LINEARLY AND CIRCULARLY POLARIZED EMISSION IN SAGITTARIUS A*  
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ABSTRACT  
We perform general relativistic ray-tracing calculations of the transfer of polarized synchrotron radiation through the relativistic accretion flow in Sagittarius (Sgr) A*. Based on a two-temperature magnetorotational instability (MRI) induced accretion mode, the birefringence effects are treated self-consistently. By fitting the spectrum and polarization of Sgr A* from millimeter to near-infrared bands, we are able to not only constrain the basic parameters related to the MRI and the electron heating rate, but also limit the orientation of the accretion torus. These constraints lead to unique polarimetric images, which may be compared with future millimeter and submillimeter VLBI observations. In combination with general relativistic MHD simulations, the model has the potential to test the MRI with observations of Sgr A*.

Subject headings: black hole physics — Galaxy: center — plasmas — polarization — radiative transfer — submillimeter

Online material: color figure

1. INTRODUCTION  
The compact radio source Sagittarius (Sgr) A* at the Galactic center is associated with a $\sim 4 \times 10^6 M_\odot$, supermassive black hole (Schödel et al. 2002; Ghez et al. 2005) and is one of the best astrophysical sources for studying the physics of black hole accretion. Since the discovery of active galactic nuclei and Galactic X-ray binaries, it has been suggested that black hole accretion is responsible for the high luminosity and radiation efficiency of these sources. To release the gravitational energy of materials falling into black holes, the angular momentum must be transported to large radii in the accretion processes. The nature of the viscosity responsible for the angular momentum transport and energy dissipation is not well understood. Shakura & Sunyaev (1973) proposed that the viscous stress is induced by turbulence and is proportional to the pressure of the accretion flow.

MHD simulations have made the magnetorotational instability (MRI) the primary mechanism for generation of turbulence and viscous stress in accretion flows (Balbus & Hawley 1991, 1998). Although there are indications that the viscous stress is proportional to the magnetic field energy density, the ratio of the magnetic field energy density to the gas pressure depends on the initial magnetic field configurations of these simulations (Pessah et al. 2006). King et al. (2007) argue that, at least for some astrophysical accretion systems, the MRI may not be able to generate high enough viscosity to account for the observations. Nevertheless, the MRI is believed to play dominant roles in some black hole accretion systems. It has been studied extensively through MHD simulations and is awaiting observational tests.

Sgr A* plays a crucial role in this aspect. Since its discovery in 1974 (Balick & Brown 1974), the source has been observed extensively due to its association with the supermassive black hole (Melia & Falcke 2001). Radio imaging reveals a compact radio source with its intrinsic size marginally uncovered in the millimeter band (Bower et al. 2004; Shen et al. 2005). The centimeter spectrum can be fitted with a power-law function, and there is clear evidence for a millimeter and submillimeter spectral bump (Falcke et al. 1998). Radio ($\leq 43$ GHz) emission of Sgr A* has unusual polarization characteristics with no detection ($\leq 0.4\%$) of linear polarization (LP) and weak circular polarization (CP) of up to $\sim 1.0\%$ in the quiescent state (Bower et al. 2002). Weak LP ($\leq 1\%$) was detected recently at 43 and 22 GHz during radio outbursts (Yusef-Zadeh et al. 2007a, 2007b). Macquart et al. (2006) reported a mean LP of $2.1\% \pm 0.1\%$ at 3.5 mm. In the submillimeter band, high LP of $\sim 10\%$ has been routinely measured (Aiiken et al. 2000; Marrone et al. 2007) without any firm indication of detectable CP (Marrone et al. 2006). These observations suggest that Sgr A* is powered by accretion of the supermassive black hole with the millimeter and submillimeter emission originating within $10R_s$, where $R_s \sim 1.2 \times 10^{-2}[M/(4 \times 10^6 M_\odot)]$ cm is the Schwarzschild radius for a nonspinning black hole with mass $M$. The high LP implies a very low accretion rate (Quataert & Gruzinov 2000). Based on the MRI, a small ($<10R_s$) accretion torus can account for the spectrum and polarization of the submillimeter emission (Melia et al. 2000, 2001; Bromley et al. 2001). Liu et al. (2007, hereafter L07) recently generalize the model to a two-temperature MRI-driven accretion flow, which naturally explains the 90° flip between the electric vector position angles (EVPA) of the polarized submillimeter and near-infrared (NIR) emissions (Eckart et al. 2006; Meyer et al. 2007; Marrone et al. 2007). Besides the basic parameters related to the MRI, the model introduces only one more parameter to characterize the not-well-understood electron heating processes.

However, all models proposed so far have ignored the birefringence effects of synchrotron radiation propagating through the relativistic accretion flow in Sgr A* and have treated the general relativistic (GR) effects with simplified disk structures. The Faraday rotation of the birefringence effects can depolarize low-frequency emission, and the GR light bending effect can complicate the transfer of polarized emission near the black hole. Based on the model proposed by L07, in this Letter we present a more complete and self-consistent treatment of radiation from the relativistic accretion flow. The onset fre-
frequency of LP from the centimeter to millimeter bands, emission spectrum, and polarization are very sensitive to the ratio of the magnetic field to gas pressure, electron heating rate, and disk inclination angle, respectively. A fitting to these observations leads to unique images of fractional LP and CP. The combined effects of GR and birefringence produce distinct LP and CP features between the submillimeter and NIR bands. These predictions can be tested with future polarimetric VLBI observations. More importantly, in combination with MHD simulations, observations of Sgr A* can be used to test the MRI. In § 2 we briefly discuss our treatment of the transfer of polarized synchrotron radiation near a Schwarzschild black hole. Our main results are presented in § 3. We draw conclusions and propose future work in § 4.

2. TRANSFER OF SYNCHROTRON RADIATION

Following Landi Degl’Innocenti & Landolfi (2004), we define four vectors \( \mathbf{p}, \mathbf{e}, \eta, \) and \( \rho \) to treat the transfer of polarized emission in a magnetized plasma. Here \( \mathbf{p} \) represents three normalized Stokes parameters \( (Q, U, V)/I \), and \( \epsilon, \eta, \) and \( \rho \) correspond to three normalized emission coefficients \( (\epsilon_0, \epsilon_v, \epsilon_e) = (e_0, e_v, e_e)/I \), absorption coefficients \( (\eta_0, \eta_v, \eta_e) \), and Faraday conversion and rotation \((\rho_0, \rho_v, \rho_e)\), respectively. For the total specific intensity \( I \), the corresponding emission and average absorption coefficients are indicated by \( e_i = \epsilon_i I \) and \( \eta_i \), respectively. Thus we have

\[
\frac{dI}{ds} = -(\eta_0 + \eta_v \mathbf{p} - \epsilon_0) I, \tag{1}
\]

\[
\frac{d\rho}{ds} = -\eta + (\eta_0 \mathbf{p}) \rho + \rho_v \times \mathbf{p} + \epsilon - \epsilon_v \mathbf{p}, \tag{2}
\]

where \( s \) is the distance along a light ray.

Synchrotron radiation is elliptically polarized with a dominant extraordinary emission component, whose electric field is perpendicular to both the magnetic field vector and the wave propagation direction, and the emission coefficients satisfy \( \epsilon_v > (\epsilon_0^2 + \epsilon_e^2)^{1/2} \gg \epsilon_e \) (Bekefi 1966; Melrose 1971). In the coordinate system with the ordinary and extraordinary components as the axes, the \( U \) (or \( Q \)) components of \( \epsilon, \eta, \) and \( \rho \) vanish, and \( \rho_0 \) (or \( \rho_v \)) and \( \rho_e \) correspond to the Faraday conversion and rotation coefficients, respectively (Melrose 1997; Kennett & Melrose 1998). We use formulas of Melrose (1971) to obtain \( \epsilon_v \) and \( \epsilon_e \). Then \( \eta_0 \) and \( \eta_v \) can be derived from the Kirchhoff’s law. The polarization of the natural wave modes of a magnetized plasma determines \( \rho \). Because there are no birefringence effects for the natural wave modes, \( \rho \) needs to be parallel to \( \epsilon \) (Melrose 1997; Kennett & Melrose 1998; Landi Degl’Innocenti & Landolfi 2004).

For the thermal plasmas we are interested in here, Melrose (1997) has given expressions for the Faraday rotation coefficients in cold and relativistic limits. We extrapolate these results \( \rho_v = (\epsilon^2 n B \cos \theta_e / \pi \gamma / m_e^2 c^2 v^2)(1 - \ln \gamma_e / \ln \gamma_e / \ln \gamma_e / \ln \gamma_e) \), where \( e, m_e, \gamma, m_e, c^2 l / m_e, n, k_B, c, B, \theta_e, \) and \( \nu \) are the electron charge, mass, temperature, density, Boltzmann constant, speed of light, magnetic field, angle between the magnetic field and wave propagation direction, and wave frequency, respectively. The Faraday conversion coefficient is then given by \( \rho_v (\epsilon_0^2 + \epsilon_e^2)^{1/2} / \epsilon_e \).

We simulate the pseudo-Newtonian disk structure in a Schwarzschild metric with all quantities treated as measured by a locally static observer. Because the emission frequencies in the millimeter to NIR bands are much higher than the cyclotron and plasma frequencies of the thermal electrons in the accretion flow, the plasma effects on photon propagation discussed by Broderick & Blandford (2003) can be neglected and all photons travel along the null geodesics. To perform the above radiation transfer of polarized synchrotron emission with the ray-tracing code discussed in Huang et al. (2007) one only needs to take into account the rotation of EVPAs along a null geodesic due to the space curvature (Bromley et al. 2001). The effects of magnetic field and GR have been taken care of in the radiation transfer equations (Broderick & Blandford 2004).

MHD simulations of the MRI show that the magnetic field is dominated by its azimuthal component due to the shearing of the Keplerian accretion flow. Previous modeling of the submillimeter to NIR polarizations of Sgr A* assumed that the magnetic field does not have other components (see Melia et al. 2000; Bromley et al. 2001; L07). Here we consider two more realistic configurations: one with the magnetic and velocity field parallel to each other in the upper half of the disk and antiparallel in the lower half, and the other with the magnetic field direction just reversed. The averaged Stokes parameters of these two configurations are used to fit observations. Sgr A* is highly variable with variation timescales decreasing with observing frequency. In millimeter and submillimeter bands, both flux and LP vary on hourly timescales (Marrone et al. 2006), while the accretion timescale near the black hole is a few days. Shorter timescale variations are likely associated with turbulent fluctuations in the accretion flow, which may be addressed with MHD simulations. The results of our stationary accretion model should be compared with measurements averaged over days.

3. SPECTRAL FITTING AND POLARIMETRIC IMAGES

The thick solid lines in Figures 1a and 1b represent the best fit to the spectrum and polarization data of Sgr A*. The data points are summarized in L07, except the two CP measurements at 112 GHz (Bower et al. 2002) and 340 GHz (Marrone et al. 2006). We fit the data around the millimeter to submillimeter bump, including the EVPAs at 86 GHz averaged over five epochs in 2004 (Macquart et al. 2006), and at 230 GHz that remained unchanged during observations from 2002 October to 2004 January (Bower et al. 2005). In our fitting, neither the NIR measurement (Eckart et al. 2006) nor the detected LP at 22 and 43 GHz (Yusef-Zadeh et al. 2007a, 2007b) is used simply because these are related to flaring activity. However, the EVPAs at 2.2 \( \mu \)m remained unchanged from 2004 to 2006 and may be explained by a temporary disk with toroidal magnetic field (Eckart et al. 2006; Meyer et al. 2007). The corresponding five model parameters are the ratio of viscous stress to magnetic field energy density \( \beta_v = 0.7 \), the ratio of magnetic field energy density to gas pressure \( \beta_e = 0.4 \), the electron heating rate indicated by the dimensionless parameter \( C_e = 0.44 \), the accretion rate \( M = 0.95 \times 10^{-8} M_{\odot} \text{ yr}^{-1} \), and the inclination angle of the accretion flow \( i = 40^\circ \). The most prominent GR effect is the amplification of the emission area due to the light bending near the black hole (see Fig. 2). At higher frequencies the disk becomes optically thin and lower frequency emission is produced at relatively larger radii, so the flux density enhancement by the GR effects is significant near the spectral peak and vanishes in the low- and high-frequency limits as can be seen by comparing with the thin solid line of the pseudo-Newtonian spectrum.

The GR effects also change the LP. The thin solid line in Figure 1b shows the corresponding pseudo-Newtonian result. The LP is significantly suppressed \((\leq 1\%)\) by birefringence effects at low frequencies. Another radio component introduced
by L07, which is presumably associated with an outflow, may fit the centimeter CP and flux density spectra (Beckert & Falcke 2002; Zhao et al. 2004) and explain the polarization observed during radio outbursts (Yusef-Zadeh et al.2007a, 2007b). The dotted line shows the GR calculation without the birefringence effects, i.e., \( \rho = 0 \). We see that there is a weak level of LP (∼2%–3%) at low frequencies. The birefringence effects reduce the LP in millimeter and submillimeter bands by up to ∼50%. At very high frequencies, the birefringence effects are weak and the results approach that of the fiducial model. As shown by L07, the flux density is very sensitive to the inclination angle of the disk. In practice, the fitting to the emission spectrum does not give a unique set of model parameters. With the additional LP measurements, the best-fit model parameters can be fixed. This is because the synchrotron emissivity is proportional to \( B^2 n \) while the Faraday rotation measure is proportional to \( B_n \). For a \( B^2 n \) constrained by the emission spectrum, the Faraday rotation increases with the decrease of \( B \), which is determined by \( \beta_n \). Therefore the onset frequency of LP increases with the decrease of \( \beta_n \). Two magnetic field configurations give almost identical LP but different EVPA and CP. So we plot only one LP (dashed line in Fig. 1b) but two sets of EVPA and CP (dashed lines in Figs. 1c and 1d). The average of Stokes parameters over the two configurations reduces the LP. One may use the difference between the average and individual configurations to estimate the variability of the corresponding quantities.

The most interesting prediction of our model is oscillations of LP and CP with frequency ∼1 THz (see Figs. 1b and 1d). This is due to the combined effects of GR and birefringence.
\( (i \geq 60^\circ) \) cases overestimate the LP in the submillimeter band and fail to produce a 90° EVPVA flip from the submillimeter to NIR bands because the near side of the torus dominates the emission.

Figure 2 shows the intrinsic polarimetric images of the fiducial model at 230 GHz with the disk axis in the vertical direction. Panels \( a, b, c, \) and \( d \) are the images of the total specific intensity \( I \), fractional LP \( \left( \left[ (Q^2 + U^2)^{1/2}/I \right] \right) \), fractional CP \( (V/I) \), and the residual \( (\Delta \phi_{\text{res}}) \) of the Faraday rotation angle \( \chi \) divided by \( 2\pi \), respectively. The results shown in Figure 1 are obtained by integrating the Stokes parameters over the disk to compare with spatially unresolved observations. Doppler boosting enhances \( I \) on the left-hand side of Figure 2a. The white dashes in Figure 2b represent the intrinsic EVPVA of each ray. In Figure 2c positive (red) and negative (black) values of CP correspond to right- and left-hand polarization, respectively. The strips in Figures 2b and 2c are clearly correlated with the Faraday conversion shown in Figure 2d. In our case, the Faraday conversion coefficient is always much greater than the Faraday rotation coefficient, and its integrations along the rays are much larger than \( 2\pi \) at 230 GHz; the Stokes \( V \) therefore changes its sign over one period of EVPVA rotation (Kennett & Melrose 1998; Landi Degl’Innocenti & Landolfi 2004). The strips will be less distinct in a realistic turbulent flow. The observed images will always be smoothed by finite spatial resolutions and interstellar scattering effects (Shen et al. 2005). However, rough contours may be resolved by future polarization-sensitive VLBI observations since the scattering effect is less important in the submillimeter band. The black hole shadow may be detected in the \( I \) image, as evidence for the GR theory. We also expect high levels of LP with the EVPVA mostly in the radial direction, as evidence for toroidal component dominated magnetic fields.

### 4. Discussion and Conclusion

Tremendous efforts have been made to image shadows caused by the strong gravity of black holes (Krichbaum et al. 2006; Shen et al. 2005; Bower et al. 2004). Sgr A* has been the best candidate for such studies. The polarization measurements of Sgr A* have put strong constraints on the physical processes near the black hole. Based on the two-temperature MRI-driven Keplerian accretion model, which has only five parameters, we calculate the synchrotron emission from the accretion flow with both the GR and birefringence effects taken into account self-consistently. The model can reproduce the spectrum and polarization of Sgr A* in millimeter and shorter wavelength bands and further predicts distinct LP and CP at \( \sim 1 \) THz. Polarimetric images of the relativistic accretion flow of the best-fit model are made, which can be verified with future polarization-sensitive VLBI observations.

The MRI has become the leading mechanism for the generation of viscous stress required to drive the accretion flow of plasmas into black holes. The theory has been well established through both theoretical analyses and numerical investigations. The model presented here makes it possible for the first time to put the theory into observational tests. If the scaling laws for the MRI discovered by Pessah et al. (2006) are applicable to the accretion flow in Sgr A*, the fiducial model parameter \( \beta_{\text{r}} = 0.4 \) requires the Alfvén velocity induced by the large-scale vertical magnetic field be \( \sim 20 \) times smaller than the sound velocity. With this large-scale magnetic field configuration, one can carry out GR MHD simulations for Sgr A* (Noble et al. 2006). The outflows from these simulations should not only reproduce low-frequency radio flux, CP, and source size measurements, but also induce a Faraday rotation measure external to the disk studied here, which can improve the fit to the observed EVPVAs in millimeter and submillimeter bands. NIR and X-ray flares and the high variability of this source, which cannot be studied with the current model, may also be addressed with such simulations (Baganoff et al. 2001; Gillessen et al. 2006; Hornstein et al. 2007; Meyer et al. 2007).

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### REFERENCES

Aitken, D. K., et al. 2000, ApJ, 534, L173
Baganoff, F. K., et al. 2001, Nature, 413, 45
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
—. 1998, Rev. Mod. Phys., 70, 1
Balick, B., & Brown, R. L. 1974, ApJ, 194, 265
Becker, T., & Falcke, H. 2002, A&A, 388, 1106
Bekefi, G. 1966, Radiation Processes in Plasmas (New York: Wiley)
Bower, G. C., Falcke, H., Wright, M. C. H., & Backer, D. C. 2002, ApJ, 571, 843
Bower, G. C., Falcke, H., Wright, M. C. H., & Backer, D. C. 2005, ApJ, 618, L29
Bower, G. C., et al. 2004, Science, 304, 704
Broderick, A. E., & Blandford, R. 2003, MNras, 342, 1280
—. 2004, MNras, 349, 994
Broderick, A. E., & Loeb, A. 2006, MNras, 367, 905
Bromley, B., Melia, F., & Liu, S. 2001, ApJ, 555, L83
Eckart, A., et al. 2006, A&A, 455, 1
Falcke, H., et al. 1998, ApJ, 499, 731
Ghez, A. M., et al. 2005, ApJ, 620, 744
Gillessen, S., et al. 2006, ApJ, 640, L163
Hornstein, S. D., et al. 2007, ApJ, 667, 900
Huang, L., Cai, M., Shen, Z.-Q., & Yuan, F. 2007, MNRAS, 379, 833
Kennett, M., & Melrose, D. 1998, Publ. Astron. Soc. Australia, 15, 211
King, A., Pringle, J. E., & Livio, M. 2007, MNRAS, 376, 1740
Krichbaum, T. P., et al. 2006, J. Phys. Conf. Ser., 54, 328
Landi Degl’Innocenti, E., & Landolfi, M. 2004, Polarization in Spectral Lines (Dordrecht: Kluwer)
Liu, S., Qian, L., Wu, X.-B., Fryer, C. L., & Li, H. 2007, ApJ, 666, L127 (L07)
Macquart, J.-P., et al. 2006, ApJ, 646, L111
Marrone, D. P., Moran, J. M., Zhao, J.-H., & Rao, R. 2006, ApJ, 640, 308
—. 2007, ApJ, 654, L57
Melia, F., & Falcke, H. 2001, ARA&A, 39, 309
Melia, F., Liu, S., & Coker, R. 2000, ApJ, 545, L117
—. 2001, ApJ, 553, 146
Melrose, D. B. 1971, Ap&SS, 12, 172
—. 1997, J. Plasma Phys., 58, 735
Meyer, L., et al. 2007, A&A, 473, 707
Noble, S. C., et al. 2006, ApJ, 641, 626
Pessah, M. E., Chan, C. K., & Psaltis, D. 2006, Phys. Rev. Lett., 97, 221103
Quataert, E., & Gruzinov, A. 2000, ApJ, 545, 842
Schödel, R., et al. 2002, Nature, 419, 694
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 737
Shen, Z.-Q., et al. 2005, Nature, 438, 62
Yusef-Zadeh, F., et al. 2007, ApJ, 668, L47
—. 2007b, ApJ, submitted (arXiv:0712.2882)
Zhao, J. H., et al. 2004, ApJ, 603, L85