Stability of Excited Dressed States with Spin-Orbit Coupling

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We study the decay behaviors of ultracold atoms in metastable states with spin-orbit (SO) coupling, and demonstrate that there are two SO-coupling-induced decay mechanisms. One arises from the trapping potential and the other is due to interatomic collision. We present general schemes for calculating decay rates from these two mechanisms, and illustrate how the decay rates can be controlled by experimental parameters. We experimentally measure the decay rates over a broad parameter region, and the results agree well with theoretical calculations. This work provides an insight for both quantum simulation involving metastable dressed states and studies on few-body problems with SO coupling.

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Recently, synthetic magnetic field and a restricted class of spin-orbit (SO) coupling have been successfully realized in ultracold Bose gases and degenerate Fermi gases. In addition, many schemes have been proposed to create general gauge fields, particularly the isotropic Rashba SO coupling; see Ref. [3] for a review. These will bring about novel quantum systems of SO coupled atoms that display many interesting phases, and provide a new and powerful tool for quantum simulation. However, in many of these proposals, one or more metastable states, e.g., dark states, play essential roles. Thus, the lifetime of atoms in the metastable states becomes crucial in practical implementation of these schemes.

In this Letter, we carry out a thorough and quantitative study of the decay behavior of excited dressed states with SO coupling. We find that, due to the SO coupling, decay mechanisms can arise both from single-atom motion in inhomogeneous trap potential and from two-body collisions. The trap-induced decay rate is determined by the trap frequency, while the collisional decay rate is controlled by atomic density and scattering length. Further, we present rigorous methods for calculating decay rate for each mechanism. Our method for the rate of collisional decay can be applied to both elastic and inelastic scattering of two atoms with a general type of SO coupling and arbitrary scattering length. Finally, we experimentally investigate the decay behavior of a SO coupled $^{87}$Rb BEC prepared in a metastable state for a broad parameter region. The experimental results are compared to theoretical calculations, and the excellent agreement supports the validity of our theory.

Our work provides a comprehensive understanding of the SO-coupling-induced decay and thus can serve as a valuable reference for experimental realization of proposals involving metastable dressed states. For a given system of interest, one can use our theory to figure out which is the dominant mechanism, and then apply appropriate approaches to control the stability of those metastable states. In addition, our method for the exact calculation of interatomic scattering amplitude sheds lights on few-body problems with SO coupling.

The trap-induced decay. The Hamiltonian of a single atom with SO coupling takes form $H_{1b} = \hat{p}^2/(2m) + M(\hat{p}) + \hat{V}(\hat{r}) \equiv H_0(\hat{p}) + \hat{V}(\hat{r})$, with $m$ the atomic mass, $\hat{p}$ and $\hat{r}$ the atomic momentum and position operator, respectively, and $\hat{V}(\hat{r})$ the trap potential. The SO coupling is described by the $\hat{p}$-dependent operator $M(\hat{p})$. For instance, for effective spin-1/2 systems in Refs. [2,3,24,8], one has $M(\hat{p}) = \delta \hat{\sigma}_z/2 + \Omega \hat{\sigma}_r/2 + 2k_r \hat{\sigma}_z$, where $\hat{\sigma}$ is the Pauli operators, $\delta$ is the two-photon detuning, $k_r$ is the recoil momentum and $\Omega$ is the Raman-coupling strength.

It is clear that the eigen-state of $H_0$ is $|k\rangle |\alpha(k)\rangle$, where $|k\rangle$ satisfies $\hat{p}|k\rangle = k|k\rangle$ and the state $|\alpha(k)\rangle$ in the spin space is the eigen-state of $M(\hat{p})$. If there were no SO coupling, both $M(\hat{k})$ and $|\alpha(k)\rangle$ are $\hat{k}$-independent. In this case, since the trap potential $\hat{V}$ is independent of the atomic spin, it cannot induce the transition between two eigen-states with different $\alpha$, or the decay from the

![FIG. 1: (a and b): Sketch of the trap-induced decay mechanism (a) and the collisional decay mechanism (b). (a) illustrates two steps of the trap-induced decay, and (b) illustrates the two typical processes in the collisional decay.](image-url)
excited spin state. In the presence of SO coupling, both $M(k)$ and $|α(k)⟩$ depend on $k$. Thus, $|α(k)⟩$ and $|α′(k)⟩$ with $k \neq k′$ can overlap with each other even if $α \neq α′$. Due to this fact, $V(δ)$ will couple two dressed states with either different $α$, or different $k$, or both. Therefore, excited dressed states can decay to lower-energy branch via the single-atom motion in trap $V$.

Here the trap-induced decay process can be understood as two steps as illustrated in Fig. 1(a). First atoms tunnel from the initial state to the energy-conserved states in the lower branch (the solid arrow in Fig. 1(a)). The rate $Γ_{1b}$ of this process can be calculated by Fermi’s golden rule (FGR). Second, due to the dissipation effects given by the environment, the atoms further decay to states with lower energy (the dashed arrow in Fig. 1(a)). Usually, the second process is much faster than the first, and thus the total rate is given by $Γ_{1b}$.

Now we calculate $Γ_{1b}$ in the momentum representation. The atomic state $|ψ(t)⟩$ at time $t$ is described by the spinor wave function $|ψ(k, t)⟩ \equiv (k|ψ(t)⟩)$, and the position operator $\hat{r}$ is proportional to the gradient of the momentum $k$, i.e., $\hat{r} = i\nabla_k$. The harmonic trap potential can be written as $V = -\sum_{j=x,y,z}(m/2)ω_j^2k_j^2$, which behaves as the “kinetic energy” of the atom motion in the $k$-space. Furthermore, $|ψ(k, t)⟩$ can be expressed as $|ψ(k, t)⟩ = \sum_α ψ_α(k, t)|α(k)⟩$. Then the Schrödinger equation $i\hbar|ψ(k, t)⟩/dt = [\hat{H}_0 + U_0]|ψ(k, t)⟩$ can be rewritten as the equations for each component $ψ_α(k, t)$: $i\hbar\dot{ψ}_α/dt = \sum_β T_{αβ}ψ_β + E_αψ_α$, where $E_α(k)$ is the eigen-energy of $\hat{H}_0$ for $|k⟩|α(k)⟩$, and $T_{αβ}$ is defined as $T_{αβ} = \sum_{j=x,y,z}(mω_j^2/2)\sum_γ X_γ^{(j)}X_γ^{(β)} - X_α^{(j)} = iδ_{αβ}\nabla_β + i(iα(k)\nabla_β + i(α(k)\nabla_β + \beta(k))$. It is clear that the terms $i(α(k)\nabla_β + \beta(k))$ play the similar role as the effective gauge field in the Born-Oppenheimer approximation.

The decay of the atoms from the excited dressed state, or the transition between dressed states with different $α$, is induced by these terms with $α \neq β$.

Suppose that the initial wave function of the atom is $|ψ_i(k)⟩ = ϕ(k)|α(k)⟩$, with $ϕ(k)$ satisfying $[T_{αα} + E_α]|ϕ(k)⟩ = ϵϕ(k)$. Then $Γ_{1b}$ can be given by FGR as

$$Γ_{1b} = 2\pi\sum_{β \neq α} ρ_β(ε)|\int d\phi β(k)T_{βα}\phi(k)|^2,$$  \hspace{1cm} (1)

where $ϕ(k)$ satisfies $[T_{ββ} + E_β]|ϕ(k)⟩ = ϵϕ(k)$ and $ρ_β(ε)$ is the associated density of states. Since $\hat{V} \propto \omega_2^2$, the decay rate $Γ_{1b}$ is proportional to $ω_2^2$, and thus can be controlled by tuning the trap frequency.

The collisional decay. For two SO-coupled atoms under scattering, the general Hamiltonian takes form $H_{2b} = H_0(1) + H_0(2) + U(\hat{r}_{12}) \equiv \hat{H}_F + \hat{U}$, with $H_0(i)$ for the free motion of the ith atom ($i = 1, 2$) and $U(\hat{r}_{12})$ the interaction potential of the two atoms. Here $\hat{r}_{12}$ is the relative position operator of the two atoms. We shall consider the simple case where $\hat{U}$ is independent of the atomic spin. If there were no SO coupling, $\hat{U}$ cannot induce transition between different spin states or the atomic decay from excited spin states. In the presence of the SO coupling, the free-motion state of the two atoms, or the eigen-state of $\hat{H}_F$, becomes the two-atom dressed state $|c⟩ \equiv |k_1⟩_1|α_1(k_1)⟩_1|k_2⟩_2|α_2(k_2)⟩_2$. Here we define $c = (k_1, α_1, k_2, α_2)$ as the set of the four quantum numbers. Similarly as in our above discussion, states $|α(k)⟩$ and $|α′(k′)⟩$ can overlap with each other when $α \neq α′$. Then we have $|c|\hat{U}|c⟩ \neq 0$ even if $(α_1, α_2) \neq (α_1′, α_2′)$, and $\hat{U}$ can introduce inelastic collisions or the transitions between the states with different quantum number $(α_1, α_2)$. This leads to the decay of atoms from excited dressed states (Fig. 1(b)).

The above discussions are applicable to both bosonic and fermionic systems. Hereby we shall focus on a system of bosonic atoms condensed in an initial dressed state $|k_0⟩|α_0(k_0)⟩$ with atomic density $n_0$. The characteristic decay rate $Γ_{2b}$ for the collisional decay is defined as $Γ_{2b} = n_0K$. Parameter $K$ is the inelastic-collision rate defined as $K = 2συ$, where $σ$ is the total cross-section of the inelastic collision, $υ$ is the relative velocity of the two atoms before collision, and the factor 2 comes from the bosonic statistics. According to the standard scattering theory [13], the factor $K$ is given by

$$K = \frac{8}{m^2} \sum_{(α′, α′′)\neq(α_1, α_2)} \int d\omega_2d\omega_3δ(β′, 0)|f(c′, c_0)|^2 \hspace{1cm} (2)$$

with $c_0 = (k_0, α_0, k_0, α_0)$ and $f(c′, c)$ is the scattering amplitude between the incident state $|c⟩$ and the output state $|c′⟩$ with $c′ = (k_1′, α_1′, k_2′, α_2′)$. The Dirac function $δ(β′, 0)$ means that the total energy is conserved during the scattering process, and $δ(β′, 0) = δ(k_1 + k_2 - k_1′ - k_2′)$ is for the momentum conservation.

In Eq. (2), the scattering amplitude $f$ is defined as $f(c′, c) = -2\pi m^2/c|\hat{U}|c⟩$, with $|c⟩$ the scattering state given by the Lippman-Schwinger equation (LSE) $|c⟩ = |c⟩ + G_0|c⟩$. Here the free Green’s operator $G_0$ is defined as $G_0 = [E_{α_1}(k_1) + E_{α_2}(k_2) + i0^+ - \hat{H}_F]^{-1}$. We assume $U(\hat{r}_{12})$ is a short-range potential and becomes negligible when $|r_{12}|$ is larger than an effective range $R_*$. In this case, when there were no SO coupling, the wave function $⟨r_{12}|c⟩$ of the scattering state should satisfy the Bethe-Perel boundary condition (BPC) $⟨r_{12}|c⟩ \propto (1/|r_{12}| - 1/a)$ in the short-range region $R_* < |r_{12}| << 1/k$, where $k = |k_1 - k_2|/2$, $a$ is the scattering length and $|r_{12}|$ given by $|r_{12}| = r_{12}|r_{12}|$. Therefore, one can first derive the expression of $⟨r_{12}|c⟩$ in the short-range region with the LSE, and then substitute the result into the BPC and obtain $Φ$ as well as $f$.

In the presence of SO coupling, the Hamiltonian of two colliding atoms is revised in the whole range of the interatomic distance $|r_{12}|$, and thus the few-body properties are also strongly affected by SO coupling [3, 14, 23]. In particular, the Bethe-Perel boundary condition needs
The single-atom Hamiltonian is described in Ref. [4]. A BEC of $2^{133}$Rb atoms in our experiment. The measured fraction $R(t)$ of condensate versus different hold time $t_h$ for cases with $\Omega = 0.6E_r$, $\delta = 6E_r$ (a) and $\Omega = 0.9E_r$, $\delta = -6E_r$ (b), with the dispersion relationships shown in the insets of (a) and (b), respectively. The blue curve is obtained by fitting the experimental data with our theoretical function of $R(t)$.

To be modified [24] and the free Green’s function will be re-calculated. Nevertheless, the above procedure is still valid. With this, we find that $f(c', c)$ is given by [24]

$$f = \frac{1}{\pi} \frac{1}{\mathcal{F}(\mathbf{k})} \frac{1}{\Delta^2} \left| \alpha_1^\dagger(\mathbf{k}_1) \right| \left| \alpha_2^\dagger(\mathbf{k}_2) \right|_2$$

with $a$ the scattering length for the absence of SO coupling. The operator $\mathcal{F}$ can be expressed as $\mathcal{F} = \left( \begin{array}{cc} i & m/(4\pi a) \sqrt{E - (k_1^2 + k_2^2)/4m} - (2\pi)^{-3} \sum_{\alpha, \beta} \int dp \left| \Delta \right|^2 \right| \left| \beta^\dagger_\alpha(p_2) \right| \left| \beta^\dagger_\beta(p_1) \right| \right| \mathcal{F}(\mathbf{k}) | \alpha \rangle \langle \beta |$, with $\Delta = E_{\alpha}(k_1) + E_{\beta}(k_2)$, $p_{1,2} = (k_1 + k_2)/2 \pm p$, $\Delta = E + i0^+ - \langle \beta^\dagger_\beta(p_1) \rangle - \langle \beta^\dagger_\beta(p_2) \rangle$ and $\delta_{0} = E + i0^+ - (p_1^2 + p_2^2)/(2m)$. When $m/(4\pi a)$ is much larger than eigenvalues of $\mathcal{F}$, one has $f = -a \left( \alpha_1^\dagger(\mathbf{k}_1) \right| \alpha_2^\dagger(\mathbf{k}_2) \right| \alpha_1^\dagger(\mathbf{k}_1) \rangle \langle \alpha_2(\mathbf{k}_2) |$, i.e., the result given by the FGR approximation [6]. For large $a$, contribution from $\mathcal{F}$ becomes significant, and the FGR fails. With the expression (3) of the scattering amplitude, one can obtain the factor $K$ and the decay rate $\Gamma_{20} = n a K$.

**Experiment.** Our experimental layout has been described in Ref. [1]. A BEC of $2.5 \times 10^5$ $^{87}$Rb atoms in the $F = 1$ manifold is created in an optical dipole trap with frequencies $\{\omega_x, \omega_y, \omega_z\} = 2\pi \times \{30, 30, 50\}$Hz. The single-atom Hamiltonian in the $x$-direction is given by $\hat{H}_x = \hat{H}_{0x} + \hat{V}$, with

$$\hat{H}_{0x} = \left( \begin{array}{ccc} \frac{(\hat{p}_x + 2k_x)^2}{2m} & -\frac{\delta}{2} & 0 \\ \frac{\delta}{2} & \frac{\Omega}{2} & 0 \\ 0 & 0 & \frac{(\hat{p}_x - 2k_x)^2}{2m} + \frac{\Omega}{2} + \epsilon \end{array} \right),$$

and $\hat{V} = m\omega_x^2\hat{x}^2/2$ the dipole trap potential. Here $\Omega$ is the strength of Raman coupling, $\delta$ is the two-photon Raman detuning and $\epsilon$ is the quadratic Zeeman shift given by a homogeneous bias magnetic field. Symbol $k_r$ represents the recoil momentum, and $E_r = k_r^2/(2m) = 2\pi \times 2.21$kHz is the recoil energy. Diagonalization of the Hamiltonian $\hat{H}_{0x}$ leads to three momentum-dependent eigen-states $|k_x\rangle \alpha(k_x)\rangle$ $\alpha = 0, \pm 1$ with eigen-energies $E_{-1}(k_x) < E_0(k_x) < E_{+1}(k_x)$. Two examples of the dispersion curves are shown in the insets of Fig. 2(a,b).

For experiments with $\delta > 0$, the BEC is first prepared in the bare state $|F = 1, m_F = -1\rangle$, and transformed to the $|m_F = 0\rangle$ state with a $\pi$-pulse. Then we adiabatically turn on the SO coupling, so that the BEC is prepared in the middle dressed state with $\alpha = 0$ and $k_x$ around some value $k_0$, corresponding to the global minima of the $E_0(k_x)$ curve for $\delta > 0$ (the inset of Fig. 2(a)). In the experiments with $\delta < 0$, the SO coupling is adiabatically applied to the BEC in the state $|F = 1, m_F = -1\rangle$, and then the system is prepared in the dressed state with $\alpha = 0$ and $k_x$ around the local minima for $\delta < 0$ (the inset of Fig. 2(b)). The Raman coupling is held for a variable duration $t_h$. During this time interval, the atoms can decay from the initial dressed state with $\alpha = 0$ to those with $\alpha = -1$ $\tilde{\alpha}$ $\tilde{\alpha}$. At $t = t_h$, the Raman lasers and the dipole trap are suddenly turned off. With the Stern-Gerlach technique, a time-of-flight image is taken to measure the number of atoms remained in the BEC after the decay process. As an example, the fraction of remaining atoms is shown in Fig. 2 as a function of $t_h$.

**Data Analysis.** The numerical calculations with Eqs. (13) show that in our system the rate $\Gamma_{1b}$ and $\Gamma_{2b}$ of the trap-induced and collisional decay are of the order $10^{-5}$Hz and $10$Hz, respectively. Therefore, the trap-induced decay in our experiments is negligible, and we
The lifetime $L$ of both scattering amplitude.

Experimental results is excellent [25]. This confirms our blue curves. The agreement between the theoretical and estimated values of the lifetime $L$.

In this letter we have shown the two mechanisms of decay in the ultracold gases with SO coupling, carried out the calculation of two decay rates, and presented a comparison with experiments. This guides us how to control the stability of excited dressed state in current setup. For instance, as shown in Fig. 4, we plot characteristic times $1/\Gamma_{1b}$ and $1/\Gamma_{2b}$ of the trap-induced and collisional decay as functions $\omega_x$ and $a$ for the SO coupling realized in current experiment. When $\omega_x > (2\pi)60$Hz, as shown in Fig 4 (a), the decay is dominated by the trap-induced decay, and then the decay rate can be controlled by the trap frequency. As the trap frequency decreases, the lifetime of excited state gets longer. However, when $\omega_x < (2\pi)60$Hz, the collisional decay becomes dominating. In this region, the decay rate is no longer sensitive to the trap frequency, but it can be controlled by the atomic density and the scattering length. For instance, as shown in Fig 4(b), when $\omega_x = (2\pi)50$Hz, the life time determined by the collisional decay can be increased by reducing the scattering length $a$ or atomic density.

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Discussion on the control of stability. In this letter we have shown the two mechanisms of decay in the ultracold gases with SO coupling, carried out the calculation of two decay rates, and presented a comparison with experiments. This guides us how to control the stability of excited dressed state in current setup. For instance, as shown in Fig. 4, we plot characteristic times $1/\Gamma_{1b}$ and $1/\Gamma_{2b}$ of the trap-induced and collisional decay as functions $\omega_x$ and $a$ for the SO coupling realized in current experiment. When $\omega_x > (2\pi)60$Hz, as shown in Fig 4 (a), the decay is dominated by the trap-induced decay, and then the decay rate can be controlled by the trap frequency. As the trap frequency decreases, the lifetime of excited state gets longer. However, when $\omega_x < (2\pi)60$Hz, the collisional decay becomes dominating. In this region, the decay rate is no longer sensitive to the trap frequency, but it can be controlled by the atomic density and the scattering length. For instance, as shown in Fig 4(b), when $\omega_x = (2\pi)50$Hz, the life time determined by the collisional decay can be increased by reducing the scattering length $a$ or atomic density.

As emphasized before, our analysis and calculation method are very general and can be applied to ultracold gases with any kind of SO coupling. Thus, similar stability analysis as discussed above for current experimental system can be straightforwardly carried out for other realizations of SO couplings. It will help the implementation of those schemes involving metastable states, and therefore open up more possibilities for rich physics with synthetic gauge field.

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