A MAP/PH/1 Production Inventory Model with Perishable Items and Dependent Retrials

K P Jose\textsuperscript{1} and P S Reshmi\textsuperscript{1}

\textsuperscript{1} P. G. \& Research Dept. of Mathematics, St.Peter’s College, Kolenchery - 682 311, Kerala, India
E-mail: kpjspc@gmail.com

Abstract. This article studies a perishable inventory system with a production unit. The production process is governed by \((s, S)\) policy and it is exponentially distributed. The primary arrival follows Markovian arrival process (MAP) and the service time is phase-type distributed random variable. The inventoried items are subject to decay exponentially with a linear rate. A newly arriving customer realizes that system is running out of stock or server busy either moves to infinite waiting space with a pre-assigned probability or exit system with complementary probability. Customers in the waiting space make retrials to access the free server at a linear rate. If the system is running out of stock or the server is busy upon retrial, customers go back to orbit with different pre-defined probabilities according to the level of inventory or exit the system with corresponding complementary probabilities. The system is analysed using Matrix Analytic Method (MAM) and the findings are numerically illustrated.

1. Introduction

Baek and Moon [1] analysed an inventory system attached to multiple server queue in which items are produced internally. The Poisson process characterizes the arrival of demand and production of items. If the system is out of stock, the unsatisfied customer wait in the system, and all new arrivals are lost. The proposed model is analysed through the regenerative process and product form solution is derived for probability vectors. Ravithammal et al. [2] dealt with a production inventory system of fixed lifetime product. The deterministic demand model considered both fixed back-orders and linear back-orders. Ioanidis et al. [3] considered a perishable inventory system with production and impatient customers. Impatient customers may balk upon arrival or renege while stock out period. The lifetime of items and patience time of customers are random variables with general distributions. Also, service time and inter-arrival time are assumed as exponential random variables. Chakraborty et al. [4] introduced a generalised economic manufacturing quantity model with a production machine which undergoes breakdown. Arbitrary probability distributions are assigned to time for breakdown, corrective, and preventive repair. Krishnamoorthy and Narayanan [5] studied a system to which arrivals according to a Poisson process and the production unit is controlled by \((s, S)\) policy. The server needs a random time to serve the inventory for demanding customers and which is exponentially distributed. If the production is on, it adds single item to inventory successively and the time required to add items is exponentially distributed. A product form solution is obtained by assuming that the system loses customers during stock out period.
Krishnamoorthy and Jose [6] distinguished three retrial inventory systems with a production unit. In all the three models, the demand process follows a Poisson distribution and exponential production time of items. The unsatisfied customer in each model join the orbit with pre-determined probability and make retrials to achieve their demands. The inter-retrial and service time are exponentially distributed. The unsatisfied retrial customers re-join the orbit in accordance with a pre-assigned value or loss system with complementary probability. Jose and Nair [7] differentiated two \((s,S)\) production inventory systems with retrial of customers. The production unit runs at two different rates per cycle according to the inventory level. That is, the rate is higher during the beginning of a cycle then it is lowered when the inventory level crosses a pre-defined level. The rate of loss of customers from the system is reduced through that assumption. All the underlying distributions assumed are exponential. Nair and Jose [8] proposed another work in which the variation of service rate is considered. The authors changed the service rate into two different rates; normal rate and reduced rate. The time for producing a single item follows an exponential distribution.

In this paper, the production inventory model proposed by Nair and Jose [8] is extended to perishable items and inventory dependent rejoining of customers. In real life situations, nearly all inventoried items are subject to decay and it leads us to the assumption of random lifetime of items. The lifetime of each item is distributed as exponential with a linear rate. Also, the model provides an orbit for retrial facility in which retrial customers rejoin the orbit depending on the inventory level. In existing models, the lifetime of items stored is assumed as infinite, and customers are allowed to orbit independent of the inventory level. We assume the modelling tools as Markovian Arrival Process and Phase type distribution for arrival process and service time distribution respectively. Thus, Matrix Analytic Method is well suited for analysing a practical situation considering non exponential inter-arrival and service time distribution. Also, the method reduces the problem of numerical intractability of the model. The paper is constructed follows: section 2 model the system mathematically and stability of the system is discussed in section 3. Section 4 and 5 deal with computation of stationary probability vector and performance measures respectively. In section 6, a profit function is constructed and results are numerically illustrated.

2. Mathematical model

\[\text{Figure 1. Representation of the system}\]
Consider an inventory system of perishable items governed by continuous review \((s, S)\) policy. If the inventory diminishes to switch on level \(s\), the production unit is switched on and it adds one by one item to the inventory. The manufacturing time of an item follows exponential distribution with rate \(\beta\) and the production stops when the inventory level attains its maximum capacity \(S\). The primary arrivals follow Markovian arrival process having representation \((D^p, D^d)\) with \(m\) phases. A single server is activated in the service area and service time is phase type distributed with representation \((\eta, T)\) having \(n\) phases. If the server busy or inventory zero upon primary arrival, customers occupy the infinite waiting space called orbit with probability \((1 - \gamma)\). The unsatisfied primary customers wait in the orbit and make retrials to access the server. If the inventory is zero upon retrial, customers reoccupy the orbit with probability \(p_0\). If the server is busy upon retrial with inventory level \(j; 1 \leq j \leq s\), then the customers reoccupy the orbit with probability \(p_j\) such that \(p_j > p_j-1(1 \leq j \leq s)\). If the inventory level is greater than switch on level \(s\) then the customers reoccupy the orbit with probability \(p\). In all cases, retrial customers exit the system with corresponding complementary probabilities. The lifetime of an item and inter-retrial times of consecutive retrials are exponentially distributed with linear rate \(j\omega\) and \(i\theta\) respectively. Assume that the item does not perish when the server is providing service on the last item left in the inventory.

- \(N_t\): Number of customers in the system at time \(t\)
- The server status, \(K_t = \begin{cases} 0, \text{server idle at time } t \\ 1, \text{server busy at time } t \end{cases}\)
- The production status, \(J_t = \begin{cases} 0, \text{production unit is off at time } t \\ 1, \text{production unit is on at time } t \end{cases}\)
- \(I_t\): Level of inventory at time \(t\)
- \(H_{1t}\): arrival phase at time \(t\)
- \(H_{2t}\): service phase at time \(t\)

Then \(\{X_t|t \geq 0\}\) is a level dependent quasi birth-death process on the state space \(E\), where

\[
X_t = (N_t, K_t, J_t, I_t, H_{1t}, H_{2t}) \quad \text{and} \quad E = (i,0,0,j) \cup (i,0,1,j) \cup (i,1,0,j) \cup (i,1,1,j)
\]

\[
(i,0,0,j) = \{i,0,0,j,k) : i \geq 0, s + 1 \leq j \leq S, 1 \leq k \leq m\},
\]

\[
(i,0,1,j) = \{(i,0,1,j,k) : i \geq 0, 0 \leq j \leq S - 1, 1 \leq k \leq m\},
\]

\[
(i,1,0,j) = \{(i,1,0,j,k,l) : i \geq 0, s + 1 \leq j \leq S, 1 \leq k \leq m, 1 \leq l \leq n\},
\]

\[
(i,1,1,j) = \{(i,1,1,j,k,l) : i \geq 0, 1 \leq j \leq S - 1, 1 \leq k \leq m, 1 \leq l \leq n\}.
\]

The generator matrix has the following structure

\[
Q = \begin{bmatrix}
B_{10} & C & & \\
A_{21} & B_{11} & C & \\
& A_{22} & B_{12} & C \\
& & A_{23} & B_{13} & C \\
& & & \ddots & \ddots & \ddots \\
& & & & \ddots & \ddots & \ddots \\
\end{bmatrix}
\]

in which each matrix has order \((2S - s)m + (2S - s - 1)mn\). The entries of \(A_{2i}, B_{1i}\) and \(C\) are transitions from the level \(i\) to \(i - 1, i\) and \(i + 1\) respectively. Transitions are given in table 1, where
The system under consideration is stable.

### 3. Stability of the system

**Theorem 3.1.** The system under consideration is stable.
Proof. For any state \( r \) in the level \( i \), \( \phi(r) = i \) be the Lyapunov test function. Then the mean drift of the state \( r \) in level \( i \geq 1 \) is,

\[
d_r = \begin{cases} 
-i\theta(1-p_0) + \gamma[D^1e_m]_k, & r = (i,0,1,0,k), 1 \leq k \leq m \\
-i\theta, & r = (i,0,0,j,k), s + 1 \leq j \leq S, \\
-i\theta, & r = (i,0,1,j,k), 1 \leq j \leq S - 1, \\
-i\theta(1-p) + [\gamma D^1e_m \otimes e_n]_{(k-1)m+l}, & r = (i,1,0,j,k,l), s + 1 \leq j \leq S, \\
-i\theta(1-p) + [\gamma D^1e_m \otimes e_n]_{(k-1)m+l}, & r = (i,1,1,j,k,l), 1 \leq j \leq S - 1, \\
-i\theta, & r = (i,0,1,0,k), 1 \leq k \leq m \\
-i\theta(1-p) + [\gamma D^1e_m \otimes e_n]_{(k-1)m+l}, & r = (i,1,0,j,k,l), s + 1 \leq j \leq S, \\
-i\theta(1-p) + [\gamma D^1e_m \otimes e_n]_{(k-1)m+l}, & r = (i,1,1,j,k,l), 1 \leq j \leq S - 1 
\end{cases}
\]

Since \((1-p_0),(1-p_j),(1-p) > 0\), for arbitrary \( \epsilon > 0 \), there exist \( N' \) such that \( d_r < -\epsilon \) for all \( r \) in level \( i \geq N' \). Now, the stability follows from Tweedi’s result \([9]\).

4. Computation of stationary probability vector

Let \( \Phi = (\varphi_0, \varphi_1, \varphi_2, ... \) be the stationary probability vector of \( Q \), where each \( \varphi_i(i \geq 0) \) has \((2S-s)m + (2S-s-1)mn\) elements:

\[
\varphi_i = (\psi_{i,0,0,0,1}, \ldots, \psi_{i,0,0,1,0}, \ldots, \psi_{i,0,1,0,1}, \ldots, \psi_{i,1,0,0,1}, \psi_{i,0,0,1,0}, \ldots, \psi_{i,1,1,1,1}) \\
\psi_{i,0,0,j} = (\psi_{i,0,0,j,1}, \ldots, \psi_{i,0,0,1,m}), s + 1 \leq j \leq S \\
\psi_{i,0,1,j} = (\psi_{i,0,1,j,1}, \ldots, \psi_{i,0,1,1,m}), 0 \leq j \leq S - 1 \\
\psi_{i,1,0,j} = (\psi_{i,1,0,j,1}, \ldots, \psi_{i,1,0,1,m}, \psi_{i,1,0,j,1}, \ldots, \psi_{i,1,0,1,m}), s + 1 \leq j \leq S; \\
\psi_{i,1,1,j} = (\psi_{i,1,1,j,1}, \ldots, \psi_{i,1,1,1,m}, \psi_{i,1,1,j,1}, \ldots, \psi_{i,1,1,1,m}), 1 \leq j \leq S - 1
\]

Matrix Analytic method is used to obtain the probability vectors. According to Neuts \([10]\), \( \Phi \) satisfies the relation

\[
\varphi_{N+k} = \varphi_{N-1}R^{k+1}, \ k \geq 0
\]

where the matrix \( R \) is the minimal non-negative solution of the equation \( R^2A_2 + RB_1 + C = 0 \) with \( B_1 = B_{1,N}, A_2 = A_{2,N}; \) \( N \) is obtained by Neuts -Rao truncation \([11]\). For \( 1 \leq i \leq N-1; \) \( \varphi_{N-1} = \varphi_{N-1}R_{N-1} \), where \( R_{N-1} = -C(B_{1,N-1} + R_{N-1}A_{2,N-1})^{-1} \). Now, \( \varphi_0 \) is obtained by solving \( \varphi_0(B_{10} + R_{1}A_{21}) = 0 \). The stationary probability vector of the truncated system is then obtained by normalizing each \( \varphi_i \) by \((\sum_i \varphi_i)e\).

5. Performance measures

a) Expected Inventory level,

\[
E_{inv} = \sum_{i=0}^{\infty} \sum_{j=s+1}^{S} \sum_{k=1}^{m} \left( \psi_{i,0,0,j,k} + \sum_{l=1}^{n} \psi_{i,1,0,j,k,l} \right) + \sum_{i=0}^{\infty} \sum_{j=1}^{m} \sum_{k=1}^{1} \left( \psi_{i,0,1,j,k} + \sum_{l=1}^{n} \psi_{i,1,1,j,k,l} \right)
\]

b) Expected perishable rate, \( E_p = \omega \times E_{inv} \)

c) Expected switching rate,

\[
E_{sw} = (s+1)\omega \sum_{i=0}^{\infty} \sum_{k=1}^{m} \left( \psi_{i,0,0,s+1,k} + \sum_{l=1}^{n} \psi_{i,1,0,s+1,k,l} \right) + \sum_{i=0}^{\infty} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,0,s+1,k,l}T_i^0
\]
d) Expected number of orbiting customers,

\[ E_{\text{orbit}} = \sum_{i=1}^{\infty} \sum_{j=s+1}^{S} \sum_{k=1}^{m} \left( \psi_{i,0,0,j,k} + \sum_{l=1}^{n} \psi_{i,1,0,j,k} \right) + \sum_{i=1}^{\infty} i \left( \sum_{j=s+1}^{S} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,0,j,k,l} + \sum_{j=1}^{S-1} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,1,j,k,l} \right) \]


e) Expected departures after service,

\[ E_{ds} = \sum_{i=0}^{\infty} \sum_{j=s+1}^{S} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,0,j,k,l} T_{l}^{0} + \sum_{i=1}^{\infty} \sum_{j=1}^{S-1} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,1,j,k,l} T_{l}^{0} \]

f) Expected number of primary loss,

\[ E_{la} = (1 - \gamma) \sum_{i=0}^{\infty} \left( \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,0,1,0,k,l} D_{kk}^{1} + \sum_{j=s+1}^{S} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,0,j,k,l} D_{kk}^{1} \right) \]

\[ + \sum_{j=1}^{S-1} \sum_{k=1}^{m} \sum_{l=1}^{n} \psi_{i,1,1,j,k,l} D_{kk}^{1} \]


g) Expected number of retrial customer loss,

\[ E_{lr} = \theta \sum_{i=1}^{\infty} i \left( \sum_{k=1}^{m} (1 - p_{0}) \psi_{i,0,1,0,k} + \sum_{j=s+1}^{S} \sum_{k=1}^{m} (1 - p) \psi_{i,1,0,j,k,l} \right) + \sum_{j=1}^{S-1} \sum_{k=1}^{m} \sum_{l=1}^{n} (1 - p) \psi_{i,1,1,j,k,l} \]

h) Overall rate of retrial, \( \theta_{1}^{*} = \theta \times E_{\text{orbit}} \)

i) Successful rate of retrial,

\[ \theta_{2}^{*} = \theta \left( \sum_{i=1}^{\infty} \sum_{j=s+1}^{S} \sum_{k=1}^{m} i \psi_{i,0,0,j,k} + \sum_{i=1}^{\infty} \sum_{k=1}^{m} i \psi_{i,0,1,j,k} \right) \]

j) Ratio of successful rate of retrial, \( \theta^{*} = \frac{\theta_{2}^{*}}{\theta_{1}^{*}} \)

6. Profit analysis

We define the total profit as

\[ E_{\text{profit}} = r_{1} E_{ds} - c_{1} E_{sr} - c_{2} E_{inv} - c_{3} E_{\text{orbit}} - c_{4} (E_{la} + E_{lr}) - c_{5} E_{p}, \]

where \( r_{1} = \text{revenue from unit purchase per unit time}, \)

\( c_{1} = \text{production unit running cost per unit per unit time}, \)

\( c_{2} = \text{inventory holding cost per unit per unit time}, \)

\( c_{3} = \text{waiting space maintenance cost per unit per unit time}, \)

\( c_{4} = \text{penalty cost of customer losing per unit per unit time}, \)

\( c_{5} = \text{disposing cost of decayed items per unit per unit time}. \)
6.1. Numerical illustrations

In this section, we investigate the effect of positive and negative correlations on overall rate of retrial, successful rate of retrial and ratio of successful retrial. The variation is compared by making the rejoin probabilities independent of the inventory level, that is, \( p_j = p \) for all \( j \) (II).

Assume that the maximum inventory level, \( S = 20 \) and the production switch on level, \( s = 5 \).

Further, the remaining parameters are chosen artificially as \( T = \begin{bmatrix} -6 & 3 \\ 1 & -4 \end{bmatrix} \), \( T_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \), \( \eta = [0.4 \ 0.6] \) and the probabilities

\[
p_0 = 0.15, p_1 = 0.25, p_2 = 0.3, p_3 = 0.36, p_4 = 0.42, p_5 = 0.57, p = 0.7 \tag{I}
\]

Again, two types of MAP are given in following table:

| D₀ | D¹ | Average arrival rate | Correlation coefficient |
|----|----|----------------------|------------------------|
| MAP(+) \[\begin{bmatrix} -4.6 & 2.5 \\ 3 & -6 \end{bmatrix}\] | \[\begin{bmatrix} 1.1 & 1 \\ 1.0 & 2 \end{bmatrix}\] | 2.52 | +0.00076 |
| MAP(−) \[\begin{bmatrix} -5.49 & 3.49 \\ 2.12 & -5.17 \end{bmatrix}\] | \[\begin{bmatrix} 1.50 & 0.5 \\ 1.95 & 1.1 \end{bmatrix}\] | 2.52 | −0.00076 |

If the rejoining of retrial customers to orbit is controlled by probabilities depending on the inventory level then it will reduce the congestion in the orbit and increases the fraction of successful retrials. As expected, table 2, 3, 4 and 5 shows that the ratio of successful rate of retrial \( \theta^* \) is greater for (I) compared to (II) having all probabilities \( p_j = p \), for all \( j \) in both MAP(+) and MAP(−). That is, rejoining of retrial customers to orbit in accordance with probabilities depending on inventory level makes retrial more successful.

Table 2. Variation of \( \theta \) on \( \theta^* \)

| \( \theta \) | MAP(+) | I | II |
|----------|--------|---|---|
| \( \theta_1^* \) | 1.9 | 1.7100 | 0.1635 | 0.0956 | 3.5819 | 0.2326 | 0.0649 |
| \( \theta_2^* \) | 2.0 | 1.7088 | 0.1612 | 0.0944 | 3.5856 | 0.2299 | 0.0641 |
| \( \theta^* \) | 2.1 | 1.7077 | 0.1590 | 0.0931 | 3.5891 | 0.2272 | 0.0633 |
| \( \theta_1^* \) | 2.2 | 1.7066 | 0.1569 | 0.0919 | 3.5926 | 0.2246 | 0.0625 |
| \( \theta_2^* \) | 2.3 | 1.7056 | 0.1548 | 0.0907 | 3.5960 | 0.2221 | 0.0618 |

| \( \theta \) | MAP(−) | I | II |
|----------|--------|---|---|
| \( \theta_1^* \) | 1.9 | 1.6996 | 0.1617 | 0.0952 | 3.5671 | 0.2309 | 0.0647 |
| \( \theta_2^* \) | 2.0 | 1.6984 | 0.1595 | 0.0939 | 3.5705 | 0.2283 | 0.0639 |
| \( \theta^* \) | 2.1 | 1.6972 | 0.1574 | 0.0927 | 3.5737 | 0.2257 | 0.0631 |
| \( \theta_1^* \) | 2.2 | 1.6961 | 0.1553 | 0.0916 | 3.5770 | 0.2231 | 0.0624 |
| \( \theta_2^* \) | 2.3 | 1.6950 | 0.1532 | 0.0904 | 3.5801 | 0.2207 | 0.0616 |

\( \beta = 1.9, \omega = 0.6, \gamma = 0.8 \).
### Table 3. Variation of $\beta$ on $\theta^*$

|       | MAP(+) |       |   | MAP(-) |       |   |
|-------|--------|-------|---|--------|-------|---|
| $\theta_1^*$ | $\theta_2^*$ | $\theta^*$ | $\theta_1^*$ | $\theta_2^*$ | $\theta^*$ |
| 1.7   | 1.7468 | 0.1307 | 0.0748 | 3.8069 | 0.1882 | 0.0494 |
| 1.8   | 1.7262 | 0.1447 | 0.0838 | 3.6949 | 0.2076 | 0.0562 |
| 1.9   | 1.7077 | 0.1590 | 0.0931 | 3.5891 | 0.2272 | 0.0633 |
| 2.0   | 1.6913 | 0.1737 | 0.1027 | 3.4895 | 0.2470 | 0.0708 |
| 2.1   | 1.6769 | 0.1886 | 0.1124 | 3.3958 | 0.2669 | 0.0786 |

$\omega = 0.6, \gamma = 0.8, \theta = 2.1$.

### Table 4. Variation of $\omega$ on $\theta^*$

|       | MAP(+) |       |   | MAP(-) |       |   |
|-------|--------|-------|---|--------|-------|---|
| $\theta_1^*$ | $\theta_2^*$ | $\theta^*$ | $\theta_1^*$ | $\theta_2^*$ | $\theta^*$ |
| 0.4   | 1.6828 | 0.2083 | 0.1238 | 3.3799 | 0.2830 | 0.0837 |
| 0.5   | 1.6936 | 0.1801 | 0.1063 | 3.4932 | 0.2517 | 0.0721 |
| 0.6   | 1.7077 | 0.1590 | 0.0931 | 3.5891 | 0.2272 | 0.0633 |
| 0.7   | 1.7228 | 0.1427 | 0.0828 | 3.6721 | 0.2075 | 0.0565 |
| 0.8   | 1.7378 | 0.1296 | 0.0746 | 3.7452 | 0.1913 | 0.0511 |

$\beta = 1.9, \gamma = 0.8, \theta = 2.1$. 

$\omega = 0.6, \gamma = 0.8, \theta = 2.1$.
6.2. Optimization of total profit

Here, a numerical optimization of the total profit is carried out using a three dimensional plot. In Figure 2, the $E_{\text{profit}}$ is plotted for various combination of $s$ and $S$. For that, fix the other parameters value as $D_0 = \begin{bmatrix} -5.49 & 3.49 \\ 2.12 & -5.17 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1.50 & 0.5 \\ 1.95 & 1 \end{bmatrix}$, $T = \begin{bmatrix} -6 & 3 \\ 1 & -4 \end{bmatrix}$, $T_0 = \begin{bmatrix} -0.4 & 0.6 \end{bmatrix}$, $\beta = 2.5$, $\omega = 0.4$, $\theta = 2$, $\gamma = 0.8$, $p = 0.8$ and different costs assumed are $r_1 = 1000$, $c_1 = 105$, $c_2 = 5$; $c_3 = 10$, $c_4 = 2$, $c_5 = 3$. Again, the probabilities $p_j; (0 \leq j \leq s)$ for various $s$ are given below:

| $s$ and corresponding probabilities |
|-------------------------------------|
| $s = 3; p_0 = 0.15, p_1 = 0.25, p_2 = 0.30, p_3 = 0.45$ |
| $s = 4; p_0 = 0.15, p_1 = 0.25, p_2 = 0.30, p_3 = 0.40, p_4 = 0.51$ |
| $s = 5; p_0 = 0.15, p_1 = 0.25, p_2 = 0.30, p_3 = 0.40, p_4 = 0.50, p_5 = 0.60$ |
| $s = 6; p_0 = 0.12, p_1 = 0.21, p_2 = 0.29, p_3 = 0.36, p_4 = 0.45, p_5 = 0.60, p_6 = 0.70$ |
| $s = 7; p_0 = 0.13, p_1 = 0.21, p_2 = 0.27, p_3 = 0.35, p_4 = 0.44, p_5 = 0.56, p_6 = 0.64, p_7 = 0.72$ |
| $s = 8; p_0 = 0.12, p_1 = 0.22, p_2 = 0.29, p_3 = 0.35, p_4 = 0.43, p_5 = 0.50, p_6 = 0.57, p_7 = 0.64, p_8 = 0.75$ |

From figure 2, the optimum value of $E_{\text{profit}} = 1675.6$ is obtained at $s = 6$, $S = 21$. Therefore the optimal $(s, S)$ pair is $(6,21)$.

**Concluding remarks**

In this model, we considered a MAP/PH/1 perishable inventory system with a production unit presided by $(s, S)$ policy. The unsatisfied primary customers move to orbit for further retrials
Figure 2. Variation of $E_{\text{profit}}$ on $(s, S)$

with pre-determined probability. The retrial customers join back to the orbit with different probabilities in accordance with items present in the inventory. The lifetime of inventoried items and inter-retrial times of customers in the orbit is characterized by independent exponential distributions in linear rate. A profit function was constructed and the results were numerically studied. One can extend this work by considering the Batch Markovian Arrival Process (BMAP) instead of MAP. Further, another way of extension is possible if the random lifetime of the item is replaced by a common lifetime.

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