ANALYTICAL INVESTIGATION of CURVATURE DUCTILITY of REINFORCED CONCRETE COLUMNS

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Abstract: In this study; the effect of axial load; longitudinal reinforcement diameter; transverse reinforcement diameter and transverse reinforcement spacing were investigated on the moment curvature relationships of reinforced concrete columns. Reinforced concrete columns having different geometries were designed considering the regulations of Turkish Building Earthquake Codes (2018). Investigations of the effect of axial load; longitudinal reinforcement diameter; transverse reinforcement diameter and transverse reinforcement spacing on the behavior of the concrete columns are the main purpose of this study. The behavior of the columns was investigated from the moment-curvature relation; by considering the nonlinear behavior of the materials taken into account. The moment-curvature relationships of the reinforced concrete column cross-sections having different axial load levels have been obtained by considering Mander model which considers the lateral confined concrete strength. The examined effects of the parameters on the column behavior were evaluated in terms of curvature ductility and the moment capacity of the cross-section. In the designed column cross-sections; different parameters effecting the moment-curvature relationships were calculated and compared. It is observed that the variation of the axial load; longitudinal reinforcement diameter; transverse reinforcement diameter and transverse reinforcement spacing have an important effect on the moment-curvature behavior of the reinforced concrete columns.

Keywords: Transverse reinforcement; nonlinear behavior; confined concrete strength; axial load; moment-curvature; curvature ductility.

Betonarme Kolonlarin eğrilik sünekliğinin analitik olarak araştırılması

Öz: Bu çalışmada; eksenel yük; boyuna donatı çapı; sargı donatı çapı ve sargı donatı aralığının değişiminin betonarme kolonların moment-egrilik ilişkisine olan etkisi incelenmiştir. Farklı geometriye sahip betonarme kolon modelleri Türkiye Bina Deprem Yönetmeliği (2018) hükümlerine uyularak tasarlanmıştır. Farklı geometri ve alana sahip betonarme kolon modellerinin davranışına eksenel yük; boyuna donatı çapı; sargı donatı çapı ve sargı donatı aralığının etkisinin araştırılması bu çalışmının amacı olmuştur. Betonarme kolonların davranışları; malzemelerin doğrusal olmayan davranışları göz önünde alınarak moment-egrilik ilişkisinden elde edilmiştir. Betonarme kolon kesitlerinin moment-egrilik ilişkileri farklı eksenel yük seviyeleri için yana sargı basıncını göz önüne alınan Mander modeli ile elde edilmiştir. Incelenen parametrelerin kolon davranışı etkileri; eğrilik sünekliği ve kesit dayanımı açısından değerlendirilmiştir. Tasarlanan betonarme kolon kesitlerde; farklı parametrelerin moment eğrilik ilişkisindeki etkisi hesaplanarak karşılaştırılmıştır. Eksenel yük; boyuna donatı çapı; sargı donatı çapı ve sargı donatı aralığının değişiminin, betonarme kolonların moment-egrilik davranışını üzerinde önemli bir etkiye sahip olduğu görülenmiştir.

Anahtar Kelimeler: Sargı donatısı; doğrusal olmayan davranış; sargılı beton dayanımı; eksenel yük; moment-egrilik; eğrilik sünekliği.

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1. INTRODUCTION

In reinforced concrete structures, reinforced concrete columns are one of the most crucial elements under earthquake loads. Column mechanisms are very critical to prevent total collapse in earthquakes. The objective performance levels of reinforced concrete structures could not be ensured due to the failure of some critical reinforced concrete columns. Because of this, determining the behavior of the structures should be known well to design earthquake-resisting structures (Dok et al., 2017). In seismic zones, it is important to design structures, with power ranging deformation beyond the elastic deformations without losing its ability to stay in service, in other words designing structures with ductile behavior. The current philosophy used in the seismic design of reinforced concrete frames auto-stable is based on the hypothesis of the formation of plastic hinges at critical sections, the ability of the latter to resist several cycles of inelastic deformations without significant loss in bearing capacity is evaluated in terms of available ductility. The ductility of a reinforced concrete beam section is measured by the expression $\mu = \frac{C_u}{C_y}$, where $C_u$ represents the curvature of the section when the concrete reaches its ultimate limit state and $C_y$, is the curvature of the section when steel in tension reaches the elastic limit state (Yousef and Chemrouk, 2012). Ductility of reinforced concrete structures is a desirable property where resistance to brittle failure during flexure is required to ensure structural integrity. Ductile behavior in a structure can be achieved through the use of plastic hinges positioned at appropriate locations throughout the structural frame. These are designed to provide sufficient ductility to resist structural collapse after the yield strength of the material has been achieved. The available ductility of plastic hinges in reinforced concrete is determined based on the shape of the moment-curvature relations (Olivia and Mandal, 2005). Ductility may be defined as the ability to undergo deformations without a substantial reduction in the flexural capacity of the member (Park and Ruitong, 1988). According to Xie et al., (1994) this deformability is influenced by some factors such as the tensile reinforcement ratio, the amount of longitudinal compressive reinforcement, the amount of lateral tie and the strength of concrete. The behavior of reinforced concrete elements are determined by the cross-sectional behavior of elements. Cross-sectional behavior depends on the materials designed of the cross-section and the loading on that particular cross-section. The behavior of a reinforced concrete cross-section under bending moment or bending moment plus axial force can be monitored from moment-curvature relationship.

The aim of Olivia and Mandal (2005) study is to examine the influence of three variables on curvature ductility of reinforced concrete beams. A computer program was developed to predict moment-curvature and available curvature ductility of reinforced concrete beams with or without axial loads. Ten beams with different variables were analyses using the program. The variables measured are concrete strength, amount of longitudinal reinforcement and spacing of transverse reinforcement. The input consists of beam geometry, material properties and loading. A confined stress-strain curve for concrete proposed by Saatcioglu and Razvi (1992) is applied in the program, while, steel stress-strain model is adopted from BS 8110 (British Standard Institution, 1985). Computer analysis indicates that the curvature ductility increases with the increase of longitudinal reinforcement and concrete strength. On the other hand, the spacing of transverse reinforcement does not have any significant influence on the curvature ductility.

Bedirhanoglu and Ilki (2004) obtained the analytical moment-curvature relationships for reinforced concrete cross-sections by using three different models for confined concrete. The theoretical moment-curvature relationships were then compared with experimental data reported in the literature. The results showed that the theoretical moment-curvature relationships obtained by all of these three models were in quite good agreement with experimental data. In the second part, a parametric investigation was carried out for examining the effects of various variables on the moment-curvature relationships, such as quality of concrete, level of axial load, amount and arrangement of transverse reinforcement.
İlki et al. (2005a) carried out non-linear seismic analyses of a six-story reinforced concrete building, representing many common characteristics of typical existing buildings in Istanbul, before and after retrofitting the columns of the building by carbon fiber reinforced polymer (CFRP) composite sheets in the transverse direction. For this purpose, moment-curvature relationships of original and retrofitted columns were obtained by cross-sectional fiber analysis, during which material models such as stress-strain curves of reinforcing steel, unconfined concrete, and concrete confined by CFRP sheets were utilized. According to the results of the analyses, significantly larger lateral displacement and relatively higher lateral strength with respect to original performance are possible by jacketing the columns of the building with CFRP sheets.

İlki et al. (2005b) carried out an analytical cross-sectional moment-curvature analysis for FRP jacketed reinforced concrete columns using the available stress-strain models for FRP (fiber reinforced polymer) confined concrete. In the analysis rupture of FRP composite sheets, the interaction of FRP jacket and internal transverse reinforcement and strain hardening of longitudinal reinforcement are taken into account. The analytical procedure is used for predicting the moment-curvature relationships of several specimens tested by different researchers. After verifying the accuracy of the analytical predictions with experimental data, a parametric study is carried out to investigate the effects of different variables on flexural behavior. These variables are the level of axial load, the thickness of FRP jacket, cross-section shape, corner radii and compressive strength of unconfined concrete.

Foroughi and Yuksel (2020) investigated the effect of the material model, axial load, longitudinal reinforcement ratio, transverse reinforcement ratio and transverse reinforcement spacing on the behavior of square reinforced concrete cross-sections. The effect of axial load, transverse reinforcement diameter and transverse reinforcement spacing on the behavior of reinforced concrete column models have been analytically investigated. The moment-curvature relationships for different axial load levels, transverse reinforcement diameter and transverse reinforcement spacing of the reinforced concrete column cross-sections were obtained considering the Mander confined model (Mander et. al, 1988). It was examined behavioral effects of the parameters were evaluated by comparing the curvature ductility and the cross-section strength. It has been found that transverse reinforcement diameters and transverse reinforcement spacing are effective parameters on the ductility capacities of the column sections. Axial load is a very important parameter affecting the ductility of the section. It has been observed that the cross-sectional ductility of the column sections increases with the decrease in axial load.

In this study, reinforced concrete columns having different cross sections were designed and the effects of the longitudinal reinforcement diameter, axial loads level, transverse reinforcement diameter and transverse reinforcement spacing on the behavior of these models were investigated. The behavior of the reinforced concrete column models was investigated through the relation of moment-curvature. Eighteen different square, circular and rectangular reinforced concrete columns having different longitudinal and transverse reinforcements were analyzed. Moment-curvature relations were obtained and presented in graphical form using SAP2000 Software (CSI, V.20.1.0) which takes nonlinear behavior of materials into consideration. The designed reinforced concrete cross section models are considered to be composed of three components: cover concrete, confined concrete and reinforcement steel. The SAP2000 Software material models are defined considering the Mander unconfined concrete model for cover concrete, and the Mander confined concrete model for core concrete. A concrete model proposed by Mander et al. (1988) which is widely used, universally accepted and mandated in Turkish Building Earthquake Code (TBEC, 2018) has been used to determine the moment-curvature relationships of reinforced concrete members. For reinforcement modeling stress-strain relationship given in TBEC (2018) was used. The examined behavioral effects of the parameters were evaluated by the curvature ductility and the cross-section
strength. Curvature ductility of square, rectangular and circular columns get different values when longitudinal reinforcement diameter, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels were changed. Each cross sectional analysis is compared according to criteria which can change the curvature ductility of square, rectangular and circular reinforced concrete column cross sections. The moment-curvature curves were drawn for different models and were interpreted by comparing the curves.

2. MATERIAL and METHOD

The aim of this paper is to examine the influence of four parameters on curvature ductility of reinforced concrete columns. SAP2000 software was used to predict moment-curvature and available curvature ductility of reinforced concrete columns having different axial load levels \( (N/N_{\text{max}}) \). Eighteen reinforced concrete column models with different parameters were analyzed using the program. The parameters investigated in the moment-curvature relations and ductility of the reinforced concrete column models are the longitudinal reinforcement diameter, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels. By using the Mander model (Mander et. al, 1988), the moment-curvature relationships of the reinforced concrete columns are obtained by using the SAP2000 software, which performs non-linear analysis for different models designed. For all column models, C30 was chosen as concrete grade and S420 was selected as reinforcement for the reinforcement behavior model, the stress-strain curves given in TBEC, (2018) were used.

![Figure 1: Cross-section of reinforced concrete column models](image)

Reinforced concrete column models having different cross-sections were created in order to examine the effects of cross-sectional behavior of design parameters of reinforced concrete column sections. In order to investigate the effect of longitudinal reinforcement diameter, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels, the column models having dimensions of 500mm×500mm square cross section, 300mm×550mm rectangular cross section and 565mm diameter circular cross sections were designed (Fig. 1). The details of the cross-sections with different transverse reinforcement diameters and transverse reinforcement spacing are given in Table 1.

| Table 1. Details for the designed column model cross-sections |
|-------------------------------------------------------------|
| **Cross-sectional dimensions (mm)**                         |
| Square | Circular | Rectangular | Longitudinal reinforcement | Transverse reinforcement | Axial load level |
| No | b | h | No | D | No | b | h | Diameter (mm) | Spacing (mm) |
| S1 | 500 | 500 | CR1 | REC1 | 300 | 550 | Φ20 |
| S2 | 500 | 500 | CR2 | REC2 | 300 | 550 | Φ22 |
| S3 | 500 | 500 | CR3 | REC3 | 300 | 550 | Φ24 |
| S4 | 500 | 500 | CR4 | REC4 | 300 | 550 | Φ26 |
| S5 | 500 | 500 | CR5 | REC5 | 300 | 550 | Φ28 |
| S6 | 500 | 500 | CR6 | REC6 | 300 | 550 | Φ30 |
|       |       |       |       |       |       |       |       | Φ8  |
|       |       |       |       |       |       |       |       | Φ10 |
|       |       |       |       |       |       |       |       | 50  |
|       |       |       |       |       |       |       |       | 75  |
|       |       |       |       |       |       |       |       | 100 |
3. NUMERICAL STUDY

Theoretical moment-curvature analysis for reinforced concrete columns indicating the available bending moment and ductility can be constructed providing that the stress-strain relations for both concrete and steel are known. The objective of this study is to analyze the curvature ductility of eighteen reinforced concrete columns with four different parameters. Moment-curvature relationships were obtained by SAP2000 Software which takes the nonlinear behavior of materials into consideration. The numerical model was employed to analyses the square, circular and rectangular cross-sectional columns. The calculation of moment-curvature curves and ductility of reinforced concrete columns having different geometries are calculated in the following paragraphs.

Six different longitudinal reinforcement diameters and three different transverse reinforcement diameters are used for each concrete column models. In order to examine the effect of longitudinal reinforcement diameter on cross-sectional behavior, six different longitudinal reinforcement diameters (Φ20mm, Φ22mm, Φ24mm, Φ26mm, Φ28mm ve Φ30mm) were selected. The diameters of the reinforcements and reinforcement ratios used in the cross-sections have been determined by considering the limitations given in TS500 (2000) and TBEC (2018). The effects of longitudinal reinforcement diameters on the moment-curvature relationship for reinforced concrete column sections are given in Fig. 3 (square column section), Fig. 5 (circular column section) and Fig. 7 (rectangular column section). As shown in Figures, for the constant transverse reinforcement diameter, moment-curvature graphs of four different axial load levels are presented comparatively. The bending moment capacity and ductility of the reinforced concrete column sections are clearly seen from the moment-curvature relations.

The parameters affecting the ductility of the different reinforced concrete columns were investigated. Different transverse reinforcement diameters (Φ8mm, Φ10mm and Φ12mm) were selected in order to investigate the effect of the transverse reinforcement ratio and the transverse reinforcement spacing on the cross-section behavior. For each model, three different transverse reinforcement spacing have been selected (50mm, 75mm and 100mm). The moment-curvature relationships obtained from the analytical results are presented in graphical form. Fig. 2 (square column section), Fig. 4 (circular column section) and Fig. 6 (rectangular column section) show the moment-curvature comparisons for different transverse reinforcement spacing and different axial loads for the designed columns cross sections. In Figures, moment-curvature relationships are given for different transverse reinforcement spacing and axial loads according to the transverse reinforcement diameters of 8mm, 10mm and 12mm, respectively.

The combined effect of vertical and seismic loads (N_{dm}), gross section area of column shall satisfy the condition A_c ≥ N_{dmax}/0.40f_{ck} (TBEC, 2018). In this section, the moment-curvature relationships of the column sections were investigated for the values of N/N_{max} ratios of 0.10, 0.20, 0.30 and 0.40. To investigate the effect of axial force on the cross-section behavior; square and circular columns models were investigated under four different axial loads (750kN, 1500kN, 2250kN and 3000kN). The rectangular reinforced concrete columns models were investigated under axial loads of 495kN, 990kN, 1485kN and 1980kN. Moment-curvature relationships for the designed column cross-sections are presented for different axial load level, transverse reinforcement diameter and transverse reinforcement spacing. Using the yield curvatures (Φ_y), ultimate curvatures (Φ_u) the curvature ductility (μ) values are calculated.

4. RESULTS and DISCUSSION

In this study, design parameters of reinforced concrete members are investigated to determine the behavior of reinforced concrete square, rectangular and circular columns. The sections designed according to TBEC (2018) and TS500 (2000) were analyzed by SAP2000 to generate moment-curvature relationship. In this part of the study, the moment-curvature
relations are obtained by changing the longitudinal reinforcement diameter, transverse reinforcement diameter, transverse reinforcement spacing and axial load level. The numerical model was employed to analyses the curvature ductility of eighteen reinforced concrete columns with different parameters. The moment-curvature relationships of reinforced concrete square, rectangular and circular columns were determined and the results were prepared in graphs. The effects of the major variables on moment-curvature curves are discussed in the following paragraphs.

a. Moment-Curvature Relationship of Square Columns

The moment-curvature relationships obtained from the analytical results are presented in graphical form. Moment-curvature relationships of square columns for different transverse reinforcement diameter, spacing and axial load levels show in Fig. 2 (longitudinal reinforcement diameter is constant). Fig. 3 shows the moment-curvature comparisons for different longitudinal reinforcement diameter and different axial load levels in the designed square column cross-sections (transverse reinforcement diameter and spacing is constant).

![Graphs of moment-curvature relationships for square columns with different transverse reinforcement diameters and axial load levels.](image_url)

**Figure 2:**

*Moment-curvature relationships of square columns for different transverse reinforcement diameter, spacing and axial load levels*
Figure 3:
Moment-curvature relationships of square columns for different axial load levels and longitudinal reinforcement diameters (transverse reinforcement $\Phi_{10/50\text{mm}}$)

b. Moment-Curvature Relationship of Circular Columns

Moment-curvature relationships of circular columns for different transverse reinforcement diameter, transverse reinforcement spacing and axial load levels are shown in Fig. 4 (longitudinal reinforcement diameter is constant). Fig. 5 shows the moment-curvature comparisons for different longitudinal reinforcement diameter and different axial load levels in the designed circular column cross-sections (transverse reinforcement diameter and spacing is constant).
Foroughi S., Yüksel S.B.: Analyt. Invest. of Curv. Duct. of Reinf. Conc. Columns

**Figure 4:**
Moment-curvature relationships of circular columns for different transverse reinforcement diameter, spacing and axial load levels

**Figure 5:**
Moment-curvature relationships of circular columns for different axial load levels and longitudinal reinforcement diameters (transverse reinforcement $\Phi 10/50$mm)

c. **Moment-Curvature Relationship of Rectangular Columns**

Moment-curvature relationships of rectangular columns for different transverse reinforcement diameter, transverse reinforcement spacing and axial load levels are shown in Fig. 6 (longitudinal reinforcement diameter is constant). Fig. 7 shows the moment-curvature comparisons for different longitudinal reinforcement diameter and different axial load levels in the designed rectangular column cross-sections (transverse reinforcement diameter and spacing is constant).
Figure 6:
Moment-curvature relationships of rectangular columns for different transverse reinforcement diameter, spacing and axial load levels

Figure 7:
Moment-curvature relationships of rectangular columns for different axial load levels and longitudinal reinforcement diameters (transverse reinforcement $\Phi_{10}/50$mm)
In the square, circular and rectangular reinforced concrete columns, Influence of different parameters on the ductility are shown in Fig. 8, 9 and 10 respectively. As it shall be seen from influence of different parameters on these ductility relations, ductility decreases with the increase of axial load \((N/N_{\text{max}} \geq 0)\) ratio where the transverse reinforcement is constant. However, when the axial load is small in the same cross-sections, (transverse reinforcement is constant) the ductility in the cross-section is high. The increase in transverse reinforcement diameter increases the ductility of the cross-section.

**Figure 8:**
Influence of different parameters on the ductility of the square column cross-sections

**Figure 9:**
Influence of different parameters on the ductility of the circular column cross-sections

**Figure 10:**
Influence of different parameters on the ductility of the rectangular column cross-sections
The following results were obtained from the moment curvature analyses of the square, circular and rectangular columns:

When the analysis results are examined, it is observed that the variation of the axial load and transverse reinforcement have an important effect on the moment-curvature behavior of the reinforced concrete cross-sections. The axial load level is an effective parameter on the moment-curvature results. With increasing axial load values yield curvature, yield moment and ultimate moment values increase, however, the ultimate curvature and curvature ductility values decrease as the axial load values increase for the constant transverse reinforcement diameters and spacing. As can be seen from the moment-curvature relationships, the curvature ductility values of the reinforced concrete columns for constant transverse reinforcement and longitudinal reinforcement ratios decrease with increasing axial load ratio. Axial load is very important parameter affecting the ductility of the section. Significant reductions in ductility capacities of the column sections under increasing axial force have been observed. As can be seen from the moment-curvature graphs, it is observed that the cross-section ductility decreases when the transverse reinforcement spacing is increased under constant axial load. It is observed that the cross-section ductility and the curvature increase significantly with the reduction of the transverse reinforcement spacing. It is observed that the ratio of transverse reinforcement is effective in cross-section behavior of reinforced concrete cross-section. The increase in transverse reinforcement diameter increases the ductility of the cross-section and the maximum moment bearing capacity. The increase in the transverse reinforcement diameter increases the ultimate moment, ultimate curvature and curvature ductility values, but yield moment and yield curvature values remain almost constant (transverse reinforcement spacing and axial load levels are the constant). Yield moment, yield curvature, ultimate moment and ultimate curvature values increases however, curvature ductility values decreases as the longitudinal reinforcement diameter increases while other parameters kept constant.

5. CONCLUSION

When the analysis results are examined, it is observed that the variation of the axial load, longitudinal reinforcement diameter, transverse reinforcement diameter and transverse reinforcement spacing have an important effect on the moment-curvature behavior of the reinforced concrete columns. Axial load, transverse reinforcement diameter and spacing are very important parameters affecting the ductility of the cross-section. Yielding and ultimate moment capacities of the sections increase when the transverse reinforcement spacing decreases. It is observed that the cross-section ductility and the curvature ductility increase significantly with the reduction of the transverse reinforcement spacing.

The ratio of transverse reinforcement is an effective parameter on cross-section behavior of reinforced concrete columns. The ductile behavior for reinforced concrete column sections is observed due to the increment of curvature ductility with the increase of transverse reinforcement diameter. As the diameter of the transverse reinforcement increases, the moment capacity of the cross-section increases. The increase in the axial load level causes curvature values to decrease. In cases where the axial load is low, reinforced concrete sections have a ductile behavior. Yield and ultimate moment capacities of the members increase with the increment of longitudinal reinforcing ratio for columns section. Moreover, with increase of transverse reinforcement ratio, the more ductile behavior is achieved due to increment of curvature ductility on reinforced concrete columns. In order to see the real behavior of a reinforced concrete cross-section, a concrete model that takes the transverse reinforcement ratio into consideration should be used.
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