On the Erasure and Regeneration of the Primordial Baryon Asymmetry by Sphalerons

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Abstract
We show that a cosmological baryon asymmetry generated at the GUT scale, which would be destroyed at lower temperatures by sphalerons and possible new $B$- or $L$-violating effects, can naturally be preserved by an asymmetry in the number of right-handed electrons. This results in a significant softening of previously derived baryogenesis-based constraints on the strength of exotic $B$- or $L$-violating interactions.
It was first noticed by Fukugita and Yanagida (FY) \cite{FY} that, if the baryons comprising us were created early enough in cosmological history, then lepton number must be at least a fairly good approximate symmetry. They reasoned that sphalerons, which tend to destroy any net excess in $B + L$ number \cite{FV}, would ensure the baryon and lepton asymmetries were equal and opposite, roughly speaking. But if lepton violating processes were significant, then $B$ and $L$ would both be driven to zero. Their analysis yielded a rather weak bound of 50 keV on the majorana mass of a neutrino. Since then a great deal of effort has gone into strengthening and generalizing their result. References \cite{K,S} significantly improved the FY bound by requiring the $L$-violating interactions responsible for seesaw neutrino masses to be out of equilibrium at very high temperatures where the sphalerons go out of equilibrium, rather than the much lower electroweak phase transition temperature used by FY. The improved results were then applied to general operators which violate either $B$ or $L$ \cite{K,S}. Although the strongest bounds on non-renormalizable interactions of this type come from considering their effects at the sphaleron decoupling temperature, $T_m \sim 10^{12}$ GeV, in the supersymmetric case it was realized \cite{GS} that above a certain scale associated with supersymmetry breaking, $T_s \sim 10^8$ GeV, the presence of new anomalies would cause the baryon number to be encoded in supersymmetric particles, saving it from erasure until temperatures below $T_s$. Moreover, in generic models of inflation, equilibrium is restored at the relatively low temperature of $T_r \sim 10^5$ GeV and further softens this bound \cite{G}. 

In this letter we point out a surprise: most of the efforts to strengthen the original Fukugita-Yanagida bound are invalidated by a rather mundane feature of the Standard Model, namely the smallness of the Yukawa coupling of the right-handed electron, which causes interactions that change the net number of $e_R$’s to come into equilibrium only at very late times. The authors of ref. \cite{K} investigated whether the equilibration was late enough so that a baryon excess in a universe with $B - L = 0$ could be preserved by an $e_R$ asymmetry, despite sphaleron effects. Whereas their answer was “no,” here we make the startling observation that the answer becomes “yes” if one includes some explicit $L$ or $B$ violation, such as Majorana neutrino masses. This is contrary to the usual prejudice toward the strength of lepton violation vis-a-vis baryogenesis, where it was always assumed that smaller is better. If on the other hand $B - L$ is nonzero, lepton number violation is not
necessary, but it is much more permissible than was previously thought.

We emphasize that it is the right-handed electrons that play the role of protector of the baryon asymmetry. The key observation is that any asymmetry in their own numbers remains untouched until relatively low temperatures around $T^* = 10$ TeV, when the small Yukawa interactions with left-handed electrons and Higgs bosons finally become fast enough to convert the $e_R$’s into $e_L$’s. Because sphalerons interact only with the left-handed particles, they can only directly deplete the latter. Therefore, as long as any additional lepton violating interactions have gone out of thermal equilibrium before the right-handed electrons come into equilibrium, the initial $e_R$ asymmetry is protected from being washed out. When the temperature eventually falls below $\sim 10$ TeV, sphalerons will be able to convert a sizable fraction of the initial $e_R$ asymmetry into the baryon excess that exists today. The condition that lepton (or baryon) violating effects freeze out before 10 TeV leads to new, relaxed bounds on the sizes of various symmetry-breaking operators.

We will for the most part concentrate on the $\Delta L = 2, D = 5$ operator considered by FY. The major result is that the FY’s original 50 keV bound on the neutrino mass matrix elements can only be strengthened to a few keV. The difficulty of arranging masses and mixing angles to conflict with this bound will be explored, leading to the strong conclusion that sphalerons, together with the $L$-violating effects of the dimension five operator $LLHH$, generically cannot destroy any primordial baryon asymmetry, which might be produced in grand unified theories! The bounds on other $B$ and $L$ violating operators, though still interesting, are also weakened relative to previous expectations. In the remainder of our letter we elaborate upon and prove these statements. To avoid burying the simplicity of our idea in an avalanche of details, we give a rough derivation, whose result will be corroborated more rigorously in a future publication [10].

Let us consider first the effective operator $\frac{\lambda^2}{2M}LLHH$ generated by adding right-handed neutrinos $N_k$ to the standard model Lagrangian, with large Majorana masses $M_k$ and Yukawa couplings to the Higgs and lepton doublets $H$ and $L_j$,

$$\frac{1}{2} \bar{N}_k (i\gamma^\mu - M_k) N_k - \frac{1}{2} \lambda_{jk} \bar{L}_j H (1 - \gamma^5) N_k + \text{h.c.}$$  \hspace{1cm} (1)

Below the electroweak phase transition, the SU(2) doublet neutrinos acquire Majorana
masses \( m_{ij} = \sum_k \lambda_{ik} \lambda_{kj}^\dagger v^2 / M_k \) and \( v \) is the Higgs vacuum expectation value of 174 GeV. Therefore the dimension five operator induced by \( N_k \) exchange may be written

\[
\frac{1}{2v^2} \sum_{ij} m_{ij} (\bar{L}_i H)(H^T L_j^c). \tag{2}
\]

Figure 1 illustrates the sequence of events that would lead to the destruction of the baryon asymmetry, and suggests how to avoid that outcome. At very high temperatures, lepton-violating (\( \Delta L \)) annihilation processes due to the interaction (2) are in equilibrium because their rates scale as \( T^3 \), whereas the Hubble expansion rate goes like \( T^2 \). Slowest of all are the processes shown in fig. 2 that equilibrate the left- and right-handed electrons, scaling like \( T \). These come into equilibrium at a temperature \( T_\star \). If \( T_\star \) is much greater than \( T_f \), the freezeout temperature of the \( \Delta L \) processes, then baryon number is completely washed out by the sphalerons. But in the opposite case, \( T_f > T_\star \), sphalerons are able to destroy baryon number only until \( T_f \). At this point, they begin to regenerate baryon number out of the \( e_L \) asymmetry, which in turn came from the primordial \( e_R \) asymmetry, once the left-right equilibrium was established at \( T = T_\star \).

We are interested in the rate \( \Gamma_{\Delta e_L} \) at which \( e_L \)-type lepton number is violated, since electrons have the smallest Yukawa coupling and hence the lowest temperature for reaching right-left equilibrium:

\[
\Gamma_{\Delta e_L} = \frac{11}{32\pi^3} \frac{\mu^2 T^3}{v^4} \equiv 2|m_{ee}|^2 + |m_{e\mu}|^2 + |m_{e\tau}|^2. \tag{3}
\]

This is just the thermally averaged rate of \( \Delta L = 2 \) annihilations \( L_e L_i \rightarrow HH \), summed over families \( i \).\(^1\) Note that the mass matrix is generally not diagonal, as we are in the flavor eigenstate basis. Comparing this rate to that of the Hubble expansion \( (\approx 17T^2/M_P) \) yields a freezeout temperature of

\[
T_f = 120 \left( \frac{\text{keV}^2}{\mu^2} \right) \text{TeV}. \tag{4}
\]

\(^1\)In arriving at the coefficient, e.g. for the \( ee \)-part, we considered the channels \( e_L e_L \leftrightarrow h^- h^- \), \( e_L \nu e_L \leftrightarrow h^- h^0 \), \( e_L h^+ \leftrightarrow e^\nu e^- \) and \( e_L h^+ \leftrightarrow h^0 \nu e^- \). We used initial state Maxwell-Boltzmann distribution functions but ignored those of the final states. The factor of 2 difference between the \( ee \) and \( e\mu \) or \( e\tau \) contributions is due to the fact that the \( ee \) processes change \( e_L \) lepton number by two units, twice as much as the other processes.
On the other hand, the rate of $e_R\bar{e}_L$ equilibration is determined by the Higgs decays and inverse decays, shown in fig. 2. There will also be scattering processes such as $t_R\bar{t}_L \rightarrow e_R\bar{e}_L$, but we have found that for Higgs boson masses above the experimental limit they are subdominant to the decays, whose rate is

$$\Gamma_d = \frac{\pi h_e^2}{192\zeta(3)} \frac{m_h^2(T)}{T},$$  \hspace{1cm} (5)$$

where $h_e = 2.9 \times 10^{-6}$ is the electron Yukawa coupling. The thermal Higgs mass can be parametrized in the form $m_h(T) \simeq xT$, with $x = m_0/2T_{EW}$, in terms of the vacuum Higgs mass $m_0$ and the critical temperature of the electroweak phase transition $T_{EW}$. Laboratory bounds on $m_0$ thus imply that $x > 0.4$. The thermal masses of the electrons are much smaller and have been neglected. Again comparing to the expansion rate, we find that the decays come into equilibrium at a temperature

$$T_* = 80x^2 \text{ TeV}.$$  \hspace{1cm} (6)$$

This temperature scale is significantly below the sphaleron scale $T_m$ and even the supersymmetric scale $T_s$. Though it appears to be only a modest improvement over the inflationary scale $T_r$, our determination of $T_*$ carries none of the uncertainties necessarily carried by the inflationary bound.

Previous work took as the criterion for preserving the baryon asymmetry that $T_f$ must exceed some very high temperature, where the baryons were first produced or the sphalerons first came into equilibrium. Our criterion is much weaker; we demand only that $T_f > T_*$, assuming that the asymmetry of $e_R$’s is of the same order as the original baryon asymmetry, which generally happens in GUT baryogenesis scenarios. This condition on the freezeout temperatures translates into a bound on the neutrino mass matrix elements,

$$\mu \equiv (2|m_{ee}|^2 + |m_{e\mu}|^2 + |m_{e\tau}|^2)^{1/2} < 1.2 \times 1^{-1} \text{ keV}.$$  \hspace{1cm} (7)$$

Because we have allowed for mixing between the neutrino flavors, we do not obtain a direct bound on one of the neutrino masses as in ref. [1]. To rephrase (7) in terms of potentially measurable quantities, first note that $m_{ee}$ is precisely the quantity constrained to

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be less than 1 eV by searches for neutrinoless double beta decay \[11\], and so it is negligible in (7). It is also convenient to assume that the mixing angles $\theta_{i\alpha}$ and $CP$-violating phases that arise from the diagonalization of $m_{ij}$ are small, and that there is a hierarchy of mass eigenvalues. (If any of the mixing angles was large, the corresponding mass eigenvalue would have to be so small as to be irrelevant in equation (7).) Then $\mu$ is simply related to $\theta_{e\alpha}$ and the masses $m_\alpha$ by

$$\mu^2 \cong \theta_{e2}^2 m_2^2 + \theta_{e3}^2 m_3^2,$$

plus corrections of order $\theta_{e\alpha}^4$. One quickly realizes that the bound (7) guaranteeing preservation of the baryon asymmetry must already be satisfied due to other constraints, unless the standard model is supplemented with some way for heavy neutrinos to decay much faster. If $m_2$ or $m_3$ is in the keV region, the corresponding neutrino will make too large a contribution to the energy density of the universe, since only the radiative decay mode $\nu \rightarrow \gamma \nu_1$ is available, and this is far too slow. If $m_3 > 2m_e$ then the weak decay $\nu_3 \rightarrow e^+ e^- \nu_1$ can occur, with a lifetime of

$$\tau_3 \cong 9 \times 10^{-4} \left( \frac{31 \text{ MeV}}{m_3} \right)^5 \theta_{e3}^{-2} \text{s}. \quad (9)$$

The experimental limit on this mixing angle is $\theta_{e3}^2 < 10^{-6}$ for a 31 MeV neutrino, and it relaxes to $\theta_{e3}^2 < 10^{-4}$ if $m_3 = 3.5$ MeV \[12\]. The respective lifetimes are therefore 900 s and $5 \times 10^5$ s. Although these are compatible with the expansion of the universe, they are in strong conflict with nucleosynthesis \[13\] and cosmic microwave background constraints.

A remarkable feature of our result, which provides a new means for generating the baryon asymmetry, is that it is independent of the initial values of $B$ and $L$, and it applies so long as three requirements are satisfied: (1) the existence of a primordial $e_R$ asymmetry comparable to the present baryon asymmetry, (2) the absence of any exotic interactions involving $e_R$ which would wash out this asymmetry before the electroweak phase transition, and (3) additional $B$- or $L$-violating interactions which must have frozen out between $10^{12}$ GeV, when sphalerons reached equilibrium, and $T_*$, the temperature at which $e_R$-$e_L$ equilibrium began.\footnote{To be precise, this statement applies specifically to baryon or electron-type lepton number. Violation of other varieties $L$ can remain in equilibrium even below $T_*$, resulting in a different, yet nonzero final baryon asymmetry, as will be shown in \[10\].}
Consider again the case of the dimension five $\Delta L = 2$ operator. Under the above conditions, the $L$-violating and sphaleron reactions will establish equilibrium between all interacting species, with the boundary condition that the $e_R$ asymmetry is conserved. Because $e_R$ carries charge, this constraint carries over to the interacting species because the universe is charge-neutral. One can easily show that

$$B = 12\mu_{qL};$$
$$L = 3(\mu_{\mu L} + \mu_{\tau L}) + 2\mu_{eL} + \mu_{eR} - 2\mu_h;$$
$$Q = 6\mu_{qL} - 2(\mu_{\mu L} + \mu_{\tau L}) - \mu_{eL} - \mu_{eR} + 13\mu_h,$$

where $\mu_{qL}$ and $\mu_h$ are the chemical potentials for left handed quarks and Higgs bosons and $Q$ is the total charge. Imposing $Q = 0$ along with constraints coming from the sphaleron processes, $9\mu_{qL} + \sum_{i=1}^{3}\mu_i = 0$, and lepton number violation, $\mu_i = -\mu_h$ ($i \equiv e_L, \mu_L, \tau_L$), one finds that just above $T_*$, $B$ and $L$ are given in terms of the primordial $e_R$ asymmetry $\mu_{eR,p}$

$$B_* = \frac{1}{3}\mu_{eR,p};$$
$$L_* = \frac{1}{2}\mu_{eR,p},$$

regardless of the primordial values of $B$ and $L$. Assuming there was no lepton number violation below $T_*$ would then lead to a final baryon number completely determined by $\mu_{eR,p}$,

$$B_f = \frac{28}{79}(B_* - L_*) \cong -0.11\mu_{eR,p},$$

Note that for the particular case that $B - L$ was initially zero, lepton (or baryon) number violation at some intermediate scale is actually helpful for generating a baryon asymmetry, rather than being constrained. The strength of $L$-violation needed is quite modest: all we require is that either an $L$-violating decay or scattering remain in equilibrium to temperatures below $T_m \sim 10^{12}$ GeV. This could be accomplished by a tau neutrino mass of at least $10^{-2}$ eV [14] in the case of decays, or a slightly larger mass of 0.1 eV in the case that $\Delta L = 2$ scatterings dominate over decays [3].

Although we have focused on the $\Delta L = 2$ operator $LLHH$, the same condition must be applied to other $B$ or $L$ violating operators, namely those interactions must go out of equilibrium at $T > T_*$. Indeed there is a large number of potentially dangerous operators
in grand unified or supersymmetric theories, bounds on which were considered in detail in refs. [3, 4, 5, 15]. Our arguments above modify the results for all dimension $D > 5$ operators. For renormalizable operators, e.g., those found in supersymmetric models with $R$-parity violation, the interaction rate increases slower than the Hubble rate as a function of temperature, and the bound is derived using the lowest temperature at which sphalerons are in equilibrium, $T \sim 100$ GeV. For a nonrenormalizable operator of the form $M^{-n}O_D$ with $D = 4 + n$, the strongest bound is based on the highest temperature the two interactions ($B$ or $L$ violation, and $e_L \leftrightarrow e_R$) are both in equilibrium [5],

$$M^{2n} \gtrsim \frac{1}{17} M_P T^2_{*}^{2n-1}$$

(13)

Because of the smallness of the electron Yukawa coupling, we must take $T_*$ as the maximum temperature, giving a weaker condition than previous derivations that overlooked this fact.

An interesting application of this limit is to a $\Delta B = 2$, $D = 9$ operator that would induce neutron-antineutron oscillations [3]. Using $T_* \sim 10$ TeV from (3) we find that the limit on the heavy mass scale $M$ suppressing the operator becomes $M \gtrsim 3 \times 10^5$ GeV, which is substantially weaker than the stronger of the two bounds quoted in [3] and is comparable to the current experimental bound of $M > 10^5 - 10^6$ GeV.

Our mechanism for the generation of a baryon asymmetry also applies to general $B$ or $L$ violating operators. For example the operator $M^{-5}(u_Rd_Rd_R)^2$, which would induce $n-\bar{n}$ oscillations, imposes the condition $\mu_{u_R} + 2\mu_{d_R} = 0$ on the chemical potentials while it is in equilibrium. Supposing this was true only for $T > T_*$, so that the limit (13) was satisfied, one would find the final baryon number to be proportional to the difference of the primordial $e_L$ and $e_R$ asymmetries,

$$B_f = \frac{8}{79}(\mu_{e_L,p} - \mu_{e_R,p}).$$

Similar results would be obtained for other operators. It could also happen that interactions of more than one operator were in equilibrium simultaneously, but since the interpretation of such situations is somewhat more convoluted, we will postpone their discussion until the forthcoming publication [16].

We conclude by stressing that our mechanism for regenerating the baryon asymmetry from an $e_R$ asymmetry is generic, in the sense that it is natural for decaying GUT gauge

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3 Because of the additional residual conserved quantities $L_e - L_\mu$ and $L_e - L_\tau$, this difference is also conserved.
bosons or supersymmetric condensates to produce as large an excess of $e_R$’s as of any other particle. Although there are many possible effective interactions that could destroy the $e_R$ asymmetry, we have noted that the most popular one is already so constrained by laboratory experiments that it cannot interfere with our scenario. This is the dimension five operator (2) that would give seesaw masses to neutrinos. More generally, we have shown that other interactions threatening the baryon asymmetry are impotent unless they stay in equilibrium below temperatures around 10 TeV, the temperature at which an $e_R$ excess becomes vulnerable to being erased, so that these interactions are less tightly constrained than was formerly supposed.

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Fig. 1. Lepton-violating and left-right equilibrating rates.

Fig. 2. Decay of Higgs boson into $e_R$ and $L_e$ doublet.