High-resolution laser spectroscopy of a Bose–Einstein condensate using the ultranarrow magnetic quadrupole transition

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\textbf{Abstract.} High-resolution laser spectroscopy of a Bose–Einstein condensate (BEC) was performed by using the ultranarrow magnetic quadrupole ($6s^2\overset{1}{S}_0 \leftrightarrow (6s6p)^3P_2$) transition of ytterbium (Yb). The transition from the Doppler-broadened spectrum of thermal atoms to the asymmetric spectrum, reflecting the inhomogeneous density distribution of a BEC in a harmonic trap, was observed. The role of the inter-atomic interaction was highlighted by performing spectroscopy of a BEC loaded in a one-dimensional (1D) optical lattice.
1. Introduction

Studies of extremely narrow optical transitions have shown rapid progress. In metrology, studies of an ultraprecise atomic clock using an ultranarrow optical transition have been performed to achieve an uncertainty of $10^{-18}$ [1]. In quantum information science, atomic states are precisely manipulated with the use of an ultranarrow optical transition and such atoms are used as a quantum bit (qubit) due to their large coherence times [2, 3]. In studies of quantum degenerate gases, an ultranarrow transition has been utilized as a highly sensitive probe of the weak interatomic interactions. For example, the difference between the mean field energy in the excited and ground states results in a frequency shift [4] and asymmetry of the lineshape of the probe transition, which were first observed in a hydrogen Bose–Einstein condensate (BEC) and were used to identify the formation of the BEC [5, 6].

So far, in the experiment using alkali metals, frequency shift and broadening, which reflect the mean-field energy of a BEC, were observed using a radio-frequency (RF) transition. However, in such experiments, small signals in the spectrum are easily smeared out due to the line broadening caused by a finite interaction time [7]. On the other hand, spectroscopy in the optical region can overcome this difficulty. This is because the mean-field shift for an optical transition is generally larger than that for an RF transition, whereas frequency resolutions are comparable. For example, the mean field shift for the two-photon optical transition of the hydrogen atoms is 9.5 kHz for an atomic density of $1 \times 10^{14} \text{cm}^{-3}$, which is larger than that of $^{23}\text{Na}$ atoms (RF transition) by a factor of 50 for the same atomic density.

In this work, we observed the mean field shift and broadening of the spectrum of a BEC by using the ultranarrow $^1S_0 \leftrightarrow ^3P_2$ optical transition in ytterbium (Yb). A distortion of the lineshape due to the mean-field energy of a BEC in a harmonic trap was observed, which has never been seen in the RF spectroscopy of alkali metals [7, 8]. To highlight the role of the inter-atomic interaction, we also performed spectroscopy of a BEC in a one-dimensional (1D) optical lattice and detected the on-site interaction of atoms.

We note that the spectroscopy of a BEC using the $^1S_0 \leftrightarrow ^3P_2$ transition has the following advantages over that using the so-called ‘clock’ transition $^1S_0 \leftrightarrow ^3P_0$, which is also an optical and ultranarrow transition [9, 10]. Firstly, in bosonic isotopes, the strength of the $^1S_0 \leftrightarrow ^3P_2$ transition is much stronger than that of the $^1S_0 \leftrightarrow ^3P_0$ transition, which is only weakly allowed.
Figure 1. Relevant energy levels for Yb. The wavelength and the natural linewidth are presented for relevant transitions. The resonance frequency \( f \) of the \( ^1S_0 \leftrightarrow ^3P_2 \) transition can be estimated by \( f = f_0 + f_1 - f_2 \).

by the application of a strong magnetic field \([11]–[13]\). Secondly, the polarizability of the \( ^3P_2 \) state can be widely tuned, not by changing the wavelength of the trapping beam (\( \lambda = 532 \) nm) but only by changing the relative alignment between an applied magnetic field and a polarization of the trapping beam.

2. Methods

2.1. Experimental methods

In order to prepare ultracold Yb atoms \([15, 16]\), we collect typically \( 2 \times 10^7 \) Yb atoms in the magneto-optical trap (MOT) using the \( ^1S_0 \leftrightarrow ^3P_1 \) transition with a 10 s loading. These atoms are transferred to a crossed far-off-resonance trap (FORT; figure 2). The wavelength of the FORT laser is 532 nm. By performing evaporative cooling, we cool Yb atoms down to \( < 1 \mu \text{K} \) and obtain a pure BEC of \( \sim 10^5 \) atoms. The typical peak density of a BEC reaches \( 1 \times 10^{14} \text{ cm}^{-3} \).

The excitation laser of the \( ^1S_0 \leftrightarrow ^3P_2 \) transition (\( \lambda = 507 \) nm) is described in detail in \([17]\) and is briefly summarized here. The output of an external cavity laser diode at 1014 nm is amplified by a tapered amplifier. Then its frequency is doubled by a periodically poled lithium niobate (PPLN) crystal. The frequency of the fundamental laser is stabilized to a high-finesse cavity that is made of an ultralow expansion (ULE) glass. The finesse and the free spectral range (FSR) of the cavity are 50 000 and 1.5 GHz, respectively, at 1014 nm. As a result, the laser linewidth becomes \( \sim 1 \) kHz and the long-term frequency drift is reduced to \( 0.4 \text{ Hz s}^{-1} \). In all of the data in this paper, this frequency drift is corrected.

In order to detect \( ^1S_0 \) atoms, we use the absorption imaging technique using a strong \( ^1S_0 \leftrightarrow ^1P_1 \) (399 nm) transition (see figure 1). The absorption imaging also allows us to detect the \( ^1S_0 \leftrightarrow ^3P_2 \) transition as follows. Due to the long lifetime (15 s) of the \(^3P_2 \) state compared to the duration of the absorption imaging stage \( (< 1 \text{ ms}) \), once atoms are excited to the \(^3P_2 \) state, we
can consider that they remain there. Therefore, the ultranarrow $^1S_0 \leftrightarrow ^3P_2$ transition is detected as a decrease of the number of atoms in the $^1S_0$ state.

2.2. Observation of the $^1S_0 \leftrightarrow ^3P_2$ transition

The observation of the $^1S_0 \leftrightarrow ^3P_2$ transition of Yb has not been reported before this study. Therefore, first we estimated the resonance frequency $f$ of the $^1S_0 \leftrightarrow ^3P_2$ transition. This was done by combining three frequencies: $f_0(^1S_0 \leftrightarrow ^3P_0, 578 \text{ nm})$, $f_1(^3P_0 \leftrightarrow ^3S_1, 649 \text{ nm})$ and $f_2(^3P_2 \leftrightarrow ^3S_1, 770 \text{ nm})$. As shown in figure 1, $f$ is given by

$$f = f_0(^1S_0 \leftrightarrow ^3P_0) + f_1(^3P_0 \leftrightarrow ^3S_1) - f_2(^3P_2 \leftrightarrow ^3S_1).$$

Here, $f_0$ is the frequency of the clock transition and was precisely measured in [10]. Frequencies $f_1$ and $f_2$ were measured in this study by an optical frequency comb (FC8003, Menlo Systems GmbH). Both the $^3P_0 \leftrightarrow ^3S_1$ and $^3P_2 \leftrightarrow ^3S_1$ transitions are dipole-allowed transitions. Therefore we can easily observe them by saturation spectroscopy using an optogalvano cell in which the $^3P_0$ and $^3P_2$ states are populated by electric discharge [19, 20]. In the measurement, we tuned the laser frequency at the peak of the spectrum and measured its frequency. Both the repetition rate and the carrier-envelope offset frequency of the frequency comb were locked to an Rb frequency standard (PRS10, Stanford Research Systems). While the observed spectra suffer from collisional shift and broadening, we can estimate the resonance frequency $f$ within several tens of MHz, which is accurate enough for the present purpose.

In addition, in order to make it easier to find the $^1S_0 \leftrightarrow ^3P_2$ transition, we broadened the spectrum in the following ways. Firstly, we used relatively hot ($\sim 50 \mu\text{K}$) atoms in the MOT in order to utilize their Doppler broadening ($\sim 200 \text{ kHz}$). Secondly, we rapidly switched on and off the MOT laser during the excitation in order to broaden the spectrum by the ac Stark effect.
Figure 3. Selective excitation to five magnetic sublevels of the $^3P_2$ state in $^{174}$Yb. Here, we used atoms that are cooled by evaporative cooling in a crossed FORT and excited them in a FORT. (a) The spectrum obtained by roughly scanning the frequency of the excitation laser. (b–f) Detailed spectra of each magnetic sublevel. The zero frequency in (b–f) is defined by the center frequency of the fitting. Any two neighboring spectra in (b–f) separate with the constant interval of 1.5 MHz due to the magnetic field of 0.7 G.

induced by the MOT laser. The broadening due to the Zeeman effect induced by the MOT magnetic field gradient was also used.

Thanks to these broadening effects and the estimated frequency, we could observe the $^1S_0 \leftrightarrow ^3P_2$ transition in bosonic isotope $^{174}$Yb and in fermionic isotopes: $^1S_0 (F = 5/2) \leftrightarrow ^3P_2 (F = 5/2)$ in $^{173}$Yb and the $^1S_0 (F = 1/2) \leftrightarrow ^3P_2 (F = 5/2)$ in $^{171}$Yb.

In figure 3, the spectra of five magnetic sublevels of the $^3P_2$ state in $^{174}$Yb are shown. Here, we used atoms that were cooled by evaporative cooling in a crossed FORT and excited them
in a FORT. The atomic temperature was 2 µK. The spectrum in figure 3(a) was obtained by roughly scanning the frequency of the excitation laser. The linewidths of the spectra are 58, 43, 68, 54 and 47 kHz for the \( m = +2, +1, 0, -1 \) and \(-2\) sublevels, respectively. Here, five components are observed with the interval of \( \sim 1.5 \text{ MHz} \) due to the magnetic field of 0.7 G. The signal strength of each component reflects the selection rule of the \( ^1S_0 \leftrightarrow ^3P_2(M2) \) transition.

In figures 3(b)–(f), the spectra of each magnetic sublevel are shown in detail and their spectral width is determined by Doppler broadening.

3. Measurement of the polarizability

Next, in order to estimate the trap depth of the FORT for \(^3P_2\) atoms, we measured the polarizability (light shift) of the \(^3P_2(|m| = 2, 1, 0)\) state. The resonance frequency \( \nu \) is related to the polarizability \( \alpha_S \) and \( \alpha_P, m \) of the \(^1S_0\) and \(^3P_2(m)\) states, respectively, and the FORT intensity \( I \) by \( \nu = \nu_0 - (I/4)(\alpha_P, m - \alpha_S) \), where \( \nu_0 \) is the transition frequency between the unperturbed atomic states. Since \( \alpha_S \) has been separately measured \([18]\), we could estimate \( \alpha_P, m \) from the dependence of the resonance frequency \( \nu \) on the FORT intensity \( I \). In this measurement, the light shift due to the vertical FORT on the spectroscopy was negligibly small.

In figure 4(a), the observed frequency shift of the \(^1S_0 \leftrightarrow ^3P_2(|m| = 2, 1, 0)\) transition in the case of \( B \parallel \vec{e}_{\text{FORT}} \) is plotted as a function of the FORT power. The solid lines are linear fits to the data. By subtracting the polarizability of the \(^1S_0\) state (\( \alpha_S = 32.34 \text{ mHz (mW cm}^{-2})^{-1} \)), the polarizability \( \alpha_P \) of the \(^3P_2(|m| = 2, 1, 0)\) state is determined to be 38.3 \( \pm 0.1 \), 40.6 \( \pm 0.3 \) and 41.8 \( \pm 0.3 \) mHz (mW cm\(^{-2}\))\(^{-1}\) for \( |m| = 2, 1 \) and 0, respectively.

The polarizability of the \(^3P_2\) state in a FORT can be changed by the external magnetic field. This is because five magnetic sublevels of the \(^3P_2\) atoms in a FORT can be mixed with each other by the external magnetic field \([14]\). In our experiment, when we changed the direction of the magnetic field to satisfy \( B \perp \vec{e}_{\text{FORT}} \) instead of \( B \parallel \vec{e}_{\text{FORT}} \), the polarizability of the \(^3P_2(|m| = 0)\) state changed to 33.1 \( \pm 0.1 \) mHz (mW cm\(^{-2}\))\(^{-1}\), as shown in figure 4(b). This result means

**Figure 4.** (a) Measurement of the polarizability of the \(^3P_2(|m| = 2, 1, 0)\) state in the case of \( B \parallel \vec{e}_{\text{FORT}} \). The resonance frequencies are plotted as a function of the FORT power. (b) The polarizability of the \(^3P_2(|m| = 0)\) state in the case of \( B \parallel \vec{e}_{\text{FORT}} \) is compared with that in the case of \( B \perp \vec{e}_{\text{FORT}} \). In the latter case, we can prepare the nearly ‘magic wavelength’ condition. The error bars are typically smaller than the size of the symbols in both figures.
Figure 5. Observed spectra of the $^1S_0 \leftrightarrow ^3P_2$ ($m = 0$) transition in ultracold $^{174}$Yb atoms and a BEC in a FORT ($B \perp \vec{e}_{\text{FORT}}$). The solid lines in spectra of thermal atoms are fits to the data by the lineshape in [21]. A sudden change in the spectrum below the BEC transition is due to the mean field energy of a BEC. The solid line in the BEC data is a fit to the spectrum by equation (1). The zero frequency is defined as the high-frequency edge of the BEC spectrum ($\nu = \nu_0$ in equation (1)) after the subtraction of the light shift due to the FORT laser and the recoil shift. The error bars are typically smaller than the size of the symbols. For clarity, spectra for the atoms at 1.2, 0.5 and 0.3 $\mu$K are vertically offset by $1 \times 10^4$, $3 \times 10^4$ and $3 \times 10^4$, respectively.

$V_{^3P_2(m=0)}/V_{^1S_0} = 1.02 \pm 0.01$, where $V_i$ is the trap depth of atoms in state $i$ [14]. The following experiments in this paper were carried out in this condition in order to eliminate the light shift in the spectroscopy as much as possible.

Based on the result of this measurement, the following point should be mentioned. This measurement indicates that atoms in all magnetic sublevels of the $^3P_2$ state can be optically trapped by a 532 nm FORT laser when $B \parallel \vec{e}_{\text{FORT}}$, which is important for future studies of the $^3P_2$ atoms. Note that it is difficult to theoretically predict the polarizabilities because such a calculation requires a lot of unknown transition probabilities between high-lying energy levels and the $^3P_2$ state.

4. Spectroscopy of a Bose–Einstein condensate (BEC)

4.1. Spectroscopy of a BEC in a harmonic trap

Next, we carried out the high-resolution spectroscopy of a BEC ($^{174}$Yb) in a FORT. Relative orientations of a magnetic field, a polarization of the FORT laser ($B \perp \vec{e}_{\text{FORT}}$) and the excitation laser are shown in figure 2.

Observed spectra of the $^1S_0 \leftrightarrow ^3P_2(m = 0)$ transition of thermal atoms and a BEC are shown in figure 5. The spectra corresponding to the $^1S_0 \leftrightarrow ^3P_2(m = \pm 2, \pm 1)$ transitions were separately observed (not shown in figure 5) with the equal interval of 1.6 MHz due to the Zeeman
effect by the external magnetic field of 0.8 G as observed in figure 3(a). The solid lines in the spectra of thermal atoms in figure 5 are fits to the data by the absorption lineshape introduced in [21]. As for the BEC, we use the Thomas–Fermi approximation because the mean field energy dominates over the kinetic energy. Thus, the inhomogeneous density distribution in a trap determines the spectrum of a BEC, which is given by [6, 7, 22]

$$I(v) \propto \frac{15h(v - v_0)}{4n_0\Delta U} \sqrt{1 - \frac{h(v - v_0)}{n_0\Delta U}}. \quad (1)$$

Here, $v$ is the laser frequency, $v_0$ is the resonance frequency of the non-interacting atoms in the FORT, $n_0$ is the peak density and $h$ is Planck’s constant. The peak density $n_0$ of a BEC is separately estimated from the trap frequency, the ground state scattering length and the atom number under the Thomas–Fermi approximation: $\Delta U = 4\pi \hbar^2 (a_P^{1S_0} - a_{S_0})/M$, where $M$ is the $^{174}$Yb mass and $a_{S_0}$ is the scattering length between two atoms in states 1 and 2. By fitting equation (1) to the BEC spectrum with $a_{S_0} = 5.53$ nm [23, 24], we could determine the scattering length $a_P^{1S_0} = -33 \pm 10$ nm. Here, the fitting error is about 2 nm and the residual error mainly originates from the error of the peak density.

The zero frequency in figure 5 is defined as the high-frequency edge of the BEC spectrum ($v = v_0$ in equation (1)) after the subtraction of the light shift due to the FORT laser, which is determined in the measurement of the polarizability, and the recoil shift. Therefore, the shifted position of the spectral center from the zero frequency in figure 5 can be regarded as the mean field shift. In fact, one can see that the compensation of the mean field shift of thermal atoms, which can be separately estimated to be $-3.1$, $-2.5$ and $-3.8$ kHz for 1.2, 0.5 and 0.3 $\mu$K, respectively, by using the scattering length $a_P^{1S_0}$, makes the centers of the spectra coincide with the high-frequency edge of the BEC spectrum. In figure 5, the subtracted light shifts were 5.9, 2.9, 2.0 and 1.4 kHz for thermal atoms at 1.2, 0.5 and 0.3 $\mu$K and BEC with the corresponding FORT laser intensities of $1.7 \times 10^8$, $8.4 \times 10^7$, $5.8 \times 10^7$ and $4.0 \times 10^7$ W m$^{-2}$, respectively. The recoil shift has also been subtracted from all spectra in figure 5. The contribution of the differential ac Stark shift is estimated to be smaller than 0.2 kHz. Note that, for clarity, spectra for the atoms at 1.2, 0.5 and 0.3 $\mu$K are vertically offset by $1 \times 10^4$, $3 \times 10^4$ and $3 \times 10^4$, respectively.

In order to interpret the observed BEC spectrum, the following contributions were taken into account, such as (I) elastic and inelastic collisions between the excited $^3P_2$ state atoms and and the remaining $^1S_0$ atoms, (II) elastic and inelastic collisions between the excited $^3P_2$ state atoms and also (III) a relatively large loss of atoms, typically 30–50%, at the peak of the spectrum. As for (I), the recent measurement of the inelastic collision rate between the $^3P_2$ atom and the $^1S_0$ atom indicates that the inelastic collision between them does not explain the observed broadening. As for (II), it is noted that we have measured the rapid two-body inelastic loss rate of $1.0 \times 10^{-11}$ cm$^3$ s$^{-1}$ at the $^3P_2$ state in a separate experiment [18]. At the atomic density of $10^{14}$ cm$^{-3}$ in the present experiment, this inelastic decay rate becomes only $10^3$ s$^{-1}$, which accounts for at most 1 kHz broadening of the spectrum. Since the inelastic decay time of 1 ms is shorter than the typical excitation time of 50 ms, this inelastic process causes a negligible fraction of the $^3P_2$ atoms. Thus, we neglect the contribution from collisions between the two $^3P_2$ atoms. These facts and the sudden shift of the spectrum observed across the BEC transition suggest the contribution of the elastic collisional shift. Therefore, in this measurement, we carried out an analysis of the observed spectrum by assuming that the main contribution is the elastic collisional frequency shift between the $^3P_2$ state atom and the $^1S_0$ atom. As for (III),
the inhomogeneous density distribution in a trap determines the spectrum of a BEC under the local density approximation (the Thomas–Fermi approximation). The most critical factor in the determination of the scattering length \( a_{3P_2(m=0) - 1S_0} \) from equation (1) is the frequency position of the low-frequency edge of the distorted spectrum of the BEC in figure 5. Around this frequency, the atom loss is not large and thus the estimation of \( a_{3P_2(m=0) - 1S_0} \) based on equation (1) would not be a serious problem. Although the atom loss is large at the peak of the spectrum, this affects the signal intensity, which is a fitting parameter of the observed spectrum with equation (1).

4.2. Spectroscopy of a BEC in a one-dimensional (1D) optical lattice potential

To highlight the role of the inter-atomic interaction, we carried out the spectroscopy of a BEC in a 1D optical lattice potential at the ‘magic wavelength’ condition mentioned above. If the inter-atomic interaction is negligibly small, the tight confinement of the lattice potential makes the spectrum very narrow, regardless of the atomic motion in a trap. This is known as a Lamb–Dicke effect [25]. In other words, this configuration is sensitive to the inter-atomic interaction [4].

In our measurements, after the evaporative cooling to make a BEC, the 1D optical lattice potential was ramped up to the potential depth of \( 9.6E_R \) for 100 ms where \( E_R = \frac{\hbar k^2}{2M} \) is the recoil energy with \( k \) being the magnitude of the wave vector of the lattice beams. By observing the interference pattern between BEC arrays, we confirmed that the BEC was in the superfluid region [26, 27].

Figure 7 shows the observed spectrum of the \( 1S_0 \leftrightarrow 3P_2(m = 0) \) transition. The small signal observed at the higher frequency side of the carrier resonance is attributed to the heating sideband, that is, the transition from the vibrational ground state \( v = 0 \) in the \( 1S_0 \) state to the vibrationally excited state \( v = 1 \) in the \( 3P_2(m = 0) \) state. Compared to the very narrow linewidth (\( \sim 20 \) Hz) of the spectrum of thermal atoms reported in [12], the observed spectrum is broadened.

In order to analyze the observed broadened spectrum, we need to know the density distribution of condensates in each site of the lattice potential. To this end, based on the discussions in [28], we assume that the wave function of a BEC in a 1D optical lattice potential is given by

\[
\Phi_0(r) = \sum_{k=0, \pm 1, \ldots, k_M} f_k(y) \Phi_k(r_\perp),
\]

where \( k \) labels each site and \( 2k_M + 1 \) is the number of lattice sites. \( f_k(y) \) and \( \Phi(r_\perp) \) are the wave functions along the axial direction (confined direction, \( y \)-axis) and the radial direction (\( x \)- and \( z \)-axis), respectively. Along the confinement direction in each well, we assume the Gaussian function

\[
f_k(y) = e^{-((y-kd)^2/2\sigma^2)} \],
\]

where \( d \) is the lattice interval and \( \sigma \) characterizes the width of the condensates. \( \sigma \) is given by \( \sigma = d/\sqrt{\frac{\pi}{s^{1/4}}} \) with \( s = (\hbar \omega_y)/(2E_R) \), \( \omega_y \) is the trap frequency along the confinement direction and \( E_R \) is the recoil energy of the FORT laser. On the other hand, the radial direction should be treated by the Thomas–Fermi approximation. Thus, the wave function of the \( k \)th site in the radial direction is given by

\[
|\Phi_k(x, z)|^2 = \frac{m}{2\sqrt{2\pi\hbar^2a}} \mu_k \left( 1 - \frac{r_\perp^2}{(R_\perp)^2} \right),
\]
Figure 6. Chemical potentials of condensates in each lattice site are compared to the energy of motion along the axial axis (left) and the radial axis (right).

where $\mu_k$ and $(R_\perp)_k$ are the chemical potential and the Thomas–Fermi radius of the $k$th condensate, respectively. Here, $m$ is the atomic mass, $a$ is the s-wave scattering length, $r_\perp = \sqrt{x^2 + z^2}$ and $\omega_{(\perp)}$ is the radial trap frequency of the harmonic potential. $\mu_k$ satisfies

$$\mu_k = \frac{1}{2} m \omega_{(y)}^2 d^2 (k_M^2 - k^2),$$

where $\omega_{(y)}$ denotes the axial trap frequency of the harmonic potential. From the normalization condition $N = \sum N_k$, the $k_M$ is given by

$$k_M = \sqrt{\frac{2 \hbar \bar{\omega}}{m \omega_{(y)}^2 d^2} \left( \frac{15}{8} \sqrt{\pi} N \frac{a}{a_{ho} \sigma} \right)^{1/5}}.$$

Here, $\bar{\omega} = (\omega_{(y)} \omega_{(\perp)}^2)^{1/3}$; $a_{ho} = \sqrt{\hbar / m \bar{\omega}}$ is the oscillator length. The number of atoms in each site is given by

$$N_k = \frac{15}{16} N \frac{k^2}{k_M^2} \left( 1 - \frac{k^2}{k_M^2} \right)^2.$$

In the above discussions, we assume that the confinement along the axial direction is so strong that $\sigma$ is not affected by interaction energies in each site. To confirm this assumption, we calculate the $\mu_k$ of each lattice site and compare them to $h\omega_{(y)}$, both for the axial and the radial directions, as shown in figure 6. In this measurement, $\omega_{(y)} = 2\pi \times 140$ Hz, $\omega_{(\perp)} = 2\pi \times 99$ Hz, $N = 5 \times 10^4$, $a/a_{ho} = 7.3 \times 10^{-3}$, $s = 9.6$ and thus $k_M = 13$. As shown in figure 6, $\hbar \omega_{(y)} \gg \mu_k$ is satisfied for every site in the axial direction. Thus, we can justify the approximation that we can determine the Gaussian width by neglecting the two-body interaction at each site. As for the radial direction, $\hbar \omega_{\perp} < \mu_k$ for all $k$ justifies the application of the Thomas–Fermi approximation.

We estimated the spectral shape as a summation of equation (1) of each lattice site both for the carrier and for the sideband peaks. The ratio of the transition strength between the carrier and the sideband is given by $1 : \delta^2$ [25], where $\delta$ is the Lamb–Dicke parameter. Here, we assume...
that the trap depth of the $^1S_0$ state is the same as that of the $^3P_2$ state. In this measurement, $\delta$ can be estimated to be 0.4 from the separation between the carrier and sideband peaks. The observed frequency separation between the two peaks is consistent with that expected from the band structure. An atomic peak density at each lattice site could be determined from equation (2). The result of this calculation is shown in figure 7 as a solid line. The definition of the zero frequency is defined as the high-frequency edge of the carrier spectrum. The agreement between the observed spectrum and the solid line indicates that the broadening is caused by the mean-field energy of a BEC in an optical lattice. In other words, we measured the on-site interaction of the lattice potential, which is one of the important parameters for the lattice experiment.

5. Conclusion and future prospects

In conclusion, we have observed the $^1S_0 \leftrightarrow ^3P_2$ transition in $^{171}$Yb, $^{173}$Yb and $^{174}$Yb. Using this ultranarrow optical transition, the polarizabilities of magnetic sublevels of the $^3P_2$ state were determined. In addition, high-resolution spectroscopy of a $^{174}$Yb BEC in a harmonic trap and in a 1D optical lattice potential has been performed. The frequency shift and broadening due to the mean-field energy of a BEC were observed.

The spectroscopic technique developed in the present experiment could be a promising tool in the future study of ultracold two-electron atoms. Such applications would include the implementation of a quantum computation in a magnetic field gradient [29], high-resolution probe of the superfluid–Mott insulator transitions [8], [30]–[33] and the coherent transfer of the BEC in the ground state to the $^3P_2$ state. In addition, high-resolution spectroscopy of the magnetic-field-sensitive transition $^1S_0 \leftrightarrow ^3P_2(m \neq 0)$ may enable us to access an individual atom in a lattice potential. In such a system, different Zeeman shifts at each atom in a magnetic field gradient allow us to distinguish the individual atoms in a frequency region [34]–[36]. This technique can be applied also to the high-spatial-resolution magnetic-resonance imaging of a
BEC. Note that, in such experiments, the applied fields never affect the atomic behavior because the spinless ground state $^1S_0$ is completely insensitive to the magnetic field [30].

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