Cosmic Radiation Constraints on Low String Scale and Extra Dimension Cross Sections

Günter Sigl
GReCO, Institut d’Astrophysique de Paris, C.N.R.S., 98 bis boulevard Arago, F-75014 Paris, France

The observed interaction energy of cosmic rays with atmospheric nuclei reaches up to a PeV in the center of mass. We compute nucleon-nucleon and nucleon-neutrino cross sections for various generic parton cross sections appearing in string and brane world scenarios for gravity and compare them with cosmic ray data. Scenarios with effective energy scales in the TeV range and parton cross sections with linear or stronger growth with the center of mass energy appear strongly constrained or ruled out. String-inspired scenarios with infinite-volume extra dimensions may require a fundamental scale above \( \simeq 100 \text{eV} \) for which they are probably in conflict with gravity on parsec scales.

PACS numbers: PACS numbers: 12.60.-i, 98.70.Sa, 04.50.+h

Introduction: The idea that new physics will appear at an energy of \( \simeq 1 \text{TeV} \) is currently very popular. Many of the scenarios discussed in the literature involve extra dimensions and aim at unifying gravity with the Standard Model interactions, see, e.g. Ref. [1]. Scenarios with a quantum gravity/string scale as small as in the sub-eV range have been proposed to explain the smallness of the observed cosmological constant in consistency with collider experiments, cosmology, and gravity measurements [2].

On the other hand cosmic ray interactions have been observed up to a PeV in the center of mass (CM) [3], about a factor thousand higher than reached in accelerator experiments. Although rather indirect, these observations are consistent with interactions extrapolated from their Standard Model description and thus suggest the absence of dramatic new effects at a TeV. Since new interactions above a TeV are usually weak, this does not necessarily put strong constraints on new physics. However, there are two effects that can act as "magnifiers" of weak individual interactions: First, when probed at very high energies, the nucleons appear to consist of partons each of which acts as a target for new interactions and whose numbers grow with energy roughly as \( E^{0.4} \). Second, theories in which the fundamental constituents are not described as point particles but as extended objects such as strings, often predict numbers of excitable states that increase as power laws or even exponentially with energy.

In the present paper we parametrize new parton level cross sections by a mass scale \( M_{\text{eff}} \), an integer \( n \) characterizing the spin of possible new states produced in the interactions, and a maximal squared 4-momentum transfer \( t_{\text{max}} \). We then establish constraints on these parameters from data on ultra-high energy (UHE) cosmic rays (CR) and neutrinos.

New Fundamental Cross Sections: We parametrize the fundamental cross section between Standard Model elementary particles of type \( i \) and \( j \) involving other Standard Model particles and at least one bulk state by

\[
\sigma_{ij}(s) \sim \frac{t_{\text{max}}}{s^2} \left( \frac{s}{M_{\text{eff}}^2} \right)^{1+n/2}.
\] (1)

Here, \( n \) is a constant, \( M_{\text{eff}} \) is the effective mass scale, and \( s \) and \( t \) with \( |t| \leq t_{\text{max}} \) are the usual Mandelstam variables.

There are several scenarios for which expressions such as Eq. (1) appear. In the case of \( n \) large compact extra dimensions [4] Kaluza-Klein (KK) gravitons can be produced with cross sections \( \sigma_t \sim M_{\text{Pl}}^2 \), where \( M_{\text{Pl}} \simeq 1.72 \times 10^{18} \text{GeV} \) is the reduced Planck mass [5]. For flat extra dimensions the total number of KK states is \( \sim s^{n/2}V_n \), where \( V_n \) is the volume of the extra dimension. Using the relation \( M_{\text{Pl}}^2 = M_{\text{eff}}^n V_n \), this leads to Eq. (1) with \( n \) interpreted as the number of extra dimensions and \( M_{\text{eff}} \) the 4 + \( n \) dimensional fundamental gravity scale. Above \( M_{\text{eff}} \) the growth of the cross section Eq. (1) for gravitational scattering is softened by unitarity effects [6] within scenarios involving point particles. To obtain conservatively low cross sections we will assume an exponential cut-off \( \exp(-s^{1/2}/M_{\text{eff}}) \) in such scenarios, whereas \( t_{\text{max}} \simeq s \).

It has been realized that in extra dimension scenarios the largest contribution to the cross section for collisions above \( M_{\text{eff}} \) may be the production of 3 + \( n \) dimensional black holes [6]. The corresponding cross sections have been estimated by the geometric cross section \( \pi r_s^2(s)^2 \) defined by the Schwarzschild radius in 3 + \( n \) spatial dimensions \( r_s(s) \propto s^{1/(1+n)} \). This is also of the form Eq. (1) with \( n \) substituted by \( 2/(1+n) \). It has further been argued that semi-classical p-branes completely wrapped around \( p \) small extra dimensions leads to cross sections with the scaling \( s^{3+n} \) where \( n \) is now interpreted as the number of large extra dimensions around which the p-brane is not wrapped [7].

Recently scenarios have been discussed where the fundamental quantum gravity scale is as low as \( M_s \sim 10^{-3} \text{eV} \) [8]. Such scenarios can be realized, for example, if the Standard Model lives on a 3-brane and is coupled to
gravity propagating in 5 or more infinite-volume space-time dimensions with a fundamental scale $M_s$. Thus, gravity is strong in the bulk but non-gravitational interactions on our 3-brane induce the observed Planck gravity is strong in the bulk but non-gravitational interactions on our 3-brane induce the observed Planck scale $M_{Pl}$ and thus shield strong gravity from the bulk. Above the fundamental scale $M_s$ gravity is assumed to be regularized and thus has its effective coupling unaffected\cite{2}. This is the case in string theory where $M_s$ is identified with the string scale and excitations of mass $m^2 = 4(N-1)M_s^2$ appear at integer levels $N \geq 1$\cite{3} with spin $j \leq 2N$ (for closed strings). Interactions of Standard Model particles on the brane can then lead to stringy bulk states of spin $j$ with $2 \leq j \lesssim s/M_s^2$. The cross section for each of these states is $\sigma \sim (t_{\text{max}}/s^2)(s/M_{Pl}^2)^{-1}$ [see Eq. (6.6) in Ref. \cite{2}], but the number of states grows as $(s/M_s^2)^{j-1}$. Thus, the intermediate scale in Eq. (6) arises as

$$M_{\text{eff}} = (M_{Pl}M_s)^{1/2},$$

with $n = 4j - 6$, and this scaling reflects the usual Regge behavior. To recover the correct graviton zero-mode coupling we have to use $t_{\text{max}} = s$ for $n = j = 2$ in Eq. (2). In the string theory context the scaling Eq. (6) is not expected to be strongly modified above $M_{\text{eff}}$. The usual unitarity bound for point particles, $\sigma_{ij} \lesssim s^{-1}$, does not apply to extended strings.

**Nucleon-Nucleon and Neutrino-Nucleon Cross Sections:** Total cross sections are obtained by folding Eq. (1) with the parton distributions $f_i(x, Q)$ in the nucleon. We neglect factors from the spin structure and follow an approach similar to Ref. \cite{11}, leading to

$$\sigma_{NN}(s) \sim \sum_{ij} \int_0^1 dx_1 dx_2 f_i(x_1, Q) f_j(x_2, Q) \sigma_{ij}(x_1 x_2 s),$$

$$\sigma_{\nu N}(s) \sim \sum_i \int_0^1 dx f_i(x, Q) \sigma_{\nu i}(x).$$

(3)

Here, the sums run over gluons and quarks. Note that in principle there is another integral $\int_0^1 dQ^2 (d\sigma_{ij} / dQ^2) \cdots$ over the four-dimensional energy-momentum transfer $Q^2 = -t$, where $\hat{s} = x_1 x_2 s$ or $\hat{s} = x s$, respectively. We assume $t_{\text{max}} \approx s$ in Eq. (1) and have approximated this integral by the total parton cross section $\sigma_{ij}(\hat{s})$ multiplied by the parton distributions taken at $Q^2 = \hat{s}$ which is justified within an order of magnitude estimate in the absence of concrete models. For the parton distributions we use the CTEQ6 distributions in electronic form from Ref. \cite{11}.

**Phenomenological Consequences:** Fig. 1 implies neutrino-nucleon cross sections significantly higher than in the Standard Model at energies above $\approx 1$ PeV which would make neutrinos easier to detect. Specifically, cross sections $\sigma_{\nu N} \lesssim 1$ mb would give rise to deeply penetrating atmospheric air showers. Neutrinos are expected to be produced by UHECR which produce pions on the cosmic microwave background every few Mpc above $\approx 4 \times 10^{19}$ eV\cite{12}. The observation of such energetic UHECR which are believed to have an extragalactic origin\cite{13}, allows to estimate the resulting secondary flux of “cosmogenic” neutrinos between $\approx 10^{17}$ eV and $\approx 10^{19}$ eV within a factor $\approx 100$ (14, 15, 16). For a differential neutrino flux $\phi_{\nu}(E)$, a detector whose sensitivity to deeply penetrating showers of energy $E$ effectively corresponds to $N(E)$ target nucleons would see a rate of such showers per solid angle given by $R(E) \approx \phi_{\nu}(E)\sigma_{\nu N}(E)N(f_{\nu} E)$. No such showers consistent with weakly interacting primaries have been observed yet which, for a given cosmogenic flux $\phi_{\nu}(E)$, lead
to upper limits on $\sigma_{ij}(E)$ [10, 17, 18]. Fig. 1 shows the resulting excluded range of cross sections, assuming $f_\nu \sim 1$, for a conservative and an optimistic neutrino flux estimate [10, 18]. The optimistic flux assumes that the neutrino energy fluence is comparable to the isotropic $\gamma$-ray energy fluence in the Universe [10].

Thus, cross sections that are larger than the Standard Model cross section by a factor 10-1000 and smaller than $\sim 1$ mb for $10^{17}$ eV $\lesssim E \lesssim 10^{19}$ eV would imply neutrino fluxes smaller than expected. The projected sensitivity of future experiments [10] indicate that these limits could be lowered down to the Standard Model cross section. We conclude that parton cross sections with $M_{\text{eff}} \lesssim 1$ TeV, $t_{\text{max}} \simeq s$, and $n \gtrsim 2$ in Eq. (1) are most likely in conflict with the non-observation of deeply penetrating air showers, especially in the stringy context where no strong unitarization cut-off above $M_{\text{eff}}$ is expected. In this latter scenario, Eq. (3) implies the limit $M_\pi \gtrsim 10^{-3} E$ independent of similar limits from other considerations [2].

In the string context the $j = n = 2$ (graviton) contribution considered above in Fig. 1 provides a lower limit to the total cross section. The contribution of spin $j \gg 2$ state production leads to a threshold behavior: The behavior of the Veneziano amplitude in the hard scattering limit, $s, t \gg M_\pi^2, t/s = \text{const.}$, implies that individual amplitudes for $j \gg 1$ are usually suppressed by $\sim \exp(-|t|/M_\pi^2)$, and thus $t_{\text{max}} \simeq M_\pi^2$ in Eq. (1). Now summing over all $j \lesssim s/M_\pi^2$ results in an exponential growth of the form $\sigma_{ij}(s) \sim s^{-1} \exp[(s^{1/2} - M_{\text{eff}})/M_\pi]$ [19], and similarly for cross sections involving nucleons. This leads to Hagedorn-like saturation at $s \simeq M_{\text{eff}}^2$, both at the parton level and after folding in Eq. (3). However, air showers with energies up to a few times $10^{20}$ eV, or $s \simeq (1$ PeV)$^2$ have been observed [10]. The shape and starting point in the atmosphere of these showers are consistent with nucleon primaries and cross sections with atmospheric nuclei that are within about a factor 2 of the expected Standard Model hadronic cross sections. Therefore, models with $M_\pi \ll M_{\text{eff}}$ and $M_{\text{eff}} \lesssim 1$ TeV should be ruled out. Furthermore, observation of an air shower of energy $E$ would require a primary of energy $E/f_\nu$. If visible particles are only produced for $Q^2 \lesssim M_{\text{eff}}^2$, for $M_{\text{eff}} \sim 1$ TeV Eq. (3) leads to $f_\nu \lesssim 10^{-3}$ for an incoming nucleon energy $E \gtrsim 10^{20}$ eV, and thus much higher energy primaries would be necessary to explain the observations.

This has important phenomenological ramifications for scenarios with a quantum gravity/string scale in the sub-eV range which has recently been proposed to explain the smallness of the observed cosmological constant [3]. There, the above argument together with Eq. (3) implies $M_\pi \gtrsim 100$ eV. In these scenarios Newtonian gravity is modified on scales $r \lesssim M_\pi^{-1} \sim 2 \mu$m ($M_\pi/100$eV$)^{-1}$, where massive graviton exchange becomes relevant. This is consistent with gravity measurements on sub-mm scales [20]. At very small momenta $p \lesssim M_\pi^2/M_{\text{Pl}}$, the propagator for the zero-mode graviton in four dimensions is modified to $\sim (M_\pi^2 p^2 + a M_\pi^4)^{-1}$ in case of at least two infinite-volume extra dimensions, where $a$ is a constant of order unity. This makes gravity appear higher-dimensional, i.e. decreasing with a higher power of distance than Newtonian gravity, on scales $r \gtrsim M_{\text{Pl}}^2/M_\pi^2 \sim 1$ pc ($M_\pi/100$eV$)^{-2}$ [21]. The above bound would thus imply modifications of gravity on parsec scales which seems phenomenologically unviable. This problem can be avoided for only one extra dimension where gravity is modified only at much larger scales $r \gtrsim M_{\text{Pl}}^2/M_\pi^2$ [21], however, this does not allow to explain the smallness of the cosmological constant [3]. An unlikely loop-hole consists of UHECR primaries consisting of $\gtrsim 10^9$ nucleons.

**Conclusions:** Parton cross sections involving an effective mass scale $M_{\text{eff}} \lesssim 1$ TeV and growing linearly or faster in the squared CM energy are likely in conflict with the non-observation of deeply penetrating air showers. Extra-dimensional black hole or p-brane production cross sections increase more slowly than $s$ and are thus not strongly constrained yet above a TeV, consistent with findings in the literature [10]. We further argued that in scenarios with a Hagedorn-like saturation at $M_{\text{eff}}$ due to an exponential increase of states, this mass scale should be larger than the largest CM energies observed in interactions with hadronic or smaller cross sections, which is currently about a PeV.

In possible string theoretic descriptions of quantum gravity with a scale $M_\pi \ll 1$ TeV, the lower limit on $M_{\text{eff}}$ translates into $M_\pi \gtrsim 100$ eV via Eq. (3). This bound could further increase proportional to the maximal UHECR energy that may be seen with much bigger experiments now under construction such as the Pierre Auger project [22].

In scenarios which also explain the smallness of the cosmological constant with more than two infinite-volume extra dimensions the lower bound on $M_\pi$ would lead to modifications of Newtonian gravity on scales $r \gtrsim 1$ pc ($M_\pi/100$eV$)^{-2}$. Such scenarios thus appear to be in conflict either with UHECR observations or with gravity on parsec scales.

**Acknowledgements:** We thank Gia Dvali, David Langlois, Karim Malik, and Lorenzo Sorbo for illuminating discussions.

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B 429 (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B 436 (1998) 257; N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D 59 (1999) 086004.
[2] G. Dvali, G. Gabadadze, M. Kolanović, and F. Nitti, Phys. Rev. D 65 (2001) 024031.
[3] G. Dvali, G. Gabadadze, and M. Shifman, e-print hep-th/0202174.
[4] for a recent review of the experimental situation, see, e.g., M. Nagano and A. A. Watson, Rev. Mod. Phys. 72 (2000) 689.
[5] G. F. Giudice, R. Rattazzi, and G. D. Wells, Nucl. Phys. B544 (1999) 3.
[6] see, e.g., P. Jain, D. W. McKay, S. Panda, and J. P. Ralston, Phys. Lett. B484 (2000) 267; R. Emparan, M. Masip, and R. Rattazzi, Phys. Rev. D 65 (2002) 064023.
[7] see, e.g., T. Banks and W. Fischler, e-print hep-th/9906038; S. Dimopoulos and R. Emparan, Phys. Lett. B526 (2002) 393.
[8] E.-J. Ahn, M. Cavaglia, and A. V. Olinto, e-print hep-th/0201042.
[9] see, e.g., J. Polchinski, String Theory (Cambridge University Press, Cambridge, England, 1998) Vol. 1.
[10] A. Ringwald and H. Tu, Phys. Lett. B 525 (2002) 135.
[11] J. Pumplin et al., e-print hep-ph/0201195.
[12] K. Greisen, Phys. Rev. Lett. 16 (1966) 748; G. T. Zatsepin and V. A. Kuzmin, Pis’ma Zh. Eksp. Teor. Fiz. 4 (1966) 114 [JETP. Lett. 4 (1966) 78].
[13] for a review see, e.g., P. Bhattacharjee and G. Sigl, Phys. Rept. 327 (2000) 109.
[14] F. W. Stecker, Astrophys. J. 228 (1979) 919.
[15] S. Yoshida and M. Teshima, Prog. Theor. Phys. 89 (1993) 833; R. J. Protheroe and P. A. Johnson, Astropart. Phys. 4 (1996) 253; S. Yoshida, H. Dai, C. C. Jui, and P. Sommers, Astrophys. J. 479 (1997) 547; R. Engel and T. Stanek, Phys. Rev. D 64 (2001) 093010.
[16] for a recent detailed discussion see, e.g., O. E. Kalashiev, V. A. Kuzmin, D. V. Semikoz, and G. Sigl, e-print hep-ph/0205054.
[17] D. A. Morris and A. Ringwald, Astropart. Phys. 2 (1994) 43.
[18] C. Tyler, A. V. Olinto, and G. Sigl, Phys. Rev. D 63 (2001) 055001.
[19] G. Domokos and S. Kovesi-Domokos, Phys. Rev. Lett. 82 (1999) 1366.
[20] C. D. Hoyle et al., Phys. Rev. Lett. 86 (2001) 1418.
[21] G. Dvali, G. Gabadadze, X. Hou, and E. Sefusatti, e-print hep-th/0111266.
[22] I am grateful to G. Dvali for pointing this out.
[23] J. W. Cronin, Nucl. Phys. B (Proc. Suppl.) 28B (1992) 213; The Pierre Auger Observatory Design Report (ed. 2), March 1997; see also http://www.auger.org.