General relativistic massive vector field effects in Gamma-Ray Burst production

Fabrizio Tamburini,1 Mariafelicia De Laurentis,2,3 Lorenzo Amati,4,5 and Bo Thide6,7

1Scientist in Residence at ZKM – Centre for Art and Technologies for Media, Lorentzstraße 19, D-76135 Karlsruhe, Germany
2Institute for Theoretical Physics, Goethe University, Max-von-Laue-Str. 1, D-60438 Frankfurt, Germany
3Frankfurt Institute for Advanced Studies, Goethe University, Ruth-Moufang-Str. 1, D-60438 Frankfurt, Germany
4Istituto Nazionale di Astrofisica — IASF Bologna, via P. Gobetti 101, I-40129 Bologna, Italy
5International Center for Relativistic Astrophysics, Piazzale della Repubblica 2, I-65122, Pescara, Italy
6Swedish Institute of Space Physics, Ångstrom Laboratory, P. O. Box 537, SE-75121, Sweden
7Acro Swedish ICT AB, P. O. Box 1070, SE-16425 Kista, Sweden

To explain the extremely high energy release, > 10^{53} \text{erg}, suggested by the observations of some Gamma-Ray Bursts (GRBs), we propose a new energy extraction mechanism from the rotational energy of a Kerr-Newman black hole (BH) by a massive photon field. Numerical studies show that this mechanism is stable with respect to the BH rotation parameter, a, with a clear dependence on the BH mass, M, and charge, Q, and can extract energies up to 10^{54} \text{erg}. The controversial "energy crisis" problem of GRBs that does not show evidence for collimated emission may benefit from this energy extraction mechanism. With these results we set a lower bound on the coupling between electromagnetic and gravitational fields.

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The exact Gamma-Ray Burst (GRB) energy production mechanism is still a matter of debate. One possible explanation is the energy release during the formation of a rotating black hole (BH) surrounded by the matter of a rapidly collapsing massive star and, to account for the high energy observed, one has to find very efficient energy extraction mechanisms from the BH [1–3]. Other possible mechanisms can be high-energy phenomena of energy pulse from electron-positron pair production and the photon plasma fluid created by vacuum polarization in the dyadosphere [4, 5], a region around a charged BH that extends from the exterior event horizon \( r_+ \) to a given radius \( r_a \) that depends on the mass and charge of the BH [6–9].

Vacuum polarization effects require strong magnetic fields that mimic the effects of a charged and rotating BH described by the Kerr-Newman solution [10]. However, because typical astrophysical systems show a strong tendency to eliminate any net electric charge, a charged BH is not a realistic physical solution and cannot be generated by the gravitational collapse of a core larger than the neutron star critical mass endowed with an electromagnetic field. A reasonable physical solution is to consider energy extraction mechanisms where a BH that "acquires" a temporary fictitious net charge because of vacuum polarization effects and/or generates an electrostatic field that extends towards the neighborhoods of the event horizon through a selective capture of charged particles. This temporary fictitious charge is expected to dissipate on time scales \( \tau < 10^7 \text{s} \), long enough to set up the GRB energy extraction mechanisms occurring on much shorter time scales during the collapse of the star and thus justify the use of Kerr-Newman based spacetimes [4, 9].

As described in Ref. [11], photons can acquire mass due to gravitational and electromagnetic field coupling. Exploiting this possibility, we here describe a new mechanism of energy extraction from black-hole rotational energy made possible by vacuum polarization effects occurring in the dyadosphere around a Kerr-Newman BH.

The main motivation for this work arises from observations of GRBs that do not show evidence of collimated emission [12–16]. As shown, e.g., by Amati and Della Valle [17], GRBs have a distribution of released energy, in terms of equivalent isotropic radiation peaking at about \( E_{\text{iso}} \sim 10^{53} \text{erg} \), with a clear dependence on the BH mass, \( M \), and charge, \( Q \). The "energy crisis" problem of GRBs that does not show evidence for collimated emission may benefit from this energy extraction mechanism. With these results we set a lower bound on the coupling between electromagnetic and gravitational fields.

The collimated emission scenario often called for to solve this "energy crisis" implies the detection in the light curves of the afterglow of a sudden "achromatic" change in the slope (break) at the time when the relativistic beaming angle becomes larger than the physical jet opening angle as a consequence of the slowing down of the ejected shells [19]. Based on this method, beaming angles between 4 and 9 degrees can be estimated for a number of GRBs [14, 20–22], decreasing their energy budget to below \( \sim 5 \times 10^{52} \text{erg} \). This is comparable to the kinetic energy of so called hypernovae, a sub-class of SNe-Ibc often associated with GRBs [23].

However, in some cases this change in the slope is not observed [13–16], suggesting that some GRBs could be characterized by poorly collimated emission. A quasi-isotropic emission is indeed often observed in low-luminosity GRBs [24], but in this case the energy crisis is avoided by the low amount of energy associated with \( E_{\text{iso}} \sim 10^{50} \text{erg} \), as shown in Fig. 1. However the existence of high-luminosity GRBs [24, 25] that do not exhibit an "achromatic" change in the slope of the afterglow lightcurves, poses the question to pinpoint an alternative mechanism of energy production in GRBs other than the ones normally adopted, such as "failed supernovae" [27].

We find that the energy extraction from a Kerr-Newman BH by a massive photon (vector) field can be as large as \( \sim 10^{54} \text{erg} \), which is indeed the upper limit of the energy budget observed...
The electromagnetic (EM) potential enters actively into the mathematical structure of the space-time manifold described by the Proca-Maxwell equations beyond the scope of this Letter and would need a deep understanding of a possible unification between electromagnetism and gravity [36, 37] as a hidden gauge invariance of the Proca-Maxwell equations preserving the Lorentz invariance [38]. The Proca-Maxwell Lagrangian density $\mathcal{L}$ describing a massive electromagnetic (EM) field contains a mass term $\mu_γ^2 A_μ A^μ/2$ because of the gauge invariance [see Ref. 39 Eq. (3)]

$$\mathcal{L} = -\frac{1}{4} F^μν F_μν - j_μ A^μ + \frac{1}{2} \mu_γ^2 A_μ A^μ,$$

where $\mu_γ^{-1}$ is the reduced Compton wavelength associated with the photon rest mass and $j_μ = (ρ, -j)$ is the 4-current. $F^μν$ is the electromagnetic field tensor and $A_μ$ the 4-vector potential. From the Lagrangian density, one obtains the covariant form of the Proca-Maxwell equations

$$\frac{∂ F_μν}{∂ x_ν} + \mu_γ^2 A_μ = 4π j_μ ,$$

that lead to the Proca wave equation for the 4-potential $A_μ$

$$(□ - \mu_γ^2) A_μ = -4π j_μ ,$$

with the constraint derived from the massive photon in a medium $(∂^μ A_μ = 0)$. Proca-Maxwell equations include a mass-like term for the photon due to light-matter interaction, viz., a term encompassing the photon’s interaction with the plasma (photon-plasmon interactions).

Photons and plasmas themselves gravitate, but not their coupling, unless a non-minimal coupling of the EM vector potential to gravity is present in the Lagrangian, $\mathcal{L}_R = \xi RA^\nu A_ν$, where $R$ is the curvature scalar and $\xi$ a coupling constant. Because of this, photons carry a finite rest mass, and an additional term appears in Einstein’s equations. This coupling is thought to occur, e.g., in the presence of a charge asymmetry [11] as is supposed to happen in regions near the forming BH of a GRB. An exhaustive theoretical description of this coupling is beyond the scope of this Letter and would need a deep understanding of a possible unification between electromagnetism and gravitation.

The stress-energy tensor of Proca-Maxwell equations would not be traceless like that of Maxwell’s equations, but due to symmetry breaking and EM and gravitational coupling, the total photon “Proca mass” would introduce an additional mass term, being intrinsically gravitating and thus included as a source in Einstein’s equations. When photons acquire such an additional mass term, the spacetime curvature is unavoidably modified and the environment around the BH is endowed with a massive vector field described by the Proca-Maxwell equations of the EM field. See, e.g., Refs. 40-45 for more details.

If the plasma is turbulent, then one has to consider an additional effect due to the spatial structure of the plasma itself at the plasma resonance frequencies. In fact, unlike the mathematical structure of the space-time manifold described by the
Lorentz group in which space is homogeneous and isotropic and time homogeneous, a plasma may exhibit peculiar spatial/temporal structures that break the space-time symmetry and generate the conversion of a fraction of the Proca mass into photon orbital angular momentum (OAM) that acts as a mass reducing term in the photon Proca mass \[ \ell \]. In this case the OAM term changes the mass in the Proca-Maxwell equations and the Yukawa potential of the space-time curvature \[ \frac{\mu}{\gamma} \], preserving, in any case, the sign of the norm of the metric tensor, \[ \| g \| \] as the acquired mass never becomes negative \[ \ell \].

Accretion of a massive neutral field in the dyadosphere is much more efficient than accretion involving charged particles and can supply or even replace the electromagnetic pulse that is expected to occur to generate the GRB. Moreover, the presence of these massive photons modify the original Kerr-Newman metric by introducing a Yukawa potential that depends on the electrostatic charge \( Q \) of the BH, via the term \( \Xi(Q) \) describing the charge asymmetry. For the simplest model of charged BH, one obtains \( \Xi(Q) = Q^2 / 4\pi \).

The modified Kerr-Newman metric in natural units where \( e = G = c = 1 \), and in the Boyer-Lindquist coordinates \( (t, r, \theta, \phi) \), with the presence of the massive photon field also carrying OAM, becomes

\[
ds^2 = -(1 - \frac{2Mr + \Xi(Q)e^{-\mu\sigma r}}{\rho^2} + \mu_\gamma I_\Sigma)dt^2 + \left(\frac{r^2 + a^2}{\rho^2} - \frac{2Mr + \Xi(Q)e^{-\mu\sigma r}}{\rho^2} + \mu_\gamma I_\Sigma\right)^{-1}dr^2 + \rho^2d\theta^2 + \left[\rho^2\sin^2\theta + \left(\frac{2Mr + \Xi(Q)e^{-\mu\sigma r}}{\rho^2} - \mu_\gamma I_\Sigma\right)a^2\sin^2\theta\right]d\phi^2 + 2a\sin^2\theta \left(\frac{2Mr + \Xi(Q)e^{-\mu\sigma r}}{\rho^2} - \mu_\gamma I_\Sigma\right)d\phi dt \quad (4)
\]

where \( \rho^2 = r^2 + a^2 \cos^2\theta \) and \( I_\Sigma = \int_r^\infty \Xi(Q)(e^{-\mu\sigma r}/\rho^2) \, dr \).

To the first order, when \( h = 1 \), the metric shows a dependence on the absolute value of the OAM acquired by photons \[ \ell \], viz.,

\[
\mu_\gamma \sim 2\pi \left( P_\mu - \ell \frac{D_\mu \tilde{\sigma}\sin(qr)}{2P_\mu} \right), \quad (5)
\]

and the parameters characterizing the Proca mass are

\[
P_\mu = \mu_G + \sqrt{B_\mu + C_\mu - D_\mu \cos(qr)}, \quad (6)
\]

where

\[
B_\mu = E\omega_\mu^2 \frac{1 + \varepsilon}{E + \hat{v} \cdot \nabla \phi}, \quad (7a)
\]

\[
C_\mu = \frac{4\pi \delta \nu_\mu - 4\pi \hat{v} \cdot \nabla \phi}{E + \hat{v} \cdot \nabla \phi}, \quad (7b)
\]

\[
D_\mu = \frac{4\pi \phi \delta \nu_\mu}{E + \hat{v} \cdot \nabla \phi}. \quad (7c)
\]

If the EM field and the plasma density tend to zero, together with \( \ell \), the virtual and actual photon mass, \( \mu \) and \( \mu_T \) go to zero thus recovering the Kerr-Newman spacetime.

For a BH with mass in the range of \( 3 \sim 5 M_\odot \), electric charge \( Q/M \sim 7 \times 10^{-3} \sim 7 \times 10^{-2} \) and rotation parameter \( a \leq 1 \), we find that the Yukawa potential is always confined between the radii \( r_1 = 2.95 \times 10^6 \, \text{cm} \) and \( r_2 = 2.81 \times 10^7 \, \text{cm} \). The numerical results indicate that energy produced by the Penrose extraction process of the massive photon field provides the lacking of energy budget required to reach the \( 10^{54} \, \text{erg} \) needed for isotropic emission, having the same order of magnitude as in the GRB process, \( 10^{47} \sim 10^{54} \, \text{erg} \), as reported in Tab.\[ \ell \] The lower bounds on photon mass values obtained by varying \( \ell \) as a free integer parameter, are those derived from massive vector fields, according to Refs.\[ \ell \] and \[ \ell \] in the range of \( \mu_T \sim 10^{-53} \sim 10^{-51} \, \text{g} \).

We observe a strong dependence on the BH mass \( M \) and charge \( Q \), whereas the BH rotation parameter, \( a \), plays a minor rôle. These values allow us to set the lower bound for the coupling constant between the EM and gravitational field to \( \tilde{\xi} \sim 10^{-38} \).

Accretion onto a Kerr-Newman black hole of massive neutral vector fields can explain the “energy crisis” problem of GRBs. Photons propagating in a structured plasma and in a strong gravitational field acquire mass and orbital angular momentum because of the hidden gauge invariance due to the Anderson-Higgs mechanism in Proca-Maxwell equations when there is an effective coupling between the gravitational and electromagnetic fields. The interplay between BH mass, charge, and rotation for the energy extraction from the

| Table I. Contribution of a massive photon field to the energy produced by the Penrose extraction process from a rotating charged black hole. |
| --- |
| \( \mu_T \) (g) | \( M_\odot \) (g) | \( a \) | \( Q/M \) | Energy |
| \( 1.78 \times 10^{-51} \) | 3 | 0.1 | \( 7 \times 10^{-3} \) | \( 2.88 \times 10^{31} \) |
| \( 1.78 \times 10^{-53} \) | 5 | 0.5 | \( 7 \times 10^{-3} \) | \( 5.77 \times 10^{32} \) |
| \( 1.78 \times 10^{-53} \) | 3 | 0.9 | \( 7 \times 10^{-2} \) | \( 2.88 \times 10^{35} \) |
neutrally-charged massive photon field gives the correct order of magnitude of the energy expected for a GRB, showing a strong dependence on the charge $Q$, with the result of diminishing the importance of the rotation parameter in the energy extraction from the BH. This effect, similar to that provided by charged particles falling into the BH [33], could flatten the possible differences in the energy distribution due to differently rotating BHs.

Superradiant instabilities because of the small mass of the BH in the GRB precursor considered in our calculations do not play a crucial rôle in the too rapid formation of the GRB. The effects induced by the plasma turbulence on the photon mass, effective at the frequencies where the plasma is resonant, seem not to play a significant rôle in the energy extraction. Moreover, this study could also be used in the energy budget in massive neutrino-antineutrino fireball models if the couples so generated experience both a Penrose process and viscous dissipation [47] or in the accretion of dark matter vector fields.

From our results we can set a lower bound for the coupling constant between the EM and gravitational field to $\xi \sim 10^{-38}$ that might contribute to electromagnetic emission also in those scenarios where EM waves are not expected, such as in BH-BH collisions, as hypothesized for the recent LIGO GW150914 gravitational wave detection and possible Fermi event coincidence observed after 0.4 s in the electromagnetic spectrum. [48][50].

The effect of this field coupling might also be accompanied with other non-linear gravitational wave interactions with plasmas [51]. No direct transfer of OAM from Kerr metric lensing to photons [52] has been considered because of the physical properties inside the dyadosphere.

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1. fabrizio.tamburini@gmail.com
2. laurents@th.physik.uni-frankfurt.de
3. amat@iasbo.inaf.it
4. bt@irfu.se

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