Direct observation and simulation of the diffractive saturable loss mechanism in a Kerr-lens mode-locked laser

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Abstract

Passive mode-locking critically relies on a saturable loss mechanism to form ultrashort pulses. However, in Kerr-lens mode-locking (KLM), no actual absorption takes place, but rather losses appear due to diffraction, and actual light must escape the cavity. The Kerr-lens effect works to generate through diffraction an effective instantaneous saturable absorber that delicately depends on the interplay between the spatial and temporal profiles of the pulse. Despite the importance of KLM as a technique for generating ultrafast pulses and the fundamental role of the diffraction losses in its operation, these losses were never directly observed. Here, we measure the light that leaks out due to diffraction losses in a Kerr-lens mode-locked Ti:Sapphire laser, and compare the measured results with a numerical theory that explicitly calculates the spatio-temporal behavior of the pulse.

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A mode-locked laser oscillates on many longitudinal modes together in a synchronized manner such that the modes interfere constructively to create a short pulse in time. Mode-locking is achieved by incorporating in the oscillator cavity a saturable loss mechanism, e.g. saturable absorber (SA), whose loss is reduced at high intensities [1, 2]. The common intuition is that the laser continuously optimizes to find the most efficient mode of operation, which in the case of a SA, corresponds to mode-locked operation, where the high peak-intensity of the pulses mitigates the absorption losses. Dynamically, the SA losses vary in time, indicating that the SA is more absorptive at the leading and trailing edges of the pulse, which leaves more net gain to the center part (peak) of pulse. This works to shorten the pulse, until pulse-widening effects like dispersion and finite gain bandwidth become substantial and the pulse reaches a steady-state. Understanding the dynamics that lead to minimum-loss solutions is an important fundamental problem in the study of laser physics in general and mode-locking in particular [3]. Mode-locking is one very important example of mode-competition, where through the intricate dynamics of gain, loss and non-linearities a laser finds the most efficient mode of operation, rendering it a very powerful optimization device. Using the principles of mode competition, lasers have been used as real-time optical solvers [4] and even simulators for spin-networks [5, 6]. In mode-locking, the additional non-linearity of the loss, which acts in both space and time, completely changes the most-efficient mode from a single frequency continuous-wave mode to a single highly localized temporal mode, i.e. a pulse. Understanding the spatio-temporal dynamics of mode-locking in a laser, aiming to identify what it can and cannot achieve, how, and under what conditions, is a complicated and important contemporary problem in laser physics.

We present a detailed experimental study of the spatio-temporal dynamics of a KLM Ti:Sapphire laser. Despite it being very well known that KLM is a spatio-temporal effect that involves losses exclusively due to light diffracting outside of the cavity, it was never actually considered in detail how and where it happens. Kerr-lens mode-locking (KLM) is the major technique for producing ultrafast pulses, enabling the generation of the shortest state-of-the-art pulses [7]. KLM works by creating an effective saturable absorber through the non-linear index of refraction of the Ti:Sapphire, or any other non-linear media inside the cavity [8, 9]. The intensity dependent refractive index \( n(I) = n_0 + n_2I \), together with the spatial profile of the beam generates an effective non-linear lens inside the cavity, whose focal length depends on the instantaneous beam radius \( w(t) \) and beam power \( P(t) \) at the
FIG. 1. In KLM, the cavity is operated at the point where CW operations is unstable. For high-intensity pulses, the non-linear medium acts as an effective non-linear lens, making the cavity stable under mode-locked operation.

Kerr medium according to

\[ \frac{1}{f_{nl}} = \kappa_{nl} \frac{P(t)}{w^4(t)}, \]  

where \( \kappa_{nl} \) is the non-linear response of the Kerr medium. This additional instantaneous non-linear lensing can mitigate diffraction losses for high intensities, acting as an effective ultrafast saturable absorber [10]. The virtually instantaneous nature of the Kerr-lens is key to achieve extremely short pulses, that are otherwise impossible. While the core function of the Kerr-lens is the same as a saturable absorber, clearly no actual absorption takes place. KLM is an inherently spatial effect and losses are entirely due to diffraction, indicating that light must actively leak outside the cavity. In fact, at the advent of KLM, the role of the
Kerr-lens effect and diffraction losses was not understood \cite{11,12}, and in older literature, KLM is called self mode-locking, due to the apparent lack of a visible absorber. Even today, the fully coupled spatio-temporal dynamics of KLM is not fully understood or modeled, and the effectiveness of the saturable absorber is in many cases approximated with a description in terms of Ginzburg-Landau equations \cite{13}, where the spatial nature of KLM is neglected, or analyzed in isolation from the temporal behavior of the pulse in a cavity stability analysis \cite{14,15}.

Since light is not absorbed in KLM, only diffracted out, it can be directly measured to reveal the spatio-temporal behavior of the pulse. In this work, we calculate, simulate and directly observe the diffraction losses in KLM. We use a novel theoretical approach to numerically simulates the KLM dynamics both in time and space, taking into account the interplay between the spatial and temporal parts, which is usually neglected in the analysis of KLM. In order to make the calculation tractable, we approximate the spatial mode of the laser to be a single transverse Gaussian $TEM_{00}$ mode, whose waist is intensity dependent and varies in time with the pulse. Under the Gaussian approximation, the spatial part can be fully represented by a time-dependent complex beam parameter $q_n(t)$ that is

**FIG. 2.**

**Top:** Calculated temporal profile of the main pulse (blue) and of the leaking light (red). Since the light leaks at the edges of the main pulse, the leaking light appears as a double pulse. **Bottom:** Calculated instantaneous beam radius $w(t)$ on the end mirror, showing the strong time dependence of the spot-size.
propagated through the cavity by standard linear ABCD propagation with a time-dependent propagation matrix, with time-dependence due to the Kerr-lens effect \cite{16}. Our analysis incorporates both the temporal and spatial evolution of the intra-cavity field. The temporal profile of the field $E_n(t)$ is divided into two time-scales - the fast time within a round trip $t$ and the round-trip time $\tau_n$. The evolution of the field is then calculated in a split-step Fourier method \cite{17}, where dispersion and linear gain/loss are applied in frequency-domain, while saturation and the Kerr-effect are applied in the time-domain, where they are more naturally described. Our reduced model successfully captures the dynamics of KLM, and allows us to go beyond the steady-state analysis to observe the real-time dynamical behavior of the oscillation. A more detailed description of our theory and simulation will be reported in a future publication. Our analysis yields the instantaneous relation between the pulse power and the beam radius, as well as the temporal profile of the leaking light and the pulse, as shown in figures 2 and 4.

In our experiment, we measured the diffraction losses due to light leaking outside of the cavity, well separated from the main pulse as explained hereon. We employ a standard Ti:Sapphire oscillator in an X-folded four-mirror configuration \cite{10}. Our gain medium is a 3mm Ti:Sapphire crystal located at a cavity focal point. The Ti:Sapphire crystal also provides the non-linearity for the Kerr-lens effect. Our cavity comprises of two spherical mirrors with focal length $f = 7.5$cm, two planar end-mirrors, and a prism pair (BK7 glass) to compensate for intra-cavity group delay dispersion (GDD). Since the laser beam passes near the edge of the prisms, the prism pair also acts as an effective slit for the beam that allows to measure the light that leaks out of the cavity due to diffraction, as shown in 3. The distance between the mirrors is offset by $\delta$ from a perfect telescope, corresponding to different stable (or unstable) ray configurations, with our cavity operated just beyond the cavity stability zone, which induces substantial diffraction losses for CW operation, as illustrated in figure 1. Beyond the stability zone, continuous-wave (CW) operation is unstable and suffers diffraction losses and the cavity stability becomes extremely susceptible to changes in the optical power inside the cavity due to the Kerr-lens effect. Consequently, the Kerr-lens counteracts the diffraction losses and pushes the cavity back into stable operation, leading to differences in the loss between high and low intensity operation, inducing the saturable diffraction losses that act as an effective fast saturable absorber. As clear from that description, KLM is a spatio-temporal effect - the instantaneous spatial and temporal
profiles are closely linked and directly affect one another. Explicitly in our configuration, when the instantaneous intensity is high, the non-linear lens is strong, resulting in a smaller beam waist radius on the prisms and no leakage out of the cavity.

As mentioned above, the first prism acts as a strong aperture - when the beam radius is large, some of the light will miss the prism and diffract out, but when it is small, the beam will be diffracted entirely into the other prism and be fully contained within the cavity. This allows for effective isolation of the diffraction losses from the main pulse. The dynamics of the Kerr-lens is clearly captured by the spatio-temporal profile of the leakage pulse, shown in figure 2. At peak intensity the beam size is small and barely any light leaks out of the prism, whereas at the leading and trailing edges of the pulse, the beam size is big and a fraction of the light leaks out of the cavity, but since the overall power is rapidly decreasing, not much is leaked from the cavity. However, between the peak and the edges there are
substantial diffraction losses that still carry power - both the power and beam size can be appreciable, and measurable power leaks outside. Consequently, the light that leaks outside of the cavity should have the shape of a double pulse, as shown in figure 4. Treating the stable beam radius as a circular aperture with radius $w_{\text{min}}$, the instantaneous leakage power is related to the main pulse power by

$$P_{\text{leak}}(t) = P(t) \left( \frac{w^2(t) - w_{\text{min}}^2}{w_{\text{min}}^2} \right).$$

(2)

In words, the power leakage from the cavity at a time $t$ corresponds to the fractional area of the beam outside of the stable beam area (at the peak of the pulse when the Kerr-lens is effective).

FIG. 4. Theory (left) vs. measured (right) light leakage out of the cavity in both time (top - auto-correlation) and frequency (bottom - spectrum). The experimental results show very good agreement with the simulated one (left), where the only fit parameter was the roundtrip time of the cavity. The simulation assumed leakage of the intra-cavity field according to equation $2$:

$$E_{\text{leak}}(t) = E(t) \sqrt{\frac{w^2(t) - w_{\text{min}}^2}{w_{\text{min}}^2}}$$

with a minimal radius the matched the experimental radius of the cavity beam on the Kerr medium.

We measured the light leaking from the prisms in both frequency (spectrum) and time (auto-correlation), and compared them to the same measurement of the main pulse out of the output coupler, as shown in figure 4. Indeed, the temporal auto-correlation shows a triple-peaked profile, indicative of a double pulse, as predicted by our theoretical analysis (note
that the experimental auto-correlation profile is somewhat distorted by the slow response at high gain of the auto-correlator’s photo-detector). The spectrum of the leaking pulse matches nicely that of a double pulse, as predicted theoretically as well. We further confirm

![Graph](image)

FIG. 5. Main pulse power measured at the output coupler vs. the leakage power as measured near the first prism. In orange we plot the total power as function of leakage power. The dashed line indicates the total maximal leakage power diffracted out due to the Kerr-lens effect.

that the power leakage we observe is not just standard linear loss of the main pulse, but indeed power leaking out due to the saturable diffraction losses, by observing both the leakage power and the main output power as the prism is shifted further in/out of the beam (see 3). While shifting the prism varies the effective slit size, letting more/less light to leak out of the cavity hardly affects the main output. Figure 5 shows the the main output power as a function of the leakage power, indicating clearly that the power of the main pulse is independent from the leaking light (up to the physical limit of clipping the main beam).

In conclusion, we directly observed and simulated the diffraction losses in a KLM mode-locked laser in the form of the light leaking out a dispersion compensating prism pair. The temporal profile of the leaking light inherently couples the spatial and temporal of the pulse, allowing us a direct glimpse into their mutual dynamics. We simulated these dynamics by a split-Fourier method directly in time, showing good agreement with our experimental results. Our experimental apparatus provides very good isolation between the main pulse and the light lost due to diffraction. We show that the leaking light and the main pulse are distinct, and have very different temporal profiles. The ideas and methods we presented can
be developed as a tool to study the spatio-temporal dynamics of lasers. This opens a new window towards studying the ultrafast spatial dynamics of Kerr-lens mode-locked lasers in particular and spatio-temporal mode-locking in general.

I. DISCLOSURES

Disclosures. The authors declare no conflicts of interest.

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