Learning the relation between quadrilateral using geometry’s puzzle for blind students

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Abstract. Geometry is one of the fields in mathematics that deals with the shape, size, and properties of space that doesn't require prior skills like basic arithmetic. Although the model of geometry's object familiar and found in everyday life, it doesn't mean that geometry is easily learned by students, especially to blind students who have limited visualization. A quadrilateral is a geometry matter which in its learning is often emphasized in the use of formulas and memorization so that students don't understand the concept of each shape and its relation with the other ones. So, this underlies the researcher to design a learning trajectory of the relation between quadrilaterals using geometry's puzzle based on direct experience of blind students. The purpose of this research was determining the role of learning the relation between quadrilaterals using geometry's puzzle to help students understand learning quadrilateral concepts. The method used is design research with Gravemeijer and Cobb's model that through three stages, i.e. preliminary design, teaching experiments, and retrospective analysis. This research describes how geometry's puzzle in learning the relation between quadrilaterals contribute to the understanding of quadrilateral concepts for blind students of MTs Yaketunis Yogyakarta. The result of this research shows that geometry's puzzles can help increase students’ understand the concept of each quadrilateral shapes through the construction of knowledge about the relation between quadrilaterals so that students don't need to memorize the definitions or properties of each quadrilateral shapes and achieve the defined minimum completeness criteria, that is 70.

Keywords: learning, quadrilateral, relation, geometry's puzzle, blind students.

1. Introduction

Geometry is one branch of mathematics that in its learning concept implementation used many physical models, especially concepts related to the measurement of length, surface area and volume [1]. In mathematics learning, geometry is well known to students than another mathematics branch. The introduction of geometric basic concepts is indirect knowledge mediated obtained through the environment of its informal education, especially related to the playing object during the level of its concrete development. However, there are still many students believe that geometry is a complicated subject because of geometry has ideal material [2, 3]. In addition, not many students can optimize their abilities in learning geometry, including blind students who have limitations in obtaining sensory vision [4]. The limitations or loss of facilities which integration all objects of observation can affect cognitive development and the conception of a blind student [5].

The constraints in geometry learning also experienced by teachers in which difficulty in providing instructions to blind students about how to learn geometry although they already used physical models
The difficulty of teaching geometry is because blind students need a long time to develop their mental representations mainly spatial concepts [7]. In line with that, research result [8] also show that students with visual impairments have an obstacle to understanding the geometric pictures. The factor of the lack of tactical material for students to learn, also be another obstacle for blind students [9]. Some of the results of the study showed that there were some obstacles to blind students and teachers in geometry learning, both from internal factors such as abnormalities in students' physiological structures and external factors that is the limitation of learning devices in accordance with student disabilities.

Although there are obstacles in learning geometry, it doesn’t mean that geometry can be removed from learning the mathematics of visual impaired because concepts contained in geometry can’t be separated from other branches of mathematical concepts. Some concepts such as division, measurement, estimation, and trigonometry contain spatial elements and use object manipulation on certain spaces in geometry. In Minister of Education and Culture Regulation Number 157 of 2014 concerning the special education curriculum, also stated that the special education curriculum for the visually impaired is equated to the regular education curriculum. This means that geometry material must also be taught in mathematics subjects for blind students, including quadrilateral. Furthermore, in subsection 12 Minister of Education and Culture Regulation was also regulated about the learning of students with special needs that refers to the principles and approaches to regular learning that are appropriated to the characteristics learn of students with special needs. Thus, geometry learning in blind students should refer to the principle and regular approach according to the characteristics learn and uniqueness of the visually impaired.

Related this, in her research [10] showed that there is a uniqueness of thinking representation against two-dimensional shapes. The uniqueness of visually impaired students is seen when he understands the concept of a two-dimensional figure by illustrating the characteristics of sides and angles according to his experience before. From the result of her research, [11] also showed that blind students can develop the concept of angles in their minds and not only respond to the definition syntax but also try to give the imagined meaning to the angle' definition. They do a series of horizontal mathematics through prior experience and manipulation of real objects using tactile to the process of transformation into the language of mathematics, then do vertical mathematics to the language of mathematical symbols using their own terms.

His tactile abilities possessed by blind people to replace his visualization in understanding 2-dimensional shapes are important to be optimized in learning geometry. Because previous studies showed that blind students are also able to learn geometry objects, including quadrilaterals. They can able changed horizontal mathematization to vertical mathematization, even though the formal definition constructed have not fully met the formal definition of mathematics. They can also relate to quadrilateral concepts and it is prerequisite concepts. These results basic the researcher to design the learning trajectory of the relation between quadrilateral using geometry's puzzle based on direct experience of blind students. The use of geometry’s puzzle important done because according to Mandola [in 12], giving a model of objects that difficult to reach blind people is predicted to help him explain the abstract assumptions of a concept to concrete. This geometry’s puzzle can also be a learning media related to the quadrilateral model that can be explored by blind students using tactile. Geometry puzzle is an educational game tool that is played with dismantling the puzzle pieces based on its pair and used to develop skills recognizing geometry shapes, especially quadrilateral. By designing a learning trajectory of the relation between quadrilateral, it can be seen the role of the geometry's puzzle media used in learning the relation between quadrilateral to help students understanding of the quadrilateral concept.

2. Method
The research method used in this study is design research that aims to develop a local instrumental theory (LIT) to improve the quality of learning through collaboration between researchers and teachers.
Design research defined into three stages, namely preparing for the experiment, the design experiment, and retrospective analysis [14]. At the stage of preparing for the experiment, the researcher conducted a literature study on the material of the relation between quadrilaterals, then adjusted to the literature the use of geometry’s puzzle and abnormalities of the physiological structure of the blind as the basis for formulating initial strategies in learning the relation between quadrilaterals. In addition, the researcher also examined the student's initial abilities by conducting interviews and observations with several students related to the student's understanding of the learning prerequisite material. The results are used to design a series of learning activities that contain hypothetical learning trajectory (HLT). Hypothetical establishes a cyclic process that is dynamic during the teaching experiment process.

In the design experiment stage, there are two cycles namely the pilot experiment cycle and the teaching experiment cycle. In the cycle of the pilot experiment, researchers acted as teachers by involving a totally blind student and a low vision student. Data obtained from this cycle is used to revise the initial HLT so that a new HLT is obtained which is used in the teaching experiment cycle. In the teaching experiment cycle, the teacher acts as the model teacher to teach the material to all students in grade eight class MTs Yaketunis which amount to six, while the researcher acts as an observer of learning activities.

In the retrospective analysis stage, the researcher developed the design of the next learning activity with analysis of data obtained from the teaching experiment stage. The purpose of retrospective analysis is to develop local instructional theory. At this stage, HLT is compared to the actual learning trajectory to answer the research problem formulation which is to produce a description of the learning trajectory of the relation between quadrilateral using geometry's puzzle.

Data collection is through observation, making video recording related to events in the classroom, giving an initial test and final test, as well as conducting task-based interviews. HLT which has been designed is then compared with the actual learning trajectory during learning by retrospective analysis. Data analysis was followed by researchers and adviser to improve validity and reliability. Validity is done to see the quality of data sets that influence the conclusion, while reliability is illustrated through a clear description of how the data is collected so that the conclusion can be inferred.

3. Result and Discussion

Learning in this study was conducted to produce learning trajectories of the relation between quadrilateral using geometry's puzzle. The research process was carried out in three stages, namely preparing for the experiment, the design experiment, and retrospective analysis. At the stage of preparing for the experiment, the researcher conducted a literature study, initial test, observations and task-based interviews conducted to produce learning trajectories of the relation between quadrilaterals. The researcher interviewed the subject to find out the students' initial abilities about the quadrilateral. The result of the interview shows that students have studied quadrilateral in elementary school and seventh grade of junior high school, but students haven't studied the relation between quadrilaterals.

The researcher also gave a preliminary test to find out students’ understanding of the quadrilateral concept. From the result pre-test, it is known that students still use informal definitions related to the fourth types of a quadrilateral, namely parallelogram, square rectangle, and rhombus. Students relate the characteristics of the sides and angles contained in the shape to define third these shapes. This in line with the result of [4] research which shows that blind students interpreted quadrilateral based on shape attributes related to the number of sides and angles. The students begin definition construction of fourth these types with quadrilateral. While the quadrilateral itself is defined as a shape that has four sides. Students experience confusion in defining the rhombus which is considered the same as a parallelogram in the standing position. Likewise, the rectangle is called square which is more length.

Based on the students' answers, it is known that in fact, the students have thought of the relation between quadrilaterals even though students still make mistakes constructing the concept of the relation. After giving the pre-test, the researcher designed the HLT which can change and develop during the teaching experiment process.
In the design experiment stage, the researcher tested HLT in a series of learning activities so that supporting data was obtained to evaluate the conjectures in the learning activity and revise the HLT if needed. The experiment of learning activities carried out in the cycle of pilot experiment with researcher acting as a teacher. The result of this cycle is used to revise the initial HLT adjusted to the finding during the learning activities, then the HLT will be applied to the teaching experiment cycle with the researcher as an observer of learning activities and the teacher as the model teacher.

A series of learning activities designed in HLT and carried out at the design experiment stage are explained as follows.

1. Identify the types and properties of each quadrilateral using geometry's puzzle

In the first activity, students are asked to identify what the type and quadrilateral properties are in the geometry's puzzle. The purpose of this activity provides understanding to students that each quadrilateral has certain properties also both side and angle characteristics. From the identification of shape's properties, students are expected to be able to find similarities and differences in the properties possessed between quadrilaterals by touching the model of a quadrilateral on geometry's puzzle. Students explore quadrilateral models on geometry's puzzle randomly. The interesting thing from this activity that students use their tactual abilities to investigate the shape parts to obtain the sum of sides, length of sides and types of angles as observation results to one of the students as the following figure 1-4.

**Figure 1.** Students show a measure of the length of the upper side of the parallelogram

**Figure 2.** Students show the high of a parallelogram

**Figure 3.** Students show an acute angle
Based on the above results it can be seen that students identify quadrilateral based on its routine attributes, namely sides, and angles using their tactual abilities to replace their sensory vision. This in line with the results of the study [15] and [4] that blind students explored the shape and length of the quadrilateral by using tactual abilities through touching instead of the sense of sight. In addition, students can also mention the properties of rectangles by using their own terms and using non-standard measure that is finger segment. For example, parallelogram properties include 1) the opposite sides are the same length, that is the upper and lower sides while the slant sides on the right and left; 2) the opposite angles are equal, that is a pair of acute angles and a pair of obtuse angles.

2. Find the similarities and differences in properties between quadrilateral constructs using geometry's puzzle

The second activity is students finding similarities and differences in the properties possessed between quadrilaterals construction by listing the properties of a quadrilateral. The purpose of students identifying the similarities and differences in the properties possessed relation between quadrilaterals is to find the relation between quadrilateral through the similarities and differences in the properties possessed by each shape.

Students list the properties of quadrilateral and mention the similarities and differences in the properties possessed by the quadrilateral. Students said that the parallelogram, rectangle, square and rhombus have as many as four sides that the opposite sides (sides of left-right, top-down) are equal in length, the opposite angles are equal (for example a pair of acute angles and a pair of obtuse angles). The differences between the fourth of quadrilateral shapes are the sum of same length sides and type of angle.

3. Determine quadrilaterals that have the most common shape's properties

The third activity is students determining the quadrilateral that has most common shape's properties by choosing quadrilaterals whose properties are most often contained in other quadrilateral properties. The aim is to review and determine the closest type (genus proximum) and special differentiator (differentia specifica) which will be used as the basis for analytic definition construction.

Students choose quadrilateral whose properties are most often contained in other quadrilateral properties to determine parallelogram which has the most common shape's properties. Students mentioned that parallelogram is a quadrilateral that has the properties most contained in the other quadrilateral, which is as many as two sides whose opposite (sides of left-right, top-down) are equal in length and opposite angles are equal.

4. Changed the rectangle shape into a parallelogram

The fourth activity was students changed the rectangle shape into a parallelogram. The aim is students can change the rectangle shape into a parallelogram and understanding the definition of a square if students considering the closest type (genus proximum) and special differentiator (differentia specifica) with the strategy of changing shapes and saw the properties of shapes.
Students change the rectangle shape into a parallelogram to construct the rectangle definition according to the closest type and special differentiator that is a parallelogram. According to students, changing the rectangular shape into a parallelogram is done by cutting both the left and right sides of the rectangular model so the sides to be slant and it has an acute angle and obtuse angle. But there is also a student who is looking for a pair of congested right triangle models that have an altitude side equal with the length of one of the rectangle sides. Then, the student makes a new rectangle of the two congruent triangles that have altitude side equal with the length of one of the last rectangle sides so that a new rectangle with a larger size was obtained. The student says that for example, he has a new rectangle like what he made before, he can turn it into a parallelogram by taking off a pair of triangles like in figure 5. (a). Next, students arranged each altitude side of the right triangle to the opposite side of the rectangle as shown in figure 5. (b) - (d) below.

![Figure 5. Students change the rectangle shape into a parallelogram](image)

5. Changed the rhombus shape into a parallelogram
The fifth activity was students changed the rhombus shape into a parallelogram. The aim that students can change the rhombus shape into a parallelogram and understand the definition of a rhombus if students considering the closest type (genus proximum) and special differentiator (differentia specifica) with the strategy of changing shapes and saw the properties of shapes.

Students changed the rhombus shape into a parallelogram to construct a definition of rhombus according to the closest type and special differentiator that is a parallelogram. According to students, changed the rhombus shape into a parallelogram by positioning parallelogram in standing. But there is also a student who is looking for four models of right-angled triangles that have hypotenuse side with a length equally and congruent, and then these fourth of congruent triangles be to arranged it so it's established a rhombus as figure 6. (a) - (b). Student says that for example, there had a rhombus as he had made, he could turn it into a parallelogram with dismantling it back into two pair of isosceles triangles. Each of these isosceles triangles composed a pair of right-angled triangles. Next, the student composed the two triangles isosceles with the opposite position of the two isosceles triangles. Series of the activity of two isosceles triangles showed in figure 6. (c) - (d).
Figure 6. Students change the rhombus shape into a parallelogram

6. Changed the square shape into a rectangular
The sixth activity was students changed the square shape into a rectangle. The aim that students can change the square shape into a rectangular and understand the definition of a square if students considering the closest type (genus proximum) and special differentiator (differentia specifica) with the strategy of changing shapes and saw the properties of shapes.

Students changed the square shape into a rectangle to construct the square definition according to the closest type and special differentiator that is a rectangle. According to students, changed the square shape into a rectangle by dividing the square into two rectangular pieces which have equal area, then one of the shortest sides rectangles coincide with one of the shortest side rectangles other. But there is also a student who says he will cut a square into four right triangles, then the four right triangles will be arranged so that he established a rectangle like in figure 7 below.

Figure 7. Student makes a rectangle of four right triangles
7. Changed the square shape into a rhombus

According to students, changed the square shape into a rectangle by dividing the square into two rectangular pieces which have equal area, then one of the shortest sides rectangles coincide with one of the shortest side rectangles other. But there is also a student who says he will cut a square into four right triangles, then the four right triangles will be arranged so that he established a rectangle like in figure 7 below.

The seventh activity was students changed the square shape into a rhombus. The aim that students can change the square into a rhombus and understand the definition of a square if students considering the closest type (genus proximum) and special differentiator (differentia specifica) with the strategy of changing shapes and saw the properties of shapes.

According to students, changed the square shape into a rhombus by positioning the square slant. But there is also a student who says he will cut a square into four right triangles like figure 8. (a) - (b), then the fourth of right triangles will be arranged into two pairs of isosceles triangles as shown in 8. (c), and both of it are paired so that he makes a rhombus as shown in figure 8. (d).

![Figure 8. Students change the square shape into a rhombus](image)

From the results of a series of learning activities the relation between quadrilateral using geometry's puzzle in the pilot experiment above, it can be seen that blind students can understand the relation between quadrilateral well even though there is still a student who have misconceptions when constructing the relation between rhombus and parallelogram. Student interpreted a rhombus as a parallelogram with stand position. Result of the study on the pilot experiment also showed that the actual learning trajectory corresponds to the HLT that has been designed by the researcher. Learning the relation between quadrilateral using geometry's puzzle can help students interpreted a concept of the relation between quadrilateral based on the similarity of the properties possessed by the shape of a quadrilateral to construct its analytic definition. The results of this study add empirical evidence which states that giving a model of objects that are difficult to reached blind people as possible to help them
explain the abstract assumptions of a concept to more the concrete [12]. The use of geometry's puzzle as quadrilateral learning media is in suitable with the limitations of visual of blind students who will use more tactile or sense of touch. This in line with the result of the study [16], that to understand the concepts related to the exploration of the shape of a particular object blind people use their sense of touch more.

Furthermore, the researcher also gave post-tests to blind students with questions that were identical to the pre-test questions related to an understanding of the concept and construction of quadrilateral definitions. From the test result, it is known that there is an increasing understanding of the concept of the relation between quadrilateral and construction of quadrilateral definitions, so students don't need to remember definition or properties of each quadrilateral, even they reach minimum completeness criteria that is 70.

4. Conclusion
The uses of geometry's puzzles can help blinds students understand the concept of quadrilateral shapes through the construction of knowledge about the relation between quadrilaterals so that students don't need to remember the definition and quadrilateral's properties. By understanding the relation between quadrilateral, students can construct an analytical definition of a quadrilateral by considering the closest type (genus proximum) and special differentiator (differentia specifica). From the result of the post-test given by the researcher, it knows that students have also reached the minimum completeness criteria which are 70. This shows that the learning trajectory designed in the learning of the relation between quadrilateral using geometry's puzzle has a significant influence in improving students' understanding of quadrilateral. The trajectory includes: identify the type and properties of each quadrilateral using geometry’s puzzle; find the similarities and differences in properties between quadrilateral constructs using geometry's puzzle; determine quadrilaterals that have the most common shape's properties; changed the rectangle shape into parallelogram; changed the rhombus shape into parallelogram; changed the square shape into rectangle; and changed the square shape into rhombus.

References
[1] Hwang W Y, Su J H, Huang Y M and Dong J J 2009 Educational Technology & Society 12 229-47
[2] Afthina H, Mardiyana and Pramudya I 2017 Journal of Physics: Conf. Series 895 1-6
[3] Bhagat K K and Chang C Y 2014 Eurasia Journal of Mathematics, Science & Technology Education 11 77-86
[4] Andriyani 2018 5 th ICRIEMS Proceedings ME-1-6
[5] Andriyani and Maulana M 2017 Journal of Physics: Conference Series 1188 1-12
[6] Pritchard C K and Lamb J H 2012 Journal of Mathematics Teacher 106 22-7
[7] Thinus-Blanc C and Gauinet F 1997 Psychological Bulletin 121 20-42
[8] Vianna C S, Barbosa, P M, Rocha D F and Silva B 2006 International Congress on Mathematical Education
[9] Argyropoulos V and Stamouli M 2006 British Journal of Visual Impairment 24 128- 34
[10] Andriyani, Budayasa I K and Juniati D 2018 Journal of Physics: Conference Series 947 1-6
[11] Andriyani, Juniati D and Budayasa K 2018 International Journal of Engineering and Technology 7 1-5
[12] Efendi M 2006 Pengantar Psikopedagogik Anak Berkelainan (Jakarta: Bumi Aksara)
[13] Gravemeijer K and Van Eerde D 2009 The Elementary School Journal 109 5
[14] Gravemeijer and Cobb 2006 Design Research from the Learning Design Perspective (Netherland: SLO)
[15] McLaughlin M, Hesperha J, and Sukhatme G 2002 Touch in Virtual Environments: Haptics and the Design of Interactive Systems (Upper Saddle River, NJ: Prentice-Hall)
[16] Andriyani 2018 Prosiding Seminar Nasional Pendidikan Matematika (SENPIKA) 17-20