(SUB-SYSTEMIC) QUANTUM ZENO EFFECT AND SINGLE PHOTON INTERFERENCE AT BEAM SPLITTER

Vladan Panković

Department of Physics, Faculty of Sciences,
21000 Novi Sad, Trg Dositeja Obradovića 4, Serbia
vladan.pankovic@df.uns.ac.rs

Abstract

In this work we consider quantum dynamical interaction of single photon with beam splitter after which well-known superposition between reflected and transmitted photon appears. Later reflected photon is absorbed by one and transmitted photon is absorbed by other, distant of two two-level systems of the same type (prepared initially in ground states). It implies entanglement between mentioned two two-level systems. Any two-level system in excited state represents an unstable system that quantum dynamically evolves in a superposition of the initial excited (non-decayed) and ground (decayed) state. We consider frequent detections at only one of two two-level systems and we prove that here a modification of the usual quantum Zeno effect called sub-systemic quantum Zeno effect appears. Namely after mentioned frequent detections two two-level systems are in a specific entangled state with two terms. First term (term of the “frozen” evolution) includes detected two-level system in initial excited state and non-detected two-level system in the ground state. Second term (term of free evolution) includes detected system in ground state and non-detected system in free evolved excited state.

Key words: quantum Zeno effect, single photon interference, beam splitter
PACS number: 03.65.Ta
In this work we consider quantum dynamical interaction of single photon with beam splitter after which well-known superposition between reflected and transmitted photon appears [1]. Later reflected photon is absorbed by one and transmitted photon is absorbed by other, distant of two two-level systems of the same type (prepared initially in ground states). It implies entanglement between mentioned two two-level systems. Any two-level system in excited state represents an unstable system that quantum dynamically evolves in a superposition of the initial excited (non-decayed) and ground (decayed) state. We consider frequent detections at only one of two two-level systems and we prove that here a modification of the usual quantum Zeno effect (at simple quantum system [2] or at entangled quantum systems [3]) called sub-systemic quantum Zeno effect appears. Namely after mentioned frequent detections two two-level systems are in a specific entangled state with two terms. First term (term of the “frozen” evolution) includes detected two-level system in initial excited state and non-detected two-level system in the ground state. Second term (term of free evolution) includes detected system in ground state and non-detected system in free evolved excited state.

In well-known experiment of the single photon interference at beam splitter [1] source emits single photon P with wave-length \( \lambda \) that propagates toward beam splitter. After quantum dynamical interaction between photon and beam splitter photon is described by the following quantum superposition state

\[
|F,P> = i \ 2^{-1/2} |R(t),P> + 2^{-1/2} |T(t),P>
\]

where \(|R(t),P>\) represents reflected quantum state and \(|T(t),P>\) - transmitted quantum state. (Suppose, for example, that reflected state describes photon that propagates along reflected trajectory in y-axis direction while transmitted state describes photon that propagates along transmitted trajectory in x-axis direction.) Superposition (1) means in fact that photon simultaneously propagates along both trajectories, reflected and transmitted.

As it is well-known superposition state (1) stands conserved till realization of the detection by appropriate detection device, e.g. photo-plate. After detection superposition state (1) turns out probabilistically either in reflected state (reflected trajectory) with probability

\[
W_R = |i \ 2^{-1/2}|^2 = 1/2
\]

or in transmitted state (transmitted trajectory) with probability

\[
W_T = |2^{-1/2}|^2 = 1/2
\]

Total detector includes two photo-plates at practically the same distance in respect to beam splitter, first one (perpendicular at y-axis) that detects reflected state and second one (perpendicular at x-axis) that detects transmitted state. Photo-plates can be mutually sufficiently distant so that detection time is much smaller than time of light propagation between photo-plates. When one plate does positive detection other plate does negative detection and vice versa. For this reason total detection can be realized with only one detection plate.
Consider now such change of the experimental arrangement in which mentioned detection plates are changed by two same quantum two-level systems, SR placed at y-axis and ST placed at x-axis. It will be supposed that propagation of both two-level systems in respect to photon can be approximately neglected. Any such two-level system holds ground state \(|G\rangle\) and excited state \(|E\rangle\). (In more accurate notation SR holds ground state \(|G,SR\rangle\) and excited state \(|E,SR\rangle\) while ST holds ground state \(|G,ST\rangle\) and excited state \(|E,ST\rangle\) ) Roughly speaking ground state is quantum dynamically stable and it stands conserved during time, while excited state is quantum dynamically non-stable and it decays during time in ground state with spontaneous emission of single photon with wave-length \(\lambda\) identical to wave-length of \(P\).

Suppose that both two-level systems before quantum dynamical interaction with \(P\) are prepared in the ground state. It implies that before quantum dynamical interaction between photon and both two-level systems quantum super-system \(P+SR+ST\) that includes photon and both two-level systems is described by non-entangled quantum state

\[
(4) \quad |F,P\rangle |G,SR\rangle |G,ST\rangle
\]

Then immediately after strictly deterministic quantum dynamical interaction between photon and both two-level systems (in time moment chosen to be initial) \(P+SR+ST\) is exactly described by the following entangled state

\[
(5) \quad i \ 2^{-1/2} |A(0),P+SR\rangle |G,ST\rangle + 2^{-1/2} |G,SR\rangle |A(0),P+SR\rangle
\]

Here \(|A(0),P+SR\rangle\) represents initial quantum state of the quantum super-system \(P+SR\) (immediately after quantum dynamical interaction between \(P\) and \(SR\)), while \(|A(0),P+ST\rangle\) represents initial quantum state of the quantum super-system \(P+ST\) (immediately after quantum dynamical interaction between \(P\) and \(ST\)). Mentioned quantum states can be formally presented in the following form

\[
(6) \quad |A(0),P+SR\rangle \approx |1(SR),P\rangle |G, SR\rangle
\]

\[
(7) \quad |A(0),P+ST\rangle \approx |1(ST),P\rangle |G, ST\rangle
\]

Here initial excited state of \(SR\) is formally presented by \(|1(SR),P\rangle |G, SR\rangle\) where \(|1(SR),P\rangle\) represents quantum state with single photon localized “inside” \(SR\), while initial excited state of \(ST\) is formally presented by \(|1(ST),P\rangle |G, ST\rangle\) where \(|1(ST),P\rangle\) represents quantum state with single photon localized “inside” \(ST\).

As it is well-known in quantum state (5) it is not exactly determined which two-level system absorbed photon, i.e. which is in excited state. For this reason mentioned quantum dynamical interaction between photon and both two-level systems cannot be treated as the photon trajectory detection.

Quantum states (6), (7) quantum dynamically deterministically evolve during time in the following way

\[
(8) \quad |A(t),P+SR\rangle = \exp \left((H(P+SR)t)/(i\hbar) \right) |A(0),P+SR\rangle
\]
\[ |A(t),P+ST> = \exp \left( \frac{H(P+ST)t}{(ih)} \right) |A(0),P+ST> \]

where \( H \) represents total Hamiltonian observable of the photon and corresponding two-level system. It implies that quantum state (5) quantum dynamically deterministically evolves during time in the entangled quantum state

\[ i \frac{1}{\sqrt{2}} |A(t),P+SR> |G,ST> + \frac{1}{\sqrt{2}} |G,SR>|A(t),P+ST> \]

More accurately, quantum states (6), (7) can be presented in form

\[ |A(t),P+SR> = (a(t) |1(SR),P> + b(t) |\Phi(SR),P>) |G,SR> \]

\[ |A(t),P+ST> = (a(t) |1(ST),P> + b(t) |\Phi(ST),P>) |G,ST> \]

Here quantum state \( |\Phi(SR),P> \) describes single photon “outside” SR, i.e. photon spontaneously emitted from SR (it is in fact a superposition of all possible photon trajectories that start from SR), while quantum state \( |\Phi(ST),P> \) describes single photon “outside” ST, i.e. photon spontaneously emitted from ST (it is in fact a superposition of all possible photon trajectories that start from ST). It implies that photon described by mentioned two quantum states cannot be completely detected by corresponding photo plates but by corresponding detection chambers. Also here \( a(t) \) and \( b(t) \) represent superposition coefficients corresponding respectively to non-decayed (without photon emission) and decayed (with photon emission) quantum state strictly determined by quantum dynamical interaction between photon and corresponding two-level system.

Concrete forms of mentioned superposition coefficients need extremely complex calculations. Nevertheless mentioned superposition coefficients satisfy simple initial conditions

\[ a(0) = 1 \]

\[ b(0) = 0 \]

Moreover mentioned superposition coefficients satisfy simple asymptotic final conditions

\[ a(0) \to 0 \quad \text{for} \quad t \to \infty \]

\[ b(0) \to 1 \quad \text{for} \quad t \to \infty \]

or, in a satisfactory approximation,

\[ a(0) \to 0 \quad \text{for} \quad t \geq \tau \]

\[ b(0) \to 1 \quad \text{for} \quad t \geq \tau \]

where \( \tau \) represents characteristic mean life time strictly determined by quantum
dynamical interaction between electromagnetic field and corresponding two-level system.

All this means that, if till time moment \( t \) for \( t \geq \tau \) no detection of the spontaneously emitted photon is realized, quantum state (10) in this moment has form of the following non-entangled state

\[
(19) \quad i \ 2^{-1/2} \ |\Phi(SR),P> \ |G,SR> |G,ST> + 2^{-1/2} |\Phi(ST),P> |G,SR> |G,ST> = \\
= (i \ 2^{-1/2} |\Phi(SR),P> + 2^{-1/2} |\Phi(ST),P>) |G,SR> |G,ST>
\]

Here is no-more entanglement between photon and two distant two-level systems, but there is following exact quantum superposition between two distant spontaneously emitted photon arms

\[
(20) \quad i \ 2^{-1/2} |\Phi(SR),P> + 2^{-1/2} |\Phi(ST),P>
\]

Further, for sufficiently small \( t \) expressions (8), (9) can be approximately presented by the following expressions

\[
(21) \quad |A(t),P+SR> \approx (1 - i \ H(P+SR) t/\hbar - \ H^2(P+SR)t^2/(2\hbar^2)) |1(SR),P> |G,SR> \\
(22) \quad |A(t),P+ST> \approx (1 - i \ H(P+SR) t/\hbar - \ H^2(P+SR)t^2/(2\hbar^2)) |1(ST),P> |G,ST>
\]

It implies that in situation when quantum super-system \( P+SR \) is certainly described by quantum state (21) no-decay probability by decay detection at \( SR \) in time moment \( t \) equals

\[
(23) \quad W_{ND} (SR) (t) = |<G,SR|<1(SR),P |A(t),P+SR>|^2 = 1 - (\Delta H(P+SR))^2 t^2/\hbar^2
\]

where

\[
(24) \quad \Delta H(P+SR) = (<G,SR|<1(SR),P |H^2(P+SR) |1(SR),P> |G,SR> \\
- <G,SR|<1(SR),P |H(P+SR) |1(SR),P> |G,SR> ^2)^{1/2}
\]

Then

\[
(25) \quad |a (t)| = |<G,SR|<1(SR),P |A(t),P+SR>| = (1 - (\Delta H(P+SR))^2 t^2/\hbar^2 )^{1/2} \\
= 1 - \Delta H(P+SR)^2 t^2/(2\hbar^2)
\]

and

\[
(26) \quad |b (t)| = (1 - |a (t)|^2)^{1/2} = \Delta H(P+SR) t/\hbar
\]

so that superposition coefficients are determined (till constant phase factor).

Moreover, if time interval \( t \) is divided in \( n \) equal time sub-intervals \( t/n \) and if at the end of any such sub-interval decay detection at \( SR \) is realized probability for no-decay in all \( n \) time sub-intervals equals
\[ W^{n}_{\text{ND}}(\text{SR})(t/n) = (1 - (\Delta H(\text{P+SR}))^2 \frac{t^2}{(\hbar^2)}))^n \]

It in limit when \( n \) tends toward infinity tends toward 1 that represents typical quantum Zeno effect [2].

Quite analogous results follow in case when quantum super-system \( \text{P+ST} \) is certainly described by quantum state (22) and when at \( \text{ST} \) single or frequent detections in while time interval \( t \) are realized. (As it is not hard to see, since \( \text{SR} \) and \( \text{ST} \) represent the same type two-level system corresponding expressions for \( \text{ST} \) have the same forms as the expressions for \( \text{SR} \). For this reason corresponding expressions for \( \text{ST} \) will not be given explicitly here.)

Consider now such situation in which quantum super-system \( \text{P+SR+ST} \) is really described by entangled state (10)-(13) for sufficiently small \( t \) so that expressions (21)-(27) are satisfied. Then mentioned quantum super-system is described by the following quantum state

\[
\begin{align*}
&\text{for superposition coefficients determined by (25), (26).} \\
\text{Suppose that there is only one detection chamber that should detect decay of SR, i.e. spontaneous emission of single photon by SR in sufficiently small time moment } t. \text{ For this reason expression (28) can be transformed in the following expression}\n\end{align*}
\]

\[ W^{n}_{\text{SRP}}(t) = | i 2^{-1/2} b(t) |\Phi(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > + 2^{-1/2} (a(t)|1(\text{ST})\rangle | G,\text{SR} >| G,\text{ST} > + i 2^{-1/2} a(t) |1(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > + 2^{-1/2} (a(t)|1(\text{ST})\rangle | G,\text{SR} >| G,\text{ST} >
\]

\[ W^{n}_{\text{SRND}}(t) = 1 - W^{n}_{\text{SRP}}(t) = 1 - | i 2^{-1/2} b(t) |\Phi(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} >\]

realizes quantum state \( |\Phi(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > \) of quantum super-system \( \text{P+SR+ST} \).

Positive detection of \( \text{SR} \) decay done by detection chamber which occurs with probability

\[ W^{n}_{\text{SRP}}(t) = | i 2^{-1/2} b(t) |\Phi(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > + 2^{-1/2} (a(t)|1(\text{ST})\rangle | G,\text{SR} >| G,\text{ST} > + i 2^{-1/2} a(t) |1(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > + 2^{-1/2} (a(t)|1(\text{ST})\rangle | G,\text{SR} >| G,\text{ST} >
\]

Here first term, i.e. \( |\Phi(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > \) corresponds to decay of \( \text{SR} \) (after absorption of the initial photon that propagated along reflected trajectory in respect to beam splitter toward \( \text{SR} \)). Second term, i.e. \( |1(\text{SR})\rangle | G,\text{SR} >| G,\text{ST} > \) corresponds to no-decay of \( \text{SR} \) (after absorption of the initial photon that propagated along reflected trajectory in respect to beam splitter toward \( \text{SR} \)). Third term, i.e. \( (a(t)|1(\text{ST})\rangle | G,\text{SR} >| G,\text{ST} > \) corresponds to superposition of the decayed and non-decayed quantum state of \( \text{ST} \) (after absorption of the initial photon that propagated along transmitted trajectory in respect to beam splitter toward \( \text{ST} \)).
of quantum super-system P+SR+ST. Here $|1(SR),P >|G,SR >|G,ST>$ represents initial excited state of P+SR+ST. Also, $(a(t) \ 1(ST),P > + b(t) \ |\Phi(ST),P>)|G,SR> |G,ST>$ represents quantum state of P+SR+ST with ST in evolved excited state (in time moment t after absorption of initial photon by ST). Simply speaking mentioned negative detection done by detection chamber at SR does not break entanglement between SR and ST. But mentioned negative detection changes entanglement realized quantum dynamically (which includes evolution of excited states of both two-level systems) in entanglement in which detected two-level system, i.e. SR, stands “frozen” in initial excited state while other, non-detected two-level system, i.e. ST, is in quantum dynamically evolved (without any limitation ) excited state.

Moreover, if time interval t is divided in n equal time sub-intervals $t/n$ and if at the end of any such sub-interval mentioned decay by detection chamber is realized probability for negative detection in all n time sub-intervals equals

\[
W^n_{SRND} (t) (t/n) = (1-1/2 \ (\Delta H(P+SR))^2 \ t^2/(n^2\hbar^2)) \ n
\]

It in limit when $n$ tends toward infinity tends toward 1 like by usual quantum Zeno effect. But here quantum state of P+SR+ST after frequent detection has the following form

\[
i 2^{-1/2}|1(SR),P >|G,SR >|G,ST> + 2^{-1/2} |G,SR> |A(t),P+ST>
\]

in any time moment. It means that here not all, but only one superposition term is “frozen” in initial state by frequent detections, while other superposition term quantum dynamically deterministically evolves without any limitation. For this reason we can here speak about sub-systemic quantum Zeno effect.

For sufficiently large t , i.e. for $t \geq \tau$ quantum state (34) turns out in

\[
i 2^{-1/2} |1(SR),P >|G,SR >|G,ST> + 2^{-1/2} |\Phi(ST),P> |G,SR >|G,ST>
= (i 2^{-1/2} \ 1(SR),P > + 2^{-1/2} |\Phi(ST),P>) |G,SR >|G,ST>
\]

which formally means that here superposition between photon localized in SR and photon spontaneously emitted by ST appears.

As it is not hard to see, sub-systemic quantum Zeno effect can be realized analogously at ST by corresponding frequent detections at ST only by corresponding detection chamber and without any detection at SR. Of course, as it is not hard to see too, simultaneous frequent detections at SR by one detection chamber and at ST by other detection chamber realize total Zeno effect at P+SR+ST “frozen” in quantum state (5)-(7).

In conclusion the following can be repeated and pointed out. In this work we consider quantum dynamical interaction of single photon with beam splitter after which well-known superposition between reflected and transmitted photon appears. Later reflected photon is absorbed by one and transmitted photon is absorbed by other, distant
of two two-level systems of the same type (prepared initially in ground states). It implies entanglement between mentioned two two-level systems. Any two-level system in excited state represents an unstable system that quantum dynamically evolves in a superposition of the initial excited (non-decayed) and ground (decayed) state. We consider frequent detections at only one of two two-level systems and we prove that here a modification of the usual quantum Zeno effect called sub-systemic quantum Zeno effect appears. Namely after mentioned frequent detections two two-level systems are in a specific entangled state with two terms. First term (term of the “frozen” evolution) includes detected two-level system in initial excited state and non-detected two-level system in the ground state. Second term (term of free evolution) includes detected system in ground state and non-detected system in free evolved excited state.

References

[1] V. Scarani, A. Suarez, Am. J. Phys. 66 (1998) 718 and references therein
[2] B. Misra, C. J. G. Sudarshan, J. Math. Phys. 18 (1977) 756
[3] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, F. Plastina, Protecting entanglement via the quantum Zeno effect, arXiv (quant-ph) 0710.3914 (2007)