On total edge irregularity strength of some cactus chain graphs with pendant vertices

I Rosyida, D Indriati

1 Department of Mathematics, Universitas Negeri Semarang, Semarang, Indonesia
2 Department of Mathematics, Universitas Sebelas Maret, Surakarta, Indonesia

E-mail: iisisnaini@gmail.com; diari_indri@yahoo.co.id

Abstract. Let $G(V, E)$ be a graph. A function $f$ from $V(G) \cup E(G)$ to the set $\{1, 2, \ldots, k\}$ is called an edge irregular total $k$-labeling of $G$ if the weights of any two different edges $ux$ and $vy$ in $E(G)$ satisfy $w_f(ux) \neq w_f(vy)$ where the weight $w_f(uv)$ is equal to the sum of label of $u$, label of $v$ and label of edge $uv$. The total edge irregularity strength of the graph $G$, denoted by $tes(G)$, is the minimum number $k$ for which $G$ has an edge irregular total $k$-labeling. In this paper, we investigate the exact value of $tes$ of triangular cactus chain graph and para square cactus chain graph. We get the total edge irregularity strength of triangular cactus chain with length $r$ and $r + 1$ pendant vertices as follows: $tes(TC_{r+1}) = \lceil \frac{4r+3}{3} \rceil$. Further, the total edge irregularity strength of para square cactus chain graph with length $r$ and $r$ pendant vertices is $tes(Q_r) = \lceil \frac{5r+2}{3} \rceil$.

1. Introduction

Given a simple graph $G = (V, E)$. A labeling of $G$ is a mapping from a set of graph elements into a set of integers, that are called labels. If the domain is a union of vertex and edge sets, then the labeling is called total labeling (Wallis [17]; Marr and Wallis [11]).

Bača et al. [3] introduced an edge irregular total $k$-labeling of $G$, that is a function $f : V \cup E \rightarrow \{1, 2, \ldots, k\}$ such that the weights $w_f(uv) \neq w_f(xy)$ for every two different edges $uv$ and $xy$ in $E(G)$, where $w_f(uv) = f(u) + f(v) + f(uv)$ and the minimum number $k$ used in the labeling $f$ is called a total edge irregularity strength of $G$, denoted by $tes(G)$.

Bača et al. [3] found the lower bound of $tes$ for any graph $G = (V, E)$:

$$\left\lceil \frac{|E(G)| + 2}{3} \right\rceil \leq tes(G) \leq |E|.$$  \hspace{1cm} (1)

Based on the lower bound (1), Ivančo and Jendrol [9] gave a conjecture for $tes$ of any graph $G$ as follows:

$$tes(G) = \max \left\{ \left\lceil \frac{|E(G)| + 2}{3} \right\rceil, \left\lceil \frac{\Delta(G) + 1}{2} \right\rceil \right\}.$$  \hspace{1cm} (2)

Some researchers have tried to prove the conjecture 2 for some special graphs, such as: Ivančo dan Jendrol [9] proved the conjecture for any tree; Jendrol, Miskuf, and Sotak [10] proved the conjecture for complete graph and complete bipartite graph; also Miskuf and Jendrol [12] found a total edge irregularity strength of grid graph. Further, Indriati et al. [8] investigated $tes$ of...
generalized web graph and generalized helm graph [7]. Furthermore, Nurdin et al. [13] obtained tes of the corona product of paths with some graphs and Nurdin et al. [14] also investigated total edge irregularity strength of subdivision of star. Meanwhile, Siddiqui [16] obtained tes of subdivision of star. Moreover, Ahmad et al. [1] verified the conjecture for strong product of two paths, and others. For more research on irregularity strength of graphs, we may find in Gallian [5].

This paper was motivated by the result from Imran et al. [6] that gave an edge irregularity strength (es) of cycle chain graphs by means the concept of vertex labeling. This work was also motivated by the result from Ahmad et al. [2] which proposed an edge irregularity strength (es) of some chain graphs. Different to the result in [6] and [2], we investigate a total edge irregularity strength (tes) of triangular cactus chain and para square cactus chain graphs based on the concept of total labeling. Especially, we investigate the total edge irregularity strength of triangular cactus chain graph TC\(_r\) and para square cactus chain graph Q\(_r\) with pendant vertices.

2. Main Results

In this paper, we get some results on the total edge irregularity strength of triangular cactus chain graph and para-squares cactus chain graphs with pendant vertices.

2.1. Total edge irregularity strength of triangular cactus chain graph with pendant vertices

In this section, we discuss an exact value of the total edge irregularity strength of the chain triangular cactus graph with pendant vertices as presented in Theorem 1. The concept of triangular cactus chain graph is cited from [4] and [15].

**Definition 1** A graph is said to be a cactus graph if it is a connected graph such that each edge is only located in one cycle. Therefore, a cactus graph contains some blocks such that each block is an edge or a cycle. The cactus is called \(m\)-uniform if all blocks have the same size \(m\). A graph that is a 3-uniform cactus is called a triangular cactus. Two or more triangles share a common vertex which is called a cut-vertex. In a triangular cactus graph, if each triangle has cut vertices at most two and each of two triangles has one common cut vertex then it is called a chain of triangular cactus graph. The length of the chain is indicated by the number of triangle in the chain. The notation \(TC_r\) stands for a triangular cactus chain graph with length \(r\).

It is obvious that the triangular cactus chain graph \(TC_r\) has \(2r+1\) vertices, \(3r\) edges, and it consists of \(r\) blocks \(B_1, B_2, \ldots, B_r\) where each block is a cycle \(C_3\). In this paper, we denote a triangular cactus chain graph with length \(r\) and \(r+1\) pendant vertices by \(TC_r^{r+1}\) where it has \(4r+1\) edges.

**Theorem 1** Given a triangular cactus chain graph \(TC_r^{r+1}\) with length \(r\) and \(r+1\) pendant vertices \((r \geq 3)\). The total edge irregularity strength of \(TC_r^{r+1}\) is

\[
\text{tes}(TC_r^{r+1}) = \left\lceil \frac{4r+3}{3} \right\rceil.
\]

**Proof.** The triangular cactus chain graph \(TC_r^{r+1}\) consists of \(4r+1\) edges. Let \(u_i, v_i, v_{i+1}\) be vertices located on each triangle for \(i = 1, 2, 3, \ldots, r\). Let \(y_i\) be pendant vertices connected to the vertices \(v_i\). The bound for tes of the triangular cactus chain \(TC_r^{r+1}\) is as follows [3]:

\[
\left\lfloor \frac{4r+3}{3} \right\rfloor \leq \text{tes}(G) \leq (4r+1).
\]

Further, we prove the upper bound \(\text{tes}(TC_r^{r+1}) \leq \left\lceil \frac{4r+3}{3} \right\rceil\).
We construct a total edge irregular $k$-labeling $f : V \cup E \rightarrow \{1, 2, \ldots, k\}$ where the value $k$ is $\lceil \frac{4r+3}{3} \rceil$. Let $k = \lceil \frac{4r+3}{3} \mod(r) \rceil$, $k_j = \lceil \frac{4r_j+3}{3} \rceil \mod(r_j)$ where $j = 1, 2, 3$ and $r_1 < r_2 < r_3$. The labels of vertices $u_i, v_i$ and $y_i$ for $i = 1, 2, \ldots, r$ are as follows:

$$f(u_i) = \begin{cases} 
  2, & \text{if } i = 1, \\
  3, & \text{if } i = 2, \\
  2i - 1, & \text{if } 3 \leq i \leq k, \\
  i + k, & \text{if } k + 1 \leq i \leq r - 1, \\
  \lceil \frac{4r+3}{3} \rceil, & \text{if } i = r,
\end{cases}$$

$$f(v_i) = \begin{cases} 
  i + 1, & \text{if } 2 \leq i \leq 2k_1 - 2, \\
  i + 1, & \text{if } 2 \leq i \leq 2k_2 - 1, \\
  i + 1, & \text{if } 2 \leq i \leq 2k_3, \\
  r - k + 6, & \text{if } i = r - k + 4, \\
  r - k + 8, & \text{if } i = r - k + 5, \\
  r + k - 2, & \text{if } i = r, \\
  \lceil \frac{4r+3}{3} \rceil, & \text{if } i = r + 1, \\
  f(y_i), & \text{if } 1 \leq i \leq r + 1.
\end{cases}$$

Further, the labels of edges $u_iv_i$ are defined as follows:

$$f(u_iv_i) = f(u_iv_{i+1}) = \begin{cases} 
  1, & \text{if } i = 1, \\
  i, & \text{if } 2 \leq i \leq k + 1, k \geq 2.
\end{cases}$$

If $\lceil \frac{4r_1+3}{3} \rceil \mod(r_1) = k_1$ and $k_1 \geq 2$, then

$$f(u_iv_i) = f(u_iv_{i+1}) = \begin{cases} 
  i + 1, & \text{if } i = k_1 + 2, \\
  i + 2, & \text{if } i = k_1 + 3, \\
  i + (k_1 - 4), & \text{if } i = 2k_1 - 3, \\
  i + (k_1 - 3), & \text{if } 2k_1 - 2 \leq i \leq r.
\end{cases}$$

If $\lceil \frac{4r_2+3}{3} \rceil \mod(r_2) = k_2$ with $k_2 \geq 2$ and $r_2 > r_1$, then

$$f(u_iv_i) = f(u_iv_{i+1}) = \begin{cases} 
  i + 1, & \text{if } i = k_2 + 2, \\
  i + 2, & \text{if } i = k_2 + 3, \\
  i + (k - 4), & \text{if } i = 2k_2 - 3, \\
  i + (k - 3), & \text{if } i = 2k_2 - 2, \\
  i + (k_2 - 2), & \text{if } 2k_2 - 1 \leq i \leq r.
\end{cases}$$
If \( \left\lceil \frac{4r+3}{3} \right\rceil \mod(r_3) = k_3 \) with \( k_3 \geq 2 \) and \( r_3 > r_2 > r_1 \), then

\[
f(u_i v_i) = f(u_i v_{i+1}) = \begin{cases} 
  i + 1, & \text{if } i = k_3 + 2, \\
  i + 2, & \text{if } i = k_3 + 3, \\
  i + (k_3 - 4), & \text{if } i = 2k_3 - 3, \\
  i + (k_3 - 3), & \text{if } i = 2k_3 - 2, \\
  i + (k_3 - 2), & \text{if } i = 2k_3 - 1, \\
  i + (k_3 - 1), & \text{if } 2k_3 \leq i \leq r.
\end{cases}
\]

Furthermore, the labels of edges \( v_i v_{i+1} \) are defined as follows:

Let \( k = \left\lceil \frac{4r+3}{3} \right\rceil \mod(r) \). If \( 2 \leq k \leq 3 \), then

\[
f(v_i v_{i+1}) = \begin{cases} 
  2i - 1, & \text{if } 1 \leq i \leq r - (k - 2) \\
  f(v_{i-1} v_i), & \text{if } i = r - (k - 3).
\end{cases}
\]

Meanwhile if \( k \geq 4 \), then

\[
f(v_i v_{i+1}) = \begin{cases} 
  2i - 1, & \text{if } 1 \leq i \leq r - (k - 2) \\
  \left\lceil \frac{4r+3}{3} \right\rceil, & \text{if } r - (k - 3) \leq i \leq r.
\end{cases}
\]

The labels of edges \( v_i y_i \) are defined as follows:

(i) \( f(v_i y_i) = 2i - 3; 2 \leq i \leq r, \) for \( k = \left\lceil \frac{4r+3}{3} \right\rceil \mod(r) = 2 \)

(ii) \( f(v_i y_i) = \begin{cases} 
  2i - 3, & \text{if } 2 \leq i \leq r \\
  f(v_{i-1} y_{i-1}), & \text{if } i = r + 1
\end{cases} \) for \( k = 3 \),

(iii) \( f(v_i y_i) = \begin{cases} 
  2i - 3, & \text{if } 2 \leq i \leq r_1 \\
  f(v_{i-1} y_{i-1}), & \text{if } i = r_1 + 1
\end{cases} \) for \( k_1 = \left\lceil \frac{4r_1+3}{3} \right\rceil \mod(r_1) = 4 \).

(iv) \( f(v_i y_i) = \begin{cases} 
  2i - 3, & \text{if } 2 \leq i \leq r_j - 1 \\
  f(v_{i-1} y_{i-1}), & \text{if } r_j \leq i \leq r_j + 1,
\end{cases} \) for \( k_j = \left\lceil \frac{4r_j+3}{3} \right\rceil \mod(r_j) = 4, j = 2, 3 \) and \( r_1 < r_2 < r_3 \).

(v) \( f(v_i y_i) = \begin{cases} 
  2i - 3, & \text{if } 2 \leq i \leq r - (k - 3) \\
  f(v_{i-1} y_{i-1}), & \text{if } r - (k - 4) \leq i \leq r + 1
\end{cases} \) for \( k = \left\lceil \frac{4r+3}{3} \right\rceil \mod(r) \geq 5 \).

Moreover, we obtain the weights of edges under the labeling \( f \) as follows:

\[
W(v_1 y_1) = 3; W(u_1 v_1) = 4; W(v_1 v_2) = 5, \text{ and } W(u_1 v_2) = 6.
\]

\[
W(u_i v_i) = 4i, \text{ for } 1 \leq i \leq r.
\]
Let \( k = \left\lceil \frac{4r+3}{3} \right\rceil \mod(r) \).

(i) For \( k \geq 5 \):
\[
W(u_i v_{i+1}) = \begin{cases} 
4i + 1, & \text{if } 2 \leq i \leq r - (k - 2), \\
4i + 2, & \text{if } r - (k - 3) \leq i \leq r.
\end{cases}
\]

(ii) For \( k_j = \left\lceil \frac{4r+3}{3} \right\rceil \mod(r_j) = 4 \) and \( j = 1, 2, 3 \) where \( r_3 > r_2 > r_1 \):
\[
W(u_i v_{i+1}) = \begin{cases} 
4i + 1, & \text{if } 2 \leq i \leq r_1 - 1 \\
4i + 2, & \text{if } i = r_1 \\
4i + 1, & \text{if } 2 \leq i \leq r_j - 2, \quad j=2,3 \\
4i + 2, & \text{if } r_j - 1 \leq i \leq r_j, \quad j=2,3
\end{cases}
\]

(iii) For \( k = 3 \):
\[
W(u_i v_{i+1}) = \begin{cases} 
4i + 1, & \text{if } 2 \leq i \leq r - 1 \\
4i + 2, & \text{if } i = r.
\end{cases}
\]

(iv) for \( k = 2 \):
\[
W(u_i v_{i+1}) = 4i + 1; \quad 2 \leq i \leq r.
\]

Let \( \left\lceil \frac{4r+3}{3} \right\rceil \mod(r_j) = k_j \) with \( j = 1, 2, 3 \) and \( r_3 > r_2 > r_1 \). We get the weight of edges \( v_i v_{i+1} \) as follows:

(i) For \( k_j \geq 5 \):
\[
W(v_i v_{i+1}) = \begin{cases} 
4i + 2, & \text{if } 2 \leq i \leq r_j - (k_j - 2), \quad j=1,2,3 \\
4i + 1, & \text{if } r_j - (k_j - 3) \leq i \leq r_j \quad j=1,2,3.
\end{cases}
\]

(ii) For \( k_j = 4 \):
\[
W(v_i v_{i+1}) = \begin{cases} 
4i + 2, & \text{if } 2 \leq i \leq r_j - 1, \quad j=1, \\
4i + 1, & \text{if } i = r_j, \quad j=1, \\
4i + 2, & \text{if } 2 \leq i \leq r_j - 2, \quad j=2,3, \\
4i + 1, & \text{if } r_j - 1 \leq i \leq r_j, \quad j=2,3.
\end{cases}
\]

(iii) For \( k_j = 3 \):
\[
W(v_i v_{i+1}) = \begin{cases} 
4i + 2, & \text{if } 2 \leq i \leq r_j - 1, \quad j=1,2,3, \\
4i + 1, & \text{if } i = r_j, \quad j=1,2,3.
\end{cases}
\]

(iv) For \( k_j = 2 \): \( W(v_i v_{i+1}) = 4i + 2, \) if \( 2 \leq i \leq r_j; \) \( j=1,2,3. \)

Finally, we have the weights of edges \( v_j y_j \) as follows:
\[
W(v_j y_j) = \begin{cases} 
3, & \text{if } j = 1, \\
4j - 1, & \text{if } 2 \leq j \leq r + 1.
\end{cases}
\]

We can prove that all labels of vertices and edges are less then or equal to \( \left\lceil \frac{4r+3}{3} \right\rceil \) and the weights of edges under the labeling \( f \) are all different. Thus, the total edge irregularity strength of the chain triangular cactus graph is
\[\text{tes}(TC_r^{r+1}) = \left\lceil \frac{4r+3}{3} \right\rceil. \]
An example of the edge irregular total 12-labeling of $TC_8^9$ is given in Figure 1. A red colored number represents a weight of each edge.

![Figure 1](image_url)

**Figure 1.** The edge irregular total 12-labeling of $TC_8$ with 9 pendant vertices.

2.2. Total edge irregularity strength of para-squares cactus chain graph with pendant vertices

A para-squares cactus graph is a 4-uniform cactus. It is a graph where each of its block is a cycle $C_4$. Two or more square share a cut vertex. The para-squares cactus chain graph is a cactus graph such that each of two square has one common cut vertex. The length of the para-squares cactus chain is indicated by the number of square in the chain. The para-squares cactus chain graph of length $r$ is denoted as $Q_r$.

In this section, we discuss a total $k$-labeling of a para-squares cactus chain graph of length $r$ with $r$ pendant vertices, denoted as $Q_r^r$, and find an exact value of the total edge irregularity strength of $Q_r^r$ as presented in Theorem 2.

**Theorem 2** Given a para-squares cactus chain graph of length $r$ with $r$ pendant vertices $Q_r^r$ ($r \geq 2$). The total edge irregularity strength of $Q_r^r$ is

$$\text{tes}(Q_r^r) = \left\lceil \frac{5r + 2}{3} \right\rceil.$$ 

**Proof.** The para-squares cactus chain graph $Q_r^r$ consists of $r$ blocks $B_1, B_2, \ldots, B_r$ where each block is cycle $C_4$ and has $5r$ edges. Let $u_1, u_2, \ldots, u_r, v_1, v_2, \ldots, v_{r+1}$, and $x_1, x_2, \ldots, x_r$ be vertices of the para-squares cactus chain $Q_r^r$. Meanwhile, $y_1, y_2, \ldots, y_r$ are pendant vertices connected to the vertices $x_1, x_2, \ldots, x_r$, respectively.

It follows from Baca [3] that the bound for the total edge irregularity strength of the para-squares cactus chain graph with $r$ pendant vertices $Q_r^r$ is as follows:

$$\left\lceil \frac{5r + 2}{3} \right\rceil \leq \text{tes}(Q_r^r) \leq 5r.$$ 

Further, it is proved that the upper bound is $\text{tes}(Q_r^r) \leq \left\lceil \frac{5r + 2}{3} \right\rceil$. We construct a total edge irregular $k$-labeling: $f : V \cup E \to \{1, 2, \ldots, k\}$ where the value $k$ is $\left\lceil \frac{5r + 2}{3} \right\rceil$. 
The labels of the vertices $u_i$ are defined as follows:

$$f(u_i) = \begin{cases} 
2i & \text{if } 1 \leq i \leq r, \\
2i - j & \text{if } 1 \leq i \leq r - k, \quad r \geq 5; \quad k = \left\lceil \frac{5r+2}{3} \right\rceil \mod (r-k), \\
2i - j & \text{if } r - k < i \leq r, \quad j = i - (r-k). 
\end{cases}$$

Meanwhile, labeling for the vertices $v_i$ is the following:

$$f(v_i) = \begin{cases} 
i + 1 & 1 \leq i \leq 3 & \text{for } r \geq 2 \\
2i - 2 & 4 \leq i \leq 5 & \text{for } 2 \leq r \leq 4 \\
2i - 2 & 4 \leq i \leq r - (k - 1) & \text{for } r \geq 5,
\end{cases}$$

with $k = \left\lceil \frac{5r+2}{3} \right\rceil \mod (r-k)$. Further,

$$f(x_{r-k+j}) = \left\lceil \frac{5r+2}{3} \right\rceil - (k - j + 1); 1 \leq j \leq k,$$

$$f(y_i) = f(x_i) \text{ for all } i.$$

Moreover, labeling for each edge of $Q_r^+$ is defined as follows:

**Case 1.** For $2 \leq r \leq 4$:

$$f(u_1v_1) = f(x_1v_1) = 1; f(u_1v_2) = f(x_1v_2) = 2,$$

$$f(u_1v_3) = f(x_1v_3) = i + 1; i = 2,$$

$$f(u_2v_3) = f(x_2v_3) = 4.$$

If $\left\lceil \frac{5r+2}{3} \right\rceil \mod (r-k) = k$, then we have:

$$f(u_i v_i) = f(x_i v_i) = \begin{cases} 
i + 2 & \text{if } 3 \leq i \leq r - k, \\
r - k + 1 + 3j & \text{if } i = r - k + j, 1 \leq j \leq k; k \geq 1.
\end{cases}$$

$$f(u_i v_{i+1}) = f(x_i v_{i+1}) = \begin{cases} 
i + 2 & \text{if } 3 \leq i \leq r - k, \\
r - k + 2 + 3j & \text{if } i = r - k + j; 1 \leq j \leq k; k \geq 1.
\end{cases}$$

$$f(x_i y_i) = \begin{cases} 
i & \text{if } 1 \leq i \leq r - k, \\
r - k + 3j & \text{if } i = r - k + j; 1 \leq j \leq k; k \geq 1.
\end{cases}$$

We can see that all labels of vertices and edges are less than or equal to $\left\lceil \frac{5r+2}{3} \right\rceil$.

Finally, we verify the weight of edges under the labeling $f$ as follows:

$$W(x_i y_i) = 5i - 2;$$

$$W(x_i v_i) = 5i - 1; W(x_i v_{i+1}) = 5i + 1;$$

$$W(u_i v_i) = 5i; W(u_i v_{i+1}) = 5i + 2;$$

with $1 \leq i \leq r$. 

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It is clear that all edges have different weights. Thus, the total edge irregularity strength of para-squares cactus chain with \( r \) pendant vertices \( Q_r^T \):

\[
tes(Q_r^T) = \left\lceil \frac{5r + 2}{3} \right\rceil.
\]

An example of the edge irregular total 13-labeling of \( Q_7^T \) is given in Figure 2. The weight of each edge is represented in the red colored number.

![Figure 2. The edge irregular total 13-labeling of \( Q_7^T \).](image)

3. Conclusions

In this paper, we have obtained the exact value of the total edge irregularity strength of triangular cactus chain and para square cactus chain graphs with pendant vertices. We found that \( tes(\text{TC}_r^{r+1}) = \left\lceil \frac{4r+3}{3} \right\rceil \) and \( tes(Q_r^T) = \left\lceil \frac{5r+2}{3} \right\rceil \).

As further research, we will investigate a total vertex irregularity strength of triangular cactus chain and para square cactus chain graphs (in progress). Also, we will verify a total irregularity strength of triangular cactus chain and para square cactus chain graphs. Moreover, we give an open problem for further research.

**Open Problems:** what are the total edge irregularity strength, the total vertex irregularity strength, and the total irregularity strength of generalized cactus chain graphs?

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