Mathematical modeling of electrothermal processes using the example of high-frequency welding of a batch of symmetric polymer workpieces

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Abstract. The article substantiates the necessity and practical relevance of the mathematical modeling of processes of the high-frequency electrothermy of polymers. A mathematical model of high-frequency welding of polymeric and composite products has been developed. A mathematical model of high-frequency welding of a batch of polymeric and composite products has been developed, which is a system of differential equations of unsteady-state heat conduction with internal heat sources, which allows studying the heat distribution in the entire volume of the processed product of any geometric shape. The paper presents the testing results for the developed mathematical model at high-frequency welding of a batch of polymer parts. This made it possible to identify the effect of a non-insulated electrode heating up during sequential processing of a batch of parts on the position of the coordinate of the maximum heating point and to develop a technique for its search and maintenance of the required position. This, in turn, is necessary for organizing control of the high-frequency processing operation.

1. Introduction and setting work objectives
Research and analysis of the processes of the high-frequency (HF) electrothermy of polymeric and composite products is one of the prevailing scientific tasks in this area. Its solution hinges on the performance of the developed operating procedures (OP) of the HF processing, as well as the quality of the products obtained [1, 2]. Since HF processing involves intensive heating of the processed product [3, 4], the study and analysis of the thermal processes occurring in the product is of greatest interest [1, 6, 7]. In turn, only mathematical modeling is able to reflect the most plausible picture of the required processes for today [8-14].

The capabilities of HF electrothermy make it possible to implement a wide range of operating procedures of processing [1, 14]. These procedures include: drying non-metallic materials, restoring their strength properties, welding fatigue cracks, gluing workpieces, hot stamping, welding, diagnostics, etc. [1, 14, 15-17]. In this paper, HF welding of polymer workpieces is considered as the most commonly used operating procedure of HF electrothermy to date [2, 6, 13]. Based on the foregoing, the main purpose of this work is to develop a mathematical model of high-frequency welding, which allows us to study the distribution of heat in the entire volume of a product from a polymer and composite material of any geometric shape.
2. Development of a mathematical model

Traditionally, HF electrothermy is considered in the form of a flow chart (FC) (Figure 1), consisting of the main elements (the movable high-potential electrode of the operating capacitor 1; the fixed low-potential electrode of the operating capacitor 3; the workpiece 4 and additional elements (heat-insulating gaskets 2) [15]. The processed product 4 emits thermal energy under the influence of the electric field 5.

![Figure 1. The flow chart of high-frequency electrothermy.](image1)

To achieve this goal, a five-layer flow chart has been modeled (Figure 2), the temperature distribution in which is described by the system of equations (1).

![Figure 2. The 3D model of a five-layer operating procedure of the HF welding of polymer workpieces 3 and 4, isolated from the metal electrodes of the operating capacitor 1 and 5 with an insulating gasket 2 only on one side, where \( h_i \) is the thickness of the \( i \)th element of the flow chart; \( i \) is the flow chart element number (\( i = 1 \ldots 5 \)); \( Z_0 \) is the boundary of the welded polymer workpieces.](image2)
The system of equations (1) is set up to solve for the temperature distribution in the flow chart.

\[
\begin{aligned}
    \rho_1 C_{\rho_1}(T_1) \frac{\partial T_1}{\partial \tau} &= \frac{\partial}{\partial x} \left( \lambda_1(T_1) \frac{\partial T_1}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_1(T_1) \frac{\partial T_1}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_1(T_1) \frac{\partial T_1}{\partial z} \right), \\
    \rho_2 C_{\rho_2}(T_2) \frac{\partial T_2}{\partial \tau} &= \frac{\partial}{\partial x} \left( \lambda_2(T_2) \frac{\partial T_2}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_2(T_2) \frac{\partial T_2}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_2(T_2) \frac{\partial T_2}{\partial z} \right), \\
    \rho_3 C_{\rho_3}(T_3) \frac{\partial T_3}{\partial \tau} &= \frac{\partial}{\partial x} \left( \lambda_3(T_3) \frac{\partial T_3}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_3(T_3) \frac{\partial T_3}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_3(T_3) \frac{\partial T_3}{\partial z} \right), \\
    \rho_4 C_{\rho_4}(T_4) \frac{\partial T_4}{\partial \tau} &= \frac{\partial}{\partial x} \left( \lambda_4(T_4) \frac{\partial T_4}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_4(T_4) \frac{\partial T_4}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_4(T_4) \frac{\partial T_4}{\partial z} \right), \\
    \rho_5 C_{\rho_5}(T_5) \frac{\partial T_5}{\partial \tau} &= \frac{\partial}{\partial x} \left( \lambda_5(T_5) \frac{\partial T_5}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_5(T_5) \frac{\partial T_5}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_5(T_5) \frac{\partial T_5}{\partial z} \right),
\end{aligned}
\]

where \( \rho, \lambda_i(T), C_{\rho_i}(T) \) is the material density, thermal conductivity, specific heat capacity of the \( i \)th element, respectively; \( T_i \) is the temperature of the \( i \)th element; \( P_i(T_i) \) is the specific power of the \( i \)th internal heat source; \( \tau \) is time; \( i \) is the element number: 1, 5 are high-potential and low-potential electrodes of the operating capacitor, respectively, 2 is the insulating gasket, 3, 4 are weldable polymer workpieces.

To solve the system of equations (1), the authors set initial and boundary conditions. The initial conditions are presented in equation (2)

\[
T_{\tau=0} = T_i, \quad 0 \leq x \leq X, \quad 0 \leq y \leq Y, \quad 0 \leq z \leq z_i,
\]

where \( T_i, T \) is the initial and current temperature of the material of the elements of the flow chart during processing, respectively; \( x, y, z \) is the current coordinate of the considered element (body) of the flow chart (Figure 3).

The boundary conditions of the third kind [18], according to equations (3) and (4), describe the relationship between the heat fluxes of the side walls of the layers of the flow chart and the environment

\[
- \lambda_i(T_i) \frac{\partial T_i}{\partial x} \bigg|_{x=0,X} = \alpha_i(T_i) \cdot \Delta T_i, \quad (3)
\]

\[
- \lambda_i(T_i) \frac{\partial T_i}{\partial y} \bigg|_{y=0,Y} = \alpha_i(T_i) \cdot \Delta T_i, \quad (4)
\]

and equation (5) reflects the boundary conditions at the external boundaries of the electrodes of the operating capacitor (Figure 2, position 1 and 5).
$$-\lambda_{1.5}(T_{1.5}) \frac{\partial T_{1.5}}{\partial z} \bigg|_{z=\bar{z}_i, \bar{z}_5} = \alpha_{1.5}(T_{1.5}) \cdot \Delta T_{1.5},$$

where $\Delta T_i$ is the temperature difference between the surface of the flow chart element and the environment; $\alpha_i(T_i)$ is the heat transfer coefficient of the material of the $i$th element of the flow chart with the environment; $\bar{z}_o$ and $\bar{z}_5$ are the coordinates of the external boundaries of the electrodes of the operating capacitor with the environment.

The fourth-kind boundary conditions describe the thermal interaction between elements of the flow chart. At the same time, as presented in the system of equations (6), temperatures and thermal flows at the borders of layers are equal to

$$\begin{cases}
T_j = T_{j+1} \\
\lambda_j(T_j) \frac{\partial T_j}{\partial z} = \lambda_{j+1}(T_{j+1}) \frac{\partial T_{j+1}}{\partial z}
\end{cases} \quad \text{with } z = \bar{z}_j$$

where $T_j$ is the temperature at the boundary of the $i$th elements, $j$ is the boundary of the $i$th elements.

The developed mathematical model allows us to analyze the volumetric heat distribution in a five-layer flow chart during high-frequency welding of a batch of polymer products, taking into account the temperature-dependent properties of the flow chart elements.

3. Experimental part. Results and consideration.

The solution of the developed mathematical model of the process of high-frequency polymer heating during welding, presented in the form of a system of differential equations (1), was implemented by the mesh method in the MSC Sinda software package [1]. Its solution consists of several stages (Figure 4).

The material and dimensions of the flow chart elements are as follows:

- the electrodes of the operating capacitor (low and high potential) are made of stainless steel with dimensions of $0.01 \times 0.05 \times 0.01$ m;
- insulating gasket from electrical insulation cardboard GOST 2824-86, with dimensions of $0.01 \times 0.05 \times 0.0015$ m;
- weldable polymer workpieces, in the capacity of which specimens from the polyamide PA6 with dimensions of $0.01\times0.05\times0.002$ m were used.

The initial temperature of each flow chart element, when processing the first part $n_1$ from the batch, was taken to be equal to the temperature of the normal conditions $T_o$ [20]. In the sequential processing of subsequent polymer parts from the batch, the temperature of each workpiece was also taken to be equal to $T_o$, and the temperature of the remaining elements of the flow chart was taken to be equal to the temperature of the corresponding element obtained during the welding of the previous polyamide part.

Given the low welding temperature of the studied polymer (210–220 °C [2, 6, 13]), we accept that it is only necessary to set its temperature-dependent properties (Figure 5), and for the remaining elements of the flow chart, these properties are constant ($\lambda_i = \text{const}$, $C_i = \text{const}$). Also, since high heating dynamics of processed polar polymers is prevalent during HF heating, the heat exchange of the flow chart elements with the environment can be neglected, so $\alpha_i(T_i) = 0$.

The powers of $P_3$ and $P_4$ are determined by computational and experimental means [1, 2, 6, 10]. For PA6, it is of a temperature-dependent nature, presented in the form of a graph in Figure 6.

The setting of the estimated time $\tau$ implied the indication of the time to reach the welding temperature in the polymer that corresponds to the melting temperature. $\tau$ was determined experimentally and, for a batch of parts, amounted to the values shown in Figure 7. As shown in Figure 7, the entire high-frequency processing was followed by a decrease in the time to attain the welding temperature $\tau$ in the workpieces when passing from one part to another.
In [6, 8, 12, 13] the authors point out the necessity to determine and control the coordinates of the maximum heating point $Z_s$ from the thickness of the weldable workpieces. Indeed, to increase the efficiency of HF welding and to obtain high-quality welded joints, it is necessary to organize maximum heating at the boundary of the weldable workpieces with minimal heating of their peripheral zones. On the basis on the foregoing, to solve the system of equations (1), operations to establish the current coordinate $Z_s$ from the thickness of the weldable workpieces and control its displacement from the required position $Z_o$, were included in the fifth stage of the algorithm presented above (figure 4).

At the same time, the real operating procedure of HF welding of a batch of polymer products is reduced to achieving and complying with the following condition: $Z_s \approx Z_o$. The fulfillment of this condition is ensured by the controlled and variable thermal influence of the flow chart elements. In other words, in order that $Z_s \approx Z_o$ it is necessary to determine and maintain the required initial temperature value $T_o$ of an uninsulated electrode through cooling or heating (Figure 2, position 5).

![Figure 4. PA6 polyamide dependence graphs $\lambda = f(T)$ (1) and $C_p = f(T)$ (2).](image-url)
Figure 5. The graph of dependency of $P = f(T)$ a specimen from polyamide PA6 with dimensions of $0.01 \times 0.05 \times 0.002$ m, at the voltage of the operating capacitor $U_p = 900$ V.

Figure 6. The graph of variance of time $\tau$ for heating of each polyamide part from batch $n$ to $216$ °C during high-frequency welding.
Creating a 3D model

Construction of a finite-difference mesh

Division of the finite-difference mesh of the 3D model into groups corresponding to the elements of the process scheme

Setting material properties:
heat capacity $C_p(T)$, thermal conductivity $\lambda(T)$, density $\rho$

Setting the initial conditions: $T_o$

Setting boundary conditions:
heat transfer coefficient $\alpha$,
ambient temperature $T_e$

Setting internal heat sources: $P(T)$

Setting the estimated time $\tau$ and interval of the output parameters of calculation $\Delta\tau$

Stage No. 1

Stage No. 2

Stage No. 3

Stage No. 4

Stage No. 5

Figure 7. The block schematic diagram of the solution of the system of equations (1), which describes the volumetric heat distribution in the flow chart.
Thus, in the present paper we studied the thermal effect of the non-insulated operating capacitor electrode on the displacement of the coordinate \( Z_s \) relative to the thickness of the weldable workpieces from the required value \( Z_o \) (Figure 8).

**Figure 8.** Illustration of the thermal effect of the non-insulated high-potential electrode of the operating capacitor on the displacement of the coordinate of the point \( Z_s \) relative to \( h_2 \) and \( h_3 \).

Figure 8 shows that the condition \( Z_s \approx Z_o \), excluding the width of the plateau of reaching the welding temperature is fulfilled only during the HF welding of the eighth \( n_8 \) and the ninth \( n_9 \) parts from the batch. When processing other parts, the effect of the temperature of the uninsulated electrode (Figure 9) that is heating during welding leads to an unsatisfactory effect. So, at relatively low temperatures of the uninsulated electrode (up to 55 degrees °C) the coordinate \( Z_s \) shifts towards the insulation gasket, and with the increase in its temperature, the direction of displacement changes. It should be noted here that the produced mathematical modelling revealed a very uncharacteristic correlation of the change in the temperature of the insulation gasket during the process of high-frequency welding of a batch of polymer parts. So, Figure 10 shows the scale of the initial temperature conditions of the boundary nodes of the insulating gasket during the processing of each part, obtained by calculation and experimental means. It follows from the presented dependence (Figure 10) that, when processing the first five parts from a batch, the insulating gasket fully performs its task, while continuously heating up, but then gradually begins to give off heat and, starting from processing the tenth part, it loses it intensively. This phenomenon contributes to an increase in the processing speed (Figure 7) and directly affects the displacement of coordinate \( Z_s \).
A mathematical experiment performed by a numerical method shows that, in order to increase the efficiency of high-frequency welding and to obtain a batch of high-quality products, using the example of polymer workpieces of equal thickness in accordance with the flow chart shown in Figure 2, it is necessary to maintain the temperature of an uninsulated operating capacitor electrode equal to $55 \pm 5 \degree C$.

### 4. Conclusions

The present article substantiates the necessity and practical relevance of the mathematical modeling of processes of the high-frequency electrothermy of polymers and composites. A mathematical model of HF electrothermy in the form of a system of differential equations of unsteady-state heat conduction with an internal heat source has been developed. Each of the equations describes the volumetric distribution of the thermal field in a separate element of the flow chart. The solution of the developed mathematical model was implemented by the mesh method in the MSC Sinda software package according to an algorithm consisting of five stages. It should be noted that the HF processing of a batch of polymer parts was simulated for the first time in the framework of this study. The simulation of the HF welding of a batch of polymer products, isolated only from the low-potential electrode, allowed us to determine the effect of thermal interaction of the flow chart elements on the process of HF welding of the polymer batch. The study determined a method of finding and maintaining the required coordinate of the maximum heating point needed for polymer welding. This, in turn, eliminates incomplete melting products and thus improves the quality of processing. The results of mathematical modelling can be applied in the construction of automated systems for the management of high-frequency processing procedures of a batch of polymer parts with automatic compliance with the condition of $Z_s \approx Z_o$.

### References

[1] Butorin D V 2018 Automation of control of processes of high-frequency processing of polymeric materials of different degree of polarity (Irkutsk: Irkutsk State Transport University) p 174
[2] Romanian S N 2005 *Automated control system for high-frequency welding of polyamide products* (St. Petersburg: St. Petersburg State Technological Institute (Technical University)) p 133

[3] Markov A V, Yulenets Yu P and Rumynsky S N 2005 *Welding Production* 4 45

[4] Tsirkina O G 2015 *Theoretical and experimental substantiation of increasing the effectiveness of textile finishing technologies using a high frequency current field* (Ivanovo: Ivanovo State University of Chemistry and Technology) p 133

[5] Markov A V and Yulenets Yu P 2007 *Russian Electrical Engineering* 7 390

[6] Livshits A V 2016 *Automated control of technological processes of high-frequency electrothermy of polymers* (Irkutsk: Irkutsk State Transport University) p 351

[7] Butorin D 2018 *MATEC Web of Conferences* 02003

[8] Livshits A V 2014 *Science and education* 6 31

[9] Trofimov N V, Yulenets Y P and Markov A V 2011 *Welding International* 3 196

[10] Yulenets Yu P and Markov A V 2012 *Fundamental and Applied Problems of Technics and technology* 2 44

[11] Trofimov N V and Markov A V 2009 *Materials XXII International. sci. conf. MMTT-21* B10 71

[12] Livshits A V, Larchenko A G and Filippenko N G 2014 *Transport Infrastructure of the Siberian Region* 423

[13] Kargapol'tsev S K, Livshits A V, Filippenko N G, Homenko A P, Gozbenko V E and Dambaev Z G 2017 *JP Journal of Heat and Mass Transfer* B 14(2) 219

[14] Butorin D V 2018 *Colloquium-journal* 7 14

[15] Aleksandrov A A, Butorin D V and Filippenko N G 2020 *Bulletin of Voronezh state technical university* 1 122

[16] Filippenko N G, Butorin D V and Livshits A V 2017 *Automation. Modern technologies* 4 171

[17] Popov M S, Filippenko N G, Butorin D V, Livshits A V and Gozbenko V E 2017 *Modern technologies. System analysis. Modeling* 1 96

[18] Kuznetsov G V and Sheremet M A 2007 *Difference methods for solving heat conduction problems* (Tomsk: Tomsk Polytechnic University) p 172

[19] Popov S I 2013 *Automation of control of technological processes of restoration of operational properties of polymers* (Irkutsk: Irkutsk State Transport University) p 150

[20] *Russian Federation Standard GOST 8.395-80 Normal conditions of measurement during verification* (Moscow: Publishing Standards) p 150