Numerical Investigation on Laminar Cross-flow Around a Top Surface Heated Square Cylinder

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Abstract. This numerical study attempts to discuss about the effects in vortex shedding and aerodynamic parameters due to heating the top surface of a square cylinder for various Ri values (0 ≤ Ri ≤ 1) at Pr = 0.71 and Re = 100. Numerical calculations are carried out using a finite volume method based on the PISO (Pressure-Implicit with Splitting of Operators) algorithm. We have used ANSYS Fluent for the numerical simulation. The results in this study are promising and we found that the surface heating would rather changes or modifies the flow field and the vortex shedding phenomenon behind the square cylinder. Also it is observed that heating plays a critical role in the integral quantities such as aerodynamic coefficients, Strouhal number etc.

1. Introduction
Fluid flow around the square cylinder has brought attention of researchers due to its applicability and scope in both academics and industrial fields. Researchers noticed the formation of vortices and its shedding for various Reynolds numbers (Re) [1] [2]. The Reynolds number is a non-dimensional quantity defined as the ratio between inertial force and viscous force.

\[ Re = \frac{\rho v L}{\mu} \]  

(1)

\( \rho \) represents the density, \( v \) represents the velocity, \( L \) represents the characteristic length, and \( \mu \) represents the viscosity.

Enormous studies have been reported which focus on vortex shedding and associated wake dynamics. Heating the cylinder surface adds more complexity to the wake dynamics and the vortex shedding because of buoyancy effects. Understanding this complexity has received lot of scope and are used in many applications such as cooling towers, design of bridges, designing electric components, etc.

The dependency of fluid flow and aerodynamic coefficients with Re for flow past a square cylinder is reported in Bai and Alam [1], Chatterjee and Mondal [3], Sharma and Eswaran [4]. It is established that the surface heating substantially influences the vortex shedding pattern and the shedding frequency [5]. Bhattacharyya and Mahapatra [6] studied a heated cylinder at a low Reynolds number. They found that while heating, the centreline symmetry of the cylinder wake was lost. A mean downward lift was experienced by the cylinder when the buoyancy effect is considered. The non-dimensional shedding frequency, and Strouhal number (St) were found increasing with Richardson number (Ri). The Strouhal number is defined as follows.
Nomenclature

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $C_D$  | Coefficient of drag                              |
| $C_L$  | Coefficient of lift                              |
| $f$    | Frequency of vortex shedding $[1/s]$             |
| $g$    | Acceleration due to gravity $[m/s^2]$            |
| $L$    | Characteristic length $[m]$                      |
| $p$    | Pressure $[Pa]$                                  |
| $p^*$  | Non-dimensional pressure                         |
| $Pr$   | Prandtl number                                   |
| $Re$   | Reynolds number                                   |
| $Ri$   | Richardson number                                 |
| $St$   | Strouhal number                                   |
| $T_{cold}$ | Temperature of cold wall $[K]$             |
| $T_{hot}$ | Temperature of hot wall $[K]$                  |
| $T_{ref}$ | Reference temperature $[K]$                |
| $U$    | Free stream velocity $[m/s]$                     |

Greek Symbols

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $\alpha$ | Thermal diffusivity $[m^2/s]$                   |
| $\beta$ | Coefficient of thermal expansion $[1/K]$        |
| $\mu$  | Dynamic viscosity $[PaS]$                        |
| $\rho$ | Density $[kg/m^3]$                               |
| $\tau$ | Non-dimensional time                             |
| $\Theta$ | Non-dimensional temperature                     |

$$St = \frac{fL}{U} \quad (2)$$

$f$ represents the frequency of vortex shedding, $L$ represents the characteristic length, and $U$ represents the free stream velocity.

Richardson number is a non-dimensional quantity defined as the ratio between buoyant force and the inertia force.

$$Ri = \frac{g\beta(T_{hot} - T_{ref})L}{V^2} \quad (3)$$

g represents the acceleration due to gravity, $T_{hot}$ represents the temperature of hot wall, $T_{ref}$ represents the reference temperature, $\beta$ represents the coefficient of thermal expansion, $L$ represents the characteristic length, and $V$ represents the characteristic velocity.

Dulhani et al [7] had done a numerical investigation by changing the angle of incidence ($\alpha$) of flow from 0° to 45° for $Re = 100$ and $Pr = 0.71$. Prandtl number ($Pr$) is a non-dimensional quantity defined as the ratio between momentum diffusivity and thermal diffusivity. They found that as the angle of incidence increases, the lateral wake width of the square cylinder decreases thus heat transfer rate increases. Chatterjee and Mondal [3] investigated the effect of buoyancy and heat transfer across a square cylinder subjected to upward laminar flow for $50 \leq Re \leq 150$. A decreasing trend was observed for $St$ while cooling the cylinder below the free stream temperature for all $Re$ studied. Dhiman et al [8] studied the effects of blockage ratio ($B$) in the mixed convection from a heated square cylinder in a horizontal flow for $1 \leq Re \leq 30$ and $0 \leq Ri \leq 1$ for blockage ratio 0.125, and 0.25. They observed that coefficient of drag increases with an increase in blockage ratio for distinct values of $Re$ and $Ri$.

From the literature it observed that heating the square cylinder increases the drag coefficient ($C_D$) and $St$. However, the effect of heating the top surface of a square cylinder is not clearly investigated in the literature. The present study attempts to explain the effect of buoyancy in
vortex shedding and aerodynamic parameters while heating the top surface of a square cylinder. We have created a two-dimensional rectangular flow domain with a square cylinder in mixed convection regime which is a combination of natural and forced convection. If the heat transfer occurs due to forced fluid flow over the surface by external means it is called a forced convection. In contrast, if the heat transfer occurs due to motion of fluid caused by buoyancy forces it is called natural convection. In this work, we are conducting a numerical study on a top surface heated cylinder for five different $Ri$, ranges from 0 to 1 at $Re = 100$.

2. Numerical Methodology

2.1. Geometrical Modelling and Domain Discretisation

A domain independent study is conducted in order to identify the optimum domain geometry. Computations are carried out for distinct domain geometries, and the $C_D$, $St$ values are noted, as shown in Table 1. The optimum domain size was found out to be $50L$ and $25L$.

![Figure 1: Representation of domain geometry with boundary conditions.](image)

Table 1: Domain independence study at $Ri = 0$, $Pr = 0.71$ and $Re = 100$. The percentage deviation of the calculated values from the literature is noted in brackets.

| Length | Width | Elements | $C_D$ (% Error) | $St$ (% Error) |
|--------|-------|----------|----------------|----------------|
| $35L$  | $10L$ | 15,254   | 1.608          | 0.154          |
| $40L$  | $15L$ | 23,498   | 1.533 (4.66%)  | 0.148 (3.89%)  |
| $45L$  | $20L$ | 33,514   | 1.512 (1.37%)  | 0.147 (0.68%)  |
| **$50L$** | **$25L$** | **45,144** | **1.505 (0.46%)** | **0.146 (0.68%)** |
| $55L$  | $30L$ | 58,286   | 1.503 (0.13%)  | 0.146 (0.00%)  |

Following this a grid independent study is conducted in order to obtain the optimum grid size required. A two-dimensional model was created in ANSYS SpaceClaim. Unstructured quadrilateral grid is created inside the domain using ANSYS Workbench. Figure 1 represents the domain geometry with boundary conditions. The domain variables are non-dimensionalized...
with the length of the square cylinder. The top surface of the square cylinder is heated and is highlighted in the figure.

Figure 2 shows the square adapted quad mesh. Calculations are done for four different grid structures. The $C_D$ and $St$ values are compared and the percentage deviations are noted in Table 2. The grid having 45,144 elements was found to be optimum.

![Figure 2: Mesh obtained after the grid independence study](image)

| Elements  | $C_D$ (% Error) | $St$ % Error |
|-----------|-----------------|--------------|
| 25,088    | 1.530           | 0.145        |
| 35,567    | 1.511 (1.24%)   | 0.145 (0.00%)|
| **45,144**| **1.505 (0.39%)**| **0.146 (0.69%)**|
| 55,159    | 1.501 (0.26%)   | 0.146 (0.00%)|

2.2. Mathematical Modelling

The conservation of mass, momentum and energy equations for incompressible fluid flow under the Boussinesq approximation are given by,

\[
\frac{\partial u^*}{\partial \tau} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \left( \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2} \right) \tag{5}
\]

\[
\frac{\partial v^*}{\partial \tau} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re} \left( \frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2} \right) + Ri \cdot \Theta \tag{6}
\]

\[
\frac{\partial \Theta}{\partial \tau} + u^* \frac{\partial \Theta}{\partial x^*} + v^* \frac{\partial \Theta}{\partial y^*} = \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 \Theta}{\partial x^*^2} + \frac{\partial^2 \Theta}{\partial y^*^2} \right) \tag{7}
\]

$u^*$ and $v^*$ represent the non-dimensional velocities in the $x$ and $y$ directions, $\Theta$ and $p^*$ represent the non-dimensional temperature and pressure.

\[
\Theta = \frac{T_{ref} - T_{cold}}{T_{hot} - T_{cold}}, \quad p^* = \frac{pL^2}{\rho \alpha^2} \tag{8}
\]
\( \alpha \) represents the thermal diffusivity, \( T_{cold} \) and \( T_{hot} \) represent the temperatures at cold and hot walls, and \( p \) represents the pressure.

The equations govern fluid flow for the two-dimensional laminar flow are discretize and solved using a CFD tool, ANSYS 20.1. PISO algorithm is used and \( 10^{-6} \) is assigned as the convergence criteria.

2.3. Boundary Conditions

The boundary conditions of our present study are as follows.

- At the inlet, the boundary conditions are as follows.
  \[ u^* = 1, \quad v^* = 0, \quad \Theta = 0 \]  \( (9) \)

- At the exit, the boundary conditions are as follows.
  \[ \frac{\partial u^*}{\partial x^*} = 0, \quad v^* = 0, \quad \Theta = 0 \]  \( (10) \)

- The top and bottom boundary walls are considered as adiabatic slip walls and are mathematically expressed as follows.
  \[ v^* = 0, \quad \frac{\partial u^*}{\partial y^*} = 0, \quad \Theta = 0 \]  \( (11) \)

- The cylinder surface is considered as no-slip and is mathematically expressed as follows.
  \[ u^* = 0, \quad v^* = 0, \quad \Theta = 1 \]  \( (12) \)

3. Code Validation

The present study is validated with the flow around a non-heated and heated cylinder at \( Re = 100 \). The \( C_D \) and \( St \) values obtained from the present study are compared with literature.

**Table 3:** Comparison of \( C_D \) and \( St \) at \( Ri=0, \ Pr=0.71 \) and \( Re=100 \) with literature. The percentage deviation of the calculated values from the literature is noted in brackets.

| Literature                  | \( C_D \) (% Error) | \( St \) (% Error) |
|-----------------------------|----------------------|--------------------|
| Sohankar et al. [9]         | 1.500 (0.33%)        | 0.150 (2.67%)      |
| Sahu et al. [10]            | 1.490 (1.01%)        | 0.149 (2.01%)      |
| Sharma and Eswaran [4]      | 1.490 (1.01%)        | 0.150 (2.67%)      |
| Bai and Alam [1]            | 1.500 (0.33%)        | 0.143 (2.09%)      |
| Singh et al. [2]            | 1.510 (0.33%)        | 0.147 (0.68%)      |
| Sen et al. [11]             | 1.520 (0.99%)        | 0.145 (0.69%)      |
| **Present Study**           | **1.505**            | **0.146**          |

Table 3 represents the comparison of present value of \( C_D \) and \( St \) at \( Ri = 0, \ Pr = 0.71 \) and \( Re = 100 \) with Sohankar et al [9], Sahu et al [10], Sharma and Eswaran [4], Bai and Alam [1], Singh et al [2], and Sen et al [11] with maximum deviation less than 1.01% and 2.67% respectively, which shows a fair agreement with the literature.
Figure 3: Comparison of (a) $St$ and (b) $C_D$ for various $Ri$ at $Pr = 0.71$ and $Re = 100$.

Figure 3 (a) shows the comparison of $St$ at $0 \leq Ri \leq 1$, $Pr = 0.71$ and $Re = 100$. The deviation is less than 0.64%, which shows a good agreement with Rashid and Hasan [5]. Figure 3 (b) shows the comparison of $C_D$ at $0 \leq Ri \leq 1$, $Pr = 0.71$ and $Re = 100$ with Arif and Hasan [12], Rashid and Hasan [5] and Sharma and Eswaran [13]. The maximum deviation is less than 4.951%, which shows a fair agreement with the literature.

4. Results and Discussion

Figure 4: Variation of $C_L$ for top surface heated cylinder at various $Ri$, $Pr = 0.71$ and $Re = 100$.

Studies have been conducted by heating the top surface of the square cylinder for $0 \leq Ri \leq 1$ at $Pr = 0.71$ and $Re = 100$. It was found that the pressure difference between the upper and
lower surface is increasing with the increase in $Ri$, which causes a negative lift on the cylinder, as shown in Figure 4.

Figure 5: Vortex structure for various $Ri$ at $Pr = 0.71$ and $Re = 100$. Shedding is symmetric to the horizontal axis for $Ri = 0$ and looses its symmetry for other $Ri$ values.

![Vortex Structure Diagrams](image)

Figure 6: Variation of (a) $C_D$ and (b) $St$ for top surface heated cylinder at various $Ri$, $Pr = 0.71$ and $Re = 100$.

![Variation of CD and St](image)

While heating the cylinder surface, the fluid that passes over that surface will get heated, and it becomes less dense. As a result of this, the fluid will move upwards along the direction of buoyancy. Figure 5 shows that for $Ri = 0$, the shedding is symmetric about the horizontal axis of the flow. However, when the top surface is heated, the vortex shedding became asymmetric.
When the top surface is heated, the vortices move upward, causing a widening of wake width. As a result, a reduction of pressure occurs near the base point, which increases the pressure difference between the stagnation point and base point, causing an increment in $C_D$, as shown in Figure 6 (a).

While heating the top surface, the vortices which sheds from the top surface of the cylinder gets dissipated earlier than vortices from the bottom surface due to unstable density stratification caused by heating. This acceleration in shedding from the top surfaces causes an increment in shedding frequency, as observed in Figure 6 (b).

5. Conclusion
A numerical investigation is conducted on flow over top surface heated square cylinder for $0 \leq Ri \leq 1$ at $Pr = 0.71$ and $Re = 100$. Variation of drag coefficient ($C_D$), lift coefficient ($C_L$), and $St$ with $Ri$ is presented. Major findings are,

- An asymmetry in vortex shedding found while heating top surface of the square cylinder.
- Heating the top surface of square cylinder increased the wake width and caused an increment in drag.
- An increment in shedding frequency was observed while heating due to unstable density stratification.
- The coefficient of the lift is found to be negative while heating due to modified pressure distribution.

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