Cracks in upper road layer at negative temperature change: modelling and forecasting

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Abstract. Low-temperature cracking is one of the main reasons for the deterioration of the upper layer of roads in regions with cold climate. The material of the top layer can be, e.g., asphalt concrete or frozen soil. In this paper it is assumed that the temperature of the underlying layer is non-negative, i.e. a model in the form of a two-layer structure is used. A large number of works aimed at preventing wear are known, but the interdisciplinary problem of low-temperature cracking in seasonal freezing of soils remains relevant. In publications, as a rule, consider the destruction of freezing, that is, with a decrease in temperature. Much less attention is paid to the modeling and forecasting of damage to the upper layer of roads, if its temperature rises, but remains negative. In this case, the layer is compressed and the appearance of wave-like irregularities on the day surface (buckling) is possible. The objective of the current study was modeling the conditions of occurrence and prediction of damage to the upper layer of roads when the changes negative temperature. The paper uses methods of mathematical modeling of mechanical systems, as well as adaptation of the known results of other authors. The estimation of the critical wavelength at a bend of the compressed upper layer is given. The results of numerical simulation are consistent with the experimental data known in the literature.

1 Introduction

In regions with a cold climate, frost cracks are common, which appear in the upper layer of frozen soil [1]. The same cracks are formed in the upper layers of roads during seasonal freezing [2]. A large number of works focused on deterioration prevention are known, however the interdisciplinary problem of low-temperature cracking remains relevant. Low-temperature cracks, also called frost cracks, are formed in the upper layers of soils during their freezing [1], in asphalt concrete pavement of the road [2] and airfields [3]. In publications destruction at freezing that is at decrease in temperature is rather often considered [2-6]. Much less attention is paid to modeling and predicting damage to the upper layer of roads if its temperature rises but remains negative. In this case, the layer is compressed by horizontal forces, which can cause the loss of stability of the plane form of

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equilibrium, which is visualized as the appearance of wave-like irregularities on the day surface of the road (buckling). As a result, tensile stresses and cracks may appear in the material of the upper layer near the top of the wave. An analogue of this case is the evolution of wrinkles in a thin film on the substrate [7]. In this paper, only the loss of stability of the plane form of equilibrium of the compressed upper layer is considered as the cause of irregularities. Other possible causes, such as frost heaving, are discussed, for example, in [7, 8]. Analysis of the literature [3-18] showed that an important, but insufficiently studied quantitative characteristic of the considered damage is the assessment of the distance between cracks [2]. In this regard, the objective of the current study was modeling the conditions of the appearance of cracks of the considered type and predicting the distance between the cracks depending on the change negative temperature, the interaction of layers, the thickness of the upper layer, the coefficient of thermal expansion of the upper layer material and its modulus of elasticity (Young's modulus).

2 Materials and Methods

In accordance with the objective of the work, consider one of the possible solutions to the above problem on the example of a simplified model of the road section. The model is an idealized two-layer structure in which the upper layer of frozen soil mechanically interacts with the base, i.e. with the soil with non-negative temperature. Consider the case where the temperature of the upper layer increases, for example, from $t_0 = -15^\circ C$ to $t = -5^\circ C$, which leads to an increase in the size of the upper layer. However, the tangential forces of friction and adhesion, acting in the contact zone with the base, prevent the free increase in the horizontal dimensions of the upper layer, which is why this layer is compressed by normal stresses $\sigma$. As is known, the stresses $\sigma$ are proportional to the temperature change $\Delta t$. If $\Delta t$ and, therefore, $\sigma$ are sufficiently large, there will be a loss of stability and a zone with tensile stresses, as noted above. If the tensile strength of the upper layer is insufficient, a crack will appear (Fig. 1).

\[
\Delta t = t - t_0 - \text{the change in the temperature of the upper layer (in the problem under consideration } \Delta t > 0); \ k - \text{the coefficient (N / m}^3), \text{characterizing the friction and adhesion force distributed over the contact area of the layers [6]; } B, H \text{ and } L \text{ - respectively, the width, thickness and length of the upper layer of the segment (Fig. 1); } \alpha - \text{coefficient of thermal expansion [18]. It can be shown that in the stage preceding buckling}
\]

\[
\sigma_{\text{max}} = -k \alpha \Delta t L^2 / 8H.
\]  

(1)

With increasing $\Delta t$, the stresses grow and can reach a critical value $\sigma_c$ when the plane form of equilibrium becomes unstable. Then the curvilinear form of equilibrium becomes stable (Fig. 1). In this case, taking into account the limitations on strength, it is legitimate to
predict the appearance of cracks in the upper layer. To determine the value \( \sigma_c \), we will use the results of the works known from the literature [1, 7, 18].

It is known that the destruction of frozen soil and the appearance of frost cracks is preceded by elastic state [1], so it is correct in the use of the formulas of elasticity theory for determining of the above the critical value \( \sigma_c \). We assume that similar phenomena occur in the upper layer of roads at sufficiently high humidity. Assuming that the soil layer with non-negative temperature cannot accumulate elastic energy, we use (adapt) the known results [7, 19], according to which the critical value can be determined by the formula

\[
\sigma_c = -\pi^2 H^2 E / 3L^2,
\]

Here \( E \) is the elasticity modulus of the layer material. In (2), \( L \) is interpreted as the wavelength (Fig. 1). In the cited works [7, 19] it is shown that folds with any wavelength exceeding the critical value \( L_c \) can be formed,

\[
L_c = \pi H (-E / 3\sigma)^{0.5}
\]

The increase in the amplitude of the folds occurs at different speeds. At the initial stage of deformation, the rate of amplitude growth is maximal for folds with a wavelength \( L_c = \pi H (-E / \sigma)^{0.5} \) [7, 19].

### 3 Results

The ratios (2) and (3) do not take into account the adhesion and friction forces of the layers. Consider the impact of these forces on the size of the \( L_c \).

As noted above, the buckling of the compressed layer can lead to cracks. From the physical meaning of the problem, it follows that the distance between the cracks depends on the value of \( L_c \) (Fig. 1). The dependence of the \( L_c \) on the above-mentioned characteristics of the upper layer \( \Delta t, k, H, \alpha, \) and \( E \) can be found from the condition \( \sigma_c = \sigma_{\text{max}} \). Then, using (1) and (2) we obtain:

\[
L_c = (8\pi^2 E H^3 / 3k \alpha \Delta t)^{0.25}.
\]

It is necessary to take into account that the coefficient \( \alpha \) significantly depends on the temperature, e.g. as by Fig. 2 [18]. Therefore, the product \( (\alpha \Delta t) \) in (4) is recommended to be determined numerically, e.g. by the formula trapezoids.

![Fig. 2. The dependence \( \alpha(t) \) for one of the varieties of frozen clays on temperature [18].](image-url)
Consider a numerical example of $L_c$ (4) determination [20].

Let $E = 100$ MPa, $t_0 = -15$ C, $t = -5$ C. The interval $[t, t_0]$ we divided into two sections: at $t = -5.0, -10.0$ and $-15.0$ °C by Fig. 2 we find, respectively, $\alpha(t) = 1.9, 1.2$ and $0.8$ °C$^{-1}$. Then by the formula trapezoids: $\alpha(t) \cdot \Delta t = (1.9+2 \cdot 1.2+0.8) \cdot 5/2=12.75$. The results of $L_c$ (4) calculations are shown in Fig. 3.

![Graph showing the dependence of $L_c$ on $H$ and $k$.](https://example.com/graph.png)

**Fig 3.** The dependence $L_c$ on $H$ and $k$.

**4 Discussion**

For practice, it is important to eliminate the causes of frost cracks [2, 5]. The present study showed that a decrease in $k$ leads to a decrease in $\sigma_{\text{max}}$ (1) and an increase in $L_c$ (4), i.e. to a decrease in the prerequisites for the appearance of cracks and an increase in the distance between them, if cracks still appear due to buckling ending (Fig. 1). It follows from (1)–(4) that an increase in $H$ also leads to a decrease in of the road deterioration. Similar qualitative conclusions are justified in the work [3] in relation to aerodrome surfaces. When freezing, the thickness of the upper layer (Fig. 1) increases. In the considered model example $L_c$ varies from 12 to 213 m, depending on $k$ and $H$ (Fig. 3).

To assess the adequacy of the model (4), experimental data [2] were used, according to which the average number of temperature cracks per 1000 m of asphalt pavement of roads with a service life of 1 to 12 years was in the range from 1 to 45. Accordingly, the estimates of the distance between the cracks are in the range from 22 to 1000 m, which does not contradict the results in Fig. 3. Thus, the presented results are quite reliable.

It should be noted that the more likely cause of the appearance of frost cracks is, presumably, not buckling, but stretching of the upper layer with a decrease in the negative temperature [1-4, 14, 16]. However, the phenomenon of buckling occurs quite often and therefore cannot be ignored. Application of the presented results will expand the possibilities of increasing the crack resistance of the upper layer of road surfaces during their seasonal freezing.

**5 Conclusions**

In the present article with the use of mathematical modeling of mechanical systems and adaptation of the results known in the considered area, the technique of modeling and forecasting of damages of the upper layer of roads at increase of their temperature in the
area of negative values is offered. The estimation of the critical wavelength at the buckling of the compressed upper frozen layer of the road with increasing temperature in the interval of negative values is proposed (4).

Since the coefficient $\alpha$ significantly depends on the temperature [18], the product $\alpha \cdot \Delta t$ in the calculation formulas (e.g. (4)) is recommended to be determined numerically, e.g. by the trapezoid formula. In this case, the volume of calculations almost does not increase, but at the same time, the results of numerical modeling will be more adequate. The results of numerical modeling are adequate from the physical point of view, which is confirmed by their internal consistency and compliance with the experimental data known from the literature. The practical significance of the study results lies in the possibility of their use in the justification of new technical solutions in road construction. The prospects of the study can be focused on clarifying the dependence of the modulus of elasticity on the temperature and coefficient of friction and adhesion of the pavement layers.

References

1. V. Merzlyakov, 53, 221-225 (2016). DOI: 10.1007/s11204-016-9389-1
2. B. Teltayev, Reports of the national Academy of sciences of the Republic of Kazakhstan, 5, 35-57 (2015)
3. A. Popov, A. Kochetkov, and A. Masalykin, Russian Journal of Building Construction and Architecture, 2, 65-74 (2015)
4. P. Rajbongsh, and A. Das, Journal of Transportation Engineering, 135, 745-752 (2009)
5. P. Pereira, J.A. Pais, Journal of Traffic and Transportation Engineering, 4, 316-346 (2017) https://doi.org/10.1016/j.jtte.2017.06.001
6. G. Chen, and G. Baker, Advances in Structural Engineering, 8, 595-602 (2005) http://dx.doi.org/10.1260/136943305776318338
7. S. Im, and R. Huang, Journal of applied mechanics, 72, 955-961 (2005)
8. D. Sheng, S. Zhang, F. Niu, and G. Cheng. Geotechnique, 64, 144-154 (2014)
9. E. Liu, Y. Lai, and H. Wong J. Feng. Int. Journal of Plasticity, 107, 246-285 (2018)
10. D. Rys, J. Judycki, M. Pszczola, et.al. Construction and Building Materials, 147, 478-487 (2017) https://doi.org/10.1016/j.conbuildmat.2017.04.179
11. J. Zhang, and V.C. Li, Journal of Transportation Engineering, 127, 455-462 (2001)
12. M. Pszczola, and J. Judycki, Baltic Journal of Road and Bridge Engineering, 10, 39-45 (2015) DOI: 10.3846/bjrbre.2015.05
13. M. Zavyalov, A. Kirillov, Magazine of Civil Engineering, 54, 70-76 (2015). DOI: 10.18720/MCE.70.5
14. J. Judycki, International Journal of Pavement Engineering, 19, 24-36 (2018) https://doi.org/10.1080/10298436.2016.1149840
15. G. Levashov, and V. Sirotuk, Vestnik SibADI, 22, 23-26 (2011)
16. G. Kirukhin, Roads and Bridges (Dorogi i mosty), 30, 309-328 (2013)
17. I. Dmitriev, and A. Kirillov, Construction of Unique Buildings and Structures, 53, 7-28 (2017)
18. V. Merzlyakov, Goeokologiya, 2, 159-167 (2012)
19. A. Shugurov, and A. Panin, Physical Mesomechanics, 12, 23-32 (2009)
20. T. Gavrilov, K. Khoroshilov, G. Kolesnikov, JARITS, 11 (2018). Available: http://srcms.ru/issue3.html