Shock Heating of Directly Transmitted Ions

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Abstract

Collisionless shock heats ions very efficiently. Ion heating is a nonadiabatic process and the downstream temperature is not proportional to the upstream temperature. Ion distributions just behind the shock transition are gyrophase dependent and gradually gyrotropize. With the increase of the Mach number the contribution of reflected ions into ion heating gradually increases. Yet, directly transmitted ions may remain responsible for most of the downstream temperature. Direct transmission allows analytical treatment within the approximation of a narrow shock. We determine the gyrophase-dependent distribution just behind the magnetic jump, the gyrotrropic distribution farther behind the shock, and establish the relation with the magnetic compression and the maximum overshoot magnetic field.

Unified Astronomy Thesaurus concepts: Shocks (2086); Planetary bow shocks (1246); Interplanetary shocks (829)

1. Introduction

Collisionless shocks (de Hoffmann & Teller 1950) convert the energy of the directed flow into thermal energy of the plasma species, energy of the accelerated particles, and energy of the enhanced magnetic field. Observations in the heliosphere show that magnetized collisionless shocks are very efficient at ion heating (Montgomery et al. 1970; Scckopke et al. 1983; Thomsen et al. 1985; Burgess et al. 1989; Scckopke et al. 1990; Li et al. 1995). In collisionless plasmas temperature is defined via the variance of the velocity \((v − \langle v \rangle)^2\) for any distribution function, not necessarily Maxwellian, isotropic, or even gyrotrropic. Ion-heating efficiency is not sensitive to the upstream temperature. This efficiency was initially thought to be solely due to ion reflection (Gosling et al. 1982; Scckopke et al. 1983; Burgess et al. 1989; Gosling et al. 1989), while heating of the directly transmitted ions was attributed to the initial spread of velocities (Balikhin & Wilkinson 1996; Zank et al. 2010). However, heating is strong in low-Mach-number shocks as well without a noticeable presence of reflected ions (Thomsen et al. 1985; Scckopke et al. 1990). Observations of subcritical shocks with pronounced downstream magnetic oscillations (Balikhin et al. 2008; Russell et al. 2009) have revealed that ion heating occurs due to the gyration of ions passing through the shock front (Gedalin 1997). The theory of ion heating due to gyration was further developed by Gedalin et al. (2015) and Gedalin (2015, 2016a, 2016b), and confirmed by direct observations (Pope et al. 2019). Analytical studies so far provided partial estimates of the parameters of the downstream distributions (Gedalin 1997; Gedalin et al. 2015; Gedalin 2015, 2016a). In the present paper we extend the analysis in the approximation of a narrow shock transition and low upstream temperature. We derive the pressure just behind the ramp and in the gyrotropy region and establish the relation to the overshoot and downstream magnetic field. These relations may be used as proxies for estimating the shock Mach number when reliable measurements of density are not available.

2. Heating of Directly Transmitted Ions

In a simplified description an ion crosses a narrow magnetic jump across which a potential electric field is applied toward upstream. The shock is assumed to be planar and stationary, so that all fields depend only on the coordinate \(x\) along the shock normal. The normal component of the magnetic field, \(B_n\), is constant throughout the shock. The upstream-to-downstream magnetic jump is due to the change of the main component, \(B_z\), while the noncoplanar component, \(B_\perp\), is nonzero only inside the transition layer. This is an idealized description of a shock which does not take into account possible rippling, nonstationarity, and waves around the transition. We carry the analysis in the normal incidence frame (NIF), in which the upstream plasma flow is along the shock normal. Unless the shock is unusually broad and the noncoplanar magnetic field \(B_\perp\) is unusually strong, the magnetic deflection inside the transition layer is small (Gedalin 1996, 1997, 2016b; Gedalin et al. 2018). The main effect is the motion in the NIF cross-shock potential from the upstream to the downstream. The crossing occurs within a narrow transition so that in a first order approximation

\[
\frac{v_x}{v_y} = \sqrt{\frac{V_{u,z}^2 - s}{V_{d,z}}}, \quad \frac{v_y}{v_{d,z}} = V_{y,z}, \quad \frac{v_z}{v_{d,z}} = V_{z,u},
\]

where the normalized NIF cross-shock potential is

\[
\phi_{NIF} = \frac{2e\phi}{m_p}
\]

and all velocities are normalized on the upstream flow speed in NIF, \(V_u\). The other notation used in this paper is as follows: \(B_u\) and \(B_d\) are the magnitudes of the upstream and downstream magnetic field, respectively, \(R = B_d/B_u\) is the magnetic compression ratio, and \(\theta_u\) and \(\theta_d\) are the upstream and downstream angles between the magnetic field vector and the shock normal, respectively.

In contrast with the previous analyses (Gedalin 1997; Gedalin et al. 2018), here we are interested in the ion distribution in the region where the distribution function becomes gyrotrropic. This is most conveniently done in the de Hoffman–Teller frame (HT), in which the upstream and downstream plasma velocities are along the shock normal. In HT, once an ion crosses the shock and proceeds in the
downstream region with a uniform magnetic field and no electric field, the parallel velocity, \( v_0 = v \cdot B / |B| \), and the perpendicular speed, \( v_\perp = \sqrt{v^2 - v_0^2} \), are integrals of motion. Therefore, once we know these quantities just behind the shock, in the gyrophase-dependent region, we know them in the gyrotropic region too. Nonrelativistic transformation to HT is simple:

\[
v_0^{(HT)} = v_0, \quad v_\parallel^{(HT)} = v_\parallel + \tan \theta_u \quad (3)
\]

\[
v_\perp = \sqrt{|v_\parallel^{(HT)}|^2 - v_\parallel^{(HT)}} \quad (4)
\]

\[
v_\perp = v_\parallel \cos \theta_u + (v_\parallel + \tan \theta_u) \sin \theta_u \quad (5)
\]

\[
v_\perp = \sqrt{|v_\parallel \sin \theta_u - (v_\parallel + \tan \theta_u) \cos \theta_u|^2 + v_\parallel^2} \quad (6)
\]

where \((v_\parallel, v_\parallel, v_\perp)\) is the velocity of the ion in the NIF. The above relations are equally valid both in the upstream and downstream regions. For each region \( \theta \) should be replaced with the corresponding angle, \( \theta_u \) or \( \theta_d \). Note the addition of \( \tan \theta_u \) in both cases.

In the nongyrotropic region the distribution function of ions is \( f(v_{\parallel}, v_{\perp}, \varphi, x) \), where \( \varphi \) is the gyrophase. Gyrotropization due to the gyrophase mixing removes the dependence on \( \varphi \) and \( x \), so that the distribution function reduces to \( f(v_{\parallel}, v_{\perp}) \), in terms of the velocity of the guiding center along the magnetic field, \( v_{\parallel} \), and the speed of the gyration around the magnetic field, \( v_{\perp} \). Once the downstream gyrotropic distribution function \( f_d(v_{\parallel}, v_{\perp}) \) is established, the relevant moments are determined as follows:

\[
N = \langle v_{\parallel} \rangle = \int f_d(v_{\parallel}, v_{\perp}) d\Omega \quad (7)
\]

\[
J_{\parallel} = \langle v_{\parallel} \rangle = \int v_{\parallel} f_d(v_{\parallel}, v_{\perp}) d\Omega \quad (8)
\]

\[
P_{\parallel} = \int v_{\parallel}^2 f_d(v_{\parallel}, v_{\perp}) d\Omega \quad (9)
\]

\[
P_{\parallel,\perp} = \frac{1}{2} \int v_{\perp}^2 f_d(v_{\parallel}, v_{\perp}) d\Omega \quad (10)
\]

Here \( d\Omega = 2\pi dv_{\parallel} dv_{\perp} \) is the volume element in the velocity space, \( N = n_a / n_0 \) is the normalized downstream density, and \( J_d \) is the particle flux normalized on \( n_a \). The flux along the shock normal is conserved across the magnetic field, therefore \( J_{\parallel,x} = J_{\parallel,y} \cos \theta_y = 1 \). This relation can be used as a check point. The pressures \( P \) are normalized on \( n_a m_p V_u^2 \). The parallel pressure \( P_{\parallel} = p_{\parallel} + p_{\parallel,e} \) includes both dynamic pressure \( p_{\parallel} = n_d V_{\parallel}^2 = J_{\parallel,y} / n_a \) and kinetic pressure \( p_{\parallel,e} = n_d T_{\parallel} \). The perpendicular pressure is kinetic only, and \( p_{\parallel,e} = n_d T_{\parallel,\perp} \).

Let us consider an ion crossing the shock from upstream to downstream. In the approximation of a narrow shock, combining (1) with (5) and (6) one has

\[
v_{\parallel,d} = \sqrt{v_{\parallel}^2 - s \cos \theta_d + (v_\parallel + \tan \theta_u) \sin \theta_d} \quad (11)
\]

\[
v_{\parallel,\perp,d} = \sqrt{v_{\parallel}^2 - s \sin \theta_d - (v_\parallel + \tan \theta_u) \cos \theta_u^2 + v_\perp^2} \quad (12)
\]

where \( \cos \theta_d = \cos \theta_u / R \). The conservation of the particle flux means

\[
v_{\parallel,d} \cos \theta_d f_d(v_{\parallel,d}, v_{\parallel,\perp,d}) d\Omega = v_{\parallel} f_d(v_{\parallel}, v_{\parallel,\perp}) d\Omega, \quad (13)
\]

Mapping \((v_{\parallel}, v_{\parallel,\perp}) \rightarrow (v_{\parallel,d}, v_{\parallel,\perp,d})\) is not single valued; different initial velocities result in the same downstream parallel and perpendicular velocities. However, for the calculation of moments this relation is sufficient and allows us to replace

\[
f_d(v_{\parallel,d}, v_{\parallel,\perp,d}) d\Omega \rightarrow \frac{v_{\parallel} f_d(v_{\parallel}, v_{\parallel,\perp}) d\Omega}{v_{\parallel,d} \cos \theta_d} \quad (14)
\]

so that the condition \( J_{\parallel,s} = 1 \) is satisfied automatically and

\[
n_d = \int v_{\parallel} f_d(v_{\parallel}, v_{\parallel,\perp}) d\Omega \quad (15)
\]

\[
P_{\parallel} = \int v_{\parallel}^2 f_d(v_{\parallel}, v_{\parallel,\perp}) d\Omega \quad (16)
\]

\[
P_{\parallel,\perp} = \frac{1}{2} \int v_{\perp}^2 f_d(v_{\parallel}, v_{\perp}) d\Omega \quad (17)
\]

where \( v_{\parallel,n} \) and \( v_{\parallel,\perp} \) are given by (11) and (12). In principle, these expressions give the downstream density and pressure of directly transmitted ions after gyrophase-mixing completion, in the approximation of a narrow shock.

### 3. Cold Upstream Ions

Here we consider the case of low upstream temperature that can be neglected. In this case the upstream distribution is \( f_d(v_{\parallel}, v_{\parallel,\perp}) = \delta(v_{\parallel} - 1) \delta(v_{\perp} \theta) \) and all ions have the same downstream parallel velocity and the same perpendicular speed

\[
v_{\parallel,n} = \sqrt{1 - s \cos \theta_d + \tan \theta_n \sin \theta_d} \quad (18)
\]

\[
v_{\parallel,\perp,n} = \sqrt{1 - s \sin \theta_d - \tan \theta_n \cos \theta_d} \quad (19)
\]

so that

\[
N = (n_a \cos \theta_d)^{-1}
\]

\[
= \frac{R}{\sqrt{1 - s \cos \theta_d \cos \theta_n + s \sin \theta_n \sin \theta_d}} \quad (20)
\]

\[
T_{\parallel,n} = \frac{1}{2} (\sqrt{1 - s \sin \theta_d - \tan \theta_n \cos \theta_d})^2 \quad (21)
\]

\[
P_{\parallel,n} = n_a V_u^2 \quad (22)
\]

\[
P_{\parallel,\perp,n} = n_a V_\perp^2 \quad (23)
\]

The conservation of the momentum component along the shock normal (pressure balance) reads

\[
n_a m_p V_u^2 + n_a T_u + n_{e,a} T_{e,a} = \frac{B_u^2}{8 \pi} \quad (24)
\]

The pressure balance in the dimensionless form looks as follows

\[
1 + \frac{1}{2M^2} + \beta_x + \beta_e = P_{\perp} + P_{\parallel} + \frac{R^2}{2M^2}, \quad (25)
\]

where \( M \) is the Alfvénic Mach number, \( \beta \) and \( \beta_e \) are the upstream electron and ion kinetic pressure to magnetic pressure ratios, respectively, \( P_{\perp} \) is the downstream normalized xx component of the ion pressure, and \( P_{\parallel} \) is the downstream normalized electron pressure. For low \( \beta / M^2 \) and moderate
electron heating several terms can be neglected and one gets the following approximation:

\[ 1 - P_{\perp} = \frac{R^2 - 1}{2M^2} \hspace{1cm} (26) \]

\[ P_{\perp} = \frac{1}{N} + P_\perp \sin^2 \theta_d. \hspace{1cm} (27) \]

Note that \( N \) and \( P_\perp \) depend on the magnetic compression \( R \), Alfvénic Mach number \( M \), shock angle \( \theta \), and the cross-shock potential \( s \). The relation (26) can be used as a proxy for the Mach number determination if, for example, density measurements are difficult. The relation is approximate and not applicable for substantial \( \beta \), large overshoots, and/or a significant fraction of reflected ions. Yet, it could provide a useful estimate of the Mach number if other methods fail. Figure 1 illustrates the dependence of \( R_d \) on the Mach number, given by (26), for various cross-shock potentials and \( \theta_u = 70^\circ \).

4. Overshoot and Direct Transmission

In the case of a substantial overshoot, the magnetic jump and the cross-shock potential affecting the ion dynamics are those at the overshoot and not farther downstream. Let \( R_m = B_{\text{max}}/B_u \) be the magnetic compression at the overshoot, and \( s_m \) be the cross-shock potential at the overshoot. In the same approximation of a narrow shock, the distribution function of the directly transmitted ions at the overshoot peak is

\[ f_0(v_{0,x}, v_{0,y}, v_{0,z}) = f_u(v_x, v_y, v_z) \hspace{1cm} (28) \]

\[ v_{0,x} = \sqrt{v_x^2 - s_m}, \quad v_{0,y} = v_y, \quad v_{0,z} = v_z. \hspace{1cm} (29) \]

The corresponding pressure \( P_{0,xx} \) is

\[ P_{0,xx} = \int v_x^2 f_0(v_{0,x}, v_{0,y}, v_{0,z})dV_{0,x}dV_{0,y}dV_{0,z} \]

\[ \approx \int (v_x^2 - s_m)^{3/2} v_x f_u(v_x, v_y, v_z) dV_x dV_y dV_z. \hspace{1cm} (30) \]

For a cold upstream plasma one has

\[ P_{0,xx} = \frac{1}{N_u} = \sqrt{1 - s_m}. \hspace{1cm} (32) \]

The pressure balance, in the same approximation as above, reads

\[ R_m^2 = 1 + 2M^2(1 - \sqrt{1 - s_m}). \hspace{1cm} (33) \]

Note that this relation does not include the angle and is therefore less prone to errors of the determination of the normal. Thus, it can be used as a proxy for the Mach number determination. The presence of reflected ions would not critically affect the validity of the relation since their pressure does not change significantly at the ramp crossing. Indeed, considering for simplicity a beam of reflected ions entering the ramp with \( v_{ur,x} \approx 1.5 \), their post-ramp velocity would be \( v_{dr,x} \approx 1.3 \) for \( s = 0.5 \). For the directly transmitted ions in the same conditions this change is from \( v_{ur,x} = 1 \) to \( v_{dr,x} = 0.7 \). The ratio of the change of the pressure of the reflected ions to the change of the pressure of the directly transmitted ions at the ramp crossing would be less than \((0.2/0.3)(n_e/(n_u - n_e))\), where \( n_e \) is the number of reflected ions. Thus, for \( n_e/n_u \sim 0.2 \) the relative effect of the reflected ions is less than 20%.

Figure 2 illustrates the dependence of \( R_m \) on the Mach number, given by (33), for various cross-shock potentials.

The cross-shock potential is the result of different ion and electron dynamics within the shock front. It can be related to the electron heating via the relation (Gedalin 1996)

\[ \varphi = \int \frac{1}{n_e e} dx \left( p_e + \frac{B^2}{8\pi} \right) dx. \hspace{1cm} (34) \]

While the electron density is almost equal to the ion density, because of the quasineutrality, there is no good theory providing \( p_e \) within the shock transition. We, therefore, do not know what values of the potential are appropriate and what is the dependence of the potential on the Mach number. Heliospheric observations (Schwartz et al. 1988) show substantial scatter.

5. Conclusions

In the present paper an analytical approximation for the downstream gyrotropic distribution function of directly transmitted ions is derived. A narrow shock approximation has been applied. The relation between the upstream and downstream
distributions depends on the magnetic compression, shock angle, and cross-shock potential. A closed expression is derived for a cold upstream distribution. Usage of this expression in the pressure balance allows us to suggest a proxy for the observational Alfvénic Mach number determination when density measurements are not sufficiently good. A better proxy is derived using pressure balance at the ramp and overshoot. The last one relates the maximum magnetic compression to the Mach number and cross-shock potential at the overshoot. Dependence on the latter is not strong. Thus, the derived relation \((33)\) can be potentially used as an estimate of the Alfvénic Mach number in observations where particle measurements are not of sufficient quality. Corresponding analysis of the observational data will be published separately.

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Figure 2. Dependence of \(R_m\) on the Mach number, given by \((33)\), for various cross-shock potentials.