Analysis Social Sparsity AudioDeclipper

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Abstract—We develop the analysis (cosparse) variant of the popular audio declipping algorithm of Siedenburg et al. Furthermore, we extend it by the possibility of weighting the time-frequency coefficients. We examine the audio reconstruction performance of several combinations of weights and shrinkage operators. We show that weights improve the reconstruction quality in some cases; however, the best scores achieved by the non-weighted methods are not surpassed with the help of weights. Yet, the analysis Empirical Wiener (EW) shrinkage was able to reach the quality of a computationally more expensive competitor, the Persistent Empirical Wiener (PEW). Moreover, the proposed analysis variant incorporating PEW slightly outperforms the synthesis counterpart in terms of an auditory-motivated metric.

Index Terms—audio declipping; cosparse; sparse; social sparsity; weighting

I. INTRODUCTION

We deal with the problem of recovering signal degraded by hard clipping. In such a case, the input signal \( x = [x_1, \ldots, x_N] \) passes through a system as follows:

\[
y_n = \begin{cases} 
x_n & \text{for } |x_n| < \theta_c, \\
\theta_c \cdot \text{sgn}(x_n) & \text{for } |x_n| \geq \theta_c,
\end{cases}
\]

making the output sample saturated if its original value exceeds the clipping threshold \( \theta_c \). This system is nonlinear but acts elementwise. The related inverse problem is called declipping.

Audio declipping has been studied for a long time, leading to a number of proposed methods. State-of-the-art approaches rely on prior signal assumptions such as sparsity \([1],[2]\), compressibility by the non-negative matrix factorization \([3]\), or they model the signal as an autoregressive process \([5],[6]\). For a more thorough review, see the survey \([7]\) and references therein. Deep learning approaches still appear rarely today \([8]\).

The declipping method of Siedenburg et al. \([1]\) ranks consistently high in various comparisons. It is based on signal sparsity with respect to a Gabor representation system \([9]\). In contrast to other methods, it employs neighborhoods of time-frequency coefficients, leading to time persistent shrinkage \([10]\), in effect reducing musical noise reconstruction artifacts. Nevertheless, the high restoration quality is achieved also thanks to using hand-crafted shrinkage operators.

The study \([11]\) was devoted to the inclusion of auditory modeling into the audio declipping problem. Herein, we concluded that involving straightforward time-frequency coefficient weights, quadratically increasing with frequency, leads to significantly better performance in sparsity-based declipping than delicate modeling of psychoacoustic masking curves. This surprising success was the first source of inspiration for the present paper: Does the coefficient weighting lead to any performance increase also in social-sparsity-based declipping?

Lately, reports are available showing that the analysis (aka cosparse) audio processing methods perform slightly better than their synthesis (sparse) counterparts \([12],[13],[14]\). In some cases, analysis outperforms the synthesis approach significantly, see for example \([15]\). The described fact motivated us to develop the analysis social sparsity declipper and examine its behavior on audio.

Section II sums up the core knowledge about social sparsity-based declipping. Section III then describes the proposed analysis method and presents the respective numerical algorithm. Section IV introduces the reader to the experiments (both with and without coefficients weighting), reports the results, and discusses them.

II. SYNTHESIS SOCIAL SPARSITY

The background and main ingredients of the synthesis social sparsity declipping were already presented in \([1]\). To be more precise, the estimate of the original, non-clipped signal is sought by solving an optimization problem of the form

\[
\min_{z} \left\{ \frac{1}{2} \|M_R Dz - M_R y\|^2 + \frac{1}{2} \|h(M_H Dz - M_H \theta_c 1)\|^2 + \frac{1}{2} \|h(-M_L Dz - M_L \theta_c 1)\|^2 + \lambda R(z) \right\},
\]

where \( h \) is the hinge function, a piecewise linear function for which it holds \( h(u) = -\text{ReLU}(-u) \). Furthermore, \( y \in \mathbb{R}^N \) is the clipped signal, \( z \in \mathbb{C}^P \) represents the signal coefficients and \( D: \mathbb{C}^P \rightarrow \mathbb{R}^N \) is the time-frequency synthesis operator. Formulation (2) penalizes solutions inconsistent with the reliable, unclipped samples (the first term using the mask \( M_R \)) and solutions inconsistent with the clipping model \([1]\) (the second and third term using masks \( M_H \) and \( M_L \)). The last term is the regularizer \( R \) with a balancing parameter \( \lambda \); this function forces sparse or structured-sparse solutions in...
the time-frequency domain, which is a proper prior on audio
signals.

If variations of the $\ell_1$ norm are put in place of $R$, the
algorithm solving (2) contains simple shrinkages such as the
soft thresholding. Since the quadratic terms in (2) are
Lipschitz-differentiable, the FISTA algorithm [16] can be used as
the numerical solver.

The authors of [1] rely on FISTA but show improved
performance when basic shrinkages are replaced by emprical
shrinkages, which, by the way, do not have a counterpart in
terms of $R$, as proved in [17]. Specifically, they employ
four types of shrinkages. Let $z_{ft}$ represent a particular time-
frequency coefficient. The shrinkage functions applied to each
element in iteration of FISTA can involve:

\[
L: S_{\lambda, w_{ft}}(z_{ft}) = z_{ft} \cdot \max \left(1 - \frac{\lambda \cdot w_{ft}}{|z_{ft}|}, 0 \right), \tag{3a}
\]
\[
WGL: S_{\lambda, w_{ft}}(z_{ft}) = z_{ft} \cdot \max \left(1 - \frac{\lambda \cdot w_{ft}}{|N(z_{ft})|}, 0 \right), \tag{3b}
\]
\[
EW: S_{\lambda, w_{ft}}(z_{ft}) = z_{ft} \cdot \max \left(1 - \frac{\lambda^2 \cdot w_{ft}}{|z_{ft}|^2}, 0 \right), \tag{3c}
\]
\[
PEW: S_{\lambda, w_{ft}}(z_{ft}) = z_{ft} \cdot \max \left(1 - \frac{\lambda^2 \cdot w_{ft}}{|N(z_{ft})|}, 0 \right), \tag{3d}
\]
where WGL and PEW work with a coefficient neighbor-
hood $N$, as illustrated in Fig. 1. EW and PEW are the empirical
shrinkages.

The weights $w_{ft}$ affect the actual shrinkage thresholds.
For example, (3a) implements the soft thresholding with the
threshold $\lambda \cdot w_{ft}$. There are actually no weights used in [1],
which corresponds to the special case of (3) with $w_{ft} = 1$
for every pair of $f$ and $t$. However, we include the weights,
because a part of the experiments utilizes them. In algorithm
listings, shrinkages with weights are denoted in a compact
form as $S_{\lambda, w}$, where $w \in \mathbb{R}^P, w > 0$. Alg. 1 presents the
particular numerical steps leading to the solution of (2).

It is worth noting that any approach stemming from (2)
produces solutions that are generally inconsistent with the
clipping model. Article [18] shows that a simple replacement
of reliable samples of the solution of (2) can significantly
improve performance, with negligible costs.

III. ANALYSIS VARIANT OF SOCIAL SPARSITY

Among other conclusions, the audio declipping survey [17]
revealed the fact that analysis (cosparse) variants of the recon-
struction problems tend to perform slightly better than their
synthesis counterparts. Outstanding results of the synthesis-
based social sparsity declipper described in Sec. III together
with its acceptable computation cost, motivated our effort to
develop the analysis variant of the algorithm.

The analysis reformulation of the declipping problem (2) is
fairly straightforward and reads

\[
\min_x \left\{ \frac{1}{2} \|M_R x - M_R y\|_2^2 + \frac{1}{2} \|h(M_H x - M_H \tau_1)\|_2^2 + \frac{1}{2} \|g(M_L x - M_L \tau_1)\|_2^2 + \lambda R(A x) \right\}, \tag{4}
\]
with $x \in \mathbb{R}^N$ is the sought time-domain signal and
$A: \mathbb{R}^N \to \mathbb{C}^P$ is the analysis operator.

The composition of the analysis operator with $R$ prevents us
from using the FISTA algorithm. Nevertheless, there are
algorithms capable of minimizing optimization problems with
composed linear operators [19], [20], [21]. We stick to the
Loris–Verhoeven (LV) algorithm [21], [22], [23]. The LV
algorithm adapted to solving the analysis-based declipping
problem (4) is proposed in Alg. 2.

\begin{algorithm}
\caption{ISTA-type algorithm solving (2)}
\hspace*{0.02in} \textbf{Input:} $y, \lambda > 0, \theta_c, M_R, M_H, M_L, D$; weights $w \in \mathbb{R}_+^P$;
\hspace*{0.02in} the shrinkage operator $S (L/WGL/EW/PEW)$
\hspace*{0.02in} \textbf{Parameters:} $\gamma \in \mathbb{R}$, $\delta = \|D^*D\|
\hspace*{0.02in} \textbf{Initialization:} $z_t^{(0)}, \lambda_t^{(0)} \in \mathbb{C}^P$ \hspace*{0.02in} \\
\hspace*{0.02in} \textbf{for} $i = 0, 1, \ldots$ \textbf{until convergence do} \\
\hspace*{0.04in} $g_t^{(i)} = D^* M_R^* (M_R D z_t^{(i)} - M_R y)$ \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $g_t^{(i)} = D^* M_H^* h(M_H D z_t^{(i)} - M_H \theta_1)$ \hspace*{0.02in} \hspace*{0.02in} \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $g_t^{(i)} = D^* M_L^* h(-M_L D z_t^{(i)} - M_L \theta_1)$ \hspace*{0.02in} \hspace*{0.02in} \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $g_t^{(i)} = g_t^{(i)} + g_t^{(i)} + g_t^{(i)}$ \hspace*{0.02in} \hspace*{0.02in} \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $\lambda_t^{(i)} = \lambda_t^{(i)} + \gamma (\lambda_t^{(i)} - \lambda_t^{(i)})$ \hspace*{0.02in} % shrinkage \hspace*{0.02in} \\
\hspace*{0.04in} $\lambda_t^{(i+1)} = \lambda_t^{(i+1)} = \lambda_t^{(i+1)}$ \hspace*{0.02in} % extrapolate \hspace*{0.02in} \\
\hspace*{0.02in} return $\lambda_t^{(i+1)}$
\end{algorithm}

\begin{algorithm}
\caption{Loris–Verhoeven algorithm solving (4)}
\hspace*{0.02in} \textbf{Input:} $x, \lambda > 0, \theta_c, M_R, M_H, M_L, A$; weights $w \in \mathbb{R}_+^P$;
\hspace*{0.02in} the shrinkage operator $S (L/WGL/EW/PEW)$
\hspace*{0.02in} \textbf{Parameters:} $\sigma, \tau \in \mathbb{R}, \rho \in [0, 2 - \tau/2]$
\hspace*{0.02in} \textbf{Initialization:} $x^{(0)} \in \mathbb{R}^N, u^{(0)} \in \mathbb{C}^P$ \hspace*{0.02in} \\
\hspace*{0.02in} \textbf{for} $i = 0, 1, \ldots$ \textbf{until convergence do} \\
\hspace*{0.04in} $g_t^{(i)} = M_R^* (M_R x^{(i)} - M_R y)$ \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $g_t^{(i)} = M_H^* h(M_H x^{(i)} - M_H \theta_1)$ \hspace*{0.02in} \hspace*{0.02in} \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $g_t^{(i)} = M_L^* h(-M_L x^{(i)} - M_L \theta_1)$ \hspace*{0.02in} \hspace*{0.02in} \hspace*{0.02in} % gradients \hspace*{0.02in} \\
\hspace*{0.04in} $\lambda_t^{(i)} = \lambda_t^{(i)} + \gamma (\lambda_t^{(i)} - \lambda_t^{(i)})$ \hspace*{0.02in} % shrinkage \hspace*{0.02in} \\
\hspace*{0.04in} $\lambda_t^{(i+1)} = \lambda_t^{(i+1)}$ \hspace*{0.02in} % extrapolate \hspace*{0.02in} \\
\hspace*{0.02in} return $x^{(i+1)}$
\end{algorithm}
Line 7 in Alg. 2 corresponds to the proximal operator of a Fenchel–Rockafellar conjugate of $S$, and we evaluate it indirectly using the Moreau identity [24]:

$$\text{prox}_{\alpha f}(x) = x - \alpha \text{prox}_{f/\alpha}(x/\alpha) \quad \text{for} \quad \alpha \in \mathbb{R}^+.$$  (5)

The algorithm convergence is guaranteed if $\sigma \tau \|A\|^2 \leq 1$, and therefore, as suggested in [24], we keep $\tau$ as the only tunable parameter and let $\sigma$ be inferred as $1/(\tau \|A\|^2)$. Since the Lipschitz constant of the gradients of the smooth terms of (4) is one, the step size $\tau$ is limited to $\tau \in (0, 2)$. Another condition is put on the parameter $\rho$; the respective interval is $[0, 2 - \tau/2]$.

IV. EXPERIMENTS AND RESULTS

The experiments were designed to compare the proposed analysis variant of the algorithm with its synthesis counterpart [1] in terms of the quality of restored audio.

The dataset used for the experiments was extracted from the EBU SQAM database [1] and consists of 10 various music excerpts in mono, sampled at 44.1 kHz, with an approximate length of 7 seconds. The clipping threshold $\theta_c$ was determined individually for each audio excerpt based on the input signal-to-distortion ratio (SDR), which is defined for two signals $u$ and $v$ as

$$\text{SDR}(u, v) = 20 \log_{10} \frac{\|u\|_2}{\|u - v\|_2}.$$  (6)

The audio excerpts were clipped according to (1) using 7 clipping levels ranging from 1 dB to 20 dB input SDR.

The algorithms were run in MATLAB 2019a using the LTFT toolbox [25] for signal synthesis and analysis. The oversampled STFT with 8,192 samples long Hann window, 75% overlap and 16,384 frequency channels was used as the signal transform ($A$ and $D$ operators). The size of the neighborhood $N(z_{ft})$ was set to $3 \times 7$, (3 coefficients in the frequency direction and 7 in the time direction), which seems to be one of the top-performing setups [1].

Both algorithms used an identical setup of common parameters to ensure a fair comparison. They both exploited the adaptive restart strategy [26], which was proven in [1] to significantly accelerate the overall convergence. It consists in gradually decreasing the value of the balance parameter $\lambda$ every few hundred iterations until the target value of $\lambda$ is reached. In our experiments, we used 500 inner iterations and 20 outer iterations with $\lambda$ logarithmically decreasing from $10^{-1}$ to $10^{-4}$. To slightly accelerate the computations, we also added a parameter $\varepsilon$, which is used to break a current outer iteration if the $\ell_2$-norm of the difference between the time-domain solution from the current and previous inner iteration is smaller than $\varepsilon$. The value of $\varepsilon$ was 0.001.

Other individual parameters of the algorithms were tuned and set such that the respective algorithms deliver the best possible results. The parameter $\gamma$ of the ISTA-type declipper (Alg. 1) changes according to $(k - 1)/(k + 5)$, where $k$ is the inner iteration counter, which corresponds to the FISTA acceleration [15]. The parameters of the Loris–Verhoeven-based declipper (Alg. 2) were set to $\tau = 1.5, \sigma = \frac{2}{\tau}$, and $\rho = 1$.

A. Comparison of FISTA and LV algorithms

The quality of the reconstruction is evaluated by two metrics. The physical similarity of the declipped and ground-truth waveforms is expressed using the $\Delta\text{SDR}_c$, which represents the SDR improvement over the clipped signal, computed only on clipped samples of the waveforms. However, similarity of waveforms may not necessarily imply perceptual quality. Therefore, the restoration quality is also evaluated using PEMO-Q [27], which is a perceptually motivated metric for evaluation of audio quality. The output of PEMO-Q is a number on the objective difference grade (ODG) scale ranging from $-4$ (worst quality) to 0 (best quality).

The achieved $\Delta\text{SDR}_c$ values are depicted in Fig. 2 in the form of box charts, for shrinkage operators presented in Equations (4). The results show that the proposed analysis approach is slightly worse in the case of L (see Fig. 2a) and WGL (see Fig. 2b) shrinkage types. However, in the case of EW, it is better by almost 6 dB on average. With respect to the PEW shrinkage operator, the analysis variant performs slightly better for higher input SDR values but worse for lower input SDR values, especially for 1 dB.

The PEOO-Q results shown in Fig. 3 differ from the results obtained using the $\Delta\text{SDR}_c$ metric. While the analysis variant in combination with the WGL shrinkage operator still remains marginally worse than its synthesis counterpart, for both L and PEW operators it slightly outperforms the synthesis variant in most of the test cases. The LV algorithm using the EW shrinkage even marginally overcomes the FISTA algorithm utilizing PEW, while being about 6% faster. Nevertheless, note that the computer implementation used is general for all

1https://tech.ebu.ch/publications/sqamcd

Fig. 2. Comparison of the synthesis and analysis approaches to audio declipping for 4 different shrinkage operators using $\Delta\text{SDR}_c$. [24]
shrinkage types, thus EW is run as a special case of PEW, the size of the neighborhood $N(z_{ft})$ being $1 \times 1$. Therefore, the computational complexity of EW could be further reduced by omitting the unnecessary steps related to neighborhood processing.

B. Coefficients weighting

As mentioned in the introduction, motivated by the quality of results from [11] obtained by penalizing higher frequencies using quadratic (parabolic) weights, we examined such a weighting also in current experiments, i.e. when shrinkage operators other than standard soft thresholding are used. Note that the determination of these weights is computationally very cheap and therefore the weighting does not introduce any additional computational complexity.

Formally, the vector of weights is computed as $w = m \odot m$, where $m = [1, \ldots, \left\lfloor \frac{M}{2} \right\rfloor + 1]$ and $M$ is the number of frequency bins of the STFT. The weights are subsequently normalized to fit the range $[0, 1]$. The weights are subsequently scaled to fill the interval $[0, 1]$.

The results comparing the parabola-weighted and non-weighted variants of FISTA and LV declippers are illustrated in Fig. 4. Herein, Fig. 4a shows the average $\Delta$SDR values obtained from the nonweighted algorithms and Fig. 4b represents the parabola-weighted algorithms. Different shrinkage operators are differentiated using colors, and algorithms are distinguished by line types (solid lines for FISTA, dashed lines for LV).

To summarize results, it is possible to claim that parabola weighting improves the results for L and WGL. However, such an improvement is more pronounced in the synthesis case. Even though the synthesis variant with parabola-weighted WGL produces very good results, they are still worse by approx. 2 dB compared to the results using PEW. In the case of EW, the weighting slightly improves the results in the synthesis case but deteriorates the results in the analysis case. The analysis variant, nevertheless, remains better even when the weighting is utilized. For PEW, the results obtained when weighting is introduced are worse in both the synthesis and analysis cases. Almost an identical conclusion can be drawn from the PEMO-Q results, displayed in Figs. 4c and 4d.

The most probable reason for the above-mentioned behavior is that both EW and PEW are designed to promote the sparsity of the solution by suppressing smaller coefficients. Since the energy of coefficients in audio signals generally decrease with frequency (see Fig. 5 for comparison of magnitudes of the STFT coefficients), these shrinkages already suppress higher frequencies. Additional weighting to suppress higher frequencies even more may yield situations where the middle frequencies are not thresholded enough. The average thresholds for each frequency bin for the violin audio excerpt are depicted in Fig. 6.
We have proposed the analysis variant of the successful audio declipper. For acquiring a numerical solution, the Loris–Verhoeven algorithm has been adopted. We have run experiments showing that the analysis variant brings improvement for some choices of the shrinkage operators. The case of the Empirical Wiener shrinkage enjoyed significant gain in the reconstruction quality this way. Most importantly, evidence shows that the analysis variant outperforms the synthesis counterpart uniformly (though slightly).

We furthermore generalized the social-sparsity declippers by the inclusion of weighting of the time-frequency coefficients. The evaluation revealed that parabolic weights enhance the reconstruction quality in some setups (mostly L and WGL). However, the best performing non-weighted choices were not beaten. This effect has been explained.

The final recommendation of algorithms which are expected to deliver top auditory quality within the examined family are: analysis PEW, analysis EW and synthesis PEW, all without weighting (see Fig. 6). Analysis EW is moreover advantageous for its lower computational complexity.

The presented results are appended to the declipping survey website https://rajmic.github.io/declipping2020/, through which the audio experts and Matlab implementations are available.

V. CONCLUSION

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