A Position Sensorless Control Strategy for SPMSM Based on an Improved Sliding Mode Observer

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Abstract. Due to chattering phenomenon, a low-pass filter (LPF) is commonly used to suppress chattering in conventional sliding mode observer (SMO). However, the phase lag caused by LPF will degrade the performance of sensorless control for SPMSM, especially in high-speed range. To solve this problem, an improved SMO (ISMO) is proposed in this paper, which adds the flux as a state variable based on conventional SMO. Then, the observer will filter out the chattering signal while observing the magnetic flux accurately.

1. Introduction
High-speed permanent magnet synchronous motors (PMSM) have many advantages such as small size, light weight, low loss, and wide speed range. The conventional control of PMSM relies on the application of some sensors. Position and speed information are acquired by sensors such as rotary transformers, photoelectric encoders and hall sensors [1-3]. However, the cost and volume of the system will be increased and the assembly process will be complicated with the installation of position sensors [4-5]. In addition, the installation deviation of the mechanical sensors would decline the performance of drive system [6]. To solve these problems, many different kinds of sensorless methods have been investigated in the last decades.

Generally, sensorless control methods for PMSM could be classified into two categories: high frequency signal injection methods and methods based on back-EMF model [7]. The former methods are applied in low-speed range, and they are more suitable for interior-mounted permanent magnet synchronous motors (IPMSM). The latter methods are applied in high-speed range. The methods based on back-EMF model mainly includes disturbance observer [8], model adaptive method [9], Kalman filtering method [10], and sliding mode observer (SMO) [11], etc. Because of high robustness, fast dynamic response, and easy engineering implementation, the position estimation methods based on a SMO have been widely used in high-speed PM motor sensorless control. However, the inherent chattering phenomenon of the conventional SMO degrades the system performance [12-13].

In order to improve the accuracy of estimated rotor position, various chattering elimination methods for the SMO used in the PMSM sensorless control have been studied. A sigmoid function instead of a signum function is used for the SMO to reduce chattering significantly [14]. However, it is very hard to regulate the sigmoid function parameters in practice. In Ref. [15], an iterative sliding mode observer is also proposed, which replaces the low-pass filter by four observer to suppress
chattering. However, the experiments are conducted at low speed 2000r/min. In Ref. [16], synchronous frequency-extract filter are proposed to reduce chattering in back-EMF.

2. Analysis of Conventional SMO
Fig. 1 shows the field-oriented sensorless control scheme block of a PMSM motor based on conventional sliding mode observer.

**Figure 1.** Field-oriented control scheme block of a PMSM motor based on conventional SMO.

Assuming that the three-phase windings of the motor are symmetrical and no salience, the voltage equation of the PMSM in the two-phase static coordinate system can be represented as:

\[
u_s = i_s R_s + L_s \frac{di_s}{dt} + e\
\]

Where \(i_s\) is the stator current, \(i_s = \begin{bmatrix} i_s & i_{s\beta} \end{bmatrix}^T\), \(u_s\) is the stator voltage, \(u_s = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T\), \(e\) is the back-EMF, \(e = \begin{bmatrix} e_{\alpha} & e_{\beta} \end{bmatrix}^T\), \(R_s\) and \(L_s\) are the stator-winding resistance and inductance respectively, matrix \(A = -\frac{R_s}{L_s} I\), \(B = \frac{1}{L_s} I\), \(I\) is the second-order identity matrix.

SMO is established to estimate the back-EMF as:

\[
\frac{di_s}{dt} = A \hat{i}_s + Bu_s - k \text{sgn}(\hat{i}_s - i_s)
\]

Where \(k\) is the sliding-mode gain, \(\hat{i}_s\) represents estimated current, \(i_s = \begin{bmatrix} \hat{i}_{s\alpha} & \hat{i}_{s\beta} \end{bmatrix}^T\). The sliding surface can be represented as:

\[
S_a = \begin{bmatrix} s_{\alpha} \\ s_{\beta} \end{bmatrix} = \begin{bmatrix} \hat{i}_{s\alpha} - i_{s\alpha} \\ \hat{i}_{s\beta} - i_{s\beta} \end{bmatrix} = 0
\]

According to Lyapunov theorem, the system can be asymptotically stabilized by selecting an appropriate \(k\). Therefore, when the state converge to \(S_a = 0\), back-EMF can be represented as:

\[
e = k \text{sgn}(S_a) = \omega_s \varphi_f \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}
\]
Where $\theta$ and $\omega_e$ are rotor angle and electric angular speed, $\varphi_f$ is the magnetic flux of the PM, $\varphi_f = \begin{bmatrix} \varphi_{fa} \\ \varphi_{fb} \end{bmatrix}$. Conventionally, $\theta$ can be obtained by a simple trigonometric operation

$$\theta = - \arctan(e_a / e_b) \quad (5)$$

The block diagram of the conventional SMO is shown in the Fig. 2.

**Figure 2.** The block diagram of conventional SMO.

### 3. Analysis of Improved SMO

The relationship between the rotor flux $\varphi_f$ and the rotor electrical angle $\theta$ is represented as:

$$\varphi_f = \begin{bmatrix} \varphi_{fa} \\ \varphi_{fb} \end{bmatrix} = \varphi_m \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \quad (6)$$

Where $\varphi_m$ is the amplitude of flux.

Based on (6), the relationship between the magnetic flux and its rate of change can be represented as:

$$\frac{d}{dt} \begin{bmatrix} \varphi_{fa} \\ \varphi_{fb} \end{bmatrix} = \begin{bmatrix} 0 & -\omega_e \\ \omega_e & 0 \end{bmatrix} \begin{bmatrix} \varphi_{fa} \\ \varphi_{fb} \end{bmatrix} \quad (7)$$

Therefore, the extended state equation with stator current and magnetic flux of the PM as the state variables can be represented as:

$$\frac{d}{dt} \begin{bmatrix} i_s \\ \varphi_f \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} i_s \\ \varphi_f \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_s \quad (8)$$

Where matrices in (8) are represented as follows: $A_{11} = -(R_s / L_s)I = a_{11}I$, $A_{22} = \omega_e J = a_{22}J$, $B_1 = (1 / L_s)I = b_1I$, $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

According to the above equation, the extended SMO is proposed as:

$$\frac{d}{dt} \begin{bmatrix} \hat{i}_s \\ \hat{\varphi}_f \end{bmatrix} = \begin{bmatrix} A_{11} & \hat{A}_{12} \\ 0 & \hat{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \hat{\varphi}_f \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_s - \frac{L_s}{FL_i} \sgn(\hat{i}_s - i_s) \quad (9)$$

Where $\hat{A}_{12}$ and $\hat{A}_{22}$ represent speed-related matrices, $\hat{A}_{12} = -(\dot{\omega}_e / L_s)J$, $\hat{A}_{22} = \dot{\omega}_e J$, $L_1$ is the switching gain matrix, $L_s = k_s I$, $k_s$ is a constant, $F$ is the feedback matrix.

Considering the speed estimation error, (9) can be written as:
\[
\frac{d}{dt} \begin{bmatrix} \dot{i}_s \\ \dot{\phi}_f \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} i_s \\ \phi_f \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_s - \begin{bmatrix} L_i \\ F_{Ld} \end{bmatrix} \text{sgn}(\dot{i}_s - i_s) + H
\]  

(10)

Where \( H \) is the disturbance causing by speed estimation error, and

\[
H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} (-1/L_i)\Delta \omega_f J \\ \Delta \omega_f J \end{bmatrix} \dot{\phi}_f
\]  

(11)

Where \( \Delta \omega_f \) is speed estimation error, \( \Delta \omega_f = \dot{\omega}_f - \omega_f \).

Therefore, it’s necessary to select proper matrices, \( L \) and \( F \). The block diagram of the ISMO is shown in the Fig. 3.

![Block Diagram of ISMO](image)

**Figure 3.** The block diagram of the ISMO.

Based on (9) and (10), the error equation of ISMO can be represented as

\[
\frac{d}{dt} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} - \begin{bmatrix} L_i \\ F \end{bmatrix} \text{sgn}(e_1) + H
\]  

(12)

Where \( e_1 \) is the current estimation error, \( e_1 = \dot{i}_s - i_s = [e_{is} \ e_{i\beta}]^T \), \( e_2 \) is the flux estimation error, \( e_2 = \dot{\phi}_f - \phi_f = [e_{\phi s} \ e_{\phi \beta}]^T \).

To prove that the system can reach the sliding surface in a finite time from any initial state, a Lyapunov function is defined as:

\[
V = \frac{1}{2} e_1^T e_1
\]  

(13)

Then, the stability condition for the ISMO can be represented as:

\[
\dot{V} = e_1^T \dot{e}_1 = e_{is} \dot{e}_{is} + e_{i\beta} \dot{e}_{i\beta} = e_{is}(a_{11} e_{is} - a_{12} e_{\phi \beta} - k_1 \text{sgn}(e_{is}) + H_{is}) + e_{i\beta}(a_{11} e_{i\beta} + a_{12} e_{\phi s} - k_1 \text{sgn}(e_{i\beta}) + H_{i\beta}) < 0
\]  

(14)

Where the observer gain can be derived to satisfy the inequality condition as

\[
k_1 > \max \left[ |a_{11} e_{is}| + |a_{12} e_{\phi \beta}| + |H_{is}|, \ |a_{11} e_{i\beta}| + a_{12} e_{\phi s} + |H_{i\beta}| \right]
\]  

(15)
In order to satisfy the system stability condition, it can be known from (15) that the sliding mode gain must be large enough. However, if the value is too large, the control state of the variable near the sliding surface will switch at high frequency, which may cause strong chattering, reduces the dynamic quality of the system and is not conducive to system stability. So when designing the sliding mode observer, this paper chooses a moderate $k_i = 500$.

When the ISMO is in sliding mode, $e_1 = \dot{e}_1 = 0$, then, (12) can be transformed to (16)

$$A_{12}e_2 + H_1 = L_1 \text{sgn}(e_1)$$

Defining the equivalent control for sliding mode control, it can be represented as:

$$C = \begin{bmatrix} C_\alpha \\ C_\beta \end{bmatrix} = L_1 \text{sgn}(e_1)$$

Substituting (16) into (12), and $H$ is given in (11), flux error equation can be transformed as

$$\frac{d}{dt} e_\gamma = \omega_\gamma(J + \frac{1}{L_s}FJ)e_\beta + \Delta \omega_\gamma(J + \frac{1}{L_s}FJ)e_\alpha$$

In the case that there is no error in the speed estimation, decouple the $e_{\gamma\beta}$ and $e_{\gamma\alpha}$ by choosing feedback matrix $F$ so that $\hat{\phi}_{\gamma\alpha}$ and $\hat{\phi}_{\gamma\beta}$ can be decoupled, selected $F$ satisfy the following equation:

$$J + \frac{1}{L_s}FJ = pI$$

Where $p$ is a constant, therefore $F$ can be obtained as

$$F = \begin{bmatrix} -L_s & pL_s \\ -pL_s & -L_s \end{bmatrix}$$

When only the effect of chattering on the flux estimation is considered, ignoring the speed estimation error, that is $H_1 = H_2 = 0$, and set the chattering signal as $C'$, redefine the equivalent control as:

$$C = L_1 \text{sgn}(e_1) = -A_{12}e_2 + C'$$

Substituting the above equation into flux observation equation in Eq. (9), we can get:

$$\frac{d}{dt} e_\gamma = A_{12}e_\gamma + F(C' - A_{12}e_\gamma) = p\omega_\gamma e_\beta + FC'$$

Laplacian transformation of the above equation can be represented as:

$$\frac{C'F}{s - p\omega_\gamma} = e_\gamma$$

Obviously, when $p < 0$, the chattering signal is processed by first-order LPF to obtain the value of flux estimation error, which can effectively suppress chattering. The chattering signal is filtered and
added to the actual flux as its estimation. At the same speed, from the perspective of the cutoff frequency, the smaller the $p$, the better the filtering effect. From the perspective of amplitude, $p$ is too small to cause the amplitude of chattering to increase. The larger the cutoff frequency is, the worse the filtering effect is, but the observer responds faster. When $p$ is actually selected, the requirements for chattering suppression and observer response are selected according to different systems. In this paper, parameter $p$ equals -3. Based on the analysis of chattering suppression, Fig. 3 can be equivalently represented as Fig. 4. Then, the flux estimation can be directly used to calculate rotor position, avoiding the phase lag caused by the extra filter.

$$\begin{aligned}
    \dot{i}_s &= A_i \dot{i}_s + A_i \phi_s + B_i u_i - L_i \text{sgn}(i_s - i_s) \\
    \phi_f &= \phi_f' + e_s \\
    e_s &= \frac{1}{s + \rho o} \\
    \rho &= p_o
\end{aligned}$$

**Figure 4.** The block diagram of analysis of chattering suppression.

The purpose of flux observation is to estimate the rotor angle and speed. In field-oriented control, coordinates are converted according to electrical angle to achieve torque control. In this paper, phase-locked loop (PLL) method is used to obtain the rotor angle and angular speed directly from the estimated flux.

4. Experimental Results
The proposed method is applied based on FOC to control molecular pump driven by a high-speed PMSM. A TMS320F28335 module is used for the main arithmetic control. Table 1 shows the PMSM specifications used for experiment. The physical TMP experimental platform and the motor controller are shown in Fig. 5.

| Variable                  | Value   |
|--------------------------|---------|
| Rated power (kW)         | 1.4     |
| Rated speed (r/min)      | 2100    |
| Number of pole pairs     | 1       |
| Phase resistance (Ω)     | 0.28    |
| Phase inductance (mH)    | 0.39    |
| Magnetic flux amplitude (Wb) | 0.0229 |

**Table 1.** Specifications of the SPMSM

**Figure 5.** Left: The motor controller. Right: Experimental platform with a high-speed SPMSM.
Firstly, the conventional SMO is applied to the experimental TMP at rated speed. Fig. 8 shows the analysis of the raw signals of the estimated back-EMF from conventional SMO. Fig. 6(a) shows the estimated signal of back-EMF. It can be seen that the estimated back-EMF is distorted due to high-order harmonics. The estimated rotor position compared with actual one is shown in Fig. 6(b), (c) and (d), when motor runs at 6000r/min, 12000r/min and 21000r/min respectively. It is found that the estimated position fluctuates sharply because of chattering problem. Besides it also introduces a significant phase lag, when the cut-off frequency of the LPF is 500Hz. Fig. 6(b) shows that the rotor position estimation error caused by chattering is beyond 11.2°. Fig. 6(c) shows that the rotor position estimation error caused by chattering is beyond 20.8°. And Fig. 6(d) shows that the rotor position estimation error caused by chattering is beyond 35.6°. As the speed increases, the phase lag caused by LPFs increases. We can also find that the chattering in estimated position is serious. The value of motor current will be influenced by the accuracy of estimated position. With more current harmonics, the motor loss will be increased. The phase current is shown in Fig. 7. The peak-to-peak value of the phase current is about 30A. The estimated speed with conventional SMO is shown in Fig. 8 and the estimated speed fluctuation is about 560r/min.

**Figure 6.** Experimental results with conventional SMO.

**Figure 7.** The motor phase current with conventional SMO.
Figure 8. Estimated speed with conventional SMO.

With the ISMO, the motor is also driven to rated speed. Fig.9 shows the analysis of the raw signals of the estimated flux from ISMO. The estimated fluxes are shown in Fig.9 (a). It can be seen that the chattering signal is suppressed. The estimated rotor position compared with the actual one is shown in Fig. 9(b), (c) and (d), when motor runs at 6000r/min, 12000r/min and 21000r/min respectively too. It is found that there is almost no phase lag. Especially at 21000r/min, the peak-to-peak value of estimated position error is reduced from 35.6° to 2.4° shown in Fig. 9(c). The chattering in estimated position is very small. With the proposed ESMO, the peak-to-peak value of motor drive phase current value decreases from 30.0A to 28.8A shown in Fig. 10. This means that the proposed method brings higher efficiency and less loss than the conventional SMO. Therefore, the proposed sensorless control method is more suitable for a poor heat dissipation condition such as high vacuum application. The peak-to-peak value of estimated speed is reduced from 560r/min to 280r/min shown in Fig. 11.
5. Conclusion
In this paper a novel SMO has been proposed for sensorless control for PMSM. The chattering problem in the conventional SMO is resolved by constructing a structure with filtering function to suppress chattering signals by selecting suitable parameters and the rotor angle is calculated through a PLL. The following conclusions are obtained through the analysis and experiments.

1) The LPFs of conventional SMO would introduce phase lag, which increase estimation error of rotor position and speed, especially in high-speed rang.

2) The proposed ESMO method can effectively suppress chattering with no need of LPFs, avoiding the phase lag.

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