Relevance of the mathematics teacher’s specialized knowledge model in the planning and interpretation processes at the spatial thinking

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Abstract. When analyzing the relationships between knowledge and the teacher, some relationships related to the planning of teaching practices during professional practice are identified. One of the known reasons for these knowledge needs in planning is the lack of recognition of other elements in school mathematics. Under the theoretical perspective of the model of the specialized knowledge of the teacher who teaches mathematics, this qualitative research aims to characterize the elements of specialized knowledge in a geometry class and the subsequent reflection of the tasks of planning teaching. Among the main results of this research show the need to deepen the knowledge of the "basic" concepts of geometry, such as the cube, its characteristics and its representation from different perspectives. Therefore, it is necessary to motivate in teachers the need to develop their own spatial visualization skills, so that they apply them to the school mathematics they teach in their daily lives.

1. Introduction

In prospective mathematics teachers’ education, didactic and disciplinary aspects of mathematics are basic tools that converge to taking part in their classes’ plantings. However, it is during the professional practice when arise some questions about interactions, questioning and actions to develop by the students; characteristics of the teaching and learning processes [1]. As a result, it is necessary for teachers to propose activities that improve the processes to develop through school mathematics. But, it does not necessarily happen that way, because teaching practice goes beyond the relationships between knowledge, teaching and learning.

This indicates that there is a need to strengthen the teacher's mastery of the knowledge of school mathematics, without ignoring the pedagogical and didactic. For this, it is necessary to propose a continuous and constant training, in which teachers are able to identify a suitable school mathematics, and some necessary and sufficient knowledge to develop mathematical thinking in their students.

It is not enough to know the mathematical processes and thoughts. It is also necessary to see how this converges in the reality of the classroom. This without ignoring the reality that mathematical thoughts are not taught with equal intensity in educational institutions, the conceptual domains of teachers in charge of the subject are an important factor in classrooms. Thus, despite recognizing that geometric thinking contributes to the development of the ability to visualize, critical thinking, intuition, problem solving, demonstration, etc., less time is devoted to this thought with respect to that dedicated to the teaching of thought numerical. Therefore, this proposal has as its main objective: Identify the relevance of the mathematics teacher specialized knowledge (MTSK) model in the planning and interpretation processes at the spatial thinking.
2. Theoretical framework
Since 1998, the “Ministerio de Educación Nacional (MEN)” of Colombia, designed a line of teaching of different thoughts (numerical, spatial, metric, variational and random), processes (reasoning, exercise, modeling, problem solving and communication) and contexts (mathematical, interdisciplinary and everyday) of primary, secondary and middle school education, at the national level. This document, curricular guidelines of mathematics, conceives Space Thought as part of geometric systems, and in turn, is defined as the set of cognitive processes by which the mental representations of space objects are constructed and manipulated, the relations between them, their transformations, and their various translations to material representations [2]. Consequently, spatial thinking must be achieved from activities of “visual perception through the manipulation of materials that constitute mediators that facilitate creative learning, allow rediscovering geometric properties, and prepare the student” [3]. From the above, the student manages to arrive at arguments about the constructions.

Subsequently, in the basic competency standards, the mathematical objects to be learned in a set of grades are described. Later, in the Basic Learning Rights, the minimum knowledge that a student should have in each grade, and the learning curricula, which group the thoughts according to affinities between them, linking the metric thinking with Space thinking. In particular, currently, spatial thinking is conceived through two articulating axes: (i) Forms and their relationships, and, on the other hand, (ii) Location in space and trajectory traveled [4].

In this order, knowledge is considered, from the theoretical perspective of the specialized knowledge of the teacher who teaches mathematics (MT), as a set of pedagogical-didactic and disciplinary knowledge typical of mathematics that tend to improve professional practices. In addition, the model, called MTSK [5], is a conceptualization that assumes the specialization of the knowledge of MT as a central element of teaching knowledge; In other words, all knowledge that comes from the teacher is considered as specialized.

However, in the MTSK model, represented in the hexagon of Figure 1, the domain distribution is presented: mathematical knowledge (MK) and pedagogical content knowledge (PCK). In addition to the knowledge of topics (KoT), knowledge of structure of mathematics (KSM) and knowledge of practices in mathematics (KPM) subdomains of the MK domain. And, from the PCK domain are Knowledge of mathematics teaching (KMT), Knowledge or Features of learning mathematics (KFLM) and Knowledge of mathematics learning standards (KMLS). At the center of this scheme, in a hexagonal way, are beliefs about mathematics, besides the way of teaching and learning them.

According to the above, the specialized knowledge of the MT is used as a theoretical reference in mathematics education works, also as a form of analysis of various reasoning of teachers, both in training and in exercise, for example, when performing and solving tasks of school mathematics [6-8]. This type of knowledge is characterized as part of the specialized content knowledge (SCK). On the contrary, the specialized knowledge of the MT is more than the classroom management of the MT. In other words, that knowledge with which the teacher makes sense (mathematical) of the students' responses, differs from common knowledge (common content knowledge), which represents pedagogical knowledge (pedagogical content knowledge), because with specialized knowledge (specialized content knowledge), is with which students use other reasons for the resolution of tasks, being that with this type of thinking students constitute mathematical thinking. In this way, he refers to this different type of reasoning as a space of solutions that incorporates the “knowledge of examples, strategies and representations for solving problems that allows them to make sense not only for solutions similar to their own, but also make sense of the answers, reasoning and strategies of the students” [7].

In this order, it is allowed, together with the solutions that come from experience as a teacher, and not only the solutions of mathematics, the possibility of giving other senses to the answers, which requires: “a complex mathematical knowledge, deeper than he knows for himself, or even think of multiple possible solutions/approaches” [7]. Likewise, the exploration of specialized knowledge of MT requires flexibility, breadth of knowledge, and thinking beyond the response that is perceived before your eyes. In the same way, school mathematics “learned” by students is based (limited) on the teacher's knowledge about the different topics, contents and competences [9]. In addition, the exploration-
research carried out by teachers who teach mathematics in the initial stage of schooling will enhance, or condition, the future mathematical learning of students, since this exploration is implicitly related to the knowledge of the MT of this stage [10].

Figure 1. The mathematics teacher’s specialized knowledge model from [5].

From an international perspective, the specialized knowledge of the teacher, with results that strongly relate what the teacher knows, how he knows it and what he can do in a teaching context [11], over time, has occupied a prominent place in the investigation, in turn, play a fundamental role in student learning in later educational stages [12]. In this way, it is proposed as fundamental and priority to focus on the knowledge and mathematical training of teachers who teach mathematics in school, since researchers have not received it completely, because they are not specialists in mathematics [13-15], or assume (erroneously) that the mathematics of the early years are elementary and, therefore, easy to teach [16]. To achieve this goal, it is necessary to have a broad knowledge of how teachers can be helped so that, through knowledge of school mathematics, they improve their practices and allow them to improve long-term goals, not short-term ones. medium term. deadlines (as appropriate, meet objectives in standardized tests). All of the above implies a deep understanding of the relationships between mathematical objects and not only obtaining "correct" answers to student exercises.

3. Method
This research is carried out under a qualitative perspective, with an interpretative approach of an instrumental case study [17]. In this way, multiple instrumental case studies will be developed with the interest of obtaining, through these, a deeper understanding of the observed phenomena and, thus, refining previous proposed works from the MTSK perspective [6,18,19]. The data is collected during the meetings in which six MT participate voluntarily, who teach in primary and secondary basic education. These meetings are designed for 3 hours, 1 per month, during the first half of 2019. Each session is videotaped (from the second session); The sessions seek to promote discussion around tasks that develop spatial thinking, which could be used with students in the different levels of schooling in which they perform.
The tasks proposed for each session are considered by their exploratory-investigative nature and organized, at the same time, with a view that they may be situations used in the classroom; On the other hand, it is also expected that it can contribute, from the mathematical knowledge involved in the development of the task, in the search for possible difficulties and / or errors of the participants, which could, when put into practice, be reflected in the classroom situation [8]. In this order, during the first session, it was intended to carry out the construction of different models of tetrahedral that simulated a house, adaptation of the set of tasks proposed as rich situations in [20]; however, only the first three tasks were accomplished, namely, (i) Use of objects with cubic form and exemplification; (ii) cube drawing; (iii) cut-out of a cube; and, (iv) construction of hexaminoes (figures formed by six squares so that each of them have one side in common) and selection of the 11 that build a cube after folding it.

4. Results
Next, the analysis of the task (i) developed with five MT is presented. Finally, this proposal is analyzed from the perspective of analyzing the narratives of the professors in practice [21], based on the elements discussed in the theoretical reference of the MTSK model.

When analyzing the written productions of the graduates for the activity (i), it was possible to observe that all the teachers made the flat representation of a cube in the same position with respect to an observer in front of it.

In Figure 2, Figure 3 and Figure 4 you can see how in the flat representations of the cube, drawn by Rafael, Maria R, and Johnny, respectively, all 6 visible faces are considered, as if it were a cube constructed of transparent material. The cubic representation built by Michelle (Figure 5) reveals only three faces, in the same way, in the perspective presented, this maintains the parallelism between the planes of the opposite faces.

Finally, in Figure 6, Maria Ch. presents a cube from another perspective. In this interesting representation, she reveals the steps she followed to reach the final representation. Like her, in Figure 2, Rafael shows other possible perspectives that the cube initially represented could have but considering not visible three of the six faces that he took into account in his initial representation. Such considerations that teachers had when constructing the flat representation of the cube account for the Knowledge of Topics (KoT), when identifying basic characteristics of three-dimensional objects in the absence of the physical cubic representation.

However, when considering the possibility of representing the cube, in a different position, for example, on the floor, and maintaining the position of the observer from the place of the auditorium where they were, most of the participants found it difficult to perform a representation other than the one already proposed. Feasibly, because this is due to a single form of representation response, known to them, that limits the multiple forms of representation of the same object by changing the position. In other words, possibly, teachers continue to reproduce, in their records, the model that was taught years ago by their own professors, which obey a single form of representation of the cube as indicated by the step by step presented by Maria Ch. (Figure 6). In this representation, first, a “square” is constructed, and behind it a square congruent with it, so that, finally, it only remains to project lines that will correspond to the edges of the cube, which could be confirmed by socializing the response with peers when explaining: How each one did do it? According to [7], they refer to this different type of reasoning as a solution space that incorporates the “knowledge of examples, strategies and representations for problem solving that allows them to make sense not only for solutions similar to their own, but also make sense of the answers, reasoning and strategies of the students”. This demonstrates the need to deepen the knowledge of the characteristics of the object called "cube" and the correspondence with its flat representation. Therefore, the need to propose other tasks that seek in-depth knowledge of the representations of a cube is recognized, this being the basis of, for example, the description of positions and relationships of the object with respect to an observer, as indicated in the basic learning rights [4], for the development of students’ spatial thinking since the initial years of their training. These types of tasks are developed and are intended as a possible utility for the teaching of geometry in grades of primary basic education. In fact, it is necessary to motivate in teachers the need to develop their own
spatial visualization skills, so that they apply them to the school mathematics they teach in their daily lives. As a reflection, as stated in [22], this first session allows us to recognize that the mathematics Teachers should be motivated to make sense (mathematical) of the students' responses, to be updated and adapted to what the students have within their own reasoning for problem solving.

5. Conclusions

In conclusion, this work constitutes a basis for the development of research on the model of the specialized knowledge of the mathematics teacher (MTSK) in Colombia. This model tries to deepen the "finished" knowledge at the end of the professional training of the graduate in mathematics and related areas. That is, this finished knowledge is being updated and teachers continue to reproduce in their ways of teaching, even in the records on the blackboards, that model that was learned years ago from their own teachers, which prevents the development of specialized knowledge in MTSK (Figure 1), and, consequently, the transformation of teaching practices. In this sense, it is also intended to deepen in other forms of characteristics of the learning of mathematics (KFLM), in the same way that it is inquired about other teaching practices, for example, of the flat representations of three-dimensional objects. This makes it difficult for students to recognize the positions and relationships of space occupied by a 3D figure (i.e. cube representation), which makes it difficult for students to develop other spatial visualization skills such as identifying shapes, features, invariants, as well as visual memory, and preserving perception.

The practice of visualization skills, at an early age, contributes to the development of a solid foundation for learning geometry. Thus, if these skills are adequately developed, the results will be reflected in the breadth of solution strategies in situations involving graphic records. That is, the student will be able to develop differentiated strategies to identify appropriate information in problem situations in the context of mathematics in general.

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