A Comparative Study of Gauge Fixing
Procedures
on the Connection Machines CM2 and CM5

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Abstract

Gauge fixing is a frequent task encountered in practical lattice
gauge theory calculations. We review the performance character-
istics of some standard gauging procedures for non-Abelian gauge theories, implemented on the parallel machines CM2 and CM5.

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1 Introduction

Large-scale computer simulations have become an important tool in the theory of elementary particles. There is a very rich empirical material to test quantum chromodynamics (QCD), which is held to be the basic theory of strong interactions [1]. Most of it is related to low energy data such as the spectrum and the structure of hadrons, where nonperturbative methods such as lattice gauge theory (LGT) in four dimensions are indispensable [2].

In many practical situations of LGT it is necessary to perform gauge fixing on the discrete lattice. This is in particular the case if one is interested in the computation of gauge-noninvariant quantities such as gluon propagators [3] [7], Bethe-Salpeter amplitudes [4] or monopole densities [5]. Gauge fixing might also be helpful to reduce noise in the measurement of gauge invariant quantities (such as Polyakov lines or Wilson loops) [6], or in the context of preconditioning [10].

Various methods have been proposed to achieve gauge fixing [10] [13]. They are based on iterative schemes, which might need many iteration steps and thus tend to be rather time consuming. It is therefore of considerable interest to study their convergence behaviour and the efficiency of their implementations on parallel computers.

In this paper we will present an introduction into the issues and compare two such algorithms for Landau gauge fixing in lattice QCD, on our local connection machines, CM2 and CM5. We will also consider some variants of these basic algorithms, that aim at possible acceleration gains.

2 Gauge fixing on the lattice

QCD gauge fields are defined on the lattice in terms of parallel transport operators, $U_\mu(x) \in SU(3), \mu = 1..4$, that live on the links between neighbouring sites $x, x + \hat{\mu}$ of a fourdimensional hypercubic lattice and are related to the continuum fields $A_\mu(x)$ by

$$A_\mu = \frac{1}{2i} \left( U_\mu(x) - U_\mu^+(x) \right) \big|_{\text{traceless}},$$

(1)

$U_\mu(x)$ are matrices in a three-dimensional ‘color’ space, and the set $\{U_\mu\}$ is called a configuration. Under a local gauge transformation $U_\mu(x)$ transforms like

$$U_\mu(x) \longrightarrow U_\mu(x)^{(G)} = G(x)U_\mu(x)G(x + \hat{\mu}),$$

(2)

where the matrices $G$ are elements of the gauge group $SU(3)$ and are associated to the lattice sites.
In the continuum formulation the local Landau gauge fixing condition, $\partial_\mu A^\mu = 0$, can be viewed as the solution to the global variational problem of minimizing the integral $\int d^4x A_\mu A^\mu$. The gauge fixing condition in its global form can easily be transformed into a lattice relation, in terms of $U_\mu(x)$:

$$F(G) := \text{Re Tr} \sum_\mu \sum_x U_\mu(x)^{(G)} = \text{Extremum}. \quad (3)$$

Obviously, $F(G)$ has an upper and a lower bound. Accordingly, there exist both an absolute minimum and maximum. Notice that both maximizing and minimizing $F(G)$ will satisfy condition (3). Hence we have at least two Landau configurations which differ by gauge transformation. This is due to the fact that, for $SU(3)$ gauge theory, the Landau condition is not sufficient to fix the gauge completely, which leads to the notorious phenomenon of Gribov copies [9].

We consider next a given configuration to be cast into Landau gauge. This will be achieved by driving $F$ to an extremum, in an iterative process. For constructing $G(x)$, we will follow the algorithms introduced in refs. [10][13], which will be referred to as Cornell [10] and Los Alamos [13] methods.

a. The Cornell Method

In a convergent scheme, the local gauge transformation $G(x)$ is expected to approach unity with increasing iteration number $i$. The Cornell method therefore starts off from the ansatz

$$G(x) = e^{-i\alpha \sum_\mu \partial_\mu A^\mu(x)}, \quad (4)$$

which is expanded to first order:

$$G(X) \approx 1 - i\alpha \sum_\mu \partial_\mu A^\mu(x). \quad (5)$$

This approximation requires a subsequent projection of $G(x)$ back into the group space. $A_\mu(x)$ is given by eq. (1) and the lattice form of $\partial_\mu A^\mu(x)$ is

$$\partial_\mu A^\mu(x) = A_\mu(x) - A_\mu(x - \hat{\mu}). \quad (6)$$

The quantity $\alpha$ is a real parameter to be suitably chosen, depending on the underlying lattice.

b. The Los Alamos Method

One introduces

$$w(x) = \sum_\mu U_\mu(x) + U_\mu^+(x - \hat{\mu}) \quad (7)$$

$w(x)$...
and rewrites eq.(3) as

\[ F(G) = \frac{1}{2} \text{Re} \text{Tr} \sum_x \sum_{\mu} U_{\mu}^{(G)}(x) + U_{\mu}^{(G)^+}(x - \hat{\mu}) \]

(8)

\[ =: \frac{1}{2} \text{Re} \text{Tr} \sum_{x \in \text{red or black}} w^G(x). \]

(9)

The basic idea is now to construe \( G(x) \) such that \( F \) changes its size monotonically from one iteration step to the next. This construction starts from a checkerboard (red-black) subdivision of the lattice, with \( G(x) \) being equal to unity on the red (black) sites at even (odd) iteration steps. The local gauge transformation now simplifies to

\[ U_{\mu}(x) \rightarrow G(x).U_{\mu}(x).1 \]

(10)

\[ U_{\mu}(x - \hat{\mu}) \rightarrow 1.U_{\mu}(x - \hat{\mu}).G^+(x) \]

(11)

and can be carried out in parallel on the entire lattice. This iteration step can be recast into the simple form

\[ w(x) \rightarrow w^G(x) = G(x) w(x). \]

(12)

The non-unity part of \( G(x) \) is chosen to be a projection of \( w(x) \) onto the group manifold that obeys

\[ \text{Re tr} G(x)w(x) \geq \text{Re tr} w(x). \]

(13)

Note that the change in \( F \) is due to independent local changes in \( w(x) \). For this reason one has to interchange the role of red and black points after each step.

In the following we will study the convergence behaviour of these basic methods in conjunction with suitable acceleration techniques, implemented on the massively parallel machines CM2 and CM5. We present results obtained on 8 gauge configurations of a \( 8^4 \) hypercubic lattice, equilibrated at \( \beta = 5.7 \), as well as 2 configurations of a \( 16^4 \) lattice, at \( \beta = 6.0 \). These lattices were generated using a combination of heat bath and overrelaxation sweeps. The configurations are separated from each other by 1000 sweeps.

3 Techniques

3.1 Reunitarization

We have seen that proper unitarization is an important feature of gauge fixing iterative schemes. The Gram-Schmidt orthonormalization scheme (GS) works very
well for the Cornell method but not at all for the Los Alamos algorithm, as it does not ensure the validity eq. (13). This can be achieved by the projection method of maximal trace (MaxTr), which is based on the Cabbibo-Marinari trick:\[11\]:

The projection \( G(x) \) of a 3x3 matrix \( w(x) \) onto the \( SU(3) \) group is computed iteratively with the recursive step

\[
G_i(x) \rightarrow G_{i+1}(x) = A_1^i A_2^i A_3^i,
\]

and the initial condition \( G_0(x) = w(x) \), where

\[
A_1^i = \begin{pmatrix}
\tilde{G}_{i1}^* + \tilde{G}_{i2}^* & -\tilde{G}_{i1}^* + \tilde{G}_{i2}^* & 0 \\
\tilde{G}_{i1}^* - \tilde{G}_{i2}^* & \tilde{G}_{i1}^* + \tilde{G}_{i2}^* & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

\[
A_2^i = \begin{pmatrix}
\tilde{G}_{i1}^* + \tilde{G}_{i3}^* & 0 & -\tilde{G}_{i3}^* + \tilde{G}_{i3}^* \\
0 & 1 & 0 \\
\tilde{G}_{i1}^* - \tilde{G}_{i3}^* & 0 & \tilde{G}_{i1}^* + \tilde{G}_{i3}^*
\end{pmatrix},
\]

\[
A_3^i = \begin{pmatrix}
1 & 0 & 0 \\
0 & \tilde{G}_{i2}^* + \tilde{G}_{i3}^* & -\tilde{G}_{i2}^* + \tilde{G}_{i3}^* \\
0 & \tilde{G}_{i2}^* - \tilde{G}_{i3}^* & \tilde{G}_{i2}^* + \tilde{G}_{i3}^*
\end{pmatrix}.
\]

Denoting,

\[
N_1^i = \sqrt{|G_{i1}^* + G_{i2}^*|^2 + |G_{i1}^* + G_{i2}^*|^2},
\]

\[
N_2^i = \sqrt{|G_{i1}^* + G_{i3}^*|^2 + |G_{i3}^* + G_{i3}^*|^2},
\]

\[
N_3^i = \sqrt{|G_{i2}^* + G_{i3}^*|^2 + |G_{i2}^* + G_{i3}^*|^2},
\]

the elements \( \tilde{G}_{mn}^i \) in each matrix \( A_k^i \) equal \( G_{mn}^i \), divided by the corresponding scaling factor \( N_k^i \).

By construction, \( G_i \in SU(3), i \geq 1 \) and \( Tr(G_{i+1}G_0) \geq Tr(G_0) \). We achieve maximal trace (within our 64 bit machine accuracy) after about \( N \sim 5 - 7 \) steps, and use \( G_N \) for the projection of \( w \).

### 3.2 Convergence criteria

We have to define an appropriate quantity that can serve as a monitor for the quality of gauge fixing achieved during the iteration process. Our aim is to minimize the quantity \( \delta = | \text{extremum}(F) - F_i | \). As \( \text{extremum}(F) \) is not known during the
iterative procedure, we have to use other quantities than \( \delta \) to judge the convergence. In the literature two possibilities have been considered:

1. One can use

\[
\sigma_1 = \frac{1}{n_c L^4} \sum Tr (\partial_\mu A^\mu)(\partial_\mu A^\mu)^+ \tag{14}
\]

as a direct measure of fulfillment of the Landau gauge condition. \( L \) is the lattice size, and, for the \( SU(3) \) gauge theory, \( n_c = 3 \).

2. As \( G_i \rightarrow \) unit operator with \( i \rightarrow \infty \) the average trace of the gauge matrices

\[
\sigma_2 = 1 - \frac{1}{n_c L^4} Re Tr \sum G(x) \tag{15}
\]

can serve as an alternative monitor for convergence.

As \( F \) is quadratic in all variables, we propose here as a third possibility to employ the rate of change in the iteration step \( F_i \rightarrow F_{i+1} \) as a criterium for convergence achieved at step \( i \)

\[
\sigma_3 = F_{i+1} - F_i. \tag{16}
\]

Notice that \( F \) behaves monotonically.

![Figure 1](image1.png)

**Figure 1:**
left: The convergence behaviour in maximizing \( F \), using the Cornell method and reunitarization procedure of maximal trace. right: The same for Los Alamos method.

\(^1\)In, extremum(\( F \)) is apparently estimated by the last \( F_N \) where \( N \) is the last iterative step. This is however a very misleading quantity.
We see from Fig. 1 that the three signals differ considerably in size. One must thus be careful when comparing the quality of gauge fixing quoted by different authors. Nevertheless, to the degree that the signals $\sigma_j$ provide useful measures for the distance $\delta$, we would expect them to show similar shapes during the iterative process, i.e. coincident positions of maxima and minima. This is indeed the case, as the signals can nearly be made to coincide by appropriate rescaling (see Fig. 2).

So far, we have considered gauge transformations for maximizing $F(G)$. In order to drive the system into a minimum of $F$, we simply revert the previous construction, using $G^{-1} = G^+$ instead of $G$. We see in Fig. 3 that the convergence behaviour of the maximizing and minimizing procedures is very similar during the first several hundred iterations. After that, the minimizing iterative scheme starts to oscillate badly, with poor convergence compared to the maximizing case. Recall, however, that the minimization procedure yet renders a monotonic behaviour in $F$.

4 Accelerating Procedures

The discussed algorithms, which are basically local, perform sufficiently on small lattices, such as $8^4$. On larger lattices, however, they show poor efficiency, due to the phenomenon of critical slowing down which occurs when the convergence modulating matrix carries a large range of eigenvalues. Various methods have been proposed to speed up relaxation, like preconditioning in the Fourier space[10], overrelaxation[12] or multigrid schemes[13][16]. We want to comment here shortly on some of these methods as implemented in data-parallel computing.
Figure 3: $\sigma_1$ obtained by maximizing and minimizing $F$ using different reunitarisation methods. After several hundred iterations, it begins to oscillate widely.

**Fourier acceleration**\(^\text{[10]}\) We consider the convergence behaviour of the Cornell method which it is controlled by the matrix $\sum_\mu \partial_\mu A^\mu$. In momentum space, the modes of this matrix converge with relaxation times proportional to $\frac{1}{p^2}$. Thus the overall relaxation $\tau$ time will be determined by the smallest momentum states: $\tau \simeq \frac{1}{p_{\text{min}}^2} \simeq L^2$. To overcome this critical slowing down, one modifies the gauge transformation matrix $G(x)$ in Fourier space such that all modes converge to zero at the same rate (as fast as the fastest mode):

$$G(x) = e^{-i\alpha W} \rightarrow e^{-i\tilde{F}^{-1}\left(\frac{ap_{\text{max}}^2}{p^2}\tilde{F}(W)\right)} \quad (17)$$

where $\tilde{F}$ denotes the Fourier transform.

In Fig. 4 we see that this preconditioning reduces the required number of iterations by a factor of about two\(^\text{[10]}\).

Note however, that the cost of the Fourier transformation is non-negligible on parallel machines, due to its wide range communication requirements. We used the fast Fourier transform (FFT) subroutine, as provided by the scientific subroutine\(^2\) Nevertheless this result is not as expected from ref\(^\text{[10]}\) which quotes a gain factor of 7!
library (CMSSL) on the CM2. This subroutine requires a special ordering of parallel data, the so called SEND ordering. Our axis are, however, NEWS-ordered. In this case FFT carries out an internal reordering from NEWS to SEND data structure, prior to the actual computation of FFT\(^{18}\). As a result, the iteration step on the CM2 is slowed down by a factor 15, through the two FFT steps, Eq. 17.

![Figure 4: The effect of Fourier acceleration on the convergence behaviour for the 8\(^4\) lattices.](image)

**Overrelaxation** Overrelaxation methods are widely used in improving convergence of iterative methods, as well as in overcoming critical slowing down in Monte Carlo updating. In ref.\[12\] Mandula and Ogilvie applied overrelaxation ideas to lattice gauge fixing. They replaced the gauge transformation matrix \(G(x)\) by an infinite series

\[
\tilde{G} = \sum_{n=0}^{\infty} \frac{[\omega]_n}{n!} (G - 1)^n,
\]

where

\[
[\omega]_n = \frac{\Gamma(\omega + 1)}{\Gamma(\omega + 1 - n)}.
\]

The overrelaxation parameter \(\omega\) can take values between 1 and 2. The optimal choice is to be made empirically\[1\]. For our 8\(^4\) lattices we found that the best value for \(\omega\) lies near 1.45 (cf. Fig. 5). We truncated the series after two terms (this corresponds to the original form introduced by Adler\[17\]). Calculations with higher order terms showed similar behaviour. It is worth mentioning that the overhead of the overrelaxation is very small.

**Stochastic overrelaxation**\[13\] In this method one applies a local gauge transformation \(G(x)^2\) with probability \(p\), instead of always applying \(G(x)\). For \(p = 1\) the

\[\omega_{opt} \sim \frac{2}{1 + \frac{2}{\omega}},\]

where \(L\) is the lattice size.\[16\][12]
procedure definitely diverges. This “go wrong once in a while” principle has the capability, however, to render a considerable speed up of convergence of the two gauging methods treated in this work. The actual acceleration gain turns out to depend strongly on $p$. For our $8^4$ lattices we found best convergence to occur at $p \sim 0.9$.

Figure 5: $\sigma_1$ for different overrelaxation parameters.

Figure 6: Efficacy of stochastic overrelaxation as a function of the probability $p$.

5 Performance on CM2 and CM5

The Connection Machines in Wuppertal The CM2 at the University of Wuppertal is an 8k machine with 256 64-bit floating point accelerators and 256 MByte memory. Data IO is performed on a disk parallel storage system (Data Vault), with
10 Gbyte capacity. The CM5 is configured as a 32 processor nodes machine with 4 vector units to the node, and a 16 Gbyte scalable disk array (SDA). The CM5 can be used in both message passing and data parallel models. Given the 4 dimensional hypercubic structure of our lattices, we implement the gauging procedures in the data parallel model.

**Implementation** In the following we will describe some features of our CMFORTRAN implementation on the above CM2 and CM5 machines.

First, we have to establish a proper distribution of our data (gauge fields \( \{ U \} \)) among the processors. This has to be chosen to minimize data traffic. The data layout has of course a direct influence on the subgrid structure, that the CMF compiler produces. This is especially important on the CM5 because its performance is heavily constrained by communication. Physically, we are dealing with nearest neighbour interactions. Technically, on the CM5, our nearest neighbour interactions are mapped – by the cshift operation within CMF – onto data movements which occur in-processor, on-chip and between-chips, in decreasing order in speed. It must therefore be the main goal to attain a layout that minimizes the amount of off-chip cshift operations. That implies a subgrid geometry, for which most cshifts are to be performed on the longest axis. This is achieved by assigning suitable weights to each axis.

The Cornell method, for example, is obviously isotropic in the sense, that all space time directions are equivalent with respect to the amount of shift operations required. It is therefore natural to select the “canonical” layout for the gauge field \( \{ U \} \), with all space time axis declared to be parallel with equal weights. The two matrix (color) indices of \( \{ U \} \) and the Lorentz index \( \mu \) are chosen to be serial, which leaves us finally with the array structure

\[
U^{(\text{cornell})} = U(n_c, n_c, n_l, L, L, L, L) \\
\text{cmf\$layout} \quad U(: \text{serial}, : \text{serial}, : \text{serial}, : \text{news}, : \text{news}, : \text{news}, : \text{news}).
\]

\( n_c = 3 \) is the dimension of color space, \( n_l = 4 \) corresponds to the range of the Lorentz index and \( L \) is the linear size of our hypercubic lattice.

The Los Alamos method, on the other hand, induces an asymmetry into the code, due to the red black splitting. We map the entire lattice onto the red (black) part using a restricted lattice with a geometry of the form \((L/2, L, L, L)\). In order to store all the links on such a lattice we double the range of one particular serial index, which we choose to be the Lorentz index, \( \mu \). Due to the resulting asymmetric geometry, one expects to enhance the performance in assigning appropriate weights to the four parallel axis. We found that the best performance is achieved by assigning a relative weight of two to the short axis. As a result, we work with the array

\[
U^{(\text{LosAlamos})} = U(n_c, n_c, 2n_l, L, L, L) \\
\text{cmf\$layout} \quad U(: \text{serial}, : \text{serial}, : \text{serial}, 2 : \text{news}, : \text{news}, : \text{news}, : \text{news}).
\]
Performance Data For clarity, we restrict ourselves in the following to quoting measurements from codes produced by the CMF compiler (release cmf 2.1 beta 0.1 on the CM5), using complex arithmetic in double precision. On the CM2, we are running the slicewise CMF compiler (cmf 1.2). It goes without saying, that there is room for improvement of the pure FORTRAN code by resorting to lower level language programming in some kernel routines.

We find that the local SU$(3)$ multiplication has a performance of about 1.2 Gflops (370 Mflops) on our CM5 (CM2) for a $16^4$ lattice. On the CM5, this corresponds to about 30% of the peak rate of 4 Gflops. The reason for this rather low performance lies in the fact that the present CMF compiler does not yet produce optimal complex arithmetic for the vector units: it translates a series of complex number multiplications into a code with too many add-load and mul-load commands, rather than mul-adds.

An important issue is the additional degradation of these performance characteristics through communication. Two features have a large impact on this latter loss: 1. The size and shape of the subgrids residing on the individual processors – the relevant surface effects in communication can be influenced by the programer’s layout; 2. the latency of the cshift operations during run time. As a result we find, on the CM5, the performance $P(8^4)$ for the $8^4$ lattice to be only 65% of $P(16^4)$. For this reason the data presented here refer to $16^4$ lattices only.

In table 1 we compare performance figures from CM2 and CM5, as measured on the Cornell algorithm. Notice that interprocessor-communication is needed when calculating the quantity $\sum \partial_{\mu} A_{\mu}$ according to Eq. 6 configuration \{U_{\mu}\}, and when performing the gauge transformation

$$U_{\mu}(x) \rightarrow G(x)U_{\mu}(x)G(x + \hat{\mu}).$$

In table 2 we present the corresponding performance data from the Los Alamos algorithm. Within this method, the gauging step proper is carried out locally. After each such step, however, one must rearrange the links from red (or black) into black (or red) ordering. This gather/scatter stage involves the communication and deteriorates the performance from the pure gauging step, which runs at 1 Gflops on the CM5. The communication overhead being nearly 60% of run-time on both machines, the flopate is finally degraded to an average of 304 Mflops on the CM5, which is merely 8% of the theoretical peak performance. Note, that the computing of $\sigma_1$ is more expensive in this case, due to the data structure.

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4For SU$(3)$ matrix multiplication, e.g., a performance gain of up to 30% may be reached by programming in DPEAC (CDPEAC) on the CM2 (CM5), respectively.

5The present implementation of CMF on the CM5 suffers particularly from the large latency time of 300 msec.
The situation appears to be more favorable for the Cornell method, where the time required for communication is 37% on the CM5, and 27% on the CM2, which leads to an overall performance of 530 MFlops for the CM5 (198 MFlops for the CM2).

| Subroutine              | MFlops | time in % | MFlops | time in % |
|------------------------|--------|-----------|--------|-----------|
| \( \sum \partial_\mu A_\mu \) | 166.5  | 18.45     | 409.7  | 20.1      |
| \( \sigma_1, \sigma_2, \sigma_3 \) | 556.8  | 1.1       | 1481.8 | 1.1       |
| SU(3)-projection       | 301.6  | 45        | 918.5  | 39.5      |
| transformation         | 68     | 34.75     | 163.1  | 38.8      |

Table 1: Benchmarking the Cornell Method

| Subroutine                          | MFlops | time in % | MFlops | time in % |
|-------------------------------------|--------|-----------|--------|-----------|
| Computation of G                    | 343.85 | 16.6      | 1026   | 16.5      |
| \( \sigma_1 \)                      | 84.5   | 17.6      | 210    | 20        |
| \( \sigma_2, \sigma_3 \)           | 256.3  | 0.4       | 579.8  | 0.55      |
| transformation                       | 367    | 8.86      | 1202   | 7.8       |

Table 2: Benchmarking the Los Alamos Method

We should mention, that the results presented in this paper are based upon a test version of the software where the emphasis was on providing functionality and the tools necessary to begin testing the CM5 with vector units. The CM5 software release has not had the benefit of optimization or performance tuning and, consequently, is not necessarily representative of the performance of the full version of this software.
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References

[1] see, e.g., Halzen & Martin, Quarks and Leptons, John Wiley & Sons, 1984. QCD’90, Nucl. Phys. B (Proc. Suppl.) 23 (1991).

[2] see, e.g., M. Creutz, Quarks, Gluons and Lattices, Cambridge University Press, 1983.
   H.J. Rothe, Lattice Gauge Theories, World Scientific, 1992.
   M. Creutz (edt.), Quantum Fields on the Computer, World Scientific, 1992.

[3] C. Bernard, A. Soni and K. Yee, Nucl. Phys. B (Proc. Suppl.) 20 (1991), 410.

[4] see, e.g. S. Gottlieb in “Advances in Lattice Gauge Theory”, D. Duke and J. Owens (eds.), World Scientific, 1985.
   M. Hecht and T. DeGrand, Phys. Rev D46(1992) 2155.

[5] see, e.g., F. Brandstaeter, G. Schierholz and U.J. Wiese, Phys. Letters B272(1991) 319.

[6] M. Creutz, Phys. Rev. D15(1977) 1128.

[7] J.E. Mandula and M. Ogilvie, Phys. Lett. B185(1987) 127.

[8] T.A. DeGrand and M. Hecht, Phys. Lett. B275(1992) 435.

[9] V.N. Gribov, Nucl.Phys. B139(1978) 1.

[10] C.T.H. Davies, G.G. Batrouni, G.R. Katz, A.S. Kronfield, G.P. Lepage, K.G. Wilson, P. Rossi and B. Svetitsky, Phys. Rev. D37(1988) 1581.

[11] N. Cabbibo and E. Marinari, Phys. Lett. 119B(1982) 387.

[12] J.E. Mandula and M. Ogilvie, Phys. Lett. B248(1990) 156.

[13] Ph. de Forcrand and R. Gupta, Nucl. Phys. B (Proc. Suppl.) 9(1989) 516.

[14] R. Gupta, G. Guralnik, G. Kilcup, A. Patel, S. Sharpe, and T. Warnock, Phys.
    Rev. D36(1987) 2813.
[15] M.L. Paciello, C. Parrinello, C. Petrarca, B. Taglienti and A. Vladikas, Phys. Lett. B 276(1992) 163.

[16] A. Hulsebos, M.L. Laursen and J. Smit, Phys. Lett. B291(1992) 431.

[17] S.L. Adler, Phys. Rev. D23(1981) 2901.

[18] CMSSL communications, Fall 1991, Vol 1, No. 3, Thinking Machines Documentations.