Terahertz emission from asymmetric, doped quantum wells under resonant pumping

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Abstract. When the intersubband transitions of a doped quantum well are strongly driven by a coherent optical pump, the bare energy levels are split into dressed doublets, a phenomenon known as the ac Stark (or Autler-Townes) effect. In time domain this can be understood as Rabi oscillations of the confined electrons between two subbands. Here we show how, by employing asymmetric quantum wells, the selection rule preventing emission from an upper to a lower state of the same doublet can be lifted. The fluorescence spectrum of such strongly-driven system is then marked by an additional peak that would appear at the Rabi frequency, in the terahertz portion of the electromagnetic spectrum.

1. Introduction
In the terahertz THz region of the electromagnetic spectrum the increasing demand for sources of radiation is challenged by a lack of compact, tunable and efficient devices— the so-called “THz gap”– while potential applications span several fields, from medical imaging to gas detection and product-quality control[1]. Here we illustrate how THz light with tunable frequency could be obtained by strongly driving with a coherent optical pump the intersubband transitions of a doped quantum well (QW)[2]. Asymmetry in the emitters wavefunction, has recently been explored also in other schemes[3 4 5 6 7 8].

2. Driven Dynamics
In a 2DEG confined in the x-y plane, the bulk electronic bands are split into parabolic subbands, as shown in Figure (1) (a). Doping of the QW can be used to raise the Fermi energy, \( \hbar \omega(k_F) \), where \( k_F \) is the Fermi wavevector, up to partially filling the first conduction subband. The ground state of the electronic system is then given by \( |G\rangle = \bigotimes_{|k|<k_F} c_{1,k}^{\dagger} |0_k\rangle \), where \( c_{j,k}^{(l)} \) destroys (creates) an electron in the \( j \)-th subband, \( j = \{1, 2\} \), with in-plane momentum \( k \), the standard fermionic commutation relations holding, \( \{c_\alpha, c_\beta^{\dagger}\} = \delta_{\alpha,\beta} \), and where \( |0_k\rangle \) is the vacuum state. The Hamiltonian of the free system is given by

\[
H_0 = \hbar \sum_k \omega_{1,k} c_{1,k}^{\dagger} c_{1,k} + \omega_{2,k} c_{2,k}^{\dagger} c_{2,k}
\]

and the optically active transition is the intersubband transition (ISBT) between the first and the second conduction subband occurring at the frequency \( \omega_{12} = \omega_{2,k} - \omega_{1,k} \). If a coherent
Figure 1. (a) Dispersion in $k$-space of the two conduction subbands of the doped QW. (b) Dipole transitions at the Rabi frequency $\Omega$ are allowed in a broken-symmetry two-level system (thick arrows). (c) Asymmetric QW structure and wavefunctions, with the asymmetric dipole $e(z_{22} - z_{11})$ highlighted; $L_a + L_b + L_c = L_{QW}$, $V_0$ is the central barrier potential energy.

optical pump of frequency $\omega_L$ shines on the system, it can drive the ISBTs. The Hamiltonian in the rotating wave approximation thus reads

$$H = H_0 + \frac{\hbar \Omega}{2} \sum_k e^{-i\omega_L t} c_{2,k+q'} c_{1,k} + e^{i\omega_L t} c_{1,k} c_{2,k+q'},$$

(2)

where $\Omega$, known as Rabi frequency, is the pump-ISBT coupling strength, that depends linearly upon the pump amplitude; the transitions are tilted by the in-plane photonic wavevector contribution $q'$, which is fixed by the pump. Given a state $|\varphi\rangle$ subject to the action of $H$, we can define $|\varphi\rangle \rightarrow U|\varphi\rangle$, where $U$ is a unitary transformation, that satisfies the Schrödinger equation with respect to the new Hamiltonian $H'$, $H' = UHU^\dagger + i\hbar U^\dagger U$. By choosing $U = e^{iH_0 t/\hbar}$, the Hamiltonian in Eq. (2) at resonance $\omega_L = \omega_{12}$ becomes

$$H' = \frac{\hbar \Omega}{2} \sum_k (c_{2,k+q'}^\dagger c_{1,k} + c_{1,k}^\dagger c_{2,k+q'}).$$

(3)

In this rotating frame $H'$ has become completely time independent and, as all transitions now occur between resonant states, the free energies have disappeared. Indeed, we approximated $\omega_{1,k+q} \simeq \omega_{1,k}$, which is a reasonable assumption since the photonic momentum is much smaller than the electronic one. Under strong pumping $\Omega > \gamma$, with $\gamma$ being the subband line width, the system becomes dressed, the bare energy levels of the two subbands being split by $\hbar \Omega$ into two doublets, as shown in Fig. (1) (b).

3. Interaction Hamiltonian

Incoherent photon emission can scatter electrons between states belonging to different Rabi doublets, as shown in Fig. (1) (b) by the dashed arrows. Under strong pumping, this effect gives rise to the “Mollow triplet” in the frequency fluorescence spectrum. The effect can be described considering a two-level system, $|\varphi\rangle = \{|\varphi_1\rangle, |\varphi_2\rangle\}$, interacting with the vacuum electric field

$$V = -d \cdot E,$$

(4)
where \( \mathbf{d} \) is the dipole moment and \( \mathbf{E} \) is the vacuum electric field. In symmetrical systems, only the off-diagonal terms \( V_{ij} = \langle \varphi_i | V | \varphi_j \rangle \) with \( i \neq j \) are non-zero. Recently however, it has been pointed out that the development in controlling the design of quantum emitters can be used to engineer asymmetric systems, where the parity of the eigenfunction is not well defined\[4\]. Then the optical transitions between the upper and lower state of each doublet are no more forbidden \( \langle \varphi_i | V | \varphi_i \rangle \neq 0 \), as shown in Figure (1) (b) (thick arrows). The dipole operator \( \mathbf{d} = e \mathbf{r} \), where \( e \) is the electron charge, and \( \mathbf{r} = (x, z) \), can be written in second quantisation by projecting it on the electron states in the two subbands

\[
\mathbf{r} = \sum_{k,k'} \sum_{j,j'} f_{j,k}^{j',k'} e_{j,k}^{c_{j,k}^{c_{j',k'}}.} \tag{5}\]

The dipole matrix elements are determined by the in-plane symmetry of the system. The spatial wavefunction of an electron in the \( j \)-th subband is \( \varphi_j (x, z) = e^{i k x} \psi_j (z) \) and thus

\[
\mathbf{x}_j = \mathbf{x}_k \delta_{j,j'} \quad \mathbf{z}_j = \mathbf{z}_k \delta (k - k') \tag{6}\]

with

\[
\mathbf{x}_k = \int e^{i(k - k') \cdot x} \mathbf{x} dx \quad \mathbf{z} = \int \psi^* (z) \mathbf{z} \psi_j (z) dz. \tag{7}\]

From Eq. (6) we can see that only the \( z \) component of the field can see the asymmetry, and as such, as usual for ISBTs, we can consider just a transverse magnetic (TM) polarised field, whose \( z \) component is given by

\[
\mathbf{E}_z = \sum_{q,q_z} E_0 (q, q_z) e^{i q \cdot x} (a_{q_z}^{\dagger} + a_{q_z}) \sin \theta e_z, \tag{8}\]

where \( e_z \) is a unit vector, \( \theta \) is the angle with the \( z \)-axis, \( E_0 = (\hbar \omega_{q_z} / 2c_0 e, V)^{1/2} \) is the field amplitude, \( \omega_{q_z} = c \sqrt{q_z^2 + q_z^2} \), where \( c \) is the speed of light, \( V \) is the quantisation volume, and \( c_0 \) and \( \epsilon_r \) are the absolute and relative permittivities. The interaction of Eq. (4), in the same rotating frame used for Eq. (3) is then rewritten using Eq. (5), Eq. (8), and using the \( z \) component in Eq. (7) as

\[
V' = - \sum_{k,q,q_z} \lambda_{12,q,q_z} (e^{i q_z t} c_{2,k+q}^\dagger c_{1,k} + e^{-i q_z t} c_{1,k+q}^\dagger c_{2,k-q}) + \sum_{j=\{1,2\}} \lambda_{jj,q,q_z} (c_{j,k+q} c_{j,k}) \tag{9}\]

where \( [a, a^\dagger] = \delta_{a,\beta} \) and \( \lambda_{ij,q,q_z} = c z_{ij} E_0 (q, q_z) \sin \theta. \)

4. THz emission rates

The rate of photon emission per unit time, up to first order in perturbation, is given by Fermi’s golden rule, which depends upon the matrix element of \( V' \) in Eq. (9). The diagonalisation of Eq. (3) and the derivation of the different scattering processes leading to THz emission can be found in Ref. [2]. Here we just give a qualitative analysis: in Eq. (9) the terms in \( \lambda_{12,q,q_z} \) correspond to intersubband transitions that will give rise to Mollow-like peaks in the fluorescence frequency spectrum, similarly to the case of a symmetrical two-level system. As the properties of the spectrum in many-body systems can be richer than in a single two-level system...
they are subject to current investigation. On the other hand, the terms in $\lambda_{11,q,q}$ and $\lambda_{22,q,q}$ correspond to intrasubband transitions and are peculiar of asymmetric QWs in which $\Delta z = z_{11} - z_{22} \neq 0$, as the one shown in Fig. 1 (c).

We obtain that the rate of emission per electron due to the latter (asymmetric) part of the interaction in Eq. (9) is

$$
\Gamma_{\text{Asym}} = \left| \Delta z \right|^2 \Omega^2,
$$

(10)

where $\gamma_0 = \frac{e^2}{120 \pi \hbar \epsilon_0 \epsilon_r}$. This THz emission line, centered at the Rabi frequency $\Omega$, can be Purcell-enhanced by employing a subwavelength microcavity to tailor the photonic density of modes. For example with a planar microcavity of length $L_{\text{cav}}$,

$$
\Gamma_{\text{Asym}}^{\text{cav}} = \frac{3\pi c \gamma_0 |\Delta z|^2 \Omega^2}{8\sqrt{\epsilon_r} L_{\text{cav}}}.
$$

(11)

Since $\Omega = e z_{12} E / \hbar$, where $E$ is the optical pump amplitude, with achievable values of the order of $10^6 - 10^7$ V m$^{-1}$, and $z_{12}$ that is of the order of the QW length $L_{\text{QW}}$, $L_{\text{QW}} \simeq 10$ nm, the emission can be tuned to span the THz range.

5. Conclusions

We have considered the Rabi oscillations occurring between the two conduction subbands of a 2DEG confined in a doped QW under resonant, coherent optical pumping. When the QW confining structure is not symmetrical, the interaction with the vacuum electromagnetic field leads to the emission of light centered at the Rabi frequency, which is tuned by the field strength of the optical pump. This nonlinear effect could thus empower solid-state THz emitters of tunable frequency for optoelectronic applications.

6. Acknowledgments

S.D.L. acknowledges financial support as a Royal Society Research Fellow. S.D.L. and N.S. acknowledge support from the Engineering and Physical Sciences Research Council (EPSRC), research Grant No. EP/L020335/1.

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