PION–NUCLEON SCATTERING AND ISOSPIN VIOLATION

Ulf-G. Meißner
Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany
E-mail: Ulf-G.Meissner@fz-juelich.de

I discuss low–energy pion–nucleon scattering in the framework of chiral perturbation theory. I argue that using this theoretical method one is able to match the in some cases impressive experimental accuracy (for the low partial waves). I then show how strong and electromagnetic isospin violation can be treated simultaneously. Some first results for neutral pion scattering and the $\sigma$–term are given.

1 Introduction

Arguably the biggest success of chiral symmetry encoded in the current algebra of the sixties was the prediction for the isovector and isoscalar S–wave pion–nucleon scattering lengths, $\tilde{a}^- = M_\pi^2/8\pi F_\pi^2 = 8.96$, $\tilde{a}^+ = 0$, in units of $10^{-2}/M_\pi$ (which will be used throughout). Here, $M_\pi$ and $F_\pi$ are the charged pion mass and the pion decay constant, in order. The agreement with the then accepted empirical values of $\tilde{a}^- = 9 \pm 1$, $\tilde{a}^+ = 0 \pm 1$ is quite impressive.

However, while there have been many attempts to calculate the corrections to these current algebra predictions by invoking hard pion techniques, unitarization, super–propagator methods and so on, no definite conclusions on the size and even sign of these corrections could be drawn. In fact, these lowest order predictions can now be found in many textbooks on particle physics and quantum field theory (QFT) as an example how symmetries can be used to make model–independent predictions without a full understanding of the non–perturbative dynamics. For the case at hand, this symmetry is the spontaneously broken chiral symmetry of the strong interactions. The not yet solved underlying QFT is, of course, QCD. With the advent of chiral perturbation theory (CHPT), it has become possible to readdress the question concerning the corrections to the abovementioned predictions. Also, impressive progress has been made in the measurements of the level shifts and broadening in pionic hydrogen and deuterium, leading to much improved values for the zero energy scattering amplitude, alas the S–wave scattering lengths. In this talk, however, I will mostly be concerned with the theoretical developments without going into technical details, rather I will address a collection of objections, questions and misconceptions frequently encountered.
2 Pion–nucleon scattering to one loop in CHPT

CHPT is a systematic low–energy expansion for any strong interaction process. This goes along with an expansion in pion loops supplemented by all local contact interactions allowed by chiral and other symmetries. The latter terms are accompanied by the so–called low–energy constants (LECs). The general strategy is to fit these by some data and then move on to make predictions. It is often claimed that the increasing number of these LECs makes CHPT useless beyond some low order. For example, the number of LECs of the most general two flavor effective chiral pion–nucleon Lagrangian coupled to external fields (in the heavy fermion formulation) at dimension two, three and four is $7^2, 31^3$ and $\sim 160^5$ in order. A variety of calculations performed so far have shown that with some exceptions one has to go to fourth order to achieve a good theoretical precision. So let us see how many terms can contribute to $\pi N$ scattering. The pertinent $T$–matrix is most conveniently described in terms of the standard invariant amplitudes

$$T_{\pi N}^\pm = A_{\pm} + q^2 B_{\pm}, \quad (1)$$

in a highly symbolic notation, with $q^2$ the squared momentum transfer. The invariant amplitudes are functions of two variables, which one can choose to be $\nu$ and $t$. Note also that $\nu$ and $t$ count as $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$, respectively, with $p$ denoting a genuine small momentum. The most general polynom for the four amplitudes $A_{\pm}, B_{\pm}$ to fourth order commensurate with crossing and the other symmetries thus takes the form

$$A_{pol}^+ = a_1 + a_2 t + a_3 \nu^2 + a_4 t^2 + a_5 t \nu^2 + a_6 \nu^4, \quad A_{pol}^- = \nu (b_1 + b_2 t + b_3 \nu^2),$$

$$B_{pol}^+ = c_1 \nu, \quad B_{pol}^- = d_1 + d_2 t + d_3 \nu^2, \quad (2)$$

so that in total we have 14 LECs since at third order there is one more related to the Goldberger–Treiman discrepancy, i.e. a local term with a LEC which allows to express the axial–vector coupling $g_A$ in terms of the pion–nucleon coupling $g_{\pi N}$. While the former appears naturally in the effective Lagrangian, the latter is more suitable to discuss $\pi N$ scattering. So if one calculates to orders $p^2, p^3$ and $p^4$, one has to pin down 5, 9 and 14 LECs, respectively. This pattern is quite different from the total number of terms allowed at the various orders, but it is a general rule that simple processes do not involve any exorbitant number of LECs. In case of $\pi N \to \pi N$, given the large body of data, one clearly has predictive power. Some additional general remarks are in order here. First, one can only expect to get precise predictions from CHPT for

\[\text{For dimension four, the number given refers to the LECs needed for renormalization only.}\]
the low partial waves in the threshold region. This is simply related to the fact that because of angular momentum barrier factors, the higher the partial wave, the higher the power in $p$ is where it starts to have a nonvanishing contribution. Also, if one attempts to describe the partial waves with pronounced resonances, like e.g. the $P_{33}$ one dominated by the $\Delta(1232)$, one either has to stay well below the resonance energy or to include these resonances in a systematic fashion. In what follows, I will exclusively discuss the first option.

Consider now CHPT in the isospin limit ($m_u = m_d$) and in the absence of electromagnetism ($\epsilon = 0$), i.e. in the isospin symmetric world. The first calculation of the chiral corrections to the S–wave scattering lengths goes back to ref. and was sharpened in ref. Further aspects, including the one loop contributions to $O(p^3)$, were discussed in ref. The complete $\pi N$ amplitude to third order was first discussed in ref. (note that in relativistic nucleon CHPT, this amplitude was already given in ref.). The appearing nine LECs were fitted to the Karlsruhe–Helsinki threshold parameters, the pion–nucleon $\sigma$–term and the Goldberger–Treiman discrepancy. However, since there is some debate about the actual value of the $\sigma$–term and also some of the threshold parameters are not that precisely known, an alternative way was followed in
There, the two S– and six P–waves from three different partial wave solutions were fitted for pion lab momenta between 40 and 100 MeV, i.e. in the region where data exist, cf. Fig.1. In particular, this allows to predict the partial waves at lower and at higher energies. Using these three different inputs (which are still under hot debate between the various protagonists), one can get a handle on the theoretical uncertainty within the order one calculates. The predictions for the S–wave scattering lengths obtained in ref. are (notice that a kinematical factor \((1 + \mu)^{-1}\), with \(\mu\) the ratio of the pion to the nucleon mass, has been included in the definition of \(a^\pm\))

\[
8.3 \leq a^- \leq 9.3, \quad -1.0 \leq a^+ \leq 0.6,
\]

as shown by the box labelled “CHPT” in Fig.2. The theoretical uncertainty reflects entirely the variation within the order calculated, but does not account for possible effects due to higher orders. Also shown are the current algebra (CA) prediction and the empirical bands from the measurements of the level shift in pionic hydrogen (the band labelled “H”) and pionic deuterium (the band denoted “D”). The width of the 1S level in pionic hydrogen, which gives another bound on \(a^-\), has also been measured but the analysis is not yet finished due to some complications related to Doppler broadening effects.

Figure 2: S–wave scattering lengths. Shown are the results obtained from the level shift in pionic hydrogen (H–band) and pionic deuterium (D–band) compared to the current algebra (CA) and order \(p^3\) CHPT prediction.

\(^b\)Clearly, the dispersion theoretical analysis employed by the Karlsruhe–Helsinki group is the best method. One would therefore like to see such a calculation repeated using also the more modern data and an improved treatment of the electromagnetic corrections.
The present bound from the width (not shown in Fig.2) is \( a^- = 8.8 \pm 0.4 \), consistent with the CHPT prediction. The most important observation here is that to move from the CA point to the empirical region requires pion loops, i.e. the shift from the CA point to the CHPT box is largely a loop effect, as first noted in ref.7. Another constraint comes from charged pion photoproduction. The threshold value of the electric dipole amplitude \( E_{0^+}(\gamma n \rightarrow \pi^- p) \) is directly proportional to \( a^- \). Taking the dispersion theoretical result of ref.13 for \( E_{0^+} \) and the measured Panofsky ratio, \( P = \sigma(\pi^+ p \rightarrow \pi^0 n)/\sigma(\pi^- p \rightarrow \gamma n) = 1.543 \pm 0.008 \), this leads to \( a^- = 8.5 \pm 0.2 \). There has also been a direct measurement of the inverse process at TRIUMF. Its final analysis is eagerly awaited for.

Concerning the P-waves, the situation is less satisfactory, which to some extent is related to the \( \Delta \) resonance being integrated out in this approach and thus being hidden in the LECs of certain operators. Still, when we are mostly concerned with the physics encoded in the S-wave scattering lengths, this apparent deficiency vanishes since in an approach with explicit \( \Delta \)'s, one would have to worry about its mass and the \( \Delta N \pi \) coupling constant. The latter is frequently derived from the width \( \Gamma(\Delta \rightarrow N\pi) \), but a dispersive sum rule for the \( P_{33} \) partial wave, which allows to separate the resonant and non-resonant contributions, leads to a by 40\% reduced value\(^4\).

Of course, these calculations need to be improved. First, the convergence of the chiral expansion is not very rapid to third order, so one definitively has to calculate to order \( p^4 \). Second, the chiral expansion consists of even and odd powers in small momentum. It was argued in ref.15 that in fact the even and the odd series do not talk to each other, which makes a one-loop \( \mathcal{O}(p^4) \) calculation even more mandatory, since it gives the first correction to the dimension two tree graphs. The fourth order calculation is in progress\(^1\).

### 3 Isospin violation in pion–nucleon scattering

Let me start with some general remarks. Since a large body of elastic scattering and charge exchange data exists, one has the possibility of deducing bounds on isospin violation from simple triangle relations, which link e.g. the processes \( \pi^\pm p \rightarrow \pi^\pm p \) and \( \pi^- p \rightarrow \pi^0 n \). Care has, however, to be taken since there are two sources leading to isospin violation. One is the “trivial” fact that electromagnetism does not conserve I-spin, since it couples to the charge. The other one is a strong effect, linked to the difference of the light quark masses, \( m_d - m_u \). This is essentially the quantity one is after. In terms of the symmetry breaking part of the QCD Hamiltonian, i.e. the quark mass term, we have

\[
\mathcal{H}_{QCD}^{ab} = m_u \bar{u}u + m_d \bar{d}d = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) ,
\]
so that the strong $I$–spin violation is entirely due to the isovector term. This
observation lead Weinberg\textsuperscript{17} to address the question of $I$–spin violation in the
pion and the pion--nucleon sector, with the remarkable conclusion that in neu-
tral pion scattering off nucleons one should expect gross violations of this
symmetry, as large as 30%. Only recently an experimental proposal to mea-
sure the $\pi^0p$ scattering length in neutral pion photoproduction off protons
below the secondary $\pi^+n$ threshold has been presented and we are still far
away from a determination of this elusive quantity\textsuperscript{18}.

At present, there exist two phenomenological analysis\textsuperscript{19,20} which indicate isospin breaking as large as
7% in the $S$–waves (and smaller in the $P$–waves). Both of these analysis
employ approaches for the strong interactions, which allow well to fit the existing
data but can not easily be extended to the threshold or into the unphysical
region. What is, however, most disturbing is that the electromagnetic correc-
tions have been calculated using some prescriptions not necessarily consistent
with the strong interaction models used. One might therefore entertain the
possibility that some of the observed $I$–spin violation is caused by the mis-
match between the treatment of the em and strong contributions. Even if that
is not the case, both models do not offer any insight into the origin of the
strong isospin violation, but rather parametrize them. In CHPT, these prin-
ciple problems can be circumvented by constructing the most general effective
Lagrangian with pions, nucleons and virtual photons\textsuperscript{21},

\begin{equation}
\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N, \text{str}}^{(2)} + \mathcal{L}_{\pi N, \text{em}}^{(2)} + \mathcal{L}_{\pi N, \text{str}}^{(3)} + \mathcal{L}_{\pi N, \text{em}}^{(3)} + \ldots \quad (5)
\end{equation}

where the superscript gives the chiral dimension. It is important to note that
the electric charge counts as a small momentum, based on the observation that
$e^2/4\pi \sim M_{\pi}^2/(4\pi F_{\pi})^2 \sim 1/100$. Since a virtual photon can never leave a dia-
gram, the local contact terms only start at dimension two. One has four and 17
terms at second and third order, respectively. Many of these are simple renor-
amalization of masses and coupling constants and can be absorbed accordingly.

At leading order, the virtual photons modify the covariant derivative and the
axial operator $u_{\mu}$, but these contributions can only appear in loop graphs. For
the construction of this effective Lagrangian and a more detailed discussion of
the various terms, see ref.\textsuperscript{21}.

Let me just present the pertinent results of that paper here. First, Wein-
berg’s finding concerning the scattering length difference $a(\pi^0p) - a(\pi^0n)$,
which is entirely given by a dimension two term $\sim m_u - m_d$, could be con-
firm. This is not surprising because for the case of neutral pions there is no
contribution from the em Lagrangian of order two and three. To third order
there is no term, because the charge matrix has to appear quadratically and
never two additional pions can appear. This will change at fourth order. Sec-
ond, it was noted that the I–spin breaking terms in $a(\pi^0p)$ can be as large as the I–spin conserving ones,

$$a(\pi^0p) = a(\pi^0p)_{\text{str,IC}} + a(\pi^0p)_{\text{str,IV}} + a(\pi^0p)_{\text{em}} = (-0.48 - 0.11 - 0.29), \quad (6)$$

for the values of the LECs as determined in ref. It is also important to note that the em effects are entirely due to the pion mass difference, cf. Fig.3, given by the operator $C\langle QUQU^\dagger \rangle$.

Another quantity of interest is the pion–nucleon $\sigma$–term. Of course, one now has to differentiate between the one for the proton and the one for the neutron, whose values to third order differ by the strong neutron–proton mass difference,

$$\sigma_n(0) = \sigma_p(0) + 4B_0(m_u - m_d)c_5 = \sigma_p(0) + (m_n - m_p)_{\text{str}}, \quad (7)$$

with $B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$ proportional to the scalar quark condensate and $c_5 < 0$ a dimension two LEC (for more details, see e.g. [21]). For the proton and the same LECs used before, one finds

$$\sigma_p(0) = \sigma_p^{\text{IC}}(0) + \sigma_p^{\text{IV}}(0) = 47.2 \text{ MeV} - 3.9 \text{ MeV} = 43.3 \text{ MeV}, \quad (8)$$

which means that the isospin–violating terms reduce the proton $\sigma$–term by $\sim 8\%$. The electromagnetic effects are again dominating the isospin violation since the strong contribution is just half of the strong proton–neutron mass difference, 1 MeV. Furthermore, one gets $\sigma_p(2M^2_\pi) - \sigma_p(0) = 7.5 \text{ MeV}$, which differs from the result in the isospin limit (7.9 MeV) by 5% and is by about a factor two too small when compared to the dispersive analysis of ref. This small difference of 0.4 MeV is well within the uncertainties related to the so–called remainder at the Cheng–Dashen point Again, these corrections

Figure 3: Graphs contributing to isospin violation in $\pi^0$–proton scattering. Solid and dashed lines denote nucleons and pions, in order. The heavy dot and the box refer to the em counterterm at order $e^2$, i.e. the term proportional to $C$, and the dimension two strong insertion $\sim m_u - m_d$, respectively. Diagram a) has previously been considered by Weinberg.
should be considered indicative since a fourth order calculation is called for. Furthermore, the channels involving charged pions need to be investigated for the reasons discussed above. It is worth to emphasize again that we have a consistent machinery at hand to simultaneously calculate the strong and the em isospin violating effects.

4 Outlook

I have presented first steps towards a systematic analysis of electromagnetic and strong isospin breaking effects in the pion–nucleon system at low energies. To third order in small momenta, a consistent machinery exists and first results have been obtained. In ref. isospin violation in the context of charged and neutral pion photoproduction was discussed. Specifically, to third and fourth order the calculations by Bernard et al. include the pion mass difference and it was argued that this is the dominant isospin breaking effect. Indeed, to third order using the effective Lagrangian obtained in ref. it can be shown for $\gamma N \rightarrow \pi^0N$ that the only em isospin violation comes from the rescattering graph with an insertion $\sim C$ on the internal pion line (cf. fig.3b with the incoming $\pi^0$ line substituted by a photon), which is nothing but the pion mass difference. This means that the important cusp effect due to the opening of the $\pi^+n$ threshold appears already at third order, lending credit to the calculations of ref. Nevertheless, fourth order terms are important to understand the absolute magnitude of the electric dipole amplitude $E_{0+}(\gamma N \rightarrow \pi^0N)$, whereas the energy dependence of this quantity is only mildly affected by higher order terms.

Clearly, only by combining the precise machinery with the accurate data from pionic atoms and pion photoproduction we can hope to get another bound on the quark mass difference $m_u - m_d$. In a more ambitious step to follow one should consider the near threshold data for $\pi N \rightarrow \pi\pi N$, which encode (among other things) information on the elusive $\pi\pi$ scattering lengths (see e.g.). Unfortunately, now that so much progress has been made on the theoretical side, most of the meson factories are closing down. It remains to be seen whether or not the presently available data for the various processes mentioned can be understood to sufficient accuracy and whether these data themselves are accurate enough to allow to pin down such fine effects like strong isospin violation.

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