Effects of Majorana bound states on dissipation and charging in a quantum resistor-capacitor circuit

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Abstract

We investigate the effects of Majorana bound states on the ac response of a quantum resistor-capacitor circuit which is composed of a topological superconducting wire whose two ends are tunnel-coupled to a lead and a spinless quantum dot, respectively. The Majorana states formed at the two ends of the wire are found to suppress completely or enhance greatly the dissipation, depending on the strength of the overlap between two Majorana modes and/or the dot level. We compare the relaxation resistance and the quantum capacitance of the system with those of non-Majorana counterparts to find that the effects of the Majorana state on the ac response are genuine and cannot be reproduced in ordinary fermonic systems.

1. Introduction

Topological superconductors [1] has drawn a great interest of solid-state physics society for last decade because they can host quasiparticle excitations behaving like the Majorana fermions that are their own anti-particles [2, 3, 4, 5, 6, 7]. Due to the inherent topological protection, Majorana quasiparticles, being formed at the edges of the topological superconductors, can behave as nonlocal qubits being resistant to decoherence phenomena [2]. In addition, they can perform non-Abelian statistics, making them the fundamental basis for the realization of topological quantum computation [8]. Since the realization of the elusive particles in solid-state systems was proposed, many experimental implementations for the Majorana systems have been reported. Most of them were based on a nanowire with strong spin-orbit interaction put in proximity to a $s$-wave superconductor and exposed to a magnetic field [9, 10, 11, 12, 13, 14, 15], where the evidence for Majorana states were given by the appearance of the zero-bias anomaly in tunnel spectroscopy or the fractional ac Josephson effect [16]. Another experimental setup used a magnetically ordered atomic chain on a superconducting surface and the spatially-resolved spectroscopy revealed the existence of zero energy states at its ends [17, 18]. While the unambiguous detection of the Majorana state is still questioned, it is time to implement Majorana-based circuits from the topological
superconductors and to investigate their operation in the presence of the interaction with environment. A recent study \cite{19} investigated the quantum resistor-capacitor (RC) circuit in which a quantum dot is coupled to chiral Majorana modes formed around the edge of a two-dimensional topological superconductor. The Majorana modes are found to open a dissipative channel inside the dissipationless superconductor and to be able to enhance or completely suppress the relaxation resistance in a non-trivial way, distinguishing it from normal fermionic channels \cite{20, 21}. One can expect that the similar effects on the dissipation may be observed in topological superconducting wires (TSWs) which have localized Majorana bound states at their ends. Unlike the two-dimensional case, however, the discrete Majorana bound states alone cannot dissipate, so an additional dissipative channel is required to form a RC circuit.

In this paper we investigate the ac response of a quantum RC circuit in which a TSW is inserted between a spinless quantum dot and a spinless lead [see Fig. 1], where the latter undertakes the dissipation. We scrutinize the effect of the Majorana bound states on the dissipation and the charging by analyzing relaxation resistance and the quantum capacitance of the RC circuit. We have found that when two localized Majorana modes do not overlap, no dissipation takes place. The vanishing relaxation resistance is attributed to the exact cancellation between charge-conserving and Cooper-pair tunneling processes. When the overlap is finite, the relaxation resistance becomes finite and can be greatly enhanced at the resonance condition of the quantum dot. For comparison, the same analysis is applied to non-Majorana counterparts, and we have found that the dependence of the relaxation resistance and the quantum capacitance on temperature and the values of parameters is clearly different between the Majorana and non-Majorana systems.

This paper is organized as follows. Section 2 is devoted to the introduction of the model Hamiltonian and the derivation of the formulas for the relaxation resistance and the quantum capacitance. In Secs. 3 and 4, we analyze them for the cases of zero and finite overlaps between two Majorana bound states, respectively, and compare with those of non-Majorana systems. We make a conclusion in Sec. 5, summarizing our findings.

2. Quantum RC Circuit and Admittance

2.1. Model

Figure 1 shows the schematic configuration of the system of our interest, in which two ends of a topological superconducting wire are tunnel-coupled to a spinless quantum dot and a lead, respectively. In this study we are interested in the low-energy physics below the bulk gap of the superconducting wire, so the Hamiltonian for the TSW is described by the two Majorana modes $\gamma_i = \gamma_i^\dagger$ ($i = 1, 2$) localized at the ends as

$$H_M = 2i\epsilon_m \gamma_1 \gamma_2 = 2\epsilon_m (f^\dagger f - 1/2),$$  \hspace{1cm} (1)

where $\epsilon_m$ is the overlap between the Majorana modes and $f = (\gamma_1 + i\gamma_2)/\sqrt{2}$ is the nonlocal fermion operator composed of two Majorana operators. One end of the TSW is tunnel-coupled to a spinless quantum dot, which is spin-polarized due to a
sufficiently strong magnetic field which is required to induce the topologically superconducting state in the semiconductor wire. The quantum dot with a single spinless level $\epsilon_d$ and its coupling to the TSW are described by

$$H_{\text{QD}} = [\epsilon_d + e(U(t) - V(t))] n_d$$

$$H_{\text{QD-M}} = t_m (d^\dagger \gamma_1 + \gamma_1 d) = \frac{t_m}{\sqrt{2}} (d^\dagger f + d^\dagger f^\dagger + (\text{h.c.})),$$

respectively. Here $n_d = d^\dagger d$ is the occupancy operator, and the ac voltage $V(t)$ upon the gate coupled to the quantum dot via a geometrical capacitance $C$ induces the polarization charge on the dot and eventually the internal potential $U(t)$. The latter is to be determined self-consistently under the charge conservation condition. The dot level is coupled to only one of the Majorana mode $\gamma_1$ with a tunneling strength $t_m$. The second Majorana mode $\gamma_2$ at the other end of the TSW is coupled to the lead. Since the Majorana mode has only a single spin component, only one of two spin channels in the lead is coupled to the TSW. Hence, like the quantum dot, the lead is considered to be spinless:

$$H_{\text{L}} = \sum_k \epsilon_k c_k^\dagger c_k$$

$$H_{\text{L-M}} = \sum_k t (c_k^\dagger \gamma_2 + \gamma_2 c_k) = \sum_k \frac{t}{\sqrt{2}} \left( i f^\dagger c_k - i f^\dagger c_k^\dagger + (\text{h.c.}) \right),$$

where the spinless conduction-electron operator $c_k$ for momentum $k$ defines a dispersion $\epsilon_k$ and is coupled to $\gamma_2$ via the tunneling amplitude $t$, which is assumed to be independent of momentum and energy, for simplicity. The tunneling induces the hybridizations $\Gamma = \pi \rho |t|^2$ between the TSW and the lead, where $\rho$ is the density of states at the Fermi level in the lead.

2.2. Relaxation Resistance and Quantum Capacitance

We regard our system as a RC circuit [19, 20, 21, 22, 23, 24] with respect to a weak time-dependent external gate voltage $V(t) = V_{ac} \cos \omega t$ applied on the quantum dot. The ac voltage $V(t)$, in the mean-field approximation, induces the polarization charges $N(t)$ between the dot and the gate, which in turn leads to the time-dependent
potential \( U(t) = |e|N(t)/C \) inside the dot. Consequently, the applied voltage not only generates a current \( I(t) \) between the TSW and the dot, but also induces a displacement current \( I_d(t) = e(dN/dt) = -C(dU/dt) \) between the gate and the dot. The relation between these two currents is set by the charge conservation: \( I(t) + I_d(t) = 0 \).

Assuming that the gate-invariant perturbation, \( V(t) - U(t) \), is sufficiently small, the linear response theory leads to the relation, \( I(\omega) = g(\omega)(V(\omega) - U(\omega)) \), where the \( g(t) = (ie/\hbar) \langle [I(t), n_d] \rangle \Theta(t) \) is the equilibrium correlation function between the occupation operator \( n_d \) and the current operator \( I = e(dn_d/dt) \). Then, the dot-lead impedance \( Z(\omega) = V(\omega)/I(\omega) \), which is experimentally accessible, can be expressed as \( Z(\omega) = 1/(i\omega C) + 1/g(\omega) \), by using the self-consistent condition, \( I(\omega) = -I_d(\omega) = -i\omega CU(\omega) \). The quantum correction to the impedance then gives rise to the relaxation resistance and the quantum correction to the capacitance:

\[
R_q(\omega) = \Re \left[ \frac{1}{g(\omega)} \right] \quad \text{and} \quad C_q(\omega) = \Im \left[ \frac{\omega}{g(\omega)} \right]^{-1}.
\]

In order to calculate the admittance, we adopt the Wingreen-Meir formalism [25, 26] which derives directly the current formula for arbitrary gauge-invariant perturbation \( V_g(t) - U(t) \) and then obtains the admittance by considering the linear response only. We refer the details of the derivation to Ref. [19]. The system studied in Ref. [19], being different from ours, shares the key features: (1) The ac voltage is applied to the spinless quantum dot and (2) the dot is tunnel-coupled to a single reservoir (directly or indirectly) with no other channel for dissipation. Therefore, the general expression of the admittance in terms of the dot Green’s function, derived in Ref. [19], applies to our case as well:

\[
g(\omega) = \frac{\omega}{R_Q} \int d\omega' f(\omega') \left[ G^R_d(\omega' - \omega) + \{G^R_d(\omega', \omega') + \omega \} \right]
\]

\[
\quad \times \left[ G^A_d(\omega') \sigma_3 G^R_d(\omega' - \omega) \sigma_3 \{G^R_d(\omega', \omega') - G^A_d(\omega') \} \right]_{11},
\]

where \( R_Q \equiv \hbar/e^2 \), \( f(e) \) is the Fermi distribution function at temperature \( T \), and \( \sigma_i (i = 0, 1, 2, 3) \) are the Pauli matrices in Nambu space. The dot Green’s function over the Nambu space is defined by

\[
G^R_d/t \equiv \pm i \Theta(\pm(t-t')) \left[ \langle [d(t), d(t')] \rangle \langle [d(t), d(t')] \rangle \langle [d(t), d(t')] \rangle \langle [d(t), d(t')] \rangle \right].
\]

Since our system is effectively non-interacting, it is quite straightforward to calculate the dot Green’s function, which is given by

\[
G^R_d(\omega) = [G^R_d(\omega)]^{-1} = [g^R_d(\omega) - \Sigma^R(\omega)]^{-1}
\]

with the unperturbed dot Green’s function \( g^R_d(\omega) = (\omega + i\eta - \sigma_3 \epsilon_d/\hbar)^{-1} \) and the self-energy

\[
\Sigma^R(\omega) = \frac{|t_m|^2}{\hbar^2} \frac{\omega + 2i\Gamma}{(\omega + i\eta)(\omega + 2i\Gamma) - (2\epsilon_m/\hbar)^2}(\sigma_0 - \sigma_1),
\]
where $\eta$ is positive infinitesimal number. As expected, the Green’s function has finite off-diagonal components which reflects the presence of the superconductivity.

In our study, in order to distinguish the physical features peculiar to Majorana fermions from non-Majorana ones, we consider non-Majorana counterpart systems in which the Majorana bound modes are replaced by an ordinary fermionic mode. Specifically, two systems, $N_i$ ($i = 1, 2$) are examined: In the $N_1$ system, the dot and the lead is connected via a single (local) fermionic level $f$ in the wire, whose level energy is now given by $2\epsilon_m$ [see Eq. (4)]. This system is implemented by eliminating the abnormal terms such as $d^\dagger f^\dagger$ and $\epsilon^\dagger_k f$ in both $H_{QD-M}$ and $H_{L-M}$, which turns off the superconductivity in the wire. The $N_2$ system further assumes that the localized bound state $f$ is coupled only to the dot, while being disconnected from the lead. The two systems are implemented by using the self-energies

$$
\Sigma^R_{N1}(\omega) = \frac{|t_m|^2/\hbar^2}{\omega - 2\epsilon_m\sigma_3/\hbar + i\Gamma/2} \quad \text{and} \quad \Sigma^R_{N2}(\omega) = \frac{|t_m|^2/\hbar^2}{\omega + i\eta - 2\epsilon_m\sigma_3/\hbar}, \quad (9)
$$

respectively, in the dot Green’s function, Eq. (7).

2.3. Excitation Spectrum of Dot-Wire Subsystem

For later use, we diagonalize the dot-wire subsystem disconnected from the lead and obtain the excitation spectrum for our system and the $N_{1/2}$ system. By diagonalizing $H_{QD} + H_{QD-M} + H_M$, one obtains four eigenenergies, $\epsilon_1^\pm$ and $\epsilon_2^\pm$ defined as

$$
\epsilon_1^M \equiv (\epsilon_d \pm \sqrt{(\epsilon_d - 2\epsilon_m)^2 + 2|t_m|^2})/2 \quad \text{and} \quad \epsilon_2^M \equiv (\epsilon_d \pm \sqrt{(\epsilon_d + 2\epsilon_m)^2 + 2|t_m|^2})/2
$$

for the Majorana system, and $\epsilon_1^N = \epsilon_1^M$, $\epsilon_2^N = \epsilon_2^M + \epsilon_m$, and $\epsilon_2^N = -\epsilon_m$ for the $N_{1/2}$ system. The eigenstates for the eigenenergies $\epsilon_1^\pm$ are built from the linear combinations of $d^\dagger \ket{0}$ and $f^\dagger \ket{0}$, while those for $\epsilon_2^\pm$ are from $\ket{0}$ and $d^\dagger f^\dagger \ket{0}$. Hence the excitation energies with respect to the $d$-particle are

$$
\pm \delta \epsilon_1 = \epsilon_2^\pm - \epsilon_1^\pm \quad \text{and} \quad \pm \delta \epsilon_2 = \epsilon_2^\pm - \epsilon_1^\mp.
$$

Typical plots of $\delta \epsilon_1^{M/N}$ are drawn in Fig. 2 for later use.
3. Relaxation Resistance and Quantum Capacitance: $\epsilon_m = 0$ Case

First, we investigate the case in which the two Majorana modes have no overlap between them, that is, $\epsilon_m = 0$. It corresponds to the ideal case for Majorana braiding and Majorana-based quantum computation because no time-dependent phase change of Majorana states occurs. In this case, the self energy, Eq. (8), is quite simplified to

$$\Sigma^R(\omega) = \frac{|t_m|^2}{\hbar^2} \frac{1}{\omega + i\eta} (\sigma_0 - \sigma_1).$$  \hspace{1cm} (11)

The key characteristics of the above self energy is that the effect of the lead hybridization disappears completely: it has no dependence on $\Gamma$. There are two arguments to explain it. The first one is simple: The condition $\epsilon_m = 0$ breaks the coupling between two Majorana modes so that the system is divided into two independent parts. The dot coupled to one of the Majorana modes, therefore, is completely disconnected from the lead coupled to another Majorana mode, leaving no dependence on $\Gamma$ in $G^R(\omega)$. The second argument, instead, interprets it in terms of the fermionic mode $f$, not the Majorana modes $\gamma_i$. As can be seen in Eqs. (2) and (3), the $f$-particle tunneling to both the dot and the lead is possible, and the wire is simply on resonance for $\epsilon_m = 0$. The dot, in this view, is not apparently decoupled from the lead. However, the coupling of $f$-particle to the dot and the wire involves the Cooper-pairing tunneling such as $df$ and $c_k f$ as well as the particle-conserving one such as $d^\dagger f$ and $c_k^\dagger f$. The point is that these two tunnelings can cancel out each other. This cancellation can be more easily understood in the perturbation language. In the weak wire-lead tunneling limit, the perturbation theory gives rise to correction terms related to two tunneling terms and their weights are proportional to $1/(\epsilon_k - \epsilon_m)$ and $1/(-\epsilon_k - \epsilon_m)$ for the charge-conserving ($c_k^\dagger f$) and the Cooper-pair ($c_k f$) tunnelings. For $\epsilon_m = 0$, the two terms are same in magnitude and have opposite signs, leading to complete cancellation. This vanishing effect is common in Majorana systems [19], because the Majorana modes, being anti-particles of themselves, have particle and hole components in equal magnitude.

Using the self-energy, Eq. (11) and the corresponding Green’s function, the expression for the admittance, Eq. (5) is given by

$$g(\Omega) = \frac{2}{\pi} \frac{|t_m|^2}{R_Q} \frac{\hbar\omega}{\epsilon_w(\epsilon_w^2 - (\hbar\omega)^2)} \tanh \frac{\beta\epsilon_w}{2}$$

where $\epsilon_w \equiv \sqrt{\epsilon_d^2 + 2|t_m|^2}$ is the excitation energy due to the hybridization of the dot and the Majorana mode and $\beta \equiv 1/k_B T$. The admittance is purely imaginary, meaning that there is no dissipation, $R_q(\omega) = 0$ for all frequencies $\omega$, because no dissipation channel is connected. The quantum capacitance is then

$$C_q(\omega) = -\frac{\epsilon^2}{\epsilon_w(\epsilon_w^2 - (\hbar\omega)^2)} \tanh \frac{\beta\epsilon_w}{2}.$$  \hspace{1cm} (13)

It should be noted that its temperature dependence is exponential, which reflects the fact that the dot-TSW system defines sharp energy eigenstate with no broadening.

Now we consider the non-Majorana systems and identify the features unique to the Majorana systems. The $N_1$ system which has no Cooper-paring tunneling opens a
dissipation channel, so $R_q(\omega)$ is finite and depends on the values of parameters. For example, for $|t_m| \ll \Gamma$, the low-frequency resistance $R_{q0} \equiv R_q(\omega \to 0)$ is $R_q/2$, the universal value of $R_q$ when only a single dissipation channel is involved [20, 21]. In the $N_2$ system, the dot is still disconnected from the lead so as in the Majorana case no dissipation happens: $R_q(\omega) = 0$. However, the temperature dependence of the quantum capacitance is different from that in the Majorana case, Eq. (13):

$$C_q(\omega) = -e^2 \frac{|t_m|^2}{\epsilon_w(\epsilon_w^2 - (\hbar \omega)^2)} \frac{\tanh \frac{\beta \epsilon_m}{2}}{1 + \cosh \frac{\beta \epsilon_w}{2} / \cosh \frac{\beta \epsilon_m}{2}}. \tag{14}$$

This difference can be understood by comparing the excitation spectra of the Majorana and non-Majorana systems. By diagonalizing the dot-wire subsystems, the excitation energies, Eq. (10) are obtained as $\delta \epsilon_1 = 0$ (doubly degenerate), $\delta \epsilon_2 = \epsilon_w$ for the Majorana systems and $\delta \epsilon_1 = (\epsilon_d - \epsilon_w)/2$ and $\delta \epsilon_2 = (\epsilon_d + \epsilon_w)/2$ for the $N_2$ system. Since the charging and discharging via the degenerate zero-energy excitation cancel out each other, $C_q \propto f(\epsilon_w) - f(-\epsilon_w)$ for the Majorana system, while $C_q \propto f((\epsilon_d + \epsilon_w)/2) - f((\epsilon_d - \epsilon_w)/2)$ for the $N_2$ system, resulting in Eqs. (13) and (14), respectively. Therefore, the difference is attributed to the pinning to the zero-energy excitation of the Majorana system.

4. Relaxation Resistance and Quantum Capacitance: $\epsilon_m \neq 0$ Case

Finite $\epsilon_m$ connects the dot and the lead via the non-local Majorana modes. In terms of $f$-particle tunnelings, the exact cancellation between the charge-conserving and Cooper-pairing processes is lift for non-zero $\epsilon_m$. Figure 3 shows the dependence of $R_{q0}$ on the value of $\epsilon_m$ at zero temperature. While $R_{q0}$ vanishes at $\epsilon_m = 0$, it becomes non-zero for finite values of $\epsilon_m$. Figure 3 (a) clearly shows that $R_{q0}$ has two symmetric side peaks at $\epsilon_m = \pm \epsilon_{m,\text{max}}$ with the height $R_{q0,\text{max}}$, where the peak position $\epsilon_{m,\text{max}}$ and its height $R_{q0,\text{max}}$ vary with the values of $\epsilon_d/\Gamma$ and $t_m/\Gamma$, as shown in Figs. 3 (b) and (c). As explained in details in Ref. [23, 19], the relaxation resistance measures the dissipation due to the relaxation of the particle-hole (p-h) pairs generated via the dot-lead tunneling. In the language of the perturbation, the weight of the generated p-h pairs depends on the energy of the intermediate virtual states. More specifically, it is proportional to $1/(|\epsilon_k| + |\epsilon_m|)$. Therefore, the more p-h pairs are generated for the smaller $\epsilon_m$. However, as explained in the previous section, the cancellation between the two types of tunnelings is pronounced for small values of $\epsilon_m$. As a result, $R_{q0}$ exhibits a narrow dip inside a central wider peak around $\epsilon_m = 0$. The height $R_{q0,\text{max}}$ of the side peak increases with $t_m$ and is maximized at resonance $\epsilon_d = 0$, since the more p-h pairs are generated in these conditions. The peak position $\epsilon_{m,\text{max}}$ or the width of the dip also shows qualitatively similar features since for larger $t_m$ and smaller $|\epsilon_d|$ the effect of the Majorana coupling $\epsilon_m$ becomes relatively smaller and the cancellation becomes stronger, widening the dip. It should be noted that the enhancement of $R_{q0}$ on the dot resonance condition is also observed in the RC circuit made of chiral Majorana edge modes [19], indicating that this enhancement comes from the nature of the Majorana fermions.
The dependence of the low-frequency quantum capacitance $C_{q0} \equiv C_q(\omega \to 0)$ on $\epsilon_m$ at zero temperature is displayed in Fig. 3 (d). Finite $\epsilon_m$ lifts the degeneracy at the otherwise zero-energy excitations, making $\delta \epsilon_1$ finite [see Fig. 2 (a)]. It means that the dot density of states $\rho_F$ at the Fermi level decreases with increasing $|\epsilon_m|$. Since the quantum capacitance at zero temperature is, though not exactly, proportional to $\rho_F$, $C_{q0}$ also decreases with increasing $|\epsilon_m|$, as shown in Fig. 3 (d). For $t_m > |\epsilon_d|$, on the other hand, the strong hybridization between the dot and the wire makes $\delta \epsilon_1$, though being finite, remain small even for large $\epsilon_m$ [see Fig. 2 (a)]. So, the rapid decrease of $C_{q0}$ with increasing $|\epsilon_m|$ is alleviated. In addition, as shown in Fig. 3 (d), a weak side-peak structure appears, which is attributed to the lead-driven broadening and the shift of the spectral weights.

We have found that the temperature dependence of $R_{q0}$ and $C_{q0}$ follows the Fermi-liquid-like behavior: The correction due to small thermal fluctuations is found to be proportional to $T^2$ as predicted by the Sommerfeld expansion. One exception is when the dot is exactly at resonance which induces the $\delta$-peak in the dot density of states, resulting in exponential dependence.

Now we examine the non-Majorana counterpart systems with finite $\epsilon_m$ for comparison. First, since in the N$_2$ system the dot remains disconnected from the lead even for finite $\epsilon_m$, no dissipation takes place, that is, $R_q = 0$. It is clearly distinguished from
finite \( R_{q0} \) of the Majorana system [see Fig. 3]. Next, we consider the \( N_1 \) system whose \( R_{q0} \) and \( C_{q0} \) are shown in Fig. 4. There are two clear differences when compared to the Majorana system. (1) Both \( R_{q0} \) and \( C_{q0} \) are asymmetric with respect to the change \( \epsilon_m \to -\epsilon_m \), while they are symmetric in the Majorana system. The excitation energies \( \pm \epsilon_1^{1/2} \) of the Majorana system is clearly invariant under \( \epsilon_m \to -\epsilon_m \), while \( \pm \epsilon_1^{N/2} \) are not. It is because the Majorana system is inherently particle-hole symmetric. (2) \( R_{q0} \) decreases with increasing \( t_m \) in the \( N_1 \) system [see Fig. 4 (a)] while the Majorana system exhibits the opposite behavior [see Fig. 3 (c)]. It is also attributed to the difference in the excitation structure: For the Majorana case, \( \delta \epsilon_1 \) remains small even for large \( t_m \) [see Fig. 2 (b)], which indicates that the energy cost for the p-h pair generation does not change so much. Instead, the larger \( t_m \) enhances the tunneling of electrons between the dot and the lead so that the p-h generation in the lead is enhanced. On the other hand, in the \( N_1 \) system, the excitation energy \( \delta \epsilon_1 \) increases with \( t_m \) (though not monotonically) [see Fig. 2 (b)], increasing the energy cost for the p-h pairs and suppressing its generation.

5. Conclusion

We have investigated the effect of the Majorana bound states on the charging and the dissipation of the quantum-dot system. It is found that the relaxation resistance vanishes completely for \( \epsilon_m = 0 \) and can be quite enhanced on the dot resonance condition for \( \epsilon_m \neq 0 \). The quantum capacitance is found to follow the different dependence on systems parameters and temperature when compared to that of non-Majorana systems. In order to identify clearly the physical features related to the Majorana mode, we have considered two non-Majorana systems. The \( N_2 \) system, having a localized bound states permanently decoupled from the lead, does not dissipate, which is distinguished from the Majorana case having finite dissipation for \( \epsilon_m \neq 0 \). On the other hand, the \( N_1 \) system always opens a dissipation channel and has asymmetric dependence of the relaxation resistance and the quantum capacitance on \( \epsilon_m \) since it lacks the particle-hole symmetry which is inherent in the Majorana case. Therefore, the ac
response of the Majorana RC circuit can be used as another unambiguous method of detecting the elusive Majorana fermions.

Similar ac responses such as the complete vanishing or resonance-induced-enhancing of the relaxation resistance have been predicted in the RC circuit composed of chiral Majorana edge modes around the two-dimensional topological superconductor [19]. It should be noted that in our system the dissipation does not take place in the superconductor while it does in the two-dimensional case. It means that the Majorana-related features are quite common whether or not the Majorana modes take part in the dissipation. In the experimental point of view, the topological superconducting wire is much easier to implement than the two-dimensional one. So we expect that our system is more adequate to observe the non-trivial ac response of the Majorana modes.

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