In this study, novel robust navigation system for aerial robot in GPS and GPS-denied environments is proposed. Generally, the aerial robot uses position and velocity information from Global Positioning System (GPS) for guidance and control. However, GPS could not be used in several environments, for example, GPS has huge error near buildings and trees, indoor, and so on. In such GPS-denied environment, Laser Detection and Ranging (LIDER) sensor based navigation system have generally been used. However, LIDER sensor also has an weakness, and it could not be used in the open outdoor environment where GPS could be used. Therefore, it is desired to develop the integrated navigation system which is seamlessly applied to GPS and GPS-denied environments. In this paper, the integrated navigation system for aerial robot using GPS and LIDER is developed. The navigation system is designed based on Extended Kalman Filter, and the effectiveness of the developed system is verified by numerical simulation and experiment.

1. Introduction

In recent years, great success of aerial robot, which is also called drone, attracts attention all over the world [1]. The aerial robot could be used for various tasks under various environments. For example, it is used for information gathering at the time of a disaster, observation service in a height, inspection of large infrastructure like bridges and dam, security, and many others. To apply the aerial robot to such tasks easily, it is desired that the robot has autonomous capacity, and it is necessary for autonomous flight to obtain the position and velocity information of the robot.

In general, the aerial robot uses Global Positioning System (GPS) to measure the position and velocity for autonomous flight. However, GPS could not be used in several environments, for example, GPS has huge error near buildings and trees, indoor, and so on. To realize the autonomous flight of the aerial robot in such GPS-denied environments, several sensors are used to measure the position and velocity of the robot instead of GPS. Optical motion capture system[2][3] is commonly used for the researches of autonomous flight of aerial robot in indoor environment. The motion capture system could measure precise position and attitude of the robot. Therefore, the system is used for the research about precise autonomous flight[4][5]. However, the system needs large-scale facilities and it is only applied to limited area. On the other hand, vision sensor is also used for the autonomous flight in several researches[6][7]. The monocular or stereo vision system is useful because they could be applied in various environment.
However, they are easily affected by change of light environment, and the robustness is not enough. The Laser Detection and Ranging (LIDER) sensor is mostly used for the research about the autonomous flight in GPS-denied environment\cite{9}\cite{10}. LIDER-based Simultaneous Localization and Mapping (SLAM) is matured technology in the field of robotics, and accurate position could be obtained from the system. However, LIDER sensor also could not be used in several environments, for example, an environment where less laser reflections are only obtained.

As mentioned above, every sensors have their own weakness. And there are boundaries among the environment where each sensor could be used. Therefore, it is desired to develop the integrated navigation system which is seamlessly applied to various environment. In this paper, the integrated navigation system based on GPS and LIDER sensor is proposed. The integrated navigation system could be applied to both GPS and GPS-denied environments.

The rest of the paper is organized as follows. Section 2 will introduce sensor system which is used for integrated navigation. Coordinate system and measurement equation of each sensor are defined. Especially, the equation for absolute measurement of GPS, and relative measurement of LIDER will be introduced. The integrated navigation system is designed in section 3. The process model for estimation is defined, and the navigation system is developed based on Extended Kalman Filter. A numerical simulation and experiment are carried out and the result is shown in section 4 and section 5 respectively. Finally, conclusion of this work will be presented in section 6.

2. Sensor system

In this section, the sensor system used for integrated navigation and its measurement equations are introduced. First, coordinate system used for navigation is defined. The overview of coordinate systems are shown in Fig. 1. The first coordinate system is called global frame and noted $g$-frame; its origin is fixed at an arbitrary point on the ground. The $X_g$ axis indicates true north; the $Y_g$ axis, east direction; and the $Z_g$ axis, the gravity direction. This frame is also known as world frame in the field of robotics. The absolute sensor measurement from GPS and compass are expressed on this frame. The second coordinate system is called local frame and noted $l$-frame; its origin is fixed at the origin of the local navigation system of LIDER sensor. The frame is used to express the relative measurement. The last coordinate system is called body frame and noted $b$-frame; its origin is at the center of the gravity of the robot. The $X_b$ axis is in the forward direction of the body; the $Y_b$ axis, in the rightward direction; and the $Z_b$ axis, in the downward direction. The output of a Inertial Measurement Unit (IMU) is expressed as the vector on this frame. The state of the aerial robot which is estimated by navigation...
system is defined as follows:
\[
x = \begin{bmatrix} p^g T & v^g T & \Phi^g T \end{bmatrix}^T 
\] (1)

Here, \( p^g \) and \( v^g \) is 3-axis position and velocity of the robot on \( g-frame \). \( \Phi^g \) is the attitude of the robot which is expressed by appropriate method such as euler angle or quaternion[10].

Next, the sensors used for navigation and its sensor equation which expresses the relation between sensor output and the state of the robot is introduced. IMU is used to measure the acceleration and angular velocity. Considering the measured 3-axis acceleration as \( \dot{a}^b \), 3-axis angular velocity as \( \dot{\omega}^b \), and the attitude \( \Phi^b \) as the quaternion \( q^b_g \), the following equation is obtained.

\[
\dot{q}^b_g = \frac{1}{2} q^b_g \otimes \begin{bmatrix} 0 \
\dot{\omega}^b \end{bmatrix} 
\] (2)

\[
\dot{v}^g = q^b_g \otimes \begin{bmatrix} 0 \
\dot{a}^b \end{bmatrix} \otimes q^b_g + \begin{bmatrix} 0 \
0 \
\dot{g} \end{bmatrix} 
\] (3)

Here, \( \otimes \) denotes product of quaternion, \( q^b_g \) denotes conjugate of the quaternion. Additionally, \( g \) is the gravitational acceleration. GPS is used to obtain the 3-axis position and velocity in outdoor environment. The output of GPS is considered as \( p_{gps} \) and \( v_{gps} \), then the relation between these outputs and state is expressed as follows:

\[
p_{gps} = p^g + \delta p_{gps} 
\] (4)

\[
v_{gps} = v^g + \delta v_{gps} 
\] (5)

Here, \( \delta p_{gps} \) and \( \delta v_{gps} \) are the measurement error of GPS. LIDER sensor is also used to obtain the position. However, the position obtained by LIDER is not absolute measurement, but also relative measurement. Now, considering relative attitude between \( g-frame \) and \( l-frame \) as \( q^l_g \), the position of the origin of \( l-frame \) as \( p_{lo}^g \), relation between the output of LIDER sensor \( p_{lider} \) and the state as follows:

\[
p_{lider} = q^l_g \otimes \begin{bmatrix} 0 \
p^g - p_{lo}^g \end{bmatrix} \otimes q^l_g 
\] (6)

Additionally, the magnetic compass measures the heading angle, which is the part of the attitude, on the \( g-frame \).

### 3. Integrated navigation system

In this section, novel integrated navigation system is designed based on Extended Kalman Filter.

#### 3.1. Process model

A process model of the navigation is derived. The estimate of the state of the robot is defined as follows:

\[
\begin{bmatrix} \dot{p}^g \ T & \dot{v}^g \ T & \dot{\Phi}^g \ T & \dot{\omega}^b_a & \dot{\omega}^b_\omega \end{bmatrix}^T 
\] (7)

Here, \( \dot{b}_a \) and \( \dot{b}_\omega \) are the bias error of the accelerometer and gyro sensor respectively. These bias errors is used to reduce the drift of estimated position, velocity, and attitude.

First, state space equation of the system is derived. Using the bias error, (2) and (3) are rewritten as follows:

\[
\dot{q}^b_g = \frac{1}{2} q^b_g \otimes \begin{bmatrix} 0 \
\omega^b - \dot{b}_\omega \end{bmatrix} 
\] (8)

\[
\dot{v}^g = q^b_g \otimes \begin{bmatrix} 0 \
a^b_a - \dot{b}_a \end{bmatrix} \otimes q^b_g + \begin{bmatrix} 0 \
0 \
g \end{bmatrix} 
\] (9)
Based on (8) and (9), discrete time state propagation model with system noise is derived as follows:

\[
\begin{align*}
\tilde{x}_{t+1} &= f_t(\tilde{x}_t, u_t, w_t) \\
u_t &= \begin{bmatrix} a_t \\ \omega_t \end{bmatrix}, \quad w_t = \begin{bmatrix} w_a \\ w_\omega \\ w_b \\ w_{b_a} \end{bmatrix}
\end{align*}
\] (10) (11)

Here, \(a_t\) and \(\omega_t\) are \(t\)th measure of accelerometer and gyro sensor. \(w\) is the process noise of sensors and sensor bias. Additionally, the model of bias error is defined as simple random walk model as follows:

\[
\begin{align*}
\tilde{b}_{a_{t+1}} &= \tilde{b}_{a_t} + w_{b_a} \\
\tilde{b}_{\omega_{t+1}} &= \tilde{b}_{\omega_t} + w_{b_\omega}
\end{align*}
\] (12) (13)

Next, measurement equation of each sensor is derived. The output of the GPS is absolute position and velocity on \(g\)-frame, and the sensor output is only related to current state. Then, the output of GPS is considered as \(z_t = [p_{gps} \ v_{gps}]^T\), the measurement equation of GPS is derived by using (5) as

\[
z_t = h_t(\tilde{x}_t, v_{gps})
\] (14)

Here, \(v_{gps}\) is measurement noise of GPS. Additionally, the output of the compass is also absolute measurement. Therefore, the measurement equation of compass is same style.

On the other hand, the output of LIDER system is related to both current and past state, because the origin of \(l\)-frame in (6) is depend on the past absolute position of the robot. Therefore, the measurement equation of LIDER is obtained as follows:

\[
z_t = h_t(\tilde{x}_t, \tilde{x}_{t-T}, v_{lider})
\] (15)

Here, \(\tilde{x}_{t-T}\) is \(T\) step past state from current state.

3.2. Extended Kalman Filter

Based on the derived process model, Extended Kalman Filter is designed. General Extended Kalman Filter algorithm is shown as follows:

\[
\begin{align*}
F_t &= \left( \frac{\partial f_t}{\partial x_t} \right)_{x_t = \tilde{x}_{t/1}} \quad G_t = \left( \frac{\partial f_t}{\partial u_t} \right)_{x_t = \tilde{x}_{t/1}} \\
H_t &= \left( \frac{\partial h_t}{\partial x_t} \right)_{x_t = \tilde{x}_{t/1}} \\
K_t &= P_{t/\cdot} H_t^T (H_t P_{t/\cdot} H_t^T + R_t)^{-1} \\
\tilde{x}_{t/\cdot} &= \tilde{x}_{t/\cdot-1} + K_t [z_t - h_t(\tilde{x}_{t/\cdot-1})] \\
P_{t/\cdot} &= P_{t/\cdot-1} - K_t H_t P_{t/\cdot-1} \\
\tilde{x}_{t+1/\cdot} &= f_t(\tilde{x}_{t/\cdot}) \\
P_{t+1/\cdot} &= F_t P_{t/\cdot} F_t^T + G_t Q_t G_t^T
\end{align*}
\]

Here, \(K_t\) is kalman gain, \(P_t\) is covariance matrix of estimation error, \(R_t\) is covariance matrix of measurement noise, and \(Q_t\) is covariance matrix of process noise. However, it is difficult to deal with the past state directly. Therefore, augmented state is introduced to handle both the current state and past state. In [11], the augmented state is defined as follows:

\[
\tilde{x}_t = \begin{bmatrix} \tilde{x}_t^T \\ \tilde{x}_1^T \\ \vdots \\ \tilde{x}_f^T \end{bmatrix}
\] (16)
Here, $\hat{x}_i \in \mathbb{R}^{n_i}$ is $i^{th}$ augmented state corresponding to the past measurement of each sensor. For example, in the case of LIDER sensor, $\hat{x}_i$ is related to past position which is used to express the origin of $l$-frame. Besides, covariance matrix $P$ is also augmented as

$$ P = \begin{bmatrix} p^{x,x}_1 & p^{x,x}_2 & \cdots & p^{x,x}_i \\ p^{x,x}_i & p^{x,x}_1 & \cdots & p^{x,x}_i \\ \vdots & \vdots & \ddots & \vdots \\ p^{x,x}_i & p^{x,x}_1 & \cdots & p^{x,x}_i \end{bmatrix} \tag{18} $$

When the origin of $l$-frame is reset, it is necessary to remove old augmented state, and add new augmented to consider the absolute measurement which involved new origin of $l$-frame. Such operation could be done by following matrix operation[11].

$$ \hat{x}_i^+ = M^+ \hat{x}_1, \quad M^+ = \begin{bmatrix} I \\ B_{i+1} \end{bmatrix} \tag{19} $$

$$ \hat{x}_i^- = M^- \hat{x}_1, \quad M^- = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \tag{20} $$

$$ \dot{P}^\pm = M^\pm P M^{\pm T} \tag{21} $$

(19) represents addition of augmented state, (20) represents removal of the state, and (21) is augmentation of covariance matrix. In the equation, $B$ means a binary selection matrix for $i^{th}$ augmented state such as $x_i = B_i \hat{x}$.  

### 4. Numerical simulation

The numerical simulation was carried out to verify the proposed integrated navigation system. At first, the sensor data was collected in flight by using the aerial robot shown in Fig. 2. The main specification of the robot is shown in Table 1. The aerial robot has six rotors, and it is also known as multi-rotor helicopter or Drone. The robot has large payload capability compared with its dry weight. The sensor and computer system mounted on the robot is shown in Fig. 3. The system consists of GPS, LIDER, IMU, and embedded computer. The flight data was collected in several environment shown in Fig. 4.

Next, the numerical simulation of integrated navigation was carried out by using collected data. The simulation was implemented by using MATLAB. The results are shown in Fig. 5-Fig. 6.
Fig. 5 shows the horizontal position and Fig. 6 shows the horizontal velocity. In each figure, dashed line represents the data from GPS, solid line represents the estimates of integrated navigation. From the figures, it is shown, especially for velocity data, that the integrated navigation could suppress the large sensor noise of GPS. Additionally, around 370[sec] in Fig. 5 (a), GPS data has large drift because the robot enters the indoor environment at that time. Same error is also appeared in Fig. 6 (a). On the other hand, estimated position and velocity of integrated navigation are still continuous and stable. The reason is that the integrated navigation could compensate the position and velocity by using the LIDER system. The position data from LIDER system used in the simulation is shown in Fig. 7. In contrast to GPS, the position of LIDER system around 370[sec] is quite stable because large number of laser reflection could be obtained in the indoor environment. Thus, the LIDER system could output the accurate position. The integrated navigation could compensate estimated position and velocity by using such accurate position from LIDER system. From the results, the efficiency of the integrated navigation system was verified.

Table 1. Specifications of the aerial robot

| Source of power | Electric motor |
|----------------|----------------|
| Rotor diameter | 330 [mm]       |
| Body length    | 900 [mm]       |
| Dry weight     | about 3.0 [kg] |
| Payload        | about 6.0 [kg] |
| Endurance      | 20 [min]       |
5. Experiment

Integrated navigation system was implemented on embedded computer and navigation experiment in flight was carried out. The experimental result is shown in Fig. 8. The figure shows the flight trajectory obtained by GPS and integrated navigation system. In the figure, blue line represents the trajectory from GPS, and red line represents the trajectory from integrated navigation system. From the figure, it is shown that the trajectory of the integrated navigation is more smooth than the trajectory of GPS. Especially, when the robot enter narrow space between buildings, there are large error in the trajectory of GPS. On the other hand, the
integrated navigation system still could estimate precise and continuous trajectory. From the result, the efficiency of designed integrated navigation system was shown.

6. Conclusion
In this paper, integrated navigation system applied to GPS and GPS-denied environment. The navigation system consist of GPS and LIDER sensor, and designed based on Extended Kalman Filter. The results of numerical simulation and experiment shows the efficiency of proposed navigation system.

In future works, the navigation system will be extended to used vision sensor, barometer, and ultrasonic sensor to realize more robust navigation in various environment.

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