Que será será?
The uncertainty estimation of feature-based time series forecasts

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Abstract

Interval forecasts have significant advantages in providing uncertainty estimation to point forecasts, leading to the importance of providing prediction intervals (PIs) as well as point forecasts. In this paper, we propose a general feature-based time series forecasting framework, which is divided into “offline” and “online” parts. In the “offline” part, we explore how time series features connect with the prediction interval forecasting accuracy of different forecasting methods by exploring generalized additive models (GAMs), which makes our proposed framework interpretable in the effects of features on the interval forecasting accuracy. Our proposed framework is in essence a model averaging process and we introduce a threshold ratio for the selection of individual forecasting methods in this process. In the “online” part, we calculate the point forecasts and PIs of new series by pre-trained GAMs and the corresponding optimal threshold ratio. We illustrate that our feature-based forecasting framework outperforms all individual benchmark forecasting methods on M3 competition data, with an improved computational efficiency.

Keywords: Prediction intervals, Uncertainty estimation, Model averaging, Generalized additive models, Time series features, Threshold ratio

1. Introduction

With the advent of the big data era, a large amount of time series data are continuously collected, which leads to explosive demand for time series forecasting methods. Time series forecasting has played a pivotal role in the development of many fields, including finance (Kim, 2003), meteorology (Rolph et al., 2017) and signal processing (Lu et al., 2009). The vast majority of time series forecasting literature aims to improve point forecasting accuracy, and they mainly forecast the mean (Makridakis and Hibon, 2000; Zhang, 2003; Talagala et al.,...
or the median (Freeland and McCabe, 2004) of the distributions for future observations. However, it is also important to quantify the uncertainty of the prediction, which measures the reliability of the forecasting results. As a result, there is great demand for forecasting methods that can provide a comprehensive outlook of the expected future values and the future uncertainty in many fields of research (Gneiting and Katzfuss, 2014; Taieb et al., 2015).

Petropoulos et al. (2014) point out that the specific features of data play a great role in improving the performance of forecasting methods. There are *horses for courses*, and it is unlikely that a single forecasting method works for all data. Appropriate features extracted from time series data may provide a great perceptiveness of time series analysis (Hyndman, 2001). The idea of characterizing time series with features has been widely used in various time series analyses. Fulcher and Jones (2014) classify time series based on extracted interpretable features. Wang et al. (2006) and Guo et al. (2008) present a feature-based method for time series clustering. Kang et al. (2017) apply principle component analysis to the dimensionality reduction of time series features and visualise forecasting algorithm performance in a two-dimensional instance space. Most recently, Kang et al. (2019) propose GRATIS (GeneRAting TIme Series with diverse and controllable characteristics) based on mixture autoregressive (MAR) models. They also demonstrate the feature diversity and coverage of the generated data. Moreover, Talagala et al. (2018) propose FFORMS (Feature-based FORecast-model Selection) framework that identifies the best forecasting model using time series features based on the random forest.

However, compared to point forecasting, the literature on the uncertainty estimation of feature-based time series forecasting is highly limited, although there has been a rapid expansion of probabilistic forecasting methods in various fields, such as energy (Wan et al., 2014; Hong et al., 2016; Jeon et al., 2019), hydrology (Krzysztofowicz, 2001) and finance (Groen et al., 2013). The M4 competition (Makridakis et al., 2018, 2019) is a continuation of M1 and M3 competitions (Makridakis et al., 1982; Makridakis and Hibon, 2000), which are a series of time series competitions and enable a comparison of various forecasting methods across time series with different periods. Unlike the other two competitions, the M4 competition requires participants to provide not only the point forecasting results, but also the prediction intervals. Montero-Manso et al. (2018) compute the point forecasts by FFORMA (Feature-based FORecast Model Averaging), and get the prediction intervals by a linear combination of the 95% bounds of naïve, theta and seasonal naïve methods. This approach ranks the second in the M4 competition. The approach has two obvious drawbacks: (i) the interval forecasting for all time series is the simple linear combination of naïve, theta and seasonal naïve methods,
which does not satisfy the *horses for courses*; and (ii) it is impossible to clarify the impacts of the selected features on the time series interval forecasting performances. Taieb et al. (2015) explore the traditional regression and quantile regression probabilistic forecasting methods based on boosted additive models to obtain better and more interpretable probabilistic forecasts. Nonetheless, they have not carried out probabilistic forecasting from the perspective of time series features, and still cannot explain the impact of features on probabilistic prediction. Originally developed by Hastie and Tibshirani (1990), generalized additive models (GAMs) have the virtues of interpretability, flexibility and reduction of overfitting. In this paper, we apply the GAMs to the feature-based interval forecasting, which makes interval forecasts interpretable regarding time series features.

There are three mainstream strategies for time series forecasting: (i) forecasting all the time series by a single method; (ii) choosing the most appropriate method for each time series; and (iii) averaging forecasting results from different methods. It is almost impossible to have a forecasting method that performs well on all time series (Petropoulos et al., 2014). Clemen (1989) argues that the average value of the forecasts is superior to the individuals used for averaging in forecasting accuracy. Atiya (2019) illustrates graphically why forecast combination works well and points out that we should exclude the forecasts perform poorly in the pool when combining forecasts (see also Kourentzes et al., 2019). Lichtendahl Jr et al. (2013) show that averaging quantiles outperforms averaging probabilities in some cases, which provides us with the theoretical basis for averaging the prediction intervals. Moreover, the M4 competition, the latest M-series competition, shows that the combination of methods is the winner: 12 of the 17 most accurate submission methods are combinations of forecasting models (Makridakis et al., 2018, 2019). However, this does not mean that increasing number of methods used for model averaging leads to better forecasting performance (Fildes and Petropoulos, 2015). Therefore, in this paper, we design a threshold ratio to select a plurality of apposite forecasting methods for each time series for model averaging.

The explosion of massive time series data leads to the demand for a fast automatic learning algorithm (Talagala et al., 2018), which is able to quickly identify suitable methods used for model averaging for each individual time series. Inspired by Talagala et al. (2018) and Montero-Manso et al. (2018), we propose a general feature-based time series forecasting framework in this paper, which is divided into “offline” and “online” parts. In the “offline” part, we train GAM of interval forecasting accuracy on features for each individual candidate forecasting method. Furthermore, we explore the optimal threshold ratio trained from the reference dataset. In all the pre-defined threshold ratios, the optimal threshold ratio should perform
best in choosing the number of candidate methods used for model averaging.

The reference dataset used for training algorithm in offline part should cover the new time series need to be predicted. The common methods of determining the reference dataset in vast literature focus on simulating series from candidate models, directly using the existing competition dataset and splitting from the observations of the test dataset. Talagala et al. (2018) produce reference data by simulating from exponential smoothing models and ARIMA models. Montero-Manso et al. (2018) propose a feature-based forecast model averaging framework and apply it to forecast M4 competition data. In their work, they split each series of M4 dataset into a training period and a test period, then consider the dataset that comprised by part of the training period for each series as the reference data. However, it is difficult to evaluate whether the reference dataset obtained by the above methods works for diverse algorithms. In this paper, the reference dataset we select in the offline of our proposed framework should cover the test dataset in feature space.

In the “online” part, we only have to extract the features from the new time series data and put them into the trained GAMs to obtain the predicted interval forecasting accuracy for each forecasting method. According to the optimal threshold obtained offline, we may match the appropriate number of methods (the methods are also selected) at the aim of model averaging for each new sample. Most time-consuming calculations are accomplished offline, which enables the time series forecasting framework proposed in this paper to overcome the computational challenges.

The rest of the paper is organized as follows. Section 2 introduces the general feature-based time series forecasting framework proposed in this paper. We elaborate on the components and details of this forecasting framework. Section 3 applies the proposed framework to the M3 competition data. We explore how time series features affect the interval forecasting accuracy of different forecasting methods by trained GAMs. We illustrate that our forecasting framework provides the best prediction interval forecasts compared to all individual benchmark methods on M3 data, with improved computational efficiency. Moreover, although the proposed general framework is designed for calculating prediction intervals, we also get a higher accuracy of point prediction than the benchmark methods. Section 4 concludes the paper.

2. Methodology

We propose a general framework for feature-based time series forecasting, as shown in Figure 1, which consists of the “offline” and “online” parts. We train the model averaging al-
Figure 1. Feature-based time series forecasting framework. The offline and online parts are shown in blue and orange, respectively.

gonym in the offline part and apply this pre-trained algorithm to new time series forecasting in the online process. The objective of our proposed framework is to discriminate the appropriate forecasting methods for each time series data and to use these selected methods for model averaging to improve the interval forecasting accuracy.

The effectiveness of our proposed framework rests on a fundamental assumption that the reference dataset and the test dataset come from the same population. In other words, the reference dataset and test dataset are sampled from one population and have a similar data generating process (Montero-Manso et al., 2018). This assumption ensures that the pre-trained algorithm based on the reference dataset can be used to the test dataset. Specifically, our proposed framework focus on the feature space, so the reference dataset with features as diverse as the test dataset contributes to improving the prediction accuracy.

In our proposed framework, most of the time-consuming calculations are achieved offline. Firstly, we split each time series of the reference dataset into a training period and a testing period. From each training period, we extract features (see in Section 3.2) that reveal the intrinsic nature of time series, such as length, trend and seasonality. Secondly, we have a pool of forecasting methods (e.g., Naïve, ARIMA, etc.) as benchmark methods, as described in Section 3.2. Thirdly, applying each of the candidate forecasting methods to the reference dataset, we obtain the point forecasts and the prediction intervals of all the benchmark methods. Finally, we train GAM (see in Section 2.2) for each benchmark method to link the calculated features with
the scores that reflect interval forecasting accuracy (see Section 2.1 for the details). We also utilize the fitted values from the trained GAMs to search the optimal threshold ratio for model averaging, as described in Section 2.3. In the online part, we extract features from a new time series and put them into the pre-trained GAMs. Then the optimal threshold obtained from the offline process contributes to the identification of advantageous candidate methods used for model averaging.

The proposed framework is a general procedure, we specifically pick out the appropriate features and candidate prediction methods for the time series need to be predicted. Moreover, we can consciously choose apposite models for the time series being analysed to link the time series features with the interval forecasting accuracy in our proposed framework. In this paper, we apply GAMs to achieve this goal, so we can refer to the framework as GAMMA (GAM-based Model Averaging). We then describe the details for the GAMMA framework in the following sections.

2.1. Interval forecast evaluation

When referring to the “forecast” in the time series analysis, we commonly intend the point forecast. In general, point forecast is expressed as the mean (Makridakis and Hibon, 2000; Zhang, 2003; Talagala et al., 2018) of the forecast distribution and occasionally as the median (Freeland and McCabe, 2004). However, point forecast is merely a specific value that is in a position to depict the forecast distribution. Interval forecast is a good choice when we have to measure the uncertainty of the point forecast (Chatfield, 1993). In this paper, we apply prediction intervals (PIs) to assess future uncertainty of our feature-based time series forecasts.

Gneiting and Raftery (2007) consider a scoring rule for evaluating the accuracy of prediction intervals, named the interval score. The M4 competition requests participants to submit point predictions as well as prediction intervals. The M4 competition adopts the mean scaled interval score (MSIS) as the scoring rule to evaluate the accuracy of the prediction intervals, which is an extension of the interval score.

In this paper, we adopt the central $(1 - \alpha) \times 100\%$ prediction intervals for estimating the uncertainty in point forecasts. Thus, the calculation of MSIS that can evaluate the performance of the generated PIs can be stated as follows:

$$
MSIS = \frac{1}{h} \sum_{t=n+1}^{n+h} (U_t - L_t) + \frac{\alpha}{n-m} \sum_{t=m+1}^{n} |Y_t - Y_{t-m}| \{ Y_t < L_t \} + \frac{\alpha}{n-m} \sum_{t=m+1}^{n} |Y_t - Y_{t-m}| \{ Y_t > U_t \},
$$

where $Y_t$ are the true values of the future, $[L_t, U_t]$ the generated prediction intervals, $h$ the forecasting horizon, $n$ the length of the historical data, and $m$ the time interval symbolizing the length of the time series periodicity, for example, the yearly, quarterly, and monthly data.
are 1, 4, and 12, respectively. \( 1 \) is the indicator function, which returns 1 when the judgment function inside is true, otherwise returns 0. \( \alpha \) is the level of significance of the generated prediction intervals.

Equation (1) illustrates the logic and calculations of MSIS. The numerator is a penalty for the width of the generated prediction interval and the cases that the generated prediction interval does not cover the true value of the future. The denominator is the scale of the scoring rule by historical data, which makes the MSIS scale-independent.

Another supplemental scoring rule we utilize is the absolute coverage difference (ACD), which is also a measure used in the M4 competition for evaluating accuracy of prediction intervals (Makridakis et al., 2018). ACD does not directly determine the quality of the forecasting method, but is a supplementary evaluation indicator. It reflects the absolute difference between the actual coverage of the prediction interval generated by the forecasting method and the confidence level \((1 - \alpha)\) of the expected prediction interval.

2.2. Linking time series features with interval forecasting accuracy

A critical step in our proposed time series forecasting framework is to capture how time series features affect the interval forecasts of each benchmark method. In the past three decades, generalized linear models (GLMs) and generalized additive models (GAMs) have been continuously developed and become the focus of statistical regression analysis. GLMs are extensions of classical linear models. Compared to classical linear models, what GLMs have greatly improved are: (i) no longer limited to the assumption of Gaussian distributed, the distribution of the response variable in GLMs may be any one of exponential family distributions (Guisan et al., 2002); and (ii) GLMs establish the relationships between the expected value of response variable and the linear combination of predictors by link function \( g \), this can also constrain the predictions to the range of possible values of response variable.

Furthermore, GAMs are semi-parametric generalizations of GLMs. Compared to GLMs that can only cope with linear and monotonic data structures, GAMs only have to assume that the function terms are additive and smooth. In general, the GAM has the form (Wood, 2017):

\[
g(E(Y)) = A\theta + s_1(X_1) + s_2(X_2) + ... + s_p(X_p),
\]

where \( Y \) is the response variable, \( X = (X_1, X_2, ..., X_p) \) is a matrix of \( p \) explanatory variables, \( g \) is the link function used to establish the relationship between \( E(Y) \) and the set of explanatory variables \( X \), \( A \) is a matrix of strictly parametric model components, \( \theta \) is a vector of corresponding parameters, and the terms \( s_1(\cdot), s_2(\cdot), ..., s_p(\cdot) \) are smooth, non-parametric functions, one for each covariate \( X_k \).
Therefore, we first establish GAMs to characterize the relationship between interval forecasting accuracy and time series features in the offline part. Since MSIS values are all positive, we take the logarithm form of the MSIS scores to expand their range to the real number set \( \mathbb{R} \). Considering \( p \) extracted features and \( M \) pre-prepared candidate forecasting methods, the GAM we trained for the \( i \)-th benchmark method can be written as:

\[
g(E(\log(\text{MSIS}_i))) = \beta_{i0} + \beta_{i1}F_1 + \ldots + \beta_{ik}F_k + s_{i1}(F_{k+1}) + \ldots + s_{i(p-k)}(F_p),
\]

where \( i = 1, 2, \ldots, M \), MSIS\(_i\) is the score vector of \( i \)-th method, \( F = \{F_1, \ldots, F_p\} \) denotes a predictor matrix consisting of extracted features, \( F_1, \ldots, F_k \) are linear predictors with dummy features, \( F_{k+1}, \ldots, F_p \) are predictors that can be modelled non-parametrically in addition to linear terms, \( g \) is the link function used to establish the relationship between mean of the response variable and the set of predictors, \( \beta_{i0} \) denotes the intercept of the regression, \( \beta_{i1}, \ldots, \beta_{ik} \) are regression coefficients of linear terms, and the terms \( s_{i1}(\cdot), \ldots, s_{i(p-k)}(\cdot) \) are smooth, non-parametric functions.

The key merits of GAM are: (1) Interpretability. The marginal effects of each predictor on the response variable are not interfered by other predictors because the model is additive. Therefore, we can apply GAM to explore the partial effects of each predictor on the response variable. (2) Flexibility. In GAM, we can specify the form of the smooth function according to the different influence of each predictor on the response variable. Besides, smooth functions are no longer restricted to linear and polynomial relationships. (3) Automation. Predictor functions are automatically derived during model estimation (Larsen, 2015). It is not necessary to specify the form of smooth function in advance. (4) Regularization. The model is able to prevent over-fitting by controlling the smoothness of the predictor functions.

The balance between model flexibility and computational complexity is the main challenge for GAMs. It is necessary both to determine the form of smooth functions and to control smoothness of these functions. In this paper, we estimate the GAMs by penalized iterative least squares (PIRLS), introduced in R package mgcv (Wood and Wood, 2015). By minimizing the generalized cross validation score (GCV), the package synchronously identifies the degrees of freedom for each smooth function in the process of model fitting. In mgcv, smooth functions in Equation (2) are determined by selecting the appropriate penalty for each pre-prepared basis function, thus controlling its degrees of freedom through a single smoothing parameter (Wood, 2001).

It is worth mentioning that GAM is just a tool to characterize the relationship between the time series features and interval forecasting accuracy in the offline part of our proposed framework. We can consciously choose an apposite model to replace GAM for the time series
being analysed.

2.3. Optimal threshold ratio search

Vast literature has focused on forecast combination of time series. It’s worth mentioning that we have to control the maximum number of methods used for combining (Fildes and Petropoulos, 2015). In this section, we define the threshold ratio in the offline part and find the optimal threshold. Threshold ratio is a solution proposed to identify the appropriate methods for time series, and then apply these methods to model combination. In this way, the number of methods in the combination pool for each series is different.

In mathematics, softmax function maps a \( K \)-dimensional arbitrary real vector to another \( K \)-dimensional probability vector. In general, the softmax function can be expressed as:

\[
p(x_j) = \frac{e^{x_j}}{\sum_{k=1}^{K} e^{x_k}},
\]

where \( j = 1, 2, ..., K \), and \( X = (x_1, ..., x_K) \) is a \( K \)-dimensional vector of arbitrary real values. The softmax function has two appealing properties: (i) each element in the converted probability vector takes a value between 0 and 1; and (ii) the sum of all the elements in the probability vector is equal to 1.

Based on the properties of softmax function, we can transform the fitted values obtained from the pre-trained GAMs for each time series into probabilities. However, it is apparent from the softmax function that a large value of \( x_j \) in \( X \) corresponds to a larger probability than other elements. Therefore, we define the adjusted softmax function for \( i \)-th time series as:

\[
P_{ij} = \frac{\exp \left( \frac{\mu_i - \log(MSIS_{ij})}{\sigma_i} \right)}{\sum_{k=1}^{M} \exp \left( \frac{\mu_i - \log(MSIS_{ik})}{\sigma_i} \right)}, \tag{3}
\]

where \( i = 1, ..., N \) and \( j = 1, ..., M \), \( \mu_i \) and \( \sigma_i \) denote the mean and standard deviation of the fitted values obtained by the \( M \) pre-trained GAMs for \( i \)-th time series, respectively.

According to the adjusted softmax function, the method with a smaller MSIS value in the \( M \) candidate methods for each time series has a larger probability than the other methods. Consequently, we can pick the appropriate methods for each time series based on the obtained probabilities. A similar approach has been suggested in the literature to combine models based on Akakie’s information criteria weights (Kolassa, 2011). The pseudo code for searching the optimal threshold ratio obtained from the reference dataset is presented in Algorithm 1. The process of searching the optimal threshold ratio demonstrates that the threshold ratio determines the number of candidate methods selected for model averaging. The threshold ratio quantifies as a number between 0 and 1, where 0 indicates all the benchmark methods
are selected and 1 indicates only the benchmark method with the minimal fitted log(MSIS) is
selected.

**Algorithm 1** The optimal threshold ratio search

**Input:**

- \( O = \{x_1, x_2, \ldots, x_N\} \): the collection of \( N \) time series in the reference dataset.
- \( Tr = \{Tr_1, Tr_2, \ldots, Tr_q\} \): the set of \( q \) pre-set threshold ratios.
- \( M \): the number of benchmark forecasting methods.

**Output:**

The optimal threshold ratios for yearly, quarterly and monthly data.

1: \( \text{for } i = 1 \text{ to } q \text{ do} \)

2: \( \text{for } j = 1 \text{ to } N \text{ do} \)

3: Obtain the fitted log(MSIS) of \( x_j \) from the \( M \) pre-trained GAMs in the offline.

4: Apply the Equation (3) to calculate the adjusted softmax transformation \( P \) for \( x_j \).

5: Calculate the ratio of \( P \): \( R_k = P_k / \max_{1 \leq k \leq M} (P_k) \).

6: Select the benchmark methods that satisfy \( R_k \geq Tr_i \) for \( x_j \) and utilize these methods
   for forecast combination (see Section 2.4 for the details).

7: Calculate the MSIS value of \( x_j \).

8: \( \text{end for} \)

9: Calculate the average MSIS values of yearly, quarterly and monthly data.

10: \( \text{end for} \)

11: The optimal threshold ratios are pre-set threshold ratios with minimal MSIS for the yearly,
    quarterly and monthly series in \( O \), respectively.

**2.4. Interval combination methods**

After the attempt to choose the appropriate methods for each time series by setting thresh-
hold ratio, the next question we have to solve is how to combine the prediction intervals calculated
from the selected methods? Inspired by the previous studies on quantities combination
(Hora, 2004; Lichtendahl Jr et al., 2013), we consider two interval combination methods in
this paper. The two combination methods considered are the simple average and the weighted
average.

Specifically, we illustrate the forecasting combination calculation of point forecasts. Assuming \( T \) benchmark forecasting methods are selected for a time series according to a pre-
defined threshold ratio, the $h$-step point forecast for the weighted average is defined as:

$$f_{\text{weighted}} = \frac{1}{T} \sum_{k=1}^{T} P_k f_k,$$

where $k = 1, 2, ..., T$, $f_{\text{weighted}}$ is combined point forecast of the weighted average, $f_k$ is the point forecast for one of the selected benchmark methods, $P_k$ denotes the probability of the $k$-th method being selected and is calculated from the adjusted softmax function. At this time, $P_k$ represents the weight in the weighted average method. If $P_k = 1$, $f_{\text{weighted}}$ reduces to the simple average.

Compared to point forecast, the prediction interval is related to an associated probability. We calculate the “lower radius” and “upper radius” of the prediction intervals for the selected benchmark methods. Therefore, the lower bound and upper bound of the model averaged prediction interval are obtained by subtracting the averaged lower radius and adding the averaged upper radius on the basis of the combined point forecast respectively, as shown in Equation (5).

$$f_{\text{weighted}}^l = f_{\text{weighted}} - \frac{1}{T} \sum_{k=1}^{T} P_k (f_k - f_k^l),$$

$$f_{\text{weighted}}^u = f_{\text{weighted}} + \frac{1}{T} \sum_{k=1}^{T} P_k (f_k^u - f_k),$$

where $k = 1, 2, ..., T$, $f_k^l$ and $f_k^u$ are the lower bound and upper bound of the $h$-step prediction interval for one of the selected benchmark methods. Therefore, in our proposed feature-based framework, we denote the $h$-step prediction interval calculated by the weighted average combination as $[f_{\text{weighted}}^l, f_{\text{weighted}}^u]$. Same as the combined point prediction, $[f_{\text{weighted}}^l, f_{\text{weighted}}^u]$ reduces to the simple average combination when $P_k = 1$.

3. Application to the M3 competition data

3.1. The reference dataset and testing dataset

As we reasoned in Section 2, a reference dataset that comes from the same population with the test dataset need to be prepared in advance. In this regard, the reference dataset we selected should cover the M3 data in feature spaces since our proposed framework is based on features. More recently, Kang et al. (2019) propose GRATIS that generates time series by mixture autoregressive models and demonstrates the diversity and coverage of generated time series in feature spaces. They also investigate that the time series generated by GRATIS cover the M3 data in the feature space. Therefore, in this paper, we follow Kang et al. (2019) and
generate 10000 yearly, quarterly and monthly time series that have the same forecast horizon with the M3 data based on GRATIS. We consider a random sample from the distributions of sample size on the M3 data (see Figure 2) as the length of the generated time series, making the length of our generated time series consistent with that of the M3 data. Then we consider the generated time series as the reference dataset. In this way, the algorithm trained by the reference dataset in the offline part can be extended to M3 dataset.

To evaluate the benefits of our proposed feature-based time series forecasting framework, we consider the yearly, quarterly and monthly series on M3 dataset as test dataset. The M3 dataset originates from the M3 competition (Makridakis and Hibon, 2000) devoted to exploring methods with high prediction accuracy, and captures various types of series such as microeconomic, industry and macroeconomic. The test dataset contains 2829 time series: the yearly data include 645 series with sample size ranging from 14 to 41 observations and forecast horizons of 6 periods; the quarterly data have 756 series, 8 forecast horizons and sample size ranging from 16 to 64 periods; the monthly data contain 1428 time series with a constant horizon of 18 periods, ranging from 48 to 126 sample observations. As shown in Figure 2, the sample sizes of the yearly, quarterly and monthly data in M3 competition are not constant, but vary in different distributions. The details of the reference and test dataset are summarized in Table 1.
3.2. Experimental setup

In this section we outline the features and benchmark methods that we use in the application to forecast M3 dataset, as well as analyse the partial effects of features on the interval forecasting accuracy of each benchmark method based on the pre-trained GAMs. Besides, we present the optimal threshold ratios captured in reference data.

The features used in this experiment are the same with the features in Montero-Manso et al. (2018). These 42 features capture the characteristic of time series from various aspects, such as trend that describes the strength of the trend of time series, e-acf1 that represents the first ACF value of reminder series, and peak which indicates the location of the maximum value in the seasonal component and STL decomposition of the series. The features nperiods and seasonal-period are categorical variables: nperiods takes the value 0 or 1, and seasonal-period takes the value 1, 4, or 12. Multiple dummy variables need to be created from the feature seasonal-period: seasonal-period-q (takes a value of 1 when the value of seasonal-period is 4, otherwise 0) and seasonal-period-m (takes a value of 1 when the value of seasonal-period is 12, otherwise 0). Therefore, we actually have 43 features in this experiment, and should add the dummy features as linear predictors to the GAM (see Equation (2) for the details) in the offline part.

We consider eight forecasting methods as our benchmark methods, as shown in Table 2, and these methods can be implemented in R package forecast (Hyndman et al., 2018a). From the forecast package, we can obtain the point forecasts, as well as the prediction intervals for each benchmark method. For yearly series, the forecasting results of the naive method essentially coincide with that of naïve. Therefore, there are actually seven forecasting methods in the pool of methods for yearly series.

Given the features and candidate methods, we first calculate the point forecasting accuracy by MASE (Hyndman and Koehler, 2006) and the interval forecasting accuracy measured by MSIS for all benchmark methods on reference dataset. Figure 3 shows that the distributions of point prediction accuracy for different benchmark methods are obviously similar to that of interval forecasting accuracy. For example, for the yearly series the median and variance of the point and density prediction accuracy of stlm-ar method are significantly larger than that of auto-arima, ets and tbats. Moreover, auto-arima and ets perform well in both point and density prediction for yearly, quarterly and monthly series in reference data, while stlm, naive and naive methods perform poorly compared to other benchmark methods. Therefore, a prediction method with higher point forecasting accuracy is easier to get higher interval forecasting accuracy.
Table 2. Benchmark methods considered in the application to the M3 competition data.

| Benchmark method | Description |
|------------------|-------------|
| auto-arima       | The best autoregressive integrated moving average model that automatically selected by either AIC, AICc or BIC value. |
| ets              | Exponential smoothing state space model proposed by Hyndman et al. (2002). |
| tbats            | The exponential smoothing state space model with Trigonometric, Box-Cox transformation, ARMA errors, Trend and Seasonal components. |
| stlm-ar          | Time series is decomposed by STL method proposed by Cleveland et al. (1990), then we fit an AR model for the seasonally adjusted series. |
| rw-drift         | Random walk with drift. |
| thetaf           | A univariate forecasting model, proposed by Assimakopoulos and Nikolopoulos (2000), and perform well in M3 data, especially for monthly data. |
| naive            | The simplest time series forecasting method. The point forecasts of all the forecast horizons are equal to the last observation in the training period. |
| snaive           | Seasonal naive. The point forecast is equal to the most recent value of same season. |
Figure 3. Boxplots of point and density prediction accuracy over reference dataset for benchmark methods auto-arima, ets, tbats, stlm-ar, rw-drift, thetaf, naïve and snaïve. The boxplots of yearly, quarterly and monthly series are shown in the first, second and third column, respectively. The top row subplots the boxplots of point prediction accuracy measured by MASE, while the bottom row shows the boxplots of interval forecasting accuracy evaluated by MSIS. In all the plots, outlying values are removed.
Furthermore, we get the feature matrix \( F_{N \times p} \) and accuracy matrix \( MSIS_{N \times M} \) for \( N \) time series in reference dataset and \( M \) prepared benchmark methods. Consequently, GAMs are modelled for all eight benchmark methods, characterizing the relationship between \( F_{N \times p} \) and \( MSIS_{N \times M} \). These trained GAMs give a comprehensive description of the partial effects of features on accuracy of forecasting methods. From the perspective of one benchmark method, the trained GAMs help us to analysis how each feature affects the prediction accuracy of the forecasting method. From the perspective of one feature, the trained GAMs play a great role in comparing the effects of one feature on different candidate methods.

For demonstration purpose, we analyse the partial effects of time series features on the interval forecasting accuracy of all benchmark methods. Figure 4 combines the partial effect plots of all benchmark methods and contributes to our analysis of comparing partial effects of different methods. On the one hand, different features have different partial effects on the same benchmark method. Taking auto-arima method as an example, if we keep other features fixed, the interval forecasting accuracy generally shows an increasing trend as the value of \( x\text{-acf}_1 \) increases. As the values of nonlinearity and seasonal-strength increase, the prediction accuracy of auto-arima method decreases first and then increases. However, peak that indicates the location of the maximum value in the seasonal component and STL decomposition of the series has no significant impact on the prediction accuracy of auto-arima method. On the other hand, the partial effects of the same feature on each benchmark method are different, while they behave similarly in some cases. As the value of \( x\text{-acf}_1 \) increases, the interval forecasting performance of the eight benchmark methods gets a sequential improvement under the condition of keeping other features fixed. The MSIS values of auto-arima, ets and tbats are increasing first and then decreasing by a similar way with the increasing of seasonal-strength, while MSIS values of other benchmark methods show a overall trend of constantly increasing. Therefore, when we have to forecast a time series with a large value of seasonal-strength, we prefer to choose auto-arima, ets and tbats methods. Only the prediction accuracy of the naïve method is obviously affected by the feature peak among all the eight benchmark methods. These partial effects analysis give the prospect for the selection of the appropriate forecasting method used for interval forecasting based on the time series features.

The time series features used for selecting appropriate methods should perform discriminatively on the partial effects of different benchmark methods. The features with the similar growth path of partial effects on all benchmark methods play a weak role in the model selection process. Figure 4 shows that arch-acf, alpha, beta, non-linearity, seasonal-strength, peak, trough, hw-beta, hw-gamma may have a significant impact on our appropriate model
Figure 4. The partial effects of features (x-axis) on log(MSIS) (y-axis) from trained GAM models for reference data with benchmark methods auto-arima, ets, tbats, stlm-ar, rw-drift, thetaf, naive and snaive. Plots contain 40 features and 3 dummy features are removed.
selection used for model averaging. For example, the partial effects of nonlinearity on eight benchmark methods have different growth path. As the value of nonlinearity increases, the MSIS values of rw-drift, stlm-ar and thetaf increase first and then decrease with a slightly change. The MSIS value of naïve method decreases first and then increases with the increasing of nonlinearity. Nonlinearity has no effect on the performance of naïve. However, the performance of auto-arima, tbats and ets varies significantly with the value of nonlinearity. We prefer to auto-arima, tbats and ets when we have a series with a large value of nonlinearity, as shown in Figure 4.

We then apply all the pre-trained GAMs to find the optimal thresholds that perform best on selecting appropriate methods for yearly, quarterly and monthly series on the reference dataset. As we introduced in Section 2.4, two interval combination methods are considered in this paper: the simple average and the weighted average. Figure 5 depicts the search process for the optimal threshold ratios on the reference dataset. A larger threshold value means that fewer methods are selected for model averaging, while a smaller threshold value means that much more methods are used for model averaging. In particular, a threshold value of 1 means only the benchmark method that takes the minimum value of fitted log(MSIS) is selected, that is to say, the algorithm becomes a model selection process instead of a model averaging process. A threshold value of 0.1 indicates that the benchmark methods with a ratio of $P$ (see Section 2.3) greater than 0.1 are selected, allowing more methods in the pool selected for model averaging. It can be seen from the left panel that the averaged MSIS of the yearly, quarterly, and monthly series are decreasing first and then increasing as the pre-set threshold increasing. This indicates that it is not the more the number of benchmark methods used for model averaging is, the better the forecast performance is. As shown in Figure 5, the optimal thresholds for yearly, quarterly and monthly series are all set to 0.4 for the simple average method. For the weighted average combination, we set the optimal thresholds for yearly, quarterly, and monthly series as 0.2, 0.2 and 0.1, respectively.

### 3.3. Forecasting results

In this section, we present the forecasting results of our proposed GAMMA framework on the M3 dataset and compare it with the performances of benchmark methods. Moreover, we replace GAMs with the linear regression models and set threshold ratio as 1 (actually a model selection process) to illustrate the positive role of GAMs in our framework. We refer to this framework based on the linear regression models as LMMS (LM-based Model Selection).

For each time series in M3 competition data, appropriate benchmark methods selection is performed according to the smallest predicted logarithm of MSIS values. In this paper, we
Figure 5. Optimal threshold ratio search path for yearly, quarterly and monthly series on the reference dataset. The left panel shows the optimal threshold search of the simple average combination method, while the right panel shows that of the weighted average combination method. The points with minimal MSIS for yearly, quarterly and monthly series are marked with a cross.

adopt a 95% prediction interval for quantifying the uncertainty of the prediction. Table 3 presents the MSIS values calculated from the eight benchmark methods and our proposed GAMMA framework on each group of time series in M3. In general, our proposed GAMMA framework outperforms all individual benchmark methods on M3. When we set threshold ratio as 1, the performance of GAM based framework (GAMMS) outperforms that of linear regression based framework, indicating that GAM plays a positive role in our framework. Applying threshold to our model averaging process, GAMMA with the simple average combination and GAMMA with the weighted average combination rank in the top most accurate forecasting methods on M3 data. For monthly series, GAMMA with the weighted average combination even performs better than GAMMA with all benchmark methods weighted combined. Therefore, the optimal threshold ratio contributes to the improvement in interval forecasting performance. Compared to the minimum MSIS value (6.34) of benchmark methods on monthly data, the MSIS value of GAMMA with the weighted average combination is reduced by 4.57%. For yearly and quarterly series, GAMMA with all benchmark methods weighted combined ranks best for short, medium, and long term horizons. The MSIS value of GAMMA with all benchmark methods weighted combined is overall reduced by 6.17% compared to that of the optimal benchmark method on M3 data.

Furthermore, we calculate the ACD for a supplemental scoring rule, as shown in Table 4. ACD is a supplementary evaluation indicator that reflects the absolute difference between the actual coverage of the prediction interval and the confidence level of the expected prediction.
Table 3. Comparison of the MSIS values of the feature-based time series forecasting framework and other eight benchmark methods on M3. The top panel shows the MSIS values of eight benchmark methods and the best values among these benchmark methods. The bottom panel shows the MSIS values of LMMS (model selection based on the classical linear model), GAMMS (model selection based on GAM), and our proposed framework with simple combination, weighted combination and weighted combination with all benchmark methods. In each column, values smaller than the minimum value of benchmark methods are marked in **bold** and the best MSIS value is marked in †.

| Method   | Yearly       | Quarterly    | Monthly     | All         |
|----------|--------------|--------------|-------------|-------------|
|          | 1-2          | 3-4          | 5-6         | Total       |
|          | 1-2          | 3-5          | 6-8         | Total       |
|          | 1-6          | 7-12         | 13-18       | Total       |

**Benchmark methods**

| Method   | Yearly       | Quarterly    | Monthly     | All         |
|----------|--------------|--------------|-------------|-------------|
| auto-arima | 18.28        | 45.02        | 62.61       | 41.97       |
| ets       | 11.75        | 32.07        | 48.02       | 30.62       |
| tbats     | 14.99        | 45.06        | 72.51       | 44.19       |
| stlm-ar   | 32.92        | 62.17        | 92.92       | 62.67       |
| rw-drift  | 12.96        | 31.07        | 47.24       | 30.42       |
| thetaf    | 12.68        | 31.12        | 49.90       | 31.23       |
| naive     | 14.96        | 40.16        | 64.80       | 39.98       |
| naïve     | —            | —            | —           | —           |
| Min       | 11.75        | 31.07        | 47.24       | 30.42       |

**Results**

| Method     | Yearly       | Quarterly    | Monthly     | All         |
|------------|--------------|--------------|-------------|-------------|
|            | 1-2          | 3-4          | 5-6         | Total       |
|            | 1-2          | 3-5          | 6-8         | Total       |
|            | 1-6          | 7-12         | 13-18       | Total       |
| LMMS       | 13.65        | 36.67        | 54.06       | 34.79       |
| GAMMS      | 13.26        | 36.38        | 55.13       | 34.92       |
| GAMMA(mean) | 11.83        | 32.91        | 48.35       | 31.03       |
| GAMMA(weighted) | 11.33       | 31.28        | 45.51       | 29.37       |
| GAMMA(all weighted) | 11.07†  | 30.29†       | 44.10†      | 28.49†      | 5.58†  | 8.92†       | 12.97†       | 9.60†       | 4.43  | 5.92        | 7.90        | 6.09†       | 9.12†       |

† Values smaller than the minimum value of benchmark methods.
bold Best MSIS value.
Table 4. Comparison of the ACD values of the feature-based time series forecasting framework and other eight benchmark methods on M3.

| Method       | Yearly |         |         |         | Quarterly |         |         |         |         | Monthly | September |          |          |          |          | All       | Benchmark methods |
|--------------|--------|---------|---------|---------|-----------|---------|---------|---------|---------|----------|------------|---------|---------|---------|---------|-----------|-------------------|
|              | 1-2    | 3-4     | 5-6     | Total   | 1-2       | 3-5     | 6-8     | Total   | 1-6     | 7-12     | 13-18      | Total   |          |         |          |          |          |
| auto-arima   | 0.112  | 0.177   | 0.203   | 0.164   | 0.104     | 0.126   | 0.151   | 0.130   | 0.024   | 0.025    | 0.052     | 0.033   | 0.064   |          |          |          |          |          |
| ets          | 0.053  | 0.125   | 0.141   | 0.107   | 0.052     | 0.073   | 0.102   | 0.078   | 0.017   | 0.019    | 0.054     | 0.030   | 0.046   |          |          |          |          |          |
| tbats        | 0.124  | 0.230   | 0.271   | 0.208   | 0.092     | 0.134   | 0.184   | 0.142   | 0.049   | 0.058    | 0.105     | 0.071   | 0.098   |          |          |          |          |          |
| stlm-ar      | 0.174  | 0.256   | 0.301   | 0.244   | 0.025     | 0.092   | 0.211   | 0.120   | 0.035   | 0.061    | 0.135     | 0.077   | 0.102   |          |          |          |          |          |
| rw-drift     | 0.074  | 0.143   | 0.147   | 0.121   | 0.076     | 0.038   | 0.049   | 0.052   | 0.013   | 0.010    | 0.007     | 0.010   | 0.017   |          |          |          |          |          |
| thetaf       | 0.047  | 0.124   | 0.151   | 0.107   | 0.060     | 0.068   | 0.099   | 0.078   | 0.038   | 0.045    | 0.071     | 0.052   | 0.062   |          |          |          |          |          |
| naive        | 0.090  | 0.193   | 0.214   | 0.165   | 0.048     | 0.027   | 0.056   | 0.043   | 0.013   | 0.009    | 0.030     | 0.017   | 0.036   |          |          |          |          |          |
| Min          | 0.047  | 0.124   | 0.141   | 0.107   | 0.023     | 0.027   | 0.049   | 0.043   | 0.013   | 0.009    | 0.007     | 0.010   | 0.017   |          |          |          |          |          |
| LMMS         | 0.091  | 0.156   | 0.179   | 0.142   | 0.094     | 0.111   | 0.136   | 0.116   | 0.029   | 0.024    | 0.048     | 0.034   | 0.060   |          |          |          |          |          |
| GAMMS        | 0.092  | 0.166   | 0.192   | 0.150   | 0.091     | 0.104   | 0.120   | 0.107   | 0.026   | 0.026    | 0.049     | 0.034   | 0.059   |          |          |          |          |          |
| GAMMA(mean)  | 0.073  | 0.147   | 0.160   | 0.127   | 0.048     | 0.072   | 0.085   | 0.071   | 0.015   | 0.012    | 0.035     | 0.020   | 0.040   |          |          |          |          |          |
| GAMMA(weighted) | 0.061  | 0.125   | 0.152   | 0.113   | 0.031     | 0.052   | 0.064   | 0.052   | 0.010   | 0.006    | 0.015     | 0.010   | 0.023   |          |          |          |          |          |
| GAMMA(all weighted) | 0.058 | 0.120   | 0.142   | 0.107   | 0.019     | 0.036   | 0.047   | 0.036   | 0.012   | 0.009    | 0.010     | 0.011   | 0.015   |          |          |          |          |          |

interval. Similar to the results in Table 3, GAMMA with the weighted average combination performs best on monthly series, having a minimum absolute coverage difference (0.010) with the confidence level (0.95). GAMMA with all benchmark methods weighted combined generally performs quite well, especially for yearly and quarterly series.

As we reasoned in Section 3.2, a prediction method with higher point forecasting accuracy is easier to get higher interval forecasting accuracy. We calculate MASE values to check whether our proposed framework performs well on point forecasting. Table 5 compares the MASE values of our feature-based time series forecasting framework with eight benchmark methods on M3. For yearly series, there are small difference between the average MASE values of our proposed GAMMA framework across all forecast horizons and the minimum MASE value of the benchmark methods. For quarterly and monthly series, our proposed GAMMA outperforms all individual benchmark methods in terms of point forecasting. Hence, this indicates that our proposed framework not only performs well in interval forecasting, but also obtains convincing results than the benchmark methods in point forecasting.

We also explore the benchmark methods and their selection frequency by the optimal threshold for each time series in M3. Figure 6 depicts the boxplot of the number of selected benchmark methods on M3 dataset. It shows that the median number of yearly and quarterly series is equal to 5, while that of monthly series is 7. This demonstrates that the number of benchmark methods used for monthly series is more than that for yearly and quarterly data.
Table 5. Comparison of the MASE values of the feature-based time series forecasting framework and other eight benchmark methods on M3.

| Method      | Benchmark methods | Yearly | Quarterly | Monthly | All |
|-------------|-------------------|--------|-----------|---------|-----|
|             |                   | 1-2    | 3-4       | 5-6     | Total|
|             |                   | 1-2    | 3-5       | 6-8     | Total|
|             |                   | 1-6    | 7-12      | 13-18   | Total|
| auto-arima  |                   | 1.50   | 3.10      | 4.40    | 3.00 |
| ets         |                   | 1.44   | 2.97      | 4.17    | 2.86 |
| tbats       |                   | 1.53   | 3.18      | 4.68    | 3.13 |
| stlm-ar     |                   | 3.37   | 5.02      | 6.56    | 4.99 |
| rw-drift    |                   | 1.36   | 2.75      | 3.79    | 2.63 |
| thetaf      |                   | 1.47   | 2.89      | 3.96    | 2.77 |
| naive       |                   | 1.68   | 3.28      | 4.56    | 3.17 |
| naive       |                   | —      | —         | —       | —   |
| Min         |                   | 1.36   | 2.75      | 3.79    | 2.63 |
| LMMS        |                   | 1.42   | 3.03      | 4.36    | 2.94 |
| GAMMS       |                   | 1.42   | 2.97      | 4.33    | 2.90 |
| GAMMA(mean) |                   | 1.37   | 2.81      | 3.98    | 2.72 |
| GAMMA(weighted) |              | 1.35†  | 2.77      | 3.93    | 2.68 |
| GAMMA(all weighted) |          | 1.35†  | 2.76      | 3.88    | 2.66 |

Furthermore, It is enough for some time series to select one or two methods in the pool of methods used for model averaging. Table 6 gives a more detailed description of the selection rate of each benchmark method according to the optimal threshold for yearly, quarterly and monthly series in M3. It shows that the auto-arima, ets, thetaf, and tbats methods are selected at a high rate in the yearly, quarterly and monthly series, while the stlm-ar and naïve methods are selected at a small rate. This results is consistent with the MSIS values of eight benchmark methods.

4. Conclusions

In this paper, we have proposed a general framework for interval forecasting based on time series features, which provides the uncertainty estimation of point forecasts. Our proposed feature-based time series forecasting framework is comprised by the offline part used for training algorithm on a collection of time series and the online part used for forecasting new time series. In our proposed framework, most of the time-consuming calculations are achieved offline and the only thing we need to do with a new time series is to extract features and put them into pre-trained algorithm, making our method have an advantage of allowing for a computationally cheap. We have illustrated that our proposed framework outperforms all the individual benchmark methods on M3 dataset. Our proposed framework not only per-
Figure 6. Number of benchmark methods used for model averaging according to the optimal threshold in the feature-based time series forecasting framework with the weighted average combination.

Table 6. The selection rate of each benchmark method according to the optimal threshold in the feature-based time series forecasting framework with the weighted average combination.

| Method   | Yearly | Quarterly | Monthly |
|----------|--------|-----------|---------|
| auto-arima | 0.929  | 0.865     | 0.989   |
| ets      | 0.823  | 0.791     | 0.961   |
| tbats    | 0.834  | 0.816     | 0.941   |
| stlm-ar  | 0.205  | 0.262     | 0.648   |
| rw-drift | 0.716  | 0.540     | 0.471   |
| thetaf   | 0.848  | 0.813     | 0.838   |
| naïve    | 0.397  | 0.310     | 0.658   |
| snaïve   | —      | 0.776     | 0.903   |
forms better than all benchmark methods in interval forecasting, but also obtains convincing results than the benchmark methods in point forecasting.

A key advantage of our proposed framework is its interpretability of the effects of features on the interval forecasting accuracy. We apply GAM to our framework, allowing us to analyse the effect of each feature on the interval forecasting accuracy of each benchmark methods by the partial effects plot. Partial effects analysis also gives prospect to the selection of the appropriate interval forecasting method based on the time series features.

Furthermore, we define a threshold ratio in the offline part and find the optimal threshold to select a plurality of appropriate forecasting methods for each time series for model averaging. In this way, the number of methods in the combination pool for each series is different. Therefore, instead of using all the benchmark methods for model averaging, we purposefully select the appropriate benchmark methods for each time series. The optimal threshold performs well for monthly series in M3 competition data. For yearly and monthly series, the model averaging algorithm based on the optimal threshold selection does not outperform that with all benchmark methods combined, but it outperforms all individual benchmark methods.

Our proposed framework can serve as a general framework for time series point forecasting and interval forecasting. In this framework, we can specifically select the time series features, benchmark forecasting methods, and even the model for linking time series features with interval forecasting accuracy for the time series to be predicted.

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