Bayesian inference of the properties of compact binary sources detected by gravitational wave detectors is a computationally challenging task. For the twin advanced LIGO detectors operating at design sensitivity, it is estimated to take several weeks to months of wall clock time to reconstruct a single binary neutron star source using current approaches. In this context, we present a new, computationally efficient way of rapidly reconstructing the source properties using a combination of numerical linear algebra and meshfree interpolation techniques. For a canonical binary neutron star system, we show that the method proposed in this Letter is \( \sim 4000 \) times faster than traditional algorithms, at a negligible loss of accuracy of \( \mathcal{O}(10^{-5}) \) across the sample space. This implies that the properties of such sources can be accurately measured within a few minutes of their detection in upcoming science runs, which will have significant ramifications for their prompt electromagnetic follow-up. The blueprint of this idea can be applied to Bayesian inference in other domains.

**Introduction.** The detection of gravitational waves (GW) from the GW170817 \([1]\) binary neutron star (BNS) system, followed by the prompt multi-wavelength (gammarays to radio) observation of its electromagnetic (EM) counterpart, has led to several fundamental discoveries; and is hailed as a significant breakthrough in astronomy. These discoveries include the validation of long held hypotheses that BNS mergers are ideal sites for r-process nucleosynthesis and produce short gamma-ray bursts, the first GW-based constraints on the equation of state of nuclear matter in such stars and the measurement of Hubble constant independent of the cosmic distance ladder. The current generation of detectors were almost insensitive to the post-merger signals \([2]\) of GW170817 giving rise to several open questions on the nature of the merger remnant. One hopes that these can be resolved in future observations when the advanced detectors reach design sensitivity.

The merger of a neutron star and a stellar mass black hole (NSBH systems) are also thought to be possible progenitors of gamma-ray bursts \([3]\) and are high on the priority list for EM followups. The inevitable improvement of the detectors’ sensitivity in future observation runs is likely to have a two-fold impact on the prospects of multi-messenger observations: firstly, the increased bandwidth of the detectors (especially at low frequencies) will result in a tremendous increase in the computational cost of Bayesian inference of source parameters, including sky localization essential for prompt observation of EM counterparts. For BNS sources, it is estimated that generating the posterior samples may take several weeks to months at advanced-LIGO (aLIGO) \([4]\) sensitivities assuming a seismic cut-off at 10Hz. Although the BAYESTAR \([5]\) algorithm can be used to produce rapid sky maps, it has been recently shown \([6]\) that coherent parameter estimation (PE) can localize the sources better by being able to reduce the uncertainty in sky localization by an average of 14 deg\(^2\), underlining the importance of developing fast PE algorithms. Second, the reach of the terrestrial network of GW detectors will extend out to several Gpc to the effect that one would have far too many detections of BNS/NSBH signals to contend with whilst generating prompt sky-location maps. While there were just three such signals observed in eleven months of O3 science run, it is estimated \([7]\) that \( \sim 154 \) of them may be detected in the upcoming O4 run (a fifty fold increase!). An additional order of magnitude increase in the detections can be expected once the detectors achieve their design sensitivity. As EM observational resources are limited, one may have to prioritize the GW sources for EM followup based on the prospects of pushing the envelope for astrophysics from such an exercise. In this regard, Margalit & Metzger \([8]\) have introduced the idea of a "Multi-Messenger Matrix", which maps GW events to likely EM signatures based on an estimate of the chirp masses. They advocate in favor of releasing the estimated chirp mass values first, which will help EM observatories to use resources optimally. Thus, a rapid estimation of the mass and spin components using fast PE methods can be very useful in prioritizing the EM followup of GW sources.

In this Letter, we propose a fast and efficient method for evaluating the likelihood function: a key ingredient for reconstructing source parameters using Bayesian inference. We have combined the meshfree interpolation methods with dimension reduction techniques to directly interpolate the likelihood function over the parameter space, bypassing the generation of templates altogether. Our scheme can calculate the log-likelihood function thousands of times faster than the traditional approach used in PyCBC inference \([9]\), and produce statistically indistinguishable posteriors over source parameters. This
could enable the multi-wavelength follow-up detection of EM counterparts of such systems detected in future science runs.

Since both the GstLAL [10] search framework and the meshfree method use the idea of dimension reduction using SVD, it may be prudent to incorporate this method with the low-latency GstLAL search pipeline for rapid, automated follow-ups of the detected events.

In the next section, we outline the Bayesian inference method and briefly touch upon previous algorithms for fast computation of the likelihood function. This is followed by a section where we introduce the meshfree method for accurately estimating the likelihood function, followed by a demonstration for a canonical BNS system. We then present the projected speed-up factors expected in future observation runs, for different compact binary systems. Finally, we highlight the advantages of the meshfree scheme and indicate directions of future investigations.

**Bayesian inference.** Given data \( \mathbf{d} = \mathbf{h}(\tilde{\mathbf{\Lambda}}_{\text{true}}) + \mathbf{n} \) recorded at a detector containing an astrophysical GW signal \( \mathbf{h}(\tilde{\mathbf{\Lambda}}_{\text{true}}) \) embedded in additive Gaussian noise \( \mathbf{n} \), one is interested in solving the inverse problem to estimate the source parameters. Bayesian inference is a stochastic inversion method where the posterior probability density \( p(\tilde{\mathbf{\Lambda}} \mid \mathbf{d}) \) over the source parameters is related to the likelihood function \( \mathcal{L}(\mathbf{d} \mid \tilde{\mathbf{\Lambda}}) \) of observing the data through the Bayes’ theorem:

\[
p(\tilde{\mathbf{\Lambda}} \mid \mathbf{d}) = \frac{\mathcal{L}(\mathbf{d} \mid \tilde{\mathbf{\Lambda}}) p(\tilde{\mathbf{\Lambda}})}{p(\mathbf{d})},
\]

where \( p(\tilde{\mathbf{\Lambda}}) \) is the prior distribution over the model parameters \( \tilde{\mathbf{\Lambda}} = \{ \alpha, \delta, \psi, \iota, D, \tilde{\Lambda} \} \). In our notation, \( \tilde{\mathbf{\Lambda}} \) denotes the intrinsic parameters such as component masses and spins whereas the extrinsic parameters of the source namely its right ascension, declination, polarization, inclination angle and distance from the detector are denoted by \( \alpha, \delta, \psi, \iota \) and \( D \) respectively. We are also interested in estimating the fiducial time of coalescence \( t_c \) at which the two masses merge to form the remnant. \( t_c \) is also an extrinsic parameter but treated in a special way in our analysis and will be mentioned explicitly wherever required.

The forward generative frequency-domain restricted waveform model for non-precessing compact binaries can be expressed as \( \mathbf{h}(\tilde{\mathbf{\Lambda}}) = \mathcal{A} h_+(f; \tilde{\mathbf{\Lambda}}) \), where the complex amplitude \( \mathcal{A} \) depends only on the extrinsic parameters and \( h_+(f; \tilde{\mathbf{\Lambda}}) \) is the ‘+’ polarization of the signal that depends only on the intrinsic parameters. Using this model, the posterior \( p(\tilde{\mathbf{\Lambda}} \mid \mathbf{d}) \) can, in principle, be directly evaluated at every point in \( \tilde{\mathbf{\Lambda}} \) using Eq. (1). However, in view of the high-dimensionality of \( \tilde{\mathbf{\Lambda}} \), it is more efficient to sample the posterior using stochastic sampling algorithms such as Nested Sampling [11], or Markov chain Monte Carlo (MCMC) [12].

From Eq. (1), it is evident that for a quick estimation of the posterior distribution, it is imperative to rapidly evaluate the likelihood function. We work with the phase-marginalised log-likelihood function:

\[
\ln \mathcal{L}(\tilde{\mathbf{\Lambda}}, t_c) = \ln I_0(|\langle \mathbf{d} \mid \mathbf{h}(\tilde{\mathbf{\Lambda}}, t_c) \rangle|) - \frac{1}{2} \| \mathbf{h}(\tilde{\mathbf{\Lambda}}) \|^2
\]

where \( I_0(\cdot) \) is the 0-th order modified Bessel function of the first kind, \( \| \mathbf{h}(\tilde{\mathbf{\Lambda}}) \|^2 = \mathcal{A}^2 \| h_+(\tilde{\mathbf{\Lambda}}) \|^2 \) is the squared signal norm and \( |\langle \mathbf{d} \mid \mathbf{h}(\tilde{\mathbf{\Lambda}}, t_c) \rangle| = |\mathcal{A}| z(\tilde{\mathbf{\Lambda}}, t_c) \) where \( z(\tilde{\mathbf{\Lambda}}, t_c) \) is the frequency-domain overlap-integral:

\[
z(\tilde{\mathbf{\Lambda}}, t_c) = 4 \Delta f \sum_{k=0}^{N_s/2} \frac{d^*(f_k) h_+(f_k, \tilde{\mathbf{\Lambda}})}{S_h(f_k)} e^{-2\pi if_k t_c}.
\]

In Eq. (3), \( S_h(f_k) \) is the detector’s one-sided noise power spectral density (PSD); \( d(f_k) \) and \( h_+(f_k, \tilde{\mathbf{\Lambda}}) \) respectively denote the data and template waveform in the Fourier domain, sampled at positive frequencies \( \{ f_k \}_{k=0}^{N_s/2} \). Note that Eq. (2) consists of two parts - one that depends explicitly on the overlap integral and the other, that depends only on the template norm.

The complexity of evaluating the overlap integral scales directly with the number of data samples \( N_s \), which in turn scales with the seismic cut-off frequency roughly as \( N_s \sim f_{\text{low}}^{-8/3} \). This implies that as we progress from observational run O4 \( (f_{\text{low}} = 20 \text{ Hz}) \) to run O5 at design sensitivity \( (f_{\text{low}} = 10 \text{ Hz}) \), evaluating the posterior probability (at one parameter set) is likely to take at least \( \times 6.3 \) longer. In addition, there will be further costs incurred in constructing longer templates at proposal points in \( \tilde{\mathbf{\Lambda}} \). It is easy to see that this undermines the prospects of rapid PE in future science runs: PE runs for BNS systems that are likely to take a few days to a week in O4 will clearly take weeks to months given the same computational resources.

Several attempts have been made in the past to compute the likelihood quickly and accurately by speeding up the overlap integral. These include reduced-order models (ROMs) [13, 14], machine-learning aided ROMs [15], Gaussian process regression based interpolation [16] and relative binning [17] algorithms. Finstad & Brown [6] have further extended the relative binning method for a fully coherent detector network. Our approach takes inspiration from the grid-based likelihood interpolation method [18] wherein relevant functions were expanded in the basis of orthonormal Chebyshev polynomials. The problem with such grid-based techniques is that the number of nodes required for accurate interpolation scales exponentially with the dimension of the intrinsic parameter space - which impacts the speed of likelihood interpolation and eventually slows down the PE run. We use non-orthonormal radial basis functions (RBF)
(Gaussian kernels) centred at interpolation nodes that can be randomly scattered over the volume of the intrinsic parameter space. In this manner, we have an effective control on their number in higher dimensional parameter space.

**Meshfree likelihood interpolation.** The computational cost of Bayesian inference comprises of two parts: the first is incurred in waveform generation followed by likelihood evaluation at a point proposed by the sampler. Typically, a sampler proposes a large number ($\sim 10^6 - 10^7$) of points to adequately capture the posterior distribution - which makes this part computationally expensive. The other part of the total computational cost can be attributed to the overheads of the sampling method itself. Since the latter cost depends on the efficiency of the sampling algorithm using (and its software implementation) and significantly less in comparison to the overall cost of PE, we ignore it in our discussions.

From Eq. (2) and (3), it is evident that we need to evaluate two pieces: $z(\tilde{\lambda}, t_c)$ and $||h(\tilde{\lambda})||^2$ to calculate the log-likelihood function at a given ‘query’ point $\tilde{\lambda}$ and a particular given value of $t_c$. We treat them in slightly different ways as explained below. These are combined with the amplitude $A$ calculated separately at the query point in extrinsic parameter space to give the log-likelihood value.

The squared norm of a template $||h_+(\tilde{\lambda})||^2$ is a smoothly varying scalar field over $\tilde{\lambda}$. We can interpolate its value at a query point $\tilde{\lambda}$ by first evaluating the values explicitly at a number of randomly distributed interpolation nodes $\tilde{\lambda}^\alpha$, and then expressing $||h_+(\tilde{\lambda})||^2$ at an arbitrary point as a linear combination of Gaussian RBF kernels centred at these nodes. The unknown coefficients of this linear combination can be uniquely found by enforcing the interpolation criteria as explained in the next section.

On the other hand, as the overlap integral has to be evaluated at an arbitrary point $(\tilde{\lambda}, t_c)$, it will turn out to be more convenient to interpolate it as a vector. In this case, a set of overlap-integral vectors $\tilde{z}_\alpha$ are first constructed at the randomly strown interpolation nodes $\tilde{\lambda}^\alpha$ each of which are sampled on a uniformly spaced grid over $t_c$. The vector $\tilde{z}_\alpha$ has elements $z_\alpha[k] \equiv z(\tilde{\lambda}^\alpha, k \Delta t)$, where $t_c$ is the sampling interval, $k$ is an integer $\in \text{int}((t_c^* \pm \tau/\Delta t)), t_c^*$ represents the ‘reference’ coalescence time as triggered by the search pipeline and $\tau$ is a sufficiently long interval of time from where samples of $t_c$ are drawn by the sampling algorithm. Since the bulk of the support for the posterior distribution comes from near the peak of these time series, we choose its samples that are centred around the triggered value. From Eq. (3) it is clear that $\tilde{z}_\alpha$’s can be constructed efficiently using FFT correlations. Once the set of vectors $\{\tilde{z}_\alpha\}$ is available, they can be projected over a suitable set of basis vectors $\{\tilde{u}_\mu\}$ with linear coefficients $\{C_\mu(\tilde{\lambda}^\alpha)\}$. It is easy to see that each of the coefficients are smoothly varying scalar fields over $\tilde{\lambda}$ whose values are known at the interpolation nodes. Just like the case for $||h_+(\tilde{\lambda})||^2$ above, we can once again use meshfree methods to interpolate the values of the coefficients at any arbitrary query point. This allows us to combine the interpolated coefficients with the basis vectors to obtain an approximate $\tilde{z}_\alpha$ at the query point.

The meshfree scheme can be divided into two stages: (i) a preparatory, start-up stage where we train the interpolants to learn the likelihood pattern from a given set of input values at the interpolation nodes and (ii) an online stage where the likelihood values are predicted on the fly at query points.

**Start-up stage.**

1. **Identify the sample space:** We assume that the Bayesian inference is seeded by the most significant event (compared to the background) generated by the GW search pipelines [10, 19]. The sample space, from which the sampling algorithm draws new proposals, is taken to be a reasonably small hyper-rectangle in the $\tilde{\lambda}$ space, around the detection template. The interpolation nodes are uniformly distributed over the sample space, taken to be sufficiently large so as to contain the bulk of the posterior distribution. The Fisher matrix [20] could be also used as a guide to identify the boundary of this region. More sophisticated distributions of the nodes [21] can be explored to further optimize the algorithm.

2. **SVD basis:** We are interested in finding a suitable set of basis vectors that span the space $\{\tilde{z}_\alpha\}$ calculated at the $n$-interpolation nodes $\{\tilde{\lambda}^\alpha\}$ distributed over the sample space. This is conveniently performed by stacking these vectors and performing a singular-value decomposition (SVD) of the resultant matrix:

$$\tilde{z}_\alpha = \sum_{\mu=1}^{n} C_\mu^\alpha \tilde{u}_\mu,$$  \hspace{1cm} (4)

where $C_\mu^\alpha$ are the coefficients for the set of orthonormal basis vectors $\tilde{u}_\mu$ in decreasing order of their relative importance as determined from the spectrum of singular values. A strong correlation between the $\tilde{z}_\alpha$’s implies that only the top-$\ell$ basis vectors ($\ell < n$) are able to reconstruct the overlap-integral vectors at the nodes with negligible error. This is also the reason for surmising that the vector $\tilde{z}_\alpha$ at a random query point in the sample space can also be spanned by the same set of basis vectors. For a fixed index $\mu$, the coefficients $C_\mu^\alpha$ represents a surface whose values are known only at the input nodes. Along with Eq. (4), this implies that the interpolation of the inner-product vector at an arbitrary query point essentially boils down to interpolating the value of the coefficients $C_\mu^\alpha \equiv C_\mu(\tilde{\lambda})$. 


3. Creating meshfree interpolants: In this step we create and train ‘meshfree’ interpolants for each of the coefficient surfaces (that appear in Eq. (4)) independently. The interpolant for the coefficient corresponding to the $\mu$th basis vector can be expressed [22] as a linear combination of RBF kernels centred on the scattered, distinct nodes $\vec{x}^\alpha$, augmented by monomials ranging up to a specific order:

$$C_\mu^q = \sum_{\alpha=1}^{n} r_\alpha \phi(\|\vec{x}^\alpha - \vec{x}_q\|_2) + \sum_{j=1}^{M} b_j f_j(\vec{x}_q), \quad (5)$$

where $\phi$ is the Gaussian kernel centred on $\vec{x}^\alpha \in \mathbb{R}^d$, and $\{f_j\}$’s are monomials that span the space of polynomials of some preset target degree $\nu$ in $d$ variables. Also, $r = [r_1, r_2, \ldots, r_n]^T$ and $b = [b_1, b_2, \ldots, b_M]^T$ are the set of $(n + M)$ coefficients that needs to be uniquely determined to train the interpolant.

Since $C_\mu^q$ are known at the interpolation nodes, it allows us to enforce $n$ interpolation conditions. $M$ additional conditions $\sum_{k=1}^{n} r_k f_j(\vec{x}^\alpha) = 0$, $j = 1, \ldots, M$ are added to ensure a unique solution. Together, these lead to a system of equations:

$$\begin{bmatrix} K & F \\ F^T & O \end{bmatrix} \begin{bmatrix} r \\ b \end{bmatrix} = \begin{bmatrix} C_\mu^q \\ 0 \end{bmatrix} \quad (6)$$

where the matrices $K$ and $F$ have components $K_{ij} = \phi(\|\vec{x}^i - \vec{x}^j\|_2)$ and $F_{ij} = f_j(\vec{x}^i)$ respectively; $O_{M \times M}$ is a zero-matrix and $0_{M \times 1}$ is a zero-vector. Eq. (6) can be solved uniquely for the unknown coefficients $r$ and $b$, thus completely determining the meshfree interpolant in Eq. (5). The solution for $r$ and $b$ can be shown to be unique if $F$ has full column rank.

A set of $n \geq \binom{\nu+d}{\nu}$ interpolation nodes are required to be uniformly distributed over the $d$-dimensional intrinsic parameter space. While Euclidean distances between pairs of points seem to works well for gravitational-wave PE problems, one could also use parameter-space metric based distances.

A similar procedure is followed to create a separate meshfree interpolant for $\|h_+(\vec{x}^\nu)\|^2$.

Note that by this construction we have eliminated the need for (a) generating the forward signal model and (b) explicitly calculating the overlap integral at every query point proposed by the sampling algorithm.

**Online stage.** In this stage, the $(\ell + 1)$ interpolants (prepared in the offline stage earlier) are evaluated on the fly at arbitrary query points $(\vec{x}_q, t_c)$ proposed by the sampling algorithm.

The interpolant for the square of the template norm can be directly evaluated to get the interpolated value $\|h_+(\vec{x}_q)\|^2$. Similarly, the $\ell$ interpolants for SVD coefficients (Eq. (5)) are evaluated to get a set of interpolated coefficients, which are then combined with the corresponding top-$\ell$ basis vectors $\vec{u}_\mu$ (see Eq. (4)) to obtain the interpolated overlap-integral $\vec{z}_q$.

By construction, $\vec{z}_q$ is uniformly sampled over the interval $[t_c^\pm \tau]$. As such, it is possible that the query $t_c$ does not coincide with the discrete time-samples of $\vec{z}_q$. In such a case, we use a one-dimensional cubic-spline interpolation to evaluate $z(\vec{x}_q, t_c)$ using a few ‘nearby’ grid samples of $\vec{z}_q$ as input. This implies that $\vec{z}_q$ has to be reconstituted only at a few $(\sim 10)$ consecutive sample points which considerably accelerates the matrix-vector multiplication in Eq. (4).

Combining the interpolated values $z(\vec{x}_q, t_c)$ and $\|h_+(\vec{x}_q)\|^2$ with the extrinsic-parameter dependent complex amplitude $A$ (see Eq. (2)), we finally obtain the log-likelihood ratio $\ln Z$ at the arbitrary point $(\vec{x}_q, t_c)$.

**Numerical experiments.** The method presented here is applicable for Bayesian inference of any transient compact binary source regardless of its nature. However, the maximum benefit of this construction can be realized for BNS/NSBH signals that last for a long duration in the detectors’ sensitive band. For such sources, the meshfree method can effectively offset the steep computational cost of evaluating the forward model and computing the overlap integral at a large number of query points.

We now demonstrate the accuracy and speed of the meshfree method in reconstructing the parameters of a canonical BNS system in a single-detector. In this example, we vary four intrinsic parameters (component masses and aligned-spin magnitudes) and two extrinsic parameters (luminosity distance and coalescence time). Other parameters namely, sky location, source inclination, and polarization are kept constant.

A synthetic 360 sec long data $d$, sampled at 4096 Hz was prepared by injecting a simulated GW signal $h$ from a canonical BNS system with component masses $m_{1,2} = 1.4M_\odot$ and mass-weighted effective dimensionless spin $\chi_{\text{eff}} = 0.05$ in colored Gaussian noise using the noise power spectral density model [23] for aLIGO detectors. The distance to the source was adjusted for a moderate matched-filtering SNR of 10. The injected BNS signal was generated using the IPOLPhenomD [24] signal model. The seismic cut-off frequency was chosen to be 20 Hz to mimic data from the upcoming O4 science run.

We performed Bayesian inference on this simulated data using both (a) direct likelihood calculation used in PyCBC inference and (b) and by using the proposed meshfree likelihood interpolation scheme outlined in earlier sections. We used publicly available software [25] for radial basis functions and the Dynasty [26] nested-sampling package for carrying out the Bayesian inference analysis. For this exercise we used $n = 800$ input nodes distributed uniformly within a 4D hyper-rectangle,
FIG. 1. The figure shows the marginalised PDF for the chirp mass $\mathcal{M}$, symmetric mass ratio $\eta$, and coalescence time $\Delta t_c$ parameters of a simulated BNS event at a seismic cutoff of 20 Hz. The injected parameters are shown as red lines. The 50% and 90% contours for meshfree method (dashed cyan trace) and PyCBC (solid white trace) are also shown. The plot-overlaid marginalised PDF obtained from the proposed meshfree method (dashed orange trace) and standard PyCBC inference (black line) are virtually indistinguishable. The meshfree method was about 359 times faster (wall-clock time) with 676 times faster evaluations of the log-likelihood function.

keeping the injection parameters at its centre. We used the top $\ell = 120$ basis vectors and a polynomial order $\nu = 6$ with corresponding nominal median relative error $\sim 10^{-5}$ across the sample space in approximating the log-likelihood function. The meshfree reconstruction completed in 5.3 min in comparison to 31.7 h taken by the direct calculation. The likelihood evaluations were 676 times faster using the meshfree method.

Some of the estimated parameters have been compared in Table I which show identical values obtained by both the methods. The marginalised PDF over three parameters $\mathcal{M}$, $\eta$ and $\Delta t_c$ are shown in the corner plot Fig. 1. The figure also contains cumulative density (CDF) profiles of these distributions obtained from both PyCBC inference and the meshfree method plotted together.

While both the PDF and CDF profiles look virtually indistinguishable, we also calculate statistical measures of similarity [27] using the PDFs obtained from the two methods for further validation. The Kolmogorov-Smirnov statistic (0.0130) and the Bhattacharyya distance (0.0006) between the chirp-mass PDF profiles support the fact that the distributions are nearly identical. We get similar numbers for posterior distributions of other parameters.

| Seismic cut-off | $M/M_\odot$ | $t_{\text{tot}} /\text{ms}$ | $t_{\text{pycbc}} /\text{ms}$ | speed-up |
|----------------|-----------|-----------------|-----------------|--------|
| $f_{\text{low}} = 10$ Hz | 2.8 | 0.90 | 3604.76 | 4005.2 |
| | 4.0 | 1.06 | 1405.84 | 1326.2 |
| | 20.0 | 0.88 | 29.27 | 33.2 |
| $f_{\text{low}} = 20$ Hz | 2.8 | 0.67 | 452.93 | 676.0 |
| | 4.0 | 0.81 | 207.35 | 256.0 |
| | 20.0 | 0.63 | 3.32 | 5.3 |

TABLE II. Median time (in ms) taken for a single evaluation of the log-likelihood function using standard PyCBC method and the meshfree approach at a nominal relative error of $\sim 10^{-5}$. The low-mass systems with large number of in-band cycles would benefit most from the meshfree method.

This numerical example shows that the meshfree method can generate a statistically indistinguishable replica of posterior distributions in a GW Bayesian inference problem at a small fraction of the computational cost.

Speed-up analysis. To quantify the computational advantage in using the meshfree scheme over the traditional method, we calculated the ratio of (average) time taken to compute the log-likelihood by these techniques at a fixed accuracy. Several simulated data sets were used in this study, which were generated by injecting signals having different parameters in colored Gaussian noise using the aLIGO noise model at a matched-filtering SNR of 10. Seismic cutoff frequencies at 20 Hz (10 Hz) were considered to mimic upcoming O4 (O5) science runs.

There is an obvious trade-off between accuracy and speed-up of the meshfree method which are determined by the choice of $(n, \ell, \nu)$ parameters. Larger values can lead to more accurate likelihood estimates albeit at a higher computational cost and vice-versa. We used a heuristic combination of $(n, \ell, \nu)$ to guarantee median (relative) errors $\lesssim 9 \times 10^{-5}$ in the estimated log-likelihood values across the entire sample space for all the systems. Ideally, one should take the combination of these parameters based on a preset error tolerance. However, it is hard to establish an explicit relation between these parameters and the relative error in interpolating the likelihood.

TABLE I. Reconstruction of a canonical BNS event.

| $M/M_\odot$ | $\eta$ | $\chi_{\text{eff}}$ | SNR |
|---------|------|--------|-----|
| Injection | 1.2187 | 0.25 | 0.05 | 10.00 |
| Standard | 1.2187 | 1.2185 | 0.2487 | 0.2498 | 0.2456 | 0.050 | 0.051 | 0.049 | 9.67 |
| Meshfree | 1.2187 | 1.2185 | 0.2487 | 0.2498 | 0.2456 | 0.050 | 0.051 | 0.049 | 9.67 |
The log-likelihood function was evaluated and timed for $10^5$ points uniformly distributed over the sample space for the 10 Hz cut-off. For the 20 Hz case, $5 \times 10^4$ points were used for comparing the accuracy across the two methods and estimating the speed-up.

Table II summarises the speed-ups corresponding to two different seismic cutoff frequencies 10, 20 Hz for three different compact binary systems having equal component masses. This is further elucidated in Fig. 2 where the speed-up comparison is drawn between a larger number of equal-mass binary systems with component masses between $1.4 - 100M_\odot$ and (dimensionless) spin magnitude $0.05(0.2)$ for the BNS (BBH) systems. The data duration varies from $100 - 2200$ (1 - 270) sec for BNS (BBH) systems. From Table II and Fig. 2, it is clear that the likelihood computation could be sped-up $\sim 4000$ times faster for BNS systems at a nominal error of $\sim 10^{-5}$ using the meshfree method in aLIGO detectors. This would be found very useful in rapidly reconstructing the parameters of such sources in upcoming observation runs. It is also clear that the time for a single standard evaluation of the log-likelihood depends strongly on the chirp-time of the signals. On the contrary, it is relatively unaffected for the meshfree method at a fixed relative error. All the tests were performed on a single core AMD EPYC 7542 CPU@2.90GHz CPU.

**Conclusion and Outlook.** We have presented a new and computationally efficient method for accurately evaluating the log-likelihood function using meshfree approximation methods. This new method can be integrated into known sampling algorithms (e.g., MCMC, Nested Sampling) to accelerate the Bayesian inference of source parameters of coalescing compact binary sources, seeded by the most significant trigger from an upstream detection pipeline. This could be useful in triggering the prompt observation of the EM counterparts of BNS and NSBH systems in future. Furthermore, the meshfree scheme is easy to be integrated into the PyCBC [9] and Bilby [28] GW Bayesian inference pipelines. The parameters $(n, \ell, \nu)$ that affect the speed and accuracy of the meshfree approximation should be carefully chosen. Although simulated data from a single GW detector was used for demonstrations in this Letter, it is straight-forward to extend the idea over to a network of detectors.

We have demonstrated the accuracy of the proposed method by performing Bayesian inference of a canonical BNS system where the log-likelihood function was evaluated using meshfree approximation. The posterior distributions, when compared to those obtained using a traditional approach used in PyCBC inference, were found to be statistically identical. For BNS systems, the likelihood function can be calculated $\sim 4000$ times faster at any point proposed by the sampling algorithm.

It would be interesting to incorporate this method with the low-latency search pipeline for rapid, automated follow-ups of the detected events as both methods rely on SVD for dimension reduction. We need to explore a common set of pre-computed basis vectors that span the search templates and the space of overlap-integral vector to substantially speed-up the start-up stage of the meshfree method.

In the current implementation, we need to train the interpolants from scratch for each new event triggered by the search pipelines. However, this task is embarrassingly parallel and can benefit from multiple CPU cores which could expedite the preparatory stage.

Finally, the techniques of dimension reduction and meshfree approximation could be applied to situations where the likelihood function varies smoothly over the sample space. It is thus possible to adapt this idea to Bayesian inference in fields as diverse as cosmology, biochemical kinetic processes, and systems biology.

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