Evolution of low-lying M1 modes in germanium isotopes

S. Frauendorf and R. Schwengner

Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA
Helmholtz-Zentrum Dresden-Rossendorf, 01328 Dresden, Germany

(Dated: March 3, 2022)

Magnetic dipole strength functions are determined for the series of germanium isotopes from \( N = Z = 32 \) to \( N = 48 \) on the basis of a large number of transition strengths calculated within the shell model. The evolution of the strength with increasing neutron number in the \( 1g_{9/2} \) orbital is analyzed. A bimodal structure comprising an enhancement toward low transition energy and a resonance in the region of the scissors mode is identified. The low-energy enhancement is strongest near closed shells, in particular at the almost completely filled \( 1g_{9/2} \) orbital, while the scissorslike resonance is most pronounced in the middle of the open shell, which correlates with the magnitude of the also deduced electric quadrupole transition strengths. The results are consistent with previous findings for the shorter series of iron isotopes and prove the occurrence and correlation of the two low-lying magnetic dipole modes as a global structural feature.

PACS numbers: 21.10.Pc, 21.60.Cs, 23.20.Lv, 27.50.+e

I. INTRODUCTION

The excitation and deexcitation of the nucleus by electromagnetic radiation at high excitation energy and high level density are described by means of \( \gamma \)-ray strength functions (\( \gamma \)-SF) which represent average transition strengths in a certain energy range. The experimental determination and the theoretical understanding of the properties of \( \gamma \)-SF has attracted increasing interest because of their importance for the accurate description of photonuclear reactions and the inverse radiative-capture reactions, which play a central role in the synthesis of the elements in various stellar environments \cite{Got09, Dem11}. The region of low transition energies is important for these processes. Traditionally, the \( \gamma \)-SF has been associated with electric dipole (E1) transitions. Recently a strong component of M1 character has been observed as an upbend of the \( \gamma \)-SF toward transition energy \( E_\gamma = 0 \).

First observed in \( 56,57 \)Fe \cite{Got09}, the upbend was found also in various nuclides in other mass ranges, such as in Mo isotopes \cite{Men14}, in \( 105,106 \)Cd \cite{Men14}, and in Sm isotopes \cite{Men14a, Men14b}. The experiments used light-ion induced reactions such as \((^3\text{He},\text{He})\), and the data were analyzed with the so-called Oslo method to extract level densities and \( \gamma \)-SFs. This method was also applied in connection with \( \beta \) decay to \( 76 \)Ga \cite{Men14b}. A dominant dipole character of the low-energy strength was demonstrated in Ref. \cite{Got09}, and an indication for a magnetic dipole (M1) character was discussed for the case of \( 60 \)Ni \cite{Men14a}.

Shell-model calculations revealed that a large number of M1 transitions between excited states produce an exponential increase of the \( \gamma \)-ray strength function that peaks at \( E_\gamma \approx 0 \) and describes the low-energy enhancement of dipole strength observed in Mo isotopes around the neutron shell closure at \( N = 50 \) \cite{Men14b}. In these calculations, large reduced transition strengths \( B(M1) \) appear for transitions linking states with configurations dominated by both protons and neutrons in high-\( j \) orbitals, the spins of which recouple. The low-energy enhancement was confirmed in shell-model calculations for \( 56,57 \)Fe \cite{Men14a, Men14b}, \( 46,50,54 \)Ti \cite{Men14a} and \( 44 \)Sc \cite{Men14a}. In the latter work, also the electric dipole (E1) strength function was calculated, which does not show an upbend comparable to that of the M1 strength. A correlation between the low-energy M1 strength (LEMAR - Low Energy Magnetic Radiation) and the scissors resonance (SR), a fundamental M1 excitation occurring in deformed nuclei around 3 MeV \cite{Men14b}, was found in shell-model calculations for the series of isotopes from \( 60 \)Fe to \( 68 \)Fe \cite{Men14b}. It was found that the low-energy M1 strength decreases and the scissors strength develops when going into the open shell. The simultaneous appearance of the two M1 modes is in accordance with experimental findings in Sm isotopes \cite{Men14a, Men14b}.

Later on, M1 strength functions were calculated for isotopic series in various mass regions \cite{Men14b, Men14b}. The study in Ref. \cite{Men14b} confirmed that the low-energy M1 strength is strongest in nuclides near shell closures.

In the present work we study the low-energy \( \gamma \)-SFs for the chain of the Ge isotopes. The relatively small configuration space allows us to carry out shell model calculations covering completely the open neutron shell \( 32 \leq N \leq 48 \). We demonstrate for the first time that the low-energy M1 strength is concentrated in the LEMAR spike at the bottom of the shell, it is partially moved into the SR in the middle of the shell, and is again concentrated in the LEMAR spike at the top of the shell.

II. SHELL-MODEL CALCULATIONS

The shell-model calculations for the germanium isotopes were carried out in the \( jj44 \) model space with the \( jj44 \) Hamiltonian \cite{Fra01, Fra02} using the code NuShellX@MSU \cite{Bau97}. The model space included the
proton and neutron orbitals \((1f_{5/2}, 2p_{1/2}, 2p_{3/2}, 1g_{9/2})\).

At first, we calculated the energies of the yrast states and the reduced transition strengths of the linking electric quadrupole \((E2)\) transitions for varied limitations of occupation numbers to test at which numbers the \(B(E2)\) values do not change further and a convergence is achieved. This is in particular important for the mid-shell isotopes. For example, an increase of the allowed maximum occupation number (upper limit) in the neutron \(1g_{9/2}\) orbital from four to six in \(^{70}\text{Ge}\) does not change the \(B(E2, 2^+_1 \rightarrow 0^+_1)\) value and, thus, the application of an upper limit of four is appropriate. In \(^{74}\text{Ge}\), a change of this number from six to eight changes the \(B(E2, 2^+_1 \rightarrow 0^+_1)\) value from 357 to 369 e\(^2\)fm\(^4\), while the further increase to ten neutrons does not cause any further change. At the same time, the allowed minimum occupation numbers (lower limits) in the neutron \(2p_{3/2}\) and \(1f_{5/2}\) orbitals were set to two. A decrease of these lower limits to zero results in \(B(E2, 2^+_1 \rightarrow 0^+_1)\) = 371 e\(^2\)fm\(^4\). In the full calculations including all transition strengths, the following limits of occupation numbers were applied.

The reduced transition strengths \(B(E2)\) limits to zero results in \(\Delta\) values were deduced in an analogous way, but include the \(\Delta\) values were deduced in an analogous way, but include the level densities derived from the present calculations. The strength functions \(f_{M1}(E_\gamma)\) were obtained by averaging step-by-step over \(E_i, J_i,\) and \(\pi\).

### III. RESULTS FOR THE YRAST REGION

To check the reliability of the shell-model calculations, we studied the yrast regions of the Ge isotopes. The calculated energies of the ground-state and first excited bands in \(^{74}\text{Ge}\) are compared with the experimental ones \(^{24}\) in Figs. 1 and 2. They represent the results obtained with the just discussed upper limits of six and eight \(1g_{9/2}\) neutrons, respectively. In both cases, the experimental bands are well described by the calculations. A similarly good description of the experimental yrast and yrare bands by the present calculations is achieved for all other isotopes, which are presented in Figs. 11 and 12 of the appendix.

In accordance with the experiment, the energies \(E(J)\)
TABLE I: Experimental and calculated reduced transition probabilities for the $2^+_n \rightarrow 0^+_1$ transitions in $^{64,66,70,74,78,80}$Ge.

| $E(2^+_n)$ (keV) | $B(E2, 2^+_n \rightarrow 0^+_1)$ (e²fm⁴) | $\Sigma B(E2, 2^+_n \rightarrow 0^+_1)$ (e²fm⁴) | EXP a | CALC | EXP a | CALC | CALC |
|----------------|---------------------------------|---------------------------------|-------|------|-------|------|------|
| $^{64}$Ge a | 902 | 883 | 296 | 308 | | | |
| $^{66}$Ge a | 957 | 828 | 190(36) | 300 | 321 | | |
| $^{70}$Ge a | 1040 | 649 | 356(7) | 336 | 382 | | |
| $^{74}$Ge a | 596 | 704 | 609(7) | 369 | 384 | | |
| $^{78}$Ge a | 619 | 782 | 455(79) | 320 | 343 | | |
| $^{80}$Ge a | 659 | 871 | 279(55) | 233 | 280 | | |

a The values for $^{64,66,70,74,78,80}$Ge were taken from Refs. [25,32], respectively.

increase in a regular way, forming quasi-rotational bands. The crossing of the ground-state band with an excited band at $J = 6$ is reproduced. The energy ratios $E(J)/E(2^+)$ deviate substantially from the rotor rule $J(J + 1)$. The calculated ratios $E(4^+)/E(2^+_1)$ of 2.44, 2.59, 2.59, 2.38, 2.32, 2.22 for $^{64,66,70,74,78,80}$Ge compare well with experimental ratios of 2.28, 2.27, 2.07, 2.46, 2.54, 2.64, respectively. They characterize the Ge isotopes as soft nuclei in the transitional region between spherical and deformed shapes, because they are well below the rotor ratio of 3.33. The calculated $B(E2)$ values increase toward high spin up to the crossing region. For $^{74}$Ge, the experimental ratio $B(E2, 4^+_1 \rightarrow 2^+_1)/B(E2, 2^+_1 \rightarrow 0^+_1) = 1.33(8)$ is reproduced by the calculated one of 1.35.

The results for the $2^+_n$ states are compared with the experimental values in Table I. The energies of the $2^+_n$ states are reproduced within 100 keV, except the high experimental value for $^{70}$Ge. The experimental $B(E2, 2^+_1 \rightarrow 0^+_1)$ values indicate a maximum of the quadrupole collectivity in the middle of the shell, which is reproduced by the calculations. However, the calculated peak is much shallower than in the experiment. A similar shallow peak is obtained for $J = 4$ and 6 (see Figs. 11 and 12 of the appendix).

One should be aware that the determination of the quadrupole collectivity from the $B(E2, 2^+_1 \rightarrow 0^+_1)$ values only is based on the assumption of a rotational behavior of the yrast states, which is not realized for the soft nuclei under consideration. Instead, the sums of the $B(E2, 0^+_1 \rightarrow 2^+_1)$ values of all transitions from the ground state are more appropriate [25,26] and also given in Table I. These sums of all $E2$ transitions into the ground state are however only little larger and follow the trends of the $B(E2, 2^+_1 \rightarrow 0^+_1)$ values. An even more comprehensive indicator of the collectivity may be the consideration of average $B(E2)$ values between all the states considered here. The further discussion of $B(E2)$ values in Sec. IV therefore takes into account these values.

IV. RESULTS FOR THE STRENGTH FUNCTIONS

The average $B(M1)$ and $B(E2)$ values for positive parity states and the $M1$ strength functions including both parities are shown for all considered Ge isotopes in Figs. 3, 5, 7, 8. The $N = Z = 32$ nucleus $^{64}$Ge shows a fluctuating, but on average flat distribution of the $B(M1)$ strength as a function of $E_x$, with an even slight decrease toward $E_x = 0$, which is similar to predictions for the $N = Z$ nuclei $^{48}$Ca [13] and $^{108}$Xe [18]. This seems to point to a more general feature of the low-energy $M1$ strength in $N = Z$ nuclei. It is suggested below that isospin conservation quenches the LEMAR spike. In the $N = Z + 2$ nuclide $^{66}$Ge, a gradual enhancement of the $M1$ strength toward $E_x = 0$ is seen. The behavior resembles the one in $^{60}$Fe, but is less pronounced. Both these nuclei are localized near the bottom of the neutron $(fp)$ shell. For $^{72}$Ge, the bimodal structure of a LEMAR peak at $E_x = 0$ and a broad SR peak around $E_x = 3$ MeV appears. A similar bimodal distribution is seen in $^{74}$Ge. This bimodal strength distribution is characteristic for nuclei located well in the open shell, as for example also in $^{64,66}$Fe [38,42] and in nuclides with $A > 100$ [18]. The SR peak becomes weak in $^{78}$Ge and disappears in $^{80}$Ge, when approaching the top of the neutron shell. A similar suppression of the SR peak toward the next higher neutron shell was also found in $N \approx 80$ nuclei [18]. The present calculations find a maximum of the SR strength in the middle of the neutron shell that correlates with the clear maximum of the experimental $B(E2, 2^+_1 \rightarrow 0^+_1)$ values. Unlike the experiment, the calculated $B(E2, 2^+_1 \rightarrow 0^+_1)$ values have a very shallow mid-shell maximum, as was also obtained in the calculations for the Fe isotopes [10].

However, a different behavior is seen for the average $B(E2, J \rightarrow J - 2)$ values shown in the panels (c) of Figs. 3, 5, 7, 8, 10. Here, a peak around 1 MeV develops toward the middle of the shell ($^{70,74}$Ge). This peak indicates enhanced collectivity in the $\Delta J = 2$ sequences and clearly correlates with the maximum of the $M1$ strength in the middle of the shell. It can be interpreted as the appearance of damped rotational transitions (see e.g. Ref. [22]) as a consequence of building-up quadrupole collectivity. The calculations include states up angular momentum 10 $h$ with equal weight. The average transition energy of 1 MeV and the average angular momentum of 5 $h$ correspond to a moment of inertia of about 10 $h^2$/MeV, which is somewhat smaller than the rigid-body value of 14 $h^2$/MeV for $A = 70$. In the same way, from Fig. 4 of Ref. 10 one derives a moment of inertia of 8 $h^2$/MeV for $^{68}$Fe. In Ref. [6], the increased SR strength of the $\gamma$-SF of $^{151,153}$Sm could be reproduced by replacing the ground state moment of inertia by the rigid-body value in the phenomenological expression of Ref. [22], which was developed for the excitation of the SR from the ground state. We calculated the total $B(M1)$ strengths in certain
energy ranges by a numerical integration of the $M1$ strength functions:

$$B(M1)_{int} = 9/(16\pi) \langle hc \rangle^3 \sum f_{M1}(E_\gamma)\Delta E_\gamma.$$  \hspace{1cm} (2)

The results for the LEMAR region ($E_\gamma < 2$ MeV), the SR region (2 MeV $\leq E_\gamma < 5$ MeV) and their sums are compiled in Table II. As visualized in the panels (b) of Figs. 3 and 4, one also quantitatively observes a shift of strength to the SR region when going into the open shell ($^{70,74}$Ge$_{34-50}$) and a shift back to the LEMAR region when approaching the $N=50$ shell closure ($^{78,80}$Ge$_{46-58}$), while the sum of the two remains roughly constant. Only the $N=Z$ nuclide $^{64}$Ge does not fit the systematics for reasons discussed below. In the calculations of Ref. [10], a similar redistribution of the $M1$ strength was found for the isotopes $^{60,64,66}$Ge$_{34-38,42}$ when moving into the open shell by adding neutrons. However, the integrated strength up to 5 MeV, which is the sum of the LEMAR and SR strength, is about 1 $\mu_N^2$ for the Ge isotopes and about 10 $\mu_N^2$ for the Fe isotopes. The authors of Ref. [11] suggested that the low-energy $M1$ radiation is generated by the reorientation of the valence nucleons on high-$j$ orbitals. This mechanism is particular efficient if protons are hole-like and neutrons are particle-like (or vice versa).

Then the transverse magnetic moments add up, which generates strong $M1$ radiation. An analogous mechanism generates the "shears bands" manifesting "magnetic rotation" [34]. In the case of the Fe isotopes one has active 1$f_{7/2}$ proton holes, which favorably combine with the active 1$g_{9/2}$ neutrons. In the case of the Ge isotopes the active 1$g_{9/2}$ neutrons combine with the 1$f_{5/2}$ protons, which have a small magnetic moment, and 1$g_{9/2}$ protons, which have a magnetic moment with the opposite sign. The factor of 10 in the integrated low-energy $M1$ strength reflects the different valence proton configurations of the Fe and Ge isotopes. In case of the $Z > 50$, $N \geq 80$ nuclides the integrated strength up to 4 MeV is $0.5 - 1 \mu_N^2$ (see Fig. 3 of Ref. [18]). The small number is expected because active protons and neutrons are particle-like and do not occupy the high-$j$ orbitals.

Also shown in Table III are the integrated $B(E2)$ strengths up to $E_\gamma = 5$ MeV, which were determined analogously to the integrated $B(M1)$ strengths. The values are maximal at the mid-shell nuclei $^{70,74}$Ge in accordance with the SR strengths, which proves the correlation of SR strength and collectivity.

The excitation of the SR from the ground state, which appears as a bunch of $1^+$ states around 3 MeV, has been extensively studied and reviewed in Ref. [13]. Therein,
FIG. 5: As Fig. but for $^{70}$Ge.

FIG. 6: As Fig. but for $^{74}$Ge.

FIG. 7: As Fig. but for $^{78}$Ge.

FIG. 8: As Fig. but for $^{80}$Ge. Note the different vertical scale in (a) compared with the corresponding graphs for the other isotopes.
the built-up of a SR around 4 MeV when moving into state deformation in a similar way, though being a fac-
tions for the Mo and Fe isotopes as displayed in Figs. 9, 10. An equal reduction appeared in our earlier calcula-
to the quenching of the pair correlations with increasing spin, i.e. the thermal quenching of pairing.

According to the collective model of Ref. [36] the $M1$ strength of the SR on the ground state scales $\propto A\delta^2$, where $\delta$ is the deformation parameter and $A$ the mass number. The sums $\sum B(M1, 1^+ \rightarrow 0^+_1)$ of transition down to the ground states of the Ge, Fe, Te, and Sm isotopes roughly follow the scaling (cf. Table II of Ref. [16]). In Ref. [11], the enhancement was attributed to the quenching of the pair correlations with increasing excitation energy. The simple collective behavior seems to be caused by the pair correlations. The integrated $M1$ strengths for the transitions in the quasicontinuum do not obey it. Once the pair correlations are quenched, the bimodal LEAMAR–SR structure appears in the strength function, the total strength of which depends strongly on the individual magnetic properties of the valence nucleons. This is in analogy to the moments of inertia. At low spin, when the the pair correlations are strong, the moments of inertia behave in a systematic manner, being $\propto A^{5/3}\delta^2$. At high spin, when the Coriolis force overcomes the pair correlation, the individuality of the valence nucleons comes to light.

The experimental summed strengths for the transitions from $1^+$ states to the ground state given in Table II are smaller than the calculated ones. However, the experimental values represent only a lower limit. In the calculations, a large number of weak transitions contributes to the summed strengths, whereas experiments as the ones in Ref. [37] detect only the strongest transitions. It has been demonstrated that a large number of weak transitions, which are hidden in a quasicontinuum, may substantially enlarge the $M1$ strength function, as seen for example in Ref. [38].

The $M1$ operator has approximately isospin $T = 1$ character. In $N = Z$ nuclei the low-lying states have $T = 0$. The $T = 1$ states lie substantially higher. In case of $^{64}$Ge, the experimental energy of the lowest $T = 1$
state is 6.2 MeV. Thus, the sum \( \sum B(M1, 1^+ \rightarrow 0^+) \) = 0.001 \( \mu_N^2 \) in Table II includes only transitions between \( T = 0 \) states, which are isospin forbidden. The very small value for \( ^{64}\text{Ge} \) reflects that isospin conservation nearly quenches \( M1 \) transitions between the \( T = 0 \) states. For \( N = Z + 2 \) nuclei the low-lying states have \( T = 1 \). Transitions between \( T = 1 \) states are allowed, which results in \( \sum B(M1, 1^+ \rightarrow 0^+) = 0.25\mu_N^2 \) for \( ^{66}\text{Ge} \). One expects that the same mechanism works for higher excitation energies. Transitions between the \( T = 0 \) states are nearly forbidden. The \( T = 1 \) states lie on the average substantially above the \( T = 0 \) states that are connected by the \( M1 \) operator, which prevents transition energies close to zero. For \( N > Z \) nuclides, the transitions between the states with the same isospin \( T = 0 \) are allowed and the LEMAR spike appears. The author of Ref. [18] suggested an alternative explanation: \( N = Z \) nuclei have a particular large deformation that moves \( M1 \) strength from the LEMAR spike to the SR, which results in a flat distribution. At variance, the \( N \) dependences of the \( E2 \) strength in Figs. 3 to 8 and Table II indicate little \( E2 \) collectivity for \( ^{64}\text{Ge} \).

V. SUMMARY

Shell-model calculations were performed for the series of germanium isotopes with neutron numbers from \( N = 32 \) to \( N = 48 \). Average \( B(M1) \) and \( B(E2) \) strengths were determined from a large number of transitions linking states of spins from 0 to 10. The average \( B(M1) \) strengths and the associated \( M1 \) strength functions are strongly enhanced near zero transition energy, which is the LEMAR spike observed before. The LEMAR spike develops with increasing neutron number and is strongest at \( N = 80 \). It is suppressed at \( N = Z \), which is attributed to isospin conservation. In the mid-shell nuclei, a bump around 3.5 MeV appears, which is interpreted as the scissors resonance. The strength of the SR correlates with the quadrupole collectivity, as reflected by the experimental \( B(E2, 2_1^+ \rightarrow 0^+) \) values and the integrated average \( E2 \) strength of quasiuminum transitions. The sum of the LEMAR and SR strengths depends only weakly on the neutron number. These characteristics are consistent with those found for the series of iron isotopes and with the experimental observation of LEMAR and SR strengths in samarium isotopes as well. They exhibit the important role of high-\( j \) orbitals, such as \( 1g_{9/2} \) and \( 1h_{11/2} \), for the evolution of the low-lying modes. Spin and orbital contributions to the \( M1 \) strength appear nearly equal at low energy in most isotopes, while there are stronger orbital contributions above 4 MeV of transition energy in the mid-shell isotopes \( ^{70,74}\text{Ge} \). The present systematic analysis of low-lying \( M1 \) strength in a relatively long isotopic series demonstrates that the correlated appearance of the two \( M1 \) modes is a phenomenon that occurs across various mass regions.

VI. ACKNOWLEDGMENTS

We thank B. A. Brown for his support in using the code NuShellX@MSU. The allocation of computing time through the Centers for High-Performance Computing of Technische Universität Dresden and of Helmholtz-Zentrum Dresden-Rossendorf are gratefully acknowledged. S. F. acknowledges support by the DOE Grant DEFG02-95ER4093.

TABLE II: Summed \( B(M1) \) strengths in ranges of transition energy (LEMAR: \( E_\gamma < 2 \) MeV, Scissors: \( 2 \) MeV \( \leq E_\gamma < 5 \) MeV, \( \Sigma \): the sum of the two), summed \( B(E2) \) strengths for \( E_\gamma < 5 \) MeV, and summed strengths of all discrete transitions from \( 1^+ \) states to the ground states in \( ^{64,66,70,74,78,80}\text{Ge} \).

| Nuclide | LR | \( \sum B(M1, 1^+ \rightarrow 0^+) \) | \( \sum B(E2, 2^+ \rightarrow 2^+) \) |
|---------|----|----------------------------------|----------------------------------|
| \( ^{64}\text{Ge} \) | 0.30 | 0.54 0.84 155 | 0.001 |
| \( ^{66}\text{Ge} \) | 0.35 | 0.35 0.70 185 | 0.25 |
| \( ^{70}\text{Ge} \) | 0.54 | 0.62 1.16 219 | 0.04(1) 0.49 |
| \( ^{74}\text{Ge} \) | 0.44 | 0.50 0.94 241 | 0.30(3) 0.57 |
| \( ^{78}\text{Ge} \) | 0.63 | 0.49 1.12 181 | 0.48 |
| \( ^{80}\text{Ge} \) | 0.84 | 0.28 1.12 123 | 0.31 |

\( ^{a} \)Integrated \( M1 \) strength calculated according to Eq. 4.

\( ^{b} \)Integrated \( E2 \) strength calculated for positive-parity states in analogy to Eqs. 1 and 2.

\( ^{c} \)Summed \( M1 \) strength of transitions from the \( 1^+ \) states below 5 MeV to the ground state.

\( ^{d} \)Value taken from Ref. [37].

[1] M. Arnould, S. Goriely, and K. Takahashi, Phys. Rep. 450, 97 (2007).
[2] F. Käppeler, R. Gallino, S. Bisterzo, and W. Aoki, Rev. Mod. Phys. 83, 157 (2011).
[3] A. Voinov, E. Algin, U. Agvaanluvsan, T. Belgya, R. Chankova, M. Gutormsen, G. E. Mitchell, J. Rekstad, A. Schiller, and S. Siem, Phys. Rev. Lett. 93, 142504 (2004).
[4] M. Gutormsen, R. Chankova, U. Agvaanluvsan, E. Algin, L. A. Bernstein, F. Ingebritsen, T. Lonroth, S. Messel, G. E. Mitchell, J. Rekstad, A. Schiller, S. Siem, A. C. Sunde, A. Voinov, and S. Odegard, Phys. Rev. C 71, 044307 (2005).
[5] A. C. Larsen, I. E. Rund, A. Burger, S. Goriely, M. Gutormsen, A. Görgen, T. W. Hagen, S. Harissopulos, H. T. Nylhus, T. Renström, A. Schiller, S. Siem, G. M. Tveten, A. Voinov, and M. Wiedeking, Phys. Rev. C 87, 014319 (2013).
[6] A. Simon, M. Gutormsen, A. C. Larsen, C. W. Beausang, P. Humby, J. T. Burke, R. J. Casperson, R. O.
Hughes, T. J. Ross, J. M. Allmond, R. Chyzh, M. Dag, J. Koglin, E. McCleskey, M. McCleskey, S. Ota, and A. Saastamoinen, Nucl. Phys. C 93, 034303 (2016).

[7] F. Naqvi, A. Simon, M. Guttormsen, R. Schwengner, S. Frauendorf, C. S. Reingold, J. T. Burke, N. Cooper, R. O. Hughes, S. Ota, and A. Saastamoinen, Phys. Rev. C 99, 054331 (2019).

[8] A. Spyrou, S. N. Liddick, A. C. Larsen, M. Guttormsen, K. Cooper, A. C. Dombos, D. J. Morrissey, F. Naqvi, G. Perdikakis, S. J. Quinn, T. Renstrom, J. A. Rodriguez, A. Simon, C. S. Sumithrarachchi, and R. G. T. Zegers, Phys. Rev. Lett. 113, 232502 (2014).

[9] A. C. Larsen, N. Blasi, A. Bracco, F. Camera, T. K. Eriksen, A. G"orgen, M. Guttormsen, T. W. Hagen, S. Leoni, B. Million, H. T. Nyhus, T. Renstrom, S. J. Rose, I. E. Rund, S. Siem, T. Tornyi, G. M. Tveten, A. V. Voinov, and M. Wiedeking, Phys. Rev. Lett. 111, 242504 (2013).

[10] A. Voinov, S. M. Grimes, C. R. Brune, M. Guttormsen, A. C. Larsen, T. N. Massey, A. Schiller, and S. Siem, Phys. Rev. C 81, 024319 (2010).

[11] R. Schwengner, S. Frauendorf, and A. C. Larsen, Phys. Rev. Lett. 111, 232504 (2013).

[12] B. Alex Brown and A. C. Larsen, Phys. Rev. Lett. 113, 252502 (2014).

[13] K. Sieja, EPJ Web of Conferences 146, 05004 (2017).

[14] K. Sieja, Phys. Rev. Lett. 119, 052502 (2017).

[15] K. Heyde, P. von Neumann-Cosel, and A. Richter, Rev. Mod. Phys. 82, 2365 (2010).

[16] R. Schwengner, S. Frauendorf, and B. A. Brown, Phys. Rev. Lett. 118, 092502 (2017).

[17] S. Karampagia, B. A. Brown, and V. Zelevinsky, Phys. Rev. C 95, 024322 (2017).

[18] K. Sieja, Phys. Rev. C 98, 064312 (2018).

[19] J. E. Midtbø, A. C. Larsen, T. Renstrom, F. L. Bello Garrote, and E. Lima, Phys. Rev. C 98, 064321 (2018).

[20] M. Honma, T. Otsuka, T. Mizusaki, and M. Hjorth-Jensen, Phys. Rev. C 80, 064323 (2009).

[21] B. A. Brown and A. F. Lisetskiy, unpublished.

[22] A. F. Lisetskiy, B. A. Brown, M. Horoi, and H. Grawe, Phys. Rev. C 70, 044314 (2004).

[23] B. A. Brown and W. D. M. Rae, Nucl. Data Sheets 120, 115 (2014).

[24] J. J. Sun, Z. Shi, X. Q. Li, H. Hua, C. Xu, Q. B. Chen, S. Q. Zhang, C. Y. Song, J. Meng, X. G. Wu, S. P. Hu, H. Q. Zhang, W. Y. Liang, F. R. Xu, Z. H. Li, G. S. Li, C. Y. He, Y. Zheng, Y. L. Ye, D. X. Jiang, Y. Y. Cheng, C. He, R. Han, Z. H. Li, C. B. Li, H. W. Li, J. L. Wang, J. J. Liu, Y. H. Wu, P. W. Luo, S. H. Yao, B. B. Yu, X. P. Cao, and H. B. Sun, Phys. Lett B 734, 308 (2014).

[25] K. Kumar, Phys. Rev. Lett. 28, 249 (1972).

[26] A. Poves, F. Nowacki, and Y. Alhassid, Phys. Rev. C 101, 054307 (2020).

[27] B. Singh, Nucl. Data Sheets 108, 197 (2007).

[28] E. Browne and J. K. Tuli, Nucl. Data Sheets 111, 1093 (2010).

[29] G. G"urdal and E. A. McCutchan, Nucl. Data Sheets 136, 1 (2016).

[30] B. Singh and A. R. Farhan, Nucl. Data Sheets 107, 1923 (2009).

[31] A. R. Farhan and B. Singh, Nucl. Data Sheets 110, 1917 (2009).

[32] B. Singh, Nucl. Data Sheets 105, 223 (2005).

[33] M. Matsuo, T. Dossing, E. Vigezzi, R. A. Broglia, and K. Yoshida, Nucl. Phys. A 617, 1 (1997).

[34] S. Frauendorf, Rev. Mod. Phys. 73, 463 (2001).

[35] S. Raman, C. W. Nestor Jr., and P. Tikkanen, At. Data Nucl. Data Tables 78, 1 (2001).

[36] J. Enders, P. von Neumann-Cosel, C. Ranacharyulu, and A. Richter, Phys. Rev. C 71, 014306 (2005).

[37] A. Jung, S. Lindeinstruth, H. Schacht, B. Starck, R. Stock, C. Wesselborg, R. D. Heil, U. Kneissl, J. Margraf, H. H. Pitz, and F. Steiper, Nucl. Phys. A 584, 103 (1995).

[38] R. Massarczyk, G. Rusev, R. Schwengner, F. D"onau, C. Bhatia, M. E. Gooden, J. H. Kelley, A. P. Toncev, and W. Tornow, Phys. Rev. C 90, 54310 (2014).

[39] E. Farnea, G. de Angelis, A. Gadea, P. G. Bizzeti, A. Dewald, J. Eberth, A. Algora, M. Axiotis, D. Bazzacco, A. M. Bizzeti-Sona et al., Phys. Lett. B 551, 56 (2003).

[40] E. A. Stefanova, I. Stefanescu, G. de Angelis, D. Curien, J. Eberth, E. Farnea, A. Gadea, G. Gersch, A. Jungclaus, K. P. Lieb, T. Martinez, R. Schwengner, T. Steinhardt, O. Thelen, D. Weisshaar, and R. Wyss, Phys. Rev. C 67, 054319 (2003).

[41] B. Mukherjee, S. Muralithar, G. Mukherjee, R. P. Singh, R. Kumar, J. J. Das, P. Sugathan, N. Madhavan, P. V. M. Rao, A. K. Sinha, A. K. Pande, L. Chaturvedi, S. C. Pancholi, and R. K. Bhowmik, Acta Phys. Hung. N. S. 11, 189 (2000); Erratum Acta Phys. Hung. N. S. 13, 253 (2001).

[42] A. M. Forney, W. B. Walters, C. J. Chiara, R. V. F. Janssens, A. D. Ayangeakaa, J. Sethi, J. Harker, M. Albert, C. Pancholi, and R. K. Bhowmik, Acta Phys. Hung. N. S. 71, 123 (2004).

[43] Zs. Podolyak, S. Mohammad, G. de Angelis, Y. H. Zhang, M. Axiotis, D. Bazzacco, P. G. Bizetti, F. Brandolini, R. Broda, D. Bucurescu, E. Farnea et al., Int. J. Mod. Phys. 13, 123 (2004).
Appendix A: Yrast properties of the remaining isotopes

![Graph showing excitation energies versus spin of experimental and calculated yrast states in Ge isotopes.](image)

**FIG. 11:** Excitation energies versus spin of experimental (red circles) and calculated (black squares) yrast states in $^{64,66,70}$Ge, and of experimental (red triangles up) and calculated (blue triangles down) states built on the second $2^+$ states in $^{66,70}$Ge. The lines represent the linking $E2$ transitions. The numbers at the lines are calculated $B(E2)$ values in $e^2$fm$^4$. The experimental data were taken from Refs. [39–41], respectively.

![Graph showing excitation energies versus spin of experimental and calculated yrast states in Ge isotopes.](image)

**FIG. 12:** Excitation energies versus spin of experimental (red circles) and calculated (black squares) yrast states in $^{78,80}$Ge, and of experimental (red triangles up) and calculated (blue triangles down) states built on the second $2^+$ state in $^{78}$Ge. The lines represent the linking $E2$ transitions. The numbers at the lines are calculated $B(E2)$ values in $e^2$fm$^4$. The experimental data were taken from Refs. [42, 43], respectively.