Aspects of finite electrodynamics in $D = 3$ dimensions

Patricio Gaete$^1$, José Helayel-Neto$^2$ and Euro Spallucci$^3$

$^1$ Departamento de Física and Centro Científico-Tecnológico de Valparaíso, Universidad Técnica Federico Santa María, Valparaíso, Chile
$^2$ Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, Brazil
$^3$ Dipartimento di Fisica Teorica, Università di Trieste and INFN, Sezione di Trieste, Italy

E-mail: patricio.gaete@usm.cl, helayel@cbpf.br and spallucci@ts.infn.it

Received 26 January 2012, in final form 22 March 2012
Published 10 May 2012
Online at stacks.iop.org/JPhysA/45/215401

Abstract

We study the impact of a minimal length on physical observables for a three-dimensional axionic electrodynamics. Our calculation is done within the framework of the gauge-invariant, but path-dependent variable formalism which is an alternative to the Wilson loop approach. Our result shows that the interaction energy contains a regularized Bessel function and a linear confining potential. This calculation involves no $\theta$ expansion at all. Once again, the present analysis displays the key role played by the new quantum of length.

PACS numbers: 14.70.−e, 12.60.Cn, 13.40.Gp

1. Introduction

One of the most actively pursued areas of research in high energy physics consists of the investigation of extensions of the standard model (SM). This is primarily because the SM does not include a quantum theory of gravitational interactions. As is well known, in the search for a more fundamental theory going beyond the SM string theories are the only known candidate for a consistent, ultraviolet finite quantum theory of gravity, unifying all fundamental interactions. It should, however, be noted here that string theories apart from the metric also predict the existence of a scalar field (dilaton), an antisymmetric tensor field of the third rank which is associated with torsion and noncommutativity. This has led to an increasing interest in the possible physical effects of noncommutativity in quantum field theories, which have been studied using the Moyal star product [1–6]. Mention should be made, at this point, to a novel way to formulate noncommutative quantum field theory (or quantum field theory in the presence of a minimal length) [7–9] which clearly leads to an ultraviolet finite field theory, and the cutoff is provided by the noncommutative parameter $\theta$. In this connection, it may be recalled that the essential idea of this development is to define the fields as a mean value over coherent states of the noncommutative plane, such that a star product does not need to be introduced. We further note that recently it has been shown that the coherent state approach can be summarized through the introduction of a new multiplication rule which is known as
the Voros star product \cite{10–14}. Nevertheless, and most importantly, the physics turns out to be independent of the choice of the type of product \cite{15}.

On the other hand, it is well known that a full understanding of the QCD vacuum structure and color confinement mechanism from first principles still remain elusive. However, phenomenological models have been of importance in our present understanding of confinement, and can be considered as effective theories of QCD. It is worth recalling here that many approaches to the problem of confinement rely on the phenomenon of condensation. For example, in the illustrative scenario of dual superconductivity \cite{16–18}, the condensation is due to topological defects originating from quantum fluctuations (monopoles). Accordingly, the color electric flux linking quarks is squeezed into strings (flux tubes), and the nonvanishing string tension represents the proportionality constant in the linear, quark confining, potential. In this respect, it is appropriate to recall that Abelian gauge theories also possess a confining phase, by including the effects due to the compactness of the $U(1)$ group, which dramatically changes the infrared properties of the model \cite{19}. These results, first found in \cite{19}, have been recovered by many different techniques \cite{20–22} where the key ingredient is the contribution of self-dual topological excitations.

With these ideas in mind, in a previous paper \cite{23}, we have studied axionic electrodynamics from this new noncommutative approach (coherent state approach), in the presence of a nontrivial constant expectation value for the gauge field strength. In particular, in the case of a constant magnetic field strength expectation value, we have obtained an ultraviolet finite static potential which is the sum of a Yukawa-type and a linear potential, leading to the confinement of static charges. We note that this theory experiences mass generation due to the breaking of rotational invariance induced by the classical background configuration of the gauge field strength. Interestingly, it should be noted that this calculation involves no $\theta$ expansion at all. By following this line of reasoning, this work is aimed at studying the stability of the above scenario for the three-dimensional case. The main purpose here is to reexamine the effects of this new noncommutativity on a physical observable, and to check if a linearly increasing gauge potential is still present whenever we go over into three dimensions.

At this point, we would like to recall that three-dimensional theories are interesting because of their connection to the high-temperature limit of four-dimensional theories \cite{24–27}, as well as their applications to condensed matter physics \cite{28}. Most recently, three-dimensional physics has been raising a great deal of interest in connection with the study of branes, namely issues like self-duality, and new possibilities for supersymmetry breaking as induced by 3-branes are of special relevance.

Thus, as already mentioned, the main purpose here is to examine the effects of this new noncommutativity on a physical observable for the three-dimensional case. To do this, we will work out the static potential for axionic electrodynamics by using the gauge-invariant but path-dependent variable formalism along the lines of \cite{29–31}. According to this formalism, the gauge-fixing procedure corresponds to a path choice. Nevertheless, the point we wish to emphasize is that this approach offers a natural setting to examine aspects of screening and confinement in gauge theories, because it involves the use of strings to carry electric flux \cite{32}. As we will see, there are two generic features that are common in the four-dimensional case and its lower extension studied here. First, the existence of a linear potential, leading to the confinement of static charges. The second point is related to the correspondence among diverse effective theories. In fact, in the case of a constant magnetic field strength expectation value, we obtain that the interaction energy is the sum of a regularized Bessel function and a linear potential. Incidentally, the above static potential profile is analogous to that encountered in: a Lorentz-and CPT-violating Maxwell–Chern–Simons model \cite{33}, a Maxwell-like three-dimensional model induced by the condensation of topological defects driven by quantum
fluctuations [34], a Lorentz invariant violating electromagnetism arising from a Julia–Toulouse mechanism [35] and three-dimensional gluodynamics in curved spacetime [36].

2. Three-dimensional finite electrodynamics

2.1. Maxwell case

As already mentioned, our principal purpose is to calculate explicitly the interaction energy between static point-like sources for noncommutative axionic electrodynamics. However, before going into this theory, we shall discuss the interaction energy for noncommutative electrodynamics, through two different methods. The first approach is based on the path-integral formalism, whereas the second one makes use of the gauge-invariant but path-dependent variable formalism. This would not only provide the setup for our subsequent work, but also fix the notation.

The starting point is the three-dimensional, imaginary time-rotated, Euclidean Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu}. \quad (1)$$

The effects of a minimal length $l_0 \equiv \sqrt{\theta}$ induced by ‘fluctuating’ noncommutative coordinates $[x^\mu, x^\nu] = \theta_{\mu \nu}$, where $\theta_{\mu \nu} = \text{diag}(\theta, \theta, \theta)$ can be described in various ways. The most common approach is to shift noncommutativity from coordinates to the product of functions (fields) by introducing the so-called star product. Unfortunately, once star multiplication is introduced, the only way to carry out calculations is through perturbative expansion in $\theta$ leading to inconsistent results (see the appendix in [23]). An alternative, non-perturbative approach, avoiding expansion in $\theta$, has been introduced in [7–9] and turned out to introduce a simple modification in the Feynman propagators as a final result. Let us then write down the functional generator of Green’s functions, that is,

$$Z[J] = \exp \left( -\frac{1}{2} \int d^3x \int d^3y J^\mu(x) D_{\mu \nu}(x, y) J^\nu(y) \right). \quad (2)$$

Next, adding to (1) the gauge-fixing term $\mathcal{L}_{\text{GF}} = -\frac{1}{2} (\partial_\mu A^\mu)^2$ (Feynman gauge), and noting that no Faddeev–Popov ghosts are required in this case, we obtain the propagator in momentum space

$$D_{\mu \nu}(k) = \frac{1}{k^2} \left\{ e^{-\theta k^2} \delta_{\mu \nu} + (1 - e^{-\theta k^2}) \frac{k_\mu k_\nu}{k^2} \right\}. \quad (3)$$

Equation (3) shows that only short wavelength is suppressed by the underlying space(time) quantum fluctuations. The exponential damping factor in the propagator can be seen as a friction effect experienced by the photon at length scale comparable with $\sqrt{\theta}$, where the classical model of space(time) as a smooth manifold breaks down. Deviations from classical behavior can be seen as an increasing degree of ‘roughness’, or fuzzyness, opposing to photon propagation.

By means of expression $Z = e^{-W[J]}$, and employing equation (3), $W[J]$ takes the form

$$W[J] = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} J^*_\mu(k) \left[ \frac{e^{-\theta k^2}}{k^2} \delta_{\mu \nu} - \frac{1 - e^{-\theta k^2}}{k^2} \frac{k_\mu k_\nu}{k^2} \right] J^\nu(k). \quad (4)$$

A bonus of our approach is that, even if $\theta$ has a dimension of length squared, or mass to power $-2$, the way it enters the propagator does not break gauge invariance and the current $J^\mu(k)$ is still divergence-free. Thus, expression (4) becomes

$$W[J] = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} J^*_\mu(k) \left( \frac{e^{-\theta k^2}}{k^2} \right) J^\mu(k). \quad (5)$$
Next, for \( J_\mu (x) \equiv [Q \delta^{(2)}(x - x^{(1)}) + Q' \delta^{(2)}(x - x^{(2)})] \delta_\mu^0 \), and using standard functional techniques \([37]\), we obtain that the interaction energy of the system is given by

\[
V(r) = Q Q' \int \frac{d^2 k}{(2\pi)^2} \frac{e^{-\imath k \cdot r}}{k^2} e^{i \frac{4}{r}},
\]

where \( r \equiv x^{(1)} - x^{(2)} \).

Now, we move on to calculate the integral (6). To this end it is advantageous to introduce an infrared regulator \( \mu \). This allows us to obtain a form more comfortable to handle the integral. Hence, we evaluate \( \lim_{\mu \to 0} I \), that is,

\[
I = \lim_{\mu \to 0} I = \lim_{\mu \to 0} (\mu^2)^{-\varepsilon/2} \int \frac{d^2 k}{(2\pi)^2} \frac{e^{-\imath k \cdot r}}{k^2} e^{i \frac{4}{r}}.
\]

We may further simplify equation (7) by doing the integration of \( k \) and \( s \), which leads immediately to the result

\[
I = \frac{1}{4\pi} \lim_{\mu \to 0} (\mu^2 r^2)^{-\varepsilon/2} \gamma(\varepsilon/2, r^2/4\theta).
\]

Here \( \gamma(\varepsilon/2, r^2/4\theta) \) is the lower incomplete Gamma function defined by the following integral representation:

\[
\gamma \left( \frac{\alpha}{\beta}, x \right) = \int_0^x \frac{du}{u^{1/\beta}} e^{-u}.
\]

Next, we use \( \gamma(\varepsilon/2, r^2/4\theta) \rightarrow \frac{1}{2} \left[ \gamma(\varepsilon/2, r^2/4\theta) + \gamma(1 + \varepsilon/2, r^2/4\theta) \right] \), \((\mu^2 r^2)^{-\varepsilon/2} \rightarrow 1 - \frac{1}{2} \ln(\mu^2 r^2), \quad (\frac{\mu^2}{\theta})^{\varepsilon/2} \rightarrow 1 + \frac{1}{2} \ln \left( \frac{\mu^2}{\theta} \right) \) and \( \gamma(1 + \varepsilon/2, r^2/4\theta) \rightarrow \gamma(1, r^2/4\theta) = 1 - e^{-\frac{r^2}{4\theta}} \), to examine the behavior of expression (8) as \( \varepsilon \to 0 \). Expression (8) then becomes

\[
I = -\frac{1}{2\pi} \left[ \ln(\mu r) - e^{-r^2/4\theta} \ln \left( \frac{r}{2\sqrt{\theta}} \right) \right].
\]

Combining equations (6) and (10), together with \(-Q = Q'\), the interaction energy reduces to

\[
V(r) = \frac{Q^2}{2\pi} \left[ \ln(\mu r) - e^{-r^2/4\theta} \ln \left( \frac{r}{2\sqrt{\theta}} \right) \right].
\]

It is interesting to note that unlike the Coulomb potential which is singular at the origin, \( V \) is finite there: \( V(0) = Q^2/2\pi \ln(2\mu\sqrt{\theta}) \). The fact that the potential is finite for \( r \to 0 \), it is a clear evidence that the self-energy and the electromagnetic mass of a point-like particle are finite in this noncommutative version of electrodynamics. However, when \( r \) is large, \( V \) reduces to the Coulomb potential (figure 1).

Next we compute the interaction energy from the viewpoint of the gauge-invariant but path-dependent variable formalism, along the lines of \([29–31]\). Within this framework, we shall compute the expectation value of the energy operator \( \hat{H} \) in the physical state \( |\Phi\rangle \), which we will denote by \( \langle \hat{H} \rangle_\Phi \).

By proceeding in the same way as in \([23]\), we obtain the static potential for two opposite charges, located at \( y \) and \( y' \):

\[
V(L) = \frac{Q^2}{2\pi} \left[ \ln(\mu L) - e^{-L^2/4\theta} \ln \left( \frac{L}{2\sqrt{\theta}} \right) \right]
\]

with \( |y - y'| = L \). It is remarkable that two quite different methods have led to the same expression for the effective three-dimensional potential. This astonishing result seems to indicate that to lower orders, the two approaches might be equivalent order by order.
Figure 1. The potential $V$ (in units of $Q^2/2\pi$), as a function of the distance $r$. The dashed line represents the Coulomb potential (in units of $Q^2/2\pi$).

2.2. Maxwell–Chern–Simons case

We now consider the calculation of the interaction energy between static point-like sources in a topologically massive gauge theory. In such a case, the Lagrangian reads \[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{\sigma}{4} \varepsilon^{\mu\nu\rho\sigma} A_{\mu} F_{\nu\rho} - A_0 J^0, \] (13)
where $J^0$ is the external current and $\sigma$ is the topological mass.

The above Lagrangian will be the starting point of the Dirac constrained analysis. The canonical momenta following from equation (13) are $\pi_{\mu} = -F_{\mu 0} + \frac{\sigma}{2} \varepsilon_{\mu \nu} A_{\nu}$, which results in the usual primary constraint $\pi^0 = 0$ and $\pi^i = F^{0i} + \frac{\sigma}{2} \varepsilon^{ij} A_j$ ($i, j = 1, 2$). So the canonical Hamiltonian is
\[ H_c = \int d^2 x \left( \pi_i \partial^i A_0 - \frac{1}{2} F_{00} F^{00} + \frac{1}{4} F^{ij} F_{ij} - \frac{\sigma}{2} \varepsilon^{ij} A_0 \partial_0 A_j + A_0 J^0 \right). \] (14)

Time conservation of the constraint $\pi^0$ leads to the secondary constraint (Gauss law) $\Omega_1 \left( x \right) = \partial_i \pi^i + \frac{\sigma}{2} \varepsilon_{ij} \partial^i A^j - J_0 = 0$, and the time stability of the secondary constraint does not induce more constraints, which are first class. It should be noted that the constrained structure for the gauge field remains identical to the Maxwell theory. Thus, the quantization can be done in a similar manner to that in the previous subsection. In view of this situation, and in order to illustrate the discussion, we now write the equations of motion in terms of the magnetic ($B = \varepsilon_{ij} \partial^j A^i$) and electric ($E_i^L = \pi^i - \frac{\sigma}{2} \varepsilon^{ij} A_j$) fields as
\[ \dot{E}_i \left( x \right) = -2\sigma \varepsilon_{ij} E^j \left( x \right) - \varepsilon_{ij} \partial^j B, \] (15)
\[ \dot{B} \left( x \right) = -\varepsilon_{ij} \partial^j E^i. \] (16)

In the same way, we write the Gauss law as
\[ \partial_i E^i_L + \sigma B - J^0 = 0, \] (17)
where $E^i_L$ refers to the longitudinal part of $E^i$. This implies that for a static charge located at $x^i = 0$, the static electromagnetic fields are given by
\[ B = -\sigma \frac{J^0}{\nabla^2 + \sigma^2}. \] (18)
where $\nabla^2$ is the two-dimensional Laplacian. For $J_0(x) = q e^{\sigma V^2} \delta^{(2)}(x)$, expressions (18) and (19) reduce to

$$B = q\sigma \frac{e^{\sigma^2 \theta}}{2\pi} \left\{ K_0(\sigma r) - \frac{1}{2} \int_{r/2\sigma}^{\infty} dy \frac{1}{y} e^{-\frac{\sigma^2}{2}(y^2 + \frac{1}{4})} \right\},$$

$$E_i = q\sigma \frac{e^{\sigma^2 \theta}}{2\pi} \partial_i \left\{ K_0(\sigma r) - \frac{1}{2} \int_{r/2\sigma}^{\infty} dy \frac{1}{y} e^{-\frac{\sigma^2}{2}(y^2 + \frac{1}{4})} \right\},$$

where $r = |x|$ and $K_0$ is the modified Bessel function.

Having made these observations, we can write immediately the following expression for the physical scalar potential [23]:

$$A_0(t, x) = \int_0^1 d\lambda x' E_i(t, \lambda x) = \int_0^1 d\lambda x' \partial_i \left( -\frac{J_0(\lambda x)}{\nabla^2 - \sigma^2} \right).$$

For $J'(x) = q e^{\sigma V^2} \delta^{(2)}(x - \mathbf{a})$, expression (22) then becomes

$$A_0(x) = q\sigma \frac{e^{\sigma^2 \theta}}{2\pi} \left\{ K_0(\sigma |x - \mathbf{a}|) - \frac{1}{2} \int_{|x - \mathbf{a}|/2\sigma}^{\infty} dr \frac{1}{r} e^{-\frac{\sigma^2}{2}(r^2 + \frac{1}{4})} \right\}$$

$$- q\sigma \frac{e^{\sigma^2 \theta}}{2\pi} \left\{ K_0(\sigma |\mathbf{a}|) - \frac{1}{2} \int_{|\mathbf{a}|/2\sigma}^{\infty} dr \frac{1}{r} e^{-\frac{\sigma^2}{2}(r^2 + \frac{1}{4})} \right\}.$$  

As we have explained in [23], the interaction energy for a pair of static point-like opposite charges at $\mathbf{y}$ and $\mathbf{y}'$ is given by

$$V(|\mathbf{y} - \mathbf{y}'|) = -q^2 \frac{e^{\sigma^2 \theta}}{2\pi} \left\{ K_0(\sigma |\mathbf{y} - \mathbf{y}'|) - \frac{1}{2} \int_{|\mathbf{y} - \mathbf{y}'|/2\sigma}^{\infty} dr \frac{1}{r} e^{-\frac{\sigma^2}{2}(r^2 + \frac{1}{4})} \right\},$$

which is ultraviolet finite (figure 2). Note that in figure 2 we defined $V(|\mathbf{y} - \mathbf{y}'|) = q^2 \frac{e^{\sigma^2 \theta}}{2\pi} V(x)$. 

![Figure 2. Shape of the potential (equation (24)).](image)
3. Three-dimensional axionic electrodynamics

We turn now to the problem of obtaining the interaction energy between static point-like sources for the three-dimensional version of the model studied in [23]. To do this, we shall start from the four-dimensional spacetime Maxwell theory with a term that couples the dual electromagnetic tensor to a fixed \( v^{\mu} \) [39–43]:

\[
\mathcal{L}^{(3+1)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} \epsilon^{\mu\nu\lambda} v_{\mu} A_{\lambda} F_{\nu\lambda} + \frac{1}{2} m^2 A_{\lambda} A^{\lambda},
\]

(25)

with the additional presence of a mass term for the gauge field. Here the Greek letters run from 0 to 3. This model was considered in [44], where the Proca mass stems from a Higgs scalar sector. It was shown that this model is unitary just for space-like background, while it presents ghost states for a timeline or light-like background.

Next, to study this model in three-dimensional spacetime dimensions, we perform its dimensional reduction along the lines of [33]. In other words, we use the prescription: \( A^{\mu} \rightarrow (A^{\mu}; \phi) \), \( v^{\mu} \rightarrow (v^{\mu}; s) \) and \( \partial_3(\text{anything}) = 0 \). Carrying out this prescription in equation (25), we then obtain

\[
\mathcal{L}^{(2+1)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \epsilon^{\mu\nu\lambda} v_{\mu} \phi (\partial_\nu A_\lambda) + \frac{s}{2} \epsilon^{\mu\nu\lambda} A_\nu (\partial_\mu A_\lambda) + \frac{m^2}{2} A_\mu A^\mu - \frac{m^2}{2} \phi^2,
\]

(26)

where \( \mu, \nu, \lambda = 0, 1, 2 \). Accordingly, there appear two scalars, that is, the scalar field \( \phi \) which exhibits dynamics, and \( s \), a constant scalar. Then, by discarding the scalar field \( s \) and the mass term for the gauge field, we arrive at

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2,
\]

(27)

which represents the three-dimensional analogue of the model studied previously [23]. In addition, a preliminary study of this model was considered in [45].

Following our earlier procedure [23], we restrict ourselves to static scalar fields, a consequence of this is that one may replace \( \Delta \phi = -\nabla^2 \phi \), with \( \Delta \equiv \partial_\mu \partial^\mu \). It also implies that, after performing the integration over \( \phi \), the induced effective Lagrangian density is given by

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{8} \epsilon^{\mu\nu\lambda} v_{\mu} F_{\nu\lambda} \frac{1}{\nabla^2 - m^2} \epsilon^{\gamma\beta} v_{\gamma} F_{\beta\lambda}.
\]

(28)

By introducing \( V^{\nu\lambda} \equiv \epsilon^{\mu\nu\lambda} v_{\mu} \), expression (28) then becomes

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{8} V^{\nu\lambda} F_{\nu\lambda} \frac{1}{\nabla^2 - m^2} V^{\beta\gamma} F_{\beta\gamma}.
\]

(29)

Note that (29) has the same form as the corresponding effective Lagrangian density in four-dimensional spacetime. This gives us the starting point for the examination of the effects of the Lorentz violating background on the interaction energy.

It is once again straightforward to apply the gauge-invariant formalism discussed in the preceding section in the \( V^{0i} \neq 0 \) and \( V^{ij} = 0 \) (\( u_0 \neq 0 \)) case (referred to as the space-like background in what follows). In such a case, the Lagrangian reads

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} V^{0\phi} F_{0\phi} \frac{1}{\nabla^2 - m^2} V^{0\phi} F_{0\phi},
\]

(30)

where \( (\mu, \nu = 0, 1, 2) \) and \( (i, k = 1, 2) \). This leads us to the canonical Hamiltonian

\[
H_C = \int d^2 x \left\{ -A_0 (\partial_0 \Pi^0) + \frac{1}{2} \Pi^2 + \frac{1}{2} (\mathbf{V} \cdot \mathbf{Pi})^2 + \frac{1}{2} \mathbf{B}^2 \right\},
\]

(31)
where $M^2 \equiv m^2 + V^2$ and $B$ is the magnetic field. We skip all the technical details and refer to [23] for them. The corresponding static potential for two opposite charges located at $y$ and $y'$ turns out to be

$$V = -\frac{q^2}{2\pi} e^{M\theta} \left[ K_0(ML) - \frac{1}{2} \int_{U(2M)} dt e^{-\frac{ML}{4}(t+\frac{1}{t})} \right] + \frac{q^2 m^2 e^{M\theta}}{4M} L, \quad (32)$$

where $L \equiv |y - y'|$. Again, this result explicitly displays the effect of including a smeared source in the form of an ultraviolet finite static potential. It is interesting to note that the rotational symmetry is restored in the resulting form of the potential, although the external background breaks the isotropy of the problem in a manifest way. It should be remarked that this feature is also shared by the corresponding four-dimensional spacetime interaction energy.

Here, an interesting matter comes out. The result (32) agrees with that of Polyakov [46] based on the monopole plasma mechanism, except that this result shows a regularized Bessel function. In this way, the above analysis reveals that, although both models are different, the physical content is identical in the short distance regime. This behavior is also obtained in the context of the condensation of topological defects [34, 35].

4. Concluding comments

To conclude, this work is a sequel to [23], where we have considered a three-dimensional extension of the recently proposed finite axionic electrodynamics. To do this, we have exploited a crucial point for understanding the physical content of gauge theories, namely the correct identification of field degrees of freedom with observable quantities. Again, our calculations involve no $\theta$ expansion at all and, as in [23], the above analysis displays the key role played by the new quantum of length.

It is worth mentioning that our result of section 2.1 is finite for a special combination of both ultraviolet and infrared regulators. This is different to what happens in $(3 + 1)$-dimensions. Another interesting point we mention here is that, according to the results of the papers [47, 48], a spin-charge $SU(2) \times U(1)$ gauged (planar) model can be written down which is exactly equivalent to the original $t$–$J$ model [49] for strongly correlated electronic systems. In this treatment, the dynamics of both the $SU(2)$ and $U(1)$ gauge potentials is described by Chern–Simons actions. Considering this frame, we believe that our approach to extract potentials between static sources could be pushed forward and it would be relevant to understand how to obtain a low-energy effective action for the self-generated gauge field of the residual $U(1)$ gauge interaction induced by the spin-charge separation [50]. Our study of section 2.2 could be extended to take into account this gauge-invariant scenario which describes interesting physical properties of the $t$–$J$ model.

We further note that, based on the investigation we have pursued in our work, we are faced with the prospect of going deeper into the study of axionic-like electromagnetic models in $(2 + 1)$-dimensions. We have here considered an axionic action that stems from the dimensional reduction of a Lorentz-symmetry violating electrodynamical action. The scalar $\phi$-field that mixes with the electromagnetic field strength is the $(2 + 1)$-dimensional descent of the $A_3$-component of the four-dimensional potential. We intend, as a step forward, to write down the $(2 + 1)$-dimensionally reduced version of the true $\phi F_{\mu\nu} \tilde{F}^{\mu\nu}$ of $(3 + 1)$-dimensions and, then, analyze the planar counterpart of the Witten effect [51] and place our approach for calculating potentials in a situation such that we may study a sort of planar topological insulator [52–54] as an axionic medium.

Finally, it seems a challenging work to extend the above analysis to the non-Abelian case as well as to three-dimensional gravity. We expect to report on progress along these lines soon.
Acknowledgments

PG was supported in part by Fondecyt (Chile) grant 1080260. One of us (PG) wants to thank the Abdus Salam ICTP for hospitality and support. JH-N expresses his gratitude to CNPq-Brasil and FAPERJ-Rio de Janeiro for the invaluable support.

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