Gravitational lensing and polarization in astrophysics

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Abstract. We discuss astrophysical problems connected with gravitational lensing where current and future polarimetric observations are very important to clarify theoretical models. Namely, we consider polarization observations for exoplanet searches with gravitational microlensing, optical polarization for gravitational lens systems, polarization signatures of cosmological gravitational waves.

1. Basics of gravitational lensing

There are a number of books and reviews on gravitational lensing and microlensing [1, 2, 3, 4, 5, 6, 7, 8], however we have to remind basics of gravitational lensing.

A science in a modern sense started since I. Newton investigations and corresponding monographs. In particular, in his book [9], he described a number of experiments to establish laws of optics and he listed queries which have to be confirmed (or disprove) with future experiments. His first query (hypothesis) was the following one [9]. “Query 1. Do not not bodies act upon light at a distance and by their action bend its rays, and is not this action strongest at the least distance?” Since in the Newton’s times scientists (and people) do not use a fundamental system of units, sometimes each county (or country) used their own system of units. After reading a classical book [10], one can understand that based on his corpuscular theory of light, Newton proved a correctness of Query 1 in a mathematical way.\textsuperscript{1} However, Newton thought that physics is an experimental science and physical laws have to be confirmed with experiments. Therefore, clearly Newton could derive the following relation for a deflection angle $\theta$\textsuperscript{2}

$$\delta \theta = \frac{2GM}{c^2 p},$$

where $M$, $p$, $c$ and $G$ are the gravitating body mass, the impact parameter, the speed of light and the Newton constant, respectively. If $M = M_\odot$, $p = R_\odot$, then $\theta_\odot = 0.87''$.

\textsuperscript{1} The relation was derived but not published by Cavendish [11, 12] and at the first time it was published by J. Soldner [13, 14].
In 1915 Einstein found that in the framework of general relativity the deflection angle of light has to be in two times greater in comparison with Newtonian gravity \cite{15}, namely
\[
\delta \theta = \frac{4GM}{c^2p}.
\]
As it was mentioned earlier \cite{1}, in Newton’s times the estimate for speed of light was much smaller \( c = 2 \times 10^{10} \text{ cm/c,} \) therefore, assuming that Newton had an opportunity to check Eq. (1), he could confirm the relation, in spite of the fact that the factor 2 has been lost because now we know that the prediction of general relativity Eq. (2) has been confirmed at the first time in 1919 \cite{17, 18}. One of the first astrophysical applications of light deflection or gravitational lensing has been considered by O. Chwolson \cite{19}.

2. Caustics and magnification

As is shown \cite{2}, the gravitational lens equation may be written in the following form: when the distance between a source and an observer is \( D_s \), the distance between gravitational lens and an observer is \( D_d \), \( D_{ds} \) is the distance between the gravitational lens and a source. If we suppose the small angle of deflection then we have the following simple expression for lens equation
\[
\eta = D_s \xi / D_d - D_{ds} \beta(\xi),
\]
where the vectors \((\eta, \xi)\) define the coordinates in lens plane and in source plane respectively,
\[
\beta(\xi) = \frac{4G}{c^2} \int \rho(R) \frac{(r-R)}{|r-R|^2} dX \, dY,
\]
where \( R = \{X, Y\} \) is the point vector in lens plane, \( \rho(R) \) is the surface mass density of the gravitational lens. We introduce the following variables \cite{2}
\[
x = \xi / R_0, \quad y = D_s \eta / (R_0 D_d),
\]
where \( R_0 = \sqrt{2r_g D_d D_{ds} / D_s} \) is the Einstein - Chwolson radius. We also introduce the following notation for scaled \cite{2} angle
\[
\alpha = \beta D_{ds} D_d / (D_s R_0).
\]
In gravitational lensing, the surface mass density is normalized with the critical surface mass density \cite{23}
\[
\rho_{cr} = \frac{c^2 D_s}{4\pi GD_d D_{ds}}.
\]
For typical lensing situations the critical surface mass density is of the order \( \rho_{cr} = 10^4 M_\odot pc^{-2} \) \cite{23}. Therefore, if we define the scaled surface mass density by the following expression
\[
\sigma = \rho / \rho_{cr},
\]
\[2\) Earlier, Einstein derived Eq.(1) and asked his assistant E. Freundlich to check the prediction in 1914 \cite{2}. The Soldner’s result has been practically forgotten for a century.
\[3\) Chwolson described circular images \cite{19} and Einstein obtained basic expressions for gravitational lensing \cite{20}. Moreover, it was found that Einstein analyzed gravitational lens phenomenon in his unpublished notes in 1912 \cite{20, 22}.
Figure 1. Cusp type singularity as a result of projection of two-dimensional surface onto plane. A touching the two folds on the plane gives the cusp.

then we have the expression for the angle $\alpha$

$$\alpha(x) = \int \frac{\sigma(x')|x-x'|^2}{|x-x'|^2} d^2x.$$  \hfill (7)

As it was shown [24], we may introduce the scalar potential $\psi$, such, that

$$\alpha(x) = \nabla \psi(x),$$  \hfill (8)

where

$$\psi(\xi) = \int \sigma(x') \ln |x-x'| d^2x.$$  \hfill (9)

Then

$$y = x - \alpha(x).$$  \hfill (10)

If we introduce

$$\phi(x, y) = \frac{(x-y)^2}{2} - \psi(x).$$  \hfill (11)

Then we can write the lens equation in the expression [24]

$$\nabla \phi(x, y) = 0.$$  \hfill (12)

It is easy to see that mapping $x \mapsto y$ is Lagrange’s mapping [25], since that is gradient mapping [26]. Really, if we consider the function

$$S = \frac{x \cdot x}{2} - \psi(x),$$  \hfill (13)

then $y = \nabla S$ Singularities of Lagrange’s mappings are described [27, 26], and their bifurcations are described [28]. Stable singularities for two-dimensional Lagrange mappings are only folds and cusps [27, 29]. Therefore, a point singularity arisen in point mass model of gravitational lens (the Schwarzschild gravitational lens) is unstable and disappeared under small perturbations [5]. A formation of a cusp singularity as a projection of two dimensional surface on plane is shown in Fig. 1.
Figure 2. Contours for magnification near cusp type singularities. Contours are \(10 \times 2^i, \forall i \in [-1, 14], i \in \mathbb{Z}[30].\)

Eq. (10) defined the mapping of points on the lens plane into points on the source plane. Using the Jacobian matrix we define the local mapping \(y = A \delta x\), where

\[
A = \begin{pmatrix}
\frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\
\frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2}
\end{pmatrix} = \begin{pmatrix}
1 + \psi_{11} & \psi_{12} \\
\psi_{21} & 1 + \psi_{22}
\end{pmatrix},
\]  

(14)

where

\[
\psi_{ij} = \frac{\partial^2 \psi}{\partial x_i \partial x_j}(i, j = 1, 2).
\]  

(15)

The magnification of an image at \(x\) is

\[
\mu(x) = 1 / \det A(x)
\]  

(16)

since of square for transformation is determined by the Jacobian matrix. The singular points of mapping are characterized by \(\det A(x) = 0\) (where the mapping is not one-to-one). The set is formed by so-called critical curves [2]. Images of critical curves are caustics. The most amplified images are for sources located on caustics because an amplification for these cases is infinite, however, since sources have finite sizes but not infinitesimal ones amplifications for finite sources located on (or near) caustics is high but not infinite (corresponding integrals from singular functions are convergent). One can find asymptotic relations for magnification near folds [2] and cusps [30].

3. Searches of exoplanets with gravitational microlensing

3.1. Light curve analysis

In October 1995 the first extrasolar planet has been found [31]. In October 2015 almost 2000 exoplanets (1969 planets in 1249 planetary systems including 490 multiple planetary systems) have been discovered.\(^4\) Majority of exoplanets have been found searching transits with space borne telescope Kepler. Exoplanet search are one of the hottest topic of modern astronomy because here we have an intersection of many different branches of science including not only astronomy and astrophysics but also astrobiology including abundance and origin of life [32].

\(^4\) http://exoplanet.eu/catalog/.
Already before the discovery of first exoplanets Mao and Paczynski showed how efficient is gravitational microlensing as a tool to search for extrasolar planets, including the low mass ones, even at relatively large distances from their host stars [33]. Since there is a proper motion in a system consisting of a source, a lens and an observer, a source is passing areas with different amplification of a gravitational lens (sometimes a source is passing areas with high amplifications near caustics), therefore there are variation of light curves. Later on, observations and simulations gave the opportunity to confirm the robustness of these conclusions. Exoplanets near the snow line may be also detected with this technique as it was shown, for instance, in Fig. 8 presented in [34]. In 2006 one of the lightest exoplanet with mass around 5.5 $M_{\oplus}$ has been found [35] with gravitational microlensing and later it has been proven that in average each star has one or more exoplanet [36] (practically such an assumption has been used for simulations of possible detection of exoplanets with pixel lensing [37]).

3.2. Polarization curve analysis

For extended sources, the importance of polarization measurements was pointed out for point-like lens in [39] and for binary lens in [40]. For point-like lens polarization could reach 0.1% while for binary lens it could reach a few percent since the magnification gradient is much greater near caustics. It has been shown that polarization measurements could resolve degeneracies in theoretical models of microlensing events [40]. Calculations of polarization curves for microlensing events with features in the light curves induced by the presence of an exoplanet and observed towards the Galactic bulge have been done [41, 42, 43, 44, 45]. Here we emphasize that measurements of then polarization angle could give additional information about the gravitational microlensing model.\(^5\) If polarization measurements are possible, in principle, one could measure polarization as a function of direction for an orientation of polarimeter and an instant for microlensing event. If a polarimeter is fixed one has an additional function of time to explain observational data, but if a polarimeter could be rotated, polarization is an additional function of direction at each instant. Such an information could help to resolve degeneracies and confirm (or disprove) microlensing models for observed phenomena.

4. Polarization in CMB and gravitational waves

CMB (cosmic microwave background) radiation is polarized. The CMB radiation is polarized because it was scattered off of free electrons during decoupling. The quadrupole anisotropy (which produced CMB polarization) could arise from three types of perturbations: Scalar (due to density fluctuations), Vector (due to vorticity induced by defects/strings), Tensor (due to gravity waves). The polarization pattern in the sky can be decomposed into two components: Curl-free component, called ”E-mode” (electric-field like) or ”gradient-mode”, with no handedness Grad-free component, called ”B-mode” (magnetic-field like) or ”curl-mode”, with handedness. The E-mode may be due to both the scalar and tensor perturbations, but the B-mode is due to only vector or tensor perturbations because of their handedness. One should mention that gravitational lensing can transform E-mode into B-mode. In 2014 BICEP2 collaboration claimed the discovery of primordial gravitational waves (or initial tensor perturbations) at a level $r \approx 0.2$ [46], however, joint analysis of BICEP2 and Planck collaborations showed that $r < 0.12$ and the initial result of BICEP2 collaboration has been caused by cosmic dust [47]. The dust originates B-mode of CMB and initially this systematic effect was underestimated.

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\(^5\) We call polarization angle the angle which corresponds to a direction with the maximal polarization.
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