Large-angle Polarization of the
Cosmic Microwave Background Radiation
and Reionization

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We discuss the effect of matter reionization on the large-angular-scale anisotropy and polarization of the cosmic microwave background radiation (CMBR) in the standard CDM model. We separate three cases in which the anisotropy is induced by pure scalar, pure tensor, and mixed metric perturbations respectively. It is found that, if reionization occurs early enough, the polarization can reach a detectable level of sequentially 6%, 9%, and 6.5% of the anisotropy. In general, a higher degree of polarization implies a dominant contribution from the tensor mode or reionization at high redshift. Since early reionization will suppress small-scale CMBR anisotropies and polarizations significantly, measuring the polarization on few degree scales can be a direct probe of the reionization history of the early universe.

Subject headings: cosmology: cosmic microwave background — cosmology: observations
1. INTRODUCTION

The large-scale anisotropy of the cosmic microwave background radiation (CMBR) measured by the DMR onboard the Cosmic Background Explorer satellite (Smoot et al. 1992) may be induced by density perturbations (scalar mode) and primordial gravitational waves (tensor mode) via the Sachs-Wolfe (SW) effect (Sachs & Wolfe 1967). If the anisotropy is mainly due to the scalar mode, the COBE result combined with observations of the large-scale structure would provide us with an important clue for discriminating between different cosmological models. However, it was recently argued that this anisotropy might be dominated by the tensor mode (Krauss & White 1992). Furthermore, it was shown that the tensor-mode dominance actually occurs in certain inflation models (Davis et al. 1992). Therefore, distangling the scalar from the tensor contributions is not only very important for testing the inflation model but is also needed to understand the formation of large-scale structure. It was then suggested that by comparing large- and small-scale anisotropy measurements, one can separate the scalar- and tensor-mode contributions (Davis et al. 1992; Crittenden et al. 1993).

However, early reionization of the universe may influence the formation of structures (Couchman & Rees 1986) as well as damp out small-scale CMBR anisotropies (Vittorio & Silk 1984; Bond & Efstathiou 1984). The Gunn-Peterson test indicates that the universe must have been highly reionized by a redshift of five or greater (Gunn & Peterson 1965). Recently, the likelihood of early reionization by radiation emitted from young galaxies has been investigated in detail. In a standard cold dark matter (CDM) model, typical parameter values predict that reionization occurs at a redshift around 50 (Tegmark, Silk, & Blanchard 1994); similar results were also obtained by numerical simulation (Fukugita & Kawasaki 1994). When normalized to COBE observations, the CDM model very likely has reionization at redshifts around 28-69 (Liddle & Lyth 1994). With reionization at redshift $\sim 50$, the Doppler peak on degree scales may be damped almost completely away while large-scale anisotropies remain reasonably unaffected (Sugiyama, Silk, & Vittorio 1993). Indeed, it was attempted to have early reionization to smooth out excessive temperature fluctuations on degree scales predicted in the CDM model, in order to reconcile the model with the lowest limits from South-Pole 91 data (Gaier et al. 1992). However, recent small-scale
anisotropy measurements from different groups have already hinted that the Doppler peak seems to be present at around the correct height for models based on adiabatic density perturbations without reionization. For nonstandard CDM models, some of them could have reionization occurring at redshifts early enough (Liddle & Lyth 1994) to suppress degree-scale anisotropies (Kamionkowski, Spergel, & Sugiyama 1993; Tegmark & Silk 1994a). Therefore, future anisotropy measurements on degree and subdegree angular scales would be crucial for determining the reionization history of the universe.

Polarization of the CMBR is another clue that could have a great potential of probing the reionization history of the universe. Anisotropic radiation acquires linear polarization when it is scattered with free electrons (Rees 1968). There have been several works on calculating the small-angular-scale (≤ 1°) r.m.s. polarization of the CMBR induced by adiabatic density perturbations in an universe with standard recombination (Kaiser 1983; Bond & Efstathiou 1984), and in a reionized universe (Bond & Efstathiou 1984; Nasel’skii & Polnarev 1987). It was shown that roughly 10 – 20% of the CMBR anisotropy is polarized on arc-minute scales. A thorough numerical calculation for both large- and small- angular-scale polarization of the CMBR induced by adiabatic and isocurvature density perturbations with standard recombination has been performed (Bond & Efstathiou 1987). Their calculations have confirmed earlier small-scale results and shown that large-scale polarization is insignificant.

An analytic estimation of the quadrupole polarization induced by scalar and tensor metric perturbations was made in various cosmological models including matter reionization (Ng & Ng 1993). The r.m.s. temperature anisotropy and polarization of the CMBR induced by the tensor mode perturbation of arbitrary wavelength was also computed (Polnarev 1985; Frewin, Polnarev, & Coles 1993; Ng & Ng 1994). It was shown that large-scale polarization is greatly enhanced by early reionization.

Similarly, the large-scale polarization of the CMBR induced by scalar and tensor modes within inflationary models was investigated under the assumption of no reionization (Harari & Zaldarriaga 1993). A detailed numerical calculation of scalar and tensor contributions to the CMBR polarization power spectrum in inflationary models has also been carried out (Crittenden, Davis, & Steinhardt 1993). Their calculations show that the polarization can reach a 10% level of the anisotropy in an universe with no hydrogen recombination, and may
be a useful discriminant for determining the ionization history of the universe.

In this paper, we will investigate in detail the CMBR polarization induced by scalar and tensor modes in the presence of early reionization at redshift around 30 ~ 90. In this scenario, CMBR anisotropies on degree and subdegree scales are suppressed significantly. Hence, although the degree of polarization is still about 10%, the absolute polarization would be suppressed. In contrary, large-scale anisotropies measured by COBE remain reasonably unaffected while the degree of large-scale polarization is greatly enhanced. Thus, measuring large-scale CMBR polarization would become more important.

In our previous paper (Ng & Ng 1994), we have given a detailed numerical calculation of the polarization of CMBR induced by pure tensor modes in an universe with and without matter reionization. It was shown that future polarization measurements with windows for the power spectrum from \(l = 2\) to \(l = 50\), at sensitivity of 10% level of the anisotropy, might have a chance to detect the CMBR polarization if early reionization took place at redshifts \(\geq 90\). Here we will combine these tensor-mode results with the scalar-mode contribution. The logic of this paper is more or less similar to the work by Crittenden et al. (Crittenden, Davis, & Steinhardt 1993). The difference is that we will not pursue models with no recombination, since they are disfavored (Tegmark & Silk 1994b) by the COBE FIRAS data (Mather et al. 1994).

2. METHODOLOGY

We shall use the units \(c = \hbar = 1\) throughout. Our calculations are based on the standard CDM model with a flat metric: \(ds^2 = a^2(\eta) (d\eta^2 - dx^2)\), where \(a(\eta)\) and \(d\eta = dt/a(t)\) are the scale factor and conformal time respectively. Here we normalize the conformal time to unity today. In this metric, \(\Omega_{\text{total}} = \Omega_{CDM} + \Omega_B = 1\), where \(\Omega_{CDM}\) and \(\Omega_B\) denote respectively the cold dark and baryonic matter. The Hubble constant is \(H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}\) with \(h = 0.5\), and the baryon density according to nucleosynthesis is given by \(\Omega_Bh^2 = 0.0125\). We will approximate the ionization history by a step function: the universe is completely ionized before hydrogen recombination at redshift \(z_{\text{rec}} \simeq 1375\), so is after early reionization occurring at redshift \(z_i\). Between is an universe with no optical opacity. This sudden approximation for the recombination and reionization processes is sufficient for the present consideration as long as their widths in redshift are reasonably narrow. The width will affect only calculations
on small-scale anisotropies. Since we concentrate on large-scale (> 1°) effect of scalar and tensor perturbations on CMBR, we include only the dominant SW effect (Sachs & Wolfe 1967).

To study how polarized photons propagate in the expanding universe, one need to solve the equation of transfer for photons (Chandrasekhar 1960). In general, arbitrarily polarized photons are characterized by four Stokes parameters, \( n = (n_l, n_r, n_u, n_v) \), where \( n = n_l + n_r \) is the distribution function for photons with \( l \) and \( r \) denoting two directions at right angle to each other. The equation of transfer for an arbitrarily polarized photon is governed by the collisional Boltzmann equation,

\[
\left( \frac{\partial}{\partial \eta} + \mathbf{e} \cdot \frac{\partial}{\partial \mathbf{x}} \right) n = -\frac{1}{2} \frac{\partial n}{\partial \ln \nu} \frac{\partial h_{ij}}{\partial \eta} \epsilon^i \epsilon^j - \sigma_T N_e a \left[ n - \frac{1}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} P(\mu, \phi, \mu', \phi') n d\mu' d\phi' \right],
\]

where \( \sigma_T \) is the Thomson scattering cross section, \( N_e \) is the number of free electrons per unit volume, \((\mu = \cos \theta, \phi)\) are the polar angles of the propagation direction \( \mathbf{e} \) of the photon with a comoving frequency \( \nu \), and \( P \) is the phase matrix for Thomson scattering.

The first term on the right-hand-side of equation (1) is the SW effect. For scalar modes, we have \( h_{ij} = \int d\mathbf{k} \delta_k \mathbf{e}^{i\mathbf{k} \cdot \mathbf{x}} k_i k_j \), where \( \mathbf{k} \) is the wave vector and \( \delta_k \) is the fluctuation amplitude which satisfies the time evolution equation for density perturbations. Since we are considering large-scale effect, we take \( \delta_k \propto k^2 \eta^2 \), which is a good approximation for long-wavelength modes. As to tensor modes, we have \( h_{ij} = \int d\mathbf{k} h_k \mathbf{e}^{i\mathbf{k} \cdot \mathbf{x}} \epsilon_{ij}^\lambda \), where \( \epsilon_{ij}^\lambda \) denote the gravitational wave polarization tensors, \( \epsilon_{ij}^+ = \epsilon_i \epsilon_j - \epsilon^*_i \epsilon^*_j \), and \( \epsilon_{ij}^\times = \epsilon_i \epsilon^*_j + \epsilon^*_i \epsilon_j \); where \( \epsilon_i \) and \( \epsilon_i^* \) are two mutually orthogonal unit vectors perpendicular to the wave vector \( \mathbf{k} \). Then, the gravitational wave amplitude \( h_k \) is governed by the equation of motion: \( \ddot{h}_k + 2 \dot{a} h_k / a + k^2 h_k = 0 \), where dot means \( d/d\eta \). The scalar and tensor power spectra are \( P^{(S)}(k) \propto T(k) k^{n_s-4} \) and \( P^{(T)}(k) \propto k^{n_t-3} \) respectively, where the power indices \( n_s = 1 \) and \( n_t = 0 \) correspond to strict scale invariance. For cold dark matter the scalar-mode transfer function is (Bardeen et al. 1986)

\[
T(k) = \frac{\ln(1 + 0.146 k \eta_{eq})/(0.146 k \eta_{eq})^2}{\left[ 1 + 0.242 k \eta_{eq} + (k \eta_{eq})^2 + (0.340 k \eta_{eq})^3 + (0.417 k \eta_{eq})^4 \right]^\frac{3}{2}},
\]

where \( \eta_{eq} \) is the time at which the energy density of radiation is equal to that of matter.
The solution $n$ for the equation of transfer assumes the form $n = n_0 + n_0 \delta n/2$, where $n_0$ and $\delta n$ are the unperturbed solution and perturbation respectively. We expand $\delta n = \int dk \ n'(k)e^{i k \cdot x}$, where $n' = \alpha a + \beta b$. For the scalar-mode solution, the Stokes components $n_u$ and $n_v$ both decouple from $n_l$ and $n_r$, and it suffices to consider only the first two components of $n$ with $a = (1, 1)$ and $b = (1, -1)$. Substituting the solution $n$ and the Fourier expansion for $h_{ij}$ into equation (1), and expanding $\alpha$ and $\beta$ in terms of Legendre polynomials,

$$
\alpha(\mu) = \sum_l (2l + 1) \alpha_l P_l(\mu),
\beta(\mu) = \sum_l (2l + 1) \beta_l P_l(\mu),
$$

we obtain two coupled differential equations for $\alpha$ and $\beta$,

$$
\dot{\alpha} + ik \mu \alpha = \frac{1}{3} T_3^4(k) k^2 \eta (1 + 2 P_2) - q(\alpha - \alpha_0 - \frac{1}{2} P_2(\alpha_0 - \beta_0 + \beta_2)),
\dot{\beta} + ik \mu \beta = -q(\beta - \frac{1}{2}(1 - P_2)(\beta_0 - \alpha_0 - \beta_2)),
$$

where $q = \sigma_T N_e a$. These equations are equivalent to equations (1a) and (1b) of Bond & Efstathiou 1984 after making the approximations that the metric perturbations $\dot{h} = \dot{h}_{33} \propto \eta$ and the baryon velocity $v = 0$. These approximations are valid as long as large-angular-scale calculation is concerned.

Equations (4) and (5) can be casted into a system of coupled differential equations,

$$
\dot{\alpha}_0 = -ik \alpha_1 + \frac{1}{3} T_3^4(k) k^2 \eta \\
\dot{\alpha}_1 = \frac{i}{3} k(\alpha_0 + 2 \alpha_2) - q \alpha_1 \\
\dot{\alpha}_2 = \frac{i}{5} k(2 \alpha_1 + 3 \alpha_3) + \frac{2}{15} T_3^4(k) k^2 \eta - \frac{q}{10}(9 \alpha_2 + \beta_0 - \beta_2) \\
\dot{\beta}_0 = -ik \beta_1 - \frac{q}{2}(\beta_0 + \beta_2 + \alpha_2) \\
\dot{\beta}_1 = \frac{i}{3} k(\beta_0 + 2 \beta_2) - q \beta_1 \\
\dot{\beta}_2 = \frac{i}{5} k(2 \beta_1 + 3 \beta_3) - \frac{q}{10}(9 \beta_2 + \beta_0 - \alpha_2)
$$

for $l \geq 3$,

$$
\dot{\alpha}_l = -q \alpha_l - \frac{ik}{2l + 1}[l \alpha_{l-1} + (l + 1) \alpha_{l+1}] \\
\dot{\beta}_l = -q \beta_l - \frac{ik}{2l + 1}[l \beta_{l-1} + (l + 1) \beta_{l+1}].
$$

(6)
The coefficients $\alpha_l$ and $\beta_l$ are then solved by numerical method.

In case of tensor mode, only $n_v$ decouples, and thus we choose the basis (Polnarev 1985):
$$a = \frac{1}{2}(1 - \mu^2)\cos 2\phi (1, 1, 0) \quad \text{and} \quad b = \frac{1}{2}(1 + \mu^2)\cos 2\phi, -(1 + \mu^2)\cos 2\phi, \ 4\mu\sin 2\phi),$$
for the $+$ mode solution. The $\times$ mode solution is given by the same expressions with $\cos 2\phi$ and $\sin 2\phi$ interchanged. Defining $\xi = \alpha + \beta$, we obtain a system of coupled differential equations in a similar way,

$$\dot{\xi}_0 = -q\xi_0 - ik\xi_1 + \dot{h}_k$$
$$\dot{\beta}_0 = -\frac{3}{10}q\beta_0 - ik\beta_1 + q\left(\frac{5}{7}\beta_2 + \frac{3}{35}\beta_4 - \frac{1}{10}\xi_0 + \frac{1}{7}\xi_2 - \frac{3}{70}\xi_4\right)$$
for $l \geq 1$,

$$\dot{\xi}_l = -q\xi_l - \frac{ik}{2l+1}[l\xi_{l-1} + (l+1)\xi_{l+1}]$$
$$\dot{\beta}_l = -q\beta_l - \frac{ik}{2l+1}[l\beta_{l-1} + (l+1)\beta_{l+1}],$$

(7)

where the source term $\dot{h}_k$ is obtained by solving numerically the equation of motion for $h_k$.

To describe the degrees of anisotropy and polarization, we compute the power spectra for the anisotropy, $C_\alpha^\alpha$, and polarization, $C_\beta^\beta$. To evaluate these functions, we expand the photon fluctuation distribution function in terms of spherical harmonic functions, i.e.,

$$\delta n = \sum_{l,m} a_{lm} Y_{lm}, \quad a_{lm} = \int \delta n Y_{lm}^* d\Omega.$$  

(8)

The total power spectrum is then given by $\langle \sum_m a_{lm}^* a_{lm} \rangle = C_\alpha^\alpha + C_\beta^\beta$, where $\langle \rangle$ denotes the average over all observation positions in the universe. For the scalar mode, it is straightforward to obtain (Bond & Efstathiou 1987)

$$C_\alpha^{\alpha(S)} = 8\pi (2l+1) \int dk P^{(S)}(k)|\alpha_l^{(S)}|^2,$$
$$C_\beta^{\beta(S)} = 8\pi (2l+1) \int dk P^{(S)}(k)|\beta_l^{(S)}|^2.$$  

(9)

The anisotropy and polarization power spectra for the tensor mode, $C_\alpha^{\alpha(T)}$ and $C_\beta^{\beta(T)}$ respectively, can be deduced in a similar way (Crittenden et al. 1993; Ng & Ng 1994). In deriving $C_\beta^{\beta(S)}$ and $C_\beta^{\beta(T)}$, we have neglected the rotation from the $k$-dependent basis to a fixed (laboratory) basis (see Bond & Efstathiou 1987 for details about the basis rotation). We have solved our transfer equations relative to this $k$-basis and summed up all mode contributions.
to the polarization power spectra in the same basis. Rigorously, this summation would not make any sense unless one has rotated the Stokes parameters from the $k$-basis to the fixed basis. However, for Gaussian random perturbations, on the average, the expressions for $C_{l}^{\beta(S)}$ and $C_{l}^{\beta(T)}$ could well represent the degree of polarization in the laboratory (Bond & Efstathiou 1987). Having the power spectra, we can construct the correlation function,

$$C^{\alpha,\beta}(\Theta) = \frac{1}{4\pi} \sum_{l} C_{l}^{\alpha,\beta} W_{l} P_{l}(\cos \Theta), \quad (10)$$

where $W_{l}$ is the window function for detector, and $\Theta$ is the separation angle. In actual observations the lower end of $l$ is excluded by limited sky coverage, whereas the high-$l$ cutoff is fixed by the finite beam width.

### 3. NUMERICAL RESULTS

In this section, we present the scalar and tensor contributions to the CMBR anisotropy and polarization on large angular scales. We separate three cases in which the anisotropy is induced by pure scalar, pure tensor, and mixed metric perturbations respectively. To ensure that we capture the dominant contributions in the $k$ integration when computing the multipoles in equation (1), we have investigated the $k$ dependence of certain $l$th multipole. Roughly speaking, the dominant contribution to the $l$th multipole comes from modes with $k \sim l$ (note that $\eta_{0} = 1$). It is found that for $l \leq 100$, it is sufficient to consider the contributions from modes with $k \leq 2l$, except for the tensor-induced polarization power spectrum. In the scalar-mode calculation, the $k$ integration must be cut off at the point where linear perturbation theory breaks down. We thus set the cut-off at $k_{\text{rec}}$, where $k_{\text{rec}}$ is the wavenumber which enters the horizon during the recombination era, since shorter-wavelength fluctuations probably have gone nonlinear (Abbott & Wise 1984). We refer the interested reader to Appendix for further discussion.

Figure 1 shows the normalized CMBR anisotropy power spectra due to scale-invariant scalar ($n_{s} = 1$) and tensor ($n_{t} = 0$) mode perturbations respectively. The constant behavior of $l(l+1)C_{l}^{\alpha(S),\beta(T)}/(2l+1)$ for small $l$, with a scale-invariant spectrum in an universe with standard recombination ($z_{i} = 0$), is evident from the figure. For the scalar mode, we have compared our numerical calculation for $z_{i} = 0$ with another alternative approach which makes use of the SW integral formula (Sachs & Wolfe 1967). Using this formula, one can
easily derive an analytic free-streaming solution for the anisotropy power spectrum for \( l \geq 0 \) (Abbott & Wise 1984),

\[
C_{l}^{(S)}(S) \propto (2l + 1) \int \frac{dk}{k} \left| \delta_0\delta_{l}^{0} + \frac{i}{3} k \eta_{0} \delta_{l}^{1} - i^{l} \eta_{l} \right| k (\eta_{0} - \eta_{rec}) \right] \nonumber \\
- i^{l} k \eta_{rec} \left[ \frac{l}{2l + 1} j_{l-1} k (\eta_{0} - \eta_{rec}) - \frac{l + 1}{2l + 1} j_{l+1} k (\eta_{0} - \eta_{rec}) \right] \right|^{2}, \tag{11}
\]

where \( \eta_{rec} \) and \( \eta_{0} \) are respectively the recombination time and the present time. Note that we have used \( \eta_{rec} \) as the lower integration limit in the SW formula instead of the usual decoupling time \( \eta_{de} \). Under our assumption of the ionization history, they are in fact identical. We have plotted the power spectrum \( (\cdot) \) represented by a long dashed curve in the figure. We see that, for small \( l \), the result agrees very well with the numerical result. The last term in equation \( (\cdot) \) is essentially due to a Doppler-shift correction for the world velocity of the source at the last scattering surface (Sachs & Wolfe 1967). In the numerical calculation, Thomson scatterings during the ionized stage before photon decoupling can effectively damp out this term. This effect is prominent mostly on small angular scales. That is why the high-\( l \) \( C_{l}^{(S)} \)'s in the numerical result are lower than that in equation \( (\cdot) \). For the tensor case, similarly, we have plotted the power spectrum for \( z_{i} = 0 \) caused by pure SW effect by putting the numerical solution of the evolution equation for gravitational waves in the SW formula (Ng & Speliotopulos 1994), and found good agreement. Besides, late-time reionization \( (z_{i} = 30 - 90) \) reduces the CMBR temperature anisotropy on small-angular scales, and shift the \( l(l + 1)C_{l}^{(S), (T)}/(2l + 1) \) curves from scale-invariance.

Figure 2 shows the normalized CMBR anisotropy power spectra due to tilted scalar \( (n_{s} = 0.85) \) and tensor \( (n_{t} = -0.15) \) spectra respectively. It is evident that the constancy of \( l(l + 1)C_{l}^{(S), (T)}/(2l + 1) \) for small \( l \) is broken, however, the relative behavior of curves with different \( z_{i} \) values is similar to the scale-invariant case. Furthermore, the normalized magnitude of the power spectrum for large \( l \) is smaller when compared to the scale-invariant case. This can be easily explained by referring to the \( k \) dependence of the power spectrum in equation \( (\cdot) \).

Figure 3 shows the polarization multipole to anisotropy multipole ratio for scalar perturbations, \( C_{l}^{(S)}/C_{l}^{(S)} \), as a function of \( l \) with \( n_{s} = 1 \) and 0.85. The ratio increases significantly for an universe which underwent an early reionization phase. In general, the
earlier the reionization takes place, the larger is the polarization-to-anisotropy ratio. Note that the ratio with early reionization has a peak around $l \sim 10 - 40$, which corresponds to a few degree angular scales. This makes the search for large-angular-scale polarizations more interesting. In particular, the peak of the $z_i = 90$ curve at $l \sim 20$ corresponds to about 10% polarization in the CMBR fluctuations. Our results indicate that the ratio is rather insensitive to the variation in $n_s$. In fact, there is almost no discernible difference between the $n_s = 1$ and $n_s = 0.85$ results. To estimate the r.m.s. polarization-to-anisotropy ratio, $[C^\beta(0)/C^\alpha(0)]^{1/2}$, we sum $l$ from 5 up to 50 with $W_l = 1$ in equation (1), for spectra with $n_s = 1, 0.85$ and different reionization redshifts. This should correspond to typical large-scale CMBR polarization measurements. The ratios are listed in Table 1. In an universe with standard recombination, the polarization is much less than 1% of the anisotropy. However, the degree of polarization with early reionization at redshift $\sim 90$ is enhanced to 6.1%.

Figure 4 shows the polarization multipole to anisotropy multipole ratio for tensor perturbations, $C_l^\beta(T)/C_l^\alpha(T)$, as a function of $l$ with $n_t = 0$ and $-0.15$. The curves are similar to the scalar case, and the ratio is again insensitive to the variation in $n_t$. Table 2 lists the r.m.s. large-scale polarization-to-anisotropy ratio. The polarization with early reionization at redshift $\sim 90$ for a scale-invariant spectrum is 9% of the anisotropy.

So far, we have considered only pure scalar- or tensor-mode contributions to CMBR fluctuations. In general, these fluctuations can be induced by both scalar and tensor modes. In fact, inflation can generate both types of metric perturbations. In inflationary models, $n_s$ is related to $n_t$ by $n_s = n_t + 1$, and there is a nearly model-independent relation between their induced anisotropy quadrupole moment: $C_2^{\alpha(T)}/C_2^{\alpha(S)} \simeq -7n_t$. For $n_s = 0.85$ and $n_t = -0.15$, $C_2^{\alpha(T)} \simeq C_2^{\alpha(S)}$.

In Figure 5, we have plotted the total (scalar plus tensor) polarization multipole to anisotropy multipole ratio, $C_l^{\beta(S)+T}/C_l^{\alpha(S)+T}$, versus $l$ for $n_s = 0.85$ and $n_t = -0.15$, assuming $C_2^{\alpha(T)} = C_2^{\alpha(S)}$. Also, in the figure, we use short-dashed and long-dashed lines to represent the scalar and tensor portions respectively. The tensor contribution is dominant for almost all $l$ in the standard recombination model, however, the dominance is taken by the scalar mode as $z_i$ increases. Table 3 lists the r.m.s. large-scale total polarization-to-anisotropy ratio. The polarization with early reionization at redshift $\sim 90$ for this mixed
model is 6.5% of the anisotropy.

4. COMPARISONS

When comparing our result for the pure scalar mode with \( z_i = 0 \) in Figure 3 to Figure 7 of Bond & Efstathiou 1987 and Figure 4 of Crittenden, Davis & Steinhardt 1993, we find that our ratio is about two orders of magnitude lower than theirs for small \( l \) and an order of magnitude lower for large \( l \). This may be due to the vanishing residual ionization subsequent to the hydrogen recombination that we have assumed in our whole calculation. In fact, we found that a residual ionization could raise the quadrupole polarization by two orders of magnitude (Ng & Ng 1993). This also explains why our r.m.s. polarization-to-anisotropy ratio (0.06\%, see Table 1) for the standard recombination model with \( n_s = 0.85 \) pure scalar modes is much less than Crittenden et al.’s 0.4\%. These discrepancies should be removed if we consider a more accurate model for the standard thermal history of the universe. Since the degree of polarization in the standard recombination model is already less than 1\% which is well below the present detectable level, we will not pursue along this line and only concentrate on the reionization model. The 6.1\% result with \( z_i = 90 \) in Table 1 is, however, consistent with their 7.9\% result for the no recombination model.

In Figure 4, the high-\( l \) part of the curve with \( z_i = 0 \) for the pure tensor mode is again an order of magnitude lower than the result in Figure 4 of Crittenden, Davis & Steinhardt 1993. But, the low-\( l \) part is four orders of magnitude lower. Once again, the residual ionization can account for two orders of magnitude difference in the low-\( l \) part. For the other two orders of magnitude difference, we suspect that it may be due to different definitions for the polarization power spectrum. Unfortunately, we cannot make an explicit comparison, since their paper did not show explicitly how to calculate the polarization spectrum. We see from Figure 4 that the polarization to anisotropy ratio for \( l = 2 \) is \( 6.3 \times 10^{-6} \). In fact, this value is close to our earlier analytic calculation for the tensor mode contribution to the polarization quadrupole moment in the instantaneous recombination model with vanishing residual ionization, which is about \( 9 \times 10^{-6} \) (Ng & Ng 1993).

As to the mixed-mode results, we compare our Figure 5 with Figures 1 & 3 of Crittenden, Davis & Steinhardt 1993. Once again, the solid curve with \( z_i = 0 \) in Figure 5 is well below their result probably due to the reasons that we have mentioned above. The shape of the
solid curve with \( z_i = 90 \) agrees quite well with their no-recombination result. In particular, our value for the ratio of the total polarization quadrupole moment to the total anisotropy quadrupole moment (\( \approx 5 \cdot 10^{-5} \)) agrees fairly well with their result (\( \approx 8 \cdot 10^{-5} \)). Our r.m.s. total polarization-to-anisotropy ratio for the standard recombination is 0.13\% (see Table 3) whereas theirs is 0.5\%. With reionization at redshift equal to 90, we have 6.5\%, which is comparable to their 7.4\% for the no-recombination model. Although the general trend that the tensor contribution becomes subdominant as \( z_i \) increases is the same in both works, the details on how the dominance taken by the scalar contribution are different. In our work, we observe that when \( z_i = 0 \), there exists a window for \( l \) where the tensor mode is dominant. As \( z_i \) increases, the width of this window becomes narrow and disappears for \( z_i \geq 90 \) at \( l \approx 5 - 10 \).

5. CONCLUSIONS

We have considered the effects of matter reionization on the large-angular-scale anisotropy and polarization of the cosmic microwave background radiation (CMBR) induced by scalar and tensor metric perturbations, with scale-invariant or tilted spectra. The results are rather insensitive to the power indices of fluctuation spectra. We separate three cases in which the anisotropy is induced by pure scalar, pure tensor, and mixed modes respectively. It is found that the polarization is insignificant in the standard recombination model. But, if reionization occurs early enough, the polarization can reach sequentially 6\%, 9\%, and 6.5\% of the anisotropy. In general, a higher degree of polarization implies a dominant contribution from the tensor mode or reionization at high redshift. A 1\% level sensitivity measurement could set constraint on the reionization redshift value, as well as provide information on whether the metric perturbation consists of a tensor component in models with a high redshift reionization value. For instance, if polarization is detected to be 5\%, then reionization occurs at a redshift between 45 and 90, rather independent of the types of the metric perturbation and the power spectrum index. For the cases where \( z_i \geq 80 \), measurements with 1\% sensitivity can distinguish the scale-invariant tensor perturbation from the mixed mode metric perturbation.

CMBR fluctuations have a 10\% level polarization on angular scales less than 1° rather model-independently. In the CDM model with standard recombination, the predicted r.m.s.
$\Delta T/T \sim 10^{-5}$ at 1°. Hence, to detect small-scale polarizations would require sensitivity at a level of $\Delta T/T \sim 10^{-6}$. In fact, this signal level can be achieved by using new technology and instrument design (Timbie, P. T. private communication). It is known that the universe has been reionized. Theoretical sides also predict an early reionization in dark matter models. Our calculations show that, if reionization did occur at redshift $\sim 90$, the polarization to anisotropy ratio for either scalar, tensor, or mixed mode would have a peak of height of 10% around $l \sim 20$. This peak corresponds to an angle of about $9^\circ$ ($\theta \sim \pi/l$). When normalized to COBE/DMR anisotropy signals, this large-angle polarization is at a level of $\Delta T/T \sim 10^{-6}$ and would be detectable in the near future. Unfortunately, the early reionization would suppress small-scale anisotropies by an order of magnitude (Sugiyama, Silk, & Vittorio 1993), thus making small-scale polarization measurements elusive. It appears that measuring CMBR polarization on both large and small angular scales is a promising method for determining the ionization history of the universe.

**APPENDIX**

In this Appendix, we show how the $l$th multipole depends on the range of $k$ integration. In Figures 6 and 7, we plot for the scalar mode the spectral power spectrum $C_{l}^{\alpha(S)}(k)$, where $C_{l}^{\alpha(S)} = \int dk C_{l}^{\alpha(S)}(k)$, as a function of $k$ with $n_s = 1$, for $z_i$ equal to 0 and 90 respectively. In Figure 6, we also plot the dipole and quadrupole moments by using the SW formula (equation ()), and find very well agreements. The dipole moment ($l = 1$) is sensitive to short-wavelength fluctuations. To ensure that we do not go beyond the linear regime, we set the cut-off $k = k_{rec}$ in our calculation. We also investigate the $k$ dependence of higher multipoles, and find that the dominant contributions arise from modes with $k \leq 2l$ for the $l$th multipole (however, it does not apply very well for small $l$). We illustrate this dependence for $l$ equal to 2 and 20 in Figures 6 and 7. We see that reionization does not affect much on the magnitude of the spectra but enrich structures at their high-$k$ tails. For $l \sim 100$, the moments will be damped out by reionization. Similarly, in Figure 8, we plot $C_{l}^{\beta(S)}(k)$ with $n_s = 1$, for $l = 20, 40$ and $z_i = 0, 90$. Obviously, reionization enhances the spectra by several orders of magnitude. The $k \leq 2l$ rule applies very well for $l \geq 20$. Roughly speaking, the contributions drop by an order of magnitude when $k$ is increased to $2k$.

In Figure 9, we plot $C_{l}^{\alpha(T)}(k)$ for the tensor mode as a function of $k$ with $n_t = -0.15$,
for $z_i = 0$ and 90. We notice that the $k \leq 2l$ rule applies for high multipoles. The $l = 20$ moment is damped by reionization. In Figure 10, we plot $C_\ell^{\beta(T)}(k)$ with $n_t = -0.15$, for $z_i = 0$ and 90. Again, reionization greatly enhances the magnitude of the spectra. For an universe with no reionization, the spectrum falls off with $k$ less abruptly. In this case, to capture the main contributions to the $l$th multipole, one might need to include modes with $k \leq 10l$ in the calculation. However, this range of $k$ is getting narrower for large $l$. At $l \sim 100$, it is sufficient to apply the $k \leq 2l$ rule again.

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TABLES

**TABLE I. R.m.s. Polarization-to-Anisotropy Ratio for Scalar Mode**

| $z_i$ | $\left[ \frac{C_{\gamma}(0)}{C_{\alpha\alpha}(0)} \right]^{\frac{1}{2}}$ | $\left[ \frac{C_{\gamma}(0)}{C_{\alpha\alpha}(0)} \right]^{\frac{1}{2}}$ |
|-------|-----------------|-----------------|
|       | $(n_s = 1)$     | $(n_s = 0.85)$  |
| 0     | 0.0006          | 0.00057         |
| 30    | 0.016           | 0.016           |
| 60    | 0.040           | 0.041           |
| 90    | 0.061           | 0.061           |

**TABLE II. R.m.s. Polarization-to-Anisotropy Ratio for Tensor Mode**

| $z_i$ | $\left[ \frac{C_{\beta}(0)}{C_{\alpha\alpha}(0)} \right]^{\frac{1}{2}}$ | $\left[ \frac{C_{\beta}(0)}{C_{\alpha\alpha}(0)} \right]^{\frac{1}{2}}$ |
|-------|-----------------|-----------------|
|       | $(n_t = 0)$     | $(n_t = -0.15)$ |
| 0     | 0.0025          | 0.0019          |
| 30    | 0.022           | 0.021           |
| 60    | 0.058           | 0.053           |
| 90    | 0.090           | 0.077           |

**TABLE III. R.m.s. Total Polarization-to-Anisotropy Ratio for Mixed Mode**

| $z_i$ | $\left[ \frac{C_{\beta}(0)}{C_{\alpha\alpha}(0)} \right]^{\frac{1}{2}}$ |
|-------|-----------------|
|       | $(n_s = 0.85, n_t = -0.15)$ |
| 0     | 0.0013          |
| 30    | 0.018           |
| 60    | 0.044           |
| 90    | 0.065           |
References
Abbott, L., & Wise, M. 1984, Phys. Lett. 135B, 279
Bardeen, J. M., et al. 1986, ApJ, 304, 15
Bond, J. R., & Efstathiou, G. 1984, ApJ, 285, L45
Bond, J. R., & Efstathiou, G. 1987, MNRAS, 226, 655
Couchman, H. M. P., & Rees, M. J. 1986, MNRAS, 221, 53
Chandrasekhar, S. 1960, Radiative Transfer (New York: Dover)
Crittenden, R., et al. 1993, Phys. Rev. Lett., 71, 324
Crittenden, R., Davis, R. L., & Steinhardt P. J. 1993, ApJ, 417, L13
Davis, R. L., et al. 1992, Phys. Rev. Lett., 69, 1856
Frewin, R., Polnarev, A., & Coles, P. 1994, MNRAS, 266, L21
Fukugita, M., & Kawasaki, M. 1994, MNRAS, 269, 563
Gaier, T., et al. 1992, ApJ, 398, L1
Gunn, J. E., & Peterson, B. A. 1965, ApJ, 142, 1633
Harari, D., & Zaldarriaga, M. 1993, Phys. Lett. B, 319, 96
Kaiser, N. 1983, MNRAS, 202, 1169
Kamionkowski, M., Spergel, D. N., & Sugiyama, N. 1994, ApJ, 426, L57
Krauss, L., & White, M. 1992, Phys. Rev. Lett., 69, 869
Liddle, A., & Lyth, D. 1994, SUSSEX-AST 94/9-2 preprint, astro-ph/9409077
Mather, J. C., et al. 1994, ApJ, 420, 439
Nasel’skii, P., & Polnarev, A. 1987, Astrofizika, 26, 543
Ng, K. L., & Ng, K.-W. 1993, astro-ph/9305001 to appear in Phys. Rev. D
Ng, K. L., & Ng, K.-W. 1994, astro-ph/9406076, to appear in ApJ
Ng, K.-W., & Speliotopoulos, A. D. 1994, IP-ASTP-07-94 preprint, astro-ph/9405043
Polnarev, A. G. 1985, Astron. Zh., 62, 1041 [Sov. Astron., 29(6), 607]
Rees, M. J. 1968, ApJ, 153, L1
Sachs, R. K., & Wolfe, A. M. 1967, ApJ, 147, 73
Smoot, G., et al. 1992, ApJ, 396, L1
Sugiyama, N., Silk, J., & Vittorio, N. 1993, ApJ, 419, L1
Tegmark, M., & Silk, J. 1994a, CfPA-94-th-24 preprint, astro-ph/9405042
Tegmark, M., & Silk, J. 1994b, ApJ, 423, 529
Tegmark, M., Silk, J., & Blanchard, A. 1994, ApJ, 420, 484
Vittorio, N., & Silk, J. 1984, ApJ, 285, L39
Figure Captions

Fig. 1 Normalized anisotropy power spectra. The solid and short-dashed curves correspond to the scale-invariant scalar \( n_s = 1 \) and tensor \( n_t = 0 \) modes respectively, with reionization at \( z_i = 0, \ 30, \ 60, \ 90 \). The long-dashed curve is drawn by using the SW formula in equation (). The long-short-dashed curve is also drawn by using SW formula. For all curves in this figure and figures below, \( \Omega_B = 0.05 \) and \( h = 0.5 \).

Fig. 2 Normalized anisotropy power spectra. The solid and dashed curves correspond to the tilted scalar \( n_s = 0.85 \) and tensor \( n_t = -0.15 \) spectra respectively, with reionization at \( z_i = 0, \ 30, \ 60, \ 90 \).

Fig. 3 Ratio of polarization multipole to anisotropy multipole as a function of \( l \) due to scalar mode perturbations. The solid and dashed curves correspond to the scale-invariant \( (n_s = 1) \) and tilted \( (n_s = 0.85) \) cases respectively.

Fig. 4 Ratio of polarization multipole to anisotropy multipole as a function of \( l \) due to tensor mode perturbations. The solid and dashed curves correspond to the scale-invariant \( (n_t = 0) \) and tilted \( (n_t = -0.15) \) cases respectively.

Fig. 5 Total (scalar plus tensor) polarization multipole to anisotropy multipole ratio versus \( l \), assuming equal scalar- and tensor-induced anisotropy quadrupole moments. The solid curves denote the total contribution. The short- and long-dashed curves correspond to the scalar and tensor portions respectively.

Fig. 6 Spectral anisotropy power spectrum for scalar mode, \( C^{\alpha(S)}_l(k) \), as a function of \( k \) with \( n_s = 1 \) and \( z_i = 0 \), for \( l = 1, \ 2, \ 20 \). SW denotes the results obtained by using SW formula.

Fig. 7 Spectral anisotropy power spectrum for scalar mode with \( n_s = 1 \) and \( z_i = 90 \), for \( l = 1, \ 2, \ 20 \).

Fig. 8 Spectral polarization power spectrum for scalar mode \( C^{\beta(S)}_l(k) \) with \( n_s = 1 \) and \( z_i = 0, \ 90 \), for \( l = 20, \ 40 \).

Fig. 9 Spectral anisotropy power spectrum for tensor mode with \( n_t = -0.15 \) and \( z_i = 0, \ 90 \), for \( l = 2, \ 20 \).

Fig. 10 Spectral polarization power spectrum for tensor mode with \( n_t = -0.15 \) and \( z_i = 0, \ 90 \), for \( l = 2, \ 20 \).