As a candidate for dark matter in galaxies, we study an $SU(3)$ triplet of complex scalar fields which are non-minimally coupled to gravity. In the spherically symmetric static spacetime where the flat rotational velocity curves of stars in galaxies can be explained, we find simple solutions of scalar fields with $SU(3)$ global symmetry broken to $U(1) \times U(1)$, in an exponential scalar potential, which will be useful in a quintessence model of the late-time acceleration of the Universe.

1. Introduction

It is well known that the flat rotational velocity curves (FRVC) of stars in galaxies suggest the existence of non-luminous (or dark) matter in the galactic halo. If the energy density of dark matter in the galactic halo is proportional to $1/r^2$, then one can account for the FRVC of stars in galaxies. In the global monopole solution found by Barriola and Vilenkin, scalar fields with global $O(3)$ broken symmetry are minimally coupled to gravity and the background spacetime has deficits of angle. Nucamendi et al. suggested that the global monopole solution could explain the FRVC in galaxies because its energy density is proportional to $1/r^2$. But Harari and Lousto showed that the monopole core mass is negative and that there are no bound orbits.

Matos, Guzmán, and Ureña-López obtained solutions for metric coefficients of the spacetime in which the FRVC of stars in galaxies could be explained. In the spacetime they found a solution of the scalar field in a singlet in an exponential potential. However its sign is negative, $V \propto -e^{-\phi}$, and it is necessarily opposite to that of the exponential potentials that had been considered in quintessence cosmologies. We generalize them to the theory of complex scalar fields with $SU(3)$ global symmetry, coupled non-minimally to gravity. Complex scalar fields with $U(1)$ global symmetry were considered by Boyle, Caldwell, and Kamionkowski as "spintessence" models for dark matter and dark energy. Difficulty of this spinning complex scalar field to be dark energy was discussed by Kasuya. A new quintessence model in which scalar fields possess a global $O(N)$ internal symmetry was studied.
Both Boyle et al. and Li et al. considered spinning (time-dependent) scalar fields. We study in this paper complex scalar fields which depend on $r$ only.

We consider $SU(3)$ scalar fields which are non-minimally coupled to gravity. In the spherically symmetric static spacetime in which the FRVC of stars in galaxies can be explained, we find simple solutions of scalar fields with $SU(3)$ global symmetry broken to $U(1) \times U(1)$, in an exponential scalar potential $V = \exp(-\Phi^a\Phi^a)$, choosing a special value of non-minimal coupling parameter $\xi$ as in Eq. (32). The potential will be useful in a quintessence model for the late-time acceleration of the Universe.

2. $SU(3)$ Scalar Fields Non-Minimally Coupled to Gravity.

The action for complex scalar fields, with an $SU(3)$ global symmetry, which are non-minimally coupled to gravity is given by

$$S = -\int d^4x \sqrt{-g} [g^{\mu\nu} \partial_{\mu} \Phi^a \partial_{\nu} \Phi^a + \xi R \Phi^a \Phi^a + V(\Phi^a \Phi^a)],$$

(1)

where $\Phi^a (a = 1, 2, 3)$ are complex scalar fields, $R$ is a scalar curvature, $\xi$ is a non-minimal coupling parameter, and $V(\Phi^a \Phi^a)$ is a scalar potential. Using the variational method, we obtain following equations for scalar fields $\Phi^a$:

$$\frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g}g^{\mu\nu} \partial_{\nu} \Phi^a) - \xi R \Phi^a - \frac{\partial V}{\partial \Phi^a} = 0.$$  

(2)

Using the standard definition of the energy-momentum tensor

$$T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}},$$

(3)

we have

$$T_{\mu\nu} = \partial_{\mu} \Phi^a \partial_{\nu} \Phi^a - \frac{1}{2} g_{\mu\nu} [g^{\alpha\beta} \partial_{\alpha} \Phi^a \partial_{\beta} \Phi^a + V(\Phi^a \Phi^a)] + \xi \Phi^a \Phi^a G_{\mu\nu},$$

(4)

where $G_{\mu\nu}$ is the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

(5)

with Ricci tensor $R_{\mu\nu}$.

Assuming the spherically symmetric static spacetime with the line element

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu = -B(r) dt^2 + A(r) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

(6)

and the hedgehog Ansatz for scalar fields,

$$\Phi^a = F(r) \frac{x^a}{r},$$

(7)
with a complex function \( F(r) \) and \( r = (x^a x^a)^{1/2} = (x^2 + y^2 + z^2)^{1/2} \), we get the equation for the complex function, \( F \):

\[
F'' + \frac{F'}{2} \left( \frac{B'}{B} - \frac{A'}{A} + \frac{4}{r} \right) - \frac{2AF}{r^2} - \xi ARF - A \frac{\partial V}{\partial F^*} = 0, \tag{8}
\]

where \( F' = \frac{dF}{dr} \), \( F'' = \frac{d^2F}{dr^2} \), ..., and the scalar curvature is given by

\[
R = -\frac{B''}{AB} + \frac{B'}{2AB} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{r}{2A} \left( \frac{A'}{A} - \frac{B'}{B} \right) + \frac{2}{r^2} \left( 1 - \frac{1}{A} \right). \tag{9}
\]

In next section we solve both the field equation in Eq. (8) and the Einstein’s equation given by

\[
G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{10}
\]

or

\[
R_{\mu\nu} = \kappa (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} T_{\alpha\beta}), \tag{11}
\]

with \( \kappa = 8\pi G \). Instituting Eq. (4) into the Einstein’s equation given in Eq. (11), we have

\[
R_{\mu\nu} = \kappa (\partial_\mu \Phi^* a \partial_\nu \Phi^a + \frac{1}{2} g_{\mu\nu} V + \xi R_{\mu\nu} \Phi^* a \Phi^a), \tag{12}
\]

which can be reduced to

\[
R_{\mu\nu} (1 - \kappa \xi F^* a F^a) = \kappa (\partial_\mu \Phi^* a \partial_\nu \Phi^a + \frac{1}{2} g_{\mu\nu} V). \tag{13}
\]

When elements of Ricci tensor calculated from metric coefficients in Eq. (6) and the Ansatz for \( \Phi^a \) in Eq. (7) are substituted into Eq. (13), we obtain the following equations:

\[
(1 - \kappa \xi F^* F) \left[ \frac{B''}{2A} - \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{B'}{rA} \right] = -\kappa \frac{B}{2} V; \tag{14}
\]

\[
(1 - \kappa \xi F^* F) \left[ -\frac{B''}{2B} + \frac{B'}{4B} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{A'}{rA} \right] = \kappa [F'' + \frac{A}{2} V], \tag{15}
\]

\[
(1 - \kappa \xi F^* F) \left[ 1 - \frac{1}{2A} + \frac{r}{2A} \left( \frac{A'}{A} - \frac{B'}{B} \right) \right] = \kappa [F^* F + \frac{r^2}{2} V]. \tag{16}
\]

The first in above equations is the \((t, t)-\)component of Eq. (10), the second is \((r, r)-\)component, and the last is \((\theta, \theta)-\) (or equivalently, \((\phi, \phi)-\) component, respectively.

### 3. FRVC of Stars in Galaxies and Solutions for them

Circular motions of stars in a galaxy are determined by the geodesic equations in the spacetime with metric coefficients in Eq. (6). When we consider the case \( \theta = \frac{\pi}{2} \), dividing Eq. (6) by \( d\tau^2 \) and defining

\[
i \equiv \frac{dt}{d\tau}, \quad \dot{r} \equiv \frac{dr}{d\tau}, \quad \dot{\phi} \equiv \frac{d\phi}{d\tau}, \tag{17}
\]

...
we get the equation
\[ G(r, \dot{r}, \dot{\phi}, \dot{t}) \equiv -B(r)\dot{t}^2 + A(r)\dot{r}^2 + r^2 \dot{\phi}^2 = -1, \]  
where it is used that $ds^2 = -d\tau^2$ in the unit system with $c = 1$. Since $\frac{\partial G}{\partial \phi} = 0$ and $\frac{\partial G}{\partial t} = 0$, we have constants of motion:
\[ \frac{\partial G}{\partial \dot{\phi}} = 2r^2 \dot{\phi} = 2L, \]  
\[ \frac{\partial G}{\partial \dot{t}} = -2B\dot{t} = -2E, \]
where $E$ and $L$ are constants. Using Eqs. (19) and (20), Eq. (18) reads
\[ \dot{r}^2 + V_{rad}(r) = 0, \]
where
\[ V_{rad}(r) = \frac{1}{A(r)}(1 + \frac{L^2}{r^2} - \frac{E^2}{B(r)}). \]
Conditions for stars in a galaxy to have circular motions are:
\[ \dot{r} = 0, \]  
\[ \frac{\partial V_{rad}}{\partial r} = 0, \]  
\[ \frac{\partial^2 V_{rad}}{\partial r^2} > 0. \]
From above equations, we get the rotational velocity
\[ v(\phi) \equiv \frac{r}{\sqrt{B}} \frac{d\phi}{dt} = \sqrt{\frac{rB'}{2B}}. \]
The condition which the metric coefficient $B$ in Eq. (6) satify for the flat rotation curves of stars in galaxies to be explained is given by Guzmán and so on.
\[ \sqrt{\frac{rB'}{2B}} = v = \text{constant}. \]
A solution to the above equation is
\[ B(r) = B_0\left(\frac{r}{r_0}\right)^{2\alpha^2}. \]
We note that the metric coefficient function $B(r)$ in Eq. (26) is determined independently of the other function $A(r)$ in Eq. (6), and that $B$ becomes singular for $r \to \infty$ and for $r \to 0$, but it is not seriously problematic since $v$ is very small(of the order $10^{-4} - 10^{-3}$ and Eq. (26) is valid only for the region where the galactic halo
exists \( r_i < r < r_o \). For example, for our galaxy, \( r_o \simeq 230 \text{ Kpc} \) and for others, \( r_o > 400 \text{ Kpc} \). Centers of galaxies are much more complicated and therefore \( B \) in Eq. (26) should be substituted by other functions for the region \( r < r_i \).

When Eq. (26) is used, Eq. (14) reads

\[
\frac{A'}{A} = \frac{2(v^2 + 1)}{r} - \frac{\kappa rAV}{v^2(1 - \kappa \xi F^*F)},
\]

which can be solved with the actual form of a potential \( V(F^*F) \).

In a very simple case when

\[
V \simeq 0,
\]

Eq. (27) can be easily integrated and we get

\[
A(r) \simeq A_o \left( \frac{r}{r_o} \right)^{2v^2 + 2}.
\]

With \( B(r) \) and \( A(r) \) given in Eq. (26) and Eq. (27), respectively, the following function \( F \) of scalar fields satisfies scalar field equations and Einstein’s equation:

\[
F = \eta \exp(i\sqrt{4v^2 + 2} \ln r),
\]

when

\[
\eta = \text{constant}
\]

and

\[
\xi \simeq -1.
\]

Actually the above condition in Eq. (28) can be satisfied if we consider the potential

\[
V = \exp(-\Phi^a\Phi^a) = \exp(-F^*F) = e^{-\eta^2},
\]

for \( \eta \to \infty \), which is natural in quintessence models for the late-time acceleration of the Universe.\( \text{L}\)

4. Summary and Discussions

In the spherically symmetric static spacetime, we have determined metric coefficients to explain the FRVC of stars in galaxies, with a simple scalar potential \( V \simeq 0 \). With the metric coefficients in Eq. (26) and Eq. (29) we have solved equations of scalar fields non-minimally coupled to gravity and got simple solutions given in Eq. (7) and Eq. (30), which have \( SU(3) \) global symmetry broken to \( U(1) \times U(1) \). If this \( SU(3) \) is the flavor symmetry of light quarks, \( u, d, \) and \( s \), then the \( SU(3) \) triplet of complex scalar fields we have considered as a candidate for galactic dark matter might be mesons, \( Q\bar{q}_a \), which are bound states of a heavy quark \( Q \) and light anti-quark \( \bar{q}_a \), where \( Q \) is one of heavy quarks, \( t, b, \) or \( c \) quark, and \( \bar{q}_1 = \bar{u}, \bar{q}_2 = \bar{d} \) and \( \bar{q}_3 = \bar{s} \).

Since the metric function \( B(r) \) given in Eq. (26) is valid only for \( r_i < r < r_o \), it will be interesting to build more realistic models which are valid for all distance
from the origin, $0 \leq r < \infty$, and astronomical data of rotational velocities of stars in galaxies can be compared with these: Beyond the region to which the galactic halo is pervaded, $r > r_o$, $B(r)$ should be substituted by, for example, one of Schwarzschild spacetime, and for the region $0 \leq r < r_i$, $B(r)$ by a more appropriate function for explaining complicated dynamics of centers of galaxies. These functions will have to be matched with $B(r)$ in Eq. (26) at boundaries, $r = r_i$ and $r = r_o$.

The assumption of Eq. (28) for a scalar potential can be realized when we consider an exponential potential as in Eq. (33). Exponential scalar potentials would be very useful in cosmological models for the recently observed cosmic acceleration. The late-time acceleration of the Universe could be explained by scaling solutions for the scale factor of the Universe, driven by exponential potentials of quintessence. It thus will be more interesting to construct an cosmological model of the accelerating Universe, by using our solutions of scalar fields with the exponential potential in Eq. (33).

Acknowledgements

T. H. Lee thanks Drs. P. Oh, J. Lee, J. Yoon, and J. M. Kim for useful discussions. This work was supported by Korea Research Foundation Grant(KRF-2001-015-DP0091).

References

1. U. Nucamendi, M. Salgado, and D. Sudarsky, gr-qc/0011049
2. M. Bariola and A. Vilenkin, Phys. Rev. Lett. 63 (1989) 341.
3. U. Nucamendi et al, gr-qc/0002001.
4. D. Harari and C. Lousto, Phys. Rev. D 42 (1990) 2626
5. T. Matos, F. S. Guzmán, and L. A. Ureña-López, astro-ph/0102419.
6. T. Matos, F. S. Guzmán, and D. Núñez, astro-ph/0003398.
7. L. A. Boyle, R. R. Caldwell, and M. Kamionkowski, astro-ph/0105318.
8. S. Kasuya, astro-ph/0105408.
9. X. Li, H. Hao, and D. Lin, astro-ph/0107171.
10. S. Perlmutter et al. Ap. J. 483 (1997) 565, astro-ph/9608192; A. G. Riess et al., Astron. J. 116 (1998) 1009, astro-ph/9805201; P. M. Garnavich et al., Ap. J. 509 (1998) 74, astro-ph/9806304; S. Perlmutter et al. Ap. J. 517 (1999) 565, astro-ph/9812153; A. G. Riess et al., astro-ph/0104453.
11. F. S. Guzmán, astro-ph/0003103.
12. T. Matos, F. S. Guzmán, and L. A. Ureña-Lópoz, astro-ph/0102413.
13. A. S. Kulesa and D. Lynden-Bell, Mon. Not. R. Astr. Soc. 255 (1992) 105; C. S. Kochanek, Astrophys. J. 457 (1996) 228.
14. D. Zaritsky, R. Smith, C. S. Frenk and S. D. M. White, astro-ph/9611199.
15. B. Ratra and P. J. E. Peebles, Phys. Rev. D37 (1988) 3406; P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. 325 (1988) L17; C. Wetterich, Astron. Astrophys. 301 (1995) 321; C. Wetterich, Nucl. Phys. B302 (1998) 668; E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D57 (1998) 4686; P. G. Ferreira and M. Joyce, Phys. Rev. D58 (1998) 023503; J. C. Fabris, S. V. B. Goncalves, and N. A. Tomimura, Class. Quantum Grav. 17 (2000) 2983; T. Barreiro, E. J. Copeland, and N. J. Nunes, astro-ph/9910214.