Understanding the wicked world that surrounds you

Luis A. Anchordoqui\textsuperscript{1,2,3} and Gabriela V. Anchordoqui\textsuperscript{4}

\textsuperscript{1}Department of Physics & Astronomy, Lehman College, City University of New York, NY 10468, USA
\textsuperscript{2}Department of Physics, Graduate Center, City University of New York, NY 10016, USA
\textsuperscript{3}Department of Astrophysics, American Museum of Natural History, NY 10024, USA
\textsuperscript{4}Sociedad Odontológica La Plata, Calle 13 No. 680, La Plata 1900, Argentina

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There is a new urgency for wider segments of the non-expert population to join in shaping the future direction of our technological society. Certain decisions being made now in the development of energy resources will profoundly affect the lives of the next few generations. This course is oriented toward providing an educational base for participating in these decisions. At this level, there is no need for any mathematics beyond simple arithmetic, nor for any previous knowledge of physics or chemistry.

I. CONSERVATION OF ENERGY

§ What is energy?

Energy = \{ makes things happen \}
\{ does work \}
\{ does not change as things happen \} . \quad (1)

The last statement in (1) is a profound and subtle truth, a great discovery of the 19th century; namely, that energy is not only an intuitive vagary (“I have lots of energy”) but can be measured, and once you have a certain amount in a close system (e.g., Earth) you keep it, but it does change form. This is called the first law of thermodynamics: energy changes in form but not in amount \cite{1}. Our world is full of transformation of energy, e.g.,

- Stored chemical energy in muscles ⇒ mechanical energy of movement and heat energy in muscles.
- Stored gravitational energy of water in a dam ⇒ energy of falling water ⇒ mechanical energy of generators ⇒ electrical energy.
- Stored nuclear energy in uranium ⇒ kinetic energy of fission fragments ⇒ heat energy in water ⇒ electrical energy.

In all these processes, there is a quantity whose amount is the same before and after the process. That quantity is energy.

The second law is more subtle. It says that not only can’t you increase the amount of energy in the system, but in any process you always degrade part of it to a form which is less useful \cite{2}. For example, the chemical energy stored in the muscles is the source of some useful work, but part of it is also converted to heat which can never be entirely recovered for useful work. (Usually, it is entirely wasted.) Many times these losses are due to friction.

§ If energy is a quantity, how do you measure it?

This is a long story, and as a result of the length of the story, we have many different units. In the U.S., a common unit for electrical energy is a kilowatt-hour (kWh), and for heat energy either the British thermal unit (Btu), or kilocalorie (kcal or Cal). The latter is the one used in diet planning. Since heat and electrical energy can be changed into one another without loss, there is a fixed relation between the units; namely, 1 kWh = 860 Cal, 1 kWh = 3,412 Btu, 1 Cal = 3,967 Btu.

§ What do these units mean? A Btu raises the temperature of a pint (1 lb) of water 1°F. So to bring 1 quart (2 lb) of H\textsubscript{2}O from 42°F to boiling (212°F) takes

\[ 2 \times (212 - 42) = 340 \text{ Btu} . \quad (2) \]

This by itself is not very enlightening; but when you are told that every time a cubic foot of natural gas is burnt, 1,000 Btu of heat (photon energy) are released, and therefore 3 quarts of water can be boiling; it is only then that you can plan how many cubic feet of gas you need to power a city and for better or worse, planning is absolutely essential in our modern density populated society. It is no longer possible for everyone to go out and chop a tree on a moment’s notice. A Cal raises the temperature of 1 liter of water 1°C.

The kWh is most often used as a measure of electrical energy, and is the energy supplied by a 10 ampererecurrent from a 100 volt line running for 1 hour. This is the energy released in an hour’s use of a toaster, or 10 hours’ use of a 100 watt bulb.

A 1,000 watt heater, running for one hour uses 1 kWh, and shows up as a 12¢ cost on your month bill. Burning a gallon of gasoline releases 36 kWh of heat, or 125,000 Btu or about 31,000 Cal. Your daily food consumption (about 2,200 Cal) is about 2.5 kWh. If you didn’t get rid of the heat, you would boil in 3 days. Compare the price of 1 kWh of electricity to that of food, which is about $3.00. The kWh is incredibly cheap for what it does. The trouble is we don’t really appreciate what it does. Imagine having no electric lighting, and then someone offers you the use of a 60 watt bulb for two hours a night for a week, all for a nickel – you take it!

A kWh is about 3,400 Btu. This means that the 100 watt bulb going for 10 hours submerged under 100 lb of water will raise the temperature of that water by 34°F.
If energy is used at 1 kWh/hour, we simply say the power usage is 1 kW (1 kilowatt). So an electric bulb burns at 0.1 kW (100 watts).

§ Relative motion interlude. Everything is in motion. Even the stuff that appears to be motionless moves, but (of course) the motion is relative. For example, while you are reading these notes, you are moving at about 107,000 kilometers per hour relative to the Sun, and you are moving even faster relative to the center of the Galaxy. When we discuss the motion of something we describe the motion with respect to something else. In other words, to describe the motion of something we need a reference point, sometimes called the origin, and a reference of time, e.g.: A long time ago in a galaxy far, far away...

The position defines an object’s location in space with respect to the origin. To determine the object’s location you need a measuring stick. The displacement defines the change in position that occurs over a given period of time. Note that it is not the same to move 3 meters north from the origin than 3 meters south. To distinguish a displacement to the north of the origin from that to the south, or the east, or the west, or up, or down, we say the displacement has a magnitude (3 meters in the example above) and a direction (north, south, east, west, up, or down). Objects with magnitude and direction are generically called vectors.

Consider the situation of the cartoon in Fig. 1. One morning, on his way to work, Harry left home on time, and he walk at 3 m/s east towards downtown. After exactly one minute, when Harry met Sally, he realized that he had left his computer at home, so he turns around and runs, at 6 m/s, back to get it. He was running twice as fast as he walked, so it took half as long (30 seconds) to get home again.

There are several ways to analyze those 90 seconds between the time Harry left home and the time he arrived back again. One number to calculate is his average speed, which is defined as the total distance covered divided by the time. If he walked for 60 seconds at 3 m/s, he covered 180 m. Then he covered the same distance on the way back, so he went 360 m in 90 seconds,

\[
\text{average speed} = \frac{\text{distance}}{\text{elapsed time}} = 4 \text{ m/s}. \quad (3)
\]

The average velocity, on the other hand, is given by:

\[
\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}}. \quad (4)
\]

Note that for the situation described above Harry’s average velocity for the round trip is zero, because he was back where he started, so the displacement was zero.

We usually think about speed and velocity in terms of their instantaneous values, which tell us how fast, and,
for velocity, in what direction an object is traveling at a particular instant. The instantaneous velocity is defined as the rate of change of position with time, for a very small time interval. The instantaneous speed is simply the magnitude of the instantaneous velocity.

An object accelerates whenever its velocity changes. The acceleration vector measures the rate at which an object speeds up, slows down, or changes direction. Any of these variations constitutes a change in velocity. Going back to the example we used above, let’s say instead of instantly breaking into a run the moment Harry turned around, he steadily increased his velocity from 3 m/s west to 6 m/s west in a 10 second period. If his velocity increased at a constant rate, he experienced a constant acceleration of 0.3 m/s². We can figure out the average velocity during this time. If the acceleration is constant, which it is in this case, then the average velocity is simply the average of the initial and final velocities. The average of 3 m/s west and 6 m/s west is 4.5 m/s west. This average velocity can then be used to calculate the distance he traveled during the acceleration period, which was 10 seconds long. The distance is simply the average velocity multiplied by the time interval, so 45 m.

Similar to the way the average velocity is related to the displacement, the average acceleration is related to the change in velocity: the average acceleration is the change in velocity over the time interval (in this case a change in velocity of 3 m/s in a time interval of 10 seconds). The instantaneous acceleration is defined as the rate of change of velocity with time, for a very small time interval. As with the instantaneous velocity, the time interval is very small (unless the acceleration is constant, and then the time interval can be as big as we feel like making it).

On the way out, Harry traveled at a constant velocity, so his acceleration was zero. On the trip back his instantaneous acceleration was 0.3 m/s² for the first 10 seconds, and then zero after that as he maintained the top speed. Just as Harry arrived back at his front door, his instantaneous acceleration would be negative, because his velocity dropped from 6 m/s west to zero in a small time interval. If he took 2 seconds to come to a stop, his acceleration was \( a = -3 \text{ m/s}^2 \). To return, Harry decided to take the subway to ensure he was going to arrive on time.

§ Types of energy. There are two types of energy: potential and kinetic. Potential energy is energy that is stored in an object due to its position. This type of energy is not in use, but is available to do work. For example, a book possesses potential energy when it is stationary on the top of the bookshelf. Kinetic energy is energy that is possessed by an object due to its motion. For example, if the book were to fall off the shelf, it would possess kinetic energy as it fell. The kinetic energy of an object depends on the mass \( m \) of the object as well as its speed \( v \),

\[
\text{kinetic energy } K = \frac{1}{2}mv^2. \tag{5}
\]

All energy is either potential or kinetic. The fantastic thing about this commodity, energy, is that when the conversion from potential to kinetic takes place it occurs in a definite, predictable way. For example, a ton of water dropping over Niagara Falls always yields the same amount of electrical energy. Or the combustion of a gallon of oil always gives the same amount of heat (thermal energy). Thus, all forms of energy have the same measure.

II. FORMS OF ENERGY

§ Mechanical energy. It represents the energy that is possessed by a mechanical system or device due to its motion or position. Stated differently, mechanical energy is the ability of an object to do work. Mechanical energy can be either kinetic (energy in motion) or potential (energy that is stored). The sum of an object’s kinetic and potential energy equals the object’s total mechanical energy.

The difference between kinetic and mechanical energy is that kinetic is a type of energy, while mechanical is a form that energy takes. For instance, a bow that has been drawn and a bow that is launching an arrow are both examples of mechanical energy. However, they do not both have the same type of energy. The drawn bow is an example of potential energy, because the energy necessary to launch the arrow is only being stored in the bow; whereas the bow in motion is an example of kinetic energy, because it is doing work. If the arrow strikes a bell, some of its energy will be converted to sound energy. It will no longer be mechanical energy, but it will still be kinetic energy.

§ Thermal energy. This is the kinetic energy of molecules moving in a random way. The faster they move, the higher the temperature. The thermal energy may be transferred from one body (say the ocean) to another (say the air). This is called heating something. The transfer occurs through the collision of the speedy molecules in the warm body (the ocean) with the sluggish molecules in the cold body (the air), resulting in a rise of temperature of the air.

§ Radiant energy (photons). This is a form of kinetic energy carried by gamma rays, X-rays, ultraviolet (UV) rays, light, infrared (IR) rays, microwaves, and radio waves. The energy comes in tiny packets called photons, and the energy in each packet depends on the type of radiant energy: X-ray photons carry higher energy than UV photons, which are in turn more energetic than light photons, etc. Light photons themselves differ in the amount of energy they carry: blue photons more
than yellow, yellow more than red, etc. The spread is enormous: an X-ray photon is about 10,000 times as energetic as a blue photon, which in turn is about 10 billion times as energetic as a photon carrying an AM signal. Single photons are detectable by the eye only under very special conditions. Typically, a 60 watt light bulb is emitting \(10^{19}\) (10 billion billion) photons per second of visible light. Even at a distance of 100 yards, your eye would be intercepting about 5 billion of these per second. Only at a distance of about 600 miles from the light bulb does your eye intercept an average of only one of these photons per second.

§ Newtonian dynamics interlude. The newtonian idea of force is based on experimental observation. Experiment tells us that everything in the universe seems to have a preferred configuration: e.g., (i) masses attract each other; (ii) magnets can repel or attract one another. The concept of a force is introduced to quantify the tendency of objects to move towards their preferred configuration. If objects accelerate very quickly towards their preferred configuration, then we say that there is a big force acting on them. If they don’t move (or move at constant velocity), then we say there is no force.

We cannot see a force; we can only deduce its existence by observing its effect. More specifically, forces are defined through Newton’s laws of motion:

0. A particle is a small mass at some position in space.
1. When the sum of the forces acting on a particle is zero, its velocity is constant.
2. The sum of forces acting on a particle of constant mass is equal to the product of the mass of the particle and its acceleration:

\[
\text{force} = \text{mass} \times \text{acceleration}. \tag{6}
\]

3. The forces exerted by two particles on each other are equal in magnitude and opposite in direction.

The standard unit of force is the newton, given the symbol N. The newton is a derived unit, defined through Newton’s second law of motion: one newton is the force needed to accelerate one kilogram (kg = 2.2 lb) of mass at the rate of one meter per second squared in direction of the applied force.

Now, a point worth noting at this juncture is that forces are vectors, which evidently have both magnitude and direction. For example, the gravitational force is a force that attracts any two objects with mass. The magnitude of this force is directly dependent upon the masses of both objects and inversely proportional to the square of the distance that separates their centers,

\[
\text{gravitational force} = F_g = \frac{GMm}{r^2}, \tag{7}
\]

where \(G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2\) is the proportionality constant. The direction of the force is along the line joining the two objects. Near the Earth’s surface, the acceleration due to gravity is approximately constant,

\[
\text{gravitational acceleration} = g = \frac{GM_b}{R_b^2} \approx 9.8 \text{ m/s}^2, \tag{8}
\]

where \(M_b = 1.3 \times 10^{25} \text{ lb}\) is the mass of the Earth and \(R_b = 3,959 \text{ miles}\) its radius.

So, the Earth pulls on the Moon because of gravity? Why doesn’t the moon get pulled into the Earth and crash? To answer this provocative question we first note that an object can move around in a circle with a constant speed yet still be accelerating because its direction is constantly changing. This acceleration, which is always directed toward the center of the circle, is called centripetal acceleration. The magnitude of this acceleration is written as

\[
\text{centripetal acceleration} = \frac{v^2}{r}, \tag{9}
\]

where \(v\) is the speed of the object and \(r\) the radius of the circle. If a mass is being accelerated toward the center of a circle, it must be acted upon by an unbalanced force that gives it this acceleration. The centripetal force is the net force required to keep an object moving on a circular path. It is always directed inward toward the center of the circle. So, we can say that the Moon continuously falls towards Earth due to gravity, but does not get any closer to Earth because its motion is an orbit. In other words, the Moon is constantly trying to fall upon the Earth, due to the force of gravity; but it is constantly missing, due to its tangential velocity.

§ Gravitational energy. This is energy stored whenever two masses are separated, and is recoverable as kinetic energy when they fall together. Examples are: (i) water to go over a waterfall (water is separated from earth); (ii) high jumper at top of jump; (iii) interstellar dust before it comes together to form a star. The general expression for gravitational potential energy arises from the law of gravity and is equal to the work done against gravity to bring a mass to a given point in space.

Indeed, the general form of the gravitational potential energy follows from (7) and, for a particle of mass \(m\), is given by

\[
\text{gravitational potential energy} = U = -\frac{GMm}{r}, \tag{10}
\]

where \(M\) is the mass of the attracting body, and \(r\) is the distance between their centers. Note that the gravitational force approaches zero for large distances, and consequently it makes sense to choose the zero of gravitational potential energy at an infinite distance away. This means that the gravitational potential energy near a planet is negative. This negative potential energy is indicative of a bound state; once a mass is near a large body, it is trapped until something can provide enough energy to allow it to escape.
Since the zero of gravitational potential energy can be chosen at any point (like the choice of the zero of a coordinate system), the potential energy at a height \( h \) above that point is equal to the work which would be required to lift the object to that height with no net change in kinetic energy. We define the work done by a constant force as the product of the force and the distance moved in the direction of the force. With this in mind, the gravitational potential energy of an object of mass \( m \) near the Earth’s surface is found to be

\[
U = \text{gravitational force} \times \text{distance} = mgh , \tag{11}
\]

where \( h \) is the height above the zero-point energy.

With regard to the sign difference in (10) and (11) we might note in passing that it relates to the choice of zero-point energy, which is taken at infinity in (10) and at the surface of the Earth in (11). This choice is completely arbitrary because the important quantity is the difference in potential energy, and this difference will be the same regardless of the choice of zero level. However, once this position is chosen, it must remain fixed to describe a given physical phenomenon.

In summary, the gravitational energy is stored energy. It is not at all obvious that it is present. However, it can be called on and used when needed, mostly to be converted to some form of kinetic energy.

§ **Nuclear energy.** This is energy which is stored in certain (almost) unstable nuclei (such as uranium), and which is released when the unstable nucleus is disturbed (in much the same way as a stretched rubber band which is snipped). This is called fission and takes place in nuclear reactors. It is also energy which is stored when two nuclei which want to come together are allowed to do so. (Such as a stretched rubber band which is allowed to contract.) This is called fusion and takes place in stars.

§ **Chemical energy.** This is a repeat of the nuclear story, but with much lower energy content per gram of material. Some chemicals release energy when they are disturbed (TNT, nitroglycerin) some when they combine (carbon and oxygen). The potential energy is said to be stored in the carbon (oil, coal, etc.). More correctly, it is stored in the electric field between the carbon and the oxygen. When the C and O come together, the electric field gives up its energy in the form of a photon (kinetic energy).

§ **Forms of energy in a steady flow.** Water is everywhere around us, covering 71% of the Earth’s surface. The water content of a human being can vary between 45% and 70% of body mass. Water can exist in three states (a.k.a. phases) of matter: solid (ice), liquid, or gas. Matter in the solid state has a definite shape and size. Matter in the liquid phase has a fixed volume, but can be of any shape. Matter in the gas phase can be of any shape and also can be easily compressed. Liquids and gases both flow, and this is why they are called fluids.

\[
\text{liquid pressure} = P = \rho gh , \tag{13}
\]

because \( F_g = mg \) and \( m = \rho Ah \), i.e., \( P \) depends on the area \( A \) over which the force \( F_g \) is distributed. The unit for pressure is the Pascal, 1 Pa = 1 N/m².

Anyone who has ever lifted a heavy submerged object out of the water is familiar with the concept of buoyancy, which is the apparent loss of weight experienced by objects submerged in a liquid. This is because, when an object is submerged, the water exerts an upward force on it that it is exactly opposite to the direction of gravity’s pull. This upward force is called the buoyant force, and it is a consequence of pressure increasing with depth. The macroscopic description of the buoyant force, which results from a very large number of collisions of the fluid
molecules, is called Archimedes’ principle [5]. It is stated as follows: An immersed object is buoyed up by a force equal to the weight of the fluid it displaces.

So far we have only discussed fluids at rest. In studying fluid dynamics we focus our attention on what is happening to various fluid particles at a particular point in space at a particular time. The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time. This does not mean that the velocity at different points in space is the same. The velocity of a particular particle may change as it moves from one point to another. That is, at some other point the particle may have a different velocity, but every other particle which passes the second point behaves exactly as the previous particle that has just passed that point. Each particle follows a smooth path, and the path of the particles do not cross each other. The path taken by a fluid particle under a steady flow is called a streamline.

Consider a steady flow of a fluid through an enclosed pipe. Because no fluid flows in or out of the sides, the mass flowing past any point during a short period of time must be the same as the mass flowing past any other point, so

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2,$$

where the subscript 1 and 2 refer to two different points along the pipe shown in Fig. 2. This equation is called the continuity equation. If the fluid is incompressible, then the density is the same at all points along the pipe and (14) becomes

$$A_1 v_1 = A_2 v_2.$$

We see that if the cross sectional area is decreased, then the flow rate increases. This is demonstrated when you hold your finger over the part of the outlet of a garden hose. Because you decrease the cross sectional area, the water velocity increases.

The general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change) was first derived by Bernoulli and is given by

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2;$$

see Fig. 3 [6]. Since 1 and 2 refer to any two locations along the pipeline, we may write the expression in general as

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}.$$  

Bernoulli’s equation can be considered to be a statement of the conservation of energy principle appropriate for a steady flow: The work done by the pressure forces on the fluid particle is equal to the increase in the kinetic and gravitational potential energy of the particle.

III. THERMODYNAMICS

§ What is heat? The subjective sensation, familiar to everyone, is that when you touch something hot, heat goes into your hand and burns it. We talk of the transfer of heat from one object (the radiator) to another (your hand). The fact that the radiator is hot is sometimes expressed as “the radiator is at high temperature.”

In the olden days (before 1800), people thought of heat as a fluid (called caloric), which runs from a hot body to a cooler body. Each body has a fixed amount of heat, or caloric, and the temperature was a measure of the concentration of heat (a small body could be at a higher temperature than a larger body, but could have less heat, because the small body has a higher concentration of heat content, say more per gram of material).

This theory, although in some ways appealing, fell apart with the observation of Count Rumford: that by grinding away at the cannon barrel he was able to produce unlimited amount of heat (that is, as long as he kept grinding away!) [7]. So it wasn’t true that there was a fixed amount of caloric, or heat, in the cannon: there was a way of creating it by doing work [8].

Heat and work are mutually convertible: you can do work and get heat, or use heat and do work (a car engine or steam engine). The catch is, that when you go from heat to useful work, you never achieve 100% conversion: there is always some heat wasted. The reason will be given in later discussion.

§ Absolute zero. As mentioned in Sec. II, the random motion of molecules in a sample of matter constitutes a form of kinetic energy, called thermal energy. The temperature is a measure of the average kinetic energy/molecule. Temperatures are usually measured in degrees Fahrenheit (°F) [9], or degrees Celsius (°C) [10]. The temperature at which water freezes is defined as 0°C. The temperature at which water boils is defined as 100°C. The relation between the Fahrenheit (°F) and Celsius (°C) temperature scales is given by $F = 9C/5 + 32$. 
It will be convenient to talk about temperature in absolute scale. That is, we would like the absolute zero of temperature to correspond to that point where all molecular motion ceases. It turns out that if we plot the pressure of a given volume of gas as a function of the temperature, we find it looks like the one shown in Fig. 4. The line, extrapolated to zero pressure, gives a value of $-273^\circ C$ (or $-460^\circ F$) for absolute zero. So let us make absolute zero read 0 K (or 0 Kelvin), and therefore the freezing point of water is 273 K [11]. Room temperature ($68^\circ F$) is $20^\circ C = (273 + 20) K = 293 K$. We can never reach absolute zero (0 K), but temperatures as low as $5 \times 10^{-10} K$ have been reached [12]. Liquid nitrogen at atmospheric pressure is at 75 K (about $-200^\circ C$, or $-330^\circ F$).

The coldest natural temperature ever directly measured on Earth is $-89.2^\circ C = -128.6^\circ F = 184.0 K$, which was recorded at the Russian Vostok station in Antarctica (78.5°S, 106.9°E) on 21 July 1983 [13]. On 10 August 2010, satellite observations measured a surface temperature of $-93.2^\circ C = -135.8^\circ F = 180.0 K$ along a ridge between Dome Argus and Dome Fuji (81.8°S, 59.3°E) [14].

§ So, what is heat? Heat is also kinetic energy (moving energy), but it is the kinetic energy of billions of atoms in a material vibrating (in a solid) or zapping around (in a gas) in a random way, going nowhere in particular. Heat is the random energy of atoms and molecules. This is very different from organized energy. It is like throwing a bag of hot popcorn (organized energy) vs. letting them pop around inside the popper (random energy). Popcorn popping very quickly has a lot of heat energy.

Now, the total heat energy of a material depends on two things: (i) how many atoms there are, and (ii) how energetic each atom is (on average). Then the heat energy $Q$ can be expressed as the product of the number of atoms $N$ and the average random energy of an atom $\langle \varepsilon \rangle$,

$$Q = N \langle \varepsilon \rangle.$$  \hspace{1cm} (18)

The temperature is a direct measurement of the second factor, the average energy of an atom. High temperature means peppy atoms; low temperature means sluggish atoms. When the atoms are so sluggish that they do not move, then you have hit bottom in the temperature scale: absolute zero (which is $T = -459^\circ F$, very cold.)

§ How do you get heat from mechanical work? Well, when you push your hand across the table (mechanical energy), you set the molecules at the surface of the table into random vibration and create heat energy (friction, in this case).

§ How do you transfer heat? You touch the hot poker; the madly vibrating atoms on the surface of the poker bombard the molecules of your skin; these then start vibrating, causing all kinds of neurochemical reactions which tell your brain, “It’s hot, let go!”

§ How much work makes how much heat? The important thing to know here is that the same work makes the same amount of heat always. The equivalence will be given by example: for each 50 pounds of force which are pushed or dragged through one foot (say, across the table), you make enough heat to raise the temperature of 1 ounce of water by about 1° F. You can also push with a force of 5 lb through 10 ft, or with 10 lb through 5 ft, as long as force $\times$ distance $= 50$ lb ft. If you push twice as hard (100 lb) through one foot, you make twice the heat. If this last part confuse you, then forget it for now and just remember that heat and mechanical work are interconvertible in definite proportions. The proportions are $1,000$ ft lb $= 1.28$ Btu $= 1/3$ Cal, or $1$ Btu $= 779$ ft lb.

§ First law of thermodynamics. We have seen in Sec. I that this is an expression of the conservation of energy. Energy can cross the boundaries of a closed system in the form of heat $Q$ or work $W$. By system, we mean a well-defined group of particles or objects. Energy transfer
across a system boundary due solely to the temperature difference between a system and its surroundings is called heat. For a closed system, the first law of thermodynamics is expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$  \hspace{1cm} (19)$$

where $E_{\text{in}} = Q_{\text{in}} + W_{\text{in}}$ is the total energy entering the system, $E_{\text{out}} = Q_{\text{out}} + W_{\text{out}}$ is the total energy leaving the system, and $\Delta E_{\text{system}} = E_{\text{final}} - E_{\text{initial}} = E_f - E_i$ is the change in the total energy of the system.

§ Second law of thermodynamics. Each cubic meter of air at room temperature has about $3 \times 10^{25}$ molecules zipping around in a random fashion. The average kinetic energy of each atom in the air is

$$\langle \varepsilon \rangle = \frac{3}{2} k_B T,$$  \hspace{1cm} (20)$$

without reference to individual (atomic or subatomic) constituents. A complete specification of the system requires a description not only of its contents but also of its boundary and the interactions with its surroundings (a.k.a. the environment) permitted by the properties of the boundary. Boundaries need not be impenetrable and may permit passage of matter or energy in either direction or to any degree. An isolated system exchanges neither energy nor mass with its environment. A closed system can exchange energy with its environment but not matter, while open systems also exchange matter.

where $k_B = 1.380 \times 10^{-23} \text{ J} \text{K}^{-1}$ is the Boltzmann constant, and one Joule (J) is the work done by a force of one newton when its point of application moves one meter in the direction of action of the force (equivalent to one 3,600th of a watt-hour). The total kinetic energy of all these molecules is about 347 Btu. The room may then be thought of as a reservoir of thermal energy at temperature 70°F (or 295°K). The question we may well ask is the following: Is it possible to extract some of this thermal energy and change it entirely into useful work (such as turning a generator)? There is no contradiction here with the conservation of energy, but it is a fact that no such process is known. This negative statement, which is the result of everyday experience constitutes the second law of thermodynamics: It is impossible to construct an engine that, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work [2].

Reduced to its simplest terms, the important characteristics of heat-engine cycles may be summed up as follows: (i) There is some process during which there is an absorption of heat from a reservoir at high temperature (called simply the hot reservoir). (ii) After some work is done, there is some process during which heat is rejected to a reservoir at lower temperature (called simply the cold reservoir). For a heat engine, $Q_1$ is the heat taken from the boiler, $W$ is the work done, and $Q_2$ is the
heat transferred to the condenser, with \( Q_1 = W + Q_2 \); see Fig. 5. For a refrigerator (or air conditioner), \( Q_2 \) is the heat taken from the refrigerator by coolant, \( W \) is the energy pumped in by the motor, and \( Q_1 \) is the heat given up to the surrounding air, with \( Q_1 = W + Q_2 \); see Fig. 6.

Now, using the idea of a refrigerator, we can show that a violation of the second law would make it impossible to construct a device which, operating in a cycle, will produce no effect other than transfer of heat from a cooler body to a warmer body. Since this never happens (for statistical reasons), we can then see why the second law works. The proof is as follows: Suppose we had an engine (on the left in Fig. 7) which violates the second law (rejects no heat to the cold reservoir). Then we could use the work liberated by the engine to run a refrigerator which operates between the same two reservoirs and takes heat \( Q_2 \) from the cold, and puts \( W + Q_2 = Q_1 + Q_2 \) into the hot. The net result is a transfer of heat from the cold to the hot reservoir, with nothing else occurring. This is impossible so the engine of the left of Fig. 7 is impossible.

§ Thermal efficiency. So now we come to a practical question: Given a hot reservoir at temperature \( T_1 \), and a cold reservoir at temperature \( T_2 \), what is the highest efficiency with which a heat engine may operate between the two? By efficiency one simply means

\[
\frac{\text{work out}}{\text{heat extracted from hot reservoir}} = \frac{W}{Q_1}. \tag{21}
\]
FIG. 8: A steam engine is a heat engine that performs mechanical work using steam as its working fluid such as railway steam locomotives.

It has been shown that the maximum efficiency is

$$\eta_{\text{max}} = \frac{T_1 - T_2}{T_1} = \frac{670 - 370}{670} \approx 45\%.$$  

(22)

where $T_1$ and $T_2$ are in °K.

For example, the steam engine shown in Fig. 8 with a boiler heating the steam to 400 °C (= 670 °K), and a condenser operating at 100 °C (= 370 °K) will have a maximum theoretical efficiency of

$$\eta_{\text{max}} = \frac{670 - 370}{670} \approx 45\%.$$  

(23)

Similarly the coefficient of performance of a refrigerator, defined as

$$\text{CoP} = \frac{\text{heat removed from cold reservoir}}{\text{work done (electrical energy used)}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$  

(24)

has a maximum value of

$$\text{CoP}_{\text{max}} = \frac{T_2}{T_1 - T_2}.$$  

(25)

For an air conditioner operating between a room at 70°F (= 295 °K) and the outdoors at 85°F (= 395 °K), the maximum theoretical coefficient of performance is

$$\text{CoP}_{\text{max}} = \frac{295}{309 - 295} = \frac{295}{14} = 20.$$  

(26)

In practice, this turns out to be about 7 to 10. Commonly, one rates air conditioners in terms of Btu/kWh. For a coefficient performance of 10, this is 10 kWh (heat)/1 kWh (electric) = 34,000 Btu/kWh.

IV. WAVES AS ENERGY TRANSFER

§ Wave basics. A wave is a type of energy transmission that results from a periodic disturbance (vibration). Waves transfer energy from one place to another without transferring matter. They are composed of a series of repeating patterns. There are two classes of waves: transverse and longitudinal. In the former the vibration is perpendicular to the direction of motion of the wave, whereas in the latter the vibration is in the same direction as the direction of the wave.

Everyone has seen waves on the surface of water. You made them in the bathtub when you were a child, and you have seen them in the ocean. In the open ocean the water waves are transverse. A water wave can travel hundreds of kilometers over the ocean, but the water just moves up and down as the waves pass. Energy is transferred from one water molecule to the next by the forces that hold the molecules together. Near the shore the waves become also longitudinal: the small distance from the surface to the bottom of ocean makes the difference, see Fig. 9.

We live surrounded by waves. Some are visible, others are not. By observing the visible waves (e.g., in water) we can describe some characteristics that all waves, including invisibles ones, have in common.

§ Periodic motion. Most waves originate from objects that are vibrating so rapidly that they are difficult to observed with our unaided senses. For the purposes of describing the properties of vibrating objects, we need a slowly moving device such as a mass bouncing on a spring or a pendulum; see Fig. 10.

When an object repeats a pattern of motion, as a bouncing springs does, we say the object exhibits periodic motion. The vibration, or oscillation, of the object is repeated over and over with the same time interval each time. When we describe the motion of a vibrating object, we call one complete oscillation a cycle. The number of cycles per second is called the frequency $\nu$. The unit used to measure frequency is the hertz (Hz). Another term used in describing vibrations is the period $T$, which is the time required for one cycle. The period is usually measured in seconds. Frequency and period are reciprocals, i.e.,

$$\nu = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{\nu}$$  

(27)

If the frequency is 60 Hz, then the period is 1/60 s (or 0.017 s). If the period is 0.01 s, the frequency is 100 Hz.

As a pendulum of length $\ell$ swings, it repeats the same
FIG. 9: Evolution of waves in the surface of the ocean. In the upper panel we show the circular path of particles in the open ocean due to oscillations from passing waves. In the lower panel we show the waves of transition. The frictional interaction of the wave with the seabed causes orbital particle circles to stretch as the wave is dissipated. After a wave breaks, the remains of the wave moves as a chaotic surf until it spreads onto the beach as swash.

FIG. 10: Periodic motion of a vibrating spring (left) and pendulum (right). The rest position is where the object will remain at rest. The object can move through its rest position. The distance in either direction from the rest position to a maximum displacement is called the amplitude.

motion in equal time intervals,

\[ T = 2\pi \sqrt{\ell/g}. \]  \hspace{1cm} (28)

We say it exhibits periodic motion. Observing successive swings, we find that the distances reached by the pendulum on either side of the rest position are almost equal. In the same way a vertically bouncing mass on a spring exhibits periodic motion, and it too moves almost the same distance on either side of the rest position. This is a property of all objects oscillating with periodic motion. The distance in either direction from the rest position to maximum displacement is called the amplitude \( A \).

§ What is sound? We have seen that a wave is a transfer of energy, in the form of a disturbance usually through a material, or medium. Sound is a pressure wave, which is created by a vibrating object. This vibrations set particles in the surrounding medium (typical air) in vibrational motion, thus transporting energy through the medium. Since the particles are moving in parallel direction to the wave movement, the sound wave is referred to as a longitudinal wave. The result of longitudinal waves is the creation of compressions and rarefactions within the air, as shown in Fig. [11]
A sound wave, as any other wave can be characterized by its: (i) amplitude \( A \), which is the distance from the midpoint of the wave to a crest or trough (maximum displacement from equilibrium); (ii) frequency \( \nu \), which is the number of repeating patterns (cycles) per unit of time; (iii) period \( T \), which is the time for one cycle; (iv) wavelength \( \lambda \), which is the distance (shown in Fig. 9) from one crest (or trough) to another crest (or trough), and (v) speed, 

\[
v = \lambda \nu = \lambda / T.
\]  

(29)

The wavelength has distance units, such as meters, and so the speed is measured in m/s. The sound requires an elastic medium (solid, liquid, or gas) for transmission. The speed of sound in dry air is about 330 m/s at 0°C, and increases 0.60 m/s for every °C increase, i.e.,

\[
v_{\text{sound}} = \left[ 331.5 + 0.6 \left( \frac{T}{\circ C} \right) \right] \text{m/s}. \]  

(30)

The amount of work done to generate the energy that sets the particles in motion is reflected in the degree of displacement which is measured as the amplitude of a sound. The frequency is measured as the number of complete back-and-forth vibrations of a particle of the medium per unit of time. The human ear can hear from 20 to 20,000 Hz. Infrasonic is below this frequency and ultrasonic above. The softest audible sound modulates the air pressure by around \( 10^{-6} \) Pa, whereas the loudest (pain inflicting) audible sound does it by \( 10^2 \) Pa.

The sound can bounce off of objects, and the angle of incidence is equal to the angle of reflection. The sound reflection gives rise to echoes. Multiple echoes are called reverberations. The study of the sound properties (especially reflections) is called acoustics. The change of the speed of sound in different mediums can bend a wave if it hits the different medium at a non 90° angle. This is called refraction. The wave bends toward a slower medium or away from a faster medium. Ultrasound imaging, bats, and dolphins all use reflection and refraction of sound waves.

\[\text{§ Doppler effect.}\] When we observe a sound wave from a source at rest, the time between the arrival wave crests at our instruments is the same as the time between crests as they leave the source. However, if the source is moving away from us, the time between arrivals of successive wave crests is increased over the time between their departures from the source, because each crest has a little farther to go on its journey to us than the crest before. The time between crests is just the wavelength divided by the speed of the wave, so a wave sent out by a source moving away from us will appear to have a longer wavelength than if the source were at rest. Likewise, if the source is moving toward us, the time between arrivals of the wave crests is decreased because each successive crest has a shorter distance to go, and the waves appear to have a shorter wavelength.

A nice analogy was put forward by Weinberg [15]. He compared the situation with a travelling man that has
to send a letter home regularly once a week during his travels: while he is travelling away from home, each successive letter will have a little farther to go than the one before, so his letters will arrive a little more than a week apart; on the homeward leg of his journey, each successive letter will have a shorter distance to travel, so they will arrive more frequently than once a week.

The Doppler effect is the change in the observed frequency of a source due to the relative motion between the source and the receiver. The relative motion that affects the observed frequency is only the motion in the line-of-sight between the source and the receiver.

We first consider the relative motion of the receiver. If a source is stationary, as the one exhibited in Fig. 12, it will emit sound waves that propagate out from the source as shown in the figure. As the receiver moves towards the source with velocity $V_{\text{receiver}}$, it will detect the sound coming from the source, but each successive sound wave will be detected earlier than it would have if the receiver were stationary, due to the motion of the receiver in the line-of-sight. Hence, the frequency with which each successive wave front would be detected will be changed by this relative motion according to

$$\Delta \nu = \frac{V_{\text{receiver}}}{\lambda_{\text{emitted}}} \nu_{\text{emitted}}$$

where $\Delta \nu = \nu_{\text{received}} - \nu_{\text{emitted}}$ is the change in the observed frequency and $\lambda_{\text{emitted}}$ is the original wavelength of the source. Since the original frequency of the source can be expressed in terms of the wavelength as $\nu_{\text{emitted}} = \frac{\nu_{\text{sound}}}{\lambda_{\text{emitted}}}$, the observed frequency becomes:

$$\nu_{\text{received}} = \nu_{\text{emitted}} + \Delta \nu = \frac{\nu_{\text{sound}}}{\lambda_{\text{emitted}}} + \frac{V_{\text{receiver}}}{\lambda_{\text{emitted}}} \nu_{\text{emitted}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}} + V_{\text{receiver}}}{\nu_{\text{sound}}} \right).$$

Note that (31) only works if the relative velocity of the receiver, $V_{\text{receiver}}$, is towards the source. If the motion is away from the source, the relative velocity would be in the opposite direction and (31) would become:

$$\nu_{\text{received}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}} - V_{\text{receiver}}}{\nu_{\text{sound}}} \right).$$

(31) and (32) are usually combined and expressed as:

$$\nu_{\text{received}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}} \pm V_{\text{receiver}}}{\nu_{\text{sound}}} \right).$$

If the source is moving towards the receiver, the effect is slightly different. The spacing between the successive wave fronts would be less, as seen in Fig. 13. This would be expressed as: $\Delta \lambda = V_{\text{source}}/\nu_{\text{emitted}}$, where $V_{\text{source}}$ is the relative velocity of the source. To calculate the observed frequency

$$\nu_{\text{received}} = \frac{\nu_{\text{sound}}}{\lambda_{\text{emitted}} + \Delta \lambda} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}}}{\nu_{\text{sound}} - V_{\text{source}}} \right).$$

Note that this is only when the source is moving towards the receiver. If the source is moving away, (34) would be changed to:

$$\nu_{\text{received}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}}}{\nu_{\text{sound}} - V_{\text{source}}} \right).$$

When (34) is combined with (35), we have:

$$\nu_{\text{received}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}} \pm V_{\text{receiver}}}{\nu_{\text{sound}} \mp V_{\text{source}}} \right).$$

Notice that this time, the plus/minus symbol is inverted because the sign on top is to be used for relative motion of the source towards the receiver.

By combining (33) and (36), we obtain

$$\nu_{\text{received}} = \nu_{\text{emitted}} \left( \frac{\nu_{\text{sound}} \pm V_{\text{receiver}}}{\nu_{\text{sound}} \mp V_{\text{source}}} \right).$$

We stress again two important points. Firstly, that the quantities for the velocity of the receiver $V_{\text{receiver}}$ and the velocity of the source $V_{\text{source}}$ are only the magnitudes of the relative velocities in (or along) the line of sight. In other words, the component of the velocity of the source and the receiver, that are perpendicular to the line of sight do not change the received frequency. Secondly, that the top sign in the numerator and the denominator are the sign convention to be used when the relative velocities are towards the other. If the source were moving towards the receiver, the sign to use in the denominator would be the minus sign. If the source were moving away from the receiver, the sign to use would be the plus sign.

One interesting application of the Doppler effect is the active sonar. This is a system in which pulses of acoustic energy are launched into the water for the purpose of producing echoes. By examining the echoes of transmitted pulses, it affords the capability of both detecting the presence of and estimating the range of an underwater target. We must carefully define the source and receiver for both the outgoing active pulse and the returning signal. For the outgoing active pulse, the Doppler shifted frequency of the active pulse when it hits the target (that is the receiver) would be:

$$\nu_{\text{target received}} = \frac{\nu_{\text{emitted}}}{\nu_{\text{sound}} \pm \nu_{\text{source}} \pm \nu_{\text{target}}}.$$
FIG. 14: Force acting on a charged particle inside an electric field.

Again, the velocities are only the magnitudes of the velocity in the line of sight, and we must take care to pick the correct sign to use in front of each velocity.

V. ELECTRICITY AND MAGNETISM

§ Electric charge. There are two types of observed electric charge, which we designate as positive and negative. The convention was derived from Franklin's experiments [17]. He rubbed a glass rod with silk and called the charges on the glass rod positive. He rubbed sealing wax with fur and called the charge on the sealing wax negative. Like charges repel and opposite charges attract each other. The unit of charge is called the Coulomb (C). The smallest unit of free charge known in nature is the charge of an electron or proton, which has a magnitude of \( e = 1.602 \times 10^{-19} \) C.

Charge of any ordinary matter is quantized in integral multiples of \( e \). An electron carries one unit of negative charge \( -e \), whereas a proton carries one unit of positive charge \( +e \). In a closed system, the total amount of charge is conserved since charge can neither be created nor destroyed. A charge can, however, be transferred from one body to another.

Consider a system of two point charges, \( q_1 \) and \( q_2 \), separated by a distance \( r \) in vacuum. The magnitude of the force exerted by \( q_1 \) on \( q_2 \) is given by

\[
F = k_e \frac{q_1 q_2}{r^2},
\]

where \( k_e = 8.9875 \times 10^9 \) N m\(^2\)/C\(^2\) is the Coulomb's constant [18].

§ Electric field. An electric charge \( q \) produces an electric field everywhere. To quantify the strength of the field created by that charge, we can measure the force a positive test charge \( q_0 \) experiences at some point. We must take \( q_0 \) to be infinitesimally small so that the field \( q_0 \) generates does not disturb the "source charges." With this in mind, the magnitude of the electric field \( E \) is defined as \( E = F/q_0 \).

An electric field \( \vec{E} \) would then exert a force on a charge, as shown in Fig. 14. The direction of the force is along the field if the charge is positive (e.g., a proton) and opposite to the field if the charge is negative (e.g., an electron).

§ Electric current. The current \( i \) is the rate of flow of the electric charge in a wire. Seen through some supermicroscope, a copper wire carrying an electrical current looks like the illustration in Fig. 15. The \( + \) charges are fixed atoms arranged on a regular array. They vibrate in place, but do not flow along the wire. The \( \ominus \) charges are electrons which flow along the wire, bumping into the fixed atoms, losing their energy in this way (i.e., "heating the wire"). The electrons are pushed by the battery or generator, and the current can be thought as being produced by the electrical pressure, or voltage of the power source.

We have seen that the electric charge is measured in coulombs. A coulomb contains about 6 billion billion electrons. More conveniently, we measure the rate of flow of electric charge, or current. If one coulomb of charge past some point in a circuit in one second, then the current is one ampere (A). Typically, the current through a light bulb is 0.7 to 1 ampere.

§ Electromotive force. The energy given to each coulomb by the power source is called the electromotive force (emf), or the voltage \( V \). Therefore, if there are \( q \) coulombs, the total energy handed out is proportional to \( V \) times \( q \). The

\[
\text{power} = \frac{\text{energy}}{\text{time}}
\]

is proportional to

\[
V \left( \frac{q}{1} \right) = \text{volts} \times \text{amperes}.
\]

The units are such that the power (in watts) equals the voltage (in volts) times the current (in amperes). A watt is a small amount of power: it is 1/1000 of a kilowatt, and the kilowatt is the power generated at the rate of 1 kWh/hour.

Here is one example. A light bulb rated at 100 W (or watts) is connected to a line voltage of 110 V (or volts).
(i) What is the current through the light bulb?

\[
\text{power (watts)} = \text{voltage (volts)} \times \text{current (amperes)}.
\]

This implies that

\[
100 \text{ watts} = 110 \text{ volts} \times \text{current}, \tag{44}
\]

and so

\[
\text{current} = \frac{100 \text{ watts}}{110 \text{ volts}} = 0.9 \text{ amperes}. \tag{45}
\]

(ii) How much energy (in kWh) is used by the light bulb in 24 hours?

\[
\text{energy (kWh)} = \text{power (kilowatts)} \times \text{time (hours)}. \tag{46}
\]

Now, since

\[
100 \text{ watts} = \frac{1 \text{ kilowatt}}{10^3}, \tag{47}
\]

we have

\[
\text{energy} = \frac{1}{10} \text{ kW} \times 24 \text{ hours} = 2.4 \text{ kWh}. \tag{48}
\]

At 12¢/kWh this is 29¢. (iii) If the light bulb were immersed in a bathtub full of water (500 lb, or 500 pints) initially at 60°F, what would be the temperature of the water after 24 hours? The bulb releases 2.4 kWh in 24 hours, which is 2.4 kWh × 3,412 Btu/kWh = 8,189 Btu. Each Btu will heat 1 lb H\text{2}O 1°F. Therefore, to heat 500 lb 1°F takes 8,189 Btu. 8,189 Btu will raise the temperature of the water in the tub by 8,189/500 = 16.4°F.

Another example is as follows. A 100 MW electric power station serves a metropolitan area of 2 × 10⁶ people. How much electrical energy (in kWh) is supplied per capita per month? Since

\[
1,000 \text{ MW} = 1,000 \times 10^6 \text{ watts} = 10^6 \times (1,000 \text{ watts}) = 10^6 \text{ kilowatts}. \tag{49}
\]

the energy generated in a month is

\[
\text{power (kW)} \times \text{time (hr)} = 10^6 \text{ kW} \times 24 \text{ hr} \times 30 \text{ days} = 7.2 \times 10^8 \text{ kWh}. \tag{50}
\]

Per capita, this is 7.2 × 10⁶/2 × 10⁶ = 360 kWh per month.

§ Electric circuit. Any path along which electrons can flow is a circuit. For a continuous flow of electrons, there must be a complete circuit with no gaps. A gap is usually provided by an electric switch that can be opened or closed to either cutoff or allow energy flow. Most circuits have more than one device that receives electric energy. These devices are commonly connected in a circuit in one of two ways: series or parallel. When connected in series, they form a single pathway for electron flow between the terminals of the battery, generator, or wall socket (which is simply an extension of these terminals). When connected in parallel, they form branches, each of which is a separate path for the flow of electrons. Both series and parallel connectors have their own distinctive characteristics.

A simple example of a series circuit is shown in Fig. 16. All devices, lamps in this case, are connected end to end, forming a single path for electrons. The same current exists almost immediately in all three lamps, and also in the battery, when the switch is closed. The greater the current in a lamp, the brighter it glows. Electrons do not "pile up" in any lamp but flow through each lamp, simultaneously. Some electrons move away from the negative terminal of the battery, some move toward the positive terminal, and some move through the filament of each lamp. Eventually, the electrons may move all the way around the circuit. This is the only path of the electrons through the circuit. A break anywhere in the path results in an open circuit, and the flow of electrons ceases. Burning out one of the lamp filaments or simply opening the switch could cause such a break.

A simple example of a parallel circuit is shown in Fig. 17. Three lamps are connected to the same two points: A and B. Electrical devices directly connected to the same two points of an electric circuit are said to be connected in parallel. The pathway from one terminal of the battery to the other is completed if only one lamp is lit. The circuit shown in Fig. 17 branches into three separate pathways from A to B. A break in any one path does not interrupt the flow of charge in the other paths.
§ How do voltage and current relate? The relationship among voltage and current is summarized by Ohm’s law, which states that, at a constant temperature, the electrical current flowing through a wire between two points is directly proportional to the voltage across the two points [19]. Introducing the constant of proportionality, the resistance $R$, one arrives at the usual expression that describes this relationship

$$i = \frac{V}{R}, \quad (49)$$

where $i$ is the current through the wire in units of amperes, $V$ is the voltage measured across the wire in units of volts, and $R$ is the resistance of the wire in units of ohms ($\Omega$).

If resistors are connected in series, each resistor has the same current $i$. Each resistor has voltage $iR$, given by Ohm’s law. Then, for the circuit shown in Fig. [16], the total voltage drop across all three resistors is

$$V_{\text{total}} = iR_1 + iR_2 + iR_3 = i(R_1 + R_2 + R_3). \quad (50)$$

When we look at all three resistors together as one unit, we see that they have the same $i$ vs. $V$ relationship as one resistor, whose value is the sum of the resistances. So we can treat these resistors as just one equivalent resistance,

$$R_{\text{eq}} = R_1 + R_2 + R_3,$$  \quad (51)

as long as we are not interested in the individual voltages. Their effect on the rest of the circuit is the same, whether lumped together or not.

Resistors in parallel carry the same voltage. All of the resistors in Fig. [17] have voltage $V$. The current flowing through each resistor could definitely be different. Even though they have the same voltage, the resistances could be different. If we view the three resistors as one unit, with a current $i$ going in, and a voltage $V$, this unit has the following $i$ vs. $V$ relationship:

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \quad (52)$$

Thus, to the outside world, the parallel resistors look like one satisfying

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \quad (53)$$

for the equivalent resistance $R_{\text{eq}}$.

§ Magnetic field. We have seen that a charged object produces an electric field $\vec{E}$ at all points in space. In a similar fashion, a bar magnet is a source of a magnetic field $\vec{B}$. This can be readily demonstrated by moving a compass near the magnet. The compass needle will line up along the direction of the magnetic field produced by the magnet, which is shown in Fig. [18].

Notice that the bar magnet consists of two poles, which are designated as the north $N$ and the south $S$. Magnetic fields are strongest at the poles. The magnetic field
lines leave from the north pole and enter the south pole. When holding two bar magnets close to each other, the like poles will repel each other while the opposite poles attract, see Fig. 19.

Unlike electric charges which can be isolated, the two magnetic poles always come in a pair. When you break the bar magnet, two new bar magnets are obtained, each with a north pole and a south pole, as shown in Fig. 20. In other words, magnetic monopoles have not been seen in isolation, although they are of theoretical interest.

A magnetic field \( \vec{B} \) exerts a force on a moving charge. The direction of the force is perpendicular to both the field and the motion of the charge. The magnitude of the force depends on the velocity of the charge \( v \), the magnetic field strength \( B \), and the angle between the direction of \( \vec{v} \) and \( \vec{B} \),

\[
F_B = |q| v B \sin \theta,
\]

where \(|q|\) is the absolute value of the charge. The magnetic force \( F_B \) vanishes when \( \vec{v} \) is parallel to \( \vec{B} \). However, when \( \vec{v} \) makes an angle \( \theta \) with \( \vec{B} \), the direction of \( \vec{F}_B \) is perpendicular to the plane formed by \( \vec{v} \) and \( \vec{B} \), and the magnitude of \( \vec{F} \) is proportional to \( \sin \theta \). The unit of magnetic field is the tesla (T)

\[
\text{tesla} = \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})} = \frac{\text{newton}}{\text{(ampere)(meter)}},
\]

We have just seen that a charged particle moving through a magnetic field experiences a magnetic force \( \vec{F}_B \). Since electric current consists of a collection of charged particles in motion, when a current-carrying wire is placed in a magnetic field, it will also experience a magnetic force, see Fig. 21. Moreover, Oersted noticed that an electric current flowing through a wire can cause a compass needle to deflect perpendicular to the wire, showing that a current also creates a magnetic field \( \vec{B} \). The lines describing the \( \vec{B} \) direction surround the current, as shown in Fig. 22.

In closing this section, it is important to note that the configuration of the Earth’s magnetic field (shown in Fig. 23) is very much like the one that would be achieved by burying a gigantic bar magnet deep in the interior of the Earth. When we speak then of a compass magnet having a north pole and a south pole, we should say more properly that it has a “north-seeking” pole and a “south-seeking” pole. By this we mean that one pole of the magnet seeks, or points to, the north geographic pole of the Earth. Because the north pole of a magnet is attracted toward the north geographic pole of the Earth, we conclude that the Earth’s south magnetic pole is located near the north geographic pole, and the Earth’s north magnetic pole is located near the south geographic pole.

VI. HOW LIGHT WORKS

\$\textbf{Electromagnetic waves.}\$ In the mid-1800’s, Faraday observed (see Fig. 24) that a changing magnetic field creates an electric field. Exhilarated by this discovery, Maxwell hypothesized that a changing electric field creates a magnetic field, and putting all this together he predicted that if you began changing \( \vec{E} \) and \( \vec{B} \) in any region in space, a wave of these changing fields propagates at the speed of light \( c \approx 3 \times 10^8 \text{ m/s} \), outward from the region where the change first took place; see
FIG. 23: A pictorial representation of the Earth’s magnetic field lines that is very useful in visualizing the strength and direction of the magnetic field. The $\vec{B}$ lines describe the direction of the magnetic force on a north monopole at any given position.

Fig. 25 [23]. For example, moving an electron causes a change in $\vec{E}$, which causes a change in $\vec{B}$, etc. These changing fields zip over to a second electron, which was jiggled by the electric field which arrives at a microscope time later. It was not until 1888 that Maxwell’s prediction passed an important test when Hertz generated and detected certain types of electromagnetic waves in the laboratory [24]. He performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Light itself consists of electric and magnetic fields of this kind. But what about photons? Good question. We will deal with this soon. But meanwhile, we note the very important fact that we have here: a means of transporting energy through empty space, without transporting matter. The electromagnetic waves (that’s what they are called) propagate any time an electron is jiggled.

§ Ray optics. We just learned that light is a wave. Unlike particles, waves behave in funny ways, e.g., they bend around corners (think of sound coming through a doorway). However, the smaller the wavelength $\lambda$ is, the weaker these funny effects are, and so for light (tiny $\lambda$), no one noticed the “wave nature” at all, for a long time. The wavelength of light is about 100 times smaller than the diameter of a human hair! This means that for most physics phenomena of everyday life, we can safely ignore the wave nature of light, because light waves travel through and around obstacles whose transverse dimensions are much greater than the wavelength, and the wave nature of light is not readily discerned. Under these circumstances the behavior of light can be adequately described by rays obeying a set of geometrical rules. This model of light is called ray optics. Strictly speaking, ray optics is the limit of wave optics when the wavelength is infinitesimally small.

To study the more classical aspects of how light travels we will take into account the following considerations:

- We will ignore time oscillations/variations ($10^{14}$ Hz is too fast to notice, generally!)
- We will assume light travels through a transparent medium in straight line (at 186,282 miles per second, super fast).²
- Light can change directions in 3 main ways:

² Fermat’s principle states that when a light ray travels between any two points, its path is one that requires smallest time interval. An obvious consequence of this principle is that paths of light rays traveling in a homogeneous medium are straight lines, because a straight line is shortest distance between two points.
that in the medium, defined as the ratio of the speed of light in vacuum to $c$, the "speed of light", in vacuum. The index of refraction is determined by how fast light travels through the material. In materials, it is always slowed down. The bigger the $n$, the slower the light travels. The index of refraction is defined as the ratio of the speed of light in vacuum to that in the medium,

$$\text{index of refraction} = \frac{\text{speed of light (in vacuum)}}{\text{speed of light (in medium)}}. \quad (56)$$

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as shown in Fig. 26, part of the energy is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be refracted. The incident ray, the reflected ray, and the refracted ray all lie in the same plane. Experiments and theory show that the angle of reflection $\theta'_1$ equals the angle of incidence $\theta_1$,

$$\theta'_1 = \theta_1. \quad (57)$$

This relationship is called the law of reflection. The angle of refraction depends on the properties of the two media and on the angle of incidence through the relationship

$$\sin \theta_2 = \frac{v_2}{n} \sin \theta_1 = \text{constant}, \quad (58)$$

where $v_1$ is the speed of light in the first medium and $v_2$ is the speed of light in the second medium. If we replace the $v_2/v_1$ term with the ratio of the refractive indexes $n_1/n_2$ we can express (58) in an alternative form:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \quad (59)$$

The experimental discovery of this relationship, known as law of refraction, is usually credited to Snell [26].

The path of a light ray through a refracting surface is reversible. For example, the ray shown in Fig. 26 travels from point $A$ to point $B$. If the ray originated at $B$, it would travel to the left along line $BA$ to reach point $A$, and the reflected part would point downward and to the left in the glass.

Light rays can pass through several boundaries. For example, you might have a sheet of glass: a light ray will enter (going from small $n_1$ to larger $n_2$) and then exit (large $n_2$ to small $n_1$). At each boundary, Snell’s law will hold. At the left boundary we have $n_1 \sin \theta_{in} = n_2 \sin \theta_2$ (light bends toward the normal – convince yourself of the equation and the physics). At the right boundary we have $n_2 \sin \theta_3 = n_1 \sin \theta_{out}$ (light bends away from the normal – again, convince yourself).

But geometry tells us (if the walls are parallel) that $\theta_2 = \theta_3$ (do you see why?), which means $\sin \theta_2 = \sin \theta_3$. So $n_1 \sin \theta_{in} = n_2 \sin \theta_2 = n_2 \sin \theta_3 = n_1 \sin \theta_{out}$ (can you follow all the steps required to write that last line down?) That means (compare the far left with the far right of the equation) that $\sin \theta_{in} = \sin \theta_{out}$, which says $\theta_{in} = \theta_{out}$.

What if you have glass with walls that are not parallel? This is the idea behind lenses. As light enters, it is bent, and rays come out different depending on where and how they strike. The focal length of a given optical system is a measure of how strongly the system converges or diverges light. For an optical system in air, it is the distance over which initially collimated (parallel) rays are brought to a focus. A system with a shorter focal length has greater optical power than one with a
long focal length; that is, it bends the rays more sharply, bringing them to a focus in a shorter distance. The lens geometry usually looks complicated (and it is!) but for thin lenses, the result is relatively simple. For a given object position, the focal length defines where the image will appear:

\[
\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{\text{focal length}}. \tag{60}
\]

(60) is known as the thin lens equation.

Convex lenses (lenses which bulge outwards instead of curving inwards) have the property of converging parallel light rays to a single point, see Fig. 27. This point is called the focus of the lens, and the distance of the focus from the center of the lens is called the focal length. Light rays are parallel if the distance to the object is very large, effectively infinity compared to the size of the lens. If (60) yields a negative image distance, then the image is a virtual image on the same side of the lens as the object. If it yields a negative focal length, then the lens is a concave (curving inwards) diverging lens rather than a convex converging lens, see Fig. 27. The linear magnification relationship,

\[
\text{magnification} = -\frac{\text{image distance}}{\text{object distance}}, \tag{61}
\]

allows one to predict the size of the image. The sign of the magnification indicates whether the image is a real and inverted (negative) or a virtual and upright (positive).

Now, we are in the position to answer two questions that have been intriguing people since the time of Ptolemy. How do you know where objects are? How do you see them? You deduce the location (distance and direction) in complicated ways, but it arises from the angle and intensity (wavelength-weighted power emitted by a light source) of the little bundle of light rays that make it into your eye. The eye is an adaptive optical system. Unlike most optical systems, the crystalline lens of the eye changes its shape to focus light from objects over a great range of distances.

§ True colors shining through. It is a common experience to use a piece of shaped glass, a triangular cut of glass called a prism, to produce a rainbow of color from sun light. This is basically a refractive phenomenon and a simple extension of Fermat's least time principle can be used to describe it. A narrow beam of white light incident at a non-normal angle on one surface of the glass is refracted; the beam changes direction. The spread of color appears because the different colors in the light have different speeds in the glass, with the red being faster than the blue and all colors slower than for light in air. Hence, the red is bent less than the blue. The separated rays then emerge from the other interface of the glass spread in a familiar rainbow pattern. This spread of color can be seen by placing a piece of paper after the second interface. It was Newton who introduce the idea that white light was a complex phenomenon composed of an internal structure – the colors [27]. Prior to Newton's interpretation, the understanding was that the different colors in the prism came from the glass and was not an intrinsic property of the light. To show otherwise Newton placed a prism in the path of a narrow beam of sunlight. As expected, the beam was spread over a band
of angles. He then inserted a second prism and allowed the spread beam to enter it. When arranged carefully, he found that the second prism was able to reconstitute the original beam in the original direction. He labeled the different colors with a continuously varying parameter that had the units of a time, now identified with the reciprocal of the frequency. The length $\lambda$ and time $T$ characterizing a given color are connected by the speed of light in the medium according to $\lambda/T = c/n$ where $n$ is the index of refraction.

§ The basic building blocks of matter. In the early fifth century BC, the Greek philosopher Democritus proposed that matter consists of small indivisible particles that he named atomos (meaning uncuttable). Democritus’ vision did not gain much favor with his contemporaries and almost 2,300 years passed before Dalton reintroduced the idea of atoms in the early 1800s [28]. Despite the fact that Democritus’ conception of atoms does not reflect our modern understanding of atoms, the idea that matter is composed of indivisible particles remains a simple but powerful idea.

We now know that all matter is composed of molecules, and these molecules consist of one or more atoms. The molecules retain characteristics of the substance (e.g., water molecules); they separate whereas atoms do not (e.g., hydrogen and oxygen separately have nothing to do with water), unless the substance is one of the elements. The molecules of an element contain only one kind of atom (e.g., hydrogen, oxygen, uranium, iron). There are about 100 elements, and therefore 100 different kinds of atoms. Substances which are not elements are called compounds; at present we know of about one million compounds.

Although Dalton conceived atoms to be indivisible, subsequent experiments by Thomson [29] and Rutherford [30] revealed a more complex structure to the atom. Between 1900 and 1932, we had essentially answered the question: “What are atoms themselves made off?” Atoms are made from three smaller subatomic particles: the electron, the proton, and the neutron. The atom, however, remains the smallest division of matter with distinct chemical properties.

The 99.97% of the mass of the atom is concentrated in a very small nucleus at its center, consisting of two kinds of heavy particles: protons and neutrons. The other 0.03% of the mass consists of the very light electrons, which buzz around in fixed orbits very far from the nucleus, see Fig. 28. The nucleus of the atom has a radius of approximately $5 \times 10^{-15}$ m. The remainder of the atom, which has radius of approximately $10^{-10}$ m, is mostly empty space in which the electrons buzz around. The scale of the atom is such that if the nucleus were the size of a golfball, the electron orbits would be at a distance of half a mile.

![Diagram of the early-20-th-century Bohr’s model of the carbon-12 atom, with a central nucleus and orbiting electrons, much like a solar system with orbiting planets. The $^{12}\text{C}$ atom consists of 6 protons and 6 neutrons that are found in the nucleus at the center of the atom, and 6 electrons that are outside the nucleus.](image-url)

**TABLE I: Mass and charge of subatomic particles.**

| Particle | Mass (g) | Coulomb charge | Charge unit |
|----------|----------|----------------|-------------|
| electron | $9.10939 \times 10^{-28}$ | $-1.6022 \times 10^{-19}$ | $-1$ |
| proton   | $1.67262 \times 10^{-24}$ | $+1.6022 \times 10^{-19}$ | $+1$ |
| neutron  | $1.67493 \times 10^{-24}$ | $0$ | $0$ |

Now, besides mass, the electron and proton have something called electric charge, or charge for short. Although the electron is 2,000 times lighter than the proton, it has the same charge as the proton or, more precisely, it has an equal and opposite charge. This is because, as we have seen in Sec. 5, electric charges come in two kinds: positive (or plus, +) and negative (or minus, −). Like charges repel one another, whereas opposite charges attract, and these forces become much stronger when the charges are closed together. The characteristic properties of electrons, protons, and neutrons are summarized in Table 1.

**TABLE II: Stable isotopes of hydrogen, helium, and carbon.**

| Element | # of protons | # of neutrons | # of electrons |
|---------|-------------|---------------|---------------|
| hydrogen| 1           | 0,1,2         | 1             |
| helium  | 2           | 1,2           | 2             |
| carbon  | 6           | 6,7           | 6             |

3 The neutron is neutral; it has no charge.
4 The elementary (pointlike) particle model accepted today views quarks and leptons as the basic constituents of ordinary matter. By pointlike, we understand that quarks and leptons show no evidence of internal structure at the current limit of our resolution, which is about $2 \times 10^{-20}$ m. Leptons include electrons (along with muons, taus, and neutrinos). Quarks are the fundamental particles that
The electrons are held in orbit by the electrical attraction of the protons. The whole atom is neutral, because there are always equal numbers of electrons and protons. The reason the protons in the nucleus do not fly apart (due to their strong electrical repulsion) is that at these small distances, a much stronger attractive force (a hundred times as strong) comes into play between neutrons and protons: it is called the nuclear force.

We can now address the following question: "Why is an atom of carbon different from an atom of hydrogen or helium?" One possible explanation is that carbon, hydrogen, and helium have different numbers of electrons, protons, or neutrons. Table II provides the relevant numbers. Note that atoms of helium and carbon each have two possibilities for the numbers of neutrons, and that it is even possible for a hydrogen atom to exist without a neutron. Clearly the number of neutrons is not crucial to determining if an atom is carbon, hydrogen, or helium. Although hydrogen, helium, and carbon have different numbers of electrons, the number is not critical to an element’s identity. For example, it is possible to strip an electron away from helium forming a helium ion, with a charge of +1 that has the same number of electrons as hydrogen. What makes an atom carbon is the presence of 6 protons, whereas every atom of hydrogen has 1 proton and every atom of helium has 2 protons.

We describe the atomic structure in terms of: (i) the atomic number \( Z \), which equals the number of protons inside the nucleus; (ii) the baryon number \( N \), which equals the number of nucleons inside the nucleus. The nuclei of all atoms of a particular element contain the same number of protons but often contain different numbers of neutrons. Nuclei that are related in this way are called isotopes. Now, a bit of nomenclature. For an isotope with baryon number \( A \) of an element with symbol \( X \), it is common writing \( \overset{A}{X} \). For example, when we write \( ^{238}_{92}U \) we mean the isotope of uranium which has a total of 238 neutrons plus protons, and 92 protons. (How many neutrons?)

§ The microworld \( \rightarrow \) macroworld connection. Individual atoms weigh very little, typically about \( 10^{-24} \) g to \( 10^{-22} \) g. This amount is so small that there is no easy way to measure the mass of a single atom. To assign masses to atoms it is necessary to assign a mass to one atom and report the masses of other atoms relative to that absolute standard. By agreement, atomic mass is stated in terms of atomic mass units (u), where 1 u is defined as \( \frac{1}{12} \) of the mass of an atom of carbon-12. The atomic mass of carbon-12, therefore, is exactly 12 u. The atomic mass of carbon-13 is 13.00335 u because the mass of an atom of carbon-13 is 1.0836125 times greater than the mass of an atom of carbon-12. Because carbon exists in several isotopes, the atomic mass of an “average” carbon atom is neither 12.0 u nor 13.00335 u. Instead it is 12.0107 u, a value that is closer to 12.0 u because 98.90% of all carbon atoms are carbon-12. The average atomic masses for the various elements are given in Table III.

Although the atomic mass unit provides a scale for comparing the relative masses of atoms, it is not a useful unit when working in the laboratory because it is too small (approximately \( 10^{-24} \) g). Additionally, the atomic mass unit applies to a single atom, whereas we work with gazzillions of atoms at a time. To get around this problem we introduce another unit that is better suited for samples containing enormous numbers of atoms. Basically, the idea is to define a unit that represents a particular number of objects, just as we use a dozen to represent a collection of 12 eggs and a baker’s dozen to represent 13 cookies.

To scale up from the microscopic level to the macroscopic world, we use a unit called mole that has the unit symbol mol. The mole is defined as the amount of a substance containing the same number of objects as there are atoms in exactly 12 g of carbon-12. This number has been determined experimentally to be

\[
N_A = 6.0220943 \times 10^{23} \text{ mol}^{-1},
\]

and is known as Avogadro’s number. For our purposes, we usually round Avogadro’s number to \( 6.022 \times 10^{23} \) particles/mol. A mole of zinc contains \( 6.022 \times 10^{23} \) atoms of zinc and a mole of jellybeans contains \( 6.022 \times 10^{23} \) jellybeans.

The advantage of defining a mole in this way is that an element’s average atomic mass is identical to its molar mass. Why is the true? A single atom of carbon-12 has an atomic mass of exactly 12 u and a mole of carbon-12 atoms has a molar mass of exactly 12 g. A single atom of carbon-13 has an atomic mass of 13.00335 u because its mass is 1.0836125 times greater than the mass of an atom of carbon-12. A mole of carbon-13, therefore, will have a mass of 13.00335 g.

A straightforward calculation shows that the two mass scales, grams and atomic mass units, are related by

\[
1 \text{ g} = 6.022 \times 10^{23} \text{ u}.
\]

The proton and neutron each have a mass of approximately 1 u, and the electron has a mass that is only a small fraction of an atomic mass unit. More specifically, \( m_p = 1.00725 \text{ u}, m_n = 1.00864 \text{ u}, \) and \( m_e = 0.0005486 \text{ u}. \)

§ Photons. Two atoms (such as C and O; see Fig. 19) combine to form a molecule (CO, or carbon monoxide; see Fig. 30) as follows. In one way or another (which we will discuss soon) the C and O atoms are driven up against one another. The negative electrons repel each other, and the atoms fly apart. But once in a while, a pair of atoms come together so hard that the
TABLE III: List of the elements with their symbols, atomic numbers, and average atomic masses. Approximate atomic masses for radioactive elements are given in parentheses.

| Element       | Symbol | Atomic Number | Atomic Mass | Element       | Symbol | Atomic Number | Atomic Mass |
|---------------|--------|---------------|-------------|---------------|--------|---------------|-------------|
| Actinium      | Ac     | 89            | (227)       | Mendelevium   | Md     | 101           | (256)       |
| Aluminum      | Al     | 13            | 26.981538   | Mercury       | Hg     | 80            | 200.59      |
| Americium     | Am     | 95            | (243)       | Molybdenium   | Mo     | 42            | 95.94       |
| Antimony      | Sb     | 51            | 121.760     | Neodymium     | Nd     | 60            | 144.24      |
| Argon         | Ar     | 18            | 39.948      | Neon          | Ne     | 10            | 20.1797     |
| Arsenic       | As     | 33            | 74.92160    | Neptunium     | Np     | 93            | (237)       |
| Astatine      | At     | 85            | (210)       | Nickel        | Ni     | 28            | 58.6934     |
| Barium        | Ba     | 56            | 137.327     | Niobium       | Nb     | 41            | 92.90638    |
| Berkelium     | Bk     | 97            | (247)       | Nitrogen      | N      | 7             | 14.00674    |
| Beryllium     | Be     | 4             | 9.012182    | Nobelium      | No     | 102           | (253)       |
| Bismuth       | Bi     | 83            | 208.98038   | Osmium        | Os     | 76            | 190.23      |
| Bohrium       | Bh     | 107           | (262)       | Oxygen        | O      | 8             | 15.9994     |
| Boron         | B      | 5             | 10.811      | Palladium     | Pd     | 46            | 106.42      |
| Bromine       | Br     | 35            | 79.904      | Phosphorous   | P      | 15            | 30.973762   |
| Cadmium       | Cd     | 48            | 112.411     | Platinum      | Pt     | 78            | 195.078     |
| Calcium       | Ca     | 20            | 40.078      | Plutonium     | Pu     | 94            | (242)       |
| Californium   | Cf     | 98            | (249)       | Polonium      | Po     | 84            | (210)       |
| Carbon        | C      | 6             | 12.0107     | Potassium     | K      | 19            | 39.0983     |
| Cerium        | Ce     | 58            | 140.116     | Praseodymium  | Pr     | 59            | 140.90765   |
| Cesium        | Cs     | 55            | 132.90545   | Promethium    | Pm     | 61            | (147)       |
| Chlorine      | Cl     | 17            | 35.4527     | Protactinium  | Pa     | 91            | 231.03588   |
| Chromium      | Cr     | 24            | 51.9961     | Radium        | Ra     | 88            | (226)       |
| Cobalt        | Co     | 27            | 58.933200   | Radon         | Rn     | 86            | (222)       |
| Copper        | Cu     | 29            | 63.546      | Rhenium       | Re     | 75            | 186.207     |
| Curium        | Cm     | 96            | (247)       | Rhodium       | Rh     | 45            | 102.90550   |
| Dubnium       | Db     | 105           | (260)       | Rubidium      | Rb     | 37            | 85.4678     |
| Dysprosium    | Dy     | 66            | 162.50      | Ruthenium     | Ru     | 44            | 101.07      |
| Einsteinium   | Es     | 99            | (254)       | Rutherfordium | Rf     | 104           | (257)       |
| Erbium        | Er     | 68            | 167.26      | Samarium      | Sm     | 62            | 150.36      |
| Europium      | Eu     | 63            | 151.964     | Scandium      | Sc     | 21            | 44.95591    |
| Fermium       | Fm     | 100           | (253)       | Seaborgium    | Sg     | 106           | (263)       |
| Fluorine      | F      | 9             | 18.9984032  | Selenium      | Se     | 34            | 78.96       |
| Francium      | Fr     | 87            | (223)       | Silicon       | Si     | 14            | 28.0855     |
| Gadolinium    | Gd     | 64            | 157.25      | Silver        | Ag     | 47            | 107.8682    |
| Gallium       | Ga     | 31            | 69.723      | Sodium        | Na     | 11            | 22.98977    |
| Germanium     | Ge     | 32            | 72.61       | Strontium     | Sr     | 38            | 87.62       |
| Gold          | Au     | 79            | 196.96655   | Sulfur        | S      | 16            | 32.066      |
| Hafnium       | Hf     | 72            | 178.49      | Tantalum      | Ta     | 73            | 180.9479    |
| Hassium       | Hs     | 108           | (265)       | Technetium    | Tc     | 43            | (99)        |
| Helium        | He     | 2             | 4.002602    | Tellurium     | Te     | 52            | 127.60      |
| Holmium       | Ho     | 67            | 164.93032   | Terbium       | Tb     | 65            | 158.92534   |
| Hydrogen      | H      | 1             | 1.00794     | Thallium      | Tl     | 81            | 204.3833    |
| Indium        | In     | 49            | 114.818     | Thorium       | Th     | 90            | 232.0381    |
| Iodine        | I      | 53            | 126.90447   | Thallium      | Tm     | 69            | 168.93421   |
| Iridium       | Ir     | 77            | 192.217     | Tin           | Sn     | 50            | 118.710     |
| Iron          | Fe     | 26            | 55.845      | Titanium      | Ti     | 22            | 47.867      |
| Krypton       | Kr     | 36            | 83.80       | Tungsten      | W      | 74            | 183.84      |
| Lanthanum     | La     | 57            | 138.9055    | Uranium       | U      | 92            | 238.0289    |
| Lawrencium    | Lr     | 103           | (257)       | Vanadium      | V      | 23            | 50.9415     |
| Lead          | Pb     | 82            | 207.2       | Xenon         | Xe     | 54            | 131.29      |
| Lithium       | Li     | 3             | 6.941       | Ytterbium     | Yb     | 70            | 173.04      |
| Lutetium      | Lu     | 71            | 174.967     | Yttrium       | Y      | 39            | 88.90585    |
| Magnesium     | Mg     | 12            | 24.3050     | Zinc          | Zn     | 30            | 65.39       |
| Manganese     | Mn     | 25            | 54.938049   | Zirconium     | Zr     | 40            | 91.224      |
| Meitnerium    | Mt     | 109           | (266)       |               |        |               |             |
FIG. 29: Simplified view of a carbon (up) and oxygen (down) atoms. The $^{12}$C nucleus consists of 6 protons and 6 neutrons, while the $^{16}$O nucleus contains 8 protons and 8 neutrons.

Electrons are driven past each other, and a new situation comes being: The negative electrons of the carbon get far enough past the negative electrons of the oxygen, and begin to experience the attractive force of the positive oxygen nucleus. Same history for the oxygen electrons. When this happens, the C and O combine into a stable CO molecule.

But as they snap together, something very important happens: a small packet of energy, called a photon, is emitted (sort of like when two magnets snap together, a little heat is generated). For each molecule of CO formed, a photon is emitted and each of the photons has the same energy. So every time 3 grams of C combines with 4 grams of O, the same amount of energy (6 Cal) is released (in the form of a million billion billion photons). The reason C and O always combine in the proportions 3 g of C to 4 g of O is that 3 g of C contain the same number of atoms as 4 g of O (convince yourself of this relation), and the atoms just combine one-to-one.

§ Burning. Now we come to a crucial consideration: the chain reaction. Let us suppose the following two considerations are fulfilled: (i) there are many C and O molecules together; (ii) the C and O molecules are well interspersed. Then the photon which originates during the combination of one pair of C and O has a good chance of hitting another C (or O) and driving it with enough force so that it will combine with an O (or C), and so on. Thus, the combination will go on without outside energy, so long as there are enough C and O atoms close together, and they are interspersed enough [conditions (i) and (ii)]. This is what we call burning. Condition (i) can be translated to mean “we need enough fuel” and condition (ii) “give the fire some air!”

How do we ignite the reaction? We must really agitate the C-O mixture so that the C's really slam into the O's, overcoming the repulsion of the outer electrons. Agitating a group of molecules to a larger average velocity is the definition of raising the temperature of the group. We can raise the temperature of the mixture by introducing another source of photons (a light match), or by physically agitating the C's (friction), e.g., as a rocket nose cone burns when it rushes through the air.

A question which may have arisen at this point is, “Where does the photon come from?” Unfortunately, a complete revelation of these secrets would require you to take a graduate physics course; at this point we can simply provide the following explanation. The photon does not exist in the atoms before it appears. But when it appears, something else does disappear: mass. The Planck's constant [35].
mass of the CO molecule is less than the sum of the masses of the C and O atoms. Einstein discovered the relationship between the energy of the photon, $E$, and the disappearing mass $m$,

$$E = mc^2,$$  \hspace{1cm} (64)

with $c$ being the speed of light \[36\]. In the energy units you have learned, the appearance of 2 Cal of energy in photons is associated with the disappearance of $10^{-10}$ g of mass. Roughly speaking, $10^{-18}$-th of the mass of fuel burned disappears.

By the way, if you calculate the masses of carbon-12 and carbon-13 by adding up the masses of each isotope’s electrons, neutrons, and protons you will obtain a mass ratio of 1.08336, not 1.0836125. The reason for this is that the masses given in Table I are for free electrons, protons, and neutrons; that is, for electrons, protons, and neutrons that are not in an atom. When an atom forms, some of the mass can be converted to energy according to \[64\], and the “lost” mass is the nuclear binding energy that holds the nucleus together.

§ Why the Sun is not burning chemical fuel. The rate at which solar energy reaches the Earth’s upper atmosphere is 1.5 kilowatts/square meter (= 1.5 kW/m²). Since 1kW ≈ 1 Btu/second, this means an energy flux of 1.5 Btu/second per $m^2$. We can use this number to obtain the total rate at which energy is being radiated by the Sun.

Imagine a sphere drawn with the Sun at the center and with the Earth at the surface; see Fig. 31. Since we know the rate at which radiation arrives at 1 $m^2$ of the sphere, we could obtain the total radiation emitted by the Sun by multiplying 1.5 Btu/(m² s) by the area of the sphere. So let’s find that.

The distance from the Sun to the Earth is $93 \times 10^6$ miles. Now, 1 mile = 1.6 km = 1600 m, so

$$\text{the earth – sun distance} = 93 \times 10^6 \times 1600 \text{ m} = 150 \times 10^8 \text{ m}.$$  \hspace{1cm} (65)

The area of the whole sphere is

$$A = 4\pi r^2 = 4\pi(150 \times 10^8)^2.$$  \hspace{1cm} (66)

Now, $\pi \approx 3$, so $4\pi \approx 12$, and $(150)^2 = 22500 = 2.25 \times 10^4$. Thus, the area of the sphere is

$$A \approx 12 \times 2.25 \times 10^4 \times 10^{18} \approx 27 \times 10^{22} \text{ square meters}.$$  \hspace{1cm} (67)

The total radiation arriving at the sphere is

$$L_0 \approx 1.5 \text{ Btu/s/m}^2 \times 27 \times 10^{22} \text{ m}^2 \approx 40 \times 10^{22} \text{ Btu/s} \approx 4 \times 10^{23} \text{ Btu/s}.$$  \hspace{1cm} (68)

This must be the rate at which energy is being radiated by the Sun. We know that the Sun is not heating up, so $4 \times 10^{23}$ Btu/s must also be the rate at which energy is being created at the Sun.

Now, suppose the Sun were burning oil, or some other chemical with energy content of the order of 20,000 Btu/lb. If the entire mass of the Sun (which is $M_\odot = 4 \times 10^{30}$ lb) were composed of such fuel, and it were all to burn, then the total energy release would be $4 \times 10^{30} \text{ lb} \times 2 \times 10^4 \text{ Btu/lb} = 8 \times 10^{34}$ Btu. Since the rate of energy release is $4 \times 10^{23}$ Btu/s, we use the general formula

$$\text{amount} = \text{rate} \times \text{time}$$  \hspace{1cm} (69)

to obtain

$$8 \times 10^{34} \text{ Btu} = 4 \times 10^{23} \text{ Btu/s} \times \text{time},$$  \hspace{1cm} (70)

yielding

$$\text{time} = \frac{8 \times 10^{34}}{4 \times 10^{23}} \text{ s} = 2 \times 10^{11} \text{ s}$$  \hspace{1cm} (71)

as the time available for burning. Since a year has $3 \times 10^7$ seconds, the time in (71) is equivalent to

$$\frac{2 \times 10^{11}}{3 \times 10^7} \text{ yr} = \frac{2}{3} \times 10^4 \text{ yr} = \frac{2}{3} \times 10,000 \text{ yr} = 6,666 \text{ yr}.$$  \hspace{1cm} (72)

Way too short. To last $6 \times 10^9$ yr (or so), the fuel must have an energy content of $10^6$ time greater than that of oil. But that is another story and shall be told another time.
VIII. RADIOACTIVITY

§ Nuclear structure. The size and structure of nuclei were first investigated in Rutherford’s scattering experiments [30, 37]. In these experiments, positively charged nuclei of helium atoms (α particles) were directed towards a thin piece of metal foil. As the particles moved through the foil, they often passed near a metal nucleus. Because of the positive charge on both the incident particles and the nuclei, particles were deflected from their straight-line paths by the Coulomb repulsive force. In fact, some particles were even deflected backward, through an angle of 180° from the incident direction. Those particles were apparently moving directly toward a nucleus in a head-on collision course.

Using energy conservation it is straightforward to derive an expression for the distance $d$ at which a particle approaching a nucleus is turned around by Coulomb repulsion. Namely, in a head-on collision, the kinetic energy of the incoming α particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around, see Fig. 32. If we equate the initial kinetic energy of the α particle to the electrical potential energy of the system (α particle plus target nucleus), we have

$$\frac{1}{2}m_\alpha v_\alpha^2 = k_e \frac{q_1 q_2}{r} = \frac{k_e (2e)(Z e)}{d}.$$  \hspace{1cm} (73)

Solving for $d$, the distance of closest approach, we get

$$d = \frac{4k_eZe^2}{m_\alpha v_\alpha^2}. \hspace{1cm} (74)$$

From (74) it follows that α particles with kinetic energy $K_\alpha = 1.12 \times 10^{-12}$ J would approach nuclei to within a distance of $3.2 \times 10^{-14}$ m when the foil is made of gold. Thus, the radius of the gold nucleus must be less than $3.2 \times 10^{-14}$ m. For silver atoms, the distance of closest approach is found to be $2 \times 10^{-14}$ m. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere with a radius of approximately $10^{-14}$ m, which he called the nucleus. Because such small dimensions are common in particle physics, a convenient length unit is the femtometer (fm), almost always called the fermi, defined as $1 \text{ fm} = 10^{-15}$ m.

Modern particle colliders are essentially a repeat of the Rutherford scattering type experiment, but at a much higher energy. We now know from many such experiments that the nucleons are all made up of elementary particles called quarks, which glue together via the strong interaction. The CERN Large Hadron Collider has directly probed distance scales well inside the proton, as short as 2 $\times 10^{-15}$ fm.

Some nuclei are unstable. An unstable nucleus tries to achieve a balanced state by given off neutrons or protons and this is done via radioactive decay. It is this that we now turn to study.

§ Radioactive decay. When $^{238}\text{U}$ (we many times omit the “92”) undergoes radioactive decay, it emits an α-particle which consists of 2 protons and 2 neutrons [38]. This leaves an atom with 90 protons (92 minus 2) and 234 nucleons (238 minus 4), which is called thorium-234. $^{234}\text{Th}$ then decays again and we have a chain of decays ending up with lead ($^{206}\text{Pb}$).

Beta decay takes place by the emission of an electron (or β-ray) and a neutrino, from the nucleus at the same time that one of the neutrons changes to a proton. \(^8\) So when a carbon-14 (6 protons, 8 neutrons) β-decays it changes to nitrogen-14 (7 protons, 7 neutrons – ordinary nitrogen!).

There is a law governing the way a substance undergoes radioactive decay: a constant fraction of the atoms in the sample disintegrate per second [39]. The time it takes for half of the sample to decay is called the half-life. For example, if 100 atoms are initially present and the half life is 10 years, then after ten years, 50 atoms will remain undecayed, after another 10 years, 25 will be left, after 30 years, 12 will be left, etc. Since the radioactivity level depends directly on the number of undecayed

\(^8\) Neutrinos are very weird teeny, tiny, nearly massless particles that do not carry electric charge.
atoms remaining, we can also say that after each half-life time the radioactivity is cut in half.

Knowing the half-life, we know how to draw the graph shown in Fig. [33] which will allow us to find the radioactivity level (or the amount of substance left) after any amount of time. So, for example, to find out how much long substance must decay in order that its radioactivity level drop by 25%, we look at the graph for the time which corresponds to 75% activity. This is point A on the time axis, and is about 0.4 of a half-life.

§ Radioactivity carbon dating. Because of cosmic ray bombardment, a tiny fraction (1 millionth of 1%) of the carbon in the atmosphere (in CO2) is carbon-14. The CO2 is breath by plants. Hence all plants have a tiny bit of radioactive 14C in them.

Through radioactive decay, the 14C in living things changes to stable nitrogen, but because living plants breathe, the decayed 14C is replenished, and there is a constant ratio of 14C to 12C (the ordinary carbon). The equilibrium is such that there is a radioactive level of 15 disintegrations per second for every gram of the carbon mixture. When a plant dies, however, the replenishment stops.

The percentage of 14C steadily decreases with a half-life of 5,730 years. Since we know the radioactivity of plants today, we are able to determine the ages of ancient objects by measuring their radioactivity. If we extract a small but precise quantity of carbon from an ancient papyrus scroll, for example, and find it has 1/2 as much radioactivity as the same amount of carbon extracted from a living tree, then the papyrus must be 5,730 years old. If it is 75% as radioactive, then 0.4 of one half-life, or 2,292 years have elapsed since the papyrus was alive.

§ Age of the elements. (i) 238U: The half-life of 238U is 4.5 × 10^9 years. Since this isotope is not markedly less abundant than the other heavy elements (bismuth, mercury, gold, etc.), we conclude that these elements were formed not much longer than 4.5 × 10^9 years ago (like maybe 5 × 10^9 or 6 × 10^9). (ii) 235U: On the other hand, 235U is only 1/140 as abundant as 238U, and has a half-life of 0.9 × 10^9 years. If 238U and 235U were formed in roughly equal amounts, it must have taken about 7 half-lives to get them to the present ratio (since (1/140)^7 = 1/128 close to 1/140). So we estimate that both these elements were formed 7 × 0.9 × 10^9 = 6.3 × 10^9 years ago (6.3 billion).

§ Age of the Earth. (i) Age of the rocks. When 238U undergoes radioactive disintegration the final products of the sequence of decays are an isotope of lead (206Pb), 8 helium atoms, and various electrons and neutrinos. When the 238U became encased in rock, the lead (and helium, to some extent) was locked into close proximity to the 238U. As time passes the ratio 206Pb/238U increases. By knowing this ratio, and the half-life of 238U (4.5 × 10^9 years), one can estimate the time which has passed since the 238U was encased in rock. The same procedure can be used with other “mother-daughter” pairs (232Th → 208Pb and 235U → 207Pb). Using this method, the oldest rocks found on Earth have been dated 4 billion years. More sophisticated methods eventually date the formation of the earth’s crust at 4.5 × 10^9 years. (ii) Age of the oceans. The oceans have become salty as a result of minerals being washed into them by rivers flowing to the sea. The evaporation of water from the ocean (leaving behind the brine) and its subsequent return to the rivers as fresh-water rain leads to an increase in the salinity from year to year. From the knowledge of the total volume of the oceans, the rate of fresh water flow to the sea, and the mineral content of the river water, one may estimate that the salinity of the oceans has been increasing at the rate of one billionth of a percent per year (10^-9% per year). Thus, to reach a concentration of 3%, it must have taken about 3 billion years.

FIG. 34: Contraction of a gas cloud. As the collapse due to gravitational attraction proceeds it speeds up. The gas cloud heats up, in the center most of all. Nuclear fusion starts. The contraction stops and a balance is established between pressure in the gas cloud and gravity.

IX. BIRTH AND DEATH OF THE SUN

§ In the beginning ···! there is a huge cloud of hydrogen and helium, about a trillion miles in diameter (this is a million times the sun’s diameter). If the cloud has a mass greater than about 10^30 lb, the attractive forces due to gravitation will be sufficient to overcome the dispersive effects of the random motion of the atoms in the cloud, and the cloud will begin to contract; see Fig. [54].

As the cloud contracts, one can think of all the atoms falling in toward the center, and just like any other kind of falling, they pick up speed. Their speed gets randomized through collisions, so that the net effect is a large increase in the temperature (like the rise in temperature of a falling brick when it strikes the ground).

§ Formation of the plasma. After about 10 million (10^7) years of contraction, the hydrogen cloud shrunk to a diameter of 100 million miles (100D⊙ or 100 times the diameter of the sun). This is like shrinking the entire Lehman campus down to an inch. The temperature of the cloud has gone from about 100°K (representing an
average speed of 1 mile per second for the hydrogen atoms) to about 50,000 K (representing an average speed of 20 miles per second). The density of the cloud is about $1/1000$ of the density of air. At this speed and density the collisions are frequent enough (each atom collides about a billion times/second) and violent enough to ionize all the atoms in the cloud (that is, remove the electrons from the atoms). So at this stage, the star-to-be (called a protostar) consists of gas of positively charged protons and helium nuclei, and negatively charged electrons in equal balance. This kind of hot gas is called a plasma.

§ Further contraction. The plasma continues to contract under the influence of the gravitational force, getting hotter and denser. The temperature rises because the surface area of the star is not large enough in relation to its volume to get rid of the heat as it is produced. After another 10 million years the plasma has shrunk by another factor of 100 to the size of the sun ($D_\odot = 1$ million miles), the density near the center has risen to 10 million °K (corresponding to an average speed of 280 miles per second for the protons). At this stage the plasma is contracting so quickly that is about one hour from a total collapse into a point. But in the nick of time, we get ...
FIG. 37: \(D + p \rightarrow ^{3}\text{He} + \gamma\).

FIG. 38: \(^{3}\text{He} + ^{3}\text{He} \rightarrow ^{4}\text{He} + 2p\).

FIG. 39: Outward expansion into the red giant phase.

The net effect of this sequence, which is called the pp-cycle, is for four protons to combine to form one \(^{4}\text{He}\) nucleus, plus two positrons, two neutrinos, and two gamma rays.\(^9\) Now, nothing new happens for a long time except for the accumulation of helium at the center of the star.

§ Red giant phase. As the helium accumulates, it starts undergoing gravitational contraction, heating up in the process (just like the hydrogen did before it). This heating and contracting proceeds at a rather rapid rate (over a period of tens of millions of years) leading to a great increase in temperature of the hydrogen surrounding the helium core, and a consequent acceleration of its burning. It also expands outward as shown in Fig. 39.

The end result is that the star becomes huge, the core becomes small and dense (20,000 miles across, 1 ton per in\(^3\)). The large size of the star allows the surface to remain “cool” (i.e., only red hot) so that the star at this stage is visible as a “red giant.” An example is Betelgeuse (pronounced “beetle juice”) which is visible as Orion’s left shoulder in the night spring sky, looking south.

§ Burning of the helium. When the temperature of the core reaches 100 million °K, the helium begins to burn, undergoing the reaction

\[ ^{4}\text{He} + ^{4}\text{He} \rightarrow ^{8}\text{Be} \text{ (beryllium-8)}. \] (75)

Now, there is another of those “just so” accidents. \(^{8}\text{Be}\) is not stable; although it can exist for a short time, the nucleons out of which it is made do not attract each other strongly enough, and in less than \(10^{-12}\) s it breaks up into separate helium nuclei again:

\[ ^{8}\text{Be} \rightarrow ^{4}\text{He} + ^{4}\text{He}. \] (76)

However, at the temperature and density of the helium core, each helium nucleus undergoes about \(10^{12}\) collisions per second. So it is not unlikely that \(^{8}\text{Be}\) will be hit by another helium-4 before it breaks apart; see Fig. 40. This leads to the formation of carbon, and a \(\gamma\)-ray (which is energy).

§ On to the white dwarf. So now a carbon core begins to form. The heat generated by these interactions and the subsequent collapse of the carbon core lead to a blow-off of most of the outer gas layers. The star has become a hot carbon core, about 20,000 miles across, with a density of 10 tons per in\(^3\). The electron gas exerts enough outward pressure to keep the carbon core from contracting much in diameter, if the star is less than one and a half times as massive as the Sun. But the inward pressures still generate a great deal of heat, and the

\(^9\) The theory of the pp cycle as the source of energy for the Sun was first worked out by Bethe [12]. Interestingly, the same set of nuclear reactions that supply the energy of the Sun’s radiation also produce neutrinos that can be searched for in the laboratory [13][14].
A carbon star glows with a white heat. This is a white dwarf. After a few million years, the dwarf cools some, becomes yellow, then red, then cools completely and the fire goes out. The star has met its death as a massive, dense lump of coal.

§ Heavy stars. An entirely different end awaits the 5% of stars whose masses exceed 1.5 times the mass of the sun [45]. For such masses, the inward gravitational pressure of the carbon core can generate a temperature sufficiently large (600 million °K) to ignite the carbon. The carbon burns to form magnesium and other elements.

The pressure and temperature mount, nuclear reactions continue, until finally the iron is reached. At this point, the process stops, because iron is a very special element. Any reaction that takes place involving an iron nucleus will use up energy. The iron, instead of providing more fuel to burn, puts the fire out. The center of the star commences to collapse again, but this time, because of the presence of the iron nuclei, the fire cannot be rekindled; it has gone out for the last time, and the entire star commences its final collapse.

The collapse is a catastrophic event. The materials of the collapsing star pile up at the center, creating exceedingly high temperatures and pressures. Finally, when all the nuclei are pressed against each other, the star can be compressed no further. The collapsed star, compressed like a giant spring, rebounds instantly in a great explosion. About half the material within the star disperses into space, the other half remains as a tiny core (about 10 miles in diameter), containing a mass equal to that of the Sun. This is now a pulsar or neutron star (for reason to be explained soon). The entire episode (collapse and explosion) lasts a few minutes.

The exploding star is called a supernova. The most famous supernova was recorded by Chinese astronomers in A.D. 1054, and its remnants are now visible as the Crab Nebula.

At the very high temperatures generated in the collapse and explosion, some of the nuclei in the star are broken up, and many neutrons and protons freed. These are captured by other nuclei, building up heavier elements, such as silver, gold, and uranium. In this way the remaining elements of the periodic table, extending beyond iron, are perhaps manufactured in the final moments of the star’s life [46]. Because the time available for making these elements is so brief, they never become as abundant as the elements up to including iron.

The core of the collapsed star then contracts until all the nuclei are touching, and then stops. The forces are so great that all the nuclei (iron, etc.) disintegrate into their constituents (neutrons and protons); the protons combine with electrons to leave a dense core of neutrons – a nucleus about as large as Boston [47]. This object, a neutron star, rotates madly about its axis, emitting energy at a billion times the rate at which the sun does so. We see it in the sky as a pulsar.

If the mass of the neutron star is greater than about three solar masses, then the star further contracts under gravity and eventually collapses to the point of zero volume and infinite density [48]. As the density increases, the paths of light rays emitted from the star are bent and eventually wrapped irrevocably around the star. The “star” with infinite density is completely enclosed by a boundary known as the event horizon, inside which the gravitational force of the star is so strong that light cannot escape [49]. This is called a black hole, because no light escapes the event horizon.
X. NUCLEAR PROCESSES

§ Nuclear masses and binding energies. Two distinct processes involving the nuclei of atoms can be harnessed, in principle, for energy production: fission (the splitting of a nucleus) and fusion (the joining together of two nuclei). In Fig. 41, we show the mass per nucleon for the different elements. We see that we have a minimum for $^{56}$Fe. This means that $^{56}$Fe is relatively more tightly bound than the other nuclei, i.e., the average binding energy per nucleon is greater for $^{56}$Fe than for other nuclei. This also means that for lighter elements (with less than 56 nucleons), the mass per nucleon decreases when combining nuclei to form more heavier elements. Thus, for lighter elements, energy is usually released in a fusion reaction. For elements heavier than iron however, the mass per nucleus increases with increasing number of nucleons and energy is liberated in a fission reaction. For any given mass or volume of fuel, nuclear processes generate more energy than can be produced through any other fuel-based approach.

§ Nuclear fission The fission of a uranium or plutonium nucleus liberates a large amount of energy, which comes from the changes in the internal energy of the nuclei involved in the reaction. The splitting of uranium-235 shown in Fig. 42, for example, releases over $3 \times 10^{-11}$ J, i.e., 50 times more than the alpha decay of the same nucleus.$^{10}$ Only the $^{235}$U part of uranium (0.7%) undergoes fission when irradiated with slow neutrons; the $^{238}$U (99.3%) will absorb fast neutrons (becoming $^{239}$U, but it will not fission. The neutrons must be slow because they have to stay around a nucleus a while in order to be captured. They don’t really knock the $^{235}$U apart during fission; it is really more like they enter the nucleus to form $^{236}$U, which is very unstable and undergoes fission. During each fission, about 3 neutrons (on the average) are produced. The possibility of a chain reaction is then present, except that these fission neutrons are fast. As a consequence, not only do they fly past $^{235}$U without causing further fission, but they also get captured by the $^{238}$U. So we need some way of slowing down the neutrons produced in fission (moderating them) and then letting them hit more uranium.

The modus operandi is as follows. Arrange a series of uranium rods sitting in the moderator, as shown in Fig. 43. You must make sure that:

- moderator moderates (slow the neutrons);
- moderator does not absorb too many neutrons;
- moderator does not become radioactive, because it is also used for cooling.

For the moderator to slow down neutrons, it must consist of light atoms (neutrons would just bounce off heavy atoms without losing speed). Hydrogen (as part of water) is excellent except that it absorbs neutrons,

$$
\text{hydrogen + neutron} = \text{deuterium}
$$

So to use ordinary water, one must eliminate some of the absorption of neutrons by $^{238}$U by making the uranium rod richer in $^{235}$U ("enrichment"). This is a very expensive process. Carbon (graphite) does not absorb neutrons, but it is also pretty heavy so it doesn’t moderate as well. So, if you use graphite, you must enrich the uranium. Heavy water ($\text{D}_2\text{O}$) is excellent: it moderates almost as well as ordinary water and it doesn’t absorb neutrons; reactors using heavy water do not need to have the uranium enriched – they use the natural mixture of $^{238}$U and $^{235}$U. Neither water, graphite, or heavy water become radioactive when neutrons pass through them.

§ The bomb and all that... In December 1942, Fermi constructed the first nuclear reactor. It used natural uranium (not enriched), a graphite moderator (to slow neutrons) and boron control rods (to control the reaction by absorbing neutrons) $^{50,51}$. The stage was then set for building the uranium bomb.

For the bomb, one has no moderator, no control rods, but pure $^{235}$U (very expensive). Two pieces of $^{235}$U are brought together through a small explosion or a spring, and are irradiated with neutrons from a small radioactive source (in the bomb). Each piece by itself is not enough to “go critical”, but together they can sustain a chain reaction using the small number of slow neutrons that are emitted during fission, and the small probability

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*Footnote:* 10 The mass deficit for the reaction $^{235}$U + n → $^{141}$Ba + $^{92}$Kr + 3n is $140.91441 \, \text{u} + 91.92616 \, \text{u} + 2 \times 1.008665 \, \text{u} - 235.04394 \, \text{u} = -0.18604 \, \text{u}$. From $^{33}$ we have that $1 \, \text{u} = 1.66058 \times 10^{-27} \, \text{kg}$, and so using $^{34}$ it is straightforward to see that the annihilation of 3.08934 $\times 10^{-26} \, \text{kg}$ of mass produces $2.78041 \times 10^{-13} \, \text{J}$ of energy.
that a fast one will be captured by a $^{235}\text{U}$ to induce fission. Unless you have almost pure $^{235}\text{U}$, this won’t work. (That is why we have moderators in reactors, which do not use pure $^{235}\text{U}$.)

The first (test; code name Trinity) bomb was exploded in July 1945. The first $^{235}\text{U}$ bomb was dropped on Hiroshima on August 6, 1945, killing 80,000 people. The second bomb (Plutonium) was dropped on Nagasaki two days later with similar results. Plutonium ($^{239}\text{Pu}$) also undergoes fusion with slow neutrons. It is not found naturally, but is made whenever $^{238}\text{U}$ (the common isotope) is radiated with fast neutrons:

$$^{238}\text{U} + n \rightarrow ^{239}\text{U} \xrightarrow{\beta^{-} \text{ decay}} ^{239}\text{Np} \xrightarrow{\beta^{-} \text{ decay}} ^{239}\text{Pu},$$

where the times are respectively half-lives of uranium-239 and neptunium-239. Plutonium is highly radioactive, but has a long half-life of 24,100 years, so it can be used. It is lethal in microgram quantities (much less than a speck), due to both its radioactive and chemical properties.\textsuperscript{11} \textit{Terrible stuff!}

§ Nuclear power plants. There are 61 commercially operating nuclear power plants with 99 nuclear reactors in 30 states of the U.S. Though there are several types of reactor that convert the kinetic energy of fission fragments to electrical energy, the most common type in use in the U.S. is the pressurized-water reactor shown in Fig. 44. Its main parts are common to all reactor designs. Fission events in the reactor core supply heat to the water contained in the primary (closed) loop, which is maintained at high pressure to keep it from boiling. This water also serves as the moderator. The hot water is pumped through a heat exchanger, and the heat is transferred to the water contained in the secondary loop. The hot water in the secondary loop is converted to steam, which drives a turbine-generator system to create electric power. The secondary loop has a condenser, where steam from the turbine is condensed by cold water (from a lake or a river). Note that the water in the secondary loop is isolated from the water in the primary loop to prevent contamination of the secondary water and steam by radioactive nuclei from the reactor core.

\textsuperscript{11} A 5 kg mass of $^{239}\text{Pu}$ contains about $12.5 \times 10^{24}$ atoms. With a half-life of 24,100 years, about $11.5 \times 10^{12}$ of its atoms decay each second by emitting an alpha particle of $8.3 \times 10^{-17}$ J. This amounts to 9.68 watts of power.
§ Dangers of operating nuclear reactors. The 1986 accident at the Chernobyl reactor in the Ukraine rightfully focused attention on reactor safety. Despite of their advantage (energy content of fuel very high, negligible atmospheric pollution), there are some serious potential hazards associated with the use of nuclear power. These may be catalogued as follows:

(1) Loss of coolant accident. Possible failure of a pump or pipe would prevent the water from cooling. Unless the emergency core cooling system went into effect, the core would melt down through the casing of the reactor in about 30 seconds, with the possible release of an enormous amount of radioactivity to the outside world. To control the power level, control rods are inserted into the reactor core (see Fig. 43). These roads are made of material such as cadmium that absorb neutrons very efficiently.

(2) Radioactive waste disposal. After a certain amount of time, the $^{235}$U concentration in the fuel rod becomes too low (it gets used up in fission!) and the road must be replaced. Unfortunately, it now contains a large amount of radioactive material, including the products of the fission:

$$n + ^{235}\text{U} \rightarrow \text{fission} \rightarrow \text{cesium-137} \quad (^{137}\text{Cs})$$
$$\text{strontium-90} \quad (^{90}\text{Sr})$$
$$\text{krypton-85} \quad (^{85}\text{Kr}). \quad (79)$$

The $^{85}\text{Kr}$ escapes as a gas – not too dangerous. But $^{137}\text{Cs}$ and $^{90}\text{Sr}$ take about 60 years (two half-lives) to lose 75% of their activity, so they create a storage problem. Strontium-90 is very dangerous because if release to the natural surroundings, it finds its way into cow’s milk in very high concentrations, and from there into children’s bones. The wastes, in liquid, form are usually stored in stainless steel casks buried in concrete vaults. They need constant surveillance because of leakage and the possibility of corrosion. This method is regarded as a temporary expedient.

Unfortunately, the problem does not die with the Cs and Sr, because wastes also include some very long-lived materials, such as plutonium-239. Since a large amount of $^{239}\text{Pu}$ is present in every commercial reactor, and fast neutrons are released during the fission of $^{235}\text{U}$, some of these fast neutrons (the ones that aren’t slowed by the moderator) will react with the $^{238}\text{U}$ to form plutonium in the same fuel road. So, a fair amount of plutonium is formed every present day commercial reactor. $^{239}\text{Pu}$ undergoes fission just like $^{235}\text{U}$, so more fuel for fission has been created in the reactor. This is called converting. When the reactor makes more fuel than it consumes, it is called a breeder. Breeders were at first found attractive because their fuel economy was better than light water reactors, but interest declined after the 1960s as more uranium reserves were found, and new methods of uranium enrichment reduced fuel costs.

§ Nuclear fusion. The reactions that take place in the Sun cannot take place on Earth. To hold the hydrogen cloud together by gravitation (against the expanding forces) until the slow interaction $pp \rightarrow D + e^+ + \nu$ takes place, requires a mass of material equal to the mass of the Sun. So if we would attempt to burn “protons” here, the “fire” would keep going out.

However, there may be other nuclear fusion reactions which release enormous amount of energy. These play no role in the nuclear chemistry of the Sun because of the negligible amounts of the necessary materials present there. We will discuss two of these. They involve the isotopes of hydrogen, deuterium (D), and tritium (T). The deuterium-tritium reaction, shown in Fig. 45, is

$$D + T \rightarrow ^4\text{He} + n + 6 \text{ billion Btu/oz of (D + T)}. \quad (80)$$

(Compare to an energy release of 1,000 Btu/oz of oil burnt.)

The D-D reaction is more complicated, and it releases about 1/4 as much energy. Nevertheless, in the very long run, it is very important (as we will see). For now, we will concentrate on the D-T reaction.

First, why D-T? Well, to make a nuclear fusion reaction go, the nuclei must be brought close enough together to
touch. All nuclei are positively charged, so that one must accelerate them to high speeds in order to overcome the effects of the electric repulsion. Since the positive charge of on a nucleus increases with its mass (more protons), it is then clear why it is easiest to overcome the electrical repulsion for the lightest nuclei. We have seen this in the case of the Sun’s life cycle, where the fusion of helium (charge +2 each) or carbon (+6 each) require a higher temperature than the fusion of hydrogen. So, we want to fuse hydrogen, if we can. But we have seen that the fusion of protons goes only via the weak interaction, so we then may inquire if the fusion of deuterium or tritium (same 1 + charge) can go via strong interaction. The answer is yes!

What about availability? Deuterium is stable and very plentiful: one out of every 6,000 atoms of hydrogen in water is $^2$H (a.k.a. deuterium). So, a gallon of H$_2$O yields about a spoonful of heavy water D$_2$O. It is relatively easy to separate off. Tritium is another problem. It is a radioactive gas, decaying with a half-life of 12 years (to $^3$He + electron + neutrino) so it doesn’t occur naturally. It can be produced in small quantities at ordinary nuclear reactors through the bombardment of lithium with neutrons

$$n + ^6\text{Li} \rightarrow ^4\text{He} + T. \quad (81)$$

To overcome the difficulty, the usual design of the future fusion reactor involves the following general procedure:

1. Start the reaction (in one way or another) using D and T.

2. Surround the reaction region with a liquid lithium blanket. This serves two processes:
   (a) The neutrons emerging from the D-T reaction will react with $^6\text{Li}$ to generate tritium and
   (b) the liquid lithium will absorb energy of the reaction, become hot and can be used to boil water, make steam, etc.

So two things are apparent:

1. One must deal with technology of liquid lithium (similar to that of liquid sodium) and

2. the limiting fuel resource is lithium. At present, the world supply of lithium is equivalent (in eventual Btu) to that of all the fossil fuels (a few hundred years). By that time, one will have learned to run the D-D reaction, where the resources are almost infinite (sea-water!).

The basic fact is, that to overcome the electrical repulsion, the D and T must approach one another at a relative speed of about 750 miles per second. As we have seen in the case of the Sun, this involves heating the D-mixture to temperatures of tens of millions of degrees K (about 50 million, to be exact). Such high temperatures are attainable on Earth in the proximity of a nuclear explosion.

Indeed, by surrounding a $^{235}$U bomb with D-T mixture, you can ignite all at once, and it explodes: Presto, a hydrogen bomb! But this is not a controllable source of power.

Given this, the basic problem is the following: Once the D-T mixture is heated to $50 \times 10^6\text{K}$, how you get the reaction going before the mixture is cooled through contact with the environment? There are two basic possibilities, and both are being actively pursued: Firstly, raise the temperature so quickly that the fuel mixture is all consumed before the heat escapes. Secondly, (and this at first sounds impossible), keep the mixture away from all contact with the outside world.

The first process (look back!) is essentially what we mean by an explosion. In the D-T case, this means setting off a series of tiny hydrogen bombs; at present this is being researched under the heading of laser-induced fusion. The scheme is basically very simple: a small pellet (about 1/10 to 1 cm in diameter) of frozen D-T (at 4 K) is injected into an explosion vessel. A massive laser pulse is focussed onto the pellet, heating it “instantaneously” to 100 million”K, causing the D-T reaction to consume the pellet, giving off the fast neutrons which heat the lithium blanket and make more tritium. Then another pellet is dropped, the process repeated and so on. Each explosion is equivalent to the detonation of a small stick of dynamite.

Let’s just list some of the problems:

- The projected power required from the laser is about 100 times that envisaged in the next generation of giant lasers. At this point, the Russians are in a distinct lead in this field.

- Through computer studies, it looks like there will be a problem in heating the pellet fast enough and evenly enough to cause the reaction to go to completion before the D-T evaporates.
There may be a cost problem: the combined cost of pellet and energy for laser may be excessive.

However, the combination of military applicability (lasers for antimissile defense, the fusion process itself as a simulation for weapon testing) and the payoff in the event of success provide a distinct impetus for the program.

The second way of preventing the cooling of the mixture before the burning takes place depends on our old friend, the magnetic field. In essence, one can keep charged particles out of touch with their environment by getting them to circle around magnetic field lines. The idea is one we have already dealt with: if you create a strong magnetic field, charged particles will be confined to describe tight circles about the field lines.

If the field is strong enough, the plasma will be kept from all contact with the wall of the container. One of the most promising setups at present was first developed in Russia and goes under the name of Tokamak [52]. In this machine the magnetic field takes a toroidal, or doughnut, shape, and the plasma is squeezed into very tight circles about the field lines; see Fig. 46. This serves not only as a containment device, but also as a heating agent for the plasma.

The main problem with magnetically contained fusion is the following: for “break-even”, that is, to get more fusion energy out than you put in, the plasma must be contained at a high density for a long enough time, at a temperature of about 40 million°K. On May 2011, the Experimental Advanced Superconducting Tokamak (EAST) was able to sustain hydrogen plasma to about 50 million°K for 30 s [53]. EAST is one of the precursors of the International Thermonuclear Experimental Reactor (ITER), a full-scale nuclear fusion power plant currently being built in France [54]. Thereby the ITER machine aims to demonstrate the principle of producing more energy from the fusion process than is used to initiate it, something that has not yet been achieved in any fusion reactor.

XI. SPACETIME

§ Foundations of special relativity. In the 19th century, it was thought that just as water waves must have a medium to move across (water), and audible sound waves require a medium to move through (e.g., air), so also light waves require a medium, which was called the “luminiferous” (i.e. light-bearing) “aether.” If this were the case, as the Earth moves in its orbit around the Sun, the flow of the aether across the Earth’s surface could produce a detectable “aether wind.” Unless for some reason the aether were always stationary with respect to the Earth, the speed of a beam of light emitted from a source on Earth would depend on the magnitude of the aether wind and on the direction of the beam with respect to it. In 1881, Michelson designed an experiment to measure the speed of light in different directions in order to measure the speed of the aether relative to Earth, thus establishing its existence [55]. The Michelson-Morley experiment became what might be regarded as the most famous failed experiment to date and is generally considered to be the first strong evidence against the existence of the luminiferous aether [56].

To explain nature’s apparent conspiracy to hide the aether drift, in 1905 Einstein advanced the principle of relativity based on the following two postulates [57]:

1. All laws of nature are the same in all uniformly moving reference frames.

2. The speed of light in free space has the same measured value for all observers, regardless of the motion of the source or the motion of the observer, i.e. the speed of light is a constant.

The first postulate recollects the idea that all motion is relative, and not to any stationary hitching post in the universe, but to arbitrary reference frames. This implies that a spacecraft cannot measure its speed with respect to empty space, but only with respect to other objects. In other words, if Harry’s rocket-ship drifts past Sally’s rocket-ship in empty space, spaceman and spacewoman will each observe the relative motion, and from this observation, each will be unable to determine who is moving and who is not, if either. The second postulate introduces the idea that in empty space the speed of light is the same in all reference frames. Einstein’s postulates describe in simple and clear terms the Michelson-Morley experiment, which cannot be explained otherwise.

§ Relativity of simultaneity. We experience time as a succession of events with certain regularities. The days pass one after another. We and others grow older according to the number of days we have lived through. Time was traditionally measured in days, which added up to months, and then to years. Slowly other types of regularities came to be used to measure time: spring motion, pendulum motion, the flow of sand, etc. These days we use electronic oscillators (very accurate), atomic vibrations (incredibly accurate), and other things. All of these things could be used as clocks. In some way, the flow of time governs them all.

Once we accept Einstein’s 2nd postulate it turns out that our commonsense notion of time needs to be profoundly modified in several ways. The first modification is to the notion of simultaneity. We expect, based on everyday experience, that two events that are simultaneous for observer S will also be simultaneous for observer S’, even if observers S and S’ are moving with respect to one another. This is certainly true in Newtonian mechanics. In other words, two events being simultaneous is an invariant. Now let’s check if this invariance carries over to the theory of special relativity. This time, observer S’ (Harry) in a rocket-ship takes a picture with a flashbulb, and the flash goes out in all directions.
FIG. 47: From Harry’s viewpoint the light from the source travels equal distances to both ends of the spacecraft, and hence strikes both ends simultaneously. The events of striking the front and the end of the spacecraft are not simultaneous in Sally’s reference frame. Because of the rocket’s motion, light that strikes the back end doesn’t have as far to go and strikes sooner than light that strikes the front end.

We will focus on just the east and west directions, shown in Fig. 47. Harry is sitting in the middle of the spaceship. The vessel is not moving in his reference frame. The camera is at rest in his frame, as are the front and back of the spacecraft. The flash, which travels at the speed $c$ in both directions, reaches the front of the spacecraft and the rear of the spacecraft at the same time since it started in the middle. Nothing strange about that. Those two arrival events are simultaneous in his frame.

But what about in Sally’s frame, where the rocket is moving at the velocity $v$? Sally also sees the light flashes moving forwards and backwards at the speed $c$. But there is a time lapse between the time when the flash is generated at the middle of the rocket and the time when the flash hits the front of the rocket. During that admittedly short time the front of the rocket has moved to the east, and so has the rear of the rocket. So the flash takes longer to reach the front of the rocket than it does to reach the rear of the rocket. The arrival time at the front is later than the arrival time at the back. Two events that are simultaneous in Harry’s frame happen
FIG. 48: Harry and the mirror are in a spaceship at rest in the $S'$ frame. The time it takes for the light pulse to reach the mirror and return is measured by Harry to be $2d'/c$. In the frame $S$, the spaceship is moving to the right with speed $v$. For Sally, the time it takes for the light to reach the mirror and return is longer than $2d'/c$.

FIG. 49: A right triangle for computing the time $\Delta t$ in the $S$ frame.

at different times in Sally’s frame. This is called the relativity of simultaneity, since whether two events are simultaneous depends on the observer. Simultaneity is not an invariant.

But really, if two events that happen at the same time in one frame happen at different times in another frame, how can we make sense of motion at all? We defined velocity as length traveled divided by elapsed time. The elapsed time did not depend on which frame we were talking about, and it seems like this is the only way we can have a sensible theory of motion. To answer this question we have to think carefully about how we actually do measurements of position and time. Einstein pointed out that when we speak of the time that something happens we must be present at the place that it happens and we must have a clock to record the time. Up until now it was enough for us to have a measuring stick for each reference frame, a rigid body that defined units of a coordinate system. But we could all depend on just one clock, a master timepiece that was used by all observers. Now what we need is a measuring stick with clocks all along it, maybe a little clock every centimeter or so, so that when something happens we can record both the time and the place. All the clocks on our measuring stick in one frame must be synchronized with one another. Any such fancy measuring stick then defines the observations in one reference frame. Then, the description of an event 1 in frame $S$ is given by a statement that looks like this: event 1 happens at point $x_1$ at a time $t_1$ according to the measuring-stick-plus-clocks of frame $S$. Note that space and time are somehow intertwined together, so we can take this a little further and speak of an event happening at a point in spacetime.

§ Time dilation. Having established our measuring system we now turn to derive predictions of the principle of special relativity using some thought experiments devised by Einstein. Einstein’s thought experiments involve an idealized clock in which a light wave is bouncing back and forth between two mirrors. The clock “ticks” when the light wave makes a round trip from mirror $A$ to mirror $B$ and back, that is the time that passes as the light travels from one mirror to the other and returns is the unit of time. Let’s go back to frame $S$, where Sally is standing on a planet with measuring-stick-plus-clocks. She is watching Harry that goes by in his rocket as before.

Assume the mirrors $A$ and $B$ are separated by a distance $d'$ in Harry’s rest frame. In that frame a light wave will take a time

$$\Delta t' = 2d'/c$$  \hspace{1cm} (82)

for the round trip $A \rightarrow B \rightarrow A$. This is the proper time interval between two consecutive ticks of the clock. Let $\Delta t$ be the interval between two consecutive ticks of the clock in Sally’s frame, in which the mirrors move with
velocity \( v \), as shown in Fig. 48. It is noted that when the light wave is bounced back at the mirror \( B \), the latter has already moved a distance \( v\Delta t/2 \), as shown in Fig. 49. Since light has velocity \( c \) in all directions
\[
d^2 + \left( \frac{v \Delta t}{2} \right)^2 = \left( \frac{c \Delta t}{2} \right)^2,
\]
or
\[
\Delta t = \frac{2d'}{\sqrt{c^2 - v^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}.
\]
Hence the ticking of the clock in Harry’s frame, which moves with velocity \( v \) in a direction perpendicular to the separation of the mirrors, is slower by a factor
\[
\gamma = \sqrt{\frac{1}{1 - v^2/c^2}},
\]
usually refer to as the Lorentz factor \([58]\).

A number of experiments support the time dilation predicted by special relativity. One such experiments, conducted by Ross and Hall in 1941, revealed that muons decay more slowly while falling \([59]\). Muons are sub-atomic particles generated when cosmic rays strike the upper levels of our atmosphere. They have a half-life of about 2.2 microseconds (\( \mu s \)) meaning that every 2.2 \( \mu s \), their population will reduce by half. By observing the concentration of muons at both the summit and base of Mount Washington, Ross and Hall were able to measure what proportion of them have decayed and to compare this result with the predictions of special relativity. The experiment was carried out using detectors that only count muons traveling within a certain speed range, 0.9950\( c \) < \( v \) < 0.9954\( c \). Mount Washington has a height difference of 1.9 km between the summit and the base. Flying 1.9 km through the atmosphere at the above speed takes about 6.4 \( \mu s \).

Based on the stated half-life, we should thus expect that only 13% of the original concentration of muons should arrive. However, it is observed that about 82% of the muons arrive below. This percentage would correspond to a half-life of 22 \( \mu s \), i.e. ten times greater than the original. A factor of ten, however, corresponds to what \([63]\) would give for a speed of 0.995\( c \).

§ Length contraction. Now, suppose that Harry rotates the clock by 90° before being set in motion, so that it has velocity \( v \) parallel to the separation between the mirrors, see Fig. 50. According to Sally (in reference frame \( S \)) the length of the clock (distance between the mirrors) is \( d \). As we will see, this length is different from the length \( d' \) measured by Harry in \( S' \), relative to whom the clock is at rest.

The flash of light emitted from mirror \( A \) reaches the mirror \( B \) at time \( \Delta t_1 \) later. In this time interval, the light travels a distance \( c\Delta t_1 \), which is equal to the length \( d \) of the clock plus the additional distance \( v\Delta t_1 \) that the mirror moves forward in this interval; namely,
\[
d + v\Delta t_1 = c\Delta t_1 \quad \text{or} \quad \Delta t_1 = \frac{d}{c-v}.
\]

Assuming the light wave, after bouncing at \( B \), takes time \( \Delta t_2 \) to reach \( A \) again, we have by the same reasoning
\[
d - v\Delta t_2 = c\Delta t_2 \quad \text{or} \quad \Delta t_2 = \frac{d}{c+v}.
\]

Hence the interval between two consecutive ticks in the moving frame is
\[
\Delta t = \Delta t_1 + \Delta t_2 = \frac{2d}{c(1 - v^2/c^2)}
\]
\[
= \left( \frac{d}{d'} \right) \frac{\Delta t'}{\sqrt{1 - v^2/c^2}},
\]
where we have used \([82]\). Substituting \([84]\) into \([88]\) we have
\[
d = \left( \frac{1 - v^2}{c^2} \right)^{1/2} d'.
\]

Such a length contraction was independently proposed by Lorentz and FitzGerald to explain the result of Michelson’s experiment \([60, 61]\).

Direct confirmation of length contraction is hard to achieve in practice since the dimensions of the observed particles are vanishingly small. However, there are indirect confirmations; for example, the behavior of colliding heavy nuclei can only be explained if their increased density due to Lorentz contraction is considered. The implications of special relativity have been widely tested over the past century. All of the experiments support Einstein’s theory and are in complete disagreement with non-relativistic predictions. Consequently, both time dilation and length contraction must be considered when conducting experiments in particle colliders.

§ What is gravity? It turns out that Newton’s universal law of gravity is not compatible with special relativity. In particular, having learned special relativity we now know that it should not be possible to send messages faster than the speed of light. However, Newton’s relation \([7]\) would allow us to do so using gravity. The point is that Newton stated that the gravitational force depends on the separation between the objects at a given instant of time.

To further convince yourself that \([7]\) is not compatible with special relativity consider the following situation. The Earth is about eight light-minutes from the Sun. This means that, at the speed of light, a message would take eight minutes to travel from the Sun to the Earth. However, suppose that, unbeknownst to us, some aliens are about to move the Sun. Then, based on our understanding of relativity, we would expect it to take eight minutes.
for us to find out! But Newton would have expected us to find out instantly because the force on the Earth would shift (changing e.g., the tides).

Now, it is important to understand how Maxwell’s light waves get around this sort of problem. That is to say, what if the Sun were a positive electric charge, the Earth were a big negative electric charge, and they were held together by an electromagnetic field? We know that Maxwell’s light waves are consistent with relativity, so we can investigate what would these waves tell us happens when the aliens move the Sun.

The point is that the positive charge does not act directly on the negative charge. Instead, the positive charge sets up an electric field which tells the negative charge how to move. When the positive charge is moved, the electric field around it must change, but it turns out that the field does not change everywhere at the same time. Instead, the movement of the charge modifies the field only where the charge actually is. This makes a “ripple” in the field which then moves outward at the speed of light.

Thus, the basic way that Maxwell’s electromagnetic waves get around the problem of instant reaction is by having a field that will carry the message to the other charge (or, say, to the planet) at a finite speed. What we see is that the field concept is the essential link that allows us to understand electric and magnetic forces in relativity.

Something like this must happen for gravity as well. Let’s try to introduce a gravitational field by breaking Newton’s law of gravity up into two parts. The idea will again be than an object should produce a gravitational field \( \vec{g} \) in the spacetime around it, and that this gravitational field should then tell the other objects how to move through spacetime. Any information about the object causing the gravity should not reach the other objects directly, but should only be communicated through the field. We rewrite (7) to indicate the force on \( m \) produced by \( M \) as follows:

\[
\vec{F}_g = \vec{g} m,
\]

where \( \vec{g} = \frac{GM}{r^2} \). Einstein used this idea to develop the general theory of relativity, in which the gravitational field is related to the geometry of spacetime [62][65]. A major forecast of general relativity is that when two massive objects crash into each other there should be a release of gravitational waves, which transport energy as gravitational radiation [66].

The first direct detection of gravitational waves was made on 14 September 2015 by the LIGO and Virgo collaborations [67]. The observed gravitational waves were emitted during the final moments of the merger of a pair of black holes. Previously gravitational waves had only been inferred indirectly, via their effect on the timing of pulsars in binary star systems [68]. More recently, the
LIGO and Virgo detectors observed “ripples” in the geometry of spacetime originating in the violent collision of two distant neutron stars [69]. When these two objects combined, they spiraled around each other rapidly before smashing into one another, creating a gigantic fireball of light also visible to telescopes on Earth [70]. The substantial ejecta masses inferred from observations at optical and infrared wavelengths suggest that the accumulated nucleosynthesis from neutron star mergers could account for all the gold, platinum, and many other heavy elements around us [71].

**XII. ACROSS THE UNIVERSE**

§ Stars and galaxies. A look at the night sky provides a strong impression of a changeless universe. We know that clouds drift across the Moon, the sky rotates around the polar star, and on longer times, the Moon itself grows and shrinks and the Moon and planets move against the background of stars. Of course we know that these are merely local phenomena caused by motions within our solar system. Far beyond the planets, the stars appear motionless. In today’s class we are going to see that this impression of changelessness is illusory.

According to the ancient cosmological belief, the stars, except for a few that appeared to move (the planets), where fixed on a sphere beyond the last planet. The universe was self contained and we, here on Earth, were at its center. Our view of the universe dramatically changed after Galileo’s first telescopic observations: we no longer place ourselves at the center and we view the universe as vastly larger [72,74].

The astronomical distances are so large that we specify them in terms of the time it takes the light to travel a given distance. For example,

one light second = $3 \times 10^5 \text{m} = 300,000 \text{ km}$,

one light minute = $1.8 \times 10^7 \text{ km}$,

and

one light year = 1 ly = $9.46 \times 10^{15} \text{ m} \approx 10^{13} \text{ km}$.

For specifying distances to the Sun and the Moon, we usually use meters or kilometers, but we could specify them in terms of light. The Earth-Moon distance is 384,000 km, which is 1.28 ls. The Earth-Sun distance is 150,000,000 km; this is equal to 8.3 lm. Far out in the solar system, Pluto is about $6 \times 10^9 \text{ km}$ from the Sun, or $6 \times 10^{-4} \text{ ly}$. The nearest star to us, Proxima Centauri, is about 4.2 ly away. Therefore, the nearest star is 10,000 times farther from us that the outer reach of the solar system.

On clear moonless nights, thousands of stars with varying degrees of brightness can be seen, as well as the long cloudy strip known as the Milky Way. Galileo first observed with his telescope that the Milky Way is comprised of countless numbers of individual stars. A half century later Wright suggested that the Milky Way was a flat disc of stars extending to great distances in a plane, which we call the Galaxy [73].

Our Galaxy has a diameter of 100,000 ly and a thickness of roughly 2,000 ly. It has a bulging central nucleus and spiral arms. Our Sun, which seems to be just another star, is located half way from the Galactic center to the edge, some 26,000 ly from the center. The Sun orbits the Galactic center approximately once every 250 million years or so, so its speed is

$$v = \frac{2\pi}{2.5 \times 10^8 \text{ yr}} \frac{26,000 \times 10^3 \text{ km}}{3.16 \times 10^7 \text{ s/yr}} = 200 \text{ km/s}. \quad (94)$$

The total mass of all the stars in the Galaxy can be estimated using the orbital data of the Sun about the center of the Galaxy. To do so, assume that most of the mass is concentrated near the center of the Galaxy and that the Sun and the solar system (of total mass $m$) move in a circular orbit around the center of the Galaxy (of total mass $M$),

$$\frac{GMm}{r^2} = m \frac{v^2}{r}, \quad (95)$$

where we recall that $a = v^2/r$ is the centripetal acceleration. All in all,

$$M = \frac{r \nu^2}{G} \approx 2 \times 10^{11} \text{ kg}. \quad (96)$$

Assuming all the stars in the Galaxy are similar to our Sun ($M_\odot \approx 2 \times 10^{30} \text{ kg}$), we conclude that there are roughly $10^{11}$ stars in the Galaxy.

In addition to stars both within and outside the Milky Way, we can see with a telescope many faint cloudy patches in the sky which were once all referred to as nebulae (Latin for clouds). A few of these, such as those in the constellations of Andromeda and Orion, can actually be discerned with the naked eye on a clear night. In the XVII and XVIII centuries, astronomers found that these objects were getting in the way of the search for comets. In 1781, in order to provide a convenient list of objects not to look at while hunting for comets, Messier published a celebrated catalogue [76]. Nowadays astronomers still refer to the 103 objects in this catalog by their Messier numbers, e.g., the Andromeda Nebula is M31.

Even in Messier’s time it was clear that these extended objects are not all the same. Some are star clusters, groups of stars which are so numerous that they appeared to be a cloud. Others are glowing clouds of gas or dust and it is for these that we now mainly reserve the word nebula. Most fascinating are those that belong to a third category: they often have fairly regular elliptical shapes and seem to be a great distance beyond the Galaxy. Kant seems to have been the first to suggest that these latter might be circular discs, but appear elliptical.
because we see them at an angle, and are faint because they are so distant [77]. At first it was not universally accepted that these objects were extragalactic (i.e. outside our Galaxy). In 1920, Sir Hubble’s observations revealed that individual stars could be resolved within these extragalactic objects and that many contain spiral arms [28]. The distance to our nearest spiral galaxy, Andromeda, is over 2 million ly, a distance 20 times greater than the diameter of our Galaxy. It seemed logical that these nebulae must be galaxies similar to ours. Today it is thought that there are roughly $4 \times 10^{10}$ galaxies in the observable universe – that is, as many galaxies as there are stars in the Galaxy.

Sir Hubble also observed a persistent redshift in the spectra of known elements and that the shift was greater the greater the distance of the galaxy from the Earth. It was Hubble himself who explained the redshift as indicating that distant galaxies were radially moving away from the Earth [29]. In every direction, these vast accumulations of stars and interstellar matter were moving outward at enormous speeds. He called this motion, recession. He showed that the velocity of recession was greater at greater distances. Hubble’s law of cosmic expansion states that an observer at any point in the universe will observe distant galaxies receding from him/her with radial velocities $V$ proportional to their distance $d$ from the observer,

$$V = H_0 d$$  \hspace{1cm} (97)

where $H_0$ is the Hubble’s proportionality constant. Hubble’s initial determination of $H_0$ was approximately 160 km/s per million-light-years. Most recent observations indicate that $H_0 \approx 22.4$ km/s per million-light-years [80, 83].

Hubble’s law is consistent with a general expansion of the space between galaxies (or galactic clusters), and is not a particular characteristic of the galaxies (clusters) themselves. This statement means that the galaxies themselves are not changing in any way; only the regions between them are expanding with time. If the expansion is run backward (as can be done with mathematics), then it would appear that, very long ago, all the matter of the universe was once compacted into a relatively small volume from which it was hurled outward by some titanic force. This idea is the basis for the hot Big Bang model [84–87].

§ Are we alone? Of course we are not alone! We now know that our Sun is but one of ten billions of stars in the Milky Way, and our Galaxy is but one of ten billions of galaxies in the universe. Over the last decade, the search for life in the universe has been transformed from speculation to a data-driven science.

It is well known that all organisms living on Earth require carbon-based chemistry in liquid water. Therefore, according to the hot Big Bang model, life (as we know it) could not have appeared earlier than $t \sim 10$ Myr after the Bang, since the entire Universe was bathed in a thermal radiation background above the boiling temperature of liquid water. After a while ($10 < t/\text{Myr} < 17$) though, the Universe cooled down to habitable comfortable temperatures: $273 < T/\text{°K} < 373$ [88].

Phase diagrams show the preferred physical states of matter at different temperatures and pressure. Within each phase, the material is uniform with respect to its chemical composition and physical state. At typical temperatures and pressures enforced by the earth atmosphere water is a liquid, but it becomes solid (that is, ice) if its temperature is lowered below 0°C and gaseous (that is, water vapor) if its temperature is raised above 100°C. This means that water would remain liquid only under the external pressure in an atmosphere, which can be confined gravitationally on the surface of a planet. To keep the atmosphere restrained against evaporation depends upon the strong surface gravity of a rocky planet, with a mass above that of the Earth [89].

Life needs stars for at least two reasons. We have seen that stars are required to synthesize heavy elements (such as carbon, oxygen,···, iron) out of which rocky planets and the molecules of life are made. We have also seen that stars maintain a source of heat to power the chemistry of life on the surface of their planets.

Each star is surrounded by an habitable zone. Since water is essential for life as we know it, the search for biosignature gases naturally focuses on planets located in the habitable zone of their host stars, which is defined as the orbital range around the star within which surface liquid water could be sustained. To understand this we need to take a quick side trip into how one estimates temperature. The total energy flux $F$ (energy per unit area per unit time) passing through a region can be related to the effective temperature $T$ according to

$$F = \sigma_{SB} T^4,$$  \hspace{1cm} (98)

where $\sigma_{SB} \approx 5.67 \times 10^{-8}$ watt per meter squared per kelvin to the fourth is the Stefan-Boltzmann constant [90, 91]. We have seen in Sec. VII that the luminosity (energy per unit time) of a star is $L$, and that $L$ and the flux at a distance $r$ from the star are related by

$$F = \frac{L}{4\pi r^2},$$  \hspace{1cm} (99)

because the area of a sphere of radius $r$ is $A = 4\pi r^2$ and the flux is the luminosity divided by the area. A quick estimate of the effective temperature at a given radius from a star can proceed by combining (98) and (99), yielding

$$\sigma_{SB} T^4 = F = \frac{L}{4\pi r^2}.$$  \hspace{1cm} (100)

When we consider our solar system, then not only $\sigma_{SB}$ and $4\pi r$, but also the luminosity $L_{\odot}$, are constants with distance. This tells us that

$$T^4 \propto \frac{1}{r^2} \Rightarrow T \propto r^{-1/2}.$$  \hspace{1cm} (101)
Therefore, if we calculate the effective temperature at any radius (say, one astronomical unit \(1 \text{ AU} = \text{earth-sun distance}\)), we can use the proportionality in \(\text{(101)}\) to calculate the temperature at any other radius. For example, if the temperature is \(300^\circ K\) at \(1 \text{ AU}\), then four times farther away the temperature is \(4^{-1/2} = 1/2\) times as low, or \(150^\circ K\). Similarly, the radius where the temperature is \(600^\circ K\) would be given by \((600/300)^{-2} \times 1 \text{ AU} = 0.25 \text{ AU}\).

The habitable zone of the solar system looks like a ring (with habitable temperatures \(273 \lesssim T/^\circ K \lesssim 373\)) around the Sun. Rocky planets with an orbit within this ring may have liquid water to support life. The habitable zone around any other star in the cosmos looks similar to the habitable zone in our Solar System. The only difference is the size of the ring. If the star is bigger than the Sun it has a wider zone, if the star is smaller it has a narrower zone. It might seem that the bigger the star the better. However, the biggest stars have relatively short lifespans, so the life around them probably would not have enough time to evolve. The habitable zones of small stars face a different problem. Besides being narrow they are relatively close to the star. A hypothetical planet in such a region would be tidally locked. That means that one half of it would always face the star and be extremely hot, while the opposite side would always be facing away and freezing. Such conditions are not very favorable for life \(\text{(92)}\).

As of today, we only know of life on Earth. The Sun formed about \(4.6\) Gyr ago and has a lifetime comparable to the current age of the Universe. Tiny zircons (zirconium silicate crystals) found in ancient stream deposits indicate that Earth developed continents and water – perhaps even oceans and environments in which microbial life could emerge – \(4.3\) billion to \(4.4\) billion years ago, remarkably soon after our planet formed \(\text{(93, 94)}\). The presence of water on the young Earth was confirmed when the zircons were analyzed for oxygen isotopes and the telltale signature of rocks that have been touched by water was found: an elevated ratio of oxygen-18 to oxygen-16. If we insist that life near the Sun is typical and not premature, we can assume that the evolution of an advanced civilization requires approximately \(4\) Gyr \(\text{(95)}\).

The Fermi paradox is the discrepancy between the strong likelihood of alien intelligent life emerging (under a wide variety of assumptions) and the absence of any visible evidence for such emergence \(\text{(96)}\). By adopting the starting point of a first approximation to the answer of this intriguing unlikeness, we can write the number of intelligent civilizations in our galaxy at any given time capable of releasing detectable signals of their existence into space using a quite simple functional form,

\[
N = \langle \xi_{\text{astro}} \rangle \langle \xi_{\text{biotec}} \rangle L_T, \tag{102}
\]

where \(\langle \xi_{\text{astro}} \rangle = R_* f_p n_e f_I f_c L_T\) represents the production rate of habitable planets with long-lasting ecoshell (determined through astrophysics) and \(\langle \xi_{\text{biotec}} \rangle = f_l f_f f_c\) represents the product of all chemical, biological and technological factors leading to the development of a technological civilization. \(\langle \cdot \cdot \cdot \rangle\) indicates average over all the multiple manners civilizations can arise, grow, and develop such technology, starting at any time since the formation of our Galaxy in any location inside it. This averaging procedure must be regarded as a crude approximation because the characteristics of the initial conditions in a planet and its surroundings may affect \(f_l, f_f, f_c\) with high complexity.

The star formation rate in the Galaxy is estimated to be \(3 \lesssim R_\ast / \text{yr}^{-1} \lesssim 12\) \(\text{(98)}\). Now, only \(10\%\) of these stars are appropriate for harboring habitable planets. This is because the mass of the star \(M_\ast < 1.1 M_\odot\) to be sufficiently long-lived and \(M_\ast > 0.7 M_\odot\) to possess circumstellar habitable zones outside the tidally locked region. The frequency \(\eta_{\odot}\) of terrestrial planets in and the habitable zone of solar-type stars can be determined using data from the Kepler mission. Current estimates suggest \(0.15^{+0.13}_{-0.06} < \eta_{\odot} < 0.61^{+0.07}_{-0.15}\) \(\text{(99, 100)}\). An Estimate of the rate of planetary catastrophes that could threaten the evolution of life on the surface of these worlds suggests that the survival probability of the planet’s atmosphere for about \(4\) Gyr is roughly \(5\%\) \(\text{(101)}\). The production rate of habitable planets with long-lasting ecoshell is then

\[
\langle \xi_{\text{astro}} \rangle \sim 3 \text{ yr}^{-1} \times 0.1 \times 0.15 \times 0.05 = 0.002 \text{ yr}^{-1}. \tag{104}
\]

Now, since \(\langle \xi_{\text{biotec}} \rangle < 1\), it is easily seen that if the communicative phase is smaller than \(500\) years there would be no paradox. Then, if we assume again that \textit{humans} typify the population of advanced civilizations in the Galaxy we would not have yet expected any contact with intelligent beings from extraterrestrial worlds, but we are on the verge of discovering evidence of alien life in the cosmos.

\section{XIII. FIFTY QUESTIONS TO PONDER}

1. If a household consumes 300 kWh of electrical energy in a 30 day month, what is the average actual power usage, assuming utilization for 16 hour/day?

2. During a normal day of activities, the average person consumes about 2,200 Cal (2.5 kWh). (i) If 300 Cal are released by burning 1 oz fat, how much
weight (not including initial water loss) will a person lose by going on a starvation diet for 2 weeks? (ii) How long would it take to lose 15 lb by reducing intake to 1,600 Cal?

3. A typical Detroit automobile averages about 15 miles/gallon in the city. If the energy content of gasoline is about 32,000 Cal/gal (126,944 Btu/gal), compare the energy used in driving a mile to that used in walking a mile (about 20 Cal), and to that used as electrical energy (per day) for an average household (about 10 kWh).

4. For an astronaut sealed inside a space suit, getting rid of body heat can be difficult. Suppose an astronaut is performing vigorous physical activity, expending 200 watts of power. An energy of 47.8 Cal is enough to raise the body temperature by 1°C. If none of the heat can escape from the space suit, how long will it take before the body temperature rises by 6°C (11°F), an amount sufficient to kill. Express your answer in units of minutes.

5. The cost of home-heating fuel oil is about 40c/gallon. If the cost of heating a certain home during the winter is $60/month, calculate the cost of suppling electric heat to the same home. Assume the thermal energy content of fuel oil to be 140,000 Btu/gal, and the cost of electricity to be 12c/kWh.

6. In 2016, the U.S. imported crude oil at the rate of 7.8 Mbbl/d (7.8 million barrels/day), at a price of about $50/bbl. (i) What was the annual cost to a family of 4 for imported oil? (Take the number of such families to be about 1/4 of the U.S. population, which is about 322 × 10^6). (ii) The price of gasoline at the pump, $2.5 per gallon, is approximately given by 2.2 times the price of a gallon of domestic crude oil, plus taxes. As of 2017, taxes on gasoline amount to 18.4¢ per gallon. If taxes remain unchanged and the price for domestic crude were to rise to present import prices, what would be the price of gasoline at the pump? Do you think this price would appreciably influence American driving habits? [Hint: 1 bbl = 42 gal.]

7. The rocket shown in Fig. 51 has an acceleration of 30 m/s^2 until its engines burnout at 100 s. After ignition the rocket will coast and slowly lose speed: (i) what is its instantaneous speed at burnout? (ii) what is its average speed during the burnout phase? (iii) what is the altitude at burnout? (iv) what is the altitude at the top of its trajectory. Assume the acceleration of gravity is constant, g = 9.8 m/s^2, up to the maximum height.

8. The unnecessary use of electric lights in the home is often cited as source of energy wastage. Suppose each person in the U.S. is responsible for the unnecessary use of a 100 watt (0.1 kW) light bulb for 1 hour each day. (i) What is the amount of wasted electrical energy each day? Give your answer in kWh. At 12c/kWh, what is the added electrical cost per year to a family of 4? Does the saving of this money provide sufficient incentive to "turn off lights when not in use"? (iii) How many kWh electric are expended in the U.S. homes each year on unnecessary lighting? (iv) If each kWh electric demands 3 kWh of thermal heat energy, how many barrels of oil/day are burn in power plants to provide for this wasted energy? (1 bbl oil → 1,700 kWh). Is it worth thinking about from a societal point of view?

9. Every few hundred years most of the planets line up on the same side of the Sun. Calculate the total force on the Earth due to Venus, Jupiter, and Saturn, assuming all four planets are in a line. The masses are \( M_V = 0.815M_\oplus \), \( M_J = 318M_\oplus \), \( M_S = 95.1M_\oplus \), and their mean distances from the Sun are \( r_{SV} = 108 \) million km, \( r_{SE} = 150 \) million km, \( r_{SJ} = 778 \) million km, and \( r_{SS} = 1,430 \) million km. What fraction of the Sun's force on the Earth is this.

10. Given that the acceleration of gravity at the surface of Mars is 0.38 of what it is on Earth, and that Mars radius is 3400 km, determine the mass of Mars.

11. At a depth of 10.9 km, the Challenger Deep (in the Marianas Trench of the Pacific Ocean) is the deepest known site in any ocean. Yet, in 1960, Walsh and Piccard reached the Challenger Deep in the bathyscaphe Trieste. Assuming that seawater has a uniform density of 1,025 kg/m^3, calculate the force the water would exert in the ocean floor on a Plexiglas viewing round observation window of 24 cm diameter.

12. Only a small part of an iceberg protrudes above the water, while the bulk lies below the surface. The density of ice is 917 kg/m^3 and that of seawater is 1,025 kg/m^3. Find the percentage of the iceberg’s volume that lies below the surface.

13. In humans, blood flows from the heart into the
aorta, from which it passes into the major arteries. These branch into small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. For an illustration, see Fig. 52. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capilar has a radius of about $4 \times 10^{-4}$ cm, and blood flows through it at speed of about $5 \times 10^{-4}$ m/s. Estimate the number of capillaries that are in the body.

14. Acrobat Sally of mass $m_S$ stands on the left end of a seesaw. Acrobat Harry of mass $m_H$ jumps from a height $h_H$ onto the right end of the seesaw, thus propelling Sally into the air. (i) Neglecting inefficiencies (that transform energy into heat), how does the potential energy of Sally at the top of his trajectory compares with the potential energy of Harry just before he jumps? (ii) Shows that ideally Sally reaches a height $m_H h_H / m_S$.

15. Harry and Sally are at a ski resort with two ski runs, a beginner’s run and an expert’s run. Both runs begin at the top of the ski lift and end at finish line at the bottom of the same lift. Let $h$ be the vertical descent for both runs. The beginner’s run is longer and less steep than the expert’s run. Harry and Sally, who is a much better skier than him, are testing some experimental frictionless skis. To make things interesting, Harry offers a wager that if she takes the expert’s run and he takes the beginner’s run, her speed at the finish line will not be greater than his speed at the finish line. Forgetting that Harry study physics, Sally accepts the bet. The conditions are that both Harry and Sally start from rest at the top of the lift and they both coast for the entire trip (i.e., there is no external work done on the system). Who wins the bet? (Assume air drag, which may dissipate energy via heat, is negligible).

16. The escape velocity is the minimum speed needed for an object to “break free” from the gravitational attraction of a massive body. More particularly, the escape velocity is the speed at which the sum of an object’s kinetic energy and its gravitational potential energy is equal to zero. Show that the escape velocity from Earth is about 25,020 miles per hour.

17. The energy gained by a person climbing to the top of Mt. Washington is about 300 calories (0.35 kWh). This also happens to be the amount of energy released by burning about 1 ounce of oil. (i) If for every calorie of useful work done the body expends 5 calories in heat, what is the weight loss resulting from the climb? (ii) If you were to perform this climb, would you expect to regain weight during the trip down? Why not?

18. A car engine derives its thermal energy from a “reservoir” of burning fuel whose average temperature is about 4,000°F. The release of the exhaust takes place at a “cold” temperature of about 180°F. What is the theoretical efficiency of the car engine? Note that in the absolute scale for Fahrenheit degrees, we use Rankine (°R) temperatures. Zero on both the Kelvin and Rankine scales is absolute zero, but the Rankine degree is defined as equal to one Fahrenheit degree, rather than the Celsius degree used on the Kelvin scale. A temperature of −459.67°F is exactly equal to 0° R.

19. Biological energy. (i) Suppose a person were entirely made of water (not a bad approximation). If all the heat generated by the body during the day (3,000 Cal = 12,000 Btu) were used to warm up the body, what would be the body temperature at the end of the day for a person weighing 120 lb? Assume an initial temperature of 98.6°F. (ii) If 1,600 Cal were used to work
at 25% efficiency, how high could a 120 lb person climb in the day? (1 Btu = 800 ft lb.)

20. It is well known that a heavy rhinoceros is harder to stop than a small dog at the same speed. We state this fact by saying that the rhino has more momentum than the dog. And if two dogs have the same mass, the faster one is harder to stop than the slower one. So we also say that the faster moving dog has more momentum than the slower one. By momentum we mean the product of the mass of an object and its velocity

\[
\text{momentum} = \text{mass} \times \text{velocity}. \quad (105)
\]

When the direction is not an important factor, we can say

\[
\text{momentum} = \text{mass} \times \text{speed}. \quad (106)
\]

Conservation of momentum is a fundamental law of physics, which states that the momentum of a system is constant if there are no external forces acting on the system. Momentum conservation is especially useful for collisions.\(^{12}\) The forces involved may be so complicated that you cannot practically use Newton’s second law of motion to figure out what happens. And, since some energy may go into heat, sound, or deformation in a collision, energy conservation may also not be useful. But as long as there are no outside forces involved (only internal ones, no matter how big!) then momentum conservation often lets you figure out the outcomes! For example, a crater in Arizona is thought to have been formed by the impact of a meteorite with the Earth over 20,000 years ago. The mass of the meteorite is estimated at 5 \(\times 10^{10}\) kg and its speed 7,200 m/s. (i) Judging from a frame of reference in which the Earth is initially at rest, what speed would such a meteor impart to the Earth in a head-on collision? Assume the pieces of the shattered meteor stayed with the Earth as it moved. (ii) What fraction of meteor’s kinetic energy was transformed to kinetic energy of Earth? (iii) By how much did Earth kinetic energy change as a result of this collision? The mass of the Earth is \(M_{\text{E}} = 6 \times 10^{24}\) kg.

21. (i) If your heart is beating at 76.0 beats per minute, what is the frequency of your heart’s oscillations in hertz? (i) What is the oscillating period of your heart when the frequency increases by a factor of 1.3? (iii) A sewing machine needle moves in simple harmonic motion with a frequency of 2.5 Hz and an amplitude of 1.27 cm; how long does it take the tip of the needle to move from the highest point to the lowest point in its travel? (iv) How long does it take the needle tip to travel a total distance of 11.43 cm?

22. Astronaut Harry has landed on Pluto and conducts an experiment to determine the acceleration due to gravity on the dwarf planet. He uses a simple pendulum that is 0.640 m long and measures 10 complete oscillations in 63.8 s. What is the acceleration of gravity on Pluto?

23. If you drop a stone into a mineshaft that is \(\ell = 122.5\) m deep, how soon after you drop the stone do you hear it hit the bottom of the shaft? The temperature in the mineshaft is 10°C.

24. Two submarines are underwater and approaching each other head-on as shown in Fig. 53. Sub A has a speed of 10 knots and sub B has a speed of 30 knots. Commander Harry on Sub A is pinging on B with an active sonar frequency of 10 kHz. The speed of sound underwater is 2,912 knots, that is roughly 4.3 times as fast as in air. (i) What frequency will receive Sally on submarine B from Harry’s sonar? (ii) What is the frequency of the echo Harry receives from Sally’s submarine?

25. The security alarm on a parked car goes off and produces a frequency of 960 Hz. The speed of sound is 343 m/s. As you drive toward this parked car, pass it, and drive away, you observe the frequency to change by 95 Hz. At what speed are you driving?

26. (i) Compare the electric force holding the electron in orbit \((r = 0.53 \times 10^{-10}\) m) around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton. What is the ratio of these two forces? (ii) Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions?

27. A typical 1.5 volt flashlight battery can deliver a current of 1 ampere for 1 hour. (i) What is the power (in watts) when the current is 1 ampere? (ii) What is the total energy (in kWh) delivered by the battery in 1 hour.

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\(^{12}\) A collision refers to two objects hitting one another, interacting with (probably very large) forces for some (probably very short) amount of time, and then continuing along (probably in radically altered paths, and maybe pretty squashed!) In a perfectly elastic collision the kinetic energy is conserved (no energy is lost to the surroundings or participants), whereas in an inelastic collision the kinetic energy is not conserved (some kinetic energy is converted to heat, or sound, or deformation).
(ii) If the cell costs $0.50, what is the cost for 1 kWh of this form of chemical energy?

28. Kerosene has a fuel value of 1,400 Btu/oz. At what rate (i.e., how many oz/hr) must it be burned in order to give off as much heat as a 1,000 watt electric heater?

29. An air conditioner operating on a 110 volt line is rated at 750 watts. (i) What is the current (in amperes) drawn by this appliance? (ii) At $0.12/$kWh, what is the cost of running the air conditioner for 8 hours?

30. Calculate the power delivered to each resistor in the circuit shown in Fig. 54.

31. (i) The distance to the North Star, Polaris, is approximately $6.44 \times 10^{18}$ m. If Polaris were to burn out today, in what year would we see it disappear? (ii) How long does it take for sunlight to reach the Earth? (iii) How long does it take for a microwave radar signal to travel from the Earth to the Moon and back? (iv) How long does it take for a radio wave to travel once around the Earth in a great circle, close to the planet’s surface? (v) How long does it take for light to reach you from a lightning stroke 10 km away?

32. (i) As a result of his observations, Roemer concluded that eclipses of Io by Jupiter were delayed by 22 min during a 6 month period as the Earth moved from the point in its orbit where it is closest to Jupiter to the diametrically opposite point where it is farthest from Jupiter. Using $1.5 \times 10^8$ km as the average radius of the Earths orbit around the Sun, calculate the speed of light from these data; see Fig. 55. (ii) The Apollo 11 astronauts set up a panel of efficient corner-cube retroreflectors on the Moon’s surface. The speed of light can be found by measuring the time interval required for a laser beam to travel from Earth, reflect from the panel, and return to Earth. If this interval is measured to be 2.51 s, what is the measured speed of light? Take the center-to-center distance from Earth to Moon to be $3.84 \times 10^8$ m, and do not ignore the sizes of the Earth and Moon. The Earth radius is $R_{\oplus} = 3,959$ miles and $R_{\text{moon}} = 1,079$ miles.

33. The light beam shown in Fig. 56 makes an angle of $20.0^\circ$ with the normal line $NN'$ in the linseed oil. Determine the angles $\theta$ and $\theta'$. (The index of refraction of air is 1.00029, the one of water is 1.33, and that of linseed oil is 1.48.)

34. A (thin) converging lens of focal length 10 cm forms images of objects placed at (i) 30 cm, (ii) 10 cm, (iii) 5 cm from the lens. In each case, find the image distance and describe the image characteristics. A (thin) diverging lens of focal length 10 cm forms images of objects placed at (iv) 30 cm, (v) 10 cm, (vi) 5 cm from the lens. Repeat the calculations to find the image distance and describe the image.

35. Faraday’s law states that the induced voltage in a coil is proportional to the product of its number of
Actually, the modern view of electromagnetism states that the induced voltage in a loop when it is rotated in the constant magnetic field drives the turbine, which provides the mechanical input.

loops, the cross-sectional area of each loop, and the rate at which the magnetic field lines change within those loops. For example, when one end of a magnet is repeatedly plunged into and back out of a coil of wire, the direction of the induced voltage alternates. As the magnetic field strength inside the coil is increased (as the magnet enters the coil), the induced voltage in the coil is directed one way. When the magnetic field strength diminishes (as the magnet leaves the coil), the voltage is induced in the opposite direction. The frequency of the alternating voltage that is induced equals the frequency of the changing magnetic field within the loops. It is more practical to induce voltage by moving a coil than by moving a magnet. This can be done by rotating the coil in a stationary magnetic field, as shown in Fig. 57. This arrangement is called a generator. The voltage induced in the loop of area \( A \) when it is rotated in the magnetic field \( B \) varies with time \( t \) according to

\[
V = B A \omega \sin(\omega t) ,
\]

where \( \omega = 2\pi v \) is known as the angular frequency. Assume the circular loop of Fig. 57 (with radius of \( r = 0.25 \) meters) is rotated about the axis at a constant rate of 120 revolution per minute in a uniform magnetic field that has a magnitude of 1.3 Tesla. (i) Calculate the maximum value of the emf induced in the loop. (ii) Determine the times for which the voltage will be at the maximum. (iii) This is another example of periodic motion. What is the period?

36. (i) How many moles of copper atoms are in a copper penny, which has a mass \( \approx 3.1 \) g? (ii) How many copper atoms are in the penny? (iii) What is the mass in grams of \( 10^{15} \) (a trillion) gold atoms? (iv) How many moles of carbon atoms and oxygen atoms are in 0.25 mol of \( \text{CO}_2 \) (or carbon dioxide)? (v) If \( 5 \times 10^9 \) bbl oil are burned in the U.S./yr, and each barrel has a mass of about 300 lb, how many pounds of matter disappear/yr due to oil consumption in the U.S.?

37. Feynman pointed out that if two persons stood at arm’s length from each other and each person had 1% more electrons than protons, the force of repulsion between the two people would be enough to lift a “weight” equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.

38. If 1 gram of carbon extracted from the soot on a cave wall is 40% as radioactive as 1 gram of carbon extracted from a living tree, estimate the age of the soot. Before proceeding you must convince yourself that \( 0.513 = 0.4 \).

39. The average solar energy falling on each square meter of a roof top is about 700 watts (= 0.7 kW). If a house has 10 m\(^2\) of collector, how long would it take to bring 60 gallons of water (1 bathtub-full) to 150\(^\circ\)F from an initial temperature of 50\(^\circ\)F. Remember: 1 kW = 1kWh/hour = 3,412 Btu/hour, 1 gallon of \( \text{H}_2\text{O} \) = 8 lb, and to heat 1 lb of \( \text{H}_2\text{O} \) 1\(^\circ\)F takes 1 Btu.

40. On the 21st August 2017, a giant shadow moving west to east temporarily removed a large amount of the photovoltaic resources from the U.S. This was the first total solar eclipse to darken the skies of the country in a generation, and forced utilities to draw up contingency plans for an electric grid increasingly powered by the Sun. The previous total solar eclipse crossed the U.S. in 1979, when president Jimmy Carter bemoaned an energy crisis and renewable technology was in its infancy. The state of California lost on average about 3,400 MW of output during the event, a big chunk of the 10,000 MW of solar power that currently provides one-tenth of the states electricity; see Fig. 58. At 12c/kWh, what was the total electrical cost during the eclipse?

41. Consider a source which emits energy at a rate of \( L \) units per second (the type of source, and the units of \( L \) are actually irrelevant for this discussion). This situation is shown in the diagram of Fig. 59. Consider a sphere centered on the source, and surrounding it at a radius \( r \). If we assume the energy flows out isotropically (this means the flux is the same in all directions) from
the energy (energy per unit area) must reduce as \(1/r^2\) as the surface area of each sphere increases as \(r\) of its 10 billion year lifetime. Note that 1 Btu may be as much as the total output of our Sun during supernova delivered over the course of a few seconds in joules.

(iii) For a distant galaxy is observed to have a redshift \(Z = \frac{V}{c}\), where \(V\) is the recess. 44. Compare how much \(^{235}\text{U}\) is required to fission and how much gasoline is required to burn in order to boil a bathtub (which is approximately 280 liters of water). Consider the initial temperature of the water at \(15\,^\circ\text{C}\). Recall that in 1 gram of \(^{235}\text{U}\) there are \(N_A/235 = 2.6 \times 10^{21}\) atoms.

45. The escape velocity from Earth is \(4 \times 10^4\) km/h. What would be the percent decrease in length of a 95.2 m long spacecraft traveling at that speed?

46. At what speed do the relativistic formulas for length and time intervals differ from the classical values by 1%?

47. Space explorer Harry sets off at a steady 0.95c to a distant star. After exploring the star for a short time, he returns at the same speed and gets home after a total absence of 80 yr (as measured by earth-bound observers). How long do Harry’s clocks say he was gone, and by how much has he aged as compared to his twin Sally who stayed behind on Earth. [Note: This is the famous “twin paradox.” It is fairly easy to get the right answer by judicious insertion of a factor of \(\gamma\) in the right place, but to understand it, you need to recognize that it involves three uniformly moving reference frames: the earth-bound frame \(S\), the frame \(S’\) of the outbound rocket, and the frame \(S''\) of the returning rocket. Write down the time dilation formula for the two halves of the journey and then add. Noticed that the experiment is not symmetrical between the two twins: Sally stays at rest in the single uniformly moving frame \(S\), but Harry occupies at least two different frames. This is what allows the result to be unsymmetrical.]

48. A distant galaxy is observed to have a redshift \(V/c = 0.1\), where \(V\) is the recession velocity of the galaxy, and \(c\) is the speed of light. (i) What is the recession velocity of the galaxy in units of \(\text{km/s}\)? (ii) Using the Hubble expansion formula calculate the distance to the galaxy in units of \(\text{ly}\)? (iii) How long ago was the light we are now seeing from the galaxy emitted?

49. The electromagnetic spectrum includes a wide range of light waves, some that we cannot see. Some of the non-visible types of waves are radio waves, microwaves, infrared rays, and X-rays. In the visible spectrum of light, the color of the light depends on the fre-
frequency. We have seen in Sec. [VI] that the visible spectrum is always the same for a rainbow, or the separated light from a prism. The order of colors is red, orange, yellow, green, blue, indigo, and violet. Wien’s Law tells us that objects (such as stars) of different temperature emit spectra that peak at different wavelengths. [107]. Hotter objects emit most of their radiation at shorter wavelengths; hence they will appear to be bluer. Cooler objects emit most of their radiation at longer wavelengths; hence they will appear to be redder. The wavelength of the peak of the spectrum emitted by a star gives a measure of the temperature,

$$\lambda_{\text{peak}} = \frac{2897}{T} \text{ cm} \times K.$$  \hspace{1cm} (108)

(i) Use (68) and (100) to determine the temperature of the sun, which has a radius $R_\odot = 432,288$ miles. (ii) Using (108) convince yourself that while the sun does emit ultraviolet radiation, the majority of solar energy comes in the form of light in the visible regions of the electromagnetic spectrum, $390 \leq \lambda/nm \leq 700$, where one nanometer (nm) equals $10^{-9}$ m.

50. We have seen that the average translational kinetic energy of molecules in a gas is directly proportional to the temperature of the gas. We can invert (20) to find the average speed of molecules in a gas as a function of the temperature,

$$v_{\text{avg}} = \sqrt{\frac{3k_B T}{m}}.$$  \hspace{1cm} (109)

If the average speed of a gas is greater than about 15% to 20% of the escape speed of a planet, virtually all the molecules of that gas will escape the atmosphere of the planet. (i) At what temperature is average speed for $O_2$ equal to 15% of the escape speed of the Earth? (ii) At what temperature is the average velocity for $H_2$ equal to 15% of the escape speed for Earth? (iii) Temperatures in the upper atmosphere reach 1,000 K. How does this help account for the low abundance of hydrogen in Earth’s atmosphere? (iv) Compute the temperatures for which the average speeds of $O_2$ and $H_2$ are equal to 15% of the escape speed at the surface of the moon, where the mass of the moon is $1.6 \times 10^{27} \text{ lb}$ and its radius 1,080 miles. How this account for the absence of an atmosphere on the moon? The average mass of a hydrogen atom is $1.674 \times 10^{-24} \text{ g}$ and that of an oxygen atom is $2.657 \times 10^{-23} \text{ g}$. 

Acknowledgments

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1. The total time-of-use is 480 hours, so the average power is 625 watts.

2. (i) In 14 days a person will consume 30,800 Cal, which corresponds to about 103 oz. Now, since 1 oz = 0.0625 lb, the person will lose about 6.4 pounds. (ii) Now if the intake is 1,600 Cal, and it burns 2,200 Cal per day, it will lose 2 oz per day, or equivalently 600 Cal. So if the person loses 0.125 lb per day, to lose 15 lb it will take 120 days.

3. The energy used in driving a mile is 2,133 Cal and that used as electrical energy (per day) for an average household is 8,600 Cal.

4. To raise the temperature 6°C the astronaut will need to burn 286.8 Cal, or equivalently 0.333 kwh. If the astronaut is expending 200 watts of power, in 333/200 hr which is about 100 minutes, would reach critical damage temperatures.

5. If the cost of heating in a house is $60 per month, this means that in a month the family uses 150 gallons of oil, or equivalently 2.1 × 10^8 Btu. Now, since 1 kWh = 3,412 Btu, it is easily seen that the family uses 6,155 kWh. Therefore to heat the same house with electricity will cost $739.

6. (i) The annual cost per family is $50 × 7.8 × 10^8 × 365 × 4/(3.22 × 10^6) = $1,768. (ii) The price would be $50 × 2.2/42 + 0.184 = $2.8 per gallon. This would not change substantially the American driving habits.

7. (i) The burnout instantaneous speed = 30 m/s^2 × 100 s = 3,000 m/s. (ii) The average speed = 1,500 m/s. (iii) The height at burnout = 1,500 m/s × 100 s = 150 km. (iv) The coasting time = (3,000 m/s)/(9.8 m/s^2) = 306.12 s, and the average coasting speed = 1,500 m/s, yielding a coasting height = 1,500 m/s × 306.12 s = 459,180 m. The maximum height is then 609.18 km.

8. (i) The energy wastage per day is 32 MkWh. (ii) The added electrical cost per year is $17. (iii) The energy wastage per year is 10^{10} kWh. (iv) This corresponds to 48,348 bbl per day.

9. To calculate the force on Earth we need the distance of each planet from Earth: r_EV = (150 – 108) × 10^6 km = 4.2 × 10^{10} m, r_EH = (778 – 150) × 10^6 km = 6.28 × 10^{11} m, and r_ES = (1430 – 150) × 10^6 km = 1.28 × 10^{12} m. Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as...
positive, and so force due to the planets on the Earth is

\[ F_{Ep} = G \frac{M_\oplus M_I}{r_{Ei}^2} + G \frac{M_\oplus M_S}{r_{ES}^2} - G \frac{M_\oplus M_V}{r_{EV}^2} \]

\[ = GM_\oplus \left( \frac{318}{6.28 \times 10^{11} \text{ m}^2} + \frac{95.1}{(1.28 \times 10^{12} \text{ m})^2} - \frac{0.815}{(4.2 \times 10^{10} \text{ m})^2} \right) \]

\[ = (6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2) \times (5.97 \times 10^{24} \text{ kg})^2 \times (4.02 \times 10^{-22} \text{ m}^{-2}) = 9.56 \times 10^{17} \text{ N}. \]

The force of the Sun on Earth is as follows

\[ F_{ES} = GM_\oplus \frac{M_i}{r_{SE}^2} = (6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2) \times \frac{(5.97 \times 10^{24} \text{ kg}) \times (1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 3.52 \times 10^{22} \text{ N}. \]

Finally, the ratio is \( F_{Ep}/F_{ES} = 9.56 \times 10^{17} \text{ N}/3.52 \times 10^{22} \text{ N} = 2.71 \times 10^{-5}, \) which is 27 millionths.

10. The expression for the acceleration due to gravity at the surface of the Earth is given by [8]. For Mars, \( g_M = 0.38g \)

and so \( GM/M_r = 0.38GM_\oplus/R^2_\oplus. \) It follows that \( M_M = 0.38M_\oplus(R_M/R_\oplus)^2 = 6.4 \times 10^{23} \text{ kg}. \)

11. The pressure at the bottom of the ocean is \( P = \rho gh = 10^8 \text{ N/m}^2, \) and so the force on the window is \( F = P\pi r^2 = 5 \times 10^8 \text{ N}. \) The descent into the Challenger Deep took nearly five hours. Once the Bathyscaphe Trieste reached the sea floor, Walsh and Piccard observed their surroundings. The ship's light allowed them to see what they described as a dark brown "diatomaceous ooze" covering the sea floor, along with shrimp and some fish that appeared to resemble flounder and sole. Since the Plexiglas viewing window had cracked during the descent, the men were only able to spend about twenty minutes on the sea floor. Then, they unloaded the ballasts (nine tons of iron pellets, and tanks filled with water) and began to float back to the ocean's surface. The ascent was much quicker than the dive, taking only three hours and fifteen minutes.

12. Because the iceberg is in equilibrium, the buoyant force equals its weight, \( B = F_g, \) and so \( \rho_{ice} V_g = \rho_{sea-water} V_{sub}. \)

It follows that the fraction which is submerged is \( f = V_{sub}/V = \rho_{ice} / \rho_{sea-water} = 0.89, \) i.e. 89%.

13. From [15] we have \( v_{cap}N \pi r_{cap}^2 = v_{aorta} \pi r_{aorta}^2, \) and so \( N = v_{aorta} r_{aorta}^2 / (v_{cap} r_{cap}^2) = 7 \times 10^9. \)

14. (i) Neglecting inefficiencies (that transform energy into heat) we can consider the system to be isolated. Then, the entire potential energy of Harry before he drops goes into the potential energy of Sally rising to her peak, that is at Sally's moment of zero kinetic energy. (ii) The potential energy of Harry equals the potential energy of Sally, i.e., \( U_H = U_S. \) This means that \( m_H gh_H = m_S gh_S, \) and so \( h_S = m_H h_H/m_S. \) (iii) Since 1 lb = 0.45 kg, we have \( h_S = (70 \text{ kg} / 40 \text{ kg}) \times 4 \text{ m} = 7 \text{ m}. \)

15. The final speed is related to the total kinetic energy at the bottom of the mountain, which in turn is related to the total gravitational potential energy at the top of the hill, as the system is isolated. The potential energy Harry has at the top of the mountain is \( U_H = m_H g h_H, \) where \( m_H \) is Harry's mass. The initial potential energy of Sally is \( U_S = m_S g h_S, \) where \( m_S \) is her mass. The final kinetic energies of Harry and Sally are \( K_H = m_H v_H^2/2 \) and \( K_S = m_S v_S^2/2, \) respectively. By equating the initial potential energy to the final kinetic energy it is easily seen that both Harry and Sally will have the same speed at the bottom of the hill. Harry wins, because the bet was that she would not be going faster than him.

16. The escape velocity on Earth is the minimum velocity with which a rocket (of mass \( m \)) has to be projected vertically upwards from the earth's surface so that it just crosses the earth's gravitational field and never returns. Just after the rocket is launched, the potential energy of the system is \( U_i = -GmM_\oplus /R_\oplus \) and its kinetic energy is \( K_i = mv_i^2/2. \) When it escapes the earth's gravitational field (at an infinite height above the earth's surface) the potential energy is zero. At the critical escape velocity \( v_{esc}, \) the velocity of the spacecraft at this point drops to zero. The total energy at escape is therefore zero. By energy conservation we know that the initial total energy of the system must equal the final total energy, that is \( U_i + K_i = U_f + K_f = 0. \) Therefore, \( mv_{esc}^2/2 = GmM_\oplus /R_\oplus. \) One thing we notice is there is a single \( m \) on both sides of the equation, so they cancel. What does this tell us? The escape velocity does not depend upon the mass of the rocket: \( v_{esc}^2 = \sqrt{2GM_\oplus /R_\oplus}. \)

The escape velocity at Earth is 11.2 km/s.

17. (i) Since 1 oz = 0.062 lb, the weight loss is 0.31 lb. (ii) No, because the process is irreversible.

18. The theoretical efficiency is \( \eta_{max} = 0.85. \)
19. (i) A Btu raises the temperature of a 1 lb of water 1°F. Then 12,000 Btu will raise 120 lb of water 100°F. Since the initial temperature is 98.6°F the final temperature would be 198.6°F. (ii) Since 1 Cal = 4 Btu, it follows that if 1,600 Cal with 25% efficiency amounts to 1,600 Btu. Now, 1 Btu = 779 ft lb, and so the total high = 1,600 × 779/120 ≈ 10,387 feet ≈ 3,166 m.

20. (i) The initial momentum of the system is \( p_i = m_m v_m \), where \( m_m \) and \( v_m \) are the mass and velocity of the meteor, respectively. The final momentum of the system is \( p_f = (m_m + M_E)v_f \). Since the momentum during the collision is conserved \( p_i = p_f \) and so the recoil speed is \( v_f = 6 \times 10^{-11} \text{ m/s} \). (ii) The fraction of the meteor’s kinetic energy that was transformed to kinetic energy of the Earth is equal to \( \frac{K_{\text{Earth}}}{K_{\text{meteor}}} \approx 8 \times 10^{-15} \). (iii) The change in the Earth’s kinetic energy is \( \Delta K_{\text{Earth}} = K_{f} - K_{i} = M_E v_f^2 / 2 = 10,800 \text{ J} \).

21. (i) The frequency is \( \nu = 1.27 \text{ Hz} \). (ii) The period is \( T = 0.60 \text{ s} \). (iii) It takes 0.20 s. (iv) It takes 0.90 s.

22. The oscillating period is \( T = 6.38 \text{ s} \), so from (28) we have \( g_{\text{atm}} = \ell (2\pi / T)^2 = 0.62 \text{ m/s}^2 \).

23. The total time is the sum of the time it takes the rock to reach the bottom and the time it takes the sound wave to reach you. The average velocity of the rock is \( \langle v_{\text{rock}} \rangle = g_{\text{atm}} / 2 \), so the time it takes is \( t_{\text{rock}} = \sqrt{2H / g} = 5 \text{ s} \). The time for the sound wave to go up is \( t_{\text{sound}} = \ell / v_{\text{sound}} = 0.36 \text{ s} \), where using (30) we have taken \( v_{\text{sound}} = (331.5 + 0.610) \text{ m/s} \). The total time is 5.36 seconds.

24. Because both the source and observer are in motion, there will be two Doppler shifts: first for the emitted sound with the sub A as the source and sub B as the observer, and then the reflected sound with sub B as the source and sub A as the observer. (i) Using (38) we have

\[
\nu_{\text{target received}} = \nu_{\text{emitted}} \frac{v_{\text{sound}} + V_B}{v_{\text{sound}} - V_A} = 10.14 \text{ kHz}.
\]

(ii) Using (39) we have

\[
\nu_{\text{echo}} = \nu_{\text{target received}} \frac{(v_{\text{sound}} + V_A)}{(v_{\text{sound}} - V_B)} = \nu_{\text{emitted}} \frac{(v_{\text{sound}} + V_B)}{(v_{\text{sound}} - V_A)} = 10.28 \text{ kHz}.
\]

25. The frequency heard when you move towards a sound source at rest is \( \nu_{\text{towards received}} = \nu_{\text{emitted}} (1 + V_{\text{receiver}} / v_{\text{sound}}) \), where \( \nu_{\text{emitted}} \) is the emitted frequency, \( V_{\text{receiver}} \) is the velocity of receiver, and \( v_{\text{sound}} \) is the speed of sound. The frequency heard when moving away from the sound source is \( \nu_{\text{away received}} = \nu_{\text{emitted}} (1 - V_{\text{receiver}} / v_{\text{sound}}) \). The difference between these frequencies \( \nu_{\text{towards received}} - \nu_{\text{away received}} = 95 \text{ Hz} \) and so \( V_{\text{receiver}} = 16.97 \text{ m/s} \).

26. (i) Take the ratio of the electric force divided by the gravitational force, that is

\[
\frac{F_e}{F_g} = \frac{k_f e^2}{G m_p m_e} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times (1.602 \times 10^{-19} \text{ C})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \times 9.11 \times 10^{-31} \text{ kg} \times 1.67 \times 10^{-27} \text{ kg}} \approx 2.3 \times 10^{39}.
\]

The electric force is about \( 2.3 \times 10^{39} \) times stronger than the gravitational force for the given scenario. (ii) No. Life would be no different if electrons were positively charged and protons were negatively charged. Opposite charges would still attract, and like charges would still repel. The designation of charges as positive and negative is merely a definition.

27. (i) Power = 1.5 volts × 1 ampere = 1.5 watts = 1.5 × 10^{-3} kW. (ii) Energy = Power × time = 1.5 × 10^{-3} kW × 1 hr = 1.5 × 10^{-3} kWh. (iii) Cost/kWh = $0.5/1.5 \times 10^{-3} = $333.

28. 1,000 W = 1 kW = 1 kWh/hr = 3410 Btu/hr. At 1,400 Btu/oz we have 3,410/1,400 = 2.4 oz/hr.

29. (i) Current = power/volts = 750/150 amperes = 6.8 amperes. (ii) Energy used = power × time = 0.75 kWh × 8 hr = 6 kWh. Now, at 12¢/kWh we have 72¢.

30. \( R_3 \) and \( R_4 \) are connected in parallel with equivalent resistance given by \( R_{eq}^{-1} = R_3^{-1} + R_4^{-1} \), yielding \( R_{eq} = 0.75 \Omega \). The total resistance is \( R_{total} = R_1 + R_2 + R_{eq} = 6.75 \Omega \). The current circulating through \( R_1 \) and \( R_2 \) is \( i = V/R_{total} = 1.18 \text{ A} \). This current splits into \( i_3 \) and \( i_4 \) satisfying \( i_3 R_3 = i_4 R_4 \) and \( i = i_3 + i_4 \). Therefore, \( i_3 = R_4 / (R_3 + R_4) \approx 0.295 \) A and \( i_4 = R_3 / (R_3 + R_4) \approx 0.885 \) A. The power delivered to \( R_1 \) is \( i^2 R_1 = 2.78 \text{ watts} \), to \( R_2 \) is \( i^2 R_2 = 5.57 \text{ watts} \), to \( R_3 \) is \( i_3^2 R_3 = 0.26 \text{ watts} \), and to \( R_4 \) is \( i_4^2 R_4 = 0.78 \text{ watts} \).
31. The speed of light is \( c = 3 \times 10^8 \) m/s and since \( t = d/c \): (i) using the distance to Polaris \( d = 6.44 \times 10^{13} \) meters we obtain \( t \approx 2.15 \times 10^10 \) seconds \( \approx 5.972 \times 10^6 \) hours = 248,840 days = 682 years; (ii) the distance from the Sun to the Earth is \( d = 150 \times 10^9 \) meters and therefore \( t = 500 \) seconds = 8 minutes and 20 seconds. The distance from the Earth to the Moon is \( d = 3.84 \times 10^8 \) meters, thus \( t = 2.56 \) seconds. (iii) The Earth radius is \( R_e = 6,371 \) km, and the distance of maximum circle around the Earth is \( d = 4 \times 10^7 \) meters, so \( t = 0.13 \) seconds. (iv) For \( d = 10 \) kilometers, we have \( t = 3 \times 10^{-5} \) seconds.

32. (i) According to Roemer measurements the speed of light is \( c = 2 \times \text{distanceSE}/(22 \) minutes) = \( 2.27 \times 10^8 \) m/s. (ii) A more precise determination of the speed of light gives \( c = 2 \times \text{distanceME}/(2.51 \) seconds) = \( 2.99516 \times 10^8 \) m/s.

33. Using \[ \sin \theta = 1.48 \sin 20^\circ \] and so \( \theta = 30^\circ \). Likewise, \( 1.48 \sin 20^\circ = 1.33 \sin \theta' \), and so \( \theta' = 22^\circ \).

34. (i) The thin lens equation \[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \] can be used to find the image distance, \( \frac{1}{30 \text{ cm}} + \frac{1}{\text{image distance}} = \frac{1}{10 \text{ cm}} \Rightarrow \text{image distance} = 15 \text{ cm} \).

The positive sign for the image distance tells us that the image is indeed real and on the back side of the lens. From \[ \theta = 30^\circ \] we obtain the magnitude of the image = \( 0.5 \). Hence, the image is reduced in height by half, and the negative sign for the magnification tells us that the image is inverted. (ii) No calculation is necessary for this case because we know that, when the object is placed at the focal point, the image is formed at infinity. This is readily verified by substituting an object distance of 10 cm into the thin lens equation. (iii) We now move inside the focal point. In this case the lens acts as a magnifying glass; that is, the image is magnified, upright, on the same side of the lens as the object, and virtual. Because the object distance is 5 cm, the thin lens equation gives \[ \frac{1}{5 \text{ cm}} + \frac{1}{\text{image distance}} = \frac{1}{10 \text{ cm}} \Rightarrow \text{image distance} = -10 \text{ cm} \]

and the magnification of the image = 2. The negative image distance tells us that the image is virtual and formed on the side of the lens from which the light is incident, the front side. The image is enlarged, and the positive sign for the magnification tells us that the image is upright. (iv) We use the thin lens equation to find the image distance, \[ \frac{1}{30 \text{ cm}} + \frac{1}{\text{image distance}} = \frac{1}{10 \text{ cm}} \Rightarrow \text{image distance} = -7.5 \text{ cm} \]

The magnification of the image = 0.25. We conclude that the image is virtual, smaller than the object, and upright. (v) When the object is at the focal point we have \[ \frac{1}{10 \text{ cm}} + \frac{1}{\text{image distance}} = -\frac{1}{10 \text{ cm}} \Rightarrow \text{image distance} = -5 \text{ cm} \]

The magnification of the image = 0.5. Notice the difference between this situation and that for a converging lens. For a diverging lens, an object at the focal point does not produce an image infinitely far away. (vi) When the object is inside the focal point we have \[ \frac{1}{5 \text{ cm}} + \frac{1}{\text{image distance}} = -\frac{1}{10 \text{ cm}} \Rightarrow \text{image distance} = -3.33 \text{ cm} \]

and the magnification of the image = 0.667. In this case the virtual image is upright and shrunken.

35. (i) \( V_{\text{max}} = B \alpha \omega = B n^2 \pi \omega = 1.3 \times \pi \times (0.25 \text{ m})^2 \times 4 \times \pi \times 1 \text{ s}^{-1} = 3.2 \text{ V} \).

(ii) The maxima occur when the argument of the sine equals \( (4 \kappa + 1)/2 \), with \( \kappa = 0, 1, 2, 3, \ldots \). In other words, \( \omega t = (4 \kappa + 1)\pi/2 \) and so \( t = (4 \kappa + 1)/8 \) s.

(iii) The periodic motion has a frequency \( \nu = 2 \text{ Hz} \) and so the period is \( T = 0.5 \text{ s} \).

36. (i) Since we have 63,546 g/mol of \( ^{63}\text{Cu} \) there are 0.0488 mol. (ii) We know there are \( 6.022 \times 10^{22} \) atoms/mol, then in 0.0488 mol we have 2.94 \times 10^{22} \) atoms. (iii) Since 1 mol has 6.022 \times 10^{22} \) atoms, in 10^{12} \) atoms there are 1.66 \times 10^{12} \) mol. There are 196,967 g/mol of \( ^{197}\text{Au} \), so the mass of a trillion gold atoms is 3.27 \times 10^{-10} \) g.

(iii) Since one molecule of \( \text{CO}_2 \) contains one carbon atom, one mole of \( \text{CO}_2 \) molecules will contain one mole of carbon atoms. If we have 0.25 mol of \( \text{CO}_2 \), there will be 0.25 mol of carbon atoms present. Since there are two oxygen atoms in one \( \text{CO}_2 \) molecule, there are \( 2 \times 0.25 \) mol = 0.5 mol of oxygen atoms present in this amount.

(v) \( 1.5 \times 10^{12} \) pounds of oil are burned in the U.S. per year, from which 15,000 pounds of mass disappears.
37. If each person has a mass of about 70 kg and is (almost) composed of water, then each person contains

\[ N = \left( \frac{70,000 \text{ grams}}{18 \text{ grams/mol}} \right) \times \left( 6.023 \times 10^{23} \text{ molecules/mol} \right) \times \left( 10^{-3} \text{ protons/molecule} \right) \approx 2.3 \times 10^{28} \text{ protons}. \]

With an excess of 1% electrons over protons, each person has a charge \( q = 0.01 \times 1.602 \times 10^{-19} \text{ C} \times 2.3 \times 10^{38} \approx 3.7 \times 10^7 \text{ C}, \) and so the electric force is

\[ F_e = k_e \frac{|q|^2}{r^2} = 9 \times 10^9 \times \frac{(3.7 \times 10^7)^2}{0.6^2} N = 4 \times 10^{35} \text{ N} \approx 10^{26} \text{ N}. \]

This force is almost enough to lift a weight equal to that of the Earth \( M_{\text{Earth}} = 6 \times 10^{24} \text{ kg} \times 9.8 \text{ m/s}^2 = 6 \times 10^{25} \text{ N} \approx 10^{26} \text{ N.} \)

38. Since \( (1/2)^{1.3} = 0.4 \) it must have taken about 1.3 half-lives to get them to the present ratio. So we conclude the soot is 7,449 years old.

39. The total heat falling on a 10 m\(^2\) roof is equal to 7 kW = 23,884 Btu/hr. We want to raise the temperature of 480 lb of water by 100°F. To heat 1 lb of water 1°F takes 1 Btu. A straightforward calculation pinpoints that we need 48,000 Btu, and so the time required is about 2 hours.

40. The cost of electricity is $120/MWh. From Fig. 57 we see that the event lasted about 5 hours or 18,000 s, and so about 3,400 MW \( \times \) 18,000 s = 6.12 \times 10^{13} \text{ J} = 17,000 \text{ MWh} \) came off the system during the eclipse. This gives a cost of 17,000 MWh \( \times \$120/MWh \approx 2 \text{ million dollars}. \)

41. The flux density of neutrinos at Earth is

\[ \mathcal{F}_\nu = \frac{1.6 \times 10^{38} \text{ neutrinos/s}}{4\pi d^2} = 6 \times 10^{10} \text{ neutrinos/cm}^2 \text{s}, \]

where \( d \) is the Sun-Earth distance. Thus, the flux of neutrinos passing through the brain per second is

\[ \frac{\Delta N_\nu}{\Delta t} = \mathcal{F}_\nu A_{\text{brain}} = 6 \times 10^{10} \frac{\text{neutrinos}}{\text{cm}^2 \text{s}} \times \frac{\pi D_{\text{brain}}^2}{4} \approx 10^{13} \frac{\text{neutrinos}}{\text{s}}, \]

where we have assumed that the diameter of the brain is \( D_{\text{brain}} = 15 \text{ cm}. \)

42. (i) The number of neutrons = mass of neutron star/neutron mass = \( 2 \times M_\odot/m_n = 2 \times 2 \times 10^{33} \text{ g}/(1.67 \times 10^{24} \text{ g}) = 2.4 \times 10^{57}. \) (ii) The total energy output of the supernova = (number of neutrons) \( \times \) (energy per neutron) = \( 2.4 \times 10^{57} \times 3.037 \times 10^{-16} \text{ Btu} = 7.3 \times 10^{41} \text{ Btu} = 7.7 \times 10^{44} \text{ J}. \) (iii) From (68) we know that the rate at which the sun emits energy (its luminosity) is around \( 4 \times 10^{33} \text{ Btu/s}. \) Assuming a constant rate during the estimated lifetime of \( 10^{10} \text{ yr} \) we find that the total energy output of the sun = \( 1.3 \times 10^{44} \text{ J}. \)

43. The escape velocity from the surface (i.e., the event horizon) of a black hole is exactly \( \sqrt{2GM/\text{S}} \). Particularizing this to the black hole candidate at the center of our Galaxy we have \( v_{\text{esc}}^\text{BH} = c = \sqrt{2GM_{\text{BH}}/R_\text{S}}, \) where \( M_{\text{BH}} = 4 \times 10^6 M_\odot \) is the black hole mass and \( R_\text{S} \) is the Schwarzschild radius. The size of the black hole is characterized by \( R_\text{S} = 2GM/c^2 = 7.4 \text{ million miles}. \)

44. We have seen that a Cal raises the temperature of 1 liter of water 1°C. Then, the heat required to raise the temperature from 15°C to 100°C is \( Q = \text{(variation of temperature)} \times \text{(number of liters)} = 21,000 \text{ Cal} \approx 9 \times 10^7 \text{ J}. \) Now, since the energy released by the fission of one uranium-235 nucleus is approximately \( 3 \times 10^{-11} \text{ J} \) and there are roughly \( 2.6 \times 10^{21} \text{ atoms} \) in 1 gram of \( ^{235}\text{U}, \) the amount of energy released when 1 gram of uranium-235 undergoes fission is about \( 8 \times 10^{10} \text{ J}. \) Thus, the heat required to boil the bathtub can be obtained through fission of \( 10^{-3} \text{ grams}, \) or 1 mg (a speck) of \( ^{238}\text{U}. \) Recalling that the energy content of gasoline is about 32,000 Cal/gal = \( 1.32 \times 10^8 \text{ J/gal} \) and that 1 gallon = 3.79 liters, we have that the energy content of gasoline = \( 3.4 \times 10^7 \text{ J/l.} \) This means we need about 3 liters of gasoline. Since the mass of 1 liter of water is 1 kg, by comparison we see that burning gasoline would require \( 10^6 \) times more mass.

45. The fractional decrease in length of the spacecraft is \( \delta d' = (d' - d)/d', \) where \( d' \) is the length measured by observers relative to whom the vessel is at rest and \( d \) is the contracted length of the vessel along the direction of its motion. For a spacecraft moving at speed of \( 1.1 \times 10^3 \text{ m/s}, \) we have \( \delta d' = 1 - d/d' = 1 - \sqrt{1 - v^2/c^2} = 7 \times 10^{-10}, \) where we have used (89). This corresponds to \( 7 \times 10^{-8}\% \).
46. For a 1% change, $\sqrt{1 - v^2/c^2} = 0.99$, which gives $v = 0.14c$.

47. For $v/c = 0.95$, the Lorentz factor for both the outward and return trip is $\gamma = (1 - v^2/c^2)^{-1/2} = 3.20$. The times for the two halves of the journey satisfy $\Delta t_{\text{out}}^\gamma = \gamma \Delta t_{\text{out}}$ and $\Delta t_{\text{back}}^\gamma = \gamma \Delta t_{\text{back}}$, so by addition, the times for the whole journey satisfy the same relation. Thus $\Delta t_H = \Delta t_S/\gamma = 25 \text{ yr}$, which is the amount by which Harry has aged.

48. (i) The recession velocity is $V = 0.1c = 0.1 \times 3 \times 10^5 \text{ km/s} = 30,000 \text{ km/s}$. (ii) $d = V/H_0 = 1,339 \text{ million light years}$. (iii) The light we are seeing today was emitted 1,339 million years ago.\(^{14}\)

49. (i) The temperature at the surface of the Sun is found to be $T = [L_\odot / (4 \pi \sigma T_\odot^4)]^{1/4} = 5,914 \text{ K}$. (ii) The solar radiation has a peak wavelength $\lambda = 0.29 \text{ cm}/5,914 = 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$, which is in the visible part of the electromagnetic spectrum.

50. (i) The escape velocity from Earth is $v_{\text{esc,Earth}} = \sqrt{2GM_\odot/R_\odot} \approx 11 \text{ km/s}$ and the average speed of oxygen molecules is given by $109$. By equating $109$ to 15% of $v_{\text{esc,Earth}}$ we obtain the critical temperature,

$$\text{temperature}_{\text{crit, O}_2}^{\text{Earth}} = \frac{2}{3} \times (0.15)^2 \times G \times M_\odot \times \frac{1}{R_\odot} \times \text{average mass O}_2 \text{ molecule} \times \frac{1}{k_B} \approx 3,628 \text{K},$$

where average mass O\(_2\) molecule $= 5.314 \times 10^{-26} \text{ kg}$. (ii) Duplicating the procedure for H\(_2\) we have

$$\text{temperature}_{\text{crit, H}_2}^{\text{Earth}} = \frac{2}{3} \times (0.15)^2 \times G \times M_\odot \times \frac{1}{R_\odot} \times \text{average mass H}_2 \text{ molecule} \times \frac{1}{k_B} \approx 228 \text{K},$$

where average mass H\(_2\) molecule $= 3.348 \times 10^{-27} \text{ kg}$ (iii) At a temperature of 1,000\(^{\circ}\)K the average speed of H\(_2\) molecules is much larger than the escape velocity from Earth. (iv) The escape velocity from the Moon is $v_{\text{esc, Moon}} = \sqrt{2GM_{\text{Moon}}/R_{\text{Moon}}} \approx 2.4 \text{ km/s}$, where we have taken $M_{\text{Moon}} = 7.3 \times 10^{22} \text{ kg}$ and $R_{\text{Moon}} = 1,737 \text{ km}$. The critical temperature for oxygen is

$$\text{temperature}_{\text{crit, O}_2}^{\text{Moon}} = \frac{2}{3} \times (0.15)^2 \times G \times M_{\text{Moon}} \times \frac{1}{R_{\text{Moon}}} \times \text{average mass O}_2 \text{ molecule} \times \frac{1}{k_B} \approx 162\text{K},$$

and that of hydrogen is

$$\text{temperature}_{\text{crit, H}_2}^{\text{Moon}} = \frac{2}{3} \times (0.15)^2 \times G \times M_{\text{Moon}} \times \frac{1}{R_{\text{Moon}}} \times \text{average mass H}_2 \text{ molecule} \times \frac{1}{k_B} \approx 10^6\text{K}.$$ The mean surface temperature of the Moon is 107\(^{\circ}\)C during the day and −153\(^{\circ}\)C during night, so molecules of O\(_2\) and H\(_2\) would escape the attraction of the Moon.

\(^{14}\) Relativistic effects must be taken into account for the optical Doppler effect. Besides the ordinary Doppler effect, of order $V/c$, a relativistic correction of order $(V/c)^2$ contribute to the Doppler signal. For source and observer moving away from each other, the relativistic correction leads to

$$\lambda' = \lambda \sqrt{1 + V/c} / (1 - V/c)$$

$$\approx \lambda \left[ 1 + \frac{V}{c} + O\left( \frac{V^2}{c^2} \right) \right] \frac{1}{1 - \left( \frac{V}{c} \right) + O\left( \frac{V^2}{c^2} \right)}$$

$$\approx \lambda \left[ 1 + \frac{V}{c} + O\left( \frac{V^2}{c^2} \right) \right],$$

where $\lambda$ is the emitted wavelength as seen in a reference frame at rest with respect to the source and $\lambda'$ is the wavelength measured in a frame moving with velocity $V$ away from the source along the line of sight. Note that in the second rendition we used the binomialial expansion. For relative motion toward each other, $V < 0$ in $110$. For the problem at hand, relativistic corrections are of order $(V/c)^2 = 0.01$, and can be safely neglected.