Nonlinear transport of Cosmic Rays in turbulent magnetic field

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Abstract. Recent advances in both the MHD turbulence theory and cosmic ray observations call for revisions in the paradigm of cosmic ray transport. We use the models of magnetohydrodynamic turbulence that were tested in numerical simulation, in which turbulence is injected at large scale and cascades to to small scales. We shall present the nonlinear results for cosmic ray transport, in particular, the cross field transport of CRs and demonstrate that the concept of cosmic ray subdiffusion in general does not apply and the perpendicular motion is well described by normal diffusion with $M_A^4$ dependence. Moreover, on scales less than injection scale of turbulence, CRs' transport becomes super-diffusive. Quantitative predictions for both the normal diffusion on large scale and super diffusion are confronted with recent numerical simulations. Implication for shock acceleration is briefly discussed.

1. Introduction

The propagation and acceleration of cosmic rays (CRs) is governed by their interactions with magnetic fields. Astrophysical magnetic fields are turbulent and, therefore, the resonant and non-resonant (e.g. transient time damping, or TTD) interaction of cosmic rays with MHD turbulence is the accepted principal mechanism to scatter and isotropize cosmic rays (see Schlickeiser 2002). In addition, efficient scattering is essential for the acceleration of cosmic rays. For instance, scattering of cosmic rays back into the shock is a vital component of the first order Fermi acceleration (see Longair 1997). At the same time, stochastic acceleration by turbulence is entirely based on scattering. The dynamics of cosmic rays in MHD turbulence holds one of the keys to high energy astrophysics and related problems.

At present, the propagation of the CRs is an advanced theory, which makes use both of analytical studies including nonlinear formalism and numerical simulations. However, these advances have been done within the turbulence paradigm which is being changed by the current research in the field. Instead of the empirical 2D+slab model of turbulence, numerical simulations suggest anisotropic Alfvén and slow modes (an analog of 2D, but not an exact one, as the anisotropy changes with the scale involved) + fast modes (Cho & Lazarian 2002).

Propagation of CRs perpendicular to mean magnetic field is another important problem for which one needs to take into account both large and small scale interactions in tested models of turbulence. Indeed, if one takes only the diffusion along the magnetic field line and field line random walk (FLRW [Jokipii 1966; Jokipii & Parker 1969; Forman et al. 1974]), compound (or subdiffusion) would arise. Whether the subdiffusion is realistic in fact depends on the models of turbulence chosen (Yan & Lazarian 2002).
In this paper we again present our understandings to this problem within the domain of numerically tested models of MHD turbulence.

In what follows, we discuss the nonlinear cosmic ray transport in §2. In §3, we shall study the perpendicular transport of cosmic rays. We shall also discuss the issue of super-diffusion and its implication in §4. Summary is provided in §5.

2. Cosmic ray scattering in nonlinear theory

2.1. Model of MHD turbulence

Obviously the model of turbulence determines the CR transport properties. Recent years have seen substantial progress in the understanding of MHD turbulence, which is different from the conventionally adopted slab model and isotropic Kolmogorov model. It has been demonstrated by a number of studies that MHD turbulence can be decomposed into three modes: Alfvén, slow and fast modes. Among them, Alfvén and slow modes can be well described by the (Goldreich & Sridhar 1995, henceforth GS95) model and exhibit scale-dependent anisotropy with respect to the local magnetic field (Lazarian & Vishniac 1999; Cho & Lazarian 2002). Fast modes, however, are isotropic (Cho & Lazarian 2002).

The anisotropy leads to strong suppression of gyroresonance of CRs by the Alfvénic turbulence, which was the favorite modes in earlier literatures. The studies in Yan & Lazarian (2002, 2004) demonstrated with quasilinear theory (QLT), nevertheless, that the scattering by fast modes dominates in most cases in spite of the damping.

2.2. Nonlinear theory

The conclusion that fast modes dominates the gyroresonance interaction over Alfvénic turbulence is also confirmed in nonlinear theory (NLT, see YL08). Efficient gyroresonance itself does not ensure finite mean free path/diffusion coefficient for the CRs because particles cannot be scattered through gyroresonance at 90°. This is the longstanding 90° problem, which is intrinsically associated with the QLT approach.

Indeed, many attempts have been made to improve the QLT and various non-linear theories have been attempted (see Dupree 1966, Völk 1973, 1975, Jones, Kaiser & Birmingham 1973, Goldstein 1976) (see Matthaeus et al. 2003). An important step was taken in Yan & Lazarian (2008), where non-linear effect was included in treating CR scattering in the type of MHD turbulence that are tested by numerical simulations. The results have been applied to both solar flares (Yan, Lazarian & Petrosian 2008) and grain acceleration (Hoang et al. 2012). Below, we introduce the nonlinear theory and their applications to particle transport in turbulence based on the results from Yan & Lazarian (2008).

The basic assumption of QLT is that the particles’ orbit is unperturbed until a scattering event occurs. In reality, however, particle pitch angle does change because of magnetic field compression according to the first adiabatic invariant \( v_\bot^2 / B \), where \( B \)

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1 On the basis of weak turbulence theory, Chandran (2005) has argued that high-frequency fast waves, which move mostly parallel to magnetic field, generate Alfvén waves also moving mostly parallel to magnetic field. We expect that the scattering by thus generated Alfvén modes to be similar to the scattering by the fast modes created by them. Therefore we expect that the simplified approach adopted in Yan & Lazarian (2004) and the papers that followed to hold.
is the total strength of the magnetic field (see Landau & Lifshitz 1975). Since $B$ varies in turbulent field, so are the projections of the particle speed $v_\perp$ and $v_\parallel$. This results in broadening of the resonance from the delta function $\pi \delta(k_\parallel v_\parallel - \omega \pm n\Omega)$ in QLT to

$$R_n(k_\parallel v_\parallel - \omega \pm n\Omega) \simeq \frac{\sqrt{\pi}}{|k_\parallel v_\perp \sqrt{M_A}|} \exp \left[ - \frac{(k_\parallel v_\mu - \omega + n\Omega)^2}{k_\parallel v_\perp^2 M_A} \right] \quad (1)$$

where $M_A \equiv \delta V/v_A = \delta B/B_0$ is the Alfvénic Mach number and $v_A$ is the Alfvén speed. Note that Eqs. (1) are generic, and applicable to both incompressible and compressible medium.

For gyroresonance ($n = \pm 1, 2, ...$), the result is similar to that from QLT for $\mu \gg \Delta \mu = \Delta v_\parallel/v$. In this limit, Eq.(1) represents a sharp resonance and becomes equivalent to a $\delta$-function. On the other hand, the dispersion of the $v_\parallel$ means that CRs with a much wider range of pitch angle can be scattered by the compressible modes through the TTD interaction ($n = 0$), which is marginally affected by the anisotropy and much more efficient than the gyroresonance. In QLT, the projected particle speed should be comparable to phase speed of the magnetic field compression according to the $\delta$ function for the TTD resonance. This means that only particles with a specific pitch angle can be scattered. With the resonance broadening, however, wider range of pitch angle can be scattered through TTD, including $90^\circ$. Finite mean free path is therefore ensured in NLT (Yan & Lazarian 2008).

2.3. Comparison with test particle simulations

We run test particle simulations with turbulence generated from 3D MHD simulations. Test particle simulation has been used to study CR scattering and transport (Giacalone & Jokipii 1999; Mace et al. 2000). The aforementioned studies, however, used synthetic data for turbulent fields, which have several disadvantages. Creating synthetic turbulence data which has scale-dependent anisotropy with respect to the local magnetic field (as observed in Cho & Vishniac 2000 and Maron & Goldreich 2001) is difficult and has not been realized yet. Also, synthetic data normally use Gaussian statistics and delta-correlated fields, which is hardly appropriate for description of strong turbulence.

Using the results of direct numerical MHD simulations as the input data, Xu & Yan (2013) performed test particle simulations. The results show good correspondence with the analytical predictions. As shown in Fig[1h], particles’ trajectories are traced in the turbulent magnetic field. The scattering coefficient shows the same pitch angle dependence as that predicted in Yan & Lazarian (2008), namely the scattering is most efficient for large pitch angles due to the TTD mirror interaction and dominated by gyroresonance at small pitch angles (see Fig[1h]). Both interactions are necessary and complementary to each other in scattering CRs.

3. Perpendicular transport

In this section we deal with the diffusion perpendicular to mean magnetic field. Both observation of Galactic CRs and solar wind indicate that the diffusion of CRs perpendicular to magnetic field is comparable to parallel diffusion (Giacalone & Jokipii 1999; Maclennan et al. 2001).
The perpendicular transport is slow if particles are restricted to the magnetic field lines and the transport is solely due to the random walk of field line wandering (see Kóta & Jokipii 2000). In the three-dimensional turbulence, field lines are diverging away due to shearing by Alfvén modes (see Lazarian & Vishniac 1999, Lazarian 2006). Since the Larmor radii of CRs are much larger than the minimum scale of eddies $l_{\perp,\min}$, field lines within the CR Larmor orbit are effectively diverging away owing to shear by Alfvénic turbulence. The cross-field transport thus results from the deviations of field lines at small scales, as well as field line random walk at large scale ($>\min[L/M_A^3,L]$), where $L,M_A$ are the injection scale and Alfvénic Mach number of the turbulence, respectively.

3.1. Perpendicular diffusion on large scale

High $M_A$ turbulence: High $M_A$ turbulence corresponds to the field that is easily bended by hydrodynamic motions at the injection scale as the hydro energy at the injection scale is much larger than the magnetic energy, i.e. $\rho V_p^2 \gg B^2$. In this case magnetic field becomes dynamically important on a much smaller scale, i.e. the scale $l_A = L/M_A^2$. If the parallel mean free path of CRs $\lambda_\parallel \ll l_A$, the stiffness of B field is negligible so that the perpendicular diffusion coefficient is the same as the parallel one, i.e., $D_\perp = D_\parallel$. If $\lambda_\parallel \gg l_A$, the diffusion is controlled by the straightness of the field lines, and $D_\perp = D_\parallel \approx 1/3l_A v_M A > 1$, $\lambda_\parallel > l_A$. The diffusion is isotropic if scales larger than $l_A$ are concerned. In the opposite limit $\lambda_\parallel < l_A$, naturally, a result for isotropic turbulence, namely, $D_\perp = D_\parallel = 1/3\lambda_\parallel v_M$ holds.

Low $M_A$ turbulence: For strong magnetic field, i.e. the field that cannot be easily bended at the turbulence injection scale, individual magnetic field lines are aligned with the mean magnetic field. The diffusion in this case is anisotropic. If turbulence is injected at scale $L$ it stays weak for the scales larger than $LM_A^2$ and it is strong at smaller scales. Consider first the case of $\lambda_\parallel > L$. The time of the individual step is $L/v_\parallel$, then $D_\perp \approx 1/3l_A v_M A, M_A < 1, \lambda_\parallel > L$. This is similar to the case discussed in the FLRW model (Jokipii 1966). However, we obtain the dependence of $M_A^4$ instead of...
their $M_A^2$ scaling. In the opposite case of $\lambda_\parallel < L$, the perpendicular diffusion coefficient is $D_\perp \approx D_\parallel M_A^4$, which coincides with the result obtained for the diffusion of thermal electrons in magnetized plasma (Lazarian 2006).

As shown in Fig.2, the test particle simulations conducted by Xu & Yan (2013) confirm the dependence of $M_A^4$ instead of the $M_A^2$ scaling in, e.g., Jokipii (1966). This is exactly due to the anisotropy of the Alfvénic turbulence. In the case of sub-Alfvénic turbulence, the eddies become elongated along the magnetic field from the injection scale of the turbulence (Lazarian 2006; YL08). The result indicates that CR perpendicular diffusion depends strongly on $M_A$ of the turbulence, especially in magnetically dominated environments, e.g., the solar corona.

4. Superdiffusion on small scales

The diffusion of CR on the scales $\ll L$ is different and it is determined by how fast field lines are diverging away from each other. The mean separation of field lines $\bar{\delta x}$ is proportional to the 3/2 power of distance $[\delta z]^3/2$ (Lazarian & Vishniac 1999; Lazarian 2006), same as Richardson diffusion in the case of hydrodynamic turbulence (see Eyink et al. 2011). Following the argument, we showed in Yan & Lazarian (2008) that the cosmic ray perpendicular transport is superdiffusive. The reason is that there is no random walk on small scales up to the injection scale of strong MHD turbulence ($L M_A^2$ for $M_A < 1$ and $l_A$ for $M_A > 1$). In the case of sufficiently strong scattering, particles’ mean free path becomes smaller than the scale of interest so that they diffuse along the field lines which are diverging away $[\delta z]^3/2$ and the separation of particle trajectories therefore goes as $\propto t^{3/4}$ (see Fig.3b).
4.1. Superdiffusion and shock acceleration

The super diffusion on small scales has important consequences for shock acceleration. For shock acceleration, the crucial element is the confinement of particles by magnetic perturbations at upstream and downstream regions. The first order Fermi process can be only realized if particles can be scattered back and forth consecutively before they escape the shock front. Ultimately both the acceleration rate and the maximum energy attainable are determined by the diffusion properties of particles in the local turbulence.

There have been arguments that the perpendicular shock is much more efficient accelerator than the parallel one based on the suppressed diffusion perpendicular to the magnetic field. And perpendicular shock thereby has been invoked to remedy the problem of insufficient acceleration by parallel shocks at places like termination shock (Jokipii 1987). In the presence of turbulence and superdiffusion, nevertheless, the difference between the parallel shock and perpendicular shock becomes diminished. Only when local turbulence is generated at the shock with sufficiently small integral scale, the super diffusion which only happens on scales smaller than the injection scale can be neglected and the shock acceleration can be more effective. We refer the readers to Lazarian & Yan (2013) for the detailed discussions.

5. Summary

In the paper above, we presented current understanding of cosmic ray transport in tested model of turbulence. Nonlinear approach was employed along with the numerical testings. We showed that

- Compressible fast modes are most important for CR scattering. CR transport therefore varies from place to place.
- Mirror interaction is essential for pitch angle scattering (including 90 degree).
- Subdiffusion does not happen in 3D turbulence.
• On large scales, CR perpendicular diffusion is suppressed by $M_A^4$.
• On small scales, CR transport is super-diffusive, has a dependence of $M_A^4$ in sub-Alfvénic turbulence.
• Implications are wide, from shock acceleration to turbulent reconnection.

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