Phenomenology of IR-renormalons in inclusive processes

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We have compared the existing experimental data on the leading power corrections to the structure functions $F_2(x, Q^2)$, $F_3(x, Q^2)$, and $F_L(x, Q^2)$ with the IR-renormalon model predictions for higher-twist contributions. Our analysis shows that the model properly describes the $x$-dependence, but typically falls short by a factor 2 or 3 as far as the magnitude of higher twist corrections is concerned.
Measurements of QCD observables now have reached such high precision that power corrections to the structure functions can often be extracted with a reasonable accuracy from the existing data. The situation on the theoretical side is much less clear. In the best understood case of deep inelastic scattering, the relevant contributions can be attributed in the framework of operator product expansion (OPE) to matrix elements of higher twist operators [1], but their determination in QCD is ambiguous due to occurrence of power divergences [2]. From the phenomenological point of view, however, attempts to compute these matrix elements using e.g., QCD sum rules or the bag model have provided results which seem to have at least the right order of magnitude [3, 4, 5, 6] as compared with available experimental estimates.

Recently there has been some interest in another phenomenological approach to power-suppressed corrections in QCD [7], based on the fact that the only possibility to interpret the higher-order radiative corrections in a consistent manner (i.e. as asymptotic series) requires the existence of power suppressed terms [8]. The IR-renormalon contributions occur because certain classes of higher order radiative corrections to twist-2 are sensitive to large distances, contrary to the spirit of OPE. Although these divergences have to cancel with UV power divergencies in matrix elements of twist-4 operators i.e., they are totally spurious, there are arguments which suggest that they reflect the correct shape, if not the magnitude, of the power-suppressed terms [9, 10, 11]. In this sense we define the prediction of the renormalon model of higher-twist corrections to a DIS structure function as the power-suppressed uncertainty which occurs in the perturbative expansion of the Wilson coefficient to the twist-2 contribution.

Commonly the divergent series of radiative corrections is regarded as an asymptotic series and defined by its Borel integral. The actual calculations are done in a large-$N_F$ limit which allows to resum the fermion bubble-chain to all orders yielding the coefficient of the $\alpha_S^2 N_F^{-1}$ - term exactly. Subsequently, it is converted into the exact coefficient of the $\alpha_S^2 \beta_0^{-1}$ - term by the substitution $N_F \to N_F - 33/2 = -6\pi\beta_0$, known as the ’Naive Non-Abelianization’ (NNA) (see the last two references of [8]). The asymptotic character of the resulting perturbative series leads to resummation ambiguities - a singularity in the Borel integral destroys the unambiguous reconstruction of the series and shows up as a factorial divergence of the coefficients of the perturbative expansion. The general uncertainty in the perturbative prediction can be estimated to be of the order of the minimal term in the expansion, or by taking the imaginary part (divided by $\pi$) of the Borel integral. Both procedures lead to resummation ambiguities of the form $C \times (\Lambda^2/Q^2)^r$, with $r = 1$ for the leading IR renormalon, and a numerical coefficient $C$ which either can be taken as it comes out from the NNA calculation, see below, or can be fitted as a free parameter.

The unpolarized hadronic scattering tensor for leptons scattering off nucleons can be divided into three structure functions [12]

\[
W_{\mu\nu}(p, q) = \left( g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) \frac{F_L(x, Q^2)}{2x} + \left( -\frac{p_{\mu} p_{\nu}}{(p \cdot q)^2} q^2 + \frac{p_{\mu} q_{\nu} + p_{\nu} q_{\mu}}{p \cdot q} - g_{\mu\nu} \right) \frac{F_2(x, Q^2)}{2x} \nonumber \\
- i\epsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{p \cdot q} F_3(x, Q^2). \tag{1}
\]
Here $x = Q^2/(2p \cdot q)$ and $Q^2 = -q^2$. The structure functions $F_i, i = L, 2, 3$ can be generally decomposed in the following manner

$$F_i(x, Q^2) = F_{i-2}^t(x, Q^2) + \frac{1}{Q^2} h_{i \text{TMC}}(x, Q^2) + \frac{1}{Q^2} h_i(x, Q^2) + \mathcal{O}(\frac{1}{Q^4}) \; ,$$

(2)

where $F_{i-2}^t$ describes the leading twist-2 contribution. $h_{i \text{TMC}}$ describes the target mass corrections which are directly related to twist-2 matrix elements. $h_i$ contains the genuine twist-4 contribution which is in principle sensitive to multiparton correlations within the hadron, and which we want to estimate using the renormalon model. Note that $h_i$ has dimension 2 (if radiative corrections are neglected) and hence it has to be proportional to a certain mass scale $\mu^2$. In the analysis of the experimental data used in this paper the coefficients $h_i$ were extracted by accounting for target mass corrections up to order $\mathcal{O}(M_N^2/Q^4)$, except for the case of $h_L$ where target mass corrections have been accounted for to the order $\mathcal{O}(M_N^2/Q^2)$ only. In the analysis of the twist-4 contributions it is common to neglect the $Q^2$ dependence of $h_i$, which is due to radiative corrections, and to sum target mass corrections and the twist-2 contributions into a leading-twist (LT) structure function thus arriving at the notation

$$F_i(x, Q^2) = F_{i \text{LT}}(x, Q^2) + \frac{1}{Q^2} h_i(x) = F_{i \text{LT}}(x, Q^2) \left(1 + \frac{C_i(x)}{Q^2}\right) \; .$$

(3)

Now we shall shortly summarize the main features of the model. In the renormalon approach the expression for the higher twist correction $h_i(x)$ to a structure function $F_i$ has the form of a Mellin convolution

$$h_i = P_i \otimes F_{i-2}^t$$

(4)

of a coefficient $P_i(z)$, and the twist-2 part $F_{i-2}^t(x)$ of the structure function $F_i$. The mass scale $\mu^2$ which enters $P_i(z)$ equals to $\mu_R^2 = 8\pi C_F \Lambda^2 e^{-C_s/\beta_0}$ where $C_F = 4/3$, $\Lambda$ is the QCD scale parameter in a given renormalization scheme $s$, and $C_s$ is determined by the finite part of the fermion loop, $C_{\text{MS}} = -5/3$. This identification, which follows from the NNA prescription, is RG invariant only in the strict large-$N_f$ limit. Another difficulty stems from the observation that within the logic of the model one cannot distinguish between LO and NLO parametrizations of twist-2 structure functions $F_{i-2}^t(x)$. In the following we will consequently use the LO GRV parametrizations of parton distributions, but we have checked that qualitatively the same conclusions follow from a calculation which incorporates the NLO parametrizations. In the present analysis we always refer to a region of $Q^2$ around 2 GeV$^2$ i.e., below the charm threshold, and accordingly we adopt the $N_F = 3$ value of the $\Lambda$ parameter from the LO GRV fit, $\Lambda = 232$ MeV. We mention also that due to the two possible contour deformations above or below the pole in the Borel integral, as discussed above, in principle the overall sign of the coefficients $P_i(z)$ cannot be uniquely determined.
We shall state from the very beginning that because the IR-renormalon result is proportional to the twist-2 contribution, it cannot be expected to describe the complete twist-4 correction which contains a genuine multiple field correlation, and which depends therefore on the exact internal hadron wave function. This distinction has a real physical meaning as can be seen from the following argument. Let us assume that the same structure functions $F(x, Q^2)$ are measured for different hadrons and that the lowest-order (in $\alpha_s$) leading twist ($F^{t=2}(x)$) and $1/Q^2$ parts ($F^{t=4}(x)$) can be separated experimentally. Then the renormalon contribution cancels for the difference of the ratio of moments

$$\left( \frac{M_n(F^{t=4})}{M_n(F^{t=2})} \right)_{\text{hadron 1}} - \left( \frac{M_n(F^{t=4})}{M_n(F^{t=2})} \right)_{\text{hadron 2}}$$

which is, however, still sensitive to the genuine higher-twist corrections, which depend on the exact specific internal hadron wave function. We hope that this argument elucidates the fundamental limitations of the renormalon approach. Nevertheless, to achieve a better understanding of the renormalon contributions it is important to study as many different cases as possible. On the basis of these results we hope to develop a physical interpretation which explains why this phenomenological approach is successful in some cases, but fails in others [19].

So far the renormalon model was used to calculate the power corrections for the non-singlet part of the structure functions $F_L$ [17] and $g_1$ [20]. The physically equivalent approach based on dispersion relations of [9, 21] was used to calculate the higher twist contribution to the non-singlet part of the structure functions $F_2$ and $F_3$ [22].

In the case of the structure function $F_2(x)$ an experimental analysis of higher twist corrections exists for proton and deuteron targets [15]. Assuming $Q^2$ to be low enough, i.e., below the charm threshold, one obtains

\[
C_d = \frac{1}{F_2^{d,t=2}} \left( \frac{2}{9} P_S \otimes F_2^{0,t=2} + \frac{1}{18} P_{NS} \otimes F_2^{8,t=2} + P_G \otimes G \right)
\]

\[
C_{(p,n)} = \frac{1}{F_2^{(p,n),t=2}} \left( \frac{2}{9} P_S \otimes F_2^{0,t=2} + \frac{1}{6} P_{NS} \otimes (\pm F_2^{3,t=2} + \frac{1}{3} F_2^{8,t=2}) + P_G \otimes G \right),
\]

where $F_i^{i,t=2}$, $i = 0, 3, 8$ denote corresponding SU(3) combinations of parton densities, and $G$ is the gluon twist-2 structure function of the nucleon. The corresponding expression for the $F_3(x)$ structure function reads

\[
C_3 = \frac{1}{F_3^{t=2}} (P_3 \otimes F_3^{t=2}).
\]

As already mentioned, so far only the coefficients $P_{NS}$ and $P_3$ are known. For a first try, we set in (6) the coefficient $P_G$ to zero, and approximate $P_S \sim P_{NS}$, which results in

\[
C_d = \frac{1}{F_2^{d,t-2}} (P_{NS} \otimes F_2^{d,t-2})
\]

\[
C_{(p,n)} = \frac{1}{F_2^{(p,n),t-2}} (P_{NS} \otimes F_2^{(p,n),t-2}).
\]
Equations (6), (7), and (8) define the renormalon model of higher twist corrections. In the comparison of the model predictions with the data we have used the NMC analysis of $F_2(x, Q^2)$ [14] for $C_{2, \text{proton}}$ and $C_{2, \text{deuteron}}$ as well as the more precise one for $C_{2, \text{proton}} - C_{2, \text{neutron}}$. For $F_3$ we take the new fit of [16]. We have also confronted our results with the output of the computer code kindly provided to us by B. Webber [22], but with the LO GRV [18] parametrization consistently used to represent twist-2 structure functions.

Fig. 1 shows our results for $C_p(x)$ and $C_d(x)$ of $F_2$, with twist-2 structure functions parametrized according to the leading order GRV-parametrization [18], as well as the experimental results. Obviously the renormalon model underestimates the experimental data. On the other hand the proton and deuteron data differ only slightly, indicating the dominance of the flavor singlet contribution.

The $x$-dependence of the higher twist correction can be, however, reproduced very well if one adopts the value of $\mu^2$ a factor of 3 larger than what follows from the renormalon model, $\mu^2 \approx 3\mu_R^2$, as it was done in Ref. [22]. Strictly speaking, the authors of [22] used twist-2 MRS(A) [23] parton distributions normalized at the scale $Q^2 = 10 \text{ GeV}^2$ and hence they obtained $\mu^2 \approx 2.4\mu_R^2$. Note that as this scale dependence is governed by anomalous dimensions of twist-2 rather than twist-4 operators, it differs from what one would expect in QCD.

It is interesting to speculate how taking into account the yet unknown coefficients $P_S$ and $P_G$ may influence the renormalon model predictions. First, note that the characteristic rise of the renormalon model prediction in Figure 1, which clearly follows the trend seen in the data, results from the term in $P_{NS}$ proportional to $N$ in the Mellin space [20, 22]. If one assumes that $P_S$ and $P_G$ follow the same pattern, i.e. they do not contain terms proportional e.g. to $N^2$, then the gluonic contribution should be always negligible at large $x$. Indeed, the twist-2 gluon distribution should fall at least one power of $1 - x$ faster than the quark one [24], and therefore it cannot be responsible for the discrepancy observed in Figure 1. One could then try to constrain the singlet coefficient $P_S$ taking into account the experimental fact that $C_d(x) \sim C_p(x)$, and known relations between various parton distributions when $x \to 1$ [25], but it leads to the relation $P_S = P_{NS}$, which has been discussed already above. Finally, we note that the deuteron and proton contributions to Eq. (6) contain the unknown combination $\frac{2}{9}P_S \otimes F_2^{0,t-2} + P_G \otimes G$ which therefore can be determined independently from $C_p$ and $C_d$ data:

$$\frac{2}{9}P_S \otimes F_2^{0,t-2} + P_G \otimes G = C_d F_2^{d,t-2} - \frac{1}{18}P_{NS} \otimes F_2^{8,t-2},$$

(9)

and

$$\frac{2}{9}P_S \otimes F_2^{0,t-2} + P_G \otimes G = C_p F_2^{p,t-2} - \frac{1}{6}P_{NS} \otimes (F_2^{3,t-2} + \frac{1}{3}F_2^{8,t-2}).$$

(10)

We have explicitly checked that both equations (9) and (10) are indeed consistent with each other, see Figure 2, and so one can take e.g. the arithmetic average between both estimates as the approximation for the unknown flavor-singlet contribution. Note that the curve is negative for $x \leq 0.35$ in accordance with the fact that the renormalon model prediction for $C_p$ and $C_d$ is higher than the data in this region, see Figure 1. It turns out, however, that when one wants to use this approximation for comparison...
with the difference $C_{2,\text{proton}} - C_{2,\text{neutron}}$, which has been extracted in [14] with much smaller experimental errors, one encounters a problem. The results are shown in Figure 3 together with the now fixed renormalon model prediction (including the fitted quark singlet and gluonic parts). The agreement with the data becomes better, but the model still underestimates the higher twist corrections for $x$ between 0.2 and 0.4.

In addition we have compared the renormalon model prediction for the difference of coefficients $(h_{2p}(x) - h_{2n}(x))/x$ as given in Ref. [15], see Figure 4. As far as the $x$-dependence is concerned, the general trend seems to be reproduced by the model, but the scale has again to be taken a factor of 3 or larger than the $\mu^2_R$. We note that as $(h_{2p}(x) - h_{2n}(x))/x$ is free from flavor-singlet contributions, it is an ideal observable for models of higher-twist corrections, and that one should try to get better data (smaller error bars) for this quantity.

The case of the purely non-singlet structure function $F_3(x)$ is illustrated on Figure 5. Again, although the renormalon model prediction seems to have the correct shape, it falls short by a factor of 2 or 3 with respect to the magnitude of the higher-twist effects, as required by the data according to the LO analysis of [16]. Note that while the analysis of the magnitude of the coefficient $xh_3(x)$ in [16] shows that it slightly decreases from LO to NNLO fits, its characteristic shape, indicating an increase from a negative to a positive value somewhere at $0.2 \leq x \leq 0.65$, remains stable. Finally, an analysis along the lines of Ref. [17] shows that the renormalon model underestimates the magnitude of higher-twist corrections to $F_L$ by a factor of 2, see figure 6. Adjusting the scale $\mu^2$ to the higher twist corrections to $F_2(x, Q^2)$, as in [22], results this time in a prediction which is larger than what follows from the present data.

Summarizing, we conclude that the model in its present form can certainly be considered as a useful phenomenological guide for estimates of higher-twist contributions to DIS, but one has still to be careful in claiming a phenomenological success of this approach. Our analysis shows that although the renormalon model does in fact a good job as far as the $x$-dependence of the higher-twist correction is concerned, the mass scale it predicts is by a factor 2 - 3 too low, depending on the observable. In other words, the NNA estimates of the leading powered-suppressed corrections to DIS turn out to be a factor 2 or 3 smaller than what seems to be required by the data. Other estimates, for example the results presented in [20], should be seen from the same perspective. Another possibility would be to fit the mass scale independently to each observable, a point of view which has been recently advocated in [14]. One should also keep in mind that the analysis of $F_2(x)$ data still reveals some discrepancies and it is difficult to say now whether taking into account the singlet part, possibly with its specific mass scale, will improve the situation. We hope that these observations will help to understand successes and limitations of the renormalon phenomenology.

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Figure 1: Renormalon contribution to the coefficient $C_p$ and $C_d$ according to Eq. (8). The solid line shows the renormalon model predictions for $C_p$ and $C_d$, which are indistinguishable, obtained with the LO GRV parametrization [18]. The dashed line shows the fit, as in Ref. [22], which corresponds to the same $x$ dependence, but the value of the scale parameter $\mu^2$ about 3 times larger than the one which follows from the renormalon model. The filled and empty circles display the data for $C_p$ and $C_d$ according to Ref. [15], respectively.
Figure 2: Determination of the unknown flavor-singlet component of the renormalon model from $C_p$ (filled circles) and $C_d$ (empty circles), see Eqs. (9) and (10) respectively. The LO GRV parametrizations [18] normalized at $Q^2 = 2$ GeV$^2$ have been used to generate the twist-2 parton distributions. The solid line is the arithmetic average.
Figure 3: The renormalon model prediction for $C_p - C_n$ using the LO GRV parametrization [18], solid line, where $C_p$ and $C_d$ have been calculated according to Eq. (8). Note that due to its definition, see Eq. (3), $C_p - C_n$, is not a pure non-singlet quantity. The dashed line shows the prediction for $C_p - C_n$ after the unknown flavor-singlet contribution has been adjusted to reproduce the data for $C_p$ and $C_d$, see Eqs. (9) and (10). The dashed-dotted line shows the result of the same procedure, but with the scale $\mu^2$ adjusted, like in [22], to the description of the coefficients $C_p$ and $C_d$, see Figure (1). The data points are from Ref. [14].
Figure 4: The renormalon model prediction for \((h_{2p}(x) - h_{2n}(x))/x\) using the LO GRV parametrization \([18]\), dashed line. The dot-dashed line shows the prediction with the scale \(\mu^2\) adjusted to the description of the coefficients \(C_p\) and \(C_d\), as in \([22]\). The data points are from Ref. \([15]\). Error bars for data points lying below \(x = 0.2\) have been cut to fit into the figure.
Figure 5: Renormalon prediction for $x h_3(x)$ using the LO GRV [18] parametrization (solid line). The data points taken from Ref. [16] correspond to the LO analysis. The dashed line shows the prediction with the scale $\mu^2$ adjusted to the description of the coefficients $C_p$ and $C_d$, as in [22]. The dot-dashed line shows the original prediction of [22], obtained using the MRSA parametrization [23] normalized at $Q^2 = 10$ GeV$^2$. 
Figure 6: Coefficient $h_{Lp} - h_{Ln}$, obtained from the phenomenological parametrizations to $R(x, Q^2)$ [26] and $F_2$ [27] (solid line). The renormalon model prediction for $h_{Lp} - h_{Ln}$ using the LO GRV parametrizations is depicted by the dashed line. The dot-dashed line shows the prediction with the scale $\mu^2$ adjusted to the description of the coefficients $C_p$ and $C_d$, as in [22].