The Ferroelectric Point Contact

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We formulate a scattering theory of polarization and heat transport through a ballistic ferroelectric point contact. We predict a polarization current under either an electric field or a temperature difference that depends strongly on the direction of the ferroelectric order and can be detected by its magnetic stray field and associated thermovoltage and Peltier effect.

Orifices such as Maxwell/Sharvin point contacts [1, 2], micro- and nano-structured constrictions such as semiconductor quantum point contacts [3, 4] and atomic-scale break junctions [5] etc. are important instruments to study transport properties in condensed matter physics on small length scales. The quantization of transport of electrons [6, 7], light [8], super- [9, 10], and spin [11, 12] currents as well as photonic [13, 14], electronic [15–17] and phononic [18–20] heat currents are some of the important breakthroughs in this field. To the best of our knowledge, the transport through constrictions formed by ferroelectrics has never been addressed, neither theoretically nor experimentally.

Ferroelectricity refers to the electrically switchable macroscopic order of electric dipoles or persistent polarization that forms spontaneously below a Curie temperature [21] with some analogies with magnetism [22]. In so-called displacive ferroelectrics, the phase transition is accompanied by a symmetry-breaking structural phase transition: ferroelectricity is then caused by short-range elastic forces, while magnetism is caused by the short-range exchange interaction. The dipolar interaction is much larger in ferroelectrics than in ferromagnets and causes important secondary effects.

A magnon, the quantum of a spin wave, carries energy, linear, and spin angular momentum. In high quality magnetic insulators, coherently excited magnons with long wave lengths travel ballistically over large distances [23]. Thermal magnons, on the other hand, propagate diffusely under a gradient of temperature, magnetic field, or chemical potential [24] and cause the spin Seebeck [25] and spin Peltier effects [26, 27]. We previously addressed the diffuse polarization transport in ferroelectric capacitors [28].

In this Letter, we formulate ballistic transport through constrictions of a displacive ferroelectric by the scattering theory of transport [19, 29, 30]. In contrast to the findings for diffuse systems [28], we predict a dc polarization current that generates observable stray magnetic fields as well as a dc polarization Peltier effect.

We consider a monolithic ferroelectric sample in which a narrow wire adiabatically connects two reservoirs with perpendicular ferroelectric order at temperatures sufficiently below the phase transition. The two reservoirs are at thermal equilibrium but at possibly different temperature $T_1$ and $T_2$. Top and bottom gates allow application of different electric fields $E_1$ and $E_2$ as sketched in Fig. 1(a). We focus on steady-state transport, which requires sufficiently large reservoirs. In linear response, currents and forces are related by a matrix of transport coefficients [28]

$$
\begin{pmatrix}
-J_p \\
J_q
\end{pmatrix} = G
\begin{pmatrix}
1 & S \\
\Pi & K/G
\end{pmatrix}
\begin{pmatrix}
\Delta E \\
-\Delta T
\end{pmatrix}
$$

where the $J_p$ and $J_q$ are the polarization and heat currents flowing from reservoir 1 to 2, while $\Delta E = E_2 - E_1$ and $\Delta T = T_2 - T_1$ are the electric field and temperature differences between the two reservoirs, respectively. $G$ is the polarization conductance, $S$ (the polarization Seebeck (Peltier) coefficient with Kelvin-Onsager relation $\Pi = ST$, and $K$ the thermal conductance. We now derive all transport coefficients by the scattering theory of transport.

Phonon model: A phonon mode in a wire along the $x$-axis with wave number $k$ reads

$$
u_{nks}(l,s) = \frac{1}{\sqrt{NM_s}}e^{ikx_1-k\omega_{nks}\tau}$$

where $\nu_{nks}(l,s)$ is the displacement of the $s$-th ion in the $l$-th ferroelectric unit cell, with the ionic mass of $M_s$, $N$ the number of unit cells. Here $\omega_{nks}$ is the phonon frequency dispersion, $e_{nks}(s)$ the polarization vector of phonon with polarization index $\sigma = 1, \cdots, 3m$, where $m$ is the number of ions in each ferroelectric unit cell, with band index $n$ and $\rho$ the transverse coordinate. The orthogonality relations are $\sum_s e_{nks}(s)\cdot e_{nks}(s)^* = \delta_{\sigma\sigma'}$

A phonon $\nu_{nks}$ originating in the left reservoir (R1) with positive group velocity $v_{nks} = \partial\omega_{nks}/\partial k > 0$ can be elastically reflected or transmitted by the constriction, as illustrated in Fig. 1(b), and the amplitude in both leads then reads

$$
\nu_{nks}^{(1)} = \begin{cases}
\nu_{nks} + \sum_{n'k's'} \nu_{n'-k's'}^{(1)} \rho_{n's'}^{(1)} \sigma_{n's'}^{(1)} & \text{in lead 1} \\
\sum_{n'k's'} \nu_{n'k's'}^{(1)} \rho_{n'k's'}^{(1)} \sigma_{n'k's'}^{(1)} & \text{in lead 2}
\end{cases}
$$

where $\rho_{n's'}^{(1)}$ and $\sigma_{n's'}^{(1)}$ are scattering coefficients for $\nu_{nks}^{(1)}$.
FIG. 1. (a) Polarization transport between two ferroelectric reservoirs (R1 and R2) connected by a ferroelectric lead (left panel). The polarization current induces a magnetic field (right panel). In accordance with the polarization current direction as in (a), there are an electric field applied on the R1 along the direction of ferroelectric order and a temperature difference (ΔT > 0) on the R2. (b) The ferroelectric lead is divided by the scattering region into two parts of lead 1 and 2 that are in adiabatic contact with the R1 and R2, respectively.

where \( r_{n's'ns} \) is the reflection amplitude from mode \( ns \) to \( n's' \) in lead 1, while \( t_{n's'ns} \) is the transmission amplitude from mode \( ns \) in lead 1 to \( n's' \) in lead 2 and \( \omega_{nk\sigma} = \omega_{n'k's'} \) for the elastic scattering. Analogously, assuming that the junction is symmetric, an incoming phonon \( u_{nk\sigma} \) from the right reservoir (R2) generates the following amplitudes in both leads

\[
\begin{align*}
\underline{u}_n^{(2)} &= \{ \sum \sum \sum \mathbf{u}_{n'k's'}^{(2)} \mathbf{u}_{n'k's'}^{(1)} \} \\
&= \mathbf{u}_{n-k\sigma}^{(2)} + \mathbf{u}_{n-k\sigma}^{(2)} \quad \text{leads 1 and 2.}
\end{align*}
\]

The total displacement field operator in the leads can then be written as [19]

\[
\mathbf{u}(l,s) = \sum_{n,s,k>0} \sqrt{\frac{\hbar}{2\omega_{nk\sigma}}} [\mathbf{u}_{nk\sigma}^{(1)}(l,s) + \mathbf{u}_{nk\sigma}^{(2)}(l,s)] + \text{h.c.}
\]

where \( \mathbf{u}_{nk\sigma}^{(1)} \) and \( \mathbf{u}_{nk\sigma}^{(2)} \) are the creation operator of phonons originating in R1 (R2) and h.c. denotes the hermitian conjugate of the previous term.

We adopt the standard rigid-ion approximation for displacive ferroelectrics [31, 32] with polarization fluctuations around the ground state,

\[
\delta \mathbf{P}_l = \sum_s Q_s \mathbf{u}_s(l,s),
\]

where \( Q_s \) is the ionic charge of the \( s \)-th ion in the ferroelectric unit cell with index \( l \) and \( \sum_s Q_s = 0 \). The polarization projection along the ferroelectric order in the \( l \)-th unit cell \( \hat{P}_l = (P_0 + \hat{P}_l) \cos \hat{\theta}_l \), where \( \hat{P}_l \) is the oscillation of the polarization modulus relative to \( P_0 = |\mathbf{P}_0| \) in the ground state and \( \hat{\theta}_l \) a small angle of rotation. We now make the ferron approximation that the fluctuations in \( \hat{P}_l \) and \( \hat{\theta}_l \) are uncorrelated, which is justified when the longitudinal elastic constant of the dipole is sufficiently larger than the transverse one and excellent for the order-disorder type of ferroelectrics with stable molecular dipoles [33]. To leading order in small Cartesian transverse and longitudinal fluctuations, i.e.,

\[
\hat{P}_l = P_0 \left[ 1 - \frac{\langle \hat{P}_l^2 \rangle^2}{2P_0^2} \right] + \hat{P}_l^{\perp} + O([\hat{P}_l^2]_2, \hat{P}_l^{\perp}_2).
\]

The first term captures the reduced polarization projection along the ferroelectric order by the transverse fluctuations and we disregard \( \hat{P}_l^{\perp} \) and higher-order terms.

When the polarization is conserved on the length scale of the constriction, the operator for the coarse-grained polarization density \( \hat{P}_l/\Omega \rightarrow \hat{p}(r,\tau) \) per unit cell with volume \( \Omega \) is related to the polarization current density \( \hat{j}_p(r,\tau) \) through

\[
\partial_\tau \hat{j}_p(r,\tau) = -\partial_r \hat{p}(r,\tau),
\]

where \( r \equiv (x,\rho) \) and the operators are in the Heisenberg picture. The partial Fourier transform \( \hat{p}(\rho, q, \omega) = \int d\tau \int d\mathbf{p} \hat{p}(\mathbf{p}, x, \tau) e^{i\mathbf{p}\cdot\mathbf{x} - iq\omega} \) leads to

\[
\hat{j}_p(r,\tau) = \int \frac{dq}{2\pi} \int \frac{d\omega}{2\pi} \frac{\omega}{q} \hat{p}(\rho, q, \omega) e^{-i\mathbf{p}\cdot\mathbf{x} + iq\omega}.
\]

Substituting Eq. (7) and to leading order in the fluctuations Eqs. (6), the statistical average of the total polarization current

\[
J_p = \int d\rho \langle \hat{j}_p(\rho,\tau) \rangle
\]

is

\[
J_p = \frac{\hbar}{2P_0} \sum_{n',s',\sigma} \sum_{n,k\sigma} \int_{BZ} d\mathbf{k}_{n'k's'}^\perp \left| \mathbf{F}_{n'k's'}^\perp \right|^2 \left| \mathbf{E}_{n'k's'}^{\perp} \right|^2 \left| \mathbf{v}_{n'k's'}^{\perp} \right|^2 \left| \mathbf{v}_{n's'}^{\perp} \right|^2 \left| \mathbf{v}_{n's'}^{\perp} \right|^2
\]

\[
\times \left[ \langle \hat{a}_{nk\sigma}^{\dagger} \hat{a}_{nk\sigma} \rangle - \langle \hat{a}_{nk\sigma}^{\dagger} \hat{a}_{nk\sigma} \rangle \right]^2,
\]

where BZ indicates the Brillouin zone boundary. Here \( \mathbf{F}_{nk\sigma} = \mathbf{F}_{nk\sigma} - (\mathbf{F}_{nk\sigma} \cdot \mathbf{P}_0)/P_0 \) is the transverse component of

\[
\mathbf{F}_{nk\sigma} = \sum_s \frac{Q_s}{\sqrt{M_s}} \mathbf{e}_{nk\sigma}(s)
\]

and \( \langle \cdots \rangle^{(i)} \) is a thermal average in reservoir \( i \):

\[
\langle \hat{a}_{nk\sigma}^{\dagger} \hat{a}_{nk\sigma} \rangle^{(i)} = N \left( \hbar \omega_{nk\sigma} - F_{nk\sigma}^{(i)} T^{(i)} \right).
\]

The parameters in the Planck distribution function \( N(\hbar \omega, T) \equiv 1/[e^{\hbar \omega/k_BT} - 1] \) are the temperature \( T^{(i)} \).
and effective chemical potential \( \mu_{nk}\sigma = \xi_{nk}\sigma E^{(i)} \), where \( E^{(i)} \) is the electric field of reservoir \( i \) and polarization

\[
\xi_{nk}\sigma = -\frac{\hbar}{2P_0} \left| \mathbf{F}_{nk}\sigma \right|^2 \omega_{nk}\sigma. \tag{13}
\]

In the above calculations we adopted the conventional assumption of an adiabatic connection between the reservoirs and the leads. The electric fields are applied on the reservoirs where they affect the equilibrium phonon statistics. In linear response, transport is then only governed by the differences in \( E^{(i)} \) and \( T^{(i)} \) in the reservoirs and the transmission coefficients through the constriction. We consider here two different configurations, viz. with equilibrium polarization in the constriction normal to the plane (parallel to those in the reservoirs) and along the wire. They can in principle be switched by the electric fields of local gates, but we assume here for simplicity that the field in the constriction vanishes.

The energy or heat current reads

\[
J_q = \sum_{n'\sigma'} \sum_{n\sigma} \int_{0}^{BZ} \frac{dk}{2\pi} \hbar \omega_{n'k'\sigma'} \left| t_{n'\sigma'\sigma} \right|^2 \left[ N \left( \hbar \omega_{nk}\sigma - \mu_{nk}\sigma^{(1)} \right) - N \left( \hbar \omega_{nk}\sigma - \mu_{nk}\sigma^{(2)} , T^{(2)} \right) \right], \tag{14}
\]

In the ballistic limit, \( t_{n'\sigma'\sigma} = \delta_{n'\sigma} \delta_{\sigma'\sigma} \)

\[
J_q = -\sum_{n\sigma} \int_{\omega_{\min \sigma}}^{\omega_{\max \sigma}} \frac{d\omega}{2\pi} \hbar \omega \left[ N \left( \hbar \omega - \mu_{n\sigma}^{(1)} \left( \omega \right) , T^{(1)} \right) - N \left( \hbar \omega - \mu_{n\sigma}^{(2)} \left( \omega \right) , T^{(2)} \right) \right], \tag{15}
\]

where \( \omega_{\min \sigma} \) and \( \omega_{\max \sigma} \) are the band edges of the \( n\sigma \) phonon mode, which in the absence of an electric field or ferroelectric order reduces to the conventional phonon heat current expression \([18, 19]\). We obtain the transport coefficients in Eq. (1) by linearizing the distribution functions:

\[
G = -\frac{1}{\hbar} \sum_{n\sigma} \int_{\omega_{\min \sigma}}^{\omega_{\max \sigma}} \xi_{n\sigma}^2 \left( \omega \right) \left( \frac{\partial N}{\partial \omega} \right)_T \frac{d\omega}{2\pi},
\]

\[
\Pi = ST = G^{-1} \sum_{n\sigma} \int_{\omega_{\min \sigma}}^{\omega_{\max \sigma}} \omega \xi_{n\sigma} \left( \omega \right) \left( \frac{\partial N}{\partial \omega} \right)_T \frac{d\omega}{2\pi},
\]

\[
K = \sum_{n\sigma} \int_{\omega_{\min \sigma}}^{\omega_{\max \sigma}} \hbar \omega \left( \frac{\partial N}{\partial T} \right)_\omega \frac{d\omega}{2\pi}, \tag{16}
\]

which (for simple ferroelectrics) are positive since \( \partial N(\omega, T)/\partial \omega < 0 \).

**Diatomic ferroelectric chain:** For concreteness, we model transport properties at temperatures sufficiently below the ordering transition by a one-dimensional dimer chain, with two ions of opposite charges \( \pm Q \) and same mass \( M \) in the unit cell, as sketched in Fig. 2 for the polarization perpendicular and parallel to the chain. The ground state permanent electric dipole in each unit cell is \( P_0 = Q\delta \), where \( \delta \) is the symmetry breaking deformation and \( Q \) the ionic charge. For strongly anisotropic systems this model holds by multiplying the results with the number of parallel wires.

In the perpendicular configuration, only two optical phonons with polarization vector transverse to the ferroelectric order carry an average polarization of \( \xi_{\parallel k} = -(hQ^2)/(MP_0\omega_{\parallel k}) \). Using the results from the Supplementary Material (SM) \([34]\)

\[
G = \frac{2\xi_P^2}{\hbar} \int \sqrt{2\epsilon_0} \frac{e^2e^*}{c^2(e^*-1)^2} \]

\[
\Pi = \left( \frac{2\xi_P^2}{G} \right) C\delta^2 \int \frac{e^*}{\hbar} \left( \frac{1}{3\sqrt{3}} - \frac{1}{3\sqrt{3}} \right) \frac{k_BT}{\hbar \omega_0} \text{ for } k_BT \ll \hbar \omega_0, \tag{17}
\]

\[
K \approx \frac{k_BT}{h} \left( \int \frac{e^*}{\hbar} \left( \frac{3e^2e^*}{(e^*-1)^2} + \frac{e^*}{\hbar} \left( \frac{e^*}{\hbar} \right) \right) \right) \]

\[
\left\{ \begin{array}{lc}
\frac{\pi k_BT}{h} & \text{for } k_BT \ll \hbar \omega_0 \\
\left( 3 + \frac{6}{\sqrt{2}} \right) \frac{k_BT}{\hbar} & \text{for } k_BT \gg \hbar \omega_0 \end{array} \right., \tag{19}
\]

where \( \omega_0 = \sqrt{C/M} \) is the characteristic frequency of lattice vibration with \( C \) the shear force constant, \( \xi_P = \hbar \omega_0 P_0/(C\delta^2) \) is a quantum polarization, analogous to the Bohr magneton for magnetization with \(-\xi_P E/\hbar \) equivalent to the Rabi frequency, \( \epsilon_0 = \hbar \omega_0/k_BT \) and \((0,2\epsilon_0)\) and \((\sqrt{2}\epsilon_0, \sqrt{6}\epsilon_0)\) are the band edges in unit of \( k_BT \) for three acoustic and two transverse optical phonon modes, respectively \([34]\). The ratio between the elastic energy and electric dipole \( C\delta^2/P_0 \) also governs transport in the diffuse model \([28]\). In Fig. 2(c), we plot the polarization conductance and Peltier coefficient as a function of temperature (see Fig. S2 in SM \([34]\) for the heat conductance). Since the polarization transport is contributed by gapped optical phonons, \( G \) is exponentially small when \( k_BT \ll \hbar \omega_0 \). In contrast to \( \Pi = \pi k_BT/h \), the well-known quantum of phononic heat conductance \([18-20]\), the polarization Peltier quantum \( \Pi = C\delta^2/P_0 \) is not universal, but depends on the material parameters.

Aligning the polarization with the wire axis drastically changes the polarization transport that is then carried by both transverse acoustic and optical phonon modes. According to the SM \([34]\)

\[
\xi_{\parallel k} = -\frac{\hbar \omega_{\parallel k} P_0}{4C\delta^2}, \tag{20}
\]
FIG. 2. Diatomic ferroelectric chains with spontaneous polarization (a) perpendicular and (b) parallel to the chain axis, respectively. The polarization conductance \( G \) and Peltier coefficient \( \Pi \) as a function of temperature for the perpendicular (c) and parallel (d) configurations, where TA and TO in (d) represent the contributions from the transverse acoustic and optical phonons, respectively.

where \( \sigma = TA, TO \) denotes the transverse acoustic and optical phonons, respectively, and Eq. (16) reduces to

\[
K = \frac{k_B T}{\hbar} \int_0^{2\epsilon_0} \frac{d\epsilon}{(e^\epsilon - 1)^2} = \begin{cases} \frac{\epsilon^2 k_B^2 T}{3k_B \hbar^2} & \text{for } k_B T \ll \hbar \omega_0, \\ \frac{2K T}{3} \Pi^2 & \text{for } k_B T \gg \hbar \omega_0 \end{cases}
\]

(21)

\[
G = \frac{\epsilon^2}{3h} \int_0^{2\epsilon_0} \frac{d\epsilon}{\epsilon_0(e^\epsilon - 1)^2} = \frac{2}{3} K T \Pi^2,
\]

(22)

\[
\Pi = \frac{4C \delta^2}{P_0}.
\]

(23)

Here \( \Pi \) is constant and the conductance \( G \) vanishes quadratically with temperature since polarization transport by the transverse acoustic phonons is massless at low energies. The figure of merit of thermal polarization transport turns out to be constant as well:

\[
ZT \equiv \frac{G\Pi^2}{K T} = \frac{2}{3}
\]

(24)

Detection of polarization current: In the steady state the polarization current from the high-field to the low field region is accompanied by a heat current. We assume in the derivations above that the reservoirs are such large that on the time scale of the transport process the bias is constant. Finite reservoirs react parametrically to these currents on a larger time scale. The Peltier effect can be observed by an increased temperature in the high-field regime and cooling of the low-field terminal, while the “battery” becomes depleted. When the two reservoirs are not electrically biased but subject to a temperature difference, a heat current flows, accompanied by a Seebeck polarization current. Both currents contribute to an increase (decrease) of the polarization on the hot (cold) side that charges the capacitors by the pyroelectric as well as Seebeck effect, i.e., generates thermovoltages in both reservoirs. With the parameters above we can compute the time dependence for given sample geometries parametrically.

The dc transport of electric polarization is accompanied by dc stray magnetic fields and, vice versa, an applied magnetic field can affect the polarization current. When flowing along the \( x \) with polarization along \( z \) as in Fig. 1(a), the magnetic flux density at a position \( r = (0, y, z) \) follows from the Lorentz transformation

\[
B_p = \frac{\mu_0 J_p}{2\pi \rho} (0, \cos 2\phi, \sin 2\phi)
\]

(25)

where \( J_p \) is the polarization current, \( \rho = \sqrt{y^2 + z^2} \) the probing distance, \( \cos \phi = y/\rho \) and \( \sin \phi = z/\rho \). For \( N_0 \) uncoupled parallel chains \( J_p = N_0 G[\Delta E + \Pi(\Delta T/T)] \), where \( G \) and \( \Pi \) are the polarization conductance and Peltier coefficient for the single chain, respectively. In the
perpendicular polarization configuration at $T = 300$ K, with $C = 25 \text{J/m}^2$, $\omega_0 = 10\text{THz}$, $\delta = 0.03 \text{nm}$, $P_0 = 2 \times 10^{-29} \text{Cm}$, we arrive at $G = 9.74 \times 10^{-28} \text{m}^2/\Omega$, $I = 3.52 \times 10^3 \text{V/m}$. The induced magnetic field for $N_p = 1$ at a distance $\rho = 10 \text{nm}$ is $B_p \approx 200 \mu\text{T}$ for either $\Delta E = 10^3 \text{V/m}$ or $\Delta T = 10 K$, which can be detected by single diamond-NV center magnetometry enhanced by spin-to-charge NV readout protocols [35]. The stray magnetic fields generated by the polarization current in thicker wires and tuned by the sample geometry may become large enough to be detectable by conventional sensors.

**Conclusions:** We derive expressions for the steady state polarization and heat transport through a ferroelectric constriction driven by temperature and electric field differences. We find drastic effects of rotating the polarization direction, such as an algebraic vs. exponential suppression of the polarization current. The results can be extended to include, e.g., the effects of ferroelectric domain walls. The polarization current can be detected indirectly via the polarization Peltier effect and a thermovoltage or, more directly, by the stray magnetic field that accompanies the streaming dipoles. Our formulation for the polarization transport is not limited to this simple chain model but is accessible to first-principles calculations. The thermally and electrically induced transport of electric polarization opens alternative strategies for thermal management using ferroelectric materials.

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[1] J. C. Maxwell, A treatise on electricity and magnetism, Volume 1 (Oxford: Clarendon Press, 1873).
[2] Y. V. Sharvin, Zh. Ekserper. i Teor. Fiz. 48 (1965).
[3] H. van Houten, L. W. Molenkamp, C. W. J. Beenakker, and C. T. Foxon, Semiconductor Science and Technology 7, B215 (1992).
[4] H. van Houten and C. W. J. Beenakker, Physics Today 49, 22 (1996).
[5] N. Agrait, A. L. Yeyati, and J. M. Van Ruitenbeek, Physics Reports 377, 81 (2003).
[6] B. J. Van Wees, H. Van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. Van der Marel, and C. T. Foxon, Physical Review Letters 60, 848 (1988).
[7] D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C Jones, Journal of Physics C: Solid State Physics 21, L209 (1988).
[8] E. A. Montie, E. C. Cosman, G. W.’t Hooft, M. B. Van der Mark, and C. W. J. Beenakker, Nature 350, 594 (1991).
[9] C. W. J. Beenakker and H. Van Houten, Physical Review Letters 66, 3056 (1991).
[10] H. Takayanagi, T. Akazaki, and J. Nitta, Physical Review Letters 75, 3533 (1995).
[11] P. Debray, S. M. S. Rahman, J. Wan, R. S. Newrock, M. Cahay, A. T. Ngo, S. E. Ulloa, S. T. Herbert, M. Muhammad, and M. Johnson, Nature Nanotechnology 4, 759 (2009).
[12] F. Meier and D. Loss, Physical Review Letters 90, 167204 (2003).
[13] M. Meschke, W. Guichard, and J. P. Pekola, Nature 444, 187 (2006).
[14] T. Ojanen and T. T. Heikkilä, Physical Review B 76, 073414 (2007).
[15] O. Chiatti, J. T. Nicholls, Y. Y. Proskuryakov, N. Lumpkin, I. Farrer, and D. A. Ritchie, Physical Review Letters 97, 056601 (2006).
[16] S. Jezouin, F. D. Parmentier, A. Anthore, U. Gennser, A. Cavanna, Y. Jin, and F. Pierre, Science 342, 601 (2013).
[17] L. Cui, W. Jeong, S. Hur, M. Matt, J. C. Klöckner, F. Pauly, P. Nielaba, J. C. Cuevas, E. Meyhofer, and P. Reddy, Science 355, 1192 (2017).
[18] L. G. C. Rego and G. Kirezenow, Physical Review Letters 81, 232 (1998).
[19] M. P. Blencowe, Physical Review B 59, 4992 (1999).
[20] K. Schwab, E. A. Henriksen, J. M. Worlock, and M. L. Roukes, Nature 404, 974 (2000).
[21] Y. H. Xu, Ferroelectric materials and their applications (Elsevier, 2013).
[22] N. A. Spaldin, Physics of ferroelectrics, Pages 175–218 (2007).
[23] Y. Kajiwara, K. Harri, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takahashi, S. Maekawa, and E. Saitoh, Nature 464, 262 (2010).
[24] L. J. Cornelissen, K. J. H. Peters, G. E. W. Bauer, R. A. Duine, and B. J. van Wees, Physical Review B 94, 014412 (2016).
[25] K. Uchida, J. Xiao, H. Arachi, Jun-ichiho Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai, G. E. W Bauer, S. Maekawa, and E. Saitoh, Nature Materials 9, 894 (2010).
[26] J. Flipse, F. K. Dejene, D. Wagenaar, G. E. W. Bauer, J. B. Youssef, and B. J. Van Wees, Physical Review Letters 113, 027601 (2014).
[27] S. Daimon, R. Iguchi, T. Hikoi, E. Saitoh, and . Uchida, Nature Communications 7, 1 (2016).
[28] G. E. W. Bauer, R. Iguchi, and K. Uchida, Physical Review Letter 126, 187603 (2021).
[29] M. Büttiker, Physical Review B 46, 12485 (1992).
[30] S. Datta, Electronic transport in mesoscopic systems (Cambridge University Press, 1997).
[31] M. Born and K. Huang, Dynamical theory of crystal lattices (Clarendon Press, 1954).
[32] J. D. Freire and R. S. Katiyar, Physical Review B 37, 2074 (1988).
[33] J. Pouget, A. Aşkar, and G. A. Maugin, Physical Review B 33, 6304 (1986).
[34] See the Supplementary Material for technical details.
[35] T.I. Andersen, B.L. Dwyer, J. D. Sanchez-Yamagishi, J.F. Rodriguez-Nieva, K. Agarwal, K. Watanabe, T. Taniguchi, E.A. Demler, P. Kim, H. Park, and M.D. Lukin, Science 364, 154 (2019).
In this Supplementary Material, we derive the phonon dispersions and transport coefficients for the configurations with ferroelectric polarization perpendicular (Section A) and parallel (Section B) to the chain axis as used in the main text [1]. Fig. S1 summarized the results for the phonon band structure. S2 on the thermal conductance and figure of merit supplement the figures given in the main text.

In the perpendicular configuration, only the transverse optical modes carry polarization, while in the parallel configuration the transverse acoustic modes also contribute. Here the parameters are taken as $C_0(x) = C_0(y) = C_1(x) = C_1(y) = C$ and $C_0(z)/C \gg 1$ for (a) and $C_0(x) = C_0(y) = C_1(x) = C_1(y) = C, C_1(z) = 2C$ and $C_0(z)/C \gg 1$ for (b). The high-frequency longitudinal optical mode ($Oz$) lies outside the plotting range of both (a) and (b).
FIG. S2. (a) The heat conductance ($K$) and (b) figure of merit ($ZT \equiv G\Pi^2/KT$) as a function of temperature for ferroelectric order perpendicular and parallel to the chain. $K$ is in units of $K_0 = \pi k_B\omega_0^2/6$.

A. PERPENDICULAR POLARIZATION CONFIGURATION

The energy in a displacive ferroelectric has both mechanical and electrostatic contributions. The elastic energy cost of small deformations from equilibrium in our chain model with a perpendicular configuration is

$$\Phi_{el} = \frac{1}{2} \sum_{l\alpha} C^{(\alpha)}_0 [u^{(\alpha)}_l - v^{(\alpha)}_l]^2 + \frac{1}{2} \sum_{l\alpha} C^{(\alpha)}_1 [u^{(\alpha)}_l - u^{(\alpha)}_{l+1}]^2 + \frac{1}{2} \sum_{l\alpha} C^{(\alpha)}_2 [v^{(\alpha)}_l - v^{(\alpha)}_{l+1}]^2,$$

where $\alpha = x, y, z$ denotes the Cartesian components and $u^{(\alpha)}_l$ and $v^{(\alpha)}_l$ the displacement of positive and negative ions of unit cell $l$ in the $\alpha$-direction, respectively. $C^{(\alpha)}_0$ is the corresponding $\alpha$-th component force constant between the positive and negative ions in the unit cell and $C^{(\alpha)}_1$ that between the same type of ions in neighboring unit cells, for simplicity assumed to be the same for the positive and negative ions.

In a ferroelectric, we should also consider the depolarization cost of the dipolar energy

$$\Phi_{dp} = \frac{1}{8\pi \varepsilon} \sum_{l \neq l'} \frac{P_l \cdot P_{l'} - 3(P_l \cdot \hat{r}_{ll'}) (P_{l'} \cdot \hat{r}_{ll'})}{R_{ll'}^3},$$

(S.A1)

where $\varepsilon$ the dielectric constant, $R_{ll'}$ the distance between $l$-th and $l'$-th unit cell with direction vector $\hat{r}_{ll'}$ and $P_l = [1 - \delta_{l,0}] P_0 + \delta_{l,0} \hat{r}$ the polarization of each unit cell with $P_0$ the polarization at the ground state and $\delta P_l = Q(u_l - v_l)$ the polarization fluctuations [1]. The dipolar interaction contains terms that are linear in the ion displacement and affect the equilibrium position of the atoms. These can be removed by defining a new ground state and subsequently disregarded.

With chain axis along $x$ and $P_0$ along $z$ direction, a plane wave ansatz with wave number $k$ leads to a dynamical matrix in the $\alpha$-direction

$$D^{(\alpha)}(k, s, s') = \frac{1}{M} \sum_{l'} \left( \begin{array}{c} \frac{\partial^2 \Phi}{\partial u^{(\alpha)}_l \partial u^{(\alpha)}_{l'}}, \frac{\partial^2 \Phi}{\partial v^{(\alpha)}_l \partial v^{(\alpha)}_{l'}} \end{array} \right) e^{ik(l-l')s'}

= \frac{1}{M} \left( \begin{array}{cc} 2C^{(\alpha)}_1(1 - \cos ka) + \tilde{C}^{(\alpha)}_0 + C^{(\alpha)}_d(k), & \tilde{C}^{(\alpha)}_0 - C^{(\alpha)}_d(k) \end{array} \right)_{ss'},$$

(S.A2)

where $\Phi = \Phi_{el} + \Phi_{dp}$ is the total energy cost by the lattice deformation, $s, s' = \pm$ denotes the type of ions, $M$ the ionic mass and $a$ the lattice constant; $\tilde{C}^{(\alpha)}_0 = C^{(\alpha)}_0 [1 - 2\zeta(3)\eta^{(\alpha)}(\delta_{\alpha,x} + \delta_{\alpha,y})]$ is the dipolar-interaction modified force constant for the ions in the same unit cell, and

$$C^{(\alpha)}_d(k) = 2(1 - 3\delta_{\alpha,z})\eta^{(\alpha)} \left[ \sum_{n=1}^{\infty} \cos(nka)/n^3 \right] C^{(\alpha)}_0$$
is generated by the dipolar interaction and \( \eta(\alpha) = Q^2/(4\pi \varepsilon a^3 C(\alpha)) \) is a dimensionless constant representing the relative strength of the dipolar interaction and \( \sum_{n=1}^{\infty} n^{-3} = \zeta(3) \). For \( a = 0.4 \text{ nm}, Q = e, C_0 = 25 J/m^2 \) and a dielectric constant \( \varepsilon/\varepsilon_0 = 10^3 \) in a typical perovskite ferroelectric, \( \eta(\alpha) \sim 10^{-4} \), where \( \varepsilon_0 \) is the vacuum dielectric constant. We therefore may discard the effect of the dipolar interaction on the phonon dispersions. We note that in ferromagnets the exchange interaction vanishes for low-frequency spin waves like \((ka)^2\) while the dipolar scales like \(ka\). In ferroelectrics the leading elastic energy is a large constant for the electrically active optical modes.

The eigenvalues and eigenvectors of the dynamical matrix are the phonon dispersion relation and polarization vectors, respectively,

\[
\omega_{kO}^{(\alpha)} = \sqrt{\frac{2C_1^{(\alpha)}}{M} (1 - \cos ka)} + \frac{2C_0^{(\alpha)}}{M}, \quad \mathbf{e}_{kO}^{(\alpha)} = \frac{1}{\sqrt{2}} (1, -1),
\]

\[
\omega_{kA}^{(\alpha)} = \sqrt{\frac{2C_1^{(\alpha)}}{M} (1 - \cos ka)}, \quad \mathbf{e}_{kA}^{(\alpha)} = \frac{1}{\sqrt{2}} (1, 1), \quad (\text{S.A3})
\]

where \(O\) and \(A\) are the optical and acoustic modes, respectively. The two components of \( \mathbf{e}_{kO}^{(\alpha)} \) represent the relative phase of the two ions in the unit cell (rather than the spatial direction \( \alpha \)). The vector \( \mathbf{F}_{kO}^{(\alpha)} = \sum Q_s/\sqrt{P_0} \mathbf{e}_{kO}^{(\alpha)}(s) \) (defined in the main text) determines the contribution of phonon to the polarization dynamics with

\[
|\mathbf{F}_{kO}^{(\alpha)}|^2 = \frac{2Q^2}{M}, \quad |\mathbf{F}_{kA}^{(\alpha)}|^2 = 0. \quad (\text{S.A4})
\]

Thus, the acoustic phonons do not contribute to the polarization dynamics. At a given \( k \) two optical phonons \( \alpha = x, y \) with polarization vector transverse to the ferroelectric order (along \( z \)-direction) carry a polarization of \([1]\)

\[
\xi_{kO}^{(\alpha)} = \frac{\hbar}{2P_0} |\mathbf{F}_{kO}^{(\alpha)}|^2 = - \frac{\hbar Q^2}{MP_0 \omega_{kO}^{(\alpha)}}. \quad (\text{S.A5})
\]

We now assume that all the components of force constants are the same except for the longitudinal one between the positive and negative ions in the same unit cell that is taken to be much larger, which implies that the modulus of the polarization does not change, i.e., \( C_0^{(x)} = C_0^{(y)} = C_1^{(x)} = C_1^{(y)} = C \) and \( C_0^{(z)}/C \gg 1 \). Eq. (S.A3) then reduces to

\[
\omega_{kO}^{(x)} = \omega_{kO}^{(y)} = 2\omega_0 \sqrt{1 - \frac{1}{2} \cos ka},
\]

\[
\omega_{kO}^{(z)} \approx \sqrt{\frac{2C_0}{C} \omega_0} \gg \omega_0,
\]

\[
\omega_{kA}^{(x)} = \omega_{kA}^{(y)} = \omega_{kA}^{(z)} = \sqrt{2\omega_0} \sqrt{1 - \cos ka}, \quad (\text{S.A6})
\]

where \( \omega_0 = \sqrt{C/M} \). Using Eq. (S.A5) and Eq. (S.A6), we semi-analytically obtain the transport coefficients in the main text:

\[
G = -\frac{1}{\hbar} \sum_{\alpha=x,y} \int_{\sqrt{2\omega_0}}^{\sqrt{\omega_{kO}^{(\alpha)}}} d\omega_{kO}^{(\alpha)} \frac{\partial N(\omega_{kO}^{(\alpha)}, T)}{\partial \omega_{kO}^{(\alpha)}} \frac{d\omega_{kO}^{(\alpha)}}{2\pi} = \frac{2\xi_P^2}{\hbar} \int_{\sqrt{\omega_{\xi}}}^{\sqrt{\omega_{\xi}}} \frac{d\epsilon}{\sqrt{\omega_{\xi}}} \frac{\epsilon^2 e^\epsilon}{(e^\epsilon - 1)^2}
\]

\[
\Pi = ST = G^{-1} \sum_{\alpha=x,y} \int_{\sqrt{\omega_{kO}^{(\alpha)}}}^{\sqrt{\omega_{kO}^{(\alpha)}}} \omega_{kO}^{(\alpha)} \frac{d\omega_{kO}^{(\alpha)}}{2\pi} \frac{\partial N(\omega_{kO}^{(\alpha)}, T)}{\partial \omega_{kO}^{(\alpha)}} \frac{d\omega_{kO}^{(\alpha)}}{2\pi} = \frac{CS^2}{P_0} \left( \frac{2e_P^2}{G\hbar} \right) \int_{\sqrt{\omega_{\xi}}}^{\sqrt{\omega_{\xi}}} \frac{d\epsilon}{\sqrt{\omega_{\xi}}} \frac{\epsilon^2 e^\epsilon}{(e^\epsilon - 1)^2}
\]

\[
K \approx \sum_{\alpha=x,y} \int_{\sqrt{2\omega_0}}^{\omega_{kO}^{(\alpha)}} \frac{h\omega_{kO}^{(\alpha)}}{\partial T} \frac{d\omega_{kO}^{(\alpha)}}{2\pi} + \sum_{\alpha=x,y} \int_{\sqrt{\omega_{kO}^{(\alpha)}}}^{\sqrt{\omega_{kO}^{(\alpha)}}} h\omega_{kO}^{(\alpha)} \frac{\partial N(\omega_{kO}^{(\alpha)}, T)}{\partial T} \frac{d\omega_{kO}^{(\alpha)}}{2\pi}
\]

\[
= \frac{k_B^2 T}{\hbar} \left[ \int_{0}^{2\omega_0} d\epsilon \frac{3\epsilon^2 e^\epsilon}{(e^\epsilon - 1)^2} + \int_{\sqrt{\omega_{\xi}}}^{\sqrt{\omega_{\xi}}} \frac{d\epsilon}{\sqrt{\omega_{\xi}}} \frac{2\epsilon^2 e^\epsilon}{(e^\epsilon - 1)^2} \right] \quad (\text{S.A7})
\]

where \( \epsilon_0 = h\omega_{0}/k_BT, \xi_P = h\omega_{0}P_0/(CS^2) \) with \( \delta = P_0/Q \) the ionic deformation, \((0, 2\epsilon_0)\) and \((\sqrt{2\epsilon_0}, \sqrt{6\epsilon_0})\) are the band edges in unit of \( k_BT \) for the three acoustics and two transverse optical phonon modes, respectively. Assuming \( \omega_{kO}^{(z)} \gg \omega_0 \forall k \), we disregarded the contribution of the longitudinal optical phonon band \((Oz)\) to the heat conductance.

The polarization transport by the two transverse optical modes is gapped and therefore exponentially suppressed at low temperatures.
B. PARALLEL POLARIZATION CONFIGURATION

When the electric dipoles point along the chain axis, the deformation costs the energy

$$\Phi = \frac{1}{2} \sum_{i, \alpha} C_{i}^{(\alpha)} [u_{i}^{(\alpha)} - v_{i}^{(\alpha)}]^{2} + \frac{1}{2} \sum_{i, \alpha} C_{i}^{(\alpha)} [u_{i}^{(\alpha)} - v_{i+1}^{(\alpha)}]^{2},$$

with notation as above. Here we discarded the dipolar interaction, as discussed in the previous section. Without loss of generality, we set both the electric polarization ordering and chain axis along the $z$-direction, and the lattice dynamical matrix reads,

$$D^{(\alpha)}(k, s, s') = \frac{1}{M} \begin{pmatrix}
C_{0}^{(\alpha)} + C_{1}^{(\alpha)}, & -C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb}, & C_{0}^{(\alpha)} + C_{1}^{(\alpha)} \\
-C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{-ikb}, & C_{0}^{(\alpha)} + C_{1}^{(\alpha)}, & C_{0}^{(\alpha)} + C_{1}^{(\alpha)} \\
C_{0}^{(\alpha)} + C_{1}^{(\alpha)}, & -C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb}, & C_{0}^{(\alpha)} + C_{1}^{(\alpha)}
\end{pmatrix}_{ss'},$$

where the lattice constant $b = 2a$ since the dipolar symmetry breaking doubles the size of the unit cell. This implies that the phonon bands are folded back with gaps at the Brillouin zone boundaries $k = \pi/b$. The eigenvalues and eigenvectors of the dynamic matrix are now

$$[\omega_{kO}^{(\alpha)}]^{2} = \frac{C_{0}^{(\alpha)} + C_{1}^{(\alpha)}}{M} \left(1 + \frac{1 - 2C_{0}^{(\alpha)} C_{1}^{(\alpha)}}{(C_{0}^{(\alpha)} + C_{1}^{(\alpha)})^{2}} (1 - \cos kb) \right)$$

$$[\omega_{kA}^{(\alpha)}]^{2} = \frac{C_{0}^{(\alpha)} + C_{1}^{(\alpha)}}{M} \left(1 - \frac{1 - 2C_{0}^{(\alpha)} C_{1}^{(\alpha)}}{(C_{0}^{(\alpha)} + C_{1}^{(\alpha)})^{2}} (1 - \cos kb) \right)$$

and

$$e_{kO}^{(\alpha)} = \frac{1}{\sqrt{2}} \begin{pmatrix} C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \\ C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \end{pmatrix}, \quad e_{kA}^{(\alpha)} = \frac{1}{\sqrt{2}} \begin{pmatrix} C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \\ C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \end{pmatrix},$$

where $O$ and $A$ indicate again the optical and acoustic modes, respectively. The vector relevant to the polarization dynamics reads now,

$$F_{kO}^{(\alpha)} = \frac{Q}{\sqrt{2M}} \begin{pmatrix} C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \\ C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \end{pmatrix}, \quad F_{kA}^{(\alpha)} = \frac{Q}{\sqrt{2M}} \begin{pmatrix} C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \\ C_{0}^{(\alpha)} + C_{1}^{(\alpha)} e^{ikb} \end{pmatrix},$$

where $e_{\alpha}$ is the unit vector along $\alpha$-direction. In the long-wavelength limit ($k \rightarrow 0$), $|F_{kO}^{(\alpha)}|^{2} \rightarrow 2Q^{2}/M$ and $|F_{kA}^{(\alpha)}|^{2} \rightarrow 0$. However, in contrast to the perpendicular configuration, the transverse acoustic phonons with finite $k$ now carry polarization because the phase difference in the displacements of the two ions in the same unit cell.

As above, we simplify the expressions by adopting $C_{0}^{(x)} = C_{0}^{(y)} = C_{1}^{(x)} = C_{1}^{(y)} = C$, $C_{1}^{(z)} = 2C$ and $C_{0}^{(z)}/C \gg 1$. the phonon dispersion now read

$$\omega_{kO}^{(x)} = \omega_{kO}^{(y)} = \sqrt{2\omega_{0}} \left[1 + \sqrt{1 + \frac{1 + \cos kb}{2}}\right]^{-1/2},$$

$$\omega_{kO}^{(z)} \approx \sqrt{\frac{C_{0}^{(z)}}{C}} \omega_{0} \left[1 + \sqrt{1 - \frac{2C_{0}^{(z)}}{C} (1 - \cos kb)}\right]^{-1/2} \gg \omega_{0},$$

$$\omega_{kA}^{(x)} = \omega_{kA}^{(y)} = \sqrt{2\omega_{0}} \left[1 - \sqrt{1 + \frac{1 + \cos kb}{2}}\right]^{-1/2},$$

$$\omega_{kA}^{(z)} \approx \sqrt{\frac{C_{0}^{(z)}}{C}} \omega_{0} \left[1 - \sqrt{1 - \frac{2C_{1}^{(z)}}{C_{0}^{(z)}} (1 - \cos kb)}\right]^{-1/2} \rightarrow \sqrt{2\omega_{0}} \sqrt{1 - \cos kb}.$$
This approximation restores translational symmetry for the transverse modes and the gaps at \( k = \pi/b \) vanish (see Fig. S1). The polarization carried by the transverse acoustic and optical phonons reads

\[
\xi_{kO}^{(a)} = -\frac{\hbar}{2P_0} \frac{|F_{kO}^{(a)}|^2}{\omega_{kO}^{(a)}} = \frac{Q^2 \hbar \omega_{kO}^{(a)}}{4P_0 C},
\]

\[
\xi_{kA}^{(a)} = -\frac{\hbar}{2P_0} \frac{|F_{kA}^{(a)}|^2}{\omega_{kA}^{(a)}} = \frac{Q^2 \hbar \omega_{kA}^{(a)}}{4P_0 C},
\]

(S.B7)

where \( \alpha = x, y \). In contrast to the perpendicular case, the polarization carried by the transverse optical and acoustic modes is proportional to their frequencies. With Eq. (S.B6) and Eq. (S.B7), the transport coefficients are then given by

\[
G = -\frac{1}{\hbar} \sum_{\alpha=x,y} \left[ \int_0^{\sqrt{2}\omega_0} \left[ \xi_{kA}^{(a)} \right]^2 \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial \omega_{kA}^{(a)}} d\omega_{kA}^{(a)} \right] + \int_0^{2\omega_0} \left[ \xi_{kA}^{(a)} \right]^2 \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial \omega_{kA}^{(a)}} d\omega_{kA}^{(a)}
\]

\[
\Pi = ST = G^{-1} \sum_{\alpha=x,y} \left[ \int_0^{\sqrt{2}\omega_0} \omega_{kA}^{(a)} \xi_{kA}^{(a)} \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial \omega_{kA}^{(a)}} d\omega_{kA}^{(a)} \right] + \int_0^{2\omega_0} \omega_{kA}^{(a)} \xi_{kA}^{(a)} \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial \omega_{kA}^{(a)}} d\omega_{kA}^{(a)}
\]

\[
K \approx \sum_{\alpha=x,y} \left[ \int_0^{\sqrt{2}\omega_0} \hbar \omega_{kA}^{(a)} \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial T} d\omega_{kA}^{(a)} \right] + \int_0^{2\omega_0} \hbar \omega_{kA}^{(a)} \frac{\partial N(\omega_{kA}^{(a)}, T)}{\partial T} d\omega_{kA}^{(a)} + \int_0^{2\omega_0} \hbar \omega_{kA}^{(z)} \frac{\partial N(\omega_{kA}^{(z)}, T)}{\partial T} d\omega_{kA}^{(z)}
\]

(S.B8)

where \((0, \sqrt{2}\omega_0), (0, 2\omega_0)\) and \((\sqrt{2}\omega_0, 2\omega_0)\) are the band edges for \( Ax \ (Ay) \), \( Az \) and \( Ox \ (Oy) \) modes, respectively. In contrast to the case of perpendicular configuration, the massless transverse acoustic phonons contribute significantly to the polarization transport. We disregarded again the contribution of the high-frequency longitudinal optical mode \( Oz \) to the heat conductance.

[1] Ping Tang, Ryo Iguchi, Ken-ichi Uchida and G.E.W. Bauer, The Ferroelectric Point Contact, submitted.