Accounting Method of the Spatial Shape of the Cross Section of URM Walls in the Analysis of Stiffness Under the Action of Horizontal Seismic Loads

O V Kabantsev¹, O A Simakov¹
¹Moscow State University of Civil Engineering, Yaroslavskoye Shosse, 26, Moscow, 129337, Russia

E-mail: ovk531@gmail.com, simakovoa@mgsu.ru

Abstract. The working of buildings with load-bearing walls made of masonry has a significant difference from the working of buildings made of monolithic and precast concrete: the degree of damage, coefficient of variation, and overall stiffness differ. One of the most difficult issues in calculating load-bearing masonry structures for seismic impacts is determining the relative shear deformations of walls of rigid stone buildings. The calculation of these deformations on a straight line depends on the cross-section shape coefficient—a coefficient that takes into account the influence of uneven tangential stresses on the deformation of the bent element. This article presents the results of analytical studies of the dependencies of the cross-section shape coefficient. In conclusion, we present an analytical method for determining this coefficient, which allows us to determine the rigidity of a wall structure of any shape with a sufficient level of accuracy and, as a result, adequately take into account the shear forces in the vertical joints of the wall interface.

1. Introduction
Buildings with masonry load-bearing systems make up a significant part of the existing stock of buildings in earthquake-prone areas. Stone structures, especially unreinforced stone (URM) are widely used in the design and construction of new earthquake-resistant buildings and structures.

The analysis of the seismic resistance of stone buildings, performed on the basis of generalization and comparison of the actual seismic response of buildings of different structural systems, shows significantly different levels of damage (for example, work [1, 2, 3]) – see table 1.

| №  | Structural type of building                          | Average degree of damage $d$ | Coefficient of variation $\nu$ |
|----|------------------------------------------------------|------------------------------|-------------------------------|
| 1  | Multistory large panel buildings                    | 1.1                          | 0.18                          |
| 2  | Multistory frame and link buildings (AIS-04 series) | 2.3                          | 0.37                          |
| 3  | Stone buildings with monolithic reinforced concrete belts | 2.8                          | 0.43                          |

Table 1. Level of damage to buildings.
Studies [4, 5, 6, 7] have shown that the seismic reaction of stone buildings is characterized not only by a high average degree of damage $d$, but also by large deviations of partial values from the average value, as evidenced by the highest (relative to other types of load-bearing systems) value of the coefficient of variation "$v$". It is logical to assume that load-bearing systems made of masonry have some specific features that are not fully taken into account by modern approaches to assessing and predicting seismic resistance.

Buildings with masonry load-bearing systems are constructed in several main types of structural schemes: box-shaped structural scheme; with transverse load-bearing walls; with longitudinal load-bearing walls. Analysis of structural schemes of buildings with load-bearing structures made of masonry shows that the load-bearing systems consist of a limited number of types of vertical load-bearing structures-walls, piers and bridge sections. For such structures, a large number of studies have been conducted to determine how such structures work under seismic influences (for example, [5, 6]).

It is obvious that the assessment and forecast of the stress-strain state of structures of stone buildings under seismic impacts can not be correct without taking into account the features of the spatial operation of load-bearing systems made of masonry.

A common feature of stone buildings erected in both normal and earthquake-prone areas is the presence of a system of cross walls United by an overlap disk. Joint operation of the cross-wall system is possible if sufficient load-bearing capacity of the vertical joint of the intersecting walls is provided. The value of the bearing capacity of a vertical wall joint is determined by the values of the shear forces formed in the joint under the action of horizontal loads, including seismic ones. Ensuring reliable operation of the butt joint of cross walls makes it possible to include in the spatial section of the wall adjacent sections of walls perpendicular to the plane of the calculated wall, which allows us to consider and include in the calculation model a stone wall structure of a complex spatial section - an I-beam, channel or other more complex shape (see Fig. 1).

Figure 1. Schemes of spatial sections of walls of stone buildings.

2. Statement of the problem research

Assessment of the spatial performance of the load-bearing system of stone buildings should be based on the analysis of the complex stress-strain state of both the main structures of the building (walls, piers and lintels) and junctions (joints) of these elements.

The issues of strength and deformability of the main structural elements of stone buildings (walls, piers and lintels) under seismic impacts are well studied (for example, [8, 9, 10]). However, the work of vertical butt joints of these wall structures and the influence of strength and deformability of such joints on the joint work of individual planar structural elements has not been fully studied.

The vertical joint of the walls ensures the compatibility of the work of planar structures erected in different planes, which allows you to include in the calculation analysis not only a wall with dimensions corresponding to the wall plane, but also adjacent sections of walls involved in joint work located in a different (relative to the wall under consideration) plane. This spatial structure has not only increased rigidity, but also increased load-bearing capacity under horizontal, including seismic, impacts [8, 9]. But for the formation of a spatial structure, the necessary load-bearing capacity of the vertical joint must be provided, which is not taken into account by a number of studies – the research [9] shows only the joint work of complex walls in terms of plan. However, this most important factor
is usually not considered either in the formation of computational models or in performing computational analysis.

When horizontal seismic impacts occur, significant shear forces are formed in the vertical joint of the walls. To ensure the functionality of the joint, the latter must have the necessary (and sufficiently high) bearing capacity according to the shear criterion. But to perform a joint with a certain load-bearing capacity, it is necessary to know the amount of shear forces along the contact line of the walls in the vertical joint.

The codes on the calculation of stone structures contain the necessary equations for calculating shear forces in the vertical joints of walls. In accordance with these known formulas, the shear forces in the vertical joint of walls located in different planes depend on the value of the stiffness characteristic of the spatial wall console structure \( (C) \), which, in turn, is a function of the deformations of such a console from the action of a single evenly distributed horizontal load \( q = 1 \):

\[
C = \frac{1}{\delta}
\]

\[
\delta = \delta_M + \delta_Q
\]

where \( \delta_M \) is the deflection of the console from bending deformations at 2/3 of the building height from the action of a uniformly distributed load \( q = 1 \);

\( \delta_Q \) - deflection of the console from shear deformations by 2/3 of the building height from the action of a uniformly distributed load \( q = 1 \).

As part of analytical methods for solving the problem of rigidity of a console with a cross section of a spatial shape, standard methods of structural mechanics are used, which after transformations and simplifications can be represented as follows:

\[
\delta_M = 0,07 \frac{H^4}{EI}
\]

\[
\delta_Q = \frac{4KH^2}{9GA}
\]

where \( H \) is the height of the wall;

\( EI \) is the exural rigidity of the console (wall);

\( G \) - the masonry shift modulus - is taken from the survey data and is equal to \( G = 0.4 \ E \);

\( A \) – cross-sectional area of the console (wall);

\( K \) - coefficient of the cross-section shape, which takes into account the influence of uneven tangential stresses on the deformation of the bent element.

As a result, the total deflection of the console with a cross section of the spatial shape, taking into account both bending and shear deformations, is determined by the formula:

\[
\delta = \frac{H^2}{E} \left( 0,07 \frac{H^2}{I} + 1,1 \frac{K}{A} \right)
\]

Determining the shear deformations of walls of rigid stone buildings is difficult, because when calculating these deformations, the calculation formulas contain a cross-section shape coefficient \( (K) \), which takes into account the influence of uneven tangential stresses on the deformation of the bent element. The value of this coefficient is generally expressed by the formula (6):
where \( A \) and \( I_0 \) are the cross-section area and the moment of inertia relative to the \( x \)-axis passing through the center of gravity of the cross-section;

\( y_1 \) and \( y_2 \) - ordinates of the upper and lower section borders.

\( S(y) \) – static moment of a part of the cross-section area from the \( y \)-level to the edge of the cross-section relative to the cross-section axis.

\( b(y) \) – width of the section at the \( y \) level.

The study of literature sources shows that analytical methods for calculating the cross-section shape coefficient that are suitable for practical application are not available today. At the same time, failure to take into account or incorrect account of the cross-section shape coefficient when predicting the distribution of seismic forces in a stone building can lead to significant errors in the estimation of the bearing capacity reserves of the prosten. Thus, the task of developing scientifically based and correct methods for determining the cross-section shape coefficient is in demand in the practice of analytical solutions for forecasting and evaluating the seismic resistance of stone buildings.

3. The results of the study

The method for calculating the cross-section shape coefficient \( K \) for arbitrary sections (see Fig. 2) is generally represented by the expression (6)

\[
K = \frac{A}{I_0} \int_{y_1}^{y_2} S(y) dy
\]

Figure 2. Schemes of a cross section of a wall of any shape.

Taking into account the fact that the sections of the load-bearing walls are mostly arbitrary in shape, for the class of problems under consideration, these sections can be decomposed into rectangular components. This, in turn, allows us to numerically integrate expression (6) using the rectangle method with the required accuracy. If the lower face of the section is taken as the origin, then expression (6) takes the form:

\[
K = \frac{A}{I_0} \sum_{i=1}^{i=n} \frac{S_i^2 dy_i}{b_i}
\]

where is the cross section area \( A \):

\[
A = \sum_{i=1}^{i=n} b_i dy_i
\]

is the moment of inertia about an axis passing through the center of gravity of the cross section:
The static moment of the area of the cross section from the $i$-th layer to the edge section relative to the axis of the cross section passing through the center of gravity:

$$ S_i = \sum_{j=1}^{i} b_j (y_j - y_c) dy_j $$

- coordinate axis from the bottom face of the cross section passing through the center of gravity of a section:

$$ y_c = \sum_{i=1}^{n} b_i y_i' dy_i $$

$y_i, y_j$ - the top coordinate of the $i$-th or $j$-th layer from the bottom face section;
- coordinate of the center of gravity of the $i$-th layer from the lower face of the section;
$y_c$ is the width of the $i$-th or $j$-th layer;
$dy_i$ - height of the $i$-th or $j$-th layer;
n - the number of layers that the section on the $y$-axis is divided into.

As a result of numerical studies, it was found that the cross-section coefficient $K$ has the same values for an arbitrary cross-section shape, provided that the geometric dimensions of the composite rectangular sections are directly proportional to the direction of the Central axes. This property allows you to significantly generalize the range of values of the $K$ coefficient. To confirm this, and as an example, consider the section shown in figure 2 and integrate it along the directions of the Central axes, i.e. calculate the $K$ coefficient for its value when the axes "$x$" and "$y$" are shifted in the direction (which coincide with the directions of the Central axes of the section under consideration).

Table 2 shows the results of calculating the section under consideration, provided that its dimensions in the direction of the "$x$" axis are directly proportional to the size "$b$", and the dimensions in the direction of the "$y$" axis are directly proportional to the size "$g$" (see Fig.3).

| a    | b    | c    | d    | e    | f    | g    | H    | i    | j    |
|------|------|------|------|------|------|------|------|------|------|
| 1,50 | 1,00 | 0,50 | 2,00 | 2,00 | 3,30 | 6,00 | 1,00 | 9,00 | 2,40 |
| 1,50 | 1,00 | 0,50 | 2,00 | 2,00 | 3,30 | 6,00 | 1,00 | 9,00 | 2,40 |
| 3,00 | 1,50 | 1,00 | 4,00 | 6,60 | 18,00| 1,00 | 9,00 | 2,40 | 2,00 |
| 0,75 | 0,50 | 0,25 | 1,00 | 1,65 | 30,00| 5,00 | 45,00| 12,00| 10,00|
| 0,50 | 0,33 | 0,17 | 0,66 | 1,09 | 3,00 | 0,50 | 4,50 | 1,20 | 1,00 |
| 1,35 | 0,90 | 0,45 | 1,80 | 2,97 | 6,00 | 1,00 | 9,00 | 2,40 | 2,00 |
| 3,90 | 2,60 | 1,30 | 5,20 | 8,58 | 60,00| 10,00| 90,00| 24,00| 20,00|
| 7,65 | 5,10 | 2,55 | 10,20| 16,83| 36,00| 6,00 | 54,00| 14,40| 12,00|
| 0,15 | 0,10 | 0,05 | 0,20 | 0,33 | 5,40 | 0,90 | 8,10 | 2,16 | 1,80 |

Analysis of table 2 shows that the $K$ coefficient has the same value for arbitrary sections obtained as a result of arbitrary movement of individual components of the section along the normal to the direction of the section shift. This conclusion can also be drawn directly from the analysis of expression (6), since it integrates in one direction. As an illustration of the obtained conclusions, figure 3 shows a number of cross sections that have the same value of the $K$ coefficient when shifting in a given direction.
4. Conclusion
A correct analytical assessment of the seismic resistance of masonry buildings should be based on both the parameters of elastic-plastic deformation of the structural material and the spatial operation of the load-bearing system. Joint spatial operation of the cross-wall system is possible if sufficient load-bearing capacity of the vertical joint of the intersecting walls is provided. The value of the bearing capacity of a vertical wall joint is determined by the values of the shear forces formed in the joint under the action of horizontal loads, including seismic ones.

The result of the research developed a scientifically based and valid analytical method for determining the shape factor of the cross-section \( (K) \), that allows to accurately determine the rigidity of an arbitrary shape, and hence, shearing force in the vertical joints of this section. The method is of practical value in the practice of forecasting and evaluating the seismic resistance of stone buildings.

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