Inter-rater reliability (IRR) has been the prevalent quality and precision measure in ratings from multiple raters. However, applicant selection procedures based on ratings from multiple raters usually result in a binary outcome. This final outcome is not considered in IRR, which instead focuses on the ratings of the individual subjects or objects. In this work, we outline how to transform the selection procedures into a binary classification framework and develop a quantile approximation which connects a measurement model for the ratings with the binary classification framework. The quantile approximation allows us to estimate the probability of correctly selecting the best applicants and assess error probabilities when evaluating the quality of selection procedures using ratings from multiple raters. We draw connections between the inter-rater reliability and the binary classification metrics, showing that binary classification metrics depend solely on the IRR coefficient and proportion of selected applicants. We assess the performance of the quantile approximation in a simulation study and apply it in an example comparing the reliability of multiple grant peer review selection procedures.

Keywords: Error rate; mixed-effect models; rating, type I error, type II error.
1 Introduction

Applicant selection procedures typically use ratings to assess applicant quality, but they generally result in a binary outcome: an applicant is either hired or refused, a grant proposal is either funded or dismissed, and an article is either accepted or rejected (e.g., Cicchetti, 1991). However, the most commonly used metric for evaluating the quality of selection procedures, inter-rater reliability (IRR), does not focus on the outcome of selection procedures: whether the best applicants, grants proposals, or articles (further referred to as applicants) were selected. Instead, IRR focuses on scores or ratings of the applicants which form an important but not the final step of the selection procedures.

This is, of course, not a novel observation and the disconnect between evaluating raters’ agreement and whether the best applicants were selected was noted earlier (e.g., Kraemer, 1991; Mayo et al., 2006; Nelson, 1991). In this work, we try to seal this gap by developing a quantile approximation that links the ratings’ measurement model to the expected classification probabilities of selecting the best applicants. Subsequently, the selection procedures can be characterized as binary classification and evaluated via well-known, easy-to-interpret, and tangible metrics such as sensitivity or false positive/negative rates. Furthermore, the binary classification framework allows researchers and stakeholders to evaluate and improve selection procedures while incorporating the costs of incorrect decisions, increasing the number of raters or modifying the rating procedure.

While our approach is related to univariate classification under measurement error, this specific use case is different. In comparison to typical classification tasks which aim to separate subjects into different categories (Duda, Hart, et al., 2006), we assume the existence of a single continuous latent trait measured by the ratings. Then, we select a proportion of the best applicants defined by the latent trait and evaluate the (miss)classification probabilities due to the measurement error contained in the observed ratings.

The paper proceeds as follows: First, we focus on the proposed methods: we define a minimal measurement model for ratings in the selection procedures, we characterize the selection procedures in terms of their outcome – the binary classification, we propose a quantile approximation connecting the measurement model with the binary classification, and we show the relationship between IRR and the binary classification metrics. Second, we evaluate the quality of the quantile approximation in a simulation study, comparing the empirical true positive rate to the estimated true positive rate. Third, we demonstrate the outlined methodology on an example of multiple grant peer review, where we compare multiple selection procedures in terms of their false positive rate and show how it changes with increasing the number of raters and IRR. We close with a discussion of limitations and extensions.

2 Methods

2.1 Measurement Model

We consider a minimal selection procedure that evaluates \( N \) applicants with the goal of estimating their latent ability \( \gamma \). In the selection process, each applicant is rated \( J \) times, resulting in a vector of observed scores \( y_{ij} \) for \( i = 1, \ldots, N \) and \( j = 1, \ldots, J \). The resulting measurement model can be written as

\[
y_{ij} = \gamma_i + \epsilon_{ij},
\]

with the usual assumptions that (a) the measurement error \( \epsilon_{ij} \) of each applicant is normally distributed with mean zero and variance \( \sigma^2_\epsilon \) and (b) the latent ability \( \gamma \) is normally distributed with mean \( \mu \) and variance \( \sigma^2_\gamma \).

The quality of the selection procedure is usually measured as inter-rater reliability (sometimes also referred to as single-rater IRR), which is the inter-class correlation coefficient (ICC(1,1) McGraw & Wong, 1996, Shrout & Fleiss, 1979)

\[
\text{IRR}_1 = \frac{\sigma^2_\gamma}{\sigma^2_\gamma + \sigma^2_\epsilon}.
\]

When the final decisions are based on the average of \( J \) raters, the multiple-rater IRR can be calculated using the Spearman-Brown formula as

\[
\text{IRR}_J = \frac{\sigma^2_\gamma}{\sigma^2_\gamma + \frac{\sigma^2_\epsilon}{J}} = \frac{J \, \text{IRR}_1}{J \, \text{IRR}_1 + 1 - \text{IRR}_1}.
\]

\[\text{IRR}_1\] This is a different goal than measuring agreement of multiple raters performing binary decisions using e.g., Cohen’s Kappa (Cohen, 1960), that, again, measures agreement and not whether the best applicants were selected.
2.2 Binary Classification

The endpoint of the selection procedure is, however, not estimating the latent abilities \( \gamma_i \) per se but selecting \( k \) applicants with the highest latent ability \( \gamma \). Consequently, we can divide the applicants into two groups: high ability applicants \( A \) that consist of \( k \) applicants with the highest latent ability \( \gamma \) and “not high ability applicants” \( \neg A \) that consist of the remaining \( N - k \) applicants. Denoting the selected applicants by \( S \) and the remaining applicants by \( \neg S \), we can characterize the selection procedure by probabilities of the different types of (mis)classification (Table 1).

|                | Selected (S) | Not Selected (\( \neg S \)) |
|----------------|--------------|-----------------------------|
| High Ability Applicants (A) | \( P(S \cap A) \) | \( P(\neg S \cap A) \) |
| Not High Ability Applicants (\( \neg A \)) | \( P(S \cap \neg A) \) | \( P(\neg S \cap \neg A) \) |

Table 1: Overview of the selection procedure as a binary classification.

Since the unconditional probability of an applicant being selected corresponds, by definition, to the unconditional probability of an applicant having high ability, \( P(S) = P(A) = \frac{k}{N} \), the whole classification process is determined by the probability of correctly selecting the high ability applicants \( P(S \cap A) \), i.e., by the probability of true positives classifications (Table 2).

\[
\begin{array}{ccc}
\text{High Ability Applicants (A)} & \text{Selected (S)} & \text{Not Selected (\( \neg S \))} \\
\text{Not High Ability Applicants (\( \neg A \))} & \frac{k}{N} - P(S \cap A) & P(S \cap \neg A) + \frac{(N-k)}{N} \\
\end{array}
\]

Table 2: Simplification of the binary classification in selection procedures.

Subsequently, if we knew the probability of true positive classification, we could use standard metrics for evaluating binary classification (e.g., Pepe, 2003, pp. 14-33). Since the off-diagonal probabilities of the classification outcomes are equal (Table 2), many commonly specified metrics become equal or simplify to a constant. For example, true positive rate (TPR, aka sensitivity) and positive predictive value (PPV, aka precision) are equal,

\[
\text{TPR} = \text{PPV} = \frac{P(S \cap A)}{\frac{k}{N}},
\]

and true negative rate (TNR, aka specificity) is equal to \( \frac{1}{2} \). We can also compute the false positive rate (FPR, corresponding to type I error rate),

\[
\text{FPR} = \frac{\frac{k}{N} - P(S \cap A)}{\frac{k}{N}},
\]

or false negative rate (FNR, corresponding to type II error rate),

\[
\text{FNR} = \frac{\frac{k}{N} - P(S \cap A)}{\frac{(N-k)}{N}}.
\]

Alternatively, the classification probabilities can be used with a utility function specifying the cost of each type of error (e.g., Metz, 1978).

2.3 Quantile Approximation

The probability of true positive classification is, unfortunately, not directly estimable from the observed scores \( y_{ij} \), unless we know the true latent ability \( \gamma_i \) of each applicant. To deal with this hindrance, we propose a quantile approximation that allows us to estimate the true positive classification probability indirectly. Specifically, we leverage distributional assumptions of the measurement model (Equation 1), our ability to estimate the populational parameters of the measurement model, and one peculiarity of the selection procedure: the fact that we set the unconditional probability of the applicant with high ability, \( P(A) \) and the unconditional probability of the applicant selected, \( P(S) \), when designing the selection procedure.

In selection procedures the high ability applicants are defined by their ranking on the latent ability \( \gamma \). However, we only measure the observed scores \( y_{ij} \) when performing the selection. According to the measurement model, the observed scores \( y_{ij} \) are an imprecise reflection of the latent abilities \( \gamma_i \) which is disturbed by the measurement error \( \sqrt{\sigma_e} \). The \( J \) observed scores of each applicant are usually aggregated into a mean score \( y_i \) with the resulting measurement error
Importantly, the latent abilities $\gamma$ and the random variable of the aggregated scores $\bar{Y}$ are jointly distributed with bivariate normal density:

$$\begin{bmatrix} \gamma_i \\ \bar{y}_i \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \sigma_\gamma^2 + \sigma_\gamma^2/J \\ \sigma_\gamma^2 & \sigma_\gamma^2 / J \end{bmatrix} \right),$$

(e.g., Searle et al., 2006, pp. 258-259).

Since the goal of selection procedures is selecting the high ability $k$ applicants and since the desirability of applicants is defined by their latent ability $\gamma$, the high ability applicants are defined as those with the highest $k$ latent ability scores $\gamma$. Then, we can think about a cut-score on the latent ability $\gamma_c$ that separates the high ability applicants from the remaining applicants (with the high ability applicants satisfying the condition $\gamma_i > \gamma_c$). Similarly, since the selection procedure is performed with the observed aggregated scores $\bar{y}_i$ and since we select the high ability $k$ applicants according to their observed aggregated scores $\bar{y}_i$, the selected applicants are defined as those with the highest $k$ observed scores $\bar{y}_i$. Then, again, we can think about a cut-score on the observed aggregated scores $\bar{y}_c$ that separates the high ability applicants from the remaining applicants (with the high ability applicants satisfying the condition $\bar{y}_i > \bar{y}_c$).

In any particular data set, the cut-scores $\gamma_c$ and $\bar{y}_c$ are dependent on the actual latent abilities $\gamma_i$ and observed aggregated scores $\bar{y}_i$ of applicants participating in the given selection procedure. However, under the assumption of the measurement model, we can approximate the cut-scores with marginal quantile functions of the joint distribution (Equation 8), yielding the quantile approximated cut-scores

$$\bar{y}_c = \mu + \sqrt{\frac{\sigma_\gamma^2}{\mathbf{I} + \frac{\sigma_z^2}{\sigma_\gamma^2}} \Phi^{-1} \left( \frac{N - k}{N} \right)}$$

and

$$\gamma_c = \mu + \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\gamma^2/J} \Phi^{-1} \left( \frac{N - k}{N} \right),$$

with $\Phi$ corresponding to a cumulative distribution function of a standard normal distribution.

Subsequently, the true positives classification probability in Table 2 defining the binary classification metrics, can be approximated using the latent ability $\gamma$, its imperfect measurement via the random variable of aggregated observed scores $\bar{Y}$, and the quantile approximated cut-scores $\gamma_c$ and $\bar{y}_c$,

$$P(\bar{S} \cap A) \approx P(\gamma > \gamma_c, Y > \bar{y}_c).$$

In other words, the probability of an applicant being amongst both the high ability and selected applicants can be approximated by the probability of both having latent ability higher than the approximated cut-score on the latent ability and having the observed aggregated score higher than the approximated aggregated cut-score.

Finally, the true positives classification probability can be approximated solely with the populational parameters and integrating over the joint bivariate normal density

$$P(\gamma > \gamma_c, Y > \bar{y}_c) = \int_{\gamma_c}^{\infty} \int_{\bar{y}_c}^{\infty} N \left( \begin{bmatrix} \gamma \\ \bar{Y} \end{bmatrix} \mid \begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \sigma_\gamma^2 + \sigma_\gamma^2/J \\ \sigma_\gamma^2 & \sigma_\gamma^2 / J \end{bmatrix} \right) d\bar{Y} d\gamma.$$

### 2.4 Relationship between IRR and Binary Classification

We can further utilize the quantile approximation to draw connections between IRR and the true positives classification probability of the binary classification.

We subtract the grand mean $\mu$ from both the latent abilities $\gamma$ and the observed aggregated scores $\bar{Y}$ and standardize the random variables by total variance of the observed aggregated scores $\sqrt{\sigma_\gamma^2 + \sigma_\gamma^2/J}$, transforming the latent abilities into $\zeta = (\gamma - \mu)/\sqrt{\sigma_\gamma^2 + \sigma_\gamma^2/J}$ and the observed aggregated scores into $Z = (\bar{Y} - \mu)/\sqrt{\sigma_\gamma^2 + \sigma_\gamma^2/J}$. Subsequently, the bivariate normal density of the transformed abilities and the random variable of transformed observed aggregated scores simplifies to

$$\begin{bmatrix} \zeta_i \\ \bar{z}_i \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \text{IRR}_J & \text{IRR}_J \\ \text{IRR}_J & 1 \end{bmatrix} \right),$$

and yields the quantile approximated cut-scores of the transformed abilities and observed aggregated measures

$$\tilde{\zeta}_c = \text{IRR}_J \Phi^{-1} \left( \frac{N - k}{N} \right),$$

$$\tilde{\bar{z}}_c = \Phi^{-1} \left( \frac{N - k}{N} \right).$$
Therefore, the true positives classification probability can be directly estimable with only $\text{IRR}_j$ and the proportion of selected applicants $k$,

$$P(\zeta > \tilde{\zeta}, Z > \tilde{Z}) = \int_{\tilde{\zeta}}^{\infty} \int_{\tilde{Z}}^{\infty} N\left(\begin{bmatrix} \zeta \\ Z \end{bmatrix} | \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \text{IRR}_j & \text{IRR}_j \\ \text{IRR}_j & 1 \end{bmatrix} \right) dZ d\zeta.$$

This allows us to retrospectively evaluate selection procedures in the binary classification framework without access to the primary data.

### 3 Simulation

We conducted a simulation study to assess the quality of the quantile approximation to the true positives classification probability using binary classification. Specifically, we compared the $P(S \cap A)$ based on the empirical (true) classification labels comparing the simulated abilities $\gamma_i$ vs the aggregated ability estimates $\hat{y}_i$ with the true positives classification probability obtained by quantile approximation $P(\gamma > \tilde{\gamma}, Y > \tilde{y})$ based on $\hat{\sigma}_\gamma^2$ and $\hat{\sigma}_\gamma^2$ estimates from a linear mixed model.

We simulated data from the measurement model defined by Equation (1) with settings inspired by Martinková et al. (2018). We manipulated the inter-class correlation coefficient, $\text{IRR}_1 = \{0.15, 0.30, 0.45\}$ (fixing the overall variance $\sigma^2$ to 1), number of applicants, $N = \{100, 300, 1000\}$, and the number of ratings, $J = \{3, 5, 10\}$. We replicated each simulation condition 1000 times and estimated the variance parameters $\sigma_\gamma^2$ and $\sigma_{\gamma}^2$ using linear models implemented in the lme4 R package (Bates et al., 2015). In the remainder of the section, we only discuss results based on $N = 100$ applicants but see (Appendix A) for similar results with $N = 300$ and $N = 1000$.

Figure 1 visualizes the mean empirical true positive classification probabilities $P(S \cap A)$ (full red line) vs the mean quantile approximated true positive classification probabilities $P(\gamma > \tilde{\gamma}, \gamma > \tilde{\gamma})$ (dashed blue line) across all possible proportions of selected applicants ($\hat{\gamma}/N$, $x$-axis). The results are presented for different inter-rater reliability coefficients ($\text{IRR}_1$, rows), the number of ratings ($J$, columns), and 100 applicants. The $x$-axis corresponds to the proportion of selected applicants ($\hat{\gamma}/N$). The diagonal red dotted line corresponds to the maximum probability of the true positive classification probability at a given proportion of selected applicants. The mean of the quantile approximation closely resembles the mean of the true positive classification probabilities – making the two lines essentially indistinguishable – for all but the smallest number of raters and the lowest $\text{IRR}_1$ (which resulted in a slight positive bias for a low proportion of selected applicants and a slight negative bias for a high proportion of selected applicants). However, the performance of the approximation improved with increasing number of ratings and inter-rater reliability as well as with the increasing number of applicants.

Importantly, the quantile approximation estimates the expected classification probabilities for a selection procedure with given populational characteristics. In other words, the quantile approximation might provide noisy estimates of the true values for any particular selection procedure, especially when the classification probabilities depend on a small number of selected applicants. Figure 2 visualizes the root mean square error (RMSE) of the quantile approximation to the true positive classification probabilities across all possible proportions of selected applicants ($\hat{\gamma}/N$, $x$-axis). The results are presented for different inter-rater reliability coefficients ($\text{IRR}_1$, rows), number of ratings ($J$, columns), and 100 applicants. The $x$-axis corresponds to the proportion of selected applicants ($\hat{\gamma}/N$). Note that the true positive classification probability is bounded at the endpoints – it is zero if no candidate is selected and it is one if all candidates are selected. As such, the RMSE of the true positive classification probability is necessarily higher above the middle proportion of selected applicants since it can attain the widest range of values. We, again, see a high RMSE for a condition with a low number of ratings and the inter-rater reliability ($J = 3$ and $\text{IRR}_1$) with an odd shape at the maximum proportion of selected applicants (due to the bias). However, the RMSE of the approximation, again, improves with the increasing number of ratings and inter-rater reliability as well as with the increasing number of applicants.

#### 3.1 Confidence and Prediction Intervals

Dependency on any particular selection procedure can be further illustrated by visualizing the empirical vs quantile approximated results for a single trial. For example, consider the false positive rate, the probability an applicant is selected although they are not part of the high ability group (a transformation of the previously summarized true positive

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2 Whereas the measurement model is defined in terms of the grand mean $\mu$ and variances $\sigma_\gamma^2$ and $\sigma_{\gamma}^2$, Equation (8) shows that $\text{IRR}$ is the only relevant quantity for the quantile approximation.
Figure 1: Mean empirical true positive classification probabilities for 100 applicants (full red line) vs the mean quantile approximated true positive classification probabilities (dashed blue line) across all possible proportions of selected applicants (x-axis). The diagonal shows the maximum true positive classification probability for a given proportion of selected applicants. Results are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).

classification probability, Equation 3). Figure 3 compares the quantile approximated false positive rate (full blue line) to a random simulation’s empirical false positive rate (full black line) with \( J = 5 \), \( IRR_1 = 0.30 \), and \( N = 100 \).

The empirical false positive rate wildly oscillates between 0 and 1, especially when considering the selection of only a small number of applicants (x-axis). These wild oscillations, accompanied by the extremely wide 95% prediction interval (blue dotted line; based on quantile function of binomial distribution), are an inherent property of summarizing the proportion of successes of a binomial outcome with a small number of trials. In contrast, 95% confidence intervals of the false positive rate (blue dashed lines) which quantifies the uncertainty of the false positive rate estimate (based on non-parametric bootstrap) is wider in the lower proportion of the selected candidates and shorter around the endpoints. This is a result of the false positive rate estimate being bounded at the endpoints, with higher uncertainty in the lower proportion of selected applications – as it is a transformation of the true positive classification probability.
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Figure 2: Root mean square error of quantile approximation to the true positive classification probabilities for 100 applicants across all possible proportions of selected applicants (x-axis). Results are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).

4 Example: Grant Peer Reviews

We illustrate the methodology by estimating binary classification metrics for several grant peer reviews. See Table 3 for characteristics of the grant peer reviews as summarized in Table 1 of Erosheva et al. (2021) and extended by the results reported therein. It is important to note that the reported values are averages, often aggregated across disciplines and years (sometimes accompanied by a range of values). The results should, therefore, be taken as an illustration showcasing the possible interpretation and inferences rather than an evaluation of the funding agencies’ grant review process.

Figure 4 visualizes the false positive rate estimates for each grant peer review procedure (different colors) based on quantile approximation. Thin full lines visualize the computed false positive rate across the range of possible proportions of selected applicants, thick full lines (and points) highlight the ranges (and values) of the actual proportion of selected applicants. We see the estimated false positive rate ranges from 0.75 for the lowest proportion of selected applicants (5%), lowest inter-rater reliability (0.14), and two rates in the AIBS 2009-11 grant proposals data to 18% for

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We focus only on studies that estimated IRR using all applicants to prevent bias from the restricted range (Erosheva et al., 2021).
The highest proportion of selected applicants (0.51), high inter-rater reliability (0.37) and more than four raters in the NSF & COSPUP (1985) grant proposals data.

It is important to highlight that most of the differences in false positive rate can be ascribed to the difference in the proportion of selected applicants. Although the false positive rates differ by 58 percentage points across the example data sets (37 percentage points excluding the lower IRR estimates of NSF & COSPUP and AIBS), comparing the grant proposal selection procedures at an equal proportion of selected applicants (across the 5% – 53% range of selected applicants) reduces the maximum difference to 30 percentage points (15 percentage points excluding the lower IRR estimates of NSF & COSPUP and AIBS; both at 5% of selected applicants).

4.1 In Detail Assessment of NIH (2014-16)

We can take a closer look at the NIH data set of Erosheva et al. (2021) where the reported IRR$_1 = 0.34$ was accompanied by 95% CI $[0.31, 0.37]$. The corresponding estimate of the false positive rate is 0.397, and as in the simulation study, we can transform the 95% CI of IRR$_1$ to obtain the 95% CI interval $[0.380, 0.416]$ and the 95% prediction interval $[0.331, 0.465]$ for the false positive rate. We can also compute the false negative rate estimate (i.e., type II error rate, Equation 4) of 0.087, 95% CI $[0.083, 0.091]$.

We might consider redesigning the selection procedure to achieve lower false positive rate. This can be achieved by increasing the number of raters (left panel of Figure 5) or modifying the rating protocol to improve the raters agreement.
Figure 4: Estimates of the false positive rate for grant peer review procedures based on quantile approximation for published estimates of IRR and number of raters $J$ of individual studies (different colors). Thin full lines visualize the estimated false positive rate across the range of possible proportions of selected applicants, and thick full lines (and points) highlight the ranges (and values) of the actual proportion of selected applicants. For NSF & COSPUP and AIBS, the lower IRR$_J$ estimate, resulting in a higher false positive rate, is shown as dotted lines / empty circles, whereas the higher IRR$_J$ estimate is shown as full lines / full circles.

Figure 5: Change in false positive rate when increasing the number of raters for different levels of IRR$_1$ (left) or increasing the IRR$_1$ for different number of raters (right) for a selection procedure based on NIH (2014-16) grant peer review. The original trial selected 18% out of 2076 applicants with $J = 2.79$ raters (black line in the right panel) and IRR$_1 = 0.34$ (black line in the left panel). Join increase of IRR$_1$ and the number of raters is depicted in orange and red color.
IRR\textsubscript{1} (right panel of Figure 5). Even though both changes lead to an increase in the overall inter-rater reliability, IRR\textsubscript{1}, and consequently lower false positive rate, each change corresponds to a different modification of the peer review process (i.e., hiring more raters vs training the raters).

The black line in Figure 5 visualizes changes in the false positive rate when increasing the number of raters (left) or the IRR\textsubscript{1} (right). The added orange and red lines show an additional change in the false positive rate when altering both factors simultaneously. We see that for a selection procedure resulting in the selection of 18% applicants, the false positive error rate remains relatively high even when increasing both the number of raters and IRR\textsubscript{1}. Similar analysis can further incorporate the cost of each modification and weight it against the benefit of decreasing the false positive rate of the selection procedure.

## 5 Discussion

We outlined an approach for evaluating ratings based selection procedures in a binary classification framework. This approach allows researchers and stakeholders to assess the quality of selection procedures by linking rating agreement to the actual selection decisions. As a result, the quality of selection procedures does not need to be evaluated only via a coefficient of the inter-rater agreement but also via the desired false (true) positive / negative rates and their associated costs. These results can be used in more complex utility functions also featuring costs of increasing the number of raters, modifying the rating guidelines, and changing the proportion of selected applicants.

The approach is based on a minimal measurement model, commonly used for assessing inter-rater reliability, and linked to the binary decisions via a quantile approximation. We showed how to compute the probability of correct classification, other binary classification metrics, and showed how the binary classification relates to inter-rater reliability via the quantile approximation relying only on the number of ratings and proportion of selected applicants.

We evaluated the quantile approximation in a simulation study and found little bias. Importantly, the estimated classification probabilities and resulting binary classification metrics correspond to the expectation of the selection procedure. The actual classification probabilities for any given selection procedure might be far from the expectations, especially when considering only a small proportion of selected applicants – as the results are dependent on a few binary events that are inherently noisy.

We showed the methodology in an example comparing false positive rates across multiple grant peer reviews. The results of our example should be interpreted with great caution. First, we based our results on averages across different fields that often vary in the inter-rater reliability, number of raters per proposal, and the number of selected applicants. Second, the grant agencies often base the funding decisions on a combination of the overall rating score and additional information (which is not included in our calculation). While acknowledging the limitations, the results still indicate a relatively high false positive rate, especially in selection procedures selecting a small portion of the applicants.

The outlined approach only evaluates the reliability of selecting the high ability applicants due to the measurement error, while assuming that the rating process is valid and indeed measuring the relevant latent abilities (Bornstein, 1991). Furthermore, one might argue against optimizing raters’ agreement as the disagreement between raters might indicate that raters were selected on diverse bases and focused on different aspects of applicants (e.g., Bailar, 1991; Hargens, 1991; Kiesler, 1991; Lee, 2012).

The outlined approach could be further expanded in multiple directions. First, we used a minimal measurement model for assessing the inter-rater reliability of continuous ratings. However, each parameter of the measurement model might differ across groups of applicants (e.g., Bartoš et al., 2019; Martinková et al., 2018; Martinková et al., 2022; Mutz et al., 2012). Additional variance components might be used for modeling dependencies in the data (e.g., the effect of raters Martinková et al., 2018). Different applicants might be rated a different number of times, the measured rankings might not be assumed to be normally distributed (e.g., see Pearce & Erosheva, 2022 for joint modeling of continuous and ordinal data), and the latent abilities might follow a different distribution as well. All these possibilities can be included in the measurement model and further propagated through the quantile approximation, either by using a mixture of distributions for the latent abilities based on the proportion of groups and group specific parameters, specifying a different type of distributions and measurement error for the observed rankings, and different types of distributions for the latent abilities.

Second, we focused on classification probabilities and the resulting binary classification metrics themselves. Under the specified measurement model, most erroneous classifications happen close to the selection boundary. Consequently, evaluating selection procedures solely via a false positive rate does not reflect the loss associated with the degree of the
errors. A further extension might consider weighting the classification probabilities by a utility function accounting for the degree of the error (i.e., mistakenly refusing an applicant who is right above the classification threshold is less costly than refusing an applicant much higher on the latent ability).

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Supplementary material

The analysis and simulation scripts are available at https://osf.io/674fk/.

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Appendix A: Further Simulation Results

Figure 6: Mean empirical true positive classification probabilities for 300 applicants (full red line) vs the mean quantile approximated true positive classification probabilities (dashed blue line) across all possible proportions of selected applicants (x-axis). The diagonal shows the maximum true positive classification probability for a given proportion of selected applicants. Results are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).
Figure 7: Mean empirical true positive classification probabilities for 1000 applicants (full red line) vs the mean quantile approximated true positive classification probabilities (dashed blue line) across all possible proportions of selected applicants (x-axis). The diagonal shows the maximum true positive classification probability for a given proportion of selected applicants. Results are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).
Figure 8: Root mean square error of quantile approximation to the true positive classification probabilities across all possible proportions of selected applicants (x-axis). Results shown for 300 applicants and are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).
Figure 9: Root mean square error of quantile approximation to the true positive classification probabilities across all possible proportions of selected applicants ($x$-axis). Results shown for 1000 applicants and are presented for different inter-rater reliability coefficients (rows) and the number of ratings (columns).