Gravitational waves from chiral phase transition in a conformally extended standard model

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Received October 18, 2019
Revised February 8, 2020
Accepted March 9, 2020
Published April 1, 2020

Abstract. The gravitational wave (GW) background produced at the cosmological chiral phase transition in a conformal extension of the standard model is studied. To obtain the bounce solution of coupled field equations we implement an iterative method. We find that the corresponding $O(3)$ symmetric Euclidean action $S_3$ divided by the temperature $T$ has a simple behavior near the critical temperature $T_C$: $S_3/T \propto (1 - T/T_C)^{-\gamma}$, which is subsequently used to determine the transition’s inverse duration $\beta$ normalized to the Hubble parameter $H$. It turns out that $\beta/H \gtrsim 10^3$, implying that the sound wave period $\tau_{\text{sw}}$ as an active GW source, too, can be much shorter than the Hubble time. We therefore compute $\tau_{\text{sw}}H$ and use it as the reduction factor for the sound wave contribution. The signal-to-noise ratio (SNR) for Deci-Hertz Interferometer Gravitational Wave Observatory (DECIGO) and Big Bang Observer (BBO) is evaluated, with the result: $\text{SNR}_{\text{DECIGO}} \lesssim 1.2$ and $\text{SNR}_{\text{BBO}} \lesssim 12.0$ for five years observation, from which we conclude that the GW signal predicted by the model in the optimistic case could be detected at BBO.

Keywords: particle physics - cosmology connection, gravitational waves / theory

ArXiv ePrint: 1910.05025
1 Introduction

One of the central questions in particle physics today is: How to go beyond the standard model (SM), see, e.g., [1]. Indeed many theoretical suggestions have been made since ever [2]. The fact that the Higgs mass term is the only dimensionful parameter in the SM and the theory is perturbative — no Landau pole below the Planck scale [3–6] — may be regarded as a hint of how to go beyond the SM [1]. Even before the SM was proposed, John Wheeler [7] wished to remove all the dimensionful parameters from the fundamental equations. If we start with a theory, which at the classical level contains no dimensionful parameter such as mass parameter at all, an energy scale has to be generated by quantum effects. A quantum generation of the Higgs mass term from “nothing” would be along the line of John Wheeler’s thought. There are two known mechanisms of “scalegenesis”: One is the Coleman-Weinberg mechanism [8] that is based on improved perturbation theory and works thanks to scale anomaly [9, 10]. The other one is the dynamical scale symmetry breaking by strong dynamics in nonabelian gauge theories, e.g., Quantum Chromodynamics (QCD). We recall that about 99 % of the energy portion of the ordinary matter in the Universe — baryon — is generated by the nonperturbative effect in QCD [11], dynamical chiral symmetry breaking [12–14]. Several realistic models using the strong dynamics have been suggested in [15–20]: It has been found that not only the Higgs mass term, but also the dark matter mass [15–17, 19–22], contributing to 27 % of the total energy of the Universe [23], as well as the Planck mass [24] can be generated by dynamical scale symmetry breaking.

At finite temperature the real QCD does not undergo a phase transition (PT), rather a continuous change of crossover type [25]. However, for sufficiently small current quark
masses, the system can undergo a first-order PT \[26–29\], and such a situation can be realized in hidden sector models \[17, 30, 31\] (see also \[32\] and references therein), in which dynamical breaking of scale symmetry takes place at energies higher than the SM scale. If the coupling of the hidden sector to the SM is very small, a chief signal from the hidden sector is the gravitational wave (GW) background produced at a first-order PT in a certain epoch of the Universe \[33\], see e.g. refs. \[34, 35\] for reviews.\(^1\) This has been even more the case since the GWs have been detected on the earth \[42–44\].

In this paper we consider the model \[21, 22\], in which a robust energy scale, created by the chiral symmetry breaking in a strongly interacting QCD-like hidden sector, transmits via a SM singlet real scalar mediator \(S\) to the SM sector and generates the Higgs mass term to trigger electroweak (EW) symmetry breaking. We are particularly interested in the GW background produced at the cosmological chiral PT of the model.\(^2\) The present work is an extension of ref. \[31\], where a few benchmark points in the parameter space have been chosen to study the GW background spectrum. We have decided to extend the analysis of ref. \[31\] from the following reasons:

a) The GW energy density depends strongly on the ratio of the duration time \(\tau_{\text{PT}} = 1/\beta\) of the first-order PT to the Hubble time \(1/H\), i.e., \((\beta/H)^{-1}\) \[33, 58–60\]. Using effective field theories it has been shown \[61\] that, in contrast to the commonly assumed value of \(\beta/H \sim O(10^2)\) \[33, 62\] (see also \[34\]), it is of order \(10^4\) in QCD like theories if the coupling to the SM is neglected, i.e., in the absence of the mediator \(S\). This means a large suppression of the GW energy density. Here we will systematically look for a parameter space with smaller \(\beta/H\), which leads to larger GW energy densities.

b) It turns out that the influence of the mediator \(S\) is an important factor to decrease \(\beta/H\); the quartic self-coupling of \(S\), \(\lambda_S\), should be of order \(10^{-3}\), which is much smaller than the Higgs self-coupling \(\lambda_H \sim O(10^{-1})\). Consequently, the mass of \(S\) denoted by \(m_S\) can become comparable with — or even smaller than — the Higgs mass \(m_h\), and consequently the mixing of the Higgs \(h\) and \(S\) is no longer negligible, i.e., subject to the LHC constraint (see e.g. refs. \[63, 64\]). We will here take into account this LHC constraint.

c) To compute \(\beta/H\) one has to solve classical equations of motion and obtain the so-called bounce solution that describes a bubble appearing during a first-order PT \[65\]. In the model in question there are two fields that are involved in the problem, \(\sigma\) for the chiral condensate and \(S\), so that we have to deal with a system of coupled differential equations. In ref. \[31\] we have employed a (modified) path-deformation method \[66\] to solve them. However, it has turned out that this method suffers from a large uncertainty and does not yield trustful results. Here we will employ another iterative method to realize a faster convergence of the iterative process.

d) The sound wave contribution to the GW spectrum will be the most dominant contribution in the model we will consider. A large \(\beta/H\) means a short duration of the first-order cosmological PT and hence a short sound wave period \(\tau_{\text{sw}}\) compared with \(1/H\). However, the formula for the sound wave contribution to the GW spectrum has

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\(^1\)The crossover transition in the real QCD can influence the spectrum of the inflationary GW \[36–41\]. The frequency band of the damped GWs is what has been predicted by Witten \[33\].

\(^2\)The GWs produced during a cosmological first-order PT in classically scale invariant models have been recently studied in refs. \[45–57\].
been derived from the numerical simulations for which a long-lasting source of the GW, i.e., $\tau_{sw}H > 1$, is assumed [67]. If $\tau_{sw}H < 1$, the sound wave is an active GW source only for a period shorter than the Hubble time. The above-mentioned formula therefore overestimates the sound wave contribution. Following refs. [68, 69] along with ref. [70], we calculate $\tau_{sw}H$ and use it as the reduction factor for the sound wave contribution.

e) The signal-to-noise ratio (SNR) is an important measure to evaluate the detectability of the GW background of the model [71]. We will calculate the SNR for Deci-Hertz Interferometer Gravitational Wave Observatory (DECIGO) [72–74] and Big Bang Observer (BBO) [75–77].

In section 2 we outline the basic feature of the model; dynamical generation of the Higgs mass term, mass spectrum, the LHC constraint of the Higgs-$S$ mixing, and dark matter (DM). Since the hidden sector of the model is strongly interacting, we use an effective theory for the dynamical chiral symmetry breaking — the Nambu-Jona-Lasinio (NJL) model [12–14] — as in refs. [17, 21, 22, 31], where our approximation method, the self-consistent mean-field approximation (SCMF) of refs. [78, 79], is also briefly elucidated in this section.

After a short review on the chiral PT in the hidden sector of the model we present, in section 3, our iterative method to obtain the bounce solution. We narrow the parameter space with smaller $\beta/H$. Two benchmark points are chosen for an orientation of the parameter space that we consider. In section 4 we discuss the GW spectrum. The above-mentioned reduction factor $\tau_{sw}H$ for the sound wave contribution is computed in this section. We then calculate the SNR to evaluate the detectability of the GW signal at DECIGO and BBO. We also compare the GW spectrum for two chosen benchmark points with the power-law integrated sensitivity [71] of DECIGO and BBO. Section 5 is devoted to summary and conclusion.

2 The model

We consider a classically scale invariant extension of the SM studied in refs. [21, 22]. The model consists of a hidden SU($n_c)_H$ gauge sector coupled to the SM sector via a real singlet scalar $S$. The hidden sector Lagrangian $\mathcal{L}_H$ of the total Lagrangian $\mathcal{L}_T = \mathcal{L}_H + \mathcal{L}_{SM+S}$ of the model is given as

$$\mathcal{L}_H = -\frac{1}{2} \text{Tr} F^2 + \text{Tr} \bar{\psi}(i\gamma^\mu \partial_\mu + g_H \gamma^\mu G_\mu + g' Q \gamma^\mu B_\mu - y S) \psi,$$

(2.1)

where $G_\mu$ is the gauge field for the hidden QCD, $B_\mu$ is the U(1)$_Y$ gauge field,

$$B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu, \quad g' = e / \cos \theta_W,$$

(2.2)

and the hidden vector-like fermions $\psi_i$ ($i = 1, \ldots, n_f$) belong to the fundamental representation of SU($n_c)_H$. The $y$ is an $n_f \times n_f$ Yukawa coupling matrix which can be taken as a diagonal matrix without loss of generality, i.e. $y = \text{diag}(y_1, \ldots, y_{n_f})$. Here the diagonal entries $y_i$ are assumed to be positive. The $\mathcal{L}_{SM+S}$ part contains the SM gauge and Yukawa interactions along with the scalar potential

$$V_{SM+S} = \lambda_H (H_1 H)^2 + \frac{1}{4} \lambda_S S^4 - \frac{1}{2} \lambda_{HS} S^2 (H_1 H),$$

(2.3)
where the portal coupling $\lambda_{HS}$ is assumed to be positive, and $H^T = (H^+, (h + iG^0)\sqrt{2})$ is the SM Higgs doublet field with $H^+$ and $G^0$ as the would-be Nambu-Goldstone (NG) fields. The (tree-level) stability condition for the scalar potential is given by

$$\lambda_H > 0, \quad \lambda_S > 0, \quad 2\sqrt{\lambda_H \lambda_S} - \lambda_{HS} > 0.$$  \hspace{1cm} (2.4)

Following refs. [17, 21, 22] we consider $n_f = n_c = 3$. In this case, the hidden chiral symmetry $SU(3)_L \times SU(3)_R$ is dynamically broken to its diagonal subgroup $SU(3)_V$ by the nonzero chiral condensate $\langle \bar{\psi}\psi \rangle$, which implies the existence of 8 NG bosons. At the same time of the dynamical chiral symmetry breaking, the singlet scalar field $S$ acquires a nonzero vacuum expectation value (VEV) due to the Yukawa interaction $-y_S\bar{\psi}\psi$ in $\mathcal{L}_H$, generating an explicit-chiral-symmetry-breaking mass term. Consequently, the NG bosons acquire their masses and can become DM candidates due to the remnant unbroken flavor group $SU(3)_V$ (or its subgroup, depending on the choice of $y_f$) that can stabilise them. Finally, with the nonzero $v_S = \langle S \rangle$, the EW symmetry breaking is triggered by the Higgs mass term $+ \frac{1}{2} \lambda_{HS} v_S^2 H^H$. 

### 2.1 Nambu–Jona-Lasinio description

In order to analyze the strongly interacting hidden sector, we replace the Lagrangian $\mathcal{L}_H$ (2.1) by the NJL Lagrangian that serves as an effective Lagrangian for the dynamical chiral symmetry breaking [12–14]:

$$\mathcal{L}_{NJL} = \text{Tr} \bar{\psi}(i\gamma^\mu \partial_\mu + g'Q\gamma^\mu B_\mu - y S)\psi + 2G \text{Tr} \Phi^\dagger \Phi + G_D (\text{det} \Phi + \text{h.c.}),$$  \hspace{1cm} (2.5)

where

$$\Phi_{ij} = \bar{\psi}_i (1 - \gamma_5) \psi_j = \frac{1}{2} \sum_{a=0}^8 \lambda^a_{ij} [ \bar{\psi} \lambda^a (1 - \gamma_5) \psi ],$$  \hspace{1cm} (2.6)

and $\lambda^a (a = 1, \ldots, 8)$ are the Gell-Mann matrices with $\lambda^0 = \sqrt{2/3} \cdot 1$. The dimensionful parameters $G$ and $G_D$ have canonical dimensions of $-2$ and $-5$, respectively. In order to deal with the nonrenormalizable Lagrangian (2.5) we work in the SCMF approximation of refs. [78, 79]. The mean fields $\sigma_i$ ($i = 1, 2, 3$) and $\phi_a$ ($a = 0, \ldots, 8$) are defined in the “Bardeen-Cooper-Schrieffer” vacuum as

$$\sigma_i = -4G \langle \bar{\psi}_i \psi_i \rangle, \quad \phi_a = -2iG \langle \bar{\psi}_i \gamma_5 \lambda^a \psi_i \rangle,$$  \hspace{1cm} (2.7)

where the CP-even mean fields corresponding to the non-diagonal elements of $\langle \bar{\psi}_i \psi_j \rangle$ are suppressed, because they do not play any role for our purpose. Splitting the NJL Lagrangian $\mathcal{L}_{NJL}$ into two parts as $\mathcal{L}_{NJL} = \mathcal{L}_{MFA} + \mathcal{L}_I$, where $\mathcal{L}_I$ is normal ordered (i.e., $\langle 0 | \mathcal{L}_I | 0 \rangle = 0$), we find the Lagrangian in the SCMF approximation $\mathcal{L}_{MFA}$ in the $SU(3)_V$ limit as³

$$\mathcal{L}_{MFA} = \text{Tr} \bar{\psi}(i\partial^\mu - g'Q\gamma^\mu B_\mu)\psi - i\text{Tr} \bar{\psi} \gamma_5 \phi \psi - \frac{1}{8G} \left( 3\sigma^2 + 2 \sum_{a=1}^8 \phi_a \phi_a \right) + \frac{G_D}{8G^2} \left( -\text{Tr} \bar{\psi} \phi^2 \psi + \sum_{a=1}^8 \phi_a \phi_a \text{Tr} \bar{\psi} \psi + i\sigma \text{Tr} \bar{\psi} \gamma_5 \phi \psi + \frac{\sigma^3}{2G} + \frac{\sigma}{2G} \sum_{a=1}^8 (\phi_a)^2 \right),$$  \hspace{1cm} (2.8)

³The mean-field Lagrangian $\mathcal{L}_{MFA}$ in the case of broken $SU(3)_V$ can be found in ref. [22].
with $\phi = \sum_{a=1}^{8} \phi_a \lambda^a$ and $\sigma = \sigma_1 = \sigma_2 = \sigma_3$. Here $\phi_0$ has been suppressed and the constituent fermion mass $M$ is given by

$$M(S, \sigma) = \sigma + yS - \frac{G_D}{8G^2} \sigma^2, \quad y = y_1 = y_2 = y_3.$$  

(2.9)

The one-loop effective potential obtained from $\mathcal{L}_{\text{MFA}}$ (2.8) can be obtained by integrating out the hidden fermions:

$$V_{\text{NJL}}(S, \sigma) = \frac{3}{8G} \sigma^2 - \frac{G_D}{16G^3} \sigma^4 - 3n_c I_0(M, \Lambda_H).$$  

(2.10)

Here the function $I_0$ is given by

$$I_0(M, \Lambda) = \frac{1}{16\pi^2} \left[ A^4 \ln \left( 1 + \frac{M^4}{A^2} \right) - M^4 \ln \left( 1 + \frac{\Lambda^2}{M^2} \right) + \Lambda^2 M^2 \right]$$  

(2.11)

with a four-dimensional momentum cutoff $\Lambda$, where we denote the cutoff in the hidden sector by $\Lambda_H$. For a certain interval of the dimensionless parameters $G^{1/2}\Lambda_H$ and $(-G_D)^{1/5}\Lambda_H$ we have $\langle \sigma \rangle \neq 0$ and $\langle S \rangle \neq 0$ [17, 21, 22]. It is then meant that the dynamics of the hidden sector creates a nonvanishing chiral condensate $\langle 0|\bar{\psi}\psi|0 \rangle \neq 0$. One can see that the potential $V_{\text{NJL}}(S, \sigma)$ is asymmetric in $\sigma$ owing to the last term in the NJL Lagrangian (2.5) and also from the constituent mass $M$ (2.9), which is the reason that the chiral PT at finite temperature can become of first order. It is noted that the mean fields $\sigma$ and $\phi_a$ are nonpropagating classical fields at the tree level. Therefore, their kinetic terms are generated by integrating out the hidden fermions at the one-loop level, which will be seen in section 2.2 where two point functions are calculated.

The NJL parameters for the hidden QCD sector are obtained by scaling-up the values of $G, G_D$ and the cutoff $\Lambda$ from QCD hadron physics. Following refs. [17, 21, 22] we assume that the dimensionless combinations

$$G^{1/2}\Lambda_H = 1.82, \quad (-G_D)^{1/5}\Lambda_H = 2.29,$$  

(2.12)

which are satisfied for the real-world hadrons, remain unchanged for a higher scale of $\Lambda_H$. Therefore, the free parameters of the (effective) model are: $\lambda_H, \lambda_S, \lambda_{HS}$ and $\Lambda_H$. Once these parameters are fixed, the VEVs of $\sigma, S$ and $h$ can be obtained through the minimization of the scalar potential $V_{\text{SM}+S} + V_{\text{NJL}}$ where we choose these parameters so as to satisfy $m_h = 125 \text{ GeV}$ and $\langle h \rangle = 246 \text{ GeV}$.

### 2.2 Mass spectrum

Once the VEVs of $\sigma, S$ and $h$ are obtained, the scalar mass spectrum can be calculated from the corresponding two point functions at one-loop in which the hidden fermions are circulating. The CP even scalars $h, S$ and $\sigma$ mix with each other. The two point functions at the one-loop level $\Gamma_{AB}(A, B = h, S, \sigma)$ in the SU(3)$_V$ flavor symmetry limit are given by

$$\Gamma_{hh}(p^2) = p^2 - 3 \lambda_H \langle h \rangle^2 + \frac{1}{2} \lambda_{HS} \langle S \rangle^2, \quad \Gamma_{hS} = \lambda_{HS} \langle h \rangle \langle S \rangle, \quad \Gamma_{h\sigma} = 0,$$  

$$\Gamma_{SS}(p^2) = p^2 - 3 \lambda_S \langle S \rangle^2 + \frac{1}{2} \lambda_{HS} \langle h \rangle^2 - y^2 3n_c I_{\varphi^2}(p^2, M, \Lambda_H),$$(2.13)

$$\Gamma_{S\sigma}(p^2) = -y \left( 1 - \frac{G_D \langle \sigma \rangle}{4G^2} \right) 3n_c I_{\varphi^2}(p^2, M, \Lambda_H),$$(2.14)

$$\Gamma_{\sigma\sigma}(p^2) = -\frac{3}{4G} + \frac{3G_D \langle \sigma \rangle}{8G^3} - \left( 1 - \frac{G_D \langle \sigma \rangle}{4G^2} \right)^2 3n_c I_{\varphi^2}(p^2, M, \Lambda_H) + \frac{G_D}{G^2} \left( 3n_c I_{\varphi^2}(M, \Lambda_H) \right).$$
Here the loop functions are defined as

\[ I_{\chi^2}(p^2, M, \Lambda) = \int d^4k \frac{\Tr(k + \not{p} + M)(k + M)}{i(2\pi)^4 ((k + p)^2 - M^2)(k^2 - M^2)}, \] (2.15)

\[ I_V(M, \Lambda) = \int d^4k \frac{M}{i(2\pi)^4 (k^2 - M^2)} = -\frac{1}{16\pi^2} M \left[ \Lambda^2 - M^2 \ln \left( 1 + \frac{\Lambda^2}{M^2} \right) \right]. \] (2.16)

The flavor eigenstates \((h, S, \sigma)\) and the mass eigenstates \(h_i (i = 1, 2, 3)\) are related by

\[
\begin{pmatrix}
 h \\
 S \\
 \sigma
\end{pmatrix} =
\begin{pmatrix}
 \xi_h^{(1)} & \xi_h^{(2)} & \xi_h^{(3)} \\
 \xi_S^{(1)} & \xi_S^{(2)} & \xi_S^{(3)} \\
 \xi_{\sigma}^{(1)} & \xi_{\sigma}^{(2)} & \xi_{\sigma}^{(3)}
\end{pmatrix}
\begin{pmatrix}
 h_1 \\
 h_2 \\
 h_3
\end{pmatrix}.
\] (2.17)

The squared masses \(m_{h_i}^2\) are determined by the zeros of the two point functions at the one-loop level, i.e. \(\Gamma_{AB}(m_{h_i}^2)\xi_B^{(i)} = 0\).

In this model the DM candidates are the NG bosons in the hidden sector which are CP-odd scalars \(\phi_a\) in eq. (2.7), i.e. the dark mesons. The two point function at the one-loop level for the DM candidate is (in the SU(3)_V flavor symmetry limit)

\[ \Gamma_{DM}(p^2) = -\frac{1}{2G} + \frac{G_D \langle \sigma \rangle}{8G^3} + \left( 1 - \frac{G_D \langle \sigma \rangle}{8G^2} \right)^2 2n_c I_{\phi^2}(p^2, M, \Lambda_H) + \frac{G_D}{G^2 n_c} I_V(M, \Lambda_H), \] (2.18)

where the loop function \(I_{\phi^2}(p^2, M, \Lambda)\) is given by

\[ I_{\phi^2}(p^2, M, \Lambda) = \int d^4k \frac{\Tr(k - \not{p} + M)\gamma_5(k + M)\gamma_5}{i(2\pi)^4 ((k - p)^2 - M^2)(k^2 - M^2)}. \] (2.19)

The mass of the DM is obtained from \(\Gamma_{DM}(m_{DM}^2) = 0\).

2.3 LHC constraint on \(\lambda_{HS}\)

The size of the portal coupling \(\lambda_{HS}\) controls the \(h - S\) mixing. Since in the parameter space we will consider the Yukawa coupling \(y\) is small, i.e. of order \(10^{-3}\) (see eq. (4.12)), the mixing \(\Gamma_{S\sigma}\) (2.14) is also small, so that we will neglect it in the following discussions. (We will also neglect the last term of \(\Gamma_{SS}\) (2.13), because it is proportional to \(y^2\).) Therefore, the \(h - S\) mixing can be written as

\[
\begin{pmatrix}
 h_1 \\
 h_2
\end{pmatrix} =
\begin{pmatrix}
 \cos \theta & \sin \theta \\
 -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
 h \\
 S
\end{pmatrix}.
\] (2.20)

with \((\cos \theta, -\sin \theta, 0) \simeq (\xi_h^{(1)}, \xi_h^{(2)}, \xi_h^{(3)})\) which is defined in eq. (2.17). Here we identify \(h_1\) with the SM Higgs having mass \(m_h \simeq 0.125\) TeV, i.e., \(m_h = m_{h_1}\) and \(m_S = m_{h_2}\). The \(h - S\) mixing is constrained by LHC data (see e.g. refs. [63, 64] and references thererin). In the left panel of figure 1 we plot \(|\sin \theta|\) versus \(m_S\) and in the right panel \(m_S\) (purple) and \(m_{DM}\) (green) versus \(\lambda_{HS}\) both at \(\lambda_S = 0.001\). We vary \(\lambda_{HS}\) between 0.0001 and 0.018 and \(y\) between 0.001 and 0.00172. (Why we consider \(y\) in this interval will be explained in section 3.) As we see from the left panel, there are two branches: \(m_S < m_h\) and \(m_S > m_h\), and on each branch there exist a (blue) region, I and II, that is allowed by LHC. The band of \(m_{DM}\)
in the right panel can be seen, because it sensitively depends on $y$, while $m_S$ is insensitive against $y$. From each allowed region we choose a representative point, BP1 and BP2, to get an orientation in the parameter space, especially when discussing the GW spectrum later on:

$$\begin{align*}
\text{BP1: } & \lambda_S = 0.001, \lambda_{HS} = 0.00485, y = 0.00172, \lambda_H = 0.1238, \Lambda_H = 4.322 \text{ TeV}, \\
\text{BP2: } & \lambda_S = 0.001, \lambda_{HS} = 0.00230, y = 0.00170, \lambda_H = 0.1325, \Lambda_H = 6.606 \text{ TeV}.
\end{align*}$$

### 2.4 Dark matter

Due to the vector-like flavor symmetry (i.e. SU(3)$_V$ or its subgroup), the dark mesons are good DM candidates. As we see from figure 1, the mass of the real singlet $m_S$ is smaller than the DM mass $m_{DM}$, so that the DM can annihilate into two $S$s in principle. However, this annihilation cross section is negligibly small because it is $\propto y^4 \lesssim 10^{-11}$ in the parameter space of interest. To explain the observed value for the relic DM abundance in this circumstance, we assume a hierarchy in the Yukawa couplings: $y_1 = y_2 < y_3$ (which breaks SU(3)$_V$ down to SU(2)$_V$ $\times$ U(1) explicitly), where $y_3$ should not differ very much from $y_2$ [22]. Under this assumption, the dark mesons fall into three categories, $\tilde{\pi} = \{\tilde{\pi}^\pm, \tilde{\pi}^0\}$, $\tilde{K} = \{\tilde{K}^\pm, \tilde{K}^0, \tilde{K}'^0\}$ and $\tilde{\eta}$. Here the dark mesons are given like the real-world mesons:

$$\begin{align*}
\tilde{\pi}^\pm &\equiv (\phi_1 \mp i\phi_2)/\sqrt{2}, & \tilde{\pi}^0 &\equiv \phi_3, \\
\tilde{K}^\pm &\equiv (\phi_1 \mp i\phi_3)/\sqrt{2}, & \tilde{K}^0(\tilde{K}'^0) &\equiv (\phi_6 + (-)i\phi_7)/\sqrt{2}, & \tilde{\eta}^8 &\equiv \phi_8,
\end{align*}$$

where $\tilde{\eta}^8$ will mix with $\tilde{\eta}^0$ to form the mass eigenstates $\tilde{\eta}$ and $\tilde{\eta}'$. The states in the same category have the same mass, $m_{\tilde{\pi}^0} = m_{\tilde{\pi}^\pm} (\equiv m_{\tilde{\pi}})$ and $m_{\tilde{K}^\pm} = m_{\tilde{K}^0} = m_{\tilde{K}'^0} (\equiv m_{\tilde{K}})$, with $m_{\tilde{\pi}} < m_{\tilde{K}} < m_{\tilde{\eta}}$, where the differences among $m_{\tilde{\pi}}$, $m_{\tilde{K}}$ and $m_{\tilde{\eta}}$ are small because of the small difference between $y_1 = y_2$ and $y_3$. The heavier state $\tilde{\eta}$ is an unstable NG boson which can mainly decay into two $\gamma$s. On the other hand, the $\tilde{\pi}$ and $\tilde{K}$ are stable due to the SU(2)$_V$ flavor symmetry and become the DM. Since the mass difference among $\tilde{\pi}$, $\tilde{K}$ and $\tilde{\eta}$ are small, the DM annihilation into a pair of heavier DMs and/or $\tilde{\eta}$s, which are kinematically forbidden at zero temperature, can become operative. In ref. [22] it has been shown that the
inverse conversion $\tilde{\pi}\tilde{\pi}, \tilde{K}\tilde{K} \rightarrow \tilde{\eta}\tilde{\eta} \rightarrow \gamma\gamma\gamma$ can play a significant role to make the DM relic abundance realistic. This mechanism works only if the SU(3)$_V$ flavor symmetry is broken into its subgroup.\footnote{In the model considered in ref. [31] the flavor group SU(3)$_V$ is unbroken and the hidden fermions have no coupling with the U(1)$_Y$ gauge boson.}

In figure 2 we show the total DM relic abundance $\Omega h^2 = \Omega_\pi h^2 + \Omega_K h^2$ with the U(1)$_Y$ hypercharge $Q = 1/3$ as a function of $y_3$, where the other parameters are chosen for the benchmark point BP1 defined in eq. (2.21), and $h$ is the dimensionless Hubble parameter. We see from this figure that the DM relic abundance at $y_3 \simeq 0.0024$ can coincide with the experimentally observed value [23].\footnote{Though the portal coupling $\lambda_{HS}$ is very small $\sim 10^{-3}$, the singlet scalar $S$ decays via the $h - S$ mixing into the SM particles much before Big Bang Nucleosynthesis (BBN): $\Gamma(S \rightarrow \text{SM particles})/H \simeq \sin^2 \theta \Gamma(h \rightarrow \text{SM particles})/H \sim \sin^2 \theta \times 10^{22}$ at $T = 1 \text{ MeV}$.}

3 Chiral phase transition and bounce solution

3.1 Effective potential and chiral phase transition

The EW and chiral PTs in our model (2.1) have been studied in refs. [17, 31] in some detail. For a phenomenologically viable region of the parameter space, the EW PT occurs — with decreasing temperature — after the chiral PT takes place in a hidden sector. Therefore, the VEV of $h$ vanishes during the chiral PT, so that we set $\langle h \rangle = 0$ in investigating the chiral PT. Accordingly, we analyze the following scalar potential at finite temperature:

\begin{equation}
V_{\text{EFF}}(S, \sigma, T) = V_{\text{SM+SY}}(S) + V_{\text{NJL}}(S, \sigma) + V_{\text{CW}}(S) + V_{\text{FTB}}(S, T) + V_{\text{FTF}}(S, \sigma, T),
\end{equation}

where $V_{\text{SM+SY}}$ and $V_{\text{NJL}}(S, \sigma)$ are given in eqs. (2.3) and (2.10), respectively,

\begin{align*}
V_{\text{CW}}(S) &= \frac{m^4_S}{64\pi^2} [\ln(S^2/\langle S \rangle^2) - 1/2], \\
V_{\text{FTB}}(S, T) &= \frac{T^4}{2\pi^2} \left[ J_B(M_S(T)) - J_B(\lambda S/4 - \lambda_{HS}/6)^{1/2} \right],
\end{align*}

Figure 2. The total DM relic abundance $\Omega h^2$ with the U(1)$_Y$ hypercharge $Q = 1/3$ versus $y_3$, where we have fixed other parameters at the benchmark point BP1 defined in eq. (2.21).
\[ V_{\text{FTF}}(S, \sigma, T) = -6n_c \frac{T^4}{\pi^2} \left[ J_F(M(S, \sigma)/T) - J_F(0) \right], \]  
\[ M^2_S = m_S^2(S) + \left( \frac{\lambda_S}{4} - \frac{\lambda_{HS}}{6} \right) T^2 \] with \( m_S^2(S) = 3\lambda_SS^2 \),

and \( M(S, \sigma) \) is given in (2.9). The thermal functions are
\[ J_{F/B}(u) = \int_0^\infty dx x^2 \ln \left(1 \pm e^{-\sqrt{x^2 + u^2}}\right) \] 
\[ = \sum_{j=1}^{\infty} (\mp 1)^{(1+j)}(u^2/j^2)K_2(ju), \]

where \( K_2(ju) \) is the modified Bessel function of the second kind of order two, and we will truncate the sum at \( j = 10 \). In \( V_{\text{FTF}}(S, T) \) and \( V_{\text{FTF}}(S, \sigma, T) \) we have subtracted the (temperature-dependent) constant terms such that \( V_{\text{FTF}}(0, T) = V_{\text{FTF}}(0, 0, T) = 0 \). We first note that the role of the singlet scalar \( S \) becomes more important for smaller \( \lambda_S \). To see this, we will consider \( V_{\text{NJL}}(S, \sigma) \) for small \( y \):
\[ V_{\text{NJL}}(S, \sigma) = V_{\text{NJL}}(0, \sigma) - \frac{3n_c\Lambda_H^2\sigma^2}{4\pi^2}yS + O((yS)^2). \]

Since, neglecting the portal coupling \( \lambda_{HS} \), the scalar potential \( V_{\text{SM}+S} \) becomes \( \lambda_H(H^1H)^2 + (1/4)\lambda_SS^4 \), we find
\[ v_S^3 = \langle S \rangle^3 \simeq \frac{3n_c\Lambda_H^2\sigma^2}{4\pi^2} \left( \frac{y}{\lambda_S} \right). \]

Therefore, the deviation from the pure NJL model (i.e. without the singlet scalar \( S \)) is larger for smaller \( \lambda_S \) and larger \( y \). The above feature remains at finite temperature, as we can see from figure 3, where we show \( v_S/v_\sigma \) against \( \lambda_S \) at the critical temperature \( T_C \) for \( y = 0.001, \lambda_{HS} = 0 \). In figure 4 we plot \( v_\sigma/T \) (blue) and \( v_S/T \) (red) as a function of \( T/\Lambda_H \) for \( \lambda_S = 0.005, \lambda_{HS} = 0 \) and \( y = 0.001 \) (upper left panel) and 0.007 (upper right panel), respectively. We see that the chiral PT is no longer a strong first-order PT at \( y = 0.007 \). The lower panels show the case for \( \lambda_S = 0.001, \lambda_{HS} = 0 \) and \( y = 0.00172 \) (left) and 0.0045 (right).

### 3.2 Bounce solution

One of the main quantities in discussing the stochastic GW background produced by a first-order PT in the expanding Universe is the duration time of the first-order PT, \( \tau_{\text{pt}} = \beta^{-1} \), which should be compared with the inverse rate of the expansion \( H^{-1} \) [33, 58–60]. In fact

\[ \text{The error of the approximate function } J_{F/B}(u); j_{\text{max}} \text{ with the truncation at } j = j_{\text{max}} \text{ is } \Delta J_{F/B} = \left| J_{F/B}(u) - J_{F/B}(u; j_{\text{max}}) \right|. \] For \( j_{\text{max}} = 10, \Delta J_F < 5 \times 10^{-5} \text{ and } \Delta J_B < 3 \times 10^{-4} \text{ are satisfied. So, with } j_{\text{max}} = 10, \text{ the error in } \Delta(V_{\text{EFF}}/T^4) \text{ is less than } 10^{-4}. \] Since we are interested in the behavior of \( V_{\text{EFF}} \) near the critical temperature \( T_C \), we obtain \( \Delta(V_{\text{EFF}}/\Lambda_H^2) < 10^{-8} \text{ because } T_C/\Lambda_H \sim 0.1. \) This accuracy is sufficient for our purpose.

\[ \text{The absolute scale of the hidden sector (i.e. } \Lambda_H \text{) can be anything when the hidden sector has no coupling with the SM sector. This happens, for instance, when } \lambda_{HS} \text{ is set equal to zero. Nevertheless, dimensionless quantities have their meaning. As we see from eqs. (2.3) and (3.1) the } \lambda_{HS} \text{ dependence of the effective potential is very small if } \langle h \rangle = 0: \text{ It enters only in the thermal mass of } S \text{ as one can see in eq. (3.5).} \]
Figure 3. $v_S/v_\sigma$ at the critical temperature $T_C$ against $\lambda_S$. We see that the smaller $\lambda_S$ is, the larger the ratio $v_S/v_\sigma$ becomes, which means more deviation from the pure NJL (i.e. without the singlet scalar $S$).

Figure 4. Upper (lower) left: $v_\sigma/T$ (blue) and $v_S/T$ (red) against $T/\Lambda_H$ for $\lambda_S = 0.005 (0.001), \lambda_HS = 0, y = 0.001 (0.00172)$, showing a strong first-order PT. Upper (lower) right: The same with $y = 0.007 (0.0045)$, showing a transition of cross-over type.
the GW energy density increases — depending on the nature of its source — linearly or even quadratically with $(\beta/H)^{-1}$ (see eqs. (4.21), (4.24) and (4.27)), while its peak frequency increases linearly with $\beta/H$ (see eqs. (4.23), (4.26) and (4.29)). In ref. [61] it has been found that $\beta/H$ is of order $10^4$ in the pure NJL model, so that $\Omega_{GW}$, the spectral GW energy density normalized to the critical energy density, is considerably suppressed. Therefore, we consider here a parameter space in which the deviation from the pure NJL model is large. From the discussion of section 3.1 we can infer that the area with small $\lambda_S$ and large $y$ is an optimistic parameter space, where $y$ should not be too large for a strong first-order chiral PT to be realized. It turns out that $\lambda_S \sim O(10^{-3})$ is an optimistic magnitude for $\lambda_S$, and in the following discussions we concentrate on the parameter space with $\lambda_S = 0.001$.

To obtain $\beta/H$ we have to compute the value of the corresponding $O(3)$ symmetric Euclidean action $S_3$ [65]. The mean field $\sigma$, introduced as an auxiliary field at the tree level in the mean-field approximation, is a driving force for the chiral PT. As it has been discussed in section 2.2 the mean field $\sigma$ can be promoted to a propagating quantum field at one loop, which also applies at finite temperature. The kinetic term for $\sigma$ at finite temperature has been correctly computed in ref. [61]. Quoting the result of ref. [61], the $O(3)$ symmetric action $S_3$ can be written as

$$S_3(T) = 4\pi \int drr^2 \left[ \frac{Z_\sigma^{-1}(S, \sigma, T)}{2} \left( \frac{d\sigma}{dr} \right)^2 + \frac{1}{2} \left( \frac{dS}{dr} \right)^2 + V_{\text{EFF}}(S, \sigma, T) \right],$$

(3.9)

where $r$ is the radial coordinate of the 3-dimensional space, and $V_{\text{EFF}}(S, \sigma, T)$ is given in eq. (3.1). $Z_\sigma(S, \sigma, T)$ is the wave function renormalization “constant” at finite temperature [61]:

$$Z_\sigma^{-1}(S, \sigma, T) = \frac{n_c n_f}{2\pi^2} \left[ 1 - \frac{G_D}{4G^2} \right]^2$$

$$\times \left\{ -\frac{\Lambda_H^2}{4(\Lambda_H^2 + M^2)} + \frac{1}{4} \ln(1 + \Lambda_H^2/M^2) + \frac{\Lambda_H^4}{8(\Lambda_H^2 + M^2)^2} - \frac{\Lambda_H^4(\Lambda_H^2 + 3M^2)}{6(\Lambda_H^2 + M^2)^3} \right. $$

$$+ \int_0^\infty dx x^2 \left[ \frac{1 + e^{\omega/T}(1 + \omega/T)}{(1 + e^{\omega/T})^2(\omega/T)^3} - (M/T)^2 \left( \frac{3 + e^{\omega/T}(6 + 3\omega/T - (\omega/T)^2)}{4(1 + e^{\omega/T})^3(\omega/T)^5} \right) 

- (M/T)^2 \frac{2e^{\omega/T}(3 + 3\omega/T + (\omega/T)^2)}{4(1 + e^{\omega/T})^3(\omega/T)^5} 

+ (M/T)^4 \frac{15 + e^{\omega/T}(45 + 15\omega/T - 6(\omega/T)^2 + (\omega/T)^3) + e^{2\omega/T}(45 + 30\omega/T - 4(\omega/T)^2)}{6(1 + e^{\omega/T})^4(\omega/T)^7} 

+ (M/T)^4 \frac{e^{3\omega/T}(15 + 15\omega/T + 6(\omega/T)^2 + (\omega/T)^3)}{6(1 + e^{\omega/T})^4(\omega/T)^7} \right]\right\},$$

(3.10)

where

$$\omega/T = [x^2 + (M/T)^2]^{1/2},$$

(3.11)

and $M$ is given in eq. (2.9).

The equations of motion for the action (3.9) read

$$\frac{d^2 \sigma}{dr^2} + \frac{2}{r} \frac{d\sigma}{dr} + \frac{1}{2} \frac{\partial \ln Z_\sigma^{-1}(S, \sigma, T)}{\partial \sigma} \left( \frac{d\sigma}{dr} \right)^2 = Z_\sigma(S, \sigma, T) \frac{\partial V_{\text{EFF}}(S, \sigma, T)}{\partial \sigma},$$

(3.12)

$$\frac{d^2 S}{dr^2} + \frac{2}{r} \frac{dS}{dr} - \frac{1}{2} \frac{\partial Z_\sigma^{-1}(S, \sigma, T)}{\partial S} \left( \frac{dS}{dr} \right)^2 = \frac{\partial V_{\text{EFF}}(S, \sigma, T)}{\partial S},$$

(3.13)
and the boundary conditions for the bounce solution are given by \[65\]

\[
\frac{d\sigma}{dr}\Big|_{r=0} = 0, \quad \frac{dS}{dr}|_{r=0} = 0, \quad \lim_{r \to 0} \sigma(r) = 0, \quad \lim_{r \to \infty} S(r) = 0. \tag{3.14}
\]

The bounce solution describes a bubble, where \( r = 0 \) is the center of the bubble, inside of which the chiral symmetry is broken. The bubble however has no sharp boundary, but \( \sigma(r) \) and \( S(r) \) at \( r \approx r_w \) drop sharply from a finite value to a small value (see figure 5) so that \( r_w \) can be understood as the position of the bubble wall: We may say in a less precise way that inside of the wall the chiral symmetry is broken and in the outside of the wall it is unbroken. In the one-dimensional case, in which there exists only one scalar degree of freedom as an order parameter, we can obtain a bounce solution by using the so-called overshooting/undershooting method [80]. However, this is an extremely cumbersome method in a multi-dimensional case, because a set of certain initial conditions have to be simultaneously fine tuned. An appropriate method is the path deformation method [66].

But to minimize the problem associated with the complicated structure of the wave function renormalization (3.10), we here use another iterative method, which we will describe below.

One round of the calculation consists of two steps. At the first step in the \( n \)th round, we solve the differential equation (3.12) for \( \sigma(r) \) with \( S(r) = S_{(n-1)}(r) \), where \( S_{(n-1)}(r) \) is obtained in the \( (n-1) \)th round. The solution is denoted by \( \sigma_{(n)}(r) \). At the second step in the \( n \)th round, we solve the differential equation (3.13) for \( S(r) \) with \( \sigma(r) = \sigma_{(n)}(r) \) to obtain \( S_{(n)}(r) \). Then, using \( \sigma_{(n)}(r) \) and \( S_{(n)}(r) \) we compute \( S_3/T \) in the \( n \)th round and denote it by \( (S_3/T)_{(n)} \). Since each step is a one-dimensional problem, we apply the overshooting/undershooting method. Of course, there is no mathematical warranty that the iterative process converges: It depends strongly on \( S(0) \) that is needed to carry out the first step in the first round, i.e., to obtain \( \sigma(1)(r) \). Here we assume that \( S(0) \) is a function of \( \sigma \) and choose it as a straight line linking the origin of the field space \( (\sigma = S = 0) \) and the position \( (v_\sigma, v_S) \) of the minimum of \( V_{\text{EFF}}(S, \sigma, T = T_C) \):

\[
S_{(0)}(\sigma) = \frac{v_S}{v_\sigma} \sigma. \tag{3.15}
\]

In the upper (lower) left panel of figure 5, we show \( \sigma_{(n)}(r) \) and \( S_{(n)}(r) \) with \( n = 1 \) (blue), 2 (red), 3 (black) for \( \lambda_S = 0.003 \) (0.001), \( \lambda_{HS} = 0 \), \( y = 0.001 \) (0.00172), \( T_C/\Lambda_H = 0.0796 \) (0.0843), \( T/\Lambda_H = 0.0769 \) (0.0798). We find also

\[
(S_3/T)_{(1)} = 144.4 \ (153.5), \quad (S_3/T)_{(2)} = 140.9 \ (140.0),
\]

\[
(S_3/T)_{(3)} = 141.5 \ (144.7), \quad (S_3/T)_{(4)} = 140.8 \ (142.2). \tag{3.16}
\]

So, the convergence of the iterative process, described above, is quite fast. We have calculated \( S_3/T \) for several values of \( x = T/T_C \) in the second round and found that \( S_3/T \) for \( x < 1 \) can be nicely fitted with a simple function [61, 62]

\[
\frac{S_3}{T}(x) = b \ (1 - x)^{-\gamma}. \tag{3.17}
\]

This is shown in the upper (lower) right panel of figure 5 for \( \lambda_S = 0.003 \) (0.001), \( \lambda_{HS} = 0 \), \( y = 0.001 \) (0.00172). The blue dotted line is the function (3.17) with \( b = 0.1833 \) (2.232) and \( \gamma = 1.961 \) (1.418).
Figure 5. Upper (lower) left: The bounce solutions $\sigma_{(n)}(r)$ and $S_{(n)}(r)$ for $n = 1$ (blue), 2 (red) and 3 (black), where we have used the parameters: $\lambda_S = 0.003 (0.001), \lambda_{HS} = 0, y = 0.001 (0.00172), T_C/\Lambda_H = 0.0796 (0.0843), T/\Lambda_H = 0.0769 (0.0798)$. Upper (lower) right: $S_3/T$ (in the second round) against $x = T/T_C$ with the same $\lambda_S, \lambda_{HS}$ and $y$ as in the upper (lower) left panel. The black points are obtained by applying our iterative method, while the blue dotted line is the fitting function defined in eq. (3.17) with $b = 0.1833 (2.232)$ and $\gamma = 1.961 (1.418)$.

There is a limitation of our iterative method. As we have discussed in section 3.1, the chiral PT turns into a cross-over type for large $y$. We have found that our iterative process does not converge for large $y$ even much before the chiral PT turns into a cross-over type. The reason is that a new local minimum, other than the true and false minima, develops near the origin $\sigma = S = 0$. The bounce solution passes near the new local minimum to arrive at the origin, as one can see in figure 6. The depth of the new local minimum becomes deeper and deeper with an increasing value of $y$, and around a certain value of $y$ the new local minimum starts to affect the iterative method in such a way that the iterative process does not converge. At the moment we are not able to find a bounce solution beyond this value of $y$.

4 Gravitational wave spectrum

There are three production mechanisms of the stochastic GW background at a strong first-order PT: Bubbles are nucleated and grow, and then the collisions of the bubble walls take place, producing a GW background [58–60, 81–83]. We denote by $\Omega_\varphi$ its contribution to the total GW spectrum $\Omega_{GW}$. After the bubble-wall collisions sound waves surrounding the
bubble walls [67, 84–87] and magnetohydrodynamic (MHD) turbulence [88–94] in the plasma become the source of the GW background. Their contributions to $\Omega_{GW}$ are denoted by $\Omega_{sw}$ and $\Omega_{turb}$, respectively:

$$\Omega_{GW}(f) h^2 = \left[ \Omega_{\phi}(f) + \Omega_{sw}(f) + \Omega_{turb}(f) \right] h^2,$$

(4.1)

where $h$ is the dimensionless Hubble parameter, and $f$ is the frequency of the GW at present. To calculate $\Omega_{GW}$ for a given model we first have to find out the nucleation temperature $T_n$. Then we compute the duration time of the first-order PT at $T = T_n$ and the released vacuum energy density at $T = T_n$. The released vacuum energy is indeed the source for the GW energy density, but only its part is effectively used as the source. The corresponding efficiency is expressed by the efficiency coefficients that again depend on the released vacuum energy density. In the following we start by computing $T = T_n$.

### 4.1 Nucleation temperature $T_n$

The cosmological tunneling process is quantum mechanical transition from a false vacuum state to the true vacuum state in the expanding Universe and has been studied in refs. [33, 62, 65, 95, 96]. The probability of the decay rate of the false vacuum per unit volume per unit time at a finite temperature $T$ is given by [65]

$$\Gamma(T) \simeq T^4 \left( \frac{S_3}{2\pi T} \right)^{3/2} \exp(-S_3/T),$$

(4.2)

where $S_3$ is the three dimensional Euclidean action and is given in eq. (3.9) for our model. The first-order PT proceeds via the tunneling process in the expanding Universe, in which the bubbles of the true vacuum are nucleated. Since after each tunneling process we have one bubble nucleation, $\Gamma(T)$ is also the nucleation rate of the bubbles. The nucleation temperature
$T_n$ is defined as the temperature, at which one bubble for Hubble time and Hubble volume is nucleated, i.e., $\Gamma(T_n)/H(T_n)^4 = 1$, which leads to the approximate expression \[ [33, 62] \]

\[
\frac{S_3(T_n)}{T_n} \simeq 2 \ln \left( \frac{90}{g_s \pi^2} \frac{M_{Pl}^2}{T_n^2} \right), \tag{4.3}
\]

where we have ignored the slowly varying factor $(S_3/(2\pi T))^{3/2}$ on the r.h.s. of eq. (4.2), $g_s$ is the relativistic degrees of freedom in the Universe at $T = T_n$, and $M_{Pl} = 2.435 \times 10^{18}$ GeV is the reduced Planck mass. Then the nucleated bubbles expand and collide. Note that the absolute scale of $T_n$ (and also $T_C$) depends crucially on $\lambda_{HS}$ and $y$, because in the absence of both couplings the scale in the hidden sector can be anything; no information about the energy scale in the SM sector, e.g. $m_h \simeq 0.125$ TeV, can be transferred to the hidden sector.

### 4.2 Duration of the phase transition

The temperature $T$ and time $t$ in the expanding Universe is related through

\[
\frac{dT}{dt} = -H(t)T. \tag{4.4}
\]

The nucleation time $t_n$ is the time, at which the temperature $T$ is equal to the nucleation temperature $T_n$. Since the nucleation time $t_n$ is now defined, we can compute the duration of the phase transition. To this end, we consider the four-dimensional Euclidean action $S_E(t) = S_3(T)/T$ and expand it around $t_n$:

\[
S_E(t) = S_E(t_n) - \beta \Delta t + O((\Delta t)^2), \tag{4.5}
\]

where $\Delta t = (t - t_n) > 0$. Then the nucleation rate for $t \sim t_n$ can be written as

\[
\Gamma(T) \simeq \Gamma(T_n)e^{\beta \Delta t}. \tag{4.6}
\]

Clearly, the larger $1/\beta$ is, the longer is the time for which $\Gamma(T)$ stays close to $\Gamma(T_n)$. Therefore, $\beta^{-1}$ is the duration time and can be computed from \[ [58–60] \]

\[
\beta = -\frac{dS_E}{dT} \bigg|_{t=t_n} = \frac{1}{\Gamma} \frac{d\Gamma}{dT} \bigg|_{t=t_n} = H(t_n)T_n \frac{d}{dT} \left( \frac{S_3}{T} \right) \bigg|_{T=T_n}, \tag{4.7}
\]

where eqs. (4.2) and (4.4) are used. This means that we need to compute the derivative of $S_3/T$, which is a cumbersome task in the presence of many scalar fields involved in the bounce equation like in our case. To overcome this problem we use the fact that $S_3/T$ can be well approximated by the fitting function defined in eq. (3.17). Since $b$ and $\gamma$ are independent of $x = T/T_C$, we determine them from the actual calculation of $S_3/T$ for some $x$ and obtain $\beta/H$ from

\[
\beta/H = \frac{d}{dT} b(1 - T/T_C)^{-\gamma} \bigg|_{T=T_n} = b\gamma x_n(1 - x_n)^{-1-\gamma}, \tag{4.8}
\]

where $x_n = T_n/T_C$. The quantities, $b$ and $\gamma$, do not depend very much on $\lambda_{HS}$, because not only they are dimensionless, but also $\lambda_{HS}$ enters into the chiral PT only through the thermal mass of $S$ as one can see in eq. (3.5). In contrast to this, they and hence $\beta/H$ depend considerably on $y$, because it is the origin of the explicit breaking of the chiral symmetry. In figure 7 (left)
we show $\beta/H$ for several values of $y$ with $\lambda_S$ and $\lambda_{HS}$ fixed at 0.001 and 0, respectively. Since $\beta/H \simeq 1.4 \times 10^4$ in the pure NJL model [61], we see from figure 7 that the larger $y$ is, the more deviation from the pure NJL model we can expect. The right panel shows the $y$-dependence of $T_n/T_C$ for $\lambda_S = 0.001$ and $\lambda_{HS} = 0$, from which we see that in contrast to $\beta/H$ the value of $T_n/T_C$ does not change very much as $y$ changes. Therefore, the iterative method breaks down (as it is explained in section 3.2), so that we stop at $y = 0.00172$ for this example.

4.3 Released vacuum energy

As we see from figure 7, $\beta/H$ is large $\sim 10^3$. Therefore, the scalar contribution $\Omega_\phi$ to the GW spectrum, being proportional to $(\beta/H)^{-2}$, is much more suppressed than the other contributions $\Omega_{\text{sw}}$ and $\Omega_{\text{turb}}$, because they are proportional to $(\beta/H)^{-1}$ (see eqs. (4.21), (4.24) and (4.27)). Furthermore, as we will see, the turbulence contribution $\Omega_{\text{turb}}$ is suppressed, compared with $\Omega_{\text{sw}}$, because the relevant GW frequency $f$ is much larger than $h_\text{n}$, the Hubble parameter at $T_n$, which is red-shifted today. Therefore, we here focus on the sound-wave contribution and follow the treatment of ref. [70]. It should be noted that the definition of $\alpha$ in ref. [70] is not the ratio of the latent heat released at the PT to the radiation energy of the Universe. Instead, they use the trace of the energy momentum tensor of the plasma, leading to

$$\alpha = \frac{1}{\rho_{\text{rad}}(T_n)} \left( \frac{\Delta V(T_n)}{4} T \frac{\partial \Delta V(T)}{\partial T} \right|_{T=T_n} \right),$$

where $\Delta V(T) = V_{\text{EFF}}(0,0,T) - V_{\text{EFF}}(\langle \sigma \rangle, T)$, and $\rho_{\text{rad}}(T) = \pi^2 g_\ast T^4/30$. According to ref. [70], if the speed of the wall $\xi_w$ is larger than $\xi_f$, we may identify the vacuum energy density

\footnote{\(\beta/H\) computed in ref. [31] does not seem to approach the pure NJL value, $\sim 10^4$, as the Yukawa coupling $y$ goes to zero (see, e.g. the result for the case C in TABLE I and II: $y = 1.07 \times 10^{-4}$ but $\beta/H = 7.15 \times 10^2$.) Therefore, we suspect that the modified path deformation method of ref. [31] to obtain the bounce solution of a coupled system fails to yield trustful results.}
sity, which enters into the definition of $\alpha$, with the vacuum energy density outside of the bubble (as we have already done so above), where $\xi_j$ is the wall speed for the Jouguet detonation

$$\xi_j = \frac{\sqrt{\alpha(2 + 3\alpha)} + 1}{\sqrt{3(1 + \alpha)}}. \quad (4.10)$$

Correspondingly, $g_*$ in $\rho_{\text{rad}}$ is the relativistic degrees of freedom in the symmetry phase, which is not necessarily the same as that at the GW production. So, in our case

$$g_* = 106.75 + 1 + 8 \times 2 + \frac{7}{8} \times 3 \times 3 \times 2 \times 2 = 155.25. \quad (4.11)$$

When calculating the GW spectrum later on, we will be considering an optimistic parameter space given by

$$\lambda_S = 0.001, \quad \lambda_{HS} \in [0.0001, 0.018], \quad y \in [0.0008, 0.00172]. \quad (4.12)$$

In this parameter space, $\alpha$ does not change very much:

$$0.0242 \lesssim \alpha \lesssim 0.0250. \quad (4.13)$$

### 4.4 Reduction of the sound-wave contribution

As we mentioned above, the sound-wave contribution $\Omega_{\text{sw}}$ will be the most dominant one in our model. The formula for $\Omega_{\text{sw}} \hbar^2$ (see eq. (4.24)) has been derived from the numerical simulations for which a long-lasting source of the GW, i.e., $\tau_{\text{sw}} H > 1$, is assumed [67], where

$$\tau_{\text{sw}} \simeq (8\pi)^{1/3} \frac{\xi_w}{\bar{U}_f \beta} \quad (4.14)$$

is the duration of the sound-wave period, $\bar{U}_f$ is the root-mean four-velocity of the plasma, and $\xi_w$ stands for the speed of the wall ($\beta$ is defined in eq. (4.8)). That is, $\tau_{\text{sw}} H \propto (\beta/H)^{-1}$, so that $\tau_{\text{sw}} H > 1$ is unlikely satisfied in our model, because $\beta/H \gtrsim O(10^3)$. In refs. [68, 69] it has been suggested, for the case that $\tau_{\text{sw}} H < 1$, to use this quantity as a reduction factor for $\Omega_{\text{sw}}$ to take into account the fact that the sound wave is an active GW source only for a period shorter than the Hubble time. Here we follow ref. [68] along with ref. [70] to calculate $\tau_{\text{sw}}$ and consider throughout the case of detonations of the plasma motion.

The root-mean four-velocity $\bar{U}_f$ can be calculated from [70]

$$\bar{U}_f^2 = \frac{3}{\xi_w} \int_{c_s}^{\xi_w} d\xi \frac{\xi^2 v^2(\xi)}{1 - v^2(\xi)}, \quad (4.15)$$

where $v(\xi)$ is the velocity profile of the plasma in the frame of the bubble center, and $c_s$ is the speed of sound in the plasma (we assume here $c_s = 1/\sqrt{3}$). The velocity profile $v(\xi)$ satisfies the first-order differential equation [70]

$$\frac{v}{\xi} = \frac{1}{2} \left( \frac{1 - v \xi}{1 - v^2} \right) \left( \frac{\mu^2(\xi, v)}{c_s^2} - 1 \right) \frac{dv}{d\xi}, \quad \text{where } \mu(\xi, v) = \frac{\xi - v}{1 - \xi v}. \quad (4.16)$$

To solve the differential equation (4.16) uniquely, we use $v(\xi_w)$ as an initial value, i.e., the plasma speed just behind the wall. Since we focus on the detonations, the plasma in front of the wall is at rest in the bubble center frame, i.e., $v_+ = \xi_w$, where $v_+$ is the speed of the
plasma in front of the wall in the wall frame ($\xi_w$ is the speed of the wall in the bubble center frame). Therefore, the speed of the plasma just behind the wall in the wall frame, denoted by $v_-$, can be obtained by the Lorentz transformation

$$-v_- = \frac{v(\xi_w) - \xi_w}{1 - \xi_w v(\xi_w)},$$

(4.17)

(The minus sign is introduced, because the plasma velocity in the wall frame has the opposite direction compared with the wall velocity in the bubble center frame.) Eq. (4.17) can be used to obtain

$$v(\xi_w) = \frac{v_+ - v_-}{1 - v_+ v_-} \text{ with } v_+ = \xi_w,$$

(4.18)

where $v_\pm$ are constrained by the matching equations between the plasma states in front of and behind the wall:

$$\xi_w = v_+ = \frac{1}{1 + \alpha} \left[ \frac{v_-}{2} + \frac{1}{6v_-} \right] + \left\{ \left( \frac{v_-}{2} + \frac{1}{6v_-} \right)^2 + \alpha^2 + \frac{2}{3} \alpha - \frac{1}{3} \right\}^{1/2}. \quad (4.19)$$

So, we obtain $v_-$ from eq. (4.19) for a given set of $\xi_w$ and $\alpha$ and insert it into the r.h.s. of eq. (4.18) to obtain the initial value $v(\xi_w)$. Since the minimum value of $v_-$ is $c_s$ for the detonations to be realized [70], we find that the minimum value of $\xi_w$ is just the Jouguet speed $\xi_J$ defined in eq. (4.10). To obtain an idea on the size of $\tau_{sw}H$, we show $\tau_{sw}H$ in figure 8 as a function of $\xi_w (\geq \xi_J)$ for $\alpha = 0.0245$ and $\beta/H = 5 \times 10^3$. As we see from figure 8, the reduction factor for $\Omega_{sw}$ is of order $10^{-2}$ in our model.

### 4.5 Gravitational wave spectrum

Now we are in position to present the GW spectrum $\Omega_{GW}$ of our model. As we have argued, the area (4.12) is an optimistic choice of the parameter space, and we expect that $\Omega_{GW}$ will be smaller in other regions of the parameter space. The relativistic degrees of freedom in the expanding Universe enters in the following expressions. It is the relativistic degrees of freedom...
freedom \( g' \) at the time, at which the GW background is produced. Therefore, \( g' \) varies with the time, because the tunneling process takes place for a finite period of time. It is certainly not \( g^\ast \) that is the one in the symmetric phase (4.11) and has been used for the computation of \( \alpha \) in eq. (4.9). In the following we assume that \( g' \) can be approximated by the relativistic degrees of freedom in the broken phase:

\[
g' = 106.75 + 8 + 1 + 1, \tag{4.20}
\]

where 8 comes from the NG bosons, and 1 is from \( \sigma \) as well as from \( S \).

Numerical simulations and analytic estimates [58–60, 67, 81–94, 97] of the individual contributions to \( \Omega_{GW} \) lead to the following formula:

- Scalar field contribution \( \Omega_{\varphi} \) [83]:

\[
h^2 \Omega_{\varphi}(f) = 1.67 \times 10^{-5} (\beta/H)^{-1} \left( \frac{\kappa_{\varphi} \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g^\ast} \right)^{1/3} \left( \frac{0.11 \xi_w^3}{0.42 + \xi_w^2} \right) S_{\varphi}(f), \tag{4.21}
\]

where

\[
S_{\varphi}(f) = \frac{3.8 (f/f_{\varphi})^{2.8}}{1 + 2.8 (f/f_{\varphi})^{3.8}} \tag{4.22}
\]

with the peak frequency

\[
f_{\varphi} = 16.5 \times 10^{-6} (\beta/H) \left( \frac{0.62}{1.8 - 0.1 \xi_w + \xi_w^2} \right) \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g^\ast}{100} \right)^{1/6} \text{ Hz.} \tag{4.23}
\]

- Sound-wave contribution \( \Omega_{sw} \) [84, 85]:

\[
h^2 \Omega_{sw}(f) = (\tau_{sw} H) 2.65 \times 10^{-6} (\beta/H)^{-1} \left( \frac{\kappa_{sw} \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g^\ast} \right)^{1/3} \xi_w S_{sw}(f), \tag{4.24}
\]

where

\[
S_{sw}(f) = (f/f_{sw})^3 \left( \frac{7}{4 + 3(f/f_{sw})^2} \right)^{7/2} \tag{4.25}
\]

with the peak frequency

\[
f_{sw} = 1.9 \times 10^{-5} \xi_w^{-1} (\beta/H) \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g^\ast}{100} \right)^{1/6} \text{ Hz.} \tag{4.26}
\]

According to refs. [68, 69], the reduction factor \( \tau_{sw} H \) (calculated in section 4.4) is multiplied in eq. (4.24).

- MHD turbulence contribution \( \Omega_{turb} \) [93]:

\[
h^2 \Omega_{turb}(f) = (1 - \tau_{sw} H) 3.35 \times 10^{-4} (\beta/H)^{-1} \left( \frac{\kappa_{sw} \alpha}{1 + \alpha} \right)^3 \left( \frac{100}{g^\ast} \right)^{1/3} \xi_w S_{turb}(f), \tag{4.27}
\]
where
\[
S_{\text{turb}}(f) = \frac{(f/f_{\text{turb}})^3}{[1 + (f/f_{\text{turb}})]^{2/3}(1 + 8\pi f/h_n)}
\] (4.28)
with the peak frequency
\[
f_{\text{turb}} = 2.7 \times 10^{-5} \xi_w^{-1} \beta \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g^*_{\nu}}{100} \right)^{1/6} \text{Hz},
\] (4.29)
and
\[
h_n = 16.5 \times 10^{-6} \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g^*_{\nu}}{100} \right)^{1/6} \text{Hz},
\] (4.30)
which is the value (redshifted to today) of the Hubble parameter at the production of the GW. We have introduced the enhancement factor \((1 - \tau_{\text{sw}} H)\) in eq. (4.27) and used the same efficiency coefficient as for the sound-wave contribution [69].

As we have mentioned in various places and we can see now from eq. (4.21), the scalar contribution \(\Omega_\phi\) is, due to \(\beta/H \sim 10^3\), about 2 orders of magnitude smaller than \(\Omega_{\text{sw}}\). Furthermore, the case at hand corresponds to a nonrunaway scenario, in which the friction between the bubbles in the surrounding plasma prevents the acceleration of the bubble expansion [70, 98]. To see this, we estimate \(\alpha_\infty\) according to ref. [70]:
\[
\alpha_\infty \simeq \frac{30}{24\pi^2 g, T_n^2} \left[ \frac{1}{2} n f n_c M^2((S), (\sigma)) + M^2_S((S)) \right] \in (0.078, 0.098)
\] (4.31)
for the parameter space (4.12), where \(M((S, \sigma))\) and \(M_S(S)\) are given in eqs. (2.9) and (3.5), respectively. Therefore, \(\alpha_\infty < \alpha \simeq 0.024\) (see (4.13)), so that we have a nonrunaway scenario (4.21) in the following discussion.

We use the efficient coefficient \(\kappa_{\text{sw}}\) given in ref. [70] for \(\Omega_{\text{sw}}\) and also for \(\Omega_{\text{turb}}\):
\[
\kappa_{\text{sw}}(\xi_w \gtrsim \chi_J) \simeq \frac{\chi_J^3 (\xi_J/\xi_w)^{5/2} \kappa C \kappa D}{(\chi_J^3 - \chi_w^3) (\xi_J/\xi_w)^{5/2} \kappa C + \chi_w^3 \kappa D},
\] (4.32)
where
\[
\chi_J = \xi_J - 1, \quad \chi_w = \xi_w - 1,
\]
\[
\kappa C \simeq \frac{\alpha^{1/2}}{0.135 + 0.981^{1/2} + \alpha}, \quad \kappa D \simeq \frac{\alpha}{0.73 + 0.083 \alpha^{1/2} + \alpha},
\] (4.33)
and \(\xi_J\) is given in eq. (4.10). Although \(\Omega_{\text{sw}}\) is reduced by the reduction factor \(\tau_{\text{sw}} H\) and \(\Omega_{\text{turb}}\) is enhanced by \((1 - \tau_{\text{sw}} H)\) and also by the identification \(\kappa_{\text{turb}} = \kappa_{\text{sw}}\), the turbulence contribution \(\Omega_{\text{turb}}\) is about one order of magnitude smaller than \(\Omega_{\text{sw}}\), because \(f_{\text{turb}}/h_n \sim \beta/H \sim 10^3\) that is in the denominator of eq. (4.28): \(f_{\text{turb}}/h_n \sim (\xi_w/\xi_J)^{-1}/(\tau_{\text{sw}} H) \sim 0.1\).

As we see from eq. (4.26) the scale of the GW frequency is fixed by the nucleation temperature \(T_n\). Note that the absolute scale of the critical temperature \(T_C\) and hence \(T_n\) is fixed through the coupling with the SM sector, i.e., \(\lambda_{HS}\) and \(y\). In the left panel of figure 9 we show \(T_C\) [TeV] and \(f_{\text{sw}}\) [Hz] against \(\lambda_{HS}\). Obviously, the smaller \(\lambda_{HS}\) is, the larger is \(\Lambda_H\).
Figure 9. Left: The critical temperature $T_C$ [TeV] (purple) and the peak frequency of the sound-wave contribution $f_{sw}$ [Hz] (green) against $\lambda_{HS}$. The area I and II are allowed by LHC (see figure 1). Right: SNR$^{BBO}$ against $\lambda_{HS}$ with five years observation. The SNR$^{BBO}$ (5 yrs) of the benchmark points, BP1 (purple star) and BP2 (green star), are also plotted.

(the scale of the hidden sector), and consequently higher $T_C$ and $f_{sw}$. The band of $f_{sw}$ is wider than that of $T_C$, because $\beta/H$ depends on $y$ (see figure 7) more than $T_C$ does. As we also see from this figure that the GW frequencies in our model are $\gtrsim 0.3$ Hz, which can be covered by DECIGO [72–74] and BBO [75–77].

We calculate the SNR according to ref. [71],

$$\text{SNR} = \sqrt{2t_{\text{obs}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left( \frac{\Omega_{\text{GW}}(f) h^2}{\Omega_{\text{noise}}(f) h^2} \right)^2},$$

where $t_{\text{obs}}$ stands for the duration of an observation in seconds, and $(f_{\text{min}}, f_{\text{max}})$ is the frequency range of a given experiment. The quantity $\Omega_{\text{noise}}(f) h^2$ represents the effective strain noise power spectral density for a given detector network, expressed as energy density parameter [99]. For the space-based observatories mentioned above, we adopt the strain noise power spectral densities from refs. [100–102]. (We use the sky-averaged sensitivity [101].) The result,$^9$ SNR against $\lambda_{HS}$ for BBO, is shown in the right panel of figure 9,$^{10}$ where we assume that $t_{\text{obs}} = 5$ years and the speed of the wall $\xi_w$ is equal to the Jouguet speed $\xi_J$ given in eq. (4.10). The SNR$^{BBO}$ (5 yrs) of the benchmark points, BP1 and BP2 defined in (2.21), are 11.8 and 5.7, respectively, while for DECIGO we find SNR$^{DECIGO}$ (5 yrs) = 1.1 and 0.5, respectively. Therefore, there is a good chance that the GW signals of our model can be detected by BBO, where the area I and II are allowed by LHC (see figure 1).

In the left panel of figure 10 we present the GW spectra for BP1 (purple) and BP2 (green) with $\xi_w = \xi_J$, which should be compared with the power-law-integrated sensitivity [71] of BBO (red dashed curve) and DECIGO (blue dashed curve), where we assume that the threshold SNR is 5 ($\rho_{\text{thr}} = 5$) with five years observation for both detectors. Since a part of the spectral curves for BP1 and BP2 runs over the sensitivity curve of BBO, we see once again that their signals could be detected at BBO, while for DECIGO it would be very difficult. For comparison we also present the GW spectra (dotted purple and green

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$^9$The SNR is computed including the turbulence contribution.

$^{10}$The effect of unresolvable astrophysical foregrounds from black hole, neutron star and white dwarf mergers on the signal significance are ignored.
Figure 10. Left: The GW spectrum for the benchmark points BP1 (purple), BP2 (green) and the power-law-integrated sensitivity of BBO (red dashed curve) as well as DECIGO (blue dashed curve), where we assume that the threshold SNR is 5 ($\rho_{\text{thr}} = 5$) with five years observation for both detectors. The GW spectrum is computed including the turbulence contribution, which is about one order of magnitude smaller than that of the sound-wave contribution. The dotted purple and green lines present, respectively, the GW spectrum of BP1 and BP2, for which the reduction factor $\tau_{\text{sw}} H$ due to the short sound-wave period is ignored. Right: The $\xi_w$ dependence of SNR$^{\text{BBO}}$ (5 yrs). The Jouguet speed $\xi_J$ is the minimum speed of $\xi_w$ for detonations. At this speed the SNR becomes maximal.

As the last task we consider the dependence of the wall speed $\xi_w$, because we have assumed so far that it is equal to the Jouguet speed $\xi_J$. In the right panel of figure 10 we show the $\xi_w$ dependence of SNR$^{\text{BBO}}$ (5 yrs). In fact, SNR$^{\text{BBO}}$ (5 yrs) assumes the maximal value at $\xi_w = \xi_J$, which follows from the fact that the reduction factor $\tau_{\text{sw}} H$ decreases as $\xi_w$ increases (see figure 8). But there is still a sufficient range in the parameter space, in which the detectability threshold is exceeded.

5 Summary and conclusion

In this paper we have studied the stochastic GW background produced at the cosmological chiral PT in a conformal extension of the SM [21, 22] and extended the analysis of ref. [31]. In particular, we have re-calculated $\beta/H$, because $\beta/H$ in ref. [31] does not approach the pure NJL value, $\sim 10^4$, as the Yukawa coupling $y$ decreases and for this reason we have suspected that the modified path deformation method of ref. [31] to obtain the bounce solution of a coupled system fails to yield trustful results.

Therefore, we have adopted an iterative method (with a reasonable convergence property) and found that $S_3/T$ can be fitted with a simple function (3.17). Using this fitting function for the determination of $\beta/H$ we have obtained $\beta/H \simeq (4 - 9) \times 10^3$ in the optimistic parameter space. We also have found that the benchmark point values of $\beta/H$ presented in ref. [31] are about one order of magnitude smaller than those calculated by using the new method.

There are, in the SU(3)$_V$ flavor symmetry limit, five independent parameters, $\lambda_H, \lambda_S, \lambda_{HS}, y$ and $g_H$ (or the hidden sector scale $\Lambda_H$), where effectively two of them are used to obtain $m_h = 125$ GeV and $\langle h \rangle = 246$ GeV. We have systematically narrowed the parameter space, giving smaller values of $\beta/H$ than that of the pure NJL model and hence larger (dimensionless) spectral GW energy density $\Omega_{\text{GW}}$. Obviously, $\Omega_{\text{GW}}$ will be smaller in other
regions of the parameter space. In this optimistic parameter space (with $\lambda_S \sim 10^{-3}$) the singlet scalar $S$ can become as light as the Higgs $h$, and therefore we have taken into account the LHC constraint on their mixing: There are two allowed regions for $\lambda_S = 0.001$ that are denoted by I (for $m_S < m_h$) and II (for $m_S > m_h$). We remark that for this optimistic parameter space in the SU(3)$_V$ flavor symmetry limit no realistic DM relic abundance can be obtained because the resonant condition ($m_S \approx 2m_{\text{DM}}$) in the s-channel of the DM annihilation can not be realized [17]. (In the model studied in ref. [31] the flavor group SU(3)$_V$ is unbroken and the hidden fermions have no U(1)$_Y$ charge.) This is why we have considered the model with a finite U(1)$_Y$ charge for the hidden fermions and have explicitly broken SU(3)$_V$ down to SU(2)$_V \times U(1)$ to apply the mechanism of ref. [22] to obtain a realistic DM relic abundance. But for the analyses of the GW background spectrum we have considered the SU(3)$_V$ limit, because it is only marginally broken and we would have had to deal with three variables (instead of two) to find a bounce solution.

The fact, $\beta/H \approx (4 - 9) \times 10^3$, implies a short duration time of the first-order chiral PT, much shorter than the Hubble time, and consequently a short sound wave period $\tau_{\text{sw}}$ as an active GW source; $\tau_{\text{sw}} H \approx 10^{-2}$. Then following refs. [68, 69] we have used $\tau_{\text{sw}} H$ as the reduction factor for the sound wave contribution $\Omega_{\text{sw}}$, which is nevertheless the most dominant contribution to $\Omega_{\text{GW}}$. We have evaluated the SNR for DECIGO and BBO and found that $\text{SNR}_{\text{DECIGO}} \lesssim 1.2$ and $\text{SNR}_{\text{BBO}} \lesssim 12.0$ with five years observation, from which we conclude that the GW signal predicted by the model in the optimistic case could be detected at BBO.$^{11}$

At last we recall that the results obtained by using effective theory methods to study the GWs produced at a first-order PT in strong-interacting QCD-like theories agree with each other only qualitatively [61] and for a more precise determination of the GWs we need first-principle calculations (like lattice simulations) which may become available in future [104].

Acknowledgments

We thank Alexander J. Helmboldt and Susan van der Woude for useful discussions. J.K. thanks for kind hospitality at the Max-Planck-Institute for Nuclear Physics, Heidelberg, where a part of this work has been done. The work of M.A. is supported in part by the Japan Society for the Promotion of Sciences Grant-in-Aid for Scientific Research (Grant No. 17K05412). J.K. is partially supported by the Grant-in-Aid for Scientific Research (C) from the Japan Society for Promotion of Science (Grant No.19K03844).

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