Investigation of Micro- and Nanosized Particle Erosion in a 90° Pipe Bend Using a Two-Phase Discrete Phase Model

Research Article

M. R. Safaei, O. Mahian, F. Garoosi, K. Hooman, A. Karimipour, S. N. Kazi, and S. Gharekhani

Correspondence should be addressed to S. N. Kazi; salimnewaz@um.edu.my

Received 14 July 2014; Accepted 22 July 2014; Published 14 October 2014

Copyright © 2014 M. R. Safaei et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper addresses erosion prediction in 3-D, 90° elbow for two-phase (solid and liquid) turbulent flow with low volume fraction of copper. For a range of particle sizes from 10 nm to 100 microns and particle volume fractions from 0.00 to 0.04, the simulations were performed for the velocity range of 5–20 m/s. The 3-D governing differential equations were discretized using finite volume method. The influences of size and concentration of micro- and nanoparticles, shear forces, and turbulence on erosion behavior of fluid flow were studied. The model predictions are compared with the earlier studies and a good agreement is found. The results indicate that the erosion rate is directly dependent on particles’ size and volume fraction as well as flow velocity. It has been observed that the maximum pressure has direct relationship with the particle volume fraction and velocity but has a reverse relationship with the particle diameter. It also has been noted that there is a threshold velocity as well as a threshold particle size, beyond which significant erosion effects kick in. The average friction factor is independent of the particle size and volume fraction at a given fluid velocity but increases with the increase of inlet velocities.

1. Introduction

Erosion-corrosion, defined as the accelerated corrosion following the damage of surface films, is a common cause of failure in a large amount of power plant equipment like pipes, pumps, compressors, vessels, and turbines. It can often be assumed that corrosion is controlled by adjusting the mass transfer while erosion is under the flow of a particulate second phase. This is a credible assumption as corrosion films are brittle-like materials and therefore are eroded easily by impacting particles [1, 2]. This phenomenon has been investigated experimentally in a number of pioneering studies; see [3–7], for instance. Despite recent advances in computational techniques, erosion-corrosion process is yet to be fully resolved with reasonable accuracy. A multitude of reasons for this rather slow development of simulation techniques applied to this problem can be mentioned. For modeling mass transfer near the solid boundaries, it is necessary to solve the governing equations across the mass transfer boundary layer. In aqueous flows this layer may be an order of magnitude shorter than the viscous sublayer. This requires fine meshes in the near-wall region. Utilizing fine near-wall grids with the support of appropriate near-wall turbulence models, the required mass transfer data for corrosive species can be evaluated [8].

Chen et al. [9] studied erosion prediction approach and its usage in oilfield fittings, especially 3-D elbows and plugged tees, using CFX which is a commercially available CFD package. They used RNG k-ε turbulence model along with DPM to track the particles. The results demonstrated that particle rebound and erosion profile have the most significant roles in particles motion inside oilfield geometries. The
comparisons also indicated that CFD predictions for erosion are in good agreement with experimental data.

An erosion prediction approach for specifying wear profiles for a 2-D jet impingement test has been developed by Gnanavelu et al. [10]. This prediction model was according to material wear data achieved from laboratory experiments and CFD modeling. They found an appropriate relationship between predicted and experimental data. Although they found that due to some assumptions about particle size and shape, material hardening, numerical errors, and so forth, some essential errors always exist in the calculation.

Mohyaldin et al. [11] have used three methods (empirical, semiempirical, and computational fluid dynamics, i.e., CFD) to model 2-D sand erosion in a pipe, a problem with significant practical application in oil and gas industry. The results of this study have shown that the direct impingement model (semiempirical model) agrees with the results achieved from the discrete phase model (DPM) implemented in CFD whereas the CFD results dramatically underpredict the empirical ones.

Particles, in an erosion problem, can be external to fluid flow; that is, they may be removals from the walls or upstream flow processes. There are, on the other hand, cases where particles are internal to flow like nanofluids. Nanofluids are synthesized by adding highly conductive solid materials to the base fluid, such as water, ethylene glycol, and oil, all with relatively lower thermal conductivity, usually due to some assumptions about particle size and shape, material hardening, numerical errors, and so forth, found that due to some assumptions about particle size and shape, material hardening, numerical errors, and so forth, some essential errors always exist in the calculation.

In particular, erosion of nanofluids in turbulent flow regime are compared with those in the literature for validation purpose. Special attention was paid to micro- and nanosized copper particles of different solid volume fractions and Reynolds numbers in a commercial elbow.

2. Governing Equations of Turbulent Micro- and Nanofluids Erosion

The underlying physical assumption in this study is that the particles are carried by the flowing fluid. Therefore, continuity, momentum, DPM, and turbulent equations are used to analyze the flow. The spherical particles' velocity is assumed to be the same as those of flowing fluid. Assuming constant thermophysical properties for fluid and particles, the governing equations are as follows [21–23].

Continuity equation:

$$\frac{\partial}{\partial t}\left( \rho \right) + \nabla \cdot \left( \rho \vec{V} \right) = 0. \quad (1)$$

Momentum equation:

$$\frac{\partial}{\partial t} \left( \rho \vec{V} \right) + \nabla \cdot \left( \rho \vec{V} \vec{V} \right) = -\nabla P + \nabla \cdot \left( \mu \left( \nabla \vec{V} + \nabla \vec{V}^T \right) \right) + \rho g. \quad (2)$$

Standard k-ε turbulence model is as follows.

Turbulent kinetic energy transport equation:

$$\frac{\partial (pk)}{\partial t} + \nabla \cdot (\rho \vec{V} k) = \nabla \cdot \left( \left[ \mu + \frac{\mu_t}{\sigma_k} \right] \nabla k \right) + G_k - \rho \varepsilon. \quad (3)$$

Dissipation of turbulent kinetic energy transport equation:

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \nabla \cdot (\rho \vec{V} \varepsilon) = \nabla \cdot \left( \left[ \mu + \frac{\mu_t}{\sigma_\varepsilon} \right] \nabla \varepsilon \right) + \frac{\varepsilon}{k} \left( C_{\varepsilon 1} G_k - \rho \varepsilon C_{\varepsilon 2} \right). \quad (4)$$

The turbulent eddy viscosity obtained from Prandtl-Kolmogorov relation:

$$\mu_t = C_{\mu} \rho \frac{k^2}{\varepsilon}. \quad (5)$$
The turbulence kinetic energy production of the mean velocity gradients, \( G_k \), is given as:

\[
G_k = \mu_t \nabla \cdot (\nabla \mathbf{V} + (\nabla \mathbf{V})^T) - \frac{2}{3} \nabla \cdot (3\mu_t \nabla \cdot \mathbf{V} + \rho k).
\]  

(6)

The constants for the standard \( k-\varepsilon \) turbulence model in the above formula are represented in Table 1 [24, 25].

DPM is as follows:

\[
m_p \frac{dV_p}{dt} = \sum \mathbf{F},
\]

where \( \mathbf{F} \) is an external force acting on the particles which for fine particles with high density ratio (more than one) is drag and buoyancy forces [26].

Therefore, the equation of motion can be simplified to the following form:

\[
\frac{dV_p}{dt} = F_D (\bar{V} - \bar{V}_p) + \frac{g (\rho_p - \rho)}{\rho_g},
\]

(8)

where [27]

\[
F_D = \left( \frac{18\mu}{\rho_p d_p^2} \right) \left( \frac{C_D Re_p}{24} \right),
\]

(9)

wherein \( Re_p \) is the particle Reynolds number and is given as [28–30]

\[
Re_p = \left( \frac{\rho d_p \bar{V}_p - \bar{V}}{\mu} \right).
\]

(10)

The drag coefficient, \( C_D \), as a function of the particle Reynolds number is defined by [31, 32]

\[
C_D = \frac{24}{Re} \left( 1 + 11.2355Re^{-0.653} \right) + \left( -0.8271 \right) \frac{Re}{8.8798 + Re}.
\]

(11)

The solid particle erosion rates are defined as [33, 34]

\[
R_{erotion} = \sum_{p=1}^{N} \left( m_p C \left( d_p \right) f(\alpha) b(\nu) \right) \frac{v}{A_f},
\]

(12)

where \( C(d_p) \) is a function of particle diameter, \( f(\alpha) \) is a function of impact angle, \( \alpha \) is the angle between the particle trajectory and wall, \( v \) is the relative velocity among particles, \( b(\nu) \) is a function of relative velocity among particles, and \( A_f \) is the cell face area at the wall [33].

3. Boundary Conditions

Figure 1 illustrates the schematic of the problem which is analyzed in the present study. The boundary conditions are also indicated in this figure.
5. Numerical Procedure Validation

5.1. Validation with Numerical Study. In order to verify the present simulation, the results from this work were compared with those of [11] where sand erosion in a 2-D elbow was simulated. The geometry was a 50 mm diameter elbow with two 100 mm straight pipes protruded from both sides. The two-phase (air/sand) dilute slurry flow with sand as the dispersed phase was injected at 0.000886 kg/s to the continuous phase, here air, with an inlet velocity of 20 m/s. The variations of total erosion rate and maximum erosion rate with velocity were compared with the results reported by Mohyaldin et al. [11], as shown in Figures 2(a) and 2(b), to observe an excellent agreement between the results.

5.2. Validation with Experimental Study. The numerical predictions based on our work were also compared with numerical and experimental results reported by Chen et al. [9] for erosion in elbows and plugged tees. Comparisons were performed for a 2.54 cm (diameter) elbow with a curvature ratio of 1.5 where sand particles of 150-micron diameter are injected at $2.08 \times 10^{-4}$ kg/s over a range of air/sand velocities: 15.24, 30.48, and 45.72 m/s. The computed average mass loss for elbow was successfully compared with measurements reported in Chen et al. [9], as shown in Figure 3.

6. Grid Independence

The computational zone was discretized through structured, nonuniform hexahedral grid distributions. The refined grid was used at the vicinity of the walls where sharp gradients are expected. Several grid distributions were examined as Table 3 indicates. As seen, the effect of grid refinement beyond 61440 grids on the average erosion rate is insignificant implying grid independence of our results.

7. Results and Discussion

In this work, the turbulent fluid flow of water and copper micro- and nanoparticle suspensions through a 90° elbow has
Table 3: Grid independence tests.

| Number of grids ($V = 20$ m/s, $\varphi = 2\%$) | 30720 | 61440 | 122880 |
| Average erosion rate for 100 $\mu$m particles | $6.9523 \times 10^{-6}$ | $6.7833 \times 10^{-6}$ | $6.6965 \times 10^{-6}$ |
| Number of grids ($V = 20$ m/s, $\varphi = 2\%$) | 30720 | 61440 | 122880 |
| Average erosion rate for 10 nm particles | $2.6789 \times 10^{-6}$ | $2.5029 \times 10^{-6}$ | $2.4351 \times 10^{-6}$ |
| Number of grids ($V = 20$ m/s, $\varphi = 4\%$) | 30720 | 61440 | 122880 |
| Average erosion rate for 100 $\mu$m particles | $1.5270 \times 10^{-5}$ | $1.3857 \times 10^{-5}$ | $1.2994 \times 10^{-5}$ |
| Number of grids ($V = 20$ m/s, $\varphi = 4\%$) | 30720 | 61440 | 122880 |
| Average erosion rate for 10 nm particles | $4.3001 \times 10^{-6}$ | $4.1646 \times 10^{-6}$ | $4.0843 \times 10^{-6}$ |

Figure 4: The variation of total erosion rate with velocity.

been investigated. The material of the 0.0032 m (1/8 inches) diameter elbow was aluminum (3003 Alloy). The length of the two attached pipe pieces at the beginning and the end of the elbow was 0.016 m (5/8 inches) long (5 times pipe diameter). The ratio of the bend radius to pipe inside diameter is equal to 1.5. Water was allowed to flow through the pipe at different velocities (5 m/s, 10 m/s, and 20 m/s). It was assumed that the solid particles are spherical and flow at the same velocity as that of water. Different particle diameters (10, 50, and 100 microns as well as 10, 50, and 100 nanometers) and particle volume fractions (2% and 4%) in the suspension were examined.

7.1. The Influence of Velocity on Erosion Rate. To investigate the impact of velocity on the maximum erosion rate and total erosion rate, several inlet velocities were simulated. The impact of inlet flow velocity on the total erosion rate is demonstrated in Figures 4(a) and 4(b) for different particle sizes. One notes that the total erosion rates are near zero for inlet velocity less than 5 m/s and particle volume fraction of 2%. For volume fraction of 4%, this quantity is still negligible for inlet velocity less than 5 m/s and particle diameters below 10 microns. This inlet velocity value of 5 m/s can be considered as a “threshold limit” for total erosion rate beyond which the total erosion rate rockets up with an increase in the inlet flow velocity for each particle diameter. These figures also indicate that, with the increase of particle volume fraction, the total erosion rate increases. The maximum of this erosion increase for $\varphi = 4\%$ is around 4.9 times at $V = 20$ m/s and $d_p = 100$ microns, compared to that of $\varphi = 2\%$.

Similar trends are observed in Figures 5(a) and 5(b) for the maximum erosion rate at six various particle diameters. As seen, the maximum erosion rate is amplified with the particle diameter and velocity increment. This augmentation is negligible at velocities less than 5 m/s, but the difference between the values is more pronounced with an increase in the inlet velocity. Thus, when velocity is increased from 10 m/s to 20 m/s, the maximum erosion rate increases by about an order of magnitude, in fact, by around 7.5 times and 9 times at $\varphi = 2\%$ and $\varphi = 4\%$, respectively.
7.2. The Effect of Particle Dimension on Erosion Rate. It is significant to study the effect of particle diameter on fluid-solid interaction as particles’ size in different systems varies to a large extent from nanometer to centimeter. The particle diameter has direct influence on the drag force and, therefore, affects the flow behavior. The influence of particle diameter on maximum erosion rate, total erosion, pressure drop, and friction factor was studied by changing the particle diameter from 10 nm to 100 μm.

The influence of particle size on the maximum erosion rate was represented in Figures 6(a) and 6(b). As seen, the maximum erosion rate is closely related to the fluid velocity.
where a threshold velocity as well as a threshold particle size can be identified below which erosion is negligible. These figures also indicate that the rate of erosion augments linearly with particle diameter. One also notes that increasing the volume fraction of the particles, with other parameters fixed, will cause higher maximum erosion rate. The average of this increment is around 4.5 times.

Similar trends are observed in Figures 7(a) and 7(b) for total erosion rate where higher erosion rate is observed when the particle diameter and inlet fluid velocity are increased. This is expected as the particle impact velocity grows with the increase of the inlet flow velocity and particle size (see (12)). However, our numerical results can be used to quantify this increment. Note that the increase in the total erosion rate is around 8.5 times for the increase of velocity from 10 m/s to 20 m/s at \( \varphi = 2\% \) and 9.5 times at \( \varphi = 4\% \). The influence of volume fraction enhancement on total erosion rate is also around 8 times when the volume fraction is increased from 2\% to 4\%.

The declining impact of particle size on the maximum pressure was shown in Figures 8(a) and 8(b). This can be attributed to the reduction in drag forces as a result of an increase in the particle size. Consequently, with the same particle volume fraction, particle numbers are lowered compared to the case with smaller particles. The figures also indicate that there is a direct relationship between the velocity and increase of maximum pressure. It is also clear from the figures that an increase in particle volume fraction leads to higher maximum pressure. As a result, the maximum pressure value is observed when 10 nm particles at 4\% volume fraction flow with water at 20 m/s.

Interestingly, according to Figures 9(a) and 9(b), the average friction factor—which has been calculated based on Fanning equation—is insensitive to either the particle size or volume fraction. However, one observes that the average friction factor increases with inlet velocity unlike a single-phase flow.

Figure 10 illustrates the erosion contour inside the elbow for \( V = 20 \text{ m/s} \), particle size = 100 microns, and the volume fractions of (Cu) 2\%. As seen, the maximum erosion is observed near the midpoint, along the symmetry plane of the pipe bend, which is the location where velocity profiles begin an inverse behavior and the pressure is maximum.

Finally, for engineering applications and presentation of the physical influence of the parameters, the following single nonlinear correlation is derived from Figures 11(a) and 11(b) to estimate the average erosion rate as a function of particles’ concentration, diameter, and inlet velocity, valid for the range of parameters in this work; that is, \( 0.02 \leq \varphi \leq 0.04 \), \( 5 \text{ m/s} \leq V \leq 20 \text{ m/s} \), and \( 10 \text{ nm} \leq d_p \leq 100 \text{ microns} \). The average deviation of this correlation is 9.5\%. Consider the following:

\[
\text{Average erosion rate (AER)} = 3.6667 	imes 10^{-8} \left( \varphi^{1.0024} V^{3.4953} d_p^{0.1399} \right). \tag{13}
\]

8. Conclusion

A numerical study of erosion in turbulent water-based/copper (Cu) micro- and nanosized fluid flow through a 90° elbow has been conducted. Different solid volume fractions, particle sizes, and velocities were considered along with
the maximum erosion rate, total erosion rate, average erosion rate, friction factor, and maximum pressure.

The conclusions are summarized as follows.

(i) There is a threshold velocity as well as a threshold particle size, beyond which erosion is significant.

(ii) The maximum erosion rate, average erosion rate, and total erosion rate increase with particle diameter, volume fraction, and inlet fluid velocity.

(iii) Increase of the particle diameter decreases the maximum pressure.
(iv) An increase in particle volume fraction or velocity augments the maximum pressure.

(v) The average friction factor does not depend on particle size and/or volume fraction for a given flow rate.

(vi) With the increase of the inlet velocity, the average friction factor enhances.

The usage of nanofluids in heat transfer has an obvious benefit from the thermal efficiency point of view. Nonetheless, care
must be taken as depending on particle size, fluid velocity, particle shape, particle sedimentation, particle agglomeration, and surface erosion adverse effects can negate the benefits associated with heat transfer augmentation.

**Nomenclature**

\(x, y\): Cartesian coordinates (m)  
\(d\): Diameter (m)  
\(G_k\): Generation of turbulent kinetic energy (m² s⁻²)  
\(g\): Gravitational acceleration (m s⁻²)  
\(m_p\): Particle mass (kg)  
\(\bar{v}_p\): Particle velocity (m s⁻¹)  
\(P\): Pressure (N m⁻²)  
\(Re\): Reynolds number \((V D \rho / \mu)\)  
\(d_p\): Solid particle diameter (m)  
\(t\): Time (sec)  
\(k\): Turbulence kinetic energy (m² s⁻²)  
\(V\): Velocities vector (m s⁻¹).

**Greek Symbols**

\(\rho\): Density (kg m⁻³)  
\(\varepsilon\): Dissipation rate of turbulent kinetic energy (m² s⁻³)  
\(\mu\): Dynamic viscosity (Pa s)  
\(\sigma_k\): Effective Prandtl number for \(k\)  
\(\sigma_\varepsilon\): Effective Prandtl number for \(\varepsilon\)  
\(v\): Kinematics viscosity (m² s⁻¹)  
\(\nu_t\): Turbulence eddy viscosity (m² s⁻¹)  
\(\phi\): Volume fraction of particles.

**Subscripts**

\(D\): Drag  
\(p\): Particle  
\(t\): Turbulent.

**Conflict of Interests**

The corresponding author declares that there is no conflict of interests regarding the publication of this paper.

**Acknowledgments**

The authors gratefully acknowledge the High Impact Research Grant UM.C/HIR/MOHE/ENG/45, UMRG Grant RP012D-13AET, and Faculty of Engineering, University of Malaya, Malaysia, for support in conducting this research work.

**References**

[1] A. Keating and S. Nesic, “Prediction of two-phase erosion-corrosion in bends,” in *Proceedings of the 2nd International Conference on CFD in the Minerals and Process Industries (CSIRO ’99)*, pp. 229–236, Melbourne, Australia, 1999.

[2] S. Nešić, “Using computational fluid dynamics in combating erosion-corrosion,” *Chemical Engineering Science*, vol. 61, no. 12, pp. 4086–4097, 2006.

[3] J. Southard, R. A. Young, and C. D. Hollister, “Experimental erosion of calcareous ooze,” *Journal of Geophysical Research*, vol. 76, no. 24, pp. 5903–5909, 1971.

[4] P. Lonsdale and J. B. Southard, “Experimental erosion of North Pacific red clay,” *Marine Geology*, vol. 17, no. 1, pp. M51–M60, 1974.

[5] M. E. Gulden, “Correlation of experimental erosion data with elastic—plastic impact models,” *Journal of the American Ceramic Society*, vol. 64, no. 3, pp. C59–C60, 1981.

[6] R. A. Saravanam, M. K. Surappa, and B. N. Pramila Bai, “Erosion of A356 Al-SiCp composites due to multiple particle impact,” *Wear*, vol. 202, no. 2, pp. 154–164, 1997.

[7] G. T. Burstine and K. Sasaki, “Effect of impact angle on the slurry erosion-corrosion of 304L stainless steel,” *Wear*, vol. 240, no. 1-2, pp. 80–94, 2000.

[8] W. H. Ahmed, M. M. Bello, M. El Nakla, and A. Al Sarkhi, “Flow and mass transfer downstream of an orifice under flow accelerated corrosion conditions,” *Nuclear Engineering and Design*, vol. 252, pp. 52–67, 2012.

[9] X. Chen, B. S. Mclaury, and S. A. Shirazi, “Application and experimental validation of a computational fluid dynamics (CFD)-based erosion prediction model in elbows and plugged tees,” *Computers and Fluids*, vol. 33, no. 10, pp. 1251–1272, 2004.

[10] A. Gnanavelu, N. Kapur, A. Neville, J. F. Flores, and N. Ghorbani, “A numerical investigation of a geometry independent integrated method to predict erosion rates in slurry erosion,” *Wear*, vol. 271, no. 5–6, pp. 712–719, 2011.

[11] M. E. Mohyaldin, N. Elkhatib, and M. C. Ismail, “Evaluation of different modelling methods used for erosion prediction,” in *Proceedings of the NACE Corrosion Shanghai Conference & Expo*, pp. 1–19, Shanghai, China, 2011.

[12] M. H. Esfe, M. Akbari, D. Toghrhaie, A. Karimipour, and M. Afrand, “Effect of nanofluid variable properties on mixed convection flow and heat transfer in an inclined two-sided lid-driven cavity with sinusoidal heating on sidewalls,” *Heat Transfer Research*, vol. 45, no. 5, pp. 409–432, 2014.

[13] M. R. Safaei, H. Togun, K. Vafai, S. N. Kazi, and A. Badarudin, “Investigation of thermal conductivity and rheological properties of nanofluids containing graphene nanoplatelets,” *Numerical Heat Transfer A*, vol. 66, no. 12, pp. 1321–1340, 2014.

[14] H. Togun, M. R. Safaei, R. Sadri et al., “Numerical simulation of laminar to turbulent nanofluid flow and heat transfer over a backward-facing step,” *Applied Mathematics and Computation*, vol. 239, pp. 153–170, 2014.

[15] M. Goodarzi, M. R. Safaei, K. Vafai et al., “Investigation of nanofluid mixed convection in a shallow cavity using a two-phase mixture model,” *International Journal of Thermal Sciences*, vol. 75, pp. 204–220, 2014.

[16] J. Routbort, D. Singh, W. Yu et al., “Effects of nanofluids on heavy vehicle cooling systems,” in *Proceedings of the VT Annual Merit Review Meeting*, pp. 1–16, Argonne National Laboratory, February 2008.

[17] J. Routbort, D. Singh, E. Timofoeva, W. Yu, and R. Smith, *Erosion of Radiator Materials by Nanostructures*, Argonne National Laboratory, Vehicle Technologies—Annual Review, 2010.

[18] J. Routbort, D. Singh, E. Timofoeva, W. Yu, and R. Smith, *Erosion of Radiator Materials by Nanostructures*, Vehicle Technologies—Annual Review, Argona National Lab., 2011.
[47] M. R. Safaei, M. Goodarzi, and M. Mohammadi, “Numerical modeling of turbulence mixed convection heat transfer in air filled enclosures by finite volume method,” *The International Journal of Multiphysics*, vol. 5, no. 4, pp. 307–324, 2011.