Testing the CKM Picture of Flavour and CP Violation in Rare K and B Decays and Particle-Antiparticle Mixing

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We summarize briefly the CKM picture of flavour and CP violation that governs the models with minimal flavour violation (MFV). We then describe how this framework can be efficiently tested through particle-antiparticle mixing and rare K and B decays. In particular we provide a list of theoretically clean tests that the simplest version of the MFV framework, the constrained MFV hypothesis, has to face in the coming years. Finally we offer a brief look at the most popular SM extensions that go beyond the CKM framework like the general MSSM, Little Higgs model with T-parity and Randall-Sundrum models with bulk fermions.

§1. Preface

It is a great honour to be able to contribute to this volume that celebrates the 2008 Nobel Prize in Physics awarded to Kobayashi and Maskawa for their seminal 1973 paper\(^1\) on flavour and CP violation in the Standard Model (SM). However, I would like to emphasize that the recognition of this field by the Nobel Committee is not only the success of these two renowned Japanese physicists but also a great success of Nicola Cabibbo whose seminal paper of 1963\(^2\) had a tremendous impact on the field of flavour violation. Therefore this article pays tribute also to his work and I hope, together with many of my colleagues, that one day he will be awarded the Nobel Prize as well.

§2. Introduction

The understanding of flavour dynamics is one of the most important goals of elementary particle physics. Because this understanding will likely come from very short distance scales, the loop induced processes like flavour changing neutral current (FCNC) transitions will for some time continue to play the crucial role in achieving this goal. They can be best studied in K and B decays but D decays and hyperon decays can also offer useful information in this respect. There is also a good chance that we will learn a lot from FCNC processes in the top quark sector once the LHC will start producing results.

Within the Standard Model, FCNC processes are governed by

- the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix\(^1, 2\) that parametrizes the weak charged current interactions of quarks,
- the Glashow-Iliopoulos-Maiani (GIM) mechanism\(^3\) that forbids the appearance of FCNC processes at the tree level with the size of its violation at the one loop level depending sensitively on the CKM parameters and the masses of exchanged particles,
• the asymptotic freedom of QCD\textsuperscript{4,5} that allows to calculate the impact of strong interactions on weak decays at sufficiently short distance scales within the framework of renormalization group improved perturbation theory,

• the operator product expansion (OPE)\textsuperscript{6} with local operators having a specific Dirac structure and their matrix elements calculated by means of non-perturbative methods or in certain cases extracted from experimental data on tree level decays with the help of flavour symmetries.

The present data on rare and CP violating $K$ and $B$ decays are consistent with this structure but as flavour physics enters only now a precision era and many branching ratios still have to be measured, it is to be seen whether some modifications of this picture will be required in the future when the data improve.

In order to appreciate the simplicity of the structure of FCNC processes within the SM, let us realize that although the CKM matrix was introduced in connection with charged current interactions of quarks, its departure from the unit matrix is the origin of all flavour violating and CP-violating transitions in this model. Out there, at very short distance scales, the picture could still be very different. In particular, new complex phases could be present in both charged and neutral current interactions, the GIM mechanism could be violated already at the tree level, the number of parameters describing flavour violations could be significantly larger than the four parameters present in the CKM matrix and the number of operators governing the decays could also be larger. We know all this through extensive studies of complicated extensions of the SM to which we will return briefly at the end of this writing.

In view of New Physics (NP) waiting for us at very short distance scales the SM is considered these days only as a low energy effective quantum field theory, based on the spontaneously broken gauge symmetry

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_C \otimes U(1)_Q,$$

that describes low energy phenomena in terms of 28 parameters. The latter have to be determined from experiment. Two of these parameters ($\alpha_{\text{QCD}}$, $\theta_{\text{QCD}}$) are related to strong interactions and four to the electroweak gauge boson and Higgs sector. The remaining 22 parameters reside in the flavour sector: six quark masses, six lepton masses, four parameters of the CKM matrix\textsuperscript{1,2} and six parameters of the PMNS matrix.\textsuperscript{7,8}

At first sight it would appear that while the success of the SM in describing the data in the strong interaction sector and electroweak gauge boson sector is very profound, the corresponding success in the flavour sector is rather obvious in view of so many free parameters. Yet in the case of the CKM picture of flavour changing interactions in the quark sector, combined with the GIM mechanism\textsuperscript{3} that governs FCNC processes in the SM, such a view would totally misrepresent the facts. Indeed, once all quark and lepton masses are determined, there are only the four free parameters of the CKM matrix to our disposal and in terms of them all existing data in the quark flavour sector can be properly described within experimental and theoretical uncertainties. Indeed, bearing in mind a few hints for the departures from the CKM picture of flavour and CP violation, to which we will return later on, all leading decays of $K$, $D$, $B_d^0$ and $B_s^0$ mesons, that have been measured, are
correctly described,
- suppressed transitions in the SM, like $K^0 - \bar{K}^0$ mixing, $B_d^0 - \bar{B}_d^0$ mixing and $B_s^0 - \bar{B}_s^0$ mixing have not only been found at the suppressed level, but even at the predicted order of magnitude and in fact even better than that,
- CP-violating observables in $K_L$, $K^\pm$, $B_d^0$ and $B^\pm$ decays agree well with the existing data and
- the best measured semi-rare (radiative) B-decays: $B \to X_s \gamma$, $B \to X_s l^+ l^-$ and $B_d \to K^* \gamma$ all turned out to have branching ratios close to the SM predictions.

| $B_s \to \mu^+ \mu^-$ | $K_L \to \pi^+ \nu \bar{\nu}$ | $K_L \to \mu e$ | $\mu \to e \gamma$ | $d_n$ |
|-----------------------|--------------------------|----------------|----------------|------|
| SM                    | $3 \cdot 10^{-9}$         | $6 \cdot 10^{-11}$ | $10^{-10}$     | $10^{-34}$ |
| Exp Bound             | $4 \cdot 10^{-8}$         | $6 \cdot 10^{-8}$  | $10^{-12}$     | $10^{-11}$ |
|                       |                          |                 | $5 \cdot 10^{-26}$ e cm. |      |

Table I. Approximate SM values and experimental upper bounds for selected branching ratios and the neutron electric dipole moment $d_n$.

But this is not the whole story, as many very strongly suppressed branching ratios within the SM are also consistent with experiment: the corresponding decays have not been observed yet. Examples are collected in Table I where we compare approximate SM values with the experimental upper bounds. Clearly there is still a lot of room for NP contributions.

However, one of the very suppressed decays has been seen. It is $K^+ \to \pi^+ \nu \bar{\nu}$ which in the SM is predicted to have the branching ratio $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (8.5 \pm 0.7) \cdot 10^{-11}$. Seven events have been found implying $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (17 \pm 11) \cdot 10^{-11}$ on the high side but still consistent with the SM value.

In spite of all these successes the situation is certainly not satisfactory. Indeed,
- the neutral Higgs boson has not been found yet,
- the Higgs mass $m_H$ is plagued by quadratic divergences present in the one-loop contributions to the Higgs propagator with internal top quark, gauge boson and Higgs exchanges. Within the SM there is no protective symmetry that would keep $m_H = O(v_{ew})$ and if we want to assure this in the presence of a cut-off as high as $\Lambda_{Planck}$, a fantastic fine tuning of SM parameters has to be made, which is obviously very unnatural,
- the hierarchical structures of quark and lepton masses and of their flavour violating interactions parametrized by the CKM and PMNS matrices remain a mystery, which at least from my point of view has not been satisfactorily uncovered in spite of intensive efforts during the last 30 years. But there are some interesting advances which we will mention briefly later on.

There are clearly other issues like the quantization of electric charge, the number of quark and lepton generations, the number of space dimensions, the baryon-antibaryon asymmetry in the universe, dark matter and dark energy, but I do not have space to address them here. Similarly, I do not have space to address in detail the tests of various extensions of the SM like general supersymmetric models, Little Higgs models and Randall-Sundrum models. What I would like to do primarily here, in view of the recent Nobel Prize, is to summarize briefly the present status of the CKM picture of flavour and CP violation that goes beyond the SM itself and en-
compares all models with constrained minimal flavour violation (CMFV) and more general models with MFV. In particular we will summarize stringent tests through rare $K$ and $B$ decays that hopefully will tell us one day whether the CKM picture is indeed the whole story.

The next pages recall briefly the CKM matrix and the related unitarity triangle (UT) and describe the concepts of CMFV and MFV. Subsequently we will discuss the most stringent tests of these frameworks. We next collect signals of NP beyond CMFV and MFV that seem to be present in a number of experimental results. Before closing with an outlook we will briefly summarize the most prominent non-MFV extensions of the SM and their implications for FCNC processes.

§3. CKM Matrix and the Unitarity Triangle

The unitary CKM matrix connects the weak eigenstates $(d', s', b')$ and the corresponding mass eigenstates $d, s, b$:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= 

\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
= \hat{V}_{\text{CKM}} \begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}. 
\tag{3.1}
$$

Many parametrizations of the CKM matrix have been proposed in the literature. While the so called standard parametrization

$$
\hat{V}_{\text{CKM}} = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
  -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -s_{23} c_{12} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix},
\tag{3.2}
$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ ($i, j = 1, 2, 3$) and the complex phase $\delta$ necessary for CP violation, should be recommended for any numerical analysis, the Wolfenstein parametrization and its more accurate generalization to higher orders in $\lambda = |V_{us}|$ are more transparent than the standard parametrization and allow a fast estimate of different contributions to a given decay amplitude.

To this end we make the following change of variables in the standard parametrization

$$
\begin{align*}
  s_{12} &= \lambda, & s_{23} &= A \lambda^2, & s_{13} e^{-i \delta} &= A \lambda^3 (\varrho - i \eta) \\
  c_{ij} &= \cos \theta_{ij} & s_{ij} &= \sin \theta_{ij}
\end{align*}
\tag{3.3}
$$

where

$$
\lambda, \quad A, \quad \varrho, \quad \eta
\tag{3.4}
$$

are the Wolfenstein parameters with $\lambda \approx 0.225$ being the expansion parameter. We find then

$$
\begin{align*}
  V_{ud} &= 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4, & V_{cs} &= 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 (1 + 4 A^2), \\
  V_{tb} &= 1 - \frac{1}{2} A^2 \lambda^4, & V_{cd} &= -\lambda + \frac{1}{2} A^2 \lambda^5 [1 - 2 (\varrho + i \eta)], \\
  V_{us} &= \lambda + \mathcal{O}(\lambda^7), & V_{ub} &= A \lambda^3 (\varrho - i \eta), & V_{cb} &= A \lambda^2 + \mathcal{O}(\lambda^8).
\end{align*}
\tag{3.5}
$$
\[ V_{ts} = -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\varrho + i\eta)], \quad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}), \tag{3.8} \]

where terms \( \mathcal{O}(\lambda^6) \) and higher order terms have been neglected. A non-vanishing \( \eta \) is responsible for CP violation in the MFV models. It plays the role of \( \delta \) in the standard parametrization. Finally, the barred variables in (3.8) are given by

\[ \bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \tag{3.9} \]

Now, the unitarity of the CKM-matrix implies various relations between its elements. In particular, we have

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{3.10} \]

The relation (3.10) can be represented as a “unitarity” triangle in the complex \((\bar{\varrho}, \bar{\eta})\) plane. One can construct five additional unitarity triangles\(^{15,16}\) corresponding to other unitarity relations.

Noting that to an excellent accuracy \( |V_{cd}V_{cb}^*| = A\lambda^3 + \mathcal{O}(\lambda^7) \) and rescaling all terms in (3.10) by \( A\lambda^3 \) we indeed find that the relation (3.10) can be represented as the triangle in the complex \((\bar{\varrho}, \bar{\eta})\) plane as shown in fig. 1. Let us collect some useful formulae related to this triangle:

- We can express \( \sin(2\phi_i) \), \( \phi_i = \beta, \alpha, \gamma \), in terms of \((\bar{\varrho}, \bar{\eta})\). In particular:
  \[ \sin(2\beta) = \frac{2\bar{\eta}(1 - \bar{\varrho})}{(1 - \bar{\varrho})^2 + \bar{\eta}^2}. \tag{3.11} \]

- The lengths \( CA \) and \( BA \) are given (respectively) by
  \[ R_b \equiv \frac{|V_{ud}V_{ub}|}{|V_{cd}V_{cb}|} = \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \tag{3.12} \]
  \[ R_t \equiv \frac{|V_{td}V_{tb}|}{|V_{cd}V_{cb}|} = \sqrt{(1 - \bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|. \tag{3.13} \]

- The angles \( \beta \) and \( \gamma = \delta \) of the unitarity triangle are related directly to the complex phases of the CKM elements \( V_{td} \) and \( V_{ub} \), respectively, through
  \[ V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}. \tag{3.14} \]
• The unitarity relation \((3.10)\) can be rewritten as
\[ R_b e^{i\gamma} + R_t e^{-i\beta} = 1. \] (3.15)

• The angle \(\alpha\) can be obtained through the relation
\[ \alpha + \beta + \gamma = 180^\circ. \] (3.16)

Formula \((3.15)\) shows transparently that the knowledge of \((R_t, \beta)\) allows to determine \((R_b, \gamma)\) through
\[
R_b = \sqrt{1 + R_t^2 - 2R_t \cos \beta}, \quad \cot \gamma = \frac{1 - R_t \cos \beta}{R_t \sin \beta}. \] (3.17)

Similarly, \((R_t, \beta)\) can be expressed through \((R_b, \gamma)\) by
\[
R_t = \sqrt{1 + R_b^2 - 2R_b \cos \gamma}, \quad \cot \beta = \frac{1 - R_b \cos \gamma}{R_b \sin \gamma}. \] (3.18)

These relations are remarkable. They imply that the knowledge of the coupling \(V_{td}\) between \(t\) and \(d\) quarks allows to deduce the strength of the corresponding coupling \(V_{ub}\) between \(u\) and \(b\) quarks and vice versa.

The triangle depicted in fig. 1 together with \(|V_{us}|\) and \(|V_{cb}|\) gives the full description of the CKM matrix. Looking at the expressions for \(R_b\) and \(R_t\), we observe that within the MFV models the measurements of four CP conserving decays sensitive to \(|V_{us}|\), \(|V_{tb}|\), \(|V_{cb}|\) and \(|V_{td}|\) can tell us whether CP violation \((\bar{\eta} \neq 0\) or \(\gamma \neq 0, \pi)\) is present or not in the MFV models. This property is often used to determine the angles of the unitarity triangle without the study of CP-violating quantities. It constitutes a very important test of the CMFV and MFV frameworks.

\section*{§4. Theory of CMFV}

4.1. Master Formula

As already stated at the beginning of this writing the physics at very short distances could in principle deviate profoundly from the CKM picture of flavour and CP violation. However, it is also possible that this picture will dominantly describe all the data available in the future. This would not imply that there is no NP beyond the SM but only that the flavour and CP violation in the quark sector is governed even beyond the SM by the CKM matrix or equivalently by the structure of quark Yukawa couplings. This is the MFV hypothesis. Let us state this hypothesis in explicit terms by using the master formula for weak decays that follows from the operator product expansion and renormalization group approach.

The master formula in question reads\(^{17}\)
\[
A(\text{Decay}) = \sum_i B_i \eta_{i\text{QCD}}^i V_{i\text{CKM}}^j F_i(v), \tag{4.1}
\]
where \(B_i\) are non-perturbative parameters representing hadronic matrix elements of the contributing operators, \(\eta_{i\text{QCD}}^i\) stand symbolically for the renormalization group
QCD factors, $V_{i \mathrm{CKM}}^i$ denote the relevant combinations of the elements of the CKM matrix and finally $F_i(v)$ denote the loop functions that in most models result from box and penguin diagrams but in some models can also represent tree level diagrams if such diagrams contribute. The variable $v$ collects all parameters in addition to $m_t$, in particular the set of new gauge couplings $g_{i \mathrm{NP}}$, masses of new particles $m_{i \mathrm{NP}}$, and new flavour and CP violating couplings $V_{i \mathrm{NP}}^{ij}$. It turns out to be useful to factor out $V_{i \mathrm{CKM}}^i$ in all contributions in order to see transparently the deviations from MFV. In writing (4.1) we did not show explicitly the internal charm contributions that cannot be neglected in certain $K$ decays but are usually very small in $B$ decays.

Now, in the SM only a particular set of parameters $B_i$ is relevant, the functions $F_i$ are real and the flavour and CP violating effects enter only through the CKM factors $V_{i \mathrm{CKM}}^i$. This implies that the functions $F_i$ are universal with respect to flavour so that they are the same in the $K$, $B_d$ and $B_s$ systems. Consequently a number of observables in these systems are strongly correlated in the SM.

The simplest class of extensions of the SM are models with constrained Minimal Flavour Violation (CMFV). They are formulated as follows:

- All flavour changing transitions are governed by the CKM matrix with the CKM phase being the only source of CP violation,
- The only relevant operators in the effective Hamiltonian below the weak scale are those that are also relevant in the SM.

This implies that relatively to the SM only the values of $F_i$ are modified but their universal character remains intact. Moreover, in cases where $F_i$ can be eliminated by taking certain combinations of observables, universal correlations between these observables for this class of models result. We will list them below.

The SM, the Two Higgs Doublet Model II with a moderate $\tan \beta$, the SM with one extra universal flat dimension and the Littlest Higgs model without T parity are prominent members of this class of models.

More generally, as formulated elegantly with the help of global symmetries and the spurion technique, the operator structure in MFV models can differ from the SM one if two Higgs doublets are present and bottom and top Yukawa couplings are of comparable size. A well known example is the MSSM with MFV and large $\tan \beta$. In these models new parameters $B_i$ and $\eta_{i \mathrm{QCD}}$, related to new operators enter the game but the functions $F_i(v)$ still remain real quantities as in the CMFV framework and do not involve any flavour violating parameters, so that the CP and flavour violating effects are again governed by the CKM matrix. However, the presence of new operators makes this approach less constraining than the CMFV framework.

In the simplest non-MFV models, the basic operator structure of CMFV models remains unchanged but the functions $F_i$ in addition to real SM contributions can contain new flavour parameters and new complex phases. Consequently the CKM matrix ceases to be the only source of flavour and CP violation. The Littlest Higgs model with T-parity and the SM extended to four generations are prominent members of this class of models.

Finally, in most general non-MFV models, new operators (new $B_i$ parameters)
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contribute and the functions $F_i$ in addition to real SM contributions can contain new flavour parameters and new complex phases. Here the prominent members are the general MSSM, Randall-Sundrum models and generally models with FCNC transitions at the tree level. Also the NMFV framework\textsuperscript{25} can be classified here.

We postpone the discussion of these non-MFV models to the last section and we will first concentrate on the properties of the CMFV models.

4.2. Master Functions in the CMFV

The CMFV models can be formulated to a very good approximation in terms of 11 parameters:\textsuperscript{20} four parameters of the CKM matrix and seven values of the real master functions $F_i(v)$ that enter the master formula (4.1) and parametrize the short distance contributions. In a given CMFV model, the $F_i$ can be calculated in perturbation theory and are generally correlated with each other but in a model independent analysis they must be considered as free parameters. Explicit calculations indicate that five or even only four of these functions receive significant new physics contributions.

The master functions $F_i(v)$ originate from various penguin and box diagrams. In order to find these master functions we first express the penguin vertices (including electroweak counter terms) in terms of the functions $C (Z^0 \text{ penguin}), D (\gamma \text{ penguin}), E (\text{gluon penguin}), D' (\gamma-\text{magnetic penguin})$ and $E' (\text{chromomagnetic penguin})$. Similarly we can define the box diagram function $S (\Delta F = 2 \text{ transitions})$, as well as $\Delta F = 1$ box functions $B_{\nu \bar{\nu}}$ and $B_{\mu \bar{\mu}}$ relevant for decays with $\nu \bar{\nu}$ and $\mu \bar{\mu}$ in the final state, respectively. The result of this exercise exists.\textsuperscript{26}

While the $\Delta F = 2$ box function $S$ and the penguin functions $E$, $D'$ and $E'$ are gauge independent, this is not the case for $C$, $D$ and the $\Delta F = 1$ box diagram functions $B_{\nu \bar{\nu}}$ and $B_{\mu \bar{\mu}}$. In phenomenological applications it is more convenient to work with gauge independent functions\textsuperscript{27} given by

\begin{equation}
X(v) = C(v) + B_{\nu \bar{\nu}}(v), \quad Y(v) = C(v) + B_{\mu \bar{\mu}}(v), \quad Z(v) = C(v) + \frac{1}{4} D(v). \quad (4.2)
\end{equation}

Indeed, the box diagrams in the CMFV framework have the Dirac structure $(V - A) \otimes (V - A)$, the $Z^0$ penguin diagram has the $(V - A) \otimes (V - A)$ and $(V - A) \otimes V$ components and the $\gamma$ penguin is pure $(V - A) \otimes V$. The $X$ and $Y$ functions correspond then to linear combinations of the $(V - A) \otimes (V - A)$ component of the $Z^0$ penguin diagram and box diagrams with final state quarks and leptons having weak isospin $T_3 = 1/2$ and $T_3 = -1/2$, respectively. The $Z$ function corresponds to the linear combination of the $(V - A) \otimes V$ component of the $Z^0$ penguin diagram and the $\gamma$ penguin.

Then the set of seven gauge independent master functions which govern the FCNC processes in the CMFV models is given by

\begin{equation}
S(v), \ X(v), \ Y(v), \ Z(v), \ E(v), \ D'(v), \ E'(v). \quad (4.3)
\end{equation}

Generally, several master functions contribute to a given decay, although decays exist which depend only on a single function. We have the following correspondence between the most interesting FCNC processes and the master functions in the CMFV models:
\[ K^0 - \bar{K}^0\text{-mixing (}\varepsilon_K) \]
\[ B^0_{d,s} - \bar{B}^0_{d,s}\text{-mixing (}\Delta M_{s,d}) \]
\[ K \rightarrow \pi \nu \bar{\nu}, B \rightarrow X_{d,s} \nu \bar{\nu} \]
\[ K_L \rightarrow \mu \bar{\mu}, B_{d,s} \rightarrow \bar{u} \bar{l} \]
\[ K_L \rightarrow \pi^0 e^+ e^- \]
\[ B \rightarrow X_{s} \gamma \]
\[ B \rightarrow X_{s} \text{ gluon} \]
\[ B \rightarrow X_{d,s} l^+ l^- \]

This table means that the observables like branching ratios, mass differences \( \Delta M_{s,d} \) in \( B^0_{d,s} - \bar{B}^0_{d,s} \)-mixing and the CP violation parameters \( \varepsilon \) und \( \varepsilon' \), all can be to a very good approximation entirely expressed in terms of the corresponding master functions and the relevant CKM factors. The remaining entries in the relevant formulae for these observables are the renormalization group QCD factors \( \eta_i \) and the non-perturbative parameters \( B_i \) that can be calculated by lattice methods in the SM or in certain cases extracted from experiment.

We know from the study of FCNC processes that not all master functions are important in a given decay. In fact plausible arguments exist\(^{20}\) that only the functions
\[ S(v), C(v), D'(v), E'(v) \] (4.4)
are significantly affected by NP contributions so that the remaining entries in (4.3) can be to first approximation replaced by their SM values if one does not aim at high precision. In this approximation the CMFV models depend effectively on eight parameters.

4.3. Model Independent Relations

The simple structure of CMFV models allows to derive a number of relations between various observables that do not depend on the functions \( F_i(v) \) and consequently are universal within this class of models. Let us list the most important relations.

1. There exists a universal unitarity triangle (UUT)\(^{19}\) common to all these models and the SM that can be constructed by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in the extensions of the SM. The UUT can be constructed, for instance, by using \( \sin 2\beta \) from the mixing induced CP asymmetry \( S_{\psi K_s} \) and the ratio \( \Delta M_s/\Delta M_d \).

The relevant formulae are given in (4.8) and (4.10).

2. Next we have
\[ \frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d} \bar{B}_d \bar{F}_{B_d}^2}{m_{B_s} B_s F_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2, \] (4.5)
\[ \frac{Br(B \rightarrow X_{d} \mu \nu)}{Br(B \rightarrow X_{s} \nu \nu)} = \left| \frac{V_{td}}{V_{ts}} \right|^2, \] (4.6)
\[ \frac{Br(B_d \rightarrow \mu^+ \mu^-)}{Br(B_s \rightarrow \mu^+ \mu^-)} = \frac{\tau(B_d) m_{B_d} F_{B_d}^2}{\tau(B_s) m_{B_s} F_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2. \] (4.7)
that all can be used to determine $|V_{td}/V_{ts}|$ without the knowledge of $F_r(v)$. In particular, the relation (4.5) with the precisely measured mass differences $\Delta M_{d,s}$ offers a powerful determination of the length of one side of the unitarity triangle, denoted usually by $R_t$. One finds\(^{21}\)

$$(R_t)_{\text{CMFV}} \approx 0.90 \left( \frac{\xi}{1.21} \right) \sqrt{\frac{17.8/\text{ps}}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.507/\text{ps}}},$$

(4.8)

where

$$\xi = \frac{\sqrt{\hat{B}_{Bs} F_{Bs}}}{\sqrt{\hat{B}_{Bd} F_{Bd}}} = 1.21 \pm 0.04, \quad \xi = 1.258 \pm 0.033$$

(4.9)

as summarized by Lubicz and Tarantino\(^{28}\) and by the HPQCD collaboration,\(^{29}\) respectively. The expression (4.8) and

$$(\sin 2\beta)_{\text{CMFV}} = (\sin 2\beta)_{\text{ψK}}$$

(4.10)

allow to construct the UUT.

3. Eliminating $|V_{td}/V_{ts}|$ from the three relations above allows to obtain three relations between observables that are universal within the CMFV models. In particular from (4.5) and (4.7) one finds the first “golden” relation of CMFV\(^{30}\)

$$\frac{\text{Br}(B_s \to \mu \bar{\mu})}{\text{Br}(B_d \to \mu \bar{\mu})} = \frac{\hat{B}_d \tau(B_s) \Delta M_s}{\hat{B}_s \tau(B_d) \Delta M_d},$$

(4.11)

that does not involve the decay constants $F_{B_q}$ and consequently contains substantially smaller hadronic uncertainties than the formulae considered above. Indeed the present lattice values from\(^{28}\) read

$$F_{B_s} \sqrt{\hat{B}_{B_s}} = 270(30) \text{ MeV}, \quad F_{B_d} \sqrt{\hat{B}_{B_d}} = 225(25) \text{ MeV},$$

(4.12)

while the HPQCD collaboration\(^{29}\) finds similar values but smaller errors

$$F_{B_s} \sqrt{\hat{B}_{B_s}} = 266(18) \text{ MeV}, \quad F_{B_d} \sqrt{\hat{B}_{B_d}} = 216(15) \text{ MeV}.$$  

(4.13)

Note that the simple relation in (4.11) involves only measurable quantities except for the ratio $\hat{B}_s/\hat{B}_d$ that is known with respectable precision\(^{28}\)

$$\frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03, \quad \hat{B}_d = 1.22 \pm 0.12, \quad \hat{B}_s = 1.22 \pm 0.12.$$  

(4.14)

The formulae given above imply three universal results for CMFV models:

$$\frac{\text{Br}(B_s \to \mu^+ \mu^-)}{\text{Br}(B_d \to \mu^+ \mu^-)} = 32.5 \pm 1.7$$

(4.15)

$$\frac{\text{Br}(B \to X_s \nu \bar{\nu})}{\text{Br}(B \to X_d \nu \bar{\nu})} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_d}}{m_{B_s}} \frac{1}{\xi^2} \frac{\Delta M_s}{\Delta M_d} = 22.8 \pm 2.2,$$

(4.16)
\[
\frac{|V_{td}|}{|V_{ts}|} = 0.210 \pm 0.011 ,
\]

where we have used the most recent values for the relevant quantities.\(^{28}\) When \(\xi\) from HQPQCD in (4.9) is used, the central values in (4.16) and (4.17) are changed to 21.1 and 0.218, respectively. The three numbers in (4.15)–(4.17) are universal magic numbers of CMFV. Non-confirmation of these numbers in future experiments would signal non-CMFV contributions. Only the last number can be tested at present and as seen in (6.3) CMFV survives this test.

4. Next the relation (4.11) allows to predict the branching ratios for \(B_{s,d} \to \mu^+ \mu^-\) within the SM and any CMFV model with much higher accuracy than it is possible without \(\Delta M_{s,d}\). In the SM one has\(^{30}\)

\[
Br(B_q \to \mu^+ \mu^-) = \frac{\tau(B_q)}{B_q} \frac{Y^2(x_t)}{S(x_t)} \Delta M_q, \quad (q = s,d) \tag{4.18}
\]

with

\[
C = 6\pi \frac{\eta^2}{\eta_B} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \frac{m^2_H}{M^2_W} = 4.39 \cdot 10^{-10} \tag{4.19}
\]

and \(S(x_t) = 2.32 \pm 0.07\) and \(Y(x_t) = 0.94 \pm 0.03\) being the relevant top mass dependent one-loop functions. More generally we have in CMFV models

\[
\frac{Br(B_q \to \mu \bar{\mu})}{\Delta M_q} = 4.4 \cdot 10^{-10} \frac{\tau(B_q)}{B_q} F(v), \quad F(v) = \frac{Y^2(v)}{S(v)}. \tag{4.20}
\]

Using these expressions one finds in the SM rather precise predictions

\[
Br(B_s \to \mu^+ \mu^-) = (3.6 \pm 0.3) \cdot 10^{-9}, \quad Br(B_d \to \mu^+ \mu^-) = (1.1 \pm 0.1) \cdot 10^{-10}. \tag{4.21}
\]

These predictions should be compared to the 95% C.L. upper limits from CDF\(^{31}\)

\[
Br(B_s \to \mu^+ \mu^-) \leq 6 \cdot 10^{-8}, \quad Br(B_d \to \mu^+ \mu^-) \leq 2 \cdot 10^{-8}. \tag{4.22}
\]

While the bounds from D0\(^{32}\) were a bit weaker, the 2009 bounds will be similar to CDF ones. It is clear that a lot of room is still left for NP contributions.

5. Next it is possible to derive a very accurate formula for \(\sin 2\beta\) that depends only on the two \(K \to \pi \nu \bar{\nu}\) branching ratios and a calculable parameter \(P_c(X)\) that represents charm contribution to the decay \(K^+ \to \pi^+ \nu \bar{\nu};\(^{33}\)

\[
\sin 2\beta = \frac{2r_s}{1 + r_s^2}, \quad r_s = \sqrt{\sigma} \frac{\sqrt{\sigma(B_1 - B_2) - P_c(X)}}{\sqrt{B}} \tag{4.23}
\]

where \(\sigma = 1/(1 - \lambda^2/2)^2\) and we have assumed \(X > 0\). The corresponding formula valid also for \(X < 0\) exists.\(^{34}\) Here we have defined the “reduced” branching ratios

\[
B_1 = \frac{Br(K^+ \to \pi^+ \nu \bar{\nu})}{5.27 \cdot 10^{-11}}, \quad B_2 = \frac{Br(K_L \to \pi^0 \nu \bar{\nu})}{2.27 \cdot 10^{-10}}. \tag{4.24}
\]
with the numerical factors obtained from.\textsuperscript{35)} Reviews on $K \to \pi \nu \bar{\nu}$ decays can be found in.\textsuperscript{36,37)}

It should be stressed that this formula is valid for the full class of MFV models and $\sin 2\beta$ determined this way depends only on two measurable branching ratios and on the function $P_c(X)$ which is dominated by perturbative contributions and is known with NNLO accuracy.\textsuperscript{9,38)} Also the small hadronic contributions are known.\textsuperscript{39)} The theoretical uncertainties in the determination of $\sin 2\beta$ in this manner amount to at most 1\% and with the measurements of $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ with 5 – 10\% accuracy a useful determination of $\sin 2\beta$ should be possible.

6. Moreover, as in CMFV models there are no phases beyond the KM one, we also expect that\textsuperscript{33,34)}

$$ (\sin 2\beta)_{\pi \nu \bar{\nu}} = (\sin 2\beta)_{J/\psi K_S}, \quad (\sin 2\beta)_{\phi K_S} \approx (\sin 2\beta)_{J/\psi K_S}.$$ \textsuperscript{(4.25)}

with the accuracy of the last relation at the level of a few percent.\textsuperscript{40,41)} The confirmation of these two relations is a very important test for the MFV idea. Indeed, in $K \to \pi \nu \bar{\nu}$ the phase $\beta$ originates in the $Z^0$ penguin diagram, whereas in the case of the $B \to \psi K_S$ CP asymmetry from the $B^0_d - \bar{B}^0_d$ box diagram. The CP asymmetry in $B_d \to \phi K_S$ originates also in $B^0_d - \bar{B}^0_d$ box diagram but the second relation in (4.25) could be spoiled by new physics contributions in the decay amplitude for $B \to \phi K_S$ that is non-vanishing only at the one loop level. We will return to the second relation in (4.25) below. The first relation being very clean is the second golden relation of CMFV.

An important consequence of (4.23) and (4.25) is the following one. For a given $(\sin 2\beta)_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu \bar{\nu})$ only two values of $Br(K_L \to \pi^0 \nu \bar{\nu})$, corresponding to two signs of $X$, are possible in the full class of CMFV models, independently of any new parameters present in these models.\textsuperscript{34)} Consequently, measuring $Br(K_L \to \pi^0 \nu \bar{\nu})$ will either select one of these two possible values or rule out all CMFV models.

7. Last but certainly not least the CP asymmetry $S_{\psi \phi}$ is predicted in the CMFV and also MFV models to be small: $S_{\psi \phi} = 0.04$. This leaves a lot of room for non-MFV contributions. We will return to this important observable below.

The stringent correlations between various observables in the CMFV framework listed above imply rather strong upper bounds on the branching ratios of rare $K$ and $B$ decays.\textsuperscript{42)} Including the additional constraint from $Z \to bb$ makes these bounds even stronger.\textsuperscript{43)} Typically departures from SM expectations by more than 50\% are not allowed in this framework any longer.

§5. Theory of MFV

When one considers MFV at large, new operators that are strongly suppressed in the SM and CMFV models enter the game modifying or even removing the correlations present in the latter models. Consequently larger NP effects are allowed in these models. A recent model independent analysis of $\Delta F = 1$ processes\textsuperscript{44)} finds the most interesting effects of this type in the $B_{s,d} \to \mu^+ \mu^-$ decays, where the presence of scalar operators can enhance their branching ratios up to the existing experimental
bounds in [47,22]. Also the branching ratios for \( K \to \pi \nu \bar{\nu} \) decays and the forward-backward asymmetry in \( B \to K^* l^+ l^- \) can be sizably modified. The corresponding analysis for \( \Delta F = 2 \) processes in\(^{45,46}\) shows that in this sector this framework is already rather constrained.

It should be emphasized that it will be a great challenge to prove that MFV is the whole story at low energies. It will be much easier to disprove it through the violation of the correlations listed above and through non-SM CP-violating effects, provided such non-MFV effects are really present. Also the pattern of the CKM matrix with strongly suppressed couplings of the third generation of quarks to the first two is very characteristic for the MFV models and could even be tested at the LHC. Similar tests of MFV at the LHC have been discussed in the literature\(^{47,48}\). There further references to collider tests of MFV can be found.

\section*{§6. Status of CMFV and MFV}

Presently both CMFV and MFV are in a good shape even if some signals of departures from these simplest frameworks exist. We will list them in the next section.

The parameters of the CKM matrix have been already strongly constrained through the measurements of tree level processes and of loop induced observables like the \( B^0_{d,s} - \bar{B}^0_{d,s} \) mixing mass differences \( \Delta M_{d,s} \), \( \varepsilon_K \) in \( K_L \to \pi \pi \) decays and very importantly through the mixing induced CP asymmetry \( S_{\psi K_S} \). We have

\[
|V_{us}| = 0.2255 \pm 0.0010, \quad |V_{cb}| = (41.2 \pm 1.1) \cdot 10^{-3}, \quad \beta = \beta_{\psi K_S} = (21.1 \pm 0.9)^\circ, \quad (6.1)
\]

where the last number follows from\(^{49}\)

\[
\sin 2\beta = 0.670 \pm 0.023. \quad (6.2)
\]

Also the following ratio is well known\(^{49}\)

\[
\left| \frac{V_{td}}{V_{ts}} \right| = 0.207 \pm 0.001 \pm 0.006, \quad (6.3)
\]

with the given errors being experimental and theoretical errors, respectively.

It should be mentioned that the value for \( |V_{cb}| \) quoted above results from inclusive and exclusive decays that are not fully consistent with each other. Typically the values resulting from exclusive decays are below \( 40 \cdot 10^{-3} \). It would be important to clarify this difference which has been with us already for many years. Hopefully, the future Super B facilities in Italy and Japan and new theoretical ideas will provide more precise values. The ratio in (6.3) could still be polluted by new physics. To get the true value one would need precise values of \( |V_{ub}| \) and \( \gamma \) as discussed below.

Next, the angle \( \alpha \) is already well determined from \( B_d \to \rho \rho \) and \( B_d \to \rho \pi \) decays\(^{49}\)

\[
\alpha = (91.4 \pm 4.6)^\circ. \quad (6.4)
\]

A specific analysis employing the mixing induced CP asymmetries \( S_{\psi K_S} \), \( S_{\rho \rho} \) and the QCDF approach finds\(^{50}\) \( \alpha = (87 \pm 6)^\circ \). Summaries of other determinations of \( \alpha \) exist\(^{51}\).
On the other hand the status of $|V_{ub}|$ and $\gamma$ from tree level decays is not as impressive:

$$|V_{ub}| = \begin{cases} (4.0 \pm 0.3) \times 10^{-3} \text{ (inclusive)}, \\ (3.6 \pm 0.4) \times 10^{-3} \text{ (exclusive)}. \end{cases}$$

$$\gamma = \begin{cases} (78 \pm 12)^{\circ} \text{ (UTfit)}, \\ (76^{+16}_{-23})^{\circ} \text{ (CKMfitter)}. \end{cases}$$

It is very important to precisely measure $|V_{ub}|$ and $\gamma$ in the future as they determine the so-called reference UT that is free from NP pollution. The angle $\gamma$ should be measured at LHCb to better than $5^\circ$ accuracy in the first half of the next decade. A precise measurement of $|V_{ub}|$ will require better understanding of hadronic uncertainties and can only be performed at a Super-B facilities in Italy and Japan.

The unitarity triangle fits are shown in Fig. 2. The parameters $\tilde{\rho}$ and $\tilde{\eta}$ corresponding to these plots are given as follows

$$\tilde{\rho} = \begin{cases} 0.156 \pm 0.020 \text{ (UTfit)}, \\ 0.139^{+0.025}_{-0.027} \text{ (CKMfitter)}. \end{cases}$$

$$\tilde{\eta} = \begin{cases} 0.342 \pm 0.013 \text{ (UTfit)}, \\ 0.341^{+0.016}_{-0.015} \text{ (CKMfitter)}. \end{cases}$$

We emphasize that these results do not include new corrections to $\varepsilon_K$ discussed below so that the tension between the values of $\varepsilon_K$ and $S_{\psi K_S}$ within the SM is not visible in these plots.

Fig. 2. Unitarity triangle fits by CKMfitter (left) and UTfit (right) collaborations in 2009.
§7. Puzzles

The CMFV and MFV frameworks appear at first sight compatible with all the existing data. On the other hand, a closer look at several CP violating observables indicates that the CKM phase might not be sufficient to simultaneously describe CP violation in $K$, $B_d$ and $B_s$ decays. In particular:

- Some modes dominated by penguin diagrams, such as $B \rightarrow (\phi, \eta', \pi^0, \omega)K_S$ that, similarly to the golden mode $B \rightarrow \psi K_S$, allow the determination of $\sin 2\beta$, result in $\sin 2\beta$ visibly lower than $(\sin 2\beta)_{\psi K_S} = 0.670 \pm 0.023^{49}$ from $B \rightarrow \psi K_S$. For the theoretical cleanest modes it is experimentally found that $(\sin 2\beta)_{\phi K_S} = 0.44 \pm 0.17$ and $(\sin 2\beta)_{\eta' K_S} = 0.59 \pm 0.07^{49}$.

- With the decreased value of the non-perturbative parameter $\hat{B}_K$ from lattice simulations and the inclusion of additional negative contributions to $\epsilon_K$ that were neglected in the past, CP violation in the $B_d - \bar{B}_d$ system, represented by $(\sin 2\beta)_{\psi K_S}$, appears insufficient to describe the experimental value of $\epsilon_K$ within the SM if the $\Delta M_d/\Delta M_s$ constraint is taken into account. In fact we find $|\epsilon_K|_{SM} / |\epsilon_K|_{exp} = 0.80 \pm 0.11$.

If confirmed by more precise values of $\hat{B}_K$ and more precise values of the CKM parameters, in particular $|V_{cb}|$, which enters roughly as $|V_{cb}|^4$ in $\epsilon_K$, this could signal new physics in $\epsilon_K$. Alternatively, no new physics in $\epsilon_K$ would imply $\sin 2\beta = 0.88 \pm 0.11^{56,58}$. This could only be made consistent with the measured value of $S_{\psi K_S}$ by introducing a new phase $\phi_{new}$ in $B_d - \bar{B}_d$ mixing. Other possibilities are discussed in.

- There are some hints for the very clean asymmetry $S_{\psi\phi}$ to be significantly larger than the SM value $S_{\psi\phi} \approx 0.04^{59-62}$. Theoretical papers related to these results can be found in.

- The rather large difference in the direct CP asymmetries $A_{CP}(B^- \rightarrow K^- \pi^0)$ and $A_{CP}(B^- \rightarrow K^- \pi^+)$ observed by the Belle and BaBar collaborations has not been expected but it could be due to our insufficient understanding of hadronic effects rather than NP. Similar comments apply to certain puzzles in $B \rightarrow \pi K$ decays which represent additional tensions that decreased with time but did not fully disappear.

- Finally, there is the muon anomalous magnetic moment anomaly. Most recent analyses converge towards a $3\sigma$ discrepancy in the $10^{-9}$ range: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} \approx (3 \pm 1) \times 10^{-9}$ where $a_\mu = (g - 2)_\mu / 2$. Despite substantial progress both on the experimental and on the theoretical sides, the situation is not completely clear yet. However, the possibility that the present discrepancy may arise from errors in the determination of the hadronic leading-order contribution to $\Delta a_\mu$ seems to be unlikely, as stressed in. Recent reviews can be found in.
§8. Beyond CMFV and MFV

8.1. Most Popular Non-MFV Extensions of the SM

Having possible signals of non-MFV interactions at hand, let us consider briefly the most popular non-MFV extensions of the SM that are connected with the hierarchy problem related to quadratic divergences in the Higgs mass and the disparity of the electroweak, GUT and Planck scales. The three most promising and most popular directions which aim to solve at least some of these problems are as follows:

a) Supersymmetry. In this approach the cancellation of divergences in $m_H$ is achieved with the help of new particles of different spin-statistics than the SM particles: supersymmetric particles. For this approach to work, these new particles should have masses well below 1 TeV, otherwise fine tuning of parameters cannot be avoided. As none of the supersymmetric particles has been seen so far, the MSSM became a rather fine tuned scenario even if much less than the SM in the presence of the GUT and Planck scales. One of the important predictions of the simplest realization of this scenario, the MSSM with R-parity, is light Higgs with $m_H \leq 130$ GeV and one of its virtues is its perturbativity up to the GUT scales. The ugly feature of the MSSM is a large number of parameters residing dominantly in the soft sector that has to be introduced in the process of supersymmetry breaking. Constrained versions of the MSSM can reduce the number of parameters significantly. The same is true in the case of the MSSM with MFV. An excellent introduction to the MSSM can be found in.

Concerning the FCNC processes let us recall that in addition to a light Higgs, squarks, sleptons, gluinos, charginos and neutralinos, also charged Higgs particles $H^\pm$ and additional neutral scalars are present in this framework. All these particles can contribute to FCNC transitions through box and penguin diagrams. New sources of flavour and CP violation come from the misalignment of quark and squark mass matrices and similar new flavour and CP-violating effects are present in the lepton sector. Some of these effects can be strongly enhanced at large $\tan \beta$ and the corresponding observables provide stringent constraints on the parameters of the MSSM. In particular $B_s \rightarrow \mu^+\mu^-$ can be enhanced up to its experimental upper bound, branching ratios for $K \rightarrow \pi\nu\bar{\nu}$ can be much larger than their SM values and the CP asymmetry $S_{\phi\psi}$ can also strongly deviate from the tiny SM value.

There is a very rich literature on FCNC processes in general supersymmetric models and the large number of parameters present in these models, while allowing to make numerous phenomenological analyses, precludes often clear cut conclusions.

The flavour blind MSSM (FBMSSM) scenario$^{76}$–$^{80}$ having new complex phases that are flavour conserving belongs actually to the class of MFV models but as the functions $F_i$ become complex quantities we mention this model here. The FBMSSM has fewer parameters than the general MSSM(GMSSM) implying striking correlations between various observables. In particular the desire to explain the suppression of $S_{\phi K_S}$ below $S_{\psi K_S}$ implies a direct CP asymmetry in $B \rightarrow X_s\gamma$ that is by one order of magnitude larger than its SM value. Also $d_n$, the electric dipole moment of the neutron, is found to be as high as $10^{-28}$ e.cm for the same reason. The FBMSSM
b) Little Higgs Models. In this approach the cancellation of divergences in $m_H$ is achieved with the help of new particles of the same spin-statistics. Basically the SM Higgs is kept light because it is a pseudo-Goldstone boson of a spontaneously broken global symmetry. Thus the Higgs is protected from acquiring a large mass by a global symmetry, although in order to achieve this the weak gauge group has to be extended and the Higgs mass generation properly arranged (collective symmetry breaking). The dynamical origin of the global symmetry in question and the physics behind its breakdown are not specified. But in analogy to QCD one could imagine a new strong force at scales $O(10−20\text{ TeV})$ between new very heavy fermions that bind together to produce the SM Higgs. In this scenario the SM Higgs is analogous to the pion. At scales well below 5 TeV the Higgs is considered as an elementary particle but at 20 TeV its composite structure should be seen. Possibly at these high scales one will have to cope with non-perturbative strong dynamics and an unknown ultraviolet completion with some impact on low energy predictions of Little Higgs models has to be specified. Concrete perturbative completions, albeit very complicated, have been found. The advantage of these models, relative to supersymmetry, is a much smaller number of free parameters but the disadvantage is the presence of new matter and new interactions on the way to the GUT scale so that Grand Unification in this framework is rather unlikely. Excellent reviews can be found in.

Concerning the FCNC processes let us recall that in contrast to the MSSM, new heavy gauge bosons $W_H^\pm$, $Z_H$, and $A_H$ in the case of the so-called littlest Higgs model without and with T-parity\cite{88,87} are present. Restricting our discussion to the model with T-parity (LHT), the masses of $W_H^\pm$ and $Z_H$ are typically $O(700\text{ GeV})$. $A_H$ is significantly lighter with a mass of a few hundred GeV and, being the lightest particle with odd T-parity, it can play the role of a dark matter candidate. Concerning the fermion sector, there is a new heavy $T$-quark necessary to cancel the quadratic divergent contribution of the ordinary top quark to $m_H$ and a copy of all SM quarks and leptons, required by T-parity. These mirror quarks and mirror leptons interact with SM particles through the exchange of $W_H^\pm$, $Z_H$ and $A_H$ gauge bosons that in turn implies new flavour and CP-violating contributions to decay amplitudes. These new contributions are governed by new mixing matrices in the quark and lepton sectors which can have a structure very different from the CKM and PMNS matrices. The mirror quarks and leptons can have masses typically in the range 500-1500 GeV and could be discovered at the LHC. Their impact on FCNC processes can be sometimes spectacular. Reviews on flavour physics in the LHT model can be found in\cite{89,90} and selected papers containing details of the pattern of flavour violation in these models can be found in.\cite{91,92} In particular the asymmetry $S_{\psi\phi}$ can be much larger than its SM value, the rare decays $K \rightarrow \pi\nu\bar{\nu}$ can be strongly enhanced\cite{94,92} and the effects in lepton flavour violating decays like $\mu \rightarrow e\gamma$ can be very large.\cite{93,95}

Recently also CP violation in $D^0 - \bar{D}^0$ mixing has been analyzed in this framework.\cite{88} Observable effects at a level well beyond anything possible with CKM dynamics have been identified. Comparisons with CP violation in $K$ and $B$ systems
should offer an excellent test of this NP scenario and reveal the specific pattern of flavour and CP violation in the $D^0 - \bar{D}^0$ system predicted by this model.

c) Extra Space Dimensions. When the number of space dimensions is increased, new solutions to the hierarchy problems are possible. Most ambitious proposals are models with a warped extra dimension first proposed by Randall and Sandrum (RS)\(^\text{99}\) which provide a geometrical explanation of the hierarchy between the Planck scale and the EW scale. Moreover, when the SM fields, except for the Higgs field, are allowed to propagate in the bulk,\(^\text{100} - \text{102}\) these models naturally generate the hierarchies in the fermion masses and mixing angles\(^\text{100} - \text{102}\) while simultaneously suppressing FCNC transitions with the help of the so-called RS-GIM mechanism.\(^\text{103}, \text{104}\) Yet, in these models FCNC processes appear already at tree level\(^\text{103} - \text{106}\) and in the case of $\varepsilon_K$ which receives tree level KK gluon contributions some fine-tuning of parameters in the flavour sector is necessary in order to achieve consistency with the data for KK scales in the reach of LHC.\(^\text{106}, \text{107}\)

Moreover, to avoid problems with electroweak precision tests (EWPT) and FCNC processes, the gauge group is generally larger than the SM gauge group and similarly to the LHT model new heavy gauge bosons are present. However, even in models with custodial symmetries,\(^\text{108} - \text{110}\) these gauge bosons must be sufficiently heavy (2 – 3 TeV) in order to be consistent with EWPT.

In the case of rare $K$ and $B$ decays the RS-GIM mechanism\(^\text{103}, \text{104}\) combined with additional custodial protection of flavour violating $Z$ couplings\(^\text{107}, \text{111}\) allows to achieve agreement with existing data without a considerable fine tuning of parameters\(^\text{107}, \text{111}\) and still produce interesting effects in observables that should be measured in the coming years. Most recent reviews on the latter work can be found in.\(^\text{112}, \text{113}\) One finds in these models a clear pattern of flavour violation: large effects in $\Delta F = 2$ transitions and rare $K$ decays but small effects in rare $B$ decays except for $B \to X_s \gamma$.\(^\text{114}\) However, simultaneous large effects in $\Delta F = 2$ processes and rare $K$ decays are rather unlikely. Large effects are also found in $\mu \to e\gamma$\(^\text{115}\) and electric dipole moments. A detailed presentation of a particular model with custodial protection including Feynman rules exists.\(^\text{116}\) Very recently possible flavour protections in warped Higgsless models have been presented.\(^\text{117}\) On the other hand various aspects of flavour physics in a model without custodial protections have been discussed by the Mainz group.\(^\text{118}, \text{119}\)

8.2. The Flavour Matrix

After the discussion of CMFV, MFV and various non-MFV extensions of the SM let us compare them from the point of view of the presence of new operators and/or new sources of flavour violation with respect to the SM. Our discussion of Section 4 results in a $2 \times 2$ matrix shown in Fig.\(^\text{8}\). Let us briefly describe the four entries in this matrix.

The element (1,1) or the class A represents the models with CMFV discussed in detail in Section 4. We have seen above that this class of models does not allow for large deviations from the SM predictions.

The elements (1,1) and (1,2) or classes A and B taken together, the upper row of the flavour matrix, represent the class of models with MFV at large that we
discussed briefly in Section 5. Basically the new effect in the (1,2) entry relative to (1,1) alone is the appearance of new operators with different Dirac structures that are strongly suppressed in the CMFV framework but can be enhanced if $\tan \beta$ is large or equivalently if $Y_d$ cannot be neglected. The presence of new operators, in particular scalar operators, allows to lift the helicity suppression of certain rare decays like $B_s \rightarrow \mu^+\mu^-$, resulting in very different predictions than found in CMFV models.

A very interesting class of models is the one represented by the (2,1) entry or the class C. Relatively to CMFV it contains new flavour violating interactions, in particular new complex phases, forecasting novel CP-violating effects that may significantly differ from those present in the CMFV class. As there are no new operators relatively to the SM ones, no new $B_i$-factors and consequently no new non-perturbative uncertainties relative to CMFV models are present. Therefore predictions of models belonging to the (2,1) entry suffer generally from smaller non-perturbative uncertainties than models represented by the second column in the flavour matrix in Fig. 3.

When discussing the models in (2,1), it is important to distinguish between models in which NP couples dominantly to the third generation of quarks, basically the top quark, and models where there is a new sector of fermions that can communicate with the SM fermions with the help of new gauge interactions. Phenomenological approaches with enhanced Z-penguins,\(^{66,120,121}\) some special $Z'$-models\(^{122}\) and the fourth generation models\(^{123-125}\) belong to the first subclass of (2,1), while the LHT model belongs to the second subclass.

Finally there is the most complicated class of models represented by the (2,2) entry or the class D in which not only new flavour violating effects but also new

![Fig. 3. The Flavour Matrix](image-url)
operators are relevant. The MSSM with flavour violation coming from the squark sector and RS models are likely to be the most prominent members of this class of models. In the RS models FCNC transitions take place already at tree level and the pattern of flavour violation in these models generally differs from the LHT model and the MSSM.\textsuperscript{111} NMFV\textsuperscript{25} and left-right symmetric models belong also to this class. A spurion technology to classify these models has been developed by Feldmann and Mannel.\textsuperscript{126}

§9. Outlook

The frameworks of CMFV and MFV in which the CKM matrix is the only source of flavour and CP violation in the quark sector could turn out to be the correct description of the full class of flavour violating processes. On the other hand many new branching ratios and many CP-violating observables will be measured in the coming years with high precision. Therefore we should be prepared for surprises and the studies of various extentions of the SM with non-MFV interactions indicate that these surprises could still be spectacular.

In particular:
- The measurement of $S_{\psi\phi}$ with a value above 0.2 would signal a clear violation of both CMFV and MFV.
- The measurement of $Br(B_s \to \mu^+\mu^-)$ above $5 \cdot 10^{-9}$ would be inconsistent with CMFV, signalling the presence of new (scalar) operators, but would still be consistent with MFV at large.
- The measurement of electric dipole moment of the neutron at the level of $10^{-27}$ e.cm would be inconsistent with the CKM picture by several orders of magnitude.

These are just three out of many possible prominent signals that would definitely imply NP beyond the SM and beyond the CKM picture. Later $K \to \pi\nu\bar{\nu}$ and $K_L \to \pi^0 l^+l^-$ decays could be used to obtain a deeper insight into the flavour structure at very short distance scales. Correlations between $B$ and $K$ decays will play an important role in this context. Also CP violation in $D^0 - \bar{D}^0$ mixing\textsuperscript{98,127,128} and searches for FCNC processes in the up-quark sector at the LHC will be very helpful in this respect. Nice recent reviews of the physics of CP violation and flavour violation have been presented by Fleischer\textsuperscript{129} and Nierste,\textsuperscript{130} where further references can be found.

If no spectacular deviations from the SM will be observed, there is still a multitude of correlations between various observables, in particular the two golden relations discussed in Section 4 that eventually could tell us how precise the CKM picture is. In any case the next decade should be very exciting for flavour physics resulting hopefully in new Nobel Prizes for advances in this fascinating field.

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