Determination of the Kobayashi-Maskawa-Cabibbo matrix element $V_{us}$ under various flavor-symmetry breaking models in hyperon semileptonic decays

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Abstract

We study the success to describe hyperon semileptonic decays of four models that incorporate second-order $SU(3)$ symmetry breaking corrections. The criteria to assess their success is by determining $V_{us}$ in each of the three relevant hyperon semileptonic decays and comparing the values obtained with one another and also with the one that comes from $K_{l3}$ decays. A strong dependence on the particular symmetry breaking model is observed. Values of $V_{us}$ which do not agree with the one of $K_{l3}$ are generally obtained. However, in the context of chiral perturbation theory, only the model whose corrections are $O(m_s)$ and $O(m_s^3/2)$ is successful. Using its predictions for the $f_1$ form factors one can quote a value of $V_{us}$ from this model, namely, $V_{us} = 0.2176 \pm 0.0026$, which is in excellent agreement with the $K_{l3}$ one.

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I. INTRODUCTION

From the theoretical point of view, hyperon semileptonic decays (HSD) are considerably more complicated than pseudoscalar-meson semileptonic decays. The participation of vector and axial-vector currents in the former leads to the appearance of many more form factors. While in the latter not only less form factors appear, because only the vector current can participate, but also the theoretical approach to compute such form factors is under quite reasonable control [1]. These facts allow that the Kobayashi-Maskawa-Cabibbo matrix element $V_{us}$ be more reliably determined in the decays $K^+ \rightarrow \pi^0 l^+ \nu_l$ and $K^0 \rightarrow \pi^- l^+ \nu_l$ than in HSD. An analysis of $K_{l3}$ decays [1] yields $V_{us} = 0.2196 \pm 0.0023$. The inclusion of more refined $SU(2)$ symmetry-breaking corrections leads to [2]

$$V_{us} = 0.2188 \pm 0.0016.$$  \hspace{1cm} (1)

It is difficult to assess the success of the many calculations of $SU(3)$ symmetry-breaking corrections to the form factors of HSD. Predictions that vary substantially from one another are obtained. An important selection of such calculations can be found in references [3–6]. These are refined calculations that incorporated second-order symmetry breaking corrections to the leading vector form factor $f_1$. However, a reliable knowledge of $V_{us}$ provides an opportunity to establish some criteria to discriminate between the several such calculations. If one uses them to determine $V_{us}$ from HSD, then –in addition to reproducing the experimental data reasonably well– the following two criteria must be satisfied:

(i) one must obtain a consistent value of $V_{us}$ in the relevant HSD, and

(ii) this latter value of $V_{us}$ must also be consistent with its value of $K_{l3}$ decays, Eq. (1).

With the currently available experimental information the relevant HSD to determine $V_{us}$ are $\Lambda \rightarrow p e \nu, \Sigma^- \rightarrow n e \nu$, and $\Xi^- \rightarrow \Lambda e \nu$ [7]. This information in the form of decay rates, angular correlations, and spin asymmetries is collected in Table I. An alternative set of experimental data is constituted by the rates and the measured $g_1/f_1$ ratios. However, this latter set is not as rich as the former and will not be used here.

In this paper we shall perform a detailed analysis of the success of the predictions of references [3–6] for HSD through the values obtained for $V_{us}$, as explained above. In Sec. II we shall briefly review the predictions of these references and we shall make the first determination of $V_{us}$. In Sec. III we shall study the effect upon $V_{us}$ of the induced vector and axial-vector form factors $f_2$ and $g_2$, respectively. This study will give us a more precise determination of $V_{us}$. Sec. IV will be reserved for discussions and conclusions. Our main result will be that only the predictions of Ref. [6] satisfy criteria (i) and (ii), in accordance with the findings of a model independent analysis performed before [8].

II. A FIRST DETERMINATION OF $V_{us}$

We shall refer to the calculations of references [3–6] as Models I, II, III, and IV, respectively. Our interest in them arises from the fact that in each of them not only first order but second-order $SU(3)$ symmetry-breaking corrections to the leading vector form factor $f_1$ were calculated. In models I and III the corrections to the leading axial-vector form factor $g_1$ were also produced. The approaches and/or approximations used in going from one model
to another are quite different. In Model I a relativistic quark model was used. Model II made use of chiral perturbation and included corrections of $O(m_s)$. Model III relied on the non-relativistic quark and bag models and included both wave-function mismatch and center of mass corrections. A similar approach treating solely center of mass corrections was analyzed in Ref. [8]. Model IV followed the lines of Model II but it incorporated the more refined corrections of $O(m^3_s/2)$. The corresponding predictions for $f_1$ are reproduced in Table II. They are displayed in the form of ratios $f_1/f_1^{SU(3)}$. The values of $g_1/g_1^{SU(3)}$ predicted by Models I and III are displayed in Table III. The symmetry limit values $f_1^{SU(3)}$ and $g_1^{SU(3)}$ correspond to the conserved vector current hypothesis (CVC) and the Cabibbo theory predictions, respectively. A review of this last can be found in Ref. [10].

For our analysis we shall include radiative corrections and the four-momentum transfer contributions of the form factors. The detailed expressions are given in Ref. [4]. None of the four models give the predictions for $f_2$ and $g_2$. In this section we shall assume for the several $f_2$ their CVC predictions and we shall keep each $g_2$ equal to zero, in accordance with the assumption of the absence of second-class currents. The other two induced form factors $f_3$ and $g_3$ can be safely ignored in the three decays we consider, because their contributions are proportional to the electron mass.

With this last information we can already make the first determination of $V_{us}$ with Models I and III. The values obtained are given in Tables IV and V, respectively. We show in these tables the values of $g_1$ used, but this time normalized with respect to $f_1$. The effects of considering only center of mass corrections in Model III, as discussed in Ref. [9], have been displayed in the entries within parentheses of Table IV. In the case of Models II and IV we do not have the corresponding predictions for the $g_1$’s. We shall leave each one as a free parameter and then the results of Tables VI and VII are obtained. In order to make a comparison on an equal footing of the four models we also leave the $g_1$’s as free parameters with Models I and III. Tables VIII and IX are thus obtained.

Let us now look into the results obtained. The $f_1$ and $g_1$ form factors predicted by Models I and III lead to values of $V_{us}$ that differ from one decay to another by more than three standard deviations, as can be seen in Tables IV and V. That is, criterion (i) above is not satisfied. In contrast, Models II and IV do lead to values of $V_{us}$ in Tables VI and VII that in each model are consistent with one another within a little bit more than one standard deviation. When the $g_1$’s are free parameters the new determinations of $V_{us}$ of Models I and III given in Tables VII and VIII become consistent in each model, too. The criterion (i) is satisfied by the four models when the $g_1$’s are allowed to be free parameters. The calculated $g_1$’s of Models I and III seem to be ruled out by criterion (i). This is also confirmed by the high $\chi^2$ obtained when the $g_1$’s are fixed. However, in Model III when only center of mass corrections are considered the $\chi^2$ of $\Lambda \rightarrow p e\nu$ is remarkably lowered although the value of $V_{us}$ obtained is increased with respect to the case when the wave-function mismatch corrections are included.

Concerning criterion (ii), we see that Models I, II, and III give values of $V_{us}$ that are systematically higher than the $K_{l3}$ value of Eq. (1) close to three standard deviations in some cases or more than three in other cases. In contrast, Model IV gives systematically values of $V_{us}$ that are lower than Eq. (1). These values, however, are pretty close to Eq. (1).

The high $\chi^2$ in $\Lambda \rightarrow p e\nu$ in Tables IV and V is due mainly to $\alpha_{e\nu}$ and $\alpha_{\nu}$, whereas in Tables VII and VIII it comes mainly from $\alpha_e$ and $\alpha_\nu$. In the case of $\Sigma^- \rightarrow n e\nu$ the $\chi^2$ is also high.
and comes mainly from $\alpha_\nu$ and $\alpha_B$. Even when the $g_1$’s are used as free parameters, despite the appreciable lowering of $\chi^2$, a still rather high $\chi^2$ remains in $\Lambda \to p e \nu$ and $\Sigma^- \to n e \nu$.

Before drawing conclusions, it is important to consider the effect the induced form factors $f_2$ and $g_2$ have upon the determination of $V_{us}$ and $\chi^2$. This we do in the next section.

### III. EFFECT OF THE INDUCED VECTOR AND AXIAL-VECTOR FORM FACTORS

None of the four models under consideration here produced predictions for $f_2$ and $g_2$. Nevertheless, it is necessary to study their relevance in determining $V_{us}$. We shall allow them to be free parameters, since inasmuch as they help reduce $\chi^2$ we may expect that experimental data, which certainly know of symmetry breaking corrections, will force them to move into the correct direction.

The CVC contributions of $f_2$ are already first-order symmetry breaking contributions to the experimental observables of Table I. Accordingly, one should only consider first-order symmetry breaking of such CVC predictions in order to take into account the second-order contributions of the $f_2$. It is reasonable to allow the $f_2$’s to vary only up to 20% around the CVC values. This we shall do in steps, first by changing the $f_2$’s by $\pm 10\%$ and keeping them fixed while redoing the fits of the previous section and next by changing them by $\pm 20\%$ and repeating the whole procedure.

The results of this analysis are that practically no observable effects upon the values of $V_{us}$ are seen to occur. Only the fourth digits are changed, without even affecting third digits by rounding up. There is no need to produce new tables with such negligible changes.

Due to the absence of second-class currents, the $g_2$ are all zero in the symmetry limit. They will be rendered non-zero by $SU(3)$ symmetry breaking. As in the case of the $f_2$, the first-order corrections to them will amount to second-order contributions to the observables. We shall introduce fixed values of the $g_2$’s first of $\pm 0.10$ and next by $\pm 0.20$ and redo all the fits of Sec. II. These changes seem to be of reasonable size according to the estimations of Refs. [11] and [12]. The $g_2$’s do lead observable changes.

Models I and III with $f_1$ and $g_1$ fixed at their predictions give values of $V_{us}$ in $\Lambda \to p e \nu$ that come closer to Eq. (1), but still with high $\chi^2$—meaning that the corresponding experimental data are not satisfactorily reproduced. Also the dispersion of the values of $V_{us}$ from the three decays, although somewhat mitigated is not corrected either. All this is collected in Tables XI and XII. The effect of dropping the wave-function mismatch corrections of Model III is displayed in the entries within parentheses of Table XI. Again an appreciable lowering of $\chi^2$ is seen in $\Lambda \to p e \nu$ and also in $\Sigma^- \to n e \nu$, but at the expense of increasing $V_{us}$ with respect to the corresponding values of $V_{us}$ when such corrections are included.

When the $g_1$’s are allowed to vary then Models I and III improve their agreement with experiment, the corresponding $\chi^2$’s are noticeably reduced. This can be seen in Tables XII and XIII. But the values of $V_{us}$ are increased to the extent that none is any longer compatible with Eq. (1). This situation repeats itself for Model II in Table XIV. In contrast, the values of $V_{us}$ obtained with Model IV are fairly stable with respect to changes of $g_2$. Actually, as seen in Table XV they tend to increase with respect to the corresponding entries of Table VII, which is in the right direction towards Eq. (1).
Concerning the agreement with experiment we observe that a further lowering to an acceptable value of $\chi^2$ is obtained in $\Sigma^- \to n\nu\bar{v}$ as an effect of a non-zero $g_2$. However, this lowering is not observed in the $\chi^2$ of $\Lambda \to p\nu\bar{v}$, which remains at around 10 through Tables X – XV. This effect may be due to some experimental inconsistency of the value of $\alpha_\nu$, which contributes 7 to $\chi^2$, with the other asymmetries. If this $\alpha_\nu$ is left out the same $V_{us}$ is obtained along with practically the same error bars. For example, with the $f_1$ of Model III and with variable $g_1/f_1$, one obtains $V_{us} = 0.2220 \pm 0.0035, 0.2261 \pm 0.0035,$ and $0.2302 \pm 0.0035$ for $\Delta g_2 = -0.20, 0.0,$ and $+0.20,$ respectively. The corresponding $\chi^2$'s are 4.30, 4.1, and 4.0, which represents a considerable reduction with respect to the corresponding $\chi^2$'s in Table XIII; these new $\chi^2$'s indicate a very good agreement with other four observables in $\Lambda \to p\nu\bar{v}$. The same pattern repeats itself when $\alpha_\nu$ is left out in the comparison of the other models. In view of this situation we shall keep the several tables as they are. The high $\chi^2$ of $\Lambda \to p\nu\bar{v}$ should serve as a remainder that some problem exists in this decay. It is not idle to insist that new measurements in this decay should be most welcome.

The combined effect of simultaneous changes of $f_2$ and $g_2$ leads to the same results of Tables X – XV, except for minor changes in the fourth digits of the several values of $V_{us}$. Again there is no need to produce tables to show this. Let us pass to the last section.

**IV. DISCUSSION AND CONCLUSIONS**

Throughout our study, we notice that the values obtained for $V_{us}$ are very model dependent. We also notice that, except for one model, the values of $V_{us}$ are inconsistent with each other within the same model. These observations render inadmissible to quote a consistent average value from HSD.

However, since the dispersion of the values of $V_{us}$ in each of the three decays is mitigated in all the models when one allows the $g_1$ to be free parameters, one may quote an average value of the $V_{us}$ obtained with each model by selecting the appropriate sign of $\Delta g_2$ that lowers most the corresponding $\chi^2$. That is, we accept that criterion (i) is more or less satisfied by each model. These averages are collected in Table XVI. We have also included there the averages of the case $\Delta g_2 = 0$. This last table allows us to better appreciate how criterion (ii) is satisfied or not.

Looking at the averages obtained for $V_{us}$ with each model, one readily sees that Models I, II, and III are far from satisfying criterion (ii), while Model IV satisfies it remarkably well. From this point of view, it becomes very clear that the criteria discussed in the introduction indeed serve as quite stringent discriminating tools between different models and/or approximations. Our main conclusion in this regard is that of the four models that provide second-order symmetry breaking corrections to the $f_1$’s only Model IV of Ref. [6] is acceptable.

This conclusion allows us to quote the best value of $V_{us}$ that can be obtained from Model IV, namely,

$$V_{us} = 0.2176 \pm 0.0026.$$  (2)

Since this value is statistically in very good agreement with the $K_{l3}$ one of Eq. (1), we can average both and get
\[ V_{us}^{AV} = 0.2185 \pm 0.0014. \] (3)

The determination of \( V_{us} \) in Eq. (2) is quite acceptable in the light of the model independent analysis of Ref. \[8\]. Although we have committed ourselves with the predictions of Model IV for the \( f_1 \)'s, the rest of the form factors was dealt with in a model-independent fashion.

This last remark brings us to our closing comments. One cannot yet consider the theoretical issues as closed. It is most important that within the same Model IV used to calculate the \( f_1 \)'s the other relevant form factors be also computed. The values displayed for these form factors in Tables VII and XV may provide useful guidance for this enterprise. Our analysis of Sec. III shows that detailed values of the \( f_2 \)'s are not relevant and thus these HSD do not provide useful guidance for their calculation. It should be found elsewhere. Also, as pointed out in Ref. \[13\] a viable model of \( SU(3) \) breaking should be able to predict the \( \Delta S = 0 \) modes, \( \Sigma^\pm \rightarrow \Lambda e\nu \). Only if the predictions for \( \Delta S = 0 \) and \( \Delta S \neq 0 \) decays are simultaneously correct should one consider Model IV completely successful.

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REFERENCES

[1] H. Leutwyler and M. Roos, Z. Phys. C 25, 91 (1984).
[2] G. Lópes-Castro and G. Ordaz-Hernández, Mod. Phys. Lett. A5, 755 (1990).
[3] F. Schlumpf, Phys. Rev. D 51, 2262 (1995), hereafter called Model I.
[4] A. Krause, Helv. Phys. Acta 63, 3 (1990), hereafter called Model II.
[5] J. Donoghue, B. Holstein, and W. Klimt, Phys. Rev. D 35, 934 (1987), hereafter called Model III.
[6] J. Anderson and M. A. Luty, Phys. Rev. D 47, 4975 (1993), hereafter called Model IV.
[7] Particle Data Group, J. J. Hernández et al., Phys. Rev D 50, Part I (1994).
[8] A. García, R. Huerta, and P. Kielanowski, Phys. Rev. D 45, 879 (1992).
[9] P. G. Ratcliffe, Phys. Lett. B 242, 271 (1990).
[10] A. García and P. Kielanowski, The Beta Decay of Hyperons, Lecture Notes in Physics Vol. 222 (Springer-Verlag, Berlin, 1985).
[11] L. J. Carson, R. J. Oakes, and C. R. Willcox, Phys. Rev. D 33, 1356 (1986).
[12] D. P. Rath and S. N. Jena, Pramāṇa – J. Phys. 32, 753 (1989).
[13] P. G. Ratcliffe, Phys. Lett. B 365, 383 (1996).
### TABLE I. Experimental data for the three relevant HSD. The units of $R$ are $10^6$ s$^{-1}$.

|                  | $\Lambda \rightarrow p\nu$ | $\Sigma^- \rightarrow n\nu$ | $\Xi^- \rightarrow \Lambda\nu$ |
|------------------|----------------------------|----------------------------|----------------------------|
| $R$              | $3.169 \pm 0.058$          | $6.876 \pm 0.235$          | $3.36 \pm 0.19$            |
| $\alpha_{e\nu}$ | $-0.019 \pm 0.013$         | $0.347 \pm 0.024$         | $0.53 \pm 0.10$           |
| $\alpha_e$      | $0.125 \pm 0.066$         | $-0.519 \pm 0.104$       |                            |
| $\alpha_\nu$    | $0.821 \pm 0.060$         | $-0.230 \pm 0.061$       |                            |
| $\alpha_B$      | $-0.508 \pm 0.065$        | $0.509 \pm 0.102$        |                            |
| $A$              |                            |                            | $0.62 \pm 0.10$           |
| $g_1/f_1$       | $0.718 \pm 0.015$         | $-0.340 \pm 0.017$       | $0.25 \pm 0.05$           |

### TABLE II. $SU(3)$ breaking for $f_1$. The values correspond to the ratio $f_1/f_1^{SU(3)}$.

| Decay           | Model I | Model II | Model III | Model IV |
|-----------------|---------|----------|-----------|----------|
| $\Lambda \rightarrow p\nu$ | $0.976$ | $0.943$ | $0.987$  | $1.024$  |
| $\Sigma^- \rightarrow n\nu$ | $0.975$ | $0.987$ | $0.987$  | $1.100$  |
| $\Xi^- \rightarrow \Lambda\nu$ | $0.976$ | $0.957$ | $0.987$  | $1.059$  |

### TABLE III. $SU(3)$ breaking for $g_1$. The values correspond to the ratio $g_1/g_1^{SU(3)}$. In parentheses, the breaking pattern of Model III including only center of mass corrections is given.

| Decay           | Model I | Model III |
|-----------------|---------|-----------|
| $\Lambda \rightarrow p\nu$ | $1.072$ | $1.050$ (0.9720) |
| $\Sigma^- \rightarrow n\nu$ | $1.056$ | $1.040$ (0.9628) |
| $\Xi^- \rightarrow \Lambda\nu$ | $1.072$ | $1.003$ (0.9287) |

### TABLE IV. Values of $V_{us}$ within the SB proposed by Model I. Both breaking patterns for $f_1$ and $g_1$ were used.

| Decay           | $V_{us}$          | $g_1/f_1$  | $\chi^2$  |
|-----------------|-------------------|------------|------------|
| $\Lambda \rightarrow p\nu$ | $0.2133 \pm 0.0020$ | $0.8019$   | $38.54$    |
| $\Sigma^- \rightarrow n\nu$ | $0.2318 \pm 0.0040$ | $-0.3529$  | $8.95$     |
| $\Xi^- \rightarrow \Lambda\nu$ | $0.2434 \pm 0.0068$ | $0.2221$   | $1.40$     |
TABLE V. Values of $V_{us}$ within the SB proposed by Model III. Both breaking patterns for $f_1$ and $g_1$ were used. In parentheses, below each entry, the corresponding values of $V_{us}$, $g_1/f_1$, and $\chi^2$ considering only center of mass corrections are given.

| Decay      | $V_{us}$         | $g_1/f_1$ | $\chi^2$ |
|------------|------------------|-----------|-----------|
| $\Lambda \to p(e\nu)$ | 0.2153 ± 0.0020  | 0.7767    | 25.43     |
|            | (0.2258 ± 0.0021)| (0.7190)  | (10.85)   |
| $\Sigma^- \to n(e\nu)$ | 0.2307 ± 0.0040  | −0.3433   | 7.92      |
|            | (0.2351 ± 0.0041)| (−0.3178) | (8.89)    |
| $\Xi^- \to \Lambda(e\nu)$ | 0.2429 ± 0.0068  | 0.2055    | 2.42      |
|            | (0.2449 ± 0.0069)| (0.1903)  | (3.62)    |

TABLE VI. Values of $V_{us}$ within the SB proposed by Model II, with $g_1$ as free parameter.

| Decay      | $V_{us}$         | $g_1$     | $g_1/f_1$ | $\chi^2$ |
|------------|------------------|-----------|-----------|-----------|
| $\Lambda \to p(e\nu)$ | 0.2372 ± 0.0037  | −0.8250   | 0.7142    | 10.79     |
| $\Sigma^- \to n(e\nu)$ | 0.2320 ± 0.0049  | 0.3312    | −0.3356   | 7.70      |
| $\Xi^- \to \Lambda(e\nu)$ | 0.2396 ± 0.0108  | 0.3264    | 0.2784    | $6 \times 10^{-3}$ |

TABLE VII. Values of $V_{us}$ within the SB proposed by Model IV, with $g_1$ as free parameter.

| Decay      | $V_{us}$         | $g_1$     | $g_1/f_1$ | $\chi^2$ |
|------------|------------------|-----------|-----------|-----------|
| $\Lambda \to p(e\nu)$ | 0.2183 ± 0.0034  | −0.8974   | 0.7155    | 10.77     |
| $\Sigma^- \to n(e\nu)$ | 0.2082 ± 0.0044  | 0.3694    | −0.3358   | 6.73      |
| $\Xi^- \to \Lambda(e\nu)$ | 0.2165 ± 0.0098  | 0.3611    | 0.2784    | $6 \times 10^{-3}$ |

TABLE VIII. Values of $V_{us}$ within the SB proposed by Model I, with $g_1$ as free parameter.

| Decay      | $V_{us}$         | $g_1$     | $g_1/f_1$ | $\chi^2$ |
|------------|------------------|-----------|-----------|-----------|
| $\Lambda \to p(e\nu)$ | 0.2291 ± 0.0036  | −0.8545   | 0.7148    | 10.78     |
| $\Sigma^- \to n(e\nu)$ | 0.2349 ± 0.0049  | 0.3271    | −0.3355   | 7.82      |
| $\Xi^- \to \Lambda(e\nu)$ | 0.2349 ± 0.0106  | 0.3328    | 0.2784    | $6 \times 10^{-3}$ |

TABLE IX. Values of $V_{us}$ within the SB proposed by Model III, with $g_1$ as free parameter.

| Decay      | $V_{us}$         | $g_1$     | $g_1/f_1$ | $\chi^2$ |
|------------|------------------|-----------|-----------|-----------|
| $\Lambda \to p(e\nu)$ | 0.2265 ± 0.0035  | −0.8643   | 0.7149    | 10.78     |
| $\Sigma^- \to n(e\nu)$ | 0.2320 ± 0.0049  | 0.3312    | −0.3356   | 7.70      |
| $\Xi^- \to \Lambda(e\nu)$ | 0.2323 ± 0.0105  | 0.3366    | 0.2784    | $6 \times 10^{-3}$ |
### TABLE X. Values of $V_{us}$ within the SB proposed by Model I. $f_1$ and $g_1$ are fixed. $g_2$ are non-zero.

| Decay     | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\Lambda \rightarrow pe\nu$ | 0.2163 ± 0.0020 | 19.7 | 0.2148 ± 0.0020 | 27.7 | 0.2118 ± 0.0019 | 52.1 | 0.2103 ± 0.0019 | 68.1 |
| $\Sigma^- \rightarrow ne\nu$ | 0.2266 ± 0.0039 | 19.9 | 0.2292 ± 0.0040 | 12.2 | 0.2343 ± 0.0040 | 9.9 | 0.2368 ± 0.0041 | 15.0 |
| $\Xi^- \rightarrow \Lambda e\nu$ | 0.2409 ± 0.0068 | 0.8 | 0.2421 ± 0.0068 | 1.0 | 0.2445 ± 0.0069 | 1.8 | 0.2457 ± 0.0069 | 2.3 |

### TABLE XI. Values of $V_{us}$ within the SB proposed by Model III. $f_1$ and $g_1$ are fixed. $g_2$ are non-zero. In parentheses, below each entry, the corresponding values of $V_{us}$ and $\chi^2$ considering only center of mass corrections are given.

| Decay     | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\Lambda \rightarrow pe\nu$ | (0.2290 ± 0.0021) | 17.0 | (0.2274 ± 0.0021) | 12.2 | (0.2241 ± 0.0021) | 12.8 | (0.2225 ± 0.0020) | 17.9 |
| $\Sigma^- \rightarrow ne\nu$ | 0.2256 ± 0.0039 | 15.2 | 0.2282 ± 0.0039 | 9.5 | 0.2331 ± 0.0040 | 10.5 | 0.2355 ± 0.0041 | 16.9 |
| $\Xi^- \rightarrow \Lambda e\nu$ | (0.2301 ± 0.0040) | 7.2 | (0.2327 ± 0.0040) | 6.0 | (0.2375 ± 0.0041) | 15.6 | (0.2398 ± 0.0041) | 25.9 |
|         | (0.2406 ± 0.0068) | 1.5 | (0.2418 ± 0.0068) | 1.9 | (0.2440 ± 0.0069) | 3.0 | (0.2450 ± 0.0069) | 3.6 |

### TABLE XII. Values of $V_{us}$ within the SB proposed by Model I. The $g_1$ are free and $g_2$ are non-zero. In parentheses, below the entries for $V_{us}$, the corresponding $g_1$ are also given.

| Decay     | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\Lambda \rightarrow pe\nu$ | 0.2248 ± 0.0036 | 11.6 | 0.2270 ± 0.0036 | 11.2 | 0.2312 ± 0.0036 | 10.4 | 0.2333 ± 0.0036 | 10.0 |
| $\Sigma^- \rightarrow ne\nu$ | (0.2377 ± 0.0049) | 4.4 | (0.2364 ± 0.0049) | 6.0 | (0.2333 ± 0.0050) | 9.8 | (0.2316 ± 0.0050) | 12.0 |
| $\Xi^- \rightarrow \Lambda e\nu$ | (0.2349 ± 0.0104) | 0.0 | (0.2349 ± 0.0105) | 0.0 | (0.2349 ± 0.0108) | 0.0 | (0.2349 ± 0.0109) | 0.0 |

### TABLE XIII. Values of $V_{us}$ within the SB proposed by Model III. The $g_1$ are free and the $g_2$ are non-zero. In parentheses, below the entries for $V_{us}$, the corresponding $g_1$ are also given.

| Decay     | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\Lambda \rightarrow pe\nu$ | 0.2233 ± 0.0035 | 11.6 | 0.2244 ± 0.0036 | 11.2 | 0.2286 ± 0.0035 | 10.4 | 0.2307 ± 0.0035 | 10.0 |
| $\Sigma^- \rightarrow ne\nu$ | (0.2348 ± 0.0048) | 4.4 | (0.2335 ± 0.0049) | 5.9 | (0.2305 ± 0.0049) | 9.7 | (0.2288 ± 0.0049) | 11.8 |
| $\Xi^- \rightarrow \Lambda e\nu$ | (0.2323 ± 0.0103) | 0.0 | (0.2323 ± 0.0104) | 0.0 | (0.2322 ± 0.0106) | 0.0 | (0.2322 ± 0.0108) | 0.0 |
TABLE XIV. Values of $V_{us}$ within the SB proposed by Model II. The $g_1$ are free and the $g_2$ are non-zero. In parentheses, below the entries for $V_{us}$, the corresponding $g_1$ are also given.

| Decay      | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|------------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|
| $\Lambda \to p\nu\bar{\nu}$ | $0.2325 \pm 0.0037$ | 11.6     | $0.2349 \pm 0.0037$ | 11.2     | $0.2394 \pm 0.0037$ | 10.4     | $0.2417 \pm 0.0037$ | 9.9     |
| $\Sigma^- \to n\nu\bar{\nu}$ | $0.2348 \pm 0.0048$ | 4.4      | $0.2335 \pm 0.0049$ | 5.9      | $0.2305 \pm 0.0049$ | 9.7      | $0.2288 \pm 0.0049$ | 11.8    |
| $\Xi^- \to \Lambda\nu\bar{\nu}$ | $0.2396 \pm 0.0106$ | 0.0      | $0.2396 \pm 0.0107$ | 0.0      | $0.2395 \pm 0.0110$ | 0.0      | $0.2395 \pm 0.0111$ | 0.0     |

TABLE XV. Values of $V_{us}$ within the SB proposed by Model IV. The $g_1$ are free and the $g_2$ are non-zero. In parentheses, below the entries for $V_{us}$, the corresponding $g_1$ is also given.

| Decay      | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ | $V_{us}$  | $\chi^2$ |
|------------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|
| $\Lambda \to p\nu\bar{\nu}$ | $0.2144 \pm 0.0034$ | 11.5     | $0.2164 \pm 0.0034$ | 11.2     | $0.2200 \pm 0.0037$ | 10.4     | $0.2222 \pm 0.0034$ | 10.0    |
| $\Sigma^- \to n\nu\bar{\nu}$ | $0.2104 \pm 0.0043$ | 3.9      | $0.2093 \pm 0.0044$ | 5.2      | $0.2070 \pm 0.0044$ | 8.4      | $0.2056 \pm 0.0044$ | 10.2    |
| $\Xi^- \to \Lambda\nu\bar{\nu}$ | $0.2165 \pm 0.0096$ | 0.0      | $0.2165 \pm 0.0097$ | 0.0      | $0.2165 \pm 0.0099$ | 0.0      | $0.2164 \pm 0.0100$ | 0.0     |

TABLE XVI. Values of $V_{us}$ obtained within different $SU(3)$ SB models with changes in $g_2$. The rates and angular coefficients were used.

| $\Delta g_2$ | Model I       | Model II      | Model III     | Model IV      |
|--------------|---------------|---------------|---------------|---------------|
| $= 0$        | $0.2314 \pm 0.0028$ | $0.2356 \pm 0.0028$ | $0.2286 \pm 0.0027$ | $0.2147 \pm 0.0026$ |
| $\neq 0$     | $0.2348 \pm 0.0028$ | $0.2392 \pm 0.0028$ | $0.2321 \pm 0.0027$ | $0.2176 \pm 0.0026$ |