Universal fluctuations in tropospheric radar measurements

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Abstract – Radar data collected at an experimental facility arranged on purpose suggest that the footprint of atmospheric turbulence might be encoded in the radar signal statistics. Radar data probability distributions are calculated and nicely fitted by a one-parameter family of generalized Gumbel (GG) distributions, \(G_a\). A relation between the wind strength and the measured shape parameter \(a\) is obtained. Strong wind fluctuations return pronounced asymmetric leptokurtic profiles, while Gaussian profiles are eventually recovered as the wind fluctuations decrease. Besides stressing the crucial impact of air turbulence for radar applications, we also confirm the adequacy of \(G_a\) statistics for highly correlated complex systems.

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Radar sensors are currently employed in a large variety of applications, e.g. surveillance and target identification [1], weather observations and forecasting [2], and geophysical investigations [3]. It is therefore of paramount importance to have a clear control on all possible sources of external disturbances, which could significantly alter the measurements response.

In 2006, within the activities of the GALAHAD project [4], a large collection of ground-based radar data was recorded, and soon realized to constitute a unique opportunity for investigating the impact of external perturbations on the sought response. It was in particular observed that occasionally, but in coincidence with hard windy conditions, the radar interferograms were affected by an overall decorrelation. Motivated by this unexpected finding, it was hypothesized that the atmospheric turbulence could drastically influence the radar propagation even over short ranges, an observation which however rested on purely qualitative ground. No further experimental realizations were in fact subsequently developed to scrutinize the process and reach an unambiguous proof of concept.

At variance, on the theoretical side, pioneering studies on electromagnetic signal propagation through a turbulent atmosphere [5,6] date back to the 1970s. Punctual fluctuations of air refractive index materialize in a recorded imprint, which could therefore encode an indirect signature of turbulence. Indeed, the embedding medium can be ideally segmented in neighboring volume cells, each contributing to the signal according to an overall trend and its own statistics. The signal impinging on the junction between two adjacent cells is partly reflected and partly transmitted and the data collected at the receiver is sensitive to the sequence of scattering events occurred along the propagation path. In this sense, the radar echo is a global quantity indirectly representing the specific state of the crossed medium. A comprehensive interpretative framework for such phenomena is however still lacking, following the inherent difficulties to accommodate for the intimate, highly complex nature of radar/air interactions. In particular, the characteristic of air-induced fluctuations remain to be fully elucidated.
Starting from this setting, and to gain a conclusive insight into this crucial issue, we have arranged an outdoor experiment, realized in a controlled environment. A $k_u$-band radar was mounted on a plateau looking at an artificial target at fixed distance. The location was chosen in order to reproduce as close as possible an ideal situation, in which the radar signal fluctuations would solely have depended on the propagation medium. An anemometer, located near the target, was used to register the wind velocity.

Then, the probability distribution function of the collected signal was obtained and studied, focusing on the shape modification as a function of the air wind condition. This allowed us to return a statistical characterization of the radar fluctuations due to the atmospheric propagation, up to momenta of arbitrarily high order. As we shall demonstrate, strong wind conditions are associated to asymmetric leptokurtic profiles, standard Gaussian distributions being instead recovered as the wind strength decreases. This observation can be cast in the form of a phase transition, the wind strength acting as the external control parameter. To establish a clear causality relation between the air turbulence, here encapsulated in the wind velocity, and the radar signal statistics constitutes the primary goal of the letter. Equally important, as an additional result of our analysis, we will confirm the adequacy of the so-called generalized Gumbel (GG) distribution as the paradigmatic PDF of global quantities in correlated, spatially extended systems [7].

In our experiment, a portable continuous-wave stepped-frequency $k_u$-band radar was installed over the excavation area of a disused quarry of Pietra Serena located near Firenzuela (Florence, Italy)\textsuperscript{1}. It operated with range resolution of 1 meter and unambiguous range of 600 meter. Details on the radar system can be found in [8]. The observed scene was essentially a rectangular rock plateau with three sides ending with a cliff\textsuperscript{2}. The site was equipped with an artificial radar target, a trihedral corner reflector, posed at the edge of the cliff opposite to the radar side (see fig. 1). The radar-target distance was about 60 m. An ultrasonic anemometer, working at 4 Hz sampling rate, installed near the radar, measured the wind velocity fluctuations.

Radar data were collected at a rate of about 50 Hz, (sampling a 6 hours long recording window). Mainly they represent the amplitude of the backscattered signal from the corner reflector after propagation to the corner and back. The data to be analyzed were arranged in a time series $A(t)$ of about one million points.

To just focus on air’s impact on the propagation, all other possible sources of disturbances need to be identified and subsequently excluded. In the current case, because of the bareness of the scene, signal fluctuations may depend on the oscillations of the radar tripod and/or the corner reflector’s support, and on the intrinsic instrumental noise. Mechanical oscillations can be straightforwardly characterized in the frequency domain and their contribution to the PDF shape was found to be negligible. The instrumental noise is thermal noise, which is known to be white ($SNR = 60$ dB). The residual signal as departing from the expected, a priori constant output, is imputed to atmospheric propagation.

Different scale processes occur within the atmosphere, generally referred to as synoptic-scale processes (dozen of days), meso-scale processes (from day to hour) and micro-scale processes (less than an hour). Turbulent phenomena concern the micro-scale [9], corresponding to distances shorter than 1 km. In order to remove the components due to meso-scale and longer scale processes from the series $A(t)$, a multiplicative detrending was applied. The “detrended” series $A_{\text{det}}(t)$ is determined as

\begin{equation}
A_{\text{det}}(t) = \frac{A(t)}{A_0(t)},
\end{equation}

where $A_0(t)$ is the “trend” obtained by a moving average filter, which in this case is equivalent to a low-pass filter $A(t)$ with a threshold frequency of $5.5 \cdot 10^{-4}$ Hz. Once detrended, only micro-scale fluctuations are expected to contribute to the PDF.

Non-stationary modulation can still affect the detrended series, e.g., varying Reynolds regimes. A 60 minutes long temporal window was applied sequentially, in order to analyze finite portions of the series, that are as stationary as possible, while still containing a meaningful number of samples for statistical analysis. Each selected window consists of about $10^5$ points, which renders of some reliability the study of rare fluctuations. Different windows overlapped partially, for about one-third of their length.

\footnotesize
\textsuperscript{1}The data here presented were recorded from 22 : 45 LT on March 25, 2009, to 04 : 15 LT on March 26, 2009. Clear air conditions were fulfilled during the measurement period.

\textsuperscript{2}It would be worth to emphasize the crucial role played by the extremely favorable conditions under which the experiment was realized. The location was chosen to minimize any other possible source of fluctuations, as those due to the vegetation. Indeed, from previous experimentation activities, we observed that vegetation may produce further unwanted scattering, blurring away any beautiful footprint of clear air turbulence in the radar signal statistics.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig.1.png}
\caption{The radar sensor at the outdoor experimental facility. The site was equipped with a trihedral corner reflector (highlighted on the right side of the picture), posed at about 60 m from the radar, and an ultrasonic anemometer (highlighted on the left).}
\end{figure}
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Turbulent-wind conditions associated to the different analysis windows, is quantified via the turbulent kinetic energy (TKE) indicator $K_t$ [9]. This latter is calculated from the fluctuations of the three wind velocity components $v_x$, $v_y$, and $v_z$, revealed by the anemometer and reads

$$K_t = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle,$$

where the symbol $\langle \cdot \rangle$ stands for a time average over the inspected window.

To obtain an exhaustive representation of the signal fluctuations, beyond the second moment, we reconstructed the full PDF, within each selected time domain. Imagine to assume an ideal setting where interference of the radar signal with the wind can be neglected. Then, having eliminated other possible sources of systematic errors, one expects to recover the theoretically predicted PDF for a pointwise scatterer (the corner reflector) suffering from thermal noise. This is the well-known Rice distribution [10], a right skewed continuous curve, which approaches the Gaussian for high signal-to-noise ratio. Surprisingly, and at variance with what is customarily believed, a quite different distribution appears from our data analysis: the PDF of $\Delta_{\text{det}}$ is an asymmetric, left skewed, distribution characterized by an exponential tail on the one side and a rapid falloff on the other. Such a significant discrepancy can be ultimately imputed to the non-trivial interplay between the propagating signal and the turbulent dynamics of the embedding atmospheric medium, an effect which is not accommodated for in the theoretical scenario yielding to the Rice profile.

Motivated by this working ansatz, we set down to analyze more closely the experimental PDFs. To this end, and aiming at a quantitative representation of the data, we considered a non-Gaussian, one-parameter family of distributions, recently invoked to describe the fluctuations of global quantities in a turbulent context [11]. These are the so-called generalized Gumbel (GG) distributions, originating from the study of extreme-values statistics [12,13] and more recently proposed to universally describe the fluctuation of global quantities in correlated many-degrees-of-freedom systems [7,14]. In this respect the GG constitutes the natural extension of the Gaussian distribution which instead apply to uncorrelated variables, as a consequence of the central-limit theorem. In the last years, GG distributions proved to adequately adapt to distinct realms of applications, ranging from e.g. the magnetization of the two-dimensional XY-model close to the Kosterlitz-Thouless transition [15–17]3, to the statistics of the level of the Danube river [18], passing through the power injected in a turbulent flow and in electroconvection [19]. Complexity and high correlation would be the common feature of these systems, ending up with a GG distribution.

The GG distributions family of curves $G_a$ is defined as

$$G_a(x) = \frac{a \exp \left\{ b(a) \left( x - s(a) \right) - \frac{b(a)}{\sigma} \left( x - s(a) \right) \right\}}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \frac{a}{\sigma^2} \ln \Gamma(a) \right\},$$

where $a \in \mathbb{R}^+$, $b(a) = \frac{1}{\sigma} \sqrt{\frac{2}{\pi}} \ln \Gamma(a)$, $s(a) = \langle x \rangle + \frac{1}{\sqrt{\pi \sigma^2}} \ln \Gamma(a)$.

When the real positive parameter $a$ varies from large to small values, $G_a(x)$ changes its shape from that of a Gaussian distribution towards a more and more skewed PDF. This feature of the distribution family $G_a$ rendered it very popular in complex dynamics, when the statistics of global quantities are dealt with. The parameter $a$ has been proposed to be inversely related to the correlation length of the system [18], and it has been used for measuring the number of its effective degrees of freedom [20]. As a side observation, it is worth noting here that the GG distribution with $a = \frac{a}{\pi}$ approaches the well-known Bravais-Holdsworth-Pintond distribution that was found for the fluctuations of turbulent power consumption [11].

Back to the data collected in our experiment, properly rescaled fluctuations are calculated and the normalized experimental histograms reconstructed. These latter are then interpolated via $G_a(x)$, $a$ being adjusted as the best-fit parameter. In fig. 2 three representative curves are displayed which testified on the adequacy of the proposed ansatz. The general agreement between data and GG distributions is remarkable. It is worth noting that we did not find any correspondence between the statistical behavior of the radar signal and TKE, approaching the distribution of TKE a Rayleigh distribution on every analyzed window.

Even more interestingly, the values of $a$ as obtained from the fitting procedure appear to be strongly correlated to the quantity $K_t$, representing the turbulent-wind intensity within the crossed medium, for every chosen (statistically stationary) data segment. This finding, reported in the main panel of fig. 2, materializes in a genuine transition for $a^{-1}$ vs. $K_t$, showing the direct footprint of turbulence in the medium on the non-Gaussian nature of the radar fluctuations.

The transition between Gaussian and a non-Gaussian regime is rather sharp and takes place for a specific value of $K_t = K_t^c$ (≈ 6 m s⁻², in this case). The robustness of the scenario here depicted is also tested vs. modulating the size of the windows, in which the full time series is being decoupled: the transition point is not smeared over a wider $K_t$ interval, but conversely, it remains strictly positioned in $K_t^c$. Notice however, that the value of $K_t^c$ was obtained keeping fixed the geometry of the experiment, in particular the radar-target distance: it would be interesting to vary this distance and monitor the consequent changes on the estimated value of $K_t^c$. An experimental study on this specific point is already being planned, and results will be reported in a further publication. These
parameter, themainpanelofthefigurereportstheinverseoftheshape values, and the wind fluctuations intensity. The scatterplot in 

ted according to non-Gaussian PDFs. Moreover, they 

theatmosphere, even over short ranges (less than 100 m) 

oppositelimit, when the wind turns weak. 

as seen in [21]. Qualitativeagreementisalsofoundinthe 

onesreportedinthepanelsarefoundforTKEvalueswithin 

the intervals highlighted below the panels. This suggests a 

shapeparameter to three different analysis windows. The value of the shape 

GGdistribution (solid line) shown in the upper panels refer 

value of the shape parameter $a$ as resulted from the fitting procedure is also 

parameter 

function, which limits the maximum accessible value for the 

shape parameter to $a = 143$). The shape of the distribution varies from Gaussian, for low TKE values (upper panel, left), to 

Bramwell-Holdsworth-Pinton distribution [11], for high TKE values (upper panel, right). The general agreement confirms 

the adequacy of the GG distribution for describing the radar 

data fluctuations. Distributions with shape similar to the 

Bramwell-Holdsworth-Pinton distribution [11], for high TKE 

values (upper panel, right). The general agreement confirms 

varies from Gaussian, for low TKE values (upper panel, left), to 

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TKE of the crossed medium. All these facts clearly support 

the wind strength. The developing of a 

consistent model for radar signal transmission through a 

windy atmosphere, according to which the GG distribution 

emerges from the physical mechanisms of scattering, 

will be the focus of forthcoming works.

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