A new approach to assessment of vehicular traction dynamics

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Abstract. The work shows the inaccuracy of the statement that the traction moment on the traction wheel is an internal force and the tangential reaction on this wheel is an external force. The author proves that the rolling resistance moment of the vehicle wheels is an internal force, and the losses caused by this moment should be covered by the transmission efficiency factor. The paper revises the equation for traction dynamics of the translational motion of a vehicle, which can be used in the analysis of its traction-speed properties.

1. Introduction

Vehicular traction dynamics depicted in the classical works of domestic and foreign scientists require clarification and harmonisation with the provisions of classical mechanics.

One of the erroneous statements is the statement that the traction moment on the traction wheel is an internal force and the tangential reaction on this wheel is an external force. In terms of mechanics, it should be just the opposite.

This article shows that the rolling resistance moment of the vehicle wheels is also an internal force, and the losses caused by this moment should be covered by the transmission efficiency factor. This requires a revision of the equations of vehicular traction dynamics, which was done by the authors of the proposed study.

2. Analysis of the latest achievements and publications

The issues of vehicular traction dynamics are considered in the works of E.A. Chudakov [1,2], N.A. Yakovlev [3], B.S. Falkevich [4], V.A. Ilarionov [5] and other authors.

In the work of E.A. Chudakov [1], an equation was proposed showing the distribution of the traction force $P_k$ by individual types of resistance, which was named the vehicular traction balance

$$ P_k = P_f + P_w + P_i + P_j $$

where $P_f$ – rolling resistance force;
$P_w$ – aerodynamic drag force;
$P_i$ – resistance force when driving uphill (climbing resistance);
$P_j$ – inertial force during uneven movement.

This equation is also given in works [2-5]. However, in [6] it is shown that the equation (1) cannot be considered the traction balance formula, since balance means equilibrium, while the inertial force $P_j$
is a manifestation of the imbalance of forces. The work [6], proposes presenting the terminology and form of the equation (1) as:

$$m_a \frac{dV_a}{dt} = P_k - P_f \pm P_i - P_w, \quad (2)$$

where $m_a$ – vehicle weight;

$dV_a/dt$ – linear equation of translational motion of a vehicle.

Equation (2) is the equation for the dynamics of translational motion. The left side of the equation determines the lack of balance of forces acting on the vehicle. When the left side is equal to zero (at $dV_a/dt=0$), the traction balance occurs. The traction balance can be static, where $dV_a/dt=0$ at any moment of time, and dynamic, with periodically occurring positive and negative accelerations [7]. A dynamic traction balance exists in a steady motion, since a uniform motion of the vehicle is possible only in theoretical analysis [7].

The second conditional assumption made in [1-5] is the introduction of the rotational inertia coefficient $\delta$ in the left side of equation (2), i.e.

$$\delta_j m_a \frac{dV_a}{dt} = P_k - P_f \pm P_i - P_w \quad (3)$$

The $\delta_j$ rotational inertia coefficient for the transmission and engine was obtained from the equation of vehicular traction force [4].

$$P_k = \frac{M_e U_k U_o \eta_{tr} - Y_e U_k^2 \eta_{tr} \frac{dV_a}{dt}}{r_\delta}, \quad (4)$$

where $M_e$ – effective engine torque;

$U_k, U_o$ – transmission forces of the gearbox and axle drive gear of the vehicle;

$\eta_{tr}$ – instantaneous efficiency of the transmission;

$\Sigma Y_k$ - resultant moment of inertia of all wheels of the vehicle;

$r_\delta$ – effective radius of wheels of the vehicle;

$Y_e$ – reduced moment of inertia of the motor.

The resultant tractive resistance force of the vehicle (in equation (2))

$$\Sigma P_c = P_f \pm P_i + P_w, \quad (5)$$

After substituting expression (4) into equation (2), taking into account (5), we obtain:

$$m_a \left(1 + \frac{Y_e U_k^2}{w_a r_\delta^2} \cdot \frac{U_o^2}{\eta_{tr}} + \frac{\Sigma Y_k}{w_a r_\delta^2} \frac{dV_a}{dt} \right) = \frac{Y_e U_k^2 \eta_{tr} \frac{dV_a}{dt}}{r_\delta} - \Sigma P_c, \quad (6)$$

The authors of [1-5] called the part highlighted in the left side of equation (6) “the rotational inertia coefficient of the engine and transmission.”

$$\delta_j = 1 + \frac{Y_e U_k^2}{w_a r_\delta^2} \cdot U_o^2 \cdot \eta_{tr} + \frac{\Sigma Y_k}{w_a r_\delta^2} = 1 + \sigma_1 + \sigma_2 U_k^2 \quad (7)$$

where $\sigma_1; \sigma_2$ – coefficients depending on the vehicle design and proportional moments of inertia $Y_e$ and $\Sigma Y_k$,

$$\sigma_1 = \frac{\Sigma Y_k}{w_a r_\delta^2}; \quad (8)$$

$$\sigma_2 = \frac{Y_e U_k^2 \eta_{tr}}{w_a r_\delta^2}. \quad (9)$$

After processing the statistical data, an empirical formula was proposed in [8]:

$$\delta_j = 1.03 + 0.05 U_k^2. \quad (10)$$
When analysing the dynamics of vehicle acceleration, introduction of rotational inertia coefficient of the engine and transmission \( \delta_j \) provides for taking into account the energy loss for the acceleration of the engine and transmission masses. However, in this case, an error is introduced when determining the adhesion-limited vehicular traction force of the driving wheels to the road. To prevent this error, the concept of instantaneous dynamic efficiency of the transmission was introduced in [9] \( \eta_{tr}^{din} \).

Expression (4) in [9] was transformed to the form:

\[
P_k = \frac{M_e U_k U_0 \eta_{tr}}{r_0} \left[ 1 - \frac{dV_a}{d\tau} \left( Y_e \cdot U_k \cdot U_0 + \frac{\Sigma Y_k}{u_k u_0 \eta_{tr}} \right) \right]
\]

The expression of equation (11) in square brackets [9] is precisely the instantaneous dynamic efficiency of the transmission \( \eta_{tr}^{din} \), which determines the transmission’s loss of the engine energy due to rotational inertia:

\[
(\eta_{tr}^{din})_{inst} = 1 - \frac{dV_a}{d\tau} \left( Y_e \cdot U_k \cdot U_0 + \frac{\Sigma Y_k}{u_k u_0 \eta_{tr}} \right) \]

In work [9], an expression was also obtained for the efficiency factor, taking into account losses due to dry, viscous friction:

\[
(\eta_{tr})_{inst} = \eta_{tr}^{st} + (\eta_{tr}^{kin})_{inst} - 1, \tag{13}
\]

where \( \eta_{tr}^{st} \); \( \eta_{tr}^{kin} \); – components of transmission efficiency factor that take into account losses for dry and viscous friction, respectively. Taking into account the dynamic efficiency of the transmission, the following formula was proposed in [9]:

\[
(\eta_{tr})_{inst} = (\eta_{tr}^{st})_{inst} + (\eta_{tr}^{kin})_{inst} + (\eta_{tr}^{din})_{inst} - 2, \tag{14}
\]

Equation (12) with due consideration of (13) takes the form:

\[
(\eta_{tr}^{din})_{inst} = 1 - \frac{dV_a}{d\tau} \left[ Y_e \cdot U_k \cdot U_0 + \frac{\Sigma Y_k}{u_k u_0 (\eta_{tr}^{st} + \eta_{tr}^{kin} - 1)} \right] \]

In work [10], it is proved that from the standpoint of classical mechanics the rolling resistance moment of the wheels is an internal force in the mechanism of the vehicle's running gear. In this case, the instantaneous efficiency of the wheel can be defined as [10]

\[
\eta_{k}^{inst} = \eta_{k}^{inst}_{force} \eta_{k}^{inst}_{elast}, \tag{16}
\]

where \( \eta_{k}^{inst}_{force} \) – instantaneous wheel force efficiency factor;

\[
\eta_{k}^{inst}_{force} = 1 - \frac{M_f}{M_k}, \tag{17}
\]

\( M_f \) – wheel rolling resistance moment;

\( M_k \) – wheel torque;

\( \eta_{k}^{inst}_{elast} \) – instantaneous elastic efficiency of the wheel, with due account of losses on circumferential strain of the tire;

\[
\eta_{k}^{inst}_{elast} = \tau_k / \tau_0, \tag{18}
\]

\( \tau_k \) – kinematic radius of the wheel.

In well-known studies [1-10], the influence of the instantaneous efficiency of a wheel on the instantaneous efficiency of a transmission is not taken into account. Taking into account the instantaneous efficiency of the wheel provides for a different approach to the analysis of the dynamics of vehicle's translational motion.
3. Goal and formulation of research objectives
The aim of the study is to improve the method for assessing the dynamic properties of a vehicle transmission by specifying the instantaneous efficiency factor by taking into account the rolling resistance of wheels and circumferential deformation of tires.
To achieve the stated goal, it is necessary to obtain the dependence of the instantaneous efficiency of the transmission from its design parameters, the rolling resistance of the wheels and the circular stiffness of the tires.

4. Presentation of new material
In the further consideration of the material, we will distinguish between the wheel traction force $P_k$ and vehicle traction force $P_x$. The vehicular traction force $P_x$ is applied to the vehicle frame and is a part of the the equation of translational motion:

$$m \frac{dv_a}{dt} = P_x - P_u,$$  \hspace{1cm} (19)

The [11] presents the equation of the force balance of vehicle’s driving wheel:

$$R_{zk} = M_{kk} - R_{xk} r_D,$$  \hspace{1cm} (20)

where $R_{zk}$ – normal road reaction to wheel of the vehicle;
$M_{kk}$ – wheel torque;
$a$ – rolling friction coefficient,
$R_{xk}$ – tangential reaction on the wheel.

Vehicular traction force

$$P_x = \sum_{i=1}^{n} R_{xki} - \sum_{j=1}^{m} R_{zkj},$$  \hspace{1cm} (21)

where $n$ – number of driving wheels of the vehicle;
$m$ – number of driven wheels of the vehicle;
$f$ – rolling resistance coefficient,

$$f = \frac{a}{r_D},$$  \hspace{1cm} (22)

Dividing the left and right sides of equation (20) by the dynamic radius of the wheel $r_D$, we obtain:

$$P_{fk} = P_{kk} - R_{fk},$$  \hspace{1cm} (23)

From where we define:

$$P_{sk} = P_{kk} - R_{fs},$$  \hspace{1cm} (24)

After substituting (24) into (21), we get:

$$P_\xi = \sum_{i=1}^{n} R_{kki} - P_{fki} - \sum_{j=1}^{m} R_{zkj} = P_k - f m_ag,$$  \hspace{1cm} (25)

where $g$ – acceleration of gravity, $g = 9.81 \text{ m/s}^2$.

Instantaneous efficiency of vehicle’s driving wheels:

$$\eta_k^{\text{inst}} = \frac{P_k \cdot v_a}{M_k \cdot w_k} = \frac{r_k}{r_0} \left(1 - \frac{M_f}{M_k}\right),$$  \hspace{1cm} (26)

Thus, the revised transmission efficiency can be defined as:

$$\left(\eta_{tr}\right)_{\text{inst}} = \left[(\eta_{tr}^s)_{\text{inst}} + (\eta_{tr}^{kin})_{\text{inst}} + (\eta_{tr}^{din})_{\text{inst}} - 2\right] \cdot \eta_k^{\text{inst}}.$$  \hspace{1cm} (27)

After substituting expressions (14), (15) and (26) into equation (27) we get:

$$\left(\eta_{tr}\right)_{\text{inst}} = \left[(\eta_{tr}^s)_{\text{inst}} + (\eta_{tr}^{kin})_{\text{inst}} - 1 - \frac{dv_a/dt}{M_k \cdot r_0} \left[v_a \cdot U_k + \frac{\sum v_k}{U_k \cdot U_0 (\eta_{tr}^{din} - 1)}\right]\right] \cdot \frac{r_k}{r_0} \left(1 - \frac{\sum M_f}{M_k}\right),$$  \hspace{1cm} (28)

where $M_k$ – resultant torque on the driving wheels of the vehicle;
\[ \sum M_f \] – resultant rolling resistance moment on all wheels of the vehicle;

\[ \sum M_f = m_a \cdot g \cdot f \cdot r_\theta, \] (29)

Analysis of equation (28) shows that for

\[ M_k = \sum M_f \] (30)

transmission efficiency factor \( \eta_{tr} \) is zero. When implementing the adhesion-limited tangential road reaction on the driving wheels

\[ M_k = K_{adh} \cdot m_a \cdot g \cdot \phi_x \cdot r_\theta, \] (31)

where \( K_{adh} \) – coefficient of use of the adhesion weight of the vehicle (for a four-wheel drive vehicle \( K_{adh} = 1 \));

\( \phi_x \) – longitudinal coefficient of adhesion of wheels to the road.

The instantaneous efficiency factor of the driving wheels \( \eta_k^{\text{inst}} \) is equal to the product of the instantaneous force \( \eta_k^{\text{inst, force}} \) and elasticity \( \eta_k^{\text{inst, elast}} \) efficiency factors (see dependence (16) [10]). To find out the instantaneous elastic efficiency of driving wheels, we will use the formula for determining the kinematic \( r_k \) and dynamic \( r_\theta \) radii of the wheel [10]

\[ r_k = \left[ 1 - \frac{M_k}{\sum C_{angl}} \cdot \eta_k^{\text{inst, force}} \right] \left( r_0 - \frac{P_z}{2C_2} \right), \] (32)

\[ r_\theta = r_0 - \frac{P_z}{2C_2}, \] (33)

where \( C_{angl}; C_z \) – torsional and radial stiffness coefficients of driving wheels;

\( P_z \) – normal wheel load, \( P_z = R_z \),

\( r_0 \) – free wheel radius.

The instantaneous elastic efficiency of the wheel (dependence (18), taking into account relations (32) and (33)) will take the form:

\[ \eta_k^{\text{inst, elast}} = 1 - \frac{M_k}{\sum C_{angl}} \cdot \eta_k^{\text{inst, force}}, \] (34)

Thus, the instantaneous efficiency of the wheel, taking into account relations (16), (17) and (34), will have the following form:

\[ \eta_k^{\text{inst}} = \left( 1 - \frac{M_f}{M_k} \right) \left[ 1 - \frac{M_k}{\sum C_{angl}} \left( 1 - \frac{M_f}{M_k} \right) \right] \] (35)

With multiple driving wheels:

\[ \eta_k^{\text{inst}} = \left( 1 - \frac{\sum M_f}{M_k} \right) \left[ 1 - \frac{M_k}{\sum C_{angl}} \left( 1 - \frac{\sum M_f}{M_k} \right) \right], \] (36)

where \( \sum C_{angl} \) – resultant torsional rigidity of driving wheels.

Taking into account (36), the equation (28) can be represented as:

\[ (\eta_{tr})_{\text{inst}} = A - B \cdot \frac{dv_a}{dt} \left( 1 - \frac{2M_f}{M_k} \right) \left[ 1 - \frac{M_k}{\sum C_{angl}} \left( 1 - \frac{M_f}{M_k} \right) \right], \] (37)

where \( A, B \) – generalised coefficients,

\[ A = (\eta_{tr}^{\text{inst}} + (\eta_{tr}^{\text{kin}})_{\text{inst}} - 1), \] (38)

\[ B = \frac{1}{M_{eff}} \left\{ Y_a \cdot U_k \cdot U_0 + \frac{\sum Y_k}{U_0 \left[ \eta_{tr}^{\text{inst}} + (\eta_{tr}^{\text{kin}})_{\text{inst}} - 1 \right]} \right\}, \] (39)
Figure 1 shows the diagrams of the dependence of the instantaneous efficiency of the transmission from the vehicle acceleration, at various values of the coefficient $B$. The coefficient $A$ as a function of the product $U_k U_o$ has a minimum.

Conditions for obtaining $A_{\text{min}}$ are as follows:

$$\frac{\partial B}{\partial (U_k K_o)} = 0; \quad (40)$$

$$\frac{\partial^2 B}{\partial (U_k K_o)^2} > 0; \quad (41)$$

Figure 1. Dependencies $(\eta_{tr})_{\text{inst}} = F \left( \frac{dV_o}{dt} \right)$ at different values $B(B_1 < B_2 < B_3)$.

Condition (40) has the form:

$$Y_e - \frac{\Sigma Y_k}{u_k^2 u_o^2 [(\eta_{tr})_{\text{inst}} + (\eta_{tr})_{\text{inst}}]^{-1}} = 0; \quad (42)$$

From which we define

$$U_k U_o = \frac{\Sigma Y_k}{\sqrt{Y_e} \left( (\eta_{tr})_{\text{inst}} + (\eta_{tr})_{\text{inst}} \right)^{-1}}; \quad (43)$$

We define

$$\frac{\partial^2 B}{\partial (U_k U_o)^2} = 2 \frac{\Sigma Y_k}{M_r r \dot{\theta}} \left( \frac{u_k^2 u_o^2 [(\eta_{tr})_{\text{inst}} + (\eta_{tr})_{\text{inst}}]^{-1}}{2} \right) > 0. \quad (44)$$

Thus, expression (43) determines the value of the gear ratio of the transmission $U_k U_o$, where $B = B_{\text{min}}$.

The results obtained allow us to write the equation of the dynamics of translational motion of a vehicle in the following form:

$$m_r \frac{dV_o}{dt} = \frac{m_r U_k U_o}{r \dot{\theta}} (\eta_{tr})_{\text{inst}} \pm P_l - P_w \quad (45)$$

The resulting equation (45) can be used to analyse the traction-speed properties of vehicles.

5. Conclusions

1. A wheel with an elastic tire is the final element of the vehicle's transmission. Taking into account energy losses in tires when determining the transmission efficiency simplifies the analysis of the traction and speed properties of the vehicle.

2. The results obtained became possible when considering the rolling resistance moment of the wheels as internal, rather than external forces.
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