To know the quantum mechanical state of a particle implies restrictions on the results of measurements of its momentum

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Abstract. In the framework of the standard axioms of quantum mechanics, the wave function of a particle in a one-dimensional configuration space (OCS) determines not only the probability of its detection in the vicinity of the OCS points, but also the local average values of the observables at these points, which form the fields of the (operators) first initial moments of the observables. The momentum operator field also determines the probability flow rate in the OCS. Together with the field of the kinetic energy operator, it defines two fields of particle momentum values. Their half-sum (the field of local average values of the momentum) is determined in terms of the phase of the wave function, and their half-difference (the deviation of the momentum from the local average momentum) is determined in terms of its amplitude. The average (over the ensemble) value of local momentum deviations at the OCS points is equal to zero, and the average value of these deviations, multiplied by the deviations of the coordinates of these points from the average (over the ensemble) coordinate value, is equal to $\pm \hbar/2$.

1. Introduction

At the very beginning of his book [1] John Bell writes “To know the quantum mechanical state of a system implies, in general, only statistical restrictions on the results of measurements.” This idea is supplemented by a phrase from the article [2]: “...the absence of definite values of measurable quantities before the moment of measurement is a fundamental conclusion of quantum theory in the Copenhagen interpretation”. In fact, these two statements reflect the root cause, due to which the physical interpretation of quantum theory, in its modern form, is impossible.

Indeed, in this state of affairs, researchers are faced with an insoluble dilemma — either (as is done in the ‘hydrodynamic’ formulation of quantum mechanics [3, 4, 5]) introduce the ‘definite values of measurable quantities’ as hidden variables, or to assume (as is done in the orthodox approach) that they appear at the moment of measurement. But the former contradicts Bell’s theory of hidden variables (see [1]), while the latter raises a natural question as to what relation these ‘measurement-generated’ values have...
to a closed microsystem. So David Mermin’s catchphrase “Shut up and calculate” is quite appropriate for this situation.

At the same time, such a position is unacceptable in principle: quantum mechanics, as a successful physical theory, presupposes a physical interpretation, and a single one at that. As for the current situation, it is a consequence of the fact that the existing formulation of the Born rule [6] reflects only a small part of the restrictions that are laid down in the axioms of quantum mechanics and wave function. As will be shown in this article, knowing the wave function and the form of the operator of the observable in the coordinate representation, one can uniquely determine, on the basis of these axioms, not only the probability field in the OCS, but also the field of local averages values of the observable – the field of the first the initial moment of the observable (or, for short, a field of the observable’s operator).

As shown in the 2 section, the key role in the unambiguous determination of the fields of the momentum, kinetic and total energy operators of a particle is played by the constraints that follow from the Schrodinger equation, as well as the fact that the field of the kinetic energy operator cannot take negative values. The procedure for determining the fields of these operators involves the simultaneous elucidation of the physical meaning of the fields themselves, the equations for the amplitude and phase of the wave function, which follow from the Schrodinger equation, as well as the wave-particle duality relations that arise for a (localized) wave function.

In Section 3, it is shown that the fields of the momentum and kinetic energy operators determine two fields of particle momentum values in the OCS: their sum depends only on the phase of the wave function, and the difference depends only on its amplitude. An important role is played by the value equal to the deviation of the particle momentum at a given point of the OCS from the local average value of the momentum, multiplied by the deviation of the coordinate of this point from the average (over the ensemble) value of the coordinate: the average (over the ensemble) value of this product is equal to $\pm \hbar/2$. The closed system of equations for the probability field and both momentum fields is presented in section 4).

2. Fields of observables’ operators in the Schrodinger formalism

According to Born, the probability of finding a particle in the interval $[x, x + dx]$ is given by $w(x,t)dx$, where $w = |\psi(x,t)|^2$ is the probability density ($\int_{-\infty}^{\infty} w(x,t)dx = 1$), and the mean value of any observable $f(x,t)$ whose operator commutes with the operator $\hat{x}$, is defined as the first initial moment of a random variable in classical probability theory:

$$\langle f \rangle = \int_{-\infty}^{\infty} f(x,t)w(x,t)dx.$$  (1)

As for the observable $O$, whose (self-adjoint) operator $\hat{O} = O(\hat{x}, \hat{p})$ does not commute with the $\hat{x}$ operator, its average value is defined as the average value of the operator $\hat{O}$:

$$\langle O \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\hat{O}\psi(x,t)dx.$$  (2)
It is assumed that rule (2) for such operators cannot in principle be reduced to the rule (1). In particular, the existence of the function $p(x, t)$, which could describe the field of momentum values in the OCS, is not assumed.

It would seem that there is no reason to doubt this formulation of the Born rule, since it has passed a long experimental verification. In addition, this rule is fully consistent with the idea of Bohr, according to which the introduction of the particle momentum as a function of its position in the OCS contradicts the Heisenberg uncertainty relation (according to Bohr, this principle prohibits the simultaneous measurement of the position and momentum of a particle, and this idea was fixed by Bohr in the principle of complementarity).

Nevertheless, the Bornian rule of calculating the average value of the particle momentum in the $x$-representation, as well as the Bohr interpretation of the uncertainty relation, misrepresent the essence of quantum mechanics. On the one hand, the Born rule misses an important part of those restrictions on the results of measurements that are contained in the wave function and the axioms of quantum mechanics. On the other hand, the uncertainty relation, as a restriction on the standard deviations of momentum and particle position, does not prohibit the introduction of momentum as a function of particle position (and time); instead, it imposes a restriction on the form of this function in OCS (see Section 3).

The main idea of our approach is that for any normalized wave function and any observable $O$ whose operator $\hat{O}$ does not commute with the operator $\hat{x}$, there exists such a function $O(x, t)$ that

$$\int_{-\infty}^{\infty} \psi^*(x, t) \hat{O} \psi(x, t) dx = \int_{-\infty}^{\infty} O(x, t) w(x, t) dx. \tag{3}$$

It follows from a comparison of the left and right parts of this equality that the function $O(x, t)$ is defined by the expression

$$O(x, t) = \frac{\text{Re} \left[ \psi^*(x, t) \hat{O} \psi(x, t) \right]}{\psi^*(x, t) \psi(x, t)} \tag{4}$$

(in the following, any function defined on the basis of this rule will be referred to as “the field of the corresponding operator”).

Of course, from a mathematical point of view, such fields are not uniquely determined by this rule, since the modified field

$$\tilde{O}(x, t) = O(x, t) + \frac{1}{w(x, t)} \frac{\partial f(x, t)}{\partial x}, \tag{5}$$

with an arbitrary function $f(x, t)$ equal to zero at infinity, leads to the same mean value $\langle O \rangle$ as the original field $O(x, t)$. However, since we are dealing with the fields of operators of observables, which must obey physical requirements and restrictions (including those arising from the Schrodinger equation), the definition of these fields becomes unique. This will be used below for introducing the fields of the momentum, kinetic and total energy operators.
Let us write the wave function in the form
\[ \psi(x, t) = \sqrt{w(x, t)} \exp \left( \frac{iS(x, t)}{\hbar} \right), \] (6)

where \( S/\hbar \) is the phase of the wave function. As in the case of the wave \( e^{i(kx-\omega t)} \), where \( k \) and \( \omega \) are constants to be, respectively, the wavenumber and frequency of the wave, we introduce the wavenumber field \( k(x, t) \) and the frequency field \( \omega(x, t) \) for the wave function (wave packet) of a general form:
\[ k(x, t) = \frac{1}{\hbar} \frac{\partial S(x, t)}{\partial x}, \quad \omega(x, t) = -\frac{1}{\hbar} \frac{\partial S(x, t)}{\partial t}. \] (7)

We will also need the equations for the phase and the amplitude square of the wave function, which follow from the Schrodinger equation
\[ i\hbar \frac{\partial \psi(x, t)}{\partial t} = \hat{H} \psi(x, t); \]
they will play a key role in finding the "physical" fields of the operators.

We write these equations in the form
\[ \frac{\partial w}{\partial t} + \frac{1}{m} \frac{\partial}{\partial x} \left( w \frac{\partial S}{\partial x} \right) = 0, \] (8)
\[ \frac{\partial S}{\partial t} + \frac{1}{2m} \left( \frac{\partial S}{\partial x} \right)^2 + U + V = 0; \] (9)

where \( U \) is the so-called "quantum-mechanical" potential (see [3, 4, 5]), in which we will distinguish two contributions (each of which will play its own role in our approach):
\[ U = K_w + U_w; \] (10)
\[ K_w = \frac{\hbar^2}{8m} \cdot \frac{1}{w^2} \left( \frac{\partial w}{\partial x} \right)^2; \] (11)
\[ U_w = -\frac{\hbar^2}{4m} \cdot \frac{1}{w} \frac{\partial^2 w}{\partial x^2}. \] (12)

As is known, that these equations were first obtained in the work of Madelung [3], which laid the foundation for the so-called 'hydrodynamic' formulation of quantum mechanics. Later this approach was developed in the works of de Broglie [4] and Bohm [5] (see equations (5) and (6)). However, in all three cases, the question of the physical interpretation of the equations (8) and (9) is solved on the basis of assumptions that take the 'hydrodynamic' formulation out of the scope of the (standard) quantum mechanics.

In particular, according to Bohm’s interpretation, this system of equations describes an ensemble of one-particle trajectories coinciding with the probability flow lines in the OCS: the equation (8) is treated as a continuity equation describing the conservation of probability on these trajectories; the equation (9) is interpreted as a modified Hamilton-Jacobi equation describing the motion of a particle along these trajectories in an external field \( V \) and in the field of the “quantum mechanical” potential (10), and the field \( p(x, t)/m \), where \( p(x, t) = \partial S/\partial x \), is treated as the velocity of the particle on these
trajectories. However, such a momentum field \[ \mathbf{5} \] belongs to the category of 'hidden variables', because it is defined outside the axiomatics of quantum mechanics. As a consequence, the question of interpretation of the equations \[ \mathbf{8} \] and \[ \mathbf{9} \] within the framework of (standard) quantum mechanics remains open.

In this paper, this issue is solved inextricably with the definition of 'physical' fields of operators. In this procedure, the field \( p(x,t) \) of the momentum operator \( \hat{p} = -i\hbar \frac{\partial}{\partial x} \) must be defined primarily. According to (14),

\[
p(x,t) = \frac{\text{Re} (\psi^*(x,t) \hat{p} \psi(x,t))}{\psi^*(x,t) \psi(x,t)} = \frac{\partial S(x,t)}{\partial x} \equiv \hbar k(x,t).
\] (13)

The field \( p(x,t) \) defined in this way can be considered as a required 'physical' field: firstly, in this case Eq. (8) takes the form of a continuity equation

\[
\frac{\partial w}{\partial t} + \frac{\partial}{\partial x} \left( w \frac{p}{m} \right) = 0,
\] (14)

where the field \( p(x,t)/m \) is the velocity of the probability flow at the point \( x \) at time \( t \); secondly, the equality (13) is nothing but a 'wave-packet' analogue of the de Broglie relation.

The next 'physical' field to be defined is the field of the kinetic energy operator. According to (14), the operator \( \hat{K} = \hat{p}^2/2m \) should be associated (taking into account the definition of (13)) the field

\[
\hat{K}(x,t) = \frac{1}{2m} \frac{\text{Re} (\psi^*(x,t) \hat{p}^2 \psi(x,t))}{\psi^*(x,t) \psi(x,t)} = \frac{1}{2m} [p(x,t)]^2 + U(x,t),
\] (15)

where \( U \) is the 'quantum mechanical potential' mentioned above \( \mathbf{11} \). However, the function \( \hat{K}(x,t) \) can take negative values, which is unacceptable for the 'physical' field of the kinetic energy operator. To find the 'physical' field, we must use the equality

\[
\int_{-\infty}^{\infty} \psi^*(x,t) \hat{p}^2 \psi(x,t) dx = \int_{-\infty}^{\infty} |\hat{p} \psi(x,t)|^2 dx,
\] (16)

the right side of which is associated with the field

\[
K(x,t) = \frac{1}{2m} \left| \frac{\hat{p} \psi(x,t)}{\psi(x,t)} \right|^2 = \frac{p^2}{2m} + K_w,
\] (17)

where \( K_w(x,t) \) is defined by Exp. \( \mathbf{11} \).

Two admissible fields arise also for the Hamiltonian operator \( \hat{H} = \hat{p}^2/2m + V(x,t) \), where \( V(x,t) \) is the potential energy of a particle in the external field: \( \hat{H} = K + V \) and \( H = K + U_w + V \),

\[
H(x,t) = \hbar \omega(x,t).
\] (19)
Thus, for any potential $V(x)$ that admits bound stationary states (in this case, $k(x, t) = 0$ and $\hbar \omega(x, t) = E$, where $E$ – constant), there appear two alternative forms of Eq. (9) (written for the amplitude $R = \sqrt{\omega}$):

$$\tilde{H} \equiv \frac{\hbar^2}{2mR^2} \left( \frac{\partial R}{\partial x} \right)^2 + V = \frac{\hbar^2}{2mR^2} \left[ \left( \frac{\partial R}{\partial x} \right)^2 + R \frac{\partial^2 R}{\partial x^2} \right] + E;$$

$$H \equiv \frac{\hbar^2}{2mR^2} \left( \frac{\partial R}{\partial x} \right)^2 - \frac{\hbar^2}{2mR^2} \left[ \left( \frac{\partial R}{\partial x} \right)^2 + R \frac{\partial^2 R}{\partial x^2} \right] + V = E;$$

whence it follows that in the case of bound stationary states, only Eq. (19) takes the form of the stationary Schrödinger equation

$$H(x) \cdot R(x) \equiv -\frac{\hbar^2}{2m} \frac{d^2 R(x)}{dx^2} + V(x)R(x) = ER(x).$$

Therefore, the 'physical' field of the total energy operator is given by (18). The role of the 'quantum mechanical potential' is played here by the field $U_w$, rather than the Bohmian 'potential' $U = K_w + U_w$, which contains the contribution $K_w$ of the kinetic energy $K$.

The 'physicality' of the field (18) is also supported by the fact that Eq. (19) is nothing but an analogue of the Planck-Einstein relation for wave packets. Together with Eq. (13) that defines the momentum operator field, this equation is the mathematical formulation of wave-particle duality for any normalized wave function that satisfies the Schrödinger equation. These two exact copies of the well-known relations indicate that the inseparable connection between the corpuscular and wave properties of a quantum particle exists not only when its state is described by the de Broglie wave, but also in the general case when its state is described by a wave packet localized in space. However, the physical meaning of the corpuscular characteristics ('momentum' and 'total energy') changes, when moving from a de Broglie wave to a wave packet, since the meaning of the corresponding wave characteristics (of 'wave number' and 'frequency') changes in this case.

3. Particle momentum fields and uncertainty relation

We have already mentioned that in Bohm’s theory [5] the probability streamlines are treated as particle trajectories, and the probability flow velocity $p(x, t)/m$ in the continuity equation (14) is treated as particle velocity on these trajectories. However, this interpretation is erroneous, since the velocity of the probability flow at point $x$ at time $t$ is the mean velocity of the particle. This is due to the fact that the field of the kinetic energy operator (17) contains not only by the field of the momentum operator $p(x, t)$, but also by the field $K_w(x, t)$, which is nonzero when $p(x, t) = 0$.

A “hydrodynamic” analogy is useful to understand this situation. After all, if the liquid is at rest, that is, if the speed of the flow of the liquid at all points is zero, then this does not mean at all that the speed of the particles of the liquid is zero. It is the average particle velocity is zero in a fluid at rest; while their average kinetic energy
in this case is not zero due to thermal motion. Of course, in a quantum one-particle ensemble, the cause of the "thermal" motion is different, but the conclusion is the same - in a quantum ensemble, the state of which is described by a wave packet localized in space, the velocity \( p(x, t)/m \) of the probability flow, determined by the field of the momentum operator, is not the particle velocity. Ultimately, this is due to the fact that the wavenumber of any localized wave packet, through which the field of the momentum operator is determined (see (13)), is not the wavenumber of any de Broglie wave.

Let us illustrate this by the example of the basic stationary state of a particle in the infinitely deep potential well located in the interval \( 0 \leq x \leq L \),

\[
\psi_1(x, t) = \sqrt{\frac{2}{L}} \sin \left( k_1 x \right) e^{-i\omega_1 t}, \quad k_1 = \frac{\pi}{L}, \quad \omega_1 = \frac{\pi^2 \hbar}{2mL^2}.
\] (20)

The phase \( \phi(x, t) \) of \( \psi_1(x, t) \) depends only on the time \( t \) and hence \( p(x, t) = \hbar k(x, t) \equiv 0 \). Thus, if the field \( p(x, t)/m \) determined the velocity of a particle, then this would mean that the wave function \( \psi_1(x, t) \) described an ensemble of motionless particles. But then it would be unclear what motion describes the non-zero contribution \( K_w \) in Exp. (17), thanks to which

\[
K(x, t) = K_w(x, t) = \frac{\pi^2 \hbar^2}{2mL^2}.
\] (21)

To understand this paradoxical situation, let’s write the wave function (20), in the region \( 0 \leq x \leq L \), as a superposition of two de Broglie waves:

\[
\psi_1(x, t) = -\frac{i}{\sqrt{2L}} \left[ e^{i(k_1 x - i\omega_1 t)} - e^{-i(k_1 x + i\omega_1 t)} \right].
\] (22)

According to de Broglie, the first wave corresponds to an ensemble of particles with momentum \( p_1 = \hbar k_1 = \sqrt{2mK_w} \), and the second wave corresponds to an ensemble of particles with momentum \( p_2 = -\sqrt{2mK_w} \). At the same time, the kinetic energy of the particles is the same in both cases, and it coincides with (21): \( K_1 = K_2 = K_w \). In other words, the particle of the quantum ensemble whose state is described by a superposition (22) has, at each point \( x \) at each instant of time \( t \), two possible values of momentum, \( p_1(x, t) \) and \( p_2(x, t) \), which follow from the fields \( p(x, t) \) and \( K(x, t) \):

\[
\frac{1}{2}(p_1 + p_2) = p \equiv 0, \quad \frac{1}{2} \left( \frac{p_1^2}{2m} + \frac{p_2^2}{2m} \right) = K_w = \frac{\pi^2 \hbar^2}{2mL^2}.
\]

So, the 'hydrodynamic' analogy and this particular case suggest one why the interpretation of the de Broglie relation, which is valid for the de Broglie wave, cannot be transferred one to one to the de Broglie relation for the wave packet. It follows from the above arguments that in the case of a wave packet of a general form, the field of the momentum operator (or, equally, the field of the first initial moment of momentum) \( p(x, t) \) and the field of the operator of the kinetic energy, multiplied by \( 2m \) (or, equally, the field of the second initial moment of momentum) \( 2mK(x, t) \) uniquely determine the pair of the momentum fields \( p_1(x, t) \) and \( p_2(x, t) \), which predict the existence of two definite values of the particle momentum for each OCS point:

\[
\frac{1}{2}(p_1 + p_2) = p, \quad \frac{1}{2} \left( \frac{p_1^2}{2m} + \frac{p_2^2}{2m} \right) = K = \frac{p^2}{2m} + K_w.
\] (23)
Taking into account (11), we write the roots \( p - \sqrt{2mK_w} \) and \( p + \sqrt{2mK_w} \) of these equations as

\[
p_1 = p - \frac{\hbar}{\sqrt{2w}} \frac{\partial w}{\partial x}, \quad p_2 = p + \frac{\hbar}{\sqrt{2w}} \frac{\partial w}{\partial x}.
\] (24)

Thus, the de Broglie wave describes an ensemble of particles with the same momentum at all points of the OCS, while the wave packet (6) describes an ensemble of particles that have at time \( t \) two possible values of momentum at each point of the OCS: \( p_1(x, t) \) and \( p_2(x, t) \). Note that

\[
\int_{-\infty}^{\infty} [p_{1,2}(x, t) - p(x, t)] w(x, t) \, dx = 0, \quad \Rightarrow \quad \langle p_1 \rangle = \langle p_2 \rangle = \langle p \rangle.
\]

In addition, integrating by parts, for both fields we find

\[
\int_{-\infty}^{\infty} (x - \langle x \rangle) [p_{1,2}(x, t) - p(x, t)] w(x, t) \, dx = \mp \frac{\hbar}{2},
\] (25)

where \( \langle x \rangle = \int_{-\infty}^{\infty} x w(x, t) \, dx \). This equality should be considered as an exact 'uncertainty' relations which differ from exact uncertainty relations presented in [7, 8, 9].

As for the Heisenberg inequality \( \Delta x \Delta p \geq \frac{\hbar}{2} \). The fields \( K(x, t) \) and \( p(x, t) \) that determine the standard deviation of momentum \( \Delta p = \sqrt{2m\langle \dot{K} \rangle - \langle \dot{p} \rangle^2} \), does not violate it for any state (6), since the average values of the fields of operators coincide, by definition (see (3)), with the average values of the corresponding operators. Let us illustrate this by the example of a quantum harmonic oscillator

\[
\psi(x, t) = \sqrt{\frac{m \omega}{\hbar \pi}} \exp \left( - \frac{m \omega x^2}{2\hbar} - \frac{\omega t}{2} \right).
\] (26)

In this case \( p(x, t) \equiv 0 \),

\[
V = \frac{m \omega^2 x^2}{2}, \quad K = K_w = \frac{m \omega^2 x^2}{2}, \quad U_w = \frac{\hbar \omega}{2} - m \omega^2 x^2.
\] (27)

Consequently,

\[
\langle \dot{x} \rangle = 0, \quad \langle \dot{p} \rangle = 0, \quad (\Delta x)^2 = \langle \dot{x}^2 \rangle = \frac{\hbar}{2m\omega}, \quad (\Delta p)^2 = 2m\langle \dot{K} \rangle = \frac{m\hbar \omega}{2}; \quad \Delta x \Delta p = \hbar/2.
\]

4. Equations for the probability field and two momentum fields

Let’s write Eqs. (14) and (19) as a closed system of equations for the momentum fields \( p_1, p_2 \) and the probability field \( w \). For this purpose we have to differentiate the left and right sides of Eq. (19) with respect to \( x \), and take into account the definition (7):

\[
\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial x}.
\]

Taking into account (24), we write it, together with the continuity equation, in the form

\[
\frac{\partial w}{\partial t} + \frac{1}{m} \frac{\partial (wp)}{\partial x} = 0, \quad \frac{\partial p}{\partial t} + \frac{\partial}{\partial x} \left( \frac{p^2}{2m} + \frac{P_w^2}{2m} \right) = -\frac{\partial V}{\partial x} - \frac{\partial U_w}{\partial x},
\] (28)

\[
p = \frac{p_2 + p_1}{2} = \hbar \frac{\partial \phi}{\partial x}, \quad P_w = \frac{p_2 - p_1}{2} = -\frac{\hbar}{2w} \frac{\partial w}{\partial x}.
\]
It is easy to show that Eqs. (28) for the fields of observables guarantee the fulfillment of the Ehrenfest equations. To get the first equation, we calculate the time derivative of the average value of the particle coordinate:

$$\frac{d\langle x \rangle}{dt} = \frac{d}{dt} \int_{-\infty}^{\infty} x w(x,t) dx = \int_{-\infty}^{\infty} x \frac{\partial w}{\partial t} dx.$$  \hspace{1cm} (29)

Given the first equation in (28) and integrating by parts, we get the first Ehrenfest equation:

$$\frac{d\langle x \rangle}{dt} = \frac{\langle \hat{p} \rangle}{m}.$$  \hspace{1cm} (30)

Further, taking into account the first two equations in (28), we obtain

$$\frac{d}{dt} \int_{-\infty}^{\infty} pw dx = \int_{-\infty}^{\infty} \left( w \frac{\partial p}{\partial t} + p \frac{\partial w}{\partial t} \right) dx = \int_{-\infty}^{\infty} \left( w \frac{\partial V}{\partial x} + \frac{\partial Q}{\partial x} \right) dx,$$  \hspace{1cm} (31)

where

$$Q = \left( \frac{p^2}{m} + 2K_w + U_w \right) w.$$

Which leads to the second Ehrenfest equation

$$\frac{d\langle p \rangle}{dt} = -\langle \frac{dV}{dx} \rangle.$$  

5. Conclusion

So, knowledge of the quantum mechanical state implies not only statistical restrictions on the results of measurements. As a consequence, we consider erroneous the main conclusion of the "quantum theory in the Copenhagen interpretation" about the absence of definite values of the measurable quantities before the moment of measurement. Using the example of the quantum dynamics of a particle in an OCS, it is shown that the standard rule for calculating the average values of the momentum, kinetic and total energy operators defines the fields of observables' operators as functions in the configuration space. Together with the physical requirements and constraints that these observables must satisfy (including the constraints that follow from the Schrodinger equation), this rule determines these fields uniquely.

The presented field formulation of quantum mechanics leads to the following conclusions regarding the quantum dynamics of a particle in the OCS, the state of which is described by a given (normalized to unity) wave function:

1. the probability field and operator fields of one-particle observables, which are calculated on the basis of the wave function, are the "visiting card" of the quantum one-particle ensemble;

2. the probability flow lines in the SSC cannot be considered as particle trajectories, since the field of the momentum operator that determines the probability flow velocity at the points of the SSC is the local average velocity of the ensemble's particles at these points; a quantum one-particle ensemble whose state is described
by a wave function cannot be considered as an ensemble of one-particle trajectories (this conclusion does not apply to a quantum one-particle ensemble described by a de Broglie wave);

(3) the wave function predicts not only the probability \( w(x, t) dx \) of detecting a particle in the interval \([x, x + dx]\) at time \(t\), but also two (with equal probability) possible momentum values – \(p_1(x, t)\) and \(p_2(x, t)\): the sum of these two fields is determined by the phase of the wave function, and their difference is determined by the amplitude;

(4) for any wave function, the deviations \(x - \langle x\rangle\) and \(p_{1,2}(x, t) - p(x, t)\) are such that the mean value of their product, weighted by the probabilities of the possible values of the \(x\) coordinate, are equal to \(-\hbar/2\) and \(\hbar/2\), respectively; this suggests that the prediction of two (with equal probability) possible values of the momentum for each point of the SCS is in full agreement with the Heisenberg uncertainty principle;

(5) the dependence of the predicted values of the particle momentum at each point of the OCS on the amplitude and phase of the wave function suggests that a particle, associated with a single-particle quantum ensemble, exhibits wave properties even in a single experiment with one particle.

The last conclusion can be seen as an answer to the question posed in the title of the article [10] “Does a Single Electron Have Wave Properties?”. The answer to it within the framework of “quantum theory in the Copenhagen interpretation”, with its (erroneous) conclusion about the absence of definite values of the measured quantities before the moment of measurement, is in principle impossible.

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