Second order quantum decoherence in the boson system

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The second order quantum decoherence (SOQDC) is proposed as a novel description for the loss of quantum coherence only reflected by second order quantum correlations. By calculating the two-time correlation function, the phenomenon of SOQDC is studied in details for a simple model, a two boson system interacting with a reservoir composed of one or many bosons. The second order quantum decoherence effects can be observed in the sketched cavity QED experiment.

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Embodying the wave nature of particles in the quantum world, the coherence of a quantum system is manifested in the observation of interference fringes reflected both by the first order correlation functions and by higher order ones introduced by Glauber many years ago [1]. In the presence of external quantum system (i.e., a reservoir) interacting with the studied system, the quantum decoherence of the system happens with the disappearance of interference fringes. This decoherence mechanism provides essential elements in the understanding of quantum measurement [2] and the transition from quantum to classical mechanics [3]. However, to our best knowledge, in both aspects from experiments [4-8] and theories [9,10], most studies focus on the first order quantum decoherence, in which the quantum coherence is described by the superposition of two or several single states and reflected by the first order correlation function. In this Letter we will present a novel description of decoherence phenomenon reflected by the second order quantum correlation functions.

Actually the higher order effect of quantum coherence is manifested in the multi-particle picture [2]. For instance, for a single particle, the coherent superposition $|s\rangle = \frac{1}{\sqrt{2}}(|k\rangle + |\Delta k\rangle)$ of two momentum states $|k\rangle$ and $|\Delta k\rangle$ with opposite wave vectors explicitly shows the first order quantum coherence reflected by the interference fringes $I(x) = |\langle x|s\rangle|^2 = \sin^2(kx)$. However, there indeed exists such a situation e.g., a single two particle state $|k, -k\rangle$, in which there is not the first order quantum coherence, but we can see the second order effect through the second order quantum correlation functions. This fact even enjoyed by Hanbury-Brown-Twiss experiment [9] is exactly necessary to building quantum optics. Therefore, it is natural to extend the investigations of decoherence to the case of second order and to probe how an external system destroys the interference fringes due to high order coherence.

Studying the high order decoherence quantitatively without much technical disturbance, we first restrict ourselves to a simple model: the two mode bosonic system coupling to a reservoir composed of one or many bosons. In the case of one reservoir boson being in the Fock state, the second order decoherence factor measuring the loss of second order coherence is found to be a fast oscillating function when the particle number becomes larger. In the case of many reservoir bosons being in vacuum state, the second order decoherence factor manifests the quantum revival-collapse phenomena of long time. This kind of quantum jump even was predicted by one (CPS) of the authors for the first order quantum decoherence with the universal factorization structure [11]. In this letter it is found that, in the continuous mode limit with some given spectral distributions, the second order decoherence factor will decay following an exponential law. To test this theoretical model, a Gedanken experiment is proposed: two bosonic atoms pass through a one-mode cavity or many-mode (leaky) cavity. The possibility of the realization of the experiment will be only briefly discussed. We take $\hbar = 1$ in this letter.

The model Hamiltonian $H = H_0 + V$ is defined by

$$H_0 = \omega_e \hat{b}_e^{\dagger} \hat{b}_e + \sum_j \omega_j \hat{a}_j^{\dagger} \hat{a}_j,$$

$$V = \sum_j d(\omega_j)(\hat{a}_j \hat{b}_e + \hat{a}_j^{\dagger} \hat{b}_g),$$

where $H_0$ is the free Hamiltonian of the system plus the reservoir, $V$ the interaction between the system and the reservoir and $\hat{b}_e^{\dagger}(\hat{b}_e), \hat{b}_g^{\dagger}(\hat{b}_g)$ the creation (annihilation) operators of two modes labeled by index $e$ and $g$. Their frequencies are $\omega_e$ and $\omega_g = 0$ respectively. The operators $\hat{a}_j^{\dagger}(\hat{a}_j)$ are creation (annihilation) operators of the modes which labeled by index $j$ and the frequency of each mode is denoted by $\omega_j$. The frequency-dependent constant $d(\omega_j)$ measures the coupling constant between the system and the $j$ mode of the reservoir. This model can be physically implemented as a cavity QED system, in which many identical bosonic atoms of two levels $(|e\rangle$ and $|g\rangle$ are placed in a cavity with some modes of frequencies $\omega_j$. $H$ defines the second quantization Hamiltonian for the atom-cavity system.

In our model the interaction is turned on only in the time interval $t \in [0, T]$. This time period can be understood as the time of atom passage in the cavity. The coupling with the reservoir—the multi-mode cavity field...
will change the second order coherence of atoms. After time $T$, an ensemble measurement in term of the second order correlation function is made on the system so that the decoherence effect in the above time period can be observed directly. The second order correlation function

$$G[t,t',\hat{\rho}_S(T)] = Tr_S(\hat{\rho}_S(T)\hat{B}^\dagger(t)\hat{B}^\dagger(t')\hat{B}(t')\hat{B}(t)),$$

(3)

is defined as a functional of the the reduced density operator $\hat{\rho}_S(T)$ of the system for a given time $T$. Here, the atomic field operator

$$\hat{B}(t) = \exp(i\hat{H}_0t)[c_1\hat{b}_g + c_2\hat{b}_e]\exp(-i\hat{H}_0t)$$

(4)

describes a specific quantum measurement with respect to the superposition state $|+\rangle = c_1|e\rangle - c_2|g\rangle$ where $c_1$ and $c_2$ satisfy the normalization relation $|c_1|^2 + |c_2|^2 = 1$. Without loss of generality, we take $c_1 = c_2 = 1/\sqrt{2}$ standing for a measurement in terms of $\hat{B}(t) = 1/\sqrt{2}[\hat{b}_g + \hat{b}_e]\exp(-i\omega_c t)$ as follows.

Before entering the main issue, let us consider what kind of atomic states can demonstrate the second order quantum decoherence of the identical atom system directly. To this end we temporarily analyze the case free of interaction, i.e., we assume $d(\omega_j) = 0$ for the moment. We denote the Fock states for the two-level atom by $|n_g, n_e\rangle$. Obviously, the two particle states $|0_g, 2_e\rangle, |1_g, 1_e\rangle$ and $|2_g, 0_e\rangle$ span a 3-dimensional invariant subspace. With respect to the states $|0_g, 2_e\rangle$ and $|2_g, 0_e\rangle$, in each of which two particles occupy the same position, the two second order correlation functions take the same constant $1/2$ and thus do not show quantum interference. However, a direct conclusion illustrates that only in the state $|1_g, 1_e\rangle$, is the second order correlation function $G(t,t') = \cos^2\left[\frac{1}{2}\omega_c(t-t')\right]$ just an interference term in time domain. The crucial point to be emphasized is that for the general case with non zero coefficients the result of this conclusion is invariant when any other measurement is taken on the system under the condition that $c_1c_2 \neq 0$. So, in the following discussions, the second order decoherence is studied for the initial state each component of which possesses one particle. To purify the central idea, we will distinguish the two cases with one mode reservoir and many mode reservoir respectively.

In the one mode case, the whole system is initially prepared in the initial state

$$|\psi(0)\rangle = |1_g, 1_e, N\rangle,$$

(5)

It is easy to see that there is an invariant subspace $V^N$ with the basis $\{ |0_g, 2_e, (N-1)\rangle, |1_g, 1_e, N\rangle, |2_g, 0_e, (N+1)\rangle \}$. Since these vectors are determined completely by the particle number in the reservoir mode and thus a simple notation can be introduced without confusion: $|N-1\rangle \equiv |0_g, 2_e, N-1\rangle$, $|N\rangle \equiv |1_g, 1_e, N\rangle$, $|N+1\rangle \equiv |2_g, 0_e, N+1\rangle$. Because the invariant subspace $V^N$ is also closed for the evolution operator $\hat{U}(T) = \exp(-i\hat{H}T)$, the time evolution of the whole system can be determined as a sub-evolution $|\psi(T)\rangle = \hat{U}(T)|N\rangle$ in $V^N$. Correspondingly, the reduced density operator of the system can be calculated as

$$\hat{\rho}_S(T) = |1_g, 1_e\rangle\langle 1_g, 1_e|\langle N|\hat{U}(T)|N\rangle^2 + |0_g, 2_e\rangle\langle 0_g, 2_e|\langle N-1|\hat{U}(T)|N-1\rangle^2 + |2_g, 0_e\rangle\langle 2_g, 0_e|\langle N+1|\hat{U}(T)|N+1\rangle^2$$

(6)

By making use of the normalization condition

$$\sum_{M=N-1}^{N+1} |\langle M|\hat{U}(T)|N\rangle|^2 = 1$$

(7)

the second order correlation function can be calculated in a compact form

$$G[t,t',\hat{\rho}_S(T)] = \frac{1}{2}[1 + |\langle N|\hat{U}(T)|N\rangle|^2\cos(\omega_c(t-t'))].$$

(8)

Obviously, the decoherence effect is completely characterized by the decoherence factor $|\langle N|\hat{U}(T)|N\rangle|^2$. When this factor is equal to 1, the interference fringe is clearest; As it becomes zero, the interference fringe disappear completely.

Now, we adopt the resolvent method to calculate the decoherence factor $\langle N|\hat{U}(T)|N\rangle$. Its Fourier transformation is written as

$$\langle N|\hat{U}(T)|N\rangle = \frac{i}{2\pi} \int_{-\infty}^{\infty} dz e^{-izT} \langle N|\frac{1}{z - H + i0^+}|N\rangle.$$

(9)

A straightforward non-perturbative calculation gives the diagonal element

$$\langle N|\frac{1}{z - H}|N\rangle = \frac{1}{z - E_N - \frac{2N\omega^2}{z-E_N+\Delta} - \frac{2(N+1)d^2}{z-E_N-\Delta}}$$

(10)

of the resolvent $G(z) = \frac{1}{z-H}$ for $E_N \equiv \omega_e + N\omega_j$, $\Delta \equiv \omega_j - \omega_e$ and $d = d(\omega_j)$.

In the resonant case, i.e., $\Delta = 0$, there exist two first order poles of $G(z)$ at $z = E_N \pm d\sqrt{4N+1}$. Using the Cauchy’s integral theorem, the decoherence factor can be explicitly obtained as a oscillating function

$$|\langle N|\hat{U}(T)|N\rangle|^2 = \cos^2[(4N+1)d^2T],$$

(11)

of the passage time $T$ with the frequency $2(4N+1)d^2$. This perfect oscillation is exactly the manifestation of the complete collapse- revival of the second order coherence, which is very similar to that in the first order quantum decoherence.

For the off-resonant case $\Delta \neq 0$, the analytical result for $|\langle N|\hat{U}(T)|N\rangle|^2$ can also be obtained by the same method, but it will be verbose to demonstrate the physical meaning explicitly. Therefore, we numerically calculate the decoherence factor in FIG. 1 for $N = 0$, $N = 1$, $N = 2$, $N = 7$ respectively.
From FIG. 1, we observe that when one mode reservoir is just in the vacuum state, the decoherence factor makes a sinusoidal oscillation. As the photon number $N$ in the cavity becomes larger and larger, the decoherence factor oscillates faster and faster regularly. From the point of view of the second order quantum decoherence, this numerical result without decaying of coherence re-proves that the Fock state of photon is basically quantum even for very large photon number $N$ forming a macroscopic quantum state. Unlike the quasi-classical state or a factorized macroscopic state, e.g., the coherent state, it can not decohere its coupling quantum system into a classical one.

To examine whether the macroscopic feature of the reservoir or the cavity causes the second order decoherence or not, we consider the same many mode cavity -atom system in an initial state $|\psi(0)\rangle = |1_g, 1_e, \{0_j\}\rangle \otimes |\{0_j\}\rangle$ where $|\{0_j\}\rangle$ is the vacuum state of the cavity. We denote the general Fock states of the many mode field by $|\{n_j\}\rangle \equiv |n_1, n_2, \ldots\rangle$.

Similar to the case with single mode cavity, there is still an invariant subspace spanned by $\{|1_g, 1_e, \{0_j\}\}, |2_g, 0_e, 1_j, \{0_m\} \rangle_{m \neq j}\}$ conserving the total atomic number. These basis vectors are also determined completely by the particle number in one reservoir mode and its index, and similar notation will be adopted for them: $|0\rangle \equiv |1_g, 1_e, \{0_j\}\rangle, |1_j\rangle \equiv |2_g, 0_e, 1_j, \{0_m\} \rangle_{m \neq j}$.

As in the case of one mode reservoir, we can verify that the second order correlation function is still in the same form except that $|\langle N|\hat{U}(T)|N\rangle|^2$ is replaced by $|\langle 0|\hat{U}(T)|0\rangle|^2$. Obviously, the decoherence effect is similarly determined by the second order decoherence factor $|\langle 0|\hat{U}(T)|0\rangle|^2$ completely.

In this case, the resolvent has the expectation value

$$\langle 0|G(z)|0\rangle = \frac{1}{z - \omega_e - \sum_j \frac{2d^2(\omega_j)}{z - \omega_j}},$$

In the limit of continuous mode, the Wigner-Weiskoff approximation

$$\sum_j \frac{2d^2(\omega_j)}{z - \omega_j + i0^+} = \Delta_e - i\frac{\Gamma_e}{2},$$

deduces the exponential decaying decoherent factor

$$|\langle 0|\hat{U}(T)|0\rangle|^2 = \exp(-\Gamma_e T).$$

for the decaying rate $\Gamma_e = 4\pi \rho(\omega_e) d^2(\omega_e)$, the renormalization of frequency

$$\Delta_e = 2P \int d\omega_j \rho(\omega_j) \frac{d^2(\omega_j)}{\omega_e - \omega_j},$$

and $\rho(\omega_j)$ is the mode density of the reservoir. Obviously, it represents an irreversible decay of the second order coherence. Compared with the usual result of the spontaneous radiation decay rate for one excited particle, this decay rate $\Gamma_e$ is two times of that which manifests the bosonic stimulation effect, that is, the existence of one boson in the ground state increases the transition amplitude of the other boson from the excited state to the ground one.

In fact, the continuous mode limit is an extreme situation with infinite modes of reservoir. Like the studies for the first order quantum coherence in our previous work [14,15], it is very interesting to consider how the quantum system graduates into the second order decoherence as the mode number of the coupled external system becomes larger step by step. This problem possibly reveals the physical process extrapolating quantum to classical in the case that there does not exist the first order coherence, but there exists the second order coherence. In our model we study the case that the number of the modes of the reservoir is finite. The numerical results of the decoherence factors $|\langle 0|\hat{U}(T)|0\rangle|^2$ are given in FIG. 2 for different numbers of the modes of the reservoir. With the number of the modes increasing, the quantum revival-collapse phenomena in the decoherence factor is clearly illustrated in FIG. 2. It should be emphasized that the decoherence rate becomes shorter and the quantum revival time becomes longer when the mode density increases. This implies the irreversibility of the decoherence process when the modes become continuous.
Before concluding this letter, we sketchily discuss the possibility to observe this second order decoherence process in a cavity QED experiment. The experiment is arranged as follows in three steps. The first step is to prepare the bosonic atoms in the initial state $|1_g, 1_e\rangle$ while the cavity is in an appropriated state. Then, we control the two atoms passing through the cavity within a time period $T$, which is determined by the velocity of the atoms. In the third step, to show the second order coherence, a coincidence measurement for the two atoms in two different locations is made by two detectors at $t$ and $t'$. The sketched experiment setup is illustrated in FIG. 3. One of the possible difficulties in experiments is the preparation of the initial quantum state of the two atoms and the cavity field. With better controlling of the atomic motion and the state of the cavity field in experiments, we believe, this is not a big problem in the near future.

In sum, we have proposed the dynamic description of the loss of the second order coherence by studying the simplest case: the considered system is composed of two bosons in two modes and the reservoir composed of one or many bosons, and we have suggested a possible cavity QED experiment to detect this process of the second order quantum decoherence.

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\[ \text{FIG. 2. The horizontal axe denotes time period } T, \text{ the vertical axe denotes the decoherent factor } |\langle 0| \hat{U}(T)|0 \rangle|^2, \text{ parameter } d(\omega_j) = 0.17, \text{ the number of the modes of the reservoir is different: (a)} N_{mod} = 3, \text{ (b)} N_{mod} = 5, \text{ (c)} N_{mod} = 7, \text{ (d)} N_{mod} = 9. \]

\[ \text{FIG. 3. Two bosonic atoms pass through a cavity in a time period } T \text{ and are detected coincidentally by two detectors in different time } t \text{ and } t'. \]