Influence of technological deviations on the basic operational characteristics of hydrodynamic bearings

Y Tarasevych\textsuperscript{1}, I Savchenko\textsuperscript{1}, N Sovenko\textsuperscript{1}

\textsuperscript{1}General Mechanics and Machine Dynamics Department, Sumy State University, 2 Rymskogo-Korsakova st., 40007 Sumy, Ukraine

E-mail: tyulia@ukr.net

Abstract. The main purpose of this paper is to define a methodology to determine the analytical approximate closed-form expression of probabilistic characteristics of load capacity of liquid-lubricated journal bearings in the case of a steady flow regime. Influence of random change of basic geometrical parameters of journal bearings (gap value and eccentricity) on the load capacity of bearing is considered. It is shown that the actual value of hydrodynamic force in bearing can substantially differ from a calculation one. The got results confirm the necessity of application of such approach at consideration both geometrical and operating parameters of bearing, for example, axes misalignment, deviation of surface form, roughness and others, which have casual nature too.

1. Introduction and statement of the problem

The operational characteristics of rotary machines are largely determined by the efficiency of the supporting and sealing units. A number of specific requirements are imposed on the work of the supporting units. This requirements different from those that should be satisfied by the sealing units, namely: sufficient bearing capacity with small dimensions, high vibration stability under all operating modes, minimal friction and wear of the working surfaces for a given life-time, low rate of lubricating - cooling material, the possibility of using a working medium as a lubricant, easiness of installation and operation.

At present, complex tribotechnical devices with various physical principles of creating bearing capacity, including rolling bearings, magnetic bearings, fluid friction bearings and various combinations thereof, are used as rotor supports. The using of rolling bearings as supports for high-speed rotors is limited by their extreme speed and durability. Therefore, the use of sliding bearings makes it possible to provide reliable operation of the rotary machine in a wide range of speeds and loads. A variety of types of sliding bearings can be conditionally divided according to the following criteria:

- by the principle of operation (hydrodynamic, hydrostatic, combined);
- by the direction and nature of the load (radial, axial and radial-axial);
- by type of lubricant (liquid, gas). Moreover, under operation in liquid lubrication conditions, in some cases, it is possible to transfer part of the volume of the lubricant from the liquid to the gaseous phase and vice versa;
- by design.
The variety of these categories of sliding bearings is united by one feature: they are all non-standard elements of machines, in the design and calculation of which in each specific case additional experimental and theoretical studies are needed. Despite the fact that a significant number of works have been published in the area of calculating and designing of sliding bearings, created mathematical models and proposed methods of calculating characteristics of bearings [1-6] mainly do not take into account the fact that the majority of both the geometric and operational parameters of these complex devices are random. There are only a few studies considering stochastic nature of some parameters. For example, the effect of a spatially randomly varying component of roughness has been studied in [7] using the stochastic finite element method. The potential inaccuracies in calculating the dynamic coefficients using the ratio of pressure change due to a film thickness change are reported in [8].

The thickness of the lubricating layer, which is one of the main operating parameters, is determined by the appropriate tolerances for the manufacture of parts and assembly of the machine, the quality of the surface treatment (the presence of macro and microroughness), which are random variables. In addition, possible misalignment of the main bearing surfaces and their variation under operation are also random. Therefore, the study of the probability characteristics of sliding bearing is not only an actual scientific, but also an important practical problem.

2. Main equations and the solution of the problem
A mathematical model of a full journal sliding bearing (Figure 1) is considered.

![Figure 1. Mathematical model of sliding bearing.](image)

The determination of the pressure field in the lubricating layer is based on the solution of the modified Reynolds equation in the isothermal setting \( \mu = \text{const} \). It has the form:

\[
\frac{\partial}{\partial x} \left( h \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h \frac{\partial p}{\partial z} \right) = 6 \mu \frac{\partial}{\partial x} (Uh) - 12 \mu V
\]

where \( U, V \) – components of shaft speed in the direction of the axes \( x \) and \( y \), respectively.

To solve the problem in the first approximation the following assumptions are assumed. The lubricating fluid is incompressible, i.e. \( \rho = \text{const} \), \( \frac{\partial p}{\partial t} = 0 \). The flow of the lubricant is steady, \( U = \text{const} \). As the changes in the characteristics of the fluid flow in the axial direction (along the \( O_z \) axis) are significantly less than their changes in the circumferential direction, the second term in Eq.
can also be neglected. Taking into account the assumed assumptions, the Reynolds equation for determining the pressure distribution in the bearing gap takes the form:

\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = 6 \mu h \frac{\partial h}{\partial x}
\]  

(2)

After integrating equation (2) once it is obtained:

\[
h^3 \frac{\partial p}{\partial x} = 6 \mu U h + C
\]

The constant \( C \) can be determined from the condition: under \( h = h_\ast \), \( p = p_{\text{max}} \) and \( \frac{\partial p}{\partial x} \bigg|_{h=0} = 0 \). So, \( C = -6 \mu U h_\ast \), and equation (2) is transformed to the form: \( h^3 \frac{\partial p}{\partial x} = 6 \mu U (h - h_\ast) \).

According to Figure 1, the gap in an arbitrary bearing cross-section can be presented by the formula \( h = R - r + e \cos \varphi = H(1 + e \cos \varphi) \). The speed of the shaft wall is \( U = \omega r \).

Further solution of the problem is conducted in the movable system of coordinates, one axes of that passes through the line of centers \( OO_1 \). Passing to the cylindrical coordinate \( x = r \varphi \) and after integration, it is obtained

\[
\frac{\partial p}{\partial x} = \frac{6 \mu \omega r^2 e (\cos \varphi - \cos \varphi_\ast)}{H^2 (1 + e \cos \varphi)}
\]

where \( H \) is a random variable, the measurement limits of which depend on the tolerances accepted. For values with two-sided tolerances, the normal distribution law is applied, so \( H \) can be assumed to have the normal distribution. As a mathematical expectation of this parameter can be taken its calculated value. The actual value of the gap deviation from its design value, due to the landing chosen for the bearing, can be commensurable with the value of the gap itself. For example, for a shaft diameter of 140 mm and for one of the recommended landings of H7 / c8, its tolerance is 0.103 mm, and the radial clearance caused by the deviations in the dimensions of the shaft and sleeve can vary from 0.1 to 0.15 mm.

The random variables \( H, e \) are independent. Therefore, the complex probability density can be written in the form \( f_j(H, e) = f_j(H) f_j(e) \). For eccentricity, all directions are equally possible, and proceeding from physical properties, this random variable can take only positive values, so for its description the truncated Rayleigh law can be used:

\[
f_j(e) = \frac{C e}{\sigma^2_e} \exp \left[ - \frac{e^2}{2\sigma^2_e} \right]
\]

where the constant \( C \) is determined from the normalization condition for the probability density:

\[
C = \left( 1 - \exp \left[ - \frac{1}{2\sigma^2_e} \right] \right)^{-1/2}
\]

Since the probability that the eccentricity takes values close to zero and close to unity is equally small, it can be assumed: \( \sigma_e = 0,288 \).

Under steady mode the external loading on bearing is balanced by hydrodynamic force, acting in a bearing gap:

\[
F = \int_{\varphi_\ast}^{\varphi_\ast} p(\varphi) \frac{ld}{2} \cos \left[ \pi - (\varphi - \varphi_\ast) \right] d\varphi =
\]
\[
\frac{3\mu \omega' \varepsilon}{H^2} \int_{\phi_1}^{\phi_2} \cos[\pi - (\phi + \phi_1)] \int_{\phi_1}^{\phi_2} \cos \varphi - \cos \varphi_1 \int_{\phi}^{\phi_2} \left(1 + \varepsilon \cos \varphi\right)^{-1} d\varphi d\phi, \quad (4)
\]

Where \(\phi_1\) is a current angle, \(\pi - \phi\) is angle under which the pressure distribution in a bearing gap is maximum, \(\phi_1, \phi_2\) are angles which determining beginning and the end of bearing load zone. Under assumed problem statement (Fig. 1): \(\phi_1 = 0, \phi_2 = \pi + \phi\). The angle \(\phi\) is determined numerically, depending on the bearing geometry and the required bearing capacity.

The range of variation in pressure force of the bearing gap can be defined by the inequality:

\[
p < \frac{6 \mu \omega' \varepsilon}{H^2} \int_{\phi_1}^{\phi_2} f(\varepsilon, \varphi, \varphi) d\phi
\]

Then the density of the probability of pressure in the bearing clearance can be represented as:

\[
g(P) = \int_{0}^{\infty} f_1(\varepsilon) f_2((\gamma(P, \varepsilon)) \frac{\partial \gamma(P, \varepsilon)}{\partial P} d\varepsilon \quad (5)
\]

where \(\gamma(P, \varepsilon)\) is the inverse function by variable \(H\), \(f_1(H) = \frac{1}{\sigma_H \sqrt{2\pi}} \exp \left(-\frac{(H - m_H)^2}{2\sigma_H^2}\right)\), \(m_H\) is the mean value of the gap, \(\sigma_H\) is the standard deviation of the gap.

The basic characteristics of the bearing operation are: volume flow of the lubricant, the power loss due to friction, load capacity.

The effect of the random variation of \(H\) and \(\varepsilon\) on the load capacity of bearing is considered. The standard deviation of the gap value in the bearing can be represented as: \(\sigma_H = \Delta m_H\). Using the obtained probability density (5), the basic moment characteristics of the hydrodynamic pressure force is determined: mean value and standard deviation.

\[
\begin{align*}
P & \quad 1.5\times10^4 & \quad 1.2\times10^4 & \quad 9\times10^3 & \quad 6\times10^3 & \quad 3\times10^3 \\
\Delta & \quad 0 & \quad 0.06 & \quad 0.12 & \quad 0.18 & \quad 0.24 & \quad 0.3 & \quad 0.36 \\
& \quad m_P & \quad m_P + 3\sigma_P & \quad m_P & \quad m_P - 3\sigma_P & \quad m_P
\end{align*}
\]

**Figure 2.** Dependence of mean value of pressure forces in the bearing gap and interval of possible values of radial force on the relative value of the standard deviation of the gap.

The change in the mean value as a function of the change in the root-mean-square deviation of the value of \(H\) and interval of possible values of hydrodynamic force (without taking into account the effect of a random change in the eccentricity) is shown in Figure 2.

As undertaken studies show with increasing of standard deviation value of geometric gap of bearing the actual value of radial force can be substantially differ from the calculation one.
3. Conclusions
As a rule the deterministic methods of basic characteristics calculation of sliding bearing are used. These methods do not take into account stochastic nature of geometric parameters of bearings. Lately a lot of different studies are devoted to the determination the effect of micro and macro roughnesses of bearing surfaces on its loading characteristic. However, taking into account admitted in engineering tolerances, the effect of these parameters can be negligible as compared to indetermination (stochastic) of calculated geometrical parameters. For example, under value of root-mean-square deviation of gap \( H_{m1} \) (initial value of gap is \( 1.4 \times 10^{-4} \)) the interval of possible gap values is \( (9.8 \times 10^{-3} ; 1.82 \times 10^{-4}) \) and deviation of calculation value from its mean value of bearing load capacity is 14%. And under value of root-mean-square deviation of gap \( 0.3m_{\mu} \) difference in calculation and possible value is 31%. As undertaken studies show with increasing of standard deviation value of geometric gap of bearing the interval of possible values of radial force is substantially increases. And as the expected value of bearing capacity decreases, the probability of bearing failure raises.

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