Neutrino Masses Induced by $R$-Parity Violation in a SUSY SU(5) Model with Additional $\bar{5}'_L + 5'_L$

Yoshio Koide

Department of Physics, University of Shizuoka, 52-1 Yada, Shizuoka 422-8526, Japan
E-mail address: koide@u-shizuoka-ken.ac.jp

Abstract

Within the framework of an SU(5) SUSY GUT model, a possible general form of the neutrino mass matrix induced by $R$-parity violation is investigated. The model has matter fields $\bar{5}'_L + 5'_L$ in addition to the ordinary matter fields $\bar{5}_L + 10_L$ and Higgs fields $H_u + \bar{H}_d$. The $R$-parity violating terms are given by $\bar{5}_L 5_L 10_L$, while the Yukawa interactions are given by $\bar{H}_d 5'_L 10_L$. Since the matter fields $\bar{5}_L$ and $\bar{5}_L'$ are different from each other at the unification scale, the $R$-parity violation effects at a low energy scale appear only through the $\bar{5}_L \leftrightarrow \bar{5}_L'$ mixings. In order to make this $R$-parity violation effect harmless for proton decay, a discrete symmetry $Z_3$ and a triplet-doublet splitting mechanism analogous to that in the 5-plet Higgs fields are assumed.

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1 Introduction

As an origin of the neutrino masses, the idea of the radiative neutrino mass [1] is very interesting as well as the idea of the neutrino seesaw mechanism [2]. However, currently, the latter idea is influential, because it is hard to embed the former model into a grand unification theory (GUT). For example, a supersymmetric (SUSY) model with $R$-parity violation can provide radiative neutrino masses [3], but the model cannot be embedded into GUT, because the $R$-parity violating terms induce proton decay inevitably [4].

Recently, the author [5] has proposed a model with $R$-parity violation within the framework of an SU(5) SUSY GUT: we have quark and lepton fields $\bar{5}_L + 10_L$, which contribute to the Yukawa interactions as $H_u 10_L 10_L$ and $\bar{H}_d \bar{5}_L 10_L$; we also have additional matter fields $\bar{5}'_L + 5'_L$ which contribute to the $R$-parity violating terms $\bar{5}'_L \bar{5}'_L 10_L$. Since the two $\bar{5}_L$ and $\bar{5}_L'$ are different from each other, the $R$-parity violating interactions are usually invisible. The $R$-parity violating effects become visible only through $\bar{5}_L \leftrightarrow \bar{5}_L'$ mixings in low energy phenomena.

In the previous model [3], a discrete symmetry $Z_3$ has been assumed, and their quantum numbers have been assigned as $\bar{5}_L(-) + 10_L(+)$ and $\bar{5}'_L(+)$, and $\bar{H}_d(0) + H_u(+)$, where we have denoted fields with the transformation properties $\Psi \rightarrow \omega^+ \Psi$, $\Psi \rightarrow \omega^0 \Psi$ and $\Psi \rightarrow \omega^{-1} \Psi$ ($\omega = e^{i2\pi/3}$) as $\Psi(+)$, $\Psi(0)$ and $\Psi(-)$, respectively. Therefore, in the set $\bar{5}_L + 10_L$, the fields $\bar{5}_L(-)$ and $10_L(+)$ have different transformation properties each other. In contrast to the previous model, in the present paper, we will propose a model with alternative assignments
\[
(\bar{5}_L + 10_L)(+) + (\bar{5}'_L + 5'_L)(0) + \bar{H}_d(-) + H_u(+). \tag{1.1}
\]
Although the mechanism of the harmless $R$-parity violation is the same as the previous model, since
the $Z_3$ quantum number assignment is different from the previous one, the structure of the model is
completely different from the previous one.

In the present paper, we will investigate not only the radiatively-induced neutrino masses, but also the
contributions from the vacuum expectation values (VEV) of the sneutrinos, $(\tilde{\nu})$, although in the previous
paper the estimate of $\langle \tilde{\nu} \rangle$ was merely based on an optimistic speculation.

## 2 Harmless $R$-parity violation mechanism

Under the $Z_3$ quantum number assignment (1.1), the $Z_3$ invariant tri-linear terms in the superpotential
are only the following three terms:

$$W_{\text{tri}} = (Y_u)_{ij} H_u^{(+)} 10_L^{(a)} i 10_L^{(+)} j + (Y_d)_{ij} \overline{H}_d^{(-)} \overline{\Phi} L^{(0)} i 10_L^{(+)} j + \lambda_{ijk} \Phi^{(0)} L^{(a)} j \overline{\Phi} L^{(+)} j 10_L^{(+)} k . \quad (2.1)$$

Similarly, the $Z_3$ invariant bi-linear terms are only two: $\overline{H}_d^{(-)} H_u^{(+)}$ and $\overline{\Phi} L^{(0)} H_L$. In order to give
doublet-triplet splitting, we assume the following “effective” bi-linear terms

$$W_{\text{bi}} = \overline{H}_d^{(-)} (\mu + g_H \langle \Phi(0) \rangle) H_u^{(+)} + \overline{\Phi} L^{(0)} i (M_5 - g_5 \langle \Phi(0) \rangle) 5_L^{(a)} i + M_i^{SB} \Phi^{(0)} i 5_L^{(a)} i , \quad (2.2)$$

where $\Phi(0)$ is a 24-plet Higgs field with the VEV $\langle \Phi(0) \rangle = v_{24} \text{diag}(2, 2, 2, -3, -3)$, so that, for example,
the effective masses $M^{(a)}$ in the term $\overline{\Phi} L^{(0)} 5_L^{(a)}$ ($5_L^{(2)}$ and $5_L^{(3)}$ denote doublet and triplet components of
the fields $5_L$, respectively) are given by

$$M^{(2)} = M_5 + 3g_5 v_{24} , \quad M^{(3)} = M_5 - 2g_5 v_{24} . \quad (2.3)$$

The last term in Eq. (2.2) has been added in order to break the $Z_3$ symmetry softly. We define the $5_L \leftrightarrow 5_L^{\prime}$ mixing as follows:

$$\overline{5}_L^{(0)} i = c_i \overline{5}_L^{\prime} i + s_i \overline{5}_L^{\text{heavy}} i ,$$

$$\overline{5}_L^{(+)} i = -s_i \overline{5}_L^{\prime} i + c_i \overline{5}_L^{\text{heavy}} i , \quad (2.4)$$

where $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. Then, we can rewrite the second and third terms in Eq. (2.2) as

$$\sum_{a=2,3} \sqrt{(M^{(a)})^2 + (M_i^{SB})^2} \left( \overline{5}_L^{\text{heavy}} \right)^{(a)} (\overline{5}_L^{\text{heavy}})^{(a)}, \quad (2.5)$$

where $\overline{5}_L^{\text{heavy}} = \overline{5}_L^{(0)}$ and

$$s_i^{(a)} = \frac{M^{(a)}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} , \quad c_i^{(a)} = \frac{M_i^{SB}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} . \quad (2.6)$$

The fields $\overline{5}_L^{\text{heavy}(a)}$ have masses $\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}$, while $\overline{5}_L^{\ell(a)}$ are massless. We regard $\overline{5}_L^{\ell} + 10_L^{(+)} i$ as the observed quarks and leptons at low energy scale ($\mu < M_{GUT}$). (Hereafter, we will simply denote $\overline{5}_L$ and $10_L^{(+)} i$ as $\overline{5}_L$ and $10_L$, respectively.)
Then, the effective $R$-parity violating terms at $\mu < M_{GUT}$ are given by

$$W^\text{eff}_R = s^{(a)}_i \bar{s}^{(b)}_j \lambda_{ijk} \overline{\tilde{\psi}}^{(a)}_{Li} \psi^{(b)}_{Lj} 10_{Lk}.$$  

(2.7)

In order to suppress the unwelcome term $d_R^c d_R^c u_R^c$ in the effective $R$-parity violating terms (2.7), we assume a fine tuning

$$M^{(2)} \sim M_{GUT}, \quad M^{(3)} \sim m_{\text{SUSY}}, \quad M^S_{i} \sim M_{GUT} \times 10^{-1},$$

(2.8)

where $m_{\text{SUSY}}$ denotes a SUSY breaking scale ($m_{\text{SUSY}} \sim 1$ TeV), so that

$$s^{(2)}_i = 1 - O(10^{-2}), \quad c^{(2)}_i \sim \frac{M^S_{i}}{M^{(2)}} \sim 10^{-1}; \quad s^{(3)}_i \sim \frac{M^{(3)}}{M^S_{i}} \sim 10^{-12}; \quad c^{(3)}_i = 1 - O(10^{-24}).$$

(2.9)

Note that in the present model the observed down-quarks $d_R^c = (\tilde{\nu}^{(3)}_{Li})$ are given by $(\tilde{\psi}^{(2)}_{L}) \sim (\tilde{\psi}^{(0)}_{L(0)i})^{(3)},$ while the observed lepton doublets $(\nu_L, e_L) = (\tilde{\nu}^{(2)}_{Li})^{(2)}$ are given by $(\tilde{\psi}^{(2)}_{L}) \sim - (\tilde{\psi}^{(2)}_{L(i)})^{(2)}.$

From Eq. (2.9), the $R$-parity violating terms $d_R^c d_R^c u_R^c$ and $d_R^c (e_L u_L - \nu_L d_L)$ are suppressed by $s^{(3)}_i$ and $s^{(2)}_i \sim 10^{-24}$ and $s^{(3)}_i$ and $s^{(2)}_i \sim 10^{-12},$ respectively. Thus, proton decay caused by terms $d_R^c d_R^c u_R^c$ and $d_R^c (e_L u_L - \nu_L d_L)$ is suppressed by a factor $(s^{(3)}_i)^3 s^{(2)}_i \sim 10^{-36}.$ On the other hand, radiative neutrino masses are generated by the $R$-parity violating term $(e_L u_L - \nu_L d_L e_R^c)$ with a factor $s^{(2)}_i s^{(2)}_j \sim 1.$

The up-quark masses are generated by the Yukawa interactions (2.1), so that we obtain the up-quark mass matrix $M_u$ as $(M_u)_{ij} = (Y_u)_{ij} v_u,$ where $v_u = (H^0)_{u(+)i}.$ We also obtain the down-quark mass matrix $M_d$ and charged lepton mass matrix $M_e$ as

$$M^d = C^{(3)} Y_d v_d \quad M_e^c = C^{(2)} Y_d v_d,$$

(2.10)

where

$$C^{(a)} = \text{diag}(c^{(a)}_1, c^{(a)}_2, c^{(a)}_3),$$

(2.11)

so that

$$M^d = \left(C^{(3)} C^{(2)} - 1\right)^* M_e,$$

(2.12)

where $v_d = (\tilde{T}_d^{(0)})$. Note that $M^d_T$ has a structure different from $M_e,$ because the values of $c^{(2)}_i (i = 1, 2, 3)$ can be different from each other. (The idea $M^d_T \neq M_e$ based on a mixing between two $\tilde{\psi}_L$ has been discussed, for example, by Bando and Kugo in the context of an $E_6$ model.)

### 3 General form of the neutrino mass matrix

First, we investigate a possible form of the radiatively-induced neutrino mass matrix $M_{\text{rad}}.$ In the present model, since we do not have a term which induces $\tilde{\nu}_R^c \leftrightarrow \tilde{T}_d^{(0)}$ mixing, there is no Zee-type diagram, which is proportional to the Yukawa vertex $(Y_{d})_{ij}$ and $R$-parity violating vertex $\lambda_{ijk}.$

Only the radiative neutrino masses in the present scenario come from a charged-lepton loop diagram: the radiative diagram with $(\nu_L)_i \rightarrow (\nu_R)_i(\tilde{\nu}_L^c)_n$ and $(\nu_L)_k \rightarrow (\nu_L^c)_m \rightarrow (\nu_L^c)_n.$ The contributions $(M_{\text{rad}})_{ij}$ from the charged lepton loop are given, except for the common factors, as follows:

$$(M_{\text{rad}})_{ij} = s_i s_j s_k s_n \lambda_{ikm} \lambda_{jnl} (M_e)^{\mu \nu}_{kl} (M_{eLR})^{\nu \mu}_{mn} + (i \leftrightarrow j),$$

(3.1)
where \( s_i = s_i^{(2)} \), and \( M_e \) and \( \tilde{M}_{eLR}^2 \) are charged-lepton and charged-slepton-LR mass matrices, respectively. (In the present paper, we define the charged lepton mass matrix \( M_e \) and the neutrino mass matrix \( M_\nu \) as \( e_L M_e e_R \) and \( \bar{\nu}_L M_\nu \bar{\nu}_R \), respectively, so that the complex conjugate quantities \( \lambda_{ijk}^\ast \) and so on have appeared in the expression (3.1).) Since \( \tilde{M}_{eLR}^2 \) is proportional to \( M_e \), i.e.

\[
\tilde{M}_{eLR}^2 = (A + \mu^{(2)} \tan \beta) M_e
\]

\( (\mu^{(2)} = \mu - 3g_H v_{24} \), and \( A \) is the coefficient of the soft SUSY breaking terms \( (Y_d)_{ij}(\tilde{\nu}, \tilde{e})^T_L \bar{e}_L \bar{H}_d \) with \( A \sim 1 \text{ TeV} \)), we obtain

\[
(M_{\text{rad}})_{ij} = 2(A + \mu^{(2)} \tan \beta) s_i s_j s_k s_n \lambda_{ikm}^\ast \lambda_{jnl}^\ast (M_e)^\ast_{kl} (M_e)^\ast_{nm} .
\]

Since the coefficient \( \lambda_{ijk} \) is antisymmetric in the permutation \( i \leftrightarrow j \), it is useful to define

\[
\lambda_{ijk} = \varepsilon_{ijl} L_{lk} ,
\]

and

\[
K = (SM_e L^T)^\ast ,
\]

where \( S = \text{diag}(s_1, s_2, s_3) \). Then, the radiative neutrino mass matrix is given by

\[
(M_{\text{rad}})_{ij} = m_{0}^{-1} s_i s_j \varepsilon_{ikm} \varepsilon_{jln} K_{ml} K_{nk} .
\]

The coefficient \( m_{0}^{-1} \) is calculated from one-loop diagram (Fig.1) as

\[
m_{0}^{-1} = \frac{2}{16\pi^2} (A + \mu^{(2)} \tan \beta) F(m_{\tilde{e}_R}^2, m_{\tilde{e}_L}^2) ,
\]

where

\[
F(m_a^2, m_b^2) = \frac{1}{m_a^2 - m_b^2} \ln \frac{m_a^2}{m_b^2} .
\]

Next, let us investigate the contributions from the VEVs of sneutrinos \( \langle \tilde{\nu}_i \rangle \). In general, the sneutrinos \( \tilde{\nu}_i \) can have VEVs \( v_i \equiv \langle \tilde{\nu}_i \rangle \neq 0 \), if there are one or more of the following terms: \( \mu_i \tilde{\nu}_L H_u \) in superpotential \( W \), and \( B_i \tilde{\nu}_L H_u + m_{\tilde{H}_L} \tilde{\nu}_L \tilde{H}_d \) in the bilinear soft SUSY breaking terms \( V_{\text{soft}} \). In the present model, there is no such a term at tree level, because these terms are forbidden by the \( Z_3 \) symmetry.
However, only an effective $m^2_{HLi}$-term can appear via the loop diagram $\text{H}_d \rightarrow (\overline{\text{H}}_L^j)^c + (10_L)^c \rightarrow \overline{\text{H}}_L^i$ (Fig. 2). The contribution $m^2_{HLi}$ is proportional to

$$s_is_j\lambda_{ijk}(M_e)_{jk} = s_i\varepsilon_{ijk}K^*_j\ .$$

(3.8)

On the other hand, the contribution $M_{VEV}$ from $\langle \tilde{\nu}_i \rangle \neq 0$ to the neutrino mass matrix is proportional to

$$\begin{pmatrix}
  v_1^2 & v_1v_2 & v_1v_3 \\
v_1v_2 & v_2^2 & v_2v_3 \\
v_1v_3 & v_2v_3 & v_3^2
\end{pmatrix},$$

(3.9)

and $v_i \equiv \langle \tilde{\nu}_i \rangle$ are proportional to the values $(m^2_{HLi})^*$, so that the mass matrix $M_{VEV}$ is given by

$$(M_{VEV})_{ij} = \xi m_0^{-1}s_is_j\varepsilon_{ikl}\varepsilon_{jmn}K_{kl}K_{mn} \ ,$$

(3.10)

where $\xi$ is a relative ratio of $M_{VEV}$ to $M_{rad}$.

![Figure 2: Effective $\overline{\text{H}}_L^j\overline{\text{H}}_d$ term](image)

In conclusion, the neutrino mass matrix $M_{\nu}$ in the present model is given by the form

$$(M_{\nu})_{ij} = m_0^{-1}s_is_j\varepsilon_{ikl}\varepsilon_{jmn}(K_{kn}K_{ml} + \xi K_{kl}K_{mn}) \ ,$$

(3.11)

i.e.

$$M_{\nu} = m_0^{-1}S \left\{ \left[ (K - K^T)(K - K^T) - I \text{Tr}(KK - KK^T) \right] (1 + \xi) + \left[ (K + K^T) - I \text{Tr}K \right] \text{Tr}K - (KK + K^TK^T) + I \text{Tr}(KK) \right\} S$$

(3.12)

where $I$ is a $3 \times 3$ unit matrix.

### 4 General features of the neutrino mass matrix

In the present model, if the charged lepton mass matrix $M_e$ and the structure of $\lambda_{ijk}$ (i.e. $L_{ij}$) are given, then we can obtain $K = (SM_eL^T)^*$, so that we can predict neutrino masses and mixings. However, at present, we have many unknown parameters, so that in order to give explicit predictions of the neutrino masses and mixings, we must put a further assumption on the parameters $K_{ij}$. In the present section,
we investigate general features of the neutrino mass matrix (3.11) [or (3.12)] without making any explicit assumptions about flavor symmetries.

So far, the expression of \( M_\nu \), (3.12), has been given in the initial flavor basis, where \( \vec{\nu}_{L(\pm)} \leftrightarrow \vec{\nu}_{L(0)} \) mixings have been taken place a diagonal form

\[
S^{(a)} = \text{diag}(s_1^{(a)}, s_2^{(a)}, s_3^{(a)}) , \quad C^{(a)} = \text{diag}(c_1^{(a)}, c_2^{(a)}, c_3^{(a)}) , \tag{4.1}
\]

and the matrix \( K \) has been defined by Eq. (3.4), \( K = (SM_L L^T)^* \). Since \( S, M_\nu \) and \( L \) are transformed as

\[
\begin{align*}
M_\nu &\to M'_\nu = U_5^\dagger M_\nu U_{10}^\ast , \\
L &\to L' = U_5^\dagger L U_{10} , \\
S &\to S' = U_5^\dagger S U_5 ,
\end{align*}
\tag{4.2}
\]

under a rotation of the flavor basis

\[
10_L \to 10'_L = U_{10}^\dagger 10_L , \quad \tilde{5}_L^l \to (\tilde{5}_L^l)' = U_5^\dagger \tilde{5}_L^l , \tag{4.3}
\]

the matrix \( K \) transforms as

\[
K \to K' = U_5^T K U_5 . \tag{4.4}
\]

We have a great interest in the form of \( M'_\nu \) in the flavor basis with \( M'_\nu = D_\nu \equiv \text{diag}(m_\mu, m_\tau, m_\tau) \). Hereafter, we denote the quantities \( M'_\nu, K' \), and so on in the \( M'_\nu = D_\nu \) basis as \( \hat{M}_\nu, \hat{K} \) and so on, respectively. The matrix \( \hat{K} \) is expressed as

\[
\hat{K} = \tilde{S} D_\nu \hat{L}^\dagger \simeq D_\nu (U_{\nu}^\dagger L^\dagger U_L^\ast) , \tag{4.5}
\]

where \( U_5 = U_{\nu}^L \) and \( U_{10} = U_{\nu}^R \), and we have put \( \tilde{S} \simeq 1 \) because of \( S \simeq 1 \) as we have assumed in Eq. (2.9).

Here, let us summarize general features of the present neutrino mass matrix (3.12).

(i) If the matrix \( K \) defined by Eq. (3.4) satisfies \( K^T = K \) in the initial basis, the matrix \( K' \) in the arbitrary basis also satisfies \( K'^T = K' \), so that the present model gives \( \langle \tilde{\nu}_1^l \rangle = 0 \) in the arbitrary basis. For such a case, the neutrino mass matrix is simply given by

\[
M_\nu = -m_0^{-1} S \left[ 2KK - 2K \text{Tr} KK - \text{Tr}(KK^2) + (\text{Tr}K)^2 \right] S . \tag{4.6}
\]

(ii) When \( K \) is symmetric under the flavor 2 \( \leftrightarrow \) 3 permutation, the neutrino mass matrix \( M_\nu \) is also symmetric under the 2 \( \leftrightarrow \) 3 permutation. It is well-known \[8\] that when the neutrino mass matrix \( \hat{M}_\nu \) is symmetric under the 2 \( \leftrightarrow \) 3 permutation, the mass matrix \( \hat{M}_\nu \) gives a nearly bimaximal mixing, i.e. \( \sin^2 2\theta_{23} = 1 \) and \( |U_{13}|^2 = 0 \), which are favorable to the observed atmospheric \[9\], K2K \[10\] and CHOOZ \[11\] data. In the present model, the 2 \( \leftrightarrow \) 3 symmetry of \( \hat{M}_\nu \) means that the parameters

\[
\hat{K}_{ij} = K_{ki}(U_{\nu}^L)_{ki}(U_{\nu}^c)_{lj} , \tag{4.7}
\]

are symmetric under the 2 \( \leftrightarrow \) 3 permutation. In other words, the 2 \( \leftrightarrow \) 3 symmetry of \( \hat{M}_\nu \) is due to special structures of \( U_{\nu}^c \) and \( K \). For example, when \( K \) and \( U_{\nu}^c \) are given by the textures

\[
K = \begin{pmatrix}
K_{11} & 0 & 0 \\
0 & K_{22} & K_{23} \\
0 & K_{32} & K_{33}
\end{pmatrix} . \tag{4.8}
\]
the matrix $\hat{K}$ is $2 \leftrightarrow 3$ symmetric:

$$\hat{K} = \begin{pmatrix} f & a & a \\ a' & g & b \\ a' & b & g \end{pmatrix},$$  \hfill (4.10)

so that the neutrino mass matrix $\hat{M}_\nu$ is also $2 \leftrightarrow 3$ symmetric:

$$(\hat{M}_\nu)_{11} = -2(g^2 - b^2)m_\nu^{-1},$$

$$(\hat{M}_\nu)_{12} = (\hat{M}_\nu)_{13} = (\hat{M}_\nu)_{21} = (\hat{M}_\nu)_{31} = (a + a')(g - b)m_\nu^{-1},$$

$$(\hat{M}_\nu)_{22} = (\hat{M}_\nu)_{33} = [(a - a')(1 + \xi) + 2(aa' - fg)]m_\nu^{-1},$$

$$(\hat{M}_\nu)_{23} = (\hat{M}_\nu)_{32} = -[(a - a')(1 + \xi) + 2(aa' - fb)]m_\nu^{-1},$$  \hfill (4.11)

and $K$ in the initial basis is given by

$$K_{11} = g - b,$$

$$K_{22} = (g + b)c^2 - \sqrt{2}(a + a')cs + fs^2,$$

$$K_{33} = (g + b)s^2 + \sqrt{2}(a + a')cs + fc^2,$$

$$K_{23} = \sqrt{2}(-as^2 + a'c^2) + (g + b - f)cs,$$

$$K_{32} = \sqrt{2}(ac^2 - a's^2) + (g + b - d)cs.$$  \hfill (4.12)

Finally, let us show a simple example which is suggested by above comments (i) and (ii). We assume that \(M_eM_e^\dagger\) on the initial basis is $2 \leftrightarrow 3$ symmetric:

$$M_eM_e^\dagger = \begin{pmatrix} F & A & A \\ A & G & B \\ A & B & G \end{pmatrix},$$  \hfill (4.13)

so that $U_L^\dagger$ has a form of a nearly bimaximal mixing. For simplicity, we assume that $U_L^\dagger$ is given by the full bimaximal mixing form

$$U_L^\dagger = (U_L^\dagger)^T = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix},$$  \hfill (4.14)
which demands the constraint \( F = B + G \) on the matrix (4.13). Then, the eigenvalues \( D_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \) are given by

\[
\begin{align*}
m_e^2 &= G - B, \\
m_\mu^2 &= G + B - \sqrt{2}A, \\
m_\tau^2 &= G + B + \sqrt{2}A, \\
\end{align*}
\]

(4.15)

On the other hand, we assume that \( K \) in the initial basis is given by the form (4.8) with \( K_{23} = K_{32} \), so that we obtain \( a = a' \) and

\[
\hat{M}_\nu = 2m_0^{-1} \begin{pmatrix}
-(g^2 - b^2) & a(g - b) & a(g - b) \\
 a(g - b) & a^2 - fg & -(a^2 - fb) \\
 a(g - b) & -(a^2 - fb) & a^2 - fg \\
\end{pmatrix}.
\]

(4.16)

Note that the mass matrix (4.16) does not include the contributions (\( \xi \)-terms) from nonvanishing sneutrino VEVs because of \( K^T = K \). The mass matrix (4.16) gives the following eigenvalues and mixings:

\[
\begin{align*}
m_{\nu 1} &= (g - b) \left[ \sqrt{9(g + b)^2 + 2f(g + b) + f^2} - (g + b + f) \right] m_0^{-1}, \\
-m_{\nu 2} &= -(g - b) \left[ \sqrt{9(g + b)^2 + 2f(g + b) + f^2} + g + b + f \right] m_0^{-1}, \\
m_{\nu 3} &= -2 \left[ 2a^2 - (g + b)f \right] m_0^{-1},
\end{align*}
\]

(4.17)

\[
\hat{U}_\nu = \begin{pmatrix}
c_\nu & s_\nu & 0 \\
\frac{1}{\sqrt{2}} s_\nu & -\frac{1}{\sqrt{2}} c_\nu & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} c_\nu & \frac{1}{\sqrt{2}} s_\nu & \frac{1}{\sqrt{2}} \\
\end{pmatrix},
\]

(4.18)

\[
s_\nu = \sqrt{\frac{m_{\nu 1}}{m_{\nu 1} + m_{\nu 2}}}, \quad c_\nu = \sqrt{\frac{m_{\nu 2}}{m_{\nu 1} + m_{\nu 2}}},
\]

(4.19)

so that we obtain

\[
\tan^2 \theta_{solar} = \frac{m_{\nu 1}}{m_{\nu 2}},
\]

(4.20)

together with \( \sin^2 2\theta_{atm} = 1 \) and \( |U_{13}|^2 = 0 \). For a further simple case with \( f = 0 \), which demands

\[
K_{23} = K_{32} = \frac{1}{2} \left( K_{33} + K_{22} \right),
\]

(4.21)

we obtain \( m_{\nu 1} = m_{\nu 2}/2 = 2(g^2 - b^2)m_0^{-1} \), so that

\[
\tan^2 \theta_{solar} = \frac{1}{2},
\]

(4.22)
\[
R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{3}{4} \frac{(g^2 - b^2)^2}{a^4 - (g^2 - b^2)^2},
\]
(4.23)

where we have considered
\[
a^2 = \frac{1}{8} (K_{33} - K_{22})^2 \gg g^2 - b^2 = K_{11}^2 (K_{33} + K_{22})^2.
\]
(4.24)

The result (4.22) is favorable to the recent solar \[12\] and KamLAND data \[13\]. By using the best fit values \(\Delta m^2_{\text{solar}} = 7.2 \times 10^{-5} \text{ eV}^2\) \[12, 13\] and \(\Delta m^2_{\text{atm}} = 2.4 \times 10^{-3} \text{ eV}^2\) \[9, 10\], we obtain
\[
\frac{m_{\nu 2}}{m_{\nu 3}} = \frac{|g^2 - b^2|}{a^2} = \sqrt{\frac{4R}{3 + 4R}} = 0.20,
\]
(4.25)

where \(R = \Delta m^2_{\text{solar}}/\Delta m^2_{\text{atm}}\), and
\[
m_{\nu 1} = 0.0049 \text{ eV}, \quad m_{\nu 2} = 0.0098 \text{ eV}, \quad m_{\nu 3} = 0.050 \text{ eV},
\]
(4.26)

where we have used the relation \(m_{\nu 1}/m_{\nu 2} = 1/2\) and \(\Delta m^2_{\text{atm}} = 3m_{\nu 2}^2/4\). Of course, this is only an example, and the result (4.22) is not a prediction which is inevitably driven from the general form of \(M_{\nu}\).

## 5 Summary

In conclusion, within the framework of a SUSY GUT model, we have proposed an \(R\)-parity violation mechanism which is harmless for proton decay and investigated a general form of the neutrino mass matrix \(M_{\nu}\). As we have given in Eq. (3.12), the form of \(M_{\nu}\) is described in terms of the matrix \(K\) defined in Eq. (3.4). (i) If \(K^T = K\), the VEVs of sneutrinos are exactly zero, \(\langle \tilde{\nu}_i \rangle = 0\), in the arbitrary basis, so that \(M_{\nu}\) is given only by the radiative contributions. (ii) If \(\hat{K}\) is \(2 \leftrightarrow 3\) symmetric, then \(\hat{M}_{\nu}\) is also \(2 \leftrightarrow 3\) symmetric, so that \(\hat{M}_{\nu}\) can predict \(\sin^2 2\theta_{\text{atm}} = 1\) and \(|U_{13}|^2 = 0\).

In order to demonstrate that the general form indeed has a phenomenologically favorable parameter range, we have given a simple example of \(K\) and \(M_{\nu}M_{\nu}^\dagger\) in the last part of the section 4. Although such a simple form of \(K\), (4.8), with the constraint (4.23) is likely, the investigation of the origin of the possible form \(K\) will be our next task. The purpose of the present paper is not to give a special model for neutrino phenomenology, and it is to demonstrate that it is indeed possible to build a neutrino mass matrix model with \(R\)-parity violation, i.e. without a seesaw mechanism, even if the model is within a framework of GUT.

The present model has assigned \(Z_3\) quantum numbers to the superfields differently from those in the previous model \[14\] with \(\tilde{\psi}_L \leftrightarrow \tilde{\psi}_L'\) mixing: we have been able to assign the same \(Z_3\) quantum number to the matter fields \(\tilde{\psi}_L\) and \(10_L\) (and also to \(\tilde{\psi}_L\) and \(5_L'\)). This re-assignment will give fruitful potentiality for a further extension of the present model.

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