Contingency-based Voltage Stability Monitoring via Neural Network with Multi-level Feature Fusion

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Abstract: To monitor the voltage stability state of complex power grid, a four-category stability classification problem that incorporates a set of serious contingencies is posed. Quick decision-making and high accuracy are critical for the safety operation of power system. However, this problem involves feature of different types, levels and dimensions and is hard to be handled by the traditional classifier. This paper utilizes the deep learning technique and proposes a multi-level deep neural network (ML-DNN) that achieves feature fusion of the electrical parameter measurements, topology and contingency information. Experiments are implemented on IEEE-39 system, the ML-DNN performs better in four main evaluation indices comparing with five existing models, which demonstrates its advantage for online voltage stability monitoring.

Keywords: voltage stability, contingency, deep neural network, multi-level feature fusion.

1. INTRODUCTION

The continuous increment of various loads in modern power grid aggravates the situation of supply/demand imbalance. The large variations in electrical load and generation pose considerable threats to the voltage stability of the whole power system. Therefore, a monitoring system that provides accurate online situational assessment of voltage stability condition is necessary. For operators, one of the key measurement indices of the stability state is the voltage stability margin (VSM) (Zhou et al., 2010), which is defined as the difference of active power between the current operating point (OP) and the voltage collapse point. A large VSM indicates that the power system can hold heavy loads and afford large variations. Conversely, the reduction of VSM signifies the degradation of stability condition and proper preventive measures are supposed to be implemented.

Given the load increment directions (LIDs), VSM can be obtained by searching the maximal total transfer capability of the power system. The most frequently used method is the continuous power flow (CPF) (Chiang et al., 1995). CPF increases the load demand step by step until the Jacobian matrix of power flow equations becomes singular and the total active power increment in this process is equivalent to VSM. CPF is a powerful tool but suffers from high complexity due to the iterative computation. For online applications with rapid load variations, its responses might delay. To solve this problem, researchers use CPF offline to generate training data for online machine learning-based model. The objective is to build a precise VSM estimation model for quantitative analysis or classification model for qualitative analysis (Zhou et al., 2010). The latter categorizes the voltage stability state into several patterns according to the convergence of power flow computation, voltage stability indices or VSM (Li et al., 2018). The advantage of qualitative classification model lies in its ability to incorporate possible contingencies. VSM is affected directly by the topological structure of power grid. Failures on transmission lines, electrical generators, etc. change the grid topology, destroy the balance between power supply and demand and reduce VSM. Owing to the uncertainty of contingency, a qualified monitoring system should store an anticipated contingency set and check them one by one for each OP. With the development of wide-area measurement system (WAMS), the so-called contingency-based voltage stability monitoring employs machine learning model to map the acquired data of the current OP into their corresponding categories with consideration of the major contingencies. Approaches including decision trees (DT) (Mohammadi and Dehghani, 2015), random forest (RF) (Ngnevitsky et al., 2015), support vector machine (SVM) (Kalyani and Swarup, 2010), various neural networks (NN) (Javan et al., 2013), ensemble learning (Zhukov et al., 2019) are adopted by researchers to construct the key classification model based on either real measurements from phasor measurement units (PMUs) (Venkatesh and Jain, 2018) or software simulation. Categories of voltage stability state are defined according to the range of evaluation indices like loading index, voltage deviation index and others. However, due to the complexity of power grid dynamics, the performance and reliability of current monitoring approaches are still unsatisfactory. On the one hand, voltage instability has different manifestations or subtypes. It may stem from regional abnormal distribution of power supply and demand or overload of the whole system. Therefore, the classification model should focus on both regional and global characteristics. On the other hand, contingencies lead to topology change of power grid and may directly alter the category of stability state, thus the topology information should also be taken into consideration.
To solve the difficulties, the deep learning approaches are adopted due to its powerful layer-wise feature extraction and fusion ability. In this paper, a multi-level deep neural network (ML-DNN) architecture is proposed to achieve information fusion of both topology feature and electrical parameter measurements in different regions. The layers in ML-DNN incorporate only the same level features thus the architecture can handle the subtype problem as well as integrate topology information.

The major contributions of this paper are presented in the following three aspects. (1) A grid partition strategy is designed to extract regional characteristics based on the energy transmission path. (2) The betweenness centrality index is utilized to describe the change of topological structure caused by the contingencies. (3) A ML-DNN-based voltage stability state classification model is proposed to integrate features with different levels, include regional electrical characteristics, degree of topology change and the contingency information.

This paper is organized as follow. Section 2 introduces the concept of contingency-based voltage stability monitoring. Section 3 elaborates the proposed ML-DNN model in details. Experiments on the IEEE-39 system are implemented and analysed in Section 4 and Section 5 summarizes the paper.

2. PROBLEM STATEMENT

For online contingency-based voltage stability monitoring, a problem of building a four-category classifier is considered. Given an input variable set \( X = \{x_1, x_2, ..., x_n\} \in \mathbb{R}^{3\times n} \) and an output label set \( Y = \{y\} \in \mathbb{N}^{1\times 1} \), the classifier creates a mapping \( f \) that achieves \( f(X) \rightarrow y_i \), where \( N \) refers to the number of samples, \( n \) refers to the number of input variables and \( X = \{x_1, x_2, ..., x_n\} \). The label \( y_i \) is a categorical variable that satisfies \( y_i = \{0, 1, 2, 3\} \). These numbers are the indicators of voltage stability state. From \( y_i = 0 \) to \( y_i = 3 \), the stability state shifts from unstable to stable. The stability criterion of power system is the convergence of load flow, which determines whether \( y_i \) equals to 0 or other. The degree of voltage stability is determined by VSM. In Fig. 1, the P-V curves of both normal and contingency conditions are illustrated. VSM is the difference between the current OP and the critical OP, namely a \( \Delta P \) that fulfils:

\[
\Delta P = P_{\max} - P_0
\]

where \( P_{\max} \) and \( P_0 \) are the maximal and current total transfer active power. The stability state is further classified into three categories according to VSM: “less stable”, “stable” and “more stable”, which correspond with \( y_i = \{1, 2, 3\} \). Note that the contingency will reduce the current \( \Delta P \) from \( \Delta P' \) to \( \Delta P'' \) in most cases. In Fig. 1, the stability state drops from “more stable” to “less stable” due to the occurrence of failures on two transmission lines. Therefore, the difficult of this problem is that even with identical load and generation distribution, the categories of pre and post contingencies are different. It requests the classifier to utilize available information including active/reactive power measurements and grid topology to achieve high performance. For the “unstable” state, the classifier should monitor both regional and global power distribution to discriminate the possible reasons. The effects towards VSM of contingencies are measured by removing the relevant components and recalculating VSM. Those with serious impact, frequencies and vulnerabilities form the anticipated contingency set. In this paper, the line faults that lead to interruptions of one or more buses (fault 1 in Fig. 1) are included in the contingency set.

![Fig. 1 Illustration of voltage stability state](image1)

3. PROPOSED APPROACH

The development of deep learning enables flexible information fusion by designing appropriate network structure and end-to-end training. In this paper, a ML-DNN model is proposed to integrate regional measurements and topology information for contingency-based voltage stability monitoring. The framework is shown in Fig. 2, which can be basically divided into three parts.

![Fig. 2 The framework of proposed model](image2)
with similar energy transmission path will be assembled in a cluster. Next, the load nodes will be assigned to the closest generator cluster to form a local region.

The second part is topology representation. It aims to design or select appropriate index that can reflect the topology change of power grid. In this paper, the betweenness centrality (BC) index of each node is calculated and form a global feature vector to describe the change of topological structure of power grid.

The third part is contingency selection. Only the serious contingency which creates isolated grid is considered and added into the anticipated contingency set. The one-hot encoding is adopted to represent these contingencies.

The aforementioned three parts are connected through the ML-DNN model. The measurements of active/reactive power of generators and loads in their respective regions are regarded as the initial “level-1” input feature. The topological feature vector, as the “level-2” feature, are integrated with the concatenated measurements. The one-hot representation of contingencies are taken as the “level-3” features and merged with the previous fused features. The output of ML-DNN is the stability state, namely \( y_i \). The “L0” to “L3” in Fig. 2 are equivalent to \( y_i = \{0,1,2,3\} \).

3.1 Grid partition

Power grid can be simply divided into three parts: source, connection network and load. The electrical energy is created by the source, transmitted along the connection network and consumed by the load. Although the process of energy flow is complex, the power generators will give priorities to the loads with shorter electrical distance. These generators and loads form a local region that basically realize the balance of supply and demand. The out-of-balance problem of any local regions may affect the whole power grid, thus need to be monitored separately.

Let the generator and load set be \( V_g \) and \( V_l \), the number of generators and loads be \( n_g \) and \( n_l \), an electrical distance matrix \( D \in \mathbb{R}^{n_g \times n_l} \) between each “generator-load” pair is firstly calculated using the Dijkstra’s algorithm. A power grid can be regarded as a graph \( G = (V,E,A) \), where \( A \) is a weight adjacent matrix of the branch connecting impedance. The Dijkstra’s algorithm find a path \( p \) between a generator node \( v_i \) and load node \( v_j \) with minimal total weights:

\[
D_{g,l} = \min L(p) = \sum_{(v_i,v_j,p)} A_{ij}
\]

This algorithm is a successive approximation scheme. It uses two types of labels: tentative \( T \) and permanent \( P \). From \( v_i \), all neighbours will be searched and given \( T \). The label of node on the shortest path will changed to \( P \) and its neighbour will be searched. This process will iterate until a path that is composed of all \( P \) label nodes is found.

Next, the generator node (G-node) will be clustered via correlation analysis. The row vectors in matrix \( D \) represent the shortest distance between each G-node and all load node (L-node) in the same order. On the basis of a principle that the arrangement of L-nodes on the energy transmission path of similar G-nodes should be similar, the Spearman correlation coefficient (SCC) is adopted to measure their relationships. For two variable sequences \( r \) and \( s \) with length \( n_r \), SCC is defined as:

\[
\rho_{r,s}(r,s) = \frac{\sum_{i=1}^{n_r} (r_i - \bar{r})(s_i - \bar{s})}{\sqrt{\sum_{i=1}^{n_r} (r_i - \bar{r})^2 \sum_{i=1}^{n_r} (s_i - \bar{s})^2}}
\]

For general continuous variable sequence, the grade sequence can be formed by its ordinal number after sorting. A threshold \( T \) is used to determine whether two G-nodes are similar. A G-node cluster \( c_g \) is defined as:

\[
c_g = \{v | v \in V_g, \forall v_i \in c_g, \rho_i(v,v_j) > T \}
\]

All \( c_g \) form the G-node cluster set. The L-nodes will be assigned to their neighbour \( c_g \), which is measured by the minimal average distance between each G-node and the target L-node. The partitioned region, namely the merged G-node and L-node cluster is defined as:

\[
c_c = c_g \cup \{v | v \in V_l, \forall v_i \in c_g \cap V_l, \text{arg min} \left\{ \frac{1}{|c_g|} \sum_{v \in c_g} \sum_{v_j \in V_l} D_{i,j} \right\} \}
\]

Measurements from all \( c_c \) will be collected and utilized as the “level-1” feature for ML-DNN.

3.2 Topology Representation

The betweenness centrality index is adopted in this paper for topology representation. Likewise, BC assumes that the electrical energy transmits along the shortest path and is defined as the ratio of the number of the shortest path between \( v_i \) and \( v_j \) that contains \( v \) and the total number of the shortest path:

\[
b(v) = \sum_{v_i,v_j,v \in P} \frac{\delta(v)}{\delta_{ij}}
\]

where \( \delta_{ij} \) is the total number of the shortest path between \( v_i \) and \( v_j \), \( \delta(v) \) is the shortest path that contains \( v \).

BC is a global index for node importance evaluation. No matter where a contingency occurs, it will affect the BC of every nodes in the power grid. Two other common used centrality index are degree centrality (DC) and closeness centrality (CC). DC is a local index that considers the first order neighbour of nodes. If a line contingency occurs, only the value of the direct connected nodes will be influenced. CC is defined as the average shortest distance between a target node and other nodes. It cannot well describe the topology change because the averaging operation reduces its impacts. Comparing with them, BC takes account of both local changes and their global impacts. A global feature
vector $B$ is formed by the BC value of all nodes in the power grid:

$$B = \left[ b(v_1), b(v_2), ..., b(v_V) \right]$$  \hspace{1cm} (7)

The vector $B$ is employed to monitor the topology change of the whole power grid and utilized as the “level-2” feature of ML-DNN.

3.2 Contingency selection

The anticipated contingency set is critical for the voltage stability monitoring. In general, if a contingency causes the formation of isolated grid, it is considered serious. The isolated grid refers to the case where the only transmission line between a local region and the rest part of power grid is broken. If the isolated grid contains G-nodes, the active and reactive power supplement as well as the VSM will be reduced. If the isolated region contains L-nodes, the electricity customers will be affected. Therefore in this paper, these serious contingencies are selected and represented by one-hot encoding. The encoded contingencies are utilized as the “level-3” feature of ML-DNN because they have direct impact to the stability state of the power grid.

3.4 Architecture of ML-DNN

After obtaining multi-level features, the ML-DNN is employed to achieve feature fusion and construct the transformation between them and the stability state. The proposed ML-DNN is an essentially a multi-layer perceptron (MLP) composed of many fully-connected layers according to a particular architecture, which is shown in Fig. 3.

![Fig. 3 The architecture of ML-DNN](image)

DNN owns the ability of hierarchical feature transformation and layer-wise joint feature learning, thus can achieve effective feature fusion under the supervision of the given labels. For input like image pixels, speech signals, etc., features are at the same level and can be arranged in a vector. DNN can extract high-order representation of such type of data successively. For the problem of voltage stability classification, features are at different levels. For example, the measurements acquired from a region cannot be processed together with the topology vector. Therefore, the level of data should match the level of layer in the DNN.

The basic idea of ML-DNN is to design an architecture that allows multi-level input via concatenation operation. In Fig. 3, the “level-1” input measurements are connected with their respective neurons and the outputs are concatenated after feature transformation. The concatenated vector goes through another transformation and its output will be integrated with the “level-2” input. Likewise, the higher the feature level, the deeper the network structure.

4. EXPERIMENTS

The IEEE-39 New-England power system (Fig. 4) is used to test the model performance. There are total 39 buses (nodes) and 46 branches in this system. The number of nodes are marked in Fig. 4. The symbol “G” represents the G-nodes and the triangle symbol represents the L-nodes. Except node 31 and 39, the system contains 10 G-nodes and 17 L-nodes. The node 31 is the slack bus with constant voltage magnitude and phase angle.

![Fig. 4 The IEEE-39 system](image)

4.1 Experimental data

The MATPOWER (Zimmerman et al., 2011) power flow and continuous power flow tools are used to generate experimental data. In this paper, it is assumed that the active power of all generators, the active and reactive power of all loads varies from -20% to 100% of their base values, namely:

$$P_{L,i} = P_{L,i}^0 \left(0.8 + 1.2 \epsilon_{P_{L,i}} \right)$$  \hspace{1cm} (8)

$$Q_{L,i} = Q_{L,i}^0 \left(0.8 + 1.2 \epsilon_{Q_{L,i}} \right)$$  \hspace{1cm} (9)

$$P_{G,j} = P_{G,j}^0 \left(0.8 + 1.2 \epsilon_{P_{G,j}} \right)$$  \hspace{1cm} (10)

where $\epsilon_{P_{L,i}}$, $\epsilon_{Q_{L,i}}$, and $\epsilon_{P_{G,j}}$ are random variables under uniform distribution from 0 to 1. The symbol $i$ and $j$ refer to the number of L-nodes and G-nodes. The superscript “b” indicates the base value.

The selected contingencies are 11 single line contingencies between node number 2-30, 6-31, 10-32, 19-33, 20-34, 22-35, 23-36, 25-37, 29-38, 16-19, and 19-20. For each generated normal cases, all 11 contingencies are simulated. In this paper, 3000 normal cases and 33000 contingency cases form the experimental data set.
For each cases, the power flow test is implemented. If the power flow does not converge, then $y_i = 0$. For other situations, the CPF tool is employed to calculate VSM and determines the labels, namely $y_i = \{1, 2, 3\}$. The generated data set contains 5670, 9894, 11152 and 9284 cases for the corresponding labels and is randomly divided into training, validation and testing set with a proportion of 10:1:1.

4.2 Experimental Results

(1) Results of grid partition

The shortest distance matrix $D \in \mathbb{R}^{10 \times 17}$ between all “G-L” pair is firstly calculated. Next, the Spearman correlation analysis is implemented based on $D$. The result is illustrated in Fig. 5. Through manual selection, the threshold $T$ is set to 0.8. From the figure, apparently four G-node clusters: $c^1_g = \{30, 37, 39\}$, $c^2_g = \{31, 32\}$, $c^3_g = \{33, 34, 35, 36\}$, and $c^0_g = \{38\}$ are obtained. The special node 39 should have formed a new cluster E, whereas none of the L-nodes will be assigned to E if so. Therefore, it is manually classified as $c^4_g$ according to the topological structure in Fig. 4.

(2) Results and comparison of stability classification

The active and reactive power of all L-nodes as well as the active power of G-nodes are selected as input measurements. On this foundation, the architecture and parameters of the proposed ML-DNN is listed in Table 2.

The min-max normalization is used to transform the range of the original measurements to 0 and 1. Measurements from 4 regions are concatenated at layer 4 after one fully connected layer. For each contingency, the BC of each node $v$ in the IEEE-39 system is calculated. The global topological feature vector $B$ is added at layer 8 and contingency information is added at layer 12. The categorical cross entropy is selected as loss function and “Adam” is selected as optimizer. The 256 epochs training process is shown in Fig. 6.

During model training, the learning rate of the optimizer reduces twice at 112th epoch and 178th epoch to accelerate convergence and stabilize the performance.

For the purpose of performance assessment, the following evaluation indices including macro-precision (macro-P), macro-recall (macro-R), macro-F1-score (macro-F1) and micro-F1-score (macro-F1) are introduced:

$$\text{macro-P} = \frac{1}{n_c} \sum_{i=0}^{n_c-1} \frac{TP_i}{TP_i + FP_i}$$ (11)

$$\text{macro-R} = \frac{1}{n_c} \sum_{i=0}^{n_c-1} \frac{TP_i}{TP_i + FN_i}$$ (12)

$$\text{macro-F1} = \frac{1}{n_c} \sum_{i=0}^{n_c-1} \frac{2 \times TP_i}{2 \times TP_i + \sum_{i=0}^{n_c-1} FP_i + \sum_{i=0}^{n_c-1} FN_i}$$ (13)

where true positive (TP), false negative (FN) and false positive (FP) are the three basic elements. $n_c = 4$ is the number of label type. Five existing classifier including the classification and regression tree (CART), support vector machine (SVM), random forest (RF), adaptive boosting (AdaBoost) and deep neural network (DNN) are compared with the proposed model.

Table 2. Architecture and parameters of ML-DNN

| No. | Layers | Parameters |
|-----|--------|------------|
| 1   | Input  |            |
| 2   | Fully connected (ReLU) | 27 10 60 15 |
| 3   | Batch normalization | \ / |
| 4   | Concatenation | 132 |
| 5   | Fully connected (tanh) | 40 |
| 6   | Batch normalization | \ / |
| 7   | Dropout | 0.2 |
| 8   | Input Concatenation | 40 39 |
| 9   | Fully connected (tanh) | 40 |
| 10  | Batch normalization | \ / |
| 11  | Dropout | 0.2 |
| 12  | Input Concatenation | 40 12 |
| 13  | Output (softmax) | 4 |

Fig. 5 Results of Spearman correlation analysis

The L-nodes with the shortest average distance will be assigned to their corresponding $c_g$ to form $c_r$. The grid partition results is shown in Table 1.

Table 1. Partitioned regions

| Partitioned region | G-Nodes | L-Nodes |
|--------------------|---------|---------|
| A                  | 30, 37, 39 | 3, 25, 26 |
| B                  | 31, 32    | 4, 7, 8, 12 |
| C                  | 33, 34, 35, 36 | 15, 16, 18, 20, 21, 23, 24, 27 |
| D                  | 38       | 28, 29  |

Fig. 6 Training of ML-DNN
The performance comparison is shown in Table 3. For all four indices, the proposed ML-DNN outperforms the other models. To better demonstrate the results, the confusion matrices are shown in Fig. 7. The horizontal and vertical axes indicate the number of the predicted and real samples, which increase as the colour deepens.

| Model     | macro-P | macro-R | macro-F1 | micro-F1 |
|-----------|---------|---------|----------|----------|
| CART      | 0.5878  | 0.5839  | 0.5858   | 0.5720   |
| SVM       | 0.7579  | 0.7343  | 0.7441   | 0.7270   |
| RF        | 0.6980  | 0.6719  | 0.6810   | 0.6673   |
| AdaBoost  | 0.6755  | 0.6614  | 0.6641   | 0.6533   |
| DNN       | 0.9319  | 0.9182  | 0.9240   | 0.9217   |
| ML-DNN    | 0.9560  | 0.9536  | 0.9546   | 0.9517   |

Table 3. Performance comparison

According to the table and figure, CART is unable to achieve effective classification due to the complexity of input. The number of estimators are set to 300 for RF and AdaBoost. It is understandable that the ensemble operation raises the performance. SVM is the best performed traditional model that even gets 94.72% precision in distinguishing L0 and L1. However, results are not satisfactory for other categories. The introduction of deep learning is a huge progress. DNN raises the value of indices to 0.9 and reduces the error rate of all categories because of the feature fusion ability. Based on it, the ML-DNN considers both local and global characteristics and matches the layer of the network with the level of different features, thus obtains the best performance.

5. CONCLUSION

In summary, aiming to solve the classification problem for contingency-based voltage stability monitoring, a ML-DNN model is proposed to integrate multi-level and multi-type feature. First, the power grid is partitioned to extract the local supply-demand relationship information. The “level-1” electrical parameter measurements in all regions are transformed and merged in the network. Second, the BC is employed to describe the topology change caused by the contingencies. As the “level-2” feature, the BC vector is concatenated with the merged measurements. Finally, the representation of contingency is regarded as “level-3” feature to strength the classification effect. Performance and comparison on IEEE-39 system suggests that ML-DNN improves at least 2.41%, 3.54%, 3.06% and 3% of macro-P, macro-R, macro-F1 and micro-F1 comparing with the other models. We have reasons to believe that ML-DNN can increase the security of power system depend on its classification ability. Future research will concern about involving real PMU measurements and testing with more complex power grid structure.

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REFERENCES

Chiang, H.-D., Flueck, A. J., Shah, K. S. and Balu, N. (1995). CPFLOW: a practical tool for tracing power system steady-state stationary behavior due to load and generation variations. IEEE Transactions on Power Systems, 10 (2), 623-634.

Javan, D. S., Mashhadi, H. R. and Rouhani, M. (2013). A fast static security assessment method based on radial basis function neural networks using enhanced clustering. International Journal of Electrical Power & Energy Systems, 44 (1), 988-996.

Kalyani, S. and Swarup, K. S. (2010). Classification and assessment of power system security using multiclass SVM. IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews), 41 (5), 753-758.

Li, S., Ajjarapu, V. and Djukanovic, M. (2018). Adaptive Online Monitoring of Voltage Stability Margin via Local Regression. IEEE Transactions on Power Systems, 33 (1), 701-713.

Mohammadi, H. and Dehghani, M. (2015). PMU based voltage security assessment of power systems exploiting principal component analysis and decision trees. International Journal of Electrical Power & Energy Systems, 64, 655-663.

Negnevitsky, M., Tomin, N., Kurbatsky, V., Panasetsky, D., Zhukov, A. and Rehtanz, C. (2015). A random forest-based approach for voltage security monitoring in a power system. 2015 IEEE Eindhoven PowerTech, 2015. IEEE, 1-6.

Venkatesh, T. and Jain, T. (2018). Synchronized measurements-based wide-area static security assessment and classification of power systems using case based reasoning classifiers. Computers & Electrical Engineering, 68, 513-525.

Zhou, D. Q., Annakkage, U. D. and Rajapakse, A. D. (2010). Online monitoring of voltage stability margin using an artificial neural network. IEEE Transactions on Power Systems, 25 (3), 1566-1574.

Zhukov, A., Tomin, N., Kurbatsky, V., Sidorov, D., Panasetsky, D. and Foley, A. (2019). Ensemble methods of classification for power systems security assessment. Applied Computing and Informatics, 15 (1), 45-53.

Zimmerman, R. D., Murillo-Sanchez, C. E. and Thomas, R. J. (2011). MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education. IEEE Transactions on Power Systems, 26 (1), 12-19.