T_{c} and Pauli limited critical field of Sr$_2$RuO$_4$: uniaxial strain dependence

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Variations of critical temperature $T_{c}$ and in-plane critical field $H_{c2}$ of Sr$_2$RuO$_4$ under uniaxial stress have recently been reported. We compare the strain dependence of $T_{c}$ and $H_{c2}$ in various pairing channels ($d$-wave, extended $s$-wave and $p$-wave) with the experimental observations, by studying a three-band tight-binding model that includes effects of spin-orbit and Zeeman couplings and a separable pairing interaction. Our study helps narrow down the possibility of pairing channels. The importance of the multi-band nature of Sr$_2$RuO$_4$ is also highlighted.

I. INTRODUCTION

Sr$_2$RuO$_4$ has long been one of the best characterized materials in which unconventional superconductivity condenses out of a Fermi liquid. Thus, it presents an almost unique opportunity, where well-controlled theoretical approaches can play a key role in deducing superconducting properties, starting from the underlying electronic structure. Nevertheless, several basic phenomenological aspects, including the symmetry of the superconducting order parameter itself, remain unresolved. The results of early NMR spectroscopy measurements and spin-polarized neutron scattering studies, together with evidence for time-reversal symmetry breaking (TRSB) were all taken as consistent with a chiral $p_x + ip_y$ state. However, the chiral $p + ip$ state was recently excluded as a possibility due to newly reported measurements of the $^{17}$O Knight shift, which revealed a reduced spin susceptibility in the superconducting state. Moreover, the observation has been confirmed in independent NMR studies as well as in a spin-polarized neutron scattering study.

The measurements reported in Ref. 7 are among several new experimental studies, in which the application of uniaxial ($[100]$) stress has placed further constraints on the nature of the Sr$_2$RuO$_4$ order parameter. The induced strain in these experiments acts as a tetragonal symmetry breaking perturbation. Thus, it is a sensitive probe of multi-component order parameters that in turn are required for spontaneous TRSB in the superconducting state, and can be exploited to reveal more details of its nature. With these recent developments in mind, we are led to reconsider the phenomenological consequences and to see how distinct order parameters behave in the presence of strain.

The particular focus here is on recent experiments of critical temperature $T_{c}$ and in-plane critical field $H_{c2}$ in strained crystals. We compute the strain response of $T_{c}$ and $H_{c2}$ in different pairing channels and compare them with the observations. At the so-called Van Hove strain ($\varepsilon_{aa} = \varepsilon_{v}$), one of the Fermi sheets, customarily labeled $\gamma$ in the literature, crosses the Van Hove singularity (vHs) at the boundary of the first Brillouin zone. This Fermi sheet consists of quasiparticles built predominantly from electrons in the $d_{xy}$ orbital, with weak mixing of $d_{xz}$, $d_{yz}$ orbitals in the presence of atomic spin-orbit coupling. Since the $\gamma$ sheet has little dispersion in the $c$-direction, the density of states is expected to diverge logarithmically in the neighborhood of the vHs. Therefore, tuning $E_F$ to the vHs results in an expected enhancement of both the transition temperature and the upper critical field. Further, the enhancement of in-plane ($H/|b|$) critical field was observed to be stronger than that in the critical temperature. Here, we compare and contrast the observed behavior to expectations for selected order parameter symmetries.

More specifically, in this work, we analyze the ratio $H_{c2}/T_{c}$ as a function of uniaxial $\varepsilon_{xx} - \varepsilon_{yy}$ strain, by studying BCS theory on a 3-band tight binding model for different pairing channels, including $d$-wave, $p$-wave and extended-$s$-wave pairing channels, and compare the results with experimental observations. Our study points out a new direction for narrowing down the possible choices of order parameters for Sr$_2$RuO$_4$, and the methods are readily applied to other systems. Besides comparing results obtained for different pairing channels, the importance of the multi-band nature of this material is highlighted by comparing results with/without atomic spin-orbit coupling (SOC) and orbital Zeeman effects. Guided by the observation of a field-induced first order transition from the (low-field) superconducting state, we consider the possibility of an inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state. Our study of the strain-dependent $H_{c2}/T_{c}$ in Sr$_2$RuO$_4$ may provide a new way to search for the FFLO state elsewhere.

As we explain in more detail below, our key finding is that (1) the strain-dependence of the ratio $H_{c2}/T_{c}$ in $d$-extended-$s$-wave pairing channel is consistent with the experimental observations, while $p$-wave pairing channel (analogs of B-phase of $^3$He) is not. (2) The multi-band nature of Sr$_2$RuO$_4$, including spin-orbit coupling and the
Sec. IV, we extend our study to the FFLO state. In Sec. III, we present the numerical results for different pairing channels with/without multi-band effects. In Sec IV we extend our study to the FFLO state.

II. THE MODEL

We consider an effective three-band tight-binding-Hamiltonian for $t_{2g}$ ($d_{yz}, d_{xz}, d_{xy}$) electrons of Sr$_2$RuO$_4$ with tetragonal symmetry under Zeeman effect. The Hamiltonian is given by $H_0 + H_Z + H_{BCS}$, where:

$$H_0 = \sum_{\vec{k},a,b,\sigma} h_0(\vec{k}) c_\sigma^{\dagger} \sum_{a} c_{\sigma a} + H_{SOC}$$

and:

$$h_0(\vec{k}) = \begin{pmatrix} \epsilon^{yz} & \epsilon^{off} & \epsilon^{xy} \\ \epsilon^{off} & \epsilon^{xz} & 0 \\ \epsilon^{xy} & 0 & \epsilon^{zz} \end{pmatrix}$$

$$\epsilon^{yz} = 2t_2 \mu \cos k_x - 2t_1 \mu \cos k_y - \mu$$

Here, $c_{\sigma a}^{\dagger}$ ($c_{\sigma a}$) are creation(annihilation) operators for electrons in $a = d_{yz}$, $d_{xz}$, or $d_{xy}$ orbitals for spin state $\sigma = \uparrow, \downarrow$, and $h_0$ is a $3 \times 3$ Hamiltonian in orbital space. The parameters are obtained in Ref. [10] by fitting the above TBH with experimental data, and the resulted fitting parameters are listed here: $(t_1, t_2, t_3, t_4, t_5, t_6, \mu) = (0.145, 0.016, 0.081, 0.039, 0.005, 0.122) eV$. Note that the off-diagonal term $\epsilon^{off}$ that couples $d_{yz}$ and $d_{xz}$ orbitals is zero from fitting. Here, the three $t_{2g}$ orbitals ($d_{yz}$, $d_{xz}$, $d_{xy}$) transform as a vector under point-group symmetry operations. Hence, the angular momentum operator in this internal coordinate representation is $\hat{L}_{bc} = -\epsilon_{abc}$, where $\epsilon_{abc}$ is the totally anti-symmetric tensor, while spin operators are the standard Pauli matrices. Thus, the spin-orbit coupling is

$$H_{SOC} = \vec{L} \cdot \vec{S} = \lambda \begin{pmatrix} 0 & i\sigma^z & -i\sigma^y \\ -i\sigma^z & 0 & i\sigma^x \\ i\sigma^y & -i\sigma^x & 0 \end{pmatrix}$$

The strength of spin-orbit coupling is taken to be $\lambda = 0.032 eV$ [10].

We introduce the “hopping ratio” $\tau$ to incorporate the effect of uniaxial strain, which modifies the hopping strength along x and y direction (nearest neighbor hopping strength $t_1$ and $t_2$) in Eq. [2]. Under the above settings, zero uniaxial strain corresponds to hopping ratio $\tau = 1$, while Van Hove strain is around $\tau = 1.055$. Fermi surfaces at zero strain and Van Hove strain are plotted in Fig. 1.

The Zeeman field couples to both the spin and orbital, and the resulting Zeeman term is

$$H_Z = -\vec{H} \cdot \vec{\sigma} \otimes \tau_0 + \sigma^0 \otimes \begin{pmatrix} 0 & iH_z & -iH_y \\ -iH_z & 0 & iH_x \\ iH_y & -iH_x & 0 \end{pmatrix}$$

Here, we have assumed that the system is strongly type-II, so that $\vec{H}$ is the external magnetic field. $\tau_0$ is the identity matrix in orbital space, while $\sigma^0$ is the identity matrix in spin space.

In this work, we will consider an approximation, that the order parameters are purely on the $d_{xy}$ orbital of Sr$_2$RuO$_4$, which are closest to the $\gamma$ band and most sensitive to the Van Hove strain. It should be noted that order parameters on other bands also contribute to the total gap function. However, they are much less strain-sensitive, so variations of $T_c$, $H_{c2}$ due to pairings on the other bands would otherwise be smooth and analytic. Experimentally, which band contributes mostly to superconductivity is still being investigated [2]. The general form of the BCS interaction on the $d_{xy}$ orbital can be written as

$$H_{BCS} = \sum_{\vec{k}, \vec{k}', \{\sigma_i\}} V_{\sigma_1 \sigma_2 \sigma_3 \sigma_4} (\vec{k}, \vec{k}') C_{-\vec{k} \sigma_1} C_{\vec{k} \sigma_2} C^{\dagger}_{\vec{k}' \sigma_3} C^{\dagger}_{-\vec{k}' \sigma_4}$$

Here, $\sigma_i$ denotes spin up/down. In the following calculations, we assume for simplicity that the above BCS interaction is separable, i.e. it is of the following form,

$$V_{\sigma_1 \sigma_2 \sigma_3 \sigma_4} (\vec{k}, \vec{k}') = g \delta (\vec{k} - \vec{k'}) \delta (\vec{k}' - \vec{k})$$

Then the BCS gap equation can be simplified to

$$\Delta = \sum_{\{\sigma_i\} \in E_D, \sigma_1 \sigma_2} g \delta (\vec{k} - \vec{k}_1) \delta (\vec{k}_1' - \vec{k})$$

FIG. 1: Fermi surfaces of the three band tight binding Hamiltonian in Eq. [2]. (Left) Zero strain $\tau = 1$, where system has tetragonal symmetry. (Right) Van Hove strain around $\tau = 1.055$, where the $\gamma$ band touches the boundary of the first Brillouin zone.
Here \(\langle \ldots \rangle\) denotes thermal averaging, and \(\varepsilon_k\) is the normal state eigenenergy. \(E_D\) is an energy cutoff, analogous to the Debye temperature in conventional BCS theory. Given a pairing channel \(f(\vec{k})\) and pairing strength \(g\), we can numerically diagonalize the Bogoliubov de Gennes (BdG) Hamiltonian and solve for the gap magnitude \(\Delta\) self-consistently, at arbitrary temperature \(T\), magnetic field \(H\) and hopping ratio \(\tau\). The critical temperature can then be determined by the standard procedure. When calculating the response to an in-plane critical field, we neglect the \(c\)-axis warping of the Fermi surface, and consider a 2-dimensional Fermi surface. We thus neglect the orbital effect of the in-plane magnetic field, and the resulting \(H_{c2}\) is the Pauli-limited critical field.

In order to better compare the results from different pairing channels, we would like to fix the gap magnitude at zero temperature, zero magnetic field and zero uniaxial strain to be the same in all channels, i.e.

\[
\Delta(T=0, B=0, \tau=1) = \Delta_0 \equiv 2.8 \times 10^{-4} \text{eV}. \quad (8)
\]

The magnitude of \(\Delta_0\) is chosen, such that \(T_C\) is \(O(1)\) Kelvin, which is on the same order as the experimental values. The above fixing is achieved by tuning the BCS interaction strength \(g\) in each channel. Those interaction strengths will then be fixed throughout the calculation. That is, we have assumed that the BCS interaction strength \(g\) is strain-independent. A strain-dependent interaction strength will change the magnitude of \(T_C\) and \(H_{c2}\). However, the interaction strength will not affect the ratio \(T_C/H_{c2}\). This is well-known in the standard BCS theory without disorder, where \(T_C\) and Pauli-limited \(H_{c2}\) are both proportional to the gap magnitude at zero temperature and zero field, while the proportionality constant only depends on the band structure and type of pairing channel, rather than on the interaction strength. The energy cutoff of the interaction (analogous to Debye temperature for BCS theory) is taken to be \(E_D = 10\Delta_0\).

#### III. RESULTS

In the first subsection below, we present numerical results for \(d\)-wave, \(p\)-wave and \(d+s\)-wave pairing channels, in the absence of spin-orbit coupling and the orbital Zeeman effect. In the next subsection, we will perform the same calculations, but with SOC and orbital Zeeman effects.

The direct comparison between various order parameters helps narrow down the possible pairing channels in \(\text{Sr}_2\text{RuO}_4\). We will also highlight the importance of the multi-band nature of \(\text{Sr}_2\text{RuO}_4\), as we compare the results between these two subsections. It is worth noting that, in the absence of SOC and orbital Zeeman effect, the problem becomes effectively single-orbital.

##### A. single-band

In this subsection, we remove the spin-orbit coupling and orbital Zeeman effect in the Hamiltonian. Now the \(d_{yx}\) and \(d_{xz}\) orbitals will not affect the calculations, and effectively we end up with the problem on the \(d_{xy}\) orbital. It should be noted that the Van Hove strain is shifted to a larger hopping ratio, \(\tau = 1.08\).

1. \(d\)-wave pairing channel

Let us start with the \(d\)-wave pairing channel

\[
f(\vec{k})_{\sigma_1, \sigma_2} = i\sigma_2 \gamma \cos k_x - \cos k_y. \quad (9)
\]

\(T_C\) and Pauli-limited critical field \(H_{c2}\) as a function of uniaxial strain (hopping ratio \(\tau\)) are shown in the left panel of Fig. 2. Both quantities have been normalized to unity at zero strain (\(\tau = 1\)).

At the Van Hove strain \(\tau = 1.077\), \(T_C\) (blue solid line) is clearly enhanced more significantly than \(H_{c2}\) (red dotted line). Therefore, the ratio \(H_{c2}/T_C\) decreases as approaching the Van Hove strain, which is inconsistent with the experimental observations.

![FIG. 2: Critical temperature \(T_C\) and Pauli-limited critical field \(H_{c2}\) as a function of \(d\)-wave and \(p\)-wave pairing on single-orbital model, as a function of uniaxial strain (hopping ratio \(\tau\)). Both quantities have been normalized to unity at zero strain (\(\tau = 1\)).](image)

2. \(p\)-wave pairing channel

For spin-triplet superconductors without spin-orbit coupling, Pauli-limited critical field cannot be obtained, if the \(d\)-vector of the pairing state is perpendicular to the magnetic field. With the presence of SOC (in next subsection), the above scenario no longer holds, but the
resulting $H_{c2}$ could be much bigger than the maximal gap magnitude if SOC is small. Experimentally, $H_{c2}$ is found to be of the same order of the maximal gap magnitude, $\Delta / g\mu_B$. Noting also that the strain lifts the $p_x$, $p_y$ degeneracy, states with $d$-vector parallel to the magnetic field are, in principle, possible. That is, the $p$-wave pairing channel of the form:

$$f(\vec{k})_{\sigma_1, \sigma_2} = i(\sigma^x \sigma^y)_{\sigma_1, \sigma_2} \sin k_x$$  \hspace{1cm} (10)

, arising on the $d_{xy}$ orbital, is considered. The $\hat{d}$-vector is along $x$-axis, and we calculate the corresponding $H_{c2}$.

Critical temperature $T_c$ and Pauli-limited critical field $H_{c2}$ as a function of uniaxial strain (hopping ratio $\tau$) are shown in the right panel of Fig. 2. Both quantities are normalized to unity at zero strain ($\tau = 1$). $T_c$ (blue solid line) is clearly enhanced more significantly than $H_{c2}$ (red dotted line) at the Van Hove strain $\tau = 1.077$. Therefore, the ratio $H_{c2}/T_c$ decreases as approaching the Van Hove strain, which is inconsistent with the experimental observations.

Under the assumption of strain-independent BCS interaction strength, the critical temperature and critical field in the $p$-wave pairing channel are not sensitive towards the Van Hove strain, and we do not observe any peak in Fig. 2 at the Van Hove strain. This is because the $p$-wave gap function $f(\vec{k})$ vanishes at the Van Hove singularity $(k_x, k_y) = (0, \pi)$.

3. $d+$extended-$s$-wave pairing channel

We now consider mixture between $d$-wave $f(\vec{k})_{\sigma_1, \sigma_2} = i\sigma^y_{\sigma_1, \sigma_2} \cos k_x - \cos k_y$ and extended-$s$-wave $f(\vec{k})_{\sigma_1, \sigma_2} = i\sigma^y_{\sigma_1, \sigma_2} \cos k_x + \cos k_y$ pairing channel. When applying uniaxial strain, the tetragonal symmetry is broken, and these two pairing channels belong to the same irreducible representation, and hence are allowed to mix. In the following calculations, we again only consider pairing on the $d_{xy}$ orbital. We now introduce two BCS interaction strengths $g_d$ and $g_s$ for the two channels, and assume they are strain-independent. We choose the strengths $g_d$ and $g_s$, such that the $d$-wave gap magnitude satisfies Eq. and the gap magnitude of extended-$s$-wave pairing channel vanishes at zero strain.

The gap magnitude as a function of uniaxial strain is plotted in the left panel of Fig. 2. Thus, in this calculation, $d$-wave pairing dominates over the extended-$s$-wave pairing. We solved the two gap equations, and obtained critical temperature and critical field.

At the Van Hove strain $\tau = 1.077$, $T_c$ (blue solid line) is clearly enhanced more significantly than $H_{c2}$ (red dotted line). Therefore, the ratio $H_{c2}/T_c$ decreases on approaching the Van Hove strain, which is inconsistent with the experimental observations.

In this subsection, we effectively removed $d_{yz}$ and $d_{xz}$ orbitals from the Hamiltonian, and obtained $H_{c2}$ and $T_c$ for a single orbital ($d_{xy}$ orbit) system. We have tried $d$-wave, $p$-wave and $d+$extended-$s$-wave pairing channels, but none trend similarly to the experimentally observed ratio $H_{c2}/T_c$.

B. three-band

1. $d$-wave pairing channel

We start with $d$-wave only pairing channel $f(\vec{k})_{\sigma_1, \sigma_2} = i\sigma^y_{\sigma_1, \sigma_2} \cos k_x - \cos k_y$ for the 3-band system. The critical temperature $T_c$ and Pauli-limited critical field $H_{c2}$ as a function of uniaxial strain (hopping ratio $\tau$) are shown in the left panel in Fig. 2. Both quantities have been normalized to unity at zero strain (at $\tau = 1$). $H_{c2}$ (red dotted line) is clearly enhanced more significantly than $T_c$ (blue solid line). Therefore, the ratio $H_{c2}/T_c$ increases as approaching the Van Hove strain, which is consistent with the experimental observations.

2. $p$-wave pairing channel

For reasons mentioned in Sec. III A 2, in order to calculate the Pauli-limited critical field, the $p$-wave pairing state with $f(\vec{k})_{\sigma_1, \sigma_2} = i(\sigma^x \sigma^y)_{\sigma_1, \sigma_2} \sin k_x$ is considered. The $d$-vector is along $x$ direction, and we calculate critical field also in this direction.

$T_c$ and Pauli-limited critical field $H_{c2}$ as a function of uniaxial strain (hopping ratio $\tau$) are shown in the right panel of Fig. 2. Both quantities have been normalized to unity at zero strain (at $\tau = 1$). The enhancement in $H_{c2}$ (red dotted line) and $T_c$ (blue solid line) are almost the same. Therefore, the ratio $H_{c2}/T_c$ does not change on approach to the Van Hove strain, which is inconsistent with the experimental observations.

For the same reasons as in Sec. III A 2, we did not observe any peak in $T_c$ or $H_{c2}$ near the Van Hove strain,
since the gap function for the p-wave pairing state vanishes at the Van Hove singularity. Again, one could get the correct shape of peak by introducing strain-dependent BCS interaction strengths, but this will not affect the ratio $H_{c2}/T_c$.

3. d+extended-s-wave pairing channel

We now turn to mixture between d-wave $f(\mathbf{k})_{\sigma_1 \sigma_2} = i \sigma_{\sigma_1 \sigma_2} (\cos k_x - \cos k_y)$ and extended-s-wave $f(\mathbf{k})_{\sigma_1 \sigma_2} = i \sigma_{\sigma_1 \sigma_2} (\cos k_x + \cos k_y)$ pairing channel. Similar to the single-orbital case in Sec. IIIA3, we choose the strength $g_d$ and $g_s$, such that the d-wave gap magnitude satisfies Eq[8] and the gap magnitude of extended-s-wave pairing channel vanishes at zero strain. Thus, in this calculation, d-wave pairing dominates over the extended-s-wave pairing. The gap magnitude as a function of uniaxial strain is shown in the left panel of Fig[5]. We solved the two gap equations, and obtained the critical temperature and critical field.

$T_c$ and $H_{c2}$ as a function of uniaxial strain are summarized in the right panel of Fig[5]. Enhancement of $H_{c2}$ is notably stronger than of $T_c$. Further, under the strain-independent BCS interaction, enhancement in $T_c$ and $H_c$ at the Van Hove strain agrees quantitatively with experimental observations, with an maximal enhancement around 2.5 to 3 times. The peak position matches with the Van Hove strain, at around $\tau = 1.055$.

It is worth noting that $g_s/g_d$ is a free parameter in the calculation. In the calculation of d+extended-s-wave pairing channel, we have chosen $g_d/g_s = 1$, and the calculation of d-wave only pairing channel in Sec. IIIA1 can be thought as special case with $g_s/g_d = 0$. Choices of $g_s/g_d$ do not qualitatively change the results; in the calculation of stronger extended-s-wave pairing channel with $g_s/g_d = 6.7$, where $g_s$ is taken such that the extended-s-wave gap magnitude satisfies Eq[8] $H_{c2}/T_c$ ratio also increases from zero strain to Van Hove strain by about 30%.

In this subsection, we have illustrated our numerical results on the ratio $H_{c2}/T_c$ as a function of uniaxial strain, for different pairing channels. By comparing the results with experimental observations, we found that d+extended-s-wave pairing channel can provide the correct strain-dependence in the ratio $H_{c2}/T_c$ while p-wave pairing channel cannot.

Comparing the results in these two subsections, we summarize that the multi-band nature of Sr$_2$RuO$_4$ and d-wave type pairing channels are the key to explain the strain dependence of the ratio $H_{c2}/T_c$.

IV. SINGLE-BAND FFLO STATE

An inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, with non-zero momentum Cooper pairs, may appear as an intermediate phase in strong applied fields under conditions that the Zeeman Effect dominates over orbital suppression of the superconducting state. Such conditions otherwise apply in very anisotropic organic superconductors for in-plane field. And since the applicability of an otherwise isotropic Zeeman effect implies singlet-pairing, the evidence for such an intermediate state has been searched for in the case of parallel fields in Sr$_2$RuO$_4$. In only one case that we know of is there a suggestion for a field-induced intermediate phase. Nevertheless, we would like to study the possibility of the d-wave FFLO state on the $d_{xy}$ orbital considered here.

Following[18], we extend the BCS interaction to

$$H_{BCS} = - \sum_{\mathbf{k}, \sigma} V_{\sigma_1 \sigma_2 \sigma_3 \sigma_4} (\mathbf{k}, \mathbf{k}') c_{\mathbf{k} \sigma_1} c_{\mathbf{k}+\mathbf{q} \sigma_2} c_{\mathbf{k'}+\mathbf{q} \sigma_3} c_{-\mathbf{k'} \sigma_4}$$

(11)
In this section, we choose interaction strength \( \vec{q} \) possible for the \( d \) wave pairing state. We removed the \( d_y \) orbital as a function of uniaxial strain points out a new direction to search for FFLO states in other materials.

We also studied the ratio \( H_{c2}/T_c \) for the FFLO state. Due to Fermi surface nesting, \( d \) wave FFLO state on \( d_{xy} \) orbital is not sensitive to the Van Hove strain. However, our study points out a new way to test FFLO state for a broader system.

Lastly, we discuss our results within the context of the broader phenomenological paradoxes presented by \( \text{Sr}_2\text{RuO}_4 \). Prior to the NMR spectroscopy results in Ref. \(^7\), the key phenomenological issues involved rationalizing various experimental observations within the hypothesis of a chiral \( p_x + ip_y \) superconducting ground state. The experimental results in Ref. \(^7\) have ruled out this scenario. Instead, the focus has shifted towards reconciling the NMR measurements with observations of TRSB in Kerr and muon spectroscopy studies.

On the one hand, TRSB requires having two distinct and degenerate order parameters. This can be ensured by symmetry, if the order parameter belongs to a multi-dimensional irreducible representation (irrep). Such states however, exhibit a split transition in the presence of the uniaxial strain considered in this paper: the absence of such split transitions casts significant doubt on the viability of such explanations. TRSB can also occur in a fine-tuned situation where two distinct irreps become degenerate (see for instance the recent proposal in Ref. \(^21\)). Such degeneracy, if present, would be sensitive to perturbations, and may well be lifted by strain. Indeed, a recent mu-SR experiment in the presence of uniaxial strain shows the absence of TRSB at the superconducting transition from the normal state\(^23\). It is thus reasonable to start with a simpler setting of a single pairing channel when study the strain effects, which is precisely what we have done here.

V. CONCLUSION AND DISCUSSION

We studied the ratio \( H_{c2}/T_c \) as a function of uniaxial strain for \( \text{Sr}_2\text{RuO}_4 \), and tried to match the experimentally observed increased ratio near the Van Hove strain. We considered a three-band tight-binding Hamiltonian with separable and strain-independent BCS interaction on the \( d_{xy} \) orbital. We tried different pairing channels and found that the experimental observation can be explained with \( d+ \) extended s-wave, rather than \( p \)-wave pairing state. We removed \( d_{yz} \) and \( d_{zx} \) orbitals, and then found that none of pairing channels could match the experimental results. Therefore, we concluded that the multi-band nature of \( \text{Sr}_2\text{RuO}_4 \) and \( d \) -wave type pairing channels are the key to explain the strain dependence of the ratio \( H_{c2}/T_c \).

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