Measurement of Time-Dependent CP-Violating Asymmetry in $B^0 \to K^0_S \pi^0 \gamma$ Decay

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We present a new measurement of CP-violation parameters in $B^0 \rightarrow K_S^0 \pi^0 \gamma$ decay based on a sample of $275 \times 10^6$ $B \bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB energy-asymmetric $e^+e^-$ collider. One of the $B$ mesons is fully reconstructed in the $B^0 \rightarrow K^0_S \pi^0 \gamma$ decay. The flavor of the accompanying $B$ meson is identified from its decay products. CP-violation parameters are obtained from the asymmetry in the distribution of the proper-time intervals between the two $B$ decays. We obtain $S_{K^0_S \pi^0} = -0.58^{+0.49}_{-0.38}$(stat) $\pm 0.11$(syst) and $A_{K^0_S \pi^0} = +0.03 \pm 0.34$(stat) $\pm 0.11$(syst), for the $K^0_S \pi^0$ invariant mass covering the full range up to 1.8 GeV/$c^2$. We also measure the CP-violation parameters for the case $B^0 \rightarrow K^{*0} \rightarrow K^0_S \pi^0 \gamma$ and obtain $S_{K^{*0}} = -0.79^{+0.63}_{-0.50}$(stat) $\pm 0.10$(syst) for $A_{K^{*0}}$ fixed at 0.

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In the Standard Model (SM), CP violation arises from an irreducible phase, the Kobayashi-Maskawa (KM) phase $\theta$, in the weak-interaction quark-mixing matrix. The phenomena of time-dependent CP violation in decays through radiative penguin processes such as $b \rightarrow s \gamma$ are sensitive to physics beyond the SM. Within the SM, the photon emitted from a $B^0$ ($\bar{B}^0$) meson is dominantly right-handed (left-handed). Therefore the polarization of the photon carries information on the original b-flavor and the decay is, thus, almost flavor-specific. As a result, the SM predicts a small asymmetry and any significant deviation from this expectation would be a manifestation of new physics. Recently it was pointed out that in decays of the type $B^0 \rightarrow P^0 Q^0 \gamma$, where $P^0$ and $Q^0$ represent any CP eigenstate spin-0 neutral particles (e.g. $P^0 = K_S^0$ and $Q^0 = \pi^0$), many new physics effects on the mixing-induced CP violation do not depend on the resonant structure of the $P^0Q^0$ system. In this Letter, we describe two measurements of CP asymmetries: one with a $K^0_S \pi^0$ mass range restricted around the $K^0_S \pi^0$ mass, and the other with an extended $K^0_S \pi^0$ mass range. Whenever necessary, we distinguish these two by referring to them as $K^{*0}\gamma$ and $K^0_S \pi^0 \gamma$, respectively; otherwise the analysis procedure is common to both measurements since it was first optimized for the $K^{*0}\gamma$ and was extended for the $K^0_S \pi^0 \gamma$ later.

At the KEKB energy-asymmetric $e^+e^-$ (3.5 on 8.0 GeV) collider, the $\Upsilon(4S)$ is produced with a Lorentz boost of $\beta\gamma = 0.425$ along the $z$ axis defined as antiparallel to the $e^+$ beam direction. In the decay chain $\Upsilon(4S) \rightarrow B^0\bar{B}^0 \rightarrow f_{\pi \gamma}$, where one of the $B$ mesons decays at time $t_{\pi \gamma}$ to a final state $f_{\pi \gamma}$, which is our signal mode, and the other decays at time $t_{\pi \gamma}$ to a final state $f_{\pi \gamma}$ that distinguishes between $B^0$ and $\bar{B}^0$, the decay rate has a time dependence given by

$$P(\Delta t) = e^{-|\Delta t|/\tau_{B^0}} \left\{ 1 + q \left[ S \sin(\Delta m_d \Delta t) + A \cos(\Delta m_d \Delta t) \right] \right\}.$$  (1)

Here $S$ and $A$ are CP-violation parameters, $\tau_{B^0}$ is the $B^0$ lifetime, $\Delta m_d$ is the mass difference between the two $B^0$ mass eigenstates, $\Delta t$ is the time difference $t_{\pi \gamma} - t_{\pi \gamma}$, and the b-flavor charge $q = +1$ ($-1$) when the tagging $B$ meson is a $B^0$ ($\bar{B}^0$). Since the $B^0$ and $\bar{B}^0$ mesons are approximately at rest in the $\Upsilon(4S)$ center-of-mass system (c.m.s.), $\Delta t$ can be determined from the displacement in $z$ between the $f_{\pi \gamma}$ and $f_{\pi \gamma}$ decay vertices: $\Delta t \simeq (z_{\pi \gamma} - z_{\pi \gamma})/(\beta\gamma)$.

The Belle detector is a large-solid-angle magnetic spectrometer. A 2.0 cm radius beam pipe and a 3-layer silicon vertex detector (SVD1) were used for a 140 $fb^{-1}$ data sample containing $152 \times 10^6$ $B\bar{B}$ pairs, while a 1.5 cm radius beampipe, a 4-layer silicon detector (SVD2) and a small-cell inner drift chamber were used for an
additional 113 fb$^{-1}$ data sample that contains 123 $\times 10^6$ $B \bar{B}$ pairs for a total of 275 $\times 10^6$ $B \bar{B}$ pairs.

For high energy prompt photons, we select an isolated cluster in the electromagnetic calorimeter (ECL) that has no corresponding charged track, and has the largest energy in the c.m.s. We require the shower shape to be consistent with that of a photon. In order to reduce the background from $\pi^0$ or $\eta$, we exclude photons compatible with $\pi^0 \to \gamma \gamma$ or $\eta \to \gamma \gamma$ decays; we reject photon pairs that satisfy $L_{\pi^0} \geq 0.18$ or $L_{\eta} \geq 0.18$, where $L_{\pi^0(\eta)}$ is a $\pi^0 (\eta)$ likelihood described in detail elsewhere [10].

The polar angle of the photon direction in the laboratory frame is restricted to the barrel region of the ECL ($33^\circ < \theta < 128^\circ$), but is extended to the end-cap regions ($17^\circ < \theta < 150^\circ$) for the second data sample due to the reduced material in front of the ECL.

Neutral kaons ($K_S^0$) are reconstructed from two oppositely charged pions that have an invariant mass within $\pm 6$ MeV/c$^2$ (2$\sigma$) of the $K_S^0$ nominal mass. The $\pi^+\pi^-$ vertex is required to be displaced from the interaction point (IP) in the direction of the pion pair momentum [11].

Neutral pions ($\pi^0$) are formed from two photons with the invariant mass within $\pm 16$ MeV/c$^2$ (3$\sigma$) of the $\pi^0$ mass. The photon momenta are then recalculated with a $\pi^0$ mass constraint and we require the momentum of $\pi^0$ candidates in the laboratory frame to be greater than 0.3 GeV/c. The $K_S^0\pi^0$ invariant mass, $M_{K_S^0\pi^0}$, is required to be less than 1.8 GeV/c$^2$.

$B^0$ mesons are reconstructed by combining $K_S^0, \pi^0$ and $\gamma$ candidates. We form two kinematic variables: the energy difference $\Delta E \equiv E_{B}^{c.m.s.} - E_{beam}^{c.m.s.}$ and the beam-energy constrained mass $M_{bc} \equiv \sqrt{(E_{beam}^{c.m.s.})^2 - (p_B^{c.m.s.})^2}$, where $E_{c.m.s.}$ is the beam energy, $E_B^{c.m.s.}$ and $p_B^{c.m.s.}$ are the energy and the momentum of the candidate in the c.m.s. Candidates are accepted if they have $M_{bc} > 5.2$ GeV/c$^2$ and $|\Delta E| < 0.5$ GeV.

We reconstruct $B^+ \to K_S^0\pi^0\gamma$ candidates in a similar way as the $B^0 \to K_S^0\pi^0\gamma$ decay in order to reduce the cross-feed background from $B^+ \to K_S^0\pi^+\gamma$ in $B^0 \to K_S^0\pi^0\gamma$. The $B^+ \to K_S^0\pi^+\gamma$ events are also used for various cross-checks. For a $\pi^+\gamma$ candidate, we require that the track originates from the IP and that the transverse momentum is greater than 0.1 GeV/c. We also require that the $\pi^+$ candidate cannot be identified as any other particle species ($K^+, p^+, e^+$, and $\mu^+$).

Candidate $B^+ \to K_S^3\pi^+\gamma$ and $B^0 \to K_S^3\pi^0\gamma$ decays are selected simultaneously; we allow only one candidate for each event. The best candidate selection is based on the event likelihood ratio $R_{s/h}$ that is obtained by combining a Fisher discriminant $\mathcal{F}$ [12], which uses the extended modified Fox-Wolfram moments [10] as discriminating variables, and $\cos \theta_{B}$ defined as the angle between the $B$ meson momentum and the daughter $K_S^0$ momentum in the rest frame of the $K_S^0\pi$ system. We select the candidate that has the largest $R_{s/h}$. For the exclusive $B^0 \to K^{*0}\gamma$ study, we further require $0.8$ GeV/c$^2 < M_{K^{*0}\gamma} < 1.0$ GeV/c$^2$ after the best candidate selection. The signal region is defined as $-0.2$ GeV $< \Delta E < 0.1$ GeV and $5.27$ GeV/c$^2 < \Delta E < 5.29$ GeV/c$^2$.

We use events outside the signal region as well as large Monte Carlo (MC) samples to study the background components. The dominant background is from continuum light quark pair production ($e^+e^- \to q\bar{q}$ with $q = u,d,s,c$), which we refer to as $q\bar{q}$ hereafter. We require $R_{s/h} > 0.5$ to reduce the $q\bar{q}$ background; after applying all other selection criteria, this rejects 79% of the $q\bar{q}$ background while retaining 88% of the signal. Background contributions from $B$ decays, which are considerably smaller than $q\bar{q}$, are dominated by cross-feed from other radiative $B$ decays including $B^+ \to K_S^3\pi^+\gamma$.

The $b$-flavor of the accompanying $B$ meson is identified from inclusive properties of particles that are not associated with the reconstructed signal decay. The algorithm for flavor tagging is described in detail elsewhere [14]. We use two parameters, $q$ defined in Eq. [11] and $r$, to represent the tagging information. The parameter $r$ is an event-by-event flavor-tagging dilution factor that ranges from 0 to 1; $r = 0$ when there is no flavor discrimination and $r = 1$ implies unambiguous flavor assignment. It is determined by using MC data and is only used to sort data into six $r$ intervals. The wrong tag fraction $w$ and the difference $\Delta w$ in $w$ for the $B^0$ and $\bar{B}^0$ decays are determined for each of the six $r$ intervals from data [11].

The vertex position of the signal-side decay is reconstructed from the $K_S^0$ trajectory with a constraint on the IP; the IP profile ($\sigma_x \simeq 100$ $\mu$m, $\sigma_y \simeq 5$ $\mu$m, $\sigma_z \simeq 3$ $\mu$m) is convolved with the finite $B$ flight length in the plane perpendicular to the $z$ axis. Both pions from the $K_S^0$ decay are required to have enough hits in the SVD in order to reconstruct the $K_S^0$ trajectory with high resolution. The reconstruction efficiency depends not only on the $K_S^0$ momentum but also on the SVD geometry. The efficiency with SVD2 (55%) is higher than with SVD1 (41%) because of the larger detection volume. The other (tag-side) $B$ vertex determination is the same as that for the $B^0 \to \phi K_S^0$ analysis [11].

Figure [11] shows the $M_{bc}$ ($\Delta E$) distribution for the reconstructed $K_S^0\pi^0\gamma$ candidates within the $\Delta E (M_{bc})$ signal region after flavor tagging and vertex reconstruction. The signal yield is determined from an unbinned two-dimensional maximum-likelihood fit to the $\Delta E$-$M_{bc}$ distribution. The fit region for the $B^0 \to K^{*0}\gamma$ analysis is $5.20$ GeV/c$^2 < M_{bc} < 5.29$ GeV/c$^2$ and $-0.5$ GeV $< \Delta E < 0.5$ GeV. For the $B^0 \to K_S^0\pi^0\gamma$ analysis, the $\Delta E$ fit region is narrowed to $-0.4$ GeV $< \Delta E < 0.5$ GeV in order to avoid other $B$ background events that populate the low $\Delta E$ region. The signal distribution is represented by a smoothed histogram obtained from MC simulation that accounts for a small correlation between $M_{bc}$ and $\Delta E$. The background from $B$ decays is also modeled with a smoothed histogram obtained from MC events. For the
FIG. 1: (a) $M_{bc}$ distributions within the $\Delta E$ signal region and (b) $\Delta E$ distributions within the $M_{bc}$ signal region for $B^0 \to K_S^0 \pi^0 \gamma$. Solid curves show the fit to signal plus background distributions. Lower (upper) dashed curves show the background contributions from $q\bar{q}$ ($q\bar{q}$ and $B$ decays).

FIG. 2: $M_{K_S^0 \pi^0 \gamma}$ distribution of the $B^0 \to K_S^0 \pi^0 \gamma$ signal yield obtained from the fit to the $\Delta E$-$M_{bc}$ distribution. Background shape parameters are fixed at the values obtained with the whole $M_{K_S^0 \pi^0 \gamma}$ region.

$q\bar{q}$ background, we use the ARGUS parameterization for $M_{bc}$ and a second-order polynomial for $\Delta E$. Normalizations of the signal and background distributions and the $q\bar{q}$ background shape are the five free parameters in the fit. We observe 501 (215) $K_S^0 \pi^0 \gamma$ ($K^{*0} \gamma$) candidates in the signal box, which decrease to 227 (92) after the flavor tagging and the $B$ vertex reconstruction, and obtain 105 $\pm$ 14 (57 $\pm$ 9) signal events by the fit. Figure 2 shows the signal yields in different $M_{K_S^0 \pi^0 \gamma}$ regions. For each bin, a fit to the $\Delta E$-$M_{bc}$ distribution is performed, and the obtained signal yield is plotted.

We determine $S$ and $A$ from an unbinned maximum-likelihood fit to the observed $\Delta t$ distribution. The probability density function (PDF) expected for the signal distribution, $P_{\text{sig}}(\Delta t; S, A, S, \Delta \nu)$, is given by the time dependent decay rate [Eq. 1] modified to incorporate the effect of incorrect flavor assignment. The distribution is convolved with the proper-time interval resolution function $R_{\text{sig}}$, which takes into account the finite vertex resolution. The parametrization of $R_{\text{sig}}$ is the same as that used for the $B^0 \to K_S^0 \pi^0$ decay [11]. $R_{\text{sig}}$ is first derived from flavor-specific $B$ decays [14] and modified by additional parameters which rescale vertex errors to account for the fact that there is no track directly originating from the IP.

For each event, the following likelihood function is evaluated:

$$P_i = (1 - f_{\text{col}}) \int_{-\infty}^{+\infty} \left[P_{\text{sig}}(\Delta t') R_{\text{sig}}(\Delta t_i - \Delta t') + (1 - f_{\text{sig}}) P_{\text{bkg}}(\Delta t') R_{\text{bkg}}(\Delta t_i - \Delta t')\right] d(\Delta t') + f_{\text{col}} P_{\text{col}}(\Delta t_i),$$

where $P_{\text{col}}$ is a Gaussian function that represents a small outlier component with fraction $f_{\text{col}}$ [17]. The signal probability $f_{\text{sig}}$ is calculated on an event-by-event basis from the function which we obtained as the result of the two-dimensional $\Delta E$-$M_{bc}$ fit for the signal yield extraction. A PDF for background events, $P_{\text{bkg}}$, is modeled as a sum of exponential and prompt components, and is convolved with a sum of two Gaussians which represent the resolution function $R_{\text{bkg}}$ for the background. All parameters in $P_{\text{bkg}}$ and $R_{\text{bkg}}$ are determined by a fit to the $\Delta t$ distribution of a background-enhanced control sample, i.e. events outside of the $\Delta E$-$M_{bc}$ signal region. We fix $\tau_{B^0}$ and $\Delta M_{bd}$ at their world-average values [15].

The only free parameters in the final fit for the $B^0 \to K_S^0 \pi^0 \gamma$ decay are $S_{K_S^0 \pi^0 \gamma}$ and $A_{K_S^0 \pi^0 \gamma}$, which are determined by maximizing the likelihood function $L = \prod_i P_i(\Delta t_i; S, A)$ where the product is over all events. For the $B^0 \to K^{*0} \gamma$ decay, we fix $A_{K^{*0} \gamma}$ at zero and perform a single parameter fit for $S_{K^{*0} \gamma}$. This is based on the previous measurement [10] of the partial rate asymmetry in $B \to K^{*0} \gamma$ decays, which yielded $A_{K^{*0} \gamma}$ consistent with zero. The error of the previous measurement of $A_{K^{*0} \gamma}$ is included in the systematic error of $S_{K^{*0} \gamma}$. We obtain

$$S_{K^{*0} \gamma} = -0.79^{+0.63}_{-0.50} \text{(stat)} \pm 0.10 \text{(syst)},$$
$$S_{K_S^0 \pi^0 \gamma} = -0.58^{+0.46}_{-0.38} \text{(stat)} \pm 0.11 \text{(syst)},$$
$$A_{K_S^0 \pi^0 \gamma} = +0.03 \pm 0.34 \text{(stat)} \pm 0.11 \text{(syst)}.$$
$A_{K^0_S\pi^0\gamma}$ and ±0.01 for $S_{K^0\cdot\pi^0\gamma}$. Also included are effects from uncertainties in the wrong tag fraction and physics parameters ($\Delta m_d$, $\tau_B$, and $A_{K^0\cdot\pi^0\gamma}$). A possible bias in the fit is estimated by using MC data. The total systematic error is obtained by adding these contributions in quadrature.

Various cross-checks of the measurement are performed. We apply the same fit procedure to the $B^0 \rightarrow J/\psi K_S^0$ sample without using $J/\psi$ daughter tracks for the vertex reconstruction. We obtain $S_{J/\psi K_S^0} = +0.68 \pm 0.10$ (stat) and $A_{J/\psi K_S^0} = +0.02 \pm 0.04$ (stat), which are in good agreement with the world average values [18]. We perform a fit to $B^+ \rightarrow K^0_S\pi^+\gamma$ ($B^0 \rightarrow K^{*+}\gamma \rightarrow K^0_S\pi^+\gamma$), which is a good control sample of the $B^0 \rightarrow K^0_S\pi^0\gamma$ ($B^0 \rightarrow K^{*0}\gamma \rightarrow K^0_S\pi^0\gamma$) decay, without using the primary $\pi^+$ for the vertex reconstruction. The result is consistent with no $CP$ asymmetry, as expected. Lifetime measurements are also performed for these modes, and values consistent with the world-average are obtained. Ensemble tests are carried out with MC pseudo-experiments; we find that the statistical errors obtained in our measurements are all within the expectations from the ensemble tests. A fit to the $B^0 \rightarrow K^0_S\pi^0\gamma$ subsample that excludes the $K^{*0}$ mass region yields $S = -0.39^{+0.63}_{-0.52}$ (stat) and $A = +0.10 \pm 0.51$ (stat). The result is consistent with the results for $B^0 \rightarrow K^{*0}\gamma$ and for the full $B^0 \rightarrow K^0_S\pi^0\gamma$ sample.

In summary, we have performed a new measurement of the time-dependent $CP$ asymmetry in the decay $B^0 \rightarrow K^0_S\pi^0\gamma$ based on a sample of 275 $\times 10^6$ $B\overline{B}$ pairs. Two regions of the $K^0_S\pi^0$ invariant mass are used: between 0.8 GeV/$c^2$ and 1.0 GeV/$c^2$ for the $K^{*0}$ resonance, and the full region up to 1.8 GeV/$c^2$ including other resonances and non-resonant contributions. We obtain $CP$-violation parameters in the $B^0 \rightarrow K^0_S\pi^0\gamma$ decay $S_{K^0_S\pi^0\gamma} = -0.58^{+0.46}_{-0.38}$ (stat) ± 0.11 (syst) and $A_{K^0_S\pi^0\gamma} = +0.03 \pm 0.34$ (stat) ± 0.11 (syst), and in $B^0 \rightarrow K^{*0}(\rightarrow K^0_S\pi^0\gamma)$ decay $S_{K^0\cdot\pi^0\gamma} = -0.79^{+0.63}_{-0.50}$ (stat) ± 0.10 (syst) with $A_{K^0\cdot\pi^0\gamma}$ fixed at zero. The value of $S_{K^0\cdot\pi^0\gamma}$ is consistent with the measurement by BABAR [2]. $CP$-violation parameters in the three-body $B^0 \rightarrow K^0_S\pi^0\gamma$ decay are measured for the first time. The two results are consistent with each other and with no $CP$ asymmetry, while the statistical error is considerably reduced by the inclusion of these additional events in the $B^0 \rightarrow K^0_S\pi^0\gamma$ analysis.

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