Several comparison result of two types of equilibrium (Pareto Schemes and Stackelberg Scheme) of game theory approach in probabilistic vendor – buyer supply chain system with imperfect quality

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Abstract. In this paper, Economic Order Quantity (EOQ) of the vendor-buyer supply-chain model under a probabilistic condition with imperfect quality items has been analysed. The analysis is delivered using two concepts in game theory approach, which is Stackelberg equilibrium and Pareto Optimal, under non-cooperative and cooperative games, respectively. Another result is getting a comparison of the optimal result between integrated scheme and game theory approach based on analytical and numerical result using appropriate simulation data.

1. Introduction
Supply chain analysis is one of the interesting research topics in operation research in which industrial engineering problem as a primary reference. The objective of supply chain analysis is to find the mathematical model, often called by inventory model, that represents the relationship between a firm in supply chain system and to find the optimal condition for all of the decision variables. Based on previous references until the present, supply chain management analysis is lead to multiplayer in such system. Firstly, two-level supply chain, known as a vendor–buyer system. In two level supply-chain system, refer to the literature, is known about the integrated scheme which is used by many authors in that field. In the integrated scheme, there is only one possible strategy between each party in the system that is cooperative policy, cooperative in price coordination and other policy like a discount. The aim of the integrated scheme is to get long-term mutual benefits. Goyal [11] introduced an integrated scheme for getting EOQ of inventory problem in his paper. He considered the joint optimization problem of a single vendor and a single buyer, in which he assumed that the vendor’s rate is infinite. Then, after the Goyal’s work, many authors are also interested in integrated analysis. For example, Banerjee [4] assumed a finite rate of production and developed a joint economic-lot-size (JELS) model for a product with a lot-for-lot shipment policy. He proved that through JELS model, that combine both jointly ordering policy and appropriate price adjustment. Furthermore, there are several results in integrated vendor – buyer system like Goyaland Szendrovits [12], Goyal [15], Goyal and Gupta [13], Goyal [14], Hill (1997), Hill [18], Hoque and Goyal [19], Setiawan and Triyanto [28]. In general, the integrated scheme only applicable in a system which each firm agree to join their economic move.
In order to get the more realistic model, the development of inventory model always concerns about the crucial topics that represent the actual problem in an industrial process. One of those concerning topic is defective items in a lot of production. Some researcher argues that product quality is not always perfect in quality, but is directly affected by the reliability of the production process used. Therefore, the process may deteriorate and produce defectives or poor-quality item. There are many author that explain the effect of imperfect quality items in various assumption of inventory model in their result, for example Salameh and Jaber [27], Huang [21] in his result that extended the integrated vendor – buyer inventory model by accounting for imperfect quality items and considered the situation where the delivery quantity sent to the buyer was identical for each shipment, Goyal et al. [16], Huang [21] developed an inventory model to determine an optimally integrated vendor –buyer inventory policy for unreliable production process in a JIT (Just In Time) manufacturing environment. Another result in supply chain management which includes imperfect quality is proposed by several authors like Lo et al. [24], Wee et al [30], Yoet al. [32], Chang and Ho [6], Sana [26], Hsu and Hsu [20]. Another concerning in current analysis of inventory model is uncertainty condition. One condition that under consideration with uncertainty condition is lead time demand length. In the face of reality, the information about that time is quite limited and most often provided in first two moments of demand only. Lead time length in supply chain management maybe is one of the crucial problems that inflict many other problems in supply chain process, cost, and shortage condition, if they not handled properly. According to the Lin [22], lead time can be reduced by additional crushing cost, customers service level improved, inventory in safety stocks reduced, and the competitive edge in business increased, in other words, it is controllable. Lin [22] study an integrated vendor-buyer inventory policy for a continuous review model with a random number of defective items and screening process gradually. Shortages are allowed and partially backlogged on the buyer’s side and that the lead time demand distribution is unknown, except its first two moments.

In recent of industrial competition, only thinking about the integrated scheme is no longer suitable for any case of supply chain management. Each member in supply chain system may play a different strategy rather than integrated scheme. That chosen one depends on the economic purposes of each member. They can choose the cooperative or non-cooperative type of strategy to each other. According to the current literature, games theory is the best mathematical tool for getting an analysis of this condition and also finding the optimal result. There are two types of games related to the strategy analysis in supply chain management that cooperative (centralized approach) and non-cooperative (decentralized approach) games. In cooperative games condition, there are no dominant players, the players choose their strategies simultaneously, for instances, an integrated scheme in the inventory system. While, in a non-cooperative game, this is not exactly true. There are some circumstances that each member in inventory system can dominate each other. It can be classified as leader and follower. There are two types of non-cooperative games that are dynamics non-cooperative games and static non-cooperative games. According to the literature by Alaei et al [1], Stackelberg game is a non-cooperative game. Each firm makes an optimal decision given the behavior of the other firm, and therefore, none has an incentive to deviate unilaterally from the equilibrium. On the other hand, for the cooperative game, according to literature by Peters [25], the results of non-cooperative approaches are not Pareto efficient. In order to achieve this solution, which is Pareto-efficient, a cooperative game can be applied by Abad and Jaggi [2]. In this game, the weighted sum of the player’s objectives which agree on the result of the Nash equilibrium is optimized. There are many kinds of literature about the optimal analysis of supply chain model using game theory approach. But, according to current literature, all of them are still limited to the deterministic model and most of them did not consider the model with the probabilistic condition and imperfect process condition such as lead time demands, defective items, inspection error rate, shortage and partial back-ordering. According to the history of game theory literature, the first complete analysis and basic theory regarding the application of game theory in the supply management system were analyzed by Cachon and Netessine [7]. They focus on basic result about various equilibrium solution properties of cooperative and non-cooperative games in supply chain model. Another literature about the
application of game theory in order to strategic behavior modeling in the industrial and economic field had been written by GeÇkil and Anderson [10]. Esmaelli et al [9] have analyzed two level seller and buyer model, they use a basic deterministic model that proposed by Abad [2]. They use Stackelberg strategy solution concept to analyze the noncooperative game and Pareto optimal concept to analyze the cooperative game. But, this paper still meets with the assumption that shortage is not permitted and no back ordering or partial back ordering process allowed. Elyasi et al [8] study modified EOQ for deterministic two – echelon (a manufacturer and a supplier) model with imperfect quality items. They use just in time (JIT) concept (centralized model) in inventory problem to formulate total cost of inventory. In their paper, the optimal solution also analyzed by game theory approach: non-cooperative static game using and cooperative game that first introduced by Abad and Jaggi [2]. A cooperative game is applied in order to get Pareto – efficient solution which refers to literature by Peters [25]. In their result, they proved that the result of the cooperative model is the nearest one to the result of the centralized model. Wu et al. [31] consider a supply chain with one supplier and two competing retailers and develop six different scenarios of power imbalance that leads to different sequences of moving among the members of the supply chain. They implement both Nash and Stackelberg equations and compare the gained results.

2. Material and Method

2.1. General Assumptions

There are single-vendor and single-buyer. A single type of product is considered. Both the vendor and the buyer play the non-cooperative game under the monopolistic condition, which addresses to the vendor’s move. Because of that condition, then the Buyer becomes the leader and both the buyer as a follower in that system. According to the current theory in applied game theory in strategic behavior, we will use Stackelberg Equilibrium approach to analyzing that situation. It’s assumed that defective items only occur in the production process. Follow to the random number of defective items in retailer arriving order lot. In the buyer’s side, the defective items exist in an arriving lot of size $Q$ with defective percentage $\gamma$ and probability function $f(\gamma)$. Vendor’s production rate for non-defective items is greater than buyer’s demand rate. Shortage conditions are allowed in the relationship between the vendor and the buyer. Shortages are partially back ordered with a fraction of the demand $\beta, \beta \in [0,1]$ during the stock out period that will be backordered. We ignore the inspecting time in buyer’s side, so defective items can be inspected immediately by 100 % screening process of the lot. The Buyer will return all defective items to the manufacturer upon receipt of the next lot. The reorder point $r = \text{expected demand during lead time} + \text{security stock (SS)}$ and $\text{SS} = kx$ (standard deviation of lead time), so $r = DL + k\sigma\sqrt{L}$, where $k$ is a safety factor.

2.1.1. Notations

In this paper, we use some following parameter with some specific notation

- $Q$: The size of the shipments from the vendor to the buyer, a decision variable
- $N$: The number of lots in which the product is delivered from the vendor to the buyer, a positive integer, a decision variable.
- $D$: Expected demand per unit time on the Buyer for deteriorating items
- $P$: The production rate at the vendor
- $F$: The freight (transportation) cost per shipment.
- $S_p$: Buyer’s ordering cost per order.
- $S_v$: Vendor’s set up cost per production.
- $h_{b1}$: Buyer’s holding cost for non-defective item per unit per unit time.
- $h_{b2}$: Buyer’s holding cost for defective item per unit per unit time.
- $h_v$: Vendor’s holding cost per unit time.
- $\pi$: Buyer’s shortage cost per unit short.
- $\gamma$: The probability that an item produced is defective.
- $\pi_0$: Buyer’s marginal profit (cost of demand lost) per unit.
\( \beta \): The fraction of the demand during the stock-out period.

\( c_{w0} \): The vendor’s unit warranty cost per defective items.

\( c_{ib} \): Screening cost at the buyer.

\( R \): Reorder point for deteriorating items.

\( X \): The lead time demand which has a p.d.f. \( f(x) \) with finite demand \( DL \) and standard deviation \( \sigma \sqrt{L} \).

\( DL \) is an expectation of demand when the lead time occurs. Then, \( \sigma \) is standard deviation of demand per unit time.

\( x \): Buyer’s unit screening cost.

3. Result and Discussions

3.1. Buyer Total Cost

The buyer inventory cost per cycle per unit time is consists of components due to placing an order, transportation cost, screening cost, holding cost, expected shortage cost and lead time crushing cost. The mathematical formula of buyer’s total cost is given by the following equation:

\[
TC_b(Q, k, n, L) = S_b + nF + c_{ib}Q + [\pi + \pi_0(1 - \beta)]E[(X - r)'] + C(L)
\]

\[
= h_{b1} Q(1 - \gamma)(1 - \theta) \left[ \frac{Qy\theta}{2x(1 - \gamma)(1 - \theta)} + \frac{Q(1 - \gamma)(1 - \theta)}{2} + k\sigma \sqrt{L} + (1 - \beta)E[(X - r)'] \right]
\]

\[
+ h_{b2} \left[ \frac{Q^2(1 - \gamma)(1 - \theta)}{D} - \frac{Q^2y\theta}{2x} \right] \quad (3.1)
\]

Using renewal-reward theorem, the expected average total cost per unit time for the buyer follows the following formula:

\[
ETC_b(Q, k, n, L)
\]

\[
= \frac{D(S_b + nF + c_{ib}Q + [\pi + \pi_0(1 - \beta)]E[(X - r)'] + C(L))}{Q(1 - \gamma)(1 - \theta)}
\]

\[
+ h_{b1} \left[ \frac{Qy\theta}{2x(1 - \gamma)(1 - \theta)} + \frac{Q(1 - \gamma)(1 - \theta)}{2} + k\sigma \sqrt{L} + (1 - \beta)E[(X - r)'] \right]
\]

\[
+ h_{b2} \left[ \frac{Q^2(1 - \gamma)(1 - \theta)}{D} - \frac{Q^2y\theta}{2x} \right] \quad (3.2)
\]

or equivalent with

\[
ETC_b(Q, k, n, L)
\]

\[
= \frac{D(S_b + nF + c_{ib}Q + [\pi + \pi_0(1 - \beta)]E[(X - r)'] + C(L))}{Q(1 - \gamma)(1 - \theta)}
\]

\[
+ h_{b1} \left[ \frac{Q(1 - \gamma)(1 - \theta)}{2} + k\sigma \sqrt{L} + (1 - \beta)E[(X - r)'] \right] + h_{b2}Qy\theta
\]

\[
+ \left( h_{b1}D - h_{b2}QDy\theta \right) \frac{2x(1 - \gamma)(1 - \theta)}{2x(1 - \gamma)(1 - \theta)} \quad (3.3)
\]

where \( \pi + \pi_0(1 - \beta) = \bar{\pi} \). Based on the equation (3.2), we get the worst distribution of \( ETC_b(Q, k, L) \)
3.2. Vendor Total Cost

The vendor’s inventory production per unit time can be obtained by subtracting the accumulated buyer’s inventory level from the accumulated vendor’s inventory level as follows:

\[
ETC_v(Q, k, n, L) = \frac{D}{Q(1-\gamma)(1-\theta)} \left[ S_v + nc_vQ + \frac{\bar{n}_c\sqrt{T}}{2} \sqrt{1 + k^2 - k} + C(L) \right] + h_vQ + h_vDy\theta + \frac{(h_vD - h_{v2})QDy\theta}{2x(1-\gamma)(1-\theta)}
\]  

(3.4)

3.3. EOQ by Integrated Scheme

In this paper, we will use JIT (Just in Time) system concept to analyse the optimum policy in the integrated inventory system. In practically, JIT system is suitable for the integrated inventory system. This system focuses primarily on purchasing and manufacturing required for immediate consumption. It requires a spirit of cooperation and same long-term strategic partnership between the buyer and the vendor. They can coordinate their product, inventory strategies and share information with each other to determine the best policy and also profit sharing for both parties. Following the work by Banerjee [4] and Lin [23] about concept of joint optimization for the buyer and the vendor, we get the joint total expected average cost per cycle per unit time as follows.

Thereby, holding cost for the vendor is formulated by

\[
h_v\left( \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-\theta)}{2D} \right)
\]  

(3.6)

\[
ETC_v(Q, n) = \frac{D}{nQ(1-\gamma)(1-\theta)} \left[ S_v + nc_vQ + \frac{\bar{n}_c\sqrt{T}}{2} \sqrt{1 + k^2 - k} \right] + h_v\left( \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-\theta)}{2D} \right)
\]  

(3.7)
It is easy to prove the JETC is a convex function for $Q$ and $k$. Thereby, for fixed $n$ and $L \in [L_i, L_{i-1}]$, the minimum value of $JETC(Q, k, n, L)$ occurs at the critical point $(Q, k)$. Using Karush-Kuhn-Tucker (KKT) conditions for optimal condition, e.g. first derivative criteria, we get EOQ and optimal value of the others as follow:

\[
Q^* = \sqrt{\frac{2h_1(1-\gamma)(-\gamma)}{2\gamma^2 + h_2\gamma^2 + \frac{nQ^2}{2p} + \frac{DnQ^2}{n(1-\gamma)^2}}}, \quad \text{where} \quad A_1 = \frac{2h_1}{Q(1-\gamma)^2h_1} - \frac{D}{h_1}
\]

\[k^* = 1 + \frac{1}{A_1^2 + 2A_2}, \quad \text{where} \quad A_2 = \frac{2h_1}{Q(1-\gamma)^2h_1} - \frac{D}{h_1}
\]

### 3.4. EOQ by Stackelberg Scheme

The second analysis of our research is about the non-cooperative result of our inventory model. We consider the situation that the buyer and the vendor prefer to play non-cooperative strategies. Mathematically, those conditions lead to Stackelberg scheme in games theory approach. The vendor as leader chooses his move first and then buyer as follower reacts by observing respective decision. The buyer gets his best own move consistent with available information. Finally, the leader determines his optimal move as Stackelberg’s optimal move consider the optimal reaction from the follower. According to the Basar and Olsder in Esmaelli, et al. [9], the objective of the leader in inventory system is to design his move in such ways as to maximize his revenue after considering all rational moves that the follower can devise. By applying Karush-Kuhn-Tucker (KKT) conditions for optimum condition, we get expected average total cost of buyer as follower for fixed value of lead time $L \in [L_{i-1}, L_i]$.

\[
Q_{fl} = \sqrt{\frac{D(S_b + nF + c_{ib}Q + \frac{\pi\sqrt{L}}{2}(\sqrt{1 + k^2} - k) + C(L))}{\frac{1}{2}h_1(1-\gamma)^2(1-\theta)^2 + \frac{1}{2\gamma}(h_{b1}D - h_{b2})\gamma^2\theta + h_{b2}\gamma^2(1-\gamma)(1-\theta)}}}
\]

On the other hand, if we apply KKT condition to the vendor’s expected average total cost, we get the vendor optimal move:

\[
Q_{vl} = \sqrt{\frac{2DS_v}{n_{vl}(n_{vl} - 1)(1-\gamma)(1-\theta)}}
\]

\[
n_{vl} = \sqrt{\frac{2(DS_v + c_{vw}Q\gamma\theta)}{h_vQ^2(1-\gamma)(1-\theta)}}
\]
Thereby, optimal decision of the leader, that called by Stackelberg optimal solution, is determined based on all reasonable reaction of the buyer as follower \( Q^S = Q^*_d \), \( n^S = n^*_d \), \( k^S = \frac{1-A^2}{2A^2} \)

where,

\[
A_2 = \frac{4DS_S b_1}{n_d(n_d-1)(1-\gamma)(1-\theta)} - 2D(S_b + nF + c_{ib}Q + C(L)) \]

\[
B_1 = \frac{1}{2} h_{b_1}(1-\gamma)^2(1-\theta)^2 + \frac{1}{2\lambda}(h_{b_1}D - h_{b_2})D\gamma\theta + h_{b_2}\gamma\theta(1-\gamma)(1-\theta) \tag{3.12}
\]

Equation (3.12) is proposed in implicit form. Using this result is often difficult and not applicable in practice. According to the almost researcher in supply chain management, numerical simulation for the set of equation (3.12) using suitable procedure algorithm and appropriate data may be the best way to describe the approximation value of the optimal solution in a numerical sense and tell us about the condition in the inventory system. Thereby, solution procedure algorithm and also numerical simulation will be provided in next chapter.

3.5. EOQ by Pareto Optimal Solution

According to the Abad and Jaggi [2] and Elyasi et al. [8], the optimal result of the non-cooperative approaches is not Pareto – efficient. To get the Pareto-efficient solution, a cooperative game is applied. We use the cooperative game introduced by Abad and Jaggi [2] and also used by Elyasi et al. [8]. The weighted sum of the players’ objectives is optimized with assumption that the players (vendor and buyer) agree on the result of the Nash equilibrium

\[
Z = \lambda_p (ETC_b) + (1-\lambda_p)ETC_v, \quad 0 < \lambda < 1 \tag{3.13}
\]

so

\[
Z = \lambda_p \left[ \frac{D}{Q(1-\gamma)(1-\theta)} \left( S_b + nF + c_{ib}Q + \frac{\bar{n}\sigma\sqrt{L}}{2}\sqrt{1+k^2-k} + C(L) \right) + h_{b_1}\sigma\sqrt{L} \left( k + \frac{(1-\beta)(\sqrt{1+k^2-k})}{2} \right) + h_{b_2}Q\gamma\theta + \frac{(h_{b_1}D - h_{b_2})D\gamma\theta + h_{b_2}Q(1-\gamma)(1-\theta)}{2} \right] + (1 - \lambda_p) \left[ \frac{D}{nQ(1-\gamma)(1-\theta)} \left( S_v + c_{iv}Q\gamma\theta \right) + h_v \left[ \frac{n(n-1)Q^2(1-\gamma)(1-\theta)}{2D} \right] \right] \tag{3.14}
\]

To get optimal solution of decision variables, we take the first partial derivative of \( Z \) respect to \( n \) and then set this equal to zero to get optimal value of parameter \( \lambda_p \) as follow
For fixed value of \( \lambda_i \in [L_{i-1}, L_i], i = 1, 2, 3, \ldots, N \), optimal shipment lot size \( Q \) and its safety factor, which yield the minimum of \( Z \) are solutions to the set of following equations \( \frac{\partial Z}{\partial Q} = 0, \frac{\partial Z}{\partial k} = 0 \) corresponding to the optimal value of \( \lambda_p^* \). Thereby, we have

\[
\lambda_p^* = \frac{1}{\left(1 + \frac{P}{D}\right)} \tag{3.16}
\]

\[
n^* = \sqrt{\frac{(1 - \lambda_p^*)D(S_p + c_{up}Q^*\gamma \theta)}{Q^*(1 - \gamma)(1 - \theta)} \left(\frac{\lambda_p^*F}{Q^*(1 - \gamma)(1 - \theta)} + \frac{(1 - \lambda_p^*)h_bQ^*(1 - \gamma)(1 - \theta)}{2D}\right)} \tag{3.15}
\]

\[
Q^* = \sqrt{\frac{\lambda_p^*D}{(1 - \gamma)} \left( S_b + n^*F + \frac{n^*\sqrt{T}}{2} \left(\sqrt{1 + k^*^2} - k^*\right) + C(L)\right) - \frac{(1 - \lambda_p^*)D\theta}{n^*(1 - \gamma)(1 - \theta)}} \tag{3.17}
\]

\[
k^* = \frac{\left(B_2 - 1\right)^2}{\sqrt{(1 - (B_2 - 1)^2)}}, \quad B_2 = \frac{2h_{b_k}}{\bar{h}_{b_1}(1 - \beta) + \left(\frac{D}{2q(1 - \gamma)}\right)} \tag{3.18}
\]

As we know from (3.17) and (3.18), an explicit form of \( k, Q \) and \( n \) are not possible, so we use a numerical example to get an optimal approximation of decision variables.

### 3.6. Numerical Simulation and Comparison Result

We use that proposed by Lin [22] to find the approximation of the optimal value decision variables and also the total cost of inventory system by Pareto optimal concept, Stackelberg’s scheme and integrated system. For numerical example, we consider an integrated inventory with single vendor, single buyer and single product with following example data: \( D = 1200 \) unit/year, \( \sigma = 8 \) unit/week, \( S_p = $400/\text{order}, S_p = $3000/\text{setup}, P = 3000 \) units/year, \( h_p = $14/\text{unit/year}, h_{b_1} = $15/\text{unit/year}, h_{b_2} = $11/\text{unit/year}, F = $20/\text{shipment}, c_{up} = $5/\text{unit}, c_{ib} = $0.5/\text{unit}, \pi_0 = $50/\text{unit}. The lead time has three components. We follow the lead time data that proposed in reference Lin [21]. We propose the numerical result of simulation data with vary in parameter \( \beta \) and \( \gamma \) value. Then we investigate the effects of parameters \( \beta \) and \( \gamma \) to the to the inventory total cost and decision variables. The optimum value result of decision variable and total cost function value as objective function by Pareto optimal scheme and also integrated scheme as comparison result. According to the numerical example result, we get some valuable information for our analytical result. Total cost of inventory system based on Pareto optimal scheme is more economically profitable to each party rather than integrated scheme. Although, they are included in the same category of strategy that is cooperative strategy, according to the numerical result about the effect of defective item rate \( (\gamma) \), the larger value of defective item rate, then it will be result in smaller value of optimum order quantity \( Q \) and it’s total cost. But, there is an interesting result about the value of defective item rate, we can say that an “bifurcation value” in the value of about 0.05. If the defective item rate is greater than those critical point, then the total value will increase than before. From this result, it’s recommended that tolerable value of defective item rate of this supply chain model is less than 0.05. In the other-hand, for fixed value of defective item, then the changes value of \( \beta \) also resulted in changes in the optimal value of total inventory cost. Based on numerical example, overall, in average, the greater value fraction of the demand in the stock out period \( \beta \) then it will result to the smaller value of total cost of inventory system.

\[
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4. Conclusion and Remarks
In this research, we propose the mathematical model of probabilistic inventory under probabilistic condition using three different schemes of the optimum solution. First, one is integrated scheme, the second one is non-cooperative game using Stackelberg optimum and then the third one is cooperative schemes using Pareto Optimal scheme. Because of the complexity of the model, the result all of the optimal solution from all of the scheme is proposed in implicit form. We investigate the optimal order quantity under two condition of game theory. While the vendor and the buyer agree to play cooperative strategy, then Pareto optimal scheme is used to get the optimal decision of each party.

Based on the analytical result, it’s difficult to get an explicit form of each decision variable. To get more information about that variable we can use numerical simulation with some appropriate data. Regarding the numerical example result, we get the result that total cost of inventory system based on Pareto Optimal scheme is more economically profitable to each party rather than integrated scheme. On the other hand, if both of vendor and buyer does not agree with cooperative schemes, or rather prefer to determine their own optimal solution, then Stackelberg games is used to analyze this situation. Based on the numerical result for optimal result, the Stackelberg Scheme is also more economically profitable than integrated scheme.

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