Thermal Behavior of the Natural Convection of Air Confined in a Trapezoidal Cavity

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Authors’ contributions

This work was carried out in collaboration among all authors. Author WNDK designed the study, wrote the protocol and wrote the first draft of the manuscript. Authors IO and NB managed the analyses of the study. Author PFK managed the literature searches. All the authors read and approved the final manuscript.

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ABSTRACT

The thermal behavior of air by natural convection in a confined trapezoidal cavity, one of the walls of which is subjected to a constant heat flow in hot climates, has been analyzed numerically. The heat and mass transfers are carried out by the classical equations of natural convection. These equations are discretized using the Finite Difference Method and the algebraic systems of equations thus obtained are solved with the Thomas and Gauss algorithms. We analyze the influence of the number $Ra = 10^3 - 10^5$ on the current and isothermal lines as well as the effects of the aspect ratio $A = l / H$ and the angle of inclination $\phi$. In particular, we have shown that convective exchanges in the cavity are preponderant for high Ra numbers. Also we have watches the increase in the values of the isothermal lines and the decrease in the intensity of the streamlines for the low values of $A$ and of the angle $\phi$.

Keywords: Natural convection; thermal; trapezoidal cavity.
Global warming and energy problems are now part of the major challenges for humanity and in particular for developing countries, such as Burkina Faso. Indeed, the issue of energy consumption in buildings in hot and dry tropical climates also constitutes a second challenge. The control of this energy consumption has been at the heart of government strategies since decade.

To cope with population growth, habitats are often designed without taking into account environmental constraints and regional specificities. This results in still high temperatures (thermal discomfort) inside habitats, requiring the use air conditioners to improve thermal comfort. It is therefore necessary to design habitats that reduce energy consumption while providing thermal comfort to occupants.

The improvement of thermal comfort by natural means in habitats located in hot and dry climates where electrical energy generates exorbitant expenses for air conditioning and ventilation has been the subject of many studies. These include studies of the phenomena generated by natural convection, the impact of habitat geometry, and the thermal properties of the fluids involved. Numerical analyzes of heat and mass transfers by natural convection in confined cavities have been the subject of an abundant literature [1,2,3,4]. The results show the influence of adimensional numbers such as Gr, Nu on heat and mass transfers in various trapezoidal cavities [5,6,7,8].

Thus, we propose in this scientific article to study numerically the transfers of heat and mass in natural convection in a trapezoidal cavity confined in hot and dry climates. The studied physical model is assimilated to a trapezoidal shape of which the small base constitutes the upper part and the large base the lower part. In the literature we have encountered physical models of which the large base constitutes the upper part [9]. We can say that our physical model is less studied numerically and experimentally.

2. DESCRIPTION OF THE PHYSICAL MODEL

Consider, as shown in Fig.1, a confined trapezoidal cavity whose upper (small base) and lower (large base) horizontal walls are maintained at constant temperatures (\( T_{sup} > T_{inf} \)). One of the inclined walls is subjected to a...
heat flux density $Q = 100 W \cdot m^{-2}$ and the other wall maintained adiabatic. The principle of operation of our roof model is that the air confined in the trapezoidal shape exchanges with the different walls inside and outside (with the surrounding environment).

Thus, to explain the physical phenomena of heat and mass transfers that occur inside the trapezoidal cavity, we use mathematical equations. To formulate these equations, we assume the following assumptions:

- The flow is laminar, stationary and two-dimensional,
- The fluid is Newtonian and incompressible,
- The radiative exchanges between the walls are negligible,
- Enthalpy diffusion is neglected,
- The dissipation of viscous energy is neglected,
- The Dufour and Soret effects are negligible,
- No condensation on the walls.

3. MATHEMATICAL FORMULATION OF THE PHYSICAL MODEL STUDIED

The heat and mass transfer equations applied to the model are written [10,11]:

- Continuity equation:

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0$$

- Momentum equation:

Along the x axis:

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = -\frac{1}{\rho_i} \frac{\partial p_i}{\partial x} + \nu_i \left( \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right) - g \beta_T (T_i - T_a) + \beta_c (C_i - C_a)$$

Along the y axis:

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial y} = -\frac{1}{\rho_i} \frac{\partial p_i}{\partial y} + \nu_i \left( \frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} \right) - g \beta_T (T_i - T_a) + \beta_c (C_i - C_a)$$
- heat equation
\[
\frac{\partial T_i}{\partial t} + \frac{u_i}{\partial x} + v_i \frac{\partial T_i}{\partial y} = \frac{\lambda_i}{\rho_i C_p} \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right)
\]  
(4)

- diffusion equation
\[
\frac{\partial C_i}{\partial t} + \frac{u_i}{\partial x} + v_i \frac{\partial C_i}{\partial y} = D_i \left( \frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} \right)
\]  
(5)

We reduce the number of parameters in the conservation, energy and diffusion equations by introducing adimensional numbers. So equations (1-5) can be write:

\[
\frac{\partial \mathbb{U}^*_i}{\partial X} + \frac{\partial \mathbb{V}^*_i}{\partial Y} = 0
\]  
(6)

\[
\frac{\partial U^*_i}{\partial X} + U^*_i \frac{\partial U^*_i}{\partial X} + V^*_i \frac{\partial U^*_i}{\partial Y} = \frac{\partial \mathbb{U}^*_i}{\partial X} + \frac{\partial \mathbb{U}^*_i}{\partial Y}
\]  
(7)

\[
\frac{\partial V^*_i}{\partial X} + U^*_i \frac{\partial V^*_i}{\partial X} + V^*_i \frac{\partial V^*_i}{\partial Y} = \frac{\partial \mathbb{V}^*_i}{\partial X} + \frac{\partial \mathbb{V}^*_i}{\partial Y} - \frac{\partial \mathbb{C}^*_i}{\partial Y}
\]  
(8)

\[
\frac{\partial T^*_i}{\partial X} + U^*_i \frac{\partial T^*_i}{\partial X} + V^*_i \frac{\partial T^*_i}{\partial Y} = 1 \frac{\partial \mathbb{T}^*_i}{\partial X} + \frac{\partial \mathbb{T}^*_i}{\partial Y}
\]  
(9)

\[
\frac{\partial C^*_i}{\partial X} + U^*_i \frac{\partial C^*_i}{\partial X} + V^*_i \frac{\partial C^*_i}{\partial Y} = 1 \frac{\partial \mathbb{C}^*_i}{\partial X} + \frac{\partial \mathbb{C}^*_i}{\partial Y}
\]  
(10)

To simplify the difficulty posed by the boundary conditions to be imposed on the walls to solve the momentum conservation equations, this equation are transformed using the formulation as a function of current \((\psi(X,Y))\) - vorticity \((\omega)\).

The function of current and vorticity are defined by the following expressions:

\[
\mathbb{U}^*_i = \frac{\partial \psi^*_i}{\partial X}; \quad \mathbb{V}^*_i = -\frac{\partial \psi^*_i}{\partial Y}; \quad \omega^*_i = \frac{\partial \mathbb{V}^*_i}{\partial X} - \frac{\partial \mathbb{U}^*_i}{\partial Y}
\]

By introducing the current function into equation (6-10) we obtain:

\[
\frac{\partial^2 \mathbb{U}^*_i}{\partial X^2} + \frac{\partial^2 \mathbb{V}^*_i}{\partial Y^2} = -\omega^*_i
\]  
(11)

\[
\frac{\partial \mathbb{T}^*_i}{\partial t} + \mathbb{U}^*_i \frac{\partial \mathbb{T}^*_i}{\partial X} + \mathbb{V}^*_i \frac{\partial \mathbb{T}^*_i}{\partial Y} = 1 \frac{\partial \mathbb{U}^*_i}{\partial X} + \frac{\partial \mathbb{U}^*_i}{\partial Y}
\]  
(12)

\[
\frac{\partial \mathbb{C}^*_i}{\partial t} + \mathbb{U}^*_i \frac{\partial \mathbb{C}^*_i}{\partial X} - \mathbb{V}^*_i \frac{\partial \mathbb{C}^*_i}{\partial Y} = 1 \frac{\partial \mathbb{C}^*_i}{\partial X} + \frac{\partial \mathbb{C}^*_i}{\partial Y}
\]  
(13)

4. ADIMENTIONAL BOUNDARY CONDITIONS

For:

- \(0 \leq X \leq \frac{l}{H}\) et \(Y = 0,06\)

\[
\mathbb{U}^*_i = \mathbb{V}^*_i = 0
\]

\[
\psi^*_i = 0
\]

\[
\frac{\partial^2 \psi^*_i}{\partial X^2} + \frac{\partial^2 \psi^*_i}{\partial Y^2} = -\omega^*_i
\]

\[
\frac{\partial \mathbb{T}^*_i}{\partial t} = 0.479 \left( \frac{l}{H} \right)^{3/4} \frac{\lambda_Y}{\lambda_Y^2} \frac{T_i^{15/4}}{T_i^{5/4}} \frac{\partial \mathbb{C}^*_i}{\partial Y} = 0
\]  
(14)

- \(-\tan \varphi \leq X \leq 0; \quad l/H \leq X \leq l/H + \tan \varphi\) et \(Y = 0\)

\[
\mathbb{U}^*_i = \mathbb{V}^*_i = 0
\]

\[
\psi^*_i = 0
\]

\[
\frac{\partial^2 \psi^*_i}{\partial X^2} + \frac{\partial^2 \psi^*_i}{\partial Y^2} = -\omega^*_i
\]

\[
\frac{\partial \mathbb{T}^*_i}{\partial Y} = 0.479(\tan \varphi)^{3/4} \frac{\lambda_Y}{\lambda_Y^2} \frac{T_i^{15/4}}{T_i^{5/4}} \frac{\partial \mathbb{C}^*_i}{\partial Y} = 0
\]  
(15)

- \(0 \leq X \leq \frac{l}{H}\) et \(Y = 0\);

\[
\mathbb{U}^*_i = \mathbb{V}^*_i = 0
\]

\[
\psi^*_i = 0
\]

\[
\frac{\partial^2 \psi^*_i}{\partial X^2} + \frac{\partial^2 \psi^*_i}{\partial Y^2} = -\omega^*_i
\]

\[
\frac{\partial \mathbb{T}^*_i}{\partial Y} = 0.479 \left( \frac{l}{H} \right)^{3/4} \frac{\lambda_Y}{\lambda_Y^2} \frac{T_i^{15/4}}{T_i^{5/4}} \frac{\partial \mathbb{C}^*_i}{\partial Y} = 0
\]  
(16)

- \(-\tan \varphi \leq X \leq 0; \quad 0 \leq Y \leq 0,06\)

\[
\mathbb{U}^*_i = \mathbb{V}^*_i = 0
\]

\[
\psi^*_i = 0
\]

\[
\frac{\partial^2 \psi^*_i}{\partial X^2} + \frac{\partial^2 \psi^*_i}{\partial Y^2} = -\omega^*_i
\]

\[
\frac{\partial \mathbb{T}^*_i}{\partial X} = 0, \quad \frac{\partial \mathbb{C}^*_i}{\partial X} = 0
\]  
(17)
5. NUMERICAL METHODOLOGY

Equations (11-13) are discretized using the Finite Difference Method (FDM). The method of discretization consists in transforming these equations into a system algebraic. The resulting algebraic equations are expressed as tridiagonal matrices and solved row by row using Thomas's algorithm for the energy and diffusion equations, Gauss for the equation movement. The numerical computer code developed to simulate the behavior of the air in the trapezoidal cavity is under natural and laminar regime. We consider that natural convection is laminar and stationary in the confined trapezoidal cavity. For our numerical calculation needs, we have developed a numerical calculation code with the Comsol Multiphysics 4.2. software.

6. Discussion Of Results

We present the numerical results from the computer code developed to simulate the behavior of the air in the trapezoidal cavity by natural convection. We compared our results to those obtained by Ahmed Kadhim Hussein [12]. As shown in Fig. 2, our results agree well with those obtained by author Ahmed Kadhim Hussein. Indeed, the average Nusselt number \( \bar{Nu} \) calculated and those obtained by the author are in good agreement with a difference estimated at between 5%.

The streamlines and isothermal lines within the trapezoidal enclosure compared with those of Ahmed Kadhim Hussein show good agreement and the error is estimated at 5% by the author.

We observe that there is a little uncertainty due to the relative.

Fig.5 shows the influence of the Rayleigh number of which one of the walls has a flux density on the streamlines and the isothermal lines respectively. The application of \( Q = 100 \text{W/m}^2 \) on one of the inclined walls combined with the convective exchanges between the horizontal walls and the confined air generates natural convection in the trapezoidal enclosure. The streamlines of which the number is higher in the proximity of the wall subjected to the flux density.

- \( l/H \leq X \leq \tan \phi ; \ 0 \leq Y \leq 0.06 \)
- \( U_1^* = V_1^* = 0 \)
- \( \psi_1^* = 0 \)
- \( \frac{\partial^2 \psi_1^*}{\partial X^2} + \frac{\partial^2 \psi_1^*}{\partial Y^2} = -\omega_1^* \)
- \( \frac{\partial T_1^*}{\partial X} = 0, \ \frac{\partial C_1^*}{\partial X} = 0 \quad (18) \)
characterize the air moving in the trapezoidal enclosure. This distribution is due to an upward movement of the fluid along the two inclined walls. As the Rayleigh number increases, the temperature gradient between the wall and the confined air becomes important, which leads to the appearance of new cells in the proximity of the wall subjected to the heat flux density, thus translating an intensification of natural convection in the cavity (Fig.5(a),(c),(e)). For better
understand the influence of the effect of the increase in the Rayleigh number on the structure of the air flow, we have made the number $Ra$ vary between $10^3$ to $10^5$. We note that for values $Ra = 10^3$ and $10^4$, the structure of the air flow is presented by a multicellular regime consisting of a main cell of low intensity occupying the entire cavity inside. We also observe counter-rotating secondary cells located in different corners of this cavity. For $Ra = 10^5$, we note the disappearance of some secondary cells; the airflow is characterized by a main cell undergoing a deformation which tends to occupy the part of the cavity near the adiabatic wall. For an increasing number $Ra$, the intensity of the current functions also increases in the cavity. Thus, we note that the increase in the Rayleigh number therefore reflects an intensification of the natural convection which takes place in the trapezoidal cavity.

We also notice that the values of isotherms decrease with increasing $Ra$ (Fig. 5(b), 5(d), 5(f)). This distribution of isotherms is obtained for $Q = 100W.m^{-2}$. The increase in $Ra$ causes a uniform distribution of isotherms in certain areas of the enclosure. When $Ra = 10^3$, the isotherms are concentrated near the wall subjected to the heat flow. However, as $Ra$ increases these isotherms detach from each other and propagate in both directions (horizontal and inclined) and both at the lower and upper level of the enclosure. In this case, the temperature distribution decreases from the upper horizontal wall to the lower horizontal wall. For a $Ra = 10^5$ the isotherms deform more and tend to follow the shape of the trapezoidal enclosure. And also, the isothermal lines are concentrated on the horizontal walls (upper and lower). This results in very intense transfers in these areas. Also, we observe that there is a decrease in temperature values as, on the one hand, we move away from the wall subjected to the flow (tilted left) and on the other hand, when we get closer to the adiabatic wall where the temperatures reach their minimum values. This shows that the intensification of the Rayleigh number reflects an intensification of the natural convection that takes place there. We also see the variation in temperature values in the cavity with increasing Rayleigh number.

Figs. 6 and 7 show the influence of the ratio of the length (small base) on the height, therefore the angle of inclination $\varphi$ of the trapezoidal enclosure on the streamlines and isothermal lines. So we denote by $A = l / H$ [13-16]. We find that the decrease in the length of the small base leads to the decrease in the angles of inclination $\varphi$ of the inclined walls with respect to the large base. We observe the influence of the ratio $A$ therefore of the angles of inclinations $\varphi$ on the intensity and the shape of the streamlines for a
Fig. 5. Distribution of the streamlines (left) and isothermal lines (right) in the trapezoidal cavity with a wall subjected to $Q = 100\text{W/m}^2$ according to the number of $Ra=10^5$ and a heat flow $Q=100\text{W/m}^2$ applied to one of the walls inclined. The curves in Fig. 6 show that the intensity of the streamlines increases as the aspect ratio decreases. Also, we find that the decrease in the length ($l$) of the small base with respect to the height leads to an increase in the values of the current lines in the trapezoidal enclosure and the disappearance of counter-rotating cells. This increase in the values of the current lines is due to the fact that the
convective exchanges between the inclined wall subjected to the flows \(Q=100\text{W/m}^2\) and the air becomes more important with the decrease in the length \((l)\) or the angles \(\phi\) of inclinations. We see in Fig. 7, the values of isotherms in the proximity of the inclined wall subjected to the heat flow \(Q=100\text{W/m}^2\) increase on the one hand, and on the other hand the transformation of their shape. Momentum transfers, heat transfers are characterized by the Rayleigh number \(Ra\) are higher the greater the density of the heat flow.

The average temperature of the air in the trapezoidal enclosure increases when the aspect ratio decreases because, for the same density of heat flow applied to an inclined wall, the volume of air is all the smaller as the ratio of form is small. These results also that the increase in the values of the isothermal lines is due to the fact that the temperature gradients due to the convective exchanges between the wall and the air become more important with the decrease of the aspect ratio.

Fig. 6. Distribution of the streamlines as a function of the aspect ratio \(A\) or of the angle of inclination \(\phi\) in the trapezoidal cavity with an inclined wall subjected to \(Q=100\text{W/m}^2\) for \(Ra=10^5\)
7. CONCLUSION

We have studied numerically the heat and mass transfers by natural convection in a trapezoidal cavity. At the end of our simulation calculations, we first analyzed the influence of the Rayleigh number $Ra$ on the current lines and the isothermal lines in a trapezoidal cavity of which one of the walls is subjected to a density flux constant $Q = 100W/m^2$. Then, we also analyzed the influence of the aspect ratio $\lambda = l/H$ and the "tilt angles" on the streamlines and isothermal lines in our trapezoidal confined cavity.

The results obtained lead to a number of conclusion:

- for heat transfers, it has been observed that natural convection becomes predominant in the cavity with high values of Rayleigh number $Ra = 10^5$,
- the aspect ratio $\lambda = l/H$ (therefore the angle of inclination $\varphi$) is also very...
important on heat and mass transfers such as streamlines and isothermal lines in the enclosures. The heat and mass transfers are all the more important as the ratio $A$ and the angle $\varphi$ decrease.

We believe that we have well explained the thermal behavior of the air confined in the trapezoidal cavity in hot and dry climates.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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