Critical behaviors of a black hole in an asymptotically safe gravity with cosmological constant

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Abstract
We study the $P-V/r_c$ criticality and phase transition of a quantum-corrected black hole in asymptotic safety (AS) gravity in the extended phase space. For a black hole, the cosmological constant is dependent on the momentum cutoff or energy scale; therefore, one can naturally treat it as a variable and connect it with thermodynamic pressure. We find that for a quantum-corrected black hole there is a first-order phase transition similar to that of the van der Waals liquid–gas system. We also analyze the types of phase transition between the smaller and larger black hole phases according to Ehrenfest’s classification. It is shown that they are second-order phase transitions.

Keywords: black hole thermodynamics, phase transition, critical exponents

(Some figures may appear in colour only in the online journal)

1. Introduction
Like ordinary thermodynamic matter, black holes also have temperature, entropy, and energy. The laws of black hole mechanics are similar to those of thermodynamics [1]. Therefore, we can treat black holes as thermodynamic systems. In fact, between black holes and conventional thermodynamic systems, there are other similarities, such as phase transition and critical behaviors. The pioneering work of Davies [2] and the well-known Hawking–Page phase transition [3] are both proposed to elaborate these points. The phase transitions and
critical phenomena in anti-de Sitter (AdS) black holes have been studied extensively [4–10]. Some interesting works show that phase transition similar to the van der Waals liquid–gas phase transition exists for some black holes [11–19]. Even for black holes in dS space critical behaviors can also be studied by considering the connections between the black hole horizon and the cosmological horizon [20, 21].

Recently some physicists reconsidered the critical phenomena of AdS black holes by treating the cosmological constant $\Lambda$ as a variable and connecting it with thermodynamic pressure [22–29]. In some models, the cosmological constant may be considered to be a time-variable quantity [30, 31] or certain thermodynamic quantities, such as thermodynamic pressure [32, 33], which should be a conjugate quantity of thermodynamic volume. Inclusion of the variation of $\Lambda$ can make the first law of black hole thermodynamics consistent with the Smarr formula for some black holes.

In this paper, we study the critical behaviors of a kind of black hole derived in asymptotic safety gravity. The asymptotic safety scenario for quantum gravity was put forward by Weinberg [34]. It is based on a nontrivial fixed point of the underlying renormalization group (RG) flow for gravity. This theory has been studied extensively and applied to a number of different subjects related to quantum gravity [35–39]. Bonanno and Reuter [40] derived the RG-improved black hole metrics by replacing the Newtonian coupling constant with a ‘running’ one. In [41, 42], the authors employed a different cutoff identification, which is a linear relation between the renormalization scale and the curvature, to discuss the cosmological dynamics. Cai et al [43] have found a spherically symmetric vacuum solution to the field equation derived from AS gravity with higher derivative terms and with a cosmological constant. In this theory, the cosmological constant is no longer constant but dependent on a momentum cutoff. Therefore it is reasonable to include the variation of $\Lambda$ in the first law of black hole thermodynamics as thermodynamic pressure $P$. The quantum correction of AS gravity for conventional Schwarzschild–AdS black holes makes their thermodynamic quantities and critical behaviors very different. A quantum-corrected black hole can also show a phase transition analogous to the liquid–gas phase transition in the van der Waals system. In accordance with Ehrenfest’s classification we also consider the Gibbs free energy, isothermal compressibility, and the expansion coefficient. It is shown that the type of phase transition for a black hole at the divergent points of these quantities is second-order, or continuous.

This paper is arranged as follows: In the next section we simply introduce the AS gravity model and its quantum-corrected black hole solution. In section 3 we study the $P = V/r_+$ criticality by considering the cosmological constant as thermodynamic pressure. We also calculate the critical exponents here. In section 4 we analyze the type of phase transition of the quantum-corrected black hole in the extended phase space according to Ehrenfest’s classification. We make some concluding remarks in section 5.

2. Quantum-corrected black hole in AS gravity

We start with a generally covariant effective gravitational action with higher derivative terms involving a momentum cutoff $p$ [43]:

$$\Gamma_p\left[g_{\mu\nu}\right] = \int d^4x \sqrt{-g} \left[ p^4 g_0(p) + p^2 g_1(p)R + g_{2a}(p)R^2 + g_{2b}(p)R_{\mu\nu}R^{\mu\nu} + g_{2c}(p)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + O(p^{-2}R^3) + \ldots \right],$$

(2.1)

where $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $R$ is the Ricci scalar, $R_{\mu\nu}$ is the Ricci tensor, and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor. The coefficients $g_i (i = 0, 1, 2a, \ldots)$ are
dimensionless coupling parameters and are functions of the dimensionful UV cutoff. The couplings satisfy the RG equations:

$$\frac{d}{d \ln p} g_i(p) = \beta_i [g_i(p)].$$

(2.2)

Assuming a static spherically symmetric metric ansatz and choosing the Schwarzschild gauge

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_2^2,$$

(2.3)

and then substituting it into the generalized Einstein field equations

$$\tilde{G}^{\mu\nu} \equiv \frac{\delta I_F}{\delta g_{\mu\nu}} = 0,$$

(2.4)

one can derive a Schwarzschild–(anti)-de Sitter-like solution

$$f(r) = 1 - \frac{2G_p M}{r} \pm \frac{r^2}{l_p^2},$$

(2.5)

where $G_p$ and $l_p$ are the gravitational coupling and the radius of the asymptotically (A)dS space, and both depend on the momentum cutoff $p$.

It is shown in [43] that there are a Gaussian fixed point in the IR limit and a non-Gaussian fixed point in the UV limit. A central result is

$$g_0 \approx \left( \Lambda_{IR} + \eta p^2 G_N \right) \left( 1 + \xi p^2 G_N \right),$$

(2.6)

$$g_1 \approx \frac{1 + \xi p^2 G_N}{16\pi^2 G_N},$$

(2.7)

where $G_N$ and $\Lambda_{IR}$ are the values of the gravitational coupling and the cosmological constant in the IR limit, which should be determined by astronomical observations. The parameters $\xi$ and $\eta$ are both related to the running couplings $\lambda(p)g_{2\nu}$, $\lambda(p)g_{2\Omega}$, $\lambda(p)g_{2\nu\Omega}$ at the non-Gaussian fixed point. The coefficient $\lambda$ has the familiar logarithmic form which approaches asymptotic freedom:

$$\lambda(p) = \frac{1}{1 + \frac{133}{160\pi^2} \xi \ln p / M_p},$$

(2.8)

where $\lambda_0$ is a fixed value of the coefficient $\lambda$ on the Planck scale and $M_p$ is the Planck mass.

It is shown that the running gravitational coupling $G_p$ is related the Newtonian gravitational coupling constant $G_N$ by

$$G_p = \frac{G_N}{1 + \xi p^2 G_N}.$$  (2.9)

On the high-energy scale,

$$p(r) \approx 2.663 \left( \frac{M_p^4}{\lambda_0} \right)^{1/8} r^{-3/4}.$$  (2.10)
Thus,

$$f(r) \approx 1 - \frac{625}{512\pi} \left[\sqrt{\xi} (Mr)^{1/2}\right]$$

(2.11)

which is singular-free at $r = 0$. However, the curvature singularity still exists due to divergent $R_{\mu\nu}R^{\mu\nu}$.

On the low-energy scale, the momentum cutoff drops to the infrared (IR) limit, and $p \approx \frac{1}{r}$. At this time,

$$f(r) \approx 1 - \frac{2Mr}{r^2 + \xi} \pm \frac{\xi}{l_p^2}$$

(2.12)

where $G_N = 1$ has been set for simplicity. The parameter $\xi$ represents the quantum correction to the conventional Schwarzschild–AdS black hole. Obviously, when $\xi = 0$, the corrected metric returns to that of the Schwarzschild–AdS black hole. In the following paragraphs, we study the thermodynamics of quantum-corrected black holes based on equation (2.12). It is shown that owing to the correction, the thermodynamic quantities are also corrected.

3. $P-V$ criticality of a quantum-corrected black hole

In this paper we are concerned with only the asymptotic AdS black hole. First we identify the pressure by

$$P = \frac{3}{8\pi l_p^2}. \quad (3.1)$$

From equation (2.12), one can easily obtain the mass

$$M = \frac{(8\pi Pr_r^2 + 3)(\xi + r_r^2)}{6r_r}. \quad (3.2)$$

where $r_r$ is the radius of the black hole event horizon.

The Hawking temperature of the black hole can be easily derived:

$$T = \frac{f'(r_r)}{4\pi} = \frac{-3\xi + 24\pi Pr_r^2 + 8\pi^2 Pr_r^2 + 3r_r^2}{12\pi r_r^3 + 12\pi \xi r_r}. \quad (3.3)$$

When $\xi = 0$, this gives the temperature of a Schwarzschild–AdS black hole.

Obviously, $\xi$ is a dimensionful parameter with $[\xi] = L^2$. We can take it as a new parameter in black hole thermodynamics. Thus, the first law of black hole thermodynamics should be written as

$$dM = TdS + VdP + \phi d\xi. \quad (3.4)$$

Here the mass of a black hole is no longer internal energy but should be interpreted as thermodynamic enthalpy, namely $H = M(S, P, \xi)$ [14, 22, 44, 45]. The first law of black hole thermodynamics represented by the internal energy $U(S, V)$ reads

$$dU = TdS - PdV + \phi d\xi, \quad (3.5)$$

where $U = H - PV$. 

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According to the first law equation (3.4), one can derive the entropy
\[ S = \int \frac{\Delta M}{T} = \int \frac{1}{T} \frac{\partial M}{\partial r_+} \, dr_+ = \pi r_+^2 + 2\pi \xi \ln \frac{r_+}{\sqrt{\xi}} + S_0 \] (3.6)
where \( S_0 \) is an integration constant which can be decided by the boundary conditions. The additional logarithmic term in the expression of entropy indicates the quantum gravitational correction. Because the parameter \( \xi \to 0 \), the standard Bekenstein–Hawking area law returns. It is interesting that no \( P \) exists in the expression of \( S \), although it is included in \( M \) and \( T \).

The conjugate thermodynamic volume can be calculated:
\[ V = \frac{\partial M}{\partial P} \] (\( S_0 \))
\[ V = \frac{4}{3} \pi r_+^3 (\xi + r_+^2). \] (3.7)

And the conjugate quantity of \( \xi \) is
\[ \phi = \frac{\partial M}{\partial \xi} \] (\( S_0 \), P)
\[ \phi = \frac{3\xi + 40\pi P r_+^4 + 3r_+^2 (8\pi \xi P + 3) - 2 \left( -3\xi + 24\pi P r_+^4 + r_+^2 (8\pi \xi P + 3) \right) \ln \left( \frac{r_+}{\sqrt{\xi}} \right)}{12r_+ (\xi + r_+^2)}. \] (3.8)

Omitting \( S_0 \), one can easily verify that a generalized Smarr formula
\[ M = 2TS - 2VP + 2\phi \xi \] (3.9)
can be constructed.

This is an unexpected result. According to the scaling argument in [14], one can indeed obtain \( M = 2 \frac{\partial M}{\partial A} A + 2 \frac{\partial M}{\partial \xi} \xi - 2 \frac{\partial M}{\partial P} P \), where \( A = \pi r_+^2 \) is the area of the horizon. Because of the existence of an additional logarithmic correction term, the entropy of the black hole is not proportional to \( A \). Therefore, equation (3.9) is not an obvious result.
The heat capacity at constant pressure can be given by
\[
C_P = \frac{\partial M}{\partial T} \bigg|_{r_{\xi}} = \frac{2\pi \left(\xi + r_+^2\right)^2 \left(-3\xi + 24\pi P r_+^4 + 8\pi^2 P r_+^2 + 3 r_+^2\right)}{3\xi^2 + 24\pi P r_+^6 + 64\pi^2 P r_+^4 + 8\pi^2 P r_+^2 - 3 r_+^4 + 12\xi r_+^2},
\] (3.10)
The qualitative behaviors of the temperature \(T\) and the heat capacity \(C_P\) are depicted in figure 1. Obviously, owing to the existence of \(\xi\), the temperature will not blow up as the radius of the event horizon approaches zero but tends toward zero at a finite radius, where the black hole becomes an extremal one. For a fixed pressure there is a critical value of \(\xi\), below which there are both a local maximum and a local minimum for the temperature, and above which no local extremum exists. At the critical value, the maximum and minimum coincide. From figure 1(b), one can see that when \(\xi < \xi_c\), \(C_P\) suffers discontinuities at two points. The divergences of the heat capacity appear precisely at the extrema of the temperature. Smaller and larger black holes with positive heat capacity can be local stable, whereas the intermediate black holes with negative heat capacity are instable.

From equation (3.3), one can derive the equation of state of a black hole:
\[
P = \frac{3\left(\xi + 4\pi r_+^3 T - r_+^2 + 4\pi r_+ T\right)}{8\pi r_+^2 \left(\xi + 3 r_+^2\right)},
\] (3.11)
One can take the specific volume as \(\nu \propto V/N\), with \(N = A/l_p^2\), counting the number of degrees of freedom associated with the black hole horizon [25]. \(l_p\) here is the Planck length. If we take \(\nu = 6V/N\), the specific volume can be expressed as
\[
\nu = 2\left(r_+ + \frac{\xi}{r_+}\right).
\] (3.12)
Obviously, when \(\xi = 0\), the result is similar to that in [23, 24]. Replacing \(r_+\) in equation (3.11) with \(\nu\), one can obtain the equation of motion, \(P = P(\nu, T, \xi)\). As is done in [24], one can also expand \(P(\nu, T, \xi)\) in powers of \(\xi\) in the small \(\xi\) limit and take the first several terms approximately.
\[
P = \frac{T}{\nu} - \frac{1}{2\nu^2} + \frac{4(5\pi T - 1)\xi}{3\pi^4} + \frac{8(68\pi T - 1)\xi^2}{9\pi^6} + O(\xi^3).
\] (3.13)
One can see that the preceding equation of state indeed exhibits \(P - \nu\) criticality. However, in this paper we want to treat equation (3.11) exactly as is. We will use the horizon radius in the equation of state instead of the specific volume hereafter. In figure 2 we show the \(P - r_+\) criticality.

The critical point can be obtained according to
\[
\frac{\partial P}{\partial r_+} = 0, \quad \frac{\partial^2 P}{\partial r_+^2} = 0
\] (3.14)
which leads to
\[
r_c = \sqrt{c^2 - 3}, \quad T_c = \frac{3c^2 - 6c - 1}{2\pi (3c^2 + 8c + 1)\sqrt{c^2 - 3}}, \quad P_c = \frac{3\left(c^2 - 4c - 1\right)}{8\pi c (3c^2 + 8c + 1)\xi}
\] (3.15)
where the constant \(c = 3 + \frac{2}{3\sqrt{3}} \left[39 + i\sqrt{15}\right]^{1/3} + \left(39 - i\sqrt{15}\right)^{1/3}\), which is a real number. Numerically \(c \approx 9.53\). These critical values can lead to the following universal ratio:
Obviously, it is independent of the quantum-corrected constant $\xi$. Note that for the van der Waals gas, the universal ratio is $\rho_e = 3/8$, whereas for some actual gas, such as water, it is $\rho_e = 0.230$. 

\begin{equation}
\rho_e = \frac{P_c r_e}{T_c} = \frac{3\left(c^2 - 4c - 1\right)}{4\left(3c^2 - 6c - 1\right)} \approx 0.181.
\end{equation} 

Figure 2. $P - r_s$ diagram for the AS-improved black hole. We choose $\xi = 0.1$ here. The critical radius, temperature, and pressure are respectively $r_c = 0.976$, $T_c = 0.0999$, $P_c = 0.0185$.

Figure 3. The Gibbs free energy as a function of temperature for different pressures for the quantum-corrected black hole. We also choose $\xi = 0.1$ here.
Furthermore, one can analyze the Gibbs free energy: \( G = G(T, P) = H - TS = M - TS \). As is shown in figure 3, the Gibbs free energy develops a ‘swallowtail’ for \( P < P_c \), which is a typical feature in a first-order phase transition. Above the critical pressure \( P_c \), the ‘swallowtail’ disappears.

Next we will calculate the critical exponents at the critical point for the quantum-corrected black hole. For a van der Waals liquid–gas system, the critical behaviors can be characterized by the critical exponents as follows [46]:

\[
C_v \sim \left(\frac{T - T_c}{T_c}\right)^\alpha, \quad \frac{v_g - v_l}{v_c} \sim \left(\frac{T - T_c}{T_c}\right)^\beta, \quad \kappa_T \sim \left(\frac{T - T_c}{T_c}\right)^\gamma, \quad P - P_c \sim (v - v_c)^\delta.
\]

Here \( v_g \) and \( v_l \) refer to the specific volume for the gas phase and liquid phase respectively. For the quantum-corrected black hole, we use \( r_g \) and \( r_l \) instead.

Defining
\[
t = \frac{T}{T_c} - 1, \quad x = \frac{r_g}{r_c} - 1, \quad p = \frac{P}{P_c},
\]
and replacing \( r_c, T, \) and \( P \) in equation (3.11) with the new dimensionless parameters \( x, t, \) and \( p \) and then expanding the equation near the critical point approximately, one can obtain

\[
p = 1 + At + Btx + Cx^3 + O(t^2, x^4).
\]

where \( A, B, \) and \( C \) are all complicated expressions composed of \( c = 3 + \frac{2}{3^{2/3}} \left[ (39 + i\sqrt{15})^{1/3} + (39 - i\sqrt{15})^{1/3} \right] \). Numerically, \( A \approx 2.95, \ B \approx -3.31, \) and \( C \approx -1.20 \).

Equation (3.19) has the same form as that for the van der Waals system and an RN–AdS black hole [23]. Therefore, we can derive the critical exponents \( \beta = 1/2, \ \gamma = 1, \) and \( \delta = 3 \) in the same way. In addition, according to (3.6) entropy is independent of \( T \). Thus, \( C_V = T \left. \frac{\partial S}{\partial T} \right|_V = 0 \). Therefore, we also have the critical exponent \( \alpha = 0 \). Obviously, these obey scaling symmetry as in ordinary thermodynamic systems:

\[
\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(\delta + 1) = 2, \quad \gamma(1 + \delta) = (2 - \alpha)(\delta - 1), \quad \gamma = \beta(\delta - 1).
\]

### 4. The second-order phase transition at the critical point

In this section, we study the types of phase transition for a quantum-corrected black hole at the divergent points of heat capacity. It should be noted that the divergent points depend on the values of the pressure \( P \) or the temperature \( T \) for a positive \( \xi \). When \( P = P_c \) or \( T = T_c \), there is only one divergent point; when \( P < P_c \) or \( T < T_c \), there are two divergent points; no divergent point exists when \( P > P_c \) or \( T > T_c \).

Ehrenfest had always attempted to classify phase transitions. Phase transitions connected with an entropy discontinuity are called discontinuous or first-order phase transitions, and phase transitions where the entropy is continuous are called continuous or second-/higher-order phase transitions. More precisely, for a first-order phase transition the Gibbs free energy \( G(T, P, ...) \) should be continuous and its first derivative with respect to the external fields:
S = - \frac{\partial G}{\partial T} \bigg|_{(P, \ldots)} , \quad V = \frac{\partial G}{\partial P} \bigg|_{(T, \ldots)} \tag{4.1}

is discontinuous at the phase transition points.

For the second-order phase transition the Gibbs free energy $G(T, P, \ldots)$ and its first derivative are both continuous, but the second derivative of $G$ diverges at the phase transition points, such as the specific heat $C_P$, the compressibility $\kappa$, and the expansion coefficient $\alpha_P$:

$C_P = T \frac{\partial S}{\partial T} \bigg|_P = -T \frac{\partial^2 G}{\partial T^2} \bigg|_P , \quad \kappa_T = \frac{1}{V} \frac{\partial V}{\partial P} \bigg|_T = \frac{1}{V} \frac{\partial^2 G}{\partial P^2} \bigg|_T , \quad \alpha_P = -\frac{1}{V} \frac{\partial V}{\partial T} \bigg|_P = -\frac{1}{V} \frac{\partial^2 G}{\partial P \partial T} \bigg|_P \tag{4.2}

In accordance with equations (3.3) and (3.6), one can easily obtain the $S - T$ plot, as is shown in figure 4. Obviously, entropy is a continuous function of temperature.

One can easily calculate $\kappa_T$ and $\alpha_P$:

$\kappa_T = -\frac{1}{V} \frac{\partial V}{\partial r_a} \bigg|_T = \frac{8 \pi r_+^2 \left( \xi + 3 r_+^2 \right)^2}{3 \xi^2 + 24 \pi P r_+^6 + 64 \pi \xi^2 P r_+^8 + 8 \pi \xi^2 P r_+^2 - 3 r_+^4 + 12 \xi r_+^2} \tag{4.3}$

$\alpha_P = -\frac{1}{V} \frac{\partial V}{\partial r_a} \bigg|_P = \frac{12 \pi r_+ \left( \xi + r_+^2 \right) \left( \xi + 3 r_+^2 \right)}{3 \xi^2 + 24 \pi P r_+^6 + 64 \pi \xi^2 P r_+^8 + 8 \pi \xi^2 P r_+^2 - 3 r_+^4 + 12 \xi r_+^2} \tag{4.4}$

They diverge when the denominator vanishes. It is clear that the denominators of $\kappa_T$, $\alpha_P$ are the same as that of $C_P$. As shown in figure 5, there are two divergent points for both $\kappa_T$ and $\alpha_P$ for $P < P_c$. Only one divergent point is left when $P = P_c$. Owing to the divergence of $\kappa_T$, $\alpha_P$, phase transitions at these critical points are all second-order.

When $\xi = 0$, the quantum-corrected black hole returns to the Schwarzschild–AdS black hole, for which there is still a point where $C_P$, $\kappa$, $\alpha_P$ diverge. However, in this case, only one divergent point exists. One can also analyze the types of phase transition at the divergent points by means of the Ehrenfest scheme employed in [47]. That can give the same result. Generally, thermodynamic geometry can also be employed to study phase transitions [48–50]. However, for the quantum-corrected black hole this does not work. This is because mass/enthalpy is linear at the pressure $P$, which leads to a degenerate thermodynamic metric.
5. Concluding remarks

In this paper we studied the thermodynamics and critical behaviors of a kind of quantum-corrected black hole obtained in the asymptotically safe gravity theory with higher derivatives and a cosmological constant. The asymptotic safety scenario includes the scale-dependent Newtonian ‘constant’ $G_p$ and a cosmological ‘constant’. $G_p$ leads to the correction to the conventional Schwarzschild–AdS black hole. The running cosmological ‘constant’ can naturally be treated as a variable. We can identify it by thermodynamic pressure and include its variation in the first law of black hole thermodynamics.

Based on a quantum-corrected black hole, we studied the $P - V/r$ criticality at the critical point and plotted the isotherm curves. It is shown that the $P - V/r$ phase diagram is the same as that of the van der Waals liquid–gas system. Furthermore, we calculated the critical exponents at the critical point, which all coincide with those of the van der Waals system and an RN–AdS black hole. From the critical parameters we can also construct the universal ratio $\rho_* = \frac{P r_*}{T_*} \approx 0.181$. We analyzed the types of phase transition at the divergent points of heat capacity using Ehrenfest’s classification. The Gibbs free energy and entropy are both continuous functions of temperature. The heat capacity at constant pressure $C_P$, the compressibility $\kappa_T$, and the expansion coefficient $\alpha_P$ all suffer discontinuities at some points when the pressure or the temperature is not greater than its critical value. Therefore, we conclude that the phase transitions at these points are second-order.

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References

[1] Bardeen J M, Carter B and Hawking S 1973 Commun. Math. Phys. 31 161
[2] Davies P 1977 Proc. R. Soc. Lond. A 353 499-521
[3] Hawking S and Page D N 1983 Commun. Math. Phys. 87 577
[4] Hut P 1977 *Mon. Not. R. Astron. Soc.* **180** 379–89
[5] Sokolowski L M and Mazur P 1980 *J. Phys. A: Math. Gen.* **13** 1113
[6] Lousto C O 1995 *Phys. Rev.* D **51** 1733
[7] Peča C and Lemos J P S 1999 *Phys. Rev.* D **59** 124007
[8] Banerjee R, Modak S K and Samanta S 2011 *Phys. Rev.* D **84** 064024
[9] Banerjee R and Roychowdhury D 2012 *Phys. Rev.* D **85** 044040
[10] Myung Y S 2012 *Eur. Phys. J.* C **72** 2116
[11] Chamblin A, Emparan R, Johnson C and Myers R 1999 *Phys. Rev.* D **60** 064018
[12] Chamblin A, Emparan R, Johnson C and Myers R 1999 *Phys. Rev.* D **60** 104026
[13] Wu X N 2000 *Phys. Rev.* D **62** 124023
[14] Kastor D, Ray S and Traschen J 2009 *Class. Quantum Grav.* **26** 195011
[15] Cvetic M, Gibbons G, Kubiznak D and Pope C 2011 *Phys. Rev.* D **84** 024037
[16] Banerjee R, Modak S K and Roychowdhury D 2012 *J. High Energy Phys.* JHEP10(2012)125
[17] Wei S-W and Liu Y-X 2013 *Phys. Rev.* D **87** 044014
[18] Poshteh M B J, Mirza B and Sherkatghanad Z 2013 *Phys. Rev.* D **88** 024005
[19] Dolan B P 2011 *Class. Quantum Grav.* **28** 235017
[20] Kubiznak D and Mann R B 2012 *J. High Energy Phys.* JHEP2012(7)1–25
[21] Gunasekaran S, Kubiznak D and Mann R B 2012 *J. High Energy Phys.* JHEP2012(11)1–43
[22] Wei S-W and Liu Y-X 2013 *Phys. Rev.* D **87** 044014
[23] Kubiznak D and Mann R B 2012 *JHEP10(2012)125*
[24] Wei S-W and Liu Y-X 2013 *Phys. Rev.* D **87** 044014
[25] Horvat R 2004 *Phys. Rev.* D **70** 087301
[26] Brown J D and Teitelboim C 1987 *Phys. Lett.* B **195** 177
[27] Brown J D and Teitelboim C 1988 *Nucl. Phys.* B **297** 787
[28] Weinberg S 1979 *General Relativity* ed S W Hawking and W Israel (Cambridge: Cambridge University Press)
[29] Niedermaier M and Reuter M 2006 *Living Rev. Relativ.* **9** 5
[30] Niedermaier M 2007 *Class. Quantum Grav.* **24** R171
[31] Reuter M and Saueressig F 2012 *New J. Phys.* **14** 055022
[32] Koch B and Saueressig F 2014 *Int. J. Mod. Phys.* A **29** 08
[33] Koch B and Saueressig F 2014 *Class. Quantum Grav.* **31** 015006
[34] Bonanno A and Reuter M 2000 *Phys. Rev.* D **62** 043008
[35] Bonanno A 2012 *Phys. Rev.* D **85** 081503(R)
[36] Hindmarsh M and Saltas I D 2012 *Phys. Rev.* D **86** 064029
[37] Dolan B P 2011 *Phys. Rev.* D **84** 127503
[38] Dolan B P 2011 *Class. Quantum Grav.* **28** 125020
[39] Stanley H E 1987 *Introduction to Phase Transitions and Critical Phenomena* (New York: Oxford University Press)
[40] Banerjee R and Roychowdhury D 2011 *J. High Energy Phys.* JHEP2011(11)1–13
[41] Ruppeiner G 2008 *Phys. Rev.* D **78** 024016
[42] Quevedo H 2007 *J. Math. Phys.* **48** 013506
[43] Quevedo H and Sánchez A 2008 *J. High Energy Phys.* JHEP2008(09)34