g-2 in composite models of leptons

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ABSTRACT

Based on the bound state description of the muon and general relativistic covariant quantum field theory, we illustrate with a simple composite model that the observed deviation of $(g - 2)_{\mu}$ can be a demonstration of the substructure of the muon and give the constraints on the radius of the muon in different cases of light constituents and heavy constituents.

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1 Introduction

Recently, the E821 experiment at the Brookhaven National Laboratory [1] announced their latest measurement of the anomalous magnetic moment of the muon

\[ a_\mu = \frac{g - 2}{2} = 11659202(14)(6) \times 10^{-10}. \]  

(1)

In the Standard Model (SM), the anomalous magnetic moment, \( a_\mu^{SM} = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW} \), is estimated to be [2]

\[ a_\mu = 116591597(67) \times 10^{-11}, \]  

(2)

where the error is mainly from \( a_\mu^{Had} \). An estimate of hadronic contributions has been renewed recently [3]. From these results one obtains present deviation from SM

\[ \Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (4.3 \pm 1.6) \times 10^{-9}. \]  

(3)

The size of the deviation is hotly debated due to the uncertainties in the hadron vacuum polarization in \((g - 2)_\mu\) [4]. The deviation may give a hint of new physics. It is well-known that the progress to investigate physics beyond SM, new physics, is essentially along two different directions. One is to extend the symmetry of SM, such as supersymmetry, the fourth generation, more Higgs doublets, grand unification etc. or to assume the presence of extra dimensions, while keeping the assumption that all the SM particles as well as new particles are fundamental. The other is to assume the substructure of SM particles, such as composite models of lepton, quark and weak gauge boson, technicolor models, etc. Inspired by the exciting 2.6 \( \sigma \) excess, people have suggested many possible explanations of the deviation, mostly along the first direction [5, 6].

In particular, supersymmetric loop effects with \( m_{SUSY} \approx 55\sqrt{\tan \beta} \) Gev and the radiative mass mechanism at 1 - 2 TeV scale have been reviewed in [2, 4]. The reason of the deviation has also been probed, along the second direction, in technicolor models [7] and preonic models [8, 9]. In ref.[8], the deviation is attributed to the presence of heavy exotic colored lepton and extra Z-boson states arising in a preonic model and essentially the bound state description of a lepton is not touched in their calculations. In ref.[9] the model-independent limit on muon substructure has been given by using the previous formula given in ref.[12]. The constraint from the anomalous magnetic moments of leptons on possible substructure of leptons has been analyzed in terms of a general formalism for describing a bound state in early 80’s [11, 12]. In the non-relativistic theory of a bound state, where the binding energy and the inverse radius \( R^{-1} \) are much smaller than the mass of the bound state, the magnetic moment of the bound state is the vectorial sum of the magnetic moments of the constituents. This would contradict the precise experimental data for \( a_\mu \) if \( \mu \) is such a bound state [11]. It has been shown that this is not the case for the relativistic bound state [12]. In this paper, we will consider the theoretical implication of the E821 new result in a composite model of the lepton, based on the bound state description of the muon and the general relativistic covariant quantum field theory. We generalize the previous investigations to higher orders in \( \alpha \) and including both the heavy and light constituent cases. We shall repeat some previous derivations in order to make the paper self-contained. For the sake of simplicity we calculate the anomalous magnetic moment of a muon in a simple composite model in which the lepton is assumed to be a bound state
composed of a fermion and a scalar boson. We show that the deviation, \( \delta \theta_\mu \), can be a signal for compositeness of the muon and poses a constraint on the charge radius of the muon (and the masses of constituents if constituents are heavy).

Our paper is organized as follows. In Sec. II, we present the general form of the Bethe-Salpeter (BS) bound state wave function of a lepton, calculate the lowest order matrix element of the electric current of a composite particle and estimate the order of magnitude for the anomalous magnetic moment of a composite lepton at the lowest order in \( \alpha \). Using the spectrum representation of the wave function and the general properties of the four-point Green function containing the lepton bound state pole, the \( O(\alpha) \) correction to the anomalous magnetic moment is estimated in Sec. III. The last section is devoted to conclusions and discussions.

2 The matrix element of the electric current of a composite particle at the lowest order in \( \alpha \)

For simplicity, we assume that the lepton is the bound state composed of a charged fermion and a neutral scalar boson which we shall also call as preons for convenience. We shall confine us in this article to the relativistic tightly bounded states with \( R^{-1} \gg m_t \) and \( R^{-1} \approx m_F \), where \( R \) is the charge radius of the composite lepton and \( m_F \) is the mass of the charged fermionic preon. We shall consider two cases. In the case A, the mass \( m_F \) is of the same order as \( R^{-1} \) and much heavier than \( m_t \). In the case B, \( m_F \) and \( m_l \) are both much smaller than \( R^{-1} \). In this case, if the force between the preons is some confining gauge interaction, there may be an approximate global chiral symmetry which naturally results in the small lepton mass \( m_l \). Essentially \( R^{-1} \) is also the confinement scale of the gauge interaction.

The general Lorentz and space inversion invariant B-S wave function for a composite lepton composed of a fermionic and a scalar preon is

\[
\chi^s_P(p) = \int d^4x e^{ipx} < 0 | T(\psi(x/2), \phi(-x/2)) | P, s > = (f_1 + i\hat{p}/M f_2) u^s(P), \tag{4}\]

where \( \psi \) and \( \phi \) are fields of the fermion and bosonic preon respectively, \( \hat{p} = p \cdot \gamma \), \( P = p_1 + p_2 \) is the momentum of the lepton, \( p = \frac{m_B m_F - m_B m_l}{m_B - m_F} \) is the relative momentum between preons, \( m_B \) and \( m_F \) are the masses of the bosonic preon and fermionic preon respectively. The two cases mentioned above are: A. \( m_F = O(R^{-1}) \gg m_l \). B. \( m_F, m_l \ll R^{-1} \). In Eq. (4), the constant \( M \) with the mass dimension is introduced for convenience, \( f_i = f_i(P \cdot p, \frac{1}{2} p^2) = f_i(P, p) \), \( (i = 1, 2) \), are real functions corresponding to the S wave state, and \( u^s(P) \) is the Dirac spinor with spin component \( s_z = s \). It is straightforward to derive from invariance under the space-time inversion that the BS wave function for a lepton final state is

\[
\tilde{\chi}_P^s = \tilde{u}^s(P)(f_1 + i\hat{p}/M f_2). \tag{5}\]

The simplest diagram for the lowest order electro-magnetic interaction of composite lepton is illustrated in Fig. 1. The corresponding bound state matrix element is

\[
\Gamma^{(0)}_{\mu}(P, q) = < P + \frac{q}{2}, s | J_\mu(0) | P - \frac{q}{2}, s > \tag{6}\]

\[
= -\frac{1}{(2\pi)^4} \int d^4k \tilde{\chi}_{P+q/2}(k + \frac{q}{4}) \cdot \gamma_\mu \tilde{\chi}_{P-q/2}(k - \frac{q}{4})i\Delta_F^{-1}(\frac{P}{2} - k) \]
\[
\Gamma_\mu^{(0)} = \bar{u}(P + q/2) \left\{ \gamma_\mu F_1^{(0)}(q^2) - \frac{i}{2} [\gamma_\mu, \hat{q}] F_2^{(0)}(q^2) \right\} \frac{}{} u(P - q/2),
\]

where

\[
F_1^{(0)}(q^2) = S_1 + \frac{2T}{M^2} - \frac{2m_l}{M} V_{1P} + \frac{1}{M^2} (m_l^2 - \frac{q^2}{4}) T_{PP} + \frac{q^2}{M^2} T_{qq}
\]

\[
F_2^{(0)}(q^2) = \frac{m_l}{M^2} T_{PP} + \frac{1}{2M} S_3 - \frac{1}{M} V_{1P} + \frac{2}{M} V_{iq} - \frac{m_l}{2M^2} V_{2P}.
\]

In eqs. (8) and (9) the Lorentz invariant functions \(S_i\) etc. are defined by

\[
S_1 = \frac{-i}{(2\pi)^4} \int d^4k f'_1 f_1 \Delta_F^{-1}((P/2 - k)^2),
\]

\[
S_2 = \frac{-i}{(2\pi)^4} \int d^4k f'_2 f_2 \Delta_F^{-1},
\]

\[
S_3 = \frac{-i}{(2\pi)^4} \int d^4k f'_2 f_1 \Delta_F^{-1},
\]

\[
-\frac{i}{(2\pi)^4} \int d^4k f'_2 f_1 \Delta_F^{-1} k_{\mu} = V_{1P} P_{\mu} + V_{iq} q_{\mu},
\]

\[
-\frac{i}{(2\pi)^4} \int d^4k f'_2 f_2 \Delta_F^{-1} k_{\mu} = V_{2P} P_{\mu},
\]

\[
-\frac{i}{(2\pi)^4} \int d^4k f'_2 f_2 \Delta_F^{-1} k_{\mu} k_{\nu} = T_{PP} P_{\mu} P_{\nu} + T_{qq} q_{\mu} q_{\nu} + T_{\delta_{\mu\nu}}.
\]

The left hand side of the last two formulae in Eq. (10) is even in \(q\), hence there is no term linear in \(q_{\mu}\) on the right hand side.

It follows from Eq. (8) that the normalization condition of the electric charge is

\[
S_1(0) + \frac{2}{M^2} T(0) - \frac{2m_l}{M} V_{1P}(0) + \frac{m_l^2}{M^2} T_{PP}(0) = 1,
\]

which can approximately be written as

\[
S_1(0) + \frac{2}{M^2} T(0) = 1
\]

if \(m_l/M \ll 1\). In Eq. (12)

\[
T(0) = \frac{1}{4} \frac{-i}{(2\pi)^4} \int d^4k f'_2 f_2 \Delta_F^{-1} k^2.
\]
Let us choose $M$ to make the integral of $f_i f_j \Delta F^{-1}$ to be of the same order for $i, j = 1, 2$. From (13) we have

$$T(0) = O(R^{-2}) S_1(0).$$

(14)

Since $V_{1p}(0)$ and $V_{1q}(0)$ are of the same order of $S_1(0)$, which (as well as Eq. (14)) has been checked by using the spectra representation (25) given below, we obtain from (8),(9),(12) and (14)

$$\frac{F_2(0)}{F_1(0)} = \frac{C_2 M}{M^2 + C_1 R^{-2}}$$

(15)

where $C_1$ and $C_2$ are constants of the order one.

Eq. (15) is in agreement with the result in Eq. (25) of [12] derived with a different approach. Authors of [12] asserted that the constant $M$ with the dimension of mass should be equal to $m_F$. In our opinion, $M = m_F$ is likely to hold in theories with vector or pseudo-vector interactions as a chirality flip along the fermion line is required for the $F_2$ term. However, it may not be true in theories with scalar or pseudo-scalar interactions which change the chirality. In the latter case the more natural possibility is $M = O(R^{-1})$. Thus, in general we have $M = m_F$ or $O(R^{-1})$. In case A, $m_F$ and $R^{-1}$ are of the same order, from Eq. (15) the anomalous magnetic moment of the muon arising from compositeness at the leading order of $\alpha$ is

$$a_\mu = O(\frac{m_l}{R^{-1}}).$$

(16)

In case B, $m_F \ll R^{-1}$, from Eq. (15) one has

$$a_\mu = O(\frac{m_l}{R^{-1}}),$$

(17)

for $M = O(R^{-1})$, or

$$a_\mu = O(\frac{m_l m_F}{R^{-2}})$$

(18)

for $M = O(m_F)$.

Therefore, $a_\mu$ can be suppressed linearly or quadratically by $O(R^{-1})$ depending on whether $m_F = O(R^{-1})$ or $m_F \ll R^{-1}$ and also on different internal dynamics.

From [10], [17] and [18], the magnetic moment of the lepton bound state is found to approximately be the Dirac magnetic moment $e/2m_l$ at the zeroth order of $\alpha$ provided that the charge radius of the lepton is small enough.

By drawing lines corresponding to the “very strong” interactions which bind preons into a lepton in Feynman diagrams one can obtain diagrams more complex than Fig. 1, an example for such diagrams is shown in Fig. 3(a). The contributions from such diagrams do not change the estimate obtained above. We shall discuss such diagrams in the next section.

3 Radiative corrections at the $\alpha$ order

3.1 The contribution corresponding to the simplest diagram

It is necessary to examine the radiative corrections at higher orders of $\alpha$ in the composite model in view of the agreement between the experimental data
and the standard model prediction for \(a_\mu\) up to the order \(10^{-8}\) which is of the order \((\frac{\alpha}{\pi})^3\). In this section, we will consider radiative corrections to the anomalous magnetic moment of the composite lepton at the \(\alpha\) order.

To illustrate how the corrections at the \(\alpha\) order in the composite model is suppressed let us consider at first the case \(A\), \(m_F = O(R^{-1}) \gg m_l\). For simplicity we set \(m_F = m_B = m\) in this section. The simplest diagram at this order is shown in Fig. 2 and the corresponding matrix element is given by

\[
\Gamma^{(1)}_\mu = \frac{-i}{(2\pi)^4} \int d^4p \frac{1}{p^2} \gamma_\nu \frac{-i}{4} \lambda_{\mu} \chi_p \frac{-i}{4} \lambda_{\nu} \Delta_F^{-1} \cdot \frac{P}{2} \cdot \frac{p}{2} - q \cdot \frac{P}{2},
\]

where

\[
\lambda_{\mu} = \frac{ie^2}{(2\pi)^4} \int d^4k \frac{1}{k^2} \gamma_\nu \frac{-i}{4} \lambda_{\mu} \chi_p \frac{-i}{4} \lambda_{\nu} \Delta_F^{-1} \cdot \frac{P}{2} \cdot \frac{p}{2} - q \cdot \frac{P}{2} + \frac{q}{2}.
\]

Firstly, let us consider the contribution of the \(f_1\) term in the BS wave function [4]. Using both the Dirac equation and the symmetry of integrand under permuting Feynman parameters, it is easy to show that [4] can be transformed into the same form as Eq. (7), which is expected from the conservation of electric current.

In calculating the anomalous magnetic moment \(q^2\) can be put to zero. The correction to \(F_1(0)\) from Eq. (19) is absorbed by the normalization condition \(F_1(0) = 1\). We know by examining Eq. (19) that the contribution to \(F_2(0)\) comes from the term

\[
\frac{\alpha}{\pi} m\bar{u}(P + q/2)u(P - q/2) \frac{-i}{(2\pi)^4} \int d^4p f_1^2(P, p, q = 0) \Delta_F^{-1} \frac{P}{2} - p \cdot \frac{P}{2},
\]

\[
\times \int_{x_1 + x_2 \leq 1} dx_1 dx_2 \frac{2(1 - x_1 - x_2)}{(x_1 + x_2)} \frac{(P + p)}{(1 - x_1 - x_2)} \frac{(P + p)}{(P + p)} + m^2.
\]

\(\bar{q}\)From the normalization condition [12] of the electric charge and Eq. (14), we have

\[
S_1(0) = O(1)
\]

because of \(M = O(R^{-1}) = O(m)\) in the case \(A\).

Since \(m = O(R^{-1}) \gg m_l\), \(O(m_l/m)\) terms can be neglected. When the momentum of the internal preon \(\frac{P}{2} + p\) is in the Euclidean region, the term in the square brackets in the denominator of the integrand in the second line of (21) is larger than \(m^2\). Therefore, once the wave function can be continued into the Euclidean region, as verified by Wick [14], the contribution of (21) to the anomalous magnetic moment is

\[
O(\alpha \frac{e}{2m}) = O(\alpha \frac{e}{2R^{-1}}).
\]

The conclusions will not change when the contribution from the \(f_2\) term of the BS wave function [4] is included.

Similar analysis can be carried out for the case \(B\), \(m \ll R^{-1}\), in which \(m^2\) in the denominator of (21) can be neglected. The same result as Eq. (23) is obtained when \(M = O(R^{-1})\). Nevertheless, when \(M = O(m)\), the \((\frac{m^2}{m})^2\) term dominates and instead of (22) we have

\[
\frac{T(0)}{M^2} = O(1).
\]

The contribution of the \(f_2^2\) term can be obtained by replacing \(f_1^2\) by \((\frac{m}{2})(\frac{m}{2})^2\) in (21). Using (24) we find that the contribution of Fig. 2 to the anomalous magnetic moment is \(O(\alpha \frac{m^2}{m})\).
3.2 The contributions corresponding to complex diagrams

In order to discuss more complex diagrams, we assume that the wave function has the following spectrum representation

\[
f_i(P, p) = \int_{\mu_n}^{\infty} d\mu^2 \int \frac{dy}{|p^2 - 2yp \cdot p + \mu^2 - i\epsilon|^n}, \quad i = 1, 2, \tag{25}
\]

where \( \mu_n^2 = m^2 - \frac{m^2}{4} \). The advantage of this representation is that the dependence of \( f_i \) on the momenta \( P \) and \( p \) appear directly only in the propagator-like denominator. In calculating diagrams containing B-S wave functions and additional propagators one can use Feynman parameters to combine all factors containing the momenta into a single factor and carry out the integration over the internal momenta of the diagram. This makes the dimensional estimate clear. For the purpose of illustration we use the spectrum representation to the simplest diagram, Fig. 2, discussed in last subsection. Substituting Eq. (25) into Eq. (14) and using the approximation

\[
\Delta P^{-1} \simeq \Delta P^{-1} = \left(\frac{1}{2} P - p\right)^2 + m^2, \tag{26}
\]

the contribution of \( f_1 \) to the anomalous magnetic moment becomes

\[
\alpha \int d\mu^2 d\mu^2 dy_1 dy_2 g_1(y_1, \mu_1^2)g_1(y_2, \mu_2^2)\int \prod_{i=1}^{5} dx_i \delta(1 - \sum_{i=1}^{5} x_i)
\]

\[
\times \frac{m b_1(y_1, x_1)}{a(y_1, x_1)q^2 + x_1 \mu_1^2 + x_2 \mu_2^2 + (x_3 + x_4)\mu^2 |^{2n_1 - 2}} + \frac{m^3 b_2(y_1, x_1)}{a(y_1, x_1)q^2 + x_1 \mu_1^2 + x_2 \mu_2^2 + (x_3 + x_4)\mu^2 |^{2n_1 - 1}}
\]

where \( a, b_1 \) and \( b_2 \) are rational functions of \( x_i \) and \( y_i \) whose explicit expressions are omitted here.

Using the spectrum representation (25), it is easy to obtain

\[
S_1(q^2) = 2^{-3} \pi^{-2} \frac{(2n_1 - 4)!(n_1 - 1)!}{[n_1 - 1]!} \int d\mu^2 d\mu^2 dy_1 dy_2 g_1(y_1, \mu_1^2)g_1(y_2, \mu_2^2) \int dx_1 dx_2 \delta(1 - x_1 - x_2)
\]

\[
\times \left\{ \frac{2n - 3}{2} \left[ x_1 x_2 \left( \frac{1}{2} - y_1 \right) \left( \frac{1}{2} - y_2 \right) q^2 + x_1 \mu_1^2 + x_2 \mu_2^2 \right] + 1 \right\}
\]

\[
\times \frac{1}{x_1 x_2 \left( \frac{1}{2} - y_1 \right) \left( \frac{1}{2} - y_2 \right) + x_1 \mu_1^2 + x_2 \mu_2^2 |^{2n_1 - 3}}.
\]

Comparing (27) with (28), the conclusion that the \( \alpha \) order correction of the anomalous magnetic moment from the diagram in Fig. 2 is given by (23) in the case A is obtained once again, as expected.

Similar analysis can be carried out for the case B, \( M_F \ll O(R^{-1}) \). The difference is that in the case B the contribution of the \( f_2 \) term dominates if \( M = M_F \).

When the "very strong" interaction binding preons into a lepton is considered, it will bring corrections to the electro-magnetic vertex of the lepton. Accordingly, there will appear much more complex diagrams. If we assume that the constituent fermion interacts with the constituent boson through exchanging bosons, we get complex diagrams, for example, those in Fig. 3 at the lowest
order of $\alpha$. Such diagrams can be calculated with the procedure similar to that used to obtain (23). Combining the denominators of the integrand in the electromagnetic vertex with Feynman parameters and integrating out the internal momentum, there always appear terms with a factor $(x_1\mu_1^2 + x_2\mu_2^2 + \sum_{i=3}^{N} x_im_i^2)^L$ in the denominator in the approximation $\mu_i^2 \gg m_i^2, q^2$ or $m^2 \gg m_i^2, q^2$. The corresponding numerator is a polynomial in $m_i, m$ and $\mu_i$ with the appropriate dimension. Since the effective value of $\mu_i^2$ is of the order $R^{-2}$, the order of magnitude of the denominator is controlled by the larger one of $R^{-2}$ and $m^2$. Since $L$ is increased by 1 for one additional propagator and correspondingly the order of magnitude of the polynomial in the nominator is increased by 2 from the dimensional reason, the contributions from all these diagrams to the anomalous magnetic moment are at most of the order of that from the diagram with the least number of propagators which is Fig.1 at the zeroth order of $\alpha$ and is Fig.2 at the first order of $\alpha$. If the exchanged boson is a scalar, we assume that the coupling constant $\lambda$ of three scalar particles is of $O(m)$. If the exchanged boson is a gauge boson, the derivative coupling does not change the result of the dimensional analysis here.

The above discussion can also be applied to the diagrams in which a bosonic preon is converted into a pair of fermionic preons. Under the assumption that all preons have the same mass $m$, which is used in the above discussion for simplicity, the result is of the same form as that described above.

From the above discussions we conclude that the contribution to $F_2(0)$ from any individual diagram, which contains BS wave functions and additional finite number of propagators of the preons or the particles mediating the “very strong” interaction, is suppressed by $O(\alpha R)$ or $O(\alpha m R^2)$ at the order $\alpha$. However, can the QED result for $a_\mu$ at the $\alpha$ order approximately be obtained in the composite model and how large is the deviation from the QED result? This question is answered as follows.

Now we return to the general case. Adding infinite diagrams, we will get the pole term contribution of the bound state (see Fig. 4). The four point Green function $K$ in Fig. 4 is the Fourier transform of $<0|T(\bar{\psi}(x_1)\phi(x_2)\bar{\psi}(y_1)\phi(y_2))|0>$. It can be written as

$$K(K;p',p) = \frac{\chi\bar{\chi}K(p)\bar{\chi}(p')}{K^2 + m^2} + \cdots$$

From the above discussions we conclude that the contribution to $F_2(0)$ from any individual diagram, which contains BS wave functions and additional finite number of propagators of the preons or the particles mediating the “very strong” interaction, is suppressed by $O(\alpha R)$ or $O(\alpha m R^2)$ at the order $\alpha$. However, can the QED result for $a_\mu$ at the $\alpha$ order approximately be obtained in the composite model and how large is the deviation from the QED result? This question is answered as follows.

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$$K(K;p',p) = \frac{\chi\bar{\chi}K(p)\bar{\chi}(p')}{K^2 + m^2} + \cdots$$

The terms which have not been written explicitly in Eq. (29) correspond to contributions from possible excited states of the lepton or states of free preons.

In the case A of heavy preons, these terms can be neglected when $K^2 \ll m^2 = O(R^{-2})$. In case B of light preons, these terms can be neglected when $K^2$ is much smaller than the scale of confinement, which is essentially equivalent to $K^2 \ll R^{-2}$. (If a lepton has high excited states accessible through the electromagnetic transition, it is required that $K$ is much smaller than the masses of excited states which are of the same order as $R^{-1}$.) Therefore, the contributions of these omitted terms to the anomalous magnetic moment are of the order $O(R^2)$ or $O(m^{-2})$. The sub-diagram $T$ in Fig. 4 represents a five point vertex function with two fermionic preons, two bosonic preons and the electromagnetic current. Each sub-diagram $T$ in Fig. 4 combining with two factors $f_i, f'_j$, which are associated with the two adjacent $K$ diagrams according to (29) forms an on-shell electro-magnetic vertex $\Gamma_\mu^{(0)}$ at the zeroth order in $\alpha$. Since in $F_1^{(0)}$, $F_1^{(0)} \approx 1$
and $F_2^{(0)}$ can be neglected when the photon momentum squared is much smaller than $R^{-2}$, and the contribution from the region of large momentum squared $k^2$ of the virtual photon in Fig. 4 is highly suppressed, we get the electro-magnetic vertex at the $\alpha$ order corresponding to Fig. 4.\(^{[11]}\)

$$
\Gamma^{(\alpha)}_{\mu} = \frac{ie^2}{(2\pi)^4} \int \bar{u}(P') \gamma_{\nu} \frac{-i(\hat{P}' + \hat{k}) + m_I}{(P' + k)^2 + m_I^2} \frac{-i(\hat{P} + \hat{k}) + m_I}{(P + k)^2 + m_I^2} \gamma_{\nu} \frac{d^4k}{k} u(P) + O(\alpha R).$$

(30)

Eq. (30) is the same as the radiative correction at the $\alpha$ order of the QED vertex to a point particle when $R$ is small enough.

### 4 Conclusions and discussions

The above calculations show that the anomalous magnetic moment of a composite lepton up to $e^3$ order is

$$
\mu = \frac{e}{2m_I} \left[ \frac{\alpha}{2\pi} + O(\alpha^2) + O\left(\frac{m_I}{R^{-1}}\right) \right]
$$

(31)

in the case A and it is

$$
\mu = \frac{e}{2m_I} \left[ \frac{\alpha}{2\pi} + O(\alpha^2) + O\left(\frac{m_I}{R^{-1}}\right) \right]
$$

(32)

or

$$
\mu = \frac{e}{2m_I} \left[ \frac{\alpha}{2\pi} + O(\alpha^2) + O\left(\frac{m_I m_F}{R^{-2}}\right) \right].
$$

(33)

depending on the internal dynamics in the case B.

Apparently, these results may be generalized to arbitrary order in $\alpha$ and we have

$$
a_\mu = a_{\mu}^{SM} + O\left(\frac{m_I}{R^{-1}}\right)
$$

(34)

for the case A, and

$$
a_\mu = a_{\mu}^{SM} + O\left(\frac{m_I}{R^{-1}}\right)
$$

(35)

for theories without a chiral symmetry or

$$
a_\mu = a_{\mu}^{SM} + O\left(\frac{m_I m_F}{R^{-2}}\right)
$$

(36)

for theories of the confining gauge interactions with an approximate global chiral symmetry in the case B. In Eq. (34), (35) and (36) we have assumed $R^{-1} \gg m_Z$, which is necessary when including the electro-weak corrections. In the recent literature, e.g., the refs. [2] and [3], it is stated that the corrections from compositeness to $a_\mu$ is of the order $O(\frac{m_I}{R})$. This corresponds to the case (33) if $m_F$ is of the same order as $m_I$.

Similar conclusions can be obtained if the scalar preon is charged. In contrast with the non-relativistic loose bound state, the magnetic moment of a relativistic tight bound state is not of the vectorial sum of the magnetic moments of its constituents. When the radius of the bound state is sufficiently small, the bound state behaves as a whole in electro-magnetic field, like a point particle does. Although our conclusions are obtained in a simple composite model, they depend only on the dimensional analysis and the analytical property of the BS wave function. So it is expected that the conclusions will stand also in other more complex models.
It is obvious from Eqs. (34), (35) and (36) that the observed deviation of $a_{\mu}$ can be a demonstration of the substructure of the muon and give a constraint on the radius of the muon or the masses of preons. From the theoretical Eqs. (34), (35), (36) and the experimental results, we get

$$R^{-1} \geq 10^8 m_{\mu}$$  \hspace{1cm} (37)

for theories without an approximate chiral symmetry, or

$$R^{-1} \geq 10^4 m_{\mu}$$  \hspace{1cm} (38)

for composite leptons bound by gauge interactions with an approximate global chiral symmetry if $m_F$ is of the same order as $m_l$.

Surely, when the E821 result is regarded as the consequence of the substructure of leptons, it is necessary to re-analyze the features of leptons, especially their decay characters, in composite models and probe the fundamental theory which governs the dynamic of composite models in the deeper layer of the structure of matter. One has started the research in the direction [17].

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Figure 1: The lowest order diagram of the electro-magnetic interaction of a composite lepton in the composite model of leptons. The solid (dashed) line represents the fermionic (bosonic) preon and the wave line represents the photon.
Figure 2: The $e^3$ order diagram of the electro-magnetic interaction of a composite lepton in the composite model of leptons.

Figure 3: Some examples of complex diagrams. The dotted line represents the boson which mediates the "very strong" interaction.
Figure 4: The diagram corresponding the contribution of the bound state pole term. The circles with "T" and "K" (or "K'") inside it denote the electromagnetic vertex of the bound state at the lowest order in $\alpha$ when $K^2 (or K'^2) = m^2$ and the 4-point Green function respectively. $K = P + k$ and $K' = P' + k$ in the diagram.