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Reduction of structure to parabolic subgroups. (English) Zbl 07581369
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Summary: Let $G$ be an affine group over a field of characteristic not two. A $G$-torsor is called isotropic if it admits reduction of structure to a proper parabolic subgroup of $G$. This definition generalizes isotropy of affine groups and involutions of central simple algebras. When does $G$ admit anisotropic torsors? Building on work of J. Tits, we answer this question for simple groups. We also give an answer for connected and semisimple $G$ under certain restrictions on its root system.

MSC:
11E72  Galois cohomology of linear algebraic groups
20G15  Linear algebraic groups over arbitrary fields
20G07  Structure theory for linear algebraic groups
11E39  Bilinear and Hermitian forms
16W10  Rings with involution; Lie, Jordan and other nonassociative structures

Keywords:
Galois cohomology; algebraic geometry; rings and algebras

Software:
MathOverflow

Full Text: DOI arXiv

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