Accelerated expansion of the Universe in the Presence of Dark Matter Pressure

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Abstract

Expansion dynamics of the Universe is one of the important subjects in modern cosmology. The dark energy equation of state determines this dynamics so that the Universe is in an accelerating phase. However, the dark matter can also affect the accelerated expansion of the Universe through its equation of state. In the present work, we explore the expansion dynamics of the Universe in the presence of dark matter pressure. In this regard, applying the dark matter equation of state from the observational data related to the rotational curves of galaxies, we calculate the evolution of dark matter density. Moreover, the Hubble parameter, history of scale factor, luminosity distance, and deceleration parameter are studied while the dark matter pressure is taken into account. Our results verify that the dark matter pressure leads to the higher values of the Hubble parameter at each redshift and the expansion of the Universe grows due to the DM pressure.

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I. INTRODUCTION

Type Ia supernova observational data confirm an accelerating phase for the Universe \[1\]. Accelerated expansion of the Universe can be explained considering an energy component named dark energy (DE) in Friedmann equation derived from general relativity. Using the multiwavelength observation of blazars, it is possible to measure the expansion rate of the Universe \[2\]. The capability of different configurations of the space interferometer eLISA to probe the late-time background expansion of the Universe using gravitational wave standard sirens has been studied \[3\]. Different cosmological models have been constrained to understand the expansion dynamics of the Universe from galaxy cluster scales \[4\].

Different aspects of the accelerated expansion of the Universe have been investigated \[5–21\]. The influence of inhomogeneities on the global expansion factor by averaging the Friedmann equation has been calculated \[5\]. The effect of various particles such as massless fermions, gauge bosons, and conformally coupled scalars on the cosmic expansion rate relative to that of the graviton has been explored \[6\]. The vacuum energy of a free quantized field of very low mass may alter the recent expansion of the Universe \[7\]. An empirical evidence relating to the Friedmann equation and the dynamical relation in general relativity between the expansion rate of the Universe and the energy density have been presented \[8\]. The precision of distance-redshift observations indicating the acceleration-deceleration transition and the components and equations of state of the energy density have been studied \[9\]. The acceleration of the expanding Universe can be explained by Gauss-Bonnet gravity with negative Gauss-Bonnet coefficient and without a cosmological constant \[10\]. The second order in perturbation variables the expansion rate of an inhomogeneous Universe and the corrections to the evolution of the expansion rate have been demonstrated \[11\]. A model of interacting DE where the DE density is related by the holographic principle to the Hubble parameter is in agreement with the observational data supporting an accelerating Universe \[12\]. Nonminimal Yang-Mills theory in which the field couples to a function of the scalar curvature can realize both inflation and the late-time accelerated expansion of the Universe \[13\]. Considering a pressureless fluid with a constant bulk viscosity driving the present accelerated expansion of the Universe, a bulk viscous matter-dominated Universe model has been presented \[14\]. Accelerated expansion of the Universe can be driven by traditional matter with positive pressure because of the back-reaction of the gravity field \[15\]. Considering a
non-adiabatic-like accelerated expansion of the Universe in entropic cosmology shows that the increase of the entropy for the simple model is uniform \[16\]. Distinct behaviors of the scalar and vector fields together with the real valued mass gained by the Stueckelberg mechanism lead the Universe to go through the two different accelerated expansion phases with a decelerated expansion phase between them \[17\]. General conditions for the acceleration in the expansion of the Universe and a solution for the Weyl scalar field describing a cosmological model for the present time have been presented \[18\]. Higher dimensional steady state cosmologies with constant volume for which the three dimensional external space is expanding at an accelerated rate but the internal space is contracting have been explored \[19\]. Effective Friedmann equation from the dynamics of group field theory shows the occurrence of an era of accelerated expansion without the need to introduce an inflaton field \[20\]. The evolution solutions of the FRW Universe have been derived by combination of the Friedmann acceleration equation based on the thermodynamics of the Hubble horizon and the evolution equation of the Universe based on the energy balance relation \[21\].

Dark matter (DM) in the Universe alters the accelerated expansion of the Universe \[22\]–\[24\]. A bulk viscous matter-dominated Universe with a pressureless fluid characterizes both the baryon and DM components has been considered to explain the present accelerated expansion of the Universe \[22\]. Applying the Archimedean-type coupling of the DM with DE to study the late-time accelerated expansion confirms that the Archimedean-type coupling provides the Universe evolution to be a quasiperiodic and/or multistage process \[23\]. Studying the dynamical characteristics of a spatially-flat cosmological model in which instead of DE the DM possesses some sort of fluid-like properties indicates that the pressure becomes negative enough, so that the Universe accelerates its expansion \[24\].

However, the DM as one of the significant portion of the Universe can affect the astrophysical systems through its pressure \[25\]–\[39\]. The observations of rotation curves together with the gravitational lensing can determine the equation of state (EOS) of DM \[25\]. The constraints on the EOS of DM have been presented employing CMB, supernovae Ia, and large scale structure data in a modified ΛCDM cosmology \[26\]. The observations of galaxy rotation curves and gravitational lensing give the density and pressure profiles of the galactic fluid \[27\]. A decelerated-accelerated transition considering the present value of the deceleration parameter can occur considering the non-vanishing DM pressure and negative small values of the coupling constant \[28\]. Degeneracy pressure of fermionic DM affects the flat-
top column density profile of clusters of galaxies \[29\]. The non-ideal fluid EOS for the DM halo can be obtained from the observations of gravitational lensing deflection angle \[30\]. Extended theories of DM considering self-interaction, non-extensive thermostatistics, and boson condensation can be explained using the polytropic EOS of DM haloes \[31\]. Pressures related to an EOS parameter of total energy of the same value as for weak fields in solar-relativistic ranges can be the result of the DM dominance in the scalar-field excitations of induced gravity with a Higgs potential \[32\]. Inner slope of halo density profile and the mass and the annihilation cross-section of DM particles into electron-positron pairs are influenced by the pressure from DM annihilation \[33\]. The contribution of accreted DM to the supermassive black hole growth is affected by the DM with non trivial pressure near a supermassive black hole \[34\]. The radial and tangential pressures of anisotropic DM have been studied considering a mixture of two different non-interacting perfect fluids \[35\]. The energy density and the radial pressure of the DM halos have a general r-dependent functional relationship \[36\]. The effective pressure of the DM component, originated from the non-minimal coupling between gravity and DM, reduces the growth of structures at galactic scales \[37\]. The DM EOS obtained using the observational data from the rotation curves of galaxies has a functional dependence universal for all galaxies \[38\]. The DM pressure has also been constrained by the large-scale cosmological observations \[39\]. Since the accelerated expansion of the Universe is influenced by the properties of DM, it is necessary to study the effects of DM pressure on the expansion dynamics of the Universe. Here, we investigate the accelerated expansion of the Universe in the presence of the DM pressure.

II. DARK MATTER DENSITY EVOLUTION IN THE PRESENCE OF DARK MATTER PRESSURE

Starting with a homogeneous and isotropic cosmology, the Friedmann equations are,

$$\dot{a}^2 + kc^2 = \frac{8\pi G}{3} \rho a^2,$$

(1)

$$\dot{a}^2 + kc^2 + 2a\ddot{a} = -\frac{8\pi G}{c^2} Pa^2.$$

(2)

In the above equations, \(a(t)\) is the scale factor and also the total density, \(\rho\), is related to the DM density, \(\rho_{DM}\), and DE density, \(\rho_{DE}\), by \(\rho = \rho_{DM} + \rho_{DE}\). In addition, the total pressure, \(P\),
is expressed in terms of the DM pressure, $P_{DM}$, and DE pressure, $P_{DE}$, as $P = P_{DM} + P_{DE}$. Besides, $k = -1, 0,$ and $1$ for an open, flat, and closed Universe, respectively. With the DM EOS $P_{DM} = 0$ and the DE EOS $P_{DE} = -c^2 \rho_{DE}$ in $\Lambda$CDM model, we know that the combination of Eqs. (1) and (2) leads to the following density evolution for DM,

$$\rho_{DM}(a) = \rho_{DM0} a^{-3}. \quad (3)$$

In Eq. (3), $\rho_{DM0}$ denotes the DM density at the present time. However, in this work, we are interested in the effects of the DM pressure on the DM density evolution as well as the accelerated expansion of the Universe. Therefore, we first calculate the density evolution of DM assuming the DM pressure is not zero, i.e. $P_{DM} \neq 0$. To do this, we multiply Eq. (1) by $a$ and differentiate both its sides with respect to $t$. This gives,

$$\dot{a}(\dot{a}^2 + 2a\ddot{a} + kc^2) = \frac{8\pi G}{3} d \left( \rho a^3 \right). \quad (4)$$

Using Eq. (2), the above equation leads to,

$$\frac{da}{dt} \left( -\frac{8\pi G}{c^2} P a^2 \right) = \frac{8\pi G}{3} d \left( \rho a^3 \right), \quad (5)$$

which this also results in,

$$\frac{d(\rho a^3)}{da} = -\frac{3}{c^2} P a^2. \quad (6)$$

We consider the Eq. (6) for the DM with the EOS, $P_{DM} = P_{DM}(\rho_{DM})$,

$$\frac{d(\rho_{DM} a^3)}{da} = -\frac{3}{c^2} P_{DM} a^2. \quad (7)$$

In this paper, we employ the DM EOS obtained from the rotational curves of galaxies [38]. Applying some simple calculations, Eq. (7) gives,

$$\frac{d\rho_{DM}}{\rho_{DM} + P_{DM}/c^2} = -\frac{3}{a} \frac{da}{a}, \quad (8)$$

which can be integrated as follows,

$$\int_{\rho_{DM0}}^{\rho_{DM}} \frac{d\rho_{DM}}{\rho_{DM} + P_{DM}/c^2} = -3 \int_{a_0}^{a} \frac{da}{a}, \quad (9)$$

in which $a_0 = 1$ is the scale factor at the present time. The left side of Eq. (9) can be calculated after inserting the DM EOS, i.e. $P_{DM} = P_{DM}(\rho_{DM})$. We present the result of
this integration by \( B(\rho_{DM}) \) to emphasis its dependency on the DM density. Therefore, Eq. (9) gives,

\[
B(\rho_{DM}) = \ln(a^{-3}).
\]  

(10)

Solving the above equation for the value of \( \rho_{DM} \) leads to the \( a \) dependency of the function \( \rho_{DM}(a) \). In fact, \( \rho_{DM}(a) \) describes the DM density evolution. Eq. (10) is solved by the fixed point method. In our calculations, the maximum error in the value of \( B_{DM} \) is \( 10^{-7} \). It should be noted that for the case \( P_{DM} = 0 \), the function \( \rho_{DM}(a) \) leads to Eq. (3).

In this work, we apply the DM EOS obtained using observational data of the rotation curves of galaxies [38]. The pseudo-isothermal model results in a mass density profile with the property of regularity at the origin. Using the velocity profile, geometric potentials, and gravitational potential, the DM EOS obtained applying the pseudo-isothermal density profile has the following form,

\[
P_{DM}(\rho_{DM}) = \frac{8p_0}{\pi^2} - \left[ \frac{\arctan(\sqrt{\frac{\rho_0}{\rho_{DM}}})}{\sqrt{\frac{\rho_0}{\rho_{DM}}} - 1} - \frac{1}{2}(\arctan(\sqrt{\frac{\rho_0}{\rho_{DM}}}) - 1)^2 \right],
\]  

(11)

in which \( \rho_{DM} \) and \( P_{DM} \) denote the density and pressure of DM and the free parameters, \( \rho_0 \) and \( p_0 \), are the central density and pressure of galaxies. This EOS has a functional dependence universal for all galaxies with free parameters which are related with the evolution history of the galaxy. This universality makes it possible to describe the DM EOS in other scales, e.g. dark-matter admixed neutron stars [40] with Eq. (11). Here, we suppose that the universality allows the DM EOS to be used in large scale structure in studying the cosmological accelerated expansion. The DM EOS which we have used is related to the galaxy U5750 which is the result of one of the best fit with \( \chi^2_{\text{min}}/\text{d.o.f.} = 0.01 \) [38]. In Fig. 1 we have presented this DM EOS.

Fig. 2 gives the DM density evolution for both cases of zero pressure DM (ZPDM) and non zero pressure DM (NZPDM). The evolution of DM density in NZPDM case differs from the ZPDM one. For lower values of the scale factor, i.e. \( a < 1 \), the existence of the DM pressure leads to larger DM density. This enhancement is more significant at \( a < 0.5 \). Besides, at lower values of the cosmological scale factor, the DM density in the case of NZPDM reduces with scale factor more rapidly than the case of ZPDM. This is while for \( a > 1 \), the density of NZPDM decreases with scale factor similarly to ZPDM one.
FIG. 1: Dark matter EOS related to the galaxy U5750 with the parameters $\rho_0 = 0.31 \text{ GeV/cm}^3$ and $p_0 = 1.1 \times 10^{-8} \text{ GeV/cm}^3$ and $\chi^2_{\text{min}}/d.o.f. = 0.01$, [38]. $\rho_c$ denotes the critical density of the Universe.

FIG. 2: Evolution of DM density, $\rho_{DM}(a)$, versus the cosmological scale factor, $a$, for two cases of zero pressure DM (ZPDM) and non zero pressure DM (NZPDM).
III. DYNAMICAL EXPANSION OF THE UNIVERSE WITH DARK MATTER PRESSURE

Considering a spatially flat universe in which \( k = 0 \) and the Hubble parameter, \( H(a) \equiv \dot{a}/a \), Eq. (1) is written as follows,

\[
H^2(a) = \frac{8\pi G}{3}(\rho_{DM}(a) + \rho_{DE}),
\]

in which \( \rho_{DM}(a) \) is given by Eqs. (3) and (10) for the cases of \( P_{DM} = 0 \) and \( P_{DM} \neq 0 \), respectively. This equation describes the dynamical expansion of the Universe. Fig. 3 shows the Hubble parameter as a function of scale factor in two cases of ZPDM and NZPDM. NZPDM predicts higher values for the Hubble parameter, especially at smaller scale factors. In addition, the rate at which the Hubble parameter decreases with \( a \) is affected by the DM pressure. For \( a < 1 \), the Hubble parameter reduction with scale factor is more considerable when the pressure of DM is taken into account.

Figs. 4-6 present the Hubble parameter versus the redshift, \( z \). The redshift is related to the scale factor by \( 1 + z = a^{-1} \). Our results have been also compared with the observational data from the median \( D4000_n - z \) relations in Fig. 4 from the upper envelope in Fig. 5, and the observational data from Refs. 4 and 42 in Fig. 6. At each redshift the Hubble parameter is higher for the case of NZPDM. Besides, the DM pressure leads
FIG. 4: Hubble parameter versus the redshift, $z$, in two cases of ZPDM and NZPDM and the observational data from the median $D4000_n - z$ relations [1].

to the increase in the growth rate of the Hubble parameter with the redshift. The Hubble parameter is affected by the DM pressure more significantly at higher values of the redshift. Our results for the accelerating expansion of the Universe with the DM pressure also agree with the observational data from the median $D4000_n - z$ relations and the upper envelope [1] and the observational data from Refs. [4] and [42]. Interestingly, for the most observational data, the Hubble parameter is higher than the theoretical result with ZPDM. This difference can be explained by the DM pressure which increases the Hubble parameter at each redshift.

In the following, we study the properties of the Universe with the DM pressure.

A. Scale factor $a$

The history of scale factor, $a(t)$, is given by

$$\int_{1}^{a} \frac{da}{aH(a)} = \int_{t_0}^{t} dt,$$

(13)
FIG. 5: Same as Fig. 4 but for the observational data from the upper envelope [1].

FIG. 6: Same as Fig. 4 but for the observational data from Refs. [4] and [42].
in which \( t \) denotes the cosmic time and also \( t_0 \) shows the cosmic time today. Using the definition \( \tau = H_0(t - t_0) \), Eq. (13) has the following form

\[
\int_1^a \frac{H_0 da}{aH(a)} = \tau, \tag{14}
\]

where \( H(a) \) has been given in Eq. (12). The scale factor, \( a(\tau) \), the first derivative of scale factor, \( da(\tau)/d\tau \), and the second derivative of scale factor, \( d^2a(\tau)/d\tau^2 \), are presented in Figs. 7-9. The increase of the scale factor with the cosmic time is faster if the DM pressure is considered. This effect is more considerable for \( \tau > 0 \), i.e. after the present time. Therefore, the expansion of the Universe grows due to the DM pressure. Considering each cosmic time, the value of \( da(\tau)/d\tau \) is higher when the DM pressure is taken into account. The effects of NZPDM on the \( da(\tau)/d\tau \) are more important when \( |\tau| \) is larger. Moreover, the DM pressure results in the increase of the slope of \( da(\tau)/d\tau \). It can be seen from Fig. 9 that depending on the sign of \( d^2a(\tau)/d\tau^2 \), the effects of NZPDM on this quantity are different. Considering the cosmic times at which \( d^2a(\tau)/d\tau^2 < 0 \), the second derivative of scale factor decreases with the DM pressure leading to more negative values for this quantity. However, for the cosmic times with \( d^2a(\tau)/d\tau^2 > 0 \), the second derivative of scale factor is higher for the
FIG. 8: Same as Fig. 7 but for $da(\tau)/d\tau$.

FIG. 9: Same as Fig. 7 but for $d^2a(\tau)/d\tau^2$. 
case of NZPDM. Fig. 9 also confirms that the DM pressure affects the $d^2a(\tau)/d\tau^2$ more significantly when $\tau < 0$, i.e. the past time, compared to $\tau > 0$.

B. Luminosity distance $d_L$

One of the important parameters in studying the expansion of the Universe which can be compared with the observational results is the luminosity distance, $d_L$. This quantity is calculated as follows [43],

$$d_L(z) = c(1 + z) \int_0^z \frac{dz}{H(z)}.$$  \hspace{1cm} (15)

In addition, considering the same absolute magnitude $M$ for the supernovae, the extinction-corrected distance moduli is as follows [12],

$$\mu(z) = 5 \log_{10}(d_L(z)/\text{Mpc}) + 25.$$  \hspace{1cm} (16)

Figs. 10 and 11 show the $z$ dependency of the luminosity distance and extinction-corrected distance moduli, respectively. The supernova data [44] for the distance moduli are also presented in Fig. 11. The results for the luminosity distance and distance moduli are not significantly affected by the DM pressure. Fig. 11 confirms that our results for the distance moduli in the case of NZPDM agree with the supernova data points.

C. Deceleration parameter $q$

The deceleration parameter, $q$, is defined by

$$q(a) = -\frac{\ddot{a}}{a} \frac{1}{H^2}.$$  \hspace{1cm} (17)

Fig. 12 presents the deceleration parameter as a function of the scale factor, $a$, in two cases of ZPDM and NZPDM. At lower scale factors, the deceleration is higher if the NZPDM is considered. However, for $a > 1$, the DM pressure leads to more negative values for the deceleration parameter which this corresponds to more acceleration. The deceleration parameter is more affected by the DM pressure for $a < 1$. 

FIG. 10: Luminosity distance versus the redshift, $z$, in two cases of ZPDM and NZPDM.

FIG. 11: Distance moduli versus the redshift, $z$, in two cases of ZPDM and NZPDM and the supernova data [44].
IV. SUMMARY AND CONCLUSIONS

Dark matter (DM) equation of state from the observational data of the rotational curves of galaxies has been employed to investigate the accelerated expansion of the Universe in the presence of the DM pressure. The results verify that at lower values of the scale factor, the existence of the DM pressure leads to the larger DM density. The Hubble parameter also has higher values when the DM pressure is considered. Our calculations confirm that the DM pressure results in the increase of the growth rate of the Hubble parameter with the redshift. The growth of the scale factor versus the cosmic time is more significant when the DM pressure is present. In addition, we have shown that the luminosity distance and distance moduli are not considerably influenced by the DM pressure. Besides, our results indicate that the DM pressure affects the deceleration parameter.
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[1] M. Moresco, et al., JCAP 8, 6 (2012).
[2] A. Dominguez and F. Prada, Astrophys. J. Lett. 771, L34 (2013).
[3] N. Tamanini, C. Caprini, E. Barausse, A. Sesana, A. Klein, and A. Petiteau, JCAP 1604, 002 (2016).
[4] D. Wang and X.-H. Meng, Phys. Dark Univ. 18, 30 (2017).
[5] H. Russ, M. H. Soffel, M. Kasai, and G. Borner, Phys. Rev. D 56, 2044 (1997).
[6] N. C. Tsamis and R. P. Woodard, Phys. Lett. B 426, 21 (1998).
[7] L. Parker and A. Raval, Phys. Rev. D 60, 063512 (1999); Erratum-ibid. D 67, 029901 (2003).
[8] S. M. Carroll and M. Kaplinghat, Phys. Rev. D 65, 063507 (2002).
[9] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003).
[10] M. H. Dehghani, Phys. Rev. D 70, 064009 (2004).
[11] E. W. Kolb, S. Matarrese, A. Notari, and A. Riotto, Phys. Rev. D 71, 023524 (2005).
[12] M. S. Berger and H. Shojaei, Phys. Rev. D 74, 043530 (2006).
[13] K. Bamba, S. Nojiri, and S. D. Odintsov, Phys. Rev. D 77, 123532 (2008).
[14] A. Avelino and U. Nucamendi, JCAP 0904, 006 (2009).
[15] A. B. Balakin and H. Dehnen, Phys. Lett. B 681, 113 (2009).
[16] N. Komatsu and S. Kimura, Phys. Rev. D 87, 043531 (2013).
[17] O. Akarsu, M. Arik, N. Katirci, and M. Kavuk, JCAP 07, 009 (2014).
[18] R. Aguila, J. E. M. Aguilar, C. Moreno, and M. Bellini, EPJC 74, 3158 (2014).
[19] O. Akarsu, T. Dereli, and N. Oflaz, Class. Quantum Grav. 32, 215009 (2015).
[20] M. de Cesare and M. Sakellariadou, Phys. Lett. B 764, 49 (2017).
[21] F.-Q. Tu, Y.-X. Chen, B. Sun, and Y.-C. Yang, Phys. Lett. B 784, 411 (2018).
[22] A. Avelino and U. Nucamendi, JCAP 1008, 009 (2010).
[23] A. B. Balakin and V. V. Bochkarev, Phys. Rev. D 83, 024035 (2011).
[24] K. Kleidis and N. K. Spyrou, A and A 576, A23 (2015).
[25] S. Bharadwaj and S. Kar, Phys. Rev. D 68, 023516 (2003).
[26] C. M. Muller, Phys. Rev. D 71, 047302 (2005).
[27] T. Faber and M. Visser, MNRAS 372, 136 (2006).
[28] J. B. Binder and G. M. Kremer, Gen. Rel. Grav. 38, 857 (2006).
[29] T. Nakajima and M. Morikawa, Astrophys. J. 655, 135 (2007).
[30] K.-Y. Su and P. Chen, Phys. Rev. D 79, 128301 (2009).
[31] C. J. Saxton and I. Ferreras, MNRAS 405, 77 (2010).
[32] N. M. Bezares-Roder, H. Nandan, and H. Dehnen, JHEP 1010, 113 (2010).
[33] M. Wechakama and Y. Ascasibar, MNRAS 413, 1991 (2011).
[34] F. S. Guzman and F. D. Lora-Clavijo, MNRAS 415, 225 (2011).
[35] T. Harko and F. S. N. Lobo, Phys. Rev. D 83, 124051 (2011).
[36] T. Harko and F. S. N. Lobo, Astropart. Phys. 35, 547 (2012).
[37] D. Bettoni, V. Pettorino, S. Liberati, and C. Baccigalupi, JCAP 07, 027 (2012).
[38] J. Barranco, A. Bernal, and D. Nunez, MNRAS 449, 403 (2015).
[39] M. Kunz, S. Nesseris, and I. Sawicki, Phys. Rev. D 94, 023510 (2016).
[40] Z. Rezaei, Astrophys. J. 835, 33 (2017).
[41] S. Weinberg, Cosmology, Oxford University Press, (2008).
[42] Y. Chen et al., JCAP 02, 010 (2015).
[43] A. G. Riess et al., Astrophys. J. 659, 98 (2007).
[44] A. G. Riess et al., Astrophys. J. 607, 665 (2004).