Abstract

A jet of heavy fluid is injected upwards, at time $t = 0$, into a lighter fluid and reaches a maximum height at time $t = t_i$ and then flows back around the upward flow. A similar flow situation occurs for a light fluid injected downward into a heavy one. In this paper an exact analytical expression for $t_i$ is derived. The expression remains valid for laminar and turbulent buoyant jets with or without swirl.

1 Introduction

The study of jets with reversing buoyancy has found many applications in engineering. A heavy fluid jet injected upward into a lighter fluid is referred to as a negatively buoyant jet or fountain. This type of flow situation often occurs in disposal of industrial effluent. Typical examples are brine and gypsum discharged into the ocean through multi-port diffusers.

Similarly, a positively buoyant jet occurs when a light fluid is injected downwards into a heavier one. This jet reaches a maximum depth and then turns upward and rises around the downward flow. These types of jets are also referred to as inverted fountains. Typical examples are heating of a large open structure by fan-driven heaters on the ceiling and mixing of a two-layer water reservoir with propellers [1]. In both cases the jets also possess some swirl.

One of the first experimental studies on negatively buoyant jets was conducted by Turner [2]. He used a nozzle to inject a salt solution of density $\rho_0$ upward into a tank of stationary fresh water of density $\rho_a < \rho_0$. He measured the maximum height, $z_i$, attained by the jet and the mean height, $z_m < z_i$,

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at which the jet finally settles down. He related the two parameters $z_i/D$ and $z_m/D$ as proportional to the densimetric Froude number

$$F_r = \left( \frac{U_0^2 \rho_0}{g D (\rho_0 - \rho_a)} \right)^{\frac{1}{2}}$$

by using a dimensional analysis. Here, $U_0$ is the uniform velocity at the nozzle outlet, $g$ is the acceleration due to gravity and $D$ is diameter of the nozzle.

Later, Abraham [3] studied the problem theoretically and confirmed the result of Turner. Recently Baines et al. [1, 4] made an extensive study, both experimentally and theoretically on negatively buoyant jets. None of these studies provide an expression for $t_i$. Here a simple analysis is presented to obtain an exact analytical expression for $t_i$.

2 Analysis

A schematic diagram of the jet and control volume at time $t = t_i$ is shown in Figure 1.
Consider a flow situation in which a fluid of density $\rho_0$ is injected, at time $t = 0$ in the upward direction at a constant flow rate, from a nozzle at $z = 0$ into an ambient fluid of density $\rho_a < \rho_0$. Let the cross sectional area of the nozzle be $A$ and the fluid velocity distribution at the nozzle to be $u$. The jet continuously looses its momentum due to the reverse buoyancy force and reaches a maximum height $z_i$ at time $t = t_i$ after which the jet turns downwards.

For times $t < t_i$ the $z$-component of the momentum in the control volume increases continuously until time $t = t_i$ from which time it decreases due to the downward flow. Therefore, at $t = t_i$, the rate at which the momentum is changing inside the control volume is equal to zero. Application of the momentum theorem to the flow situation at time $t = t_i$ implies that the net force $F_b$ (in this case the buoyancy force in the $z$-direction) acting on the control volume is equal to the rate at which the net momentum $M$ is leaving the control volume, that is to say,

$$ F_b = M $$

If we take the control volume to be sufficiently large then the net rate at which the momentum is leaving the control volume can be written as

$$ M = -\rho_0 \int_A u^2 \, dA' \mathbf{k}, $$

where $\mathbf{k}$ is the unit vector in the $z$-direction.

The total volume of the fluid jet of density $\rho_0$ entering the control volume in time $t_i$ is

$$ V = t_i \int_A u \, dA'. $$

This is also equal to the volume of fluid of density $\rho_a$ displaced by the jet in time $t_i$. Therefore, the net buoyancy force acting on the fluid inside the control volume is

$$ F_b = -t_i (\rho_0 - \rho_a) g \int_A u \, dA' \mathbf{k}. $$

From equations (1), (2) and (3) we obtain an analytic expression for $t_i$ as

$$ t_i = \frac{\rho_0 \int_A u^2 \, dA'}{(\rho_0 - \rho_a) g \int_A u \, dA'}, $$

which reduces to

$$ t_i = \frac{U_0 \rho_0}{|\rho_0 - \rho_a| g}, $$

for $u = U_0 = \text{constant}$ over the area $A$. The modulus is introduced so that equation (5) is valid for positively and negatively buoyant jets. It is worth
mentioning that we have not made any assumptions about the regime of flow. Therefore the expression for \( t_i \) remains valid for laminar and turbulent buoyant jets with reversing buoyancy. Also, as any swirl added to the fluid at the nozzle does not affect the \( z \)-momentum of the fluid, equation (4) remains valid for this case too.

### 3 Notation

The following symbols are used in this paper:

- \( t \) time
- \( t_i \) time at which the jet reaches its maximum height
- \( \rho_0 \) density of the fluid in jet
- \( \rho_a \) density of the ambient fluid
- \( z_i \) maximum height attained by the jet
- \( z_m \) mean height at which the jets settles
- \( F_r \) Froude number
- \( U_0 \) uniform fluid velocity as the nozzle outlet
- \( D \) diameter of the nozzle outlet
- \( g \) acceleration due to gravity
- \( A \) cross sectional area of nozzle outlet
- \( u \) fluid velocity distribution at the nozzle outlet
- \( F_b \) buoyancy force
- \( M_b \) rate at which momentum is leaving the control volume
- \( M_b \) rate at which momentum is leaving the control volume
- \( k \) unit vector in the \( z \)-direction
- \( V \) total volume of jet fluid

### References

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