Disorder and Simulations (World Scientific, Singapore 1992); K. Binder and A. P. Young, Spin Glasses: Experimental Facts, Theoretical Concepts and Open Questions Rev. Mod. Phys. 58, 801 (1986).

[2] W. L. McMillan, Scaling Theory of Ising Spin Glasses J. Phys. C 17 (1984) 3179; A. J. Bray and M. A. Moore, The Nature of the spin-glass phase and Finite-Size effects J. Phys. C 18 (1985) L699; D. S. Fisher and D. A. Huse, Equilibrium Behavior of the spin-glass ordered phase Phys. Rev. B 38 (1988) 386; D. S. Fisher and D. A. Huse, Non-Equilibrium dynamics of spin-glasses Phys. Rev. B 38 (1988) 373.

[3] I. Kondor, On Chaos in spin glasses J. Phys. A 22 (1989) L163.

[4] F. Ritort, Static Chaos and Scaling Behavior in the Spin-Glass Phase Phys. Rev. B. 50 (1994) 6844.

[5] E. Marinari, G. Parisi and F. Ritort, On the 3d Ising spin glass J. Phys. A 27 (1994) 2687 and refernces therein

[6] A. J. Bray and M. A. Moore in Heidelberg Colloquium in Spin Glasses, Springer Lecture Notes in Physics, Vol. 275 (1986).

[7] N. Kawashima, N. Hatano and M. Suzuki, Critical Behavior of the two-dimensional EA model with a Gaussian bond distribution J. Phys. A 25 (1992) 4985.

[8] S. Franz and M. N. Nifle, On Chaos in mean-field spin glasses cond-mat 9412083.

[9] F. Ritort, unpublished results.

[10] M. Cieplak and J. R. Banavar, Scaling and Phase Transitions in Random Systems in Statistical Physics (StatPhys 18) (1992) North Holland.

[11] Y. Y. Goldschmidt and C. De Dominicis, Replica Symmetry Breaking in the spin-glass model on lattices with finite connectivity Phys. Rev. B 41 (1990) 2184 and references therein.

[12] M. Nifle and H. J. Hilhorst, J. Phys. A 24 (1991) 2397.

[13] M. Nifle and H. J. Hilhorst, New Critical-Point Exponent and New Scaling Laws for Short-Ranged Ising Spin Glasses Phys. Rev. Lett. 68 (1992) 2992.

[14] D. Sherrington and S. Kirkpatrick, Infinite Ranged Models of Spin Glasses Phys. Rev. B 17 (1978) 4384.
[15] D. Badoni, J. C. Ciria, G. Parisi, J. Pech, F. Ritort and J. J. Ruiz, *Numerical Evidence of a Critical Line in the 4d Ising Spin Glass* Europhys. Lett. **21** (1993) 495.
**Figure Captions**

Fig. 1 Chaos in the SK model at the critical point $T_c = 1$.

Fig. 2 Chaos in the SK model at $T = 0.7$ in the spin glass phase.

Fig. 3 Chaos in the FC model with $c = 8$ at the critical point $T_c \simeq 2.76$.

Fig. 4 Chaos in the FC model with $c = 8$ at $T = 2.0$ in the spin glass phase.

Fig. 5 Chaos in the 4d Ising spin glass at the critical temperature $T_c \simeq 2.05$. We obtain $\lambda \sim 2/3$ for the chaos exponent.

Fig. 6 Chaos in the 4d Ising spin glass at $T = 1.7$ in the spin-glass phase. The mean-field chaos exponent $\lambda = \frac{1}{2}$ fits data very well.

Fig. 7 Chaos in the 4d Ising spin glass at $T = 1.7$ compared to the FC model with $c = 8$. The abcissa $x$ corresponds to $N r^2$ (with $N = L^4$ in four dimensions). This scaling suggests that four dimensions is the upper critical dimension for the criticality of chaos.
$a(N, r)$

- $N$
- $32$
- $64$
- $96$
- $256$
- $320$
- $480$
The graph shows the relationship between $a(r,N)$ and $N r^2$. The data points correspond to different values of $N$: 32, 64, 96, 128, 256, 320, and 480. The axes are labeled $a(r,N)$ on the y-axis and $N r^2$ on the x-axis.
