Some Implications of Charge and Color Breaking in the MSSM

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Abstract
We examine some physically relevant implications of potentially dangerous charge and color breaking minima for supersymmetric models. First, we analyze the stability of the corresponding constraints with respect to variations of the initial scale for the running of the soft breaking terms, finding that the larger the scale is, the stronger the bounds become. In particular, by taking $M_P$ rather than $M_X$ for the initial scale, which is a more sensible election, we find substantially stronger and very important constraints. Second, we find general bounds on the universal gaugino mass, the universal scalar mass, $m \geq 55$ GeV, and the Higgs bilinear coefficient, $|B| \lesssim 3m$. Finally, we study the infrared fixed point solution of the top quark mass. Again, the constraints on the parameter space turn out to be very important, including the analytically derivable bound $|M/m| \lesssim 1$. 
1 Introduction

Recently there has been some activity in trying to constrain the soft parameter space of supersymmetric (SUSY) models [1–7] through the possible existence of dangerous charge and color breaking minima [8–11]. In ref.[2] a systematic discussion of all the potentially dangerous directions in the field space of the minimal supersymmetric standard model (MSSM) was carried out. Imposing that the SUSY standard vacuum should be deeper than the charge and color breaking minima, the corresponding constraints are very strong. Important bounds, not only on the value of the trilinear scalar term \( A \), but also on the values of the bilinear scalar term \( B \) and the scalar and gaugino masses \( m, M \) respectively, are produced on these grounds\(^1\).

In the present paper we discuss some issues related to this kind of restrictions, which are relevant for supersymmetric model-building, since they offer a more precise realization of the constraints under very common circumstances.

The analysis of ref.[2] was performed assuming universality of the soft breaking terms at the unification scale, \( M_X \), as it is usually done in the MSSM literature. However, in the standard supergravity (SUGRA) framework, where SUSY is broken in a “hidden” sector, the natural initial scale to implement the boundary conditions for the soft terms is \( M_P \equiv M_{\text{Plank}}/\sqrt{8\pi} \) rather than \( M_X \) (\( M_P \) is the scale in which the standard SUGRA Lagrangian is written). Hence, it is natural to wonder how much the standard charge and color breaking analysis (and in particular the results of ref.[2]) will get modified by taking \( M_P \) instead of \( M_X \) for the initial scale of the soft terms\(^2\).

This is one of the purposes of the present paper which will be carried out in section 2. The second one is to obtain explicit bounds on the supersymmetric parameters. We will show that the charge and color breaking constraints put important bounds not only on the value of \( A \), but also on the values of \( M, B \) and \( m \). Concerning the latter, in ref.[2] was mentioned the fact that in the limiting case \( m = 0 \) the whole parameter space turns out to be excluded. Now, we will be more precise, obtaining a general lower bound on \( m \). These tasks will be accomplished in section 3. Finally, in section 4, we apply the charge and color breaking constraints to a particular region of the parameter space of the MSSM, namely that corresponding to the infrared fixed point solution for the top quark mass. The conclusions are left for section 5.

Of course, we are aware of the possibility of living in a metastable vacuum, provided that its lifetime is longer than the present age of the universe [4, 5, 6]. Although this possibility poses some cosmological questions, as discussed in ref.[5], it is clear that could help to rescue some regions of the parameter space. This means that the bounds we consider here are the most conservative ones (in the sense of safe ones). Besides, the identification of the dangerous charge and color breaking minima is the first necessary step for the cosmological analysis. On the other hand, there are many

\(^1\)Very strong bounds can also be obtained in a SO(10) GUT \[7\]. In the context of String Theory, in particular when SUSY is broken by the dilaton, the whole parameter space is essentially excluded \[5\].

\(^2\)This question was first pointed out in ref.[12] for a different type of analysis. In particular the authors studied the modifications on the low-energy predictions of a SUSY GUT when non-universal corrections to the soft parameters arise from their evolution between \( M_P \) and \( M_X \).
possible cosmological scenarios, which requires separate analyses. In particular one can consider scenarios where the initial conditions are dictated by thermal effects (see e.g. refs. [4, 6]) or inflationary scenarios. The latter situation may be much more dangerous and involved due to the large fluctuations of all the scalar fields, that could be driven in this way to the dangerous minima.

Let us review the constraints associated with the existence of dangerous directions in the field space. As was mentioned above, a complete analysis of this issue, including in a proper way the radiative corrections to the scalar potential, was carried out in ref. [2]. The most relevant results obtained there for our present task are the following.

There are two types of constraints: the ones arising from directions in the field-space along which the (tree-level) potential can become unbounded from below (UFB), and those arising from the existence of charge and color breaking (CCB) minima in space along which the (tree-level) potential can become unbounded from below (UFB), ref. [2]. The most relevant results obtained there for our present task are the following.

Concerning the UFB directions (and corresponding constraints), there are three of them, labelled as UFB-1, UFB-2, UFB-3 in [2]. It is worth mentioning here that in general the unboundedness is only true at tree-level since radiative corrections eventually raise the potential for large enough values of the fields, thus developing minima. Still these minima can be deeper than the realistic one (i.e. the SUSY standard model vacuum) and thus dangerous. The UFB-3 direction, which involves the scalar fields $\{H_2, \nu_{L_i}, e_{L_j}, e_{R_j}\}$ with $i \neq j$ and thus leads also to electric charge breaking, yields the strongest bound among all the UFB and CCB constraints. For the sake of later discussions, let us briefly expose the explicit form of this bound. By simple analytical minimization it is possible to write the value of all the relevant fields along the UFB-3 direction in terms of the $H_2$ one. Then, for any value of $|H_2| < M_P$ satisfying

$$|H_2| > \sqrt{\frac{\mu^2}{4\lambda_{e_j}^2} + \frac{4m_{L_i}^2}{g'^2 + g_2^2} - \frac{|\mu|}{2\lambda_{e_j}}},$$

the value of the potential along the UFB-3 direction is simply given by

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2) |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} (m_{L_j}^2 + m_{e_j}^2 + m_{L_i}^2) |H_2| - \frac{2m_{L_i}^2}{g'^2 + g_2^2}. \quad (2)$$

Otherwise

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2) |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} (m_{L_j}^2 + m_{e_j}^2) |H_2| + \frac{1}{8} (g'^2 + g_2^2) \left[ |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} |H_2| \right]^2. \quad (3)$$

In eqs. (2), $\lambda_{e_j}$ is the leptonic Yukawa coupling of the $j$–generation and $m_2^2$ is the sum of the $H_2$ squared soft mass, $m_{H_2}^2$, plus $\mu^2$. Then, the UFB-3 condition reads

$$V_{\text{UFB-3}}(Q = \hat{Q}) > V_{\text{real min}}, \quad (4)$$

where $V_{\text{real min}} = -\frac{1}{8} (g'^2 + g_2^2) (v_1^2 - v_2^2)^2$, with $v_{1,2}$ the VEVs of the Higgses $H_{1,2}$, is the realistic minimum evaluated at $M_S$ (see below) and the $\hat{Q}$ scale is given by $\hat{Q} \sim \text{Max}(g_2 |e|, \lambda_{\text{top}} |H_2|, g_2 |H_2|, g_2 |L_i|, M_S)$ with $|e| = \sqrt{|H_2| |\mu| / \lambda_{e_j}}$ and $|L_i|^2 = |H_2|^2 + ...
\[ |e|^2 - \frac{4m_L^2}{g^2 + g_s^2} \]. Finally, \( M_S \) is the typical scale of SUSY masses (normally a good choice for \( M_S \) is an average of the stop masses, for more details see refs.\([11, 13, 2]\)). Notice from (3,4) that the negative contribution to \( V_{UFB-3} \) is essentially given by the \( m_2^2 - \mu^2 \) term, which can be very sizeable in many instances. On the other hand, the positive contribution is dominated by the term \( \propto 1/\lambda e_j \), thus the larger \( \lambda e_j \) the more restrictive the constraint becomes. Consequently, the optimum choice for the \( e \)-type slepton in eqs.\((2,3)\) is the third generation one, i.e. \( e_j = stau \).

Concerning the CCB constraints, let us mention that the “traditional” CCB bounds\([8]\), when correctly evaluated (i.e. including the radiative corrections in a proper way), turn out to be extremely weak. However, the “improved” set of analytic constraints obtained in ref.\([2]\), which represent the necessary and sufficient conditions to avoid dangerous CCB minima, is much stronger. It is not possible to give here an account of the explicit form of the CCB constraints used in the present paper. This can be found in section 5 of ref.\([2]\), to which we refer the interested reader.

2 CCB and UFB Constraints and the Initial Scale

As mentioned in the introduction, the initial boundary conditions for the running MSSM soft terms are usually understood at a scale \( M_X \). As we will see now, bigger initial scales, as for example \( M_p \), will imply stronger charge and color breaking constraints. This can be understood, for example, from our discussion about the UFB-3 direction above: the larger the initial scale for the running is, the more important the negative contribution \( m_2^2 - \mu^2 \) to the potential (see eqs.\((2,3)\)) becomes.

On the other hand, as discussed in the introduction, the natural initial scale for the soft SUSY-breaking terms triggered by the spontaneous breaking of SUGRA is \( M_p \). In Fig. 1 we show the UFB and CCB constraints using \( M_p \) as the initial scale. More precisely, we plot the case\([3]\) \( B = 2m \), with \( m = 100, 300 \) and \( 500 \) GeV, but similar conclusions will be obtained for other values. We take \( M_{top}^{phys} = 174 \) GeV as the physical (pole) top mass. As a matter of fact, it is not always possible to choose the boundary condition of the top Yukawa coupling \( \lambda_{top} \) so that the physical (pole) mass is reproduced because the renormalization group (RG) infrared fixed point of \( \lambda_{top} \) puts an upper bound on the running top mass \( M_{top} \), namely \( M_{top} \leq 197 \sin \beta \) GeV\([14]\), where \( \tan \beta = v_2/v_1 \). The corresponding restriction in the parameter space (black region in Fig. 1) is certainly substantial. The region excluded by the CCB bounds is denoted in the figure by circles. The restrictions coming from the UFB constraints (small filled squares) are very strong in all the cases. Most of the parameter space is in fact excluded by the UFB-3 constraint. Finally, we have also plotted in Fig.1 the region excluded by the experimental bounds on SUSY particle masses (filled diamonds).

Quite conservatively, we have imposed

\[
\begin{align*}
M_\tilde{g} &\geq 120 \text{ GeV} , \quad M_\tilde{\chi}^\pm \geq 45 \text{ GeV} , \quad M_\tilde{\chi}^0 \geq 18 \text{ GeV} , \\
M_\tilde{q} &\geq 100 \text{ GeV} , \quad M_\tilde{l} \geq 45 \text{ GeV} , \quad M_\tilde{t} \geq 45 \text{ GeV} , \quad \text{(5)}
\end{align*}
\]

\[3\]This value of \( B \) is particularly interesting since it is obtained in several SUGRA theories. See e.g.\([2, 5]\) and references therein.
in an obvious notation. The ants indicate regions which are excluded by negative squared mass eigenvalues. Notice from Fig.1 that there are areas that are simultaneously constrained by different types of bounds. At the end of the day, the allowed region left (white) is quite small.

This result is to be compared with the one of ref.\[2\], where the assumption of universal soft terms at $M_X$ was taken. From Fig. 3 of that paper we see that the constraints are now substantially increased and in fact regions of large $M$ which were previously allowed become now completely excluded (see Fig. 1 of this paper). Note also that the UFB bounds are the most sensible ones to modifications of the initial scale (for the reasons commented at the beginning of this section), while the CCB bounds are almost insensitive to them.

3 Bounds on MSSM parameters

From Fig. 1 we see that for a given value of $m$ the values of $A$ and $M$ are both bounded from below and above in a correlated way. E.g. for $-M = m = 500$ GeV, we get $1 \leq A/m \leq 3.5$; while for $A/2 = m = 500$ GeV, we get $-1.75 \leq M/m \leq 1$. This restriction of $M/m$ to a finite and rather narrow range is a novel fact\[4\].

We will show now that the CCB and UFB constraints put important bounds not only on the value of $A$ and $M$, but also on the values of $B$ and $m$, which is also an interesting novel fact.

In Fig. 2 we generalize the previous analysis by varying the value of $B$ for different values of $m$, namely $m = 100$ GeV, $m = 300$ GeV. The final allowed regions from all types of bounds in the parameter space of the MSSM are shown. Both figures exhibit a similar trend. For a particular value of $m$, the larger the value of $B/m$ is, the smaller the allowed region becomes. More precisely, the maximum allowed value of $B$ for $m = 100$ GeV is $B = 2.8m$, while for $m = 300$ GeV is $B = 2.9m$. For negative values of $B$ the corresponding figures can easily be deduced from the previous ones, taking into account that they are invariant under the transformation $B, A, M \rightarrow -B, -A, -M$.

In general, for $m < \sim 500$ GeV, $B$ has to satisfy the bound

$$|B| \lesssim 3m.$$ \(6\)

This behaviour comes mainly from the enhancement of the forbidden areas by the UFB-3 constraint and the requirement of $M_{\text{top}}^{\text{phys}} = 174$ GeV. Both facts are due to the decreasing of $\tan \beta$ as the low-energy value of $B$ grows. Then higher top Yukawa couplings are needed in order to reproduce the experimental top mass. On the one hand, this cannot be always accomplished due to the infrared fixed point limit on the top mass. On the other hand, the larger the top Yukawa coupling is, the stronger the UFB-3 bound becomes. The same two effects are obtained when $M_{\text{top}}^{\text{phys}}$ is increased and therefore, the larger the top mass is, the stronger the constraints become.

On the other hand, figures 1, 2 show a clear trend in the sense that the smaller the value of the soft scalar masses, $m$, the more restrictive the constraints become. This is

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\[4\]This has obvious implications for gaugino dominance scenarios (see e.g. ref.\[13\]), where a large ratio $M^2/m^2$ is found to be useful in order to solve the FCNC problem.
mainly due to the effect of the UFB-3 constraint (note the almost exact $m$ invariance of the CCB bounds).

In fact, doing the same type of analysis as in Fig. 2 it is possible to find a value of $m$ for which the whole parameter space turns out to be excluded. This interesting lower bound on $m$ is

$$m \geq 55 \text{ GeV} .$$

From the above discussion it is evident that the limiting case $m = 0$ is also excluded. Of course, this has obvious implications for no-scale models [11]. As an example we have plotted in Fig. 3 the case $B = A$.

From all the figures, it is clear that the CCB and UFB bounds put also important constraints on $A$ (as has been traditionally considered) and $M$, although the precise form of these constraints depends on the values of the other MSSM parameters and cannot be cast in simple general formulae as eqs.(6, 7).

Finally, let us mention that the previous bounds would be weaker if one uses $M_X$ as initial scale (see ref. [2]). In particular, eq.(7) would then change to $m \geq 50 \text{ GeV}$.

## 4 Infrared Fixed Point Model

Let us now apply the UFB and CCB constraints to a particular and attractive MSSM scenario, namely when the top Yukawa coupling at high energy is large enough to be in the infrared fixed point regime\(^5\). Then the running top mass is approximately given by $M_{\text{top}} \simeq 197 \sin \beta$ and therefore $\tan \beta$ is fixed once a particular value for $M_{\text{top}}^{\text{phys}}$ is chosen (in our case $M_{\text{top}}^{\text{phys}} = 174 \text{ GeV}$). Thus the number of independent soft parameters is reduced since for given values of $m$, $M$ and $A$, the value of $B$ is fixed through $\tan \beta$. In addition, the value of $A_t$ at low energies is also fixed in an infrared fixed point value (this is not the case for the other trilinear couplings), and therefore is not an independent parameter for the electroweak breaking process. Consequently, the required value of $\mu$ in order to get the correct amount of breaking is determined only by the values of $m$ and $M$.

As a consequence of the previous characteristics, it is easy to check that the UFB-3 constraint, eqs.(6, 7), does not depend on the value of $A$ in the strict fixed point limit, while the CCB bounds do.

In Fig. 4 we show the UFB and CCB constraints for the cases $m=100$ and $500 \text{ GeV}$. The figure corresponds to positive sign of $\mu$ which corresponds to negative value of $B$ at low energy. The results for negative sign of $\mu$ can be deduced by taking into account the invariance of all the results under the transformation $B, A, M \rightarrow -B, -A, -M$.

Notice the almost exact independence of the UFB constraints on the value of $A$, as announced. Actually, the slight departure from this independence is mainly due to the contribution of the UFB-1 and UFB-2 constraints in addition to the UFB-3 one. It is clear from the figure that the UFB-3 constraint is very well approximated by

$$|M/m| \lesssim 1 .$$

\(^5\)For a review see e.g. [7] and references therein.
It is interesting to note that an approximate form of this bound can be obtained in an analytical way as follows.

Although the UFB-3 bound must be satisfied for any possible value of $H_2$ (see eqs. (1–4)) a weaker bound arises by fixing $H_2$ (and thus $\hat{Q} \simeq |H_2|$) at a convenient value. To do this, notice from eq.(2) that since $m^2 + m^2_{L_i} > 0$, $V_{\text{UFB}-3}$ can only take negative values for $|H_2| \gtrsim O(10^2) \times m_\ell^2/\mu$. A particular simple form for the bound is obtained for a large enough value of $|H_2|$. For example, taking $|H_2| = 10^6$ GeV (which corresponds to $t = \log(M_P^2/|H_2|^2) = 57$) $V_{\text{UFB}-3}$ becomes

$$V_{\text{UFB}-3} \simeq m^2|H_2| \left\{ 0.5 \times 10^6 \left( 1 - 0.67 x^2 \right) + 3 \left| \frac{\mu}{\lambda_T} \right| \left( 1 + 0.32 x^2 \right) \right\}, \quad (9)$$

where we have applied the RG equations of $m^2_{H_2} = m^2_2 - \mu^2$, $m^2_L$ and $m^2_e$ to write their values at $\hat{Q}$, and $x = M/m$. Taking into account that for most of the parameter space $|\mu/\lambda_T| \ll 10^6$ GeV, the second term within brackets in eq.(9) is negligible and the UFB-3 bound becomes simply $x^2 = (M/m)^2 < 1/0.67 = 1.49$, i.e. $|M/m| < 1.22$.

A more refined bound comes by carefully adjusting $|H_2|$ and $\hat{Q}$ to lower values. However, then the bound becomes slightly dependent on the value of $\mu$, and thus on the value of $m$ since both are correlated, as it is apparent in Fig.4.

Concerning the experimental bounds on SUSY particle masses, let us mention that the region excluded in Fig. 4 is mainly due to charginos and neutralinos. Since their masses are independent of the value of $A$, so the corresponding experimental constraints are. At the end of the day, the allowed region (white) left is quite small.

Finally, let us point out that Fig. 4 was obtained imposing the soft terms boundary conditions at the most sensible (Planck) scale. Had we chosen $M_X$ instead of $M_P$, the constraints would have been less strong. For example, for $m = 500$ GeV the empirical form of the UFB-3 bound would be $|M/m| \lesssim 1.6$ instead of $|M/m| \lesssim 1$.

5 Conclusions

In this paper we have analyzed some physically relevant implications of the possible existence of dangerous charge and color breaking minima for supersymmetric models. First, we have noted that the strength of the corresponding (CCB and UFB) constraints is quite sensitive to the value of the initial scale for the running of the soft breaking terms. The larger the scale is, the stronger the bounds become. In particular, by taking $M_P$ rather than $M_X$ for the initial scale, which is a more sensible election, we find substantially stronger and very important constraints.

We have also shown that the CCB and UFB constraints put important bounds not only on the value of $A$ but also on the values of $M$, $B$ and $m$, which is an interesting novel fact. More precisely, the values of $A$ and $M$ are both bounded from below and above in a correlated way and we get the bounds $|B| \lesssim 3 m$ and $m \geq 55$ GeV. The lower bound on $m$ has obvious implications for no-scale models.

Finally, we have studied the interesting case of the infrared fixed point solution for the top quark mass. Again, the constraints on the parameter space turn out to be very important, including the analytically derivable bound $|M/m| \lesssim 1$. 

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Figure Captions

Fig. 1 Excluded regions in the parameter space of the MSSM with $B=2m$. The black region is excluded because it is not possible to reproduce the experimental mass of the top. The small filled squares indicate regions excluded by UFB constraints. The circles indicate regions excluded by CCB constraints. The filled diamonds correspond to regions excluded by the experimental lower bounds on SUSY-particle masses. The ants indicate regions excluded by negative scalar squared mass eigenvalues.

Fig. 2 Contours of allowed regions in the parameter space of the MSSM, for different values of $B$ and $m$, by the whole set of constraints.

Fig. 3 The same as Fig.1 but with $m=0$, $B=A$.

Fig. 4 Excluded regions, with the same conventions than in Fig. 1, in the parameter space of the infrared fixed point model.
Fig. 1
Fig. 2
Fig. 3
Fig. 4