Parameters optimization design of tape spring considering fatigue performance

Xuesong Shi, Hongling Ye and Bowen Li
Institute for Advanced Mechanics in Engineering, College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, 100124

1 Corresponding author, Tel./Fax: +86-13520355438, E-mail: yehongl@bjut.Edu.cn

Abstract. As a crucial part of micro-satellite, the spatial deployable mechanism has the direct effects on satellite performance and success or failure of flight mission. With requirements of lightless and low cost for satellite, the tape spring hinges are widely used for their simple structural form, high reliability of deployment, high-driving performance and accurate positioning in deployable mechanism. Strain energy stored in the folding process of tape spring hinge is transformed into kinetic energy to achieve the deployment of solar panels. When in full deployment, the tape spring hinge can realize self-locking based on its critical moment. Tape spring is the crucial component of a novel space deployable structure that applied to solar panels. Its full deployment determines the normal function of satellite in space. Mechanical properties, steady-state moment and fatigue life, are observed in folding and deploying processes, which are affected by geometric parameters. Influences of thickness, subtended angle, radius and length on the fatigue life of tape spring are carried out to through numerical simulations so as to ensure a multi folding and deploying cycles for tape spring hinge. Results show that thickness has the greatest influence on the fatigue life, subtended angle and radius are second, and the influence of length on fatigue life is relatively small. On the basis, the optimal model is established based on response surface methodology with the thickness and radius as design variables, steady moment as constraint, and fatigue life as optimization objective. By using genetic algorithm to solve the model, optimal design is obtained within the constraint, and then the optimal design is remodeled and simulated, and the obtained results indicate the accuracy of the optimal design. This research could be a reference for the stability design of space deployable structures.

1. Introduction
As the development of the aerospace industry, it is an important research project that carrying large structures in the limited confined spaces of launch vehicles and ensuring that they can be deployed and remain locked in subsequent orbits for space-deployable structures. A novel spatial deployment structure [1-2] formed by the tape spring that is a thin-walled, open cylindrical structure, through various combinations. This advanced tape-spring has received extensive attention since it can be elastically folded and automatically deployed through the release of stored energy. Domestic and foreign scholars have analysed and studied the mechanical property of tape spring.

Pellegrino et al [1], first conduct a preliminary study of the tape spring, and investigated the bending process of single tape spring and have an experimental verification. Seffen et al. [2-3], analyze the difference of mechanical properties such as critical moment and steady moment in forward and reverse
bending of tape spring by analytical, numerical, and experimental methods. Soykasap et al \cite{4}, analyze the mechanical properties in deploying process of four different types of tape-spring’s deployment structure and fitted an empirical formula. Kwok et al \cite{5}, use theoretical derivation, numerical simulation, experimental testing to analyze the influence of geometric parameters on the mechanical properties of a tape spring. Wang et al \cite{6}, investigate the bending process of single tape spring and tape spring hinge, and analyze the influence of thickness and length of the tape spring on steady moment and maximum strain energy. Khan et al \cite{7}, experimentally analyzed the dynamic deploying of tape spring that made of three-layer carbon fiber reinforced braided composites, and compared the experimental measurement with the finite element model. In summary, it can be seen that research on the spatial deployable structures is mostly focused on its mechanical properties, while the limitations of extant research are that few attention is paid to structural fatigue. Practice shows that under normal service conditions, the destruction of mechanical parts or structures is mostly caused by fatigue. Tape spring is a core components in spatial deployable structures that applied to the solar panel of the satellite, if the tape spring cannot deploy steadily, the whole satellite will not operate normally. Therefore, it is very important to study the fatigue performance of tape spring.

In this research, we take tape spring as the research object, and carry out the numerical simulation analysis for the fatigue performance of tape spring. The influence of the length, thickness, subtended angle and radius on the fatigue performance of the tape spring are studied. In addition, the optimal model with maximum steady moment subject to minimum fatigue life limit is established with the thickness and radius of a tape-spring as design variables. Response Surface Method (RSM) is used to define the explicit relationship between responses and design variables. The Genetic Algorithm (GA) is employed to solve the optimization problem.

2. The Fatigue properties analysis of tape spring

2.1. Geometry and material definition
Tape-spring is a thin-walled, open cylindrical structure, as shown in figure 1. The geometric parameters include length \( L \), radius \( R \), subtended angle \( \theta \), thin shell thickness \( T \). The geometric parameters of the tape spring studied in this paper are \( L=110\text{mm}, \ R=16\text{mm}, \ \theta=90^\circ, \ T=0.2\text{mm} \). The material is spring steel, Elastic modulus \( E=210\text{GPa} \) and Poisson's ratio \( v=0.3 \). The S-N curve is shown in figure 2.

![Figure 1. Geometrical model.](image1)

![Figure 2. S-N curve.](image2)

2.2. Mechanical property analysis of tape spring
In this paper, the finite element software Abaqus/Standard is used to build a geometric and finite element models of tape spring. The folding model of tape spring is shown in figure 3. The folding process of the tape spring is achieved by applying a rotation angle at both ends of the tape spring. The figure 3 (a) is forward folding of tape spring, figure 3 (b) is reverse folding of tape spring.
The main mechanical properties of tape spring include critical moment, steady moment and maximum strain energy. Critical moment represents the locking ability of tape spring in space, while steady moment and maximum strain energy represent the deploying ability. Figure 4 show variation of the strain energy and bending moment of the tape spring with rotation angle during the folding process. Figure 4(a) shows the variation of the bending moment with the rotation angle in the forward folding and reverse folding. The peak value of the bending moment is the critical moment, and the bending moment value after the folding is the steady moment. It can be seen from figure 4(a) that critical moment and steady moment in the forward folding are smaller than the reverse folding; figure 4(b) shows the variation of the strain energy with the rotation angle. The strain energy at the completion of the folding is the maximum strain energy. It is found from figure 4(b) that maximum strain energy in forward folding is smaller than that in the reverse folding.

2.3. Fatigue analysis of tape spring

We take once folding and deploying process of tape spring as a cycle, and use the principal stress method to analyze the fatigue life of tape spring. The fatigue analysis results are shown in figure 5 and figure 6. Figure 5 is fatigue life cloud contour of tape spring in forward folding, and figure 6 is the fatigue life cloud contour of tape spring in reverse folding.

It can be seen from the fatigue life cloud contour that when the tape spring in forward folding, the minimum life of tape spring is 66 times, and the minimum life region is at the edge of the middle region of tape spring; when the tape spring in reverse folding, the minimum life of tape spring is 133 times, and the minimum life region at the whole middle section of tape spring. It can be seen that there are obvious differences in the fatigue life of tape spring in forward folding and reverse folding. So we will...
analyze the influence of parameters on the fatigue life of tape spring in forward folding and reverse folding respectively.

**Figure 5.** Fatigue life cloud map in forward folding

**Figure 6.** Fatigue life cloud map in reverse folding

### 3. Parameters effects analysis of fatigue life

In order to study the main factors affecting the fatigue life of tape spring, this paper explores the influence of geometric parameters on the fatigue life of tape spring by changing the geometric parameters. The geometric parameters of tape spring include length, thickness, subtended angle and radius. The length of tape spring is between 80 mm and 140 mm, thickness is between 0.05 mm and 0.35 mm, subtended angle is between 60° and 120° and the radius is between 10 mm and 22 mm.

#### 3.1. Length effect

The effects of tape spring length on the fatigue life are investigated in forward and reverse folding. The variation trend of the fatigue life of tape spring with the length is shown in figure 7 and figure 8.

**Figure 7.** Variation tendencies of fatigue life for different length in forward folding

**Figure 8.** Variation tendencies of fatigue life for different length in reverse folding

As can be seen from figure 7 and figure 8, in forward folding, the fatigue life of tape spring decreases with the increase of length, and then continues to decrease after a slight rise within the range of 100mm-130mm. When tape spring is reverse folding, the fatigue life increases with the increase of length, and then continues to increase after a slight decrease within the 110mm-120mm range. Form figure 7 and figure 8, the influence trend of length of tape spring on fatigue life is opposite variations in forward and reverse folding. In forward folding, length of tape spring is negatively correlated with fatigue life as a whole. While in reverse folding, length is positively correlated with fatigue life as a whole.

#### 3.2. Thickness effect
The effects of tape spring thickness on the fatigue life are investigated in forward and reverse folding. The variation trend of the fatigue life of tape spring with thickness is shown in figure 9 and figure 10.

![Figure 9](image1.png)  ![Figure 10](image2.png)

**Figure 9.** Variation tendencies of fatigue life for different thickness in forward folding  
**Figure 10.** Variation tendencies of fatigue life for different thickness in reverse folding

As can be seen from figure 9 and figure 10, in forward folding, the fatigue life decrease sharply with the increase of the thickness at first, and then slowly decrease with the increase of the thickness; when tape spring is reverse folding, the fatigue life increases with increase of thickness at first, and then decreases with increase of thickness. Form figure 9 and figure 10, with the increase of thickness, the influence of thickness on fatigue life gradually decreases.

3.3. **Subtended angle effect**

The effects of subtended angle on the fatigue life are investigated in forward and reverse folding. The variation trend of the fatigue life of tape spring with subtended angle is shown in figure 11 and figure 12.

![Figure 11](image3.png)  ![Figure 12](image4.png)

**Figure 11.** Variation tendencies of fatigue life for different subtended angle in forward folding  
**Figure 12.** Variation tendencies of fatigue life for different subtended angle in reverse folding

It can be seen from figure 11 and figure 12, when tape spring is forward folding, the fatigue life decreases with the subtended angle increases, and then the fatigue life have a rise at the subtended angle range of 90°-110°, after that fatigue life continue decreases with increase of subtended angle; In forward folding, the fatigue life decreases with the increase of subtended angle, and the decreasing trend is stable and linear. By comparing and observing figure 11 and figure 12 it can be found that when tape spring in forward and reverse folding, the influence of subtended angle on the fatigue life is obviously different. In forward folding, the influence of subtended angle on fatigue life is fluctuant, while in the reverse folding, the influence of subtend angle on fatigue life is relatively stable.
3.4. Radius effect

The effects of radius on the fatigue life are investigated in forward and reverse folding. The variation trend of the fatigue life of tape spring with radius is shown in figure 13 and figure 14.

Figure 13. Variation tendencies of fatigue life for different radius in forward folding

Figure 14. Variation tendencies of fatigue life for different radius in reverse folding

It can be seen from figure 13 and figure 14, in forward folding, the fatigue life increases with the increase of the radius, and the increasing speed is nonlinear and gradually increases, when the radius reaches 20mm, the increasing speed becomes slower; when tape spring is reverse folding, the fatigue life increases with the increase of the radius when the radius is less than 16mm, after radius reaches 16 mm, the fatigue life decreases with the increase of the radius. Form figure 13 and figure 14, the influence of radius on fatigue life has obviously different at forward and reverse folding; before R=16 mm, both forward and reverse folding, fatigue life is increase with the increase of the radius. After the radius reaches 16 mm, the fatigue life increases with increase of radius in forward folding, while decreases with increase of radius in reverse folding.

4. Optimization investigations

4.1. Establishment of optimization model

In order to obtain the tape spring that satisfies the mechanical properties of tape spring in service and has a long enough life, here fatigue life is taken to be a constraint on the optimization problem, the steady moment as the objective. Thickness and radius of tape spring are used as design variables. The optimization problem for tape spring is mathematically represented by (1);

\[
\begin{align*}
\text{find} & \quad x \in \mathbb{E}^N \\
\text{make} & \quad sm \rightarrow \text{max} \\
\text{s.t.} & \quad \underline{life} \leq Life \leq \overline{life} \\
& \quad x_i \leq x_i \leq \overline{x}_i \quad (i = 1, \ldots, n)
\end{align*}
\]

where \( x \) is design variables vector, including thickness and radius, \( sm \) is steady moment of tape spring in the folding process, \( Life \) is fatigue life of tape spring; \( \underline{life}, \overline{life} \) is lower and upper bounds of Fatigue life, \( \underline{x}_i, \overline{x}_i \) is lower and upper bounds of design variables.

4.2. Response surface methodology

Response Surface Methodology (RSM) is a collection of mathematical and statistical techniques and applied widely in engineering optimization process, which is used to obtain the relationship between the input and output to build an explicit expression for design variables and objects. An explicit cubic polynomial is selected here to represent the implicit function of the independent variables.
In this paper, we study the parameter optimization design of the tape spring that in reverse folding. The RSM model is constructed on design variable $T$ ranging from 0.1 mm to 0.2 mm, $R$ ranging from 10 mm to 20 mm, the length of tape spring is 110 mm, $\theta$ is 90°. 25 sample points are collected by Full Factorial design. Through numerical simulation the test points and responses show in Table 1, and the fatigue life of the tape spring is rounded downward to an integer.

| Item | Radius (mm) | Thickness (mm) | Steady Moment(N-mm) | Fatigue Life |
|------|-------------|----------------|---------------------|--------------|
| 1    | 10          | 0.1            | 36.66               | 205          |
| 2    | 12.5        | 0.1            | 38.02               | 268          |
| 3    | 15          | 0.1            | 41.85               | 301          |
| 4    | 17.5        | 0.1            | 47.59               | 223          |
| 5    | 20          | 0.1            | 54.23               | 150          |
| 6    | 10          | 0.125          | 70.93               | 183          |
| 7    | 12.5        | 0.125          | 73.01               | 187          |
| 8    | 15          | 0.125          | 79.32               | 213          |
| 9    | 17.5        | 0.125          | 89.27               | 203          |
| 10   | 20          | 0.125          | 101.34              | 155          |
| 11   | 10          | 0.15           | 121.56              | 103          |
| 12   | 12.5        | 0.15           | 124.46              | 136          |
| 13   | 15          | 0.15           | 133.93              | 162          |
| 14   | 17.5        | 0.15           | 149.63              | 154          |
| 15   | 20          | 0.15           | 169.14              | 145          |
| 16   | 10          | 0.175          | 191.52              | 84           |
| 17   | 12.5        | 0.175          | 195.99              | 109          |
| 18   | 15          | 0.175          | 209.03              | 130          |
| 19   | 17.5        | 0.175          | 231.96              | 127          |
| 20   | 20          | 0.175          | 261.09              | 117          |
| 21   | 10          | 0.2            | 283.97              | 70           |
| 22   | 12.5        | 0.2            | 290.22              | 88           |
| 23   | 15          | 0.2            | 307.73              | 130          |
| 24   | 17.5        | 0.2            | 339.55              | 104          |
| 25   | 20          | 0.2            | 380.66              | 97           |

The response surface functions of steady moment and fatigue life of tape spring with respect to thickness and radius are obtained by Least Square Method. The response surface functions are as follows.

$$sm(R,T) = -0.0114R^3 + 8.1675R^2T + 374.744RT^2 + 34456.92T^3 - 0.2084R^2$$
$$-278.405RT - 4211.44T^2 + 15.6336R + 2168.955T - 137.94$$ (2)

$$Life(R,T) = -0.028R^3 + 26.5376R^2\theta - 1924R^2T + 94910RT^2 - 4.6734T^3 - 126.586R^2$$
$$-5485.75T^2 + 105.4824R - 2060.56T - 117.219$$ (3)

To evaluate the accuracy of the fitted functions, the complex correlation coefficient ($R^2$) and the modified complex correlation coefficient ($R_{adj}^2$) are introduced. The fitted function is deemed more...
accurate when each coefficient is closer to one. In this paper $R^2$ and $R_{adj}^2$ of steady moment function are all equal to one; $R^2$ and $R_{adj}^2$ of fatigue life function are all equal to 0.955. So, the response surface functions meet the accuracy requirements.

4.3. **Optimal solution**

We take fatigue life greater than 150 cycles as constraint, and Genetic Algorithm is used to solve the optimization model. In order to ensure the correctness of the optimization results, the optimization results are remodelled and analysed. The results of tape spring optimization and reconstruction model analysis are listed in Table 2.

| Item                  | Radius (mm) | Thickness (mm) | Steady Moment (N·mm) | Fatigue Life |
|-----------------------|-------------|----------------|-----------------------|--------------|
| Optimization Result   | 17.28       | 0.16           | 179.4728              | 150          |
| FEM verification      | 17.28       | 0.16           | 180.47                | 145          |
| Tolerance             |             |                | 0.32%                 | -3.64%       |

From the table, we can see that there is a relative error between the optimization result and finite element verification, which is within 5% meeting the engineering requirements. So the optimization model is reliable.

In this paper, the fatigue life of the finite element verification is slightly different from the constraint value of the optimization model. Therefore, in engineering optimization, the fatigue life constraint should be appropriately larger than the engineering requirements to avoid losses.

5. **Conclusions**

In this paper, the parametric effects and optimization design of tape spring are studied considering fatigue life, and the following conclusions are drawn:

1). Tape spring has different fatigue life at forward folding and reverse folding, and its fatigue life cloud contour has obvious difference. When tape spring is forward folding, the minimum area of fatigue life is located on both edges of the middle region of tape spring, while when tape spring folded reversely, the minimum area of fatigue life is located in the whole middle region of tape spring.

2). With the increase of length, fatigue life of tape spring decreases in forward folding, but increases in reverse folding. And when thickness increase, fatigue life decreases in both forward folding and reverse folding. With the increase of subtended angle, in forward folding the fatigue life is nonlinear variation; in reverse folding fatigue life decreases. With the increase of radius, the fatigue life increases when in forward folding and decreases gradually when in reverse folding.

3). RSM is used to construct response surface functions of steady moment and fatigue life. Optimal model is established with thickness and radius as design variables, with steady moment as optimization objective, and fatigue life as constraint. Finally, the obtained optimal results of tape spring with maximum fatigue life satisfy the requirements of steady moment constraint.

6. **References**

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