Joint Processing of DOA Estimation and Signal Separation for Planar Array Using Fast-PARAFAC Decomposition

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1. Introduction

Signal separation and direction of arrival (DOA) estimation are significant themes in signal processing and have been investigated in various engineering fields including wireless communication, navigation, radar, and sonar [1–5]. As fundamental issues for signal processing, they have sparked considerable attention of researchers for decades. These two problems involve multiple signals and sensors which receive a mixture of signals [6]. The goal of DOA estimation is to find the source signal location, while the signal separation is aimed at extracting desired source signals. Through the years, many classical methods have been developed to solve these problems. For DOA estimation, subspace-based methods like MUSIC and ESPRIT have been widely adopted [7, 8]. The conventional nonparametric Fourier-based methods have also been further developed [9], and the emerging sparse reconstruction-based methods like orthogonal matching pursuit (OMP) and sparse Bayesian inference (SBI) are introduced into DOA estimation [10, 11]. For signal separation, the researches focus primarily on blind source separation (BSS) methods, where independent component analysis (ICA) and Joint Approximative Diagonalization of Eigen matrix (JADE) method are the most famous among these methods [12, 13] and have been widely applied in the separation of speech and medical signals.

Compared with conventional DOA methods, BSS methods do not require much waveform prior information and are capable of identifying the transmission parameters based on the mixture signals, which has aroused an amount of attention of researchers. Many researchers study to apply blind separation algorithms into array signal model and have made lots of works. A combined complex blind source separation DOA estimation and signal recovery method was proposed for uniform linear array (ULA) in [14], which obtains better performance by exploiting BSS to estimate the array manifold. In [15], a blind DOA estimation method based on the JADE algorithm was proposed, which introduces fourth-order cumulant and has great performance in multi-path environment. In [16], the chaotic adaptive firework algorithm was applied for solving the problem of radar emitter mixed signal. In [17], a new EM-based method for broadband DOA estimation and BSS was proposed, which reduces the complexity of traditional methods. In [18], a method based
on eigenvalue decomposition for DOA estimation and blind separation of narrow-band independent signals was presented. The above studies were discussed for ULA geometry and did not involve more complex array structures like planar arrays which are more practical in actual applications.

In recent years, tensor technique has taken off in the field of data analysis and signal processing [19], in which trilinear decomposition or the parallel factor (PARAFAC) technique has been extensively investigated in radar and wireless communication fields especially [20–24]. PARAFAC is a common model for low-rank decomposition of a tensor, whose computation can be completed by alternating least squares (ALS). Many models in array signal processing can be represented as trilinear models, which enables us to utilize the trilinear alternating least square (TALS) algorithm to achieve parameter estimation and signal separation. The authors in [25, 26] studied multiparameter estimation in bistatic multiple-input multiple-output (MIMO) radar and proposed a joint direction of departure (DOD) and direction of arrival (DOA) estimation using the PARAFAC model. In [27], the authors proposed a novel 2D-DOA estimation for trilinear decomposition-based monostatic cross MIMO radar. In [28], a joint DOA and carrier frequency estimation of narrow-band sources was proposed using the unitary PARAFAC method. These methods based on TALS can separate source signals and obtain automatically paired parameters without spectral peak search, but have relatively high computational complexity. It can be seen that PARAFAC has a great potential in DOA estimation and signal separation, but its shortcoming is the standard PARAFAC method has slow speed on convergence.

Motivated by the works mentioned above, in this paper, under the basic framework of PARAFAC, we propose a method of joint two-dimensional DOA estimation and signal separation for planar arrays. We first model the output of a planar array as the PARAFAC model and then utilize the PARAFAC model until convergence. Finally, acquire the 2D-DOA separation, but its shortcoming is the standard PARAFAC method has slow speed on convergence.

The proposed method has better performance of DOA estimation than 2D-PM and 2D-ESPRIT and can accurately separate the source signals with lower complexity compared with the standard PARAFAC approach.

(4) The proposed method can obtain separated signals and corresponding DOA estimates without an additional pairing procedure.

The outline of this paper is given as follows. We discuss the data model for uniform planar array and introduce the PARAFAC model briefly in Section 2. In Section 3, the proposed algorithm is described in detail. In Section 4, the complexity analysis and advantages of the proposed method are provided. The results of numerical simulations are given in Section 5, and conclusions are drawn in Section 6.

1.1. Notation. Lower-case and upper-case boldface letters denote vectors and matrices. \( C \) denotes the sets of complex numbers. The superscripts \((\cdot)^T\), \((\cdot)^*\), and \((\cdot)^H\) represent the transpose, complex conjugate, and conjugate transpose of a vector or matrix, respectively. \( \text{diag}(\cdot) \) denotes a diagonal matrix that consists of the elements of the matrix. \( \odot \) denotes the Kronecker product. \( D_m(\cdot) \) denotes a diagonal matrix whose diagonal elements are defined with the \( m \)-th row of the matrix. angle(\( \cdot \)) denotes phase angle operator. \(|\cdot|\) and \( \|\cdot\|_F \) denote the \( \mathcal{E} \) and Frobenius norms. \((\cdot)^{-1}\) and \((\cdot)^*\) stand for the inverse and pseudo-inverse of a matrix.

2. Data Model
Consider a uniform rectangular array (URA) containing \( N \times M \) sensors as depicted in Figure 1, where \( N \) and \( M \) are the numbers of elements along the \( x \)-axis and \( y \)-axis. The interelement spacings along both the \( x \)-axis and \( y \)-axis of the array are taken as half the wavelength of the waves, \( d_1 = d_2 = \lambda/2 \).

Assume that \( K \) uncorrelated far-field signals individually impinge on the array from \( \left\{ (\theta, \phi) | k = 1, 2, \ldots, K \right\} \), where \( \theta_k \) and \( \phi_k \) are the corresponding elevation and azimuth angles of the \( k \)-th signal (\( K < N \times M \), \( \theta_k \in (0, 90^\circ) \), and \( \phi_k \in (0, 180^\circ) \)). The output of the rectangular array can be represented as follows [29]:

\[
\mathbf{X} = \mathbf{A} \mathbf{S} + \mathbf{N},
\]  

where \( \mathbf{X} \in \mathbb{C}^{N \times M} \) is the output data with noise; \( L \) denotes the number of snapshots; \( \mathbf{S} = [s_1, s_2, \ldots, s_K]^T \in \mathbb{C}^{K \times L} \) is the signal matrix of \( L \) snapshots; \( \mathbf{N} \in \mathbb{C}^{N \times M} \) is the additive white Gaussian noise matrix. The array manifold matrix \( \mathbf{A} \in \mathbb{C}^{N \times M} \) consists of the steering vectors and is given by [29]

\[
\mathbf{A} = [\mathbf{a}_1(u_1) \odot \mathbf{a}_x(u_1), \mathbf{a}_1(u_2) \odot \mathbf{a}_x(u_2), \ldots, \mathbf{a}_1(u_K) \odot \mathbf{a}_x(u_K)],
\]  

where \( u_k = \sin \theta_k \cos \phi_k \) and \( v_k = \sin \theta_k \sin \phi_k \); \( \mathbf{a}_x(u_k) \) and \( \mathbf{a}_y(v_k) \) are the steering vectors of the array, which can be represented as [30]
Based on the PARAFAC model, the noiseless received data for URA can be written as [22]

\[
X_m = A_x D_m(A_y) S. \tag{6}
\]

Due to the symmetry of the PARAFAC model, the other two slice matrices can be obtained.

\[
Y_n = S^T D_n(A_x) A_y^T, \tag{7}
\]

\[
Z_l = A_y D_l(S^T) A_x^T. \tag{8}
\]

Define \( X, Y, \) and \( Z \) as the results of the concatenation of matrices \( X_m, Y_n, \) and \( Z_l \), respectively, and then, the noise-free received signal matrices \( X, Y, \) and \( Z \) can be represented as follows:

\[
\begin{align*}
X &= \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_M
\end{bmatrix}, \\
Y &= \begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_N
\end{bmatrix}, \\
Z &= \begin{bmatrix}
Z_1 \\
Z_2 \\
\vdots \\
Z_L
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
X &= \begin{bmatrix}
A_x D_1(A_y) \\
A_x D_2(A_y) \\
\vdots \\
A_x D_M(A_y)
\end{bmatrix} S, \\
Y &= \begin{bmatrix}
S^T D_1(A_x) \\
S^T D_2(A_x) \\
\vdots \\
S^T D_N(A_x)
\end{bmatrix} A_y^T, \\
Z &= \begin{bmatrix}
A_y D_1(S^T) \\
A_y D_2(S^T) \\
\vdots \\
A_y D_L(S^T)
\end{bmatrix} A_x^T.
\end{align*}
\]

Note that in this paper, we assume that there is no mutual coupling across the sensors. In fact, mutual coupling will degrade the performance of the algorithms. The recent researches in the presence of mutual coupling can be found in [31].

### 3. The Proposed Algorithm

We show how to perform signal separation and DOA estimation using our proposed algorithm in this section. The standard PARAFAC suffers from expensive computation cost due to slow convergence. To handle this problem, we introduce the propagator method (PM) to accelerate TALS by providing the initial angle estimates. Then, alternately update the LS estimates of \( S, A_y, \) and \( A_x \) until they converge.
Finally, obtain the separated signals and the corresponding DOA estimates.

Note that in practice, we need to estimate the number of sources from the received signal first. In this study, we assume that the number of sources is known in advance.

3.1. Initialization with Propagator Method. By exploiting the property of rotational invariance, the propagator method can achieve the angle estimation with relatively low complexity [32, 33].

First, compute the data covariance matrix \( \hat{R} \) using the received signal data in (1) and partition it as follows [33]:

\[
\hat{R} = [\hat{G}, \hat{H}],
\]

where \( \hat{G} \in \mathbb{C}^{MN \times K} \) is the first column to the \( K \)-th column of \( \hat{R} \) and \( \hat{H} \in \mathbb{C}^{MN \times (MN-K)} \) stands for the remaining columns.

Then, we can estimate the propagator \( \hat{P} \) by

\[
\hat{P} = (G^H G)^{-1} G^H H,
\]

Define [33]

\[
\hat{P} = \begin{bmatrix}
I_K \\
A_x \Phi_y \\
\vdots \\
A_y \Phi_x^{N-1}
\end{bmatrix}
\]

where \( \Phi_y = \text{diag}\{\exp(-j2\pi d_1 u_1 / \lambda), \ldots, \exp(-j2\pi d_M u_M / \lambda)\} \) and \( I_K \) denotes a \( K \)-order identity matrix.

The estimates \( \hat{u}_k \) of \( u_k \) can be obtained by partitioning the matrix \( \hat{P} \) and eigenvalue decomposition.

After reconstructing the matrix \( \hat{P}_e \), another matrix \( \hat{P}_{es} \) can be obtained by

\[
\hat{P}_{es} = \begin{bmatrix}
A_y \\
A_y \Phi_x \\
\vdots \\
A_y \Phi_x^{N-1}
\end{bmatrix}
\]

where \( \Phi_x = \text{diag}\{\exp(-j2\pi d_1 u_1 / \lambda), \ldots, \exp(-j2\pi d_M u_M / \lambda)\} \). The estimates \( \hat{u}_k \) of \( u_k \) can also be obtained by a similar method.

3.2. Trilinear Alternating Least Square. Trilinear alternating least square (TALS) is the most common method for trilinear model decomposition [21, 22]. The standard TALS algorithm utilizes random matrices as the initial load matrices, which usually converges slowly. In this part, the initial estimates \( \hat{u}_k \) and \( \hat{v}_k \) provided by PM are used to construct the matrices \( A_x^{(0)} \) and \( A_y^{(0)} \) as initial matrices.

Recall that we assume noise is additive Gaussian noise, and it is reasonable to employ the least square principle to estimate \( S \), \( \hat{A}_x \), and \( \hat{A}_y \). The estimation of the matrix \( S \) can be conducted by minimizing the following quadratic cost function [21]:

\[
\min_{S^{(n)}} \left\| \begin{bmatrix}
\hat{X}_1 \\
\hat{X}_2 \\
\vdots \\
\hat{X}_M
\end{bmatrix} - \begin{bmatrix}
A_x^{(n-1)} D_1 (A_y^{(n-1)}) \\
A_x^{(n-1)} D_2 (A_y^{(n-1)}) \\
\vdots \\
A_x^{(n-1)} D_M (A_y^{(n-1)})
\end{bmatrix} S^{(n)} \right\|_F,
\]

where \( \hat{X}_m \) denotes the data matrix \( X_m \) with noise, \( m = 1, 2, \ldots, M \); \( A_x^{(n-1)} \) and \( A_y^{(n-1)} \) denote the estimates of \( A_x \) and \( A_y \) obtained from \((n-1)\)-th iteration.

Then, the LS estimate of \( S \) can be obtained as [21]

\[
S^{(n)} = \begin{bmatrix}
A_x^{(n-1)} D_1 (A_y^{(n-1)}) \\
A_x^{(n-1)} D_2 (A_y^{(n-1)}) \\
\vdots \\
A_x^{(n-1)} D_M (A_y^{(n-1)})
\end{bmatrix}^* \begin{bmatrix}
\hat{X}_1 \\
\hat{X}_2 \\
\vdots \\
\hat{X}_M
\end{bmatrix}.
\]

The LS fitting for \( A_y \) is similar to \( S \).

\[
\min_{A^{(n)}_y} \left\| \begin{bmatrix}
\hat{Y}_1 \\
\hat{Y}_2 \\
\vdots \\
\hat{Y}_N
\end{bmatrix} - \begin{bmatrix}
S^{(n)} D_1 (A_x^{(n-1)}) \\
S^{(n)} D_2 (A_x^{(n-1)}) \\
\vdots \\
S^{(n)} D_N (A_x^{(n-1)})
\end{bmatrix} A^{(n)}_y \right\|_F,
\]

where \( \hat{Y}_n \) denotes the data matrix \( Y_n \) with noise, \( n = 1, 2, \ldots, N \); \( S^{(n)} \) denotes the estimate of \( S \) according to (15).

Then, the LS estimate of \( A_y \) can be represented as

\[
\hat{A}_y^{(n)} = \begin{bmatrix}
S^{(n)} D_1 (A_x^{(n-1)}) \\
S^{(n)} D_2 (A_x^{(n-1)}) \\
\vdots \\
S^{(n)} D_N (A_x^{(n-1)})
\end{bmatrix}^* \begin{bmatrix}
\hat{Y}_1 \\
\hat{Y}_2 \\
\vdots \\
\hat{Y}_N
\end{bmatrix}.
\]
Similarly, the LS fitting for \( A_x \) is

\[
\min_{A_{\gamma}^{(n)}} \left\| \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \vdots \\ \tilde{Z}_L \end{bmatrix} - \begin{bmatrix} A_{\gamma}^{(n)} D_1 \left( S^{(n)} \right) \\ A_{\gamma}^{(n)} D_2 \left( S^{(n)} \right) \\ \vdots \\ A_{\gamma}^{(n)} D_L \left( S^{(n)} \right) \end{bmatrix} A_{\gamma}^{T(n)} x \right\|_F,
\]

where \( \tilde{Z}_l \) denotes the data matrix \( Z_l \) with noise, \( l = 1, 2, \ldots, L \), and \( A_{\gamma}^{(n)} \) is the estimate of \( A_x \) according to (17).

The estimate of \( A_x \) can be expressed as

\[
A_{\gamma}^{T(n)} = \begin{bmatrix} A_{\gamma}^{(n)} D_1 \left( S^{(n)} \right) \\ A_{\gamma}^{(n)} D_2 \left( S^{(n)} \right) \\ \vdots \\ A_{\gamma}^{(n)} D_L \left( S^{(n)} \right) \end{bmatrix}^{+} \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \vdots \\ \tilde{Z}_L \end{bmatrix}.
\]

According to (15), (17), and (19), we can repeatedly update the estimates of \( S_x, A_y, \) and \( A_x \) until convergence. Because of the utilization of PM, the proposed algorithm fast converges to the final estimates of \( S_x, A_y, \) and \( A_x \), noted as \( \hat{S}_f, \hat{A}_{f_x}, \) and \( \hat{A}_{f_y} \). At this point, the task of signal separation is complete. The last part is to perform DOA estimation.

It is worth noting that the TALS algorithm outlined above contains only the simplest steps. Some techniques, like line search [34, 35], can be coupling with the basic TALS algorithm, which may improve the rate of convergence further. There is no universally accepted most efficient TALS algorithm for all of the problems. We use the basic implementation of TALS for our issues and compare the complexity of our method with line search schemes [34, 35] in Section 5.

3.3. DOA Estimation. First, we normalize the column vectors of \( \hat{A}_{f_x} \) and \( \hat{A}_{f_y} \) and make the first element of the column to equal one. Then, compute the phase vector \( r_x \) by

\[
r_x = \text{angle}(a_{sk}) = \left[ 0, \frac{2\pi d_x}{\lambda}, \ldots, \frac{2\pi(N-1)d_x}{\lambda} \right]^T u_k = B_x u_k,
\]

where \( a_{sk} \) denotes the \( k \)-th column vector of \( \hat{A}_{f_x} \) after normalization.

According to LS criterion, calculate the estimates of \( u_k \) by

\[
\hat{u}_k = B_x^* r_x.
\]

In a similar way, we can also get the estimates \( \hat{v}_k \) of \( v_k \) by the following expressions:

\[
\hat{v}_k = B_y^* r_y,
\]

where \( r_y \) is another phase vector defined as

\[
r_y = -\text{angle}(a_{sk}) = \left[ 0, \frac{2\pi d_y}{\lambda}, \ldots, \frac{2\pi(M-1)d_y}{\lambda} \right]^T u_k = B_y u_k,
\]

where \( a_{sk} \) denotes the \( k \)-th column vector of \( \hat{A}_{f_y} \) after normalization.

Finally, the estimates of \( \theta_k \) and \( \phi_k \) can be calculated by

\[
\theta_k = \arcsin (|\hat{u}_k + j\hat{v}_k|),
\]

\[
\phi_k = \text{angle}(\hat{u}_k + j\hat{v}_k),
\]

where \(|\cdot|\) denotes the modulus of the complex number and \( \hat{\theta}_k \) and \( \hat{\phi}_k \) are the estimates of the elevation and azimuth angles of the \( k \)-th signal.

Note that there are the same permutation effects for the estimation of \( \hat{S}_f, \hat{A}_{f_x}, \) and \( \hat{A}_{f_y} \) during the TALS decomposition, so the final estimates, \( \hat{\theta}_k \) and \( \hat{\phi}_k \), are automatically paired.

3.4. The Procedure of the Proposed Algorithm. We summarize the major steps of our algorithm as follows:

Step 1. Exploit the propagator method to calculate the initial estimates \( \hat{u}_{k0} \) and \( \hat{v}_{k0} \) of \( u_k, v_k \).

Step 2. According to the PARAFAC models (6)-(8), reshape the received signal data to acquire the data matrices \( \hat{X}, \hat{Y}, \) and \( \hat{Z} \).

Step 3. Construct the direction matrices \( \hat{A}_x \) and \( \hat{A}_y \) with \( \hat{u}_{k0} \) and \( \hat{v}_{k0} \) and use them as initial matrices.

Step 4. According to (15), (17), and (19), update the estimates of \( S, A_y, \) and \( A_x \) alternately from the data matrices \( \hat{X}, \hat{Y}, \) and \( \hat{Z} \) until convergence.

Step 5. According to (20)-(25), calculate the DOA estimates of separated signals, \( \hat{\theta}_k \) and \( \hat{\phi}_k \).

4. Performance Analysis

4.1. Complexity Analysis. Since complex multiplication requires the most computation time and resources, we use the time of complex multiplication to evaluate the complexity of the algorithm. The algorithm proposed in this paper adopts PM for initial estimation, whose complexity is \( O(5K^3 + 2KL + 3K^2N(M-1) + 3K^2M(N-1) + (NM-K)KL) \). The complexity of the TALS method is related to the number of iterations and the complexity of a single iteration, and the complexity of each iteration is easily obtained as \( O(3K^3 + 3NMKL + 2K^2(NM + NL + ML)) \). The number of iterations is affected by many factors such as array
size, iteration accuracy, and signal type. Define the number of iterations is \( T \), and the total computational complexity is \( O(T (3K^3 + 3NMKL + 2K^2(NM + NL + ML))) \). Therefore, the complexity of the proposed method is \( O(5K^3 + 2K^2L + 3K^2N(M - 1) + 3K^2M(N - 1) + (NM - K)KL + T(3K^3 + 3NMKL + 2K^2(NM + NL + ML))) \). Due to PM initialization, the iterations of the proposed method are greatly reduced compared with the standard PARAFAC method, which we can see in the next section.

4.2. Advantages. The advantages of the proposed algorithm are as follows:

1. The proposed method has lower computational cost than the standard PARAFAC method due to introducing PM.
2. The proposed method outperforms 2D-ESPRIT and 2D-PM in the aspect of angle estimation performance for planar array.
3. The proposed method can obtain separated signals and corresponding DOA estimation without an additional pairing procedure.

5. Simulation Results

In this section, we employ a URA equipped with \( 8 \times 8 \) sensors to illustrate the improvement of the performance of 2D DOA estimation and signal separation of the proposed algorithm.

Suppose there are \( K = 3 \) typical modulated signals impinging on the array simultaneously, which are single-frequency signal \( s_1(t) = \cos (2\pi \times 3 \times 10^2t) \), linear frequency modulated signal \( s_2(t) = \cos (\pi \times 10^{12}t^2 + 2\pi \times 2 \times 10^4t) \), and amplitude modulated signal \( s_3(t) = \cos (2\pi \times 3 \times 10^5t) \sin (2\pi \times 5 \times 10^6t) \). The DOAs of the signals are \((\theta_1, \phi_1) = (10^\circ, 15^\circ), (\theta_2, \phi_2) = (20^\circ, 25^\circ), \) and \((\theta_3, \phi_3) = (30^\circ, 35^\circ)\), and the sampling frequency is 100 MHz. The noiseless source signal waveforms are demonstrated in Figure 2.

To assess the performance of DOA estimation, root mean square error (RMSE) is used,

\[
\text{RMSE} = \sqrt{\frac{1}{Ck} \sum_{c=1}^{C} \sum_{k=1}^{K} (\alpha_{k,c} - \alpha_k)^2},
\]
where \( C \) is the total number of Monte-Carlo trials, \( \alpha_k \) is the true value of the elevation or azimuth angle of \( k \)-th signal, and \( \hat{\alpha}_{k,c} \) is the estimate of the angle \( \alpha_k \) in the \( c \)-th trial. For each simulation, we set \( C = 1000 \). The signal to noise ratio is defined by \( \text{SNR} = 10 \log_{10}(\|X\|_F^2/\|N\|_F^2) \), where \( X \) is noiseless received data matrix and \( N \) is zero-mean white Gaussian noise matrix.

Besides, to qualify the performance of signal separation, the average similar coefficient between the source signal and the separated signal is adopted,
schemes as respectively. Besides, we also compare our method with line search fi PARAFAC without modification and our algorithm, respec-

SNR = 10 of DSSR, where iteration. Figure 4 shows typical curves of the evolution of the signal-to-noise ratio, and the types of signals. Although a

method is related to many factors, like the scale of the array, the mean CPU time and the mean number of iterations required are 

 SSR = 800

5.1. Convergence Analysis. In Figure 3, we give the result of the complexity of the TALS method. Note that the original 2D-PM and 2D-ESPRIT do not have the capability of signal separation, so we use their results of DOA estimation to construct the matrix A in (1) and compute the LS estimate of S by \( \hat{S} = A^*X \).

As shown in Figure 5, it is evident that our method has the same angle estimation performance as the standard PARAFAC method, which surpasses 2D-ESPRIT and 2D-PM.

Figure 6 shows the performance of signal separation of different algorithms with different SNR. From Figure 6, the signal separation performance of our algorithm, standard PARAFAC method, and 2D-ESPRIT are approximately the same with different SNR, while the 2D-PM algorithm has a slightly weaker signal separation performance with low SNR. Figure 7 is the separated signal diagram by the proposed algorithm. From Figure 7, it can be seen that the error between the source signal and the separated signal is very small, which is consistent with the high average similarity coefficient observed in Figure 6.

Figure 8 illustrates the DOA estimation performance as a function of SNR with different numbers of snapshots. It is seen from Figure 8 that when the number of snapshots \( L \) increases, angle estimation performance can be improved.

\[
\rho = \frac{1}{CK} \sum_{c=1}^{C} \sum_{l=1}^{L} \frac{|s_k \hat{S}_{k,c}^H|}{|s_k||\hat{S}_{k,c}|_2}, \tag{27}
\]

where \( s_k \) is the \( k \)-th source signal and \( \hat{s}_{k,c} \) is the corresponding estimate in the \( c \)-th trial.

5.2 Comparison of Performance. To verify the improvement of the proposed algorithm, the proposed method is compared with 2D-PM, 2D-ESPRIT, and the standard PARAFAC method. Figure 6 shows the performance of signal separation of different algorithms with different SNR.

As shown in Figure 8, the complexity of the TALS method is related to many factors, like the scale of the array, the signal-to-noise ratio, and the types of signals. Although a slight change in configurations can cause a significant difference in the execution time, the proposed method can always reduce the computational cost compared with the standard TALS method.
versus SNR.

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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