Lepton Anomalous Magnetic Moments in an $S_4$ Flavor-Symmetric Extra U(1) Model

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Abstract

We study supersymmetric extra U(1) model with $S_4$ flavor symmetry. The flavor symmetry not only stabilizes proton but also suppresses the flavor changing processes without raising the supersymmetry breaking scale. After the flavor symmetry is broken, the Yukawa hierarchy is realized by the Froggatt-Nielsen mechanism. The relevant Peccei-Quinn scale for axion dark matter: $f_a/M_P \sim 10^{-5}$ accounts for small up quark mass. The muon mass scale: $m_\mu/M_W \sim 10^{-3}$ is related to the $O(10^{-6})$ mass degeneracy of right-handed neutrinos, from which we can identify the relevant scale of right-handed neutrino mass for baryon asymmetry of the Universe as TeV. Due to the existence of the extra higgsinos, the discrepancies of the anomalous magnetic moments of the muon and electron between the standard model predictions and the observations are explained by the chargino-sneutrino contributions simultaneously.

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I. INTRODUCTION

The standard model (SM) is a successful theory of gauge interactions; however, there are many unsolved issues, such as how to generate the Yukawa hierarchy, the tiny mass of neutrino and the baryon asymmetry of the Universe (BAU), how to stabilize the electroweak scale \( M_W \sim 10^2 \text{GeV} \) against the Planck scale \( M_P \sim 10^{18} \text{GeV} \) quantum corrections, why the strong interaction conserves CP and what dark matter is. With introducing heavy right-handed neutrino (RHN), the smallness of neutrino mass and the origin of baryon asymmetry are accounted for simultaneously by the seesaw mechanism [1] and leptogenesis [2] respectively. The well known solution of the Yukawa hierarchy problem is the Froggatt-Nielsen mechanism [3]. The strong-CP problem is solved by the Peccei-Quinn mechanism [4] which accommodates a candidate for dark matter; axion [5].

The elegant solution of the large scale hierarchy problem is supersymmetry (SUSY) [6] which is the main target of the LHC. The existence of a light Higgs boson (such as 125GeV [7]) supports the idea of SUSY. Furthermore, it is well known that the long-standing problem of the discrepancy of muon anomalous magnetic moment \( (g - 2)_\mu (\equiv 2a_\mu) \) between the SM prediction and the experimental value [8]:

\[
\Delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM}) = (27.9 \pm 7.6) \times 10^{-10},
\]

(1)
can be explained by SUSY [9]. If this explanation is true, then some light sparticles should exist, hence SUSY is verifiable for the LHC or a future collider.

In the minimal supersymmetric standard model (MSSM), as the Higgs superfields \( H^U \) and \( H^D \) are vector-like under the SM gauge symmetry \( G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_W \times \text{U}(1)_Y \), we can introduce a \( \mu \) term:

\[
\mu H^U H^D,
\]

(2)
into the superpotential. The natural size of the parameter \( \mu \) is \( O(M_P) \), however, \( \mu \) must be \( O(M_W) \) in order for the electroweak gauge symmetry to break. This is the so-called \( \mu \) problem, which is solved by making the Higgs superfields chiral under a new U(1) symmetry. Based on the \( E_6 \)-inspired extra U(1) model [10], we can eliminate a fundamental \( \mu \) term from the superpotential and introduce a trilinear term:

\[
\lambda S H^U H^D,
\]

(3)
which is converted into an effective $\mu$ term when the singlet $S$ develops an $O(1\text{TeV})$ vacuum expectation value (VEV) [11]. To prevent the squared masses of sleptons from receiving large D-term contribution of the U(1), we introduce the U(1) under which the lepton doublet $L$ is neutral. The new gauge symmetry requires additional Higgs superfields which contribute to the lepton anomalous magnetic moments.

Though we can account for $(g - 2)_\mu$ in this framework, some problems remain. The extra U(1) symmetry requires colored Higgs superfilelds $G, G^c$ to cancel gauge anomaly and replaces the baryon- and lepton-number violating terms in the MSSM by single G interactions:

$$GQQ + G^c U^c D^c + G^c U E^c + G^c Q L + G N^c D^c,$$

which induce too rapid proton decay. Successful leptogenesis requires $10^{11}\text{GeV}$ scale RHN, which contradicts with the low reheating temperature as $T_{RH} < 10^7\text{GeV}$ which is required to avoid an overproduction of the gravitino [12]. Moreover, such a large mass scale is not testable for a collider. The light slepton induces too large flavor changing processes such as $\mu \rightarrow e + \gamma$ when the soft SUSY breaking squared masses are non-universal as is naively expected [13]. The extra Higgs bosons induce additional contributions to flavor changing process [14]. These problems give additional information about the flavor structure.

To solve these problems, we introduce a flavor symmetry. There are many candidates of continuous flavor symmetries such as U(1) [15] SU(2) [16], SU(3)[17]. In this paper, we adopt a discrete $S_4$ flavor symmetry [18, 19]. Assigning the RHNs to singlet and doublet, the resonant leptogenesis is realized, which resolves the contradiction between the reheating temperature and the RHN mass scale by reducing later [20]. Assigning the left-handed leptons and quarks to singlets and doublets respectively, the flavor violating processes are suppressed by a degeneracy of the sfermion mass [21][22]. The Yukawa hierarchies of the quarks and the charged leptons are realized by assigning the right handed fermions to singlets. In this case, the discrepancy of the size of representations between left-handed and right-handed fermions causes suppression of mass in the same manner as SU(2)$_W$. We should keep in mind that the hierarchy between fermion mass scale and the Planck scale is generated by the discrepancy of the size of the representations of SU(2)$_W$ between the left-handed and right-handed fermions. We adopt this manner for suppressing single G-interactions, too. Assigning $G, G^c$ to triplets, the single G interactions are forbidden. The
existence of $S_4$ triplets compels all fermions to consist of three generations to cancel gauge
anomaly, which is the possible answer to the question, why three generations exist, (or to
the Rabi’s question, “Who ordered that?”). The number of generations is fixed to the size of
the flavor representation of $G, G^c$ which we call “G Higgs” (generation-number-imprinted-
colored-Higgs). If the G Higgs decays dominantly to the RHN, the seesaw mechanism and
leptogenesis may be verified directly at a collider experiment. The G Higgs is a Rosetta
stone to decipher the undeciphered part of flavor physics. The LHC or a future collider may
reveal what is imprinted in the particle.

Recent observation of the fine structure constant [23] gives new prediction to the electron
$(g - 2)_e$, which has small discrepancy with experimental value [24]:

$$\Delta a_e = a_e(\text{exp}) - a_e(\text{SM}) = -(8.7 \pm 3.6) \times 10^{-13}.$$  \quad (5)

Several papers studying both $g - 2$ anomalies based on SUSY can be found in the literature
[25]. As the universal new physics contribution gives wrong prediction:

$$\frac{m^2_{\mu}a_{\mu}(\text{NP})}{m^2_{\mu}a_{\mu}(\text{NP})} \simeq 1,$$  \quad (6)

for the experimental value:

$$\frac{m^2_{\mu}\Delta a_{\mu}}{m^2_{\mu}\Delta a_{\mu}} \simeq -14,$$  \quad (7)

there should be a non-trivial flavor structure. Several attempts to explain it based on flavor
symmetry or on other frameworks can be found in the literatures ([26] [27]). In this paper
we explain both $g - 2$ anomalies simultaneously based on our model.

II. SYMMETRY BREAKING

A. Gauge symmetry

We extend the gauge symmetry from $G_{\text{SM}}$ to $G_{\text{32111}} = G_{\text{SM}} \times U(1)_S \times U(1)_Z$, and
introduce new superfields $N^c, S, G, G^c$ which are embedded in the 27 representation of $E_6$
with quark and lepton superfields $Q, U^c, D^c, L, E^c$ and Higgs superfields $H^U, H^D$. Here, $N^c$
is the RHN, $S$ is the $G_{\text{SM}}$ singlet, and $G, G^c$ are colored Higgses. The two U(1) charges, $X$
and $Z$ are defined by the linear combinations of $Q_\psi$ and $Q_\chi$ as

$$X = \frac{\sqrt{15}}{4} Q_\psi + \frac{1}{4} Q_\chi, \quad Z = -\frac{1}{4} Q_\psi + \frac{\sqrt{15}}{4} Q_\chi,$$  \quad (8)
TABLE I: $G_{32111}$ assignment of superfields. Here the $x$, $y$, $z$ and $q_S$ are charges of $U(1)_X$, $U(1)_Y$, $U(1)_Z$ and $U(1)_S$, and $Y$ is hypercharge. The charges of $U(1)_\psi$ and $U(1)_\chi$ ($Q_\psi$ and $Q_\chi$) are also given.

where $E_6 \supset SO(10) \times U(1)_\psi \supset SU(5) \times U(1)_\chi \times U(1)_\psi$.

The definition of $U(1)_S$ charge under which the left-handed lepton is neutral is given by

$$Q_S = \sqrt{\frac{2}{5}} Y + \sqrt{\frac{3}{5}} X,$$

which is automatically anomaly free. Note that it is impossible to embed $G_{32111}$ in $E_6$. The charge assignment of the superfields are given in Table 1. To break $U(1)_Z$, we add new vector-like superfields $\Phi, \Phi^c$, where $\Phi^c$ is the same representation as the RHN $N^c$ under the $G_{32111}$, and its anti-representation $\Phi$ originates from $27^*$. To discriminate between $N^c$ and $\Phi^c$, we introduce $Z_2^\Phi$ symmetry under which $\Phi^c$ and $\Phi$ are odd. The invariant superpotential under these symmetries is given by

$$W_{32111} = W_0 + W_S + W_G + W_\Phi,$$

$$W_0 = Y^U H^U Q^c U^c + Y^D H^D Q D^c + Y^E H^D L E^c + Y^N H^U N^c + \frac{1}{M_P} Y^M \Phi \Phi N^c N^c,$$

$$W_S = k S G G^c + \lambda S H^U H^D,$$

$$W_G = Y^Q Q G Q Q + Y^{UD} G^c U^c D^c + Y^{UE} G U^c E^c + Y^{QL} G^c Q L + Y^{DN} G D^c N^c,$$

$$W_\Phi = M_\Phi \Phi \Phi^c + \frac{1}{M_P} (\Phi \Phi^c)^2 + \cdots ,$$
where $M_P = 2.4353 \times 10^{18}\text{GeV}$ is the reduced Planck scale. Since the interactions in $W_S$ drive the squared mass of $S$ to be negative through renormalization group equations, the $U(1)_S$ symmetry is broken spontaneously and the $U(1)_S$ gauge boson $Z'$ acquires the mass

$$m(Z') \simeq \frac{\sqrt{5}}{2} g_S \langle S \rangle,$$

where $\langle H^D \rangle \ll \langle S \rangle$ is assumed based on the experimental constraint for $Z'_\psi$ [28]:

$$m(Z'_\psi) > 3900\text{GeV}. \tag{16}$$

The constraint for the $Z'$ mass is not far from this bound. In this paper we assume $\langle H^{U,D} \rangle / \langle S \rangle \sim O(10^{-2})$.

If $M_\Phi = 0$ in $W_\Phi$ and the origin of the potential $V(\Phi, \Phi^c)$ is unstable, then $\Phi, \Phi^c$ develop large VEVs along the D-flat direction of $\langle \Phi \rangle = \langle \Phi^c \rangle = V$, $U(1)_Z$ is broken, and the $U(1)_Z$ gauge boson $Z''$ acquires the mass

$$m(Z'') = \frac{4}{3} \sqrt{\frac{5}{2}} g_Z V. \tag{17}$$

After the gauge symmetry breaking, since the R-symmetry defined by

$$R = Z_2^\Phi \exp \left[ \frac{i\pi}{4} (g_S - 2y + 3z) \right], \tag{18}$$

remains unbroken, the lightest SUSY particle (LSP) is stable.

**B. Flavor symmetry**

The superpotential defined in Eq.(10)-(14) has the following problems. As the interaction $W_G$ induces a proton decay that is too fast, it must be strongly suppressed. The mass parameter $M_\Phi$ in $W_\Phi$ must be forbidden in order to allow for $U(1)_Z$ symmetry breaking. In $W_0$, the contributions to flavor-changing processes from the extra Higgs bosons must be suppressed. These problems should be solved by flavor symmetry.

If we introduce $S_4$ flavor symmetry and assign $G, G^c$ to be triplets, then $W_G$ [defined in Eq. (13)] is forbidden. This is because any products of doublets and singlets of $S_4$ do not contain triplets. However, as the existence of a $G$ Higgs with a lifetime longer than $0.1\text{s}$ spoils the success of big bang nucleosynthesis (BBN), the $S_4$ symmetry must be broken. Therefore we assign $\Phi^c$ to be a triplet in order to break $U(1)_Z$ and $S_4$ at the same time and
assign $\Phi$ to be a doublet and a singlet in order to forbid $M_\Phi \Phi \Phi^c$. With these assignments, $S_4$ symmetry is broken due to the VEV of $\Phi^c$ and the effective trilinear terms which correspond to $W_G$ are induced from nonrenormalizable terms. The size of the VEV of $\Phi$ is fixed by the
superpotential

\[ W_{X\Phi} = \frac{1}{M_P} \left[ X^{14} + Y^X X^{10}(\Phi\Phi^c)^2 + X^6(\Phi\Phi^c)^3 + X^6(\Phi\Phi^c)^4 + X^4(\Phi\Phi^c)^5 + X^2(\Phi\Phi^c)^6 + (\Phi\Phi^c)^7 \right], \]

(19)

and the soft SUSY-breaking terms as follows:

\[ \langle X \rangle = \langle \Phi \rangle = 10^{-\frac{5}{12}}, \]

(20)

where \( X \) is a gauge singlet. We assume that the global minimum of the potential \( V(X, \Phi, \Phi^c) \) is at the \( S_3 \)-symmetric vacuum and along with the D-flat direction of \( U(1) \) as follows:

\[ \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = 0, \quad \langle X \rangle = \langle \Phi_3 \rangle = \sqrt{3} \langle \Phi_1^c \rangle = \sqrt{3} \langle \Phi_2^c \rangle = \sqrt{3} \langle \Phi_3^c \rangle = V \equiv 10^{-\frac{5}{12}} M_P. \]

(21)

The assignments of the other superfields are determined based on the following criteria:

1) the mass matrices of quarks and leptons are consistent with the observed mass hierarchies and the Cabibbo-Kobayashi-Maskawa (CKM) and Maki-Nakagawa-Sakata (MNS) matrices;
2) the third-generation Higgses \( H^U_3, H^D_3 \) are specified as the dominant component of MSSM Higgses;
3) the experimental constraints for the flavor violating processes are satisfied;
4) the resonant leptogenesis mechanism works;
5) an accidental Peccei-Quinn \( U(1) \) global symmetry is included;
6) two anomalies in \( (g - 2)_e \) and \( (g - 2)_\mu \) are accounted for. The representation of all superfields under the flavor symmetry is given in Table 2.

In order to realize the Yukawa hierarchies, we introduce gauge singlet flavon superfields \( F_{1,2,3}^{A,B,C}, R_i, T_i \) and fix their VEVs as follows:

\[ \frac{\langle F_{1,2,3}^A \rangle}{M_P} = \epsilon^3 (c_a, s_a, 1), \quad \frac{\langle F_{1,2,3}^B \rangle}{M_P} = \epsilon^3 (\alpha c_b, \beta s_b, 1), \quad \frac{\langle F_{1,2,3}^C \rangle}{M_P} = \epsilon^3 (c_c, s_c, 1), \]
\[ \frac{\langle R_i \rangle}{M_P} = \epsilon^5 (c_R, s_R), \quad \frac{\langle T_i \rangle}{M_P} = \epsilon^5 (c_T, s_T), \quad \epsilon = 0.1, \quad |\alpha| = |\beta| = 1, \]
\[ c_x \equiv \cos \theta_x, \quad s_x \equiv \sin \theta_x \quad (x = a, b, c, R, T, \cdots), \]

(22)

where \( \alpha \) and \( \beta \) are complex. In this paper, we assume that the original Lagrangian has CP symmetry and all parameters in it are real. Therefore the complex VEVs given in Eq. (22) induce spontaneous CP violation.

The superpotentials of \( F_{1,2,3}^X \) are given by

\[ W_{FX} = \frac{1}{M^4} \left[ (E_2^X)^2 + (F_3^X)^2 E_2^X + (F_3^X)^4 \right] E_3^X, \]

(23)
for $X = A, B, C$, respectively, where $E_2 X = (F_1 X)^2 + (F_2 X)^2$ and $E_3 X = 3(F_1 X)^2 F_2 X - (F_2 X)^3$ are $S_4$ invariants. The $S_4$ invariants $E_2^3 E_3, F_3^3 E_2 E_3, F_3^4 E_3$ have nine different spurions $a_{1,2,\ldots,9}$:

$$E_2^3 E_3 = a_1 F_1^6 F_2 + a_2 F_1^4 F_2^3 + a_3 F_1^2 F_2^5 + a_4 F_2^7, \quad (24)$$

$$F_3^2 E_2 E_3 = a_5 F_1^4 F_2 F_3^2 + a_6 F_1^2 F_2^3 F_3^2 + a_7 F_2^5 F_3^2, \quad (25)$$

$$F_3^4 E_3 = a_8 F_1^2 F_2 F_3^4 + a_9 F_2^3 F_3^4, \quad (26)$$

where $a_1 = 3, a_2 = 5, a_3 = 1, a_4 = -1, a_5 = 3, a_6 = 2, a_7 = -1, a_8 = 3, a_9 = -1$ and $F_i$ is the $S_4$ doublet and $F_3$ is the singlet. Since the number of spurions ($= 9$) is larger than the dimension ($= 3$) of the vector space spanned by the nine charge vectors as follows:

$$a_1 : (6, 1, 0), \quad a_2 : (4, 3, 0), \quad a_3 : (2, 5, 0), \quad a_4 : (0, 7, 0), \quad a_5 : (4, 1, 2), \quad a_6 : (2, 3, 2), \quad (27)$$

$$a_7 : (0, 5, 2), \quad a_8 : (2, 1, 4), \quad a_9 : (0, 3, 4); \quad \text{for} \quad F_1 (-1, 0, 0), \quad F_2 (0, -1, 0), \quad F_3 (0, 0, -1),$$

spontaneous CP violation is not forbidden (see [30]).

The scale of VEV is fixed as follows. If the superpotential of the gauge singlet superfield $\Psi$ is given by

$$W = \frac{\Psi^n}{n M_P^{n-3}}, \quad (28)$$

then the potential of $\Psi$ is given by

$$V(\Psi) = m_\Psi^2 |\Psi|^2 - \left( \frac{A \Psi^n}{M_P^{n-3}} + h.c. \right) + \left( \frac{|\Psi^{n-1}|^2}{M_P^{2n-6}} \right), \quad (29)$$

where $m_\Psi \sim A \sim m_{\text{SUSY}} \sim O(1) \text{TeV}$ is assumed. At the global minimum $\langle \Psi \rangle \neq 0$, because each of the terms in the potential should be balanced, the scale of the VEV is fixed by

$$\frac{\langle \Psi \rangle}{M_P} = \left( \frac{m_{\text{SUSY}}}{M_P} \right)^{1/(n-2)}. \quad (30)$$

We assume that the effect of SUSY breaking in the hidden sector is mediated by gravity and induces soft SUSY-breaking terms in the observable sector. Since these terms are non-universal in general, large flavor-changing processes are induced by the sfermion exchange. From the experimental constraints on them, the assignments of quarks and leptons under the flavor symmetry are restrictive.

After the flavor symmetry breaking, the soft breaking scalar squared mass matrices become non-diagonal. For the Higgs scalars, this gives the mixing mass terms

$$V \supset m_{UB}^2\epsilon^3(H_3^U)^*(c_c H_1^U + s_c H_2^U) + m_{DB}^2 \epsilon^3(H_3^D)^* H_i^D (c_c H_1^D + s_c H_2^D) + m^2 \epsilon^5 (S_1)^* (c_s S_2 + s_s S_3) + h.c., \quad (31)$$
which compel the extra Higgs scalars to develop VEVs as

\[
\langle H_i^U \rangle = N_U \epsilon^3 (c_c, s_c) v_u, \quad \langle H_i^D \rangle = N_D \epsilon^3 (c_c, s_c) v_d, \quad \langle S_1 \rangle = O(\epsilon^5) v_s, \quad (32)
\]

where we put

\[
\langle H_3^U \rangle = v_u = 150.7 \text{GeV}, \quad \langle H_3^D \rangle = v_d = 87.0 \text{GeV}, \quad \sqrt{v_u^2 + v_d^2} = v = 174.0 \text{GeV},
\]

\[
\langle S_2 \rangle = c_s v_s, \quad \langle S_3 \rangle = s_s v_s, \quad \langle S \rangle = \sqrt{\langle S_2 \rangle^2 + \langle S_3 \rangle^2} = v_s \geq 9581 \text{GeV}. \quad (33)
\]

The constraint for \(v_s\) is derived from Eq.(16) and the assumption: \(g_S(1 \text{TeV}) = g_Y(1 \text{TeV}) = 0.3641\). As the same effects affect the flavons, the VEV directions given in Eq.(21) are perturbed as follows:

\[
\langle \Phi_1 \rangle \sim \langle \Phi_2 \rangle \sim O(\epsilon^6) V, \quad \sqrt{3} \langle \Phi_a^c \rangle = (1 + O(\epsilon^6)) V, \quad \langle F_A^4 \rangle = \epsilon^3 (c_a + O(\epsilon^6)) M_P, \quad \cdots \quad (34)
\]

and so on. Note that the dominant parts of the scalar squared mass matrices of the extra Higgs and G Higgs are diagonal and degenerated. Due to the smallness of VEVs of the extra Higgs bosons, the superpartners of the extra Higgs and G Higgs also have diagonal and degenerated mass matrices. Therefore the trace of \(S_4\) flavor symmetry is imprinted in their mass spectra which may be testable for the LHC or a future collider.

C. Accidental Peccie-Quinn symmetry

In this paper, we adopt axion-flavon unification scenario [31] which can be embedded into the non-abelian flavor symmetric models, for example: \(A_4\) [32], \(D_6\) [33], \(T'\) [34], \(SL_2(F_3)\) [35] and into the supersymmetric model: [36] too. As the cut off scale of our model is the Planck scale \(M_P\), the way to embed the axion into flavon is restrictive. For the allowed region of the PQ scale: \(10^8(\text{SN1987A}) < f_a < 10^{12}(\text{Dark Mater}) \text{GeV}\), the order of the effective Yukawa coupling is given by

\[
Y_{\text{eff}} = \frac{f_a}{M_P} \sim 10^{-10} \sim 10^{-6}, \quad (35)
\]

which is not sizable for the second and third generation Yukawa couplings. In this paper we identify \(Y_{\text{eff}}\) as the up quark Yukawa coupling [37].

At the leading order, the potential of flavons \(R_i, T_i\) is given by

\[
V = m^2_R |R|^2 + m^2_T |T|^2 - \frac{A}{M_P} (R^4 T + \text{h.c.}) + \frac{1}{M_P} \left[ |R|^4 + |4R^3 T|^2 \right], \quad (36)
\]
where $m_{R} \sim m_{T} \sim m_{SUSY}$ is assumed and unimportant $S_{4}$ indexes are omitted. As this potential is invariant under the redefinition of fields as
\begin{equation}
R \rightarrow e^{i\theta} R, \quad T \rightarrow e^{-4i\theta} T, \quad U_{1}^{c} \rightarrow e^{-i\theta} U_{1}^{c},
\end{equation}
it has an accidental Peccei-Quinn U(1) symmetry [38], hence the strong-CP problem is solved by the Peccei-Quinn mechanism. We assume that the F-flat direction: $R = 0, T = \infty$ is stabilized due to the positive squared mass $m_{T}^{2}$. The phase of flavon $a$ defined by
\begin{equation}
R = f_{a} e^{ia/f_{a}}, \quad T = f_{a} e^{-4ia/f_{a}}, \quad f_{a} = 10^{13} \text{GeV},
\end{equation}
takes the role of axion, where $f_{a}$ is a $U(1)_{PQ}$ breaking scale.

Taking account of next leading order superpotential
\begin{equation}
W_{PQB} = \frac{(RT^{4})^{3}(F^{B})^{3}X^{4}}{M_{P}^{19}} = \frac{\epsilon^{13}}{M_{P}^{12}}(RT^{4})^{3},
\end{equation}
this accidental U(1) symmetry is explicitly broken and the low energy axion potential is modified as
\begin{equation}
V_{a} = -\Lambda_{QCD}^{4} \cos \left(\theta_{0} + \frac{a}{f_{a}}\right) - \frac{\epsilon^{13} m_{SUSY} f_{a}^{15}}{M_{P}^{12}} \cos(45a/f_{a}),
\end{equation}
which sifts the global minimum from $\theta_{QCD} = 0$ to
\begin{equation}
\theta_{QCD} = \frac{45\epsilon^{13} m_{SUSY} f_{a}^{15}}{M_{P}^{12} \Lambda_{QCD}} \sim 10^{-27}.
\end{equation}
As the experimental constraint on the neutron electric moment: $\theta_{QCD} < 10^{-10}$ is satisfied, this U(1) symmetry has sufficient quality.

The density parameter of the coherent oscillation of the axion is evaluated as
\begin{equation}
\frac{\Omega_{a}}{\Omega_{CDM}} \sim \theta_{i}^{2} \left(\frac{f_{a}}{10^{12} \text{GeV}}\right)^{7},
\end{equation}
where $\theta_{i}$ is the initial value of the strong-CP phase when the QCD potential of the axion is switched on. In this paper, we assume $\theta_{i} \sim 0.3$ and dark matter is dominated by the axion. As the domain wall number of this model is $N_{DW} = 1$, our model is free from a domain wall problem of the axion. Furthermore, we assume that the flavor symmetry is not recovered both during inflation (e.g. due to the negative Hubble induced mass terms) and after reheating (due to the low reheating temperature) hence a domain wall problem of the discrete flavor symmetry is avoided [39]. As the flavon multiplets, including the axino which is the superpartner of the axion, have TeV scale mass, the thermal production of them is suppressed due to the low reheating temperature.
III. QUARK SECTOR

The superpotential of the quark sector is given by

\[ W = H_3^U Q Y^U U^c + H_3^D Q Y^D D^c + H_i^U Q Y_{i1}^U U^c + H_i^D Q Y_{i1}^D D^c, \]  

(43)

where the Yukawa matrices are given by

\[
Y^U = \begin{pmatrix}
\epsilon^6 Y_1 U^c_R & -\epsilon^3 Y_2 s_c & \epsilon^4 Y_3 U^c_b \\
\epsilon^6 Y_1 s_R & \epsilon^4 Y_2 c & \epsilon^4 Y_3 U^c_b \\
\epsilon^{25} Y_5 U^c & \epsilon^{22} Y_6 U^c & Y_9 U^c
\end{pmatrix}, \quad Y_i^U = \begin{pmatrix}
\epsilon^9 Y_{i11} U^c & \epsilon^6 Y_{i12} U^c & \epsilon^9 Y_{i13} U^c \\
\epsilon^9 Y_{i21} U^c & \epsilon^6 Y_{i22} U^c & \epsilon^9 Y_{i23} U^c \\
\epsilon^{28} Y_{i31} U^c & \epsilon^{25} Y_{i32} U^c & \epsilon^3 Y_{i33} U^c
\end{pmatrix}, \quad (44)
\]

\[
Y^D = \begin{pmatrix}
\epsilon^5 Y_1 D s_a & \epsilon^4 Y_2 D c_a & \epsilon^4 Y_3 D c_b \\
-\epsilon^5 Y_1 D c_a & \epsilon^4 Y_2 D s_a & \epsilon^4 Y_3 D s_b \\
\epsilon^{24} Y_5 D & \epsilon^{23} Y_6 D & \epsilon^2 Y_9 D
\end{pmatrix}, \quad Y_i^D = \begin{pmatrix}
\epsilon^{23} Y_{i11} D & \epsilon^{22} Y_{i12} D & \epsilon^{22} Y_{i13} D \\
\epsilon^{23} Y_{i21} D & \epsilon^{22} Y_{i22} D & \epsilon^{22} Y_{i23} D \\
\epsilon^{42} Y_{i31} D & \epsilon^{41} Y_{i32} D & \epsilon^{20} Y_{i33} D
\end{pmatrix}. \quad (45)
\]

As the Kähler potential receives the effect of the flavor violation, the superfields must be redefined as

\[
U^c \rightarrow V_K(U) U^c, \quad D^c \rightarrow V_K(D) D^c, \quad Q \rightarrow V_K(Q) Q, \quad (46)
\]

\[
V_K(U) = \begin{pmatrix}
1 & \epsilon^9 k_{12} U s_R & \epsilon^{10} k_{13} U a^U \\
\epsilon^9 k_{12} s_R & 1 & \epsilon^7 k_{23} U a^U \\
\epsilon^{10} k_{13} (a^U_1)^* & \epsilon^7 k_{23} (a^U_2)^* & 1
\end{pmatrix}, \quad (47)
\]

\[
V_K(D) = \begin{pmatrix}
1 & i \epsilon^7 k_{12} D s_{2b} & \epsilon^7 k_{13} D a^D \\
-\epsilon^7 k_{12} s_{2b} & 1 & \epsilon^6 k_{23} D a^D \\
\epsilon^7 k_{13} (a^D_2)^* & \epsilon^6 k_{23} (a^D_2)^* & 1
\end{pmatrix}, \quad (48)
\]

\[
V_K(Q) = \begin{pmatrix}
1 & \epsilon^6 k_{12} Q a_Q & \epsilon^4 k_{3} Q c_b a \\
\epsilon^6 k_{12} (a_Q)^* & 1 & \epsilon^4 k_{3} Q s_b \beta \\
\epsilon^4 k_{3} c_b a^* & \epsilon^4 k_{3} s_b \beta^* & 1
\end{pmatrix}. \quad (49)
\]
in order to get canonical kinetic terms [40]. As the result, the quark Yukawa matrices are redefined as

\[
(Y^U)' = V_K^T(Q)Y^UV_K(U), \quad (Y^{U1})' = V_K^T(Q)Y^{U1}V_K(U),
\]

\[
(Y^D)' = V_K^T(Q)Y^DV_K(D), \quad (Y^{D1})' = V_K^T(Q)Y^{D1}V_K(D),
\]

\[
(Y^U)' = \begin{pmatrix}
Y_1^U c_R e^6 & -Y_2^U s_R e^3 & O(e^4) \\
Y_1^U s_R e^6 & Y_2^U c_c e^3 & O(e^4) \\
O(e^{10}) & O(e^7) & Y_3^U
\end{pmatrix}, \quad (Y^U_i)' = \begin{pmatrix}
e^6 & e^6 & e^7 \\
e^6 & e^6 & e^7 \\
e^{13} & e^{10} & e^3
\end{pmatrix},
\]

\[
(Y^D)' = \begin{pmatrix}
Y_1^D s_a e^5 & Y_2^D c_a e^4 & Y_4^D \alpha_c e^4 \\
-Y_1^D c_a e^5 & Y_2^D s_a e^4 & Y_4^D \beta_s e^4 \\
O(e^9) & O(e^8) & Y_3^D e^2
\end{pmatrix}, \quad (Y^D_i)' = \begin{pmatrix}
e^{23} & e^{22} & e^{22} \\
e^{23} & e^{22} & e^{22} \\
e^{27} & e^{26} & e^{20}
\end{pmatrix}.
\]

Likewise, the Higgs superfields must be redefined as

\[
H^U \rightarrow V_K(H^U)H^U, \quad H^D \rightarrow V_K(H^D)H^D,
\]

\[
V_K(H^U) = \begin{pmatrix}
1 & \rho_1 e^6 & k_U c_c e^3 \\
\rho_1^* e^6 & 1 & k_U s_c e^3 \\
k_U c_c e^3 & k_U s_c e^3 & 1
\end{pmatrix},
\]

\[
V_K(H^D) = \begin{pmatrix}
1 & \rho_2 e^6 & k_D c_c e^3 \\
\rho_2^* e^6 & 1 & k_D s_c e^3 \\
k_D c_c e^3 & k_D s_c e^3 & 1
\end{pmatrix},
\]

hence the order of the elements in \((Y^{D1})'\) is modified to

\[
(Y_i^{D1})'' = (Y_i^{D1})' + O(e^3)(Y^D)' = \begin{pmatrix}
e^8 & e^7 & e^7 \\
e^8 & e^7 & e^7 \\
e^{12} & e^{11} & e^5
\end{pmatrix}.
\]

On the other hand, the changes of \((Y^U)'\), \((Y^D)'\), \((Y^{U1})'\) are negligible. Since the contributions to the quark mass matrices from the extra Higgs bosons through \((Y^{U1})'\), \((Y^{D1})'\) are negligible, the quark mass matrices are approximated as

\[
M'_U = (Y^U)'v_u, \quad M'_D = (Y^D)'v_d.
\]

These matrices are diagonalized by the superfields redefinitions

\[
U \rightarrow L_U U, \quad D \rightarrow L_D D, \quad U^c \rightarrow R_U U^c, \quad D^c \rightarrow R_D D^c,
\]
from which we get

\[
(L_U)^T = \begin{pmatrix}
  c_e & s_e & \epsilon^4 \\
  -s_e & c_e & \epsilon^4 \\
  \epsilon^4 & \epsilon^4 & 1
\end{pmatrix},
\]

\[
R_U = \begin{pmatrix}
  1 & (Y^U_1/Y^U_2)s_U\epsilon^3 & \epsilon^{10} \\
  -(Y^U_1/Y^U_2)s_U\epsilon^3 & 1 & \epsilon^7 \\
  \epsilon^{10} & \epsilon^7 & 1
\end{pmatrix}, \quad \theta_U = \theta_R - \theta_c, \tag{60}
\]

\[
(L_D)^T = \begin{pmatrix}
  s_a & -c_a & -(Y^D_4/Y^D_3)\epsilon^2 \\
  c_a & s_a & -(Y^D_4/Y^D_3)\epsilon^2 \\
  (Y^D_4/Y^D_3)\epsilon^2 & (Y^D_4/Y^D_3)\epsilon^2 & 1
\end{pmatrix},
\]

\[
R_D = \begin{pmatrix}
  1 & \epsilon^5 & \epsilon^5 \\
  \epsilon^5 & 1 & \epsilon^4 \\
  \epsilon^5 & \epsilon^4 & 1
\end{pmatrix}, \tag{62}
\]

\[
L^T_U M'_U R_U = \text{diag}(m_u, m_c, m_t) = \text{diag}(Y^U_1 c_U\epsilon^6, Y^U_2\epsilon^3, Y^U_3\epsilon^1)v_u, \tag{63}
\]

\[
L^T_D M'_D R_D = \text{diag}(m_d, m_s, m_b) = \text{diag}(Y^D_1\epsilon^5, Y^D_2\epsilon^4, Y^D_3\epsilon^2)v_d, \tag{64}
\]

\[
V_{CKM} = L^T_U L_D = \begin{pmatrix}
  s_{a-c} & c_{a-c} & (c_c b_c \alpha^* + s_c b_c \beta^*)r_D \epsilon^2 \\
  -c_{a-c} & s_{a-c} & -(s_c b_c \alpha^* - c_c b_c \beta^*)r_D \epsilon^2 \\
  -(s_a c_b s_c - c_a s_b b_c)r_D \epsilon^2 & -(c_a c_b \alpha^* + s_a s_b \beta^*)r_D \epsilon^2 & 1
\end{pmatrix}, \tag{65}
\]

\[
s_{a-c} = \sin(\theta_a - \theta_c), \quad r_D = Y^D_4/Y^D_3.
\]

The experimental values of the CKM matrix elements:

\[
\begin{pmatrix}
  |V_{ud}| & |V_{us}| & |V_{ub}| \\
  |V_{cd}| & |V_{cs}| & |V_{cb}| \\
  |V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix} = \begin{pmatrix}
  0.974 & 0.227 & 0.361 \times 10^{-2} \\
  0.226 & 0.973 & 4.05 \times 10^{-2} \\
  0.854 \times 10^{-2} & 3.98 \times 10^{-2} & 1
\end{pmatrix}, \tag{66}
\]

\[
J = \text{Im}(V_{us} V_{cb} V_{cs}^*) = 3.00 \times 10^{-5}, \tag{67}
\]

are realized by tuning the five parameters: \(\theta_{a,b,c}, r_D, \arg(\alpha\beta^*)\). The experimental values of quark running masses at 1 TeV [28][41]:

\[
m_u = 1.17 \times 10^{-3}, \quad m_c = 0.543, \quad m_t = 148.1, \tag{68}
\]

\[
m_d = 2.40 \times 10^{-3}, \quad m_s = 4.9 \times 10^{-2}, \quad m_b = 2.41 \text{ (GeV)}, \tag{69}
\]

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are realized by the Planck scale boundary values:

\[
\begin{align*}
|Y_{1U}| &= 1.5, \\ |Y_{2U}| &= 0.71, \\ |Y_{3U}| &= 0.28, \\ |Y_{1D}| &= 0.38, \\ |Y_{2D}| &= 0.78, \\ |Y_{3D}| &= 0.38,
\end{align*}
\]

(70)

where we have used the renormalization factors given in Ref. [21]. As these parameters are consistent with the assumption that all of the factor \( Y \) are \( O(1) \), the quark mass hierarchy is realized without fine-tuning. In our definition, \( Y = O(1) \) means that \( 10^{-0.5} < Y < 10^{0.5} \) is satisfied. The soft SUSY-breaking squared mass matrices of the squarks are given by

\[
\begin{align*}
\frac{m_{U}^{2}}{m^{2}} &= \begin{pmatrix} O(1) & \epsilon^{9} & \epsilon^{10} \\ \epsilon^{9} & O(1) & \epsilon^{7} \\ \epsilon^{10} & \epsilon^{7} & O(1) \end{pmatrix}, \\
\frac{m_{D}^{2}}{m^{2}} &= \begin{pmatrix} O(1) & \epsilon^{7} & \epsilon^{7} \\ \epsilon^{7} & O(1) & \epsilon^{6} \\ \epsilon^{7} & \epsilon^{6} & O(1) \end{pmatrix}, \\
\frac{m_{Q}^{2}}{m^{2}} &= \begin{pmatrix} 1 & \epsilon^{6} & \epsilon^{4} \\ \epsilon^{6} & 1 & \epsilon^{4} \\ \epsilon^{4} & \epsilon^{4} & O(1) \end{pmatrix},
\end{align*}
\]

(71) (72) (73)

and the squark A-term matrices are given by

\[
\begin{align*}
V &\supset -v_{u}UA_{U}U^{c} - v_{d}DA_{D}D^{c} + h.c. \\
A_{U} &= \begin{pmatrix} A_{1}^{U}c_{R}\epsilon^{6} & -A_{2}^{U}s_{c}\epsilon^{3} & A_{3}^{U}O(\epsilon^{4}) \\ A_{1}^{U}s_{R}\epsilon^{6} & A_{2}^{U}c_{c}\epsilon^{3} & A_{3}^{U}O(\epsilon^{4}) \\ A_{5}^{U}O(\epsilon^{10}) & A_{6}^{U}O(\epsilon^{7}) & A_{3}^{U} \end{pmatrix}, \\
A_{D} &= \begin{pmatrix} A_{1}^{D}s_{a}\epsilon^{5} & A_{2}^{D}c_{a}\epsilon^{4} & A_{3}^{D}\alpha_{b}\epsilon^{4} \\ -A_{1}^{D}c_{a}\epsilon^{5} & A_{2}^{D}s_{a}\epsilon^{4} & A_{3}^{D}\beta_{b}\epsilon^{4} \\ A_{5}^{D}O(\epsilon^{9}) & A_{6}^{D}O(\epsilon^{8}) & A_{3}^{D}\epsilon^{2} \end{pmatrix},
\end{align*}
\]

(74) (75) (76)

where these matrices are defined for canonically normalized superfields. The sizes of parameters \( m, A_{n}^{X} \) are assumed to be \( O(\text{TeV}) \). After the diagonalization of the Yukawa matrices,
the squared mass and A-term matrices are given by

\[
(m_{U})_{SCKM} = R_{U}^{\dagger} m_{U}^{2} R_{U} = m^{2} \begin{pmatrix}
O(1) & e^{3} & e^{10} \\
 e^{3} & O(1) & e^{7} \\
e^{10} & e^{7} & O(1)
\end{pmatrix},
\]

(77)

\[
(m_{D})_{SCKM} = R_{D}^{\dagger} m_{D}^{2} R_{D} = m^{2} \begin{pmatrix}
O(1) & e^{5} \\
 e^{5} & O(1) & e^{4} \\
e^{5} & e^{4} & O(1)
\end{pmatrix},
\]

(78)

\[
(m_{Q})_{SCKM} = L_{(U, D)}^{\dagger} m_{Q}^{2} L_{(U, D)} = m^{2} \begin{pmatrix}
1 & e^{(6, 4)} & e^{(4, 2)} \\
e^{(6, 4)} & 1 & e^{(4, 2)} \\
e^{(4, 2)} & e^{(4, 2)} & O(1)
\end{pmatrix},
\]

(79)

\[
(A_{U})_{SCKM} = L_{U}^{T} A_{U} R_{U} = \begin{pmatrix}
A_{U}^{c} e^{6} & A e^{9} & A e^{4} \\
 A e^{6} & A_{U}^{c} e^{3} & A e^{4} \\
 A e^{10} & A e^{7} & A_{U}^{c}
\end{pmatrix},
\]

(80)

\[
(A_{D})_{SCKM} = L_{D}^{T} A_{D} R_{D} = \begin{pmatrix}
A_{D}^{c} e^{5} & A e^{8} & A e^{4} \\
 A e^{9} & A_{D}^{c} e^{4} & A e^{4} \\
 A e^{7} & A e^{6} & A_{D}^{c}
\end{pmatrix},
\]

(81)

The off-diagonal elements of the squark mass matrices contribute to the flavor and CP violation through the squark exchange, on which severe constraints are imposed. With the mass insertion approximation, the most stringent bound for the squark mass $M_{Q}$ is given by $\epsilon_{K}$ as

\[
\sqrt{\frac{\text{Im}[(m_{Q})_{12}(m_{D})_{12}]}{M_{Q}^{4}}} = \epsilon_{K}^{4.5} < 4.4 \times 10^{-4} \left(\frac{M_{Q}}{\text{TeV}}\right) \rightarrow M_{Q} > 72\text{GeV},
\]

(82)

where $M_{Q} = M(\text{gluino}) = M(\text{squark})$ is assumed [13]. This bound is very weak and the SUSY flavor-changing-neutral-current (FCNC) problem is solved. The contribution to the FCNC from the extra Higgs is also suppressed enough due to the small $Y_{UI}, Y_{DI}$. For example, the constraint from $D^{0} - \bar{D}^{0}$ mixing gives very weak constraint: $m_{H} > 2.5\text{GeV}$ for the extra Higgs boson mass[14].
IV. LEPTON SECTOR

The superpotential of the lepton is given by

\[ W = H_3^U LY^N N^c + H_3^D LY^E E^c + H_4^D L Y_{EI} E^c + M_N N^c Y_M N^c. \]  

(83)

After the redefinition of superfields:

\[ E^c \rightarrow V_K(E)E^c, \quad N^c \rightarrow V_K(N)N^c, \quad L \rightarrow V_K(L)L, \quad H^U \rightarrow V_K(H^U)H^U, \quad H^D \rightarrow V_K(H^D)H^D, \]  

(84)

the kinetic terms are canonically normalized and the Yukawa matrices are given by

\[ (Y^N)' = \epsilon^6 \begin{pmatrix} \alpha c_b c_Y^1 N + \beta s_b s_Y^2 N + Y_5^N & \alpha c_b s_c Y_4^N + \beta s_b c_c Y_3^N & Y_7^N (\alpha c_b s_c + \beta s_b c_c) \\ \alpha c_b s_Y^3 N + \beta s_b c_c Y_4^N & \alpha c_b c_c Y_2^N + \beta s_b s_c Y_1^N + Y_5^N & Y_7^N (\alpha c_b c_c - \beta s_b s_c) \\ Y_6^N (\alpha c_b s_a + \beta s_b c_a) & Y_6^N (\alpha c_b c_a - \beta s_b s_a) & Y_9^N (\alpha c_b c_a + \beta s_b s_a) + Y_0^N \end{pmatrix}, \]  

(85)

\[ (Y^E)' = V_K^T (L) Y^E V_K (E) = \begin{pmatrix} \epsilon^6 Y_1^E & \epsilon^8 & \epsilon^6 \\ \epsilon^6 Y_2^E & \epsilon^8 & \epsilon^6 \\ \epsilon^6 & \epsilon^8 & \epsilon^6 \end{pmatrix}, \]  

(86)

\[ (Y_1^{EI})' = V_K^T (L) Y_1^{EI} V_K (E) = \begin{pmatrix} \epsilon^8 & Y_2^{EI} & \epsilon^6 \\ -\epsilon^2 Y_1^{EI} & \epsilon^8 & \epsilon^6 \\ \epsilon^8 & \epsilon^6 & \epsilon^8 \end{pmatrix}, \]  

(87)

\[ (Y_2^{EI})' = V_K^T (L) Y_2^{EI} V_K (E) = \begin{pmatrix} \epsilon^8 & Y_2^{EI} & \epsilon^6 \\ \epsilon^8 & \epsilon^6 & \epsilon^6 \end{pmatrix}, \]  

(88)

\[ (Y^M)' = \begin{pmatrix} Y_1^M & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & Y_1^M & \epsilon^6 \\ \epsilon^6 & \epsilon^6 & Y_3^M \end{pmatrix}. \]  

(89)

The charged lepton mass matrix is given by

\[ M'_E = \langle H_3^D \rangle (Y^E)' + \langle H_4^D \rangle (Y_4^{EI})' = \begin{pmatrix} \epsilon^5 (Y_1^E + Y_1^{EI} N_D) s_c & \epsilon^3 (Y_2^E + Y_2^{EI} N_D) c_c & \epsilon^8 \\ -\epsilon^5 (Y_1^E + Y_1^{EI} N_D) c_c & \epsilon^3 (Y_2^E + Y_2^{EI} N_D) s_c & \epsilon^8 \\ \epsilon^{11} & \epsilon^9 & \epsilon^8 \end{pmatrix} v_d. \]  

(90)
This matrix is diagonalized by the superfields redefinition

$$E \rightarrow L_E E, \quad E^c \rightarrow R_E E^c,$$

$$\begin{align*}
(L_E)^T &= \begin{pmatrix} s_c & -c_c & \epsilon^6 \\ c_c & s_c & \epsilon^6 \\ \epsilon^6 & \epsilon^6 & 1 \end{pmatrix}, \\
R_E &= \begin{pmatrix} 1 & \epsilon^{14} & \epsilon^9 \\ \epsilon^{14} & 1 & \epsilon^7 \\ \epsilon^9 & \epsilon^7 & 1 \end{pmatrix},
\end{align*}$$

from which we get

$$L_E^T M'_E R_E = \text{diag}(m_e, m_\mu, m_\tau) = \text{diag} \left((Y_1^E + Y_1^{EI} N_D)\epsilon^5, (Y_2^E + Y_2^{EI} N_D)\epsilon^3, Y_3^E \epsilon^2\right) v_d. \quad (94)$$

The experimental values of the charged lepton running masses at 1 TeV [41]:

$$m_e = 4.895 \times 10^{-4} \text{ (GeV)}, \quad m_\mu = 0.1033 \text{ (GeV)}, \quad m_\tau = 1.757 \text{ (GeV)}, \quad (95)$$

are realized by setting the parameters at 1 TeV as

$$Y_1^E + Y_1^{EI} N_D = 0.56, \quad Y_2^E + Y_2^{EI} N_D = 1.19, \quad Y_3^E = 2.02. \quad (96)$$

The seesaw neutrino mass matrix is given by

$$M_\nu = (L_E)^T (Y_N)^\dagger [M_N (Y_M)^\dagger]^{-1} [(Y_N)^\dagger]^T L_E, \quad (97)$$

which is diagonalized as

$$U_{\text{MNS}}^T M_\nu U_{\text{MNS}} = \text{diag}(m_1, m_2, m_3), \quad (98)$$

where $U_{\text{MNS}}$ is the Maki-Nakagawa-Sakata matrix. Since there are too many parameters in $M_\nu$, we cannot give any prediction for the neutrino mass and the elements of MNS matrix except for the mass scales

$$M_N = M_P \left(\frac{\langle X \rangle}{M_P}\right)^{10} \left(\frac{\langle \Phi_3 \rangle}{M_P}\right)^2 = 10^{-15} M_P = 1 \text{ TeV}, \quad m_\nu = \frac{(\epsilon^6 v_u)^2}{M_N} \sim 10^{-2} \text{ eV}. \quad (99)$$

The soft SUSY-breaking squared-mass matrices of the sleptons are given by

$$\frac{m_E^2}{m^2} = \begin{pmatrix} O(1) & \epsilon^8 & \epsilon^9 \\ \epsilon^8 & O(1) & \epsilon^{11} \\ \epsilon^9 & \epsilon^{11} & O(1) \end{pmatrix}, \quad \frac{m_L^2}{m^2} = \begin{pmatrix} 1 & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & 1 & \epsilon^6 \\ \epsilon^6 & \epsilon^6 & O(1) \end{pmatrix}, \quad (100)$$
and the slepton A-term matrices are given by

\[
V \supset - \langle H^D_3 \rangle E A^E E^c - \langle H^D_1 \rangle E A^E_I E^c + h.c., \quad (101)
\]

\[
A^E = \begin{pmatrix}
A_E^1 c_5 c c^3 & A^8 \\
-A^1_E c_5 c c^3 & A^8 \\
A^1 E^{11} & A^9 & A^2 E^{c_2}
\end{pmatrix}, \quad (102)
\]

\[
A^E_{1I} = \begin{pmatrix}
A^8_E & A^2 E^{11} \\
-\epsilon^2 A^1 E^I & A^6 E^{11} \\
A^8 E & A^6 E^c
\end{pmatrix}, \quad A^E_{2I} = \begin{pmatrix}
\epsilon^2 A^1 E^I & A^6 E^{11} \\
A^8 E & A^2 E^{11} \\
A^8 E & A^6 E^c
\end{pmatrix}, \quad (103)
\]

where these matrices are defined for canonically normalized superfields. We define the effective A-term matrix as follows:

\[
(A^E)'_{v_d} = A^E \langle H^D_3 \rangle + A^E_{1I} \langle H^D_1 \rangle + A^E_{2I} \langle H^D_2 \rangle, \quad (104)
\]

\[
(A^E)' = \begin{pmatrix}
(A^1_E)' s c_5 & (A^2_E)' c c^3 & A^8 \\
-(A^1_E)' c c^3 & (A^2_E)' s c^3 & A^8 \\
A^1 E^{11} & A^9 & A^2 E^{c_2}
\end{pmatrix}, \quad (105)
\]

After the diagonalization of the charged lepton Yukawa matrix, the squared-mass matrices and the effective A-term matrix are given by

\[
(m^2_{E})_{\text{SMNS}} = R^T_E m^2_E R_E = m^2 \begin{pmatrix}
O(1) & e^8 & e^9 \\
e^8 & O(1) & e^7 \\
e^9 & e^7 & O(1)
\end{pmatrix}, \quad (106)
\]

\[
(m^2_{L})_{\text{SMNS}} = L^T_E m^2_L L_E = m^2 \begin{pmatrix}
1 & e^6 & e^6 \\
e^6 & 1 & e^6 \\
e^6 & e^6 & O(1)
\end{pmatrix}, \quad (107)
\]

\[
(A^E)_{\text{SMNS}} = L^T_E (A^E)' R_E = \begin{pmatrix}
A^E_{1I} e^5 & A^9 E^c \\
A^e^{11} & A^2 E^c e^3 \quad A^8 \\
A^e^{11} & A^9 & A^2 E^{c_2}
\end{pmatrix}. \quad (108)
\]

As the (1, 1) element of $(A^E)_{\text{SMNS}}$ is real at leading order, the SUSY contribution to the electric dipole moment of the electron is negligible. Based on the consideration of the lepton
flavor violation, the most stringent bound for the slepton mass $M_L$ is given by $\mu \rightarrow e + \gamma$ as

$$\frac{v_d}{M_L} < 1.4 \times 10^{-6} \left( \frac{M_L}{300 \text{GeV}} \right) \sqrt{\frac{\text{Br}(\mu \rightarrow e\gamma)_{\text{exp}}}{4.2 \times 10^{-13}}} \rightarrow M_L > 4.3 \text{GeV},$$

(109)

where $M_L = M(\text{slepton}) = M(\text{photino})$ is assumed.

For the canonically normalized superfields, the RHN mass matrix is given by

$$M_R = M_N(Y^M)' ,$$

(110)

whose eigenvalues $M_1, M_2, M_3$ give the degenerated mass spectrum of RHNs as follows:

$$M_1 \simeq M_2 = M_1(1 + \epsilon^6) \rightarrow \delta_N = \frac{M_2 - M_1}{M_1} \sim \epsilon^6.$$

(111)

In this paper, we assume $N^c_3$ is the heaviest RHN, hence $M_2 < M_3$. The right-handed sneutrinos have the same spectrum. In the early Universe, the out-of-equilibrium decay of $n^c_1$ and $N^c_1$ generates a lepton asymmetry which is transformed into a baryon asymmetry by the electroweak sphaleron process. Following Ref.[42], the baryon asymmetry is given by

$$B_f \sim -\frac{\kappa \epsilon_{CP}}{3 g_*},$$

(112)

where $g_* = 340$ is the degree of freedom of radiation, $\kappa$ is the dilution factor given by

$$\kappa \sim \left( \frac{1}{K \text{ln } K} \right) ,$$

$$K = \left( \frac{\Gamma(M_1)}{2H(M_1)} \right), \Gamma(M_1) = \frac{K_{11}M_1}{8\pi}, \quad H(M_1) = \sqrt{\frac{\pi^2 g_* M_1^4}{90 M_P^2}}, \quad K_{ij} = \frac{1}{3} \sum \text{Tr} (Y^N_{li} ' Y^N_{lj} ),$$

(113)

and $\epsilon_{CP}$ is given by

$$\epsilon_{CP} = -\frac{1}{2\pi} \frac{\text{Im}(K^2)}{K_{11}} \left( \frac{2\sqrt{x} + \sqrt{x} \ln \left( 1 + \frac{x}{x} \right)}{x - 1} \right) \simeq -\frac{\text{Im}(K^2)}{2\pi K_{11} \delta_N}, \quad x = \frac{M_2^2}{M_1^2} \sim 1 + 2\delta_N.$$

(114)

From the order estimations

$$K_{12} \sim K_{11} \sim \epsilon^{12}, \quad K \sim 6 \left( \frac{\text{TeV}}{M_1} \right), \quad \epsilon_{CP} \sim 10^{-6}, \quad M_1 \sim 1000 \text{GeV},$$

(115)

we get the observed baryon asymmetry, $B_f \sim 10^{-10}$. This mechanism works even at the low reheating temperature as $T_{RH} < 10^7 \text{GeV}$ which is required for avoiding gravitino overproduction.
V. HIGGS SECTOR

A. Higgs bosons

For the canonically normalized superfields, the superpotential of the Higgses up to $O(\epsilon^3)$-terms is given by

\[
W = \lambda_2 S_2 H_3^U H_3^D + \lambda_3 S_3 H_3^U H_3^D \\
+ \lambda_4 S_2 (H_1^U H_1^D + H_2^U H_2^D) + \lambda_5 S_3 (H_1^U H_1^D + H_2^U H_2^D) \\
+ \epsilon^3 \lambda_6 S_2 (c_c H_1^U + s_c H_2^U) H_3^D + \epsilon^3 \lambda_7 S_3 (c_c H_1^U + s_c H_2^U) H_3^D \\
+ \epsilon^3 \lambda_8 S_2 H_3^U (c_c H_1^D + s_c H_2^D) + \epsilon^3 \lambda_9 S_3 H_3^U (c_c H_1^D + s_c H_2^D),
\]

from which we get the Higgs potential as

\[
V = -m_{H_3}^2 |H_3^U|^2 + m_{H_3}^2 |H_3^D|^2 + m_{S_1}^2 |S_1|^2 - m_{S_2}^2 |S_2|^2 - m_{S_3}^2 |S_3|^2 \\
+ m_{H_3}^2 (|H_1^U|^2 + |H_2^U|^2) + m_{H_3}^2 (|H_1^D|^2 + |H_2^D|^2) - m_{S_2}^2 (S_2 S_3 + S_2 S_3^*) \\
- \epsilon^3 m_{D_U}^2 [(c_c H_1^U + s_c H_2^U)^* H_3^U + h.c.] - \epsilon^3 m_{D_U}^2 [(c_c H_1^D + s_c H_2^D)^* H_3^D + h.c.]
\]

\[
- A_2 [S_2 H_3^U H_3^D + h.c.] - A_3 [S_3 H_3^U H_3^D + h.c.] \\
- A_4 [S_2 (H_1^U H_1^D + H_2^U H_2^D) + h.c.] - A_5 [S_3 (H_1^U H_1^D + H_2^U H_2^D) + h.c.] \\
- \epsilon^3 A_6 [S_2 (c_c H_1^U + s_c H_2^U) H_3^D + h.c.] - \epsilon^3 A_7 [S_3 (c_c H_1^U + s_c H_2^U) H_3^D + h.c.] \\
- \epsilon^3 A_8 [S_2 H_3^U (c_c H_1^D + s_c H_2^D) + h.c.] - \epsilon^3 A_9 [S_3 H_3^U (c_c H_1^D + s_c H_2^D) + h.c.]
\]

\[
+ \lambda_2 H_3^U H_3^D + \lambda_4 (H_1^U H_1^D + H_2^U H_2^D) \\
+ \epsilon^3 \lambda_6 (c_c H_1^U + s_c H_2^U) H_3^D + \epsilon^3 \lambda_8 (c_c H_1^D + s_c H_2^D)^2 \\
+ \lambda_3 H_3^U H_3^D + \lambda_5 (H_1^U H_1^D + H_2^U H_2^D) \\
+ \epsilon^3 \lambda_7 (c_c H_1^U + s_c H_2^U) H_3^D + \epsilon^3 \lambda_9 (c_c H_1^D + s_c H_2^D)^2 \\
+ \lambda_2 S_2 H_3^U + \lambda_3 S_3 H_3^D + \epsilon^3 \lambda_8 S_2 (c_c H_1^D + s_c H_2^D) + \epsilon^3 \lambda_9 S_3 (c_c H_1^D + s_c H_2^D)^2
\]
\[
\begin{align*} 
&+ |\lambda_2 S_2 H_3^U + \lambda_3 S_3 H_3^U + \epsilon^3 \lambda_6 S_2 (c_c H_1^U + s_c H_2^U) + \epsilon^3 \lambda_7 S_3 (c_c H_1^U + s_c H_2^U)|^2 \\
&+ |\lambda_4 S_2 H_1^D + \lambda_5 S_3 H_1^D + \epsilon^3 \lambda_6 S_2 (c_c H_2^D + \epsilon^3 \lambda_7 S_3 H_3^D)|^2 \\
&+ |\lambda_4 S_2 H_2^D + \lambda_5 S_3 H_2^D + \epsilon^3 \lambda_6 S_2 (c_c H_3^D + \epsilon^3 \lambda_7 S_3 H_3^D)|^2 \\
&+ |\lambda_4 S_2 H_1^U + \lambda_5 S_3 H_1^U + \epsilon^3 \lambda_6 S_2 (c_c H_2^U + \epsilon^3 \lambda_7 S_3 H_3^U)|^2 \\
&+ |\lambda_4 S_2 H_2^U + \lambda_5 S_3 H_2^U + \epsilon^3 \lambda_6 S_2 (c_c H_3^U + \epsilon^3 \lambda_7 S_3 H_3^U)|^2 \\
&+ \frac{1}{8} g_y^2 \left[ |H_a^U|^2 - |H_a^D|^2 \right]^2 + \frac{1}{8} g_2^2 \sum_{A=1}^{3} \left[ (H_a^U)^* \sigma_A H_a^U + (H_a^D)^* \sigma_A H_a^D \right]^2 \\
&+ \frac{9}{2} g_s^2 \left[ |S_a|^2 - |H_a^D|^2 \right]^2, \tag{118}
\end{align*}
\]

where

\[ g_s = \frac{\sqrt{10}}{12} g_S. \tag{119} \]

With the definition of the charged, the CP-even neutral, and the CP-odd neutral Higgs bosons

\[
H_a^U = \begin{pmatrix} u_a^+ \\ u_a + i p_a \end{pmatrix}, \quad H_a^D = \begin{pmatrix} d_a + i q_a \\ d_a \end{pmatrix}, \quad S_a = s_a + i r_a, \tag{120}
\]

their mass matrices are defined as follows:

\[
\begin{align*} 
V & \ni (u_a^+, d_a^+) \\
&+ (p_a, q_a, r_a) \begin{pmatrix} m_{ab}(U^+U^-) & m_{ab}(U^+D^-) \\ m_{ab}(D^+U^-) & m_{ab}(D^+D^-) \end{pmatrix} \begin{pmatrix} u_b^- \\ d_b^- \end{pmatrix} \\
&+ (u_a, d_a, s_a) \begin{pmatrix} m_{ab}(U^0U^0) & m_{ab}(U^0D^0) \\ m_{ab}(D^0U^0) & m_{ab}(D^0D^0) \end{pmatrix} \begin{pmatrix} u_b \\ d_b \end{pmatrix} + \begin{pmatrix} m_{ab}(S^0S^0) \end{pmatrix} \begin{pmatrix} s_b \end{pmatrix}. \tag{121}
\end{align*}
\]
After imposing the constraints which are derived from the potential minimum condition on the elements of matrices, we get

\[
m_{ab}^2(U^+ U^-) = m_{ab}^2(U U) = m_{ab}^2(U^0 U^0) = \\
\begin{pmatrix}
m_H^2 + \lambda_{23}^2 v_s^2 & 0 & \epsilon^3 M_{BU}^2 c_c \\
0 & m_H^2 + \lambda_{45}^2 v_s^2 & \epsilon^3 M_{BU}^2 s_c \\
\epsilon^3 M_{BU}^2 c_c & \epsilon^3 M_{BU}^2 s_c & A_{23} v_s v_u / v_d
\end{pmatrix}, \tag{122}
\]

\[
m_{ab}^2(D^+ D^-) = m_{ab}^2(D D) = m_{ab}^2(D^0 D^0) = \\
\begin{pmatrix}
m_{DD}^2 + \lambda_{45}^2 v_s^2 - D_s & 0 & \epsilon^3 M_{BD}^2 c_c \\
0 & m_{DD}^2 + \lambda_{45}^2 v_s^2 - D_s & \epsilon^3 M_{BD}^2 s_c \\
\epsilon^3 M_{BD}^2 c_c & \epsilon^3 M_{BD}^2 s_c & A_{23} v_s v_u / v_d
\end{pmatrix}, \tag{123}
\]

\[
m_{ab}^2(U^+ D^-) = m_{ab}^2(U D) = -m_{ab}^2(U^0 D^0) = \\
\begin{pmatrix}
A_{45} v_s & 0 & \epsilon^3 A_{67} v_s c_c \\
0 & A_{45} v_s & \epsilon^3 A_{67} v_s s_c \\
\epsilon^3 A_{89} v_s c_c & \epsilon^3 A_{89} v_s s_c & A_{23} v_s
\end{pmatrix}, \tag{124}
\]

\[
m^2(D^+ U^-) = m_{ab}^2(D U) = -m_{ab}^2(D^0 U^0) = [m_{ab}^2(U^+ D^-)]^T \tag{125}
\]

\[
m_{ab}^2(S S) = \\
\begin{pmatrix}
m_{S_1}^2 + D_s & 0 & 0 \\
0 & m_{S_4}^2 (s_s / c_s) & -m_{S_4}^2 \\
0 & -m_{S_4}^2 & m_{S_4}^2 (c_s / s_s)
\end{pmatrix}, \tag{126}
\]

\[
m_{ab}^2(S^0 S^0) = \\
\begin{pmatrix}
m_{S_1}^2 + D_s & 0 & 0 \\
0 & 18 g_s^2 c_s^2 v_s^2 + m_{S_4}^2 (s_s / c_s) & 18 g_s^2 c_s s_s v_s^2 - m_{S_4}^2 \\
0 & 18 g_s^2 c_s^2 s_s v_s^2 - m_{S_4}^2 & 18 g_s^2 s_s^2 v_s^2 + m_{S_4}^2 (c_s / s_s)
\end{pmatrix}, \tag{127}
\]

\[
D_s = 9 g_s^2 v_s^2, \tag{128}
\]

\[
\lambda_{nm} = \lambda_n c_s + \lambda_m s_s, \tag{129}
\]

\[
A_{nm} = A_n c_s + A_m s_s, \tag{130}
\]

\[
M_{BU}^2 = (\lambda_{23} \lambda_{67} + \lambda_{45} \lambda_{89}) v_s^2 - m_{BU}^2, \tag{131}
\]

\[
M_{BD}^2 = (\lambda_{23} \lambda_{89} + \lambda_{45} \lambda_{67}) v_s^2 - m_{BD}^2, \tag{132}
\]
where \( O(v_{u,d}) \) contributions are neglected. After the field redefinitions as

\[
\begin{pmatrix}
X_1 \\
X_2 \\
X_3
\end{pmatrix} = \begin{pmatrix}
cc -sc \\
sc \\
0 
0 
0 
1
\end{pmatrix} \begin{pmatrix}
X_1' \\
X_2' \\
X_3'
\end{pmatrix}, \quad X = (H^U, H^D),
\]

\begin{equation}
(133)
\end{equation}

we get approximately diagonalized mass matrices

\[
\begin{pmatrix}
m^2_{ab}(U^+ U^-) & \\
m^2_{ab}(U^+ D^-) & \\
m^2_{ab}(S^0 S^0)
\end{pmatrix}' = \begin{pmatrix}
m^2_{HU} + \lambda^2_{45} v^2_s & 0 & e^3 M^2_{BU} \\
0 & m^2_{HU} + \lambda^2_{45} v^2_s & 0 \\
e^3 M^2_{BU} & 0 & A_{23} v_u v_d / v_u
\end{pmatrix},
\]

\begin{equation}
(135)
\end{equation}

\[
\begin{pmatrix}
m^2_{ab}(U^+ D^-) & \\
m^2_{ab}(D^+ D^-) & \\
m^2_{ab}(S^0 S^0)
\end{pmatrix}' = \begin{pmatrix}
m^2_{HD} + \lambda^2_{45} v^2_s - D_s & 0 & e^3 M^2_{BD} \\
0 & m^2_{HD} + \lambda^2_{45} v^2_s - D_s & 0 \\
e^3 M^2_{BD} & 0 & A_{23} v_u v_u / v_d
\end{pmatrix},
\]

\begin{equation}
(137)
\end{equation}

\[
\begin{pmatrix}
m^2_{ab}(SS) & \\
m^2_{ab}(S^0 S^0)
\end{pmatrix}' = \begin{pmatrix}
m^2_{S_1} + D_s & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & m^2_{S_1} / c_s s_s
\end{pmatrix},
\]

\begin{equation}
(138)
\end{equation}

\[
\begin{pmatrix}
m^2_{ab}(S^0 S^0)
\end{pmatrix}' = \begin{pmatrix}
m^2_{S_1} + D_s & 0 & 0 \\
0 & 18 g_s^2 v^2_s & 0 \\
0 & 0 & m^2_{S_4} / c_s s_s
\end{pmatrix}.
\]

\begin{equation}
(139)
\end{equation}

In order to get the mass of the lightest neutral Higgs boson, we take account of \( O(v_{u,d}) \) contributions and diagonalize the mass matrix by the field redefinition as

\[
\begin{pmatrix}
u_3 \\
d_3 \\
s_2 \\
s_3
\end{pmatrix} = \begin{pmatrix}
v_u / v & -v_d / v & 0 & 0 \\
v_d / v & v_u / v & 0 & 0 \\
0 & 0 & c_s - s_s \\
0 & 0 & s_s & c_s
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\phi_4
\end{pmatrix},
\]

\begin{equation}
(140)
\end{equation}
then we get the squared masses of $\phi_{1,2,3,4}$ as follows:

\begin{align}
    m_1^2 &= \frac{1}{2}(g_Y^2 + g_2^2)(v_u^2 - v_d^2)/v^2 + 4(\lambda_2^2 + \lambda_3^2)v_u^2v_d^2/v^2 = [0.0687 + 0.75(\lambda_2^2 + \lambda_3^2)]v^2, \quad (141) \\
    m_2^2 &= A_{23}v_s v_d^2/v_u v_d + O(v^2), \\
    m_3^2 &= 18 g_s^2 v_s^2, \\
    m_4^2 &= m_{S_4}^2/c_s s_s. \\
\end{align}

where $g_Y = 0.357, g_2 = 0.65$ are substituted into Eq.(141). Here, $m_1$ is a tree level contribution to the mass of the lightest CP-even neutral Higgs boson $\phi_1$. While the two contributions from $\lambda_2$ and $\lambda_3$ to $m_1$ are additive, the contributions from them to the higgsino mass: $\lambda_{23}v_s$ could be destructive, and if so, which enhances the lepton $g - 2$ through reducing the higgsino mass. The experimental value 125GeV is realized by adding the one-loop contribution from the stop [43]

\begin{equation}
    \Delta m_1^2 \simeq \frac{3m_T^2}{4\pi^2} \ln \frac{m_T^2}{m_i^2}, \quad (145)
\end{equation}

where $m_T$ is stop mass. The one-loop contributions from the G Higgses assist to push up the Higgs mass [44].

**B. G Higgs**

The mass terms for the G Higgses are derived from the superpotential:

\begin{equation}
    W = k_2 S_2 G_a G^c_a + k_3 S_3 G_a G^c_a, \quad (146)
\end{equation}

from which mass terms are given by

\begin{align}
    V &\supset m_G^2 |G|^2 + m_{G^c}^2 |G^c|^2 - A_{k_{23}} v_s G_a G^c_a + h.c. \\
    &+ |k_2 G_a G^c_a + \lambda_2 v_u v_d|^2 + |k_3 G_a G^c_a + \lambda_3 v_u v_d|^2 + k_{23}^2 |v_s G|^2 + |v_s G^c|^2) + D-terms, \quad (147) \\
    A_{k_{23}} &= A_{k_2} c_s + A_{k_3} s_s, \quad k_{23} = k_2 c_s + k_3 s_s. \quad (148)
\end{align}

Due to the $S_4$ symmetry, the triplets $G, G^c$ have unified mass matrices as follows:

\begin{equation}
    V \supset (G_a, G^c_a) M_G^2 \begin{pmatrix} G_a \\ (G^c_a)^{*} \end{pmatrix}, \quad (149)
\end{equation}

\begin{equation}
    M_G^2 = \begin{pmatrix} m_G^2 + (k_{23} v_s)^2 - \frac{1}{3} m_{Z^c}^2 & (k_2 \lambda_2 + k_3 \lambda_3) v_u v_d - A_{k_{23}} v_s \\
    (k_2 \lambda_2 + k_3 \lambda_3) v_u v_d - A_{k_{23}} v_s & m_{G^c}^2 + (k_{23} v_s)^2 - \frac{1}{3} m_{Z^c}^2 \end{pmatrix}. \quad (150)
\end{equation}
The interaction terms for the G-Higgses are given by

\[ W_G = GQY^Q Q + G^c U^c Y^{UD} D^c + GU^c Y^{UE} E^c + G^c QY^Q L + GD^c Y^{DN} N^c, \]  

(151)

where the coupling matrices are given by

\[ Y^{QQ} = \begin{pmatrix} \epsilon^{21} & \epsilon^{21} & \epsilon^{19} \\ \epsilon^{21} & \epsilon^{21} & \epsilon^{19} \\ \epsilon^{19} & \epsilon^{19} & \epsilon^{17} \end{pmatrix}, \quad Y^{UD} = \begin{pmatrix} \epsilon^{28} & \epsilon^{27} & \epsilon^{27} \\ \epsilon^{25} & \epsilon^{24} & \epsilon^{24} \\ \epsilon^{24} & \epsilon^{23} & \epsilon^2 \end{pmatrix}, \]  

(152)

\[ Y^{UE} = \begin{pmatrix} \epsilon^{24} & \epsilon^{22} & \epsilon^{42} \\ \epsilon^{21} & \epsilon^{36} & \epsilon^{39} \\ \epsilon^{20} & \epsilon^{35} & \epsilon^{38} \end{pmatrix}, \quad Y^{QL} = \begin{pmatrix} \epsilon^{25} & \epsilon^{25} & \epsilon^{25} \\ \epsilon^{25} & \epsilon^{25} & \epsilon^{25} \\ \epsilon^{23} & \epsilon^{23} & \epsilon^{23} \end{pmatrix}, \quad Y^{DN} = \begin{pmatrix} \epsilon^{24} & \epsilon^{24} & \epsilon^{24} \\ \epsilon^{23} & \epsilon^{23} & \epsilon^{23} \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \end{pmatrix}. \]  

(153)

As \( Y^{QQ}, Y^{UE}, Y^{QL} \) do not cause any observable effect, they are out of our consideration. At the SCKM basis, \( Y^{UD}, Y^{DN} \) are redefined as follows:

\[ (Y^{UD})_{SCKM} = R^T_U V^T_K (U) Y^{UD} V_K (D) R_D = \begin{pmatrix} \epsilon^{17} & \epsilon^{16} & \epsilon^{12} \\ \epsilon^{14} & \epsilon^{13} & \epsilon^9 \\ \epsilon^7 & \epsilon^6 & \epsilon^2 \end{pmatrix}, \]  

(154)

\[ (Y^{DN})_{SCKM} = R^T_D V^T_K (D) (Y^{DN}) V_K (N) = \begin{pmatrix} \epsilon^7 & \epsilon^7 & \epsilon^7 \\ \epsilon^6 & \epsilon^6 & \epsilon^6 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \end{pmatrix}. \]  

(155)

As the large elements in the \((3, 3)\)-entry of \( (Y^{UD})_{SCKM} \) and the third row of \( (Y^{DN})_{SCKM} \) induces very fast decay of \( G, G^c \), the success of BBN is not spoiled. Further more, if \( G \to n + b \) is the dominant decay mode of \( G \), we can verify the RHN directly at a collider experiment. These couplings also open the dangerous channel to the proton decay. The dominant contributions to the proton decay are induced by the couplings

\[ (Y^{us}) (Y^{dn}) = (\epsilon^{16})(\epsilon^7) = \epsilon^{23}, \quad (Y^{ud}) (Y^{sn}) = (\epsilon^{17})(\epsilon^6) = \epsilon^{23}. \]  

(156)

Multiplied by the factor

\[ \frac{\epsilon^6 v_u}{M_R} \sim \epsilon^6, \]  

(157)

which comes from \( N^c - \nu \) mixing, the dimension-less coefficient of the 4-Fermi operator

\[ \mathcal{L} \supset \frac{c_{ud\nu}}{M^2(G)} \bar{u}d\bar{\nu}s, \]  

(158)
is estimated as

\[ c_{uds\nu} \sim \epsilon^{29}, \]  

(159)

which is consistent with the experimental bound for \( p \rightarrow K^+ + \nu \):

\[ c_{uds\nu} < 10^{-27}. \]  

(160)

As the single G interactions violate \( B + L \) while they conserve \( B - L \), they may assist the \( B + L \) violating process which converts a lepton number to a baryon number. The dominant terms which contribute to this process are given by

\[
W = \epsilon^2 Y_{UD} (G_1^c + G_2^c + G_3^c) U_3^c D_3^c \\
+ \epsilon^2 Y_{DN}^D [\sqrt{3}(G_2 - G_3)N_1^c + (G_1 + G_2 - 2G_3)N_2^c] D_3^c \\
+ \epsilon^8 Y_{DN}^D (G_1 + G_2 + G_3) D_3^c (c_N N_1^c + s_N N_2^c).  
\]  

(161)

The contributions from the first line and the second line in Eq.(161) to the term

\[
(\epsilon^2 Y_{UD}^D)(\epsilon^2 Y_{DN}^D) U_3^c D_3^c N_i^c D_3^c, 
\]  

(162)

is canceled due to the mass degeneracy in G Higgs [45]. As any linear combinations of \( G_a \) which are gotten by unitary transformation are assumed to be mass eigenstates, we can move to more convenient view point. Up to \( O(\epsilon^2) \), since we can assign the \( S_3 \) singlet

\[ G_D = \frac{G_1 + G_2 + G_3}{\sqrt{3}} \]  

(163)

to a diquark and the \( S_3 \) doublet

\[ G_{Li} = \left( \frac{G_2 - G_3}{\sqrt{2}}, \frac{G_1 + G_2 - 2G_3}{\sqrt{6}} \right) \]  

(164)

to a leptoquark, the baryon number and the lepton number are conserved respectively. However they are violated by including the \( O(\epsilon^8) \) terms. The contribution to Eq.(162) is induced by the first line and third line in Eq.(161). Requiring the process \( n + \bar{t} \rightarrow b + b \) is in equilibrium, we get the constraint

\[
1 < \frac{\Gamma(n + \bar{t} \rightarrow b + b)}{H(m_N)} \sim 10^{12}\epsilon^{10}(Y_{UD}^D Y_{DN}^D)^2 \frac{m_N^4}{m_G^4} \sim 10^{-8}(Y_{UD}^D Y_{DN}^D)^2, \]  

(165)

which is difficult to be satisfied. Therefore the terms in Eq.(161) do not have significant impact on leptogenesis.
C. LSP

As the R-parity is conserved in this model, the LSP is stable. We identify the singlino $s_1$ as the LSP which has a tiny mass

$$m(s_1) \sim \frac{(\epsilon^6 v_u)(\epsilon^6 v_d)}{\lambda_{23} v_s} \sim 10^{-2}\text{eV}.$$  \hfill (166)

Although $s_1$ is not the dominant component of dark matter, it may help to explain the delay of structure formation [46]. Furthermore, $s_1$ behaves as an extra neutrino, which changes the effective number of neutrinos to ([47])

$$N_{\text{eff}} = 3.097,$$  \hfill (167)

where $m_{Z'} < 4700\text{GeV}$ is assumed. This extra contribution softens the Hubble tension between the distance ladder method [48] and the CMB data [49].

The interaction of the bino which is the LSP of MSSM is given by

$$\mathcal{L} \supset ig_Y \frac{\epsilon^6 v_d}{m_{\text{SUSY}}} (H_3^U)^* \lambda_Y s_1,$$  \hfill (168)

from which the bino lifetime is calculated as follows:

$$\Gamma(\lambda_T \rightarrow H + s_1) \sim \frac{a_Y m_{\text{SUSY}}}{\epsilon^6 v_d} \left(\frac{m_{\text{SUSY}}}{m_{\text{SUSY}}}\right) \sim 10^{-4}\text{eV} \rightarrow \tau(\lambda_Y) \sim 10^{-11}\text{sec},$$  \hfill (169)

which is consistent with the standard cosmology. The NLSP in our model is the lighter of two linear combinations of two singlinos $s_{2,3}$ which must be heavier than 100MeV to avoid the longer lifetime than 1sec [50]. It is easy to give such a tiny mass to the NLSP.

VI. LEPTON ANOMALOUS MAGNETIC DIPOLE MOMENTS

Here, we evaluate the lepton anomalous magnetic moments. The $(g - 2)_\mu$ has 3.7$\sigma$ discrepancy between the SM prediction and the experimental value as given in Eq.(1). For the $(g - 2)_e$, there is 2.4$\sigma$ discrepancy between the new SM prediction and the experimental value as given in Eq.(5). These gaps are filled by the SUSY contributions. While the flavor blind contribution gives Eq.(6), the experimental observation Eq.(7) does not obey it. This discrepancy reflects non-trivial flavor structure of new physics. In our model, this comes from the structure of charged lepton Yukawa matrices.
In the basis that the charged lepton and Higgs mass matrices are diagonalized, the Yukawa interactions of the charged lepton are given by

$$W_E = H_A^D L \begin{pmatrix} e^2 Y^E_1 & e^6 & e^7 \\ e^8 & Y^E_2 & e^7 \\ e^8 & e^6 & e^5 \end{pmatrix} E^c + H_B^D L \begin{pmatrix} e^8 & -Y^E_1 & e^7 \\ e^2 Y^E_1 & e^6 & e^7 \\ e^8 & e^6 & e^{13} \end{pmatrix} E^c + H_C^D L \begin{pmatrix} e^5 Y^E_1 & \epsilon^{15} & e^8 \\ e^{17} & e^3 Y^E_2 & e^8 \\ e^{11} & e^9 & e^2 Y^E_3 \end{pmatrix} E^c,$$

(170)

where $H_A^D = (H_1^D)'$ and $H_B^D = (H_2^D)'$ are mass eigenstates defined by Eq.(133). These interactions induce $\mu \to e + \gamma$ process and the experimental constraint for the branching ratio

$$BR(\mu \to e \gamma) = \frac{48\pi^3 \alpha_{em}}{G_F^2} \left( \frac{e^6 Y^E_2}{192\pi^2 m_{A,B}^2} \right)^2 < 4.2 \times 10^{-13},$$

(171)

which gives the lower mass bound for $H_{A,B}^D$ as

$$\frac{m_{A,B}}{\sqrt{Y^E_2}} > 15\text{GeV},$$

(172)

where $\alpha_{em} = 1/137$ and $G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$ are used. This constraint is easily satisfied.

The chargino and the neutralino mass matrices are given by

$$\mathcal{L} = -\chi_+^T M_C \chi_- - \frac{1}{2} \chi^T M_N \chi - \lambda_{45} v_s (h_B^U)^+ (h_B^D)^- - \lambda_{45} v_s (h_B^D)^0 (h_B^U)^0 + h.c.,$$

(173)

$$M_C = \begin{pmatrix} \lambda_{45} v_s & e^3 \lambda_{67} v_s & g_2 N_U e^4 v_u \\ e^3 \lambda_{87} v_s & \lambda_{23} v_s & g_2 v_u \\ g_2 N_D e^4 v_d & g_2 v_d & M_2 \end{pmatrix},$$

(174)
which are diagonalized by bi-unitary translation and unitary translation respectively as

$$\chi_T = (h_A^D)^-, (h_3^D)^-, w^-),$$

$$\chi_T = (h_A^U)^+, (h_3^U)^+, w^+),$$

$$w^\pm = \mp i\lambda_1 - \lambda_2^\pm/\sqrt{2},$$

$$\chi_T = (h_A^U, h_3^U, h_A^D, h_3^D, i\lambda_Y, i\lambda_2^3),$$

which are diagonalized by bi-unitary translation and unitary translation respectively as

$$\chi_+ = U\chi^T_+, \quad \chi_- = D\chi^T_-, \quad U^T M_3 D = \text{diag}(\mu_1, \mu_2, \mu_3),$$

$$\chi = V\chi^T, \quad V^T M_N V = \text{diag}(\xi_1, \xi_2, \cdots, \xi_6),$$

from which we calculate the one-loop contributions to the muon and electron $g-2$ as follows:

$$a_{\mu,e}(\text{SUSY}) = a_{\mu,e}(\chi^\pm) + a_{\mu,e}(\chi^0) + a_{\mu,e}(h_B),$$

$$a_{\mu}(\chi^\pm) = \frac{m_\mu}{16\pi^2 m^2(N)} \sum_{a=1,2,3} \left\{ \frac{1}{3} m_\mu (|C_{\mu,a}^L|^2 + |C_{\mu,a}^R|^2) f_C(x_a) - 3 \mu_a \text{Re}[C_{\mu,a}^L (C_{\mu,a}^R)^*] g_C(x_a) \right\},$$

$$a_e(\chi^\pm) = \frac{m_e}{16\pi^2 m^2(N)} \sum_{a=1,2,3} \left\{ \frac{1}{3} m_e (|C_{e,a}^L|^2 + |C_{e,a}^R|^2) f_C(x_a) - 3 \mu_a \text{Re}[C_{e,a}^L (C_{e,a}^R)^*] g_C(x_a) \right\},$$

$$f_C(x) = \frac{1}{(1-x)^4} \left( 1 + \frac{3}{2} x - 3 x^2 + \frac{1}{2} x^3 + 3 x \ln x \right),$$

$$g_C(x) = \frac{1}{(1-x)^3} \left( 1 - \frac{4}{3} x + \frac{1}{3} x^2 + \frac{2}{3} \ln x \right),$$

$$x_a = \frac{m_a^2}{m^2(N)}, \quad m^2(N) = m_L^2 + \frac{g_3^2 + g_2^2}{4} (v_d^2 - v_u^2),$$

$$C_{\mu,a}^L = Y_2^E D_1 a + \epsilon Y_2^F D_2 a,$$

$$C_{\mu,a}^R = - g_2 U_{3a},$$
\[ C_{e,a}^L = \epsilon^2 Y_1^E D_{1a} + \epsilon^5 Y_1^E D_{2a}, \]  
\[ C_{e,a}^R = -g_3 U_{3a}, \]  
(190)  
(191)

\[ a_\mu(\chi^0) = -\frac{m_\mu}{16\pi^2 m^2(E)} \sum_{a=1}^{6} \left\{ \frac{1}{6} m_\mu (|N_{\mu,a}^L|^2 + |N_{\mu,a}^R|^2) f_N(y_a) + \xi_a \text{Re}[N_{\mu,a}^L N_{\mu,a}^R]^* g_N(y_a) \right\}, \]  
(192)

\[ a_e(\chi^0) = -\frac{m_e}{16\pi^2 m^2(E)} \sum_{a=1}^{6} \left\{ \frac{1}{6} m_e (|N_{e,a}^L|^2 + |N_{e,a}^R|^2) f_N(y_a) + \xi_a \text{Re}[N_{e,a}^L N_{e,a}^R]^* g_N(y_a) \right\}, \]  
(193)

\[ f_N(x) = \frac{1}{(1-x)^4} (1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x), \]  
(194)

\[ g_N(x) = \frac{1}{(1-x)^3} (1 - x^2 + 2x \ln x), \]  
(195)

\[ y_a = \frac{\xi_a^2}{m^2(E)}, \quad m^2(E) = m_L^2 + \frac{g_Y^2 - g_2^2}{4} (v_d^2 - v_u^2), \]  
(196)

\[ N_{\mu,a}^L = -Y_2^E V_{3a} - \epsilon^3 Y_2^E V_{4a}, \]  
(197)

\[ N_{\mu,a}^R = \frac{g_2}{\sqrt{2}} V_{6a} + \frac{g_Y}{\sqrt{2}} V_{5a}, \]  
(198)

\[ N_{e,a}^L = -\epsilon^2 Y_1^E V_{3a} - \epsilon^5 Y_1^E V_{4a}; \]  
(199)

\[ N_{e,a}^R = \frac{g_2}{\sqrt{2}} V_{6a} + \frac{g_Y}{\sqrt{2}} V_{5a}; \]  
(200)

\[ a_\mu(h_B) = \frac{m_\mu |Y_2^E|^2}{16\pi^2} \left( \frac{f_C(x_B) - f_N(y_B)}{3m^2(N) - 6m^2(E)} \right), \]  
(201)

\[ a_e(h_B) = \frac{m_e |Y_1^E|^2}{16\pi^2} \left( \frac{f_C(x_B) - f_N(y_B)}{3m^2(N) - 6m^2(E)} \right), \]  
(202)

\[ x_B = \frac{\lambda_{45} v_s^2}{m^2(N)}, \quad y_B = \frac{\lambda_{45} v_s^2}{m^2(E)}, \]  
(203)

where \(m_L\) is the \(S_1\)-doublet left-handed slepton mass. In calculating the neutralino contributions, we omitted the negligible contributions from the right-handed slepton: \(E_{1,2}\), the singlino, and the \(U(1)_S\) gaugino.

At the degenerated mass and large \(N_U\) limit:

\[ m_L = \mu_1 = \mu_2 = \mu_3, \quad N_U \gg 1, \]  
(204)

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then we get
\[ a_\mu = \frac{g_2^2 v_\mu N_U v_u m_\mu}{32\pi^2 m_L^2} = 27 \times 10^{-10} \left( \frac{200\text{GeV}}{m_L} \right)^2 \left( \frac{N_U}{13} \right) \left( \frac{Y_{EI}}{0.4} \right), \]  
(205)

\[ \frac{m_\mu^2 a_e}{m_e^2 a_\mu} = \frac{\epsilon^2 Y_{EI}^2 m_\mu}{Y_{EI}^2 m_e} \approx \frac{2Y_{EI}^1}{Y_{EI}^2}. \]  
(206)

The experimental values given in Eq.(1) and Eq.(5) are realized by putting by hand as
\[ m_L = 200\text{GeV}, \quad N_U = 13, \quad Y_{EI}^1 = -2.8, \quad Y_{EI}^2 = 0.4. \]  
(207)

Assuming \( N_D = 1 \) and imposing Eq.(96), we get
\[ N_D = 1, \quad Y_1^E = 3.36, \quad Y_2^E = 0.79. \]  
(208)

The values of coupling constants at the Planck scale:
\[ Y_1^E(M_P) = 1.77, \quad Y_{EI}^1(M_P) = -1.47, \quad Y_2^E(M_P) = 0.42, \quad Y_{EI}^2(M_P) = 0.21, \]  
(209)

are consistent with the \( O(1) \) criterion. The enhancement of the \((g - 2)_e\) compared to the \((g - 2)_\mu\) is originated from a large cancellation between two terms in the electron mass:
\[ m_e = (Y_1^E + Y_{EI}^1 N_D)\epsilon v_d. \]

We give the numerical estimations of both \( g - 2 \) as follows. We define three parameter sets:

\begin{align*}
\text{Model A} & : \quad 200 < \min(|M_2|, |\lambda_{23} v_s|) < 1000\text{GeV}, \quad \max(|M_2|, |\lambda_{23} v_s|) = 1.1 \times \min(|M_2|, |\lambda_{23} v_s|), \\
& \quad |\lambda_{45} v_s| = 2 \times \min(|M_2|, |\lambda_{23} v_s|), \\
\text{Model B} & : \quad 200 < |\lambda_{45} v_s| < 1000\text{GeV}, \quad \min(|M_2|, |\lambda_{23} v_s|) = 1.2 |\lambda_{45} v_s|, \\
& \quad \max(|M_2|, |\lambda_{23} v_s|) = 1.4 |\lambda_{45} v_s|, \\
\text{Model C} & : \quad 700 < |M_2|, |\lambda_{23} v_s|, |\lambda_{45} v_s| < 1000\text{GeV},
\end{align*}
(210)

and the common parameter set:
\[ 200 < |\lambda_{67} v_s|, |\lambda_{89} v_s|, m_L < 1000\text{GeV}, \quad |M_Y| = 0.5 |M_2|, \]  
(213)

\[ 0.2 < |Y_{1,2}^E| < 3.0, \quad 0.2 < |Y_{1}^{EI}| < 3.0, \quad 0.2 < |Y_{2}^{EI}| < 0.4, \quad 0.5 < |N_{U,D}| < 10 \]  
(214)

Taking account of the RGE factor
\[ \frac{Y^E(M_S)}{Y^E(M_P)} = 1.9, \]  
(215)
for $Y_{1,2}^{E}, Y_{1}^{EI}$, we have imposed the $O(1)$ criterion on the Yukawa couplings at $M_{P}$. In Model A, as the extra higgsinos are decoupled, the advantage of large $Y_{1,2}^{EI}$ is not available unlike in the cases of Model B and Model C. In Model C, an accidental degeneracy of the mass parameters could enhance the mixing angle of the higgsinos, which is prevented in Model A and Model B. Note that such a enhancement needs fine-tuning and so is unnatural. We focus on the lightest charged SUSY particle in the loop whose mass is defined as

$$\mu_{L} = \min(|\mu_{1}|, |\mu_{2}|, |\mu_{3}|, m(E)).$$ (216)

The results are shown in Figure 1. The constraints for both $g-2$ give the upper bounds of $\mu_{L}$ as follows:

$$\mu_{L} < 375\text{GeV} (\text{Model A}), \quad \mu_{L} < 660\text{GeV} (\text{Model B}), \quad \mu_{L} < 940\text{GeV} (\text{Model C}).$$ (217)

While the contributions to both $g-2$ from the extra higgsino are suppressed in Model A, this contributions are not suppressed and raise the upper bound of $\mu_{L}$ in Model B. In Model C, an accidental degeneracy of the diagonal elements of the chargino mass matrix enhances the off-diagonal elements of the mixing matrices $U, D$, which raises the upper bound of $\mu_{L}$ further. Eq.(7) and Eq.(206) give the condition:

$$\frac{Y_{1}^{EI}}{Y_{2}^{EI}} \simeq -7;$$ (218)

which is satisfied for the numerical calculation as shown in Figure 2. There is a tendency that the condition $Y_{1}^{EI}/Y_{2}^{EI} > -4$ (green points) gives the smaller $|\Delta a_{e}|$ and the condition $Y_{1}^{EI}/Y_{2}^{EI} < -10$ (red points) gives the smaller $|\Delta a_{\mu}|$. The allowed region in $\Delta a_{e} - \Delta a_{\mu}$ plane is dominated by the blue points which satisfy the condition $-10 \leq Y_{1}^{EI}/Y_{2}^{EI} \leq -4$.

VII. CONCLUSIONS

We have considered an $S_{4}$ flavor symmetric extra $U(1)$ model that accounts for dark matter and a baryon asymmetry. In this model, we assume that dark matter is dominated by an axion so that the smallness of the up quark mass is understood by the smallness of the Peccei-Quinn scale. Furthermore, we assume that the muon mass is induced by
FIG. 1: Lepton anomalous magnetic moments of electron (left) and muon (right) for Model A(top), Model B(middle) and Model C(bottom). Blue points satisfy both Eq.(1) and Eq.(5), green points satisfy only Eq.(1), and red points satisfy only Eq.(5).

the result of the symmetry breaking of the $S_3$ subgroup. In this case, successful resonant leptogenesis requires TeV scale RHNs, therefore it may be possible to verify the nature of the RHN by a future collider. As the TeV scale seesaw mechanism requires very small neutrino Yukawa couplings, the most relevant interaction of the RHN for a collider experiment is the interaction with the G-Higgs. Our model also accounts for two lepton $g-2$ anomalies.
FIG. 2: Anomalous magnetic moments of electron and muon for Model B. Green, blue and red points correspond to $Y_{1}^{EI}/Y_{2}^{EI} > -4$, $-10 \leq Y_{1}^{EI}/Y_{2}^{EI} \leq -4$ and $Y_{1}^{EI}/Y_{2}^{EI} < -10$ respectively. The vertical (horizontal) dotted lines mean the 1σ bounds for the muon (electron) anomalous magnetic moment.

without causing too large flavor violation. Our numerical estimation shows that the typical upper mass bound of the lightest charged SUSY particle in the loop is about 660 GeV. We can expect to prove the existence of supersymmetry and the flavor symmetry by a future collider.

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