Probing $\Xi N$ interaction through inversion of spin-doublets in $\Xi N\alpha\alpha$ nuclei

E. Hiyama,1,2 M. Isaka,3 T. Doi,4 and T. Hatsuda4

1Department of Physics, Tohoku University, Sendai, Japan, 980-8578
2Nishina Center for Accelerator-Based Science, RIKEN, Wako, 351-0198, Japan
3Science Research Center, Hosei University, Tokyo 102-8160, Japan
4Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, Wako 351-0198, Japan

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A new way to study the spin-isospin dependence of the $\Xi N$ interaction is explored through the energy levels of $\Xi N\alpha$ and $\Xi N\alpha\alpha$ systems with a being a spectator to attract the $\Xi N$ pair without changing its spin-isospin structure. By using the Gaussian expansion method (GEM) with the state-of-the-art $\Xi N$ potential obtained from lattice QCD calculations, it is found that $\Xi N\alpha\alpha$ has spin-doublet bound states with $J^P = 1^-$ and $2^-$ in both isospin triplet and singlet channels. The inversion of the $1^-\to 2^-$ spin-doublet between the iso-triplet and the iso-singlet is found to be strongly correlated with the relative strengths of the $\Xi N$ interaction in the $^{11}\text{S}_0, ^{13}\text{S}_1, ^{31}\text{S}_0$ and $^{33}\text{S}_1$ channels. The $(K^+, K^+)$ and $(K^-, K^0)$ reactions on the $^{10}\text{B}$ target are proposed to produce those bound states.

Introduction. – In recent years, considerable progress has been made in studying the unknown hyperon interactions in the $S = 1$ channel, especially the $\Xi N$ interaction. Experimentally, a bound $^{15}$C($\Xi^+, ^{14}$N) hypernucleus was observed using the emulsion detector [1–4], which indicates that the $\Xi$-nucleus potential is attractive and the $\Xi N$-AA coupling is weak. Also, the femtoscopic analysis of the $\Xi N$ momentum correlation in $p$-$\text{Pb}$ and $p$-$p$ collisions by ALICE Collaboration at LHC [5, 6] shows that the spin-isospin averaged $\Xi N$ interaction is attractive at low energies.

Experimental studies so far have limited access to the spin-isospin decomposition of the $\Xi N$ interaction, and it is important to explore theoretically possible ways to make the decomposition. In Ref. [7], it was pointed out, by using the Gaussian expansion method (GEM) with the Nijmegen $\Xi N$ potentials, that the $^7\text{H}(\Xi^+, ^6\text{He})$ and $^{10}\text{Li}(\Xi^+, ^9\text{Be})$ hypernuclei may have bound states and are suited to extract the information on the spin-isospin independent part of the $\Xi N$ interaction. More recently, in Ref[8], the binding energies of $\Xi NN$ and $\Xi NNN$ hypernuclei were examined by GEM and the first principles lattice QCD interaction (the HAL QCD potential) [9], where it is found that a shallow bound state exists with $T = 0, J^P = 1^+$ in the $\Xi NN$ system due to the moderately large attraction of the $\Xi N$ interaction in the $\Xi N(1^{1}\text{S}_0)$ channel[10]. Subsequently, several bound states for light $\Xi$ hypernuclei ($A = 4, 5$ and $7$) were suggested by using the no-core shell model [10] with a possible strong attraction of the chiral effective field theory interaction in the $\Xi N(3^{3}\text{S}_1)$ channel.

The purpose of the present paper is to explore a robust and unambiguous way to extract the spin-isospin component ($^{11}\text{S}_0, ^{13}\text{S}_1, ^{31}\text{S}_0$ and $^{33}\text{S}_1$) of the $\Xi N$ interaction by considering the systems, $\Xi N\alpha$ and $\Xi N\alpha\alpha$: Since $\alpha$ is a spin-isospin saturated system, the $\Xi N$ interaction is directly linked to the spin-isospin structure of these systems. In particular, we calculate their binding energies within the framework of three- and four-body cluster models using the HAL QCD $\Xi N$ potential, and propose possible experiments to produce such states through the $K^-$ induced reactions.

Few-body method. – For $\Xi N\alpha$ and $\Xi N\alpha\alpha$, the total Hamiltonian is given by

$$H = K + \sum_{a,b} V_{ab} + V_{\text{Pauli}},$$

(1)

where $K$ is the kinetic-energy operator, $V_{ab}$ is the interaction between the constituent particle $a$ and $b$, and $V_{\text{Pauli}}$ is the Pauli projection operator to be defined below.

The total wavefunctions of $\Xi$ hypernuclei with $A = 6, 10$ satisfying the Schrödinger equation, $(H - E)\Psi_{JM,TTz}^{(A=6,10)} = 0$, are described as sums of amplitudes of all rearrangement channels with the $\alpha$ cluster(s) ($1$–$3$ ($1$–$9$) channels for $A = 6$ ($10$)) in the $LS$ coupling scheme. Shown in Fig.1 is the case for $A = 10$ with the wave function,

$$\Psi_{JM,TTz}^{(A=10)} = \sum_{c=1}^{9} \sum_{n,N,\nu,L,\lambda,S,I,K} C^n_c \times S_{\alpha\alpha} \times \left[ \Phi(\alpha_1)\Phi(\alpha_2)\chi_2(N)\chi_2(\Xi)\right]_S \times \left[ [\phi^{(c)}_{nl}(r_c)\psi^{(c)}_{NL}(r_c)]_J S_{\alpha\lambda}(\rho_c) \right]_K \times \eta_2(N)\eta_2(\Xi)_{TTz}.$$  

(2)

Here $S_{\alpha\alpha}$ stands for the symmetrization operator for exchange of two $\alpha$ clusters, $\chi_2(N)$ and $\chi_2(\Xi)$ are the spin wavefunction of the nucleon and $\Xi$ particle, respectively. $\eta_2(N)$ and $\eta_2(\Xi)$ are the isospin wavefunction of the nucleon and $\Xi$, respectively. Throughout this paper, we

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1 Here, we employ the spectroscopic notation $^{2S+1}J^P$ to classify the S-wave $\Xi N$ interaction where $T$, $s$, and $J$ stand for total isospin, total spin, and total angular momentum, respectively.
use the isospin-averaged nucleon mass and that of the Ξ mass. The Coulomb interaction is fully taken into account. The mixing of the wave functions between the T = 1 and T = 0 states is the second-order effect in isospin symmetry breaking and is not considered.

Following GEM [11], we take the functional forms, \( \phi_{\alpha \ell m}(\mathbf{r}) = r^\ell e^{-(r/a_\ell)^2} Y_{\ell m}(\hat{r}) \), \( \psi_{NLM}(\mathbf{R}) = R^L e^{-(R/a_N)^2} Y_{LM}(\hat{R}) \), and \( \xi_{\alpha \lambda \mu}(\rho) = \rho^{\lambda} e^{-(\rho/a_\rho)^2} Y_{\lambda \mu}(\hat{\rho}) \), where the Gaussian range parameters are chosen to follow the geometrical progressions: \( r_\alpha = r_1 u^{n-1} \) (\( n = 1 - n_{\text{max}} \)), \( R_N = R_1 u^{N-1} \) (\( N = 1 - N_{\text{max}} \)), and \( \rho_\alpha = \rho_1 u^{\nu-1} \) (\( \nu = 1 - \nu_{\text{max}} \)). The eigenenergy \( E \) and the coefficient \( C \) in Eq. (2) are to be determined by the Rayleigh-Ritz variational method.

As for \( V_{\alpha \alpha} \) and \( V_{N\alpha} \), we employ the potentials which reproduce reasonably well the low-energy \( \alpha \alpha \) and \( N\alpha \) scattering phase shifts [12,13]. An in-medium fudge factor, 0.955, is multiplied to \( V_{N\alpha} \) when it is used in the systems containing \( \alpha \alpha \), with the factor determined from the binding energy (1.57 MeV) of \(^{9}\)Be. The Coulomb potentials involving \( \alpha \) cluster(s) are constructed by folding the proton distribution in the \( \alpha \) cluster.

The Pauli principle for the nucleons belonging to \( \alpha \) and \( j = N, \alpha \) is taken into account by the orthogonality condition model (OCM) [14] where the Pauli projection operator in Eq. (1) is given by

\[
V_{\text{Pauli}} = \gamma \sum_f |\phi_f(\mathbf{r}_{j\alpha})\rangle\langle \phi_f(\mathbf{r}'_{j\alpha})| .
\]

The Pauli-forbidden relative wavefunction between \( \alpha \) and \( j \) is denoted by \( \phi_f(\mathbf{r}_{j\alpha}) \), where \( f = 0S \) for \( j = N \) and \( f = 0S, 1S, 0D \) for \( j = \alpha \) are chosen according to the standard OCM procedure. The Gaussian range parameter \( b \) of the single-particle 0s orbit in the \( \alpha \) particle ((0a)\(^4\)) is taken to be \( b = 1.358 \) fm as in the literature [15]. In the actual calculation, the strength \( \gamma \) is taken to be \( 10^4 \) MeV, which is large enough to push the unphysical forbidden state to high energies while keeping the physical states unchanged.

**ΞN potential.**— As for the ΞN interaction, we employ the results based on the isospin symmetric (2+1)-flavor lattice QCD simulations in a spacetime volume \( L^4 = (8.1 \text{ fm})^4 \) with a lattice spacing \( a = 0.0846 \) fm and at the nearly physical quark masses \( (m_\pi, m_K) = (146, 525) \) MeV [9]. HAL QCD Collaboration derived the ΞN-ΛΛ coupled-channel potentials from the simulation data at the Euclidean time \( t/a = 11, 12, 13 \). As discussed in Ref. [8], the lattice QCD data of the coupled-channel ΞN-ΛΛ system are fitted with multiple Gaussians and a Yukawa form with \( (m_\pi, m_K) = (146, 525) \) MeV, and the results are extrapolated to the isospin symmetric physical point \( (m_\pi, m_K) = (138, 496) \) MeV. In the \(^{11}\)S\(_0\) channel, an effective single-channel ΞN potential is introduced by renormalizing the coupling to ΛΛ into a single range Gaussian whose parameter is chosen to reproduce the ΞN phase shift obtained by the channel coupling.

Key properties of the resulting potentials are that the \(^{11}\)S\(_0\) channel is most attractive, \(^{13}\)S\(_1\) and \(^{33}\)S\(_1\) channels are weakly attractive, and the \(^{31}\)S\(_0\) channel is weakly repulsive (See Fig.2(a)). In Ref. [8], the lightest bound Ξ hypernucleus was found to be ΞNNN system using this potential. We use this single-channel ΞN potential throughout this paper for all partial waves, which is valid for the long-range part of the interaction relevant to the present study. In the following, central values of the observables such as the binding energy are obtained from the data at \( t/a = 12 \) with systematic errors estimated from those at \( t/a = 11 \) and 13. Uncertainties associated with the statistical errors of lattice QCD data are comparable to these systematic errors.

**Ξα potential.**— The Ξα potential is obtained by folding the above ΞN interaction based on the (0s)\(^4\) configuration of \( \alpha \). Since the isospin-spin averaged ΞN potential reads \( V_0 = [V(1^1S_0) + 3V(1^3S_1)] + 3V(3^1S_1) + 9V(3^3S_1)]/16 \), the Ξ-α interaction is dominated by the \(^{33}\)S\(_1\) channel. The attraction of the HAL QCD potential from lattice QCD in this channel is considerably weaker than those of the Nijmegen ESC08c potential (Fig.1 of [8]) and of the chiral NLO potential in [10]. Thus, we find that the binding energies \( B_{\Xi\alpha} \) with respect to \( \frac{1}{2} \)H with the Coulomb interaction are as small as 0.64, 0.45, 0.25 MeV for \( t/a = 11, 12, 13 \), respectively, with the width \( \Gamma \sim 5 \) keV, so that \( \Xi\alpha \) is likely to be the Coulomb assisted bound state. In contrast, \( B_{\Xi\alpha} \) is about 2.16 MeV.
in the chiral NLO potential [10] where the attraction in the $^{33}_{11}S_1$ channel is assumed to be large.

**ΞNα system.**—Let us first discuss the ΞNα system with $T = 1$ and $T = 0$. Since the ground state of $^5\text{He}$ has $J^\pi = 3/2^-$, total isospin-spin states of possible ΞNα nuclei are $(T, J^\pi) = (1, 1^-), (1, 2^-), (0, 1^-)$, and $(0, 2^-)$, where $J^\pi = 2^- (1^-)$ corresponds to the spin-parallel (anti-parallel) ΞN pair. Thus, ΞN interactions in $^{33}_{11}S_0$, $^{33}_{11}S_1$, $^{31}_{11}S_0$, and $^{13}_{11}S_1$ channels are primarily (but not entirely) related to ΞNα systems with $(1, 1^-)$, $(1, 2^-)$, $(0, 1^-)$, and $(0, 2^-)$, respectively, as summarized in Table I. In the HAL QCD potential, $^{31}_{11}S_0$ is repulsive and $^{33}_{11}S_1$ is only weakly attractive (Fig.2(a)), so that $T = 1$ ΞNα bound states do not appear in $1^-$ and $2^-$ states. It is unlikely to find bound states, unless the strength of the potential in the $^{33}_{11}S_1$ channel is artificially increased by a factor of 2. In $T = 0$, there generally arise no bound states too in both $1^-$ and $2^-$ states, since the attractions in $^{11}_{11}S_0$ and $^{33}_{11}S_1$ channels are not large enough. (Only when we use the HAL QCD potential at $t/a = 11$, there remains a possibility of a very shallow bound state in $1^-$ state.) Experimentally, one may try to produce ΞNα with $T = 1$ and $T = 0$ by the $(K^-, K^+)$ and $(K^-, K^0)$ reactions on the $^6\text{Li}$ target, respectively. However, it would be difficult to find bound states in Λ=6 systems according to the ΞN potential based on lattice QCD.

**ΞNαα system.**—Let us now study possible four-body bound states by adding an extra α to ΞNα. Such a four-body system has three-body bound states; $^9\text{Be}(T = 1/2, J^\pi = 3/2^-)$ with the experimental binding energy of 1.57 MeV, and Ξ $αα(T = 1/2, J^\pi = 1/2^-)$ with the theoretical binding energy of 2.08$^{+0.7}_{-0.67}$ MeV. The first number is reproduced by the three-body calculation by GEM, while the second number is obtained by GEM with HAL QCD potential. Then possible ΞNαα nuclei with $T = 1$ would have $J^\pi = 2^- (1^-)$ corresponding to the spin-parallel (anti-parallel) ΞN pair. Accordingly, the ΞN channels primarily contributing to these states are those given in Table I.

| $(T, J^\pi)$ | $(1, 1^-)$ | $(1, 2^-)$ | $(0, 1^-)$ | $(0, 2^-)$ |
|--------------|------------|------------|------------|------------|
| **primary ΞN channel** | $^{33}_{11}S_0$ | $^{33}_{11}S_1$ | $^{31}_{11}S_0$ | $^{13}_{11}S_1$ |

By considering the state $|\Xi^-ααα\rangle$ for $T = 1$ and $|\Xi^-ααα + Ξ^-ααα\rangle$ for $T = 0$ by the four-body calculation with GEM and the HAL QCD potential. we found the bound states with $(T, J^\pi) = (1, 1^-), (1, 2^-), (1, 3^-)$ and $(0, 1^-), (0, 2^-)$. Summarized in Table I are the energy levels $E$ relative to the four-body breakup threshold $Ξ + N + α + α$ with systematic errors. Note that the states in Table I are located below the lowest two-body

**Table I. Possible spin-doublets in ΞNα and ΞNαα together with the ΞN channel primarily related to each state.**

**Table II. Energy levels of ΞNα for $T = 1$ and 0 with the HAL QCD potential.** $E$ is measured relative to the $Ξ + N + α + α$ four-body breakup threshold. $Γ$ is the decay width through the process $ΞN → ΛΛ$. Central values are evaluated by the HAL QCD potential at $t/a = 12$ and the systematic errors are estimated by the potential at $t/a = 11$ and 12.

| $T$ | $E (\text{MeV})$ | $Γ (\text{MeV})$ | $Γ (\text{MeV})$ |
|-----|-----------------|-----------------|-----------------|
| 1   | $-4.56^{+0.94}_{-0.80}$ | $-4.70^{+0.94}_{-0.83}$ | $-2.47^{+0.94}_{-0.84}$ |
| 2   | $0.02^{+0.01}_{-0.01}$ | $0.02^{+0.01}_{-0.01}$ | $0.02^{+0.01}_{-0.01}$ |
| 3   | $-0.02$ | $-0.02$ | $-0.02$ |

**Table II. Energy levels of ΞNα for $T = 1$ and $9\text{Be} + Ξ$ for $T = 0$. Also shown in Table I are the widths $Γ$ estimated perturbatively by using the HAL QCD ΞN-ΛΛ coupling potential.**

**Inversion of spin-doublets.**—In Fig.2(b), we show the level structure of the lowest spin-doublets, $1^-→^-2$, for $t/a = 12$. For $T = 1$, the ground (1st excited) state has $2^-(1^-)$ in which dominant ΞN interaction is in the $^{33}_{11}S_1$ channel being very weakly attractive (weakly repulsive) in the HAL QCD potential as seen from Fig.2(a). Thus, the ordering of the $2^- → 1^-$ spin-doublet directly reflects the strength of the ΞN interactions. For $T = 0$, the ground (1st excited) state has $1^-(2^-)$ in which dominant ΞN interaction is in the $^{11}_{11}S_0$ ($^{33}_{11}S_1$) channel being moderately attractive (weakly attractive) in the HAL QCD potential as seen from Fig.1(a). An interesting feature in Fig.2(b) is that we have “spin-doublet inversion” between $T = 1$ and $T = 0$, which imprints the relative strength of ΞN interactions in different channels.

We remark here that ΞNαα$(T = 0, J^\pi = 1^-)$ is most bound relative to the two-body breakup threshold with the binding energy of 2.74 MeV ($= 4.31$ MeV $− 1.57$ MeV) as seen in Fig.2(b). This is due to the largest attraction in the $^{11}_{11}S_0$ ΞN channel among other channels as shown in Fig.2(a). Also, we find that the states in Table I have very small decay widths, $Γ = 20 − 40$ keV. This is due to the fact that the ΞN-ΛΛ interaction appears only at a short distance in the HAL QCD potential [9], so it causes only a small coupling at low energies. This is also in accordance with the recent emulsion data of Ξ hypernuclei [3] [4].

To see if Ξ and Ν interact with each other while loosely coupled to αα core, we plot the density distributions of Ξ, Ν and α inside the ground state of ΞNαα with $T = 1$ and $T = 0$ in Fig.3(a) and Fig.3(b), respectively. In both cases, we find that Ξ and Ν have extended distributions outside the αα core; root mean square distances $\sqrt{<R^2>}$ of α, Ξ and Ν, from the center of mass of αα are 1.74 fm (1.74 fm), 3.83 fm (3.53 fm) and 3.33 fm (3.24 fm), re-
FIG. 2. (a) $\Xi N$ phase shifts obtained from the HAL QCD potential. The figure is adapted and modified from [9]. (b) The calculated energy level of $\Xi N\alpha\alpha$ system. The energy is measured with respect to $\Xi N\alpha\alpha$ four-body breakup threshold.

FIG. 3. Density distributions of $\alpha, \Xi$ and the valence nucleon $N$ with respect to the center of the mass of $\alpha-\alpha$ in the ground states of the $\Xi N\alpha\alpha$ system with $T = 1$ (a) and with $T = 0$ (b).

Summary and concluding remarks.— We explored a new possibility to study the two-body $\Xi N$ interaction, especially its spin-isospin dependence, through the energy levels of the $\Xi N\alpha$ and $\Xi N\alpha\alpha$ systems where $\alpha$ plays a role to attract $\Xi N$ pair without changing its spin-isospin structure. By using the Gaussian expansion method (GEM) for three- and four-body cluster model and the $\Xi N$ potential obtained from the state-of-the-art lattice QCD calculations, we found that $\Xi N\alpha\alpha$ has spin-doublet bound states, $J^\pi = 1^-, 2^-$ in each isospin channel, while it is unlikely that $\Xi N\alpha$ has a bound state.

The ordering of the bound state levels of $\Xi N\alpha\alpha$ has a characteristic structure associated with the spin-isospin dependence of the $\Xi N$ interaction (Fig.2(a)): We found the inversion of the $1^- - 2^-$ spin-doublet between $T = 1$ and $T = 0$ (Fig.2(b)). Also, the largest $\Xi N$ attraction in the $^1S_0$ channel is reflected in the largest binding energy of the $(T,J^\pi) = (0, 1^-)$ state relative to the two-body breakup threshold.

These level structures of $\Xi N\alpha\alpha$ bound states can be studied experimentally by producing $\Xi N\alpha\alpha$ in $T = 1$ and $T = 0$ states through $(K^-, K^+)$ and $(K^-, K^0)$ reactions on the $^{10}$B target, respectively. If the level ordering between the $1^-$ and $2^-$ states is determined by the reaction cross section, information on the relative strengths of the $\Xi N$ interactions in $^{33}S_1$ and $^{31}S_0$, $^{13}S_1$ and $^{11}S_0$ channels can be extracted. At J-PARC facility, such experiments could be pursued after the planned experiment.
to produce $^{12}$Be and $^{7}$H using $^{12}$C and $^{7}$Li target by the $(K^-, K^+)$ reaction \[14\]. For an accurate comparison with future experimental data, theoretical calculations that fully account for isospin symmetry breaking are needed.

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