Anomaly in decay of $^8$Be and $^4$He - can an observed light boson mediate low energy nucleon-nucleon interactions?

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Abstract. We present a hypothesis that the reported anomaly in the folding angle distribution of electron-positron pairs, emitted in the decay of the excited levels of nucleus $^8$Be and $^4$He, can be related to the cluster structure of the decaying state. In particular, we suggest that the potentially reported boson with rest mass $m_X=17$ MeV can mediate the nucleon-nucleon interaction at the low-energy regime of QCD, in the weakly bound cluster state $p^+ ^7$Li, and $p^+ ^3$H. We explore a possible equations of state of symmetric nuclear matter, corresponding to the vector meson mass $m_v=17$ MeV, obtained using relativistic mean field theory of nuclear force using QHD-I. We find several relations for boson masses, based on concepts of chiral symmetry breaking. Both model approaches, thus point towards apparent restoration of chiral symmetry in nucleon-nucleon interaction at large distances, possibly via bounce into false instanton vacuum. A possible existence of a boson with rest mass $m_X=17$ MeV in the decay of high lying excited states of $^8$Be and $^4$He, could shed new light in one of the long lasting open questions in nuclear physics.

Keywords: Beryllium Anomaly, Instantons, QCD, QHD-I, Chiral Symmetry

1. Introduction

In 2016 an article of Krasznahorkay et al. appeared [1], where an anomaly in the angular correlation of electron-positron decay of the $1^+$ excited level of $^8$Be nucleus at 18.15 MeV, specifically reported enhancement at the folding angles close to 140 degrees, is interpreted as a signature of a decay via emission of neutral boson with the mass around $m_X=17$ MeV. Since initial article [1], similar effect was reported by the same group also for the lower $1^+$ excited state of the $^8$Be at 17.6 MeV [2] and more recently in the $0^-$ excited state of the $^4$He at 21.01 MeV [3]. The interpretation of the anomaly
by introducing a new bosonic particle is rather far-reaching, especially in the domain of low energy nuclear spectroscopy, which immediately leads to question why such particle would not be reported by particle physicists already before. Modern accelerators provide enough energy to produce it copiously. Even if it would be a weakly interacting (sterile) particle, still it should be detectable in the energy balance of the reported event. In principle such particle is not excluded (see e.g. \cite{4}) but possibly such nuclear decay could still be explained within nuclear physics domain.

First of all, one needs to understand the structure of the light nuclei like $^8\text{Be}$ and $^4\text{He}$. The nucleus $^8\text{Be}$ is rather unique case, practically unstable, which can be considered as consisting of two $\alpha$-particles. In fact, the ground state is located above the sum of masses of two $\alpha$-particles and it is stabilized only by the Coulomb barrier (maximum 1.5 MeV for $l=0$). Basic mode of decay of the ground state and of excited states up to 17 MeV is the decay into two $\alpha$-particles. Ratio of energies of first two excited states suggests that it is a nuclear rotator. However, starting from the levels above 17 MeV basic mode of decay becomes emission of protons, while emission of gamma-particles and electron-positron pairs is also reported, and decay into two $\alpha$-particles is suppressed.

This sounds quite strange, since these states are well above the Coulomb barrier and there should be no obstacle for $\alpha$-decay, unless something happens to structure of these excited states. What could happen there? The answer can be possibly derived from the fact, that the proton separation energy of $^8\text{Be}$ is 17.25 MeV and the $1^+$ excited states at 17.6 and 18.15 MeV can be cluster configurations such as proton and $^7\text{Li}$, again stabilized only by Coulomb barrier. Thus the excited states above 17 MeV are possibly molecular states, stabilized mainly by Coulomb barrier of a given cluster configuration. Recent reported observation of similar effect in the excited state of the $^4\text{He}$ \cite{3} appears to support the role of cluster structure in the decay of $1^+$ excited states of $^8\text{Be}$ at 17.6 and 18.15 MeV. In the case of $0^-$ excited state of the $^4\text{He}$ at 21.01 MeV, again above proton separation energy of around 20.5 MeV, the $^3\text{H}+p$ cluster structure appears to play a role, thus further supporting the comparable role of cluster configurations in the case of $^8\text{Be}$.

2. Phenomenological analysis

As discussed above, namely the cluster structure can be the key for emergence of the effect suggesting a possible existence of 17 MeV boson. If this is the case, the strong interaction of the proton with remaining cluster may be the source of the reported boson. Compared to scattering experiments, the nuclear molecule like $^7\text{Li}$ or $^3\text{H}+p$ can exist much longer by several orders of magnitude, up to the lifetime of the excited state. Under such conditions low energy degrees of freedom of QCD can be possibly manifested. Thus the reported signal suggesting the 17 MeV boson can provide evidence for long sought for and now almost forgotten exchange boson mediating specifically strong interaction among nucleons. This explanation can provide alternative to presently
preferred explanations in terms of dark matter or exotic particles. In particular, a possible existence of the effect in the decay of 1+ excited states of 8Be lead the authors [3] to possible interpretation in terms of a dark photon particle [5], a vector boson. Since until today a relevant theory of nucleon-nucleon interaction such as low energy QCD does not exist it is hard to judge on constraints concerning rest mass of such bosonic particle in nucleon-nucleon interaction. Below we discuss possible scenarios and resulting quantitative relations on both nucleonic and quark level.

2.1. Quantum hadrodynamics

One of the models of nuclear forces used in nuclear theory is the QFT model of quantum hadrodynamics (QHD), which assumes in its simplest variant QHD-I an interaction of nucleons with massive scalar and vector bosons. This model does not provide constraint of the mass of vector boson, but the ratio of its coupling g_v to its rest mass m_v is constrained by saturation density and binding energy. Typically the mass of ω-meson is chosen as m_v, which leads to large values of g_v around 10-15. In this respect, the rest mass of m_v=17 MeV would lead to g_v around or below unity. Concerning the range of interaction mediated by such vector boson, using the uncertainty principle and assuming massless particle, the range would be around 5 fm. This of course would be much less for particle with a rest mass at low energy and thus the typical range of strong force around 1-2 fm can be surely reproduced. The choice of vector particle reflects properties of QHD model, where contribution of pseudoscalar particle cancels out due to odd parity. However the obtained coupling of vector boson can be close also to the coupling of a pseudoscalar particle - obtained from the chiral symmetry breaking - as it is in the case of ω and π-mesons. In order to verify above mentioned possibility, the energy density of symmetric nuclear matter can be calculated using the formula [6]

\[
\epsilon = \frac{g_v^2}{2m_v^2}\rho_N^2 + \frac{g_v^2}{2m_s^2}(m_N - m_N^*)^2 + \frac{\kappa}{6g_s^3}(m_N - m_N^*)^3 + \frac{\lambda}{24g_s^4}(m_N - m_N^*)^4 + \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \sqrt{k^2 + (m_N^*)^2}
\] (1)

where g_s,m_s are coupling and rest mass of scalar boson, g_v,m_v are coupling and rest mass of vector boson, κ,λ are couplings of cubic and quartic self-interaction of scalar boson, m_N,m_N^* are rest mass of nucleon and its effective mass, \(\rho_N\) is the nucleonic density, \(k_F\) is Fermi momentum of nucleons at zero temperature and \(\gamma\) is the degeneracy (with value \(\gamma=4\) for symmetric nuclear matter and \(\gamma=2\) for neutron matter). In the simplest variant, without scalar boson self-interaction, the energy density is sensitive only to the ratios g_s/m_s and g_v/m_v. Usually, the mass of vector boson is chosen as the mass of experimentally observed ω-meson (m_ω=783 MeV) and the mass of the scalar boson is a free parameter, with the typical values around 500 MeV. The corresponding couplings g_s and g_v are constrained using the experimental values of binding energy of symmetric
nuclear matter \((E/A=-16 \text{ MeV})\) and nuclear saturation density \((\rho_0=0.16 \text{ fm}^{-3})\), which are deduced from the masses and radii of finite nuclei. The final constraints are \[\frac{(m_N)^2}{m_s} g_s^2 = 357.4\]
\[\frac{(m_N)^2}{m_v} g_v^2 = 273.8\]
(2)

which guarantee proper values of both properties of nuclear matter. Typical values of couplings \(g_s\) and \(g_v\) with the choice of \(m_v=783 \text{ MeV}\) are between 10 and 20, what obviously excludes perturbative treatment. The choice of vector boson mass \(m_v=17 \text{ MeV}\) allows to describe symmetric nuclear matter using values of couplings around 0.3, what could allow perturbative treatment in principle. In order to test the choice \(m_v=17 \text{ MeV}\) one can start with cubic scalar boson self-interaction. A wide range of equations of state, reproducing the binding energy and saturation density of symmetric nuclear matter, was obtained using the range of values of coupling \(\kappa\) between -0.0032 and 0.0032 MeV. Outside of this interval properties of the global energy minimum of symmetric nuclear matter can not be described. In particular, the incompressibility of the softest equation of state obtained using \(m_v=17 \text{ MeV}, m_s=32.22 \text{ MeV}, g_v=0.249, g_s=0.589,\) \(\) and \(\kappa=0.0032 \text{ MeV}\) reaches down to incompressibility \(K_0=330 \text{ MeV}\), which is however higher than the values, given by experimental constraints from nuclear reactions and astrophysical data, in particular from recent data of binary neutron star mergers.

In order to obtain more realistic values of incompressibility, it is necessary to implement also quartic self-interaction of scalar meson. Using as a starting point the most promising values of \(\kappa\) from scan with cubic self-interaction term only, a range of the values of \(\lambda\) was examined and ultimately regions of parameters were identified, allowing to obtain equations of state with the values of incompressibility in the range \(K_0=240-260 \text{ MeV}\), which was constrained e.g. in our recent study on fusion hindrance in reactions, leading to production of superheavy elements [7]. Similar values were extracted recently [8] from analysis of the binary neutron star merger event GW170817. Further constraints were applied based on the values of binding energy [9] and pressure [10] of symmetric nuclear matter at double and triple of saturation density. The resulting parameter sets for equations of state with incompressibilities \(K_0=245, 250, \) and \(260 \text{ MeV}\), respectively, are shown in the Table [1].

In this respect it is worthwhile to mention that the value of second derivation of single particle density, from which is the value of \(K_0\) derived, must not necessarily represent the behavior at high densities. This is true only if the shape of equation of state can be fitted using polynomial function. Then the relation between global minimum (saturation density) and behavior at high densities can be established. Fortunately this is the case here and the obtained equations of state can be described up to triple of saturation density using the quartic polynomial and the values of \(K_0\), extracted around the saturation density are consistent with the values obtained from the global fit of
equation of state up to triple of saturation density. Concerning the mass of scalar boson, the value around $m_s=25.5$ MeV was obtained. Until now, no such particle was reported, however also for usual value of $m_s=500-520$ MeV, corresponding to the choice $m_v=783$ MeV, this question is not solved conclusively, so the scalar boson with mass $m_s=25.5$ MeV can be considered as an artefact of the model.

Concerning bosonic particle with mass $m_v=17$ MeV, it can be considered as specific to low-energy nucleon-nucleon interaction with no obvious relation to dark matter, introduced in cosmological models, or to new type of interaction. This bosonic particle can represent low-energy degrees of freedom of QCD, where however no conclusive theory exists at present. In this respect, an intuitive candidate for mediator of interaction between nucleons would be a complex of gluons, called usually glueball. However, mass of such elementary particle is estimated around 1.5-1.7 GeV. Still, such estimates possibly include mechanism of confinement, which generates more than 95% of the nucleonic mass. A possible observation of particle with mass around 17 MeV, comparable to quark current masses, may suggests that mechanism of confinement is not applicable and that the nucleons exchange virtual particles consisting of unconfined gluons or quarks, generated from QCD vacuum. In any case, the corresponding value of $g_v=0.245$ appears compatible with such explanation and thus may provide some encouragement concerning possibility to formulate corresponding theory.

Of course, the analysis, performed here does not exclude the role of heavier (confined) bosonic particles as mediators of nucleon-nucleon interaction. Both lighter and heavier bosonic particles can contribute to low-energy nucleon-nucleon interaction, but the interaction via lighter virtual bosons may be pronounced especially when nucleons interact at large distance, what would be the case if nuclear molecule is formed, as can be expected for highly excited states of $^4$He and $^8$Be. In any case, possible existence of light bosonic particle opens possibility to understand better the long standing open question of nuclear physics.

Realistic relativistic mean field calculations of finite nuclei, like ones based on point-like couplings [11] and meson exchange [12] effective Lagrangians, are complex tasks which goes far beyond the scope of the present work. We tested the obtained equations of states on less computation-complex finite object such as neutron star. We observe [13] that the obtained simple equation of state, allows reasonable description of mass-radius diagram, even allowing for the existence of a neutron star up to 2.5 Solar masses, for which a possible candidate was observed in recent gravitational wave event GW190814 [14].

Being aware of connection between neutron star radius and neutron skin of nuclei such as $^{208}$Pb, we would anticipate that introduction of a light boson acting at longer distance would allow to harmonize the relativistic mean field results for neutron skin with recent experimental observations of PREX-2 [15].
Table 1: Constrained parameter sets for equations of state with incompressibilities $K_0=245$, 250, and 260 MeV.

| $K_0$ | $m_v$ | $m_s$ | $g_v$ | $g_s$ | $\kappa$ | $\lambda$ |
|-------|-------|-------|-------|-------|----------|----------|
| MeV   | MeV   | MeV   | MeV   | MeV   | MeV      | MeV      |
| 245   | 17.58 | 25.58 | 0.2407| 0.4666| 0.0039   | -0.001396|
| 250   | 17.58 | 25.58 | 0.2417| 0.4703| 0.00398  | -0.001316|
| 260   | 17.58 | 25.58 | 0.2417| 0.4684| 0.00374  | -0.001204|

2.2. Instantons and chiral symmetry

The analysis, based on the model of QHD, is motivated by a possible existence of the signal of bosonic particle in the decay of $1^+$ excited states of $^8$Be at 17.6 and 18.15 MeV and thus can suggest that it is a vector particle. However, recent reported observation of analogous effect in the decay of $0^-$ excited state of the $^4$He at 21.01 MeV allows explanation in terms of pseudoscalar particle, while the case of $^8$Be can be possibly explained by emission of such particle with orbital angular momentum $l=1$. There exists theory of nuclear force based on exchange of pseudoscalar particle, a pion. Pion is a hadronic particle observed in particle physics, and it is also related to both spontaneous and explicit breaking of chiral symmetry, with its mass of $m_\pi=135$ MeV being a measure of the latter effect. In this respect, the reported observation of pseudoscalar bosonic particle with rest mass of $m_X=17$ MeV might suggest that for whatever reason, the chiral symmetry is restored in the case of low energy nucleon-nucleon interaction at large distances.

This is quite surprising possibility, which could be possibly related to properties of the QCD vacuum. There exists a theory of gluonic solutions of QCD equations called instantons [16]. Such QCD vacuum consists of granular configuration, where instantons are regions with gluonic field with radius of 1/3 fm, with typical distance around 1 fm. These gluonic formations can be understood as tunneling events where quarks can be hopping between instantons, in analogy with conducting electrons in metals. The model of random instanton liquid allows to describe phenomenologically the experimental values of gluonic and quark condensates, and leads to chiral symmetry breaking (for review see e.g. [17], [18]). It can be also used to derive the properties of mesons, in particular of the pseudoscalar meson. The formula for mass of pion derived using the instanton model reads [19]

$$m^2_\pi = \frac{m}{f}$$

(3)

where $m$ can be related directly to the mass of fermionic particle (quark) [20] and $f$ is the expectation value of scalar field in the minimum of the Mexican hat potential.

In traditional theory of chiral symmetry such as linear $\sigma$-model (see e.g. review [21]), $m$ is interpreted as a measure of explicit violation of chiral symmetry, and the value
of $f$ is taken equal to pion decay constant $f_\pi = 93$ MeV, which is also equal to vacuum expectation value (VEV) of scalar field. Compared to the experimental pion mass around $m_\pi = 135$ MeV the mass of pseudoscalar particle $m_X = 17$ MeV means that, as two opposite extremes, either the $m$ (and thus possibly the quark mass $m_q$) gets 63 times smaller or the value of $f$ is 63 times larger. In this respect, it is remarkable that the ratio of dynamical quark mass around $m_{q,dyn} = 310$ MeV to current quark mass of around $m_{q,curr} = 5$ MeV fits this value almost perfectly:

$$\frac{m_X^2}{m_{q,curr}} \simeq \frac{m_\pi^2}{m_{q,dyn}}$$  \hspace{1cm} (4)

This appears to signal that as explicit breaking of chiral symmetry is restored from dynamical mass scale down to current mass scale, and also the properties of pseudoscalar particle get closer to Goldstone boson. In theory of chiral symmetry the axial current of pion is usually assumed to counterbalance the axial current of nucleon, so that total axial current is conserved (see also [21]). The relation (1) then suggests that the same outcome is reached when considering sum of axial current of current quark and axial current of 17 MeV boson. The 17 MeV boson thus appears to restore the symmetry of QCD in the vacuum, where current quarks are considered as a degree of freedom. Based on this, it is possible to derive an analogue of the Goldberger-Treiman relation [22]

$$g_{Xqq} = \frac{g_A m_{q,curr}}{f_\pi}$$  \hspace{1cm} (5)

where $g_{Xqq}$ is the coupling of $X$-boson to current quark, $g_A$ is renormalization factor of axial current and $f_\pi$ was adopted as proportionality factor also for axial current of $X$-boson. The value of coupling $g_{Xqq} = 0.07$ is obtained, what suggests the coupling with nucleon $g_{XNN} = 0.21$. In this respect, it is apparent that also the results using relativistic mean field theory of nuclear force, presented above, point practically in the same direction, since the value of coupling corresponding to vector mesonic mass of 17 MeV is $g_v = 0.24$, while the value of coupling $g_v = 10-20$, corresponding to vector meson mass of 783 MeV is in good agreement with value $g_{\pi NN} = 13$ for experimental coupling of pion to nucleon, confirmed also by original Goldberger-Treiman relation. Similar value of coupling for low-energy QCD is actually obtained also in the calculation using the model of spaghetti vacuum [23]. When adopting the value of $f_\pi$ in the denominator of eq. (3), the value of $m$, a quark condensate, drops from the value of $m = (120$ MeV$)^3$ for $m_\pi = 135$ MeV down to the value of $m = (30$ MeV$)^3$ for $m_X = 17$ MeV. Since the value of $m$ scales with quark mass, the relation in eq. (4) depends only on vacuum expectation value $f_\pi$, what explains the validity of this relation.

This situation depicted in the Fig.1, which compares two trends of mesonic versus quark masses. First trend combines formulas (2) and (5) under assumption that the obtained coupling correspond to $g_{XNN} = 3g_{Xqq}$. Second trend represents formula (4). Both trends cross around the value of $m_X = 17$ MeV and $m_q = 5$ MeV.
Possible observation of a boson with mass $m_X=17$ MeV and the relations (4) and (5) thus suggests existence of a new scale in QCD, with chiral symmetry practically restored. The reason for apparent restoration of chiral symmetry suggested above is a matter of discussion. It might be related to the interaction between nucleons at distance, mediated by instantons, where possibly the density of instantons mediating interaction in a region between nucleons becomes smaller and possibly a bounce into a false instanton vacuum (a quantum mechanical tunneling into a metastable vacuum and back), a mechanism based on instanton model [24], can occur. Such a state might obviously differ from a stable QCD vacuum, which is satisfactorily described by the lattice QCD (for review see e.g. [25]). The relations (4) and (5) will characterize such state and the mechanism of explicit chiral symmetry breaking may be substantially weaker or nonexistent. Assumption of a bounce into a false vacuum state might sound as speculation, but one should remember that a long-lived practically unbound system of nucleons and clusters - a nuclear molecule - with sufficient excitation energy to produce particle with rest mass of 17 MeV, is a unique system, reported exclusively in highly excited states of light nuclei like $^4\text{He}$ and $^8\text{Be}$. This might be practically the only opportunity to study long range interaction between nucleons on long timescale (compared to typical nuclear time of the order of $10^{-23}$ s), which was not explored in detail yet and thus surprises can not be excluded.

Figure 1: (Color online). Trends of mesonic versus quark masses. First trend combines formulas (2) and (5) as explained in the text. Second trend represents formula (4).
3. Conclusions

To summarize, we present a hypothesis that the anomaly in the folding angle distribution of electron-positron pairs, emitted in the decay of the excited level of nucleus $^8$Be at 18.15 MeV \cite{1}, can be related to the cluster structure of the decaying state. Furthermore, based on subsequent reported observation of similar anomaly in decay of excited state of $^4$He \cite{2}, which can be represented as a cluster state $p+^3$H, we present a hypothesis that the potentially reported boson with rest mass $m_X=17$ MeV can be an exchange boson mediating the nucleon-nucleon interaction at the low-energy regime of QCD. We also present a range of possible equations of state of symmetric nuclear matter based on this hypothesis and obtained using relativistic mean field theory of nuclear force, QHD-I in particular, including the equations of state of physical interest with incompressibilities $K_0=240-260$ MeV. The value of coupling $g_v$, corresponding to the meson masses $m_v=17$ MeV is lower than unity. Based on concepts of instanton liquid model with resulting chiral symmetry breaking, we show that reduction of the rest mass of pseudoscalar particle from physical value $m_\pi=135$ MeV to $m_X=17$ MeV is equivalent to reduction of the quark mass from dynamical value around 310 MeV down to current quark mass $\frac{m_X^2}{m_{q,curr}} \simeq \frac{m_{q,dyn}^2}{m_{q,dyn}}$. Assumption of conservation of axial current and resulting variant of Goldberger-Treiman relation leads to the value of coupling close to the results from relativistic mean field theory of nuclear force. Both model approaches thus point towards apparent restoration of chiral symmetry in nucleon-nucleon interaction at large distances, possibly via bounce into state of a false QCD vacuum, a mechanism based on the instanton model. Possible existence of boson with rest mass $m_X=17$ MeV in the decay of high lying excited states of $^8$Be and $^4$He, could shed new light in one of the long lasting open questions in nuclear physics.

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