Investigation on the effects of suspension stiffness using experimental modal analysis and finite element model updating

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Abstract. The model validation process known as the finite element model updating method aims to improve the accuracy of the predicted results by reducing the discrepancies between the initial finite element model and the experimental data. The objective of this paper is to investigate the effects of the suspension spring stiffness to the experimental data. Experimental modal analysis with free-free boundary conditions is conducted on the slender beam to measure the modal parameters. The finite element analysis is performed to obtain the natural frequencies and the mode shapes. The results from both methods are compared to investigate the discrepancy of the finite element model of the suspension spring. Firstly, the initial model of the slender beam is updated to reduce the discrepancy between the experiment data and the initial finite element model of the slender beam. Finally, model updating method is performed to the suspension stiffness of the spring and the result shown effectiveness finite element updating method in reducing the discrepancies between the initial model of the stiffness of the suspension spring and the measured result from 33.23 % to 3.05 %.

Keywords: suspension stiffness, boundary condition, experimental modal analysis, finite element model updating

1. Introduction

Structural engineering analysis requires accurate finite element model to produce high confidence result to be used for the subsequent analysis. Model updating is a systematic adjustment method than can be used to improve the finite element model as close as possible to a real structure or system. It is based on modifying the properties assign in the model so that the result from experiment data match the numerical results [1]. The accuracy is depend on the type of finite element model used to represent the structure and the properties assigned to the elements in the model [2].

In the finite element model updating, the measured data of a structure is used as a reference parameters. The modal parameters such as natural frequencies, damping ratios and mode shapes can be
obtained by finite element analysis and experimental modal analysis [3]. Analytic simulations through finite element analysis in the free-free boundary conditions are more preferable than the boundary conditions with fixed conditions [4-8]. However, in the experimental modal testing set-up, the structure must be supported in some manner to approximate the free-free boundary conditions. It is execute by using such as soft spring to support the structure [9]. Understandably, the dynamic characteristics of the test structures are significantly sensitive to the suspension system [10-14]. The effects of suspension stiffness towards the natural frequencies may lead to significant errors in the measured modal frequencies of the test structures [15].

In this present paper, model updating is performed to produce accurate model of the finite element model of the slender beam. Suspension system for free-free boundary conditions in the experimental modal testing is applied by using the soft springs and the effects of the suspension springs are investigated. Comparison is made between the initial finite element with and without suspension stiffness consideration.

2. Experimental modal analysis
The test set-up as shown in Figure 1 is designed to investigate the effects of suspension stiffness to the modal parameters, namely natural frequencies and mode shapes. The slender beam used in this study is made from stainless steel. Table 1 shows the detail dimensions of the slender beam. In the experimental work the slender beam is suspended at both ends using soft springs to represent free-free boundary conditions and by considering the bending modes of the slender beam.

In the the experimental work the impact hammer Dytran model 5800 was used to excite the slender beam and the excitation direction was performed in the Z-direction. This vertical direction is referred to the direction of the deflection of the soft springs. This experimental modal analysis was performed based on the roving accelerometers method. It is functions to avoid mass loading issue to the test slender beam. The location of the excitation is fixed to one point only. Meanwhile the acceleration of the slender beam as an output response was measured using two accelerometers Dytran model 3032A. The average measurement for each node considered in this experimental modal testing is 10 readings.

![Suspension location and Soft rubber band](image)

**Figure 1.** Experimental modal analysis set-up for the slender beam

| Material | Stainless steel |
|----------|----------------|
| Length (mm) | 1024 |
| Width (mm) | 32 |
| Thickness (mm) | 1.5 |

**Table 1.** Geometry properties of the slender beam

3. Finite element modelling and model updating
The finite element model of the slender beam has been created using HyperMesh simulation software. Figure 2 shows the finite element model of the slender beam that are consisted of 3813 elements and 8350 nodes. The determination of the modal parameters, which are the natural frequencies and the mode shapes was performed using the normal modes analysis that is available in HyperMesh. The natural frequencies and mode shapes of the beam were obtained by solving the equation of motion for an undamped free vibration [16-17] as in Eq. (1).
\[ \mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{K} \mathbf{x}(t) = 0 \]  

(1)

where \( M \) and \( K \) are the mass and stiffness matrices. These matrices are the square \( n \times n \) system. \( \ddot{x} \) and \( \mathbf{x} \) are the \( n \times 1 \) acceleration and displacement vectors, respectively. The results from normal mode analysis are the modal parameters which are the natural frequencies and mode shapes of the structure using \( \omega \) and \( \varphi \) denoting the undamped eigenfrequency and eigenvector [18].

\[ (\mathbf{K} - \omega^2 \mathbf{M}) \varphi = 0 \]  

(2)

The purpose of model updating is to obtain a better finite element model by modifying the model parameters which are mass, stiffness and damping matrices based on the experimental data in term of the modal characteristics (natural frequencies and mode shapes) [19]. The optimisation algorithm, which is a sensitivity-based iterative procedure, allows the objective function \( J \) to be minimised by adjusting the eigenvalues of the initial finite element model until the objective function is converged. The objective function based on eigenvalues is defined as follows [20],

\[ J = \sum_{i=1}^{n} W_i \left( \frac{\lambda_i^{exp}}{\lambda_i^{pre}} - 1 \right)^2 \]  

(3)

where \( \lambda_i^{exp} \) is the \( i^{th} \) experimental eigenvalue and \( \lambda_i^{pre} \) is the \( i^{th} \) predicted eigenvalue from the finite element model and \( n \) is the number of eigenvalues involved in the updating procedure. The similarity between the experimental mode shape \( \varphi_m \) and the numerical mode shape \( \varphi_a \) was obtained by using Modal Assurance Criterion [21] as in Eq (4).

\[ \text{MAC} = \langle \varphi_m, \varphi_a \rangle = \frac{\left| \varphi_m^T \varphi_a \right|^2}{\left( \varphi_m^T \varphi_m \right) \left( \varphi_a^T \varphi_a \right)} \]  

(4)

**Figure 2.** Finite element model (a) isometric view of the CAD model (b) meshed model

4. Results and discussion

4.1 Experimental and simulation results

Experimental modal analysis was conducted to obtain the natural frequencies and the mode shapes of the slender beam. Finite element analysis by using normal mode has been performed to calculate the natural frequencies and the mode shapes by employing the equation of motion for an undamped free vibration as shown in Eq. (2). Table 2 presents the results of the first five natural frequencies and MAC values from the experimental modal analysis and the initial finite element analysis. As can be seen, there are considerable differences between the two results. By comparison, the total error (column III) is 33.23
percent and the average MAC value is above 0.88 (column IV). It is also found that the largest error is in the first mode, which is 16.89 percent.

Table 2. Comparison of the results between the measured and the initial FE.

| Mode | Experiment Frequency (Hz) | Initial FE Frequency (Hz) | Relative Error [%] | MAC | [I-II/I] |
|------|---------------------------|---------------------------|--------------------|-----|---------|
| 1    | 8.82                      | 7.33                      | 16.89              | 0.95|         |
| 2    | 21.35                     | 20.22                     | 5.29               | 0.96|         |
| 3    | 41.25                     | 39.64                     | 3.90               | 0.91|         |
| 4    | 68.02                     | 65.55                     | 3.63               | 0.93|         |
| 5    | 101.51                    | 97.95                     | 3.51               | 0.89|         |
|      |                           |                           | Total error        | 33.23|        |

4.2 Model updating the material properties

In order to improve the initial finite element model of the slender beam, the initial model need to be updated as close as possible to the experimental counterpart. This can be performed by adjusting systematically the uncertainties of the finite element modelling parameters, which potentially influences the modal properties of the slender beam [22]. The material properties that influenced the modal properties are Young’s modulus, density and Poisson’s ratio, namely. For this experiment, the density parameter was calculated manually from the measured mass and volume of the slender beam. Therefore, the Young’s modulus and Poisson’s ratio were selected as an updating parameters. These two potential parameters were used as the updating process (Young’s modulus and Poisson’s ratio) which are based on the most sensitive parameters listed in the sensitivity analysis via NASTRAN optimisation code (SOL 200). Element of the sensitivity matrix was calculated until the updating parameter is converged to the final value.

Table 3. Comparison of the results between the measured and the updated FE model (Young’s modulus and Poisson’s ratio).

| Mode | Experiment Frequency (Hz) | Initial FE Frequency (Hz) | Relative Error [%] | MAC | Updated FE Frequency (Hz) | Error (%) | Updated FE MAC |
|------|---------------------------|---------------------------|--------------------|-----|---------------------------|-----------|----------------|
| 1    | 8.82                      | 7.33                      | 16.89              | 0.95| 7.62                      | 13.61     | 0.96           |
| 2    | 21.35                     | 20.22                     | 5.29               | 0.96| 20.99                     | 1.69      | 0.97           |
| 3    | 41.25                     | 39.64                     | 3.90               | 0.91| 41.15                     | 0.24      | 0.93           |
| 4    | 68.02                     | 65.55                     | 3.63               | 0.93| 68.04                     | 0.03      | 0.93           |
| 5    | 101.51                    | 97.95                     | 3.51               | 0.89| 101.67                    | 0.16      | 0.90           |
|      |                           |                           | Total error        | 33.23| 15.72            |

The discrepancies between the updated finite element model of the slender beam and the measured data is shown in Table 3. It is found that the total error was reduced from 33.23 to 15.72 percent. In this first updating, the initial value of the Young’s modulus has increased from 193 GPa to 208 GPa while the initial value of the Poisson’s ratio has decreased from 0.31 to 0.30. It is note that by updating these two material properties alone is not capable to reduce the discrepancies especially the error in the first mode in (Column VI). The natural frequency in the first mode still contributes to the highest error in the updated results. The values of the updated parameters are listed in Table 4.
Table 4. Parameters before and after updated FE model

| Parameter          | Initial Value | Updated Value | Unit  |
|--------------------|---------------|---------------|-------|
| Young’s modulus    | 193           | 208           | (GPa) |
| Poisson’s ratio    | 0.31          | 0.30          | -     |

4.3 Model updating the suspension stiffness

In this paper, free-free boundary conditions are applied for the modal parameters measurement. For the experimental modal testing, the soft springs were used to support the slender beam to approximate the free-free boundary conditions. Impulse excitation was performed by using an impact hammer while the acceleration responses of the slender beam were recorded by using accelerometers with roving accelerometers method. It is generally known that the different type of boundary conditions affect the natural frequencies and the mode shapes of the structure or system. Therefore, in the second model updating stage, the suspension stiffness of the soft springs was considered in the sensitivity analysis.

Figure 3 shows a stiffness parameter that is available in the simulation software known as PELAS was included in the finite element model of the slender beam. The assigned location of PELAS in the model is refers to the same location of the soft springs used to support the slender beam in the experimental modal testing as described in Figure 1. PELAS is the elastic property of CELAS elements to represent the suspension stiffness of the soft springs.

Figure 3. Finite element modelling of the slender beam with the suspension springs

Normal mode analysis has been performed to obtain the modal parameters in free-free boundary conditions. The initial value of PELAS stiffness (suspension soft spring) assigned to the model is 0.01 N/mm. By the sensitivity analysis during the model updating process, the final value for PELAS stiffness was increased to 0.015 N/mm. Table 5 lists the natural frequencies and MAC values after updating the PELAS stiffness. A significant reduction was found at the first mode of the slender beam from 8.82 Hz to 8.61 Hz. The initial error for mode 1 which is 16.89 percent was reduced to 2.38 percent. By statistical calculation, the total error between the updated and the measured natural frequencies (column VI) was reduced from 33.23 percent to 3.05 percent. From this result, it indicates that the significant improvement has been achieved as a result of including the suspension stiffness of the soft springs in the finite element model. The mode shapes and the corresponding natural frequencies for the updated model and experimental modal analysis are displayed in Table 7.
### Table 5. Comparison of the results between the measured and the updated FE model (Young’s modulus, Poisson’s ratio and spring stiffness).

| Mode | Experiment Frequency (Hz) | Initial FE Frequency (Hz) | Relative Error (%) [I-II/I] | IV Initial FE MAC (Hz) | V Updated FE MAC (Hz) | VI Error (%) [I-IV/I] | VII Updated FE MAC |
|------|---------------------------|---------------------------|-----------------------------|------------------------|-----------------------|------------------------|-------------------|
| 1    | 8.82                      | 7.33                      | 16.89                       | 0.95                   | 8.61                  | 2.38                   | 0.98              |
| 2    | 21.35                     | 20.22                     | 5.29                        | 0.96                   | 21.36                 | 0.05                   | 0.97              |
| 3    | 41.25                     | 39.64                     | 3.90                        | 0.91                   | 41.33                 | 0.19                   | 0.93              |
| 4    | 68.02                     | 65.55                     | 3.63                        | 0.93                   | 68.15                 | 0.19                   | 0.93              |
| 5    | 101.51                    | 97.95                     | 3.51                        | 0.89                   | 101.75                | 0.24                   | 0.90              |
| Total error | 33.23                  |               |                            |                       |                       |                        | 3.05              |

### Table 6. Parameters before and after updated FE model

| Parameter       | Initial Value | Updated Value | Unit  |
|-----------------|---------------|---------------|-------|
| Spring stiffness| 0.01          | 0.015         | N/mm  |

### Table 7. Comparison of the mode shapes between the experimental and the updated FE model.

| Mode | Experimental Modal Analysis | Updated FE Model | MAC value |
|------|----------------------------|------------------|-----------|
| 1    | ![Image](image1.png)       | ![Image](image2.png) | 0.98      |
| 2    | ![Image](image3.png)       | ![Image](image4.png) | 0.97      |
| 3    | ![Image](image5.png)       | ![Image](image6.png) | 0.93      |
5. Conclusions
The present work investigates the effects of the suspension spring stiffness in experimental work of the slender beam in the free-free boundary conditions. The modal parameters of the slender beam, which are the natural frequencies and the mode shapes are measured by numerical simulation and experimental modal analysis. Next, the finite element model updating is performed by modifying the material properties, which are Young’s modulus and Poisson’s ratio in order to match the results between the finite element analysis and the experimental data. However, the obtained results show a large discrepancy. Therefore the second model updating processes is then performed, it is found that a better correlation is obtained by including the effect of the suspension springs in the updating parameters. The CELAS element is used as the updating parameter to represent the suspension springs as a support for free-free boundary conditions in the experimental modal testing. A significant improvement in the finite element model of the slender beam by the reduction of the total error from 33.23 percent to 3.05 percent. It is also revealed that the suspension spring stiffness influenced the low frequency of the slender beam.

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