The Pyramid Scheme: Oblivious RAM for Trusted Processors

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Abstract

Modern processors, e.g., Intel SGX, allow applications to isolate secret code and data in encrypted memory regions called enclaves. While encryption effectively hides the contents of memory, the sequence of address references issued by the secret code leaks information. This is a serious problem because these leaks can easily break the confidentiality guarantees of enclaves.

In this paper, we explore Oblivious RAM (ORAM) designs that prevent these information leaks under the constraints of modern SGX processors. Most ORAMs are a poor fit for these processors because they have high constant overhead factors or require large private memories, which are not available in these processors. We address these limitations with a new hierarchical ORAM construction, the Pyramid ORAM, that is optimized towards online bandwidth cost and small blocks. It uses a new hashing scheme that circumvents the complexity of previous hierarchical schemes.

We present an efficient x64-optimized implementation of Pyramid ORAM that uses only the processor’s registers as private memory. We compare Pyramid ORAM with Circuit ORAM, a state-of-the-art tree-based ORAM scheme that also uses constant private memory. Pyramid ORAM has better online asymptotical complexity than Circuit ORAM. Our implementation of Pyramid ORAM and Circuit ORAM validates this: as all hierarchical schemes, Pyramid ORAM has high variance of access latencies; although latency can be high for some accesses, for typical configurations Pyramid ORAM provides access latencies that are 8X better than Circuit ORAM for 99% of accesses. Although the best known hierarchical ORAM has better asymptotical complexity, Pyramid ORAM has significantly lower constant overhead factors, making it the preferred choice in practice.

1 Introduction

Intel SGX [21] allows applications to isolate secret code and data in encrypted memory regions called enclaves. Enclaves have many interesting applications, for instance they can protect cloud workloads [3][5][22][46] in malicious or compromised environments, or guard secrets in client devices [21].

While encryption effectively hides the contents of memory, the sequence of memory address references issued by enclave code leaks information. If the attacker has physical access to a device, the addresses could be obtainable by attaching hardware probes to the memory bus. In addition, a compromised operating system or a malicious co-tenant in the cloud can obtain the addresses through cache-probing and other side-channel attacks [31][42][44]. Such leaks effectively break the confidentiality promise of enclaves [7][9][18][29][35][47][54].

In this paper, we explore Oblivious RAM (ORAM) designs that prevent these information leaks under the constraints of modern SGX processors. ORAMs completely eliminate information leakage through memory address traces, but most ORAMs are a poor fit for enclave, because they have high constant overhead factors [10][39][49] or require large private memories [50], which are not available in the enclave context. We address these limitations with a new hierarchical ORAM, the Pyramid ORAM. Our construction differs from previous hierarchical schemes as it uses simpler building blocks that incur smaller constants in terms of complexity. We instantiate each level of the hierarchy with our variant of multilevel adaptive hashing [8], that we call Zigzag hash table. The structure of a Zigzag hash table allows us to avoid the kind of expensive oblivious sorting that slowed down previous schemes [14][15][25]. We propose a new probabilistic routing network as a building block for constructing a Zigzag hash table obliviously. The routing network runs in $n \log n$ steps and does not bear “hidden constants”.

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Hierarchical ORAMs, despite being sometimes regarded inferior to tree-based schemes due to complex rebuild phases, have specific performance characteristics that make them the preferred choice for many applications. Inherently, most accesses to a hierarchical ORAM are relatively cheap (i.e., when no or only few levels need to be rebuilt), while a few accesses are expensive (i.e., when many levels need to be rebuilt). In contrast, in tree-based schemes, all accesses cost the same. Since, in hierarchical ORAMs, the schedule for rebuilds is fixed, upcoming expensive accesses can be anticipated and the corresponding rebuilds can be done earlier, e.g., when the system is idle, or concurrently [16, 37]. This way, high-latency accesses can be avoided. To reflect this, we report online and amortized access costs. The former measures access time without considering the rebuild and the latter measures the online cost plus the amortized rebuild cost. Furthermore, hierarchical schemes can naturally support resizable arrays and do not require additional techniques compared to tree-based schemes (as also noted in [34]).

We present an efficient x64-optimized implementation of Pyramid ORAM that uses only CPU registers as private memory. We compare our implementation with our own implementation of Circuit ORAM [51], a state-of-the-art tree-based ORAM scheme that also uses constant memory, on modern Intel Skylake processors. Although the predecessor of Circuit ORAM, Path ORAM [50], has been successfully used in the design of custom secure hardware and memory controllers [11, 13, 30, 43], Circuit ORAM has better asymptotical complexity when used with small private memory.

Pyramid ORAM has better online asymptotical complexity and outperforms Circuit ORAM in practice in many benchmarks, even without optimized scheduling of expensive shuffles. For typical configurations, Pyramid ORAM is at least 8x faster than Circuit ORAM for 99% of accesses. Finally, although the best known hierarchical ORAM [25] has better asymptotical complexity, Pyramid ORAM has significantly lower constant overhead factors, making it the preferred choice in practice.

In summary, our key contributions are:

- A novel hierarchical ORAM, Pyramid ORAM, that asymptotically dominates tree-based schemes when comparing online overhead of an oblivious access;
- Efficient implementations of Pyramid ORAM and Circuit ORAM for Intel SGX;
- The first experimental results of a hierarchical ORAM in Intel SGX. Our results show that Pyramid ORAM outperforms Circuit ORAM on small datasets and has significantly better latency for 99% of accesses.

2 Preliminaries

Attacker Model Our model considers a trusted processor and a trusted client program running inside an enclave. We assume the trusted processor’s implementation is correct and the attacker is unable to physically open the processor package to extract secrets. The processor has a small amount of private trusted memory: the processor’s registers; we assume the attacker is unable to monitor accesses to the registers. The processor uses an untrusted external memory to store data; we also refer to the provider of this external memory as the untrusted server as this terminology is often used in ORAM literature; in our implementation this is simply the untrusted RAM chips that store data. We assume the attacker can monitor all accesses of the client to the data stored in the external memory. We assume the processor encrypts the data before sending it to external memory (this is implemented in SGX processors [32]), but the addresses where the data is read from or written to are available in plain-text. We assume the processor’s caches are shared with programs under the control of the attacker, for instance malicious co-tenants running on the same CPU [44, 47] or a compromised operating system [54]. Hence, we assume that an attacker can observe client memory accesses at cache-line granularity. This model covers scenarios where SGX processors run in an untrusted cloud environment where they are shared amongst many tenants, the cloud operating system may be compromised, or the cloud administrator may be malicious. All side channels that are not based on memory address traces, including power analysis, accesses to disks and to the network, as well as channels based on shared microarchitectural elements inside the processor other than caches, (e.g., the branch predictor [27]) are outside the scope of this paper.

Definitions An element $e$ is an index-value pair where $e.key$ and $e.val$ refer to its key and value, respectively. A dummy element is an element of the same size as a real element. Elements, real or dummy, stored outside of the client’s private memory are encrypted using semantically secure encryption and are re-encrypted whenever the client reads them to its private memory and writes back. Hence, encrypted dummy elements are indistinguishable from real elements. Our construction relies on hash functions that are instantiated using pseudo-random functions and modeled as truly random functions. We say that an event happens with negligible probability if it happens with probability at most $\frac{1}{s}$, for a sufficiently large $s$, e.g., 80. An event happens with very high probability (w.v.h.p.) if it happens with probability at least $1 - \frac{1}{2^s}$.
Data-oblivious Algorithm To protect against the above attacker, the client can run a data-oblivious counterpart of its original code. Informally, a data-oblivious algorithm appears to access memory independent of the content of its input and output.

Let \( A \) be an algorithm that takes some input and produces an output. Let \( \rho \) be the public information about the input and the output (e.g., the size of the input). Given the above attacker model, we assume that all memory addresses accessed by \( A \) are observable. We say that \( A \) is data-oblivious, i.e., secure in the outlined setting, if there is a simulator \( \text{Sim}_A \) that takes only \( \rho \) and produces a trace of memory accesses that appears to be indistinguishable from \( A \)'s. That is, a trace of a data-oblivious algorithm can be reconstructed using \( \rho \).

Oblivious RAM ORAM is a data-oblivious RAM [14]. We will distinguish between virtual and real accesses. A virtual access is an access the client intends to perform on RAM, that is, an access it wishes to make oblivious. The real access is an access that is made to the data structures implementing ORAM in physical memory, i.e., those that appear on the trace produced by an algorithm.

Performance We assume that the client uses external memory (i.e., memory at the server) to store elements of size \( D \) (e.g., 64 bytes) and it can fit a constant number of such elements in its private memory. Similar to other work on oblivious RAM [50,51], we assume that \( D \) is of size \( \Omega(\log N) \), that is, it is sufficient to store a tag of the element. During each access the client can read a constant number of elements into its private memory, decrypt and compute on them, re-encrypt, and write them back. When analyzing the performance of executing an algorithm that operates on external memory, we are interested in counting the number of bits of the external memory the client must access. Similar to previous work on ORAM [50,51], we call this measurement bandwidth cost.

For ORAM schemes, we distinguish between online and amortized bandwidth cost. The former measures the number of bits to obtain a requested element, while the latter measures the amortized bandwidth for every element. This distinction is important especially for hierarchical schemes that periodically stall to perform a reshuffle of the external memory before they can serve the next request. However, such reshuffles happen after a certain number of requests are served and their cost can be amortized over preceding requests that caused the reshuffle.

In Table 1 we compare the overhead of Pyramid ORAM with other ORAM schemes that use constant private memory. Though Pyramid ORAM uses more space at the server than tree-based schemes, it has better asymptotical online performance for blocks of size \( O((\log N / \log \log N)^2) \). Pyramid ORAM is less efficient than hierarchical schemes by Kushilevitz et al. [25] and Goodrich and Mitzenmacher [15]. However, as we explain in Section 9 our scheme relies on primitives that are more efficient in practice than those used in [15,25]. For example, we do not rely on the AKS sorting network.

| Scheme                        | Bandwidth cost | Server Space |
|-------------------------------|----------------|--------------|
|                               | Online         | Amortized    | Overhead  |
| Binary Tree ORAM [10]         | \( D(\log N)^2 \) \( + (\log N)^3 \) | \( \log N \) |
| Path ORAM (SCORAM) [52]      | \( D \log N + (\log N)^3 \log \log N \) | 1           |
| Circuit ORAM [51]            | \( D \log N + (\log N)^3 \) | 1           |
| Hierarchical, GO [14]        | \( D(\log N)^2 \) | \( D(\log N)^3 \) | \( \log N \) |
| Hierarchical, GM [15]        | \( D(\log N) \) | \( D(\log N)^2 \) | 1           |
| Hierarchical, Kushilevitz et al. [25] | \( D(\log N)^2 / \log \log N \) | 1           |
| Pyramid ORAM                 | \( D(\log N)(\log \log N)^2 \) | \( D(\log N)^2(\log \log N)^2 \) | \( (\log \log N)^2 \) |

3 Hierarchical ORAM (HORAM)

Our Pyramid ORAM scheme follows the same general structure as other hierarchical ORAM (HORAM) constructions [14,15,17,25,53]. We outline this structure below and describe how we instantiate it using our own primitives, Probabilistic Routing Network (PRN) and Zigzag Hash Table (ZHT) in the next sections.

Data Layout HORAM abstractly arranges the memory at the server into \( O(\log N) \) levels, where \( N \) is the number of elements one wishes to store remotely and access obliviously later. Though \( N \) does not have to be specified a priori and the structure can grow to accom-
moderate more elements, for simplicity, we assume \( N \) to be a fixed power of 2.

The first two levels, \( L_0 \) and \( L_1 \), can store up to \( p \) elements each, where \( p \geq 2 \). The third level \( L_2 \), can store up to \( 2p \) elements and the \( i \)th level stores up to \( 2^{i−1}p \). The last level, \( l \), can fit all \( N \) elements (i.e., \( l = \log N / p \)). Note that we only mentioned the number of real elements that each level can fit. In many HORAM constructions, each level also contains dummy elements that are used for security purposes. Hence, the size of each level is of some fixed size and is often larger than the number of real elements the level can fit. The data within each level is stored encrypted (implicitly done by the hardware in the case of SGX), hence, real and dummy elements are indistinguishable.

The first level \( L_0 \) is implemented as a list. The search over \( L_0 \) is a simple scan. Each consequent level, \( L_{i≥1} \), is implemented as a hash table to ensure fast search time (e.g., a cuckoo hash table \([15, 17, 25, 40]\) or a hash table with buckets \([14]\)).

**Setup** During the setup, the client inserts \( N \) elements into the last level \( L_0 \) and uploads the first \( l − 1 \) empty levels and \( L_0 \) to the server. As mentioned before, this is not required and the client can populate the data structure by writing elements obliviously one at a time.

**Access** During a virtual access for an element with \( e.key \), the client performs the following search procedure across all levels. If level \( i \) is empty, it is skipped. If the element was not found in levels \( 0, \ldots, i − 1 \), then the client searches for \( e.key \) in \( L_i \). If \( L_i \) has the element, it is removed from \( L_i \), cached, and the client continues accessing levels \( i + 1, i + 2, \ldots \) but for dummy elements. If \( L_i \) does not have the element, the client searches for \( e.key \) in level \( i + 1 \) and so on. After all levels are accessed, the cached \( e \) is appended to \( L_0 \) (even if \( e \) was found in some earlier location in \( L_0 \) from which it was removed).

**Rebuild** After \( p \) accesses, \( L_0 \) contains up to \( p \) elements, where some may be dummy if the same element was accessed more than once. This triggers the rebuild phase: \( L_0 \) is emptied by inserting its real elements into \( L_1 \). After the next \( p \) accesses, \( L_0 \) becomes full again. Since \( L_1 \) is already full, real elements from both \( L_0 \) and \( L_1 \) are inserted into \( L_2 \). In general, after every \( 2^i p < N \) accesses levels \( 0, 1, \ldots, i \) are emptied into \( L_{i+1} \). After \( N \) accesses levels \( 0, 1, \ldots, l − 1 \) are all full. In this case, a new table, \( L_i' \), of the same size as \( L_i \), is created. Elements from \( L_0, L_1, \ldots, L_i \) are inserted into \( L_i' \) and emptied, and \( L_i \) is replaced with \( L_i' \). The same process repeats for the consequent accesses. Since the rebuild procedure is a deterministic function of the number of accesses performed so far, it is easy to determine which levels need rebuilding for any given access.

The insertion of real elements from levels \( 0, 1, \ldots, i \) to \( i + 1 \) has to be done carefully in order to hide the number of real and dummy elements at each level. For example, knowing that there was only one real element reveals that all past accesses were for the same element. The data-oblivious insertion (or rebuild) process depends on the specific HORAM construction and the data structure used to instantiate each level. Many constructions rely on an oblivious sorting network \([2, 4]\) for this.

**Security** A very high-level intuition behind the security of an HORAM relies on the following: (1) level \( i \) is searched for every element at most once during the accesses between two sequential rebuilds of level \( i \); (2) searches for the same element before and after a rebuild are uncorrelated; (3) the search at each level does not reveal which element is being searched for.

**Pyramid ORAM Overview** HORAM’s performance and security rely on two critical parts: the data structure used to instantiate levels \( L_1, \ldots, L_{l} \) and the rebuild procedure. In this paper we propose Pyramid ORAM, an HORAM instantiated with a novel data structure and a rebuild phase tailored to it. In Section \([5]\) we devise a new hash table, a Zigzag Hash Table, and use it to instantiate each level of an HORAM. To avoid using an expensive oblivious sorting network when rebuilding a Zigzag Hash Table, in Section \([4, 5]\) we design a setup phase using a Probabilistic Routing Network (Section \([4]\)). The combination of the two primitives and their small constants result in an HORAM that is more efficient in practice than previous HORAM schemes.

### 4 Probabilistic Routing Network (PRN)

In this section, we describe a Probabilistic Routing Network that we use in the rebuild phase of our HORAM instantiation. However, the network as described here, with its data-oblivious guarantees and efficiency, may be of an independent interest.

A PRN takes a table of \( n \) buckets with \( c \) slots in each bucket. The bucket size is sufficiently small compared to \( n \). For example, in Pyramid ORAM \( c \) is set to \( O(\log \log n) \). The input table has at most \( n \) real elements labeled with indices \( 0, 1, \ldots, n − 1 \) denoting their destination buckets. The labels do not have to be stored alongside the elements and can be determined using a hash...
Figure 1: Routing network for $n = 8$ elements where each element is indicated in the input table by its destination position (e.g., the destination of the element in the first bucket is 5 and destinations of the two elements in the second bucket are 6 and 7). The elements that could not be routed in the second stage (i.e., tagged false by the network) are greyed out.

function, for example. There can be more than one element in the input bucket and, since a hash function is not a permutation function, two or more elements may be assigned to the same destination bucket.

A PRN’s goal is to route a large subset of elements to the correct output buckets. In the output, elements in that subset are associated with the tag true, others with false. Given the destination labels, the network’s output can be used for efficient element lookups. Such a lookup is successful, if an element was routed to its correct destination (i.e., has tag true).

For the security purposes of HORAM, we wish to design a network that is data-oblivious in the following sense. The adversary that observes the routing as well as element accesses to the output learns nothing about the placement of the elements in the output as long as it does not learn whether accesses are successful or not.

Algorithm We now describe a routing network with the above properties. The routing network consists of $\log n$ stages where the input at each stage, except the first stage, is the output of the previous stage. The output of the last stage is the output of the network. Besides the destination bucket indices, every element also carries a Boolean tag that is set to true for every real element in the original input. (However, depending on the use case, input elements may be tagged false, for example, if they are dummy elements.) Once the tag is set to false, it does not change throughout the network. Crucially, only elements that are in their correct destination buckets in the output are tagged true.

At each stage $i$, for $1 \leq i \leq \log n$, we perform a re-partition operation between each pair of buckets with indices that differ only by the bit in position $i$ (starting from the lowest bit). (Note that here indices correspond to indices of the input buckets, not the indices of the destinations of the elements inside.) For example, for $n = 8$, when $i = 1$ we re-partition the following pairs of buckets: $(000,001), (010,011), (100,101)$ and $(110,111)$ and when $i = 3$, buckets: $(000,100), (001,101), (010,110)$ and $(011,111)$. The re-partition operation itself moves items between the two buckets (number them one and two) and works as follows: For elements with tag true, we consider the $i$th bit of their bucket index. If this bit is set to 0, the element goes to the first bucket, and to the second bucket, otherwise. As there may be more than $c$ elements assigned the same bit at position $i$, the network chooses $c$ of them at random, and retags the extra/spilled elements with false. All elements tagged false, including those that arrived with the false tag from the previous stage, are then allowed to go in either bucket (note that there is always space for all elements). The effect of the network is that at stage $i$, every element still tagged true will have the first $i$ bits match that of the destination bucket. Therefore, after $\log n$ stages elements will either be tagged false, or be in the correct bucket. (An illustration of the routing network for $n = 8$ is given in Figure 1)

Bucket Partitioning We now describe how to perform the re-partition of two buckets. If the client memory is $2c$ then the re-partition is trivial. Otherwise, we use Batcher’s sorting network as follows. The two buckets are treated as one contiguous range of length $2c$ that needs to be sorted ordering the elements as follows: (1) elements with bit 0 at the $i$th position, (2) empty elements and elements tagged false, and (3) elements with bit 1 at the $i$th position. Sorting will therefore cause a partition, with empty and false-tagged elements in the middle, and other elements in their correct buckets.

Analysis In this section we summarize the performance and security properties of the network.

Theorem 1. The probabilistic routing network operates in $(n/2) \log n \times \text{Sort}(2c)$ steps, where Sort$(c)$ is time to sort $c$ elements obliviously.

The theorem follows trivially: the network has $\log n$ stages where each stage consists of repartitioning of $n/2$ pairs of buckets.

Corollary 1. The probabilistic routing network operates in $O(c(\log c)^2 n \log n)$ steps using constant private memory and Batcher’s sorting network for repartitioning bucket pairs.
The PRN is more efficient than Batcher’s sorting network, that runs in \( O(n \log n)^2 \) steps, at the expense of not being able to route all the elements. Though the network cannot produce all \( n! \) permutations, it gives us important security properties. As long as the adversary does not learn which output elements are tagged true and false it does not learn the position of the elements in the output. Hence, when using the network in the ORAM setting we will need to ensure that the adversary does not learn whether an element reached its destination or not. We summarize this property below.

**Theorem 2.** The probabilistic routing network and lookup accesses to its output are data-oblivious as long as the adversary does not learn whether lookups were successful or not.

**Proof.** The accesses made by the routing network to the input are independent of the input as they are deterministic and the re-partition operation is either done in private memory or using a data-oblivious sorting primitive. Hence, we can build the simulator \( \text{Sim}_\text{route} \) that, given a table of dummy elements, performs accesses according to the network. The adversary cannot distinguish \( \text{Sim}_\text{route} \) from the real routing even if it knows the location of the elements in the input.

During the search, the adversary is allowed to request lookups for the elements from the original input and observe the accessed buckets. For data-obliviousness, the lookup simulator is notified about the lookup but not the element being searched. The simulator then accesses a random bucket of the output generated by \( \text{Sim}_\text{route} \).

The accesses of the simulator and the real algorithm diverge only in the lookup procedure. The accesses generated by the simulator are from a random function, while real lookups access elements according to a hash function used to route the elements. Since the adversary is not notified whether a lookup was successful or not, it cannot distinguish whether simulator’s function could have been used to successfully allocate the same elements as the real routing network. Hence, the distribution of lookup accesses by the simulator is indistinguishable from the real routing network.

We defer the analysis of the number of elements that are routed to their output bucket to the next section as it depends on how elements are distributed across input buckets. For example, we assume random allocation of the elements in the input in Pyramid ORAM.

## 5 Zigzag Hash Table (ZHT)

In this section we describe a data structure, called Zigzag Hash Table (ZHT), that we use to instantiate the levels of our hierarchical ORAM. A ZHT is a variation of multilevel adaptive hashing [8] but instantiated with tables of fixed size and buckets of size \( c \) (see Section 9 for a detailed comparison). We begin with the non-oblivious version of a ZHT and then explain how to perform search and rebuild obliviously.

A ZHT is a probabilistic hash table that can store up to \( n \) key-value elements with very high probability. We refer to \( n \) as the capacity of a ZHT, that is, the largest number of elements it can store. ZHT consists of \( k \) tables, \( H_1, H_2, \ldots, H_k \). Each table contains \( n \) buckets, with each bucket containing at most \( c \) elements. Each table is associated with a distinct hash function \( h_i \) that maps element keys to buckets. The hash functions are fixed during the setup, before the elements arrive.

An element \( e \) is associated with an (abstract) zigzag path defined by the series of bucket locations in each table: \( h_1(e.\text{key}), h_2(e.\text{key}), \ldots, h_k(e.\text{key}) \). Elements are inserted into the table by accessing the buckets along the path, placing the element in the first empty slot. If the path is full, the insertion procedure has failed.

![Zigzag hash table](image)

**Figure 2:** A Zigzag hash table for \( n = 8 \) elements with \( k = 3 \) tables where each table has 8 buckets of size \( c = 2 \). Elements 0–7 were inserted in order, with hash functions defined as: \( h_1(1) = 4 \), \( h_1(2) = 1 \), \( h_1(3) = 5 \), \( h_1(4) = 1 \), \( h_1(5) = 6 \), \( h_1(6) = 1 \), \( h_1(7) = 1 \), \( h_1(0) = 5 \), \( h_2(5) = 2 \), \( h_2(6) = 2 \), \( h_2(7) = 5 \), and \( h_3(7) = 3 \).

The accesses of the simulator and the real algorithm differ only in the lookup procedure. The accesses generated by the simulator are from a random function, while real lookups access elements according to a hash function used to route the elements. Since the adversary is not notified whether a lookup was successful or not, it cannot distinguish whether simulator’s function could have been used to successfully allocate the same elements as the real routing network. Hence, the distribution of lookup accesses by the simulator is indistinguishable from the real routing network.

We defer the analysis of the number of elements that are routed to their output bucket to the next section as it depends on how elements are distributed across input buckets. For example, we assume random allocation of the elements in the input in Pyramid ORAM.

### Table 2: Notation

| Symbol | Description |
|--------|-------------|
| \( N \) | Number of elements in ORAM |
| \( n \) | Capacity of a Zigzag Hash Table (ZHT) |
| \( k \) | Number of tables in a ZHT |
| \( c \) | Bucket size of tables \( H_1, \ldots, H_k \) in a ZHT |
referring to the insertion of several elements, we use
throw\((A, H_1, H_2, \ldots, H_t)\) that takes elements in the ar-
ray \(A\) and inserts them according to their zigzag paths.
Once inserted, elements are found by scanning the path
and comparing indices of elements in the buckets until
the element is found. (See Figure 2 for an illustration of
a ZHT.)

**Definition 5.1** (Valid zigzag paths). Let \(p_1, p_2, \ldots, p_n\) be
\(n\) zigzag paths. If every element \(e_j\) can be inserted into a
ZHT according to locations in the path \(p_j\), then the paths
are valid.

If \(n\) elements are successfully inserted in a ZHT, we
say that the zigzag paths associated with the elements
are valid. We will use \(ZHT_{n,k,c}\) to denote a ZHT with
parameters \(k\) and \(c\) sufficient to avoid overflow with high
probability when inserting \(n\) elements. We summarize
the parameters and the performance of a ZHT below.

**Theorem 3** (ZHT Performance). Insertion and search of
an element in a \(ZHT_{n,k,c}\) takes \(O(kc)\) accesses. Insertion
of \(n\) elements fails with negligible probability when \(k =
O(\log \log n)\) and \(c = O(\log \log n)\).

**Proof.** (Sketch.) The analysis follows Theorem 5.2
from \([1]\), adopted for our scenario. We note that the analysis
in \([1]\) is for re-throwing elements into the same table
and not into an empty table, as is the case in our data
structure.

We refer to an element of a table as **overflowed** if the
bucket it was allocated to already had \(c\) elements in it.

For each table \(H_i\), we are interested in measuring \(i_n\),
the number of incoming elements, and \(o_{it}\), the
number of outgoing elements. The former comprises the ele-
ments that did not fit in tables \(1, 2, \ldots, i - 1\). The number
of outgoing elements comprises elements that did not fit
into table \(i\) when throwing \(i_n\). Except for the first phase,
the number of incoming elements into \(H_i\) is determined
by the number of outgoing elements of \(H_{i-1}\). Hence, to
determine \(k\) we need to bound the number of outgoing
elements at each level.

We first show that one round of throw is required to
bound the number of overflowed elements to be at most
\(n/(2c)\) (Lemma 2). We then show that as long as the
number of elements arriving at the table is less than
\(n/(2c)\) only \(O(\log \log n)\) tables are required to fit all \(n\)
elements (Lemma 1). \(\square\)

We note that buckets that hold at most one element
(i.e., \(c = 1\)) are sufficient to bound the number of ta-
bles, \(k\), to be \(\log \log n + O(1)\) \([1, 3]\). However, as will
be explained in the following sections, in order to in-
sert elements into a ZHT obliviously we require \(c\) to be
\(O(\log \log n)\).

As described so far, the insertion procedure of a
ZHT does not provide security properties one desires for
ORAM (e.g., observing insertion of \(n\) elements and a
search for element \(e\) reveals when \(e\) was inserted). In
the next section, we make small alternations to the inser-
tion and search procedures of a ZHT and summarize the
security properties a ZHT already has. In particular, we
will show that one can obliviously search for elements in
a ZHT, as long as it was constructed privately. Then, in
Section 5, we show how to insert \(n\) elements into a ZHT
obliviously using small private memory.

### 5.1 Data-oblivious properties of ZHT

We make the following minor changes to a ZHT and
its insertion and search procedures. We fill all empty
slots of a ZHT with dummy elements. Recall that in
the ORAM setting the data structure is encrypted and
dummy elements are indistinguishable from real. Then,
when inserting an element, instead of stopping once the
first empty slot is found, we access the remaining buckets
on the path, pretending to write to all of them. Similarly,
during the search, we do not stop when the element is
found and access all buckets on its zigzag path. Hence,
anyone observing accesses of an insert (or search) ob-
serves the zigzag path but do not know where on the path
the element was inserted (or found). We define dummy
counterparts of insertion and search procedures that sim-
ply access a random bucket in each table of a ZHT, per-
forming fake writes and reads.

Given the above changes, the search for \(n\) elements is
data-oblivious as long as every element is searched for
only once and assuming that an adversary does not ob-
serve how the table was built.

**Lemma 1** (ZHT Data-Oblivious Search). Searching
for \(n\) distinct elements in a \(ZHT_{n,k,c}\) is data-oblivious.

**Proof.** We can build a simulator that given \(n, k\) and \(c\)
produces a sequence of accesses to its tables indistin-
guishable from locations accessed by a ZHT algorithm.

Consider an algorithm (with large private memory)
that takes \(n\) elements and inserts them one by one into
an empty ZHT. If it cannot insert all \(n\) elements, it fails.
Otherwise, during a search request for an element key,
e.key, it accesses the locations that correspond to the zigzag
path defined by e.key and the hash functions of the ZHT.

The simulator, Sim\(_{search}\), sets up \(k\) tables with \(n\) buck-
ets of size \(c\) each. For every search, the simulator per-
foms a dummy search. We say that a simulator fails if
after simulating searches for \(n\) elements, it has produced
accesses to ZHT tables that do not form \(n\) valid zigzag
paths.
In each case the adversary chooses \( n \) original elements. It then requests a search for one of the \( n \) elements that it has not requested before. A ZHT algorithm is given the key of the element, while the simulator is invoked to perform a dummy search. In both cases, the adversary is given the locations accessed during the search. If neither the simulator nor the ZHT algorithm fail, the accessed locations are indistinguishable as long as ZHT’s hash functions are indistinguishable from truly random functions. The failure cases of the algorithms allow the adversary to distinguish the two as failures happen at different stages of the indistinguishability game. However, both failures happen with negligible probability due to the success rate of a ZHT \( n,k,c \). 

**Lemma 2.** Searching for \( n \) distinct elements in a ZHT \( n,k,c \) is data-oblivious even if the adversary knows the order of insertions.

**Proof.** The proof follows the proof of Lemma 1 except now the adversary controls the order in which the ZHT algorithm inserts the elements. As before, we use Sim\(_{\text{search}}\) that is not given the actual elements and the adversary does not observe accesses of the insertion phase.

Let \( p_i \) be the zigzag path of element \( e_i \). Notice that \( k \) hash functions that determine zigzag paths are chosen before the elements are inserted into an empty ZHT. Hence, the order in which \( e_i \) is inserted in the ZHT influences where on the path \( e_i \) will appear (e.g., if \( e_i \) is inserted first, then it will be placed into \( H_1 \), on the other hand, if it is inserted last it may not find a spot in \( H_1 \) and could even appear in \( H_k \) but not the path it is on. Since the adversary does not learn where on the path an element was found, the rest of the proof is similar to the proof of Lemma 1.

We capture another property of a ZHT that will be useful when used inside of an ORAM: locations accessed during the insertion appear to be independent of the elements being thrown. Moreover, as we show in the next lemma, the adversary cannot distinguish whether thrown elements were real or dummy.

**Lemma 3.** The algorithm throw\((A, H_1, H_2, \ldots, H_k)\) is data-oblivious where \( H_1, H_2, \ldots, H_k \) are hash tables of ZHT \( n,k,c \) and the size of \( A \) is at most \( n \).

**Proof.** The adversary chooses up to \( n \) distinct elements and requests throw. The algorithm fails if it cannot insert all elements in \( k \) tables. The simulator Sim\(_{\text{throw}}\) is given \( A \) that contains only dummies. It accesses a random bucket in each of the \( k \) tables for every element in \( A \).

The adversary can distinguish the two algorithms only when throw fails or when Sim\(_{\text{throw}}\) generates invalid zigzag paths for \( A \). According to Theorem 3 this happens with negligible probability.

**Lemma 4.** Consider throw\((A, H_1, H_2, \ldots, H_k)\) where \( A \) contains \( n_{\text{real}} \leq n \) real elements and \( n_{\text{dummy}} \) dummy elements, where a real element is inserted as usual while on a dummy element each table is accessed at a random index. The algorithm throw remains data-oblivious.

**Proof.** We define an adversary that chooses a permutation of \( n_{\text{real}} \) real elements and \( n_{\text{dummy}} \) dummy elements, that is, it knows the locations of the real elements. We build a simulator that is given an empty ZHT \( n,k,c \) table. The simulator accesses a random bucket in each table of the ZHT on every element in \( A \).

The adversary can distinguish between the simulator and the throw algorithm if (1) the algorithm fails, which happens when the algorithm cannot insert all real elements successfully or (2) the simulator generates invalid zigzag paths when performing a fake insertion for the input locations that correspond to real elements (recall that the simulator does not know the locations of real elements). Both events correspond to the failure of the ZHT construction, which happens with negligible probability (Theorem 3). 

### 5.2 Data-oblivious ZHT Setup

A Zigzag hash table allows one to search elements obliviously if insertion of these elements was done privately. Since the private memory of the client is much smaller than \( n \) in our scenario, we develop an algorithm that can build a ZHT relying on small memory and accessing the server memory in an oblivious manner. That is, the server does not learn the placement of the elements in the final ZHT.

The algorithm starts by placing elements in the tables \( H_1, H_2, \ldots, H_k \) using throw procedure from Section 5 according to random hash functions. The algorithm then proceeds in \( k \) phases. In the first phase, \( H_1 \) is given as an input to the probabilistic routing network where the destination bucket of every element in \( H_1 \) is determined by \( h_1 \). The result is a hash table with some fraction of the elements in the right buckets and some overflowed elements (tagged as false). The algorithm then pretends to throw all elements of \( H_1 \) into tables \( H_2, H_3, \ldots, H_k \) using yet another set of random functions. However, only overflowed elements, that is elements that were not routed to their correct bucket in \( H_1 \), are actually inserted into tables \( H_2, H_3, \ldots, H_k \). For empty entries and elements that are labeled true in \( H_1 \), the algorithm accesses random locations in \( H_2, H_3, \ldots, H_k \) and only pretends to write to them. In the end of this step every element that did not fit into \( H_1 \) during the routing network appears in one of \( H_2, H_3, \ldots, H_k \) at a random location. In the second phase, \( H_2 \) goes through the network where destination buckets are determined using \( h_2 \), and so on. In general, every \( H_i \) goes through the network using \( h_i \), then all elements of \( H_i \)
are thrown into tables $H_{i+1}, H_{i+2}, \ldots, H_k$, where only elements that were not routed correctly are inserted. (See Figure 12 in Appendix for an illustration of the oblivious setup of a ZHT.)

The algorithm fails if an element cannot find a spot in any of the $k$ tables. This happens either during one of the throws of the elements or during the final phase where not all elements of $H_i$ can be routed according to $h_k$.

**Performance** We now analyze the parameters and the performance of the ZHT setup and then describe the security properties.

**Theorem 4.** Insertion of $n$ elements into a ZHT of $n$, as described in Section 5.2, succeeds with very high probability if $k = O(\log \log n)$ and $c = O(\log \log n)$.

**Proof.** (Sketch.) The analysis is similar to the analysis in Theorem 3 with the difference that the number of elements going into level $H_{i+1}$ is higher due to additional overflows that happen during routing. In Lemma 8 we show that if $n$ is the number of input elements in hash table $H_i$, then the number of overflows from routing is bounded by $\log n \times$ the number of overflows of a throw into a single hash table. Then, following similar proof arguments as Theorem 3, we use Lemmas 10 and 12 to bound the spill at each phase and show that if $k = O(\log \log n)$, then the insertion of $n$ elements succeeds w.v.h.p.

**Theorem 5 (Oblivious ZHT Setup Performance).** If oblivious ZHT insertion of $n$ elements does not fail, it takes $O(nkc((\log c)^2 \log n + kc))$ steps.

**Proof.** The initial throw takes $nkc$ and $(n(k-1)c^2, n(k-2)c^2, \ldots, nc^2)$ steps for the subsequent throws. The complexity of applying a routing network to a single table is $O(c(\log c)^2 n \log n)$. Since the algorithm performs $k$ routing networks, the total cost is $O(kc((\log c)^2 n \log n + n(kc)^2)) = O(nkc((\log c)^2 \log n + kc))$.

When used in Pyramid ORAM, a ZHT is constructed from all elements in previous levels of the hierarchy. The levels include real elements as well as many dummy elements. Hence, the initial throw of the setup algorithm may contain additional dummy elements. Throwing dummy elements does not change the failure probability of the setup. However, the initial throw becomes more expensive than that in Theorem 5 as summarized below.

**Corollary 2 (Oblivious ZHT Setup Performance).** Oblivious ZHT insertion of $n$ real elements and $(\alpha - 1)n$ dummy elements takes $O(nkc((\log c)^2 \log n + \alpha kc))$, for $\alpha \geq 1$.

**Security** We show that an adversary that observes the ZHT setup does not learn the final placement of the elements across the tables.

**Theorem 6.** The insertion procedure of ZHT of $n$ is data-oblivious.

**Proof.** An adversary is allowed to interact with the algorithm and a simulator as follows: choose up to $n$ elements and request the oblivious setup of ZHT of $n$, then request a search of at most $n$ elements (real or dummy) on ZHT of $n$. It then repeats the experiment either for the same set of $n$ elements or a different one.

As before, the real algorithm fails if it cannot build ZHT of $n$. The simulator simulates the algorithm as $n$ dummy elements and parameters $k$ and $c$. It sets up a dummy and runs $\text{Sim}_{\text{throw}}(A, H_1, H_2, \ldots, H_k)$. It then calls $\text{Sim}_{\text{route}}$ from Theorem 2 on $H_1$. After that the throw of the spilled elements is imitated via $\text{Sim}_{\text{throw}}(H_1, H_2, H_3, \ldots, H_k)$ from Lemma 1. It then again calls $\text{Sim}_{\text{route}}$ but on its own version of $H_2$. Once finished, on every search request from the adversary, $\text{Sim}_{\text{search}}$ that is not aware of the actions of $\text{Sim}_{\text{throw}}$, is called.

If the algorithm does not fail during the insertion phase and the simulator produces valid zigzag paths during $\text{Sim}_{\text{throw}}$ and $\text{Sim}_{\text{search}}$, then the adversary obtains only traces that are produced either by a hash function (i.e., a pseudo-random function) or by a random function. We showed that $\text{ZHT}_{\text{throw}}$ and $\text{Sim}_{\text{throw}}$ fail with negligible probability. The real search and $\text{Sim}_{\text{search}}$ are also indistinguishable even if the adversary controls the order of the inserted elements as long as the adversary does not learn element’s position in its zigzag path. Since there is $\text{Sim}_{\text{route}}$ which simulates route, the adversary does not learn element positions after routing. Since every element participates in the throw, we showed in Lemma 4 that the adversary does not learn which elements are dummy. Hence, the adversary does not learn positions of the elements across the tables and search can be simulated successfully.

6 **Pyramid ORAM**

We are now ready to present Pyramid ORAM by instantiating the hierarchical construction in Section with data-oblivious Zigzag hash tables.

**Data Layout.** Level $L_0$ is a list as before while every level $L_i, 1 \leq i \leq l$, is a Zigzag hash table with capacity $2^{i-1}p$ and $k_i, c_i$ chosen appropriately for Theorem 3 to hold. We denote the bucket size and table number used for the last level with $c$ and $k$. That is, $L_i$ is a ZHT of $n, k, c$.

**Setup.** The setup is as described in Section i.e., elements can be placed either in $L_i$ or inserted obliviously one by one causing the initially empty ORAM to grow.
Access. The access to each level \( i \) is an oblivious ZHT search, if the element was not found in previous levels, or a dummy ZHT access.

Rebuild. The rebuild phase involves calling the data-oblivious ZHT setup as described in Section 5.2. In particular, at level \( i \), a new ZHT for \( 2^{i-1} P \) elements is set up. Then, all elements from previous levels, including dummy, participate in the initial throw.

We first determine the amortized rebuild cost of level \( i \) for an HORAM scheme and then analyze the overall performance of Pyramid ORAM.

Lemma 5. If the rebuild of level \( i \geq 1 \) takes \( O(2^{i-1} P C_i) \) steps, for \( C_i > 0 \), then the amortized cost of the rebuild operation for every access to ORAM is \( \sum_{i=1}^{\log N/p} O(C_i) \).

Proof. Since the rebuild of level \( i \) happens every \( 2^{i-1} P \) accesses, the amortized cost of rebuilding level \( i \) is \( O(C_i) \). Each access eventually causes a rebuild of every level, that is, \( \log N/p \) rebuilds of different costs. Hence, total amortized cost is:

\[
\sum_{i=1}^{\log N/p} O(C_i).
\]

Theorem 7 (Performance). Pyramid ORAM incurs \( O(kc \log N) \) online and \( O(kc (\log N)^2) \) amortized bandwidth overheads, \( O(kc) \) space overhead, and succeeds with very high probability.

Proof. The complexity of a lookup in a ZHT is \( kc \). The complexity of a lookup in the first level is \( p \). There are \( \log N/p \) levels. So the cost of a lookup is \( p + kc \log N/p \). Since \( p \) is usually set to \( \log N \), the access time between table rebuilds, that is the online bandwidth overhead, is \( O(kc \log N) \).

The rebuild of level \( i \) takes \( O(2^{i-1} P k c ((\log c)^2 \log (2^{i-1} P) + kc)) \) steps since \( \alpha = kc \) (see Corollary 2) and \( k \geq k_i \) and \( c \geq c_i \), for every \( i \). Setting \( C_i \) to \( kc ((\log c)^2 \log (2^{i-1} P) + kc) \) and \( p \) to be at least 2 in Lemma 5 we obtain:

\[
\sum_{i=1}^{\log N/p} k c ((\log c)^2 \log (2^{i-1} P) + kc)
\]

\[
= kc \sum_{i=1}^{\log N/p} ((\log c)^2 \log (2^{i-1} P) + kc)
\]

\[
= kc \left( (\log c)^2 \sum_{i=1}^{\log N/p} (i - 1 + \log p) + kc \log N/p \right)
\]

\[
= kc ((\log c)^2 (\log N)^2 + k c \log N)
\]

\[
= O(kc (\log N)^2)
\]

Pyramid ORAM fails when the rebuild of some level \( i \) fails, that is, the number of tables allocated for level \( i \) was not sufficient to accommodate all real elements from the levels above. Since, the number of real elements in tables at levels \( 0, 1, \ldots, i - 1 \) is at most the capacity of the ZHT at level \( i \), ZHT fails with negligible probability as per analysis in Theorem 4. Hence, the overall failure probability of Pyramid ORAM can be bounded using a union bound.

Theorem 8 (Security). Pyramid ORAM is data-oblivious with very high probability.

Proof. The adversary chooses \( N \) elements of its choice and requests oblivious insertion in \( L_1 \). It then makes a polynomial number of requests to elements in the ORAM. We use \( \text{Sim}_{\text{build}} \) from Theorem 6 and \( \text{Sim}_{\text{search}} \) from Lemma 7 for every level except \( L_0 \), which simulates naive scanning.

Between two rebuilds the search operation at each level is performed to distinct elements and every table uses a new set of hash functions after its rebuild. The hash functions are also different across the levels. Hence, \( \text{Sim}_{\text{search}} \) can simulate the search at every level independently.

\( \text{Sim}_{\text{build}} \) for level \( i \) takes as input the number of dummy elements proportional to the total number of elements, real and dummy, in \( L_0, L_1, \ldots, L_{i-1} \). We showed that we can construct \( \text{Sim}_{\text{build}} \) even if the adversary knows the original elements inserted in the table. Hence, \( \text{Sim}_{\text{build}} \) succeeds when the adversary controls the elements inserted in the ORAM as well as the access sequence.

7 Implementation

We implemented Pyramid ORAM and Circuit ORAM [51] in C++ with oblivious primitives written in x64 assembly. As ORAM block size, we use the x64 architecture’s cache-line width of 64 bytes. Given that our assumed attacker is able to observe cache-line granular memory accesses, this block size facilitates the implementation of data-oblivious primitives. A block comprises 56 bytes of data, the corresponding 4-byte index, and a 4-byte tag, which indicates the element’s state (see Section 4).

To be data-oblivious, our implementations are carefully crafted to be free of attacker-observable secret-dependent data or code accesses. We avoid secret-dependent branches on the assembly level either by employing conditional instructions or by ensuring that all branch’s possible targets lie within the same 64-byte cache line. For data, we use registers as private memory and align data in memory with respect to cache-line boundaries. Similar to previous work [36][41], we create
a library of simple data-oblivious primitives using conditional move instructions and CPU registers. These allow us, for example, to obliviously compare and swap values. Hash functions used in Pyramid ORAM are instantiated with our SIMD implementation of the Speck cipher.\footnote{https://en.wikipedia.org/wiki/Speck_(cipher)}

7.1 Pyramid ORAM

For bucket sizes of $c > 4$, our implementation of Pyramid ORAM uses a Batcher’s sorting network for the re-partitioning of two buckets within our probabilistic routing network described in Section 4.

For $c = 4$, we employ a custom re-partition algorithm that is optimized for the x64 platform and leverages SIMD operations on 256-bit AVX2 registers. The implementation is outlined in the following. For most operations, we use AVX2 registers as vectors of four 32-bit components. To re-partition two buckets $A$ and $B$, we maintain the tag vectors $t_A$ and $t_B$ and position vectors $p_A$ and $p_B$. Initially, we load the four tags of bucket $A$ into $t_A$ and those of bucket $B$ into $t_B$. A tag indicates an element’s state. Empty elements and elements that were spilled in the previous stages of the network are assigned -1; elements that have not been spilled and need to be routed to either bucket $A$ or $B$ are assigned 0 and 1, respectively.

$p_A$ and $p_B$ map element positions in the input buckets to element positions in the output buckets. We refer to the elements in the input buckets by indices. The elements in input bucket $A$ are assigned the indices $1 \ldots 4$, those in input bucket $B$ the indices $5 \ldots 8$. Correspondingly, we initially set $p_A = (1, 2, 3, 4)$ and $p_B = (5, 6, 7, 8)$.

We then sort the tags by performing four oblivious SIMD compare-and-swap operations between $t_A$ and $t_B$. Each such operation comprises four 1-to-1 comparisons. (Hence, a total of 16 1-to-1 compare-swap-operations are performed.) Every time elements are swapped between $t_A$ and $t_B$, we also swap the corresponding elements in $p_A$ and $p_B$. In the end, $t_A$ and $t_B$ are sorted and $p_A$ and $p_B$ contain the corresponding mapping of input to output positions. Finally, we load the two input buckets entirely into the 16 available AVX2 registers (which provide precisely the required amount of private memory) and obliviously write their elements to the output buckets according to $p_A$ and $p_B$.

**Instantiation** In our implementation, the ORAM’s first level holds 1024 cache-lines, because for smaller array sizes scanning performs better. All other levels use buckets of size $c = 4$ and the number of tables is set to $\lceil \log \log N \rceil$ of the capacity of the level. We empirically verified parameters $k = c = 4$ for tables of size $2^{11}$ and $2^{15}$ when re-throwing elements $2^{23}$ and $2^{20}$ times, respectively. In both cases, we have not observed spills into the third table (in any of the levels), hence, the last two tables of every level were always empty.

7.2 Circuit ORAM

For comparison, we implemented a recursive version of Circuit ORAM\footnote{https://github.com/wangxiao1254/oram_simulator} with a deterministic eviction strategy. Circuit ORAM proposes a novel protocol to implement the eviction of tree-based ORAMs using a constant amount of private memory. This makes Circuit ORAM the state-of-the-art tree-based ORAM protocol with optimal circuit complexity for $O(1)$ client memory. Similar to other tree-based schemes Circuit ORAM stores elements in a binary tree and maintains a mapping between element indices and their position in the tree in a recursive position map. We refer the reader to [51] for the details of Circuit ORAM.

We implemented Circuit ORAM from scratch using our library of primitives, as existing implementations were built for simulation purposes and only concern with high-level obliviousness. This makes these implementations not fully oblivious and vulnerable to low-level side-channel attacks based on control flow. Similar to Pyramid ORAM, we use a cache line as a block for Circuit ORAM. This favors Circuit ORAM since up to 14 indices can be packed in each block, requiring only $\log_{14} N$ levels of recursion to store the position map. Hence, the bandwidth cost of our implementation of Circuit ORAM is only 64 bytes $\times O(\log_{14} N \log N)$. Similar to Pyramid ORAM, scanning is faster than accessing binary trees for small datasets. To this end, we stop the recursion when the position map is reduced to 512 indices or less and store these elements in an array that is scanned on every access. We chose 512 since it gave the best performance.

**Instantiation** We use a bucket size of 3 with two stash parameters: 12 and 20, that is every level of recursion uses the same stash size since these parameters are independent of the size of a tree, and a bucket size of 4 and stash size 12. We note that buckets of size 2 were not sufficient in our experiments to avoid overflows. In the figures we denote the three settings with Circuit-3-12, Circuit-3-20 and Circuit-4-12.

8 Evaluation

We experimentally evaluate the performance of Pyramid ORAM in comparison to Circuit ORAM and naive scanning when accessing arrays of different sizes, when used
in two machine learning applications, and when run inside Intel SGX enclaves. The key insight is that, for the given datasets, Pyramid ORAM considerably outperforms Circuit ORAM (orders of magnitude) in terms of online access times and is still several $x$ faster for 99.9% of accesses when rebuilds are included in the timings.

All experiments were performed on SGX-enabled Intel Core i7-6700 CPUs running Windows Server 2016. All code was compiled with the Microsoft C/C++ compiler v19.00 with `/O2`. In cases where we ran code inside SGX enclaves, it was linked against a custom SGX software stack. We report all measurements in CPU cycles.

**ORAM vs. Scanning** Figure 3 shows the break-even point of Pyramid and Circuit ORAM compared to the basic approach—called Naive in the following—of scanning through all elements of an array for each access. Our implementation of Naive is optimized using AVX2 SIMD operations and is the same as the one we use to scan the first level in our Pyramid implementation. Hence, for up to 1024 elements (the size of Pyramid’s first level), Naive and Pyramid perform identically. We also set Circuit ORAM to use scanning for small arrays. Pyramid and Circuit start improving over Naive at 4096 (229 KB) and 8192 elements (459 KB), respectively.

**Pyramid vs. Circuit** Figures 6 and 7 depict the performance of Pyramid and Circuit when sequentially accessing arrays of 1-byte and 56-byte elements. Pyramid dominates Circuit for datasets of size up to 14 MB. Though Circuit ORAM should perform worse for smaller elements than larger ones, it is not evident in the 1-byte and 56-byte experiments since our implementation uses (machine-optimized) large block sizes favoring Circuit ORAM.

**Intel SGX** We run several benchmarks inside of an SGX enclave. Due to memory restrictions of around 100 MB in the first generation of Intel SGX, we evaluate the algorithms on small datasets. Since both Circuit and Pyramid have space overhead, the actual data that can be loaded in an ORAM is at most 12 MB. Compared to the results in the previous section, SGX does not add any noticeable overhead. Figures 4 and 5 show the results for arrays with 1-byte and 56-byte elements.

**Variation of access overhead** As noted earlier, the overhead of accessing a hierarchical ORAM depends on the number of previous accesses. For example, an access that causes a rebuild of the last level is much slower than the subsequent access (for which no rebuild happens). Hence, Pyramid ORAM exhibits different online and amortized cost as noted in Table 1. To investigate this effect, we accessed Pyramid and Circuit $3N$ times and measured the overhead of every request for $N = 2^{20}$ and $N = 2^{25}$. For both schemes the measured time includes the time to access the element and to prepare the data structure for the next request, that is, it includes rebuild and eviction for Pyramid ORAM and Circuit ORAM, respectively. Figures 8 and 9 show the corresponding overhead split across 99.9% of the fastest among $3N$ accesses. As can be seen, the fraction of expensive accesses in Pyramid is very small since large rebuilds are infrequent. For $N = 2^{20}$, almost 40% of all Pyramid accesses are answered on average in 16.6K CPU cycles, almost 10× faster than Circuit. Except for the few accesses that cause a rebuild, Pyramid answers 99.9% of all accesses on average in 26.8K CPU cycles, compared to 178.7K cycles for Circuit. A similar trend can be observed for the larger array in Figure 9 for 35% of all accesses, Pyramid
Figure 5: Average CPU cycle overhead (normalized to Pyramid ORAM) of obliviously reading 56-byte elements from arrays of size 4096 to 32768 (229KB–1.8MB) from an Intel SGX enclave.

Figure 6: Average CPU cycle overhead (normalized to Pyramid ORAM) of obliviously reading 1-byte elements from arrays of size $2^{17}$ to $2^{24}$ (131KB–16MB).

Figure 7: Average CPU cycle overhead (normalized to Pyramid ORAM) of obliviously reading 56-byte elements from arrays of size $2^{11}$ to $2^{24}$ (114KB–939MB).

Figure 8: Cumulative distribution of access times in CPU cycles when requesting elements from an array of $2^{20}$ 56-byte elements (58 MB) where 99.9% of fastest accesses are plotted. For example, Pyramid ORAM (Circuit ORAM) answers 50% and 99% of all requests under 23K (159K) and 82K (340K) CPU cycles.

is at least $10 \times$ faster than Circuit and still $8 \times$ faster for 99% of all accesses, while, on average, Circuit performs better. We stress that for many applications the online cost is the significant characteristic, as expensive rebuilds can be anticipated and performed during idle times or, to some extent, can be performed concurrently.

Sparse Vectors Dot product between a vector and a sparse vector is a common operation in machine-learning algorithms. For example, in linear SVM [23], a model contains a weight for every feature while a sample record may contain only some of the features. Since relatively few features may be present in a sample record, records are stored as (feature index, value) pairs. The dot product is computed by performing random accesses into the model based on the feature indices, in turn, revealing the identities of the features present in the record.

We use ORAM to store and access the model vector of 4-byte floating feature values in order to protect record content during classification. To measure the overhead, we choose three datasets with sparse sample records from LIBSVM Tool[4] news20.binary (1,355,191 features and 19,996 records), rcv1-train (47,236 features and 20,242 records) and rcv1-test (47,236 features and 677,399 records). These datasets are common datasets for text classification where a feature is a binary term signifying the presence of a particular trigram. As a

[4]https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/
result the datasets are sparse (e.g., if the first dataset were to be stored densely, only 0.0336% of its values would be non-zero). In Figure 10 we compare the overhead of the ORAM schemes over insecure baseline when computing dot product on each dataset. The insecure baseline measures sparse vector multiplication without side-channel protection. As expected, Pyramid performs better than Circuit, as model sizes are 5.4MB for news20.binary and 188KB for rcv1. Comparing to the baseline, the average ORAM overhead is four and three orders of magnitude, respectively.

**Decision Trees** Tree traversal is also a common task in machine learning where a model is a set of decision trees (a forest) and a classification on a sample record is done by traversing the trees according to record’s features. Hence, accesses to the tree can reveal information about sample records. One can use an ORAM to protect such data-dependent accesses either by placing the whole tree in an ORAM or by placing each layer of the tree in a separate ORAM. The latter approach suits balanced trees, however, for unbalanced trees it reveals the height of the tree and the number of nodes at each layer.

Our experiments use a pre-trained forest for a large public dataset[^5]. The forest consists of 32 unbalanced trees with 30K to 33K nodes each. A node comprises four 8-byte values and hence fits into a single 56-byte block. We load each tree into a separate ORAM and evaluate 290,506 sample records; the performance results are given in Figure 11. Pyramid’s low online cost shows again: Pyramid and Circuit incur three orders of magnitude overhead on average, while 99% of all classifications on average have only two orders of magnitude overhead for Pyramid.

In summary, Pyramid ORAM can give orders of magnitude lower access latencies than Circuit ORAM. In particular, Pyramid’s online access latencies are low. This makes Pyramid the better choice for many applications in practice.

### 9 Related work

**Oblivious RAM** The hierarchical constructions were the first instantiations proposed for oblivious RAM [14]. Since then, many variations have been proposed which alter hash tables, shuffling techniques, or assumptions on the size of private memory [15][17][40][53]. The hierarchical construction by Kushilevitz *et al.* [25] has the

[^5]: The Covertype dataset is available from the UCI Machine Learning Repository: [https://archive.ics.uci.edu/ml/datasets/covertype](https://archive.ics.uci.edu/ml/datasets/covertype) (accessed 18/05/2017)
best asymptotical performance among ORAMs for constant private memory size. It uses the oblivious cuckoo hashing scheme proposed by Goodrich and Mitzenmacher [15]. In particular, the scheme relies on the AKS sorting network [2], which is performed $O(\log N)$ times for every level in the hierarchy. The AKS network, though an optimal sorting network, in simplified form has a high hidden constant of 6,100 [39]. Moreover, until the hierarchy reaches the level that can hold at least $(\log N)^7$ elements, every level is implemented using regular hash tables with buckets of size $\log N$. Thus, AKS only becomes practical for large data sizes.

Tree-based ORAM schemes, first proposed by Shi et al. [10], are a fundamentally different paradigm. There have been a number of follow-up constructions [26, 28, 50] that improve on the performance of ORAM schemes. Most of these works focus on reducing bandwidth overhead, often relying on non-constant private memory and elements of size $\Omega(\log N)$ or $\Omega((\log N)^2)$. We note that such schemes can be modified to rely on constant private memory by storing its content in an encrypted form externally and accessing it also obliviously (e.g., a scanning for every access). However, this transformation is often expensive. For example, it increases the asymptotic overhead of Path ORAM, because stash operations now require using oblivious sorting [51, 52].

ORAMs have long held the promise to serve as building blocks for multi-party computation (MPC). Hence some recent works focus on developing protocols with small circuit complexity [52, 55], including the Circuit ORAM scheme [51] discussed in previous sections. Some of the schemes that try to decrease circuit size also have small private memory requirements, since any “interesting” operation on private memory (e.g., sorting) increases the size. However, not all ORAM designs that target MPC can be used in our client-server model. For example, the work by Zahur et al. [55] optimizes a square-root ORAM [14] assuming that each party can store a permutation of the elements.

**Memory Side-channel Protection for Hardware**

Rane et al. [41] investigated compiler-based techniques for removing secret-dependent accesses in the code running on commodity hardware. Experimentally, they showed that Path ORAM is not suitable in this setting due to its requirements on large private memory, showing that naive scanning performs better for small datasets.

Ring ORAM [28] optimizes parameters of Path ORAM and uses an XOR technique on the server to decrease the bandwidth cost. As a case study, Ren et al. estimate the performance of Ring ORAM if used on secure processors assuming 64-byte element size. The design assumes non-constant size on-chip memory that can fit the stash, a path of the tree (logarithmic in $N$) and the position map at a certain recursion level. Ascend [11] is a design for a secure processor that accesses code and data through Path ORAM. As a result, Ascend relies on on-chip private memory that is large enough to store the stash and a path internally (i.e., $O(\log N)$).

PHANTOM [50] is a processor design with an oblivious memory controller that also relies on Path ORAM, storing the stash in trusted memory and using 1KB–4KB elements. Fletcher et al. [12] improve on this using smaller elements, but they also assume large on-chip private memory. Other hardware designs optimized for Path ORAM exist [13, 20, 43]. Sanctum [9] is a secure processor design related to Intel SGX. Other than SGX, it provides private cache partitions for enclaves, stores enclave pages in enclaves, and dispatches corresponding page faults and other exceptions directly to in-enclave handlers. This largely prevent malicious software (including the OS) from inferring enclave memory access patterns. However, it does not provide additional protection against a hardware attacker.

Recent work also addresses SGX’s side channel problem in software: Shih et al. [48] proposed executing sensitive code within Intel TSX transactions in order to suppress and detect leaky page faults. Gruss et al. [19] preload sensitive code and data within TSX transactions in order to prevent leaky cache misses. Ohrimenko et al. [36] manually protect a set of machine learning algorithms against memory-based side-channel attacks, applying high-level algorithmic changes and low-level oblivious primitives. DR.SGX [6] aims to protect enclaves against cache side-channel attacks by frequently re-randomizing data locations (except for small stack-based data) at cache-line granularity. For this, DR.SGX augments a program’s memory accesses at compile time. DR.SGX incurs a reported mean overhead of 3.39x–9.33x, depending on re-randomization frequency. DR.SGX does not provide strong guarantees; it can be seen as an ad-hoc ORAM construction that may induce enough noise to protect against certain software attacks. ZeroTrace [45] is an oblivious “memory controller” for enclaves. It implements Path ORAM and Circuit ORAM to obliviously access data stored in untrusted memory and on disk; only the client’s position map and stash are kept in enclave memory and are accessed obliviously using scanning and conditional x86 instructions.

**Routing and Hash Tables**

The probabilistic routing network in Section 4 belongs to the class of unbuffered delta networks [38]. The functionality of our network is different as instead of dropping the elements that could not be routed, we label them as spilled (i.e., tagged false) and carry them to the output. Probably the closest to ours, is the network based on switches with multiple links studied by Kumar [24]. The network assumes that
each switch of the first stage (input buckets in our case) receives one packet, which is not the case in our work where buckets may contain up to $c$ elements. Moreover, the corresponding analysis, similar to other analyses of delta networks, focuses on approximating the throughput of the network, while we are interested in the upper-bound of the number on elements that are not routed.

The Zigzag hash table can be seen as a combination of the load balancing strategy called Threshold$(c)$ [1] and Multilevel Adaptive Hashing [8]. In the former, elements spilled from a table with buckets of size $c$ are re-thrown into the same table, discarding the elements from the previous throws. In Multilevel Adaptive Hashing, $k$ tables of different sizes with buckets of size 1 are used to store elements. The insertion procedure is the same as that for non-oblivious ZHT, that is elements that collide in the first hash table are re-inserted into the next table. Different from ZHTs, multilevel hashing non-obliviously rebuilds its tables when the $k$th table overflows.

10 Conclusions

We presented Pyramid ORAM, a novel ORAM design that targets modern SGX processors. Rigorous analysis of Pyramid ORAM, as well as an evaluation of an optimized implementation for x64 Skylake CPUs, shows that Pyramid ORAM provides strong guarantees and practical performance while outperforming previous ORAM designs in this setting either in asymptotical online complexity or in practice.

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Figure 12: Oblivious Zigzag Hash Table Setup from Section 5.2. During the initial throw elements are allocated using random hash functions. In phase 1 elements in $H_1$ are routed with probabilistic routing network using hash function $h_1$. Element 1 could not be routed to its bucket and, hence, is greyed out (tagged false by the PRN). Hence, it is thrown into the next table. In the next phases elements in $H_2$ and $H_3$ are routed according to $h_2$ and $h_3$, correspondingly.

Appendix

A Additional lemmas

Lemma 6. When throwing $m$ elements in $n$ buckets, each with capacity $c$, the probability that a bucket receives $c$ or more elements is at most \( \left( \frac{m^c}{c} \right) \frac{1}{n^c} \).

Proof. The probability that a bucket gets assigned $c$ or more elements is

\[
\sum_{i=c}^{m} \frac{m}{i} \frac{1}{n^i} \left( \frac{1}{n} \right)^{m-i} \]

\[
= \left( \frac{m}{c} \right) \frac{1}{n^c} \sum_{i=c}^{m} \frac{c!}{(m-i)! n^{m-c}} \left( \frac{1}{n} \right)^{m-i}
\]

\[
= \left( \frac{m}{c} \right) \frac{1}{n^c} \sum_{i=0}^{m-c} \frac{c!}{(c+i)! (m-c-i)!} \frac{1}{n^c} \left( \frac{1}{n} \right)^{m-c-i}
\]

To prove the lemma, we show that the sum expression is at most 1. Note that as long as the first component of the sum is at most \( \left( \frac{m^c}{c} \right) \frac{1}{n^c} \), we can use the binomial formula:

\[
\sum_{i=0}^{m-c} \left( \frac{m-c}{i} \right) \frac{1}{n^i} \left( \frac{1}{n} \right)^{m-c-i} = 1
\]

We now show that \( \left( \frac{m^c}{c} \right) \frac{1}{n^c} \leq \frac{c!}{(c+i)! (m-c-i)!} \) for $i \geq 0$ and $c \geq 1$:

\[
\frac{(m-c)!}{i!(m-c-i)!} \leq \frac{c!}{(c+i)! (m-c-i)!} \]

\[
\frac{1}{i!} \leq \frac{c!}{(c+i)!} \]

\[
= \frac{1}{(c+1) \ldots (c+i)}
\]

Lemma 7. The average number of elements that do not get assigned to a bucket during the throw of $m$ elements into $n$ buckets of size $c$ is at most \( \frac{1}{c} \frac{1}{n^c} \).

Proof. Let $X_j$ be a random variable that is 1 if there is a collision on the $j$th inserted element, and 0 otherwise. For the first $c+1$ elements, $\text{Pr}[X_1] = \text{Pr}[X_2] = \ldots = \text{Pr}[X_c] = 0$. For $c+1 \leq j \leq n$, we need to estimate the probability that $j$th element is inserted in a bucket with at least $c$ elements. The probability of a given bucket having $c$ or more elements after $j-1$ elements have been inserted is given in Lemma 6. Hence,

\[
\text{Pr}[X_j] \leq \left( \frac{j-1}{c} \right) \frac{1}{n^c}
\]
Note that $X_j$ are independent. Then, expected number of collisions is
\[
\mu = E \left[ \sum_{j=1}^{m} X_j \right] \leq \sum_{j=c+1}^{m} \Pr[X_j] \leq \frac{m-1}{n^c} \frac{1}{n^c} + \frac{1}{n^c} \sum_{j=c}^{m-1} \frac{1}{n^c}
\]

Using the fact that $\sum_{j=c}^{m-1} \binom{m}{c} = \binom{m}{c+1}$:
\[
\mu \leq \frac{1}{n^c} \left( \frac{m}{c+1} \right)
\]

\[\square\]

**Lemma 8.** The number of elements that are spilled during the $i$th stage of the routing network of size $n$ on input of size $m \leq n$ is at most the number of elements that do not fit into $H_1$ of a ZHT when throwing $m$ elements.

**Proof.** We are interested in counting the number of elements whose tag changes from true to false during the $i$th stage of the probabilistic routing network and refer to such elements as spilled during the $i$th stage.

Let $h_{\text{start}}$ be the hash function that was used to distribute elements during the throw of $n$ elements into the input table of the network (i.e., the elements that did find a spot in the table appear in buckets according to $h_{\text{start}}$) and $h_{\text{end}}$ be the hash function of the network. Let $||$ denote a concatenation of two bit strings.

Consider stage $i$ of the network, $1 \leq i \leq \log n$. After stage $i$, element $e$ is assigned to a bucket $a||b$ where $a$ are the first $i$ bits of $h_{\text{start}}(e)$ and $b$ are the first $\log n - i$ bits of $h_{\text{end}}(e)$. Now consider, a throw phase of $m$ elements into a table of $n$ buckets using the same hash function. Let us compare the number of elements assigned to a particular bucket in each case. We argue that the number in the former case is either the same as in the latter case or smaller. Note that the elements that can be assigned to a bucket is the same in both cases as elements are distributed across buckets using the same hash function. However, some of the elements of the PRN may have been lost in earlier stages (those that arrive to the $i$th stage with the false tag) and have already been counted towards the spill of an earlier stage. Since elements tagged false are disregarded by the network during routing, $i$th stage may spill the same number of elements as the hash table or less since there are less “live” (tagged true) elements that reach their buckets. \[\square\]

**Lemma 9.** Consider the following allocation of $n$ elements into tables, each with $n$ buckets of size $c = \log \log n$. The elements are allocated uniformly at random in the first table. The elements that do not fit into the first table are allocated into the second table.

The number of elements that arrive into the second table is at most $n/(2e)$.

**Proof.** In Lemma 7 we showed that average number of outgoing elements at level $i$ is $\mu_i = \frac{m}{n} \binom{m}{c+1}$. Hence, with $i_1 = n$
\[
\mu_1 \leq \frac{n^{c+1}}{n^{c+1}!} \leq \frac{n}{(c+1)!} \leq \frac{n}{2e \log n}
\]

where we rely on $(c+1)! \geq 2e \log n$ (see below). If $n/(2e \log n)$ is sufficiently large (e.g., $n/(2e \log n) \geq 3s$ where $s$ is the security parameter), we can use Chernoff bounds to show that with probability $\exp(-\mu_1/3)$ the number of spilled elements from the first level is no more than $2\mu_1$. Hence, in total the number of elements that spill is at most:
\[
\frac{n}{e \log n} \leq \frac{n}{2e}
\]

If $n$ is of moderate size (e.g., $n \geq 2^{10}$), we can use another form of Chernoff bounds [33, Chapter 4] to show that with probability $2^{-k}$ the number of spilled elements from the first level is no more than $R \geq 6\mu_1$, where $6\mu_1 \geq s$ and $6\mu_1 \leq n/2e$.

Finally we show that for any $a \geq 10$, $(\log a + 1)! \geq 2ea$:
\[
(\log a + 1)! \geq \sqrt{2\pi(\log a + 1)} \frac{e^{\log a + 1}}{\log a + 1} \geq \frac{\sqrt{2\pi(\log a + 1)}(\log a + 1)^{\log a + 1}}{3^{\log a + 1}} \geq 2ea
\]

\[\square\]

**Lemma 10.** Consider the process in Lemma 9 but after the first throw, the first table is routed through the routing network from Section 4 and elements spilled from the network are also thrown into the second table. The elements of the second table are routed as well and spilled elements are thrown into the third table, and so on. The number of elements that arrive into the third table is at most $n/(2e)$.

**Proof.** From Lemma 8 we know that the total overflow from throwing the elements and routing them through the network is at most $\log n$ times the overflow from the original throw. Consider the proof of Lemma 9. The second table receives at most
\[
2n \log n \frac{n}{2e \log n} = \frac{n}{e}
\]
number of elements. Then the average overflow according to Lemma 7 is
\[\frac{1}{n^e} \left( \frac{n}{e+1} \right) \leq \frac{n^{e+1}}{2e\log n} = \frac{n}{2e \log ne^{e+2}} \leq \frac{n}{4e \log n}\]

We can again use Chernoff bound to show that the number of elements that arrive into the next level will be at most \(2\log n\) factor away from the above expression. Hence, the number of elements that arrive into the third table is
\[\frac{2n \log n}{4e \log n} = \frac{n}{2e}\]

Lemma 11. Consider the following allocation of \(m \leq n/(2e)\) elements into \(k\) tables, each with \(n\) buckets of size \(c = \log\log n\). The elements are allocated uniformly at random in the first table. The elements that do not fit into the first table are allocated into the second table and so on, until there are no elements left. Then, \(k = O(\log\log n)\) is sufficient to allocate all elements.

Proof. We analyze the process using Poisson random variables as an approximation of the balls-and-bins approximation process [33, Chapter 5]. Recall that as long as event’s probability is monotonically decreasing or increasing in the number of elements, the event that happens with probability \(\epsilon\) in the Poisson case has probability at most \(2\epsilon\) in the exact case. Let \(Z_i\) be a Poisson random variable that measures the number of elements in a bucket after a random throw. Let \(\alpha\) be the mean of \(Z_i\). Recall that
\[\Pr(Z_i \geq c) \leq \frac{e^{-\alpha}(e\alpha)^c}{c^c}\]

Hence, the number of \(n\) buckets that will have at least \(c\) elements is \(ne^{-\alpha}(e\alpha)^c/c^c\). It is known, that with very high probability no bucket has more than \(O(\log n)\) elements, hence, the number of overflow elements is \((n \log n)e^{-\alpha}(e\alpha)^c/c^c\). We now show that as long as \(\alpha \leq 1/(2e)\), we require \(O(\log\log n)\) rounds until no overflow occurs. After the first throw, the overflow is
\[(n \log n)\frac{(e\alpha)^{c+1}}{(c+1)^{c+1}} \leq \frac{n}{2c+1 \epsilon \log n}\]

where with \(c = \lfloor \log\log n \rfloor\), \((c+1)^{c+1} \leq 1/e\) (see below). Hence, in the next round:
\[(n \log n)\frac{(e/(2c+1 \epsilon \log n))^{c+1}}{(c+1)^{c+1}} \leq \frac{n}{2(c+1)^2(\log n)^{c+2} e}\]

Then for the \(i\)th round the overflow is:
\[\frac{n}{2^{(c+1)^2(\log n)^{c+1} e}}\]

Hence, after at most \(O(\log\log n)\) rounds the above expression is less than 1.

We are left to show that as long as \(a \geq 8\), \(a/(\log a + 1)^{\log a + 1} \leq 1/e\). In the following lemma we require a similar result but for \(a^2\) in the numerator. Hence, instead we show that \(a^2/(\log a + 1)^{a(\log a + 1)} \leq 1/e:\)
\[\frac{a^2}{(\log a + 1)^{a(\log a + 1)}} \leq \frac{1}{2log(\log a + 1)(\log a + 1) - 2log a} \leq \frac{1}{\log a + 1} \leq \frac{1}{4} \leq \frac{1}{e}\]

Lemma 12. Consider the process in Lemma 11 but after the first throw, the first table is routed through the routing network from Section 4 and elements spilled from the network are also thrown into the second table. The elements of the second table are routed as well and spilled elements are thrown into the third table, and so on. Then, \(O(\log\log n)\) rounds are still sufficient to avoid overflow in the last round.

Proof. From Lemma 8 we know that the total overflow from throwing the elements and routing them through the network is at most \(\log n\) times the overflow from the original throw. Consider the proof of Lemma 11. In Equation 1 we can replace \(n\) with \(n \log n\) to account for the buckets in each round of the network. This would result in the spill of
\[\frac{n}{2c+1 \epsilon \log n}\]

in the first round and
\[\frac{n}{2^{(c+1)^2 \epsilon}}\]
in the \(i\)th round. Hence, \(O(\log\log n)\) rounds are still sufficient to fit all the elements.