The large CP phase in $B_s - \bar{B}_s$ mixing and Unparticle Physics

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In this work we investigate the contribution to $B_d,s$ mixing from both scalar and vector unparticles in a number of scenarios. The emphasis of this work is to show the impact of the recently discovered $3\sigma$ evidence for new physics found in the CP phase of $B_s$ mixing. Here we show that the inclusion of the CP phase constraints for both $B_d$ and $B_s$ mixing improves the bounds set on the unparticle couplings by a factor of $2 \sim 4$, and one particular scenario of scalar unparticles is found to be excluded by the $3\sigma$ measurement of $\phi_s$.

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I. INTRODUCTION

It has recently been suggested [1, 2] that there may exist a nontrivial scale invariant sector at high energies, known as unparticle stuff. These new fields with an infrared fixed point are called Banks-Zaks fields [3], interacting with standard model fields via heavy particle exchange,

$$\frac{1}{M_{ Ut}^{ 2}} O_{ SM} O_{ BZ}.$$ (1)

Here $O_{ SM}$ is a standard model (SM) operator of mass dimension $d_{ SM}$, $O_{ BZ}$ is a Banks-Zaks($BZ$) operator of mass dimension $d_{ BZ}$, with $k = d_{ SM} + d_{ BZ} - 4$, and $M_{ Ut}$ is the mass of the heavy particles mediating the interaction. At a scale denoted by $\Lambda_{ U}$ the $BZ$ operators match onto unparticle operators with a new set of interactions,

$$C_{ U} \frac{\Lambda_{ d_{ BZ} - d_{ U}}}{M_{ Ut}^{ 2}} O_{ SM} O_{ U}$$ (2)

where $O_{ U}$ is an unparticle operator with scaling dimension $d_{ U}$ and $C_{ U}$ is the coefficient of the low-energy theory. Unparticle stuff of scaling dimension $d_{ U}$ looks like a nonintegral number $d_{ U}$ of invisible massless particles. In this work we shall study both scalar unparticles ($O_{ U}$) and vector unparticles ($O_{ U}^{ \mu}$ ) with couplings to the SM quarks as follows,

Scalar unparticles :

$$\frac{c_{ L}^{ S, q ' q} }{ \Lambda_{ Ut}^{ d_{ U} - 1} } \bar{q} \gamma_{ \mu} (1 - \gamma_5) q \partial^{ \mu} O_{ U} + \frac{c_{ R}^{ S, q ' q} }{ \Lambda_{ Ut}^{ d_{ U} - 1} } \bar{q} \gamma_{ \mu} (1 + \gamma_5) q \partial^{ \mu} O_{ U}$$ (3)

Vector unparticles :

$$\frac{c_{ L}^{ V, q ' q} }{ \Lambda_{ Ut}^{ d_{ U} - 1} } \bar{q} \gamma_{ \mu} (1 - \gamma_5) \gamma^{ \mu} \partial_{ U} + \frac{c_{ R}^{ V, q ' q} }{ \Lambda_{ Ut}^{ d_{ U} - 1} } \bar{q} \gamma_{ \mu} (1 + \gamma_5) \gamma^{ \mu} O_{ U}$$ (4)

Here we assume that the left-handed and right-handed flavour-dependent dimensionless couplings $c_{ L}^{ S, q ' q}$, $c_{ R}^{ S, q ' q}$, $c_{ L}^{ V, q ' q}$ and $c_{ R}^{ V, q ' q}$ are independent parameters. We analyze a number of scenarios, in each case determining the allowed parameter space and placing bounds on the unparticle couplings.

The propagators for scalar and vector unparticle fields are as follows [2, 4],

$$\int d^{ 4} x e^{ i P . x} \langle 0 | O_{ U} ( x) O_{ U} ( 0) | 0 \rangle = \frac{ i A_{ Ut}}{ 2 \sin d_{ Ut} \pi} \frac{1}{ ( P^{ 2} + i \epsilon)^{ 2 - d_{ U}}} e^{ - i \phi_{ U}}$$ (5)

$$\int d^{ 4} x e^{ i P . x} \langle 0 | O_{ U}^{ \mu} ( x) O_{ U}^{ \nu} ( 0) | 0 \rangle = \frac{ i A_{ Ut}}{ 2 \sin d_{ Ut} \pi} \frac{ ( - \partial^{ \mu} + P^{ \mu} P^{ \nu}/P^{ 2})}{ ( P^{ 2} + i \epsilon)^{ 2 - d_{ U}}} e^{ - i \phi_{ U}}$$ (6)
where

$$A_{dU} = \frac{16\pi^{3/2}}{(2\pi)^{2dU}} \frac{\Gamma(dU + 1/2)}{\Gamma(dU - 1)\Gamma(2dU)}, \quad \phi_{dU} = (dU - 2\pi)$$  \hspace{1cm} (7)

Since the publication of the first theoretical papers on this new subject [1, 2] there has been a huge interest in unparticle phenomenology, for example, Collider signatures [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34], CP violation [36, 37, 38, 39, 40, 41], meson mixing [5, 36, 42, 43, 44, 45, 46, 47], lepton flavour violation [48, 49, 50, 51, 52, 53, 54], consequences in astrophysics [55, 56, 57, 58, 59, 60, 61, 62], in neutrino physics [63, 64, 65, 66, 67] and in supersymmetry [68, 69, 70]. In this work we shall study the constraints coming from the measurements of $B_{s,d}$ meson mass differences $\Delta M_{s,d}$ and also their CP violating phases $\phi_{s,d}$. In the $B_d$ system these quantities have been well measured for some time and show only small deviations from the SM expectation. In the $B_s$ system recent measurements have also found small discrepancies between the SM expectation for $\Delta M_s$ [71], but now the CP violating phase $\phi_s$, measured by the DØ [72] and CDF [73] collaborations reveals a deviation of 3$\sigma$ [74, 75] [89]. This is the first evidence for new physics in $b \leftrightarrow s$ transitions.

Studying both vector and scalar unparticles, we study the constraints imposed by these latest measurements on the coupling between SM fields and unparticles, with particular interest on the impact of $\phi_s$.

In Sec. II we discuss unparticle contributions to general meson-antimeson mixing and in particular, to the case of $B$ mixing. Section III contains the results of our numerical analysis where we first consider scalar and then vector unparticle effects in $B_{d,s}$ mixing. In each case we set bounds on the allowed parameter space using constraints from measurements of $\Delta M_{d,s}$ and from $\phi_{d,s}$. Finally, Sec. IV concludes our results.

II. MESON-ANTIMESON MIXING FROM UNPARTICLES

With unparticle operators coupling to standard model operators as in Eq. (3,4) it is possible for unparticle physics to contribute to meson-antimeson mixing via the $s$- and $t$-channel processes shown in Fig. 1.

![Fig. 1: s- and t-channel unparticle contributions to meson mixing.](image)

Using the interactions listed in Eq. (3,4) to evaluate the $s$- and $t$-channel contributions to meson mixing as shown in Fig. 1 we obtain the effective Hamiltonian for scalar unparticles as,

$$H_{\text{eff}}^{S,q'q} = \frac{2\sin dU\pi}{\Lambda_{dU}^2} e^{-i\phi_{dU}} \left( \frac{1}{t^2 - dU} + \frac{1}{s^2 - dU} \right) \times \left[ Q_2 \left( m_q c_{L,q'} S_{q'q} - m_q c_{L,q} S_{q,q'} \right) \left( m_q c_{R,q'} S_{q'q} - m_q c_{R,q} S_{q,q'} \right) \\
+ \tilde{Q}_2 \left( m_q c_{R,q'} S_{q'q} - m_q c_{L,q} S_{q,q'} \right) \left( m_q c_{L,q'} S_{q'q} - m_q c_{R,q} S_{q,q'} \right) \\
+ 2 Q_4 \left( m_q c_{L,q'} S_{q'q} - m_q c_{R,q} S_{q,q'} \right) \left( m_q c_{L,q'} S_{q'q} - m_q c_{R,q} S_{q,q'} \right) \right]$$  \hspace{1cm} (8)
and for vector unparticles we obtain,
\[
\mathcal{H}_{\text{eff}}^{V,q'q} = \frac{A_{dL}}{2 \sin d_{\mu}\pi} \frac{e^{-i\phi_{dL}}}{\Lambda_{dL}^2} \times \left\{ \frac{1}{t^3-d_{\mu}} + \frac{1}{s^3-d_{\mu}} \right\} \left[ Q_2 \left( m_q c_L^{q'q} - m_q c_R^{q'q} \right) \left( m_q c_L^{q'q} - m_q c_R^{q'q} \right) 
+ \hat{Q}_2 \left( m_q c_R^{q'q} - m_q c_L^{q'q} \right) \left( m_q c_R^{q'q} - m_q c_L^{q'q} \right) 
+ 2 Q_4 \left( m_q c_L^{q'q} - m_q c_R^{q'q} \right) \left( m_q c_L^{q'q} - m_q c_R^{q'q} \right) \right] 
- \left( \frac{1}{t^2-d_{\mu}} + \frac{1}{s^2-d_{\mu}} \right) \left[ Q_1 \left( c_L^{q'q} \right)^2 + \hat{Q}_1 \left( c_R^{q'q} \right)^2 - 4 Q_5 \left( c_L^{q'q} c_R^{q'q} \right) \right] \right\} \tag{9}
\]

In the final line we have used a Fierz identity to rearrange the operator \((V - A) \odot (V + A)\) into the scalar operator \(Q_5\). Here \(\mathcal{H}_{\text{eff}}^S\) describes the effective Hamiltonian for the case of scalar unparticles and \(\mathcal{H}_{\text{eff}}^V\) for vector unparticles, hence the total effective Hamiltonian is simply,
\[
\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^S + \mathcal{H}_{\text{eff}}^V \tag{10}
\]

Above we have defined the quark operators \(Q_1 - Q_5\) as follows,
\[
\begin{align*}
Q_1 &= \bar{u}_L^{\alpha} \gamma_{\mu} q_{L}^{\mu} \bar{u}_R^{\beta} q_{L}^{\beta} \\
Q_2 &= \bar{u}_R^{\alpha} \gamma_{\mu} q_{L}^{\mu} \bar{u}_R^{\beta} q_{L}^{\beta} \\
Q_3 &= \bar{q}_{R}^{\alpha} q_{R}^{\beta} \bar{q}_{R}^{\beta} q_{R}^{\alpha} \\
Q_4 &= \bar{q}_{R}^{\alpha} q_{R}^{\beta} \bar{q}_{R}^{\beta} q_{R}^{\alpha} \\
Q_5 &= \bar{q}_{R}^{\alpha} q_{R}^{\beta} \bar{q}_{L}^{\beta} q_{L}^{\alpha} 
\end{align*} \tag{11-15}
\]

Writing the effective Hamiltonian in terms of these operators we have,
\[
\mathcal{H}_{\text{eff}}^q = \sum_{i=1}^{5} (C_i^S + C_i^V) Q_i + \sum_{i=1}^{3} (\tilde{C}_i^S + \tilde{C}_i^V) \tilde{Q}_i \tag{16}
\]

Here the operators \(\tilde{Q}_{1,2,3}\) are obtained from \(Q_{1,2,3}\) by the exchange \(L \leftrightarrow R\).

The hadronic matrix elements, taking into account for renormalization effects, are defined as follows,
\[
\begin{align*}
\langle \tilde{M}^0 \rangle (Q_1(\mu)) &\langle M^0 \rangle = \frac{1}{3} M_M f_M^2 B_1(\mu) \tag{17} \\
\langle \tilde{M}^0 \rangle (Q_2(\mu)) &\langle M^0 \rangle = -\frac{5}{24} M_M f_M^2 R(\mu) B_2(\mu) \tag{18} \\
\langle \tilde{M}^0 \rangle (Q_3(\mu)) &\langle M^0 \rangle = \frac{1}{24} M_M f_M^2 R(\mu) B_3(\mu) \tag{19} \\
\langle \tilde{M}^0 \rangle (Q_4(\mu)) &\langle M^0 \rangle = \frac{1}{4} M_M f_M^2 R(\mu) B_4(\mu) \tag{20} \\
\langle \tilde{M}^0 \rangle (Q_5(\mu)) &\langle M^0 \rangle = \frac{1}{12} M_M f_M^2 R(\mu) B_5(\mu) \tag{21} \\
\text{where, } R(\mu) &= \left( \frac{M_M}{m_q(\mu) + m_{q'}(\mu)} \right)^2 \tag{22}
\end{align*}
\]

From Eq. (8-9), it is straightforward to calculate the Wilson coefficients for scalar unparticles which are as follows,
\[
\begin{align*}
C_2^S &= \frac{A_{dL}}{\sin d_{\mu} \pi} \left( \frac{M_M^2}{\Lambda_{dL}^2} \right)^{\frac{d_{\mu}}{d_{\mu}}} \left( m_q c_L^{S,q'q} - m_q c_R^{S,q'q} \right) \left( m_q c_R^{S,q'q} - m_q c_L^{S,q'q} \right) \tag{23} \\
C_4^S &= 2 \frac{A_{dL}}{\sin d_{\mu} \pi} \left( \frac{M_M^2}{\Lambda_{dL}^2} \right)^{\frac{d_{\mu}}{d_{\mu}}} \left( m_q c_L^{S,q'q} - m_q c_R^{S,q'q} \right) \left( m_q c_L^{S,q'q} - m_q c_R^{S,q'q} \right) \tag{24} \\
\tilde{C}_2^S &= \frac{A_{dL}}{\sin d_{\mu} \pi} \left( \frac{M_M^2}{\Lambda_{dL}^2} \right)^{\frac{d_{\mu}}{d_{\mu}}} \left( m_q c_R^{S,q'q} - m_q c_L^{S,q'q} \right) \left( m_q c_L^{S,q'q} - m_q c_R^{S,q'q} \right) \tag{25} \\
C_1^S &= C_3^S = C_5^S = \tilde{C}_1^S = \tilde{C}_3^S = 0 \tag{26}
\end{align*}
\]
and for vector unparticles we find the following Wilson coefficients,

\[ C_1^V = - \frac{A_{d_L}}{\sin d_L \pi} \left( \frac{M_M^2}{\Lambda_{d_L}^2} \right)_{d_L}^{-1} \left( c_{L,q}^{V,q} \right)^2 \]  

\[ C_2^V = \frac{A_{d_L}}{\sin d_L \pi} \left( \frac{M_M^2}{\Lambda_{d_L}^2} \right)_{d_L}^{-1} \left( m_{q'} c_{R,q'}^{V,q'} - m_q c_{L,q'}^{V,q'} \right) \left( m_q c_{L,q'}^{V,q'} - m_q c_{R,q'}^{V,q'} \right) \]  

\[ C_3^V = 2 \frac{A_{d_L}}{\sin d_L \pi} \left( \frac{M_M^2}{\Lambda_{d_L}^2} \right)_{d_L}^{-1} \left( c_{L,q}^{V,q} c_{R,q}^{V,q'} \right) \]  

\[ C_3^V = \tilde{C}_3^V = 0 \]  

where we have approximated \( t = s \sim M_M^2 \). From these two sets of Wilson coefficients it is clear that the case of vector unparticles not only includes more contributions as \( C_1^V \neq 0, C_2^V \neq 0 \), but also that their Wilson coefficients are enhanced by a factor \( \Lambda_{d_L}/M_M^2 \) compared to the scalar unparticle case. As a result the vector unparticle parameter space shall be suppressed by the same factor.

These Wilson coefficients will mix with each other as a result of renormalization group (RG) running down to the scale of \( M_M \). For the \( B \) system, with a scale of new physics \( \Lambda_{d_L} = 1 \) TeV, these Wilson coefficients at the scale \( \mu_b = m_b \) may be approximated as \[ 77 \].

\[ C_1(\mu_b) \approx 0.805 C_1(\Lambda_{d_L}) \]  

\[ C_2(\mu_b) \approx 1.988 C_2(\Lambda_{d_L}) - 0.417 C_5(\Lambda_{d_L}) \]  

\[ C_3(\mu_b) \approx -0.024 C_2(\Lambda_{d_L}) + 0.496 C_5(\Lambda_{d_L}) \]  

\[ C_4(\mu_b) \approx 3.095 C_4(\Lambda_{d_L}) + 0.725 C_5(\Lambda_{d_L}) \]  

\[ C_5(\mu_b) \approx 0.086 C_4(\Lambda_{d_L}) + 0.884 C_5(\Lambda_{d_L}) \]  

The \( \Delta F = 2 \) transitions are defined as,

\[ \langle M^0 | H_{\text{eff}}^{\Delta F=2} | M^0 \rangle = M_{12} \]  

with the meson mass eigenstate difference defined as,

\[ \Delta M \equiv M_{HL} - M_L = 2|M_{12}| \]  

We can define in a model independent way the contribution to meson mixings in the presence of new physics (NP) as,

\[ M_{12} = M_{12}^{\text{SM}} (1 + R) \]  

where \( M_{12}^{\text{SM}} \) denotes the SM contribution and \( R \equiv r e^{i \phi} = M_{12}^{\text{NP}} / M_{12}^{\text{SM}} \) parameterizes the NP contribution.

The associated CP phase may then be defined as,

\[ \phi \equiv \text{arg}(M_{12}) = \phi^{\text{SM}} + \phi^{\text{NP}} \]  

where \( \phi^{\text{SM}} = \text{arg}(M_{12}^{\text{SM}}) \) and \( \phi^{\text{NP}} = \text{arg}(1 + r e^{i \phi}) \).

At this point it is important to make a further remark regarding the unparticle like model derived from a scale invariant theory of continuous mass fields as discussed in \[ 78 \]. Although this model contains no fixed point or dimensional transmutation and therefore does not correspond to unparticles as defined in \[ 2 \], it does contain unparticle-like local operators coupling to the SM. In this model it can be shown that the scale invariance will be broken by interactions with SM fields, resulting in a mass gap which could be rather large \[ 79 \]. In this setting we may then expect that there are unparticle states lying below the scale of 1 TeV. Therefore the above approach of integrating out
unparticle effects at the scale of 1 TeV would not be valid in this special case, rather the Wilson coefficients should be instead directly calculated at the low-energy scale. Such a calculation would offer its own challenges, in particular the $t$-channel exchange would involve a nonlocal hadronic matrix element. In this work we shall not consider such a case and instead assume that our unparticles do not derive from a scale invariant theory of continuous mass fields. Rather, we follow closely along the general framework set out by Georgi’s original work in \cite{2} where there is dimensional transmutation at the scale $\Lambda_U$.

A. $B_d,s$ mixing and unparticle physics

In this work we shall focus on the constraints imposed on unparticle physics couplings from $B_{d,s}$ mixing. Therefore we set $q' = b$, $q = s$, $d$ and $M^0 = B^0_d$, $B^0_s$.

In the $B$ system, the standard model contribution to $M_{12}$ is given by,

$$M_{12}^{q,\text{SM}} = \frac{G_F^2 M_W^2}{12\pi^2} M_{B_q}\hat{\eta}^R f_{B_q}^2 B_B (V_{tq} V_{tb})^2 S_0(x_t)$$  \hspace{1cm} (43)

where $G_F$ is Fermi’s constant, $M_W$ the mass of the $W$ boson, $\hat{\eta}^R = 0.551$ \cite{30} is a short-distance QCD correction identical for both the $B_s$ and $B_d$ systems. The bag parameter $\hat{B_B}$ and decay constant $f_{B_q}$ are nonperturbative quantities and contain the majority of the theoretical uncertainty. $V_{tq}$ and $V_{tb}$ are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix \cite{81,82}, and $S_0(x_t = \bar{m}_t^2/M_W^2) = 2.34 \pm 0.03$, with $\bar{m}_t(m_t) = 164.5 \pm 1.1$ GeV \cite{83}, is one of the Inami-Lim functions \cite{84}.

We can now constrain both the magnitude and phase of the NP contribution, $r_q$ and $\sigma_q$, through the comparison of the experimental measurements with SM expectations. From the definition of Eq. (41) we have the constraint,

$$\rho_q \equiv \frac{\Delta M_q}{\Delta M_q^{\text{SM}}} = \sqrt{1 + 2r_q \cos \sigma_q + r^2_q}$$  \hspace{1cm} (44)

The values for $\rho_q$ given by the UTfit analysis \cite{74,76} at the 95% C.L. are,

$$\rho_d = [0.53, 2.05]$$  \hspace{1cm} (45)

$$\rho_s = [0.62, 1.93]$$  \hspace{1cm} (46)

These constraints on $\rho_q$ encode the CP conserving measurements of $\Delta M_{d,s}$. The phase associated with NP can also be written in terms of $r_q$ and $\sigma_q$,

$$\sin \phi_q^{\text{NP}} = \frac{r_q \sin \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r^2_q}},$$

$$\cos \phi_q^{\text{NP}} = \frac{1 + r_q \cos \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r^2_q}}$$  \hspace{1cm} (47)

Here \cite{74,76} gives the 95% C.L. constraints,

$$\phi_d^{\text{NP}} = [-16.6, 3.2]^{\circ}$$  \hspace{1cm} (48)

$$\phi_s^{\text{NP}} = [-156.90, -106.40]^{\circ} \cup [-60.9, -18.58]^{\circ}$$  \hspace{1cm} (49)

theses constraint represent those of the CP phase measurements of $\phi_{d,s}$.

In order to consistently apply the above constraints all input parameters are chosen to match those used in the analysis of the UTfit group \cite{74,76} with the nonperturbative parameters,

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 262 \pm 35 \text{ MeV}$$  \hspace{1cm} (50)

$$\xi = 1.23 \pm 0.06 \text{ MeV}$$  \hspace{1cm} (51)

$$f_{B_d} = 230 \pm 30 \text{ MeV}$$  \hspace{1cm} (52)

$$f_{B_s} = 189 \pm 27 \text{ MeV}$$  \hspace{1cm} (53)
III. NUMERICAL ANALYSIS

In our analysis we shall first consider the case of scalar and vector unparticles separately and further subdivide each into four classes as follows,

- One real coupling; \( c_L \neq 0 \) and \( c_R = 0 \), with \( c_L \in \mathbb{R} \)
- Two real couplings; \( c_L \neq 0 \) and \( c_R \neq 0 \), with \( \{c_L, c_R\} \in \mathbb{R} \)
- One complex coupling; \( c_L \neq 0 \) and \( c_R = 0 \), with \( c_L \in \mathbb{C} \)
- Two complex couplings; \( c_L \neq 0 \) and \( c_R \neq 0 \), with \( \{c_L, c_R\} \in \mathbb{C} \)

Hence we have a total of eight scenarios to consider in the following analysis.

For our analysis we shall take the unparticle scale \( \Lambda_U = 1 \text{ TeV} \). In the literature [5, 36, 42, 43, 45, 46, 47] it has been common to perform an analysis with a fixed value of the scaling dimension, \( d_U = \frac{3}{2} \). It was recently shown [85, 86] that from unitarity considerations [87] there exists a bound on the scaling dimension, \( d_U \geq j_1 + j_2 + 2 - \delta_{j_1,j_2,0} \). Here \( (j_1, j_2) \) are the operators Lorentz spins. In particular this leads to the bounds \( d_U \geq 2(3) \) for our scalar(vector) unparticle operators, which conflicts with the choice \( \frac{3}{2} \). First it should be mentioned that these bounds are derived from a conformal field theory(CFT) point of view. Here we are considering a scale invariant sector at high energies, and in general scale invariant theories need not also be invariant under the conformal group. The specific example of Banks-Zaks fields though, is in fact invariant under the full conformal group [85, 86].

![FIG. 2: Constraints on the \( d_U \) versus \( c^S_L \) parameter space from \( B_d \) mixing(left) and \( B_s \) mixing(right) for the case of a single real coupling \( c^S_L \neq 0 \) and \( c^S_R = 0 \). Black points indicate the \( \Delta M_{d,s} \) allowed regions, while grey points indicate the regions are in agreement with both \( \Delta M_{d,s} \) and the CP phase \( \phi \).](image)

In order to make a connection with the literature, and to quantify the impact of the recent measurement of the CP phase \( \phi \), we choose to fix the scaling dimension at \( d_U = \frac{3}{2} \). Through simple scaling of the coefficient \( S_{d_U} \), it is also possible to make a connection to a value of the scaling dimension in the region suggested by the above mentioned unitarity bound. In general the bounds on unparticle couplings with the scaling dimension in the region \( d_U \geq 2(3) \) shall be much weaker than for \( d_U = \frac{3}{2} \), unless the value is close to an integer, where there is a pole in the unparticle propagator. For example, if we write the scaling dimension for scalar unparticles as \( d_U = 2 + \epsilon_S \), with \( \epsilon_S \) a small positive number, then we have \( |S_{d_U=3/2}| = |S_{d_U=2+\epsilon_S}| \) for a value of \( \epsilon_S \approx 0.00021 \). Hence the bounds derived here from the CP conserving quantities \( \Delta M_{d,s} \) shall also apply to the case \( d_U \approx 2.00021 \). The situation is not so straightforward for the bounds derived from CP violating constraints of \( \phi \). For vector unparticles we have a similar situation for \( d_U = 3 + \epsilon_V \), with \( \epsilon_V \approx 2 \times 10^{-11} \). The above unitarity bound shall be considered in more detail elsewhere [88].

A. Scalar unparticles

In this section we shall analyze the contribution to \( B_{d,s} \) mixing from scalar unparticles taking a number of example scenarios. In each case we shall use the measurements of \( \Delta M_{d,s} \) as well as the CP phases \( \phi_{d,s} \) to constrain the unparticle parameter space.
In this case the Wilson coefficients at the scale $\Lambda_U$ simplify to,

\[ C^S_2 = - \frac{A_{dU}}{\sin d_U \pi} \frac{e^{-i\phi_U}}{M^4_M} \left( \frac{M^2_M}{\Lambda^2_U} \right)^{d_U} m_b^2 \left( c^{S,bq}_L \right)^2 \]

(54)

\[ C^S_4 = 2 \frac{A_{dU}}{\sin d_U \pi} \frac{e^{-i\phi_U}}{M^4_M} \left( \frac{M^2_M}{\Lambda^2_U} \right)^{d_U} m_b m_q \left( c^{S,bq}_L \right)^2 \]

(55)

\[ \tilde{C}^S_2 = - \frac{A_{dU}}{\sin d_U \pi} \frac{e^{-i\phi_U}}{M^4_M} \left( \frac{M^2_M}{\Lambda^2_U} \right)^{d_U} m_b^2 \left( c^{S,bq}_L \right)^2 \]

(56)

\[ C^S_5 = C^S_5 = \tilde{C}^S_5 = \tilde{C}^S_5 = 0 \]  

(57)

where $q = d, s$ correspond to $B_d$ or $B_s$ mixing.

In this first case, in order to have two free parameters, we shall allow the scaling dimension $d_U$ to vary. In general the unparticle effects shall be smaller for larger $d_U$. In this case the scaling dimension $d_U$ also uniquely determines the NP CP phase and as such the small CP phase allowed by $\phi_d$ and the large CP phase required by the recent measurement of $\phi_s$ will act as a stringent constraint on the $d_U - c^S_L$ parameter space.

Fig. 2 displays the constraint on the $d_U - c^S_L$ parameter space from the $B_d$ mixing(left panel) and $B_s$ mixing(right panel), including CP phase constraints. As expected the general feature of these plots is that for small $d_U$ the mixing contribution from unparticles is large and so the couplings $c^{S,bq}_L$ are strongly constrained. For larger values of $d_U$ the opposite is true with $d_U > 2$ resulting in no bound on $c^{S,bq}_L$ from either $\Delta M_d$ or $\Delta M_s$.

\[ \text{FIG. 3: Plot of the variation of the phase and magnitude of the quantity } S_{dU} \text{ with the scaling dimension } d_U. \]

In the case of scalar unparticles the Wilson coefficients have the common quantity,

\[ S_{dU} = \frac{A_{dU}}{\sin d_U \pi} \frac{e^{-i\phi_U}}{M^4_M} \left( \frac{M^2_M}{\Lambda^2_U} \right)^{d_U} \]

(58)

Here the NP CP phase is determined solely by $\arg(S_{dU})$ such that $\sigma_q = \arg(S_{dU}/M^{SM}_{12})$. The magnitude of $S_{dU}$ also determines the size of the unparticle contribution to $B$ mixing and hence determines the level of constraint on the coupling $c^S_L$. Fig. 3 shows the variation with $d_U$ of the phase and magnitude of $S_{dU}$, from which we can see that the phase of $S_{dU}$ is constrained to be in the range $(-\pi, 0)$. The magnitude of $S_{dU}$ is generally decreasing for increasing $d_U$ except for when $d_U$ approaches 2 where there is a singularity due to the factor $1/\sin d_U \pi$. The general features of Fig. 2 are now easy to understand. The large value of $S_{dU}$ for small $d_U$ restricts the allowed values of $c^S_L$ to a very narrow region. For example, for $d_U = \frac{3}{2}$ we have the bounds,

\[ |c^{S,bd}_L| < 0.067 \]

(59)

\[ |c^{S,bs}_L| < 0.23 \]

(60)
The difference in the level of the above constraints on the couplings $c_{S,bd}^{S,L}$ and $c_{S,bs}^{S,L}$ are due to the ratio of CKM matrix elements $V_{td}/V_{ts} \approx \lambda$. Increasing $d_U$ causes $S_{d_U}$ to decrease sharply, resulting in an increase in the allowed size of the couplings $c_{S}^{S,L}$ as seen in Fig. 2. Approaching $d_U = 2$ causes a rapid increase in $S_{d_U}$ and thus the allowed values of $c_{S}^{S,L}$ are again heavily constrained. For $d_U > 2$ the quantity $S_{d_U}$ becomes increasingly small, resulting in no bound on the couplings $c_{S}^{S,L}$.

FIG. 4: Plot of the allowed $c_{S,bd}^{S,L}$ parameter space for $B_d$ mixing (left) and $B_s$ mixing (right) in the case of two real couplings. Black points show regions which agree with the measurement of $\Delta M_{d,s}$ while grey points show additional agreement with the measurement of the CP phases $\phi_{d,s}$.

We can see from Fig. 2 that there are windows in the allowed parameter space of $d_U - c_{S}^{S,L}$. From Eq. (44) we can determine $r_q$ in terms of $\sigma_q$ as,

$$r_q = -\cos \sigma_q \pm \sqrt{\rho_q^2 - \sin^2 \sigma_q}$$

(61)

and for $\rho < 1$ the phase $\sigma_q$ is constrained to be in the region,

$$-\pi - \arcsin \rho_q \leq \sigma_q \leq -\pi + \arcsin \rho_q$$

(62)

Taking the minimum allowed value of $\rho_q$ we can determine these windows as corresponding to,

$$-\pi - \arcsin \rho_q^{min} + \phi_{SM} \leq \arg(S_{d_U}) \leq -\pi + \arcsin \rho_q^{min} + \phi_{SM}$$

(63)

with $\phi_{SM} = -2\beta_s = -0.0409$ and $\phi_{SM}^{SM} = 2\beta = 0.781$, determines these windows to be,

$$B_d : \quad 1.56 < d_U < 1.92$$

(64)

$$B_s : \quad 1.80 < d_U < 2.00$$

(65)

To the inside of these windows, with smaller values of the coupling $c_{S}^{S,L}$, the unparticle contribution is small enough to satisfy the mixing constraint. To the outside of these windows, with larger values of $c_{S}^{S,L}$, the unparticle contribution is larger than the SM contribution. When the phase $\sigma_q$ is in the range $(-\pi, -\pi/2)$ the quantity $\cos \sigma_q$ in Eq. (44) is negative, allowing larger values of $r_q$. This solution corresponds to the case when the unparticle contribution carries the opposite sign to the SM and turns over the sign of $M_{12}$.

The black points of Fig. 2 correspond to the region of parameter space allowed by the measurement of the $B$ mixing parameters $\Delta M_{d,s}$, while grey points also satisfy the constraint from the measurement of the CP phases $\phi_{d,s}$ respectively. The left panel of Fig. 2 shows that the allowed region is strongly constrained by the measurement of $\phi_d$ in addition to the constraint of $\Delta M_d$. This additional constraint restricts the parameter space of $c_{S,bd}^{S,L}$ considerably. For example, for $d_U = \frac{3}{2}$ and including the CP constraint we have the improved bound,

$$|c_{S,bd}^{S,L}| < 0.024$$

(66)
FIG. 5: Variation of allowed parameter space of the real coupling \( c^S_{\mathrm{L}} = c^S_{\mathrm{R}} \) with scaling dimension \( d_{dU} \) for \( B_d \) mixing (left) and \( B_s \) mixing (right). Black plotted points agree with the CP conserving mixing quantities \( \Delta M_{d,s} \), while grey points also agree with the CP phases \( \phi_{d,s} \).

which is almost three times smaller than the bound from \( \Delta M_d \) alone. As shown in Eq. (49), in the \( B_s \) system there is still a two fold ambiguity in the measurement of the CP phase \( \phi_s \). This two fold ambiguity is then seen in the right panel of Fig. 2 as two distinct pairs of grey regions. The pair of grey regions with larger \( c_{\mathrm{L},bs} \), and consequently larger \( r_s \), corresponds to \( \phi_{s,\mathrm{NP}} = [-156.90, -106.40]^\circ \). This region is clearly disfavoured as it corresponds to the coupling \( c_{\mathrm{L},bs} \) being outside of the perturbative region. On the other hand the pair of grey regions with smaller \( c_{\mathrm{L},bs} \) and \( r_s \), corresponds to \( \phi_{s,\mathrm{NP}} = [-60.9, -18.58]^\circ \). In this case the coupling \( c^S_{\mathrm{L}} \) is constrained to be in a narrow band away from zero. For example with \( d_{dU} = \frac{3}{2} \) we have a two-sided bound,

\[
0.106 < |c^S_{\mathrm{L},bs}| < 0.23
\]

The small \( c^S_{\mathrm{L},bs} \) region also indicates that a value of the scaling dimension in the range \( 1.22 < d_{dU} < 1.87 \) is preferred.

Despite the two fold ambiguity in \( \phi_s \) it is clear that values of the scaling dimension in the range \( d_{dU} < 2 \) are preferred by the measurement of \( \phi_s \). Combining these \( \phi_s \) preferred values for \( d_{dU} \) with the constraints from \( \phi_d \) implies a general bound on \( c^S_{\mathrm{L},bd} \) as,

\[
|c^S_{\mathrm{L},bd}| < 0.13
\]

2. Two real coupling: \( c^S_{\mathrm{L}} \neq 0, c^S_{\mathrm{R}} \neq 0 \)

In this second case we shall allow the left and right unparticle couplings \( c^S_{\mathrm{L}} \) and \( c^S_{\mathrm{R}} \) to both be real and nonzero. For this analysis we shall fix the scaling dimension \( d_{dU} = \frac{3}{2} \). The allowed \( c^S_{\mathrm{L}} - c^S_{\mathrm{R}} \) parameter space is plotted in Fig. 4 with black points constrained by \( \Delta M_{d,s} \) and grey points further constrained by the CP phase \( \phi_{d,s} \). The allowed region extends out along two lines where there is a cancellation between \( c^S_{\mathrm{L}} \) and \( c^S_{\mathrm{R}} \). As a result no direct bound can be set on the unparticle couplings.

Starting from the definition of \( R_q = M^{\text{NP}}_{12}/M^{\text{SM}}_{12} \) and taking the limit \( m_q \to 0 \) we arrive at an approximate relation,

\[
6 \left[ (c^S_{\mathrm{L},bq})^2 + (c^S_{\mathrm{R},bq})^2 \right] - 32 c^S_{\mathrm{L},bq} c^S_{\mathrm{R},bq} \approx r_q \epsilon_q
\]

where the small quantity \( \epsilon_q \) is defined as,

\[
\epsilon_q = \frac{|M^{\text{SM}}_{12}|}{M_{B_q} f_{B_q} |S_{dU}|}
\]

Here it is clear that a large cancellation between the couplings \( c^S_{\mathrm{L}} \) and \( c^S_{\mathrm{R}} \) is possible. From Eq. (61) we find that \( (\rho_q - 1)^2 \leq r_q^2 \leq (\rho_q + 1)^2 \) which leads to,

\[
-0.1 x_q \leq r_q \epsilon_q \leq x_q
\]
with \( x_s = 1 \) and \( x_d = \lambda^2 \). The solution to these equations is then a parabola in the \( c_L^S - c_R^S \) parameter space described by,

\[
c_L^{S,bq} = \frac{16}{5} c_R^{S,bq} \pm \frac{1}{6} \sqrt{220 \left( \frac{c_R^{S,bq}}{c_L^{S,bq}} \right)^2 + 6 \epsilon_q}
\]

(72)

If we set \( \epsilon_q \approx 0 \), then these allowed regions follow along the lines,

\[
c_L^{S,bq} \approx 5 c_R^{S,bq}, \quad c_L^{S,bq} \approx \frac{1}{5} c_R^{S,bq}
\]

(73)

in good agreement with the results of the full calculation shown in Fig. 4.

FIG. 6: Plot of the allowed \(|c_L^S| - \phi_L^S\) parameter space for \( B_d \) mixing (left) and \( B_s \) mixing (right) for the case of a single complex coupling \( c_L^S \neq 0 \) and \( c_R^S = 0 \). Black plotted points agree with the CP conserving mixing quantities \( \Delta M_{d,s} \), while grey points also agree with \( \phi_{d,s} \).

In this case it is possible to have very large values of the couplings, as long as they lie approximately on the lines shown in Eq. (73). These solutions with large cancellations are clearly highly fine-tuned. If we consider the case of \( c_L^S = c_R^S \) then we have no such fine-tuning and we can extract the bounds,

\[
c_L^{S,bd} = c_R^{S,bd} < 2.5 \times 10^{-2}
\]

(74)

\[
c_L^{S,bs} = c_R^{S,bs} < 0.14
\]

(75)

from the constraints of \( \rho_{d,s} \). From the grey regions of Fig. 4 we again see that the measurement of the CP phases \( \phi_{d,s} \) further constrains the allowed parameter space. In the \( B_d \) system the allowed region is reduced by more than a half. For the case \( c_L^S = c_R^S \) the CP constraint provides a much improved bound,

\[
c_L^{S,bd} = c_R^{S,bd} < 6.7 \times 10^{-3}
\]

(76)

In the \( B_s \) system the new CP measurement of \( \phi_s \) prefers points away from the origin. It is also interesting to notice that the case, \( c_L^{S,bs} = c_R^{S,bs} \) is disfavoured by the latest measurement of \( \phi_s \) at the 3\( \sigma \) level.

The case of equal left and right couplings is shown in more detail in Fig. 5 where the allowed parameter space is plotted as a function of the scaling dimension \( d_{U} \). Fig. 5 shows the allowed parameter space for both \( B_d \) (left panel) and \( B_s \) (right panel). From the left panel we can see that the allowed parameter space is rather similar to that shown in Fig. 2. This time the windows in the plots have disappeared for the case of \( B_d \) and have moved in the case of \( B_s \). When we have \( c_L^S = c_R^S \) we get a new contribution from \( C_4^S \) not present in the case of Sec. IIIA. The introduction of \( C_4^S \) changes the overall sign of \( M_{12}^{NP} \) and so in this case these holes obey a modified relation similar to Eq. (63) except there is no factor of \( -\pi \). This means that for the case of \( B_d \) there is no such window and for \( B_s \) the window is now at \( 2 < d_{U} < 2.23 \), as shown in Fig. 5. In this case the impact of the CP phase measurement \( \phi_d \) is to again reduce the allowed parameter space. The impact of the recently measured \( \phi_s \) is much more profound, as can be seen from the right panel of Fig. 5 where there are no grey points and only black regions. The lack of any grey regions in this plot indicates that this case cannot satisfy the 3\( \sigma \) measurement of \( \phi_s \) for any choice of the scaling dimension \( d_{U} \).
FIG. 7: Plot of the allowed $|c^S_L|$, $|c^S_R|$ parameter space for $B_d$ mixing (left) and $B_s$ mixing (right) for the case of two complex couplings $c^S_L \neq 0$ and $c^S_R \neq 0$. Black plotted points agree with the CP conserving quantities $\Delta M_{d,s}$, while grey points also agree with $\phi_{d,s}$.

3. One complex coupling: $c^S_L \neq 0$, $c^S_R = 0$

In this case we have $c^S_R = 0$ and one complex coupling $c^S_L$, with the scaling dimension also fixed at $d_{dq} = \frac{3}{2}$. In this case the Wilson coefficients again take the simplified form of Eq. (56), this time with a complex $c^S_{L,bq}$. As a result the phase of $M_{12}^{NP}$ is not only determined by the scaling dimension, but also by the phase of the coupling $c^S_{L,bq}$.

Due to the hierarchy $m_b \gg m_q$ the dominant contribution here comes from the Wilson coefficient $C^S_{2,bq}$. For the purpose of simplicity we can make an approximation of the unparticle contribution as, 

$$ R_q = \frac{M_{12}^{NP}}{M_{12}^{SM}} = \delta \left( c^S_{L,bq} \right)^2 $$

where, $\delta \approx \frac{5}{24} m_b^2 M_{B_d} f_{B_d}^2 B_1 S_{d_{dq}} / M_{12}^{SM}$, with $|\delta| \approx 9.45$. This then leads to,

$$ r_q = |\delta| \left| c^S_{L,bq} \right|^2 $$

$$ \sigma_q = \text{arg} \left( \left( c^S_{L,bq} \right)^2 S_{d_{dq}} / M_{12}^{q,SM} \right), $$

For $d_{dq} = \frac{3}{2}$, we have $\sigma_q = 2 \phi^S_{L,bq} - \frac{\pi}{2} - \phi^{SM}_{q}$, where $\phi^S_{L,bq} = \text{arg} \left( c^S_{L,bq} \right)$. From Eq. (44) we then have,

$$ \rho_q = \sqrt{1 + 2|\delta| \left| c^S_{L,bq} \right|^2 \cos(2\phi^S_{L,bq} - \phi^{SM}_{q} - \frac{\pi}{2})} + |\delta|^2 \left| c^S_{L,bq} \right| \left| c^S_{L,bq} \right|^4 $$

The solutions to this equation are shown in Fig. 6 plotted in the $|c^S_L| - \phi^S_L$ plane. Again we can see from Eq. (62) that there are holes in the parameter space corresponding to, $-38.63^\circ < \phi^S_{L,bq} < -6.62^\circ$ and $-65.33^\circ < \phi^S_{L,bq} < -27.01^\circ$, as determined by Eq. (62). Using only the black points of Fig. 6 we can extract a bound on the magnitude of the couplings as follows,

$$ \left| c^S_{L,bd} \right| < 7.1 \times 10^{-2} $$

$$ \left| c^S_{L,bs} \right| < 0.31 $$

From the grey regions plotted in Fig. 6 we can again see that the inclusion of the CP phase constraints leads to a much reduced parameter space and improved bounds on the unparticle couplings,

$$ \left| c^S_{L,bd} \right| < 4.3 \times 10^{-2} $$

$$ 0.10 < \left| c^S_{L,bs} \right| < 0.31 $$

$$ |c^S_{L,bd}| < 7.1 \times 10^{-2} $$

$$ |c^S_{L,bs}| < 0.31 $$
FIG. 8: Plot of the allowed $|c_L^S| - \phi_L^S$ parameter space for $B_d$ mixing (left) and $B_s$ mixing (right) for the case of one complex coupling $c_L^S = c_R^S$. Black plotted points agree with the CP conserving mixing quantities $\Delta M_{d,s}$, while grey points also agree with $\phi_{d,s}$.

In addition we may also constrain the phase of the unparticle coupling $c_V^{bq}$ in the case of $B_s$ mixing as,

$$-41.77^\circ < \phi_L^{S,bq} < 25.55^\circ$$ (85)

4. Two complex couplings: $c_L^S \neq 0$, $c_R^S \neq 0$

In this final scalar unparticle case we take two complex couplings, $c_L^{S,bd}$ and $c_R^{S,bd}$, with the scale dimension fix at $d_U = \frac{3}{2}$. Fig. 7 displays the allowed parameter space for this scenario. In a similar way to the case with both couplings real, see Fig. 4, the allowed regions extend along the lines $|c_L^{S,bd}| = 5|c_R^{S,bd}|$ and $|c_L^{S,bd}| \approx \frac{1}{5}|c_R^{S,bd}|$. No general bound can be placed on the size of the unparticle couplings as large cancellations may occur even for very large values of the couplings. These solutions with large cancellations are however very highly tuned and unattractive. Taking the subset of this case with the left and right-handed couplings equal, we get the bound,

$$|c_L^{S,bd}| = |c_R^{S,bd}| < 0.039$$

$$|c_L^{S,bq}| = |c_R^{S,bq}| < 0.19$$ (87)

In this case the additional constraints from the CP phase measurements of $\phi_{d,s}$ have very little effect on the allowed parameter space. From the left panel of Fig. 7 it is clear from the clustering of grey points near the origin that the CP phase $\phi_d$ prefer smaller values of $|c_L^{S,bd}|$. On the other hand from the right panel we see that the opposite is true for $|c_L^{S,bq}|$ where there are only black points near the origin.

In Fig. 8 we study in more detail the case of equal and complex couplings $c_L^S = c_R^S$. Fig. 8 shows the experimentally allowed $|c_L^S| - \phi_L^S$ parameter space plotted for scaling dimension $d_U = \frac{3}{2}$.

The left and right panels of Fig. 8 display exactly the features just described. The grey allowed regions in the left panel corresponding to points satisfying both $\Delta M_d$ and $\phi_d$ constraints prefer lower values of the coupling in the range,

$$|c_L^{S,bd}| = |c_R^{S,bd}| < 0.024$$ (88)

From the right panel we can see that the phase $\phi_s$ indeed prefers regions with a larger coupling in the region,

$$0.06 < |c_L^{S,bq}| = |c_R^{S,bq}| < 0.18$$ (89)

In Fig. 8 we again see holes in the $|c_L^S| - \phi_L^S$ parameter space. In the case of $c_L^S = c_R^S$ we have $\sigma_d = 2\phi_L^{S,bq} + \pi + \arg(S_{d_L}) - \phi_{SM}^S$, where the extra $\pi$ is from a change in sign of $M_{12}^{NP}$ relative to the case in Sec. III A 3. Using Eq. (62) we find that these holes correspond to,

$$24.67^\circ < \phi_L^{S,bd} < 62.99^\circ$$ (90)

$$51.37^\circ < \phi_L^{S,bq} < 83.38^\circ$$ (91)
B. Vector unparticles

Now let us consider the case of vector unparticles. In general the contributions from vector unparticles to meson mixing is larger than from scalar unparticles due to the enhancement of the Wilson coefficients by the factor $\Lambda^2 U / M^2 B q$. As a result the experimentally allowed parameter space for vector unparticles will in general also be suppressed by this same factor.

1. One real coupling: $c^y_L \neq 0$, $c^y_R = 0$

First we shall consider the case of a single real vector unparticle coupling, $c^{V,bq}_L$. Assuming $c^{V,bq}_R = 0$ we then choose to allow the scaling dimension $d_U$ to vary and study the experimentally allowed parameter space. This case is rather similar to that of scalar unparticle case discussed in Sec. III A 1. The allowed regions of parameter space for this case are shown in Fig. 9 with black points indicating the regions of parameter space allowed by the measurement of $\Delta M_{d,s}$, while grey points indicate in addition agreement with measurements of the CP phase $\phi_{d,s}$. Fig. 9 again shows that the measurement of the CP phases $\phi_{d,s}$ improves greatly on the constraints set by $\Delta M_{d,s}$. For example with $d_U = \frac{3}{2}$ the constraint from $\Delta M_{d,s}$ alone gives,

$$|c^{V,bd}_L| < 6.8 \times 10^{-4}$$  \hspace{1cm} (92)

$$|c^{V,bs}_L| < 2.2 \times 10^{-3}$$  \hspace{1cm} (93)

Adding the restriction from the measurement of the CP phases $\phi_{d,s}$ we get the improved bounds,

$$|c^{V,bd}_L| < 2.4 \times 10^{-4}$$  \hspace{1cm} (94)

$$1 \times 10^{-3} < |c^{V,bs}_L| < 2.2 \times 10^{-3}$$  \hspace{1cm} (95)

Looking at the grey allowed regions of the right panel of Fig. 9 it is clear that the recently measured phase in $B_s$ mixing, $\phi_s$, prefers values of the scaling dimension in the range $1.22 < d_U < 1.96$. With the scaling dimension $d_U$ in the range $1 < d_U < 2$ we may extract the general bounds,

$$|c^{V,bd}_L| < 1.3 \times 10^{-3}$$  \hspace{1cm} (96)

$$2.4 \times 10^{-4} < |c^{V,bs}_L| < 3.1 \times 10^{-2}$$  \hspace{1cm} (97)

2. Two real couplings: $c^y_L \neq 0$, $c^y_R \neq 0$

In this case we fix the scaling dimension $d_U = \frac{3}{2}$ with two real nonzero couplings $c^{V,bq}_L$ and $c^{V,bq}_R$. Fig. 10 shows the allowed regions of the $c^{V,bq}_L$-$c^{V,bq}_R$ parameter space. Here the unparticle contribution to the $\Delta F = 2$ effective
FIG. 10: Plot of the allowed $c_V^L-c_V^R$ parameter space for $B_d$ mixing (left) and $B_s$ mixing (right) in the case of two real couplings $c_V^L \neq 0$ and $c_V^R \neq 0$. Black plotted points agree with the CP conserving mixing quantities $\Delta M_{d,s}$, while grey points also agree with $\phi_{d,s}$.

Hamiltonian satisfies the approximate relation,

$$2 \left[ (c_{V,bq}^L)^2 + (c_{V,bq}^R)^2 \right] + 12 c_{V,bq}^S c_{V,bq}^S \approx r_q e_q \frac{M_{Rq}^2}{\Lambda_U^2}$$

(98)

where the small quantity $e_q$ is as defined in section III A 2. Again solutions extend along two lines of the approximate form,

$$c_{V,bq}^L \approx -0.3 c_{V,bq}^R$$

(99)

$$c_{V,bq}^L \approx -3.3 c_{V,bq}^R$$

(100)

as can be seen in Fig. [10]. In general no bound can be placed on the size of the unparticle coupling in this case due to the possibility of large cancellations. Let us then again consider the simplified case with left and right unparticle couplings set equal to each other, $c_{V,bq}^L = c_{V,bq}^R$. First using the constraints from $\Delta M_{d,s}$ we may set the bounds,

$$|c_{V,bd}^L| = |c_{V,bd}^R| < 2.7 \times 10^{-4}$$

(101)

$$|c_{V,bs}^L| = |c_{V,bs}^R| < 9.2 \times 10^{-4}$$

(102)

The grey plotted points of Fig. [10] show the effect of the additional constraint from the CP phase measurements of $\phi_{d,s}$. The allowed parameter space is once again much reduced by the inclusion of these CP constraints. The CP phase $\phi_d$ improves the above bound on $c_{V,bd}^L$ to,

$$|c_{V,bd}^L| = |c_{V,bd}^R| < 9.6 \times 10^{-5}.$$  

(103)

while the CP phase constraint of $\phi_s$ improves the above bound on $c_{V,bs}^L$ to,

$$4.2 \times 10^{-4} < |c_{V,bs}^L| = |c_{V,bs}^R| < 9.2 \times 10^{-4}.$$  

(104)

when $d_U = \frac{3}{2}$.

The case of $c_V^L = c_V^R$ is displayed in Fig. [11] which very much resembles the case of section III B 1 with just one real coupling $c_V^L \neq 0$ and $c_V^R = 0$. Here with two identical real couplings $c_V^L = c_V^R$ the allowed parameter space is less than half that found in section III B 1. As such all bounds on the couplings found in section III A 1 may be restated for this case by simply multiplying by 0.42 for $B_s$ and 0.4 for $B_d$. 


FIG. 11: Plot of the variation of the allowed parameter space with the scaling dimension \(d_U\) for \(B_d\) mixing (left) and \(B_s\) mixing (right) in the case of one real coupling \(c_L^d = c_R^d\). Black plotted points agree with the CP conserving mixing quantities \(\Delta M_{d,s}\), while grey points also agree with \(\phi_{d,s}\).

FIG. 12: Plot of the allowed \(|c_L^d| - \phi_V\) parameter space for \(B_d\) mixing (left) and \(B_s\) mixing (right) in the case of one complex coupling \(c_L^d \neq 0, c_R^d = 0\). Black plotted points agree with the CP conserving mixing quantities \(\Delta M_{d,s}\), while grey points also agree with \(\phi_{d,s}\).

3. One complex coupling: \(c_L^d \neq 0, c_R^d = 0\)

Next we consider the case of a single complex coupling and fix the scaling dimension to be \(d_U = \frac{3}{2}\). This case is similar to that analyzed in section [II A 3]. The allowed parameter space for this case is shown in Fig. 12. The black points in Fig. 12 show the regions of parameter space which agree with experimental measurements of \(\Delta M_{d,s}\) while grey points indicate the additional agreement with the CP phases \(\phi_{d,s}\). From the black points we can extract the bound,

\[
|c_L^{S, bd}| < 7.2 \times 10^{-4} \tag{105}
\]
\[
|c_L^{S, bs}| < 2.9 \times 10^{-3} \tag{106}
\]

It is clear from the grey plotted points of Fig. 12 that the CP phase constraints have an important role to play in this scenario also. From these grey regions we may again extract improved bounds in the magnitude of the couplings,

\[
9.7 \times 10^{-4} < |c_L^{S, bs}| < 2.9 \times 10^{-3} \tag{108}
\]
In addition we may also constrain the phase of the unparticle coupling $c_{V,bs}^L$ as,

$$-41.80^\circ < \phi_{L}^{V,bs} < 25.52^\circ$$  \hspace{1cm} (109)
For our numerical analysis we have chosen to study the cases of scalar unparticles and vector unparticles separately. Each case has further been subdivided into a number of scenarios depending on the type unparticle couplings. In each scenario we have been careful to understand and discuss the specific characteristics of the allowed parameter space and, in particular, to understand the impact of the CP phase constraints on this parameter space and, where possible, to set bounds on the unparticle couplings. Our main goal here has been to identify the impact of the phase constraints on the allowed parameter space in each scenario and, in particular, that of the recently measured phase of $B_s$ mixing, $\phi_s$. A summary of the bounds placed on the unparticle couplings from the mixing parameters $\Delta M_{d,s}$ and for $\Delta M_{d,s}$, together with $\phi_{d,s}$, are shown in Table I.

![FIG. 14: Plot of the allowed $|c_L^d| - \phi_L^d$ parameter space for $B_d$ mixing(left) and $B_s$ mixing(right) for the case of one complex coupling $c_L^d = c_R^d$. Black plotted points agree with the CP conserving mixing quantities $\Delta M_{d,s}$, while grey points also agree with $\phi_{d,s}$.

TABLE I: Summary of bounds on unparticle couplings with scaling dimension $d_\ell = \frac{3}{2}$ from 95\% C.L. measurement of $\Delta M_{d,s}$ alone and also from a combination of the 95\% C.L. measurements of both $\Delta M_{d,s}$ and $\phi_{d,s}$.

| Scenario | Scalar unparticles | Vector unparticles |
|----------|-------------------|--------------------|
| $c_L^d \in \mathbb{R}$, $c_R^d = 0$ | $|c_{L^d}^{bd}| < 0.067$ | $|c_{L}^{bd}| < 0.067$ |
| $c_L^d$, $c_R^d \in \mathbb{R}$ | no constraint | no constraint |
| $c_L^d = c_R^d \in \mathbb{R}$ | $|c_{L}^{bd}| < 0.25$ | $|c_{L}^{bd}| < 0.25$ |
| $c_L^d, c_R^d \in \mathbb{C}$, $c_R^d = 0$ | $|c_{L}^{bd}| < 0.31$ | $|c_{L}^{bd}| < 0.31$ |
| $c_L^d$, $c_R^d \in \mathbb{C}$ | no constraint | no constraint |
| $c_L^d = c_R^d \in \mathbb{C}$ | $|c_{L}^{bd}| < 0.039$ | $|c_{L}^{bd}| < 0.039$ |

The main conclusion of this work is that the CP phase constraints do indeed improve markedly on the CP conserving bounds set on unparticle couplings in every scenario studied by a factor of 2 $\sim 4$, see table [1]. Of particular interest is the case of scalar unparticles with real and equal couplings $c_{L}^{bd} = c_{R}^{bd}$ for the $B_s$ system. Using only the measurement of $\Delta M_s$ we have a bound in the region of 0.14, but by adding the constraint from the 95\% C.L. measurement of $\phi_s$ this case is excluded.

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It should be noted that the combination of DΦ and CDF results for φs were made without full knowledge of the likelihoods, which could have an effect on the significance of this SM deviation. These are now being made available and a new experimental combination is due to be released soon. Until that time this present hint of new physics is the best available and remains a very exciting prospect.