A “VERTICAL” ANALYSIS OF CRISES AND INTERVENTION: FEAR OF FLOATING AND EX-ANTE PROBLEMS

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ABSTRACT

Emerging economies are prone to crises triggered by external shocks. During these crises, should the central bank stabilize the currency or domestic interest rates? If the choice is outside the central bank’s control, as in a currency board, are there good policy substitutes? We argue that these questions are best analyzed in a “vertical” framework, where the supply of external funds faced by the country is inelastic during the crisis and monetary policy affects mostly the domestic cost of scarce international liquidity. This is in contrast to the standard “horizontal” framework where supply is elastic at the (now higher) international interest rate. In this vertical view, raising domestic interest rates during a crisis has relatively limited output consequences, while not doing so causes a sharp exchange rate overshooting. This asymmetry naturally leads to the widely observed fear of floating. However, while this response is ex-post rationalizable, it has negative ex-ante consequences as it exacerbates the structurally insufficient private sector incentives to insure against crises. Ex-ante, optimal monetary policy is countercyclical, and increasingly so as financial development falls. The silver lining for countries with limited financial development that cannot (or should not) overcome this conservative-central-bank time inconsistency problem, is that since the main role of monetary policy in the vertical view is one of incentives, it can be substituted by ex-ante measures to induce the private sector to insure against crises.

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1 Introduction

Emerging economies are prone to crises triggered by external shocks. During these crises, should the central bank stabilize the value of its currency or domestic interest rates? If the choice is outside the central bank’s control, as in a currency board, are there good policy substitutes?

Uniformly, the analysis of these questions begins by describing the external shock as an upward shift in the interest parity condition: a rise in the country-premium or international interest rate. We depart at the outset and argue that for most emerging economies, this “horizontal” approximation of the external supply of funds (unlimited funds available at a high and fixed price) is misleading for the questions at hand. The proposition, implicit in this view, that an emerging economy mired in an external crisis could attract capital flows by adopting an expansionary monetary policy, seems counterfactual at best.

Instead, what is needed is a “vertical” approximation, where the supply of funds is inelastic during the crisis as the country faces an international liquidity constraint. In this context, monetary policy predominantly affects the domestic cost of the scarce international liquidity. Expansionary monetary policy brings about a sharp overshooting in the exchange rate depreciation, without any substantial gain in terms of real activity. In contrast, conservative monetary policy stabilizes the exchange rate, with little additional output loss. A modern central bank concerned with its inflation target will respond to this asymmetry by tightening during the crisis. The vertical view thereby accounts naturally for the widely observed “fear of floating” among emerging economies.

While a contractionary monetary policy may appear as optimal during the crisis, it is not from an ex-ante perspective as it exacerbates the structural underinsurance problem that afflicts emerging economies. Quite the contrary, we show that if the central bank could commit to a monetary policy, it should in most circumstances pledge expansionary policy during external crises. Importantly, the optimality of the latter stems not from the impact of monetary policy during a crisis, as the standard argument has it, but from the ex-ante effect the policy has on the incentive to insure against episodes of international liquidity scarcity.

When firms in need of international resources face domestic financial constraints, their demand for international liquidity is constrained as well. In equilibrium, these domestic financial constraints generate a wedge between the marginal value of the international re-

\[1\] See Calvo and Reinhart (2000) and Hausmann et al (2001) for extensive documentation of fear of floating among emerging economies with flexible exchange rate regimes.
sources to the domestic firms and the market price of these resources. This underpricing reduces the private sector’s incentive to carry international liquidity into crisis-states and hence insure against crises (see Caballero and Krishnamurthy (2001a)). Expansionary monetary policy, while not directly alleviating the international financial constraint —the main constraint during an external crisis— increases domestic liquidity and hence the private reward of maintaining international liquidity. Thus in the vertical view, the only role of monetary policy is to affect the private sector’s incentives to manage international liquidity.

The time consistency problem that arises in the vertical context, coupled with the institutional conservatism of modern independent central banks, suggests that in practice monetary policy will be rarely used for incentive purposes. This bias increases the underinsurance problem, in particular in those economies with more limited financial development (i.e., where firms face tighter financial constraints). The silver lining for these economies is that there are substitutes for the international liquidity management role of monetary policy. Since the primitive problem is one of underinsurance, either direct or indirect ex-ante measures that induce the economy to carry more international liquidity into crises states reduce the underinsurance problem. For example, an active international reserve management, capital controls, or procyclical international liquidity ratio requirements, can all substitute for monetary policy. Conversely, a credible commitment to a countercyclical monetary policy during crises is a good substitute for costly capital controls and ex-ante measures.

The distinction we draw between international and domestic liquidity, coupled with the association of monetary policy to the latter, also offers a different perspective on several issues related to liquidity policy within different exchange rate regimes. The loss of monetary policy in a currency board or other inflexible exchange rate systems, somewhat paradoxically, is not a serious impediment during crises. Monetary policy does little to relax the binding international financial constraint and thus is largely irrelevant. The problem with a currency board is instead due to the perverse ex-ante effects that the lack of credit expansion during a crisis generates; as before, these can be handled with ex-ante measures. Within our perspective, international contingent credit lines are desirable regardless of the exchange rate regime, and thus should not be thought of as a substitute for domestic monetary policy in a dollarized economy. They are primarily about relaxing international rather than

\footnote{Our analysis is meant to isolate the liquidity and financial market aspects of monetary policy. In so doing, we eliminate the important but better understood goods-labor markets dimensions of monetary and exchange rate policy. All our remarks must be understood in this context. We briefly return to these issues in the conclusion.}
domestic financial constraints, and it is the latter rather than the former that differentiates flexible and fixed exchange rate regimes from a liquidity management perspective.3

Section 2 presents the basic model and highlights the differences between the traditional horizontal and the vertical analyses, including the natural emergence of exchange rate overshooting and the related fear of floating. Our model is designed to establish a clear distinction between two forms of liquidity (collateral): international and domestic. The vertical view highlights the fact that during external crises it is the former that is binding and hence measures to relax the latter —such as monetary policy— cannot have a significant immediate impact. The horizontal perspective, on the other hand, draws no distinction between these two forms of liquidity.

Section 3 asks how ex-ante private sector financing decisions are affected by ex-post policy measures and, in turn, how this answer leads to the design of optimal policies. We show that domestic financial underdevelopment implies that in equilibrium the private sector undervalues international liquidity and insurance. A credible commitment to a countercyclical monetary policy reduces the extent of this undervaluation. While this would appear to be a damaging criticism of dollarized regimes, we argue that this need not be the case since the primitive problem is one of distorted incentives rather than one of insufficient domestic liquidity, and the former can be resolved by alternative means.

Section 4 summarizes our message in the context of a standard political economy discussion of rules-versus-discretion. We show that while in the horizontal perspective there is a standard tension between inflation bias and ex-post inflexibility, in the vertical perspective there is neither an inflation bias nor an advantage of ex-post flexibility. Quite the contrary, if it were at all possible, commitment in the vertical region would require the central bank to be less conservative than it will be inclined to be during a crisis.

Section 5 concludes and is followed by two appendices. The first one contains the proofs of our main propositions. The second one is more substantive. While the interaction between financial market development and international liquidity is explicitly modeled in the main text, the monetary channel we superimpose on it is reduced-form. The second appendix sketches a debt-deflation model of this monetary channel.

3See the recent work by Diamond and Rajan (2001) for a related perspective on financial crises based on two potentially binding constraints: a solvency and a liquidity constraint. Their analysis focuses on the ex-post effect of interventions during crises and, in particular, on the perils of policies that fail to identify the binding constraint during the crisis.
2 The Vertical View and the Fear of Floating

In this section we describe the environment and discuss the difference between horizontal and vertical views of crises. We show that when domestic financial markets are illiquid, the vertical view implies that the exchange rate overshoots in response to monetary policy and central banks are naturally led to adopt a fear of floating strategy.

2.1 Basic setup

We study an economy exposed to an external financial crisis, triggered by the rise of the country’s risk-adjusted international cost of capital. The crisis occurs at date 1, and is followed by a final date 2 when firms’ repay their outstanding debts. We start time with a date 0, which is a fully flexible period when agents make investment, financing, and precautionary decisions. The periods are indexed by \( t = 0, 1, 2 \), and there is a single (tradeable) good.

There is a unit measure of domestic firms, each endowed with \( w \) units of collateral, in the form of receivables arriving at date 2. These date 2 goods have collateral value to foreigners (e.g., prime exports), who are willing to lend against it at dates 0 and 1 at the rate \( i^*_0 \) and \( i^*_1 \) from period 0 to 1, and 1 to 2, respectively. Foreigners play no other role in our model.

Domestic firms also have access to a production technology. Building a plant of size \( k \) at date 0 requires them to invest \( c(k) \) — with \( c(\cdot) \geq 0, c' > 0 \) and \( c'' > 0 \) — which yields date 2 output proportional to the size of the plant (see below). Since domestic firms have no resources at date 0, they must import the capital goods and borrow from foreigners, \( d_{0,f} \), to finance this investment. The financing and investment decisions are taken to maximize expected plant profits at date 2. To keep matters simple, we shall assume that each firm is run by a domestic entrepreneur/manager who has risk neutral preferences over date 2 consumption of the single good.

Firms face significant financial constraints. Neither the plants nor their expected output are valued as collateral by foreigners. When real investments are undertaken, firms mortgage a part of their international collateral in securing foreign funds. All financing is done via fully collateralized debt contracts, thus, \( d_{0,f} \leq w \).

\(^4\)Nearly the same analysis applies to a sharp decline in terms of trade that makes financial constraints binding.
2.2 Date 1 financing needs and Crises

For the remainder of the section, let us take as given all date 0 decisions — $k$ and $d_{0,f}$ — and focus on the crisis period. We define a crisis as an event in which a rise in the international lending rate, $i_1^*$, causes financial constraints to bind for firms. Let us now turn to defining the financing need and explaining how financial constraints may come to bind.

In our economy the financing need stems from the normal ongoing maintenance of the productive structure. We capture this feature by simply assuming that the plants of one-half of the firms receive a production shock at date 1 that lowers output per plant from $A$ to $a$. This productivity decline can be offset by reinvesting $\theta k$ ($\theta \leq 1$) goods, to give date 2 output of,

$$\tilde{A}(\theta)k = (a + \theta \Delta)k \leq Ak,$$

where $\Delta \equiv A - a$.

We assume that the return on reinvestment exceeds the international interest rate: $\Delta - 1 > i_1^*$. This means that firms will borrow as much as possible to finance reinvestment. We shall say that a crisis occurs if firms are curtailed in their date 1 reinvestment, $\theta < 1$, despite the fact that $\Delta - 1 > i_1^*$. If this is the case, then firms are financially constrained at date 1. Our assumptions on parameters are such that this is the case in equilibrium (see the appendix).

A firm that receives an idiosyncratic shock is said to be distressed. To cope with the shock, the firm first borrows against its net international collateral:

$$w^n \equiv w - d_{0,f}$$

directly from foreigners. After this, it must turn to the domestic firms that did not receive a shock (“intact firms”) for funds. Intact firms have no output at date 1 either, so they must borrow from foreigners if they are to finance the distressed firms. This they can do up to their $w^n$ of financial slack.

But why would intact firms lend to distressed firms any more than foreigners? We assume that domestics value as collateral the firm’s installed assets as well. However, since a perfectly functioning domestic financial market is hardly a good description of an emerging economy either, and this departure has central implications for our analysis, we assume that only a fraction of the output from domestic investment can be pledged to other domestics. It will simplify the formulae, without loss of insight, to make this a fraction of minimum output: $\lambda a k$. Since firms can use this collateral to borrow up to this amount, we refer to $\lambda a k$ as domestic liquidity.

Likewise, since at date 1 firms can borrow from foreigners up to

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5Since we do not want insurance markets to undo the ex-post heterogeneity, we assume that idiosyncratic shocks are non-observable and non-contractible. Moreover, we assume that the coordination-fragile ex-ante pooling equilibrium is not feasible (see Caballero and Krishnamurthy (2001b)).
of international collateral, we refer to $w^n$ as the *international liquidity* during the crisis.

After pledging $w^n$ to foreigners, distressed firms pledge their $\lambda ak$ to intact domestics, who in turn pledge their $w^n$ to foreigners to access foreign funds. All direct borrowing from foreigners is done at the interest rate of $i^*_1$.

### 2.3 The (standard) Horizontal View

In the standard horizontal view, distressed firms are constrained in meeting their financing needs only to the extent that they have limited collateral. They have total collateral of $\lambda ak + w^n$ which they borrow against at the interest rate of $i^*_1$.

Translated into our context, the horizontal view implicitly assumes that the country as a whole, at the margin, has an international liquidity slack. In other words, a foreigner would be willing to extend another loan at $i^*_1$ to some domestic firm. But since distressed firms have limited collateral, it happens that the worthy firm is not distressed.

In our model, since intact firms borrow from foreigners against $w^n$ and lend to distressed firms against $\lambda ak$, there is excess international liquidity if,

$$\frac{1}{2} w^n > \frac{1}{2} \lambda ak. \tag{1}$$

Since intact firms are not saturating their international financial constraint, the interest rate they charge on the loan to a distressed firm, against domestic collateral of $\lambda ak$, is simply determined by the arbitrage condition:

$$i^d_1 = i^*_1.$$

Total reinvestment is then determined by the individual firms’ financial constraints:

$$\theta^h_k = \frac{w^n + \lambda ak}{1 + i^*_1} < \frac{2w^n}{1 + i^*_1}, \quad \theta^h < 1, \tag{2}$$

where the superscript $h$ denotes the horizontal equilibrium. The inequality in the main expression reflects that the economy has not used all its international liquidity, while $\theta^h < 1$ indicates that the economy is in a crisis: distressed firms are unable to meet all financing needs because of the binding financial constraint.

We refer to this as the horizontal view, because the price of loans is not affected by the quantity of them. A distressed firm could in principle continue borrowing at the given interest rate $i^*_1$, as long as its domestic financial constraint is relaxed. Thus, if we imagine an experiment where $\lambda$ is raised slightly, we would relax the domestic collateral constraint, leading to increased loans at $i^*_1$ and increased reinvestment. We return to this discussion after introducing the vertical view.
2.4 The Vertical View

In this view, the international supply of funds that is faced by emerging economies during external crises is vertical – i.e. inelastic. The country has enough domestic collateral to aggregate the international liquidity available to agents in the economy, so that inequality (1) is reversed:

\[ \frac{1}{2} w^n < \frac{1}{2} \lambda a k. \]

However, there is insufficient international liquidity in the aggregate to raise finance for all of the economy’s needs so that the economy is still in a crisis:

\[ \frac{k}{2} > \frac{w^n}{1 + i^*}. \]

When the above conditions hold, the interest rate on loans against domestic collateral departs from \( i^*_1 \) (see the appendix for restrictions on primitives). Since intact firms, and not foreigners, accept as collateral the \( \lambda a k \), and intact firms are borrowing up to their maximum capacity from foreigners and lending to distressed firms, the domestic price of a dollar-loan, \( i^d_1 \), rises above \( i^*_1 \):

\[ i^d_1 > i^*_1. \]

Importantly, the fact that the international liquidity constraint is binding at the margin does not mean that the domestic collateral problems that dominate the horizontal approach disappear. This observation will be central in understanding the desirability and impact of monetary policy. As we show below, as long as domestic firms continue to be credit constrained, the domestic (dollar) interest rate is less than the marginal product of investment for the distressed firm:

\[ i^d_1 < \Delta - 1. \]

This is the rate at date 1 on a one-period domestic loan against a unit of domestic collateral, and is both the dollar cost of capital for firms in need of funds, as well as the expected return on loans for domestic lenders. It is determined by both, the amount of collateral of distressed firms and the amount of international liquidity of intact ones. The aggregate collateral of distressed firms is \( \lambda a k / 2 \). Thus they pledge \( \lambda a k / 2 \) of date 2 goods to intact firms in exchange for date 1 goods of \( w^n / 2 \). If domestic collateral is not too limited, there is a scarcity price for the international liquidity, and the price needed to clear the domestic loan market, \( i^d_1 \), will exceed the international interest rate, \( i^* \):

\[ i^d_1 = \frac{\lambda a k}{w^n / (1 + i^*_1)} - 1 > i^*_1. \]
This expression highlights the effect of domestic collateral and international liquidity on date 1 cost of capital. A shortage of the latter means that \( i^d_1 > i^*_1 \); while a shortage of the former means that the cost of capital will generally be less than the marginal product of investment at date 1 (\( \Delta \)): \( i^d_1 < \Delta - 1 \).

In other words, in the aggregate, a shortage of international liquidity yields a spread between domestic marginal product and international cost of capital. Domestic collateral, on the other hand, determines the sharing of this spread between domestic lenders and borrowers of a marginal dollar.

In this region, domestic collateral plays no role in determining aggregate reinvestment. In equilibrium, all international liquidity is aggregated, so that reinvestment is:

\[
\theta^v k = \frac{2w^n}{1 + i^*_1},
\]

where the superscript \( v \) stands for vertical equilibrium.

### 2.5 Date 1 monetary policy, overshooting, and fear of floating

As an introduction to our discussion of monetary policy, the main distinction between the vertical and horizontal views can be seen by considering the experiment of increasing
\( \lambda \) (boosting domestic collateral) by a small amount. Figure 1 illustrates this difference. On the vertical axis we measure the domestic dollar-interest rate on loans against domestic collateral. On the horizontal axis we capture domestic loans or domestic reinvestment. The solid flat-and-then-vertical curve correspond to the supply of loans from intact firms. \( d \) and \( d' \) represent effective (collateralized) loan demand from distressed firms in regions where the supply is horizontal (\( d \)) and vertical (\( d' \)).

Note that in the horizontal case a shift in demand (say by increasing \( \lambda \)) raises date 1 investment, while leaving the domestic price of loans unaffected. On the other hand, in the vertical region, an increase in \( d' \) has no effect on equilibrium investment, and instead only pushes up the domestic interest rate of \( i^d_1 \).

Let us take a reduced form approach to monetary policy whereby the central bank can indeed affect \( \lambda \) (see the appendix for an explicit model of this channel). The central bank chooses a combination of peso-interest rates, \( i^p_1 \), and expected appreciations, \( (e_1 - e_2) \), traced by the (domestic) interest parity condition: \[ i^d_1 = i^p_1 + (e_1 - 1), \]

where \( \lambda \) can be written as \( \lambda(i^p_1, e_1) \), and without loss of generality, we have set \( e_2 = 1 \).

Most models with a monetary channel yield \( \lambda_{i^p} < 0 \), while the sign of \( \lambda_e \) varies across models and scenarios. However, as long as the latter is positive, or not too negative, an expansionary monetary policy (i.e., a reduction in domestic peso-rates) also expands domestic collateral by raising \( \lambda \).

We will take the above configuration as our reference case, although in the conclusion we discuss briefly the case of \( \lambda_e << 0 \) —perhaps capturing an extensive dollarization of domestic liabilities— as this is one of the main reasons given in the literature to prefer constrained monetary regimes over more flexible monetary arrangements.\(^8\) It is important to notice, nonetheless, that our main argument is very distinct from the issues raised in this debate.

The tradeoff of the monetary authority in the horizontal perspective can be understood in this context. An expansionary monetary policy relaxes the domestic financial constraint (increases \( \lambda \)), and by doing so it increases investment (see (2)):

\[ \frac{\partial \theta^h k}{\partial \lambda} = \frac{ak}{1 + i^*_1}. \]

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\(^7\)By domestic interest parity condition we mean the relationship between the return on peso and dollar instruments backed by domestic collateral. See appendix A3.

\(^8\)See, e.g., Aghion et al. (2000), Gertler et al. (2001), Cespedes et al. (2000), and Christiano et al. (2000).}
The standard tradeoff is that this investment increase must be weighed against the inflationary costs brought about by the monetary expansion and exchange rate depreciation. In the midst of a crisis, and within a reasonable range, the latter costs are likely to be dominated by the output gains of the expansionary policy.\textsuperscript{9}

Our main results in this section follow directly from noticing that the weights in the above tradeoff change abruptly in the vertical region. This happens for two important and closely related reasons. First, since the main binding constraint is the international as opposed to the domestic collateral constraint, relaxing the latter does not affect reinvestment and output (see (4)):

\[ \frac{\partial \theta v k}{\partial \lambda} = 0. \]

Second, while all choices of \( i^d_1 \) and \( e_1 \) that leave the economy in the vertical region yield the same real investment decisions, they do not all lead to the same asset prices. In fact, from (3) we see that \( i^d_1 \) is increasing in \( \lambda \):

\[ \frac{\partial i^d_1}{\partial \lambda} = \frac{ak}{w^n/(1 + i^*_1)}. \]

Thus a key difference from the horizontal region is that an increase in \( \lambda \) results in a shift in the (domestically determined) interest parity condition. That is, in the vertical region there are two effects of an expansionary monetary policy on the exchange rate that need to be considered. There is the standard interest parity effect: Fixing \( i^d_1 \) in,

\[ i^p_1 + e_1 - 1 = i^d_1 > i^*_1, \]

the exchange rate weakens by the same amount as the peso-interest rate is lowered. But at this new point, \( \lambda \) rises as well. Since \( i^d_1 \) is increasing in domestic collateral, the (domestic) interest parity condition shifts upwards, and the exchange rate depreciates by a larger amount. We refer to this phenomenon as exchange rate overshooting because, relative to the horizontal region, in the vertical region peso-interest rate reductions lead to proportionately larger depreciations in the exchange rate.

With little direct real consequences, monetary policy in the vertical region may be dictated by secondary considerations (e.g., inflation targets). We take the widely observed fear of floating to be an example of this. The central bank wishes to protect the exchange at date 1 from the shock to \( i^*_1 \). At date 1, the output costs to raising interest rates are minimal.

\textsuperscript{9}In this sense, the monetary policy problem with financial frictions in the horizontal perspective is no different from the standard closed economy problem applied to developed economies as in, e.g., Bernanke, Gertler and Gilchrist (2000).
(beyond the direct costs of the external shock), and instead the action has a substantial effect on the exchange rate. The logical conclusion is to raise interest rates, lowering $\lambda$ to the point where $i_d^1 = i^*_1$, and thereby defending the exchange rate.

Does this mean that emerging economies should abandon countercyclical monetary policy, making explicit what they have de-facto adopted through their fear of floating? This is one of the main questions we turn to in the next section.

3 Optimal Policy Regime: The menu of date 0/date 1 measures

In the vertical view, international liquidity and hence the supply of external resources to the country is predetermined during a crisis. There is nothing that a central bank, or private agent, can do to alter this reality.

Date 0 actions, on the other hand, can affect the date 1 external position of the country. In particular, we show that the anticipation of an expansionary date 1 monetary policy leads to more efficient date 0 private sector decisions. But what if monetary policy is constrained — as in a currency board? We argue that explicit date 0 measures can substitute for the loss of monetary policy.

This section offers a menu of date 0/date 1 policies that are optimally taken together. We also show that the conclusions one draws regarding optimal policy and the choice of exchange rate regimes hinges crucially on separately identifying domestic and international liquidity.

3.1 Structural Underinsurance and Optimal Monetary Policy Commitment

Let us revisit the private sector’s date 0 investment and financing decisions, taking the date 1 policy of the central bank as given. Moreover, let us for now summarize the central bank’s policy by the value $\lambda$ takes, and disregard any decision that may arise from the specific combination of $(i^p_1, e_1)$ that achieves this $\lambda$.

To make our points, we do not depend on the presence of aggregate shocks. We assume that date 1 aggregate conditions are fully anticipated to be those of an external crisis — that is, the economy will be in the vertical region, with binding financial constraints (see the appendix for parameter conditions to arrive at this scenario).

Let us consider firms’ incentives to precaution against the date 1 crisis and, particularly,
how these incentives are affected by the domestic cost of capital, \( i^d_1 \). Precautioning is an ex-ante decision to reduce date 0 investment, borrow less of \( d_{0,J} \) and save this debt capacity for the date 1 crisis. Equivalently this is a date 0 private decision to hoard some international liquidity.

The net return on investing an extra unit of \( k \) for an intact firm is composed of two pieces. On the gross return side, the firm obtains \( Ak \) goods at date 2. On the cost side, it sacrifices \( (1 + i^*_0)c'(k) \) units of international liquidity which would have yielded a return \( i^d_1 \) in the domestic financial market at date 1. Thus, at the margin increasing \( k \) yields an ex-post return, net of opportunity cost, of

\[
A - c'(k)(1 + i^d_1)(1 + i^*_0).
\]

(5)

It follows immediately from (5) that the opportunity cost of increasing date 0 indebtedness is undervalued by an intact firm as long as \( i^d_1 < \Delta - 1 \). Since the demand for date 1 external funds is constrained by the distressed firms’ limited domestic collateral, the value of holding on to a unit of international liquidity in order to supply it to distressed firms at date 1 is depressed relative to the socially efficient —and the distressed firm’s— valuation, \( \Delta \).

Now consider the same net return for a distressed firm. On the gross return side, at the margin the firm obtains \( a \) unit of goods directly. But because a fraction \( \lambda \) of these goods can be pledged as collateral in the domestic loan market at date 1, that fraction must be multiplied by the private return that each generates (its value as a collateral asset). A loan secured by \( \lambda a \) goods at date 2, generates proceeds of \( \frac{\lambda a}{1 + i^d_1} \) goods at date 1, which each can be reinvested at a gross return of \( \Delta \). Thus the gross benefit to increasing \( k \) is \((1 - \lambda)a + \lambda a \frac{\Delta}{1 + i^d_1}\). On the cost side, it sacrifices \( (1 + i^*_0)c'(k) \) units of international liquidity which yield a private return of \( \Delta \) to a distressed firm. Thus, at the margin increasing investment in \( k \) yields a net return of:

\[
a \left( (1 - \lambda) + \lambda \frac{\Delta}{1 + i^d_1} \right) - c'(k) \Delta (1 + i^*_0).
\]

(6)

It follows immediately from (6) that while the cost of sacrificing a unit of international liquidity is properly valued by a distressed firm, the domestic investment is not. As long as \( i^d_1 < \Delta - 1 \), a distressed firm is able to keep some of the surplus from reinvestment by selling its overvalued domestic collateral. A central planner, on the other hand, realizes that during an external crisis only international liquidity generates social surplus, and hence discounts domestic assets at \( \Delta \) rather than \((1 + i^d_1)\).\(^{10}\)

\(^{10}\)Since the analysis above takes date 0 decisions as given, the expected return on domestic loans splits the
Although for different reasons, both intact and distressed firms overvalue (from a social point of view) domestic investment relative to its opportunity cost — namely its opportunity cost in terms of the international liquidity used. Since the ex-ante decision is based on the average of these two outcomes, it follows that firms will overinvest, overborrow and under-precaution for the date 1 shock.\(^{11}\)

The over-borrowing problem is an equilibrium problem. It arises only in the vertical region, as the supply of international funds faced by distressed firms is independent of others’ actions in the horizontal region, and it stems from distorted asset prices due to financial constraints — limited \(\lambda ak\) constrains demand so that \(i^d_1 < \Delta - 1\).

The optimal monetary policy commitment in the vertical region should be apparent by now. Since increasing \(\lambda\) at date 1 increases \(i^d_1\) and reduces the spread between \(\Delta - 1\) and \(i^d_1\), the distortion caused by limited domestic collateral falls as \(\lambda\) rises. Thus the optimal monetary policy commitment is to ex-post (i.e., during the crisis) choose the maximum possible \(\lambda\).\(^{12}\)

Aside from the standard inflationary concerns associated to the need for a very active monetary policy, the contrast between ex-ante (countercyclical) and ex-post (pro-cyclical) optimal policy during crises highlights an unusual commitment problem. The central bank should certainly commit to not “fear floating,” but should ideally commit to exacerbate the exchange rate depreciation during a crisis. If this is not credible, the incentive benefit —its only benefit in the vertical world— of countercyclical monetary policy vanishes and the cure for the structural underinsurance problem must be sought elsewhere. We return to such an alternative after introducing constrained monetary regimes.

\(^{11}\)This opens the door to international liquidity management policies, as we study in Caballero and Krishnamurthy (2000a). See also Harberger (1985) and Aizenman (1989), for alternative models of over-borrowing based on the underevaluation of the country’s monopsony power in international financial markets.

\(^{12}\)See the appendix for a formal proposition and proof of this result.
3.2 Constrained Monetary Regimes

One of the main criticisms of dollarization and other hard fixed exchange rate systems is that the central bank is unable to implement countercyclical monetary policy. This criticism would appear to be all the more damaging when financial accelerator mechanisms are appended to standard arguments.\textsuperscript{13} As we have argued, however, this policy option has little value at the time of a vertical crisis.

Under the vertical view, the concern with these type of exchange rate regimes is close to that of the free insurance criticism of fixed exchange rate systems, whereby the latter is perceived as a central bank subsidy on dollar-borrowing.\textsuperscript{14} However in our model underinsurance does not stem from the fixed value of the exchange rate itself, but from the central bank’s inability to optimally expand $\lambda$ during crises and reward the hoarding of scarce international liquidity. Thus, for example even in a flexible exchange rate system, if policy is not sufficiently counter-cyclical, there will be the underinsurance problem.

There is a “silver-lining” for constrained monetary regimes in the above discussion. As the problem of dollarization is shifted from an ex-post to an ex-ante distortion, a new set of policy options emerges. Rather than seeking hard-to-find substitutes for monetary policy during crises, the policymaker can introduce measures to solve the ex-ante underinsurance problem directly.\textsuperscript{15}

Recall that the problem induced by not being able to commit to expand credit at date $1$ is that $\Delta - (1 + i_d^1)$ remains high, and thus the return to hoarding international liquidity until date $1$ remains undervalued. While in practice this undervaluation of international liquidity may take many forms, in our simple model it is just high external leverage (high $d_{0,f}$) or, equivalently, excessive investment in domestic firms (high $k$).\textsuperscript{16} In our environment, there are two obvious ex-ante policy measures that can deal with the underinsurance problem: capital inflows taxation during normal times (date 0), and international liquidity requirements at date 0.

These ex-ante options, of course, are also available in a flexible exchange rate system.

\textsuperscript{13}See, e.g., Gertler et al (2001) and Cespedes et al (2001).
\textsuperscript{14}See, e.g., Dooley (1999).
\textsuperscript{15}Note, however, that even hard currency board systems can effectively implement some degree of monetary policy by, e.g., temporarily allowing domestic Treasury instruments denominated in dollars to count as international reserves, or relaxing the banks international liquidity ratios, as Argentina has done over the last decade.
\textsuperscript{16}With a slightly richer model, in Caballero and Krishnamurthy (2000a,2000b) we show that this undervaluation also leads to increased dollarization of liabilities, increased short term debt, and insufficient contingent lines.
To the extent that $\lambda$ cannot move sufficiently at date 1 in order to align the date 0 problems of the central planner and private sector, perhaps as a result of the commitment problems discussed earlier, these measures are desirable. More generally, let us return to the analysis of the previous subsection and characterize the relationship between the optimal ex-ante tax and $\lambda$.

At the aggregate level, building a marginally larger plant always generates date 2 output of $A + \frac{a}{2}$. The opportunity cost of doing this is to save this international liquidity until date 1 at which point, since one of the distressed or intact firms will be reinvesting, the return is $c'(k)(1 + i_{0}^{*})\Delta$. In total, the social return is,

$$A + \frac{a}{2} - c'(k)(1 + i_{0}^{*})\Delta.$$  \hfill (7)

Aligning the date 0 private and social incentives is a matter of choosing a tax/transfer policy. Suppose that the central bank levies a tax $\tau$ per unit of $k$, which is returned to firms in a lump sum fashion. Then, from (6) and (5), the private sector return to hoarding a unit of international liquidity as opposed to investing it, is:

$$\frac{A + a}{2} + \frac{a}{2} \left( (1 - \lambda) + \lambda \frac{\Delta}{1 + i_{1,\tau}^{d}} \right) - c'(k)(1 + i_{0}^{*})\frac{\Delta + 1 + i_{1,\tau}^{d}}{2} - \tau,$$  \hfill (8)

where $i_{1,\tau}^{d}$ represents the new (after taxes) equilibrium cost of a domestic dollar-loan.

As we concluded earlier, if domestic financial markets are well developed or monetary policy is powerful enough so that in the absence of taxes $i_{1}^{d} = \Delta - 1$, (7) and (8) coincide for $\tau = 0$ and there is no reason for intervention at date 0. If that is not the case, the optimal tax must equate these two expressions. A few steps of algebra, and identifying the social planner’s quantities with a hat, gives us that the optimal tax is:

$$\tau = \frac{1}{2} \left( \frac{\hat{w}}{\hat{k}} + c'\left(\hat{k}(1 + i_{0}^{*})\right) \right) \left( \Delta - (1 + i_{1,\tau}^{d}) \right),$$  \hfill (9)

which is clearly decreasing with respect to $\lambda$ since:

$$i_{1,\tau}^{d} = \lambda \frac{\hat{k}}{\hat{w}},$$

and all the social planner’s quantities are independent of $\lambda$.

We think of the above as an “iso-international liquidity” menu: a schedule of $(\tau, \lambda)$ such that the private sector and social incentives are aligned. Thus, for example, in the case of a $\lambda$ that cannot respond to external shocks, as is the case of a dollarized system, a positive $\tau$ would be beneficial.
Of course, in practice taxes come with their own sets of distortions — deadweight costs of taxation, costs of enforcement, evasion, etc. Moreover, a significant drawback of date 0 measures in our model is that if they are not fully reversed at date 1, the reduction in date 1 international liquidity will more than undo the date 0 benefit of having the private sector hoard liquidity. Thus these measures do require the authority to be very responsive to economic conditions.

These issues, along with the credibility (see above) and inflationary problems of an active monetary policy, need to be weighed in deciding which is the optimal exchange and monetary policy arrangement in a specific country. Our main purpose in this section is to point out the existence of a menu of options. In contrast to prevailing views, a fixed exchange rate system is not strictly at a liquidity disadvantage to the flexible system. This contrast arises from the fact that the prevailing wisdom is based on a domestic liquidity perspective, while we argue that during severe crisis it is international rather than domestic liquidity that matters the most.

3.3 International Credit Lines and Reserves

International credit lines are often perceived as a necessary supplement to constrained monetary regimes. In a vertical framework, however, these lines are desirable regardless of the exchange rate regime, and thus should not be thought of as a substitute for domestic monetary policy in a dollarized economy. Since in a crisis the main binding constraint is the international one, any effort to loosen this constraint is desirable.

Indeed, the usual view that credit lines are valuable in a currency board because they allow for some ability to expand credit in crises and prevent bank runs is just a version of this. This argument makes little sense in a horizontal view because, if it could, the country would simply borrow the dollars to expand banking credit in crises. There would be no need to have contracted ex-ante for a credit line. On the other hand, in the vertical view, the country is internationally constrained in a crisis, thus the only policy that can alleviate the problem are ex-ante measures to ease this constraint.\textsuperscript{17} The same logic applies to international reserves management considerations.

\textsuperscript{17}International credit lines should in principle be contracted directly by the private sector, but the same underinsurance problem we have highlighted in the core of the paper will limit the extent to which they will do so (see Caballero and Krishnamurthy (2000a) for a related argument in the context of a sterilization of capital inflows). Of course, if the central bank or government has access to international contingent instruments that the private sector does not, it should use it. Again, however, this is true regardless of the particular exchange rate arrangement in place, as it is justified by the vertical constraint rather than by the lack of domestic monetary policy.
4 Political Economy

In the standard discussion of rules versus discretion in monetary policy, there is a fundamental tension between the inflation-bias costs and the ex-post flexibility benefits of discretion (e.g., Kydland and Prescott (1977), Barro and Gordon (1983), Rogoff (1987)). This logic is often extrapolated to the debate on fixed versus flexible exchange rates. In this context, the optimal system typically involves a central bank that is left with some ex-post discretion but that commits to do less smoothing than it would be tempted to do ex-post.\(^\text{18}\)

In this section we show that while this logic is applicable in the horizontal region, it fails when external crises are of the vertical type. As we discussed in previous sections, in the vertical region the ex-post incentive for the central bank is to tighten excessively, so there is neither an inflationary bias nor an ex-post smoothing advantage. In a sense, the commitment problem is reversed in this case. Partly as a result of institutional design to prevent the traditional inflationary problem and partly due to the time inconsistency issue we have described, central banks are likely to behave too conservatively during crises. If so, the exchange rate/monetary policy combination is unlikely to solve the structural incentive problem, and the solution may have to be looked for among ex-ante measures.

To analyze these issues, let us introduce two modifications to our basic model. First, let \(\lambda\) now depend not on the level of peso-interest rates but on the gap between actual and expected rates. For simplicity, let us also make this function linear and drop the exchange rate from it:

\[
\lambda(i^p_1 - E_0 i^p_1) = \bar{\lambda} - \eta(i^p_1 - E_0 i^p_1), \quad \bar{\lambda}, \eta > 0
\]

Second, suppose that the central bank objective at date 1 covers both real investment, \(\theta_k\), as well as meeting its inflation target. For this purpose, let the gap between actual and target inflation be proportional to the depreciation of the nominal exchange: \((e_1 - 1)\), and let the central bank’s objective be to maximize,

\[
\theta_k - \frac{\alpha}{2}(e_1 - 1)^2,
\]

with \(\alpha > 0\).

4.1 Horizontal region

At date 1 the central bank takes as given expected interest rates and maximizes:

\[
\max_{i^p_1} \frac{u^n + \lambda(i^p_1 - E_0 i^p_1)\alpha k}{1 + i^*_1} \quad \frac{\alpha}{2}(e_1 - 1)^2
\]

\(^{18}\text{The discussion of fixed with escape clauses, or flexible exchange rate systems with tight inflation targets, and so on, is largely rooted in this view.}\)
This yields as a first order condition (where the superscript \(H\) denotes horizontal):

\[ i_{1}^{H} = i_{1}^{s} - \frac{\eta}{\alpha} \frac{ak}{1 + i_{1}^{s}}, \]  

(10)

which in the absence of aggregate shocks is equal to the expected interest rate. Thus we obtain the standard inflation bias of the non-commitment solution:

\[ e_{1} - 1 = \frac{\eta}{\alpha} \frac{ak}{1 + i_{1}^{s}} > 0. \]

Without aggregate shocks there is no advantage of keeping the option to devalue at date 1 since the latter is fully anticipated and results on higher inflation but no expansion of real investment. The role for discretion comes from the presence of aggregate shocks and the fact that the underlying contracts cannot fully insure these shocks away, so the expected inflation that enters into the \(\lambda\) expression is the unconditional rather than the state contingent one.

Let us introduce aggregate shocks, so now there is a good state of the world where the international interest rate is low enough that all firms can fully finance their investment needs. The bad state, on the other hand, leads the economy to be in the constrained horizontal region.

Starting from the good state, it is apparent that since the central bank gains nothing in terms of real activity from lowering interest rates, it will set \(e_{1} = 1\) to meet its inflation target. In the bad state, on the other hand, the problem is exactly as above, and the interest rate is set as in (10). The important difference with the full certainty (as of date 0) case, is that the expected interest rate is now lower than \(i_{1}^{pH}\), which means that \(\lambda > \bar{\lambda}\), and reinvestment is enhanced by the expansionary ex-post monetary policy.

The latter establishes the standard tradeoff between the stabilization role of discretionary monetary policy and the inflationary bias that such option generates.

4.2 Vertical

These tradeoffs change when the bad state of the world brings about a vertical constraint. At date 1, regardless of whether this was fully anticipated or not, the central bank sees no real reward in lowering interest rates as reinvestment no longer depends on \(\lambda\). Thus it will be only concerned with its inflation target and set \(e_{1} = 1\) in all states of the world.

In this case, there is no inflation bias as there is no advantage of ex-post opportunistic behavior by the central bank. Leaving the central bank with ex-post discretion does not help smoothing real fluctuations.
Quite the contrary, as we described above, the optimal commitment solution is to force the central bank to be expansionary during the bad state of the world even if that does not have any ex-post reward. If such commitment exists, then the inflationary bias re-emerges but in exchange for it date 0 incentives to precaution are improved. That is, rather than by the ex-post impact of monetary policy, real fluctuations are smoothed by inducing the private sector to do something about them.

In practice, however, it is highly unlikely that a modern independent central bank — especially those still affected by the standard reputation issues of and inflation prone past—will be willing to follow this countercyclical recipe very actively. If so, fear of floating must be recognized as a positive statement on policy, in which case central bank discretion has little advantages and the exchange rate discussion becomes more or less moot from a liquidity provision perspective. The underinsurance problem has to be resolved by ex-ante means such as the capital taxation of the previous section.

To summarize this section, the standard debate of rules versus discretion, and its application to the exchange rate selection debate, applies to countries that are reasonably well integrated to international financial markets \(w \) large and hence have crises that fall into the horizontal region. It does not apply well to countries that are frequently affected by sudden stops. Under the modern rules of independent central banking, with concerns for narrow inflation targets, inflationary bias is not the main concern but fear of floating is. The latter is problematic because it exacerbates the structural underinsurance problems of these economies.

5 Final Remarks

There are three main insights highlighted by our analysis: First, monetary policy during a sudden-stop crisis has limited real effects since it primarily affects domestic liquidity, while the main problem is one of international liquidity shortages. The dual of this real-ineffectiveness is the large sensitivity of the exchange rate to monetary policy. Fear of floating follows naturally from this asymmetry. Second, private sector decisions in anticipation of a sudden-stop crisis are affected by the expected actions of the monetary authority during the crisis. In particular, anticipation of increased domestic liquidity during a crisis will induce the private sector to preserve more international liquidity for the eventual crisis. Third, since the role of monetary policy in this context is one of incentives rather than one of (international) liquidity provision, loss of monetary discretion can be substituted for with ex-ante measures that induce the private sector to save international liquidity for crises.
Of course there are many caveats that a stylized model like ours is subject to. For example, in practice expanding domestic liquidity during crises will have some contemporaneous positive effects. Similarly, we have assumed that once in a crisis, there are no tools to expand international liquidity. We can relax this assumption and even connect it directly to monetary policy. The standard argument that a devaluation helps the export sector when nominal wages are sticky applies here as well if much of the country’s international collateral is linked to export firms. Conversely, if the prime companies of a country are in the non-tradeable sector (e.g., energy and telecommunications) and their debt is dollarized, then a devaluation may reduce international collateral during the crisis.

Equally stylized is our assumption that all crises are either vertical or horizontal. In many instances crises build up, going first through a horizontal phase, where domestic financial conditions tighten and external borrowing becomes gradually more expensive, before falling into a sharp vertical sudden stop phase. A central question for policymakers in this context is what to do with monetary policy at the early stages of the crisis, when the environment is still fairly horizontal but there is a real concern that a sudden stop may be around the corner. At this stage, tightening monetary policy destroys financially constrained projects but saves international liquidity for the potential sudden stop. We conjecture that this tradeoff can be analyzed in terms similar to those we have used throughout: If the commitment to an aggressive countercyclical monetary policy were the sudden stop to arrive is credible, then there is little need to tighten during the horizontal phase. But if the commitment is not credible or feasible, then the appropriate response is to tighten during the early phase to protect international liquidity, very much as taxing capital flows at date 0 was advisable in our simplified model in such case. In fact, the costs in terms of the additional financial distress imposed upon the domestic private sector, is to a large extent comparable in nature to that of the ex-ante measures we already discussed.

Our goal has been not to provide a policy recommendation but to identify the nature of the tradeoffs involved and, in particular, to highlight the contrast between these tradeoffs and those identified in the traditional horizontal view.

Nevertheless, in closing, it is worth speculating on the relevance of our perspective for the question of what is the optimal exchange rate system for emerging economies. We conjecture that for this purpose our distinctions are most relevant for an intermediate range countries. In fact, for countries with a history of inflation problems, the gains of currency boards probably outweigh the costs. Looking at crises as vertical rather than horizontal does not change the calculation. At the other extreme, for countries with no credibility

In addition to those we mention in the text and to the strong credibility anchor it offers, some speculate
problems and a precedent of good central banking, floating is probably the best choice, as long as fear of floating does not become the perceived rule. Quite the opposite, a credible commitment to a countercyclical monetary policy during crises, is a good substitute for costly capital controls and ex-ante measures.

The countries that lie in between are those with good central banking, but which — perhaps for historical reasons — are still concerned with establishing a reputation for containing inflation. Whether in a currency board or not, the evidence is that these countries fear floating and hence de facto give up ex-post monetary discretion. Our analysis suggests that these countries ought to look toward ex-ante measures to balance out the incentive-based need for active monetary policy that will not take place. Indeed, for those countries with very limited financial development, which require overly-active monetary policy in order to restore adequate incentives, this advice is particularly pertinent. They may be best served by adopting a currency board and focusing efforts on improving international liquidity management in the private and public sectors.

that the advantages include a lower interest rates and inflation risk premia due to the enhanced credibility in controlling inflation, and the potential positive impact that the latter may have on financial deepening. On the cost side, there is the loss of seignorage.
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A Appendix

A.1 Detailed programs, definitions and assumptions

There are three main assumptions we have made in the model:

**Assumption 1 (Non-observability of Production Shock)**

The production shock at date 1 is idiosyncratic. The identity of firms receiving the shock is private information.

**Assumption 2 (Domestic Borrowing Constraint)**

A domestic lender can only be sure that a firm will produce \( \lambda a k \) units of goods at date 2. Any excess production based on physical reinvestment at date 1 is neither observable nor verifiable.

**Assumption 3 (Liquidity Bias)**

Foreigners lend to domestic firms only against the backing of \( w \). Domestics lend against both \( w \) and \( \lambda a k \).

This gives a date 0 debt constraint with respect to foreigners of \( d_{0,f} \leq w \). At date 1, if a firm takes on additional debt with foreigners, the date 1 debt constraint is:

\[
d_{0,f} + d_{1,f} \leq w.
\]

Since domestics lend against \( \lambda a k \) the debt constraint for domestic lending at date 1 is,

\[
\frac{d_{1,d}}{1+i^*_1}(1+i^*_1) \leq w + \frac{\lambda a k}{1+i^*_1}(1+i^*_1) - d_{1,f} - d_{0,f}
\]

The program for a distressed firm at date 1 is:

\[
(P1) \quad V_i \equiv \max_{d_{1,d},x_{1,d}} w + \tilde{A}(\theta)k - d_{0,f} - d_{1,f} - d_{1,d}
\]

\[\text{s.t.} \]

\[
(i) \quad d_{1,f} + d_{0,f} \leq w
\]

\[
(ii) \quad \frac{d_{1,d}}{1+i^*_1}(1+i^*_1) + d_{1,f} + d_{0,f} \leq w + \frac{\lambda a k}{1+i^*_1}(1+i^*_1)
\]

\[
(iii) \quad \theta k = \frac{d_{1,d}}{1+i^*_1} + \frac{d_{1,d}}{1+i^*_1}
\]

\[
(iv) \quad \theta \leq 1.
\]

Constraints \((i)\) and \((ii)\) are balance sheet constraints (net marketable assets greater than liabilities), while constraint \((iii)\) reflects that new investment must be fully paid with the resources received by the firm at date 1 in taking on debts of \( d_{1,f} \) and \( d_{1,d} \). Constraint \((iv)\) is purely technological.

An intact firm at date 1 has only one decision: how much finance will it extend to the distressed firm. Suppose that the firm accepts claims at date 1 of \( x_{1,d} \) (face value of date 2 goods) in return for making a date 1 contribution of \( \frac{x_{1,d}}{1+i^*_1} \) which is financed with new external debt \( d_{1,f} \). Then,

\[
(P2) \quad V_i \equiv \max_{x_{1,d}} w + Ak + x_{1,d} - d_{0,f} - d_{1,f}
\]

\[\text{s.t.} \]

\[
\frac{x_{1,d}}{1+i^*_1} \leq \frac{d_{1,f}}{1+i^*_1}
\]

\[
d_{0,f} + d_{1,f} \leq w
\]
Date 0 problem. At date 0, a firm looking forward to date 1 can expect to find itself as either distressed or intact. Thus the decision at date 0 is,

\[
(P3) \quad \max_{k, d_{0,f}} \quad \frac{(V_s + V_i)}{2} \\
\text{s.t.} \quad d_{0,f} \leq w \\
c(k) = \frac{d_{0,f}}{(1+i_0)(1+i_1)}
\]

Equilibrium. Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

\[
D_{1,d} = \frac{1}{2} d_{1,d} \\
X_{1,d} = \frac{1}{2} x_{1,d}
\]

Therefore, market clearing,

\[
D_{1,d} = X_{1,d}, \quad (11)
\]
determines the domestic dollar-cost of capital, \(i_d^1\).

An equilibrium of this economy consists of date 0 and date 1 decisions, \((k, d_{0,f})\) and \((\theta, d_{1,f}, d_{1,d}, x_{1,d})\), respectively, and prices \(i_d^1\). Decisions are solutions to the firms’ problems (P1), (P2), and (P3) given prices. At these prices, the market clearing condition (11) holds.

Let us now study equilibrium in more detail. Starting from date 1, consider financing and investment choices of the distressed firm given \((k, d_{0,f})\). First, if \(\Delta - 1 \geq i_d^1\), then the distressed firm would choose to save as many of its production units as it can. It may borrow up to its international debt capacity,

\[
d_{1,f} = w - d_{0,f}. \quad (12)
\]

If the amount raised from international investors, \(\frac{w - d_{0,f}}{1+i_1}\), is less than the funds needed for restructuring, \(k\), the firm will have to access the domestic debt market to make up the shortfall. It will choose to do this as long as \(\Delta - 1 \geq i_d^1\), or the return on restructuring exceeds the domestic cost of capital. If the firm borrows fully up to its domestic debt capacity, it will issue debt totalling,

\[
d_{1,d} = \lambda a k, \quad (13)
\]

and raise funds with which to pay for imported goods of \(\frac{\lambda a k}{1+i_1}\). As long as the sum of \(\frac{\lambda a k}{1+i_1}\) and the right hand side of (12) is more than the borrowing need, the firm is unconstrained in its reinvestment
at date 1 and all production units will be saved. In this case, the firm will borrow less than its domestic debt capacity (and perhaps less than the international debt capacity).

Intact firms can tender at most their excess international debt capacity of \( w - d_{0,f} \) in return for purchasing domestic debt. They will choose to do this as long as the return on domestic loans exceeds the international rate, \( \delta^d_i \geq \delta^*_{1} \).

Assume for a moment that \( \Delta - 1 \geq \delta^d_i \geq \delta^*_{1} \) so that distressed firms borrow as much as they can, and intact firms lend as much as they can. Then, in total the economy can import \( \frac{w-d_{0,f}}{1+\delta^*_{1}} \) goods, which is directed to the distressed firms. A necessary condition for all production units to be saved is that,
\[
\frac{k}{2} \leq \frac{w-d_{0,f}}{1+\delta^*_{1}} \tag{14}
\]

We shall refer to this constraint as the international liquidity constraint. When neither (13) nor (14) binds, all production units are saved. Since there is excess supply of funds from intact firms relative to domestic demand for funds, there is no international liquidity premium, and \( \delta^d_i \) is equal to the international interest rate.

The other extreme case is when both (13) and (14) bind. Equilibrium in the domestic debt market requires that,
\[
\frac{\lambda a k}{1+\delta^*_{1}} = \frac{w-d_{0,f}}{1+\delta^*_{1}}.
\]
Since (13) binds, distressed firms borrow fully up to their debt capacity. As (14) binds, intact firms purchase this debt with all of their excess funds. Solving for \( \delta^d_i \), yields
\[
\delta^d_i = \frac{\lambda a k}{w-d_{0,f}}(1+\delta^*_{1}) - 1 > \delta^*_{1}. \tag{15}
\]

That is, in this case \( \delta^d_i \) is above the international interest rate in order to clear the domestic market for scarce international liquidity. One half times the numerator in (15) corresponds to the transferable domestic resources owned by distressed firms.

Define the index of domestic illiquidity as the difference between the marginal profit of saving a distressed production unit and the domestic interest rate of \( \delta^d_i \). When (14) binds, this is,
\[
s_d = \Delta - \delta^d_i - 1,
\]

Equilibrium at date 1 can place the economy in one of four regions, classified according to which of the two (domestic and international) liquidity constraints are binding. The horizontal view corresponds to the case where the domestic constraint binds, and the international one does not. The vertical view corresponds to the case where the international constraint binds and the domestic
one may or may not. $s_d > 0$ if and only if both constraints bind, and this is the scenario we focused on in the text. At the aggregate level, the economy is liquidity constrained with respect to foreigners; at the individual level, firms are liquidity constrained with respect to other domestics since they are selling all of their domestic liquidity in aggregation; real investment is constrained; domestic spreads are positive; and the domestic cost of capital of $i^*_1$ is above the international interest rate.

**Technical Assumption 1 (Conditions for Crisis)**

(13) and (14) bind in equilibrium as long as the following conditions are met. Define,

$$k = c^{-1} \left( \frac{A + a}{2(1 + i_0^d)\Delta} \right)$$

and,

$$\bar{k} = c^{-1} \left( \frac{A + a((1 - \lambda) + \lambda \frac{\Delta}{1 + i_0^d}(1 + i_1^* + \Delta))}{(1 + i_0^d)(1 + i_1^* + \Delta)} \right).$$

To ensure that in equilibrium $\Delta - 1 > i_1^d > i_1^*$, we need,

$$\lambda a\frac{1 + i_1^d}{\Delta} + (1 + i_0^d)(1 + i_1^*)c(k) > w > \lambda a\bar{k} + (1 + i_0^d)(1 + i_1^*)c(\bar{k}).$$

This comes from noting that $k < k < \bar{k}$, and using the equilibrium expression that

$$i_1^d = \frac{\lambda a}{w - (1 + i_0^d)(1 + i_1^*)c(k)}(1 + i_1^*) - 1.$$  

It is satisfied, for example, by choosing $\Delta - 1$ high relative to $i_1^*$. Given this assumption, $\theta < 1$ is guaranteed if $a < \frac{1 + i_1^*}{2\lambda a}$.

**Proposition: (λ and Welfare)** In the case that $s_d > 0$, welfare is increasing in $\lambda$, and $k$ is decreasing in $\lambda$.

Proof: First we show that $\frac{\partial h(k, \lambda)}{\partial \lambda} < 0$. Then we show that welfare is decreasing in $k$. To arrive at the first step consider the first order condition,

$$h(k, \lambda) \equiv (1 + i_0^d)(1 + i_1^d + \Delta)c'(k) - A - a(1 - \lambda) - \frac{\Delta}{1 + i_1^*}\lambda a = 0,$$

where,

$$i_1^d = \frac{\lambda a}{w - (1 + i_0^d)(1 + i_1^*)c(k)}(1 + i_1^*) - 1.$$  

Implicitly differentiating the first order condition,

$$\frac{\partial k(\lambda)}{\partial \lambda} = -\frac{\frac{\partial h(k, \lambda)}{\partial k}}{\frac{\partial h(k, \lambda)}{\partial k}}$$

>From the first order condition we can sign,

$$\frac{\partial h(k, \lambda)}{\partial k} = c''(k)(1 + i_0^d)(1 + i_1^* + \Delta) + \left( (1 + i_0^d)c'(k) + \frac{\Delta}{1 + i_1^* + \lambda a} \right)\frac{\partial i_1^d}{\partial k} > 0.$$
and,
\[
\frac{\partial h(k, \lambda)}{\partial \lambda} = a (1 - \frac{\Delta}{1 + \rho^2}) + \left((1 + \rho^2)c'(k) + \frac{\Delta}{(1 + \rho^2)^2} \lambda a\right) \frac{1 + \rho^2}{\lambda} > 0.
\]
Thus we conclude that \( \frac{\partial h(\lambda)}{\partial \lambda} < 0 \).

Consider welfare next. This can be written as,
\[
U = \frac{1}{2} \left( \left( \frac{w}{1 + \rho^2} - c(k)(1 + \rho^2) \right) (\Delta + 1 + \rho^2) + \frac{1}{2} k \left(A + a(1 - \lambda) + \frac{\lambda a}{1 + \rho^2} \Delta \right) \right).
\]
Now substituting in the market clearing condition for \( \rho^2 \) simplifies this to,
\[
U = \left( \frac{w}{1 + \rho^2} - c(k)(1 + \rho^2) \right) \Delta + \frac{1}{2} k (A + a).
\]
By comparing the first order condition for \( k \) in this expression to the previous one, it is straightforward to show that this function is decreasing in \( k \) as long as \( s_d > 0 \). Thus we can also conclude that welfare is increasing in \( \lambda \) in the vertical region.

A.2 Monetary policy and domestic collateral

In this appendix we sketch a model connecting monetary policy to domestic collateral, and thereby provide one explicit justification for linking \( \lambda \) to monetary policy, as we did in the main text. The mechanism we illustrate is based on debt-deflation and the resulting transfer from lenders to borrowers. The lenders will be large, less constrained firms, while the borrowers will be small, constrained, firms. We do not introduce banks into the model, although in practice these institutions are surely affected by debt-deflation. In this model, a monetary system that tightens during crises has the interpretation of corresponding to an environment where there is too little domestic insurance. The latter is equivalent to a decline in \( \lambda \) in the text.

We begin with the observation that tight monetary policy affects small firms more severely than large firms. There is a measure \( S \) of small firms, and a unit measure of large ones (exactly as before). Date 0 investment of both types builds \( ak^i \) (for \( i = s, l \) - small and large, respectively) units of domestic collateral. However small firms differ from large ones in two ways. First, they have no international collateral so that they are reliant on large firms for all investment needs. Second, to simplify matters, we shall assume that all small firms are distressed at date 1 (this asymmetry with large firms ensures that in equilibrium there is a reason for small firms to be (partially) insured against aggregate shocks by large firms).

As before, large firms borrow directly from the rest of the world to setup their plants. Small firms, on the other hand, finance their date 0 plants by borrowing from large firms, who in turn borrow from international markets.
We introduce a role for monetary policy by assuming that all domestic borrowing at date 0 is done in non-contingent (on aggregate conditions) pesos and is one period.\(^{20}\) Denote \(b\) as the face value of one period domestic debt that each small firm takes at date 0. At date 1 there is an external shock to the foreign interest rate of \(i^*_1(\omega)\), where \(\omega \in \{L, H\}\) is the state of the world. This, in conjunction with monetary policy, results in a nominal exchange rate of \(e_1(\omega)\) that satisfies the domestic interest parity condition,

\[
i^*_1(\omega) + e_1(\omega) - 1 = i^*_1(\omega).
\]

Thus at date 1 the net worth of a small firm in terms of domestic collateral is,

\[
ak^s - b/e_1(\omega)
\]

Since large firms are at the other side of this transaction, and there is a unit measure of large firms, the domestic collateral of large firms at date 1 is,

\[
ak^l + Sb/e_1(\omega).
\]

To save notation, we have substituted the small firm’s debt in the latter expression. Of course, domestic debt is a private decision of the small and large firms respectively.

Monetary policy is the choice of \(e_1(\omega)\) given the domestic interest parity condition. Note that if either there is no aggregate uncertainty, or if the debt is fully contingent on \(e_1\), there is no role for monetary policy in this setup. The rigidity we introduce is one on the debt repayment, resulting in a debt-deflation channel for monetary policy that transfers resources from large firms (domestic date 0 lenders) to small firms (domestic date 0 borrowers).\(^{21}\)

At date 1, the total investment need of both small and large firms is \(Sk^s + \frac{1}{\theta}k^l\). The international liquidity of the country is \(w - d_{0,f}\). We assume that in the \(H\)-state, \(i^*_1\) is low enough so that these investment needs are met by each firm. That is,

\[
Sk^s + \frac{1}{2}k^l \leq \frac{w - d_{0,f}}{1 + i^*_1(H)}, \quad \text{and} \quad \lambda a \geq 1 + i^*_1(H).
\]

In the \(L\) state, on the other hand, we shall assume the economy is in the “crisis” equilibrium: \(\theta^*, \theta^s < 1\), with\(^{22}\)

\[
S\theta^*k^s + \frac{1}{2}\theta k^l = \frac{w - d_{0,f}}{1 + i^*_1(L)}.
\]

\(^{20}\)As in much of the debt-deflation monetary policy literature, as well as in much of monetary economics, we do not address the issue of why the private sector does not write contingent contracts rather than relying on —or suffering from— the central bank. We are currently working on this issue and conjecture that adding an extra layer of segmentation, now among domestics, will allow us to address this deficiency more adequately.

\(^{21}\)See, e.g., Lorenzoni (2000) for a model of debt deflation in the context of an interbank market.

\(^{22}\)S = 0 corresponds to the model we analyzed in the main text. It is always possible to choose a small \(S\) so that parameters lead to the situation we analyze.
Since both large and small firms sell all of their domestic collateral in the market, the analogue of (3) gives us market clearing of,

\[ i_1^{d} = \frac{S(ak^s - b/e_1(L)) + \frac{i}{2}(ak^L + Sb/e_1(L))(1 + i_1^L) - 1}{\frac{h}{L}(w - d_{0f})} \]
\[ = \frac{2Sak^s + ak^L - Sb/e_1(L)(1 + i_1^L) - 1}{w - d_{0f}} \]  \hspace{1cm} (16)

From the last expression we can clearly see that \( i_1^d \) is increasing in the amount of depreciation in the \( L \) state. This dependence is equivalent to our \( \lambda \) function in the main text.

What has happened? As monetary policy tightens (i.e., \( e_1(L) \) is not allowed to depreciate) during crises, the amount of effective insurance from large to small firms declines. In contrast, if policy is loose, \( e_1(L) \) falls, and resources are transferred from large intact firms to small distressed firms.\(^{23}\) Since on net these resources loosen the financial constraint on investing firms, in equilibrium, \( i_1^d \) rises toward the marginal product of \( \Delta - 1 \).

In this model the exchange rate is a domestic insurance mechanism. Since date 0 borrowing from small to large firms is based on the expectations of exchange rate in both \( H \) and \( L \) states, optimal policy will call for a strong exchange rate in the \( H \)-state, while a depreciated one in the \( L \)-state. If policy is not sufficiently expansionary in the \( L \)-state versus the \( H \)-state, there is limited domestic insurance. The latter depresses the effective demand for international collateral and hence, at date 0, it reduces the incentive to precaution against shortages in aggregate international illiquidity.

### A.3 The interest parity condition

Throughout the paper we have not been explicit about the mechanism through which the central bank affects interest rates/exchange rates. For the most part the connection is fairly standard – indeed many of the papers exploring the accelerator mechanism are not explicit about it either (e.g., Gertler et al (2001), or Chang et al. (2000)). However, the one deviation is that in our model, the fact that there is limited collateral in the vertical region does affect \( i_1^d \) and in turn the exchange rate/domestic interest rate. We draw this association because the price of the nominal asset (money) is determined by domestics and their demand for money. Since domestics only hold domestic collateral and money at date 1, the price of money is linked to the price of domestic collateral. To see this, consider adding an infinitesimal amount of an agent that demands money, lends against domestic collateral, and has international collateral of \( w \) (say a bank). This agent solves at date 1,

\[ \max_{m,x} \{v(m/e_1) + m/e_2 + x_i M \} \quad s.t. \quad x + m/e_1 \leq w \].

\(^{23}\)Resources are also transferred away from large distressed firms, but since these firms have the same marginal product as the small firms at date 1, this is a zero-sum transfer.
Where $v(\cdot)$ is demand for real money balances. Normalizing $e_2 = 1$, this gives the F.O.C.,

$$v'(m/e_1) + e_1 = i_1^d.$$  

Monetary policy is the choice over $m$, which from this expression results in a unique $e_1$. Setting $e_1^p = v'(m/e_1) - 1$ gives the parity condition we have used throughout the text.