Behaviour prediction of closed-loop HTS coils in non-uniform AC fields

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Abstract
Field decay rate is the key characteristic of superconducting magnets based on closed-loop coils. However, in Maglev trains or rotating machines, closed-loop magnets work in external AC fields and will exhibit an evidently accelerated field decay resulting from dynamic resistances, which are usually much larger than joint resistance. Nevertheless, there has not been a numerical model capable of systematically studying this behaviour, which is the main topic of this work. The field decay curves of a closed-loop high-temperature-superconducting (HTS) coil in various AC fields are simulated based on $H$-formulation. A non-uniform external field generated by armature coils is considered. Reasonable consistence is found between experimental and simulation results. In our numerical model, the impact of current relaxation, which is a historical challenge, is analysed and subsequently eliminated with acceptable precision. Our simulation results suggest that most proportion of the field decay rate is from the innermost and outermost turns. Based on this observation, a magnetic shielding pattern is designed to reduce the field decay rate efficiently. This work has provided magnet designers with an effective method to predict the field decay rate of closed-loop HTS coils in external AC fields, and explore various shielding designs.

Keywords: closed-loop HTS magnet, field decay, AC field, dynamic resistance, $H$-formulation

1. Introduction
Various high-temperature superconducting (HTS) magnet applications may require closed-loop coils to work in persistent current mode (PCM) to reduce heat losses and generate stable magnetic field [1–3]. Among these applications, field decay performance is a key factor for their operation. In a stationary operation, the field decay rate is determined by the joint resistance and index loss (if with a large working current). However, during the dynamic operation, e.g. rotating machines, Maglev trains and other potential applications [1, 2], the closed-loop coils are exposed to external AC magnetic field. Therefore, dynamic resistance, which arises from the interactions between the transport DC current and moving fluxons [4], is likely to be generated in the closed-loop. In these cases, the field decay of the closed-loop coil is accelerated and exhibits a significant deviation from its original design.

Researchers have studied the experimental field decay of closed-loop magnets under AC field [5–7], as well as the angular dependence of the external field [8]. Although the experimental results suggested that external AC field would accelerate the field decay much more significant than joint resistance [5], this behaviour is still unpredictable.

Dynamic resistance for single or stacked tapes, as a key feature of HTS flux pump devices, have been studied by both analytical expressions [4, 9–13] and numerical models [14, 15], which have been proven to be in good agreement with experimental results. Zhang et al has tried to calculate the AC loss of HTS coils including dynamic loss [16]. However, dynamic resistance for coils, as the acceleration mechanism of the field decay of closed-loop magnets, has...
not been systematically studied by numerical method and experiment.

This paper presents a modelling technique for simulating the field decay of a closed-loop GdBCO magnet under AC field, and our experimental verification. The $H$-formulation and $E$-$J$ power law assumption are utilised. The challenges are: (a) for coil cases, it is found that the historical challenge involving the eddy current model, current relaxation [17–20], is very influential to the results and must be properly considered; and (b) the impact of induced current circulating in a closed-loop coil, which is eliminated in this work.

2. Numerical simulation

2.1. Geometry and governing equations

A double-pancake (DP, series-connected by two single pancakes) HTS coil and two copper coils were established in a 2D axisymmetric model based on the $H$-formulation and $E$-$J$ power law in COMSOL$^\text{TM}$, which is originally developed by Hong et al [21] and Brambilla et al [22]. The copper coils served as armature coils to generate AC fields. The general model geometry is shown in figure 1(a). The coil parameters and positions are identical with the experimental configuration and introduced in more detail in section 4. The resistivity ($\rho$) was set to $2 \times 10^{-5}$ $\Omega$ m for copper coil regions and $100$ $\Omega$ m for air region.

The measured self-field critical current of the single tape at $77$ K, $I_{c0}$, is $213$ A with the criterion set as $1 \times 10^{-3}$ V m$^{-1}$. The $n$-value is set as $25$. The anisotropy of our experimental GdBCO tape at $77$ K was calculated by equation (1) introduced by [23]. The explanation of $P1(B)$, $P2(B)$, and $G(\theta)$ is illustrated in figure 2.

$$J_c(B, \theta) = J_{c0} \times \{P1(B) + [P2(B) - P1(B)] \times G(\theta)\}. \quad (1)$$

The homogenisation technique [24] is used to improve the computation speed. Any tape layer (including the metal, substrate, insulation and superconducting layers) is artificially homogenised and regarded as superconducting volume. The self-field critical current density in the homogenised tape, $J_{c0}$, of equation (1), is set accordingly to keep $I_{c0}$ unchanged.

2.2. Simulation by direct field cooling and decay in an AC field

The excitation and decaying procedures of our experimental $2 \times 28$ closed-loop coils are simulated. Pointwise constraints are applied only to keep the transport current, $I_{\text{DC}}$, in HTS coil identical among each turn. The copper coils are used for both field-cooling excitation [3] and demagnetisation for the HTS coil. As is shown in figure 3, the results of field-cooling excitation and current decay curves under AC fields are shown. The field decay curves can be obtained by simply multiplying the coil constant. It is clear that a greater decay occurs after a 50 Hz AC field (represented by the root mean square (RMS) current in each copper coil, $I_{\text{em}} = 69.23$ A) is applied at $6$ s. With this pure 2D finite element method (FEM) model, the impact of the induced AC current circulating the closed-loop coil on the field decay can also be investigated.

However, considering that the experimental field decay process usually continues for hundreds of seconds (≈10$^4$ cycles), this simulation is computational consuming. Thus, it is proposed that the field decay curves are simulated by the discrete calculation of dynamic resistance.

2.3. Simulation by the discrete calculation of dynamic resistance

In figure 1(c), the closed-loop coil is regarded as an $L$-$R$ circuit. The current decay curve can be calculated by modelling the instantaneous resistances in the HTS coil, which come from the dynamic resistance, $R_{\text{dyn}}$ and joint resistance, $R_j$. For each step in the circuit model, the interval of $I_0 \rightarrow I_1$ is artificially
selected by the required accuracy. Then, the decay time, $\Delta t$, is calculated by the $R_{\text{dyn}}$ resulting from the FEM model.

To calculate the $R_{\text{dyn}}$ value at a certain point of a current decay curve, the intended transport currents in the HTS coil and copper coils are applied using pointwise constraints [25]. In FEM model, $I_{\text{DC}}$ in the HTS coil is reached after undergoing a $1 \text{s}$ ramping function and kept constant for $2 \text{s}$. Then, the external AC field is generated by two copper coils.

This study calculates $R_{\text{dyn}}$ within the HTS coil using the average electric field. For 2D axisymmetric coils, the resistive voltage ($V$) can be calculated by only one-time integration of the entire HTS volume [26]:

$$U_{\text{coil}}(t) = \frac{\sum_{n=1}^{N} 2\pi r_n \int E_\phi(t) \, dv}{s} = \frac{\int \Omega_{\text{SC}} E_\phi(t) \, dV}{s}$$  \hspace{1cm} (2)

where $s$ and $r_n$ are the cross-sectional area and radius of each homogenised turn, $E_\phi$ is the azimuthal electric field and $\Omega_{\text{SC}}$ is the entire superconducting volume. Hence, the dynamic resistance ($\Omega$) can be calculated as:

$$R_{\text{dyn}} = f \cdot \int_0^T \frac{U_{\text{coil}}(t)}{I_{\text{DC}}(t)} \, dt$$  \hspace{1cm} (3)

where $T$ is the period and $f$ is the frequency of AC field. For each $R_{\text{dyn}}$ value, 100 cycles are carried for current relaxation, and the reason is illustrated in section 3.

2.4. Basic feasibility test

In section 2.3, the $I_{\text{DC}}$ for the $R_{\text{dyn}}$ calculation in FEM model, is applied after undergoing a ramping function. However, in real field decay experiment, $I_{\text{DC}}$ is supplied by the inductance of the HTS coil and is a decaying function. Therefore, the influence of the differing history of the $I_{\text{DC}}$ on the calculation of current decay curves must be investigated.

3. Current relaxation

3.1. Descending $R_{\text{dyn}}$ and current relaxation

The $R_{\text{dyn}}$ values of our experimental $2 \times 28$ HTS coil are calculated. $I_{\text{DC}}$ in HTS coil and $I_{\text{em}}$, which is the RMS current of each copper coil, are applied and kept their values of 55.38 A and 69.3 A, respectively. In figure 5, the coil voltage and $R_{\text{dyn}}$ values in the HTS coil are plotted as a function of the cycle number of $50 \text{ Hz}$ AC field. However, a problematic phenomenon occurs: the voltage peaks and $R_{\text{dyn}}$ values undergo an endless decay as the solution evolves over time. The decay of $R_{\text{dyn}}$ did not stop after 100 simulation cycles, and $R_{\text{dyn}}$ of the 100th cycle was only 43.8% of the 2nd cycle.

Considering that the experimental field decay process continues for hundreds of seconds ($\approx 10^4$ cycles), we assume that simulation results from larger cycles or steady state will be more relevant to the experimental conditions. Thus, two questions are posed: (a) is there a lower limit for the descending $R_{\text{dyn}}$? (b) is there an effective method to determine the stabilised value of $R_{\text{dyn}}$ with a reasonable error over limited cycles?

This phenomenon is concerned with a historical simulation challenge involving the $E$-$J$ power law model: current relaxation [17–20]. After applying a pure DC current, the

Figure 3. Simulated results of the field-cooling excitation and current decay of our experimental $2 \times 28$ closed-loop coil under a $50 \text{ Hz}$ field.

Figure 4. The current decay curves calculated by the methods in sections 2.2 and 2.3, respectively.
current distribution will always lead to minimal ohmic heat generation. If there is a non-zero \( J \), the power law resistivity will never be zero. Same with normal conductors, DC current will flow from high-resistivity branches to low-resistivity branches. Therefore, current density will penetrate from the edge to the centre of tape and tends to present a homogeneous distribution [17–20], lowering the DC resistance of the entire tape.

In this study, the DC current density under external AC field also tends to penetrate toward the centre of the coil cross-section, causing \( R_{\text{dyn}} \) values descending with cycles. However, in this case, the current relaxation can be somewhat different from the pure DC current cases. The DC current penetration under AC field is driven by not only the power law resistivity, but also dynamic resistivity.

### 3.2. How to deal with current relaxation

Although the decay of resistance is endless, the impact of current relaxation and descending \( R_{\text{dyn}} \) on the final results can be very inconsequential by carrying enough number of AC cycles.

The simulation presented in section 3.1 was extended to 100 cycles. In figure 6, the current distribution of the cross-section of HTS coil at the 3/2\( \pi \) phase of the last cycle is presented. The direction of the relaxation of DC current is illustrated by the black arrows. And the dashed boxes denote the selected innermost/outmost (figure 6(a)), and middle part turns (figure 6(b)), respectively. Their calculated \( R_{\text{dyn}} \) as functions of simulation cycles are presented in the right two figures, respectively. In figure 6(a), for the innermost/outmost turns, the decay of the \( R_{\text{dyn}} \) in the first few cycles originates from the superconductors’ transitional phase of initial-magnetisation. Particularly, in the subsequent cycles, \( R_{\text{dyn}} \) will stabilise after the current density saturates the cross-section of tape, because the current relaxation has stopped. However, in figure 6(b), for the middle part turns, the \( R_{\text{dyn}} \) curve still has a negative derivative at the 100th cycle, because the current virgin region (where \( J \rightarrow 0 \)) for further penetration is still large. At this time, the descending of \( R_{\text{dyn}} \) and variation of current profile is extremely slow, and achieving a completely stable state may require thousands of simulation cycles and unimaginable computational loads.

Thus, the lower limit of the \( R_{\text{dyn}} \) of the coil can be defined by the stabilised \( R_{\text{dyn}} \) of the turns shown in figure 6(a). Meanwhile, the descending portion of \( R_{\text{dyn}} \) of the coil is only contributed by the middle part turns shown in figure 6(b), which determines the error between the descending \( R_{\text{dyn}} \) and stabilised \( R_{\text{dyn}} \) of the coil. The magnitude of the error caused by these turns must be investigated.

In figure 6(b), for the turns that \( R_{\text{dyn}} \) still under descending, a current shielded region (\( J \rightarrow 0 \)) exists in the centre of the tape. Thus, the flux flow cannot cross the central region and exits the conductor from the same side as it enters. Then, the electrical potential caused by flux moving will cancel over a cycle, so the DC electric field would not occur. This indicates that these turns have not reached the threshold states for generating dynamic resistance [4, 15]. In contrast, in figure 6(a), for the turns that \( R_{\text{dyn}} \) has stabilised, the central region is no longer shielded, indicating that a net flux flow would move across the entire DC region, leading to the development of \( R_{\text{dyn}} \) [15]. The 100th cycle’s \( R_{\text{dyn}} \) value of the 48 turns in figure 6(b) is only 1.6% of the 8 turns in figure 6(a). So, although the resistance of the turns in figure 6(b) continues to descend, its influence on the resistance of the whole coil is insignificant. The \( R_{\text{dyn}} \) values of the entire coil calculated by the 100th cycle are very close to the completely stabilised values, thus proving its effectiveness.

### 4. Experimental verification

#### 4.1. Coil parameters

The experimental HTS coil was wound with GdBCO tape with a width of 6 mm and a thickness of 190 \( \mu \text{m} \), provided by Shanghai Superconductor Technology Co., Ltd (SSTC). The thickness of the tape with Kapton insulation was 440 \( \mu \text{m} \) and the total length was about 20 m. Thickness and width of the HTS layer were about 1 \( \mu \text{m} \), 4.75 mm, respectively. The DP coil, with inner/outer diameters of 10 cm/12.46 cm and a height of 2.1 cm, has 28 turns for each pancake. The coil constant at the centre is about 0.62 mT A\(^{-1}\), and the coil

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**Figure 5.** Voltage peaks and \( R_{\text{dyn}} \) values of the multi-turn coil obtained as the simulation cycles increase, when keeping \( I_{\text{DC}} = 55.38 \text{ A} \) and \( I_{\text{em}} = 69.3 \text{ A} \) at 50 Hz.

**Figure 6.** (a) Innermost and outermost turns (8 turns); (b) middle part turns (48 turns). Left figures: current density distribution of the cross-section of the HTS coil at the 100th cycle; Right figures: calculated \( R_{\text{dyn}} \) as a function of the cycle number, when \( I_{\text{DC}} = 55.38 \text{ A}, I_{\text{em}} = 69.3 \text{ A} \) at 50 Hz. The black arrows denote the direction of current relaxation.
\( I_c \) is 135 A. The two terminations of the coil are soldered together by a thin tin layer to form a joint with a resistance of \( R_j = 12.5 \, \text{m} \Omega \). The terminals are arranged close to the coil surface to avoid non-axisymmetric geometry that deviates from our model.

4.2. Field decay rate measurements

The geometry of the field decay system is shown in figure 7. First, a current of 90 A was excited in the closed-loop coil by a heat-triggered persistent current switch. Then the HTS coil was operated in the PCM. A Hall sensor was fixed at the centre of the HTS coil to monitor the field and current. Then, the external 50 Hz magnetic field was generated by two identical copper coils. Each copper coil had 317 turns and was powered by a single-phase transformer.

To study the acceleration mechanism of the field decay caused by pure AC field, the influence of the induced AC current circulating in the closed-loop coil is eliminated. Two copper coils were connected anti-parallel to the transformer and symmetrically placed on both sides of the HTS coil. In this way, the net axial magnetic fluxes variation in the HTS coil caused by copper coils, \( \Delta \Phi_z \), was zero. Besides, an epoxy resin sealed XPS box was used to fix the HTS coil and isolates it from the temperature and mechanical disturbances from copper coils.

Figure 8 shows the measured field decay results under ten different AC fields, which are represented by the RMS currents of each copper coils, \( I_{em} \). The calculated radial field \( B_z \) that is perpendicular to the tape surface generated by unit, \( I_{em} \), was 0.98 mT A\(^{-1}\) at the innermost turn and 1.04 mT A\(^{-1}\) at the outermost turn. It is clear that the field decay rates increase with the rise of the amplitude of AC fields.

With the calculated coil inductance \( L = 0.51 \, \text{mH} \), the instantaneous \( R_{dyn} \) during decays can be calculated by assuming the HTS coil to be an L-R circuit [8]:

\[
R_{dyn}(t) = -\frac{dB_z(t)}{dt}L/B_z(t) - R_j. \tag{4}
\]

5. Results and discussion

5.1. Dynamic resistance of closed-loop coil

The experimental \( R_{dyn} \) is calculated by (4). Five transport currents, \( I_{DC} \), were randomly selected from the experimental current decay curves, and then, examined in the simulation. Figures 9(a)–(e) display \( R_{dyn} \) as a function of the external AC field (represented by \( I_{em} \)), at different \( I_{DC} \) in HTS coil, including both the simulated and experimental results. Good agreement is found between the measured and simulated results. It is evident that there exist threshold magnetic fields, \( B_{th} \), above which \( R_{dyn} \) is generated. The external AC field would have little effect on accelerating the field decay rate if their amplitudes are below \( B_{th} \).

Figure 10 shows the calculated 100th cycle’s \( R_{dyn} \) distribution among the turns of the upper pancake of our experimental HTS coil, when \( I_{DC} = 69.23 \, \text{A} \) under different AC fields. Interestingly, most proportion of \( R_{dyn} \) is distributed in the innermost and outermost turns, while \( R_{dyn} \) of the middle part turns are less than \( 2 \times 10^{-8} \, \Omega \). With a portion of the DC and AC fields shielded by the innermost and outermost turns, the middle part turns have higher \( I_c \) and are able to shield their DC transport current from external field variations; therefore, little dynamic resistance generated.

5.2. Field decay curve of closed-loop coil

The current decay curves are calculated using the technique presented in section 2.3. The initial current of \( I_{DC} \) is 89 A, and the interval of \( I_0 \rightarrow I_1 \) for each step is set as 1 A. To reduce the computational load, \( R_{dyn} \) is calculated only at \( I_{DC} \) from 89 A to the final value of each decay with an interval of 5 A, and \( R_{dyn} \) at other \( I_{DC} \) values is obtained by linear interpolation. The experimental conditions in figure 8(a) are examined in simulation. The simulated current decay curves and their comparisons with the experimental results are shown as the red dashed lines and black lines in figures 9(f)–(j), respectively.

Reasonable agreement is found and the relative error of final field decay rate is within 6.6%–16.3%. The difference is likely due to a few factors. First, dynamic resistance of a coil and induced field decay are sensitive to the \( I_c \) uniformity along the coil length. As is concluded in section 5.1, dynamic resistance is mainly decided by the innermost and outermost turns. In simulation, the outermost two turns of each pancake were artificially assumed as the weak part, for which \( I_{c0} \) was set
lower as 188 A or 150 A. The simulated field decay curves are shown as the blue and green dashed lines in figures 9(f)–(j). A notable difference compared to previous calculation with uniform $I_{c0} = 213$ A can be observed, even though this weak part accounts for only 7.8% of the length of the whole coil. Second, vibration of copper coils is observed in experiment. It would introduce a travelling feature for the external field in the HTS coil. Travelling field penetrates $\sqrt{2} - \sqrt{3}$ times further than a standing AC field of the same amplitude and frequency [27, 28]. Therefore, in the case that the AC field amplitude is less than $B_{th}$, travelling field might be still able to reach the penetration depth and interact with central DC current. The simulated field decay has reached a relatively steady-state in lower current regions, but experimentally, the travelling feature continues to generate dynamic resistance and accelerate the field decay rate for more time.

5.3. Magnetic shielding design method

Section 5.1 suggests the field decay rates are mainly caused by the $R_{dy}$ of the innermost and outermost turns. Accordingly, higher $I_c$ tapes or particular magnetic shield should be equipped at these locations to reduce the field decay rates.

The effectiveness of this magnetic shielding design was tested in simulation. In figure 11(a), two unclosed turns of superconductor tapes are added in the innermost and outermost parts of the $2 \times 28$ coil, respectively. In figure 11(b), the significant reduction of field decay rate caused by this shielding design is about 25%. With this model, more efficient shielding (either ferro-magnetic material [7] or HTS tapes) can be designed by varying the number of shielding turns and their space distributions. Even more, HTS tapes with larger width may have better efficiency to shield external field variations. The effectiveness of the shielding technique in the outermost layer of the HTS coil has been tested experimentally [8].

6. Conclusion

A numerical model is reported to predict the field decay performance of closed-loop HTS coil in AC fields. In this work, $H$-formulation is adopted and $\sim 10^3$ s simulation of
decay is available by integrating the discrete dynamic resistances into a circuit model. Feasibility of this model on field decay calculation was verified by comparing with a pure FEM model of direct field cooling and decay under AC field. The impact of current relaxation is proven to be able to be eliminated with acceptable precision by simulation of a limited number (∼100) of AC cycles, at which state the turns under relaxation have been already below the threshold states for generating dynamic resistance. The simulated field decay rates and $R_{\text{dyn}}$ values are considered to be in reasonable consistence with the experimental results. This model also suggests that the field decay in AC fields is mainly attributed to the dynamic resistance of innermost and outermost turns. Based on this observation, a targeted and more efficient magnetic shielding can be designed to reduce the field decay rates. In summary, by this model, the behaviour of closed-loop HTS coil in non-uniform external AC fields is predictable.

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