On accretion of dark energy onto black- and worm-holes

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Abstract

We review some of the possible models that are able to describe the current Universe which point out the future singularities that could appear. We show that the study of the dark energy accretion onto black- and worm-holes phenomena in these models could lead to unexpected consequences, allowing even the avoidance of the considered singularities. We also review the debate about the approach used to study the accretion phenomenon which has appeared in literature to demonstrate the advantages and drawbacks of the different points of view. We finally suggest new lines of research to resolve the shortcomings of the different accretion methods. We then discuss future directions for new possible observations that could help choose the most accurate model.

1 Introduction

The discovery of the cosmic acceleration indicated by the observational data \cite{1,2,3} has caused a break in the belief of what could be the matter content of the universe and what might be its possible future evolution. The interpretation of this data in the framework of General Relativity implies that the majority of the content in the universe should be new stuff, which has been called dark energy, possessing anti-gravitational properties, i.e. the equation of state parameter of the dark energy must be $w \leq -1/3$ ($w = p/\rho$). It even seems to be possible that the equation of state parameter is less than $-1$. In that case the new stuff is known as phantom energy \cite{4} and the consideration of this fluid could lead the universe to a catastrophic end by the appearance of a future singularity. The

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most popular of such singularities is the so-called big rip \[5\], which is a possible doomsday of the universe where both of its size and its energy density become infinitely larger. However, the big rip is not the only possibility which has been suggested as the catastrophic end to the universe in the new phantom models. It has also been argued that the universe could finish its evolution at a time where its energy density becomes infinitely larger maintaining the scale factor as a finite value, known as the big freeze singularity \[6, 7\].

It is well known that dark energy should be accreted onto black holes in a different way that ordinary matter does, since that new fluid covers the whole space. Therefore, the study of dark energy accretion onto black holes becomes an interesting field of study which could lead to surprising effects as the possible disappearance of black holes in phantom environments. As we will show in this chapter the mentioned accretion phenomenon was originally studied by Babichev et al. \[21\], although a great number of works have been done to improve the method used by those authors \[28, 31, 33\].

On the other hand, the accretion process could also imply unexpected consequences in the case that one considers the evolution of cosmological objects even stranger than black holes, wormholes\[1\]. Traversable wormholes are short-cuts between two regions of the same universe or between two universes, which could be used to construct time-machines \[16\]. The reason for its strangeness is not related to its bridge character but to that, in order to be traversable and stable, the wormholes must be surrounded by some kind of exotic matter which do not fulfil the null energy condition. Nevertheless, the consideration of phantom models as the possible current description of the universe has caused a revival of interest in traversable wormholes, since this fluid would also violate the null energy condition. Even more, it has been shown that an inhomogeneous version of phantom energy can be the exotic stuff which supports wormholes \[14\]. In some cosmological models the accretion of phantom energy onto a wormhole could lead to an enormous growth of its mouth, engulfing the whole universe which would travel through it in a big trip \[25\], avoiding the big rip \[19\] or big freeze \[6\] singularity in the corresponding cases.

In the present chapter we show the method of how to treat the accretion phenomenon onto black- and wormholes and its cosmological consequences in some models. In Sec. II, we review some candidates that could be responsible for the current cosmological acceleration and which pay special attention to the possible future singularities appearing in some of them. In Sec. III, the procedure of the study of the dark energy accretion onto black holes based on the Babichev et al. method is shown and its application to the models included in Sec. II is presented. The corresponding study in the case of accretion onto wormholes is shown in Sec. IV, where the possible avoidance of the future singularities is highlighted. Since the study of dark energy accretion onto black- and wormholes is still an open issue, we refer in Sec. V some interesting works which have produced a debate about the used method and we also outline some

\[1\] For information about the historical development of wormholes and a deep study of their spacetime, please see Ref. [17].
possible lines for future research to solve the shortcomings. Finally, in Sec. VI, the results are discussed and further comments are added.

2 Brief review of some candidates to cosmic acceleration.

The origin of the current accelerating expansion of the universe is one of the most interesting challenges in cosmology. Therefore, a plethora of cosmological models have been developed in recent years in order to take into account such acceleration. Although modifications of the Lagrangian of General Relativity or considerations of more than four dimensions could explain the current phase of the universe, the acceleration can also be modelled in the framework of General Relativity theory, which has shown agreement with the observational tests up to now. Nevertheless, the consequences of using the Einstein’s theory is that most part of the universe’s content must be some kind of fluid with anti-gravitational properties, called dark energy.

In order to show the necessity of the inclusion of dark energy as a new component of a universe described by General Relativity, we must consider an homogeneous, isotropic and spatially flat universe, i.e. a Friedmann-Lemaître-Robertson-Walker (FLRW) model with \( k = 0 \). As an approximation, one can consider that this model is only filled with one fluid\(^2\). Throughout this chapter we shall use natural units so that \( G = c = 1 \). The Friedmann equations can be expressed in the usual way\(^3\)

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi\rho, \tag{1}
\]

\[
3 \frac{\ddot{a}}{a} = -4\pi(\rho + 3p), \tag{2}
\]

where \( a(t) \) is the scale factor, which can be used to define the Hubble parameter \( H = \dot{a}/a \). \( \rho \) and \( p \) are the energy density and the pressure of the fluid, respectively. Eq. (2) shows that, in order to obtain an accelerating universe, the dark energy must have an energy density and pressure such that \( \rho + 3p < 0 \), violating at least the strong energy condition. If one wants to minimise the strange character of the dark energy, then \( w > -1 \) \((w = p/\rho)\) could be imposed, but it must be noted that such restriction is not based in direct observations but in theoretical wishes.

In fact, as we have already mentioned in the introduction, the observational data indicates that \( w \) must be around \(-1\) and, therefore, values less than \(-1\) are

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\(^2\)It must be noted that such an approximation is justified because the contribution of the current dark energy density is around 74% of the total energy density of the universe and, from the evolution of the energy density in terms of the scale factor, it is expected that dark energy density decays slower than the ordinary matter energy density when the scale factor increases, being therefore the future dynamic of the universe governed by the dark component. Even more, in the phantom case the phantom energy density would increase with the scale factor, so the same conclusions can be, of course, recovered in this case.
not excluded. In that case the fluid is called phantom energy\(^4\) and violates even the dominant energy condition.

The limiting case, \(w = -1\), is also possible and is equivalent to the introduction of a positive cosmological constant. We shall not pay much attention to this case because it is the well-known de Sitter solution and more importantly, the cosmological constant would be accreted neither by black- nor by worm-holes, as it could be expected and we show in the next sections.

In this section we present the simplest dark and phantom energy models, where the equation of state parameter is considered to be closely constant. As we shall see, such a phantom energy model implies the occurrence of a big rip\(^5\). Since it could seem that phantom energy implies the occurrence of a big rip singularity, we want to show that such a singularity is not an inherent property of that fluid. So we shall consider Phantom Generalized Chaplygin Gas (PGCG) models, in order to clarify that phantom models could present no future singularities\(^8\) or future singularities of other kinds\(^6, 7\).

2.1 Quintessence with a constant equation of state parameter.

The most popular candidate to describe dark energy, allowing a dynamic evolution for this unknown component, is the quintessence model. In this model a spatially homogeneous massless scalar field is considered, which can be interpreted as a perfect fluid with negative pressure, taking the equation of state parameter values on the range \(-1 < w < -1/3\). Supposing that the equation of state parameter is approximately constant, the conservation law of the fluid in a FLRW spacetime, \(\dot{\rho} + 3H(p + \rho) = 0\), can be integrated to obtain

\[
p = w\rho = w\rho_0 [a(t)/a_0]^{-3(1+w)},
\]

which can be introduced in Eq. (1) leading

\[
a(t) = a_0 \left(1 + \frac{3}{2} (1 + w)C(t - t_0)\right)^{-2/[3(w+1)]},
\]

with \(C = (8\pi\rho_0/3)^{1/2}\) and the subscript \(0\) denoting the value at the current time \(t_0\). Therefore, a universe described with that model would accelerate forever, decreasing the dark energy density in the process.

2.2 Phantom quintessence with a constant equation of state parameter.

Phantom energy can be considered to be a fluid with an equation of state parameter less than \(-1\) which would, therefore, violate not only the strong energy condition but also the dominant energy condition\(^3\). But, since the observational

\(^3\)It must be noted that if we want to express the phantom fluid by using a scalar field like in the quintessence dark energy case, then it will possess a negative kinetic term.
data suggests that it could be responsible for the current accelerating expansion therefore, such a pathological fluid should be seriously considered as the possible dominating matter content of our universe.

Analogous to the quintessence case, one can easily obtain an expression for the scale factor in this model considering that \( w \) is approximately constant. One has

\[
a(t) = a_0 \left( 1 - \frac{3}{2}(|w| - 1)C(t-t_0) \right)^{-2/3(|w|-1)},
\]

with \( \rho = \rho_0 \left[ a(t)/a_0 \right]^{3(|w|-1)} \) and \( C \) taking the already mentioned value. Therefore, the scale factor (5) increases with time even faster than the scale factor of a de Sitter universe (which has an exponential behaviour) up to

\[
t_{br} = t_0 + \frac{2}{3(|w|-1)C} > t_0.
\]

At this time both the scale factor and the energy density of the fluid blow up in which is known as the big rip singularity [5]. If our Universe is described by this model, then an observer located on the Earth would see how progressively it could be ripped apart the galaxies, the stars, our solar system and finally, the atoms and nuclei, up until the moment when every component of the universe would be out of the Hubble horizon of the other components.

### 2.3 Phantom Generalized Chaplygin Gas.

It could seem that the consideration of a universe filled with phantom energy implies the occurrence of a future big rip singularity, but this is not necessarily the case. We support this claim with the example taken from the Phantom Generalized Chaplygin Gas (PGCG) [6, 7, 8].

The Generalized Chaplygin Gas (GCG) is a fluid with an equation of state of the form [9]

\[
p = -\frac{A}{\rho^\alpha},
\]

where \( A \) is a positive constant and \( \alpha > -1 \) is a parameter. In the particular case \( \alpha = 1 \) we recover the equation of state of a Chaplygin gas. Inserting Eq. (7) in the conservation of the energy momentum tensor, one obtains

\[
\rho = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{\alpha+1}},
\]

with \( B \) a constant parameter. It can be seen that, maintaining \( A > 0 \), such a fluid fulfills the dominant energy condition for \( B > 0 \) and it is violated otherwise. The rather strange equation of state expressed through Eq. (7) has been considered firstly in cosmology because the GCG could reproduce a transition from a dust dominated universe at early time to de Sitter behaviour at late time.
On the other hand, PGCG are fluids with an equation of state of the GCG type, Eq. (7), with the parameters $A$, $B$ and $\alpha$ taking values in intervals such that the energy density and pressure of the fluid fulfil the requirements $\rho > 0$ and $p + \rho < 0$. It can be seen that such requirements lead to four classes of PGCG with

- type I: $A > 0$, $B < 0$ and $1 + \alpha > 0$.
- type II: $A > 0$, $B < 0$ and $1 + \alpha < 0$.
- type III: $A < 0$, $B > 0$ and $(1 + \alpha)^{-1} = 2n > 0$.
- type IV: $A < 0$, $B > 0$ and $(1 + \alpha)^{-1} = 2n < 0$.

We want to emphasise that when PGCG is considered, the sign of the parameters $A$ and $B$ must not to be necessarily positive and also $\alpha$ can be bigger or less than $-1$.

It can be seen that for type I the scale factor is bounded from below by $a_{\min} = |B/A|^{1/[3(1+\alpha)]}$ and, therefore, it takes values in the interval $a_{\min} \leq a < \infty$, which corresponds to $0 \leq \rho < |A|^{1/(1+\alpha)}$, approaching the energy density a finite constant value when the scale factor tends to infinity. The Friedmann Eq. (1) can be analytically integrated for the energy density (8) to lead a functional dependence of the cosmic time and the scale factor in terms of a hypergeometric series [8]. Since this expression is rather complicated and can be found in the literature, [8], we consider that it is enough to comment that the resulting expression implies that when the scale factor diverges the cosmic time also blows up, that is, there is no a singularity at a finite time in the future. The future normal behaviour is also indicated by the Hubble parameter which approaches a constant finite non-vanishing value for large scale factors and, therefore, we can conclude that the late time evolution of such a model is asymptotically de Sitter. It can be seen [7] that the type III model has a similar evolution for late times than the type I, although both behaviours can differ greatly at early times[7].

Although we have just discussed that the consideration of phantom models could avoid the occurrence of a future big rip singularity, another kind of doomsday could appear in phantom models. In order to show this fact, let us consider the type II PGCG. In this case the scale factor is bound from above by $a_{\max} = |B/A|^{1/[3(1+\alpha)]}$ taking, therefore, values in the interval $0 < a \leq a_{\max}$ which correspond to $A^{1/(1+\alpha)} \leq \rho < \infty$. As in the type I an analytical expression for the scale factor depending on the cosmic time can be found in terms of hypergeometric series [7]. Such an expression can be approximated close to the

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4The quantised $\alpha$ parameter in the type IV model eliminates possible past singularities that could appear in type I. A discussion about the evolution of these models at early times is out in the scope of the present chapter, so we refer the interested reader to Ref. [7].
maximum scale factor value and inverted leading to [6, 7]

\[
a \simeq a_{\text{max}} \left\{ 1 - \left( \frac{8\pi}{3} \right)^{\frac{1+\alpha}{2(1+\alpha)}} \left[ 1 + 2\alpha \right]^{\frac{2(1+\alpha)}{1+2\alpha}} \frac{1}{3(1+\alpha)} \left( t_{\text{max}} - t \right)^{\frac{2(1+\alpha)}{1+2\alpha}} \right\},
\]

which implies that the cosmic time elapsed since the universe has a given scale factor \( a \) (closed to \( a_{\text{max}} \)) until it reaches its maximum value is finite, i.e., \( t_{\text{max}} - t < \infty \). Therefore, this model would end at a finite singularity where its energy density blows up whereas the scale factor remains finite, called big freeze singularity [6]. The type IV model would have a similar behaviour and it exhibits a singularity of the same kind, [7]. So, both models describe a universe which would expand accelerating until it freezes its evolution at a finite time where it is infinitely full of phantom energy.

It must be pointed out that from a classical point of view, as such we are considering through this chapter, a singularity would break down the spacetime. Nevertheless, it has been argued that the consideration of quantum effects could avoid the big rip [22] and the big freeze singularity [10].

3 Dark energy accretion onto black holes.

As we have pointed out in the previous section, dark energy is filling our Universe and, therefore, it could be expected that it would interact with different cosmological objects like black- and worm-holes. This section is dedicated to the study of the accretion process onto black holes by using the most accepted model dealing with this phenomenon. Nevertheless, it must be pointed out that some controversy has originated around this method and as such other approaches have been proposed, which will be discussed in Sec. V.

The standard method treating the dark energy accretion onto black holes was firstly presented by Babichev et al. [21] and is based on the consideration of a black hole described by the Schwarzschild metric, surrounded by a perfect fluid which represents dark energy. In such a framework they considered the zero component of the energy-momentum conservation equation and the projection of this equation along the four-velocity, to derive the dynamical evolution of the black hole mass. We want to summarise a generalisation of this method, presented in Ref. [28], which allows an internal nonzero energy-flow component \( \Theta^0_0 \) by the consideration of the simplest non-static generalisation of the Schwarzschild metric, in which the black hole mass can depend generically on time. That is, the metric can be given by

\[
ds^2 = \left( 1 - \frac{2M(t)}{r} \right) dt^2 - \left( 1 - \frac{2M(t)}{r} \right)^{-1} dr^2 - r^2 \left( d\theta^2 + \sin^2 d\phi^2 \right),
\]

where \( M(t) \) is the black hole mass.
The zero component of the conservation law for energy-momentum tensor and its projection along the four-velocity can be integrated in the radial coordinate considering a surrounding perfect fluid. On the other hand, it is known that the rate of change of the black hole mass due to accretion of dark energy can be derived by integrating over the surface area the density of momentum $T_{r0}$. Taking into account these considerations one gets the following equation

$$\dot{M} = 4\pi A_M M^2 (p + \rho) e^{\int_\infty^r f(r,t) dr}, \quad (11)$$

relating the temporal rate of change of the mass with the pressure and energy density of the perfect fluid which is considered to describe dark energy.

For the relevant physical case of an asymptotic observer, i.e. $r \to \infty$, the previous equation simplifies to

$$\dot{M} = 4\pi A_M M^2 (p + \rho), \quad (12)$$

where $A_M$ is a positive constant of order unity. It must be pointed out that this expression is the same as that obtained by Babichev et al. [21] using the usual Schwarzschild metric, with the difference that in the current case it is only valid for asymptotic observers.

It can be noted that Eq. (12) shows that the black hole mass, and with it its size, must decrease when the black hole accretes a fluid which violates the dominant energy condition, i.e. a phantom fluid. This would increase when the dominant energy condition is preserved and it remains constant in the cosmological constant case, since a cosmological constant can be modelled by a perfect fluid with $p + \rho = 0$. It must be emphasised that Eq. (12) has been obtained for a general perfect fluid, therefore it would be valid in a great variety of dark energy models, since a load of them are based on a perfect fluid.

Since accretion of dark energy onto black holes would increase the black hole size in dark energy models fulfilling the dominant energy condition, an interesting question is whether this growth could be large enough to that the black hole might engulf the whole universe. In order to find a possible answer for this question, one can take into account the Friedmann equations to integrate Eq. (12), obtaining the temporal evolution of the black hole mass. That is, [12]

$$M(t) = \frac{M_0}{1 + \sqrt{\frac{8\pi}{3} A_M M_0 \left( \rho_{1/2} - \rho_0^{1/2} \right)}}. \quad (13)$$

This equation can also be expressed in terms of the Hubble parameter in the following way

$$M(t) = \frac{M_0}{1 + D M_0 \left[ H(t) - H_0 \right]}. \quad (14)$$

Therefore, as mentioned in Ref. [28], a black hole capable of engulfing the whole universe as shown in a model with an equation of state parameter bigger than minus one, should have a current mass which, roughly speaking, is bigger than...
all the matter content of the current observable universe, making the occurrence of such a phenomenon impossible.

In the following subsections we consider the models reviewed in the previous section to study the evolution of a black hole living in a universe filled with dark energy.

### 3.1 Application to a quintessence model.

We assume that dark energy is modelled by a quintessence model satisfying the equation of state \( w > -1 \) constant. As we have already mentioned, Eq. (12) implies that the rate of mass change is positive in this model, therefore the black hole mass grows along time due to accretion of quintessence. In order to obtain the dynamical behaviour of the black hole mass, one can introduce the equation of state (3) in Eq. (13) which leads to

\[
M = \frac{M_0 \left[ 1 + \frac{3}{2} (1 + w) \right] C (t - t_0) \right]}{1 + \frac{3}{2} (1 + w) C (t - t_0) - 4\pi A_M \rho_0 M_0 (1 + w) (t - t_0)}. \tag{15}
\]

There are something very interesting in this expression for the black hole mass, because it allows the occurrence of a bizarre fate for our universe as we have already pointed out. That is, Eq. (15) expresses the possibility that our universe might be engulfed by a black hole, since accretion of dark energy could make the mass of the black hole increase so quickly as to yield a black hole size that would eventually exceed the size of the universe in a finite cosmic time. In fact, the time in which the black hole might reach a infinite size would be

\[
t_{bs} = t_0 + \frac{1}{(1 + w) \left( 4\pi A_M \rho_0 M_0 - \sqrt{6\pi \rho_0} \right)}. \tag{16}
\]

This time is finite but, although a universe filled with quintessence has no future singularity, present observational data seems to imply that \( w \) is not constant and suggests values less than -1, making it unlikely that the occurrence of the considered bizarre phenomenon at any time in the far future would happen, at least in principle. However, if \( \dot{w} > 0 \) then the black holes would undergo a larger growth due to accretion of dark energy.

Nevertheless, if one studies in deeper detail Eq. (15), then one obtains (see Ref. [28]) that in order to have a black hole able to reach an infinite mass in an infinite time, this black hole must possess an initial mass such as \( \frac{8\pi \rho_0 / 3}{1/2} A_M M_0 = 1 \) which means, taking into account the observational data, \( M \sim 10^{23} M_\odot \) (where \( M_\odot \) is the Sun’s mass). Therefore, in order to have an infinitely large black hole in a finite time in the future, the current black hole mass should be bigger than \( 10^{23} M_\odot \), which is an extremely large value even for black holes in the galaxies centres. Even more, one can estimate the current matter content of the universe assuming that the observable Universe expands at the speed of light, obtaining a total of \( 10^{23} \) starts [28]. Therefore, it seems that in order to have a black hole able to engulf the whole universe,
it should have a current mass equal to the mass of all the observable universe, which would not be possible.

Finally, we want to point out that, even in the case that the accretion phenomenon of dark energy onto black holes could not produce cosmological consequences in terms of a catastrophic end, it could help in the determination of the correct dark energy model. So, if astronomers were able, in practice, to observe a growth bigger than expected of those black holes living in the centre of the galaxies, then this could be an observational measure of the effects of dark energy with $w > -1$.

### 3.2 Application to a phantom quintessence model.

As we have already pointed out, the observational data not only allows that the equation of state parameter takes a value less than $-1$ but even they seem to suggest it, acquiring therefore, special interest in the study of the evolution of a black hole in a universe filled with phantom energy. So, now we consider $w < -1$ and constant, consequently (12) shows that the black hole mass decreases with time. More precisely, inserting Eqs. (3) and (5) in Eq. (13), we get an accurate expression of the evolution of the black hole mass, i.e.

$$
M = M_0 \frac{1 - \frac{3}{2} (|w| - 1) C (t - t_0)}{1 - \frac{3}{2} (|w| - 1) C (t - t_0) + 4\pi A_M \rho_M (|w| - 1) (t - t_0)}.
$$

(17)

Taking into account that in a universe filled with phantom energy with $w$ constant a big rip singularity will take place in the future, one can introduce the time of occurrence of the big rip, $t_{br}$, in (17) to get that the black hole mass vanishes at $t_{br}$, independently of the current black hole mass, $M_0$; that is, all black holes disappear at the big rip [21]. It can be noted, by inspection of Eq. (13), that this is not only an interesting property of this phantom model but it can be found in all models which present a singularity with a divergence of the Hubble parameter at a finite time in the future.

Finally, the decrease of black holes due to the phantom energy accretion phenomenon could provide us with a possible observational test of these models. Therefore, if future observations of black holes in the centre of galaxies (or other possible black holes) indicate a growth of those objects less than expected, then it could be associated to accretion of phantom energy, providing us with another measure able to discriminate between different dark energy models, which would complete those that come from GRB [44, 48], supernova, or other observational data.

### 3.3 Application to a Generalized Chaplygin model.

Let us now study the evolution of a black hole in a universe filled with a Generalized Chaplygin Gas. Since the consideration of dark energy modelled by some kinds of Phantom Generalized Chaplygin Gas could prevent the occurrence of a big rip [8], one could expect the avoidance of the weird behaviours that appear.
in quintessence or phantom models in these frameworks. In order to see if that is the case, one must take into account Eq. (13).

As we have mentioned in Sec. II, a Generalized Chaplygin Gas, with \( A > 0 \) and \( \alpha > -1 \), preserves dominant energy condition when the parameter \( B > 0 \) and violated otherwise. So when \( B > 0 \) the black hole mass increases with cosmic time up to a constant value, but there are a set of parameters where if \( \alpha \) is close to \(-1\) then it would seem that the black hole mass could eventually exceed the size of the universe at finite time in the future [24]. When dominant energy condition is violated, \( B < 0 \), black hole mass decreases along cosmic time, tending to a nonzero constant value, therefore black holes do not disappear in this model.

On the other hand, it can be seen [12] that performing a deeper study of the mentioned four types of PGCG (where the sign of \( A \) and the range on \( \alpha \) is not previously fixed) at late times, where the phantom fluid would drive the dynamical evolution of the universe, the results can be summarised as follows:

- Type I and III. Black holes decrease with time, where the mass tend to a nonzero value when the time goes to infinity.
- Type II and IV. Black holes masses decrease, but now all black holes disappear at big freeze, i.e., the mass of all black holes tend to zero when the universe reaches the big freeze singularity with independence of their initial mass (as it should be expected by inspection of equation (14)).

To end this section, we consider that it would be quite interesting to explore the region of parameter space \((\alpha, A, H_0, \Omega_K, \Omega_\phi)\) allowed by current observations in order to determine whether there exists any allowed sections leading to a big freeze or a big rip. However, all available analyses [37, 38, 39, 40, 41, 42, 43, 44, 45, 46] are restricted to the physical region where no dominant energy condition is violated. Therefore, the section described by the interval implied by a PGCG necessarily is outside the analysed regions. One has to extend the investigated domains to include values of parameter \( A > 1, A < 0 \) or \( \alpha < -1 \) to probe the parameters space where the dominant energy condition is violated.

### 3.4 Consideration to other black holes.

Up to now, we have shown the evolution of a Schwarzschild black hole in a universe filled with dark energy. In this subsection, we study the accretion of dark energy onto charged or rotating black holes, to show whether charge or angular momentum have some influence in their evolution.

Let us continue by considering another black hole metric in order to understand better the application of the dark energy accretion mechanism. In [34], Babichev et al. apply a generalisation of the accretion formalism to a Reissner-Nordsröm black hole. In this case, the metric is given by

\[ B \] is the constant parameter that appears in the energy momentum tensor conservation law [2] for a GCG.
\[
\frac{\mathrm{d}s^2}{\mathrm{d}t^2} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) \frac{\mathrm{d}t^2}{\mathrm{d}t^2} - \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} \frac{\mathrm{d}r^2}{\mathrm{d}r^2} - r^2 \left(\sin^2 \theta \frac{\mathrm{d}\theta^2}{\mathrm{d}\theta^2} + \sin \theta \cos \theta \frac{\mathrm{d}\phi^2}{\mathrm{d}\phi^2}\right),
\]

where \( m^2 > e^2 \). It can be noted that if \( m < |e| \), then the solution would represent a naked singularity, corresponding \( m = |e| \) to the extreme case. By integrating the conservation laws for momentum-energy and its projection along four-velocity for the case for a perfect fluid, and taking into account that the rate of change of the black hole mass due to accretion of dark energy can be derived by integration over the surface area the density of momentum \( T^0_r \), Babichev et al. get again the same expression (12) which relates the temporal rate of change of the black hole mass to the pressure and the energy density of the perfect fluid. So, also in a Reissner-Nordström black hole, the black hole mass increases when it is accreting dark energy holding the dominant energy condition and its mass decreases when phantom energy is considered in the accretion process.

At this point the next question arises again, if phantom energy is getting involved then black hole mass decreases, vanishing at big rip singularity. Therefore, since the electric charge \( e \) remains constant due to phantom energy accretion, then in a finite time, the black hole must reach the extreme case, transforming the black hole into a naked singularity. Nevertheless, the authors of Ref. \[34\] perform a more detailed study about this possible transformation in a naked singularity, concluding that there is no accretion of the perfect fluid onto the Reissner-Nordström naked singularity when \( m^2 < e^2 \) and that, in this situation, a static atmosphere of the fluid around the naked singularity would be formed.

It must be emphasised that, although when one is considering cases far from the extreme case, the back reaction can be neglected and the perfect fluid approximation appears to be valid, it seems that this approximation breaks down close to the extremal case, where one has to take into account the back reaction of the perfect fluid onto the background metric. Even more, the same consideration about the avoidance of transformation of a black hole into a naked singularity, can also be applied to a Kerr black hole. Nevertheless, if the back reaction does not prevent the process of phantom accretion onto a charged black hole or rotating black hole, then this process could be a way to violate the cosmic censorship conjecture \[51\].

4 Dark energy accretion onto wormholes.

The first solution of the Einstein’s equations describing a traversable wormhole was found by Morris and Thorne in their seminal work \[15\]. That solution, obtained under the assumption of staticity and spherical symmetry, describes a throat connecting two asymptotically flat regions of the spacetime without any
horizon and can be expressed as
\[
\text{d}s^2 = -e^{2\Phi(r)}\text{d}t^2 + \frac{\text{d}r^2}{1 - K(r)/r} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right),
\]
(19)
where \(\Phi(r)\) and \(K(r)\) are the shift and shape functions, respectively, both tending to a constant value when the radial coordinate \(r \to \infty\) in order to have asymptotic flatness. It must be noted that, in these coordinates, two coordinate patches are needed to cover the two asymptotically flat regions, each with \(r_0 \leq r \leq \infty\), with \(r_0\) the minimum radius which corresponds to the throat radius, where \(K(r_0) = r_0\).

It can be seen [15] that solution (19) must fulfil some additional requirements in order to describe a traversable wormhole. In particular the outward flaring condition imposes \(K'(r_0) < 1\) what, through the Einstein’s equations, implies that \(p_r(r_0) + \rho(r_0) < 0\) (where here \(p_r\) denotes the radial component of the pressure). Therefore, the wormhole must be surrounded by some material with unusual characteristics, called exotic matter, which could lead to the neglect of such spacetime.

Nevertheless, as we have already mentioned in the introduction, the discovery of the current accelerated expansion of the Universe and the consideration of phantom energy as a possible candidate for its origin has produced a more natural consideration of the properties of exotic matter, since it seems that phantom energy could be precisely the exotic stuff which supports wormholes. Gonzalez-Diaz [19] considered that similarly to black holes accrete dark energy, wormholes could accrete phantom energy producing a great increase of their size, in such a way that the size of a wormhole could be infinitely larger before the universe reaches the big rip singularity. Such a process would produce the moment that the size of the wormhole equals the size of the universe, the universe boards itself in a travel through the wormhole, called big trip. Even more, the notion of phantom energy has been extended to inhomogeneous spherically symmetric spacetimes showing that it can be in fact the exotic material which supports wormholes [14], which backs up the mentioned idea.

In order to study such a process one can follow a similar method to the one used by Babichev et al. for the black hole case. So, considering the non-static generalisation of Eq. (19) obtained by the consideration of an arbitrary dependence of the shape function on the time, \(K(r, t)\), and an energy momentum-tensor of a perfect fluid, one can find the equivalent of Eq. (12) for the wormhole case, that is the temporal mass rate evolution as measured by an asymptotic observer, which is [30]
\[
\dot{m} = -4\pi Qm^2(p + \rho),
\]
(20)
with \(Q\) a positive constant. That expression shows that the wormhole mass, and with it, its size must increase when the wormhole accretes phantom energy, it decreases by the accretion of dark energy, remaining constant in the cosmological constant case. It must be remarked that in the achieving of Eq. (20) no assumption about the possible dependence on the energy density or on the pressure of the fluid have been done, allowing an arbitrary dependence with the
time and with the radial component, therefore, such an equation is general and take into account the possible back reaction in an asymptotically flat wormhole spacetime.

If one now considers as an approximation that the fluid which surrounds the wormhole is a cosmological one, i.e., an homogeneous and isotropic fluid fulfilling the Friedmann equations (1) and (2) and the conservation law, then Eq. (20) can be integrated to obtain

\[ m(t) = \frac{m_0}{1 - Qm_0[H(t) - H_0]} \]  

(21)

This expression shows that in phantom models, where \( H(t) \) is an increasing function, the wormhole throat could become infinitely big if the Hubble parameter reaches the value \( H_* = H_0 + 1/(Qm_0) < \infty \) at some time \( t_* \) in the future. It can be seen that this would be the case at least in models which show a future singularity in a finite time in the future characterised by a divergence of the Hubble parameter, because in those models one has \( H_0 < H_* < H_{\text{sing}} = \infty \) and, since the Hubble parameter is a strictly increasing and continuous function before the time of the singularity, this implies \( t_0 < t_* < t_{\text{sing}} \). Therefore, the size of a wormhole would be bigger than the size of the universe before the occurrence of the future singularity in all models possessing a future singularity where the Hubble parameter blows up, i.e., in such models the universe would travel through a big trip.

In this section we show the implications of the phenomenon of dark and phantom energy accretion onto wormholes in the models presented in Sec. II. That procedure lead, as it is expected, to the decrease of the wormhole size when it accretes dark energy with \( w > -1 \) and to a growth of the wormhole mouth in phantom cases. Even more, the big rip and big freeze singularities, in the corresponding models, can be avoided by a big trip phenomenon since, although these singularities present a different behaviour of the scale factor, at both singularities the Hubble parameter blows up.

### 4.1 Application to a quintessence model.

Let us consider that the dark energy is modelled by a quintessence model, satisfying the equation of state (3) with \( w > -1 \) constant. Eq. (20) tells us that the rate of mass change is negative, so the wormhole mass would decrease with time due to the accretion of quintessence. Furthermore, taking into account the equation of state (3), one can solve (20) getting the following expression which relates the wormhole mass to the cosmic time [25],

\[ m = \frac{m_0}{1 + \frac{4\pi Qm_0(1+w)(t-t_0)}{1+\frac{2}{3}(1+w)C(t-t_0)}} \]  

(22)

---

\( ^6 \)It must be emphasised that, whereas in the case of the study of the dark energy accretion onto black hole phenomenon, the solution is not able to take into account the back reaction of the spacetime, in this case we are treating with a non-vacuum solution and allowing arbitrary time dependence on the involved functions and, therefore, up to now we are taking into account any possible back reaction.
This expression shows us how a wormhole loses mass due to the accretion of quintessence. Even more, if due to any additional hypothetical process this wormhole would have a macroscopic size, then it would be subjected to chronology protection [36]; therefore, vacuum polarisation created particles would catastrophically accumulated on the chronology horizon of the wormhole, letting the corresponding normalised stress-energy tensor to diverge which, at the end of the day, would imply the disappearance of the wormhole.

4.2 Application to a phantom quintessence model.

Now, we are interested in study the evolution of a wormhole in a universe filled with phantom energy with $w < -1$ constant. In order to obtain the temporal evolution of the wormhole, one can introduce the equation of state of phantom energy into the r.h.s. of Eq. (20), getting [25],

$$m = \frac{m_0}{1 + \frac{2\pi Q m_0 (|w|-1)(t-t_0)}{1+\frac{8}{3}(|w|-1)C(t-t_0)}}. \quad (23)$$

This expression implies that the exotic mass of the wormhole diverges at the time

$$t_{bt} = t_0 + \frac{t_{br} - t_0}{1 + \frac{8\pi Q m_0 3(|w|-1)/2^3}{C}}. \quad (24)$$

where $t_{br}$ is the finite time at which the big rip singularity takes place. Since the wormhole size diverges before that the universe reaches the big rip singularity, it would be a previous time at which the size of the wormhole would be bigger than the universe, being at this time where properly starts the travel of the universe through the wormhole.

The huge growth of the wormhole throat poses the following two problems. On the one hand, since the wormhole spacetimes are usually considered to be asymptotically flat, when the wormhole increases more than the universe it is impossible to place the wormhole on this universe. On the other hand, since the universe is travelling through the wormhole, one can ask where is the universe travelling to? The solution of these equivalent problems requires the consideration of a multiverse scenario. In such a framework the wormhole could be re-infixed in another universe where the wormhole would be asymptotically flat to, giving also a final destination to the universal travel.

\[It is worth noticing that in models showing one big trip, as it is the considered case, the universe would travel through the time of the arrival universe being, in this case, not a proper time travel. On the other hand, in models which present more than one big trip phenomenon the consideration of a multiverse framework would be not more necessary, since the wormhole mouth at the moment that it is bigger than the universe could be connected to the other infinitely large wormhole mouth, travelling in this case the universe along its own time from future to past. The reader interested on this topic is advised to consult Ref. [49].\]
4.3 Application to a Generalized Chaplygin Gas model.

Finally, we will study the evolution of a wormhole in the case of a universe filled with a Generalized Chaplygin Gas. Since type I and III of PGCG avoid the occurrence of a future singularity \cite{7, 8}, it is of special interest to study the possible occurrence of a big trip phenomenon in these models. Following this line of thinking, in Ref. \cite{24} it is analysed the phantom energy accretion phenomenon onto wormholes when the phantom energy is modelled by a type I PGCG. In order to perform this study, let us temporarily fix $A > 0$ and $\alpha > -1$, therefore, solving Eq. (21) for the equation of state of a GCG, one obtains

$$m = \frac{m_0}{1 - Qm_0 \sqrt{\frac{8\pi}{3} \left( \frac{\rho_1}{2} - \rho_0^{1/2} \right)}}.$$  \hspace{1cm} (25)

For the case where the dominant energy condition is violated, i.e. $B < 0$, we obtain that $m$ increases with time and tends to a maximum, nonzero constant value. If the dominant energy condition is assumed to be hold, i.e. $B > 0$, then $m$ decreases with time, with $m$ tending to nonzero constant values.

It can be seen that, when the cosmic time goes to infinity, then the exotic mass of wormhole approaches to

$$m = \frac{m_0}{1 - Qm_0 \sqrt{\frac{8\pi}{3} \left( A^{\frac{1}{1+\alpha}} - \rho_0^{1/2} \right)}},$$  \hspace{1cm} (26)

that is a generally finite value both for $B > 0$ and $B < 0$. Thus, it could be thought that the presence of a Generalized Chaplygin Gas prevents the eventual occurrence of the big trip phenomenon. However, such a conclusion cannot be guaranteed as the size of the wormhole throat could still exceed the size of the universe during its previous evolution. The question is whether the wormhole would grow rapidly enough or not to engulf the universe during the evolution to its final classically stationary state. To avoid a big trip one needs that the radius of the wormhole does not exceed the size of the universe. It can be checked \cite{24} that GCG generally prevents the occurrence of a big trip when $\alpha$ does not reach values sufficiently close to $-1$, but when $\alpha$ is inside the interval

$$-1 < \alpha < \frac{\ln A}{\ln \left( \sqrt{\frac{3}{8\pi} \frac{1}{m_0 D} + \rho_0^{1/2}} \right)^2} - 1,$$  \hspace{1cm} (27)

a big trip would still take place.

It is worth noticing that, when there is no big trip phenomenon, the wormhole size tends to become constant at the final stages of its evolution being a rather a macroscopic object. So, the wormhole at this stage would be subjected to chronology protection \cite{36} and vacuum polarisation created particles would catastrophically accumulate on the chronology horizon of the wormhole making the corresponding renormalised stress-energy tensor to diverge and hence the wormhole would disappear.
On the other hand, one can study the wormholes’ evolution living in a universe with phantom energy modelled by a type II or IV PGCG \cite{12}, where a future big freeze singularity is predicted. Since a big freeze singularity implies the divergence of the Hubble parameter at this singularity, as we have mentioned in the introduction of the present section, this implies that the wormhole size would blow up before the occurrence of the singularity, implying a big trip phenomenon. That can be easily proved taking into account Eqs. (1), (8) and (21) for type II PGCG, which yields

\[ m(x) = \frac{m_0}{1 + \sqrt{\frac{8\pi}{3} \frac{Q m_0}{A^{2(1+\alpha)}} \left[ \frac{1}{(1-x^0_0)^{2(1+\alpha)} - \frac{1}{(1-x^0_0)^{2(1+\alpha)}}} \right]}} \]

(28)

where \( x = a/a_{\text{max}} \) (0 \( \leq x \leq 1 \)) and a similar expression can be obtained for type IV replacing \( A \) with \( |A| \). In order to study the behaviour of the wormhole mass, one can define the function \( F(x) = m_0/m(x) \) which is continuous in the interval \([x_0, 1)\). This function takes a value \( F(x_0) = 1 > 0 \) and tends to minus infinity when \( x \) goes to 1 (which corresponds to \( a \to a_{\text{max}} \)), which implies that \( F(x) \) vanishes at some \( x_\ast \) with \( x_0 < x_\ast < 1 \). Therefore, \( m(x) \) blows up at \( x_\ast \) being the throat size infinitely large before the universe reaches the big freeze singularity (at \( x = 1 \)). So the whole universe will travel through the wormhole before the occurrence of the doomsday.

The results can be summarised as follows

- Types I and III. The evolution of the wormhole is the same for GCG.

- Types II and IV. A big trip phenomenon would prevent the expected cosmological doomsday, i.e., the big freeze.

5 Debate and new lines of research.

In the present chapter we have used methods based in the Babichev et al. one, in order to consider the dark and phantom energy accretion onto black- and worm-holes. Our intention is not to claim that the study of these processes is a closed issue, on the contrary, it remains open up to now and a lot of discussion has been originated in this way.

In order to point out the shortcomings of the current available methods which deal with the mentioned accretion phenomenon and suggest possible new lines of research. In this section we also include in chronological order some comments which have appeared in the literature supporting, improving or criticizing the Babichev et al. method \cite{11}. We alternate works regarding the accretion onto black holes with others dealing with the accretion onto wormholes, because both phenomena can be studied following the same procedure. Nevertheless, as it has been and will be pointed out, the wormhole case is free of some shortcomings which affect the black hole one, since the first one can never be considered as a vacuum solution (at least if one restricts oneself to the traversable wormhole case).
The first work about accretion of dark energy onto black holes was due to E. Babichev, V. Dokuchaev and Yu. Eroshenko [11]. They considered the spherically symmetric accretion of dark energy onto black holes adjusting the analytic relativistic accretion solution onto the Schwarzschild black hole developed by Michel [18], eliminating from the equations the particle number density. So they obtain the expression for the black hole temporal mass rate

\[ \dot{M} = 4\pi A_M M^2 \left[ \rho_{\infty} + p(\rho_{\infty}) \right], \]

showing that the black hole mass could decrease by the accretion phenomenon. The authors pointed out that such a decrease of the black hole size is due to the violation of the dominant energy condition, since this condition is assumed to be fulfilled in the derivation of the black hole non-decrease area theorem. On the other hand, by integrating (29) in a phantom Friedmann universe, they found that the masses of all black holes tend towards zero when the universe approaches the big rip, independently of their initial masses.

Soon after, P. F. Gonzalez-Diaz [19] considered the spherically symmetric accretion of dark and phantom energy onto Morris-Thorne wormholes. He assumed that, since the mass of the spherical thin shell of the exotic matter in a Morris-Thorne wormhole,

\[ \mu = -\pi b_0/2 \] (where \( b_0 \) is the radius of the spherical wormhole throat), is approximately just the negative of the amount of the mass required to produce a Schwarzschild wormhole, then the rate of change of the wormhole throat radius should be similar to that obtained by Babichev et al. [11] for the black hole mass but with a minus sign, i.e.

\[ b'_0 = -2\pi^2 Q b_0^2 (1 + w) \rho, \]

with \( Q \simeq A_M \). He concluded, therefore, that the wormhole throat should increase by the accretion of phantom energy. Even more, he showed, by integrating Eq. (30) in a phantom model with constant equation of state parameter, that the wormhole increases even faster than the universe itself, engulfing the whole universe before it reaches the big rip singularity. Therefore, the universe would embarks itself in a big trip.

Later on, P. F. Gonzalez-Diaz and C. L. Siguenza, [20], obtained that the phantom energy accretion onto black holes leads to the disappearance of the black holes at the big rip even when Eq. (29) is integrated in more complicated models and Babichev et al. recovered their previous result in a more detailed work [21] where they also included a deeper study of two dark energy models admitting analytical solutions. On the other hand, works containing relevant implications of the result of Babichev et al. were also published during that time, like the influence of the accretion phenomenon on the black hole and phantom thermodynamics, Ref. [20], and the possible survival of black holes at the big rip due to the same phenomenon which could smooth the big rip singularity when quantum effects are taken into account, Ref. [22].

But the previous mentioned works did not had the final say about this topic. In 2005 V. Faraoni and W. Israel [23] considered the time evolution of a wormhole in a phantom Friedmann universe finding no big trip phenomenon.
In that work they commented that the way in which Gonzalez-Diaz applied the Babichev et al. method to the wormhole case in Ref. [19] could be wrong, since \( \mu = -\pi b_0/2 \) must not to be necessarily valid for a time-dependent wormhole embedded in a FLRW universe and, therefore, the wormhole mass time rate due to the accretion phenomenon would not be simply the analogous negative of the black hole mass time rate.

Soon after, Gonzalez-Diaz [25] followed a similar procedure as it has been done by Babichev et al. [11], adjusting the Michel theory to the case of Morris-Thorne wormholes, in order to study the dark energy accretion onto wormholes. He obtained Eq. (20) for the case of an asymptotic observer (which is equivalent to Eq. (30) with a re-definition of the constants). He also claimed that the results obtained by Faraoni and Israel [23] just take into account the inflationary effects of the accelerated expansion of the universe on the wormhole size (also considered by himself years ago in another work [26]) and do not include the superposed effects due to the accretion phenomenon, which existence is clarified in that work [25].

On the other hand, a number of difficulties related to the big trip process were treated also by Gonzalez-Diaz in Ref. [27]. First, he showed how the corrections appearing in the expressions of the study of a wormhole metric with a non-static shape function applying the Babichev et al. method [11] should disappear on the asymptotic limit, coinciding with those corresponding calculated expressions in the static case in that regime. Even more, he considered explicitly a metric able to describe a wormhole in a Friedmann universe, arguing that it would ultimately imply the occurrence of a big trip phenomenon. Second, since wormhole spacetimes are usually considered to be asymptotically flat, then when the wormhole increases more than the universe, this object can neither be placed on it nor be asymptotically flat to it. He proposed that in such a situation a multiverse context must be considered, what would allow to re-insert the wormhole in another universe, recovering the meaning of the asymptotic regime where the accretion process has been calculated. In the third place, he considered the possible instability of wormholes due to the quantum creation of vacuum particles on the chronology horizon when the wormhole throat grows at a rate smaller than or nearly the same as the speed of light. However, in a phantom model the accreting wormhole would clearly grow at a rate which exceeds the speed of light asymptotically and so the vacuum particles would never reach the chronology horizon where they have being created, keeping the wormhole stability. Moreover, although it was known that quantum effects could affect the big rip singularity [22], Gonzalez-Diaz showed that those effects have no influence in the big trip, which would take place before that singularity. In the fourth place, he argued that the big trip phenomenon would not imply any contradiction with the holographic bound, since wormholes able to connect regions after and before the big rip extend the evolution of the universe up to infinite time.

Following that line of thinking, the authors of [28] applied the Babichev et al. method to the simplest non-static generalisation of the Schwarzschild metric, in order to study the dark energy accretion onto black holes with arbitrary accre-
tion rates. Although they are still using a test fluid approach and, therefore, the validity of their result on arbitrary accretion rates is debatable, the non-static metric is enough to take into account internal non-zero energy flow $\Theta^r_0$. As it was suggested by Gonzalez-Diaz in the case of wormholes [27], a study of the accretion phenomenon using a non-static metric recovers the result obtained in the static case, Eq. (29), for asymptotic observers.

Later on, Faraoni published “No “big trips” for the universe”, [29], where a sceptical attitude about the big trip phenomenon is adopted, based on some shortcomings of the works of Gonzalez-Diaz [19, 25, 27] in particular and the method of Babichev et al. [11] in general. His principal objection regarding the mentioned works of Gonzalez-Diaz was that the use of a static metric can never produce a non-zero radial energy flow onto the hole, i.e. static metrics always imply $\Theta^r_0 = 0$. Even more, the solution of Gonzalez-Diaz (and the corresponding of Babichev et al. in the black hole case) cannot be adjusted to satisfy the Einstein’s equations, as the used conservation laws would only strictly correspond to vacuum solutions. On the other hand, he also showed that if the phantom fluid is modelled by a perfect fluid, as it is done in the Babichev et al. method and its application to wormholes, the proper radial velocity of the fluid is $v \sim a^{3(1+w)/2}$ which vanishes at the big rip stopping the accretion phenomenon.

Soon after, Gonzalez-Diaz et al., [30], applied the method of Babichev et al. to a non-static generalisation of the Morris-Thorne metric, introducing a shape function with an arbitrary dependence on time, $K(r, t)$. They recovered again Eq. (20) for the temporal mass rate of a wormhole in the asymptotic limit. It must be noted that in their derivation of such expression they allowed an arbitrary dependence of the energy density, pressure and the four velocity of the fluid on both the time and radial coordinates. Therefore, the wormhole mass rate expression Eq. (20) must be valid in general for asymptotically flat wormholes. It must be emphasised that a wormhole is a non-vacuum solution and that the consideration of a time dependence in the shape function leads also to a non zero $\Theta^r_0$, which take into account the non-zero energy flow onto the hole, therefore taking into account the back reaction. Nevertheless, the authors of Ref. [30] considered that the most crucial argument against the big trip included in the paper of Faraoni [29] is the vanishing of the proper radial velocity at the big rip and its quickly decrease close to it, since it is in the point of introducing an explicitly equations of state for the fluid in order to integrate Eq. (20) where they were considering an approximation. First of all, the authors noted that, besides the fact that the time where the universe is engulfed by a wormhole is not only before the big rip but even before the divergence of the wormhole mouth, the important quantity which refers to an accretion process is the proper radial flow, which is approximately $\rho v \sim a^{-3(1+w)/2}$ which increases with time for $w < -1$ and diverges at the big rip, what guarantees the process would not be stopped. In the second place, they point out that the accretion of dark and phantom energy onto astronomical objects differs from the accretion of usual energy concentrated in a given region of space onto those objects, because in the first case the energy pervades the whole space being, therefore,
a phenomenon not based on any fluid motion, but on increasing more and more space filled with such kind of energy inside the boundary of the considered object.

Later on, C. Gao, X. Chen, V. Faraoni and Y. G. Shen, [31], emphasising that the method of Babichev et al. applied to black holes is not taking into account the backreaction of the fluid on the background, claimed that the results obtained by using that method can only be valid in a low matter density background. In that spirit, they used a generalised McVittie metric and inserted a radial heat flux term in the energy-momentum tensor, in order to show that a cosmological black hole (non-asymptotically flat) should increase by the accretion of phantom energy. The authors also pointed out any difficulties to compare their solution with the corresponding of Babichev et al. [11], arguing that this fact could be due to the simplifications taken in both cases.

The work of Gao et al. originated some interesting comments. First, in Ref. [32], X. Zhang pointed out the shortcomings of the Babichev et al. method, in particular the use of a non-cosmological metric and the exclusion of the backreaction of the phantom fluid on the black hole metric. Nevertheless, Zhang found the results of Ref. [31] highly speculative, giving the example of the use of a hypothesised metric. Therefore, he decided to use for the moment the method of Babichev et al. in order to extract at least some tentative conclusions, lacking a complete method to the study of the dark energy accretion phenomenon. Second, in [33] where a first attempt to include cosmological effects in the study of the accretion of dark energy onto black holes was done by the consideration of a Schwarzschild-de Sitter spacetime, it was noted that the results achieved in [31] were obtained under the assumption of a premise (contradictory with all studies of this problem in the literature) in which their desired result is contained, making circular their whole argument, and their result invalid. Third, in [34], Babichev et al. included some comments expressing their doubts regarding the conclusions of [31]. They claimed that the heat flux term is introduced in an unnatural way in the solution of Gao et al. to support their configuration, because the perfect fluid is not accreted in the mentioned solution and that such an introduction could be lead to instabilities to small perturbations. They also pointed out that the temperature of the fluid blows up at the event horizon.

In summary, regarding the phenomenon of dark and phantom energy accretion onto black holes the method developed by Babichev et al. [11] has been improved to take into account the backreaction on the black hole size in an asymptotically flat space and also in a cosmological one by using a non-static generalisation of the Schwarzschild and Schwarzschild-de Sitter metric, respectively. Nevertheless, the study still lacks the consideration of the complete backreaction of the dark or phantom fluid in an asymptotically dark or phantom universe.

Although the method used to study the dark and phantom energy accretion onto wormholes is similar to the one treating the above mentioned phenomenon, in the case of the phantom energy accretion onto wormholes the backreaction originated by the consideration of the phantom fluid is automatically taken into
account by using a non-static generalisation of the Morris-Thorne metric \[30\]. In this case there are no studies, up to our knowledge, considering rigorously the wormhole accretion phenomenon in a cosmological spacetime.

We want however to point out, that the increase (decrease) of the black hole size by the accretion of dark (phantom) energy, and the contrary in the case of wormholes, seems considerably well supported. Such affirmation can be also understood taken into account a different method. It is well known that the formalism developed by Hayward for spherically symmetric spacetimes \[35\] implies that a dynamical black hole (characterised by a future outer trapping horizon) would increase if it is considering in an environment fulfilling \( p + \rho > 0 \) and decrease if \( p + \rho < 0 \), phenomenon which is due to a flow of such surrounding material into the hole. Therefore, this totally independent study confirms the results presented in this chapter about black holes at least in a qualitative way. The question would be, how large are the quantitative differences which could appear by the consideration of the backreaction and the cosmological space?

On the other hand, regarding wormholes it has been shown \[50\] that in order to recover using the Hayward formalism the results obtained by the accretion method where the backreaction is included and by the very basis of wormhole physics, a wormhole must be characterised by a past outer trapping horizon. Since it seems that there would be no reason to change the local characterisation of an astronomical object because of the consideration of such an object in a space with a different asymptotically behaviour, the qualitative increase (decrease) of wormholes by the accretion of phantom (dark) energy should be recovered by considering cosmological wormholes spacetimes. Nevertheless, whether a wormhole would suffer a so huge increase to include the whole universe, occurring a big trip, is still an open question.

We want to point out that there are several interesting opened questions concerning accretion onto black holes. More improvement in the accretion theory is needed to take into account the backreaction of the space-time and study the situation where a perfect fluid approximation is not valid, what would clarify whether a black hole can become a naked singularity? A more detailed study about the spin or charge super-radiance is also needed, in order to show whether these processes could concur in such a way that finally the cosmic censorship conjecture would keep hold. Preliminary results indicate that this is the case, but the possibility of violation of the cosmic censorship conjecture produced by the process of dark energy accretion onto a charged or rotating black holes is still open.

Finally, it must be worth noticing that recent papers, \[52, 53\], consider the Babichev et al. method to propose a new observational dark energy test. The main idea is based on the black hole mass change induced by the dark energy accretion process which, as we have shown in this chapter, is proportional to \( 1 + w \). Although a direct observation of this change is beyond our current detection possibilities, because the time scale to produce a change in the black hole mass measurable with our present devices would be too long, it would cause observable modifications in the orbital radius of the supermassive black hole binaries, since the black hole binaries would either merge in a more acceler-
ated way than expected if $1 + w > 0$ or the merging would be progressively stopped if the dominant energy condition is violated, being also possible that the binaries would rip apart in the second case. At this moment there are two candidates, Galaxy 0402+379 and Radio Galaxy OJ287, for the observation of the mentioned phenomenon what could provide us with more information about the nature of dark energy, and probably more interesting candidates can be expected in the future.

6 Conclusions and further comments.

In the present chapter we have shown that if the current accelerating expansion of the universe is explained in the framework of General Relativity, then the consideration of a dynamical dark energy fluid would produce other effects besides the modelling of that acceleration. In this sense, the consideration of dark energy would not be simply the consideration of an ether covering the whole space, since footprints of such a fluid could appear in our Universe by observing the evolution of black- and worm-holes. Whether such effects are measurable in practice, is a question related to the accuracy of the observational data.

The effects in question regarding the dynamical evolution of black- and worm-holes, would be an additional increase (decrease) of the black hole size in the case that the dark energy fulfils (violates) the dominant energy condition, and the contrary in the wormhole case. Even more, these effects would produce changes in the orbital radius of black hole binaries which could be large enough to be detected in practice, helping us to get new constrains to dark energy equation of state parameter.

If one considers the used test fluid approximation to hold in phantom models possessing a singularity at a finite time in the future, then the black holes would tend to disappear at that singularity, which would never be reached since a big trip phenomenon may take place before. On the other hand, in dark energy models with $w > -1$ black holes would not engulf the whole universe, since the current mass of a hole able to exceed the universe size in a finite time should be so huge that it would be bigger than the mass of the observable Universe.

Although, by the arguments presented in this chapter, the qualitative evolution of the considered astronomical objects by the accretion of dark energy seems to be a solid result, the quantitative dynamical behaviour could differ from the mentioned results, since at the final step we are considering the approximation that the surrounding fluid is a cosmological one. In the black hole case this approximation is even stronger, since the non-static generalisation of the Schwarzschild metric, though taking into account the radial flow, lacks of a consideration of the backreaction.

Finally, whether or not the above features studied in this chapter can be taken to imply that certain dark energy models are more consistent than others is a matter that will depend on both the intrinsic consistency of the different models and the current and future observational data.

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8The interested reader can look up in [52, 53] for more details.
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