Space-Time Geometry of Electromagnetic Field in the System of Photon

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ABSTRACT

In the concept of general relativity gravity is the space-time geometry. Again, a relation between electromagnetic field and gravitational field is expected. In this paper, space-time geometry of electromagnetic field in the system of photon has been introduced to unify electromagnetic field and gravitational field in flat and curvature space-time.

Keywords: space-time geometry; unified field; electromagnetic field

1. INTRODUCTION

In physics, a unified field theory is a type that allows all fundamental forces and elementary particles to be written in terms of a single field.

The term was proposed by Einstein, who attempted to unify the general theory of relativity with electromagnetism. According to Einstein’s general relativity [2,3], gravity is the space-time geometry. Also, he suggested [4] the field equation for the gravity of an electromagnetic wave as 

\[ G_{ab} = -K T(E)_{ab} \]

where, \( G_{ab} \) is the Einstein tensor, and \( K \) is the coupling constant. But, the problem of the unification of fundamental fields into a single theory has not been solved until now in a satisfactory manner, although, in different time, a lot of papers have been published which attempt to unify the fundamental fields.

Recently, in [1], a relation between electromagnetic field and gravitational field has been introduced by considering a super system in photon. In this paper a trial has been made to introduce a geometrical relation between electromagnetic field and gravitational field.

2. SPACE-TIME GEOMETRY OF SYSTEMS

In [1], to clarify two simultaneous superimposed motion (either linear or rotational), three types of system has been assumed which are L-L system, S-S system and S-L system; depending upon the S-L system SSP picture of photon has been considered; also, using this picture (SSP) a connection between electro-magnetic field \( (\psi_o (r,t)) \) and gravitational field \( (G'_o (r',t')) \) has been introduced by the relation
\[ \psi_{\alpha}(r,t) = \gamma, \quad (r',t') \]  
(1)

where, \(Z_{ij}\) are transformation matrix in the picture of SSP.

It is also pointed out that to clarify L-L or S-S or S-L system, four reference frames \((S, S_1, S_2, S_3)\) has been considered in a simultaneous superimposed form.

Relation for co-ordinate transformation from \(S_3\) to \(S\) in S-L system \([1]\) is

\[ X(x, y, z, t) = \bar{Z}_{i j} X'(x', y', z', t') \]  
(2)

where, \(\bar{Z}_{ij}\) is co-ordinate transformation matrix and the co-ordinates of an event in \(S_3\) be

\[
X'(x', y', z', t') = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} \quad \text{which would be} \quad X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \quad \text{with respectively in} \ S.
\]

Now, following the space-time geometry as in \([5]\), one can introduced the Space-time geometry of the said system as stated below

From (2) we obtain

\[
\begin{align*}
&dx^2 = (\bar{Z}_{11} dx' + \bar{Z}_{12} dy' + \bar{Z}_{13} dz' + \bar{Z}_{14} dt')^2 \\
&dy^2 = (\bar{Z}_{21} dx' + \bar{Z}_{22} dy' + \bar{Z}_{23} dz' + \bar{Z}_{24} dt')^2 \\
&dz^2 = (\bar{Z}_{31} dx' + \bar{Z}_{32} dy' + \bar{Z}_{33} dz' + \bar{Z}_{34} dt')^2 \\
&dt^2 = (\bar{Z}_{41} dx' + \bar{Z}_{42} dy' + \bar{Z}_{43} dz' + \bar{Z}_{44} dt')^2
\end{align*}
\]  
(3)

Now, we have Cartesian Co-ordinate geometry in flat space-time

\[ ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \]  
(4)

Using (3) we obtain from (4) the space-time geometry in S-L system

\[
\begin{align*}
&ds^2 = P_1 dx'^2 + P_2 dy'^2 + P_3 dz'^2 + P_4 dt'^2 + 2(Q_1 dx' dy' + Q_2 dx' dz' \\
&\quad + Q_3 dx' dt' + Q_4 dy' dz' + Q_5 dy' dt' + Q_6 dz' dt')
\end{align*}
\]  
(5)

where,

\[
P_1 = \bar{Z}_{11}^2 + \bar{Z}_{21}^2 + \bar{Z}_{31}^2 - \bar{Z}_{41}^2, \quad P_2 = \bar{Z}_{12}^2 + \bar{Z}_{22}^2 + \bar{Z}_{32}^2 - \bar{Z}_{42}^2
\]
Again, relation for co-ordinate transformation from $S_3$ to $S$ in S-S system \[1\] is

\[
X(x, y, z, t) = \bar{S}_{ij}X'(x', y', z', t')
\] 

(6)

where, $\bar{S}_{ij}$ is co-ordinate transformation matrix.

From (6) we obtain

\[
dx'^2 = (\bar{S}_{1i}dx' + \bar{S}_{1j}dy' + \bar{S}_{1k}dz' + \bar{S}_{1l}dt')^2
\]

\[
dy'^2 = (\bar{S}_{2i}dx' + \bar{S}_{2j}dy' + \bar{S}_{2k}dz' + \bar{S}_{2l}dt')^2
\]

\[
dz'^2 = (\bar{S}_{3i}dx' + \bar{S}_{3j}dy' + \bar{S}_{3k}dz' + \bar{S}_{3l}dt')^2
\]

\[
dt'^2 = (\bar{S}_{4i}dx' + \bar{S}_{4j}dy' + \bar{S}_{4k}dz' + \bar{S}_{4l}dt')^2
\]

(7)

Using (4) and (7) we obtain the space-time geometry in S-S system as in \[5\] where,

\[
P_1 = \bar{S}_{1i}^2 + \bar{S}_{1j}^2 + \bar{S}_{1k}^2 - \bar{S}_{1l}^2,
\]

\[
P_2 = \bar{S}_{2i}^2 + \bar{S}_{2j}^2 + \bar{S}_{2k}^2 - \bar{S}_{2l}^2
\]

\[
P_3 = \bar{S}_{3i}^2 + \bar{S}_{3j}^2 + \bar{S}_{3k}^2 - \bar{S}_{3l}^2
\]

\[
P_4 = \bar{S}_{4i}^2 + \bar{S}_{4j}^2 + \bar{S}_{4k}^2 - \bar{S}_{4l}^2
\]

\[
Q_1 = \bar{S}_{1i}\bar{S}_{1j} + \bar{S}_{1k}\bar{S}_{1l} - \bar{S}_{1i}\bar{S}_{1k} - \bar{S}_{1i}\bar{S}_{1l} - \bar{S}_{1j}\bar{S}_{1k} - \bar{S}_{1j}\bar{S}_{1l}
\]

\[
Q_2 = \bar{S}_{2i}\bar{S}_{2j} + \bar{S}_{2k}\bar{S}_{2l} - \bar{S}_{2i}\bar{S}_{2k} - \bar{S}_{2i}\bar{S}_{2l} - \bar{S}_{2j}\bar{S}_{2k} - \bar{S}_{2j}\bar{S}_{2l}
\]

\[
Q_3 = \bar{S}_{3i}\bar{S}_{3j} + \bar{S}_{3k}\bar{S}_{3l} - \bar{S}_{3i}\bar{S}_{3k} - \bar{S}_{3i}\bar{S}_{3l} - \bar{S}_{3j}\bar{S}_{3k} - \bar{S}_{3j}\bar{S}_{3l}
\]

\[
Q_4 = \bar{S}_{4i}\bar{S}_{4j} + \bar{S}_{4k}\bar{S}_{4l} - \bar{S}_{4i}\bar{S}_{4k} - \bar{S}_{4i}\bar{S}_{4l} - \bar{S}_{4j}\bar{S}_{4k} - \bar{S}_{4j}\bar{S}_{4l}
\]

\[
Q_5 = \bar{S}_{5i}\bar{S}_{5j} + \bar{S}_{5k}\bar{S}_{5l} - \bar{S}_{5i}\bar{S}_{5k} - \bar{S}_{5i}\bar{S}_{5l} - \bar{S}_{5j}\bar{S}_{5k} - \bar{S}_{5j}\bar{S}_{5l}
\]

\[
Q_6 = \bar{S}_{6i}\bar{S}_{6j} + \bar{S}_{6k}\bar{S}_{6l} - \bar{S}_{6i}\bar{S}_{6k} - \bar{S}_{6i}\bar{S}_{6l} - \bar{S}_{6j}\bar{S}_{6k} - \bar{S}_{6j}\bar{S}_{6l}
\]

3. SPACE-TIME GEOMETRY OF ELECTROMAGNETIC FIELD IN PHOTON

Since picture of SSP depends upon the S-L system so, following (1) and using \(dx' = dx^g\), \(dy' = dy^g\), \(dz' = dz^g\), \(dt' = dt^g\) we obtain from (5), the space-time geometry of electromagnetic field in the SSP

\[
(ds_{em}^2)^2 = P_1(dx^g)^2 + P_2(dy^g)^2 + P_3(dz^g)^2 + P_4(dt^g)^2 + 2(Q_1dx^gdy^g + Q_2dx^gdz^g + Q_3dx^gdt^g + Q_4dy^gdz^g + Q_5dy^gdt^g + Q_6dz^gdt^g)
\]

(8)

where, superscript ‘g’ represents the gravitational system and superscript \(em\) represents electromagnetic system.
Following the convention as in (1), one may assume a relation between electromagnetic field and gravitational field as

\[ \psi_a(r, t) = \bar{\gamma}_i \bar{S}_y G_a'(r', t') \]  

(9)

where, \( \bar{\gamma}_i \) is a constant and \( \bar{S}_y \) is transformation matrix in S-S system.

This means that, in S-S system, gravitational field of frame \( S_3 \) would be electromagnetic field with respect to frame \( S \). For this system space-time geometry of electromagnetic field would also be as in (8) where, transformation matrix would be \( \bar{S}_y \).

However, equation (8) would be the space-time geometry of electromagnetic field connecting gravitational field and electromagnetic field in the system of photon.

4. CONCLUSION

Equation (8) represents a picture of space-time geometry of the electromagnetic field in the system of photon. This implies that a geometrical relation is existed in between electromagnetic field and gravitational field.

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