An Improved P-DPC for A New Inverter Based on MLD Model

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Abstract. Building an accurate mathematical model for inverter is the key to achieving a precise control. Contrasted to the traditional switching function model of power electronic circuits, this paper built the mixed logical dynamic (MLD) model for a new inverter, and the MLD model was used as a prediction model, then a predictive direct power control (P-DPC) method was researched for the new inverter. A symmetrical 4+4 voltage vector sequence was employed to obtain constant switching frequency and lower THD of output voltage, the action time of vector sequence was calculated by minimizing objective function. The feasibility and effectiveness were proved by simulations.

1. Introduction
Constraint conditions and nonlinear factors of the control object could be fully considered by model predictive control (MPC), which is also suitable to the multivariable system control, multiple targets can be reached by minimizing objective function, therefore MPC is appropriate for power electronic circuits control [1]. While a challenge we faced with is the problem of mixed integer quadratic programming (MIQP) [2].

There are actually two approaches for solving MIQP, one is solving MIQP problem off-line, the results are stored in formulated table, circuit states are monitored to select the optimal solution by searching table, but this method would occupy computer storage, and an effective searching algorithm is needed[3-5]. Another is using on-line optimization algorithm, for example, Branch and Bound, which needs a large amount of calculation [6].

The document [7-10] researched Finite Control Set Model Predictive Control (FCS—MPC) for circuits, the discrete properties of power electronic circuits were fully used by FCS-MPC, each possible combination of switch states was considered, and the switch state, which minimized objective function was used as circuit control, the complex MIQP problem was avoided, but there still had two problems:
1) Switching frequency was inconstant, which increased the difficulties to design filter;
2) Each switch state should be considered, which was not suitable for more switch states circuits.

P-DPC, which combined direct power control and MPC, had both advantages, documents [11] adopted 3+3 voltage vector sequence to decrease switching loss.

An improved 4+4 voltage vector sequence was selected by this paper to obtain constant switching frequency and lower THD of output voltage, the action time of vector can be got by minimizing objective function.
2. The prediction model of inverter based on MLD model

The inverter shown at figure 1 is improved from [12], switch S1-S4 are increased to balance the voltage of neutral point O. The inverter could be used in high reliable system for its simple control and fine fault tolerant.

![Figure 1: New inverter topology](image)

The switch states of one phrase of inverter is shown in table 1, similarly to the rest two phrases. $U_{ao}$ is voltage between a and g, $U_{Ag}$ is voltage between A and g, $s_1-s_6$ are control signals of switch $S_{a1}-S_{a6}$, $s_7-s_{12}$ are control signals of switch $S_{A1}-S_{A6}$. The control of neutral point voltage is achieved by setting switches $S_1-S_4$ on and off.

**Table 1: Switch states of one phrase**

| $(s_1, s_4, s_7, s_10)$ | $(U_{ao}, U_{Ag})$ |
|--------------------------|---------------------|
| (1,0,1,0)                | $(V_{dc}, V_{dc}/2)$ |
| (0,1,0,1)                | $(V_{dc}/2, 0)$     |
| (0,1,1,0)                | $(V_{dc}/2, V_{dc}/2)$ |
| (1,0,0,1)                | $(V_{dc}, 0)$      |

The direction of filter current $i_a$ is shown in Figure 1, for phase a, operating models are shown in (1), when “1” represents “on”, “0” represents “off”.

\[
\begin{align*}
    i_a > 0: \quad s_1 = 0, s_4 = 1 & \Rightarrow u_{ao} = U_{dc}/2; s_7 = 1, s_{10} = 0 & \Rightarrow u_{ao} = U_{dc}; s_7 = 0, s_{10} = 0 & \Rightarrow u_{ao} = U_{dc}/2 \\
    i_a < 0: \quad s_1 = 0, s_4 = 1 & \Rightarrow u_{ao} = U_{dc}/2; s_7 = 1, s_{10} = 0 & \Rightarrow u_{ao} = U_{dc}; s_7 = 0, s_{10} = 0 & \Rightarrow u_{ao} = U_{dc}
\end{align*}
\]

Logical operators are introduced [13], “∧” represents conjunction, “∨” represents disjunction, “↔” represents equivalence, “¬” represents not, meanwhile logical variable $a$ is also introduced, the discrete event $i_a > 0$ and $i_a < 0$ are respectively shown by $a = 1$ and $a = 0$, as (2).

\[
\begin{align*}
    [a = 1] & \leftrightarrow [i_a > 0] \\
    [a = 0] & \leftrightarrow [i_a < 0]
\end{align*}
\]
The discrete events in (1) are replaced by logical variables, then mixed logical relation of phase a is shown in (3).

\[
\begin{align*}
[s_i = 0, s_k = 1, \sigma_a = 1] \lor [s_i = 0, s_k = 0, \sigma_a = 1] \lor \\
[s_i = 0, s_k = 1, \sigma_a = 0] \leftrightarrow [u_\alpha = U_{dc} / 2] \\
[s_i = 1, s_k = 0, \sigma_a = 1] \lor [s_i = 0, s_k = 0, \sigma_a = 0] \lor \\
[s_i = 1, s_k = 0, \sigma_a = 0] \leftrightarrow [u_\alpha = U_{dc}] \\
\end{align*}
\]

(3) is expressed as one equation, and the mixed logical expression of \( u_\alpha \) is shown in (4).

\[
u_\alpha = U_{dc} [s_k (s_i + \sigma_a) + \frac{1}{2} s_k (s_i + \sigma_a)]
\]

The expression of phase ‘b’ and ‘c’ can also be obtained by the same way

\[
\begin{align*}
u_{\beta} = V_{dc} [s_k (s_i + \sigma_a) + \frac{1}{2} s_k (s_i + \sigma_a)] \\
u_{\gamma} = V_{dc} [s_k (s_i + \sigma_a) + \frac{1}{2} s_k (s_i + \sigma_a)]
\end{align*}
\]

If three phase of inverter are balanced, then the MLD expressions of \( u_{a0}, u_{b0} \) and \( u_{c0} \) are shown in (6).

\[
\begin{bmatrix}
u_{a0} \\
v_{b0} \\
v_{c0}
\end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \\ \end{bmatrix} \begin{bmatrix} s_k (s_i + \sigma_a) \\ s_k (s_i + \sigma_b) \\ s_k (s_i + \sigma_c) \\ \end{bmatrix}
\]

Then, (6) is transformed into space vector form, have:

\[
u_a = \frac{2}{3} (u_{a0} + \alpha u_{b0} + \alpha^2 u_{c0})
\]

Where: \( \alpha = e^{j(2\pi/3)} \).

By the same way, inductance current, \( i_{af}, i_{bf}, i_{cf} \) and \( i_{cf} \), capacitance voltage \( u_{ac}, u_{bc} \) and \( u_{ce} \), are all transformed into space vector form, which are shown in (8).

\[
\begin{align*}
i_f = \frac{2}{3} (i_{af} + \alpha i_{bf} + \alpha^2 i_{cf}) \\
u_c = \frac{2}{3} (u_{ac} + \alpha u_{bc} + \alpha^2 u_{ce})
\end{align*}
\]

\( u_a, u_c, i_f \) are transformed into \( \alpha, \beta \), coordinate system, which can obtained \( u_{af}, u_{f0}, u_{ac}, u_{cf}, i_{fa}, i_{fb} \) the active and reactive power are shown in (9)

\[
\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} u_{af} & u_{c0} \\ u_{f0} & -u_{af} \end{bmatrix} \begin{bmatrix} i_{fa} \\ i_{fb} \end{bmatrix}
\]

Then, (9) is derived to (10)
\[
\begin{bmatrix}
\frac{dP}{dt} \\
\frac{dQ}{dt}
\end{bmatrix} =
\begin{bmatrix}
\frac{di_{fa}}{dt} + i_{fa} + \frac{du_{ca}}{dt} + u_{ca} + \frac{di_{f\beta}}{dt} + i_{f\beta} + \frac{du_{\beta}}{dt}
\\
\frac{di_{f\beta}}{dt} + i_{f\beta} + \frac{du_{\alpha}}{dt} - u_{\alpha} + \frac{di_{f\alpha}}{dt} - i_{f\alpha} + \frac{du_{\alpha}}{dt}
\end{bmatrix}
\tag{10}
\]

Ignored the filter resistance, Considered:
\[
\frac{di_{f\alpha}}{dt} = \frac{1}{L} \left( u_{n\alpha} - u_{c\alpha} \right)
\tag{11}
\]

Then the (12) can be obtained by (11)
\[
\begin{align*}
\frac{di_{f\alpha}}{dt} &= \left( u_{n\alpha} - u_{c\alpha} \right) / L \\
\frac{di_{f\beta}}{dt} &= \left( u_{n\beta} - u_{c\beta} \right) / L
\end{align*}
\tag{12}
\]

The target of this control is to get a standard sinusoidal voltage from the input of transformer. So that \( u_{c\alpha} \) and \( u_{c\beta} \) can be described as (13)
\[
\begin{align*}
u_{c\alpha} &= U_s \sin \omega t \\
d\frac{du_{c\alpha}}{dt} &= U_s \omega \cos \omega t = -\omega u_{c\beta}; \\
u_{c\beta} &= -U_s \cos \omega t \\
d\frac{du_{c\beta}}{dt} &= U_s \omega \sin \omega t = \omega u_{c\alpha}
\end{align*}
\tag{13}
\]

Where: \( U_s \) is the amplitude which is expected. Substituting (10) with (12) and (13), have:
\[
\begin{align*}
\frac{dP}{dt} &= u_{c\alpha} \left[ \left( u_{n\alpha} - u_{c\alpha} \right) / L + \omega i_{f\beta} \right] + u_{c\beta} \left[ \left( u_{n\beta} - u_{c\beta} \right) / L - \omega i_{f\alpha} \right] \\
\frac{dQ}{dt} &= u_{c\alpha} \left[ \omega i_{f\alpha} - \left( u_{n\alpha} - u_{c\alpha} \right) / L \right] + u_{c\beta} \left[ \left( u_{n\beta} - u_{c\beta} \right) / L + \omega i_{f\beta} \right]
\end{align*}
\tag{14}
\]

It is clear that the variation rate of power is related with the filter inductance and switch status from (14).

Under the condition of a certain voltage vector \( u_{n} = \left[ u_{n\alpha} \ 0 \right]^T \) and a large switch frequency, the \( u_{c} = \left[ u_{c\alpha} \ u_{c\beta} \right]^T \) can be regarded as a constant. Considering under that condition, the \( i_{f\alpha} \) and \( i_{f\beta} \) only have little change, and in the period when the voltage vector effect, the variation rate of power can be approximately regarded as a constant.
\[
k_P = \frac{dP}{dt} \bigg|_{u_{n} = u_{n}} \quad k_Q = \frac{dQ}{dt} \bigg|_{u_{n} = u_{n}}
\tag{15}
\]

Where: \( u_{n} \) is one of the eight space voltage vectors (\( i=0 \ 1 \ 2 \ldots \ldots \ 7 \)). The space voltage vectors are shown in figure 2.
Through (15) the track of active and reactive power can be predicted.

\[
P_i = P_{i-1} + k_P \cdot t_m \\
Q_i = Q_{i-1} + k_Q \cdot t_m
\]  

(16)

Where: \( t_m \) is the effecting period of voltage vector \( u_{n_i} \); \( P_{i-1}, Q_{i-1} \) are the power before the vector effecting, and \( P_i, Q_i \) are the after.

3. Symmetrical 4+4 voltage vector sequence

3.1. The principle of 4+4 vector sequence

Symmetrical 4+4 vector sequence is improved from 3+3 vector sequence [14], as Figure 2 shown, two voltage vectors which near reference voltage vector are selected as effective vectors, in addition zero vectors \( u_{n_0} \) and \( u_{n_7} \) are selected, the sequence is constituted by 4 vectors, and these vectors are used as inverter control.

The sequence is divided into two subsequences, and the first subsequence is completely symmetrical to the second one, the last vector and its action time of the first subsequence are the same as the first vector and its action time of second subsequence, the other vectors of the two subsequences also follow the same symmetrical law.

When the reference voltage vector located in different 6 sections, the 4+4 vector sequences of inverter are shown in table.2.

**Table.2 4+4 voltage vector sequence**

| Section | The first sequence | The second sequence |
|---------|--------------------|---------------------|
| Section 1 | \( u_{n_0}, u_{n_1}, u_{n_2}, u_{n_7} \) | \( u_{n_7}, u_{n_2}, u_{n_3}, u_{n_0} \) |
| Section 2 | \( u_{n_0}, u_{n_1}, u_{n_2}, u_{n_7} \) | \( u_{n_7}, u_{n_2}, u_{n_3}, u_{n_0} \) |
| Section 3 | \( u_{n_0}, u_{n_4}, u_{n_5}, u_{n_7} \) | \( u_{n_7}, u_{n_5}, u_{n_6}, u_{n_0} \) |
| Section 4 | \( u_{n_0}, u_{n_4}, u_{n_5}, u_{n_7} \) | \( u_{n_7}, u_{n_5}, u_{n_6}, u_{n_0} \) |
| Section 5 | \( u_{n_0}, u_{n_5}, u_{n_6}, u_{n_7} \) | \( u_{n_7}, u_{n_6}, u_{n_7}, u_{n_0} \) |
| Section 6 | \( u_{n_0}, u_{n_5}, u_{n_6}, u_{n_7} \) | \( u_{n_7}, u_{n_6}, u_{n_7}, u_{n_0} \) |
3.2. Vector action time computation
If reference voltage vector locates $i^{th}$ section at k moment, adjacent vectors $u_{0i}, u_{0i+1}$ and $u_{0i+2}$ are selected as effective vectors. From (14) and (15) the $k_P, k_Q, k_R$ can be obtained.

$$P_1 = P_k + k_P \cdot t_0, Q_1 = Q_k + k_Q \cdot t_0$$
$$P_2 = P_1 + k_P \cdot 2t_m, Q_2 = Q_1 + k_Q \cdot 2t_m$$
$$P_3 = P_2 + k_P \cdot 2t_n, Q_3 = Q_2 + k_Q \cdot 2t_n$$
$$P_{k+1} = P_3 + k_P \cdot t_1, Q_{k+1} = Q_3 + k_Q \cdot t_1$$
$$T_s = t_0 + 2t_m + 2t_n$$

Where $t_0, t_m, t_n, t_1$ respectively are the action time when $u_{0i}, u_{0i+1}, u_{0i+2}$, effect. $P_k, Q_k$ are the active power and reactive power at the k moment, and $P_{k+1}, Q_{k+1}$ are the k+1 moment. $T_s$ are the sampling time.

If the action time of $u_{0i}$ is equal to the action time of $u_{0i+2}$, (17) can be simplified as:

$$P_1 = P_k + k_P \cdot 2t_0, Q_1 = Q_k + k_Q \cdot 2t_0$$

(18)

The objective function is selected as (20) in order to compute vector action time.

$$g = (P_{k+1} - P_{k+1}')^2 + (Q_{k+1} - Q_{k+1}')^2$$

(19)

Where: $P_{k+1}', Q_{k+1}'$ are the reference power at ‘k’ moment.

To minimize the objective function, the effective vectors action time should satisfy:

$$\frac{\partial g}{\partial t_m} = 0, \quad \frac{\partial g}{\partial t_n} = 0$$

(20)

Thus, combined (20)-(18) the action time of each vector can be obtained, then the inverter can be controlled by these 4 vectors.

4. The structural design for P-DPC

The block diagram of P-DPC is shown in figure 3, which are constituted by sampling module, prediction module, PLL module, reference voltage prediction module, objective function module, the inductance current and capacitor voltage is sampled, and $i_{fa}(k), i_{fb}(k)$, $u_{ca}(k), u_{c\beta}(k)$ are obtained by $abc/\alpha\beta$ transformation, the phase angle $\theta$ is calculated by PLL module, adjacent two voltage vectors is selected as effective vectors by judging the section of reference voltage vector [15,16].

Figure 3 The block diagram of P-DPC
5. Simulation
According to the inverter topology shown in Figure 1, the strategy researched in this paper was proved by MATLAB/Smulink, main simulation parameters were provided as following. \( V_{dc} = 270V \), filter inductance \( L = 2mH \), filter capacitance \( C = 780mF \), rated frequency is 400Hz, sampling period \( T_s = 10\mu s \).

The results of P-DPC based on switching function model and 6 voltage sequence were shown in figure 4, THD=5.98\%, the output voltage had a dispersive harmonic seen from spectrogram, which was difficult for filter design. figure 5 were results which based on MLD model and 3+3 voltage vector sequence, THD=4.26\%, the THD was reduced compared to the results in figure 4, the reason was the setting up of MLD model for inverter, which was more accurate than switching function model, but the harmonic was still dispersive. figure 6 were results based on MLD model and 4+4 voltage vector sequence, THD=2.43\%, and the harmonic concentrated on low-frequency, which was easy to clear up.

The inverter transient properties from no-load to full-load was shown in figure 7, a resistance with 200\( \Omega \) was installed at 0.0015s, the output voltage can adjust to reference voltage quickly, after full-load, the output voltage THD=3.44\%.

6. Conclusions
MLD prediction model was built for a new inverter, then the paper researched P-DPC strategy based on 4+4 voltage vector sequence, the low THD and fixed harmonic of output voltage were obtained, which was easy for filter design, and the favorable steady state and transient properties of the strategy was proved by simulations.

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