On the Standard Model Predictions for Rare K and B Decay Branching Ratios: 2022

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Abstract

In this decade one expects a very significant progress in measuring the branching ratios for several rare $K$ and $B$ decays, in particular for the decays $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \pi^0 \nu \bar{\nu}$, $B_s \to \mu^+ \mu^-$ and $B_d \to \mu^+ \mu^-$. On the theory side a very significant progress on calculating these branching ratios has been achieved in the last thirty years culminating recently in rather precise SM predictions for them. It is then unfortunate that some papers still cite the results for $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ presented by us in 2015. They are clearly out of date. Similar comments apply to predictions for $B_{s,d} \to \mu^+ \mu^-$. In this note I want to stress again that, in view of the tensions between various determinations of $|V_{cb}|$ in tree-level decays, presently, the only trustable SM predictions for the branching ratios in question can be obtained by eliminating their dependence on the CKM parameters with the help of $|\varepsilon_K|$, $\Delta M_s$, $\Delta M_d$ and $S_{\psi K}$, evaluated in the SM, and setting their values to the experimental ones. This is supported by the fact that presently NP is not required to describe simultaneously the very precise data on $|\varepsilon_K|$, $\Delta M_s$, $\Delta M_d$ and $S_{\psi K}$. This strategy for obtaining true SM predictions for rare decay branching ratios is moreover not polluted by hadronic uncertainties and observed anomalies in semi-leptonic decays used often in global analyses as stressed recently in a longer paper by the present author [1].
1 Introduction

In this decade and the next decade one expects a very significant progress in measuring the branching ratios for several rare \( K \) and \( B \) decays, in particular for the decays \( K^+ \to \pi^+\nu\bar{\nu}, \) \( K_L \to \pi^0\nu\bar{\nu}, \) \( B_s \to \mu^+\mu^- \) and \( B_d \to \mu^+\mu^- \). On the theory side a very significant progress on these branching ratios has been achieved in the last thirty years, culminating recently in rather precise SM predictions on the four branching ratios in question. It is then unfortunate that some papers and conference presentations still cite the results for \( K^+ \to \pi^+\nu\bar{\nu} \) and \( K_L \to \pi^0\nu\bar{\nu} \) presented by us in 2015. While this paper contains several useful expressions that could play an important role when the data on these decays improves, the final results for the branching ratios presented there are clearly out of date. This is also the case of more recent predictions of Brod, Gorbahn and Stamou. Similar comments apply to predictions for \( B_{s,d} \to \mu^+\mu^- \) quoted recently in the literature and at conferences. In these analyses either specific assumptions on the values of \( |V_{cb}| \) and \( |V_{ub}| \) from tree-level decays have been used or as in the case of \cite{9} the results of an out-of-date global CKM fit have been inserted in the otherwise correct SM formulae. The latter authors will soon update their result using most recent global fit of PDG.

In this updated note, that can be considered as an overture to the very recent paper, I want to stress again, as done already in \cite{6,7,11,12}, that in view of the tensions between exclusive and inclusive determinations of \( |V_{cb}| \) and \( |V_{ub}| \) from tree-level decays used often in global analyses.

Let me make also clear that I am not against new citations to my 2015 paper because there are several useful results in that paper, but to quote our 2015 results as best results in 2022, is equivalent to stating that theorists made no progress in the last seven years. It is like referring in 2022 to the Brookhaven experiment for the best measurement of \( K^+ \to \pi^+\nu\bar{\nu} \) branching ratio, instead to the last impressive result from the NA62 collaboration.

We will next exhibit these statements in numbers.

\[ |\varepsilon_K|, \quad \Delta M_s, \quad \Delta M_d, \quad S_{\psi K_S}, \]  

\[ (1) \]

\[ B(K^+ \to \pi^+\nu\bar{\nu})_{\text{exp}} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}. \]  

\[ (2) \]
2 The Past

There are two estimates of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ SM branching ratios presented by us in 2015.\(^3\)

\[
B(K^+ \to \pi^+\nu\bar{\nu})_{SM} = (8.4 \pm 1.0) \times 10^{-11}, \quad B(K_L \to \pi^0\nu\bar{\nu})_{SM} = (3.4 \pm 0.6) \times 10^{-11},
\]

(3)

usually cited by the experimentalists, and

\[
B(K^+ \to \pi^+\nu\bar{\nu})_{SM} = (9.11 \pm 0.72) \times 10^{-11}, \quad B(K_L \to \pi^0\nu\bar{\nu})_{SM} = (3.00 \pm 0.30) \times 10^{-11}.
\]

(4)

The result in (3) is based on the 2015 average of inclusive and exclusive values of $|V_{cb}|$ and $|V_{ub}|$

\[
|V_{cb}|_{avg} = (40.7 \pm 1.4) \times 10^{-3}, \quad |V_{ub}|_{avg} = (3.88 \pm 0.29) \times 10^{-3}.
\]

(5)

The result in (4) is based on the 2015 determination of $|V_{cb}|$ and $|V_{ub}|$ from $\Delta F = 2$ observables in (1) that implied

\[
|V_{cb}|_{loop} = (42.4 \pm 1.2) \times 10^{-3}, \quad |V_{ub}|_{loop} = (3.61 \pm 0.14) \times 10^{-3}.
\]

(6)

Finally the authors of \(^9\) found in 2019

\[
B(K^+ \to \pi^+\nu\bar{\nu})_{SM} = (7.7 \pm 0.6) \times 10^{-11}, \quad B(K_L \to \pi^0\nu\bar{\nu})_{SM} = (2.6 \pm 0.3) \times 10^{-11}.
\]

(7)

The three results in (3), (4) and (7) are out of date for the following different reasons

- Result in (3) uses some averages of inclusive and exclusive values of $|V_{cb}|$ and $|V_{ub}|$ that are hardly consistent with each other. As demonstrated in (6) the standard route to SM predictions for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ used in (8) has to be avoided because of the known tensions in the determinations of $|V_{cb}|$ from tree-level decays.

- Result in (4) uses 2015 hadronic matrix elements from Lattice QCD relevant for $\Delta M_s$ and $\Delta M_d$ that changed significantly since then.

- The authors of \(^9\) made in an earlier paper \(^17\) significant progress in the reduction of theoretical uncertainties in $\varepsilon_K$. Unfortunately, in predicting the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ they used the values of the CKM parameters adopted by PDG in 2020 \(^18\). These values are out of date because in obtaining them the 2016 values of hadronic matrix elements relevant for $\Delta M_d$ and $\Delta M_s$ have been used\(^7\). As demonstrated in (19)\(^20\) and recently in (7) there are significant inconsistencies in the SM description of $\Delta M_d$ and $\varepsilon_K$ if the hadronic input of 2016 is used.

In the case of $B_{s,d} \to \mu^+\mu^-$ branching ratios the common values quoted in the literature and conference presentations are the ones from Beneke, Bobeth and Szafron \(^21\)\(^22\).

\[
\overline{B}(B_s \to \mu^+\mu^-)_{SM} = (3.66 \pm 0.14) \times 10^{-9}, \quad \overline{B}(B_d \to \mu^+\mu^-)_{SM} = (1.03 \pm 0.05) \times 10^{-10}.
\]

(8)

This is an important last detailed direct calculation of these branching ratios that further reduced theoretical uncertainties, but in reality subject to larger $|V_{cb}|$ uncertainties than quoted above as pointed out in \(^23\) with the participation of one of the authors of \(^21\)\(^22\). To circumvent this difficulty the 2003 proposal of the present author \(^24\), see below, has been used in \(^23\) and also in \(^6\)\(^7\).

\(^3\)Private communication from one of the CKMfitters.
3  2022

For completeness let me finish this note by recalling the basis formulae which were used in [6,7] to obtain the most accurate predictions for the four branching ratios in question to date.

These are the following four $|V_{cb}|$-independent ratios that are valid only within the SM:

\[
R_{11}(\beta, \gamma) = \frac{\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})}{|\varepsilon_K|^{0.82}} = (1.31 \pm 0.05) \times 10^{-8} \left( \frac{\sin \gamma}{\sin 67^\circ} \right)^{0.015} \left( \frac{\sin 22.2^\circ}{\sin \beta} \right)^{0.71}, \tag{9}
\]

\[
R_{12}(\beta, \gamma) = \frac{\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})}{|\varepsilon_K|^{1.18}} = (3.87 \pm 0.06) \times 10^{-8} \left( \frac{\sin \gamma}{\sin 67^\circ} \right)^{0.03} \left( \frac{\sin \beta}{\sin 22.2^\circ} \right)^{0.98}, \tag{10}
\]

\[
R_q = \frac{\mathcal{B}(B_q \to \mu^+\mu^-)}{\Delta M_q} = 4.291 \times 10^{-10} \frac{\tau_{B_q}(Y_0(x_i))^2}{S_0(x_i)}, \quad q = d, s. \tag{11}
\]

The notation is known to any flavour practitioner. The first two have been found in [6], the last two already in 2003 in [24]. Note that the only relevant CKM parameter in these $|V_{cb}|$-independent ratios is the UT angle $\beta$ and this is the reason why we need the mixing induced CP-asymmetry $S_{\nu\bar{K}_0}$ to obtain predictions for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. While $\gamma$ also enters these expressions, its impact on final results is practically irrelevant. Changing $\gamma$ in this expression from $67^\circ$ to $64.6(16)^\circ$ quoted in (16) has practically no impact on the four ratios in question. This is still another advantage of this strategy over global fits in addition to the independence of $|V_{cb}|$.

Setting the values of the four $\Delta F = 2$ observables in (1) to their experimental values and including all experimental and theoretical uncertainties one finds [6]

\[
\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\text{SM}} = (8.60 \pm 0.42) \times 10^{-11}, \quad \mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\text{SM}} = (2.94 \pm 0.15) \times 10^{-11}, \tag{12}
\]

and [7]

\[
\bar{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\text{SM}} = (3.78^{+0.15}_{-0.10}) \times 10^{-9}, \quad \mathcal{B}(B_d \to \mu^+\mu^-)_{\text{SM}} = (1.02^{+0.05}_{-0.03}) \times 10^{-10}. \tag{13}
\]

Note that relative to the usually quoted values by experimentalists in (3) the central values for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ did not change by much but the uncertainties decreased by factors 2.5 and 4.0, respectively. Therefore, referring to (3) as the present best values instead of (12) totally misrepresents the situation and should be abandoned as already emphasized in [6,7,11,12]. The values in (13) are rather close to those in (8), although the value for $B_s \to \mu^+\mu^-$ is by one $\sigma$ larger.

Indeed, the most interesting at present is the SM prediction for the $B_s \to \mu^+\mu^-$ branching ratio that exhibits a $2.7\sigma$ anomaly when confronted with its experimental value [25,27]

\[
\bar{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\text{EXP}} = 2.86(33) \times 10^{-9}. \tag{14}
\]

In terms of the ratio $R_s$ this anomaly is given by

\[
\left[ \frac{\mathcal{B}(B_s \to \mu^+\mu^-)}{\Delta M_s} \right]_{\text{SM}} = 2.13(7) \times 10^{-10} \text{ps}, \quad \left[ \frac{\mathcal{B}(B_s \to \mu^+\mu^-)}{\Delta M_s} \right]_{\text{EXP}} = 1.61(18) \times 10^{-10} \text{ps}. \tag{15}
\]
Unfortunately the recent messages from CMS are expected to decrease or even remove this anomaly. We are looking forward to the average from CMS, LHCb and ATLAS.

The support for this procedure comes from the analysis in [7]. It turns out that the simultaneous description of the data for $\Delta M_d$, $\Delta M_s$, $\varepsilon_K$ and $S_{\psi K_S}$ can be made without any participation of NP. Indeed, as seen in the first $|V_{cb}| - \gamma$ plot in Fig. 8 of that paper, the SM predictions for $\varepsilon_K$, $\Delta M_d$, $\Delta M_s$ and $S_{\psi K_S}$ turn out to be remarkably consistent with each other and with the data for the following values of the CKM parameters [7]

$$|V_{cb}| = 42.6(4) \times 10^{-3}, \quad \gamma = 64.6(16)^\circ, \quad \beta = 22.2(7)^\circ, \quad |V_{ub}| = 3.72(11) \times 10^{-3}. \quad (16)$$

As emphasized in [7] this agreement is only found using the hadronic matrix elements with $2 + 1 + 1$ flavours from the lattice HPQCD collaboration [15]. These values are consistent with the inclusive determination of $|V_{cb}|$ in [13] and the exclusive ones of $|V_{ub}|$ from FLAG [14].

Please note that our strategy in [6], similarly to the second 2015 strategy in [8], used as constraints the experimental values for $\Delta F = 2$ observables in (1). However, the elimination of $|V_{cb}|$ with the help of the ratios (9) and (10), not done in the 2015 paper [8], simplified the subsequent numerical analysis significantly.

The points made in this note are discussed in greater detail in [1], where using this strategy we obtain SM predictions for 26 branching ratios for rare semileptonic and leptonic $K$ and $B$ decays with the $\mu^+\mu^-$ pair or the $\nu\bar{\nu}$ pair in the final state. Most interesting turn out to be the anomalies in the low $q^2$ bin in $B^+ \to K^+\mu^+\mu^-$ ($5.1\sigma$) and $B_s \to \phi\mu^+\mu^-$ ($4.8\sigma$).

Let me also emphasize that in any global fit of CKM parameters the SM expressions for the four observables in (1) are set to their experimental values like in our strategy. But in usual global fits other observables are included which we intentionally leave out in our strategy to avoid the tension in $|V_{cb}|$ and $|V_{ub}|$ determinations from tree-level decays and the danger of NP infection of the resulting CKM parameters and consequently of SM predictions for rare branching ratios. Moreover, the pollution by hadronic uncertainties in other decays used in the fit that are larger than in many rare decay branching ratios considered can be avoided in this manner. We have elaborated on this in [1].

Finally, let me stress that this is a novel route to SM predictions for rare decay branching ratios that has not been explored by anybody to date. Only time will show whether this new strategy will be more successful to find NP than the usual global fits.

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