The Higgs sector of a 3-3-1 model with right-handed neutrinos to be tested at the LHC

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Abstract

We explore in this paper certain phenomenological consequences - to be tested at the LHC - regarding the scalar sector of a $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge model with right-handed neutrinos. Our analysis is performed in a particular theoretical approach of treating gauge models with spontaneous symmetry breaking in which a single free parameter $a$ finally remains to be tuned, once all the Standard Model phenomenology is recovered. It is also proved that this particular method is flexible enough as to accommodate the traditional approach in which three VEVs supply masses for gauge bosons and fermions, while three accompanying neutral scalars survive the SSB and take part in various interactions. Two of them exhibit a hierarchy $m(H_3) \simeq 2m(H_2)$ with masses below the SM scale $\langle \phi \rangle_{SM} = 246$ GeV (independently of the parameter $a$) and the third one coming out very heavy (depending on $a$), at a mass comparable to the overall breaking scale $\langle \phi \rangle$. A plausible scenario implying $\langle \phi \rangle \in 1 - 10$ TeV is then exploited.

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1 Introduction

The Standard Model (SM) \cite{1} - \cite{3} - based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ undergoing a spontaneous symmetry breakdown (SSB) in its electro-weak sector - has established itself as a successful theory in explaining the strong, weak and electromagnetic forces. Nevertheless, some recent evidences - regarding mainly the neutrino oscillation (see \cite{4} and references therein for an excellent review) - definitely call for certain extensions of the SM. In order to cover this new and richer phenomenology, any realistic theoretical model must conceive a consistent device responsible for generating masses of both fermion and boson sectors. In the SM this role is accomplished by the so called Higgs Mechanism \cite{5} - \cite{9} which - up to date - seems to be the paradigmatic procedure to give particles their appropriate masses, while the renormalizability of the model is kept valid. The Higgs mechanism enforces a suitable SSB up to the electromagnetic $U(1)_{em}$ group regarded as the residual symmetry of the model.
However, this procedure implies not only a great number of Yukawa coupling coefficients (undetermined on theoretical ground) in the fermion sector, but also the existence of a still elusive neutral scalar particle - namely, the Higgs boson.

Among the possible extensions of the SM, the so called “3-3-1” class of models [10] - [14] emerged two decades ago and has meanwhile earned a wide reputation through a systematic and compelling study of its phenomenology. It is based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge group that undergoes a SSB up to the universal electromagnetic $U(1)_{em}$ symmetry, as in the SM. The discrimination among various models in this class [15] - [17] can be done on the particle content criterion, each model supplying in its own right some new and distinct phenomenological consequences. We deal here with a particular model [13, 14] that includes both left-handed and right-handed neutrinos along with the left-handed charged lepton in triplet representations of the fermion sector. Besides recovering all the particles coming from the SM (six quarks and four gauge bosons), it predicts the occurrence of three new exotic quarks and five new gauge bosons. Apart from other versions [10, 11] that claim the existence of exotic electric charges (quarks with $\pm 5e/3$, $\pm 4e/3$ or bosons with $\pm 2e$), the version under consideration here implies only ordinary electric charges (even for the exotic particles).

A few words about the method we have employed to "solve" this class of models. Proposed initially by Cotăescu [18], it essentially consists of a general algebraical procedure in which electro-weak gauge models with high symmetries ($SU(N)_L \otimes U(1)_X$) achieve their SSB in only one step up to $U(1)_{em}$ by means of a special Higgs mechanism. This supplies a single physical scalar remaining in the spectrum and the exact expressions for the masses and neutral currents (charges) of all particles involved in the model. Here we work out the modified original version and prove that the procedure can accommodate the traditional approach with three neutral Higgs scalars surviving the SSB. The proper parametrization of the scalar sector is paired by an orthogonal restriction among scalar multiplets that warrants for only three Higgs scalars surviving the SSB, while all other degrees of freedom (Goldstone bosons) are eaten by the gauge bosons to become massive. The advantage of this new minimal Higgs mechanism resides in the fact that a realistic boson mass spectrum appears to be simply a matter of tuning a single remaining free parameter $a$. Consequently, the decay widths of these three Higgs scalars can be expressed in terms of this parameter.

The purpose of this paper is to give an estimate of the properties of the surviving neutral Higgs bosons from a 3-3-1 model with right-handed neutrinos (331RHN) based on this particular approach of finally tuning a single free parameter [19, 20]. We focus especially on the Higgs bosons couplings such as $HW^+W^-$, $HZZ$, $HZZ'$, $HXX^*$, $HY^+Y^-$ (where capital letters denote bosons of the model), in view of obtaining their possible signatures at the LHC and finally narrowing its mass estimate around the most plausible values.

The paper is organized as follows. In Sec.2 we offer a brief review of the gauge model under consideration here. Possible Higgs boson decays and other phenomenological consequences are sketched in Sec.3, while in Sec.4 certain numerical estimates in our scenario are given. Sec.5 is reserved for sketching our conclusions and suggestions for experimental search in the Higgs sector at LHC.
2 Brief review of the model

The study of the 331RHN models has revealed a rich phenomenology \[21\] - \[33\] (FCNC processes, \(Z\)'-boson phenomenology, exotic \(T\)-quark properties etc.) including some suitable solutions for the neutrino mass issue \[34\] - \[40\]. With regard to the scalar sector and Higgs phenomenology a series of papers \[41\] - \[44\] were published too.

However, we consider it worthwhile presenting the main features of constructing a 331RHN model. It is based on the gauge group \(SU(3)_c \otimes SU(3)_L \otimes U(1)_X\) and the main pieces are the irreducible representations which correspond to fermion left-handed multiplets. The fermion content is the following:

**Lepton families**

\[
\begin{align*}
 f_{\alpha L} &= \begin{pmatrix} \nu_c^\alpha \\ \nu_\alpha \\ e_\alpha \end{pmatrix}_L \sim (1, 3, -1/3) \\
 e_{\alpha R} &\sim (1, 1, -1)
\end{align*}
\]

(1)

**Quark families**

\[
\begin{align*}
 Q_{iL} &= \begin{pmatrix} D_i \\ -d_i \\ u_i \end{pmatrix}_L \sim (3, 3^*, 0) \\
 Q_{3L} &= \begin{pmatrix} U_3 \\ u_3 \\ d_3 \end{pmatrix}_L \sim (3, 3, +1/3)
\end{align*}
\]

(2)

\[
\begin{align*}
 d_{iR}, d_{3R} &\sim (3, 1, -1/3) \\
 u_{iR}, u_{3R} &\sim (3, 1, +2/3)
\end{align*}
\]

(3)

\[
\begin{align*}
 U_{3R} &\sim (3, 1, +2/3) \\
 D_{iR} &\sim (3, 1, -1/3)
\end{align*}
\]

(4)

with \(i = 1, 2\).

In the representations displayed above one has to assume that two generations of quarks transform differently from the third one in order to cancel all the axial anomalies (by an interplay between families, although each one remains anomalous by itself). In this way one prevents the model from compromising its renormalizability by triangle diagrams. The capital letters denote the exotic quarks included in each family. Many authors consider that \(U_{3R} = T\) and \(D_{iR} = D, S\) as a possible explanation of the unusual heavy masses of the third generation of quarks, but we restrict ourselves here to make no particular choice.

**Gauge bosons** The gauge bosons of the model are connected to the generators of the \(su(3)\) Lie algebra, expressed by the usual Gell-Mann matrices \(T_\alpha = \lambda_\alpha/2\). So, the Hermitian diagonal generators of the Cartan sub-algebra are

\[
\begin{align*}
 D_1 &= T_3 = \frac{1}{2} \text{Diag}(1, -1, 0), \\
 D_2 &= T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(1, 1, -2).
\end{align*}
\]

(5)
In this basis the gauge fields are $A^0_\mu$ (corresponding to the Lie algebra of the group $U(1)_X$) and $A_\mu \in su(3)$, that can be put as

$$A_\mu = \frac{1}{2} \begin{pmatrix} A^0_\mu + A^a_\mu / \sqrt{3} & \sqrt{2}X_\mu & \sqrt{2}Y_\mu \\ \sqrt{2}X^*_\mu & -A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2}W_\mu \\ \sqrt{2}Y^*_\mu & \sqrt{2}W^*_\mu & -2A^6_\mu / \sqrt{3} \end{pmatrix},$$  

(6)

where $\sqrt{2}W^*_{\mu} = A^0_\mu + i A^1_\mu$, $\sqrt{2}Y^*_{\mu} = A^1_\mu + i A^5_\mu$, and $\sqrt{2}X_{\mu} = A^1_\mu - i A^2_\mu$, respectively.

One notes that apart from the charged Weinberg bosons ($W^\pm$) from SM, there are two new complex boson fields, $X$ (neutral) and $Y$ (charged).

The diagonal Hermitian generators are associated to the neutral gauge bosons $A^{em}_\mu$, $Z_\mu$, and $Z'_\mu$. On the diagonal terms in Eq. (6) a generalized Weinberg transformation ($gWt$) must be performed in order to consequently separate the massless electromagnetic field from the other two neutral massive fields. The details of this procedure can be found in Ref. [18] and its concrete realization in the model of interest here in Refs. [19, 20].

### 3 Scalar sector

In the general method [18], the scalar sector of any gauge model must consist of $n$ Higgs multiplets $\phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(n)}$ satisfying the orthogonal condition $\phi^{(i)+} \phi^{(j)} = \phi^2 \delta_{ij}$ in order to eliminate unwanted Goldstone bosons that could survive the SSB. Here, $\phi$ is a gauge-invariant real field variable acting as a norm in the scalar space and $n$ is the dimension of the fundamental irreducible representation of the gauge group.

The parameter matrix $\eta = (\eta_0, \eta_1, \eta_2, \ldots, \eta_n)$ with the property $Tr \eta^2 = 1 - \eta_0^2$ is a key ingredient of the method: it is introduced in order to obtain a non-degenerate boson mass spectrum. Obviously, $\eta_0, \eta_i \in [0, 1)$. Then, the Higgs Lagrangian density (Ld) reads:

$$\mathcal{L}_H = \frac{1}{2} \eta_0^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_{i=1}^n \eta_i^2 \left( D_\mu \phi^{(i)} \right)^+ \left( D^\mu \phi^{(i)} \right) - V(\phi^{(i)})$$  

(7)

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig A_\mu \phi^{(i)} / 2$ act as covariant derivatives of the model, and $g$ and $g'$ the coupling constants of the groups $SU(N)_L$ and $U(1)_X$ respectively. Real characters $y^{(i)}$ stand as a kind of hyper-charge of the new theory.

For the particular 331RHN model under consideration here the most general choice of parameters is given by the matrix $\eta^2 = (1 - \eta_0^2) \text{Diag} \left[ 1 - a, \frac{1}{2}(a - b), \frac{1}{2}(a + b) \right]$. It obviously meets the trace condition required by the general method for any $a, b \in [0, 1)$. After imposing the phenomenological condition $M^2_Z = M^2_W / \cos^2 \theta_W$ (confirmed at the SM level) the procedure of diagonalizing the neutral boson mass matrix [19, 20] eliminates a parameter and thus the parameter matrix becomes $\eta^2 = (1 - \eta_0^2) \text{Diag} \left[ 1 - a, a \frac{1 - \tan^2 \theta_W}{2}, a \frac{1}{2 \cos^2 \theta_W} \right]$. 


3.1 Scalar fields redefinition

In the following we accommodate our method with the traditional approach in which there are 3 distinct VEVs resulting from the potential minimum condition. For this purpose we redefine the scalar triplets as following

\[ \phi^{(1)} \rightarrow \eta_1 \phi^{(1)} \equiv \rho, \quad \phi^{(2)} \rightarrow \eta_2 \phi^{(2)} \equiv \chi, \quad \phi^{(3)} \rightarrow \eta_3 \phi^{(3)} \equiv \phi. \]  

or in an equivalent notation (with the upper index showing the electric charge of the filed it labels):

\[
\begin{align*}
\rho &= \begin{pmatrix} \rho^0 \\ \rho^0 \\ \rho^- \end{pmatrix}, \\
\chi &= \begin{pmatrix} \chi^0 \\ \chi^0 \\ \chi^- \end{pmatrix}, \\
\phi &= \begin{pmatrix} \phi^+ \\ \phi^+ \\ \phi^0 \end{pmatrix}.
\end{align*}
\]  

(9)

Obviously, these new fields obey orthogonal relations in a new form, namely:

\[ \rho^+ \rho = \eta_1^2 \varphi^2, \quad \chi^+ \chi = \eta_2^2 \varphi^2, \quad \phi^+ \phi = \eta_3^2 \varphi^2. \]  

(10)

The simplest potential that preserves renormalizability can be put now in the following form:

\[
V = -\mu_1^2 \rho^+ \rho - \mu_2^2 \chi^+ \chi - \mu_3^2 \phi^+ \phi + \lambda_1 (\rho^+ \rho)^2 + \lambda_2 (\chi^+ \chi)^2 + \lambda_3 (\phi^+ \phi)^2 \\
+ \lambda_4 (\rho^+ \rho) (\chi^+ \chi) + \lambda_5 (\rho^+ \rho) (\phi^+ \phi) + \lambda_6 (\phi^+ \phi) (\chi^+ \chi).
\]  

(11)

One can easily observe that the SSB is accomplished in the unitary gauge by three VEVs, as follows:

\[
\begin{align*}
\begin{pmatrix} \eta_1 \langle \varphi \rangle + H_\rho \\ 0 \\ 0 \end{pmatrix}, \\
\begin{pmatrix} 0 \\ \eta_2 \langle \varphi \rangle + H_\chi \\ 0 \end{pmatrix}, \\
\begin{pmatrix} 0 \\ 0 \\ \eta_3 \langle \varphi \rangle + H_\phi \end{pmatrix}
\end{align*}
\]  

(12)

with

\[
\langle \varphi \rangle = \frac{\sqrt{\mu_1^2 \eta_1^2 + \mu_2^2 \eta_2^2 + \mu_3^2 \eta_3^2}}{\sqrt{2} (\lambda_1 \eta_1^2 + \lambda_2 \eta_2^2 + \lambda_3 \eta_3^2) + \lambda_4 \eta_1^2 \eta_2^2 + \lambda_5 \eta_1^2 \eta_3^2 + \lambda_6 \eta_2^2 \eta_3^2}
\]  

(13)

resulting from the minimum condition applied to the above potential (11).

\( H_1, H_2, H_3 \) are the physical Higgs fields surviving the SSB. Let’s look for their couplings. To this end one can write explicitly the terms in the potential \( V \) after SSB took place:
\[ V = - \left[ \mu_1^2 \langle \phi \rangle + H_\rho \right]^2 + \mu_2^2 \langle \phi \rangle + H_\chi \right]^2 + \mu_3^2 \langle \phi \rangle + H_\phi \right]^2 \\
+ \left[ \lambda_1 \langle \phi \rangle + H_\rho \right]^4 + \lambda_2 \langle \phi \rangle + H_\chi \right]^4 + \lambda_3 \langle \phi \rangle + H_\phi \right]^4 \\
+ \left[ \lambda_4 \langle \phi \rangle + H_\rho \right]^2 \langle \phi \rangle + H_\chi \right]^2 + \lambda_5 \langle \phi \rangle + H_\rho \right]^2 \langle \phi \rangle + H_\phi \right]^2 \\
+ \lambda_6 \langle \phi \rangle + H_\chi \right)^2 \langle \phi \rangle + H_\phi \right)^2 . \tag{14} \]

### 3.2 Scalar fields couplings

The next step is to identify for each Higgs its own coupling terms. These are in order.

**(i) linear terms** (must be absent - as in the SM - so one gets three constraints on the parameters):

\[ H_\rho : -\mu_1^2 + (2\lambda_1 \eta_1^2 + \lambda_4 \eta_2^2 + \lambda_5 \eta_3^2) \langle \phi \rangle^2 = 0 \]

\[ H_\chi : -\mu_2^2 + (2\lambda_2 \eta_2^2 + \lambda_4 \eta_1^2 + \lambda_6 \eta_2^2) \langle \phi \rangle^2 = 0 \tag{15} \]

\[ H_\phi : -\mu_3^2 + (2\lambda_3 \eta_3^2 + \lambda_5 \eta_2^2 + \lambda_6 \eta_2^2) \langle \phi \rangle^2 = 0 \]

**(ii) mass terms:**

\[ H_\rho H_\rho : -\mu_1^4 + (6\lambda_1 \eta_1^2 + \lambda_4 \eta_2^2 + \lambda_5 \eta_3^2) \langle \phi \rangle^2 = 4\lambda_1 \eta_1^2 \langle \phi \rangle^2 \]

\[ H_\chi H_\chi : -\mu_2^4 + (6\lambda_2 \eta_2^2 + \lambda_4 \eta_1^2 + \lambda_6 \eta_2^2) \langle \phi \rangle^2 = 4\lambda_2 \eta_2^2 \langle \phi \rangle^2 \tag{16} \]

\[ H_\phi H_\phi : -\mu_3^4 + (6\lambda_3 \eta_3^2 + \lambda_5 \eta_2^2 + \lambda_6 \eta_2^2) \langle \phi \rangle^2 = 4\lambda_3 \eta_3^2 \langle \phi \rangle^2 \]

\[ H_\rho H_\chi : 4\lambda_4 \eta_1 \eta_2 \langle \phi \rangle^2 , \ H_\rho H_\phi : 4\lambda_5 \eta_1 \eta_3 \langle \phi \rangle^2 , \ H_\phi H_\chi : 4\lambda_6 \eta_2 \eta_3 \langle \phi \rangle^2 . \tag{17} \]

**(iii) H H H trilinear terms:**

\[ H_\rho H_\rho H_\rho : \lambda_1 \eta_1 \langle \phi \rangle , \ H_\rho H_\chi H_\chi : 2\lambda_4 \eta_1 \langle \phi \rangle , \ H_\rho H_\phi H_\phi : 2\lambda_5 \eta_1 \langle \phi \rangle , \]

\[ H_\chi H_\rho H_\rho : 2\lambda_4 \eta_2 \langle \phi \rangle , \ H_\chi H_\chi H_\chi : 4\lambda_2 \eta_2 \langle \phi \rangle , \ H_\chi H_\phi H_\phi : 2\lambda_6 \eta_2 \langle \phi \rangle , \]

\[ H_\phi H_\rho H_\rho : 2\lambda_5 \eta_3 \langle \phi \rangle , \ H_\phi H_\chi H_\chi : 2\lambda_6 \eta_3 \langle \phi \rangle , \ H_\phi H_\phi H_\phi : 4\lambda_3 \eta_3 \langle \phi \rangle . \tag{18} \]
(iv) $HHHH$ quartic terms:

\[ H_\rho H_\rho H_\rho H_\rho : \lambda_1, \quad H_\chi H_\chi H_\chi H_\chi : \lambda_2, \quad H_\phi H_\phi H_\phi H_\phi : \lambda_3. \]  

(19)

3.3 Higgs masses

From the above expressions one can identify the Higgs mass matrix as:

\[
M_H^2 = 4 \begin{pmatrix}
\lambda_1 \eta_1^2 & \lambda_1 \eta_1 \eta_2 & \lambda_5 \eta_1 \eta_3 \\
\lambda_4 \eta_1 \eta_2 & \lambda_2 \eta_2^2 & \lambda_6 \eta_2 \eta_3 \\
\lambda_5 \eta_1 \eta_3 & \lambda_6 \eta_2 \eta_3 & \lambda_3 \eta_3^2
\end{pmatrix} \langle \phi \rangle^2 \]  

(20)

In the phenomenological case of interest here, as we will see in Sec.4, $\langle \rho \rangle \gg \langle \chi \rangle, \langle \phi \rangle$ that is $\eta_1 \rightarrow 1$ and $\eta_2, \eta_3 \rightarrow 0$ in our parametrization, in order to ensure a correct boson mass spectrum \[19\,20\]. Consequently, the Higgs mass matrix can be computed by eluding the very small entries in its texture and considering the mass of the first Higgs boson - $H_1 \cong H_\rho$, as:

\[m_1^2 \cong 4 \lambda_1 \eta_1^2 \langle \phi \rangle^2\]  

(21)

Assuming this Higgs ($H_1$) does not mix with the two remaining ones, their physical basis can be reached by a simple $2 \times 2$ rotation:

\[
H_2 \cong \frac{\lambda_5 \eta_2 H_\chi - \lambda_4 \eta_3 H_\phi}{\sqrt{\lambda_4^2 \eta_3^2 + \lambda_5^2 \eta_2^2}} \]  

(22)

\[
H_3 \cong \frac{\lambda_4 \eta_3 H_\chi + \lambda_5 \eta_2 H_\phi}{\sqrt{\lambda_4^2 \eta_3^2 + \lambda_5^2 \eta_2^2}} \]  

(23)

Hence, their corresponding masses are:

\[m_2^2 \cong 2 \eta_2^2 \left( \frac{\lambda_3 \lambda_4 - \lambda_5 \lambda_6}{\lambda_4} \right) \langle \phi \rangle^2\]  

(24)

\[m_3^2 \cong 2 \left( \lambda_3 \eta_2^2 + \frac{\lambda_4 \lambda_6}{\lambda_5} \eta_3^2 \right) \langle \phi \rangle^2\]  

(25)

For the sake of simplicity here is the point where one can make certain assumptions, namely considering, $\lambda_1 \simeq \lambda_2 \simeq \lambda_3 \equiv \lambda$ and $\lambda_4 \simeq \lambda_5 \simeq \lambda_6 \equiv \lambda'$. By inserting these notations into Eqs. (20), (25) and (26) one can get the following expressions:

\[m_1^2 \cong 4 \lambda \eta_1^2 \langle \phi \rangle^2\]  

(26)

\[m_2^2 \cong 2 \eta_2^2 \left( \lambda - \lambda' \right) \langle \phi \rangle^2\]  

(27)

\[m_3^2 \cong 2 \left( \lambda \eta_2^2 + \lambda' \eta_3^2 \right) \langle \phi \rangle^2\]  

(28)
Obviously $\lambda'$ has to range in $[0, \lambda)$ in order to keep meaningful the whole procedure of identifying Higgs masses. We roughly inspect three cases, accounting certain particular values of the ratio $\lambda'/\lambda : .0, .5, .1$.

The heaviest Higgs gets in all three cases its mass: as $m_1 \equiv 2\sqrt{\lambda(1-a)} \langle \varphi \rangle$.

**Case 1:** If $\lambda' = 0$ one gets $m_2 = m_3 \equiv \sqrt{\lambda a \left(1 - \tan^2 \theta_W\right)} \langle \varphi \rangle$, two small but degenerate masses for the lighter Higgs bosons. This setting is less probable since it means that there are suppressed quartic terms like $HHHH$.

**Case 2:** If $\lambda' = \lambda$ one gets $m_2 = 0$ and $m_3 \equiv \sqrt{2\lambda a} \langle \varphi \rangle$. This setting also has to be ruled out, since a massless Higgs which couples to SM bosons causes logarithmically divergent contributions in 1-loop corrections to $\rho$ parameter and W boson mass, spoiling thus the renormalizability of the model.

**Case 3:** If $\lambda' = \lambda/2$ some plausible numerical estimates can be performed. First of all, $H_3$ can be seen as the SM-like Higgs boson. Since the custodial symmetry of the SM is no more valid here, the second SM-like Higgs doublet is missing, so that the new $H_2$ takes the role of giving quarks their masses. $H_1$ and $H_2$ are the new Higgs bosons specific to this 331RHN model. The three masses are:

$$m_1 \equiv 2\sqrt{\lambda(1-a)} \langle \varphi \rangle$$  \hspace{1cm} (29)

$$m_2 \equiv \sqrt{\frac{1}{2} \lambda a \left(1 - \tan^2 \theta_W\right)} \langle \varphi \rangle$$  \hspace{1cm} (30)

$$m_3 \equiv \sqrt{\frac{1}{2} \lambda a \left(\frac{4 \cos^2 \theta_W - 1}{\cos^2 \theta_W}\right)} \langle \varphi \rangle$$  \hspace{1cm} (31)

The resulting expressions for Higgs masses in Case 3 suggest that $m_2 \simeq 2m_3$ (both are in quite the same range - the SM scale - since $\sqrt{\left(1 - \tan^2 \theta_W\right)} \simeq 0.845$ and $\sqrt{\frac{4 \cos^2 \theta_W - 1}{\cos^2 \theta_W}} \simeq 1.65$ for $\sin^2 \theta_W \simeq 0.223 \ [45]$, and $m_1$ lies in TeV domain, as it will be seen more clearly in the next section, when the parameters will be properly tuned.

### 3.4 Higgs interactions

In order to analyze the possible phenomenological consequences regarding the Higgs sector and its likely processes (decays, pair production etc) one has to observe the terms that provide us with the couplings of the physical Higgs bosons to the gauge bosons of the model (HBB). They can be read from the resulting Ld in unitary gauge after SSB, namely:
\[ \mathcal{L} = \frac{g^2}{4} \left[ (\eta_1 \langle \varphi \rangle + H_\rho)^2 + (\eta_2 \langle \varphi \rangle + H_\phi)^2 \right] X_{\mu}^+ X^\mu \\
+ \frac{g^2}{4} \left[ (\eta_1 \langle \varphi \rangle + H_\rho)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] Y_{\mu}^+ Y^\mu \\
+ \frac{g^2}{4} \left[ (\eta_2 \langle \varphi \rangle + H_\chi)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] W_{\mu}^+ W^\mu \\
+ \frac{g^2}{8 \cos^2 \theta_W} \left[ (\eta_2 \langle \varphi \rangle + H_\chi)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] Z_{\mu} Z^\mu \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_1 \langle \varphi \rangle + H_\rho)^2 Z_{\mu} Z_{\mu} \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_2 \langle \varphi \rangle + H_\chi)^2 Z_{\mu} Z_{\mu} \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_3 \langle \varphi \rangle + H_\phi)^2 Z_{\mu} Z_{\mu} \right] \tag{32} \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_1 \langle \varphi \rangle + H_\rho)^2 Z_{\mu} Z_{\mu} \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_2 \langle \varphi \rangle + H_\chi)^2 Z_{\mu} Z_{\mu} \\
+ \frac{g^2}{8} \left( \frac{1}{3 - 4 \sin^2 \theta_W} \right) (\eta_3 \langle \varphi \rangle + H_\phi)^2 Z_{\mu} Z_{\mu} \right]. \]

3.4.1 Boson mass spectrum

From the above expression the boson mass spectrum can be inferred, by simply identifying the proper terms as the mass \( L_{\text{mass}} \):

\[ \mathcal{L}_{\text{mass}} = (2 M_W^2 W_{\mu}^+ W^\mu + M_Z^2 Z_{\mu} Z^\mu \\
+ 2 M_X^2 X_{\mu}^+ X^\mu + 2 M_Y^2 Y_{\mu}^+ Y^\mu + M_{Z'}^2 Z'_{\mu} Z'^\mu) \tag{33} \]

A rapid calculus drives straightforwardly from \( L_d \) to the boson mass spectrum previously obtained with our method in Refs. [19, 20], namely:

- \( M_W^2 = m^2 a \)
- \( M_Y^2 = m^2 \left( 1 - a/2 \cos^2 \theta_W \right) \)
- \( M_X^2 = m^2 \left[ 1 - a(1 - \tan^2 \theta_W)/2 \right] \)
- \( M_Z^2 = m^2 a/ \cos^2 \theta_W \)
- \( M_{Z'}^2 = m^2 \left[ 4 \cos^2 \theta_W - a \left( 3 - 4 \sin^2 \theta_W + \tan^2 \theta_W \right) \right] / (3 - 4 \sin^2 \theta_W) \)

We have made the notation: \( m^2 = g^2 \langle \varphi \rangle^2 (1 - \eta_0^2)/4 \). The mass scale is now just a matter of tuning the parameter \( a \) in accordance with the possible values for \( \langle \varphi \rangle \). One can set parameter \( \eta_0^2 \) (of the original method) very small so that, for our purpose here, \( m^2 \approx g^2 \langle \varphi \rangle^2 / 4 \).

One can note for the neutral bosons sector that the diagonalization of the resulting mass matrix [19] has been performed by imposing the specific relation between \( M_W \) and \( m_Z \), namely \( M_Z^2 = M_W^2 / \cos^2 \theta_W \). That is why one finally remains with...
a single free parameter to be tuned $a$. Moreover, the rotation matrix doing the diagonalization job has established the mixing angle $\sin \phi = 1/2 \sqrt{1 - \sin^2 \theta_W}$. The traditional approach in the literature assumes $\phi$ as a free parameter restricted on experimental ground. Here it is fixed, the role of ensuring the experimentally observed gap between $m(Z')$ and $m(Z)$ being realized exclusively by the free parameter $a$. In addition, we mention that the correct coupling match is recovered through our method, namely $g' = g \sqrt{3} \sin \theta_W / \sqrt{3 - 4 \sin^2 \theta_W}$. All the couplings in the neutral currents of the model (or, in other words, the neutral charges of the fermions) are exactly obtained and need no approximation. They also reproduce for the SM fermions their established values (for the detailed list, the reader is referred to the Table in Ref. [20]).

3.4.2 Higgs fields couplings

From (32) combined with Eqs. (22) - (23) one can get the $HBB$ couplings for the real Higgs fields. Their general expressions are put in the first two columns of the Table 1, while their numerical values in the scenario considered in Sec.4 are displayed in the last column of the same Table 1.

$$g (H_1 BB) \simeq g (H_\rho BB)$$

$$g (H_2 BB) \simeq \left[ g (H_\chi BB) \sqrt{1 - \tan^2 \theta_W / 2} - g (H_\phi BB) \sqrt{1 / 2 \cos^2 \theta_W} \right]$$

$$g (H_3 BB) \simeq \left[ g (H_\chi BB) \sqrt{1 / 2 \cos^2 \theta_W} + g (H_\phi BB) \sqrt{1 - \tan^2 \theta_W / 2} \right]$$

The couplings of the form $HHBB$ can be obtained from the ones in Eqs. (35) - (36) by simply dividing by $2 \langle \phi \rangle$.

3.4.3 Higgs decay rates

The most general decay scenario is the one in which each Higgs comes out heavier than double mass of the heaviest boson to which it couples. so all channels are kinematically allowed.

$$H_1 \rightarrow X^+ X \quad H_1 \rightarrow Y^+ Y \quad H_1 \rightarrow Z'Z'$$

$$H_2 \rightarrow W^+ W \quad H_2 \rightarrow ZZ$$

$$H_3 \rightarrow W^+ W \quad H_3 \rightarrow ZZ$$

The general formula for the partial width of the Higgs decay into two any gauge bosons is given in the Born approximation (at tree level) by the well-known formula:
Table 1: HBB couplings

| Couplings HBB | \((m^2/\langle \phi \rangle)\) | \((2M_W^2/\langle \phi \rangle_{SM})\) |
|---------------|--------------------------|-------------------------|
| \(H_1 X_\mu^+ X^\mu\) | \(2\eta_1\) | \(\sqrt{\frac{1-a}{a}}\) |
| \(H_1 Y_\mu^+ Y^\mu\) | \(2\eta_1\) | \(\sqrt{\frac{1-a}{a}}\) |
| \(H_1 Z_\mu^+ Z^{\mu\nu}\) | \(\frac{4\cos^2\theta_W}{3-4\sin^2\theta_W}\eta_1\) | \((\frac{1}{2})\) \(\sqrt{\frac{1-a}{a}}\) |
| \(H_2 X_\mu^+ X^\mu\) | \(2\eta_2^2 \frac{1}{\sqrt{a}}\) | \((\frac{1}{2}) (1 - \tan^2 \theta_W) = 0.36\) |
| \(H_2 Y_\mu^+ Y^\mu\) | \(-2\eta_3^2 \frac{1}{\sqrt{a}}\) | \((-\frac{1}{2}) \frac{1}{\cos^2 \theta_W} = -0.64\) |
| \(H_2 W_\mu^+ W^\mu\) | \(2 (\eta_2^2 - \eta_3^2) \frac{1}{\sqrt{a}}\) | \(-\frac{1}{\sqrt{a}} \tan^2 \theta_W = -0.28\) |
| \(H_2 Z_\mu^+ Z^{\mu\nu}\) | \(\eta_2^2 \eta_3^2 \frac{1}{\cos \theta_W} \frac{1}{\sqrt{a}}\) | \((-\frac{1}{2}) \frac{\tan^2 \theta_W}{\cos^2 \theta_W} = -0.37\) |
| \(H_2 Z_\mu^+ Z^{\mu\nu}\) | \(\left[\frac{(1-2\sin^2\theta_W)^2\eta_2^2-\eta_3^2}{(3-4\sin^2\theta_W \cos^2 \theta_W)}\right] \frac{1}{\sqrt{a}}\) | \((-\frac{1}{2}) \frac{\tan^2 \theta_W (3-6\sin^2 \theta_W + 4\sin^4 \theta_W)}{(3-4\sin^2 \theta_W \cos^2 \theta_W)} = -0.12\) |
| \(H_3 Y_\mu^+ Y^\mu\) | \(2\eta_2 \eta_3 \frac{1}{\sqrt{a}}\) | \(\sqrt{\frac{1-2\sin^2 \theta_W}{2 \cos^2 \theta_W}} = 0.47\) |
| \(H_3 X_\mu^+ X^\mu\) | \(2\eta_2 \eta_3 \frac{1}{\sqrt{a}}\) | \(\sqrt{\frac{1-2\sin^2 \theta_W}{2 \cos^2 \theta_W}} = 0.47\) |
| \(H_3 W_\mu^+ W^\mu\) | \(4\eta_2 \eta_3 \frac{1}{\sqrt{a}}\) | \(\sqrt{\frac{1-2\sin^2 \theta_W}{\cos^2 \theta_W}} = 0.95\) |
| \(H_3 Z_\mu^+ Z^{\mu\nu}\) | \(2\eta_2 \eta_3 \frac{1}{\cos \theta_W \sqrt{a}}\) | \(\sqrt{\frac{1-2\sin^2 \theta_W}{2 \cos^2 \theta_W}} = 0.61\) |
| \(H_3 Z_\mu^+ Z^{\mu\nu}\) | \(\eta_2^2 \eta_3 \left[\frac{(1-2\sin^2 \theta_W)^2+1}{(3-4\sin^2 \theta_W \cos^2 \theta_W)}\right] \frac{1}{\sqrt{a}}\) | \(\sqrt{\frac{1-2\sin^2 \theta_W + 2\sin^4 \theta_W}{2(3-4\sin^2 \theta_W \cos^2 \theta_W)}} = 0.19\) |
\[
\Gamma(H \to BB) = g_{HBB}^2 \frac{m(H)^2 \alpha}{32\sqrt{2} \langle \phi \rangle^2} \sqrt{1 - \frac{4m(B)^2}{m(H)^2}} \left( 4 - \frac{16m(B)^2}{m(H)^2} + \frac{48m(B)^2}{m(H)^4} \right)
\]

with \( \alpha = 1 \) for neutral bosons and \( \alpha = 2 \) for charged ones and \( B \) denoting any gauge boson in the model. Noting the ratio \( x = \frac{4M_W^2}{m_H^2} \), the concrete functions can be computed as depending only on the couplings \( g_{HBB} \), ratio \( x \) and parameter \( a \).

4 Results and numerical estimates

4.1 Plausible scenarios

Up to this point, our approach has been a pure theoretical exercise stemming from the fertile soil of the SM. At this moment one can test some plausible scenarios beyond SM by choosing certain orders of magnitude for the overall VEV \( \langle \phi \rangle \). Hence, some rough estimates are obtained for the resulting phenomenology. We work out here the case of interest in which \( \langle \phi \rangle \in (1 - 10) \) TeV with the three VEVs aligned as:

- \( < \rho > \in (\sqrt{1 - a \div 10} \sqrt{1 - a}) \) TeV,
- \( < \chi > \simeq \sqrt{\frac{1}{2} \tan^2 \theta_W} \langle \phi \rangle_{SM} = 147.6 \) GeV
- \( < \phi > \simeq \sqrt{\frac{1}{2} \cos^2 \theta_W} \langle \phi \rangle_{SM} = 197 \) GeV

implying \( a \in (0.0006 - 0.06) \) as it results from \( \sqrt{a} \langle \phi \rangle = \langle \phi \rangle_{SM} \) in order to ensure \( m(W) = 80.4 \) GeV and \( m(Z) = 91.1 \) GeV.

Before entering the discussion of the Higgs phenomenology and its restrictions, let’s estimate the implications of some verified phenomenological aspects [45]. For instance, the ”wrong muon decay” gives at a 98% CL the result

\[
R = \frac{\Gamma(\mu^- \to e^- \bar{\nu}_\mu \nu_e)}{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)} = \left( \frac{M_W}{M_Y} \right)^4 \leq 1.2\%
\]

Hence \( M_Y \geq 240 \) GeV or equivalently - in our approach - to \( a \leq 0.123 \), which is already fulfilled.

With the allowed range of the parameter \( a \), one can compute the allowed domain for boson masses. These are, at the presumed breaking scales, those presented in Table 2.

4.2 Perturbativity

Now, in order to keep the Higgs phenomenology in the perturbative regime, the numerical values of the couplings in Table 1 must not overcome those in SM. That obviously happens, since each of them (except for those involving \( H_1 \)) exhibit couplings less than
Table 2: Masses of the gauge bosons in 331RHN model

| Mass         | at $\langle \phi \rangle = 1\text{TeV}$ | at $\langle \phi \rangle = 5\text{TeV}$ | at $\langle \phi \rangle = 10\text{TeV}$ |
|--------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $m(Y)$       | 321.8GeV                               | 1.64TeV                                 | 3.28TeV                                 |
| $m(X)$       | 324.7GeV                               | 1.64TeV                                 | 3.28TeV                                 |
| $m(Z')$      | 389.2GeV                               | 1.99TeV                                 | 3.98TeV                                 |

those in SM, as one can read from the last column of Table 1. For $H_1$ that requirement enforces a lower bound on parameter $a$. For the considered domain of the breaking scale, the lower bound is $a \geq 0.0027$ in the case $\langle \phi \rangle = 1\text{TeV}$, $a \geq 0.00052$ in the case $\langle \phi \rangle = 5\text{TeV}$, and respectively $a \geq 0.00013$ in the case $\langle \phi \rangle = 10\text{TeV}$, that are automatically satisfied. So, there are no problems with perturbativity due to HBB couplings or HHBB.

By inspecting trilinear and quartic couplings of the Higgs bosons - $g(\text{HHH})$ and $g(\text{HHHH})$ from Eqs. (18) and (19) - one can derive an upper bound on their masses, if they are set up to keep perturbativity. That is, the couplings must also remain below 1 at the considered breaking scale.

$$g(\text{HHH}) = 4\lambda n_h \langle \phi \rangle, \quad g(\text{HHHH}) = \lambda, \quad (40)$$

Consequently, one obtains $\lambda < 1/4$. Assuming that $H_3$ is the SM Higgs boson, its experimental constraints \[46, 47\] impose $m_3 \geq 114.4\text{GeV}$. If we take the upper limit for $\lambda = 1/4$, then in order to get a safe behavior concerning perturbativity, the Higgs masses become:

$$m_1 \cong \frac{1}{\sqrt{a}} \sqrt{(1-a) \langle \phi \rangle_{\text{SM}}} \quad (41)$$

$$m_2 \cong \frac{1}{2} \sqrt{\frac{1}{2} (1 - \tan^2 \theta_W) \langle \phi \rangle_{\text{SM}}} \quad (42)$$

$$m_3 \cong \frac{1}{2} \sqrt{\frac{4 \cos^2 \theta_W - 1}{\cos^2 \theta_W}} \langle \phi \rangle_{\text{SM}} \quad (43)$$

Numerical estimates yield precisely $m_2 = 73.44\text{GeV}$ and $m_3 = 143.25\text{GeV}$. The new Higgs develops distinct masses, in the following cases: $m_1 = 973.7\text{GeV}$ when $\langle \phi \rangle = 1\text{TeV}$, $m_1 = 5.01\text{TeV}$ when $\langle \phi \rangle = 5\text{TeV}$ and $m_1 = 10.03\text{TeV}$ when $\langle \phi \rangle = 10\text{TeV}$ respectively.

This state of affairs leads - as expected - to the conclusion that $H_2, H_3 \to Z'Z'$, $H_2, H_3 \to YY$, and $H_2, H_3 \to XX$ are completely forbidden. In addition, neither $H_2, H_3 \to ZZ$ nor $H_2, H_3 \to W^+W^-$ occur. Therefore, no decay event with regard to those two “lighter” Higgs to vector bosons is expected to be observed.
4.3 Loop corrections

Furthermore, it is interesting to investigate if such Higgs bosons do alter somehow - by means of radiative corrections - the parameter $\rho$, the masses of the SM bosons $W$ and $Z$. We restrict ourselves here to inspect the 1-loop corrections. First of all, one notices that the biggest Higgs $H_1$ does not interact with SM bosons, so its contribution to 1-loop corrections will be identical zero. The other two Higgs have slightly different couplings to SM bosons, so their contributions will be different, since $m_3 > M_W$ and $m_2 < M_W$. The formula giving the 1-loop contribution to $\rho$ of a neutral scalar field interacting with $W$ and $Z$ was computed decades ago in [48] - [53]. It is:

$$(\Delta \rho)^{1-\text{loop}} = -\frac{3G_F M_W^2}{8\sqrt{2}\pi^2} \left[ (-0.28) f \left( \frac{m_2^2}{M_W^2} \right) + (0.95) f \left( \frac{m_3^2}{M_W^2} \right) \right]$$

(44)

where we introduced the actual couplings $g (H_2 W W) = -0.28 \times (2m_W^2 / \langle \phi \rangle_{SM})$ and $g (H_3 W W) = 0.95 \times (2m_W^2 / \langle \phi \rangle_{SM})$. The function $f$ is

$$f(x) = x \left[ \frac{\ln c_W^2 - \ln x}{\ln c_W^2 - x} + \frac{\ln x}{\ln c_W^2 (1-x)} \right]$$

(45)

Assuming the above order of magnitude for the Higgs masses, the 1-loop radiative correction to $\rho$ parameter due to Higgs contribution yields: 0.008. Furthermore, if one wants to calculate the 1-loop contribution of the Higgs sector to the mass of the $W$ boson, one can use the celebrated formula obtained in Refs. [54] - [57]

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_F} (1 + \Delta r)$$

(46)

with $(\Delta r)^{1-\text{loop}}$ as in Refs. [54] - [60] but taking into consideration our specific couplings:

$$$(\Delta r)^{1-\text{loop}} \simeq \frac{G_F M_W^2}{8\sqrt{2}\pi^2} \left[ (-0.28) \left( \log \frac{m_2^2}{M_W^2} - \frac{5}{6} \right) + (0.95) \left( \log \frac{m_3^2}{M_W^2} - \frac{5}{6} \right) \right]$$

(47)

This yields, in the case of interest here, a negligible amount $(\Delta r)^{1-\text{loop}} \simeq 0.0009$.

4.4 Higgs production

On the experimental level, at the LHC the Higgs "hunting" is currently in the run and has raised big expectations. In the 331RHN model there are three distinct kinds of producing the SM-like Higgs boson. The processes to be watched are in order: (a) $pp \rightarrow Z H_3$, (b) $pp \rightarrow Z' H_3$ and respectively (c) $pp \rightarrow Z'$ and then following the decay modes of $Z'$ such as $Z' \rightarrow H_3 B$ (where $B$ denotes a neutral gauge bosons). Some numerical analyses have been performed for such processes in Ref. [43] in slightly different scenarios, therein assuming the exotic quarks with masses similar to the heaviest Higgs ($M(Q) \simeq m_1$). However, roughly speaking, the (c) way gives less hope in
our scenario since the resulting total width of the $Z'$ seems to be greater than that in Ref. [43], as our $M_{Z'}$ is significantly greater when $\langle \phi \rangle$ goes to 10 TeV, so consequently the branching ratio $\Gamma(Z' \rightarrow HZ)/\Gamma(Z' \rightarrow \text{all})$ diminishes. At the same time, the (b) route can be ignored, as the total cross section of such $pp$ processes is negligible too, even for lighter $Z'$ (Fig.6 in Ref. [43] proves this in the case $M_{Z'} \in 1 - 2$ TeV), while our $M_{Z'}$ reaches even 3.9 TeV). So, the remaining process to be thoroughly investigated with numerical accuracy is the Higgs production via $Z$ boson exchange in $pp$ collisions and it will be performed in a future work. However, from Fig.4 in Ref. [43] one can read a rough estimate for our SM-like Higgs bosons. This indicts a total cross section of about 1 pb from $Z$ exchange, and at most $10^{-3}$ pb from $Z'$ exchange, if we assume an average 2 TeV mass for the heavy $Z'$. Yet, if $M_{Z'}$ is greater, the (c) channel’s cross section diminishes even more. Therefore, (a) remains the most relevant process to be sought-after at the LHC and to be work out in a separate paper.

5 Concluding remarks

We have discussed here the Higgs sector of a 331RHN gauge model and suggested a plausible scenario supplied by an overall breaking scale $\langle \phi \rangle \in 1 - 10$ TeV. Our work primarily proves that the particular method conceived by Cotăescu and developed by the author in previous papers can be successfully accommodated with the traditional approach in the literature, by simply redefining the scalar multiplets, so that instead of one surviving Higgs field there are three such physical fields in the end. Yet, the advantage of tuning a single free parameter is kept here and it is exploited in order to make some phenomenological predictions such as: boson masses $M_X = M_X(a)$, $M_Y = M_Y(a)$ and $M_{Z'} = M_{Z'}(a)$ and Higgs masses $m_1 \approx \langle \phi \rangle$ TeV, $m_2 = 73$ GeV, $m_3 = 143$ GeV - all independently of the free parameter $a$, while the SM phenomenology is entirely recovered. It remains to be analyzed the Higgs contributions in higher loops diagrams (of $\rho$ parameter and SM bosons mass), in order to fulfill the renormalizability requirement for such theories and work out the details of the Higgs production from $Z$ exchange processes.

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