Study of the AC machines winding having fractional $q$

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Abstract. The winding schemes with a fractional numbers of slots per pole and phase $q$ have been known and used for a long time. However, in the literature on the low-noise machines design there are not recommended to use. Nevertheless, fractional $q$ windings have been realized in many applications of special AC electrical machines, allowing to improve their performance, including vibroacoustic one. This paper deals with harmonic analysis of windings having integer and fractional $q$ in permanent magnet synchronous motors, a comparison of their characteristics is performed, frequencies of subharmonics are revealed. Optimal winding pitch design is found giving reduce the amplitudes of subharmonics. Distribution factors for subharmonics, fractional and high-order harmonics are calculated, results analysis is represented, allowing for giving recommendations how to calculate distribution factors for different harmonics when $q$ is fractional.

1. Introduction

Modern electric drives with AC motors are often frequency controlled. Thanks to the inverter supply a machine has different speed regardless of a pole number $2p$, which in its turn is a result of optimal design. The best mass-dimension, say synchronous permanent magnet motors, have at $2p \geq 6\div10$ fractional $q$ winding.

Fractional $q$ is used first of all in limited size micromachines [1]. Also, it is good in powerful low speed hydrogenators having large pole number, because great demands to dimensions and tooth saturation do not allow to use integer $q$ winding. Besides, fractional $q$ windings often have been used in the course of modernization – electric machines redesign to the lower rotation speed at the stator having odd slots number. When the new series RA of induction machines was under development [2, 3], symmetrical 3-phase fractional $q$ winding has been applied in spite of their lower efficiency. It is worth mentioning quite successful use of fractional $q$ (with $q<1$) windings in reluctance machines thanks to linear load increase because of stator number of slots decrease [4].

Fractional $q$ is one of the most effective mean to depress tooth-order harmonics. However, in the specialized literature on low-noise electrical machines, one can find recommendation to refrain from this windings application [5]. This is bound up, first of all, with high probability of low-order force waves development, which cause increased vibrations. This phenomenon is explained by appearance and interaction of harmonics with number of poles non-multiple to number of poles of the main harmonic $p_v \neq k p_f$, having orders less than main harmonic ones. Such harmonics have been called subharmonics or low-order ones. Besides of them, in vibration spectrum, there may be fractional and high-order harmonics.
2. Simulation and harmonic analysis

In order to reveal space harmonics in spectra of winding MMF and air gap flux density, a model was developed in FEMM program, which allows simulating a two-dimensional stationary magnetic field by the finite element method. Stator winding is made of copper, nonlinear magnetic core – of the steel 2412. Boundary conditions are: Dirichlet – on the outer bound of the machine active zone which is a stator outer diameter, Neumann – on all the rest bounds dividing mediums. Number of slots have been taken 36, \( q = 1\frac{1}{5} \). Windings current density is given as

\[
J = \frac{\sqrt{2} I_p w_x}{S_s}
\]

where \( I_p \) – phase current, \( w \) – phase number of turns, \( S_s \) – stator slot area.

Phase currents are given for the time corresponding phase \( A \) maximum current, taking into account their directions of flow, that is \( I_a = I_m \), \( I_b = \frac{I_m}{2} \), \( I_c = \frac{I_m}{2} \). Number of finite elements is equal to 250 thousands. Stator slots have trapeziform. To exclude slots influence rotor surface is considered to be smooth.

As a result of the simulation, flux density distribution \( B_\delta \) is obtained along the imaginary line dividing the machine’s air gap by two equal parts, for the integer and fractional \( q \) (Figure 1). As well seen, for both cases positive and negative half-waves have similar forms with the shift \( \pi \) radians. That is why flux density curve \( B_\delta \) contains only odd harmonics, it is a second type symmetry.

![Figure 1](image)

**Figure 1.** Flux density in the air gap of PM synchronous motor with: (a) integer \( q \), (b) fractional \( q \).

Fractional \( q \) windings possess two types of asymmetry. First, within each pole flux density curve is not symmetrical with respect to the pole axis, that is the first type symmetry is broken. The result is explained only by the fact that odd high-order harmonics intersect the abscissa axis not simultaneously with the main wave and no new harmonics are formed at the same time. Secondly, integral meanings of flux density \( B_\delta \) (that is magnetic flux) are different under poles resulting different forces amplitudes acting on poles. This may give rise to the additional vibration components.

Results of field simulation are analyzed with Fourier series of field harmonic components (Figures 2a and 4a). Flux density in the gap is represented as the sum of terms of the harmonic series

\[
B = \sum_{v=1}^{\infty} B_{vm} \cos \frac{\pi}{T} x
\]
where \( B_{sm} = \frac{m \sqrt{2} I_p W_k \mu_0 \lambda_\delta}{\pi v p} \); \( T \) - pole pitch of the main space harmonic, m; \( v \) - space harmonic order; \( x \) - space coordinate, m; \( m \) - phases number; \( k \) - winding factor; \( \mu_0 = 4\pi 10^{-7}, \) H/m – space permeability; \( p \) - number of pole pairs of fundamental harmonic; \( \lambda_\delta \) - air gap specific magnetic permeance.

![Figure 2](image-url)  
**Figure 2.** Flux density harmonics of induction motor with integer \( q \): (a) \( T = 2\pi \), (b) \( T = 6\pi \).

![Figure 3](image-url)  
**Figure 3.** Flux density harmonics of induction motor with fractional \( q \): a) \( T = 2\pi \), b) \( T = 6\pi \).

In this case, the period of the fundamental harmonic is equal to \( T = 2\pi \). When considering subharmonics, their period will be \( c \) times greater \( T_c = cT \). Therefore, it is possible to identify subharmonics by increasing the period of the fundamental harmonic by a factor of \( c \) (Figures 2b and 3b).

The spectrum of the harmonics of the machine, as expected, does not contain even harmonics by virtue of the symmetry of the flux density curve in the air gap relative to the abscissa axis under the shift of half waves.
The non-tooth orders caused by the nonuniformity of the distribution of the MMF curve and the different air gap permeance are present in the windings with integer and fractional $q$:

$$v = \frac{2m}{d_q}k + 1$$  \hspace{1cm} (3)

where $k = \pm 1; \pm 2\ldots$

There is a wide range of flux density tooth-order harmonics in machine with integer $q$:

$$v = \frac{Z}{p}k + 1$$  \hspace{1cm} (4)

where $k = \pm 1; \pm 2\ldots$

For our example $v = 11, 13, 23, 25, 35, 37, 47, 49\ldots$ In the flux density curve of winding having fractional $q$, these harmonic components disappear or considerably decline. However, if main harmonic period increase, one may note subharmonic components and that maximal amplitude has harmonic of order $v = \frac{1}{5}$. In spite of the low order, its amplitude reaches a value commensurable to the main component. Namely this harmonic causes flux density asymmetry between poles in the gape and consequently unequal distribution of electromagnetic forces acting on the machine rotor. Subharmonic effect can be brought down through the rational choice of winding pitch $y$.

Air gap field spectra for different winding pitch $y$ are shown in the table 1. Subharmonic amplitude may reach 60% at $y = 6$. As can be seen, the amplitudes of the subharmonics for different $y$ acquire different values. For a given winding layout, it is possible to reduce subharmonics thanks to the choice of proper $y$. In our calculation $y$ is varied from 1 to 6. For $y = 1$ and $y = 6$ main harmonic amplitude goes down considerably but subharmonics and high-order harmonics amplitude grow. Fundamental amplitude maximum takes place at $y = 3$ and $y = 4$. In addition, their characteristics are better than at the rest variants. Within the considered range, an optimal harmonics minimum is at $y = 3$. It means that if we design a low-noise machine, one must consider not only influence of high-order harmonics, but subharmonics as well, since their amplitudes may reach high values. However, it should be noted if some harmonics are to be suppressed, it may be possible to choose certain interim $y$ in order to reduce undesirable components of the spectrum.

| Table 1. Air gap field spectra for different winding pitch $y$. |
|---------------------------------------------------------------|
| winding pitch $y$   | 6 | 5 | 4 | 3 | 2 | 1 |
|---------------------|---|---|---|---|---|---|
| Main harmonic amplitude, T | 0.41 | 0.63 | 0.74 | 0.74 | 0.62 | 0.35 |
| Amplitude of harmonic 1/5, T | 0.248 | 0.193 | 0.149 | 0.122 | 0.105 | 0.067 |
| Amplitude of harmonic 3/5, T | 0.01 | 0.012 | 0.007 | 0.02 | 0.002 | 0.001 |
| Subharmonic factor, % | 7.2 | 10.1 | 8.4 | 6.4 | 9.2 | 12.7 |
| High harmonic factor, % | 61.1 | 30.7 | 20.2 | 16.8 | 16.9 | 19.2 |
| Common factor of subharmonics and high-order harmonics, % | 61.6 | 32.3 | 21.9 | 18 | 19.3 | 23 |

### 3. Frequencies of electromagnetic forces

Electromagnetic vibration component mostly is calculated by a simple Maxwell formula:

$$P = \frac{B_s^2}{2\mu_0}$$  \hspace{1cm} (5)
It allows to find a radial force acting on the rotor and shaft of electrical machine. Axial and tangential components are ignored. Axial force appears only because of eccentricity and considerable defects of magnetic system, such as fluffed up rotor or stator laminated core. Tangential component causes vibrations of magnetic core teeth. Their natural frequency of the teeth usually exceeds 10 kHz, but electromagnetic forces have frequencies limited by 4 kHz. That is why only radial component of force is taken into account.

Air gap flux density is a sum of components: created by motionless stator (or armature) $B_s$ and rotating rotor (or inductor) $B_r$. In this way:

$$B_\delta = B_s + B_r$$

Each of them can be represented by a product of MMF and magnetic permeance:

$$B_\delta = F \cdot \Lambda$$

Space harmonics in a permanent magnet synchronous motor rotate together with rotor at a synchronous speed $n_\mu = n_1$. High harmonics have number of poles with respect to the fundamental harmonic $p_\mu = \mu p_1$. Therefore harmonics induce EMF with a frequency in the stator:

$$f_\mu = \frac{p_\mu n_\mu}{60} = \frac{\mu \cdot p_1 \cdot n_1}{60} = \mu \cdot f_1$$

Stator current also create space harmonics of the stator winding. Their number of poles, like rotor’s ones, is equal to the fundamental, but speed is lower. Thus, the frequency of EMF from their high harmonics is equal:

$$f_\nu = \frac{p_\nu n_\nu}{60} = \frac{\nu \cdot p_1 \cdot n_1}{60} = f_1$$

It is known that pulsating MMF wave can be represented as a sum of the forward and backward rotating waves. Taking into consideration high-order harmonics, one may have an expression for the phase MMF:

$$F = \sum_{\nu=1}^{\infty} F_\nu \sin \omega t \cos \frac{\pi}{\tau} x = \sum_{\nu=1}^{\infty} F_\nu \frac{1}{2} \left[ \sin \left( \omega t - n \frac{\pi}{\tau} x \right) + \sin \left( \omega t + n \frac{\pi}{\tau} x \right) \right]$$

(10)

Having found each flux density component, one may write the radial magnetic force as follows:

$$P = \frac{1}{2\mu_0} \left[ \left( \sum_{\nu=1}^{\infty} F_\nu \sin \left( \omega t - \frac{\pi}{\tau} x \right) + \sum_{\mu=1}^{\infty} F_\mu \sin \left( \mu \omega t - \frac{\pi}{\tau} x \right) \right) \right]^2$$

(11)

Let’s raise to the second power the first multiplier. Since we consider interaction of stator and rotor harmonics, this expression will be:

$$P = \frac{1}{2\mu_0} \left[ \left( \sum_{\nu=1}^{\infty} F_\nu \sum_{\mu=1}^{\infty} F_\mu \sin \left( \omega t - \frac{\pi}{\tau} x \right) \sin \left( \mu \omega t - \frac{\pi}{\tau} x \right) \right) \right]^2$$

(12)

Using the sines product formula we finally get...
Thus, electromagnetic forces frequencies in permanent magnet synchronous motor will be

\[
P = \frac{1}{4\mu_0} \left[ \sum_{\nu=1}^{\infty} F_\nu \sum_{\mu=1}^{\infty} F_\mu \cos \left( (1 - \mu)\omega t - (\nu - \mu)\frac{\pi}{\tau} x \right) \cos \left( (1 + \mu)\omega t - (\nu + \mu)\frac{\pi}{\tau} x \right) \right] \times \left( A_0 + \sum_{\nu=1}^{\infty} A_m \cos \nu Z_1 \frac{\pi}{\tau} x \right)^2
\]

(13)

As it is seen, for machines with a field winding and with permanent magnet, frequencies coincide. The same result one can get substituting \( Z_2 = 2p \) and \( s = 0 \) for induction machine formulas ignoring saturation:

\[
f = f_\nu \pm f_\mu = f_1 \pm \mu \cdot f_1 = f_1 \pm (1 + 2k)f_1 = \left[ \frac{2 \cdot (1 + k)f_1}{2kf_1} \right]
\]

(14)

Pay attention at subharmonics which appear mainly at double supply frequency and divisible by its frequencies.

4. Winding factor
It is one of the main criteria for choice and design of the electrical machine winding. Winding factor reflects the influence of high-order harmonics and consists of three components:

\[
k_\nu = k_{pv}k_{dv}k_{skv}
\]

(16)

where \( k_{pv} \) - pitch factor, \( k_{dv} \) - distribution factor, \( k_{skv} \) - skewing factor.

An effective way to suppress high-order harmonics (first of all, tooth-order harmonics) is a slot (teeth) skewing on the stator or rotor. However, some problems arise when realizing this measure. To skew rotor magnets, one must make them segmental, it is necessary to find a way of their mounting (fixing) that is rotor assembling gets profoundly complicated. To make stator slots skewed it is not always possible as well. Therefore, application of the fractional \( q \) becomes a priority mean to do away with high-order harmonics. For this reason we take here \( k_{skv} = 1 \). Pitch factor for each harmonic is given:

\[
k_{y\nu} = \sin \left( \nu \beta \frac{\pi}{2} \right)
\]

(17)

where \( \beta \) - relative winding pitch.

As a rule, pitch factor is not considered in the analysis of fractional \( q \) windings, since parameter \( q \) itself does affect this coefficient. However, it is worth mentioning that the winding pitch choice may considerable influence on a subharmonic amplitude, that is to be taken into account in the design procedure of low-noise electrical machines.

In the paper, analysis of distribution factor \( k_{dv} \) for a main, high-order and subharmonics. Several known formulas for \( k_{dv} \) are as follows.

Method [6] for integer and fractional \( \nu \) is:
\[ k_{pv} = \frac{\sin\left(\frac{\pi}{2m} \nu \right)}{C_q \sin\left(\frac{\pi}{2mC_q} \nu \right)} \]  

Method [7] for integer \( \nu \) is:

\[ k_{pv} = \frac{\sin(\nu \frac{\pi}{k_m})}{C_q \sin(\nu \frac{\pi}{k_mC_q})} \]  

Method [7] for fractional \( \nu \) is:

\[ k_{pv} = \frac{\sin(\nu \frac{\pi}{k_m} d_q y_i)}{C_q \sin(\nu \frac{\pi}{k_mC_q} d_q y_i)} \]  

Method [8] for integer \( \nu \) is:

\[ k_{pv} = \frac{\sin(\nu \cdot 30)}{d_q q \sin(\nu \frac{30}{d_q q})} \]  

Method [8] for fractional \( \nu \) is:

\[ k_{pv} = \frac{\sin(\nu \cdot 30 - \nu \cdot 180)}{d_q q \sin(\nu \frac{30}{d_q q} + \nu \cdot 180)} \]  

Method [9] for integer and fractional \( \nu \) is:

\[ k_{pv} = \sum \frac{\cos n\varphi}{2p} \]  

Here in \( k_z \) – zone coefficient; \( C_q \) - numerator of the improper fraction \( q \); \( d_q \) – denominator of the improper fraction \( q \); \( \varphi \) - angular coordinate of the phase conductor; \( n \) - number of conductors having the same coordinate.

Figures 4 and 5 represent distribution factor calculated by these formulas for \( q = 1 \frac{1}{5} \). According to the theorem on periodicity of pitch factors they will be repeated periodically. Therefore it is quite enough to consider them within a limited range. It is seen that distribution factor means found by formulas [6-8] coincide for high-order harmonics. Only meaning found by [9] differ, since in them all even harmonics turn into zero.

Pay attention to results of calculation of the distribution factor for subharmonics. Formulas [6-9] give different \( k_{pv} \), that can be explained as follows. Method [6] is based on the assumption that fractional \( q \) is substituted by a meaning calculated for an equivalent number of slots, what does not allow to calculate distribution factor for subharmonics and fractional-order harmonics. Besides, while calculating by methods [7, 8], even harmonics get nonzero meanings. As a matter of fact it is not quite correct result. In spite of fractional \( q \) and presence of subharmonic components, a winding is symmetrical and even harmonics must not be present in it. Nonzero meanings occur because,
substituting $q$ in formulas, its meaning changes from $q = \frac{1}{5}$ to $q = \frac{2}{10}$, that is number of elementary periods becomes twice as much and winding characteristics change. As a result winding gets unsymmetrical features and distribution factors $k_{dv}$ for subharmonics and fractional-order harmonics become nonzero.

**Figure 4.** Distribution factor for high-order harmonics.

**Figure 5.** Distribution factor for subharmonics and fractional harmonics.

Method [9] allows to calculate $k_{dv}$ on the base of space EMF distribution and angular conductor positions $\varphi$. Thanks to them, it is easy to calculate distribution factor for subharmonics and fractional components correctly. These data is confirmed by results of flux density harmonics spectra calculations:

$$B_v = \frac{1}{\nu} \frac{I_1}{I_0} \frac{k_v}{k_s}$$  \hspace{1cm} (24)

For simplicity we ignore saturation $k_s$ and consider no load operation, that is $I_1 = I_0$.

On Figure 6, we noted flux density subharmonics received by winding factor calculations with different formulas.

Comparison of calculation and simulation results (Figure 2b) reveals better convergence of the method [9]. For experimental verification of these results, it is planned to produce permanent magnet
synchronous motor with \( q = 1 \frac{1}{5} \) model, operating at subharmonic frequency \( 2f_i \).

\[ \text{Figure 6. Calculated flux density in air gap of subharmonics.} \]

5. Conclusions
1. Flux density harmonics spectrum in the air gap of permanent magnet synchronous machine, when fractional \( q \) is used, contains subharmonics and fractional harmonics which may cause heightened vibrations on certain harmonics.
2. At different values of winding pitch \( y \), subharmonic amplitudes will be different, that must be taken into account in the design procedure of low-noise machines.
3. Frequencies of subharmonics and high-order harmonics coincide with frequencies of electromagnetic vibration sources for salient pole synchronous machines and are equal to \( 2fk \), where \( k = 1, 2, 3, \ldots \)
4. Calculation methods [6-9] of distribution factor \( k_d \) differ for subharmonics and fractional harmonics. Method [9] provides the best meeting with results of simulation approach.

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