Reliability allocation method based on Intuitionistic Trapezoidal Fuzzy Number and Grey Correlation

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Abstract. The reliability allocation method of machine tools has the feature of high complexity and uncertainty. To reduce the fuzziness in the process of reliability allocation, this paper proposes a novel reliability allocation method of CNC machine tools based on Intuitionistic Trapezoidal Fuzzy Number and Grey Correlation. First, the subsystems of CNC machine tools are divided based on function, and the reliability of subsystems are also allocated. Then, expert decision matrices are obtained based on intuitionistic trapezoidal fuzzy numbers. The result is obtained by the calculation of the grey relation degree between decision matrices and ideal matrices. Finally, an example is taken to verify the effectiveness of the proposed method. Compared to the traditional method, the proposed method can reduce the calculation and more convenient to use.

1. Introduction

The CNC machine tool is very important equipment in the manufacturing industry. The machine tool equipment with good reliability guarantees the production efficiency of the enterprise. Therefore, in addition to improving the reliability of machine tools, an accurate reliability allocation method of machine tools is particularly important. However, methods based on different theories make the accuracy vary. Therefore, many scholars are developing more effective reliability allocation methods for machine tools. Zhao Yunbo¹ proposed a reliability allocation method for machine tools based on AHP and grey theory, an example is given to verify its validity. Liu Yang² introduced an allocation method based on entropy weight method.
Machine tool reliability allocation is an important part of the machine tool reliability design. It is a process of allocating the machine tools to the subsystem and components in a certain way to ensure the reliability of machine tools. At present, the commonly used reliability distribution methods of CNC machine tools are: AGREE method, ARINC method, Engineering weighted allocation method, Minimum workload allocation method, Dynamic programming allocation method, etc. ³ Li⁴ improved the AGREE method and added the concept of failure hazard level to the allocation method. Lv⁵ used the engineering weighted distribution method to assign the reliability of a new rocket launcher. Yang⁶ combined the analytic hierarchy process and the entropy weight coefficient method to complete the reliability distribution of CNC machine tools. But these methods do not consider the fuzziness of reliability allocation, therefore the research of reliability assignment problem using the fuzzy correlation mathematical model is increasing⁷. For example, Yang⁸ used the interval analysis and fuzzy comprehensive evaluation method to propose a reliability distribution method for CNC machine tools.

There are many influencing factors to consider in the reliability allocation process of CNC machine tools, so the reliability allocation of machine tools is a process of comprehensive weighing of many factors⁹. It can also be said that it is a multi-attribute decision problem, that is, to sort the weights of the reliability distribution of each subsystem under multiple attribute conditions. Common methods for multi-attribute decision-making problems include hesitant fuzzy TOPSIS method based on foreground theory, multi-attribute decision ranking method based on relative entropy, and hesitant fuzzy language group decision-making method.

Multi-attribute decision-making is widely used in research, and a large number of researchers have begun to study decision-making problems and innovate on original methods. Kuei⁷ used the combination of hesitant fuzzy linguistic term sets and the minimum variance OWGA weights to achieve flexible allocation of system reliability. Based on supplier selection, Kamran Rashidi⁰ compared two widely used methods - TOPSIS and DAE, and found that TOPSIS is superior to DAE in both computational complexity and data sensitivity. Depeng Kong¹¹ proposed a combinatorial optimization method based on decision variables for the interval-valued intuitionistic fuzzy problem of multi-attribute decision making that is difficult to calculate with traditional methods. Chunxia Yu¹² proposed a multiple decision selection methods based on interval Pythagoras fuzzy using extended TOPSIS. L. Z. Wu¹³ proposed a method of rock mass quality classification based on MCS and TOPSIS methods.

The gray correlation coefficient was proposed by Julong Deng. The traditional gray correlation coefficient describes the closeness of two variables, which is necessary for the field of decision-making. This theory is widely used in decision making, pattern recognition, and some other problems with uncertainty, especially with discrete data and fuzzy information. Li¹⁴ proposed a method of foundation pit risk assessment by combining the gray correlation model with the analytic hierarchy process. Yang¹⁵ applied the grey correlation model to the mechanical fault diagnosis method of a high voltage circuit breaker.

Intuitionistic trapezoidal fuzzy not only can express the degree of “good” and “bad” but also can express decision information of different dimensions¹⁶. It has certain advantages in solving fuzzy problems and is conducive to the reliability distribution of machine tools. The operation of machine
tools is full of complexity and uncertainty, and gray correlation is widely used to solve uncertain problems. Therefore, in this paper, a machine reliability allocation method which based on intuitionistic trapezoidal fuzzy numbers and gray correlation is proposed: through listing the expert decision matrix by using intuitionistic trapezoidal fuzzy, calculating the grey correlation between the decision matrix and the ideal matrix, the fuzzy problem in the reliability allocation process is reduced, and a reasonable reliability allocation result is obtained.

2. Intuitionistic trapezoidal fuzzy numbers and grey correlation analysis

Fuzzy numbers are often used to represent fuzzy problems that cannot be clearly determined and have certain advantages in representing the reliability of CNC machine tools.

2.1. Intuitionistic trapezoidal fuzzy numbers

definition 17: Let $A$ be an intuitionistic trapezoidal fuzzy number on a real number set $R$, its parameters can be defined as follows:

$$b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4,$$

denoted as:

$$A = \{ (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \},$$

Then its membership degree and non-membership degree are defined as:

$$\mu_A(x) = \begin{cases} 0, & x < a_1; \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2; \\ 1, & a_2 \leq x \leq a_3; \\ \frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4; \\ 0, & x > a_4; \end{cases}$$

$$\nu_A(x) = \begin{cases} 1, & x < b_1; \\ \frac{x-b_1}{b_1-b_2}, & b_1 \leq x \leq b_2; \\ 0, & b_2 \leq x \leq b_3; \\ \frac{x-b_3}{b_3-b_4}, & b_3 \leq x \leq b_4; \\ 1, & x > b_4; \end{cases}$$

where $\mu_A(x)$ is the membership degree of element $x$ to $A$; $\nu_A(x)$ is the non-membership degree of element $x$ to $A$. 

Intuitionistic trapezoidal fuzzy numbers can express decision information of different dimensions, and include non-membership and hesitation information, so intuitionistic trapezoidal fuzzy numbers are more precise and accurate and have greater flexibility for dealing with fuzzy problems.

Definition 2: Let $A_i (i = 1, 2)$ be two intuitionistic trapezoidal fuzzy numbers, where:

$$A_i = \left( (a_{i1}, a_{i2}, a_{i3}, a_{i4}), (b_{i1}, b_{i2}, b_{i3}, b_{i4}) \right)$$

then:

$$A_1 \oplus A_2 = \left( (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}), (b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}) \right)$$

$$\lambda A_i = \left( (\lambda a_{i1}, \lambda a_{i2}, \lambda a_{i3}, \lambda a_{i4}), (\lambda b_{i1}, \lambda b_{i2}, \lambda b_{i3}, \lambda b_{i4}) \right), \lambda > 0$$

Definition 3: Let $A_i (i = 1, 2, \cdots, n)$ be a set of intuitionistic trapezoidal fuzzy numbers, $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $A_i$, then the intuitionistic trapezoidal fuzzy weighted average operator (TrIFWA) is defined as:

$$\text{TrIFWA} (A_1, A_2, \cdots, A_n) = \bigoplus_{i=1}^{n} w_i A_i$$

TrIFWA operator assembly results are still intuitionistic trapezoidal fuzzy numbers. For all $i = 1, 2, \cdots, n$, if $w_i = \frac{1}{n}$, the TrIFWA operator degenerates into intuitionistic trapezoidal fuzzy arithmetic average operator (TrIFA):

$$\text{TrIFA} (A_1, A_2, \cdots, A_n) = \frac{1}{n} \bigoplus_{i=1}^{n} A_i$$

Another important concept of an intuitionistic trapezoidal fuzzy number is its expected value. According to Jun Ye[17], the expected value of the intuitionistic trapezoidal fuzzy number $A = \left( (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \right)$ can be obtained by the following equation:

$$EV(A) = \frac{1}{8} \left( \sum_{i=1}^{4} a_i + \sum_{j=1}^{4} b_j \right)$$

2.2. Grey correlation coefficient and correlation

The definition of the gray correlation coefficient is as follows:

Let there have two sequences $X_0 = (x_0(j), j = 1, 2, \cdots, k)$ and $X_1 = (x_1(j), j = 1, 2, \cdots, k)$, then:
\[
\begin{align*}
    r(x_0(j), x_i(j)) &= \frac{\min \min_i j \left| x_0(j) - x_i(j) \right| + \rho \max j \max_i \left| x_0(j) - x_i(j) \right|}{\left| x_0(j) - x_i(j) \right| + \rho \max j \max_i \left| x_0(j) - x_i(j) \right|} \\
    &= \left(1 - \frac{\max |x_0(j) - x_i(j)|}{\max_i \max_j |x_0(j) - x_i(j)|}\right)^{\rho} \\
    &= \left(1 - \left(\frac{\max |x_0(j) - x_i(j)|}{\max_i \max_j |x_0(j) - x_i(j)|}\right)^{\rho}\right) \\
    &= \left(1 - \left(\frac{\max |x_0(j) - x_i(j)|}{\max_i \max_j |x_0(j) - x_i(j)|}\right)^{\rho}\right)
\end{align*}
\]

Where \( \rho \) is the resolution factor, chosen between 0-1, Generally taken \( \rho = 0.5 \).

Besides, the gray correlation degree calculation formula:

\[
\xi_j = \frac{1}{N} \sum_{i=1}^{N} r(k)
\]

3. Allocation method based on intuitionistic trapezoidal fuzzy and grey correlation
The reliability allocation of CNC machine tools is a multi-attribute decision problem. Let \( O = \{o_1, o_2, \ldots, o_m\} \) be a scheme set consisting of \( m \) schemes; \( C = \{c_1, c_2, \ldots, c_n\} \) are \( n \) decision attributes; \( D = \{d_1, d_2, \ldots, d_l\} \) is the set of decision-makers, \( l \) is the number of decision-makers; \( R^{(k)} = \left( r_{ij}^{(k)} \right)_{mn} \) is the decision matrix of the decision-maker, Where \( r_{ij}^{(k)} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \) is the evaluation value of the attribute \( c_j \) on the scheme \( o_i \) given by the decision-maker \( d_k \) in the form of an intuitionistic trapezoidal fuzzy number. Let \( \xi_j^{(k)} \) represent the importance evaluation information about attribute \( c_j \) given by decision-maker \( d_k \).

Specific steps are as follows:

Step 1: Divide the structural composition of the system.

Step 2: Determine the system reliability requirements and reliability allocation principles.

Step 3: Decision makers give decision matrices in the form of intuitionistic trapezoidal fuzzy numbers.

\( K \) experts are invited to evaluate this issue. Decision-makers use natural language to evaluate things. By referring to Jun Ye\(^8\), corresponding decision matrices are established based on the intuitionistic trapezoidal fuzzy numbers and linguistic variables.

Step 4: Construct intuitionistic trapezoidal fuzzy decision matrices.

All individual decision information is aggregated into group decision information. The intuitionistic trapezoidal fuzzy decision matrix \( R = \left( r_{ij} \right)_{mn} \) can be assembled by \( TrIFWA \), that is:

\[
\begin{align*}
    r_{ij} &= \left\{ (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}), (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}) \right\} \\
    &= TrIFWA \left( r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)} \right) = \oplus_{k=1}^{l} w_{ij}^{(k)} r_{ij}^{(k)}
\end{align*}
\]

Step 5: Use formulas (10) and (11) to calculate the expected value of intuitionistic fuzzy numbers.
\[
\xi_j = \left\{ \left( p_{ij1}, p_{ij2}, p_{ij3}, p_{ij4} \right), \left( q_{ij1}, q_{ij2}, q_{ij3}, q_{ij4} \right) \right\}
\]
\[
= \text{TriFWA} \left( \xi_{j1}, \xi_{j2}, \ldots, \xi_{jI} \right) = \bigoplus_{k=1}^{I} \omega_{i}^{(k)} \xi_{j}^{(k)}
\]
\[
\xi_j' = \text{EV} \left( \xi_j \right) \left( \sum_{j=1}^{n} \text{EV} \left( \xi_j \right) \right)^{-1}
\]  

Step 6: Determine the ideal solution sequence.

According to the principle of machine tool reliability allocation, the larger the trapezoidal fuzzy number in the comprehensive decision matrix, the lower the reliability of the corresponding subsystem allocation, and the easier it is to improve its reliability, therefore the ideal solution should be the maximum of all expert decision values. The maximum of each column in the matrix constitutes the ideal solution.

Step 7: Calculate the gray correlation coefficient between the expected value of the trapezoidal fuzzy number and the ideal solution.

Use gray correlation degree calculation formula to calculate the gray correlation degree between the expected value and the ideal solution of the intuitionistic trapezoidal fuzzy number of each scheme in the comprehensive decision matrix.

Step 8: The grey correlation degree is used to calculate the reliability allocation weight, and the reliability of each subsystem is obtained. The reliability calculation formula of the subsystem is as follows:

\[
R_i = R_{s}^{ci} \quad i = 1, 2, \ldots, m
\]

where \( R_{s} \) —— Machine tool overall reliability index;

\( R_{i} \) —— Reliability index assigned to the ith subsystem.

4. Case Analysis

Take a kind of CNC machining center’s progress of reliability allocation as an example. It includes the following steps:

Step 1: The CNC machining center is subdivided and divided into 8 copies according to the mechanism of the machine tool, as shown in Table 1 below. These 8 components constitute the solution set of the multi-attribute decision problem, namely \( O = \{ o_1, o_2, \ldots, o_8 \} \), each is called a scheme.

| Team members | Subsystem | Subsystem abbreviation |
|--------------|-----------|------------------------|
| \( o_1 \)    | Hydraulic and pneumatic systems | HP |
Step 2: According to the user requirements, the designer determines the overall reliability of the machine tool and then determines the factors affecting the reliability allocation of the machine tool and the principle of reliability allocation in combination with the actual situation.

According to the actual situation, the overall reliability $R_s$ of this CNC machining center is determined to be 0.85. 6 factors affect the reliability allocation of machine tools. These factors are shown in Table 2 below.

**Table 2. The factors affecting reliability allocation and its principles.**

| Corresponding attribute number | Influencing factors                  | Reliability allocation principle                                                                 |
|--------------------------------|--------------------------------------|-------------------------------------------------------------------------------------------------|
| $c_1$                          | Complexity                           | The more complex the system structure, the lower the reliability of the distribution.           |
| $c_2$                          | Ease of maintenance                  | The easier the system is to repair, the lower the reliability of the distribution.               |
| $c_3$                          | Technique level                      | The higher the technology maturity, the higher the reliability of the distribution.              |
| $c_4$                          | Working environment                  | The better the environmental conditions, the more reliable the distribution.                    |
| $c_5$                          | Cost                                 | The higher the cost, the lower the reliability of the distribution.                             |
| $c_6$                          | Operating hours                      | The longer the working time, the lower the reliability of the distribution.                    |

These six factors affecting reliability allocation constitute the attribute set $C = \{c_1, c_2, \ldots, c_6\}$ of the multi-attribute decision problem, and each influencing factor is an attribute. Among them, to be consistent with the allocation principles of other influencing factors, for the influencing factors of
technical level and working environment, we require experts to make decisions based on the immature degree of technology and the harshness of the environment. In this way, when an expert makes decisions on these two influencing factors, the higher the decision value obtained by the two factors, the lower the reliability of the allocation, which is consistent with the allocation principles of other factors and facilitates subsequent calculations.

Step 3: List all expert decision matrices. Three experts are asked to make decision evaluations on the 8 subsystems under each attribute. To facilitate the expression of expert opinions during the evaluation, there are 7 types of linguistic variables for evaluation: Absolutely low, low, generally low, medium, generally high, high and absolutely high. The specific conversion criteria are shown in Table 3.

Table 3. Translation criteria of linguistic variables and intuitionistic trapezoidal fuzzy Numbers.

| Term             | Logogram | Intuitionistic trapezoidal fuzzy numbers corresponding to language values |
|------------------|----------|--------------------------------------------------------------------------------|
| Absolutely low   | AL       | \( \langle (0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0) \rangle \) |
| Low              | L        | \( \langle (0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \rangle \) |
| Generally low    | FL       | \( \langle (0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5) \rangle \) |
| Medium           | M        | \( \langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7) \rangle \) |
| Generally high   | FH       | \( \langle (0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9) \rangle \) |
| High             | H        | \( \langle (0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0) \rangle \) |
| Absolutely high  | AH       | \( \langle (1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0) \rangle \) |

Three experts are invited to make decisions on these 8 subsystems under 6 influencing factors, and three decision matrices were obtained:
Step 4: The decision matrices of various experts are assembled to construct a comprehensive intuitionistic trapezoid fuzzy decision matrix $R$.

Formula (1), (2), (3) and table 3 are used to aggregate the decision matrix of the three experts. The weight information of the three experts is 0.45, 0.3, and 0.25. The resulting comprehensive decision matrix $R$ is shown in Tables 4.

**Table 4. Comprehensive decision matrix.**

(a)

|      | $c_1$                      | $c_2$                      | $c_3$                      |
|------|----------------------------|----------------------------|----------------------------|
| $o_1$| $(0.67, 0.73, 0.80, 0.87)$ | $(0.07, 0.17, 0.27, 0.37)$ | $(0.33, 0.43, 0.53, 0.63)$ |
|      | $(0.60, 0.73, 0.80, 0.93)$ | $(0.00, 0.17, 0.27, 0.43)$ | $(0.30, 0.43, 0.53, 0.67)$ |
| $o_2$| $(0.50, 0.60, 0.70, 0.80)$ | $(0.57, 0.67, 0.77, 0.87)$ | $(0.43, 0.53, 0.63, 0.73)$ |
|      | $(0.43, 0.60, 0.70, 0.87)$ | $(0.50, 0.67, 0.77, 0.93)$ | $(0.37, 0.53, 0.63, 0.80)$ |
| $o_3$| $(0.80, 0.87, 0.93, 1.00)$ | $(0.50, 0.60, 0.70, 0.80)$ | $(0.37, 0.47, 0.57, 0.67)$ |
|      | $(0.80, 0.87, 0.93, 1.00)$ | $(0.43, 0.60, 0.70, 0.87)$ | $(0.27, 0.47, 0.57, 0.77)$ |
| $o_4$| $(0.73, 0.80, 0.87, 0.93)$ | $(0.63, 0.73, 0.83, 0.93)$ | $(0.73, 0.80, 0.87, 0.93)$ |
|      | $(0.70, 0.80, 0.87, 0.97)$ | $(0.60, 0.73, 0.83, 0.97)$ | $(0.70, 0.80, 0.87, 0.97)$ |
Step 5: Calculate the expected value of each element in the comprehensive decision matrix to form the comprehensive decision expectation matrix $U$. 

\[
\begin{array}{c}
o_5 \\
o_6 \\
o_7 \\
o_8 \\
\end{array}
\begin{array}{c}
(0.17, 0.27, 0.37, 0.47), \\
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\end{array}
\begin{array}{c}
(0.17, 0.27, 0.37, 0.47), \\
(0.07, 0.27, 0.37, 0.57)
\end{array}
Use formula (6) or (7) to calculate the expected value of the intuitionistic trapezoidal fuzzy number, and finally get the attribute's weight matrix $U$, as shown below:

$$
U = (u_{ij})_{6 \times 6} = \begin{pmatrix}
0.77 & 0.22 & 0.48 & 0.63 & 0.52 & 0.9 \\
0.65 & 0.72 & 0.58 & 0.18 & 0.65 & 0.83 \\
0.9 & 0.65 & 0.52 & 0.52 & 0.85 & 1.0 \\
0.83 & 0.78 & 0.83 & 0.42 & 0.72 & 0.95 \\
0.32 & 0.52 & 0.22 & 0.35 & 0.25 & 0.78 \\
0.22 & 0.52 & 0.15 & 0.58 & 0.22 & 0.65 \\
0.9 & 0.72 & 0.52 & 0.52 & 0.85 & 1.0 \\
0.83 & 0.78 & 0.83 & 0.42 & 0.72 & 0.95 \\
0.9 & 0.72 & 0.52 & 0.52 & 0.85 & 1.0 \\
1.0 & 0.45 & 0.32 & 0.35 & 0.85 & 1.0
\end{pmatrix}
$$

Step 6: Determine the ideal solution sequence $A$.

The element in the ideal solution sequence $A$ is $a_j$, then:

$$
a_j = \max_i \max_k u^k_{ij}, \quad k = 1, 2, \ldots, l, i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n
$$  \hspace{1cm} (15)

Calculate the expected value of each element in the ideal solution sequence $A$ to form a new ideal solution sequence $B$, whose constituent elements are $b_j$, then $b_j$:

$$
b_j = \text{EV}(a_j), \quad j = 1, 2, \ldots, n
$$  \hspace{1cm} (16)

According to equations (14) and (15):

$$
B = (1, 0.85, 1, 0.85, 1)
$$

Step 7: Use Equation (9) to calculate the gray correlation $\xi$ between the sequence of plans and the sequence of ideal solutions $B$ in the comprehensive decision matrix, as shown in Table 5.

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ |
|-------|-------|-------|-------|-------|-------|
| $a_1$ | 0.6489 | 0.4028 | 0.4497 | 0.5346 | 0.5629 | 0.8095 |
| $a_2$ | 0.5484 | 0.7658 | 0.5030 | 0.3414 | 0.6800 | 0.7143 |
| $a_3$ | 0.8095 | 0.6800 | 0.4696 | 0.4696 | 1 | 1 |
| $a_4$ | 0.7143 | 0.8586 | 0.7143 | 0.4229 | 0.7658 | 0.8947 |
| $a_5$ | 0.3846 | 0.5629 | 0.3527 | 0.3953 | 0.4146 | 0.6589 |
| $a_6$ | 0.3527 | 0.5629 | 0.3333 | 0.5030 | 0.4028 | 0.5484 |
| $a_7$ | 0.8095 | 0.7658 | 0.6028 | 0.4696 | 0.6115 | 0.3846 |
| $a_8$ | 1 | 0.5152 | 0.3846 | 0.3953 | 1 | 1 |

And according to formula (10) and Table 6, calculate the grey correlation degree $\xi$ between the sequence of plans and the ideal solution sequence $B$ in the comprehensive decision matrix, as shown in Table 6.
Table 6. Grey Relevance $\xi$ of Each Scheme.

| $\xi$ |  
|------|--------|
| $\alpha_1$ | 0.5681 |
| $\alpha_2$ | 0.5922 |
| $\alpha_3$ | 0.7381 |
| $\alpha_4$ | 0.7284 |
| $\alpha_5$ | 0.4615 |
| $\alpha_6$ | 0.4505 |
| $\alpha_7$ | 0.6073 |
| $\alpha_8$ | 0.7159 |

Step 8: According to the results shown in Table 6, combined with the formula (14) to calculate the reliability of each subsystem allocation, the results are shown in Table 7 below.

Table 7. Calculation results of each subsystem reliability.

| $\xi$ | Reliability $k$ |
|------|-----------------|
| $\alpha_1$ | 0.9812 |
| $\alpha_2$ | 0.9804 |
| $\alpha_3$ | 0.9756 |
| $\alpha_4$ | 0.9759 |
| $\alpha_5$ | 0.9847 |
| $\alpha_6$ | 0.9850 |
| $\alpha_7$ | 0.9799 |
| $\alpha_8$ | 0.9764 |

Also, this article uses the AHP method to compare the results with the method this article proposed to illustrate the effectiveness and accuracy of the method. As shown in Table 8.

Table 8. Reliability allocation results and comparison.

| Subsystem | The method proposed in this paper | AHP method |
|-----------|-----------------------------------|------------|
| HP        | 0.9812                            | 0.9976     |
| FD        | 0.9804                            | 0.9729     |
| SP        | 0.9756                            | 0.9536     |
| SO        | 0.9759                            | 0.9571     |
| LU        | 0.9847                            | 0.9955     |
| CO        | 0.9850                            | 0.9961     |
| ATC       | 0.9799                            | 0.9870     |
| CNC       | 0.9764                            | 0.9804     |

5. Conclusion
This paper proposed a new reliability allocation method for machine tools based on intuitionistic trapezoidal fuzzy and grey correlation. The principle and steps of this method are discussed in detail.
in the article. Finally, an example is taken to verify the effectiveness of the proposed method. The results show that the calculation results of this method are close to the traditional AHP method, which illustrates the accuracy and feasibility of this method. Compared to the traditional method, the proposed method can reduce the calculation and more convenient to use.

It is worth noting that regard machine tool reliability allocation problem as a multi-attribute decision problem and allocates based on expert opinion, which has possible that the accuracy may be affected by the different opinions between experts or the subjectivity of the experts is too strong. Although impact can be reduced through weight the experts, however, reducing the subjective impact of people is still a consideration of using this method and an important issue to be resolved in future development.

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