Bridging the gap between rectifying developables and tangent developables: a family of developable surfaces associated with a space curve

Brian Seguin, Yi-chao Chen and Eliot Fried

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Original submission: 30 July 2020
Revised submission: 12 January 2021
Final acceptance: 13 January 2021

Note: Reports are unedited and appear as submitted by the referee. The review history appears in chronological order.

Review History

RSPA-2020-0617.R0 (Original submission)

Review form: Referee 1

Is the manuscript an original and important contribution to its field?
Good

Is the paper of sufficient general interest?
Good

Is the overall quality of the paper suitable?
Good

Can the paper be shortened without overall detriment to the main message?
Yes

Do you think some of the material would be more appropriate as an electronic appendix?
Yes

Do you have any ethical concerns with this paper?
No
Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
From a space curve, there are two familiar constructions of a developable surface. The tangent developable whose rulings are tangent to the curve at each point and relative to the surface the absolute value of geodesic curvature of the curve is equal to the curvature of the curve. The other construction is the rectifying developable. The geodesic curvature of the curve relative to this surface vanishes. The authors show that there is a family of developable surfaces that can be generated from a curve, one surface for each function $k$ that is defined on the curve and satisfies $|k| < \kappa$, and that the geodesic curvature of the curve relative to each such constructed surface satisfies $k_{gsk}$. The theoretical background is interesting. If the authors explain the equations (2.12) and (2.13) in detail or explain them by pictures, I think it will be more acceptable.

Review form: Referee 2

Is the manuscript an original and important contribution to its field?
Good

Is the paper of sufficient general interest?
Good

Is the overall quality of the paper suitable?
Good

Can the paper be shortened without overall detriment to the main message?
Yes

Do you think some of the material would be more appropriate as an electronic appendix?
No

Do you have any ethical concerns with this paper?
No

Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
I recommend the paper "Bridging the gap between rectifying developables and tangent developables: a family of developable surfaces associated with a space curve" by S. Brian, Y.-C. Chen and E. Fried for publication after considering my comments below.

---Please consider to include two other very recent papers on developable surfaces:

O. Stein, E. Grinspun and K. Crane:
Developability of Triangle Meshes.
ACM Trans. Graph, 37(4), 2018.

M. Rabinovich, T. Hoffmann and O. Sorkine-Hornung:
The Shape Space of Discrete Orthogonal Geodesic Nets,
ACM Trans. Graph, 37(6), 2018.
---The authors use the notion "pointwise vanishing ..." throughout the paper. I would appreciate if you could rephrase it. For example, I would say that minimal surfaces have vanishing mean curvature and I would not say that minimal surfaces have *pointwise* vanishing mean curvature.

---In "classical" differential geometry, a curve, parametrized by $d$, is *regular* iff $d'$ is not vanishing. However, the authors introduce the notion of "piecewise regular" (line 40) by $d'' = 0$ only on a finite number of disjoint subintervals. That is a bit confusing notion. Can you change that?

---Line 49: I think you are talking about a ruled surface with vanishing mean curvature. However, such a surface does not necessarily have vanishing Gaussian curvature as the helicoid is a counterexample. And therefore, $S$ is not necessarily flat. Please reconsider that statement.

---Why does Struik (1961) not get a number in the Reference section?

Review form: Referee 3

Is the manuscript an original and important contribution to its field?
Excellent

Is the paper of sufficient general interest?
Good

Is the overall quality of the paper suitable?
Excellent

Can the paper be shortened without overall detriment to the main message?
Yes

Do you think some of the material would be more appropriate as an electronic appendix?
No

Do you have any ethical concerns with this paper?
Yes

Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
This is a positive review of the article entitled
Bridging the gap between rectifying developables and tangent developables: a family of developable surfaces
associated with a space curve written by Seguin, Chen and Fried.

Publication in Proc. Royal Soc. A is recommended without reservation.

This submission demonstrates the existence of a continuum of developables built from a single space curve, including the tangent developable and rectifying developable as outliers within the collection. It appears to be an infinite dimensional family, as it is parametrized by a family of functions.

There is a thorough explanation of historical context and applications, which indicates why the authors are interested in fixing the initial curve for constructing the surfaces, and why the curve
is allowed to have intervals of zero curvature, and why the surfaces are not assumed to be $C\infty$ smooth. Ambiguity for the Frenet frame results at points where the initial curve has curvature zero. Mathematicians generally resolve this with specific choices of frame (equivalently, ribbons) along the curve. The present work takes an approach that is different in spirit -- the issue is resolved by separating out the intervals on which the curvature is nonzero, and then connecting frames for those intervals by jumping along the parts where the curvature is zero.

This work is creative, interesting and a significant contribution.

This report concludes with two extremely minor suggestions for changes in the final published form:

In the main theorem starting at the bottom of page 7:

1) the first line has a misspelling "arclengh" $\longrightarrow$ "arclength"

2) the third line refers to a "script K", and perhaps it should be mentioned within the theorem that this "script K" was defined in equation (2.5)

Publication of this submission is fully recommended.

Decision letter (RSPA-2020-0617.R0)

22-Dec-2020

Dear Dr Seguin,

On behalf of the Editor, I am pleased to inform you that your Manuscript RSPA-2020-0617 entitled "Bridging the gap between rectifying developables and tangent developables: a family of developable surfaces associated with a space curve" has been accepted for publication subject to minor revisions in Proceedings A. Please find the referees' comments below.

The reviewer(s) have recommended publication, but also suggest some minor revisions to your manuscript. Therefore, I invite you to respond to the reviewer(s)' comments and revise your manuscript. Please note that we have a strict upper limit of 28 pages for each paper. Please endeavour to incorporate any revisions while keeping the paper within journal limits. Please note that page charges are made on all papers longer than 20 pages. If you cannot pay these charges you must reduce your paper to 20 pages before submitting your revision. Your paper has been ESTIMATED to be 13 pages. We cannot proceed with typesetting your paper without your agreement to meet page charges in full should the paper exceed 20 pages when typeset. If you have any questions, please do get in touch.

It is a condition of publication that you submit the revised version of your manuscript by 4th January 2021. If you do not think you will be able to meet this date please let me know in advance of the due date.

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You will be unable to make your revisions on the originally submitted version of the manuscript. Instead, revise your manuscript and upload a new version through your Author Centre.

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- Acknowledgements
- Funding statement

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Professor Achim Kempf
Board Member
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Reviewer(s)' Comments to Author:

Referee: 1

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Publication of this submission is fully recommended.

Board member pre-assessment comments (if available):

Board Member: 1
Comments to Author(s):
This paper should be acceptable after the referees' comments have been suitable addressed.

Decision letter (RSPA-2020-0617.R1)

13-Jan-2021

Dear Dr Seguin

I am pleased to inform you that your manuscript entitled "Bridging the gap between rectifying developables and tangent developables: a family of developable surfaces associated with a space curve" has been accepted in its final form for publication in Proceedings A.

Our Production Office will be in contact with you in due course. You can expect to receive a proof of your article soon. Please contact the office to let us know if you are likely to be away from e-mail in the near future. If you do not notify us and comments are not received within 5 days of sending the proof, we may publish the paper as it stands.

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On behalf of the Editor of Proceedings A, we look forward to your continued contributions to the Journal.

Sincerely,
Raminder Shergill
proceedingsa@royalsociety.org