Measurement of $|V_{ub}/V_{cb}|$ (and $|V_{ub}|$) in Exclusive Nonleptonic
Decays, $\bar{B}^0 \rightarrow D_s^{(*)-}(\pi^+,\rho^+)$ and $\bar{B}^0 \rightarrow D_s^{(*)-}D^{(*)+}$

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Abstract

We have studied extracting $|V_{ub}/V_{cb}|$ by calculating the ratios $\mathcal{B}(\bar{B}^0 \rightarrow D_s^{(*)-}(\pi^+,\rho^+))/\mathcal{B}(\bar{B}^0 \rightarrow D_s^{(*)-}D^{(*)+})$ including penguin effects within the factorization assumption. The ratios involving $\bar{B}^0 \rightarrow D_s^-D^+$ mode have considerable penguin corrections ($\sim 15\%$ at the amplitude level), but those involving $\bar{B}^0 \rightarrow D_s^-D^{**}$ mode have relatively small penguin corrections. On the other hand, the $\bar{B}^0 \rightarrow D_s^-D^+$ mode has smaller form-factor dependance. Therefore, these ratios complement each other in measuring $V_{ub}/V_{cb}$. The theoretical uncertainty from the hadronic form factors in our method is at the level of 15\%, which is comparable to the model-dependence uncertainty of about 20\% in the measurement of $|V_{ub}/V_{cb}|$ from the exclusive semileptonic $B$ decays. Using the newest upper limit on $B \rightarrow D_s\pi$ decay from CLEO, our method sets an upper limit $|V_{ub}/V_{cb}| < 0.13$ which is very close to the measured values from the semileptonic $B$ decays. We also discuss the possible breaking of factorization assumption.
I. INTRODUCTION

A precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements is one of the key issues in the study of \( B \) mesons and \( B \)-factory experiments. For instance, the value of \( V_{ub} \) being non-zero is a necessary condition for CP-violation to occur in the SM; otherwise, we have to seek for new physics explanation for the observed CP violation in the \( K_L \rightarrow \pi\pi \) decays. For a stringent test of the SM and a search for new physics, it is important to make a precise measurement of the modulus \( |V_{ub}| \) from the decays of \( B \) mesons.

Theoretical and experimental studies on \( V_{ub} \) have been mostly focused on the semi-leptonic \( B \) meson decays. Observations of inclusive and exclusive semileptonic \( b \rightarrow u \) transitions by the CLEO and ARGUS experiments confirm that \( V_{ub} \) is indeed nonzero, but these measurements suffered from large uncertainty due to model-dependence in extracting the value of \( V_{ub} \). For instance, determination of \( V_{ub} \) from the inclusive \( b \rightarrow u\ell\nu \) using the leptonic end-point momentum involves large uncertainty in determining the fraction of partial decay width for \( E_\ell \) being larger than some cut value. In particular, the dependences of the lepton energy spectrum on perturbative and non-perturbative QCD corrections as well as on the unavoidable model-specific parameters are the strongest at the end-point region, which makes the determination of \( |V_{ub}/V_{cb}| \) very difficult by this method. There have been other suggestions for avoiding these difficulties in studying the inclusive semileptonic \( b \rightarrow u \) decays, for example, by using the invariant mass of the hadronic system recoiling against \( \ell\nu \), or the invariant lepton mass. Experimental studies have been made using the invariant mass of the hadronic system to separate \( b \rightarrow u \) from \( b \rightarrow c \) decays, and measure \( |V_{ub}/V_{cb}| \). By considering the exclusive semileptonic decay modes such as \( B \rightarrow \pi\ell\nu \) and \( \rho\ell\nu \), we may reduce the dependence on the \( E_\ell \) spectrum, but we are confronted with different aspects of theoretical uncertainties. In this case, determination of \( V_{ub} \) becomes very sensitive to the fraction of such exclusive decays with respect to the inclusive \( b \rightarrow u\ell\nu \) and it has large uncertainty coming from hadronic form factors.
Although traditional difficulties with the understanding of non-leptonic weak decays have prevented their use in determination of CKM elements, the possibility of measuring $|V_{ub}|$ via non-leptonic decays of $B$ mesons to exclusive two meson final states has been theoretically explored. To avoid the theoretical difficulties of non-spectator decay diagrams, only those final states have to be chosen in which no quark and anti-quark ($q\bar{q}$) pair has the same flavor.

Within the factorization approximation and after considering the final state interactions, exclusive two body decay modes of $B$ mesons would certainly be worthy of full investigation. In Ref. [11], it was pointed out that for the extraction of $|V_{ub}/V_{cb}|$ from nonleptonic $B$ meson decay data, study of the ratio $\mathcal{B}(B^0 \to D_s^+\pi^-)/\mathcal{B}(B^0 \to D_s^+D^-)$ is useful. Since the final states $\pi^-D_s^+(D^-D_s^+)$ consist of a single isospin component $I = 1$ ($I = 1/2$), the decay amplitudes are independent of the phase shift caused by elastic rescatterings in final states. Furthermore, the decay mode $B^0 \to D_s^+\pi^-$ is caused by only one diagram, the $b \to u$ tree transition with external $W$ emission and so completely independent of the penguin-type interaction. On the other hand, in the decay mode $B^0 \to D_s^+D^-$, although the dominant contribution is from the $b \to c$ tree diagram, $b \to s$ penguin diagrams also contribute. In Ref. [11] the authors neglected the penguin effect by expecting its size to be small compared to that of the tree diagram. However, we find that the theoretical estimate of the penguin correction is not small enough to be simply neglected for $B^0 \to D_s^+D^-$ decay mode. On the other hand, we find that $B^0 \to D_s^+D^{*-}$ decay mode receives much smaller penguin corrections compared to $B^0 \to D_s^+D^-$ decay mode. In this work we investigate more thoroughly the possibility of extracting $|V_{ub}/V_{cb}|$ from study of the nonleptonic decay ratio $\mathcal{B}(\bar{B}^0 \to D_s^{(*)-}(\pi^+, \rho^+))/\mathcal{B}(\bar{B} \to D_s^{(*)-}D^{(*)+})$. 

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$^a$ For possible breaking of the factorization approach, we give a short discussion in Section II.
II. THEORY ON $B^0 \to D_s^{(*)-}(\pi^+, \rho^+)$ AND $B^0 \to D_s^{(*)-}D^{(*)+}$
AND NUMERICAL ANALYSIS

While only tree diagrams contribute to $B^0 \to D_s^-\pi^+$ decay in the SM, the decays $B^0 \to D_s^-D^{(*)+}$ receive penguin contributions as well. The relevant $\Delta B = 1$ effective Hamiltonian has the form:

$$\mathcal{H}_{\text{eff}}^d = \frac{G_F}{\sqrt{2}} \left\{ V_{tb}V_{ts}^* \left[ c_1(\mu)O_1^d(\mu) + c_2(\mu)O_2^d(\mu) \right] - V_{tb}V_{ts}^* \sum_{i=3}^{10} c_i(\mu)O_i(\mu) \right\} + \text{h.c.},$$

where $O_{1,2}$ represent QCD corrected tree-level operators and $O_{3-6}$ ($O_{7-10}$) the QCD (electroweak) penguin operators, which are defined as

$$O_1^d = -\bar{q}\gamma_\mu Lb\bar{s}\gamma^\mu Lc, \quad O_2^d = -\bar{q}\gamma_\mu Lb\bar{s}\gamma^\mu Lc, \quad O_{3(5)} = \bar{s}\gamma_\mu Lb\bar{c}\gamma^\mu L(R)c, \quad O_{4(6)} = \bar{s}\gamma_\mu Lb\bar{c}\gamma^\mu L(R)c, \quad O_{7(9)} = \bar{s}\gamma_\mu Lb\bar{c}\gamma^\mu R(L)c, \quad O_{8(10)} = \bar{s}\gamma_\mu Lb\bar{c}\gamma^\mu R(L)c,$$

with $R(L) \equiv 1 \pm \gamma_5$ and $q = u$ ($c$) for $B \to D_s\pi$ ($B \to D_sD^{(*)}$) decays.

In the factorization approximation, the decay amplitudes of our interest are expressed as

$$A(B^0 \to D_s^{(*)-}(\pi^+, \rho^+)) = \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* a_1 \langle D_s^- | \bar{s}\gamma^\mu Lc | 0 \rangle \langle \pi^+, \rho^+ | \bar{u}\gamma_\mu Lb | B^0 \rangle,$$

$$A(B^0 \to D_s^-D^+) = -\frac{G_F}{\sqrt{2}} \left\{ V_{tb}V_{ts}^* a_1 - V_{tb}V_{ts}^* \left[ a_4 + a_{10} + 2(a_6 + a_8) \frac{m_{D_s}^2}{(m_b - m_c)(m_c + m_s)} \right] \right\} \times \langle D_s^- | \bar{s}\gamma^\mu Lc | 0 \rangle \langle D^+ | \bar{c}\gamma_\mu Lb | B^0 \rangle,$$

$$A(B^0 \to D_s^-D^{*-}) = \frac{G_F}{\sqrt{2}} \left\{ V_{tb}V_{ts}^* a_1 - V_{tb}V_{ts}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{D_s}^2}{(m_b + m_c)(m_c + m_s)} \right] \right\} \times \langle D_s^- | \bar{s}\gamma^\mu Lc | 0 \rangle \langle D^{*-} | \bar{c}\gamma_\mu Lb | B^0 \rangle,$$

$$A(B^0 \to D_s^-D^+) = \frac{G_F}{\sqrt{2}} \left\{ V_{tb}V_{ts}^* a_1 - V_{tb}V_{ts}^* (a_4 + a_{10}) \right\} \langle D_s^- | \bar{s}\gamma^\mu Lc | 0 \rangle \langle D^+ | \bar{c}\gamma_\mu Lb | B^0 \rangle$$

Here $a_j$’s represent effective parameters defined as

$$a_{2i} = c_{2i}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})^{2i}}c_{2i-1}^{\text{eff}}, \quad a_{2i-1} = c_{2i-1}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})^{2i-1}}c_{2i}^{\text{eff}},$$
where $c_i^\text{eff}$ are the renormalization scheme and scale independent effective Wilson coefficients, and $N_c^\text{eff}$ is the so-called effective color number, which is supposed to include non-factorization effects as well as color suppression effect, and can thus be considered a free parameter.

Using $V_{tb}V_{ts}^* \approx -V_{cb}V_{cs}^*$, one can cast the amplitudes in more compact forms

$$A(\bar{B} \to D_s^{(*)-}D^{(*)+}) = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^* \bar{a}_1(B \to D^{(*)}D_s^{(*)}) \langle \bar{s} \gamma^\mu Lc | 0 \rangle \langle D^{(*)+} | \bar{c} \gamma_\mu Lb | \bar{B} \rangle,$$

where

$$\bar{a}_1(B \to DD_s) = a_1 \left( 1 + \frac{a_4 + a_{10}}{a_1} + 2 \frac{a_6 + a_8}{a_1} \frac{m_{D_s}^2}{(m_b - m_c)(m_c + m_s)} \right),$$

$$\bar{a}_1(B \to D^* D_s) = a_1 \left( 1 + \frac{a_4 + a_{10}}{a_1} - 2 \frac{a_6 + a_8}{a_1} \frac{m_{D_s}^2}{(m_b + m_c)(m_c + m_s)} \right),$$

$$\bar{a}_1(B \to DD_s^*) = a_1 \left( 1 + \frac{a_4 + a_{10}}{a_1} \right).$$

Using the numerical values of $a_j$'s in Ref. [13], the effective parameters $\bar{a}_1$ defined above are related to $a_1$ by

$$|\bar{a}_1(B \to DD_s)| = 0.847 a_1,$$

$$|\bar{a}_1(B \to D^* D_s)| = 1.037 a_1,$$

$$|\bar{a}_1(B \to DD_s^*)| = 0.962 a_1.$$  \hspace{1cm} (10)

From the above relations one can see that, at the amplitude level, the penguin contributions to $\bar{B} \to D_s^- D^{**}$ decay (3.7%) are much smaller than those for $\bar{B} \to D_s^- D^+$ mode (15.3%). Actually the penguin effects on $\bar{B} \to D_s^- D^+$ decay are not small enough to be simply neglected. As mentioned in the Introduction, the penguin effects are neglected in Ref. [11].

We note that for $\bar{B} \to D_s^- D^{**}$ decay mode the penguin contribution can be neglected. This difference of penguin contributions to the similar modes $\bar{B} \to D_s^- D^+$ and $\bar{B} \to D_s^- D^{**}$ is due to the different chiral structure of the final states. $B \to D^*$ transitions occur through axial vector currents, while $B \to D$ through vector currents.

Then, the ratios

$$R_{(\pi, \rho)/D^{(*)}} \equiv \frac{B(\bar{B} \to D_s^- (\pi^+, \rho^+))}{B(\bar{B} \to D_s^- D^{(*)+})},$$

$$\tilde{R}_{\pi/D} \equiv \frac{B(\bar{B} \to D_s^- \pi^+)}{B(\bar{B} \to D_s^- D^+)}.$$  \hspace{1cm} (11)
are given as

\[
\mathcal{R}_{\pi/D} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( \frac{a_1}{\bar{a}_1(B \to D D_s)} \right)^2 \left( \frac{m_B^2 - m_\pi^2}{m_B^2 - m_D^2} \right)^2 \left( \frac{p_\pi^0}{p_c^0} \right)^2 \left( \frac{F_0^{B\pi}(m_D^2)}{F_0^{BDD}(m_D^2)} \right)^2,
\]

\[
\mathcal{R}_{\rho/D} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( \frac{a_1}{\bar{a}_1(B \to D D_s)} \right)^2 \left( \frac{m_B^2 - m_\rho^2}{m_B^2 - m_D^2} \right)^2 \left( \frac{p_\rho^0}{p_c^0} \right)^2 \left( \frac{2A_0^{B\rho}(m_D^2)}{2A_0^{BD\rho}(m_D^2)} \right)^2,
\]

\[
\mathcal{R}_{\pi/D^*} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( \frac{a_1}{\bar{a}_1(B \to D^* D_s)} \right)^2 \left( \frac{m_B^2 - m_\pi^2}{m_B^2} \right)^2 \left( \frac{p_\pi^0}{p_c^0} \right)^3 \left( \frac{A_0^{B\pi}(m_D^2)}{A_0^{BD\pi}(m_D^2)} \right)^2,
\]

\[
\mathcal{R}_{\rho/D^*} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( \frac{a_1}{\bar{a}_1(B \to D^* D_s)} \right)^2 \left( \frac{p_\rho^0}{p_c^0} \right)^3 \left( \frac{A_0^{B\rho}(m_D^2)}{A_0^{BD\rho}(m_D^2)} \right)^2,
\]

\[
\tilde{\mathcal{R}}_{\pi/D} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left( \frac{a_1}{\bar{a}_1(B \to D D_s)} \right)^2 \left( \frac{p_\pi^0}{p_c^0} \right)^3 \left( \frac{F_1^{B\pi}(m_D^2)}{F_1^{BD\pi}(m_D^2)} \right)^2,
\]

where \(p_c^0\) is the c.m. momentum of the decay particle \(X\). Here the form factors follow the following parameterization \[13\]:

\[
\langle P'(p')|V_u|P(p)\rangle = \left( p_\mu + p'_\mu - \frac{m_P^2 - m_{P'}^2}{q^2} \right) F_1(q^2) + \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu F_0(q^2),
\]

\[
\langle V(p', \epsilon)|V_u|P(p)\rangle = \frac{2}{m_P + m_V} \epsilon_{\omega\alpha\beta} \epsilon^{\mu\nu} p^\alpha p'^\beta V(q^2),
\]

\[
\langle V(p', \epsilon)|A_\mu|P(p)\rangle = i \left[ (m_P + m_V) \epsilon_\mu A_1(q^2) - \frac{\epsilon \cdot p}{m_P + m_V} (p + p')_\mu A_2(q^2)
\]

\[
-2m_V \frac{\epsilon \cdot p}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \right],
\]

where \(q = p - p'\), \(F_1(0) = F_0(0), A_3(0) = A_0(0)\),

\[
A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2),
\]

and \(P, V\) denote the pseudoscalar and vector mesons, respectively.

As can be inferred from the explicit expressions in Eqs. (13) – (17), these ratios do not have much dependence on values of \(a_i\). Especially, for \(B^0 \to D_s^+ D^{*-}\) decay, the penguin contributions add up destructively and so the dependence on \(a_i\) is almost cancelled out in the ratio. However, explicit calculations for the ratios of branching fractions depend strongly
on the form factors. In the following analysis, we consider five models for the form factors of $B \rightarrow \pi$ transitions: two quark-model approaches (the Bauer-Stech-Wirbel (BSW) model [14] and Melikhov/Beyer [15]), light-cone sum rules (LCSR [16]), lattice QCD (UKQCD [17]), and relativistic light-front (LF) quark model [18]. And for $B \rightarrow D^{(*)}$ transitions, we adopt BSW, Melikhov/Stech [19], and LF models. In Table 1, we list explicit numerical values of form factors evaluated at $q^2 = m_{D^{(*)}}^2$. Then we get the theoretical predictions

\[ R_{\pi/D} \equiv \frac{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \pi^+)}{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} D^+)} = 0.424 \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left[ \frac{F_0^{B\pi}(m_{D_s}^2)}{0.319} \right]^2 \left[ \frac{0.740}{F_0^{BD}(m_{D_s}^2)} \right]^2 \]

\[ = [0.424 \pm 0.041] \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (19) \]

\[ R_{\rho/D} \equiv \frac{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \rho^+)}{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} D^+)} = 0.443 \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left[ \frac{A_0^{B\rho}(m_{D_s}^2)}{0.398} \right]^2 \left[ \frac{0.740}{F_0^{BD}(m_{D_s}^2)} \right]^2 \]

\[ = [0.443 \pm 0.063] \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (20) \]

\[ R_{\pi/D^{*}} \equiv \frac{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \pi^{*+})}{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} D^{*+})} = 0.459 \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left[ \frac{F_0^{B\pi}(m_{D_s}^2)}{0.319} \right]^2 \left[ \frac{0.793}{A_0^{BD^{*}}(m_{D_s}^2)} \right]^2 \]

\[ = [0.459 \pm 0.076] \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (21) \]

\[ R_{\rho/D^{*}} \equiv \frac{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \rho^{*+})}{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} D^{*+})} = 0.480 \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left[ \frac{A_0^{B\rho}(m_{D_s}^2)}{0.398} \right]^2 \left[ \frac{0.793}{A_0^{BD^{*}}(m_{D_s}^2)} \right]^2 \]

\[ = [0.480 \pm 0.094] \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (22) \]

\[ \bar{R}_{\pi/D} \equiv \frac{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \pi^{*+})}{\mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} D^{*+})} = 0.456 \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left[ \frac{F_1^{B\pi}(m_{D_s}^2)}{0.367} \right]^2 \left[ \frac{0.817}{F_1^{BD}(m_{D_s}^2)} \right]^2 \]

\[ = [0.456 \pm 0.038] \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (23) \]

where the errors are originated only from the dependence on the hadronic form-factors.

Considering the current experimental results [20,21]

\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^{-} \pi^+) < 5.1 \times 10^{-5}, \]
\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^- \rho^+) < 7.0 \times 10^{-4}, \]
\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^- D^+) = (8.0 \pm 3.0) \times 10^{-3}, \]
\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^- D^{*+}) = (9.6 \pm 3.4) \times 10^{-3}, \]
\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^- D^+) = (1.0 \pm 0.5) \times 10^{-2}, \]
\[ \mathcal{B}(\overline{B}^0 \rightarrow D_s^- \pi^+) < 7.5 \times 10^{-5}, \]  

(24)

we then estimate

\[
\left| \frac{V_{ub}}{V_{cb}} \right| < \begin{cases} 
0.13 & \text{from } \mathcal{R}_{\pi/D}, \\
0.12 & \text{from } \mathcal{R}_{\pi/D^*}, \\
0.14 & \text{from } \tilde{\mathcal{R}}_{\pi/D}, \\
0.48 & \text{from } \mathcal{R}_{\rho/D}, \\
0.44 & \text{from } \mathcal{R}_{\rho/D^*}.
\end{cases}
\]  

(25)

We note that with the new preliminary upper limits on \( B \rightarrow D_s \pi^+ \) from CLEO [21], the upper limit on \( |V_{ub}/V_{cb}| \) is already very close to the current estimate \( |V_{ub}/V_{cb}| = 0.09 \pm 0.025 \) [22].

Next we consider direct extraction of \( |V_{ub}| \) from the \( \overline{B}^0 \rightarrow D_s^- \pi^+ \) decay rate. As mentioned earlier, this mode is one of the cleanest processes without any penguin corrections. Main uncertainties come from the form factor \( F_{p/B}^{B \pi} \) and Wilson coefficients or the effective parameter \( a_1 \). The decay rate is given as

\[
\Gamma(\overline{B}^0 \rightarrow D_s^- \pi^+) = \frac{G_F^2}{2} |V_{ub}| V_{cs}^* \frac{p_\pi^2}{8 \pi m_B^2} (m_B^2 - m_\pi^2)^2 |a_1 f_{D_s} F_{p/B}^{B \pi}(m_{D_s}^2)|^2
\]

\[
= (1.065 \times 10^{-12}) |V_{ub}|^2 \frac{a_1}{1.059} \left[ \frac{f_{D_s}}{0.240 \text{ GeV}} \right]^2 \frac{\left[ F_{p/B}^{B \pi}(m_{D_s}^2) \right]^2}{0.319} \text{ GeV}. \]  

(26)

In order to estimate theoretical uncertainty from the effective coefficient \( a_1 \), we choose different values for \( N_c^{\text{eff}} \), the effective number of colors appearing in Eq. (7), \( N_c^{\text{eff}} = 2, 3 \) and \( \infty \), and we use for the renormalization scheme independent Wilson coefficients \( c_1^{\text{eff}} = 1.149 \) and \( c_2^{\text{eff}} = -0.325 \) [13]. Considering the new preliminary upper limits on \( \overline{B}^0 \rightarrow D_s^- \pi^+ \) in Eq. (24), we then estimate
\[ |V_{ub}| < (0.450 \pm 0.020 \pm 0.021) \times 10^{-2}, \]  

where the first error is due to the effective coefficient \( a_1 \) and the second one is from the dependence on the form factor \( F_{0}B\pi \).

Finally we note on the possible breaking \( \text{[23]} \) of the factorization assumption, on which our previous results are based; we now generalize Eqs. \( \text{[13]} \) - \( \text{[17]} \) by including the factor for breaking of factorization,

\[
\mathcal{R}_{i/J} \equiv \frac{\mathcal{B}(\bar{B} \to D_{s}^{(*)\leftarrow D_{s}^{(*)}\pi})}{\mathcal{B}(\bar{B} \to D_{s}^{(*)\leftarrow s})} = \frac{|V_{ub}|}{|V_{cb}|} \frac{a_{1}}{\bar{a}_{1}} \frac{\left| K_{i/J} \right|^{2}}{F_{B \to i}(m_{D_{s}^{(*)}})^{2}} \left( 1 + (F - B)_{i/J} \right),
\]

where \( i = \pi, \rho \) and \( J = D, D^{*} \). \( K_{i/J} \) is the kinematic phase space factor, and \( a_{1}/\bar{a}_{1} \) is the ratio of effective Wilson coefficients, for which we used values estimated within factorization assumption. Because the decay \( \bar{B} \to D_{s}^{(*)\leftarrow D_{s}^{(*)}\pi} \) is only through tree diagrams and \( \bar{B} \to D_{s}^{(*)\leftarrow s} \) is polluted by penguin diagrams, the ratios \( \mathcal{R}_{i/J} \) would possibly be the best observables to measure the breaking of factorization, \( (F - B)_{i/J} \). Instead of measuring \( |V_{ub}/V_{cb}| \) as proposed, if we use the value of \( |V_{ub}/V_{cb}| \) measured from the semileptonic decays of \( B \) mesons, then we can systematically estimate the \( (F - B)_{i/J} \) through experimental values of \( \mathcal{R}_{i/J} \).

**III. DISCUSSIONS ON EXPERIMENTAL FEASIBILITY AND SUMMARY**

We have investigated the possibility of extracting \( |V_{ub}/V_{cb}| \) and \( |V_{ub}| \) from non-leptonic exclusive decays of \( B \) meson into two meson final states. In particular, we calculated the ratios \( \mathcal{B}(\bar{B} \to D_{s}^{(*)\leftarrow D_{s}^{(*)}\pi})/\mathcal{B}(\bar{B} \to D_{s}^{(*)\leftarrow s}) \) including penguin effects in the factorization assumption. By taking the ratios, some model-dependence on the coefficients \( a_{i} \) and hadronic form-factors is reduced. We found that the \( \bar{B} \to D_{s}^{-}D^{+} \) mode has considerable penguin corrections \( (\sim 15\% \) at the amplitude level) which cannot be simply ignored as it was done in Ref. [11]. We also found that the \( \bar{B} \to D_{s}^{-}D^{(*)} \) mode has very small penguin corrections. On the other hand, the \( \bar{B} \to D_{s}^{-}D^{+} \) mode has smaller form-factor dependence than \( \bar{B} \to D_{s}^{-}D^{(*)} \). Therefore, these modes can complement each other in measuring \( V_{ub}/V_{cb} \), in conjunction with \( B \to D_{s}^{(*)}\pi \) and \( B \to D_{s}\rho \).
We have shown that the relevant hadronic form factor uncertainties in our methods are typically at the level of 15% (Eqs. (19) – (23)). On the other hand, the model-dependence uncertainties in the measurement of $|V_{ub}|$ from exclusive semileptonic $B$ decays are currently at the level of 20% [5]. Therefore, if we can contain the experimental uncertainty of our method within 15%, the method described in this paper becomes competitive with the semileptonic analyses. This implies that we have to determine the decay branching ratios of $B \rightarrow D_s^{(*)}\pi$, etc. with experimental uncertainties being less than 30%.

Current experimental uncertainties for the branching ratios of $B \rightarrow D_s D^*$ decays are at the level of approximately 30%. The recent CLEO measurements [24] of $B(B^0 \rightarrow D_s^- D^*) = (8.4 \pm 3.0) \times 10^{-3}$ and $B(B^0 \rightarrow D_s^- D^{*+}) = (9.0 \pm 2.7) \times 10^{-3}$ are based on the event sample of $(2.19 \pm 0.04) \times 10^6$ $B \bar{B}$ pairs. With high-statistics event sample expected from the $B$-factories, we anticipate reducing the uncertainties of these decays to below 10% level in the very near future. As for the decay $B \rightarrow D_s^{(*)}\pi$ and $D_s \rho$, none of them have been experimentally measured yet. Recently, CLEO has presented the following preliminary upper limits: $B(B \rightarrow D_s\pi^+) < 8.9 \times 10^{-5}$ and $B(B \rightarrow D_s^{*}\pi^+) < 7.5 \times 10^{-5}$ [24], based on an event sample of $9.7 \times 10^6$ $B \bar{B}$ pairs. We expect to have larger amount of data from each $B$-factory experiment very soon. This, combined with much improved hadron identification and vertexing capabilities of the $B$-factory experiments, will improve the sensitivity of $B \rightarrow D_s^{(*)}\pi$ and $D_s \rho$ searches down to $B \approx 10^{-5}$ level and discovery of the $B \rightarrow D_s^{(*)}\pi(\rho)$ reactions might be possible in the near future.

Using the current estimate of $|V_{ub}/V_{cb}| = 0.09 \pm 0.025$ [22], along with the world-average value of $B(B^0 \rightarrow D_s^- D^+) = (8.0 \pm 3.0) \times 10^{-3}$ [20], we take $B(B^0 \rightarrow D_s^- \pi^+) = 2.7 \times 10^{-5}$ as the expected branching ratio. Based on this, we estimate experimental conditions to achieve 30% statistical uncertainty in $B(B^0 \rightarrow D_s^- \pi^+)$ from the $B$-factories. Consider we measure $B \rightarrow D_s \pi$ decay through $D_s \rightarrow \phi \pi$ and $\phi \rightarrow K^+ K^-$. In terms of signal detection efficiency $\varepsilon$ and integrated luminosity $\mathcal{L}$ (in fb$^{-1}$), the expected number of reconstructed $B \rightarrow D_s \pi$ events are

$$N = \varepsilon \mathcal{L} \times 10^6 \times 2.7 \times 10^{-5} \times 0.018$$
where 0.018 is the decay branching ratio of the particular $D_s$ decay mode that we consider. Assuming $\varepsilon = 17\%$ [25] we need $L \approx 120 \text{ fb}^{-1}$ to obtain 10 signal events. We can further improve the experimental sensitivity by several factors, if we include other decay channels of $D_s$ and if we also analyze related modes such as $B \rightarrow D_s^* \pi$, $B \rightarrow D_s^{(*)} \rho$, $B \rightarrow D_s^{(*)} \omega$, etc. In this case, with $L \approx 50 \text{ fb}^{-1}$ of data from the $B$-factories, it may be possible to obtain a competitive, independent measurement of $V_{ub}$ from exclusive non-leptonic $B$ decays.

There have been other studies of using nonleptonic $B$ decays for measuring $|V_{ub}/V_{cb}|$. For example, there were studies of utilizing fully inclusive nonleptonic $b \rightarrow u\bar{c}s'$ decays [26] or exploiting semi-inclusive nonleptonic $B$ decays $B \rightarrow D_s^{+}X_u$ [27]. The method suggested in Ref. [26] is clean in theoretical calculation. But it is based on the assumption that we can experimentally separate, without introducing much model-dependent uncertainty, $b \rightarrow u\bar{c}s'$ decays from the dominant $b \rightarrow c\bar{c}s'$ decays which is two-orders-of-magnitude larger in size. With existing experimental capabilities of $B$-factory experiments, this assumption is not justified. Ref. [27] considers more carefully the experimental backgrounds, but ignores the significance of the continuum background which can be statistically dominant in the interesting range of $2.0 < p_{D_s} < 2.5 \text{ GeV/c}$ [28].

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TABLES

Table 1. Numerical values of form factors at $q^2 = m_{D_s}^2$ in various form-factor models

|                  | $F_0^{B\pi}(m_{D_s}^2)$ | $F_1^{B\pi}(m_{D_s}^2)$ | $F_0^{BD}(m_{D_s}^2)$ | $F_1^{BD}(m_{D_s}^2)$ | $A_0^{BP}(m_{D_s}^2)$ | $A_0^{BD^*}(m_{D_s}^2)$ |
|------------------|--------------------------|--------------------------|-----------------------|-----------------------|-----------------------|-------------------------|
| BSW              | 0.377                    | 0.395                    | 0.753                 | 0.776                 | 0.326                 | 0.690                   |
| Beyer/Melikhov   | 0.299                    | 0.357                    | ·                     | ·                     | 0.381                 | ·                       |
| LCSR             | 0.322                    | 0.382                    | ·                     | ·                     | 0.457                 | ·                       |
| UKQCD            | 0.301                    | 0.359                    | ·                     | ·                     | 0.469                 | ·                       |
| Melikhov/Stech   | ·                        | ·                        | 0.722                 | 0.806                 | ·                     | 0.818                   |
| LF               | 0.295                    | 0.349                    | 0.746                 | 0.869                 | 0.357                 | 0.872                   |
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