A study of a complex model of opinion dynamics in social networks

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Abstract. Many micro-level models of information processes in social networks consider either a change in the information-psychological state of agents (opinions, beliefs, attitudes) or a change in their observable behavior (actions). The goal in this paper is to develop and analyze a complex agent-based model of opinion dynamics that describes the dynamics of agents’ beliefs and the process of performing actions by agents. The issues of reaching consensus and polarizing opinions of agents are investigated.

1. Introduction

In the theory of the dynamics of information processes in social networks, micro- and macro-level models are traditionally developed and studied. The former consider either a change in the “internal” information-psychological state of individuals (e.g., opinions) or a change in their “external” behavior (e.g., actions). To date, many models have been proposed and analytically investigated, both within the first line of research (the DeGroot model, the Friedkin–Johnsen model, the Hegselmann–Krause model, etc. [1,2,3,4,6]) and the second one (threshold models, epidemiological models, etc. [5,6]).

Meanwhile, for an adequate study of informational control and confrontation in society, complex models of individuals in a social network are important, which describe their behavior and control of their behavior. In such models, the interrelated parameters reflecting both the internal state and external behavior are explicitly taken into account. As was shown in the paper [7], such an approach can be used to simulate key socio-psychological effects in social networks: the opinion consensus, the persistence of differences in beliefs, and the growth of disagreements. Modeling the latter effect seems to be an important problem in various fields of application. In sociology and political science, it is supposed that the formation of isolated communities (echo chambers) and the accompanying polarization effect pose a threat to modern society.

This paper introduces and investigates a nonlinear model of individuals' behavior in a social network. The model considers interrelated processes of activity, namely, the process of the dynamics of individuals' opinions and the process of performing actions by them. Section 2 describes this model briefly, suggests an asymmetric measure of polarization, and states an informational control problem. In Section 3, we study via simulation modeling the dependence of individuals' behavior on the model parameters and the control actions for particular model cases.
2. Model of Behavior of Individuals in Social Network

2.1. Model description
Consider the set \( N = \{1, 2, \ldots, n\} \) of interacting individuals (agents). The interaction of agents is carried out within a network structure \( G = (N, E) \), where \( E \) is the set of relations between agents.

Participants in a social network exchange information and try to eliminate uncertainty about some issue (as a rule, the state of nature), forming their own beliefs about it. Assume that there are several polar positions from the set \( \{1, 2, \ldots, m\} \) on the issue discussed in the network.

Introduce the parameters of the internal state of agent \( i \in N \), reflecting his essential individual characteristics: an opinion on the issue \( x_i \in \mathbb{R}_+^m \), representing a “mixture” of polar positions \( \sum_{i=1}^m x_{i1} = 1 \), and the readiness level for action \( p_i \in [0, 1] \).

Let agent \( i \) be activated, \( y_i \in \{0, 1\} \), depending on his readiness level for action. In case of activation, the agent performs an action \( s_i \in S \), which reflects his opinion to some extent.

Assume that each agent always knows his state, and his action is observable for his entire neighborhood. The actions of neighbors observed by each agent can change his internal state.

![Figure 1. Behavior of agent in social network.](image-url)

Following [7], describe in brief the formation of opinions and the actions of agents in this social network; see figure 1.

2.1.1. Formation of agents’ opinions. At each time step \( t = 1, 2, \ldots \), the opinion of agent \( i \) evolves in accordance with the recursive equation

\[
x_i^{(t)} = \beta x_i^{(0)} + (1 - \beta) \left( b_{ii}^{(t-1)} x_i^{(t-1)} + \sum_{j \in N_i} b_{ij}^{(t-1)} s_j^{(t-1)} \right),
\]

where a coefficient \( 0 \leq \beta \leq 1 \) reflects the agent’s adherence to his initial opinion, a matrix \( B^{(t)} = (b_{ij}^{(t)}) \) expresses the agent’s trust in his neighbors, and \( N_i = \{k \in N: a_{ik} > 0\} \) is the set of all his neighbors (adjacent nodes) in the network. In other words, the opinion of agent \( i \) at each time step depends on his own initial opinion and on the actions of his environment at the previous time step.

The trust matrix is variable and reflects the trust degree of agent \( i \) in the actions of other agents depending on their content:

\[
b_{ij}^{(t)} = \begin{cases} \phi \left( d_{ij}^{(t)} \right), & j \in N_i \land y_j^{(t)} = 1 \\ 1, & j = i \\ 0, & \text{otherwise} \end{cases}
\]

where \( d_{ij}^{(t)} = 1 - \sum_{l=1}^m \min(x_{il}^{(t)}, s_{jl}^{(t)}) \) is a measure of dissimilarity between the opinion of agent \( i \) and the action of agent \( j \), and \( \phi \) is some function defined on the half-interval \([0, +\infty)\). Assume that the row normalization condition holds, i.e., the matrix \( B^{(t)} \) is row stochastic.

If \( \phi(d) \) is a monotonically decreasing function on the half-interval \([0, +\infty)\), agent \( i \) has maximum trust in the neighbors whose actions coincide with his opinion. Some examples of such functions are \( \phi(d) = \exp(-\gamma d) \), \( \phi(d) = 2/(\exp(2d) + 1) \), and \( \phi(d) = \max\{0, 1 - d/\gamma\} \).
2.1.2. **Activation of agents and actions performed by them.** At the initial time step, agent \(i\) has a readiness level for action inherent to him. At all subsequent time steps \(t = 1, 2, \ldots\), the agent’s readiness level also depends on the activity of his neighbors in the network:

\[
p_i^{(t)} = \alpha p_i^{(0)} + (1 - \alpha) \sum_{j \in N} a_{ij} y_j^{(t-1)},
\]

where a coefficient \(0 \leq \alpha \leq 1\) can be treated as the agent’s personality: some tend to act more often and others less often, regardless of the behavior of their environment; \(A = (a_{ij})\) is an influence matrix, stochastic in rows. In a practical interpretation, the agent’s readiness is related to the activity of his environment.

Accordingly, at a time step \(t\) agent \(i\) becomes active, \(y_i^{(t)} = 1\), with a probability \(p_i^{(t)}\), and then chooses an action \(s_i^{(t)}\) from the set \(S\). If the set of admissible actions is continuum, suppose that the action completely reflects the agent’s opinion (is informative for all observers in the network). If the set of admissible actions is finite, suppose that his action reflects one of the polar positions on the issue he mostly adheres to:

\[
P \left( s_i^{(t)} = I_{lt} \mid y_i^{(t)} = 1 \right) = x_{lt}^{(t)},
\]

where \(I_{lt}\) denotes the \(l\)th column of an identity matrix.

2.2. **Measure of polarization**

In the paper [8], an approach was earlier proposed to construct a measure of polarization in the case of several poles of opinions \((m \geq 2)\). It is based on dividing the set of agents into two subsets, the ones gravitating to pole 0 and the ones gravitating to pole 1. In the multidimensional case, the two poles can be distinguished in different ways, each corresponding to a partition of the set \(M = \{1, \ldots, m\}\) into two non-empty subsets. Under a fixed a partition \(M = M_0 \cup M_1, M_0 \cap M_1 = \emptyset\), to agent \(i\) with an opinion vector \(x_i = (x_{i1}, \ldots, x_{im})\) assign his aggregated opinion characterizing the location of the agent between the two poles \(M_0\) and \(M_1\), i.e., the number \(y_i = \sum_{j \in M} x_{ij}\). The proposed measure of polarization depends on the partition \(M = M_0 \cup M_1\):

\[
\pi(M_0, M_1) = \frac{4}{n^2} \max_{k \in [n]} \sum_{i \in N_1(k), j \in N_0(k)} (y_i - y_j),
\]

where, without loss of generality, let the aggregated opinions be arranged in the non-decreasing order \((y_1 \leq \ldots \leq y_n)\) and maximum be calculated over all possible partitions of the agents into two subsets \(N_0(k) = \{1, \ldots, k\}\) and \(N_1(k) = \{k + 1, \ldots, n\}\).

2.3. **Control of polarization in social network**

A problem of informational control [3] is managing the polarization in a social network. Consider the case when a control authority (Principal) seeks to minimize the degree of polarization in a network by a time step \(T\), affecting the initial readiness levels of agents and, hence, the dynamics (1)–(3). (For example, in a practical interpretation, the Principal provides network agents with some new information.) Then the control problem can be formulated as follows:

\[
\pi^T(M_0, M_1) \rightarrow \min, \tag{5}
\]

where \(U\) is the set of all possible informational controls of the Principal. The solution of the control problem (5) is the optimal informational impact of the Principal in the described situation.

3. **Results of Simulation Modeling**

The model considered above is rather complicated; within this model, it is not easy to obtain solutions of the problems of analysis and design of optimal controls in explicit form. The solutions can be obtained and evaluated using simulation modeling. Also, simulation modeling allows checking the hypotheses about the behavioral principles of participants in a social network adopted during the development and analysis of the model, that is, allows assessing the adequacy of the model.

In this section, assume that a social network is a complete graph, and the set of network participants is finite: \(N = \{1, \ldots, 20\}\). The set of polar positions on the issue under consideration is
also finite: \( M = \{1, 2\} \) \( (M_0 = \{1\}, M_1 = \{2\}) \). The beliefs of individuals at the initial time step are uniformly distributed on the interval \([0.1, 0.9]\). The trust function is \( \phi(d, \gamma) = \max\{0, 1-d/\gamma\} \).

Below, the dependence of polarization in this network on the model parameters will be studied for each of the following specialized models.

- **Model A**, in which agents act at each time step and the actions of agents completely reflect their opinions (the actions are perfectly informative).
- **Model B**, in which agents decide to act based on their internal readiness for action and the actions of agents completely reflect their opinions.
- **Model C**, in which agents decide to act based on their internal readiness for action and the actions of agents partially reflect their opinions (the actions are imperfectly informative).

**Model A.** Consider two cases with the same initial conditions but different values of the parameter \( \gamma \). In a practical interpretation, this parameter can be treated as a threshold of perception: the agent perceives actions from his neighbors if the differences in opinions do not exceed a given threshold.

If the value of the parameter is small (\( \gamma = 0.1 \)), sets of agents with the same opinions are formed in the network; see figure 2a. The final degree of polarization is moderate: \( \pi = 0.38 \).

**Figure 2.** Opinion dynamics (first component of vector \( x_i^0(0) \)).

For a higher threshold \( \gamma = 0.3 \), the agents in the network reach a consensus (figure 2b), and the final degree of polarization is \( \pi = 0 \).

**Model B.** Consider the case with the basic parameter values \( \gamma = 0.3 \), \( p^0 = 0.1 \), and \( \alpha = 1.0 \). Experiments show that a high initial activity of agents leads to the rapid formation of stable opinions. At the same time, differences in the initial activity can lead to the formation of various communities in the network; see figure 3. A high activity of “extremists” \( (x^0 < 0.3, p^0 = 1.0) \) allows them to maintain their opinions and influence their neighbors; however, the “distant” agents form a separate community.

**Figure 3.** Opinion dynamics (points correspond to active agents).
An increase in the sensitivity of agents to the neighbors' activity (\(\alpha = 0.5\), see figure 3b) leads to (a) an increase in the activity of the whole society and (b) the rapid achievement of equilibrium.

Model B is non-deterministic: the examples presented above are indicative, but still some particular realizations of the model. Therefore, for various combinations of the input parameters, a series of realizations was obtained, and then the influence of the model parameters (input factors) on the output function (the degree of polarization) was analyzed. According to the calculation of the global sensitivity indices [6], the parameters \(\beta (0.449 \pm 0.041)\), in the first place,\(^3\) and \(\gamma (0.310 \pm 0.028)\), in the second place, have the greatest effect on the final degree of polarization. In addition, the paired interaction of the parameters \(\gamma\) and \(\beta\) (0.158 \pm 0.048) is significant. At the same time, the parameters \(p^0\) (0.039 \pm 0.013) and \(\alpha\) (0.010 \pm 0.007) are insignificant for the steady state of the network.

**Model C.** The actions of agents partially reflect their opinions; this leads to a sharp polarization of the network. In particular, in the case of the parameter values \(\gamma = 0.3\), \(p^0 = 0.1\), and \(\alpha = 0.5\), the society has the degree of polarization \(\pi = 0.61\); see figure 4a. A high initial activity of agents with moderate opinions (one third of the agents have \(p^0 = 1.0\)) accelerates the achievement of equilibrium (and, accordingly, the process of polarization, although the degree of polarization does not change). At the same time, moderate agents may not change their opinion but influence other agents; see figure 4b.

![Figure 4. Opinion dynamics (each type of action has its own marker of points).](image)

An increase in the value \(\gamma\) first leads to an increase in polarization (for \(\gamma = 0.6\), the degree of polarization is \(\pi = 0.99\); see figure 5a) and then, starting from some threshold, to its decrease (for \(\gamma = 0.7\), the degree of polarization is \(\pi = 0.84\); see figure 5b). This can be explained by the fact that one of the groups of agents, due to its activity, attracts most of the other agents.

![Figure 5. Opinion dynamics.](image)

\(^3\)Note that 95% confidence intervals are built for the indices. Also, each of the parameters \(\alpha\), \(\beta\), \(\gamma\), and \(p^0\) is assumed to have the same value for all agents.
According to the calculation of the global sensitivity indices [9], the parameters $\gamma$ (0.436±0.047) and $\beta$ (0.204±0.041) have the greatest effect on the final degree of polarization. Compared to model B, in model C the parameter $\gamma$ has become dominant. The paired interaction of the parameters $\gamma$ and $\beta$ (0.272 ± 0.081) is also significant. The parameters $p^0$ (0.039±0.014) and $\alpha$ (0.009±0.006) are insignificant for the steady state. However, see the examples above, they play an important role in the transient process.

In view of the conditions given at the beginning of Section 3, the following conclusions are. First, if the Principal seeks to reduce the final degree of polarization in society, he should primarily influence the values of the parameters $\gamma$ and $\beta$. Second, if the Principal seeks to reduce the degree of polarization in the short-term perspective, he should also pay attention to the parameters $p^0$ and $\alpha$.

4. Conclusions

A nonlinear micro-level model of opinion dynamics in social networks has been considered. This model explicitly takes into account the interrelated parameters of an individual, reflecting both the internal state and external behavior. An asymmetric measure of polarization has been studied, and a corresponding informational control problem has been stated. For particular cases of the model, using simulation modeling, the dependence of the behavior of individuals on the model parameters has been analyzed, and the influence of the parameters on the final degree of polarization in the network has been assessed. The possibility of reaching a consensus of opinions and of growing disagreements in the network has been illustrated.

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