Tunnelling for Large N

Yun-Song Piao

College of Physical Sciences, Graduate University of Chinese Academy of Sciences, Beijing 100049, China

In this brief note, by applying the stochastic approach to multiple fields, we estimate the probability of tunneling between vacua in a landscape with \( N \) fields. We find that the probability can be enhanced by large \( N \). When \( N \) saturates the dS entropy of some vacuum, the corresponding vacuum will be extremely unstable and be expected to rapidly decay. We discuss the implications of this result.

PACS numbers:
denotes the equivalent radial field. The slow roll equation for each field is \(3h\dot{\varphi} + V' \approx 0\), where \(h\) is the Hubble parameter. Thus after multiplying \(d\varphi_i\) to this equation and then making the sum for equations of all fields, we have \(3h\dot{\varphi} + V' \approx 0\), in which \(V\) is the potential in field space and corresponds to the sum of potential \(V_i\) of all fields. This means the radial motion in field space can be described equally by an effective radial field obeying the same slow roll equation with that of each field. Thus the motion of this radial field can be given by one dimension stochastic equation, which corresponds to that of Eq. (1) with only one freedom and equals to that used in Refs. [10, 11].

When there is only one freedom, \(D^{(1)}\) and \(D^{(2)}\) will be not vector and tensor any more, respectively. In this case, \(D^{(1)}(\varphi)\) may be called the drift coefficient of radial motion, which here equals to \(\dot{\varphi}\), and \(D^{(2)}(\varphi)\) is the diffusion coefficient of radial motion, which can be determined as follows. In slow roll approximation, each field contributes the fluctuation \(\delta\varphi_i \approx \frac{h}{2\pi}\), thus the radial random walk in field space has the step length \((\delta\varphi)^2 \approx N(h/2\pi)^2\) for each time interval \(\Delta t \sim 1/h\). In this case, similar to the calculations in Ref. [12],

\[
D^{(2)}(\varphi) \approx N\frac{h^3(\varphi)}{8\pi^2},
\]

(2)
can be found. It can be noticed that compared with single field, here the diffusion coefficient \(D^{(2)}\) is markedly enhanced by the number \(N\) of fields, which means that compared with single field, the step length of effective random walk in field space is amplified by \(N\).

The stationary solution can be found by taking \(\frac{\partial P}{\partial \varphi} = 0\). Thus the probability finding \(\varphi\) in some position is

\[
P \sim \exp \int \frac{D^{(1)}}{D^{(2)}} d\varphi \approx \exp \int \frac{8\pi^2 \varphi}{Nh^3(\varphi)} d\varphi,
\]

(3)
where Eq. (2) has been applied. The probability of \(\varphi\) jumping the top of its potential barrier can be obtained by making the integral for \(\varphi\) from its original background vacuum to the top of its potential barrier, which is

\[
P \sim \exp \left( -\frac{3}{8N} \frac{1}{V_b} - \frac{1}{V^*} \right),
\]

(4)
where the gravitational scale \(G = 1\) has been set, and the subscript ‘b’ and ‘*’ denote the values of original background vacuum and the top of its corresponding potential barrier, respectively. For \(N = 1\), Eq. (4) is exactly same with that for single field [11, 12], which is suppressed exponentially. While for large \(N\), the result given by Eq. (4) is enhanced. This means that the probability tunneling to adjacent vacua is actually heightened by large \(N\). This result is not surprised, since it is straightly leaded by Eq. (2), in which the diffusion coefficient has a large \(N\) enhancement. In contrast, if the diffusion coefficient has a suppression, the probability of tunneling will be depressed, see Ref. [14] for studying in noncommutative eternal inflation.

When \(N \approx \frac{1}{V_b}\), the probability \(P \sim 1\), since generally \(V_b \ll V_i\). Further, noticing the dS entropy of background vacuum \(S \approx \frac{1}{3} \approx \frac{1}{V^*}\), thus we obtain that when \(\sqrt{V_b} \sim S\), \(P \sim 1\). This means when the number of fields saturates the dS entropy of vacuum which all these fields builded together, the corresponding vacuum will be overly unstable, and will rapidly tunnel to other neighboring vacua by quantum jump. This result is consistent with that obtained in Ref. [12, 13] for Inflation. In Refs. [12, 13], it is found that there is a critical point for large \(N\) transition, in which the number of fields saturates the dS entropy, beyond which the quantum effect will strong so that the slow roll inflation phase disappears. Here the enhancement of quantum effect is reflected in the increase of the diffusion coefficient \(D^{(2)}\). It seems that when \(N \approx S\), \(D^{(2)}\) completely overwhelms the drift coefficient \(D^{(1)}\), and the latter is negligible. In this case, it is interesting to notice that in unit of Hubble time the diffusion distance in field space \(\sqrt{D^{(2)}\Delta t} \sim 1\), where \(N \approx S \sim 1/h^2\) and \(\Delta t \sim 1/h\) have been applied. This means this diffusion distance in a landscape of field space is in Plank order, which is enough large for a jump to the top of potential barrier and thus the tunneling. In another point of view, it can be intuitively thought that there is at least a freedom degree for every field, thus the total freedom degree of \(N\) fields system, i.e. the entropy, should be at least \(N\). Thus a vacuum state with \(N > S\) will be obviously impossible, since \(S\) is the dS entropy which is the maximal entropy of corresponding system. This reflect the fact again why such vacua should rapidly decay.

For Eq. (4), if \(N > \frac{1}{V_b}\), then the corresponding vacuum will be unstable and rapidly tunnel to other vacua. Thus in this sense, the number \(N\) of fields can be thought as a division to violently unstable and metastable vacua. For fixed \(N\), only the vacua with the energy density small than \(1/N\) are metastable, while those large than \(1/N\) are violently unstable, which will experience a or a series rapid tunneling till a metastable vacuum is arrived. The value \(1/N\) corresponds to set a critical energy density, beyond which the vacuum will be not possible to be stable or metastable. For Eq. (4), the critical energy density for single field, i.e. one dimension landscape, is \(1/N \sim 1\), which is Planck order. This in some sense is also the reason why the tunneling is exponentially suppressed. In order to make a critical energy density slightly larger than the cosmological constant value observed currently, \(N \sim 10^{125}\) is required, which seems not realistic. However, it is plain that a lower critical energy density can be acquire by considering a larger \(N\). When the energy density of successive vacua satisfies \(V_b \geq 1/N\), the tunnelings will be expected to occur one after the other with the probability \(P \sim 1\), up to the vacuum with \(V_b < 1/N\) in which the rapid tunneling ends. In this case, the scenario will be slightly similar to that of chain inflation [20, 21, 22]. However, here the tunneling is regarded in a different viewpoint. Thus the relevant scenario will be different, and need to be explored.

The inclusion of multiple fields, in some sense, corre-
sponds to reduce the effective gravitational scale in corresponding model with single field [14]. Thus the stochastic approach and the method of HM instanton [23] seems possibly give same results for large $\mathcal{N}$, which is valid for single field [11, 18]. However, recently, it was showed in [9, 24] that with multiple fields, in many classes of potentials there do not exist the HM points, since it is not easy to find a top of barrier which is an extremal point for all fields. It is generally thought that the HM instanton occur when the gravitational effect is dominated. Thus it seems that there may not be the tunneling in such a strong gravitational regime. This seem conflicted with the result obtained here, in which the tunneling will not only occur but be enhanced, especially the stronger the gravitational effect is, which is characterized by the large number of fields, the easier the tunneling is. The underlying reason of this conflict might be that the application of the stochastic approach only requires there are the barriers separating the vacua, and is not dependent of whether there is a top of barrier behaving the extrema for all fields, as the HM instanton requires.

It can be also noticed that, even if there is a top of barrier behaving the extrema for all fields, in the stochastic approach the random walk of fields also must not pass through this top, since in multiple dimension field space the paths for fields striding over the barrier between both vacua is certainly not only one. The different paths, i.e. different top $\frac{1}{\mathcal{N}}$, will give different probability, though $\frac{1}{\mathcal{N}}$ is actually dominated. In principle, the fields will most possibly pass through the top giving the largest probability, which must not coincide with that behaving the extrema for all fields. Thus in these cases whether the results from the stochastic approach and the instanton method is consistent seems still open. These results and observations indicate that in a field space with large $\mathcal{N}$ fields, the tunneling between vacua seems more subtle than imagined.

In summary, by applying the stochastic approach to the landscape with $\mathcal{N}$ fields, we found that the tunneling probability between vacua can be enhanced by large $\mathcal{N}$. The more the number of fields is, the larger the probability is. When $\mathcal{N}$ saturates the dS entropy of some vacuum, the corresponding vacuum will has the decay probability $P \sim 1$, and thus will be extremely unstable and be expected to rapidly decay. The implications of this result were discussed. It should be mentioned that in order to have an estimate for the probability we identify the problem be effectively one dimensional. Thus it is inevitable that the result obtained is slightly rough, which, however, might have captured some essentials of full answer. We hope this work could be an interesting step towards a comprehensive understanding to the tunneling in the landscape with large $\mathcal{N}$ dimensions. There are many open issues left to study.

Acknowledgments The author thank Y.F. Cai for helpful discussions and comment. This work is supported in part by NSFC under Grant No: 10405029, 10775180, in part by the Scientific Research Fund of GU-CAS(NO.055101BM03), in part by CAS under Grant No: KJCX3-SYW-N2.

[1] L. Susskind, arXiv:hep-th/0302219
[2] A. Vilenkin, Phys. Rev. D27, 2848 (1983).
[3] A. Linde, Phys. Lett. B175, 395 (1986).
[4] A. Vilenkin, J. Phys. A40, 6777 (2007).
[5] S. Winitzki, Lect. Notes Phys. 738, 137 (2008).
[6] P.J. Steinhardt, in “The Very Early Universe”, ed. by G.W. Gibbons, S.W. Hawking and S.T.C. Siklos (Cambridge University Press, 1983).
[7] A.H. Guth, E.J. Weinberg, Nucl. Phys. B212, 321 (1983).
[8] A. Aguirre, S. Gratton, M.C. Johnson, Phys. Rev. Lett. 98, 131301 (2007).
[9] M.C. Johnson, M. Larfors, arXiv:0809.2604.
[10] A.A. Starobinsky, in “Current Topics in Field Theory, Quantum Gravity and Strings,” edited by H.J. de Vega and N. Sanchez, Lecture Notes in Physics, Vol. 26 (Springer, Heidelberg, 1986), 107.
[11] A.S. Goncharov, A.D. Linde, V.F. Mukhanov, Int. J. Mod. Phys. A2, 561 (1987); A.D. Linde, Nucl. Phys. B372, 421 (1992); A.D. Linde, “Particle Physics and Inflationary Cosmology”, (Harwood, Chur, Switzerland, 1990); Contemp. Concepts Phys. 5, 1 (2005), arXiv:hep-th/0503203.
[12] I. Ahmad, Y.S. Piao, C.F. Qiao, JCAP 0806, 023 (2008).
[13] I. Ahmad, Y.S. Piao, C.F. Qiao, arXiv:0809.3333.
[14] G. Dvali, arXiv:0706.2050; G. Dvali and M. Redi, arXiv:0710.4344.
[15] Q.G. Huang, Phys. Rev. D77, 105029 (2008).
[16] L. Leblond and S. Shandera, JCAP 0808, 007 (2008).
[17] H. Risken, ”The Fokker-Planck Equation: Methods of Solutions and Applications”, 2nd edition, Springer Series in Synergetics.
[18] A. Linde, D. Linde, A. Mezhulman, Phys. Rev. D49 (1994) 1783.
[19] Y.F. Cai, Y. Wang, JCAP 0706, (2007) 022; JCAP 0801, (2008) 001.
[20] K. Freese, D. Spolyar, JCAP 0507, (2005) 007; K. Freese, J.T. Liu, D. Spolyar, arXiv:hep-th/0612056.
[21] Q.G. Huang, JCAP 0705, (2007) 009.
[22] D. Chialva, U.H. Danielsson, arXiv:0804.2846; arXiv:0809.2707.
[23] S.W. Hawking, I.G. Moss, Nucl. Phys. B224, 180 (1983).
[24] M.C. Johnson, M. Larfors, arXiv:0805.3705.