1. INTRODUCTION

The oldest known stellar systems in the Milky Way are the globular clusters. As such, their nature reflects the conditions under which our Galaxy first formed and offers a unique window into the high-redshift universe. Age determination of globular clusters by fitting models to the main-sequence turnoff (MSTO) allows a long period of time, the results of these studies led to a so-called cosmological crisis, in which the estimated cluster ages (~16–20 Gyr) were larger than the age of the universe, based on a variety of cosmological tests (e.g., Bolte & Hogan 1995). However, the last decade, improvements in the distance measurements to globular clusters, particularly using the Hipparcos satellite, have resulted in a lower estimate for the mean age of the metal-poor (hence oldest) globular clusters (~10.4 Gyr at 95% confidence; Krauss & Chaboyer 2003), which is now consistent with the age of the universe estimated from microwave background measurements (Spiegel et al. 2003). As befits a mature method, the accuracy of this MSTO method is currently limited by a variety of systematic errors (distance and metallicity uncertainties, atmosphere models that do not fit the shape of the turnoff). This method has been carried as far as it can go with current technology and further significant improvements must await technical advances, such as improvements in distance measurement using the Space Interferometry Mission (Chaboyer et al. 2002).

In the past several years, we have embarked on a program to measure the ages of globular clusters by an entirely different method—measuring the white dwarf cooling sequence (WDCS) and determining the age by modeling the rate at which they cool (Chaboyer et al. 1999). This method also has a distinguished history when applied to the stellar population in the solar neighborhood (Winget et al. 1987; Wood 1992; Hernanz et al. 1994; Hansen et al. 2002, 2004). This method also has a distinguished history when applied to the stellar population in the solar neighborhood (Winget et al. 1987; Wood 1992; Hernanz et al. 1994; Hansen et al. 2002, 2004). This method also has a distinguished history when applied to the stellar population in the solar neighborhood (Winget et al. 1987; Wood 1992; Hernanz et al. 1994; Hansen et al. 2002, 2004).
WFPC2 camera on the Hubble Space Telescope, which has since been superseded by the Advanced Camera for Surveys (ACS). The larger field and better resolution of ACS thus offers the possibility of improving on this result. Furthermore, the original M4 result was considered controversial by some (de Marchi et al. 2004), although unreasonably so in our opinion (Richer et al. 2004; Hansen et al. 2004). Nevertheless, an opportunity to repeat the experiment would serve to dispel any lingering doubts.

Consequently, we have observed a field in the second closest globular cluster to the Sun, NGC 6397, using ACS on HST, with the goal of measuring the white dwarf cooling sequence to unprecedented depth and precision. In addition, the lower reddening (relative to M4) of this cluster and the inclusion of a series of short exposures (to avoid saturation of bright stars) allows us to perform a fit to the MSTO (Richer et al. 2007) simultaneously while fitting the WDCS—a measurement never before possible. In § 2 we describe the observations and the reduction process, including characterization and removal of background galaxies as well as the characterization of our observational biases, using extensive artificial star tests. In § 3 we describe our modeling procedure and construct simulated cooling sequences, which we then compare to the data. In § 4 we summarize our result and its implications.

2. OBSERVATIONS

The observational data consist of a series of images taken with ACS of a single field in NGC 6397, over the course of 126 orbits (4.7 days). The field is located 5° southeast of the cluster core, overlapping that of several previous data sets taken by the Wide Field Planetary Camera 2 (WFPC2) in 1994 and 1997. All told the data set consists of 252 exposures (totalling 179.7 ks) in the F814W filter and 126 exposures totalling 93.4 ks in F606W. The F814W exposures were taken at the beginning and end of each visibility period, so that the F606W exposures were taken with as low a sky background as was possible. The images were typically dithered by 15 pixels, with additional subpixel dithers applied. Apart from a variety of short exposures (durations ranging from 1 to 40 s) used to treat the brighter stars, the deep exposures were of similar duration and were thus assigned equal weight in the analysis. The full details of the analysis will be described elsewhere (Anderson et al. 2007), but we review here the aspects thereof that are directly relevant to the identification of the faint point sources that make up the white dwarf cooling sequence.

Essentially, our approach uses the fact that the only influence the faintest stars will have on the image is to push their central pixels up above the noise in some number of the images. If a star is not bright enough to do this, then there is no way it will be found. However, if it does manage to generate a local maximum in a statistically significant number of images, then one can find it by searching for coincident peaks, peaks that occur in the same place in the field in different exposures. This is the preferred procedure because the peak map is less sensitive to point-spread function (PSF) features and other artifacts than a stacked image. After much experimentation (including artificial star tests), it was determined that a star must generate a peak in at least 90 out of the 252 F814W images to qualify as a 99% significance detection (the inclusion of the F606W images did not increase the significance levels).

Thus, we generate a list of possible sources by identifying every place in the field where a peak was found in at least 90 F814W images. To suppress artifacts around bright stars, we also require that the source be the most significant concentration of peaks within 7.5 pixels. This initial pass results in a catalog of 48,785 potential sources. Inspection of these sources on a stacked image shows that while this procedure identifies everything that is visibly a star, it also nets plenty of artifacts and galaxies.

We measure each of these sources as if it was a star, using a careful PSF-fitting procedure to get a position and a flux in F606W and F814W. This is based on the least-squares fitting procedure originally introduced by Anderson & King (2000) and generalized for the WFC in Anderson & King (2006). It takes into account the variation in the PSF with position across the detector, due both the optical aberrations and charge diffusion (Anderson et al. 2007). We then use a three-pronged weeding procedure to remove the nonstellar objects. The first two procedures target the PSF artifacts. While the 7.5 pixel halo removes a lot of such artifacts, there are many PSF features that extend beyond this radius. A spike filter is applied to the stacked image to identify the likely contribution of diffraction spikes to every point in the field. We require that each identified source be much brighter than the expectation value of spikes at its location. The second weeding step examines the brightness of a source in relation to its brighter neighbors. Artifacts exhibit a clear relationship between their distance from a bright star and their own brightness. We create a halo around each bright star that tells us how bright a faint star must be to be believable. Both of these tests are detailed in Anderson et al. (2007).

The above cuts remove the instrumental artifacts but still include real astrophysical objects (i.e., galaxies) that we wish to discount. In order to restrict our sample to point sources we make two further morphological cuts.

2.1. Star-Galaxy Separation

The image of a galaxy differs from that of a star in two quantifiable ways. The first is that it has more light in the envelope, relative to the core, than does the PSF of a star. The second is that it may be elongated. We thus derive two quantities to measure sharpness and roundness (Anderson et al. 2007). The sharpness parameter CENXS measures the central pixel excess or deficit relative to the stellar PSF. A deficit indicates that the object is partially resolved and thus extended. A similar measure (ELONG) can be made for the residual asymmetry in the image within 1.5 pixels radius. Larger asymmetries again suggest that the object is resolved. Using these parameters, we can remove all the partially resolved objects, leaving us with a sample of 8399 potential point sources.

The last cut made is to exclude a region of radius 180 pixels around an apparent concentration of faint blue sources located at \((x, y) = (725, 720)\) on the chip. It turns out that there is a large elliptical galaxy located near that position. Thus these objects are most likely not white dwarfs but rather globular clusters associated with the galaxy. Most of these fall outside the color-magnitude range of the white dwarfs, but not all of them, and so we exclude sources in this region from our sample.

2.2. Magnitude Calibration

The conversion from instrumental to Vega magnitudes was done according to the prescription of Sirianni et al. (2005). We measure the bright stars on specific reference images in F814W (\(j970101bbq\,drz\)) and F606W (\(j970101bdq\,drz\)), using the standard procedure and 10 pixel radius aperture. Because this is a crowded field, we reject some of these stars for which too large a fraction of the flux was found beyond 5 pixels. This controls errors that might be introduced by either cosmic rays or other stars. These are then compared to the fluxes measured using our PSF-fitting procedure to determine the zero point for our calibration. The
Natural text: Artificial stars were chosen with magnitudes and colors drawn from the canonical white dwarf cooling sequence, and added to the data on a grid, with mutual separation of 10 pixels, plus a random offset of 0.5 pixels relative to the grid position. This is large enough that any given star does not affect the detectability of its neighbors. The new data set is then analyzed as before and the recovery fraction measured. Figure 3 shows the resulting recovery fraction as a function of F814W. Also shown are the measured dispersions of the recovered magnitudes as a function of input magnitude. We see that, down to magnitude F814W = 28, the recovery fraction is >50% and the scatter is <0.25 mag.

2.4. Residual Galaxy Contamination

In order to model the distribution of white dwarfs in color and magnitude, we bin the data in the manner shown in Figure 4. As described in Hansen et al. (2004), we find that the distribution of white dwarfs above the luminosity function jump (F814W ∼ 26 in this case) contain very little age information, so these are added together into a single large bin. However, at fainter magnitudes, the distribution in magnitude and in color is affected by the mass and age of the underlying white dwarfs and so there is useful information to be obtained by quantifying this distribution. Thus we bin our data on a grid in color-magnitude space from F606W − F814W = 0.9−1.5 and from F814W = 26−28.

Our definition of our data product thus far involves the cut on the CENXS and ELONG parameters to exclude extended sources, followed by binning the data as described. Before we continue to the modeling we also require a measure of the systematic error contributions to the counts in each bin. The first such contribution is from the remaining galaxy contamination in each bin. Figure 5 shows the distribution of CENXS parameter for all objects in the range 26 < F814W < 28 and 0.9 < F606W − F814W < 1.5. Hence this data set represents the distribution of all objects that fall within the color-magnitude bounds of interest in describing the white dwarf cooling sequence. We model this as a sum of two Gaussians. One, the narrow peak, represents the stellar component, and the second represents the broad distribution of CENXS parameters appropriate to the partially resolved galaxy contribution. We use this second distribution to estimate the potential contribution of unresolved galaxies to our model fits. A second contribution to systematic error comes from the fact that the counts will differ slightly if we change the particular values of the cuts on CENXS and ELONG, so we include a contribution in each bin due to a reasonable variation in the actual values of the cuts.

Table 1 shows the number counts of sources with |CENXS| < 0.02 in each bin along with the estimated error in each bin, including both statistical and systematic error.

2.5. An Empirical Cooling Sequence

Before we proceed to the full modeling description, we pause to describe a procedure whereby we can derive an empirical cooling sequence, without reference to any modeling. Armed with the data and the artificial star tests, we can derive an empirical relationship between color and magnitude under the simple assumption that the underlying relationship is monotonic—that, for any given magnitude, there is a unique color appropriate to the underlying cooling sequence. The observed scatter is assumed to be the result of photometric errors, quantified by the artificial star tests.

Our derivation, briefly described in Richer et al. (2006), proceeds as follows. For each magnitude bin defined in Figure 4, our data consists of number counts as a function of color. If we assume a value for the true underlying color of the white dwarfs at this magnitude, we can then use the results of the artificial star comparison to model the observed number counts.

Sigma-clipped average of this comparison yields ZP(F606W) = 33.321 and ZP(F814W) = 32.414, with accuracy of 0.01 mag. No indication of color dependence was found.

Figure 1 shows the final color-magnitude diagram that results. The main sequence of the cluster is clearly seen, from above the turnoff to where the contrast with the background population fades near the hydrogen-burning limit. Most exciting for the purposes of this paper is the very clear cooling sequence of the cluster white dwarfs, beginning near F814W ∼ 22.5 and extending down to an observed truncation at F814W ∼ 27.6. The presence of a population of bluer and fainter objects (mostly the remaining unresolved galaxies but perhaps also a few background white dwarfs) in the CMD indicates that the truncation is real, and not a result of observational incompleteness. Figure 2 shows a zoom into the faint, blue region of this diagram—where white dwarfs and galaxies lie. The two panels show the effect of our galaxy-star separation. Due to the excellent image quality of the ACS data, we can identify most of the galaxies from the photometry alone. While there is certainly still some level of galaxy contamination in the point source sample, it is now at a level that can be securely modeled and included in the errors. As we will show in subsequent sections, with this data set it is possible to do model comparisons even without a proper-motion separation.

2.3. Artificial Stars

An important part of the modeling effort is the need to subject the models to the same kind of observational scatter and incompleteness as the data. This is particularly important for a data set such as this, where even relatively bright white dwarfs can be lost if they happen to be projected close to a bright main-sequence star. Thus, we have performed a detailed series of artificial star tests, to measure not only the recovery fraction, but also the degree of correlation between input magnitude and observed magnitude.
tests to predict the final distribution of colors expected after accounting for observational scatter. We may then characterize how well this fits with the true observed color distribution using the $\chi^2$ statistic. By varying the value of the true underlying color until we find the minimum of $\chi^2$ at each magnitude, we may then derive the best-fit color as a function of magnitude—an empirical cooling sequence.

This is shown in Figure 6. This may seem superfluous given that, in subsequent sections we fit model atmosphere colors to the data. However, there are still several issues outstanding in the chemical evolution and atmospheric modeling of white dwarfs, which means that there is some uncertainty in the final model colors (Bergeron et al. 1997; Bergeron & Leggett 2002). So, it is of interest to see what kind of relationship between color and magnitude fits the data independent of theoretical models. In particular we see evidence in Figure 6 (in the form of a turn toward the blue) for the deviation from blackbody trends (Hansen 1998; Saumon & Jacobsen 1999) expected due to collisionally induced absorption by molecular hydrogen in hydrogen-rich white dwarf atmospheres (Mould & Liebert 1978; Bergeron et al. 1995a; Borysow et al. 1997). Below we will see that this empirical relationship is similar to that found from theoretical hydrogen atmosphere models, suggesting that our sample is dominated by hydrogen atmosphere dwarfs.

Furthermore, this empirical sequence should allow other groups to compare their models to the data. Table 2 gives the best-fit colors as a function of magnitude.

2.6. Distance and Extinction

In both the MSTO and WDCS methods, the determination of the distance to the cluster is a fundamental aspect of the age measurement. The traditional method for globular clusters is to compare the main sequence with local, metal-poor subdwarfs with known parallaxes to determine the distance. We take our default distance to NGC 6397 to be $d = 12.13 \pm 0.15$ by Reid & Gizis (1998), who used Hipparcos distances for the subdwarfs in $V$ and $I$ for their main-sequence fit. This also assumes a reddening $E(B-V) = 0.18$ for this line of sight. We chose this determination because most other main-sequence distance determinations to NGC 6397 use the $B$ and $V$ bandpasses, so that the Reid & Gizis work is a more direct comparison to the bandpasses used here. We examine this further in § 4.2.

Although we shall later compare to the distance and extinction derived from the main sequence, we prefer to initially determine
these quantities directly from the white dwarf sequence. There are several reasons for this preference—for a cluster as metal-poor as NGC 6397, there are very few appropriate subdwarfs and color transformations are necessary; our observations use the HST bandpasses rather than the F555W, so that there are non-negligible color transformations between the HST bandpasses and the ground-based bandpasses used for the distance and extinction transformation, and finally there is the fact that cool white dwarfs can have spectral shapes not well represented by the (usually much hotter) stars used to determine the reddening.

To determine the distance and extinction, we follow the same method as used in Hansen et al. (2004), essentially a variation of the distance determination method of Renzini et al. (1996). An empirical fit to the apparent magnitude on the upper part of the observed NGC 6397 cooling sequence is

\[
F_{814W} = 3.00 \frac{F_{606W}}{C_0} + 23.37
\]

Using the atmosphere models of Bergeron et al. (1995b), but corrected slightly to be at fixed radius rather than fixed gravity, we can obtain a corresponding fit to the absolute magnitude for a model white dwarf,

\[
(F_{814W})_0 = 2.77(F_{606W} - F_{814W})_0 + 11.51 - 5 \log R_9,
\]

where \(R_9\) is the white dwarf radius in units of \(10^9\) cm and the subscript 0 indicates the color before reddening is applied. Taking reddening into account and assuming \(A_{814} = 0.65A_{606}\) (Sirianni et al. 2005), we can thus express the model apparent magnitude as

\[
F_{814W} = 2.77(F_{606W} - F_{814W})
+ 11.51 - 5 \log R_9 + \mu_0 - 0.49A_{814},
\]

where we have used the assumed reddening to convert the color term to reflect the observed (reddened) quantity. Equating equations (1) and (3), we can now express the true distance modulus in terms of the other parameters such as model radius and observed color:

\[
\mu_0 = 11.86 + 0.49A_{814} + 5 \log R_9 + 0.23(F_{606W} - F_{814W}).
\]
The last term indicates that there is some small color dependence in the fit (resulting from the fact that there is a residual difference in the slopes between eqs. [1] and [3]). The other two parameters are the extinction, and the white dwarf radius (related directly to the mass). Thus, one can trade off the distance, extinction, and white dwarf mass to maintain a good fit to the observed cooling sequence. This means that the constraints on these three quantities are interrelated. Using an extinction of $A_{V} = 0.3$ (explained below), and performing the fit at $F606W - F814W = 0.6$, we find the distance and radius are related by

$$
\mu_0 = 12.15 + 5 \log R_0.
$$

(5)

For a 0.5 $M_{\odot}$ white dwarf at the top of the cooling sequence, we get $\mu_0 = 12.08$, which is well within the range of acceptable distance moduli derived by Reid & Gizis. If we consider the lower limit from Reid & Gizis to be $\mu_0 > 12.0$, this restricts the range of allowed radii and hence places an upper limit on the masses at the top of the cooling sequence $M < 0.53 M_{\odot}$.

Of course, this just means that the models and observations agree at the bright end. Much more information emerges when we consider the data at the fainter end. In the following sections, we get constraints on the distance, extinction, and masses directly from the fit to the full cooling sequence. But we can get a preview of these results in a simple way by performing an operation similar to the one above, but now for the fainter white dwarfs. In this case, the color shift at faint magnitudes provides a feature in the cooling curve which must be fit by the observations. The most transparent way to do this is to compare our empirical cooling curve with the model curve of color versus magnitude. Fitting a 0.5 $M_{\odot}$ model curve to the empirical sequence (Fig. 6) suggests a reddening in the observed passbands of $E(F606W - F814W) = 0.16$. If we infer from this an extinction $A_{V} = 0.30$, then fitting a 0.5 $M_{\odot}$ white dwarf model to the data requires $\mu_0 = 12.0$, which is slightly smaller than the value derived in equation (5). The two independent measures can be made consistent if we allow the mass to increase along the cooling sequence, since the difference in the inferred $\mu_0$ is then offset by the more negative value of the $5 \log R_0$ term for a larger mass white dwarf. By requiring that our model fit both constraints simultaneously, we are thus able to place limits on the initial-final mass relation. The fact that the change is not large also suggests that there is not a lot of variation in mass between the bottom and top of the cooling sequence, as larger mass models have smaller radii and would result in a smaller inferred $\mu_0$.

We examine all of this in more detail in §3, but the above analysis demonstrates the origin of some of our forthcoming constraints on cluster distance and white dwarf masses. It also suggests

### Table 1

| Bright bin | $\Delta$ | $\mu_0$ | $R_0$ |
|------------|---------|---------|-------|
| Bright bin | 92 (13) | 12.15   | 5     |
| 26.00–26.25 | 36 (11) | 2       | 2     |
| 26.25–26.50 | 34 (13) | 5       | 5     |
| 26.50–26.75 | 20 (10) | 65 (14) | 45 (10) |
| 26.75–27.00 | 13 (8)  | 33 (10) | 83 (12) |
| 27.00–27.25 | 20 (10) | 65 (14) | 45 (10) |
| 27.25–27.50 | 30 (14) | 28 (13) | 28 (11) |
| 27.50–27.75 | 14 (10) | 8 (7)   | 4 (3)  |
| 27.75–28.00 | 30 (14) | 28 (13) | 28 (11) |

**Note.**—Errors (statistical and systematic) in each bin are listed in parentheses.
our measurements are not particularly sensitive to many of the details of the initial-to-final mass relation (which emerge mostly at the high-mass end). This will also be examined in §4.3.

3. MODELS

The final data product is shown in Figures 7 and 8. In Figure 7 we show the white dwarf luminosity function using a cut |CENXS| < 0.02 and ELONG < 0.02. Figure 8 shows the binned data from the Hess diagram in color-magnitude space using the same cuts. In Figure 7 we also show the galaxy luminosity function, which rises smoothly well beyond the observed truncation in the white dwarf luminosity function.

3.1. Default Models

We are now in a position to begin our investigation of fitting models to the full cooling sequence. Our initial fit will be to the same default model as in Hansen et al. (2004). We use the white dwarf cooling models described in Hansen (1999), with a hydrogen layer mass fraction of 10^{-4} and a helium layer mass fraction of 10^{-2}. We use model colors based on the atmosphere models of Bergeron et al. (1995a). The position of the photosphere in the atmosphere models is well matched to that assumed in the boundary conditions of the cooling models (see Fig. 3 of Hansen 1999), so that the combination is robust. We furthermore adopt an initial-final mass relation of the form (M_{TO} is the turnoff mass at a given age)

$$M_{WD} = A e^{B(M - M_{TO})},$$

and we determine A and B, along with cluster distance μ₀ and extinction A_{814}, by minimizing the χ² of the model fit to the observations, in tandem with the determination of cluster age and progenitor mass function slope. To begin with, we take the lifetimes of main-sequence progenitors from the same models as in Hansen et al. (2004). We will investigate the effect of other main-sequence models in §3.2.

Using this model we generate a Monte Carlo realization of the white dwarf population. For each white dwarf, we then add a photometric error chosen from the distribution of output magnitudes (given the intrinsic value as input magnitude) as determined from the artificial star tests. We do this for both F606W and F814W. In this manner, each realization includes a realistic level of photometric scatter. The data is then binned in two ways. The first measure is to simply bin the F814W luminosity function in the traditional way. The second measure is to also bin the data in two dimensions (sometimes referred to as the Hess diagram) the same way as the observations, using the grid shown in Figure 4. In each case, we then perform a χ² fit between model and observed populations to determine the goodness of fit. To completely specify each model, we need to choose values for the age, the progenitor mass function slope x, the distance and reddening, and the two parameters A and B in equation (6). If we marginalize over these last four parameters, we find the best fit (to the Hess diagram) if we adopt the values μ₀ = 12.05, A_{814} = 0.37 and

$$M_{WD} = 0.502 M_\odot e^{0.101(M - M_{TO})}.$$  

Keeping these parameters fixed, we show, in Figure 9, the confidence intervals for the fit to age and main-sequence mass function slope x for this improved solution. The solid contours represent the 1, 2, and 3 σ ranges obtained by fitting to the two-dimensional Hess diagram, while the dotted contours result from the fit to the luminosity function. The value of χ²_{min} = 41.5 for the Hess diagram fit corresponds to 1.34 per degree of freedom (dof), as there are 34 total bins used in the fit. The 2 σ age constraint is thus 11.7 ± 0.3 Gyr for this particular set of parameters. The comparison of the best-fit model (T = 11.61 Gyr, x = 0.95) and observations is shown in Figure 10, and is in excellent agreement with the observations.
agreement overall. The one obvious disagreement that remains appears to be that the models produce a weaker turn to the blue than is found in the observations. This can be seen in the F814W = 27.25 panel in Figure 10. For a visual comparison, we plot in Figure 11 the observations along with both a cooling curve\(^{13}\) for the best-fit model and the resulting simulated population (after accounting for photometric scatter). The location and width of the observed and modeled sequences are very similar, except for the weaker turn to the blue at the faint end in the models.

Also shown in Figure 9 are confidence intervals corresponding to a fit of the models to the observed luminosity function. Although the Hess fit is a more accurate measure, the historical use of the luminosity function makes this a comparison of interest. Figure 12 shows the observed luminosity function compared to the best-fit model (\(x = 0.59, T = 11.69 \text{ Gyr}\)). Also shown is the region used to perform the fit. Once again, we sum the bins with F814W < 26 to use as a single bin in the constraint. The constraints from the Hess diagram and the luminosity function overlap considerably, although the luminosity function, not surprisingly, offers a weaker 95\% constraint on the age (11.6 ± 0.6 Gyr).

### 3.2. Internal Chemical Composition

One advantage the WDCS method has over the MSTO method is that the colors are not metallicity dependent in any obvious way, which means the subtleties of cluster-to-cluster variations and of the transformation from observational to theoretical planes is less fraught with peril. However, the white dwarf method is not completely free of metallicity effects. In particular, the main-sequence lifetime of a given progenitor star is shorter (lower metallicity stars burn at higher temperatures and hence consume their fuel more rapidly). As a result, the models used in Hansen et al. (2004) likely overestimate the main-sequence lifetimes of stars as metal-poor as those in NGC 6397.

To address this problem, we use the models of Dotter & Chaboyer (Dotter et al. 2007, based on the models of Chaboyer et al. 2001), for metallicities [Fe/H] = −2 ± 0.1. We have also performed fits of these same models to the main-sequence turnoff (Richer et al. 2007), so that our white dwarf cooling age is self-consistent with the estimates from the MSTO models (at least, to the extent that they are coupled through the progenitor lifetimes). If we repeat the procedure of § 3.1 but use these models instead, the best-fit distance, extinction, and mass values are given by \(\mu_0 = 12.05, A_{814} = 0.36\) and

\[
M_{\text{WD}} = 0.5 M_\odot e^{0.169(M - M_{\text{TO}})}.
\]

Thus, the best-fit distance and extinction are quite consistent, although the initial-final mass relation is steeper than before. The resulting fit for age and \(x\) is shown in Figure 13, yielding a 2 \(\sigma\) range of 11.52 ± 0.23 Gyr from the Hess fit. The best-fit Hess model has \(\chi^2 = 39.6\), at \(T = 11.51 \text{ Gyr}\). This is shown in Figure 14, and a Monte Carlo realization of this model is shown in Figure 15. The best-fit luminosity function (at \(T = 11.46 \text{ Gyr}\)) is shown in Figure 16. The use of these new models results in a shift to a slightly lower age (although by less than the difference between the main-sequence lifetimes at fixed mass in the two models), but otherwise the fits are of similar quality to before, although slightly better in this case.

Thus, the age is reduced somewhat if we use the metal-poor models of Dotter & Chaboyer, as were used in the study of the MSTO in this cluster (Richer et al. 2007). However, the variation between different main-sequence models is also a source of systematic error, so let us now expand our parameter study to use both these models and those of Hurley et al. (2000), from which our default model was drawn, but now with the appropriate metallicity. In this case, we will marginalize over all the other parameters including distance, extinction, the initial-final mass relation, and also \(x\), to obtain a final, all-encompassing constraint on cluster age. The \(\chi^2\) curve is shown in Figure 17, along with the 2 \(\sigma\) age constraint \(T = 11.43 ± 0.46 \text{ Gyr}\).

Progenitor metallicities also can result in changes in the cooling models, as the nuclear burning history will affect the ratio of carbon to oxygen (and thus the heat capacity) in the resulting white dwarfs. The above calculations use the white dwarf compositional profiles from Hernanz et al. (1994). We have also performed fits using profiles resulting from the models of Hurley et al. (2000), but these result in lower \(\chi^2\) and worse fits to the data. Another possibility is that some fraction of the white dwarfs may possess helium cores (Hansen 2005), as suggested by the strange white dwarf luminosity function of NGC 6791 (Bedin et al. 2005). However, a significant population of such white dwarfs would lead to a peak in the luminosity function at much brighter magnitudes (F814W ~ 25.7), which is not seen in our data. We thus conclude that NGC 6397 contains a negligible population of helium-core white dwarfs.

### 3.3. Hydrogen Layer Mass

The amount of hydrogen on the surface of a white dwarf is one of the parameters that has to be specified for a white dwarf cooling model. The other models in this paper assume a hydrogen surface layer with a total mass fraction \(q = 10^{-4}\) for each white dwarf. This is the canonical value expected from standard stellar evolution, but there have been claims in the past for lower surface values. To that end, we have also performed a fit using the Dotter & Chaboyer main-sequence models, but now with cooling...
models that have thinner hydrogen surface masses, \( q_H = 10^{-6} \). The resulting fits yield worse \( \chi^2 \), even when we allow the distance, extinction, and initial-final mass relation parameters to float. The minimum \( \chi^2 = 55 \) (for an age of 10.55 Gyr), so that we conclude that the data supports models with a standard hydrogen layer mass.

3.4. Atmospheric Composition

In the field, some fraction of white dwarfs show evidence for atmospheres whose dominant constituent is helium, rather than hydrogen. This has a marked effect on the colors at faint magnitudes, since helium atmospheres continue to redden as they get cooler, while hydrogen atmospheres get bluer (in the infrared and optical bandpasses) because of molecular hydrogen absorption. The good fit of our empirical sequence to the hydrogen model (Fig. 6) suggests that the contribution of helium atmospheres is small in our case, but we can make this quantitative by including an extra parameter in our model, the fraction \( f_{\text{He}} \) of helium atmosphere white dwarfs. However, despite the addition of an extra parameter, this does not result in any improvement in the fit—essentially because helium atmosphere white dwarfs cool much faster than their hydrogen atmosphere counterparts and so smooth out the relatively sharp truncation in the luminosity function or Hess diagram. Furthermore, the colors of cool helium atmospheres are redder than for hydrogen atmospheres, so they do not help to alleviate the slight remaining discrepancy in the blue hook between model and observations.

3.5. Binarity

Binarity can also have an effect on the colors and magnitudes of white dwarfs. A white dwarf in a binary with a main-sequence star will be overwhelmed and excluded from our sample, but two white dwarfs in a binary will add together and potentially be brighter than a single isolated star. We model this binarity by randomly selecting the progenitors from the initial mass function and then accounting for the evolution of the two white dwarfs as isolated objects, eventually summing their light in the model. The resulting simulated samples result in degraded \( \chi^2 \), indicating that there is little to no evidence for binarity in our white dwarf sample.

To illustrate this, we show in Figure 18 the effect of a given binary fraction on the \( \chi^2 \)/dof for a model with all the other parameters held fixed at the best-fit values for the \( Z = 0.001 \) model. The open circles represent fits to the luminosity function, and the

**Fig. 10.** This model is an excellent fit overall. The models follow the observations both in color trends at fixed magnitude and as relative amplitudes as a function of magnitude. The one real discrepancy that remains is that the model color distribution at \( \text{F814W} = 27.25 \) is too red.
Fig. 11.—*Left:* The observed cooling sequence, after removal of the majority of background galaxies using CENXS cuts. *Middle:* Comparison of the data (plotted now as small crosses) with the theoretical cooling sequence for the best-fit model, before any photometric scatter is applied. *Right:* Monte Carlo realization of the white dwarf population derived from the best-fit cooling sequence, adopting the best-fit age of 11.61 Gyr, and modeling the photometric scatter in accordance with the artificial star tests.

Fig. 12.—Filled circles are the observed luminosity function. The solid histogram shows the model population, which includes both model white dwarfs and our estimate of the residual galaxy contamination per bin. The fitting region is delineated by the vertical dashed lines.

Fig. 13.—Solid contours are the confidence intervals for the Hess diagram fit but now using the [Fe/H] = −2 models from Dotter & Chaboyer. The dotted contours indicate the same but using the luminosity function.
filled triangles represent fits to the Hess diagram. We note that the luminosity function constraints are much more strict. This is because binarity results in model stars being found in some bins in the grid where they are never found otherwise (and therefore we do not usually use these bins in our $\chi^2$ fits because they contain no information). Thus, the open circles represent Hess fits with an expanded diagram (which feature three more bins at the bright, red end of the grid in Fig. 4). This results in much stronger constraints, and we use this to quote a 2/2/C27 upper limit on the binary fraction <4%. This represents the fraction of white dwarfs that have another white dwarf as a companion.

3.6. The Main-Sequence Turnoff

The traditional method for determining globular cluster ages is to fit stellar models to the main-sequence turnoff. In Richer et al. (2007) we compare our observations of the turnoff in this cluster to four sets of theoretical isochrones. In each case, good fits are obtained for a wide variety of ages, 12 ± 2 Gyr, where the accuracy is limited, as in prior studies, by the uncertainties in the distance and extinction. When the distance and extinction used in the MSTO method are restricted to the range that fits the white dwarf model (see §4.2) and ages determined using the models of Dotter & Chaboyer, we find a best-fit age of 11.6 Gyr, with a 95% confidence lower limit of 10.6 Gyr. Hence the two methods are in complete agreement, as these were the models used to determine the progenitor ages in Figure 13. As a result, this combination represents a successful end-to-end (main sequence to white dwarf) comparison of a set of stellar models with the observed stellar population.

4. DISCUSSION

The fits obtained here from the white dwarf cooling sequence result in a constraint that is quite a bit tighter than the traditional MSTO method. The reason for this comes from the fact that we have modeled the entire cooling sequence, rather than just a localized feature (as in the case of the turnoff). In principle, simply fitting a cooling model to the truncation in the observed cooling sequence leads to a similarly simple constraint, but it is subject to several unsatisfactory parameter degeneracies, including those between age, white dwarf mass, and cluster distance. By constructing a self-consistent model for the entire cooling sequence, these degeneracies are lifted because different choices of white dwarf mass and age have different consequences for distributions of white dwarfs in both color and magnitude. We illustrate this
Fig. 15.—Left-hand panel shows the observed cooling sequence, while the right-hand side shows a realization of an 11.51 Gyr old model with metal-poor progenitors.

Fig. 16.—Model luminosity function shown is for an age of 11.46 Gyr.

Fig. 17.—The $\chi^2$ curve vs. cluster age, marginalized over all other parameters for the metal-poor main-sequence models (both those of Dotter & Chaboyer and those of Hurley et al. 2000). The dashed lines indicate the 2 $\sigma$ age range and the horizontal dotted line corresponds to $\chi^2 = 1$ per degree of freedom.
by performing such a simple fit in Appendix A and contrasting it with our more detailed procedure.

To arrive at a final age constraint, we have marginalized over uncertainties in distance, extinction, the white dwarf mass (and its variation along the WDCS) and binary fraction, as well as model systematics that result from different choices of main-sequence models, white dwarf internal compositions and hydrogen layer masses. We have restricted our models to those of appropriate metallicity (so the models in §3.1 are not included in the final fit) and thus our final age constraint is that derived from Figure 17, $T = 11.47 \pm 0.47$ Gyr.

To further understand which information is important in determining this constraint, let us consider the nature of the best-fit models.

4.1. The Best-Fit Solution

The $\chi^2$ value per degree of freedom for the best-fit models is $\sim 1.27$, so it is good, but not perfect. Figure 14 shows the level of agreement between the data and the best-fit model. The model reproduces well not only the color distributions at each magnitude, but also the normalizations at different magnitudes relative to each other. The only feature of the observations that is not well reproduced by our models is the blue colors at magnitudes $F814W \sim 27.25$. This can also be seen in the visual comparison of the two panels in Figure 15, where the models clearly do not fully reproduce the clump of stars blueward of the faint end of the main locus. This most likely indicates some level of mismatch between the theoretical colors and the true colors at the faintest temperatures. The deviations are in the same sense as those seen in Figure 6, where the model colors lie a little lower than the empirical cooling sequence at the faint end. The fact that the blueward shift of the colors is driven by collisionally induced absorption of molecular hydrogen (Mould & Liebert 1978; Bergeron et al. 1995a; Borysow et al. 1997; Hansen 1998; Saumon & Jacobsen 1999) suggests that these deviations may indicate residual deficiences in the models or simply that the atmospheric composition contains an admixture of helium along with the hydrogen. Nevertheless, we emphasize again that the model fits are still good despite this residual mismatch and that improving the colors is likely to only make the age constraint even tighter.

Let us now consider the properties of the underlying white dwarf model. Figure 19 shows the mass of the white dwarf as a function of $F814W$ magnitude in our best-fit model. Note, the $F814W$ magnitude shown in this plot does not include the photometric scatter. In the top panel we show the observed luminosity function, to indicate the range over which the cutoff occurs. The white dwarfs in this part of the luminosity function range from 0.52 to 0.62 $M_\odot$. This is important, because it means the more massive white dwarfs do not contribute significantly to the luminosity function. This is because they cool faster at late times (as can be seen by the flattening in the $F814W - M_{\text{WD}}$ curve). A consequence of this is that we cannot directly interpret the cluster age as the white dwarf cooling time, because even the white dwarfs at the start of the truncation come from relatively low-mass main-sequence stars, and therefore have a small contribution to the total age from the main-sequence lifetime. This is shown explicitly in Figure 20, where we show a similar diagram to Figure 19, but where we now show the main-sequence mass, related to the white dwarf mass through the initial-final mass relation. Also shown is the corresponding main-sequence lifetime from the Dotter & Chaboyer models. We see that the truncation in the white dwarf luminosity function appears when the main-sequence age of the progenitors starts to drop precipitously, so that we get older, cooler, and fainter white dwarfs. Over a range of only 0.5 mag, we see that the cooling time of the white dwarfs at these magnitudes changes by $>5$ Gyr! This is the ultimate source of the truncation—a bin of fixed width in magnitude corresponds to a much smaller range in the progenitor mass function than higher up the cooling sequence, and so there are fewer stars in that bin. The underlying physical reason for this is that more massive white dwarfs initially cool...
more slowly because they have larger heat capacity, but they crystallize earlier because of the high central densities and thereafter cool more quickly as the heat capacity enters the Einstein-Debye regime. This means that, at any given age, there is a “leading edge mass,” above which the white dwarfs have overtaken their less massive counterparts, and cooled beyond detection.

We can now also return to the issue of how the luminosity function truncation is affected by incompleteness. Figure 21 shows the luminosity function taking into account the observational incompleteness (solid histogram) and the corresponding complete luminosity function (dashed histogram). We see that the sharp drop is a direct consequence of the model, and not strongly affected by observational incompleteness, even at F814W > 28, where the recovery fraction is only 53%. The reason for this is that we lose white dwarfs primarily through confusion with brighter stars, not because of large photometric scatter from noise in those that we do recover. To understand this better, consider Figure 22, in which we show the results of artificial star tests for stars inserted with F814W = 27.9. The fraction of stars never recovered at all is 47%, but the dispersion of those that are recovered is only ∼0.1 mag. This is not sufficient to alter the shape of the recovered luminosity function.

As a final demonstration of the properties of our modeling procedure, we show in Figure 23 the effect on the model population as we change the age. The middle panel shows the best-fit model from Figure 15, and the left and right panels show the same model but for ages of 10 Gyr and 13 Gyr, with all other parameters held fixed. The younger population is clearly distinguishable from the best-fit model, with a truncation at brighter magnitudes (the χ² = 171 for this model). The population at 13 Gyr shows no abrupt edge except that due to incompleteness at F814W > 28, and a much stronger color evolution (χ² = 185).

4.2. Distance Scale and Extinction Redux

The shaded region in Figure 24 shows the 2σ range for NGC 6397 distance modulus and reddening, for the fit to the Dotter & Chaboyer models. This is compared to the equivalent values from three studies determined by comparing the main-sequence with field M subdwarfs measured by Hipparcos. The solid point is the value obtained by Reid & Gizis (1998), which is probably the most easily compared to our value, since the comparison was made in the V and I bands. Indeed, our value is consistent with the low end from that study. The value obtained by Reid (1998), used the B and V bands, is shown as the open circle, and is somewhat
higher than our white dwarf distance. The best agreement is obtained with the value quoted by Gratton et al. (2003), obtained again using the $B$ and $V$ bandpasses, but with a variety of assumptions regarding color corrections etc., that differ from those of Reid. In fact, our distance is somewhat better constrained than the main-sequence values, yielding $d_0 = 12.03 \pm 0.06$ at 2$\sigma$, while the reddening is constrained to be $E(F606W - F814W) = 0.20 \pm 0.03$.

### 4.3. Initial-to-Final Mass Relation

One of the inputs into our model is the relationship between the white dwarf mass and the mass of its progenitor. This is a fundamental quantity that has been studied for many years, so it is natural to ask how our best model fits correspond to those determined empirically. Figure 25 shows two of our fits (using our default models and then the metal-poor Dotter & Chaboyer fits in § 3.2) compared to a well-known empirical initial-final mass relation from Weidemann (2000) and a more recent fit from Ferrario et al. (2005). It is encouraging that our best-fit model recovers a relation similar to the empirical ones, although it must be noted that our fit really only probes a limited range of masses, as inferred from Figure 19, essentially the unshaded region in Figure 25. We note also that our initial-final mass relation is in agreement with the results of Moehler et al. (2004), who found, using multicolor photometry, that the mean mass of white dwarfs at the top of the cooling sequences in NGC 6397 and NGC 6752 was $0.53 \pm 0.03 M_\odot$. This is also in agreement with theoretical expectations (Renzini & Fusi Pecci 1988).

### 4.4. Cosmological Considerations

One of the more exciting aspects of this result is that we get a finite upper limit on the age. Most other measurements, including our M4 result, yield primarily lower limits on the age of star

$\sigma_M = 0.066 M_\odot$. It is an order of magnitude larger than the prediction and larger than the scatter in the photometry at these magnitudes. We might infer a larger dispersion in mass than expected, although there are other possible contributions such as variations in hydrogen layer mass and spectral composition (DB stars have smaller radii). It is worth noting, though, that this width is narrower than the estimated width of the field sample $\sigma_M = 0.137 M_\odot$ (Bergeron et al. 1992).
Fig. 24.—Shaded region shows the 2 σ age range for distance and reddening allowed by fitting to the white dwarf cooling sequence. The filled circle with the error bar represents the distance inferred by Reid & Gizis (1998), plotted at their assumed reddening. Similarly, the inferred values from Reid (1998) and Gratton et al. (2003) are also shown. The reddenings have been converted from the ground-based colors to the ACS photometric system using the prescription of Sirianni et al. (2005). The horizontal error bars represent the effect of assuming different underlying spectral types, spanning M5 to O2, when converting from ground-based to HST magnitudes.

Fig. 25.—Solid lines indicate our best-fit initial-final mass relations for the default models and for the metal-poor models with Dotter & Chaboyer main-sequence lifetimes. The filled circles show the empirical relation inferred by Weidemann (2000), and the dashed line indicates the best-fit linear relation espoused by Ferrario et al. (2005). The open circle, with the error bar, is the mean upper cooling sequence mass inferred by Moehler et al. (2004). Our fit is only constrained for a limited range of white dwarf masses (see Fig. 19), and so we shade the range of white dwarf masses that fall beyond our magnitude limit. We see that our best-fit models overall (the DC models) are in excellent agreement with Weidemann’s empirical determination.

Fig. 26.—Solid curve indicates the relationship between cosmological redshift z and look-back time for the best-fit flat universe model from Spergel et al. (2003). The shaded regions indicate white dwarf cooling ages (2 σ range) for the Galactic disk (Hansen et al. 2002) and NGC 6397 (this paper), and the arrow indicates the lower limit on the age for M4 (Hansen et al. 2004). Above the plot we show two 95% lower limits. The limit marked KC indicates the lower limit for the age of the globular cluster system as whole, taken from Krauss & Chaboyer (2003). The limit marked G03 is the 2 σ lower limit on the age of NGC 6397, based on the results of Gratton et al. (2003). The comparison indicates that our age determination is consistent with, but also more accurate than, the best measurements using the MSTO method.

Fig. 27.—Filled circles represent the Hubble Ultra Deep Field star formation rates (corrected for extinction and surface brightness) measured by Thompson et al. (2006). The crosses give the extinction corrected values measured from the Hubble Deep Field by Madau et al. (1996). The shaded regions indicate the epoch in which the Galactic Disk and NGC 6397 formed, according to our measures of the white dwarf cooling.
clusters when the full systematics are taken into account. The reason for the excitement is that we can now start to properly constrain the cosmological epoch at which NGC 6397 formed.

Let us consider a flat universe with a cosmological constant $\Lambda$.

$T(z_c) = \frac{3}{2} \left[ 1 + \frac{\Omega_0}{1 - \Omega_0} \right]^{1/2} H_0^{-1} \left\{ \sinh^{-1} \left[ \frac{1 - \Omega_0}{\Omega_0} \right]^{1/2} \right\} - \sinh^{-1} \left[ \left( \frac{1 - \Omega_0}{\Omega_0} \right)^{1/2} (1 + z_c)^{-3/2} \right], \quad (9)

where $H_0$ is the present-day Hubble constant (which we will take as 71 km s$^{-1}$ Mpc$^{-1}$). Using the parameters from the best-fit cosmologically flat model to the microwave background anisotropy (Spergel et al. 2003), we take $\Omega_0 = 0.27$, which gives a present day age of 13.7 Gyr for the universe. We can use equation (9) to figure out the range of possible formation redshifts, given the age of NGC 6397, which we take to be $11.47 \pm 0.47$ Gyr (our 2 $\sigma$ value). The resulting formation redshift is

$$z_c = 3.1 \pm 0.6. \quad (10)$$

Figure 26 shows this result compared to the relation (9) for the WMAP cosmology. Our result places the formation epoch of NGC 6397 somewhat more recently than the reionization epoch (generally taken to be at redshifts of 6 or greater) but quite naturally associated with the copious star formation seen at redshifts $\sim 2-4$. As noted above, we are in agreement with prior age estimates for the globular cluster system based on the MSTO method but those estimates could only place a lower limit on the age of the system. The greater precision of our method now allows us, for the first time, to determine a realistic offset between the creation of the universe and the formation of the globular cluster system,
while also confirming the significant delay between globular cluster formation and the onset of star formation in the Galactic disk that we found in Hansen et al. (2002).

The age for NGC 6397 places its origin firmly near the peak of the cosmic star formation rate, as measured by various deep cosmological surveys (Madau et al. 1996; Thompson et al. 2006), shown in Figure 27. The origins of globular clusters has been a subject of debate for at least 40 years. The first proposed scenarios postulated a very early origin (Peebles & Dicke 1968; Fall & Rees 1985), although the possible role of mergers was also recognized (Searle & Zinn 1978; Schweizer 1987; Ashman & Zepf 1992). The position of globular clusters within the modern hierarchical structure paradigm is highly uncertain, and our age determination now offers the opportunity to draw a distinction between at least some of the competing proposals. With a formation redshift $z > 3$, NGC 6397 appears to have formed after the epoch of reionization, which is conservatively held to be at $z > 6$ (Fan et al. 2006). If we assume NGC 6397 is representative of the class of metal-poor globular clusters as a whole, this rules out the notions that cluster formation was triggered by radiation pressure-driven collapse in protogalactic halos during reionization (Cen 2001) or that the clusters were themselves responsible for the reionization (Ricotti 2002). Furthermore, the age of NGC 6397 seems inconsistent with the notion of Forbes et al. (1997) that the metal-poor clusters formed early and were “switched off” by some later event, possibly reionization (Santos 2003). Our results are much more consistent with scenarios that associate cluster formation with the epoch of rapid, starburst-driven star formation that occurs at $z > 6$ (Fan et al. 2006; Kalirai et al. 2007), never getting more than 3 kpc above the galactic plane. Although MISTO methods place the

| $F606W$  | $N_{in}$ | $N_{out}$ | $\Delta F606W$ |
|---------|---------|----------|--------------|
| 22.0    | 0       | 0        | $-0.40$      |
| 22.2    | 835     | 785      | $-0.35$      |
| 22.4    | 2533    | 2360     | $-0.30$      |
| 22.6    | 2624    | 2440     | $-0.25$      |
| 22.8    | 2599    | 2405     | $-0.20$      |
| 23.0    | 2552    | 2366     | $-0.15$      |
| 23.2    | 2523    | 2310     | $-0.10$      |
| 23.4    | 2224    | 2007     | $-0.05$      |
| 23.6    | 2268    | 2041     | $-0.00$      |
| 23.8    | 2266    | 2037     | $-0.15$      |
| 24.0    | 2275    | 2048     | $-0.20$      |
| 24.2    | 2195    | 1948     | $-0.25$      |
| 24.4    | 2280    | 1989     | $-0.30$      |
| 24.6    | 2119    | 1873     | $-0.35$      |
| 24.8    | 2153    | 1848     | $-0.40$      |
| 25.0    | 2246    | 1921     | $-0.45$      |
| 25.2    | 2258    | 1933     | $-0.50$      |
| 25.4    | 2186    | 1844     | $-0.55$      |
| 25.6    | 2197    | 1841     | $-0.60$      |
| 25.8    | 2159    | 1806     | $-0.65$      |
| 26.0    | 2006    | 1610     | $-0.70$      |
| 26.2    | 2098    | 1702     | $-0.75$      |
| 26.4    | 2098    | 1690     | $-0.80$      |
| 26.6    | 2095    | 1660     | $-0.85$      |
| 26.8    | 2028    | 1600     | $-0.90$      |
| 27.0    | 2070    | 1593     | $-0.95$      |
| 27.2    | 1985    | 1472     | $-1.00$      |
| 27.4    | 2144    | 1588     | $-1.05$      |
| 27.6    | 2228    | 1690     | $-1.10$      |
| 27.8    | 2266    | 1602     | $-1.15$      |
| 28.0    | 2148    | 1518     | $-1.20$      |
| 28.2    | 2223    | 1469     | $-1.25$      |
| 28.4    | 2199    | 1397     | $-1.30$      |
| 28.6    | 2091    | 1236     | $-1.35$      |
| 28.8    | 2207    | 1310     | $-1.40$      |
| 29.0    | 2238    | 1247     | $-1.45$      |
| 29.2    | 2273    | 1202     | $-1.50$      |
| 29.4    | 2184    | 1088     | $-1.55$      |
| 29.6    | 2184    | 994      | $-1.60$      |

Notes.—Each bin corresponds to the difference between recovered magnitude and input magnitude. Thus, a negative value implies that those stars were recovered as being brighter than their true magnitudes.
ages of the outer globulars as consistent with that of NGC 6397 (e.g., Salaris & Weiss 2002) the 2 σ accuracy of the method (~1.5 Gyr) means the constraint is much less stringent than our absolute age measurement.

4.5. Outlook

The age constraint we derive is more precise than those of previous studies using the MSTO method, and also more precise than our earlier result on M4 (Hansen et al. 2004). In our M4 study we also tried to calculate ages using other white dwarf models from the literature. We have not done so in this study because we now require knowledge of the models in more detail than can be gleaned from published material. We hope that other groups will perform similar analyses using their models, so that we may gain a more realistic understanding of the systematic uncertainty in the age based on using different evolutionary codes. We caution, however, that simple, heuristic comparisons between models and observations are unlikely to yield particularly interesting answers, as can be seen from Appendix A. To aid in proper comparison, we include in Tables 3 and 4 the results of our artificial star tests, in which we quantify the amount of photometric scatter that results as a function of intrinsic magnitude. Feeding theoretical models through this matrix and then comparing to the data in Table 1, one can perform statistically well-posed fits between models and data.

The prospects of extending this methodology to other clusters is somewhat limited by the extreme faintness of the coolest white dwarfs. The only other clusters for which this seems feasible are NGC 6752 and 47 Tucanae. Nevertheless, similar observations of these clusters could be very illuminating as this trio represent the archetype for the metal-poor, intermediate-metallicity, and metal-rich/thick disk/bulge cluster families. In particular, NGC 6397 and NGC 6752 are considered contemporaneous by MSTO age determinations, while 47 Tuc may be marginally younger (e.g., Sarajedini et al. 1997; Rosenberg et al. 1999; Gratton et al. 2003). The white dwarf cooling sequence offers us then the chance to test these assertions at greater accuracy. However, the observational expense involved in such projects will prevent similar tests in more distant clusters.

Further improvements in the model fitting procedure will require a more detailed understanding of the surface chemical evolution of old white dwarfs. As discussed in § 4.1, there is a residual mismatch in the colors at the faintest magnitudes. The most likely explanation for this is that we are using pure hydrogen atmospheres to calculate the model colors. Studies of old white dwarfs in both the Galactic disk (Bergeron et al. 1997) and the Galactic halo (Bergeron & Leggett 2002) suggest that the oldest white dwarfs frequently experience atmospheric helium contamination, which can affect the strength of the CIA absorption that is responsible for the blue color of faint white dwarfs. A similar level of helium contamination, most likely the result of convective dredge-up, would affect the colors at precisely those magnitudes where the models yield imperfect fits. Fortunately, the levels of helium contamination are not likely to change the cooling age significantly, because the atmospheric opacity is still dominated by hydrogen in the case of observed halo stars (e.g., Bergeron & Leggett 2002). Another observation of this cluster to similar depth, in order to repeat our analysis with a true proper-motion-selected sample, would shed further light on this question by allowing us to potentially find rare white dwarfs with very blue colors in parts of the CMD that are dominated by background galaxies. Stars with such strong infrared flux depression would provide the best benchmark for constraining mixed hydrogen and helium atmosphere models. Such a program was originally planned but was canceled with the failure of the ACS on HST.

5. CONCLUSION

In conclusion, we have performed a detailed Monte Carlo simulation of the white dwarf population in the metal-poor globular cluster NGC 6397, comparing it to the observed WDCS obtained with the ACS camera on HST. The principal conclusion that results from this comparison is that the age of the cluster is 11.47 ± 0.47 Gyr at 95% confidence. NGC 6397 is a member of the class of metal-poor clusters that are thought to be among the oldest objects in the Milky Way and so this age places the epoch of original assembly of the Galaxy at z = 3.1 ± 0.6.

Our model is also in agreement with various independent measures of several of the parameters, such as the distance and extinction along the line of sight, and the relationship between white dwarf mass and progenitor mass.

B. H., J.A., R. M. S., I. K., and M. M. S. acknowledge support from proposal GO-10424, and J. K. is supported through Hubble Fellowship grant HF-01185.01-A, all of which were provided by NASA through grants from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. H. B. R. also acknowledges the support of the US-Canada Fulbright Fellowship Committee. The research of H. B. R. and G. G. F. is supported in part by the Natural Sciences and Engineering Research Council of Canada.

APPENDIX A

A SIMPLE FIT TO THE COOLING SEQUENCE TRUNCATION

In this appendix we perform a simple comparison between our cooling models and the observed truncation of the white dwarf sequence. Although our full model comparison results in much tighter constraints, this exercise is still instructive as it further illustrates how some of the ancillary information is used in the proper models.

As a first step, we characterize the magnitude of the truncation as $F814W = 27.6 \pm 0.1$. Using the distance and extinction from Reid & Gizis (1998), we convert this into an absolute magnitude $M_{V14} = 15.15 \pm 0.15$. We can thus compare this to a model to infer an age. However, we first need to specify the mass of the white dwarf, and this is a free parameter in the absence of other constraints. Figure 28 shows the absolute magnitude as a function of age of hydrogen atmosphere models of various masses. A further constraint is included, requiring that the $F606W$-$F814W$ color of the models lie in the range $1.13 \pm 0.13$ (the dereddened color range spanned by the observations at the location of the truncation). Those parts of the curve that meet this criterion are solid, while those that do not are dashed. We see that acceptable ages are found for all masses, the cooling age increasing for masses from 0.5 to 0.65 $M_\odot$ and then decreasing thereafter (because of the faster cooling—due to core crystallization—of more massive white dwarfs at late times). Thus, in the absence of other information, we infer an age range 10.2 ± 1.4 Gyr (cooling age only).
The reason that the full models yield a precise value is because they incorporate other constraints. Most notably, the models have to fit the location of the cooling sequence at the bright end (which favor models in which the white dwarfs have masses \(<0.55 \, M_\odot\) initially). With this additional constraint, the shape of the observed cooling sequence constrains how much the mass can vary along the cooling sequence and the manner in which the numbers increase along the sequence constrains which cooling models and which masses yield good fits. Finally, this full modeling approach also incorporates the main-sequence lifetime, which contributes about 5\%–10\% to the final age for stars at F814W = 27.6 in the best-fit model (this can be seen by comparing Figs. 20 and 28.)

Thus, we can build a rough picture of where the final number comes from. Our best-fit models suggest the white dwarf mass at the truncation is \(0.6 \, M_\odot\), which corresponds to a cooling age of \(11.5 \pm 0.5\) Gyr, according to Figure 28. The initial-final mass relation emerging from the fit yields a progenitor mass \(2 \, M_\odot\), which in turn yields a main-sequence lifetime \(0.5\) Gyr. Thus, we arrive at a cluster age \(11.5 \pm 0.5\) Gyr.

APPENDIX B
PROPER MOTIONS

Approximately 54\% of our ACS field has some prior WFPC2 imaging, so proper-motion separation is, in principle, possible. However, the first epoch exposures are not deep enough to allow us to perform proper-motion separation at a level necessary to reach the end of the white dwarf cooling sequence.

Figure 29 shows the proper-motion displacement (\(\mu\), in units of ACS pixels) for all point sources (all objects in Fig. 1) as a function of F814W magnitude. This is done by matching each ACS detection to the closest 2\sigma peak in the WFPC2 data. This works well for magnitudes F814W < 26, leading to a very clear separation of the cluster (\(\Delta \mu < 1.5\) pixel) and Galactic field stars (\(1.5 < \mu < 5\) pixels). However, we can see that, at magnitudes F814W > 26.5 there is a very incomplete separation of the cluster stars. There is clearly still a clump at small \(\mu\), but the distribution of \(\mu\) is large and consistent with noise.

APPENDIX C
CLUSTER DYNAMICAL EFFECTS

NGC 6397 is a post–core-collapse cluster. At the high central densities experienced in this cluster, many exotic dynamical interactions can occur between stars. Stars in binaries can be ejected, single stars can be exchanged into binaries, and mass transfer between stars can be either halted or initiated. It is thus natural to wonder whether such processes have any effect on our parameter estimation.

The first thing to note, of course, is that our field is chosen to minimize contributions of this kind, since it is located well away from the high-density core. Nevertheless, mass segregation is an important process in globular clusters and some stars with exotic evolutionary histories may be able to migrate outwards far enough to enter our sample. To examine this possibility, we have examined an \(N\)-body simulation of NGC 6397, calculated in the manner described in Shara & Hurley (2006). The simulation began with 100,000 stars, 5\% of which were in binaries initially. The model cluster reached core collapse at 15 Gyr, with 25,000 stars remaining, so its late-time structure is qualitatively similar to that observed for NGC 6397. Our modeling procedure is already able to treat a binary population and our data also show little or no evidence for binarity. The bigger worry is that we are not modeling the age distribution of white dwarfs properly.
because of the contribution of so-called divorced white dwarfs—stars that spent some time in a binary, experienced mass transfer which altered the stellar evolution “clock,” and were then removed from the binary by a dynamical interaction. It is these stars that are not necessarily so easy to identify directly, since the only way they will stand out from the regular cooling sequence is if there is a sufficiently large mass difference with similar age white dwarfs from single star evolution (“bachelor” white dwarfs) to make a measurable difference in the photometry. Given the telescoped nature of the initial-final mass relation, this is not a very sensitive test.

The top panel of Figure 30 shows the radial distribution of bachelor and divorced white dwarfs from the Hurley & Shara simulation at an age of 13 Gyr. We see that the divorced white dwarf profile is essentially a scaled version of the bachelor radial profile. The scaling

Fig. 29.—Proper-motion displacements of all detected point sources that overlap with WFPC2 data, as a function of F814W. We can see the excellent separation for bright magnitudes, failing at F814W ~ 26.5. Brighter than this magnitude, the stars separate cleanly into cluster members (μ ~ 0) and background stars (μ ~ 2.5). Fainter than this magnitude, most real stars in the ACS data are matched to noise peaks in the WFPC2 data, resulting in large spurious displacements.

Fig. 30.—(a) Filled circles and solid line indicate the radial profile of “bachelor” white dwarfs—those which result from single star evolution and which experience no perturbations during the course of their evolution. The open circles represent the radial profile of “divorced” white dwarfs, which represent stars that spent some fraction of their life in a binary but are now single. The dotted histogram is the same as the solid histogram, but scaled by a factor 0.07. The divorced white dwarfs follow the same radial profile as the bachelors. Radii are in units of the half-mass radius of the cluster. (b) The filled circles show the distribution of divorced white dwarfs with T_{WD}—the time at which the white dwarf formed (i.e., T_{WD} = 0 corresponds to the birth of the cluster). The solid histogram represents the age distribution of the bachelor white dwarfs, once again suitably scaled. There is clearly little difference in the age distribution between the two populations. Even if the deviation at T_{WD} < 200 Myr is real, it is too small to have any effect on our modeling.
factor is ~7% i.e., at any radius, the divorced white dwarfs make up roughly 7% of the white dwarfs. The lower panel shows the age distribution of the divorced white dwarfs (points) compared to a scaled version of the bachelor age distribution. Once again, there is excellent agreement, suggesting that the degree of dynamical interaction and mass transfer is not sufficient to dramatically alter the stellar evolution clock. The one potential discrepancy is a 2σ excess of white dwarfs with T_{WD} < 200 Myr i.e., divorced white dwarfs that were born anomalously early. However, even if this is real, the excess corresponds to roughly 5% of the divorced white dwarfs, which are themselves a minority population. Overall, this corresponds to roughly three anomalously young white dwarfs per sample of 1000. This is far too little to have any effect on our results.

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