**$N\Delta$ and $\Delta\Delta$ dibaryons**

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**Abstract.** Experimental evidence for $I(J^P)=0(3^+)$ $\Delta\Delta$ dibaryon $D_{03}(2370)$ has been presented recently by the WASA-at-COSY Collaboration. Here I review new hadronic-basis calculations of $L=0$ nonstrange $N\Delta$ and $\Delta\Delta$ dibaryon candidates. In particular, $D_{03}(2370)$ is generated dynamically in terms of long-range physics dominated by pions, nucleons and $\Delta$'s. These calculations are so far the only ones to reproduce the relatively small $D_{03}(2370)$ width of $70-80$ MeV. Predictions are also given for the location and width of $D_{30}$, the $I(J^P)=3(0^+)$ exotic partner of $D_{03}(2370)$.

1. Introduction

The WASA-at-COSY Collaboration has presented recently striking evidence for a $I(J^P) = 0(3^+)$ $\Delta\Delta$ dibaryon some 80-90 MeV below the $\Delta\Delta$ threshold, with a relatively small width of $\Gamma \approx 70 - 80$ MeV, by observing a distinct resonance in the energy spectrum of $pn \rightarrow d\pi\pi$ reactions [1,2] as shown in Fig. 1–left. Isospin $I = 0$ is uniquely fixed in this particular $\pi^0\pi^0$ production reaction and the spin-parity $3^+$ assignment follows from the measured deuteron and pions angular distributions, assuming $s$-wave decaying $\Delta\Delta$ pair. The peak of the $M^2_{d\pi}$ distribution on the right panel at $\sqrt{s} \approx 2.13$ GeV, almost at the $D_{12}(2150)$ $N\Delta$ dibaryon location (see below), suggests that $D_{12}$ plays a role in forming the $\Delta\Delta$ dibaryon $D_{03}$.

![Figure 1](image_url). $D_{03}(2370)$ $\Delta\Delta$ dibaryon resonance signal on the left panel, and its $M^2_{d\pi}$ Dalitz-plot projection on the right panel, from $pn \rightarrow d\pi^0\pi^0$ measurements by WASA-at-COSY [1]. This resonance was also observed consistently in $pn \rightarrow d\pi^+\pi^-$ measurements [2]. Figures courtesy of Heinz Clement.
Further evidence supporting the $D_{03}(2370)$ dibaryon assignment comes from very recent measurements of $pn$ elastic scattering as a function of energy, taking sufficiently small steps around $\sqrt{s} = 2370$ MeV [3]. This is shown in Fig. 2–left for the Argand diagram of the $^3D_3$ partial wave, and in the right panel for the speed plot of the $^3D_3$ partial wave, within a new SAID partial wave analysis incorporating these measurements.

\[\begin{align*}
\text{Re} & \quad -0.2 \quad -0.1 \quad 0 \quad 0.1 \quad 0.2 \\
\text{Im} & \quad 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \\
3D_3\text{Argand diagram} & \\
\end{align*}\]

\[\begin{align*}
\text{Speed} & \quad 0 \quad 1 \quad 2 \\
3D_3\text{speed plot} & \\
\end{align*}\]

\textbf{Figure 2.} $D_{03}(2370)$ $\Delta\Delta$ dibaryon resonance signals in the Argand diagram on the left panel, and in the speed plot on the right panel, both for the $np$ $^3D_3$ partial wave, from recent $np$ scattering measurements by WASA-at-COSY [3]. Figures courtesy of Heinz Clement.

$N\Delta$ and $\Delta\Delta$ s-wave dibaryon resonances $D_{IS}$ with isospin $I$ and spin $S$ were proposed as early as 1964, when quarks were still perceived as merely mathematical entities, by Dyson and Xuong [4] who focused on the lowest-dimension SU(6) multiplet in the $56 \times 56$ product that contains the SU(3) $10$ and $27$ multiplets in which the deuteron $D_{01}$ and $NN$ virtual state $D_{10}$ are classified. This yields two dibaryon candidates, $D_{12}$ ($N\Delta$) and $D_{03}$ ($\Delta\Delta$) as listed in Table 1. Identifying the constant $A$ in the resulting mass formula $M = A + B[I(I+1) + S(S+1) - 2]$ with the $NN$ threshold mass 1878 MeV, a value $B \approx 47$ MeV was determined by assigning $D_{12}$ to the $pp \leftrightarrow \pi^+d$ resonance at $\sqrt{s} = 2160$ MeV (near the $N\Delta$ threshold) which was observed already during the 1950’s. This led to the prediction $M(D_{03}) = 2350$ MeV. The $D_{03}$ dibaryon was the subject of many quark-based model calculations since 1980, see Refs. [5–13] for a representative although incomplete listing. Dibaryons were reviewed recently in Ref. [14].

\textbf{Table 1.} Nonstrange s-wave dibaryon SU(6) predictions [4].

\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
dibaryon & $I$ & $S$ & SU(3) & legend & mass \\
\hline
$D_{01}$ & 0 & 1 & 10 & deuteron & $A$ \\
$D_{10}$ & 1 & 0 & 27 & $nn$ & $A$ \\
$D_{12}$ & 1 & 2 & 27 & $N\Delta$ & $A + 6B$ \\
$D_{21}$ & 2 & 1 & 35 & $N\Delta$ & $A + 6B$ \\
$D_{03}$ & 0 & 3 & 10 & $\Delta\Delta$ & $A + 10B$ \\
$D_{30}$ & 3 & 0 & 28 & $\Delta\Delta$ & $A + 10B$ \\
\hline
\end{tabular}
\end{center}

It is shown below that the pion-assisted methodology applied recently by Gal and Garcilazo [15,16] couples $D_{12}$ and $D_{03}$ dynamically in a perfectly natural way, the analogue of which has not emerged in quark-based models. Our hadronic-based calculations emphasize the long-range physics aspects of nonstrange dibaryons.
2. Pion-assisted nonstrange dibaryons

The discussion in this section is divided into two subsections, the first one specializing to \( N\Delta \) dibaryons and the second one highlighting the \( D_{03} \Delta\Delta \) dibaryon.

2.1. \( N\Delta \) dibaryons

The \( D_{12} \) dibaryon shows up experimentally as \( NN(1^D_2) \leftrightarrow \pi d(3^P_2) \) coupled-channel resonance corresponding to a quasibound \( N\Delta \) with mass \( M \approx 2.15 \text{ GeV} \), near the \( N\Delta \) threshold, and width \( \Gamma \approx 0.12 \text{ GeV} \) \cite{17,18} as shown in Fig. 3 for the Argand diagram of the \( 1^D_2 \) partial wave in \( pp \) elastic scattering.

\[
\Delta N \quad I(J^P) = 1(2^+) \text{ Dibaryon}
\]

**Figure 3.** Argand diagram of the \( 1^D_2 \) partial wave in \( pp \) elastic scattering from SAID, in agreement with past determinations of the \( D_{12} \) dibaryon resonance pole position, \( W=2148-i63 \text{ MeV} \) \cite{17} and \( W=2144-i55 \text{ MeV} \) \cite{18}.

In our recent work \cite{16} we have calculated this dibaryon and other \( N\Delta \) dibaryon candidates such as \( D_{21} \) (see Table 1) by solving Faddeev equations with relativistic kinematics for the \( \pi NN \) three-body system, where the \( \pi N \) subsystem is dominated by the \( P_{33} \Delta(1232) \) resonance channel and the \( NN \) subsystem is dominated by the \( 3^S_1 \) and \( 1^S_0 \) channels. The coupled Faddeev equations give rise then to an effective \( N\Delta \) Lippmann-Schwinger (LS) equation for the three-body \( S \)-matrix pole, with energy-dependent kernels that incorporate spectator-hadron propagators, as shown diagrammatically in Fig. 4 where circles denote the \( N\Delta T \) matrix.

**Figure 4.** \( N\Delta \) dibaryon's Lippmann-Schwinger equation \cite{16}. 

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Of the four possible $L = 0$ $N\Delta$ dibaryon candidates $D_{IS}$ with $IS = 12, 21, 11, 22$, the latter two do not provide resonant solutions. For $D_{12}$, only $^3S_1$ contributes out of the two $NN$ interactions, while for $D_{21}$ only $^1S_0$ contributes. Since the $^3S_1$ interaction is the more attractive one, $D_{12}$ lies below $D_{21}$ as borne out by the calculated masses listed in Table 2 for two choices of the $P_{33}$ interaction form factor corresponding to spatial sizes of 1.35 fm and 0.9 fm of the $\Delta$ isobar. The two dibaryons are found to be degenerate to within less than 20 MeV. The mass values calculated for $D_{12}$ are reasonably close to the value $W = 2148 - i63$ MeV [17] and $W = 2144 - i55$ MeV [18] derived in coupled-channel phenomenological analyses.

### Table 2. $N\Delta$ dibaryon $S$-matrix poles (in MeV) for $D_{12}$ and $D_{21}$, obtained by solving $\pi NN$ Faddeev equations for two choices of the $\pi N P_{33}$ form factor, with large (small) spatial size denoted $> ($<).

|       | $W^>(D_{12})$ | $W^>(D_{21})$ | $W^<(D_{12})$ | $W^<(D_{21})$ |
|-------|---------------|---------------|---------------|---------------|
|       | 2147−i60      | 2165−i64      | 2159−i70      | 2169−i69      |

#### 2.2. $\Delta\Delta$ dibaryons

![Figure 5](https://example.com/figure5.png)  
**Figure 5.** Coupled-channel fit (solid) to the SAID (dashed) $NN \ 1D_2$ phase shift $\delta$ (left panel) and inelasticity $\eta$ (right panel), see text.

Four-body $\pi\pi NN$ calculations are required, strictly speaking, to discuss $\Delta\Delta$ dibaryons. In Ref. [15] we studied the $D_{03}$ dibaryon by solving a $\pi N\Delta'$ three-body model, where $\Delta'$ is a stable $\Delta(1232)$ and the $N\Delta'$ interaction is dominated by the $D_{12}$ dibaryon. The $I(J^P) = 1(2^+) \ N\Delta'$ interaction was not assumed to resonate but, rather, it was fitted within a $NN - \pi NN - N\Delta'$ coupled-channel caricature model to the $NN \ 1D_2$ $T$-matrix, requiring that the resulting $N\Delta'$ separable-interaction form factor is representative of long-range physics, with momentum-space soft cutoff $\Lambda \lesssim 3$ fm$^{-1}$. A fit of this kind is shown in Fig. 5.

The Faddeev equations of the $\pi N\Delta'$ three-body model give rise, as before, to an effective LS equation for the $\Delta\Delta'$ $S$-matrix pole corresponding to $D_{03}$. This LS equation is shown diagrammatically in Fig. 6, where $D$ stands for the $D_{12}$ dibaryon. The $\pi N$ interaction was
assumed again to be dominated by the $P_{33}$ $\Delta$ resonance, using two different parametrizations of its form factor that span a reasonable range of the $\Delta$ hadronic size. In Ref. [16] we have extended the calculation of $D_{03}$ to other $D_{IS} \Delta\Delta$ dibaryon candidates, with $D$ now standing for both $N\Delta$ dibaryons $D_{12}$ and $D_{21}$. Since $D_{21}$ is almost degenerate with $D_{12}$, and with no $NN$ observables to constrain the input $(I, S)=(2,1)$ $N\Delta'$ interaction, the latter was taken the same as for $(I, S)=(1,2)$. The model dependence of this assumption is under study at present. The lowest and also narrowest $\Delta\Delta$ dibaryons found are $D_{03}$ and $D_{30}$.

Table 3. $\Delta\Delta$ dibaryon $S$-matrix poles (in MeV) obtained in Refs. [15,16] by using a spectator-$\Delta'$ complex mass $W(\Delta')$ (first column) in the propagator of the LS equation depicted in Fig. 6. The last two columns give calculated mass and width values averaged over those from the $>$ and $<$ columns, where $>$ and $<$ are defined in the caption of Table 2.

| $W(\Delta')$ | $W(\Delta_{03})$ | $W(\Delta_{30})$ | $W(\Delta_{03})$ | $W(\Delta_{30})$ | $W_{av}(\Delta_{03})$ | $W_{av}(\Delta_{30})$ |
|--------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 1211−i49.5   | 2383−i47         | 2412−i49         | 2342−i31         | 2370−i30         | 2363−i39         | 2391−i39         |
| 1211−i(2/3)49.5 | 2383−i41       | 2411−i41         | 2343−i24         | 2370−i22         | 2363−i33         | 2390−i32         |

Representative results for $D_{03}$ and $D_{30}$ are assembled in Table 3, where the calculated mass and width values listed in each row correspond to the value listed there of the spectator-$\Delta'$ complex mass $W(\Delta')$ used in the propagator of the LS equation shown in Fig. 6. The value of $W(\Delta')$ in the first row is that of the $\Delta(1232)$ $S$-matrix pole. It is implicitly assumed thereby that the decay $\Delta' \rightarrow N\pi$ proceeds independently of the $\Delta \rightarrow N\pi$ isobar decay. However, as pointed out in Ref. [15], care must be exercised to ensure that the decay nucleons and pions satisfy Fermi-Dirac and Bose-Einstein statistics requirements, respectively. Assuming $L=0$ for the decay-nucleon pair, this leads to the suppression factor $2/3$ depicted in the value of $W(\Delta')$ listed in the second row. It is seen that the widths obtained upon applying this width-suppression are only moderately smaller, by less than 15 MeV, than those calculated disregarding this quantum-statistics correlation.

The mass and width values calculated for $D_{03}$ [15] agree very well with those determined by the WASA-at-COSY Collaboration [1–3], reproducing in particular the reported width value $\Gamma(D_{03}) \approx 70$ MeV which is considerably below the phase-space estimate $\Gamma_\Delta \leq \Gamma(D_{03}) \leq 2\Gamma_\Delta$, with $\Gamma_\Delta \approx 118$ MeV. No other calculation so far has succeeded to do that. Similarly small widths according to Table 3 hold for $D_{30}$ which is located about 30 MeV above $D_{03}$. This is about half of the spacing found very recently in the quark-based calculations of Ref. [13]. Note, however, that the widths calculated there are considerably larger than ours. A more complete discussion of these and of other $D_{IS} \Delta\Delta$ dibaryon candidates is found in Ref. [16].
3. Conclusion

It was shown how the 1964 Dyson-Xuong SU(6)-based classification and predictions of nonstrange dibaryons [4] are confirmed in our hadronic model of pion-assisted NΔ and ΔΔ dibaryons [15,16]. The input for dibaryon calculations in this model consists of nucleons, pions and Δ’s, interacting via long-range pairwise interactions. These calculations reproduce the two nonstrange dibaryons established experimentally and phenomenologically so far, the NΔ dibaryon $D_{12}$ [17,18] and the ΔΔ dibaryon $D_{03}$ reported by WASA-at-COSY [1–3], predicting also an exotic $I = 2$ NΔ dibaryon $D_{21}$ nearly degenerate with $D_{12}$. We note that $D_{12}$ provides in our πNΔ three-body model of $D_{03}$ a two-body decay channel $\pi D_{12}$ with threshold lower than ΔΔ. Our calculations are capable of dealing with other ΔΔ dibaryon candidates [16], in particular the $I = 3$ exotic $D_{30}$ highlighted recently by Bashkanov, Brodsky and Clement [19]. These authors emphasized the dominant role that six-quark hidden-color configurations might play in binding $D_{30}$, but recent explicit quark-based calculations [13] find these configurations to play a marginal role, enhancing dibaryon binding by merely 15$\pm$5 MeV and reducing the dibaryon width from 175 to 150 MeV for $D_{03}$, still twice as big as the reported width, and from 216 to 200 MeV for $D_{30}$. Hidden-color considerations are naturally outside the scope of hadronic models and it is gratifying that the results presented here in the hadronic basis are independent of such poorly understood configurations.

Acknowledgments

Fruitful collaboration with Humberto Garcilazo and stimulating discussions with Mikhail Bashkanov and Heinz Clement are gratefully acknowledged. Special thanks are due to Güray Erkol and the other Organizers of TROIA14 for their kind hospitality.

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