Simulation of the magnetic field generated by wires with stationary current and magnets with constant magnetization applied to the mirror trap, minimum-B and zero-B

M T Murillo\textsuperscript{1,2} and O Otero\textsuperscript{2}
\textsuperscript{1} Universidad Santo Tomás, Bucaramanga, Colombia.
\textsuperscript{2} Universidad Industrial de Santander, Bucaramanga, Colombia.
E-mail: oswaldoterolarte@gmail.com

Abstract. As a contribution to the computational simulation of magnetic confinement and heating of plasmas ECR (Electron Cyclotron Resonance), this work is dedicated to the calculation and subsequent analysis of the magnetic fields generated by permanent magnets and coils required in magnetic traps between which we can mention the mirror trap, minimum-B and zero-B. To do this, we solved numerically the Biot-Savart law in the case of the coils with stationary current and the Ampere law in the case of the permanent magnets. The study includes the characterization of the ECR areas as well as the display of the vector field all of this applied to the magnetic traps mentioned above. Additionally, in the case of the mirror type trap and minimum-B trap, it is determined the ratio of the mirror, because it is important in the description of confinement.

1. Introduction
The magnetic confinement searches to capture and retain charged particles using two properties of the magnetic field; first one is that every particle tends to move in a spiral way along the field lines, so that their displacement in parallel direction to the field lines is not affected. Second one, according to the magnitude of the field, the particle is ejected or reflected to areas where the field is weak, this phenomenon is known as magnetic mirror [1]. Taking the relevance of these premises have been proposed and built many devices trying to find an appropriate configuration of the geometry and magnitude of the field and prolong the lifetime of the particles into the trap. This problem has been studied in many works from the middle of last century [2]. The research group of plasma physics and technology and corrosion (FITEK), ascribed to the school of physics at the University Industrial of Santander is working on a theoretical study of plasmas ECR confined in magnetic mirror type trap, minimum-B trap and and zero-B trap [3]. To generate these confinement fields usually is used elements such as coils and permanent magnets [2, 4, 5]. For this reason, it is necessary to develop a tool to reproduce the field generated by those elements. Using these results and physical schemes presented in [2, 4, 5], we determined the confinement fields for each one of the traps, the ECR surfaces and the mirror radio to the mirror trap and minimum-B trap [2], because it is important in the description of confinement.
2. Coils and magnets modelling

The magnetic field generated by a coil is obtained from the superposition of contributions produced by circular loops with steady current, according to that the Biot-Savart’s law [1] in this case is used as:

\[
B = \frac{I r_0}{c} \int_0^{2\pi} \left[ (z - z_s) \cos \varphi \, \hat{i} + (z - z_s) \sin \varphi \, \hat{j} + (r_0 - y \sin \varphi - x \cos \varphi) \, \hat{k} \right] d\varphi
\]

where \( r_0 \) is the radius of the loop. We have determined the field produced by a coil in the following conditions: the current flowing through the loops is \( I = 9.6 \times 10^{12} \) stat amperes, 8 longitudinal loops and 5 transversal loops, the radius of the first loop is 5cm and the wire diameter is 0.5cm. The obtained field is presented in Figure 1, which is stronger in the central region due to its proximity to the loops constituting the coil.

![Figure 1. YZ view (plane x = 0cm) of magnetic field.](image)

In the case of magnets, we consider a constant magnetization into the volume of the material, this vector is oriented radially from one pole to the opposite pole, as is shown in Figure 2. The region of interest is located outside of the magnet, the field can be calculated through the gradient of the magnetic scalar potential [6], because there is no current density in that region:

\[
\phi_m(\mathbf{r}) = \int_\mathbf{R} \frac{M(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \cdot d\mathbf{s}' - \int_\mathbf{V} \frac{\nabla' \cdot M(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{v}'
\]
Where $\phi_m$ is the magnetic scalar potential, $s(v)$ is the boundary surface of the magnet, $v$ is the volume of the magnet. The contribution made by the second term is zero because the vector $\mathbf{M}$ is constant. Therefore, we can write the expression for the potential as follows:

$$
\phi_m(r) = \int_{z_1}^{z_2} \int_{x_1}^{x_2} \frac{M_0}{|\sin \theta| |r - r'|} dx' dz' - \int_{z_1}^{z_2} \int_{x_1}^{x_2} \frac{M_0}{|\sin \theta| |r - r'|} dx' dz'.
$$

(3)

Where $x_2R$, $x_2$, $x_1R$, $y$ are the integration limits on the axis $X$ and $z_1$ and $z_2$ are the limits on the axis $Z$, as presented in the Figure 2(a). An analytical result was published in [7] for the modelling of magnets with various polarization directions, however, we solved the Equation 3 in the case when the polarization vector is constant in all magnet and can turn around an axis.

Now, we study a particular case, according to Figure 2 we define $R_1 = 4.7cm$, $R_2 = 7.1cm$, $\theta = 90^\circ$, $a = 2.5cm$, $b = 21cm$ and $M_0 = 2.21KG$. The magnet defined here is outside of the simulation area. For a better visualization of the vector field, we present the Figure 2(b), as we can see, this figure shows that in the area near to the pole of the magnet (north pole), the field is intense, but as we move away from it, the magnitude decreases. The field lines are curved at $x = 1.5cm$ because it are looking for the south pole, which it corresponds to presented in [1].

### 3. Mirror trap

The mirror trap is obtained by the superposition of the field produced by two coils [12] and [10]. Such elements are placed at the end of a cylindrical cavity with which they share the same axis of symmetry; steady currents flowing through them with equal magnitudes and directions, see Figure 3. The radius of the cylindrical cavity is 6.4cm and its length is 16.1cm, the plasma is heated by microwave whose frequency is 2.45GHz. The current flowing through the coils is $3.3 \times 10^{12}$ stat amperes, we used 10 longitudinal loops and 3 transversals, the radius of the first loop is 8.43 cm and the loop separation is 1cm.

![Figure 3. Physical schemes: mirror trap, minimum-B and zero-B.](image)

The magnetic field of the mirror trap has an axial symmetry, we present a view of the field in the $YZ$ plane (see Figure 4). In which, the magnitude of the field decreases as you move on the $Z$ axis, approaching to the transversal center plane of the cavity, placed at $z = 0cm$. Additionally, in this region the field decreases with distance from the axis of the cavity, it means, the field is weak near to the walls. The magnetic field near to the coils is 1584.3G, while in the center of the cavity is 906.63G, with a mirror ratio equal to 1.75.

![Figure 4. YZ view (plane $x = 0cm$) of the magnetic field.](image)
The ECR surface is obtained when \( B = m c \omega / e \), where \( m \) and \( e \) is the mass and charge of the electron, \( c \) is the speed of light and \( \omega \) is the frequency of microwaves with which the plasma is heated [10, 12]. In the case of 2.45GHz, the resonant field is 875.89G, the ECR surface takes the form of a one-sheet hyperboloid. The cross section of the hyperboloid in the central plane is a circle of radius 2.3cm. The ECR surface is closed and is not a regular one-sheet hyperboloid, which tends to the infinity, as we can see in the Figure 5.

![Figure 5.](image) XYZ, XY (plane \( z = 0 \)cm) and YZ (plane \( x = 0 \)cm) view of ECR surface.

4. Minimum-B trap

The minimum-B configuration is obtained adding a transversal magnetic field to the mirror trap, the complementary field increases in the radial direction, which is placed throughout the cavity [4], see Figure 3(c). The longitudinal field is generated by two coils, the current flowing through the coils is \( 9.6 \times 10^{12} \) stat amperes, the coil has 8 longitudinal loops and 5 transversals, the radius of the first loop is 7.29cm and the loop separation is 1cm. The transversal field is produced by permanent magnets (hexapole system), according to the Figure 2 we define the following parameters: the radius of the inner cap is 4.7cm and the outer 7.1cm, the dimensions bars are 2.5cm and 21cm, with a magnetization of 2.21KG. For hexapole field simulation we take the following angles \( \theta = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ \) and \( 330^\circ \). To get a better idea about the minimum-B field, we present several figures. The longitudinal field has a mirror ratio equal to 1.82. The transversal field produced by a hexapole system corrects magnetohydrodynamic plasma instabilities confined on a mirror trap because it strengthens the radial component of the field in the central region of the cavity, given that in this region the contribution generated by the coils is zero. The magnitude of the field produced by the Minimum-B configuration increases in the longitudinal direction as \( |z| \) increases, which can be seen in the color scale shown in Figure 6. Additionally, the field is minimized at the center of the cavity (see Figure 6).

![Figure 6.](image) YZ view (plane \( x = 0 \)cm) of magnetic field.
The value that take in this case the ECR magnetic field is 5005.102G and the ECR surface takes the form of an ellipsoid flattened at the ends. To determine the dimensions of the surface obtained, we present a cross-section on \( z = 0 \) cm which allows us to visualize a circle of radius 1.9cm and a longitudinal-section on \( x = 0 \) cm obtaining a value of 2.85cm, as we can see on the Figure 7.

5. Zero-B trap
In the year 2000, Valeriy Dougar proposed a new trap to confine plasmas called zero-B trap. The main feature of this new system is the absence of magnetic field in the center of the cavity and the increase in the value of the same in all directions of the space [5]. The magnetic trap zero-B is obtained by the superposition of a cusp longitudinal field and a transversal multicusp field, the first one is generated by two coils while the latter is produced by a hexapole system, see Figure 3(right). The values used for the construction of the zero-B trap are the same that we use at the previous section for the minimum-B configuration. The radial component of the longitudinal field is positive and grows when are close to the central transversal plane of the cavity. The field generated by magnets strengthens the field in the region adjacent to the cavity walls, the magnitude of the field in the region close to south poles of the magnets is strengthened, because they have the same direction of the field produced by coils, while that near to the north poles is weakened, because they have opposite directions. The Figure 8 shown how the magnitude of the zero-B trap increases as \( |z| \) grows. This view also allows us to see how the field becomes zero at the center of the cavity. The magnitude of the magnetic field at the ECR surface is 5001.07G and its shape looks like an ellipsoid flattened at the ends but noncircular cross-section like that obtained in the minimum-B trap. In this case, the dimensions of the ellipsoid on the \( X \) axis is 2.58cm, \( Y \) axis is 2.95 cm and \( Z \) axis is 7.55cm (see Figure 9).
6. Conclusion

The graphs obtained for the magnetic traps allow the researcher to know the direction of field and its magnitude gradient, which is important to determine the direction of the diamagnetic force, essential in the study of magnetic confinement. The study of the ECR surfaces is significant for plasma heating by microwaves, allowing understand the fields involved in each of the magnetic traps.

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