A Cascaded Switched-capacitor AC-AC Converter with a Ratio of $1/2^n$

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Abstract: Based on the existing AC-AC switched-capacitor (SC) converters, this paper demonstrates a new cascaded AC-AC converter circuit topology with a ratio of $1/2^n$, which only consists of power switches and capacitors. The converter consists of multi-stage converters and the pre-and post-stage circuits are independent with each other. The principle of the topology and the formula of related parameters, including equivalent resistance, equivalent capacitance and switching loss, are deduced with the port network theory, thus the equivalent circuit is gained in this paper. After the theoretical analysis, simulation models and experimental prototype were established to validate the correctness of the circuit topology. Both simulation and experimental results have verified the effectiveness of the circuit topology and the correctness of the theoretical derivation.

Keywords: AC-AC Converter, Switch-capacitor, Cascade Circuit, Voltage Transformation Ratio

1. Introduction

The switched-capacitor (SC) converter has been an important research topic for many years, particularly in non-isolated DC-DC applications [1-4]. In recent years, the research of AC-AC conversion based on switched-capacitor has caught on. The literature [5-8] extended the principle of SC from DC-DC to AC-AC converters. Publication [5] described the switched-capacitor principle in AC-AC field firstly.

As is shown in Figure 1 [6], the circuit topology includes four bidirectional power switch groups which are connected in series and represented as $S_1$-$S_4$, and three capacitors represented as $C_1$-$C_3$. In this paper the circuit is called the single-stage circuit structure. The practical implementation of a bidirectional switch using 2 MOSFETs is shown in Figure 2 ($R_{on}$ is the switch conduction resistance).

As the switching of power MOSFETs is controlled, the output voltage of all the capacitors with identical ratios of $1/2^n$ can be achieved. In a switching cycle, there are two states of the converter. In the first state the switch groups $S_1$ and $S_2$ are turned on and the capacitor $C_1$ is paralleled with $C_2$. In the second state the switch groups $S_3$ and $S_4$ are turned on and the capacitor $C_1$ is in parallel with $C_3$. The switch timing diagram of the switch groups can be seen in Figure 3 with the switching frequency of 100 kHz and duty cycle of 50%.

This converter can be operated as a step-down converter with the static gain of $1/2$. Its equivalent resistance is small enough and the efficiency is almost 95%. Experimental results verified the practicability of the switched-capacitor AC-AC converter. Literature [7] proposed an AC-AC direct static power converter aiming at providing a split-phase system from an AC single-phase voltage source. Literature [8] modified the behavior of the present switched-capacitor
AC-AC converter in each operation stage and showed that shortening the circuit by sacrificing the bi-directionality with 4 bi-directional switches instead of 8, resulted in a reduction of cost and size.

Figure 2. Bidirectional switch model and its practical implementation using two MOSFETs.

Figure 3. The drive waveform of the switch groups.

The literature [9, 10] proposed the cascaded switched capacitor for DC-DC power converter which also has some inspirations for this paper.

In this paper, inspired by the research mentioned above, a new cascaded switched-capacitor AC-AC converter based on the SC principle is proposed and investigated. The proposed converter is composed only of power MOSFETs and capacitors to achieve the step-down ratio of $1/2^n$ ($n = 1, 2, 3, \ldots$).

It has such advantages as high power density, small size, high efficiency, light weight, etc. More importantly it can realize the AC-AC Buck without the magnetic transformer. With the miniaturization of switching power supply, switched capacitor converters are used more widely in power electronic circuits and have very broad application prospects, such as the fields like the miniaturization of power chip design, PV modular design and new energy applications.

2. The Proposed Implement Method

Developed by the basic circuit unit in Figure 1, a new cascaded switched-capacitor converter with a ratio of $1/2^n$ is proposed. The cascaded circuit topology makes use of the same working principle as the circuit topology in Figure 1. It is realized by controlling the operation of power switches to achieve the stress voltages of capacitors in all stages which are equal to half of their input-side voltages respectively.

Figure 4. The SC AC-AC converter with the ratio of 1/4.

In Figure 1, when the input voltage $U_i$ is applied across capacitors $C_1$ and $C_3$, the voltages across all the capacitors are identical, being equal to 1/2 of the input voltage $U_i$.

Figure 5. The SC AC-AC converter with the ratio of 1/2$^n$.

The cascaded two-stage SC circuit is shown in Figure 4. The input of the second stage circuit is connected to the output of the first one. The voltages across all capacitors in the second stage of the Figure 3 ($C_4$, $C_5$, and $C_6$) are all identical, being equal to half of the voltages across the capacitors in the first stage and as a result, a fixed conversion ratio of 1/4 can be achieved.

It can be seen that the SC principle is used for an N-stage SC converter circuit, as shown in Figure 5, a fixed conversion ratio of 1/2$^n$ can be achieved.

3. Theoretical Analysis

3.1. Equivalent Resistance

The proposed single-stage circuit has an SC cell as shown in Figure 6 (a). Previous studies on DC-DC converters have demonstrated that this SC cell can be represented by an equivalent resistance, as illustrated in Figure 6 (b), which is calculated using (1) for different duty cycle values and (2) for a duty cycle of 50% [11, 12] (if the switching frequency become high enough, the value may change[13]). The model in Figure 6 (b) and Figure 6 (c) describes the equivalent...
resistance of the SC AC-AC converter single-stage circuit ‘seen’ through its input and output side. Nevertheless, it can also be represented through its output side employing the equation defined in (3) [6].

\[ R_{eqO} = \frac{1}{4} R_{eqI} \]  

In a single-stage circuit, the output-side voltage is 1/2 of the input-side voltage and the trunk current in the output side is twice the value of that in the input side if the power transmission efficiency is considered as 100%. When it comes to a multi-stage converter circuit, the output side in previous stage can be considered as the input side of the following stage. Thus the \( R_{eqO} \) of each stage is 1/4 of that in the previous stage. The model in Figure 7 describes the equivalent resistance of an N-stage SC AC-AC converter seen through the output side of the first stage. Equ.(4) presents the relation between the equivalent resistance \( R_{eqO(N)} \) of an N-stage SC AC-AC converter and \( R_{eqO(1)} \) of a single stage SC AC-AC Converter.

\[ R_{eqO(N)} = \left( 1 + \frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^{(N-1)}} \right) R_{eqO(1)} \]  

3.2 Equivalent Capacitance

The operation works as two equivalent capacitors, one in parallel with \( C_2 \) which has a value of DC and the other in parallel with \( C_3 \) which has a value of \((1-D)C\), as represented in Figure 8(a). The analysis of this electrical circuit (considering \( D=0.5 \) and three equal capacitors) enables an equivalent capacitance to be found for the AC-AC converter when seen through the input side which is defined in (5) and has the configuration shown in Figure 8(b). The same capacitance can be represented by the output side shown in Figure 8(c), as given by (6) [6]. The equivalent capacitance allows the reactive power flow required by the AC-AC converter and its power factor to be estimated.

\[ C_{eqO(1)} = \frac{1}{2} C \]  

\[ C_{eqO(1)} = 3C \]  

Equ. (7) can be obtained by (5) and (6):

\[ C_{eqO} = 4C_{eq1} \]  

In a single-stage circuit, the relation between the equivalent capacitance seen through the output side and that seen through the input side can be described as in (7). When it comes to a multi-stage circuit, it is true that the output side in previous stage is the input side of the following stage. Thus the value of the equivalent capacitance in each stage is 4 times of that in the previous stage. The equivalent capacitance of an N-stage converter circuit can be seen as a parallel connection of that in every stage. Equation (8) presents the relation between the equivalent capacitance of an N-stage SC AC-AC converter and that of a single-stage SC AC-AC converter when seen through the output side of the first stage.

\[ C_{eqO(N)} = \left( 1 + 4 + 16 + \cdots + 4^{(N-1)} \right) C_{eqO(1)} \]  

(9) can be obtained by (5) and (8):

\[ C_{eqO(N)} = 3 \cdot \left( 1 + 4 + 16 + \cdots + 4^{(N-1)} \right) C \]  

3.3 Switching Losses

The single-stage circuit employs bidirectional switches which are implemented by MOSFETs. The switching loss in one MOSFET due to its parasitic capacitance is given by (10) and shown as \( P_{\text{sl1s}} \). \( C_{\text{oss}} \) is the output capacitance of a MOSFET, \( f_s \) is the switching frequency and \( V_{pk} \) is the voltage across one switch defined by (11) \( (V_{pk} \) being the input-side voltage and \( V_{pk} \) being the peak value of \( V_{pk} \)). Equation (10) calculates the power supplied to the MOSFET capacitance in a switching period.

\[ P_{\text{sl1s}} = \frac{C_{\text{oss}} f_s V_{pk}^2}{2} \]  

\[ C_{\text{eqO}} = \left( 1 + 4 + 16 + \cdots + 4^{(N-1)} \right) C \]
The average of the supplied power in all switching periods during one period of input voltage provides the total power supplied to the MOSFET capacitance, which is given by (12). As the single-stage circuit uses eight switches, the switching losses \( P_{sls} \) due to parasitic capacitances of 8 MOSFETs are expressed by (13). This equation demonstrates that increasing the switching frequency of the AC-AC converter consequently decreases the switching losses [6]. An equivalent resistance \( R_{sl0} \) that describes the switching losses of the AC-AC converter on the output side can be obtained from (14).

\[
P_{sls} = \frac{1}{2} \cdot C_{oss} \left( \frac{V}{2} \right)^2 f_s \tag{10}
\]

\[
v_s = \frac{V}{2} \cdot \frac{V_{pk}}{2} \cdot \sin(\theta) \tag{11}
\]

\[
P_{sls} = \frac{1}{2} \cdot C_{oss} f_s \cdot \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{V_{pk}}{2} \right)^2 (\sin(\theta))^2 d\theta
\]

\[
= \frac{1}{16} \cdot C_{oss} f_s V_{pk}^2 \tag{12}
\]

\[
P_{sls} = 8 \cdot P_{sls} = \frac{1}{2} \cdot C_{oss} f_s V_{pk}^2 \tag{13}
\]

\[
R_{sl0} = \frac{v_s^2}{P_{sls}} \tag{14}
\]

\[
P_{s Lans} = \left( 1 + \frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^{N-1}} \right) P_{s Lans(1)}
\]

\[
= \frac{1}{2} \cdot \left( 1 + \frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^{N-1}} \right) C_{oss} f_s V_{pk}^2 \tag{15}
\]

When it comes to an N-stage circuit, there are 4N groups of switches and correspondingly 8N MOSFETs. It can be obtained by equation (13) that the loss in MOSFETs of each stage is proportional to the \( V_{pk}^2 \). Because the \( V_{pk} \) in each stage is de-escalated by the ratio of 1/2, the loss in MOSFETs of each stage decreases by the ratio of 1/4 from stage to stage. The total loss in MOSFETs in an N-stage circuit is defined by equation (15).

Thus, the equivalent resistance of an N-stage circuit seen through its output side is defined in (16) and (17).

\[
R_{sl0(N)} = \frac{v_s^2}{P_{s Lans(N)}} \tag{16}
\]

\[
R_{sl0(N)} = \frac{1}{\left( 1 + \frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^{N-1}} \right) C_{oss} f_s} \tag{17}
\]

### 3.4. Equivalent Circuit

The equivalent circuit of an N-stage circuit on the input side is shown in Figure 9 [6]:

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**Figure 9.** Equivalent circuit of an N-stage circuit.

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### 4. Simulation

Based on the theoretical basis, a series of simulations on the proposed switched-capacitor AC-AC converter with a ratio of 1/2\( ^n \) as shown in Figure 10 and Figure 11 are processed by SIMETRIX. The main parameters of simulation are presented in Table 1.

| Description            | Values |
|------------------------|--------|
| Peak Voltage \( (V_{pk}) \) | 311.1V |
| Switching Frequency \( (f_s) \) | 50kHz |
| Capacitors \( (C) \) | 20\( \mu \)F |
| Duty Cycle | 47\% |
| MOSFET | IRFP460 |

The input voltage of the simulation curves shown in Figure 10 is 220V. The input voltage and the output voltage waveforms of a single-stage circuit, a 2-stage circuit, a three-stage circuit and a 4-stage circuit are shown in Figure 10 (a) to Figure 10 (d), respectively. It can be seen that the output voltages perfectly follow the shape of the input voltage and ideal peak values are \( V_{pk} / 2, V_{pk} / 4, V_{pk} / 8 \) and \( V_{pk} / 16 \), respectively.

A three-stage circuit is used for further analysis. The simulation waveforms of the input voltage and output voltages of all stages are shown in Figure 11, where the peak value of the input voltage is 311V and output voltage of each stage is half of that in the previous stage, which matches well with the theoretical analysis. A conversion ratio of 1/2 is achieved by the first-stage circuit, a conversion ratio of 1/4 is achieved by the second-stage circuit and a ratio of 1/8 is achieved by the third-stage circuit.

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**Figure 10.** The simulation result of the proposed SC AC-AC converter.

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It can be seen from the simulation result that the implementation method of the N-stage AC-AC converter based on the SC principal is able to achieve a ratio of 1/2\( ^n \).
5. Experimental Results and Analysis

In order to verify the validity of the topology, a series of step-down prototypes were built in the laboratory. A simple drive circuit with a SG3525 PWM modulator is used to produce PWM signals to control the power switches. The circuit connections are shown in Figure 4 and Figure 5. The RMS of input voltage is 220V and the other parameters are consistent with the simulation parameters as shown in Table 1.

When the output terminal is connected across any of the three capacitors (C1, C2 and C3) in Figure 1, the experimental waveform is shown in Figure 12(a). It shows that $U_{o} = 1/2U_{i}$. When the input voltage is 220V, the voltage of each capacitor is close to 110V.

When the output terminal is connected with any of the three capacitors in the second part of the two-stage unit circuit in Figure 3 (C4, C5 and C6), the experimental waveform is shown in Figure 12(b). It shows that $U_{o} = 1/4U_{i}$. When the input voltage is 220V, the voltage of each capacitor is 54.9V.

When the output terminal is connected with any of the three capacitors in the third part of the three-stage unit circuit, the experimental waveform is shown in Figure 12(c). It shows that $U_{o} = 1/8U_{i}$. When the input voltage is 220V, the voltage of each capacitor is 27.69V.

When the output terminal is connected with any of the three capacitors in the fourth part of the four-stage unit circuit, the experimental waveform is shown in Figure 12(d). It shows that $U_{o} = 1/16U_{i}$. When the input voltage is 220V, the voltage of each capacitor is 14.4V. All above experimental results are fully consistent with the simulation results.

5.1. Equivalent Resistance Analysis

The experimental results are consistent with the theoretical and simulation results. Meanwhile, the equivalent resistance in the output side of the proposed circuit can be defined. The equivalent resistance of an N-stage SC AC-AC converter seen through the output side can be calculated using (20), in which $U_{NOL}$ is the no-load voltage of the Nth circuit, $U_{L}$ is the voltage of the $R_{eqO}$. $U_{L}$ is the load voltage of the Nth circuit and $R_{L}$ is the load.

\[
U_{NOL} = U_{L} + U_{E} \quad (18)
\]

\[
\frac{U_{L}}{R_{L}} = \frac{U_{NOL} - U_{L}}{R_{eqO}} \quad (19)
\]

\[
R_{eqO} = \frac{U_{NOL} - U_{L}}{U_{L}} \quad (20)
\]

When the input voltage is set as 220V, the experimental values of the equivalent resistance ($R_{eqO}$) of laboratory prototype with the ratio of 1/2 are 1.32Ω ($R_{eqO(1)} = 1.32Ω$, 1.64Ω ($R_{eqO(2)} = 1.64Ω$) with the ratio of 1/4, 1.73Ω ($R_{eqO(3)} = 1.73Ω$) with the ratio of 1/8, respectively.

The equivalent resistance increases with loads being larger and the increment become smaller with more stages in the circuit. As shown in Figure 13, the limit value is close to 1.77, which validates the Equ. (4). It proves that equivalent resistance between the pre-and post-stage circuits are independent with each other. A theoretical value curve is drawn compared with the experimental value curve as shown in Figure 13. The experimental value of the equivalent resistance is a little inferior to the theoretical value while they have the same variation. Considering the energy losses during the transmission and the differences on the device selection, the result is acceptable.

5.2. Equivalent Capacitance Analysis

A three-stage circuit was established to illustrate the
capacitance in the converter. The following stage circuit can be seen as the resistive-capacitive load of the previous stage circuit. So there are phase differences between voltages and currents. The resistive component of Resistive-capacitive load equals to the post-stage circuit equivalent resistance and the load. And the capacitive component equals to the post-stage circuit equivalent capacitance. Through the power factor, the ratio between the resistive and capacitive component can be calculated. When the circuit load is fixed and the resistive component is hypothesized unchanged, the ratio of the capacitive component in different stages circuit can be calculated. Ideally, the increasing ratio of equivalent capacitance in different stages is 1:5.21 as presented in (8) and (9). Using the measured average power factor from the experimental circuits shown in Table 2, considering the resistance unchanged, the calculated ratio of equivalent capacitance is 1:4.5:17 in different stages. When the number of stages increases, the parallel resistance $R_D$ that indicates the switching loss due to intrinsic capacitances of the MOSFETs decreases thus the resistive part of the circuit decreases. Considering the influence of experiment error, the theoretical and experimental results are quite similar, which validates equations (8) and (9).

### 5.3. Circuit Analysis

To verify the transfer efficiency of the circuit topology, experiments are conducted on the converter with the fixed load of 100Ω. The experimental power transmission efficiency between stages of a three-stage circuit is shown in Table 3, in which the input voltage ranges from 80V to 220V.

| Input voltage (V) | Efficiency of 1st-stage (%) | Efficiency of 2nd-stage (%) | Efficiency of 3rd-stage (%) | Overall efficiency (%) |
|-------------------|------------------------------|----------------------------|----------------------------|-----------------------|
| 80                | 91.98                        | 91.28                      | 88.74                      | 74.50                 |
| 100               | 91.58                        | 92.18                      | 88.53                      | 74.74                 |
| 120               | 91.68                        | 90.34                      | 90.33                      | 74.81                 |
| 140               | 92.22                        | 90.51                      | 89.61                      | 74.79                 |
| 160               | 92.29                        | 91.06                      | 88.89                      | 74.70                 |
| 180               | 91.80                        | 90.77                      | 89.57                      | 74.63                 |
| 200               | 91.79                        | 90.43                      | 89.60                      | 74.37                 |
| 220               | 91.86                        | 90.52                      | 89.79                      | 74.66                 |

The average transfer efficiency between the input and the output of the first-stage circuit is 91.9%, 90.9% for the second-stage circuit and 89.4% for the third-stage circuit. The overall average efficiency of the three-stage circuit is 74.65%. The losses caused by switches, which is needed for the capacitor charge/discharge process cause the efficiency drop when the number of stages increases. The efficiency of the single-stage circuit mentioned in [6] is 95% which leads to an ideal overall efficiency of a three-stage circuit as 85% as the switching and conduction losses are neglected. Considering the differences of the power MOSFETs, capacitors and other electrical elements as well as the topology between [6] and this paper, the efficiency of the proposed circuit is expected to be higher with the improvement of elements and design.

With the change of circuit load, the output power also changes. The output power of the single-stage circuit is up to 1KW [6]. Because the circuit topology of the converter is a step-down circuit, when the circuit load is fixed, the output power of the circuit is reduced with the circuit stage increasing. The output power of the two-stage circuit is nearly 200W and the three-stage circuit is nearly 50W.

### 6. Conclusion

Based on the existing AC-AC switched-capacitor (SC) converter, this paper demonstrates a new cascaded AC-AC converter circuit topology with a ratio of 1/2" which only consists of power switches and capacitors. The converter consists of multi-stage converters, and the pre-and post-stage circuits are independent with each other. By theoretical analysis, an equivalent circuit of the proposed N-stage converter is derived, which is composed of equivalent resistances and equivalent capacitances. Through the two-port network theory, the formulas of related parameters such as equivalent resistance, equivalent capacitance and switch loss are gained in the paper and proved by the experiments. Experimental results verified the correctness of the equivalent circuit and the feasibility of the converter.

The SC principle is extended to AC-AC static conversion to achieve a ratio that decreases exponentially by 1/2 with the orders, which provides a wider output voltage range. The equivalent resistance of the circuit increases with more stages. But the increasing value of resistance is about to decrease exponentially. Therefore, the limit value exists, which leads to great significance of research and practical applications. This converter consists only of capacitors and switches. This method has greatly reduced the volume of the AC-AC converter and has potential application prospects in the low-power/low-voltage field.

There are still lots of problems needed to be further researched and explored. The reduction of the equivalent resistance and switching losses on MOSFETs, improvement of the circuit efficiency and power rate are all further research focuses of this circuit topology.

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