Chaotic Search Optimal Design and Modeling of Permanent Magnet Synchronous Linear Motor

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Abstract: This paper presents an electromagnetic finite element model of permanent magnet synchronous linear motor and distortion rate of the air gap flux density waveform is analyzed in detail. By designing the sample space of the parameters, non-linear regression modeling of the orthogonal experimental design is introduced. We put forward possible air gap flux density waveform sine electromagnetic scheme. Parameters optimization of the permanent magnet synchronous linear motor is also introduced, which is based on chaotic search and adaptation function. Simulation results prove that the pole shifting does not affect the motor back electromotive symmetry based on the structural parameters, which provides a novel way for the optimum design of permanent magnet synchronous linear motor and other engineering.

1. Introduction

Linear motor has a wide range of application in the industry and plays an indispensable role in the process of pursuing automatic production, which has two major advantages of high efficiency and smooth working at the same time. Linear motor usually takes the form of permanent magnet synchronous, and performs best feature in the numerically controlled lathe of all the applications. Hellinger\(^{[1]}\) introduces finite element analysis of the permanent magnet synchronous linear motor and application of the vector control to realize speed control. With energy crisis and product competition, it has become a very significant project to reduce energy losses, cut down production cost and improve production performance. Structural optimization design can lower production cost and energy consumption, so optimization design concept gradually gets more attention. Optimization design is within a scope of restrictions to seek for the maximum or minimum of the objective function. However, restricted conditions are determined by actual demand.

Researchers have proposed several solutions, but few optimization structure scheme\(^{[2]}\) is raised. In order to meet the need of engineering, they used a variety of technologies, and the high-order heuristic search method can solve the complicated optimization problem among many optimization design theories. This is a heuristic search method, simulating natural phenomena in the natural process of trying to find the optimal solution of the stochastic simulation approach, avoiding falling into the solution of the local optimization. This kind of searching method makes use of only a small amount of
mathematics requirements, and does not need a very explicit mathematical model. Swarm intelligence\cite{3} means that most of the phenomenon simulation theories are the behavior of modeling all kinds of creatures in the nature, and here introduces a kind of chaotic search method. Chaos theory has diverse studies in the different fields, some have presented that chaos theory has already conducted local fine search again based on global search, and the search results have a good stability\cite{4}. The paper combines chaos theory, the improvement of the fitness function\cite{5}, and control parameters, to enhance the searching ability and also avoid falling into local optimal solution\cite{6-7}.

The paper firstly takes in-depth analysis and calculation to the air gap flux density waveform of the permanent magnet synchronous linear motor, and gets feasible electromagnetism scheme of the air gap density waveform sine degree. Finally make use of the chaotic search and fitness function to obtain optimization based on the nonlinear model. Utilize the method that pole group and migrate to diminish electrical machine tooth pitch to solve the back-EMF of the motor based on the structural parameters after pole shifting, by the contrast analysis, finding that this method will not affect the symmetry of the electrical machine’s Back-EMF, and simulation results prove the effectiveness of the approach.

2. Finite Element Analysis of Permanent Synchronous Linear Motor

The mathematics model of permanent synchronous linear motor is a high-order, nonlinear, strong coupling multivariable systems. Analyzing excitation function of permanent magnet by equivalent magnetization method, we can use Fourier series to express the equivalent magnetization space distribution function \( M(x)\)\cite{8}:

\[
M(x) = \sum_{n=1}^{\infty} \frac{AB_r}{\mu_0 \tau_m} \sin \left( \frac{m_n \tau}{2} \right) \sin \left( \frac{m_n \tau}{2} \right) \sin m_n x
\]

Including,

\[
m_n = \frac{(2n - 1)\pi}{\tau}
\]

In the formulas, \( B_r \) is the residual magnetization of permanent magnet; \( \mu_0 \) is air permeability; \( \tau_m \) is the width of permanent magnet; \( \tau \) is the pole pitch of permanent magnet. According to Maxwell equations, establish Poisson equations of the air gap region and the permanent magnet area\cite{9}:

\[
\begin{align*}
\frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial y^2} &= 0 \\
\frac{\partial^2 A_2}{\partial x^2} + \frac{\partial^2 A_2}{\partial y^2} &= -\mu_0 \frac{\partial (x)}{\partial x}
\end{align*}
\]

In the formulas, \( A_1 \), \( A_2 \) respectively are the vector magnetic potential of air gap region and the permanent magnet area.

Air gap region and the permanent magnet area are limited to below boundary conditions:

\[
\begin{cases}
B_y \bigg|_{x = \frac{\tau}{2}} = B_y \bigg|_{x = \frac{\tau}{2}}, \\
H_x \bigg|_{x = \frac{\tau}{2}} = H_x \bigg|_{x = \frac{\tau}{2}} \\
H_x \bigg|_{x = \frac{\tau}{2} + h} = 0, \\
H_y \bigg|_{r = 0} = 0
\end{cases}
\]

Taking the type (4) into type (3), we can get the air-gap flux density:

\[
\begin{align*}
B_x &= \sum_{n=1}^{\infty} m_n (A_{n1} \sin m_n y + B_{n1} \sin m_n y) \cos m_n x \\
B_y &= \sum_{n=1}^{\infty} m_n (A_{n1} \sin m_n y + B_{n1} \sin m_n y) \sin m_n x
\end{align*}
\]

Including,
\begin{align*}
A_{n1} &= 0 \\
B_{n1} &= T_n \frac{\sinh(m_n h_m)}{\sinh[m_n(h_e + \frac{g}{2})]}
\end{align*}

In the formulas, $h_m$ is permanent magnet thickness; $g$ is gas length;

\[ T_n = \frac{4B_p}{\tau m_n^2} \sin \frac{m_n \tau}{2} \sin \frac{m_n \tau_e}{2}. \]

Its main parameters are shown in the table 1:

| Parameter                          | Numerical value | Parameter                          | Numerical value |
|------------------------------------|-----------------|------------------------------------|-----------------|
| Voltage rating/v                   | 300             | Groove width                       | 4.7             |
| The length of primary/mm           | 80              | Notch pitch                        | 2               |
| The external diameter of primary/mm| 75              | Slot pitch                         | 6.7             |
| The external diameter of secondary/mm| 25           | Number of poles                    | 8               |
| air-gap width/mm                   | 0.55            | Numbers of stator slot             | 12              |

Considering the end effect of linear motors, take the air around the ends of the motor and secondary as a part of the model to research. Take advantage of axial symmetry of the permanent magnet synchronous linear motor, and corresponding boundary conditions to draw the geometric model of the motor, as showed in Figure 1:

![Figure 1 The model of permanent magnet synchronous linear motor](image)

After getting the 2D geometric model, set up on the grid subdivision. The specific parameters are shown in the figure 2:

| Density of setting                | Apportionment region            |
|-----------------------------------|---------------------------------|
| Length band=0.2cm                 | The area of Band                |
| Length coil=0.25cm                | The area of stator winding      |
| Length stator=0.3cm               | The stator and rotor core       |
| Length PM=0.2cm                   | Permanent magnet                |
| Length outregion=0.4cm            | The out area                    |

Specific subdivision is shown in figure 2:
After finishing the physical model of the motor, analyzing the travelling magnetic field. We must restrain harmonic in order to get current air gap flux waveform, and the harmonic has three influential factors, they respectively are space width, air-gap size and magnet thickness.

We can take the method of orthogonal experiments concretely, which means to design normative orthogonal table and propose appropriate experiment scheme and parameters, it’s a kind of scientific experiment method by comparing to attain the best conditions. The design can pick out the best parameters to analyzing the air gap flux density distribution curve definitely through arranging simulation test and making orthogonal experiment table.

Firstly arrange three levels respectively to three influential factors, and the aim is to get good air density waveform sine degrees, concrete contents are shown in the table three:

| Serial number | A: space width(mm) | B: air gap size(mm) | C: magnet thickness(mm) |
|---------------|--------------------|---------------------|-------------------------|
| Level 1       | 4.7                | 0.45                | 2.8                     |
| Level 2       | 5                  | 0.5                 | 3                       |
| Level 3       | 5.3                | 0.55                | 3.2                     |

Through the ratio of the effective value of the air gap flux density and that of the fundamental component, we can get the waveform distortion rate,

| Scheme | A | B | C | Distortion rate (%) |
|--------|---|---|---|----------------------|
| 1      | 1 | 1 | 1 | 29.31                |
| 2      | 1 | 2 | 2 | 27.83                |
| 3      | 1 | 3 | 3 | 27.53                |
| 4      | 2 | 1 | 2 | 28.63                |
| 5      | 2 | 2 | 3 | 28.80                |
| 6      | 2 | 3 | 1 | 28.07                |
| 7      | 3 | 2 | 2 | 30.22                |
| 8      | 3 | 2 | 2 | 28.38                |
| 9      | 3 | 3 | 2 | 28.17                |
We can find that aberration of scheme three is minimum, and that is the result of the air gap density distribution curve is best under the parameter of A1B2C3, concrete contents are shoe in the figure three.

When finishing the model, observe magnetic force line distribution and magnetic secret cloud, as shown in the figure four and five:

Through the comparison of figure four and figure five, part of the winding at the top of the flux density is stronger in the figure five, however, in the corresponding figure five, it’s dense for part of the magnetic line of force, which sufficiently prove this part magnetic field strength is high.

Set the speed as 2.3m/s of the permanent magnet synchronous, and observe the sliding position of the linear motor when operating, curve of A、B、C three-phase winding flux linkages are shown in the figure six and figure seven, so we can see that linear motor has good dynamic characteristics.
3. Optimization design based on chaos optimization algorithm

The structural parameter optimizations method can be applied to permanent magnet synchronous linear motor, such as genetic algorithm, simplex method and chaos optimization algorithm, etc[10-11]. However, the traditional optimization method is easy to fall into local minimum. The result of genetic algorithm is random, we can get the different optimization results through executing algorithm program more than once, so human beings also need to make a choice[4]. Select chaos optimization method for permanent magnet synchronous linear motor parameters to do optimal design. A simple function relation can represent the basic electromagnetic design. The relation between Structural parameters of the motor need to optimize and optimization objective can be expressed as:

\[
\min \text{imize} W(x) = \sum_{i=1}^{g} \frac{\gamma_i}{\sigma_i} \\
\text{subject to} 
\begin{cases} 
4.7 \leq A \leq 5.3 \\
0.45 \leq B \leq 0.5 \\
2.8 \leq C \leq 3.2 
\end{cases} 
\] (6)

In the formulas, \(\gamma_i\) is the effective value of air gap induction, and \(\delta_i\) is the effective value of fundamental component. In order to achieve the goal that permanent magnet synchronous linear motor optimize parameters, even if the air gap flux density waveform distortion rate is minimal, we can calculate a better sinusoidal air gap flux density waveform and can be reasonable to select the main design parameters (A: space width; B: air-gap size; C: magnet thickness) as independent input variables, and search the choices of space basing on the result of said electromagnetic design.

We can get the distortion rate of air gap flux from construction optimize analysis, these will enter into the penalty function calculation and compare allowable limits, according to the following formula:

\[
g_j(x) = \left| \frac{b_j}{(b_{a(j)})} - 1 \right| \quad 0 < b_j < b_a(j) 
\] (7)

In the formula, \(g_j(x)\) is limiting function, \(b_j\) represents the j-th flux density waveform distortion rate, \(b_a(j)\) is flux density distortion constraint values.

According to the objective function and constraints, and come together as an adaptation function:

\[
f(x) = (1 + \varepsilon_1 \sum g_j)^{\varepsilon_2} \times W(x) 
\] (8)

In the formulas, \(f(x)\) is adaptation function, \(\varepsilon_1\) and \(\varepsilon_2\) are penalty coefficients. \(\varepsilon_2\) is a fixed value
fifteen\textsuperscript{[12]}, but $\varepsilon_1$ can select the self-adjusting.

By means of searching the global and local parameter space, get the result tend to be the best optimization goal. Chaos optimization results are shown in table 5:

| Optimization results          | A    | B    | C    | Distortion rate (%) |
|-------------------------------|------|------|------|---------------------|
| Chaos global optimization     | 4.6799 | 0.5499 | 3.1993 | 27.5312             |
| Chaos local optimization      | 4.6992 | 0.5497 | 3.1998 | 27.5368             |
| Round the result              | 4.7  | 0.55 | 3.2  | 27.5305             |

According to the table 5, the best optimization results are consistent, and also prove the effectiveness. The best optimization results prove that structural parameters have an influence on back-EMF after pole shifting, and have a simulation to original motor model and motor model after pole shifting, simulation results are shown in the figure 8 and 9, the figure shows that back-EMF amplitude before pole shifting was 358V, but it’s 357V after pole shifting. As shown in the figure 9, the three phase back-EMF of motor after pole shifting still have better symmetry, so this method cannot affect symmetry of motors’ back-EMF, and also have little influence on its three phase back-EMF amplitude.

Finally, acquiring a more precise gradually shrinking thrust ripple, as shown in the figure ten.
4. Conclusion
Special structure of permanent magnet synchronous linear motor leads to that its inner magnetic field distribution has clear characteristic, only taking finite element method can we get the current solution. The distortion rate of permanent magnet synchronous linear motor is the most important factor that affects its air gap flux density waveform sine. This start point of this article is optimizing the structure of motor stator of the motor to minimize the distortion rate of the motor.

Modeling method based on finite element method to establish has more accurate precision and highly efficient calculation. The use of traditional orthogonal test method can attain the feasible electromagnetic scheme. Making use of chaotic search algorithm and optimization parameter of adaptation function based on the model, we can get the optimization and minimize air gap flux density waveform distortion rate of the motor under the same conditions. It has little effect on the back-EMF of the motor through simulation, and can get a more accurate thrust. The conclusion can give a help to the control system design of permanent magnet synchronous linear motor.

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