Face Hallucination with Linear Regression Model in Semi-Orthogonal Multilinear PCA Method

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Abstract. In this paper, we propose a new face hallucination technique, face images reconstruction in HSV color space with a semi-orthogonal multilinear principal component analysis method. This novel hallucination technique can perform directly from tensors via tensor-to-vector projection by imposing the orthogonality constraint in only one mode. In our experiments, we use facial images from FERET database to test our hallucination approach which is demonstrated by extensive experiments with high-quality hallucinated color faces. The experimental results assure clearly demonstrated that we can generate photorealistic color face images by using the SO-MPCA subspace with a linear regression model.

1. Introduction
In digital signal processing, video surveillance is often of small size because of the great distance between the camera and the objects. Image resolution is a potential factor affecting face recognition performance. For face identification, especially by human, it is desirable to render a high-resolution face image from the low-resolution one. This technique is called face hallucination or face super-resolution [1]. They infer the high frequency components from a parent structure by recognizing the local features from the training set, but there exists some noise in certain area. The nearest neighbor or cubic spline is the simplest way to increase image resolution because it is a direct interpolation of input images. On the other hand, the performance of direct interpolation is usually poor since no new information is added in the process. For this reason, face super-resolution methods which are based on learning from the training set are proposed [2]–[4]. Some methods with the assumption that high-resolution images are Markov random fields (MRFs) are more suitable for synthesizing local texture, and are usually applied to generic images without special consideration of the property of face images [5,6].

A novel hallucination method based on eigentransformation is proposed [7]. It is similarly related to the work in [8] and this approach is developed for sketch recognition. To go beyond the current super-resolution techniques which only consider face images under fixed imaging conditions in terms of pose, expression and illumination, these factors are crucial to face analysis and synthesis. Then, Vasilescu et al. introduce multilinear analysis to face modelling [9] and demonstrate its promising application in computer vision. In the method, equipped with tensor algebra, the multiple factors are unified in the same framework with the coordination between factors expressed in an elegant tensor product form. Next, Wu et al. propose a novel regression model to use tensor principal component analysis (PCA) subspace as the face representation [10], which is a special case of the concurrent subspace analysis. In addition, Multi-linear principal component analysis (MPCA) is a general extension of traditional linear method such as matrix singular value decomposition (SVD) or PCA.
The idea of MPCA is extended to use Tensor-to-Vector Projection (TVP) that learns low-dimensional vectors from high-dimensional tensors in a successive way [12]. We focus on color face hallucination in a SO-MPCA method. Motivated by the problem in the studies involving the hallucination method in the HSV color space [13] and we use color face images from the FERET database [14]. We propose a novel hallucination technique in color face images of HSV color space with a linear regression model in SO-MPCA formulation for tensor object. In this paper, a novel technique can capture more variance and learn more features than full orthogonality. Then, the bias can be increased and the variance of learning model be reduced. The rest of this paper is organized as follows. In Section 2, we explain the basic notation. We introduce the basic idea of the MPCA in Section 3. In Section 4, we propose a color face hallucination with SO-MPCA for HSV model. Some experimental results are shown in Section 5. Finally, conclusions are summarized in Section 6.

2. Basic Notation
In this paper, scalars are denoted by lower case letters \(a, b, \ldots\), vectors by italic upper-case case letters \(A, B, \ldots\), matrices by bold upper-case case letters \(\mathbf{A}, \mathbf{B}, \ldots\), and higher-order tensors by calligraphic upper-case letters \(\mathcal{A}, \mathcal{B}\). \((\mathbf{A})^T\) denotes the transpose of a matrix. \((\mathbf{A})^+\) denotes the pseudo-inverse.

3. Basic Idea in Multi-linear Principal Component analysis (MPCA)
The basic idea in MPCA solution to the problem of dimensionality reduction for tensor objects is introduced [11]. In Figure. 1, we show the basic idea in MPCA, an n-mode unfolding of a tensor. An \(N\)th-order tensor is denoted as \(\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}\) for \(I_n = 1, \ldots, N\). The \(n\)-mode vectors of \(\mathcal{A}\) are defined as the \(I_n\)-dimensional vectors obtained from \(\mathcal{A}\) by varying the index \(i_n\) while keeping all the other indices fixed. The unfolding of \(\mathcal{A}\) along the \(n\)-mode is denoted as \(A_{\mathcal{a}} \in \mathbb{R}^{I_n \times I_1 \times I_2 \times \ldots \times I_{n-1} \times I_{n+1} \times \ldots \times I_N}\) and the column vectors of \(A_{\mathcal{a}}\) are the \(n\)-mode vectors of \(\mathcal{A}\). Let the set of tensors be \(\{\mathcal{A}_m, m = 1, \ldots, M\}\) and the total scatter of these tensors is defined as

\[
\psi_{\mathcal{A}} = \sum_{m=1}^{M} \|A_m - \bar{A}\|^2
\]

Where \(\bar{A}\) is the mean tensor calculated as

\[
\bar{A} = (1/M) \sum_{m=1}^{M} A_m
\]

Then, the total scatter matrix of these samples can be defined as

\[
C_{\mathcal{A}} = \sum_{m=1}^{M} (A_m - \bar{A}) (A_m - \bar{A})^T
\]

Where \(A_m(n)\) is the \(n\)-mode unfolded matrix of \(A_m\).

The main objective of MPCA is to define a multi-linear transformation \(\hat{U}^{(n)}\) which denoted \(I_n \times P_n\) matrix containing the orthonormal \(n\)-mode basis vectors and the matrix \(\hat{U}^{(n)}\) is \(n\)th projection matrix, \(n = 1, \ldots, N\). It can map the original tensor space \(\mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}\) into a tensor subspace \(\mathbb{R}^{P_1 \times P_2 \times \ldots \times P_N}\) with \((P_n < I_n\), for \(n = 1, \ldots, N\)). We can define the projection of \(n\)-mode vector of \(X_m\) as

\[
y_m = \mathcal{X}_m \times_1 \hat{U}^{(n_1)} \times_2 \hat{U}^{(n_2)} \times_3 \hat{U}^{(n_3)} \ldots \times_N \hat{U}^{(n_N)}
\]

The tensor \(y_m\) can capture most of the variations observed in the original tensor objects, assuming that these variations are measured by the total tensor scatter. The objective of MPCA is the determination of the \(N\) projection matrices \(\hat{U}^{(n)}\) that maximize the total tensor scatter \(\psi_{\mathcal{A}}\) as
\[
\{ \hat{U}^{(n)}, n = 1, 2, ..., N \} = \arg \max_{\hat{U}^{(n)}, \hat{U}^{(2)}, ..., \hat{U}^{(N)}} y
\]  

Figure 1. Block diagram of the basic idea in multi-linear principal component analysis (MPCA)

4. Color Face Hallucination with linear regression model in SO-MPCA method

In this section, we explain the Tensor-to-vector projection which is the building blocks of a TVP. In (4), we denote an elementary multi-linear projections (EMPs) as \( \{ u^{(1)}, u^{(2)}, ..., u^{(N)} \} \), which consist of one unit projection vector in each mode and \( ||u^{(n)}|| = 1 \) for \( n = 1, ..., N \), where \( || \cdot || \) is the Euclidean norm for vectors. Then, we can get the projection of a tensor \( \mathcal{X} \) to a scalar \( y \) through the \( N \) unit projection vectors as:

\[
y = \mathcal{X} \times_1 u^{(1)}_p \times_2 u^{(1)}_p \times_N u^{(N)}_p.
\]  

The TVP of a tensor \( \mathcal{X} \) to a vector \( y \in \mathbb{R}^P \) consists of \( P \) EMPs \( \{ u^{(1)}_p, ..., u^{(N)}_p \}, p = 1, ..., P \), which can be written concisely as \( \{ u^{(N)}_p \}_{p=1}^P \):

\[
y(p) = \mathcal{X} \times_1 u^{(1)}_p \times_2 u^{(N)}_p = \mathcal{X} \times_N \{ u^{(N)}_p \}.
\]  

The purpose of SO-MPCA method is to find a TVP to maximize the variance of the projected samples in each projection direction, subject to the orthogonality constraint in only one mode which is defined as \( \alpha \)-mode. The variance is measured by the total scatter \( S_p \) defined as:

\[
S_p = \sum_{m=1}^M (y_{mp} - \bar{y}_p)^2,
\]  

Where \( y_{mp} = \mathcal{X} \times_{n=1}^N \{ u^{(N)}_p \} \) and \( \bar{y}_p = \frac{1}{M} \sum_{m=1}^M y_{mp} \), that is to say that the objective of SO-MPCA is to achieve the \( P \) EMPs. In addition, the \( p \)th EMP which maximized the variance in (8) can be determined as:

\[
\{ u^{(N)}_p, n = 1, ..., N \} = \arg \max_{m=1}^M (y_{mp} - \bar{y}_p)^2.
\]  

For orthogonality constraint in only one mode, each the \( p \)th EMP can be described as:

\[
u^{(n)}_p = 1 \text{ for } n = 1, ..., N
\]
And
\[ u_p^{(\alpha)}^T u_q^{(\alpha)} = 0 \text{ for } p > 1 \text{ and } q = 1, \ldots, P - 1. \] (11)

The normalization constraint in (10) is applied for all modes and the orthogonality constraint in (11) is applied only in \( \alpha \)-mode and there is no such constraint for the other modes.

However, the selection of mode \( \alpha \) is free to choose any mode \( n \) as \( \alpha \) to impose the orthogonality constraint in (11). We choose the mode with the highest dimension as \( \alpha \):
\[ \alpha = \text{argmax } I_n. \] (12)

Such that \( = \max_n I_n = I_\alpha \). In this paper, we only keep on SO-MPCA with \( \alpha \) determined by (12).

Following standard multi-linear algebra, any tensor can be expressed as the products
\[ y_l^h = x_l^h \times_1 u_l^{(1)}^T \times_2 u_l^{(2)}^T \times_3 u_l^{(3)}^T. \] (13)

And
\[ y_l^l = x_l^l \times_1 u_l^{(1)}^T \times_2 u_l^{(2)}^T \times_3 u_l^{(3)}^T. \] (14)

As \( y_l^h \) is in the index of high-resolution (HR) training set and \( y_l^l \) is the low-resolution (LR) training set respectively. In addition, the sets of \( u_l^{(1)}, u_l^{(2)}, u_l^{(3)} \) are followed in (10)-(11) and the correlation between the decomposition coefficients can be suppressed.

From the model of a mapping function \( y_l^h = f(y_l^l) \), \( f \) can be thought of as a probability distribution function. Thus, we can consider the following conditional probability \( P(y_l^h | y_l^l) \). When a new test color face image (LR) \( x^l \) is provided, and the HR tensorPCA subspace projection is given by:
\[ y^h = \text{argmax}_y P(y | y^l). \] (15)

From [13], we use the assumption of a low-correlation between the coefficients in \( y^l \), we can simplify this probability in (15) as
\[ \tilde{y}_{r,s,t}^h = \text{argmax}_{y_{r,s,t}} P(y_{r,s,t} | y^l). \] (16)

Next step, The Maximum Likelihood estimation is applied in (16) and we can express \( \tilde{y}_{r,s,t}^h \) in a linear regression model as:
\[ \tilde{y}_{r}^h = \sum_{p=1}^{Q_1 Q_2 Q_3} \left( \frac{w_{r,p}}{Q_1 Q_2 Q_3} \right) y_p^l. \] (17)

5. Experimental Results
This section evaluates the proposed methods on third-order tensor data in terms of recognition rate and ability to reconstruct the color face images. Our face hallucination with SO-MPCA in a color facial image algorithm is tested against the traditional PCA and the MPCA method. We used face images from a subset of FERET databases [14] to form two data sets for training and testing images in two color models: RGB and HSV. In our experiments, we randomly selected 300 normal expression images of different people under the same light conditions and another 30 images in Figure. 2 were used for testing. According to demand, we manually cropped the most interesting region of the faces, and we standardized the images to a size of \((30 \times 30)\). In the degradation process, each testing image (LR) was introduced with Gaussian blur with a variance of 1 and resized by down-sampling 2:1 \((15 \times 15)\). We then added Gaussian noise with a variance of \(10^{-6}\).
This section evaluates the proposed methods on third-order tensor data in terms of recognition rate and the outcome from reconstructed face images in RGB and HSV color models. In Table 1, show the color face recognition and we select $L = 1, 2, 3, 4, 5$ samples for each subject as the training data and use the rest for testing. For SO-MPCA, we set both selected mode $\alpha = 1$ and number of iterations to 30. All features are sorted according to the scatters (captured variance) in descending order for classification. The Nearest Neighbor Classifier with the Euclidean distance is applied to classify the top $P$ features which up to $P = 50$ features in face recognition. In Table 1, we shows the face recognition results for $P = 1, 5, 10, 20, 50$ and $L = 1, 2, 3, 4, 5$, including both the mean and the standard deviation (std) over ten repetitions. We can see that the SO-MPCA can give more recognition rates than other methods in all cases.

**Table 2.** PSNR of different methods for each facial image in RGB and HSV color space.

| Case | RGB first row | RGB second row | RGB third row | HSV first row | HSV second row | HSV third row |
|------|---------------|---------------|---------------|--------------|--------------|--------------|
| 1) 90 percent PCA | 23.32 dB | 29.56 dB | 32.60 dB | 22.76 dB | 28.14 dB | 25.23 dB |
| PCA/MPCA/SO-MPCA | PSNR Values |
|------------------|-------------|
|                  | 23.35 dB    | 29.51 dB    | 26.41 dB    | 22.98 dB    | 28.56 dB    | 25.36 dB    |
| 2) 95 percent PCA | 23.41 dB    | 29.62 dB    | 26.78 dB    | 23.15 dB    | 29.11 dB    | 25.87 dB    |
| 3) 100 percent PCA| 25.01 dB    | 31.46 dB    | 27.23 dB    | 24.69 dB    | 29.23 dB    | 26.88 dB    |
| 4) 90 percent MPCA| 27.62 dB    | 32.87 dB    | 28.18 dB    | 26.13 dB    | 30.45 dB    | 27.26 dB    |
| 5) 95 percent MPCA| 30.35 dB    | 33.74 dB    | 29.56 dB    | 29.84 dB    | 32.19 dB    | 28.59 dB    |
| 6) 100 percent MPCA| 25.03 dB    | 31.53 dB    | 27.29 dB    | 24.71 dB    | 29.28 dB    | 26.79 dB    |
| 7) 90 percent SO-MPCA| 27.78 dB    | 32.82 dB    | 28.23 dB    | 26.32 dB    | 30.42 dB    | 27.23 dB    |
| 8) 95 percent SO-MPCA| 30.32 dB    | 33.76 dB    | 29.54 dB    | 29.86 dB    | 32.26 dB    | 28.61 dB    |

**Figure 3.** Color face hallucination results in RGB color space.

(a) original HR images (30 × 30 pixels); (b) input LR images (15 × 15 pixels) with noise, motion and blur in LR images; (c) face hallucination result with 90 percent traditional PCA; (d) face hallucination result with 95 percent traditional PCA; (e) face hallucination result with 100 percent traditional PCA; (f) face hallucination result with 90 percent MPCA; (g) face hallucination result with 95 percent MPCA; (h) face hallucination result with 100 percent MPCA; (i) face hallucination result with 90 percent SO-MPCA; (j) face hallucination result with 95 percent SO-MPCA; (k) face hallucination result with 100 percent SO-MPCA.

**Figure 4.** Color face hallucination results in HSV color space.

(a) original HR images (30 × 30 pixels); (b) input LR images (15 × 15 pixels) with noise, motion and blur in LR images; (c) face hallucination result with 90 percent traditional PCA; (d) face hallucination result with 95 percent traditional PCA; (e) face hallucination result with 100 percent traditional PCA; (f) face hallucination result with 90 percent MPCA; (g) face hallucination result with 95 percent MPCA; (h) face hallucination result with 100 percent MPCA; (i) face hallucination result with 90 percent SO-MPCA; (j) face hallucination result with 95 percent SO-MPCA; (k) face hallucination result with 100 percent SO-MPCA.

To provide a quantitative comparison of the color face reconstruction deviations from the ground truth data, we also computed the PSNR of each method. All the PSNR results of Figure 3-4 in the different color spaces and the number of PCA are shown in Table 2. The results shown that our proposed method has the highest PSNR values compared with the traditional PCA method on all test faces. In addition, the PSNR results form both MPCA and SO-MPCA methods are quite similar in the RGB and HSV space. The experimental results from Figure 3-4. clearly demonstrated that we can
generate photorealistic color face images by using the SO-MPCA subspace with a regression model and that our hallucinated approach is suitable for the HSV color spaces.

6. Conclusion
This paper proposes a novel color face hallucination with linear regression model with algorithm under the TVP setting, named as semi-orthogonal multilinear PCA (SO-MPCA) method. This method can capture more variance and learn more features than full orthogonality. To test our proposed method, we used facial images from the FERET database to validate the algorithm. The experiments clearly demonstrated that we can generate photorealistic color face images by using the SO-MPCA subspace with a regression model and that our hallucinated approach is suitable for the RGB and HSV color spaces. The results showed that this proposed method can reconstruct reasonable HR color face images. This is the next step toward taking advantage of SO-MPCA for color image processing. Based on using the SO-MPCA in only one mode with the regression model, we will directly apply our technique to each mode across different views and under changing illumination conditions.

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