Letters and Comments

Reply to Comment on ‘The pedagogical value of the four-dimensional picture I: Relativistic mechanics of point particles’

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Abstract
In this reply, it is argued that the criticism against my derivation of the relativistic dynamical law for a point particle (Kosyakov B P 2014 Eur. J. Phys. 35 025012) given in the ‘Comment’ by Rębilas is based on mistakes.

Keywords: relativistic mechanics, education, particle dynamics

The statement of my paper [1], which is called into question by Rębilas [2], is as follows: the relativistic law of dynamics for point particles is nothing but Newton’s second law smoothly embedded into the geometry of four-dimensional spacetime. In more exact terms, Newton’s second law in its original form,

\[
\frac{dp}{dt} = f,
\]

which is a strict law as viewed by an instantaneously comoving Lorentz observer, implies

\[
\mathbf{v}(\mathbf{p} - f) = 0.
\]

It was demonstrated in [1] that (2) is the law governing the behaviour of relativistic point particles of any kind. We specify a particular kind of particle by fixing the relationship between the four-momentum \(p^\mu\) and kinematic variables which is characteristic of the given kind of particle. The general dynamical law (2) then becomes the equation of motion for just these particles. The most studied class of mechanical point object is composed of the so-called Galilean particles which are commonly referred to as simply ‘particles’. These objects are
defined by
\[ p^\mu = mv^\mu. \]  
(3)

Accordingly, the equation of motion for Galilean particles reads
\[ ma^\mu = f^\mu. \]  
(4)

Particularly striking is the absence of the projection operator \( \perp \) from (4). The reason for this is the identity \( a \cdot v = 0 \) and the requirement \( f \cdot v = 0 \) imposed on every four-force \( f^\mu \), so that \( \perp \) acts as a unit operator.

However, the classical theory also admits non-Galilean point objects whose \( p^\mu \) depends on kinematic variables differently than that shown in (3), and hence the equation of motion is not as simple as equation (4). Non-Galilean objects are best exemplified by particles which live in a realm with discrete time, and are governed by a finite-difference counterpart of Newton’s second law,
\[ m \frac{v(t + \ell) - v(t)}{\ell} = f, \]  
(5)

where \( \ell \) is a ‘quantum of time’. A smooth embedding of (5) in Minkowski space is provided by equation (2) in which
\[ p^\mu (s) = \frac{m}{\ell} [v^\mu (s + \ell) - v^\mu (s)]. \]  
(6)

One further example of a non-Galilean object is a spinning particle. For a free spinning particle in the Frenkel model, mentioned in [1], \( p^\mu \) is expressed in terms of kinematic variables as
\[ p^\mu = \frac{M^2}{m} v^\mu + \frac{S^2}{m} \dot{\alpha}^\mu. \]  
(7)

The interested reader may wish to consult [3] where other examples of non-Galilean objects (rigid particles, spinning particles, charged and colored dressed particles) with an extended discussion of the properties of these mechanical entities can be found.

Rębilas [2] asserts that the line of reasoning in [1] ‘makes no sense’. He regards the presence of the projector \( \perp \) onto a hyperplane \( \Sigma \) perpendicular to the world line in (2) as ‘unintelligible and misleading’. Furthermore, he emphasizes its ‘uselessness’.

In response to the last claim, I note that if we are to consider the dynamics universally applicable to both Galilean and non-Galilean particles, the projector \( \perp \) is a valuable tool.

As to the intelligibility of the advent of the projection structure, the expert reader will recognize that the Euler–Lagrange equation for relativistic point particles, \( \delta S/\delta z^\mu = 0 \), is endowed with this structure by reparametrization invariance of the action \( S \). Indeed, Noether’s second theorem as applied to reparametrization invariance (see, e.g., [3]) reads
\[ \frac{\delta S}{\delta z^\mu} = 0. \]  
(8)

This Noether identity implies the presence of \( \perp \) in \( \delta S/\delta z^\mu \). Unfortunately, this penetrating insight into the structure of the relativistic dynamics appears to be not quite elementary, and hence inappropriate to a paper written in a pedagogical style for a general physics audience. Therefore, I was forced to omit it from [1].

What is the origin of the criticism of Rębilas? The starting point of my paper, equation (1), was falsified by substituting it with
\[ \frac{dp}{dt} \equiv ma = f \]  \hspace{1cm} (9)

(see equation (1) in [2]). And then, by assuming that \( a \) is the spatial part of a four-vector \( a^\mu \) in the rest frame, Rešibas arrived at equation (4). Of course this is the correct equation of motion for Galilean particles. But is this result new? No. The discovery of equation (4) can be traced back over 100 years. At that time, people were unaware of spin, rigidity, self-interaction, and other things that underlie the concept of non-Galilean particles.

It is a regrettable fact that Rešibas entirely missed what was said about non-Galilean particles in [1]. In due course, this concept will be an integral part of relativistic culture.

References

[1] Kosyakov B P 2014 The pedagogical value of the four-dimensional picture: I. Relativistic mechanics of point particles Eur. J. Phys. 35 025012
[2] Rešibas K 2015 Comment on ‘The pedagogical value of the four-dimensional picture: I. Relativistic mechanics of point particles’ Eur. J. Phys. 36 048002
[3] Kosyakov B P 2003 On the inert properties of particles in classical theory Phys. Part. Nucl. 34 808–28 (Translated from Fizika Elementarnykh Chastits i Atomnogo Yadra 34 1564-609)