Cosmology with non-minimal scalar field: graceful entrance into inflation.

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Abstract

We propose a scenario of the beginning of inflation in which the non-vacuum value of the scalar field that drives inflation develops dynamically due to the non-minimal coupling to gravity. In this scenario, inflation emerges as an intermediate stage of the evolution of the Universe, well after the Planck epoch, from a fairly general initial state.

One of the intriguing aspects of inflation is its beginning. When did inflation start? Why was the inflaton, the scalar field responsible for inflation, essentially homogeneous over the initial Hubble volume? Why was the inflaton field originally placed far away from its present value, the physical minimum of its effective potential? Designing various ways to address these issues is not only of academic interest: different scenarios of the beginning of inflation may lead to different observable consequences, such as properties of the spectrum of density perturbations and/or gravitational waves, deviation of $\Omega_{\text{present}}$ from 1, etc.

In a currently popular set of scenarios (for reviews and references see, e.g., ref.[1]), inflation began immediately after the epoch governed by quantum gravity. In that case the initial conditions for the classical evolution are in principle determined by the Planck scale physics or semiclassical quantum gravity phenomena. While it is feasible that quantum fluctuations of gravitational and matter fields eventually give rise to the classical inflationary expansion, their quantitative analysis faces problems related to the lack of the detailed theory of quantum gravity.

Another possibility is that inflation is an intermediate stage of the evolution of the Universe, i.e., that it started well after the Planck epoch. This possibility is realized in the models of “old” [2] and “new” [3] inflation and their descendants, where the inflaton field is driven to a “false”, metastable minimum of its effective potential, e.g., by thermal effects. These models, however, are in potential conflict with flatness of the inflaton potential that is required for generating acceptably small density perturbations.
In this paper we suggest another mechanism for the “intermediate stage” inflation. We show that in a class of models with the inflaton field non-minimally coupled to gravity, inflationary expansion emerges automatically well after the Planck epoch from a fairly general initial state. In particular, the classical scalar field may initially be placed at the true minimum of its potential. What is required is the pre-inflationary Friedmann-like stage with deviation from radiation domination. The idea is that the non-minimal gravitational coupling of the inflaton field drives it out to non-zero value, so that the Friedmann expansion is followed by the inflationary stage. The mechanism works provided that the scalar potential is sufficiently flat and its functional form is related in a certain way to the field-dependent Planck mass describing the non-minimal coupling of the scalar field to gravity.

In our scenario, the inflationary stage itself is similar to chaotic inflation [4], as the scalar field slowly rolls down the monotonous effective potential. The difference to many models of chaotic inflation is that in our case inflation begins when the value of the inflaton potential is well below the Planck energy density. Hence, the number of $e$-foldings is not extraordinarily large. This may lead to $\Omega_{\text{present}} \neq 1$ and/or features in the observable spectrum of density perturbations, in addition to another observational consequence [3] of the non-minimally coupled inflaton — deviation of the spectral index from 1.

The models of the class that we consider in this paper contain one real scalar field $\phi$ and are described by the action

$$S = \int d^4 x \sqrt{-g} \left\{ -\frac{M_{Pl}^2}{16\pi} A(\phi) \cdot R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right\}.$$  

We will also add matter that produces non-vanishing scalar curvature $R$ at the initial Friedmann stage. In what follows we will use the units in which $3M_{Pl}^2/8\pi = 1$.

As we do not specify the particle physics origin of the inflaton field, the functions $A(\phi)$ and $V(\phi)$ are essentially arbitrary. We assume that the scalar potential has one minimum at $\phi = 0$ and the cosmological constant vanishes,

$$V(0) = 0.$$  

Then the function $A(\phi)$ behaves as follows near $\phi = 0$,

$$A(\phi) = 1 + \delta \phi + \frac{1}{2} \xi \phi^2 + \ldots$$  \hspace{1cm} (1)$$

The most favorable case for generating inflation from a state with $\phi \simeq 0$ is $\delta \neq 0$. It is this case that we consider in this paper; we define the field in such a way that

$$\delta > 0.$$  \hspace{1cm} (2)$$

To be specific, we assume that $A(\phi)$ increases monotonously at $\phi > 0$ and does not contain large or small parameters; in particular, $\delta \sim 1$ in Planck units. On the
other hand, the scalar potential $V(\phi)$ is required to contain small parameters so that $V(\phi) \ll 1$ at $\phi \ll 1$. Furthermore, we choose the functional forms of $V(\phi)$ and $A(\phi)$ in such a way that they are correlated above a certain sub-Planckian scale $\mu$,

$$V(\phi) \approx \text{const} \cdot A^2(\phi) \quad \text{at} \quad \phi \gg \mu,$$  \hfill (3)

We will see that successful inflation occurs as an intermediate stage for $\mu \lesssim 10^{-3}$ and that it is sufficient that the relation (3) holds well below the Planck scale only. In other words, we need not specify any relation between $V(\phi)$ and $A(\phi)$ at $\phi \sim 1$.

A concrete example which we will occasionally refer to is,

$$A(\phi) = \left(1 + \frac{\delta}{2} \phi\right),$$  \hfill (4)

$$V(\phi) = \lambda \frac{\phi^2}{\mu^2 + \phi^2} \left(1 + \frac{\delta}{2} \phi\right)^2,$$  \hfill (4)

where $\delta \sim 1$, $\lambda \ll 1$ and $\mu \ll 1$. We stress that the concrete form (4) will be used for illustrative purposes only, while our analysis and results apply to the general case specified by eqs. (3) and (2).

Equation (3) is easy to understand in the Einstein frame (i.e., after performing the conformal transformation $g_{\mu \nu} \rightarrow A^{-1}g_{\mu \nu}$): the effective potential relevant to this frame,

$$U(\phi) = \frac{V(\phi)}{A^2(\phi)},$$  \hfill (5)

has the form reminiscent of the potential along an approximate flat direction, as shown in Fig.1. (We will not use the Einstein frame in what follows, because the description of matter in that frame is inconvenient.)

![Figure 1: Einstein-frame effective potential.](image)

To begin with, let us consider the open FRW Universe which is initially filled with matter, and study homogeneous scalar field $\phi$. We will argue later on that the
homogeneity and/or isotropy of the initial state of the Universe are in fact irrelevant. On the other hand, the assumption of open Universe is essential: while our mechanism works even better in the spatially flat case, the closed Universe of sufficiently small size shrinks down to singularity before the conditions for the beginning of inflation set up.

The effect of matter is to produce the scalar curvature at the early stage of the expansion. As an example, we consider the equation of state $p = \gamma \epsilon$ with $0 \leq \gamma < 1/3$. Then the field equations take the following form

\[
\left( H^2 + \frac{\kappa}{a^2} \right) A(\phi) + HA'(\phi)\dot{\phi} = \frac{\rho_0}{a^\alpha} + \frac{1}{2} \phi^2 + V(\phi), \tag{6}
\]

\[
\ddot{\phi} + 3H \dot{\phi} - \left( \dot{H} + 2H^2 + \frac{\kappa}{a^2} \right) A'(\phi) = -V'(\phi), \tag{7}
\]

where $a$ is the scale factor, $H = \dot{a}/a$ is the Hubble parameter, $\alpha = 3(1+\gamma)$, and $\rho_0$ and $\kappa = -1$ characterize the initial density of matter and spatial geometry, respectively. We consider the evolution after the Planck epoch, i.e., set $a(t_i) = 1$ at the initial time $t_i$. As an example we impose the initial condition

\[
\phi(t_i) = 0, \quad \dot{\phi}(t_i) = 0. \tag{8}
\]

In fact, our results are not sensitive to the choice of the initial data provided that $|\phi(t_i)|, |\dot{\phi}(t_i)| \ll 1$, i.e., the scalar field evolves from well below the Planck scale.

At the first stage, the evolution of the Universe is essentially the conventional Friedmann expansion. As $A'(0) = \delta > 0$, the third term in eq. (6) acts as drag force, and positive scalar field develops. The scalar field continues to increase as long as the terms due to the scalar potential in eqs. (6) and (7) are negligible. At small $V(\phi)$ (e.g., small $\lambda$ in eq. (4)), the scalar field reaches its maximum value $\phi_{\text{max}}$ which depends on $\rho_0$ and $\delta$ (and, in general, on the functional form of $A(\phi)$). When the scalar potential starts to dominate the expansion, the inflationary stage begins. The scalar field slowly rolls down towards the origin (provided that the Einstein-frame effective potential $U(\phi)$ slowly increases as function of $\phi$ at $\phi \leq \phi_{\text{max}}$), and the Universe expands exponentially. Inflation ends at $\phi \sim \mu$ when the scalar field begins to oscillate about $\phi = 0$. The number of inflationary e-foldings depends on $\phi_{\text{max}}$ and the parameters entering $A(\phi)$ and $V(\phi)$.

The analysis of the first Friedmann stage is most easily performed at $\rho_0 \ll 1$, i.e., when the Universe is curvature dominated right after the Planck epoch. Let us consider this case in some detail. We will see that $\phi_{\text{max}} \sim \rho_0$, so one can approximate $A(\phi)$ by two first terms in eq. (6). Let us write

\[
a(t) = t + b(t)
\]

with $b \sim \rho_0$. Also, $\phi \sim \rho_0$. The scalar potential is negligible at the first stage, and, to the linear order in $\rho_0$, we find from eqs. (6) and (7),

\[
\frac{2}{t^2} \ddot{b} + \frac{\delta}{t} \dot{\phi} = \frac{\rho_0}{t^\alpha},
\]
\[ \ddot{\phi} + \frac{3}{t} \dot{\phi} - \delta \left( \frac{1}{t} \dddot{b} + \frac{2}{t^2} \dot{b} \right) = 0 . \]

The solution to these equations with the initial data (8) imposed at \( t_i = 1 \) (i.e., when \( a(t_i) = 1 \)) is

\[ \phi = \phi_{\text{max}} - \frac{\delta}{1 + \delta^2/2} \frac{\rho_0}{2} \left[ \frac{1}{(\alpha - 2)t^{\alpha - 2}} - \frac{1}{2t^2} \right] , \tag{9} \]

where

\[ \phi_{\text{max}} = \frac{\delta}{1 + \delta^2/2} \frac{\rho_0}{2} \left[ \frac{1}{(\alpha - 2)} - \frac{1}{2} \right] . \tag{10} \]

The solution (9) rapidly tends to \( \phi_{\text{max}} \) at \( t \gg 1 \) (recall that \( \alpha > 3 \)). Hence, the initial data for inflation are independent of the scalar potential at small \( V(\phi_{\text{max}}) \). Note that at least at small \( \rho_0 \), the maximum value of the scalar field is small in Planck units, \( \phi_{\text{max}} \ll 1 \).

The second, inflationary stage begins when \( a(t) \) becomes of the order of \( [V(\phi_{\text{max}})]^{-1/2} \) which is much greater than 1. In other words, the duration of the first Friedmann stage is fairly large.

Let us now discuss the inflationary stage. At this stage both the matter and spatial curvature terms in eq. (3) may be neglected. At small enough \( \mu \), the maximum value of the scalar field, eq. (10), is such that eq. (3) holds. Let us write at \( \phi \gg \mu \)

\[ V(\phi) = \lambda A^2(\phi) \left[ 1 + w(\phi) \right] , \]

where \( \lambda \) is a positive small constant, and \( w(\phi) \ll 1 \) at \( \mu \ll \phi \ll \phi_{\text{max}} \). The conditions of slow roll in this model are

\[ \dot{\phi} \ll H A, \quad \ddot{\phi} \ll H A', \quad \dot{\phi} \ll H \frac{A}{A'}, \quad \dddot{\phi} \ll H \dot{\phi} . \tag{11} \]

Under these conditions, the Hubble parameter has the form

\[ H = \sqrt{\frac{V}{A}} \left( 1 + h \right) , \]

where \( h \ll 1 \). We find from eq. (3)

\[ h = -\frac{1}{2\sqrt{\lambda}} \frac{A'}{A^{3/2}} \dot{\phi} , \]

which is indeed small. Equation (7) then gives

\[ \dot{\phi} = -\frac{1}{3} \sqrt{\lambda} \frac{A^{3/2}w'}{1 + A^2/2A} . \]

We find that the conditions (11) are indeed satisfied at small \( w(\phi) \), i.e., the Universe undergoes inflation in the slow roll regime.
Let us note in passing that the scalar field indeed rolls down towards \( \phi = 0 \) only if \( w'(\phi) > 0 \). The opposite case corresponds to run-away behavior without end of inflation. Generally speaking, the undesirable run-away solutions appear when \( V'(\phi) < 2V(\phi)A'(\phi)/A(\phi) \), i.e., when the Einstein-frame effective potential (3) decreases at large enough \( \phi \). In this paper we consider the physically interesting case \( V' > 2VA'/A \), i.e., \( w' > 0 \).

Finally, the number of inflationary \( e \)-foldings is

\[
N_e = \int_{\mu}^{\phi_{\text{max}}} \frac{H}{|\dot{\phi}|} d\phi = \int_{\mu}^{\phi_{\text{max}}} \frac{3(1 + A'^2/2A)}{Aw'} d\phi.
\]

This number is large at small \( w(\phi) \) and depends both on the properties of the Universe at the initial Friedmann stage (through \( \phi_{\text{max}} \) ) and on the parameters of the model. As an example, for the concrete choice (4) one has at small \( \rho_0 \)

\[
N_e = \frac{3}{8} \left(1 + \frac{1}{2}\delta^2\right) \frac{\phi_{\text{max}}^4}{\mu^2},
\]

where \( \phi_{\text{max}} \) is given by eq. (10). To get an idea of numerics, at \( \rho_0 = 1, \delta = 1, \) and \( \gamma = 0 \) (non-relativistic matter at the initial stage) one finds in this example that \( N_e \gtrsim 60 \) at \( \mu \lesssim 2 \cdot 10^{-3} \). We see that our scenario does not necessarily lead to an enormous number of \( e \)-foldings.

Until now we have considered the homogeneous and isotropic Universe filled with a specific form of matter. We argue, however, that the non-minimal scalar field with the the property (3) is capable of producing inflation from quite general initial state of the Universe (the exceptions being purely radiation dominated FRW Universe with zero scalar curvature and closed Universe of small size). In general, the Universe at the initial stage has non-vanishing scalar curvature. Even if \( R \) is relatively small at this stage, it drags the scalar field out to the plateau of Fig. 1, albeit inhomogeneously, provided that \( \mu \) is small enough[4]. As the Einstein-frame effective potential \( U(\phi) \) is essentially independent of \( \phi \) at this plateau, the inflationary stage is generic, and possible large inhomogeneities are stretched out.

To conclude, inflation may emerge as an intermediate stage of the evolution of the Universe under mild assumptions on the cosmological initial conditions. This feature, however, is inherent only in a restricted class of models of scalar fields interacting with gravity: the most essential ingredient of our scenario is the relation (3) between the scalar potential \( V(\phi) \) and inflaton–gravity coupling \( A(\phi) \). It remains to be understood whether this property of the scalar field may be motivated by models of particle physics.

\[1\] In a sense, the above case of the open Universe and small \( \rho_0 \) is particularly unfavourable for establishing the initial conditions for inflation: the Hubble parameter determining the friction term in eq. (7) is large whereas the drag force is small (proportional to \( \rho_0 \)) and rapidly decreases in time.
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