Detonation of a hydrogen-oxygen gas mixture in a plane-radial combustor with exhaustion toward the periphery in the regime of oxygen ejection

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Abstract. The results of experimental study concerning a continuous spin detonation of hydrogen-oxygen mixture in a flow-through plane-radial combustor with a diameter of 10 cm and with exhaustion toward the periphery in the regime of oxygen ejection are presented. In the experiment, the regimes of continuous spin detonation with one and two transverse detonation waves rotating with velocity \( D = 1.3 \pm 1.43 \) and \( 1.45 \pm 1.54 \) km/s, measured relative to the inner cylindrical surface of the combustor, respectively, and two near-sonic waves rotating with velocities of about 0.7 km/s have been observed. The waves of pulse combustion with frequencies of 4.4 – 3.6 kHz have been found for the first time. The structure of waves and flow in their vicinity has been studied.

1. Introduction

Previously, it was shown that the detonation combustion of acetylene-oxygen and hydrogen-oxygen mixtures in the regime of oxygen ejection [1, 2] can be carried out in a flow-through annular cylindrical combustor (C) with a diameter of 100 mm and channel expansion. Similar regimes were obtained in the C with diameter of 306 mm for hydrogen and synthesis gas-air mixtures in the regime of air ejection [3-5]. In these papers, the authors obtained the regimes of continuous spin detonation (CSD) and pulse detonation (PD), determined the conditions and limits of existence of these detonation regimes with respect to the geometry of C and the system of injection of mixture components, considered the structure of detonation waves and flow in their vicinity. In a plane-radial combustor (PRC) the detonation regimes are essentially affected by centrifugal forces acting on the products and singularities of product exhaustion in an expanding channel. The term “a plane-radial combustor” is chosen because the distance between flat walls \( \Delta \) is much less than its inner (\( d_{c1} \)) and outer diameters (\( d_{c2} \)): \( \Delta \ll d_{c1} < d_{c2} \).

The goal of this work is to continue and generalize the investigation results on detonation combustion of fuel in the regime of oxidizer ejection in the PRC of flow-through type with an inner diameter of 100 mm and exhaustion toward the periphery. Such an operation regime of the combustor is of practical importance as it allows using the combustor under steady-state conditions and to provide more intense fuel combustion. Gaseous oxygen was used as an oxidizer, and hydrogen was used as a fuel.

2. Experimental section

The investigation was carried out in the PRC 1 with exhaustion toward the periphery with the diameter of cylindrical surface \( d_{c1} = 100 \) mm and outer diameter of flat walls \( d_{c2} = 300 \) mm, and the distance between them was \( \Delta = 12 \) or 7 mm. In a series of experiments, the distance between the walls was uniformly decreased to the combustor exit from \( \Delta = 12 \) mm to \( \Delta_1 = 5.3 \) mm (Fig. 1).

The area of circular cross-section of the PRC channel at \( \Delta = 12 \) mm = const increases along the flow from the area of cylindrical surface \( S_3 = \pi d_{c1} \Delta = 37.7 \) cm\(^2\) proportionally to the increasing of the radius. The cross-section of a constricting channel also increases, and at the PRC exit it is \( S_3 = 50 \) cm\(^2\). Thus, in both cases, the PRC has a channel with the area expanding to the exit. Hydrogen was injected
from a receiver with a volume V_{rf} = 4.2 litres through injectors 2, uniformly distributed over the cylindrical combustor wall at a distance of 0.5 mm from the slot for oxygen injection and directed downward the flow at an angle of 45° to the axis and generator of cylindrical surface of the PRC. Oxygen was injected into the PRC through the annular slot with width \( \delta = 2 \text{ mm} \) (the cross-section is \( S_\delta = \pi d_1 \cdot \delta = 6.28 \text{ cm}^2 \)). It was exhausted from the receiver \( r = 40 \) litres with the opposite end of the receiver contacting the ambient air (it is not shown in Fig. 1) by the whole area of its cross-section, i.e., 415 cm². A necessary amount of oxygen was injected into the receiver (the equivalence ratio is \( \phi \leq 1 \)) during the whole experiment.

**Fig. 1.** Schematic of the PRC with exhaustion toward the periphery in the regime of ejection.

When hydrogen was injected into the PRC, a rarefaction occurred (as it does in jet-pump operation) and the first oxygen portions were exhausted. The mixture \( \text{H}_2 - \text{O}_2 \) obtained in the PRC was initiated by blasting the wire located at a distance of 20 mm from the cylindrical surface. A CSD developed approximately in 10 ms. Here, the transverse detonation waves (TDWs), namely, their rarefaction waves behaved as pistons exhausting oxygen by increasing its flow rate. The detonation products were ejected into the atmosphere. The process was photographed by a Photon FASTCAM SA5 high-speed camera with a frequency of 525000 frames per second through two Plexiglas windows 5 that are 9.5 cm long and 1.0 cm wide and located radially in a flat wall of the combustor (see Fig. 1). For illumination of detonation of hydrogen-oxygen mixture, acetylene jets were injected opposite the windows from the side of cylindrical surface. A computer processing of the frames according to a specially developed C++ program allowed using luminescent objects moving in the tangential direction to obtain the flow pattern in the wave-fitted system. The flow pattern along the radius yielded \( x, t \) – diagram. The objects moving in the tangential direction were detected with some distortions because they moved over circumferences of different diameters, whereas the photographic records were aligned linearly. Number of waves rotating in the PRC at the moment must be determined and cut from the photographic record. Therefore, to visualize a real flow pattern, it is necessary to roll the fragment into a ring. In some experiments, the frames were taken across the long side of the window and \( x, t \) – diagrams of luminescent objects moving in the tangential direction were detected. The photographic records and the \( x,t \)-diagrams were used for finding the time \( \Delta t \) when the detonation waves are moving across the window, which allowed unique determination of their frequency \( f = 1/\Delta t \) with accuracy up to ±1%. In the case of CSD, it was also possible to calculate the velocity of motion of TDWs with respect to the cylindrical surface diameter \( d_{c1} \): \( D = \pi d_{c1} / (n \cdot \Delta t) \), where \( n \) is the number of TDWs along the combustor circumference. By the inclination of the trajectories of luminescent objects, it was possible to determine their velocity \( v_l = L_w / \Delta t_w \) using \( x, t \)-diagrams, where \( L_w \) is the length or width of the window and \( \Delta t_w \) is the time of presence of the luminescent object in the window. Due to insufficiency of time resolution and fuzzy trajectories, the velocity was determined with accuracy up to ±10%.

The computer system detected the signals of gas pressure probes: in the oxygen (\( p_{ox} \)) and hydrogen (\( p_{fl} \)) receivers, at the entrance of the slot for oxygen injection (\( p_o \)) and in the fuel manifold (\( p_{mf} \)), mean static pressures at a distance of 1.0, 5 and 10 cm from the PRC cylindrical surface (\( p_{c1}, p_{c2}, p_{c3} \), respectively), and total pressures at a distance of 5 and 10 cm (\( p_{c20} \) and \( p_{c30} \)). The pressure measurements were performed by certified pressure probes with the accuracy class of 0.5% produced by the Trafag company (Switzerland). The initial hydrogen pressure in the receiver was set as \( p_{f0} = 12 \pm 1 \cdot 10^5 \text{ Pa} \). Here the initial flow rate of \( \text{H}_2 \) was \( G_{f0} = 19 \pm 2 \text{ g/s} \), and the current flow rates \( G_f \) decreased by a factor of 10 during the process (0.4 s). As a results, specific hydrogen flow rates through the PRC cross-section at
diameter $d_c$ varied within the range $g_f = G_f/S_a = 0.5 \div 7.4$ kg/(s$\cdot$m$^2$). The oxygen flow rate in the PRC was estimated depending on experimental conditions.

3. Experimental results

The regimes of detonation combustion of hydrogen in the TDWs are obtained by adjusting a slot gap $\delta = 2$ mm in the PRC. In the performed experiments, the coefficient of oxygen flow expansion was 6 or 3.5 at the PRC entrance $K_s = \Delta / \delta$.

3.1. The PRC with $\Delta = 12$ mm, $\Delta^1 = 5.3$ mm

The fragment of photographic records of the process developing from the instant of initiation (bright flash of mixture in the combustor) for $\Delta = 12$ mm and $\Delta^1 = 5.3$ mm is illustrated in Fig. 2a.

![Fig. 2. The instant of process initiation and transformation to CSD; $\Delta = 12$ mm, $\Delta^1 = 5.3$ mm; $g_f = 4.17$ kg/(s$\cdot$m$^2$), $n = 1$, $D = 1.39$ km/s (f = 4.44 kHz).](image)

It is seen that the time duration from the instant of initiation up to the TDW development is 3 ms. As the pictures were taken through two windows, two bands were recorded for each regime; these bands are separated by a dark bar, which is the nearest distance between the windows along the combustor diameter: $d_{c1} + 2 \cdot 5 = 110$ mm, where 5 mm is the distance from the PRC entrance to the edge of the window. In reconstructing the flow in the wave-fitted system by the above-mentioned special computer code, the wave rotation direction was not determined; only those luminescent points that consecutively pass opposite the window were detected. Therefore, in both windows, it seems that the waves in the photographic records are aligned in the same direction, whereas in fact the waves in the upper and lower windows have opposite orientations despite an identical tangential direction of wave rotation. Therefore, each point, e.g., in the upper window should have its mirror reflection with respect to the vertical axis.

Figure 2b shows an enlarged fragment of one detonation wave (see Fig. 2a, to the left) recorded in the lower window and reduced to the length scale with respect to the radius and cylindrical surface of the PRC. As a linear representation of the photographic records is used, a real wave structure and pattern of the flow can be obtained by rolling this fragment into a ring with a ratio of inner and outer diameters 110/300.

The detonation regime with one TDW was observed for about 100 ms within the range of hydrogen flow rate $G_f = 15.7 \rightarrow 9.75$ g/s ($g_f = 4.17 \rightarrow 2.6$ kg/(s$\cdot$m$^2$)). The arrow shows the decrease of hydrogen flow rate during the experiment. The detonation velocity with respect to the PRC end wall varied in the interval $D = 1.3 \div 1.43$ km/s ($f = 4.17 \div 4.56$ kHz). The structure of detonation and shock waves and also the flow in the vicinity of these waves are seen most clearly in the enlarged fragment (see Fig. 2b). The front of detonation wave $BC$ is oblique and its projection onto the combustor radius (with allowance for 5 mm of invisible part) is $h \approx 45$ mm, and with respect to the circumference $l = \pi d_{c1}$, it is approximately 1/7 part. The tail $CD$ (a shock wave in the products) adjacent to the front $BC$ is also strongly inclined backwards. The longitudinal compression wave $MN$ propagating upwards the flow from outside to the combustor end and decelerating the flow of products is visible. Some fragments illustrate a greater number of transverse waves and even opposite direction of the flow (e.g., see Fig. 2a, the second left TDW in the upper band) as the product flow occurs in the region of convergence of characteristics. The process of CSD is irregular enough both in the structure of the TDWs and in the flow in the vicinity of these waves.

In the region of hydrogen flow rates $G_f = 9.75 \rightarrow 5$ g/s ($g_f = 2.6 \rightarrow 1.33$ kg/(s$\cdot$m$^2$)), the TDWs were degenerated into acoustic ones with radial displacement of compression wave $MN$ and periodic ($f = 4.07 \rightarrow 3.8$ kHz) burning of the mixture after reflection of longitudinal compression wave from the PRC end (e.g., see Fig. 5). At $G_f = 5 \rightarrow 1.6$ g/s ($g_f = 1.33 \rightarrow 0.42$ kg/(s$\cdot$m$^2$)), weakening acoustic waves were observed during the combustion.

Figure 3 shows the oscillograms of pressure in the system of $\text{H}_2$ and $\text{O}_2$ injection (Fig. 3, a) and static pressure in front of and behind the annular slot $\beta$ $p_a$ and $p_c$ (Fig. 3, b) obtained in the experiment. The fragments of photographic records of the experiment are illustrated in Fig. 2.
Note that the mean value of static pressure of oxygen at the slot entrance, in the combustor and in the receiver have almost identical values: 

\[ p_o \approx p_{c1} \approx p_{c30} \approx 1 \cdot 10^5 \text{ Pa}. \]

Only the instant of initiation of the

3.2. The PRC with \( \Delta = 12 \text{ mm} \)

As the constant gap \( \Delta = 12 \text{ mm} \) was adjusted, the regime of CSD with one TDW (\( n = 1, D = 1.54 \text{ km/s} \) and \( f = 4.9 \text{ kHz} \)) existed for 2 ms immediately after initiation at \( G_t \approx 19.5 \text{ g/s} \) (\( g_f \approx 5.17 \text{ kg/(s} \cdot \text{m}^2) \)). Then with decreasing \( G_f = 19.5 \rightarrow 11.9 \text{ g/s} \) (\( g_f = 5.17 \rightarrow 3.16 \text{ kg/(s} \cdot \text{m}^2) \)) the regime with two weak nearsonic waves (\( n = 2, D = 0.75 \rightarrow 0.69 \text{ km/s} \) and \( f = 4.8 \rightarrow 4.4 \text{ kHz} \)) but sufficiently well-defined luminescent fronts (Fig. 4 a, b) was set. Sometimes, the one-wave regime of CSD was restored and lasted for several milliseconds. At \( G_f = 11.9 \rightarrow 2.74 \text{ g/s} \) (\( g_f = 3.16 \rightarrow 0.73 \text{ kg/(s} \cdot \text{m}^2) \)) the luminescent fronts moving in the tangential direction disappeared but one could observe the amplification of the compression waves MN moving radially and burning the mixture periodically with the frequency \( f = 4.4 \rightarrow 3.3 \text{ kHz} \) after they reflected from the combustor end (e.g., see Fig. 5). Further reduction of flow rate of \( G_f < 2.74 \text{ g/s} \) (\( g_f < 0.73 \text{ kg/(s} \cdot \text{m}^2) \)) in the PRC resulted in a conventional combustion (Fig. 4 c).

3.3. The PRC with \( \Delta = 7 \text{ mm} \)

As the PRC gap decreased to \( \Delta = 7 \text{ mm} \), one could observe a two-wave regime of the CSD (\( n = 2, D = 1.45 \rightarrow 1.53 \text{ km/s}, f = 9.26 \rightarrow 9.77 \text{ kHz} \)) in a narrow range of hydrogen flow rates \( G_t = 16.3 \rightarrow 15.9 \text{ g/s} \) (\( g_f = 5.7 \rightarrow 5.4 \text{ kg/(s} \cdot \text{m}^2) \)). In the region of hydrogen flow rates \( G_t = 15.9 \rightarrow 1.7 \text{ g/s} \) (\( g_f = 7.3 \rightarrow 0.77 \text{ kg/(s} \cdot \text{m}^2) \)), the TDWs were degenerated into acoustic ones, the compression waves MN began to displace radially and burned the mixture periodically with frequency \( f = 4.5 \rightarrow 3.4 \text{ kHz} \) after they reflected from the cylindrical surface of the combustor (Fig. 5).

Fig. 3. Oscillograms of pressure in the injection system of hydrogen (\( p_{r,f}, p_{m,f} \)) and oxygen (\( p_{o,ex} \)) at the entrance (a) to the slot (\( p_o \)) for oxygen and to the PRC (\( p_{c1} \) and \( p_{c30} \)) b).

Fig. 4. Process in the PRC with \( \Delta = 12 \text{ mm} \); a) nearsonic waves moving in the tangential direction, \( G_t = 13.4 \text{ g/s} \) (\( g_f = 3.55 \text{ kg/(s} \cdot \text{m}^2) \)), \( n = 2, D = 0.7 \text{ km/s} \) (\( f = 4.4 \text{ kHz} \)); b) (x-t) - diagram of the process obtained by cutting pixels across the window in the PRC; c) combustion, \( G_t = 7 \text{ g/s} \) (\( g_f = 1.86 \text{ kg/(s} \cdot \text{m}^2) \)).

Fig. 5. Process in the PRC with \( \Delta = 7 \text{ mm} \); \( g_t = 2.74 \text{ kg/(s} \cdot \text{m}^2) \), \( f = 3.97 \text{ kHz} \).
In this case, the compression wave is also indistinct as it is at the CSD (see Fig. 2b). At $G_t < 1.7$ g/s ($g_t < 0.77$ kg/(s·m$^2$)), the longitudinal compression wave gradually weakened and a rather homogeneous regime of conventional combustion was set in the channel (e.g., see Fig. 4c).

4. Analysis of results
The most important result of the work is the experimental validation of the possibility for the CSD to exist in ejectors, namely, in the PRC. The known types of ejectors [6, 7] operate due to a pressure gradient formed between the gas jet exhausted from the nozzle and ambient gas. It follows from Fig. 3, b, that in the case of one-dimensional flow for the classical ejector the presence of ejection is out of the question since static pressures both at the combustor entrance and exit are almost identical and have the ambient pressure level. Even if one assumes that the velocity of oxygen injection into the PRC $v_{ox} = (2\Delta p_0/\rho_0)^{0.5} = 1.24$ m/s can be ensured with allowance for the 1 % measurement error of pressure probes and actual oxygen pressure drop at the slot $\Delta p_o = p_0 - p_{cl} = 0.01 \cdot 10^5$ Pa, this velocity will be apparently insufficient to obtain detonation since for characteristic time of wave rotation $\sim 250$ $\mu$s, only 0.3 mm of oxygen will be injected into the PRC. Ignoring the path needed for mixing oxygen with hydrogen, this value is less than a critical size needed to obtain detonation (cell size) which is $a = 1.6$ mm [8] for hydrogen–oxygen mixture under standard conditions. Due to the fact that the CSD is obtained in the experiments described above, there is a mechanism of inhaling the outside gas into the PRC. It is found in the experiment with annular cylindrical combustors having channel expansion [1, 2] and is verified in the PRC in the present work. The radial gas velocity in the combustor $v_i = dr/dt$ determined in accordance with the photographic records of luminescent gas particles moving along the window was $v_{r,m} \approx 350$ m/s (see Fig. 2a) in front of the TDW front. According to computations on physical and mathematical models of the CSD [9], this velocity is ensured by a pressure drop in the rarefaction wave behind the detonation front BC. This pressure is 2 or 3 times less than the mean pressure in the combustor. Since the mean pressure in the combustor is $p_{cl} = 1 \cdot 10^5$ Pa, the pressure in front of BC front is not more than 0.5 $\cdot 10^5$ Pa. In this connection, in the regime of nonstationary self-oscillatory oxygen ejection, the detonation wave behaves as a pump, and the rarefaction wave adjacent to the detonation front behaves as an inhaling piston.

As the CSD occurs, the detonation front BC is near the place of reflection of compression wave MN from the cylindrical surface. The velocity of products near the PRC exit ($v_{r,p}$) is essentially subsonic: right $\sigma$ compression waves penetrating into the PRC.

Interestingly, that as the CSD and the pulse regime of combustion take place, the wave frequencies were within the narrow limits $f = 4.9 + 3.4$ kHz. Probably, the reason is a sufficiently stable velocity of the compression wave MN in products. When reflecting from the PRC end, it caused the appearance of the TDW front in one case and the mixture combustion in another. Where do these compression waves come from? First, this is due to decelerating of supersonic flow behind the TDW front, and second, due to flashes of the mixture burned incompletely outside the combustor. The second reason is real owing to the fact that hydrogen exhaustion occurs as the TDW front moves opposite the slot of oxygen injection. Thus, oxygen injection into the PRC is stopped and hydrogen injection continues. As a result, free hydrogen can be periodically exhausted outside the combustor and burn completely in the air generating the compression waves penetrating into the PRC.

The composition of mixture forming in front of the front is unknown. It can be only estimated by using the photographic records of the process. It is seen that oxygen is injected into the PRC in the region $\sigma = (1/2 - 2/3)S_0$. Taking oxygen exhaustion through the slot to be critical, we obtain the flow rates: $G_{ox} = \mu \rho_{ox} v_{ox} \sigma = 64.2 + 85.66$ g/s, where $\mu = 0.8$ is a slot flow coefficient [10]. $\rho_{ox} = 0.64 \rho_0$ [6], and $\rho_0 = 1.35$ kg/m$^3$ at pressure $p_0 = 1.013 \cdot 10^5$ Pa and $T_0 = 293$ K. $v_{ox} = (2/(\gamma(\gamma + 1))) (\rho_0/\rho_{ox})^{0.5} = 290$ m/s is a critical oxygen velocity through the slot, $\gamma = 1.4$ is a relation of specific heat capacity values at constant
pressure and volume. In the experiment, the equivalence ratio was in the interval \( \phi = 1.96 \div 1.47 \) at \( \alpha = (1/2 - 2/3)S_{\delta} \), respectively. The photographic record of the experiment is shown in Fig. 2 and the oscillogram is presented in Fig. 3. The injection of oxygen from the receiver at the entrance to the slot (see Fig. 1) was \( G_{ox} = 107 \text{ g/s} \) that was sufficient for all its possible flow rates in the combustor.

Let us compare CSD in combustors of cylindrical geometry with channel expansion from \( \Delta = 5 \text{ mm} \) at angle 8.5° \([1,2]\) and in the PRC for \( \Delta = 12 \text{ mm} = \text{const.} \) At similar degree of the PRC channel expansion (3.37 and 3.0, respectively), the fuel-lean limit for detonation with respect to the specific hydrogen flow rate is \( g_{f,\text{min}} = G_f/S_{\Delta} = 2 \) and 5.1 kg/(s·m²), respectively. The fuel-rich limit for CSD in a cylindrical combustor was in the region \( g_{f,\text{max}} = 4 \text{ kg/(s·m²)} \), but in the PRC it was not determined because of the setup capabilities. The region of the CSD existence increased, i.e., \( g_f = 2.6 \div 4.17 \text{ kg/(s·m²)} \) as the channel at the PRC exit was constricted to \( \Delta^{'1} = 5.3 \text{ mm} \) (the degree of expansion is 1.33). However, the fuel-rich limit was not determined for the same reason. The detonation velocities in the PRC turned out to be less than those in the cylindrical combustor. This seems to be caused by the centrifugal forces acting on the detonation products and reducing the pressure behind the front of the detonation wave BC. In the PRC with \( \Delta = 7 \text{ mm} = \text{const} \) (the degree of expansion is 3.0), the fuel-lean limit of the CSD existence increased up to \( g_f = 7.23 \text{ kg/(s·m²)} \). Moreover, the two-wave regime of the CSD was successfully obtained in ±1% range with respect to \( g_f \), whereas in the cylindrical combustor the regime of pulse detonation was observed for \( g_f > 4.5 \text{ kg/(s·m²)} \). The regime of pulse detonation was observed in all cases for \( g_f \) less than the fuel-lean limit. It is possible to obtain the CSD with more hydrogen flow rates by modernizing experimental setup with the PRC, i.e., by increasing the flow rates of fuel and geometry.

5. Conclusions
1. In the combustor of plane radial geometry of through-flow type with exhaustion toward the periphery, the regimes of hydrogen combustion are first performed in the transverse (spin) detonation waves and periodic pulse combustion waves in the regime of self-oscillatory ejection of oxidizer.
2. The structure of spin and pulse combustion waves and flow in their vicinity is considered.
3. In comparison with the annular cylindrical combustor, in the plane radial one it was possible to obtain the continuous spin detonation for higher values of specific flow rate of hydrogen. The constriction of exit cross-section of the combustor was sufficient to decrease the lean-fuel limits with respect to this parameter.
4. The stability of frequency of detonation wave rotation and combustion pulsations in the range from 3.6 kHz to 4.4 kHz is found.

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6. References
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