Self-Starting Micromotors in a Bacterial Bath

Luca Angelani¹, Roberto Di Leonardo², and Giancarlo Ruocco²,³

¹Research center SMC INFM-CNR, c/o Università di Roma “Sapienza”, I-00185, Roma, Italy
²Research center Soft INFM-CNR, c/o Università di Roma “Sapienza”, I-00185, Roma, Italy
³Dipartimento di Fisica, Università di Roma “Sapienza”, I-00185, Roma, Italy

Micromotors pushed by biological entities, like motile bacteria, constitute a fascinating way to convert chemical energy into mechanical work at the micrometer scale. Here we show, by using numerical simulations, that a properly designed asymmetric object can be spontaneously set into the desired motion when immersed in a chaotic bacterial bath. Our findings open the way to conceive new hybrid microdevices exploiting the mechanical power production of bacterial organisms. Moreover, the system provides an example of how, in contrast with equilibrium thermal baths, the irreversible chaotic motion of active particles can be rectified by asymmetric environments.

Ensembles of animate organisms behave in a very rich and surprising way if compared to inanimate objects, as atoms or molecules in a gas or a liquid. Everyone has been amazed by the cooperative motion of birds in a flock, fishes in a school or wildebeest in a herd [1, 2]. Also at the micrometer scale elementary living organisms, like bacterial cells, show an extraordinary variety of behaviors, such as collective motions [3–6], complex chemical-mediated motility or chemotaxis [7], spatiotemporal patterns [8], self-organized structures [9], biofilms formation [10]. An important peculiarity of animate organisms is the fact that they can be self-propelled, using a variety of different mechanisms for this purpose [11]. Motile cilia and turned flagella are two example of evolutionary tricks adopted by living organisms to accomplish the hard task of swimming at low Reynolds number [12]. One can think about such ensembles of organisms as open systems, with a net incoming flux of energy (provided by nutrients) stored and converted into mechanical work. The resulting dynamics breaks time inversion symmetry so that asymmetric environments can result in directed motions which, in equilibrated Hamiltonian systems, would be forbidden by detailed balance [13, 14]. A natural question then arises: is it possible to rectify such a non equilibrium dynamics to propel microdevices?

Biological molecular motors constitute a fascinating mechanism to generate motion at the nanoscale [12, 16]. When larger, micron sized, structures need propulsion a non equilibrium dynamics to propel microdevices?

Spinning a bundle of helical flagella, bacteria like E. coli, may swim along their body axis with speeds of order 10 body lengths per second [19]. Decorrelation of velocity may occur via four different mechanisms: tumbling, Brownian motion, mechanical interactions and hydrodynamic interactions. The first mechanism is a spontaneous tumble produced by a temporary reversal in the spinning direction of the flagellar motor [20]. Brownian motion can also be effective in producing diffusion of orientation and hence of propelling direction. Interactions with other bacteria can be mechanical, by direct contact, or hydrodynamic, via flow currents produced by the swimming motions. Trying to mimic the behavior of an elongated E. coli cell with a minimal model we only retain the two most effective mechanisms that are tumbling and mechanical interactions. Hydrodynamic interactions, occurring only through dipole or higher order multipoles, turn out to be effective only over short distances where mechanical interactions between elongated bodies are much more effective in reorienting the bacteria. We directly checked that including hydrodynamic interactions has a negligible effect on the mean squared displacements and on its crossover from ballistic to diffusive regimes.

Each cell is represented by an instantaneous position \( \mathbf{r}_i \) and an orientation \( \mathbf{e}_i \) pointing in the free swimming direction. The elongated hard body of the cells (length \( l \) and thickness \( a \)) is modeled by the sum of \( p \) short range repulsive potentials centered at equally spaced locations along the cell axis \( r_i^\beta = r_i + d^\beta \mathbf{e}_i \) with \( \beta = 1, p \) and \( d^\beta = (l - a)(2\beta - p - 1)/(2p - 2) \). The neighboring cells will then act on the ith cell with a system of forces \( \mathbf{F}_i^\beta \) applied at \( r_i^\beta \):

\[
\mathbf{F}_i^\beta = \sum_{j\neq i, \gamma} f(r_i^\beta - r_j^\beta) \quad (1)
\]

\[
f(r) = \frac{Ar}{r^{p+2}} \quad (2)
\]
To such intercellular forces we added intracellular forces consisting of a constant linear propelling force \( f_0 \) (directing along \( \hat{e}_i \)) which is only active in the running state and a random torque \( T_i \), which switches on during the tumbling state. The probability per unit time to switch in a tumbling state is constant and such as to give an average free run length of 10 cell lengths \([19]\). Introducing the state variable \( \theta_i \) which is 0 in the running state and 1 during a tumbling event, the net forces and torques acting on the \( i \)th cell read:

\[
\mathbf{F}_i = f_0 \hat{e}_i (1 - \theta_i) + \sum_\beta \mathbf{F}_i^\beta \\
\mathbf{T}_i = \mathbf{T}_i \theta_i + \hat{e}_i \times \sum_\beta \mathbf{d}_i^{\beta} \mathbf{F}_i^\beta
\]

For the subsequent motion the rigid cell body is modeled as a prolate spheroid of aspect ratio \( \alpha = a/l \). Therefore the center of mass and the angular velocity are \([21]\):

\[
\mathbf{V}_i = \mathbf{M}_i \cdot \mathbf{F}_i \\
\mathbf{\Omega}_i = \mathbf{K}_i \cdot \mathbf{T}_i
\]

where

\[
\mathbf{M}_i = m_{||} \hat{e}_i \hat{e}_i + m_\perp (\mathbf{1} - \hat{e}_i \hat{e}_i) \\
\mathbf{K}_i = k_{||} \hat{e}_i \hat{e}_i + k_\perp (\mathbf{1} - \hat{e}_i \hat{e}_i)
\]

We choose the force coefficient \( A \) in such a way that two bacteria facing head to head on the same line would be in equilibrium at a distance \( a = al \):

\[
A/a^{n+1} = f_0 \Rightarrow A \sim f_0 a^{n+1}
\]

We choose \( l \) as the unit length, \( \tau = l/v_0 \) as the unit of time (where \( v_0 = m_{||} f_0 \) is the free swimming velocity) and \( m_{||} \) as unit of mobility. When not specified physical quantities will be expressed in reduced units. A planar geometry will be investigated in a box \( L \times L \) with periodic boundary conditions. We will specialize to the case of \( N = 1092 \) bacteria with number density \( \rho = N/L^2 = 0.945 \), aspect ratio \( \alpha = 1/2 \) and potential parameters \( p = 2, n = 12 \). Mobility values are \( m_{||} = 1, m_\perp = 0.87, k_\perp = 4.8 \) \((k_{||} \) does not enter in the equation of motion, as \( \mathbf{T}_i \) is perpendicular to \( \hat{e}_i \) in the planar geometry\). We consider a micromotor immersed in the bacterial bath. The asymmetric micromotor is a gear with a sawtooth profile whose center of mass is kept fixed at the center of the box. The motor is free to rotate around its axis. Each of the \( p \) force centers, describing a single bacterium body, interacts with boundary walls through a force of the form in Eq. \([2] \) where \( \mathbf{r} \) is a vector perpendicular to the wall connecting the \( p \)-centers to a point located at a distance \( a/2 \) behind the wall.

The resulting cell-boundary forces produce further force and torque terms in Eqs.\([3,4] \) and \([5,6] \) and a net fluctuating torque on the gear motor, whose angular velocity is then

\[
\Omega_g = K_g T_g
\]

where \( T_g \) is the torque exerted by bacteria on the gear whose rotational mobility is \( K_g \). We consider a gear with 8 teeth and internal (external) radius \( R_{\text{int}} = 5 \) \((R_{\text{ext}} = 8) \). The gear mobility is estimated as that of a disk \([21]\) of radius 6.5, \( K_g = 1.9 \cdot 10^{-3} \). Equations of motion \([3,4] \) and \([10] \) where numerically integrated by Runge-Kutta methods.
The total torque on the device can be estimated from the bath is obtained when an external reversible system applies an opposing torque equal to $T_g/2$. The extracted mechanical power is then given by $T_g^2K_g/4$ and increases with $R$. Therefore in a planar geometry, a 2D array of small gears would perform better, in terms of usable power, than a single big one. The dependence on bacterial concentration is also non trivial due to the interbacterial interactions that could result in reduced motilities at high packing fractions. We note that, however, the observed directed motion of the rotor is a quite robust effect with respect to the variation of different physical parameters, as the density of bacteria, their aspect ratio, the shape of the asymmetric rotor, its
size, the boundary conditions: a quantitative discussion on the role of different parameters will appear in a forthcoming paper.

Our main point here is to demonstrate that, in contrast to thermal baths of passive particles, useful work can be extracted from the chaotic motion of a non-equilibrium suspension of active objects. This behavior reminds of the ratchet effect or Brownian motors [20], in which out-of-equilibrium systems undergo a rectification process in the presence of some asymmetric potential or device. More specifically, in an equilibrated Hamiltonian system, there’s no entropy production and time reversal symmetry guarantees that any trajectory has the same probability than its time reversed, so that no systematic directed motion can be observed on average. On the other hand when a self propelled particle collides to another (or to a boundary), the forces they exchange is not just the repulsion of their rigid bodies, but there are also the forces generated by the propelling units. Such forces are directed along the incoming directions of the two particles and therefore would change sign upon time reversal, while particles repulsion wouldn’t. Time reversed trajectories are then incompatible with the assumed dynamical laws. From a thermodynamic viewpoint such irreversible dynamics reflects the constant entropy production involved in the chemico-physical processes driving the propelling unit, such as the flagellar rotary motor of E-coli. Once time inversion symmetry does not hold a spontaneous directed motion is allowed whenever a spatial inversion symmetry is broken [27].

In conclusion, we have shown that it is possible to conceive opportunely shaped microdevices that can move in a directional way when immersed in a bath of motile microorganisms. In particular, we numerically show that a rotary micromotor, consisting of an asymmetric gear in a bath of *E. Coli* bacteria, spontaneously sets into a unidirectional rotational motion at an average speed of a few rpm. Using asymmetrically shaped boundaries also linear translatory motions could be obtained and bacterial driven transport could be achieved by self assembly of bacteria along the particle’s boundary. Remarkably, when coupled to an external reversible device, a net amount of useful energy could be extracted from the chaotic motion of a bacterial bath. Our findings can open the way to new and fascinating applications in the field of hybrid bio-microdevices engineering, and also provide new insight in the more fundamental aspects of non-equilibrium dynamics of active matter.

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