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Vortex-mediated relaxation of magnon BEC into light Higgs quasiparticles

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A magnon Bose-Einstein condensate (BEC) in superfluid $^3$He is a fine instrument for studying the surrounding macroscopic quantum system. At zero temperature, the BEC is subject to a few distinct forms of decay into other collective excitations, owing to momentum and energy conservation in a quantum vacuum. We study the vortex-Higgs mechanism: The vortices relax the requirement for momentum conservation, allowing the optical magnons of the BEC to transform into light Higgs quasiparticles. This facilitates a direct measurement of the dimensions of the $B$-phase double-core vortex, providing experimental access to elusive phenomena, such as the Kelvin wave cascade and core-bound Majorana fermions. Our paper expands the spectrum of possible interactions between magnetic quasiparticles in $^3$He-$B$ and lays the groundwork for building magnon-based quantum devices.

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One illuminating perspective to the ground state of a fermionic condensate, such as zero-temperature superfluid $^3$He, is to treat it as a quantum vacuum where moving objects interact with the excitations of the vacuum [1–5]. Various collective excitations, for example, magnetic quasiparticles (magnons), and topological defects, such as quantized vortices can be manipulated in this extremely pure environment. A Bose-Einstein condensate of optical magnons (magnon BEC), trapped within the superfluid, can be instrumented to probe objects in the system without influencing them [6–9]. This capacity has inspired suggestions to use the BEC to detect surface- or vortex-core-bound Majorana fermions [10,11] or the Kelvin wave cascade [12,13]. Both have so far remained elusive despite decades of active research. Changes in the BEC ground-state frequency as well as the population decay rate of the BEC can be devised for such purposes, provided the basic interactions between the excitations of the quantum vacuum are first thoroughly mastered. On the other hand, macroscopic quantum systems, such as BEC-based time crystals [14–16] provide a promising building block for quantum technologies, which rely on controlled nondestructive manipulation of the system. Such control can be accessed in the superfluid vacuum by coupling the BEC to and decoupling it from available excitations selectively.

In superfluid $^3$He, the spin and orbital angular momenta of Cooper pairs are equal to one. In the $B$ phase, the relative spin-orbit symmetry is broken in addition to the emergence of a coherent phase, as described by a $3 \times 3$ complex order-parameter matrix [17,18]. The macroscopic spin and orbital momentum directions are connected by the spin-orbit rotation angle $\theta$ around axis $\hat{n}$. In equilibrium, $\theta$ is equal to the Leggett angle $\theta_L \approx 104^\circ$. The fermionic thermal excitations of this system have energy gap $\Delta_B$ which at low temperatures is on the order of $k_B T_c$, where $k_B$ is the Boltzmann constant and $T_c$ is the superfluid transition temperature. At temperatures much below $T_c$, the number of thermal excitations is reduced exponentially, creating a vacuum void of fermionic quasiparticles.

Besides the fermionic quasiparticles, there are three collective spin-wave modes with a small (or zero) gap, corresponding to the combined oscillations of three spin components and three components of spin-orbit rotation [18]. Following Ref. [19], we call these modes optical magnons, acoustic magnons, and light Higgs quasiparticles. A detailed derivation of the modes can be found, e.g., in Ref. [20]. In the absence of a magnetic field, optical and acoustic magnons are gapless, corresponding to the oscillations of $\hat{n}$. That is, their frequency vanishes in the long-wavelength limit. In a magnetic field $H$, optical magnons acquire a gap equal to the Larmor frequency $2\pi f_L = \omega_L = |\gamma| H$, where $\gamma$ is the gyromagnetic ratio. The light Higgs mode corresponds to oscillation of $\theta$ around $\theta_L$ and has a gap $\Delta_B/2\pi \sim 100$ kHz ($\Omega_B$ is the Leggett frequency). The dispersion relations of these three modes are illustrated in Fig. 1(a). Direct interactions between these modes are studied in Ref. [19].

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FIG. 1. (a) Spectra of spin waves in $^3$He-B. The mass (gap) $\omega_\delta$ of optical magnons (blue line) can be tuned using the magnetic field. Direct conversion of optical magnons into light Higgs quasiparticles (solid arrow), studied in this Letter, requires balancing the momentum mismatch. The parametric conversion of optical magnons into gapless acoustic magnons (red line), and light Higgs quasiparticles (green line) can be observed when the density and mass of the optical magnons are large. Processes indicated by dashed arrows were reported in Ref. [19]. (b) Superfluid $^3$He in a cylindrical container. A BEC of optical magnons (blue blob) is trapped in the middle by the spatial distribution of the orbital order parameter ($\hat{n}$ vector, small green arrows) and by an axial minimum in the external magnetic field $H$. The coherently precessing magnetization $M$ (large magenta arrow) in the BEC is parametrized with the tipping angle $\beta_M$. In constant rotation $\Omega$ around the vertical axis, an array of vortices is created, penetrating the BEC (red vertical rods at the top). For illustrational reasons the vortex rods have been made transparent in the vicinity of the BEC and drawn only in the upper half of the container, and the order parameter distribution is only shown in the middle section. The vortex configuration obtained in modulated rotation is sketched with the magenta rods at the bottom of the container, based on Ref. [21].

The superfluid vacuum can also host topological defects [1], in particular, quantized vortices. An ordered array of vortices can be created by rotating the sample at a constant angular velocity $\Omega$. The density of the vortex array is proportional to $\Omega$. The $B$-phase vortices have a broken-symmetry core [22,23] where the low-temperature vortex studied in this Letter has a double-core structure consisting of two tightly bound half cores [24–30]. A theoretical description of the interactions of vortices and collective excitations in superfluid $^3$He can be confidently expanded by changing on the BCS theory with Fermi-liquid effects included [31].

In this Letter we study the interaction of a condensate of optical magnons with quantized vortices and light Higgs quasiparticles, which we call the vortex-Higgs mechanism: If a vortex penetrates the magnon BEC, optical magnons are scattered by the order-parameter distortion that surrounds the vortex. This interaction lifts the requirement for momentum conservation for inbound and outbound quasiparticles. We show that in such collisions the optical magnons in the condensate are converted directly into light Higgs quasiparticles. This is seen as zero-temperature relaxation of the BEC. We study this conversion in two qualitatively different vortex configurations, ordered and disordered, and find that the results are in good agreement with theory.

The magnon BEC in superfluid $^3$He consists of coherent optical magnons [32]. Their magnetization $M$ precesses around the external magnetic field $H$ and is described by a macroscopic wave function $\Psi$. The total number of magnons $N \propto \int |\Psi|^2 dV \propto \int \beta_M dV$, where $\beta_M$ is the deflection angle of $M$ from the equilibrium direction along $H$, and $V$ is the volume. Here we assumed that $\beta_M$ is small, which is satisfied in all the experiments presented in this Letter. The coherently precessing magnetization is generated and detected using nuclear magnetic resonance techniques: Transverse coils placed in the vicinity of the sample container cylinder allow driving the condensate using a radio-frequency field and detecting the condensate by measuring the induced voltage (Fig. S1a in the Supplemental Material [33]).

The magnon BEC is trapped in the middle of the superfluid sample [Fig. 1(b)] by the radial distribution of the orbital order parameter (“texture”), and an axial minimum of the external magnetic field. The resulting trap is nearly harmonic [34], characterized by the radial and axial trapping frequencies $f_r$ and $f_z$, determined from measurements of the full spectrum of states in the trap [20]. We concentrate on the ground-state magnon BEC, whose precession frequency is $f = f_r + f_z + f_z/2$. Temperature is measured using a mechanical oscillator, a quartz tuning fork. Its resonance width follows $\Delta \nu \propto \exp(-\Delta_H/k_B T)$ at temperatures $T \ll T_c \sim 1 \text{ mK}$, probing the density of thermal quasiparticles [35]. Further details of the experimental setup can be found in the Supplemental Material [33].

At a finite temperature, the relaxation of a magnon BEC is primarily caused by spin transfer via thermal fermionic quasiparticles [34,36] via the Leggett-Rice spin-rotation effect [37]. This process is usually called nonhydrodynamical spin diffusion. It results in exponential decay of the condensate $\rho_M \propto \exp(-t/\tau_{DP})$. Here $1/\tau_{DP}$ is the spin-diffusion relaxation rate. The rate is proportional to the thermal quasiparticle density and, thus, to the thermometer fork resonance width and the BEC relaxation rate $1/\tau_{DP} \propto \Delta \nu$ [red points in Fig. 2(a)] [34]. In practice there are also unavoidable losses in the measurement circuitry, but this effect can be confidently subtracted [34].

In the zero-temperature limit intrinsic decay channels are absent, and the condensate lifetime approaches infinity [7,14,16,38]. Any (extrapolated) zero-temperature dissipation in the bulk liquid [38] is, therefore, an indication of interaction with other collective modes either via parametric excitation or via direct conversion. The former is allowed assuming the density of optical magnons is high enough, and their mass is large enough [19]. Direct conversion is ruled out due to momentum conservation unless mediated by boundaries or interaction with topological defects of the superfluid vacuum, such as quantum vortices.

The interaction with a nonsingular topological defect arises due to the distortion of the order parameter distribution in the vicinity of the defect: The order parameter is not suppressed to zero in the core but changes to another superfluid phase.
The distortion. Around the double-core vortex

\[ \theta \approx L/1 \) at distances comparable to

\( \Delta \nu \) owing to spin diffusion (red line). Intrinsic

\( 0 \) measured at \( \Omega = 0 \) show an increased slope, reflecting the changing

\( \Delta \nu \) in panel (a) (points) are proportional to the radial trapping frequency

\( f_1 \) (dashed line is a linear fit through zero) as expected for spin diffusion,

\( \Omega \) that is, to the vortex density (dashed lines). (c) The temperature-independent

\( \Delta \nu = 0 \) (colored dots) is proportional to \( \Omega \), is in good agreement with that obtained

\( \tau \) increases). This affects the spin-diffusion relaxation, which

\( R_0 \) [Fig. 2(a)] and that the

\( \Lambda \nu \) and light Higgs quasiparticles are viable candidates to explain

\( 1 \) kHz spacing in

\( f_1 \) [Fig. 2(b)]. This observation implies that the temperature dependence of the relaxation rate

\( 1/\tau \) are proportional to \( \Delta \nu \) at any given \( \Omega \) [Fig. 2(a)] and that the slope

\( 1/\Delta \nu \) is proportional to the measured radial trapping frequency

\( f_1 \) [Fig. 2(b)]. This observation implies that the temperature dependence of the relaxation rate

\( 1/\tau \), contained in \( D \), is not affected by rotation, and any relaxation directly related to the vortices is temperature independent below

\( T = 0.17T_c \). We emphasize that all the relaxation signals measured were exponential in time, implying that no non-exponential contribution was added by the vortex array. The second observation is that the zero-temperature (\( \Delta \nu = 0 \)) extrapolation of the relaxation is proportional to \( \Omega \) [Fig. 2(c)]. That is, the observed temperature-independent relaxation is as follows: (i) exponential and (ii) proportional to the density of vortices (see Fig. S1b in the Supplemental Material [33]). This is in good agreement with the theoretical expectation for vortex-Higgs mechanism of BEC relaxation, Eq. (S17) of the Supplemental Material [33].

We note that peaks in the measured relaxation, associated with the presence of vortices, were observed with roughly 1-kHz spacing in \( f_1 \) on top of the vortex-Higgs dissipation described above. We account this phenomenon for resonant production of standing spin-wave modes in the sample container, mediated by the vortex array. In Ref. [19] such peaks are accounted to vortex-mediated production of acoustic magnons [see Fig. 1(a)]. We believe both acoustic magnons and light Higgs quasiparticles are viable candidates to explain this observation, but a detailed study is left for a future publication. For simplicity, in what follows we call the peaks relaxation peaks. The peak frequencies were avoided in all measurements conducted at stable rotation.

As an alternative to constant rotation, the angular velocity can be modulated. We used linear modulation in the range from 1.4 to 1.8 rad s\(^{-1}\) with |d\( \Omega \)/dt| = 0.03 rad s\(^{-1}\). In the steady state the vortex number is expected to remain constant but the vortex array is distorted. We find experimentally that this removes the resonance peaks found at constant rotation. The measured zero-temperature relaxation with and without modulation, the latter avoiding the relaxation peaks are shown in Fig. 2(d). The two vortex configurations yield the same BEC relaxation rate. This observation allows us to probe the vortex-Higgs mechanism at arbitrary magnetic fields, avoiding the relaxation peaks altogether.

We can further characterize the vortex-Higgs mechanism by varying the coupling among magnons and vortices and, thus, parameter \( C \). The core separation can be varied by changing pressure. The relaxation caused by the vortex-Higgs mechanism also depends on the magnitude on the magnetic field as derived in the Supplemental Material [33].

We compare the experiment with the theoretical model in Fig. 3. We find very good agreement in the magnetic field as well as the pressure dependence with \( C = 7.4R_0 \). Here \( R_0 \)
is a characteristic length scale on the order of the coherence length in the superfluid (see Supplemental Material [33] for details). This value is close to the theoretical value of $C/R_0 = 5.9 - 6.6$ [27,42], and, therefore, the agreement between experiment and theory is highly satisfactory without any fitting parameters. This means that the fitted $C$ agrees well with the theoretically predicted core separation. The millimeter-sized magnon BEC, therefore, correctly measured the effective vortex half-cores’ separation which is on the order of 1 μm, confirming the assumption that the BEC interacts with each individual vortex independently and providing a strong argument in support of the vortex-Higgs interpretation.

The observations presented above imply that the vortex-Higgs mechanism opens an otherwise-unavailable relaxation channel for the magnon BEC, corresponding to zero-temperature conversion of optical magnons of the BEC into light Higgs quasiparticles. This connection is mediated by the light Higgs quasiparticles. Theory lines correspond to Eq. (S17) in the Supplemental Material [33], fitted to the data using parameter $C$. All measurements were carried out at $T = 0.15T_c$. Spin-diffusion dissipation and radiation losses have been subtracted based on measured trapping frequencies $f_i$ and $f_j$ [36]. This correction is about 5% for the 4-bar data and less than 1% for the 29-bar data. The inset shows $C$ vs pressure, in good agreement with theoretical expectation $C = CR_0$ with fitted $C = 7.4$ (dashed line). The colored circles correspond to fits in the main panel, and the triangles correspond to the straight-vortex data in Figs. 2(c) and 2(d). Error bars correspond to uncertainty in removing the resonant relaxation peaks.

FIG. 3. Vortex-Higgs mechanism as a function of magnetic field and pressure: Measured BEC-relaxation field dependence at different pressures (colored dots) is in good agreement with the theoretical expectation (dashed lines) for vortex-mediated conversion of BEC magnons into light Higgs quasiparticles. Theory lines correspond to Eq. (S17) in the Supplemental Material [33], fitted to the data using parameter $C$. All measurements were carried out at $T = 0.15T_c$. Spin-diffusion dissipation and radiation losses have been subtracted based on measured trapping frequencies $f_i$ and $f_j$ [36]. This correction is about 5% for the 4-bar data and less than 1% for the 29-bar data. The inset shows $C$ vs pressure, in good agreement with theoretical expectation $C = CR_0$ with fitted $C = 7.4$ (dashed line). The colored circles correspond to fits in the main panel, and the triangles correspond to the straight-vortex data in Figs. 2(c) and 2(d). Error bars correspond to uncertainty in removing the resonant relaxation peaks.

The data that support the findings in this Letter are available from the corresponding author upon reasonable request.

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[1] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford University Press, Oxford, 2003).
[2] D. I. Bradley, S. N. Fisher, A. M. Guénault, R. P. Haley, C. R. Lawson, G. R. Pickett, R. Schanen, M. Skyba, V. Tsepelin, and D. E. Zmeev, Breaking the superfluid speed limit in a fermionic condensate, *Nat. Phys.* **12**, 1017 (2016).
[3] J. A. Kuorelahti, S. M. Laine, and E. V. Thuneberg, Models for supercritical motion in a superfluid fermi liquid, *Phys. Rev. B* **98**, 144512 (2018).
[4] S. Autti, V. Tsepelin, J. Vonka, T. Wilcox, A. J. Woods, and D. E. Zmeev, dissipation due to bound fermions in the zero-temperature limit, *Nat. Commun.* **11**, 4742 (2020).
[5] Y. M. Bunkov, S. N. Fisher, A. M. Guénault, and G. R. Pickett, Persistent Spin Precession in $^3$He-B in the Regime of Vanishing Quasiparticle Density, *Phys. Rev. Lett.* **69**, 3092 (1992).
[6] S. N. Fisher, A. M. Guénault, A. J. Hale, G. R. Pickett, P. A. Reeves, and G. Tvalashvili, Thirty-minute coherence in free induction decay signals in superfluid $^3$He-B, *J. Low Temp. Phys.* **121**, 303 (2000).
[7] S. Autti, Y. V. Dmitriev, J. T. Mäkinen, J. Rysti, A. A. Soldatov, G. E. Volovik, A. N. Yudin, and V. B. Eltsov, Bose-Einstein Condensation of Magnons and Spin Superfluidity in the Polar Phase of $^3$He, *Phys. Rev. Lett.* **121**, 025303 (2018).
[8] S. Murakawa, Y. Wada, Y. Tamura, M. Wasai, M. Saito, Y. Aoki, R. Nomura, Y. Okuda, Y. Nagato, M. Yamamoto, S. Higashitani, and K. Nagai, Surface Majorana cone of the superfluid $^3$He $^1$B phase, *J. Phys. Soc. Jpn.* **80**, 013602 (2011).
[9] B. Rosenstein, I. Shapiro, and B. Y. Shapiro, Effect of nanoholes on the vortex core fermion spectrum and heat transport in p-wave superconductors, *J. Phys.: Condens. Matter* **25**, 075701 (2013).
[10] V. B. Eltsov and V. S. L’vov, Amplitude of waves in the Kelvin-wave cascade, *JETP Lett.* **111**, 389 (2020).
[11] V. S. L’vov and S. Nazarenko, Spectrum of Kelvin-wave turbulence in superfluids, *JETP Lett.* **91**, 428 (2010).
[12] S. Autti, V. B. Eltsov, and G. E. Volovik, Observation of a Time Quasicrystal and Its Transition to a Superfluid Time Crystal, *Phys. Rev. Lett.* **120**, 215301 (2018).
[13] A. J. E. Kreil, H. Y. Musiienko-Shmarova, S. Eggert, A. A. Serga, B. Hillebrands, D. A. Bozhko, A. Pomyalov, and V. S. L’vov, Tunable space-time crystal in room-temperature magnetodielectrics, *Phys. Rev. B* **100**, 020406(R) (2019).
[14] S. Autti, P. J. Heikkinen, J. T. Mäkinen, G. E. Volovik, V. V. Zavjalov, and V. B. Eltsov, AC Josephson effect between two superfluid time crystals, *Nature Mater.* **20**, 171 (2021).
[15] A. J. Leggett, A theoretical description of the new phases of liquid He 3, *Rev. Mod. Phys.* **47**, 331 (1975).
[16] D. Vollhardt and P. Wöllle, *The Superfluid Phases of Helium 3* (Dover, New York, 2013).
[39] M. Arrayás, R. P. Haley, G. R. Pickett, and D. Zmeev, Orbital effect in superfluid $^3$He B-phase boundaries, Sci. Rep. 8, 13965 (2018).
[40] S. M. Laine and E. V. Thuneberg, Spin-wave radiation from vortices in $^3$He-B, Phys. Rev. B 98, 174516 (2018).
[41] V. B. Eltsov, R. De Graaf, M. Krusius, and D. E. Zmeev, Vortex core contribution to textural energy in $^3$He--B below 0.4T_c, J. Low Temp. Phys. 162, 212 (2011).
[42] M. A. Silaev, E. V. Thuneberg, and M. Fogelström (unpublished).
[43] M. A. Silaev, Universal Mechanism of Dissipation in Fermi Superfluids at Ultralow Temperatures, Phys. Rev. Lett. 108, 045303 (2012).
[44] N. B. Kopnin and M. M. Salomaa, Mutual friction in superfluid $^3$He: Effects of bound states in the vortex core, Phys. Rev. B 44, 9667 (1991).
[45] N. B. Kopnin and G. E. Volovik, Rotating vortex core: An instrument for detecting core excitations, Phys. Rev. B 57, 8526 (1998).
[46] E. Kozik and B. Svistunov, Kelvin-Wave Cascade and Decay of Superfluid Turbulence, Phys. Rev. Lett. 92, 035301 (2004).
[47] D. Kivotides, J. C. Vassilicos, D. C. Samuels, and C. F. Barenghi, Kelvin Waves Cascade in Superfluid Turbulence, Phys. Rev. Lett. 86, 3080 (2001).
[48] S. B. Chung and S.-C. Zhang, Detecting the Majorana Fermion Surface State of $^3$He-B Through Spin Relaxation, Phys. Rev. Lett. 103, 235301 (2009).
[49] A. J. E. Kreil, A. Pomyalov, V. S. L’vov, H. Y. Musienko-Shmarova, G. A. Melkov, A. A. Serga, and B. Hillebrands, Josephson oscillations in a room-temperature Bose-Einstein magnon condensate, arXiv:1911.07802.
[50] D. A. Bozhko, A. J. E. Kreil, H. Y. Musienko-Shmarova, A. A. Serga, A. Pomyalov, V. S. L’vov, and B. Hillebrands, Bogoliubov waves and distant transport of magnon condensate at room temperature, Nat. Commun. 10, 2460 (2019).
[51] A. J. E. Kreil, D. A. Bozhko, H. Y. Musienko-Shmarova, V. I. Vasyuchka, V. S. L’vov, A. Pomyalov, B. Hillebrands, and A. A. Serga, From Kinetic Instability to Bose-Einstein Condensation and Magnon Supercurrents, Phys. Rev. Lett. 121, 077203 (2018).
[52] D. A. Bozhko, A. A. Serga, P. Clausen, V. I. Vasyuchka, F. Heusser, G. A. Melkov, A. Pomyalov, V. S. L’vov, and B. Hillebrands, Supercurrent in a room-temperature Bose-Einstein magnon condensate, Nat. Phys. 12, 1057 (2016).
[53] P. J. Heikkinen, Magnon Bose-Einstein condensate as a probe of topological superfluid, Ph.D. thesis, Aalto University School of Science, 2016, https://aaltodoc.aalto.fi/handle/123456789/20580.
[54] M. Silveri, T. Turunen, and E. Thuneberg, Hard domain walls in superfluid $^3$He-B, Phys. Rev. B 90, 184513 (2014).
[55] E. V. Thuneberg, Hydrostatic theory of superfluid $^3$He-B, J. Low Temp. Phys. 122, 657 (2001).