Nonequilibrium Phase Transition in the Kinetic Ising model: Critical Slowing Down and Specific-heat Singularity

Muktish Acharyya*
Department of Physics
Indian Institute of Science, Bangalore-560012, India
and
Condensed Matter Theory Unit
Jawaharlal Nehru Centre for Advanced Scientific Research
Jakkur, Bangalore-560064, India

The nonequilibrium dynamic phase transition, in the kinetic Ising model in presence of an oscillating magnetic field, has been studied both by Monte Carlo simulation and by solving numerically the mean field dynamic equation of motion for the average magnetisation. In both the cases, the Debye 'relaxation' behaviour of the dynamic order parameter has been observed and the 'relaxation time' is found to diverge near the dynamic transition point. The Debye relaxation of the dynamic order parameter and the power law divergence of the relaxation time have been obtained from a very approximate solution of the mean field dynamic equation. The temperature variation of appropriately defined 'specific-heat' is found to diverge near the dynamic transition point. The specific-heat has been observed to diverge near the dynamic transition point.

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I. INTRODUCTION

The dynamic response of the Ising system in presence of an oscillating magnetic field has been studied extensively [1-3] in the last few years. The dynamic hysterisis [1-3] and the nonequilibrium dynamic phase transition [4-6] are two important aspects of the dynamic response of the kinetic Ising model in presence of an oscillating magnetic field. This kind of phase transition in the Ising model was first studied by Tome and Oliviera [4]. They solved the mean field (MF) dynamic equation of motion (for the average magnetisation) of the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. They have defined the order parameter as the time averaged magnetisation over a full cycle of the oscillating field and showed that depending upon the value of the field and the temperature, the order parameter takes nonzero value from a zero value. Precisely, in the field amplitude and temperature plane there exists a phase boundary separating dynamic ordered (nonzero value of order parameter) and disordered (order parameter vanishes) phase. They [4] have also observed and located a tricritical point (TCP), (separating the nature (discontinuous/continuous) of the transition) on the phase boundary line (see Fig.1). However, such a mean field transition is not truly dynamic in origin and exists even in the static (or zero frequency) limit. This is because, if the field amplitude is less than the coercive field (at temperature less than the transition temperature without any field), then the response magnetisation varies periodically but asymmetrically even in the zero frequency limit; the system remains locked to one well of the free energy and cannot go to the other one, in the absence of fluctuation.

Lo and Pelcovits [5] first attempted to study the dynamic nature of this phase transition in the kinetic Ising model by Monte Carlo (MC) simulation. Here, the transition disappears in the zero frequency limit; due to the fluctuations, the magnetisation flips to the direction of the magnetic field and the dynamic order parameter (time averaged magnetisation) vanishes. However, Lo and Pelcovits [5] have not reported any precise phase boundary. Acharyya and Chakrabarti [6] studied the nonequilibrium dynamic phase transition in the kinetic Ising model in presence of oscillating magnetic field by extensive MC simulation. They [6] have also identified that this dynamic phase transition is associated with the breaking of the symmetry of the dynamic hysteresis ($m-h$) loop. In the dynamically disordered (value of order parameter vanishes) phase the corresponding hysteresis loop is symmetric, and loses its symmetry in the ordered phase (giving nonzero value of dynamic order parameter). They [6] also studied the temperature variation of the ac susceptibility components near the dynamic transition point. The major observation was that the imaginary (real) part of the ac susceptibility gives a peak (dip) near the dynamic transition point (where the dynamic order parameter vanishes). The conclusions were: (i) this is a distinct signal of phase transition and (ii) this is an indication of the thermodynamic nature of the phase transition.

It is worth mentioning here that the statistical distribution of dynamic order parameter has been studied by Sides et al [7]. The nature of the distribution changes near the dynamic transition point. They have also observed [7] that the fluctuation of the hysteresis loop area becomes considerably large near the dynamic transition point.

In the equilibrium critical phenomena, both the length and time scales diverge at criticality. This gives rise to the singularities in various thermodynamic quantities,
like the specific heat and the relaxation time. Can one expect similar kind of features in the case of this nonequilibrium dynamic phase transition problem? To be specific, (i) Is there any `relaxation time', in this nonequilibrium problem which diverges at the dynamic transition point? (ii) Can there be any appropriately defined 'specific-heat', which will show singular behaviour at the transition point?

The main motivation of this paper is to get some answers of these questions at least numerically. The nonequilibrium dynamic phase transition in the kinetic Ising model in presence of an oscillating magnetic field has been studied by MC simulation. Also the MF dynamic equation has been solved numerically, to compare the results. The 'relaxation' behaviour (defined in the following section) of the dynamic order parameter [4] and the behaviour of 'specific-heat' (defined in section III) near the dynamic transition point are studied by MC simulation. It may be mentioned here that the preliminary results of 'specific-heat' singularity near the dynamic transition point were reported briefly in ref [8]. More detailed results are reported here. The 'relaxation' behaviour has also been studied here by solving numerically the mean field (MF) dynamic equation of motion of the kinetic Ising model in presence of an oscillating magnetic field. The MF equation has also been solved exactly in the linearised limit and studied the 'relaxation' behaviour of the dynamic order parameter, near the dynamic transition point. The paper is organised as follows: In section II the 'relaxation' behaviour of the order parameter near the dynamic transition point has been studied by both Monte Carlo simulation and by solving mean field dynamic equation of motion of the kinetic Ising model. In section III the temperature variation of the 'specific-heat' has been studied near the transition point only by Monte Carlo simulation. A brief summary of all the results is given in section IV.

II. 'RELAXATION' BEHAVIOUR OF THE DYNAMIC ORDER PARAMETER

A. Monte Carlo Study

1. The Model and Simulation

A ferromagnetically interacting (nearest neighbour) Ising model in presence of a time varying magnetic field can be represented by the Hamiltonian

$$H = - \sum_{<ij>} J_{ij} s_i^z s_j^z - h(t) \sum_i s_i^z$$  \hspace{1cm} (2.1)

Here, $s_i^z (\pm 1)$ is Ising spin variable, $J_{ij}$ is the interaction strength and $h(t) = h_0 \cos(\omega t)$ represents the oscillating magnetic field, where $h_0$ and $\omega$ are the amplitude and the frequency respectively of the oscillating field. The system is in contact with an isothermal heat bath at temperature $T$. For simplicity all $J_{ij}$ are taken equal to unity and periodic boundary condition is chosen.

A square lattice of linear size $L (= 100)$ has been considered. At any finite temperature $T$ and for a fixed frequency ($\omega$) and amplitude ($h_0$) of the field, the dynamics of this system has been studied here by Monte Carlo simulation using Glauber single spin-flip dynamics with the Metropolis rate of spin flip. Each lattice site is updated here sequentially and one such full scan over the lattice is defined as the time unit (Monte Carlo step or MCS) here. The initial configuration has been chosen such that the all spins are directed upward. The instantaneous magnetisation (per site), $m(t) = (1/L^2) \sum_i s_i^z$ has been calculated. From the instantaneous magnetisation, the dynamic order parameter $Q = \frac{\langle s^z \rangle}{\sqrt{L^2}} \int_0^\infty m(t) dt$ (time averaged magnetisation over a full cycle of the oscillating field) is calculated.

2. Results

Fig.1 shows the schematic diagram of the dynamic phase boundary in the field amplitude ($h_0$) and temperature ($T$) plane. For small values of $h_0$ and $T$ the dynamic order parameter $Q$ is nonzero and the corresponding dynamic hysteresis loop ($m - h$ loop) is asymmetric and for larger values of $h_0$ and $T$ the dynamic order parameter $Q$ vanishes, correspondingly, the $m - h$ loop becomes symmetric (inset of Fig.1). The dynamic transition temperature ($T_d$) is a function of field amplitude ($h_0$). The transition across the dotted line (in Fig.1) is discontinuous and that across the solid line is continuous. For very small values of $h_0$ the nature of the dynamic transition is continuous. In this paper, all studies are done in the region where $Q$ undergoes always a continuous transition.

It has been observed carefully that the dynamic order parameter $Q$ does not acquire the stable value within the first cycle of the oscillating field. It takes several cycles (of the oscillating field) to get stabilised i.e., it shows 'relaxation' behaviour. Starting from the initial (all spins are up) configuration, the $Q$ has been calculated for various number (say $n$-th) of cycles of the oscillating magnetic field and plotted (inset of Fig.2) against the number of cycles ($n$). Each value of $Q$ shown here has been obtained by averaging over 100 random Monte Carlo samples. Inset of Fig. 2 shows a typical 'relaxation' behaviour of the dynamic order parameter $Q$. This has been plotted for fixed values of $\omega = 2\pi \times 0.04$, $h_0 = 1.0$ and $T = 1.5$. It shows that the dynamic order parameter $Q$ is relaxing as the time (number of cycles) goes on. The best fit curves shows that the 'relaxation' is exponential type. So, one can write $Q \sim Q_0 \exp(-n/\Gamma)$, where $\Gamma$ is the 'relaxation' time which provides the 'time scale' for this nonequilibrium problem. The physical interpretation of $\Gamma$ is, the number of cycles required, so that $Q$ becomes $1/e$ times of its initial value (value of $Q$ at starting cycle).
From the exponential fitting, the ‘relaxation’ time ($\Gamma$) has been measured. The temperature ($T$) variation, for fixed values of $\omega$ and $h_0$, of this ‘relaxation’ time $\Gamma$ has been studied (in the disordered region of dynamic transition) and displayed in Fig. 2. The temperature ($T$) variation of $\Gamma$ are shown (Fig. 2) for two different values of $h_0 (=0.5$ and $1.0)$ and for a fixed value of $\omega = 2\pi \times 0.04$ here. From the figure (Fig 2) it is clear that the relaxation time $\Gamma$ diverges near the dynamic transition point (where $Q$ vanishes) in the both cases ($h_0 = 0.5$ and $1.0$).

B. Mean field study

1. Mean field equation of motion and numerical solution

Although as mentioned earlier, the mean field system does not undergo a true dynamic transition (as the transition exists even in the static limit), the mean field case has been considered here as a pathological one.

The time evolution of the average magnetisation (under mean field approximation) in presence of an oscillating magnetic field can be described by the equation

$$\frac{dm}{dt} = -m + \tanh \left( \frac{m(t) + h(t)}{T} \right), \quad (2.2)$$

where $m(t)$ is the instantaneous magnetisation, $h(t) = h_0 \cos(\omega t)$ is a sinusoidally oscillating magnetic field, $T$ is the temperature and $\tau$ is a constant.

This equation has been solved by fourth order Runge-Kutta method taking $\tau = 2\pi \times 0.01$ and $dt = 0.01$. The initial boundary condition is $m(0) = 1.0$. From the numerical solution for the instantaneous magnetisation $m(t)$, the dynamic order parameter $Q (= \frac{1}{2\pi} \int m(t) dt)$ has been calculated.

2. Results

The inset of Fig. 3 shows a typical ‘relaxation’ of the dynamic order parameter $Q$ for $\omega = 2\pi \times 0.02$, $h_0 = 0.4$ and $T = 0.765$. Here, also the exponential type of relaxation is observed and the ‘relaxation’ time has been measured in the same way, discussed earlier (in the MC case). Fig 3 shows the temperature variation of the ‘relaxation’ time $\Gamma$ for $\omega = 2\pi \times 0.02$ and two different values of $h_0 (=0.3$ and $0.4$). Here also from the figure (Fig.3) it is clear that the typical time scale or the ‘relaxation’ time $\Gamma$ for this nonequilibrium problem, diverges, for both the cases ($h_0 = 0.3$ and $0.4$), near the dynamic transition point (where $Q$ vanishes).

3. An approximate solution of MF equation

In the limit of $h_0 \to 0$ and $T > 1$, the equation (2.2) can be linearised (i.e., linearising $\tanh$ term) as

$$\frac{dm}{dt} = -\epsilon m + \frac{h_0 \cos(\omega t)}{T},$$

where $\epsilon = 1 - 1/T$. The solution of the above equation is

$$m(t) = \exp(-\epsilon t/\tau) + m_0 \cos(\omega t - \phi),$$

where $m_0$ and $\phi$ are two constants. The value of the dynamic order parameter $Q$ at $n$-th cycle of the oscillating field is

$$Q = \frac{\omega}{2\pi} \int m(t) dt = \frac{\omega}{2\pi} \int_{t_{n-1}}^{t_n} m(t) dt,$$

where $t_n = 2\pi n/\omega$. The value of $Q$, at the $n$-th cycle, can be written as

$$Q = Q_0 \exp\left(\frac{-2\pi n \epsilon}{\tau \omega}\right) = Q_0 \exp\left(-n/\Gamma\right)$$

$Q_0$ is a constant independent of $n$. The above form shows that $Q$ relaxes exponentially with the number of cycles ($n$) of the oscillating field. The ‘relaxation’ time $\Gamma$ is equal to $\frac{2\pi}{\omega} \epsilon^{-1}$. It should be noted here that the dynamic transition occurs at $T = 1$ in the limit $h_0 \to 0$ [4]. So, for $h_0 \to 0$ near the dynamic transition point (where the lineariisation holds good) the behaviour of relaxation time is

$$\Gamma \sim \epsilon^{-1} \sim (T - T_d(h_0 \to 0))^{-1}$$

which shows the power law (exponent is unity) divergence of the ‘relaxation’ time at the dynamic transition point.

III. BEHAVIOUR OF ‘SPECIFIC-HEAT’ NEAR THE TRANSITION POINT

The time averaged (over a full cycle) cooperative energy of the system may be defined as

$$(a) \quad E_{\text{coop}} = -\frac{\omega}{2\pi L^2} \int \left( \sum_{<ij>} s_i^z s_j^z \right) dt,$$

and the time averaged (over a full cycle) total energy (including both cooperative and field part) of the system can be written as

$$(b) \quad E_{\text{tot}} = -\frac{\omega}{2\pi L^2} \int \left( \sum_{<ij>} s_i^z s_j^z + h(t) \sum_i s_i^z \right) dt.$$

The temperature variations of $E_{\text{tot}}$ and $E_{\text{coop}}$ have been studied. The specific heat’s are defined as $C_{\text{tot}} = \frac{dE_{\text{tot}}}{dT}$ and $C_{\text{coop}} = \frac{dE_{\text{coop}}}{dT}$. The temperature variations of the ‘specific-heat’s have also been studied and found prominent divergent behaviour near the dynamic transition point (where $Q$ vanishes).

Here, again a square lattice of linear size $L = 100$ has been considered. Both $E_{\text{coop}}$ and $E_{\text{tot}}$ are calculated using MC simulation. Each data point has been obtained by averaging over 100 different MC samples.
A. Results

The temperature derivatives of $E_{\text{coop}}$ and $E_{\text{tot}}$ can be defined as the ‘specific-heat’s for this nonequilibrium problem. The temperature variations of $Q$, $C_{\text{coop}}$ ($= \frac{dE_{\text{coop}}}{dT}$) and $C_{\text{tot}}$ ($= \frac{dE_{\text{tot}}}{dT}$) have been studied. The values of $h_0$ (=0.4 and 0.8) are chosen here in such a way that $Q$ always undergoes a continuous transition. The temperature variations of $Q$, $C_{\text{coop}}$ has been shown in Fig. 4. Inset shows the variation of total cooperative energy $E_{\text{coop}}$ (per spin) with temperature ($T$). In this case the frequency ($\omega$) of the field is kept fixed ($\omega = 0.0628$). Fig. 5 shows the temperature variation (for the same values of $\omega$, $h_0$ and $T$) of $C_{\text{tot}}$ and the inset shows the temperature variation of the total (cooperative + field) energy (per spin). From the figure it is clear that the appropriately defined ‘specific-heat’s $C_{\text{coop}}$ and $C_{\text{tot}}$ diverge near the dynamic phase transition point.

IV. SUMMARY

The nonequilibrium dynamic phase transition, in the kinetic Ising model in presence of oscillating magnetic field, is studied both by Monte Carlo simulation and by solving the meanfield dynamic equation of motion.

Acharyya and Chakrabarti [6] observed that the complex susceptibility components have peaks (or dips) at the dynamic transition point. Sides et al [7] observed that the fluctuation in the hysteresis loop area grows (seems to diverge) near the dynamic transition point. In this study it is observed that the ‘relaxation time’ and the appropriately defined ‘specific-heat’ diverge near the dynamic transition point. All the results are obtained here numerically. No attempts were made to extract any exponent values from the numerical studies.

It should be mentioned that recent experiments [9] on ultrathin ferromagnetic Fe/Au(001) films have been performed to investigate the frequency dependence of hysteresis loop areas. Recently, attempts have been made [10] to measure the dynamic order parameter $Q$ experimentally, in the same material, by extending their previous study [9]. The dynamic phase transitions has been studied from the observed variation of $Q$. However, the detailed study of the dynamic phase transitions by measuring variations of associated response functions (like the ac susceptibility, ‘specific-heat’, correlations, relaxations etc) have not been studied experimentally.

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* E-mail:muktish@physics.iisc.ernet.in
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Figure Captions

Fig.1 Schematic diagram of the dynamic phase boundary in the field amplitude ($h_0$) and temperature ($T$) plane. The dotted line is the boundary of the discontinuous transition and the solid line represents the boundary of continuous transition. The small circle represents the tricritical point (TCP). Insets demonstrate the breaking of the symmetry of the dynamic hysteresis ($m-h$) loop due to dynamic transition.

Fig.2 Monte Carlo results of the temperature ($T$) variation of ‘relaxation’ time ($\Gamma$) for two different values of field amplitudes ($h_0$): the bullet represents $h_0 = 1.0$ and the diamond represents $h_0 = 0.5$. Solid lines show the temperature ($T$) variations of dynamic order parameter $Q$. Inset shows a typical ‘relaxation’ of $Q$ plotted against the number of cycles ($n$). The solid line is the best fit exponential form of the data obtained from MC simulation. Here, $L = 100$, $\omega = 2\pi \times 0.04$.

Fig.3 Mean field results of the temperature ($T$) variation of ‘relaxation’ time ($\Gamma$) for two different values of field amplitudes ($h_0$): the filled triangle represents $h_0 = 0.4$ and filled square represents $h_0 = 0.3$. Solid lines represent the temperature variation of the dynamic order parameter $Q$. Inset shows a typical ‘relaxation’ of $Q$ plotted against the number of cycles $n$. The solid line is the best fit exponential for of the data obtained from the solution of equation 22. Here, $\omega = 2\pi \times 0.02$.

Fig.4 Monte Carlo results of the temperature variations of $C_{\text{coop}}$ for two different values of $h_0$: the filled square represents $h_0 = 0.8$ and the filled triangle represents $h_0 = 0.4$. Solid lines represent the temperature variations of $Q$. Inset shows the temperature variations of $E_{\text{coop}}$ for two different values of $h_0$: (I) $h_0 = 0.8$ and (II) $h_0 = 0.4$. 

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Here, $L = 100$, $\omega = 2\pi \times 0.01$.

Fig. 5 Monte Carlo results of the temperature variations of $C_{tot}$ for two different values of $h_0$: the filled square represents $h_0 = 0.8$ and the filled triangle represents $h_0 = 0.4$. Solid lines represent the temperature variations of $Q$. Inset shows the temperature variations of $E_{tot}$ for two different values of $h_0$: (I) $h_0 = 0.8$ and (II) $h_0 = 0.4$. Here, $L = 100$, $\omega = 2\pi \times 0.01$. 


Fig. 1

TCP

Q > 0

Q = 0

h₀

T
Fig. 2

$\begin{align*}
T &= 1.5 \\
h_0 &= 1.0
\end{align*}$
Fig. 3

- $T = 0.765$
- $h_0 = 0.4$
Fig. 4
Fig. 5