Pomeron Flux Renormalization: 
A scaling Law in Diffraction

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Abstract. The pomeron flux renormalization hypothesis is reviewed and presented as a scaling law in diffraction. Predictions for soft and hard diffraction based on pomeron flux scaling are compared with experimental results.

STANDARD POMERON FLUX

The cross section for hadron dissociation on protons, \( hp \to Xp \), at large \( x_F \equiv p_{\|}^* / 2 \sqrt{s} \), where \( p_{\|}^* \) is the \( z \)-beam-component of the (leading) proton in the final state, is dominated by pomeron exchange [1]. In Regge theory, the pomeron contribution is given by the triple-pomeron amplitude

\[
\frac{d^2 \sigma_{sd}}{d \xi dt} = \frac{\beta_{pp}^2(t)}{16 \pi} \xi^{1-2\alpha_F(t)} \left[ \frac{\beta_{hh}(0) g(t)}{s'} \right] \left( \frac{s'}{s_0} \right)^{\alpha_F(0)-1}
\]

(1)

where \( \alpha_F(t) = \alpha_F(0) + \alpha' t = (1 + \epsilon) + \alpha' t \) is the pomeron trajectory, \( \beta_{pp}^2(t) \) is the coupling of the pomeron to the proton, \( g(t) \) is the triple-pomeron coupling, \( s' = M_X^2 \) is the \( IP - p \) center of mass energy squared, \( \xi \equiv 1 - x_F = s'/s = M_X^2 / s \) is the fraction of the momentum of the proton carried by the pomeron, and \( s_0 \) is an energy scale parameter not determined by the theory and usually set to \( 1 \text{ GeV}^2 \) (the hadron mass scale).

The term in brackets in (1) has the form of the \( IP - p \) total cross section. Thus, the process \( hp \to Xp \) can be viewed as a flux of pomerons emitted by the proton interacting with the hadron \( h \). The pomeron “flux factor” is represented by

\[
\begin{align*}
\alpha_F(t) &= \alpha_F(0) + \alpha' t = (1 + \epsilon) + \alpha' t \\
\beta_{pp}^2(t) &= \beta_{pp}(0) / 16 \pi \\
F(t) &= F(t)
\end{align*}
\]

where \( K \equiv \beta_{pp}^2(0) / 16 \pi \) and \( F(t) \) is the proton form factor. Ingelman and Schlein (IS) [2] proposed using this standard pomeron flux factor in calculating hard single diffraction dissociation cross sections. In such calculations, one assumes that the

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pomeron has a partonic structure and lets the partons of a $IP$ coming from the proton interact with the partons in $h$.

There are two problems with the IS method in using the standard pomeron flux to calculate hard diffraction rates:

1. The normalization of the standard flux depends on the energy scale $s_0$ through the total cross section equation, $\sigma_T^{pp} = \beta_0^2 \cdot \left(s/s_0\right)^\epsilon$. Since the scale $s_0$ is not determined by the theory, the value of $\beta_0^2$ and therefore that of the standard flux normalization, is arbitrary.

2. For any given value of the energy scale $s_0$, the diffractive cross section grows as $s^{2\epsilon}$, overtaking at high energies the total cross section, which grows as $s^\epsilon$, in violation of unitarity [3].

It is well known that the Regge theory $\sim s^\epsilon$ dependence of $\sigma_T(s)$ itself violates the unitarity based Froissart bound, which states that the total cross section cannot rise faster than $\sim \ln^2 s$. Unitarity is also violated by the $s$-dependence of the ratio $\sigma_{el}/\sigma_T \sim s^\epsilon$, which eventually exceeds the black disc bound of one half ($\sigma_{el} \leq \frac{1}{2} \sigma_T$), as well as by the $s$-dependence of the $b = 0$ value of the elastic scattering amplitude in impact parameter space, which has already reached a value close to the maximum allowed by unitarity at $\sqrt{s} = 1.8$ TeV [4]. However, for both the elastic and total cross sections, unitarization can be achieved by taking into account rescattering effects using the eikonal formalism [5,6]. Attempts to introduce rescattering in the diffractive amplitude by eikonalization [5] or by including cuts [7,8] have met with moderate success. Through such efforts it has become clear that these “shad-owing” or “screening” corrections affect mainly the normalization of the diffractive amplitude, leaving the form of the $M^2$ dependence almost unchanged. This feature is clearly present in the data, as demonstrated by the CDF Collaboration [4] in comparing their measured diffractive differential $\bar{p}p$ cross sections at $\sqrt{s} = 546$ and 1800 GeV with $pp$ cross sections at $\sqrt{s} = 20$ GeV.

Motivated by these theoretical results and by the trend observed in the data, a phenomenological approach to unitarizing the diffractive amplitude was proposed [3] based on “renormalizing” the pomeron flux by requiring its integral over all available $\xi$ and $t$ to saturate at unity. Such a normalization, which corresponds to a maximum of one pomeron per proton, leads to interpreting the pomeron flux as a probability density simply describing the $\xi$ and $t$ distributions of the exchanged pomeron in a diffractive process.

**RENORMALIZED POMERON FLUX**

The renormalization of the pomeron flux is based on a hypothesis, rather than on a calculation of unitarity corrections, and therefore can be stated as an axiom:

| The pomeron flux integrated over all phase space saturates at unity. |

Mathematically, the renormalized pomeron flux is given by
\[ f_N(\xi, t) = N^{-1}(\xi_{\text{min}}) \cdot f_{F/p}(\xi, t) \]  

(3)

The renormalization factor \( N(\xi_{\text{min}}) \) is the integral of the flux

\[ N(\xi_{\text{min}}) = \int_{\xi_{\text{min}}}^{0.1} \int_{t=-\infty}^{t=0} f_{F/p}(\xi, t) d\xi dt \]  

(4)

where the upper limit of the integration over \( \xi \) has been taken to be \( \xi_{\text{max}} = 0.1 \) (the coherence limit [1]).

The renormalized flux overcomes the two problems of the standard flux:

1. The normalization is no longer arbitrary, since the energy scale factor \( s_0 \) cancels out in dividing the standard flux by its integral.

2. The diffractive cross section now grows as

\[ \sigma_{sd} \sim \int_{\xi} \int_{t} (s\xi)^{\epsilon} f_N(\xi, t) d\xi dt \sim s^{\epsilon} \cdot \langle \xi^{\epsilon} \rangle_{f_N} \quad s \to \infty \Rightarrow \text{constant} \]  

(5)

and thus respects the unitarity bound.

The renormalization factor is a function of \( \xi_{\text{min}} \), which is process dependent. Thus, conventional factorization breaks down. The scaling of the pomeron flux to its integral can be viewed as

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which unitarizes the diffractive amplitude at the expense of factorization.

**COMPARISON OF RENORMALIZED FLUX PREDICTIONS WITH DATA**

Predictions made using the renormalized pomeron flux have been compared with data for both soft [3,9] and hard [3,10–15] diffraction. In this section we summarize briefly the results of such comparisons.

**Soft Diffraction**

- The renormalized flux prediction of the \( s \)-dependence of the total \( pp/\bar{p}p \) single diffractive cross section is in excellent agreement with the data [3].

- The differential cross section \( d^2\sigma_{sd}/dM_X^2 dt \big|_{t=0} \) for \( pp/\bar{p}p \) is independent of \( s \) and behaves as \( \sim 1/(M_X^2)^{1+\epsilon} \) [9]. This scaling behavior, which holds over six orders of magnitude, is predicted by the renormalized flux [9] and disagrees with the \( \sim s^{2\epsilon} \) standard flux expectation.
Hard Diffraction

Hard diffraction has been studied at HERA and at $\bar{p}p$ colliders. In this section we discuss results on the pomeron structure obtained at HERA and at the Tevatron and compare measured diffractive production rates with predictions based on the standard and renormalized pomeron flux.

Results from HERA

At HERA, both the H1 and ZEUS Collaborations used deep inelastic scattering (DIS) to measure the "diffractive structure function" of the proton, $F_{2}^{D(3)}(Q^2, \beta, \xi)$, where $\beta$ is the fraction of the momentum of the pomeron taken by the struck quark. Both experiments found the form

$$F_{2}^{D(3)}(Q^2, \beta, \xi) = \frac{1}{\xi^{1+n}} \cdot A(Q^2, \beta)$$

in which the variable $\xi$ factorizes out into an expression reminiscent of the pomeron flux factor. Therefore, it appeared reasonable to consider the term $A(Q^2, \beta)$ as being proportional to the pomeron structure function $F_{2}^{P}(Q^2, \beta)$. This term was found to be rather flat in $\beta$, suggesting that the pomeron has a hard quark structure. For a fixed $\beta$, $A(Q^2, \beta)$ increases with $Q^2$. By interpreting the $Q^2$ dependence to be due to scaling violations, the H1 Collaboration extracted the gluon fraction of the pomeron using the DGLAP evolution equations in a QCD analysis of $F_{2}^{D(3)}(Q^2, \beta, \xi)$. The ZEUS Collaboration determined the gluon fraction by combining information from diffractive DIS, which is sensitive mainly to the quark component of the pomeron, and diffractive dijet photoproduction, which is sensitive both to the quark and gluon contents. Both experiments agree that the pomeron structure is hard and consists of gluons and quarks in a ratio of approximately $3 \div 1$. In both cases, the extracted gluon fraction does not depend on the pomeron flux normalization.

Results from the Tevatron

Both the CDF and DØ Collaborations have reported that the jet $E_T$ distributions from non-diffractive (ND), single diffractive (SD) and double pomeron exchange (DPE) dijet events have approximately the same shape [14–16]. Since in going from ND to SD or from SD to DPE a nucleon of momentum $p$ is replaced by a pomeron of momentum $p\xi$, the similarity of the $E_T$ spectra suggests that the pomeron structure must be harder than the structure of the nucleon by a factor of $\sim 1/\xi$. Assuming a hard pomeron structure, the CDF Collaboration determined the gluon fraction of the pomeron to be $f_g = 0.7 \pm 0.2$ by comparing the measured rate of diffractive $W$ production, which is sensitive to the quark content of the
pomeron, with the rate for diffractive dijet production, which depends on both the quark and gluon contents [13]. These results, which are independent of the pomeron flux normalization, agree with the results obtained at HERA.

For a hard pomeron structure with \( f_g = 0.7 \) and \( f_q = 0.3 \), the measured \( W \) and dijet rates are smaller than the rates calculated using the standard flux by a factor \( D = 0.18 \pm 0.04 \). This flux “discrepancy” factor is consistent with the pomeron flux renormalization expectation [3,10].

The CDF Collaboration also measured the rate for DPE dijets and compared it with the rates for SD and ND dijets and with calculations using the standard pomeron flux [15]. To obtain the measured DPE/SD ratio, the standard flux in DPE must be multiplied by the factor \( D \) for both the proton and antiproton. This result supports the hypothesis that the suppression factor, relative to the standard flux calculations, is associated with the flux, rather than with “screening corrections” as proposed by other authors [5,7,8].

From HERA to the Tevatron

The rate for diffractive \( W \) production at the Tevatron can be calculated directly from \( F_2^{D(3)}(Q^2, \beta, \xi) \) [10,17]. Using conventional factorization, the expected SD to ND ratio for \( W \) production is 6.7\% [10], while by scaling the normalization of the \( 1/\xi^{1+n} \) term in (6) by the ratio of its integral at HERA (\( \xi_{\text{min}} = Q^2/\beta s \)) to its integral at the Tevatron (\( \xi_{\text{min}} = M_0^2/\beta s \), with \( M_0^2 = 1.5 \text{ GeV}^2 \)) the prediction becomes 1.24\%, in agreement with the data.

CONCLUSION

We have reviewed the pomeron flux renormalization hypothesis and compared expectations for renormalized soft and hard diffraction rates with available experimental results. In all cases considered, soft and hard, the renormalized flux predictions are found to be in excellent agreement with the data. The renormalization procedure consists in simply scaling the standard pomeron flux to its integral over all available phase space. This integral is process dependent and therefore conventional factorization breaks down. Thus, the renormalization of the pomeron flux can be viewed as a scaling law in diffraction, which unitarizes the diffractive amplitude at the expense of conventional factorization.

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