A sixteen decimal places' accurate Darcy friction factor database using non-linear Colebrook's equation with a million nodes: A way forward to the soft computing techniques

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\textbf{ABSTRACT}

The Colebrook's equation is considered as an empirical model to accurately compute the Darcy friction factor in pipes under fully-developed turbulent flow. Due to non-linearity and implicitness of the Colebrook's equation, one needs to use numerical methods to acquire reasonably good approximation to the true friction factor values. However, such idea is not preferred by practitioners as it demands use of computers — also more computational time and effort. To overcome this, explicit equations that can describe Darcy friction factor directly in terms of the Reynolds number and relative roughness are essential. Using Fixed point iteration method in the MATLAB software, we have developed a 16 decimal places' accurate friction factor database for the Darcy friction factor for a 1000 by 1000 mesh of Reynolds number and relative roughness values. The accurate dataset described in this work will serve to be basis for the construction of new and more reliable...
explicit equations using regression modeling, artificial intelligence techniques and other soft computing methods.

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1. Data

The Colebrook's equation nonlinearly relates the friction factor in a pipe fluid flows. For particular cases, and with some specified values of relative roughness and the Reynolds number, the nonlinear equation can be solved numerically for the friction factor.

To overcome the complexities due to numerical methods, explicit approximations are usually preferred. The explicit approximations can be developed using accurate database of friction factor values from the Colebrook's equation.
The data in this work comprises of the sixteen decimal places’ accurate friction factor solution of the nonlinear Colebrook’s equation for a range wider range of values of relative roughness and Reynolds number, thousand both; thus in total a million nodes. A part of this data (see supplementary material) has been used in Ref. [11] to propose a new and efficient explicit approximation to find the friction factor approximation using a pair of known values of relative roughness and Reynolds number.

2. Experimental design, materials and methods

The Colebrook’s equation [1–11] is given as:

\[
\frac{1}{\sqrt{z}} = -2 \log \left( \frac{2.51}{x \sqrt{z}} + \frac{y}{3.7} \right)
\]  

(1)

where \(z\) is the Darcy friction factor, \(x\) is the Reynolds number and \(y\) is the relative roughness.

Using numerical methods, to any desired accuracy, one can solve equation (1) for \(z\) for a given pair \((x, y)\).

Using a 1000 by 1000 mesh of values of \(x\) and \(y\), defined as:

\[x: 4000: 99996: 10^8\]  

(2)

\[y: 0: 0.00005: 0.05\]  

(3)

An accurate database can be obtained, particularly to an accuracy of 16 decimal places, using Fixed-Point iteration method [11] starting with an initial assumption \(z^{(0)} = 0.1\) as given in equation (3).

\[z^{(i+1)} = 0.25 \left[ \log \left( \frac{2.51}{x \sqrt{z^{(i)}}} + \frac{y}{3.7} \right) \right]^{-2} \text{ for } i = 0, 1, 2..\]  

(4)

Usually after 30 repeated evaluations of (4) i.e. 30 iterations as used in Ref. [11], we can get 16 decimal places accurate numerical solution to (1), however the present data has been obtained upto 50 iterations for more careful examination.

After getting the dataset i.e. values of \(z\) (a matrix of 1000 by 1000 nodes based on distribution of \(x\) and \(y\)), soft computing techniques like: regression modeling, artificial neural networks and adaptive neuro-fuzzy inference system can be employed to get best explicit relationships of \(z\) in terms of \(x\) and \(y\), like:

\[z = f(x, y)\]  

(5)

The obtained dataset has been generated in MATLAB software with double precision arithmetic using the Fixed point iteration method (4). With an initial guess of 0.1, and continuing for more than fifty iterations the sixteen decimal places’ accurate solutions to the Colebrook’s equation have been obtained for a million nodes.

The dataset is attached as supplementary material to this article in form of a Microsoft Excel file. The first three sheets, respectively, contain million values of relative roughness (2), Reynolds number (3) and the consequent Darcy friction factor values (4). The breakup is best described in Table 1.

3. Verification of accuracy of the proposed database

It is important to verify how the obtained database is accurate, and there can be many ways of checking this. The most obvious, straightforward and the primary way to investigate the 16 decimal
Table 1
Layout and breakup of the dataset included as supplementary material.

| Sheet 1 (Reynolds numbers) | Sheet 2 (Relative Roughness) | Sheet 3 (accurate friction factors) | Sheet 4 (validity of the data) |
|---------------------------|-----------------------------|-----------------------------------|-------------------------------|
| INPUT $x$                 | INPUT $y$                   | OUTPUT $z$                        | $F(x, y; z^*)$                |
| 1000 by 1000 values       | 1000 by 1000 values         | 1000 by 1000 values [2$^7$ 2$^7$ ... 2$^7$] | 1000 by 1000 values [2$^7$ 2$^7$ ... 2$^7$] |
| [$x^T$ $x^T$ ... $x^T$] using (2) | [$y^T$ $y^T$ ... $y^T$] using (3) | using (4)                         | using (7)                     |
| 50 times                  | 50 times                    |                                   | 50 times                      |

places' accuracy of the computed friction factor values if the database is to check these through the actual equation — the Colebrook equation (1). For example [12], if one claims that

$$x^* = -1.207647827130918927009$$

is a solution of the nonlinear equation (6),

$$G(x) = xe^{x^2} - \sin^2 x + 3 \cos x + 5 = 0$$  \hspace{1cm} (6)

then the validity of the claim for $x^*$ can be checked by substituting claimed $x^*$ in (6), i.e. by checking if $G(x^*) = 0$.

Similarly, accuracy check has been performed to investigate the claimed double precision accuracy of the computed database of solutions $z$'s (see supplementary material) of the nonlinear Colebrook's equation (1) for 1000 by 1000 values of Reynolds number ($x$'s) and relative roughness ($y$'s). Re-writing (1) in the form:

$$F(x, y; z) = \frac{1}{\sqrt{z}} + 2 \log \left(\frac{2.51}{x^{\sqrt{2}}} + \frac{y}{3.7}\right) = 0$$  \hspace{1cm} (7)

the used values of inputs $x$ and $y$ (from sheets 1 and 2 of the supplementary file, respectively) and the computed accurate values of output $z$ (from sheet 3 of the same file) have been substituted in (7) and for the million nodes $F(x, y; z^*)$ have been analyzed, which are available on sheet 4 of the supplementary file. For all million nodes of Reynolds number and pipe relative roughness, the values of $F(x, y; z^*)$ have been found to be exactly equal to 0 or 1E-16 (to double precision default in Ms Excel). The values on sheet 4 of the supplementary file confirm the claimed accuracy of the presented database of the Darcy friction factor values ($z$'s) corresponding to $x$'s and $y$'s. It should be noted that any other way of investigating the accuracy of database would be a secondary analysis, and mostly in the context of such numerical calculations the examination of the equality in the nonlinear equation which has been solved at the claimed solution is given top priority, see Refs. [11,12].

The presented database is a standalone accuracy checker of any explicit equation in literature, for example 16 decimal places' accurate friction factor values of this database (see Sheet 3 of the supplementary material) has been used successfully in Ref. [11] to verify the accuracy of a new explicit equation proposed therein by us and a few others by Manadilli [13], Fang et al. [14] and Brkić [15].

The dataset can be used in future for proposition of robust and highly accurate explicit approximations for the Darcy friction factor by means of many soft computing techniques, and thus, will provide a way forward for the mathematicians, engineers and data scientists in analysis of pipe flow systems. On the other hand, the ready dataset can be readily used by practitioners to know the friction factor exactly for different values of relative roughness and Reynolds numbers.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.dib.2019.104733.

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