The Definition of Double Commutators and Consistency in Free Field Theory

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Abstract

Within the framework of generalized functions a general consistent definition of double commutators is given. This definition respects the Jacobi identity even if the regularization is removed. The double commutator of fermionic currents is calculated in this limit.

We show that BJL--type prescriptions and point-splitting prescriptions for calculating double commutators fail to give correct results in free field theory.

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1 Introduction

It has been known since the days of current algebra that an iterative computation of double commutators of fermionic currents leads to a violation of the Jacobi identity in the current algebra [1, 2]. This violation occurs in anomalous gauge theories (see for example [2]-[5]) and also in the quark model, where it was observed first [1].

In recent publications the failure of iterative schemes has been claimed and prescriptions for the calculation of double commutators were presented which manifestly respect the Jacobi identity [6, 7]. However, it was still an open question, whether these prescriptions define correctly a double commutator. Since a violation of the Jacobi identity even takes place in the case of free currents [8], it is possible to study the different methods for calculating double commutators in free field theory.

In this paper we construct double commutators within a general regularization scheme. This enables us to discuss the consistency of BJL–type and point–splitting prescriptions used in the literature. We claim that there is no BJL–type prescription, which fulfills the consistency condition of a correct regularization. Although point–splitting is consistent, the averaging procedure normally used leads to inconsistencies. Since the consistent average procedure is very complicated the advantages of point splitting are spoiled.

In the second section we give a definition of double commutators within a general regularization and calculate them in the limit, in which the regularization is removed. In the third section point–splitting and BJL–type prescriptions are discussed. We show that these prescriptions fail to give correct results in free field theory. We summarize and discuss these results.

2 Double commutators within a general regularization

In this section we construct a double commutator of free fermionic currents within a general regularization in four dimensions. The explicit form of the free fermionic fields is well-known. We define a regularized version as

\[ \psi_f(x) := \int d\tilde{p} \sum_\alpha [b_\alpha(p)u^\alpha(p)e^{-ipx} + d_\alpha^+(p)v^\alpha(p)e^{ipx}] f(p^2) \]
\[ \bar{\psi}_f(x) = \int d\tilde{p} \sum_\alpha [\bar{b}^+_\alpha(p)\bar{u}^\alpha(p)e^{ipx} + d_\alpha(p)\bar{v}^\alpha(p)e^{-ipx}] f(p^2) \]

with

\[ \{b^+_\alpha(p), b_\beta(p')\} = (2\pi)^3 2p_0 \delta^3(p - p') \delta_{\alpha\beta} \]
\[ \{d^+_\alpha(p), d_\beta(p')\} = (2\pi)^3 2p_0 \delta^3(p - p') \delta_{\alpha\beta} \]

\[ d\tilde{p} := \frac{d^4p}{(2\pi)^3} \delta(p^2) \theta(p_0). \]
In eq. (2.1) \( f \) is an arbitrary function of fast decrease with
\[
\begin{align*}
f(0) &= 1 \\
\lim_{\|p\| \to \infty} \|p\|^n f(p^2) &= 0 \quad \forall n \in \mathbb{N}.
\end{align*}
\]
(2.2)

The regularized free vector and axial currents are given by
\[
\begin{align*}
V^\mu(x; f) &:= : \bar{\psi}_f(x) \gamma^\mu \psi_f(x) : \\
A^\mu(x; f) &:= : \bar{\psi}_f(x) \gamma_5 \gamma^\mu \psi_f(x) :
\end{align*}
\]
(2.3)

We define the equal–time double commutators \( \Gamma^{ij}_l \), \( l = 1, 2, 3 \) of these currents as the regularization limit \( f \to 1 \) of
\[
\Gamma^{ij}_l(x_1, x_2, x_3; f) := \langle 0 \left[ A^0(x_1; f), [V^i(x_2; f), V^j(x_3; f)] \right]_{ET} | 0 \rangle.
\]
(2.4)

The Jacobi identity is fulfilled by these regularized double commutators.
\[
J := \Gamma^{ij}_1(x_1, x_2, x_3; f) + \Gamma^{ij}_2(x_1, x_2, x_3; f) + \Gamma^{ij}_3(x_1, x_2, x_3; f) = 0.
\]
(2.5)

We have
\[
\Gamma^{ij}_l(x_1, x_2, x_3) := \lim_{f \to 1} \Gamma^{ij}_l(x_1, x_2, x_3; f).
\]
(2.6)

The equal–time double commutators are sums of products of the spatial \( \delta \)–distribution and its derivatives. The structure of the \( \Gamma^{ij}_l \), \( l = 1, 2, 3 \) is fixed by the symmetry properties of the corresponding double commutator. For example, \( \Gamma^{ij}_3 \) is given by
\[
\Gamma^{ij}_3(x_1, x_2, x_3) = 2i\epsilon^{ijk} [ a_1 \partial_k \delta(x_1 - x_2) \delta(x_3 - x_2) \\
+ a_2 \Delta \partial_k \delta(x_1 - x_2) \delta(x_3 - x_2) \\
+ a_3 \partial_k \delta(x_1 - x_2) \Delta \delta(x_3 - x_2) \\
+ a_4 \partial_k \partial_l \delta(x_1 - x_2) \partial_l \delta(x_3 - x_2) ] \\
- ((x_1 - x_2) \leftrightarrow (x_3 - x_2)).
\]
(2.7)
The factor \( a_1 \) is dependent on the choice of the regularization function \( f \) because it has not zero dimension \([3, f]\). This is already known from similar coefficients in commutators of free currents. We determine the other components by calculating the following integrals (\( k \) is fixed).

\[
\langle x_1^2(x_1)_k \rangle := \int d^3x_1 d^3x_3 \, x_1^2(x_1)_k \Gamma^3_k[x_1, x_3; f]
\]

\[
\langle x_2^2(x_2)_k \rangle := \int d^3x_1 d^3x_3 \, x_2^2(x_2)_k \Gamma^3_k[x_1, 0, x_3; f]
\]

\[
\langle (x_1)_k(x_1)_l(x_3)_l \rangle := \int d^3x_1 d^3x_3 \, (x_1)_k(x_1)_l(x_3)_l \Gamma^3_k[x_1, 0, x_3; f]
\]

with

\[
2i\epsilon^{ijk}\Gamma^3_k[x_1, x_2, x_3; f] := \Gamma^3_{ij}[x_1, x_2, x_3; f].
\]

These integrals are connected with the coefficients \( a_j, \, j \neq 1 \) by

\[
\langle x_1^2(x_1)_k \rangle = -10a_2
\]

\[
2\langle x_2^2(x_2)_k \rangle - \langle (x_1)_k(x_1)_l(x_3)_l \rangle = -10a_3
\]

\[
3\langle (x_1)_k(x_1)_l(x_3)_l \rangle - \langle x_1^2(x_3)_k \rangle = -10a_4.
\]

The \( f \)-dependence of the integrals in eq. \((2.8)\) is of the type

\[
\int d^3p \, \frac{p_i p_j}{p^2} \frac{\partial}{\partial p^2} f(p^2) = \frac{2\pi}{3} \delta_{ij} f(0) = \frac{2\pi}{3} \delta_{ij}.
\]

Thus only \( a_1 \) depends on \( f \) and it follows

\[
\Gamma^3_{ij}(x_1, x_2, x_3) = -\frac{1}{81\pi^2} \epsilon^{ijk} \left\{ c[f] \partial_k \delta(x_1 - x_3)[\delta(x_1 - x_2) + \delta(x_3 - x_2)]
\right.
\]

\[
+ \frac{22}{5} \partial_k \Delta \delta(x_1 - x_3)[\delta(x_1 - x_2) + \delta(x_3 - x_2)]
\]

\[
+ 2 \left( \Delta \delta(x_1 - x_2) \partial_k \delta(x_3 - x_2)
\right.
\]

\[
- \Delta \delta(x_3 - x_2) \partial_k \delta(x_1 - x_2)
\]

\[
+ 2 \left( \partial_k \partial_l \delta(x_1 - x_2) \partial_l \delta(x_3 - x_2)
\right.
\]

\[
- \partial_k \partial_l \delta(x_3 - x_2) \partial_l \delta(x_1 - x_2)
\}
\]

\[
(2.12)
\]
where \( c[f] \) is an \( f \)-dependent constant \( (c[f] = -162\pi^2 i a_1[f]) \). \( \Gamma_2^{ij} \) and \( \Gamma_1^{ij} \) follow similarly

\[
\Gamma_2^{ij}(x_1, x_2, x_3) = -\frac{1}{81\pi^2} \epsilon^{ijk} \left\{ c[f] \partial_k \delta(x_1 - x_2)[\delta(x_1 - x_3) + \delta(x_2 - x_3)] \\
+ \frac{22}{5} \partial_k \Delta \delta(x_1 - x_2)[\delta(x_1 - x_3) + \delta(x_2 - x_3)] \\
+ 2 \left( \Delta \delta(x_1 - x_3) \partial_k \delta(x_2 - x_3) \right) \\
- \Delta \delta(x_2 - x_3) \partial_k \delta(x_1 - x_3) \right\}
\]

and

\[
\Gamma_1^{ij}(x_1, x_2, x_3) = -\left[ \Gamma_2^{ij}(x_1, x_2, x_3) + \Gamma_3^{ij}(x_1, x_2, x_3) \right].
\]

The double commutators given in the literature [3, 7] are not compatible with these results. Neither the use of BJL-type prescriptions nor the use of point-splitting prescriptions lead to the results \(2.12, 2.13, 2.14\).

### 3 Comparison with point–splitting and BJL–type prescriptions

At the beginning we introduce an extension of the regularization given in eq. (2.4), where some of the properties of a double commutator are lost. We define a more general expression \( \Gamma_i^{ij} \) as follows. For convenience we choose as an explicit regularization

\[
f_\epsilon(p^2) := \exp\{-\epsilon\|p\|\}.
\]

In order to compare the result with BJL–type prescriptions we also introduce a regularization for the equal–time limits.

\[
\Gamma_1^{ij}(x_1, x_2, x_3; \epsilon, r) := \int \prod_{i=1}^3 \mathrm{d}x_i^0(0) [A^0(x_i; f_\epsilon), [V^i(x_2; f_\epsilon), V^j(x_3; f_\epsilon)]] |0\rangle \delta_r(x^0, t)
\]

\[
\Gamma_2^{ij}(x_1, x_2, x_3; \epsilon, r) := \int \prod_{i=1}^3 \mathrm{d}x_i^0(0) [V^j(x_3; f_\epsilon), [A^0(x_1; f_\epsilon), V^i(x_2; f_\epsilon)]] |0\rangle \delta_r(x^0, t)
\]

\[
\Gamma_3^{ij}(x_1, x_2, x_3; \epsilon, r) := \int \prod_{i=1}^3 \mathrm{d}x_i^0(0) [V^i(x_2; f_\epsilon), [V^j(x_3; f_\epsilon), A^0(x_1; f_\epsilon)]] |0\rangle \delta_r(x^0, t)
\]

\[
\delta_r(x^0, t) := \prod_{i=1}^3 \delta_{r_i}(x^0_i - t).
\]
The $\delta_r$ are regularizations of the one-dimensional $\delta$–distribution with regularization parameters $r_i$. The Jacobi identity is fulfilled by these expressions

$$J := \Gamma^{ij}_1(x_1, x_2, x_3; \epsilon, r) + \Gamma^{ij}_2(x_1, x_2, x_3; \epsilon, r) + \Gamma^{ij}_3(x_1, x_2, x_3; \epsilon, r) = 0. \quad (3.17)$$

We want to emphasize that we have introduced independent regularizations $r_i$ of the equal-time limits only to compare our results with other methods used in the literature (e.g. BJL-limit, see appendix). The symmetry properties of the double commutators (under the interchange of currents) force us to take

$$r_1 = r_2 = r_3 = \tilde{r}. \quad (3.18)$$

Furthermore, it is quite natural to calculate equal–time double commutators of the regularized currents (keeping $f$ fixed and remove $r$ first) and to take the regularization limit of $f \to 1$ after this computation. This corresponds to the usual definition of quantities in a regularized theory.

First we concentrate on the case of eq. (3.18), which still leads to an expression consistent with the symmetry properties of an double commutator. For the calculation we need an explicit expression for the regularized $\delta$–distribution,

$$\delta_{\tilde{r}}(x^0_i) = \frac{1}{\pi \tilde{r}^2 + x^0_i^2} \frac{a^4}{(a^2 + x^0_i^2)^2}. \quad (3.19)$$

Here $a$ is a free parameter with $a \neq 0$. The limit, in which the regularization is removed does not depend on $a$. It is only introduced for technical reasons. Performing the integration of $\Gamma^{ij}_3(x_1, x_2, x_3; \epsilon, r)$ as in eq. (2.8), the coefficients $a_j$, $j = 2, 3, 4$ follow with (2.10) after a long but straightforward calculation [9].

$$a_2(u) = \frac{i}{30\pi^2} \left( \frac{2 + 3u}{3 + 4u} + \frac{4 + 18u + 25u^2 + 10u^3}{(3 + 4u)^3} \right)$$

$$a_3(u) = -\frac{i}{30\pi^2} \left( \frac{u}{3 + 4u} + \frac{10 + 40u + 49u^2 + 19u^3}{(3 + 4u)^3} \right) \quad (3.20)$$

$$a_4(u) = \frac{i}{30\pi^2} \left( -2 \frac{u}{3 + 4u} + \frac{10 + 35u + 32u^2 + 2u^3}{(3 + 4u)^3} \right)$$

with

$$u = \frac{\tilde{r}}{\epsilon}. \quad (3.21)$$

The coefficients $a_j$ depend on $u$, which is a measure for the mixing of the regularization limit ($\epsilon \to 0$) with the equal–time limit ($\tilde{r} \to 0$). The convenient choice $u = 0$ (first taking the equal-time limit and then removing the regularization of the currents) has been calculated already in the previous section. However, even for arbitrary $u$ the results in the literature [6, 7] are not compatible with eq. (3.20).
Now we discuss how point-splitting and BJL–prescriptions fit into this picture. We point out that the limit procedure in a point–splitting prescription should correspond to the case $u = 0$. Taking the value of $u$ equal to zero is equivalent to a fixed–time regularization. The use of regulator functions $f$ with dependence only on spatial momenta is an introduction of point–splitting as an spatial regularization in an unambiguous way. The averaging, relevant in point–splitting

$$\frac{\epsilon_i \epsilon_j}{\epsilon^2} = \delta_{ij} \text{ for commutators}$$

$$\frac{\epsilon_i \epsilon_j \epsilon_k \epsilon_m}{\epsilon^4} = \frac{1}{3} (\delta_{ij} \delta_{km} + \delta_{ik} \delta_{jm} + \delta_{im} \delta_{jk}) \text{ for double commutators}$$

(3.22)

is replaced by the explicit calculation of these quotients (see (2.11)). The point–splitting result for the commutator is reproduced in our scheme [9]. For the double commutator the provided averaging leads to a different result as the explicit calculation of the quotient in the general regularization (eq. (2.4)). We conclude that the usual averaging procedure is not applicable to calculate double commutators.

In the following we concentrate on BJL–type prescriptions for double commutators. To reproduce the results of the BJL–type prescriptions, we have to use the extension (eq. (3.16)) of the general regularization (eq. (2.4)). The calculations involved are tedious and not illuminating, so we only discuss the results. First we reproduce the iterative BJL–scheme within eq. (3.16), for which we need a general regularization of the equal-time limits with different $r_i$ (see eq. (3.16)). The result of the iterative BJL–scheme follows by the following different limits [9] (see appendix for the definition of the BJL–limit)

$$(q_0, k_0)[A^0(x_1), V^i(x_2), V^j(x_3)] = (r_1, r_2, r_3, \epsilon)[\Gamma^{ij}_1 (x_1, x_2, x_3; \epsilon, r)]$$

formally $\rightarrow$ $\langle 0 | [A^0(x_1), [V^i(x_2), V^j(x_3)]]_{ET} | 0 \rangle$

$$(k_0, p_0)[A^0(x_1), V^i(x_2), V^j(x_3)] = (r_3, r_1, r_2, \epsilon)[\Gamma^{ij}_2 (x_1, x_2, x_3; \epsilon, r)]$$

formally $\rightarrow$ $\langle 0 | [V^i(x_3), [A^0(x_1), V^j(x_2)]]_{ET} | 0 \rangle$

$-(q_0, p_0)[A^0(x_1), V^i(x_2), V^j(x_3)] = (r_2, r_3, r_1, \epsilon)[\Gamma^{ij}_3 (x_1, x_2, x_3; \epsilon, r)]$$

formally $\rightarrow$ $\langle 0 | [V^i(x_2), [V^j(x_3), A^0(x_1)]]_{ET} | 0 \rangle$

(3.23)

with

$$(r_{i_1}, r_{i_2}, r_{i_3}, \epsilon)[\Gamma^{ij}_l] := \lim_{r_{i_1} \rightarrow 0} \lim_{r_{i_2} \rightarrow 0} \lim_{r_{i_3} \rightarrow 0} \lim_{\epsilon \rightarrow 0} \Gamma^{ij}_l.$$ 

The reason for this equality between the iterative BJL–limits and the $(r_{i_1}, r_{i_2}, r_{i_3}, \epsilon)$-limits is the following. Clearly, the regularization parameter $\epsilon$, which corresponds to a regularization of the contributing $n$–point functions is not involved explicitly in the iterative scheme (in
every space–time prescription, e.g. BJL–type prescriptions) so we set it to zero first. Then we have to perform the limits \( r_i \). Each limit \( r_i \) corresponds to the limit \( (x_i)_0 \to t \) which is equivalent to the limit \( q_i^0 \to \infty \) with \( q_i \) is the corresponding momenta of \( x_i \). Thus the limits on the right side of (3.23) reproduce the iterative BJL–type limits. However, we are not allowed to calculate each double commutators \( \Gamma_{ij}^l \) with a different limit, if the results depend on the chosen limit. This is the reason for the violation of the Jacobi identity. Furthermore the \( \Gamma_{ij}^l \) with \( r_1 \neq r_2 \neq r_3 \) loose the symmetry properties a double commutator need to have under the interchange of the currents. We take as an example \( \Gamma_{ij}^3 \) calculated in the iterative scheme

\[
\Gamma_{ij}^3_{\text{iterat}}(x_1, x_2, x_3) := (r_2, r_3, r_1, \epsilon)[\Gamma_{ij}^3(x_1, x_2, x_3; \epsilon, r)].
\] (3.24)

We get the following inequality

\[
\Gamma_{ij}^3_{\text{iterat}}(x_1, x_2, x_3) \neq (r_2, r_3, r_1, \epsilon)[\Gamma_{ji}^3(x_1, x_3, x_2; \epsilon, r)],
\] (3.25)

which should be an equality for a consistent scheme. However, whereas the left-hand side is determined by eq. (3.24) with finite coefficients \( a_i, i = 2, 3, 4 \), the coefficients \( a'_i \) at the right-hand side of eq. (3.25) are not only different but also diverging.\(^2\) If one insists nevertheless on using one of the limits \( (r_{i1}, r_{i2}, r_{i3}, \epsilon) \) in (3.23) for the calculation of the double commutators, the only prescription, which respects the Jacobi identity is to calculate all \( \Gamma_{ij}^l \) within this limit. However, two of the \( \Gamma_{ij}^l \) are not well defined in this procedure. The coefficients \( a_i, i = 2, 3, 4 \) diverge. This also shows the invalidity of the iterative scheme. Only for \( r_i = r, i = 1, 2, 3 \) the symmetry properties are maintained.

With these remarks it is possible to look at general BJL–type prescriptions to calculate double commutators \([A(x), [B(y), C(z)]]]\). All these prescriptions have to use different equal–time limits \( (r_{i1}, r_{i2}, r_{i3}, \epsilon) \) in calculating the different double commutators (see as an example the iterative scheme (3.23), appendix). If the Jacobi identity is violated by the iterative scheme this indicates that different limiting procedures do not lead to a unique result. As we have explained in this case every single result is incompatible with the symmetry properties of a double commutator. It follows that in this case all BJL–type scheme are using inconsistent limit procedures. Even a combination of limits that leads to expressions which respect the Jacobi identity provides not a consistent scheme in general because each limit itself is inconsistent.

### 4 Conclusion

We have calculated double commutators of free vector and axial currents within a general regularization (see eq. (2.4)). The Jacobi identity is fulfilled within this framework and the dimensionless coefficients \( a_i, i = 2, 3, 4 \) are independent of the chosen limit, in which\(^2\) In fact, implicitly \( \epsilon \) is involved; but it is only used to get formally definiteness of the taken equal–time limits.

\(^2\)
the regularization is removed \((f \to 1)\). Only \(a_1[f]\) depends on the regularization, which is well known from the calculation of commutators of free currents.

Neither point-splitting results nor the results of BJL-type prescriptions are compatible with the double commutators (see eq. (2.12, 2.13, 2.14)).

In the general regularization (eq. (2.4)) the averaging procedure relevant in point-splitting is replaced by an explicit calculation of the quotients

\[
\frac{\epsilon_i \epsilon_j}{\epsilon^2}, \quad \frac{\epsilon_i \epsilon_j \epsilon_k \epsilon_m}{\epsilon^4}.
\]

Thus we conclude, that the usual averaging is inconsistent in the case of double commutators. However the advantage of point-splitting is the simplicity of the involved calculations due to the averaging procedure. This advantage is spoiled, if one has to calculated the quotients explicitly.

To reproduce the results of BJL-type prescriptions used in in the literature, we have to introduce an extension of the general consistent regularization (eq. 3.16), for which consistency conditions for double commutators are violated. We have shown explicitly, how to reproduce the iterative BJL-prescription within this extension. BJL-type prescriptions have to use different equal-time limits for the calculation of different double commutators. However, if the result depends on the limit, this is inconsistent. The violation of the Jacobi identity in the iterative scheme is the result of the use of different equal-time limits for each double commutator. This violation measures the consistency of the used methods and therefore it has no physical interpretation. Furthermore it shows the general inconsistency of BJL-type methods for calculating double commutators. It is due to the fact that in every calculation of double commutators with BJL-type prescription one is forced to take different equal-time limits for each double commutator.

We conclude that the use of point-splitting-prescriptions related with the iterative scheme and BJL-type methods leads to wrong results if the iterative schemes violate the Jacobi identity. This is true also in interacting theories like chiral gauge theories. However, in these theories it is possible to use their geometric structure for an algebraic BJL-approach to compute double commutators with anomalous Schwinger terms \(\cite{9}\).

### A Appendix

The Bjorken–Johnson–Low (BJL) limit is a prescription often used to calculate commutators \(\cite{1,2,10}\). It does not suffer from averaging procedures like the fixed-time prescriptions like the point-splitting. We define a BJL-type prescription to calculate double commutators in the following way.

\[
\langle 0\left|\left[ A(x), [B(y), C(0)] \right]\right| 0 \rangle_{ET} = \lim_{p_0 \to \infty} \lim_{q_0 \to \infty} \lim_{p_0 q_0} T(x, y; p_0, q_0)
\]

\[
T(x, y; p_0, q_0) := \int dx_0 dy_0 e^{ip_0 x_0} e^{iq_0 y_0} \langle 0|TA(x)B(y)C(0)|0 \rangle
\]  

(A.1)
We can easily see that this definition provides not only one prescription to calculate double commutators \[\text{(6)}\]. We define the following different limits

\[
(g, l)[A(x), B(y), C(0)] := \lim_{g \to \infty} \lim_{l \to \infty} glT(x, y; p_0, q_0)
\]
\[
g, l \in \{p_0, q_0, -(p_0 + q_0)\}.
\]  

(A.2)

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