A Supervised Geometry-Aware Mapping Approach for Classification of Hyperspectral Images

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Abstract—The lack of proper class discrimination among the Hyperspectral (HS) data points poses a potential challenge in HS classification. To address this issue, this paper proposes an optimal geometry-aware transformation for enhancing the classification accuracy. The underlying idea of this method is to obtain a linear projection matrix by solving a nonlinear objective function based on the intrinsic geometrical structure of the data. The objective function is constructed to quantify the discrimination between the points from dissimilar classes on the projected data space. Then the obtained projection matrix is used to linearly map the data to more discriminative space. The effectiveness of the proposed transformation is illustrated with three benchmark real-world HS data sets. The experiments reveal that the classification and dimensionality reduction methods on the projected discriminative space outperform their counterpart in the original space.

Index Terms—Dimensionality reduction, geometry-aware mapping, hyperspectral classification, manifold, nonlinear objective function.

I. INTRODUCTION

The multi-path scattering of light within a pixel [1], bidirectional reflectance distribution [2], and the heterogeneity of sub-pixel constituents [3] are the major concerns in the hyperspectral (HS) data classification. These nonlinearity properties naturally place the HS data on a non-euclidean space. Handling these high dimensional redundant data in a non-euclidean space is one of the major bottlenecks in HS data analysis.

Typically, HS classification consists of dimensionality reduction (DR) and subsequent classification operation. The popular DR methods such as principal component analysis (PCA) [4] and linear discriminant analysis (LDA) [5] are linear and operate on Euclidean structures. These linear DR methods skip the curved nonlinear structures of the HS data. On the other hand, manifold learning helps in recovering compact, meaningful low dimensional structures from those complex high dimensional data from a non-euclidean space. The manifold learning methods consider the real world high dimensional data to be generated with a few degrees of freedom [6]. This leads to the projection of the data into lower dimensional space while preserving their underlying geometrical structure [7].

Several state-of-art techniques for DR use manifold learning, such as local linear embedding (LLE) [8], isometric feature mapping (ISOMAP) [9], Laplacian eigenmap (LE) [10], local tangent space alignment (LTSA) [11], local scaling cut (LSC) and semi-supervised local scaling cut (SSLSC) [12] etc. These algorithms assume that the input data lie on or close to a smooth low dimensional manifold [7] [13]. Some of these algorithms use local approaches like LE, LLE, LTSA, LSC, SSLSC etc., while others use the global approaches such as ISOMAP [14]. These local approaches use spectral embedding method to retain the local geometry of the data while projecting them to lower dimensions. The embedding task, in these algorithms, is reduced to the form of an eigen decomposition problem under unit covariance constraint [15]. This imposed covariance constraint lose the aspect ratio of the embedding data and the underlying manifold can not reflect its original global shape. On the other hand, the global approach like ISOMAP preserves the metrics at all scales and give a better embedding. However, ISOMAP is only applied to intrinsically flat structures (cylinders, cones, etc) [15]. Therefore, the majority of the manifold based DR algorithms give undesired results in HS data classification due to the above-said issues. In order to address this, mapping the data from existing space to more discriminative space appears to be a promising direction.

A majority of the manifold learning (ML) algorithms first flatten the non-euclidean structure by different methods such as tangent space computation and Hilbert space embedding [16]. The flattened non-euclidean structure then projected to a lower dimensional more discriminative space or directly apply the classification algorithms on that flattened structure. This flattening process mainly incurs distortions in the data geometry on the non-euclidean space [16]. Then this distortion imparts a negative impact on the projection operation as well as the classification which gives undesired results. Additionally, the nonlinear manifold structure is also not properly constructed with less training data points with disruptive geometry in their original data space.

In this work, we propose a geometry-aware mapping approach that maps the data from the original data space to a new more discriminative space to achieve better performance in subsequent DR and classification tasks. The mapping procedure preserves the original geometry of the data to a great extent in the discriminative space due to the linear projection. However, we employ the nonlinear cost function based on the local property by embedding a weighted graph on the data for computing the projection matrix. Moreover, the positioning of data from similar and dissimilar classes in the local neighborhood is also incorporated in the cost function making it more reliable for obtaining optimal discriminative directions. Here the class labels are used to build an affinity function to
encode the inter-class and intra-class similarity. The projection matrix is obtained using these similarity matrices without disrupting the underlying geometry. Thus, the projected data lie on a more discriminative space improve the subsequent data interpretation.

II. PROPOSED GEOMETRY AWARE MAPPING TECHNIQUE

The DR methods aim at finding a projection matrix to project the data points to a new space with less degree of freedoms. However, some information is lost in this process, which in turn affects the performance of the system. To address this issue, we aim to project the data to a new space with reduced dimension that will minimize the information loss and maximize the class discrimination by preserving the geometry of the data points. Thus, our goal is to obtain a projection matrix that will improve the classification performance in the projected space without distorting the data geometry.

Given a set of data points \( X = \{x_1, x_2, \ldots, x_p\}; x_i \in \mathbb{R}^n \) with class labels \( y_i \in \{1, 2, \ldots, c\} \), our objective is to find the mapping matrix \( U \), such that, the resulting space enhances some interesting structure of the original data with minimum information loss and improves the classification performance. Thus, we seek to learn the mapping matrix \( U \in \mathbb{R}^{n \times m} \), of a generic mapping \( f : S^n \times \mathbb{R}^{n \times m} \rightarrow S^m \), \( m < n \) and is defined as

\[
f(X, U) = U^T X \tag{1}
\]

To improve the classification accuracy, we need to determine the directions along which different class points can be discriminated easily. Class overlapping in HS data is a big issue because of the complex structure of the data. In such cases, we can focus on directions which will improve the discrimination in closely situated points from dissimilar classes. Thus, the intrinsic nonlinear geometrical structure of the data can be utilized to improve class discrimination locally.

An unsupervised way of achieving DR would consider the following cost function,

\[
\min_U L(U) = \sum_{i,j} ||U^T x_i - U^T x_j||_F^2 \quad \tag{2}
\]

However, with the knowledge of the class labels of the training data, we can formulate a non-linear cost function to achieve the optimal class discrimination in the projected space. To this end, we use the class labels along with the affinity of the data points to formulate the cost function.

We encode the underlying geometric structure of the data by an undirected graph reflected by the affinity matrix \( A \in \mathbb{R}^{n \times n} \). The element \( A_{ij} \) of affinity matrix \( A \) measures the affinity between data point \( x_i \) and \( x_j \). In particular, we use the class labels to compute the affinity function through intra-class \( g_w \) and inter-class \( g_b \) similarities.

The binary functions \( g_w \) and \( g_b \) are computed as

\[
g_w(x_i, x_j) = \begin{cases} 
1, & \text{if } x_i \in N_w(x_j) \text{ or } x_j \in N_w(x_i) \\
0, & \text{Otherwise}
\end{cases} \tag{3}
\]

\[
g_b(x_i, x_j) = \begin{cases} 
1, & \text{if } x_i \in N_b(x_j) \text{ or } x_j \in N_b(x_i) \\
0, & \text{Otherwise}
\end{cases}
\]

where \( N_w(x_i) \) is the set of \( v_w \) nearest neighbors of \( x_i \) that belong to the same class as of \( x_i \), \( N_b(x_i) \) is the set of \( v_b \) nearest neighbors of \( x_i \) that belong to different classes, other than the class of \( x_i \). The affinity function is then defined as

\[
A_{ij} = g_w(x_i, x_j) - g_b(x_i, x_j) \quad \tag{4}
\]

After obtaining \( A \), we propose to modify the cost function in (2) by incorporating the supervised local properties for computing the projection matrix. Thus, we formulate the cost function as

\[
L(U) = \sum_{i,j} A_{ij} ||U^T x_i - U^T x_j||_F^2 \quad \tag{5}
\]

An orthonormal constraint is imposed on the objective function \( L(U) \) i.e. \( U^T U = I_m \). Hence the final projection matrix can be obtained by minimizing the \( L(U) \) w.r.t \( U \) such that \( U^T U = I_m \).

That is

\[
U = \min_U \left\{ L(U) \quad \text{s.t. } U^T U = I_m \right\} \quad \tag{6}
\]

Here in (6), \( L(U) \) is the optimization function with unitary constraint. Here the objective function forces the distance between the geometrically close points of the same class to be closer, while increasing the distance between closely situated dissimilar class points. The addition of nonlinear weighted graph \( A_{ij} \) with the linear cost function of (2) make the proposed cost function nonlinear in nature. Moreover, the complexity of this cost function \( L(U) \) in (6) leads us to solve this using nonlinear optimization technique.

Taking inspiration from [16] and [19], here we use a Conjugate Gradient (CG) method to solve the optimization problem (6). In this work, we use manopt optimization toolbox [20] to implement the CG method. The CG method completely relies on the gradient on the nonlinear space. To this end, the gradient of a function is computed as

\[
\text{grad } L(U) = (I_n - U U^T) \nabla_U L \quad \tag{7}
\]

where \( \nabla_U L \) is the usual Jacobian matrix at \( U \) and the Jacobian matrix for Frobenius norm is computed as

\[
\nabla_U L = 2 U^T (x_i - x_j)(x_i - x_j)^T \quad \tag{8}
\]

Finally, we project the HS data from its original space into a new discriminative space spanned by \( U \) by retaining their geometrical structure to achieve better discriminative property for subsequent DR and classification operation. In our experiments, we found that the optimization cost is minimum when the reduced dimension in the proposed method is one less than the dimension of the data. This can be simply explained as the dimension in which the information loss will be minimum, while improving the class discrimination. Therefore, similar setting is adopted in all our experiments.

III. EXPERIMENTS

A. Dataset Description

Three HS image datasets were used in our experiments, such as Indian Pine, Botswana, and Pavia university. The size of

\[1\text{http://www.ehu.eus/ccwintco/index.php?title=Hyperspectral_Remote_Sensing_Scenes} \]
Fig. 1: Comparison of classification performances with four different DR techniques (PCA, LDA, KPCA and MFA) w.r.t various dimensions on three different HS datasets (Indian Pines (1a), Botswana (1b) and Pavia University (1c)). (Notations:- e.g. Normal PCA: PCA in original space (dotted line), Mapped PCA: PCA in projected discriminative space (solid line))

TABLE I: The effect of variation of the number of nearest neighbor (\(N_w\) and \(N_b\)) points on overall accuracy (OA)

| Datasets            | \(N_w = N_b\) | \(N_w\) | \(N_b\) |
|---------------------|----------------|---------|---------|
| Indian Pines        | Original Space | 3       | 52.86   |
|                     | Projected Space| 3       | 60.11   |
|                     | Original Space | 5       | 53.50   |
|                     | Projected Space| 5       | 57.82   |
|                     | Original Space | 7       | 52.75   |
|                     | Projected Space| 7       | 58.57   |
|                     | Original Space | 9       | 56.23   |
|                     | Projected Space| 9       | 60.31   |
|                     | Original Space | 11      | 54.87   |
|                     | Projected Space| 11      | 57.53   |
|                     | Original Space | 13      | 55.94   |
|                     | Projected Space| 13      | 59.44   |

the Indian Pine dataset is \(145 \times 145\) pixels in the spatial domain and 200 bands in spectral domain with 16 different classes. Botswana dataset consists of \(1476 \times 256\) pixels in the spatial domain and 145 bands in spectral domain with 14 different classes. Similarly, Pavia university consists of \(610 \times 610\) spatial pixels with 103 spectral bands of 9 different classes.

Fig. 2: Comparison of cost function values and mapped space dimension (n: Original dimension of the datasets).

B. Parameters Sensitivity

A few samples were randomly selected from each class for training while the rest data were used for testing purpose. Unless specified, we used 10 random data samples from each class for training. The dimensions chosen for the HS datasets are follows: Indian pine is 199, Botswana is 144 and Pavia university is 102. We employed different types of classifiers to compare the classification accuracy of HS data, such as support vector machine (SVM), K-nearest neighbor \((KNN (K = 1, 3, 5))\), linear discriminant classifier (LDC), quadratic discriminant classifier (QDC), and decision tree (TREE). We used linear kernel for SVM classifier.

Table II provides the statistics of the OA with respect to the variation of number of nearest neighbors \((N_w\) and \(N_b\)) for three HS datasets while fixing the number of train data samples to 10 and using SVM as the classifier. In practice, the number of nearest neighbors are always equal for both between-class and within-class \((N_b = N_w)\) to avoid the data imbalance. As observed from the Table II, the SVM classifier gives better accuracy when \(N_b = N_w = 9\) for two datasets out of three. Hence, we chose the value \(N_b = N_w = 9\) for subsequent experiments.

Fig. 2 shows the variations of cost function values with respect to the varied mapped space dimensions. In this experiment, we varied the dimensions of the data in the mapping space from \(n\) (original dimension) to \(n - 40\) and observe the cost function values. In Fig. 2 the cost function value is minimum for Indian pine and Pavia university data sets when the mapping dimension is one less than the original data dimensions \((n - 1)\). In case of Botswana data set, the cost function value is least when mapped space dimension is \(n - 35\) but it gives competitive results for the \(n - 1\) mapping dimension. Since, all the three datasets produce consistently better performance (cost function value) on the dimension \(n - 1\) dimensions in the mapping space, We chose that dimension \((n - 1)\) for rest of the experiments.

C. Classification Results With Different Classifiers

We compared the performances of different classification methods on original and newly mapped space without applying DR techniques.

Table III provides the statistics of the average accuracy (AA), kappa (\(\kappa\)), and overall classification accuracy (OA) on two spaces (original space and mapped space) for three HS datasets. From Table III we can observe that
TABLE II: Experimental results (AA:average accuracy, κ:kappa, and OA:overall accuracy) with different classifiers

| Classifiers | Datasets | Original Space | Projected Space | Original Space | Projected Space | Original Space | Projected Space |
|-------------|----------|----------------|----------------|----------------|----------------|----------------|----------------|
|             | Bollywood |                |                |                |                |                |                |
| SVM         | 69.16     | 56.43          | 0.5123         | 70.35          | 60.33          | 0.5629         | 89.18          | 89.12          | 0.8004         | 90.58          | 90.39          | 0.8956         | 74.47          | 66.45          | 0.5813         | 74.61          | 75.17          | 0.6373         |
| LDA         | 66.93     | 51.51          | 0.4817         | 75.87          | 77.66          | 0.7428         | 89.08          | 87.49          | 0.8642         | 96.67          | 96.45          | 0.9614         | 74.38          | 62.85          | 0.5436         | 90.03          | 89.12          | 0.8684         |
| QDC         | 63.31     | 50.30          | 0.4455         | 64.16          | 63.79          | 0.4990         | 88.52          | 86.88          | 0.8577         | 92.57          | 91.86          | 0.9116         | 72.16          | 73.80          | 0.8906         | 82.78          | 77.94          | 0.7590         |
| LDC         | 57.24     | 43.69          | 0.3736         | 58.89          | 51.70          | 0.4683         | 87.30          | 85.51          | 0.8426         | 90.37          | 89.54          | 0.8865         | 78.91          | 73.45          | 0.6004         |
| QDC         | 53.96     | 44.05          | 0.3736         | 58.14          | 45.25          | 0.3966         | 81.10          | 78.99          | 0.7722         | 82.38          | 80.70          | 0.7909         | 70.67          | 61.66          | 0.5251         | 74.12          | 67.85          | 0.6114         |
| TREE        | 50.36     | 39.64          | 0.3364         | 54.98          | 43.82          | 0.3797         | 80.00          | 79.40          | 0.7763         | 81.06          | 80.41          | 0.8173         | 61.05          | 51.88          | 0.4152         | 60.47          | 57.30          | 0.4886         |

Fig. 3: Comparison of classification performances of PCA with various classifiers (SVM, NN1, NN3, NN5, LDC, QDC and Decision Tree) on three different HS datasets (Botswana (3a), Indian Pines (3b) and Pavia University (3c)) for two spaces (original and mapped discriminative space).

Fig. 4: Comparison of classification accuracy with SVM classifier using different number of training samples per class on three different HS datasets (Botswana (4a), Indian Pines (4b) and Pavia University (4c)) for two spaces (original and mapped discriminative space).

D. Classification Results With Different DR Techniques

In our experiments, we used four different DR methods such as PCA, LDA, kernel PCA (KPCA) and marginal fisher.
analysis (MFA) followed by classification using linear SVM. The performance measurements are reported in Fig. 2. For these experiments, we used 10 samples from each class for training and the rest as testing samples. The number of reduced dimensions were varied from 20 – 60 with a step size of 5. In addition, we reported the classification performance of other classifiers on the original and projected space after reducing the dimension to 20 using PCA in Fig. 3.

Fig. 2 shows the variations of the overall accuracies of different DR methods with respect to different reduced dimensions on their original data space and newly mapped discriminative space. We can observe that all the DR methods on the proposed discriminative space outperform their counterparts on the original space for various reduced dimensions. In the case of Indian pine data in Fig. 4a all the DR methods in the proposed discriminative space significantly outperforms the DR in original space. Similar performance scores can also be observed in Fig. 4b and 4c with a few exceptions. The performance in Fig. 2 proves the robustness of the proposed discriminative space to the variation of number of reduced dimensions in DR methods.

The performance of different classifiers with PCA are shown in Fig. 3. In these experiments, we considered multiple classifiers, such as SVM, 1NN, 3NN, 5NN, LDC, QDC and decision tree on original and projected space. It can be observed in Fig. 3a that the overall classification accuracy of PCA in the proposed discriminative space mostly outperforms the accuracy on the original space for Botswana dataset. PCA with classifier LDC and QDC also achieved an equivalent performance for Botswana dataset. In the case of Indian pine dataset in Fig. 3b the accuracy on the proposed space outperforms the accuracy on the original space by a large margin for all the classifiers. Similarly, the PCA on proposed space also performs better for Pavia university dataset (Fig. 3c).

E. Classification Results With Different Train Data

We also experimented the effect of variation of training data size against the classification accuracy. The experimental results for the performance of the original and projected data space without dimensionality reduction are shown in Fig. 4. As can be seen in Fig. 4a the performance is better in the proposed discriminative space for different training sample size in Botswana dataset. Similar trends are also observed for Indian Pines (Fig. 4b) and Pavia University (Fig. 4c) dataset. This signifies the effectiveness of the proposed method irrespective of the size of the available data for training.

IV. CONCLUSION

This work proposed a novel approach in which the data are linearly transferred from their original data space to a more discriminative space to improve the classification performance. The proposed method finds a projection matrix in a supervised manner by solving the nonlinear cost function to preserve the intrinsic geometrical manifold while improving the class discrimination. The experiments show that among the classifiers the 1NN classifier gives up to 8 –10% and among DR methods KPCA with SVM classifier gives up to 5 –7% improved performance in projected space for all the three datasets. The promising experimental results of different classifiers with varying number of training data samples on the proposed discriminative space demonstrate its robustness as well as generic applicability.

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