Calculation models and analysis methods of the parametric vibration of radial gates

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Abstract. Parametric vibration is one of the main reasons for the failures of radial gates. This paper presents three calculation models of the parametric vibration of radial gates which reflect the mechanical properties and geometric features of the gates. Two most commonly used analysis methods, i.e., Bolotin’s method and the finite element method, were described to determine the boundaries of regions of the dynamic instability of radial gates. A numerical example concerning finite element method was also demonstrated.

1. Introduction
Radial gates are commonly used as working gates because of lots of advantages. Safe operation of radial gates is very important for dams or spillways. Up to now, most radial gates were found in good conditions. But there were still a few radial gates which were destroyed because of severe vibrations. Figure 1 shows two destroyed engineering applications.

(a) (b)

Figure 1. Failures of radial gates[1]
The vibration of radial gates is a complex hydro-elasticity problem, and the mechanism of which has not be fully understood. Some researchers have analyzed the vibration of radial gates from the point of view of parametric vibration. For a rod subjected to a periodic longitudinal load, if the amplitude of the load is less than that of the static buckling value, the rod experiences only longitudinal vibrations. However, a straight rod becomes dynamically unstable and transverse vibration occurs for certain relationships between the disturbing frequency and the natural frequency of transverse vibration. Such transverse vibration is called parametric vibration. The amplitude of parametric vibration rapidly increases to large values at which it approaches such a resonance (so-called parametric resonance) and the rod loses dynamic stability. The load of parametric vibration is listed on the left-hand side of the equation of motion and is called parametric load [2]. One of the main aims of parametric vibration is to determine the region of dynamic instability. If the corresponding parameters fall in to these regions, parametric resonance occurs. According to the theory of parametric vibration, Zhang and Liu studied the dynamic stability of the arms of radial gates and found out that parametric resonance occurred in model test [3]. Based on the theory of Thin-walled structures, Niu and Hu gave the region of dynamic instability of radial gates [4]. Liu et al. analyzed the dynamic stability of the arm the radial gates with the consideration of bending, torsion and warping by using the theory of parametric vibration [5]. Lian et al. discussed different kinds of vibration stability of radial gates and pointed out that radial gates may lose dynamic stability in some cases [6]. Cai et al. developed a new method to detect whether the parametric resonance of the arms of radial gates occurred by using Green function [7]. Li and Lian established the calculation model of the parametric vibration of arms of radial gates under eccentric loading [8]. Zhu et al. summered the development of dynamic stability of radial gates and pointed out that the dynamic stability of main frames should be analyzed to reflect the spatial effect of radial gates [9]. Niu and Liu determined the region of dynamic instability of main frames of radial gate by using the finite element method [10, 11].

In order to provide the comprehensive information of parametric vibration of radial gate for readers, this paper presents the calculation models and the related analysis methods of the parametric vibration of radial gates.

2. Calculation models

Letting parametric load:

\[ P = \alpha P_{cr} + \beta P_{ct} \cos \theta t \]  

(1)

where \( P_{cr} \) is the static buckling load of structures, and \( \alpha \) and \( \beta \) are the static load parameter and dynamic load parameter, respectively; \( \cos \theta t \) is a periodic function with the period \( T = 2\pi/\theta \).

Figure 2 shows three commonly used calculation models of parametric vibration of radial gates.
Which calculation model will be used depends on the natural vibration characteristics of radial gate. For example, if the natural frequency of radial gate is close to that of the arm of radial gates which can be simplified to a simply supported bar, Figure 2(a) may be selected as the calculation model of parametric vibration of radial gates. Otherwise, plan frame model (Figure 2(b)) or spatial frame model (Figure 2(c)) should be used as the calculation models.

3. Analysis method

3.1. Bolotin’s method

Bolotin did many works in the area of parametric vibrations of elastic systems containing rods, beams, frames, plates and shells and he developed lots of analytical solutions for parametric vibration [2].

According to the theory of parametric vibration developed by Bolotin, the solution of the dynamic deflection of a rod when parametric vibration occurs can be expressed as:

$$v(x,t) = \sum_{n=1}^{\infty} f_n(t) \varphi_n(x)$$  \hspace{1cm} (2)

where $v(x,t)$ is the dynamic deflection of the rod; $\varphi_n(x)$ is the vibration mode of the rod; $f_n(t)$ is unknown function of time.

The governing equation of parametric vibration (i.e. Mathieu equation) has the following form:

$$C \ddot{f} + \left[ E - (\alpha + \beta \cos \theta t) P_{cr} A \right] f = 0$$  \hspace{1cm} (3)

where

$$f = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$  \hspace{1cm} (4)

$$C = \begin{bmatrix} 1 \\ \omega_1^2 \\ \omega_2^2 \\ \vdots \\ \omega_n^2 \end{bmatrix}$$  \hspace{1cm} (5)

$$E = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$  \hspace{1cm} (6)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$  \hspace{1cm} (7)

in which $\omega_n$ is the $n^{th}$ natural frequency of an unloaded rod and
\[ a_i = \frac{1}{\alpha^2} \int_0^\alpha \frac{d\phi_i(x)}{dx} d\phi_i(x) \, \text{dx} \]  

We seek the periodic solution with a period 2T in the form:

\[ f = \sum_{k=1,3,5,\ldots}^\infty \left( a_k \sin \frac{k\theta t}{2} + b_k \cos \frac{k\theta t}{2} \right) \]  

Substituting equation (9) into equation (3) and letting the obtained determinants of homogeneous systems be equal to zero yields:

\[
\begin{bmatrix}
E - \left( \alpha \pm \frac{1}{2} \beta \right) P_{cr} A - \frac{\theta^2}{4} C & -\frac{1}{2} \beta P_{cr} A & 0 & \ldots \\
-\frac{1}{2} \beta P_{cr} A & E - \alpha P_{cr} A - \frac{9\theta^2}{4} C & -\frac{1}{2} \beta P_{cr} A & \ldots \\
0 & -\frac{1}{2} \beta P_{cr} A & E - \alpha P_{cr} A - \frac{25\theta^2}{4} C & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{bmatrix} = 0
\]  

According to equation (10), we obtain the following equation used to compute the boundaries of the principal region:

\[ E - \left( \alpha \pm \frac{1}{2} \beta \right) P_{cr} A - \frac{\theta^2}{4} C = 0 \]  

For the simple calculation models of parametric vibration (e.g. figure 2(a)), Bolotin’s method gives analytical solutions which are easy to use to determine the region of dynamic instability.

3.2. Finite element method

For complex calculation models, finite element method is usually employed to formulate Mathieu equation of parametric vibration as follows:

\[ M \dot{u} + \left[ K - (\alpha + \beta \cos \theta t) P_{cr} S \right] u = 0 \]  

where \( u \) is the vector of node displacement; \( M, K \) and \( S \) are mass matrix, stiffness matrix and geometric stiffness matrix, respectively.

Similarly, the periodic solution with a period 2T has the form:

\[ u = \sum_{k=1,3,5,\ldots}^\infty \left( a_k \sin \frac{k\theta t}{2} + b_k \cos \frac{k\theta t}{2} \right) \]  

Substituting equation (13) into equation (12) and letting the obtained determinants of homogeneous systems be equal to zero gives:

\[
\begin{bmatrix}
K - \left( \alpha \pm \frac{1}{2} \beta \right) P_{cr} S - \frac{\theta^2}{4} M & -\frac{1}{2} \beta P_{cr} S & 0 & \ldots \\
-\frac{1}{2} \beta P_{cr} S & K - \alpha P_{cr} S - \frac{9\theta^2}{4} M & -\frac{1}{2} \beta P_{cr} S & \ldots \\
0 & -\frac{1}{2} \beta P_{cr} S & K - \alpha P_{cr} S - \frac{25\theta^2}{4} M & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{bmatrix} = 0
\]  

According to equation (14), we obtain the following equation used to determine the boundaries of the principal region:

\[ K - \left( \alpha \pm \frac{1}{2} \beta \right) P_{cr} S - \frac{\theta^2}{4} M = 0 \]
4. Numerical applications

One example of application of finite element method for computing the region of dynamic instability of radial gates is given below ( \( P = \alpha P_{cr} + \beta P_{cr} \cos \theta t \)).

![Figure 3. Geometry size of main frame of a radial gate](image)

The finite element model of this frame is as follows:

![Figure 4. Finite element model](image)

According to equation (15), the region of dynamic instability is shown in figure 5 (shaded part).

![Figure 5. Dynamic instability region (\( \alpha=0 \))](image)

5. Conclusions

This paper summarizes the research progress of the parametric vibration of radial gates. Three commonly used calculation models of parametric vibration were presented. Bolotin’s method and the finite element method which are employed to analyse the parametric vibration of radial gates were also introduced. It is hoped that the study will be useful for the researchers.
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