EMBEDDING NOISE PREDICTION INTO LIST-VITERBI DECODING USING ERROR DETECTION CODES FOR MAGNETIC TAPE SYSTEMS

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ABSTRACT
A List–Viterbi detector produces a rank ordered list of the N globally best candidates in a trellis search. A List–Viterbi detector structure is proposed that incorporates the noise prediction with periodic state-metric updates based on outer error detection codes (EDCs). More specifically, a periodic decision making process is utilized for a non-overlapping sliding windows of P bits based on the use of outer EDCs. In a number of magnetic recording applications, Error Correction Coding (ECC) is adversely affected by the presence of long and dominant error events. Unlike the conventional post processing methods that are usually tailored to a specific set of dominant error events or the joint modulation code trellis architectures that are operating on larger state spaces at the expense of increased implementation complexity, the proposed detector does not use any a priori information about the error event distributions and operates at reduced state trellis. We present pre-ECC bit error rate performance as well as the post-ECC codeword failure rates of the proposed detector using perfect detection scenario as well as practical detection codes as the EDCs are not essential to the overall design. Furthermore, it is observed that proposed algorithm does not introduce new error events. Simulation results show that the proposed algorithm gives improved bit error and post-ECC codeword failure rates at the expense of some increase in complexity.

INTRODUCTION
At high recording densities, generalized partial response polynomials with real coefficients are shown to outperform Partial Response 4 (PR4) and Extended PR4 (EPR4) detectors. In particular, the idea of noise prediction is embedded into Viterbi decoding in [1] and has later become the de facto standard for many recording devices. More specifically, a Noise Predictive Maximum Likelihood (NPML) detection is proposed which uses a generalized partial response channel with a polynomial of the form

\[ G(D) = (1 - D^2)P(D) \]

where \( P(D) = 1 + p_1D + p_2D^2 + \cdots + p_LD^L \) is the transfer polynomial and \( L \) is the order of the noise whitening filter. Such a finite impulse response filter is used to approximately whiten the noise at the input of the detector. Introduction of a whitening filter increases the inter-symbol interference, the number of trellis states of the system and hence the complexity of the decoding process. However, a feedback loop can be used to reduce the complexity of the detection algorithm at the expense of slight loss in performance due to relying on the past decisions obtained from the trellis.

The performance of the NPML detection must be improved for a reliable operation of magnetic recording systems, particularly at higher user densities. There have been two major approaches in the past to improve the detector performance. One of them focused on combined detector-modulation code [2] and the noise predictive decoding architectures using joint trellises at the expense of larger number of trellis states and hence increased implementation complexity. Joint-trellis idea has been shown for some of the well known constrained codes and extended PR4 channels to improve overall performance [3]. However, an extension to a specific type of modulation code (such as
a twins constrained Maximum Transition Run (MTR) code \[4\] is not straightforward due to excessive number of states of the code that define the constraint. Although such schemes have been shown to improve performance, they are not used in tape systems mostly due to their poor performances in bursty-error scenarios. The other approach was to take the detector structure for granted and devise post-processing algorithms \[5\] to improve the performance by eliminating some of the dominant error events at the output of the detector. Such error events are determined by the recording channel that is disturbed by various types of noise sources due to mechanical components and media. They are shown to be helpful when the frequency of error event occurrences at the output of the detector is uneven and known to the post processor prior to its operation. Although, post-processing methods are shown to be low complexity, they often result in suboptimum solutions and are not robust i.e., they may attempt to correct some of the dominant error events at the cost of leading to other error events that were originally not part of the detector output. Considering the pros and cons of both approaches in this study, we have developed a List-Viterbi decoding methodology that embedded a noise prediction into the detector and shown considerable BER gains for generalized partial-response channels \[6\]. Furthermore, the proposed scheme is shown to be robust to miscorrections due to a verification stage using error detection codes.

In conventional tape recording systems, Reed Solomon (RS) codes are utilized as the error correcting code to recover residual bit errors after the detector. Since the errors at the output of the detector are correlated, different approaches are taken to estimate the post-ECC performance such as multinomial and Block Multinomial Models (BMMs) \[7\]. This study also presents the semi-analytical post-ECC performance of the proposed detector based on BMMs. We have seen that the proposed List-Viterbi decoding with noise prediction in branch metric computations might be a very important candidate for magnetic tape recording systems giving improved Bit Error Rate (BER) and post-ECC performances at the expense of some increase in complexity.

THE SYSTEM MODEL, PROPOSED DETECTOR AND THE POST-ECC PERFORMANCE

The system model

Raw user data bits are encoded using a RS code over GF(256), which can correct up to \(t\) bytes of error. RS codewords are then byte-interleaved, encoded by a precoder and a Run Length Limited (RLL) code which satisfies the run length requirements of 1s and 0s. After RLL encoding, codewords go through an EDC parity insertion stage without compromising the RLL constraints. We divide the bit stream into equal size non-overlapping windows and for every \(P\)-bit window, equal amount of EDC bits are computed and appended at the end of each window. The unconstrained positions of the RLL code are used to insert the EDC parity bits into the bit stream. In this paper, \(P\) is called the period of the proposed algorithm and chosen to be a multiple of RLL codeword length. The data is finally mapped onto the symbol sequence \(e \in \{+1, -1\}\) and written on a storage medium for readback. Read signal waveform goes through a Low Pass Filter (LPF), PR4 equalization (symbols \(e \in \{+2, 0, -2\}\), proposed detector, inverse precoder and RLL/ECC decoding to be able to recover the data.

The proposed detection algorithm

The proposed algorithm is a combination of periodic updates and a parallel noise predictive List-Viterbi detection that produces a rank ordered list of the \(N\) globally best candidates. The detector starts decoding from the first chunk of the encoded data stream. Using feedbacks from different path memories, it decodes the corresponding incoming symbol sequence and sends the possible candidate paths to the update stage. Before giving an example, let us provide first the notation we use.

Let us define \(\phi_n(j, l), 1 \leq l \leq L + 2\), to be the \(l\)-th lowest accumulated metric to reach state \(j\) \((s_j)\) at time \(n\) from some starting state at time 0. At time \(n\), we use \(\beta_n(j, l)\) to denote the state covered by the \(l\)-th best path at time \(n - 1\), which passes through state \(j\) at time \(n\). Similarly, \(r_n(j, l)\) characterizes the ranking of
The l-th best path at time n − 1, when this path passes through state j at time n. We will denote the incremental branch metric (cost) that corresponds to a state transition s_j → s_k (using the l-th best path of s_j) at time n using the notation \( \gamma_l^n(s_j, s_k) \).

An example is shown in FIGURE 1 to explain how the algorithm computes the accumulated metrics in each time step. At time n, we would like to compute the best two accumulated metrics of s_3. Based on the trellis structure, we can see that s_1 and s_2 make connections with s_3 with accumulated metrics \( \phi_{0-1}(1, 1) = 12.2, \phi_{0-1}(1,2) = 13.9 \) and \( \phi_{0-1}(2, 1) = 26.1, \phi_{0-1}(2,2) = 31.3 \), respectively. Considering the different branch metrics corresponding to the state transitions s_1 → s_3 and s_1 → s_3, it is easy to see that we will end up with four possible path metrics arriving s_3: \( \{12.5, 14.6, 26.6, 31.7\} \). We retain the smallest two in our algorithm i.e., \( \phi_n(3,1) = 12.5 \) and \( \phi_n(3,2) = 14.6 \) to be the accumulated metrics of s_3 at time n and discard the other paths. Note that both of these survival paths pass through state s_1 (i.e., \( \beta_b(3,1) = 1 \) and \( \beta_b(3,2) = 1 \)) using the first and the second best path of s_1, respectively (i.e., \( r_n(3,1) = 1 \) and \( r_n(3,2) = 2 \)).

After the computation of accumulated metrics as in the example, the detector generates the corresponding incoming symbol sequences and send the possible candidate paths to the update stage. The update stage periodically makes a decision on the correct path and updates the accumulated metrics. After making a decision on a particular path using the outcome of EDC decoding, updated accumulated metrics are forwarded to the detector for further processing i.e., they are used as the initial conditions for decoding the next window of P bits. We repeat the same set of operations for the decoding of each chunk. Since a decision is made after each update step, the algorithm continually outputs the decoded bits. For more technical details, we refer the reader to [6].

The post-ECC performance

The data recovery performance of modern tape storage systems is usually measured by the post-ECC failure rates. In our configuration, an RS code encodes a data block of size \( m_b = 1960 \) bits into \( 245 + 2t \) ECC symbols, where \( t = 5 \) is the RS code correction power. In other words, the ECC decoder can correct any combination of \( T \leq t = 5 \) symbol errors. The Codeword Failure Rate (CFR) is defined as the probability of reading a codeword not correctable by the RS decoder and related to Hard Bit Error Rate (HBER) via the expression HBER = CFR/m_b, where \( m_b \) is the codeword length in bits. Since the conventional ECC decoders might have to bring the CFR down to \( \approx 10^{-9} \sim 10^{-13} \), it is infeasible to simulate data to get to these error rates. In this study, we use the BMM for finding post-ECC CFR performance [7] particularly for low CFR values. Most disk or tape drive manufacturers report the HBER to their customers rather than the raw BER at the output of the detector. Therefore, it is reasonable to present the Post-ECC performance in terms of the CFR.

In that model, we divide the codewords into M-symbol equal size blocks. Let the weight distribution function be \( Y(D) = y_1D + y_2D^2 + \cdots + y_mD^M \), where \( y_w \) is the probability of receiving an M-symbol block, with \( w \) errors at the output of the RLL decoder. In our study, \( y_w \)'s are estimated by way of error counting through simulations. Once we estimate the probabilities \( y_j \) for a given \( M \), it is straightforward to compute the approximate CRFs using the Algorithm 1 [7].

**Algorithm 1 CFR estimation using Block Multinomial Model [7]**

**Initialization:** \( j = 1, R(D) = r_1D + r_2D^2 + \cdots + r_mD^M = Y(D) \)

**Recursion:**

for \( 1 < j \leq j_{\text{max}} \) do

\( y_{j,t + 1} = \sum_{j \geq t + 1} r_j, R(D) := R(D) * Y(D) \)

end for

**Result:** we get: \( CFR = \sum_{t \geq 1} (\frac{n}{M})_j \times y_0^{n/M-j} \times y_{j,t+1} \) where * denotes polynomial multiplication, \( y_0 = 1 - \sum_{j=1}^M y_j \) and \( j_{\text{max}} = 2t \) is called the truncation parameter.

**Numerical Results**

We assumed PR4 signalling, set the order of noise whitening filter \( L = 3 \) and considered various values of \( N \) in all the simulations. The trellis has four states and there are three bits in feedback loop. Our channel model is the first order position jitter model [8] based on a Lorentzian step response. Electronics and stationary transition noise samples are added to the signal waveform at the input of the LPF as modeled in [4], i.e., as a mixture of the electronics and transition noises. SNR is computed at the

![Figure 2](image-url)
input of the LPF and given by $2/(N_0 + N_m)$ where $N_0$ is double the two sided spectral densities of a white Gaussian noise source and $N_m$ is the spectral density of the colored/transition noise. We approximate the ratio of the transition noise to the total noise power by $\beta = N_m/(N_0 + N_m)$ [4]. We assumed perfect timing recovery, a 5th order Butterworth LPF with a 3dB cutoff frequency. PR4 equalizer is based on the minimum mean square error criterion. Whitening filter coefficients ($p_1, p_2$ and $p_3$) are selected using linear prediction in minimum least squares sense based on the channel noise samples.

FIGURE 2 shows selected results assuming Perfect Error Detection (PED) for $P = 198$ bits and various $N$. We also include the performance of the conventional NPML both for adaptive and fixed whitener coefficients ($p_1 = 0, p_2 = 0, p_3 = 0$) i.e., PR4 Viterbi Algorithm. At a BER of $10^{-4}$, using $N = 50$, a gain of 1.9dB is observed over the conventional NPML detector. In a more practical scenario with $N = 3$, an average gain of 1dB is observed at the same operating BER. Also, these performance curves show an almost 2.5dB gain at a BER of $10^{-5}$ using $N = 50$. Those ideal error correction-based performance curves may serve as benchmarks for the ultimate system performance. We have also tested our detector combined with actual EDCs such as Cyclic Redundancy Check (CRC) codes. We have seen the performance degradation is minor relative to PED case and is only clear at lower SNR and higher linear densities $D_c = PW50/T$, where PW50 is the pulse width measured at half the peak amplitude of the channel’s step response and $T$ is the bit period.

Finally, we present approximate post-ECC CFR performances in FIGURE 3. We use $D_c = 3.25$ for the List-Viterbi detector whereas, in order for a fair comparison, we assumed that $T$ increases to $T_{NPMLD} = 67 \times T/66$ for the NPML detection. This approximately corresponds to $D_c = 3.2$. We also note that SNR loss due to this density increase is minor. We choose $M = 17$ bytes for BMM and use a $(255, 245)$ RS code with $t = 5$. The overall 0.85dB pre-ECC gain at a BER of $10^{-3}$ translates to around 0.65dB post-ECC gain using PED. This gain is reduced to 0.45dB when the proposed detector is used with actual detection codes. We note that this gain will increase at higher BER operating points and is greater than the reported CFR gains in [7] for a single parity bit post processor per RLL codeword. This basically shows that although the pre-ECC system performance is slightly effected by the imperfection of actual detection codes, the post-ECC performance can severely be effected. On the other hand, with the provided flexibility, the post-ECC gains can be improved by increasing $N$ and/or having smaller $P$ as long as the proposed detector satisfies the system-specific physical constraints.

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