NuRIA: Numerical Relativity Injection Analysis of spinning binary black hole signals in Advanced LIGO data

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Abstract
The advent of gravitational wave (GW) astronomy has provided us with observations of black holes more massive than those known from X-ray astronomy. However, the observation of an intermediate-mass black hole (IMBH) remains a big challenge. After their second observing run, the LIGO & Virgo Scientific collaborations (LVC) placed upper limits on the coalescence rate density of non-precessing IMBH binaries (IMBHBs). In this article, we explore the sensitivity of two of the search pipelines used by the LVC to signals from 69 numerically simulated IMBHBs with generic spins, out of which 27 have a precessing orbital plane. In particular, we compare the matched-filter search PyCBC, and the coherent model-independent search technique cWB. We find that, in general, cWB is more sensitive to IMBHBs than PyCBC, with their difference depending on the masses and spins of the source. Consequently, we use cWB to place the first upper limits on the merger rate of generically spinning IMBH binaries using publicly available data from the first Advanced LIGO observing run.

Keywords: Intermediate-mass black holes — gravitational waves — precession — searches

1 INTRODUCTION

During its first two observing runs (respectively O1 and O2), the gravitational-wave detector network formed by Advanced LIGO Aasi et al. (2015) and Virgo Acernese et al. (2015, 2018) has detected the coalescence of one binary neutron star Abbott et al. (2017c)) and ten binary black holes (BBHs) Abbott et al. (2016c,d, 2017a,d,e,b, 2016a, 2019a). After entering its third observing run with much-improved sensitivity, the network is now reporting alerts for astrophysical signals on a weekly basis, and ~ O(100) detections are expected by the end of the run. Not only will these observations allow us to study the population and properties of these objects but will hopefully lead to the observation of new, currently unobserved sources, like neutron star-black hole mergers or supernovae.

In this work, we focus on yet another unobserved source: intermediate-mass black holes (IMBHs). These are usually defined as black holes (BHs) with masses in the range of \(10^2 \text{ to } 10^5 M_\odot\) and are a missing link between the stellar-mass black holes (SBHs) observed so far by GW detectors (roughly in \(18 M_\odot \text{ to } 85 M_\odot\) Abbott et al. (2019a)) and the supermassive black holes (SMBHs) with masses larger than \(10^5 M_\odot\) that are known to lay in the centres of most galaxies. Despite several indirect shreds of evidence for the existence of these objects from electromagnetic measurements, there is no conclusive direct observation. Such observation would set a milestone for astrophysics, shedding light on how a population of SBHs can transition to SMBHs through, for instance, a hierarchical merger channel Mezcua (2017); Koliopanos (2017).

Mergers of IMBHs (IMBHBs) are the loudest source for current GW detectors. Despite this, a dedicated search on O1-O2 data reported no detection of any IMBHBs and hence placed very constraining upper limits on their merger rate density Abbott et al. (2019c). In particular, the most stringent upper limit of \(0.2 Gpc^{-3} yr^{-1}\) was placed for the case of equal-mass binaries with individual masses \(m_1 = m_2 = 100 M_\odot\) and equal aligned spin parameters of \(\chi_{1z} = \chi_{2z} = 0.8\). To place this upper limit, simulated IMBHB signals were injected in the detector data and then recovered with the search algorithms. Abbott et al. (2019c), made use for the first time numerically simulated signals containing all the physics of the IMBHB systems but restricted to the systems with BH spins aligned to the orbital plane of the binary. This is in principle a sensible choice, as the effects of a precessing orbital plane may not be, in principle, observable for short-lived IMBHB signals, vastly dominated by the merger and ringdown emission. However, some studies have shown that the effect of precession can be observed in IMBHB systems Mapelli (2016); Calderon Bustillo et al. (2019a).

Hierarchical mergers of BHs in the dense globular clusters are

\[ \chi_i = \frac{c S_i}{G m_i^2} \] with \(m_i\) and \(S_i\) being respectively the masses and spins of the two-component objects.
one of the birth-places for IMBHBs. Studies have shown that in such a dense environment, BHs do not carry any preferential spin orientation Rodríguez et al. (2016). As a result, binaries formed from these BHs are expected to distribute isotropically in spin orientation resulting in spin-orbit precession.

In this paper, we evaluate the sensitivity of current search algorithms to sources with generic spins to place the first-ever upper limits on their coalescence rate then. We use two searches used by the LVC in Abbott et al. (2019c): the matched-filter algorithm for aligned-spin sources PyCBC Dal Canton et al. (2014b); Usman et al. (2016); Nitz et al. (2017) and the unmodelled time-frequency map-based algorithm, coherent WaveBurst (cWB) Klimenko et al. (2016). Consistently with previous work Calderón Bustillo et al. (2017a), we find cWB is more sensitive than PyCBC to signals from IMBHBs, which can strongly deviate from the “chirp” which PyCBC targets. Finally, we use cWB to place upper limits on the coalescence rate of a vast family of IMBHBs with generic spins. In section 5, we first compare the sensitivity of our searches and then report upper limits on a family of IMBHBs with generic spins. Finally, in section 6, we summarise our results.

2 SOURCE PROPERTIES AND WAVEFORM MORPHOLOGY

All confirmed gravitational-wave observations of BBHs show a very characteristic “chirp” morphology. This consists of a monotonic increase of both frequency and amplitude during the inspiral and merger stages of the binary, followed by a damped sinusoid with a constant frequency signal during the ringdown. While this is the most extended and studied signal, it is only true for the case of BBHs with similar component masses with constant (non-precessing) orbital planes, nearly facing the observer. The BBHs detected so far are consistent with this in terms of their parameters as well as the signal features and henceforth are referred to as vanilla BBH.

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2 As we will describe later, the PyCBC search is currently restricted to the so-called “quadrupole” or \((\ell, m) = (2, \pm 2)\) modes of aligned spin BBHs, omitting higher emission modes.
Generically, the two polarisations $h_+,\times$ of a GW from a BBH are expressed as a superposition of GW modes $h_{l,m}$ weighted by spin-2 spherical harmonics ($Y_{l,-2}^m$): Creighton & Anderson (2011); Maggiore (2008); Goldberg et al. (1967):

$$h = h_+ - i h_\times = \sum_{l \geq 2} \sum_{m=-l}^{m=+l} Y_{l,m}^2(\iota, \Phi) h_{l,m}(\Xi, D_L; t - t_c)$$

(1)

where the masses and spins of the individual BHs are collectively denoted by $\Xi$, $\iota, \Phi$ are the BBH orientation parameters, $D_L$ is the luminosity distance and $t_c$ denotes the time of coalescence.

For the case of a vanilla BBH, the above sum is largely dominated by the $(2,2)$ quadrupolar mode which is responsible for this characteristic “chirp”. However, for asymmetric high-mass sources with orbital plane inclinations away from face-on (i.e., $\iota = \pi/2$) configuration, higher modes are significant. This does not only lead to more complex waveform morphologies but can also significantly impact the signal loudness Pekowsky et al. (2013); Varma et al. (2014); Calderón Bustillo et al. (2016); Varma & Ajith (2017); Calderón Bustillo et al. (2017b); Graf et al. (2015); Calderón Bustillo et al. (2018); Calderón Bustillo et al. (2019b).

In addition, spin-induced precession introduces amplitude and phase modulations of the individual modes. Spin-precession is triggered by in-plane BH spin components $\chi_1^\parallel, \chi_2^\parallel$ and the total BBH energy is dominated by out-of-the plane spins $\chi_{1,2}^\perp$. The effect of the spins in the gravitational waveform is commonly modelled by the effective-spin precession parameter $\chi_p$ and the effective-spin parameter $\chi_{\text{eff}}$. These are expressed in terms of the component mass ratio $q = m_1/m_2 \geq 1$ Schmidt et al. (2015) as

$$\chi_p = \max \left( \chi_1^\perp, \frac{2 + 3q/2}{q^2} \chi_2^\perp \right), \quad \chi_{\text{eff}} = \frac{\chi_1^\parallel m_1 + \chi_2^\parallel m_2}{m_1 + m_2}.$$  

Fig. 1 shows that for an asymmetric and nearly edge-on system, an increase in $\chi_{\text{eff}}$ leads to a longer duration signal and hence louder signal. Also, a non-zero $\chi_p$ leads to a significant contribution from higher modes that leads to a complex full waveform. Since the PyCBC search implements template that model the dominant $(2,2)$ mode of aligned-spin sources, we expect it to be inefficient at detecting precessing/asymmetric sources as compared to the unmodelled cWB search.

3 SEARCH ALGORITHM

In this section, we describe the two search algorithms used in this work. The first one, PyCBC, is a matched-filter search that compares the incoming data to waveform templates for the quadrupole mode.
of aligned-spin BBHs. The second is a model-agnostic search for generic signals coherent across different detectors. Both the algorithms compute the significance of their signal candidates by ranking them together with the accidental background triggers according to a given ranking statistic. To estimate this background distribution, the data of one detector is time-shifted by an unphysical travel time which falls outside the physically viable time difference of the astrophysically coincident signal Abbott et al. (2016b). This process, known as time-sliding, is repeated until a sufficient amount of background statistic is generated. The ranking statistic depends on the actual search and is tailored to provide a clear separation between background triggers and the signals targeted by the search. The final product of these searches is a list of trigger candidates with an associated astrophysical significance which is given by their inverse false alarm rate (IFAR). Triggers above a given IFAR threshold are then recorded as detections.

3.1 Coherent WaveBurst

Coherent WaveBurst Klimenko et al. (2016) is a unmodelled, multi-detector, all-sky GW transient search based on wavelet transform which looks for excess power in the time-frequency domain. It targets a broad range of generic transient signals, with a minimal assumption about the underlying GW signal. An event is identified by clustering the time-frequency pixels with excess power as compared to the background noise level. Using the constrained maximum likelihood analysis method, the network correlation ($c_c$) measures the correlation of the signal between the detectors, and the detection statistics ($\eta_c$) measures the signal-to-noise ratio. The events are then ranked based on $c_c$ and $\eta_c$, which help to distinguish the real GW from the non-Gaussian noise transients. It uses a large number of noise vetos to distinguish GW transients from noisy transients (for more details refer to Appendix A in Gayathri et al. (2019)). The noise-based vetoes rely on the residual noise energy per time-frequency pixel per detector and the extent of localisation of the noisy event in the time-frequency plane. Also, the signal-based vetos are developed on the frequency evolution of the signal and the number of wavelets used for a given class of signal reconstruction Tiwari et al. (2016). The veto values are tuned for IMBHB signals based on simulations study. The cWB ranks candidate events that survived the cWB veto thresholds and are assigned a FAR value given by the rate of the corresponding background events with $\eta_c$ value more significant than that of the candidate event.

Figure 3. cWB vs PyCBC: Comparison of sensitive distance reach of cWB and PyCBC for our set of injections at IFAR=300yr. At large IFAR, the dominant effect is worsening of $\chi^2_r$ due to mismatch between injected signal and quadrupolar template. The effect of higher modes and precession on the search causes the unmodelled algorithm to outperform the matched-filter search.
3.2 The PyCBC search

The PyCBC search matched-filters the incoming detector data $d$ with precomputed waveform templates $h$. This filter is optimal when the template is a faithful representation of the GW signal present in the data. The output, known as signal-to-noise ratio, is given by

$$\rho^2 = 4 \left[ \mathbb{E} \int \frac{h(t) h(f)^*}{S_n(f)} df \right]^2,$$

where $\hat{h}(f)$ denotes the Fourier transform of $h(t)$ (For details, Owen & Sathyaprakash (1999); Wainstein & Zubakov (1970)). Coincident triggers across detectors with $\rho > 5.5$ are listed as signal candidate events and signal-template consistency vetoes are applied to these triggers to discriminate real GW signals from noisy transients of terrestrial origin known as glitches (Allen et al. (2012); Dal Canton et al. (2014a); Nitz (2018); Messick et al. (2017)). In particular, the PyCBC search implements a $\chi^2$ signal/glitch discriminator given by

$$\chi^2_r = \frac{1}{2N-1} \sum_{i=1}^{N} (\rho_i - \rho_e)^2,$$

where the index $n$ is normally set to 6. The significance of each “foreground” trigger is then estimated by comparing its $\hat{\rho}$ to the background distribution and is usually expressed in terms of inverse false alarm rate in $yr^{-1}$ units.

For this study, we consider the same configuration of PyCBC used for the LIGO-Virgo O1-O2 IMBHB. The template bank Dal Canton & Harry (2017) targets the $(2, \pm 2)$ modes of BBHs with total masses from $2M_\odot$ to $500M_\odot$, mass ratios up to 98 and restricted to spins aligned (anti-aligned) with the total angular momentum with maximum dimensionless aligned-spin parameter of 0.998. Additionally, it excludes templates shorter than 0.15s which are often mimicked by short glitches. Templates for BBHs heavier...
This figure shows the 90% upper limit on merger rate density \( R_{90\%} \) for our set of injections in \( Gpc^{-3}yr^{-1} \) in \( m_1 - m_2 \) plane. We see that with increase in total mass, mass ratio and \( \chi_p \) the rate increases.

Figure 5. This figure shows the 90% upper limit on merger rate density \( R_{90\%} \) for our set of injections in \( Gpc^{-3}yr^{-1} \) in \( m_1 - m_2 \) plane. We see that with increase in total mass, mass ratio and \( \chi_p \) the rate increases.

than \( 4M_\odot \) are computed with the reduced-order effective-one-body model SEOBNRv4ROM Bohé et al. (2017).

4 SIMULATION SETUP

4.1 Injection Set

We inject in the Advanced LIGO O1 data state-of-the-art numerically simulated signals for a large family of IMBHBs with generic spin configurations, described in Table 1. These has been computed by the Georgia Tech group (See Table 1 for a detailed list) using the Einstein Toolkit code Jani et al. (2016); Zilhao & Loffler (2013)) and are publicly available as part of the Georgia Tech Catalogue. The waveforms include the modes \( \{ (2, \pm 1), (2, \pm 2), (3, \pm 3), (4, \pm 2), (4, \pm 3), (4, \pm 4) \} \). We do not include further modes as these are usually very weak and dominated by numerical noise.

We consider IMBHB sources with total masses of \( M = 210M_\odot, 300M_\odot \) and \( 500M_\odot \). For each of these, we create injection sets uniformly distributed over the sky sphere, uniformly distributed in the BBH orientation parameters (\( \Phi, \cos \iota \)), and uniformly distributed in co-moving volume up to a redshift of \( z \approx 1 \).

4.2 Sensitive Distance Reach

We determine the sensitivity of a search to each of our sources by calculating the corresponding sensitive distance reach. To do that, we inject a set of \( N_{\text{tot}} \) injections distributed uniformly over the comoving volume \( VT_{\text{tot}} [Gpc^3 yr] \) into O1 data and recover them using our search algorithms. We consider as detections those recovered with significance equal or larger than a predetermined threshold Abbott et al. (2016e,f). Denoting by \( N_{\text{rec}} \) the number of detected injections, the corresponding sensitive volume and reach are computed as

\[
\langle VT \rangle_{\text{sen}} = \frac{N_{\text{rec}}}{N_{\text{tot}}} \langle VT \rangle_{\text{tot}} \tag{5}
\]

\[
D_{\langle VT \rangle_{\text{sen}}} = \left[ \frac{3 \langle VT \rangle_{\text{sen}}}{4\pi T_a} \right]^{1/3} \tag{6}
\]

where \( T_a \) is the total analysis time Abbott et al. (2019c). At a first stage, we will consider an injection we fix our significance threshold for injections to be considered as detections at an IFAR of 2.94 years. This choice is motivated the loudest IMBH-like trigger reported in Abbott et al. (2019c), which was then used to place upper limits on the coalescence rate of these objects. In addition, we will evaluate the sensitivity of our searches at a larger
Figure 6. cWB Zero spin vs generic spin: For $\chi_p = 0$ with an increase in $\chi_{eff}$, the sensitive distance reach increases while a decrement in $\chi_{eff}$ causes the sensitivity drops.

IFAR of 300yr, closer to that required to claim a confident detection.

We then compare the sensitivity of the two pipelines to our IMBH sources. We do this by computing the percent fractional difference in sensitive volume as

$$\Delta D_{(VT)_{sen}}[\%] = \left( \frac{D_{cWB}(VT)_{sen} - D_{PyCBC}(VT)_{sen}}{D_{cWB}(VT)_{sen}} \right) \times 100, \quad (7)$$

so that positive values indicate that cWB over-performs PyCBC and vice versa.

Finally, for a given search we can place astrophysical bounds on the merger rate density at the 90% confidence upper limit can be obtained by Biswas et al. (2009); Abbott et al. (2017e):

$$R_{90\%} = -\frac{\ln 0.1}{\langle VT \rangle_{sen}}. \quad (8)$$

5 RESULTS

In this section, we first compare the sensitivity of the two search algorithms using a fraction of O1 data (between September 12 - October 8, 2015) and find that cWB largely over-performs PyCBC in most cases. Next, we place upper limits on the coalescence rate density for a precessing set using O1 data using the results of the cWB search.
5.1 Comparing the searches

We compare the sensitivity of our searches at two reference significance thresholds given by IFARs of 2.94 and 300 yr. At low IFAR, the significance of the PyCBC triggers is mostly given by the recovered SNR so that a good separation of injections from the background is not required. Hence, subtle physical effects that cause a signal-template mismatch may not play a role in the search comparison.

Fig. 2 shows $\Delta D_{\langle VT \rangle, \text{sen}}$ at an IFAR threshold of 2.94 yr, for all the sources considered in this study, expressed in the $(m_1, m_2)$ plane, with varying $\chi_p$ and $\chi_{\text{eff}}$. For most cases, cWB out-performs PyCBC, so that $\Delta D_{\langle VT \rangle, \text{sen}} > 0$. In agreement with previous studies restricted to aligned-spins Abbott et al. (2019c), the difference between the two searches increases with an increase in total mass for fixed mass-ratio and spin parameters. This is partially due to the increasing contribution of higher-modes to the signals, not included in the PyCBC search templates. On the one hand, mismatches between injections and templates lead to a poor SNR recovery. On the other, it increases the $\chi^2_r$ statistic, making the search interpret the injections as glitches. Additionally, we note that even in the absence of higher-modes, it has been shown in the past that the $\chi^2_r$ discriminator performs poorly at separating short-duration signals from glitches Nitz (2018); Dhurandhar et al. (2017).

We note that, in a somewhat unexpected way, for bin $(M = 500, q = 2)$ PyCBC was not able to recover any injection while its performance improved for larger mass ratios as has been shown in Fig. 2(a). We attribute this to the fact that for the latter cases, the $(2, 2)$ mode of the system gets out of the sensitive band so that the PyCBC templates can effectively match the next mode remaining in the band, namely the $(3, 3)$ instead of having to match a combination of modes. A similar effect is also noticeable in Fig. 2(c).

At a larger IFAR, mismatches between injections and templates will importantly affect the sensitivity of PyCBC. As a consequence, there is an important reduction of its sensitivity toward high total mass and high mass ratio sources that have a strong higher mode contribution. Consistently, even at this IFAR 3) we find that the two pipelines have a comparable performance for low mass and low mass ratio systems, due to the that the impact of precession / higher modes on the signals being less important in these cases.

We conclude that, as expected, the signal morphology of IMBHB sources – higher mass, shorter signals and with high mass as well as mass ratio, short and complex signals – is better captured by the model agnostic cWB search than the PyCBC search. In the following, we report our results for cWB only over the entire duration of O1.
For this same reason, the cases with volumefor signal. For this reason, in Fig. 4 (b), we observe a larger sensitive

Table 1. Summary of Georgia Tech NR simulations used to model our target signals. The source parameters are defined at the starting frequency of 16 Hz.

| $\chi_p$ | $\chi_{\text{eff}}$ | $q$ | SIM ID |
|-------|-----------------|---|--------|
| 0     | 1,1.5,2         | 0 | GT0905,GT0477,GT0446 |
| 0     | 3,4.7           | 0 | GT0453,GT0454,GT0818 |
| 0.4   | 1,1.5,2         | 3 | GT0422,GT0558,GT0472 |
| 0.6   | 2, 3            | 1 | GT0596 |
| 0.3   | -0.520          | 2 | GT0437 |
| 0.3   | 0.5196          | 3 | GT0732 |
| 0.424 | 1,1.5,2         | 0 | GT0803,GT0873,GT0872 |
| 0.424 | 3,4.7           | 0 | GT0874,GT0875,GT0888 |
| 0.520 | -0.300          | 3 | GT0729 |
| 0.6   | 0.011           | 3 | GT0696 |

5.4 Impact of higher-order modes

Finally, similar to what was done in Calderón Bustillo et al. (2017a) for aligned-spin sources, we look at the impact of the inclusion/omission of the higher modes. To this, we compare the sensitivity of our search to injection sets including and omitting this effect. Fig. 7 shows the fractional increase of sensitive distance produced by the inclusion of higher-order modes in our injections. We observe that the sensitivity of the pipeline increases when the higher-modes are included in the injections, as this generally increases the available signal power. Since higher-modes have a larger impact on the case of large mass-ratio and large total-mass sources, the impact in the sensitive distance is larger in these cases. An increment as large as $\sim 57\%$ is observed for the system with $\chi_p = 0.4243, M = 500M_\odot$ and $q = 7$.

6 CONCLUSION

The detection of intermediate-mass black holes is a standing challenge in astronomy. Despite being one of the loudest sources for advanced gravitational-wave detectors, the shorter duration of the signals in the detector sensitive band and the prominent impact of higher modes and possibly precession (not captured by model-based searches) makes their detection more difficult than that of lighter binary black holes. In this situation, un-modeled searches have shown to be a promising method toward the detection of such objects Abbott et al. (2019c). In this work, for the first time, we present a comprehensive study on the ability of current gravitational-wave searches to detect generic spinning IMBHs. We focus on two searches used by the LIGO-Virgo collaborations in their recent second observing runs: the matched-filter algorithm PyCBC and model agnostic cWB. We find that at their current status, the latter currently offers a much better chance to observe IMBHs. Finally, we have placed the first ever upper limits on the coalescence rate of precessing IMBHs using data from the first Advanced LIGO observing run using the un-modelled search cWB.

We place our most stringent upper limit of $0.36/Gpc^3/yr$. This improves on the 0.94/Gpc$^3$/yr placed for aligned-spin IMBHs after the first Advanced LIGO observing run, indicating that generically spinning sources offer a better chance for the detection of BBHs in this mass range. While the latter has been pushed to 0.2/Gpc$^3$/yr after the second observing run, we expect that more constraining limits when these are computed using injections from generically spinning binaries, once the corresponding data becomes publicly available.

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