Performance Analysis of Sparse Array Designed by Improved Density-Tapered Method

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Abstract—The density-tapered method is a method to simulate amplitude weighting of full array by using the density of antenna element. However, the probability of the existence and non-existence of the element is 0 and 1 respectively, which makes the performance of the side lobe level of the pattern worse than that of the amplitude weighting of full array. In this paper, by improving the density-tapered method, gradual and interdependent density-tapered method is proposed to realize the design of sparse linear array, and the simulation experiments are carried out to compare and analyze the performance of peak side lobe level. The experimental results show that the improved density-tapered method can further reduce the side lobe level of the pattern.

1. INTRODUCTION
The sparse antenna array selects elements on the grid of the uniform full array and the average element spacing is greater than half wavelength, which can not only obtain larger aperture under the condition of the same number of elements, but also reduce the mutual coupling effect between elements and reduce the construction cost of the antenna array. It is always an important goal of array design to make the array as sparse as possible while maintaining the peak side lobe performance.

The design methods of sparse array can be divided into two categories, one is swarm intelligence optimization design method represented by genetic algorithm [1], particle swarm optimization algorithm, invasive weed algorithm etc., the other is based on mathematical tools represented by density tapered method [2-4], iterative Fourier transform method [5], matrix pencil method [7]. With the increase of the number of array elements, the optimization variables will also increase, resulting in a sharp increase in calculation. For the former methods, it is easier to fall into the local solution, and the convergence speed becomes very slow, so it is not feasible to use them to design large-scale sparse antenna array. When the density tapered method is used to design the transmitting array, each element uses the same transmitting power [6], and the feeding is simple. Moreover, the calculation process of the density tapered method is used to determine the position of the element is simple, which has more advantages in the design of large-scale sparse array.

The density tapered method was proposed by M. Skolnik [3]. The core idea is to use the spatial variation of the elements’ density to simulate the amplitude weighting of the uniform full array to
reduce the side lobe of the pattern. Taking the amplitude weighting of the uniform full array as a reference, the greater the amplitude excitation is, the greater the probability of the element appearing will be. However, the probability of the sparse array can only take 0 and 1, and each element is selected independently, which can not accurately control the probability of the elements on each grid. In order to reduce the performance loss, Sun Maoyou [8] proposed the interdependent density tapered method.

In this paper, an improved method of density-tapered method, the gradual and interdependent density-tapered method, is proposed. Taking the amplitude weighting function \( \cos^2(x) \) as an example, the performance of the improved method is verified by comparing with the original density-tapered method and the interdependent density-tapered method. Then, by changing the number of array elements, the effectiveness of the improved method in reducing the side lobe is analyzed.

2. PROBLEM FORMULATION

Suppose that the array works in the far-field and narrow-band conditions and that each antenna array element is the omnidirectional radiating antenna. The geometric model of uniform full-array is shown in Figure 1.

\[
F(u) = f(u) \sum_m a(m) \exp \left( \frac{2\pi}{\lambda} x_m u \right)
\]

Where \( u = \sin(\theta) - \sin(\theta_0) \), \( \theta_0 \) is the pointing angle of the array. Unless otherwise specified, \( \theta_0 = 0 \) in this paper. \( u \in [-1,1] \), \( x(m) \) is the distance between the \( m \)-th element and the center of the array.

The sparse array can be seen as a result of selecting a part of the elements in the uniform full array with half wavelength spacing, so its position is limited to the grid of uniform full array, and the result of selection leads to the array becoming non-uniform. Which is vividly illustrated in Figure 2.

\[
F(u) = f(u) \sum_m P(m) \exp \left( \frac{2\pi}{\lambda} x_m u \right)
\]

Define the variable for element selection as \( S(m) \), the pattern of the array produced by the density-tapered method can be uniformly expressed as:
\begin{equation}
P(m) = \begin{cases}
1 & \text{rand} < S(m) \\
0 & \text{rand} \geq S(m)
\end{cases} \quad (3)
\end{equation}

Where \text{rand} is random number between 0-1. The element selection variable \text{S(m)} determines the probability of the element being selected. When \text{rand} < \text{S(m)}, the \text{m}-th element will be selected, otherwise the \text{m}-th element will not be selected.

3. DESIGN ARRAYS WITH THREE DENSITY-TAPERED METHODS

3.1. Design steps
The difference between three density tapered methods is mainly reflected in the generation method of \text{S(m)}. The density-tapered method to design a sparse array mainly includes the following steps:
- Determine \text{N} that is defined as the number of elements.
- Determine the number of grids by the formula below.
\begin{equation}
M = \frac{N}{\eta} \quad (4)
\end{equation}
Where \eta is the sparsity of the array, \text{M} is the number of grids.
- Calculate \text{S(m)} at each grid point. Three calculation methods of \text{S(m)} will be introduced in the next part.
- Complete the array element selection by comparing \text{S(m)} with \text{rand}.

3.2. Three calculation methods of \text{S(m)}
When using original density-tapered method, the value of the \text{S(m)} at each grid is independent of each other. Expressed as:
\begin{equation}
S_{i}(m) = a(m) \ast k \quad (5)
\end{equation}
Where \text{a(m)} is amplitude weighted value of full array, \text{k} can be used to control the sparsity of the array. The sparsity can be expressed as:
\begin{equation}
\eta = \frac{k \times \sum_{m=1}^{M} a(m)}{M} \quad (6)
\end{equation}
Replace \text{S(m)} with \text{S_d(m)}, the pattern of the array generated by original density-tapered method can be calculated.

The original density-tapered method first calculates the selection probability when determining whether the array element is selected, and then generates a random number to compare with the probability value, and determines that the array element exists when the conditions are met. However, the probability value is greater than zero and less than one. When the state of the array element is determined to exist or not exist, the posterior probabilities obtained is 1 or 0. A feasible method is to consider both the influence of the arranged elements and the selection probability of the current grid when calculating \text{S(m)}, which is interdependent density-tapered method. The element selection variable of interdependent density-tapered method can be expressed as:
\begin{equation}
\begin{cases}
S_{i}(m) = a(m) \ast k - \sum_{i=1}^{M} [P(i) - a(i) \ast k] & 1 < m \leq M \\
S_{i}(1) = a(1) \ast k
\end{cases} \quad (7)
\end{equation}
It can be seen that the interdependent density-tapered method compensates the difference between the posterior probability and the expected probability of the elements being selected by changing the selection probability of the elements at the current grid. In this way, the element density determined by full array amplitude weighting can be approximated better, and the side lobe of the array can be reduced.

The amplitude weighting, which determines the density of the array elements, varies in the aperture, and the farther the distance between the array elements is, the greater the difference of amplitude weighting is. So when designing the array, we usually hope that the closer the array element is, the
stronger the correlation is, otherwise, the weaker the correlation is. The gradual and interdependent density-tapered method is based on this idea, and the element selection variables can be expressed as follows:

\[
S_{\beta}(m) = \begin{cases} 
  a(m) \ast k - \sum_{i=m-L}^{m-1} \left[ P(i) - a(i) \ast k \right] \ast \beta^{i-m} & \text{if } L < m \leq M \\
  a(m) \ast k & \text{if } m \leq L 
\end{cases}
\]

(8)

Where \( L \) is the number of elements related to the \( m \)-th element.

Fig 3 Characteristics of different density-tapered methods

The arrow in the figure indicates that the array is designed from left to right. The dotted line box means that the selection status of element in the current grid is to be determined. The solid line box means that the status of element has been determined. The black area in the box is used to show the relevance between these elements and the element to be determined. The larger the black area is, the stronger the relevance will be.

4. EXPERIMENT AND ANALYSIS

In this section, numerical simulation is used to analyze the peak to side lobe ratio of the sparse array designed by density-tapered method. The amplitude weighting function is \( a(m) = \cos^2(\pi(m-0.5M)/M) \), the wavelength is 0.05 m. The two parameters of the improved density-tapered method are set to \( L = 50 \), \( \beta = 0.7 \). FFT technology is used to accelerate the calculation speed when calculating the beam pattern of sparse array. It should be noted that the randomness of array element selection in the design process leads to a small difference between the final number of elements and the expected number of elements.

Set the number of array elements to 512 and the sparsity to 0.5, and compare the peak side lobe ratio (PSLR) of the three density-tapered methods. The result is shown in Figure 4.
It can be seen from the figure that the PSLR of the antenna array designed by the improved method is lower than that designed by the original density-tapered method and the interdependent density-tapered method, and the improved method has the potential to achieve better PSLR performance.

Without changing the number of array elements, the PSLL of the array is calculated by changing the sparsity of the density-tapered array. Compare the results of PSLR, we can observe the influence of the sparsity on the PSLR of sparse array. Set the number of array elements to 1024, and the results are shown in Table 1.

| Performance index          | Sparsity of density-tapered array |
|----------------------------|-----------------------------------|
|                            | 0.5  | 0.4  | 0.3  | 0.2  | 0.1  |
| PSLR of original method (dB)| -24.7 | -22.7 | -18.5 | -19.44 | -18.4 |
| PSLR of interdependent method (dB) | -23.4 | -21.8 | -19.4 | -18.64 | -19.4 |
| PSLR of proposed method (dB) | -24.5 | -23.2 | -21.0 | -20.5 | -20.3 |

In general, when the number of array elements is determined, the PSLR performance of the three methods will deteriorate with the decrease of sparsity. Compared with the other two methods, the performance of the array designed by the gradual and interdependent density-tapered method is the best.

Calculating the PSLR of the pattern by changing the number of elements under the condition that the sparsity is not changed, we can observe the influence of the number of array elements on the PSLR of sparse array. Set the sparsity to 0.1, then change the number of array elements and calculate the PSLR of the pattern. The results are shown in Figure 5.
As shown in the figure above, as the number of elements increases, the PSLR of arrays designed by various methods is getting better and better. Among them, the gradual and interdependent density-tapered method still has advantages compared with the other two methods.

5. CONCLUSIONS

By analyzing the results in the fourth part, some meaningful conclusions can be obtained.

- The density-tapered methods are suitable for the design of large-scale sparse arrays. When the sparsity is determined, the more the number of array elements, the better the PSLR performance of the designed sparse array, and the improved density-tapered method has better performance.
- When the number of array elements is fixed, the PSLR performance of the designed sparse array deteriorates when the sparsity becomes smaller. The gradual and interdependent density-tapered method can reduce the deterioration.
- The gradual and interdependent density-tapered method can improve the PSLR performance by adjusting $L$ and $\beta$, but the optimal parameter values corresponding to different number of elements and different weighting forms of full array amplitude are generally different. It takes a long time to select the appropriate parameter values for different situations, so improving the operation efficiency will be the next research direction.

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