Radiation from Global Topological Strings using Adaptive Mesh Refinement

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Global Topological Strings

- Assume complex scalar field $\phi$, e.g. global cosmic string, (QCD) axion string

- Postulate a phase transition due to e.g. temperature cooling $\rightarrow$ Lagrangian with U(1) symmetry

- Energetically favourable for field to fall into potential minimum at different spatial locations $\rightarrow$ U(1) symmetry breaking

- Traverse closed path in space $\rightarrow$ string if we want simply connected

\[
\mathcal{L} = (\partial_{\mu} \bar{\phi})(\partial^{\mu} \phi) - V(\phi)
\]

\[
V(\phi) = \frac{1}{4}\lambda(\bar{\phi}\phi - \eta^2)^2
\]

Euler-Lagrange equations

\[
\partial_{\mu} \partial^{\mu} \phi + \frac{\lambda}{2} \phi (|\phi|^2 - \eta^2) = 0
\]
Radiative Modes

• Oscillating strings emit massive (Higgs) and massless (Goldstone or `axion’) radiation

• Need robust diagnostic tools to numerically extract and analyse - radiative modes must be separated from string self-fields

• \( \phi(x^\mu) = \phi(x^\mu)e^{i\theta(x^\mu)} \) where \( \phi(x^\mu) \) and \( \theta(x^\mu) \) are real scalar fields associated with orthogonal excitations in figure

• Can show that these fields obey massive/massless wave equations respectively

\[
\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi - \lambda \phi(1 - \phi^2) = 0
\]

\[
\frac{\partial^2 \theta}{\partial t^2} - \nabla^2 \theta = 0
\]
Radiative Modes \[ \varphi = \phi_1 + i \phi_2 \]

- We can write the energy flux as massive and massless components

\[
T_{\mu\nu} = 2\partial_{(\mu} \bar{\varphi} \partial_{\nu)} \varphi - g_{\mu\nu} \left( \partial_\sigma \bar{\varphi} \partial^\sigma \varphi - \frac{\lambda}{4} (\bar{\varphi} \varphi - 1)^2 \right)
\]

\[
S_i \equiv T^{0i} = \Pi_\phi \mathcal{D}_i \phi + \Pi_\vartheta \mathcal{D}_i \vartheta
\]

\[
\Pi_\phi \equiv \frac{\phi_1 \dot{\phi}_1 + \phi_2 \dot{\phi}_2}{\phi}, \quad \mathcal{D}_i \phi \equiv \frac{\phi_1 \nabla_i \phi_1 + \phi_2 \nabla_i \phi_2}{\phi}
\]

\[
\Pi_\vartheta \equiv \frac{\phi_1 \dot{\phi}_2 - \phi_2 \dot{\phi}_1}{\phi}, \quad \mathcal{D}_i \vartheta \equiv \frac{\phi_1 \nabla_i \phi_2 - \phi_2 \nabla_i \phi_1}{\phi}
\]

- Two quantities are equivalent of EM Poynting vector

- Spatial diagnostic \( \mathcal{D} \vartheta \cdot \hat{r} \) particularly useful for distinguishing massless radiation from string self-field which can be similar order of magnitude in energy density - spatial gradient of self-field is weak
Analytic Predictions: Axions and Antisymmetric Tensors

- Duality of massless scalar $\phi$ and two-index antisymmetric tensor $B_{\mu\nu}$
  \[ \phi^2 \partial_\mu \phi = \frac{1}{2} f_a \epsilon_{\mu\nu\lambda\rho} \partial^\nu B^{\lambda\rho} \]

- Can use this analytically to predict radiation power spectrum from global string

**Predicted Radiation Power Spectrum**

\[
\frac{dP}{dz} = 2\pi \sum_{n=1}^{\infty} \omega_n \left( \sum_{|\kappa_m|<\alpha \omega_n} \int_0^{2\pi} d\varphi \tilde{J}^{\mu\nu*}(\omega_n, k^\perp, \kappa_m) \tilde{J}_{\mu\nu}(\omega_n, k^\perp, \kappa_m) \right)
\]

**Radiation Equation**

\[ \partial_\sigma \partial^\sigma B^{\mu\nu} = 4\pi J^{\mu\nu} \]

**String Source Term**

\[ J^{\mu\nu} = \frac{f_a}{2} \int \delta^{(4)}[x - X(\sigma, \tau)] d\sigma^{\mu\nu} \]

- Can rewrite axion spectrum in terms of left- and right-moving modes (Fourier transform) - analogy with gravitational radiation

\[
\frac{dP}{dz} = \frac{8\pi^3 f_a^2}{L} \sum_{n=1}^{\infty} n \sum_{\substack{|m|<n \\text{m+n even}}} \left\{ |U^\perp|^2 |V^\perp|^2 + |U^{\perp*} \cdot V^\perp|^2 - |U^\perp \cdot V^\perp|^2 \right\}
\]

Periodic axion string

Similar forms

**Local string for GWs**

\[
\frac{dP}{dz} = \frac{64\pi G \mu_0^2}{L} \sum_{n=1}^{\infty} n \sum_{\substack{|m|<n \\text{m+n even}}} \left\{ |U^\perp|^2 |V^\perp|^2 - |U^{\perp*} \cdot V^\perp|^2 + |U^\perp \cdot V^\perp|^2 \right\}
\]
Separating String Scales

- Energy per unit length of global (axion) string
  \[ \mu = 2\pi f_a^2 \log(L/\delta) \]

- String width, \( \delta \approx m_s^{-1} = (\sqrt{\lambda} f_a)^{-1} \)

- Axion strings (realistic), \( \log \left( \frac{L}{\delta} \right) \approx 70 \)

- Cosmological GUT strings, \( \log \left( \frac{L}{\delta} \right) \approx 100 \)

- Typical fixed grid simulations, \( \log \left( \frac{L}{\delta} \right) \approx 4 \)
Current Simulations

• Difficult to simulate topological strings accurately due to large separation of scales

• Two main approaches to date [Battye and Moss 2010, arXiv:1005.0479]:
  - Nambu-Goto string action - infinitely thin, no backreaction [e.g. Allen and Shellard 1990, Sakellariadou 1991]:
    \[ S = -\mu \int d^2 \zeta \sqrt{-\gamma} \]
  - Field theory - ‘fat string’ to deal with expansion of the universe [e.g. Vincent, Antunes and Hindmarsh 2008, arXiv:hep-ph/9708427]
    - E.g. axion strings (Battye and EPS, 1994; Yamaguchi, 1999; Klaer & Moore 2017; Villadoro et al, 2018)
      \[ \partial_\mu \partial^\mu \varphi + \frac{\lambda}{2} \varphi (|\varphi|^2 - \eta^2) = 0 \]

• Both approaches use a fixed grid
AMR and GRChombo

We use GRChombo adaptive mesh refinement code to bridge the gap:

www.grchombo.org

• Fully adaptive mesh refinement dynamically adapts solution grid to scale of the problem

• Dynamically tag cells according to chosen gradient criterion

• Significant OpenMP/MPI parallelism, optimised with support from Intel

• Refinement levels structured into boxes which can be distributed over processors
Radiation from Oscillating Strings

- Periodic string configuration (z-direction)

- Obtain initial conditions numerically with dissipative evolution - reduce string amplitude considerably to ensure long-range fields are relaxed

- Sinusoidal perturbations with range of amplitudes e.g. $A_0 = 1,3$

- Run range of $\lambda$ (string width/mass) with AMR

\[
\delta \approx m_s^{-1} = (\sqrt{\lambda f_a})^{-1}
\]
Harmonic Radiation Modes

\[ \vartheta(t, r, \theta, z) = \Re \sum_{p m n} A_{p m n} e^{-i \Omega [(p/\alpha) t - nz]} e^{i m \theta} \times H_{m}^{(1)} \left( \Omega \kappa_{p n} r \right) \]
$A_0 = 1$

$\lambda = 1$

$\lambda = 10$
String Radiation Backreaction

- We analyse detailed evolution of oscillating string trajectories by tracking string core.
- Focus on regimes where AMR evolution is robust and accurate.
- Analyse two specific sets of simulations with initial amplitudes $A_0 = 1, 3$.
- Vary the string width across the wide range $1 \leq \lambda \leq 100$.

### NG Sinusoidal Long String

\[
\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon_0^2} = \frac{\beta t}{\bar{\mu}L} \rightarrow \varepsilon_0 \left(1 + \frac{\beta \varepsilon_0^2 t}{\bar{\mu}L}\right)^{-1/2}
\]

### Unequal left/right-moving modes

\[
\varepsilon = \varepsilon_0 \exp \left(-\frac{\beta t}{2\bar{\mu}L}\right)
\]
String Radiation Backreaction

$A_0 = 1$

$A_0 = 3$
Inverse Square Model

- Backreaction predicts linear slope depends on energy density, independent of amplitude.
- Here $A_0 = 1$ plot is offset by -20 for clarity, slopes unchanged.
- Finite width effects for lighter strings reduce damping rate.

\[ \epsilon_0 = 0.20 \quad (A_0 = 1) \]
\[ \epsilon_0 = 0.54 \quad (A_0 = 3) \]
Inverse Square Model

- Dashed lines - analytic predictions from linear inverse square model
- Least squares best fit, analytic prediction remarkably good agreement for $\lambda > 3$
- Have applied a finite width correction of 8%; internal modes mask true oscillation amplitude
- Again, see finite width effects for lighter strings reduce damping rate
Inverse Square Model

- Measured damping rates plotted as function of inverse energy density $\mu^{-1}$ (inverse $\ln \lambda$)

- Have used an effective string radius cutoff $R = 3.75$ for which damping rate vanishes as $\mu \to \infty$

\[ R = 3.75 \]

\[ \frac{4\beta}{\mu L} \]

\[ 0 \leq 0.20 \text{ (fwc)} \]

\[ 0 \leq 0.20 \text{ (raw)} \]

\[ 0 \leq 0.54 \text{ (fwc)} \]

\[ 0 \leq 0.54 \text{ (raw)} \]
Exponential Decay Model

- Alternative logarithmic model is less accurate.
- Decay rate strongly dependent on initial amplitude.
- See clear deviation from exponential behaviour (dashed).
Summary

• Previous simulations of cosmic/axion strings do not capture sufficient dynamic range - AMR bridges the gap

• Oscillating global strings simulated on a wide variety of physical length scales by exploiting AMR GRChombo code:
  
  • Quadrupole massless radiation observed together with dipole massive radiation using quantitative Fourier analysis
  
  • Matching physical intuition: Lower lambda = larger width and lower mass implies relatively more radiation into massive channels, more massive backreaction

• Analytic inverse square model offers excellent description of oscillating radiating global string, predicting both correct power law and magnitude of damping

• Future work: Further spectral analysis of axion radiation from string configurations (including relativistic cases). AMR simulations of large-scale string networks and precision constraints on axion mass. Calculation of GW signatures.
