Non linear transport theory for negative-differential resistance states of two dimensional electron systems in strong magnetic fields.

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We present a model to describe the nonlinear response to a direct dc current applied to a two-dimensional electron system in a strong magnetic field. The model is based on the solution of the von Neumann equation incorporating the exact dynamics of two-dimensional damped electrons in the presence of arbitrarily strong magnetic and dc electric fields, while the effects of randomly distributed impurities are perturbatively added. From the analysis of the differential resistivity and the longitudinal voltage we observe the formation of negative differential resistivity states (NDRS) that are the precursors of the zero differential resistivity states (ZDRS). The theoretical predictions correctly reproduce the main experimental features provided that the inelastic scattering rate obey a $T^2$ temperature dependence, consistent with electron-electron interaction effects.

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I. INTRODUCTION

In the past few years the study of non-equilibrium magneto-transport in high mobility two-dimensional electron systems (2DES) has received much attention due to the experimental finding of intense oscillations of the magneto-resistivity and zero resistance states (ZRS). Microwave-induced resistance oscillations (MIRO) were discovered in 2DES samples subjected to microwave irradiation and moderate magnetic fields. For the MIRO the photoresistance is a function of the ratio $\epsilon^{ac} = \omega/\omega_c$, where $\omega$ and $\omega_c$ are microwave and cyclotron frequencies. This outstanding discovery triggered a great amount of theoretical work.\textsuperscript{5,6,7,8,9,10,11,12,13,14,15,16,17} Our current understanding of this phenomenon rests upon models that predict the existence of negative-resistance states (NRS) yielding an instability that rapidly drive the system into a ZRS\textsuperscript{18}. Two distinct mechanisms for the generation of NRS are known, one is based in the microwave-induced impurity scattering\textsuperscript{5,6,7,8,9,10,11,12,13}, while the second is linked to inelastic processes leading to a non-trivial distribution function\textsuperscript{8,14,15,17}.

An analogous effect, Hall field-induced resistance oscillations (HIRO) have been observed in high mobility samples in response to a dc-current excitation\textsuperscript{19,20,21}. Although MIRO and HIRO are basically different phenomena both rely on the commensurability of the cyclotron frequency with a characteristic parameter; in both cases oscillations are periodic in $1/B$. In HIRO the oscillation peaks, observed in differential resistance, appear at integer values of the dimensionless parameter $\epsilon^{dc} = \omega_H/\omega$. Here, $\hbar \omega_H \approx eE_H(2R_C)$ is the energy associated with the Hall voltage drop across the cyclotron diameter; $E_H$ is the Hall field and $R_C$ the cyclotron radius of the electron at the Fermi level. It has been found that there are two main contributions to the HIRO: the inelastic one is related to the formation of a non-equilibrium distribution function component that oscillates as a function of the energy\textsuperscript{22} and the elastic contribution is related to electron transitions between different LLs due to impurity scattering\textsuperscript{23}. The first one was shown to be dominant at relatively weak electric fields, and the latter prevails in the strong-field regime.

More recently it has been demonstrated that the effects of a direct dc current on electron transport can be quite dramatic leading to zero differential resistance states (ZDRS)\textsuperscript{24,25}. As compared with the HIRO conditions, the ZDRS are observed under dc bias at higher magnetic fields (0.5 – 1.0 $T$) and lower mobilities (70 – 85 $m^2/Vs$). At low temperature and above a threshold bias current the differential resistivity vanishes and the longitudinal dc voltage becomes constant. Positive values for the differential resistance are recovered at higher bias as the longitudinal dc voltage slope becomes positive. Bykov et al. analyzed the results following an approach similar to that of Andreev et al.\textsuperscript{18}, the presence of the ZDRS is attributed to the formation of negative differential resistance states (NDRS) that yields an instability that drives the system into a ZDRS. Similar results where obtained by Chen et al.\textsuperscript{26}.

In this paper we present a model to explain the formation of NDRS. According to our formalism both the effects of elastic impurity scattering as well as those related to inelastic processes play an important role. The model is based on the solution of the von Neumann equation for 2D damped electrons, subjected to arbitrarily strong magnetic and dc electric fields, in addition to the weak effects of randomly distributed impurities. This procedures yields a Kubo formula that includes the nonlinear response with respect to the dc electric field. Considering a current controlled scheme, we obtain a set of nonlinear self-consistent relations that allow us to deter-
the inelastic scattering rate must obey a to correctly reproduce the main experimental results for the imposed external current. It is shown that in or-
mine the longitudinal and Hall electric fields in terms of 

\[ FIG. 2: \text{Electric field } E_x \text{ as a function of the dc bias } J_x \text{ for } B = 0.784T \text{ and for fixed temperatures ranging from } T = 1K \text{ to } T = 10K. \]

\[ FIG. 1: \text{Differential resistance } r_{xx} \text{ as a function of the dc bias } J_x \text{ for } B = 0.784T \text{ temperatures from } T = 1K \text{ to } T = 10K. \]

in-plane electric field \( E = (E_x, E_y, 0) \), and the impurity scattering potential \( V \). Hence the dynamics is governed by the total Hamiltonian \( H = H_e + V \), with

\[ H_e = H_0 + eE \cdot x, \]  

(1)

here \( H_0 = \Pi^2/2m \), \( m \) is the effective mass of the electron, \( e \) is the electron’s charge, \( \Pi = p + eA \) is the velocity operator and the vector potential in the symmetric gauge is given as \( A = (-By, Bx)/2 \). The impurity scattering potential is expressed in terms of its Fourier components

\[ V(r) = e^{-\eta \tau} \sum_{N_i} \int d^2q/(2\pi)^2 V(q) \exp [iq \cdot (r - r_i)], \]  

(2)

where \( r_i \) is the position of the \( i \)th impurity and \( N_i \) is the number of impurities. The explicit form of \( V(q) \) depends on the nature of the scattering.

The motion of a planar electron in magnetic and electric fields can be decomposed into the guiding center coordinates \( Q \) and the relative coordinates \( R = (-\Pi_y, \Pi_x)/eB \), such that the position of the electron is given by \( r = Q + R \). The guiding center coordinates is written as \( Q = (Q_x, Q_y)/eB \). The commutation relations for velocity and guiding center operators are \( [\Pi_x, \Pi_y] = [Q_x, Q_y] = -ieB \), with all the other commutators being zero.

Our aim now is to compute the electric current density. In order to calculate the expectation value of the current density we need the time-dependent matrix \( \rho(t) \) which obeys the von Neumann’s equation \( i\hbar \partial \rho/\partial t = [H, \rho] \). We assume that in the absence of the impurity potential the density matrix reduces to the equilibrium density matrix given by \( \rho_0 = f(H_0) \), with \( f(E) \) given by the Fermi distribution function. In order to solve the von Neumann’s equation we apply three unitary transformations: the first two transformations exactly take into account the effects of the electric and magnetic fields, whereas the third transformation incorporates the impurity scattering effects to second order in time dependent perturbation theory. First we consider the unitary transformation

\[ W(t) = e^{-i\int dt E(t) \cdot x} e^{-i\int dt \frac{v_x(t) v_x(t)}{m} \frac{d^2q}{(2\pi)^2} e^{-i\frac{q^2}{2\hbar} \int dt f(q) \cdot x} e^{-i\frac{q^2}{2\hbar} \int dt f(q) \cdot y} \]  

(3)

where \( v_x(t), v_y(t), X(t) \) and \( Y(t) \) are solutions of the dynamical equations

\[ \dot{v}_x + \frac{1}{\tau_i} v_x + \omega_e v_y + \frac{e}{m} E_x = 0, \quad \dot{X} - \frac{E_y}{B} = 0, \]  

(4)

\[ \dot{v}_y + \frac{1}{\tau_i} v_y - \omega_e v_x + \frac{e}{m} E_y = 0, \quad \dot{Y} + \frac{E_x}{B} = 0. \]  

(5)

Except for the damping terms, these equations follow from the variation of the classical Lagrangian \( \mathcal{L}_{\text{cl}} \). The
variables \( v_x \) and \( v_y \) correspond to the electron velocity components and \( X \) and \( Y \) are the coordinates that follow the drift of the electron’s orbit. In order to incorporate dissipative effects we added the damping term \( \nu/\tau_i \) to the dynamical equations. This procedure yields a simple scheme to incorporate dissipation to the quantum system. Recent magnetoresistance experiments and theory\(^{27,28}\) suggest, that in 2DES, electron-electron interaction provide an important contribution to the inelastic scattering rate, giving rise to \( 1/\tau_i \propto T^2 \) temperature dependence. Consequently, in what follows we shall assume that the inelastic scattering rate is given by \( 1/\tau_i \approx (k_B T)^2/\hbar E_F \), where \( E_F \) is the Fermi energy.

The transformation (3) renders von Neumann equation into the following form

\[
\frac{i\hbar}{\partial t} \left( W \partial W \right) = [H_0 + V(t), W \partial W].
\]

The electric field term is conveniently removed from the Hamiltonian to produce a time-dependent impurity potential

\[
V(t) = V \left( x + X(t), y + Y(t) - v_x(t)/\omega_c \right).
\]

We proceed to switch to the interaction picture through the unitary operator \( U_0 = \exp(iH_0 t/\hbar) \) and solve the remaining equation up to second order in time dependent perturbation theory obtaining yet another simplified version of von Neumann equation

\[
\frac{i\hbar}{\partial t} \left( U_0 \frac{\partial W}{\partial t} \right)^\dagger U_0^\dagger = 0,
\]

where the time evolution operator is given by

\[
U = \frac{1}{\hbar} \int_{t_0}^{t} V_t(s_1) ds_1 - \frac{1}{\hbar^2} \int_{t_0}^{t} \int_{t_0}^{s_1} V_t(s_1) V_t(s_2) ds_1 ds_2,
\]

here \( V_t(s) = U_0 V(t) U_0^\dagger \) is the impurity potential in the interaction picture. The formal solution to (7) is given by \( \rho(t) = WU_0^\dagger W^\dagger \rho(0) U_0 W \) where \( \rho(0) = \rho_0 = f(H_0) \) is the equilibrium density matrix at the initial time \( t_0 \rightarrow -\infty \).

The density current is proportional to the thermal and time average of the velocity operator

\[
J = \frac{e}{S} \int_{-\infty}^{\infty} dt \text{Tr} \left[ \rho(t) \Pi \right],
\]

where \( S \) is the surface of the sample, and the limit \( S \rightarrow \infty \) is understood. By performing a cyclic permutation in the trace we obtain

\[
J = \frac{e}{S} \text{Tr} \left[ \rho(t_0) U_0 U_0^\dagger W^\dagger W^\dagger U_0^\dagger \right].
\]

After lengthy calculations the components of the density current is worked out as

\[
J_i = \frac{ne^2 \tau_i E_i - \omega_c \tau_\epsilon ij E_j}{m} \left( 1 + \omega_c^2 \tau_i^2 \right) + \frac{e^2}{\hbar} \sum_{\mu\nu} \int d^2 q (f_{\mu} - f_{\mu'}) G_{\mu\mu'}^{i} \left( q \right)
\]

\[
\text{(11)}
\]

where \( i, j = x, y \) and \( \epsilon_{ij} \) is the antisymmetric tensor \( (\epsilon_{12} = -\epsilon_{21} = 1 \text{ and } \epsilon_{11} = \epsilon_{22} = 0) \),

\[
G_{\mu\mu'}^{i} = \frac{N_B}{S m \hbar} \left| D_{\mu\mu'}^{i} \left( q \right) \right| ^2 \left( \frac{q_i \Delta_{\mu\mu'}^{i} + 2 |\epsilon_{ij}| q_j \omega_c \eta}{\Delta_{\mu\mu'}^{i} + 4 \omega_c^2 \eta^2} \right)
\]

\[
\text{(12)}
\]

and \( \Delta_{\mu\mu'}^{i} = (\omega_q + \omega_c(\mu - \mu'))^2 - \omega^2 + \eta^2, \omega_q = \omega_x E_x + \omega_y E_y, \omega_c = -\tau_i \omega_c(q_x + q_y \tau_\epsilon \omega_c)/B(1 + \tau_i^2 \omega_c^2) \),

\[
\omega_q = \tau_i \omega_c(-q_y + q_x \tau_\epsilon \omega_c)/B(1 + \tau_i^2 \omega_c^2) \text{ and } f_{\mu} = \frac{1}{h \omega_c (\mu + 1/2)}.
\]

The matrix elements \( D_{\mu\nu} \) are given by

\[
D_{\mu\nu}^{i} \left( q \right) = \exp \left( \frac{|z_q|^2}{2} \right) \times \left\{ \begin{array}{ll}
    z_q^{\mu'-\mu} \sqrt{\frac{\mu'}{\mu}} L_{\mu'-\mu}^{\mu}(\frac{|z_q|^2}{2}), & \mu \geq \mu', \\
    (-z_q^{\mu' \mu})^{\mu'-\mu} \sqrt{\frac{\mu}{\mu'}} L_{\mu'-\mu}^{\mu}(\frac{|z_q|^2}{2}), & \mu \leq \mu',
\end{array} \right.
\]

\[
\text{(13)}
\]

where \( z_q = (q_x - i q_y)/\sqrt{2} \) and \( L_{\mu'-\mu}^{\mu} \) denotes the associated Laguerre polynomial.

Retaining a finite value of the switching parameter \( \eta \) yields a density of states for the Landau levels with the Lorentzian form given in Eq. (12), it is distorted by the electric field through the \( \omega_q \) term. Henceforth we will consider \( \eta = \Gamma \omega_c \). The differential conductivity tensor is calculated from Eq. (11) as \( \sigma_{ij} = \partial J_i / \partial E_j \). Finally the differential resistivity tensor is obtained from the inverse of the conductivity: that is \( \sigma_{ij} = \sigma_{ij}^{-1} \).

In the limit of small bias and small magnetic field the expression for the density current reduces to \( J_x = ne^2 \tau_i E_x (1 - \alpha) / m \) where

\[
\alpha = \frac{2 \pi}{k_B T} \frac{e^{-E_F / k_B T}}{e^{E_F / k_B T} + 1} \frac{|V|^2 N_i m}{2 Sh \Gamma^2}.
\]

\[
\text{(14)}
\]

Hence the quantum scattering time and the inelastic scattering time can be related by setting \( \tau = \tau_\epsilon (1 - \alpha) \) or similarly the elastic scattering time is given by \( \tau_\epsilon = \tau_0 (1 - \alpha) / \alpha \).

The factor \( N_i |V|^2 / S \Gamma^2 \) present in the expressions for the density current can be estimated from the sample’s mobility and the inelastic scattering time.

In a current controled scheme: the longitudinal density current is fixed to a constant value \( J_0 \) while \( J_y \) should vanish. This leads to a set of two implicit equations for the density current

\[
J_x(E_x, E_y) = J_0, \quad J_y(E_x, E_y) = 0,
\]

\[
\text{(15)}
\]
To obtain the components of the electric field\(^{11}\)), then the accuracy of the solution is improved
where the explicit form of the functions \(J_i\) is given in
Eq. (11). To obtain the components of the electric field \(E_x\) and \(E_y\), we start assigning initial values \(E_x = E_{x0}\) and \(E_y = E_{y0}\) that solve these relations in the absence of
impurities \(i.e.,\) using only the first term on the R.H.S. of
Eq. (11), then the accuracy of the solution is improved
by a recursive application of Newton’s method.

III. RESULTS

Fig. 1 shows the differential resistivity \(r_{xx} = \partial E_x/\partial J_x\)
as a function of the longitudinal dc density current \(J_x\) for
a magnetic field \(B = 0.784T\) and various values of the
temperature. We use a sample mobility \(\mu = 100m^2/Vs\),
electron density \(n = 8.2 \times 10^{15}m^{-2}\) and a broadening
parameter \(\Gamma = 0.04\). As the value of the temperature is
reduced the differential resistance decrease approaching
zero. We can observe that at low temperature \((T < 2K)\)
and above a threshold bias current \((J_x > 0.44/m)\) the
differential resistivity becomes negative. Positive values
for the differential resistance are recovered at higher bias
or higher temperatures. The strong temperature depen-
dence observed in this plots, consistent with the experi-
ments, is originated mainly on the \(T^2\) dependence of
the inelastic scattering rate.

The electric field \(E_x\) is plotted as a function of the lon-
gitudinal current \(J_x\) in Fig. 2. It is important to notice
that \(E_x\) differs from the longitudinal voltage by a geo-
metrical factor. DNRS are observed below \(T = 4K\) and
above the current threshold \(J_x > 0.44/m\) in the form of
negative slope curves \(see\ inset\ of\ Fig.\ 2\) in accordance
with the \(r_{xx}\) negative values observed in Fig. 1. Accord-
ing to Bykov et al.\(^{22}\), the stability condition is simply
expressed as \(r_{xx} \geq 0\). Thus the regions in Figs. 1 and 2
that display a negative differential resistivity are unsta-
bility, and they will rapidly evolve into ZDRS to insure
stability. Accordingly in Fig. 1, we should replace the
NDRS by \(r_{xx} = 0\) and maintain a constant slope in Fig.
2 instead of the negative slope. At higher values of \(J_x\) the
differential resistivity becomes positive (Fig. 1) as well as
the longitudinal voltage slope as a result of an increase in
the impurity scattering prevalent at high electric fields.
In this regime the large electric field components, neces-
sary to maintain the strong dc bias and \(J_y = 0\), cause
the impurity terms to strongly participate\(^{22}\).

Fig. 4 display a series of plots of \(E_x\) field as a function
of the longitudinal density current \(J_x\) at \(T = 2K\) for
various fixed values of the magnetic field that corre-
spond to Shubnikov-de Haas oscillations maxima. The
thin lines indicate negative values of \(r_{xx}\) that violate the
stability condition. As the magnetic field increases the
width of the electric field plateaus increase and the posi-
tive slope is recovered for higher onset density currents.
An isolated plot of the longitudinal electric field \(E_x\) as a
function of the dc current \(J_x\) is shown in Fig. 4. In the
inset of Fig. 4, we show a nonuniform distribution current
similar to the one proposed by Bykov et al.\(^{22}\). With this
configuration not only the stability condition \(r_{xx} > 0\) is
fulfilled but the electric field is uniform throughout the
sample given that \(E_x = E_{min}\) for \(J_{x1}\) and \(J_{x2}\). The aver-
gage current density \(J_x = (J_{x1}y_1 + J_{x2}y_2)/(y_1 + y_2)\) may
be modulated by varying the sizes \(y_1\) and \(y_2\) of the dif-
frent density current domains with the restriction that
\(y_1 + y_2 = w\). Notice that more complicated schemes with
more density current modulations also fulfill this condi-
tions.
IV. CONCLUSIONS

We have presented a model for the nonlinear transport of a 2DES placed in a strong perpendicular magnetic field. The model is based on the solution of the von Neumann equation for 2D damped electrons, subjected to arbitrarily strong magnetic and dc electric fields, in addition to the weak effects of randomly distributed impurities. This procedure yields a Kubo formula that includes the non-linear response with respect to the dc electric field. Considering a current controlled scheme, we obtain a set of nonlinear self-consistent relations that allow us to determine the longitudinal and Hall electric fields in terms of the imposed external current. NDRs are found in the low temperature ($T \leq 2$) and moderate bias regime $0.4A/m < J_x < 1.6A/m$. In low dc bias (low electric field regime) the dominant mechanism is the inelastic one. The longitudinal electric field (and voltage) recover their positive slope in the high bias (high electric field regime). It is shown that in order to correctly reproduce the main experimental results the inelastic scattering rate must obey a $T^2$ temperature dependence, consistent with electron-electron Coulomb interaction as the dominant inelastic process.

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