In the cold regions of China, coarse-grained materials are frequently encountered or used as backfilling materials in infrastructure construction, such as dams, highways, railways, and mineral engineering structures. Effects of confining pressure (0.2, 0.5, and 1 MPa) and frozen temperature (−2, −5, −10, and −15°C) on the stress-strain response and elastic modulus were investigated using triaxial compression tests. Moreover, the microscale structures of a coarse-grained material were obtained by X-ray computed tomography. The coarse-grained material specimens exhibited strain-softening and significant dilatancy behaviors during shearing. A modified model considering microstructures of the material was proposed to describe these phenomena. The predicted values coincided well with the experimental results obtained in this study and other literatures. A sensitivity analysis of parameters indicated that the model can simulate the initial hardening and post-peak strain-softening behavior of soils. And the transition of volume strain from contraction to dilatancy can also be described using this model. The results obtained in this study can provide a helpful reference for the analysis of frozen coarse-grained materials in geotechnical engineering.

1. Introduction

Civil infrastructures in cold regions, such as dams, highways, railways [1], and mineral engineering structures, have been constructed more and more in recent years. Coarse-grained materials are often encountered or used as backfill. It is important to understand the mechanical features of frozen coarse-grained materials when they are used in cold regions.

Frozen soil is a special kind of geotechnical material, which is composed of ice inclusions, mineral particles, gaseous inclusions, and liquid water [2]. Compared with the unfrozen soils, the mechanical properties of frozen soils are much more complicated. Many research studies [3, 4] have been conducted to investigate the deformation behaviors and strength of frozen soil. Effects of ice content [5], strain rate [6], temperature [7], and confining pressure [8] have been studied on the mechanical features of frozen soils. Generally, those studies mainly focused on clay, sand, and other fine-grained soils. The mechanical properties of frozen soil are significantly influenced by soil types [2]. However, almost none of these works properly explores deformation behaviors and strength characteristics of frozen coarse-grained materials. The possible mechanisms for the mechanical properties of frozen coarse-grained materials are interesting to be clarified.

Many constitutive models (e.g., hyperelastic model, nonlinear elastic model, and viscoelastic-plastic model) have been established to describe the mechanical features of frozen soils [9–14]. In addition, on the basis of continuous damage theory, some constitutive models of frozen soil have also been established. The statistical-damage-based method has been applied to the constitutive response of soils
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material is shown in Figure 1. The curve coefficient (typical grain size distribution of the tested coarse-grained material) can be predicted by the statistical-damage-based model simply and correctly. However, few models have been established to predict the volume change.

The purpose of this paper is to propose a model that can describe the mechanical behavior of frozen coarse-grained material. The effects of confining pressure and temperature on the frozen coarse-grained materials were studied with triaxial tests and X-ray computed tomography (CT). Then, an empirical formula that reflects the relationship between microstructural change and residual strength is proposed. Furthermore, the stress-strain response of frozen coarse-grained material considering microstructures was predicted. At last, the parameter sensitivity of the statistical damage model was discussed.

2. Materials and Methods

2.1. Material Properties and Specimen Preparation. The typical grain size distribution of the tested coarse-grained material is shown in Figure 1. The curve coefficient ($C_A$) and uniformity coefficient ($C_u$) are 6.48 and 13.78, respectively. According to the Standard for Soil Test Method [19], the tested coarse-grained material is poorly graded gravel soil. The optimum water content and maximum dry density were determined to be 4.5% and 2.02 g/cm$^3$, respectively. And the detailed characteristics of the tested coarse-grained material are listed in Table 1.

An appropriate amount of distilled water was mixed fully with the dried coarse-grained material to obtain the optimum moisture content (Table 1). A closed container was then used to store the mixture for 12 h to ensure a uniform distribution of moisture. The cylindrical specimens with the maximum dry density of 95% were made by a cylindrical mold. The size of specimens was unified into 125 mm in height and 61.8 mm in diameter on a specially made specimen machine (Figure 2) to decrease the difference in the preparation of specimens. The machine has the advantage of easy to use with a wide operating range. Moreover, it is suitable for various sizes of specimens, and the test precision is high. In order to avoid frost heaving, the specimens fastened by three-piece split molds were placed in an incubator at −30°C for 24 h. Then, the completely frozen specimens were removed from the molds and wrapped with a rubber membrane. After that, they were placed in a refrigerator at the objective temperature for more than 12 h to make the temperature of specimens uniform.

2.2. Testing Apparatus and Scheme. The cryogenic triaxial compression tests were performed on a material test system (MTS-810) exhibited in Figure 3. It is mainly composed of two subsystems: the temperature controlling system and the loading system. In the temperature controlling system, the temperatures at the side and top of the pressure chamber are adjusted by two independent refrigeration circulators (shown in Figure 3). In the loading system, the high confining pressure is withstood by a designed steel airtight pressure cell. The axial load measured by a force sensor is applied to the top of the specimens. The parameters of the triaxial testing apparatus are as follows: the maximum axial load is 100 kN, the maximum axial deformation is 85 mm, the stable temperature ranges from −30°C to 30°C with the precision of ±0.1°C, and the experimental confining pressure is 0.1–20 MPa. To investigate the effects of temperature and confining pressure on the mechanical features of frozen coarse-grained materials, 12 triaxial tests were conducted. The temperatures ranged from −2°C to −15°C, while the confining pressures varied from 0.2 MPa to 1.0 MPa. And the detailed schemes are listed in Table 2.

2.3. Experimental Procedures. The undrained static triaxial test was conducted according to the following procedures: (1) the target temperature was kept for 10 h to ensure the uniform temperature distribution in the pressure chamber; (2) the frozen specimen was transferred from the refrigerator to the triaxial pressure chamber; (3) the confining pressure ($\sigma_3$) was set to the specified value and kept for 10 min; and (4) the axial deviatoric stresses ($q$) were imposed at a strain rate of 1%/min until the axial strain reached 15%.

In the past few years, CT has been widely applied in geotechnical studies as a tool of describing the microstructure due to the advantage of being nondestructive [20]. X-ray CT observations were conducted on some selected specimens to analyze the microstructure variations of frozen coarse-grained materials. The spatial and the density resolution of the CT scanner (PHILIPS Brilliance 16) are 0.208 mm and 0.3%, respectively. The specimens before and after the triaxial test were scanned from the bottom to top in the CT scanner with a spacing of 3 mm. As displayed in Figure 3, five typical sections of a specimen were selected to display the microstructure changes.

3. Results and Discussion

3.1. Stress-Strain Response. The typical relationships under different confining pressures and temperatures between deviator stress ($q$), axial strain ($\varepsilon_2$), and volumetric strain ($\varepsilon_v$) are illustrated in Figure 4. Generally, there are roughly three stages in the stress-strain curves, i.e., the initial linear elastic stage, the plastic deformation stage, and the softening stage. In the initial stage, there is remarkable linear relationship between deviator stress and axial strain. When the axial strain increases to larger than 1%, most stress-strain curves present the plastic behavior, which means that their slopes reduce gradually in this plastic deformation stage. Under relatively low confining pressure (i.e., $\sigma_3 = 0.2$ MPa), a strain-softening phenomenon is observed at different temperatures. This may attribute to the fact that more water turns into ice with the decrease of temperature. Additionally, ice becomes more brittle as temperature decreases [2, 21], which
results in the brittle failure of frozen coarse-grained material. At the same temperature, the deformation behavior of frozen coarse-grained material changes from strain softening to strain hardening with the increase of confining pressures. This indicates that the ductile behavior of material is enhanced at higher pressure. Similar behavior is obtained for frozen sand [22]. It can be concluded that temperature and confining pressure have a remarkable effect on the deformation characteristics of frozen soils.

It can be seen from Figure 4 that the temperature and confining pressure both have significant effects on the stress-strain curves of frozen coarse-grained materials. Under low confining pressures, the deviatoric stress rises slowly in the initial stage. However, the deviatoric stress increases rapidly with the increase of the confining pressure. With the decrease of temperature, the strength of frozen coarse-grained materials gets higher under the same confining pressure. Generally, the decrease of temperature will improve the strength frozen soil significantly. The frozen water acts as a bonding agent between the soil particles, increasing the stiffness of soil. In addition, lower temperature can significantly influence the strength of both single particle [23, 24] and soils [6, 25], usually enhancing it [26].

The key to determine the volumetric strain of the specimen is to determine its volume change, which is difficult to measure because there is no seepage for the frozen coarse-grained material. In this study, the volume changes of specimens were measured by the volume change caused by the leakage of hydraulic oil in the pressure chamber. The minus sign indicates an increase in volume. With the increase of the axial strain, the volumes of specimens reduce from a starting state to a phase transition state and then expand gradually into a residual state. As observed in Figure 4, the confining pressure has a significant influence on the volumetric strain curves of frozen coarse-grained material. The increase of confining pressure reduces the dilation of specimens.

3.2. Elastic Modulus and Strength Properties. The characteristics of the soil could be reflected by two important parameters: elastic modulus and strength properties. The stress-strain curve slope in the elastic deformation stage can be defined as the elastic modulus, according to a previous report [27]. The initial curve tangent with the axial strain of 1% was used to calculate the elastic modulus in this study [16]. As depicted in Figure 5, the increase of confining pressure and the decrease of temperature lead to the monotonic increase of elastic modulus, indicating that both the confining pressure and temperature have significant influences on stiffness. It can be found that when the tested temperature is −2°C, the elastic modulus of 0.2, 0.5, and 1 MPa is approximately 92.1, 115.3, and 163.9 MPa, respectively. When the temperature drops to −15°C, the values of elastic moduli at 0.2, 0.5, and 1 MPa increase by approximately 234%, 198%, and 126%, respectively. The relationships between elastic modulus and these two parameters can be expressed as follows:

\[
E = 76.71|T|^{0.453}\left(\frac{\sigma_3}{P_a}\right)^{0.164},
\]

where \(E\), \(|T|\), and \(P_a\) are the elastic modulus, the absolute value of negative temperature, and the atmospheric pressure \((P_a = 101 \text{ kPa})\), respectively.

![Figure 1: Grain size distribution of the coarse-grained material.](image1.png)

![Figure 2: The picture of the specimen machine.](image2.png)

![Table 1: Basic physical characteristics of the frozen coarse-grained material.](table1.png)
The variations of peak strength ($q_f$) and residual strength ($q_r$) at various confining pressures and temperatures are illustrated in Figure 6. It is evident that the variations of peak strength and residual strength are similar to those of elastic modulus. Further, their relationships are as follows:

\[
q_f = 1.241|T|^{0.264} \times \left( \frac{\sigma_3}{P_a} \right)^{0.399},
\]

\[
q_r = 0.642|T|^{0.242} \times \left( \frac{\sigma_3}{P_a} \right)^{0.647}.
\]

The strength of the frozen coarse-grained material enhances as the confining pressure increases, which can be interpreted as the following two aspects. Firstly, the rising confining pressure makes the ice particles and soil particles closer, forming a stronger interlocking friction between these particles. Secondly, the rising confining pressure increases the normal pressure on the shear plane, increasing the resistance of sliding friction.

The angle of the internal friction ($\phi$) and the cohesion ($c$) were the two shear strength parameters used to analyze the effect of temperature on the shear strength of soil. Figure 7(a) illustrates the effect of temperature on the cohesion of specimens. The cohesion of soil rises with the decrease of temperature in the form of a power function, as shown in the following equation:

Table 2: Experimental schemes of the triaxial tests.

| Samples | $\omega$ (%) | $\rho_d$ (g/cm$^3$) | $T$ (°C) | $\sigma_3$ (MPa) | CT scanning |
|---------|-------------|----------------------|---------|-----------------|-------------|
| S1      | 4.5         | 2.02                 | —       | 0.2             | —           |
| S2      | 0.5         | Sections I–V         | —       | 0.5             | —           |
| S3      | 1           | —                    | —       | 0.2             | —           |
| S4      | 0.2         | Sections I–V         | —       | 0.2             | —           |
| S5      | 0.5         | Sections I–V         | —       | 0.5             | —           |
| S6      | 1           | Sections I–V         | —       | 1               | —           |
| S7      | 4.5         | 2.02                 | —       | 0.2             | —           |
| S8      | 0.5         | Sections I–V         | —       | 0.5             | —           |
| S9      | 1           | —                    | —       | —               | —           |
| S10     | 0.2         | —                    | —       | —               | —           |
| S11     | 0.5         | Sections I–V         | —       | —               | —           |
| S12     | 1           | —                    | —       | —               | —           |

Note. “—” denotes no test.
w+he variations of internal friction angle with different frozen temperatures are presented in Figure 7(b). It can be seen that the friction angles fluctuate in a small range near the average value of 34.11°. w+he results indicate that the frozen temperature has little influence on the tangential interaction between soil and ice particles. On the contrary, the binding force between ice and soil particles is highly dependent on the frozen temperature.

3.3. X-Ray CT Observations. In order to describe the microstructure changes of a specimen before and after the triaxial test, the typical CT images of the specimen under 0.5 MPa and −2°C are presented in Figure 8. Before testing, the particles are evenly distributed, and thus the appearance of specimen slices is almost the same. After testing, from top to bottom, the soil particle distributions are from loose to tight and then to looser.

Apart from soil particles, the unfrozen water and internal ice are related to the fabric of frozen soil. The distribution of unfrozen water, internal ice, and soil particles on the cross section of the specimen can be characterized by the average density, which is defined by the CT value (AV) [28]:

\[ AV = 1000 \times \frac{u_i - u_w}{u_w}, \]  

where \( u_w \) and \( u_i \) represent the X-ray absorption coefficients of water and material, respectively. Water has a CT value of 0 HU by setting.

The relationships between CT values and section positions are shown in Figure 9. The CT values of specimens after testing are smaller than those before testing. Therefore, the deformation of the specimen belongs to the dilation type, which is coincident with its strain-volumetric behavior presented in Figure 4. It is apparent that the CT values distribute as a bow shape. It decreases to the minimum value at the middle section of the specimen. This means that the particle distributions in the middle part of specimens are

![Figure 4: Stress-strain behaviors under different testing temperatures and confining pressures: (a) \( T = -2°C \); (b) \( T = -5°C \); (c) \( T = -10°C \); (d) \( T = -15°C \).](image)

\[ c = 0.177 |T|^{0.636}. \]  

The variations of internal friction angle with different frozen temperatures are presented in Figure 7(b). It can be seen that the friction angles fluctuate in a small range near the average value of 34.11°. The results indicate that the frozen temperature has little influence on the tangential interaction between soil and ice particles. On the contrary, the binding force between ice and soil particles is highly dependent on the frozen temperature.

![Figure 8: Typical CT images of the specimen under 0.5 MPa and −2°C.](image)
looser than those in both ends of specimens, which can be interpreted as the end effect [29]. These results are consistent with the shape of the specimen after the triaxial test, as observed in Figure 3. CT values of tested specimens in five typical sections decrease gradually as the temperature decreases, which is consistent with the strain-softening phenomenon caused by the increase of brittleness from ice (Figure 10(a)). Besides, increasing the confining pressure also leads to an increase in CT values as shown in Figure 10(b).

As reported in a previous study, the gradual failure of cemented structure and interlocking structure gradually reduces the mechanical resistance, resulting in post-peak softening response. The residual strength is closely related to the structure of the shear specimen [30]. The evolution of specimen structure can be reflected by the change of CT values. And the average change ratio of CT values ($R_{AV}$) can be defined as follows:

$$R_{AV} = \frac{AV_{after} - AV_{before}}{AV_{before}}, \quad (6)$$

where $AV_{before}$ and $AV_{after}$ are the average CT values before and after the test.

Figure 11 shows the relationship between the residual strength and $R_{AV}$. It can be seen that the residual strength is positively correlated with $R_{AV}$ at $-5°C$ and different confining pressures, which can be expressed as follows:

$$q_r = 24.684R_{AV} + 5.031, \quad (7)$$

and the residual strength is negatively correlated with $R_{AV}$ under 0.5 MPa and different temperatures, which is expressed as follows:

$$q_r = -26.904R_{AV} + 0.395. \quad (8)$$

Based on the above analysis, it is speculated that the residual strength of frozen coarse-grained material is linear with $R_{AV}$, which can be expressed as follows:

$$q_r = 24.684R_{AV} + a_T, \quad (9)$$

where $a_T$ is a parameter related to frozen temperature, which is obtained by experimental result.

The relationship between parameter $a_T$ and the frozen temperature is presented in Figure 12. Case S8 used for the following validation was not considered in this figure. As depicted in Figure 12, the parameter $a_T$ is significantly affected by frozen temperature, which can be expressed as follows:

$$a_T = 3.088|T|^{0.293}. \quad (10)$$

Thereafter, a simple empirical formula is proposed by substituting equation (10) into equation (9) to predict the residual strength, which is expressed as follows:

$$q_r = 24.684R_{AV} + 3.088|T|^{0.293}. \quad (11)$$

4. Statistical Damage Model Incorporating Microscale Structure

4.1. Prediction Model of Stress-Strain-Volumetric Characteristics. It is assumed that the frozen coarse-grained material consists of microunits. And the material strength of microunits differs. By incorporating the statistical strength theory and the continuous damage theory, the damage variable is described as follows:
where $D$ is the damage variable and $N_f$ and $N$ are the failed and total number of microunits, respectively.

When the Weibull distribution is assumed to be the density function of microdamage, the probability density distribution function $f(F)$ can be expressed as follows:

$$f(F) = \frac{m}{F_0} \left( \frac{F}{F_0} \right)^{m-1} \exp \left[ -\left( \frac{F}{F_0} \right)^m \right],$$

where Weibull modulus $m$ and $F_0$ are the material parameters, which correspond to the mechanical features of the frozen coarse-grained material. The variable $F$ corresponds to the random distribution of the microunit strength [31].
The strength criterion [27, 32] or the axial strain [33, 34] of the frozen coarse-grained material can be used to describe the random distribution variable $F$ of the microunit strength. In this study, the axial strain is applied for the expression of random distribution variable of the microunit strength. It is simple and convenient for application. Therefore, the number of damaged microunits can be determined as follows:

$$N_f = \int_0^{\varepsilon_1} N f(x) dx = N \left(1 - \exp\left(-\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right). \quad (14)$$

The damage variable based on the Weibull distribution is obtained by substituting equation (14) into equation (12), which is determined as follows:

$$D = 1 - \exp\left(-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right). \quad (15)$$

Then, the stress-strain relationship is described as follows:

$$q = E \varepsilon_1 (1 - D) = E \varepsilon_1 \exp\left(-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right), \quad (16)$$

| Case | Specimen | Section I | Section II | Section III | Section IV | Section V |
|------|----------|-----------|------------|-------------|------------|-----------|
| S2   |          | ![Before testing](image1) | ![After testing](image2) |

**Figure 8:** Typical CT images of the frozen coarse-grained specimen of case S2 before and after testing.

![Figure 9: Relationship between CT value and normalized height $h/H$ for specimens before and after testing: (a) confining pressure $\sigma_3 = 0.5$ MPa; (b) frozen temperature $T = -5^\circ$C.](image3)

![Figure 9: Relationship between CT value and normalized height $h/H$ for specimens before and after testing: (a) confining pressure $\sigma_3 = 0.5$ MPa; (b) frozen temperature $T = -5^\circ$C.](image4)
where $E$ represents the elastic modulus, $\varepsilon_1$ is the axial strain, and $\varepsilon_0$ is the distribution parameter.

As shown in equation (16), the residual stress cannot be reflected [35]. To describe the failure process of frozen coarse-grained material accurately, a modified model considering residual stress is obtained in this paper. Therefore, the stress-strain relationship can be expressed as follows:

$$ q = E_e \varepsilon_1 (1 - D) + q_r D, \quad (17) $$

where $E_e$ represents the nominal elastic modulus, which is related to elastic modulus of frozen coarse-grained material.

The prediction model of the stress-strain behavior of the frozen coarse-grained material is obtained by combining equation (15) with equation (17), which is expressed as follows:

$$ q = E_e \varepsilon_1 \exp\left(-\frac{\varepsilon_1}{\varepsilon_0}\right)^m + q_r \left\{1 - \exp\left(-\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right\}. \quad (18) $$

All of the parameters are related to the mechanical properties of frozen coarse-grained material. The residual strength $q_r$ is determined directly by the stress-strain curves in Figure 4. Other three parameters ($E_e$, $\varepsilon_0$, and $m$) can neither be obtained directly from the experimental results, nor be derived by the formulas. Generally, $m$ is determined by the peak point method, which was interpreted by Li et al. in detail [36]. However, this commonly used method cannot reflect the residual stress of frozen coarse-grained material. Actually, $m$ is a parameter of constitutive relation curve, which can be determined by fitting the experimental data according to Huang et al. [37]. Thus, the results of
parameters $E_c$, $\varepsilon_0$, and $m$ in equation (18) are obtained by fitting.

As observed in Figure 13, $E$ is linear with $E_c$, which can be expressed as follows:

$$E_c = 0.758E + 15.104.$$  \hfill (19)

$$q = (0.758E + 15.104)\varepsilon_1 \exp \left[-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right] + \left(24.684R_{A'e} + 3.088|T|^{0.293}\right) \left(1 - \exp\left[-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right]\right).$$ \hfill (20)

In the binary-medium constitutive model [18], the artificial structural soil is treated as the binary-medium material composed of friction and bonding elements. The volumetric strain is defined by the strains of these two elements. And the damage variable is used to replace the damage rate. Thus, the volume change of frozen coarse-grained material can be expressed as follows:

$$\varepsilon_v = \varepsilon_v^b (1 - D) + \varepsilon_v^f D,$$ \hfill (21)

where $\varepsilon_v^b$ and $\varepsilon_v^f$ represent the strains of frictional elements and bonded elements, induced by the deviatoric stress and spherical stress, respectively.

$\varepsilon_v^b$ and $\varepsilon_v^f$ in equation (21) are defined in equations (22) and (23), respectively:

$$\varepsilon_v^b = \frac{P}{K}$$ \hfill (22)

$$\varepsilon_v^f = A_3\varepsilon_v^b + C,$$ \hfill (23)

where $P_0$ and $P$ are the spherical stresses before and during the test, respectively, with the relationship of $p = q/3 + P_0$; $A$, $B$, and $C$ are parameters related to the volume change characteristics of frozen coarse-grained material; and $K$ is the bulk modulus.

After substituting equations (15), (22), and (23) into equation (21), a prediction model of volume change can be obtained as follows:

$$\varepsilon_v = \frac{q}{3K} \exp \left[-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right] + \left(A_3\varepsilon_v^b + C\right) \left(1 - \exp\left[-\left(\frac{\varepsilon_1}{\varepsilon_0}\right)^m\right]\right).$$ \hfill (24)

4.2. Model Verification. Figure 14 shows the comparison of experimental results and the predicted results in this study. According the equations derived in the previous section, the related material parameters are obtained and summarized in Table 3. The predicted values agree well with the experimental results. It can be concluded that the proposed models can predict the stress-strain properties and the volume change characteristics of frozen coarse-grained materials.

Two kinds of frozen soils in previous studies [12, 22] are selected to validate the presented models based on the statistical damage concept. The comparison between experimental and the predicted results by equations (18) and (24) is demonstrated in Figure 15. The parameters used to predict the behaviors of two types of soils are listed in Table 4. It can be seen that the proposed model can correctly simulate the stress-strain features of frozen soils. And the plastic deformation mechanism (i.e., compressibility and expansibility) of soils can be accurately described by the proposed models. This result seems to make it possible for the proposed model to capture the nonlinear mechanical behavior of soil.

4.3. Sensitivity Analysis of the Parameters. The parametric investigation results of the prediction model are analyzed in this section. Figures 16 and 17 present the stress-strain and volume change behavior of frozen coarse-grained materials. These results indicate that the main mechanical characteristics of the frozen coarse-grained materials can be duplicated by the predicted model.

In terms of the stress-strain responses, the behavior of specimens transform from strain hardening to strain softening with the increase of $E_c$. And the peak strength increases sharply. With the increase of $\varepsilon_0$, the peak stress and the corresponding strain increase gradually. It seems that the strength and the plastic performance of frozen coarse-grained material have been enhanced. There is a slight transformation from strain hardening to strain softening with the increase of $m$. For the volumetric strain, with the increase of $A$ and $B$, the trend of dilation diminishes. Parameter $C$ determines the rate of contraction. It can be clearly observed that the model can predict the transition from initial hardening to strain softening. Moreover, the dilation response of volume strain can also be described by this model.
**Figure 12:** Relationship between parameter $a_T$ and frozen temperature.

**Figure 13:** Relationship between $E$ and $E_e$.

**Figure 14:** Continued.
Table 3: Model parameters of the frozen coarse-grained material in this study.

| Samples | T (°C) | σ3 (MPa) | qr (MPa) | Ee (MPa) | ε0 | m   | R² |
|---------|--------|----------|---------|----------|-----|------|----|
| S1      | –2     | 0.2      | 1.193   | 76.164   | 0.019 | 0.981 | 0.991 |
| S2      | –2     | 0.5      | 2.130   | 96.130   | 0.025 | 1.089 | 0.989 |
| S3      | –2     | 1        | 3.428   | 138.049  | 0.018 | 1.114 | 0.988 |
| S4      | –5     | 0.2      | 1.346   | 165.000  | 0.022 | 0.991 | 0.944 |
| S5      | –5     | 0.5      | 2.889   | 179.791  | 0.021 | 0.945 | 0.994 |
| S6      | –5     | 1        | 3.969   | 200.780  | 0.020 | 1.126 | 0.956 |
| S7      | –10    | 0.2      | 1.440   | 217.218  | 0.027 | 1.135 | 0.972 |
| S8      | –10    | 0.5      | 3.219   | 227.500  | 0.024 | 0.981 | 0.997 |
| S9      | –10    | 1        | 5.000   | 251.082  | 0.022 | 1.077 | 0.948 |
| S10     | –15    | 0.2      | 1.866   | 255.460  | 0.031 | 1.221 | 0.899 |
| S11     | –15    | 0.5      | 3.801   | 273.700  | 0.020 | 0.879 | 0.996 |
| S12     | –15    | 1        | 5.313   | 279.103  | 0.022 | 0.996 | 0.983 |

Figure 14: Predicted and experimental deviatoric stress-strain results of frozen coarse-grained materials: (a) T = –2°C; (b) T = –5°C; (c) T = –10°C; (d) T = –15°C.

Figure 15: Predicted and experimental results of frozen soils in previous studies: (a) frozen saline sandy soil [12]; (b) frozen standard sand [22].
Table 4: Model parameters of frozen saline sandy soil and frozen standard sand.

| Soil types                     | Stress-axial strain parameters | Volumetric strain-axial strain parameters |
|-------------------------------|-------------------------------|------------------------------------------|
|                               | T (°C) σ3 (MPa) qr (MPa) Ee (MPa) ε0 m R2 K (MPa) A B C R2 |                                          |
| Frozen saline sandy soil [12] | −6 2 10.92 274.1 0.056 1.09 0.984 370.839 −1.017 2.05 0.013 0.982 |                                           |
| Frozen standard sand [22]    | −15 1 4.304 352.3 0.019 0.895 0.987 489.167 −0.914 1.193 0.012 0.985 |                                           |

Figure 16: Deviatoric stress-strain curves for different values of parameters (a) $E_e$, (b) $\varepsilon_0$, and (c) $m$. 
5. Conclusions

The effects of confining pressure and temperature on the mechanical property of frozen coarse-grained material were investigated using the triaxial compression test and X-ray CT analysis. Based on the statistical damage theory, a modified model considering the microscale structure was developed. This model is able to describe the strain-softening and dilatancy behaviors of frozen coarse-grained materials. The following conclusions can be drawn:

(1) Temperature and confining pressure have significant influence on the mechanical properties of frozen coarse-grained materials. The specimens exhibit strain-softening and dilatancy behaviors during shearing. With the increase of the confining pressure and temperature, the stress-strain relationship transfers from strain softening to strain hardening.

(2) CT values of the tested material specimens decrease gradually with the decrease of the temperature. The increase of the confining pressure leads to the increase of CT values, which is consistent with the dilatancy phenomenon. Moreover, an empirical formula that reflects the relationship between microstructural and residual strength is proposed.

(3) A model based on statistical damage theory was proposed to predict the stress-strain response and volume change of frozen coarse-grained material. It was verified with the experimental results obtained by this study and other previous studies. The sensitivity analysis of the model parameters indicated that the model can predict the transition from initial hardening to strain softening. Moreover, the dilation response of volume strain can also be described.

Figure 17: Volumetric strain curves for different values of parameters (a) $A$, (b) $B$, and (c) $C$. 

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Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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