Peculiarities of Angular Distribution of Electrons at Si <100> Channeling

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Abstract. The properties of both angular and spatial distribution of 255 MeV electrons at <100> channeling in silicon crystal has been investigated experimentally at the linac injector of SAGA light source and by computer simulations. The simulation of trajectories, angular and spatial distributions of electrons on the screen monitor has been performed taking into account initial spatial as well as angular beam divergence of electron beam. Both experimental data and simulations show the brilliant effect of so-called «doughnut scattering».

1. Introduction

The «doughnut scattering» (DS) (scattering of charged relativistic particles by crystal axes in the plane perpendicular to the axes) was first observed in earlier high-energy channeling experiments (1976 - 1980) using the proton and meson beams at CERN, see, e.g. in [1]. Theoretical consideration and Monte-Carlo calculations of similar effects but for the case of electrons and positrons, was performed in [2-3]. The review of crystal-assisted processes at high energies, including DS, was given in [4]. Rather recently (2006), similar effects have been observed using 150 MeV electrons at REFER (Hiroshima University) [5]. Dependence of electrons deflection on the beam divergence, which indicated the possibility of using such a technique for beam diagnostics as well as beam collimation, was observed unless the beam divergence exceeded several times the critical channeling angle. This dependence was only qualitatively explained by computer simulations. In our work, more detailed investigation of DS for 255 MeV electrons in a thin Si crystal using another technique as well as more correct computer simulations based on BCM model [6] have been performed.

2. The experimental setup of SAGA-LS and results of experiments

The experiments were performed at the linac injector of SAGA light source (SAGA-LS) facility, see in figure 1. The electrons with an energy 255 MeV passed through a single silicon crystal of 20 μm thickness along <100> axis. The vertical and horizontal sizes of the incident beam were σ_x=0.2 mm and σ_y=0.7 mm, respectively. The spatial distribution of the electrons in the incident beam is considered as a normal and σ is the dispersion of this distribution. The horizontal and vertical angular divergences of the beam were 0.2 mrad and 0.3 mrad, respectively. The beam profile after passing through the crystal was measured by the screen monitor placed at 182 cm from the crystal.
Figure 1. SAGA light source facilities.

Figure 2. The experimental results for 255 MeV electrons penetration through <100> 20 μm Si crystal. (a) Profiles of electron beam transmitted through Si crystal for different angles of incidence $\theta$ into the crystal. (b) Vertical and horizontal beam positions $x_{\text{peak}}, y_{\text{peak}}$ and beam sizes $\sigma_x, \sigma_y$ versus angle of incidence $\theta$.

The compact two axes goniometer was installed in a way allowing the crystal to be rotated around vertical and horizontal axes in the plane perpendicular to the beam. The angular step size for both axes...
was 0.001 degree. The spatial distribution of the particles was observed on the screen monitor, see in figure 2(a). To perform the data analysis, a 2-dimensional beam profile was projected onto the vertical and horizontal directions. Than these profiles were fitted with a Gaussian function, like in Ref. [5]:

\[
I_x = I_{0x} \exp \left( -\frac{(x-x_{\text{peak}})^2}{2\sigma_x^2} \right) + a_x x + b_x, \tag{1}
\]

and the beam sizes \( \sigma_x, \sigma_y \) and the beam positions \( x_{\text{peak}}, y_{\text{peak}} \) are defined in figure 2 (b).

3. Electron trajectories simulation

For the calculation of electrons trajectories we used the classical description (a quantum approach is applied for electron energies less than \( \sim 100 \text{ MeV} \)). Under the channeling condition the longitudinal component of relativistic electrons velocity \( v_z \) is nearly equal to the speed of light. The transversal component of velocity is much less than the longitudinal one: \( v_x, v_y \approx c \approx 1/\gamma \) (\( \gamma \) denotes the relativistic factor). Under these conditions, equations of motion are reduced to the non-relativistic equations with the replacement of the particle mass \( m \) to the relativistic mass \( m \gamma \), see e.g. [7].

\[
\begin{align*}
\gamma m \ddot{x} &= -\frac{\partial U(x,y)}{\partial x}, \\
\gamma m \ddot{y} &= -\frac{\partial U(x,y)}{\partial y}, \\
\gamma m \ddot{z} &= 0,
\end{align*}
\tag{2}
\]

where \( x \) and \( y \) are the transversal coordinates, \( z \) is the longitudinal coordinate, \( U(x, y) \) here describes the potential energy of the electrons in the field of periodically arranged crystal axes.

The initial conditions for this system contain the electron entrance point into the crystal \( x(0) \equiv x_0, y(0) \equiv y_0 \) and transversal components of initial velocity:

\[
\begin{align*}
v_x(0) &= c \sqrt{1 - \frac{1}{\gamma^2}} \sin(\theta) \cos(\phi), \\
v_y(0) &= c \sqrt{1 - \frac{1}{\gamma^2}} \sin(\theta) \sin(\phi),
\end{align*}
\tag{3}
\]

where \( \theta, \phi \) are the angles between electron momentum and \( OZ \) and \( OX \) axes correspondingly.

The potential energy \( U(x,y) \) of electron interaction with crystallographic axes within the Doyle-Turner approximation [8] for single atom potential (obtained by fitting the electron scattering form factor) is periodic in \((x, y)\) plane:

\[
U(x, y) = eV(x, y) = e \sum_k V_{as}(x - kd_{ax}, y - kd_{ay}),
\tag{4}
\]

where \( V_{as}(x,y) \) is the potential of a single axis, \( d_{ax}, d_{ay} \) are the inter axes distances. The potential energy \( U(x,y) \) for Si \(<110>\) direction is shown in figure 3. The dashed line shows the region where the entrance points of electrons were located.

Thus, to calculate every individual electron trajectory \( x(t), y(t) \) it is necessary to solve the system of classical equations of motion (2). The trajectories and velocities of electrons are obtained by numerical integration of the equation of motion, at this stage without taking into account of the dechanneling process. For the numerical simulation we used a new computer code “Basic Channeling with Mathematica©” BCM–1.0 developed by the authors [6].
The potential energy \( U(x, y) \) for \(<110>\) direction in Si crystal. The dashed line shows the region where the entrance points of electrons were located.

The projections of trajectories of axially channeled electrons onto \((x, y)\) plane under penetration through a 20 \(\mu m\) thick Si crystal, for the different incident angles and entrance points: (a) – periodic motion; (b) - non periodic motion. The points depict crystallographic axes.

The projections of trajectories of axially channeled electrons onto \((x, y)\) plane strongly depend on the incident points into the crystal and incident angles with respect to the channeling axes. Several examples are shown in figure 4. The trajectory showed in figure 4 (a) represents rosette motion when electron is trapped by the potential well of crystal axis. At larger angles of incidence the electron is scattered by the axes and trajectory of electrons becomes more complicated, see in figure 4 (b).

4. Simulation of spatial and angular distributions

The procedure for calculation of both angular and spatial distributions of electrons passed through a silicon single crystal was as follows.

1. The calculation of trajectories, the entrance points of which are situated within the square shown in figure 3. For each point of entry, 10 values of incident angles \( \theta \) and \( \varphi \) were generated by a
random number generator with normal distribution. The dispersion of the normal distribution corresponds to the experimental ones (horizontal – 0.2 mrad, and vertical – 0.3 mrad). The total number of calculated trajectories was more than 10 000. Next, these data were used to calculate spatial distribution of electron beam during penetration through the crystal using the periodicity of the crystal lattice potential.

2. The calculation of initial incident points for electrons into a crystal was performed in accordance with the profile of experimental beam by the random number generator. We assume that the initial distribution of a primary electron beam was the normal one with vertical dispersion of 0.2 mm and horizontal dispersion of 0.7 mm. The calculated SAGA-LS incident beam profile is shown in figure 5.

Because of the large number of trajectories, the computer code [6] was parallelized and further calculations were performed at the supercomputer cluster of Tomsk Polytechnic University [9]. We calculated the exit angles of electrons from the crystal and spatial distribution of electrons on the screen monitor, which is located at the distance of 1.82 m from the crystal. The results of angular distributions calculation are shown in figure 6. (a). The exit angles are calculated according the formulae $\theta_x = v_x / c$, $\theta_y = v_y / c$, where $v_x$ and $v_y$ - are the transversal components of the particle velocity at the exit from the crystal. The figures illustrate that if an incidence angle is less than the critical channeling angle (0.05 degree for this condition), than the distributions are quite symmetrical. For the large angles of incidence the angular distributions became crescent-shaped. The spatial distributions shown in figure 6 (b) clear illustrate the brilliant effect of (DS).

To perform comparison with experiment, 2-dimensional beam profile was projected onto the vertical and horizontal directions. The one-dimensional beam profiles were fitted by the Gaussian function (1), and the beam size and beam position have been estimated. The dependences of the beam position $x_{peak}$, $y_{peak}$ and beam size $\sigma_x$, $\sigma_y$ on the incidence angles $\theta$ are shown in figure 7.

From figure 7, one can conclude that results of computer simulations are in rather good quantitative agreement with experimental data. It should be mentioned, that when we calculated the beam size in figure 7, we took into account the contribution to total dispersion arising due to multiple scattering of electrons, with dispersion equal to that in amorphous target $\sigma_{ms}$ [10]. Then, the total beam size was calculated according the equation $\sigma = \sqrt{\sigma_{simulation}^2 + \sigma_{ms}^2}$. 
Figure 6. Angular distribution of electrons penetrated through the crystal (a), and spatial distribution of electrons on the screen monitor for different incident angles $\theta$. The exit angles were calculated according the formulae: $\theta_x = v_x/c$, $\theta_y = v_y/c$. 
5. Conclusions
The first experiments on 255 MeV electrons channeling and DS in a thin aligned Si crystal were performed at SAGA-LS injector Linac. The simulation of trajectories at the <100> axial channeling in Si, angular and spatial distributions of electrons on the screen monitor have been performed taking into account initial spatial and angular beam divergence of the electron beam. The comparison of experimental and theoretical results shows a good quantitative agreement. Both experimental data and simulation results show the brilliant effect of doughnut scattering - DS. In addition, we may definitely conclude, that DS of a few hundred MeV electrons in aligned thin crystals can be used for the diagnostics of the incident beam angular divergence.

6. Acknowledgements
The authors are grateful to N. Shul’ga, S. Fomin and K. Korortchenko for helpful discussions. This work was supported by the Russian Fund for Basic Research within the grant No 10–02–08607 and within the Grant of the President of RF № MK-2059.2011.2.

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