BREAKING THE COSMIC DEGENERACY BETWEEN MODIFIED GRAVITY AND MASSIVE NEUTRINOS WITH THE COSMIC WEB

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ABSTRACT

In a recent work, Baldi et al. highlighted the issue of cosmic degeneracies, consisting in the fact that the standard statistics of the large-scale structure might not be sufficient to conclusively test cosmological models beyond $\Lambda$CDM when multiple extensions of the standard scenario coexist in nature. In particular, it was shown that the characteristic features of an $f(R)$ Modified Gravity theory and of massive neutrinos with an appreciable total mass $\Sigma m_\nu$ are suppressed in most of the basic large-scale structure observables for a specific combination of the main parameters of the two non-standard models. In the present work, we explore the possibility that the mean specific size of the supercluster spines – which was recently proposed as a non-standard statistics by Shim and Lee to probe gravity at large scales – can help to break this cosmic degeneracy. By analyzing the halo samples from N-body simulations featuring various combinations of $f(R)$ and $\Sigma m_\nu$, we find that – at the present epoch – the value of $\Sigma m_\nu$ required to maximally suppress the effects of $f(R)$ gravity on the specific sizes of the superclusters spines is different from that found for the other standard statistics. Furthermore, it is also shown that at higher redshifts ($z \geq 0.3$) the deviations of the mean specific sizes of the supercluster spines for all of the four considered combinations from its value for the standard $\Lambda$CDM case are statistically significant.

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1. INTRODUCTION

The clustering of the large-scale structures (LSS, hereafter) and its evolution with redshift encode information about the statistical properties of the initial conditions of the universe and the laws governing the gravitational instability processes that determine the growth of primordial density perturbations. The basic LSS statistics, such as the two-point correlation function of the density field, the mass function of galaxy clusters and the halo bias, are frequently employed to quantify the main properties of the LSS clustering, thereby providing a handle on the underlying cosmological model. Such basic statistics have played a vital role in establishing the concordance ΛCDM scenario by allowing to place tight constraints on its basic cosmological parameters.

Furthermore, the standard LSS statistics are routinely employed to test not only the key cosmological parameters of the ΛCDM cosmology (e.g., Addison et al. 2013; Di Dio et al. 2014, and references therein) but also the viability of alternative cosmological models, such as e.g. the Warm Dark Matter model (WDM, see e.g. Smith & Markovic 2011; Viel et al. 2012), the coupled Dark Energy scenario (cDE, see e.g. Moresco et al. 2013), and various Modified Gravity theories (MG, see e.g., Sutter & Ricker 2008; Song & Koyama 2009; Stril et al. 2010; Lombriser et al. 2012; Abebe et al. 2013).

The main motivation behind most of these alternative models was to overcome the observational and theoretical shortcomings of the ΛCDM cosmology. For instance, the free-streaming effect of the WDM particles has been invoked as a possible solution to the tension between the ΛCDM predictions and astrophysical observations on (sub-)galactic scales (e.g., Menci et al. 2012, and references therein), even though recent constraints from the Lyman-alpha forest seem to exclude the WDM particle mass range required to address the tension (Viel et al. 2013). Similarly, the dark sector interactions that characterise the cDE scenario can alleviate the fine-tuning problems of the cosmological constant (e.g., Wetterich 1995; Amendola 2000, 2004), while possible large-scale deviations of the laws of gravity from their standard GR form can accommodate the present acceleration of the universe without requiring dark energy with negative pressure (for a comprehensive review, see Clifton et al. 2012).

However, some cautionary remarks have been recently raised on the rosy prospects for the basic statistics of LSS as a powerful discriminator of alternative cosmologies. Wei et al. (2013) theoretically proved that the WDM, cDE and MG models are hard to be differentiated from one another just by tracing the expansion and growth history with LSS observations. Additionally, it was shown that the presence of a cosmological background of massive neutrinos might significantly suppress the main observational footprints of various cDE and MG models (e.g., La Vacca et al. 2009; Motohashi et al. 2013; He 2013).
Such early predictions based on linear observables have been recently confirmed and extended to the non-linear regime of structure formation by Baldi et al. (2014), who studied the simultaneous effect on structure formation of a $f(R)$ MG model and of massive neutrinos (the “$f(R) + \nu$” model, hereafter) by means of large $N$-body simulations. The $f(R)$ gravity is one of the viable and most widely investigated MG theories (see e.g., Lombriser 2014, and references therein) where the Ricci scalar $R$ is replaced by a function $f(R)$ in the Einstein-Hilbert action. With suitable choices of such function, the model can be tuned to match the same expansion history of the standard \( \Lambda \)CDM cosmology (Hu & Sawicki 2007). Nonetheless, the derivative of the $f$ function $df/dR$ – which represents an additional degree of freedom called “the scalaron” – is expected to mediate a fifth-force (see de Felice & Tsujikawa 2010; Sotiriou & Faraoni 2010, for a review), implying that the evolution of density perturbations will be different as compared to \( \Lambda \)CDM even for an identical expansion history. The viability of the model is ensured by the Chameleon screening mechanism (e.g. Khoury & Weltman 2004; Brax et al. 2008) that allows to recover the behaviour of standard GR in overdense regions of the Universe.

In their recent work, Baldi et al. (2014) noted that for a given $f(R)$ model there is a specific value of the total neutrino mass $\sum m_\nu$ that cancels the effects of the Modified Gravity in most of the standard basic LSS observables, thereby yielding – besides an identical expansion history – also the same LSS statistics as \( \Lambda \)CDM at the present level of observational accuracy. In other words, the free-streaming effect of massive neutrinos (Lesgourgues & Pastor 2006) effectively cancels out that of the fifth-force of the $f(R)$ gravity on the large scale structure. Calling it a “cosmic degeneracy”, Baldi et al. (2014) regarded this result as an indication of the fundamental limitation of the basic statistics of LSS as a test of alternative cosmologies, concluding that some novel independent statistics or some other independent constraints (as e.g. a laboratory determination of the neutrino mass) are necessary to break the degeneracy.

Meanwhile, Shim & Lee (2013) have recently developed a new diagnostic based on the filamentary cosmic web for testing gravity at large scales. Such new approach has shown that the filamentary pattern of the cosmic web is significantly affected by deviations from the standard gravitational behaviour, either in terms of MG or cDE models. More specifically, considering the supercluster spines (i.e. the main stems of the superclusters) as the richest filamentary structures in the cosmic web, Shim & Lee (2013) determined their specific sizes to quantify the degree of the straightness of the superclusters, finding that the specific size distributions of the supercluster spines substantially differ among various cDE models (see again Shim & Lee 2013), implying it being indeed a good indicator of cDE. Similarly, in a subsequent paper Shim et al. (2014) also investigated the effect of $f(R)$ gravity on the specific sizes of the supercluster spines and showed that the evolution trends of the specific size distributions of the superclusters differ between the cDE and the $f(R)$ gravity models,
which indicated that this new diagnostic is in principle capable of breaking the degeneracy between the two models.

In the light of the works of Shim et al. (2014) and Baldi et al. (2014), in the present study we aim to explore whether the degree of the straightness of the superclusters quantified by the specific size of the supercluster spines can be also useful for breaking the cosmic degeneracy between the ΛCDM and the $f(R)+\nu$ models. In section 2 we will describe how the supercluster samples are obtained from the simulation datasets obtained from the work of Baldi et al. (2014). In section 3 we will show how the specific sizes of the supercluster spines are affected by the simultaneous effects of the $f(R)$ gravity and the massive neutrinos. In section 4 we discuss the implications of our results as well as possible prospects for future further improvements.

2. CONSTRUCTING THE SUPERCLUSTER SAMPLES

Baldi et al. (2014) carried out large N-body simulations for the standard GR+ΛCDM and also for a $f(R)$ MG model with four possible different values of the total neutrino mass $\Sigma_i m_{\nu_i}$, namely $\{0, 0.2, 0.4, 0.6\} \text{ eV}$. The specific $f(R)$ model considered in the simulations corresponds to the widely investigated parameterization by Hu & Sawicki (2007), for $n = 1$ and for a scalar amplitude at the present epoch of $f_{R0} \equiv df/dR|_{t_0} = -10^{-4}$. Although recent observations indicate $|f_{R0}| \leq 10^{-5}$ on the cluster scale (e.g., Schmidt et al. 2009; Lombriser et al. 2012), such a large value of $|f_{R0}| = 10^{-4}$ was chosen on purpose in order to maximize the effects of MG and to emphasize how even for large deviations from standard GR the cosmic degeneracy between $f(R)$ and massive neutrinos can very effectively suppress the observational signatures of both non-standard models. Furthermore, the above-mentioned observational constraints on $f(R)$ gravity have been derived assuming massless neutrinos, and might therefore become looser (again as a result of the degeneracy between MG and massive neutrinos) if the total neutrino mass is allowed to vary as a free parameter.

The simulations have been carried out by means of a suitable combination of the MG-GADGET code by Puchwein et al. (2013) (specifically designed for MG cosmologies) and of the particle-based implementation of massive neutrinos by Viel et al. (2010). Both algorithms are independent modules of the widely-used TreePM N-body code GADGET (Springel 2005). With such combined code at hand, Baldi et al. (2014) followed the evolution of a cosmological volume of $1 h^{-3} \text{ Gpc}^3$ filled with $2 \times 512^3$ particles for the CDM and neutrino components, with standard cosmological parameters consistent with the latest results of the Planck satellite mission (Planck Collaboration et al. 2013), from $z = 99$ to $z = 0$. A catalog of CDM particle groups has been compiled on-the-fly for several different redshifts by means
of a standard Friends-of-Friends (FoF) algorithm with a linking length set to the conventional value of 20% of the mean inter-particle separation. For a more detailed description of the simulations, we refer the interested reader to the Baldi et al. (2014) paper.

From the cluster-size halos selected from the FoF catalogs at three different epochs \(z = 0, 0.3, 0.6\), we extract the superclusters following the approach described in Shim & Lee (2013) and Shim et al. (2014). First of all, a sample of the cluster-size halos is generated by choosing those FoF halos with masses above a lower threshold of \(10^{13} h^{-1} M_{\odot}\), at each redshift and for each model. The mean separation distance among such cluster-size halos is then calculated and the marginally-bound superclusters are identified by running again a FoF group finder on the positions of the cluster-size halos with the linking length parameter of 30% the mean inter-halo separation. Finally, by binning the halo mass range of the two catalogs and calculating their number densities within each mass bin, we determine both the cluster and the supercluster mass functions.

The top and the bottom panels of Figure 1 show, respectively, the cluster and the supercluster mass functions at \(z = 0\) for all the five models under investigation. As can be clearly seen in the plots, a lower value of \(\Sigma m_{\nu}\) generally exhibits a higher number density of high-mass superclusters \((M \geq 10^{15} h^{-1} M_{\odot})\). Note also that both the cluster and the supercluster mass functions for the ΛCDM model are almost identical to those for the F4+ν04 model (red dot-dashed line), which indicates that the neutrino mass required to cancel out the effect of the fifth force on both observables is of about 0.4 eV. This result is in full agreement with the findings of Baldi et al. (2014) that the standard LSS statistics including the cluster mass functions are not capable of discriminating the ΛCDM model from the F4+ν04 model.

3. SIMULTANEOUS EFFECT OF MG AND \(\nu\) ON THE SUPERCLUSTER STRAIGHTNESS

We analyze the supercluster sample for each model at each redshift to determine the mean specific sizes of the supercluster spines, by the prescriptions described in Shim & Lee (2013) and Shim et al. (2014):

1) Apply the minimum spanning tree (MST) algorithm (Barrow et al. 1985; Colberg 2007) to the supercluster sample from the simulations of Baldi et al. (2014) to determine a MST of each supercluster.

2) From each supercluster MST, prune away repeatedly the minor twigs having less than three nodes until the main stem of each supercluster (i.e. the “supercluster spine”)
is determined and count the number of halos belonging to the main stem (i.e. the “supercluster nodes”).

iii ) Select only those superclusters whose spines consist of three or more nodes.

iv ) Find a cuboid which fits the spatial distribution of the nodes of each supercluster spine and measure the length of its diagonal line as the size of each supercluster spine (see Park & Lee 2009).

v ) Determine the specific size \( \langle \tilde{S} \rangle \) of each supercluster spine by dividing the size by the node number \( N_{\text{node}} \). The supercluster spines whose shapes are closer to straight filaments are expected to have larger specific sizes.

vi ) Calculate the mean value of \( \langle \tilde{S} \rangle \) averaged over all supercluster spines.

Figure 2 shows the mean specific sizes \( \langle \tilde{S} \rangle \) of the supercluster spines for the different cosmological models under investigation. The errors are calculated as one standard deviation in the measurement of the mean value. The value of \( \langle \tilde{S} \rangle \) is shown to monotonically increase as the value of \( m_\nu \) increases from 0 to 0.6 eV. Interestingly, it is not the F4+\( \nu \)04 model but the F4+\( \nu \)02 model which predicts almost the same \( \langle \tilde{S} \rangle \) as the \( \Lambda \)CDM cosmology. In fact, between the \( \Lambda \)CDM and the F4+\( \nu \)04 we find a statistically significant difference in the value of \( \langle \tilde{S} \rangle \). Given that for the standard LSS statistics Baldi et al. (2014) found that the F4+\( \nu \)04 model appears to be hardly distinguishable from the GR+\( \Lambda \)CDM standard scenario, our results indicate that the value of \( \sum_i m_\nu \) required to compensate for the effect of the fifth force on the specific sizes of the supercluster spines and on the basic statistics of LSS are different from each other. This result leads us to expect that it may be possible to break the cosmic degeneracy between the GR+\( \Lambda \)CDM and the F4+\( \nu \) models if the basic statistics of LSS are combined with the specific size distribution of the supercluster spines.

Figure 3 displays the mean specific sizes \( \langle \tilde{S} \rangle \) of the supercluster spines versus redshifts for the five models. As for the previous figures, the (red) dot-dashed line corresponds to the most degenerate combination of the models in the standard LSS statistics (as found by Baldi et al. 2014). As the plot shows, the difference in \( \langle \tilde{S} \rangle \) among the five models becomes larger at higher redshifts, which is consistent with the result of Shim et al. (2014) who showed that the effect of \( f(R) \) gravity on the specific size distribution becomes larger at higher redshifts, and this trend appears in all the models under investigation. The F4+\( \nu \)06 model exhibits the most rapid evolution of \( \langle \tilde{S} \rangle \), which implies that for more massive neutrinos the free streaming has a stronger effect of sharpening the filamentary structures when they were more energetic at higher redshifts. In other words, although it is impossible to distinguish between the GR+\( \Lambda \)CDM and the F4+\( \nu \)02 models by measuring \( \langle \tilde{S} \rangle \) at the present epoch,
this degeneracy is broken by looking at higher redshifts. More specifically, it appears possible in principle to distinguish the two models by comparing the value of $\langle \tilde{S} \rangle$ at $z \geq 0.3$ when the F4+$\nu$02 model significantly deviates from the GR+$\Lambda$CDM standard case.

4. DISCUSSION AND CONCLUSION

This work has been inspired by two recent publications. On one hand, the work of Shim et al. (2014) demonstrated that the specific size of the supercluster spine is a powerful diagnostic not only for detecting any $f(R)$ modification of gravity but also for distinguishing the effects of $f(R)$ gravity from that of the presence of interacting Dark Energy. On the other hand, the recent work of Baldi et al. (2014) raised for the first time a cosmic degeneracy problem consisting in the fact that the basic statistics of LSS (such as e.g. the cluster mass function, the two-point correlation function of the density field and the halo bias) might inherently fail to distinguish a suitable combination of a $f(R)$ MG model and of a specific value of the total mass of neutrinos from the standard $\Lambda$CDM cosmology, since the free streaming effect of massive neutrinos effectively suppresses the extra clustering effect associated with the scalar fifth-force of $f(R)$ gravity.

Using the numerical data from the N-body simulations carried out by Baldi et al. (2014) and the techniques developed by Shim et al. (2014), we have determined the mean specific sizes of the supercluster spines at three different redshifts ($z = 0, 0.3, 0.6$) for an $f(R)$ gravity cosmology with different values of the total neutrino mass. Our investigation showed that the neutrino mass required to maximally suppress the effects of the MG fifth force on the specific size distribution of the supercluster spines at $z = 0$ is different from the value that determines the maximum degeneracy for the basic statistics of LSS. Furthermore, we have also found that at higher redshifts all the considered combined models show larger differences in the mean specific size of the supercluster spine from the standard GR+$\Lambda$CDM cosmology. Therefore, we conclude that the evolution of the specific size of the supercluster spine can in principle play a crucial role in breaking the cosmic degeneracy highlighted by Baldi et al. (2014).

Although such conclusion emerges clearly from the present study, a more thorough investigation of the power of the supercluster spines in breaking the MG-massive neutrinos degeneracy will be required in order to quantify precisely the gain of discriminating power obtained by combining this new statistics with the standard LSS probes. First of all, the datasets employed for our analysis were obtained from a suite of intermediate-resolution N-body simulations. Since the identification of the superclusters – especially at high redshifts – is likely to be affected by the resolution of the original N-body realization, a new set of
halo catalogs from higher resolution simulations is required to examine more carefully how the mean specific sizes of the supercluster spines evolve with redshifts. Secondly, in the current work we have considered only the case of a rather “extreme” (although still possibly consistent with standard LSS data for a reasonable value of the neutrino mass) $f(R)$ scenario, namely $f_{R0} = -10^{-4}$. Therefore, it will be necessary to examine whether the specific size of the supercluster spine is capable of breaking the cosmic degeneracy even for the more challenging case of less severe deviations from the standard GR gravity.

Second of all, our conclusion is based not on a realistic observational error but only on the statistical significance of the differences in the mean specific size of the superclusters spine among the models. It will be essential to estimate an observational confidence region around the fiducial model for the specific size of the supercluster, as Baldi et al. (2014) did for the LSS statistics. Another direction in which some improvements are needed is the development of an analytic model for the mean specific size of the supercluster spine. One of the reasons why the basic LSS statistics have been so widely employed as a test for cosmology is that they can be analytically (or semi-analytically) predicted. On the contrary, the mean specific sizes of the supercluster spines have so far been only numerically determined, without being guided by any analytic prescription, which is an obvious drawback that has to be overcome for its practical application in the future. We leave these three main extensions of our analysis for future work.

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Fig. 1.— Mass functions of the cluster halos and the superclusters for five different models at $z = 0$ in the top and the bottom panel, respectively. In each panel the (red) dot-dashed line corresponds to the *most degenerate* combination of the models in the standard LSS statistics.
Fig. 2.— Mean specific sizes of the supercluster spines for five different models at $z = 0$. The errors represent one standard deviation in the measurement of the mean value.
Fig. 3.— Evolution of the mean specific sizes of the supercluster spines for five different models.