On the scalar nonet lowest in mass

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The hypothesis that there exists a nonet of scalars mainly composed of a valence quark-antiquark pair and mixed according to near singlet-octet separation:

\[ f_0(980) \text{ singlet}, \quad a_0^+, a_1^+, a_0^-, a_1^- (980), \quad K_0^+, K_0^- (1430), \quad f_0(1500) \text{ octet}, \]

is put to further tests from the three body decays \( D^\pm D_s^\pm \to PS^\pm \pi^+ \pi^- \) with \( PS^\pm = \pi^\pm, K^\pm \). The analysis of decay phases supports the singlet nature of \( f_0(980) \).

1. Spectroscopy of \( q\bar{q} \) p-waves

Following the hypothesis given in the abstract we select the following 4 p-wave nonets: \( JP_{C_n} = 0^{++}, 1^{++}, 2^{++}, 1^{+-} \). They are displayed together with the pseudoscalar nonet in figure 1.

![Figure 1. \( JP_{C_n} = 0^{++}, 0^{+-}, 1^{++}, 2^{++}, 1^{+-} \) nonets.](image)

As a consequence of the adopted selection criteria we exclude the following candidate states from the PDG listings \cite{PDG}:

\[ f_0(400 - 1200), \quad f_0(1370), \quad f_1(1410) \text{ and resonances with a mass exceeding } f_0^*(1525) \text{ in particular } h_1(1595) \text{ and } f_0(1710), \]

see also \cite{ref2}.

We pose the following questions:\cite{ref2}:

- Is the flavor mixing pattern of the \( 0^{-+} \) and \( 0^{++} \) nonets similar, i.e. near singlet - octet? The two nonets would be parity doublets due to chiral symmetry, which is spontaneously and explicitly broken.

- Does the scalar gluonic meson distort through large mixing effects the scalar \( q\bar{q} \) nonet beyond (spectroscopic) recognition?

In our analysis in \cite{ref2} we started from the hypothesis, that the answer to this question is no. Inspecting the spectra of the p-wave nonets in Fig. 1 we observe no strong distortion in the scalar sector indeed.

- Can the last question be resolved through direct observation of the scalar gluonic meson as a conventionally narrow resonance and how reliable are the mass estimates from purely gluonic lattice-QCD \cite{ref3}?

\[ m_{gb} (0^{++}) = 1600 \text{ MeV} \pm 10\% \]

- Can QCD sum rules including local gluonic operators \cite{ref4} shed light on the above mass estimate? The latter two questions are addressed in the accompanying paper \cite{ref5}.

The Gell-Mann - Okubo square mass formula

According to our analysis \( f_0 \to f_\pi (980) \) represents the SU3 singlet, whereas

\[ a_0 (984.7), \quad K_0 (1412), \quad f_0 \to f_\pi (1507) \]

form the associated octet.
The Gell-Mann - Okubo (first order) mass square relation then yields (in GeV$^2$ units)

$$m^2(f_>) = m^2(a_0) + \frac{1}{3}\left(m^2(K_0) - m^2(a_0)\right)\quad (1)$$

$$2.271 = 0.970 + 1.365 = 2.335$$

the deviation amounts to 0.064/2.271 = 2.8%.

There is no sign - yet - of any major distortion.

The degeneracy in mass of $f_<$ and $a_0$, while not offending any basic principles, indicates further dynamic simplicity to be explained.

2. Further evidence for (near) octet-singlet flavor phase structure

The aim is to study and eventually confirm the nonstrange versus strange $q\bar{q}$ flavor structure of the two singlet-octet assigned isoscalar scalars :

- $f_<(980) = \cos \vartheta |0\rangle - \sin \vartheta |8\rangle$
- $f_>(1500) = \sin \vartheta |0\rangle + \cos \vartheta |8\rangle$

The following phase convention shall be chosen in the flavor basis

$$f = \sum_q c_q |q\bar{q}\rangle, \quad q = u, d, s$$

$$c_u = c_d = \frac{1}{\sqrt{2}} c_{ns}; c_{ns} = \sin \varphi, c_s = \cos \varphi$$

$$|0\rangle = \frac{1}{\sqrt{2}}(1,1,1); \quad |8\rangle = \frac{1}{\sqrt{6}}(1,1,-2)$$

$$|ns\rangle = \frac{1}{\sqrt{2}}(1,1,0); \quad |s\rangle = (0,0,1)$$

and in the ns-s basis we have

$$|0\rangle = \sin \varphi* |ns\rangle + \cos \varphi* |s\rangle$$

$$|8\rangle = \cos \varphi* |ns\rangle - \sin \varphi* |s\rangle$$

$$\varphi* = \arccot \frac{1}{\sqrt{2}} = 54.74^\circ$$

so in the flavor basis we have

- $f_<(980) = \sin \varphi |ns\rangle + \cos \varphi |s\rangle$
- $f_>(1500) = \cos \varphi |ns\rangle - \sin \varphi |s\rangle$

$$\vartheta = \varphi* - \varphi$$

As conjectured range of the singlet-octet angle $\vartheta$ we consider

$$0 \leq \vartheta \leq \vartheta_m, \quad \vartheta_m = \arcsin \frac{1}{2} = 19.47^\circ$$

\[\text{Eq. 3} \text{ is numerically a refinement with respect to ref. 4.}\]

The corresponding range for $\varphi$ becomes

$$35.26^\circ \leq \varphi \leq \varphi* = 54.74^\circ$$

Previously we have analysed the decays involving $f_0(980)$ : $J/\Psi \rightarrow \phi f_0, \omega f_0$, the radiative decays $f_0, a_0 \rightarrow \gamma \gamma$ and $f_0 \rightarrow K\pi, \pi\pi$ and concluded on a large flavor mixing similar to $\eta-\eta'$ in the pseudoscalar nonet. Now we extend our analysis to D decays.

Our analysis of the mixing pattern shall focus on the two ratios of strange to nonstrange components

$$R_\varphi = R(f_0(980)) = \cot \varphi$$

$$R_\varphi = R(f_0(1500)) = -\tan \varphi$$

If the premise of mainly singlet $f_0(980)$ is correct we infer the ranges $R_\varphi > 0$ and $R_\varphi = -1/R_\varphi < 0$. The relation $R_\varphi < R_\varphi = -1$ follows from orthogonality.

The restricted ranges are

$$-\sqrt{2} \leq R_\varphi \leq \sqrt{2}$$

We determine $\varphi$ considering the three body decays of the charmed mesons $D$ and $D_s$:

\[A\] $D^+_s \rightarrow f_0(980) \pi^+$

\[B\] $D^+ \rightarrow f_0(980) \pi^+$

\[C\] $D^+ \rightarrow K_0(1430) \pi^+$

We consider the color favored amplitudes ($\propto a$) which contribute to all processes in eq. 5, and color suppressed amplitudes ($\propto ea$) obtained by $D_1 \leftrightarrow D_2$ (see Fig. 3) and obtain

$$A = \cos \varphi V_{ud}V_{cs}^* a$$

$$B = \sin \varphi V_{ud}V_{cs}^*(1 + e)a + \cos \varphi V_{us}V_{cs}^* e a$$

$$C = V_{ud}V_{cs}^* (1 + e) a$$

We consider the two ratios of partial decay widths $A/B$ and $A/C$ :

$$R_{AB} = \frac{\Gamma(D^+_s \rightarrow f_0(980)\pi^+)}{\Gamma(D^+_s \rightarrow f_0(1500)\pi^+)}$$

$$R_{AC} = \frac{\Gamma(D^+_s \rightarrow f_0(980)\pi^+)}{\Gamma(D^+_s \rightarrow K_0(1430)\pi^+)}$$
In the approximation $V_{ud} = V_{cs} = \cos \vartheta_c$, $V_{us} = -V_{cd} = \sin \vartheta_c$ with Cabibbo angle $\vartheta_c$, we find

$$R_{A/B} = 2 \frac{\Phi_1}{\Phi_2} \cot^2 \vartheta_c \cot^2 \varphi \frac{1}{1-(\sqrt{2} \cot \varphi - 1)\epsilon}$$
$$R_{A/C} = \frac{\Phi_3}{\Phi_2} \cos^2 \varphi \frac{1}{1+\epsilon}$$

where $\Phi_{1,2,3} = (p_{\pi^+})_{1,2,3}$ denote the phase space in s-wave decays, proportional to the $\pi^+$ momentum in the decay resonance rest frame. Using the branching fractions established by the E791 Collaboration and the PDG results we find numerically

$$\cot^2 \varphi / |1 - (\sqrt{2} \cot \varphi - 1)\epsilon|^2 = 1.26 (1.0 \pm 0.4)$$
$$\cos^2 \varphi / |1 + \epsilon|^2 = 0.52 (1.0 \pm 0.3)$$

Then we obtain as solutions two bands (a and b) due to the quadratic nature of the relations in eq. 12

$$\varphi_a = 1.11^{+0.33}_{-0.20}, \quad \varphi_a = 42.14^{+5.8}_{-7.3}$$
$$\epsilon_a = (2.85 \pm 5.35) \times 10^{-2}$$

$$\cot \varphi_b = -0.34^{+0.46}_{-0.37}, \quad \varphi_b = 161.2^{+25.7}_{-16.5}$$
$$\epsilon_b = 0.31^{+0.004}_{-0.13}$$

The angle $\varphi$ is defined modulo $180^\circ$. The two solutions in eq. 14 can be distinguished through the sign of the quantity $R_{<}$ (or $R_{>}$) defined in eq. 7.

The phase ($+$ $-$) structure of $f^+, f^>$ is determined from interference with other resonances in $D^+$ and $D^+_s$ decays into $3\pi$ and $\pi K K$ [7,8]. The amplitudes $A$, $B$, $C$ in eq. 10 exhibit the $+$ $-$ phase structure shown in table 1.

3. Conclusions and outlook

It becomes clear from the results in table 1 and the assumed form of the amplitudes in eq. 10 that only the solution in band a) in eq. 14 is compatible with the data. This implies $\varphi = 42.14^{+5.8}_{-7.3}$ confirming the near singlet quark flavor mixing of $f_0(980)$, the ideal singlet corresponds to $\varphi = 54.7^\circ$.

The present analysis leaves the mixing with the scalar gluonic meson(s) completely open. Here we refer to our present results on glueballs in ref. 10. Future work will hopefully establish the full structure of the scalar nonet lowest in mass including the scalar glueball $gb(0^{++})$.

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Table 1
Decay phases (in degrees) for resonances in $D$ and $D_s$ decays as measured by experiments E687 and E791. In each line an overall phase has been fixed arbitrarily. The states marked by ($\ast$) are not directly evident in the plots. Comparison with theoretical expectations for the mixing angle $\varphi$ and predictions for $f_0(980)$ near flavour singlet and $f_0(1500)$ as octet partner ($0^\circ < \varphi < 90^\circ$) with arbitrary angle $\alpha$, standard choice is $\alpha = 0$.

| $D \to 3\pi$ | $dd \to$ | $f_0(980)$ | $f_2(1270)^*$ |
|---------------|----------|------------|--------------|
| E687          | $27 \pm 14 \pm 11$ | $197 \pm 28 \pm 24$ | $207 \pm 17 \pm 4$ |
| E791          | 0(fixed)  | $151.8 \pm 16.0$ | $102.6 \pm 16.0$ |
| Theory        | $-\varphi$ | $\varphi \sin(\varphi)$ | $\varphi$ |
|               | $\alpha + 180^\circ$ | $\alpha$ | $\alpha$ |

| $D_s \to 3\pi$ | $ss \to$ | $f_0(980)$ | $f_2(1270)^*$ | $f_0(1500)$ |
|----------------|----------|------------|--------------|
| E687           | 0(fixed)  | $83 \pm 16$ | $210 \pm 10$ |
| E791           | 0(fixed)  | $133 \pm 13 \pm 28$ | $198 \pm 19 \pm 27$ |
| Theory         | $s\bar{s} \cos(\varphi)$ | $\varepsilon s\bar{s}$ | $-s\bar{s} \sin(\varphi)$ |
|               | $\alpha$ | $\alpha$ | $\alpha + 180^\circ$ |

| $D_s \to \pi KK$ | $ss \to$ | $f_0(980)^*$ | $\phi(1020)$ |
|-----------------|----------|-------------|-------------|
| E687            | $159 \pm 22 \pm 16$ | $178 \pm 20 \pm 24$ |
| Theory          | $s\bar{s} \cos(\varphi)$ | $s\bar{s}$ |
|                 | $\alpha$ | $\alpha$ |

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