NMSGUT II : Pinning the NMSGUT@LHC

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ABSTRACT

We exhibit sample parameter sets for the SO(10) NMSGUT\textsuperscript{1,2} superpotential, and associated Susy soft spectra that match, at low energies, with SM fermion(including neutrino) precision data and are consistent with Unification constraints including $M_{B-L} \sim M_X$, besides yielding Right handed neutrino masses in the Leptogenesis relevant range $10^8 - 10^{13}$GeV. Matching the precision data requires lowering strange and down quark masses above $M_S$ via large tan $\beta$ driven supersymmetric threshold corrections: which requires characteristic soft Susy breaking spectra. Thus our solutions evade the numerical ‘no-go’ objection raised by Grimus and Lavoura and exactly realize our proposal\textsuperscript{3} that tiny $126$ couplings allow realistic neutrino masses even though $M_{B-L} \sim 10^{16}$ GeV : as is generic in single scale SO(10) Susy GUTs. The $\mu$ and trilinear soft $A_t$ parameters are positive and negative respectively and one or more third generation sfermions tend to be heavier than the first two generations: this marks out a distinctive NMSGUT experimental signature in the sfermion spectrum potentially discoverable at LHC. We estimate the $d = 5$ RRRR decay rate for the dominant $p \rightarrow K^+\bar{\nu}_r$ channel. It indicates that $d = 5$ baryon decay rates will further winnow candidate parameter sets and strongly limit sfermion mixing angles. The NMSGUT is thus reaching the stage where its parameters are subject to enough constraints to overdetermine them and thus subject it to stringent falsification tests: given sfermion data from the LHC and from on going flavour and B violation experiments.
Grand Unification has long retained its attractive status as the most audacious extrapolation of the structural logic of the Standard Model which is not so hyperbolic as to have lost every possibility of ever satisfying common and well established demands of the scientific method (such as falsifiability or even verifiability). It enjoys the support of all of the few clear clues about beyond Standard Model (SM) physics that have so far emerged. Yet only infrequently, if at all, have there been attempts to fully specify a Grand Unified model (GUM) at a level of detail approaching the SM, so as to be able to confront the fully specified model with all the experimental data in hand and to extract the consequences thereof without the aid of egregious assumptions. The commonly accepted necessity of invoking Supersymmetry in order to ensure the stability of the parametrization of physics in the extreme Ultraviolet has drawn an additional veil, woven of the plethora of unknown supersymmetry breaking parameters, over the portal to the Grand Desert at the scale ($M_S$) where supersymmetry is (softly) broken. As we enter the epoch of the 'discovery machine' LHC we still lack well defined candidate patterns of $M_S \sim 1\text{ TeV}$ scale physics which are well correlated with any specific supersymmetric GUM. In this letter we begin to delineate emergent patterns associated with the so called minimal renormalizable Supersymmetric SO(10) models\cite{4, 5, 6, 7, 9, 8, 11, 3} which are arguably the best motivated and well defined of supersymmetric GUMs. By numerically investigating to what extent the mass formulae and unification constraints, written directly in terms of the fundamental GUM parameters\cite{1, 2}, can accommodate the known fermion mass data, the constraints of unification and those from limits on exotic processes we arrive at tentative information about soft supersymmetry breaking parameters required to prevent the NMSGUT from failing to fit the available fermion mass data, unification constraints and limits on exotic processes.

The question whether the renormalizable Supersymmetric SO(10) GUT based on the Higgs set $210 \oplus 10 \oplus 120 \oplus 126 \oplus \overline{126}$ (i.e the so called Next or New Minimal Supersymmetric GUT NMSGUT\cite{11} (although a more accurate name might be the Full Minimal Supersymmetric GUM(FMSGUM)!) is capable of encompassing fermion mass-mixing data has received much attention lately \cite{11, 3, 12, 13, 14, 1, 15, 16}. In particular the demonstration that generically feasible neutrino mass patterns could not actually be realized\cite{11, 3} in the classic MSGUT\cite{4, 5} (in the context of the Babu-Mohapatra\cite{17} program of restricting the FM Higgs irreps to just the $10, \overline{126}$) dramatically highlighted the possibility and necessity of using GUM specific mass formulae formulae. In particular the generic sub-domiance\cite{11, 3} of the Type II seesaw in the MSGUT suggested\cite{3, 12, 13} that by using very small $\overline{126}$ couplings to lower the right handed neutrino masses one might enhance the Type I Seesaw masses which were otherwise generically $\sim v_W^2/M_X < 1\text{ meV}$ and thus too small to account for the measured neutrino oscillation data. This would obviously be possible only if
the CKM structure were generated by the third possible Higgs representation i.e the 120. However this idea almost immediately faced an apparently strong challenge\[18\] which showed (numerically) that the fit to charged fermion data using only 10 and 120 representations always failed badly to account for the down quark masses which always come out much too small if the 3rd generation was fitted correctly. It is worth mentioning here that till recently these discussions—in the ‘MSGUT community’ that is—proceeded under the somewhat sanguine assumption that the appropriate uncertainties for the fitting exercise could be estimated by ignoring all new threshold effects right up to the GUT scale and hence were essentially the extrapolated errors at the low scale: thus for example the lepton masses were sought to be fitted to one part in a million or better\[38\]. In fact however it has been known\[19, 20\] from the early days of large tan\(\beta\)\(b - \tau - t\) unification that the \(T_{3L} = -1/2\) charged fermions suffer large threshold corrections and the relevance of these corrections for estimating the uncertainties in the GUT scale fermion masses has recently been re-emphasized\[22, 23\].

However this was followed by other work in which we found\[12, 13\] that our scenario implied somewhat recognizable simulacra of the fermion data and the same authors\[14, 15\] even showed that generic fits with small 126 couplings but much larger 10, 120—plet ones could in fact accurately fit all the fermion data available to date: thus making their own earlier and quite cogent generic arguments somewhat moot. However the question of whether the specific structure of the NMSGUT might still, as in the MSGUT case, defeat the demonstrated generic possibility remained open. In\[2\] we showed that in order to obtain fits that were compatible with the NMSGUT it would be necessary to search for fits using the detailed NMSGUT specific formulae developed by us in\[1\] since the the generic parameters used in\[14, 15\] were in fact subject to highly non-trivial constraints (due to their origin in the NMSGUT) that were in practice unverifiable since they could not be inverted. Two recent papers\[22, 23\] re-emphasized that down type fermions could suffer very large corrections to their masses at the Supersymmetry breaking scale \(M_S\) and that the formulae for computing these effects to all orders in \(\tan\beta\) were long known to experts if not so well appreciated by the MSGUT community\[20, 19, 21\]. This reminder\[23\] came at a particularly opportune time for our numerical investigation of the fitting problem in the NMSGUT which, while always tumbling naturally towards ultra low 126 coupling values, was constrained, once it had satisfied the strong demand for accuracy in the top quark and charged lepton masses, to settle for very low \(d\) and \(s\) quark yukawa couplings at the GUT scale. This was indeed the very behaviour noticed—and judged pathological—in\[18\]. However the new emphasis on large \(\tan\beta\) uncertainties in these very masses naturally suggested to us that the NMSGUT (which like other \(SO(10)\) GUTs prefers large \(\tan\beta\) to naturally explain third generation charged fermion yukawas as nearly equal due to their domination by the 33 component of the 10—plet alone, as is possible—and highly persuasive—precisely because all of them lie in a single 16-
plet) could readily avail of this crutch that seemed almost designed to aid its frailty. Taking this new found liberative message at face value, and the difficulties of the NMSGUT with $d,s$ quark masses as a signal from the model of its character and inner necessity, we have used the diagonal threshold correction formulae to match the down and charged fermion masses to the rather low values found by our "downhill simplex" based search and fitting program (after running them down from $M_X$ using the full two loop equations of [32] to $M_S$); we also include running of the Weinberg operator [33, 34].

The analytic solution of the fitting equations, is practically inconceivable in view of their staggering nonlinearity[1, 2] even in the case of generic fits, although a perturbative approach [12, 13] may now well work(see Section 3). Thus even the generic demonstration of feasibility [14, 15] had recourse to a nonlinear numerical algorithm (the 'downhill simplex' method). As explained in [1, 2] the fitting problem becomes much more complex when formulated in terms of the actual parameters of the underlying GUM theory. Thus all treatments of this problem must resort to numerical analysis at some stage. Indeed, saving some unforeseeable miracle, realistic consideration of the fitting problem shows that the entire program of understanding fermion masses in a GUM context must finally bow before the intrinsic non-analyticity implied by the highly non linear relation between the fundamental parameters and those of the low energy theory. We have used numerical algorithms (based on the so called downhill simplex method of Nelder and Mead[25]) which incorporate our explicit (but very complicated) fermion mass formulae in terms of NMSGUT parameters and searched extensively for plausible parameter sets for the NMSGUT while demanding that the parameters be compatible with both the known fermion data and the requirements of unification, besides plausible criteria such as perturbativity.

In a sense this effort is still premature. Even with the assumption that the parameters of the NMSGUT superpotential are real (save the parameter that is fine tuned to keep one pair of doublets light), and thus only 23 in number, there are still 5 more parameters than available data \{m_{u,c,t}, m_{d,s,b}, m_{e,\mu,\tau}, \Delta m^2_{12}, \Delta m^2_{23}, \theta^p_{12,23,13}, \delta_{\text{CKM}}, \theta^p_{\text{PMNS}}\} (\theta^p_{\text{PMNS}} < 10^\circ$ is only an upper limit [26]). The presently available fermion data thus comprise only 18 parameters: three leptonic CP phases and the absolute value of one neutrino mass remain unknown for the moment. The NMSGUT, on the other hand has 23 free parameters. These are however subject to various additional structural constraints such as the requirements of unification on $\alpha_s(M_Z), M_X$ and of neutrino masses on $M_{B-L}$ (via the Seesaw mechanisms naturally operative in the SO(10) GUTs) which are in fact closely related through their dependence on the spontaneous symmetry breaking pattern in the NMSGUT. Additional constraints may be expected to emerge when one examines the compatibility of the fundamental parameters with the emergent data. Indeed in the foreseeable future we may have access to another Neutrino angle and phase as well as a neutrino mass and also obtain information about the pattern of spartner masses at LHC. Thus the day may not be too
far when the fitting problem is fully determined or even over determined. At that point at least the NMSGUT will mature as a scientific (i.e falsifiable and predictive) hypothesis. Thus although the fitting problem is both under determined and analytically insoluble the fundamental significance of the problem justifies a numerical attack to see if any trends that may inform experimental searches for supersymmetry and searches for exotic processes can be extracted when the GUM begins to be specified at this unprecedented level of detail.

From the searches we have so far carried out on PC based computer systems, certain persistent and very specific themes have emerged. In particular we find that the achievement of adequate neutrino masses and large lepton mixing require that the masses of the strange and down quark above the Susy breaking scale $M_S$ must be reduced by a factor of 3-4 while raising the $b, \tau$ masses by 5-30%. As mentioned above, such an ad hoc distortion might have seemed unacceptable to some workers in the field until the fitting effort in GUTs, and specially in the SO(10) MSGUT [38], was reminded of the large Susy threshold corrections operative at large $\tan \beta$ for the down type quarks[22, 23]. A fit of the fermion spectrum achievable at the GUT scale in the NMSGUT then seems to require a characteristic scenario for Susy breaking mass patterns. Since (accurate) unification is the main motivation for Supersymmetry itself, the need at this juncture is for GUM driven insight into the possible mass patterns of Susy partners that might be unveiled by LHC. We offer the preferred Soft parameter patterns of the NMSGUT as our ‘bet’ in the LHC-Susy ‘sweepstakes’!

In Section 2 we briefly describe the NMSGUT and the fermion fitting formulae specific to it. Complete details can be found in [1, 2]. In Section 3 we describe our numerical procedure and then give typical fits found by our search algorithms and dissect their structure. The required pattern of threshold corrections provides indications about the pattern of sfermion masses and other Susy breaking parameters and thus constitutes a first(qualitative) prediction of the NMSGUT about potential discoveries at the LHC as well as about the nature of the soft supersymmetry breaking terms at the GUT scale(after extrapolation back to that scale). In Section 4 we use the explicit sample sets of NMSGUT parameter values and Susy soft breaking data to estimate typical predictions of a fully specified NMSGUT for the RRRR diagram for $p \to K^+\bar{\nu}$ nucleon decay which is dominant at large $\tan \beta$[28]. In Section 5 we conclude with a discussion about the outlook for discovery as well as the further development of the fitting program.

2 NMSGUT fermion mass formulae

The NMSGUT [1] is a renormalizable globally supersymmetric $SO(10)$ GUT whose Higgs chiral supermultiplets consist of AM(Adjoint Multiplet) type totally antisymmetric tensors : $210(\Phi_{ijkl})$, $126(\Sigma_{ijklm})$, $126(\Sigma_{ijklm})(i,j = 1...10)$ which break the $SO(10)$ symmetry to the MSSM, together with Fermion mass (FM) Higgs $10 (H_{i})$
and $120(O_{ijk})$. The $\mathbf{126}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three $\mathbf{16}$-plets $\Psi_A (A = 1, 2, 3)$ contain the matter including the three conjugate neutrinos ($\bar{\nu}_L^T$). The superpotential (see $[6, 7, 8, 9, 11]$ for comprehensive details) contains the mass parameters

$$m : 210^2 ; \quad M : \mathbf{126} \cdot \mathbf{126} ; \quad M_H : 10^2 ; \quad m_O : 120^2 \quad (1)$$

and trilinear couplings

$$\lambda : 210^3 ; \quad \eta : 210 \cdot 126 \cdot \overline{126} ; \quad \gamma \oplus \bar{\gamma} : 10 \cdot 210 \cdot (126 \oplus \overline{126})$$

$$k : 10 \cdot 120 \cdot 210 ; \quad \rho : 120 \cdot 120 \cdot 210$$

$$\zeta : 120 \cdot 210 \cdot 126 ; \quad \bar{\zeta} : 120 \cdot 210 \cdot \overline{126} \quad (2)$$

In addition one has two symmetric matrices $h_{AB}, f_{AB}$ of Yukawa couplings of the the $10, \overline{126}$ Higgs multiplets to the $\mathbf{16}_A, \mathbf{16}_B$ matter bilinears and one antisymmetric matrix $g_{AB}$ for the coupling of the $120$ to $\mathbf{16}_A, \mathbf{16}_B$. One of the complex symmetric matrices (which, as in most previous work, we choose to be $h_{AB}$) can be made real and diagonal by a choice of SO(10) flavour basis. Thus initially complex Yukawas contain 3 real and 9 complex i.e 21 real parameters. If the Yukawas are taken to be real however one will have just 12 real yukawas. One out of the rest of the superpotential parameters (i.e $m, M_H, M, M_O, \lambda, \eta, \rho, k, \gamma, \bar{\gamma}, \zeta, \bar{\zeta}$), say $M_H$, can be fixed by the fine tuning condition to keep two doublets light. As explained in $[6, 7, 9, 11, 3, 1]$ the fine tuning fixes the Higgs fractions i.e the composition of the massless electroweak doublets in terms of the (6 pairs of electroweak) doublet fields in the GUT. A subtle point here is that even if the other parameters are taken real the fine tuned $M_H$ (which does not itself enter into the low energy lagrangian) will be complex and this obscures the issue of whether the resultant CP violation in the low energy effective theory should be considered ‘spontaneous’ or not. Since doublet triplet splitting is a common unresolved problem for most GUMs we shall take a pragmatic approach and assume all superpotential parameters except the fine tuned $M_H$ to be real. One could refer to this induction of phases in the low energy effective superpotential via the super heavy vevs and the fine tuning as “effectively spontaneous CP violation”!

MSGUTs enjoy $[6, 9, 8]$ a crucial simplification : the GUT scale vevs and therefore the mass spectrum are all expressible in terms of a single complex parameter $x$ which is a solution of the cubic equation

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1 - x)^2 \quad (3)$$

where $\xi = \frac{\lambda M}{\eta m}$.

If we ascribe the generation of phases in this theory to the high scale symmetry breaking (the fine tuning mechanism that keeps a pair of MSSM doublets light stands in for the unknown dynamics that enforces doublet triplet splitting in this theory)
then CP violation implies\cite{1} that \( x \) must lie on one of the two complex solution branches \( x_{\pm}(\xi), (\xi \in (-27.917, \infty)) \). Since \( \lambda, \eta \) are already counted as independent \( x_{+}(\xi) \) counts for \( M/m \).

As in the case of the MSGUT one imposes the fine tuning condition \( \text{Det} \mathcal{H} = 0 \) to keep a pair of Higgs doublets \( H^{(1)}, \bar{H}^{(1)} \) (defined via left and right null eigenstates of the mass matrix \( \mathcal{H} \)) light\cite{6} \cite{9} \cite{3} \cite{1}. The composition of these null eigenstates in terms of the GUT scale doublets then specifies how much the different doublets contribute to the low energy EW scale symmetry breaking. In the Dirac mass matrices we can replace \( < h_{i} > \to \alpha_{i}v_{u}, \bar{h}_{i} \to \bar{\alpha}_{i}v_{d} \). The fermion Dirac masses may be read off the decomposition of \( 16 \cdot 16 \cdot (10 \oplus 120 \oplus 126) \) given in \cite{7} \cite{9} \cite{3} and this yields we have removed \( \sin \beta, \cot \beta \) factors in \( \bar{h}, \bar{g}, \bar{f}, \bar{r}_{i} \) relative to our previous notation in the expressions for mass matrices \cite{3} \cite{1} and written the result as \( \bar{h}, \bar{g}, \bar{f}, \bar{r}_{i} \)

\[
\begin{align*}
\bar{y}^{u} &= (\bar{h} + \bar{f} + \bar{g}) \quad ; \quad \bar{r}_{1} = \frac{\bar{\alpha}_{1}}{\alpha_{1}} \quad ; \quad \bar{r}_{2} = \frac{\bar{\alpha}_{2}}{\alpha_{2}} \\
\bar{y}^{d} &= (\bar{r}_{1} \bar{h} + \bar{r}_{2} \bar{f} + \bar{r}_{6} \bar{g}) \quad ; \quad \bar{r}_{6} = \alpha_{6} + i\sqrt{3}\alpha_{5} \\
\bar{y}^{l} &= (\bar{r}_{1} \bar{h} - 3 \bar{r}_{2} \bar{f} + (\bar{r}_{5} - 3 \bar{r}_{6}) \bar{g}) \quad ; \quad \bar{r}_{5} = \frac{4i\sqrt{3}\alpha_{5}}{\alpha_{6} + i\sqrt{3}\alpha_{5}} \\
\bar{g} &= 2i\sqrt{\frac{2}{3}}(\alpha_{6} + i\sqrt{3}\alpha_{5}) \quad ; \quad \bar{h} = 2\sqrt{2}h\alpha_{1} \quad ; \quad \bar{f} = -4\sqrt{\frac{2}{3}}i\bar{f}\alpha_{2}
\end{align*}
\]

The Yukawa couplings of matter fields with \( 120 \) Higgs field give no contribution to the Majorana mass matrix of the superheavy neutrinos \( \tilde{\nu}_{A} \) so it remains \( M_{AB}^{\nu} = 8\sqrt{2}f_{AB}\bar{\sigma} \). Thus the Type I contribution is obtained by eliminating \( \tilde{\nu}_{A} \)

\[
W = \frac{1}{2}M_{\mu}^{\nu}\tilde{\nu}_{A}\tilde{\nu}_{B} + \bar{\nu}_{A}m_{\nu}^{\nu}\nu_{B} + \ldots \to \frac{1}{2}M_{AB}^{\nu(I)}\nu_{A}\nu_{B} + \ldots
\]

\[
M_{AB}^{\nu(I)} = -((m_{\nu}^{\nu})^T(M_{\mu}^{\nu})^{-1}m_{\nu}^{\nu})_{AB} \equiv 2\kappa_{AB}^{nu}v_{u}^{2}
\]

As shown in\cite{11} \cite{8} it is very likely that the Type II seesaw contribution is totally subdominant to the Type I seesaw. However the consistency of the assumption that it is negligible must be checked and quantified. Evaluating the tadpole that gives rise to the Type II seesaw (the \( 120 \) plet contributes new terms) one obtains \cite{1} the Type II contribution to the light neutrino Majorana mass :

\[
M_{\nu}^{II} = 16if_{AB} < \bar{O}_{-} > = 16i\sqrt{2}f_{AB}(i\frac{\gamma}{\sqrt{2}}\alpha_{1} + i\sqrt{6}\eta\alpha_{2} - \sqrt{3}\zeta\alpha_{6} + i\zeta\alpha_{5})\alpha_{4}(\frac{v_{u}^{2}}{M_{O}}) \\
\equiv 2\kappa_{AB}^{(II)}v_{u}^{2}
\]
where $M_O = 2(M + \eta (3\alpha - p))$. We have always considered Type I and Type II mechanisms together since in the NMSGUT one does not have any freedom to switch one off at will.

As is apparent from the fermion mass formulae the coefficients $\alpha_i, \bar{\alpha}_i$ are critical for the phenomenology of these models. They are calculated by determining the null left and right eigenvectors of $H$ (the $[1, 2, \pm 1]$ mass matrix). The explicit (and rather complicated) expressions for the $\alpha_i, \bar{\alpha}_i$ can be found in Appendix C of [1]. The only modification being that we work in a convention where $\alpha_1 = \bar{\alpha}_1$ are both real so that the contributions of the $10-$plet to all Dirac masses are real.

From the formulae in [1] one can check that for real superpotential parameters and real values of $x$ the $\alpha_i, \bar{\alpha}_i, i = 1...5$ are real while $\alpha_6, \bar{\alpha}_6$ are pure imaginary. Then it immediately follows [1] that except the trivial phase convention dependent phase values all other phases are zero and there is no CKM CP violation. The only way to get non trivial phases in the CKM matrix while retaining real superpotential parameters is thus for $x$ to be complex. Thus we constrain the value of $\xi$ to be real and greater than -27.9 and choose the complex solution branch of the cubic equation which has a positive imaginary part [1]: this is an arc in the upper half complex $x$-plane with real value between about .75 and 2.25 while the imaginary part lies between about 0 and 1 (see [1] for details about solutions of the cubic).

### 3 Numerical search and anatomy of specimens found

We use the 2-loop RG equations for the SM [30] and MSSM [31, 32] to extrapolate the central values of fermion Yukawa data at the top mass scale (including neutrino mass splitting and mixing data), which were recently recalculated and summarized in [29, 26], past the MSSM threshold (taken to be 1 TeV and defined by expressing all soft masses in units of the gluino mass fixed at 1 TeV) up to the MSSM 1 loop gauge coupling unification scale $M_U = 10^{16.3} \, GeV$ while ignoring, for the time being, complications such as righthanded neutrino thresholds. In the case of the neutrino masses we extrapolate the coefficient $\kappa_{AB} = M_{\nu_2}/(2v_u^2)$ of the $SU(2) \times U(1)$ invariant dimension 5 operator [39, 40] that gives rise to neutrino mass when the Electroweak vev $(v_u = 174 \sin \beta GeV); \text{for the SM } \sin \beta \text{ is 1}$ is substituted for the two electroweak doublets in it.

At $M_X$ we re-extract the input parameters i.e the 9 charged fermion Yukawas, the 4 CKM parameters, and the 5 neutrino mass data. Using the percentage uncertainties in fermion Yukawas (for $\tan \beta(M_S) = 50$) at $M_X = 10^{16.3} \, GeV$ recently re-estimated in [23], which include the large uncertainties due to large threshold effects on down type quarks and charged leptons at large $\tan \beta$, we form a $\chi^2$ function comparing the central values with the values of the same parameters calculated using the formulae described in Section 2 and normalized by the errors mentioned. The ‘down hill simplex’ or “amoeba” algorithm of Nelder and Mead [25] is then used to
Table 1: Two sample sets of NMSGUT couplings which pass the criteria of Section 2. Note how the strong hierarchy of $M'\nu$ weakens the strong hierarchy of $(M'_{Drc})^2$ to reproduce the weak observed light neutrino hierarchy via the Type I seesaw mechanism. Note also the utter irrelevance of Type II neutrino masses.
troll the 23 dimensional hyperspace of NMSGUT parameters searching for local minima of the $\chi^2$ function. While searching we imposed penalties to keep all the Yukawas and GUT superpotential parameters within the perturbative range. The programs were written in FORTRAN and run on PC’s for $10^6$ iterations for each random starting point (or less if it was clear that the algorithm was stuck in a local minimum with large $\chi^2$ : at which point the program was restarted). Typically our algorithm found candidate ‘solutions’ every 20 program iterations or so with $4.0 < \chi^2 < 8$(but never any lower).

The striking features of the candidate solutions (besides the inability to pass below $\chi^2 = 4.0$) were:

- Neutrino masses and large mixing angles were accurately fitted.
- The values of the $\overline{126}$ couplings were so tiny that the three righthanded neutrino masses were much lighter than $M_X$ and spanned the range $\sim 10^8 – 10^{13}$GeV, just as conjectured by us earlier [3, 12, 13]. Note that this is just the range required by most (Type I) Leptogenesis models.
- The bulk (over 85%) of the residual $\chi^2$ typically consisted of errors in the d,s quark yukawa couplings with a further 10% or so coming from the bottom mass.
- The overall scale parameter of the MSGUT symmetry breaking which suppresses the neutrino masses was typically in the range $10^{15} – 10^{16}$ GeV, so that the role of the tiny $\overline{126}$ couplings in generating viable neutrino masses is manifest.

We then subjected the candidate solutions to a highly demanding (‘ouroborotic’) consistency test between the neutrino mass fitting specified scale parameter $m$ (since this fixes the scale of the right handed $M_\nu$ mass matrices and thus the light neutrino masses) and the value of $m$ determined[11, 3, 1] by the RG flow via the specification of $M_X$. We required that they should lie within a factor of 5 of each other. In view of the many approximations and short cuts adopted in this initial survey of the huge NMSGUT parameter space this requirement is quite stringent but we even found solutions which agree to within 15% for these crucial numbers. We also demand that the correction to $\alpha_S(M_Z)$ due to the GUT scale threshold correction effects evaluated using the complete NMSGUT superheavy spectra[9, 11, 36] should lie within the range $-0.017 < \Delta \alpha_S(M_Z) < -0.004$ and that the unification scale correction should raise the unification scale (scale of $d = 6$ baryon violation). We have relaxed the upper limit to $-0.001$ for the case of Soln-1 since it was the first ‘good’ solution we found: subsequently we always use $-0.004$. These three demanding requirements (which can perhaps be thought of as adding 3 equations to the still deficient set of 18) killed practically 95% of the randomly generated solutions immediately. Nevertheless among every 15-20 candidates we would find one that seemed to be suffering only from the ‘ds-b$\tau$ disease’: which we propose is rather a legerity-adiposity attained when crossing the supersymmetric portal.
We then extrapolate these ‘rough nuggets’ back down to the summarily assumed Susy breaking scale $M_\mathrm{S} = 1 \text{ TeV}$.

Only the $d, s$ quarks were seriously in error while $b, \tau$ require minor revisions upwards from their standard model values. We use the freedom to choose Susy soft parameters and the elementary formulae matching the threshold corrected MSSM down/charged lepton yukawas to their SM analogues\cite{23} to correct the discrepancies. These corrections all have\cite{23} the form (for the $i$'th down type fermions only):

$$\frac{y_{i\mathrm{GUT}}^2(M_s)}{y_{i\mathrm{SM}}^2(M_s)} = \{\cos \beta + \epsilon_i(m_{\tilde{F}_i}, m_{\tilde{d}_i}, \bar{m}_i; M_1, M_2, M_3, \mu, A_t, \alpha_1, \alpha_2, \alpha_3) \sin \beta\}^{-1} \quad (7)$$

The explicit forms of these formulae and the consequent larger uncertainties at $M_X$ were summarized recently in\cite{23} and we used those uncertainties while locating candidate parameter sets. The equations given by\cite{23} following\cite{24} for $\epsilon_i$ are

$$\epsilon_i = \epsilon_i^G + \epsilon_i^B + \epsilon_i^W + \epsilon_i^\eta \delta_{ib}, \text{ with}$$

$$\epsilon_i^G = - \frac{2\alpha_s}{3\pi} \frac{\mu}{M_3} H_2(u_{\tilde{Q}_i}, u_{\tilde{d}_i}) \quad (8)$$

$$\epsilon_i^B = \frac{1}{16\pi^2} \left[ \frac{g^2}{6} M_1 \left( H_2(v_{\tilde{Q}_i}, x_1) + 2H_2(v_{\tilde{d}_i}, x_1) \right) + \frac{g^2}{9} \mu M_1 H_2(w_{\tilde{Q}_i}, w_{\tilde{d}_i}) \right] \quad (9)$$

$$\epsilon_i^W = \frac{1}{16\pi^2} \frac{3g^2 M_2}{2\mu} H_2(v_{\tilde{Q}_i}, x_2) \quad (10)$$

$$\epsilon_i^\eta = - \frac{y_i^2}{16\pi^2} \frac{A_t}{\mu} H_2(v_{\tilde{Q}_i}, v_{\tilde{d}_i}) \quad (11)$$

where $u_{\tilde{f}} = m_{\tilde{f}}^2/M_3^2$, $v_{\tilde{f}} = m_{\tilde{f}}^2/\mu^2$, $w_{\tilde{f}} = m_{\tilde{f}}^2/M_1^2$, $x_1 = M_1^2/\mu^2$ and $x_2 = M_2^2/\mu^2$ for $i = d, s, b$. All mass parameters are assumed to be real. The correction $\epsilon_i^\eta$ to $d, s$-quarks are neglected because of the strong hierarchy of the quark Yukawa couplings. $H_2$ is defined as

$$H_2(x, y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)}. \quad (12)$$

$H_2$ is negative for positive $x$ and $y$ and $|H_2|$ is maximal, if its arguments are minimal, and vice versa. It is a rather smooth and bounded function with typical values $\sim 10^{-2} - 10^{-1}$.

The last term in equation (13) causes an enhancement by a factor of $-9$ compared to the corresponding term in the quark sector in equation (9). The $\tau$-lepton Yukawa coupling does not have a relevant correction of the $\epsilon_i^\eta$ type because the corresponding vertex correction is suppressed by the heavy mass scale of the right handed neutrinos.

$$\epsilon_i^B = \frac{1}{16\pi^2} \left[ \frac{g^2}{2} M_1 \left( -H_2(v_{\tilde{L}_i}, x_1) + 2H_2(v_{\tilde{e}_i}, x_1) \right) - \frac{g^2}{9} \mu M_1 H_2(w_{\tilde{L}_i}, w_{\tilde{e}_i}) \right], \quad (13)$$

$$\epsilon_i^W = \frac{1}{16\pi^2} \frac{3g^2 M_2}{2\mu} H_2(v_{\tilde{L}_i}, x_2), \quad (14)$$
for $i = e, \mu, \tau$.

Taking the sign of $M_3$ positive by convention we see that the dominant $\alpha_S$ dependent corrections to the down and strange yukawas, which must be lowered at $M_S$ from their SM values since the fit values at $M_X$ are so small, will be positive only if $\mu$ is positive and large relative to the Gluino mass. Conversely the bottom yukawa coupling in the MSSM must be raised at $M_S$ from the standard model value and hence the value of $A_t$ should be negative and appreciable relative to $\mu$ (note that while $y_t$ is $\sim 1$ there is a relative factor of 6 overall in the formulae for $\epsilon_G, \epsilon_3$). On the other hand, for positive values of $M_i$ the terms in lepton $\epsilon_i^{W,B}$ are both positive and negative. Thus various solutions with both signs for $M_{1,2}$ are possible. Since the interpretation of Gaugino phases is fraught with complexities we usually prefer to display only solutions with positive signs for both $M_{1,2}$.

However the final fit is at $M_S$. The 19 (!) free parameters in these formulae make the task of trimming and padding the the down type fermion Yukawas to match at $M_S$ facile using an appropriately tuned downhill simplex program. We obtain fits accurate to a few parts in $10^{16}$ for the $d, s, b, e, \mu, \tau$ fermions. This clearly indicates that the sfermion parameters are underdetermined and there is, in principle, ample room to accommodate constraints from Flavour violation, Baryon Decay and Unification.

An interesting point in this regard was that we did not prejudge the value of $\tan \beta$ but used it as another fitting parameter. Since only the low energy values of the fermion yukawas and Weinberg neutrino mass operator coupling $\kappa$ have any sanctity while the consistent GUT data sets are taken at face value perhaps our procedure is not completely wild. A further consistency check is then that the $T_{3L} = +1/2$ quark masses and the neutrino masses should also lie within experimental errors and we retain only such soft parameter sets for which these errors are within the standard experimental errors evaluated at $m_{top}(m_{top})$. Although we also find solutions of the fit at $M_S$ which give extremely accurate fits to the lepton masses comparable to the extreme precision with which they are known at $M_Z$ it should be kept in mind always that our results carry systematic errors due to the omission of various effects such as right handed neutrino thresholds (important due to their light masses), due to the mass splitting among Susy partners at $M_S$ and due to non-diagonality in the supersymmetric threshold corrections. The right handed neutrino thresholds will particularly affect the neutrino parameters and so they should be allowed somewhat larger margins of error. The first of these is quite amenable to correction and the second can at least be estimated and the soft spectra found subjected to self-consistency. The last mentioned theoretical results can be easily calculated from the same diagrams by inserting the appropriate flavour structure and diagonalization and this is now urgently required for the next stage of our program. However the results depend on a large number of unknown sfermion mixing parameters and it

\footnote{C.S.A is obliged to Ts. Enkhbat for a discussion on this subject.}
will be necessary to make simplifying assumptions in order to proceed. Luckily, the constraints on Flavour violation will strongly suppress most of the mixing angles, the problem is mainly to disentangle the various dependencies. One may also extend the number of targets of the random fit at $M_S$ by optimizing also the up quark masses and the neutrino mass squared splittings: however the only parameter that affects these is $\tan \beta$ and we found it opportune not to overburden this formerly quite free number with too many sartorial duties! When one includes non-diagonal threshold corrections(which will presumably affect the CKM matrix as well as lead to flavour changing processes) the entire sfermion sector including the the sup and scharm quark masses will enter the calculation[49].

In general, due to the the structure of the threshold correction formulae and the opposing requirements of strong corrections of charge -1/3 quarks and minor fine tuning of charged lepton masses, the fits are easier if one allows oneself the somewhat moot luxury of negative gaugino mass parameters. Since the role of phases of the gaugino masses is obscure we will allow both signs but only real masses. When searching for the soft parameters which would cure the d-s-b quark mass disease we imposed various commonly accepted requirements (to within 10% or so) on the soft parameters such as the mass ratios $M_1 : M_2 : M_3 :: 1 : 2 : 6$, degeneracy of the first two generation sfermion masses, and the constraints from RG evolution on the ratios of the masses of the sfermions of the first two generations to the gaugino masses[27]:

$$|\tilde{m}_{i,L}/M_1| \geq 0.9$$ ; $$|\tilde{m}_{q,Q}/M_3| \geq 0.75$$ (15)

However the conventional wisdom that the sfermions of the third generation will be much lighter than the first two generations of squarks due to strong renormalization by top yukawa driven effects seems directly challenged by our spectra which seem always to include a large mass for at least one of the third generation sfermion multiplets at $M_s$. Since one can obtain solutions where this could be any of $\tilde{\tau}, \tilde{b}, \tilde{t}, \tilde{Q}_3, \tilde{L}_3$ it may still be possible to arrange that this happens in a way consistent with the expectations from the RG flow: for instance if the sfermion with the largest gauge contributions to its mass corrections(e.g $\tilde{Q}_3$)remained massive while the other third generation fermions are driven to lighter values as usual. If however we demanded that all the sfermions of the third generation are lighter than the corresponding first two generation fermions then we have, so far, failed entirely to find any fits at all! This feature thus marks out such sfermion spectra as linked to the inner logic governing the fermion mass pattern in the NMSGUT and we proffer it as a prediction regarding supersymmetric physics that may be tested by LHC or its successors. The trilinear soft parameter $A_t$ for the third generation and the $\mu$ parameter from the superpotential play critical roles in the simultaneous correction of the the $b$ quark mass downwards(as is generally required since the $b$ quark yukawa tends to drift towards
the $\tau$ yukawa in $SO(10)$ fits rather than stay at the - upto 30%- lower value indicated by the MSSM RG flow at large $\tan \beta$. Although we have set the gluino mass $M_3$ mass to 1 TeV for definiteness since the threshold correction factors depend only on ratios of masses it is clear that one may need to consider a range of values in general.

Given the large value of $\mu$ one may wonder how EWRSB(always a bit difficult to implement in large $\tan \beta$ models ) will work here. This depends strongly on the assumption for the Higgs soft masses and since it is quite possible to envisage scenarios where these masses are tachyonic there is no reason to be overly pessimistic in this regard. The drastic constraint of suppression of $d = 5$ mediated proton decay is a further dragon that the parameter sets found will need to face. Nevertheless the very fact that they came so far while satisfying all structural requirements is, in our view, highly remarkable in such a constrained and structurally unique theory as the NMSGUT.

The parameter sets found so far display some characteristic preferences which mark them out as possible signals of NMSGUT driven behaviour at LHC scales. In Tables 1,2,3,4 we give example NMSGUT parameter sets which satisfy our criteria. However this is done only as a proof of principle and feasibility and the typical form of non pathological exemplars of our fitting program. It would be highly premature to subscribe to any one parameter set as very specifically demanded by NMSGUT fits. Although the fits truly involve only the yukawa couplings and Weinberg operator coefficient $\kappa_{AB}$ (since the low energy vev is supplied by hand on both sides of the mass formulae) we display the values of the yukawas times the two loop run up of the MSSM Higgs vevs just to make the numbers somewhat recognizable via their recognizable origin the low energy mass values. We defer the study of the properties of the sfermion spectra found at the GUT scale vis a vis the requirements of unification to a later more extensive work[49]. Clearly the highly structured solution will lead to differences from the standard SUGRY scenarios and require generation dependent soft terms at $M_X$[49].

3.1 Dissecting Specimens

The basic difficulty in building a grand unified theory of fermion masses is the achievement of sufficiently large neutrino masses and the reconciliation between small CKM mixing angles and large PMNS ones. So it is interesting to dissect the solutions we have found to see how exactly they circumvent the difficulties typically encountered[11, 3, 12, 13]. The roles played in the effective matter fermion Yukawa couplings $Y_{u,d,e}, \kappa_{AB}$ by the Yukawa couplings $h_{AB}, g_{AB}, f_{AB}$ of the three FM Higgs representations are, so to speak, the vital organic functions of the NMSGUT and their dissection reveals the integral unity and ‘intelligent design’ within the fermion mass pattern. By killing the Yukawa couplings individually or in groups and noting the resultant effect one easily uncovers their roles.
| Parameter | Target | Uncert. | SOLN1 | SOLN2 |
|-----------|--------|---------|-------|-------|
| $m_t \,(GeV)$ | 76.6   | 3.06    | 75.69 | 75.96 |
| $m_c \,(MeV)$ | 194.97 | 32.2    | 207.31| 193.477 |
| $m_u$ | .4     | .15     | .41   | .39   |
| $m_b$ | 1186.8 | 616.0   | 1621.31| 1626.01 |
| $m_s$ | 17.82  | 8.4     | 4.47  | 4.92  |
| $m_d$ | .931   | .54     | .24   | .25   |
| $m_\tau$ | 1493.37| 283.74  | 1693.83| 1698.87 |
| $m_\mu$ | 74.989 | 11.248  | 75.01 | 76.65 |
| $m_e$ | 0.355  | 0.0533  | .35   | .35   |
| $\sin \theta_{12}$ | 0.22098| 0.0016  | 0.221004| 0.220974 |
| $\sin \theta_{23}$ | 0.0349 | 0.0013  | 0.0347803| 0.0349877 |
| $\sin \theta_{13}$ | 0.00297| 0.0005  | 0.00294978| 0.00305354 |
| $\delta_{CP}$ | 60.026 | 14.0    | 52.0769| 64.53 |
| $\sin^2 \theta_{12}^p$ | 0.30029 | .06    | 0.298219| 0.288079 |
| $\sin^2 \theta_{23}^p$ | .465   | .14    | 0.467765| 0.326723 |
| $\sin^2 \theta_{13}^p$ | 0.0053 | .03    | .00108116| 0.00292243 |
| $m_{12}^2 \,(\times 10^{-5} eV^2)$ | 3.46   | .37    | 3.44621| 3.4533 |
| $m_{23}^2 \,(\times 10^{-3} eV^2)$ | 1.11   | 0.22   | 1.11379| 1.10343 |
| $\chi^2$ | 0      | 18.0   | 5.71427| 6.79112 |
| $\chi_{dd}^2$ | 0      | 1      | 1.58   | 1.536 |
| $\chi_{ss}^2$ | 0      | 1      | 2.51   | 2.351 |
| $\chi_0^2$ | 0      | 1      | 0.49   | 0.508 |
| $\chi_{DOWN}^2$ | 0      | 1      | 4.58   | 4.395 |

Table 2: Values of the fermion data at $M_X$ achieved by the sample coupling sets of Table I. The values $\nu(M_X) = 136.729 \, GeV, \tan \beta(M_X) = 46.819$ obtained by a complete two loop extrapolation to $M_X$ of central values at $M_S \,[29]$ have been used to write the notional masses at the GUT scale instead of the Yukawa couplings and neutrino mass generating Weinberg dimension = $-1$ couplings $\kappa_{AB}$. 
| Parameter | SOLN1 – Set1 | S2 | S3 | SOLN2 – S1 | S2 | S3 |
|-----------|--------------|----|----|------------|----|----|
| $M_1$     | 0.17876      | 0.1687 | −1.1786 | 0.165479 | 0.1759 | 0.1677 |
| $M_2$     | 0.3260       | 0.3315 | −0.3703 | 0.33562 | 0.3326 | 0.3299 |
| $M_3$     | 1.0000       | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\tilde{A}_t$ | −6.7908   | −27.1390 | −18.9522 | −1.41918 | −7.4401 | −3.4534 |
| $\mu$     | 22.6655      | 17.8729 | 16.4992 | 9.65223 | 14.4994 | 21.4061 |
| $\tilde{m}_{Q_1}$ | 3.7317     | 2.2145 | 1.16261 | 0.82551 | 0.8975 | 2.7146 |
| $\tilde{m}_{Q_2}$ | 3.4621     | 2.1701 | 1.2201 | 0.90581 | 0.8949 | 2.9140 |
| $\tilde{m}_{Q_3}$ | 5.2620     | 18.0030 | 5.209 | 1.125382 | 20.0218 | 1.1453 |
| $\tilde{m}_{L_1}$ | 0.8036     | 0.6337 | 2.0285 | 0.812268 | 0.6688 | 1.3384 |
| $\tilde{m}_{L_2}$ | 0.7623     | 0.6406 | 2.0974 | 0.897360 | 0.7391 | 1.4735 |
| $\tilde{m}_{L_3}$ | 0.3000     | 0.4535 | 1.2242 | 0.3634 | 0.4303 | 3.6479 |
| $\tilde{m}_d$ | 0.8969     | 1.6780 | 2.6808 | 1.85571 | 2.6043 | 1.7563 |
| $\tilde{m}_s$ | 0.9840     | 1.5881 | 2.4728 | 1.80036 | 2.6405 | 1.6093 |
| $\tilde{m}_b$ | 27.6858    | 1.7006 | 7.2372 | 29.29089 | 16.9781 | 29.9958 |
| $\tilde{m}_t$ | 0.7858     | 0.3673 | 1.5447 | 0.421398 | 0.3259 | 0.5962 |
| $\tilde{m}_c$ | 1.4492     | 1.3102 | 4.6487 | 0.39730 | 0.6527 | 0.5595 |
| $\tilde{m}_\mu$ | 1.5055     | 1.3102 | 4.3451 | 0.436624 | 0.6796 | 0.6076 |
| $\tilde{m}_\tau$ | 7.3105    | 6.1334 | 0.6736 | 2.02467 | 1.6692 | 0.5264 |
| $\tan(\beta)$ | 50.3352    | 50.0910 | 45.7251 | 51.1654 | 54.3213 | 52.0464 |

Table 3: Values (TeV) in of the soft susy parameters found by the threshold fitting at scale $M_S$. Note the large values, relative to the isoquantal first two generation fermions, of at least one of the third generation sfermions in each column.
| Parameter | SM($M_S$) | Uncert. (%) | Ach. value. | Ach. value | Ach. value | Ach. value |
|-----------|-----------|-------------|-------------|-------------|-------------|-------------|
|           |           | (SM)        | Sol4(S1)    | Sol4(S2)    | Sol10(S1)   | Sol10(S2)   |
| $m_b$     | 2394.77   | 2.1         | 2394.77     | 2394.77     | 2394.77     | 2394.77     |
| $m_s$     | 45.9467   | 28.3        | 45.9466     | 45.9466     | 45.9466     | 45.9466     |
| $m_d$     | 2.43876   | 42.2        | 2.43876     | 2.43876     | 2.43876     | 2.43876     |
| $m_t$     | 145873.0  | 1.65        | 147472.0    | 147472.0    | 147303.0    | 147306.0    |
| $m_c$     | 520.549   | 13.3        | 526.254     | 526.253     | 525.652     | 525.664     |
| $m_u$     | 1.07639   | 36.5        | 1.08819     | 1.08819     | 1.08694     | 1.08697     |
| $m_r$     | 1734.45   | 1.12 × 10^{-4} | 1734.45   | 1734.45     | 1734.45     | 1734.45     |
| $m_\mu$   | 102.014   | 8.88 × 10^{-6} | 102.014   | 102.014     | 102.014     | 102.014     |
| $m_e$     | 0.483143  | 8.6 × 10^{-6} | 0.483143   | 0.483143    | 0.483143    | 0.483143    |
| $m^2_{12}(meV^2)$ | 81.3871 | 3.75        | 83.1851     | 83.1845     | 84.5943     | 84.6016     |
| $m^2_{23}(meV^2)$ | 2543.34 | 8.0         | 2630.3      | 2630.28     | 2510.63     | 2510.85     |
| sin $\theta^q_{12}$ | 0.2210 | .7           | 0.221019    | 0.221019    | .220993     | .220993     |
| sin $\theta^q_{13}$ | 0.00345 | 3.7         | 0.00351868  | 0.00351868  | 0.00364475  | 0.00364475  |
| sin $\theta^q_{23}$ | 0.04059 | 15.5        | 0.041483    | 0.041483    | 0.0417563   | 0.0417563   |
| $\delta_{CP}$ | 60.024 | 23.3        | 52.08       | 52.08       | 64.535      | 64.535      |
| sin$^2 \theta^P_{12}$ | 0.31 | 7.4         | 0.314852    | 0.314852    | 0.2904      | 0.2904      |
| sin$^2 \theta^P_{13}$ | 0.005 | ≤ 0.03      | 0.012641    | 0.012641    | 0.034       | 0.034       |
| sin$^2 \theta^P_{23}$ | 0.500 | 4.0         | 0.5082494   | 0.5082494   | 0.3637      | 0.3637      |

Table 4: Values of the precision data at $m_t(m_t)$ extrapolated to $M_S = 1$ TeV compared with the values achieved by the NMSGUT parameter sets listed in Table 1 extrapolated down to $M_S$ and translated to Standard Model values by applying threshold corrections using Susy spectra and $\tan \beta$ selected using a downhill simplex optimization. Quark masses are in units of MeV. The mixing values obtained from the rundown from $M_X$ have been listed unchanged since only diagonal threshold corrections were applied. Systematic theoretical errors of the GUT parameter estimates have not been included in the error estimates which can be only notionally as precise as quoted for the leptons.
The most dominant coupling (though not, generally, the largest) is $h_{33}$ which directly and drastically affects every mass and mixing angle and may justifiably be called the lynchpin of the Yukawa coupling pattern. The third generation masses can be fit to within 5% by this coupling alone: this is precisely the content of 'b - τ - t unification' in this context: even though the Yukawa couplings $Y_{u,d,e}(M_X)$ themselves may differ by as much as 30%. The next most important (and largest) coupling is $g_{23}$ whose value is generally larger than $h_{33}$ perhaps to compensate for the fact that it enters the mass matrix non-diagonally. It is crucial for the $\theta_{23}$ mixing and thus also for second generation masses: specially the $\mu$ mass. The importance of the couplings $f_{AB}$ arises from their smallness: since they control the mass and couplings of the superheavy right handed neutrinos which leave behind a characteristic signature in the Type I seesaw masses (where they enter in the denominator) the limit $f_{AB} \to 0$ is singular i.e decreasing $f_{AB}$ changes the effective theory (MSSM with massive left handed neutrinos) drastically. The couplings $h_{11}, h_{22}, g_{12}, g_{13}$ are relevant for the masses and mixing of the first and second generations.

If we first set $f_{AB} = 0$ then clearly all neutrino masses and mixings become unphysical. However one finds that the fractional changes in the charged fermion masses and mixing are changed by less than one part in $10^4 - 10^6$. Thus the $\overline{126}$ coupling is utterly irrelevant to the charged fermion sector in our fits. This feature clearly marks out the solutions we have found as totally distinct from the class of generic fits explored in [14, 15]. Moreover our $f_{AB}$ eigenvalues (and thus the right handed neutrino masses) are strongly hierarchical $(10^2 : 10^1 : 10^{-3})$; so much so that they are effective in compensating the large hierarchy in the neutrino Dirac masses $(10^2 : 10^1 : 10^{-2})$ (which is amplified by the square of the neutrino Dirac mass in the Type I formula) to produce the rather weak $(15 : 3 : 1)$ hierarchy of the light neutrino masses. This is again crucially different from the generic SO(10) solutions of [14, 15] which have a hierarchy of at most $300 : 60 : 1$ for the right neutrino masses (which are also much larger). There [14, 15] the dilution of the Dirac mass hierarchy relies on a strong boost of the coefficient $r_D$ of $g_{AB}$ in the Dirac neutrino mass matrix $M'_{Drc}$ to huge values $\sim 10^4 - 10^5$. Our parameters simply never venture towards such large $r_D$ : they always produce values of the coefficient $r_D$ below 500. In fact even if one removes constraints that the parameters remain perturbative it proves extremely difficult to force the downhill simplex ‘amoeba’ – in its unicellular wisdom! – to crawl towards the high $r_D$ region. Of course we have already shown [2] that generic numeric fits carry no direct implication for the NMSGUT since their parameters are constrained, but still the generic analysis propagated a scare [18] that the small $\overline{126}$ scenario was utterly flawed and then compounded it by the confusing conclusion that small (but not tiny) $f_{AB}$ solutions were in fact capable of finding accurate fits. In any case this history underlines once again [11, 3] the emphatic non sequitur with which explicit UV theories must confront such generic no-go claims unless they are founded on
something more than numeric projections.

In [12, 13] we argued that the 2-3 sector should be regarded as the core of the fermion mass hierarchy and that the complete hierarchy could and should be understood semi-analytically by perturbing around the couplings $h_{33}, g_{23}$ i.e by taking these couplings as $O(1)$ and the others as order $O(\epsilon^n), \epsilon \sim 0.1, n \geq 2$. The solutions we have found by random search bear this out fully. The couplings in the 12 sector are hierarchical and suppressed relative to the 23 sector: $h_{22} \sim \epsilon^3 h_{33}, h_{11} \sim \epsilon^6 h_{33}, g_{13} \sim \epsilon g_{23}, g_{12} \sim \epsilon^2 g_{23}$. If we keep only the couplings $h_{33}, g_{23}$ and set the rest to zero one finds that $m_t, b, \tau, \mu, \theta_{13}$ are still within 1.0% or so of their final values. Restoring $h_{22} \sim \epsilon^3$ restores also $m_c, s$ to within 5% or less of their final values and improves $\theta_{23}$, but the CKM parameters apart from $\theta_{23}$ are still 100% or more different. The switched-on set $\{h_{33}, g_{13}, h_{22}\}$ gives good values of $t, c, b, s, \tau, \theta_{13}$ but not the rest. Similar considerations apply to $\{h_{33}, g_{12}, h_{22}\}$. Finally restoring $g_{12}, g_{13}$ restores the rest of the CKM parameters i.e $\theta_{12}^q, \theta_{13}^q, \delta_{CP}$ to within .05%. Thus the $g_{AB}$ are directly correlated with the corresponding $\theta_{AB}$ angles. We can now pursue[50] our earlier proposal[12, 13] to understand the fermion parameters by an expansion around the 23 sector in a single hierarchy parameter with confidence that the solutions exist and have the required form. The phases and coefficient values we have found can be checked for their compatibility with such an expansion and thus a qualitative understanding of the hierarchy arrived at.

Of course we must keep in mind that the values of the Yukawa couplings we find match the SM precision couplings extrapolated to $M_S$ only after applying the large corrections at $M_S$ due to large $\tan \beta$ that are the underpinning of our work. Thus the fermion masses explained above are themselves significantly different from the ones that we earlier[12, 13] had only partial success in describing via the $\epsilon-$expansion. Indeed it precisely this re-scaling downwards of the target $m_d, m_s$ in the fit at $M_X$ downwards in the hierarchy by a factor of $\epsilon$ that we anticipate will allow the determination of excellent fits to the large angles of the lepton sector. Analytic (perturbative) confirmation of this proposal is thus an important topic for future work[50] since it will yield a more robust understanding of the innards of the hierarchy.

4 Nucleon decay

As was already shown generically in[1] and can also be seen from Table 1, viable RG flows in the NMSGUT have $M_X$ raised by up to two orders of magnitude. Thus $d = 6$(i.e gauge boson mediated) proton decay will be suppressed by up to 8 orders of magnitude relative to the already long life times $\sim 10^{36}yrs$ corresponding to the one-loop unification scale $10^{16.3}$ GeV. Thus question of the rate of $d = 5$ operator mediated baryon decay in the theory at hand is clearly the critical one. Since the decay rates in these channels will depend sensitively on the soft scalar spectrum and mixing they could provide a welcome additional filtration of the parameter sets that pass the
other criteria. Of course since the threshold analysis so far is all performed in the diagonal approximation the mixing parameters of sfermions of the same $SU(3) \times U(1)$ quantum numbers are arbitrary and one could at most derive upper bounds on these mixings once the variation possible due to sfermion spectra had been mapped. For the sfermion masses on the other hand we have already used the freedom to choose all but the right handed $u, c$ squark masses. Of course -given the non-uniqueness of the parameter sets we obtained– it is possible that a more general analysis could later impose the requirements of fixing the down fermion masses, of Unification(of soft masses), obtaining acceptable proton decay partial widths and flavour violation due to loop effects simultaneously.

For the moment however we will simply treat the $u, c$ squark parameters, the squark mixing, soft trilinear couplings($A_{AB}^f, f \neq t$), Higgs scalar $B$ parameter and the Higgs soft masses ($\mathcal{L}_{soft} = B\bar{H}H + m_H^2 |H|^2 + m_{\tilde{H}}^2 |\tilde{H}|^2 + ...$) as the only remaining adjustable parameters in our search for completely consistent data sets in the NMSGUT. Actually, at large $\tan \beta$ one can quite plausibly fix the last three parameters via the Electroweak symmetry breaking conditions ; in any case they affect sfermion mass-mixing structure and thus nucleon decay only through their contributions to the two loop beta functions. The dominant vev $v_u$ may be fixed (to $O(\cot \beta)$) from the leading approximation to the scalar potential which retains only $v_u$ thus fixing one soft parameter out of $B, m_H^2, m_{\tilde{H}}^2$. The minimization conditions will eliminate two more.

The suspicion may be that like others[45, 46] this GUT too will be hard put to account for evasion of the quite stringent experimental bounds on the commonly expected modes in supersymmetric GUTs. A proper analysis of this question requires extensive calculation of the large arrays of Wilson coefficients: which we have currently undertaken. However we can get a preview of the issues, techniques and results by considering the channel $N \to K^+\bar{\nu}_\tau$ since they are the dominant ones at large $\tan \beta$ anyway[42, 28].

In [9, 1] we calculated the effective $d = 5$ operators for nucleon decay in terms of the couplings $\hat{h}_{AB}, \hat{f}_{AB}, \hat{g}_{AB}$ and the (inverses of) the mass matrices for the three different triplet types relevant to proton decay in this model. Not only the $t[3, 1, -\frac{2}{3}]+t[3, 1, \frac{2}{3}]$ triplets (square brackets enclose MSSM gauge quantum numbers and the subscripts refer to the multiple copies arising from the various grand unified Higgs present in the theory : whose spectra have been completely evaluated[7, 9, 6, 8, 10, 3, 1]) but also in the novel ones from the $P[3, 3, \pm \frac{2}{3}]$ and $K[3, 1, \pm \frac{8}{3}]$ multiplet types which can contribute to baryon violation[1, 17]. In the case of the $\mathbf{126}$ the $P_1, K_1 \subset \mathbf{126}$ multiplets couple to matter fermions but $P_1, K_1 \subset \mathbf{126}$ did not. The $\mathbf{120}$ however contains $P_2, \bar{P}_2$ and $K_2, \bar{K}_2$. Since these mix with $P_1, \bar{P}_1$ and $K_1, \bar{K}_1$, a number of fresh contributions to Baryon decay appear(although they are usually suppressed relative to the usual triplets by their larger masses).

On integrating out the heavy triplet Higgs supermultiplets one obtains[9, 1] the
the MSSM

The fields in terms of which we actually require the decay operators are those of
easily computable and we spare the reader the explicit sampl e expressions.

of our parameter sets. The values of these coefficients in the MS SM basis are thus
determination of the the right handed mixing at
basis by the above standard bi-unitary transformations but with all the matrices
do not carry carets and are unitarily related to the GUT fields [11]:

which we first determined our parameter sets by a random searc h. The MSSM fields
following additional effective

and

here $S = T^{-1}$ and $T$ is the mass matrix for $[3, 1, \pm 2/3]$-sector triplets : $W = \bar{v}T^j t_j + \ldots$, while

The carets on the fields denote that they are in the $h_{AB}$ diagonal basis(at $M_X$) in
which we first determined our parameter sets by a random search. The MSSM fields
do not carry carets and are unitarily related to the GUT fields[11]:

The fields in terms of which we actually require the decay operators are those of
the MSSM $Y^{u,e}$-diagonal $Y^d$ diagonal-modulo-CKM basis. So one passes to that
basis by the above standard bi-unitary transformations but with all the matrices
determined(upto conventional phases) by diagonalization of the caret basis. The
determination of the the right handed mixing at $M_X$ underscores the completeness
of our parameter sets. The values of these coefficients in the MSSM basis are thus
easily computable and we spare the reader the explicit sample expressions.
A proper analysis, now under way\cite{49}, requires one to integrate the renormalization group flow of the above set of coefficients down to the scale $M_S$ along with the rest of the MSSM hard and soft parameter flow and there again pass to the MSSM $Y^{u,e,d}$ diagonal basis appropriate to that scale by rediagonalization. One will then dress the 2-fermion-2-sfermion operator contained in the supersymmetric $\Delta B \neq 0, d = 4$ effective superpotential by gaugino/Higgsino exchange to obtain the effective 4 fermion operators for Baryon decay at $M_S$ and finally renormalize these via SM RG flow to the nucleon mass scale. Although important for precise numbers these re-scalings do not add anything new as long as one has already allowed for an arbitrary scalar mixing matrix at $M_S$. Thus we can estimate the RRRR channel decay rates for the first solution set simply using the $\{R_{ABCD}(M_X)\}$ array values(in the MSSM basis(without carets) at $M_X$) given explicitly above but for the rest using the standard\cite{41, 42, 28} chiral Lagrangian based formulae for converting from the quark to hadron states. The sfermion fields of the $d = 5$ effective operator in the Lagrangian are in the MSSM $Y^{u,d}$ diagonal CKM basis, which is not the one that diagonalizes the sfermion mass squared matrices(which are in fact $6 \times 6$ if even $O(M_W/M_S)$ effects– which mix $SU(2)_L$ singlets and doublets –are retained). Thus their contractions(propagators) introduce the unitary diagonalization matrices ($\tilde{W}_f$ of the the sfermion mass squared terms when rewriting these dressing contractions of the 2-scalar line,one fermion line dressing triangles in terms of the sfermion mass eigenstate propagators.

The one loop triangle diagrams that dress the $d = 5$ chiral vertex from the GUT can be written in terms of a standard function:

$$f(M, \tilde{m}_1, \tilde{m}_2) = \frac{M}{\tilde{m}_1^2 - \tilde{m}_2^2} \left( \frac{\tilde{m}_1^2}{\tilde{m}_1^2 - M^2} \ln \frac{\tilde{m}_1^2}{M^2} - \frac{\tilde{m}_2^2}{\tilde{m}_2^2 - M^2} \ln \frac{\tilde{m}_2^2}{M^2} \right)$$  \hspace{1cm} (21)

Thus to account for the complete RRRR amplitude one need compute only the folding of this function - for a given sfermion spectrum- with the $R_{ABCD}$ coefficients estimated as approximately those given above at $M_X$ and the unitary sfermion diagonalization matrices. For the amplitude for the dominant $p \to K^+ \bar{\nu}_\tau$ channel this involves

$$A = \alpha (R_{ext}(1 + \frac{3F + D}{3B}) + R_{int} \frac{2m_D}{3m_B})$$ \hspace{1cm} (22)

where

$$R_{ext} = A_t(R^*_{3131} - R^*_{3311})Y^d_{33}Y^d_{22} \sum_{B=1}^{3} f(\mu, \tilde{m}_r^2, \tilde{m}_a^2) \tilde{W}_{3B}^{\tilde{u}_B} \tilde{W}_{2B}^{\tilde{u}} \hspace{1cm} (23)$$

$$R_{int} = A_t((R^*_{3132} - R^*_{3312})Y^d_{33}Y^d_{11} \sum_{B=1}^{3} f(\mu, \tilde{m}_r^2, \tilde{m}_a^2) \tilde{W}_{3B}^{\tilde{u}_B} \tilde{W}_{1B}^{\tilde{u}})$$ \hspace{1cm} (24)
The definitions of the baryon mass $m_B$ and the proton matrix elements $\alpha_p, D, F, B$ can be found in [42] and we have indicated only the dominant 3 generation contributions for the lepton side of the triangle diagram.

Since all ingredients are in hand one obtains with $A_l = 1.43; F = 0.44; D = 0.81; \alpha_p = -0.003 GeV^3$, with a standard CKM type parametrization for the mixing matrix $\tilde{W}^u$ in terms of mixing angles (we assume the phase is zero for simplicity) $(\theta = \theta_{12}, \phi = \theta_{13}, \chi = \theta_{23})$, and with the free squark masses taken to be $\tilde{m}_u = 0.98 \tilde{m}_t, \tilde{m}_c = 0.99 \tilde{m}_t$, for the proton life time in years and all masses in MeV

$$
\tau_p = 2.1 \times 10^{-29} yr - MeV \left( \frac{32\pi m_p^2 f_p^2}{(m_p^2 - m_K^2)^2} \right) \alpha_p (R_{ext} (1 + m_p (3F + D)) + R_{int} \frac{2m_p D}{(23m_B)})^{-2}
$$

$$
= 5.02565 \times 10^{18} \text{ yrs} \times (\cos^2 \phi \cos \chi \sin \chi + 1.00061(\cos \chi \sin \theta + \cos \theta \sin \phi \sin \chi)(\cos \theta \cos \chi \sin \phi - \sin \theta \sin \chi))^{-2}
$$

$$
= 1.5 \times 10^{31} \text{ yrs}
$$

(25)

where the last equality is obtained at the angle values $\{\phi = 0.001, \theta = 0.2, \chi = 0.002\}$. With only moderately different values of the parameters one can increase or (more easily) decrease this life time by factors of up to $10^5$ or more. Thus we see that the result of the answer is extremely sensitively dependent on not only the sfermion masses and their degree of degeneracy but also on their mixing. The mixing of the sfermions will be limited also by their effect on the effective CKM mixing as well as by all the constraints coming from an analysis of supersymmetric quark Flavour violation through box diagrams[43] as well as of supersymmetric lepton Flavour violation though[44]. In our approach the soft susy parameters are thus doubly constrained: by the constraints on flavour changing processes and by accurate low energy fit to fermion masses using GUT parameter sets compatible with the high energy extrapolated data but with enlarged errors. Moreover we must still incorporate the effects of right handed neutrino thresholds in the RG flow of the candidate parameter sets down to $M_S$. An in extenso calculation of all Baryon decay widths in this model using an assumed soft susy parameter spectrum as used in e.g [42] would simply have no value in the current context and it would be inane to pursue or demand it. Thus a proper analysis will be lengthy, indeed it is an extensive research program that requires many innovations to be able to handle so many diverse constraints on parameters involved in fits to low energy data through highly non linear relations comprising not only highly non linear definitions of the effective MMSM Yukawa couplings (via the fine tuning of the doublet mass matrix) but also the RG flow between $M_X$ and low energies and the experimentally bounded but otherwise completely unknown sfermion data as well. This requires an order of magnitude increase in the scale of integration of our algorithms to enable searches that are automated from beginning to end. We
are still evaluating if this is worth attempting at this stage. We therefore leave a
detailed study of the possibilities for a sequel [49].

5 Discussion

In this paper (which is the second in a triplet dedicated to formulating the falsifying
constraints to which the NMSGUT can be subjected on the basis of its structure [1],
fermion mass fits [2, 48] and exotic processes [49]) we have indicated a feasible route
to pursue if the New or Full Minimal Supersymmetric SO(10) GUT is to cross the
final hurdles and become vulnerable to falsification by the upcoming data from the
LHC and its successors and the large water Cerenkov detectors now being planned to
further raise the limits on proton decay. This route evades difficulties and confusion
that had waylaid the fermion fitting program in MSGUTs in the recent past. We have
shown that the way out is to accept the difficulties as indicative of a particular type
of sfermion spectra which will modify the $d, s$ quark Yukawa couplings significantly
at the threshold $M_S$. To our knowledge this is a novel proposal that ties the fermion
spectrum achievable by a Susy GUT unequivocally to its soft breaking terms. Such
a program is feasible and plausible precisely because of the tight constraints under
which the NMSGUT functions and the lack of freedom to add additional fermion
mass operators as is so abundantly available in SO(10) Susy theories based on low
dimensional Higgs fields and non-renormalizable operators.

If the colliders do not belie their promises they may sketch the supersymmetric
spectra for us. With these spectra in hand the NMSGUT will be have to face both
dges of the scissor we have shown operative: failure at $M_X$ to fit fermion spectra
unless the sfermion and gaugino spectrum is such as to push the down fermion Yukawa
couplings above $M_S$ to values where they match with the otherwise accurate and
consistent fits we find at $M_X$. The fits we have found rely on very small $126$
couplings to boost the Type I seesaw masses while completely killing the Type II masses.
Interestingly this has the effect of lowering the right handed neutrino masses into
just the region $10^9 - 10^{13}$ GeV where most Leptogenesis scenarios typically place
them. Moreover the same fits are consistent with a common GUT scale parameter :
something that it was feared (from the earliest days of SUSY SO(10) GUTs [5]) would
definitely suppress Type I masses completely. The no-go mechanism that the work of
[18] had seemed to have proven against our proposal for ultra small $126$ couplings $f_{AB}$
has been neatly evaded by the theory in a natural way by accepting the indication of
the reality of low down type quark masses as a token signal yielded by the model under
the pressure of the fit. This has the potential to yield information about the sfermion
masses at $M_S$ and thus about the whole pattern of Susy breaking parameters at
$M_X$. Inclusion of non-diagonal corrections and flavour change constraints dependent
on all the sfermions will significantly refine the filtration of candidates and push the
model further towards over determination. When these multiple constraints have
been applied to the viable parameter sets emerging from a comprehensive survey of the downhill simplex fits that we are now carrying out we expect that the NMSGUM will finally truly graduate from a very attractive and persuasive Model to the status of a mature Theory (ready to attempt its marriage to gravity?), if only on the day it cannot any longer escape this projected falsifying prosecution and must therefore be considered a falsified theory; that day however has yet again been deferred by our work presented here.

6 Acknowledgments

We thank Sunil Mittal for writing the first version of the 2-loop renormalization group flow program that we used. C.S.A acknowledges useful correspondence with S. Antusch and informative discussions with K. Babu and Ts. Enkhbat. C.S.A thanks G. Senjanovic for hospitality at ICTP, Trieste, where this work was completed. The work of C.S.A was supported by a grant No SR/S2/HEP-11/2005 from the Department of Science and Technology of the Govt. of India and that of S.K.G by a University Grants Commission Junior Research fellowship.

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