Variations of the Initial Mass Function in Semi-Analytical models.

Fabio Fontanot*

INAF - Astronomical Observatory of Trieste, via G.B. Tiepolo 11, I-34143 Trieste, Italy

ABSTRACT
Deviations from a universal, MW-like, Stellar Initial Mass Function (IMF) have been reported for distant galaxies, although the physical reason behind the observed variations is still matter of ongoing debate. In this paper, we present an exploratory study to assess the impact of the proposed IMF evolution on the statistical galaxy properties, as predicted by the Semi-Analytical model of galaxy formation and evolution MORGANA. In particular, we test different dependencies for the IMF shape, as a function of both model galaxy properties (such as star formation rate, velocity dispersion or stellar mass) and environment, and compare the predicted stellar mass functions and star formation rate functions with reference runs at fixed IMF. In most cases, MORGANA predictions show deviations of the order of a few tenths of dex with respect to a run assuming an Universal Kroupa IMF. Among the proposed IMF variations, an increasing Top-Heavy IMF at increasing star formation rates has the largest impact on predicted galaxy properties, while most of the models assuming an increasing Bottom-Heavy IMF at higher masses/velocity dispersion lead to variations in galaxy properties that are of the same order as the uncertainty on the mass and star formation rate determination. By comparing the predicted galaxy stellar mass functions, we conclude that the study of the high-mass end can provide useful constraints to disentangle models assuming an increasing Top-Heavy IMF in high star forming or Bottom-Heavy IMF in massive systems.

Key words: galaxies:evolution - galaxies: fundamental parameters - galaxies: star formation - galaxies: luminosity function, mass function

1 INTRODUCTION
The characterisation of the role played by the different physical processes acting on baryonic gas in shaping the observed properties of galaxy populations is still a fundamental goal for modern astrophysics. Among those mechanisms expected to impact galaxy evolution, the description of star formation is a key asset for any theoretical model of galaxy formation and evolution. However, despite the huge scientific effort in understanding the chain of events leading to the formation of single and/or clustered stars from an unstable gas cloud, a number of long-standing problems are still open (see e.g. Krumholz 2014, and references herein). In particular, the definition of the Stellar Initial Mass Function (IMF, hereafter), which regulates the relative abundance of massive versus low-mass stars per each stellar generation, has seen considerable recent debate about possible variation of its shape as a function of galaxy properties.

Historically, the IMF has been estimated via stellar counts in the solar neighbourhood and/or the Milky Way (MW), where stellar populations can be measured with enough accuracy up to low-mass stars. Different functional representations for the IMF shape in the MW have been proposed in the literature, from early suggestions of a single power-law (Salpeter 1955, S55), to a broken power-law (Kroupa 2001, K01) or lognormals with power-law tail (Chabrier 2003, C03). Relevant uncertainties are connected with both the brown dwarf regime below 0.08 $M_{\odot}$ (where star counts are expected to decrease, but the sharpness of the decline is poorly constrained) and to the cut-off at high masses between 100 and 150 $M_{\odot}$. It is very difficult to repeat such detailed analysis in distant galaxies, as only the integrated light of multiple stellar populations is observationally accessible; therefore the apparent invariance of the IMF among MW regions has been usually assumed to be an universal property of star forming regions.

Theoretical models investigating the origin of the IMF shape from first principles predict some degree of variability as a function of the physical conditions of the star forming medium. These include models trying to describe star formation in a turbulent medium (see e.g. Klessen et al. 2005, Hennebelle & Chabrier 2008, Hopkins 2012), models based on the Jeans mass argument (see e.g. Narayanan & Dav´e 2013) and models investigating the role of cosmic rays as star formation regulators (Papadopoulos 2010). The

* E-mail: fontanot@oats.inaf.it

© 0000 RAS
claim for a systematic change of IMF with the mass of parent stellar clusters led to the notion of integrated galactic IMF (see e.g. Kroupa & Weidner 2003); since the mass spectrum of stellar clusters in a galaxy is not uniform, this implies that the integrated galactic IMF is different from that of individual clusters.

From an observational point of view, several authors reported a flattening of the IMF at increasing star formation rates (SFR) in late type galaxies (Hoversten & Glazebrook 2008, Gunawardhana et al. 2011), on the base of multi-colour photometry. The study of unresolved stellar populations in external galaxies also shows indications for a non-universal IMF: Cappellari et al. (2012) compared mass-to-light ratios derived from stellar kinematics to the predictions of stellar population synthesis models, finding systematic variations of about a factor of two with respect to a MW-like IMF. Similar results have been obtained spectrosopically using spectral features sensible to the stellar effective temperature and surface gravity (van Dokkum & Conroy 2011, Conroy & van Dokkum 2012), with the data suggesting increasingly bottom-heavy IMFs for galaxies with larger velocity dispersions. It is not clear to what extent the Cappellari et al. (2012) and Conroy & van Dokkum (2012) results agree among themselves (see e.g. Smith 2014) and/or are in tension with the increasingly Top-Heavy IMFs in high star forming galaxies (see, e.g. Narayanan & Davé 2013). So far no clear evidence for a redshift evolution of the IMF shape has been found: Shetty & Cappellari (2014) studied a sample of 68 $0.7 < z < 0.9$ field galaxies, and they found an average normalisation of the IMF in massive galaxies consistent with S55 slope, with a substantial scatter.

Any IMF change will impact both the predictions of theoretical models and the reconstruction of galaxy properties from multi-wavelength photometry, i.e. through Spectral Energy Distribution (SED) fitting techniques. In the latter approach, it is still possible to shift between models at fixed IMF by means of correction factors, but the same approach does not hold for the predictions of theoretical models (see e.g. De Lucia & Blaizot 2007), as a different IMF has a larger effect than simply changing the amount of baryonic mass locked in long-lived stars. The pictures became more complicated if we allow the IMF to vary along galaxy evolution: in this case it is not possible to recover galaxy physical properties from algorithms comparing observed to synthetic photometry, without any prior knowledge of the relation between star formation history and IMF shape. Also in the case of an algorithm relaxing the universal IMF hypothesis and best-fitting also the shape of the IMF, like in the Van Dokkum & Conroy (2012) approach, the resulting shape represents the IMF of the dominant stellar population, and/or an integrated mean value along the whole galaxy lifetime.

In this paper, we present an exploratory study of the effect of IMF variations on the predictions of Semi-Analytical models of galaxy formation and evolution (SAMs, see e.g. Baugh 2006, for a review of this approach). To this aim, we will explicitly include in the model different parametrization of IMF variation, both theoretically and observationally grounded. We will focus on the SAM parameters that directly depend on the IMF definition, while keeping all other model parameters fixed, with the aim of highlighting the impact of a varying IMF shape on the predicted physical properties of model galaxies. Earlier attempts to include different IMF shapes in the SAM framework have been already presented in the literature (Baugh et al. 2005), but the novelty of our approach lies in the wide range of possible IMF variations and dependencies considered. In the following, there will be no attempt to compare model predictions to physical quantities derived under the hypothesis of universal IMF shape, as we consider such a comparison misleading. Moreover, we do not attempt a comparison with direct photometry either, as we want to highlight the modifications on the basic physical properties of galaxies and avoid any possible additional uncertainty due to the choice of a given simple stellar population library.

This paper is organised as follows. In Sec. 2 we describe the IMF variable models we implement on the the SAM model MORGANA; in Sec. 3 we present our results, while in Sec. 4 we give our conclusions.

## 2 SEMI-ANALYTICAL MODEL

Modern theoretical models of galaxy formation and evolution assume that Dark Matter (DM) haloes are the privileged sites for galaxy formation, driven by a complex network of physical processes including at least (but not only) the cooling of baryonic gas, the onset of star formation and the various feedbacks effects associated with the death of massive stars and the accretion of cold gas on to Super-Massive Black Holes. In an attempt to overcome our limited knowledge of these key physical mechanisms, SAMs describe them using simple mathematical prescriptions (either physically or observationally motivated) and then follow the time evolution of the different galaxy components (bulge, disc and halo) and gas phases (stars, cold gas, hot gas). Quantitative comparisons between the predictions of different SAMs have been extensively discussed in the literature, showing the relative strength of this approach, which provides, at least, a coherent picture of galaxy evolution in the ΛCDM concordance cosmological model (see e.g. Fontanot et al. 2009, 2012). This level of coherence was not expected a priori, given the different implementations for the relevant physical processes assumed by the different groups developing SAMs, and represents another facet of the relevant degeneracies involved in the definition of the parameter space associated with SAMs themselves.

In this paper, we consider predictions from the MOdel for the Rise of Galaxies and Agns (MORGANA); we refer the reader to Monaco et al. (2007) for a full description of the model features, including the modelling of cooling, star formation, black hole accretion and feedback processes, and to Lo Faro et al. (2009) for the description of the most recent calibration of the model, based on the C03 IMF. In MORGANA, the shape of the IMF determines the number of supernovae (SNe) per unit stellar mass formed $(f_{sn})$ which regulates the strength of SNe feedback and the matter/energy exchange between the cold and hot gas phases (see, e.g. Fontanot et al. 2013 for a comparison of different feedback schemes in SAMs). For each stellar generation, a fraction of the baryonic material involved in the star formation process is eventually given back to the hot gas phase through stellar winds, mass loss and SNe ejecta. In MORGANA, this returned fraction ($R$) regulates the mass flow between the stellar phase and the hot gas reservoir of the halo, setting the amount of recycled gas. Moreover, this material is supposed to be enriched in metals, formed in the stellar interiors as a consequence of stellar evolution. The pollution of the primordial gas with these metals has a strong impact on the predicted cooling rates, given the dependency of the cooling function

---

2 It is worth stressing that applying a rigid shift to the predictions of a theoretical model calibrated with a given IMF to compare it with data extrapolated using a different IMF is still a valid approach (e.g when several models are compared to the same dataset), even if the correct approach would require to include the shift in the data.
on the hot gas metallicity (Sutherland & Dopita 1993). As the IMF controls the relative abundance of massive versus low-mass stars in star forming events, its shape is thus fundamental for computing both the amount of baryons locked in long-lived stars and the amount of metal spread in the Inter-Stellar and Inter-Galactic Media.

In MORGANA, the process of star formation is treated separately in the disc and bulge, due to the different physical conditions in the two galaxy components (see Fontanot et al. 2013 for a detailed comparison of the MORGANA implementation of star formation with respect to other SAMs). In particular, the cold gas associated with the bulge (due to mergers, disc instabilities and/or direct infall through the cooling flow) is converted into stars on a timescale which is usually shorter with respect to the “quiescent” star formation in discs (Monaco et al. 2007). Although we consider separately IMF variations taking place in the disc and bulge components, it is worth stressing that most of the stellar mass ending up in the bulge component is formed in discs (then transferred via mergers and/or discs instabilities), with only a limited contribution from in-situ star formation.

We run all MORGANA models considered in this study on the same merger tree ensemble, extracted from a 200 Mpc box, with concordance cosmology ($\Omega_0 = 0.24$, $\Omega_{\Lambda} = 0.76$, $h = 0.72$, $\sigma_8 = 0.8$, $N_{\text{map}} = 0.96$). This cosmological realisation has been obtained using the Lagrangian code PINOCCHIO (Monaco et al. 2002) with $N=1000^3$ particles: the particle mass in this run is thus $2.84 \times 10^8 M_\odot$, with the smallest used dark matter halo being $1.42 \times 10^3 M_\odot$ (50 particles) and the smallest resolved progenitor $2.84 \times 10^3 M_\odot$ (10 particles).

In detail, we define the IMF as the number of star counts at a given mass and we choose its normalisation $A$ by requiring that the mass integral equals $1 M_\odot$:

$$A \int_{m_{\text{low}}}^{m_{\text{up}}} m \phi(m) \, dm = 1 \quad (1)$$

where $m_{\text{low}} = 0.08 M_\odot$ and $m_{\text{up}} = 120 M_\odot$ represent our assumed integration limits; $f_{\text{sn}}$ is thus:

$$f_{\text{sn}} = A \int_{M_{\text{SN}}}^{m_{\text{up}}} \phi(m) \, dm \quad (2)$$

where $M_{\text{SN}} = 8 M_\odot$ represents the minimum initial mass for a star bound to end up its evolution as a Type II Supernova. In the useful hypothesis of the Instantaneous Recycle Approximation (IRA), we assume that stellar lifetime are negligible with respect to the integration timestep of the model and we can thus estimate $R$ as:

$$R = A \int_{m_{\text{low}}}^{m_{\text{up}}} \left[ m - r(m) \right] \phi(m) \, dm \quad (3)$$

where $r(m)$ represents the mass fraction locked into stellar remnants (i.e. long-lived stars, white dwarfs, neutron stars and black holes) at the end of stellar evolution as a function of initial stellar mass. Finally, we compute the stellar yield $y_m$, i.e. the mass fraction given to the surrounding gas in the form of newly synthesised metals:

$$y_m = \frac{A}{1 - R} \int_{m_{\text{low}}}^{m_{\text{up}}} m y(m) \phi(m) \, dm \quad (4)$$

where $y(m)$ represents the stellar yield of a star of initial mass $m$, during its whole lifetime. In this paper, we consider both $y_m$ and $y(m)$ as effective yields, referring to all synthesised metals $Z$, i.e. we choose not to follow the chemical evolution of individual elements. Our choice is motivated by the fact that this paper mainly focuses on the global statistical properties of galaxy populations, and the study of detailed chemical patterns are beyond the goal of this work; however we recognise that chemical evolution is possibly a key discriminant between different models of IMF variation (see e.g. Gargiulo et al. 2014) and we devote its study to future developments.

In this paper, we adopt the $r(m)$ and $y(m)$ values estimated by Maeder (1992, in particular their Tables 4, 5, 6). In that work, grids of evolutionary stellar models ranging from 1 to 120 $M_\odot$ were used to derive chemical yields and remnant masses as a function of the initial stellar mass. We applied this results to stellar masses larger than 1 $M_\odot$ (a linear interpolation is used between the grid points): we thus assumed that lower mass stars do not contribute significantly to both $R$ and $y_m$. Results for models with both a solar $Z = 0.02$ and a lower $Z = 0.0001$ metallicity have been provided: we choose between the two options on the basis of the predicted metallicity of the cold star forming gas in model galaxies. The use of just two tabulated values for metallicity has little impact on our results. In fact, all MORGANA runs considered in this work predict cold gas metallicities around or above the the solar value for most model galaxies at $z \gtrsim 2 – 3$. In practice, we use the low-metallicity tables only at higher redshifts and at the low-end of the mass function. We explicitly test that our main conclusions do not change, using the Maeder (1992) tables relative to solar abundances for all galaxies.

### 2.1 IMF variations

As a generalised shape for the IMF we adopted the multi-power-law, 5 parameters, K01 IMF:

![Figure 1. Predicted IMF shape variations for the models considered in this work (see text for the definition and units of labelled quantities). In each panel, IMFs are normalised to the $1 M_\odot$ value. Vertical dotted lines mark the position of the two break masses ($m_1', m_2'$) in the canonical, MW-derived, IMF.](image-url)
This shape provides the required versatility to study IMF changes in well defined mass intervals, as a function of galaxy properties; the canonical, MW-derived, IMF is defined by the primed parameter choice \((\alpha_1', \alpha_2', \alpha_3') = (-1.30, -2.30, -3.30)\) for the exponents and \((m_1', m_2') = (0.5M_\odot, 1M_\odot)\) for the break stellar masses.

From a technical point of view, we integrate Eq. 1 to 4 to compute \(f_{SN}, R\) and \(y_t\) on an appropriate table covering the desired range of IMF variations, and then, for each stellar generation, select the entry closer to the required input. We consider several models for IMF variation that we summarise in the following, while we provide a visual comparison of the sensible range of IMF shapes in Fig. 1. In this figure, we mark the position of the canonical break masses \(m_1'\) and \(m_2'\) and we normalise each IMF to its value at 1 \(M_\odot\), to highlight the relative contribution of high- and low-mass stars.

- **Models SF.** A number of recent paper has pointed out a possible dependence of the IMF shape on the SFR of individual galaxies (see e.g.
[Weidner & Kroupa 2006]: Gunawardhana et al. 2011). In this class we include different parametrisation as suggested by different groups.

  - **SF-WK model** is based on the integrated galactic IMF model of
[Weidner & Kroupa 2006]: we fixed all parameters as in the canonical IMF, but \(\alpha_3\), which we assume to be a piecewise function of \(\ell_{sfr} = \log(SFR)\):

\[
\alpha_3 = \begin{cases} 
  -3.187 & \ell_{sfr} < -9 \\
  0.300 (\ell_{sfr} + 3) - 2.887 & -4 \leq \ell_{sfr} < -3 \\
  0.175 (\ell_{sfr} + 2) - 2.712 & -3 \leq \ell_{sfr} < -2 \\
  0.090 (\ell_{sfr} + 0.25) - 2.51 & -2 \leq \ell_{sfr} < 0.25 \\
  0.36 \ell_{sfr} - 2.6 & \ell_{sfr} \geq 0.25 
\end{cases}
\]

where the functional forms have been estimated from comparison with
[Weidner et al. 2013] (their Fig. 2). The SFR dependence implies that high-SF regions have IMFs that are more Top-Heavy than moderate-to-low SF regions (Fig 1 Panel (a)).

- **SF-ND model** follows the Jean mass argument for giant molecular clouds evolution as detailed in
[Narayanan & Dave 2013]. They use high-resolution hydrodynamical simulations to calibrate a parametrisation of the IMF break (or characteristic) mass as a function of the galaxy SFR:

\[
m_1(SFR) = 0.5 \left( \frac{SFR}{2M_\odot \ yr^{-1}} \right)^{0.3}
\]

As for the other parameter in Eq. 5 we fixed \(\alpha_2 = \alpha_1'\) if \(m_1(SFR) > m_2\), \(\alpha_2 = \alpha_3'\) otherwise. In practice, we reduce the K01 form to a double power shape (Fig. 1 Panel (b)). This parametrisation corresponds to a Top-Heavy IMF for high star formation events, while providing, at the same time, a bottom-heavy IMF in low-star forming galaxies, the discriminating being set at the MW value.

- **SF-PP model** moves from the theoretical work of
[ Papadopoulos 2010]. In this approach, the shape of the IMF is explained by assuming that the thermal and ionization state of dense clouds is determined by Cosmic Rays (which are able to penetrate deeply into molecular clouds) rather than optical-to-UV photons. For a reasonable range of cosmic ray energy densities this translates into a narrow range of possible temperatures and ionization states for dense gas clouds, and thus to an almost invariant IMF. However, at high cosmic rate energy densities, typical of extreme environments like compact starburst, relevant deviations from an universal IMF are expected, due to the rather different thermal state of the dense gas and to the different fragmentation of the cloud. This model thus predicts two IMF regimes (Fig 1 Panel (c)): a MW-like IMF is assumed overall, but in compact starbursts, which are associated with a Top-Heavy IMF. The latter environments are identified by assuming a critical value of the SFR-density \(\Sigma_{SFR} > 0.1M_\odot\ yr^{-1}Kpc^{-2}\), roughly about 100 times the MW value (Papadopoulos, private communication). Above this threshold we then assume a sudden change of the IMF break mass \(m_1\) from a typical MW value to \(~0.5M_\odot\) to \(~2.5M_\odot\) \[Papadopoulos et al. 2011\], and we fix \(\alpha_2 = \alpha_1'\) as in model SF-ND.

- **Models BH** This category includes different realisations, inspired from the suggestion of an increasingly Bottom-Heavy IMF at increasing velocity dispersion or galaxy stellar mass. We mimic these behaviours by considering a shape evolution of the IMF as described below. We define \(P\) as the galaxy property we assume the IMF is dependent on, and \(p_t < p_m < p_h\), three as typical values of \(P\) describing the relative shape evolution.

\[
\begin{align*}
\alpha_1 &= \begin{cases} 
  -1.30 & P \leq p_t \\
  -2.35 + \frac{0.5(P-p_m)}{P-p_m} & p_t < P \leq p_m \\
  P > p_m 
\end{cases} \\
\alpha_2 &= \begin{cases} 
  -2.35 & P \leq p_m \\
  -2.35 + \frac{0.5(P-p_h)}{P-p_h} & p_m < P \leq p_h \\
  P > p_h 
\end{cases}
\end{align*}
\]

Qualitatively (Fig.1 Panel (d)), between \(p_t\) and \(p_m\), only \(\alpha_1\) is evolving, transforming the IMF from the K01 to the S55 shape; for \(P > p_m\) the IMF is considered as a single power-law with an increasingly Bottom-Heavy IMF with a maximal slope of \(-2.80\) (see e.g.
[Cappellari et al. 2012] at \(p_h\). We test different recipes varying the key galaxy property (see Tab. 1 for a summary). We also test

| Model   | \(P\) | \(p_t\) | \(p_m\) | \(p_h\) |
|---------|-------|--------|--------|--------|
| BH-SG1  | \log(\sigma) | 1.9  | 2.2  | 2.5 |
| BH-SG2  | \log(\sigma_{cool}) | 1.9  | 2.2  | 2.5 |
| BH-SG3  | \log(V_H) | 1.9  | 2.2  | 2.5 |
| BH-MSa  | \log(M_B/M_\odot; M_D/M_\odot) | 10.75 | 11.25 | 12  |
| BH-MMb  | \log(M_B/M_\odot; M_D/M_\odot) | 9     | 10.5  | 12  |
| BH-DMa  | \log(M_H/M_\odot) | 12    | 12.5  | 14  |
| BH-DMb  | \log(M_H/M_\odot) | 10    | 12    | 14  |
| BH-z0a  | \log(M_{BH0}/M_\odot) | 12    | 12.5  | 14  |
| BH-z0b  | \log(M_{BH0}/M_\odot) | 10    | 12    | 14  |
different choices for the $p$ values, given the somehow arbitrary definition of these parameters. One $p$ set ("a" models) is chosen by requiring that $p_1$ is set around the MW-like scale, while $p_2$ represents the scale of massive galaxies and $p_{m0}$ is an intermediate scale. We then define an alternative $p$ setting by moving $p_1$ and $p_m$ to lower values, allowing IMF variations starting from lower-mass systems ("b" models).

- **BH-SG models** assume that the IMF shape depends on the velocity dispersion of model galaxies, i.e. they closely resemble the results presented by Conroy et al. (2013), and their Fig. 4). Different definition for this quantity in SAMs have been considered: the velocity dispersion of the bulge $\sigma_B = (BH-SG1)$, the velocity dispersion $\sigma_{cold}$ of the cold clouds in the bulge (BH-SG2), defined as in Sec. 7.1 of Monaco et al. (2007) and the circular velocity $V_H$ of the host halo (as a proxy for the velocity dispersion of the galaxy, BH-SG3). In BH-SG1 and BH-SG2, the changes in IMF shape are allowed only in the bulge component (i.e. the IMF is K01 invariant in the disc component), while in BH-SG3 the variation is assumed in both the bulge and the disc.

- **BH-MS models** assume that the IMF variation depends on the mass of the star forming galaxy, either by component (i.e. the mass of the disc $M_D$ and the bulge $M_B$ separately), or total ($M_* = M_B + M_D$).

- **BH-z0 models** link the IMF variation to $M_{10}$, defined as the $z = 0$ parent halo mass for central galaxies and the mass of the hosting substructure right before DM merging for satellites. The rationale behind this class is the difficulty of a straightforward comparison between Conroy & van Dokkum (2012) or Cappellari et al. (2012) results and the predictions of our models, due to our assumption of IMF variations along the galaxy lifetime, while their approach still assumes that each galaxy has a fixed, but not universal, IMF. By using $M_{10}$, as a proxy of the final mass of the galaxy under consideration, we can artificially force a fixed IMF for each model galaxy. It is worth stressing that this model is clearly idealised, as there is no basis to the hypothesis that star formation events at a given redshift should depend on the $z = 0$ environment.

These models represent a basic set of IMF variations: they include dependencies on integrated galaxy properties, such as SFR, $\sigma_B$, stellar masses of bulge and disc (Models SF, BH-SG1-2, BH-MS). They also consider IMF variations closely related to the Large Scale Structure, such as possible environmental dependencies on the properties of parent DM halo (Models BH-SG3, BH-DM and BH-z0). Despite the lack of a clear theoretical link between a local process like star formation and the properties of the Large Scale Structure as defined by the cosmological model, those former models are designed to explore the possible influence of the cosmological evolution in setting up the properties of the baryonic content of the haloes. Moreover, the models we consider include both Top-Heavy and Bottom-Heavy IMF variations, thus allowing us to study both trends.

3 RESULTS

We then compare the redshift evolution of the galaxy stellar mass function (Fig. 2) and SFR function (Fig. 3) as predicted by our models. We define a reference model with a fixed (universal) K01 IMF and we use the corresponding predictions to estimate the statistical error associated with the stellar mass/SFR functions in the simulated cosmological volume (poissonian errorbars shown as a shaded area in Fig. 2 and Fig. 3). We recall that our reference MORGANA realisation is calibrated on C03 IMF. We test that the differences in model predictions moving from a C03 to a K01 IMF, at fixed SAM parameter space, are negligible, with respect to the variations implied by the IMF models presented in this paper. At each redshift in Fig. 2 and 3 we also show the ratio between the stellar mass/SFR function in a given model and the reference K01 realisation. For a sake of clarity, in both figures we shown only 3 BH models, chosen as representative of the whole class.

SF models provide large deviations ($\sim 0.5$ dex or larger) from the reference K01 IMF run in the stellar mass function, in particular, at $M_* > 10^{10} M_\odot$. Those differences grow larger as redshift increases and high star formation events became more important, while the corresponding stellar populations dominate the mass budget of model galaxies. The effect is particularly dramatic in the SF-PP model, which shows the largest deviation from the reference IMF at all mass scales. SF-PP is also the model showing the largest differences in the SFR function, in particular at the low-SFR end and at $z > 1$, while the SF-WK and SF-ND predictions are consistent with the reference model at most SFR levels. For all SF models, some interesting deviations (of the order of a few tenths of dex as in stellar mass function case) are seen in the most extreme starburst and at higher redshifts (which are the environments where we expect the different gas recycling factors implied by the different IMF variations to have the larger impact), but in general the SF model have little effect on the SFR function shape.

In order to deepen this crucial point we consider in Fig. 4 the metallicity of the hot gas as a function of the parent DM halo mass, in order to estimate the effect of the IMF variation on cooling rates. The shaded area refers to the predictions of the reference K01 IMF run (1-$\sigma$ scatter around the mean value), while other models are marked with the same colour coding as in Fig. 2 SF runs predict enhanced metallicities of the hot gas, due to the larger $\sigma_z$ from their (mostly) Top-Heavy IMF: this in turn translates into larger cooling rates, which balance the increased feedback strength (larger $f_{\text{cool}}$), leading to the small differences in the distribution of the instantaneous SFR.

As far as the BH models are considered, our results show milder effects with respect to the SF runs, with small deviations from the reference K01 IMF run in most cases. In particular, "a" models do not differ from the reference MF by more than $\sim 0.1$ – 0.2 dex over the whole mass range (thus we choose not to show them in the figures), while in "b" models larger differences, of the order of $\sim 0.3$ – 0.5 dex, are seen at the high mass end. In terms of SFR function, BH-MS and BH-DM models predictions are completely consistent with the reference run. The overall behaviour of BH-MS and BH-DM model is due to the fact that in all these models relevant IMF variations take place only at high mass scales (both stellar or DM): galaxies reaching those stages of their evolution are usually experiencing decreasing (if not quenched, see e.g. Kimm et al. 2009) star formation histories, thus only a small fraction of stars are formed with a Bottom-Heavy IMF. This is particularly severe in "a" models, where the range of IMF variations is reduced by construction. The model showing the largest deviations from the reference run is BH-SG3, while both BH-SG1 and BH-SG2 predict negligible deviations at most masses, given the fact that only a small fraction of the stars are forming in the bulge component, where IMF variations are allowed.

These results imply that variations of the IMF should be ef-
Figure 2. Redshift evolution of the predicted stellar mass function for different IMF models. In each panel, the grey shading represents the statistical error associated with the determination of the stellar mass function in our cosmological box, computed on the reference K01 IMF run. Line types and colours correspond to the prediction of different models, as labelled.

effective in the standard “quiescent” star formation mode (associated with discs), in order to support a mass-dependent IMF scenario, as expected given the limited contribution of star formation in bulges to the total stellar mass of model galaxies. Moreover, IMF variations should also be effective in relatively low-mass galaxies: this further point is tested in BH-z0 models, which is explicitly designed to link the level of IMF variation to the final parent halo/substructure mass, taken as a proxy to the final galaxy mass. This run, despite unrealistic, is built up to maximise the level of bottom-heaviness associated with a BH approach, and enhance any evolutionary effect connected to a mass-dependent Bottom-Heavy IMF, since, at all epochs, each model galaxy uses an IMF set by its $z = 0$ properties (according to the BH-MS model). It is thus interesting to see that BH-z0 models show differences of the order of $\sim 0.2 - 0.3$ dex at most mass scales and redshift, still compatible with the intrinsic uncertainty in mass reconstruction from observed photometry. BH-z0 models show larger deviations with respect to the predictions of other BH models in terms of SFR function, especially at $z > 1$. However, also for these models deviations are of a few tenths of dex at most.
4 DISCUSSION & CONCLUSIONS

In this paper, we present an exploratory study on the impact of IMF variations in Semi-analytical models of galaxy formation and evolution. By assuming IMF variations to depend either on the physical properties of star forming galaxies (SFR, velocity dispersion, stellar mass) or on the properties of the Large Scale Structure (parent halo mass), we run different SAM realisations. Several definitions for the relevant quantities have been considered, and different parametrisations, broadly inspired by the observational results. Our results thus test the SAM robustness against the hypothesis of moderate IMF variations.

In general, typical deviations in the galaxy mass function predictions, with respect to the reference K01 IMF run, are of the order of few tenths of dex over a wide redshift range, thus of the same order as the uncertainty due to different choices of universal (fixed) IMF (i.e. between K01 and S55), while smaller effects are seen in the star formation rate functions. In BH models, these differences are comparable to the scatter between the predictions of different SAMs (once they are calibrated against the same reference sample, see e.g. Fontanot et al. 2009, 2012), and due to the
different parametrizations of the relevant physical processes. Only models assuming an increasing Top-Heavy IMF in high SFR environments predict deviations from the reference model larger than both the statistical and modelling uncertainties. The model showing the largest deviations, both in terms of stellar masses and SFRs, from the reference K01 IMF run is SF-PP. In order to get a better understanding of the processes leading to these differences we run a couple of additional runs, varying the key hypothesis of this model, namely the value of the $\Sigma_{\text{SFR}}$ threshold and/or allowing the IMF variation only in a single galaxy component (i.e. the disc or the bulge). Our tests highlight the fact that its peculiar behaviour arises as a consequence of our treatment of star formation in discs. In both SF-WK and SF-ND, low mass galaxies are affected only weakly by IMF variations, as a consequence of their mostly MW-like SFRs; MORGANA, however, predicts SFR surface densities in discs higher than the SF-PP threshold, at most galactic scales. In fact, if the IMF change is allowed only in the bulge component, SF-PP predicts smaller deviations, and a low-mass end completely consistent to the reference K01 run, given the limited amount of stars directly forming in the bulge component. While these results are indeed interesting and possibly suggesting a deep revision of our understanding of galaxy evolution, few caveats are worth discussing. The main limitation of our modelling of the Papadopoulos (2010) approach lies in the treatment of galaxy components, i.e., galactic discs are considered as single entities, with no attempt to model density and SFR gradients. Thus the SFR surface densities entering the definition of SF-PP model are mean densities, integrated over the whole disc. Alternative descriptions, as those based on the statistics of individual star forming cloud, and/or on local properties of star forming regions will be of great interest (see e.g. Fu et al. 2010, for an attempt to overcome this limitation).

Moreover, the relatively small effect seen in most of our models implies that the complex interplay between the physical properties modelled in SAMs has the net effect of smoothing out some of the differences due to the description of instantaneous star formation in a variable IMF framework. In fact, by changing the IMF shape, we are not only changing the fraction of baryons locked in long lived stars and remnants per each stellar generation, but also the amount of recycled baryons and the energetics of the multiphase gas. We explicitly show the level of self-regulation at place in our SAM, while discussing the evolution of the SFR function and we interpret its stability, in most of our models, as the result of the interplay between the stronger (weaker) stellar feedback and the enhanced (depressed) cooling rates, the latter due to the higher (lower) metal enrichment of the hot gas phase. We thus conclude that the main effect of the IMF change in MORGANA is set by the amount of baryons locked in low mass stars (responsible for the stellar mass of model galaxies). As a general caveat, we remind the reader that the distribution of metals into the different gas phases (both hot and cold) is extremely sensible to the (poorly constrained) details of the modelling of gas reheating and ejection (see Fontanot et al. 2013, for a review of the different implementations of these processes in SAMs). Additional tests with other SAMs are thus required to assess if the self-regulation of the SFR is a general property in the SAM framework.

In order to better understand the effect of IMF variations on the evolutionary tracks of individual galaxies, we compare the $z=0$ stellar masses predicted by each model to those predicted by the reference K01 IMF run, on an object-by-object basis. In Fig. 5 we show the mean mass ratios $\Delta M = \log(<M/M_{\text{K01}}>)$ for the same models considered in Fig. 2 as a function of model galaxy mass. We also report with a shaded area the 1-σ scatter in the distribution for the BH-MSb model, which is representative. Overall, mean $\Delta M = \pm 0.2$ dex: Top-(Bottom-)Heavy model have in gen-

---

**Figure 4.** Redshift evolution of the hot gas metallicities as a function of DM halo masses. In each panel, model predictions are shown with the same lines types and colours as in Fig. 2, while the shaded area represents the 1-σ scatter in the mean relation for the reference, K01 IMF, model.

**Figure 5.** Mass deviations from reference K01 IMF model. Model predictions are shown with the same lines types and colours as in Fig. 2 as labelled. Shaded area represents the 1-σ scatter in BH-SG3 model (see text for more details).
eral negative (positive) mean deviations, as expected, but the substantial scatter implies that individual model galaxies might have deviations in opposite directions. Moreover, mean trends are not monotonic: this again shows that the impact of IMF variations is different at different mass scales. A qualitative comparison with Conroy et al. (2013) is possible: while the mean $\Delta M$ is clearly lower, model galaxies with individual $\Delta M$ as high as their results are typically less than 2-$\sigma$ outliers for most of the BH models. It is worth stressing that the sources entering the analysis have a peculiar selection function and it is not clear if they are representative of the whole galaxy population.

By comparing models assuming an increasing Bottom- or Top-Heavy IMF at increasing stellar mass or SFR, we notice that the two classes provide predictions, which are clearly different and mutually exclusive at the high-mass end of the mass function, with the resulting space densities for massive galaxies being systematically higher or lower than the reference run predictions (although the separation is not that clear by considering the mass differences in single objects, as shown in Fig. 5). We can thus conclude that this mass range is the most promising for breaking the degeneracies between the different proposed IMF variations (such as in Cappellari et al. 2012), but it is important to keep in mind that these results also depend on our choice for the IMF variation rules, which prefer by construction larger variations in more massive systems. At smaller stellar masses the situation looks more confused, as a larger fraction of stars in these objects is born from a reference K01 IMF, and its difficult to disentangle the predictions of different models, with only the SF-PP model predicting a decrease of the space density of low-mass galaxies at increasing redshifts. It is also worth noting that in most cases, the predicted deviations of our IMF variable models from the reference run depend on both stellar mass and redshift, i.e., variable IMFs have a differential effect on the shape of the MFs. This may have relevant consequences in interpreting galaxy evolution from observed photometry, i.e., by means of SED fitted physical quantities and in the context of the so-called downsizing trends (Fontanot et al. 2009).

As a final remark, we want to recall the reader that despite the reference MORGANA realisation is calibrated on a C03 IMF, in the spirit of this work, which is not aimed at comparing data and models, but at exploring the effect of IMF variations on SAM predictions, we do not try any recalibration of the reference model: with our choice we thus focus on the effect of IMF variations only. A complementary approach would rely on the recalibration of each realisation on similar reference dataset: however this approach is not currently feasible. In fact, on one side, we cannot use any dataset including galaxy physical properties reconstructed by means of SED fitting algorithms: as we already discussed in the introduction, systematic IMF variations as a function of stellar (parent halo) mass, SFR or redshift represent a critical problem for any algorithm trying to recover galaxy physical parameters from observed photometry, as most of these algorithms would be sensible (at most) only to the dominant stellar component. This is coherent with the substantial intrinsic scatter found by Shetty & Cappellari (2014) when comparing dynamically derived mass-to-light ratios to the predictions of stellar population models at fixed (S55) IMF. A possible way out of this problem for this class of algorithms would then require the self-consistent fit of the most reliable star formation history for each galaxy (and the corresponding IMF evolution as well), but this choice increases the number of fitted parameters and the level of degeneracies, possibly reducing the significance of the estimated physical quantities. Therefore, under the hypothesis of a variable IMF, it is not possible to include the statistical distribution of galaxy physical properties (such as the stellar mass function) in the calibration sample for theoretical models. A more solid calibration of such models would then require the direct comparison with the photometric properties of galaxy populations like the multi-wavelength luminosity functions or number counts. This step implies a straightforward evolution of spectrophotometric codes, allowing them to mix simple stellar populations characterised by (almost) arbitrary shaped IMFs in the SAM framework. We left this project to future developments.

ACKNOWLEDGEMENTS

I warmly thank Pierluigi Monaco and Gabriella De Lucia for discussion and comments, and Padelis Papadopoulos for useful clarifications about my model. I also want to thank the anonymous referee for a constructive report that helped improving the quality of the paper. I acknowledge financial contribution from the grants PRIN MIUR 2009 “The Intergalactic Medium as a probe of the growth of cosmic structures” and PRIN INAF 2010 “From the dawn of galaxy formation”

REFERENCES

Baugh C. M., 2006, Reports of Progress in Physics, 69, 3101
Baugh C. M., Lacey C. G., Frenk C. S., Granato G. L., Silva L., Bressan A., Benson A. J., Cole S., 2005, MNRAS, 356, 1191
Cappellari M., McDermid R. M., Alatalo K., Blitz L., Bois M., Bournaud F., Bureau M., Crocker A. F., et al. 2012, Nature, 484, 485
Chabrier G., 2003, ApJ, 586, L133
Conroy C., Dutton A. A., Graves G. J., Mendel J. T., van Dokkum P. G., 2013, ApJ, 776, L26
Conroy C., van Dokkum P. G., 2012, ApJ, 760, 71
De Lucia G., Blaizot J., 2007, MNRAS, 375, 2
Fontanot F., Cristiani S., Santini P., Fontana A., Grazian A., Somerville R. S., 2012, MNRAS, 421, 241
Fontanot F., De Lucia G., Benson A. J., Monaco P., Boylan-Kolchin M., 2013, ArXiv e-prints (arXiv:1301.4220)
Fontanot F., De Lucia G., Monaco P., Somerville R. S., Santini P., 2009, MNRAS, 397, 1776
Fontanot F., Springel V., Angulo R. E., Henriques B., 2012, MNRAS, 426, 2335
Fu J., Guo Q., Kauffmann G., Krumholz M. R., 2010, MNRAS, 409, 515
Gargiulo I. D., Coral S. A., Padilla N. D., Muñoz Arancibia A. M., Ruiz A. N., Orsi A. A., Tecce T. E., Weidner C., Bruzual G., 2014, ArXiv e-prints (arXiv:1402.3296)
Gunawardhana M. L. P., Hopkins A. M., Sharp R. G., Brough S., Taylor E., Blundel-Hawthorn J., Maraston C., Tuffs R. J., et al. 2011, MNRAS, 415, 1647
Hennebelle P., Chabrier G., 2008, ApJ, 684, 395
Hopkins P. F., 2012, MNRAS, 423, 2037
Hoversten E. A., Glazebrook K., 2008, ApJ, 675, 163
Kimm T., Somerville R. S., Yi S. K., van den Bosch F. C., Salim S., Fontanot F., Monaco P., Mo H., Pasquali A., Rich R. M., Yang X., 2009, MNRAS, 394, 1131
Klessen R. S., Ballesteros-Paredes J., Vázquez-Semadeni E., Durán-Rojas C., 2005, ApJ, 620, 786
Klessen R. S., Spaans M., Jappsen A.-K., 2007, MNRAS, 374, L29
Kroupa P., 2001, MNRAS, 322, 231
Kroupa P., Weidner C., 2003, ApJ, 598, 1076
Krumholz M. R., 2014, ArXiv e-prints (arXiv:1402.0867)
Lo Faro B., Monaco P., Vanzella E., Fontanot F., Silva L., Cristiani S., 2009, MNRAS, 399, 827
Maeder A., 1992, A&A, 264, 105
Monaco P., Fontanot F., Taffoni G., 2007, MNRAS, 375, 1189
Monaco P., Theuns T., Taffoni G., Governato F., Quinn T., Stadel J., 2002, ApJ, 564, 8
Narayanan D., Davé R., 2013, MNRAS, 436, 2892
Papadopoulos P. P., 2010, ApJ, 720, 226
Papadopoulos P. P., Thi W.-F., Miniati F., Viti S., 2011, MNRAS, 414, 1705
Salpeter E. E., 1955, ApJ, 121, 161
Shetty S., Cappellari M., 2014, ArXiv e-prints (arXiv:1402.7354)
Smith R. J., 2014, ArXiv e-prints (arXiv:1403.6114)
Sutherland R. S., Dopita M. A., 1993, ApJS, 88, 253
van Dokkum P. G., Conroy C., 2011, ApJ, 735, L13
van Dokkum P. G., Conroy C., 2012, ApJ, 760, 70
Weidner C., Kroupa P., 2006, MNRAS, 365, 1333
Weidner C., Kroupa P., Pflamm-Altenburg J., Vazdekis A., 2013, MNRAS, 436, 3309