Analysis of the strong decays of the $P_c(4312)$ as a pentaquark molecular state with QCD sum rules

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Abstract

In this article, we tentatively assign the $P_c(4312)$ to be the $\bar{D} \Sigma_c$ pentaquark molecular state with $J^P = \frac{3}{2}^+$, and study its two-body strong decays with the QCD sum rules, special attentions are paid to match the hadron side with the QCD side of the correlation functions to obtain solid duality. We obtain the partial decay widths $\Gamma (P_c(4312) \to \eta_c p) = 0.255 \text{ MeV}$ and $\Gamma (P_c(4312) \to J/\psi p) = 9.296^{+19.542}_{-9.296} \text{ MeV}$, which are compatible with the experimental value of the total width, and support assigning the $P_c(4312)$ to be the $\bar{D} \Sigma_c$ pentaquark molecular state.

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1 Introduction

In 2015, the LHCb collaboration observed two pentaquark candidates $P_c(3880)$ and $P_c(4450)$ in the $J/\psi p$ mass spectrum in the $\Lambda_b^0 \to J/\psi K^- p$ decays [1]. Recently, the LHCb collaboration observed a new narrow pentaquark candidate $P_c(4312)$ in the $J/\psi p$ mass spectrum with the statistical significance of $7.3\sigma$, and confirmed the old $P_c(4450)$ pentaquark structure, which consists of two narrow overlapping peaks $P_c(4440)$ and $P_c(4457)$ with the statistical significance of $5.4\sigma$ [2]. The masses and widths are

$$
\begin{align*}
P_c(4312) & : M = 4311.9 \pm 0.7^{+6.8}_{-9.6} \text{ MeV}, \quad \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV,} \\
P_c(4440) & : M = 4440.3 \pm 1.3^{+4.1}_{-4.7} \text{ MeV}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV,} \\
P_c(4457) & : M = 4457.3 \pm 0.6^{+4.2}_{-1.4} \text{ MeV}, \quad \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.5} \text{ MeV}. 
\end{align*}
$$

The $P_c(4312)$ may be a $\bar{D} \Sigma_c$ pentaquark molecular state [3], a pentaquark state [5, 6, 7], a hadrocharmonium pentaquark state [8].

The $P_c(4312)$ lies near the $\bar{D} \Sigma_c$ threshold, which leads to the molecule assignment naturally. In Ref.[4], we perform detailed studies of the $\bar{D} \Sigma_c$, $\bar{D} \Sigma_c^*$, $\bar{D}^* \Sigma_c$ and $\bar{D}^* \Sigma_c^*$ pentaquark molecular states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 13 in a consistent way. The prediction $M_P = 4.32 \pm 0.11 \text{ GeV}$ for the $\bar{D} \Sigma_c$ molecular state supports assigning the $P_c(4312)$ to be the $\bar{D} \Sigma_c$ pentaquark molecular state with $J^P = \frac{3}{2}^-$. On the other hand, our studies based on the QCD sum rules indicate that the scalar-diquark-scalar-diquark-antiquark type pentaquark state with the $J^P = \frac{1}{2}^-$ has a mass $4.31 \pm 0.11 \text{ GeV}$, the axialvector-diquark-axialvector-diquark-antiquark type pentaquark state with the $J^P = \frac{1}{2}^-$ has a mass $4.34 \pm 0.14 \text{ GeV}$, which support assigning the $P_c(4312)$ to be a diquark-antiquark type pentaquark state [7, 9]. The $P_c(4312)$ may be a diquark-diquark-antiquark type pentaquark state, which has a strong coupling to the $\bar{D} \Sigma_c$ scattering states, the strong coupling induces some $\bar{D} \Sigma_c$ components [10]. So we can reproduce the experimental value of the mass of the $P_c(4312)$ in both scenarios of the pentaquark state and pentaquark molecular state. In Ref.[11], we choose the $[sc]^p[\bar{s}c]^s_A \leftarrow [sc]^s_A[\bar{s}c]^p$ type tetraquark current to study the strong decays of the $Y(4660)$ with the QCD sum rules based on solid quark-hadron quality. In calculations, we observe that the hadronic coupling constants $|G_Y \psi f_0| \gg |G_Y J/\psi f_0|$, which is consistent with the observation of the $Y(4660)$ in the $\psi'/\pi^+\pi^-$ mass spectrum, and favors the $\psi f_0(980)$ molecule assignment [12]. The similar mechanism maybe exist for the $P_c(4312)$.  

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In this article, we tentatively assign the $P_c(4312)$ to be the $\bar{D}\Sigma_c$ pentaquark molecular state with $J^P = \frac{3}{2}^-$, and study its two-body strong decays with the QCD sum rules. In Ref.\cite{13}, we assign the $Z_c(3900)$ to be the diquark-antidiquark type axialvector tetraquark state, study the hadronic coupling constants in the strong decays $Z_c(3900) \to J/\psi \pi$, $\eta_c \rho$, $D^* \pi$ with the QCD sum rules based on solid quark-hadron duality by taking into account both the connected and disconnected Feynman diagrams in the operator product expansion. The method works well in studying the two-body strong decays of the $Z_c(3900)$, $X(4140)$, $X(4274)$ and $Z_c(4600)$ \cite{13,14}. Now we extend the method to study the two-body strong decays of the pentaquark molecular state by carrying out the operator product expansion up to the vacuum condensates of dimension 10.

The article is arranged as follows: we derive the QCD sum rules for the hadronic coupling constants in the strong decays $P_c(4312) \to \eta_c \rho$, $J/\psi p$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

\section{QCD sum rules for the $P_c(4312)$ decays as a pentaquark molecular state}

In the following, we write down the three-point correlation functions $\Pi_5(p, q)$ and $\Pi_\mu(p, q)$ in the QCD sum rules,

$$\Pi_5(p, q) = i^2 \int d^4xd^4ye^{ip \cdot x}e^{iq \cdot y}\langle 0|T\{J_5(x)J_N(y)\bar{J}(0)\}|0\rangle,$$

$$\Pi_\mu(p, q) = i^2 \int d^4xd^4ye^{ip \cdot x}e^{iq \cdot y}\langle 0|T\{J_\mu(x)J_N(y)\bar{J}(0)\}|0\rangle,$$

where

$$J_5(x) = \bar{c}(x)i\gamma_5c(x),$$

$$J_\mu(x) = \bar{c}(x)\gamma_\mu c(x),$$

$$J_N(y) = \epsilon^{ijk}u_T^T(y)C\gamma_\alpha u_j(y)\gamma^\alpha\gamma_5d_k(y),$$

$$J(0) = \bar{c}(0)i\gamma_5u(0)\epsilon^{ijk}u_T^T(0)C\gamma_\alpha d_j(0)\gamma^\alpha\gamma_5c(0),$$

the $i, j, k$ are color indices. We choose the currents $J_5(x)$, $J_\mu(x)$, $J_N(y)$ and $J(0)$ to interpolate the $\eta_c$, $J/\psi$, $p$ and $P_c(4312)$, respectively. Thereafter we will denote the proton $p$ as $N$ to avoid confusion due to the four momentum $p_\mu$.

At the hadron side, we insert a complete set of intermediate hadron states with the same quantum numbers as the current operators $J_5(x)$, $J_\mu(x)$, $J_N(y)$ and $J(0)$ to obtain the hadronic representation \cite{15,16}. After isolating the pole terms of the ground states, we obtain the following results:

$$\Pi_5(p, q) = \frac{f_m m_p^2 \lambda_\mu \lambda_N}{2m_c} \frac{-iu(q)\langle \eta_c(p)N(q)|P(p')\rangle \bar{u}(p')}{m_\mu^2 - p'^2 (m_N^2 - q^2)} + \cdots$$

$$= \frac{f_m m_p^2 \lambda_\mu \lambda_N}{2m_c} \frac{-i(q + m_N)(p' + m_p)}{m_\mu^2 - p'^2 (m_N^2 - q^2)} \cdot \gamma_5 + \cdots,$$

$$\Pi_\mu(p, q) = \frac{f_m m_p^2 \lambda_\mu \lambda_N}{2m_c} \frac{-i\epsilon_\mu u(q)\langle J/\psi(p)N(q)|P(p')\rangle \bar{u}(p')}{m_\mu^2 - p'^2 (m_N^2 - q^2)} + \cdots$$

$$= \frac{f_m m_p^2 \lambda_\mu \lambda_N}{2m_c} \frac{-i(q + m_N)}{(m_\mu^2 - p'^2) (m_N^2 - q^2)} \cdot \gamma_5 (p' + m_p)$$

$$\left( -g_{\mu\alpha} + \frac{p_\mu p_\alpha}{p^2} \right) + \cdots,$$
where we have used the definitions,

\[
\begin{align*}
(0|J(0)|P_{\mu}(p')) &= \lambda_{P} U(p', s), \\
(0|J_{N}(0)|N(q)) &= \lambda_{N} U(q, s), \\
(0|J_{\mu}(0)|J/\psi(p)) &= f_{J/\psi} m_{J/\psi} \varepsilon_{\mu}(p, s), \\
(0|J_{\mu}(0)|\eta_{\mu}(p)) &= \frac{f_{\eta} m_{\eta}^{2}}{2m_{c}},
\end{align*}
\]

\text{(7)}

\[
\begin{align*}
\langle \eta_{\mu}(p)|N(q)|P'(p') \rangle &= ig_{5} \bar{u}(q) u(p'), \\
\langle J/\psi(p)|N(q)|P'(p') \rangle &= \bar{u}(q) \varepsilon_{\alpha}^{*} \left( g_{\nu} \gamma^{\alpha} - i \frac{g_{T}}{m_{P} + m_{N}} \sigma^{\alpha \beta} p_{\beta} \right) \gamma_{5} u(p'),
\end{align*}
\]

\text{(8)}

the \( g_{5}, \ g_{V} \) and \( g_{T} \) are the hadronic coupling constants, the \( U(p, s) \) and \( U(q, s) \) are the Dirac spinors, the \( \lambda_{P} \) and \( \lambda_{N} \) are the pole residues, the \( f_{J/\psi} \) and \( f_{\eta} \) are the decay constants, the \( \varepsilon_{\mu} \) is the polarization vector of the \( J/\psi \).

It is important to choose the pertinent structures to study the hadronic coupling constants. If \( \Pi_{5,H}(p, q) = \Pi_{5,\text{QCD}}(p, q) \) and \( \Pi_{\mu,H}(p, q) = \Pi_{\mu,\text{QCD}}(p, q) \), we expect the relations \( \text{Tr} [\Pi_{5,H}(p, q) \Gamma] = \text{Tr} [\Pi_{5,\text{QCD}}(p, q) \Gamma] \) and \( \text{Tr} [\Pi_{\mu,H}(p, q) \Gamma'] = \text{Tr} [\Pi_{\mu,\text{QCD}}(p, q) \Gamma'] \) exist, where the subscripts \( H \) and \( \text{QCD} \) denote the hadron side and QCD side of the correlation functions, respectively, the \( \Gamma \) and \( \Gamma' \) are some Dirac \( \gamma \)-matrices.

In this article, we choose \( \Gamma = \sigma_{\mu \nu}, i\gamma_{\mu}, \Gamma' = \gamma_{5}, \gamma_{5} \),

\[
\frac{1}{4} \text{Tr} [\Pi_{5}(p, q) \sigma_{\mu \nu}] = \Pi_{5}(p^{2}, p^{2}, q^{2}) i (p_{\mu} q_{\nu} - q_{\mu} p_{\nu}) + \cdots,
\]

\[
\frac{1}{4} \text{Tr} [\Pi_{5}(p, q) i\gamma_{\mu}] = \Pi_{5}(p^{2}, p^{2}, q^{2}) iq_{\mu} + \cdots,
\]

\[
\frac{1}{4} \text{Tr} [\Pi_{\mu}(p, q) \gamma_{5}] = \Pi_{A}(p^{2}, p^{2}, q^{2}) iq_{\mu} p \cdot z + \cdots,
\]

\[
\frac{1}{4} \text{Tr} [\Pi_{\mu}(p, q) \gamma_{5}] = \Pi_{B}(p^{2}, p^{2}, q^{2}) iq_{\mu} + \cdots,
\]

\text{(9)}

and choose the tensor structures \( p_{\mu} q_{\nu} - q_{\mu} p_{\nu}, q_{\mu} p \cdot z \) and \( q_{\mu} \) to study the hadronic coupling constants \( g_{5}, g_{V} \) and \( g_{T} \), respectively, where the \( z_{\mu} \) is a four vector.

Now we write down the components \( \Pi_{5}(p^{2}, p^{2}, q^{2}), \Pi_{5}(p^{2}, p^{2}, q^{2}), \Pi_{A}(p^{2}, p^{2}, q^{2}) \) and \( \Pi_{B}(p^{2}, p^{2}, q^{2}) \) explicitly,

\[
\Pi_{5}(p^{2}, p^{2}, q^{2}) = \frac{f_{\eta} m_{\eta}^{2} \lambda_{P} \lambda_{N}}{2m_{c}} \frac{g_{5}}{(m_{p}^{2} - p^{2})(m_{n}^{2} - p^{2})(m_{N}^{2} - q^{2})}
\]

\[
+ \frac{1}{(m_{p}^{2} - p^{2})(m_{n}^{2} - p^{2})} \int_{s_{N}}^{\infty} dt \frac{\rho_{P,N}^{5}(p^{2}, p^{2}, t)}{t - q^{2}}
\]

\[
+ \frac{1}{(m_{p}^{2} - p^{2})(m_{N}^{2} - q^{2})} \int_{s_{N}}^{\infty} dt \frac{\rho_{P,N}^{5}(t, p^{2}, q^{2}) + \rho_{P,N}^{5}(t, p^{2}, q^{2})}{t - p^{2}} + \cdots,
\]

\text{(10)}
where we introduce the formal functions $\rho$ to parameterize the transitions between the ground states and the excited states. The $s_{0}^{A}$, $s_{0}^{B}$, $s_{n}^{A}$, and $s_{n}^{B}$ are the threshold parameters for the radial excited states. Now we smear the indexes $\lambda, \mu, \nu,$ et al, and rewrite (components of) the correlation functions $\Pi_{H}(p^{2}, p^{2}, q^{2})$ at the hadron side as

$$\Pi_{H}(p^{2}, p^{2}, q^{2}) = \int_{s_{0}^{A}}^{s_{0}^{B}} ds' \int_{s_{n}^{A}}^{s_{n}^{B}} ds \int_{0}^{s_{0}^{A}} du \frac{\rho_{H}(s', s, u)}{(s' - p^{2})(s - p^{2})(u - q^{2})} + \int_{s_{n}^{A}}^{\infty} ds' \int_{4m_{c}^{2}}^{s_{n}^{A}} ds \int_{0}^{s_{n}^{A}} du \frac{\rho_{H}(s', s, u)}{(s' - p^{2})(s - p^{2})(u - q^{2})} + \cdots,$$
through dispersion relation, and take $\eta_c = \eta_c$, $J/\psi$ for simplicity, where the $\rho_H(s', s, u)$ are the hadronic spectral densities.

We carry out the operator product expansion at the QCD side, and write (components of) the correlation functions $\Pi_{QCD}(p'^2, p^2, q^2)$ as

$$\Pi_{QCD}(p'^2, p^2, q^2) = \int_{4m_c^2}^{s_{\eta_c}} ds \int_{0}^{s_{\eta_c}} du \frac{\rho_{QCD}(p'^2, s, u)}{(s - p^2)(u - q^2)} + \cdots, \quad (15)$$

through dispersion relation, where the $\rho_{QCD}(p'^2, s, u)$ are the QCD spectral densities, because the QCD spectral densities $\rho_{QCD}(s', s, u)$ do not exist,

$$\rho_{QCD}(s', s, u) = \lim_{t \to 0} \frac{\text{Im} \Pi_{QCD}(s' + i\epsilon, s, u)}{\pi} = 0, \quad (16)$$

we can write the QCD spectral densities $\rho_{QCD}(p'^2, s, u)$ as $\rho_{QCD}(s, u)$ for simplicity.

Now we match the hadron side with the QCD side of the correlation functions, and carry out the integral over $ds'$ firstly to obtain the solid duality [13],

$$\int_{4m_c^2}^{s_{\eta_c}} ds \int_{0}^{s_{\eta_c}} du \frac{\rho_{QCD}(s, u)}{(s - p^2)(u - q^2)} = \int_{4m_c^2}^{s_{\eta_c}} ds \int_{0}^{s_{\eta_c}} du \frac{1}{(s - p^2)(u - q^2)} \left[ \int_{(m_c + m_N)^2}^{\infty} ds' \frac{\rho_H(s', s, u)}{s' - p^2} \right]. \quad (17)$$

It is impossible to carry out the integral over $s'$ explicitly due to the unknown functions $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$, and $\tilde{\rho}_{P^\alpha N}(t, p^2, q^2)$. Now we introduce the parameters $C_5$, $C_5$, $C_A$ and $C_B$ to parameterize the net effects,

$$C_5 = \int_{s_0^p}^{s_{\eta_c}} dt \frac{\tilde{\rho}_{P^\alpha N}(t, p^2, q^2) + \tilde{\rho}_{P^\alpha N}(t, p^2, q^2)}{t - p^2},$$

$$C_5 = \int_{s_0^p}^{s_{\eta_c}} dt \frac{\tilde{\rho}_{P^\alpha N}(t, p^2, q^2) + \tilde{\rho}_{P^\alpha N}(t, p^2, q^2)}{t - p^2},$$

$$C_A = \int_{s_0^p}^{s_{\eta_c}} dt \frac{\tilde{\rho}_{P^\alpha N}(t, p^2, q^2) + \tilde{\rho}_{P^\alpha N}(t, p^2, q^2)}{t - p^2},$$

$$C_B = \int_{s_0^p}^{s_{\eta_c}} dt \frac{\tilde{\rho}_{P^\alpha N}(t, p^2, q^2) + \tilde{\rho}_{P^\alpha N}(t, p^2, q^2)}{t - p^2}. \quad (18)$$

In the following, we write down the quark-hadron duality explicitly,

$$\int_{4m_c^2}^{s_{\eta_c}} ds \int_{0}^{s_{\eta_c}} du \frac{\tilde{\rho}_{QCD}(s, u)}{(s - p^2)(u - q^2)} = \frac{f_N m_{\eta_c}^2 \lambda_P \lambda_N}{2m_c} \frac{g_5}{(m_p^2 - p^2)(m_{\eta_c}^2 - p^2)(m_N^2 - q^2)} \frac{C_5}{(m_{\eta_c}^2 - p^2)(m_N^2 - q^2)}, \quad (19)$$

$$\int_{4m_c^2}^{s_{\eta_c}} ds \int_{0}^{s_{\eta_c}} du \frac{\tilde{\rho}_{QCD}(s, u)}{(s - p^2)(u - q^2)} = \frac{f_N m_{\eta_c}^2 \lambda_P \lambda_N}{2m_c} \frac{g_5}{(m_p^2 - p^2)(m_{\eta_c}^2 - p^2)(m_N^2 - q^2)} \frac{C_5}{(m_{\eta_c}^2 - p^2)(m_N^2 - q^2)}, \quad (20)$$
\[
\int_{4m_c^2}^{s_{J/\psi}} ds \int_0^{s_N} du \frac{\rho_{QCD}^A(s,u)}{(s-p^2)(u-q^2)} = f_{J/\psi} m_{J/\psi} \lambda_p \lambda_N \frac{g_T - g_V}{(m_P^2 - p^2)(m_{J/\psi}^2 - p^2)(m_N^2 - q^2)} + \frac{C_A}{(m_{J/\psi}^2 - p^2)(m_N^2 - q^2)}, \tag{21}
\]

\[
\int_{4m_c^2}^{s_{J/\psi}} ds \int_0^{s_N} du \frac{\rho_{QCD}^B(s,u)}{(s-p^2)(u-q^2)} = f_{J/\psi} m_{J/\psi} \lambda_p \lambda_N \frac{(m_P - m_N) g_V - g_T m_{J/\psi}}{(m_P^2 - p^2)(m_{J/\psi}^2 - p^2)(m_N^2 - q^2)} + \frac{C_B}{(m_{J/\psi}^2 - p^2)(m_N^2 - q^2)}. \tag{22}
\]

We set \( p^2 = -p^2 \) and perform double Borel transform with respect to the variables \( P^2 = -p^2 \) and \( Q^2 = -q^2 \), respectively to obtain the QCD sum rules,

\[
\frac{f_{\eta_c} m_{\eta_c}^2 \lambda_P \lambda_N}{2m_c} \frac{g_5}{m_P^2 - m_{\eta_c}^2} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} \right) \right] \frac{1}{m_P^2 - m_{\eta_c}^2} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_2^2} \right) \right] \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_2^2} \right) + \frac{C_5 \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right)}{2m_c} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right), \tag{23}
\]

\[
\frac{f_{\eta_c} m_{\eta_c}^2 \lambda_P \lambda_N \left( m_P + m_N \right) g_5}{2m_c} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) + \frac{C_{5T} \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right)}{2m_c} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) + \frac{C_{5T} \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right)}{2m_c} \left[ \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{\eta_c}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right), \tag{24}
\]

\[
f_{J/\psi} m_{J/\psi} \lambda_p \lambda_N \frac{g_{VT}}{m_P^2 - m_{J/\psi}^2} \left[ \exp \left( -\frac{m_{J/\psi}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{J/\psi}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) + \frac{C_{VT} \exp \left( -\frac{m_{J/\psi}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right)}{2m_c} \left[ \exp \left( -\frac{m_{J/\psi}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) - \exp \left( -\frac{m_P^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{J/\psi}^2}{T_1^2} - \frac{m_N^2}{T_2^2} \right), \tag{25}
\]

\[
C_V = \left[ \frac{m_{J/\psi}^2}{m_P + m_N} C_A + C_B \right] \frac{m_P + m_N}{m_P^2 - m_{J/\psi}^2 - m_N^2}; \quad C_T = \left[ (m_P - m_N) C_A + C_B \right] \frac{m_P + m_N}{m_P^2 - m_{J/\psi}^2 - m_N^2}.
\]

\[
\rho_{QCD}^V(s,u) = \left[ \frac{m_{J/\psi}^2}{m_P + m_N} \rho_{QCD}^A(s,u) + \rho_{QCD}^B(s,u) \right] \frac{m_P + m_N}{m_P^2 - m_{J/\psi}^2 - m_N^2}; \quad \rho_{QCD}^T(s,u) = \left[ (m_P - m_N) \rho_{QCD}^A(s,u) + \rho_{QCD}^B(s,u) \right] \frac{m_P + m_N}{m_P^2 - m_{J/\psi}^2 - m_N^2}, \tag{26}
\]
\[
\rho_{QCD}^5(s,u) &= \frac{m_c}{4096\pi^6} \int_{x_i}^{x_f} dx u^2 - \frac{m_c^3}{36864\pi^4 T_1^4} \langle \alpha_s GG \rangle \int_{x_i}^{x_f} dx \frac{1}{x^3} u^2 \delta \left(s - \tilde{m}^2_c\right) \\
&+ \frac{m_c}{12288\pi^4 T_1^4} \langle \alpha_s GG \rangle \int_{x_i}^{x_f} dx \frac{1-x}{x^2} u^2 \delta \left(s - \tilde{m}^2_c\right) \\
&+ \frac{m_c}{24576\pi^4 T_1^2} \langle \alpha_s GG \rangle \int_{x_i}^{x_f} dx \frac{1}{x(1-x)} u^2 \delta \left(s - \tilde{m}^2_c\right) \\
&+ \frac{m_c}{2048\pi} \langle \bar{q}q \rangle \langle \bar{q}g \sigma G q \rangle \int_{x_i}^{x_f} dx \frac{1}{x(1-x)} \delta \left(s - \tilde{m}^2_c\right) \delta(u) \\
&+ \frac{m_c}{576\pi^2} \langle \bar{q}q \rangle \langle \bar{q}g \sigma G q \rangle \int_{x_i}^{x_f} dx \delta(u) \\
&+ \frac{m_c}{576\pi^2 T_2^2} \langle \bar{q}q \rangle \langle \bar{q}g \sigma G q \rangle \int_{x_i}^{x_f} dx \frac{1}{x(1-x)} \delta \left(s - \tilde{m}^2_c\right) \delta(u) ,
\]
(27)
\[ \mathbf{p}_{QCD} (s, u) = \frac{1}{2048 \pi^6} \int_{x_i}^{x_f} dx \, x^2 u + \frac{m_c (q q)}{384 \pi^4} \int_{x_i}^{x_f} dx \, u + \frac{m_c (q g, g Q)}{768 \pi^4} \int_{x_i}^{x_f} dx \, \frac{u}{x} \delta (s - \bar{m}_c^2) \]
\[ + \frac{(q q)^2}{48 \pi^2} \int_{x_i}^{x_f} dx \, x s \delta (u) + \frac{m_c^2}{18432 \pi^4 T_i} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, u^2 \left( 2 - \frac{s}{T_i^2} \right) \delta (s - \bar{m}_c^2) \]
\[ + \frac{1}{768 \pi^4} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, u s \delta (s - \bar{m}_c^2) + \frac{1}{3072 \pi^4} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, (u + 3 x s) \]
\[ + \frac{1}{12288 \pi^4} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, (1 + x) (2 u s + u^2) \delta (s - \bar{m}_c^2) \]
\[ + \frac{1}{12288 \pi^4 T_i} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, u^2 \delta (s - \bar{m}_c^2) \]
\[ - \frac{m_c^3}{3456 \pi^2 T_i} \left( \frac{q q}{\pi} \right) \int_{x_i}^{x_f} dx \, \frac{u}{x^2} \delta (s - \bar{m}_c^2) \]
\[ + \frac{m_c (q q)}{1152 \pi^2 T_i} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, \frac{1 - x}{x^2} u \delta (s - \bar{m}_c^2) \]
\[ + \frac{m_c^2 (q q)^2}{2304 \pi^2 T_i} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, \frac{1}{x (1 - x)} u \delta (s - \bar{m}_c^2) \]
\[ + \frac{m_c (q q)}{1152 \pi^2} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, \delta (u) \]
\[ + \frac{m_c^2 (q q)^2}{432 T_i^2} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, \frac{1}{x^2} \left( 2 - \frac{s}{T_i^2} \right) \delta (s - \bar{m}_c^2) \delta (u) \]
\[ + \frac{(q q)^2}{864 T_i^2} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, [4 s \delta (s - \bar{m}_c^2) + 1] \delta (u) \]
\[ + \frac{(q q)^2}{288 T_i^2} \left( \frac{\alpha_s G G}{\pi} \right) \int_{x_i}^{x_f} dx \, \frac{2 - x}{1 - x} s \delta (s - \bar{m}_c^2) \delta (u) \]
\[ - \frac{(q q) (q g, g Q)}{96 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, x s \delta (u) + \frac{(q q) (q g, g Q)}{288 \pi^2} \int_{x_i}^{x_f} dx \, s \delta (s - \bar{m}_c^2) \delta (u) \]
\[ + \frac{(q q) (q g, g Q)}{576 \pi^4} \int_{x_i}^{x_f} dx \, \frac{1}{1 - x} s \delta (s - \bar{m}_c^2) \delta (u) \]
\[ + \frac{(q g, g Q)}{384 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, s \delta (u) + \frac{(q g, g Q)^2}{2304 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, \delta (u) \]
\[ - \frac{(q g, g Q)^2}{1728 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, s \delta (s - \bar{m}_c^2) \delta (u) - \frac{(q g, g Q)^2}{13824 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, \frac{1}{1 - x} s \delta (s - \bar{m}_c^2) \delta (u) \]
\[ + \frac{(q g, g Q)^2}{13824 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, s \delta (s - \bar{m}_c^2) \delta (u) - \frac{(q g, g Q)^2}{13824 \pi^2 T_i^2} \int_{x_i}^{x_f} dx \, \frac{1}{1 - x} s \delta (s - \bar{m}_c^2) \delta (u) \]
\[ - \frac{(q g, g Q)^2}{6912 \pi^2} \int_{x_i}^{x_f} dx \, \delta (s - \bar{m}_c^2) \delta (u), \]
\[
\rho^A_{QCD}(s,u) = \frac{m_c \langle \bar{q} q \rangle^2}{48\pi^2} \int_{x_1}^{x_f} dx \delta(u) - \frac{5m_c \langle \bar{q} q \rangle \langle g g_s G q \rangle}{256\pi^2 T_s^2} \int_{x_1}^{x_f} dx \delta(u)
\]
\[
+ \frac{m_c \langle \bar{q} q \rangle \langle g g_s G q \rangle}{288\pi^2} \int_{x_1}^{x_f} dx \frac{1}{x} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
+ \frac{m_c \langle \bar{q} q \rangle}{9216\pi^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \frac{1}{x(1-x)} u \delta \left( s - \tilde{m}_c^2 \right)
\]
\[
- \frac{m_c \langle \bar{q} q \rangle^2}{432T_s^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \frac{1}{x^3} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
+ \frac{m_c \langle \bar{q} q \rangle^2}{144T_s^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \frac{1}{x^2} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
+ \frac{m_c \langle \bar{q} q \rangle^2}{288T_s^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \left( \frac{1}{x} - \frac{2}{3} \right) \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
+ \frac{m_c \langle \bar{q} q \rangle^2}{864T_s^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \frac{1-2x}{1-x} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
- \frac{m_c \langle \bar{q} q \rangle^2}{864T_s^4} \langle \alpha_s G G \rangle \int_{x_1}^{x_f} dx \frac{1}{x(1-x)} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[
- \frac{m_c \langle \bar{q} g_s G q \rangle^2}{4608\pi^2 T_s^4} \int_{x_1}^{x_f} dx \frac{1}{x(1-x)} \delta \left( s - \tilde{m}_c^2 \right) \delta(u)
\]
\[- \frac{m_c \langle \bar{q} g_s G q \rangle^2}{13824\pi^2 T_s^4} \int_{x_1}^{x_f} dx \frac{1}{x(1-x)} \delta \left( s - \tilde{m}_c^2 \right) \delta(u),
\]
(29)
\[ \rho_{QCD}^B(s, u) = \frac{1}{4096 \pi^6} \int_{x_i}^{x_f} dx \left[ xs + x (1 - x) \left( s - \bar{m}_c^2 \right) \right] u^2 \\
\left( \frac{g}{2} \right)^2 \int_{x_i}^{x_f} dx \left[ xs + x (1 - x) \left( s - \bar{m}_c^2 \right) \right] \delta(u) \\
- \frac{m_c^2}{36864 \pi^4 T_1^2} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left( \frac{x}{1 - x T_1^2} - 1 \right) u^2 \delta \left( s - \bar{m}_c^2 \right) \\
- \frac{m_c^2}{36864 \pi^4 T_1^2} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left( \frac{3 - 2x}{s} \right) \delta \left( s - \bar{m}_c^2 \right) + u \\
- \frac{1}{2304 \pi^4} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left[ \frac{4u}{9} + xs + x (1 - x) \left( s - \bar{m}_c^2 \right) \right] \\
+ \frac{1}{73728 \pi^4} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left( \frac{x}{1 - x T_1^2} - 1 \right) u^2 \delta \left( s - \bar{m}_c^2 \right) \\
+ \frac{m_c^2 (\bar{q}q)^2}{432 T_1^2} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left[ \frac{3 - 2x}{s} \right] \delta \left( s - \bar{m}_c^2 \right) + 3 \delta(u) \\
+ \frac{m_c^2 (\bar{q}q)^2}{432} \left( \frac{\alpha_G}{\pi} \right) \int_{x_i}^{x_f} dx \left( \frac{x}{1 - x T_1^2} - 1 \right) \delta \left( s - \bar{m}_c^2 \right) \delta(u) \\
+ \frac{m_c^2 (\bar{q}q, Gq)}{1152 \pi^2 T_1^2} \int_{x_i}^{x_f} dx \left[ 1 + \frac{2 + 4x}{1 - x} \delta \left( s - \bar{m}_c^2 \right) \right] \delta(u) \\
+ \frac{m_c^2 (\bar{q}q, Gq)^2}{4068 \pi^2 T_2^2} \int_{x_i}^{x_f} dx \left[ 3 + \frac{5x}{1 - x} \delta \left( s - \bar{m}_c^2 \right) \right] \delta(u) \right), \quad (30) \]

where \( x_f = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2}, \ x_i = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2} \), \( \bar{m}_c^2 = \frac{m_c^2}{\alpha_G}, \int_{x_i}^{x_f} dx \rightarrow \int_0^1 dx \), when the \( \delta \) function \( \delta(s - s_c^2) \) appears.

In this article, we carry out the operator product expansion to the vacuum condensates up to dimension-10, and assume vacuum saturation for the higher dimension vacuum condensates. As the vacuum condensates are vacuum expectations of the quark-gluon operators, we take the truncations \( n \leq 10 \) and \( k \leq 1 \) in a consistent way, the operators of the orders \( \mathcal{O}(\alpha_s^k) \) with \( k > 1 \) are neglected. Furthermore, we set the two Borel parameters to be \( T_1^2 = T_2^2 = T^2 \) for simplicity, if we take the \( T_1^2 \) and \( T_2^2 \) as two independent parameters, it is difficult to obtain stable QCD sum rules. In numerical calculations, we take the \( C_5, \overline{C_5}, C_V \) and \( C_T \) as free parameters and choose the suitable values to obtain stable QCD sum rules.

### 3 Numerical results and discussions

At the hadron side, we take the hadronic parameters as \( m_{J/\psi} = 3.0969 \text{ GeV}, m_N = 0.93827 \text{ GeV}, m_{h_c} = 2.9839 \text{ GeV}, \sqrt{s_{J/\psi}} = 3.6 \text{ GeV}, \sqrt{s_{h_c}} = 3.5 \text{ GeV}, \sqrt{s_N} = 1.3 \text{ GeV} \text{ [17],} \ m_p = 4.3119 \text{ GeV} \text{ [2],} f_{J/\psi} = 0.418 \text{ GeV,} f_{h_c} = 0.387 \text{ GeV} \text{ [18],} \lambda_N = 0.032 \text{ GeV}^3 \text{ [19],} \lambda_p = 1.95 \times 10^{-3} \text{ GeV}^6 \text{ [4].} \)
At the QCD side, we take the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{q}g\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1 \text{GeV})^2$, $\langle \alpha_s G^2 \rangle = (0.33 \text{GeV})^4$ at the energy scale $\mu = 1 \text{GeV}$ [15, 16, 20], and choose the $\overline{\text{MS}}$ mass $m_c(m_c) = (1.275 \pm 0.025 \text{GeV})$ from the Particle Data Group [17]. Moreover, we take into account the energy-scale dependence of the parameters,

$$
\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{\Lambda^2}},
$$

$$
\langle \bar{q}g_\sigma Gq \rangle(\mu) = \langle \bar{q}g_\sigma Gq \rangle(1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{\Lambda^2}},
$$

$$
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{\mu^2}{\Lambda^2}},
$$

$$
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right],
$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33 - 2n_f}{12\pi}$, $b_1 = \frac{153 - 19n_f}{2\pi}$, $b_2 = \frac{257 - 510n_f + 283\pi^2}{12\pi^2}$, $\Lambda = 210 \text{MeV}$, $292 \text{MeV}$ and $332 \text{MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [17, 21], and evolve all the parameters to the ideal energy scale $\mu$ with $n_f = 4$ to extract the hadronic coupling constants $g_5$, $g_V$ and $g_T$.

In the QCD sum rules for the mass of the $\bar{D}S_0(2460)$ pentaquark molecular state with $J^P = \frac{3}{2}^-$ or the $P_c(4312)$, the ideal energy scale of the QCD spectral density is $\mu = 2.2 \text{GeV}$ [4], which is determined by the energy scale formula $\mu = \sqrt{M_{\eta_c}^2 - m_c^2} - (2M_P)^2$ with the effective c-quark mass $M_c = 1.85 \text{GeV}$ [22]. The energy scale $\mu = 2.2 \text{GeV}$ is too large for the $N$, $\eta_c$ and $J/\psi$. In this article, we take the energy scales of the QCD spectral densities to be $\mu = \frac{m_c}{2} = 1.5 \text{GeV}$, which is acceptable for the charmonium states [23].

We choose the values of the free parameters as $C_5 = 1.18 \times 10^{-6} \text{GeV}^9$, $C_5 = 1.94 \times 10^{-5} \text{GeV}^{10}$, $C_V = -1.77 \times 10^{-5} \text{GeV}^9$, $C_T = -1.67 \times 10^{-5} \text{GeV}^9$ to obtain flat platforms in the Borel windows $T^2 = (3.1 - 4.1) \text{GeV}^2$, $(3.3 - 4.3) \text{GeV}^2$, $(4.0 - 5.0) \text{GeV}^2$ and $(3.9 - 4.9) \text{GeV}^2$ for the hadronic coupling constants $g_5$, $g_V$ and $g_T$, respectively. We fit the free parameters $C_5$, $C_5$, $C_V$ and $C_T$ to obtain the same intervals of flat platforms $T_{\text{max}}^2 - T_{\text{min}}^2 = 1.0 \text{GeV}^2$, where the $T_{\text{max}}^2$ and $T_{\text{min}}^2$ denote the maximum and minimum of the Borel parameters, respectively.

We take into account the uncertainties of the input parameters, and obtain the values of the hadronic coupling constants $g_5$, $g_V$ and $g_T$, which are shown in Fig.1,

$$
g_5 = 0.09 \pm 0.03 \quad \text{from Eq.}\ (23),
$$

$$
g_5 = 0.09 \pm 0.07 \quad \text{from Eq.}\ (24),
$$

$$
g_V = 0.40 \pm 0.50,
$$

$$
g_T = 0.10 \pm 0.40,
$$

(32)

where we have redefined the hadronic coupling constants $g_V/g_T$ in Eq. (3) with a simple replacement $g_V/g_T \rightarrow -g_V/g_T$, as the central values of the $g_V/g_T$ are negative from the QCD sum rules in Eq. (25).

Now it is straightforward to calculate the partial decay widths of the decays $P_c(4312) \rightarrow \eta_c N$, $J/\psi N$,

$$
\Gamma (P_c(4312) \rightarrow \eta_c N) = \frac{p(m_P, m_{\eta_c}, m_N)}{16\pi m_P^2} |T|^2 = 31.488 g_5^2 \text{ MeV}
$$

$$
= 0.255^{+0.198}_{-0.142} \text{MeV} \quad \text{from Eq.}\ (23),
$$

$$
= 0.255^{+0.551}_{-0.242} \text{MeV} \quad \text{from Eq.}\ (24),
$$

(33)
Figure 1: The hadronic coupling constants $g_5$, $g_V$ and $g_T$ with variations of the Borel parameters $T^2$, the values of the $g_5$ in the first diagram and second diagram come from the QCD sum rules in Eq. (23) and Eq. (24), respectively.
where
\[
T = \bar{u}(q)i\gamma_5 u(p'),
\] (34)
and
\[
p(a, b, c) = \frac{\sqrt{|a^2 - (b+c)^2||a^2 - (b-c)^2|}}{2a},
\]
\[
\Gamma (P_\epsilon(4312) \rightarrow J/\psi N) = \frac{p(m_P, m_{J/\psi}, m_N)}{16\pi m_P^2}|T|^2 = 29.699 g_V^2 - 97.554 g_V g_T + 80.633 g_T^2 \text{ MeV}
\]
\[
= 9.296^{+19.542}_{-9.296} \text{ MeV},
\] (35)
where
\[
T = \varepsilon_\alpha^* \bar{u}(q) \left( g_V \gamma_\alpha - i \frac{g_T}{m_P + m_N} \sigma^{\alpha\beta} p_\beta \right) \gamma_5 u(p').
\] (36)

The partial decay width \(\Gamma (P_\epsilon(4312) \rightarrow \eta_c N) = 0.255 \text{ MeV}\) is vary small, the total width \(\Gamma_{P_\epsilon(4312)}\) can be saturated with the strong decay \(P_\epsilon(4312) \rightarrow J/\psi N\). The predicted width \(\Gamma (P_\epsilon(4312) \rightarrow J/\psi N) = 9.296^{+19.542}_{-9.296} \text{ MeV}\) is compatible with the experimental data \(\Gamma_{P_\epsilon(4312)} = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}\) from the LHCb collaboration [2]. The present calculations support assigning the \(P_\epsilon(4312)\) to be the \(\bar{D}\Sigma_c\) pentaquark molecular state with \(J^P = \frac{1}{2}^-\). We can search for the \(P_\epsilon(4312)\) in the \(\eta_c N\) mass spectrum, and measure the branching fraction \(\text{Br}(P_\epsilon(4312) \rightarrow \eta_c N)\), which maybe shed light on the nature of the \(P_\epsilon(4312)\) and test the predictions of the QCD sum rules.

4 Conclusion

In this article, we tentatively assign the \(P_\epsilon(4312)\) to be the \(\bar{D}\Sigma_c\) pentaquark molecular state with \(J^P = \frac{1}{2}^-\), and study its two-body strong decays with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10. In calculations, special attentions are paid to match the hadron side with the QCD side of the correlation functions to obtain solid duality. We obtain the partial decay widths \(\Gamma (P_\epsilon(4312) \rightarrow \eta_c p) = 0.255 \text{ MeV}\) and \(\Gamma (P_\epsilon(4312) \rightarrow J/\psi p) = 9.296^{+19.542}_{-9.296} \text{ MeV}\), which are compatible with the experimental data \(\Gamma_{P_\epsilon(4312)} = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}\) from the LHCb collaboration. The present calculations support assigning the \(P_\epsilon(4312)\) to be the \(\bar{D}\Sigma_c\) pentaquark molecular state with \(J^P = \frac{1}{2}^-\). We can search for the decay \(P_\epsilon(4312) \rightarrow \eta_c p\) to diagnose the nature of the \(P_\epsilon(4312)\).

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