An optimized design method of 3-point support for precision horizontal machining center with T-shaped bed

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Abstract
The support point layout has an important influence on the working performance of the machine tool, when the material, manufacturing process, and internal structure of machine bed are given. In order to ensure the performance, stability, and anti-interference of precise machine tool, this paper presents an optimized design method of 3-point support for T-shaped bed of precision horizontal machining center. This article first establishes the statics model of the T-shaped bed and analyzes grillage beam model used to characterize the main static deformation trend of the bed based on the singular function method. After verifying the rationality of the model through simulation, the optimized 3-point support position can be obtained. Then this paper measured the deformation of the upper surface of a simple bed due to gravity. The deviation between the experimental results and the simulation results is less than 20%, which verifies the reliability of the simulation and theoretical results. Based on the ISIGHT multidisciplinary optimization platform, this paper completes the multi-objective optimization of the support point layout of the bed, and the optimization results prove the accuracy of the theoretical model. This paper takes the bed of M800H precision horizontal machining center as an example to illustrate the application process of the proposed method. Then, the optimization effect of the bed and the static characteristics of the whole machine by simulation are compared. The maximum deformation of the bed has reduced by 27.1%. In the whole machine status, the deformation of the spindle end has reduced by 50.8%, and the maximum deformation of the workpiece end has reduced by 50.0%. Finally, a verification experiment of the relative position deviation between the end of the spindle and the worktable on the complete machine was carried out. The error of the simulation and experiment results was about 22%, which proves the effect of the optimization of the support position.

Keywords Precision horizontal machining center · T-shaped bed · 3-point support · Optimized design

1 Introduction
With the rapid development of science and technology, the CNC machine tool industry is developing by leaps and bounds in the direction of high precision, diversified functions, and high speed. As the processing equipment used in the manufacturing industry, CNC machine tools are widely used in the manufacture of key parts and components in aerospace, shipbuilding, and military industries. Therefore, improving the machining accuracy of CNC machine tools is of great significance to enhancing the level of equipment manufacturing [1].

The main factors affecting the machining accuracy of CNC machine tools include the static and dynamic characteristics, vibration resistance and thermal deformation resistance [2]. Bed is an important basic part of the machine tool, and its static and dynamic performance has a great influence on the machining accuracy of the machine tool. The factors that affect the static and dynamic performance of the bed mainly include the topology, material properties, manufacturing processes, and support methods [3]. Support point layout has an important influence on the working performance of the machine tool, when the material properties, manufacturing process, and internal topological structure of the machine bed are determined.
Luo et al. proposed three support structure arrangements for five-axis CNC machine tools, and studied the impact of the three support structures on the static and dynamic performance of the machine through simulation. He proved that the support structure of the machine tool has a great influence on the machining accuracy of the whole machine. Improvement of the support structure of the machine tool can effectively promote the machining accuracy of the machine tool [4]. In order to enhance the performance of a horizontal machining center with 3-point support as the main support, Wang et al. provided three support layout schemes for adding auxiliary support to the relatively weak part of the bed. He analyzed the influence of auxiliary support on the static and dynamic characteristics of the bed through simulation, and proposed that a reasonable addition of auxiliary support can effectively improve the rigidity and stability of the bed [5]. Scholars' attention to support technology is not limited to specific support methods, but also includes support areas for simulation and testing of contact stiffness. Szwengier and Godunski used the test results of the support area of the machine bed to modify the support stiffness model and verify the accuracy of the support stiffness model [6]. Daisuke and Takahiro considered that the rigidity between the machine bed and the supporting surface consists of two parts: contact rigidity and solid rigidity. They established a three-dimensional stiffness model, and fitted the contact stiffness between the metal and the ground by static loading test deformation. In this regard, they applied the stiffness parameter to the modal analysis of a certain machine tool, and verified the accuracy of the stiffness model for predicting the support stiffness of the machine bed [7]. Zhang et al. postulated that the foot joint interface of the bed can be equivalent to a three-way spring damping system. They identified the equivalent stiffness and damping parameters of the foot joint interface through modal tests, and finally verified the parameter identification results through finite element simulation [8]. Huang et al. designed a composite structure based on concrete and applied it to the supporting component, and afterwards they applied this structure to the design and optimization of the machine bed [9].

Although the optimization design for the static and dynamic performance of the bed is also mainly for the optimization design of the bed structure, there are relatively few studies on the optimal design of the layout of the support points of the bed. Researchers have proposed many optimization design methods and design theories, such as BP neural network [10], sensitivity method [11], genetic algorithm [12], response surface method [13] and multi-objective optimization [14, 15].

The traditional bed is equipped with multiple supporting feet with many contact points, which is not conducive to the leveling of the bed. It will affect the vibration characteristics of the machine tool and decrease the processing quality and service life of the machine tool. The 3-point support is to use three anchor bolts to connect the machine bed and the foundation, which realizes the support of the whole machine. Due to the principle of 3 points to determine one side, the 3-point support method is convenient for the leveling of the machine tool. This support method not only enables the bed to possess less internal stress, higher stability and better accuracy retention, but also saves installation costs and shortens the installation cycle [16].

At present, the layout design method for the 3-point support on the bed is still limited to the comparative method. The designer repeatedly modified the position of the 3-point support and arranged a limited combination of parameters for numerical simulation analysis and calculation based on the design experience. They finally selected the best performance from the limited combination of parameters [16]. This method is difficult to ensure that the selected parameter combination is optimal, and it is time-consuming to select the parameter combination and perform numerical simulation analysis and calculation. In this sense, this paper presents an optimized design method of 3-point support for T-shaped bed of precision horizontal machining center. This method can target on universal T-shaped beds. This method simplifies the T-shaped bed into a grillage beam model that can characterize the static deformation trend of the bed, and cooperates with the multi-objective optimization method of ISIGHT to obtain the optimal support point layout. This method can be used to guide the support layout design for T-shaped bed of precision horizontal machining center, and such that it can complete the optimization design according to different optimization goals, and improve the efficiency of optimization. It will improve the efficiency of optimization and the accuracy of optimization results.

The skeleton of this present study is arranged as follows. Section 2 establishes a static model of the T-shaped bed such that the deformation trend of the grillage beam obtained from the static model based on the singular function method can analyzed. Section 3 introduces the verification experiment with a simple bed, which proves that the ANSYS simulation results of the bed surface deformation are reliable. Section 4 uses the M800H bed as the research object to build a multidisciplinary optimization process based on ISIGHT, and obtains the Pareto solution set. Sections 5 compares the optimization effect of the static characteristics of the bed and the whole machine in the simulation results. Section 6 introduces a verification experiment of the relative position deviation between the spindle end and the worktable on the complete machine, which proves the effect of the optimization of the support position. Section 7 is the conclusions of this paper.
2 Statics modeling of 3-point support for T-shaped bed

2.1 Static modeling of the T-shaped bed

In the machine tool coordinate system, the deformation of the bed affected by gravity is mainly manifested as the deformation in the Y-direction as shown in Fig. 1. The deformation of the bed can be divided into the deformation of the Y-direction in the X-direction straight line and the Z-direction straight line. For the bed with a Tic-shaped rib structure inside, the static deformation trend can be characterized by the static deformation trend of the X-direction ribs and Z-direction ribs inside, as shown in Fig. 2.

The center point of the contact surface between the support foot of the machine tool and the foundation called the support point. The X-direction ribs and Z-direction ribs in the bed that overlap the supporting point in the bed can be constructed, and these ribs called the main ribs. The thickness of the main rib is the same as the original rib in the bed. The gravity of the bed acts on each supporting point, which can be regarded as acting on each main ribs. Therefore, the bed can be simplified as a grillage beam model composed of support points and main ribs, as shown in Fig. 3. Then the gravity of the bed can equivalently act on the grillage beam model.

When the cross-sectional dimensions of all beams in the grillage beam model are equal, the linear uniform load \( q_c \) on the grillage beam due to the gravity of the bed is:

\[
q_c = \frac{Gb}{S} \quad (1)
\]

where \( G \) is the gravity of the bed, \( b \) is the thickness of the beam, and \( S \) is the top area of the grillage beam model.

When the cross-sectional dimensions of all beams in the grillage beam model are not equal, the relationship between the linear load distribution function on each beam and the gravity of the bed is:

\[
G = \int_0^{L_1} q_1(x)dx + \int_0^{L_2} q_2(x)dx + \cdots + \int_0^{L_n} q_n(x)dx \quad (2)
\]

where \( L_i (i = 1, 2, 3 \ldots n) \) are the length of each beam in the grillage beam model and \( q_i(x)(i = 1, 2, 3 \ldots n) \) are the linear load distribution function on the beam.

The load value of the linear load distribution function \( q_i(x) \) distributed on the beam at different sections meets the following conditions:

---

**Fig. 1** CAD model of precision horizontal machining center

**Fig. 2** The ribs layout of the T-shaped bed

**Fig. 3** The simplified grillage beam model of T-section bed
\[
\frac{q(x)}{q(x_i)} = \frac{h_i}{h_i'}
\]

(3)

where \(x_i (i = 1, 2, 3...n)\) are the coordinates of different sections of the beam and \(h_i (i = 1, 2, 3...n)\) are the height at different sections of the beam, \(h_i (i = 1, 2, 3...n)\) are the height at different sections of the beam.

According to Eqs. (2) and (3), the gravity of the bed distributed on each beam can be obtained.

### 2.2 The static deformation trend of grillage beam

The static deformation trend of the beam can be represented by the deflection curve of the beam, which can be drawn according to the deflection curve equation of the beam.

In material mechanics, there are two main methods for solving the deflection curve equation of a beam: the integral method [17] and the superposition method [18]. These two methods are very troublesome for solving statically indeterminate beams with multiple support points under complex load conditions, and they may not be able to solve the deflection curve equation of the beam. In this regard, this paper uses the singular function method to solve the beam deflection curve equation [19].

The definition of singular function is [19]:

\[
f_n(x) = \begin{cases} 
(x - a)^n & x \geq a, n \geq 0 \\
0, & x < a, n \geq 0 \\
\infty, & x = a, n < 0 \\
0, & x \neq a, n < 0 
\end{cases}
\]

(4)

where brackets < > are generally called Macaulay brackets and \(a\) is a constant.

The calculus operation rules for singular functions are:

\[
\frac{d}{dx} < x - a > \varepsilon = \begin{cases} 
<x - a>^{n-1} & n \leq 0 \\
1 < x - a >^{n-1} & n > 0 
\end{cases}
\]

(5)

\[
\int_{-\infty}^{x} < x - a > \varepsilon = \begin{cases} 
<x - a>^{n+1} & n \leq 0 \\
\frac{1}{n+1} < x - a >^{n+1} & n > 0 
\end{cases}
\]

(6)

The singular function expressions of common loads of beams are shown in Table 1. Herein, \(a\) is the starting point coordinate of the load; \(b\) is the acting length of the distributed load; \(L\) is the length of the beam; \(M\) is the moment of couple; \(F\) is the concentrated force; \(q\) is the linear uniform load; \(q_0\) is the load value at the right end of the linear load distribution range; \(q_{ij}\) is the load distribution function of the linear load. \(W_M(x), W_F(x), W_F(x), W_{qf}(x)\) are the sum of the singular function expressions of all the moment of couple, the concentrated force, the linear uniform load, and the distributed load on beam, respectively.

The beams in the simplified grillage beam model of the bed are usually of equal cross-section. We can add the load on the beam according to the above method, remove the support of the beam and add the reaction force at the support point. A load concentration function \(W(x)\) including all loads on the beam can expressed as:

\[
W(x) = W_M(x) + W_F(x) + W_{qf}(x) + W_{qf}(x)
\]

(7)

The shear force expression \(Q(x)\) of the beam can be obtained by integrating the load concentration function once:

\[
Q(x) = \int_0^L W(x)dx
\]

(8)

The bending moment expression \(M(x)\) of the beam can be obtained by integrating the shear force expression:

\[
M(x) = \int_0^L Q(x)dx
\]

(9)

According to the static balance condition of the beam, the shear force and the static moment at the end of the beam are 0, that is:

\[
Q(L) = 0
\]

(10)

\[
M(L) = 0
\]

In material mechanics, the differential equation of beam deflection curve is:

\[
EIy''(x) = M(x)
\]

(11)

where \(y(x)\) is the deflection expression of the beam, \(E\) is the elastic modulus of the beam material, and \(I\) is the inertia moment of the beam section.

The angle expression \(\theta(x)\) and the deflection expression \(y(x)\) of the beam can be obtained by integrating the deflection curve differential equation:

\[
EI\theta(x) = \int M(x)dx + C
\]

\[
EIy(x) = \int \left[ \int M(x)dx + C \right] dx + D
\]

(12)

where \(C\) and \(D\) are both integral constants.

The boundary conditions are the following:

The deflection at the supporting point is zero when the supporting point of the beam is a hinged support. The rotation angle and deflection at the supporting point are both zero when the supporting point of the beam is a fixed support.

We can calculate the integral constant through simultaneous equations of Eqs. (9), (10), and (13) and boundary conditions. Then the boundary conditions can be
substituted into Eq. (12), and the beam deflection curve equation can obtained.

In order to obtain the static deformation trend of the grillage beam model, the grillage beam model can be decomposed into X-direction beams and Z-direction beams for analysis. Each foot support point can be simplified as a fixed support, and the rest of the support points can be simplified as a hinged support.

### 2.3 Example

Figure 4 illustrates the CAD model of the bed of the M800H precision horizontal machining center. The bed is a monolithic casting structure with a Tic-shaped rib structure inside. There are three supporting horns at the bottom of the bed. The position constraint of the bed is realized by connecting three anchor bolts with the foundation, such that the bolt holes of the three supporting points are set as fixed constraints. The contact surfaces between the bottom surface of the bed and the support irons are set as support constraint to limiting the freedom degree of the bed in the direction of gravity.

The width of each rib in the bed is equal, and the height difference of the ribs is small. In order to facilitate the analysis of the static deformation trend of the bed, each rib can be regarded as a constant cross-section rib. There are 5 beams in the simplified grillage beam model of the bed, which are marked by serial numbers from 1 to 5, as shown in Fig. 5. The parameters of the grillage beam model are shown in Table 2. The initial foot position of the bed is designed by experience, and the position parameters are $K_1 = 560$ mm, $K_2 = 540$ mm, and $K_3 = 680$ mm.

The deflection expressions and related conditional expressions of each beam can be obtained by the singular function method. Simultaneous equations are calculated by MATLAB software. Then the deflection curve graph of each beam can be obtained, as shown in the blue curve in Fig. 6.

The point on the surface of the bed that overlaps the grillage beam model can indicate the actual static deformation trend of the bed. We performed static analysis on the machine tool model in ANSYS finite element analysis software and extracted the deformation of the corresponding point of the grillage beam model in the Y-direction. The deflection curve graphs of each beam are shown in the red curve in Fig. 6.

#1 beam and #4 beam have the same length and the same load conditions, so their deformation curves are the same, as shown in Fig. 6a-h. The support of the beam is considered as a rigid support, and the deformation of the support is ignored. The beam deformation curves obtained by the two analysis methods are different at the support point, as shown in the Fig. 6.

It can be seen from the above figure that the trend of the beam deformation curves obtained by these two analysis methods is almost the same. In this regard, it is reasonable to simplify the bed as a grillage beam model to analyze the static deformation trend of the bed.

### 2.4 Optimization of support position

It can be seen from Fig. 6 that #2 beam and #4 beam can reflect the maximum deformation of the joint surface of the bed and the column. At the same time, #4 beam can also reflect the deformation of the assembly joint surface of the guide rail and the bed. Therefore, for the statics model of the bed, the position of the corresponding 3-point supporting foot is the best when the deformation of #4 beam is the smallest.

We have obtained the simultaneous equations of the deflection curves of #4 beam. In one breath, the maximum deformation difference of #4 beam as the optimization goal and the foot position parameters $K_1$, $K_2$, and $K_3$ as variables, the optimal set of foot position parameters can be obtained as $K_1 = 796$ mm, $K_2 = 690$ mm, and $K_3 = 820$ mm by the linear programming.

### 3 Experimental verification of simulation results of bed surface deformation

The objective of this experiment is to obtain the $Y$-direction deformation on the surface of the T-shaped bed under the condition of 3-point support solely due to gravity.

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Fig. 4 CAD model of the bed of horizontal machining center
Table 1: Singular function expressions of common loads on beams

| Load type                          | Graphical representation | Singular function expressions |
|------------------------------------|--------------------------|-------------------------------|
| The moment of couple               | ![Diagram](https://via.placeholder.com/150) | $W_{M}(x) = M < x - a >^{-2}$ |
| The concentrated force             | ![Diagram](https://via.placeholder.com/150) | $W_{F}(x) = F < x - a >^{-1}$ |
| The linear uniform load            | ![Diagram](https://via.placeholder.com/150) | $W_{q}(x) = q < x - a >^0$    |
| The local uniform load             | ![Diagram](https://via.placeholder.com/150) | $W_{q}(x) = q < x - a >^0 - q < x - a - b >^0$ |
| The linearly distributed load      | ![Diagram](https://via.placeholder.com/150) | $W_{q}(x) = \frac{q_{0}}{L-a} < x - a >^1$ |
| The local linearly distributed load| ![Diagram](https://via.placeholder.com/150) | $W_{q}(x) = \frac{q_{0}}{b} < x - a >^1 - \frac{q_{0}}{b} < x - a - b >^1$ |
| The arbitrarily distributed load   | ![Diagram](https://via.placeholder.com/150) | $W_{q}(x) = \int_{a}^{b} q(t) < x - t >^{-1} dt$ |
### 3.1 Experimental equipment

1. A simple T-shaped bed

   This experiment uses a simple T-shaped bed model, as shown in Fig. 7. The bed is a monolithic casting structure with a Tic-shaped rib structure inside, as shown in Fig. 2. The main size parameters of this simple T-shaped bed are given in Table 3.

2. Collapex digital autocollimator

   This experiment uses a digital autocollimator for high-precision angle measurement. The instrument has a high-precision area array detector and uses the principle of self-collimation measurement. The horizontal deformation and vertical deformation can be measured by this instrument at the same time.

### 3.2 Experimental scheme

For the upper surface of the simple bed, only the joint surface of the bed and the column and the base surface of the two guide rails are finished. In order to obtain the deformation results of the bed surface in the Y-direction as much as possible, the measurement objects of this experiment are the X-direction straight line of the back bed and the Z-direction straight line where the left and right guide rails are located, as shown in Fig. 8.

Factors such as artificial errors and noise errors during the experiment will inevitably affect the results of the experiment. In order to make the Y-direction deformation of the bed due to gravity large enough under the 3-point support to facilitate measurement and comparison, the 3-point support position designed in this experiment is shown in Fig. 9a.

Ascribed to the machining error of the bed, we also measure a set of deformation experiments of the bed under the 21-point support. The 21-point support position designed in this experiment is shown in Fig. 9b. This measurement result is used to simulate the machining error of the bed.

Notes in the experiment:

1. Before the experiment, the bed must be leveled so that the surface of the bed is perpendicular to the direction of the acceleration of gravity.
2. The temperature of the experimental measurements should be kept as consistent to reduce the influence of temperature on the measurement results.

### 3.3 Experimental results

The measuring instrument cannot be at an absolute level during the experiment, and thus, the measured data needs to be processed by the two end-point method to obtain the straightness of the measured straight line.

The experiments were carried out in an environment of 23 ± 0.5°C, and thus the environmental temperature has little effect on the deformation of the bed. The measurement results of the digital autocollimator for the deformation in the Y-direction of the three straight lines are shown in Fig. 10.

### Table 2 The parameters of the grillage beam model

| Parameter            | Value  | Parameter            | Value  |
|----------------------|--------|----------------------|--------|
| The length of the front bed $L_1$ (mm) | 2535   | The length of the back bed $L_2$ (mm) | 2714   |
| The width of the front bed $W_1$ (mm)  | 1080   | The width of the back bed $W_2$ (mm)  | 1235   |
| Beam thickness $b$ (mm) | 30     | Beam height $H$ (mm) | 880    |
| The weight of the bed $m_c$ (kg) | 9060.76 | Poisson's ratio $\mu$ | 0.3    |
| Elastic modulus $E$ (MPa) | 1.73e [5] | Acceleration of gravity $g$ (N/kg) | 9.8    |
Fig. 6 The beam deformation curves obtained by two analysis methods. a Deflection curve of #1 beam obtained by static deformation trend analysis. b Deflection curve of #1 beam obtained by ANSYS simulation. c Deflection curve of #2 beam obtained by static deformation trend analysis. d Deflection curve of #2 beam obtained by ANSYS simulation. e Deflection curve of #3 beam obtained by static deformation trend analysis. f Deflection curve of #3 beam obtained by ANSYS simulation. g Deflection curve of #4 beam obtained by static deformation trend analysis. h Deflection curve of #4 beam obtained by ANSYS simulation. i Deflection curve of #5 beam obtained by static deformation trend analysis. j Deflection curve of #5 beam obtained by ANSYS simulation.
For the simulation results, the straight line deformation curves in the Z-direction of the left and right guide rails of the bed are consistent. The results of the simulation analysis are shown in Figs. 11 and 12.

It can be seen from Figs. 9, 10, 11 and 12 that when the bed is supported at 21 points, the simulation analysis result is much smaller than the measurement result of the digital autocollimator. This means that the deformation of the bed under the 21-point support due to gravity is much smaller than its own processing error. Therefore, this part of the measurement result can be approximately regarded as the machining error value of the measured linear position. For the two support methods of the bed, the difference between the measurement results of the autocollimator can be used to reflect the deformation of the bed only due to gravity.

It can be seen from Fig. 10 that due to the influence of gravity, the three straight lines all have a concave change in the Y-direction. The straightness changes of the three straight lines are 15.77 μm, 15.60 μm, and 3.84 μm, respectively. From the simulation results, the deformation trends of these three straight lines in the Y-direction are also concave, and the straightness changes are 13.21 μm, 13.20 μm, and 4.41 μm, respectively. The error between experimental results and simulation results is within 20%, which can also demonstrate that the simulation results of the surface deformation of the bed with ANSYS are reliable. Meanwhile, the experiment result can also show that it is reasonable to simplify the bed to a grillage beam model for analyzing the static deformation trend of the bed.

Fig. 7 The simple T-shaped bed

Fig. 8 Confirmatory experiment

Fig. 9 Position distribution map of supporting foot. a Position distribution of 3-point support. b Position distribution of 21-point support
Table 3  Main size parameters of the simple T-shaped bed

| Parameter          | Value  | Parameter          | Value  |
|--------------------|--------|--------------------|--------|
| The width of the front bed $W_1$ (mm) | 840    | The width of the front bed $W_2$ (mm) | 600    |
| The length of the front bed $L_1$ (mm)  | 1650   | The length of the front bed $L_2$ (mm)  | 1665   |
| Beam thickness $b$ (mm)                  | 40     | Bed height $H$ (mm)                  | 310    |

Fig. 10  Measurement results of digital autocollimator. a Deformation curve of Z-direction straight line on the left. b Deformation curve of Z-direction straight line on the right. c Deformation curve of X-direction straight line on the back bed

Fig. 11  Simulation analysis results under 3-point support. a Deformation curve of Z-direction straight line on the left/right. b Deformation curve of X-direction straight line on the back bed
Multi-objective optimization of 3-point support position based on ISIGHT

4.1 Multi-objective optimization

Multi-objective optimization problem refers to an optimization problem with two or more objective functions. Usually, the sub-objective functions are contradictory, which means that the pros and cons of one sub-objective are often restricted by other sub-objective functions [20]. In this sense, the solution of the multi-objective optimization problem is not sole. One can only take into account the requirements of each sub-objective as much as possible within the design range and obtain a series of acceptable solutions to form a relatively uniformly distributed effective solution set, called the Pareto solution set.

4.1.1 Selection of design variables

The CAD model of the bed of the M800H horizontal machining center is shown in Fig. 2. The bed is supported by three horns, and the supporting positions are distributed in an isosceles triangle. The contact surface between the bed and the horn is simplified into three rectangular areas with a length of 400 mm and a width of 200 mm. The main structural dimensions of the bed are shown in Table 2. When the size of the bed structure is determined, the 3-point support position can be uniquely determined by the three position parameters $K_1$, $K_2$, and $K_3$. Therefore, $K_1$, $K_2$, and $K_3$ are selected as the design parameters of the optimization problem. The initial values of the design variables and the design spaces are shown in Table 4.

4.1.2 Determination of the objective function

The bed is an important supporting part of the machine tool. Its main function is to support other parts of the machine tool, such as guide rails, sliding seats, and columns. It also provides operating orbits for the mechanical movement of some components, such as the Z-direction movement of the sliding seat and the worktable. In order to ensure the machining accuracy of the machine tool, the bed must have a high static rigidity. The bed is only subject to its own gravity during assembly, and hence, the T-shaped bed must meet the following design requirements:

1. The static rigidity of the bed is large enough, which means that the maximum deformation of the bed $d_{\text{max}}$ under the action of gravity is as small as possible.
2. The deformation difference of the guide rail $d_1$ is as small as possible, which means that the deformation of the guide rail is as uniform as possible under the action of gravity. The maximum deformation difference is specified to be no more than 3 μm.
3. The deformation difference of the back bed $d_2$ is as small as possible, which means that the deformation of the back bed is as uniform as possible under the action of gravity. The maximum deformation difference is specified to be no more than 3 μm.
4. The first-order natural frequency of the bed after optimization $f_1$ cannot be lower than the first-order natural frequency before optimization $f_0$.

In summary, the mathematical model of the multi-objective optimization problem is:
4.2 Optimized process based on ISIGHT

For this multi-objective optimization problem, we adopted the second-generation non-dominant ranking genetic algorithm NSGA-II provided by ISIGHT. In the process of solving the Pareto solution, this algorithm does not use a weighted method to process multiple targets into a single target but uses a direct exploration method to avoid the situation where the Pareto frontier cannot be explored. The process of solving the Pareto solution is:

1. Select multiple initial design points in the design space to form the initial population \( P_0 \).
2. The population \( P_0 \) is subjected to mutation and crossover operations in accordance with the general process of the traditional genetic algorithm to obtain a new population \( P_1 \), and then the two populations are merged.
3. Through the methods of “non-inferior ranking” and “crowding distance ranking,” the better solutions in \( P_1 \) are retained to form a new parent population \( P_2 \), and then the above process is repeated until the convergence condition is met.

In the NSGA-II optimization algorithm, we set the population number to 20, the genetic generation number to 12 generations, and the cross mutation rate to be 0.9. The flowchart of ISIGHT-based multi-objective joint optimization is shown in Fig. 13.

4.3 Optimized result

After 240 iterations, 12 sets of Pareto solutions can be obtained, which constitute the solution set of the multi-objective optimization problem, as shown in Table 5. It can be seen from the table that the value ranges of the optimal bed support position parameters obtained with the static performance as the optimization goal are \( 771 \text{mm} \leq K_1 \leq 808 \text{mm}, 574 \text{mm} \leq K_2 \leq 708 \text{mm}, 730 \text{mm} \leq K_3 \leq 831 \text{mm} \). In Sect. 2, the optimal support position

\[
\begin{align*}
\text{Min } f(x) &= [d_1, d_2, d_{\text{max}}] \\
\text{s.t.} & \quad 200 \leq K_1 \leq 1035 \\
& \quad 100 \leq K_2 \leq 1257 \\
& \quad 100 \leq K_3 \leq 2435 \\
& \quad f_1 \geq f_0
\end{align*}
\]

(13)
parameters values $K_1 = 796$ mm, $K_2 = 690$ mm, and $K_3 = 820$ mm obtained by the simplified statics model are just within the Pareto solution set interval obtained by the multi-objective optimization. This can also confirm the accuracy of the statics model of the bed.

It can be seen that there are mutual constraints between the three static performance evaluation functions of the bed. In this sense, the designer needs to weigh the various objective functions to select the optimal solution. In this paper, we select the optimal support position parameters obtained in the Sect. 2 for the T-shaped bed.

5 Optimization results under ANSYS finite element analysis

5.1 The maximum deformation of the bed

The bed models before and after optimization are simulated with only gravity. The deformation distributions obtained are illustrated in Fig. 14. It can be seen that the position of the maximum deformation of the bed before optimization is at the junction of the front bed and back bed with a value of 5.63 μm. After optimization, this value equals 4.10 μm, which appears at the end of the front bed. The maximum deformation of the bed after optimization is reduced by 27.1% compared with that before optimization, which significantly improves the static rigidity of the bed.

5.2 The deformation of the guide rail

According to the finite element analysis results of the bed, the comprehensive deformation values of the nodes on the upper surface of the guide rail are extracted. The deformation curves of the guide rail can be obtained through curve interpolation and function fitting in MATLAB, as shown in Fig. 15.

It can be seen that the maximum deformation value of the guide rail after optimization is 3.70 μm, which appears at the end of the guide rail. The minimum deformation value is 2.40 μm, which is located the support point. The deformation difference of the optimized guide rail surface is 1.30 μm, which meets the design requirements in Sect. 4. The deformation of the guide rail after optimization is more uniform line significantly, as shown in Fig. 15.

5.3 The deformation of the back bed

According to the finite element analysis results of the bed, the deformation value of the surface of the back bed is extracted to draw into a deformation contour, as shown in Fig. 16. It can be observed that the maximum deformation value of the back bed after optimization is 4.54 μm, occurring at the corner of the back bed. The minimum deformation value is 1.80 μm, located appears near the support point. The deformation difference of the back bed after optimization is 1.30 μm, which satisfies the design requirements in Sect. 4.

5.4 Dynamic characteristics of the bed

We carried out modal analysis on the beds before and after optimization. The simulation results of the first four natural frequencies and mode shapes are shown in Table 6. The first four-order mode cloud atalases of the T-shaped bed after optimization are presented in Fig. 17.
Table 6  Comparison of the dynamic characteristics of the beds before and after optimization

| The rank     | Natural frequency before optimization (Hz) | Natural frequency after optimization (Hz) | Mode shape after optimization                                      |
|--------------|------------------------------------------|------------------------------------------|------------------------------------------------------------------|
| First-order  | 134.6                                    | 159.1                                    | The torsion of the front bed around the Z-axis                    |
| Second-order | 174.1                                    | 202.9                                    | The bed swings up and down around the X-axis                      |
| Third-order  | 189.8                                    | 210.4                                    | The torsion of the back bed around the Z-axis                     |
| Fourth-order | 231.7                                    | 260.0                                    | The swing of the back bed around the Z-axis                       |

Fig. 17  The first four-order mode cloud atlases of the T-shaped bed after optimization.  

(a) The first mode shape.  
(b) The second mode shape.  
(c) The third mode shape.  
(d) The fourth mode shape

Fig. 18  Deformation contour of the whole machine before and after optimization.  

(a) Deformation contour of the whole machine before optimization.  
(b) Optimized deformation contour of the whole machine
It can be observed that the natural frequencies of each order of the optimized bed meet the constraint requirements, and the first-order natural frequency of the bed is increased by 18.2%.

5.5 The static characteristics of the whole machine

The CAD model of the M800H precision horizontal machining center is shown in Fig. 1. The bed is the main supporting body of the whole machine with three supporting irons at the bottom. The column assembly and the workbench assembly are the two parts supported by the bed. The column assembly includes column, slide, spindle box, and spindle. The column assembly is fixed on the back bed by bolt connection. The workbench assembly is fixed on the bed through guide rails and sliders. The whole machine models before and after optimization are simulated with gravity only. The deformation contour obtained are shown in Fig. 18. It can be seen that before the support position is optimized, the maximum deformation of the spindle end is 68.7 μm, and the maximum deformation of the workbench end is 15.0 μm. When the support position is optimized, the maximum deformation of the spindle end of the machine tool is 33.8 μm, and the maximum deformation of the workbench end of the machine tool is 7.5 μm, which are reduced by 50.8% and 50.0% respectively.

6 Verification experiment of the relative position deviation between the spindle end and the worktable in the whole machine

The end pose error of the machine tool, which also refers to the geometric accuracy of the machine tool, is the most important factor to ensure the accuracy of the machine tool. It reflects the relative position deviation between the tool and the point of action on the workpiece. The relative position deviation between the end of the spindle and the worktable can best reflect the position error of the end of the machine tool when only considering the structural parts of the machine tool.

Fig. 19 The test system

An M800H precision horizontal machining center was used as an experimental object to verify the effect of the optimization of the support position on the whole machine. The experiment aims to obtain the relative position deviation between the end of the spindle of the machine tool and the worktable before and after the optimization of the support position.

6.1 Experimental equipment

The test system is composed of LION X-3 M eddy current sensor, NI9205 signals collection system, and computer, as shown in Fig. 19. LION X-3 M eddy current sensor has the characteristics of non-contact, high accuracy, and high linearity, with a measurement accuracy of 0.01 μm. NI signal acquisition system is connected to a computer, and the supporting virtual instrument software can realize data real-time display and data playback.

6.2 Experimental scheme

When the support position of the whole machine is not optimized, the test stick is installed on the spindle, and then the eddy current sensors are fixed on the workbench by the magnetic stands according to the 5-point method, as shown in Fig. 20. A support block must be placed on the workbench to ensure that the eddy current sensors can be close to the test stick when the spindle is in the middle of the machine stroke.

Then the support feet of the machine tool are adjusted to the optimal position obtained in the Sect. 2. When the collected signal is stable, the relative position deviation between the end of the machine tool spindle and the worktable can be obtained before and after the optimization.
Experimental results

The environment temperature of the experiment is 21 ± 0.2°C, so the influence of the environment temperature on the deformation of the bed can be neglected. The relative position deviation data between the end of the machine tool spindle and the worktable in finite element simulation can be obtained in Sect. 5. The comparison between the experimental and the finite element simulation results of the whole machine is shown in Fig. 21.

In the whole machine experiment, when the support position of the machine tool changes from the initial position to the optimized counterpart, the relative position deviation between the spindle end and the worktable in the X-, Y-, and Z-directions are 0.12 μm, −27.64 μm, and 27.15 μm, respectively. In the simulation results, the relative position deviations of the spindle end and the worktable in the X-, Y-, and Z-directions are 0.03 μm, −23.78 μm, and 22.54 μm, respectively. The relative position deviation between the spindle end and the worktable in the X-direction is so small that it can be neglected. The main relative position deviation is found in the Y- and Z-directions, and the error between the experimental results and the simulation results is about 22%.

7 Conclusions

Based on the development process of the 3-point support of machine tools, this paper presents an optimized design method of 3-point support for T-shaped bed of precision horizontal machining center. The following conclusions can be drawn.

1. This paper establishes the static model of the T-shaped bed by simplifying the bed as a Tic-shaped rib structure inside to a grillage beam model. The static model can characterize the static deformation trend of the surface of the bed and obtain the optimal 3-point support position layout. This static model can be applied to all T-beds. In the confirmatory experiment of the simulation results of surface deformation on the bed, the error between the simulation results and the experimental results is within 20%, which confirms the authenticity of the simulation results. The static deformation trend of the bed obtained by the statics model is consistent with the simulation results, which verifies the rationality of the statics model. This static model provides a preliminary theoretical basis for the layout design of the 3-point support of the bed.

2. This paper obtains the Pareto solution set of the multi-objective optimization model for 3-point support position based on ISIGHT, which can verify the accuracy of the bed statics model. The optimization objectives of this multi-objective optimization model are the static rigidity of the bed, the deformation difference of the guide rail, and the deformation difference of the back bed. According to the design requirements and the actual production of the machine tool, the designer can select the weight of each optimization goal to obtain a 3-point support position layout that is more consistent with actual needs.

3. We take the M800H precision horizontal machining center as an example to compare the simulation results before and after the optimization. When the support position is optimized, the maximum deformation of the bed, the spindle end, and the workbench end is reduced by 27.1%, 50.8%, and 50.0%, respectively. Then the relative position deviation between the spindle end and the worktable in the whole machine is verified by experiments. The error of simulation and experiment results is about 22%. The result of the comparison shows the effectiveness of the simulation and confirms the effect of the optimization of the support position in the whole machine.

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Data Availability All data generated or analyzed during this study are included in this article.
Declarations

Conflict of interest The authors declare no competing interests.

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