New Physics with Mirror Particles

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Abstract

The introduction of mirror fermions with masses between the weak scale and 1 TeV could offer a dynamical origin to the standard-model electro-weak symmetry breaking mechanism. The purpose of this work is to study the dynamics needed in order to render models with such a fermion content phenomenologically acceptable.

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1 Physical motivation

The standard-model description of the electro-weak interactions of elementary particles is in general agreement with present experimental data. In this model, the Higgs mechanism is implemented in order to break spontaneously the $SU(2)_L \times U(1)_Y$ gauge symmetry, to give fermion masses and to unitarize the $W^+W^-$ scattering amplitude. Naturalness arguments (the hierarchy problem) have frequently led to studies where this mechanism is just an effective low-energy description of strong non-perturbative dynamics involving new gauge and matter degrees of freedom, in which case the Higgs particle is a composite state of new fermions whose condensate breaks dynamically the electro-weak symmetry.

On the other hand, the proximity of the top-quark mass to the electro-weak scale suggests an active role of the top-quark to such a condensate. However, relevant studies have shown that the top quark by itself is hardly heavy enough to reproduce the weak scale correctly, and in any case cannot eliminate the fine-tuning problem $[1]$. A solution to this issue is to introduce new particles which mix with the top quark and which are heavy enough to break the electro-weak gauge symmetry at the right scale. This not only eliminates the need for excessive fine tuning which plagues the simpler top-condensate models, but also allows for a naturally heavy top quark.

An attempt along these lines was presented some time ago $[2]$ which involved mirror fermion generations, i.e. fermions with interchanged isospin charges in a left-right symmetric context. However, that approach had several problems that will be overcome here. A first speculation for the existence of mirror fermions appeared in
the classical paper on parity violation \(^3\) that led to the V-A interaction models. Efforts to eliminate completely mirror fermions from nature are for some reminiscent of efforts several decades ago to identify the anti-electron with the proton, and amounts to not realising that particles consistent with natural symmetries could actually exist independently. Such a gauge group and fermion extension, apart from fitting nicely into unification schemata, restores in a certain sense the left-right symmetry of the matter sector missing in the standard model or in the simplest left-right symmetric models. In this way it also provides a well-defined continuum limit of the theory, something which is usually problematic due to the Nielsen-Ninomiya theorem \(^4\).

The left-right symmetric approach to standard-model extensions renders the baryon-lepton number symmetry \(U(1)_{B-L}\) more natural by gauging it, and has also been proposed as a solution to the strong CP problem when accompanied with the introduction of mirror fermions \(^5\). As will be seen in the following, the model introduced here proves further to be economical by identifying the source of the strong dynamics which break the electro-weak symmetry dynamically with a “horizontal” generation gauge group in the mirror sector which, apart from preventing the pairing-up of the standard-model generations with the mirror ones, provides also the intra-generation mass hierarchies.

Furthermore, it was recently shown \(^6\) that the gauge and fermion content of the present model is consistent with superstring-inspired unification schemata, including the mirror fermion generation group. The corresponding gauge coupling unification does not pose problems with proton decay and allows the prediction, in order of magnitude, of the QCD and weak scales. In such a superstring-inspired
unification, possibly connected to $N = 2$ supergravity, the standard-model fermions would have both mirror and supersymmetric partners. The present approach corresponds to breaking supersymmetry and leaving the supersymmetric partners close to the unification scale, and bringing the mirror partners down to the weak scale, altering thus radically the expected phenomenology.

From the experimental side, mirror fermions at low scales could already have manifested their existence indirectly. The nature of the non-perturbative interactions introduced could namely be related to the $3\sigma$ deviation from the standard-model value of the right-handed bottom-quark weak coupling extracted from the $A_b$ asymmetry [7], as we will later see. If a similar anomaly is conjectured for the top quark, it could influence substantially the values of the electroweak precision parameters $S$ and $T$ which, even though experimentally still consistent with zero, have negative mean values and can be non-negligibly negative, signaling new physics.

On the other hand, the introduction of mirror fermions could also create problems with the $S$ parameter, which can receive large positive “oblique” corrections due to 12 new chiral fermion doublets, as we will see in section 3. This parameter could of course also receive negative contributions from the vertex corrections directly related to the anomalous couplings mentioned above. Nevertheless, the same vertex effects that are taken to cancel the “oblique” corrections could potentially have a different sign, adding to these effects instead and rendering the model phenomenologically unacceptable. Their non-perturbative nature does not allow unfortunately the a priori determination of the sign and magnitude of their contributions, and the working assumption in this work will be that they take values in agreement with present bounds on the electroweak precision parameters.
In the following section, the dynamics needed to make such a mechanism phenomenologically viable are carefully analyzed. In particular, it proves necessary to review the dynamical assumptions made in Ref. [2]. In that work it was unclear why the characteristic scale of the strong group responsible for the fermion gauge-invariant masses happened to be so close to the scale where the strong interactions breaking electro-weak symmetry became critical. Furthermore, the previous model could not provide a see-saw mechanism for the standard-model neutrinos, coupling unification would be difficult, it had problems with the isospin quantum numbers of the lighter fermions, and it needed fine-tuning in order to prevent some fermions from acquiring large masses.

In the present approach, only the mirror particles are coupled strongly and dynamically involved in the breaking of $SU(2)_L$. By eventually breaking the mirror-generation symmetries, small gauge-invariant (by this we mean here and in the following gauge-invariant under the standard-model gauge group, unless otherwise stated) masses are allowed which communicate the electro-weak symmetry breaking to the standard-model fermions by mixing them with their mirror partners. This model has neither “sterile” nor $SU(2)_L$-doublet light mirror neutrinos, as in [8] for example, which would pose problems with experiment. After the mass hierarchies are computed within this context, phenomenological consequences like electro-weak precision parameters, CKM matrix elements, flavor-changing neutral currents (FCNC) and decays are discussed.
2 The model

The gauge-group structure considered is described by $SU(4)_{PS} \times SU(2)_L \times SU(3)_2G \times SU(2)_R$. The group $SU(4)_{PS}$ is the usual Pati-Salam group unifying quarks and leptons, and $SU(2)_L$ is the group of weak interactions. The group $SU(3)_2G$ is a horizontal gauge symmetry acting only on the mirror fermions, which becomes strong at around 2 TeV. All other groups are taken to have weak couplings at this energy. The corresponding symmetry for the standard-model fermions $SU(3)_{1G}$ has already been broken at higher scales, at once or sequentially, in order to avoid large FCNC.

Under the above gauge structure, the following fermions are introduced, which are left-handed gauge (and not mass) eigenstates and transform like

| Generations | Mirror generations |
|-------------|---------------------|
| $\psi_L$    | $\psi^M_L$          |
| $\psi_R$    | $\psi^M_c$          |

The superscript $M$ denotes the mirror partners of the ordinary fermions, and $c$ denotes charge conjugation.

One observes that the generation symmetries play a very important role at this stage, and this is to prevent the formation of large gauge-invariant masses. Pairing-up of standard-model and mirror generations is thus prohibited, in agreement with what is usually called “survival hypothesis” [9].

Even though this quantum number assignment is reminiscent of technicolor with a strong group $SU(N)_{TC} \approx SU(3)_2G$, there is no corresponding extended technicolor (ETC) group, there is a left-right interchange of weak isospin charges, and the new anti-particles transform under the same (and not the complex conjugate) representation of the strong group as the new particles. Even though the latter
difference is not essential for the present work, it is introduced for two reasons. The first one is that leaving a possibility to the strong mirror group to self-break even partially could be useful to the subsequent theoretical development of the model. The second is that it can fit easier in unification groups emerging in some superstring models [8]. Possible remaining anomalies related to this matter content are assumed to be cancelled by new physics, like additional fermions, at unification scales. The breaking of the strong group in the present case is an additional difference from technicolor models. One should furthermore not confuse the present model with other “mirror” fermion approaches, like in [10] for example, where all components of the new fermions are singlets under $SU(2)_L$ and interact only gravitationally or marginally with the standard-model particles, and which obviously cannot break the electro-weak symmetry dynamically.

2.1 Getting to the standard model

At high energy scales that do not enter directly in this work, the Pati-Salam group is assumed to break spontaneously like $SU(4)_{PS} \times SU(2)_R \rightarrow SU(3)_C \times U(1)_Y$, where $SU(3)_C$ and $U(1)_Y$ are the usual QCD and hypercharge groups respectively. (The particular way of the Pati-Salam-group breaking does not influence the discussion that follows. For alternative ways, see Ref.[8].) Much later, at scales on the order of $\Lambda_G \approx 2$ TeV, the mirror generation group breaks sequentially, just after it becomes strong, like $SU(3)_{2G} \rightarrow SU(2)_{2G} \rightarrow \emptyset$. It is not attempted here to investigate how exactly these breakings occur, and for simplicity it is enough to assume that a Higgs mechanism is responsible for them, effective or not.

The first spontaneous generation symmetry breaking $SU(3)_{2G} \rightarrow SU(2)_{2G}$
occurs at a scale $\Lambda_G$, with an $SU(2)_L$-singlet scalar state denoted by $\phi_3$ and transforming like a 3 under the generation symmetry acquiring a non-zero vev. Note that the group $SU(3)_{2G}$ could in principle partially self-break dynamically via the fermion-condensation channel $3 \times 3 \rightarrow 3$ if it were given the chance to become strongly coupled at this energy scale. We comment later on the possible connection of this channel to the large top-quark mass.

The fermions have the following quantum numbers under the new gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_{2G} \times U(1)_Y$:

| The 3rd & 2nd generations | The 3rd & 2nd mirror generations |
|----------------------------|---------------------------------|
| $q^3_{L,L} \quad (\bar{3}, \ 2, \ 1, \ 1/3)$ | $q^{3,2M}_{L} \quad (3, \ 1, \ 2, \ +4/3, -2/3)$ |
| $l^3_{L,L} \quad (1, \ 2, \ 1, -1)$ | $l^{3,2M}_{L} \quad (1, \ 1, \ 2, \ 0, -2)$ |
| $q^3_{R,c} \quad (\bar{3}, \ 1, \ 1, \ -4/3, +2/3)$ | $q^{3,2M}_{R,c} \quad (\bar{3}, \ 2, \ 2, -1/3)$ |
| $l^3_{R,c} \quad (1, \ 1, \ 1, \ 0, -2)$ | $l^{3,2M}_{R,c} \quad (1, \ 2, \ 2, 1)$ |

| The 1st generation | The 1st mirror generation |
|-------------------|---------------------------|
| $q^1_{L,L} \quad (3, \ 2, \ 1, \ 1/3)$ | $q^{1M}_{L} \quad (3, \ 1, \ 1, \ +4/3, -2/3)$ |
| $l^1_{L,L} \quad (1, \ 2, \ 1, -1)$ | $l^{1M}_{L} \quad (1, \ 1, \ 1, \ 0, -2)$ |
| $q^1_{R,c} \quad (\bar{3}, \ 1, \ 1, \ -4/3, +2/3)$ | $q^{1M}_{R,c} \quad (\bar{3}, \ 2, \ 1, -1/3)$ |
| $l^1_{R,c} \quad (1, \ 1, \ 1, \ 0, -2)$ | $l^{1M}_{R,c} \quad (1, \ 2, \ 1, 1)$ |

where the superscripts 1, ..., 3 indicate the fermion generations. Moreover, the letters $q$ and $l$ stand for quarks and leptons respectively. Note that $\bar{\psi}_R \psi^M_L$ mass terms are
still prohibited by the $SU(2)_G$ symmetry for the second and third generations.

At a scale quite close to $\Lambda_G$, the $SU(2)_G$ group breaks spontaneously sequentially to $U(1)_G$ and this down to $\emptyset$ by two $SU(2)_L$-singlet scalar states, denoted by $\phi_{2,1}$ and transforming like a $2$ and being charged respectively under the generation symmetries, which acquire non-zero vevs. The quantum numbers of the third and second generation mirror fermions after these breakings are given by

| Generation | $q^M_L$ | $l^M_L$ | $d^M_R$ | $l^M_R$ |
|------------|---------|---------|---------|---------|
| 2nd        | $(3, 1, -\frac{4}{3}, -\frac{2}{3})$ | $(1, 1, 0, -2)$ | $(\bar{3}, 2, -1/3)$ | $(1, 2, 1)$ |
| 3rd        | $(3, 1, +\frac{4}{3})$ | $(1, 1, 0, -2)$ | $(\bar{3}, 2, -1/3)$ | $(1, 2, 1)$ |

while the first mirror generation and all the standard-model generation quantum numbers are left unchanged.

The breakings of the mirror generation symmetries described above induce at lower energies, among others, effective four-fermion operators $F$ of the form

$$F = \frac{\lambda}{\Lambda_G^2} (\bar{\psi}_{R}^M \psi_{L}^M)(\bar{\psi}_{L}^M \psi_{R}^M)$$

for the three mirror fermion generations, where $\lambda$ are effective four-fermion couplings and the generation indices are omitted for simplicity. The fermion bilinears in parentheses above transform like doublets under $SU(2)_L$.

The next step is to assume that, in a manner analogous to top-color scenarios [11], the $SU(2)_G$ group is strongly coupled just before it breaks, and it is
therefore plausible to take the effective four-fermion couplings $\lambda$ to be critical for the corresponding mirror generations, like in the Nambu-Jona-Lasinio model (NJL). Therefore, condensates of mirror fermions like $<\bar{\psi}_L^M \psi_R^M>$ can form which break the symmetry $SU(2)_L \times U(1)_Y$ dynamically down to the usual $U(1)_{EM}$ group of electromagnetism.

2.2 The mass generation

The fermion condensates described above give to the mirror fermions symmetry-breaking masses of order $M \approx r \Lambda_F$ via the operators $F$, with $r$ a constant not much smaller than unity if one wants to avoid excessive fine-tuning of the four-fermion interactions. Effective operators of the form $\bar{\psi}_R^1 \psi_L^1 \bar{\psi}_L^2 M^2 \psi_R^M / \Lambda_G^2$ induced by the broken $SU(3)_{2G}$ interaction feed down gauge-symmetry-breaking masses to the first mirror generation. The fact that all mirror fermions get large masses of the same order of magnitude due to the critical interactions avoids fine-tuning problems that would appear if mass hierarchies were introduced by allowing only some of them to become massive, as is done in [12]. Moreover, to avoid breaking QCD and electromagnetism, it is assumed that most-attractive-channel dynamics prevent quark-lepton condensates of the form $<\bar{q}_{L}^M q_R^M>$ from appearing.

If generation symmetries were left intact, the mass matrix $\mathcal{M}$ for all the fermions would have the form

$$
\begin{pmatrix}
\bar{\psi}_L & \bar{\psi}_M \\
\psi_R & \psi_R^M
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & M
\end{pmatrix},
$$

where the 4 elements shown are blocks of $3 \times 3$ matrices in generation space and $M$ the dynamical mirror-fermion mass due to the strong generation interactions.
However, the broken generation symmetries allow the formation of gauge-invariant masses, and the mass matrix $\mathcal{M}$ takes the form:

$$
\begin{pmatrix}
\bar{\psi}_R & \psi^M_L \\
\bar{\psi}_R & \psi^M_L
\end{pmatrix} = 
\begin{pmatrix}
0 & m_1 \\
m_2 & M
\end{pmatrix},
$$

where the diagonal elements are gauge-symmetry breaking and the off-diagonal gauge-invariant.

The off-diagonal mass matrices can be generated by Yukawa couplings $\lambda_{ij}$ associated with spinor bilinears of fermions with their mirror partners which are coupled to the scalar states $\phi_{2,3}$ responsible for the spontaneous generation symmetry breakings. The corresponding gauge-invariant term in the Lagrangian has the form $\sum_{i,j} \lambda_{ij} \bar{\psi}_i R \psi_j M \phi_{2,3}$, where the indices $i, j$ count the corresponding fermions in the model. The elements of the matrices $m_{1,2}$ will be taken in general to be quite smaller than the ones in the matrix $M$, with the exception of the entries related to the top quark.

After diagonalization of the mass matrix shown above, in which the lighter mass eigenstates are identified with the standard-model fermions, a see-saw mechanism produces small masses for the ordinary fermions and larger ones for their mirror partners. A specific example for illustration purposes is produced in the next section. The situation is reminiscent of universal see-saw models, but it involves fermions having quantum-number assignments which should not in principle pose problems with the Weinberg angle $\sin^2 \theta_W$ \cite{13}, \cite{14}.

Some remarks relative to the (1,1) block entry of the mass matrix are in order. First, there are no $< \bar{\psi}_R \psi_L >$ condensates at these high energy scales. Then,
after careful inspection of the quantum numbers carried by the gauge bosons of the broken groups one observes that there are no four-fermion effective operators of the form $(\bar{\psi}_R \psi_L)(\bar{\psi}_L^M \psi_R^M)/\Lambda_G^2$ or any other gauge-invariant operators for any generation which would feed gauge-symmetry-breaking masses to the ordinary fermions at this stage.

3 Phenomenology

3.1 Masses and mixings

We start by calculating the mass hierarchies produced by the model, since they provide the basis of any phenomenological analysis. The gauge-symmetry breaking mass submatrices $M$ are hermitian because of parity symmetry. The gauge-invariant ones, denoted by $m_{1,2}$ should be symmetric due to the quantum numbers assigned to the fermions, but not necessarily real. Complex matrix elements allow therefore in general for weak CP violation. Assuming that $SU(2)_L$ effects can be neglected in the gauge-invariant mass generation process or that their effect is just homogeneously multiplicative, one also has the relation $m_2 = c m_1^\dagger$ between the gauge-invariant submatrices, with $c$ a real constant. This means that the determinant of the mass matrix $\mathcal{M}$ is real, eliminating thus the strong CP problem in this approximation, at least at tree level.

For simplicity, the mass matrices in the following are taken real and having the form

$$\mathcal{M}_i = \begin{pmatrix} 0 & m_i \\ m_i & M_i \end{pmatrix}, i = U, D, l$$

(2)

for the up-type quarks ($U$), down-type quarks ($D$) and charged leptons ($l$). We give
as a numerical example forms for the off-diagonal gauge-invariant mass submatrices of the up-type and down-type quark sectors for illustration purposes (with obvious correspondence between column and row numbers with generation indices):

\[
m_U(\text{GeV}) = \begin{pmatrix}
2.3 & 5.7 & 1.1 \\
5.7 & 20 & 1.3 \\
1.1 & 1.3 & 360
\end{pmatrix}, \quad m_D(\text{GeV}) = \begin{pmatrix}
1.6 & 1.6 & 0.51 \\
1.6 & 4 & 1.3 \\
0.51 & 1.3 & 35
\end{pmatrix}, (3)
\]

The dynamical assumption is made here that the \( SU(2)_L \)-breaking mass submatrices are diagonal and have the form

\[
M_U(\text{GeV}) = \begin{pmatrix}
360 & 0 & 0 \\
0 & 650 & 0 \\
0 & 0 & 650
\end{pmatrix}, \quad M_D(\text{GeV}) = \begin{pmatrix}
200 & 0 & 0 \\
0 & 360 & 0 \\
0 & 0 & 360
\end{pmatrix}, (4)
\]

The gauge-symmetry breaking masses of the first mirror generation are taken to be smaller than the ones of the two heavier generations corresponding to \( SU(2)_{2G} \). It should be noted here that, in principle, \( SU(2)_{2G} \) could equally well correspond to the mirror partners of the two lighter fermion generations. Such a scenario presents a particular interest since the heaviness of the top quark could be directly related to the self-breaking channel \( 3 \times 3 \rightarrow \bar{3} \) of \( SU(3)_{2G} \) mentioned before (the completion of the generation-group breaking being attributed to QCD for instance, eliminating thus the need for elementary scalars).

It is also expected that the dynamics provide some custodial symmetry breaking which is responsible for the mass difference in the up- and down-quark sectors. The \( U(1)_Y \) could be in principle the source of this difference, but we do not speculate on how this is precisely realised here. One has to further stress that the splitting of \( M_U \) and \( M_D \) is not \textit{a priori} needed to produce the top-bottom quark mass hierarchy, but it is introduced only to better fit the experimental constraints on the electro-weak parameters, as will be seen later.
These mass matrices give, after diagonalization and without the need for any fine-tuning, the following quark and mirror-quark masses (given in units of GeV):

| Standard-model quarks | Mirror quarks |
|-----------------------|--------------|
| $m_t = 160$, $m_c = 0.77$, $m_u = 0.001$ | $m_{tM} = 810$, $m_{cM} = 651$, $m_{uM} = 360$ |
| $m_b = 3.4$, $m_s = 0.07$, $m_d = 0.003$ | $m_{bM} = 363$, $m_{sM} = 360$, $m_{dM} = 200$. |

The ordinary quark masses given are slightly smaller than the ones usually quoted because the values reported here are relevant to the characteristic scale of the new strong dynamics which is around 2 TeV, and one has therefore to account for their running with energy. The formalism presents no inherent difficulty whatsoever producing larger masses for these fermions.

The generalization of the standard-model CKM quark-mixing matrix in this scenario is a unitary $6 \times 6$ matrix of the form $V_G = K_U^T K_D$, with $K_{U,D}$ the linear operators diagonalizing the mass matrices of the up and down fermion sector. The generalized CKM matrix has the form

$$V_G = \begin{pmatrix} V_{CKM} \\ V_2 \\ V_{1M} \\ V_{MCKM} \end{pmatrix},$$

(5)

and the usual standard-model CKM matrix $V_{CKM}$ is one of its submatrices given (in absolute values) by

$$|V_{CKM}| = \begin{pmatrix} 0.98 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.042 \\ 0.006 & 0.038 & 0.95 \end{pmatrix},$$

(6)

which is consistent with present experimental constraints.

The mixing between the first and second generations is larger than the one between the second and third generations, and this can be easily traced back to the relative elements of $m_{U,D}$. Furthermore, one has to be particularly cautious...
when using the flavor symbol ‘t’ and the flavor name ‘top quark’ for the heaviest standard-model-quark mass eigenstate, since $t_L(t_R^c)$ has a non-negligible $SU(2)_L$ singlet (doublet) component as expected due to the large $t_R^c t_L^M = t_R^M t_L$ mass terms, and this is reflected on the reported value of $|V_{tb}| = 0.95$. This is particularly apparent in the third-generation fermions to which correspond larger gauge-invariant masses, since the fermion-mirror fermion mixings are given roughly by the ratio $m_{ii}/M_{ii}$. Present experimental data give $|V_{tb}| = 0.99 \pm 0.15$ [15]. More precise future measurements of this quantity should show deviations from its standard-model value which is very close to 1 assuming unitarity of the mixing matrix $V_{CKM}$. Larger mirror-fermion masses can diminish this effect by reducing the corresponding mixing of the mirrors with the ordinary fermions.

In fact, indirect experimental indications for the existence of $SU(2)_L$-singlet new fermions which can mix with the third standard-model-generation charged fermions $t_L, b_L, \tau_L$, and $SU(2)_L$-doublet new anti-fermions which can mix with $t_R^c, b_R^c$, and $\tau_R^c$ could already exist in LEP/SLC precision data. One would be coming from the $S$ and $T$ parameters, which are consistent with anomalous right-handed top-quark couplings, as will be seen later, and the other coming from anomalous right-handed $b$-quark and $\tau$-lepton couplings to the $Z^0$ boson corresponding to even $3\sigma$ effects [7]. These are extracted from the current $A_{b,\tau}$ asymmetries. Note that, contrary to extended technicolor models that introduce left-handed anomalous couplings and affect mostly $R_b$, this mirror model induces anomalous right-handed couplings which affect mostly $A_b$. The actual sign of the deviations depends on the relevant interaction strength of the two isospin partners of the mirror doublets with the standard-model fermions, but more details on this are given in subsection
3.2. Deviations from the weak couplings of the lighter standard-model particles are largely suppressed, but they can be potentially large when the mirror partners are light. Bringing all the mirror partners to lower scales should be avoided nevertheless, since reproducing the weak scale would then require fine-tuning, as will be shown shortly.

The corresponding CKM matrix for the mirror sector $V_{CKM}^M$ is equal (in absolute values) to

$$|V_{CKM}^M| = \begin{pmatrix} 1 & 0.001 & 0.001 \\ 0.001 & 1 & 0.039 \\ 0.001 & 0.036 & 0.95 \end{pmatrix}. \quad (7)$$

The third generation is here the main reason why this matrix is not diagonal (The entries (1,1) and (2,2) are close to unity because of the assumed diagonal form of $M_{U,D}$, but not exactly unity, so that the unitarity character of the mixing matrix $V_G$ is preserved.) Furthermore, the matrices $V_1$ and $V_2$ mix the up-quark sector of the standard model with the down mirror-quark sector and vice-versa, and are given (in absolute values) by

$$|V_1| = \begin{pmatrix} 0.0037 & 0.0015 & 0.0002 \\ 0.0070 & 0.0202 & 0.0027 \\ 0.0005 & 0.0153 & 0.3163 \end{pmatrix}, \quad |V_2| = \begin{pmatrix} 0.0052 & 0.0060 & 0.0007 \\ 0.0059 & 0.0193 & 0.0024 \\ 0.0026 & 0.0163 & 0.3162 \end{pmatrix}. \quad (8)$$

The elements of these matrices are very small, apart from $|V_{tb}^M| = |V_{tb}^M|$ which account for the smallness of $|V_{tb}|$. This is expected, since, apart from the top quark, the gauge-invariant masses responsible for the mixing are much smaller than the gauge-symmetry breaking masses. One cannot expect therefore to find observable FCNC effects involving the first two generations, like deviations in the $K_L - K_S$ meson mass difference. Processes involving the third generation however like $b \rightarrow s\gamma$ could be affected and relevant deviations could be detectable in the
future.

One can investigate the predictive power of such a framework by comparing the number of input and output parameters required to produce the numbers presented above. With 16 different input parameters one gets 12 masses (6 for the ordinary and 6 for the mirror fermions) and 36 different mixing angles of the generalized CKM matrix, which offers an advantage to the above considerations. Deeper insights into the gauge-invariant mass-generation mechanism should in the future further reduce the initial independent parameters.

For the charged leptons, a diagonal gauge-symmetry breaking mass matrix is used again and a gauge-invariant mass matrix having the forms

\[
M_l(\text{GeV}) = \begin{pmatrix}
180 & 0 & 0 \\
0 & 200 & 0 \\
0 & 0 & 200
\end{pmatrix}, \quad m_l(\text{GeV}) = \begin{pmatrix}
0.25 & 0.25 & 0.1 \\
0.25 & 3.8 & 1 \\
0.1 & 1 & 17
\end{pmatrix}.
\] (9)

These give the following lepton and mirror-lepton mass hierarchy (at 2 TeV and in GeV units):

| Standard-model charged leptons | Mirror charged leptons |
|-------------------------------|------------------------|
| \(m_\tau = 1.45\) | \(m_\tau^M = 201\) |
| \(m_\mu = 0.07\) | \(m_\mu^M = 200\) |
| \(m_e = 3 \times 10^{-4}\) | \(m_e^M = 180\) |

The difference of the charged-lepton mass matrix with the down-quark mass matrix is attributed to QCD effects. The same mass hierarchies could have been produced with a diagonal submatrix \(m_l\) which would require less parameters, but for the sake of consistency a submatrix form similar to \(m_D\) is chosen. The neutrino mass and mixing matrix is quite interesting and is studied elsewhere [6], since the fact that neutrinos can have both Dirac and Majorana masses makes theoretical considerations and calculations more involved.

At this point it is not claimed that the mass-matrix elements given above
can be calculated explicitly within this model, since these could in principle receive important non-perturbative contributions. We just want to illustrate that it is feasible in principle within this context to generate the correct mass hierarchies and CKM angles. Having now these ingredients allows us to tackle various other phenomenological issues.

3.2 The weak scale and the electro-weak precision data

We next proceed by giving an estimate for the dynamically generated weak scale $v$. A rough calculation using the Pagels-Stokar formula gives

$$v^2 \approx \frac{1}{4\pi^2} \sum_i N_i M_i^2 \ln \left( \Lambda_G / M_i \right),$$

where $N$ is the number of new weak doublets introduced and $M_i$ their mass, where it has been assumed for simplicity that $m_{\nu_i} = m_{u_i}$ for all mirror neutrinos and where departures from pure weak eigenstates have been neglected. Consequently, for the masses found before and $\Lambda_G \approx 1.8$ TeV one gets $v \approx 250$ GeV, as is required. The mirror fermions can therefore be heavy enough to eliminate any need for excessive fine-tuning of the four-fermion interactions responsible for their masses. This numerical example should not be taken at face value of course, since moderately heavier mirror fermions are still possible and render even smaller values for $\Lambda_G$ acceptable.

The $S$ parameter could be problematic in this scenario however, since 12 new $SU(2)_L$ doublets are introduced. The main negative effect able to cancel the corresponding large positive contributions to $S$ coming from “oblique” corrections is the existence of vertex corrections stemming from 4-fermion effective interactions,
which can give rise to similar effects as the ones induced by light $SU(2)_L$-invariant scalars known as “techniscalars” \[7\].

More precisely, it is argued that the effective Lagrangian of the theory contains terms which can lead to a shift to the couplings of the top and bottom quarks to the $W^\pm$ and $Z^0$ bosons. In particular, there are four-fermion terms involving 3rd generation-quark flavor eigenstates and their mirror partners given by

$$L_{\text{eff}} = -\left(\frac{\lambda_{n1}}{\Lambda_{n1}^2} t^M_L \gamma^\mu t^M_L + \frac{\lambda_{c1}}{\Lambda_{c1}^2} b^M_L \gamma^\mu b^M_L\right) t_R^\gamma \gamma t_R - \left(\frac{\lambda_{n2}}{\Lambda_{n2}^2} t^M_L \gamma^\mu t^M_L + \frac{\lambda_{c2}}{\Lambda_{c2}^2} b^M_L \gamma^\mu b^M_L\right) b_R^\gamma \gamma b_R$$

(11)

where the $\lambda$’s and $\Lambda$’s are the effective positive couplings and scales of the corresponding operators renormalized at the $Z^0$ boson mass, and the subscripts $n,c$ indicate whether the participating fermions have the same hypercharge or not. One should at this point furthermore note that terms like $\lambda_{n3} (\frac{\Lambda_{n3}}{\Lambda_{n3}^2}) (\bar{q}_R^a \tau^a \gamma^\mu q_L^a) (\bar{q}_L \tau^a \gamma^\mu q_L)$, where $\tau^a, a = 1, 2, 3$ are the three $SU(2)_L$ generators, cannot be generated here in perturbation theory, unlike analogous terms in extended technicolor models. Anyway, such terms would produce shifts only to the left-handed fermion couplings, and these are already too much constrained from LEP/SLC data to be of any interest here.

Adopting the effective Lagrangian approach for the heavy, strongly interacting sector of the theory \[8\], the two mirror-fermion currents are expressed in terms of effective chiral fields $\Sigma$ like

$$t^M_L \gamma^\mu t^M_L = i \frac{\nu^2}{2} \text{Tr} \left( \Sigma^\dagger \frac{\tau^3}{2} D^\mu \Sigma \right)$$

$$b^M_L \gamma^\mu b^M_L = i \frac{\nu^2}{2} \text{Tr} \left( \Sigma^\dagger \frac{1 - \tau^3}{2} D^\mu \Sigma \right)$$

(12)
where the covariant derivative $D^\mu$ is defined by

$$D^\mu \Sigma = \partial^\mu \Sigma + ig \frac{\tau^a}{2} W^\mu_a \Sigma - ig' \frac{\tau^3}{2} B^\mu.$$

(13)

The $g$ and $g'$ above are the couplings corresponding to the gauge fields $W^\mu_a$ and $B^\mu$ of the groups $SU(2)_L$ and $U(1)_Y$ respectively. The chiral field $\Sigma = e^{2i\tilde{\pi}/v}$ transforms like $L \Sigma R^\dagger$ with $L \in SU(2)_L$ and $R \in U(1)_Y$ as usual, with hypercharge $Y = \tau^3/2$ and $\tilde{\pi} = \tau^a \pi^a/2$ containing the would-be Nambu-Goldstone modes $\pi^a$ “eaten” by the electro-weak bosons.

In the unitary gauge one takes $\Sigma = 1$, and the currents given above induce shifts in the standard-model Lagrangian of the form

$$\delta L = (g W^\mu_3 - g' B^\mu) \left( \delta g^t_R \bar{t}_R \gamma^\mu t_R + \delta g^b_R \bar{b}_R \gamma^\mu b_R \right)$$

(14)

with the non-standard fermion-gauge boson couplings expressed by

$$\delta g^t_R = \frac{v^2}{4} \left( \frac{\lambda_{n1}}{\Lambda_{n1}^2} - \frac{\lambda_{c1}}{\Lambda_{c1}^2} \right)$$

$$\delta g^b_R = -\frac{v^2}{4} \left( \frac{\lambda_{n2}}{\Lambda_{n2}^2} - \frac{\lambda_{c2}}{\Lambda_{c2}^2} \right).$$

(15)

After Fierz rearrangement of the terms in the effective Lagrangian $L_{\text{eff}}$, the scales $\Lambda_{n1,n2,c1,c2}$ can be seen as masses of effective scalar $SU(2)_L$-singlet spinor bilinears consisting of a mirror and an ordinary fermion. These are reminiscent of “techniscalars” as to their quantum numbers. One may observe that, unlike the present situation, the corresponding scalar effective operators induced by ETC interactions in ordinary technicolor theories would be $SU(2)_L$-doublets and would not produce shifts to the right-handed fermion couplings. The scalar operators appearing here could in principle correspond to mesons bound with the QCD force,
but their constituents are very heavy and are expected in principle to decay weakly before they have time to hadronize.

Alternatively, one may think of effective four-fermion operators of the general form $\mathcal{O}(\bar{\psi}_L^M \psi_R)(\bar{\psi}_R^M \psi_L^M)$ where the dimensionful form factors $\mathcal{O}$ are influenced by non-perturbative effects and are renormalized differently to lower scales according to the couplings and masses of the participating fermions. The operators in question are gauge-invariant and on dimensional grounds irrelevant. Therefore, the corresponding form factors related to heavier fermions are in general expected to be more seriously damped at lower scales than the ones corresponding to lighter fermions, in accordance to the decoupling theorem [19]. This fact gives the potential to $S$ to receive substantial negative contributions, as will become clear next.

It is as a matter of fact difficult to predict the values of the effective couplings of the operators that determine the fermion anomalous couplings, since they are influenced by non-perturbative dynamics. The values of the various terms are here chosen for illustration purposes to be $\frac{\lambda_{n1} v^2}{\Lambda_{n1}^2} = 1$, $\frac{\lambda_{c1} v^2}{\Lambda_{c1}^2} = 3.32$, $\frac{\lambda_{n2} v^2}{\Lambda_{n2}^2} = 0.22$, $\frac{\lambda_{c2} v^2}{\Lambda_{c2}^2} = 0.1$. If the strong mirror group is dynamically broken, one might wish to consider all these operators near criticality in the Nambu-Jona-Lasinio sense, in which case the couplings $\lambda_{n1,c1,n2,c2}$ would be of order $4\pi^2$. One then finds for example that the smallest scale entering this discussion is $\Lambda_{c1} \approx 820$ GeV.

The terms corresponding to operators involving the standard-model top quark (see subscripts $n1, c1$ above) are assumed larger than the ones involving the standard-model bottom quark (subscripts $n2, c2$). This might be related directly or not to the fact that, as was already seen in subsection 3.1, the masses corresponding to the $\tilde{t}_R t^M_L$ gauge-invariant terms, which constitute these four-fermion operators
after Fierz transformation, are much larger than the $\bar{b}_R b_L^M$ terms, because one has to reproduce the correct top-bottom mass hierarchy (recalling that $m_t/m_b \approx 35$). This is consistent with the general expectation in dynamical symmetry breaking schemes of having effects grow larger for heavier fermions, here traceable to the corresponding larger fermion-mirror fermion mixing. This would also explain why four-fermion terms involving first- and second-generation quarks are neglected in this analysis.

The possible connection of the four-fermion operators introduced above with the mass generation process is a subtle issue that could potentially shed more light on the hierarchy of the effective couplings appearing here. With regard to the difference between terms with subscripts $n_i$ and $c_i$, with $i = 1, 2$, the ones corresponding to the field $t^M_L$ are taken here to be smaller than the ones for $b^M_L$. This is motivated by the fact that the mirror-top is much heavier than the mirror bottom, and it is plausible that the relative form factors are considerably suppressed in comparison with the ones involving the mirror-bottom. This working assumption will prove to be crucial for the reported values of the electroweak parameters in this particular numerical example.

Using the values above one finds the anomalous couplings $\delta g^b_R = -0.03$ and $\delta g^t_R = -0.58$. The coupling $\delta g^b_R$ is within its best-fit experimental value $\delta g^b_R = 0.036 \pm 0.068$ (this is a combined fit including information on $\delta g_L$ and the $S$ and $T$ parameters [20]). It is already so tightly constrained that, even if it finally turns out to be positive, as suggested by [3] and which is easily achievable here by an appropriate choice of the relevant four-fermion couplings, it will not change our conclusions substantially. The coupling $\delta g^t_R$ is of course not yet tightly
constrained, and it is therefore a good candidate for a possible source of the large vertex corrections needed in this model.

One should expect therefore that, apart from the model-independent “oblique” contributions to the electro-weak precision parameters $S$ and $T = \Delta \rho / \alpha$ (where $\alpha$ is the fine structure constant), denoted by $S^0$ and $T^0$, these parameters receive also important vertex corrections $S^{t,b}$ and $T^{t,b}$ due to the top and bottom quarks, which should be given in terms of the anomalous couplings calculated above. The “oblique” positive corrections to $S$ are given by $S^0 = 0.1N$ for $N$ new $SU(2)_L$ doublets, assuming QCD-like strong dynamics. On the other hand, the mass difference between the up- and down-type mirror fermions produces a positive contribution to $T^0$. Considerations in the past literature with mirror fermions or vector-like models which can give very small or negative $S^0$ and $T^0$ do not concern us here because they are, unlike the present case, based on the decoupling theorem due to the existence of large gauge-invariant masses [4], [21].

By summing up these effects therefore, one finds for $S$ and $T$ the expressions

\[
S = S^0 + S^{t,b} = 0.1N + \frac{4}{3\pi} (2\delta g^t_R - \delta g^b_R) \ln (\Lambda/M_Z)
\]

\[
T = T^0 + T^{t,b} = \frac{3}{16\pi^2\alpha v^2} \sum_i (m_{U^M_i} - m_{D^M_i})^2 + \delta g^t_R \frac{3m^2_t}{\pi^2\alpha v^2} \ln (\Lambda/m_t),
\]

where $m_{U^M_i}, D^M_i$ denote the masses of the up- and down-type mirror quarks, $N = 12$ in the present case, and $\Lambda$ is the cut-off, which is expected to be roughly equal to the smallest scale appearing in Eq.15, namely $\Lambda \approx \Lambda_{c1} \approx 820$ GeV. The new contributions to $S$ and $T$ reflect modifications of the $W$ and $Z$ self-energies due to non-standard top- and bottom-quark vertices inserted into the relevant loop.
diagrams. Note that these expressions are valid for small anomalous couplings, but they are used in the following to illustrate the main effect of the new sector even though the top-quark anomalous coupling is taken to be quite large. Anyway, it can be assumed that these effects can be adequately absorbed in the expressions for the unknown effective four-fermion couplings. It is also noted that contributions to $S^0$ and $T^0$ from the lepton sector are calculated assuming Dirac mirror neutrinos.

Moreover, one has to stress here that no isospin splitting whatsoever is required \textit{a priori} in the mirror sector in order to get the top-bottom quark mass hierarchy, since this can be produced by differences in the gauge-invariant mass submatrices. The dynamical generation of this hierarchy does not lead to problems with the $T$ parameter, and this can be traced to the fact that the fermion condensates which break dynamically the electro-weak symmetry are distinct from the electro-weak-singlet condensates responsible for the feeding-down of masses to the standard-model fermions. This is contrary to the usual ETC philosophy. The reason this isospin asymmetry is introduced here is only to cancel the large negative contributions to the $T$ parameter coming from the vertex corrections, as will be seen in the following.

By using the fermion masses and anomalous couplings calculated above, one finds that the parameters $S$ and $T$ are given by

\[ S \approx 1.2 - 0.48 \ln (\Lambda/M_Z) \]
\[ T \approx 19.4 \times (0.88 - 0.58 \ln (\Lambda/m_t)). \]  \hspace{1cm} (17)

The present best-fit values for the electroweak parameters are (note that this is
again a combined fit including b-quark anomalous-coupling information \cite{20}

\[ S = -0.40 \pm 0.55 \]

\[ T = -0.25 \pm 0.46 \] \hspace{1cm} (18)

It is apparent that, even though these parameters are still consistent with zero, they can assume non-negligible negative values approaching even -1 (at 1\(\sigma\)), so the loop and vertex corrections do not have to cancel exactly.

One observes moreover that for cut-off scales \(\Lambda\) of about 820 GeV, values for the \(S\) and \(T\) parameters consistent with experiment are feasible, i.e. \(S \approx 0.14,\) \(T \approx -0.3,\) and this is mainly due to the large negative anomalous coupling \(\delta g_R^t.\)

Similar values for the electro-weak precision parameters could be achieved with smaller anomalous couplings accompanied with a larger cut-off \(\Lambda.\) This would lead to lighter mirror fermions in order to reproduce the weak scale correctly, something that would also automatically imply a larger fermion-mirror fermion mixing, but it would have the undesirable effect of increasing the fine tuning in the model.

It should not be forgotten nevertheless that it is attempted here to study non-perturbative theories with dynamics not easily calculable. For instance, since some of the new fermions introduced have masses close to the scale \(\Lambda,\) the effective theory is studied very close to the cut-off where it is expected to lose its accuracy, and the corresponding results could be consequently distorted. Moreover, the numbers quoted are very model-dependent and far from having general validity, since different assumptions about the effective couplings and scales involved would lead to different values for the \(S\) and \(T\) parameters.

One should furthermore note that in such a type of calculational schemes it
seems like an accident that the $T$ parameter is so close to zero, since slight deviations in the parameters can shift it to large positive or negative values due to the large parenthesis prefactor. The large negative contributions to $T$ can be traced to the large absolute value of $\delta g^f_R$ which in its turn is needed to cancel the large positive contributions to the $S$ parameter. This problem would be therefore less acute if the “oblique” positive corrections to $S$ were smaller and a smaller $\delta g^f_R$ would thus be able to accommodate the experimental data.

One way to achieve this is to note that the generation group is broken, leading to non-QCD-like strong dynamics. If this makes the mirror-fermion masses run much slower with momentum, it can reduce the positive contributions to the $S$ parameter even by a factor of two [22]. Another way is to have Majorana mirror neutrinos [6], [23]. In any case, the purpose of the numerical example presented is merely to illustrate that theories of this type may potentially produce negative $S$ and $T$ parameters, which in general is not easy in dynamical symmetry breaking scenarios. Further phenomenological consequences of the model can be found in [23].

4 Conclusions

Motivated by several theoretical arguments and possibly by some experimental indications that there are new physics around the TeV scale, we extended the gauge sector of the standard model and its fermionic content in a left-right symmetric context. We argue that doubling the matter degrees of freedom should be considered positively if, instead of just burdening the theory with more parameters, it renders it more symmetric while simultaneously solving several problems like electroweak
radiative corrections, fine-tuning, fermion mass generation and mixing, possibly absence of strong CP violation and eventual unification at very high energy scales.

It was shown that the model sets up a precise theoretical framework for the calculation of fermion mass hierarchies and mixings. It gives furthermore rise to dynamics which could potentially reconcile the $S$- and $T$- parameter theoretical estimates with their experimental values without excessive fine tuning. Moreover, the doubling of the fermionic spectrum it predicts provides decay modes which should in principle be detectable in colliders like $LHC$ and $NLC$ [23]. This fact, together with more precise future measurements of possible FCNC and anomalous couplings in the third fermion generation render the model experimentally testable.

Within the present approach, a deeper understanding of the generation of the gauge-invariant mass matrices $m$ and the effective couplings leading to anomalous third-generation standard-model fermion couplings to the $Z^0$ boson is still needed. This would settle the question on whether the large positive loop corrections to the $S$ parameter in this model can be adequately canceled by vertex corrections without unnatural fine-tuning. Furthermore, a more complete investigation on how the mirror generation groups break just after they become strong is an important open question.

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