Optimal Hermitian Wavelet Filtering and Envelope Spectrum Based Bearing Defect Diagnosis

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Abstract. Local faults in rolling bearings often give rise to periodic impulses in vibration signal. The periodic impulses can be extracted by resonance demodulation technique. Therefore, the resonance demodulation technique is a popular technique of rolling bearing defect recognition and diagnosis. However, the selection of optimal band-pass filter parameter is usually more difficult. The proposed method is the union of the continuous Hermitian wavelet transform as time frequency tool and the spectral kurtosis as transient impulse indicator. Firstly, continuous Hermitian wavelet transform is utilized to process the vibration signal and the wavelet amplitude and phase diagram are obtained. Secondly, the spectral kurtosis is calculated to select the best band-pass filter parameters. The proposed procedure and spectral kurtosis are used to optimize the parameters of continuous Hermitian wavelet for bearing defect detection. The bearing failure test results show that the proposed method can effectively extract the transient features and diagnose the bearing fault.

1. Introduction
There are many parts in rotating machinery, among which rolling bearing is a commonly used mechanical part. The health of rolling bearing has an important influence on the safe operation of rotating machinery. In many fault diagnosis and processing technologies, vibration signal analysis technology is often used, and has been extensive utilized in fault detection and identification of rotating machinery. These methods include fast Fourier transform, correlation analysis, wavelet transform, power spectrum analysis, WVD and so on. These methods have been extensively utilized in rolling bearing fault detection and diagnosis [1-3]. Among them, resonance demodulation technology is one of the most generally utilized bearing fault diagnosis and identification methods. However, in the traditional resonant demodulation technique, the choice of the parameters of the band-pass filter often depends on the experience of the operator. Before obtaining the best parameters of the band-pass filter, many attempts are needed to adapt to the roller bearing defect diagnosis. Before obtaining the optimal band-pass filter parameters, the operator needs to try many attempts, thus it is unable to adapt to the changes in the bearing fault diagnosis [4].
Wavelet transformation technique is one of the most commonly utilized fault diagnosis technique. It has strong multi-resolution ability both in frequency domain and time domain. Therefore, wavelet transform has become an effective and reliable technique to extract defect characteristics of rolling bearing [5]. However, the performance of wavelet transform mainly depends on the properties of wavelet mother function. When the shape of the mother wavelet is close to the vibration signal to be analyzed, the effect of fault diagnosis will be better. On the contrary, when the shape of mother wavelet is very different from the vibration signal to be analyzed, it is difficult to achieve the ideal
analysis effect. Therefore, in order to more effectively pick up the defect characteristics of vibration signals, it is necessary to choose the optimal wavelet mother function.

In order that improvement the reliability and accuracy of bearing fault feature extraction, an improved resonance demodulation technique based on continuous Hermitian wavelet transformation and spectral kurtosis is proposed. Firstly, a great number of continuous Hermitian wavelet coefficients are obtained by continuous Hermitian wavelet transform to represent the distance between the signal and the specific mother wavelet. Then, the spectrum kurtosis is used to select the best band-pass filter frequency band to suppress noise and highlight the transient impact characteristics of bearing with inner or outer defect. In the end, the envelope spectrum of the filtered signal of the optimal band-pass filter is calculated, and the bearing fault detection and identification are carried out by using the envelope spectrum analysis. The bearing fault test results show that the optimal Hermitian wavelet denoising technology can effectively suppress the noise, highlight the bearing fault characteristics, accurately recognize the rolling bearing defect, and can effectively improve the reliability and accuracy of bearing fault diagnosis.

2. The Optimal Filtering Technique Based on Continuous Hermitian Wavelet

2.1. Continuous Hermitian Wavelet Transformation

The Gauss function can be given by:

$$h(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$$

The first and second derivatives of a Gaussian function are given by:

$$\psi^{(1)}(t) = \frac{dh(t)}{dt} = -\frac{1}{\sqrt{2\pi}} te^{-\frac{1}{2}t^2}$$

$$\psi^{(2)}(t) = \frac{d^2 h(t)}{dt^2} = \frac{1}{\sqrt{2\pi}} (1-t^2) e^{-\frac{1}{2}t^2}$$

![Figure 1](image)

**Figure 1.** (a) The imaginary part and real part of Hermitian wavelet, (b) its spectrum. Based on Eq.(2) and Eq.(3), Hermitian mother wavelet can be defined as:
\[ \psi(t) = \psi^{(2)}(t) - i \psi^{(1)}(t) = (1 + it - t^2) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \]  

(4)

Then Fourier transform of Eq.(4) can be calculated as follows:

\[ \hat{\psi}(\omega) = (\omega^2 + \omega)e^{\frac{1}{2}i\omega^2} \]  

(5)

Figure 1 demonstrates the time domain wave shape and its spectrum of the Hermitian mother wavelet. As Hermitian mother wavelet has a single oscillation waveform and has a real frequency spectrum, it is very suitable to pick up the singularity features of bearing defect vibration signals.

The Hermitian daughter wavelet in time and frequency domain is calculated as follow, respectively:

\[ \psi_{ab}(t) = \frac{1}{a}\left(1 + i t - b - \left(\frac{t - b}{a}\right)^2\right) e^{\frac{1}{2} \left(\frac{t-b}{a}\right)^2} \]  

(6)

\[ \hat{\psi}_{ab}(\omega) = [(a\omega)^2 + a\omega]e^{\frac{1}{2}(a\omega)^2} \]  

(7)

Where \( a \) is the parameter of wavelet scale, \( b \) is the parameter of wavelet shift.

Equation (6) states clearly continuous Hermitian wavelet is complex in time domain. Equation (7) shows that continuous Hermitian wavelet transform is real in frequency domain.

In frequency domain, the Hermitian wavelet transformation of periodic signal \( x(t) \) can be computed as follows:

\[ W_{t}(t,a) = \frac{1}{a} F^{-1}[X(\omega) \cdot aW(a\omega)] \]  

(8)

Where \( X(\omega) \) and \( aW(a\omega) \) are the Fourier transform of the signal \( x(t) \) and dilated wavelet \( \psi_{ab}(t) \), respectively.

2.2. Spectral Kurtosis

According to the magnitude of the spectral kurtosis, the intensity of impulse impact transient feature in the original vibration signal is determined. The greater the kurtosis amplitude is, the more severe the transient impact feature is in the vibration signal. Otherwise, the transient impact in the signal is not obvious. This method effectively solves the problem that Fourier transform can’t detect and pick up the transient impulsive feature in the original vibration data.

Spectral kurtosis represents the peak value of the probability density function of time-frequency distribution, which is formulated as the normalized fourth-order cumulative quantity:

\[ K_{x}(f) = \frac{S_{4x}(f)}{S_{2x}^2(f)} - 2 \]  

(9)

Where \( S_{4x}(f) \) is the fourth-order cumulative quantity of the spectral density of vibration signal \( x(t) \) and \( S_{2x}^2(f) \) is the second-order cumulative quantity.

When the original vibration signal \( x(t) \) contains noise, the spectral kurtosis can be calculated by the following formula:

\[ K_{x}(f) = \frac{K_{x}(f)}{|1 + \rho(f)|^2} \]  

(10)
Where $K(f)$ is spectral kurtosis for original vibration data $x(t)$ without noise and $\rho(f)$ is the count backwards of signal to noise ratio. The magnitude of the spectral kurtosis reflects the intensity of the transient impact phenomenon of the original vibration signal. It is related to the resonance frequency band of the selected bearing defect vibration data. The premise of accurate bearing defect characteristics extraction and recognition is to search the frequency band center and its bandwidth where the maximum value of the spectral kurtosis lies.

3. Optimal Continuous Hermitian Wavelet Transformation Based Bearing Damage Recognition

The experimental ball bearing is only utilized to research one kind of localized surface defect: bearing inner or outer ring fault. The tested rolling bearing is used discharge device to machine a groove on the outer or inner ring. Then the localized fault is a formed on the outer or inner ring. The defect was 1 mm deep and 1.5 mm wide. The gearbox is driven by three-phase alternating current motor. The input spindle speed is 1500 r/min, in other words, $f_r$ is 25Hz. The bearing model is deep groove ball bearing 6208. The contact angle $\alpha = 0^\circ$. The number of balls is 10. The bearing pitch diameter D is 97.5mm and the diameter of the ball d is $\frac{55}{3}$ mm. The rolling bearing fault characteristic frequencies of inner ring and outer ring are 148.5Hz and 101.5 Hz [5].

3.1. Bearing Inner Ring Defect Detection

The sampled bearing inner ring defect vibration data is displayed in Figure 2, the traditional envelope spectrum is displayed in Figure 3. In Figure 3, there is no significant spectral peak near the bearing defect feature frequency $f_{inner}$ and its higher harmonics. Therefore, the classical envelope spectrum analysis can not accurately pick up the transient impulse characteristics of bearing defect in noise environment, so it is very hard to effectively detect bearing localized defect.
Figure 4 displays the magnitude and phase wavelets graph of continuous Hermitian wavelet transformation. The continuous Hermitian wavelet spectral kurtosis is calculated according to the kurtosis of amplitude at each wavelet scale, which is displayed in figure 5. To demonstrate the reliability and effectiveness of the provided technique compared with the traditional envelope spectrum, Figure 6 shows the optimal kurtosis filtered signal with wavelet scales of 0.7 and its envelope spectrum. In Figure 6, we can clearly see the feature frequency and higher harmonic of bearing inner ring defect, which shows that the proposed approach of spectral kurtosis de-noising and envelope spectrum method based on continuous Hermitian wavelet transformation can effectively and correctly extract bearing defect feature.

![Figure 4. Hermitian wavelet magnitude and phase graph with inner ring defect.](image1)

![Figure 5. Spectral kurtosis with inner ring defect.](image2)
Figure 6. The optimal kurtosis filtered data and envelope spectrum with inner ring defect.

3.2. Bearing Outer Ring Defect Detection

Figure 7. Vibration data with outer ring defect.

Figure 8. Envelope spectrum with outer ring defect.
The sampled bearing outer ring defect vibration signal is displayed in Figure 7. Figure 8 is the traditional envelope spectrum. In Figure 8, there is no significant spectral peak near the bearing defect feature frequency $f_{outer}$ and its higher harmonics. Therefore, the classical envelope spectrum analysis can not accurately extract the transient impact characteristics of bearing fault in noise environment, so it is difficult to effectively diagnose bearing localized fault.

Figure 9 displays the magnitude and phase wavelets graph of continuous Hermitian wavelet transformation. The continuous Hermitian wavelet spectral kurtosis is calculated according to the kurtosis of amplitude at each wavelet scale, which is displayed in Figure 10. Figure 11 shows the optimal kurtosis filtered signal with wavelet scales of 1.3 and its envelope spectrum. In Figure 11, we can clearly see the feature frequency and higher harmonic of bearing outer ring defect, which shows that the proposed approach of spectral kurtosis de-noising and envelope spectrum approach based on continuous Hermitian wavelet transformation can correctly extract bearing defect feature.
4. Conclusion
Wavelet transformation has ability of multi-resolution and spectral kurtosis is efficient to pick up the transient impulse impact characteristics of continuous impulse buried in the vibration data. Therefore, in order that effectively extraction the components of weak defect impulse hidden in the vibration signal, envelope spectrum analysis technology based on optimal continuous Hermitian wavelet transformation de-noising is proposed by combining Hermitian wavelet transform and spectral kurtosis. This technique has obvious advantages in extracting transient signals from vibration signals, which is generally the indicator of early faults of gearbox system. Experimental results demonstrate that the optimal continuous Hermitian wavelet de-noising combined with envelope spectrum technology can effectively diagnose bearing faults.

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