Extension of VIKOR Method for Multi-criteria Decision Making Problem with Triangular Fuzzy Numbers

Wan-jun MI\textsuperscript{1}, Wen-qi JIANG\textsuperscript{2,*} and Yue-wei DAI\textsuperscript{1}

\textsuperscript{1}School of Automation, Nanjing University of Science and Technology, China
\textsuperscript{2}School of economic and management, Nanjing University of Science and Technology, China

*Corresponding author

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Abstract. The VIKOR method was developed for multi-criteria optimization of complex systems. The compromise solutions integrate group utility and individual regret of the opponent, so it has been applied widely into practice. In the past years, very few papers discussed the effect of defuzzification steps on the final decision results. The purpose of the paper is to analyze the information losses by using different defuzzification methods for the multi-criteria decision-making problem in which the criteria value and criteria weights are also triangular fuzzy numbers. Based on the requirement of the VIKOR method, we explain the necessity of defuzzifying triangular fuzzy numbers, then discuss the information losses generated in the process of four operations and compare them regarding two kinds of methods. An extension of VIKOR method is introduced to decrease the information losses. Finally, a numerical example is used to illustrate the applicability of the proposed method.

Introduction

Multi-criteria decision making (MCDM) is a well-known branch of decision making, which is a hot research topic in decision making and systems engineering, and has been proven as a useful tool, and have been successfully applied into many areas such as contractor evaluation and selection, advanced repair-to-order and disassembly-to-order system and so forth.

In MCDM problems, the decision makers need provide the qualitative or quantitative assessments to identify the value of each alternative with respect to each criterion. It is, however, unrealistic to assign a crisp value for a subjective judgment because of vague and imprecise information. The fuzzy evaluation value, usually, can be depicted as interval numbers, triangular fuzzy numbers, trapezoidal fuzzy number, linguistic terms, intuitionistic fuzzy numbers, and so forth. Many papers studied the fuzzy MCDM with fuzzy criteria value evaluation. Yingdong He [1] used probability non-membership function operator, probability membership function operator and probability heterogeneous operator on intuitionistic fuzzy sets. Behnam Vahdani [2] designed a new extension of the ELECTRE for MCGDM problems based on the intuitionistic fuzzy sets. Jianqiang Wang [3] developed an intuitionistic fuzzy MCDM method based on evidential reasoning, also defined a new score function for intuitionistic fuzzy set based on the prospect value function [4]. Renato A [5] considered the membership of intuitionistic fuzzy information, and V. Lakshmana[6]ranked interval-valued intuitionistic fuzzy number. Tomas Balezentis[7]extended the method with type-2 fuzzy sets and interval-valued trapezoidal fuzzy numbers. Junhua Hu [8] proposed an approach based on possibility degree to solve the fuzzy MCDM problems with interval type-2 fuzzy criteria value. Liang-Hsuan Chen [9] analyzed the attitudinal character of decision makers in an intuitionistic fuzzy environment. Chunqiao Tan [10] proposed an intuitionistic fuzzy Choquet integral. Behnam Vahdani [11] proposed a new interval-valued fuzzy modified TOPSIS method to reflect both subjective judgment and objective information. Shu-Chen Hsu [12] used horizontal, vertical and oblique pairwise comparisons algorithm to construct MCDM with incomplete linguistic preference relations model. ShingChung Ngan[13]extended the probabilistic linguistic framework to type-2 linguistic sets.
Mentioned by the above analysis, in this paper, we will analyze the information losses for the MCDM problems in which criteria value and criteria weights are also triangular fuzzy numbers, and propose a decision method based on VIKOR. Information losses may be generated by defuzzifying triangular fuzzy numbers, which are rarely analyzed in the above papers. Which steps will produce them, and how to avoid or decrease them, have become the critical problems in MCDM problems because they may change the final decision alternatives and make difficulty to select appropriate decision alternative more correctly.

The rest of the paper is organized as follows: section 2 analyzes which steps will generate information losses, and the extension of VIKOR method is proposed to solve the fuzzy MCDM problems in section 3. In section 4, an illustrative example is given to show the process in real situation. The conclusions of the paper are given in Section 5.

Information Losses in the VIKOR Method

Information Losses Generated by Operational Laws

The computation results can be depicted as follows.

\[ m(\tilde{A}) = \frac{\int_{a_1}^{a_2} x \mu_a(x) dx}{\int_{a_1}^{a_2} \mu_a(x) dx} = \frac{a_2^2 - a_1a_2 - a_2a_1 + a_1^2 + 2a_2^2}{3(a_2 - a_1)} \]  

(1)

\[ m(\tilde{B}) = \frac{\int_{b_1}^{b_2} x \mu_b(x) dx}{\int_{b_1}^{b_2} \mu_b(x) dx} = \frac{b_2^2 - b_1b_2 - b_1b_2 + b_1^2 + 2b_2^2}{3(b_2 - b_1)} \]  

(2)

Theorem 1: The operational laws of the TFNs will generate information losses.

Proof: Based on the above definition, we can obtain:

\[ m(\tilde{A}(+)\tilde{B}) = \frac{(a_2 + b_2)^2 - (a_1 + b_1)(a_2 + b_1) - (a_1 + b_1)^2 - (a_1 + b_1)^2 + 2(a_1 + b_1)^2}{3(a_2 + b_2 - a_1 - b_1)} \]

\[ m(\tilde{A}(-)\tilde{B}) = \frac{(a_2 - b_2)^2 - (a_1 - b_1)(a_2 - b_1) - (a_1 - b_1)^2 - (a_1 - b_1)^2 + 2(a_1 - b_1)^2}{3(a_2 - b_2 - a_1 + b_1)} \]

\[ m(\tilde{A}(\times)\tilde{B}) = \frac{(a_2b_2)^2 - a_1b_2a_2 - a_1b_2a_2 - (a_1b_1)^2 + 2(a_1b_1)^2}{3(a_1b_2 - a_1b_2)} \]

\[ m(\tilde{A}(\div)\tilde{B}) = \frac{(a_2b_1b_2)^2 - a_1b_2b_1b_2 - a_1b_2b_1b_2 - (a_1b_1b_2)^2 + 2(a_1b_1b_2)^2}{3(a_1b_2b_2 - a_1b_2b_2)} \]

For the additional laws, there is \( a_2b_2 \) in the numerator of \( m(\tilde{A}(+)\tilde{B}) \), and it, however, doesn’t exist in \( m(\tilde{A}) + m(\tilde{B}) \). The numerators of the \( m(\tilde{A}(+)\tilde{B}) \) and \( m(\tilde{A}) + m(\tilde{B}) \) only multiply the same element \( a_1b_1, a_3b_3 \), it is unnecessary to calculate all elements to obtain \( m(\tilde{A}) + m(\tilde{B}) \neq m(\tilde{A}(+)\tilde{B}) \).

Similar to the additional laws, the element \( a_2b_2 \) will also affect the value in the subtractive function \( m(\tilde{A}) - m(\tilde{B}) - m(\tilde{A}(-)\tilde{B}) \), then \( m(\tilde{A}) - m(\tilde{B}) \neq m(\tilde{A}(-)\tilde{B}) \). The constant is three in the denominator of \( m(\tilde{A}(\times)\tilde{B}) \), however, the number is nine in the denominator of \( m(\tilde{A}) \times m(\tilde{B}) \). All elements are unknown in the two triangular fuzzy numbers, then \( m(\tilde{A}) \times m(\tilde{B}) \neq m(\tilde{A}(\times)\tilde{B}) \). There are
three in the denominator of $m(\tilde{A}/\tilde{B})$, and the number is one in that of $m(\tilde{A})/m(\tilde{B})$, then $m(\tilde{A}/\tilde{B}) \neq m(\tilde{A})/m(\tilde{B})$. The two kinds of operational laws show that the information losses may be produced in the synthesis process of any two TFNs, and it is essential to avoid or decrease the information losses in the Fuzzy VIKOR method to improve the accuracy of the $Q_i, S_i, R_i$.

**Analysis of information losses for $Q_i, S_i, R_i$**

Let us consider the fuzzy decision matrix $\tilde{X}$, which consists of the decision alternatives and decision criteria, described by $\tilde{X} = [\tilde{x}_{ij}]_{m \times n}$. Where there are $m$ alternatives which are $A_1, \ldots, A_m$, $n$ criteria which are $c_1, \ldots, c_n$, $\tilde{x}_{ij} = [x^L_{ij}, x^M_{ij}, x^R_{ij}]$ are the TFNs that are the rating value of alternative $A_i$ with respect to the criterion $c_j$. The TFN $\tilde{W}_j = [W^L_j, W^M_j, W^R_j]$ indicates the weight of the criterion $c_j$. The correspondent normalization decision matrix is $\tilde{R} = [\tilde{r}_{ij}]_{m \times n}$.

Let $p = 1$, then $S_i, R_i$ and comprehensive evaluation $Q_i$ with respect to $\tilde{A}$ can be depicted as follows:

$S_i = \sum_{j=1}^{n} \left[ \tilde{w}_j \left( \frac{(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})}{(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))} \right) \right]$  \hspace{1cm} (3)

$R_i = \max\left[ \tilde{w}_j \left( \frac{(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})}{(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))} \right) \right]$  \hspace{1cm} (4)

$Q_i = v \times (\tilde{S}_i \min \tilde{S}_i + (1-v) \times (\tilde{R}_i \min \tilde{R}_i) + (\max \tilde{R}_i \min \tilde{R}_i))$  \hspace{1cm} (5)

Where $\max \tilde{r}_{ij} = [\max r^L_{ij}, \max r^M_{ij}, \max r^R_{ij}]$, and $\min \tilde{x}_{ij} = [\min x^L_{ij}, \min x^M_{ij}, \min x^R_{ij}]$. $v$ is introduced as a weight of the strategy of “the majority of criteria” (or “the maximum group utility”).

To obtain $Q_i, S_i, R_i$, the main steps can be depicted as follows.

1. **Calculate $\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij})$ and $(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})$**

   For any TFN $\tilde{A} = (a_1, a_2, a_3)$, the value satisfies the feature $a_1 \leq a_2 \leq a_3$. It is very difficult to judge whether the above two values satisfy the feature because all elements and their difference are unknown in $\tilde{R} = [\tilde{r}_{ij}]_{m \times n}$. If $\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij})$ or $(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})$ isn’t a TFN, it is necessary to defuzzify them respectively, and information losses will be generated. Otherwise, we can calculate them directly with the subtraction laws of the TFNs, and not defuzzify them.

2. **Calculate the $\tilde{w}_j \left( \frac{(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})}{(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))} \right)$**

   In this step, it is necessary to judge whether $(\max(\tilde{r}_{ij}) - \tilde{r}_{ij})/(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))$ is a TFN. The specific value can be calculated by multiplication law of the TFNs when it is a TFN for some criterion $c_j$. If it is a crisp value, we will use multiplication law by a scalar number to calculate it.

3. **$S_i$ can be calculated by adding all the values in step 2. If some are TFNs, and the others are crisp values, we can consider any crisp value is also a TFN in which the value satisfies the feature**

4. **Calculate $Q_i$**. Similar to step 1, the same analysis method is used to calculate $(\tilde{S}_i - \min \tilde{S}_i)/(\max \tilde{S}_i - \min \tilde{S}_i)$ and $(\tilde{R}_i - \min \tilde{R}_i)/(\max \tilde{R}_i - \min \tilde{R}_i)$, then we can form the final $Q_i$ by addition law and multiplication law.

5. **Select the compromise solution based on $Q_i, S_i, R_i$.**
The Proposed VIKOR Method

To identify and decrease the information losses in the process of selecting the compromise solutions, in this section, we are going to propose an extended version of VIKOR that can solve the fuzzy MCDM problems in which both the criteria values and criteria weights take TFNs.

The decision matrix is $\tilde{X} = [\tilde{x}_{ij}]_{m \times n}$ as mentioned by section 3.2. The main steps of the proposed algorithm could be described as Figure 1.

Step 1: Normalize the decision matrix $\tilde{X}$. The criteria are normally classified into two types: benefit criterion set $B$ and cost criterion set $C$. Different types of criteria have different normalization methods. The fuzzy decision matrix $\tilde{X} = [\tilde{x}_{ij}]_{m \times n}$ is normalized, whose correspondent normalization fuzzy decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m \times n}$ is depicted as follows. Where $x^+ = \max(x^+), x^- = \min(x^-)$.

$$\tilde{r}_{ij} = (x^+/x_{ij}^+, x_{ij}^-/x^+_{ij}, x_{ij}^R/x_{ij}^-), c_j \in B$$ (6)

$$\tilde{r}_{ij} = (x^+/x_{ij}^R, x^-_{ij}/x_{ij}^+, x_{ij}^-/x_{ij}^+), c_j \in C$$ (7)

Step 2: Calculate $\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij})$ and $(\max(\tilde{r}_{ij}) - \bar{r}_{ij})$. Information losses may be generated in the process of defuzzifying TFNs, so it is useful to identify if $\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij})$ or $(\max(\tilde{r}_{ij}) - \bar{r}_{ij})$ is a TFN. If $\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij})$ isn’t a TFN, then we need calculate them by equation (12) for $m(\hat{A} - \hat{B}) = m(\hat{A}) - m(\hat{B})$, compared with the method given by definition 3, and then calculate $(\max(\tilde{r}_{ij}) - \bar{r}_{ij})$ with the same approach. If not, they can be calculated as TFNs by the operational laws.

Step 3: Calculate the $\tilde{w}_j = (\max(\tilde{r}_{ij}) - \bar{r}_{ij})/(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))$ directly, which is the weighted criteria value. Normalize the criteria weight $\tilde{W}_j$ given by some decision maker, then the $\tilde{W}_j$ can be normalized as

$$\tilde{w}_j = \left[ W_j^L/\sum_{j=1}^{n} W_j^L, W_j^R/\sum_{j=1}^{n} W_j^R, W_j^M/\sum_{j=1}^{n} W_j^M \right] \wedge 1.$$ (8)

Step 4: $S_j$ can be calculated by adding all the TFNs $\tilde{M}_{ij}$, which are $\tilde{w}_j (\max(\tilde{r}_{ij}) - \bar{r}_{ij})/(\max(\tilde{r}_{ij}) - \min(\tilde{r}_{ij}))$.

$R_j$ should be calculated by comparing them. For each $\tilde{M}_{ij}$ with respect to the criterion $c_j$, the degree of possibility for a convex TFN $\tilde{M}_{ij}$ to be greater than the other TFN $\tilde{M}_{ik}$ is $V(\tilde{M}_{ij} \geq \tilde{M}_{ik})$. If it is more than 0.5, then select $\tilde{M}_{ij}$ to make comparison with the other TFNs, otherwise, $\tilde{M}_{ik}$ will be selected. Repeat the process and make sure of the maximal TFN for each alternative.

Step 5: Calculate $Q_j$. Propose as a compromise solution the alternative $A_j$ which is the best ranked by the measure $Q$ (minimum) if the following two conditions are satisfied:

Condition 1: Acceptable advantage: $Q(A_j) - Q(A_k) \geq 1/(m-1)$ Where $A_k$ is the alternative with the second place in the ranking list by $Q$, $m$ is the number of the alternatives.

Condition 2: Acceptable stability in decision making. The alternative $A_j$ should be the best alternative used by $S$ and $R$. A set of compromise solutions is proposed if one of the conditions is not satisfied. (1) Alternative $A_1$ and $A_2$ if only the condition 2 is not satisfied. (2) Alternative $A_1,...,A_m$ if the condition 1 is not satisfied; $A_m$ is determined by the relation $Q(A_{m+1}) - Q(A_k) < 1/(m-1)$ for maximum $M$.
Summary
The information losses may be generated because of defuzzifying fuzzy numbers, and different methods will produce different losses. They will affect the final decision alternatives. It is very helpful for decision makers to make sure the amount and places of information losses. Based on the VIKOR method, the paper analyzes the information losses by applying the four operational laws and membership function, and gives the beneficial suggestion of how to select the appropriate defuzzification method.

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References
[1] Yingdong He, Huayou Chen, Ligang Zhou, Jinpei Liu, Zhifu Tao. Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making. Information Sciences 259 (2014) 142-159.
[2] Behnam Vahdani, S. Meysam Mousavi, R. Tavakkoli-Moghaddam, H. Hashemi. A new design of the elimination and choice translating reality method for multi-criteria group decision-making in an intuitionistic fuzzy environment. Applied Mathematical Modelling 37 (2013) 1781-1799.
[3] Jian-qiang Wang, Rong-rong Nie, Hong-yu Zhang, Xiao-hong Chen. Intuitionistic fuzzy multi-criteria decision-making method based on evidential reasoning. Applied Soft Computing 13 (2013) 1823-1831.
[4] Jian-qiang Wang, Kang-jian Li, Hong-yu Zhang. Interval-valued intuitionistic fuzzy multi-criteria decision-making approach based on prospect score function. Knowledge-Based Systems 27 (2012) 119-125.
[5] Renato A. Krohling, André G.C. Pacheco, André L.T. Siviero. IF-TODIM: An intuitionistic fuzzy TODIM to multi-criteria decision making. Knowledge-Based Systems 53 (2013) 142-146.
[6] V. Lakshmana Gomathi Nayagam, S. Muralikrishnan, Geetha Sivaraman. Multi-criteria decision-making method based on interval-valued intuitionistic fuzzy sets. Expert Systems with Applications 38 (2011) 1464-1467.

[7] Tomas Balezentis, Shouzhen Zeng. Group multi-criteria decision making based upon interval-valued fuzzy numbers: An extension of the MULTIMOORA method. Expert Systems with Applications 40 (2013) 543-550.

[8] Junhua Hu, Yan Zhang, Xiaohong Chen. Multi-criteria decision-making method based on possibility degree of interval type-2 fuzzy number. Knowledge-Based Systems 43 (2013) 21-29.

[9] Liang-Hsuan Chen, Chia-Chang Hung, Chien-Cheng Tu. Considering the decision maker’s attitudinal character to solve multi-criteria decision-making problems in an intuitionistic fuzzy environment. Knowledge-Based Systems 36 (2012) 129-138.

[10] Chunqiao Tan, Xiaohong Chen. Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making. Expert Systems with Applications 37 (2010) 149-157.

[11] Behnam Vahdani, R. Tavakkoli-Moghaddam, S. Meysam Mousavi, A. Ghodratnama. Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method. Applied Soft Computing 13 (2013) 165-172.

[12] Shu-Chen Hsu, Tien-Chin Wang. Solving multi-criteria decision making with incomplete linguistic preference relations. Expert Systems with Applications 38 (2011) 10882-10888.

[13] Shing-Chung Ngan. A type-2 linguistic set theory and its application to multi-criteria decision making. Computers & Industrial Engineering 64 (2013) 721-730.