Determination of maximum pile spacing of anti-slide pile with rectangular section considering soil arching effects

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Abstract. The reasonable determination of pile spacing is an important link in the design of anti-slide pile. At present, there are many research results on the calculation method of maximum pile spacing of anti-slide pile. Due to different assumptions, different angles of analysis, and different complexity of the formula, the calculation results are quite different, and the research results have not been widely popularized. In this study, taking into account the whole soil arching as the research object, the formulas for calculating the maximum pile spacing of the anti-slide pile based on the soil arching effect behind piles or between piles are put forward considering the overall mechanical balance condition and strength condition of the soil arching, and its rationality is verified by an engineering example. Through calculation and analysis, it is considered that the soil arching between piles controls the maximum pile spacing of anti-slide pile. Through the comparison and analysis with the previous research, the results showed that the calculation formula in this paper is reasonable, feasible and simple, and is convenient for popularization and application in actual engineering.

1. Introduction
The anti-slide pile is one of the most commonly used supporting measures in the landslide prevention and control engineering. It protects the slope by the principle of mechanical balance and is widely used in the highway slope support. The anti-slide pile is calculated in the design time zone by rigid pile and elastic pile. In the design, the anchoring depth and pile spacing are generally based on experience. The anchoring depth is generally 1/3-1/2 of the pile length, and the pile spacing is generally 3-5 times the pile diameter. Actually, the determination of pile spacing is greatly influenced by the soil arching effect between two adjacent piles, and the pile spacing affects the formation of soil arching. When the distance between anti-slide piles is small, soil arching is formed between piles, and the landslide thrust between piles is transmitted from soil arching to piles on both sides, at this time, the strength of soil arching can ensure that the soil will not slide and extrude along the piles. With the increase of pile spacing, the landslide acting on the soil arching. When the pile spacing reaches an extreme value, the soil arching cannot be formed between piles or the strength of the soil arching is not enough to resist the landslide thrust behind the pile, which makes the soil between piles extrude or slide around the pile. At this time, the supporting effect of the anti-slide pile cannot meet the design requirements. In addition, soil layer parameters and pile-soil interface parameters also have a great
influence on soil arching effect. However, when the specific slope is determined and the soil parameters remain unchanged, the reasonable design of the spacing of anti-slide piles is particularly important.

Research on the reasonable pile spacing of anti-slide piles based on the effect of soil arching has been in progress, mainly focusing on three situations: (1) Zhou et al. [1], Jiang et al. [2], Xiao and Cheng [3] and Wang et al. [4] and other researchers believe that the soil arching generated between anti-slide piles is located at the back side of the pile, that is, the soil arching foot is the triangular compression area behind the pile; (2) Wang et al. [5], Li et al. [6], Wang [7], Zhou et al. [8], Zhao et al. [9], Hu and Wang [10] and Ma and Shang [11] believe that the soil arching between anti-slide piles occurs between two piles, and the soil arching foot is the friction arch foot produced by the pile-soil contact surface; (3) Jia et al. [12], Zhang et al. [13] believe that the soil arching effect has the function of bearing the thrust together between piles and behind piles. On this basis, it is deduced that the soil arching effect is applicable to circular piles and square piles calculation method of the maximum pile spacing.

Although a lot of results have been obtained on the calculation method of the maximum pile spacing of anti-slide piles, it is found by comparison that the calculation results of the same case are quite different for different calculation models. The different assumptions of the calculation model, the different complexity of the calculation method and the different angles of the analysis result in the big difference of the calculation results. In this paper, the relevant assumptions are improved based on the previous research results, and the calculation method of the maximum pile spacing of anti-slide pile with simple calculation method is proposed.

2. Formation stage of soil arching

Vardoulakis et al. [14] put forward the concept of "transition state" of soil arching action. Based on experiment, Chevalier et al. [15] found that the formation of excavated arch mainly goes through three stages: initial stage, transition stage and final stage. Through the analysis, two conclusions are obtained [16]: 1) the formation of soil arching is mainly related to the nature of soil, the depth of soil and the width of arching; 2) in the transition stage of the formation of soil arching, there is a small main stress arching [17], and in the last stage, the soil arching is a large main stress arching. The final stage of soil arching formation is an important stage of soil arching stress analysis. The research on the maximum pile spacing of anti-slide pile mainly focuses on the stress in the last stage. The analysis shows that the soil body deforms after being stressed. With the deformation of the soil body, the soil arching is formed first behind the pile. With the increase of thrust, the deformation increases, and the soil arching is gradually formed between the piles. At this time, the soil arching behind the pile and between the piles bear the thrust together. With the further increase of thrust, the landslide thrust between the piles is gradually transformed into the soil arching between the piles and transferred to the surrounding piles. With the increase of the deformation of soil arching between piles, the soil arching between piles is destroyed and the soil is extruded.

3. Calculation of the maximum pile spacing considering the effect of soil arching behind piles

Zhou et al. [1] proposed a reasonable pile spacing discussion based on the soil arching effect behind the pile. The main assumption is that the thrust distribution situation is rectangular. Taking the soil arching of unit depth as the research object, the calculation formula of the reasonable pile spacing for anti-slide pile was established according to the static balance condition and strength condition of the soil arching. Li et al. [16] proposed the maximum anti-slide pile spacing when the thrust triangle distribution was proposed based on Zhou et al. [1]. For the calculation formula of pile spacing, the soil arching within the range of unit depth is still used for stress analysis, but the thrust value is the maximum value of triangle load, and the calculation result is conservative. There are many kinds of distribution forms of landslide thrust, which are usually simplified into three forms, namely rectangle distribution, triangle distribution and trapezoid distribution. Based on this idea, the whole soil arching is selected as the research object. On the basis of the known single width landslide thrust, the
distributed force resultant force is used as the thrust, and the mechanical balance condition and
strength condition of the whole soil arching are used as the calculation basis to deduce and establish
the soil arching effect based on the pile. The calculation formula of the maximum pile spacing of the
anti-slide pile is given.

3.1. Assumptions
(1) After the anti-slide pile, the soil arching forms a triangle compression area, namely the soil arching
foot. (2) It is assumed that the shape of soil arching is reasonable arch axis. (3) The thrust value of
single width landslide is selected for calculation, and the shape of soil arching is assumed to be
constant along the depth. (4) It is assumed that the soil arching behind the pile will not be damaged
along the pile-soil interface.

3.2. Computational model

Based on the above assumptions, the stress diagram of soil arching behind the pile is obtained, as
shown in Figure 1.

where $p$ is single width landslide thrust (kN/m); $a$ is pile section width (m); $b$ is pile section height (m);
$l_1$ is distance between two piles (m); $l_2$ is span of soil arching, which simplified net distance between
two piles (m); $f$ is arch height (m); $d$ is arch thickness (m); $\theta$ is angle between tangent direction of arch
axis and horizontal line (°); $F_x$, $F_y$, $F_x'$, $F_y'$ is the reaction force (kN) of the arch feet on both sides
in X and Y directions, respectively.

From the reasonable equation of arch axis, Equation (2) can be obtained by the derivative at $x=0$

\[ y = \frac{4fx}{l_2^2}(l_2 - x) \]  

\[ y' = \frac{4f}{l_2} = \tan \theta \]  

When the soil arching reaches the limit equilibrium state, the stress analysis is based on the three
hinge arch, and the reaction force at the arching foot is obtained

\[
\begin{align*}
F_x &= \frac{pl_2^2}{8f} \\
F_y &= \frac{pl_2}{2}
\end{align*}
\]
It can be obtained from references [1, 3, 6] that there may be a failure surface at the arch foot of the soil arching. The soil mass in the triangle compression area behind the pile is sheared along the horizontal direction, and the angle between the failure surface and the direction of the major principal stress is $\theta$. There are:

$$\tan \theta = \tan(45^\circ - \frac{\phi}{2}) = \frac{4f}{l_2} \tag{4}$$

The transformation results are as follows:

$$f = \frac{l_2 \tan \theta}{4} \tag{5}$$

From the trigonometric function relationship in Figure 1, we can get:

$$\sin \theta = \frac{a}{2d} \tag{6}$$

The transformation results are as follows:

$$d = \frac{a}{2 \sin \theta} \tag{7}$$

Under the uniform loading, the reasonable arch axis of three hinged arch is quadratic parabola, i.e. the bending moment and shear force are zero, and only under the action of axial pressure. According to the analysis of the formation stage of the earth arch, the final stage of the earth arch is a large main stress arch, which is decomposed by the stress at the arch foot in the figure,

$$\sigma_1 = \frac{T}{dh_1} \tag{8}$$

where $T$ is the value of earth arch axial pressure, which is the resultant force of and (kN); $\sigma_1$ is the large main stress of soil arching (kPa); and $h_1$ is soil arching depth (m). Because the mid-span section of the soil arching is the most unfavorable section, the leading edge point of the mid-span is more unfavorable than the trailing edge point, and because the soil arching is in a state of unidirectional compression, according to the Mohr-Coulomb criterion, it can be concluded that

$$\sigma_1 = 2c \tan \left(45^\circ + \frac{\phi}{2} \right) \tag{9}$$

When the soil arching reaches the limit equilibrium state:

$$\frac{T}{dh_1} \leq 2c \tan \left(45^\circ + \frac{\phi}{2} \right) \tag{10}$$

where $c$ is the cohesion of soil behind anti-slide pile (kPa); and $\phi$ is the internal friction angle of soil behind anti-slide pile ($^\circ$).

Combining equations (5), (7) and (10), it can be concluded

$$l_2 \leq \frac{2cah_1 \tan \left(45^\circ + \frac{\phi}{2} \right)}{p} \tag{11}$$

When the distance of the pile meets the requirements of Equation (11), the soil arching behind the pile can bear the landslide thrust behind the pile and transmit it to the anti-slide pile. When the distance of the pile does not meet the requirements of Equation (11), the soil arching behind the pile may move to the pile.
3.3. Instance validation

In order to verify the reliability of the derived formula, an example in reference [3] is selected for analysis. The project is a high cutting slope supported by cantilever anti-slide piles with $\gamma=20$ kN/m$^3$, $c=50$ kPa, $\varphi=28^\circ$. The anti-slide pile is 22 m in length, the cantilever end $h_1=11$ m, the section size $a\times b=2\times 3$ m, and the single width landslide thrust $P=1050$ kN/m.

Substituting the parameter into Equation (11), we can get:

$$l_2 \leq \frac{2cah_1 \tan(45^\circ + \varphi/2)}{p} = \frac{2 \times 50 \times 2 \times 11 \times \tan 59^\circ}{1050} = 3.48 \text{m}$$

$$l_1 = l_2 + a = 3.48 + 2 = 5.48 \text{m}$$

It can be seen that the actual pile spacing of the project is 6 m, which is greater than the value of the pile spacing calculated by the formula. After the completion of the retaining works, the slope has a good stability. It can be seen that the maximum pile spacing calculated by Equation (11) is relatively small.

4. Calculation of the maximum pile spacing considering the effect of soil arching between piles

Based on the theory of soil arching effect between piles, Wang et al. [5] proposed a formula for calculating the maximum pile spacing of anti-slide piles. It is assumed that the landslide thrust is transformed into the side friction of anti-slide piles in the form of normal stress without any loss, which is a relatively ideal state. In this section, based on the assumption that the shape of the soil arching is a reasonable arch axis, the whole soil arching along the depth range is taken as the research object. By establishing the static balance condition of the whole soil arching, the side friction resistance borne by the friction arch foot is calculated. Combined with the strength condition of the mid-span soil arching, the calculation formula of the maximum pile spacing based on the soil arching effect between piles is derived.

4.1. Assumptions

(1) The geotechnical shape is the reasonable arch axis; (2) The soil arching foot is the friction arch foot, that is, the side friction resistance provides the support; (3) The soil resistance in front of the pile is not considered; (4) The soil arching shape is evenly distributed along the depth range, and the single width landslide thrust is evenly distributed between the piles.

4.2. Computational model

From the analysis of the formation stage of soil arching, it can be seen that the soil arching between piles is the last line of defense to bear the landslide thrust behind piles. How to determine the reasonable spacing of anti-slide piles according to the theory of soil arching effect between piles, so as to make the anti-slide piles fully play its role, is an important step to ensure the rational, reliable and economic design of anti-slide piles, as shown in Figure 2.

where $a$ is pile section width (m); $b$ is pile section height (m); $l_2$ is soil arching span (m); $p$ is the single width landslide thrust within the depth range (kN/m); $F_x, F_y, F_x', F_y'$ are the components of the reaction force at the arch foot in the $x$ and $y$ directions (kN).
Figure 2. Calculation diagram of soil arching effect between adjacent piles

Based on the assumption that the shape of soil arching conforms to the reasonable arch axis, the stress analysis is carried out according to the three hinged arch, and the static balance equation is established, then the component force at the arch foot is:

\[
\begin{align*}
F_x &= \frac{pl_2^2}{8f} \\
F_y &= \frac{pl_2}{2}
\end{align*}
\]  

(12)

When the pile side friction is greater than or equal to the effective landslide thrust, the soil arching between piles can work normally, otherwise the soil arching will fail. That is to say, the condition of Equation (13) is satisfied

\[
2(F_x \tan \phi' + c' bh_1) \geq pl_2 - \gamma h l_2 \tan \phi - c bl_2
\]

(13)

where \( \phi \) is internal friction angle of soil between piles (°); \( \phi' \) is internal friction angle between anti-slide pile and soil between piles (°), and \( \phi' = \frac{\phi}{2} \); \( c \) is cohesion of soil between piles (kPa); \( c' \) is cohesion of soil between anti-slide pile and pile (kPa), and \( c' = \frac{c \tan \phi'}{\tan \phi} \); \( \gamma \) is soil weight between piles (kN/m³); \( h_1 \) is length of anti-slide pile above sliding surface (i.e. soil arching depth) (m).

There is the most unfavorable section in the soil arching span, and the leading edge point is the most unfavorable. Under the uniform loading, the arch axis is similar to a parabola, and the soil arching is in a state of unidirectional compression. Therefore, the soil arching should meet the section strength condition of Equation (14):

\[
\sigma_i = \frac{F_{x_i}}{bh_1} = 2c\tan(45^\circ + \frac{\phi}{2})
\]

(14)

Replace Equation (12) into Equation (14), it can be derived

\[
f = \frac{pl_2^2}{16 c b h_1 \tan(45^\circ + \frac{\phi}{2})}
\]

(15)
Combining Equations (12), (13) and (15), it can be derived
\[
l_2 \leq \frac{4bh_1 \tan(45^\circ + \frac{\varphi}{2}) \tan \varphi' + 2c'b \varphi}{p - \gamma h_1 \tan \varphi - cb}
\]  

(16)

4.3. Instance validation

The same engineering case is cited for analysis, and the calculation is as follows.

\[
\varphi' = \frac{\varphi}{2} = 14^\circ, \ c' = \frac{c \tan \varphi'}{\tan \varphi} = 23\text{kPa}
\]

Substituting relevant parameters into Equation (16), it can be obtained

\[
l_2 \leq \frac{4bh_1 \tan(45^\circ + \frac{\varphi}{2}) \tan \varphi' + 2c'b \varphi}{p - \gamma h_1 \tan \varphi - cb}
\]

\[
\leq \frac{4 \times 50 \times 3 \times 11 \times \tan 59^\circ \times \tan 14^\circ + 2 \times 23 \times 3 \times 11}{1050 - 20 \times 11 \times \tan 28^\circ - 50 \times 3} = 5.43\text{m}
\]

\[
l_i = l_2 + a = 5.43 + 2 = 7.43\text{m}.
\]

Therefore, the pile spacing should be less than 7.43m. The actual pile spacing of the project is 6m. After the completion of the construction, the actual operation effect of the project is good. Therefore, the value of 6 m pile spacing is reasonable. Moreover, within the range of the calculation results of the equation, the rationality of the equation proposed in this paper is further proved.

Comparing the calculation equation of pile spacing under the two soil arching effects with the case calculation results, we can know the value of the maximum pile spacing controlled by the soil arching strength between piles. In the design of anti-slide pile, the reasonable value of anti-slide pile spacing can be selected by referring to the calculation formula of the maximum pile spacing based on the soil arching effect between piles. On the premise of ensuring safety, the pile spacing of anti-slide pile can be increased appropriately to achieve the purpose of safety and economy.

5. Comparative analysis between existing research results

For the case cited in this paper, the calculation method in different literature is used to calculate the maximum pile spacing control value, and the results are shown in Table 1. It can be seen that the calculation results of different calculation methods are quite different. The calculation results in this paper are basically consistent with the research results of Xiao and Cheng [3], Ma and Shang [11], Wang et al. [4], and are consistent with the actual project. Xiao and Cheng [3] and Wang et al. [4] considered that the inverted trapezoid compression area behind the pile was the soil arching foot behind the pile, and the calculation formula of the maximum pile spacing was derived considering the strength condition of the arch foot and the strength condition of the mid-span section; Ma and Shang [11] deduced the calculation formula of the pile spacing from the anti sliding bearing capacity. In this paper, different from the consideration of literature, it is assumed that the soil arching foot behind the pile is a triangle in compression area. By comparing the calculation formula of the maximum pile spacing derived in two cases, the maximum pile spacing value of soil arching control between piles is obtained. The parameters of the calculation formula are easy to select, the calculation results are reasonable and easy to promote.
Table 1 Statistics of the maximum pile spacing for different calculation methods

| References         | Pile spacing (m) | References         | Pile spacing (m) |
|--------------------|------------------|--------------------|------------------|
| Wang et al. [5]    | 8.71             | Hu and Wang [10]   | 4.72             |
| Zhou et al. [8]    | 6.79             | Liu et al. [18]    | 9.04             |
| Li et al. [6]      | 10.01            | Ye [19]            | 12.56            |
| Zhou et al. [1]    | 4.93             | Zhang et al. [13]  | 8.65             |
| Jiang et al. [2]   | 8.97             | Ma and Shang [11]  | 7.2              |
| Zhao et al. [9]    | 4.95             | Wang et al. [5]    | 7.49             |
| Xiao and Cheng [3] | 7.57             | This paper         | 5.48/7.43        |

6. Conclusions
(1) Based on the theory of soil arching effect behind piles, certain assumptions, overall static balance condition and soil arching strength condition, the control value of maximum pile spacing considering soil arching effect behind piles is calculated and determined.

(2) Based on the theory of soil arching effect between piles and certain assumptions, the static balance equation of "reasonable arch axis" is established under the condition that the pile side friction is greater than or equal to the effective landslide thrust between piles, and the control value of anti-slide pile spacing between piles is calculated and determined in combination with the mid-span strength condition.

(3) Based on the analysis of the last stage of the soil arching, it is considered that the soil arching first occurs after the pile, and then gradually transfers to the pile. The strength of soil arching between the piles is the last line of defense. The results of the improved calculation formula also prove that the soil arching effect between piles controls the maximum spacing of anti-slide piles.

(4) Comparing and analyzing the existing research results, it is considered that the calculation equation of the distance between anti-slide piles derived in this paper is simple in calculation method, convenient in parameter value taking, reasonable and reliable in calculation result, which can be used as the reference for the value taking of the distance between piles in engineering practice.

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