Detecting the cosmological neutrino background

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Three relativistic particles in addition to the photon are detected in the cosmic microwave background (CMB). In the standard model of cosmology, these are interpreted as the three neutrino species. However, at the time of CMB-decoupling, neutrinos are not only relativistic particles but they are also freestreaming. Here, we investigate, whether the CMB is sensitive to this defining feature of neutrinos, or whether the CMB-data allow to replace neutrinos with a relativistic gas. We show that free streaming particles are highly preferred over a relativistic perfect gas with $\Delta \chi^2 \simeq 250$. We also study the possibility to replace the neutrinos by a viscous gas and find that a relativistic viscous gas with the standard values $c_{\text{eff}}^2 = c_{\text{vis}}^2 = 1/3$ cannot provide a good fit to present CMB data. It has $\Delta \chi^2 = 52$ with respect to free streaming neutrinos. Even if we allow for arbitrary $c_{\text{eff}}^2$ and $c_{\text{vis}}^2$, the best fit still has $\Delta \chi^2 = 22$, which shows that also this possibility is highly disfavoured.

I. INTRODUCTION

The cosmic microwave background (CMB) is the most precious observational dataset with which we determine the content of the Universe. Alone and in combination with other data it has been used to infer that our Universe is presently dominated by dark energy which may be in the form of a cosmological constant $\Lambda$ contributing a density parameter of $\Omega_{\Lambda} \simeq 0.7$, and pressureless matter which is dominated by cold dark matter with $\Omega_{\text{cdm}} h^2 = \Omega_{\text{b}} h^2 + \Omega_{\text{c}} h^2 = \omega_{\text{cdm}} + \omega_b \simeq 0.14$ where the contribution from baryons is $\omega_b = \Omega_{\text{b}} h^2 \simeq 0.022$, see e.g. [1]. Here $h = H_0/100 \text{km/s/Mpc}$ and $H_0$ is the present Hubble parameter, $h \simeq (0.7 \pm 0.05)$.

Furthermore, there are the photons which make up the CMB and which contribute $\Omega_{\gamma} h^2 = 2.48 \times 10^{-5}$ and there are cosmic neutrinos. In the standard model of 3 massless neutrino species, they contribute a density parameter of $\Omega_{\nu} h^2 = 1.69 \times 10^{-5}$. Taking into account neutrino masses one obtains in the minimal model with normal hierarchy and maximal neutrino mass of $0.05 \text{eV}$, $\Omega_{\nu} h^2 \simeq 0.5 \times 10^{-3}$.

These are very small numbers. Nevertheless, during the radiation dominated epoch at temperatures above about $1 \text{eV}$, neutrinos and photons are the dominant constituents of the Universe, and the neutrinos contribute a fraction $f_{\text{rad}} = \Omega_{\nu}/(\Omega_{\gamma} + \Omega_{\nu}) \simeq 0.4$ to the total energy density of the Universe. At recombination, $z_{\text{dec}} \simeq 1100$ they still contribute

$$f_{\text{dec}} \simeq \Omega_{\nu}/(\Omega_{\gamma} + \Omega_{\nu} + \Omega_{m}/(1 + z_{\text{dec}})) \simeq 0.1,$$

i.e., $10\%$ to the total energy density of the Universe.

The first indication that cosmic neutrinos are really present came from nucleosynthesis calculations. The abundance of primordial helium-4 is very sensitive to the expansion rate at temperature $T_{\text{rec}} \simeq 0.08 \text{MeV}$, which is determined via the Friedman equation by the energy density of the Universe. At this temperature the energy density is dominated by photons and neutrinos. The observed helium-4 abundance requires $N_{\text{eff}} \simeq 3 \pm 1$ species of neutrinos. Similar results have been obtained also from CMB experiments, see, e.g., [1]. However, the nucleosynthesis results only require a relativistic component with the given energy density. Neutrinos have the additional property that they are collisionless after redshift $z_{\nu} \simeq 10^{10}$ where they decouple from the cosmic fluid. In the CMB they are modelled as collisionless particles but is the data really sensitive to this property or could we also fit it with a relativistic fluid?

This is the question we address in this work. We first compare the standard CMB-anisotropy calculation with a computation where neutrinos are modelled as a perfect fluid. Awaiting the upcoming Planck-data release, we compare our models to a "fake Planck-likelihood", provided by the package Monte Python [2], using freestreaming neutrinos and standard cosmological parameters. Planck is modelled corresponding to its Blue Book [3], with 14 months of data taking, a sky fraction of $f_{\text{sky}} = 0.65$, and including polarization.

We find that treating neutrinos as collisionless particles fits the forcasted data significantly better than a simple relativistic fluid. Even though we neglect neutrino masses in our modelling, our conclusions remain valid if neutrino masses are as small as expected from oscillation experiments with normal hierarchy [4], i.e., $0.05 \text{eV}\lesssim m_3 \gg m_2 \gg m_1$, since Planck data cannot measure these neutrino masses.

Next we show that neutrinos can not be modeled as a viscous fluid. We also compare our results with a slightly different approach which is found in the present literature [5–7].

In the next section we explain our calculations and show the result. In Section III we discuss our findings and conclude.
II. NEUTRINOS IN THE CMB

In standard CMB computations one assumes that neutrinos are massless, free streaming particles and solves the Liouville equation for them, see, e.g., [8].

\[ \dot{N}_0 + kN_1 = 0, \quad (1) \]

\[ \dot{N}_1 + \frac{k}{3}(2N_2 - N_0) = \frac{k}{3}(\Phi + \Psi), \quad (2) \]

\[ \dot{N}_\ell + \frac{k}{2\ell + 1}[(\ell + 1)N_{\ell+1} - \ell N_{\ell-1}] = 0, \quad \ell > 1. \quad (3) \]

Here \( N_\ell \) are the moments of the energy integrated neutrino distribution function in Fourier space and \( \Phi \) and \( \Psi \) are the Bardeen potentials. The moments 0 to 2 are related to the neutrino density perturbation, \( \delta_\nu \), the potential of the velocity perturbation, \( V_\nu \), and the anisotropic stress, \( \Pi_\nu \), in longitudinal gauge by

\[ \delta_\nu = 4(N_0 + \Phi), \quad (4) \]

\[ V_\nu = 3N_1, \quad (5) \]

\[ \Pi_\nu = 12N_2. \quad (6) \]

One truly only needs these first three moments of the distribution function since only they enter the energy momentum tensor which couples to the gravitational field and affects the evolution of the CMB photons. Nevertheless, in the Liouville equation each mode \( N_\ell \) is coupled by free streaming to \( N_{\ell+1} \) and \( N_{\ell-1} \) and therefore to \( N_0, \ N_1 \) and \( N_2 \) with sufficient precision one usually solves the neutrino hierarchy up to \( \ell \sim 10 - 20 \) in order to minimise problems from so called numerical ‘reflections’.

In Fig. 1, we see that when cutting the neutrino hierarchy at \( \ell_{\text{max}} \), already for \( \ell_{\text{max}} = 2 \) the difference between the standard calculation setting \( \ell_{\text{max}} = 17 \) becomes very small. Nevertheless, as we shall see below, present data from the Planck satellite are so good that they easily distinguish between free streaming neutrinos and a relativistic viscous fluid corresponding to \( \ell_{\text{max}} = 2 \).

If neutrinos would not behave the way standard neutrinos do, but they would be a relativistic perfect fluid, all moments higher than \( \ell = 1 \) would be damped away by collisions and their evolution equations would be given by eqs. (1) and (2) with \( N_2 \equiv 0 \).

If they behave like a relativistic viscous fluid, i.e., a fluid with shear, all moments higher than \( \ell = 2 \) are damped away and the evolution equations are given by eqs. (1) to (3) with \( \ell = 2 \) and \( N_3 \equiv 0 \). We have investigated whether neutrinos can be modelled by such a fluid. For this, we have replaced neutrinos by a relativistic perfect fluid or a relativistic viscous fluid and run the modified CMB code CLASS [9, 10] in combination with Monte Python to find best fit values of the standard cosmological parameters from the forecasted Planck data. In Fig 2 we compare the spectra obtained in this way with the spectrum from free streaming neutrinos and in Fig. 3 we show the best fit parameters.

\[ \Delta \chi^2_{\text{ideal}} = 250, \quad \Delta \chi^2_{\text{visc}} = 52. \quad (7) \]

Not only are most of the cosmological parameters very different, see Fig. 3, but the fit is also much worse. The \( \Delta \chi^2 \) for both fluid approximations is clearly unacceptable:

\[ \Delta \chi^2_{\text{ideal}} = 250, \quad \Delta \chi^2_{\text{visc}} = 52. \quad (7) \]

This shows that cosmic neutrinos cannot be modelled by a relativistic perfect fluid or viscous fluid.
FIG. 3. The best fit parameters for neutrinos modelled as a perfect fluid (blue), neutrinos modelled as a relativistic viscous fluid (rose), neutrinos as a viscous fluid with arbitrary sound speed $c_{\text{eff}}^2$ and viscosity $c_{\text{vis}}^2$ (green), and standard free streaming neutrinos (grey) are shown. The best fit values of most parameters for the perfect fluid and the free streaming model differ significantly. The best fit values of most parameters for the viscous fluid and the free streaming model are similar, they all agree within 1.5\sigma apart from $n_s$ which for the relativistic viscous fluid model differs by more than 3\sigma.

In previous work [5–7] on neutrino clustering properties, a somewhat different standpoint has been taken. There, eqs. (1 – 3) are replaced by

$$\dot{N}_0 + kN_1 = \mathcal{H}(1 - 3c_{\text{eff}}^2)N_0$$  \hspace{1cm} (8)$$

$$\mathcal{N}_1 + \frac{k}{3}[2N_2 - 3c_{\text{eff}}^2N_0] =$$

$$-\mathcal{H}(1 - 3c_{\text{eff}}^2)N_1 + \frac{k}{3}(3c_{\text{eff}}^2 \Phi + \Psi)$$  \hspace{1cm} (9)
\[
\dot{N}_\ell + k \left[ \frac{3}{5} N_3 - 3 c_\text{vis}^2 \frac{2}{5} N_1 \right] = 0 \quad \text{(10)}
\]
\[
\dot{N}_\ell + \frac{k}{2\ell+1} [ (\ell+1)N_{\ell+1} - \ell N_{\ell-1} ] = 0 \quad \ell > 2. \quad \text{(11)}
\]

A similar, non-perfect-fluid treatment has already been suggested in Refs. [11, 12]. However, eqs. (8) to (11) describe neither a perfect nor an imperfect fluid since the higher moments, \(\ell \geq 3\) evolve like those of free streaming particles.

The advantage of this model is that it is ‘nested’ inside the standard model of free streaming neutrinos with two additional parameters which take the values \(c_\text{eff}^2 = c_\text{vis}^2 = 1/3\) in the standard model and previous work, especially [7] have found that the preferred values of these parameters are indeed close to the standard ones. Nevertheless, the physical meaning of \(c_\text{eff}\) and \(c_\text{vis}\) remains unclear since only the evolution of the first and second moment but not higher moments can be affected by collisions in this model. This seems somewhat unphysical to us.

To remove the unphysical assumption of admitting higher multipoles, we set \(N_\ell = 0\) for \(\ell \geq 3\) and fit for \(c_\text{eff}^2\) and \(c_\text{vis}^2\). As we have discussed above, this model with \(c_\text{eff}^2 = c_\text{vis}^2 = 1/3\), i.e., the relativistic shear fluid, provides an unacceptably bad fit to the observed CMB anisotropies. Before concluding that the three relativistic particles in the CMB are indeed free streaming neutrinos, we need, however, to check whether another value of \(c_\text{eff}^2\) and \(c_\text{vis}^2\) might provide a better fit. Introducing two new free parameters, will of course improve the fit, but we find that even then the difference in \(\chi^2\) for the best fit is unacceptably high: \(\Delta \chi^2 = 22\). Furthermore, the best fit values for \(c_\text{eff}^2\) and \(c_\text{vis}^2\) are close to those of the relativistic shear fluid. In Fig. 3 we compare the parameter values obtained by replacing neutrinos by a perfect fluid, a relativistic viscous fluid or by a viscous fluid with arbitrary effective sound speed \(c_\text{eff}^2\) and viscosity, \(c_\text{vis}^2\), with the results for standard neutrinos.

### III. CONCLUSIONS

We have studied how neutrinos are detected in the CMB. We have shown that they are not only relevant as additional relativistic degrees of freedom, but CMB anisotropies and polarisation are also very sensitive to their clustering properties. While the data is in good agreement with free streaming neutrinos, it cannot be fitted by neutrinos modelled as a relativistic perfect fluid. The best fit model with perfect fluid neutrinos leads to a \(\Delta \chi^2 = 250\) with respect to the best fit free streaming neutrinos. Even including anisotropic stress, i.e. allowing for a relativistic viscous fluid cannot fit the data. The increase in \(\chi^2\) with respect to the best fit models with free streaming neutrinos is \(\Delta \chi^2 = 52\). This value can be improved somewhat when allowing arbitrary values for the effective sound speed and the viscosity, \(c_\text{eff}^2\) and \(c_\text{vis}^2\).

Actually for a viscous fluid the best fit values of these parameters are

\[
c_\text{eff}^2 = 0.337 \pm 9.3 \cdot 10^{-5}, \quad c_\text{vis}^2 = 0.329 \pm 0.025. \]

But also with these two additional parameters the fit remains much worse, leading to \(\Delta \chi^2 = 22\) w.r.t. the best fit free streaming model [13]. Hence with the cosmic microwave background we have not only found that there are 3 species of light particles but we see in addition that these particles are free streaming. This is a significant additional step towards the detection, albeit indirect, of the cosmological neutrino background.

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