Recent Advances in Neural-symbolic Systems: A Survey
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Abstract—In recent years, neural systems have displayed highly effective learning ability and superior perception intelligence, but have been found to lack cognitive ability with effective reasoning. In the contrast, symbolic systems have exceptional cognitive intelligence, but their learning capabilities are poor compared to neural systems. Considering the advantages and disadvantages of both methodologies, an ideal solution is to combine neural systems and symbolic systems, a approach that produces neural-symbolic systems with powerful perception and cognition. In this paper, we survey recent advances in neural-symbolic systems from four perspectives: challenges, methods, applications, and future directions. This paper aims to advance this emerging area of research by providing researchers with a holistic and comprehensive overview of the field that highlights the state-of-the-art and identifies promising future research directions.

Index Terms—Neural-symbolic Systems, Deep Learning, Neural Networks, Symbolic Reasoning, Symbols, Logic, Knowledge Graphs.

I. INTRODUCTION

PERCEPTION and cognition are two important paradigms of artificial intelligence (AI). In humans, the representations of objective reality in the human brain constructed from data derived from sense organs, while cognition refers to the information processing process of receiving, reasoning, converting, encoding, storing, extracting, reconstructing, and forming concepts of sensory signals in the cause of cognitive activities [1]. Using an analogous approach, neural systems (or connectionism) can simulate human perception to perceive information in data, such as through image recognition [2], while symbolic systems (or symbolism) perform human-like cognition processes to encode and reason information contained in data such as in expert systems.

Various kinds of neural systems, including deep learning systems, have achieved great success on perception tasks. However, there are many circumstances such as question answering [3], medical diagnosis [4], and autonomous driving [5], where perception alone can reach its limits or lead to unsatisfactory results. For example, if the available labeled data is insufficient, it will result in poor model performance. Another important consideration is that a purely perception-based model may not meet the criteria for explainable AI [6].

Symbolic systems are more attractive in cases where labeled data is insufficient, and also provide more interpretability. Accordingly, an increasing number of researchers have focused on how to improve neural systems by integrating symbolic systems [7]–[12]. In a special NIPS 2019 lecture, Turing Award winner Yoshua Bengio drew from Dr. Daniel Kahneman’s book “Thinking Fast and Slow” [13] to point out that deep learning requires system-1-to-system-2 transformation, where system 1 represents the intuitive, fast, unconscious, nonlinguistic, and habitual, while system 2 represents the slow, logical, sequential, conscious, linguistic, algorithmic, planning-related, and reasoning-related. This comment highlights the necessity of combining neural systems and symbolic systems. Integrating both of these system types into a unified framework produces neural-symbolic systems, which can equip AI with the ability to perform perception and cognition.

Neural-symbolic systems have the combined advantages of both neural systems and symbolic systems [14]–[27]. In the below, this survey first summarize the characteristics, the advantages and disadvantages of symbolic systems and neural systems respectively (see Table I). From Table I, we can conclude that symbolic systems make good use of knowledge, while neural systems make good use of raw data; in short, they complement. Therefore, neural-symbolic systems are a good choice for scenarios in which the availability of training data is limited, or the model would benefit from more interpretability.

Furthermore, we provide an analysis of neural-symbolic systems from three perspectives: efficiency, generalization, and interpretability. As shown in Fig. 1 first, in terms of efficiency, neural-symbolic systems can reasoning rapidly, meaning that they can reduce exponential computational complexity to polynomial complexity. The powerful computation ability of neural networks can be attributed to this improved efficiency. For example, in a large-scale knowledge graph reasoning task, traditional symbolic methods would not be suitable, as their computational complexity increases exponentially with the size of the knowledge graph. Second, in terms of generalization, neural-symbolic systems are not significantly impacted by limitations on the amount of available training data, as they can leverage symbolic knowledge to compensate for this lack of data availability. For example, in a few-shot learning task, a neural-symbolic system uses symbolic knowledge as extra data to enrich the limited training samples, which enables the model to achieve the expected effect with decent generalization ability. Third, on the subject of interpretability, neural-symbolic systems can provide explicit computation processes, such as a traced reasoning process or a chain of evidence of results. For example, in medical diagnosis, machines are expected not only to make a decision, but also to show the reason for this decision in order to aid the doctor in the diagnosis. Neural-symbolic systems have become an important constitutive element of explainable AI and achieved superior performance in a variety of fields including computer vision
TABLE I
SUMMARIZE CHARACTERISTICS FOR THE SYMBOLIC SYSTEMS AND NEURAL SYSTEMS SEPARATELY.

| Systems                      | Cognitive ways | Knowledge representation | Primary algorithms | Advantages                                      | Disadvantages                      |
|------------------------------|----------------|--------------------------|--------------------|------------------------------------------------|-------------------------------------|
| Symbolic systems             | Deductive reasoning | Logical representation | Logical deduction | Strong generalization ability                  | Weak at handling unstructured data |
|                              |                 |                          |                    | Good interpretability                           | Weak robustness                    |
|                              |                 |                          |                    | Knowledge-driven                                | Slow reasoning                     |
| Neural systems (Sub-symbolic systems) | Inductive learning | Distributed representation | BP algorithms      | Strong at handling unstructured data            | Weak generalizability (adaptability) |
|                              |                 |                          |                    | Strong robustness                               | Lack of interpretability           |
|                              |                 |                          |                    | Fast learning                                   | Data-driven                        |

Fig. 1. The advantages of neural-symbolic systems with respect to model efficiency, generalization and interpretability. The neural network is a black-box system, while symbolic systems are white-box systems.

and natural language processing.

Challenges: Symbolic systems and neural systems differ in terms of the ways in which they represent data and solve problems. The former adopts discrete symbolic representation and classic search algorithms to find solutions. The latter uses continuous feature vector representation and neural cells to learn a mapping function. Therefore, the main research challenge for neural-symbolic systems is to design a framework that effectively combines both approaches. For example, a key challenge is that integrating the discrete symbol of symbolic systems with the continuous vector of neural systems. Focusing on this challenge, the present survey will introduce neural-symbolic systems further in the following sections.

Our contributions can be summarized as follows:

1) We propose a novel taxonomy of neural-symbolic systems. Neural-symbolic systems are categorized into three groups: learning for reasoning, reasoning for learning, and learning-reasoning.

2) We provide a comprehensive overview of modern neural-symbolic techniques, along with types and representations of symbols such as logic knowledge and knowledge graphs. For each taxonomy, we provide detailed descriptions of the representative methods and summarize the corresponding characteristics.

3) We discuss the applications of neural-symbolic systems and suggest four possible future research directions.

The remainder of this survey is organized as follows. Section II introduces the background knowledge. In Section III, we categorize the different methods of neural-symbolic systems. Section IV introduces the main technology of neural-symbolic systems. We summarize the main applications of neural-symbolic systems in Section V. Section VI discusses the future research directions, after which Section VII concludes this survey.

II. BACKGROUND KNOWLEDGE

In this section, we introduce background information related to symbolic knowledge. Specifically, we focus on two categories: logic knowledge and knowledge graphs. Logic knowledge can be further sub-divided into propositional logic and first-order logic.

A. Propositional logic

The propositional logic statements are declarative sentences that are either True or False. A declarative sentence is a True sentence if it is consistent with the facts; otherwise, it is a False sentence. The connectors between propositions are “∧”, “∨”, “¬” and “⇒”. Propositional logic can be expressed in the form of the following formula:

\[ P \Rightarrow Q, \]  \tag{1}

where \( P \) represents the antecedent (condition), while \( Q \) represents the consequent (conclusion).

Propositional logic is usually compiled in the form of directed acyclic graphs that are specific tasks. Conjunctive
Normal Forms (CNFs), deterministic-Decomposable Negation Normal Forms (d-DNNFs) [40], [41], and Sentential Decision Diagrams (SDDs) [42] are representative examples of knowledge representation, where SDD is a subset of d-DNNFs. For example, given a propositional logic Smoking ⇒ Cough, its CNF and d-DNNF graph are presented in Fig. 2(a) and Fig. 2(b), respectively.

**TABLE II**

| Proposition | First-order logic | Weight |
|-------------|------------------|--------|
| Smoking causes cough. | $F1 : \forall x, \text{Smokes}(x) \Rightarrow \text{Cough}(x)$ | 1.5 |
| If two people are friends, either both smoke or neither does. | $F2 : \forall x \forall y, \text{Friends}(x,y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$ | 1.1 |

Propositional logic cannot be used to describe complex problems; for these, predicate logic is required. In this paper, we only introduce first-order logic (FOL) [43]. FOL consists of four types of elements, connectors, and quantifiers. Four types of elements include constants, variables, functions, and predicates. Constants represent objects in the domain of interest (for example, father(a,b), a=Bob, b=Mara, a and b are constant). Variables range over the objects in the domain (for example, father(x,y), where x is the father of y, and the variable x is limited to the scope of the father class). Functions represent mappings from tuples of objects to objects. Predicates represent relations among objects in a given domain or attributes of these objects. The connector is the same as in propositional logic. FOL involves the combination of atoms through connectors, such that an expression can be written in the following form:

$$B_1(x) \land B_2(x) \land \cdots \land B_n(x) \Rightarrow H(x),$$  \hspace{1cm} (2)

where $B_1(x), B_2(x), \ldots, B_n(x)$ represents the rule body, which is composed of multiple atoms. $H(x)$ represents the rule head and is the result derived from the rule body.

Knowledge representation of FOL can be achieved using a Markov logic network (MLN) [44]. MLN is an undirected graph in which each node represents a variable, and the joint distribution is represented as follows:

$$P(X = x) = \frac{1}{Z} \exp \left( \sum_i w_i n_i(x) \right),$$  \hspace{1cm} (3)

where $Z$ represents the partition function, $w_i$ represents the weight of the rule, $n_i(x)$ represents the number of times that the value of the rule is true, and we use t-norm fuzzy logic [45] to calculate logical connectives.

The following introduces a simple example of an MLN. Table II shows the two rules ($F_1, 1.5, F_2, 1.1$) of this example [46]. Given a constant set $C = \{A, B\}$, the generated ground Markov logic network is shown in Fig. 3.

**C. Knowledge graph**

A knowledge graph is a directed and labeled graph. Nodes in this graph represent semantic symbols (concepts), such as animals, computers, people, etc; for their part, edges connect node pairs and express the semantic relationships between them, such as the food chain relationship between animals, friend relationship between people, etc. Knowledge graphs can be formally expressed in the form: $G = (H, R, T), \text{ here, } H = \{h_1, h_2, \ldots, h_n\}$ represents the set of head entities, and $n$ represents the number of head entities; $T = \{t_1, t_2, \ldots, t_m\}$ represents the set of tail entities, and $m$ represents the number of tail entities; moreover, $R = \{r_1, r_2, \ldots, r_k\}$ represents the set of relationships, and $k$ represents the number of relationships. Fig. 4 presents an example. For a given triplet (cat, attribute, paw), the nodes (head entity and tail entity) are cat and paw, and the relationship is an attribute.

A knowledge graph representation is used to encode discrete symbols (entities, attributes, relationships, etc.) into a low-dimensional vector space in order to obtain a distributed...
representation. Typical methods include R-GCN [47], M-GNN [48], CompGCN [49], TransE [50], TransR [51], TransH [52], RotatE [53], DisMult [54], ComplEX [55], ConvE [56], ConvR [57], GGNN [58], and GCN [59], among others.

III. Categorization and Frameworks

In this section, we first introduce the theory and definition of neural-symbolic systems, then summarize the taxonomy of neural-symbolic methods and the frameworks of different taxonomies.

Neural-symbolic systems are a hybrid model that combine neural networks and symbolic approaches in an attempt to capture the strengths of both fields. The schematic diagram in Fig. 2 illustrates the relationships and characterizes symbolic systems, neural systems, and neural-symbolic systems. In the figure, the green rectangle represents symbolic systems that are close to the ground truth from the perspective of reasoning. Symbolic systems typically take structured data as input (such as databases, logic rules, knowledge graphs, etc.), and their information processing unit is symbols. After training, symbolic systems obtain the solution space of the search algorithm for a specific task and output the reasoning results. The blue rectangle in the figure represents neural systems that are close to the ground truth from the perspective of the learning (data relevance). Neural systems typically take unstructured data as input (such as images, videos, texts, etc.), and their information processing unit is the feature vector. After training, neural systems learn a mapping function for a specific task and output the predicted results. The outer blue box represents neural-symbolic systems, which have the advantages of both systems. Neural-symbolic systems take a mixture of structured data and unstructured data as input. After training, neural-symbolic systems obtain solutions through a process of learning and reasoning.

Based on the above description, the final objective of neural-symbolic systems is to find a function $F$ that can effectively map the data $x$ in training set $D$ and symbol $s$ (which may be pre-defined or obtained via computation) to ground-truth $y$. The formal definition is as follows:

$$\forall (x, y) \in D \ F(x, s) \rightarrow y,$$

(4)

In this survey, neural systems of interest mainly utilize deep learning, while the term symbolic systems refers to symbolic knowledge [72]–[74] or symbolic reasoning techniques [42], [75], etc. The methodology of our classification is determined by the combination mode of neural systems and symbolic systems, which has three main combination methodologies.

The first category aims to search for solutions using symbolic systems (symbolic reasoning techniques) and integrates the advantages of the neural networks to assist in finding solutions, an approach referred to as learning for reasoning [12], [60], [61], [64], [76]–[84]. The second category uses neural systems to learn mapping functions and integrates the advantages of symbolic systems (symbolic knowledge) into the learning process to enhance the learning ability of neural systems, an approach referred to as reasoning for learning [28]–[31], [33], [34], [65]–[68], [85]–[88]. In the third category, neural systems and symbolic systems play equal roles and work together in a mutually beneficial way; this approach is named as learning-reasoning [8], [11], [37], [70], [71]. According to the taxonomy of neural-symbolic method presented in this paper, we summarize and analyze the existing main approaches from five dimensions in Table III: representative works, taxonomies, combination modes, symbols, and applications. Each of these methods will be described in more detail in Section IV.

A. Learning for reasoning

In Learning for reasoning, neural systems support the reasoning of symbolic systems; in short, technologies of symbolic systems are used to solve the problems in machine reasoning, and neural networks are introduced to assist in solving these problems. There are two key aspects of this approach. The first aspect is that neural networks reduce the search space of symbolic systems to accelerate computation [60], [61], [79]–[81]; for example, the search process of the symbolic reasoning technique is replaced by a neural network. The second aspect is that the neural network abstracts the unstructured data into symbols to facilitate effective symbolic reasoning [82]–[84]; for example, neural networks may abstract an unstructured image into a symbol in a reasoning task and feed it into statistical relational learning (SRL) [89] to infer relationships. The basic framework is shown in Fig. 6. As the figure shows, this type of model is characterized by serialization.

B. Reasoning for learning

Reasoning for learning involves symbolic systems helping to support the learning of neural systems. The basic idea
| Representative works | Taxonomies          | Combination modes | Symbols            | Applications                                      |
|----------------------|---------------------|-------------------|--------------------|--------------------------------------------------|
| pLogicNet [60]       | Learning for reasoning |                  | First-order logic | Knowledge graph reasoning                         |
| ExpressGNN [61]      |                     |                  |                    | Classification                                    |
| RNM [62]             | Learning for reasoning | Serialization     |                    | Knowledge graph reasoning                         |
| NMLN [63]            |                     |                  |                    | Classification and knowledge graph reasoning      |
| NLIL [64]            |                     |                  |                    | Visual question answering                         |
| NS-CL [7]            |                     |                  |                    | Object recognition                                |
| HDNN [65]            | Reasoning for learning | Parallelization  | Propositional logic| Visual relationship detection                     |
| SBR [66]             |                     |                  |                    | Classification                                    |
| SL [67]              |                     |                  |                    | Classification                                    |
| LENS [28]            |                     |                  |                    | Visual relationship detection                     |
| CA-ZSL [68]          | Reasoning for learning |                | Knowledge graph   | Few-shot classification                           |
| LSFSL [34]           |                     |                  |                    |                                                  |
| SEKB-ZSL [30]        |                     |                  |                    |                                                  |
| DGP [31]             |                     |                  |                    |                                                  |
| KGTN [33]            |                     |                  |                    |                                                  |
| PROLONETS [69]       |                     |                  | Propositional logic| Reinforcement learning                            |
| DeepProLog [8]       | Learning-reasoning  | Interaction      | First-order logic | Complex reasoning                                 |
| ABL [11]             |                     |                  |                    |                                                  |
| GABL [70]            |                     |                  |                    |                                                  |
| WS-NeSyL [37]        |                     |                  |                    |                                                  |
| BPGR [71]            |                     |                  |                    |                                                  |
centers around using the technology of neural systems to solve problems in machine learning, specifically by introducing symbolic knowledge into the training process to improve performance and interpretability [28], [65]–[67], etc. Training data and the structured symbolic knowledge are then fed into neural networks, where symbolic knowledge constrains or guides the model learning in the training process. For example, the symbolic knowledge may be represented as a loss regularization term of loss in a specific task. These approaches encode symbolic knowledge into neural network architectures, which results in a considerable loss of reasoning ability, especially in terms of interpretability. The basic principle is shown in Fig. 7. This type of model is called parallelization.

C. Learning-reasoning

The above two approaches each focus on only one direction of learning (neural systems to symbolic systems, or vice versa), and thus fail to maximize the strengths of these two paradigms. To mitigate this issue, several approaches have been developed in subsequent works that facilitate a bidirectional interaction between these two types of systems. The goal of learning-reasoning approaches is to balance the degree of participation of neural systems and symbolic systems in the problem-solving process. For example, granting symbolic reasoning characteristics such as abduction facilitates the design of a kind of connection between deep neural networks and the symbolic reasoning framework [11], [14], [71]. More specifically, the output of the neural networks is an input of the symbolic reasoning, and the output of the symbolic reasoning is as input of the neural networks. The basic principle is illustrated in Fig. 8. We can thereby determine that learning-reasoning is a mode in which both types of technologies are combined alternately.

IV. METHODS OF NEURAL-SYMBOLIC SYSTEMS

This section introduces the methods used in neural-symbolic systems in three main categories. We aim to distill the representative ideas that provide evidence for the integration between neural networks and symbols, identify the similarities and differences between different methods, and offer guidelines for researchers. The main characteristics of these representative methods are summarized in Table IV.

A. Learning for reasoning

Learning for reasoning methods leverage neural networks to accelerate the search speed of symbolic reasoning, or to abstract unstructured data for symbolic reasoning. To accelerate the search space of symbolic reasoning, we introduce approaches based on SRL, such as pLogicNet [60] and ExpressGNN [61]. These approaches use the neural networks to parameterize the posterior computation of the probabilistic graphical models, which reduces the solution search space to accelerate the process. We investigate several methods based on ILP, such as NLIL [64], which NLIL automatically induces new logic rules from the data for model learning and reasoning. Furthermore, for abstract unstructured data, we introduce several representative approaches such as NS-CL [7]. Specific details about the different models are presented below.

1) Accelerating symbolic reasoning: Markov logic networks (MLNs) can encode logic rules into an undirected graph and employ inferred technologies of graphical models to implement problem-solving. However, it is difficult to compute on large-scale graphic models; while it is easy to scale neural networks to large datasets, the logic knowledge cannot be used directly. Although a variety of approximate inference methods have been proposed [87], [90]–[94], the computational cost remains high. Under these circumstances, researchers have successively proposed probabilistic Logic Neural Networks (pLogicNet) [60] and ExpressGNN [61], respectively. Both of these models aim to frame the problem of reasoning in the knowledge graph (triplet completion problem) as an inference problem involving hidden variables in the probability graph. Both ideas adopt a combination of variational EM and neural networks to approximate the inference. The basic process is as follows:

1) Constructing a graph. Available rules are modeled an undirected graph represented by MLN, such as in the factor graph presented in Fig. 9. The circular nodes in the figure represent ground atoms of logic rules, and the square nodes represent the factors (each rule corresponds to a factor node).

2) Determining observed variables and hidden variables in an undirected graph. Triplets in a knowledge graph correspond to observed variables (which in turn correspond to white nodes), and are hidden variables otherwise (corresponding to the gray
| Approaches     | Inputs | Technology | Tools     | Mechanism/Objective                                                                 |
|---------------|--------|------------|-----------|-------------------------------------------------------------------------------------|
| pLogicNet [60] | x,s    | SRL        | MLN       | learn a joint probability distribution                                              |
| ExpressGNN [61]| x,s    | SRL        | MLN       | learn a joint probability distribution                                              |
| NLIL [64]     | x      | ILP        | Transformer | use a Transform to learn rules based ILP                                             |
| NS-CL [7]     | x      | quasi-symbolic program | concept parser | reason based on parsing symbols for images and questions                             |
| HDNN [65]     | x,s    | regularization | t-norm   | learn a student network based on knowledge                                          |
| SBR [66]      | x,s    | regularization | t-norm   | learn a model with logic knowledge as a constraint of the hypothesis space          |
| SL [67]       | x,s    | regularization | arithmetic circuits | design a semantic loss to act as a regularization term                              |
| LENS [28]     | x,s    | regularization | d-DNNF   | align distributions between deep learning and propositional logic                   |
| CA-ZSL [68]   | x,s    | regularization | GCN      | learn a conditional random field                                                    |
| SEKB-ZSR [30] | x,s    | knowledge transfer | GCN      | learn a deep learning model with powerful generalization                            |
| DGP [31]      | x,s    | knowledge transfer | GCN      | learn network embedding with semantics                                              |
| KGTN [33]     | x,s    | knowledge transfer | GGNN     | transfer semantic knowledge into weights                                            |
| PROLONETS [69]| x,s    | knowledge transfer | decision tree | transform knowledge into neural network parameters                                    |
| DeepProLog [8] | x,s    | ProbLog    | SDD       | construct an interface between probLog program and the deep learning models         |
| ABL [11]      | x,s    | abductive reasoning | SLD      | minimize inconsistency between pseudo-labels and symbolic knowledge                |
| WS-NeSyL [37]| x,s    | ProLog     | SDD       | learn an encoder-decoder constrained by logic rules                                  |
| BPGR [71]     | x,s    | SRL        | MLN       | learn a model that fits both the ground truth and FOL                               |
nodes. (3) Calculating the probability. The probability of a query hidden variable can be calculated by the joint probability distribution of the undirected graph.

Based on the above basic process, ExpressGNN improves the inference network of pLogicNet by using a graph neural network (GNN) to replace the flattened embedding table and adding a tunable part to the entity embedding to alleviate the problem of isomorphic nodes having the same embedding. The specific framework of ExpressGNN is illustrated in Fig. 9. ExpressGNN adopts variational the EM algorithm to calculate the probability that a latent variable is true, and completes the knowledge graph.

The training process of ExpressGNN is interactive: the weight of rules in symbolic space can act as known information to assist in predicting hidden variables in a continuous space. In addition, ExpressGNN uses a neural network to parameterize variational posterior, called neuro-variational inference, which greatly improves the efficiency of inference. Fig 10 illustrates the process of neuro-variational inference. Here, the entity embedding learned by GNN is fed into a relation prediction network to attain the labels of the triplets. \( \theta_1 \) represents the parameters of GNN, \( \theta_2 \) is the parameters of the relation prediction network, \( \theta \) denotes the tunable factor, and \( r_k \) represents the \( k \)-th relation.

![Fig. 9. The framework of the ExpressGNN model. The model contains continuous space and symbolic space, and the variational EM algorithm acts as a bridge that connects the continuous space and symbolic space.](image)

![Fig. 10. Schematic diagram of neural variation inference process. This process is included in the inference process of the symbolic space MLN in Fig. 2 which replaces the variational posterior calculation of the traditional EM algorithm.](image)

The preconditions of the above methods are that the logic rules are known. Relying solely on the knowledge rules constructed by experts does not allow for the knowledge to be completely captured in the data. For this reason, researchers have begun to investigate ways in which logic rules can be automatically captured from data. Marra et al. [63], [62] extended MLN and designed a general neural network to automatically learn logic rules, such that the potential functions of MLN can be learned from the original data. In addition to the above methods, some scholars have proposed differentiable ILP based on the traditional ILP [95] method, this approach combines neural networks and logic [12], [96]–[99]. For example, \( \partial \text{ILP} \) [12] is a forward reasoning method that primarily uses a set of predefined templates to construct logic rules. This method applies logic rules multiple times on background data to infer new facts for evaluation. Even when encountering noisy data, \( \partial \text{ILP} \) can still learn effective logic rules.

In order to remain computationally feasible, the current methods [100], [102] express the chain rule as a Horn clause and control the length of the search, along with the number of relationships and entities. These approaches are negatively impacted by the limited expressive power of their complex logic rules. To solve these problems, Yang et al. [64] proposed neural logic inductive learning (NLIL), which can learn complex logic rules (such as tree and conjunctive rules, etc.) and also explain the patterns in data through learned logic rules. NLIL extends the multi-hop reasoning framework for general ILP problems and is a differentiable ILP model.

In NLIL, logic rules are grounded through matrix multiplication. For example, consider the logic rule \( \text{Friends}(x, y) \Rightarrow \text{Smokes}(x) \), where constants \( C = \{A, B\} \) are one-hot vectors, such as \( V_A \) and \( V_B \), and predicates \( \text{Friends} \) and \( \text{Smokes} \) are mapped to matrices such as \( M_{\text{Friend}}(A, B) \) is a score of \( A \) and \( B \) that are related by \( \text{Friend} \). The score of grounding is \( V_B = V_A M_{\text{Friend}} \).

NLIL implements a divide-and-conquer strategy, decomposing the search space into three subspaces in a hierarchical manner. Each of these subspaces adopts an attention mechanism for effective search, then calculates the score of each logic rule parameter through the weighted summation of the attention networks. In more detail, the rule generation process is mainly carried out to run the transformer model, which consists of a stack of three transformers. Each transformer is a multi-head attention module that generates a set of weights. Finally, it uses the generated weights and hierarchical structure to execute multi-hop reasoning.

2) Abstracting unstructured data into symbols: Mao et al. [7] proposed the Neuro-Symbolic Concept Learner (NS-CL), which uses neural symbolic reasoning as a bridge to jointly learn visual concepts, words, and the semantic parsing of sentences without the need for the explicit supervision of any of them. NS-CL builds an object-based scene representation and translates sentences into symbolic programs.

NS-CL is designed to solve tasks in visual question answering (VQA). This model includes three modules: a visual perception module, semantic parsing module, and symbolic reasoning module. The visual perception module extracts object-based symbolic representation for a scene in an image. The semantic parsing module transforms a question into an executable program. Finally, the symbolic reasoning module applies a quasi-symbolic program executor to infer the answer based on the representation and an executable program.

An example is shown in Fig. 11. Given the input image and question, the visual perception module uses a pretrained mask R-CNN [103] to generate object features for all objects, then applies a similarity-based metric to classify objects. The semantic parsing module translates a natural language question
into an executable program with a hierarchy of primitive operations, represented in a domain-specific language (DSL) designed for VQA. The symbolic reasoning module is a collection of deterministic functional modules designed to realize all logic operations specified in the DSL. It takes object concepts and a program as input to derive the answer. The optimization objective of NS-CL is composed of both a visual perception module and a semantic parsing module.

Fig. 11. An illustration of NS-CL. The perception module begins by parsing visual scenes into object-based deep representations, while the semantic parser parse sentences into executable programs. A symbolic reasoning process bridges two modules.

**Conclusion:** Based on the related works discussed above, we summarize two principles underpinning learning for reasoning approaches with a particular focus on the way in which a neural network should be integrated. (1) Accelerator. Symbolic reasoning technologies need to search large spaces to arrive at an expected solution. To accelerate solution search process, neural networks can replace traditional search techniques with an accelerator, such as reinforcement learning, etc. (2) Transformer. Symbolic systems need to take symbols as input. When dealing with unstructured data, neural networks can map these data into symbols to facilitate their use as input for symbolic reasoning.

**B. Reasoning for learning**

Reasoning for learning methods can be broadly divided into regularization models and knowledge transfer models. Regularization models add symbolic knowledge in the form of the regular terms to the objective function of training model and as a kind of prior to guide the training of the model [28], [29], [31], [65]–[68], [85], [86], [88]. In addition, regularization models differ regarding the ways in which they model symbolic knowledge as regular terms, as we will discuss in more detail below. The knowledge transfer models establish the connection between the visual space and the semantic space, then transfers the symbolic knowledge of the latter to the former to support the model’s learning process [30], [31], [33].

1) Regularization models: Hu et al. [65] proposed a general framework called harnessing deep neural networks with logic rules (HDNN), which adopts the idea of knowledge distillation (Guiding the learning of small-scale networks through a large-scale and well-trained network) to support the model design. The teacher network encodes logic rules and guides the student network during training. That is to say, the teacher network learns information from the labeled data and logic rules (unlabeled data), and teaches student network by loss function. Based on the above process, the structured information encoded by logic rules can constrain the learning of the student network. The framework of HDNN is illustrated in Fig. 12.

In the figure, the rule knowledge distillation module includes the student network \( p_\theta(y|x) \) and the teacher network \( q(y|x) \). The input of the teacher network is labeled data and unlabeled data (logic rules), while the input of the student network is labeled data. The red dotted line signifies that the student network indirectly receives encoded logic rules information-that is, by balancing the output of the teacher network and the student network to update the parameter \( \theta \) of the student network. The objective function is shown in Eq. [5].

To model the logic rules used to perform the numerical calculation, HDNN uses soft logic to encode the FOL in the teacher network construction module. To introduce logic rules into the student network \( p_\theta(y|x) \), the model needs to meet the following conditions: 1) The probability distribution \( p_\theta(y|x) \) of the student network should be as close as possible to the probability distribution \( q(y|x) \) of the teacher network; 2) The teacher network should obey the logic rules to the greatest extent possible. The formal representation of the two conditions is shown in Eq. [5]. The whole model is trained iteratively. The point of difference from the original knowledge distillation model is that the teacher network and the student network can learn at the same time.

\[
\theta(t+1) = \arg\min_{\theta \in \Theta} \frac{1}{N} \sum_{n=1}^{N} \left( (1 - \pi)l(y_n, \sigma_\theta(x_n)) + \pi l(s_n^{(t)}, \sigma_\theta(x_n)) \right),
\]

\[
\begin{align*}
&\min_{q, \xi > 0} KL(q(y|x)||p_\theta(y|x)) + C \sum_{l \in G} \xi_{l,l} \left( s_l^{(t)} - q_l(x) \right) \\
&\lambda_l \left( 1 - E_q[r_l(x,y)] \right) \leq \xi_{l,l} \\
&g_l = 1, \ldots, G_l, l = 1, \ldots, L.
\end{align*}
\]

In Eq. [5], \( \pi \) is the limitation parameter used to calibrate the relative importance of the two objectives; \( x_n \) represents the training data, while \( y_n \) the label of the training data; \( l \) denotes the loss function selected according to specific applications (e.g., the cross-entropy loss for classification); \( s_n^{(t)} \) is the soft prediction vector of \( q(y|x) \) on \( x_n \) at iteration \( t \); \( \sigma_\theta(x) \) represents the output of \( p_\theta(y|x) \); the first term is the student network, and the second term is the teacher network. In Eq. [5], \( \xi_{l,l} \geq 0 \) is the slack variable for the respective logic constraint; \( C \) is the regularization parameter; \( l \) is the index of the rule; \( g_l \) is the index of te ground rule; \( \lambda_l \) is the weight of the rule.

Adopting a different approach to the knowledge distillation framework, some approaches use logical knowledge as a constraint of the hypothesis space. For example, these approaches turn a logic formula (either propositional or first-order) into a real-valued function that is used as a regularization term of the neural model. Diligenti et al. [66] proposed semantic-based regularization (SBR). SBR combines classic machine learning (with continuous feature representation learning ability) and SRL (with advanced semantic knowledge reasoning ability) to solve problems such as multi-task optimization and classifica-
tion. The method of SBR involves learning from constraints. Constraints are described prior knowledge in FOL. The paper assumes that a set of $H$ functional constraints in the form $1 - \phi_h(f) = 0$, $0 \leq \phi_h(f) \leq 1$, $h = 1,...,H$ are provided to describe how the query functions should behave. $f$ indicates the vector of the functions. Following the classical penalty approach for constrained optimization, constraint satisfaction can be enforced by adding a term that penalizes violation of these constraints into the loss of the model.

Based on SBR, Xu et al. [67] proposed semantic loss (SL). SL combines the automatic reasoning technology of propositional logic with existing deep learning architecture, such that the output of the neural network is fed into the loss function as a constraint of the learnable neural network. The propositional logic is encoded into the loss function of the neural network by a training algorithm to improve the network’s learning ability. Adopting a different approach to SBR regarding the regularization term of the loss, SL utilizes semantic loss, which is in essence another regularization term that can be directly plugged into an existing loss function. The definition of SL is given in Eq. (7); it uses an arithmetic circuit [42] to evaluate the model. Compared to SBR, the regularization term of the loss is derived from the data and therefore more precise.

$$L^*(\alpha, p) \propto -\log \sum_{x \models \alpha} \prod_{x_i \models \alpha_i} p_i \prod_{x_i \models \neg \alpha_i} (1 - p_i),$$

where $p$ is a vector of probabilities from the prediction of the neural network, $\alpha$ is a propositional logic, $x$ is the instantiation of $X$, and $x \models \alpha$ is a state $x$ that satisfies a sentence $\alpha$.

Notably, the above methods are not explicit knowledge representation methods, which results in an unclear computing process for symbolic knowledge. To solve this problem, some researchers have opted to use tools that can model symbols, such as d-DNNF, etc. Xie et al. [28] integrated propositional logic into the relationship detection model and proposed a logic embedding network with semantic regularization (LENSR) to improve the deep model’s relationship detection ability. The process of LENSR can be briefly summarized as follows: 1) The visual relationship detection model predicts the probability distribution of the relation predicate for each image; 2) The prior propositional logic formula related to the sample image is expressed as a directed acyclic graph by d-DNNF, after which GNN is used to learn its probability distribution; 3) An objective function is designed that aligns the above two distributions.

Fig. [13] presents a schematic diagram of LENSR, which uses a propositional logic of the form $P \Rightarrow Q$. In this example, the predicate $P$ represents $\text{wear(person, glasses)}$, and the predicate $Q$ represents $\text{in(glasses, person)}$. The ground truth of the input image and the corresponding propositional logic (prior knowledge) are on the left. The directed acyclic graph of propositional logic d-DNNF is then sent to the Embedder $q$ (Embedder is a graph neural network (GNN) [59] that learns the vector representation.) to obtain $q(F_x)$, which is the embedding of the propositional logic knowledge. On the right is the relation label predicted by the GNN. The predicted labels are then combined into a conjunctive normal form $h(x) = \land p_i$ to construct a directed acyclic graph d-DNNF, which is sent to the Embedder $q$ to obtain $q(h(x))$, the embedding of the predicted propositional logic. The optimization goal of LENSR is shown in Eq. (8). Here, $L_{\text{task}}$ represents the loss of a specific task, $\lambda$ is a hyperparameter that acts as a balance factor, and $L_{\text{logic}}$ is the loss of a propositional logic (that is, the distance between vector $q(Fx)$ and vector $q(h(x)))$.

$$L = L_{\text{task}} + \lambda L_{\text{logic}},$$

$$L_{\text{logic}} = \|q(f) - q(\land p_i)\|_2,$$
aware zero-shot recognition method (CA-ZSL) to solve the zero-shot detection problem. CA-ZSL builds a model based on deep learning and conditional random fields (CRF), and uses knowledge graphs (the semantic relationship between classes) to assist in identifying objects of unseen classes. The framework of the CA-ZSL is illustrated in Fig. 12. First, the individual and pairwise features are extracted from the image. Next, the instance-level zero-shot inference module adopts individual features to generate a unary potential function, and the relationship inference module uses pairwise features and knowledge graphs to generate an binary potential function. Finally, based on the conditional random field constructed by the two potential functions, the label of the unseen objects is predicted.

CA-ZSL integrates a knowledge graph, comprising the GCN-encoded embedding, into the calculation of the binary potential function of the conditional random field, which supports the learning of the model. The optimization goal is to maximize the joint probability distribution of the conditional random field, as shown in Eq. (9); here $\theta$ is the unary potential function, $\psi$ represents the binary potential function, $c_i$ represents the class, $B_i$ represents the object region in the image, $\gamma$ is the balance factor, and $N$ is the number of objects.

$$P = \frac{1}{Z} \exp \left( \sum_i \theta(c_i, B_i) + \gamma \sum_{i < j} \psi(c_i, c_j, B_i, B_j) \right)$$  \tag{10}

For its part, DGP makes certain improvements to SEKB-ZSL. First, to address the over-smoothing problem of GCN, the six-layer graph convolution in the SEKB-ZS model is reduced to two-layers. Moreover, to enhance the connection between nodes in the knowledge graph, the attention mechanism is used to calculate the connection weights between the nodes. The principle of DGP is illustrated in Fig. 15.

In more detail, the knowledge graph is fed into the dense graph propagation module, where the initial feature is the word vector of the class. In the process of performing graph convolution, the dense graph propagation module uses two aggregation modes—ancestor propagation and offspring propagation respectively—to update the features of nodes and trains a semantic classifier. The image is fed into the ResNet module to extract its visual features and train a visual classifier. To obtain the ability to recognize unseen classes in the ResNet module, DGP uses the weights of the semantic classifier to supervise the weights of the visual classifier during the training process. The entire model is trained in an end-to-end fashion. The loss function is the similarity of the weight of the two module classifiers, as shown in Eq. (11). Here, $M$ is the number of classes, $P$ is the dimension of the weight, $W_{i,j}$ represents the weights of the visual classifier, and $W'_{i,j}$ represents the weights of the semantic classifier.

$$L = \frac{1}{2M} \sum_{i=1}^{M} \sum_{j=1}^{P} (W_{i,j} - W'_{i,j})^2$$ \tag{11}

The framework of CA-ZSL is illustrated in Fig. 14. The features of individual objects as well as and pairwise features are extracted from the image and input into an instance-level zero-shot inference module and a relationship inference module respectively. In combination with the knowledge graph, the unary potential function and binary potential function of CRF are generated respectively to predict the labels of objects. The figure is from reference [31].

2) Knowledge transfer models: Knowledge transfer integrates knowledge graphs that represents semantic information into neural network models, which compensates for the lack of available data through the transfer of semantic knowledge. In this survey, we mainly introduce representative approaches in zero-shot learning [30], [31], few-shot learning [104], [105], [33] and reinforcement learning [69] for knowledge transfer.

In visual tasks, considering that semantic information can make up for the deficiencies of visual data, researchers have successively proposed several zero-shot recognition models that combine semantic representation and knowledge graphs. These are SEKB-ZSL (zero-shot recognition via semantic embeddings) [30] and DGP (dense graph propagation module) [31]. Both models use the semantic classifier weight of the knowledge graph of all seen and unseen classes to constrain or supervise the learning of the visual classifier weights, thereby achieving knowledge transfer. The basic principles are as follows: In the visual space, SEKB-ZSL uses CNN to extract the visual features of the images, and learns a visual classifier of the seen class. In the semantic space, GCN is used to learn the node features of the knowledge graph, and a class semantic classifier is obtained. In turn, the weight of the class semantic classifier is used to supervise the process of learning the visual classifier weights in order to realize the transfer of the semantic knowledge of new classes.

Transferring the correlation information between classes can help with the learning of new concepts. DGP shows that the semantic classifier should be consistent with the feature classifier. Accordingly, Chen et al. [33] opted to employ knowledge graphs to model the correlations between seen and unseen classes, then combine them with neural networks to
propose a knowledge graph transfer network model (KGTN) to address the few-shot classification problem (see Fig. [16]).

KGTN comprises three main parts: the feature extraction module, knowledge graph transfer module, and prediction module. The feature extraction module uses CNN to extract the feature vector of images. The knowledge graph transfer module uses a gated graph neural network (GGNN) to learn the knowledge graph node embedding: here, the nodes of the knowledge graph include seen and unseen classes, the edges represent the semantic relevance between nodes, and the weights on the edges represent the learnable parameters that are randomly initialized before training. When training the model, for iterations \( t = 0 \) to \( t = T - 1 \), each node aggregates information from neighboring nodes. After \( T \) iterations, the knowledge graph transfer module obtains the final weight \( w^* \), which has captured the correlation between the seen and the unseen classes. The prediction module calculates the similarity between the weight \( w^* \) and the image feature to predict the probability distribution of the label.

Domain knowledge, unlike the above approaches, can be modified dynamically. Silva et al. [69] proposed Propositional Logic Nets (PROLONETS), which directly encodes domain knowledge as a set of propositional rules into a neural network, and can rectify domain knowledge using a trained neural network. The framework is shown in Fig. [17].

PROLONETS helps to “warm start” the learning process in deep reinforcement learning. The first step is knowledge representation. Policies and actions express domain knowledge in the form of propositional rules, which are encoded in the form of a decision tree. The second step is neural network initialization. By directly transforming nodes of the tree into neural network weights, an agent can immediately begin learning productive strategies in reinforcement learning. The final step is training. When initialized, the network interacts with the environment to collect data, which is used to update parameters and rectify domain knowledge.

Let us consider the cart pole as an example. The state space of a cart pole is a four-dimensional vector: cart position, cart velocity, pole angle and pole velocity. The action space is a two-dimensional vector (left, right). Domain knowledge can be expressed as “if the cart’s position is right of center, move left; otherwise, move right.”. The decision nodes of the tree become linear layers, leaves become action weights, and the final output is a sum of the leaves weighted by path probabilities. Therefore, if \( \text{position} > -1 \), the weight of the neural network is \( w = \{1, 0, 0, 0\} \), and the bias is \( b = -1 \).

**Conclusion:** Based on the above text, we can summarize the following key factors in reasoning for learning. (1) Knowledge representation. Symbolic knowledge is a kind of discrete representations. Most methods of combining symbolic knowledge with neural networks opt to convert the symbolic knowledge into an intermediate representation, such as a graph, to obtain a continuous representation. Moreover, some approaches use fuzzy logic such as t-norm to assign soft truth degrees in the continuous set \([0, 1]\). (2) Combining approaches. One type of approach involves taking symbolic knowledge as a regularization term in the loss of the neural networks. The others involve integrating symbolic knowledge into the structure of the neural networks to improve their performance.

### C. Learning-reasoning

In learning-reasoning approaches, learning and reasoning do not work in isolation but instead closely interact. This is a development trend of neural-symbolic systems [8], [11], [70], [37], [71].

Probabilistic logic programming (ProbLog) [106] is an extension of the probabilistic logic language (ProLog). ProLog and ProbLog consist of logical facts and logical rules. Based on ProLog, the logic facts and rules of the ProbLog have given probabilities, so that the inference task is no longer a binary classification (“yes” or “no”), but instead contains values that improve the expressive ability of the model. Based on ProbLog, Robin, et al. [8] added neural facts and neural annotated disjunction (neural AD), proposing a model that organically combines probability, logic, and deep learning, called DeepProbLog. DeepProbLog is a framework that combines general neural networks with probabilistic logic in a specific way for the first time. It has the advantage of stronger expressive ability and, can achieve end-to-end training for neural networks and logic reasoning together.

DeepProbLog is a kind of probabilistic programming language that is combined with deep learning in the form of “neural predicates”. These “neural predicates” are an interface between neural networks and symbolic reasoning. For example, an image is fed into the neural network, which outputs the distribution of each class in the dataset as logical facts in the symbolic reasoning. More concretely, neural networks are used to process simple concepts or unstructured data to form the...
input of the symbolic reasoning in DeepProbLog. Symbolic reasoning uses SDD [42] to build a directed graph, which it transforms into an arithmetic circuit to infer results for queries. To achieve end-to-end training between continuous embedding and discrete symbols, DeepProbLog utilizes gradient semiring [107] as a tool to finish optimization. The framework of DeepProbLog is illustrated in Fig. 18.

In Fig. 18, the root node is the query. Given a query and knowledge in ProbLog, DeepProbLog needs to transform the knowledge in ProbLog into nodes of the directed graph, where the leaf nodes are “neural predicates”. When computing the gradient, DeepProLog calculates the loss using cross-entropy, starting from the root node and propagating the error via stochastic gradient descent. During the training process, there is no direct label information for the machine learning, which is weakly supervised learning.

Next, ABL uses ProLog as KB and adopts abductive reasoning technology to abduct the pseudo-labels and rules. That is to say, the logical reasoning minimizes the inconsistency between the symbolic representation and the KB to revise pseudo-labels, then outputs the deductive labels. For example, $\neg C$ is revised to C in Fig. 19 which is the result of deduction. Finally, a new classifier is trained by the deductive labels and the raw data, which replaces the original classifier. The above is an iterative process that continues until the classifier is no longer changed or the pseudo-labels are consistent with the KB. ABL is a special kind of weakly supervised learning, in which the supervision information comes not only from the ground-truth labels, but also from knowledge abduction.

Based on the ABL framework, Tian et al. [37] proposed a weakly supervised neural symbolic learning model (WS-NeSyL) for cognitive tasks with logical reasoning. The framework of WS-NeSyL is similar to that of ABL. The difference between them is that ABL uses a metric of minimal inconsistency in logical reasoning, while WS-NeSyL adopts sampling technology. In WS-NeSyL, to provide supervised information for the reasoning process in complex reasoning tasks, the neural network is designed as an encoder-decoder framework, which includes an encoder and two decoders (perceptive decoder and cognitive decoder). The encoder can encode input information as a vector. The perceptive decoder decodes the vector to predict labels (pseudo-labels). Based on these pseudo-labels and the sampled logic rules, the cognitive decoder reasons inclusions. To supervise the reasoning of the cognitive decoder, WS-NeSyL provides a back search algorithm to sample logic rules from the knowledge base to act as labels that are used to revise the predicted labels. To solve the sampling problem, WS-NeSyL introduces a regular term of logic rules. The whole model is trained iteratively until convergence.

![Fig. 18. Framework of DeepProLog. Machine learning is responsible for mapping the input (unstructured data or simple structured data) to the distribution of categories (if there are n categories, the output distribution is 1xn). Logical reasoning is a complex problem described by ProbLog, which is constructed as an arithmetic circuit to solve the complex problem. Here, the root node is the query, while the leaf nodes are neural predicates and other (non-neural network output) probabilistic facts.](image)

![Fig. 19. Framework of abductive learning. It combines machine learning and logical reasoning iterative learning, bring the two technologies together to solve specific problems. The figure is from reference [11].](image)

The knowledge base is an important factor in logical reasoning, and different knowledge bases are used by different reasoning technologies. The above approaches use probabilistic logic programming language (ProbLog) as their knowledge base; notably, they only consider that neural networks can provide facts for the knowledge base, and do not quantify how many logic rules should be triggered by the neural networks. To resolve this issue, Yu et al. [71] proposed a bi-level probabilistic graphical reasoning framework, called BPGR. To quantify the amount of symbolic knowledge that is triggered,
BPGR uses MLN to model all logic rules. For instance, MLN can express the time at which a logic rule is true in the form of a potential function.

BPGR includes two parts: the visual reasoning module (VRM) and the symbolic reasoning module (SRM). VRM extracts the features of objects in images and the inferred labels of objects and relationships. SRM uses symbolic knowledge to guide the reasoning of VRM in a good direction, which acts as an error correction. In terms of the model framework, more concretely, SRM is a double-layer probabilistic graph that contains two types of nodes: one is the reasoning results of the VRD model in the high-level structure, and the other is the ground atoms of logic rules in the low-level structure. When the probabilistic graphical model is constructed, BPGR can be efficiently trained in an end-to-end manner by the variational EM algorithm. An overall framework of BPGR is provided in Fig. 20.

**Conclusion:** Learning-reasoning approaches are increasingly popular in AI research, as they enjoy the advantages of both neural networks and symbolic reasoning: specifically, the neural networks provide facts with symbolic reasoning, and the symbolic reasoning constrains/helps learning of the neural networks. For instance, DeepProbLog and ABL have similar model principles: that is, the modeling of complex problems is defined in logic programming language, and the neural network is used to define simple concepts in logic programming language. These operate in a unified framework in neural-symbolic systems. BPGR uses neural networks to accelerate the search process of symbolic reasoning, along with symbolic knowledge to constrain neural network learning. This model not only characterizes the degree of matching, but also clearly states which symbolic knowledge is being fitted, as well as the learned symbols from neural networks into symbolic systems. We also summarize the characteristics that should be considered in designing neural-symbolic approaches, as follows: (1) **Uncertainty.** The output of the neural network is a distribution, not “true” or “false”. This results in a need to consider the uncertainty of triggered symbolic knowledge. (2) **Globalization.** It is necessary to consider the fit of all symbolic knowledge in the knowledge base, not just the local knowledge. (3) **Importance.** Different knowledge may have different weights, and the degree of fitting knowledge with different weights should be considered. (4) **Interpretability.** Interpretability should be explicitly considered in learning (in terms of e.g. the immediate process of learning of the result of learning).

V. APPLICATIONS

A. Object/visual-relationship detection

The goal of object/visual-relationship detection is to identify objects, or relationships between objects, in images. If a system only uses visual features to train a model, this will result in a relatively weak performance. Recently, with the emergence of neural-symbolic systems, newer works have introduced external knowledge to improve the detection performance of the model.

Donadello et al. [29] combines neural networks with first-order logic to design a logic tensor network (LTN). By leveraging logical constraints, this approach can not only reason effectively from noisy images, but also use logical formulas to reason and describe the characteristics of data. Therefore, LTN provides the interpretability of image recognition. Marszalek and Forestier et al. [108], [109] suggested adopting methods based on symbolic knowledge to improve object detection ability. Specifically, they proposed using knowledge provided by experts for remote sensing image interpretation of coastal areas and lexical semantic networks respectively. Zhu and Nyga et al. [32], [110] use MLN to model symbolic knowledge for integration into deep models, an approach that involves learning a scoring function and predicting the relations between the input image and the person. For example, given an input image of a horse, the model can predict that its relation with people is “ridable”.

B. Knowledge graph reasoning

Knowledge graphs are often incomplete, meaning that it is often necessary to perform completion (link prediction) to improve their quality. Zhang et al. [111] survey the advantages of knowledge graph reasoning on in the context of neural-symbolic systems. Wang et al. [112] transform a triplet or a ground-rule into FOLs, then score this FOL by performing certain vector/matrix operations based on the embeddings of the entities and relationships included in the FOL. Some path-based reasoning approaches [109], [113]–[116] work to extend the multi-hop neighbors around the head entity, and then predict the answers included in these neighbors, which are found by neural networks. For example, DeepPath [114] uses reinforcement learning to evaluate the sampled paths, which can reduce the search space. To extend path-based
reasoning approaches, Teru et al. [17] proposed the graph-based reasoning framework GraIL. GraIL extracts a subgraph of the k-hop neighbors of the head entry and tail entry. Subsequently, a GNN is applied to an extracted subgraph to reason the relationship between two entities.

C. Classification/ few-shot classification

Marra et al. [62] proposed Relational Neural Machines (RNM), a novel framework that enables jointly training the parameters of the learners and the first-order logic based reasoner. To solve the few-shot problem, Sikka et al. [118] integrate common sense knowledge into deep neural networks, and also use logical knowledge as a new neural-symbolic loss function to regularize visual semantic features. This method obtains information from unseen classes in model learning to improve the zero-shot learning ability. Altszuler et al. [119] incorporate logic rules into the neural network framework for multi-domain dialogue recognition tasks, thereby enabling the model to recognize labels of unseen classes without the introduction of new training data.

D. Intelligent question answering

Intelligent question answering is one of the main applications of neural-symbolic reasoning in natural language processing. When given a question (possibly a combined question), an intelligent question answering model should be able to infer the answer from context (composed of text and images).

For non-synthetic question answering tasks on open-domain text, Gupta et al. [120] extend neural module networks (NMN) [121] and propose an unsupervised auxiliary loss to help extract arguments associated with the events in the text. Specifically, this method introduces a reasoning module for text that enables symbolic reasoning (such as arithmetic, sorting, and counting) on numbers and dates in a probabilistic or differentiable way, allowing the model to output logical parsing of questions and intermediate decisions.

Hudson et al. [122] propose a fully differentiable network model (MAC) with cyclic memory, attention, and composition functions. The MAC provides strong prior conditions for iterative reasoning, transforms the black-box architecture into something more transparent, and supports interpretability and structured learning. The core concept is to decompose the image and the question into sequential units, input the recurrent network for sequential reasoning, and then store the result in the memory unit to calculate the final answer together with the question. Tran and Poon et al. [123, 124] propose a model domain common sense with MLN and use probabilistic inference methods for the query. Sun et al. [125] learned a neural semantic parser and trained a model-agnostic model based on meta-learning to improve the predictive ability of language question-answering tasks cases involving limited simple rules.

Oltramari et al. [126] proposed integrating neural language models and knowledge graphs in common-sense question answering. Based on the architecture of the language model, this work proposes an attention-based knowledge injection method. For visual question answering tasks, Hudson et al. [127] propose the neural state machine (NSM). NSM uses a supervised training method to construct a probabilistic scene graph based on the concepts in an image, then performs sequential reasoning on the probabilistic scene graph, answering questions or discovering new conclusions.

E. Reinforcement learning

Deep reinforcement learning is a trending topic in the field of artificial intelligence and methods have certain been applied in many contexts. Notably, however, current deep reinforcement learning has limitations, in particular their lack of reasoning ability. To solve this problem, Garnelo et al. [128] propose a deep symbolic reinforcement learning method (DSRL), which integrates a symbolic prior into the agent’s learning process in order to enhance its generalization ability. Garcez et al. [129] extend DSRL and propose a symbolic reinforcement learning method with common sense (SRL+CS). This method improves both the learning phase and the decision-making phase based on the DSRL. In the learning phase, to distribute the reward, the model no longer sets a fixed calculation formula to update the Q values, but instead updates the values according to the interaction between the agent and the object. In the decision-making stage, to fully aggregate the Q values, the model assigns an importance weight for each Q function according to the distance between the object and the agent.

Yang et al. [130] proposed integrating symbolic planning and hierarchical reinforcement learning (HRL) [131] to handle decision-making in a dynamic environment with uncertainties, called PEORL. Symbolic planning is used to guide the agent’s task execution and learning, and the learned experience is fed back to symbolic knowledge to improve planning. This is the first approach to use symbolic planning for option discovery in the HRL context.

VI. FUTURE DIRECTIONS

The above paper introduces the current research status and research methods of neural-symbolic systems in detail. On this basis, we discuss some potential future research directions.

A. Efficient methods

In neural-symbolic systems, symbolic reasoning technologies continue to face problems such as intractable precise inference. For example, in a probability inference based on MLN, if the number of logic rules and constants is large, the number of the grounding will increase exponentially (if the number of constants is n, and the grounding of an m-ary predicate requires n^m approaches.), resulting in a rapid decline in the speed of model inference. These methods still have certain some improved methods have been proposed to address this problem [87, 94], these methods still have certain limitations. For example, approximate inference is usually used to improve the speed of inference, which comes at the cost of reduced inference accuracy. Therefore, by combining the characteristics of the deep learning models, researchers
should design approaches that can select the most effective knowledge for the tasks in question to achieve grounding. Furthermore, researchers could also consider corresponding fast inference algorithms for SRL models such as MLN, etc. The above is a task that should be dealt with by researchers working on inference methods.

B. Automatic construction of symbolic knowledge

The symbolic knowledge discussed in this paper includes logic knowledge and knowledge graphs. Research into the automatic construction of knowledge graphs is relatively mature [132]–[134]. By contrast, the automatic learning of logic rules from data remains underexplored. The logic rules used in the above neural-symbolic approaches are usually constructed manually by domain experts; notably, this construction method is time-consuming, laborious, and not scalable. Another challenge for neural-symbolic systems is that of how to achieve end-to-end learning for rules that describe prior knowledge from the data. At present, existing works have explored and extended ILP-based methods to solve the automatic construction of the rules, but these approaches have problems when it comes to complex reasoning processes, such as founding simple rules (such as single-chain rules). Therefore, the automatic construction of rules is also an important future research direction in the field of neural-symbolic systems.

C. Symbolic representation learning

Good symbolic representation can make seemingly complex learning tasks easier and more efficient. For example, if a learned symbolic representation contains limited useful semantic information in a zero-shot image classification task, the model will be unable to competently perform complex classification tasks. Therefore, the precise semantic information of symbolic knowledge is essential to improve the performance of these models. At present, most existing symbolic representation learning methods cannot handle predicates with strong similarity (i.e., two predicates that have similar semantics, but different logical formulas, such as “next to” and “near”). Under these conditions, current symbolic representation learning methods cannot capture the same semantics, which damages the reasoning ability of these models. Therefore, the question of how to design a more robust and efficient symbolic representation learning method remains a significant challenge for neural-symbolic systems. With the development of graph representation learning, nodes are mapped into low-dimensional, dense, and continuous vectors, which can be flexibly embedded in various learning and reasoning tasks. A large amount of symbolic knowledge can be modeled as directed or undirected graphs that are heterogeneous, multi-relational, and even multi-modal. Another direction worth exploring is accordingly that of how to develop and use heterogeneous graph representation learning methods to solve the challenges faced by neural-symbolic systems.

D. Application field expansion

At present, neural-symbolic systems have been applied in fields including computer vision and natural language processing, among others. In addition, some works have also explored how to use neural-symbolic systems to solve the interpretability problem in recommendation systems [135], [136], etc. It may also be possible to apply neural-symbolic systems techniques to epidemiological studies such as COVID-19. Therefore, a very natural idea would be to apply neural-symbolic systems in other fields of application and design corresponding models and methods for solving the challenges therein.

VII. SUMMARY

In this paper, we have presented an overall framework for neural-symbolic systems. Our main contribution is to propose a novel taxonomy for neural-symbolic systems and outline three structured categorizations. We go on to describe the methods of each structured categorization, the types and representation of symbols used in related works, as well as a wide range of applications and future directions for neural-symbolic systems. We believe that a systematic and comprehensive research survey has significant value in terms of both theory and application, and is worthy of more in-depth research and discussion.

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