A homogeneous extended state estimator-based super-twisting sliding mode compensator for matched and unmatched uncertainties

Ankur Goel¹, Saleh Mobayen²,³ and Afef Fekih⁴

Abstract
In this research work, an output tracking problem of a kind of nonlinear motion control systems influenced by exogenous uncertainties using second-order super-twisting sliding mode control is studied. It is shown that when second-order super-twisting sliding mode control is implemented with finite-time convergent homogeneous extended state observer, the second-order sliding mode is achieved on the selected sliding manifold with efficient disturbance attenuation from the output. The presented control structure is tested on the air-gap control of an electromagnetic levitation suspension system using MATLAB platform. The observations prove the efficacy of the proposed algorithm providing excellent robust control efficiency along with precise attenuation of various disturbances.

Keywords
Matched and mismatched uncertainty, high-order sliding mode, extended state observer, magnetic levitation suspension system

Date received: 12 February 2020; accepted: 31 March 2020

Introduction
Modern real-world systems are generally characterized by complicated nonlinear and multi-variable dynamics with various uncertainties such as parametric perturbations, unmodeled dynamics, measurement noises, linearization and approximation errors, and external disturbances.¹ For example, see vessel dynamic positioning system,² robot manipulator,³ DC-DC power converter,⁴ missile system,⁵ spacecraft,⁶ permanent magnet synchronous motor (PMSM),⁷ hard-disk drive,⁸ magnetic levitation (MAGLEV) suspension transportation vehicle,⁹,¹⁰ and many more practical systems. These uncompensated uncertainties need to be treated properly otherwise they may become detrimental to the desired control performances which could further lead to system instability.¹¹ Hence, to obtain intelligent and high-fidelity actuated devices, the need of interference attenuation techniques such as adaptive control,¹² robust control,¹¹ $H_2/H_{∞}$ control,¹³ sliding mode control (SMC),¹⁴ backstepping control,¹⁵ and so on have received considerable attention from motion controller designers. Although these exquisite methods have been efficiently applied in various aforementioned uncertain systems, they mainly target to achieve robust stability on the price of nominal performance.¹⁶ Among these control methods, SMC is a popular robust control technique because of its simple design, easier implementation, reduced order dynamics, robustness toward the matched disturbances and the plant uncertainties.¹⁷ It should be pointed out that the SMC methods have mainly two concerns during their practical implementation such as the presence of high-frequency chattering phenomenon¹⁸ and providing complete robustness only to the matched disturbances.¹⁹

For proper and efficient treatment of the first concern of chattering phenomenon, recently, the high-order sliding mode control (HOSMC)²⁰ methods have found popularity among chattering attenuation strategies in the literature. These HOSMC methods are widely tested
on various applications such as hypersonic vehicles,\textsuperscript{21} wind turbine,\textsuperscript{22} cable-driven manipulators,\textsuperscript{23} vehicle suspension,\textsuperscript{24} quadcopters,\textsuperscript{25} and so on. However, among HOSMC family members, the super-twisting control (STC)\textsuperscript{26} has proven its effectiveness in which control input is applied in the second-order of the sliding variable. Its advantages include effective compensation of Lipschitz perturbations, requirement of only single output variable information, finite-time state focalization to the origin and continuous control signal generation for minimizing chattering phenomenon.\textsuperscript{27}

The second concern of handling unmatched uncertainty is justified because it has been observed that the major uncertainties present in many MCS such as rail track input in MAGLEV\textsuperscript{28} load torque in PMSM,\textsuperscript{29} external wind disturbances in flight control system,\textsuperscript{30} and so on do not adhere to so-called matching condition and hence robust performance out of SMC may not be achieved.\textsuperscript{31} Hence, due to the importance of nullifying mismatched uncertainties for getting precise and effective performance from MCS, many decent SMC strategies focusing the mismatched disturbance attenuation have been reported in the literature, for example, Riccati based approach,\textsuperscript{32} LMI-based,\textsuperscript{33} $H_2$ control based,\textsuperscript{34} adaptive approach,\textsuperscript{35} and so on. However, these approaches are designed to handle mismatched uncertainties if they are vanishing type, that is, $H_2$ norm-bounded which may not be the case in real-time system, for example, in MAGLEV transportation system, the considered lumped uncertainties may not have zero steady-state and hence not $H_2$ norm-bounded.\textsuperscript{36} Consequently, these methods may not be able to provide nominal performance of the overall control systems.

Another widely used method to address this issue is integral sliding mode control (I-SMC) in which integral type of sliding surface is used for guiding the states to equilibrium from the first instant of time. Because of the simple design and better robustness feature, this method is widely applied in many practical systems.\textsuperscript{37–39} However, it is commonly known that integral action brings adversity in control action such as disturbance amplification, large overshoot and extended settling time.

To address the issue of robustness of MCS in the presence of mismatched uncertainty, an alternative approach based on estimation/observer technique known as disturbance-observer-based control (DOBC)\textsuperscript{40} approach has been presented. Due to its extraordinary advantage of obtaining robustness without loosing nominal control performance, several control methods with different observers designs such as nonlinear disturbance observer (NLDO),\textsuperscript{41} equivalent-input-disturbance control,\textsuperscript{42} extended state observer (ESO),\textsuperscript{43} sliding mode based observers\textsuperscript{44} and so on have been proposed in the literature. However, it is noticed that many DOBC designs concentrate only the systems with matched disturbances; there are fewer results focusing systems with mismatched ones.\textsuperscript{45,46} Moreover, aforementioned DOBC methods are heavily dependent on the plant information for estimation and control design. Among them the ESO, originated from Active Disturbance Rejection Control (ADRC), uses very less prior information that too only the system relative degree\textsuperscript{47} which makes it popular in theoretical as well as practical studies.\textsuperscript{48,49} The conventional linear ESO (LESO) further modified as nonlinear ESO (NLESO) and Generalized Extended State observer (GESO) control\textsuperscript{50} in order to address the issue of mismatched uncertainties that are even not expressed in the standard integral chain form. Furthermore, the adaptive ESO\textsuperscript{51} also exhibit its efficiency in solving the uncertainty estimation problem. While the ESO methodology has been often used widely, the mathematical analysis of stability, convergence time and so on are not performed rigorously. Guo and Zhao\textsuperscript{52} have proposed the stability analysis of a nonlinear ESO with system modeling uncertainties, but it results in the asymptotic stability under various complex assumptions. A finite-time ESO\textsuperscript{53} has been designed for projecting mismatched uncertainties. However, it requires the high-order time derivatives of the uncertainties to be bounded with certain constant number.

Owing to the above discussions, the authors tried to address the issue of mismatched uncertainties present in the MCS using finite-time convergent ESO to obtain better convergence speed and stability performances.

The primary contributions of this paper are briefly emphasized as follows:

1. Motivated from the existing finite-time convergent observers, the conventional ESO is modified with the help of homogeneity principle and termed as homogeneous extended state observer (HESO).
2. The need of system information is kept as minimum as possible by selecting a dynamic sliding manifold which is designed with the help of system output and observer predicted nominal states.
3. A continuous and chattering free guidance law is used for the states to reach the proposed dynamic sliding manifold in finite time.
4. The overall control structure is kept simple and effective by combining the HESO with the conventional STC for counteracting the adversities of exogenous matched as well as unmatched disturbances.

The remaining article is cataloged as follows: Section “Problem description and control objective” formulates the general MCS problem and its control objective is specified. Section “The proposed controller design” proposes the HESO-based second-order STC (HESO-STC) for the systems with uncertainties. The section “Performance analysis of different ESOs” shows the comparative analysis of LESO, NLESO and HESO applied on a standard servo motor with the help of MATLAB simulation. In section “An electromagnetic suspension (EMS) vehicle example,” an application of
electromagnetic suspension transportation system is presented for testing the proposed control algorithm. To showcase the efficacy of the proposed law, simulations results are studied in section "Simulation results." Finally, the ending remarks are summarized in section "Conclusions."

**Problem description and control objective**

Consider the following general MCS with input relative degree of ρ under the presence of both matched and unmatched disturbances, represented as

\[
\dot{\eta}_i = \eta_{i+1} + d_i, \quad \forall i = 1, \ldots, \rho - 1
\]
\[
\dot{\eta}_\rho = a(\eta) + b(\eta)u + d_\rho
\]

where \( \eta = [\eta_1, \ldots, \eta_\rho]^T \) is the state vector, control input is represented by \( u \), \( y \) denotes the system output. The term \( d_i \) is \((\rho - i)^{th}\)-order differentiable and bounded mismatched disturbance and \( d_\rho \) is the matched disturbance entering in the control input channel. The functions \( a(\eta) \) and \( b(\eta) \neq 0 \) are functions of \( \eta \) which are differentiable everywhere.

**Assumption 1.** The disturbance \( d_i \) is continuous and differentiable which comply the bounded condition as \( \frac{dd_i}{dt} \leq \Delta_i \) for some positive constant \( \Delta_i, i = 1, 2 \ldots \rho \) and \( j = 0, 1 \ldots \rho + 1 - i \).

**Remark 1.** The above assumption on the disturbance \( d_i \) is reasonably justified because major disturbances in MCS are due to discontinuous nature of friction which can be modeled with piece-wise continuous models such as LuGre model.\(^{54}\)

The main intention is to construct a sturdy, finite-time HOSM-based feedback controller \( u \) which could compel the output \( y \) of the system (1) to stabilize at a pre-defined position \( \eta_d \) in the finite-time \( T_f \) that is, which ensures equation (2) under the influence of disturbances

\[
\lim_{t \to T_f} \|P_{err}\| = \lim_{t \to T_f} \|(\eta_1 - \eta_d)\| = 0
\]

where \( P_{err} = (\eta_1 - \eta_d) \) is the position error.

**The proposed controller design**

To initiate the design of the rugged, finite-time convergent HESO-based super-twisting sliding mode controller (HESO-STC) for the system (1), a novel sliding manifold is designed as

\[
\sigma = c_1 \eta_1 + \sum_{i=2}^{\rho} c_i \dot{\eta}_i
\]

where the parameters \( c_j > 0, (i = 1, \ldots, \rho) \) with \( c_\rho = 1 \) are designed such that the polynomial \( p_0(s) = s^\rho + c_{\rho-1}s^{\rho-1} + \ldots + c_2s + c_1 = 0 \) is Hurwitz.

The variables \( [\tilde{\eta}_2, \tilde{\eta}_3, \ldots, \tilde{\eta}_\rho]^T \) are the estimations of the states \( [\eta_2, \eta_3, \ldots, \eta_\rho]^T \) from an observer which will be discussed later.

**Remark 2.** From author’s point of view, the considered sliding manifold (equation (3)) is different from the conventional sliding surface because the selected manifold depends only on the information of the MCS output and estimated states from the observer/estimator. Hence, it is justified to say that equation (3) needs minimum information from the system and depends only on the outputs.

Next, the design approach of HESO—a \((\rho + 1)^{th}\)—order finite-time observer for estimating the states \( [\tilde{\eta}_2, \tilde{\eta}_3, \ldots, \tilde{\eta}_\rho]^T \) is discussed.

**HESO**

The HESO is a special type of ESO which follows the design procedure of homogeneous state observer.\(^{55}\) The general design of \((\rho + 1)^{th}\)—order HESO for the system (1) with disturbances is given as

\[
\begin{align*}
\dot{\tilde{\eta}}_1 &= \tilde{\eta}_2 + \lambda^{\rho-1}k_1\left(\frac{\eta_1 - \tilde{\eta}_1}{\kappa_0}\right)\alpha \\
\dot{\tilde{\eta}}_2 &= \tilde{\eta}_3 + \lambda^{\rho-2}k_2\left(\frac{\eta_2 - \tilde{\eta}_2}{\kappa_0}\right)2\alpha - 1 \\
&\vdots \\
\dot{\tilde{\eta}}_{\rho-1} &= \tilde{\eta}_\rho + \lambda^{1}k_{\rho-1}\left(\frac{\eta_{\rho-1} - \tilde{\eta}_{\rho-1}}{\kappa_0}\right)(\rho - 1)\alpha - (\rho - 2) \\
\dot{\tilde{\eta}}_\rho &= \tilde{\eta}_{\rho+1} + \lambda^0k_{\rho}\left(\frac{\eta_\rho - \tilde{\eta}_\rho}{\kappa_0}\right)\rho\alpha - (\rho + 1) + a(\tilde{\eta}) + b(\tilde{\eta})u \\
\dot{\tilde{\eta}}_{\rho+1} &= \lambda^{-1}k_{\rho+1}\left(\frac{\eta_{\rho+1} - \tilde{\eta}_{\rho+1}}{\kappa_0}\right)(\rho + 1)\alpha - \rho
\end{align*}
\]

where \( \alpha \in (1 - \frac{1}{\rho}, 1), \kappa_0 = \lambda^\rho, (\lambda, k_i) \in \mathbb{R}_+^\rho \) are adjustable parameters selected as per Moreno\(^{56}\) and \( u \) is the system (1) control input. Here the notation \( |\dot{d}|^{sign}(a) \) means it is equal to \( |\dot{d}|^{sign}(a) \).

The parameters \( k_i, i = 1, 2, \ldots, (\rho + 1) \) are properly tuned to made following matrix Hurwitz

\[
E = \begin{bmatrix}
-k_1 & 1 & 0 & \cdots & 0 \\
-k_2 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-k_\rho & 0 & 0 & \cdots & 1 \\
-k_{\rho+1} & 0 & 0 & \cdots & 0
\end{bmatrix}
\]
Convergence analysis

In order to establish the convergence of the HESO equation (4), definitions of the homogeneity principle and finite-time stability along with relevant assumption and lemmas are presented herewith.

Definition 1. A function \( \mathcal{V}: \mathbb{R}^{n} \rightarrow \mathbb{R} \) is termed as homogeneous with degree \( d \) relative to weights \( \{ r_{i} > 0 \}^{n}_{i=1} \), if for all \( \forall i = 1, 2, \ldots, n \)

\[
\mathcal{V}(x_{1}, x_{2}, \ldots, x_{n}) = \mathcal{V}(r_{1}, r_{2}, \ldots, r_{n})
\]

(6)

for all \( x > 0 \) and \( (x_{1}, x_{2}, \ldots, x_{n}) \in \mathbb{R}^{n} \).

A vector field \( \mathcal{G}: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n} \) is called homogeneous of degree \( d \) relative to weights \( \{ r_{i} > 0 \}^{n}_{i=1} \), if

\[
\mathcal{G}(x_{1}, x_{2}, \ldots, x_{n}) = \mathcal{G}(r_{1}, r_{2}, \ldots, r_{n})
\]

(7)

for all \( x > 0 \) and \( (x_{1}, x_{2}, \ldots, x_{n}) \in \mathbb{R}^{n} \), where \( \mathcal{G}_{i} \) is the \( i^{th} \) element of \( \mathcal{G} \).

If \( \mathcal{V} \) satisfies equation (6) and is differentiable with respect to \( x_{n} \), then the partial derivative of \( \mathcal{V} \) in \( x_{n} \) follows

\[
\frac{\partial \mathcal{V}}{\partial x_{n}}(x_{1}, x_{2}, \ldots, x_{n}) = \frac{\partial \mathcal{V}}{\partial x_{n}}(r_{1}, r_{2}, \ldots, r_{n})
\]

(8)

Definition 2. The following system

\[
\dot{x} = f(x(t)), \quad x(0) = x_{0} \in \mathbb{R}^{n}
\]

(9)

is globally finite-time stable, if it is Lyapunov stable, and \( \forall x_{0} \in \mathbb{R}^{n} \exists T(x_{0}) > 0 \) such that the solution of equation (9) satisfies \( \lim_{t \rightarrow T(x_{0})} x(t) = 0 \), and \( x(t) = 0 \forall t \in [T(x_{0}), \infty) \).

Let us denote the error variable as \( e_{1} = \eta_{1} - \hat{\eta}_{1} \) and error of the \( j^{th} \) state as \( e_{j} = \eta_{j} - \hat{\eta}_{j} \) for \( j = 2, \ldots, p \), the observer dynamics equation (4) can be represented with change in coordinates as

\[
\begin{align*}
\dot{\xi}_{1} &= e_{1} \\
\dot{\xi}_{i} &= e_{i} + \sum_{j=1}^{i-1} 2 \frac{d \hat{\eta}_{j}}{dt} \\
\dot{\xi}_{p+1} &= -\hat{\eta}_{p+1} + \hat{a} + \sum_{j=1}^{p} 2 \frac{d \hat{\eta}_{j}}{dt}
\end{align*}
\]

(11)

where, \( i = 2, \ldots, p \)

Using equation (11), the coordinates of the system (10) are transformed as

\[
(e_{1}, e_{2}, \ldots, e_{p}, \eta_{p+1}) \rightarrow (\xi_{1}, \xi_{2}, \ldots, \xi_{p}, \dot{\xi}_{p+1}),
\]

which immediately follows that

\[
\begin{align*}
\dot{\xi}_{1} &= -\lambda^{p-1} k_{1} \frac{\xi_{1}^{\alpha}}{\gamma_{1}} + \xi_{2} \\
\dot{\xi}_{2} &= -\lambda^{p-2} k_{2} \frac{\xi_{2}^{\alpha}}{\gamma_{2}} + \xi_{3} \\
\vdots \\
\dot{\xi}_{p} &= -\lambda^{0} k_{p} \frac{\xi_{p}^{\alpha}}{\gamma_{p}} + \xi_{p+1} \\
\dot{\xi}_{p+1} &= -\lambda^{0} k_{p} \frac{\xi_{p}^{\alpha}}{\gamma_{p}} + \hat{a} + \sum_{j=1}^{p} 2 \frac{d \hat{\eta}_{j}}{dt}
\end{align*}
\]

(12)

where \( \gamma_{1} = \hat{a} + \sum_{j=1}^{p} \frac{\alpha}{\gamma_{j}} \frac{d \eta_{j}}{dt} + 1 \) is the lumped disturbances.

Using the Lemma 4.2 of Bhat and Bernstein,58 Lemma 2, 3 and Theorem 2 of Guo and Zhao,52 it can be shown that error equation (12) is a disturbed representation of global finite-time stable system \( \dot{\hat{y}} = F(\hat{y}) \), \( \hat{y} \in \mathbb{R}^{n+1} \). Guo and Zhao52 have shown that for the homogeneous global finite-time stable system, there exist a positive definite, radial unbounded, differentiable function \( V_{j}: \mathbb{R}^{n} \rightarrow \mathbb{R} \) such that \( V_{j}(x) \) is homogeneous of degree \( \gamma_{j} \) w.r.t. to weights \( \{ r_{i} \}^{n}_{i=1} \) and the Lie derivative of \( V_{j} \) along the vector field \( F_{j} \) is negative definite. Moreover, \( L_{2}V_{j} \) is homogeneous of degree \( \gamma_{j} - d \) w.r.t. weights \( \{ r_{i} \}^{n}_{i=1} \).

To test the above narration and check the homogeneity of \( V_{j} / x_{n} \), we consider \( p = 2, \lambda = 1, k_{1} = 3, k_{2} = 3, k_{3} = 1 \) in equation (12), then the vector field \( F(\xi) \) is consider as

\[
F(\xi) = \begin{bmatrix}
\xi_{2} - \phi_{1}(\xi) \\
\xi_{3} - \phi_{2}(\xi) \\
-\phi_{3}(\xi)
\end{bmatrix}
\]

(13)

where, \( \phi_{1}(\xi) = 3 \frac{\xi_{1}^{\alpha}}{\gamma_{1}}, \phi_{2}(\xi) = 3 \frac{\xi_{2}^{\alpha}}{\gamma_{2}} - 1 \) and \( \phi_{3}(\xi) = \frac{\xi_{2}^{\alpha}}{\gamma_{2}} - 1 \). It is evident that the vector field \( F(\xi) \) equation (13) is homogeneous of degree \( \alpha - 1 \) with respect to the weights \( \{ 1, \alpha, 2\alpha - 1 \} \). Since the matrix \( E \) equation (5) is Hurwitz and for some \( \alpha = (\frac{3}{2}, 1) \), the system \( \dot{\hat{y}} = F(\hat{y}) \) is finite-time stable. Also, there exists a positive definite, radial unbounded function \( V: \mathbb{R}^{3} \rightarrow \mathbb{R} \) such that \( V \) is homogeneous of degree \( \gamma \) with respect to the weights \( \{ 1, \alpha, 2\alpha - 1 \} \), and \( \frac{\partial V(\hat{y})}{\partial \hat{y}}(\gamma_{2}) - g_{2}(\gamma_{2}) + \frac{\partial V(\hat{y})}{\partial \hat{y}}(\gamma_{3}) - g_{3}(\gamma_{3}) - \frac{\partial V(\hat{y})}{\partial \hat{y}}(\gamma_{1}) \) is negative definite and homogeneous of degree
Design of HESO-based second-order STC

In this section, a HESO (equation (4)) based continuous and finite-time control law relying on 2nd-order STC is designed which rejects both matched and unmatched system uncertainties and confirms that the control objective (equation (2)) is ensured. For this purpose, the following theorem is proposed:

Theorem 1. Considering the sliding (1) and the sliding surface defined as equation (3) and control law u as

\[ u = -b^{-1}(\eta)(c_1 \dot{\eta}_2 + a(\tilde{\eta}) + \sum_{i=2}^{\rho} c_i \eta_{i+1}) + \lambda \rho^{-1}K_j \left \{ e_i \right \} \hat{J}(t) + b^{-1}u_{sm1} \]

with

\[ u_{sm1} = -k_1 |\sigma| \text{sign}(\sigma) + u_{sm2} \]

\[ u_{sm2} = -k_2 |\sigma| \text{sign}(\sigma) \]

where, \( k_1 > 0 \) and \( k_2 > 0 \) are controller gains with \( \Xi \) is some positive constant, then the HOSM is initiated on the sliding surface in finite time in the presence of matched and unmatched uncertainties.

Proof. The time-derivative of the sliding surface (equation (3)) is given as

\[ \dot{\sigma} = c_1 \dot{\eta}_1 + \sum_{i=2}^{\rho} c_i \dot{\eta}_i \quad \text{with} \quad c_\rho = 1 \]

Substituting equation (1) in equation (16), we get

\[ \dot{\sigma} = c_1 \eta_2 + c_1 \dot{\eta}_1 + a(\tilde{\eta}) + b(\tilde{\eta})u \]

\[ + \sum_{i=2}^{\rho} c_i \left ( \frac{\eta_{i+1}}{\kappa_0} \eta_i \right ) \]

Since, with proper tuning, the HESO (equation (4)) will predict the actual data of the states hence substituting \( \eta_0 = \tilde{\eta}_i \) in the first equation of \( \sigma \) will allow us to rewrite the above equation as

\[ \dot{\sigma} = c_1 \eta_2 + c_1 \dot{\eta}_1 + a(\tilde{\eta}) + b(\tilde{\eta})u \]

\[ + \sum_{i=2}^{\rho} c_i \left ( \frac{\eta_{i+1}}{\kappa_0} \eta_i \right ) \]

Substituting the control input equation (14) in equation (17), we get

\[ \dot{\sigma} = c_1 \dot{d}_1 - k_1 |\sigma| \text{sign}(\sigma) + u_{sm2} \]

Defining a new variable as \( x_1 = \sigma \) and \( x_2 = c_1 \dot{d}_1 + u_{sm2} \) and rewriting the above equation, we get

\[ \dot{x}_1 = -k_1 |\sigma| \text{sign}(\sigma) + x_2 \]

\[ \dot{x}_2 = -k_2 |\sigma| \text{sign}(\sigma) + c_1 \dot{d}_1 \]

Equation (19) is a second-order super-twisting algorithm.

Using Theorem 2, Page 1036 of Moreno and Osorio\(^6\) and its proof, it can be presented that the following Lyapunov function

\[ V(x) = \xi^T P \xi \]

is quadratic, strict and robust with symmetric and positive definite matrix \( P \), will satisfy

\[ \dot{V} \leq - |x_1|^{1/2} \xi^T Q \xi \]

almost everywhere, for symmetric and positive definite matrix \( Q \). Moreover, the time taken by a trajectory originating at \( x_0 \) will meet to origin in a finite time smaller than \( \tilde{t}(x_0) \) given by

\[ \tilde{t}(x_0) = \frac{2}{\alpha} |x_1|^{1/2}(x_0); \alpha = \frac{\lambda_{\text{max}}(P) \lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(P)} \]

The gains \( k_1 > 0 \) and \( k_2 > c_1 \Delta, \Delta > 0 \) along with the matrices \( P \) and \( Q \) of the Lyapunov function can be selected in accordance with the procedure given in Moreno and Osorio.\(^6\) Consequently, it is proved that the sliding variable \( x_1 \) and \( x_2 \) will go to zero in the finite time.

Performance analysis of different ESOs

In this section, a comparative analysis on the performances of different ESOs viz. linear ESO (LESO), nonlinear ESO (NLESO) and homogeneous ESO (HESO) are demonstrated with the help of a classical servo system.
A classical servo system is represented as
\[ \dot{\theta}(t) = u(t) - \varphi \]  
\[ \text{(23)} \]
where, \( J > 0 \) is the moment of inertia, \( u(t) \) is control input, \( \theta(t) \) represents the actual angular position and \( \varphi \) is the lumped disturbances. Expressing equation (23) in an alternate form as
\[ \dot{\theta}(t) = bu(t) + f(t) \]  
\[ \text{(24)} \]
where, \( b = 1/J, f(t) = - (1/J)d(t) \) is a function with bounded derivative. For representing equation (24) in a standard canonical form, let the angular position \( x_1 = \theta \), the angular speed \( x_2 = \dot{\theta} \) then equation (24) can be written as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(t) + bu(t)
\end{align*}
\]
\[ \text{(25)} \]
Considering \( x_3 = f(t) \) as disturbance state and \( \rho(t) = \dot{f}(t) \), then equation (25) can be re-written as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + bu(t) \\
\dot{x}_3 &= \rho(t)
\end{align*}
\]  
\[ \text{(26)} \]

The initial states of the plant are assumed at \( (0.5,0) \), while that of the observers are considered at \( (0,0,0) \). The settings used during simulation tests are \( a_1 = 5.5, a_2 = 10.5, a_3 = 6.1, \varepsilon = 0.01 \) and \( k = 0.8 \). The function \( \phi() \) is a nonlinear function as used by Guo and Zhao. The settings are kept same for all three observers to have better demonstration of the comparative performances. The results show that with the same tuning parameters, the HESO observer equation (29) has outperformed others in estimating the states \( x_1, x_2 \) as well as disturbance \( x_3 \). Figure 1 depicts the faster convergence property of HESO. Figures 2 and 3 illustrate that the despite of little higher peak in HESO, still it is performing well in estimation of other states. The peak obtained during initial time of HESO estimation can be reduced by adjusting the parameter \( \varepsilon \) which governs the speed of convergence.

\[ \begin{align*}
\dot{x}_1 &= x_2 + a_1 \varepsilon^3 \left[ \frac{1}{2} \left( 1 - x_1 \right)^3 - x_1 \right] \\
\dot{x}_2 &= x_3 + a_2 \varepsilon^2 \left[ \frac{1}{2} \left( 1 - x_1 \right)^2 - x_1 \right]^2 + bu(t) \\
\dot{x}_3 &= a_3 \varepsilon^1 \left[ \frac{1}{2} \left( 1 - x_1 \right) - x_1 \right]^{3k-2}
\end{align*} \]
\[ \text{(29)} \]

The simulation using MATLAB has been performed using ESO variants given in equations (27)–(29) on the system (26) and the comparative performances results are shown in Figures 1–3. The system model parameters are taken as \( J = 10, \varphi(t) = 0.3 \sin(t), f(t) = 0.3 \sin(t) \). The initial states of the plant are assumed at \( (0.5,0) \) while that of the observers are considered at \( (0,0,0) \). The settings used during simulation tests are \( a_1 = 5.5, a_2 = 10.5, a_3 = 6.1, \varepsilon = 0.01 \) and \( k = 0.8 \). The function \( \phi() \) is a nonlinear function as used by Guo and Zhao. The settings are kept same for all three observers to have better demonstration of the comparative performances. The results show that with the same tuning parameters, the HESO observer equation (29) has outperformed others in estimating the states \( x_1, x_2 \) as well as disturbance \( x_3 \). Figure 1 depicts the faster convergence property of HESO. Figures 2 and 3 illustrate that the despite of little higher peak in HESO, still it is performing well in estimation of other states. The peak obtained during initial time of HESO estimation can be reduced by adjusting the parameter \( \varepsilon \) which governs the speed of convergence.
An electromagnetic suspension (EMS) vehicle example

Nonlinear EMS system model

The EMS vehicle system nonlinear model shown in Figure 4 is given by

\[
\begin{align*}
B &= K_b \frac{I}{G} ; F = K_f B^2 \\
\frac{dI}{dt} &= V_c - IR_c + \frac{N_c A_p K_b G_0}{G^2} \left( \frac{dZ}{dt} \right) \\
M_c \frac{d^2 Z}{dt^2} &= M_g - F \\
\frac{dG}{dt} &= \frac{dz}{dt} - \frac{dZ}{dt}
\end{align*}
\]

(30)

where \(I\) represent the current, \(Z, Z_t\) are the positions of electromagnet and the rail, \((dZ/dt), (dz_t/dt)\) are the vertical velocities of electromagnet and the rail, \(G\) is the air-gap, \(F\) is the force, \(B\) is the flux density and the coil voltage is \(V_c\). The other system parameters used in equation (30) are shown in Table 1.

**Linearized model**

To transform the nonlinear MAGLEV model (equation (30)) into the selected design formation (equation (1)), the model linearization technique described by Michail is used to apply the designed control technique. The linearized dynamic model of MAGLEV suspension vehicle operating with nominal values tabulated in Table 1 is written as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd(t) + \Delta Ax + O(x, u, d) \\
y &= Cx
\end{align*}
\]

(31)

where the states \(x = [i, z, z_t, z] \) represent the current variations, electromagnet vertical velocity, and air-gap, the input \(u = u_c\) denotes the voltage, \(d(t) = z_t\) is the disturbances generated due to vertical velocity of rail, the air gap variation \(y = z_t - z\) is the controlled variable, \(\Delta A\) is the uncertainty matrix and \(O(x, u, d)\) is the nonlinear function of high-order non-linearities. The matrices of system (31) are selected as

\[
A = \begin{bmatrix}
-R_c & \frac{-K_b N_c A_p G_0}{G^2 (L_c + K_b N_c A_p)} & 0 \\
-2K_f I_0 & 0 & 2K_f G_0 \\
0 & 0 & -1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{1}{L_c + K_b N_c A_p G_0} \\
0 \\
0
\end{bmatrix} ; B_d = \begin{bmatrix}
\frac{K_b N_c A_p G_0}{G^2 (L_c + K_b N_c A_p)} \\
0 \\
0
\end{bmatrix}
\]

(32)

\[
C = [0 \ 0 \ 1]
\]

The control requirement is to maintain the gap position in presence of the major disturbances which is the track input to the suspension originated vertically. The control specifications for the EMS under the influence of disturbances are met in the state feedback form 

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd(t) + \Delta Ax + O(x, u, d) \\
\dot{u} &= -K x
\end{align*}
\]

(33)

Table 1. Parameters of the MAGLEV suspension vehicle.

| Parameters | Meaning | Value |
|------------|---------|-------|
| \(M_s\)   | Boggie mass | 1000 Kg |
| \(K_b\)   | Flux constant | 0.0015 T.m/A |
| \(K_f\)   | Force constant | 9810 N/T^2 |
| \(R_c\)   | Resistance of coil | 10 Ω |
| \(G\)     | Gravitational const. | 9.81 m/s^2 |
| \(L_c\)   | Inductance of coil | 0.1 H |
| \(N_c\)   | coil turns | 2000 |
| \(A_p\)   | Area of pole | 0.01 m^2 |
| \(B_0\)   | Nominal flux density | 1.0 T |
| \(F_0\)   | Minimal force | 9810 N |
| \(I_0\)   | Minimal current | 10 A |
| \(G_0\)   | Minimal air-gap | 0.015 m |
| \(V_0\)   | Minimal voltage | 100 V |
of the deterministic track input are tabulated in Table 2.61

Controller design

The high-order nonlinear expression \( O(x, u, d) \) in equation (31) is considered as a lumped disturbances consisting exogenous disturbances and parametric variation, denoted as

\[
d_i = B_id + \Delta Ax + O(x, u, d)
\]  (33)

It is worth to mention that the lumped disturbances (equation (33)) are generally very feeble as compared with the strong system dynamics and hence it is reasonable to assume that such disturbances can be compensated by the presented control method. Using equations (31) and (33), the complete dynamic model of the EMS vehicle is represented as

\[
\dot{x} = Ax + Bu + Bd_i
\]  (34)

where \( B_i = I \) is a 3×3 identity matrix. Next, by coordinate transformation method using \( \eta = T \dot{x} \) with \( T = [C; CA; CA^2]^T \), the original system is transformed into Byrnes-Isidori normal form63 with both matched and unmatched disturbances. The system (34) is then represented as

\[
\dot{\eta} = A\eta + B_iu + B_id_i
\]  (35)

where \( A = TAT^{-1} \), \( B_i = TB_i \) and \( B_i = TB_i \).

Substituting equation (32) in equation (35) gives

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 + d_3 \\
\dot{\eta}_2 &= \eta_3 - d_2 \\
\dot{\eta}_3 &= CA^3T^{-1}\eta + CA^2Bu + CA^2B_id_i
\end{align*}
\]  (36)

where vector \( d_i = [d_1, d_2, d_3]^T \) represents lumped disturbances in the respective channels. It can be clearly observed from equation (36) that the EMS vehicle is affected by both matched \((CA^2B_id_i)\) and mismatched \((d_3, d_2)\) disturbances which are not possible to attenuate by traditional SMC or even by well-known I-SMC method.

The sliding surface for system (36) as per equation (3) is selected as

\[
\sigma = c_1\eta_1 + c_2\eta_2 + c_3\eta_3
\]  (37)

where \( \eta_2, \eta_3 \) represent the estimate of \( \eta_2, \eta_3 \), respectively. The fourth order HESO (equation (4)) used to obtain the estimates \( \eta_2 \) and \( \eta_3 \) in presence of disturbances is given as

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 + \lambda^2 k_3 \frac{(\eta_1 - \eta_1)}{\kappa_0} \\
\dot{\eta}_2 &= \eta_3 + \lambda^4 k_3 \frac{(\eta_1 - \eta_1)}{\kappa_0}^{2a-1} \\
\dot{\eta}_3 &= \eta_4 + \lambda^0 k_3 \frac{(\eta_1 - \eta_1)}{\kappa_0}^{3a-2} + a(\eta) + b(\eta)u \\
\dot{\eta}_4 &= \lambda^{-1} k_4 \frac{(\eta_1 - \eta_1)}{\kappa_0}^{4a-3}
\end{align*}
\]  (38)

where \( k_i, i = 1, 2, 3 \) are the gains selected according to the procedure mentioned by Guo and Zhao.59

Based on the above analysis, proposed HESO-based 2nd-order STC (equation (14)) can be tested on a EMS vehicle system without any restriction.

Simulation results

Simulation results obtained from MATLAB are presented here to corroborate the efficacy of the overall system performance. To better showcase the claim, the proposed control method is compared with the Integral sliding mode control (I-SMC),64 a popular and effective strategy to suppress the mismatched uncertainties.

The I-SMC is generally described with the sliding surface \( \sigma_{\text{con}} = c_0\eta_2 + c_1\eta_1 + c_2 \int \eta_1 \), has the control law formulated as

\[
u = -b^{-1}(\eta)[a(\eta) + c_0\eta_2 + c_1\eta_1 + c_2\eta_1 + k\text{sign}(\sigma_{\text{con}})]
\]

The simulations tests are carried out on a full nonlinear EMS system under the influence of a measurement noise environment. The tuned control parameters of both the strategies are enlisted in Table 3.

External disturbance rejection

During simulation, the track input considered is shown in Figure 5.61 It shows 5% gradient at a vehicle speed of 15 m/s, a vertical acceleration of 0.5 m/s\(^2\) and a jerk level of 1 m/s\(^3\). Practically, the track input disturbance would change continuously due to the rail variations. Furthermore, an extra time-dependent track input

---

Table 2. Control specifications for EMS vehicle.

| Constraints                              | Value            |
|------------------------------------------|------------------|
| Max. air-gap change \((z_t - z)\_p\)     | \(\leq 0.0075\) m |
| Max. coil voltage given \((w_{coil})_p\) | \(\leq 300\) V(3\(L/R\)) |
| Setting time \((t_s)\)                   | \(\leq 3\) s     |
| Steady air-gap error \((z_t - z)_{so}\)  | = 0             |

Table 3. Controller parameters of the EMS vehicle.

| Controller | Parameters          |
|------------|---------------------|
| Proposed   | \(c_1 = 100, c_2 = 10, c_3 = 1, \lambda = 0.001, \kappa = \lambda^3\) |
| I-SMC      | \(c_0 = 200, c_1 = 100, c_2 = 20, c_3 = 1, k = 80\) |
disturbance $z_t = 0.1\sin(\pi t)$ m/s is enforced on the vehicle at $t = 4$ second to resemble the real scenario. The initial states are assumed as $[i(0), z(0), z_t(0) - z(0)]^T = [0, 0, 0.003]^T$. The simulation outputs from I-SMC and proposed control method are revealed in Figures 6–10.

Figures 6–8 represent the states of the system and clearly depict that the proposed control method has demonstrated the nominal performance recovery property. From Figure 6, it is evident that in presence of high-order time varying disturbances (after $t > 4s$), I-SMC fails to stabilize the air gap distance and also unable to compensate the disturbances effectively, whereas the proposed control shows the finite-time regulation of the gap. The control input from the proposed method is smooth and chattering free as illustrated in Figure 9. The I-SMC is effective in canceling the offset caused by track input disturbances. However, the proposed scheme demonstrates the better disturbance rejection capabilities than I-SMC. Moreover, Figure 10 illustrates the capability of the HESO in estimating the
disturbances perfectly in finite time and eliminating their effects.

Conclusions

This paper investigated both matched and mismatched disturbance attenuation problem for MCSs. A new HESO has been combined with second-order STC to handle the high-order disturbances consequences on the output in the finite time. The main contribution is to design an effective strategy which incorporates the estimations of non-vanishing disturbances whose bounds are not known a priori and compel the states onto the sliding surface and the output reaches to equilibrium in the finite time even under the influence of high-order mismatched disturbances. Simulation results of a EMS vehicle system have exhibited that the presented control method is better than I-SMC in terms of improved dynamics and better normal performance in the presence of exogenous disturbances.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Ankur Goel https://orcid.org/0000-0003-3138-9066

References

1. Ferrara A, Incremona GP and Cucuzzella M. Advanced and optimization based sliding mode control: theory and applications. Philadelphia, PA: SIAM, 2019.

2. Hosseinnajad A, Danesh M and Loueipour M. Application of observer-based sliding mode controller in dynamic positioning systems. In: Proceedings of the 5th international conference on control, instrumentation, and automation (ICCIA), Shiraz, Iran, 21–23 November 2017, pp. 238–243. New York: IEEE.

3. Mohammedi A, Tavakoli M, Marquez H, et al. Nonlinear disturbance observer design for robotic manipulators. Control Eng Pract 2013; 21(3): 253–267.

4. Wang J, Li S, Yang J, et al. Extended state observer-based sliding mode control for PWM-based DC–DC buck power converter systems with mismatched disturbances. IET Control Theory A 2015; 9(4): 579–586.

5. Shima T, Idan M and Golan OM. Sliding-mode control for integrated missile autopilot guidance. J Guid Control Dyn 2006; 29(2): 250–260.

6. Song Z, Duan C, Su H, et al. Full-order sliding mode control for finite-time attitude tracking of rigid spacecraft. IET Control Theory A 2018; 12(8): 1086–1094.

7. Aihua W, Buhui Z, Jingfeng M, et al. Extended state observer based integral sliding model control for PMSG variable speed wind energy conversion system. In: Proceedings of the 34th Chinese control conference (CCC), Hangzhou, China, 28–30 July 2015, pp. 3387–3391. New York: IEEE.

8. Sun N, Fang Y and Chen H. Tracking control for magnetic-suspension systems with online unknown mass identification. Control Eng Pract 2013; 21(3): 193–201.

9. Roy S, Baldi S and Fridman LM. On adaptive sliding mode control without a priori bounded uncertainty. Automatica 2020; 111: 108650.

10. Tamhane B and Kurode S. Finite time state and disturbance estimation for robust performance of motion control systems using sliding modes. Int J Control 2018; 91(5): 1171–1182.

11. Wang W, Xie B, Zuo Z, et al. Adaptive backstepping control of uncertain gear transmission servosystems with asymmetric dead-zone nonlinearity. IEEE T Ind Electron 2018; 66: 3752–3762.

12. Du X, Fang X and Liu F. Continuous full-order nonsingular terminal sliding mode control for systems with matched and mismatched disturbances. IEEE Access 2019; 7: 130970–130976.

13. Utkin VI. Sliding modes in control and optimization. Berlin: Springer Science & Business Media, 2013.

14. Perez-Ventura U and Fridman L. Chattering comparison between continuous and discontinuous sliding-mode controllers. In: Steinberger M, Horn M and Fridman L (eds)
Variable-structure systems and sliding-mode control. Cham: Springer, 2020, pp. 197–211.

19. Liu J, Gao Y, Yin Y, et al. Basic theory of sliding mode control. In: Liu J, Gao Y, Yin Y, et al. (eds) Sliding mode control methodology in the applications of industrial power systems. Cham: Springer, 2020, pp. 11–25.

20. Utkin V. Discussion aspects of high-order sliding mode control. *IEEE T Automat Contr* 2015; 61(3): 829–833.

21. Zhang Y, Li R, Xue T, et al. An analysis of the stability and chattering reduction of high-order sliding mode tracking control for a hypersonic vehicle. *Inf Sci* 2016; 348: 25–48.

22. Xiong L, Li P, Wu F, et al. A coordinated high-order sliding mode control of DFIG wind turbine for power optimization and grid synchronization. *Int J Elec Power 2019*; 105: 679–689.

23. Wang Y, Chen J, Yan F, et al. Adaptive super-twisting fractional-order nonsingular terminal sliding mode control of cable-driven manipulators. *ISA T 2019*; 86: 163–180.

24. Ozer HO, Hacioglu Y and Yagiz N. High order sliding mode control with estimation for vehicle active suspensions. *T I Meas Control 2018*; 40(5): 1457–1470.

25. Wu W, Jin X and Tang Y. Vision-based trajectory tracking control of quadrotors using super twisting sliding mode control. *Cyber: Phys Syst* Epub ahead of print 2 March 2020. DOI: 10.1080/23335777.2020.1727960.

26. Levant A. Sliding order and sliding accuracy in sliding mode control. *Int J Control 1993*; 58(6): 1247–1263.

27. Haimovich H and De Battista H. Disturbance-tailored super-twisting algorithms: properties and design framework. *Automatica* 2019; 101: 318–329.

28. Mercado-Uribe A and Moreno JA. Homogeneous integral controllers for a magnetic suspension system. *Control Eng Prac 2020*; 97: 104325.

29. Wang T, Li J and Liu Y. Synergetic control of permanent magnet synchronous motor based on load torque observer. *P I Mech Eng I: J Sys 2019*; 233(8): 980–993.

30. Zhao L, Dai L, Xia Y, et al. Attitude control for quadrotors subjected to wind disturbances via active disturbance rejection control and integral sliding mode control. *Mech Syst Signal Pr 2019*; 129: 531–547.

31. Wulff K, Posielek T and Reger J. Compensation of unmatched disturbances via sliding-mode control. In: Steinberger M, Horn M and Fridman L (eds) Variable-structure systems and sliding-mode control. Cham: Springer, 2020, pp. 237–272.

32. Chang JL. Sliding mode control design for mismatched uncertain systems using output feedback. *Int J Control Autom 2016*; 14(2): 579–586.

33. Tapia A, Bernal M and Fridman L. Nonlinear sliding mode control design: an LMI approach. *Syst Control Lett 2017*; 104: 38–44.

34. Saad W, Sellami A and Garcia G. Robust sliding mode- \( H_{\infty} \) control approach for a class of nonlinear systems affected by unmatched uncertainties using a poly-quadratic Lyapunov function. *Int J Control Autom 2016*; 14(6): 1464–1474.

35. Mondal S, Ghomam J and Saad M. An adaptive full order sliding mode controller for mismatched uncertain systems. *Int J Autom Comput 2017*; 14(2): 191–201.

36. Yang J, Zolotas A, Chen WH, et al. Robust control of nonlinear MAGLEV suspension system with mismatched uncertainties via DOBC approach. *ISA T 2011*; 50(3): 389–396.

37. Pan Y, Yang C, Pan L, et al. Integral sliding mode control: performance, modification, and improvement. *IEEE T Ind Inform 2017*; 14(7): 3087–3096.

38. Brisilla R, Ramana KJV and Rao MM. Output voltage regulation of DC to DC buck converter using integral SMC. In: *Proceedings of the 2019 innovations in power and advanced computing technologies (i-PACT)*, vol. 1, Vellore, India, 22–23 March 2019, pp. 1–5. New York: IEEE.

39. Ferrara A, Incronona GP and Sangiovanni B. Tracking control via switched integral sliding mode with application to robot manipulators. *Control Eng Pract 2019*; 90: 257–266.

40. Li S, Yang J, Chen WH, et al. Disturbance observer-based control: methods and applications. Boca Raton, FL: CRC Press, 2014.

41. Wei X and Guo L. Composite disturbance-observer-based control and terminal sliding mode control for non-linear systems with disturbances. *Int J Control 2009*; 82(6): 1082–1098.

42. Liu RJ, Wu M, Liu GP, et al. Active disturbance rejection control based on an improved equivalent-input-disturbance approach. *IEEE/ASME T Mech 2013*; 18(4): 1410–1413.

43. Han J. The extended state observer of a class of uncertain systems. *Control Decis 1995*; 10(1): 85–88.

44. Davila J, Fridman L and Levant A. Second-order sliding-mode observer for mechanical systems. *IEEE T Automat Contr 2005*; 50(11): 1785–1789.

45. Guo BZ and Wu ZH. Output tracking for a class of nonlinear systems with mismatched uncertainties by active disturbance rejection control. *Syst Control Lett 2017*; 100: 21–31.

46. Huang J, Ri S, Fukuda T, et al. A disturbance observer based sliding mode control for a class of underactuated robotic system with mismatched uncertainties. *IEEE T Automat Contr 2018*; 64(6): 2480–2487.

47. Gao Z. Active disturbance rejection control: a paradigm shift in feedback control system design. In: *Proceedings of the 2006 American control conference*, Minneapolis, MN, 14–16 June 2006, p. 7. New York: IEEE.

48. Ren C, Li X, Yang X, et al. Extended state observer-based sliding mode control of an omnidirectional mobile robot with friction compensation. *IEEE T Ind Electron 2019*; 66(12): 9480–9489.

49. Wang J, Zhao L and Yu L. Adaptive terminal sliding mode control for magnetic levitation systems with enhanced disturbance compensation. *IEEE T Ind Electron Epub ahead of print 26 February 2020*. DOI: 10.1109/TIE.2020.2975487.

50. Li S, Yang J, Chen WH, et al. Generalized extended state observer based control for systems with mismatched uncertainties. *IEEE T Ind Electron 2012*; 59(12): 4792–4802.

51. Pu Z, Yuan R, Yi J, et al. A class of adaptive extended state observers for nonlinear disturbed systems. *IEEE T Ind Electron 2015*; 62(9): 5858–5869.

52. Guo BZ and Zhao ZL. On convergence of non-linear extended state observer for multi-input multi-output systems with uncertainty. *IET Control Theory A 2012*; 6(15): 2375–2386.
53. Shi S, Xu S, Yu X, et al. Robust output-feedback finite-time regulator of systems with mismatched uncertainties bounded by positive functions. *IET Control Theory A* 2017; 11(17): 3107–3114.
54. Wang S, Yu H and Yu J. Robust adaptive tracking control for servo mechanisms with continuous friction compensation. *Control Eng Pract* 2019; 87: 76–82.
55. Menard T, Moulay E and Perruquetti W. A global high-gain finite-time observer. *IEEE T Automat Contr* 2010; 55(6): 1500–1506.
56. Moreno JA. Lyapunov function for Levant’s second order differentiator. In: *Proceedings of the 2012 51st IEEE conference on decision and control (CDC)*, Maui, HI, 10–13 December 2012, pp. 6448–6453. New York: IEEE.
57. Han J. From PID to active disturbance rejection control. *IEEE T Ind Electron* 2009; 56(3): 900–906.
58. Bhat SP and Bernstein DS. Geometric homogeneity with applications to finite-time stability. *Math Control Signal Syst* 2005; 17(2): 101–127.
59. Guo BZ and Zhao ZL. On the convergence of an extended state observer for nonlinear systems with uncertainty. *Syst Control Lett* 2011; 60(6): 420–430.
60. Moreno JA and Osorio M. Strict Lyapunov functions for the super-twisting algorithm. *IEEE T Automat Contr* 2012; 57(4): 1035–1040.
61. Michail K. Optimised configuration of sensing elements for control and fault tolerance applied to an electro-magnetic suspension system. PhD Thesis, © Konstantinos Michail, Loughborough, 2009.
62. Wang J, Shao C and Chen YQ. Fractional order sliding mode control via disturbance observer for a class of fractional order systems with mismatched disturbance. *Mechatronics* 2018; 53: 8–19.
63. Mueller M. Normal form for linear systems with respect to its vector relative degree. *Linear Algebra Appl* 2009; 430(4): 1292–1312.
64. Vadim IU. Survey paper variable structure systems with sliding modes. *IEEE T Automat Contr* 1977; 22(2): 212–222.