New Tensor Interactions

and the $K_L$-$K_S$ Mass Difference

M. V. Chizhov

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia

and

Center of Space Research and Technologies, Faculty of Physics, University of Sofia, 1126 Sofia, Bulgaria

Abstract

The minimal standard electroweak model yields approximately half the observed value of the $K_L$-$K_S$ mass difference. Antisymmetric tensor fields incorporated into the standard model can provide the rest $0.5 \times (\Delta m_{LS})_{\text{exp}}$. The effective lepton–lepton, quark–lepton and quark–quark tensor interactions induced by the charged tensor particles are presented.

Submitted to “Phys. Lett. B”

1E-mail: physfac2@bgearn.bitnet
2Permanent address
1 Introduction

The triumph of the standard electroweak model does not leave much place for new physics when describing heavy quark interactions, still less for the description of old hadron weak decays. Most of the papers revising the calculations in the $\pi$- and $K$-meson physics try to find more accurate solutions of the problems connected with the strong interactions, but they do not change the base of the weak V–A interactions. However, the recent experimental results for the radiative $\pi^- \rightarrow e^- \bar{\nu}\gamma$ decay in a wide range of kinematic variables \cite{1} and the three-particle $K^+ \rightarrow \pi^0 e^+ \bar{\nu}$ decay \cite{2} (see also Ref.\cite{3}) point to the impossibility of their adequate description in the framework of the V–A model. In order to explain the destructive interference in the pion decay, in Ref.\cite{4} an additional tensor quark–lepton interaction was suggested

$$\mathcal{L}_T = \sqrt{2} \ G_F V_{ud} f_T \ \bar{u}_R \sigma_{\mu\nu} d_L \cdot \bar{e}_R \sigma^{\mu\nu} \nu_L,$$

where $V_{ud}$ is an element of the Kobayashi–Maskawa mixing matrix and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. This interaction provides consistency with other experimental data on this decay \cite{5}. The dimensionless constant $f_T$ describes the strength of the interaction relative to the ordinary weak coupling. The constant $f_T$ is estimated in the framework of the relativistic quark model as $f_{RQM} = -\left(4.2 \pm 1.3\right) \times 10^{-2}$. In Ref.\cite{6}, calculations of $f_T$ have been done by applying the QCD techniques and the PCAC hypothesis. The value obtained in this way $f_{QCD} = -\left(1.4 \pm 0.4\right) \times 10^{-2}$ is one third as low as estimated in the framework of the relativistic quark model. It seems that the accuracy of the calculations in Ref.\cite{6} is the usual PCAC accuracy and that the latter result is reliable.

As was noted in Ref.\cite{7}, such a naive form of interaction will lead, as a result of radiative corrections, to an anomalously large contribution to the $\pi \rightarrow e\nu$ decay, in contradiction with the experimental data. To avoid this difficulty and to describe all the meson-decay experiments \cite{1–3} simultaneously, in Ref.\cite{8} it was suggested to extend the standard model by adding two doublets of new tensor particles. Thus extended electroweak model predicts, besides the appearance of a new quark–lepton interaction like Eq.(1), also the appearance of tensor interactions in the lepton–lepton and quark–quark sectors.

The tensor interaction does not conserve chirality, and therefore, it can play an important role in the nonleptonic weak processes. Particularly, in this paper we are going to demonstrate that the effective tensor quark–quark interaction gives a contribution to the $K_L$-$K_S$ mass difference $\Delta m_{LS}$ of the order of the contribution from the standard V–A interaction. This is easy to understand because the strength of the new interaction is just an order of magnitude as low as $G_F$. On the other hand, from the PCAC hypothesis it follows that the chirality-nonconserving currents are enhanced by a factor $\chi = (m_K/m_s)^2 \sim 10$, where $m_K$ and $m_s$ are the masses of the kaon and the strange quark, respectively. It is known that the minimal standard V–A model (without taking into account large-distance contributions) can explain only half the experimental value of the mass difference ($\Delta m_{LS})_{\text{exp}}$ \cite{9}. Formally, the large-distance contribution is suppressed by $\mu^2/m_s^2$, where $\mu$ is a characteristic mass scale in the “old” hadron physics. Neglecting the large-distance contribution for the neutral kaon system, we suppose that the remaining half of ($\Delta m_{LS})_{\text{exp}}$ is due to the new tensor interactions
of quarks. This completely fixes the parameters of the new effective tensor interactions in all sectors. The effect of the chirality-nonconserving currents in the lepton–lepton sector (for example, in $\mu \to e\bar{\nu}_e\nu_\mu$ decay) is suppressed by the factor $m_e/m_\mu$ and falls into the experimental accuracy range. The new tensor interaction in the lepton–quark sector does not contradict the existing constraints from nuclear beta-decay either.

2 New effective tensor interactions

The antisymmetric second-rank tensor fields are incorporated into the standard electroweak model as two doublets $T_{\mu\nu} = (T^0_{\mu\nu}, T^+_{\mu\nu})$ and $U_{\mu\nu} = (U^0_{\mu\nu}, U^+_{\mu\nu})$ with opposite hypercharges: $Y(T) = -Y(U) = +1$. The Yukawa interaction of the charged components of the tensor fields with the first generation of leptons $\nu_e, e$ and the quarks $u, d' = d \cos \theta_C + s \sin \theta_C$ is

$$\mathcal{L}_{\text{Yukawa}} = \frac{t}{2} [\bar{\nu}_L \sigma^{\mu\nu} e_R \bar{d}'_L \cdot T^+_{\mu\nu} + \bar{u}_R \sigma^{\mu\nu} d'_L \cdot U^+_{\mu\nu} + \text{h.c.}]$$

For simplicity we accept the coupling constants for quarks and leptons to have the same value $t$. After the spontaneous symmetry breaking, a mixing of the $T$ and $U$ fields occurs. The propagator of the charged tensor fields in the static approximation $q^2 \ll m^2, M^2$ is of the form

$$\mathcal{P}(q) = \begin{pmatrix} <T(T^+T^-)_0 & <T(T^+U^-)_0 \\ <T(U^+T^-)_0 & <T(U^+U^-)_0 \end{pmatrix}$$

$$= \frac{2i}{m^2 - M^2} \begin{pmatrix} \Pi(q) & -1 \\ -1 & \tan^2 \beta \Pi(q) \end{pmatrix},$$

where $1_{\mu\nu,\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha})$, $\Pi_{\mu\nu,\alpha\beta}(q) = 1_{\mu\nu,\alpha\beta} - \frac{q_{\mu} q_{\alpha} q_{\nu} q_{\beta} - q_{\mu} q_{\beta} g_{\nu\alpha} - q_{\nu} q_{\alpha} g_{\mu\beta} + q_{\nu} q_{\beta} g_{\mu\alpha}}{q^2}$, and $\tan \beta = M/m$ is the ratio of the two mass parameters, associated with the vacuum expectation values of the two doublets of the Higgs particles. The diagonalization of Eq.(3) gives the mass matrix of the form $\mathcal{M}^2 = M^2 \text{diag}(\lambda_T, \lambda_U)$ for the fields $T'_{\mu\nu} = T_{\mu\nu} \cos \varphi + \Pi_{\mu\nu,\alpha\beta} U^\alpha T^\beta \sin \varphi$, $U'_{\mu\nu} = -\Pi_{\mu\nu,\alpha\beta} T^\alpha \sin \varphi + U_{\mu\nu} \cos \varphi$, where

$$\lambda_T = \frac{1}{2} \left[ 1 + \tan^2 \beta + \sqrt{(1 - \tan^2 \beta)^2 + 4} \right] / \tan^2 \beta,$$

$$\lambda_U = \frac{1}{2} \left[ 1 + \tan^2 \beta - \sqrt{(1 - \tan^2 \beta)^2 + 4} \right] / \tan^2 \beta,$$

and $\tan \varphi = \frac{1}{2} \left[ 1 - \tan^2 \beta + \sqrt{(1 - \tan^2 \beta)^2 + 4} \right]$. The positive definiteness of the matrix of squared masses leads to the restriction $\tan^2 \beta > 1$. The dependence of the squared masses of the fields $T'$ and $U'$ on the parameter $\cot^2 \beta$ is shown in Fig.1.
Using lagrangian (2) and propagator (3), one can easily write the effective interactions for various sectors. For example, in the case of a muon decaying into an electron and (anti)neutrinos, one must add to the usual V–A interaction an interaction of the form

\[ \mathcal{L}_{\mu e} = -\sqrt{2} G_F f_t \bar{\nu}_L \sigma_{\alpha \lambda} \mu_R \frac{q^\alpha q_\beta}{q^2} \bar{e}_R \sigma^{\beta \lambda} \nu_{eL} + \text{h.c.}, \quad (5) \]

where \( q \) is the momentum transferred from the muon to the electron pair, and the constant \( f_t \) is defined as \( f_t G_F/\sqrt{2} = t^2/(M^2 - m^2) \). The additional interaction for semileptonic weak decays has the form

\[ \mathcal{L}_{qe} = -\sqrt{2} G_F f_t \bar{u}_L \sigma_{\alpha \lambda} d_R \frac{q^\mu q_\nu}{q^2} \bar{e}_R \sigma^{\nu \lambda} \nu_{eL} + \text{h.c.} \quad (6) \]

The constant \( f_t \) can be fixed from the decays \( \pi^- \rightarrow e^- \bar{\nu} \gamma \) or \( K^+ \rightarrow \pi^0 e^+ \nu \). Its value \( f_t^{\text{RQM}} = 0.29 \pm 0.09 \) was evaluated in the framework of the relativistic quark model on the basis of the pion-decay experiment \[1\]. In what follows we are going to work in the framework of QCD and extensively use the PCAC hypothesis. Therefore, we shall take the corrected value \( f_t^{\text{QCD}} = (7.84 \pm 2.24) \times 10^{-2} \) in accordance with Ref.\[3\].

A richer interaction is obtained in the pure quark–quark sector:

\[ \mathcal{L}_{ud} = -\sqrt{2} G_F f_t \left[ \bar{u}_L \sigma_{\alpha \lambda} d'_R \cdot \bar{d}'_R \sigma^{\nu \lambda} u_L + \bar{u}_L \sigma_{\alpha \lambda} d'_R \cdot \bar{d}'_L \sigma^{\nu \lambda} u_R \right. \\
+ \bar{u}_R \sigma_{\alpha \lambda} d'_L \cdot \bar{d}'_R \sigma^{\nu \lambda} u_L + \tan \beta \bar{u}_R \sigma_{\alpha \lambda} d'_L \cdot \bar{d}'_L \sigma^{\nu \lambda} u_R \left. \right] \frac{q^\mu q_\nu}{q^2}. \quad (7) \]

The independent unknown parameter \( \tan \beta \) appears just in the last term of the quark–quark interaction (7). Therefore, in order to fix it, it is natural to investigate some weak nonleptonic process. The \( K_L - K_S \) mass difference is most sensitive to new hypothetical particles and interactions. We will prove that the introduced new interaction (7) does not contradict \((\Delta m_{LS})_{\text{exp}}\).

### 3 The kaon mass difference

In the \( K^0 - \bar{K}^0 \) mixing calculations we shall take into account only the contributions of the \( u \) and \( c \) quarks (see the box diagrams of Fig.2), because the \( t \)-quark contribution is suppressed by the tiny mixing angle \( V_{td} \). As the standard contribution that results from the \( W \)-boson exchange (Fig.1a) gives only half the experimental value \((\Delta m_{LS})_{\text{exp}}\), we suppose that the rest of \((\Delta m_{LS})_{\text{exp}}\) is due to the other two diagrams with the exchange of the tensor particles (Fig.2b,c). The effective lagrangian for the second generation of quarks, caused by the tensor particles, has the same form as Eq.(7) with obvious substitutions \( u \rightarrow c \) and \( d' \rightarrow s' = -d \sin \theta_C + s \cos \theta_C \):

\[ \mathcal{L}_{cs} = -\sqrt{2} G_F f_t \left[ \bar{c}_L \sigma_{\mu \lambda} s'_R \cdot \bar{s}'_R \sigma^{\nu \lambda} c_L + \bar{c}_L \sigma_{\mu \lambda} s'_R \cdot \bar{s}'_L \sigma^{\nu \lambda} c_R \right. \\
+ \bar{c}_R \sigma_{\mu \lambda} s'_L \cdot \bar{s}'_R \sigma^{\nu \lambda} c_L + \tan \beta \bar{c}_R \sigma_{\mu \lambda} s'_L \cdot \bar{s}'_L \sigma^{\nu \lambda} c_R \left. \right] \frac{q^\mu q_\nu}{q^2}. \quad (8) \]
Only the first terms in Eqs.(7) and (8) contribute to the interference diagrams (Fig.2b)

\[
\mathcal{L}_{WT} = -\frac{3G_F^2 f_t^2}{16\pi^2} \sin^2 \theta_C \cos^2 \theta_C \ m_c^2 \ s(1 - \gamma^5)d \cdot \bar{s}(1 + \gamma^5)d.
\]

(9)

The divergency is eliminated by the GIM mechanism, as in diagrams of Fig.2a. The matrix element

\[
< \bar{K}^0 | \bar{s}(1 - \gamma^5)d \cdot \bar{s}(1 + \gamma^5)d | K^0 > = -\left(2\chi + \frac{1}{3}\right) F_K^2 m_K^2
\]

(10)

is calculated in the approximation of the vacuum saturation, as the vacuum is inserted in all possible channels. Assuming \(F_K=160\) MeV and \(m_c=1.35\) GeV, we can estimate the contribution of the interference diagrams to \(\Delta m_{LS} = Re < \bar{K}^0 | \mathcal{L}_{\Delta S=2} | K^0 > /m_K\) as \(1.3 \times (\Delta m_{LS})_{exp}\).

Although the contribution of the diagrams in Fig.2c seems of the order of the small parameter \(f_t^2\), actually, their contribution is enhanced because the suppressing GIM mechanism does not work. Therefore, they must be taken into account. We assume that the masses of the new tensor particles are of the order of the vector-boson mass \(M_V\). Then the logarithmically divergent integral is effectively cut off at this limit. Such an assumption will not essentially change our results, as the logarithm is a very slowly increasing function. The effective lagrangian for the \(K^0-\bar{K}^0\) mixing due to exchange of the tensor particles (Fig.2c) reads

\[
\mathcal{L}_{TT} = -\frac{3G_F^2 f_t^2}{16\pi^2} \sin^2 \theta_C \cos^2 \theta_C \ m_c^2 \left\{ -\frac{1}{4}(\bar{s}_R\gamma_\mu d_R)^2 + \frac{3}{2}(\bar{s}_R\gamma_\mu d_R)(\bar{s}_L\gamma^\mu d_L)
\right. \\
- \frac{\tan^4 \beta}{4}(\bar{s}_L\gamma_\mu d_L)^2 - \ln \left. \frac{m_W^2}{m_c^2}\right\} [(\bar{s}_R d_L)^2 + \frac{1}{6}(\bar{s}_R\sigma_{\mu\nu} d_L)^2
\right. \\
+ 2\tan^2 \beta (\bar{s}_R d_L)(\bar{s}_L d_R) + (\bar{s}_L d_R)^2 + \frac{1}{6}(\bar{s}_L\sigma_{\mu\nu} d_R)^2\right]\}
\]

(11)

The matrix elements

\[
< \bar{K}^0 | \bar{s}_\gamma(1 + \gamma^5)d \cdot \bar{s}_\gamma(1 - \gamma^5)d | K^0 > = (2 + \frac{4}{3}\chi) F_K^2 m_K^2,
\]

(12)

\[
< K^0 | \left[ \bar{s}_\gamma(1 \pm \gamma^5)d \right]^2 | K^0 > = \frac{8}{3} F_K^2 m_K^2,
\]

(13)

\[
< \bar{K}^0 | \left[ \bar{s}(1 \pm \gamma^5)d \right]^2 + \frac{1}{6} \left[ \bar{s}\sigma_{\mu\nu}(1 \pm \gamma^5)d \right]^2 | K^0 > = \chi F_K^2 m_K^2,
\]

(14)

are calculated as before with the aid of the vacuum saturation hypothesis. To give the correct total mass difference \((\Delta m_{LS})_{exp}\), the contribution of the diagrams in Fig.2c must be negative \(-0.8 \times (\Delta m_{LS})_{exp}\). This is really the case: in the whole allowed range of the parameter \(\tan^2 \beta > 1\), the contribution of Fig.2c has the opposite sign relative to Figs.2a,b.

The correct value of the mass difference is obtained when \(\tan \beta = 1.51\pm0.07\). Notice that this solution \(\cot^2 \beta = 0.44 \pm 0.04\) is very close to the value 0.4 that corresponds to the maximal possible mass of the tensor field \(U_{\mu\nu}\) (see Fig.1). That would provide
the minimal effective potential between the fermions. It is curious to note that the mixing angle $\varphi$ happens to be surprisingly close to the Weinberg angle $\theta_W$: $\sin^2 \varphi = 0.229 \pm 0.027$.

Now let us discuss the effect of the two Higgs doublets on the mass difference $(\Delta m_{LS})_{\text{exp}}$. Their Yukawa coupling constants with quarks are proportional to the quark masses. As far as the model-independent lower bound on the $t$-quark mass is $m_t > 90$ GeV (i.e. the $t$ quark is very heavy), the contribution of the virtual $t$ quarks to the box diagrams may become dominant in spite of the strong suppression of the Yukawa interaction owing to the tiny mixing angle $V_{td}$. If the ratio of the vacuum expectation values of the two doublets of the Higgs particles $v_1/v_2 = M/m = \tan \beta$ were large, this could enhance the contribution of the Higgs particles to the $K_L-K_S$ mass difference (according to the analysis in Ref.[11]). However, the value of $\tan \beta \approx 1.51$ estimated above does not lead to a large ratio $v_1/v_2$, and respectively, gives no considerable enhancement of the contribution of the two Higgs doublets to the $K_L-K_S$ mass difference as compared to the case of only one doublet in the minimal standard model.

There exists a great uncertainty in the values of the Kobayashi–Maskawa matrix elements for the $t$ quark. We use here the central values $V_{td} = 0.010$ and $V_{ts} = 0.042$ [11]. In Fig.3 the relative contribution of the Higgs doublets to the actual $K_L-K_S$ mass difference is shown. The curves in the plane of two parameters — the charged-Higgs mass $M_H$ and the $t$-quark mass $m_t$ — correspond to the fixed values of the relative contribution. Obviously, the contribution of the Higgs doublets increases with the increase of the $t$-quark mass and the decrease of the Higgs mass. There exists a bound on the value of the $t$-quark mass from the Tevatron: $m_t = 174 \pm 10_{-12}^{+13}$ GeV [12]. On the other hand, it is natural to assume that the tensor particles are included into a more complete supersymmetric theory which ensures the unification of the coupling constants and of the $b-\tau$ masses [13]. Then, using the value of $\tan \beta = 1.51 \pm 0.07$ obtained above, one can estimate the $t$-quark mass as $m_t = 165 \pm 13$ GeV. In these theories the masses of the charged Higgs particles are above 260 GeV. Therefore, for the realistic choice $m_t = 170$ GeV and $M_H > 260$ GeV, the contribution to $\Delta m_{LS}$ is less than 10%. Such an accuracy is compatible with the assumptions made: the PCAC hypothesis, the vacuum-saturation approximation, disregard of the large-distance contributions, etc.

In interactions of $up$ and $down$ quarks with the charged tensor particles, two additional generation-mixing matrices of the Cabibbo–Kobayashi–Maskawa type arise: between left–right and right–left quarks. Unfortunately, because of the lack of information about these matrices, we cannot say anything more definite about the contribution of the tensor particles to the $CP$ violation in the $K^0-\bar{K}^0$ system.

4 Conclusions

The experimental data [1–3] point to an admixture of tensor currents in the weak decays. In the electroweak model extended by the doublets of tensor particles, we can fix all the parameters of effective interactions of the charged tensor currents, using the cited experiments and the $K_L-K_S$ mass difference. The model predicts neutral
tensor currents too, but it is as yet impossible to define this sector because of the lack of experimental data. At the moment we can definitely say only that there exists a requirement that the flavor-changing neutral tensor currents should be absent.

When estimating the mixing parameter $\tan \beta$ from the $K_L - K_S$ mass difference, we have neglected the QCD corrections. Taking them into account may correct the value of $\tan \beta$ — the presented value $\tan \beta \approx 1.51$ was obtained for bare quarks. As we have accepted the same coupling of the tensor fields to leptons and quarks, the effective lepton–lepton and quark–lepton tensor interactions are governed by a single common parameter $f_t$. Therefore, by fixing this parameter from the semileptonic decays, $f_t \approx 0.08$, we can make a prediction for the weak leptonic decays, where no uncertainty associated with the strong interactions is present.

Unfortunately, all previous attempts to study the violation of the V–A structure of interaction in the $\mu \to e \bar{\nu}_e \nu_\mu$ decay have been done on the basis of the derivative-free effective lagrangian [14$. The effective tensor interactions presented here essentially depend on the momentum transfer. Certainly, this will lead to a different angular and energy distribution of the outgoing electron. Nevertheless, we can estimate the contribution of the new interaction to the Michel parameter $\rho$. Its value $\delta \rho \sim 10^{-3}$ is of the same order of magnitude as the present experimental accuracy [11$.

Acknowledgements

I am grateful to L.V.Avdeev, D.I.Kazakov and M.D.Mateev for helpful discussions. I acknowledge the hospitality of the Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, where this work has been completed. The work is financially supported by Grant-in-Aid for Scientific Research F-214/2096 from the Bulgarian Ministry of Education, Science and Culture.
References

[1] V. N. Bolotov et al., Phys. Lett. B243 (1990) 308.
[2] S. A. Akimenko et al., Phys. Lett. B259 (1991) 225.
[3] H. Stainer et al., Phys. Lett. B36 (1971) 521.
[4] A. A. Poblaguev, Phys. Lett. B238 (1990) 108.
[5] A. A. Poblaguev, Phys. Lett. B286 (1992) 169.
[6] V. M. Belyaev and Ian I. Kogan, Phys. Lett. B280 (1992) 238.
[7] M. B. Voloshin, Phys. Lett. B283 (1992) 120.
[8] M. V. Chizhov, Mod. Phys. Lett. A8 (1993) 2753
[9] M. A. Shifman, Int. J. Mod. Phys. A3 (1988) 2769.
[10] L. F. Abbott, P. Sikive and Mark B. Wise, Phys. Rev. D21 (1980) 1393.
[11] Particle Data Group, Phys. Rev. D45, Part 2 (June 1992).
[12] Preprint FERMILAB-Pub-94/097-E, 94/116-E, 1994.
[13] C. Kolda, L. Roszkowski, J. D. Wells and G. L. Kane, Preprint UM-TH-94-03 
(February 1994)
[14] R. E. Marshak, R. Riazuddin and C. P. Ryan, Theory of weak interactions in 
particle physics (Wiley-Interscience, 1969); 
W. Fetscher, H.-J. Gerber and K. F. Johnson, Phys. Lett. B173 (1986) 102.
Figure 1: The dependence of the squared masses of the $T$ and $U$ particles on the mixing parameter $\cot^2 \beta$

M. V. Chizhov

*Phys. Lett. B*
Figure 2: Box diagrams for $K_L-K_S$ mass difference
Figure 3: Isolines of the relative contribution from the Higgs doublets to the $K_L-K_S$ mass difference in the plane of the charged-Higgs mass and the $t$-quark mass

M. V. Chizhov

Phys. Lett. B
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407237v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407237v1