Gauge Mediation Models with Neutralino Dark Matter

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Abstract

We study gauge mediation models of supersymmetry breaking with neutralino LSP. These models are naturally realized by embedding the usual four-dimensional gauge mediation models into a higher-dimensional spacetime such as $M^4 \times S^1/Z_2$. We calculate the relic abundance of the neutralino LSP in these models and show that there exist wide parameter regions where the neutralino LSP constitutes the dominant component of the cold dark matter. These regions evade constraints from collider experiments such as Higgs mass bounds and $b \to s\gamma$, and also provide the value for the muon anomalous magnetic moment which is consistent with the SUSY explanation of the deviation from the standard model prediction.
1 Introduction

Low-energy supersymmetry (SUSY) with dynamical SUSY breaking (DSB) is a very attractive framework for explaining the large hierarchy between the electroweak and the Planck scales. One of the interesting features of this framework is that the lightest supersymmetric particle (LSP) is completely stable in models with $R$ parity, which is well motivated by the stability of the proton. In conventional hidden-sector SUSY-breaking scenario, the LSP is believed to be the lightest neutralino, providing a good candidate for the cold dark matter in the universe\footnote{The special case that $m_0 = m_{1/2} = A/3$ can be realized in five-dimensional theories with all the standard-model fields living in the bulk\cite{five}.}. It is, indeed, suggested that the presence of a stable particle of weak-scale mass with electroweak interactions is a crucial ingredient for a natural solution to the cosmic coincidence problems\cite{coincidence}. In fact, in view of its attractiveness, the possibility of neutralino dark matter has been extensively studied within the context of the hidden-sector scenario such as constrained version of the minimal SUSY standard model (MSSM)\cite{MSSM, MSSM1, MSSM2, MSSM3, MSSM4}.

However, it is known that the hidden-sector scenario has no firm theoretical foundation. In this scenario, the Kähler potential is assumed to take a canonical form, but it can hardly be justified in supergravity. In supergravity, we expect the presence of the following non-renormalizable interactions in the Kähler potential:

$$K = \frac{\eta_{ij}}{M_G^2} Z_i^\dagger Z Q_i^j Q_j,$$

which cannot be forbidden by any symmetry of the theories. Here, $Z$ is the superfield responsible for the SUSY breaking and $Q_i$ represents generic standard-model quark and lepton superfields with $i, j = 1, \cdots, 3$ being flavor indices; $M_G \simeq 2.4 \times 10^{18}$ GeV is the gravitational scale and $\eta_{ij}$ are constants of order one. These operators invalidate the assumption of the hidden-sector scenario, say the boundary condition of the constrained MSSM. Therefore, in the absence of any specific realization of the hidden-sector scenario, the mass spectrum used in most analyses of neutralino dark matter must be regarded as an ad hoc working hypothesis\footnote{Moreover, since there is no reason for the above operators to be flavor universal, they generically induce too much flavor-changing neutral currents (FCNC’s) in the SUSY standard-model sector. In fact, suppressing FCNC is one of the most important issues in the SUSY standard model.}

Moreover, since there is no reason for the above operators to be flavor universal, they generically induce too much flavor-changing neutral currents (FCNC’s) in the SUSY standard-model sector. In fact, suppressing FCNC is one of the most important issues in the SUSY standard model.\footnote{Gauge-mediated SUSY breaking (GMSB)\cite{GMSB, GMSB1, GMSB2} provides an elegant solution to this SUSY FCNC problem. In GMSB models, the squark and slepton masses are generated...}

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by the standard-model gauge interactions and are automatically flavor universal. On the other hand, in most GMSB models, the SUSY-breaking scale is supposed to be much lower than that in the conventional hidden-sector scenario. As a result, the gravitino mass $m_{3/2}$ becomes much smaller than the electroweak scale \[1\], and the neutralino is no longer the LSP. The reason is again due to the non-renormalizable interactions Eq. \[1\]. Since these operators would violate flavor universality, they generate flavor non-universal pieces of the soft SUSY-breaking masses for the squarks and sleptons of the order of the gravitino mass $m_{3/2} \simeq F_Z/M_G$. To preserve the success of GMSB models, these flavor non-universal pieces should be much smaller than the flavor universal pieces generated by the gauge mediation, which forces the gravitino to be much lighter than the electroweak scale.

The question then is whether there are explicit models which solve FCNC problems and also accommodate neutralino dark matter. Notice that the above argument is entirely based on the presence of the non-renormalizable operators in Eq. \[1\], and it comes from the expectation in four-dimensional effective field theories that all operators consistent with the symmetries are present with coefficients of order one suppressed by some cut-off (gravitational) scale. However, this naive expectation does not necessarily hold if fundamental theories are higher dimensional. Specifically, if $Z$ and $Q_i$ fields are localized on different $(3 + 1)$-dimensional branes in higher-dimensional spacetime, there is no direct contact interaction between $Z$ and $Q_i$ fields and the flavor non-universal operators in Eq. \[1\] are exponentially suppressed by the distance of two branes \[12, 13\]. Therefore, it is possible that the situations for the hidden-sector and GMSB models are drastically changed by embedding these models into higher dimensional theories at high energy scales. However, it turns out that the hidden sector still does not work, since necessary flavor universal pieces for the squark and slepton masses are also suppressed in this case \[13, 14\]. In the GMSB case, in contrast, the squark and slepton masses are generated by the standard-model gauge interactions. Thus, if we somehow manage the separation of $Z$ and $Q_i$, it merely says that the gravitino mass no longer has to be smaller than the electroweak scale.

In fact, in a class of GMSB models \[10, 15\] the sector responsible for DSB can be fully separated from the sector that feels standard-model gauge interactions; two sectors communicate with each other only through the $U(1)_m$ gauge interaction called messenger gauge interaction. Therefore, it is interesting to interpret these GMSB models as low-energy

\[2\] One possibility is to put the standard-model gauge fields in the bulk, and generate the squark and slepton masses through renormalization-group effects below the compactification scale \[14\]. We here consider the case where the standard-model gauge and Higgs fields are also localized on the same brane as $Q_i$'s.
manifestations of the following brane-world scenario \cite{16}. All fields in the DSB sector live on the DSB brane while the messenger and the standard-model fields are localized on our observable brane. The $U(1)_m$ gauge multiplet is put in the bulk, through which two sectors on different branes can communicate with each other. Then, the SUSY breaking on the DSB brane is transmitted to the observable brane only by the $U(1)_m$ gauge and gravitational interactions across the bulk.\footnote{The non-renormalizable operators in Eq. (1) are absent (exponentially suppressed) and flavor non-universal soft masses are not generated.} The non-renormalizable operators in Eq. (1) are absent (exponentially suppressed) and flavor non-universal soft masses are not generated.

Let us see how the above prescription works explicitly. It has been claimed \cite{18, 13} that the brane separation produces the following no-scale type Kähler potential in low-energy four-dimensional effective field theories:\footnote{Another mechanism of transmitting SUSY breaking between two branes is discussed in Ref. \cite{17}.}

$$K = -3 \log \left( 1 - \frac{1}{3} f_O(\Phi_{\text{obs}}, \Phi_{\text{obs}}^\dagger) - \frac{1}{3} f_D(\Phi_{\text{DSB}}, \Phi_{\text{DSB}}^\dagger) \right).$$ \hfill (2)

Here, $\Phi_{\text{obs}}$ and $\Phi_{\text{DSB}}$ denote superfields in the observable and DSB sectors, respectively. With the above Kähler potential, all soft SUSY-breaking masses and $A$ terms in the observable sector vanish at the tree level in the limit of zero cosmological constant \cite{20, 13}. Then, the soft SUSY-breaking masses are generated only by the gauge mediation caused by the loop effects of the bulk $U(1)_m$ gauge interaction \cite{16}. On the contrary, the $\mu$ term naturally arises at the tree level \cite{21} from the Kähler potential if $f_O$ contains $f_O \supset H_u H_d$, where $H_u$ and $H_d$ are chiral superfields of Higgs doublets. This mechanism \cite{21} produces the SUSY-invariant mass $\mu$ of the order of the gravitino mass, i.e. $\mu \simeq m_{3/2}$. Therefore, if we choose the gravitino mass to be $m_{3/2} \simeq 100 \text{ GeV} - 1 \text{ TeV}$, we can correctly reproduce the electroweak symmetry breaking.\footnote{With these gravitino masses the anomaly mediation \cite{13, 22} generates too small SUSY-breaking masses in the observable sector.} Putting the $U(1)_m$ in the bulk, indeed, the transmission of the SUSY breaking becomes necessarily weak compared with purely four-dimensional GMSB models. Thus, we can naturally obtain the gravitino mass $m_{3/2} \simeq 100 \text{ GeV} - 1 \text{ TeV}$ with moderate size of extra dimensions. In the five-dimensional model of Ref. \cite{16}, for instance, the compactification length $L$ is given by $L \simeq (2 \times 10^{15} \text{ GeV})^{-1}$, which is close to the value obtained in the scenario of Ref. \cite{12}. This provides a simple solution to the $\mu$ problem in GMSB models with DSB and observable sectors geometrically separated in higher-dimensional spacetime \cite{17, 16}.

The consequence of the above brane-world GMSB models is that the gravitino has a
mass of order 100 GeV − 1 TeV, although the mass spectrum of the gauginos, squarks and sleptons is the same with that of the usual GMSB models \[17, 16\]. Thus, the lightest neutralino is most likely the LSP and can behave as a cold dark matter in the universe. In this paper, we calculate the relic abundance of the neutralino LSP in these brane-world GMSB models and show that the neutralino can actually be the dark matter in a wide range of the parameter space. Since GMSB models are highly predictive, the relic abundance $\Omega_{\tilde{\chi}}$ is calculated in terms of a few parameters: one parameter smaller than in the case of constrained MSSM. Requiring that $\Omega_{\tilde{\chi}}$ is in the cosmologically favored region, $0.1 \lesssim \Omega_{\tilde{\chi}} h^2 \lesssim 0.3$ ($h$ is the present day Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$), we obtain the constraint on the parameter space of the models and hence masses for the superparticles. We also calculate the lightest Higgs boson mass and the constraint from $b \to s\gamma$ process to identify the phenomenologically allowed region of the parameter space. The resulting region is remarkably consistent with the SUSY explanation of the recently reported 2.6$\sigma$ deviation \[23\] of the muon anomalous magnetic moment from the standard-model value. We assume the conservation of $R$ parity throughout the paper.

\section{Gauge-Mediated SUSY Breaking Model}

The mass spectrum of the superparticles has a great influence on the estimation of the LSP relic abundance. Since brane-world GMSB models have the same mass spectrum as ordinary four-dimensional GMSB models except for the gravitino, we begin with reviewing the soft masses for the gauginos and sfermions in GMSB models.

In GMSB models, the superparticle mass spectrum does not depend on the detail of the DSB sector. Thus, it is sufficient to simply consider the messenger sector which consists of $N$ pairs of vector-like messenger superfields $q_i$ and $\bar{q}_i$ ($i = 1, \cdots, N$). To preserve the gauge coupling unification, $q_i$ is supposed to form a complete grand unified theory (GUT) multiplet. We assume, in this paper, that $q_i$ and $\bar{q}_i$ transform as $5$ and $5^*$ representation under the SU(5)$_{\text{GUT}}$. Then, the superpotential for the messenger sector is written as

$$W = \sum_{i=1}^{N} \left( \lambda_i^d S d_i \bar{d}_i + \lambda_i^l S l_i \bar{l}_i \right), \quad (3)$$

where $(d_i, l_i) \in q_i$ and $(\bar{d}_i, \bar{l}_i) \in \bar{q}_i$. The messenger quark multiplets $d$ and $\bar{d}$ transform as the right-handed down quark and its antiparticle under the standard-model gauge group, respectively, and the messenger lepton multiplets $l$ and $\bar{l}$ as the left-handed doublet lepton
and its antiparticle, respectively. The singlet field $S$ is a spurion superfield parameterizing the SUSY-breaking effect, and assumed to have vacuum expectation values (VEV’s) in its lowest and highest components as $\langle S \rangle = M + \theta^2 F$.

The SUSY breaking effect in the messenger sector is transmitted to the superpartners of the standard-model particles through the standard-model gauge interactions. The gaugino masses and the sfermion squared masses are generated at the messenger scale, $M$, by one- and two-loop diagrams, respectively. In general, various coupling constants, $\lambda^d$ and $\lambda^l$, in the superpotential Eq. (3) are not equal at the messenger scale; for instance, the couplings of the messenger quarks and leptons are not the same at the scale $M$, $\lambda^d \neq \lambda^l$, even if we assume they are equal at the GUT scale. However, the effect of having different couplings on the superparticle mass spectrum does not appear at the leading order in $F/M^2$, so that we here take all the coupling constants in Eq. (3) to be equal for simplicity and absorb it into the definition of $M$ and $F$. Then, the gaugino masses $M_a$ and the sfermion squared masses $m^2_{\tilde{f}}$ are given by [24, 25]:

$$M_a = N \frac{\alpha_a}{4\pi} \frac{F}{M} \mathcal{G}\left(\frac{F}{M^2}\right),$$  

$$m^2_{\tilde{f}} = 2N F \left(\frac{\alpha_a}{4\pi}\right)^2 \sum_a C_a^{\tilde{f}} \mathcal{F}\left(\frac{F}{M^2}\right),$$

where $a = 1, 2, 3$ represents the standard-model gauge groups and $C_a^{\tilde{f}}$ is the quadratic Casimir coefficient for the representation each sfermion belongs to. Here, the functions $\mathcal{G}$ and $\mathcal{F}$ are defined by

$$\mathcal{G}(x) = \frac{1}{x^2} \left[ (1 + x) \ln(1 + x) + (1 - x) \ln(1 - x) \right],$$

$$\mathcal{F}(x) = \frac{1 + x}{x^2} \left[ \ln(1 + x) - 2 \text{Li}\left(\frac{x}{1 + x}\right) + \frac{1}{2} \text{Li}\left(\frac{2x}{1 + x}\right) \right] + (x \to -x).$$

These functions have properties that $\mathcal{G}(x) \simeq \mathcal{F}(x) \simeq 1$ when $x \ll 1$.

To obtain the mass spectrum of superparticles, we have to evolve soft masses given in Eqs. (4, 5) using renormalization-group equations (RGE). Together with the contribution from the electroweak symmetry breaking, the masses for all the superparticles are determined. The Higgs sector contains two more parameters $\mu$ and $\mu B$: the SUSY-invariant ($W = \mu H_u H_d$) and SUSY-breaking ($\mathcal{L} = \mu B h_u h_d$) mass terms for the Higgs doublets. In brane-world GMSB models, both $\mu$ and $B$ are generated of the order of the weak scale, in contrast to the minimal model discussed in Ref. [26] where $B = 0$ is assumed at the messenger scale. The electroweak symmetry breaking condition relates these two parameters to $v$. 

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and \( \tan \beta \) up to the sign of \( \mu \), where 
\[
  v \equiv \sqrt{\langle h_u \rangle^2 + \langle h_d \rangle^2} \simeq 175 \text{ GeV}
\]
and 
\[
  \tan \beta \equiv \frac{\langle h_u \rangle}{\langle h_d \rangle}.
\]
Therefore, we end up with the following set of free parameters in our analyses: \( N, F/M, M, \tan \beta \) and \( \text{sgn}(\mu) \).

With the above GMSB mass spectrum, the correct electroweak symmetry breaking requires rather large value of the \( \mu \) parameter, \( \mu \gtrsim 2M_2 \). This is because in GMSB models the colored particles are relatively heavy and, as a result, the Higgs-boson mass squared receives large negative contribution from the top squark through the top Yukawa coupling. This fact has some important consequences in the present brane-world GMSB scenario. First, since \( \mu \) is of the order of the gravitino mass, the gravitino tends to be heavier than the lightest superpartner of the standard model. Thus, the gravitino is not the LSP in contrast to the usual four-dimensional GMSB models. According to Eqs. (4, 5), then, the lightest neutralino \( \tilde{\chi} \) or the right-handed stau \( \tilde{\tau}_R \) can be the LSP. In this paper, we concentrate on the case where \( \tilde{\chi} \) is the LSP and \( \tilde{\tau}_R \) is the next to LSP (NLSP), since \( \tilde{\tau}_R \) LSP leads to the serious problem of charged dark matter. Indeed, the LSP is \( \tilde{\chi} \) in most of the parameter space, especially when \( N = 1 \).

Another important consequence of \( \mu \gg M_2 \) is that the lightest neutralino is almost purely composed of the bino. This leads to a significant simplification in understanding the neutralino relic abundance; for instance, annihilation into \( Zh \) is strongly suppressed due to the smallness of the Higgsino component in \( \tilde{\chi} \). In the next section, we list the processes relevant for determining the relic abundance of the lightest neutralino \( \tilde{\chi} \), and calculate various quantities in the present models.

3 Relic LSP Abundance

The LSP is stable in \( R \)-parity preserving models, and hence its number density can decrease only through annihilation processes. In an expanding universe, the pair-annihilation “freezes out” when the expansion rate of the universe exceeds the interaction rate of the annihilation. After the freeze out, the number density of the LSP per comoving volume is constant, so that some amount of relic LSP is left in the present universe. Neutralino LSP abundance has been well studied in the context of conventional hidden-sector SUSY breaking models \([1, 2, 3, 4, 5]\). It is easily estimated once we could determine the relevant cross sections for the annihilation of the LSP. We first briefly review the procedure of calculating the relic abundance.

The time evolution of the LSP number density is described by the Boltzmann equation.
Since in GMSB models the bino and the right-handed stau are degenerate in some parameter region, we have to take into account the coannihilation effect \[27, 28\]. The Boltzmann equation with the coannihilation effect can be written as an equation for \( n = \sum_i n_i \), where \( n_i \) are the number densities of the species \( i \), and \( i \) represents the LSP neutralino \( \tilde{\chi} \) and the right-handed charged sleptons \( \tilde{\tau}_R, \tilde{\tau}_R^*, \tilde{\mu}_R, \tilde{\mu}_R^*, \tilde{\epsilon}_R \) and \( \tilde{\epsilon}_R^* \). The equation takes the following form \[7\]:

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}}v \rangle \left[ n^2 - (n^\text{eq})^2 \right],
\]

where \( H \) is the Hubble parameter and

\[
\langle \sigma_{\text{eff}}v \rangle = \sum_{i,j} \langle \sigma_{ij}v \rangle \frac{n_i^\text{eq} n_j^\text{eq}}{n_i^\text{eq} n_j^\text{eq}}.
\]

Here, \( n_i^\text{eq} (n_i^\text{eq}) \) is the equilibrium value of \( n (n_i) \) and \( v \) is the relative velocity of the particles \( i \) and \( j \). The bracket denotes the thermal average and \( \sigma_{ij} \) is the total annihilation cross section of \( i + j \rightarrow X + X' \):

\[
\sigma_{ij} = \sum_{X,X'} \sigma(i + j \rightarrow X + X'),
\]

where \( X \) and \( X' \) represent possible standard-model particles.

There are \( 7 \times 7 \) cross sections (\( \sigma_{ij} \)’s) in Eq. \[3\], but most of them are not independent. The independent cross sections are listed in Table \[4\], where we have shown only relevant final states which are kinematically accessible and have non-negligible cross sections. Furthermore, there are some simplifications coming from the GMSB mass spectrum. For \( \tilde{\chi}\tilde{\chi} \rightarrow f\bar{f} \), for example, the cross section is dominated by \( l_R\bar{l}_R \) final states since the right-handed sleptons are much lighter than the other sfermions and have the largest value of the hypercharge.

To estimate the relic LSP abundance, we need \( \langle \sigma_{\text{eff}}v \rangle \) at the freeze-out temperature \( T_f \) of the LSP. Since typically \( T_f \approx m_{\tilde{\chi}}/25 \) where \( m_{\tilde{\chi}} \) is the LSP mass, the expansion of \( \langle \sigma_{\text{eff}}v \rangle \) in terms of \( T/m_{\tilde{\chi}} \) (partial-wave expansion) is relevant. Thermally-averaged cross section \( \langle \sigma_{ij}v \rangle \) for the process \( i + j \rightarrow k + l \) is given by \[7\]:

\[
\langle \sigma_{ij}v \rangle = \frac{1}{m_i m_j} \left[ 1 - \frac{3(m_i + m_j)}{2m_i m_j} T \right] w(s)|_{s \rightarrow (m_i + m_j)^2 + 3(m_i + m_j)T} + \mathcal{O}\left( \frac{T^2}{m_{\tilde{\chi}}^2} \right),
\]

where

\[
w(s) \equiv \frac{1}{4} \int \frac{d^3p_k}{(2\pi)^3 E_k} \frac{d^3p_l}{(2\pi)^3 E_l} (2\pi)^4 \delta^4(p_i + p_j - p_k - p_l) |T|^2.
\]
| Initial state | Final states |
|--------------|--------------|
| $\tilde{\chi}\tilde{\chi}$ | $ff$ |
| $l^+_R\tilde{\chi}$ | $l^+\gamma, l^+Z, l^+h$ |
| $l^+_R\tilde{l}^-_R$ | $\gamma\gamma, ZZ, \gamma Z, W^+W^-, Zh, \gamma h, hh, ff$ |
| $l^+_R\tilde{l}^-_R (i \neq j)$ | $l^+\bar{l}^-$ |

Table 1: Annihilation cross sections; $i, j = \tau, \mu, e$.

Here, $s = (p_i + p_j)^2$ is the Mandelstam variable, and $|T|^2$ is the transition matrix element squared summed over final state spins and averaged over initial state spins. Terms of order $(T/m_{\tilde{\chi}})^0$ and $(T/m_{\tilde{\chi}})^1$ are called s-wave and p-wave components, respectively.

The s-wave component of the thermally-averaged neutralino annihilation cross section $\langle \sigma_{\tilde{\chi}\tilde{\chi}}v \rangle$ is suppressed by tiny final-state fermion masses, so that the p-wave part is the dominant piece. While neutralino annihilation cross section is p-wave suppressed, those for the sleptons, $\langle \sigma_{i\tilde{l}^i}v \rangle$, have s-wave components as dominant pieces. Therefore, if the equilibrium number densities for the sleptons are not much smaller than that for the neutralino, the slepton annihilation processes can significantly reduce the relic LSP abundance (see Eq. (9)). This coannihilation process is effective when the slepton masses are degenerate with the neutralino mass within $\sim 20\%$. In the GMSB spectrum, it happens when $\tan \beta$ is large and/or $N$ is greater than 1.

The partial-wave expansion of the thermally-averaged cross section does not give a good approximation when the initial momentum is near the s-channel pole or the final-state threshold [27]. They could occur at $Z, h, A, H$ poles and $WW, ZZ, Zh, t\bar{t}$ thresholds in the neutralino annihilation. However, with the present GMSB mass spectrum, these cases simply do not happen or their effects are strongly suppressed. First, the $Z, h$ poles can be hit when $m_{\tilde{\chi}} \sim m_Z/2, m_h/2$, but these regions are already being excluded by the chargino search at LEP2. The situations $m_{\tilde{\chi}} \sim m_A/2, m_H/2$ also do not occur in the parameter region we are considering. As for the threshold effect, $WW$ and $ZZ$ ones are small due to $\mu \gg M_1$; $Zh$ one is also negligible due to the smallness of the Higgsino component in $\tilde{\chi}$, and $t\bar{t}$ one is strongly suppressed by the large masses for the top squarks exchanged in t-channel.

With these understandings, we can calculate the relic abundance $\Omega_{\tilde{\chi}}h^2$. In the actual cal-
culation, we have used the computer program neutdriver coded by Jungman, Kamionkowski and Griest \[5\], which contains all the (co)annihilation cross sections calculated by Drees and Nojiri \[4\]. In Fig. 1, we have shown a cosmologically favored region, \(0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3\) (light shaded regions), on \(M-F/M\) plane in the case of \((N, \tan \beta) = (1, 10)\). We scanned the region \(10^4\) GeV \(\lesssim F/M \lesssim 2 \times 10^5\) GeV, which corresponds to the soft SUSY-breaking masses \(\sim 100\) GeV – 1 TeV. The sign of \(\mu\) is taken to be positive in the standard notation (the one in which the constraint from \(b \to s\gamma\) process is weaker). The region extends from upper left to lower right directions. This is because the relic abundance is almost completely determined by the mass of the right-handed stau, which is monotonically increasing with \(M\) with a fixed value of \(F/M\) due to renormalization group effects. We have also drawn the contours of the lightest Higgs boson mass (solid lines) and the lightest chargino mass (dashed lines) in GeV, which are calculated using neutdriver. We find that some of the parameter region satisfies constraints from the lower bounds on the Higgs boson mass \(m_{h^0} \gtrsim 113.5\) GeV and the lightest chargino mass \(m_{\tilde{\chi}^\pm} \gtrsim 150\) GeV. In particular, we find that smaller messenger scale, \(M \lesssim 10^8\) GeV, is favored.

In Fig. 2, we have shown the constraint from \(b \to s\gamma\), \(2.3 \times 10^{-4} < Br(b \to s\gamma) < 4.1 \times 10^{-4}\) \[29\], in the same \(M-F/M\) plane as Fig. 1. The dark shaded region indicates the excluded region. We see that most of the parameter region which satisfies the constraints from the Higgs and chargino masses also satisfies that from \(b \to s\gamma\). We have also drawn the contour of the SUSY contribution to the muon anomalous magnetic moment, \(a_\mu\), in units of \(10^{-10}\). The SUSY contributions to \(a_\mu\) were discussed, for example, in Refs. \[30\] – \[33\]. It is interesting that the value is consistent with that required to explain the recently claimed \(2.6\sigma\) deviation of \(a_\mu\) between the observed and the standard-model values.

In Figs. 3 and 4, we have plotted various quantities in the case of \((N, \tan \beta) = (1, 50)\). In this case, the coannihilation effect is important so that the cosmologically favored region (light shaded region) is shifted to larger values of \(F/M\) in lower \(M\) region \((M \sim F/M)\). We find that the constraint from \(b \to s\gamma\) is more stringent than the \(\tan \beta = 10\) case, but the region \(M \lesssim 10^7\) GeV still satisfies the constraints. This region gives the value of \(a_\mu|_{SUSY}\) within \(1\sigma\) and \(2\sigma\) level for the SUSY explanation of the observed deviation.

We have also done the same analyses in the case of \(N = 2\), which are plotted in Figs. 5, 6, 7 and 8. The black region represents the region where the stau is the lightest SUSY particle. In the case of \(\tan \beta = 10\) there is a consistent region which reproduces cosmologically interesting abundance of the LSP, but we do not find such a region in the case of \(\tan \beta = 50\).
4 Conclusions and Discussion

In this paper we have investigated gauge mediation models with neutralino LSP. This type of models is naturally realized in the extra-dimensional setup [16]. Due to the geometrical separation between the SUSY-breaking and observable sectors, flavor non-universal contributions to the squark and slepton masses are exponentially suppressed, which makes it possible to solve the $\mu$ problem by the mechanism of Ref. [21]. The consequence is that the gravitino mass becomes of the order of the weak scale, while all the other superparticle masses are the same as those in usual GMSB models. Therefore, the lightest neutralino (mostly bino) could provide the cold dark matter of the universe.

We have calculated the relic abundance of the LSP neutralino in this brane-world GMSB model. We found that there is a wide parameter region where cosmologically favored relic densities, $0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3$, are obtained and, at the same time, the lower bounds on the Higgs and the lightest chargino masses from collider experiments are evaded. We have also found that the experimental constraint on the $b \to s\gamma$ process is satisfied in this cosmologically favored parameter region, especially when the messenger scale $M$ is low. Interestingly, this parameter region also gives the SUSY contribution to the muon anomalous magnetic moment, $a_\mu|_{\text{SUSY}}$, with the values required to explain the recently reported 2.6$\sigma$ deviation of $a_\mu$ [23] from the standard-model value. These are due to the fact that in the GMSB spectrum the colored superparticles are relatively heavy while non-colored ones are light; the constraint from $b \to s\gamma$ is easily evaded since squarks are heavy, while we obtain relatively large values of $a_\mu|_{\text{SUSY}}$ since sleptons are light. Although we have limited ourselves to the case of the minimal messenger sector in this paper, the above features are generic to GMSB models, for example, ones with more general messenger sectors [24, 34, 35]. We leave the study of the LSP relic abundance under these more general superparticle spectrums for future work.

It is interesting to compare the present situation with other SUSY-breaking models that solve the SUSY flavor problem. In anomaly mediated SUSY-breaking scenario [13, 22], the LSP is generically wino-like neutralino, and its annihilation cross section is too large to obtain cosmologically favored thermal relic density of the LSP, so that we need to consider some other source of the LSP, such as the decay of the moduli field [36]. In the decoupling scenario [37] and the focus-point scenario [38], it would be difficult to accommodate sufficiently large values of the muon anomalous magnetic moment since the smuons are assumed to be heavy in these scenarios. The gaugino mediation scenario [14] is a promising candi-
date theory, but the parameter region of the neutralino LSP is not necessarily very large, especially when the Higgs fields are localized on the observable brane. Thus, we conclude that the GMSB models with neutralino dark matter provide one of the promising SUSY-breaking scenarios which are in accordance with current phenomenological situations. It would be interesting to study further the relic abundance of the LSP neutralino and its detections in this context.

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Figure 1: GMSB parameter space $(M, F/M)$ corresponding to $0.1 \lesssim \Omega \tilde{\chi} h^2 \lesssim 0.3$ (the light shaded region) for $\tan \beta = 10$, $\text{sgn} \mu = +1$. The dashed lines 100, 200, 300, 400, 500 are the contours for the lightest chargino mass, and the solid lines 100, 110, 120 are the contours for the Higgs boson mass (in unit of GeV). The dark shaded region in the upper left corner is the stau LSP region.
Figure 2: GMSB parameter space $(M, F/M)$ corresponding to $0.1 \lesssim \Omega_\chi h^2 \lesssim 0.3$ (the light shaded region) for $\tan \beta = 10$, $\text{sgn} \mu = +1$. The dotted lines 11, 27, 43, 59, 75 are the contours of the $-2\sigma$, $-1\sigma$, central, $+1\sigma$, and $+2\sigma$ values of the SUSY contribution to the muon $(g - 2)$, $a_\mu$, in unit of $10^{-10}$. The dark shaded region in the bottom is excluded by $b \rightarrow s\gamma$. 
Figure 3: The same figure as Fig. [1] for $\tan \beta = 50$. 
Figure 4: The same figure as Fig. 2 for $\tan \beta = 50$. The narrow strip in the dark shaded region (excluded by $b \rightarrow s\gamma$) is an accidentally allowed region.
Figure 5: The same figure as Fig. [1] for \( N = 2 \). The dark shaded region in the upper left (stau LSP region) is larger than that for \( N = 1 \).
Figure 6: The same figure as Fig. 2 for $N = 2$. The dark shaded region in the upper left (stau LSP region) is larger than that for $N = 1$. 

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Figure 7: The same figure as Fig. 3 for $N = 2$. In this case the region for $0.1 \lesssim \Omega \chi^2 \lesssim 0.3$ is absent in the range of the parameters exhibited.
Figure 8: The same figure as Fig. 4 for $N = 2$. In this case the region for $0.1 \lesssim \Omega_{\chi} h^2 \lesssim 0.3$ is absent in the range of the parameters exhibited.