Primordial neutrinos, cosmological perturbations in interacting dark-energy model: CMB and LSS

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Received 24 November 2007
Accepted 3 May 2008
Published 5 June 2008

Abstract. We present the cosmological perturbation theory for neutrino-probe interacting dark-energy models and calculate the cosmic microwave background anisotropies and the matter power spectrum. In these models, the evolution of the mass of the neutrinos is determined by the quintessence scalar field, which is responsible for the cosmic acceleration today. We consider several types of scalar field potentials and put constraints on the coupling parameter between neutrinos and dark energy. Assuming the flatness of the universe, the constraint we can derive from the current observation is $\sum m_\nu < 0.87$ eV at the 95% confidence level for the sum over three species of neutrinos. We also discuss the stability issue of our model and on the impact of the scattering term in the Boltzmann equation from the mass-varying neutrinos.

Keywords: dark energy theory, cosmological neutrinos, cosmological perturbation theory, power spectrum

ArXiv ePrint: 0705.2134
1. Introduction

After SNIa [1] and WMAP [2] observations during the last decade, the discovery of the accelerated expansion of the universe is a major challenge to particle physics and cosmology. There are currently three candidates for the dark energy which results in this accelerated expansion:

- a non-zero cosmological constant [3],
- a dynamical cosmological constant (quintessence scalar field) [4],
- modifications of Einstein’s theory of gravity [5].

The scalar field model like quintessence is a simple model with time-dependent \( w \), which is generally larger than \(-1\). Because the different \( w \) lead to a different expansion history of the universe, the geometrical measurements of cosmic expansion through observations of SNIa, CMB and baryon acoustic oscillations (BAO) can give us tight constraints on \( w \). One of the interesting ways to study the scalar field dark-energy models is to investigate the coupling between the dark energy and the other matter fields. In fact, a number of models which realize the interaction between dark energy and dark matter, or even visible...
Primordial neutrinos, cosmological perturbations in interacting dark-energy model

matter, have been proposed so far [6]–[10]. Observations of the effects of these interactions will offer an unique opportunity to detect a cosmological scalar field [6,11].

In this paper, after briefly reviewing the main idea of the three possible candidates for dark energy and their cosmological phenomena in section 2, we discuss the interacting dark-energy model, paying particular attention to the interacting mechanism between dark energy with a hot dark matter (neutrinos) in section 3. In this so-called mass-varying neutrino (MVN) model [12], we calculate explicitly the cosmic microwave background (CMB) radiation and large-scale structure (LSS) within cosmological perturbation theory. The evolution of the mass of neutrinos is determined by the quintessence scalar field, which is responsible for the cosmic acceleration today. Recently, perturbation equations for this class of models have been nicely presented by Brookfield et al [13], (see also [14]) which are necessary to compute CMB and LSS spectra. A main difference here from their work is that we correctly take into account the scattering term in the geodesic equation of neutrinos, which was omitted there (see, however, [15]). We will show that this leads to significant differences in the resultant spectra and hence the different observational constraints. In section 4, we discuss three different types of quintessence potential, namely an inverse power law potential, a supergravity potential and an exponential type potential. By computing CMB and LSS spectra with these quintessential potentials and comparing them to the latest observations, the constraints on the present mass of neutrinos and coupling parameters are derived. In conclusion, we discuss two important points of this work on the impact of the scattering term of the Boltzmann equation and on the stability issue in the interacting dark-energy model. In appendix A, the explicit calculation for the consistency check of our calculations in section 3 is shown. Since we were asked to show an explicit derivation of the geodesic equation after our first draft was released, we show it in appendix B.

2. Three possible solutions for an accelerating universe

Recent observations with supernova type Ia (SNIa) and CMB radiation have provided strong evidence that we live now in an accelerating and almost flat universe. In general, one believes that the dominance of a dark-energy component with negative pressure in the present era is responsible for the universe’s accelerated expansion. However, there are three possible solutions to explain the accelerating universe. The Einstein equation in general relativity is given in the following form:

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}. \]  

Here, the \( G_{\mu\nu} \) term contains the information on geometrical structure, the energy–momentum tensor \( T_{\mu\nu} \) keeps the information on matter distributions and the last term is the so-called cosmological constant which contain the information on non-zero vacuum energy. After solving the Einstein equation, one can derive a simple relation:

\[ \frac{\dot{R}}{R} = \frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \]  

In order to get the accelerating expansion, either the cosmological constant \( \Lambda \) (\( \omega_{\Lambda} = P/\rho = -1 \)) becomes positive or a new concept of dark energy with negative pressure (\( \omega_{\phi} < -1/3 \)) needs to be introduced. Another solution can be given by the modification
of the geometrical structure which can provide a repulsive source of gravitational force. In this case, the attractive gravitational force term is dominant in the early stage of the universe; however, at a later time near the present era, a repulsive term becomes important and drives the universe to expand under acceleration. Also, we can consider extra energy density contributions from bulk space in brane-world scenario models, which can modify the Friedmann equation as $H^2 \propto \rho + \rho'$. In summary, we have three different solutions for the accelerating expansion of our universe as mentioned in section 1. Probing the origin of the accelerating universe is the most important and challenging problem in high energy physics and cosmology now. The detailed explanation and many references are in a useful review on dark energy [16].

In this paper, we concentrate on the second solution using the quintessence field. In the present epoch, the potential term becomes more important than the kinetic term, which can easily explain the negative pressure with $\omega_\phi \simeq -1$. However, there are many different versions of quintessence field: K-essence [17, 18], phantom [19], quintom [20], etc, and to justify the origin of dark energy from experimental observations is a really difficult job. The present updated value of the equation of states (EoS) is $\omega = -1.02 \pm 0.12$ without any supernova data [21].

3. Interacting dark energy with neutrinos

As explained in the previous section, it is really difficult to probe the origin of dark energy when the dark energy does not interact with other matter at all. Here we investigate the cosmological implication of an idea of the dark energy interacting with neutrinos [12, 22]. For simplicity, we consider the case that dark energy and neutrinos are coupled such that the mass of the neutrinos is a function of the scalar field which drives the late-time accelerated expansion of the universe. In previous works by Fardon et al [22] and Peccei [12], the kinetic energy term was ignored and the potential term was treated as a dynamical cosmology constant, which can be applicable for the dynamics near the present epoch. However, the kinetic contributions become important to describe cosmological perturbations in the early stage of the universe, which is fully considered in our analysis.

3.1. Cosmological perturbations: background equations

Equations for the quintessence scalar field are given by

$$\ddot{\phi} + 2\dot{\mathcal{H}}\dot{\phi} + a^2 \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0,$$

$$V_{\text{eff}}(\phi) = V(\phi) + V_1(\phi),$$

$$V_1(\phi) = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_{\nu}^2(\phi)f(q)},$$

$$m_{\nu}(\phi) = \tilde{m}_{\nu} e^{\beta(\phi/M_{\text{pl}})} \quad \text{(as an example)},$$

where $V(\phi)$ is the potential of the quintessence scalar field, $V_1(\phi)$ is an additional potential due to the coupling to neutrino particles [22, 23] and $m_{\nu}(\phi)$ is the mass of the neutrino coupled to the scalar field. $\mathcal{H}$ is $\dot{a}/a$, where the dot represents the derivative with respect to the conformal time $\tau$. 
Energy densities of the mass-varying neutrino (MVN) and quintessence scalar field are described as

\[ \rho_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2m_\nu^2}f_0(q), \]  
(7)

\[ 3P_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{\sqrt{q^2 + a^2m_\nu^2}}f_0(q), \]  
(8)

\[ \rho_\phi = \frac{1}{2a^2}\dot{\phi}^2 + V(\phi), \]  
(9)

\[ P_\phi = \frac{1}{2a^2}\dot{\phi}^2 - V(\phi). \]  
(10)

From equations (7) and (8), the equation of motion for the background energy density of neutrinos is given by

\[ \dot{\rho}_\nu + 3H(\rho_\nu + P_\nu) = \frac{\partial \ln m_\nu}{\partial \phi} \dot{\phi}(\rho_\nu - 3P_\nu). \]  
(11)

3.2. Perturbation equations

3.2.1. Perturbations in the metric. We work in the synchronous gauge and the line element is

\[ ds^2 = a^2(\tau) \left[ -d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right]. \]  
(12)

In this metric the Christoffel symbols which have non-zero values are

\[ \Gamma^0_{00} = \frac{\dot{a}}{a}, \]  
(13)

\[ \Gamma^0_{ij} = \frac{\dot{a}}{a}\delta_{ij} + \frac{a}{a}\dot{h}_{ij} + \frac{1}{2}h_{ij}, \]  
(14)

\[ \Gamma^i_{0j} = \frac{\dot{a}}{a}\delta^i_j + \frac{1}{2}\dot{h}_{ij}, \]  
(15)

\[ \Gamma^i_{jk} = \frac{1}{2}\delta^{ia}(h_{ka,j} + h_{aj,k} - h_{jk,a}), \]  
(16)

where the dot denotes a conformal time derivative. For CMB anisotropies we mainly consider the scalar type perturbations. We introduce two scalar fields, \( h(\mathbf{k}, \tau) \) and \( \eta(\mathbf{k}, \tau) \), in \( k \)-space and write the scalar mode of \( h_{ij} \) as a Fourier integral [25]:

\[ h_{ij}(\mathbf{x}, \tau) = \int d^3k e^{i\mathbf{k} \cdot \mathbf{x}} \left[ \hat{k}_i \hat{k}_j h(\mathbf{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij})6\eta(\mathbf{k}, \tau) \right], \]  
(17)

where \( \mathbf{k} = k\hat{k} \) with \( \hat{k}_i \hat{k}^i = 1 \).
3.2.2. Perturbations in quintessence. The equation of the quintessence scalar field is given by

\[ \Box \phi - V_{eff}(\phi) = 0. \] (18)

Let us write the scalar field as a sum of the background value and perturbations around it, \( \phi(x, \tau) = \phi(\tau) + \delta \phi(x, \tau) \). The perturbation equation is then described as

\[ \frac{1}{a^2} \mathcal{H} \dot{\delta \phi} - \frac{1}{a^2} \nabla^2 (\delta \phi) + \frac{1}{2a^2} \mathcal{H} \dot{\phi} + \frac{d^2 V}{d\phi^2} \delta \phi + \delta \left( \frac{dV}{d\phi} \right) = 0, \] (19)

where

\[ \frac{dV}{d\phi} = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{\partial \epsilon(q, \phi)}{\partial \phi} f(q), \] (20)

\[ \epsilon(q, \phi) = \sqrt{q^2 + a^2 m^2_\nu(\phi)}, \] (21)

\[ \frac{\partial \epsilon(q, \phi)}{\partial \phi} = \frac{a^2 m^2_\nu(\phi)}{\epsilon(q, \phi)} \frac{\partial \ln m_\nu}{\partial \phi}. \] (22)

To describe \( \delta(dV/\phi) \), we shall write the distribution function of neutrinos with background distribution and perturbation around it as

\[ f(x^i, \tau, q, n_j) = f_0(\tau, q)(1 + \Psi(x^i, \tau, q, n_j)). \] (23)

Then we can write

\[ \delta \left( \frac{dV}{d\phi} \right) = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{\partial^2 \epsilon}{\partial \phi^2} \delta \phi f_0 + a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{\partial \epsilon}{\partial \phi} f_0 \Psi, \] (24)

where

\[ \frac{\partial^2 \epsilon}{\partial \phi^2} = \frac{a^2}{\epsilon} \left( \frac{\partial m_\nu}{\partial \phi} \right)^2 + \frac{a^2 m_\nu}{\epsilon} \left( \frac{\partial^2 m_\nu}{\partial \phi^2} \right) - \frac{a^2 m_\nu}{\epsilon^2} \left( \frac{\partial \epsilon}{\partial \phi} \right) \left( \frac{\partial m_\nu}{\partial \phi} \right). \] (25)

For numerical purposes it is useful to rewrite equations (20) and (24) as

\[ \frac{dV}{d\phi} = \frac{\partial \ln m_\nu}{\partial \phi} (\rho_\nu - 3P_\nu), \] (26)

\[ \delta \left( \frac{dV}{d\phi} \right) = \frac{\partial^2 \ln m_\nu}{\partial \phi^2} \delta \phi (\rho_\nu - 3P_\nu) + \frac{\partial \ln m_\nu}{\partial \phi} (\delta \rho_\nu - 3\delta P_\nu). \] (27)

Note that perturbation fluid variables in mass-varying neutrinos are given by

\[ \delta \rho_\nu = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \epsilon f_0(q) \Psi + a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{\partial \epsilon}{\partial \phi} \delta \phi f_0, \] (28)

\[ 3\delta P_\nu = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{\epsilon} f_0(q) \Psi - a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{\epsilon^2} \frac{\partial \epsilon}{\partial \phi} \delta \phi f_0. \] (29)

The energy–momentum tensor of quintessence is given by

\[ T^\mu_\nu = g^{\mu\alpha} \phi_\alpha \phi_\nu - \frac{1}{2} (\phi_\alpha \phi_\alpha + 2V(\phi)) \delta^\mu_\nu, \] (30)
and its perturbation is
\[ \delta T_{\mu} = g_{(0)}^{\mu a} \delta \phi_{a} \phi_{\nu} + g_{(0)}^{\mu a} \partial_{a} \partial_{\nu} + \delta g^{\mu a} \phi_{a} \phi_{\nu} - \frac{1}{2} \left( \delta \phi_{a} \phi_{a} + \phi_{a} \phi_{a} + \frac{dV}{d\phi} \right) \delta \phi. \] (31)

This gives perturbations of quintessence in fluid variables as
\[ \delta \rho_{\phi} = -\delta T^{0}_{0} = \frac{1}{a^{2}} \dot{\phi} \delta \phi + \frac{dV}{d\phi} \delta \phi, \] (32)
\[ \delta P_{\phi} = -\delta T^{0}_{0} / 3 = \frac{1}{a^{2}} \dot{\phi} \delta \phi - \frac{dV}{d\phi} \delta \phi, \] (33)
\[ (\rho_{\phi} + P_{\phi}) \theta_{\phi} = i k^{i} \delta T^{i}_{0} = \frac{k^{2}}{a^{2}} \dot{\phi} \delta \phi, \] (34)
\[ \Sigma_{j} = T_{j}^{i} - \delta_{j}^{i} T^{k}_{k} / 3 = 0. \] (35)

3.3. Boltzmann equation for mass-varying neutrino

We have to consider the Boltzmann equation to solve the evolution of MVN. A distribution function is written in terms of time (\( \tau \)), positions (\( x^{i} \)) and their conjugate momenta (\( P_{i} \)). The conjugate momentum is defined as spatial parts of the 4-momentum with lower indices, i.e. \( P_{i} = m U_{i} \), where \( U_{i} = dx_{i} / (-ds^{2})^{1/2} \). We also introduce a locally orthonormal coordinate \( X^{\mu} = (t, r) \) and we write the energy and the momentum in this coordinate as \( (E, p^{i}) \). The relations of these variables in synchronous gauge are given by [25]
\[ P_{0} = -aE, \] (36)
\[ P_{i} = a(\delta_{ij} + \frac{1}{2} h_{ij}) p^{j}. \] (37)

Next we define comoving energy and momentum (\( \epsilon, q_{i} \)) as
\[ \epsilon = aE = \sqrt{q^{2} + a^{2} m_{\nu}^{2}}, \] (38)
\[ q_{i} = a p_{i}. \] (39)

Hereafter, we shall use \((x^{i}, q, n_{j}, \tau)\) as phase space variables, replacing \( f(x^{i}, P_{j}, \tau) \) by \( f(x^{i}, q, n_{j}, \tau) \). Here we have split the comoving momentum \( q_{j} \) into its magnitude and direction: \( q_{j} = q n_{j} \), where \( n_{j} n_{i} = 1 \). The Boltzmann equation is
\[ \frac{Df}{D\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^{i}}{d\tau} \frac{\partial f}{\partial x^{i}} + \frac{dq}{d\tau} \frac{\partial f}{\partial q} + \frac{dn_{i}}{d\tau} \frac{\partial f}{\partial n_{i}} = \left( \frac{\partial f}{\partial \tau} \right)_{C} \] (40)
in terms of these variables. From the time component of the geodesic equation [24]
\[ \frac{1}{2} \frac{d}{d\tau} \left( P^{0} \right)^{2} = -\Gamma_{\alpha \beta}^{0} P^{\alpha} P^{\beta} - m g^{0\nu} m_{\nu}, \] (41)
and the relation \( P^\theta = a^{-2} \epsilon = a^{-2} \sqrt{q^2 + a^2 m^2} \), we have
\[
\frac{dq}{d\tau} = -\frac{1}{2} h_{ij} q n^i n^j - a^2 m \frac{\partial m}{q} \frac{dx^i}{d\tau}.
\] (42)

Our analytic formulae in equations (41) and (42) are different from those of [13] and [14], since they have omitted the contribution of the varying neutrino mass term. We shall show later that this term also gives an important contribution in the first-order perturbation of the Boltzmann equation. We will write down each term up to \( O(h) \):
\[
\begin{align*}
\frac{\partial f}{\partial \tau} & = \frac{\partial f_0}{\partial \tau} + f_0 \frac{\partial \Psi}{\partial \tau} + \frac{\partial f_0}{\partial \tau} \Psi, \\
\frac{dx^i}{d\tau} \frac{\partial f}{\partial x^i} & = \frac{q}{\epsilon} n^i \times f_0 \frac{\partial \Psi}{\partial x^i}, \\
\frac{dq}{d\tau} \frac{\partial f}{\partial q} & = \left( -a^2 m \frac{\partial m}{q} \frac{dx^i}{d\tau} - \frac{1}{2} h_{ij} q n^i n^j \right) \times \frac{\partial f_0}{\partial q}, \\
\frac{dn_i}{d\tau} \frac{\partial f}{\partial n_i} & = O(h^2).
\end{align*}
\] (43)

We note that \( \partial f / \partial x^i \) and \( dq / d\tau \) are \( O(h) \).

3.3.1. Background equations. From the equations above, the zeroth-order Boltzmann equation is
\[
\frac{\partial f_0}{\partial \tau} = 0.
\] (44)

The Fermi–Dirac distribution
\[
f_0 = f_0(\epsilon) = \frac{g_s}{h_p^4 \epsilon_{\text{gs}}^2 + 1}
\] (45)
can be a solution. Here \( g_s \) is the number of spin degrees of freedom, and \( h_p \) and \( k_B \) are the Planck and the Boltzmann constants.

3.3.2. Perturbation equations. The first-order Boltzmann equation is
\[
\frac{\partial \Psi}{\partial \tau} + i \frac{q}{\epsilon} (\mathbf{n} \cdot \mathbf{k}) \Psi + \left( \eta - (\mathbf{k} \cdot \mathbf{n})^2 \frac{1}{2} \right) \frac{\partial \ln f_0}{\partial \ln q} + \frac{a^2 m^2 \partial \ln m}{q^2} \frac{\partial f_0}{\partial \phi} \frac{\partial f_0}{\partial \ln q} = 0.
\] (46)

Following previous studies, we shall assume that the initial momentum dependence is axially symmetric so that \( \Psi \) depends on \( q = q \mathbf{n} \) only through \( q \) and \( \mathbf{k} \cdot \mathbf{n} \). With this assumption, we expand the perturbation of the distribution function, \( \Psi \), in a Legendre series:
\[
\Psi(k, \mathbf{n}, q, \tau) = \sum (-i)^\ell (2\ell + 1) \Psi_\ell(k, q, \tau) P_\ell(\mathbf{k} \cdot \mathbf{n}).
\] (47)
Then we obtain the hierarchy for MVN:

\[ \dot{\Psi}_0 = -\frac{q}{\epsilon} k \dot{\Psi}_1 + \frac{\dot{h}}{6} \frac{\partial \ln f_0}{\ln q}, \]  

(48)

\[ \dot{\Psi}_1 = \frac{q}{15} k (\Psi_0 - 2 \Psi_2) + \kappa, \]  

(49)

\[ \dot{\Psi}_2 = \frac{1}{5} q k (2 \Psi_1 - 3 \Psi_3) - \left( \frac{1}{15} \dot{h} + \frac{2}{5} \dot{\eta} \right) \frac{\partial \ln f_0}{\partial \ln q}, \]  

(50)

\[ \dot{\Psi}_\ell = \frac{q}{\epsilon} k \left( \frac{\ell}{2\ell + 1} \Psi_{\ell-1} - \frac{\ell + 1}{2\ell + 1} \Psi_{\ell+1} \right), \]  

(51)

where

\[ \kappa = -\frac{1}{3} \frac{q a^2 m^2_\nu}{q' q^2} \frac{\partial \ln m_\nu}{\partial \phi} \frac{\partial \ln f_0}{\partial \ln q}. \]  

(52)

Here we used the recursion relation

\[(\ell + 1) P_{\ell+1}(\mu) = (2\ell + 1) P_\ell(\mu) - \ell P_{\ell-1}(\mu). \]  

(53)

We have to solve these equations with a \( q \)-grid for every wavenumber \( k \).

4. Quintessence potentials

To determine the evolution of the scalar field which couples to neutrinos, we should specify the potential of the scalar field. A variety of quintessence effective potentials can be found in the literature. In the present paper we examine three types of quintessential potentials. First we analyze what is a frequently invoked form for the effective potential of the tracker field, i.e. an inverse power law such as originally analyzed by Ratra and Peebles [30]:

\[ V(\phi) = M^4 + \alpha \phi^{-\alpha} \]  

(model I),

(54)

where \( M \) and \( \alpha \) are parameters.

We will also consider a modified form of \( V(\phi) \) as proposed by [31] based on the condition that the quintessence fields be part of supergravity models. The potential now becomes

\[ V(\phi) = M^4 + \alpha \phi^{-\alpha} e^{3\phi^2/2m^2_{pl}} \]  

(model II),

(55)

where the exponential correction becomes important near the present time as \( \phi \to m_{pl} \). The fact that this potential has a minimum for \( \phi = \sqrt{\alpha/3} m_{pl} \) changes the dynamics. It causes the present value of \( w \) to evolve to a cosmological constant much quicker than for the bare power law potential [32]. In these models the parameter \( M \) is fixed by the condition that \( \Omega_Q \approx 0.7 \) at present.

We will also analyze another class of tracking potential, namely the potential of exponential type [33]:

\[ V(\phi) = M^4 e^{-\alpha \phi} \]  

(model III).

(56)

This type of potential can lead to an accelerating expansion provided that \( \alpha < \sqrt{2} \). In figure 1, we present examples of the evolution of energy densities with these three types of potentials with vanishing coupling strength to neutrinos.
4.1. Time evolution of neutrino mass and energy density in scalar field

For an illustration we also plot examples of the evolution of energy densities for the interacting case with inverse power law potential (model I) in figure 2. In interacting dark-energy cases, the evolution of the scalar field is determined both by its own potential and the interacting term from neutrinos. When neutrinos are highly relativistic, the interaction term can be expressed as

\[
\frac{\partial m_\nu}{\partial \phi}(\rho_\nu - 3P_\nu) \approx \frac{10}{7\pi^2} (am_\nu)^2 \rho_{\nu\text{massless}},
\]  

where \(\rho_{\nu\text{massless}}\) denotes the energy density of neutrinos with no mass. The term roughly scales as \(\propto a^{-2}\), and therefore it dominates deep in the radiation-dominated era. However, because the motion of the scalar field driven by this interaction term is almost suppressed by the friction term, \(-2\dot{\phi}\). The scalar field satisfies the slow roll condition similar to the inflation models, \(-2\dot{\phi} \approx a^2(\partial m_\nu/\partial \phi)(\rho_\nu - 3P_\nu)\). Thus, the energy density in the scalar field and the mass of neutrinos is frozen there. These behaviors are clearly seen in figures 2 and 3.

4.2. Constraints on the MVN parameters

As was shown in the previous sections, the coupling between cosmological neutrinos and dark-energy quintessence could modify the CMB and matter power spectra significantly. It is therefore possible and also important to put constraints on coupling parameters from current observations. For this purpose, we use the WMAP3 [26, 27] and 2dF [28] datasets.
Primordial neutrinos, cosmological perturbations in interacting dark-energy model

Figure 2. Examples of the evolution of energy density in quintessence and the background fields in coupled cases with an inverse power law potential (model I). Model parameters taken to plot this figure are $\alpha = 1$, $\beta = 1, 3$ as indicated. The other parameters for the dark energy are fixed so that the energy densities in three types of dark energy should be the same at present.

In figures 4 and 5, we demonstrate, as examples, the CMB angular power spectra and matter power spectra with different parameter sets, respectively.

The flux power spectrum of the Lyman-$\alpha$ forest can be used to measure the matter power spectrum at small scales around $z \lesssim 3$ [34,35]. It has been shown, however, that the resultant constraint on neutrino mass can vary significantly from $\sum m_\nu < 0.2$ eV to 0.4 eV, depending on the specific Lyman-$\alpha$ analysis used [36]. The complication arises because the result suffers from the systematic uncertainty regarding the model for the intergalactic physical effects, i.e. damping wings, ionizing radiation fluctuations, galactic winds, etc [37]. Therefore, we conservatively omit the Lyman-$\alpha$ forest data from our analysis.

Because there are many other cosmological parameters than the MaVaNu parameters, we follow the Markov chain Monte Carlo (MCMC) global fit approach [29] to explore the likelihood space and marginalize over the nuisance parameters to obtain the constraint on parameter(s) we are interested in. Our parameter space consists of

$$\vec{P} \equiv (\Omega_b h^2, \Omega_c h^2, H, \tau, A_s, n_s, m_i, \alpha, \beta),$$  \hspace{1cm} (58)

where $\Omega_b h^2$ and $\Omega_c h^2$ are the baryon and CDM densities in units of the critical density, $H$ is the Hubble parameter, $\tau$ is the optical depth of Compton scattering to the last scattering surface, $A_s$ and $n_s$ are the amplitude and spectral index of primordial density fluctuations, and $(m_i, \alpha, \beta)$ are the parameters of MVN defined in section 3. We have put priors on the MVN parameters as $\alpha > 0$, and $\beta > 0$ for simplicity and to save computational time.

Our results are shown in figures 6–8. In these figures we do not observe the strong degeneracy between the introduced parameters. This is why one can put tight constraints.
Figure 3. Examples of the time evolution of neutrino mass in power law potential models (model I) with $\alpha = 1$ and $\beta = 0$ (black solid line), $\beta = 1$ (red dashed line), $\beta = 2$ (blue dashed–dotted line) and $\beta = 3$ (dashed–dotted–dotted line). The larger coupling parameter leads to the larger mass in the early universe.

Figure 4. The CMB angular power spectra for model I. The solid line is the best fit for the model (${(\alpha, \beta)} = (2.97, 0.170)$): the other lines are models with different parameter values of $\alpha$ and $\beta$, as indicated. The points are WMAP three-year data.

on MVN parameters from observations. For both models we consider, larger $\alpha$ leads to larger $w$ at present. Therefore large $\alpha$ is not allowed due to the same reason that larger $w$ is not allowed from the current observations.

On the other hand, larger $\beta$ will generally lead to larger $m_\nu$ in the early universe. This means that the effect of neutrinos on the density fluctuation of matter becomes larger, leading to larger damping of the power at small scales. A complication arises because the
Figure 5. The matter power spectra for model III. The solid line is the best fit for the model \((\alpha, \beta) = (0.78, 0.28)\): the other lines are models with different parameter values of \(\alpha\) and \(\beta\), as indicated. The points are 2dF data.

Figure 6. Contours of constant relative probabilities in two-dimensional parameter planes for inverse power law models. Lines correspond to 68% and 95.4% confidence limits.
mass of neutrinos at the transition from the ultra-relativistic regime to the non-relativistic one is not a monotonic function of $\beta$ as shown in figure 3. Even so, the coupled neutrinos give a larger decrement of small scale power, and therefore one can limit the coupling parameter from the large-scale structure data.

One may wonder why we can get such a tight constraint on $\beta$, because it is naively expected that a large $\beta$ value should be allowed if $\Omega_\nu h^2 \sim 0$. In fact, a goodness of fit is still satisfactory with a large $\beta$ value when $\Omega_\nu h^2 \sim 0$. However, the parameters which give us the best goodness of fit do not mean the most likely parameters in general. In our parameterization, the accepted total volume by MCMC in the parameter space where $\Omega_\nu h^2 \sim 0$ and $\beta \gtrsim 1$ was small, meaning that the probability of such a parameter set is low. In table 1, we show our results of global analysis with 1σ deviation for different types of quintessence potential with WMAP3 year data. Our global fit data is well agreed with WMAP3 year data for $\Lambda$CDM model except for relatively smaller value of hubble constant $H_0 \simeq 66 \pm 3$ which is closely related with a smaller value of $\Omega_Q < \Omega_\Lambda = 0.72$ in the interacting dark-energy model. (See figure 20 in [38].)

We find no observational signature which favors the coupling between MVN and the quintessence scalar field, and obtain the upper limit on the coupling parameter within 2σ ranges as

$$\beta < 1.11, 1.36, 1.53,$$

(59)
and the present mass of neutrinos is also limited to

$$\Omega_\nu h^2_{\text{today}} < 0.0095, 0.0090, 0.0084,$$

for models I, II and III, respectively. When we apply the relation between the total sum of the neutrino masses $M_\nu$ and their contributions to the energy density of the universe, $\Omega_\nu h^2 = M_\nu/(93.14 \, \text{eV})$, we obtain the constraint on the total neutrino mass: $M_\nu < 0.87 \, \text{eV} \, (95\% \, \text{C.L.})$ in the neutrino-probe dark-energy model. The total neutrino mass contributions in the power spectrum is shown in figure 9, where we can see the significant deviation from observation data in the case of large neutrino masses.

5. Summary and conclusion

Before concluding this paper we should comment on two important points of this paper: the impact of the scattering term of the Boltzmann equation in section 3 and on the stability issue in the present models.

Recently, perturbation equations for the MVN models were nicely presented by Brookfield et al [13] (see also [14]) which are necessary to compute CMB and LSS spectra. A main difference here from their work is that we correctly take into account the scattering term in the geodesic equation of neutrinos, which was omitted there (see, however, [15]). Because the term is proportional to $\partial m/\partial x$ and the first-order quantity in perturbation, our results and those of earlier works [13, 14] remain the same in the background evolution. However, as will be shown in appendix A, neglecting this term violates the energy–momentum conservation law at linear level, leading to the anomalously large ISW effect. Because the term becomes important when neutrinos become massive, the late-time ISW is mainly affected through the interaction between dark energy and neutrinos. Consequently, the differences show up at large angular scales. In figure 10, the differences are shown with and without the scattering term. The early ISW can also be affected by this term to some extent in some massive neutrino models and the height of

| Quantities       | Model I          | Model II         | Model III         | WMAP-3 data (LCDM) |
|------------------|------------------|------------------|-------------------|--------------------|
| $\Omega_B h^2$   | 2.21 ± 0.07      | 2.22 ± 0.07      | 2.21 ± 0.07       | 2.23 ± 0.07        |
| $\Omega_{CDM} h^2$ | 11.10 ± 0.62    | 11.10 ± 0.65     | 11.10 ± 0.63      | 12.8 ± 0.8         |
| $H_0$            | 65.97 ± 3.61     | 65.37 ± 3.41     | 65.61 ± 3.26      | 72 ± 8             |
| $Z_{re}$         | 10.87 ± 2.58     | 10.89 ± 2.62     | 11.07 ± 2.44      | —                 |
| $\alpha$         | <2.63            | <7.78            | <0.92             | —                 |
| $\beta$          | <0.46            | <0.47            | <0.58             | —                 |
| $n_s$            | 0.95 ± 0.02      | 0.95 ± 0.02      | 0.95 ± 0.02       | 0.958 ± 0.016      |
| $A_s[10^{10}]$   | 20.66 ± 1.31     | 20.69 ± 1.32     | 20.72 ± 1.24      | —                 |
| $\Omega_Q h^2$   | 68.54 ± 4.81     | 67.90 ± 4.47     | 68.22 ± 4.17      | 71.6 ± 5.5         |
| Age (Gyr)        | 13.95 ± 0.20     | 13.97 ± 0.19     | 13.69 ± 0.19      | 13.73 ± 0.16       |
| $\Omega_{MVN} h^2$ | <0.44           | <0.48            | <0.48             | <1.97 (95\% C.L.)  |
| $\tau$           | 0.08 ± 0.03      | 0.08 ± 0.03      | 0.09 ± 0.03       | 0.089 ± 0.030      |
the first acoustic peak could be changed. However, the position of the peaks stays almost unchanged because the background expansion histories are the same.

As shown in [39, 40], some class of models with mass-varying neutrinos suffers from the adiabatic instability at the first-order perturbation level. This is caused by an additional force on neutrinos mediated by the quintessence scalar field and occurs when its effective mass is much larger than the Hubble horizon scale, where the effective mass is defined by $m_{\text{eff}}^2 = \frac{d^2 V_{\text{eff}}}{d \phi^2}$. To remedy this situation one should consider an appropriate quintessential potential which has a mass comparable to the horizon scale at present, and the models considered in this paper are the case [13]. Interestingly, some authors have found that one can construct viable MVN models by choosing certain couplings and/or quintessential potentials [41]–[43]. Some of these models even realize $m_{\text{eff}} \gg H$.

In figure 11, masses of the scalar field relative to the horizon scale $m_{\text{eff}}/H$ are plotted. We find that $m_{\text{eff}} < H$ for almost all periods and the models are stable. We also depict in figure 11 the sound speed of neutrinos defined by $c_s^2 = \frac{\delta P_{\nu}}{\delta \rho_{\nu}}$ with a wavenumber $k = 2.3 \times 10^{-3}$ Mpc$^{-1}$.

In summary, we investigate dynamics of dark energy with mass-varying neutrinos. We show and discuss many interesting aspects of the interacting dark energy with neutrinos scenario: (1) to explain the present cosmological observation data, we do not need to tune the coupling parameters between neutrinos and the quintessence field, (2) even with

**Figure 8.** Same as figure 6, but for exponential type models.
Figure 9. Examples of the total neutrino mass contributions in power spectrum with $M_\nu = 0.9$ eV (left panel) and with $M_\nu = 0.3$ eV (right panel). Here the variable $\lambda$ is equal to $\alpha$.

Figure 10. Differences between the CMB power spectra with and without the scattering term in the geodesic equation of neutrinos with the same cosmological parameters.

A inverse power law potential or exponential type potential, which seem to be ruled out from the observation of the $\omega$ value, we can see that the apparent value of the equation of states can be pushed down to less than $-1$, (3) as a consequence of global fit, the cosmological neutrino mass bound beyond the $\Lambda$CDM model was first obtained with the value $\sum m_\nu < 0.87$ eV (95% C.L.). More detailed discussions and theoretical predictions on the equation of state and on the absolute mass bound of neutrinos from beta decays and cosmological constraints will appear in a further paper [44].
Primordial neutrinos, cosmological perturbations in interacting dark-energy model

Figure 11. Left panel: typical evolution of the effective mass of the quintessence scalar field relative to the Hubble scale, for all models considered in this paper. Right panel: typical evolution of the sound speed of neutrinos $c_s^2 = \delta P_\nu/\delta \rho_\nu$ with wavenumber $k = 2.3 \times 10^{-3}$ Mpc$^{-1}$, for models as indicated. The values stay positive starting from $1/3$ (relativistic) and neutrinos are stable against the density fluctuation.

Acknowledgments

We would like to thank L Amendola, O Seto, S Carroll and L Schrempp for useful comments and discussion. KI thanks C van de Bruck for useful communications. KI also thanks KICP and the National Taiwan University for kind hospitality where the most part of this work was done. KI’s work is supported by a Grant-in-Aid for JSPS Fellows. YYK’s work is partially supported by Grants-in-Aid for NSC in Taiwan, Center for High Energy Physics (CHEP)/KNU and APCTP in Korea.

Appendix A. Consistency check

The form of $\kappa$ can also be obtained by demanding conservations of energy and momentum, i.e. demanding that $\nabla_\mu \delta T_\nu^\mu + \nabla_\mu \delta T_\nu^\mu = 0$. Let us begin by considering the divergence of the perturbed stress–energy tensor for the scalar field:

$$\nabla_\mu \delta T_\nu^\mu = -a^{-2} \left( \ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV}{d\phi} \right) \partial_\nu \delta \phi - a^{-2} \left( \ddot{\phi} + 2\mathcal{H}\dot{\phi} + k^2 \delta \phi + a^2 \frac{d^2 V}{d\phi^2} \right) \partial_\nu \phi$$

$$= \delta \left( \frac{dV_I}{d\phi} \right) \partial_\nu \phi + \frac{dV_I}{d\phi} \partial_\nu \delta \phi,$$

where in the last line we used equations (3) and (19). The divergence of the perturbed stress–energy tensor for the neutrinos is given by

$$\nabla_\mu \delta T_\nu^\mu = -\delta \rho - (\rho + P) \partial_\nu v_i - 3\mathcal{H}(\delta \rho + \delta P) - \frac{1}{2} \dot{\mathcal{H}}(\rho + P)$$

(JCAP06(2008)005)
Primordial neutrinos, cosmological perturbations in interacting dark-energy model

for the time component and
\[
\nabla^\mu \delta T^\mu_i = (\rho + P)\dot{v}_i + (\dot{\rho} + \dot{P})v_i + 4\mathcal{H}(\rho + P)v_i + \partial_i P + \partial_j \Sigma_j^i \tag{A.3}
\]

for the spatial component. Let us check the energy flux conservation, for example, starting with the energy flux in neutrinos (in \(k\)-space):
\[
(\rho_\nu + P_\nu)\theta_\nu = 4\pi k a^{-4} \int q^2 dq \, q f_0(q)\Psi_1, \tag{A.4}
\]

where \(\theta_\nu = i k^i v_\nu^i\). Differentiating with respect to \(\tau\), we obtain
\[
(\rho_\nu + P_\nu)\dot{\theta}_\nu + (\dot{\rho}_\nu + \dot{P}_\nu)\theta_\nu = 4\pi k a^{-4} \int q^2 dq \, q f_0 \dot{\Psi}_1 - 4\mathcal{H}(\rho_\nu + P_\nu)\theta_\nu. \tag{A.5}
\]

Let us consider the first term on the right-hand side of the above equation. This gives
\[
4\pi k a^{-4} \int q^2 dq \, q f_0 \Psi_1 = 4\pi k a^{-4} \int q^2 dq f_0 \left[ \frac{1}{3} q k (\Psi_0 - 2\Psi_2) + \kappa \right] = k^2 \delta P_\nu - k^2 (\rho_\nu + P_\nu)\sigma_\nu + \frac{1}{3} 4\pi k^2 a^{-4} \int q^2 dq \, q^2 \frac{\partial \epsilon}{\partial \phi} f_0 \delta \phi - 4\pi k a^{-4} \int q^2 dq \, q f_0 \kappa,
\]

where \(\sigma_\nu\) is defined as \((\rho + P)\sigma = -(k i k_j - \frac{1}{3} \delta_{ij}) \Sigma_j^i\) and expressed by the distribution function as
\[
(\rho_\nu + P_\nu)\sigma_\nu = \frac{8\pi}{3} a^{-4} \int q^2 dq \frac{q^2}{\epsilon} f_0(q)\Psi_2. \tag{A.6}
\]

Comparing equation (A.5) with equation (A.3), we find that the divergence of the perturbed stress–energy tensor in the spatial part for the neutrinos leads to
\[
\partial^i \nabla^\mu \delta T^\mu_i = \frac{1}{3} 4\pi k^2 a^{-4} \int q^2 dq \, q^2 \frac{\partial \epsilon}{\partial \phi} f_0 \delta \phi + 4\pi k a^{-4} \int q^2 dq \, q f_0 \kappa. \tag{A.7}
\]

On the other hand, the divergence of the perturbed stress–energy tensor in the spatial part for the scalar field is, from equation (A.1):
\[
\partial^i \nabla^\mu \delta T^\mu_i^{(\phi)} = -k^2 \delta \phi \left( \frac{\partial \ln m_\nu}{\partial \phi} \right) (\rho_\nu - 3P_\nu) = -4\pi k^2 \delta \phi a^{-4} \int q^2 dq \, \frac{\partial \epsilon}{\partial \phi} f_0. \tag{A.8}
\]

These two equations imply that \(\kappa\) should take the form as in equation (52).

Next let us check the energy conservation. Density perturbation in the neutrino is (see equation (28))
\[
\delta \rho_\nu = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \epsilon f_0(q)\Psi_0 + a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{\partial \epsilon}{\partial \phi} \delta \phi f_0. \tag{A.9}
\]
Appendix B. Boltzmann equations in interacting dark energy-neutrinos scenario

By differentiating with respect to \( \tau \), we obtain
\[
\delta \dot{\rho}_\nu = -4 \mathcal{H} \delta \rho_\nu + a^{-4} \int \frac{d^3q}{(2\pi)^3} \dot{f}_0 \Psi_0 + a^{-4} \int \frac{d^3q}{(2\pi)^3} \dot{f}_0 \Psi_0 \\
+ a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{\partial}{\partial \tau} \left( \frac{\partial \epsilon}{\partial \phi} \right) \delta \phi f_0 + a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{\partial \epsilon}{\partial \phi} \delta \phi f_0,
\]
where
\[
\dot{\epsilon} = \left( \mathcal{H} a^2 m^2 + a^2 m^2 \frac{\partial \ln m_\nu}{\partial \phi} \right) / \epsilon,
\]
\[
\frac{\partial}{\partial \tau} \left( \frac{\partial \epsilon}{\partial \phi} \right) = -\mathcal{H} \frac{a^2 m^2}{\epsilon^2} \frac{\partial \epsilon}{\partial \phi} + 2 \mathcal{H} \frac{\partial \epsilon}{\partial \phi} + \frac{\partial^2 \epsilon}{\partial \phi^2}.
\]
Inserting equation (48) for \( \dot{\Psi}_0 \) in the above equation, we obtain
\[
\delta \dot{\rho}_\nu = -3 \mathcal{H} (\delta \rho_\nu + \delta \rho_\nu) - (\rho_\nu + P_\nu) \theta_\nu - \frac{1}{2} \mathcal{(h} (\rho_\nu + P_\nu) + a^{-4} \int \frac{d^3q}{(2\pi)^3} \dot{f}_0 \delta \phi.
\]\nComparing with equation (A.2), we find
\[
\nabla_\mu \delta T_\mu^{(\nu)} = -a^{-4} \int \frac{d^3q}{(2\pi)^3} f_0 \left( \frac{\partial^2 \epsilon}{\partial \phi^2} \delta \phi + \Psi_0 \frac{\partial \epsilon}{\partial \phi} \right) \dot{\phi} - a^{-4} \int \frac{d^3q}{(2\pi)^3} f_0 \frac{\partial \epsilon}{\partial \phi} \delta \phi,
\]
which is found to be equal to \(-\nabla_\mu \delta T_\mu^{(\phi)} = -\delta (dV_1 / d\phi) \dot{\phi} - (dV_1 / d\phi) \delta \dot{\phi}.
\]

Appendix B. Boltzmann equations in interacting dark energy-neutrinos scenario

From the Lagrangian \( L = -m(\phi) \sqrt{-g_{\mu\nu} x^\mu x^\nu} \), the Euler–Lagrange equation is given by
\[
\frac{d}{d\lambda} \left( \frac{\partial L}{\partial \dot{x}^\mu} \right) = \frac{\partial L}{\partial x^\mu},
\]
where
\[
\frac{\partial L}{\partial x^\mu} = P_\mu = m(x^\mu) \frac{\dot{x}^\mu}{\sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}},
\]
\[
\frac{\partial L}{\partial \dot{x}^\mu} = -m \frac{\partial m}{\partial x^\mu} \sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta} + m(x^\mu) \frac{g_{\alpha\beta\mu} \dot{x}^\alpha \dot{x}^\beta}{2 \sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta}}.
\]
Therefore equation (B.1) becomes
\[
\frac{1}{\sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta}} \frac{d}{d\lambda} \left( m(x^\mu) \frac{\dot{x}^\mu}{\sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta}} \right) + m(x^\mu) \frac{g_{\alpha\beta\mu} \dot{x}^\alpha \dot{x}^\beta}{2} = -\frac{\partial m}{\partial x^\mu}.
\]
By using the relation \( ds = \sqrt{-g_{\alpha\beta} x^\alpha x^\beta} d\lambda \), we obtain
\[
P^\mu = m(x^\mu) \frac{\dot{x}^\mu}{\sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta}} = m(x^\mu) \frac{dx^\mu}{ds},
\]
where
and equation (B.4) becomes
\[
\frac{d}{ds} \left( m(x^\nu) g_{\mu \beta} \frac{dx^\beta}{ds} \right) - \frac{m(x^\nu)}{2} g_{\alpha \beta, \mu} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = - \partial m \frac{dx^\mu}{ds}.
\] (B.6)

\[
\frac{d}{ds} (g_{\mu \beta} P^{\beta}) - \frac{1}{2} g_{\alpha \beta, \mu} P^\alpha \frac{dx^\beta}{ds} = - \partial m \frac{dx^\mu}{ds}.
\] (B.7)

With simple calculation, finally we obtain the relations:
\[
\frac{dP^\nu}{ds} + \Gamma^\nu_{\alpha \beta} P^\alpha \frac{dx^\beta}{ds} = - g^{\nu \mu} \partial m \frac{dx^\mu}{ds},
\] (B.8)
\[
P^{0} \frac{dP^{0}}{d\tau} + \Gamma^{0}_{\alpha \beta} P^{\alpha} P^{\beta} = - m g^{0 \mu} m_{\nu}.
\] (B.9)

For \( \mu = 0 \) component, equation (B.9) can be expressed as
\[
\frac{1}{2} \frac{d}{d\tau} (P^{0})^{2} + \Gamma^{0}_{\alpha \beta} P^{\alpha} P^{\beta} = - m g^{0 \mu} m_{\mu}.
\] (B.10)

Since \( P^{0} = g^{00} P_{0} = a^{-2} \epsilon \), each terms of the equation (B.10) are given by:

First term = \( -2a^{-4} Hq^{2} + a^{-4} q \frac{dq}{d\tau} - a^{-2} Hm^{2} + a^{-2} m \frac{dm}{d\tau} \),
\] (B.11)

Second term = \( 2a^{-4} Hq^{2} + a^{-2} Hm^{2} + a^{-4} \frac{1}{2} h_{ij} q^{i} q^{j} \),
\] (B.12)

Third term = \( a^{-2} m \frac{dm}{d\tau} \).
\] (B.13)

Since the first term includes the total derivative w.r.t. comoving time, we obtain finally the equation (42) in section 3.3:
\[
\frac{dq}{d\tau} = - \frac{1}{2} h_{ij} q n^{i} n^{j} - a^{2} m \frac{dm}{d\tau} \frac{dx^{i}}{q} \frac{dx^{j}}{d\tau}.
\] (B.14)

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