Power spectrum of large-scale structure cosmological models in the framework of scalar-tensor theories

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Abstract. We study the large-scale structure formation in the Universe in the frame of scalar-tensor theories as an alternative to general relativity. We review briefly the Newtonian limit of non-minimally coupled scalar-tensor theories and the evolution equations of the N-body system that is appropriate to study large-scale structure formation in the Universe. We compute the power spectrum of the universe at present epoch and show how the large-scale structure depends on the scalar field contribution.

1. Introduction
During the last two decades high precision observations have established the existence of two component of the universe. These are known as dark energy and dark matter. What are their fundamental nature? this one of the questions we need to answer[1]. One of the most successful models to explain observations is the ΛCDM but it still has some problems.

In the past we have proposed a dark matter model in the framework of the Newtonian limit of a scalar-tensor theory[2]. We have done N-body simulations using a numerical code developed to take into account the scalar field (SF) force contribution which is of Yukawa type in its Newtonian limit and we have use it to study several situations, like collision of protogalaxies[3], dynamics and collision of galaxies[4, 5], and to study cosmological simulations[6, 7, 8]. Recently we have used it to address a possible solution to two of the main problems the ΛCDM has, i.e., the tendency to over populate the formed halos of galaxies with satellites and the flatness problem[10, 11].

Several authors have also been using similar dark matter models to study large-scale structure formation, see for instance[12, 13, 14].

One of the tools to characterize the clustering properties of a universe model is the power spectrum. Constraints on cosmological models and on cosmological parameters have been derived from measuring power spectrum. Therefore, in this work we consider a dark matter model build from the Newtonian limit of a scalar-tensor theory[2] and use it to study the large-scale structure formation of the universe by analyzing the power-spectrum versus the parameters of the model.
2. Evolution equations for a $\Lambda$CMD universe within the framework of a scalar-tensor theory

Giving that we are interested in the study of the large-scale structure formation of the universe we will use the Newtonian limit of the scalar-tensor theory. This limit applies in a region small compared to the Hubble length $cH^{-1}$, with $c$ the speed of light and $H$ the Hubble factor, and large compared to the Schwarzschild radii of any collapsed objects.

In this case the Vlasov-Poisson equation in an expanding universe describes the evolution of the six-dimensional, one-particle distribution function, $f(x, p)[15]$. The Vlasov equation is,

$$\frac{\partial f}{\partial t} + \frac{p}{ma^2} \cdot \frac{\partial f}{\partial x} - m \nabla \Phi_N(x) \cdot \frac{\partial f}{\partial p} = 0 \quad (1)$$

where $x$ is the comoving coordinate, $p = ma^2 \dot{x}$, $m$ is the particle mass, and $\Phi_N$ is the self-consistent gravitational potential given by the Poisson equation,

$$\nabla^2 \Phi_N(x) = 4\pi G_N a^2 [\rho(x) - \rho_b(t)] \quad (2)$$

where $\rho_b$ is the background mass density. Eqs. (1) and (2) form the Vlasov-Poisson equation, constitutes a collisionless, mean-field approximation to the evolution of the full $N$-body distribution. An $N$-body code attempts to solve Eqs. (1) and (2) by representing the one-particle distribution function as

$$f(x, p) = \sum_{i=1}^{N} \delta(x - x_i) \delta(p - p_i) \quad (3)$$

Substitution of (3) in the Vlasov-Poisson system of equations yields the exact Newton’s equations for a system of $N$ gravitating particles.

In the Newtonian limit of STT of gravity, the Newtonian motion equation for a particle $i$ is written as[7]

$$\ddot{x}_i + 2H \dot{x}_i = -\frac{1}{a^3} \frac{G_N}{1 + \alpha} \sum_{j \neq i} \frac{m_j (x_i - x_j)}{|x_i - x_j|^3} F_{SF}(|x_i - x_j|, \alpha, \lambda) \quad (4)$$

where the sum includes all periodic images of particle $j$, and $F_{SF}(r, \alpha, \lambda)$ is

$$F_{SF}(r, \alpha, \lambda) = 1 + \alpha \left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \quad (5)$$

which, for small distances compared to $\lambda$, is $F_{SF}(r \ll \lambda, \alpha, \lambda) \approx 1 + \alpha (1 + \frac{r}{\lambda})$ and, for long distances, is $F_{SF}(r \gg \lambda, \alpha, \lambda) \approx 1$, as in Newtonian physics.

To simulate cosmological systems, the expansion of the universe has to be taken into account. Also, to determine the nature of the cosmological model we need to determine the composition of the universe, i.e., we need to give the values of $\Omega_i$ for each component $i$, taking into account in this way all forms of energy densities that exist at present.

In this work we will consider a model with only two energy density contribution. One which is a pressureless and nonbaryonic dark matter with $\Omega_{DM} \approx 0.3$ that does not couple with radiation. Other, that will be a cosmological constant contribution $\Omega_{\Lambda} \approx 0.7$. Also, the model must be consistent with a static SF according to the Newtonian limit we are using in this work. Thus, the scale factor, $a(t)$, is given by the following Friedman model[8],

$$a^3 H^2 = H_0^2 \left[ \frac{\Omega_{DM0} + \Omega_{M0} a^3}{1 + \alpha} + \left(1 - \frac{\Omega_{DM0} + \Omega_{M0}}{1 + \alpha}\right) a \right] \quad (6)$$

where $H = \dot{a}/a$, $\Omega_{DM0}$ and $\Omega_{M0}$ are the matter and energy density evaluated at present, respectively. We have also demanded that the universe be a flat one. To be consistent with the CMB spectrum and structure formation numerical experiments, cosmological constraints must be applied on $\alpha$ in order for it to be within the range $(-1, 1)$ [16, 17, 18, 19].
3. Results
In this section, we present results of cosmological simulations of a ΛCDM universe with and without SF contribution. We start our simulations with an initial distribution of $N = 2 \times 32^3$ particles in a box with sides of $L = 50h^{-1}$ Mpc at $z = 10$. This case is similar to the one that comes with Gadget 1 [20]. At present epoch, $\Omega_{m0} = 0.3$, $\Omega_{\Lambda0} = 0.7$, $H_0 = 100h$ km/s/Mpc, $h = 0.7$. We restrict the values of $\alpha$ to the interval $(-1, 1)$ [16, 17, 18, 19] and use $\lambda = 5$ Mpc, since this scale turns out to be an intermediate scale between the size of the clump groups and the separation of the formed groups. However, we have analyzed elsewhere several values of $\lambda$ [8]. Other universe models have been analyzed in [6, 7, 9].

Because the visible component is the smaller one and given our interest to test the consequences of including a SF contribution to the evolution equations, our model excludes gas particles, but all its mass has been added to the dark matter. In Fig. 1 we show the power spectrum at redshift $z = 0$ of our ΛCDM model for several values of $\alpha$. The line without symbols shows the case of a ΛCDM model without the contribution of the scalar field. The line with (+) is the case of a ΛCDM model with scalar field with $\alpha = 1$, the line with (*) uses $\alpha = -1/2$, and the line with (o) uses $\alpha = -1/4$. One notes clearly how the SF modifies the clustering of matter of the system. The most dramatic cases are the models with $\alpha = -1/2$ and $\alpha = 1$.

We now analyze the general effect the parameter $\alpha$ has on the clustering. For a given $\lambda$ the role of $\alpha$ in our approach can be seen in equations (4)-(5) and is contained in the factor $F_{SF}(r, \alpha, \lambda)$. The factor $F_{SF}$ augments (diminishes) for positive (negative) values of $\alpha$ for small distances compared to $\lambda$, resulting in more (less) structure formation for positive (negative) values of $\alpha$ compared to the ΛCDM model. In the case of the line with (+), for $r \ll \lambda$, the effective gravitational pull has been augmented by a factor of 2, in contrast to case shown with line and (*) where it has diminished by a factor of 1/2; in the model with the line and (o) the pull diminishes only by a factor of 3/4. That is why one observes for $r < \lambda$ more structure formation in case with line and (+), less in case with line and (*), and lesser in model with line and (o). The effect is then, for a growing positive $\alpha$, to speed up the growth of perturbations, then of halos and then of clusters, whereas negative $\alpha$ values ($\alpha \to -1$) tend to slow down the growth.
4. Conclusions
The general gravitational effect of the scalar-tensor dark matter model is that the interaction changes by a factor $F_S(r, \alpha, \lambda) \approx 1 + \alpha(1 + r/\lambda)$ for $r < \lambda$ in comparison with the Newtonian case. For negative values of $\alpha$ the effect is to diminish the formation of structures and the opposite occurs for positive values of $\alpha$. For $r > \lambda$ the dynamics is essentially Newtonian.

By computing the power spectrum we were able to determine the effects of the scalar field contribution to the galaxy clustering. Of course we will need to do a systematic and with greater resolution study and make detailed comparisons with observations to support or to rule out a dark model based on scalar-tensor theories.

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