Modeling an experiment to measure the speed of gravity: optimization of the quadrupole mass

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Abstract. An experiment to measure the speed of gravity is being developed. To achieve this, a tidal gravitational signal generating device is intended to be made with a signal frequency of 3200 Hz. The gravitational wave detector Mario Schenberg, developed in Brazil, is the first option as the detector of this signal. Initially the work describes the optimization process of the generating device quadrupole mass to obtain a gravitational signal of maximum amplitude. Next, the finite element modeling (FEM) results for the rotating quadrupole mass are shown, allowing the analysis of the mechanical stresses produced by the high-speed rotation. This work reveals promising results that indicate the feasibility for the continuation of the project and subsequent construction of the real device.

1. Introduction
After the first detections of gravitational waves [1–3], and especially after the detection of gravitational waves along with the electromagnetic radiation of the same astrophysical source [4], the possibility of measuring the speed of gravity was opened through the comparison between the arrival times of the two signal types, additionally with the knowledge of the astrophysical source luminosity distance. Even so, it should be taken into account that the measurement of the speed of gravity is still a very complex technical challenge, due to the difficulty of detecting new events such as those mentioned previously. It is more convenient to emit a signal from a device on Earth and detect it, which is intended to be done experimentally in the continuation of this project.

The present work shows the current status of the project of a periodic (tidal) gravitational signal generating device. Such a device must rotate a non-cylindrically symmetrical mass (quadrupole mass) at a frequency of 1,600 Hz (96,000 rpm). The rotating mass should generate a Newtonian signal compatible with the resonance frequency of the gravitational wave detector Mario Schenberg [5–7], which is the first candidate for the function of signal detector. In this work, we took advantage of the experience gained with the work in simulation by finite elements that can be seen in previous articles [5,7–10] in which the results are validated using experimental data or by properties of symmetry of the problem.

In Section 2, a brief description of the proposed experiment is done. In Section 3, the optimization process of the quadrupole mass is discussed. In Section 4, the results obtained
Figure 1. Schematic illustration of the gravitational signal emitting device with indication of its main parts (obtained from Ref. [11]). A more realistic description of the quadrupole mass geometry is shown in Figure 3.

with the use of finite element modeling (FEM) for the quadrupole mass are shown. Conclusions are presented in Section 5.

2. Proposed experiment
To directly measure the speed of gravity, the generating device emits a Newtonian signal and a detector is proposed to receive this signal. The speed will be obtained by the phase difference between the two devices weighted by distance.

Concerning the gravitational signal generating device, it is composed by the composite quadrupole mass (non-zero quadrupole term) and the control, suspension and propulsion systems (see Figure 1). The device should operate inside a vacuum chamber to minimize the energy loss by air resistance. Since the speed of the gravitational signal should be very high, the quadrupole mass must rotate compatibly at a very high speed. The first plan is to use tools like FPGA (Field Programmable Gate Array) for control, passive magnetic bearings for suspension and a magnetic reluctance driver for the propulsion. An important feature is that the rotational frequency be stable and resonant with the detector during the measurement, implying that the control is a crucial part of the device project.

3. Quadrupole mass optimization process
For the optimization of the emitter device, a simplified modeling of the emitter-detector system has been developed, as shown in Figure 2. The emitter is described as a system of two masses $M$ connected by a rod of length $2a$, which rotates with axis passing through the center of mass (c.m.) and angular frequency $\omega$. The detector is described by a system of two masses $m = m_1 = m_2$ connected by a spring of natural length $b$ and elastic constant $k$. A mathematical description of the emitter-detector system, described in detail in our previous work [11], shows the following relation for the gravitational signal amplitude $h$:

$$h \propto \frac{a^2 MQ}{\omega_0^2 r^5}, \quad (1)$$
Figure 2. Simplified modeling of the periodic gravitational signal emitting device and the gravitational signal detector. Details are explained in the text. (obtained from Ref. [11])

Figure 3. The first proposed geometry for the quadrupole mass system: (a) the cross-sectional view and (b) the perspective view. The colors indicate: green and yellow → carbon fiber; blue → Maraging steel; orange → carbon fiber laminate.

where $Q$ is the mechanical quality factor of the detector, $\omega_0$ is the resonance frequency of the detector, and $r$ is the emitter-detector distance. Therefore, the product $Ma^2$ is the quantity to be optimized in the emitter.

A rotation frequency $f = 1,600$ Hz is considered in the sizing calculations. As the gravitational signal frequency generated by the rotating quadrupole mass is twice the rotation frequency of the same mass, the expected signal frequency is 3,200 Hz. This value is compatible with the central resonant frequency of the detector Mario Schenberg.

A more realistic configuration for the emitter quadrupole mass is shown in Figure 3. It is a non-cylindrically symmetrical mass composed of Maraging steel (shown in blue), carbon fiber + epoxy (yellow and green), and carbon fiber laminate + epoxy (orange). The optimization process consists of maximizing the product $Ma^2$ for the quadrupole mass of Figure 3 through an analogy with the simplified model previously described. A detailed description of the quadrupole mass optimization process is given in Ref. [11]. The optimized solution for the quadrupolar mass is given in Table 1, where the values corresponding to the geometric variables $p$, $l$, $d$, $q$ and $s$ are shown.

4. Results obtained with FEM
A FEM simulation was performed using the software SOLIDWORKS [12] for the configuration of Figure 3 using the mechanical properties shown in the Table 2. The simulation for a rotation
Table 1. Optimized dimensions for the quadrupole mass of the gravitational signal emitting device and the corresponding product $M a^2$. $r_{\text{hole}}$ is the fixed value for the central hole radius.

| $d$ (cm) | $l$ (cm) | $q$ (cm) | $p$ (cm) | $s$ (cm) | $r_{\text{hole}}$ (cm) | $M a^2$ (kg·cm$^2$) |
|---------|---------|---------|---------|---------|-----------------------|----------------------|
| 2.85    | 2.4     | 0.4     | 4.71    | 20.73   | 1.25                  | 97.35                |

Table 2. Properties of the materials used in the composition of the quadrupole mass.

| Material (thread) + epoxy composite | Density (g/cm$^3$) | Young’s modulus (GPa) | Ultimate strength (MPa) | Yield strength (MPa) |
|-------------------------------------|--------------------|-----------------------|-------------------------|----------------------|
| Maraging Steel 2800 $^a$            | 8.00               | 210                   | 2693                    | 2617                 |
| Carbon fiber (thread) + epoxy composite $^b$ | 1.634 $^b$        | 190.3 $^c$            | 3530 $^d$               | ——                   |
| Carbon fiber (laminate) + epoxy composite $^e$ | 1.55 $^b$         | 138                   | 1550                    | ——                   |

$a$) Properties obtained from Ref. [13]

$b$) Obtained from the densities of carbon fiber [14,15] and epoxy [16]

$c$) Obtained from the Young’s moduli of carbon fiber [14] and epoxy [16]

$d$) Ref. [17]

$e$) Properties obtained from Ref. [18]

of 1,600 Hz (96,000 rpm) has indicated that the von Mises stress is satisfactory (i.e., well below the ultimate tensile strength) for the carbon fiber components; however, the von Mises stress in the vicinity of the central hole has been well above the yield strength of Maraging Steel 2800 (the simulation has registered values above 7 GPa in that region). Therefore, some variations in the geometry of the quadrupole mass were proposed, with the condition in which such variations should not significantly change the configuration proposed in Figure 3.

After some FEM simulations with small changes in relation to the original dimensions, we obtained the configuration shown in Figure 4. In this new configuration, there is no central hole and there are extensions of 4 cm in height above and below the central steel component. The proposed changes were effective in avoiding the regions with stress excess, as seen in Figure 5.

The configuration of Figure 4 is still not definitive, as there is a need to reintroduce the central hole which is important for mounting the magnetic suspension of the quadrupole mass in rotation. This stage is under development.

5. Conclusions and prospects
In conclusion, the FEM simulation allowed to show the feasibility of construction of the gravitational signal device, indicating that it is possible to avoid stress excess in the quadrupole mass at a rotation of 96,000 rpm. In addition, the FEM simulations regarding the configuration of Figure 3 demonstrated that the optimization calculation of the quadrupolar mass [11] is a good starting point for improvements in the geometry of the quadrupolar mass through FEM.
Figure 4. Proposed configuration for the quadrupolar mass after the FEM simulations. Dimensions given in mm.

Figure 5. Representation of the quadrupole mass obtained from the FEM simulation showing section in the $yz$ plane with von Mises stress color scale (given in Pa). A rotation of 96,000 rpm is considered.
The configuration of the quadrupole mass should still be modified to adapt it to the magnetic suspension system during rotation. Therefore, we intend to present the evolution of this project in future publications.

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References
[1] Abbott B P et al. (LIGO Scientific Collaboration and Virgo Collaboration) 2016 Phys. Rev. Lett. 116 061102
[2] Abbott B P et al. (LIGO Scientific Collaboration and Virgo Collaboration) 2016 Phys. Rev. Lett. 116 241103
[3] Abbott B P et al. (LIGO Scientific and Virgo Collaboration) 2017 Phys. Rev. Lett. 118 221101
[4] Abbott B P et al. (LIGO Scientific Collaboration and Virgo Collaboration) 2017 Phys. Rev. Lett. 119 161101
[5] Bortoli F S, Frajuca C, Souza S T, de Waard A, Magalhães N S and Aguiar O D 2016 Braz. J. Phys. 46 308
[6] Frajuca C, Ribeiro K L, Andrade L A, Aguiar O D, Magalhães N S and Marinho Jr R M 2004 Class. Quantum Grav. 21 S1107
[7] Frajuca C, Bortoli F S and Magalhães N S 2006 J. Phys.: Conf. Ser. 32 319
[8] Frajuca C, Bortoli F S and Magalhães N S 2005 Braz. J. Phys. 35, n. 4B 1201
[9] Frajuca C, Magalhães N S, Bortoli F S and Horiguti A M 2008 J. Phys.: Conf. Ser. 122 012029
[10] Bortoli F S, Frajuca C, Magalhães N S and Duarte E N 2010 J. Phys.: Conf. Ser. 228 012011
[11] Frajuca C, Souza M A, Coppedé D, Nogueira P R M, Bortoli F S, Santos G A and Nakamoto F Y 2018 J. Braz. Soc. Mech. Sci. Eng. 40 319
[12] SOLIDWORKS website: https://www.solidworks.com/
[13] MatWeb - The Online Materials Information Resource. Available in: http://www.matweb.com/
[14] Texiglass® 2014 Pannel Náutico. Data presented at FEIPLAR 2014 - International Fair and Congress of Composites, Polyurethane and Engineering Plastics (São Paulo, Brazil). Available in: http://feiplar.com.br/_site2014/portugues/materiais/palestras/nautico/Texiglass.pdf Retrieved June 21, 2018.
[15] Minus M and Kumar S 2005 JOM 57 52
[16] Machado M G 2004 Estudo Experimental da Ductilidade de Vigas em Concreto Armado Reforçadas à Flexão Utilizando Compósitos com Tecido de Fibras de Carbono. MSc thesis, PUC-Rio (Brazil), Chapter 2
[17] Texiglass® 2014 Pannel da Construção Civil. Data presented at FEIPLAR 2014 - International Fair and Congress of Composites, Polyurethane and Engineering Plastics (São Paulo, Brazil). Available in: http://www.feiplar.com.br/materiais/palestras/construcao_civil/TEXIGLASS.pdf Retrieved June 21, 2018.
[18] Solinas G 2012 Reforços para Náutica. Data of Texiglass® presented at FEIPLAR 2012 - International Fair and Congress of Composites, Polyurethane and Engineering Plastics (São Paulo, Brazil). Available in: http://www.tecnologiademateriais.com.br/mt/2012/cobertura_paineis/nautico/apresentacoes/Texiglass.pdf Retrieved June 21, 2018.