Fourier analysis of non-Blazhko ab-type RR Lyrae stars observed with the Kepler space telescope

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ABSTRACT

Nineteen of the \(\sim 40\) RR Lyrae stars in the Kepler field have been identified as candidate non-Blazhko (or unmodulated) stars. In this paper we present the results of Fourier decomposition of the time-series photometry of these stars acquired during the first 417 days of operation (Q0-Q5) of the Kepler telescope. Fourier parameters based on \(\sim 18400\) long-cadence observations per star (and \(\sim 150000\) short-cadence observations for FN Lyr and for AW Dra) are derived. None of the stars shows the recently discovered ‘period-doubling’ effect seen in Blazhko variables; however, KIC 7021124 has been found to pulsate simultaneously in the fundamental and second overtone modes with a period ratio \(P_2/P_0 \sim 0.59305\) and is similar to the double-mode star V350 Lyr. Period change rates are derived from O–C diagrams spanning, in some cases, over 100 years; these are compared with high-precision periods derived from the Kepler data alone. Extant Fourier correlations by Kovács, Jurcsik et al. (with minor transformations from the \(V\) to the \(K_p\) passband) have been used to derive underlying physical characteristics for all the stars. This procedure seems to be validated through comparisons of the Kepler variables with galactic and LMC RR Lyrae stars. The most metal-poor star in the sample is NR Lyr, with \([\text{Fe/H}] = -2.3\) dex; and the four most metal-rich stars have \([\text{Fe/H}]\) ranging from \(-0.6\) to \(+0.1\) dex. Pulsational luminosities and masses are found to be systematically smaller than \(L\) and \(M\) values derived from stellar evolution models, and are favoured over the evolutionary values when periods are computed with the Warsaw linear hydrodynamics code. Finally, the Fourier parameters are compared with theoretical values derived using the Warsaw non-linear convective pulsation code.

Key words: Kepler mission – stars: oscillations – stars: variables: RR Lyr stars
1 INTRODUCTION

The Kepler Mission is designed to detect (via transits) Earth-like planets around solar-type stars (Koch et al. 2010). To achieve this goal the 1.4-m Kepler telescope has been monitoring almost continuously the light variations of over 150,000 stars in a field located at (RA,DEC)$_{J2000} = (19:22:40, +44:30)$ or $(l,b) = (-76.3, +13.5)$. The mandate of the Kepler Asteroseismic Science Consortium (KASC) is to use these data to better understand the astrophysics of the stars seen in the Kepler field, with Working Group 13 being responsible for the study of the RR Lyra variable stars.

Kolenberg et al. (2010) discussed the initial selection of 28 target RR Lyr stars in the Kepler field and presented preliminary results based on the earliest Kepler observations for RR Lyr and V783 Cyg, both of which exhibit the Blazhko (1907) effect, and for the ‘non-Blazhko’ star NR Lyr. An important discovery was the phenomenon of ‘period doubling’, a cycle-to-cycle amplitude variation that occurs at certain Blazhko phases. The effect has been explained by Szabó et al. (2010) as a 9:2 resonance between the fundamental mode and the 9th-order radial overtone. More recently Kolláth, Molnár & Szabó (2011) showed that the resonance destabilises the fundamental mode limit cycle leading to period doubling. Among RR Lyr stars period doubling has been observed to date only in the Blazhko variables. Benkő et al. (2010, hereafter B10) studied a sample of 29 modulated and unmodulated RR Lyr stars using Kepler Q0-Q2 long-cadence data, and identified 14 of the 29 stars (48%) as exhibiting the Blazhko effect. Most recently, a more detailed investigation of RR Lyrae itself has been made by Kolenberg et al. (2011), and Smolec et al. (2011) have used non-linear hydrodynamic models to model Blazhko RR Lyr stars. In the present paper we look more closely at the unmodulated ab-type RR Lyr stars – that is, those stars pulsating in the fundamental mode and believed to exhibit no amplitude or phase modulations and to have the most stable light curves. According to the notation established by Alcock et al. (2000) our sample stars are of type RR0. The analysis is based primarily on the Kepler long-cadence (30-min) photometry acquired in quarters Q0-Q5 (over 18000 measurements per star made over 417 d), and the short-cadence (1-min) photometry for FN Lyr and AW Dra acquired in Q0 (over 14000 high-precision brightness measurements per star made over 9.7 d) and Q5 (∼135,000 observations per star made over ∼90 days).

We first discuss the sample selection (§2.1) and procedure for transforming the raw fluxes delivered from the Kepler Science Office to magnitudes on the $Kp$ system (§2.2). Before analyzing the Kepler data a search (§3.1) was conducted of available historical data for the stars (e.g., photometry, periods, light curves, and times of maximum light). Accurate periods were derived using all the available $Kp$ photometry (§3.2). Then, from the plotted light curves, total amplitudes and risetimes were measured (§3.3). Period change rates ($dP/dt$) and revised periods were calculated from $O-C$ diagrams constructed with the historical data combined with the Kepler data (§3.4). Fourier analysis of light curves (§4.1) was carried out for both the $Kp$ data (§4.2) and for the extant $V$ data (§4.4). The long time base and intensive sampling of the Kepler data provided an opportunity to examine the light curves for cycle-to-cycle variations. To test for such variations the data were divided into single-cycle blocks, and for each block a separate Fourier analysis was conducted (§4.3). For the Kepler stars observed correlations among the Fourier parameters (all cycles combined) were examined (§5) for both the $Kp$ and $V$ passbands (§5.1). The resulting $V$-$Kp$ offsets (§5.2) allowed comparisons to be made between the Kepler stars and RR Lyr stars in galactic and Magellanic Cloud globular clusters (§5.3) and field stars in the inner regions of the Large Magellanic Cloud (§5.4). Physical characteristics were also derived from suitably-modified extant $V$-band correlations (§6). These include iron (or metal) abundance [Fe/H], reddened colour $(B-V)_0$, effective temperature $T_{eff}$, distance $d$, absolute magnitude $M_V$, pulsational and evolutionary luminosities, $L(puls)$ and $L(evol)$, pulsational and evolutionary masses, $M(puls)$ and $M(evol)$, and location in the instability strip. Finally, new non-linear hydrodynamical models (Warsaw code) are presented and compared with the observational data (§7), and our results are summarized (§8). In Appendix A the newly discovered double-mode star KIC 7021124 is discussed, and in Appendix B the cyclic behaviour of nine non-Blazhko stars in the $(V-I, V)$-diagram (based on ASAS-North data) is discussed.

2 DATA PROCESSING

2.1 Selection of non-Blazhko stars

Approximately 40 RR Lyr stars are presently known in the Kepler field, up from the 29 stars discussed by Kolenberg et al. (2010) and B10. Four are multiperiodic c-type RR Lyr stars and are the subject of another paper (Moskalik et al. 2011, in preparation), and ~20 are Blazhko stars, many of which have quite complex modulated light curves (see B10). We are interested here in the unmodulated non-Blazhko ab-type RR Lyr stars with the most stationary light curves. When the Fourier parameters and physical characteristics of these stars are known this information will provide a useful reference for future comparisons with the more complex Blazhko stars (see Jurcsik et al. 2009).

Table 1 of B10 gives coordinates for 16 of the 19 stars considered here, and pulsation periods to five decimal places. Three stars are not in the B10: FN Lyr (KIC 6936115) and AW Dra (KIC 11802860), both of which were observed at short-cadence (SC) in Q0 with long-cadence Q1 data and short-cadence Q5 data released later; and KIC 7021124, which was discovered subsequently. The RA and DEC coordinates (J2000) of these three stars are as follows: FN Lyr (19:10:22.25, +42:27:31.6), AW Dra (19:00:48.00, +50:05:31.3) and KIC 7021124 (19:10:26.69, +42:33:37.0).

Three of the stars included in the present study are special cases: (1) V349 Lyr was identified by B10 as a Blazhko star with a small amplitude modulation and Blazhko-period longer than 127 days. It is one of the faintest RR Lyr stars

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1 Public data are available from the internet website [http://archive.stsci.edu/kepler/]. It is the policy of the Kepler Mission to make public all the data mentioned in published research papers. Thus the Q0-Q5 long and short cadence data analysed here will become publicly available upon publication of the present paper.
in the *Kepler* field, at \(<Kp> = 17.433\) mag. The detrended light curve in fig. 1 of B10 shows the amplitude decreasing slowly throughout the Q1-Q2 epochs. Classification of this star depends on the detrending procedure that is used to transform the raw fluxes to magnitudes (see next section and Fig. 1). It has been included here as a borderline non-Blazhko star. (2) V350 Lyr was discovered by B10 to be pulsating simultaneously in the fundamental and second overtone modes, and to be similar to V350 Lyr. The procedures used here for transforming the raw fluxes to *Kp* magnitudes are described next.

### Table 1. Basic data for the 19 *Kepler* non-Blazhko ab-type RR Lyrae stars

| KIC     | Star name | \(<Kp>\) (mag) | Period (d) | Freq. \([d^{-1}]\) | \(t_0\) (BJD) | RT \((Kp)\) | *Kp*-range | \(A_{tot}(Kp)\) (mag) |
|---------|-----------|---------------|------------|-----------------|--------------|-------------|------------|---------------------|
| (1)     | (2)       | (3)           | (4)        | (5)             | (6)          | (7)         | (8)        | (9)                 |
| 3733346 | NR Lyr    | 12.684        | 0.6820264(2) | 1.4662184(9) | 54964.738    | 0.144       | 12.215-12.982 | 0.767               |
| 386769  | V715 Cyg  | 16.265        | 0.4707049(4) | 2.1244731(2) | 54964.6037   | 0.136       | 15.612-16.600 | 0.988               |
| 5299596 | V782 Cyg  | 15.392        | 0.5236373(1) | 1.9097173(4) | 54964.5059   | 0.200       | 15.108-15.631 | 0.523               |
| 6070714 | V784 Cyg  | 15.370        | 0.5340941(1) | 1.8723292(4) | 54964.8067   | 0.195       | 15.036-15.670 | 0.634               |
| 6100702 | KIC 6100702| 13.458       | 0.4881457(2) | 2.0456687(8) | 54953.8399   | 0.200       | 13.140-13.715 | 0.575               |
| 6763132 | NQ Lyr    | 13.075        | 0.5877887(1) | 1.7012916(3) | 54954.0702   | 0.144       | 12.576-13.387 | 0.811               |
| 6930115 | FN Lyr    | 12.876        | 0.5273947(4) | 1.8960995(1) | 54953.2690   | 0.118       | 12.146-13.227 | 1.081               |
| 7021124 | KIC 7021124| 13.550       | 0.6224925(7) | 1.606445(2)  | 54965.6471   | 0.139       | 13.018-13.849 | 0.831               |
| 7030715 | KIC 7030715| 13.452       | 0.6856137(2) | 1.4628145(4) | 54965.8343   | 0.177       | 13.092-13.739 | 0.647               |
| 7176080 | V349 Lyr  | 17.433        | 0.5070740(2) | 1.9720987(8) | 54964.9555   | 0.130       | 16.780-17.768 | 0.988               |
| 7742534 | V368 Lyr  | 16.002        | 0.4566081(1) | 2.1906520(5) | 54964.7828   | 0.134       | 15.237-16.370 | 1.133               |
| 7988343 | V1510 Cyg | 14.494        | 0.5811436(1) | 1.7207451(3) | 54964.6695   | 0.138       | 13.686-14.848 | 0.980               |
| 8344381 | V346 Lyr  | 16.421        | 0.5768288(1) | 1.7336166(3) | 54964.9211   | 0.141       | 15.809-16.773 | 0.964               |
| 9508655 | V350 Lyr  | 15.696        | 0.5942309(1) | 1.6828305(3) | 54964.7795   | 0.143       | 15.086-16.059 | 0.973               |
| 9591503 | V894 Cyg  | 13.293        | 0.5718366(2) | 1.7501285(6) | 54953.5627   | 0.144       | 12.579-13.684 | 1.105               |
| 9947026 | V2470 Cyg | 13.000        | 0.5485984(1) | 1.8228569(3) | 54953.7808   | 0.193       | 12.971-13.570 | 0.599               |
| 10136240| V1107 Cyg | 15.648       | 0.5657781(3) | 1.7674774(3) | 54964.7532   | 0.151       | 15.151-15.969 | 0.818               |
| 10789273| V838 Cyg  | 13.770        | 0.4802799(2) | 2.0821192(4) | 54964.5731   | 0.134       | 13.040-14.138 | 1.098               |
| 118092860| AW Dra   | 13.053       | 0.6872160(2) | 1.4551466(6) | 54954.2160   | 0.165       | 12.538-13.430 | 0.892               |

### 2.2 From raw fluxes to *Kp* magnitudes

The preprocessing phases of the long-cadence (LC) *Kepler* data have been described by Jenkins *et al.* (2010a,b) and B10. Since then three more ‘seasons’ (or ∼270 days) of LC data (Q3-Q5) have become available for 18 of the 19 stars in our sample. Also available now are Q0 (∼10 days) and Q5 (∼90 days) SC photometry (see Gilliland *et al.* 2010) for FN Lyr and AW Dra. The procedures used here for transforming the raw fluxes to *Kp* magnitudes are described next.

#### 2.2.1 Long-cadence data

The present analysis of the stars in Table 1 is based on the raw fluxes. Sixteen of the stars were observed at LC (30 min) in ‘seasons’ Q1 to Q5, and of these, five were also observed in Q0. KIC 6100702 was not observed in Q5, and KIC 7021124 was observed only in Q1. The Q2-Q4 LC observations of FN Lyr were acquired as part of the *Kepler* Guest Observer Program and Drs. K. Mighell and S. Howell kindly shared their observations with us. The original data consisted of raw flux counts as a function of time, with times given as barycentric Julian date (BJD). For convenience all the times were translated by subtracting 54953 d so that the first Q0 observations occur at time \(t = 0.53\) d. Figure 1 helps to illustrate the procedures used to convert long cadence fluxes to *Kp* magnitudes. The left panels show the observed raw fluxes (log10 scale) of V894 Cyg (top) and V349 Lyr (bottom) plotted against time. Both stars were observed every 30 minutes in ‘quarters’ Q1 to Q5 and V894 Lyr was also observed in Q0. One sees zero point differences (sensitivity variations) resulting from quarterly ‘rolls’ of the *Kepler* telescope causing the light from a given star to fall on different CCD chips and pixels each quarter, and trends (both linear and non-linear, and increasing and decreasing) within a given quarter. The scalloped
Figure 1. Raw (left) and processed (right) photometry for V894 Cyg (top) and V349 Lyr (bottom). V894 Cyg was observed 18911 times at long cadence in quarters Q0-Q5 and is of intermediate brightness with raw flux density $\sim 1.6 \times 10^8$ e$^{-}$/cadence. V349 Lyr is the faintest RR Lyr star in the Kepler field and was observed 18387 times at LC:Q1-Q5 with raw flux density $\sim 2 \times 10^6$ e$^{-}$/cadence.

The pattern seen for V894 Cyg (with amplitude larger at maximum than at minimum light) is due to the 30-min interval between the LC observations going in and out of phase with the light cycle determined by the pulsation period of 0.57 d. The time ranges for each LC ‘season’ were as follows: BJD 54953.53 to 54963.24, corresponding to time 0.53 to 10.24 d (Q0), BJD 54964.51 to 54997.98, or 11.51 to 44.98 d (Q1), BJD 55002.94 to 55091.47, or 49.94 to 138.47 d (Q2), BJD 55185.92 to 55187.98, or 232.38 to 417.66 d (Q5).

The right panels of Fig. 1 show the corresponding detrended and normalized Kp apparent magnitudes. The procedure for transforming the raw fluxes into apparent magnitudes was as follows: (1) the data were separated into blocks, usually by ‘quarter’ but various blocks were tried (see the left panels of Fig. 1); (2) the raw fluxes were converted to apparent magnitudes using $\text{mag} = 33.5 - 2.5 \times \log_{10}(F_{\text{raw}})$, where the magnitude zero point is as yet only approximate but gives mean values roughly in accord with the KIC mean magnitudes; (3) within a block the magnitudes were detrended by fitting and subtracting either a linear or polynomial fit; (4) the detrended data for each block were shifted to the Kp mean magnitude level given in the KIC; (5) within each block the magnitudes were stretched or compressed by multiplying by an appropriate scale factor so that the minimum and maximum magnitudes were in agreement with an adopted reference quarter (usually either Q4 or Q5); (6) ob-
vious outliers ($\sigma > 5$) were removed – before this was done the total number of LC observations per ‘quarter’ was as follows: 476 (Q0), 1639 (Q1), 4354 (Q2), 4370 (Q3), 4397 (Q4) and 4633 (Q5); (7) the resulting light curves were fitted according to their Fourier descriptions and secondary trending (usually at the 1-2 sigma level) was removed.

This method of normalization works well for unmodulated stars but might explain the observed apparent absence of amplitude modulation for V349 Lyr. Owing to its relative faintness (V349 Lyr is the faintest star in our sample), and, if it is a Blazhko star as suggested by B10, its long Blazhko period and low Blazhko amplitude, we may have to wait another year or longer to decide if the variations are intrinsic or due to instrumental effects.

2.2.2 Short-cadence data

In addition to the LC observations, FN Lyr and AW Dra were observed at short cadence (SC, 1 minute) in Q0 and Q5. The time baseline for the Q0 observations was 9.7 d (0.528 d to 10.254 d), while the Q5 observations were made over 94.7 d (322.98 d to 417.67 d). When combined the total number of data points amounted to $150,000$ observations per star ($14280$ from Q0, and $136140$ from Q5). Processing of the data required removal of a small number of outliers ($\sigma > 5$) and three spurious dips occurring in Q0 at times centered on 3.298 d, 6.261 d and 8.342 d. The SC data were detrended in the same way as the LC observations.

3 PERIODS AND LIGHT CURVES

3.1 Previously published periods, risetimes, etc.

All but one of the 19 stars in Table 1 (the exception being KIC 7021124) has been studied previously, either from the ground or using the early Kepler photometry. In Table 2 some of this information is summarized.

The GEOS RR Lyr database (see LeBorgne et al. 2007) at [http://rr-lyr.ast.obs-mip.fr/dbrr](http://rr-lyr.ast.obs-mip.fr/dbrr) gives pulsation periods for eight of the 19 stars – these are summarized in column 2 of Table 2. Much of the historical photometry was derived from photographic plates taken through various filters and on different emulsions, often but not always approximating Johnson $V$ and $B$ magnitudes. The database also includes HJD times of maximum light, risetimes (RTs), and visual and CCD observations acquired over many decades.

Two historical studies of particular interest are the impressive 1953 and 1956 photographic investigations by W.J. Miller, S.J. The former study includes an analysis of V715 Cyg (then known as Vatican Variable 14, or VV 14), while the latter includes analyses of V782 Cyg (VV 22) and V784 Cyg (VV 24). Both studies were based on photographic magnitudes ($m_{pg}$) derived from Harvard and Vatican plates taken between 1927 (JD 2425148) and 1952 (JD 2434323). The number of individual $m_{pg}$ observations were as follows: 840 for V715 Cyg, 735 for V782 Cyg, and 741 for V784 Cyg. Observed times of maximum light are given for V782 Cyg (46 epochs) and V784 Cyg (42 epochs) but not for V715 Cyg. For all three stars the GEOS catalog adopts the pulsation periods and times of maximum light from Miller’s investigations.

For the eight stars with periods given in the GEOS database agreement with the Kepler periods is very good, the average difference amounting to only $0.4 \times 10^{-5}$ d. The biggest period discrepancies are for V715 Cyg, V784 Cyg and AW Dra. The RT differences are discussed in §3.3. Possible slowly-changing periods that may not be detectable in the Q0-Q5 Kepler data were assessed from O–C diagrams and are discussed in §3.4.

Nine of the Kepler non-Blazhko stars also were observed during the course of the ASAS-North survey (see Pigulski et al. 2009, Szczygiel et al. 2009, and the ASAS website at [http://www.astrouw.edu.pl/asas](http://www.astrouw.edu.pl/asas)). For the nine stars $\sim 80$ V and a similar number of $I$ CCD photometric measurements were made over $\sim 500$ days in 2006 and 2007. A reanalysis of the on-line ASAS data was made and risetimes and total amplitudes, Fourier-fitted $(V)$ magnitudes, and $(V \pm I)$ colours are given in the last four columns of Table 2. If the ASAS-South survey is any indication the ASAS-North RR Lyr light curves will be “close to the standard V system” (Kovacs 2005). The ASAS V data are discussed further in §4.5 below, and the cyclic behaviour of the stars in the HR-diagram is discussed in Appendix B.

Most recently, pulsation periods and preliminary Fourier analysis for 16 of the 19 stars were reported by B10. They analyzed the long cadence $Kp$ photometry from Q0 (9.7d interval), Q1 (33.5d) and Q2 (89d) and established the non-Blazhko behaviour of most of the stars. The periods (typical uncertainties of $\pm 3 \times 10^{-5}$ d) and Fourier $A_1$ amplitudes that they derived are summarized in columns 3 and 4 of Table 2.

3.2 Period estimation

Pulsation periods for the non-Blazhko stars were derived mainly from analyses of the $Kp$ data alone. Computations were performed with the package PERIOD04 (Lenz & Breger 2005), which carries out multifrequency analyses with Fourier and least-squares algorithms, and with a version of the CLEAN program (see Nemec, Walker & Jeon 2009). Details of the derived periods are given in Table 3, where the best estimates of the periods and times of maximum light are in columns 3 and 4, and column 5 describes the specific data that were analyzed. The uncertainties in the periods (given in parentheses) are average values obtained using the three methods available in PERIOD04, i.e., with the Fisher method of randomization (Lenz & Breger 2005), with Fourier and least-squares algorithms, and with a Monte Carlo routine similar to the Fisher method of randomization described by Nemec & Nemec (1985). Typically the periods derived using the Q0-Q5 LC data alone are accurate to $1 – 2 \times 10^{-7}$ d. The uncertainty is expected to decrease further as more Kepler observations become available. The most uncertain is that for KIC 7021124 which so far has been observed only in Q1. Also given in Section (a) of Table 3 are the mean $Kp$ magnitudes obtained from the KIC. In column 5 the abbreviations ‘LC’ and ‘SC’ stand for ‘long cadence’ (30-min) and ‘short cadence’ (1-min) Kepler data. Columns 6-11 contain Fourier-based results and are described below.

In general, the new periods compare favourably with the GEOS and B10 periods (see Table 2), with the newer values being more precise and more accurate. The largest differences are for V715 Cyg and AW Dra, both of which
have significantly longer periods than given in the GEOS catalog. In §3.4 the periods derived directly from the Kp data are compared with the values from O−C diagrams.

### 3.3 Light curves, amplitudes and risetimes

Phased light curves for the Kepler non-Blazhko ab-type RR Lyr stars are plotted in the left panel of Figure 2, assuming the pulsation periods and times of maximum light given in Table 3. The light curves have been ordered from smallest to largest amplitude (measured from minimum to maximum light), with each star offset from the next by 0.5 mag. This arrangement shows that there is a clear trend in light curve shape and risetime (RT), with the lowest amplitude stars having the longest RTs and the most symmetric light curves, and the largest amplitude stars having the shortest RTs and most asymmetric light curves.

The largest amplitude star is V368 Lyr, with \( A_{\text{tot}} = 1.133 \) mag. This star also has the most asymmetric light curve and the shortest period, \( P = 0.456 \) d. At the top of the stack of light curves the star with the smallest amplitude is V782 Cyg with \( A_{\text{tot}} = 0.523 \) mag; however, its period, \( P = 0.524 \) d, is not the longest. That distinction belongs to three stars with approximately equal periods: AW Dra (0.687 d), NR Lyr (0.682 d) and KIC 7030715 (0.684 d). These three stars all have intermediate amplitudes, and are shown below to have considerably lower metallicities than V782 Cyg (see §6.1 and Figs. 7-9).

The right panel of Figure 2 also shows a comparison of the total amplitudes, \( A_{\text{tot}} \) (Table 1, column 9), and the Fourier \( A_1 \) coefficients (see next §), both derived from the Kp-photometry. Although \( A_1 \) is approximately proportional to \( A_{\text{tot}} \) it is clear that the relationship is non-linear. When a cubic equation was fitted to the graph the resulting standard error of the fit amounted to only 3 mmag. It is doubtful that this (slight) curvature would have been detected without the high precision of the Kepler data. In general the relationship is roughly approximated by non-linear hydrodynamic models (see §7, in particular the top-left panel of Fig. 14).

The RTs, magnitude ranges, and total amplitudes given in Table 1 (columns 7-9) were calculated numerically from the fitted Kp light curves. The magnitudes at maximum and minimum light are the values at which the slopes of the light curves were found to be zero, from which magnitude ranges and precise total amplitudes were calculated. The phases at the light minima and maxima were used to calculate the RTs.

It is informative to compare the Kp-based RTs with those given in the GEOS database and with RTs based on the ASAS V-photometry. Since RTs are not given at the ASAS on-line website the values given in column 5 of Table 2 were derived in the same way that the Kp values were computed. For the nine stars with Kp and ASAS RTs the agreement is excellent, the mean difference being 0.002 (ASAS minus Kp, standard deviation 0.014). For Miller’s three Vatican variables (values are given in parentheses in column 5 of Table 2) there is reasonable agreement, with the mean difference being \(-0.024\) (Miller minus Kp, standard deviation 0.021). The largest differences occur for NR Lyr and V838 Cyg, where the GEOS database gives RT=0.27 and 0.40, respectively, compared with the averages of the Kp and ASAS estimates of 0.147 and 0.132. An independent estimate of 0.141 for NR Lyr comes from our analysis of the unpublished V CCD photometry of Benkő and Nuspl, which agrees well with the Kp value.

### 3.4 O−C diagrams and Period Change Rates

O−C diagrams can be used to improve the precision of estimated pulsation periods, to detect slowly changing periods, and to derive accurate period change rates. The longer the
Figure 2. (Left) Phased light curves for the 19 non-Blazhko ab-type RR Lyr stars observed with the Kepler telescope. The quantities in parentheses following the star names are the total $K_p$ amplitudes (Table 1, column 8), ranging from $A_{\text{tot}} = 0.52$ to $1.13$ mag. Almost all of the light curves contain over 18000 points with a spread of less than one milli-magnitude. (Right) Graph comparing $A_{\text{tot}}$ values and the Fourier $A_1$ coefficients (in column 8 of Table 3 below), all derived from the $K_p$-photometry. The fit to the points is described by the cubic equation $A_1 = 0.443A_{\text{tot}}^3 - 0.950A_{\text{tot}}^2 + 0.973A_{\text{tot}} - 0.120$, with fit standard error $\sigma = 0.003$ mag.

Figure 3 shows O–C diagrams for the eight Kepler non-Blazhko stars that have ephemeris information in the GEOS database. The ordinate represents the observed minus calculated phase at which maximum light occurs, assuming the GEOS period and time of maximum light given in parentheses at the top of each panel; the abscissa is the Heliocentric Julian Date. At each epoch the O–C value was calculated by taking the observed time of maximum light minus the assumed GEOS time of maximum light ($t_0$), and dividing the resultant by the assumed (constant) GEOS period ($P$). This gave the number of pulsation cycles since $t_0$, and the remainder after subtracting an integer number of cycles ($E$) is the observed fractional phase shift from the expected value. For five of the stars linear fits to the observations are given, a positive (negative) slope indicating that the assumed period is too short (long). Quadratic fits are shown for V782 Cyg, FN Lyr and AW Dra (shown twice, the second graph representing the last ∼13 years), where the upwards curvature indicates an increasing period, the period change rate, $dP/dt$, being given by $2Pc^2$ where $P$ is the assumed GEOS period, and $c$ is the curvature coefficient in the quadratic equation.

For the three Vatican Variables (V715 Cyg, V782 Cyg, V784 Cyg) there is an ∼60 year gap between the 2009-10 Kepler data and the earlier observations. Over such a long time interval one cannot hope to keep track of cycle counts and there is little hope of using the O–C diagrams to either improve the period or calculate $dP/dt$. To illustrate the cycle-count problem the panels for V715 Cyg and V784 Cyg show two (of many) possible linear solutions, where, in all the cases there is a large O–C shift between the Kepler and earlier epochs. However, comparison of the assumed periods with the periods derived from the Kepler data do give some indication of either a period change or incorrect period. For V715 Cyg, the Kepler period, 0.4707045(1) d, is significantly longer than the GEOS period, 0.47067298(43) d, suggesting a period increase. And for V784 Cyg the Kepler period, 0.5340941(1) d, is significantly shorter than that derived by Miller (1956), 0.5341026(11) d.

Miller’s (1956) data for V782 Cyg provides 46 times of maximum light at epochs between 1928 and 1954, and with these Miller derived $P = 0.5236338$ d (adopted in the GEOS database). Our analysis of the ∼18419 Q1-Q5 $K_p$ 2 Since the offset between Barycentric Julian Dates (BJDs) used by Kepler are very small compared to the HJDs usually used by ground-based observers, the two types of dates have been used interchangeably in this paper.
photometry gave $P = 0.5236377(1)$ d, which is $3.9 \times 10^{-6}$ d longer than the Miller period (see column 3 of Table 2). Using the time of maximum light given in column 5 of Table 1 the O–C phase shift of $\sim -0.20$ (nearest) is undoubtedly significant. The quadratic solution (shown in Fig. 3) gives $dP/dt = 0.06$ d/Myr, which predicts, for 2010, the period $0.5236379$ d, a value that is only $0.2 \times 10^{-6}$ d longer than the measured Kepler period. While this agreement strongly supports the increasing period it is also possible that the period changed abruptly in the 60-year gap.

For NR Lyr (Fig. 3, top left panel) the 31 historical points and the new Kepler data are consistent with a shorter period than assumed in the GEOS database. The negative slope suggests a period shorter by $0.46 \times 10^{-6}$ d than the assumed period $P_{\text{(GEOS)}}$, i.e., $P_{\text{(true)}} = 0.6820288$ d; this period is significantly longer than the estimate based on the 18358 Kepler Q1–Q5 data points, $0.6820264(2)$ d.

The 13 historical points and the Kepler data for NQ Lyr produce consistent period estimates, with a difference of only $0.1 \times 10^{-6}$ d between the two periods. Thus the assumed (GEOS) period of $0.5877887(1)$ d derived using the 18759 Kepler Q0–Q5 data points, $0.5877887(1)$ d.
tion from 1937 was considered an outlier and excluded from our analysis). The diagram includes a point from the 1966 photoelectric V-photometry of Bookmeyer et al. (1977) at $t = 39416.6457$, and a point from unpublished 2006-08 CCD V-photometry by Layden at $t = 54288.7454$ (see §4.4 for further discussion of these data). These two points, which differ in time by $\sim 40$ years, have been plotted as open circles and differ in phase by $\sim 0.055$. The O–C value derived for FN Lyr from the Kepler data (epoch 2009) is almost identical to the Layden O–C value (see Table 2). Considering that the total baseline is longer than 100 years the O–C phase shifts are very near zero. Nevertheless there appears to be a slight upward trend, and possibly a small amount of curvature. A parabolic solution gives $dP/\,dt = 0.004 \pm 0.004$ d/Myr, with the GEOS period occurring at $t = 18100$, a residual standard error of the fit of 0.011, and a predicted period 0.5237975 d for the current epoch (2010-11). Since the curvature term does not differ significantly from zero ($p$-value $= 0.27$) the question of a changing period remains unanswered. A linear solution is also consistent with the data, in which case the predicted period amounting to 0.32 d/Myr, and predicts a period variation of the diagram shows that the last three points, one of which is that derived from the Kepler data, deviate substantially from the fitted curve. If the analysis is restricted to epochs since HJD 49000 (the first point is from the Castellani et al. 1998 study of AW Dra) and if the suspected outliers identified above are excluded, then the O–C diagram is as shown in the bottom right panel of Fig. 3. In this case, the period increases smoothly at the rate $dP/\,dt = 3.76$ d/Myr, with the assumed period of 0.6871941 d having occurred at HJD 49267. For the Kepler Q1 and Q5 LC data (6117 points) the predicted period is 0.6872156(2) d. And the period from the combined Q6 and Q5 SC data (149222 points) is similar, 0.6872162(1) d, with a fit standard error 7.3 mmag. Both periods agree, are significantly longer than the GEOS period, and are consistent with AW Dra having an increasing period. Comparing this period with the predicted periods the new data favour the smaller $dP/\,dt$ value.

To summarize this section, periods based on O–C diagrams were calculated for NR Lyr and NQ Lyr, both of which seem to have constant periods, and $dP/\,dt$ values have been calculated for FN Lyr and AW Dra. AW Dra is the only star for which we can at present be confident that the period is changing (assuming that the outliers really should be omitted from consideration) – depending on the assumed data set the rate is either $dP/\,dt = 3.786$ d/Myr (for the more recent data), or 0.32 d/Myr (all data) with the possibility of variations on top of this. For FN Lyr the period either is very nearly constant or is increasing at a very slow rate. For the other four stars with GEOS times of maximum light the large gaps in the observation record prevent improvement over the GEOS or Kepler periods. The ongoing high-precision Kepler observations should remedy this lack of knowledge (for the modern era), not only for the eight Kepler non-Blazhko stars in the GEOS database but also for all the Kepler non-Blazhko stars.

4 FOURIER ANALYSIS

The use of Fourier methods to characterize the shapes of RR Lyr light curves began with the work of Simon and his collaborators (Simon & Lee 1981, Simon & Teays 1982, Simon & Clement 1993) and has continued through the more recent papers by Kovacs, Jurcsik, Morgan and others. The goal of these studies has been to establish empirical correlations employing a minimum number of observed parameters (periods, amplitude ratios, phase parameters) from which physical characteristics can be derived (see §6). Calibration of the equations follows from independent observations (high dispersion spectroscopy, parallax measurements, etc.) and from theory (e.g., hydrodynamical pulsation models, stellar evolution models). After Simon (1988) discovered that the Fourier phase parameter $\phi_{21}$ decreases with decreasing metallicity (for field RRab stars with $P < 0.575$d) Kovacs & Zsoldos (1995) proposed that correlations involving period and the Fourier phase parameter $\phi_{31}$ can be used to derive [Fe/H] values. The first application of this method to field and globular cluster RR Lyr stars was the study by Jurcsik & Kovacs (1995,1996), and subsequent calibrations have been made by Jurcsik (1998, hereafter J98) and Kovacs & Walker (1999, 2001). From the beginning Jurcsik & Kovacs (1995) warned that the Fourier $\phi_{31}$ method “is not applicable for the estimation of [Fe/H] in peculiar stars (e.g., in Blazhko variables, highly evolved stars ...”), and subsequently JK96 and Kovacs & Kanbur (1998) introduced a ‘compatibility test’ for identifying ‘peculiar’ stars. Indeed, when such stars were omitted from the ASAS-South database, agreement between Fourier-based metallicities and low-dispersion spectroscopic metallicities reached the 0.16 dex level (see Kovacs 2005).

4.1 Coefficients and fitted light curves

Fourier decomposition of the Kepler light curves was performed for each of the 19 non-Blazhko stars by fitting the following Fourier sine series to the observed photometry:

$$m(t) = A_0 + \sum_{i=1}^{F} A_i \sin[\omega_0(t - t_0) + \phi_i],$$

where $m(t)$ is the apparent magnitude (either $K_p$ for the Kepler data, or $m_V(t)$ for the ground based $V$-photometry), $F$ is the number of fitted terms, $\omega_0$ is the angular pulsation frequency of the star ($= 2\pi f_0$, where $f_0 = P_0^{-1}$), $t$ is the observed time of the observation (BJD-54953 for the Kepler data, HJD for the ground-based $V$ photometry), $t_0$ is the time of maximum light (used to phase the light curves so...
that maximum light occurs at zero phase), and the \( A_i \), and \( \phi_i \) are the amplitude and phase coefficients for the individual Fourier terms. The Fourier calculations were made with two FORTRAN programs, one kindly provided by Dr. Géza Kovács, and the other by Dr. Paweł Moskalik. The assumed pulsation periods were those derived either from the *Kepler* data or from the period change rate analysis (§3.4), and the \( t_0 \) values were calculated numerically. The derived periods and \( t_0 \) values are summarized in Table 1 (columns 4,5) and the details given in Table 3.

### 4.2 *Kepler* Kp-photometry

Initially the *Kepler* magnitudes were fitted with a Fourier series having 7-15 terms, the larger number being needed when the skewness is greatest, i.e., the risetime is shortest. Inspection of the residuals from the fitted light curves revealed that because most of the stars show a sharp rise to maximum light, with additional detailed bumps and features, many more terms were needed. **Figure 4** illustrates 38-term Fourier fits for two non-Blazhko stars, AW Dra on the left and FN Lyr on the right. These two stars have both *Kepler* photometry (top diagrams) and high-precision ground-based V-band photometry (bottom diagrams). For AW Dra the *Kepler* data are Q5 SC observations, i.e., brightness measurements every minute over 94.7 days, resulting in 135380 data points; and for FN Lyr the *Kepler* data are Q1-Q5 LC observations, i.e., a measurement every 30 minutes over 417 days, resulting in 18338 points. In both cases the standard deviation of the fit to the *Kepler* data is less than one millimag (see column 6 of Table 3). However, the residuals still show systematic variations with the largest residuals occurring on the rise to maximum light where the slope of the light curve is steepest. To achieve residuals showing normally distributed white noise perhaps 50-100 terms would be needed.

For each of the program stars epoch-independent phase differences \( \phi'_i = \phi_i - i \phi_i \), and amplitude ratios \( R_{31} = A_3/A_1 \); were computed from the Fourier coefficients derived from the *Kepler*-photometry. The superscripts “s” and “c” signify phases and phase-parameters computed with sine and cosine series, respectively (e.g., Fig.7 below). The sine and cosine \( \phi_{31} \) phase parameters differ by \( \pi \) radians, that is, \( \phi_{31}^c = \phi_{31}^s - 3.14159 \), and the \( \phi_{21} \) parameters differ by \( \pi/2 \) radians, i.e., \( \phi_{21}^c = \phi_{21}^s + 1.5708 \).

Table 3 also contains the Fourier parameters needed for computing physical characteristics. In section (a) results derived from various combinations of short- and long-cadence *Kepler* data (see column 5) are given. The \( A_1 \) coefficients (column 7) are practically identical to the \( A_1 \) coefficients calculated by B10 using the Q0-Q2 photometry (given in column 6 of Table 2). Columns 8 and 9 contain the amplitude ratios, \( R_{21} \) and \( R_{31} \); and columns 10 and 11 contain the Fourier phase parameters, \( \phi_{21}^c \) and \( \phi_{31}^c \). The accuracy of the *Kepler*-based Fourier parameters is at least one part in 1000, and higher in many cases. For example, the parameters derived for FN Lyr from a 38-term fit of the 18338 Q1-Q5 LC data points have formal values (including uncertainties) as follows: \( R_{21} = 0.44208(2) \), \( R_{31} = 0.34610(2) \), \( \phi_{21}^c = 2.32417(7) \), and \( \phi_{31}^c = 4.81916(9) \). Correlations and graphical representations of these parameters are discussed below.

### 4.3 Cycle-to-Cycle variations and stationarity

To examine cycle-to-cycle variations in the *Kepler* light curves the Fourier parameters were re-calculated after binning the data so that each bin included data for one pulsation period (i.e., each bin represented a single pulsation cycle). For the long cadence (30 min) data there were \( \sim 24 \) observations per bin for an RRab star with a period of 12 hours, and fewer (more) observations per cycle for stars with shorter (longer) periods. None of the 19 stars exhibits the recently discovered “period doubling” effect, thus confirming the result found earlier by Szabó *et al.* (2010).

For the Fourier calculations ‘direct Fourier fitting’ (dff) rather than ‘template Fourier fitting’ (tff) methods were used (see Kovács & Kupi 2007), and for each star the resulting time series were plotted for four Fourier parameters: \( A_1 \), the first term in the Fourier series (see right panel of Fig. 2); \( \phi_1 \), the phase of the first term; \( R_{21} \), the amplitude ratio \( A_2/A_1 \); and \( \phi_{31}^c \), the Fourier parameter found by Simon, Kovács and others to be one of the most significant variables for deriving physical characteristics. Typical time series are illustrated in **Figure 5**, on the left for the unmodulated RRab star NQ Lyr having intermediate amplitude \( (A_1 = 0.279 \text{mag}) \) and intermediate brightness \( (\langle Kp \rangle = 13.075 \text{mag}) \), and on the right for V783 Cyg, a low-amplitude Blazhko star with the shortest Blazhko period \( (P_{B} = 27.7 \pm 0.04 \text{d} – \text{see B10}) \) among the stars in our sample.

For NQ Lyr linear trend lines were fit to each time series and in almost every case the slope is zero to within the systematic and random uncertainties. The mean values for the four parameters are as follows: \( \phi_{31}^c = 5.0958 \pm 0.0023 \), with residual standard error, \( \sigma = 0.002 \); \( R_{21} = 0.4710 \pm 0.0006 \), with \( \sigma = 0.0006 \); \( \phi_1 = 3.961 \pm 0.001 \), with \( \sigma = 0.001 \); and \( A_1 = 0.28016 \pm 0.00014 \text{mag} \), with \( \sigma = 0.0001 \). Since the other non-Blazhko stars show random variations of the Fourier parameters similar to those shown here for NQ Lyr these means and errors provide a measure of the typical uncertainties.

Inspection of the NQ Lyr time series in Fig. 5 shows that the stability over the \( \sim 420 \text{d interval (Q0-Q5)} \) is exceptional, not only for each Fourier parameter but for the ensemble of parameters. To better illustrate the remarkable stationarity of the light curves for all 19 sample stars a set of ‘animated gif’ light curves has been prepared and these are available in the electronic version of the Journal.

For the Blazhko star V783 Cyg the time plots of the four Fourier parameters are in striking contrast to the stochastic noise exhibited by NQ Lyr and by the candidate non-Blazhko star V349 Lyr. For V783 Cyg pronounced periodic variations are seen over the 27.7 d Blazhko cycles. There does appear to be a slight discontinuity at around 140 d, but this is certainly a data processing defect. Over the 11 Blazhko cycles seen there is very little difference from cycle-to-cycle; this is not always the case for Blazhko stars where large cycle-to-cycle differences often are seen.

### 4.4 High-precision V-photometry

By comparing the light curves and Fourier parameters for the *Kepler* and *V* photometric systems it is possible to test (in a limited way) the hypothesis that the two broad-band filters
Figure 4. Light curves and residual plots for AW Dra (left) and FN Lyr (right). The top diagrams show $K_p$ photometry and the bottom diagrams $V$-band photometry, and within each diagram the light curves are on top and the residual plots on the bottom. The $K_p$ photometry is SC(Q5) data for AW Dra and LC(Q1-Q5) data for FN Lyr. The $x$-axis labels give the assumed pulsation periods ($P_0$) and times of maximum light ($t_0$). The diagram labels indicate the data plotted and the number of terms in the fitted Fourier series, and the number of data points ($N$) and standard deviations of the fits ($\sigma$) are stated in each diagram.

4.4.1 AW Dra

The light curve and residuals derived from the $V$-photometry of Castellani et al. (1998) are plotted in the lower left diagram of Figure 4. The 112 points are from CCD observations made in 1995 with the 72-cm Teramo/Scuola Normale Telescope, modelled with a 7-term Fourier series fit. The shape of the $V$ light curve is very similar to the light curve based on the $Kepler$ data, and the Fourier parameters (Table 3) support this observation. Notice also that, although the $V$-data have a residual standard error of only 0.0116 mag, the residuals are $\sim 16 \times$ larger than the $K_p$-
Table 3. Pulsation and Fourier parameters for the *Kepler* non-Blazhko ab-type RR Lyr stars

| Star        | \((Kp)\)   | Period | \(t_0\)  | \(σ\)  | \(A_1\) | \(R_{21}\) | \(R_{31}\) | \(φ_{21}^1\) | \(φ_{31}^1\) |
|-------------|------------|--------|----------|--------|--------|---------|---------|-----------|-----------|
| (1)         | (mag) | [day] | [BJD] | [mmag] | [mag] | [rad]   | [rad]   |           |           |
| NR Lyr     | 12.638   | 0.682026(2) | 54964.7381 | LC:Q1-Q5 (1833) | 0.69 | 0.266 | 0.456 | 0.352 | 2.416 | 5.115 |
| V715 Cyg   | 16.265   | 0.4707049(4) | 54964.6037 | LC:Q1-Q5 (1837) | 1.74 | 0.338 | 0.479 | 0.358 | 2.314 | 4.901 |
| V782 Cyg   | 15.392   | 0.5236377(1) | 54964.5059 | LC:Q1-Q5 (1838) | 0.79 | 0.190 | 0.488 | 0.279 | 2.777 | 5.808 |
| V874 Cyg   | 15.370   | 0.5340941(1) | 54964.8067 | LC:Q1-Q5 (1834) | 0.96 | 0.234 | 0.487 | 0.253 | 2.904 | 6.084 |
| KIC 6100702 | 13.458   | 0.4881457(2) | 54953.8399 | LC:Q0-Q4 (14404) | 0.66 | 0.209 | 0.493 | 0.279 | 2.743 | 5.747 |
| NQ Lyr     | 13.075   | 0.5877887(1) | 54954.0702 | LC:Q0-Q5 (18759) | 0.65 | 0.280 | 0.471 | 0.356 | 2.389 | 5.096 |
| FN Lyr     | 12.876   | 0.52739845(1) | 54953.2690 | SC:Q0-Q5 (149925) | 0.60 | 0.380 | 0.454 | 0.354 | 2.321 | 4.817 |
| KIC 7021124 | 13.550   | 0.5224926(7) | 54956.6741 | LC:Q1 (1396) | 1.10 | 0.283 | 0.512 | 0.351 | 2.372 | 5.060 |
| KIC 7030715 | 13.452   | 0.6863137(2) | 54955.8434 | LC:Q0-Q5 (18802) | 0.71 | 0.231 | 0.494 | 0.303 | 2.683 | 5.606 |
| V349 Lyr   | 17.433   | 0.5070740(2) | 54964.9555 | LC:Q1-Q5 (18314) | 3.24 | 0.346 | 0.450 | 0.352 | 2.328 | 4.845 |
| V368 Lyr   | 16.002   | 0.4564851(1) | 54964.7828 | LC:Q1-Q5 (18273) | 1.57 | 0.405 | 0.464 | 0.341 | 2.272 | 4.784 |
| V1510 Cyg  | 14.494   | 0.5811436(1) | 54964.6695 | LC:Q1-Q5 (18394) | 0.82 | 0.345 | 0.473 | 0.355 | 2.389 | 5.068 |
| V346 Lyr   | 16.421   | 0.5768281(1) | 54964.9211 | LC:Q1-Q5 (18362) | 2.83 | 0.330 | 0.473 | 0.352 | 2.372 | 5.060 |
| V350 Lyr   | 15.696   | 0.5942369(1) | 54964.7705 | LC:Q1-Q5 (18326) | 1.67 | 0.340 | 0.485 | 0.342 | 2.389 | 5.124 |
| V894 Cyg   | 13.293   | 0.5713866(2) | 54953.5627 | LC:Q1-Q5 (18362) | 0.91 | 0.377 | 0.490 | 0.338 | 2.364 | 5.067 |
| V2470 Cyg  | 13.300   | 0.5485894(1) | 54953.7808 | LC:Q0-Q5 (18794) | 0.79 | 0.220 | 0.488 | 0.282 | 2.745 | 5.737 |
| V1107 Cyg  | 15.648   | 0.5657781(1) | 54964.7532 | LC:Q1-Q5 (18373) | 0.99 | 0.280 | 0.495 | 0.350 | 2.421 | 5.196 |
| V838 Cyg   | 13.770   | 0.4802799(1) | 54964.5731 | LC:Q1-Q5 (18241) | 1.22 | 0.393 | 0.465 | 0.349 | 2.300 | 4.853 |
| AW Dra     | 13.057   | 0.68721356(6) | 54953.2160 | SC:Q0 (12420) | 0.55 | 0.305 | 0.527 | 0.348 | 2.731 | 5.563 |
|            | 13.053   | 0.687217(1)  | 54953.2160 | SC:Q1 (1614) | 0.69 | 0.306 | 0.524 | 0.342 | 2.730 | 5.561 |
|            | 13.053   | 0.6872158(2) | 54953.2160 | LC:Q5 (4474) | 0.79 | 0.306 | 0.524 | 0.343 | 2.728 | 5.557 |
|            | 13.053   | 0.68721632(3) | 54953.2160 | SC:Q5 (135380) | 0.71 | 0.308 | 0.527 | 0.347 | 2.729 | 5.558 |

(a) Results from analysis of the Q0-Q5 Kp-photometry

(b) Results from (re-)analysis of high-precision ground-based V photometry

(c) Results from re-analysis of the ASAS V photometry

For FN Lyr, the V photometry shown in the lower right panels of Figure 4 are from two sources: Bookmeyer et al. (1977) and Layden (2010, unpublished). The Bookmeyer et al. data comprise 71 UBV photoelectric observations made in 1966 (21) and 1969 (50), and the Layden V-photometry constitute 124 VR CCD observations made in 2006-08. When the two data sets, which are separated by ~40 years, were combined and plotted as single light curve (i.e., the same period and time of maximum light) a phase shift amounting to 5.5% of the period (0.029 d) was seen, with the Layden data having the larger phases. The shift corresponds to the offset of the two O–C values seen in Fig. 3 (middle right panel, where the O–C values for the two data sets are plotted as open circles and are labeled ‘1966-69’ and ‘2006-08’). Our period change rate analysis suggested that the shift is probably due either to the assumed period (0.5273916 d) being too small or to a slowly increasing period (dP/dt = 0.004 ± 0.004 d/Myr).

Figure 4 (lower right) shows the composite V light curve for FN Lyr, where the Layden data (solid black squares) are plotted alongside the Bookmeyer data shifted in phase by +0.055’ (red open circles). There is a reasonably good match between the Layden and Bookmeyer light curves, the chief differences being: (1) only two (very close) Lay-

residuals (Figure 4, upper left panel). On the other hand, the risetimes are quite similar: 0.165 for the Bookmeyer data and 0.174 for the Layden data. The shift in risetimes is probably due to the Bookmeyer data having the larger phases. The shift corresponds to 5.5% of the period (0.029 d).
data points were acquired near maximum light, and they are fainter than the Bookmeyer observations near maximum light; (2) the slope of the rise to maximum light is shallower for the Layden data than for the Bookmeyer data; and (3) the Layden data are uniformly distributed in phase while the Bookmeyer data show two large gaps on the descent to minimum light.

Fourier coefficients and parameters were calculated using the composite ‘Layden plus Bookmeyer’ data, and using the 124 Layden V data points only (the former were found to be less reliable than the latter). Owing to the large gaps it was not possible to obtain a reliable Fourier fit using only the Bookmeyer data. Table 3 gives the Fourier decomposition results based on the Layden-only V-photometry. The V-residuals shown in the bottom-right panel of Figure 4 were calculated with respect to the Layden-only fit. The Layden data have a residual standard error of 0.03 mag, while the fit for the Bookmeyer data is poorer (≈0.06 mag). In both cases (as for the *Kepler* data) the largest residuals occur on the rise to maximum light. The observed smaller total amplitude and longer risetime for the Layden V photometry compared with the Bookmeyer photometry might be expected if FN Lyr is a low-amplitude Blazhko variable. The available data are insufficient to check this possibility but as additional *Kepler* observations are acquired the answer may be evident.

4.4.3 NR Lyr

In 2008 B,V,R,I CCD observations of NR Lyr were made by J. Benkő and J. Nuspl using the Konkoly telescope. At present the 183 data points per filter are on the instrumental system of the telescope; however, the photometric colour constant is very small and the V data differ from the Johnson B,V system by only a zero point shift. Because the data completely cover the phase range and are of high precision the V data were Fourier analyzed (using the period and t0-value favoured by the *Kepler* data) and the results summarized in Table 3. When the Fourier parameters are compared with those for FN Lyr and AW Dra (see Figures 6, 9 and 10) all three sets of observed offsets are in excellent agreement (see §5.2 below).

4.5 ASAS V-photometry

For the nine non-Blazhko RR Lyr stars in the ASAS-North survey (see §3.1) the on-line V-photometry was re-analyzed and the results reported in section (c) of Table 3. The derived ⟨V⟩ magnitudes (column 2) have uncertainties of ±0.01 and range from 12.44 to 14.24. A comparison of the ⟨V⟩ and ⟨Kp⟩ mags shows the following trend over the observed range: ⟨V⟩ = (1.45 ± 0.24)(Kp) − (5.97 ± 3.20). Until further V photometry fainter than 14th mag is obtained it is unclear whether this trend of V mags fainter than the Kp mags will continue. Pulsation periods (with uncertainties in parentheses) derived from our re-analysis of the V data are given in column (3); in general these agree with the periods given at the ASAS website and with those derived from the *Kepler* data (see Tables 2-3). Columns 4-6 contain, respectively, the assumed time of maximum light, the ASAS name of the star, and the spread of the Fourier fit about the mean light curve (found to range from 49 to 108 mmag). Fourier parameters derived using the on-line V data are given on the right side of Table 3. These were computed using the accurate *Kepler* periods and times of maximum light. The uncertainties typically are ±0.04 for R21, ±0.01 for R31, ±0.08 for φ21, and ±0.1 for φ31, and in several cases are larger.
Figure 6. Four panels comparing various Fourier parameters derived from the $Kp$-photometry for the 19 Kepler non-Blazhko stars. The open diamonds for AW Dra, FN Lyr and NR Lyr were derived from the $V$-photometry of Castellani et al. (1998), Layden (2010, unpublished) and Benkő & Nuspl (2010, unpublished), respectively.

5 FOURIER PARAMETER CORRELATIONS

5.1 $Kp$ Correlations

Figure 6 shows four graphs relating the Fourier parameters $A_1$, $R_{31}$, $R_{21}$, $\phi_{21}^s$ and $\phi_{31}^s$, derived from the sine-series decomposition of the $Kp$-photometry of the 19 Kepler non-Blazhko stars. Each star is represented by a solid black square, and is labelled for easy identification. Also plotted in Fig. 6 are the $V$-band Fourier parameters for AW Dra, FN Lyr and NR Lyr ($\S$4.3) which are shown as open diamonds connected to the corresponding $Kp$ points by lines.

The offsets between the $Kp$ and $V$ points appear to be systematic (i.e., generally go in one direction) and are not very large, even in the case of $R_{31}$ which is the least certain of the parameters. Perhaps this relationship between $V$ and $Kp$ is not surprising because both passbands are quite wide.

The upper-left panel of Fig. 6 shows that there is very little correlation between $R_{21}$ and period (except possibly a slight increase in $R_{21}$ with increasing period). The $R_{21}$ values from the $V$ photometry are all slightly smaller than from the $Kp$ photometry, and the $R_{21}$ values for FN Lyr and NR Lyr are significantly smaller than that for AW Dra.

The upper-right panel shows 14 stars (including AW Dra, FN Lyr and NR Lyr) with $R_{31}$ between 0.24 and 0.54, and five stars with $R_{31}$ values smaller than 0.31. In particular, AW Dra, FN Lyr and NR Lyr are seen to have similar $R_{31}$ values even though the $R_{21}$ values for FN Lyr and NR Lyr are significantly smaller than that for AW Dra. Although the overall bimodal distribution is striking, we shall see below (upper right panels of Figs. 7 and 8) that RR Lyr stars with relatively low $R_{31}$ values are quite common and can be simulated with hydrodynamic models ($\S$7).

In the lower-left panel the stars separate into high- and low-$\phi_{21}^s$ stars, with a discontinuity at $A_1 \sim 0.25$ mag. AW Dra and V784 Cyg (suspected of having a relatively high metallicity) appear to stand out from the other stars; even when compared with RR Lyr stars in globular clusters ($\S$5.3) these two stars appear extreme.

Finally, the lower-right panel shows that $\phi_{21}^s$ is roughly proportional to $\phi_{31}^s$, where the linear relationship is given by $\phi_{21}^s = 0.509 \phi_{31}^s - 0.181$. For AW Dra, FN Lyr and NR Lyr
the phase parameters are smaller for the $V$ data than the $K_p$ data, and the offsets are parallel to the fitted line.

5.2 $K_p$-$V$ offsets

Since the offsets between the $K_p$ and $V$ Fourier parameters for AW Dra, FN Lyr and NR Lyr appear to be small and systematic (Figs. 6,9,10; Table 3), transformation from $K_p$ to $V$ should be possible using the high-precision $V$ parameters (inclusion of the ASAS parameters would not have been helpful owing to their relatively large uncertainties). Transformation to the $V$ system allows derivation of physical characteristics through the application of well-established $V$-band Fourier relations (see §6 below). Because the three stars exhibit a range of periods (0.53 d to 0.69 d) and light curve shapes ($\phi_{31}^s = 4.8$ to 5.6) and mean colours (see Fig. B1 below), confounding effects that depend, for example, on location in the ‘instability strip’, are likely to have been revealed. On the other hand, if the offsets depend on [Fe/H] such an effect might not have been detected because all three stars appear to have low metal abundances (see column 3 of Table 4).

Our approach to estimating the $K_p$-$V$ offsets was simply to average the observed differences for AW Dra, FN Lyr and NR Lyr, taking the standard deviation as a measure of the uncertainty. In this way we arrived at the following $K_p$-$V$
Figure 8. Four panels comparing Fourier parameters (V-band) for the Kepler non-Blazhko ab-type RR Lyr stars (large black squares) and for 24905 RR Lyr stars in the central regions of the Large Magellanic Cloud. The LMC data are from the Soszyński et al. (2009) OGLE-III study, with colour and symbol coding as follows: red filled dots (17693 RRab stars); blue filled squares (4957 RRc stars); green open triangles (986 RRd stars), and purple open diamonds (1269 RRe stars). Because of the limited x- and y-ranges not all of the LMC RR Lyr stars are represented in the graphs.

transformations:

\[
\begin{align*}
A_{\text{tot}}(V) &= A_{\text{tot}}(Kp) + (0.14 \pm 0.01), \\
RT(V) &= RT(Kp) + (0.002 \pm 0.006), \\
A_1(V) &= A_1(Kp) + (0.054 \pm 0.002), \\
A_3(V) &= A_3(Kp) + (0.018 \pm 0.003), \\
R_{21}(V) &= R_{21}(Kp) - (0.013 \pm 0.004), \\
R_{31}(V) &= R_{31}(Kp) - (0.004 \pm 0.009), \\
\phi_{21}(V) &= \phi_{21}(Kp) - (0.089 \pm 0.021), \\
\phi_{31}(V) &= \phi_{31}(Kp) - (0.151 \pm 0.026). \\
\end{align*}
\] (2)

The most uncertain of these equations are those for RT and \( R_{31} \). While the percent uncertainties are large the offsets are small; and besides, these two quantities are rarely used in correlations to derive physical properties.

A comparison of the ASAS \( A_{\text{tot}}(V) \) values (given in Table 2, column 7) with the \( Kp \) values (given in column 8 of Table 1) is not helpful since several of the stars show larger than expected differences (for example, V838 Cyg). The accuracy of the above transformation equations is expected to improve with future BVI photometry. In §6 these offset equations are used to convert the well-established V-band correlations relating Fourier parameters and physical characteristics to \( Kp \) correlations.

5.3 Kepler vs. globular cluster RR Lyr Stars

In Figure 7 the Fourier parameters for the 19 non-Blazhko stars in the Kepler field (derived using the \( Kp \) photometry but transformed to \( V \) values using the offsets given in Eq. 2) are compared with parameters derived from \( V \)-photometry for 177 RR Lyr stars in several well-studied globular clusters (from Kovács & Walker 2001). The same symbols and colour coding are used in all four panels.

The upper-left panel is analogous to the \( P-A_{\text{tot}} \) diagram and can be used to derive metallicities (see §6.1 below). The two diagonal lines represent mean relations when the globular cluster data (small black dots) are sorted into two [Fe/H]_{BB} bins (here ‘BB’ refers to the Butler-Blanco system, which approximates the Carretta-Gratton system - see below): a metal-poor bin consisting of 19 stars (surrounded by blue squares) with metallicities between \(-1.70\) and \(-1.99\) dex and average \(-1.8\) dex (BB-scale); and an intermediate-metallicity bin consisting of 39 stars (circled in red) with [Fe/H]_{BB} between \(-0.97\) and \(-1.23\) dex and average \(-1.1\) dex. The equations of the lines are: \( \phi_{21}^\alpha = 5.556 \log P + 0.2920 \)
(upper metal-poor bin) and \( \phi_{31} = 6.200 \log P + 3.615 \) (lower intermediate metallicity bin). Four of the Kepler non-Blazhko stars are clearly richer than [Fe/H]_{BL}=−1.1 dex, while the remainder are apparently more metal poor.

The axes of the other three panels in Fig. 7 are similar to those in Fig. 6. The upper-right panel shows that the Kepler stars appear to be drawn from a distribution similar to that of the globular clusters. In particular, the Kepler stars with low \( R_{31} \) values are not unusual, except possibly that they all have relatively high \( R_{21} \) values. Since the globular cluster stars of higher metallicity (red open circles) all tend to reside on the right side of the diagram this separation is probably a metallicity effect, supporting our conclusion that V784 Cyg, V782 Cyg, KIC 6100702 and V2470 Cyg are metal rich. Likewise, the lower-left panel shows that the majority of the Kepler stars do not differ from the stars in globular clusters. Note too that there is very little metallicity discrimination in this plane. The two stars located at the extreme upper edge of the envelope of the globular cluster distribution are AW Dra and V784 Cyg. Finally, the lower-right panel shows close agreement between the phase parameters of the Kepler and globular cluster RR Lyr stars, which supports the conclusion drawn earlier (lower-right panel in Fig. 8). This graph, which is similar to the upper-right panels of Figs. 6 and 7 (and again, Fig. 6), shows that Kepler stars have counterparts among the Kovács & Walker (2001) sample of globular cluster RR Lyr stars. Here we see that both the metal-rich Kepler stars (the clump of four stars at \( \log P \sim 0.3 \) and \( \phi_{21} \sim 4.3 \)) and the metal-poor Kepler stars have counterparts in the LMC, and that the metal-rich stars appear to have relatively long periods for metal-rich RRab stars (i.e., the subgroup of LMC RRab stars with shorter periods at a given \( \phi_{31} \)).

6 PHYSICAL CHARACTERISTICS

RR Lyrae stars provide fundamental insight into the late evolution of low mass stars, and in particular the instability strip (IS) region of the horizontal branch (HB). Being present in many globular clusters (and galaxies) exhibiting different HB types, their properties are related to the evolutionary and chemical histories of these systems. Thus it is desirable to derive (from photometry, spectroscopy and detailed pulsation and stellar evolution models) their physical characteristics, such as the mass \( M \), luminosity \( L \), effective temperature \( T_{\text{eff}} \), iron abundance [Fe/H], chemical composition (X,Y,Z), age, etc. An equally important goal is to understand the interdependencies of these quantities. Significant early contributions are summarized in the papers by Christy (1966), Iben (1971), Smith (1995), Sandage & Tammann (2006, hereafter ST6), and Catelan (2009). This knowledge becomes critical when RR Lyrr stars (and Cepheids) are used to derive accurate distances within and beyond our Galaxy (see Tammann, Sandage & Reindl 2008, hereafter TSR8; Sandage & Tammann 2008, hereafter ST8).

Of course high-dispersion spectroscopy permits direct measurements to be made of some of these quantities; however, such observations are time-consuming and often impractical. Thus efforts have been on-going to derive physical characteristics from photometry only, in particular from pulsation periods, mean magnitudes and colours, quantities that characterize the shapes of the light-curves (e.g., amplitudes, risetimes, Fourier phase parameters), and most recently, from fitting detailed light curve shapes to nonlinear convective pulsation models (see Bono, Castellani & Marconi 2000, Marconi & Clementini 2005, Marconi & Degl’Innocenti, 2007).

A potential difficulty for the present Fourier investigation is that almost all the available calibration relations use empirical correlations derived from \( V \)-band photometry, while for the stars studied here we have \( Kp \) photometry, and only a limited amount of high-precision \( V \) photometry and colour information. Furthermore, the mean \( Kp \) magnitudes from the KIC are somewhat uncertain for giant stars. Despite these limitations we have shown in the previous section, at least for AW Dra, FN Lyr and NR Lyr, three RR Lyr stars with quite different periods and light curve shapes, that differences between the \( V \) and \( Kp \) Fourier parameters appear to be small and systematic. For this reason we have chosen to present approximate physical characteristics based on the

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4 The HB type of a globular cluster is defined as \((B-R)/(B+V+R))\), where \( B \), \( R \) and \( V \) are the number of blue HB stars in the cluster, the number of red HB stars, and the number of RR Lyr stars, respectively (Lee 1989). A table of values is given in Lee, Demarque & Zinn (1994).
extant V-band correlations applied to Kp correlations transformed using the observed Kp-V parameter offsets. Our results are summarized in Tables 4-6 and their derivation is discussed next.

6.1 Iron Abundances

6.1.1 Background: [Fe/H] from P-shifts in P-A diagram

Starting with the papers by Oosterhoff (1939, 1944), Arp (1955) and Preston (1959), the period-amplitude (P-A) diagram, also known as the ‘Bailey diagram’, has been found to be a useful tool for deriving iron-to-hydrogen ratios for field and cluster/galaxy RR Lyr stars. The amplitude usually employed is the B or V total amplitude, A_{tot}, and often the abscissa is log P. The basic P-A_{tot}-[Fe/H] correlation is such that for a given value of A_{tot} lower-metallicity stars tend to have longer periods than higher-metallicity stars. Sandage (2004, hereafter S04) has called this the ‘OAP period-metallicity correlation.’ Other recent discussions of the P-A diagram and period shifts are given by Di Criscienzo et al. (2004), Bono et al. (2007, hereafter BCD7) and Sandage (2006 and 2010, hereafter S06 and S10).

The basic explanation for P-shifts in the Bailey diagram follows from the ‘stacked HB luminosity levels’ model first proposed by Sandage (1958), the latest refinement of which is given in S10 (see his figs. 1 and 3). Owing to the Ritter (1879) pulsation relation, Pν/Ω = Q (where Q is a constant over a wide range of L, T eff and [Fe/H]), lines of constant density (ρ), and therefore period, cut diagonally across HBs of different luminosity (see fig.13 of Sandage, Katem & Sandage 1981, hereafter SKS). When coupled with a monotonic A_{tot}-T eff correlation (in the sense that A_{tot} increases with distance from the red edge of the IS – see, for example, fig.12 of SKS, and fig.12 of Di Criscienzo et al. 2004) one expects to see a given A_{tot} more luminous HB stars having longer periods. In fact, P-shifts and correlations such as the log P-A_{tot}-[Fe/H] correlation are expected in all diagrams where the abscissa is pulsation period and the ordinate is a light curve descriptor that correlates with A_{tot}.

Two such descriptors are the risetime (RT) and the Fourier (see fig.10 of SKS). Period shifting can also be seen between 6.1.1 Background: [Fe/H] from P-shifts in P-A diagram

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6.1.2 Period shifts caused by post-ZAHB evolution

Despite the general success of the Bailey diagram as a tool for deriving [Fe/H] values it has become apparent since the early 1980’s that simple correlations of period-shift with metallicity do not exist in all cases and factors other than metal abundance play a role in the interpretation of these diagrams and correlations. Chief among these effects is evolution away from the ZAHB.

All HB stellar evolution models show tracks leading away from the ZAHB toward the asymptotic giant branch, with HB lifetimes ∼50-100 Myr (core helium burning phase). While the evolution can be either redward or blueward it almost always is in the direction of higher luminosity, with the pace quickening as the evolution progresses. As L increases the period becomes longer and larger P-shifts are seen in the P-A diagram. However, most HB stars are expected to be located near to the ZAHB, with fewer stars in advanced evolutionary phases (CC5 put the fraction of evolved RR Lyr stars in M3 at ∼14%). Indeed skewed HB and RR Lyrae luminosity distributions are observed in many globular clusters (e.g., fig.15 of Sandage 1990, hereafter S90). Such skewed distributions are also seen in simulations of HB and RR Lyr stars (see fig.10 of Marconi et al. 2003, and fig.3 of Catelan 2004), and in period shift diagrams (see fig.6 of Kunder & Chaboyer 2009).

As a first step towards identifying and correcting for such evolutionary effects Sandage (1981a,b; hereafter S81a,b) introduced the concept of ‘reduced’ period, log P' = log P+0.336Δm bol, where Δm bol = m bol minus the mean apparent magnitude of the bulk of the RR Lyr stars in a given cluster, and the multiplicative factor follows from the van Albada & Baker (1971,1973) version of the Pν/Ω = Q relation, and demonstrated the existence of a period-luminosity-amplitude relation for equal metallicity RR Lyrae stars. The net effect of making this correction is to reduce the period shifts of the evolved RR Lyr stars, resulting in tighter correlations in the A_{tot} vs. log P' diagram. In this way S81a,b identified several stars in Messier 3

5 Based on the A_{tot}-T eff correlation one expects also to see period shifting in the log T eff vs. log P diagram, and indeed fig.11 of SKS shows at ‘any given temperature’ an observed shift between M3 and M15 of ∆log P=0.070. However, when comparing model predictions with observations one must ‘let the temperature be cooler for more metal-poor RR Lyr stars at their higher luminosities’ (Simon & Clement 1993, S93, S06).
that are extra-luminous with longer periods at a given $A_B$ than the bulk of the RRab stars. Many other globular clusters are now known to contain extra-luminous large-period-shift stars, including 47 Tuc (V9 – see Storm et al. 1994), NGC 6388 and NGC 6441 (Pritzl et al. 2000, 2001, 2002; Catelan et al. 2006), Messier 5 (Kaluzny et al. 2000, CCC5), NGC 5896 (Alves et al. 2001), IC 4499 (V9 and V54 – see CCC5; Kunder et al. 2011; Walker et al. 2011), and Messier 13 (Sandquist et al. 2011).

The reduced-period concept is useful when deriving mean metallicities for ensemble populations of RR Lyra stars in globular clusters and galaxies (e.g., Dall’Orn et al. 2003), and for field stars that are near the ZAHB (which are assumed to be in the majority), but the method fails for evolved RR Lyrae stars: what looks in the Bailey diagram like a metal-poor ZAHB star might well be an evolved metal-rich star, i.e., an evolved OoI star masquerading as an un-evolved OoII star.

Thus, the amount of $P$-shifting is determined by the amount of L-evolution, which in turn depends on the the HB morphology of the parent population (LDZ90, Lee 1990). When RR Lyra stars originate from a position on the ZAHB that is within the IS, which occurs in OoI clusters with HB type near zero, such as M3 and M4 with HB types (from LDZ94) of $0.08 \pm 0.04$ and $-0.07 \pm 0.10$, respectively, the evolution manifests itself in the IS region of the HR diagram as a vertical widening of the HB with RR Lyra stars of similar mass spread over a range of $L$ and $T_{	ext{eff}}$ (see S90; and fig.1 of Jurscik et al. 2003, hereafter J03; and fig.2 of Valkarce & Catelan 2008). On the other hand, when the original location on the ZAHB was blueward and outside of the IS, which occurs in OoII clusters with HB types that are large and positive, such as M15 (see Sobek et al. 2011), M92 (see Roederer & Sneden 2011) and M2 (Lee & Carney 1999) with HB types of $0.72 \pm 0.10, 0.88 \pm 0.08$ and $0.96 \pm 0.10$ (all from LDZ94), respectively, most if not all the RR Lyra stars are evolved and few if any lower-luminosity ZAHB stars are present (see LDZ90, D00). In the case of M2 Lee & Carney (1999) concluded that its RR Lyrae stars are $-0.2$ mag more luminous than those in M3, owing to all the M2 stars having evolved away from the blue side of the HB, whilst the bulk of the M3 stars lie near the ZAHB. Additional observational support for this idea comes from the clear correlation of metallicity and HB intrinsic vertical width, with more metal-rich systems having greater vertical widths (see Fig.16 of S90). Schematic tracks (along with grid lines of constant period and amplitude) comparing the post-ZAHB evolution of stars in M2 (and M15) with the tracks for M3 (and NGC6441) are given in Fig. 3 of S10. Completing the picture, in clusters with red HBs (i.e., HB types that are large and negative, such as 47 Tuc and M107, with HB types of $-1.0 \pm 0.03$ and $-0.76 \pm 0.08$, respectively) the evolution occurs mainly redward of the IS and few, if any, RR Lyra stars are expected (exceptions to this rule include systems with red HBs and extended blue HBs, such as NGC 6388 and NGC 6441, – see references above). Thus, the luminosity of an individual RR Lyra star in a given cluster depends not only on its metallicity (usually assumed to be that of the parent cluster) but on its evolutionary state, which in turn is determined by the HB-type of the parent cluster and the amount of evolution away from the ZAHB.

In closing, a potential solution to the evolution degeneracy problem in the Bailey diagram is to identify the evolved RR Lyrae stars using some sort of light curve index, such as the JK96 ‘compatibility criterion’ (see Kovacs & Karban 1998, Clement & Shelton 1999, Nemec 2004, CCC5). Of particular interest in this regard is fig.15 of CCC5, which shows large $P$-shifts for several stars in the $A_B$ vs. $\log P$ diagram, but no such offset in the $\phi_{31}$ vs. $\log P$ diagram; whether post-ZAHB stars stand out more in the $P$-$A$ diagram than in the period-$\phi_{31}$ diagram remains an open question. In any case it is important to exercise caution when using the individual Fourier-based metallicities derived here.

### 6.1.3 Metallicity Scales

For the Kepler non-Blazhko RRab stars we have used the correlations involving all three light curve descriptors to estimate [Fe/H] values (see §6.1.4). The resulting metallicities are reported in Table 4. The metal abundances based on the S04 equations are on the Zinn & West (1984, hereafter ZW) scale, and those based on J98’s equation 1 are on the Carretta & Gratton (1997, hereafter CG) scale. For comparison purposes the latter (i.e., the $\phi_{31}$-metallicities) have been transformed to the ZW scale and to the Carretta et al. (2009, hereafter C9) scale. Although several other metallicity systems have been proposed (e.g., Layden 1994,
Figure 9. Period-amplitude and three other metallicity diagnostic diagrams for the Kepler non-Blazhko RRab stars. For all four graphs the abscissa is log\(P\), all the stars have been labelled, and the four candidate metal-rich stars are plotted as open squares. AW Dra, FN Lyr and NR Lyr are plotted twice in each panel, with the points derived from the \(K_p\) photometry (solid black squares) connected to the points derived from the high-precision \(V\)-band photometry (open diamonds) – the consistency of the \(K_p-V\) offsets seen here (and in Figs. 6 and 10) is quite striking. The diagonal dashed lines, representing the relations for the M3 RR Lyr stars (assumed to have [\(\text{Fe}/\text{H}\)]\(_{\text{ZW}}\) = -1.6 dex), are from Equation 5 (S04eq6, top left), Equation 6 (S04eq7, bottom left) and fig.15 of CCC5 (bottom right).

Jurcsik 1995, Butler & Blanco - see S04, and Kraft & Ivans 2003), the C9 scale is most extensive. It is based on a homogeneous data set derived from \(\sim 2000\) modern spectra, and is gaining considerable favour – see, for example, its adoption by Harris (1996) for his on-line globular cluster catalog at http://www.physics.mcmaster.ca/~harris/mwgc.dat. In addition, equations are provided for transformation between the various scales. When transforming between the ZW and CG systems the following equation (from S04, footnote 1) has been used: [\(\text{Fe}/\text{H}\)]\(_{\text{ZW}}\) = 1.05 [\(\text{Fe}/\text{H}\)]\(_{\text{CG}}\) - 0.20. And when transforming from the ZW and CG scales to the C9 scale we have used (from C9): [\(\text{Fe}/\text{H}\)]\(_{\text{C9}}\) = 1.137 [\(\text{Fe}/\text{H}\)]\(_{\text{CG}}\) - 0.003; and [\(\text{Fe}/\text{H}\)]\(_{\text{C9}}\) = 4.13 + 0.130 [\(\text{Fe}/\text{H}\)]\(_{\text{ZW}}\) - 0.356 [\(\text{Fe}/\text{H}\)]\(_{\text{ZW}}^2\).

6.1.4 Metallicities for the Kepler stars

Figure 9 contains, for the Kepler non-Blazhko RRab stars, four such ‘metallicity diagnostic diagrams’. The principal observation here is the similarity, to first order, of the distribution of the points in the four graphs. The top-left panel is the classical \(P-A_{\text{tot}}\) diagram, with total \(K_p\) amplitude plotted along the ordinate. Traditionally, lines of constant [\(\text{Fe}/\text{H}\)] serve as metallicity calibration lines; these run diagonally from the upper-left to the lower-right corner (as seen in three of the panels).

The top-right panel has \(A_1\) along the ordinate. As expected, because \(A_1\) and \(A_{\text{tot}}\) are highly correlated (see Fig. 2), the distribution of the points closely resembles that in the \(A_{\text{tot}}\) vs. log\(P\) diagram. In principle the small range
of $A_1$ (only ~0.2 mag) compared with that of $A_{tot}$ (nearly 0.7 mag here, but often larger) makes the $P-A_1$ diagram less useful for estimating [Fe/H]; but with such high-precision data as provided by Kepler the derived [Fe/H] results should, in principle, be similar.

The bottom panels of Fig. 9 show risetime (left) and $\phi_{31}$ (right) plotted against $\log P$, where both RT (reverse scale) and $\phi_{31}$ (reverse scale) increase with increasing $\log P$. The bottom-right panel is the ‘Kepler non-Blazhko RRab only’ version of the top-left panel of Fig. 7, and also is similar to Fig. 11. The dashed line in the bottom-right panel (from fig.15 of CCCS), the equation of which is $\phi_{31} = 3.124 + 5.128 \log P$, represents the observed V-band relation for the RRab stars in M3 – it is encouraging that the slope of the line appears to agree with the slope of the Kepler metal-poor stars. According to this calibration NR Lyr is the most metal-poor star in the sample, followed by KIC 7021124 and FN Lyr, and most of the RRab stars have metallicities similar to those in M3, which has [Fe/H] = 0.14 dex (Carretta et al. 2009 scale; W. Harris’ on-line catalog). Metallicities for the other stars, including the four stars suspected of having high [Fe/H] values, are discussed below.

Figure 10 contains the RT vs. $A_{tot}$ diagram (left panel) and the $\phi_{31}$ (which is $\pi$ larger than $\phi_{31}$) vs. $A_{tot}$ diagram (right panel) for the Kepler stars. In general, as the total amplitude increases, both RT and $\phi_{31}$ decrease, with linear fits described by the equations: $RT = -0.118 A_{tot} + 0.256$ and $\phi_{31} = -1.75 A_{tot} + 6.75$; however, owing to the large scatter about these lines, and the suspected large range in metallicities of the Kepler stars, we believe these equations to be unreliable (and for this reason they were not plotted). Theoretical models (see the middle-left panels of Figs.14 and 15 below) suggest that it is more probable that there are families of lines (or curves), one for each metallicity. Unfortunately, the present observational data are insufficient to establish these. It is noteworthy that AW Dra, and to a lesser extent V784 Cyg and V904 Cyg, have larger $\phi_{31}$ values and larger RTs than other stars of comparable $A_{tot}$. And, while it appears that all the low-amplitude stars have large RTs and large $\phi_{31}$ values, this conclusion is based on only four low-amplitude candidate metal-rich stars and two low-amplitude long-period candidate metal-poor stars (KIC 7030715, AW Dra), one of which (AW Dra) is a possible outlier in both graphs and which may be a long-period evolved star of intermediate-metallicity (if our preliminary CFHT spectra result is correct – see §6.1.5 below) rather than a near-ZAHB star of low-metallicity. In any case, both KIC 7030715 and AW Dra have considerably longer periods than the four high-metallicity stars.

In Figure 11 the Kepler data are compared with the galactic field star data analyzed by S04 (see his Fig. 3). The field RR Lyrae stars are the same as those studied by Simon & Lee (1981), Simon & Teays (1982) and Simon (1988), the details of which are given in table 1 of S04. Fig. 11 is comparable to the bottom-right panel of our Fig. 9, which shows only the Kepler non-Blazhko stars. Because the Simon et al. stars have known [Fe/H] values from Layden (1994) it is possible to sort them by metallicity. Following Sandage we plotted two subsets of the data, which have average metallicities $\langle [Fe/H] \rangle = -0.35$ and $-1.63$ dex (ZW scale); the diagonal lines were obtained by substituting these values into eq. 3 of S04. This diagram is also the galactic field star version of the upper-left panel of Figure 7, which included RR Lyrae stars in galactic and Magellanic Cloud globular clusters. The chief difference is that the calibration has now been extended to more metal-rich stars.

Fig. 11 shows that the stars KIC 6100702, V2470 Cyg, V782 Cyg and V785 Cyg, all of which have long risetimes, are solidly in the metal-rich region of the diagram, with the highest $\phi_{31}$ values and all with periods shorter than 0.55 d. The other Kepler stars are a better match to the field stars that have [Fe/H]ZW = −1.63 dex. AW Dra and KIC 7030715, with long periods, intermediate risetimes, and high $\phi_{31}$ values, are seen to have metallicities ~−1.64 dex (ZW scale). Three of the four metal-rich stars (V782 Cyg, KIC 6100702 and V2470 Cyg) have [Fe/H]ZW = −0.4 dex, and V784 Cyg is the most metal-rich star in our sample with [Fe/H]ZW = −0.2 dex. The most metal-poor stars are seen to be more metal-poor than [Fe/H]ZW = −1.6 dex.

Equation 1 of J98, which is identical to equation 3 ofJurcsik & Kovács 1996, describes lines of constant [Fe/H]CG in the period-$\phi_{31}(V)$ diagram. Using the offset formula for $\phi_{31}$ given in Eq. 2 the $Kp$ version of the J98 equation becomes

$$[Fe/H]_{CG} = -5.241 - 5.394 P + 1.345 \phi_{31}(Kp).$$  \hspace{1cm} (3)

Metallicities calculated with this formula are listed in column 2 of Table 4 under ‘J98equ1’. For comparison purposes the corresponding values on the ZW and C9 scales also are listed.

Our second estimate of [Fe/H] is derived from the [Fe/H]ZW $- \log P - \phi_{31}$ relation of S04 (his eq. 3 - note that the cosine version of the Fourier phase parameter is employed here) – see lower-right panel of Fig. 9. Applying the $\phi_{31}$ offset given in Eq. 2, Sandage’s relation becomes

$$[Fe/H]_{ZW} = -6.217 - 7.012 \log P + 1.411 \phi_{31}(Kp),$$  \hspace{1cm} (4)

where the intercept term has a total uncertainty of ±0.023, and the $\phi_{31}$ and $\log P$ coefficients have respective uncertainties of ±0.014 and ±0.071 (from S04). Metallicities derived from Eq. 4 are listed in Table 4 under ‘S98eq3’.

Implementation of the $P-A$ diagram (see upper-left panel of Fig. 9) for deriving metallicities was via eq. 6 of S04, combined with the $Kp-V$ offset for $A_{tot}$ (Eq. 2):

$$[Fe/H]_{ZW} = -1.453 A_{tot}(Kp) - 7.990 \log P - 2.348,$$  \hspace{1cm} (5)

where the errors in the coefficients (from S04) are ±0.027, ±0.091 and ±0.043, respectively. The derived [Fe/H]ZW values are listed in Table 4 under ‘S98eq6’. According to this formula the most metal-poor star is AW Dra, 0.2 dex more metal-poor than NR Lyr, and 0.6 dex more metal-poor than the estimates derived form the $\phi_{31}$ equations! Given its location in the $\phi_{31}$ vs logP diagram, and our recently derived spectroscopic metallicity of ~−1.33±0.08 dex, it is our opinion that this is an evolved RR Lyrae star; curiously, it follows the pattern of evolved RR Lyra stars seen in M3 by CCC95 (their fig.16). The metal-rich stars also are predicted to be more-metal-poor than the values given by the $\phi_{31}$ estimates.

Our fourth set of [Fe/H] estimates was based on the RT vs. $\log P$ diagram (lower-left panel of Fig. 9). The metallicities were calculated using Sandage’s relation between RT, $P$ and [Fe/H] (his eq. 7), combined with our $V-Kp$ offset (Eq.2):

$$[Fe/H]_{ZW} = 6.33 RT(Kp) - 9.11 \log P - 4.59.$$  \hspace{1cm} (6)
for the Kepler V.

where the S04 intercept has been increased by 0.01 (representing the RT conversion from V to Kp). Application of this equation gave the metallicities listed in Table 4 under 'S4eq7'.

Table 4. Metal abundances based on the Kp photometry and the equations of Jurcsik (1998) and Sandage (2004).

| Star        | [Fe/H] | \(\phi_{31}^{s}\) | \(\phi_{31}^{s}\) | \(A_{tot}\) | RT |
|-------------|--------|------------------|------------------|-------------|-----|
| (1)         |        |                  |                  |             |     |
| NR Lyr      | -2.04, -2.34, -2.32 | -2.28 | -2.13 | -2.16 |
| V715 Cyg    | -1.18, -1.44, -1.34 | -1.45 | -1.17 | -0.74 |
| V782 Cyg    | -0.25, -0.47, -0.29 | -0.51 | -0.86 | -0.76 |
| V784 Cyg    | +0.06, -0.14, +0.07 | -0.18 | -1.09 | -0.87 |
| KIC 6100702 | -0.14, -0.35, -0.17 | -0.38 | -0.69 | -0.48 |
| NQ Lyr      | -1.55, -1.83, -1.77 | -1.86 | -1.68 | -1.57 |
| FN Lyr      | -1.62, -1.90, -1.84 | -1.94 | -1.70 | -1.31 |
| KIC 7021124 | -1.79, -2.08, -2.04 | -2.08 | -1.91 | -1.83 |
| KIC 7030715 | -1.39, -1.66, -1.58 | -1.60 | -1.97 | -1.96 |
| V349 Lyr    | -1.46, -1.73, -1.66 | -1.77 | -1.47 | -1.08 |
| V368 Lyr    | -1.27, -1.53, -1.44 | -1.53 | -1.27 | -0.64 |
| V1510 Cyg   | -1.56, -1.83, -1.77 | -1.86 | -1.89 | -1.57 |
| V346 Lyr    | -1.55, -1.83, -1.76 | -1.86 | -1.84 | -1.52 |
| V350 Lyr    | -1.56, -1.84, -1.78 | -1.86 | -1.96 | -1.62 |
| V894 Cyg    | -1.51, -1.79, -1.72 | -1.82 | -2.01 | -1.46 |
| V2470 Cyg   | -0.48, -0.71, -0.55 | -0.74 | -1.13 | -0.99 |
| V1107 Cyg   | -1.30, -1.56, -1.48 | -1.60 | -1.56 | -1.38 |
| V838 Cyg    | -1.30, -1.56, -1.48 | -1.58 | -1.40 | -0.84 |
| AW Dra      | -1.47, -1.74, -1.67 | -1.68 | -2.34 | -2.06 |

\(\phi_{31}^{s}\) J99eq1 S4eq3 S4eq6 S4eq7

RT

\(s\) CG, ZW, C9 ZW ZW ZW

\(A_{tot}\) to Kp photometry and the data reductions are continuing on the other 16 stars for which spectral are in hand.

6.1.5 Discussion of the derived metallicities

In the absence of spectroscopic metal abundances which of the [Fe/H] values in Table 4 are the most reliable? One area of potential bias comes from the fact that all the metallicity correlations used in §6.1.4 assume that the stars are unevolved. This probably is true for most of the Kepler non-Blazhko stars; however, there may a few evolved stars in the sample, the most likely candidate being AW Dra. In such a case the period will have been increased by luminosity evolution and the derived [Fe/H] will be systematically smaller than the true value. Another potential bias occurs for the

where the S04 intercept has been increased by 0.01 (representing the RT conversion from V to Kp). Application of this equation gave the metallicities listed in Table 4 under 'S4eq7'.
Figure 11. Period-$\phi_{31}^c$ diagram, comparing the Kepler non-Blazhko RR Lyr stars and the Lub-Simon galactic field stars (from S04). The latter have metallicity estimates (on the ZW scale) given in columns 2 and 3 of Table 4. When the $\phi_{31}$ metallicities are compared with those calculated using $A_{tot}$ (column 4) and RT (column 5) a number of systematic differences are present. For all the low-amplitude stars (the four suspected metal-rich stars, and AW Dra and KIC 7030715) the RT and $A_{tot}$ estimates tend to be systematically more metal-poor than the $\phi_{31}$ estimates. On the other hand, for most of the suspected metal-poor stars the $A_{tot}$ and RT equations give more metal-rich estimates of [Fe/H] than those derived with the $\phi_{31}$ equations. And for several of the stars (e.g., V1510 Cyg, V346 Lyr, V350 Lyr, V894 Cyg and V1107 Cyg) the RT gives [Fe/H] values richer than the $\phi_{31}$ values, while the $A_{tot}$ formula gives [Fe/H] values more metal poor. The largest difference occurs for V784 Cyg, where $\phi_{31}$ suggests [Fe/H]$_{ZW} = -0.16$ while $A_{tot}$ and RT imply [Fe/H]$_{ZW} = -1$ dex. Our suspicion is that the slope in the RT term of Equation 6 is too steep (see M3 line in lower-left panel of Fig.9).

AW Dra is the only star for which the estimated [Fe/H] can be compared with a previously published metallicity estimate, and, unfortunately, that estimate is quite uncertain. Based on its location in the $P$-$A_V$ diagram, Castellani et al. (1998) found the metallicity of AW Dra to be in the range $-1.9$ to $-1.4$ dex. Our estimate, [Fe/H]$_{C9} = -1.67$ dex, is consistent with theirs, but should be more reliable because it is based on much more extensive and higher precision photometry (see Table 3, and footnote 8).

6.2 Reddenings and Mean Colours

6.2.1 Dereddened colours

J98 showed that the dereddened mean $B-V$ colours correlate well with pulsation period $P$ and the Fourier $A_1$ coefficient, and derived the following relationship (her eq. 3):

$$ (B-V)_0 = 0.308 + 0.163P - 0.187A_1. $$

Replacing $A_1(V)$ with $A_1(Kp)$ from Eq. 2 simply reduces the intercept from 0.308 to 0.298. Dereddened $B-V$ colours calculated with our modified equation for the $Kp$-photometry, and with the J98 equation for the $V$-photometry of FN Lyr and AW Dra, are given in column 8 of Table 5 (under $J98$). The colour range is from 0.306 (for V368 Lyr) to 0.376 (for KIC 7030715), with a mean colour of 0.343.

Another relationship for $(B-V)_0$ was derived by KW01. Their eq. 6 includes an additive $A_3$ term, and the $A_1$ and $A_3$ are derived from $V$-photometry. Using our offsets to convert to the $Kp$ scale gives:

$$ (B-V)_0 = 0.448 + 0.189 \log P - 0.313 A_1 + 0.293 A_3 $$

where $A_1$ and $A_3$ now are derived from $Kp$ photometry and the KW01 intercept has been modified accordingly. Dereddened colours derived using this equation (and the KW01 equation for the $V$ data of AW Dra and FN Lyr) are also given in column 8 of Table 5 (under ‘KW01’). These estimates are practically identical to the J98 colours and have an overall mean of 0.347. In subsequent analyses we adopt for each star the average of the two colour estimates.

Figure 12 shows four graphs for the non-Blazhko RR Lyr stars, all with dereddened mean colour, $(B-V)_0$, plotted along the x-axis. In every graph the four high metallicity stars (V782 Cyg, V784 Cyg, KIC 6100702 and V2470 Cyg), plotted as open squares, stand apart from the other RR Lyr stars. One sees that they also tend to be on the red side of the instability strip. The three stars with the reddest colours (and coolest temperatures), NR Lyr, AW Dra and KIC 7030715, are all of very low metallicity and have the longest periods, and that at a given colour the metal-poor stars have lower amplitudes than the blue metal-poor stars; the colour-total amplitude diagram (bottom left) shows that the red metal-poor stars have lower amplitudes than the blue metal-poor stars, and that at a given colour the metal-rich
stars have lower amplitudes than the metal poor stars; and the colour-absolute magnitude diagram (bottom right – see discussion in §6.4) shows that at a given colour the metal-rich stars have lower luminosity than the metal-poor stars, and that the reddest metal-poor stars are more luminous than the bluest metal-poor stars.

6.2.2 Reddenings and extinctions

Table 5 also contains \( E(B-V) \) reddenings (column 6) derived from the large-scale reddening maps of Schlegel, Finkbeiner & Davis (1998). These were estimated using the NSStEd on-line facility and the galactic \((l,b)\) coordinates given in Table 5 (columns 4-5). According to the maps the most reddened stars in our sample are V782 Cyg, V784 Cyg and V1510 Cyg. Total extinctions in the \( V \)-passband, \( A_V \), were calculated assuming an extinction-to-reddening ratio \( A_V/E(B-V) = 3.1 \) and are given in column 7 of Table 5.

For AW Dra, which has calibrated \( B,V \) photometry, the above reddenings can be compared with reddenings derived from the Fourier-based dereddened colours and the observed reddened mean \( B-V \) colours. Castellani et al. (1998) observed \( (B-V)_0 = 0.368 \), and from the \( P-A \) diagram estimated [Fe/H] = −1.4 dex. Using the Burstein & Heiles (1982) reddening maps they adopted the value \( E(B-V) = 0.04 \pm 0.06 \) mag, which gives \( (B-V)_0 = 0.31 \pm 0.33 \). This estimate of the reddening is consistent with \( 0.047 \pm 0.001 \) given in Table 5, but the dereddened colour is bluer than the average of the two Fourier based values, \( (B-V)_0 = 0.364 \) (J98, KW01). This difference explains why the Castellani et al. estimates of \( T_{\text{eff}} \) (see their table 7, and below) are higher than our estimates (column 2 of Table 6 below).

### Table 5. Galactic coordinates, reddenings, extinctions, dereddened colours and distances

| Star | \( (K_P) \) | \( (V) \) | \( l(\text{II}) \) | \( b(\text{II}) \) | \( E(B-V) \) | \( A_V \) | \( (B-V)_0 \) | \( d \) |
|------|-------------|-------------|---------------|---------------|-------------|-------------|---------------|-------------|
|      | [mag]       | [mag]       | [deg]         | [deg]         | [mag]       | [mag]       | J98, KW01     | [kpc]       |
| NR Lyr | 12.683      | 12.44       | 69.8668       | 13.55258      | 0.175(0.000) | 0.532       | 0.369, 0.372  | 3.0         |
| V715 Cyg | 16.265      | 72.92658    | 07.72890      | 0.170(0.009)  | 0.517       | 0.321, 0.327  | 14.2         |
| V782 Cyg | 15.392      | 75.14435    | 06.94725      | 0.509(0.024)  | 1.606       | 0.358, 0.363  | 15.1         |
| V784 Cyg | 15.370      | 76.41126    | 06.5689       | 0.343(0.010)  | 1.088       | 0.351, 0.353  | 11.8         |
| KIC 6100702 | 13.458    | 13.64       | 71.0171       | 17.7097       | 0.103(0.006) | 0.324       | 0.348, 0.353  | 3.9         |
| NQ Lyr | 13.075      | 13.36       | 73.11219      | 15.06421      | 0.068(0.003) | 0.207       | 0.352, 0.358  | 3.7         |
| FN Lyr | 12.876      | 12.79       | 73.45862      | 14.69286      | 0.104(0.007) | 0.320       | 0.323, 0.328  | 2.9         |
| KIC 7021124 | 13.550    | 73.5605     | 14.7208       | 0.113(0.006)  | 0.306       | 0.357, 0.361  | 4.1          |
| KIC 7030715 | 13.452    | 13.24       | 74.5488       | 12.5345       | 0.108(0.006)| 0.334       | 0.376, 0.377  | 3.8         |
| V349 Lyr | 17.433      | 72.23472    | 18.37838      | 0.102(0.013)  | 0.301       | 0.324, 0.330  | 22.7         |
| V368 Lyr | 16.002      | 74.40557    | 14.98618      | 0.061(0.003)  | 0.189       | 0.306, 0.308  | 11.0         |
| V1510 Cyg | 14.494      | 78.7859     | 07.2393       | 0.484(0.030)  | 1.480       | 0.340, 0.345  | 10.4         |
| V346 Lyr | 16.421      | 73.65339    | 19.49070      | 0.052(0.001)  | 0.160       | 0.340, 0.345  | 13.8         |
| V359 Lyr | 15.696      | 75.65435    | 19.58792      | 0.051(0.001)  | 0.158       | 0.341, 0.345  | 9.9          |
| V894 Cyg | 13.393      | 12.92       | 78.73702      | 12.60755      | 0.112(0.005)| 0.346       | 0.329, 0.331  | 3.2         |
| V2470 Cyg | 13.300      | 13.54       | 78.3633       | 14.9140       | 0.068(0.001)| 0.211       | 0.357, 0.360  | 3.7         |
| V1107 Cyg | 15.648      | 78.54873    | 15.03295      | 0.070(0.002)  | 0.219       | 0.348, 0.355  | 9.7          |
| V830 Cyg | 13.770      | 14.24       | 79.20399      | 16.33490      | 0.067(0.002)| 0.210       | 0.314, 0.318  | 5.3         |
| AW Dra | 13.053      | 12.85       | 80.23054      | 19.05392      | 0.047(0.001) | 0.146       | 0.363, 0.365  | 3.0         |

6.3 Effective Temperatures

Mean \( T_{\text{eff}} \) values for the RR Lyr stars were estimated using two prescriptions, one from KW01 and the other from S06 (where the relationship between \( (B-V)_0 \) colour, \( T_{\text{eff}} \), and [Fe/H], including the effects of such other factors as surface gravity and turbulent velocity, has been reviewed). For the colour interval \( 0.20<(B-V)_0<0.30 \) S06 adopts the formula given by Carney, Storm & Jones (1992):

\[
(B-V)_0 = -2.632 \log T_{\text{eff}} + 0.038 \frac{[\text{Fe/H}]_{\text{ZW}}}{10.423}.
\]

Inverting this equation (eq. 18 of S06) gives:

\[
\log T_{\text{eff}} = -0.380 (B-V)_0 + 0.144 \frac{[\text{Fe/H}]_{\text{ZW}}}{3.960}.
\]
Figure 12. Four diagrams for the non-Blazhko RR Lyr stars, all with dereddened mean colour \((B-V)_0\) (average of the two values given in column 8 of Table 5) along the abscissa: metallicity vs. colour (Top left), period vs. colour (Top right), total amplitude vs. colour (Bottom left), and absolute magnitude vs. colour (Bottom right). In each panel the line (and its equation) is from a least squares fit to the points for the 15 metal-poor stars (i.e., those with \([\text{Fe/H}] < -1.0\) dex). The linear correlations show that the reddest metal-poor stars (those nearest the red edge of the Instability Strip) are more metal poor, have longer periods and smaller total amplitudes, and are more luminous than the bluest metal-poor stars. The diagrams also show that at a given colour the four metal-rich stars (plotted with open squares) have shorter periods, smaller amplitudes and are less luminous than the metal-poor stars.

\[ [\text{Fe/H}]_{\text{der}} = -7.42 (B-V)_0 + 0.78 \]

\[ A_{\text{tot}} = -6.40 (B-V)_0 + 3.27 \]

\[ A_{\phi_{31}} = -6.40 (B-V)_0 + 3.27 \]

\[ M_V = 1.179 - 1.396 P - 0.477 A_1 + 0.103 \phi_{31} \]

where \(A_1\) and \(\phi_{31}\) are Fourier parameters derived from the \(K_p\)-photometry. The resulting \(M_V\) values are given in column 3 of Table 6 (under ‘J98, eq2’).

The Fernley et al. (1998) equation relating \(M_V\) and metallicity,

\[ M_V = (0.20 \pm 0.04) [\text{Fe/H}]_{\text{CG}} + (1.03 \pm 0.14) \]

assumes that RR Lyr has \(M_V = 0.78(\pm 0.29)\) at \([\text{Fe/H}]_{\text{CG}} = -1.39\) dex and is consistent with the statistical parallax solution for 84 halo RR Lyrae stars and with various Baade-Wesselink analyses. \(M_V\) values derived using this equation are listed in column 3 of Table 6 (under ‘F98, eq2’). In general the F98 and J98 values are similar, with a mean difference (J98 minus F98) of only 0.01(±0.01) mag. On the downside there has been mounting evidence that the calibration of \(M_V\) with \([\text{Fe/H}]\) is non-linear (see S06 and TSR8).

Equation 10 of Bono, Caputo & Di Criscienzo (2007)

\[ \sim 0.65-0.75 M_\odot, \text{ and luminosity } \sim 65-80 L_\odot. \] This temperature is \(\sim 400\) K hotter than the corresponding Fourier-based estimate of \(T_{\text{eff}} \sim 6300\) K. The difference is not surprising given that their assumed mass and luminosity are higher than the Fourier-based values.

6.4 Absolute magnitudes

Following on from the Jurcsik & Kovács (1996) and Kovács & Jurcsik (1996, 1997) papers, J98 gives an equation for absolute magnitude (based on \(V\)-photometry) that depends on \(A_1\) and \(\phi_{31}\) (her eq. 2). Application of our \(V\)-\(K_p\) offsets (Eq. 2) gives

\[ M_V = (0.20 \pm 0.04) [\text{Fe/H}]_{\text{CG}} + (1.03 \pm 0.14) \]

assumes that RR Lyr has \(M_V = 0.78(\pm 0.29)\) at \([\text{Fe/H}]_{\text{CG}} = -1.39\) dex and is consistent with the statistical parallax solution for 84 halo RR Lyrae stars and with various Baade-Wesselink analyses. \(M_V\) values derived using this equation are listed in column 3 of Table 6 (under ‘F98, eq2’). In general the F98 and J98 values are similar, with a mean difference (J98 minus F98) of only 0.01(±0.01) mag. On the downside there has been mounting evidence that the calibration of \(M_V\) with \([\text{Fe/H}]\) is non-linear (see S06 and TSR8).

Equation 10 of Bono, Caputo & Di Criscienzo (2007)
should be brighter and give the following equation:

\[ M_V = 1.19 (\pm 0.10) + 0.50 \, \text{[Fe/H]}_{\odot} + 0.09 \, \text{[Fe/H]}_{\odot}^2. \]  (14)

Absolute magnitudes calculated with this quadratic formula also are given in column 3 of Table 6 (under ‘BCD7’). For the metal-poor stars the BCD7 \( M_V \) values are very similar to the F98 values; however, because of the non-linear metallicity dependence they are fainter than the F98 values for the four metal-rich stars.

Most recently, Catelan & Cortes (2008) argue that revised values for the trigonometric parallax and reddening of RR Lyr imply that the luminosity scale for RR Lyr stars should be brighter and give the following equation:

\[ M_V = (0.23 \pm 0.04) \, \text{[Fe/H]}_{\odot} + (0.984 \pm 0.127). \]  (15)

Absolute magnitudes calculated with this equation (given in column 3 of Table 6 under ‘CC8’) are, on average, \(<+0.15 \, \text{mag}\) brighter than the values from F98 and BCD7, which in turn are \(<-0.05 \, \text{brighter than the J98 values}. Note that these differences are smaller than the uncertainties in the individual \( M_V \) estimates (which are \(<-0.20 \, \text{mag})\). The colour-\( M_V \) diagram plotted in Fig.12 (bottom right panel) was constructed using the \( M_V \) values calculated with eq. 2 of J98.

### 6.5 Distances

Approximate distances for the non-Blazhko RR Lyr stars are given in column 9 of Table 5. These were computed assuming: (1) the Fernley et al. (1998) \( M_V \) values (which lie between the J98 and CC08 values); (2) the ASAS (V) values (given in column 3 of Table 5) for the nine stars having ASAS V I photometry; and for the other stars the (Kp)-magnitudes brightened by 0.15 mag (the observed average \( V-Kp \) offset for FN Lyr and AW Dra); and (3) the visual extinctions given in column 7 of Table 5. With uncertainties of \(<0.15 \, \text{mag for } M_V, \sim 0.05 \, \text{mag for the } V-Kp \text{ offset}, \text{and } \sim 0.03 \, \text{mag for } A_V \), these distances are quite uncertain.

Taken at face value the nearest of the stars have distances \(<3 \, \text{kpc}, and the most distant star is V349 Lyrae at \(<22 \, \text{kpc}. These estimates will be considerably improved when calibrated BVRI-photometry becomes available.

### 6.6 Surface gravities

Mean surface gravities were calculated using eq. 15 from J98, and eq. 12 from KW99. The J98 formula, which is accurate to \(<0.004\), depends only on the period and is given by

\[ \log g = 2.473 - 1.226 \log P. \]  (16)

The KW99 formula depends on period, mass and effective temperature:

\[ \log g = 2.938 + 0.230 \log M - 0.110 \log T_{\text{eff}} - 1.219 \log P. \]  (17)

Both estimates are given in column 4 of Table 6 and the results are in good agreement.

### 6.7 Pulsational luminosities and masses

Following on from the basic equation of stellar pulsation given by the Ritter (1879) relation, \( P \sqrt{\rho} = Q \) (where \( \rho \) is...
the mean density in cgs units and $Q$ is the pulsation constant. van Albada & Baker (1971, 1973) derived an equation relating the pulsation period to mass, luminosity and $T_{\text{eff}}$. When this equation and similar more recent equations that include a dependence on metal abundance are used to derive the mass and luminosity for an RR Lyr star such quantities are referred to as pulsational mass $M(\text{puls})$ and pulsational luminosity $L(\text{puls})$. Both are expressed here in solar units.

Pulsational luminosities were calculated with two different formulae given by J98 and are reported in column 5 of Table 6. In the first case (her eq. 16) the luminosity depends only on the metallicity:

$$\log L = 1.464 - 0.106 [\text{Fe}/\text{H}]_{\text{CG}}. \quad (18)$$

The second formula (her eq. 17) also takes into account $T_{\text{eff}}$:

$$\log L = 10.260 - 0.062 [\text{Fe}/\text{H}]_{\text{CG}} - 2.294 \log T_{\text{eff}}. \quad (19)$$

In both cases the lower metallicity stars have the higher luminosities. In the second equation the KW01 effective temperatures (Table 6, column 2) were used for the calculations. With both equations one sees in the HR-diagram two approximately parallel lines, one for the metal-rich stars and one for the metal-poor stars, each with the luminosity increasing as $T_{\text{eff}}$ decreases (see the top left panel of Fig. 13). At a given $T_{\text{eff}}$ the luminosity difference between the metal-rich and metal-poor stars is greater with eq. 16 than it is with eq. 17.

Column 7 of Table 6 contains $M(\text{puls})$ values computed using eq. 14 and 22 of J98. The former is given by

$$\log M = 1.477 \log L - 1.754 \log P - 6.272 \log T_{\text{eff}} + 0.037 [\text{Fe}/\text{H}]_{\text{CG}} + 20.884. \quad (20)$$

and the latter by

$$\log M = -0.328 - 0.062 [\text{Fe}/\text{H}]_{\text{CG}}. \quad (21)$$

In both cases the adopted [Fe/H] values given in Table 4 (transformed to the CG system) were used, and in the first
equation we used the \( \log L \) from J98 eq. 17 and the \( \log T_{\text{eff}} \) from eq. 11 of KW01. The average mass for the four metal-rich stars is \( \sim 0.50 \, M_\odot \) compared with the average mass for the metal-poor stars of \( \sim 0.60 \, M_\odot \).

### 6.8 L and M from stellar evolution models

A ZAHB stellar evolution model takes as its input the mass, \( M_{\text{evol}} \), and chemical composition \( (X, Y, Z) \). A subsequent evolutionary track for a given mass and composition gives the luminosity \( L_{\text{evol}} \) and effective temperature as a function of time. Examples of such models are those by Dorman (1992), Bono \textit{et al.} (1997) and VandenBerg \textit{et al.} (2000). Based on three different sets of stellar evolution models S6 gives formulae for \( L_{\text{evol}} \) as a function of \([\text{Fe/H}]\). His eq. 8, which follows from the models of Caputo \textit{et al.} (2000), is given by

\[
\log L = 1.245 - 0.451 \, [\text{Fe/H}] - 0.097 \, [\text{Fe/H}]^2. \tag{22}
\]

His eq. 10, which is based on the alpha-enhanced ZAHB models of Catelan, Pritzl & Smith (2004), is given by

\[
\log L = 1.404 - 0.243 \, [\text{Fe/H}] - 0.043 \, [\text{Fe/H}]^2. \tag{23}
\]

Both of these equations, like the BCD7 absolute magnitude formula discussed above, assume a quadratic dependence on \([\text{Fe/H}]\). This is not the case for eq. 12 which is derived from the models of Clementini \textit{et al.} (2003) and is given by

\[
\log L = 1.538 - 1.110 \, [\text{Fe/H}]. \tag{24}
\]

Luminosities computed with these three formulae are given in column 6 of Table 6. For the metal-poor stars all three \( L_{\text{evol}} \) are systematically larger than the \( L_{\text{puls}} \) values given in column 5. For the four metal-rich stars the agreement is better but there is a wide range of \( L_{\text{evol}} \) owing to the linear or non-linear \([\text{Fe/H}]\) dependencies. Regardless of which formula was used the most luminous stars have the lowest metallicities and there is internal consistency (as was the case for the J98 luminosities). It is not clear whether the \( L_{\text{puls}} \) or \( L_{\text{evol}} \) are correct. Further comparison of the derived luminosities is given in §6.9 (after masses are discussed).

A mass equation with \( M \) varying linearly with \([\text{Fe/H}]\) is also given by S06 (his eq. 15):

\[
\log M = -0.283 - 0.066 \, [\text{Fe/H}]_{2W}. \tag{25}
\]

Since this equation was derived from the Bono \textit{et al.} (1997) horizontal branch models it gives evolutionary masses. The \( M_{\text{evol}} \) derived with this equation are given in column 8 of Table 6. A more recent mass formula by Bono, Caputo & Di Criscienzo (2007), their eq. 7, which is based on the Pietrinferni \textit{et al.} (2004, 2006) HB models, is given by

\[
\langle \log M \rangle = -0.2675 - 0.063 \, [\text{Fe/H}]. \tag{26}
\]

This formula is very similar to the S06 equation. The masses computed with this formula are given in column 6 of Table 6 and are seen to agree to within 0.01 \( M_\odot \) with the S06 masses. As was the case for the luminosities, the \( M_{\text{evol}} \) are all larger than the corresponding \( M_{\text{puls}} \) values.

### 6.9 Comparison with evolutionary models

Having computed \( M, L \) and \( T_{\text{eff}} \) values for the non-Blazhko RR Lyr stars from observed Fourier parameters using equations that derive from both pulsation and stellar evolution theory it is of interest to compare the results with the locations of model ZAHBs and evolutionary tracks.

The top two panels of Figure 13 show HR diagrams with \( \log T_{\text{eff}} \) as abscissa and \( \log L \) as ordinate, and two sets of ZAHB loci from Dorman (1992) computed for different masses along the ZAHB. The more luminous horizontal branch assumes \([\text{Fe/H}]= -1.78 \, \text{dex} \), and the less luminous branch \([\text{Fe/H}]= -0.47 \, \text{dex} \). The numbers next to the symbols are the masses for the individual models, which are seen to be higher for the low-[Fe/H] tracks than for the high-[Fe/H] tracks. For both assumed metallicities oxygen enhanced and non-enhanced ZAHBs have been plotted – the effect of increasing the oxygen to iron ratio from \([O/Fe]=0\) to 0.66 for the low-metallicity ZAHBs is to lower the luminosity and reduce the mass at a given temperature. For the high-metallicity tracks an oxygen enhancement from \([O/Fe]=0\) to 0.23 has little effect on the derived \( L \) or \( M \). Also plotted in the low-[Fe/H] case are the evolutionary paths away from the ZAHB for two masses, 0.66 and 0.68 \( M_\odot \). In both panels the non-Blazhko RR Lyr stars with low metallicities are represented by large black squares, the four high-metallicity stars are plotted with open squares, and the \( T_{\text{eff}} \) are the average of the KW01 and S06 values (column 2 of Table 6).

In the top left panel of Fig. 13 the luminosities and masses (labelled in red) of the \textit{Kepler} RR Lyr stars were calculated with eqs. 17 and 22 of J98 and thus are based on pulsation theory. The \( L_{\text{puls}} \) and \( M_{\text{puls}} \) are seen to be systematically smaller than values derived from the ZAHB tracks (for the appropriate metal abundance). The reddest non-Blazhko RR Lyr stars lie close to the fundamental mode red-edge and have the smallest amplitudes. This graph also shows blue and red edges of the instability strip for the fundamental mode (red and blue solid lines) and first-overtone mode (red and blue dashed lines). The edges were calculated with the Warsaw pulsation code (see Section 7) assuming a mass of 0.65 \( M_\odot \). The \textit{Kepler} non-Blazhko stars all lie in the fundamental mode region of the variability strip, and the smallest amplitude RR Lyr stars (the four metal-rich stars, KIC 7030715 and NR Lyr) have locations near the fundamental red edge (FRE) of the instability strip. As expected, all the stars near the FRE have low \( R_{11} \) values.

In the top right panel of Fig. 13 the luminosities were calculated with eq. 12 of S06, and the masses (labelled in red) with eq. 15 of S06; thus they are evolutionary \( L \) and \( M \) values. In this case there is very good agreement with the stellar evolution models, as one expects since they are based on stellar evolution models. Enhancing the oxygen to iron ratio by the plotted amounts makes little difference.

It is unclear which are correct, the \( L_{\text{puls}} \) and \( M_{\text{puls}} \), or the \( L_{\text{evol}} \) and \( M_{\text{evol}} \)? The mass and luminosity discrepancies go in the same direction as seen for Cepheids. Pietrzynski \textit{et al.} (2010) recently derived a dynamical mass \( M_{\text{dynam}} \) for a classical Cepheid in a well-detached, double-lined eclipsing binary in the LMC. The mass they derive is very accurate and favours \( M_{\text{puls}} \). The reason for the discrepancies may be the same, as suggested by Pietrzynski \textit{et al.} – not enough mass loss has been taken...
into account in the evolution models. For further guidance on these questions we turn to pulsation models.

7 HYDRODYNAMIC MODELS

Smolec & Moskalik (2008) recently have developed the Warsaw convective pulsation programs for studying stellar pulsation. The codes, both linear and non-linear, are one dimensional and use a single equation to describe the generation of turbulent energy; this is done according to the model proposed by Kuhfuß (1986, see also Wuchterl & Feuchtinger 1998). Even though a simple diffusion approximation is used to describe the radiation field the models are able to reproduce quite well the dynamics of RR Lyrae pulsations.

The most recent application of these programs (Smolec et al. 2011) has been to construct hydrodynamic models for the purpose of testing Stothers’ (2006) proposed explanation of the Blazhko phenomenon that is observed in about half of all RR Lyr stars. In these models the strength of the turbulent convection was modulated and the resulting models were compared in detail with the Fourier descriptions of the Kunz et al. (2001) with the hydrogen fraction fixed at 0.76. Two masses were considered, $M/M_\odot = 0.65$ (solid lines) and 0.55 (dashed lines). In Figure 15 the metallicity was varied from [Fe/H] = -0.24 dex ($Z = 0.01$), to -1.24 dex ($Z = 0.001$), to -1.93 dex ($Z = 0.0002$) while keeping the other variables constant at $L/L_\odot = 50$, $M/M_\odot = 0.65$, and $X = 0.76$.

The $A_1$ vs. $A_{tot}$ diagrams (top left panels of Figs. 14-15) show very good agreement with the estimates for the non-Blazhko stars (right panel of Fig. 2). The approximately linear trend seems to be independent of luminosity or mass or metallicity variations. However, the fit is not exact and the observed non-linearities (which are small) appear to go in opposite directions to the model predictions.

The $\phi_{21}$ vs $A_1$ graphs (middle left panels of Figs. 14-15) appear to be quite sensitive to $L$, $M$ and metallicity effects (see the lower left panels of Figs. 6-7). A star of given mass and composition will move up and to the right as its luminosity increases. Alternatively, a star of given $L$ and composition will move up and to the right if the mass decreases. And, if $L$, $M$ and $X$ are kept constant metal-rich stars ought to occupy the high $A_1$, high $\phi_{21}$ region of the diagram.

To attempt an application consider the stars AW Dra and V784 Cyg, both of which are seen in Fig. 6 with higher than average $\phi_{21}$ values at a given $A_1$. If the models are indicative, then the two stars with the largest luminosities might be expected to be AW Dra and V784 Cyg. According to column 5 of Table 6 AW Dra is one of the most luminous stars in the sample, but V784 Cyg is not – it has one of the lowest luminosities. On the other hand, V784 Cyg is one of the four metal-rich stars, suspected of having low $L$ and low mass. The high location in Fig. 6 of V784 Cyg (and the other high metallicity stars) might be explained by its low mass, whereas the location of AW Dra would seem to be due primarily to its high luminosity (either as a low-[Fe/H] star near the ZAHB or as a more metal-rich star in an advanced evolutionary state). The highest $L$ (and highest $M$) star in the sample, NR Lyr (see Table 6), also has the lowest metallicity (see Table 4), even lower than that of AW Dra; for it the lower [Fe/H] would seem to cancel out the higher $L$, thus explaining its smaller amplitude and smaller $\phi_{21}$ than AW Dra.

7.2 Predicted Fourier parameters

Three sets of models were computed with the Warsaw code, varying $L$ while holding the other variables constant, then $M$, then [Fe/H]. The resulting Fourier parameters were plotted in diagrams such as those found in Figs. 2 and 6-8. In Figure 14 the luminosity was varied ($L/L_\odot = 40, 50, 60$) while the metallicity was held constant at [Fe/H] = -1.30 dex ($Z = 0.001$) with the hydrogen fraction fixed at $X = 0.76$. Two masses were considered, $M/M_\odot = 0.65$ (solid lines) and 0.55 (dashed lines). In Figure 15 the metallicity was varied from [Fe/H] = -0.24 dex ($Z = 0.01$), to -1.24 dex ($Z = 0.001$), to -1.93 dex ($Z = 0.0002$) while keeping the other variables constant at $L/L_\odot = 50$, $M/M_\odot = 0.65$, and $X = 0.76$.

The $A_1$ vs. $A_{tot}$ diagrams (top left panels of Figs. 14-15) show very good agreement with the estimates for the non-Blazhko stars (right panel of Fig. 2). The approximately linear trend seems to be independent of luminosity or mass or metallicity variations. However, the fit is not exact and the observed non-linearities (which are small) appear to go in opposite directions to the model predictions.

The $\phi_{21}$ vs $A_1$ graphs (middle left panels of Figs. 14-15) appear to be quite sensitive to $L$, $M$ and metallicity effects (see the lower left panels of Figs. 6-7). A star of given mass and composition will move up and to the right as its luminosity increases. Alternatively, a star of given $L$ and composition will move up and to the right if the mass decreases. And, if $L$, $M$ and $X$ are kept constant metal-rich stars ought to occupy the high $A_1$, high $\phi_{21}$ region of the diagram.

To attempt an application consider the stars AW Dra and V784 Cyg, both of which are seen in Fig.6 with higher than average $\phi_{21}$ values at a given $A_1$. If the models are indicative, then the two stars with the largest luminosities might be expected to be AW Dra and V784 Cyg. According to column 5 of Table 6 AW Dra is one of the most luminous stars in the sample, but V784 Cyg is not – it has one of the lowest luminosities. On the other hand, V784 Cyg is one of the four metal-rich stars, suspected of having low $L$ and low mass. The high location in Fig. 6 of V784 Cyg (and the other high metallicity stars) might be explained by its low mass, whereas the location of AW Dra would seem to be due primarily to its high luminosity (either as a low-[Fe/H] star near the ZAHB or as a more metal-rich star in an advanced evolutionary state). The highest $L$ (and highest $M$) star in the sample, NR Lyr (see Table 6), also has the lowest metallicity (see Table 4), even lower than that of AW Dra; for it the lower [Fe/H] would seem to cancel out the higher $L$, thus explaining its smaller amplitude and smaller $\phi_{21}$ than AW Dra.

The top right and middle right diagrams of Figs.14-15 are somewhat noisy. The top-right panel of Fig. 14 suggests that the lowest $L$ stars have the smallest $R_{21}$ values (note that the scale matches that shown in the upper right panel of Fig. 6, but is much reduced from that shown in Fig. 7 for globular cluster stars). Basically, $R_{21}$ and $R_{31}$ have to be small very close to the red edge of the instability strip. At

\[ \log Z = \log Z_\odot + 1.765, \text{ where the solar metallicity was assumed to be } Z_\odot = 0.01716 \] (Sweigart & Catelan 1998).
Figure 14. Graphs showing the effect on the pulsation-model Fourier parameters of varying the \( L \), and varying \( \mathcal{M} \), while keeping the composition constant at \((X, Z) = (0.76, 0.001)\), i.e., \([\text{Fe/H}] = -1.2\) dex. The different colours represent luminosities 40, 50 and 60 \( L_\odot \). The solid lines are for \( \mathcal{M} = 0.65 \mathcal{M}_\odot \) and the dashed lines are for \( \mathcal{M} = 0.55 \mathcal{M}_\odot \). All ordinates are for the \( V \)-passband.

the red edge the amplitude goes to zero, as do \( R_{21} \) and \( R_{31} \). The middle right panels of Fig. 14 suggest that the lowest-\( L \) stars are not expected to have high \( \phi_{21} \) values. This is to be compared with the Kepler data shown in the lower right panel of Fig. 6 that suggest that there is a nearly linear relationship between \( \phi_{21} \) and \( \phi_{31} \) – this is not seen in the models.

The bottom left panels of Figs. 14-15 show that at a given \( \phi_{31} \) a shift to the right can be caused by a luminosity increase or a higher mass. The bottom left panel of Fig. 15 shows the traditional result that at a given \( \phi_{31} \) metallicity decreases as period increases; but it also seems to be suggesting that at a given period \( \phi_{31} \) increases with decreasing metallicity.

Finally, the bottom right panel in Fig. 14 shows that at a given \( A_{\text{tot}} \) and metal abundance stars of higher \( L \) for a given mass, or of lower mass for a given \( L \), have longer periods; this is as expected, and similar results are seen in fig. 6 of Dall’Ora et al. (2003). The bottom right panel of Fig. 15 shows that for a given \( L \) and \( \mathcal{M} \) varying the metallicity has a relatively small effect on the period. Taken together these two panels suggest that the main factor shifting the periods to longer values in the Bailey diagram (or any of its surrogate diagrams - see Fig.9) seems to be higher luminosities (which
8 SUMMARY

The main results of this paper are as follows:

(1) Fourier decomposition has been performed on the 19 least modulated ab-type RR Lyr stars observed with the Kepler space telescope at long cadence (every 30 min) and short cadence (every 1 minute) during the first 417 days of its operation (Q0-Q5);

(2) While none of the RRab stars shows the recently discovered ‘period-doubling’ effect seen in Blazhko variables, the star KIC 7021124 was discovered to pulsate in the fundamental and second overtone modes with a period ratio $P_2/P_0 = 0.59305$ and to have properties similar to those of V350 Lyr;

(3) Period change rates and improved periods have been derived from O–C diagrams for several of the stars that have historical data. For AW Dra, data from the last 12 years suggest that its period is increasing at the rate $dP/dt=3.79$ d/Myr, while data spanning the last 100 years suggest a

Figure 15. Graphs showing the effect on the Fourier parameters derived with the Warsaw convective pulsation code of varying [Fe/H] from $-0.24$ dex ($Z = 0.01$, green), to $-1.24$ dex ($Z = 0.001$, blue), to $-1.93$ dex ($Z = 0.0002$, red), while keeping the mass, luminosity and hydrogen content constant at $0.65 M_\odot$, $50 L_\odot$ and $X=0.76$. All ordinates are for the V-passband.

can be caused either by higher ZAHB mass or by advanced HB evolution) and not lower metallicities.

Obviously more work needs to be done to optimize the estimation of the physical variables in these observational planes, but the potential seems high.
slower rate, $dP/dt = 0.32\ d/\text{Myr}$. FN Lyr appears to have a very slowly increasing period, and the periods of NR Lyr and NQ Lyr appear to be constant.

(4) Because the differences between the $K_p$ and $V$ Fourier parameters for three stars (AW Dra, FN Lyr an NR Lyr) are found to be small and systematically different we were able to use extant $V$-band correlations (with small modifications) to derive underlying physical characteristics for the Kepler stars. This procedure seems to be validated through comparisons of the Kepler stars with other galactic and LMC field RR Lyr stars and with RR Lyr stars in galactic and LMC globular clusters.

(5) Preliminary metal abundances have been derived for all the non-Blazhko RR Lyr stars. Thirteen of the stars appear to be similar to those found in intermediate metallicity globular clusters (i.e., $[\text{Fe}/\text{H}]_{\odot} \approx -1.6$ dex); the most metal deficient star appears to be NR Lyr with $[\text{Fe}/\text{H}]_{\odot} = -2.3$ dex, and the four lowest amplitude stars (KIC 6100702, KIC 9947026, V782 Cyg and V784 Cyg) appear to be metal-rich with $[\text{Fe}/\text{H}]_{\odot}$ between $-0.55$ and $+0.07$ dex.

(6) In general the luminosities of the metal-rich RR Lyr stars are found to be lower than those of the metal-poor stars; however, the luminosities derived from stellar evolution models are systematically higher than those derived from stellar pulsation models. It is not clear which are correct.

(7) The three stars with the longest periods, AW Dra, NR Lyr and KIC 7030715 (all with periods ~0.68 d) also are the reddest and coolest stars. We suspect that AW Dra may be in an evolved state. In general, the stars with the lowest amplitudes are found to be located nearest the red edge of the instability strip.

(8) The mass range for the entire sample (approximate) is from 0.50 to 0.65 $M_\odot$ if based on pulsation theory, or from 0.56 to 0.72 if based on HB evolution models.

(9) Finally, the Fourier parameters of the stars have been compared with values newly computed with the Warsaw convective pulsation codes. We find that in the $P-A$ diagram varying the metallicity for a given $L$ and $M$ has a relatively small effect on the period-shift at a given amplitude (in fact, the small shift is toward shorter periods for more metal poor stars), and that the main factors causing the period shifts must be $L$ and $M$.

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stars. These may be found in the online version of this article, along with the data files.

APPENDIX A: KIC 7021124

During the course of our analyses the star KIC 7021124 was discovered to be a doubly-periodic RR Lyr star, similar to V350 Lyr (see B10). From an analysis of the Q1 data (1626 long cadence observations made over a 30-day period) its period was found to be \( P_0 = 0.6224925(7) \) d, corresponding to the frequency \( f = 1.606445(2) \) d\(^{-1}\). A light curve phased with this period is plotted in the upper panel of Figure A1. The bottom panel shows the Fourier transform after prewhitening with \( f_0 \) and its harmonics. In addition to residual power seen at the location of the removed primary frequency and its harmonics, we also see a family of at least six additional peaks (labelled with arrows). Adopting the feature at 2.7088 d\(^{-1}\) as the independent frequency \( f_2 \), the other frequencies correspond to \( f_2 \pm k f_0 \), where \( k \) is an integer. With this assumption the period ratio is \( P_2/P_0 = 0.59305 \), which is almost identical to that derived by B10 for V350 Lyr (\( P_2/P_0 = 0.592 \)). The two stars also are similar in their fundamental periods, in their Fourier characteristics (see Figs. 6,9,10), and in their masses and luminosities (see Table 5).

Figure A2 shows KIC 7021124 (and V350 Lyr) plotted in the \( P_2/P_0 \) vs. \( P_0 \) ‘Petersen’ diagram. The curves were computed with the Warsaw pulsation hydrocode including turbulent convection. The periods and period ratios do not depend strongly on the convective parameters entering the model (which in this case were set C, adopted in Baranowski et al. 2009). Other model parameters are: \( Z = 0.0001 \) (or [Fe/H] = -2.2); \( X = 0.76 \) (latest opacities, with new solar mixture); three masses \( 0.55 \), \( 0.65 \), \( 0.75 \) (open circles), \( 0.65 \), \( 0.75 \) (filled circles), \( 0.75 \) (crosses); and four luminosities: \( 40L_\odot \) (red), \( 50L_\odot \) (green), \( 60L_\odot \) (blue), \( 70L_\odot \) (purple). The best agreement for KIC 7021124 (and V350 Lyr) is obtained for a high luminosity and a high mass: \( L/L_\odot = 70 \) and \( M/M_\odot = 0.75 \). In Table 6 both stars are among the highest \( L \) and \( M \) stars in our sample. The high values inferred from the Petersen diagram are more in accord with the evolutionary values than the pulsation values.

APPENDIX B: ASAS V, I-PHOTOMETRY

Very little colour information is available for the non-Blazhko RR Lyr stars. Fortunately the ASAS-North survey (see Pigulski et al. 2009) includes calibrated \( V \) and \( I \) photometry for nine of the brightest Kepler non-Blazhko stars (see §4.5). Figure B1 shows the cyclic magnitude and colour behaviour of the nine stars in the (\( V-I, V \))-diagram. In every case one observes the well known trend of bluest colour when the star is brightest (i.e., when phase equals 0.0). The largest colour range is for NQ Lyr and V894 Cyg (the faintest ASAS star in the sample), and the smallest colour range is for the brightest star in the sample, NR Lyr. The apparent red mean colour for NR Lyr probably is due to its relatively large reddening (see column 5 of Table 5). The ‘wiggles’ are artificial, a result of noise and imperfect fit.
Figure B1. Looping behaviour in the H-R diagram for the nine Kepler non-Blazhko RR Lyr stars with $V, I$ photometric data in the ASAS-North catalog. The stars are ordered according to increasing period, and the points along the mean light curves occur at every $1/100$th of the phase. Total risetimes, $V$ amplitudes, $\langle V \rangle$ magnitudes and $\langle V-I \rangle$ colours for each star are given in the last four columns of Table 2.