Application of Clayton Copula to identify dependency structure of Covid-19 outbreak and average temperature in Jakarta Indonesia

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Abstract. Today Indonesia is experiencing health problems that are also being faced by all countries in the world, namely Covid-19. Jakarta, the capital of the state of Indonesia, is one of the provinces that has been the epicenter of the Covid-19 cases. Aim of study is to determine dependency between Covid-19 and maximum temperature in Jakarta, Indonesia. Data of Covid-19 cases used are daily cumulative cases, new cases, and deaths. The correlations used are Pearson, Spearman, and Kendall. The correlation coefficient only provides information on the measure of the two variable relationship and does not show the structure of dependency between these variables. One of the methods used to see the dependency structure between variables is copula. One of the copula that is widely used is the clayton copula because of its flexible characteristics. Meanwhile, to see the dependency structure between variables will be used the Copula method from Clayton Copula. The results show that maximum temperature is significantly associated with the Covid-19 pandemic. Based on clayton copula model, the small parameters indicate small dependencies between Covid-19 and maximum temperature.

1. Introduction
Corona Virus Diseases (Covid-19) is one of the health problems being faced by almost all countries in the world today, Covid-19 was first spread in Wuhan City, China since December 2019 and has been declared as a global pandemic since March, 11 2020 by the World Health Organization (WHO). WHO generally mentions the symptoms of Covid-19, namely fever, coughing, shortness of breath, until hard to breathe. According to WHO until September 4, 2020 there were 26,171,112 Covid-19 positive people spread in 216 countries and 865,154 of them died. Meanwhile, in Indonesia Covid-19 has spread rapidly to all provinces with a total of positive confirmed as many as 187,537 and as many as 7,832 people died in the same day.

Previous studies related to coronavirus, such as SARS and MERS, state that climate and weather factors influence the spread of the disease. Dalziel et al., concluded that climatic conditions were classified as the top predictors of coronavirus disease [1]. Yuan et al., stated humidity, temperature, and wind speed have a very important role in the transmission of infectious diseases [2]. In addition, Bull reports that pneumonia mortality is highly correlated with weather changes [3].

Recent studies in several countries have shown that several climatic and weather factors can contribute to the severity and level of spread associated with Covid-19. In China, Wang et al. (2020) show that environmental conditions such as humidity and temperature can affect Covid-19 transmission when compared with other respiratory viruses, indicating a decrease in the spread of
Several studies have been carried out on Covid 19, one of which is the influence of weather parameters on the Covid 19 case in DKI Jakarta Province as the epicenter in Indonesia. Tosepu tested the correlation using Spearman rho and concluded that the average temperature has a significant correlation with the Covid 19 cases [9]. Jakarta as the capital of the State of Indonesia is the epicenter and until June 25 contributed the most positive Covid-19 cases in Indonesia. On July 7, 2020, there were 12,857 positive confirmed cases, second only to East Java Province.

One of the most widely used dependency measures is the Pearson correlation coefficient. Pearson's correlation measures the magnitude of influence between two normally distributed variables. However, this correlation is only sensitive to the linear relationship of two variables. Of course, the use of correlation coefficients on non-linear type observations will give results that are not in accordance with the actual situation. In addition, Pearson's correlation coefficient is only a measure of scalar dependencies and cannot provide much information about the structure of non-linear dependencies between two variables [10]. However, there are other dependency measures that can be used for data that have non-linear relationships, such as the Kendall-tau correlation and the rank-spearman correlation. Both of these correlations have been developed with a more robust nature than the Pearson correlation and are also more sensitive to nonlinear relationships.

Ahdika, et.al [11] states that there is an alternative correlation to measure the dependency structure of several variables that can accommodate both linear and nonlinearity and this size is expressed in a function called Copula. Different from Tosepu [9] using the Spearman correlation to see the relationship between Covid-19 cases and the weather factor, this study also captures the dependency structure of this relationship by copula approach. The most widely used copula is the copula clyaton because of its flexibility in capturing strong dependencies on the left-tail. Therefore, the purpose of this study is to applying Copula method in determining the correlation and dependencies between the Covid-19 case and maximum temperature in Jakarta. The variables observed in Covid-19 cases are the number of additional cases each day (daily case), the total number of cases each day (cumulative case) and the number of deaths per day (death case). The correlations used are Pearson, Spearman, and Kendall. Meanwhile, to see the dependency structure between variables will be used the Clayton Copula method.

2. Data and method
Data are secondary data obtained from the official website of the Task Force for the Acceleration of Handling Covid-19 BNPB https://covid19.bnpb.go.id/ and the DATA ONLINE-PUSAT DATA ONLINE-BMKG on the website www.dataonline.bmkg.go.id. Data of Covid-19 taken are cumulative cases, positive cases and daily deaths from positive confirmed Covid-19 patients in Jakarta. As for the weather factor, data are the maximum temperature at the Kemayoran Jakarta Meteorological Station. The type of data is time series since March, 13th to August, 31st 2020.

The stages of data analysis are:
1. Presenting Covid-19 data and weather factors in Jakarta in the form of descriptive statistics and histograms.
2. Identifying the relationship between Covid-19 variables and weather factors visually using scatterplot and calculating the correlation value Pearson, Spearman and Kendall-tau correlation between variables.
3. Identifying the dependency structure between temperature and humidity using the copula approach.
4. Constructing copula function by determining the best parameter estimation using R.

**Correlation of two variables**

2.1. Linear and non-linear correlation

Correlation analysis is statistical method used to determine the strength of such relationships between two variables. The strength value is measured by a single number called a correlation coefficient [12]. The correlation coefficient must range -1 to 1, a value of -1 or 1 shows perfect linear relationship and a value 0 shows no linear relationship. This value refers to the estimate \( r \) as the Pearson correlation coefficient. Pearson correlation coefficient as dependency measure was developed by Pearson (1920), and previously the idea of linear relationships was set out by Galton (1889) [11].

Pearson correlation coefficient between two variables \( X \) and \( Y \) defined by

\[
\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}
\]

where \( \sigma_{xy} = \text{Cov}(X,Y) \), \( \sigma_x^2 \) and \( \sigma_y^2 \) are variance of \( X \) and \( Y \) respectively. Linear correlation is a natural measure of dependence for elliptical distributions that variables have bivariate normal distribution. The Pearson product moment correlation estimator is acceptable for normally distributed uncontaminated data and has very low output for heavier tailed or undistributed data, both for bivariate and multivariate data. [13]

It is not efficient to use Pearson correlation on non-linear observation results. To avoid misinterpretation of data which the normal assumption is not fulfilled, one of the dependence measures that can be applied is Kendall’s Tau. Kendall’s tau utilizes concordant and discordant concepts.

**Definition 1 (concordant and discordant) [14]:**

Let denote two observations from two continuous random variables \( (X,Y); (x_i, y_i) \) and \( (x_j, y_j) \). Observations \( (x_i, y_i) \) and \( (x_j, y_j) \) are concordant, if \( x_i < x_j \) and \( y_i < y_j \), or if \( x_i > x_j \) and \( y_i > y_j \). Analogously, \( (x_i, y_i) \) and \( (x_j, y_j) \) are discordant if \( x_i < x_j \) and \( y_i > y_j \), or if \( x_i > x_j \) and \( y_i < y_j \).

Concordant and discordant definition is able to write on alternative formulation as follow: \( (x_i, y_i) \) and \( (x_j, y_j) \) are concordant, if \( (x_i - x_j)(y_i - y_j) > 0 \) and discordant if \( (x_i - x_j)(y_i - y_j) < 0 \)

**Definition 2 (Kendall)[14]:**

Let \( \{(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\} \) denotes a random sample of \( n \) observations from a vector \( (X,Y) \) of continuous random variables. There are \( \binom{n}{2} \) different pairs \( (x_i, y_i) \) and \( (x_j, y_j) \) of observations in the sample, and each pair is either concordant or discordant; let \( c \) and \( d \) denote the number of concordant and discordant pairs respectively, an estimate of Kendall’s rank correlation for the sample is given by

\[
t = \frac{c - d}{c + d} = \frac{c - d}{\binom{n}{2}}
\]

(2)

For \( (x_1, y_1) \) and \( (x_2, y_2) \) independent and identically distributed random vectors, the population version \( \tau_{XY} \) is defined by:
\[
\tau_{XY} = [(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0]
\]
\[
= 2P[(X_1 - X_2)(Y_1 - Y_2) > 0] - 1
\tag{3}
\]

Kendall’s Tau is extremely useful for statistical purposes. Many datasets in practice are not multivariate normally distributed and may distribute with heavier tailed margins. In this situation the standard estimator of correlation, based on normal assumptions and maximum-likelihood theory, is both inefficient and lacks robustness. Therefore, Kendall’s Tau can be used to build a robust estimator for both linear and non-linear correlation [13].

Another measure of non-linear correlation is the Spearman-rho. It is nonparametric measure of association between two variables \(X\) and \(Y\) is given by the rank correlation coefficient. The Spearman-rho correlation test does not carry any assumptions about the distribution of the data and is the appropriate correlation analysis when the variables are measured on a scale that is at least ordinal. The following formula is used to calculate the Spearman rank correlation:

\[
\rho = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n(n^2 - 1)}
\tag{4}
\]

2.2. Bivariate Copula

Copula is a joint distribution function of several marginal distribution functions. Copula is used to analyze the dependence of random variables in the structure described joint function. Copula is a multivariate distribution function \(F\) of \(n\) random variables \(U_1, U_2, \ldots, U_n\) possessing uniform marginal distributions \(F_1, F_2, \ldots, F_n\) in a unit interval, i.e \(U_i \sim F_i\) and \(F_i \sim U(0,1), i = 1, 2, \ldots, n\). Copula denoted \(C\) is function possessing domain \(S[0,1]^n = [0,1] \times [0,1] \times \cdots \times [0,1]\) and range \([0,1]\), in other words \(C: [0,1]^n \rightarrow [0,1]\). An essential theory of copula is the Sklar’s Theorem. Sklar’s Theorem explains that copula connect any multivariate distribution and its marginal distribution.

In case with two random variables, the copula is known as the bivariate copula. As a bivariate function model, bivariate copula has the properties of a bivariate distribution function. Further discussion is restricted to the bivariate copula.

Definition (bivariate Copula) [15]:

A bivariate copula is a function \(C: [0,1]^2 \rightarrow [0,1]\) satisfying the following properties:

a. Grounded: \(C(u, 0) = C(0, v) = 0\)

b. Uniform marginals: \(C(u, 1) = u\) and \(C(1, v) = v\)

c. 2-increasing: \(S(C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0\) for all \(u_2 > u_1\) and \(v_2 > v_1\).

The definition of bivariate copula is based on the Sklar Theorem. The following is the Sklar theorem for bivariate copula:

Theorem (bivariate Sklar theorem):

Suppose that \((x, y)\) is a joint distribution function with marginal distribution functions \(F(x)\) and \(G(y)\). Then there is a copula \(C\) such that \(\forall x, y \in R\)

\[
H(x, y) = C(F(x), G(y))
\tag{5}
\]

If \(F(x)\) and \(G(y)\) continue, then \(C\) is unique. Otherwise if \(C\) is copula, \(F(x)\) and \(G(y)\) are distribution functions, then \(H(x, y)\) is a joint distribution function with marginal distribution function \(F(x)\) and \(G(y)\).

There are several methods to construct Copula, one of them is through a generator function \(\varphi(t)\). The generator function has to satisfy \(\varphi(1) = 0\) and \(\varphi(t)\) is a monotonically increasing function for \(t \in [0,1]\). Copula built through a generator function is called the Archimedean Copula. To estimate
the Archimedean copula, we only need to find functions which will serve as generators and define the corresponding copula. Archimedean copulas are capable of capturing wide ranges of dependence in the upper and lower tail dependences. In general, the form of bivariate Archimedean copulas is

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)), 0 \leq u, v \leq 1$$ (6)

where \( \varphi \) is the generator of \( C \) with \( \varphi(0) = \infty \), \( \varphi(1) = 0 \). Archimedean Copula \( C \) has the properties below [16]:

1. \( C \) is symmetric in its \( d \) arguments, for example in bivariate case \( C(u, v) = C(v, u) \), for \( \forall (u, v) \in [0,1] \).
2. \( C \) is associative, for example in trivariate case \( C(C(u_1, u_2), u_3) = C(u_1, C(u_2, u_3)) \), for \( \forall \{u_1, u_2, u_3\} \in [0,1] \).
3. If \( a > 0 \) is any constant, then \( \varphi \) is also a generator of \( C \).

The generator function defines the specifications of the Archimedean Copula. If \( \varphi(0) \) is finite, then the archimedian copula is linked using pseudo-inverse of \( \varphi \) defined by:

$$\varphi^{-1}(t) = \begin{cases} \varphi^{-1}(t), 0 \leq t \leq \varphi(0) \\ 0, \varphi(0) \leq \infty \end{cases}$$ (7)

Let \( U \) and \( V \) be uniform \([0,1]\) variables with the joint distribution function is the Archimedean Copula, then

$$K_C(t) = t - \frac{\varphi(t)}{\varphi'(t)}, 0 \leq t \leq 1$$ (8)

is univariate distribution function of the random variables \( T = C(U, V) \) [14]. This function can be used to determine the best parametric copula that fits the data. Copula parameter can be associated with the size of the Kendall’s Tau association. There is a relationship between Kendall’s Tau and Copula parameter, which Kendall’s Tau can be obtained through the copula function. Based on theorem (Kendall’s tau), Kendall’s Tau of Archimedean Copula is:

$$\tau_C = 1 + 4 \int_0^1 \frac{\varphi(t)}{\varphi'(t)} dt$$ (9)

2.3. Clayton Copula
Clayton Copula was first introduced by Copula in 1978. The Copula is an Archimedean family with a generator function:

$$\varphi(t) = \frac{t^{-\theta} - 1}{\theta}, \theta \in [-1, \infty], \theta \neq 0$$ (10)

so,

$$\varphi^{-1}(t) = (\theta t + 1)^{-\frac{1}{\theta}}, \theta > 0$$ (11)

Clayton Copula has the copula function as follow

$$C(u, v) = \max \left( u^{-\theta} + v^{-\theta} - 1, 0 \right)^{\frac{1}{\theta}}, \theta > 0$$ (12)

Copula Clayton can only model the positive dependency between two random variables. The Clayton model is strong dependence on lower values (left-tail) and weak to model the dependence of upper values (right-tail). [17]

Kendall’s Tau of Clayton Copula is given by
\[ \tau_C = \frac{\theta}{\theta + 2}, \theta \in [-1, \infty], \theta \neq 0 \]  

(13)

One of parameter estimation method in Copula is Method-of-moments estimators. As direct transpositions of moment method estimator, moments of the copula replace moments of random variables, such as Kendall’s tau or Spearman’s rho. In bivariate case and one-parameter Copula \( C \), let \( g_\tau \) be the functions defined by the estimator \( \theta \) can be given by:

\[ g_\tau(\theta) = \tau(C) \]  

(14)

Where \( \tau(C) \) is Kendall’s Tau of Archimedean Copula defined by Ferreira (2019) [15]. Method-of-moments estimators based on Kendalls’ tau can be used for the family \( C \) if the function \( g_\tau \) is one-to-one. In that case, the estimator \( \theta_n \) of \( \theta_C \) is simply given by

\[ \theta_n = g_\tau^{-1}(\tau_n) \]  

(15)

Where \( \tau_n \) is the sample version of Kendall’s tau and computed from the original sample [18]. Copula Clayton can only model the positive dependency between two random variables. The Clayton model is strong dependence on lower values (left-tail) and weak to model the dependence of upper values (right-tail). Kendall’s Tau of Clayton Copula is given by

\[ \tau_n = \frac{\theta}{\theta + 2}, \theta \in [-1, \infty], \theta \neq 0 \]

Based on relationship between Kendall’s tau and the parameter \( \theta \) of Clayton, the parameter estimation is:

\[ \theta_{(Clay)}(\tau) = \frac{2\tau_n}{1 - \tau_n} \]  

(16)

3. Result and discussion
The following are a summary of statistics and histograms on Covid-19 variables and average temperature in Jakarta:

| Variable             | Minimum | Mean    | Maximum | Standardize Deviation | Skewness | Kurtosis |
|----------------------|---------|---------|---------|-----------------------|----------|----------|
| Cumulative Cases     | 54.00   | 11370.72| 40086   | 10293.4               | 1.04     | 0.11     |
| Daily Cases          | 1.00    | 232.89  | 1094    | 213.81                | 1.54     | 2.14     |
| Death Cases          | 5.00    | 534.91  | 1197    | 312.5                 | 0.08     | -0.67    |
| Maximum temperature  | 29.00   | 32.85   | 35.60   | 1.07                  | -0.35    | 0.45     |

Each variable in Table 1 consists of 172 observations which are presented in minimum, mean, maximum, standard deviation, skewness, and kurtosis. In normally distributed data, values of skewness and kurtosis tend to be close to zero, which indicates the data is symmetrical and centered or not sloping. Based on the skewness and kurtosis values of the four variables above, it can be suspected that the data are not normally distributed. The non-normality of each variable will affect the linearity of the relationship between the two variables. The shape of the data histogram can also be seen in the following figure:
Unnormally bivariate data can lead to unfulfilled linearity assumptions. To test the linearity correlation, a test using Pearson correlation is carried out. The coefficient values and significance test of the Pearson correlation for the maximum temperature vs cumulative, daily and death cases are presented in Table 2. Based on a significant value α=1%, cumulative and daily cases do not have a linear correlation with the average maximum temperature, while death cases still have a significant linear relationship even though the linear relationship is weak. Furthermore, Spearman’s Rho and Kendall’s Tau correlation analysis will be performed.

In Table 2, the Spearman’s Rho correlation value indicates that cumulative cases and death cases have a relationship with the maximum average temperature. The correlation coefficient both of them are 16% each, and the values is included in the very low correlation. The daily case variable does not indicate any association with the temperature, as compared to the other two variables. The results of the Kendall tau correlation in Table 2 also conclude that cumulative cases and death cases have very low correlation with the maximum mean temperature, while no correlation with the maximum mean temperature is seen in daily cases. The figure 2 show scatter and linear correlation plot between the maximum temperature and Covid-19 cases.

|                         | Maximum Temperature vs Cumulative cases | Maximum Temperature vs Daily cases | Maximum Temperature vs Death Cases |
|-------------------------|----------------------------------------|-----------------------------------|-----------------------------------|
| Pearson Correlation     | 0.14                                   | 0.09                              | 0.20**                            |
| Spearman Correlation    | 0.16*                                  | 0.08                              | 0.16*                             |
| Kendall Tau             | 0.1063*                                | 0.0587                            | 0.1068*                           |

*α=5% **α=1%
Figure 2. Scatter Plot (a) Temperature vs Cumulative Cases; (b) Temperature vs Daily cases; and (c) Temperature vs Death cases.

Figure 2 shows a relatively very scattered data pattern with a very large range on the y-axis. It also gives the information that a relatively very scattered data pattern with a very large range on the y-axis. This also causes the variable relationship to have a weak relationship. Although the maximum temperature has a significant relationship with the cumulative and death case variables, the relationship is relatively small. To see the dependency between cumulative, death case variables and maximum temperature, an analysis was carried out using the copula approach.

Next, the copula approach is continued to see the dependency cumulative, death case variables, and maximum temperature. To estimate the copula parameter, the first step to be done is transforming the variables into uniform marginal distribution. The transformation results with range [0; 1] is presented in figure 3.

In accordance with the purpose of paper, the dependence structure between dependency cumulative, death case variables, and maximum temperature is identified by Clayton Copula. By Method-of-moments, parameter estimations and dependence measures for Covid-19 data and maximum temperature are presented in Table 3.

Figure 3. Scatter Plot Transformation Scale of (a) Temperature vs Cumulative Cases; and (b) Temperature vs Death cases

| Tabel 3. Copula parameter of Covid-19 data and maximum temperature |
|---------------------------------------------------------------|
| Maximum Temperature vs Cumulative cases | Maximum Temperature vs Death Cases |
| Clayton Copula | 0.24 | 0.24 |

At the maximum temperature variable with cumulative cases and death cases, parameter of clayton copula model is 0.24. Based on parameter on Table 3, cumulative and death cases vs maximum temperature have Copula distribution function as follow:

a. Maximum Temperature vs Cumulative cases

\[ C(u, v) = \max \left( (u^{-0.24} + v^{-0.24} - 1)^{\frac{1}{0.24}}, 0 \right) \]

b. Maximum Temperature vs Death Cases

\[ C(u, v) = \max \left( (u^{-0.24} + v^{-0.24} - 1)^{\frac{1}{0.24}}, 0 \right) \]
Copula Clayton is a copula that has a parameter value greater than zero. The parameter values of cumulative and death cases vs maximum temperature, 0.24, are less than 1. The small value of this parameter can be said that the dependence between cumulative and death cases of Covid-19 and the maximum temperature is weak. Based on the dependency structure with the clayton copula model, cumulative and death cases of Covid-19 data and maximum temperature show small dependence on lower values (left-tail). It can be concluded that cumulative and death cases have positive dependence on small maximum temperature or in other words, a low maximum temperature can imply an increase in cumulative and death cases.

4. Conclusion
The increase in daily cases of Covid-19 in Jakarta from day to day continues to increase. Based on the coefficient values of Pearson, only death cases of Covid-19 have the relationship to the maximum temperature, while cumulative and daily cases do not have. Moreover, based on Spearman’s Rho and Kendall’s Tau, maximum temperature are correlated with data on cumulative and death cases and are not correlated with daily cases. Based on the dependency structure with the clayton copula model, it can be concluded that a low maximum temperature can imply an increase in cumulative and death cases. The suggestion that can be recommended for further research is to examine the identification of relationships with other Archimedean copula and the Gaussian copula. This study is limited to determining the parameters of the clayton copula model, testing the significance of clayton copula can be discussed in further research.

Acknowledgment
The authors would like to thank Lembaga Pengelola Dana Pendidikan (LPDP), Ministry of Finance Indonesia, for taking part in finding financial support.

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