Critical Current Peaks at $3B_\Phi$ in Superconductors with Columnar Defects:
Recrystalizing the Interstitial Glass

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The physics of 3D vortex pinning via correlated disorder can be approximately mapped onto the problem of interacting quantum bosons in the presence of uncorrelated disorder in 2D. This has formed the basis for many computational studies via quantum Monte Carlo simulations of local repulsive bosons and exact diagonalization studies. Several pinned phases of vortex matter emerge as a function of $B/B_\Phi$, where $B_\Phi$ is the equivalent matching field where the number of vortices equals the number of defects. For the case where the pins outnumber the vortices ($B < B_\Phi$), a Bose-glass phase is formed where vortices are localized onto columnar defects and possess an infinite tilt modulus for tilts away from columnar alignment. For equal numbers of vortices and defects ($B = B_\Phi$), an analog to a Mott insulating phase exists which possesses an infinite compression modulus. This happens for low temperatures and evidence for the Mott phase has been found in recent magnetization relaxation measurements at low temperatures. For the case when the vortices outnumber the pins, ($B > B_\Phi$), the situation is much less clear. Recent work has suggested that vortices not accommodated to the columnar defects are caged by pinned vortices to form a weakly pinned “interstitial Bose” glass. It has been conjectured that the melting temperature $T_m$ decreases rapidly as the density of interstitial vortices increases for large fields, and either extends smoothly into the Bose glass phase or shows a change in slope (kink) at $B = B_\Phi$. Some experiments do not show substantial changes of the irreversibility line at $B_\Phi$, while others show only a mild kink at $B_\Phi$ and then increases and goes through a maximum near $3B_\Phi$, which is currently unaddressed by theory.

Moreover, interesting effects are seen as $B$ increases past $B_\Phi$. In analogy with the presence of Mott lobes in the dual superconducting-insulator transition in 3D, one would expect different phases at commensurate values of vortex filling. Nowak et al. found that the critical current in Tl$_2$Ba$_2$CuO$_{6+\delta}$ decreases for $B > B_\Phi$ but then increases and goes through a maximum near $3B_\Phi$, which is currently unaddressed by theory.

One of the weaknesses of previous numerical studies is the use of a short-range screened interaction which misses the softening of the shear modulus in the absence of pins in the weak field limit as noted by Fetter, Holenberg and Pincus. The purpose of this paper is to address the importance of this omission by investigating the properties of the pinned and moving phases via 3D molecular dynamics simulations of vortices interacting via a long-range potential in the presence of columnar defects. Our key result is that the softening of the shear modulus in the low field limit has important implications for the phase diagram. We confirm the existence of a weakly pinned interstitial glass which becomes more weakly pinned as the field increases in agreement with previous studies of short-range interactions. However we find a qualitatively new phase diagram for low fields where the interstitial glass melts near $B = B_\Phi$ and recrystalizes at higher fields when the matching field is below a critical value. In this regime we find strong numerical evidence for enhanced values of the critical current near three times the matching field. This questions the quantitative appropriateness of the mapping of vortex physics onto the physics of the superconductor-insulator transition without a properly detailed consideration of long-range interactions.
tance \( z \), with the applied magnetic field aligned perpendicularly to the planes. The off-lattice simulation models the motion of vortices referenced by a 2D coordinate \( r \) under the influence of pinning and repulsive vortex interactions, and driving, line bending, and thermal forces:

\[
m_l \dot{r}_l = -\eta_i r_l + f_D + f_T - \frac{\partial H((r_1, r_2, \ldots))}{\partial r_l}
\]

(1)

Here \( f_D = \Phi_0 J / c \) is the Lorentz force per unit length due to an applied current density \( J \) perpendicular to the magnetic field. \( \eta_l = \frac{\Phi_0^2}{2\pi \xi^2 m^* c^2} \) is the Bardeen-Stephen viscous drag coefficient, with \( \Phi_0 = (hc/2e) \) the flux quantum and \( \rho_n \) the normal state resistivity. The Langevin thermal force per unit length \( f_T \) is normalized to set the rms vortex velocity via the equipartition theorem. The Hamiltonian \( H \) is the sum of the line tension for bending, and vortex-vortex and vortex-disorder potentials per unit length constructed via London theory. The line tension is given by \( \epsilon_l = \epsilon \ln(\kappa) \epsilon_0 \) with \( \epsilon = (m_\epsilon/m_0)^2, \kappa = \lambda/\xi, \epsilon_0 = (\Phi_0/4\pi\lambda)^2, \) and \( \lambda \) magnetic penetration depth. The vortex interaction is given by a sum of pairwise interactions \( V_{v-v}(r) = \epsilon_0 K(r/\lambda) \) with \( K \) a Hankel function. We model the correlated extended defects as smooth parabolic traps of width \( R_p \) and uniform depth \( \epsilon_0 = \epsilon_l/4 \). Defects are randomly placed and aligned along the c-axis.

For DC transport, in most cases the vortex mass per unit length \( m_l \) is overall quite small in comparison with the other parameters in Eq. (1) and has thus usually been neglected in previous numerical studies in 3D of vortex dynamics. However, in the case of superconductors near a Mott instability, such as the underdoped cuprates, the vortex mass can be immensely enhanced as the system approaches a superconductor-insulator transition.

We thus keep this term and simply relate the vortex mass to the mass of the highly renormalized electrons confined to the vortex core region, \( m_l = m_{eff} n \pi \xi^2 \), with \( n \) the electron density and \( m_{eff} \) the in-plane effective mass of the electron renormalized by strong interactions. While we have found that the vortex mass has an impact on the magnitude of the critical current, the relative \( J_c \) for different vortex densities do not drastically depend on the choices made.

Periodic boundary conditions are imposed in the planes to maintain constant global flux density, and open boundary conditions are employed along the c-axis. Temperature is chosen which is high enough to allow individual vortices to be quickly accommodated to defects in the absence of a driving current but well below the glass temperature.

We measure all energies in units of the bare line bending energy \( \epsilon_0 \) and measure all lengths in units of \( d = 4\xi \). A natural time unit \( t_0 \) is chosen to be \( \pi \kappa \eta c^2/\epsilon_l \) and the time step is further discretized in units of \( 0.01t_0 \) for the simulations. The current is measured in units of the BCS depairing current \( J_0 = \Phi_0 c k/12\sqrt{3} \pi^2 \lambda^3 \), and resistivities in terms of the Bardeen-Stephen flux-flow resistivity \( \rho_{BS} \) is given by \( \rho_{BS} = B \Phi_0 / (c^2 \eta_l) \). The other parameters used in the simulations are dictated by values appropriate for \( \text{YBa}_2 \text{Cu}_3 \text{O}_7 \): \( m_{eff} = 5m_\epsilon, R_p = 2\xi, \xi = 17\AA, z = 12\AA, \kappa = 100, \) and \( \epsilon = 1/25 \), giving \( H_{c2} = 120\text{T} \).

Our simulations were performed using up to 40,000 vortex segments in a \( 128d \times 128d \) square periodic cell containing 80 planes, where finite size effects were investigated and found to be minimal. Measurements are taken over an interval of \( 10^6t_0 \) after the system has reached a steady state after a typical time \( 3 \times 10^5t_0 \). We typically average the results over several hundred realizations of disorder particularly at smaller driving forces. We measure the average vortex velocity in the direction of the Lorentz force corresponding to the voltage drop across the sample, and determine the resistivity \( \rho = B / J \).

Our results for the resistivity \( \rho \) versus the applied current density \( J \) are shown for a series of vortex densities below and above the matching field \( B_m \) for two different values of \( B_m \) in Fig. 1A and B. All error bars are equal to the symbol size. For the pinned Bose-glass regime \( B < B_m \) and for appreciable \( B_m \) (Fig. 1A), vortices are localized on separate columnar defects for small \( J \) until an abrupt transition to a moving regime ensues near \( J \sim 0.1J_0 \) where the vortices simultaneously become unpinned. For larger driving currents, vortices are in the flux flow regime and the resistivity approaches \( \rho_{BS} \). The de-pinning transition occurs within a narrow range of currents corresponding to single vortex pinning. Conse-
important differences however are observed for smaller values of $B_\Phi$ (Fig. 1B). For $B < B_\Phi$ in this case, the transition to depinning occurs at lower $J$ and is substantially broadened compared to Fig. (1A). A non-monotonic dependence of the resistivity appears as shown in the inset of Fig. (1B). For low driving forces the resistivity rises with increasing $B$ for $B/B_\Phi < 1.5$ and becomes greater than $0.1\rho_{BS}$ for $B > B_\Phi$ as the interstitial glass melts into an interstitial liquid as the density of vortices is increased. However at larger fields the resistivity decreases and becomes less than $0.1\rho_{BS}$ for fields near three times the matching field. This indicates that for low $B_\Phi$ the Bose glass melts into an interstitial liquid near the matching field and recrystallizes into an interstitial glass at larger values of $B/B_\Phi$, in contradiction to the phase diagram proposed previously.

We can make this more quantitative by defining a critical current density $J_c$ as the value of the current density corresponding to a resistivity of 10% of $\rho_{BS}$. For a number of different molecular dynamics runs of different $B_\Phi$, the values we obtain for $J_c$ are shown in Fig. (2) as a function of $B/B_\Phi$ normalized to the values of $J_c$ determined in the small $B/B_\Phi$ limit. For large $B_\Phi$, the critical current decreases monotonically with $B/B_\Phi$ with a gradual fall-off near the matching field. For smaller values of $B_\Phi$ the fall-off near $B_\Phi$ becomes much more abrupt, suggesting that a magnitude of the kink in the melting curve would be dependent on the value of $B_\Phi$, reconciling previous experiments with the phase diagram proposed previously.

For even smaller $B_\Phi$, the fall-off is dramatic and the interstitial glass weakens appreciably and melts ($J_c = 0$) near a critical value of $B_\Phi/H_{c2} \sim 1.9 \times 10^{-3}$. Remarkably, the critical current resurrects for larger values of the magnetic field and has a broad peak near $B = 3B_\Phi$ for $B_\Phi/H_{c2} \times 10^3 = 1.906$ before falling off again at still larger fields. This reproduces the peak near three times the matching field seen in experiments on Tl$_2$Ba$_2$CuO$_{6+\delta}$. However we see that still smaller values of $B_\Phi$ yields a critical current peak at larger values of $B/B_\Phi$, suggesting that this peak would be dependent on the number of defects. While we expect that the actual value of the critical field $B^*$ might depend on our choice of defining $J_c$, we do not expect the rentrant behavior into the interstitial glass to be qualitatively changed. A proposed phase diagram which encompasses our results for different $B_\Phi$.
is given in Figure 3.

Using renormalization group arguments it was shown by Nelson and Seung that for clean systems the interactions between vortex lines are renormalized to zero near $H_{c1}$ and leads to the melting of the Abrikosov lattice as $H \to H_{c1}$.[2, 17] Thus we might expect that the melting of the interstitial glass for $B - B_\Phi \geq \Phi^*$ is similar to the melting of the vortex lattice near $H_{c1}$.[2, 17] However for $B < B_\Phi$ vortex localization onto unoccupied columnar defects helps to reduce vortex fluctuations and one might expect the Bose glass to be stable near $H_{c1}$ as vortices occupy the strongest defects. Yet once $B > B_\Phi$ the interstitial vortices feel a much reduced and screened defect interaction and suffer the downward renormalization of the shear modulus as in clean systems. This is borne out in our simulations for small defect concentrations. As the field increases our simulations indicate that the interstitial glass recrystallizes. This can be viewed as a hardening of the shear modulus for increasing fields as the role of interactions increases.

What leads to the recrystallization of the melted interstitials at larger values of $B$ for small defect concentrations? At large vortex concentrations the Coulomb potential is effectively short-range due to screening and the effects of disorder are also screened by flux line collisions, and indeed our results are consistent with prior simulations for contact repulsive potentials.[4]. The system favors forming dislocations at larger fields and the domains are vortices are easily depinned. For smaller vortex concentrations, new physics arises as screening is no longer effective and the bare long-range repulsive forces encourage long-range ordering concomitant with decreased prevalence of dislocation and subsequent enhanced vortex pinning.

To check these ideas, we plot in Figure 4 time-averaged structure plots for $B = 3B_\Phi$ at two values of $B_\Phi$. While long-range order is not seen even on the length scale of our simulations, it is clear that orientational short-range order is more prevalent for the small $B_\Phi$ results than the large ones, with subsidiary structure peaks at the reciprocal lattice vectors up to 1/2 the height of the central ($Q = 0$) peak. When averaged over many disorder configurations, the orientational order is lost for both values of $B_\Phi$, yet it is clear that the effective range of the vortex repulsion is longer for smaller $B_\Phi$. We would expect that for still smaller values of $B_\Phi$ positional order would grow for systems of fixed finite size $L$ with the possible formation of Bragg-like peaks. Most likely the positional order would not be quantitatively relevant for $L \to \infty$.

In summary, we have presented numerical simulations of interacting vortex dynamics in the presence of columnar disorder as a function of $B/B_\Phi$ and $B_\Phi/H_{c2}$. We find a monotonic decrease in the critical current for large values of $B_\Phi$ with a sharp drop-off near the matching field, consistent with prior notions of the weakly pinned interstitial glass emerging at higher fields. For smaller values of $B_\Phi$ however, we see abrupt melting at $B = B_\Phi$ and recrystallization near $B = 3B_\Phi$ of the interstitial glass, suggesting modifications to the phase diagram due to the role of long-range interactions even for columnar disorder.

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