Deconfined Quantum Criticality among Grand Unified Theories

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This article overviews the recent developments in applying the idea of deconfined quantum criticality in condensed matter physics to understand quantum phase transitions among grand unified theories in high energy physics in the 4-dimensional spacetime. In particular, dictated by a mod 2 class nonperturbative global mixed gauge-gravitational anomaly, there can be a gapless deconfined quantum critical region between Georgi-Glashow and Pati-Salam models — not only the Standard Model but also beyond the Standard Model phenomena (such as deconfined dark gauge force and fractionalized/fragmentary fermionic partons) emerge near the critical region. An invited contribution to Professor Chen-Ning Yang Centenary Festschrift.

I. INTRODUCTION

The Standard Model (SM) [1–4] of particle physics summarizes our current knowledge of the fundamental building blocks of our universe in the 4-dimensional spacetime (4d). It describes three generations of fermionic matter particles interacting with gauge forces and an electroweak Higgs field. In each generation, if we assume that there exists a right-handed neutrino, there are sixteen Weyl fermion matter particles, containing left- and right-handed quarks (three colors and two flavors) and leptons (two flavors), as summarized in Fig. 1(a). One of the open questions about the SM is whether the three fundamental gauge forces (electromagnetic, weak, and strong) are actually manifestations of a single all-encompassing force. This is the dream of the Grand Unified Theory (GUT). Throughout the history, many GUT models has been proposed, including the Pati-Salam (PS) model (or the so(6) \(\otimes\) so(4) GUT) [5], the Georgi-Glashow (GG) model (or the su(5) GUT) [6], and the so(10) GUT [7]. The pattern that matter particles are unified under these GUTs are illustrated in Fig. 1(b-d) respectively.

Despite the theoretical elegance of GUTs, experiments (such as proton decay [8]) have ruled out many (non-supersymmetric) GUTs as the high-energy completion of the SM, meaning that these GUTs might not be achieved only by raising up to a higher energy scale (or higher temperature scale) from the SM through thermal phase transitions, as shown in Fig. 2(a). However, experiments did not rule out the possibilities that these GUTs can be accessed by tuning model parameters away from the SM through quantum phase transitions, as shown in Fig. 2(b). From the high energy physics perspective, different GUTs are just different quantum field theories describing how gauge forces can be unified at higher energy scales. However, in condensed matter physics, different quantum field theories are often considered low-energy effective descriptions of different quantum phases of matter. From this perspective, the SM and various GUT vacua may be viewed as competing quantum phases of our universe, such that it becomes natural to discuss quantum phase transitions (and the associated quantum criticality) among different GUT vacua. By

![FIG. 1. The first generation of matter particles in (a) the Standard Model (SM), (b) the Pati-Salam (PS) model, i.e. the so(6) \(\otimes\) so(4) GUT, (c) the Georgi-Glashow (GG) model, i.e. the su(5) GUT, and (d) the so(10) GUT. Y denotes the electroweak hypercharge, and X = 5(B − L) + 3Y with (B − L) as the baryon minus lepton number.](http://example.com/fig1.png)
quantum criticality, we mean to emphasize the near zero-temperature quantum physics of gapless, scale or conformal invariant critical phenomena \([9]\). The quantum vacua (phases) tuning parameters appear perturbatively irrelevant at the SM fixed point in the renormalization group (RG) setup, hence their non-perturbative effects are largely overlooked in the previous literature. Exploring the effects of these tuning parameters and investigating quantum phase transitions among the SM and GUTs has motivated a series of works recently \([10–12]\).

The major differences between these field theories are their gauge groups. Therefore, the phase transition between GUTs (and the SM) are generally gauge-Higgs transitions, which is driven by the condensation of certain GUT Higgs field (distinct from the electroweak Higgs field in the SM) to Higgs down the gauge group. (An important remark: The present work only focuses on the GUT Higgs fields and their induced phase transitions; we do not consider the electroweak Higgs field or electroweak symmetry breaking below the electroweak energy scale within SM. So whenever Higgs fields are mentioned, they stand for the GUT Higgs fields for GUTs instead of the electroweak Higgs field within the SM.) If one conceptually suppresses the gauge fluctuation and treats the gauge group \(G\) to be a global internal symmetry in \(G\), the gauge-Higgs transitions will be closely analogous to spontaneous symmetry breaking transitions. Therefore below we will switch between the gauge group \(G\) and the global internal symmetry group \(G\) perspectives, based on the Weyl’s principle of “gauging” or “ungauging” via “summing over” or “fixing” the \(G\)-bundle with \(G\)-connections in the field theory path integral, depending on the context. Besides, in this article, we intentionally simplify the discussion of the global symmetry by focusing only on the internal symmetry \(G\) but omitting the spacetime symmetry (see the full spacetime-internal symmetry in \([10–12]\)).

The study of symmetry-breaking transitions has a long history in condensed matter physics \([13]\). The conventional symmetry breaking phases and their phase transitions are successfully described by the Landau-Ginsburg-Wilson (LGW) paradigm in terms of order parameters \([14]\). However, the condensed matter literature has also seen much discussion \([15–22]\) on exotic quantum phase transitions beyond the LGW paradigm, one notable example is the deconfined quantum criticality (DQC) \([15–17]\). The DQC occurs in a quantum system with a global symmetry \(G\) that has two spontaneous symmetry breaking phases which break \(G\) to its distinct subgroups \(G_1\) and \(G_2\) respectively. When the low-energy effective theory has a ’t Hooft anomaly of \(G\), the two symmetry breaking phases can not share a trivially gapped \(G\)-symmetric intermediate phase, resulting in a gapless critical point (or a gapless critical region as an intermediate new phase) between the two symmetry breaking phases. The theory for the quantum critical point/phase is usually not expressed in terms of order parameters in \(G/G_1\) or \(G/G_2\), but in terms of “fractionalized” degrees of freedom (such that the order parameters are composite of deconfined fractionalized particles, i.e. partons), and hence the terminology “deconfined”.

This article reviews the recent progress \([10]\) in exploring the possibility of DQC in quantum phase transitions between different GUT vacua. The study reveals the SM as a neighboring phase of two competing GUT phases (i.e. PS and GG). The competition could give rise to an intermediate DQC phase where the GUT Higgs field fractionalizes to deconfined particles and emergent gauge fields beyond the SM. Some of the fractionalized particles may persist in the SM phase, which contributes to dark matter candidates of our universe.

### II. GUT HIGGS CONDENSATION TRANSITIONS AMONG GUT PHASES

The Yang-Mills gauge theory \([23]\), or more generally the non-Abelian gauge theory, provides the basis for the theoretical formulation of the SM and GUTs. In this framework, the matter particles are excitations of Weyl fermion fields, and the gauge forces are mediated by non-Abelian gauge fields. The action of the Yang-Mills theory coupled to Weyl fermions in the 4d spacetime generally takes the form of

\[
S_{\text{YM}} = \int_{M^4} (d^4 x \, \bar{\psi} ((i \partial^\mu D_{\mu,A}) \psi) + \text{Tr}(F \wedge *F), \tag{1}
\]

where \(\psi\) denotes the Lorentz spinor Weyl fermion field, the covariant derivative \(D_{\mu,A} = \partial_\mu - i g A_\mu\) contains the Lie algebra-valued gauge field \(A_\mu\), and \(F = dA - igA \wedge A\) is the gauge curvature 2-form (given \(A = A_\mu dx^\mu\) with the Yang-Mills gauge coupling \(g\). All Weyl fermions in the theory can be taken to have the left-handed helicity, with the choice of \(\sigma^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)\), as right-handed fermions can always be conjugated to their left-handed antiparticles. Each generation of matter particles consists of sixteen or fifteen Weyl fermions. In particular, the 16th Weyl fermion is the right-handed neutrino \([24, 25]\) which is sterile to the SM gauge force and has not been observed in experiments so far. However, there is a good reason to include the sterile neutrino, as it pro-
vides the simplest way to cancel a $Z_{16}$ global anomaly of the theory [26–30]. Otherwise, with only fifteen Weyl fermions, the anomaly must be canceled in more exotic ways [31–33], either by additional 4d or 5d gapped topological quantum field theories (TQFTs) or by 4d gapless interacting conformal field theories (CFTs).

Assuming sixteen Weyl fermions in each generation, which couple to gauge fields that mediate gauge interactions, the main difference among different GUT models relies on their different choices of the gauge group $G$ (or the gauge algebra $g$), summarized as follows:

- **SM**: $G_{SM} = (SU(3) \times SU(2)_L) \times Z_6 \times U(1)_Y \times Z_2 \times U(1)_X$ (where $SU(3)$ denotes the color group, $SU(2)_L$ denotes the weak isospin group, $U(1)_Y$ denotes the electroweak hypercharge group, and $U(1)_X$ denotes the $X \equiv 5(B-L) - \frac{1}{2} Y$-charge [34] group which is a variant of baryon minus lepton $(B-L)$-like symmetry), corresponding $g_{SM} = su(3) \oplus su(2)_L \oplus u(1)_Y \oplus u(1)_X$. Note that we introduce this extra $u(1)_X$ into $g_{SM}$.

- **PS**: $G_{PS} = Spin(6) \times Z_2 \times Spin(4)$, correspondingly, $g_{PS} = so(6) \oplus so(4) = su(4) \oplus su(2)_L \oplus su(2)_R$.

- **GG**: $G_{GG} = SU(5) \times Z_{5/2} \times U(1)_X = U(5)_2$ (where the subscript 2 indicates that the $Z_5$ center of $SU(5)$ is identified with the $Z_5$ subgroup of $U(1)_X$ by a double covering, or more precisely defined by the short exact sequence $1 \to SU(5) \to U(5)_m \overset{det}{\to} U(1)/Z_{SM} \to 1$ [11]), correspondingly, $g_{GG} = su(5) \oplus u(1)_X$.

- **$so(10)$ GUT**: $G_{so(10)} = Spin(10)$, correspondingly, $g_{so(10)} = so(10)$.

The notation $G_1 \times_N G_2 \equiv \frac{G_1 \times G_2}{N}$ denotes the product of two groups $G_1$ and $G_2$ quotient their common normal subgroup $N$. One may as well treat these groups as global symmetry groups by "un-gauging" the gauge theory, i.e. by considering the dynamical gauge field as a static background probe. This allows us to understand the transitions between these models from the perspective of spontaneous symmetry breaking.

A phase with a larger symmetry group $G_{\text{large}}$ can undergo a symmetry breaking transition to a phase with a smaller symmetry group $G_{\text{small}}$, if $G_{\text{small}}$ is a subgroup of $G_{\text{large}}$, or equivalently if $G_{\text{small}}$ can be embedded in $G_{\text{large}}$ (denoted by $G_{\text{small}} \hookrightarrow G_{\text{large}}$). The group of the SM and various GUTs can be embedded as [35]

$$G_{GG} \hookrightarrow G_{so(10)}$$

$$G_{SM} \hookrightarrow G_{PS}.$$  \hfill (2)

The embedding structure indicates that the $so(10)$ GUT can go through symmetry-breaking transitions from the $so(10)$ GUT phase to the adjacent PS or GG phases. The PS and GG phases can further break symmetry to their common adjacent SM phase. The Lie group embedding structure in Eq. (2) also applies to their corresponding Lie algebras, as illustrated in Fig. 3. It turns out that $g_{SM}$ is the unique common subalgebra of $g_{PS}$ and $g_{GG}$, while $g_{so(10)}$ is the minimal algebra that contains both $g_{PS}$ and $g_{GG}$:

$$g_{PS} \cap g_{GG} = g_{SM},$$

$$g_{PS} \cup g_{GG} = g_{so(10)}. \hfill (3)$$

More than just Lie algebra intersection/union, there are also corresponding statements in Lie groups [10, 11, 35]. The stringent relations between these symmetry groups (or algebras) suggest that there might be a unifying theory that describes the phase transitions among the SM and different GUTs.

It is worth mentioning that the subalgebra $u(1)_Y \oplus u(1)_X \equiv u(1)_{3,R} \oplus u(1)_{(B-L)}$ inside $g_{SM}$ is shared between $g_{PS}$ and $g_{GG}$, but it can have different choices of generator basis. Let $Y \in Z$ be the weak hypercharge (at triple-scale), $X \in Z$ be the $X$-charge, $Q_{3,R} = 2I_{3,R} \in Z$ be the right-isospin (at double-scale), $(B-L) \in Z/3$ be the baryon minus lepton number. They are related by the linear relation

$$ \begin{bmatrix} Y \\ X \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} Q_{3,R} \\ (B-L) \end{bmatrix}. \hfill (4)$$

The electromagnetic $u(1)_{em}$ charge is given by $Q_{em} = I_{3,L} + \frac{1}{2} Y = I_{3,L} + I_{3,R} + \frac{1}{2} (B-L)$, where $I_{3,L} \in Z/2$ is the left-isospin (the weak isospin).

Within the Landau-Ginzburg-Wilson paradigm, the symmetry-breaking transition from a larger symmetry group $G_{\text{large}}$ (or its algebra $g_{\text{large}}$) to a smaller symmetry group $G_{\text{small}}$ (or its algebra $g_{\text{small}}$) can be formulated as the condensation of the order parameter field, whose target manifold is $G_{\text{large}}/G_{\text{small}}$. If the symmetries are dynamically gauged, the symmetry-breaking transition can be reinterpreted as the Higgs transition, in which the order parameter field is promoted to the Higgs field.
The Higgs field that drives the transition between GUT (and SM) phases will be called the GUT Higgs fields to distinguish from the electroweak Higgs field in the SM. In order for the Higgs condensation to break \( g_{\text{large}} \) but not \( g_{\text{small}} \), it must be in a non-trivial representation of \( g_{\text{large}} \) but in the trivial representation of \( g_{\text{small}} \). One can look up such cases from Lie algebra branching rules.

For example, to break \( g_{\text{SO}(10)} = \mathfrak{so}(10) \) to \( g_{\text{PS}} = \mathfrak{su}(4) \oplus \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \), the branching rule \( 54 \rightarrow (1, 1, 1) + \cdots \) suggests that one could condense a GUT Higgs field \( \Phi_{54} \) in the \( 54 \) representation of \( g_{\text{SO}(10)} \) to the \((1, 1, 1) \) representation of \( g_{\text{PS}} \) to achieve the desired symmetry breaking. Similarly, to break \( g_{\text{SU}(5)} = \mathfrak{so}(10) \) to \( g_{\text{SU}(3)} \oplus \mathfrak{u}(1)_X \), the branching rule \( 45 \rightarrow (1, 0) + \cdots \) suggests that one could condense a GUT Higgs field \( \Phi_{45} \) in the \( 45 \) representation of \( g_{\text{SU}(5)} \) to the \( (1, 0) \) representation of \( g_{\text{SU}(3)} \) to achieve the desired symmetry breaking. Furthermore, if both GUT Higgs fields \( \Phi_{45} \) and \( \Phi_{54} \) are condensed, the internal symmetry will be broken down to \( g_{\text{SM}} \), as \( g_{\text{SM}} \) is the intersection of \( g_{\text{PS}} \) and \( g_{\text{GG}} \) that survives both symmetry breaking. More concretely, starting from the PS phase where the GUT Higgs field \( \Phi_{54} \) is already condensed, to further break \( g_{\text{PS}} \) to \( g_{\text{SM}} = \mathfrak{su}(3) \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_X \), the branching rules \((1, 1, 3) \rightarrow (1, 1, 0) + \cdots \) and \((15, 1, 1) \rightarrow (1, 1)_{0,0} + \cdots \) in the \( 45 \) sector indicates that the GUT Higgs field \( \Phi_{45} \) needs to be condensed as well (more precisely, a certain combination of \((1, 1, 1) \) and \((15, 1, 1) \) should be condensed, whose specific form will be discussed later). On the other hand, starting from the GG phase where the GUT Higgs field \( \Phi_{45} \) is already condensed, to further break \( g_{\text{GG}} \) to \( g_{\text{SM}} \), the branching rule \( 24 \rightarrow (1, 1)_{0,0} + \cdots \) in the \( 54 \) sector indicates that the GUT Higgs field \( \Phi_{54} \) needs to be condensed as well. Both analyses point to the same conclusion that the SM phase can be achieved from the \( g_{\text{SO}(10)} \) GUT phase by simultaneously condensing the two GUT Higgs fields \( \Phi_{45} \) and \( \Phi_{54} \).

To connect different GUT phases, one can start with the largest internal symmetry (or gauge) group \( G_{\text{SO}(10)} = \text{Spin}(10) \) and access lower-symmetry phases by Higgs condensation. The Higgs condensation can be described by

\[
S_H = \int_{M^4} d^4x \left( \sum_{R} (D_{\mu,A} \Phi_R)^2 + V(\Phi_R) \right),
\]

where \( \Phi_R \) is the Lorentz scalar GUT Higgs field in the \( R \) representation of \( \mathfrak{so}(10) \), and the Higgs potential takes the form of \( V(\Phi_R) = r_R \Phi_R^2 + \lambda_R \Phi_R^4 + \cdots \). The \( \Phi^4 \) coefficient \( \lambda_R > 0 \) is always assumed to be positive. The \( \Phi^2 \) coefficients \( r_R \) is the tuning parameter that drives the transition between GUT phases. At the mean-field level, when \( r_R < 0 \), the corresponding Higgs field \( \Phi_R \) will condense, i.e., \( \langle \Phi_R \rangle \neq 0 \). Based on the above discussion, a schematic phase diagram can be concluded as in Fig. 4 (on the \( r_1 > 0 \) side).

The GUT Higgs field in both the \( 45 \) and \( 54 \) representation motivates us to include an additional GUT Higgs field \( \Phi_{1} \) in the \( 1 \) (trivial) representation, such that \( 1 + 45 + 54 \) can be unified as the \( \mathfrak{so}(10) \) bivector representation \( 10 \times 10 \), while the \( \mathfrak{so}(10) \) vector representation \( 10 \) can be further composed by the product of spinor representations as \( 16 \times 16 \rightarrow 10 + 120 + T_{26} \). This enables us to write down the most natural Yukawa-like coupling between the GUT Higgs field and the matter fermion field (in the \( 16 \) representation),

\[
S_Y = \int_{M^4} d^4x \frac{1}{2} \left( \phi^T \dot{\Phi} \phi + \sum_{a=1}^{5} \left( \psi^T i \sigma^2 (\phi_{2a-1}\Gamma_{2a-1} - i\phi_{2a}\Gamma_{2a}) \psi + h.c. \right) \right).
\]

Here the GUT Higgs field \( \Phi \) is written in the \( \mathfrak{so}(10) \) bivector representation as a \( 10 \times 10 \) matrix field (with totally 100 real scalar components). The \( \mathfrak{so}(10) \) vector field \( \phi \) has 10 real scalar components, which corresponds to the 10 orthogonal Majorana masses (pairing terms) among 16 Weyl fermions. \( \Gamma_a \) (for \( a = 1, 2, \cdots, 10 \)) are ten rank-16 real symmetric matrices satisfying \( \{ \Gamma_{2a-1}, \Gamma_{2b-1} \} = 2\delta_{ab} \), \( \{ \Gamma_{2a}, \Gamma_{2b} \} = 2i\delta_{ab} \), \( \{ \Gamma_{2a-1}, \Gamma_{2b} \} = 0 \) (for \( a, b = 1, 2, \cdots, 5 \)). In Eq. (6), \( i \sigma^2 \) acts in the 2-dimensional spacetime spinor subspace and \( \Gamma_a \) acts in the 16-dimensional flavor subspace. Unlike the electroweak Higgs field in the SM, which gives quadratic mass to the matter fermions, the GUT Higgs field \( \Phi \) does not generate fermion-bilinear mass, but rather serves as a four-fermion interaction. This can be seen by treating the \( \mathfrak{so}(10) \) vector field \( \phi \) as a Lagrangian multiplier. Integrating out \( \phi \) will lead to the quartic coupling of the form \( \Phi \psi \psi \psi \psi \). As four-fermion interactions are perturbatively irrelevant for 4d Weyl fermions, perturbative vacuum expectation value of the GUT Higgs field (i.e., \( \langle \Phi \rangle \neq 0 \) but small) will not affect the matter fermion spectrum.

As an \( \mathfrak{so}(10) \) bivector representation \( 10 \times 10 \), the GUT Higgs fields \( \Phi \) can be decomposed into \( 1 + 45 + 54 \) sectors,
which defines
\[
\Phi_1 : \text{Tr} \Phi = \sum_a \Phi_{a\alpha}, \\
\Phi_{45} : \Phi_{[a,b]} = \frac{1}{2} (\Phi_{ab} - \Phi_{ba}), \\
\Phi_{54} : \Phi_{(a,b)} = \frac{1}{2} (\Phi_{ab} + \Phi_{ba}).
\] (7)

Starting from the so(10) GUT, the condensation of \(\Phi_{45}\) or \(\Phi_{54}\) will drive symmetry breaking transitions to various lower-symmetry phases, as summarized in Fig. 4. In particular, if \(\Phi\) condenses to a specific configuration \((\Phi)\), the original algebra \(\mathfrak{g}_{\text{large}}\) will be broken to its subalgebra \(\mathfrak{g}_{\text{small}} = \{x \in \mathfrak{g}_{\text{large}} | (x, \langle \Phi \rangle) = 0\}\). It can be verified that the following condensation configurations
\[
\langle \Phi_{54} \rangle \propto \begin{bmatrix} -3 & -3 \\ 2 & 2 \end{bmatrix} \otimes [1, 1], \\
\langle \Phi_{45} \rangle \propto \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \otimes [0, 1],
\] (8)
induce the following symmetry breaking
\[
\mathfrak{g}_{\text{ps}} \rightarrow \mathfrak{g}_{\text{so}(10)} \rightarrow \mathfrak{g}_{\text{gg}} \rightarrow \mathfrak{g}_{\text{sm}} (\Phi_{54} \neq 0)
\] (9)

\(\langle \Phi_{54} \rangle\) explicitly distinguishes the first four-dimensional subspace from the last six-dimensional subspace of the \(\text{so}(10)\) vector, therefore breaking \(\mathfrak{g}_{\text{so}(10)} = \text{so}(10)\) down to \(\mathfrak{g}_{\text{ps}} = \text{so}(6) \oplus \text{so}(4)\). \(\langle \Phi_{45} \rangle\) is proportional to the \(\text{u}(1)_X\) generator, which effectively requires the unbroken generators to commute with \(\text{u}(1)_X\) generator, which singles out \(\mathfrak{g}_{\text{gg}} = \text{su}(5) \oplus \text{u}(1)_X\). When both of them condense, the symmetry is broken to \(\mathfrak{g}_{\text{ps}} \cap \mathfrak{g}_{\text{gg}} = \mathfrak{g}_{\text{sm}}\).

The condensation of \(\Phi_1\) in the trivial sector will not break the \(G_{\text{so}(10)}\) symmetry, and a perturbative vacuum expectation value of \(\langle \Phi_1 \rangle < \Phi_{1,c}\) below a critical value also has no effect on the fermion spectrum, so the theory remains in the \(\text{so}(10)\) GUT phase. However, when the condensation \(\langle \Phi_1 \rangle > \Phi_{1,c}\) exceeds a critical value, the quartic interaction induced by \(\langle \Phi_1 \rangle\) can have non-perturbative effects: it can generate an interacting gap for all fermions without breaking the \(G_{\text{so}(10)}\) symmetry. The mechanism is known as the symmetric mass generation (SMG), and has been explored in various situations, as some selective examples, for fermion zero modes in 1d [36], for chiral fermions in 2d [37, 38], for Dirac fermions in 3d [39, 40], and most notable for Weyl fermions in 4d [26–28, 41–46]. To consider the possibility of SMG, one can investigate the strongly interacting limit of \(\langle \Phi_1 \rangle \rightarrow \infty\), where the matter fermion dynamics is dominated by the interaction \(S_Y\) in Eq. (6). In this limit, the field theory decouples in each spatial location and can be solved exactly. The solution shows that the many-body ground state at each location is unique with a finite excitation gap [27], indicating all fermions are gapped by the interaction without spontaneous symmetry breaking.

III. DECONFINED QUANTUM CRITICALITY

The phase diagram in Fig. 4 resembles the phase diagram of deconfined quantum criticality in \((2+1)\)-D quantum magnet [15, 16, 22], where two distinct symmetry breaking phases – the antiferromagnetic Néel phase and the valence bond solid (VBS) phase – are connected by a direct continuous quantum phase transition [19]. The Néel phase spontaneously breaks the spin \(\text{SO}(3)\) rotation symmetry and the VBS phase spontaneously breaks the lattice rotation \(\mathbb{Z}_4\) symmetry (or \(\text{SO}(2)\) symmetry in the continuum). The direct transition between these two quantum phases is exotic, which involves fractionalized excitations and emergent gauge fields. The deconfinement of the fractionalized excitations happens at and only at the Néel-VBS critical point, therefore the critical point is also called the deconfined critical point (DQCP). In the phase diagram Fig. 4, the PS and GG phases also breaks the \(G_{\text{so}(10)} = \text{Spin}(10)\) symmetry differently, which motivates the investigation of the possible DQCP physics along the phase boundary between the PS and GG phases.

The key mechanism of DQCP is that the topological defect of one order parameter carries the charge of the other order parameter. In this case, the two order parameters are said to be intertwined orders. For example, the vortex of VBS order parameter carries a spin-1/2 degree of freedom that is charged under the spin \(\text{SO}(3)\) symmetry in a fractionalized and projective representation [18]. To explore similar scenarios in the GUTs, one should first understand the possible topological defects of the order parameters in the PS and GG phases. When the GUT Higgs fields \(\Phi_{45}\) and \(\Phi_{54}\) condense to a finite amplitude, their orientational fluctuations belongs to the following target manifolds
\[
\Phi_{45} \in \text{SO}(10)/\text{U}(5), \quad \Phi_{54} \in \text{SO}(10)/\text{SO}(6) \times \text{SO}(4). \] (10)

These manifolds have non-trivial second homotopy groups
\[
\pi_2(\text{SO}(10)/\text{U}(5)) = \mathbb{Z}, \quad \pi_2(\text{SO}(10)/\text{SO}(6) \times \text{SO}(4)) = \mathbb{Z}_2, \] (11)
which indicates that the \(\Phi_{45}\) order parameter admits \(\mathbb{Z}\)-classified topological point defects in the 3D space, and the \(\Phi_{54}\) order parameter admits \(\mathbb{Z}_2\)-classified topological point defects in the 3D space. The intertwinement of these two symmetry-breaking orders is equivalently characterized by the non-trivial mutual braiding statistics of their topological defects, i.e. the \(\Phi_{45}\) defect and the \(\Phi_{54}\) defect will see each other as a monopole emitting totally \(2\pi\) statistical Berry phase flux.
The braiding statistics of topological defects can be more precisely defined by extending the 4d spacetime to 5d with an auxiliary dimension. As shown in Fig. 5, the topological defect world-lines in the 4d spacetime will extended to pseudo world-sheets in 5d. The defect braiding statistics corresponds to the effect that each pair of linked 2-spheres of the world-sheets of two different types of defects will contribute a \((-1)\) sign to the partition function weight [47, 48]. On the field theory level, this effect can be described by a Wess-Zumino-Witten (WZW) term, defined in the 5d extension,

\[
e^{iS_{\text{WZW}}[\Phi]} = \exp \left( i \pi \int_{M_5} B(\Phi_{54}) - \delta B(\Phi_{45}) \right) \bigg|_{M_5 = \partial M_5},
\]

where \(B\) and \(B'\) are two-cocycles also two-cohomology class of the second cohomology group that evaluate the defect topological number when they are integrated over a 2-sphere enclosing the point defect in the 3d space,

\[
B \in H^2\left( \frac{\text{SO}(10)}{\text{SO}(6) \times \text{SO}(4)}, \mathbb{Z}_2 \right) = \text{Hom}\left( \pi_2\left( \frac{\text{SO}(10)}{\text{SO}(6) \times \text{SO}(4)} \right), \mathbb{Z}_2 \right),
\]

\[
B' \in H^2\left( \frac{\text{SO}(10)}{\text{U}(5)}, \mathbb{Z} \right) = \text{Hom}\left( \pi_2\left( \frac{\text{SO}(10)}{\text{U}(5)} \right), \mathbb{Z} \right).
\]

Here we use the fact that if \(K\) is a \((n - 1)\)-connected topological space, then \(H^n(K, A) = \text{Hom}(\pi_n(K), A)\) for some abelian \(A\), by the Hurewicz theorem and universal coefficient theorem.

With the additional WZW term, the field theory (which has the spacetime-internal symmetry \(\text{Spin} \times_{\mathbb{Z}_2} \text{Spin}(10)\) structure) takes the form of

\[
S = S_{\text{YM}}[\psi, A] + S_{\text{H}}[\Phi, A] + S_Y[\psi, \Phi] + iS_{\text{WZW}}[\Phi].
\]

This field theory has a \(w_2 w_3\) \(\mathbb{Z}_2\)-class mixed gauge-gravitational global anomaly [49–51] captured by a 5d bulk invertible topological quantum field theory (invertible TQFT)

\[
\exp \left( i \pi \int_{M_5} w_2(V_{\text{SO}(10)}) w_3(V_{\text{SO}(10)}) \right) = \exp \left( i \pi \int_{M_5} w_2(TM) w_3(TM) \right),
\]

where \(w_2, w_3\) denotes the Stiefel-Whitney classes of either the associated \(V_{\text{SO}(10)}\) vector bundle or the \(TM\) tangent bundle. This anomaly originated purely from the GUT Higgs sector \(\Phi\). The matter fermions \(\psi\) does not contribute to it. In the presence of this anomaly, the GUT Higgs field should be viewed as the 4d boundary modes of a 5d invertible TQFT. Due to the anomaly, the \(\text{Spin}(10)\) symmetry cannot be dynamically gauged on the 4d boundary alone. The \(\text{Spin}(10)\) gauge field must propagates into the 5d bulk.

The \(w_2 w_3\) anomaly only exists in the presence of the \(G_{\text{GUT}} = \text{Spin}(10)\) symmetry. It is lifted when the \(\text{Spin}(10)\) symmetry is broken to any subgroups of interest: \(G_{\text{PS}}, G_{\text{GG}}\) or \(G_{\text{SM}}\). Thus condensing either \(\Phi_{45}\) or \(\Phi_{54}\) GUT Higgs fields will break the \(\text{Spin}(10)\) symmetry and cancels the \(w_2 w_3\) anomaly. The resulting PS, GG, and SM phases are just ordinary spontaneous symmetry breaking phases (or Higgs phases if symmetries are gauged). What becomes non-trivial is the \(\text{Spin}(10)\) symmetric \(\text{SO}(10)\) GUT phase. Due to the anomaly introduced by the WZW term, the GUT Higgs fields can not be trivially disordered (namely, trivially and featurelessly gapped without ground state degeneracy associated with topological order or symmetry breaking order). One possibility is that the GUT Higgs field enters a “spin liquid” fractionalized phase when the \(\text{Spin}(10)\) symmetry is restored, which happens either at the PS-GG transition point or in the \(\text{SO}(10)\) GUT phase.

![FIG. 5. Illustration of defect world-sheets in extended 5d spacetime. The two colors indicate the \(\Phi_{45}\) or the \(\Phi_{54}\) topological defects. Each pair of mutually linked 2-spheres of the two types of topological defects will contribute a \((-1)\) sign to the path integral weight.](image)

![FIG. 6. Phase diagram in the presence of the WZW term. The dashed circle denotes the confine-deconfinement transition of the emergent \(U(1)\) gauge field. The solid-line phase boundaries between two neighbor phases all are described by Higgs-condensation continuous quantum phase transitions.](image)
field theory. The WZW term can be replaced by quantum electrodynamics (QED) theory in 4d spacetime, in terms of fractionalized fermionic partons $\xi$ coupled to an emergent $U(1)'$ gauge field (this $U(1)'$ is beyond the SM, not any of the $U(1)$ gauge group in the SM or GUTs),

$$S_{\text{QED}'_4} = \int_{\mathcal{M}^4} d^4 x \bar{\xi} (i\gamma^\mu D_{\mu} - \Phi_{54} - i\gamma^5 \Phi_{45}) \xi. \quad (16)$$

The fermionic parton field $\xi$ contains 10 Dirac fermions forming the representation $10$ of SO(10), which also carries 1 unit of an emergent $U(1)'$ gauge charge. The gauge field $a$ is neutral under SO(10). The emergent $U(1)'$ gauge bosons can be called “dark” photons as they do not couple to any of the SM particles. The dark photons mediate the gauge interaction between fermionic partons and bind them back into the GUT Higgs field $\Phi$. The GUT Higgs field $\Phi$ is in the $10 \times 10$ bivector representation of SO(10), which is fractionalized as the fermionic parton bilinear mass terms

$$\Phi_{54} \sim \bar{\xi} \xi; \quad \Phi_{45} \sim \bar{\xi} i \gamma^5 \xi. \quad (17)$$

The phase described by the $\text{QED}'_4$ parton theory is similar to the $U(1)$ Dirac spin liquid [52, 53] discussed in the condensed matter physics context.

To check that the proposed parton theory Eq. (16) reproduces the same $w_2w_3$ anomaly as the WZW term, one can temporally turn off the GUT Higgs coupling to partons, which allows the gauge group to be enlarged to SU(2)' (with $U(1)'$ being its subgroup). Now the theory $\int_{\mathcal{M}^4} d^4 x \bar{\xi} (i\gamma^\mu D_{\mu} - \Phi_{54} - i\gamma^5 \Phi_{45}) \xi$ contains $2 \times 10 = 20$ Weyl fermions, transforming as $(2,10)$ in SU(2)' × SO(10). The anomaly can be matched by tracking the fermion representations

$$U(1)' \times SO(10) \rightarrow SU(2)' \times SO(10) \rightarrow Sp(10) \sim \begin{pmatrix} 2,10 \\ (10) \end{pmatrix} \sim \begin{pmatrix} 4,1 \oplus (1,16) \\ (4,1) \oplus (1,16) \end{pmatrix} \sim 20 \quad (18)$$

The twenty Weyl fermions can be further embedded in the 20-dim representation of Sp(10), which transforms the fermion fields as ten quaternion Grassmann numbers. On the other hand, it can be split to Sp(2) × Sp(8) that separately rotate the first two and the last eight quaternion components $(4,1) \oplus (1,16)$. The Sp(2) group has an SU(2)'' subgroup as its maximal special embedding. It is known that odd number of fermions in the 4-dim (spin-3/2) representation of SU(2) can generate the new SU(2) anomaly [51], described by the $w_2w_3$ anomaly term. Using this fact, the SU(2)'' sector will give rise to the $w_2w_3$ anomaly and the Sp(8) sector will not (as it contains an even number of spin-3/2 fermions when broken down to its SU(2) subgroup similarly). By anomaly matching, one can conclude that the fermions will generate the $w_2w_3$ anomaly of the Sp(10) bundle. However, the emergent $U(1)'$ sector of fermionic partons alone does not give rise to the $w_2w_3$ anomaly, so the anomaly must be contained in the SO(10) sector of fermionic partons, which reproduces the anomaly of the WZW term.

The anomaly matching argument indicates that the gapless fermionic partons can induce the desired braiding statistics between the topological defects of $\Phi_{45}$ and $\Phi_{54}$, which provides a possible state that saturates the anomaly. In this state, the Higgs field $\Phi$ deconfines into fermionic partons coupled by the emergent $U(1)'$ gauge field governed by the QED'_4 theory in Eq. (16). Naïvely and intuitively, the GG and PS phases can then be connected by a continuous transition with a deconfined quantum critical (DQC) point, as the white dot in Fig. 6. However, since the deconfined $U(1)'$ gauge theory is stable in 4d, the critical point will extend into a critical phase in the $so(10)$ GUT phase (in the presence of the WZW term), as indicated by the DQC region in Fig. 6. The DQC region (including the DQC point) is described by the QED'_4 theory.

Starting from the DQC phase, condensing the $\Phi_{45}$ or $\Phi_{54}$ Higgs fields will generate bilinear masses for the fermionic partons and break the Spin(10) symmetry at the same time. When the fermionic partons become fully gapped, the emergent $U(1)'$ dark photons can remain deconfined and gapless, as the Maxwell theory has a stable deconfined phase in 4d. Therefore, the resulting phases are PS, GG, or SM theories coexisting with a decoupled pure $U(1)'$ gauge theory. They are denoted as PS*, GG* and SM* in Fig. 6 (where the superscript * indicates the coexistence of dark photons in the low-energy spectrum).

However if the $U(1)'$ gauge coupling is strong enough, the $U(1)'$ gauge theory can also be driven to its confined phase. The confinement transition is indicated by the dashed circle in Fig. 6. After $U(1)'$ confinement, the DQC physics will disappear. The PS*, GG* and SM* phases will fall back to the conventional PS, GG, and SM phases. But the $so(10)$ GUT phase with the WZW term (even after the $U(1)'$ confinement) will remain nontrivial due to the $w_2w_3$ anomaly. The anomaly may also be saturated by other CFT (differed from the QED'_4 CFT proposed here), or by some TQFT.

The quantum numbers of the ten Dirac fermionic partons in the DQC phase are summarized in Fig. 7. In terms of the $g_{SM}$ representations, the fermionic partons have charge assignments that are very different from those of the matter fermions in the SM. Some fermionic partons (call colorons $c, c'$) carries the $su(3)$ color charge but not the weak $su(2)_L$ flavor charge, while other fermionic partons (called flavorons $f, f'$) carries the weak $su(2)_L$ flavor...
charge but not the color $su(3)$ charge. This color-flavor separation on the fermionic partons is one important feature of these additional fermions that are fractionalized from the GUT Higgs field. This is analogous to the spin-charge separation [54–56] in condensed matter physics.

Suppose the DQC phase once existed in the early history of our universe, now we have entered the SM$^*$ phase, where the GUT Higgs field $\Phi$ has condensed. All these fermionic partons will be gapped by the Higgs condensation at the GUT scale. The $U(1)'$ dark photons are decoupled from the fermionic matter particles in the SM, and remain as dark cosmic background radiation, which could provide a potential dark matter candidate.

IV. SUMMARY

Standard lore ritualizes our quantum vacuum in the 4-dimensional spacetime governed by the Standard Models, while lifting towards one of Grand Unifications (GUTs) at higher energy scales. In contrast, in recent studies [10–12], we introduce an alternative view that the SM is a low-energy quantum vacuum arising from various neighbor GUT vacua competition in an immense quantum phase diagram. We study the quantum phase transition between the SM and various GUTs, including the Georgi-Glashow model, the Pati-Salam model, and the $SO(10)$ GUT. We found that the competition between the GG and PS phases could lead to a gapless quantum critical phase, similar to the deconfined quantum critical point in quantum magnets of condensed matter, where the GUT Higgs field fractionalizes into fermionic partons coupled by an emergent gauge field. This fractionalization is entailed by a $w_2w_3(TM) = w_2w_3(\mathcal{V}_{SO(10)})$ nonperturbative global anomaly in the presence of a Wess-Zumino-Witten-like term for the GUT Higgs field. The quantum critical phase features fragmentary low-energy excitations of Color-Flavor separation of fermionic partons and emergent Dark Gauge force, which may leave observable consequences even away from the critical phase.

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[1] S. L. Glashow, Partial Symmetries of Weak Interactions, Nucl. Phys. 22, 579 (1961).
[2] A. Salam and J. C. Ward, Electromagnetic and weak interactions, Phys. Lett. 13, 168 (1964).
[3] A. Salam, Weak and Electromagnetic Interactions, Conf. Proc. C 680519, 367 (1968).
[4] S. Weinberg, A Model of Leptons, Phys. Rev. Lett. 19, 1264 (1967).
[5] J. C. Pati and A. Salam, Lepton Number as the Fourth Color, Phys. Rev. D10, 275 (1974), [Erratum: Phys. Rev.D11,703 (1975)].
[6] H. Georgi and S. L. Glashow, Unification of All Elementary Particle Forces, Phys. Rev. Lett. 32, 438 (1974).
[7] H. Fritzsch and P. Minkowski, Unified Interactions of Leptons and Hadrons, Annals Phys. 93, 193 (1975).
[8] P. Nath and P. Fileviez Pérez, Proton stability in grand unified theories, in strings and in branes, Phys. Rept. 441, 191 (2007), arXiv:hep-ph/0601023 [hep-ph].
[9] S. Sachdev, Quantum Phase Transitions, 2nd ed. (Cambridge University Press, 2011).
[10] J. Wang and Y.-Z. You, Gauge Enhanced Quantum Criticality Beyond the Standard Model, arXiv e-prints, arXiv:2106.16248 (2021), arXiv:2106.16248 [hep-th].
[11] J. Wang and Y.-Z. You, Gauge Enhanced Quantum Criticality Between Grand Unifications: Categorical Higher Symmetry Retraction, arXiv e-prints, arXiv:2111.10369 (2021), arXiv:2111.10369 [hep-th].
[12] J. Wang, Z. Wan, and Y.-Z. You, Cobordism and Deformation Class of the Standard Model, arXiv e-prints, arXiv:2112.14765 (2021), arXiv:2112.14765 [hep-th].
[13] P. W. Anderson, More Is Different, Science 177, 393 (1972).
[14] L. D. Landau and E. M. Lifshitz, Statistical Physics - Course of Theoretical Physics Vol 5 (Pergamon, London, 1958).
[15] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Deconfined Quantum Critical Points, Science 303, 1490 (2004), arXiv:cond-mat/0311326.
[16] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm, Phys. Rev. B 70, 144407 (2004), arXiv:cond-mat/0312617 [cond-mat.str-el].
[17] O. I. Motrunich and A. Vishwanath, Emergent photons and transitions in the O(3) sigma model with hedgehog suppression, Phys. Rev. B 70, 075104 (2004), arXiv:cond-mat/0311222 [cond-mat.str-el].
[18] M. Levin and T. Senthil, Deconfined quantum criticality and Néel order via dimer disorder, Phys. Rev. B 70, 220403 (2004), arXiv:cond-mat/0405702 [cond-mat.str-el].
[19] A. W. Sandvik, Evidence for deconfined quantum criticality in a two-dimensional heisenberg model with four-spin interactions, Phys. Rev. Lett. 98, 227202 (2007).
[20] S. Sachdev and X. Yin, Quantum phase transitions beyond the Landau-Ginzburg paradigm and supersymmetry, arXiv e-prints, arXiv:0808.0191 (2008), arXiv:0808.0191 [cond-mat.str-el].
[21] S. Sachdev, Exotic phases and quantum phase transitions: model systems and experiments, arXiv e-prints, arXiv:0901.4103 (2009), arXiv:0901.4103 [cond-mat.str-el].
[22] A. Vishwanath and T. Senthil, Physics of three-dimensional bosonic topological insulators: Surface-deconfined criticality and quantized magnetoelectric ef-
