Degenerate Neutrinos and Supersymmetric Inflation

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Abstract

A moderate extension of the minimal supersymmetric standard model which includes a $U(1)_{B-L}$ gauge group ($B$ and $L$ being the baryon and lepton number) and a Peccei-Quinn symmetry, $U(1)_{PQ}$, is presented. The hybrid inflationary scenario is automatic and ‘natural’ in this model. The $\mu$ problem of the minimal supersymmetric standard model is solved by coupling the electroweak higgses to fields which break $U(1)_{PQ}$. Baryon number conservation and, thus, proton stability are automatic consequences of a R-symmetry. Neutrinos are assumed to acquire degenerate masses $\approx 1.5$ eV by coupling to $SU(2)_L$ triplet superfields, thereby providing the hot dark matter of the universe. The inflaton system decays into these triplets which, via their subsequent decay, produce a primordial lepton asymmetry later converted into the observed baryon asymmetry of the universe. The gravitino and baryogenesis constraints can be satisfied with ‘natural’ values ($\sim 10^{-3}$) of the relevant coupling constants.

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It is well-known that the minimal supersymmetric standard model (MSSM), despite its compelling properties, leaves a number of fundamental questions unanswered. This clearly indicates that MSSM must be part of a larger scheme. One shortcoming of MSSM is that inflation cannot be implemented in its context. Also, there is no understanding of how the supersymmetric $\mu$ term, with $\mu \sim 10^2 - 10^3$ GeV, arises. It has become increasingly clear that a combination of both cold and hot dark matter provides a good fit to the data on large scale structure formation in the universe. Although the lightest supersymmetric particle of MSSM is a promising candidate for cold dark matter, hot dark matter cannot be accommodated with purely MSSM fields. Finally, the observed baryon asymmetry of the universe cannot be generated easily in MSSM through the electroweak sphaleron processes.

A moderate extension of MSSM based on the gauge group $G_S \times U(1)_{B-L}$ ($G_S$ being the standard model gauge group, and $B$ and $L$ the baryon and lepton number) provides a suitable supersymmetric framework for inflation. Indeed, the hybrid inflationary scenario is ‘naturally’ realized in this context. Inflation is associated with the breaking of $U(1)_{B-L}$ at a superheavy scale. The $\mu$ problem of MSSM could be solved by coupling the inflaton to the electroweak higgses. However, in this case, the inflaton predominantly decays into higgs superfields, after the end of inflation, and the gravitino constraint on the reheat temperature restricts the relevant dimensionless coupling constants to be ‘unnaturally’ small ($\sim 10^{-5}$). We will, thus, choose here an alternative solution to the $\mu$ problem. This relies on the coupling of the electroweak higgses to superfields causing the breaking of a Peccei-Quinn symmetry rather than to the inflaton.

The hot dark matter of the universe, needed for explaining its large scale structure, may be provided by light neutrinos. This possibility can be made compatible with the atmospheric and solar neutrino oscillations, within a three neutrino scheme, only by assuming almost degenerate neutrino masses. These masses can be generated by including $SU(2)_L$ triplet pairs of superfields with intermediate scale masses. The inflaton decays into these triplet superfields (rather than into higgses) and much bigger dimensionless coupling constants are allowed. The subsequent decay of the triplet superfields produces a primordial lepton asymmetry which is later converted into the observed baryon asymmetry of the universe by electroweak sphaleron effects.
Let us now describe, in some detail, the moderate extension of MSSM based on the
gauge group $G = G_S \times U(1)_{B-L}$. The spontaneous breaking of $U(1)_{B-L}$ at a mass scale
$M \sim 10^{16}$ GeV is achieved through the renormalizable superpotential

$$W = \kappa S (\phi \bar{\phi} - M^2),$$

(1)

where $\phi$, $\bar{\phi}$ is a conjugate pair of standard model singlet left handed superfields with
$B-L$ charges equal to 1, -1 respectively, and $S$ is a gauge singlet left handed superfield.
The coupling constant $\kappa$ and the mass parameter $M$ can be made real and positive by
suitable redefinitions of the phases of the superfields. The supersymmetric minima of the
scalar potential lie on the D-flat direction $\phi = \bar{\phi}^*$ at $\langle S \rangle = 0$, $|\langle \phi \rangle| = |\langle \bar{\phi} \rangle| = M$.

It has been well documented [2,3,5,7,15] that hybrid inflation [4] is automatically
and ‘naturally’ realized in this supersymmetric scheme. The scalar potential possesses
a built-in inflationary trajectory at $\phi = \bar{\phi} = 0$, $|S| > M$ with a constant tree level
potential energy density $\kappa^2 M^4$ which is responsible for the exponential expansion of the
universe. Moreover, due to supersymmetry breaking by this constant energy density,
there are important radiative corrections [3] which provide a slope along the inflationary
trajectory necessary for driving the inflaton towards the supersymmetric vacua. At one
loop, the cosmic microwave quadrupole anisotropy is given by [3][5]

$$\left( \frac{\delta T}{T} \right)_Q \approx 8\pi \left( \frac{N_Q}{45} \right)^{1/2} \frac{x_Q}{y_Q} \left( \frac{M}{M_P} \right)^2.$$  

(2)

Here $M_P = 1.22 \times 10^{19}$ GeV is the Planck scale and $N_Q \approx 50 - 60$ denotes the number
of e-foldings experienced by the universe between the time the quadrupole scale exited
the horizon and the end of inflation. Also,

$$y_Q^2 = \int_1^{x_Q} \frac{dz}{z((z-1) \ln(1-z^{-1}) + (z+1) \ln(1+z^{-1}))}$$

$$= x_Q^2 \left( 1 - \frac{7}{6x_Q^2} + \cdots \right), \quad y_Q \geq 0,$$

(3)

with $x_Q = |S_Q|/M$ ($x_Q \geq 1$), $S_Q$ being the value of the scalar field $S$ when the scale which
evolved to the present horizon size crossed outside the de Sitter (inflationary) horizon.
Note that Eq.(2) holds, to a good approximation, provided \( x_Q \) is not ‘unnaturally’ close to 1. The superpotential parameter \( \kappa \) can be evaluated \([3,15]\) from
\[
\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N_Q}} y_Q \frac{M}{M_P}.
\]

One interesting possibility for generating the \( \mu \) term of MSSM has been proposed in Ref. [5]. It relies on the extension of the above scheme by adding to it the superpotential coupling \( \lambda S H^{(1)} H^{(2)} \) (\( \lambda > \kappa \)), where \( H^{(1)}, H^{(2)} \) are the chiral higgs superfields which couple to the up and down type quarks respectively (and carry zero \( B - L \) charge). It has been shown [5] that, after gravity-mediated supersymmetry breaking, \( S \) develops a vacuum expectation value (vev) \( \langle S \rangle \approx -m_{3/2}/\kappa \), where \( m_{3/2} \approx (0.1 - 1) \) TeV is the gravitino mass. This generates a \( \mu \) term with \( \mu = \lambda \langle S \rangle \approx - (\lambda/\kappa) m_{3/2} \). The cosmic microwave quadrupole anisotropy, evaluated (see Ref. [7]) in the limit \( x_Q \to 1 \) which is the relevant one here, is given by
\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{32\pi^{5/2}}{3\sqrt{3}} \left( \frac{M}{M_P} \right)^3 \frac{1}{\kappa (\epsilon^2 \kappa^{-2} + \ln 2)},
\]
where \( \epsilon \approx 2 \ln 2 \) for \( \lambda = \kappa \), and \( \epsilon \approx 1 \) for \( \lambda \gg \kappa \). (Notice that here we had to replace the contribution of the conjugate pair of \( SU(2)_R \) doublet superfields of Ref. [7] by the contribution of the standard model singlets \( \phi, \bar{\phi} \), which is smaller by a factor 2.) After the end of inflation, the inflaton (oscillating system), which consists of the two complex scalar fields \( S \) and \( \theta = (\delta \phi + \delta \bar{\phi})/\sqrt{2} \) (\( \delta \phi = \phi - M, \delta \bar{\phi} = \bar{\phi} - M \)) with mass \( m_{\text{infl}} = \sqrt{2} \kappa M \), predominantly decays, in this case, into ordinary higgsinos and higgses with a decay width \( \Gamma = (1/8\pi) \lambda^2 m_{\text{infl}} \). This can be easily deduced from the coupling \( \lambda S H^{(1)} H^{(2)} \) and the superpotential in Eq.(4). The reheat temperature is given [15] by
\[
T_r \approx \frac{1}{\Gamma M_P} \left( \Gamma M_P \right)^{1/2},
\]
for MSSM spectrum. Using Eqs.(2) and (3) with \( (\delta T/T)_Q \approx 6.6 \times 10^{-6} \) from the Cosmic Background Explorer (COBE) [16], the gravitino constraint [3] \( (T_r < 10^9 \text{ GeV}) \) becomes
\[
\lambda \kappa^{2/3} \left( \frac{\lambda^2}{\kappa^2} + \ln 2 \right)^{1/6} \lesssim 3.7 \times 10^{-8}.
\]
which, for $\lambda = \kappa$ say, gives $\kappa \lesssim 3.2 \times 10^{-5}$. In the specific model of Ref. [7], $\lambda \approx 3.95\kappa$ and the bound is even stronger, i.e., $\kappa \lesssim 1.2 \times 10^{-5}$. Moreover, in this model which employs hierarchical neutrino masses from the seesaw mechanism, the requirement of maximal $\nu_\mu$-$\nu_\tau$ mixing, deduced from the recent results of the SuperKamiokande experiment [11], further reduces the coupling constant $\kappa$ to become of order $10^{-6}$. We conclude that, within the context of the supersymmetric hybrid inflationary model, the solution of the $\mu$ problem of MSSM via the coupling of the higgs superfields to the inflaton system is not totally satisfying. The reason is that this solution together with the gravitino constraint require the presence of ‘unnaturally’ small coupling constants.

We will follow here an alternative solution [8] of the $\mu$ problem of MSSM constructed by coupling the electroweak higgses to superfields causing the breaking of a Peccei-Quinn [10] symmetry ($U(1)_{\text{PQ}}$) which solves the strong CP problem. D- and F-flat directions in field space, appearing in the supergravity extension of MSSM, can generate an intermediate scale $M_I$ which, in the simplest case, is given by

$$M_I \sim (m_3/2m_P)^{1/2} \sim 10^{11} \text{ GeV},$$

where $m_P = M_P/\sqrt{8\pi} \approx 2.44 \times 10^{18} \text{ GeV}$ is the ‘reduced’ Planck mass. This scale can be identified with the symmetry breaking scale $f_a$ of $U(1)_{\text{PQ}}$. A $\mu$ term with $\mu \sim m_3/2 \sim f_a^2/m_P$ can then be easily generated [8] via an appropriate non-renormalizable coupling of the electroweak higgses to a field which breaks $U(1)_{\text{PQ}}$.

We will assume almost degenerate (rather than hierarchical) light neutrino masses. Under this assumption, neutrinos can provide the hot dark matter of the universe which is needed for explaining [1] its large scale structure. In the hierarchical case, atmospheric and solar neutrino oscillations imply that, within a three neutrino scheme, the neutrinos have too small masses to be of any cosmological significance. Non-zero neutrino masses can be generated by introducing into the scheme standard model singlet right handed neutrinos and/or by including [1,2] $SU(2)_L$ triplet pairs of superfields. The former possibility, which is based on the well-known seesaw mechanism, cannot naturally lead to degenerate neutrino masses. We, thus, adopt here the latter option. The $SU(2)_L$ triplets acquire intermediate masses via non-renormalizable superpotential couplings to the inflaton. These same couplings, which are automatically suppressed by a factor $M/m_P$, cause the decay of the inflaton predominantly into these triplet superfields (rather than into higgses). This allows much bigger dimensionless coupling constants, thereby solving
the ‘naturality’ problem of the previous scheme. The subsequent decay of the triplet superfields produces a primordial lepton asymmetry which is later converted into the observed baryon asymmetry of the universe by electroweak sphaleron effects. For this to work, we need at least two pairs of triplets.

We now proceed to the description of the full model. We supplement the spectrum of the moderate extension of MSSM in Eq. (1), which incorporates hybrid inflation, with a pair of gauge singlet left handed superfields $N$, $\bar{N}$. Their vevs will break $U(1)_{PQ}$ at an intermediate scale. We also add two pairs of $SU(2)_L$ triplets $T_a$, $\bar{T}_a$ ($a=1,2$) with hyper-charges 1, -1 and $B-L$ charges 2, 0 respectively. They will be responsible for neutrino masses and the generation of the primordial lepton asymmetry. The superpotential $W$ contains, in addition to the terms in Eq.(1), the following couplings:

$$H^{(1)}QU^c, H^{(2)}QD^c, H^{(2)}LE^c, N^2H^{(1)}H^{(2)}, N^2\bar{N}^2, TLL, \bar{T}H^{(1)}H^{(1)}, \bar{\phi}T\bar{T}. \quad (8)$$

Here $Q_i$ denote the $SU(2)_L$ doublet quark superfields, $U^c_i$ and $D^c_i$ are the $SU(2)_L$ singlet quark superfields, while $L_i$ ($E^c_i$) stand for the $SU(2)_L$ doublet (singlet) lepton superfields ($i=1,2,3$ is the family index). The $B-L$ charges of these fields are defined in the obvious way. The quartic terms in Eq.(8) carry a factor $m_P^{-1}$ which has been left out. Also, the dimensionless coupling constants as well as the family and triplet indices are suppressed.

The continuous global symmetries of this superpotential are $U(1)_B$ (and, consequently, $U(1)_L$) with the extra chiral superfields $S$, $\phi$, $\bar{\phi}$, $N$, $\bar{N}$, $T$, $\bar{T}$ carrying zero baryon number, an anomalous Peccei-Quinn symmetry $U(1)_{PQ}$, and a non-anomalous R-symmetry $U(1)_R$. The PQ and R charges of the various superfields are as follows:

$$PQ : H^{(1)}(1), H^{(2)}(1), Q(-1), U^c(0), D^c(0), L(-1), E^c(0), S(0), \phi(0), \bar{\phi}(0), N(-1), \bar{N}(1), T(2), \bar{T}(-2) ,$$

$$R : H^{(1)}(0), H^{(2)}(0), Q(1/2), U^c(1/2), D^c(1/2), L(1/2), E^c(1/2), S(1), \phi(0), \bar{\phi}(0), N(1/2), \bar{N}(0), T(0), \bar{T}(1) ,$$

with $W$ carrying one unit of $R$ charge.

It is important to note that $U(1)_B$ (and, consequently, $U(1)_L$) is automatically implied by $U(1)_R$ even if all possible non-renormalizable terms are included in the superpotential.
Indeed, by extending the $U(1)_R$ symmetry to higher order terms, one can show that $U(1)_B$ follows as a consequence. To see this, observe that the $R$ charges of the only baryon number violating combinations of fields $3 \cdot 3 \cdot 3$ or $\bar{3} \cdot \bar{3} \cdot \bar{3}$ ($3, \bar{3}$ denote color triplet and antitriplet fields) exceed unity and cannot be compensated since there are no negative $R$ charges available in the model. In particular, the troublesome dimension five operators $QQQL$ and $U^cU^cD^cE^c$ are eliminated and proton is stable.

Lepton number is spontaneously broken by the vevs of $\phi$, $\bar{\phi}$ and, consequently, some lepton number violating effective operators will emerge at lower energies (below $M$). In particular, the last term in Eq. (8) will generate the desired intermediate scale masses for the $SU(2)_L$ triplet superfields. However, undesired mixing of the higgs $H^{(2)}$ with $L$’s will also emerge from the allowed superpotential couplings $N\bar{N}LH^{(1)}\phi$ after the breaking of $U(1)_{PQ}$ by the vevs of $N, \bar{N}$. To avoid this complication, we impose an extra discrete $Z_2$ symmetry, which we will call ‘lepton parity’. Under this symmetry, $L, E^c$ change sign, while all other superfields remain unaltered. In the present model, this symmetry is equivalent with ‘matter parity’ (under which $L, E^c, Q, U^c, D^c$ change sign), since ‘baryon parity’ (under which $Q, U^c, D^c$ change sign) is also present being a subgroup of $U(1)_B$. One can show that the only superpotential terms, which are permitted by the global symmetries $U(1)_R, U(1)_{PQ}$ and ‘lepton parity’, are the ones of Eqs. (1) and (8) modulo arbitrary multiplications by non-negative powers of the combination $\phi\bar{\phi}$.

The scalar potential which is generated by the superpotential term $N^2\bar{N}^2$ in Eq. (8) after gravity-mediated supersymmetry breaking has been studied in Ref. [9]. It has been shown that, for a suitable choice of parameters, a minimum at

$$|\langle N \rangle| = |\langle \bar{N} \rangle| \sim (m_{3/2}m_P)^{1/2}$$

(10)

is preferred over the trivial one at $\langle N \rangle = \langle \bar{N} \rangle = 0$. The vevs $\langle N \rangle, \langle \bar{N} \rangle$ together break $U(1)_{PQ} \times U(1)_R$ completely. Substitution of these vevs in the superpotential coupling $N^2H^{(1)}H^{(2)}$ in Eq. (8) then generates a $\mu$ parameter for MSSM of order $m_{3/2}$ as desired. Note that $U(1)_L$ is broken completely together with the gauge $U(1)_{B-L}$ by the superheavy vevs of $\phi, \bar{\phi}$. Thus, only $U(1)_B$ and ‘matter parity’ remain exact.

As already explained, after $B-L$ (and lepton number) breaking at the superheavy scale $M$, the last term in Eq. (8) generates intermediate scale masses for the $SU(2)_L$
triplet superfields $T_a, \bar{T}_a$ (a=1,2). The dimensionless coupling constant matrix of this term can be made diagonal with positive entries $\gamma_a$ (a=1,2) by a suitable rotation on the triplets. The triplet mass eigenvalues are then $M_a = \gamma_a M^2 / m_P$ (with $\langle \phi \rangle, \langle \bar{\phi} \rangle$ taken positive by an appropriate $B - L$ transformation). It is readily checked that the scalar components of $T_a$'s acquire non-zero vevs $\sim M^2_W / M$ (with $\langle \phi \rangle, \langle \bar{\phi} \rangle$ taken positive by an appropriate $B - L$ transformation). This is due to the fact that the last two terms in Eq.(8), after electroweak breaking, give rise to terms linear with respect to $T_a$'s in the scalar potential of the theory. The vev of $T_a$ is then given by $\langle T_a \rangle = \beta_a \langle H^{(1)} \rangle^2 / M_a$, with $\beta_a$ being the dimensionless coupling constant of the term $\bar{T}_a H^{(1)} H^{(1)}$. These vevs violate lepton number and generate a non-zero mass matrix for neutrinos, $m_\nu = \sum_{a=1,2} \alpha_{aij} \beta_a \langle H^{(1)} \rangle^2 / M_a$, via the term $\alpha_{aij} T_a L_i L_j$. Note that $U(1)_B$ and ‘matter parity’ still survive as exact symmetries. The neutrino mass matrix can be diagonalized by a suitable ‘Kobayashi-Maskawa’ rotation in its standard form (involving three angles and a CP violating phase) and the complex eigenvalues can be written as

$$m_i = \sum_{a=1,2} \alpha_{ai} \beta_a \frac{\langle H^{(1)} \rangle^2}{M_a},$$ 

where $\alpha_{ai}$ are the (complex) eigenvalues of the complex symmetric matrices $\hat{\alpha}_a = (\alpha_{aij})$. Note that the $m_i$'s, being in general complex, carry two extra CP violating phases (an overall phase factor is irrelevant) which appear in some processes like double-beta decay.

For definiteness, we will adopt the model of neutrino masses and mixing discussed in Ref. [17]. This scheme has almost degenerate neutrino masses and employs the bimaximal neutrino mixing [18], which is consistent with the vacuum oscillation explanation [19] of the solar neutrino puzzle. Moreover, all three neutrino masses are real, but the CP parity of one of them (say the second one) is opposite to the CP parities of the other two. This is important for satisfying the experimental constraints [20] from neutrinoless double beta decay. Although favored by data, this scheme has not been derived so far from a simple set of symmetries. (One interesting attempt with four neutrinos appeared in Ref. [21]).

We will not undertake here the ambitious task of implementing the above scheme of neutrino masses and mixing in the context of our model. We will restrict ourselves to observing that the required neutrino mass parameters can be obtained in our model provided the coupling constants $\alpha_{ai}$ (a=1,2; i=1,2,3) satisfy the relations $\alpha_{a1} = -\alpha_{a2} =$
$\alpha_3 \equiv \alpha_a$ to a very good approximation. The precise values of mixing angles and square-mass differences turn out to be irrelevant for our purposes.

We now turn to the discussion of the decay of the inflaton, which consists of the two complex scalar fields $S$ and $\theta$. The scalar $\theta$ ($S$) can decay into a pair of fermionic (bosonic) $T_a$, $\bar{T}_a$’s, as one easily deduces from the last coupling in Eq.(8) and the coupling $\kappa S \phi \bar{\phi}$. The decay width is the same for both scalars and equals

$$\Gamma = \frac{3}{8\pi} \gamma_a^2 \left( \frac{M}{m_{Pl}} \right)^2 m_{infl}.$$  \hspace{1cm} (12)

Of course, decay of the inflaton into $T_a$, $\bar{T}_a$ is possible provided the corresponding triplet mass $M_a \leq m_{infl}/2$. The gravitino constraint on the reheat temperature, $T_r$, then implies strong bounds on the $M_a$’s which satisfy this inequality. Consequently, the corresponding dimensionless coupling constants, $\gamma_a$, are restricted to be quite small.

To minimize the number of small couplings, we then take $M_2 < m_{infl}/2 \leq M_1$ so that the inflaton decays into only one (the lightest) triplet pair with mass $M_2$. Moreover, we take $\gamma_1 = 1$, which gives $M_1 = M^2/m_{Pl}$ and allows us to maximize the parameter $\kappa$ (see below). Using Eq.(4), the requirement $m_{infl}/2 \leq M_1$ becomes $y_Q \leq \sqrt{N_Q/2\pi} \approx 1.19$, for $N_Q = 56$, and Eq.(3) gives $x_Q \leq 1.59$. To maximize $\kappa$ (and $M$), we choose $x_Q = 1.59$. Eqs.(4), (3) with $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$ from COBE then give $M \approx 5.12 \times 10^{15}$ GeV, $\kappa \approx 2.97 \times 10^{-3}$. Also, the inflaton mass is $m_{infl} \approx 2.15 \times 10^{13}$ GeV, the triplet masses are $M_1 \approx 1.07 \times 10^{13}$ GeV, $M_2 \approx 1.07 \gamma_2 \times 10^{13}$ GeV, and the reheat temperature is $T_r \approx 1.68 \gamma_2 \times 10^{12}$ GeV. The gravitino constraint ($T_r < 10^9$ GeV) then implies $\gamma_2 \leq 5.96 \times 10^{-4}$ which is somewhat small. We do not consider this bound on the coefficient of a non-renormalizable superpotential coupling unacceptable. However, larger $\gamma_2$’s can be accommodated by allowing bigger $T_r$’s which is possible provided the branching ratio of gravitinos to photons is small enough (see Ref. [24]). Larger $\gamma_2$’s can also be obtained without relaxing the gravitino constraint, but at the expense of having smaller $\kappa$’s. For example, taking $x_Q = 1.2$, we obtain $y_Q = 0.61$, $M \approx 4.22 \times 10^{15}$ GeV, $\kappa \approx 1.26 \times 10^{-3}$, $m_{infl} \approx 7.53 \times 10^{12}$ GeV, $M_1 \approx 7.31 \times 10^{12}$ GeV, $M_2 \approx 7.31 \gamma_2 \times 10^{12}$ GeV, and $T_r \approx 8.19 \gamma_2 \times 10^{11}$ GeV. The gravitino constraint then implies $\gamma_2 \leq 1.22 \times 10^{-3}$.

Baryon number is violated only by ‘tiny’ non-perturbative $SU(2)_L$ instanton effects in the present scheme. So the only way to produce the observed baryon asymmetry of the
universe is to first generate a primordial lepton asymmetry [14] which is then partially converted into the baryon asymmetry by the non-perturbative sphaleron effects of the electroweak sector. The primordial lepton asymmetry is produced via the decay of the superfields $T_2, \bar{T}_2$ which emerge as decay products of the inflaton. This mechanism for leptogenesis has been discussed in Refs. [9,13]. The $SU(2)_L$ triplet superfields decay either to a pair of $L_i$'s or to a pair of $H(1)$'s. In the absence of right handed neutrinos, the one-loop diagrams which interfere with the tree level ones are [13] of the self-energy type with a s-channel exchange of $T_1, \bar{T}_1$. The resulting lepton asymmetry is [13]

$$n_Ls \approx -\frac{3}{8\pi} \frac{T_r M_1 M_2}{M_1^2 - M_2^2} \frac{\text{Im}(\beta_1^* \beta_2 \text{Tr}(\hat{\alpha}_1^* \hat{\alpha}_2))}{\text{Tr}(\hat{\alpha}_2^* \hat{\alpha}_2) + \beta_2^* \beta_2},$$

(13)

where the $3 \times 3$ complex symmetric matrix $\hat{\alpha}_a$, after diagonalization, becomes equal to $\text{diag}(\alpha_a, -\alpha_a, \alpha_a)$. Note that the above formula holds provided [24] the decay width of $T_1, \bar{T}_1$ is much smaller than $(M_1^2 - M_2^2)/M_2$, which is well satisfied here since $M_2 \ll M_1$.

For MSSM spectrum, the observed baryon asymmetry $n_B/s$ is related [25] to $n_L/s$ by

$$n_B/s = -(28/79)(n_L/s).$$

It is important to ensure that the primordial lepton asymmetry is not erased by lepton number violating $2 \to 2$ scatterings at all temperatures between $T_r$ and 100 GeV. This requirement gives [25] $m_{\nu_\tau} \lesssim 10$ eV which is readily satisfied. Using Eqs.(6), (12) and the fact that $M_2 \ll M_1$, Eq.(13) can be simplified as

$$\left|\frac{n_L}{s}\right| \approx \frac{9\sqrt{3}}{56\pi} \frac{M}{\sqrt{m_{\nu_{fl}} M_P}} \frac{\gamma_2^2 |\text{Im}(\alpha_1^* \beta_1 \alpha_2 \beta_2)|}{3|\alpha_2|^2 + |\beta_2|^2}.$$

(14)

The parameters $\alpha_a, \beta_a, \gamma_a (a=1,2)$ are constrained by the requirement that the hot dark matter of the universe consists of neutrinos. We take the ‘relative’ density of hot dark matter $\Omega_{HDM} \approx 0.2$, which is favored by the structure formation in cold plus hot dark matter models [1], and $h \approx 0.5$, where $h$ is the present value of the Hubble parameter in units of 100 km sec$^{-1}$ Mpc$^{-1}$. The common mass of the three light neutrinos is then about 1.5 eV and Eq.(14) gives the constraint

$$\left|\sum_{a=1,2} \frac{\alpha_a \beta_a}{\gamma_a}\right| \approx \left(\frac{M}{7.02 \times 10^{15} \text{ GeV}}\right)^2 \equiv \xi,$$

(15)

where $|\langle H^{(1)}\rangle|$ was taken equal to 174 GeV. To maximize, under this constraint, the numerator of the last fraction in Eq.(14), observe that Eq.(15) implies
\[ |\text{Im}(\delta_1^* \delta_2)|^2 = \frac{1}{4} \left( \xi^2 - (|\delta_1| - |\delta_2|)^2 \right) \left( (|\delta_1| + |\delta_2|)^2 - \xi^2 \right), \quad (16) \]

where \( \delta_a = \alpha_a \beta_a / \gamma_a \) (\( a = 1, 2 \)). For \( |\delta_1| + |\delta_2| \) fixed, this expression takes its maximal value \( \xi^2 (\delta^2 - (\xi/2)^2) \) at \( |\delta_1| = |\delta_2| \equiv \delta \). Moreover, for fixed \( \delta \), the denominator of the last fraction in Eq. (14) is minimized at \( \sqrt{3} |\alpha_2| = |\beta_2| \) with minimum value \( 2 \sqrt{3} \gamma_2 \delta \). Putting all these together, we obtain

\[ \left| \frac{n_L}{s} \right| \lesssim \frac{9}{112 \pi} \frac{M}{\sqrt{m_{\text{infl}} M_P}} \gamma_1 \gamma_2^2 \xi \left( 1 - \frac{\xi^2}{4 \delta^2} \right)^{1/2}. \quad (17) \]

To further maximize \( n_L/s \), we take \( \alpha_1 = \beta_1 = 1 \) (remember \( \gamma_1 = 1 \) too). This choice maximizes \( \delta \) which becomes equal to 1. For \( x_Q = 1.59 \), \( \xi \approx 0.53 \) and the maximal lepton asymmetry becomes \( \approx 4.14 \gamma_2^2 \times 10^{-3} \). The low deuterium abundance constraint \[24\], \( \Omega_B h^2 \approx 0.025 \), can then be satisfied provided \( \gamma_2 \approx 2.57 \times 10^{-4} \). So, for \( \gamma_2 \) in the range \( (2.57 - 5.96) \times 10^{-4} \), both gravitino and baryogenesis restrictions can be met. For \( x_Q = 1.2 \), \( \xi \approx 0.36 \) and the maximal lepton asymmetry becomes \( \approx 4.02 \gamma_2^2 \times 10^{-3} \). The allowed range of \( \gamma_2 \) is now \( 2.61 \times 10^{-4} - 1.22 \times 10^{-3} \). We see that the required values (\( \sim 10^{-3} \)) of the relevant coupling constants are ‘natural’.

In conclusion, we have presented a moderate extension of MSSM by including a \( U(1)_{B-L} \) gauge group and a Peccei-Quinn symmetry \( (U(1)_{PQ}) \) which solves the strong CP problem. The hybrid inflationary scenario is automatically and ‘naturally’ realized. The \( \mu \) problem of MSSM is solved by coupling the electroweak higgses to fields which break \( U(1)_{PQ} \). Baryon number is conserved and, thus, proton is stable as automatic consequences of a R-symmetry. Light neutrinos are assumed to acquire degenerate masses equal to about 1.5 eV through their couplings to \( SU(2)_L \) triplet superfields. These neutrinos constitute the hot dark matter of the universe. After inflation, the inflaton system decays into \( SU(2)_L \) triplets which subsequently decay producing a primordial lepton asymmetry later converted into the observed baryon asymmetry of the universe by sphaleron effects. The gravitino and baryogenesis constraints can be satisfied with ‘natural’ values \( (\sim 10^{-3}) \) of the relevant coupling constants.

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