ON THE EFFECTIVE SPATIAL SEPARATIONS IN THE CLUSTERING OF FAINT GALAXIES

BOUDEWIN F. ROUKEMA
Division of Theoretical Astrophysics, National Astronomical Observatory, Mitaka, Tokyo 181, Japan; roukema@iap.fr

AND

DAVID VALLS-GABAUD
URA CNRS 1280, Observatoire de Strasbourg, 11 rue de l’Université, 67000 Strasbourg, France; Royal Greenwich Observatory, Madingley Road, Cambridge CB3 0EZ, England, UK; dvg@astro.u-strasbg.fr, dvg@ast.cam.ac.uk

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ABSTRACT

Several recent measurements have been made of the angular correlation function \( w(\theta, m) \) of faint galaxies in deep surveys (e.g., in the Hubble Deep Field, hereafter HDF). Are the measured correlations indicative of the gravitational growth of primordial perturbations, or of the relationship between galaxies and (dark matter–dominated) galaxy halos? A first step in answering this question is to determine the typical spatial separations of galaxies whose spatial correlations, \( \zeta(r, z) \), contribute most of the angular correlation.

The median spatial separation of galaxy pairs contributing to a fraction \( p \) of the angular correlation signal in a galaxy survey is denoted by \( r^\text{eff}_p \) (§ 3) and compared with the perpendicular distance, \( r_\perp \), at the median redshift, \( z^\text{med}_\perp \), of the galaxies. Over a wide range in spatial correlation growth rates, \( v \), and median redshifts, \( r^\text{eff}_50 \) is no more than about twice the value of \( r_\perp \), while \( r^\text{eff}_0 \) is typically 4 times \( r_\perp \).

Values of \( r^\text{eff}_p \) for redshift distributions representative of recent surveys indicate that many angular correlation measurements correspond to spatial correlations at comoving length scales well below \( 1 \h^{-1} \) Mpc. For \( \Omega_0 = 1 \) and \( \Lambda_0 = 0 \), the correlation signal at 4° predominant in the Villumsen et al. (1996) estimates of \( w(\theta, m) \) for faint HDF galaxies corresponds to \( r^\text{eff}_50(\text{HDF}) \approx 40 \h^{-1} \) kpc; other cosmologies and angles up to 10° can increase this to \( r^\text{eff}_50(\text{HDF}) \lesssim 200 \h^{-1} \) kpc. The proper separations are \((1+z)\) times smaller, where \( 1 \lesssim z \lesssim 2 \).

These scales are small: the faint galaxy angular correlation measurements are at scales where halo and/or galaxy existence, let alone interactions, may modify the spatial correlation function. These measurements could be used to probe the radial extent of halos at high redshift.

Subject headings: cosmology: theory — galaxies: clusters: general — galaxies: statistics

1. INTRODUCTION

The growth of primordial density fluctuations into galaxies and clusters of galaxies is an essential element in understanding how structure forms in the universe. One common way to represent this structure statistically from observed galaxies is to calculate the two-point autocorrelation function, \( \zeta(r, z) \), which can be approximately parametrized as a power law (in spatial separation of galaxy pairs) that increases in amplitude as a function of time according to a single parameter, \( \epsilon \) (e.g., Groth & Peebles 1977):

\[
\zeta(r, z) = (r_0/r)\gamma(1+z)^{\gamma-3} \epsilon^{-\gamma},
\]

where \( r \) and \( r_0 \) are expressed in comoving coordinates and \( \gamma \) represents the approach to homogeneity at larger length scales.

Observed values are typically \( r_0 \approx 5 \h^{-1} \) Mpc and \( \gamma \approx 1.7\text{–}1.8 \) (e.g., Davis & Peebles 1983; Loveday et al. 1992) for the low-redshift general galaxy population. The analysis of Davis & Peebles (1983) is consistent with this power law on scales of \( 10 \h^{-1} \) kpc \( \lesssim r \lesssim 10 \h^{-1} \) Mpc, although the correlations appear somewhat stronger (but noisy) at the small-scale end.\(^1\)

Loveday et al. (1992) find their power-law fit to \( \zeta \) of the Stromlo-APM redshift survey to be valid over \( 200 \h^{-1} \) kpc \( \lesssim r \lesssim 20 \h^{-1} \) Mpc.

Direct estimates of the evolution in this power law at low redshift (median redshifts) include \( \epsilon = 1.6 \pm 0.5 \) (Warren et al. 1993, \( z_{\text{med}} = 0.4 \)), \( \epsilon = -2.0 \pm 2.7 \) (Cole et al. 1994, \( z_{\text{med}} = 0.16 \)),\(^2\) and \( \epsilon \approx -2.2 \) (Shepherd et al. 1997, \( z_{\text{med}} = 0.36 \)). However, consideration of galaxies as a single population, i.e., adoption of \( r_0 = 50 \h^{-1} \) Mpc and the median redshift \( z_{\text{med}} = 15,200 \text{ km s}^{-1} \) of the Stromlo-APM survey (Loveday et al. 1992, 1995) for comparison with the higher \( z_{\text{med}} = 0.56 \) Canada France Redshift Survey (CFRS) estimate of \( \zeta \) (Le Fèvre et al. 1996, hereafter CFRS89) would imply that \( \epsilon = 2.8 \) (cf. § 4.1, eq. (3) of CFRS89).

This latter value of \( \epsilon \) is considerably higher than expected either for clustering fixed in comoving coordinates (\( \epsilon = \gamma - 3 \approx -1.2 \), so that the index in eq. [1] is zero);

\(^1\) The more recent estimate of Tucker et al. (1997) finds similar behavior on scales down to about \( 20 \h^{-1} \) kpc, but for the redshift-space correlation function rather than the "real" space correlation function, which makes it difficult to interpret.

\(^2\) In Cole et al. 1994, \( \epsilon \) relates to \( \epsilon \) as \( \epsilon = \gamma(1 - \epsilon) - 3 \).
clustering fixed in proper coordinates on small scales ("stable clustering," $\epsilon = 0$; since the number of "clusters" changes by $[1 + z]^2$ and the number of galaxy pairs by $[1 + z]^6$, the factor in equation [1] for $r$ in proper coordinates is $[1 + z]^{-2}$), or linear growth of density perturbations in an Einstein–de Sitter universe ($\epsilon = \gamma \approx 1 \approx 0.8$).

(Note also that the high $z$ behavior of $\xi$ may be different; e.g., Ogawa, Roukema, & Yamashita 1997.) A lower value of $\epsilon$ could be justified by hypothesizing major changes in visible galaxy populations from low to high redshifts, as suggested by indications of color dependence in the correlation function (e.g., Infante & Pritchet 1995).

The validity or otherwise of the high observational estimates of $\epsilon$ or of major changes in the general galaxy population is not the subject of this article. In fact, as will be shown below, the parameter of interest, $r_{\text{eff}}$, is only weakly sensitive to $\epsilon$.

The motivation for this article stems from attempts to indirectly measure the dependence of $\xi$ on $z$. Since estimation of spectroscopic redshifts requires many more photons than the photometric detection of a galaxy, the projection of $\xi$ onto the celestial sphere, i.e., the angular correlation function $w(\theta, m)$ for angle $\theta$ and survey "limiting apparent magnitude" $m$ can be more easily measured than $\xi$. This means that $\xi$ can be effectively measured at larger redshifts than are obtainable in redshift surveys—but at the cost of having to deduce $\xi$ from its sky projection.

Hence, investigation of the $z$ dependence of $\xi$ can be carried out to larger redshifts by measuring $w$ rather than $\xi$. Many estimates of the amplitude of $w(\theta, m)$ of faint galaxies in faint magnitude ("deep") small-angle (a few square arcminutes, pencil beam, or up to a few square degrees) surveys have been carried out recently (e.g., Efstathiou et al. 1991; Neuschaefer, Windhorst, & Dressier 1991; Pritchet & Infante 1992; Couch, Jurecekic, & Boyle 1993; Roukema & Peterson 1994; Infante & Pritchet 1995; Brainerd, Smail, & Mould 1995). These angular correlations are usually interpreted by integrating $\xi(\theta, z)$ (see eq. [1]) over an appropriate domain in $z-u$ space (see eq. [2]).

The resulting values of $\epsilon$, or hypotheses about how population transitions might justify changes in the value of $r_\text{eff}$ as a function of $z$, are frequently discussed, and an indication of the scale of galaxy pair separations is sometimes given by $r_{\text{eff}} \equiv d_{\text{prop}}(\theta \equiv r(z, u = 0)$ (for $r$ in comoving coordinates; $u$ is redshift separation of a galaxy pair at mean redshift $z$, defined below for eq. [2]) at the median redshift, $z_{\text{med}}$, of the redshift distribution of the galaxy sample.

However, the integral also includes galaxy pairs at unequal redshifts ($u \neq 0$) and galaxy pairs at mean redshifts lower and higher than $z_{\text{med}}$, so rather than accepting the usual assumption that $r_{\text{eff}}$ is representative of typical scales contributing to $w$, we prefer to analyze the integral more closely.

This is the goal of this article: to determine what pair separations contribute to $w(\theta, m)$ over a region of parameter space judged likely for observational values of $w(\theta, m)$. This is done by (1) separating out the different contributions in the double integral relating $\xi$ to $w$, and (2) defining an effective separation $r_{\text{eff}}^p$ to be the median separation of galaxy pairs that contribute to a fraction $p$ of the numerator of this integral. The effective separation depends, in principle, on both the redshift distribution and the evolution of $\xi$, so $r_{\text{eff}}^p$ is evaluated for likely ranges of relevant parameters.

In fact, the assumption that $r_{\text{eff}}$ is a typical scale of galaxy pairs contributing to $w$ turns out to be a good intuition, to better than an order of magnitude. In this paper we present quantitative justification for this intuition.

In § 2, the double integration of $\xi(r, z)$ is presented, using Sawicki, Lin, & Yee (1997) for the photometric redshift distribution of the Hubble Deep Field (HDF; Williams et al. 1996) as an illustration. The effective separation, $r_{\text{eff}}$, is defined and evaluated in § 3. In § 4, implications of the resulting $r_{\text{eff}}$ values are discussed, and § 5 presents the conclusions. All discussion is in comoving ($t = t_0$) units unless otherwise noted, and the Hubble constant is $100$ km s$^{-1}$ Mpc$^{-1}$.

2. Limber’s Equation

The angular correlation function (small-angle approximation) is given by the double integration of $\xi(r, z)$,

$$w(\theta, m) = \frac{\int dz N(z, zm)^2 \left( \frac{d\zeta(r, z)}{dz} \right)}{\int dz N(z, m)^2}, \quad (2)$$

where $\theta$ is the angle on the sky, $m$ is the apparent magnitude, $z = \left(z_1 + z_2\right)/2$ and $u = z_1 - z_2$ parametrize the redshifts of two galaxies at redshifts $z_1$ and $z_2$, $r(z, u)$ is the spatial separation between the two galaxies, and $N(z, m) = \int_0^z d\zeta N(\zeta) d\zeta$ is the redshift distribution at $m$ (Limber 1953; Phillips et al. 1978; Peebles 1980; Efstathiou et al. 1991).

One can think of $\xi(r, z)$ as the excess probability that two randomly chosen galaxies lie at a (comoving) separation $r$. By symmetry, all such galaxies separated in projection by $\theta$ can be thought of as forming the surface of a cone of opening angle $\theta/2$. Again by symmetry, dependence on the second angle $\phi$ can be dropped, so that integration is only needed over the double range of redshifts of pairs of galaxies, weighted by the numbers of galaxies at each of the two redshifts. Note that the angular correlation function $w$ is independent of the normalization of $\int_0^z d\zeta N(\zeta) d\zeta$, so only its shape is relevant to the integral (e.g. Yoshii 1993).

In order to understand the relative importance of the different factors in the integral, we separate them out in Figure 1. The redshift distribution chosen for this illustration is the photometric redshift distribution for the HDF galaxies given by Sawicki et al. (1997); see also Lanzetta, Yahil, & Fernández-Soto (1996) for the three Wide Field Camera images of the Version 2 (1996 February 29) drizzled version of the data, to $I_{AB, 8140} < 27$. The square of this distribution is shown in Figure 1a.

The redshift and distance dependence of $\xi(r(z, u), z)$ can be seen in Figure 1b. At any given $z$, $\xi$ decreases as $u$ increases, since the spatial correlation decreases for increasing $r$, quickly approaching zero. Both the use of a fixed angle $\theta$ (chosen here as $\theta = 4^\circ$, since the range of significant signal in the $w_0$ HDF estimate of Villumsen et al. 1996 is roughly $2.5$ to $10^\circ$) and the choice of "stable clustering in proper coordinates" (i.e., $\epsilon = 0$) contribute to the decrease in $\xi$ along the path for which $u = 0$. For large (fixed) $u$, $r(z, u)$ comes to be dominated by a nearly radial distance separation. Since $d^2 d_{\text{prop}}/dz^2 < 0$, these separations become smaller for increasing $z$, which would imply higher $\xi$ for clustering fixed in comoving coordinates ($\epsilon = \gamma - 3$) as $z$ increases. Indeed, Figure 1b shows that for large constant values of $u$, the increase in $\xi$ as $z$ increases is strong enough to overcome the weakening of the amplitude of $\xi$ for higher $z$. 

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luminosity function or to the use of photometric rather than spectroscopic redshifts, it is not of cosmological interest to include spatial correlations of local dwarf galaxies within a few hundred kiloparsecs of the observer in estimates of $w(z_{\mathrm{med}}) \gtrsim 1$. It is therefore important to observe that the percentile contours concentrate toward lower and lower $z$ for higher values of the integrand (lower percentiles). The combination of rapidly decreasing separations, increasing values of $\xi(r, z)$, and slowness of $N_{\xi}(z, m)$ to decrease imply that a significant fraction of the signal in $w(\theta, m)$ could be due to very local galaxies.

For the calculations presented below, a (conservative) lower limit is therefore set, such that redshifts for which the perpendicular separation $r(z, 0)$ is less than $10 h^{-1}$ kpc are excluded from the domain of integration.

2. The $N_z$ peak of Sawicki et al. (1997) at $z \approx 2.3$ makes a significant contribution between the 70% and 90% contours; roughly 10% of the numerator in equation (2) is due to this peak. With the uncertainties in the estimate of the HDF angular correlation function given by Villumsen et al. (1996) (essentially Poisson due to the small numbers of objects), this is not likely to be important for interpretation of the HDF data. However, for the brighter and more precise angular correlation function estimates (e.g., Brainerd et al. 1995) it could be more important—analytical single-peaked redshift distributions may not be precise enough.

3. THE EFFECTIVE SEPARATION $r_{\text{eff}}$

Of course, the values of the separation scales that contribute most of the value of $w(\theta, m)$ discussed above are for particular choices of the parameters in equation (1) (or on the validity of this equation at such scales), of the redshift distribution, of $\theta$, and of the cosmological model. In order to discuss the dependence of separation scales on equation (1), on the $z_{\text{med}}$ and the shape of the redshift distribution, on $\theta$, and on cosmological parameters, it is useful to define the effective separation, $r_{\text{eff}}$, to be the median value of $r(z, u)$ over the domain in $z-u$ space inside a contour of constant $N_{\xi}^2(z, m, \xi)(r(z, u), z)$ that contributes a fraction $p$ of the value of $w(\theta, m)$ (where the denominator in eq. [2] is held fixed).

3.1. Dependence on the Spatial Correlation Function

Lowering the value of $r_0$ (which is considered by, e.g., Bernstein et al. 1994; Brainerd et al. 1995, to denote a possible transition in galaxy “populations”) factors out of equation (2) as a constant; it reduces the value of $w$ but does not affect the question of which separations are relevant to the integral. Reducing the value of $\gamma$ (for fixed $\varepsilon - \gamma$, in order not to affect the redshift evolution of $\xi$) would increase the values of $r_{\text{eff}}$ slightly. In estimates of $w$ for faint galaxies, it is usually assumed that $\gamma - 1 = 0.8$ in order to correct for the

Figure 1c shows that neither one nor the other of these limits in $u$ is sufficient to cover the domain in $z-u$ space that is relevant for the total value of the double integral. The maximum values of $u$ necessary to cover 90% of the numerator in equation (2) are less than about 0.05% of a redshift “unit,” but this is still large enough that the dominance of radial separation over perpendicular separation causes $\xi$ to increase as $z$ increases at a fixed $u$, at least for $\theta = 4^\circ$.

The smallness of the $u$ required to account for even 50% of the numerator of equation (2) is the key element to interpreting observational estimates of $w(\theta, m)$. In Figure 1d, the $u$ range covered in the $z-u$ plane plots is shown in units of (comoving) $h^{-1}$ kpc. Nearly all of the 90% contour is for galaxies separated by less than 1 $h^{-1}$ Mpc; much of the integral comes from separations of less than 100 $h^{-1}$ kpc. This is the scale on which galaxy interactions and the relationship between galaxy halos dominated by dark matter and the visible galaxies containing stars are likely to be complicated.

Other points to be noted in Figure 1c are the following:

1. The redshift distribution of Sawicki et al. (1997) includes many galaxies in the lowest redshift bins. Whether this is due to a genuinely steep faint end of the galaxy luminosity function or to the use of photometric rather than spectroscopic redshifts, it is not of cosmological interest to include spatial correlations of local dwarf galaxies within a few hundred kiloparsecs of the observer in estimates of $w(z_{\text{med}}) \gtrsim 1$. It is therefore important to observe that the percentile contours concentrate toward lower and lower $z$ for higher values of the integrand (lower percentiles). The combination of rapidly decreasing separations, increasing values of $\xi(r, z)$, and slowness of $N_{\xi}(z, m)$ to decrease imply that a significant fraction of the signal in $w(\theta, m)$ could be due to very local galaxies.

2. The $N_z$ peak of Sawicki et al. (1997) at $z \approx 2.3$ makes a significant contribution between the 70% and 90% contours; roughly 10% of the numerator in equation (2) is due to this peak. With the uncertainties in the estimate of the HDF angular correlation function given by Villumsen et al. (1996) (essentially Poisson due to the small numbers of objects), this is not likely to be important for interpretation of the HDF data. However, for the brighter and more precise angular correlation function estimates (e.g., Brainerd et al. 1995) it could be more important—analytical single-peaked redshift distributions may not be precise enough.

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integral constraint, so this is the value of interest here (e.g., Hudon & Lilly 1996; though note Campos et al. 1995; Infante & Pritchett 1995).

Of more interest is the value of $\epsilon$, which, as mentioned above, has either theoretically or observationally motivated values lying in the range from $-1.2$ to nearly 3.

Figure 2a shows that in spite of the wide range in values of $\epsilon$, the dependence of $r_{\text{eff}}$ on $\epsilon$ is weak. Even for a cosmological constant-dominated flat cosmological model and $\epsilon = 3$, $r_{\text{eff}}$ for the redshift distribution of Sawicki et al. (1997) is no more than a factor of 2 lower than for the analytical distribution.

Although weak, there is an average trend toward lower values for higher $\epsilon$. Higher $\epsilon$ means a more rapid decrease in spatial correlation amplitude with $z$, so that the correlations at lower $z$ contribute relatively more to the integral. The lower the value of $z$, the lower the (comoving) perpendicular separations $r(z, 0)$ for any cosmological model. Hence, the slight decrease in $r_{\text{eff}}^0$.

In other words, independently of the growth rate of the spatial correlation function, angular correlations in deep surveys of faint galaxies only probe length scales characteristic of galactic halos.

The stronger decrease in $r_{\text{eff}}^p$ for the observational (photometric) redshift distribution than for the analytical distribution of the same $z_{\text{med}}$ can be attributed to its complex shape. Once $\epsilon$ has increased enough that the $z \approx 2.2$ peak no longer contributes much, the effective $z_{\text{med}}$ of the observational distribution will be lower than that of the analytical distribution; thus the lower $r_{\text{eff}}^p$ values.

The trend to lower values of $r_{\text{eff}}^p$ is not monotonic for the observational redshift distribution. This is attributable to an increase in the relative importance of $u$ separations to perpendicular separations when $N_2(z, m)(1 + z)^{-3 + \gamma(z)}$ becomes flatter. A complex enough redshift distribution could possibly increase the importance of this reversed dependence of $r_{\text{eff}}^p$ on $\epsilon$, as indeed is seen below.

3.2. Dependence on the Redshift Distribution

The effective separation $r_{\text{eff}}^p$ depends on redshift distribution primarily via the typical redshifts in the sample, or more quantitatively on the median redshift, $z_{\text{med}}$, of $N_z$. This dependence is analyzed here by use of the simple analytical distribution

$$z^2 \exp \left[ -\left( z/z_0 \right)^2 \right],$$

where $\beta = 2.5$ (as in Efstathiou et al. 1991; Villumsen, Freudling, & da Costa 1996). For this distribution, $z_{\text{med}}$ dependence translates directly into dependence on $z_0$, since $z_{\text{med}} \approx 0.95z_0$.

A very useful property of this dependence is shown in Figure 2b; nearly all of this $z_0$ dependence is in the (equivalent of the) angular diameter–redshift relation, shown as the dependence of the perpendicular separation $r_\perp(z) \equiv r(z, u = 0)$ on $z$ in Figure 2c.

Both $r_{\text{eff}}^{50\%} / r_\perp(z_0)$ and $r_{\text{eff}}^{90\%} / r_\perp(z_0)$ are nearly constant over a wide range in $z_0$. Moreover, $r_{\text{eff}}^{50\%}$ is only about 5%–15% greater than $r_\perp(z_0)$, i.e., about 10%–20% greater than $r_\perp(z_{\text{med}})$. Relative to the precision of present cosmological measurements, it can be said that $r_\perp(z_{\text{med}})$ is a very good estimator for $r_{\text{eff}}^{50\%}$ for such analytical distributions. In addition, since $r_{\text{eff}}^{50\%}$ is insensitive to $\epsilon$ and since the $\epsilon$ dependence of both the observational (photometric) and analytical $N_z$ curves are similar (see Fig. 2), the quality of $r_\perp(z_{\text{med}})$ as an estimator for $r_{\text{eff}}^{50\%}$ is little dependent on the value of $\epsilon$ or the precise shape of $N_z$, avoiding the need for explicit integration of Limber’s equation (eq. [2]).

The 90% percentile median separation $r_{\text{eff}}^{90\%}$ is about 4–4.5 times $r_\perp(z_0)$, depending on $z_0$ and the cosmological model to no more than about 10%. Again, the robustness of $r_{\text{eff}}^{90\%}$ relative to $\epsilon$ implies that a constant value of $r_{\text{eff}}^{90\%} / r_\perp(z_0)$ in this range can be used for practical purposes without integration of Limber’s equation being required.

The dependence of $r_{\text{eff}}^p$ on the shape of $N_z$ is not completely negligible. This can be seen to some extent in Figure 2a, which shows that for the same $z_{\text{med}}$, the values of $r_{\text{eff}}^p$ can differ by up to about a factor of 2.

To confirm this more explicitly, various published photometric redshift distributions estimated for the HDF have been adopted in place of that of Sawicki et al. (1997), and the resulting dependence of $r_{\text{eff}}^p$ on $\epsilon$ for these $N_z$ estimates are shown in Figure 3. The $N_z$ estimates adopted are shown in Figure 4.
The results are similar to those for the \( N_z \) of Sawicki et al. (1997), though the maximum difference between photometric and analytical distributions increases from about a factor of 2 to a factor of 3–4 for \( r_{\text{eff}}^{90\%} \) in the case of high \( \epsilon \) values. The bimodality of \( N_z \) in Gwyn & Hartwick (1996) and the high resolution of \( N_z \) in Mobasher et al. (1996) makes this hardly very surprising.

The large differences between the photometric and analytical distributions for \( N_z \) in Gwyn & Hartwick (1996) are mainly due to the low \( z \) peak. Obviously, there is uncertainty in the correctness of photometric redshifts, but if \( N_z \) (HDF) were really as bimodal as Gwyn & Hartwick (1996) estimated, with a strong low-\( z \) peak, then not only would many galaxy pairs that contribute to \( w_0 \) be closer to one another than would be expected from an analytical unimodal distribution, but a large part of the \( w_0 \) signal would be from noncosmological distances, at which \( r_{\perp} \sim 10 \) \( h^{-1} \) kpc. In this case, the meaning of a \( w_0 \) estimate would have to be analyzed in considerable detail, or else redshift separation would have to be used to estimate \( w(r_p) \) (see Davis & Peebles 1983), rather than \( w_0 \), in order to extract quantities with simple physical meaning.

In summary, comparison of the earlier photometric redshift distributions to that deduced by Sawicki et al. (1997) implies similar values of \( r_{\text{eff}}^{\epsilon} \) to those for Sawicki et al. (1997) to within much less than an order of magnitude.

It should also be noted that changing the limiting magnitude of the survey or changing the value of \( \beta \) is equivalent to a changing \( z_{\text{med}} \) and/or the shape of \( N_z \).

3.3. Dependence on Other Parameters

Since \( r(z, u) \) is dominated by the perpendicular separation \( r(z, u = 0) = \theta d_{\text{proj}}(z) \) (for \( \theta \ll 1 \) rad), \( r_{\text{eff}}^{\epsilon} \) is nearly proportional to \( \theta \). Indeed, although the line-of-sight galaxy separation for a given \( z \) is not directly proportional to \( \theta \) (i.e., \( r(z, u, \theta_1) = r(z, u, \theta_2) \) if \( u \neq 0 \)), the required domain in \( u \) to obtain a fraction \( p \) of the integral of \( w(\theta, m) \) expands sufficiently (but remains small enough) that the value of \( r_{\text{eff}}^{\epsilon} \) remains proportional to \( \theta \) to the precision of the calculation (for \( p = 50\% \) and \( p = 90\% \)).

Values of \( r_{\text{eff}}^{50\%} \) and \( r_{\text{eff}}^{90\%} \) for the nominal angle of the \( w \) determination of Villumsen et al. (1996) \( [w(\theta, m) \right) \right] \) is normally interpolated or extrapolated to the same angle for a large number of different surveys in order to compare the relative amplitudes of the surveys, i.e., \( \theta = 1^\circ \), imply \( r_{\text{eff}}^{\epsilon} \) values 4 times lower than those displayed in Figure 2, i.e., \( r_{\text{eff}}^{50\%} \sim 10h^{-1} \) kpc and \( r_{\text{eff}}^{90\%} \sim 40h^{-1} \) kpc. While such a nominal angle is primarily meant for comparison of amplitudes of \( w \), the physical meaningfulness or otherwise should obviously be kept in mind when examining such figures.

Brighter faint galaxy surveys having most of their signal at, for example, \( 40^\circ \), would have their \( r_{\text{eff}}^{\epsilon} \) values multiplied by 10, i.e., \( r_{\text{eff}}^{50\%} \sim 500h^{-1} \) kpc and \( r_{\text{eff}}^{90\%} \sim 1500h^{-1} \) kpc.

The effects of cosmological parameters on \( r_{\text{eff}}^{\epsilon} \) are shown in Figures 2a and 2c; since \( d_{\text{proj}}(z) \) is successively larger for hyperbolic and cosmological constant–dominated flat cosmologies, the values of \( r(z_{\epsilon}, 0) \) and consequently of \( r_{\text{eff}}^{\epsilon} \) increase respectively. The increase in \( r(z_{\epsilon}, 0) \) is in fact faster than that of \( r_{\text{eff}}^{\epsilon} \), so that Figure 2b shows a slight reversal in this dependence for the ratio \( r_{\text{eff}}^{\epsilon}/r(z_{\epsilon}, 0) \), though the main effect is in \( r(z_{\epsilon}, 0) \).

4. DISCUSSION

The results above can be summarized as order of magnitude estimates:

\[
    r_{\text{eff}}^{\epsilon} = (1 + \Delta_p) r_{\epsilon}(z_{\text{med}}(N_z)),
\]

Fig. 3.— Dependence of \( r_{\text{eff}}^{\epsilon} \) on the shape of \( N_z \) as in Fig. 2a, but replacing the \( N_z \) estimate of Sawicki et al. (1997) by those of (a) Lanzetta et al. (1996), (b) Fig. 2 of Mobasher et al. (1996), and (c) Gwyn & Hartwick (1996), and using analytical distributions with appropriate \( z_0 \) values of (a) \( z_0 = 1.15 \), (b) \( z_0 = 1.6 \), and (c) \( z_0 = 2.0 \). Axes and line styles are as in Fig. 2a.

Fig. 4.— Shapes of the photometric redshift distributions \( N_z \) estimated by Sawicki et al. (1997), Lanzetta et al. (1996), Mobasher et al. (1996), and Gwyn & Hartwick (1996) for the HDF against redshift, normalized for purposes of comparison. The dependence of \( r_{\text{eff}}^{\epsilon} \) on the different shapes is shown in Figs. 2a, 3a, 3b, 3c, as indicated.
and

\[ r_\text{med} = \frac{2c}{H_0} \left( \frac{1}{\sqrt{1 + z}} \right), \]

(5)

and

\[ 1 \leq \Delta_{z,0} \leq 2, \]
\[ 2 \leq \Delta_{z,0} \leq 7, \]

(6)

for \( \gamma = -1.8, -1.2 < \epsilon < 3, 0.1 < \Omega < 1, 0.0 < \lambda < 1 - \Omega, 1 < z_{\text{med}}(N_s) < 2.5 \) and for \( N_s \) shaped as "smooth," as in equation (3), or no "rougher" than the redshift distribution of Sawicki et al. (1997).

These relations enable \( r_{\text{eff}}^0 \) to be estimated for observational angular correlation function measurements. Many of the brighter (\( B < 24, V < 25 \), or \( R < 26 \)) faint galaxy angular correlation measurements (Neuschaefer et al. 1991; Pritchet & Infante 1992; Couch et al. 1993; Infante & Pritchet 1995) show significant power-law correlations on scales ranging from \( \sim 10' \) to \( \sim 10' \). Adopting \( z_{\text{med}} \approx 0.5 \) as a typical value for the above surveys, which should give \( r_1 \), correct to within better than an order of magnitude, equations (4)-(6) (which remain valid under extrapolation to \( z_{\text{med}} \approx 0.5 \)) imply that \( 50 \text{ h}^{-1} \) kpc \( \leq r_{\text{eff}}^0 \leq 3 \text{ h}^{-1} \) Mpc and \( 200 \text{ h}^{-1} \) kpc \( \leq r_{\text{eff}}^0 \leq 12 \text{ h}^{-1} \) Mpc for these angular ranges. Spatial correlations are well established at most of these spatial separations, so interpretation in terms of equation (1) is likely to be a good first approximation.

In this case, almost none of the signal comes from separations for which Loveday et al. (1992) found significant correlations fit by a power law, and the similarity to halo sizes is significant. Possibly one of the neatest measurements of halo extent is by Lyman \( \alpha \) and metal line absorption systems in front of quasars. For instance, Bergeron & Boissé (1991), Betchtel et al. (1994), Lanzetta et al. (1995), Fang et al. (1996), and Le Brun et al. (1996) estimate gaseous halo radii for both kinds of systems of around \( 50-200 \text{ h}^{-1} \) kpc. This range matches very closely the range in \( r_{\text{eff}}^0 \) just mentioned. (However, it should be noted that [1] the Lyman \( \alpha \) systems may not be associated with galaxies [e.g., Le Brun, Bergeron, & Boissé 1996] and [2] there is marginal rotation curve evidence that the halo of the Galaxy is no more than 15 kpc in radius [Honma & Sofue 1996; but see Binney & Dehnen 1997], implying a mass for the Galaxy about an order of magnitude smaller than most estimates.)

Is the value of \( z_{\text{med}} \) adopted here too low? While the photometric redshift analysis of Sawicki et al. (1997) uses template spectra that account for both internal reddening and high-\( z \) Lyman absorption, earlier analyses (e.g., Mobasher et al. 1996) suggested that \( z_{\text{med}} \) should be as high as \( 2.1 \). Even if the redshifts of the R-selected sample contributing to the estimates of \( w(\theta, m) \) given by Villumsen et al. (1996) (or to independent estimates of, say, an I-selected sample) were as high as \( z = 2.1 \), these effective separations would only increase by about 30%-50% (depending on cosmology), according to equations (4)-(6). The spatial separations would still correspond to typical estimates of galaxy halo radii, particularly when applying the conversion from comoving to proper coordinates. Note that the values adopted for use in equation (3) of the models of Villumsen et al. (1996), shown in their Figure 2 for \( R < 26.5 \) to \( R < 28.5 \), are \( 1.4 < z_0 < 1.8 \).

This result brings to mind the suggestion, based on morphological analysis of the HST Medium Deep Survey (MDS) and the HDF (Griffiths et al. 1994; Casertano et al. 1995; Driver et al. 1995a, 1995b; Glazebrook et al. 1995; Abraham et al. 1996; van den Bergh et al. 1996), that many
of the HDF galaxies are “building blocks” of future galaxies which later merge together. (Pascarelle et al. 1996 make a similar suggestion based on HST imaging and Multiple Mirror Telescope spectroscopic redshifts.) In this case, the HDF galaxies’ halos should be smaller than present-day galaxy halos, so the visible galaxies could be correlated without their halos necessarily overlapping.

However, Davis & Peebles’s (1983) observation of correlated galaxies down to $10^{-1}$ kpc is a local observation, not an observation of primordial galaxies. It is quite noisy, so perhaps it is already affected by the detailed nature of the galaxy-halo relationship. In either case, the angular correlation function of the HDF must be affected by, and hence offer some clues to, the relationship between galaxies and halos.

A complementary analysis to the one presented here is that of Colley et al. (1996, 1997). Colley et al. (1996) measure a very strong angular correlation for “galaxies” selected by color to be at high redshifts and point out that the galaxies which later merge together. et al. (Pascarelle 1996) is given by Matarrese (1997), for an $\Omega_0 = 1$, $\lambda_0 = 0$ CDM universe where the full linear and nonlinear evolution is given by a formula fitted to gravity-only N-body simulations (Hamilton et al. 1991; Jain, Mo, & White 1995). Figure 5 of Matarrese et al. (1997) (for no bias factor) is approximately consistent with the HDF correlations, while Villumsen et al. (1996) found that a spatial correlation growth rate of $\epsilon = 0.8$ in equation (1) was not sufficient to provide low enough correlations unless the value of $r_0$ is decreased, i.e., that a transition in populations is hypothesized. Differences at the submegaparsec level could explain this better fit. However, the CDM simulations used for Jain et al.’s formula have only 1 particle per $350 h^{-1}$ kpc$^3$ and a force resolution of $32 h^{-1}$ kpc, so they may not be valid at these scales. In addition, they cannot address the question of how to relate stellar galaxies to dark matter-dominated halos, unless this is governed by gravity alone, which seems unlikely.

One empirical, but theoretically motivated, alternative would be the analysis of Peacock (1996), based on a two power law fit to galaxy correlations of the APM (Maddox et al. 1990a, 1990b) and IRAS (Saunders, Rowan-Robinson, & Lawrence 1992) surveys. However, Peacock (1996) finds that this fit implies that for an $\Omega_0 = 1$, $\lambda_0 = 0$ universe, an extra correlation at scales of $r \lesssim 2 h^{-1}$ Mpc is needed to fit the CFRS data (CFRS8), unless a scale-dependent bias factor is used in combination with a constant bias factor. Matarrese et al.’s (1997) use of the $N$-body motivated $\xi(r, z)$ description of Jain et al. (1995) better fits the CFRS data (for no bias), so this would perhaps be more appropriate for an extension of the present work.

An alternative empirical improvement to equation (1) might be to use the $w$ estimates of Infante, De Mello, & Menanteau (1996) on very small scales (down to $\approx 1$”) in a survey covering a very large solid angle ($2.3$ deg$^2$), having enough precision to make such estimates. Infante et al. (1996) find that at the smallest separations, $2'' \lesssim \theta \lesssim 6''$, there is a correlation signal about a factor of 3 higher than that of a $\gamma = 1.8$ power law extrapolated from larger angles. To the extent that the above $r_{\rm eff}$ calculations can be extrapolated to this “highly nonlinear” case, the value $r_1 = 24 h^{-1}$ kpc (as defined in eq. [5]) implies that half of this signal comes from three-dimensional galaxy pair separations of about this size, and 40% more comes from separations up to about $100-200 h^{-1}$ kpc. (Note that Infante et al. (1996) adopt proper units. The conversion follows from using $z_{\rm med} = 0.35$.) The values of $\xi(r, z)$ for $r \lesssim 20 h^{-1}$ kpc, without contamination from “normal” correlations at larger angles, could therefore be even higher than might be expected from simple inspection of the figures in Infante et al. (1996).

5. CONCLUSIONS

In order to see what length scales and redshifts correspond to the spatial correlations $\xi(r, z)$ represented in recent measurements of the faint galaxy angular correlation function $w(\theta, m)$, Limber’s equation (eq. [2]) has been examined over a range of likely possibilities for $\xi$ and $N_s(z, m)$. The separation into $N_s(z, m)$ and $\xi(r, z)$ and the contours in the $z-u$ plane that contribute to the integral have been shown, revealing that observational $w$ measurements combine spatial correlations from a wide range in mean redshifts.

### TABLE 1

| $p$ | $\Omega_0$ | $\lambda_0$ | $z = 2.5$ | $z = 5$ | $z = 2.5$ | $z = 5$ |
|-----|------------|------------|-----|-----|-----|-----|
| 50% | 1.0 | 0.0 | 14 | 17 | 4 | 3 |
| 50% | 0.1 | 0.0 | 21 | 35 | 6 | 6 |
| 50% | 0.1 | 0.9 | 26 | 37 | 7 | 6 |
| 90% | 1.0 | 0.0 | 54 | 69 | 16 | 12 |
| 90% | 0.1 | 0.0 | 86 | 141 | 24 | 24 |
| 90% | 0.1 | 0.9 | 103 | 147 | 30 | 25 |

Note.—Values of $r_{\rm eff}$ for $1''$ for redshifts $z = 2.5$ and $z = 5$ spanning redshifts likely for the HDF objects color-selected to high redshift by Colley et al. 1996, 1997, assuming that eqs. (4)-(6) can be validly extrapolated to these objects. Effective separations are in units of $h^{-1}$ kpc, comoving or proper as indicated.
An effective separation, $r_{\text{eff}}^p$, to quantify the relevant length scales for $w$ has been defined simply as the median (comoving) separation within the contour of constant $(\partial N/\partial z^p)_{r_0}^p [r(z, \theta, \sigma)]$ contributing a fraction $p$ of the numerator of equation (2). This is similar in order of magnitude to the perpendicular distance $r_p$ at the characteristic redshift $z_0$ of simple analytical redshift distributions (eq. [3]), and hence to the median redshifts of such distributions.

More precisely, for the parameter space covered, is not tested. However, the validity of redshift dependence scaling by $(1 + z)^{(3 - \epsilon - \gamma)}$ is not tested.

The scales on which HDF galaxies are correlated (as measured by Villumsen et al. 1996) imply comoving separations of $25 \pm 400$ kpc $r_{\text{eff}}^p < r_{\text{eff}}^p < 400$ h$^{-1}$ kpc. That is, for typical redshifts over $z \geq 1$, a substantial fraction of the angular correlation signal is generated by galaxies spatially separated, in proper units, by around 10–100 $h^{-1}$ kpc. While spatial correlations of galaxies at these separations have been observed for local galaxies (Davis & Peebles 1983), this is a scale at which halo and/or galaxy existence, let alone interactions, might strongly modify the spatial correlation function.

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