New Supersymmetric Option for Two Higgs Doublets

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Abstract

If the standard electroweak gauge model is embedded in a larger theory which is supersymmetric and the latter breaks down to the former at some mass scale, then the reduced Higgs potential at the electroweak mass scale may differ from that of the well-known minimal supersymmetric extension. Specifically, if the larger theory is based on $SU(2)_L \times SU(2)_R \times U(1)$, an interesting alternative exists for two Higgs doublets.
The most studied extension of the standard SU(2) × U(1) electroweak gauge model is that of supersymmetry with the smallest necessary particle content.\cite{1} In this minimal supersymmetric standard model (MSSM), there are two scalar doublets $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$, with Yukawa interactions $\overline{(u, d)}_L d_R \Phi_1$ and $\overline{(u, d)}_L u_R \tilde{\Phi}_2$ respectively, where $\tilde{\Phi}_2 = i\sigma_2 \Phi_2^* = (\phi_2^0, -\phi_2^-)$. The Higgs potential

\begin{align*}
V &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\
&\quad + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
&\quad + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_2)^2,
\end{align*}

(1)

is subject to the constraints

\begin{align*}
\lambda_1 &= \lambda_2 = \frac{1}{4} (g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4} g_1^2 + \frac{1}{4} g_2^2, \quad \lambda_4 = -\frac{1}{2} g_2^2, \quad \lambda_5 = 0,
\end{align*}

(2)

where $g_1$ and $g_2$ are the U(1) and SU(2) gauge couplings of the standard model respectively. Hence there are only two unknown parameters in this sector and they are usually taken to be $\tan \beta \equiv v_2/v_1$, the ratio of the two scalar vacuum expectation values, and $m_A$, the mass of its one physical pseudoscalar particle. Numerous phenomenological studies\cite{2} have been made in its name.

It is generally believed that given the gauge group SU(2) × U(1) and the requirement of supersymmetry, the quartic scalar couplings of Eq. (1) must necessarily be given by Eq. (2). This is actually not the case because the SU(2) × U(1) gauge symmetry may be a remnant\cite{3} of a larger symmetry which is broken at a higher mass scale together with the supersymmetry. The structure of the Higgs potential is then determined by the scalar particle content needed to precipitate the proper spontaneous symmetry breaking and to render massive the assumed fermionic content of the larger theory. Furthermore, the quartic scalar couplings are related to the gauge couplings of the larger theory as well as other couplings appearing in its superpotential. At the electroweak energy scale,
the reduced Higgs potential may contain only two scalar doublets, but their quartic couplings may not be those of the MSSM. In particular, we consider in the following a left-right supersymmetric model based on $E_6$ particle content proposed some years ago[4] and show that its reduced Higgs potential $V$ for two scalar doublets is given by

$$\lambda_1 = \frac{1}{4} \left( 1 + \frac{4f^2}{g_2^2} \right) \left[ g_1^2 + g_2^2 - 4f^2 \left( 1 - \frac{g_1^2}{g_2^2} \right) \right],$$  \hspace{1cm} (3)

$$\lambda_2 = \frac{1}{2}g_2^2 + \frac{1}{4}(g_1^2 - g_2^2) \left( 1 - \frac{4f^2}{g_2^2} \right)^2,$$  \hspace{1cm} (4)

$$\lambda_3 = \frac{1}{4}g_2^2 - \frac{1}{4} \left( 1 - \frac{4f^2}{g_2^2} \right) \left[ g_1^2 - 4f^2 \left( 1 - \frac{g_1^2}{g_2^2} \right) \right],$$  \hspace{1cm} (5)

$$\lambda_4 = f^2 - \frac{1}{2}g_2^2, \quad \lambda_5 = 0,$$  \hspace{1cm} (6)

where $f$ is a coupling in the superpotential of the larger theory and has no analog in the MSSM. In the limit $f = 0$, it is easily seen from the above that the MSSM conditions, *i.e.* Eq. (2), are recovered as expected. The general requirement that $V$ be bounded from below puts an upper bound on $f^2$, namely

$$f^2 \leq \frac{1}{4}(g_1^2 + g_2^2) \left( 1 - \frac{g_1^2}{g_2^2} \right)^{-1}.$$  \hspace{1cm} (7)

The saturation of this upper bound turns out to imply that the left-right symmetry of the larger theory is not broken by the soft terms of the Higgs potential which break the supersymmetry.[4] In that case,

$$\lambda_1 = 0, \quad \lambda_2 = \frac{1}{2}g_2^2 - \frac{g_1^4}{g_2^2 - g_1^2},$$

$$\lambda_3 = \frac{1}{4}g_2^2 - \frac{g_1^2 g_2^2}{2(g_2^2 - g_1^2)} = -\lambda_4, \quad \lambda_5 = 0.$$  \hspace{1cm} (8)

The lesson we learn here is that even if supersymmetry exists and there are only two scalar doublets at the electroweak energy scale, the corresponding Higgs potential is not necessarily that of the MSSM.
We now describe our model. The gauge group is $SU(2)_L \times SU(2)_R \times U(1)$ but with an unconventional assignment of fermions.\cite{4} An exotic quark $h$ of electric charge $-1/3$ is added so that $(u, d)_L$ transforms as $(2, 1, 1/6)$, $(u, h)_R$ as $(1, 2, 1/6)$, whereas both $d_R$ and $h_L$ are singlets $(1, 1, -1/3)$. There are two scalar doublets $\Phi_{1,2}$ and a bidoublet

\[ \eta = \begin{pmatrix} \eta_1^- & \eta_2^+ \\ -\eta_1^+ & \eta_2^- \end{pmatrix} \]  

transforming as $(2,1,1/2)$, $(1,2,1/2)$, and $(2,2,0)$ respectively. The Yukawa interactions are such\cite{4} that $m_h$ comes from $\langle \phi_0^0 \rangle = v_2$, $m_d$ comes from $\langle \phi_1^0 \rangle = v_1$, and $m_u$ comes from $\langle \eta_1^0 \rangle = u_1$. The part of the Higgs potential related to the gauge interactions through supersymmetry is given by\cite{4}

\begin{align*}
V_D &= \frac{1}{8} G_1^2 (\Phi_1^+ \Phi_1^* - \Phi_2^+ \Phi_2^*)^2 \\
&\quad + \frac{1}{8} G_2^2 ((\Phi_1^+ \Phi_1^*)^2 + \Phi_2^+ \Phi_2^*)^2 + 2 (Tr \eta^\dagger \eta)^2 - 2 (Tr \eta^\dagger \tilde{\eta})(Tr \tilde{\eta}^\dagger \eta) \\
&\quad - 2 (\Phi_1^+ \Phi_1^* + \Phi_2^+ \Phi_2^*) (Tr \eta^\dagger \tilde{\eta}) + 4 (\Phi_1^+ \eta \tilde{\eta} \Phi_1^* + \Phi_2^+ \eta \tilde{\eta} \Phi_2^*) \]  

(10)

where $G_1$ is the U(1) gauge coupling and $G_2$ is the coupling of both $SU(2)_L$ and $SU(2)_R$, with

\[ \tilde{\eta} \equiv \sigma_2 \eta^* \sigma_2 = \begin{pmatrix} \eta_2^+ & \eta_1^* \\ -\eta_1^- & \eta_2^- \end{pmatrix} \]  

(11)

Now the superpotential of this model also contains a cubic term linking the three superfields corresponding to $\Phi_{1,2}$ and $\eta$ as already discussed by Babu et al.\cite{4} Its contribution to the Higgs potential is given by

\[ V_F = f^2 [\Phi_1^* \Phi_1^* (\Phi_2^+ \Phi_2^*) + (\Phi_1^+ \Phi_1 + \Phi_2^+ \Phi_2^*) (Tr \eta^\dagger \eta) - \Phi_1^+ \eta \tilde{\eta} \Phi_1^* - \Phi_2^+ \eta \tilde{\eta} \Phi_2^*]. \]  

(12)

To break the gauge symmetry spontaneously, we add soft terms which also break the supersymmetry:

\[ V_{soft} = m_1^2 \Phi_1^* \Phi_1 + m_2^2 \Phi_2^* \Phi_2 + m_3^2 (Tr \eta^\dagger \eta) - f A (\Phi_1^+ \tilde{\eta} \Phi_2^* + \Phi_2^+ \tilde{\eta}^\dagger \Phi_1^*). \]  

(13)
The sum is then

\[ V = V_{soft} + \frac{1}{8}(G_1^2 + G_2^2)[(\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2] \]

\[ + \frac{1}{4} G_2^2 (Tr \eta \eta^\dagger) - (Tr \eta^\dagger \eta)(Tr \eta \eta^\dagger) + \left( f^2 - \frac{1}{4} G_2^2 \right)(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)(Tr \eta \eta^\dagger) \]

\[ - \left( f^2 - \frac{1}{4} G_2^2 \right)(\Phi_1^\dagger \eta \phi_1 + \Phi_2^\dagger \eta \phi_2) + \left( f^2 - \frac{1}{4} G_2^2 \right)(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2). \]  

(14)

Note that \( V \) is invariant also under a global U(1) transformation related to lepton number as a consequence of the theory’s E\(_6\) superstring antecedent and it remains unbroken after spontaneous breaking of the gauge symmetry.[4]

Consider now the breaking of SU(2)\(_L\) \times SU(2)\(_R\) \times U(1) down to the standard SU(2)\(_L\) \times U(1)\(_Y\). This is accomplished with \( \langle \phi_2^0 \rangle = v_2 \neq 0 \). Three of the four degrees of freedom contained in \( \Phi_2 \) are then absorbed into the three massive vector gauge bosons \( W^\pm_R \) and \( Z' \), and the remaining neutral physical Higgs boson \( (\sqrt{2} \text{Re} \phi_2^0) \) picks up a mass equal to the square root of \( (G_1^2 + G_2^2) v_2^2 / 2 \). Concurrently, the exotic \( h \) quarks and the \( \eta_2 \) components of the scalar bidoublet become heavy at the same mass scale. The reduced Higgs potential involving only the \( (\phi_1^+, \phi_1^0) \) and \( (\eta_1^+, \eta_1^0) \) doublets is then of the form of Eq. (1), but with the following constraints:

\[ \mu_1^2 = m_1^2 + \left( f^2 - \frac{1}{4} G_1^2 \right) v_2^2, \quad \mu_2^2 = m_3^2 + \left( f^2 - \frac{1}{4} G_2^2 \right) v_2^2, \quad \mu_1^2 = -f A v_2, \]  

(15)

and

\[ \lambda_1 = \frac{1}{4} (G_1^2 + G_2^2) - \frac{(4f^2 - G_1^2)^2}{4(G_1^2 + G_2^2)}, \]  

(16)

\[ \lambda_2 = \frac{1}{2} G_2^2 - \frac{(4f^2 - G_2^2)^2}{4(G_1^2 + G_2^2)}, \]  

(17)

\[ \lambda_3 = \frac{1}{4} G_2^2 - \frac{(4f^2 - G_1^2)(4f^2 - G_2^2)}{4(G_1^2 + G_2^2)}, \]  

(18)

\[ \lambda_4 = f^2 - \frac{1}{2} G_2^2, \quad \lambda_5 = 0, \]  

(19)

where the second terms on the right-hand sides of the equations for \( \lambda_{1,2,3} \) come from the cubic interactions of \( \sqrt{2} \text{Re} \phi_2^0 \). Assuming that \( v_2 \) is not many orders of magnitude greater
than \( v_1 \) and \( u_1 \), then the running of the couplings is not a significant factor and we have 
\[ g_2 = G_2 \quad \text{and} \quad g_1^{-2} = G_1^{-2} + G_2^{-2}. \]
Hence \( G_1^2 = g_1^2 g_2^2 / (g_2^2 - g_1^2) \) and we obtain Eqs. (3) to (6).

Let \( x \equiv \sin^2 \theta_W = g_1^2 / (g_1^2 + g_2^2) \), then the new coupling \( f \) can in principle take on any value in the range
\[
0 \leq f^2 \leq \frac{e^2}{4x(1 - 2x)}.
\]
(20)

As pointed out earlier, the \( f = 0 \) limit corresponds to the MSSM as it must. The upper limit, on the other hand, corresponds to that of left-right symmetry, i.e. \( m_1^2 = m_2^2 \) in \( V_{soft} \), from which Eq. (8) is obtained, namely
\[
\lambda_1 = 0, \quad \lambda_2 = \frac{e^2}{2x} \left[ 1 - \frac{2x^2}{(1-x)(1-2x)} \right], \quad \lambda_3 = \frac{e^2}{4x} \left[ 1 - \frac{2x}{1-2x} \right] = -\lambda_4, \quad \lambda_5 = 0.
\]
(21)

Now because there are nonnegligible radiative corrections due to a large value of \( m_t \), \( \lambda_2 \) has a significant additional contribution given by \( g_2^2 \epsilon / 4 M_W^2 \sin^4 \beta \), where
\[
\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2} \ln \left( 1 + \frac{\bar{m}^2}{m_t^2} \right).
\]
(22)

In the above, \( \bar{m} \) is an effective mass for the two scalar supersymmetric partners of the \( t \) quark. The \( 2 \times 2 \) mass-squared matrix spanning \( \sqrt{2} \text{Re} \phi_1^0 \) and \( \sqrt{2} \text{Re} \phi_2^0 \) is then given by
\[
\mathcal{M}^2 = \begin{pmatrix}
    m_A^2 \sin^2 \beta & -m_A^2 \sin \beta \cos \beta \\
    -m_A^2 \sin \beta \cos \beta & m_A^2 \cos^2 \beta + 2\lambda_2 v_2^2 + \epsilon / \sin^2 \beta
\end{pmatrix},
\]
(23)
where
\[
m_A^2 = \frac{-\mu_{12}^2}{\sin \beta \cos \beta}.
\]
(24)

This implies
\[
m_{H_2^0}^2 \leq m_A^2 \sin^2 \beta,
\]
(25)
as well as
\[
m_{H_2^0}^2 \leq 2 M_W^2 \left[ 1 - \frac{2x^2}{(1-x)(1-2x)} \right] \sin^4 \beta + \epsilon,
\]
(26)
where $H_0^0$ is the lighter of the two mass eigenstates. Recall in the MSSM, the above two bounds are $m_A^2 \cos^2 2\beta + \epsilon/\tan^2 \beta$ and $M_Z^2 \cos^2 2\beta + \epsilon$ instead respectively. Note that at tree level, $m_{H_0^0} \leq m_A$ is required in both models, but with radiative corrections, it holds only in this model. We plot in Figs. 1 and 2 the maximum allowed value of $m_{H_0^0}$ and the minimum allowed value of $m_{H_1^0}$ as functions of $m_A$ in the MSSM and in this model respectively. In the limit $m_A = 0$, we have $m_{H_0^0}^2 \leq M_Z^2$ and $m_{H_1^0}^2 \geq M_Z^2 + \epsilon$ in the MSSM, whereas $m_{H_0^0} = 0$ and $m_{H_1^0}^2 \geq 2[2M_W^2(1-2x^2/(1-x)(1-2x))\epsilon]^2$ in this model. In the limit $m_A$ much greater than the electroweak energy scale, both models reduce to the standard model with $H_0^0$ as its one physical Higgs boson such that $m_{H_0^0}^2 \leq M_Z^2 + \epsilon \simeq (115 \text{ GeV})^2$ in the MSSM and $m_{H_0^0}^2 \leq 2M_W^2(1-2x^2/(1-x)(1-2x)) + \epsilon \simeq (120 \text{ GeV})^2$ in this model, where we have assumed $m_t = 150 \text{ GeV}$ and \( \tilde{m} = 1 \text{ TeV} \) in estimating the value of $\epsilon$. If we now consider the decay $Z \rightarrow H_0^0A$, its experimental nonobservation at LEP down to the level of $10^{-6}$ in branching fraction restricts the parameter space of $(m_A, \tan \beta)$ as shown in Fig. 3 for both the MSSM ($f=0$) and this model ($f = f_{\text{max}}$). As for the charged Higgs boson, the well-known sum rule $m_{H^\pm}^2 = m_A^2 + M_W^2$ in the MSSM becomes

$$m_{H^\pm}^2 = m_A^2 + \frac{1}{2}M_W^2 \left(1 - \frac{2x}{1-2x}\right)$$

(27)

in this model. More details regarding the phenomenological implications of this model in comparison with the MSSM will be given elsewhere.[7]

In conclusion, we have shown in this paper that the requirement of supersymmetry does not uniquely determine the self-interaction structure (and thus the mass spectrum) of the two Higgs doublets at the electroweak energy scale. The reason is that there may be one or more terms in the superpotential linking the two Higgs-doublet superfields with a heavier superfield, as allowed by a larger symmetry at a higher mass scale. The reduced Higgs potential at the electroweak energy scale remembers these couplings and would only be identical to that of the minimal supersymmetric standard model (MSSM) if these additional
couplings were zero. [The hidden assumption of the MSSM is in fact the absence of such couplings.]

We show in particular how an especially interesting version\footnote{\label{fn:1} of the supersymmetric SU(2)$_L \times$SU(2)$_R \times$U(1) model would result in two Higgs doublets whose quartic self-couplings are given by Eqs. (3) to (6), rather than by Eq. (2). For illustration, we specialize to the case $f = f_{\text{max}} = e/2\sqrt{x(1 - 2x)}$ and discuss how the Higgs mass spectrum of this model differs from that of the MSSM. If future experiments confirm the existence of two and only two Higgs doublets at the electroweak energy scale, it will be very important to know whether they are consistent with supersymmetry and we should bear in mind that the MSSM is not the only possibility.

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[6] We now redefine \((\eta_1^+, \eta_0^0)\) as \((\phi_2^+, \phi_2^0)\). We also neglect other radiative-correction terms which are proportional to only \(m_t^2\).

[7] E. Ma and D. Ng, in preparation.
Figure captions

Fig. 1. Maximum value of $m_{H_2^0}$ (solid line) and minimum value of $m_{H_1^0}$ (dash line) as functions of $m_A$ in the MSSM.

Fig. 2. Maximum value of $m_{H_2^0}$ (solid line) and minimum value of $m_{H_1^0}$ (dash line) as functions of $m_A$ in this model. See Eq. (23) of text.

Fig. 3. Contour plots for $\text{Br}(Z \rightarrow AH_2^0) = 10^{-6}$ as functions of $m_A$ and $\tan \beta$ in the MSSM (dash line) and in this model (solid line). The allowed regions are to the right of the lines.
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