Abstract

Background/Objectives: Intercept sonar of ownship is used to track a target, which is assumed to be doing active transmission for detecting a target in underwater. Methods/Statistical analysis: The ownship intercepts the active transmissions and generates bearing measurements of the target. The measurement interval between generated bearings in intercept mode is not constant and so closed loop estimators like Kalman filter is not useful to find out target motion parameters. So, sub-optimal estimator like Pseudo Linear Estimator (PLE) is used. Findings: Recursive PLE developed by S. K. Rao is modified to suit this application.

Keywords: Bearings-Only Measurements, Estimation, Intercept Measurements, Kalman Filter, Target Tracking

1. Introduction

Active mode of surveillance exposes the identity of observer whereas passive mode does not reveal the location of the observer (silent). Ownship is silent and in active mode\textsuperscript{1,2} to provide only bearing or Line of Sight measurements. Similarly, an EW receiver on an ownship intercepts the measurements radiated by radar housed on a target ship. Bearings-only target motion analysis in two dimensional Cartesian coordinate system in underwater environment is generally used. The measurements are not available at uniform time interval, as intercept sonar receives measurements only when the active sonar of the target is in transmission mode. Passive bearing require processing to find the kinematics of target which is inherently non-linear which is estimatable only after proper ownship movement\textsuperscript{2-6}.

Though the bearing measurements are similar to passive sonar measurements, there are two major differences. 1. The noise in the measurements in intercept sonar/EW ESM measurements is in the order of 2 to 50 r. m. s, whereas in case of passive sonar, it is around 0.50 r. m. s. The measurements are discrete as these are available whenever target transmits signals. Section 2 describes PLE and section 3 describes results. Section 4 describes limitations of the algorithm and finally concluded in 5.

2. Mathematical Modeling

It is desired to estimate the following target state vector\textsuperscript{8-14}. The target state equation is given by

$$X_s(k+1) = \varphi(k)X_s(k) + W(k)$$

$$\varphi(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_n(k) = \tan^{-1} \frac{x_n(k) - x_n(0)}{y_n(k) - y_n(0)} + \nu(k)$$

$$\hat{X}_s(0/k) = A^T(k,0)A(k,0)^{-1}A^T(k,0)Z(k)$$

Let Z(k), PSI and G are the vector of measurements, $A^T(k,0)A(k,0)$ and $A^T(k,0)$ respectively. PSI and G can be expanded as [7]
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\[ \text{PSI} = \begin{bmatrix} \sum ts_i \cos^2 B_{mi} & -\sum ts_i \cos B_{mi} \sin B_{mi} & \sum ts_i \cos^2 B_{mi} & -\sum ts_i \cos B_{mi} \sin B_{mi} \\ -\sum ts_i \cos B_{mi} \sin B_{mi} & \sum ts_i \sin^2 B_{mi} & -\sum ts_i \cos B_{mi} \sin B_{mi} & \sum ts_i \sin^2 B_{mi} \\ \sum ts_i \cos B_{mi} \sin B_{mi} & -\sum ts_i \sin B_{mi} \cos B_{mi} & \sum \cos^2 B_{mi} & -\sum \cos B_{mi} \sin B_{mi} \\ -\sum ts_i \sin B_{mi} \cos B_{mi} & \sum ts_i \cos B_{mi} \sin B_{mi} & -\sum \cos B_{mi} \sin B_{mi} & \sum \sin^2 B_{mi} \end{bmatrix} \]

where \( G = \sum ts_i \cos B_{mi} z_i - \sum ts_i \sin B_{mi} z_i - \sum \cos B_{mi} z_i \)

\[ G = \begin{bmatrix} G(1) \\ G(2) \\ G(3) \\ G(4) \end{bmatrix} \]

In general we have

\[ \text{PSI}(1,1) = \text{PSUMS}[1] = \text{t}(1) \cos^2 B_m(1) + T \cos^2 B_m(1) \]

After the receipt of second measurement, \( T \) and \( \text{PSUMS}[1] \) are given as

\[ T = t(1) + t(2) \]

\[ \text{PSUMS}[1] = \text{t}(1)^2 \cos^2 B_m(1) + \text{t}(1) \cos^2 B_m(1) \]

In general we have

\[ t = \text{t}(1) + \text{t}(2) + \text{t}(3) + \ldots + \text{t}(k) \]

\[ \text{PSI}(1,1) = \text{PSUMS}[1] = \text{PSUMS}[1]_{k-1} + T \cos^2 B_m(k) \]

\[ \text{PSI}(1,2) = \text{PSUMS}[2] = \text{PSUMS}[2]_{k-1} - T \cos B_m(k) \]

\[ \text{PSI}(1,3) = \text{PSUMS}[3] = \text{PSUMS}[3]_{k-1} + T \sin^2 B_m(k) \]

\[ \text{PSI}(1,4) = \text{PSUMS}[4] = \text{PSUMS}[4]_{k-1} - T \sin B_m(k) \]

\[ \text{PSI}(2,1) = \text{PSUMS}[5] = \text{PSUMS}[2]_{k-1} \]

\[ \text{PSI}(2,2) = \text{PSUMS}[6] = \text{PSUMS}[6]_{k-1} + T \sin^2 B_m(k) \]

\[ \text{PSI}(2,3) = \text{PSUMS}[7] = \text{PSUMS}[4]_{k-1} \]

\[ \text{PSI}(2,4) = \text{PSUMS}[8] = \text{PSUMS}[8]_{k-1} + T \sin B_m(k) \]

\[ \text{PSI}(3,1) = \text{PSUMS}[9] = \text{PSUMS}[3]_{k-1} \]

\[ \text{PSI}(3,2) = \text{PSUMS}[10] = \text{PSUMS}[7]_{k-1} \]

\[ \text{PSI}(3,3) = \text{PSUMS}[11] = \text{PSUMS}[11]_{k-1} + \cos^2 B_m(k) \]

\[ \text{PSI}(3,4) = \text{PSUMS}[12] = \text{PSUMS}[12]_{k-1} - \cos B_m(k) \]

\[ \text{PSI}(4,1) = \text{PSUMS}[13] = \text{PSUMS}[4]_{k-1} \]

\[ \text{PSI}(4,2) = \text{PSUMS}[14] = \text{PSUMS}[8]_{k-1} \]

\[ \text{PSI}(4,3) = \text{PSUMS}[15] = \text{PSUMS}[11]_{k-1} \]

\[ \text{PSI}(4,4) = \text{PSUMS}[16] = \text{PSUMS}[16]_{k-1} + \sin^2 B_m(k) \]

\[ G(1) = \text{GSUMS}[1] = \text{GSUMS}[1]_{k-1} + T \text{term} \cos B_m(k) \]

\[ G(2) = \text{GSUMS}[2] = \text{GSUMS}[2]_{k-1} - T \text{term} \cos B_m(k) \]

\[ G(3) = \text{GSUMS}[3] = \text{GSUMS}[3]_{k-1} + \text{term} \sin^2 B_m(k) \]

\[ G(4) = \text{GSUMS}[4] = \text{GSUMS}[4]_{k-1} - \text{term} \sin B_m(k) \]

All SUMS are initialized to zero. The target state vector is found out using eqn. (4) and then the state vector corresponding to the time instant \( k \) can be found out using transient matrix. The range, course, bearing and speed of the target are calculated using the target state vector.

### 3. Simulation and Results

The measurements are available at every second and assumed noise 2 deg. r.m.s and the first scenario shown in Table 1 is used to evaluate the algorithm for intercept sonar system. The measurements are passed on to the estimator at random intervals, using a random number generator. Similarly, an EW receiver system is assumed to be intercepting the active measurements generated by radar on a target ship. Four radars with the measurement interval 1,

| Scenario | Initial Range (m) | Initial Bearing (deg) | Target Speed (m/sec) | Target Course (deg) | Ownship Speed (m/sec) | Time to Converge (sec) |
|----------|-------------------|-----------------------|----------------------|---------------------|-----------------------|------------------------|
| 1        | 5000              | 210                   | 5.15                 | 45                  | 10.3                  | 65                     |
| 2        | 5000              | 210                   | 5.15                 | 45                  | 10.3                  | 344                    |
2, 3 and 4 seconds respectively assumed to be operating one at a time. Each radar is assumed to be operated for a period of 4 minutes duration. The same scenario used for intercept sonar is used for EW ESM system. The ownship maneuver is shown in Figure 1. The results obtained in Monte-Carlo simulation for scenario 1 and 2 are shown in Figure 2 and 3 respectively. It is observed that range, course and speed estimates with required accuracies are obtained and details about time of convergence are shown in Table 1.

4. Conclusion

In this paper, PLE is proposed for tracking applications when the measurements are available from intercept sonar or EW receiver system. Here recursive SUMS are updated whenever the new bearing measurement is available. The performance of this algorithm is evaluated in Monte-Carlo simulation and results are found to be satisfactory. Hence PLE is recommended for this application.

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Figure 1. Ownship is S-maneuver on LOS.

Figure 2. (a) R error (b) C Error (c) S Error.

Figure 3. (a) R Error (b) C Error (c) S Error.
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