ESTIMATION OF RAIL TILTING UNDER THE DESIGN LOAD IS AN IMPORTANT FACTOR IN THE RATIONAL DESIGN OF RAIL FASTENING SYSTEMS. RAIL TILTING HAS BEEN ESTIMATED USING THE PRACTICAL SOLUTION BASED ON THE TORSION THEORY. HOWEVER, A RECENT STUDY HAS REPORTED THAT RAIL TILTING ESTIMATED BY THE CONVENTIONAL SOLUTION DOES NOT AGREE WELL WITH THE EXPERIMENTAL VALUES.

THIS STUDY AIMS TO ESTABLISH A PRACTICAL AND HIGH-ACCURACY SOLUTION FOR RAIL TILTING. FIRST, A FEM MODEL FOR RAIL TITLING IS PROPOSED AND THE VALIDITY OF THE MODEL IS EXAMINED BY COMPARING THE ANALYTICAL VALUES WITH THE EXPERIMENTAL VALUES. AS A RESULT, RAIL TILTING ESTIMATED BY THE MODEL WAS IN GOOD AGREEMENT WITH THE EXPERIMENTAL VALUES. THEREFORE, THE RESULTS SHOW THAT THE MODEL IS AVAILABLE FOR PREDICTING THE RESPONSE OF A RAIL AND ITS FASTENINGS. SECOND, THE MODEL WAS APPLIED TO DETERMINE THE LOADING CONDITION OF THE PERFORMANCE TEST OF RAIL FASTENING SYSTEMS. THE FINDINGS SHOW THAT IT IS POSSIBLE TO EVALUATE THE PERFORMANCE OF RAIL FASTENINGS WITH HIGH ACCURACY AS COMPARED WITH THE CONVENTIONAL EVALUATION.

KEY WORDS: RAIL TILTING, RAIL FASTENING SYSTEM, FEM, PRACTICAL MODEL, PERFORMANCE VERIFICATION

1. INTRODUCTION

Rail fastening systems (hereafter, rail fastenings) are designed so that stress of rail clips and rail head displacements due to rail tilting fall within the specific limits under the design load determined by track conditions. Therefore, estimation of rail tilting under the design load is an important factor in a rational design of rail fastenings. Rail tilting refers to the motion of rails, which tilt and twist in the direction of loads.

Timoshenko and Langer derived the basic equations that estimate the lateral bending and non-uniform torsion of the rail continuously supported by the elastic foundations under a load applied to the rail head. On the other hand, Satoh proposed a practical solution, which estimates uniform torsion of the rail discretely supported by sleepers. Furthermore, Hoshino derived the basic equations that estimates non-uniform torsion of the rail discretely supported by sleepers and proposed an approximate solution of the equations. These solutions need an estimation of a coefficient of resistance of rail fastenings. This coefficient is called a resistance coefficient to rail tilting and is dependent on the structure of rail fastenings and its stiffness. Satoh and Hoshino calculated the coefficient under the condition that the entire surface of rail bottom is supported by a rail pad. On the other hand, Minemura, Ichikawa and Onishi, Ichikawa calculated the coefficient assuming that the end of rail base lifts from the surface of a rail pad owing to a large rail tilting. Furthermore, Yamamoto et al. proposed a modified calculation of the coefficient by adding a variation in rail clamping force due to rail tilting and they concluded that a combination of the modified calculation and the Satoh's solution mentioned above provide an appropriate solution (hereafter, the conventional solution; see Appendix A) to estimate rail tilting.
At present, rail tilting and loads applied to a single assembled rail fastening are estimated using the conventional solution. However, a recent study has reported that the rail tilting estimated by the conventional solution does not agree well with the experimental values. In addition, the conventional solution is not applicable to rail fastenings of rail joints or turnouts, because the solution is based on the assumption that the same type of rail fastenings are continuously used on straight tracks. Therefore a new solution that is flexible, practical, and highly accurate, is required.

This study aims to establish a practical solution for rail tilting with high accuracy. A FEM analytical model for rail tilting is proposed and the validity of the model is examined by comparing the analytical values and the experimental values. Furthermore, the FEM model is applied to determining the loading condition of the performance test of a single assembled rail fastening.

2. FEM MODEL FOR RAIL TILTING

(1) Outline of FEM model
This study focuses on the standard track structures composed of a rail, rail clips, springs under rail, a rail supporter, and springs under supporter as shown in Fig. 1. Figure 2 shows the proposed FEM analytical model. The FEM model simplifies the track structures as much as possible in order to analyze various types of rail fastenings and rail supporters. On the other hand, in order to ensure high accuracy, the FEM model must meet the following requirements:
(a) Deformation of the rail includes a non-uniform torsion and a lateral bending
(b) All springs show a non-linear elastic deformation
(c) The rail is discretely supported by supporters
(d) The rail clips and the rail pad are deformed by the initial rail clamping force

(e) The rail clamping force changes its values as a result of rail tilting
(f) The end of rail base lifts from the surface of the rail pad as a result of rail tilting
(g) The toe of rail clips separates from the rail surface as a result of rail tilting

The FEM model is composed of solid elements, spring elements, and plate elements. The rail is modeled by solid elements. The rail clips composed of tip springs and lateral springs are modeled by spring elements that show a non-linear elastic deformation. The springs under the rail and the rail supporter are modeled by non-linear elastic spring elements as well as the rail clips. The features and the positions of the respective springs can be selected at will according to the type of rail fastenings and rail supporters. Rail supporters are modeled by plate elements.
elements and have sufficient stiffness (Young's modulus 1.0×10^9 kN/mm) to assume rigid bodies. The rail is discretely supported longitudinally by rail supporters, which are spaced at arbitrary intervals. In this paper, the number of rail supporters is set to 27 after checking the effect of the number of rail supporters upon dispersion of the loads. In addition, the sizes of elements of the FEM model are set to those as shown in Fig.2. The validity of the sizes of the elements are verified by comparing the analytical values of the FEM model with those of a beam theory on an elastic support.

In general, rail fastenings are designed under the condition that lateral loads of equal magnitude and the opposite direction, respectively, act on the right rail and the left one located just above the target rail fastenings. This loading condition is severe because the effect of springs that support the rail supporter sideward is canceled by each other, and the effect of dispersion of lateral loads is reduced. Under this loading condition, the right rail and the left one deform symmetrically and rail supporters can move only up and down. For this reason, only a single rail is modeled and the lateral displacement and the rotation of the rail supporter can be ignored.

As shown in Fig.3, the force-displacement curves of tip springs and springs under the rail are shifted in order to take account of the initial clamping force and the initial clamping displacement. In addition, for the larger displacement, which exceeds the initial clamping displacement, the stiffness of tip springs and springs under the rail is set to zero in order to express the behavior in which the end of rail base lifts from the surface of a rail pad and toe of rail clips separates from the rail surface as a result of rail tilting.

The following are the constraining conditions of the respective elements.

All elements: \( u = 0 \) on the cross section to which loads are applied

Lateral springs: \( u = v = w = \theta_x = \theta_y = \theta_z = 0 \) on the side opposite to the rail

Rail supporters: \( u = v = \theta_x = \theta_y = 0 \) at both ends

Springs under supporters: \( u = v = w = \theta_x = \theta_y = \theta_z = 0 \) on the side opposite to the rail supporters

Here, \( u, v, \) and \( w \) indicate the displacement in \( x, y, \) and \( z \) directions and \( \theta_x, \theta_y, \) and \( \theta_z \) indicate the rotation around \( x, y, \) and \( z \) axes.

The loads are applied to the side of rail head assuming the train loads, which are composed of the wheel load and the lateral load.

FEM analyses executed in this study are non-linear elastic analyses using NASTRAN (ver. 10) software. The total values of applied loads are divided into 10 steps. The convergence calculation at each step is performed with the quasi-Newton method.

(2) FEM model and conventional solution

The rail tilting angles by the FEM model were compared with those by the conventional solution under the condition that the same stiffness is used for both calculation methods. Table 1 shows the parameters of the FEM model and the conventional solution. These parameters were set assuming the standard rail fastenings and structures used in bal-
lasted tracks and slab tracks. Lateral springs, springs under rail, and springs under supporter were set to be linear elastic, and tip springs were set to be bilinear elastic. Almost all Japanese plate rail clips have a small spring coefficient to follow small rail tilting when small load acts upon the rail, and a large one to increase its resistance when large load acts; that is, its spring coefficient is bilinear.

In this chapter the rail tilting was calculated with the parameters, which change values in the ranges given in Table 1.

Figure 4 shows the results of the calculation. The results show that the rail tilting by FEM was smaller than that by the conventional calculation. The difference in both values is caused by the assumption of the rail torsion. Since the FEM model accounts for the non-uniform torsion, the rail rotates around a shear center of rails. On the other hand, the conventional solution assumes a uniform torsion and the rail rotates around the rail base. Consequently, the rotary moments estimated by the respective methods are different from each other. The difference in resistance to a rail tilting between the two methods depends on the characteristics of the springs under rail, tip springs, and initial clamping force. Therefore, the magnitudes of rail tilting obtained by the respective methods will be reversed in response to the characteristics of the springs.

This chapter shows the results obtained by both methods using the same values of spring coefficients. In actuality, since the FEM model takes into account the non-linear spring coefficient, the difference in rail tilting between the two methods includes the effects of the spring coefficient.

3. VALIDITY OF THE FEM MODEL

Loading test of the test track was carried out and FEM results were compared with the test results in order to verify the validity of the FEM model.

(1) Spring coefficient
The spring coefficient was measured prior to the loading test of the test track and incorporated into the FEM model and the conventional solution.

This chapter focuses on the type-9 rail fastening as shown in Fig.5, which is typically used in Japanese ballasted tracks of curved sections.

The coefficient of the spring under rail (rail pad) was measured by applying statically the compression force from 0kN to 200kN to a rail pad as shown in Fig.6. The test complied with JIS E 1117.

The coefficient of the tip spring was measured by applying the vertical force to a rail and a rail fas-
tensioning as shown in Fig.7. The test procedure was done as follows: The initial displacement of rail was set to 0mm. The rail was pulled up and the rail pad was extracted from under the rail. The force applied to the rail at this time was defined as $P_{\text{max}}$. The force was decreased until 0 kN and was increased until $P_{\text{max}}$ once again. As a result, the relationship between the force and the displacement was measured and was defined as the coefficient of the tip springs. The force at the time of displacement was decreased until 0 mm was defined as the initial clamping force.

The coefficient of the lateral springs was measured by applying the lateral force from 0kN to 60kN to the rail base as shown in Fig.8.

The relationships between the force and the displacement obtained above were approximated by some lines and test measurements. The dotted lines that were used for the conventional solution are also shown in Fig.9.

The spring coefficients of the lateral springs and the tip springs are the values per rail clip. As mentioned in Fig.3, the force-displacement curve of tip springs and springs under rail was shifted in order to take into account the initial clamping force. The initial clamping force was 5kN per rail clip.

(2) Loading test of the test track

Loading test of the test track, which was composed of type-9 rail fastening, was carried out and the results of the FEM model and the conventional solution were compared with those of the loading test.

Figure 10 and Table 2 show the test track composition and Fig.11 shows a view of the test apparatus. The lateral displacement and the rail tilting angle were measured applying the load to the rail head of the test track. Nine prestressed concrete sleepers type-6 cut to the length of 700mm were aligned on the testing machine at equal intervals (610mm). The number of sleepers was restricted by the length of surface plate of the testing machine. JIS 50kgN rail was fastened by the type-9 rail fastenings. JIS 50kgN rail is standardized by Japanese industrial standards (JIS) specification and is widely used in Japan. Rail clips were fastened by the bolts with 120Nm according to the specifications while checking the fastening condition. Rubber pads were ar-
Table 3 Loading condition of test track.

| Item                  | Condition  |
|-----------------------|------------|
| Load                  | 0kN → 100kN → 0kN |
| Angle                 | 65° → 55° → 54° |

Gauge corner → Field corner

![Fig.12 Measuring point of lateral displacement of rail.](image)

Table 4 Parameters of FEM model and conventional solution.

| Item                                  | Unit            | FEM                | Conventional solution |
|---------------------------------------|-----------------|--------------------|-----------------------|
| Rail                                  |                 |                    | JIS 50kgN rail        |
| Sleeper span a                        | mm              | 150               |                       |
| Width of rail base b₁                  | mm              | 127               |                       |
| Space of contacting point between rail and rail clip b₂ | mm | 99 | |
| Space between two rigid bodies of tip spring b₃ | mm | 200 | |
| Width of each spring in longitudinal dir. b₄ | mm | 150 | |
| Length of rail supporter L             | mm              | 400               |                       |
| Young's modulus of rail                | kN/mm²          | 206               |                       |
| Poisson's ratio of rail                |                 | 0.3               |                       |
| Moment of inertia of area of rail      |                 |                   | 196±10°               |
| Vertical bending                       | mm²             |                   | 322±10°               |
| Lateral bending                       | mm²             |                   | 161±10°               |
| Torsional stiffness of rail           | kN • mm         | 5                 |                       |
| Initial clamping force (per clip)     | kN              | 5                 |                       |
| Spring coefficient of spring under rail (per clip) | kN/mm | Fig.9 (a) | 106.7 |
| Tip spring coefficient (per clip)     |                 |                   | Fig.9 (b) 0.94       |
| Downward                              |                 |                   | Fig.9 (c) 5.95       |
| Upward                                |                 |                   | Fig.9 (d) 16.9       |
| Lateral spring coefficient (per clip) | kN/mm           | Fig.9 (e)         | 48.8                  |

Table 4 shows the parameters of the FEM model and the conventional solution. The parameters of the FEM model were based on solid lines in Fig.9 and those of conventional solution were based on the inclined dotted lines in Fig.9.

In Fig.13 and Fig.14, the results of the loading test were compared with those of the FEM model and the conventional solution. Rail tilting angle and rail displacements of the conventional solution were large compared with the loading test. By comparing the conventional method with the loading test when 100kN was applied, it can be seen that the former rail tilting angle was 1.55–1.83 times of the latter one; rail head displacement, 1.55–1.81 times; and rail base displacement, 1.18–2.60 times.

On the other hand, the rail tilting angle and rail head displacement obtained from the FEM model was in good agreement with those obtained from the loading test as shown in Fig.14. By comparing the rail tilting angle by the FEM model with that by the loading test when 100kN was applied, it can be seen that the former one was 0.92–1.07 times as large as the latter one; and for the rail head displacement, 0.95–1.07 times. There was a slight difference in rail tilting angle and rail head displacement between the FEM model and the loading test on the way to loading of 100kN at an angle of 45°. The cause of the difference was not completely explained but it was supposed that the difference was influenced by the assembling position of rail or the variation in initial clamping forces of rail clips. Regarding the rail base displacement, the value of the FEM model was 1.63 times as large as that of the loading test. This difference may be caused by the coefficient of lateral spring of the FEM model. In the test for lateral spring as shown in Fig.8, since only lateral load was applied to the rail base, frictional resistance by wheel load between rail and sleeper was ignored. As a result, it seems that the coefficient of lateral spring of the FEM model became small and rail base displacement became large as compared with those of the loading test.

Regarding the inclinations in the graph shown in Fig.13 (a) and Fig.14 (a), the inclination of the FEM model becomes larger with an increase in the load, while the inclination of the conventional solution becomes smaller. The difference in the inclination is...
caused by the non-linear elasticity of the FEM model. Yamamoto et al.\(^7\) derived the resistance coefficient to rail tilting of the conventional solution under the assumption that the non-linear elasticity can be ignored because the rail pad shows elasticity in the normal load region (50kN). However, the rail pad shows non-linear elasticity at its end, which is compressed by the rail base as shown in the FEM result. Therefore, it is important to take the non-linear elasticity of rail pad into consideration in order to improve the accuracy of prediction of rail tilting.

It was observed that the result by the FEM model was in good agreement with that by the loading test compared with the conventional solution; however, there was a difference in rail base displacement between the FEM model and the loading test. The FEM model reproduced with high accuracy the loading test result of the rail tilting, which is important to evaluate the performance of rail fastenings. Therefore, it should be concluded that the FEM model is an appropriate tool for predicting the reaction of rail and rail fastenings.

4. APPLICATION OF THE FEM MODEL TO THE PERFORMANCE TEST OF A SINGLE ASSEMBLED RAIL FASTENING

The FEM model was applied to determine the loading condition of the performance test of a single assembled rail fastening.

(1) Performance test of rail fastenings

The Japanese design standard for railway structures (track structures)\(^1\) requires that the performance of rail fastenings should be checked by either of the following two loading tests:

(a) Static loading test on a test track composed of plural rail fastenings
(b) Static loading test on a single assembled rail fastening

Figure 15 shows examples of both loading tests. Here, static loading tests (a) and (b) are defined as the performance tests of rail fastenings. The two loading tests differ in the number of specimen or the magni-
tude of load.

In the loading test (a), as described in Chapter 3, the load is applied to the test track, which simulates the realistic rail track in order to estimate the response of rail fastenings. When loading test (a) is selected, the design loads shown in Table 5 are generally used as the test loads. The design load A is applied to the rail from the gauge corner and the design load B is applied from the field corner alternatively.

In loading test (b), the load is applied to a single assembled rail fastening. Loading test (b) is normally selected because the test can be performed by using a smaller number of components, simpler constitution, and less manpower than those of loading test (a). When loading test (b) is selected, the test load is applied to the rail in consideration of the load dispersion by track. As shown in Fig.16, the dispersed design loads, namely, vertical rail pressure, lateral rail pressure, and rail tilting moment should be applied to a single assembled rail fastening. Vertical and lateral rail pressures refer to the dispersed forces that act on a single assembled rail fastening when the wheel load and lateral load act on the track. Rail tilting moment refers to the moment by rail tilting that acts on a single assembled rail fastening and is defined around the central point of the rail bottom surface.

Up to now, vertical rail pressure, lateral rail pressure, and rail tilting moment have been estimated by the conventional solution. However, as already described in the Chapter 3, the conventional solution cannot sufficiently reproduce the loading test results. It is deemed that the test accuracy may deteriorate when the static loading test on a single assembled rail fastening is performed under the loading condition based on the conventional solution.

This chapter shows the method for determining the loading condition of the loading test on a single assembled rail fastening based on the FEM model.

### Table 5 Design loads of rail fastenings. (Meter-gauged line)

| Load           | Alignment       | Design load A | Design load B |
|----------------|-----------------|---------------|---------------|
| Wheel load     | Tangent and curved | 98kN          | 86kN          |
| Lateral load   | R<600m          | 60kN          | 50kN          |
|                | 600m≤R<800m     | 45kN          | 22kN          |
|                | R≥800m, Tangent | 30kN          | 15kN          |

Design load A: Extreme force applied to rail from gauge corner
Design load B: Typical force applied to rail from field corner

(2) Method for determining loading condition

When the FEM model is in the balanced position with wheel load $P$ and lateral load $Q$ acting on the model, vertical rail pressure $W$, lateral rail pressure $H$, and rail tilting moment $M$ are expressed by the following equations (wherein each symbol is as described in Fig.17).

\[
W = \sum_{i=1}^{n} (W_i) - (P_i + P_r) \quad (1)
\]

\[
H = R_i + R_r \quad (2)
\]

\[
M = \sum_{i=1}^{n} (W_i \cdot b_i) - \sum_{i=1}^{n-1} (W_i \cdot b_j)
+ (P_i \cdot d_i - P_r \cdot d_r) + (R_i + R_r) \cdot c \quad (3)
\]

where, $W$ is positive in the downward direction, $H$ is positive in the rightward direction, and $M$ is positive in the clockwise direction in Fig.17.

When static loading test on a single assembled rail fastening is performed, vertical rail pressure $W$, lateral rail pressure $H$, and rail tilting moment $M$ calculated by eqs. (1), (2), and (3) should be applied to the rail. When vertical and lateral rail pressures are applied to a regular height rail, a moment actually generated exceeds the moment $M$ calculated above. In order to prevent the excessive moment, a
Fig. 17 FEM model balanced under wheel load and lateral load.

Fig. 18 Example of low-height rail (test rail).

a) Uniaxial loading condition
In the case of uniaxial loading condition as shown in Fig. 19, test load $L$, angle of load application $\theta$, and height of load application $h$ are expressed by the following equations.

$$L = \sqrt{W^2 + H^2}$$  \hspace{1cm} (4)

$$\theta = \tan^{-1}(W/H)$$  \hspace{1cm} (5)

$$h = (M + W \cdot e)/H$$  \hspace{1cm} (6)

The height $h$ and the distance $e$ are determined by the size of test rail. Thus, one of the parameters, either $W$, $H$, or $M$, becomes an independent variable. Here, $H$ is selected as an independent variable to solve eqs. (4), (5), and (6). In this case, $H$ is calculated back from eq. (6) with $h$ and $e$ of test rail owned by a laboratory, and $L$ and $\theta$ are calculated by substituting $H$ into eqs. (4) and (5). The ideal height of the test rail can be determined when $H$ calculated back from eq. (6) becomes closest to $H$ calculated from eq. (2).

b) Bi-axial loading condition
In the case of bi-axial loading condition as shown in Fig. 20, test loads $L_A$ and $L_B$, the angles of load application $\theta_A$ and $\theta_B$, and the height of load appli-
cation $h$ are expressed by the following equations.

\[
L_A = \sqrt{(W_A - L_0 \sin \theta_B)^2 + (H_A + L_0 \cos \theta_B)^2} \tag{7}
\]

\[
L_B = \sqrt{(W_B - L_0 \sin \theta_A)^2 + (H_B + L_0 \cos \theta_A)^2} \tag{8}
\]

\[
\theta_A = \tan^{-1} \left( \frac{W_A - L_0 \sin \theta_B}{H_A + L_0 \cos \theta_B} \right) \tag{9}
\]

\[
\theta_B = \tan^{-1} \left( \frac{W_B - L_0 \sin \theta_A}{H_B + L_0 \cos \theta_A} \right) \tag{10}
\]

\[
h = \frac{M_A + e(W_A - 2L_0 \sin \theta_B)}{H_A} \tag{11}
\]

\[
h = \frac{M_B + e(W_B - 2L_0 \sin \theta_A)}{H_B} \tag{12}
\]

Here, $W_A$, $H_A$, and $M_A$ or $W_B$, $H_B$, and $M_B$ are vertical rail pressure, lateral rail pressure, and rail tilting moment calculated by eqs. (1), (2), and (3) when $L_A$ or $L_B$ are applied. The subscript A or B means the application of design load A or B. Here, $W_A$ is positive in the downward direction, $H_A$ is positive in the direction to the field corner and $M_A$ is positive in the clockwise direction. On the other hand, $W_B$ is positive in the downward direction, $H_B$ is positive in the direction to the gauge corner and $M_B$ is positive in the counterclockwise direction. $L_0$ is the minimum load applied in the opposite direction to $L_A$ or $L_B$ and is set at 10kN in consideration of stability of the test.

The height $h$ and the distance $e$ are determined by the size of test rail. Thus, any two of the parameters, $W_A$, $W_B$, $H_A$, $H_B$, $M_A$, or $M_B$ become independent variables. Here, $H_A$ and $H_B$ are selected as the independent variables to solve eqs. (7)-(12). In this case, using $h$ and $e$ of the test rail owned by the laboratory, equations can be solved by the following steps:

a) Set $\theta_A^*$ ($0^\circ \leq \theta_A^* \leq 90^\circ$) as an assumed value of $\theta_A$ and substitute $\theta_A^*$ into eq. (12) to calculate the temporary value $H_B^*$

b) Substitute $\theta_A^*$ and $H_B^*$ into eq. (10) to calculate the temporary value $H_A^*$

c) Substitute $H_A^*$ into eq. (11) to calculate the temporary value $H_B^*$

d) Substitute $H_A^*$ and $H_B^*$ into eq. (9) to calculate the value $\theta_A^*$

e) Steps a)-d) are repeated until $\theta_A^*$ coincides with the assumed value $\theta_A^*$ and then $\theta_A^*$ is determined as $\theta_A$

f) Calculate $H_B$ from eq. (12), $\theta_B$ from eq. (10), and $H_A$ from eq. (11)

g) Calculate $L_A$ and $L_B$ from eqs. (7) and (8)

The ideal height of the test rail can be determined when $H_A$ and $H_B$ calculated above become closest to $H_A$ and $H_B$ calculated from eq. (2).

(3) Loading condition and its verification

a) Calculation of loading condition

Loading condition was calculated for performance test of a single assembled type-9 rail fastening. Here, bi-axial loading condition was calculated. FEM analysis was carried out by the FEM model. As shown in Fig.21, design load A (wheel load 98kN, lateral load 60kN) and design load B (wheel load 86kN, lateral load 30kN) were applied to rail independently from gauge corner and field corner to calculate vertical rail pressures $W_A$, $W_B$; lateral rail pressures $H_A$, $H_B$; and rail tilting moments $M_A$, $M_B$. The parameters of the analysis were the same as those shown in Table 4. The loading condition was calculated by the method described above on the basis of the FEM result.

On the other hand, the loading condition on the basis of the conventional solution was also calculated for comparison with that calculated by the FEM model. The parameters used in the conventional solution were the same as those shown in Table 4.

Table 6 shows the bi-axial loading conditions calculated by the respective methods. The height of test rail is selected from those of several test rails (height 60, 70, 80, 90, 100, and 110mm) owned by the laboratory so as to be closest to the ideal height.

b) Verification of calculation of loading condition

In order to verify the validity of the calculation of loading condition based on the FEM model, the
A performance test of a single assembled type-9 rail fastening was carried out. The test result was compared with those of the other performance test of a rail fastening by the test track.

At first, the performance test of a single assembled rail fastening was carried out. Here, case 1 means the loading condition based on the conventional solution and case 2 means that based on the FEM model. Figure 22 shows the state of the test.

One concrete sleeper type-6 cut to the length of 700mm was set on the surface plate of the testing machine. JIS 50kgN test rail (height 100mm and 90mm) was set on the sleeper and was fastened to the sleeper by the type-9 rail fastenings. Rail clips were fastened by the bolts with 120Nm, the torque specified in the specifications. The sleeper was fastened to the testing machine firmly without rubber pads. Table 7 shows the process of loading. Figure 23 shows the measuring points of test rail displacement. Rail tilting angle was calculated by dividing the difference between rail head and base displacement by the distance between two measuring points (127mm). Rail base displacement was measured at point 3. Rail head displacement was converted into a value of top of JIS 50kgN rail by multiplying rail tilting angle by a height of JIS 50kgN rail (153mm) and by adding rail base displacement. The direction of lateral displacement from gauge corner to field corner was defined as positive and clockwise rail tilting angle was defined as positive.

Figure 24 and Figure 25 show the rail tilting angle and lateral rail displacement of case 1 and case 2. On the other hand, the performance test of a rail fastening by the test track was carried out. The test track was assembled (Fig.26) in the same way as that already described in Chapter 3. Table 8 shows the process of loading. The measurement items and the
measuring points were the same as those described in Chapter 3. The test loads $L_A$ and $L_B$ were applied, which satisfy the following equations:

$$L_A \sin \theta_A + L_B \sin \theta_B = 98 \quad (13)$$
$$L_A \cos \theta_A - L_B \cos \theta_B = 60 \quad (14)$$
$$L_B \sin \theta_B + L_B \sin \theta_A = 86 \quad (15)$$
$$L_B \cos \theta_B - L_B \cos \theta_A = 30 \quad (16)$$

Here, $L_0$ was the minimum load, which acted on the rail in the direction opposite to $L_A$ or $L_B$ and was set at 10kN. Figure 27 shows the lateral rail displacement and rail tilting angle obtained from the performance test of rail fastening by the test track.

Figure 28 shows the comparison of rail tilting angle and lateral rail displacement obtained from the respective tests.

Regarding rail tilting angle, that of case 1 was 0.49 times as large as that of the test track when $L_A$ was applied and was 0.18 times as large as that of the test track when $L_B$ was applied. As regards rail head displacement, that of case 1 was 0.48 times as large as that of the test track when $L_A$ was applied and was 0.32 times as large as that of the test track when $L_B$ was applied. As for rail base displacement, that of case 1 was 0.44 times as large as that of the test track when $L_A$ was applied and was 1.15 times as large as that of the test track when $L_B$ was applied. Thus, test results based on the conventional solution tended to be small in rail tilting angle and rail head displacement compared with those of the test track. This tendency can be understood from the fact that the conventional solution cannot reproduce the result of the test track as described in Chapter 3.

On the other hand, regarding rail tilting angle, that of case 2 was 1.35 times as large as that of the test track when $L_A$ was applied and was 0.70 times as large as that of the test track when $L_B$ was applied. As regards rail head displacement, that of case 2 was 1.19 times as large as that of the test track when $L_A$ was applied and was 0.79 times as large as that of the test track when $L_B$ was applied. As for rail base displacement, that of case 2 was 0.75 times as large as that of the test track when $L_A$ was applied and was 1.46 times as large as that of the test track when $L_B$ was applied. Thus, test results based on the FEM model tended to be large in rail tilting angle and rail head displacement compared with those of the test track. The cause of the difference has not been fully explained. It is deemed that the initial clamping force or torque variation owing to individual differences affected the differences in the results. Therefore only
the difference due to loading condition was not shown here. Nevertheless, the results of case 2 were closer to the results of the test track than those of case 1. Thus, loading condition based on the FEM model was much improved as compared with that of the conventional solution. This can be understood from the fact that the FEM model can reproduce the result of the test track with high accuracy as shown in Chapter 3.

From the above result, it was confirmed that test results based on the FEM model were closer to the result of the test track as compared with those of the conventional solution. Therefore, in the case of a performance test of a single assembled rail fastening, it was concluded that the FEM model can provide the loading condition that is closer to the realistic track and can evaluate the performance of rail fastenings with higher accuracy than the conventional solution.

5. CONCLUSION

In this study, the FEM model was proposed and was validated by comparing FEM results with loading test results. Furthermore, the FEM model was applied to the method of determining loading condition of performance test of a single assembled rail fastening. The obtained results can be summarized as follows:

1) The practical FEM model was proposed which simplifies the various types of rail fastenings and rail supporting structures.
2) The rail tilting angle obtained by the FEM model was in good agreement with that obtained by the loading test on ballasted test track. It was clarified that the FEM model can be an appropriate tool for predicting the reaction of rail and rail fastenings to the acted loads.
3) The method of determining loading condition based on the FEM model was proposed for performance test of a single assembled rail fastening.
4) It is therefore concluded that the FEM model can provide the loading condition that is closer to the realistic track and can evaluate the performance of rail fastenings with higher accuracy than the conventional solution.

APPENDIX A CONVENTIONAL SOLUTION

The solution for rail tilting suggested by Satoh\(^3\) is expressed as follows:

\[
\theta_i = t_i(\lambda) = \frac{a K'}{c} M_T, \quad \lambda = \frac{a K'}{c} 
\]

(17)

The meaning of each symbol in the equation is described in Fig.29 in which \(t_0(\lambda)\) is rail tilting coefficient and \(t_0(\lambda)\) at the point of application of load is expressed as follows:

\[
t_0(\lambda) = \frac{5 + 20\lambda + 21\lambda^2 + 8\lambda^3 + \lambda^4}{2 + 25\lambda + 50\lambda^2 + 35\lambda^3 + 10\lambda^4 + \lambda^5}
\]

(18)

Yamamoto et al.\(^7\) derived \(K'\), resistance coefficient of rail tilting, under the assumption of bilinear spring coefficient of rail clips as shown in Fig.30. Here, \(K'\) is classified into three classes according to the magnitude of rail tilting angle \(\theta_0\) shown in Fig.31, and is calculated as follows:

(a) In the case that the end of rail base is touching the rail pad and the clamping force is smaller than the initial clamping force \(P_0\). (0 ≤ \(\theta_0 \leq \theta_0\))

\[
K' = \frac{K_1 b_1^2}{12} + \frac{K_2 b_2^2}{2}
\]

(19)

(b) In the case that its end is touching the rail pad and its force is larger than \(P_0\). (\(\theta_0 \leq \theta_0\))

\[
K' = \frac{K_1 b_1^2}{12} + \frac{b_2^2}{4} (K_2 + K_1)
\]

(20)

(c) In the case that its end is not touching the rail pad and its force is larger than \(P_0\). (\(\theta_0 \leq \theta_0\))
The rail tilting moment at the point of application of load $\theta_0$ can be obtained by substituting $K'$ into eq. (17).

Rail tilting moment $M$ applied to a single assembled rail fastening can be calculated by $K' \times \theta_0$. Vertical and lateral rail pressures $W$ and $H$ can be obtained by the following equations based on the beam theory on an elastic support$^{(10)}$:

$$W = P \left(1 - \exp \left(-\frac{k_v \Delta_y}{4EI_x} \right) \cos \left(\frac{k_v \Delta_y}{4EI_x} \right) \right) \quad (22)$$

$$H = Q \left(1 - \exp \left(-\frac{k_h \Delta_y}{4EI_y} \right) \cos \left(\frac{k_h \Delta_y}{4EI_y} \right) \right) \quad (23)$$

Here, $E$ is the Young's modulus of rail; $k_1$ and $k_0$ are vertical and lateral spring coefficients, respectively, of elastic rail support; and $I_x$ and $I_y$ are the moments of inertia of area for vertical and lateral bending, respectively, of rail.

REFERENCES

1) Railway Technical Research Institute: Design Standards for Railway Structures and Commentary (Track Structures), pp. 52-59, Maruzen, 2012. (in Japanese)
2) Timoshenko, S. and Langer, B. F.: Stresses in railroad track, Trans. of ASME, APM 54-26, Vol. 54, pp. 277-302, 1932.
3) Satoh, U.: On the lateral strength of railway track, Railway Technical Research Report, Railway Technical Research Institute, Vol. 110, 1960. (in Japanese)
4) Hoshino, Y.: A practical solution for the torsion (tilting) of rail, Journal of Japan Society of Civil Engineering, Vol. 210, pp. 33-46, 1973. (in Japanese)
5) Minemura, Y. and Ichikawa, S.: Design and performance tests of 101 rail fastenings (for new Tokaido line), Railway Technical Research Report, Railway Technical Research Institute, Vol. 388, 1963. (in Japanese)
6) Onishi, A. and Ichikawa, S.: Design of new tie-plate, Railway Technical Research Report, Railway Technical Research Institute, Vol. 376, 1963. (in Japanese)
7) Yamamoto, T., Umeda, S. and Kanamori, T.: Relationship between spring coefficient of fastening device and rail overturning angle, Quarterly Reports, Railway Technical Research Institute, Vol. 22, No. 4, pp. 153-156, 1981.
8) Deshimaru, T., Shono, S., Kataoka, H. and Furukawa, A.: Experimental study on the response of rail fastening clamps to the application of train load, Journal of Railway Engineering, Vol. 18, pp. 95-102, 2014. (in Japanese)
9) Nagafuji, T.: Rail fastening system (1), Journal of Japan Railway Civil Engineering Association, Vol. 30, No. 1, pp. 31-34, 1992. (in Japanese)
10) NX Nastran User's Guide, Siemens PLM Software, 2014.
11) Japanese Standards Association: Prestressed concrete sleepers - Rail fastenings, JIS Handbook Railway, pp. 211-270, 2014. (in Japanese)
12) Tamagawa, S.: Tri-axial fatigue testing machine for rail fastening systems, Railway Research Review, Railway Technical Research Institute, Vol. 73, No. 5, p. 41, 2016. (in Japanese)
13) Miura, S.: Kidou Zairyou (Track structures and components), Tetsudo Gengyosha, pp. 283-284, 1981. (in Japanese)
14) Kato, Y.: Rail, Japan Railway Civil Engineering Association, pp. 144-154, 1978.
15) Nagafuji, T.: Rail fastening system (2), Journal of Japan Railway Civil Engineering Association, Vol. 30, No. 2, pp. 27-30, 1992. (in Japanese)

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