Adiabatic passage for three-dimensional entanglement generation through quantum Zeno dynamics

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Abstract: We propose an adiabatic passage approach to generate two atoms three-dimensional entanglement with the help of quantum Zeno dynamics in a time-dependent interacting field. The atoms are trapped in two spatially separated cavities connected by a fiber, so that the individual addressing is needless. Because the scheme is based on the resonant interaction, the time required to generate entanglement is greatly shortened. Since the fields remain in vacuum state and all the atoms are in the ground states, the losses due to the excitation of photons and the spontaneous transition of atoms are suppressed efficiently compared with the dispersive protocols. Numerical simulation results show that the scheme is robust against the decoherences caused by the cavity decay and atomic spontaneous emission. Additionally, the scheme can be generalized to generate \( N \)-atom three-dimensional entanglement and high-dimensional entanglement for two spatially separated atoms.

Keywords: quantum Zeno dynamics · Adiabatic passage · three-dimensional entanglement

Quantum entanglement, an interesting and attractive phenomenon in quantum mechanics, plays a significant role not only in testing quantum nonlocality, but also in a variety of quantum information tasks [1–8], such as quantum computing [9–11], teleportation [4], cryptography [1], precision measurements [12] and so on. Recently, high-dimensional entanglement is becoming more and more important since they are more secure than qubit systems, especially in the aspect of quantum key distribution. Besides, it has been demonstrated that violations of local realism by two entangled high-dimensional systems are stronger than
that by two-dimensional systems \[13\]. A lot of works have been done in generation of high-dimensional entanglement. For example, Wu et al. proposed a scheme for generating a multiparticle three-dimensional entanglement by appropriately adiabatic evolutions \[14\]. Li and Huang deterministically generated a three-dimensional entanglement via quantum Zeno dynamics(QZD) \[15\]. Chen et al. proposed a scheme to prepare three-dimensional entanglement state between a single atom and a Bose-Einstein condensate (BEC) via stimulated Raman adiabatic passage (STIRAP) technique \[16\]. In experiment, two schemes have been put forward to generate high-dimensional entanglement by the means of the spatial modes of the electromagnetic field carrying orbital angular momentum \[17,18\].

In order to realize the entanglement generation or population transfer in a quantum system with time-dependent interacting field, many schemes have been put forward. Such as π pulses, composite pulses, rapid adiabatic passage(RAP), stimulated Raman adiabatic passage, and their variants \[19–21\]. STIRAP is widely used in time-dependent interacting field because of the robustness for variations in the experimental parameters. But it usually requires a relatively long interaction time, so that the decoherence would destroy the intended dynamics, and finally lead to an error result. Therefore, reducing the time of dynamics towards the perfect final outcome is necessary and perhaps the most effective method to essentially fight against the dissipation which comes from noise or losses accumulated during the operational processes. Rencently, various schemes have been explored theoretically and experimentally to construct shortcuts for adiabatic passage \[22–29\].

On the other hand, the quantum Zeno effect is an interesting phenomenon in quantum mechanics. It stems from general features of the Schrödinger equation that yield quadratic behavior of the survival probability at short time \[30,31\]. The quantum Zeno effect, which has been tested in many experiments, is the inhibition of transitions between quantum states by frequent measurements \[32,33\]. Recent studies \[34–36\] show that a quantum Zeno evolution will evolve away from its initial state, but it remains in the Zeno subspace defined by the measurements \[31,34\] via frequently projecting onto a multidimensional subspace. This is known as QZD. Suppose that a dynamical evolution of a system can be governed by the Hamiltonian \(H_K = H_{\text{obs}} + K H_{\text{meas}}\), where \(H_{\text{obs}}\) is the Hamiltonian of the investigated quantum system and the \(H_{\text{meas}}\) is regarded as an additional interaction Hamiltonian performing the measurement, while \(K\) is a coupling constant. In the limit \(K \to \infty\), the system is governed by the evolution operator \(U(t) = \exp[-it \sum_n (K \lambda_n P_n + P_n H_{\text{obs}} P_n)]\), which is
an important basis for our following work, with $P_n$ is the eigenprojections of $H_{\text{meas}}$ with eigenvalues $\lambda_n (H_{\text{meas}} = \sum_n \lambda_n P_n)$.

In this paper, we present an effective scheme to construct an adiabatic passage for three-dimensional entanglement generation between atoms motivated by the space division of QZD. The atoms are individually trapped in distant optical cavities connected by a fiber. Compared with previous works, our scheme has the following advantages: First, the two atoms three-dimensional entanglement can be achieved in one step, which will effectively reduce the complexity for implementing the scheme in experiment. Second, our scheme is based on the resonant interaction so the evolution time is very short. Third, the scheme is very robust against the photons leakage and atoms decay since the system only evolves in the null-excitation subspace. Fourth, the scheme can be expanded to generate $N$-atom three-dimensional entanglement and high-dimensional entanglement. This paper is structured as follows: In Sec. 2, we construct the fundamental model and give the effective dynamics to generate three-dimensional entanglement of two spatially separated atoms. In Sec. 3, we analyze the robustness of this scheme via numerical simulation. In Sec. 4, we generalize this proposal to generate $N$-atom three-dimensional entanglement. Besides, in Sec. 5, we expand our scheme to generate high-dimensional entanglement between two distant atoms. The conclusion appears in Sec. 6.

FIG. 1: The schematic setup for generating two atoms three-dimensional entanglement. The two atoms are trapped in two spatially separated optical cavities connected by a fiber.

I. MODEL AND EFFECTIVE DYNAMICS GENERATION OF TWO ATOMS THREE-DIMENSIONAL ENTANGLEMENT

The schematic setup for generating three-dimensional entanglement of two atoms is shown in Fig. 1. We consider a cavity-fibre-cavity system, in which two atoms are trapped in the corresponding optical cavities connected by a fiber. Under the short fiber limit $(lv)/(2\pi c) \ll$
FIG. 2: The level configurations of atom A and B.

1, only the resonant mode of the fiber will interact with the cavity mode \([37]\), where \(l\) is the length of the fiber and \(\nu\) is the decay rate of the cavity field into a continuum of fiber modes. The corresponding level structures of atoms are shown in Fig. 2. Atom A has two excited states \(|e_L\rangle\), \(|e_R\rangle\), and five ground states \(|1\rangle\), \(|R\rangle\), \(|L\rangle\), \(|g\rangle\) and \(|0\rangle\), while atom B is a five-level system with three ground states \(|R\rangle\), \(|L\rangle\) and \(|g\rangle\), two excited states \(|e_L\rangle\) and \(|e_R\rangle\).

For atom A, the transitions \(|0\rangle \leftrightarrow |e_R\rangle\) and \(|1\rangle \leftrightarrow |e_L\rangle\) are driven by classical fields with the same Rabi frequency \(\Omega_A(t)\). And the transitions \(|R\rangle \leftrightarrow |e_R\rangle\) and \(|L\rangle \leftrightarrow |e_L\rangle\) are resonantly driven by the corresponding cavity mode \(a_{AJ}\) with \(j\)-circular polarization and the coupling strength is \(g_{AJ}\) (\(j = L, R\)). For atom B, the transitions \(|R\rangle \leftrightarrow |e_R\rangle\) and \(|L\rangle \leftrightarrow |e_L\rangle\) are driven by classical fields with the same Rabi frequency \(\Omega_B(t)\), and the transitions \(|g\rangle \leftrightarrow |e_R\rangle\) and \(|g\rangle \leftrightarrow |e_L\rangle\) are resonantly driven by the corresponding cavity mode \(a_{BJ}\) with \(j\)-circular polarization and the coupling strength is \(g_{BJ}\) (\(j = L, R\)). The whole Hamiltonian in the interaction picture can be written as (\(\hbar = 1\)):

\[
H_{\text{total}} = H_{a-l} + H_{a-c-f},
\]

\[
H_{a-l} = \Omega_A(t)(|e_L\rangle_A\langle 1| + |e_R\rangle_A\langle 0|) + \Omega_B(t)(|e_L\rangle_B\langle L| + |e_R\rangle_B\langle R|) + H.c.,
\]

\[
H_{a-c-f} = g_{AL}a_{AL}|e_L\rangle_A\langle L| + g_{AR}a_{AR}|e_R\rangle_A\langle R| + g_{BL}a_{BL}|e_L\rangle_B\langle g| + g_{BR}a_{BR}|e_R\rangle_B\langle g|
+
\eta b_L(a_{AL}^\dagger + a_{BL}^\dagger) + \eta b_R(a_{AR}^\dagger + a_{BR}^\dagger) + H.c.,
\]

where \(\eta\) is the coupling strength between cavity mode and the fiber mode, \(b_{R(L)}\) is the annihilation operator for the fiber mode with \(R(L)\)-circular polarization, \(a_{A(B)R(L)}\) is the annihilation operator for the corresponding cavity field with \(R(L)\)-circular polarization, and
$g_{A(B)R(L)}$ is the coupling strength between the corresponding cavity mode and the trapped atom.

In order to obtain the following two atoms three-dimensional entanglement:

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|R\rangle_A|R\rangle_B + |L\rangle_A|L\rangle_B + |g\rangle_A|g\rangle_B),$$  \hspace{1cm} (4)

we assume atom A in the state $\frac{1}{\sqrt{3}}(|1\rangle_A + |0\rangle_A + |g\rangle_A)$, while atom B in the state $|g\rangle_B$, both the cavity modes and the fiber mode in vacuum state $|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f$, and then demonstrate that with the help of QZD the atom state $|0\rangle_A|g\rangle_B$ can be adiabatically evolved to $|R\rangle_A|R\rangle_B$, and $|1\rangle_A|g\rangle_B$ can be adiabatically evolved to $|L\rangle_A|L\rangle_B$. It is easy to know that $|g\rangle_A|g\rangle_B$ will remain unchange since there is no excitation for all the field modes at the beginning. For the initial state $|0\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f$, the whole system evolves in the subspace spanned by

$$|\phi_1\rangle = |0\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f,$$
$$|\phi_2\rangle = |e\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f,$$
$$|\phi_3\rangle = |R\rangle_A|g\rangle_B|1\rangle_{AC}|0\rangle_{BC}|0\rangle_f,$$
$$|\phi_4\rangle = |R\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|1\rangle_f,$$
$$|\phi_5\rangle = |R\rangle_A|g\rangle_B|0\rangle_{AC}|1\rangle_{BC}|0\rangle_f,$$
$$|\phi_6\rangle = |R\rangle_A|e\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f,$$
$$|\phi_7\rangle = |R\rangle_A|R\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f.$$

(5)

Under the condition $\Omega_A(t), \Omega_B(t) \ll \eta, g_{AR(L)}, g_{BR(L)}$, the Hilbert subspace can be divided into five invariant Zeno subspaces \[35, 36\]:

$$\Gamma_{P1} = \{|\phi_1\rangle, |\phi_7\rangle, |\psi_1\rangle\},$$
$$\Gamma_{P2} = \{|\psi_2\rangle\}, \quad \Gamma_{P3} = \{|\psi_3\rangle\},$$
$$\Gamma_{P4} = \{|\psi_4\rangle\}, \quad \Gamma_{P5} = \{|\psi_5\rangle\},$$

with the eigenvalues $\lambda_1 = 0$, $\lambda_2 = -g$, $\lambda_3 = g$, $\lambda_4 = -\sqrt{g^2 + 2\eta^2} = -\epsilon$, and $\lambda_5 = \sqrt{g^2 + 2\eta^2} = \epsilon$, where we assume $g_{AR(L)} = g_{BR(L)} = g$ for simplicity. Here

$$|\psi_1\rangle = \frac{1}{\epsilon}(\eta|\phi_2\rangle - g|\phi_4\rangle + \eta|\phi_6\rangle),$$
\begin{align*}
|\psi_2\rangle &= \frac{1}{2}(-|\phi_2\rangle + |\phi_3\rangle - |\phi_5\rangle + |\phi_6\rangle), \\
|\psi_3\rangle &= \frac{1}{2}(-|\phi_2\rangle - |\phi_3\rangle + |\phi_5\rangle + |\phi_6\rangle), \\
|\psi_4\rangle &= \frac{1}{2\epsilon}(g|\phi_2\rangle - \epsilon|\phi_3\rangle + 2\eta|\phi_4\rangle) - \epsilon|\phi_5\rangle + g|\phi_6\rangle, \\
|\psi_5\rangle &= \frac{1}{2\epsilon}(g|\phi_2\rangle + \epsilon|\phi_3\rangle + 2\eta|\phi_4\rangle) + \epsilon|\phi_5\rangle + g|\phi_6\rangle,
\end{align*}
and the corresponding projection
\begin{equation}
P_i^\alpha = |\alpha\rangle \langle \alpha|, \langle \alpha | \in \Gamma_{P_i}).
\end{equation}
Under the above condition, the system Hamiltonian can be rewritten as the following form
\begin{equation}
H_{total} \simeq \sum_{i,\alpha,\beta} (\lambda_i P_i^\alpha + P_i^\alpha H_{a-t} P_i^\beta) \\
= -g|\psi_2\rangle \langle \psi_2| + g|\psi_3\rangle \langle \psi_3| - \epsilon|\psi_4\rangle \langle \psi_4| + \epsilon|\psi_5\rangle \langle \psi_5| \\
+ \frac{1}{\epsilon}\eta(\Omega_A(t)|\psi_1\rangle \langle \phi_1| + \Omega_B(t)|\psi_1\rangle \langle \phi_7| + H.c.).
\end{equation}
When we choose the initial state $|\phi_1\rangle = |0\rangle_A |g \rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f$, the Hamiltonian $H_{total}$ reduces to
\begin{equation}
H_{eff} = \Omega_{A1}(t)|\psi_1\rangle \langle \phi_1| + \Omega_{B1}(t)|\psi_1\rangle \langle \phi_7| + H.c.,
\end{equation}
where $\Omega_{A1}(t) = \frac{1}{\epsilon}\eta\Omega_A(t)$ and $\Omega_{B1}(t) = \frac{1}{\epsilon}\eta\Omega_B(t)$. Combining with adiabatic passage method, we can obtain the dark state of $H_{eff}:
\begin{equation}
|\psi_{D1}\rangle = \frac{1}{\sqrt{\Omega_{A1}(t)^2 + \Omega_{B1}(t)^2}}(-\Omega_{B1}(t)|\phi_1\rangle + \Omega_{A1}(t)|\phi_7\rangle).
\end{equation}
When the pulses shape satisfy
\begin{equation}
\lim_{t \to -\infty} \frac{\Omega_{A1}(t)}{\Omega_{B1}(t)} = 0, \quad \lim_{t \to +\infty} \frac{\Omega_{A1}(t)}{\Omega_{B1}(t)} = \infty,
\end{equation}
thus, based on the effective Hamiltonian \[(10), our proposal for population transfer from $|\phi_1\rangle$ to $|\phi_7\rangle$ can be achieved.

On the other hand, if the initial state is $|1\rangle_A |g \rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f$, the whole system evolves in the subspace spanned by
\begin{equation}
|\phi'_1\rangle = |1\rangle_A |g \rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f,
\end{equation}
\[ |\phi'_2\rangle = |e_L\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f, \]
\[ |\phi'_3\rangle = |L\rangle_A|g\rangle_B|1_L\rangle_{AC}|0\rangle_{BC}|0\rangle_f, \]
\[ |\phi'_4\rangle = |L\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|1_L\rangle_f, \]
\[ |\phi'_5\rangle = |L\rangle_A|g\rangle_B|0\rangle_{AC}|1_L\rangle_{BC}|0\rangle_f, \]
\[ |\phi'_6\rangle = |L\rangle_A|e_L\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f, \]
\[ |\phi'_7\rangle = |L\rangle_A|L\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f. \]  

(13)

In this situation, with the method mentioned above, we can easily obtain the effective Hamiltonian:

\[ H'_\text{eff} = \Omega_{A1}(t)|\psi'_1\rangle \langle \phi'_1| + \Omega_{B1}(t)|\psi'_1\rangle \langle \phi'_7| + \text{H.c.}, \]  

(14)

where \( |\psi'_1\rangle = \frac{1}{\sqrt{2}}(|\eta|\phi'_2\rangle - g|\phi'_4\rangle + \eta|\phi'_6\rangle) \), \( \Omega_{A1}(t) = \frac{1}{\epsilon}\eta\Omega_A(t) \) and \( \Omega_{B1}(t) = \frac{1}{\epsilon}\eta\Omega_B(t) \). Combining with adiabatic passage method, we can obtain the dark state of \( H'_\text{eff} \):

\[ |\psi_{D2}\rangle = \frac{1}{\sqrt{\Omega_{A1}(t)^2 + \Omega_{B1}(t)^2}}(-\Omega_{B1}(t)|\phi'_1\rangle + \Omega_{A1}(t)|\phi'_7\rangle). \]  

(15)

When the pulses shape satisfy Eq. (12), the initial state \( |\phi'_1\rangle \) involves to \( |\phi'_7\rangle \) eventually.

If the initial state is \( |g\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f \), it will not change at all during the whole evolution.

Therefore, the initial state

\[ \Psi(0) = \frac{1}{\sqrt{3}}(|0\rangle_A + |1\rangle_A + |g\rangle_A)|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f \]  

(16)

of the compound system will evolve to the state

\[ \Psi(t) = \frac{1}{\sqrt{3}}(|R\rangle_A|R\rangle_B + |L\rangle_A|L\rangle_B + |g\rangle_A|g\rangle_B)|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f, \]  

(17)

which is a product state of the two atoms three-dimensional entanglement, the cavity modes vacuum state, and the fiber mode vacuum state.

II. NUMERICAL ANALYSIS AND THE ROBUSTNESS OF THE SCHEME

In order to generate two atoms three-dimensional entanglement, the conditions of Eq. (12) should be satisfied in our scheme. For this reason, we can choose the pulses shape of the laser fields \( \Omega_A(t) \) and \( \Omega_B(t) \) in the original Hamiltonian \( H_{\text{total}} \) as:

\[ \Omega_A(t) = \Omega_0 \sin^4 \left[ \pi (t - \tau)/31t_0 \right] \]  

(18)
FIG. 3: The time dependence of the laser fields $\Omega_A(t)$ corresponding to solid line and $\Omega_B(t)$ corresponding to dashed line with $t_0 = \Omega_0^{-1}$ and $\tau = 5.27t_0$.

Here $\Omega_0$ is the pulse amplitude, $\tau$ being the time delay. Fig. 3 shows the Rabi frequencies $\Omega_A(t)$ and $\Omega_B(t)$ versus $\Omega_0t$ with $t_0 = \Omega_0^{-1}$ and $\tau = 5.27t_0$. The Rabi frequencies are two delayed but partially overlapped pulses. The population curves of $|\phi_1\rangle(|\phi_1\rangle)$, $|\phi_7\rangle(|\phi_7\rangle)$ and $|\psi_1\rangle(|\psi_1\rangle)$ versus $\Omega_0t$ are depicted in Fig. 4, where we choose $g = 20\Omega_0$, $\eta = 100g$, $t_0 = \Omega_0^{-1}$ and $\tau = 5.27t_0$. From Fig. 4 we can see that the population inverts completely when $\Omega_0t$ is
over 25. Through the above processes, we can generate two atoms three-dimensional entanglement successfully. The evolutions are governed by the effectively Hamiltonian $H_{\text{eff}}(H'_{\text{eff}})$, and $\Omega_A(t)$ and $\Omega_B(t)$ are defined by Eqs. (18) and (19), respectively.

![Graph](image)

FIG. 5: The fidelity corresponding to the target state versus $\kappa/g$ and $\gamma/g$, with $g = 20\Omega_0$, $\eta = 100g$, $t_0 = \Omega_0^{-1}$ and $\tau = 5.27t_0$.

It is well-known that whether a scheme is applicable for quantum information processing and quantum computing depends on the robustness against possible mechanisms of decoherence. To examine the robustness of our scheme described in the previous sections, we consider the effect of photon leakage and atom spontaneous decay. The corresponding master equation for the whole system density matrix $\rho(t)$ has the following form:

$$\dot{\rho}(t) = -i[H_{\text{total}}, \rho(t)] - \sum_{j=L,R} \frac{\kappa_j}{2} [b_j^+ b_j \rho(t) - 2 b_j \rho(t) b_j^+ + \rho(t) b_j^+ b_j]$$

$$- \sum_{j=L,R} \sum_{i=A,B} \frac{\kappa_{ij}}{2} [a_{ij}^+ a_{ij} \rho(t) - 2 a_{ij} \rho(t) a_{ij}^+ + \rho(t) a_{ij}^+ a_{ij}]$$

$$- \frac{\gamma_A}{2} \left\{ \sum_{h=L} \left[ \sigma_{e_L,e_h}^A \rho(t) - 2 \sigma_{e_L,h}^A \rho(t) \sigma_{e_L,h}^A + \rho(t) \sigma_{e_L,h}^A \right] \right\}$$

$$+ \sum_{k=R,0} \left[ \sigma_{e_R,e_k}^A \rho(t) - 2 \sigma_{e_R,k}^A \rho(t) \sigma_{e_R,k}^A + \rho(t) \sigma_{e_R,k}^A \right]$$

$$- \sum_{j=L,R} \sum_{i=j,g} \frac{\gamma_B}{2} \left[ \sigma_{e_j,e_j}^B \rho(t) - 2 \sigma_{e_j,j}^B \rho(t) \sigma_{e_j,j}^B + \rho(t) \sigma_{e_j,j}^B \sigma_{e_j,j}^B \right], \quad (20)$$

where $H_{\text{total}}$ is given by Eq. (1). $\kappa_f$ and $\kappa_{R(L)}$ are the photon leakage rates of the fiber mode and cavity mode $R(L)$, respectively. $\gamma_{A(B)}$ is the atom $A(B)$ spontaneous emission
rate from the excited state $|e_R⟩(|e_L⟩)$ to the ground state $|R⟩(|L⟩)$ and $|g⟩$, respectively. $\sigma_{m,n} = |m⟩⟨n| (m, n = 0, 1, L, R, g, e_L, e_R)$. For simplicity, we assume $\kappa_I = \kappa_R = \kappa_L = \kappa/2$, $\gamma_A = \gamma_B = \gamma/4$ and the initial condition $\rho(0) = |Ψ_0⟩⟨Ψ_0|$. In Fig. 5, the fidelity of the final two atoms three-dimensional entanglement is plotted versus the dimensionless parameters $\kappa/g$ and $\gamma/g$ by numerically solving the master Eq. (20). From Fig. 5 we can see that the fidelity of two atoms three-dimensional entanglement is higher than 96.5% even in the range of $\kappa$ and $\gamma$ close to $g$.

III. GENERATION OF $N$-ATOM THREE-DIMENSIONAL ENTANGLEMENT

![Diagram](image)

FIG. 6: Schematic setup for generation of $N$-atom three-dimensional entanglement.

In this section, we will show how to deterministically generate the $N$-atom three-dimensional entanglement. The schematic setup is shown in Fig. 6, where $N$ atoms are trapped in $N$ spatially separated cavities connected by optical fibers. $S_2, S_3, ..., S_N$ are the optical switch devices which are used to control the interaction between the $n$-th cavity and the first cavity. The atom in cavity 1 possesses the level structure as atom A presented in Fig. 2, and the atoms in the other ($n = 2, 3, ..., N$) cavities are the same as atom B given in Fig. 2. By only turning on the $n$-th switch $S_n (n = 2, 3, ..., N)$, and the others off, the
interaction between the 1st atom and the n-th atom can be achieved. In this case, the interaction Hamiltonian has the following form ($\hbar = 1$):

$$H_{\text{total}}^n = H_{a-l}^n + H_{a-c-f}^n,$$ 

(21)

$$H_{a-l}^n = \Omega_A(t)(|e_L\rangle_1\langle 1| + |e_R\rangle_1\langle 0|) + \Omega_B(t)(|e_L\rangle_n\langle L| + |e_R\rangle_n\langle R|) + \text{H.c.},$$ 

(22)

$$H_{a-c-f}^n = g_{1L}a_{1L}|e_L\rangle_1\langle L| + g_{1R}a_{1R}|e_L\rangle_1\langle R| + g_{nL}a_{nL}|e_L\rangle_n\langle g| + g_{nR}a_{nR}|e_L\rangle_n\langle g| + \eta b_L(a_{1L}^\dagger + a_{nL}^\dagger) + \eta b_R(a_{1R} + a_{nR}) + \text{H.c.},$$ 

(23)

where “n” represents the atom trapped in the n-th cavity, $a_{nR(L)}$ is annihilation operator corresponding to the n-th cavity with $R(L)$-circular polarization, and $g_{nR(L)}$ is coupling strength between the corresponding cavity mode and the trapped atom.

In order to obtain $N$-atom three-dimensional entanglement

$$|\varphi\rangle_N = \frac{1}{\sqrt{3}}(|R\rangle_1|R\rangle_2\ldots|R\rangle_N + |L\rangle_1|L\rangle_2\ldots|L\rangle_N + |g\rangle_1|g\rangle_2\ldots|g\rangle_N),$$ 

(24)

we assume $g_{nR} = g_{nL} = g$ for simplicity, and the initial state of the compound system is:

$$|\varphi\rangle_1 = \frac{1}{\sqrt{3}}(|0\rangle_1 + |1\rangle_1 + |g\rangle_1|g\rangle_2|g\rangle_3\ldots|g\rangle_N|0\rangle_{1C}|0\rangle_{2C}\ldots|0\rangle_{NC}|0\rangle_f.$$

(25)

(i) First, turn on $S_2$ and keep other switches off, then the first two atoms can be prepared in the state $|\Psi\rangle = \frac{1}{\sqrt{3}}(|R\rangle_1|R\rangle_2 + |L\rangle_1|L\rangle_2 + |g\rangle_1|g\rangle_2)$ with the method mentioned in Sec. II.

(ii) Then, turn off $S_2$, and apply another pulse on atom 1 to drive the transitions $|L\rangle_1 \rightarrow |1\rangle_1$ and $|R\rangle_1 \rightarrow |0\rangle_1$, which leads the compound system to the state:

$$|\varphi\rangle_2 = \frac{1}{\sqrt{3}}(|0\rangle_1|R\rangle_2 + |1\rangle_1|L\rangle_2 + |g\rangle_1|g\rangle_2|g\rangle_3|g\rangle_4\ldots|g\rangle_N|0\rangle_{1C}|0\rangle_{2C}\ldots|0\rangle_{NC}|0\rangle_f.$$

(26)

(iii) Subsequently, only $S_n$ ($n = 3, \ldots, N$) is turned on, thus the 1st atom can interact with the n-th atom. Next, perform the same operations as the first step successively to make the state in Eq. (26) evolve to

$$|\varphi\rangle_3 = \frac{1}{\sqrt{3}}(|R\rangle_1|R\rangle_2\ldots|R\rangle_n + |L\rangle_1|L\rangle_2\ldots|L\rangle_n + |g\rangle_1|g\rangle_2\ldots|g\rangle_n|g\rangle_{n+1}|g\rangle_{n+2}\ldots|g\rangle_N|0\rangle_{1C}|0\rangle_{2C}\ldots|0\rangle_{NC}|0\rangle_f.$$

(27)

(iv) Finally we can obtain the $N$-atom three-dimensional entanglement after repeating the steps (ii) and (iii) ($N - 2$) times. Thus, the state of the compound system finally evolves to
\[ |\varphi\rangle_4 = \frac{1}{\sqrt{3}} (|R\rangle_1|R\rangle_2 ... |R\rangle_N + |L\rangle_1|L\rangle_2 ... |L\rangle_N \]
\[ + |g\rangle_1|g\rangle_2 ... |g\rangle_N) |0\rangle_1 |0\rangle_2 ... |0\rangle_N |0\rangle_f. \]  \hfill (28)

That is a product state of \( N \)-atom three-dimensional entanglement, the cavity modes state, and the fiber modes state.

**IV. GENERATION OF HIGH-DIMENSIONAL ENTANGLEMENT OF TWO SPATIALLY SEPARATED ATOMS**

![Diagram of atomic levels configuration](image)

**FIG. 7:** The potential atomic levels configuration for atom A and B.

We note that the scheme can also be expanded to generate high-dimensional entanglement of two spatially separated atoms. The potential atomic configurations is plotted in Fig. 7.
We assume that the cavity supports $2N$ independent modes of photon fields. Then the Hamiltonian can be written as ($\hbar = 1$):

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{a-l} + \mathcal{H}_{a-c-f},$$  

$$\mathcal{H}_{a-l} = \Omega_A(t)(|e_L\rangle_A\langle 1| + |e_R\rangle_A\langle 0|) + \Omega_B(t) \sum_{i=1}^{N}(|e_{Li}\rangle_B\langle L_i| + |e_{Ri}\rangle_B\langle R_i|) + \text{H.c.},$$  

$$\mathcal{H}_{a-c-f} = \sum_{i=1}^{N}[(g_{ALi}a_{ALi}|e_L\rangle_A\langle L_i| + g_{ARI}a_{ARI}|e_R\rangle_A\langle R_i|) + g_{Bli}a_{Bli}|e_{Li}\rangle_B\langle g| + \eta_{BLi}(a_{ALi}^\dagger + a_{Bli}^\dagger) + \eta_{Ri}(a_{ARI}^\dagger + a_{BRI}^\dagger) + \text{H.c.}].$$

Assume the initial state of the whole system is $|1\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f$, then the system will evolve in the subsystem $\Gamma$ spanned by the vectors $\{\{\zeta_1\}, \{\zeta_2\}, \ldots, \{\zeta_{5N+2}\}\}$:

$$\Gamma = \{\{\zeta_1\} = |1\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_2\} = |e_L\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_{2+i}\} = |L_i\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_{2+N+i}\} = |L_i\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_{2+2N+i}\} = |L_i\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_{2+3N+i}\} = |L_i\rangle_A|g\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f,$$
$$\{\zeta_{2+4N+i}\} = |L_i\rangle_A|L_i\rangle_B|0_10_2\ldots 0_N\rangle_{AC} |0_10_2\ldots 0_N\rangle_{BC} |0_10_2\ldots 0_N\rangle_f.\}$$

where $i = 1, 2, 3, \ldots, N$. With the prior procedures presented in Sec. II, we can obtain the effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{1}{\sqrt{N+1}}(\Omega_{A1}(t)|X_1\rangle \langle \zeta_1| + \Omega_{B1}(t) \sum_{j=4N+3}^{5N+2} |X_1\rangle \langle \zeta_j| + \text{H.c.}),$$

with $\Omega_{A1}(t) = \frac{1}{\epsilon} \eta \Omega_A(t)$ and $\Omega_{B1}(t) = \frac{1}{\epsilon} \eta \Omega_B(t)$, where

$$|X_1\rangle = \frac{1}{\sqrt{2N+1}}(\eta|\zeta_2\rangle - g \sum_{j=N+3}^{2N+2} |\zeta_j\rangle + \eta \sum_{k=3N+3}^{4N+2} |\zeta_k\rangle).$$

With the help of adiabatic passage method, we can obtain the dark state of $\mathcal{H}_{\text{eff}}$:

$$|X_D\rangle = \frac{1}{\sqrt{(N+1)(\Omega_{A1}(t)^2 + \Omega_{B1}(t)^2)}}(-\Omega_{B1}(t)|\zeta_1\rangle + \Omega_{A1}(t) \sum_{j=4N+3}^{5N+2} |\zeta_j\rangle).$$
When Eq. (12) is satisfied, the initial state $|1\rangle_A|g\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f$ of the whole system will finally evolve to

$$|\nu\rangle = \frac{1}{\sqrt{N}} \sum_{j=4N+3}^{5N+2} |\zeta_j\rangle$$

$$= \frac{1}{\sqrt{N}} \sum_{m=1}^{N} |L_m\rangle_A|L_m\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f,$$

(36)

which is a $N$-dimensional maximally entanglement.

For the initial state $|0\rangle_A|g\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f$, by using the similar way from Eqs. (29)–(35), this initial state finally evolves to

$$|\nu_1\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |R_k\rangle_A|R_k\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f.$$

(37)

On the other hand, the initial state $|g\rangle_A|g\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f$ don’t participate in the evolution.

Therefore, if we choose the initial state of the combined system as

$$|\nu\rangle_0 = \frac{1}{\sqrt{3}}(|1\rangle_A + |0\rangle_A + |g\rangle_A)|g\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f,$$

(38)

after implementing all the operations mentioned above, the high-dimensional entanglement can be obtained as the following form:

$$|\beta\rangle = \frac{1}{\sqrt{2N+1}} [\sum_{m=1}^{N} |L_m\rangle_A|L_m\rangle_B + \sum_{k=1}^{N} |R_k\rangle_A|R_k\rangle_B$$

$$+ |g\rangle_A|g\rangle_B|0_10_2...0_N\rangle_{AC}|0_10_2...0_N\rangle_{BC}|0_10_2...0_N\rangle_f].$$

(39)

V. ANALYSIS AND DISCUSSION

We now analyze the feasibility of the experiment for this scheme. The appropriate atomic level configuration can be obtained from the hyperfine structure of $^{133}$Cs. 5S$_{1/2}$ ground level $|F = 3, m = 2\rangle(|F = 3, m = -2\rangle)$ corresponds to $|R\rangle(|L\rangle)$ and $|F = 2, m = 1\rangle(|F = 2, m = -1\rangle)$ corresponds to $|0\rangle(|1\rangle)$, respectively, while 5P$_{3/2}$ excited level $|F = 3, m = 1\rangle(|F = 3, m = -1\rangle)$ corresponds to $|e_R\rangle(|e_L\rangle)$. Other hyperfine levels in the ground-state manifold can be used as $|g\rangle$ for atom A. For atom B, the states $|R\rangle, |L\rangle$ and $|g\rangle$ correspond to $|F = 2, m = -1\rangle, |F = 2, m = 1\rangle$ and $|F = 3, m = 0\rangle$ of 5S$_{1/2}$ ground levels, respectively. And $|e_R\rangle(|e_L\rangle)$ corresponds to $|F = 3, m = -1\rangle(|F = 3, m = 1\rangle)$ of 5P$_{3/2}$ excited level.
In this paper, we choose $\Omega_{A(B)}(t)/g$ is less than 0.05, so that the Zeno condition can be satisfied well. In experiments, the cavity QED parameters $g = 2.5\text{GHz}$, $\kappa = 10\text{MHz}$, and $\gamma = 10\text{MHz}$ have been realized in [38, 39]. For such parameters, the fidelity of our scheme is larger than 99.0%, so our scheme is robust against both the cavity decay, the fiber loss and the atomic spontaneous radiation and may be very promising within current experiment technology.

In summary, we have proposed a promising scheme to generate three-dimensional entanglement with the help of QZD in the cavity-fiber-cavity system. Because the atoms are resonant interaction, so the speed of producing entanglement is very fast compared with the dispersive protocols [40, 41]. Meanwhile, the influence of various decoherence processes such as spontaneous emission and photon loss on the generation of entanglement is also investigated. Because during the whole process the system keeps in a Zeno subspace without exciting the cavity field and the fiber, and all the atoms are in the ground states, thus the scheme is robust against the cavity, fiber and atomic decay. Numerical results show the generation of entanglement can be achieved with a high fidelity. Besides, the scheme can be generalized to generate $N$-atom three-dimensional entanglement and high-dimensional entanglement. We hope our work will have a crucial role in promoting quantum information processing including implementing quantum gates, performing atomic state transfer, generating entanglement, etc.

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