Analytic Solution for Tachyon Condensation in Berkovits’ Open Superstring Field Theory

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Abstract

We present an analytic solution for tachyon condensation on a non-BPS D-brane in Berkovits’ open superstring field theory. The solution is presented as a product of $2 \times 2$ matrices in two distinct $GL_2$ subgroups of the open string star algebra. All string fields needed for computation of the nonpolynomial action can be derived in closed form, and the action produces the expected non-BPS D-brane tension in accordance with Sen’s conjecture. We also comment on how D-brane charges may be encoded in the topology of the tachyon vacuum gauge orbit.

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1 Introduction

Tachyon condensation on unstable D-branes has always been a challenging problem to study, both because the phenomenon is intrinsically “stringy”—the tachyon mass is proportional to $\frac{1}{\alpha'}$—and because the existence of a stable ground state for the tachyon is difficult to see from the perturbative S-matrix. While many techniques have been employed to tackle this problem (for a review see [1]), perhaps the most direct and complete approach uses the formalism of open string field theory. For many years, open string field theory provided mostly a numerical understanding of the tachyon ground state (the “tachyon vacuum”) in the level truncation scheme [2] [3] [4] [5] [6] [7] [8] [9]. Then, in 2005 Schnabl [10] found an exact solution for the tachyon ground state in Witten’s open bosonic string field theory [11], providing exact formulae for the infinite set of scalar expectation values that arise upon tachyon condensation. With these results it was possible to prove that the missing energy at the tachyon vacuum exactly corresponds to the tension of the unstable D-brane, and that the vacuum supports no open string excitations [12], precisely as conjectured by Sen [13] [14] [15].

Since then, there has been considerable interest in extending Schnabl’s results to the superstring. This is not just a matter of principle, but the physics of tachyon condensation is much more interesting for the superstring, revealing a rich spectrum of stable BPS and
non-BPS ground states as solitons of the string field. Progress on this front however requires an analytic solution for the tachyon vacuum in Berkovits’ nonpolynomial open superstring field theory \cite{16, 17}.\footnote{A superstring tachyon vacuum solution was found in \cite{18} using the modified cubic superstring field theory of \cite{19, 20}. This solution will play an important role in our analysis, but, for several reasons, was not considered to be a definitive solution to the problem of tachyon condensation.} Despite many attempts\footnote{The first attempt at analytic solution for the tachyon vacuum was initiated by Berkovits and Schnabl \cite{21}, followed by proposals by Fuchs and Kroyter \cite{22}, the author in collaboration with Schnabl \cite{23}, and possibly others. These solutions proved either to be singular or computationally intractable.}, no tractable analytic solution has been found. In this paper we would like to finally propose such a solution.

The solution is constructed as a product of two factors. Each factor belongs to a distinct subgroup of the open string star algebra which is isomorphic to the group of invertible $2 \times 2$ matrices. The first factor has matrix entries belonging to the abelian algebra of wedge states \cite{24}, and the second factor has matrix entries belonging to the abelian algebra of wedge states deformed by a nonconformal boundary interaction related to the condensation of the zero momentum tachyon. Like the (closely related) bosonic tachyon vacuum of \cite{25}, the solution requires no explicit regularization or phantom term, and the simple algebraic structure makes it possible to derive all nonpolynomial expressions needed for computation of the action in closed form. Evaluating the action recovers the expected tension of a non-BPS D-brane. As an added bonus, the solution gives a hint as to how D-brane charges may be encoded in the topology of the tachyon vacuum gauge orbit.

This paper is organized as follows. In section 2 we review Berkovits’ formulation of open superstring field theory with an emphasis on concepts which are important for analytic considerations. In section 3 we introduce and motivate the subalgebra of states which we will use to formulate the tachyon vacuum solution. In sections 4 and 5, we introduce the solution, discuss its basic structure, and prove the equations of motion. In section 6 we compute the nonpolynomial action to derive the tension of the non-BPS D-brane, and in section 7 we compute the expectation value of the tachyon coefficient at the tachyon vacuum. In section 8, we argue that stability of the codimension 1 kink on a non-BPS D-brane implies that the tachyon vacuum gauge orbit comes in two disconnected pieces, related by a topologically nontrivial gauge transformation. We show how the analytic solution provides some preliminary evidence in favor of this conjecture. We end with some conclusions.

## 2 Berkovits’ Superstring Field Theory

Here we review the basics of Berkovits’ open superstring field theory \cite{16, 17}. The theory uses the RNS formalism to describe the off-shell dynamics of an open superstring in the Neveu-Schwartz (NS) sector.\footnote{Extensions of the action to the Ramond sector are described in \cite{26, 27, 28}.} The string field is

$$\Phi = \text{Lie algebra element}. \quad (2.1)$$
We call this the “Lie algebra element,” for reasons which will be clear in a moment. The Lie algebra element $\Phi$ is a Grassmann even, ghost and picture number zero state the NS state space of an open superstring quantized in a specifically chosen D-brane background. As always in string field theory, the string field $\Phi$ represents fluctuations of the D-brane system relative to the chosen background. For our calculations, we will work on a non-BPS D-brane of Type II superstring theory. The extension to a brane/antibrane pair is straightforward. Note that, unlike in the bosonic string, for the superstring the endpoint of tachyon condensation on a generic unstable brane system may not be universal, and needs to be constructed in a case-by-case basis. For example, in this paper we do not construct the closed string vacuum on a separated brane/antibrane system [29].

In the Berkovits theory it is necessary to bosonize the $(\beta, \gamma)$ ghosts of the RNS formalism, following [30]:

$$\beta(z) = \partial \xi e^{-\phi}(z), \quad \gamma(z) = \eta e^{\phi}(z).$$

(2.2)

Importantly, the string field $\Phi$ is in the “large” Hilbert space, that is, the Hilbert space which includes states proportional to the zero mode of the $\xi$ ghost. In particular, this means

$$\eta \Phi \neq 0 \quad \text{(in general)},$$

(2.3)

where $\eta \equiv \eta_0$ is the zero mode of the $\eta$ ghost. Both the $\eta$ zero mode and the BRST charge $Q \equiv Q_B$ have trivial cohomology in the large Hilbert space as a result of the existence of operators satisfying

$$\eta \cdot \xi(z) = 1,$$

(2.4)

$$Q \cdot \left[ c \xi \partial \xi e^{-2\phi}(z) \right] = 1.$$  

(2.5)

Therefore in the large Hilbert space the perturbative spectrum is not given by the cohomology of $Q$. Rather, the perturbative spectrum comes from solutions to the linearized equations of motion,

$$\eta Q \Phi = 0,$$

(2.6)

modulo the linearized gauge invariance,

$$\Phi' = \Phi + Q\Lambda + \eta\Pi.$$

(2.7)

Using this gauge symmetry, we can write physical states in the standard form,

$$\Phi \sim \xi ce^{-\phi}O^m(0),$$

(2.8)

where $O^m(z)$ is a superconformal matter primary of dimension $1/2$. Note that if we drop the $\xi$, this is the same as the vertex operator for an on-shell state in the $-1$ picture.

Berkovits’ string field theory is constructed using Witten’s associative star product and open string integration [11]. Usually we will write the star product without the star

\footnote{We follow the ghost and picture number assignment conventions of [5].}
\(AB \equiv A \ast B\), and we write Witten’s integration as a trace \(\text{Tr}[\cdot]\) to avoid confusion with other integrations. The product, the trace, and the differentials \(\eta\) and \(Q\), satisfy the usual “axioms”:

**Nilpotency:** \(Q^2 = \eta^2 = [Q, \eta] = 0;\)

**Derivation:** \(Q(AB) = (QA)B + (-1)^A A(QB),\)
\(\eta(AB) = (\eta A)B + (-1)^A A(\eta B);\)

**Integration by parts:** \(\text{Tr}[QA] = \text{Tr}[\eta A] = 0;\)

**Associativity:** \(A(BC) = (AB)C;\)

**Cyclicity:** \(\text{Tr}[AB] = (-1)^{AB} \text{Tr}[BA];\) (2.9)

where \(A, B\) and \(C\) are generic NS string fields. Though it is not strictly necessary, we will also freely assume the existence of a star algebra identity (the identity string field) \(1 \equiv |I\rangle\) satisfying

\[1 \ast A = A \ast 1 = A.\] (2.10)

Since all nonzero correlators reduce to

\[\langle \xi(z) c \partial c \partial^2 c(w)e^{-2\phi(y)} \rangle \equiv 2,\] (2.11)

the trace \(\text{Tr}[\cdot]\) is only nonvanishing on states with ghost number two and picture number minus one.

Berkovits’ string field theory is defined by a Wess-Zumino-Witten-like action \([31]\) for the Lie algebra element \(\Phi\). To write the action, it is helpful to define a “group element” \(g\) by exponentiating \(\Phi:\)

\[g = e^\Phi = \text{group element}.\] (2.12)

The group element has a star algebra inverse,

\[g^{-1} = e^{-\Phi}.\] (2.13)

Usually the group element \(g\) is a more natural field variable for analytic calculations. However, unlike \(\Phi\), the group element \(g\) is constrained by the requirement that it must have an inverse. To write the WZW-like action, we need to introduce an (arbitrary) continuous 1-parameter family of group elements \(g(t), \ t \in [0,1]\) interpolating between the identity string field 1 and the dynamical field \(g:\)

\[g(0) = 1; \ g(1) = g = e^\Phi.\] (2.14)

\[^6\text{In general (2.11) should be multiplied by a normalization for the matter correlator. For our purposes it is convenient to set this normalization to one. We also set } \alpha' = 1.\]
Next we define three “connections”

\[ \Psi_Q \equiv g(t)^{-1}Qg(t), \quad \Psi_\eta \equiv g(t)^{-1}\eta g(t), \quad \Psi_t \equiv g(t)^{-1}\partial_t g(t). \]  

(2.15)

Then the (standard) WZW-like action takes the form\[\footnote{We set the open string coupling constant to 1.}^7\]

\[ S = -\frac{1}{2} \int_0^1 dt \ Tr \left( \partial_t(\Psi_\eta \Psi_Q) + \Psi_t[\Psi_\eta, \Psi_Q] \right), \]  

(2.16)

where \[\text{[}A, B\text{]} \equiv AB - (-1)^{AB} BA \] is the graded commutator. We will find it useful to work with a different form of the action, introduced by Berkovits, Okawa, and Zwiebach\[\footnote{To be precise, the action is \textit{locally} independent of the choice of interpolation \(g(t)\) between 1 and \(g\). We will ignore the possibility that there might be distinct homotopy classes of interpolations.}^8\]:

\[ S = -\int_0^1 dt \ Tr[(\eta \Psi_t)\Psi_Q]. \]  

(2.17)

Though it is not manifest, the action is independent of the choice of interpolation \(g(t)\) provided the boundary conditions (2.14) at \(t = 0\) and \(t = 1\) are held fixed.\[\footnote{Therefore, the action only depends on \(g\), or equivalently, the Lie algebra element \(\Phi\). The action is invariant under infinitesimal gauge transformations,}

\[ g' = g + v g + g u, \]  

(2.18)

where \(Qv = 0\) and \(\eta u = 0\). The finite gauge transformation takes the form

\[ g' = V g U, \]  

(2.19)

where \(V\) and \(U\) are \(Q\)- and \(\eta\)-closed group elements, respectively. Finally, the stationary points of the action satisfy

\[ \eta(g^{-1}Qg) = 0. \]  

(2.20)

These are the classical equations of motion.

It can be tricky to prove gauge invariance and derive the equations of motion from the action. Let’s briefly explain how this is done, following closely\[\footnote{Given any derivation}^3\]. Given any derivation \(D\) we can define a connection,

\[ \Psi_D \equiv g(t)^{-1}Dg(t). \]  

(2.21)

By construction this is a flat connection, so for any pair of (anti)commuting derivations \(D_1\) and \(D_2\), the associated field strength must vanish. This implies

\[ D_1 \Psi_{D_2} = (-1)^{D_1 D_2} D_2' \Psi_{D_1}, \]  

(2.22)

where the prime denotes the covariant derivative:

\[ D' \equiv D + [\Psi_D, ·]. \]  

(2.23)
Let’s denote the variational derivative by $\delta$. With a little algebra one can prove the identity:

$$\delta \{ \eta \Psi_t, \Psi_Q \} - \eta \{ \delta \Psi_t, \Psi_Q \} = -\partial_t \{ Q' \Psi_\delta, \Psi_\eta \} + Q' \{ \partial_t \Psi_\delta, \Psi_\eta \},$$  \hspace{1cm} (2.24)

where $\{ A, B \} \equiv AB - (-1)^{AB} BA$ is the graded anticommutator. Evaluating the trace and integrating $t$ from 0 to 1, the left hand side of (2.24) gives (twice) the variation of the action (2.17). The right hand side gives, after integrating the total $t$ derivative,

$$\delta S = \text{Tr} \left[ \Psi_\delta (\eta \Psi_Q) \right] \bigg|_{t=1}.$$  \hspace{1cm} (2.27)

In this way we see that the action depends only on the value of $g(t)$ at $t = 1$. The trace with $\Psi_\delta$ is nondegenerate, so setting the variation of $S$ to zero implies the equations of motion $\eta \Psi_Q |_{t=1} = 0$. Under the infinitesimal gauge transformation (2.18), $\Psi_\delta$ changes as

$$\Psi_\delta |_{t=1} = \eta(\text{something}) + Q'(\text{something}).$$  \hspace{1cm} (2.28)

Integration by parts and nilpotency of $\eta$ and $Q'$ then demonstrates gauge invariance of the action.

Let’s explain how to expand the Berkovits theory around a classical solution. We write the group element as the product of two factors:

$$g = g_0 \tilde{g}.$$  \hspace{1cm} (2.29)

Here the factor $g_0$ is a classical solution which shifts from the perturbative vacuum to our new reference background, and the factor $\tilde{g}$ describes fluctuations of the field relative to the background set by $g_0$. To plug this into the action we must choose a family of group elements $g(t)$ which interpolates from 1 to $g_0 \tilde{g}$. With a reparameterization we can expand the range of $t$ from 0 to 2, and then choose an interpolation satisfying the conditions

$$g(0) = 1; \quad g(1) = g_0; \quad g(2) = g = g_0 \tilde{g}.$$  \hspace{1cm} (2.30)

See figure 2.1. Plugging this into (2.17) gives

$$S[g] = S[g_0] - \int_1^2 dt \text{ Tr}[(\eta \Psi_t) \Psi_Q].$$  \hspace{1cm} (2.31)

The $t \in [0, 1]$ region of integration gives the action evaluated on the reference solution $g_0$. For $t \in [1, 2]$ we can further simplify by writing $g(t)$ in the form:

$$g(t) = g_0 \tilde{g}(t), \quad t \in [1, 2].$$  \hspace{1cm} (2.32)

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\[9\] This is a special case of the identity

$$R(D_1, D_2, D_3, D_4) = (-1)^{(D_1 + D_2)(D_3 + D_4)} R(D'_3, D'_4, D_1, D_2),$$  \hspace{1cm} (2.25)

where

$$R(D_1, D_2, D_3, D_4) \equiv D_1 \{ D_2 \Psi_{D_3}, \Psi_{D_4} \} - (-1)^{D_1 D_2} D_3 \{ D_1 \Psi_{D_3}, \Psi_{D_4} \}.$$  \hspace{1cm} (2.26)

Note also that $R$ is graded antisymmetric upon interchange of the first two or last two entries.
where $\tilde{g}(t)$ interpolates from the identity 1 to the fluctuation $\tilde{g}$. The $t$-connection $\Psi_t$ evaluated on $g(t)$ is the same as that on $\tilde{g}(t)$, since the constant factor $g_0$ cancels out:

$$\Psi_t[g(t)] = \Psi_t[\tilde{g}(t)].$$

(2.33)

The $Q$-connection is evaluated as

$$\Psi_Q[g(t)] = \tilde{g}(t)^{-1}(Q + \Psi_0)\tilde{g}(t) = \Psi_{Q\Psi_0}[\tilde{g}(t)] + \Psi_0.$$  

(2.34)

Here

$$\Psi_0 \equiv g_0^{-1}Qg_0$$

(2.35)

is a solution to the Chern-Simons-like equations of motion of cubic superstring field theory [19, 20]

$$Q\Psi_0 + \Psi_0^2 = 0,$$

(2.36)

and

$$Q\Psi_0 \equiv Q + [\Psi_0, \cdot]$$

(2.37)

is the kinetic operator expanded around the cubic solution $\Psi_0$. Since $g_0$ is a solution, $\Psi_0$ is in the small Hilbert space and the second term in (2.34) drops out when we plug into the action. Now (implicitly) understanding that the connections are evaluated on $\tilde{g}(t)$ rather than $g(t)$, and shifting the range of $t$ back to $t \in [0, 1]$, the action becomes

$$S = S[g_0] - \int_0^1 dt \text{Tr}[(\eta \Psi_t)\Psi_{Q\Psi_0}].$$

(2.38)

Thus the effect of expanding around $g_0$ is to add a constant to the action and to replace $Q$ with the kinetic operator $Q\Psi_0$ around the shifted background.

For states in the GSO($-$) sector the above discussion requires minor clarification. The problem is that the zero momentum tachyon vertex operator $\xi ce^{-\phi}$ is Grassmann odd,
while the string field $\Phi$ must be Grassmann even to ensure gauge invariance. We solve this problem this by multiplying the string field with the appropriate Pauli matrix—an “internal” Chan-Paton factor—determined by the vertex operator’s Grassmann parity $\epsilon$ and its worldsheet spinor number $F$ [5]:

| $\epsilon$ | $F$ | CP factor  |
|-----------|-----|------------|
| 0         | 0   | $I$        |
| 1         | 0   | $\sigma_3$|
| 0         | 1   | $\sigma_2$|
| 1         | 1   | $\sigma_1$|

For consistency, we also require that all operators acting on the string field carry their own internal CP factor according to this table. This means that $Q$ and $\eta$ must be multiplied by $\sigma_3$, though in the following we will not write this CP factor explicitly, absorbing it into the definition of $Q$ and $\eta$. We also assume that the string field trace $\text{Tr}[\cdot]$ includes an implicit factor of $1/2$ times a trace over internal CP factors. To see how this prescription solves the problem with GSO($-$) states, it is useful to introduce the concept of *effective Grassmann parity* [33]:

$$E \equiv \epsilon + F \pmod{2}. \quad (2.39)$$

Effective Grassmann parity helps keep track of signs when commuting string fields and their associated CP factors past each other. In fact, the requisite sign can be described by a “double graded” commutator,

$$[A, B] \equiv AB - (-1)^{E(A)E(B)+F(A)F(B)} BA. \quad (2.40)$$

This means that the algebra of string fields on a non-BPS D-brane is like an algebra of matrices whose entries contain two *mutually commuting* types of Grassmann number. The first type has Grassmannality measured by $E$ and the second type by $F$. Effective Grassmann parity enters the string field theory axioms (2.9) through the relations

$$\eta(AB) = (\eta A)B + (-1)^{E(A)E(B)+F(A)F(B)} A(\eta B); \quad (2.41)$$
$$Q(AB) = (QA)B + (-1)^{E(A)E(B)} A(QB); \quad (2.42)$$
$$\text{Tr}[AB] = (-1)^{E(A)E(B)} \text{Tr}[BA]. \quad (2.43)$$

Note that permuting the trace does not produce a sign from the parity of $F$, contrary to what we might expect from (2.40). This is because the half integer conformal dimension of vertex operators in the GSO($-$) sector produces an anomalous sign when permuting the conformal maps defining the Witten vertex [5]. For this reason, worldsheet spinor
number plays no role in establishing gauge invariance of the action. This means we can incorporate GSO(−) states by assuming that the string field is effective Grassmann even

\[ g = e^\Phi = \text{Effective Grassmann even}. \] (2.44)

The zero momentum tachyon \( \xi ce^{-\phi} \) is "effectively" Grassmann even, even though it’s a Grassmann odd operator. Since worldsheet spinor number \( F \) does not appear in the string field theory axioms, the WZW-like action uses a commutator which is graded only with respect to effective Grassmann parity

\[ [A, B] \equiv AB - (-1)^{E(A)E(B)} BA. \] (2.45)

This is the commutator which appears in the Wess-Zumino term of (2.16), and in the shifted kinetic operator when expanding the action around a nontrivial solution (2.37).

3 Algebra

The tachyon vacuum is constructed by taking star products of five string fields:

\[ K \rightarrow \mathbb{1} \otimes K = \mathbb{1} \otimes \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} T(z), \]

\[ B \rightarrow \sigma_3 \otimes B = \sigma_3 \otimes \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} b(z), \]

\[ c \rightarrow \sigma_3 \otimes c(z), \]

\[ \gamma \rightarrow \sigma_2 \otimes \gamma(z) = \sigma_2 \otimes \eta e^\phi(z), \]

\[ \gamma^{-1} \rightarrow \sigma_2 \otimes \gamma^{-1}(z) = \sigma_2 \otimes e^{-\phi}\xi(z). \] (3.1)

Here we use the algebraic formalism of Okawa [34], where the string fields \( K, B, c, \gamma \) and \( \gamma^{-1} \) represent corresponding operator insertions (with internal CP factors) in correlation functions on the cylinder. To review, we can visualize the definition of \( K, B, c, \gamma, \gamma^{-1} \) using the Schrödinger representation, as functionals defined by a worldsheet path integral on a semi-infinite vertical "strip" with boundary conditions on its vertical edges corresponding to the left and right halves of the open string. Specifically, \( K, B, c, \gamma \) and \( \gamma^{-1} \) are defined by a path integral on an infinitesimally thin strip containing the appropriate operator insertion, as shown in figure 3.111. Star multiplication glues the right boundary of the

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10 We learned the notation \( \gamma^{-1} \) from N. Berkovits and M. Schnabl.

11 Explicit Fock space expansions of \( K, B, c \), and by extension \( \gamma, \gamma^{-1} \), can be found from other equivalent definitions, for example those provided in [35, 36, 37].
Figure 3.1: The wedge state $\Omega^\alpha$ and the remaining four fields in (3.1) as semi-infinite strips with operator insertions. The arrows on the vertical edges indicate the direction of the parameterization of the open string $\sigma \in [0, \pi]$. The first strip to the left boundary of the second strip and the trace glues the left and right boundaries to form a correlation function on the cylinder. If we assume the sliver coordinate frame \[38\] the field $K$ generates the algebra of wedge states \[24, 40\], in that any star algebra power of the $SL(2, \mathbb{R})$ vacuum $\Omega \equiv |0\rangle$ can be written

$$\Omega^\alpha = e^{-\alpha K}, \quad \alpha \geq 0. \quad (3.2)$$

A wedge state $\Omega^\alpha$ represents a semi-infinite strip of width $\alpha$, as shown in figure 3.1. The fields $K, B, c, \gamma, \gamma^{-1}$ come with a list of quantum numbers summarized in table \[4\].

Let’s explain why this algebraic setup is relevant for the problem of tachyon condensation. The fields $K, B$ and $c$ in (3.1) appear in Schnabl’s solution \[10\] and related solutions \[34, 35, 41, 25\] for the tachyon vacuum in the bosonic string. The superstring tachyon vacuum is related to these solutions though equation (2.35), and for this reason we will need $K, B$ and $c$ as well. However, we also need two additional string fields, $\gamma$ and $\gamma^{-1}$. They live in the GSO($-$) sector, and are required to give an expectation value to the tachyon on the non-BPS D-brane. In particular, the zero momentum tachyon can be written

$$\gamma^{-1}c \sim \xi ce^{-\phi(0)}. \quad (3.3)$$

Commuting this with $B$ gives $\gamma^{-1}$. The BRST variation of $c$ generates $\gamma^2$, so once we have $\gamma^{-1}$ we need $\gamma$ as well. Therefore the fields $K, B, c, \gamma$ and $\gamma^{-1}$ give the minimum

\[12\] We use the left handed star product convention \[25\]. Operator insertions inside the correlator and internal CP factors are multiplied in the order of star multiplication. See appendix A of \[33\] for a description of various signs related to the GSO($-$) sector in the left handed convention.

\[13\] The choice of coordinate frame corresponds to the choice of parameterization of the string on the left and right boundaries of the strip \[39, 35\]. This choice is only relevant for our computation of the tachyon coefficient in section \[7\].
The fields (3.1) satisfy a number of algebraic relations:

\[ B^2 = c^2 = 0, \quad \gamma \gamma^{-1} = \gamma^{-1} \gamma = 1; \]

\[ [K, B] = 0, \quad [B, c] = 1, \quad [B, \gamma] = 0, \quad [B, \gamma^{-1}] = 0; \]

\[ [K, \text{anything}] = \partial(\text{anything}); \]

\[ \llbracket (c \text{ and/or gamma ghosts}), (c \text{ and/or gamma ghosts}) \rrbracket = 0. \quad (3.4) \]

The second to the last equation means that the commutator of \( K \) with a string field computes the worldsheet derivative of its corresponding operator insertion in correlation functions on the cylinder. The last equation means that any two string fields made from products of \( c, \gamma, \gamma^{-1} \) and worldsheet derivatives thereof always commute in the sense of

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Table 1: Table of useful quantum numbers for fields (3.1) and (3.8). “Scaling dimension” refers to the eigenvalue under the scaling generator in the sliver frame \( L \); “Reality” refers to the eigenvalue under reality conjugation \( \mathcal{L}^- \) (with real or imaginary meaning \( A^+ = \pm A \)); Twist refers to the eigenvalue under twist conjugation \( A^\delta \equiv e^{i\pi L_0} A \).
the double bracket (2.40) \footnote{The last equation seems analogous to the “auxiliary identities” in the $K, B, c$ subalgebra of the bosonic string \cite{12}. However, this analogy is imprecise since there are no automorphisms of the $K, B, c, \gamma, \gamma^{-1}$ subalgebra like those in the $K, B, c$ subalgebra \cite{14, 12} which preserve “fundamental” but not “auxiliary” algebraic relations. Such automorphisms however can be defined on a larger subalgebra generated by products of $G, B, c, \gamma^2, \alpha = -c\gamma^{-2}$, considered in \cite{33}. As discussed in \cite{45, 46, 47, 48, 49}, these automorphisms may have applications in the search for multiple brane solutions \cite{50, 51}.} We have BRST variations:

$$QK = 0,$$
$$QB = K,$$
$$Qc = c\partial c - \gamma^2,$$
$$Q\gamma = c\partial\gamma - \frac{1}{2}\partial c\gamma,$$

$$Q\gamma^{-1} = c\partial\gamma^{-1} + \frac{1}{2}\partial c\gamma^{-1}. \quad (3.5)$$

Note the order of multiplication of $c$ and $\gamma, \gamma^{-1}$ matters in these equations, since $\gamma$ and $\gamma^{-1}$ are effective Grassmann odd (despite the fact that the $\gamma$-ghost is bosonic). All fields are annihilated by the eta zero mode except for $\gamma^{-1}$:

$$\eta\gamma^{-1} \neq 0. \quad (3.6)$$

The field $\eta\gamma^{-1}$ is outside the $K, B, c, \gamma, \gamma^{-1}$ subalgebra. Note that $\eta\gamma^{-1}$ has singular OPE with $\gamma^{-1}$, so we must be careful how it appears in star products with other states.

It is useful to introduce two composite string fields:

$$\zeta \equiv \gamma^{-1}c \rightarrow i\sigma_1 \otimes \zeta(z) \equiv i\sigma_1 \otimes \gamma^{-1}c(z), \quad (3.7)$$
$$V \equiv \frac{1}{2}\gamma^{-1}\partial c \rightarrow i\sigma_1 \otimes V(z) \equiv i\sigma_1 \otimes \frac{1}{2}\gamma^{-1}\partial c(z). \quad (3.8)$$

The first field $\zeta$ is the zero momentum tachyon. The second field $V$ can be interpreted as a kind of “integrated vertex operator” associated with the zero momentum tachyon. To see why, consider the relation

$$Q\zeta = cV + \gamma. \quad (3.9)$$

If $\zeta$ were an on-shell state of the form (2.8), then the operator multiplying $c$ above would be the integrated vertex operator which generates a boundary deformation of the worldsheet action associated with this on-shell state. Of course $\zeta$ is off-shell, but $V$ can still be viewed as an “integrated vertex operator” generating a nonconformal boundary deformation on the worldsheet. In a moment we will see how this interpretation is borne out in the
solution. The fields satisfy some useful identities:

\[ [B, Q\zeta] = V, \]
\[ [B, Qc] = \partial c, \]
\[ \zeta^2 = 0 \]
\[ c\zeta = \zeta c = 0, \]
\[ \gamma\zeta = -\zeta\gamma = c, \]
\[ (Q\zeta)\zeta = -\zeta(Q\zeta) = c, \]
\[ (Qc)\zeta = \zeta(Qc) = -\gamma c, \]
\[ (Q\zeta)c = -c(Q\zeta) = \gamma c, \]
\[ (Q\zeta)^2 = -Qc. \] (3.10)

These identities follow immediately from (3.8), (3.4) and (3.5), but appear often enough in computations to be worth remembering.

4 Solution

The tachyon vacuum solution takes the form

\[ g = (1 + \zeta) \left( 1 + Q\zeta \frac{B}{1 + K} \right), \] (4.1)

or, in terms of the inverse group element,

\[ g^{-1} = \left( 1 - Q\zeta \frac{B}{1 + K + V} \right) (1 - \zeta). \] (4.2)

See figure[4.1] for a worldsheet picture of the solution. A Berkovits solution always defines a corresponding solution to the Chern-Simons equations of motion (2.36), and in our case the solution is:

\[ \Psi = g^{-1}Qg = c - Qc\frac{B}{1 + K}. \] (4.3)

This is the “simple” analytic solution for tachyon condensation found in [25]. Since \( \Psi \) is in the small Hilbert space, (4.3) implies that \( g \) satisfies the Berkovits equations of motion.

\[ \text{\footnotesize \cite{25} The solution of \cite{25} was proposed in the context of Witten’s open bosonic string field theory, but it translates to the Chern-Simons superstring essentially unchanged \cite{18} \cite{52} \cite{37}.} \]
Figure 4.1: Worldsheet picture of the solution (4.1) and (4.2) as strips appearing inside correlation functions on the cylinder.

We will give a more detailed demonstration in section 5. Note that we can automatically obtain another tachyon vacuum solution by making a parity flip \((-1)^F\) in the GSO\((-\)\) sector. This is the solution on the “other side” of the perturbative vacuum in the tachyon effective potential.

A characteristic feature of this solution is the presence of a peculiar nonconformal boundary interaction generated by \(V\), as can be seen in the expression for \(g^{-1}\) in equation (4.2). Let’s explain this in more detail. The factor \(\frac{1}{1+K}\) can be defined using the Schwinger parameterization as an integral over wedge states:

\[
\frac{1}{1+K} = \int_0^\infty d\alpha \ e^{-\alpha} \Omega^\alpha. \tag{4.4}
\]

Likewise, the factor \(\frac{1}{1+K+V}\) which appears in the inverse group element (4.2) can be defined as an integral over “deformed” wedge states:

\[
\frac{1}{1+K+V} = \int_0^\infty d\alpha \ e^{-\alpha} e^{-\alpha(K+V)}. \tag{4.5}
\]

The “deformed” wedge state \(e^{-\alpha(K+V)}\) corresponds to a strip of width \(\alpha\) carrying an infinite number of boundary insertions of \(V\). As shown in [53], the insertions arrange
themselves in such a way as to add a boundary coupling to the worldsheet action. In our case the boundary coupling inserts a nonlocal exponential insertion of the form:

\[ e^{-\alpha(K+V)} \rightarrow \mathcal{P} \exp \left[-i\sigma_1 \otimes \frac{1}{2} \int_0^\alpha ds \gamma^{-1} \partial c(s) \right]. \] (4.6)

where we assume that the strip of width \( \alpha \) has its right vertical edge aligned with the imaginary axis \( \text{Re}(z) = 0 \). The insertion in (4.6) is path ordered since \( \gamma^{-1} \partial c \) is a fermionic operator. We define the ordering in sequence of decreasing position on the real axis

\[ \mathcal{P} \gamma^{-1} \partial c(s_1) \gamma^{-1} \partial c(s_2) \ldots \gamma^{-1} \partial c(s_n) \equiv \gamma^{-1} \partial c(s_{i_1}) \gamma^{-1} \partial c(s_{i_2}) \ldots \gamma^{-1} \partial c(s_{i_n}), \]

\( (s_{i_1} > s_{i_2} > \ldots > s_{i_n}) \). (4.7)

An interesting property of this boundary interaction is that it is BRST invariant. In particular, we have the property

\[ Q e^{-\alpha(K+\lambda V)} = -\lambda(Q\zeta + \lambda c)e^{-\alpha(K+\lambda V)} + e^{-\alpha(K+\lambda V)} \lambda(Q\zeta + \lambda c), \] (4.8)

where \( \lambda \) is the coupling constant of the deformation. This means that the only contribution to the BRST variation occurs at the interface between the deformed and undeformed boundary condition. This might suggest that we could define the boundary interaction in terms of boundary condition changing operators \[53\], but this language is not quite appropriate since the boundary interaction is nonconformal. Note that the conservation of \( bc \) ghost number implies that the boundary interaction contributes at most a finite number of insertions to any particular correlator. This means that (4.2) is a manifestly finite and explicitly computable state in the Fock space expansion.

The Berkovits solution has a number of formal similarities with the “simple” solution (4.3) of the bosonic string. One curious similarity is that both solutions can be written as a linearized gauge transformation of the zero momentum tachyon expressed in a particular gauge (specifically, the dressed-Schnabl gauge of \[25\]). This can be seen from the expressions

\[ \Psi = c \frac{1}{1+K} - Q \left(c \frac{B}{1+K} \right), \]

\[ g = 1 + \zeta \frac{1}{1+K} + Q \left(\zeta \frac{B}{1+K} \right) - c \frac{B}{1+K}. \] (4.10)

Here \( c \frac{1}{1+K} \) is the zero momentum tachyon of the bosonic string, and the second term in (4.9) is BRST exact. The state \( 1 + \zeta \frac{1}{1+K} \) represents a deformation of the perturbative vacuum 1 by the zero momentum tachyon \( \zeta \frac{1}{1+K} \), and the last two terms in (4.10) are \( Q \)- and \( \eta \)-exact, respectively. Another similarity is that \( g \) and \( \Psi \) do not need to be defined with a regularization and “phantom term” \[54\], unlike Schnabl’s solution for the bosonic
string [10, 25]. While this is an advantage, the down side is that these solutions are close to being singular from the perspective of the identity string field. This observation can be formalized in the dual $\mathcal{L}^-$ level expansion [42], where $g$ takes the form
\[
g = 1 + Q \left( \frac{B}{K} \right) + \text{lower levels}. \tag{4.11}
\]
Taking the logarithm gives the dual $\mathcal{L}^-$ level expansion of the Lie algebra element:
\[
\Phi = Q \left( \frac{B}{K} \right) + \text{lower levels}. \tag{4.12}
\]
Consulting table 1, we see that the leading level in this expansion is $-\frac{1}{2}$. Since the trace of the Lie algebra element can be used to define the on-shell part of the boundary state [55, 56], this is the highest half-integer level consistent with a regular solution [42]. So, in a sense, the Berkovits solution (4.1) is as identity-like as possible given constraints of regularity.

For later analysis it will be useful to consider a slight generalization of the solution (4.1). To obtain this generalization, note that since $\Psi$ is in the GSO(+) sector, both the GSO(+) part and the GSO(−) part of $g$ satisfy
\[
Qg_+ = g_+\Psi, \quad Qg_- = g_-\Psi. \tag{4.13}
\]
This almost implies that $g_+$ and $g_-$ are separately solutions to the equations of motion, except (it turns out) that $g_+$ and $g_-$ are not invertible. Still, we can try to form a solution by taking a linear combination of $g_+$ and $g_-:
\[
Q(pg_+ + qg_-) = (pg_+ + qg_-)\Psi. \tag{4.14}
\]
Imposing regularity of $\Phi$ in the dual $\mathcal{L}^-$ expansion requires $p = 1$. Since $g_+$ is not by itself a solution, we must add some $g_-$ with $q \neq 0$. This defines a class of tachyon vacuum solutions generalizing (4.1):
\[
g = (1 + q\zeta) \left( 1 - (1 - q^2)c\frac{B}{1 + K} + qQ\zeta\frac{B}{1 + K} \right), \tag{4.15}
\]
\[
g^{-1} = \left( 1 + (1 - q^2)c\frac{B}{q^2 + K + qV} - qQ\zeta\frac{B}{q^2 + K + qV} \right)(1 - q\zeta). \tag{4.16}
\]
All of these solutions describe the tachyon vacuum, and $q$ is merely a gauge parameter which roughly corresponds to the expectation value of the tachyon. We will clarify this relation in section 7.

\[\text{16} \quad \text{The } g_+ \text{ component is Okawa’s left gauge transformation from the perturbative vacuum to the tachyon vacuum [34], and is not invertible (cf. [57]). The } g_- \text{ component is more subtle, and is possibly an interesting solution in its own right. However, the inverse of } g_- \text{ is a fairly singular state in the dual } \mathcal{L}^- \text{ level expansion [42]. Earlier collaboration with M. Schnabl [23] investigated a similar solution, but problems with the identity string field ultimately rendered it unmanageable.}\]
Let’s explain why the solution (4.1) allows an analytic proof of Sen’s conjecture, whereas other proposals have proven intractable. Consider a possible tachyon vacuum solution of the form
\[ g = 1 + \zeta B \frac{K\Omega}{1 - \Omega} + Q\left(\zeta B\Omega\right) - cB\Omega. \] (4.17)
Replacing \( \Omega \rightarrow \frac{1}{1+K} \) gives back our solution (4.1) as expressed in (4.10). Formally (4.17) satisfies
\[ g^{-1}Qg = \left(c \frac{KB}{1 - \Omega} c + B\gamma^2\right)\Omega. \] (4.18)
The right hand side is Schnabl’s solution [10], with superstring correction [18] and “security strip” placed on the right. The problem comes with defining \( g^{-1} \). Since \( g \) can be written as \( 1 + \) (something) we can try to define \( g^{-1} \) as a geometric series:
\[ g^{-1} = 1 - \text{(something)} + \text{(something)}^2 - \ldots. \] (4.19)
At each order the number of cross terms appearing in \( \text{(something)}^n \) grows exponentially with \( n \). Aside from the practical difficulty of actually computing this series, the perturbative expansion is not controlled by a parametrically small parameter, so it is not clear whether the sum meaningfully converges. And without a usable definition of \( g^{-1} \), it seems impossible to evaluate the action and prove Sen’s conjecture. This is a problem common to many tachyon vacuum solutions in the \( K, B, c, \gamma, \gamma^{-1} \) subalgebra, and has been a major obstacle to analytic solution.

While the solution we have found has many nice properties, there are several notable shortcomings:

- As far as we know, the solution is not defined by a linear gauge condition. Actually, we are not certain how to implement an acceptable gauge fixing in our framework, since the \( (\xi_0 = 0) \)-gauge used in level truncation studies [5, 6] does not fit well with the \( K, B, c, \gamma, \gamma^{-1} \) subalgebra we have been using. This means in particular that we have no natural definition of the tachyon potential, though to be honest even in the bosonic string the Fock space tachyon potential in Schnabl gauge has not yet been computed.

- The solution (4.1) fails to satisfy the string field reality condition [43, 58, 59, 33] [17]
\[ g^\dagger = g^{-1}. \] (4.20)
This is a nonlinear condition on the group element which is typically difficult to solve. Formally we can construct a real solution as a gauge transformation of (4.1) [17].

---

[17]Determining the correct sign in the reality condition on GSO(−) states is a little tricky [58, 33]. Assuming \( ^\dagger \) is defined as the composition of Hermitian followed by BPZ conjugation [33], equation (4.20) is correct in the left-handed star product convention which we have been using. In the right handed convention favored by [5, 58, 34] the correct reality condition is \( g^\dagger = (-1)^F g^{-1} \).
as follows: Take \( g \) and define a new solution \( \tilde{g} \equiv \frac{1}{\sqrt{1+K}} g \sqrt{1+K} \). Then define a third solution \([59]\)

\[
\hat{g} \equiv \frac{1}{\sqrt{\tilde{g}} g},
\]

This solution formally satisfies \( \hat{g}^2 = \tilde{g} - 1 \) as desired. The problem is that this solution is complicated, and we know little about it aside from its formal definition. Perhaps other approaches, such as \([60]\), could be adapted to find a more tractable real solution.

- As mentioned before, the solution is fairly identity-like. This suggests that the energy may not be very well behaved in the level expansion \([25]\). A related problem is that the string field

\[
g^{-1} \eta g
\]

is logarithmically divergent due to an integrated collision between \( V \) and \( \eta \zeta \). Fortunately this divergence is absent in the computation of observables.

None of these issues will prove to be fatal for our purposes. But we hope that the solution presented here is a starting point for finding other solutions which address some of these problems, or possibly have other interesting properties.

## 5 Equations of Motion

Now we will prove the equations of motion for the solution \((4.1)\). We prove the equations of motion in two steps. First we show that the expressions \((4.1)\) and \((4.2)\) are actually inverses of one another:

\[
g^{-1}g = gg^{-1} = 1.
\]

Second we will show that \( g \) satisfies

\[
Qg = g \Psi.
\]

This together with \((5.1)\) implies \((4.3)\), which implies the equations of motion.

First let’s prove that \( g \) and \( g^{-1} \) are inverses by direct computation with the identities \((3.10)\):

\[
g^{-1}g = \left(1 - Q\zeta \frac{B}{1+K+V}\right)(1 - \zeta)(1 + \zeta) \left(1 + Q\zeta \frac{B}{1+K}\right),
\]

\[
= 1 - Q\zeta \frac{B}{1+K+V} + Q\zeta \frac{B}{1+K} - Q\zeta \frac{B}{1+K+V} Q\zeta \frac{B}{1+K},
\]

\[
= 1 - Q\zeta \frac{B}{1+K+V} + Q\zeta \frac{B}{1+K} - Q\zeta \frac{B}{1+K+V} V \frac{1}{1+K}.
\]

18
Now look at the $V$ stuck in the middle of the last term. Write it as the difference of two terms:

$$V = (1 + K + V) - (1 + K).$$  \hfill (5.4)

The terms cancel against one of the two factors on either side of $V$ in (5.3)

$$g^{-1}g = 1 - Q\zeta \frac{B}{1 + K + V} + Q\zeta \frac{B}{1 + K} - Q\zeta \left( \frac{B}{1 + K} - \frac{B}{1 + K + V} \right).$$  \hfill (5.5)

What’s left cancels leaving $g^{-1}g = 1$.

While this proof is sufficient, there is another way to look at this which provides more insight. Consider a class of states

$$M = -\gamma BX_1\zeta + cBX_2 + \gamma BY_1 - cBY_2\zeta,$$  \hfill (5.6)

where $X_1, X_2$ and $Y_1, Y_2$ are any string fields which commute with $B$. It turns out that states of this form multiply like $2 \times 2$ matrices with $X_1, X_2$ and $Y_1, Y_2$ placed in the entries as follows.

$$M = \begin{pmatrix} X_1 & Y_1 \\ Y_2 & X_2 \end{pmatrix}. \hfill (5.7)$$

Multiplying out factors and calculating $Q\zeta$, we can express the solution (4.1) in the form

$$g = -\gamma B\zeta + cB(K + V) \frac{1}{1 + K} + \gamma B \frac{1}{1 + K} + cB\zeta.$$  \hfill (5.8)

Comparing with (5.6) we find that the solution can be expressed as a $2 \times 2$ matrix:

$$g = \begin{pmatrix} 1 & \frac{1}{1 + K} \\ -1 & (K + V) \frac{1}{1 + K} \end{pmatrix}. \hfill (5.9)$$

Since the entries of this matrix do not commute, computing the inverse could be difficult. Luckily $g$ can be factorized:

$$g = \begin{pmatrix} 1 & 1 \\ -1 & K + V \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{1 + K} \end{pmatrix}, \hfill (5.10)$$

and the entries in each individual matrix factor commute. Thus we have expressed the solution as the product of two factors in two noncommuting copies of the group of invertible $2 \times 2$ matrices $GL_2$. Computing the inverse of $g$ is now as easy as computing the

\[18\] This matrix structure generalizes an old idea of Schnabl [21] for building the tachyon vacuum starting from a “square root” of the identity string field: $\sqrt{1} = \zeta + B\gamma$. The author thanks him for sharing this insight.
inverse of a $2 \times 2$ matrix:

$$g^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 + K \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{1 + K + V} & -\frac{1}{1 + K + V} \\ \frac{1}{1 + K + V} & \frac{1}{1 + K + V} \end{pmatrix}. \tag{5.11}$$

With a few more steps we can obtain the more familiar expression for $g^{-1}$:

$$g^{-1} = \begin{pmatrix} 1 - \frac{1}{1 + K + V} & -\frac{1}{1 + K + V} \\ \frac{1}{1 + K + V} & \frac{1}{1 + K + V} \end{pmatrix},$$

$$= \begin{pmatrix} 1 - \frac{1}{1 + K + V} & -\frac{1}{1 + K + V} \\ 1 - \frac{1}{1 + K + V} & 1 - \frac{1}{1 + K + V} \end{pmatrix},$$

$$= -\gamma B \left(1 - \frac{1}{1 + K + V}\right) \zeta + cB \left(1 - V \frac{1}{1 + K + V}\right) - \gamma B \frac{1}{1 + K + V}$$

$$- cB \left(1 - V \frac{1}{1 + K + V}\right) \zeta,$$

$$= 1 - \zeta + \gamma \frac{B}{1 + K + V} \zeta - cV \frac{B}{1 + K + V} - \gamma \frac{B}{1 + K + V} + cV \frac{B}{1 + K + V} \zeta,$$

$$= \left(1 - \gamma \frac{B}{1 + K + V} - cV \frac{B}{1 + K + V}\right)(1 - \zeta),$$

$$= \left(1 - Q\zeta \frac{B}{1 + K + V}\right)(1 - \zeta). \tag{5.12}$$

This matrix structure is one way to understand why this solution is so simple. Once we know that $g$ can be expressed as a product of $2 \times 2$ matrices, $g^{-1}$ cannot be much more complicated.

Finally, let’s verify the second part of the equations of motion, \textbf{(5.2)}. Let’s express \textbf{(5.2)} in the form

$$Q_{\Psi} g = 0, \tag{5.13}$$

where, following the notation of \textbf{[57]}, $Q_{\Psi}$ is the kinetic operator for a stretched string connecting the perturbative vacuum and the background corresponding to the cubic solution $\Psi$:

$$Q_{\Psi} X = Q X + 0 \ast X - (-1)^X X \Psi. \tag{5.14}$$
Since this operator is nilpotent, the equations of motion follow from expressing the solution (4.1) in the form:

\[ g = Q_0 \Psi \left( \alpha + \zeta \frac{B}{1 + K} \right). \] (5.15)

Here we’ve defined the field \( \alpha \equiv -\gamma^{-2}c \), which satisfies

\[ Q\alpha = 1. \] (5.16)

This corresponds to an insertion of the operator given in (2.5). To see that (5.15) reproduces the familiar form of the solution, compute

\[
\begin{align*}
g &= Q \left( \alpha + \zeta \frac{B}{1 + K} \right) + \left( \alpha + \zeta \frac{B}{1 + K} \right) \left( c - Qc \frac{B}{1 + K} \right), \\
&= 1 + Q \left( \zeta \frac{B}{1 + K} \right) - c \frac{B}{1 + K} + \zeta \frac{B}{1 + K} c - \zeta \frac{B}{1 + K} Qc \frac{B}{1 + K}, \\
&= 1 + Q \left( \zeta \frac{B}{1 + K} \right) - c \frac{B}{1 + K} + \zeta \frac{B}{1 + K} c + \zeta \frac{B}{1 + K} \partial c \frac{1}{1 + K}. \tag{5.17}
\end{align*}
\]

Now look at the \( \partial c \) in the last term. Write it in the form

\[ \partial c = (1 + K)c - c(1 + K), \] (5.18)

and use this to cancel the factors on either side. This gives

\[
\begin{align*}
g &= 1 + Q \left( \zeta \frac{B}{1 + K} \right) - c \frac{B}{1 + K} + \zeta \frac{B}{1 + K} c + \zeta \left( Bc \frac{1}{1 + K} - \frac{B}{1 + K} c \right), \\
&= 1 + \zeta \frac{1}{1 + K} + Q \left( \zeta \frac{B}{1 + K} \right) - c \frac{B}{1 + K}, \tag{5.19}
\end{align*}
\]

which is the solution as expressed in (4.10).

Note that any Berkovits solution can be derived from the general formula

\[ g = Q_0 \Psi \beta, \] (5.20)

for the appropriate choice of cubic solution \( \Psi \) and ghost number \(-1\) field \( \beta \). Since the cubic solution \( \Psi \) for the superstring is often very similar to that of the bosonic string, (5.20) gives an almost automatic lift of a solution in Witten’s bosonic string field theory to a solution in the Berkovits superstring field theory. The challenge is choosing \( \Psi \) and \( \beta \) so that \( g^{-1} \) is not too complicated or singular. A popular choice of \( \beta \), used in many

\[ ^{19} \text{The proof is as follows. For any Berkovits solution } g \text{ we can construct a cubic solution } \Psi = g^{-1}Qg. \text{ Then by definition we have } Q_0\Psi g = 0. \text{ Further, since } Q\alpha = 1, \text{ we can write } g = Q_0\Psi(\alpha g). \]
solutions for marginal deformations \[59, 61, 22, 62, 63\], is \( \beta = \alpha \). The tachyon vacuum (4.15) comes from a slightly more complicated choice of \( \beta \) which is necessary to generate expectation values in the GSO\((-\) sector. In the \( K, B, c, \gamma, \gamma^{-1} \) subalgebra there are no nonsingular solutions for the tachyon vacuum using GSO\( (+) \) states only. This is physically expected, and is a major advantage of the Berkovits formulation since it provides a clearer understanding of the role of the tachyon and the emergence of D-brane charges upon tachyon condensation. We discuss this further in section 8.

With a short extra step we can prove the absence of open string states around the tachyon vacuum. Expanding around the solution (4.1) gives the linearized equations of motion for a fluctuation field \( \varphi \):

\[
\eta Q_{\Psi} \varphi = 0, \tag{5.21}
\]

where \( Q_{\Psi} \) is the shifted kinetic operator around the cubic solution (4.3). Solutions of this equation should be identified modulo the linearized gauge invariance

\[
\varphi' = \varphi + Q_{\Psi} \Lambda + \eta \Sigma. \tag{5.22}
\]

Note that \( Q_{\Psi} \) has a “homotopy operator” \[12, 25\]

\[
A = \frac{B}{1 + K}, \tag{5.23}
\]

satisfying

\[
Q_{\Psi} A = 1. \tag{5.24}
\]

With this we can write

\[
\varphi = Q_{\Psi}(A\varphi) + AQ_{\Psi}\varphi. \tag{5.25}
\]

The first term is manifestly \( Q_{\Psi} \) exact. The second term is actually \( \eta \) exact, since \( A \) is in the small Hilbert space and (by assumption) \( \varphi \) satisfies the linearized equations of motion. Therefore all linearized fluctuations around the tachyon vacuum are pure gauge.

### 6 Energy

Now we compute the energy. To do this we compactify all directions (including time) tangential to the brane on circles of unit circumference.20 Then, we can compute the energy by computing the action. In our conventions, Sen’s conjecture predicts

\[
E = -S = -\frac{1}{2\pi^2}. \tag{6.1}
\]

This is minus the energy of the original unstable D-brane.

\[20\]This compactification is implicit in our normalization of the correlator (2.11).
To compute the action we must choose an interpolation \( g(t), t \in [0, 1] \) connecting the identity string field to the tachyon vacuum \((4.15)\). We choose a linear interpolation:

\[
g(t) = \bar{t} + tg,
\]

where \( \bar{t} \equiv 1 - t \). This choice is convenient since it preserves the \( 2 \times 2 \) matrix structure of the solution, making it possible to derive explicit expressions for both \( g(t) \) and \( g(t)^{-1} \):

\[
g(t) = \begin{pmatrix} 1 & qt \\ -qt & \bar{t} + K + qtV \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{1 + K} \end{pmatrix},
\]

\[
= (1 + qt\zeta) \left( 1 + (t^2q^2 - t) \frac{B}{1 + K} + qtQ\zeta \frac{B}{1 + K} \right);
\]

\[
g(t)^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 + K \end{pmatrix} \begin{pmatrix} \frac{\bar{t} + K + qtV}{t + q^2t^2 + K + qtV} & -\frac{qt}{t + q^2t^2 + K + qtV} \\ \frac{qt}{t + q^2t^2 + K + qtV} & \frac{1}{t + q^2t^2 + K + qtV} \end{pmatrix},
\]

\[
= \left( 1 - (t^2q^2 - t) c \frac{B}{t + q^2t^2 + K + qtV} + qtQ\zeta \frac{B}{t + q^2t^2 + K + qtV} \right) (1 - qt\zeta).
\]

Previous studies in Berkovits’ string field theory have used the exponential interpolation \( g(t) = e^{\Phi} \), but this choice would substantially complicate the analytic calculation. Since it is not much more difficult, we compute the energy for arbitrary values of the gauge parameter \( q \) in \((4.15)\).

Next we compute the integrand of the action in \((2.17)\):

\[
\text{Tr}[\eta \Psi_t \Psi_Q].
\]

Plugging in \( g(t) \) and \( g(t)^{-1} \) produces a lengthy expression which can be simplified using the identities of section 3. The result is

\[
\text{Tr}[\eta \Psi_t \Psi_Q] = q\bar{t} \text{Tr} \left[ -\eta Q\zeta \frac{1}{t + q^2t^2 + K + qtV} + \eta Q\zeta \frac{2q^2t^2 - t}{t + q^2t^2 + K + qtV} Bc \frac{1}{t + q^2t^2 + K + qtV} + \eta Q\zeta \frac{qt}{t + q^2t^2 + K + qtV} BQ\zeta \frac{1}{t + q^2t^2 + K + qtV} \right].
\]

\[21\]Perhaps an even easier way to compute the energy is to compute the cubic action as in \([18]\), and then rely on the argument of \([64]\), based on the formalism of \([65]\), demonstrating the on-shell equivalence of the Berkovits and cubic actions. We do not know of an obvious problem following the formal steps of \([64]\) with the solution \((4.15)\).
Now expand the denominators in powers of $V$ and select the terms with total $bc$ ghost number 3. With an additional reparameterization we find

$$
\text{Tr}[(\eta\Psi_t)\Psi_Q] = \frac{q^2t\bar{t}}{(t + q^2t^2)^2}X_1 - \frac{q^2t^2\bar{t}(2q^2t - 1)}{(t + q^2t^2)^3}X_2 + \frac{q^4t^3\bar{t}}{(t + q^2t^2)^3}X_3 + \frac{q^2t\bar{t}}{(t + q^2t^2)^2}X_4,
$$

where

$$
X_1 \equiv \text{Tr} \left[ \eta Q \zeta \frac{1}{1 + K} \right];
$$
$$
X_2 \equiv \text{Tr} \left[ \eta Q \zeta B \frac{1}{1 + K} \frac{1}{1 + K} \right] + \text{Tr} \left[ \eta Q \zeta B c \frac{1}{1 + K} \frac{1}{1 + K} \right];
$$
$$
X_3 \equiv \text{Tr} \left[ \eta Q \zeta B \frac{1}{1 + K} \frac{1}{1 + K} \frac{1}{1 + K} \frac{1}{1 + K} \right] + \text{Tr} \left[ \eta Q \zeta B \frac{1}{1 + K} \frac{1}{1 + K} \gamma \frac{1}{1 + K} \frac{1}{1 + K} \right] + \text{Tr} \left[ \eta Q \zeta B \frac{1}{1 + K} \gamma \frac{1}{1 + K} V \frac{1}{1 + K} \right];
$$
$$
X_4 \equiv \text{Tr} \left[ \eta Q \zeta B c V \frac{1}{1 + K} \frac{1}{1 + K} \right].
$$

(6.6)

$X_1, ..., X_4$ are simply constants which we can compute by evaluating the respective worldsheet correlation functions. We will do this in appendix B. The result is

$$
X_1 = 0;
$$
$$
X_2 = -\frac{1}{\pi^2};
$$
$$
X_3 = 0;
$$
$$
X_4 = -\frac{2}{\pi^2}.
$$

(6.7)

(6.8)

Plugging into (6.6) gives

$$
\text{Tr}[(\eta\Psi_t)\Psi_Q] = -\frac{1}{\pi^2} \left( -\frac{q^2t^2\bar{t}(2q^2t - 1)}{(t + q^2t^2)^3} + \frac{2q^2t\bar{t}}{(t + q^2t^2)^2} \right).
$$

(6.9)

To find the energy we integrate from 0 to 1:

$$
E = -S = \int_0^1 dt \text{Tr}[(\eta\Psi_t)\Psi_Q].
$$

(6.10)
To compute this integral, write the \( q^2 t^2 \) factor in the numerator of the first term of \((6.9)\) in the form
\[
q^2 t^2 = (\bar{t} + q^2 t^2) - \bar{t},
\]
and cancel with the denominator:
\[
\text{Tr}[(\eta \Psi_t)\Psi_Q] = -\frac{1}{\pi^2} \left( \frac{(\bar{t} + q^2 t^2) - \bar{t}(2q^2 t - 1)}{(t + q^2 t^2)^3} + \frac{2q^2 \bar{t}}{(t + q^2 t^2)^2} \right),
\]
\[
= -\frac{1}{\pi^2} \left( \frac{\bar{t}^2(2q^2 t - 1)}{(t + q^2 t^2)^3} + \frac{\bar{t}}{(t + q^2 t^2)^2} \right).
\]
Now note that the quantity \( 2q^2 t - 1 \) in the numerator of the first term is the derivative of \( \bar{t} + q^2 t^2 \) in the denominator. Thus it is easy to see that
\[
\text{Tr}[(\eta \Psi_t)\Psi_Q] = -\frac{1}{\pi^2} \frac{d}{dt} \left( -\frac{1}{2} \frac{\bar{t}^2}{(t + q^2 t^2)^2} \right),
\]
and
\[
E = \frac{1}{2\pi^2} \frac{\bar{t}^2}{(t + q^2 t^2)^2} \bigg|_{t=1} = -\frac{1}{2\pi^2}
\]
in agreement with Sen’s conjecture.

Another way to detect the energy is to probe the solution with a closed string. This can be accomplished by computing the Ellwood invariant \([55]\), which is believed to describe the shift in the closed string tadpole amplitude between the perturbative vacuum and the background described by a classical solution. In Berkovits’ string field theory, the Ellwood invariant comes in three varieties:

\[
Q\text{-Ellwood Invariant} = \text{Tr}_{V_Q}[\Psi_Q|_{t=1}] ;
\]

\[
t\text{-Ellwood Invariant} = \int_0^1 dt \text{Tr}_V[\Psi_t],
\]

\[
\eta\text{-Ellwood Invariant} = \text{Tr}_{V_\eta}[\Psi_{\eta}|_{t=1}] ;
\]

where \text{Tr}_V[\cdot] is the trace with a midpoint insertion of the vertex operator \( V \), and the vertex operators in each case are
\[
V_Q = \xi V_{-2}, \quad V_t = V_{-1}, \quad V_\eta = \alpha V_0.
\]

Here \( V_{-2}, V_{-1} \) and \( V_0 \) are on-shell closed string vertex operators in the \(-2, -1\) and \( 0 \) picture respectively, killed by \( \eta \) and \( Q \) and with vanishing conformal dimension. The operators

\[22\] A formal argument relating the Ellwood invariant and the value of the on-shell action was given in \([67]\) for the bosonic string. It would be interesting to extend this argument to the superstring.
\( \xi \) and \( \alpha \) above are some combination of the left/right zero modes of \( \xi \) and \( \alpha \) in (2.5)\(^{23}\).

Though the three Ellwood invariants look different, they compute the same quantity. For example, assuming \( V_t = QV_Q \) we have

\[
\int_0^1 dt \ Tr_V [\Psi_t] = \int_0^1 dt \ Tr_{V_Q} [Q \Psi_t],
\]

\[
= \int_0^1 dt \ Tr_{V_Q} [\partial_t \Psi_Q],
\]

\[
= \int_0^1 dt \partial_t Tr_{V_Q} [\Psi_Q],
\]

\[
= Tr_{V_Q} [\Psi_Q |_{t=1}],
\]

(6.19)

Therefore it is enough to compute the \( Q \)-Ellwood invariant, for which we don’t really need the Berkovits solution—the cubic solution (4.3) is sufficient. Then the computation reduces to that of the bosonic string [25], reproducing the expected closed string tadpole amplitude of the reference D-brane. The \( t \)- and \( \eta \)-invariants will compute the same amplitude, but their first quantized interpretation will be different since the closed string vertex operator lives in a different picture. It would be interesting to compute these invariants and clarify their first quantized interpretation.

7 Tachyon Coefficient

It is useful to consider the Fock space expansion of the solution (4.15), both for the purpose of comparison with earlier numerical solutions [5, 6, 7] and in general to understand the solution’s properties in level truncation. Though we are not able to execute a high level analysis, as a first step we can compute the tachyon coefficient \( T \)

\[
T i \sigma_1 \otimes \xi ce^{-\phi}(0)|0\rangle,
\]

(7.1)

which represents the expectation value of the tachyon field at the tachyon vacuum.

Specifically, we want to compute the tachyon coefficient of the Lie algebra element

\[
\Phi = \ln g.
\]

(7.2)

Since the solution is a product of two noncommuting \( 2 \times 2 \) matrices, in principle we can compute \( \Phi \) by taking the logarithm of each matrix factor and substituting into the Campbell-Baker-Hausdorff formula. But there is a simpler way to do this. Consider an

\(^{23}\)The triplet of Ellwood invariants is reminiscent of the triplet of cubic, WZW-like, and dual cubic actions observed in [64].
interpolation \( g(t) \) which can be written as a function of the solution \( g \) and the parameter \( t \) only:

\[
g(t) = f(t, g). \tag{7.3}
\]

We call this an abelian interpolation. Assuming \( g(t) \) is abelian, we can compute \( \Phi \) with the formula\(^{24}\)

\[
\Phi = \int_0^1 dt \, \Psi_t. \tag{7.4}
\]

The proof is as follows. Let \( \delta \) represent a variation relating abelian interpolations with fixed boundary conditions at \( t = 0 \) and \( t = 1 \). Then

\[
\delta \int_0^1 dt \, \Psi_t = \int_0^1 dt \left( \partial_t \Psi_\delta + [\Psi_t, \Psi_\delta] \right), \tag{7.5}
\]

where we used (2.22). Since the variation preserves the abelian property, \( \Psi_t \) and \( \Psi_\delta \) are functions of \( g \) and \( t \) only, and commute. The integral of the total derivative vanishes since by assumption the variation vanishes at \( t = 0 \) and 1. Therefore

\[
\delta \int_0^1 dt \, \Psi_t = 0, \tag{7.6}
\]

which means that the integral (7.4) is independent of the choice of abelian \( g(t) \). Then choosing \( g(t) = e^{t \Phi} \) establishes the result.

In particular, the linear interpolation (6.2) is an abelian interpolation. Therefore substituting (6.3) into (7.4) gives a formula for the Lie algebra element of the tachyon vacuum solution (4.15):

\[
\Phi = q\zeta + cB \left( \int_0^1 dt \, \frac{q^2t - 1}{t + q^2t^2 + K + qtV} \right) - cB \left( \int_0^1 dt \, \frac{qt(q^2t - 1)}{t + q^2t^2 + K + qtV} \right) \zeta
\]

\[
+ Q\zeta B \left( \int_0^1 dt \, \frac{q}{t + q^2t^2 + K + qtV} \right) - Q\zeta B \left( \int_0^1 dt \, \frac{q^2t}{t + q^2t^2 + K + qtV} \right) \zeta. \tag{7.7}
\]

We cannot easily integrate over \( t \) in this formula since the integrand is noncommutative. However, we do not really need to perform these integrals. We simply contract with a Fock space state and leave the integration over \( t \) as a final step once the integrand has turned into an ordinary function.

\(^{24}\)This implies that the \( t \)-Ellwood invariant can be computed as \( \text{Tr}_{V_t} [\Phi] \), as described by Michishita [56]. However, the expression (6.16) is more general since it does not assume an abelian interpolation.
Now we compute the tachyon coefficient. To do this, we contract $\Phi$ with the test state dual to the zero momentum tachyon:

$$-\sqrt{\frac{\pi}{2}} \sqrt{\Omega} (\eta \gamma^{-1} c \partial c) \sqrt{\Omega}.$$  \hfill (7.8)

So the tachyon coefficient is given by

$$T = -\sqrt{\frac{\pi}{2}} \text{Tr} \left[ \sqrt{\Omega} (\eta \gamma^{-1} c \partial c) \sqrt{\Omega} \Phi \right].$$  \hfill (7.9)

With the help of the correlators in appendix A we find the result

$$T = \frac{q}{\sqrt{2}\pi} \left[ 1 + \int_0^\infty dL \int_0^1 dt e^{-L(1-t+q^2t^2)} \left( (t + 1 - (1 + L)q^2t^2) \sin \frac{\pi}{2(1 + L)} \right. \right.$$

$$\left. - \frac{\pi}{2(1 + L)} \cos \frac{\pi}{2(1 + L)} \right].$$  \hfill (7.10)

Here $1 + L$ is the circumference of the cylinder obtained upon expanding the Lie algebra element (7.7) in terms of wedge states. The integration over $t$ can be performed analytically in terms of error functions, but makes the formula look more complicated.

Thus we have determined the tachyon coefficient as a function of the gauge parameter $q$. We have plotted this in figure 7.1. The tachyon coefficient is an odd, monotonically increasing function of $q$. For $q$ not too small, the coefficient is in the ball-park of its approximate value in the Siegel-$\xi_0 = 0$ gauge condensate, $T \approx .6$. At $q = 1$ the tachyon coefficient is

$$T|_{q=1} \approx .4998$$  \hfill (7.11)

which is surprisingly close to value $T = \frac{1}{2}$ computed from the action truncated to level zero [68]. When $q \to \pm \infty$ the tachyon approaches a finite upper/lower bound

$$\lim_{q \to \pm \infty} T \approx \pm .7554.$$  \hfill (7.12)

For small $q$ the tachyon coefficient approaches zero. Though $q \to 0$ looks smooth, this is a very singular limit of the solution. This is expected since the process of tachyon condensation should generate an expectation value for the tachyon.

---

25 The normalization is chosen so that the BPZ inner product of test state (7.8) and the Fock space zero momentum tachyon (7.1) is the tachyon coefficient $T$. Since the dual test state has $L_0 = -\frac{1}{2}$ it is orthogonal to all other Fock space states. A more efficient method for extracting Fock space coefficients at higher levels would utilize the operator formalism of Schnabl [10].

26 The absence of a tachyon in the cubic solution (4.3) has been somewhat of a puzzle, and probably indicates that the cubic equations of motion do not accurately capture the nonperturbative solution space of the superstring [33].
Figure 7.1: Tachyon coefficient plotted as a function of \( q \in [-5, 5] \). The dotted lines represent the current best approximation to the tachyon coefficient of the Siegel-(\( \xi_0 = 0 \)) gauge condensate, \( T \approx \pm 0.615 \) [8].

8 D-brane Charge?

The tachyon effective potential should have a pair of global minima corresponding to two tachyon vacuum solutions,

\[ g, g' \]

related by a sign flip in the GSO(−) sector:

\[ g' = (-1)^F g. \]

These two solutions are gauge equivalent. For the analytic solution (4.15), the finite gauge transformation relating them is simply

\[ g' = (g' g^{-1}) g, \]

since the product \( g' g^{-1} \) is BRST closed.\(^{27}\) Therefore \( g \) and \( g' \) should represent the same physical state, even though they have opposite GSO(−) expectation values.

But this raises a puzzle. A non-BPS D-brane of Type II should have a codimension 1 kink solution interpolating between \( g \) and \( g' \) describing a stable BPS D-brane of one lower dimension.\(^{28}\) But since \( g \) and \( g' \) are physically equivalent, one might think that the

\(^{27}\)In the general situation, the solutions \( g \) and \( g' \) correspond to different cubic solutions, and the gauge transformation relating them requires an \( \eta \) closed factor as well.

\(^{28}\)Some attempts to derive analytic solutions for lower dimensional branes are described in [69] [70] [71] [72].
kink solution is really a lump in disguise, and could dissipate. Turning the kink into a lump requires a gauge transformation which acts as the identity on one end of the kink, and turns $g$ into $g'$ on the other end. If $x \in \mathbb{R}$ is the coordinate along the kink, we need a gauge parameter $V(x)$ satisfying

$$\lim_{x \to -\infty} V(x) = 1, \quad \lim_{x \to \infty} V(x) = g'g^{-1}. \quad (8.4)$$

Thus $V(x)$ would define a homotopy between $g'g^{-1}$ and the identity string field. We suggest that no such homotopy exists, which is why the kink solution is stable. Thus $g'g^{-1}$ is a topologically nontrivial, “large” gauge transformation.

This is a fairly ambitious statement. We will not try to prove it, but instead provide some evidence based on our analytic solution (4.15). The existence of $V(x)$ in (8.4) would imply the existence of a homotopy $g(\lambda), \lambda \in [0,1]$ of tachyon vacuum solutions connecting $g$ and $g'$:

$$g(0) = g; \quad g(1) = g'. \quad (8.5)$$

We can look for this homotopy within the class of analytic solutions we have studied, that is, assuming solutions of the form (4.15) which differ only in the choice of $q$. Thus $g(\lambda)$ corresponds to $q(\lambda), \lambda \in [0,1]$ satisfying

$$q(0) = q; \quad q(1) = -q. \quad (8.6)$$

We claim that $g(\lambda)$ must be singular for at least one $\lambda$. To see this, note that $g(\lambda)^{-1}$ involves the state

$$\frac{1}{q(\lambda)^2 + K}. \quad (8.7)$$

According to a proposal of Rastelli [73], the algebra of wedge states should correspond to the $C^*$-algebra of bounded, continuous functions on the positive real line $K \geq 0$. Applying this criterion to (8.7) implies that $q(\lambda)$ cannot be zero or imaginary. But there is no path connecting $q$ to $-q$ which does not pass through the imaginary axis. Therefore, at least within the class of gauge transformations preserving (4.15), $g'g^{-1}$ is not homotopic to the identity. This is consistent with stability of the kink.

Actually, the state (8.7) arguably becomes singular long before $q(\lambda)$ reaches the imaginary axis. Currently, the only practical way for defining states in the wedge algebra is as a Laplace transform

$$F(K) = \int_0^\infty d\alpha f(\alpha)\Omega^\alpha. \quad (8.8)$$

This representation assumes that $F(K)$ is analytic on the positive half of the complex plane $\text{Re}(K) > 0$. Applying this to (8.7) implies

$$|\text{Re}(q)| > |\text{Im}(q)|. \quad (8.9)$$

This excludes not only the imaginary axis, but also a cone of solutions around the imaginary axis as shown in figure [8.1]. It is possible that some solutions inside the cone could
Re($q$)  
Im($q$)  

Figure 8.1: Space of tachyon vacuum solutions of the form (4.15) in the complex $q$ plane. For purely imaginary $q$ the solutions are singular, and in the shaded region they are not definable in terms of superpositions of wedge states. Thus the solution space has two disconnected components corresponding to the two minima of the tachyon effective potential.

be understood with some more general definition of the algebra of wedge states [33], but this is unclear.

One might ask why this argument does not also imply stability of the kink on the brane-antibrane pair, which should represent an unstable non-BPS D-brane. The solution (4.15) can be immediately generalized to the brane-antibrane by taking $q$ to be an arbitrary off-diagonal $2 \times 2$ matrix

$$q = q_1 \sigma_1 + q_2 \sigma_2. \quad (8.10)$$

This off-diagonal matrix represents the (external) Chan-Paton factors for the GSO($-$) strings connecting the brane and antibrane. Again we look for a homotopy $g(\lambda), \lambda \in [0,1]$ connecting $g$ and $g'$ within the ansatz (4.15). This time, the relevant state which appears in $g(\lambda)^{-1}$ is

$$\frac{1}{q_1(\lambda)^2 + q_2(\lambda)^2 + K}. \quad (8.11)$$

Restricting for simplicity to real $q_1, q_2$, the only problematic point for this state is $q_1 = q_2 = 0$. This point is easily avoided in a path from $q$ to $-q$. Thus on the brane-antibrane pair $g'g^{-1}$ is homotopic to the identity, and the kink solution is unstable. Incidentally, note that the “hole” at $q_1 = q_2 = 0$ suggests the existence of codimension 2 topological solitons obtained by winding around the tachyon vacuum gauge orbit at infinity (see figure 8.2). These solitons are the expected BPS D-$(p - 2)$-branes formed by tachyon condensation on the $Dp$-$\bar{D}p$ system. We expect that higher codimension brane charges can be seen by
Figure 8.2: Space of tachyon vacuum solutions of the form (4.15) on a brane-antibrane pair, for Hermitian \( q = q_1 \sigma_1 + q_2 \sigma_2 \). The only singular solution is at \( q_1 = q_2 = 0 \). Removing this point allows homotopically nontrivial windings around the gauge orbit, which should represent the charges of BPS D-branes of codimension 2 on the brane-antibrane pair.

looking at the tachyon vacuum on multiple non-BPS D-branes or brane-antibrane pairs. The picture so far seems consistent with the intuition for D-brane charge derived from the presumed form of the tachyon effective potential.

Understanding the topological charge of stable D-branes is a long standing and fundamental problem in string field theory. But we have only looked at a small slice of the gauge orbit, and we need to demonstrate that the topological structures we’ve observed survive a more general analysis. With further development, we hope that it is possible to strengthen our argument based on analysis of the full tachyon vacuum gauge orbit in the \( K, B, c, \gamma, \gamma^{-1} \) subalgebra\(^{29}\). Another question is how this characterization of D-brane charge leads to Ramond-Ramond flux. Analysis of the boundary state \([74, 66]\) or the Ellwood invariant \([55]\) may provide insight into this question. It would also be interesting to see if these developments can shed light on the long-speculated relation between string field theory and the \( K \)-theoretic description of D-brane charge \([75, 76, 77]\). We leave these questions for future work.

9 Conclusion

In this paper we found an exact solution for tachyon condensation in Berkovits’ nonpolynomial open superstring field theory. The solution is completely explicit and very simple.

\(^{29}\)A toy model for this kind of problem appears in the analysis of the gauge orbit of “half brane” solutions in cubic superstring field theory \([33]\).
The main obstacle to finding the solution was not necessarily the equations of motion—a simple strategy for solving the equations of motion has been known since [59, 22, 78]. The main obstacle was finding a specific solution such that the various nonpolynomial expressions needed in the theory do not produce an uncontrolled proliferation of superghost insertions. Our main innovations in this respect were the introduction of the boundary interaction (4.6) in conjunction with the $2 \times 2$ matrix algebra (5.6). With these ingredients we have been able to whittle the computation of the action down to a single correlator with merely two superghost insertions. Thus we are able to explicitly prove Sen’s conjecture and provide the first nontrivial analytic computation of the nonpolynomial action.

The solution opens a number of interesting avenues for exploration. The most immediate is obtaining new solutions and a clearer understanding of the tachyon vacuum gauge orbit and its relation to D-brane charge. Next, it is desirable to incorporate the Ramond sector and understand the role of supersymmetry. In principle, it should be possible to derive BPS equations from the spontaneously broken supersymmetries on a non-BPS D-brane, and solve these equations to find lower dimensional BPS D-branes as topological solitons. Another urgent question is the role of closed strings. While the perturbative quantization of the Berkovits theory is not yet well understood, this is a topic of active research [79, 80, 81, 82]. We hope that the tachyon vacuum solution stimulates progress in these and other important problems.

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A Correlators

Here we list some correlators which are needed for our calculations. The correlators are normalized according to the convention,

$$\langle \xi(z)c\partial c\partial^2c(w)e^{-2\phi(y)} \rangle \equiv 2.$$  \hspace{1cm} (A.1)

We will give formulas for correlation functions on the cylinder of circumference $L$, which following [34] we denote with a subscript $C_L$. These are related to correlation functions on the upper half plane through the conformal transformation

$$\langle \ldots \rangle_{C_L} = \left\langle f^{-1}_{S} \frac{2}{L} \circ \ldots \right\rangle_{UHP}. \hspace{1cm} (A.2)$$
where \( f^{-1}_S(z) = \tan \frac{\pi z}{L} \) is the inverse of the sliver coordinate map \([38]\) and \( \frac{2}{L} \) is a dilatation by a factor of \( \frac{2}{L} \):

\[
f^{-1}_S \circ \frac{2}{L}(z) = \tan \frac{\pi z}{L}. \tag{A.3}
\]

The most general correlator in the \( K, B, c, \gamma, \gamma^{-1} \) subalgebra is

\[
\langle \left( \eta^{-1}(w)\gamma^{-1}(x_1)\gamma^{-1}(x_2)\ldots\gamma^{-1}(x_{n+1})\gamma(y_1)\gamma(y_2)\ldots\gamma(y_n) \right) \left( c(z_1)c(z_2)c(z_3)c(z_4)B \right) \rangle_{C_L}. \tag{A.4}
\]

Here we have already stripped away and traced over internal CP factors. Without loss of generality, we can assume that the coordinates \( w, x_i, y_i \) and \( z_i \) sit on the real axis between \( \text{Re}(z) = L \) and \( \text{Re}(z) = 0 \), since the conformal transformation \((A.3)\) identifies coordinates outside this range modulo \( z \sim z + L \). This is what it means for \((A.4)\) to be a correlation function on the cylinder. We also assume that the \( B \) contour meets the real axis somewhere between 0 and the smallest \( z_i \) representing an insertion of \( c \).

The correlator \((A.4)\) can be computed as a product of correlators in the \( \beta\gamma \) and \( bc \) conformal field theories. The \( \beta\gamma \) factor is

\[
\langle \eta^{-1}(w)\gamma^{-1}(x_1)\gamma^{-1}(x_2)\ldots\gamma^{-1}(x_{n+1})\gamma(y_1)\gamma(y_2)\ldots\gamma(y_n) \rangle_{C_L} = \frac{\pi}{L} \frac{\sin \frac{\pi (w-y_1)}{L} \ldots \sin \frac{\pi (w-y_n)}{L}}{\sin \frac{\pi (w-x_1)}{L} \ldots \sin \frac{\pi (w-x_{n+1})}{L}}. \tag{A.5}
\]

The \( bc \) factor is \([10, 34]\)

\[
\langle c(z_1)c(z_2)c(z_3)c(z_4)B \rangle_{C_L}^{bc} = \frac{L^2}{4\pi^3} \left( z_1 \sin \frac{\pi z_{23}}{L} \sin \frac{\pi z_{24}}{L} \sin \frac{\pi z_{34}}{L} - z_2 \sin \frac{\pi z_{13}}{L} \sin \frac{\pi z_{14}}{L} \sin \frac{\pi z_{34}}{L} + z_3 \sin \frac{\pi z_{12}}{L} \sin \frac{\pi z_{14}}{L} \sin \frac{\pi z_{24}}{L} - z_4 \sin \frac{\pi z_{12}}{L} \sin \frac{\pi z_{13}}{L} \sin \frac{\pi z_{23}}{L} \right), \tag{A.6}
\]

where \( z_{ij} \equiv z_i - z_j \). Another useful form is

\[
\langle c(z_1)c(z_2)c(z_3)c(z_4)B \rangle_{C_L}^{bc} = \frac{L^2}{4\pi^3} \left( z_{14} \sin \frac{2\pi z_{23}}{L} + z_{23} \sin \frac{2\pi z_{14}}{L} - z_{13} \sin \frac{2\pi z_{24}}{L} - z_{24} \sin \frac{2\pi z_{13}}{L} + z_{12} \sin \frac{2\pi z_{34}}{L} + z_{34} \sin \frac{2\pi z_{12}}{L} \right). \tag{A.7}
\]
We also list a few special cases which appear in our calculations:

\[
\langle c \partial c(z_1) c(z_2) \rangle_{cL}^{bc} = -\left( \frac{L}{\pi} \right)^2 \sin^2 \frac{\pi z_{12}}{L}; \quad (A.8)
\]

\[
\langle c \partial c(z_1) \partial c(z_2) \rangle_{cL}^{bc} = \frac{L}{\pi} \sin \frac{2\pi z_{12}}{L}; \quad (A.9)
\]

\[
\langle c \partial c(z_1) c(z_2) c(z_3) B \rangle_{cL}^{bc} = \frac{L^2}{\pi^3} \left[ \frac{\pi}{L} \left( z_{12} \sin^2 \frac{\pi z_{13}}{L} - z_{13} \sin^2 \frac{\pi z_{12}}{L} \right) - \sin \frac{\pi z_{12}}{L} \sin \frac{\pi z_{13}}{L} \sin \frac{\pi z_{23}}{L} \right]; \quad (A.10)
\]

\[
\langle c \partial c(z_1) \partial c(z_2) c(z_3) B \rangle_{cL}^{bc} = \frac{2L}{\pi^2} \sin \frac{\pi z_{13}}{L} \left( \frac{\pi z_{13}}{L} \cos \frac{\pi z_{12}}{L} - \cos \frac{\pi z_{23}}{L} \sin \frac{\pi z_{13}}{L} \right); \quad (A.11)
\]

\[
\langle c \partial c(z_1) \partial c(z_2) \partial c(z_3) B \rangle_{cL}^{bc} = -\frac{4}{\pi} \sin \frac{\pi z_{12}}{L} \sin \frac{\pi z_{13}}{L} \sin \frac{\pi z_{23}}{L}; \quad (A.12)
\]

\[
\langle c \partial c(z_1) \partial c(z_2) B \rangle_{cL}^{bc} = \frac{2L}{\pi^2} \sin \frac{\pi z_{12}}{L} \left( \sin \frac{\pi z_{12}}{L} - \frac{\pi z_{12}}{L} \cos \frac{\pi z_{12}}{L} \right). \quad (A.13)
\]

We use these correlators to compute the tachyon coefficient in (7.10). Only (A.13) appears in the computation of the action.

**B Energy Coefficients**

In this appendix we calculate the constants \(X_1, ..., X_4\) which appear in the computation of the action.

The constants \(X_1\) and \(X_3\) vanish. Actually we can prove this without calculating any correlators. For \(X_1\) we can see this as follows:

\[
X_1 = \text{Tr} \left[ \eta Q \zeta \frac{1}{1 + K} [B, Q \zeta] \frac{1}{1 + K} \right],
\]

\[
= - \text{Tr} \left[ [B, \eta Q \zeta] \frac{1}{1 + K} Q \zeta \frac{1}{1 + K} \right],
\]

\[
= \text{Tr} \left[ \eta V \frac{1}{1 + K} Q \zeta \frac{1}{1 + K} \right],
\]

\[
= - \text{Tr} \left[ V \frac{1}{1 + K} \eta Q \zeta \frac{1}{1 + K} \right],
\]

\[
= - \text{Tr} \left[ \eta Q \zeta \frac{1}{1 + K} V \frac{1}{1 + K} \right] = -X_1. \quad (B.1)
\]
Meanwhile, for $X_3$

$$X_3 = \text{Tr} \left[ \eta Q \zeta \frac{B}{1+K} Q \zeta \frac{B}{1+K} Q \zeta \frac{B}{1+K} \right] + \text{Tr} \left[ \eta Q \zeta \frac{B}{1+K} Q \zeta \frac{B}{1+K} \right],$$

$$= \text{Tr} \left[ \eta Q \zeta \frac{B}{1+K} Q \zeta \frac{B}{1+K} \right],$$

$$= \text{Tr} \left[ \eta \left( \frac{B}{1+K} Q \zeta \frac{B}{1+K} Q \zeta \frac{B}{1+K} \right) \right] = 0. \quad (B.2)$$

Now let’s compute $X_4$. To match with correlators given in appendix A, it is useful to move the $B$ insertion in $X_4$ to the right. Commuting the $B$ past the $c$ produces a term proportional to $X_1$, which vanishes as just demonstrated. Therefore we write $X_4$:

$$X_4 = - \text{Tr} \left[ \eta Q \zeta \frac{1}{1+K} cV \frac{B}{1+K} \right]. \quad (B.3)$$

Next we compute:

$$X_4 = - \int_0^\infty d\alpha_1 d\alpha_2 e^{-(\alpha_1+\alpha_2)} \text{Tr} \left[ (\eta Q \zeta) \Omega^{\alpha_1} (cV) B \Omega^{\alpha_2} \right],$$

$$= - \int_0^\infty dL \int_0^1 d\theta \text{Tr} \left[ (\eta Q \zeta) \Omega^L (cV) B \Omega^{L(1-\theta)} \right],$$

$$= - \int_0^1 d\theta \text{Tr} \left[ (\eta Q \zeta) \Omega^\theta (cV) B \Omega^{1-\theta} \right]. \quad (B.4)$$

In the first step we expanded the two factors of $\frac{1}{1+K}$ into integrals over wedge states, and in the second step we made a change of variables into the total width of the cylinder, $L = \alpha_1 + \alpha_2$, and an angular parameter $\theta = \alpha_1/L$ describing the distance between the $\eta Q \zeta$ and $cV$ insertions on the cylinder. In the third step we made an $\mathcal{L}^-$ reparameterization which scales the cylinder to unit circumference, and performed the integral over $L$. Now express the trace as a correlation function on the cylinder of unit circumference:

$$X_4 = - \int_0^1 d\theta \frac{1}{2} \text{Tr}(\sigma_3 \sigma_2 \sigma_3 \sigma_2 \sigma_3 \sigma_3) \left\langle \frac{1}{2} \eta^{-1} c \partial_c (1) \frac{1}{2} \gamma^{-1} c \partial_c (1-\theta) B \right\rangle_{c_1},$$

$$= - \frac{1}{4} \int_0^1 d\theta \left\langle [\eta^{-1}(1) \gamma^{-1}(1-\theta)] [c \partial_c (1-\theta) B] \right\rangle_{c_1}. \quad (B.5)$$
Consulting the correlation functions (A.5) and (A.13) in appendix A, this gives

\[ X_4 = -\frac{1}{2\pi} \int_0^1 d\theta \left( \sin \pi \theta - \pi \theta \cos \pi \theta \right), \]

\[ = -\frac{1}{2\pi} \int_0^1 d\theta \left( 2 \sin \pi \theta - \frac{d}{d\theta} \theta \sin \pi \theta \right), \]

\[ = -\frac{1}{2\pi} \left( \frac{2}{\pi} \cos \pi \theta - \theta \sin \pi \theta \right) \bigg|_0^1, \]

\[ = -\frac{2}{\pi^2}. \quad \text{(B.6)} \]

Next compute \( X_2 \). We can simplify the expression as follows:

\[ X_2 = \text{Tr} \left[ \eta Q \zeta \frac{B}{1 + K} V \frac{1}{1 + K} c \frac{1}{1 + K} \right] + \text{Tr} \left[ \eta Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} V \frac{1}{1 + K} \right], \]

\[ = \text{Tr} \left[ \eta Q \zeta \frac{B}{1 + K} Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} \right] + \text{Tr} \left[ [B, Q \zeta] \frac{1}{1 + K} \eta Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} \right], \]

\[ = \text{Tr} \left[ \eta Q \zeta \frac{B}{1 + K} Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} \right] + \text{Tr} \left[ Q \zeta \frac{B}{1 + K} \eta Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} \right] \]

\[ - \text{Tr} \left[ Q \zeta \frac{1}{1 + K} \eta Q \zeta \frac{B}{1 + K} c \frac{B}{1 + K} \right], \]

\[ = \text{Tr} \left[ \eta \left( Q \zeta \frac{B}{1 + K} Q \zeta \frac{B}{1 + K} c \frac{1}{1 + K} \right) \right] - \text{Tr} \left[ Q \zeta \frac{1}{1 + K} \eta Q \zeta \frac{B}{(1 + K)^2} \right], \]

\[ = - \text{Tr} \left[ \eta Q \zeta \frac{1}{1 + K} Q \zeta \frac{B}{(1 + K)^2} \right], \]

\[ = - \text{Tr} \left[ \eta Q \zeta \frac{1}{1 + K} c V \frac{B}{(1 + K)^2} \right]. \quad \text{(B.7)} \]

Then expanding this out in terms of wedge states as in (B.4) we find

\[ X_2 = -2 \int_0^1 d\theta (1 - \theta) \text{Tr} \left[ (\eta Q \zeta) \Omega^\theta (c V) B \Omega^{1-\theta} \right]. \quad \text{(B.8)} \]

Computing the correlator this becomes

\[ X_2 = -\frac{1}{\pi} \int_0^1 d\theta (1 - \theta) \left( \sin \pi \theta - \pi \theta \cos \pi \theta \right). \quad \text{(B.9)} \]
The second term vanishes by symmetry $\theta \to 1-\theta$. For the first term substituting $\theta \to 1-\theta$ simplifies the integral to

$$X_2 = -\frac{1}{\pi} \int_0^1 d\theta \sin \pi \theta = -\frac{1}{\pi^2}.$$  \hspace{1cm} (B.10)

Therefore the coefficients $X_1, \ldots, X_4$ take the values described in (6.8).

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