Effects of moving media optics in GLONASS optical segment of new generation

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Abstract. The study introduces a system of equations describing the propagation of light rays in laser location of a moving heterogeneous medium. A recurrent system of equations is obtained for the algorithm of modeling ray trajectories in a moving space microsatellite in the form of a two-dimensional Luneberg lens. A numerical calculation and analysis of the interference optical response of the moving Luneberg lens on the examples of BLITS and BLITS-M satellites is carried out. The results obtained indicate a complex influence of the effects of the moving media optics (Doppler and Fizeau effects, Snell's law violation, dispersion phenomenon, etc.) on the spatial and temporal characteristics of the reflected interference signal on the Earth's surface, namely on the signal-to-noise ratio in the reception area.

1. Introduction
The rapid development of global satellite positioning systems is approaching the millimeter limit of measurement accuracy. A promising way to solve this problem can be the use of modern laser ranging systems, as well as the launch of reference low-orbit passive laser satellites [1].
The study [2] shows that a spherical gradient Luneberg lens can be used as a reflecting satellite. Reflectors made on the basis of gradient lenses are a good alternative to cubic angle reflectors [3]. The main property of such reflectors is the spherical aberration compensation. Calculations of miniature spherical reflectors based on the Luneberg lens [4-7] indicate a good restoring capability for a large range of horizontal viewing angles in the medium and long wavelength range of the infrared emission spectrum.
The optical response of BLITZ satellite, which is a Luneberg lens in design, is comparable with the response of a point source, and can be used to improve the accuracy of determining the coordinates in GLONASS system.
However, the optical response of a moving lens has a complex spatial structure and depends on the speed of its movement. Effects of the moving media optics affect the reflected signal of the moving lens, the accuracy of determining its orbit, which in its turn can affect the reliability of satellite laser ranging system as a whole [8].
Consequently, there arises a necessity for simulating the processes of forming the interference optical response on the earth's surface, and also for calculating the intensity distribution of the reflected coherent radiation. The mathematical model must take into account:
1) effects of the moving media optics (the Doppler effect, Fizeau effect, violation of Snell's law, dispersion phenomenon);
2) kinematic effects associated with the lens displacement during the propagation of electromagnetic radiation in it;
3) characteristic features of time formation of a multipath interference response. It should be noted that in the general case the ray path bending effect in a rotating lens [9], as well as the Fermi effect [10], influences the solution of the problem. However, due to low rotation speeds, these effects can be neglected in the first approximation. This task, apart from practical, is of fundamental importance. The reference optical satellite, orbiting in the optical GLONASS segment, provides unique opportunities for experimental verification of the equations of moving media electrodynamics.

2. Refraction on a moving media interface

The formation of the reflected interference response depends on the parameters of the laser radiation passing through the Luneberg lens at all points of intersection of the ray with the optical surfaces of the two-layer lens. Since the surfaces move relative to the Earth, it is necessary to use the solution of the problem of the fall of a flat monochromatic wave onto the moving media interface [11].

![Figure 1. An electromagnetic wave with a wave vector $\vec{k}_{1,1}$ falls on the two moving media interface $S$, where media are moving at random velocities $\vec{u}_i$ and the interface is moving at normal velocity $\vec{v}$.](image)

The boundary conditions for all waves in the first and second medium are the phase equalities at the two media interface. According to this requirement, phase invariants are introduced:

$$\vec{k}_{1,1} = \vec{k}_{1,2} = \vec{k}_{2,1} = \vec{k}_{2,2} = \hat{I}_i$$

$$I_i = k_0 \sin \theta_0$$

$$\omega_{i,1} - \vec{k}_{1,1a} v = \omega_{i,2} - \vec{k}_{1,2a} v = \omega_{i,2,2} - \vec{k}_{2,2a} v = I_i,$$

where $\vec{k}_{ai}$ and $\vec{k}_{ai}$ are the tangential and normal components of the wave vector with respect to the surface $S$ at the point of intersection.

To find the frequencies and wave vectors in media, dispersion equations are used for each medium [2]:
where \( \vec{k}_i \) is the wave vector of an electromagnetic wave in the \( i \)-th medium, \( \vec{u}_i \) is the velocity of the \( i \)-th layer of the medium, \( \kappa_i = \varepsilon_i\mu_i - 1 \), \( \beta_i = \frac{\vec{u}_i}{c} \), \( i = 1, 2 \).

The solution of this equation for the spectral source of the incident radiation far from the medium absorption region looks as follows

\[
I_{i1} = \frac{1 + \kappa_i \gamma_i^2 (\beta - \beta_m)(\beta_m - \vec{d} \vec{\beta}_m \beta) \pm \beta Q_i^{1/2}}{1 - \beta^2 - \kappa_i \gamma_i^2 (\beta - \beta_m)^2} \tag{4}
\]

\[
Q_i = 1 + \kappa_i \gamma_i^2 (\beta - \beta_m^2) - d^2 \left[ 1 - \beta^2 - \kappa_i \gamma_i^2 (\beta - \beta_m)^2 \right] - \kappa_i \gamma_i^2 \vec{d} \vec{\beta}_m \left[ 2(1 - \beta_m \beta) - (1 - \beta^2) d \vec{\beta}_m \right],
\]

\[
\vec{d} = -\frac{c \vec{I}_i}{I_1}, \quad \beta = \frac{v}{c}, \quad \vec{\beta}_m = \frac{\vec{u}_m}{c}, \quad \vec{\beta}_a = \frac{\vec{u}_a}{c}, \quad \gamma_i^2 = 1 - \beta_i^2,
\]

where the sign \( \pm \) and indices 1,2 correspond to the refracted and reflected wave respectively, \( \vec{u}_a \) and \( \vec{u}_m \) are the tangential and normal components of the medium velocity with respect to the surface \( S \) at the point of intersection.

The phase invariant \( I_i \) specifies the tangential components of the wave vectors in two media. For the normal component, substituting (4) into (2), we obtain

\[
(k_m)_{i2} = \frac{1}{c} \frac{\beta + \kappa_i \gamma_i^2 (\beta - \beta_m)(1 - \vec{d} \vec{\beta}_a \beta) \pm \beta Q_i^{1/2}}{1 - \beta^2 - \kappa_i \gamma_i^2 (\beta - \beta_m)^2}. \tag{5}
\]

The refraction angle of an electromagnetic wave \( \theta_i \) at the media interface is given by

\[
\theta_i(x,y) = \arctg \left( \frac{\cos \theta_0}{c \kappa_m(x,y)} \right). \tag{6}
\]

Differential equations governing the evolution of the wave vector coordinates in a medium with a complex two-dimensional motion will have the following form:

\[
\frac{dx}{dt} = \frac{c}{n_i} \frac{1}{\sqrt{1 + \tan^2 \theta_i(x,y)}}, \quad \frac{dy}{dt} = \frac{c}{n_i} \frac{\tan \theta_i(x,y)}{\sqrt{1 + \tan^2 \theta_i(x,y)}}. \tag{7}
\]

Thus, the system of equations (5) - (7) makes it possible to calculate the radiation parameters at the points of intersection of the ray and the moving media interface along the path.

### 3. Dispersion of an optically transparent medium

The expression (3) includes the refractive index \( n = \sqrt{\mu \varepsilon} \), which in the general case is not a constant.

The medium parameters can depend on the parameters of the radiation passing through it; therefore, in order to take into account the dispersion, it is necessary to recalculate the refractive index before each media interface. In the case of a moving two-layer Luneberg lens it is important, because on each surface the Doppler effect will be observed.
As the dependence of the refractive index on the wavelength of radiation, we choose Schott’s approximation formula [12]:

\[ n = \sqrt{A_1 + A_2 \lambda^{-2} + A_3 \lambda^{-4} + A_4 \lambda^{-6} + A_5 \lambda^{-8}}. \]  

(8)

Substituting (8) into (3), we obtain a 12th degree equation with respect to frequency, which can be solved by a numerical method:

\[
-\frac{1}{256c^8(c^2-u_i^2)\pi^8} \left( A_6 \omega^{12} - 2A_6 \omega^{11}P + \left( 4A_6 \pi^2 c^2 + A_6 P^2 \right) \omega^{10} - 8A_6 \omega^9 \pi^2 c^2 P + \\
\left( 16A_4 \pi^4 c^4 + 4A_4 \pi^2 c^2 P^2 \right) \omega^8 - 32A_4 \omega^7 \pi^4 c^4 P + \left( 16A_4 \pi^4 c^4 P^2 + 64A_4 \pi^6 c^6 \right) \omega^6 - 128A_4 \omega^5 \pi^6 c^6 P + \\
\left( 64A_6 \pi^6 c^8 P^2 - 256\pi^8 c^8 P^2 + 256A_6 \pi^8 c^8 \right) \omega^4 + \left( -512A_6 \pi^8 c^8 P + 512\pi^8 c^8 P \right) \omega^3 + \\
\left( 256k^2 u_i^8 \pi^8 c^8 - 256\pi^8 c^8 P^2 + 256A_6 \pi^8 c^8 P^2 - 256k^2 \pi^8 c^{10} + 1024A_6 \pi^{10} c^{10} \right) \omega^2 - \\
2048A_6 \pi^{10} c^{10} \omega P + 1024A_6 \pi^{10} c^{10} P^2 \right) = 0,
\]

(9)

where \( P = (\vec{k}, \vec{u}_i) \).

The dispersion effect was calculated in two ways. In the first case, we used a numerical solution of the algebraic equation (9) done by the step-by-step approximation method. The calculations were carried out until the left side of the equation converged to zero with the required degree of accuracy.

In the second case, the solution was found in an iterative way. At the first stage, we solved the dispersion equation (4) for a constant \( n \) and found the frequency in the medium behind the moving interface.

Then, using formula (8), we calculated the new value \( n(\omega) \). For the given new value of the refractive index, we again solved the dispersion equation and repeated the process to ensure convergence (in practice, 2-3 iterations are sufficient).

4. Analytical description of the light ray propagation in an optical system of arbitrary configuration

Let us consider the propagation of coherent electromagnetic radiation in an arbitrary optical system in the geometric approximation. In this case, the flux of electromagnetic radiation can be considered as a beam of light rays characterized by spatial direction, amplitude, and phase. Imagine a light ray as a straight line \( p_i \) incident at a point \( M_i(x_i, y_i, z_i) \) lying on a moving surface of the second order \( P_i \) (Figure 2). The ray comes out from the point \( M_{i-1} \) with coordinates \( x_{i-1}, y_{i-1}, z_{i-1} \).
Figure 2. Propagation of an electromagnetic wave $\vec{k}_i$ in a medium considered as a set of layers, with parameters $\varepsilon_i$, $\mu_i$ and separated by surfaces $P_i$ with normals $N_i$. Each interface between two adjacent media can have a velocity $V_i$. At each media interface, a refracted and a reflected waves appear.

The parametric equations of the straight line $p_{i+1}$ have the following form:

$$x = x_{i+1} + l_{i+1}t, \quad y = y_{i+1} + m_{i+1}t, \quad z = z_{i+1} + n_{i+1}t,$$

where $l_i$, $m_i$, $n_i$ are the components of the ray propagation velocity in the $i$-th medium.

As $P_i$ is the second degree surface, the moving optical surface is governed by the second degree algebraic equation $F_i(x, y, z, t) = 0$:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{44}t^2 + a_{12}xy + a_{13}xz + a_{14}xt + a_{23}yz + a_{24}yt + a_{34}zt + a_{44}zt = 0.$$  \hspace{1cm} (11)

Substituting (10) into (11), we obtain a quadratic equation with respect to time:

$$A_i t^2 + A_i t + C_i = 0,$$  \hspace{1cm} (12)

where

$$A_i = a_{44} + 2a_{14}l_{i+1} + a_{11}l_{i+1}^2 + 2a_{24}m_{i+1} + 2a_{12}l_{i+1}m_{i+1} + a_{22}m_{i+1}^2 + 2a_{34}n_{i+1} + 2a_{13}l_{i+1}n_{i+1} + a_{33}n_{i+1}^2.$$
Then the solution of equation (12) is the time the ray travels to the \( i \)-th surface:

\[
t_i = \frac{-B_i \pm \sqrt{B_i^2 - 4AC_i}}{2A_i}.
\]  

The sign in the expression is determined by the direction of ray propagation. Substituting \( t_i \) into equations (10) we find the coordinates of the point \( M_i(x_i, y_i, z_i) \) of intersection of the line and the plane. Thus, we obtain a recurrent system of equations (10-13), which makes it possible to describe the ray path in an arbitrary moving heterogeneous medium between adjacent media interfaces. Similarly, one can obtain expressions for the incident angle of the ray onto an arbitrary surface [13].

5. Propagation of radiation in the optical microsatellite BLITS

The study [13] describes the ray path in a two-dimensional moving Luneberg lens through the example of BLITS satellite. Fig. 3 shows the image of the central section of the lens by the plane in which the ray propagates in the limit when the velocity of the lens is zero. As the speed of the lens increases, the angle of refraction changes, the ray intersection with the surfaces shifts, the surfaces themselves move along the velocity vector of the lens.
From Figure 3 it follows that the angles $\theta_i$ are the angles of incidence and refraction, and $\varphi_i$, $\theta_i$ are the auxiliary angles.

Kinematic equations of the ray and circle are as follows:

$$
\begin{align*}
\begin{cases}
x_i = x_{i-1} + \frac{c}{n_{1,2}} t \cos (\varphi_{i-1}) \\
y_i = y_{i-1} + \frac{c}{n_{1,2}} t \sin (\varphi_{i-1})
\end{cases}
\end{align*}
$$

(14)

$$
\left( x_k - v_i (t_k + T) \right)^2 + \left( y_k - v_i (t_k + T) \right)^2 = R_{1,2}^2,
$$

(15)

where $k = 1...6$, $T = \sum_{k=1}^{6} t_{k-1}$ is the total time for which the lens center shifted when considering the $k$-th point, $R_{1,2}$ and $n_{1,2}$ are the radii and refractive indices of the outer meniscus and inner ball. The “+” sign is chosen due to the direction of the ray along the Oy axis.

Substituting the ray equation (14) into the circle equation (15), we obtain a quadratic equation in terms of time:
\( A_k t_k^2 + B_k t_k + C_k = 0, \)

where

\[
A_{i-3,4-6} = \left( \frac{c}{n_{i,2}} \sin(\phi_i) - v_y \right)^2 + \left( \frac{c}{n_{i,2}} \cos(\phi_i) - v_x \right)^2
\]

\[
B_{i-3,4-6} = 2\left( T_v + x_{k-1} \right) \left( \frac{c}{n_{i,2}} \cos(\phi_i) - v_x \right) - 2\left( T_v + y_{k-1} \right) \left( v_y \pm \frac{c}{n_{i,2}} \sin(\phi_i) \right)
\]

\[
C = \left( T_v + x_{k-1} \right)^2 + \left( T_v + y_{k-1} \right)^2 - R_{i,2}^2.
\]

Coefficients with indices 1-3 correspond to the rays before reflection, coefficients with indices 4-6 correspond to the rays after reflection (respectively, they choose \( \mp \) and \( \pm \) signs), the coefficient \( C \) is not changed for both cases. Solving the equation (16), we obtain the time \( t_k \) value of the ray path between \( k-1 \) and \( k \)-points. Then we substitute the obtained value \( t_k \) into (14) and obtain the coordinates of the point where the ray intersects the surface.

To find new values of the angles \( \phi_{i-1} \), \( \theta_{i-1} \), it is necessary to know the tangential and normal components of the incident radiation wave vector with respect to the surface at points (0-6). To do this, we obtain the normal to the circle at the points (0-6) by the following formula:

\[
\vec{n} = (2x_i, 2y_i).
\]

Knowing the projection of the wave vector \( \vec{k}_i \) onto the tangent and the normal to the surface at the points (0-6), we can use the phase invariants (1), (2) and the solution of the dispersion equation (5) to find the new wave vector \( \vec{k}_{i+1} \). New angles \( \phi_{i+1}, \theta_{i+1} \) can be determined with the help of the scalar product \((\vec{k}_{i+1},\vec{n})\) and \((\vec{k}_{i+1},\vec{h})\), where \( \vec{h} \) is the horizontal \( \vec{h} = (1,0) \).

6. Calculation of the interference response in the registration plane

To obtain the ray deviation value on the earth's surface, let us find the angle \( \theta \) between the refracted ray at point 6 - \( \vec{k}_7 \) and the horizontal \( \vec{h} \). Then the ray deviation value on the earth's surface will be calculated by the formula:

\[
x = y_6 \cot(\theta) + x_6.
\]

In this case, we assume that the center of the circles is taken as the zero along the Ox axis at the moment the ray intersects the first surface.

For the phase of the ray on the earth's surface, we can write:

\[
\Phi = \sum_{i=0}^{n} w_i t_i - w_c A t - \sum_{i=0}^{n} k_i A r_i,
\]

where \( w_i, t_i, k_i, A r_i \) are respectively, the angular frequency, the ray propagation time, the wave vector modulus and the length of the geometric path in the \( i \) section of the trajectory, \( i = (0...n) \) is the number of the optical path segment in the optical system under study between the media interfaces, \( A t = t_{def} - \sum_{i=0}^{n} t_i \) is the difference in the time of passage of the reference \( t_{def} \) and calculated rays.

The intensity distribution in the observation plane of the interference pattern depends on the number of rays from the total number \( N \) falling into each coordinate interval \( Ax \).
\[ I(x) = I_0 \left( \sum_{j=1}^{N} \sin \theta_j(x) \right)^2, \]  

(20)

where \( I_0 \) is the value of the intensity of one ray, which in practice has the value of the order of \( 10^{-6} \) W/m².

7. The results of the numerical experiment for BLITZ and BLITZ-M satellites
The calculations were done for the parameters of BLITZ and BLITZ-M satellites of identical design (Table 1).

| Parameters | BLITZ | BLITZ-M |
|------------|-------|---------|
| Radius of the central ball \( R_1 \), mm | 53.5 | 63.9 |
| Radius of the outer meniscus \( R_2 \), mm | 85 | 110.4 |
| Material of the central ball | LK106 | K108 |
| Material of the external meniscus | TF105 | TF105 |
| Orbit \( h \), km | 835 | 1500 |
| Linear velocity in orbit \( V \), m/s | 7500 | 7100 |

The intensity distribution graphs are plotted for the rays \( N = 20000 \) with incidence angles \( \theta_0 = 0.7^\circ \) to the left and right of the lens axis (Figure 3). The coordinate interval \( \Delta x = 1 \) m was chosen for the summation of the phases (18).

The calculations used a two-dimensional approximation, so all the incident and reflected rays were in the same plane.

Figure 4 shows a graph of the intensity distribution on the earth's surface for a stationary satellite BLITZ. There are 3 intensity peaks on the graph. One is in the reception area (\( X = 0 \) m) and the two others are outside it (\( X = \pm 74 \) m).

![Figure 4. Dependence of the intensity \( I(x) \) of the reflected radiation from the coordinate on the earth's surface at the satellite BLITZ velocity \( V = 0 \) m/s.](image)
Figure 5 shows a similar graph at the satellite velocity $V = 7500 \text{ m/s}$. It is noteworthy that the lateral intensity peaks shifted to the sides and markedly decreased in intensity.

![Intensity distribution graph](image)

**Figure 5.** - Intensity distribution at the satellite BLITZ velocity $V = 7500 \text{ m/s}$.

To analyze the dependence of the coordinates of the peak intensity value on the satellite velocity, we introduce the value $x_m$, i.e. the coordinates of the peak intensity of the reflected radiation. The dependency graph of the coordinate $x_m(V)$ on the satellite velocity $V$ is shown in Figure 6.

![Dependency graph](image)

**Figure 6.** Dependency graph of the coordinate $x_m$ of the peak intensity of the satellite BLITZ velocity.

To achieve the peak intensity in the reception area, one can change the geometric characteristics of the optical system. Below are the results of calculations $x_m$ at different radii of the outer meniscus of satellite $R$ (Figure 7).
Figure 7. Dependence of the coordinate $x_m$ of the peak intensity on the change in the radius of the meniscus $dR$.

As a result, for $x_m = 0$ we obtain $dR = 0.66$ mm. The radiation intensity distribution graph for the radius of the external meniscus $85.66$ mm is shown in Figure 8.

Figure 8. Intensity distribution at the satellite velocity $V = 7500$ m/s and the radius of the outer meniscus $R_i = 85.66$ mm.

The change in the geometric characteristics of the satellite significantly affects the intensity distribution. From the above graph it follows that a change in the radius of the outer meniscus allows us to optimize the distribution and amplify the signal in the reception area. Intensity in $X = 0$ increased by more than 6 times. Similar results were found for BLITS-M, which has better design parameters. Therefore, calculations showed that without taking into account the velocity, i.e. at $V = 0$ m/s, the main intensity falls on a relatively small area (Figure 9).
Figure 9. Dependence of radiation intensity $I(X)$ on the coordinate on the earth's surface at the satellite velocity $V = 0$ m/s.

However, with the motion of satellite BLITZ-M, the intensity distribution also undergoes a change (Figure 10).

Figure 10. Intensity distribution at the satellite velocity $V = 7100$ m/s a) in the interval $[-300, 300]$ m
b) in the interval $[-20, 20]$ m.

Calculations show that a change in the radius of the outer meniscus makes it possible to correct the interference pattern and to achieve an increase in intensity in the reception area by 2.5 times. To control the accuracy of numerical experiments, calculations were performed with a different subinterval of the reception area.

When the subinterval was changed from $\Delta x = 1$ m to $\Delta x = 0.1$ m to calculate the interference response, the intensity in the peaks changed by no more than $\Delta I = 3 \cdot 10^{-3}$ W/m$^2$, which is much less than the measured values of the intensities. Moreover, the results of calculations were verified using the Zemax software package at the satellites velocity $V = 0$ m/s.
8. Conclusion
The study investigated the effect of the moving media optics on the moving Luneberg lenslike microsatellite.
A recurrent system of equations is obtained that allows constructing an algorithm for simulating the motion of light rays in a moving Luneberg lens with allowance for dispersion.
As a result of numerical simulation, we obtained the interference response intensity distributions for BLITS and BLITS-M satellites in the registration plane on the earth's surface at the satellites velocity $V = 7500$ m/s and $V = 7100$ m/s, respectively.
The study shows that taking into account the optical satellite velocity at simulation of the interference response leads to the appearance of lateral peaks, which in intensity can exceed the central peak.
This effect is the result of the complex influence of violation of Snell's law, the longitudinal and transverse Fizeau effect, the Doppler effect, the kinematic effect of the satellite offset during the propagation of radiation in it, and the dispersion phenomenon.
Findings of the research show that the phenomena under investigation can have a significant effect on the signal-to-noise ratio in the new GLONASS optical segment. At the same time, the results can be applied to adjust the geometric characteristics of the satellite in order to obtain peak intensity in the reception area.
In general, it can be concluded that the Luneberg cosmic lens is an exceptional experimental laboratory for studying the effects of moving media electrodynamics in the optical spectrum.

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