Gravitational waves from rapidly rotating white dwarfs

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Abstract

Rapidly rotating white dwarfs in cataclysmic variable systems may be emitting gravitational radiation due to the recently discovered relativistic r-mode instability. Assuming that the four most rapidly rotating known systems are limited in rotation rate by the instability, the amplitude of the emitted gravitational waves is determined at Earth for both known rapid rotators and for a model background caused by a galactic population of such systems. The proposed LISA and OMEGA space-based interferometer gravitational wave detectors could observe such signals and determine whether the r-mode instability plays a significant role in white dwarf systems.

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Andersson [1] has recently discovered a new relativistic instability in rapidly rotating stars. All rotating stars formed of perfect fluid possess unstable nonaxisymmetric modes (“r-modes”) which will emit gravitational radiation, carrying away angular momentum and slowing the rotation of the star [2]. Whether these modes are important dynamically in a realistic star depends on the viscosity, which tends to damp these modes, as is the case in the previously studied f-modes [3,4]. For any given stellar model, there is a critical rotation rate which may be defined, such that the star is stable due to viscous damping when rotating below this rate, but unstable above. Estimates of this critical period for neutron stars show that the r-mode instability may be of importance in the evolution of newly born hot, rapidly rotating neutron stars, spinning them down to the critical rate [5,6]. The resulting gravitational radiation may be an interesting source for ground-based interferometers, such as LIGO and GEO600 [7].

Recently, Andersson, Kokkotas, and Stergioulas (hereafter AKS) [8] have considered whether the r-mode instability might play a role in rapidly rotating white dwarfs, particularly the DQ Herculis stars. These stars are magnetized cataclysmic variables (CVs) with rotation periods as low as 30 s [9]. They accrete material from a low-mass main sequence companion in a binary system. A simple estimate of the critical rotation period made by AKS is intriguingly close to the actual rotation periods of the more rapidly rotating DQ Her stars. If the r-mode instability is active in such a system, then it represents a steady-state source of gravitational radiation. The star would be rotating at the critical limit, where all excess angular momentum accreted would be emitted as gravitational radiation. The equivalent situation for an f-mode limited neutron star was previously considered by Wagoner [10]. AKS also found that the growth timescale of the unstable r-mode is so great in a white dwarf (of order $10^8$ yr) that it appears questionable whether the mode could play a defining role in the system’s dynamics. Nevertheless, there is at present considerable uncertainty in the analysis of the r-mode instability in white dwarfs, and the DQ Her stars are complex systems. If the most rapidly rotating DQ Her white dwarfs are potentially rotating at the critical limit, it is worth considering whether the emitted gravitational waves (GW) are
potentially detectable.

The purpose of this paper is to show that if the r-mode instability is actually excited in the rapidly rotating DQ Her stars, then the gravitational radiation emitted is potentially detectable. Gravitational wave astronomy may then, in principle, be used to constrain or determine the relevance of the r-mode instability to these systems. The gravitational wave frequency of the dominant r-mode, with \( l = m = 2 \), is \( f = 4/(3P) \), where \( P \) is the rotation period of the star. For the most rapidly rotating white dwarfs, this means the emitted gravitational waves will be in the frequency band of \( 10^{-2} - 10^{-1} \) Hz, the domain of the proposed space-based laser interferometer detectors, such as LISA [11] and OMEGA [12].

The critical rotation period for a white dwarf of mass \( M \), radius \( R \), and temperature \( T \), was estimated by AKS to be

\[
P_c \simeq 27 \left( \frac{M}{M_\odot} \right)^{1/24} \left( \frac{R}{0.01R_\odot} \right)^{11/8} \left( \frac{T}{10^5 K} \right)^{1/24} \text{s}. \tag{1}
\]

There are four candidate stars with short rotation periods which are within a reasonable range of this estimated critical period. These are: WZ Sge, with a period of 28 s, AE Aqr with period 33 s, V533 Her at 63.6 s, and DQ Her at 71.1 s. For the purposes of this paper, all four of these stars will be assumed to be rotating at the critical limit, and hence be emitting GW due to the r-mode instability. The remaining known members of the DQ Her family have considerably longer rotation periods, from 206 s to 7188 s [9], and will be ignored here.

The four most rapidly rotating DQ Her stars may be treated as periodic sources of gravitational radiation, since their rotation frequencies are known \( a \text{ priori} \) with precision and are essentially constant. These four systems are also used to develop plausible values for the spatial density and gravitational wave luminosity of rapidly rotating white dwarfs throughout the Galaxy. These values are then used to determine the cumulative background of gravitational radiation due to the superposed signals of the rapidly rotating DQ Her type systems throughout the Galaxy.

The stellar systems of interest are assumed to be rotation limited by the emission of
gravitational radiation associated with the r-mode instability. All excess angular momentum carried by the matter accreted from the companion star is converted into gravitational radiation by the instability. Following Wagoner [10], this implies that

$$\frac{dJ}{dt} \simeq (MR)^{1/2} \dot{M} - \dot{J}_{GR} = 0 ,$$

where $J$ is the total angular momentum of the accreting dwarf, and $\dot{J}_{GR}$ represents the loss of angular momentum to gravitational radiation. Units have been chosen so that $G = c = 1$. The angular momentum carried off in the gravitational radiation may be related to the energy lost, and to the dimensionless amplitude of the gravitational wave, $h$, as observed at Earth, by

$$\dot{j} = -\frac{m}{\omega} \dot{E} ,$$

$$h^2 = \left( \frac{2}{\omega r} \right)^2 |\dot{E}| ,$$

where $m$ and $\omega$ are respectively the azimuthal index and frequency of the mode, and $r$ is the distance from the source to the Earth. Combining Eqs(2-4) then yields

$$h = \frac{2 (MR)^{1/4} \dot{M}^{1/2}}{m^{1/2} \omega^{1/2} r} ,$$

or, for the dominant mode, assuming the star is rotating at the critical period,

$$h = 1.88 \times 10^{-24} \left( \frac{P_c}{30 \text{s}} \right)^{1/2} \left( \frac{M}{M_\odot} \right)^{1/4} \left( \frac{R}{0.01 R_\odot} \right)^{1/4} \left( \frac{\dot{M}}{10^{-10} M_\odot/\text{yr}} \right)^{1/2} \left( \frac{\text{kpc}}{r} \right) .$$

If the four rapidly spinning white dwarfs are assumed to be rotating at $P_c$, so that they are balanced between accretion of angular momentum and emission via gravitational radiation, then the amplitude of the wave at Earth may be calculated using Eq.(5). The pertinent properties of the four stellar systems under consideration, and their resulting values of $h$, are listed in Table 1. Values for the spin period, mass, accretion rate, and distance were taken from Refs. [8,9,13]; the radii were then determined by assuming the stars are CO white dwarfs [14]. No mass value for V533 Her was found in the literature, so a value of $1M_\odot$
was assumed. The gravitational wave amplitudes for these nearby ($r < 1 \text{ kpc}$) systems are found to be in the range $h \sim 10^{-23} - 10^{-22}$.

Since the four known rapidly rotating white dwarfs are all nearby, it is reasonable to assume that these represent only the nearest (and brightest, electromagnetically) such stars. A population of such stars throughout the galaxy will create a background of gravitational waves which may be detectable. In order to determine the GW spectrum of such a background the properties of the known stars must be used to model a galactic population.

Since the properties of the four known nearby systems are the only data available to define the model of the galactic distribution of such rapidly rotating white dwarfs, the result should be recognized as being at best a crude approximation. The four known rapid rotators fall naturally into two groups of two. The two nearby systems, WZ Sge and AE Aqr, are quite rapidly rotating, with similar periods and similar (low) mass accretion rates, and hence presumably lower gravitational wave luminosity. The two more distant systems, V533 Her and DQ Her, are more slowly rotating, again have similar periods, and have higher accretion rates. It thus seems sensible to construct a model in which the luminosity and galactic density of systems depends on the rotation period. The model distribution will assume that all DQ Her stars with periods between 20s and 80s are rotating at their critical periods and emitting GW due to the r-mode instability.

The gravitational wave amplitude associated with the four known systems is well fit by the power law:

$$h = 2.0 \times 10^{-28} \left( \frac{f}{\text{Hz}} \right)^{-3} \left( \frac{\text{kpc}}{d} \right).$$  \hspace{1cm} (7)

The spatial density of rapid rotators as a function of period will be modeled by noting that the two rapid rotators (WZ Sge and AE Aqr) are within 100 pc; while the slower DQ Her and V533 Her are within 1 kpc. A density function is then chosen for the solar neighborhood which is a power law of rotation period (or GW frequency, $f$), normalized to yield a few fast rotators within 100 pc and a few slower rotators within 1 kpc,

$$\frac{d\rho_\odot}{dP} = 5 \times 10^{-7} \left( \frac{P}{30\text{s}} \right)^{-6} \text{pc}^{-3} \text{s}^{-1}.$$  \hspace{1cm} (8)
This choice of distribution implies that there should be numerous fast \((P \sim 30\text{s})\) rotators between distances of 100 pc and 1 kpc which have not yet been discovered.

The distribution of rapidly rotating white dwarfs in the galaxy is assumed to follow the galactic disk population of CVs, so that

\[
\frac{d\rho}{dP} = \frac{d\rho_0}{dP} \exp(-R/R_0) \exp(-|z|/z_0) ,
\]

in standard Galactocentric coordinates, with \(R_0 = 3.5 \text{ kpc}, z_0 = 120 \text{ pc}\), and the central density, \(d\rho_0/dP\) is given by \(d\rho_0/dP = d\rho_\odot/dP \exp(R_\odot/R_0)\) \cite{13}. The solar Galactocentric radius is taken to be \(R_\odot = 8.5 \text{ kpc}\).

The proposed space-based interferometers, LISA and OMEGA, will integrate the signal from persistent sources for a characteristic time of \(\sim 4 \text{ months}\), yielding a bandwidth of \(10^{-7} \text{ Hz}\). The rotating white dwarf sources are assumed to have periods of \(20-80 \text{ s}\), covering a range in frequency space from \(1.67 \times 10^{-2} - 6.67 \times 10^{-2} \text{ Hz}\), which will contain \(5 \times 10^5\) frequency bins of width \(df = 10^{-7} \text{ Hz}\). The number of sources per bin can be determined by integrating Eq.(9) over the galaxy, converting from period to frequency dependence, and multiplying by the width of the frequency bin, to find

\[
\frac{dN}{df} df = 2.4 \times 10^5 \left(\frac{f}{\text{Hz}}\right)^4 .
\]

Over the relevant range of frequencies the number of sources per bin then varies from 0.02 to 5; over the majority of the frequency domain, most bins are empty of any source. Since there are not large numbers of sources per bin, the background is not stochastic; after 4 months of integration most nonempty bins will be resolved into individual periodic sources.

A simulated GW spectrum from a galactic population of r-mode unstable white dwarfs which could be potentially detected by LISA or OMEGA with a 4 month integration is illustrated in Figure (1). This simulated spectrum was constructed as follows. First, the number of sources in each of the 1/2 million bins was determined from Eq.(10); if this number is one or more, then the sources in the bin are placed randomly in the Galaxy, weighted by the distribution function of Eq.(9), using a Monte Carlo routine. The GW
spectral amplitude is then calculated using Eq.(7) and the bandwidth. Finally, if there is more than one source in a bin, their amplitudes are added incoherently (square root of the sum of squared amplitudes). The four known rapidly rotating white dwarfs are indicated by the labeled points. It is important to emphasize again that most of the bins, particularly at the lower frequencies, are actually empty; this is not apparent due to the thickness of the ink lines in the figure.

The figure also illustrates the root spectral density of the instrument noise for the proposed LISA and OMEGA gravitational wave detectors. These proposed systems each use several spacecraft arranged in an equilateral triangle, whose sides form laser interferometers. Both proposed missions plan to use 1 watt lasers, 30 cm optics, and achieve a drag-free performance of order $10^{-15} \text{ m s}^{-2} \text{ Hz}^{-1/2}$. Their sensitivity to gravitational waves is determined by these three parameters and the size of the interferometer arms. The LISA plan calls for a heliocentric orbit with an arm baseline of $5 \times 10^9 \text{ m}$; OMEGA is intended to be geocentric, with a shorter arm length, only $10^9 \text{ m}$. The longer baseline would make LISA more sensitive at low frequencies, while OMEGA’s performance would be better at higher frequencies. The sensitivity curve shown for LISA does not agree with that often displayed (see, e.g., Ref. [11], page 20) at frequencies above $10^{-2} \text{ Hz}$. This is because the degradation in sensitivity at high frequencies has usually been assumed to begin at $f = (2\ell)^{-1}$, where $\ell$ is the interferometer baseline. However, careful analysis of the transfer function [16,17] shows that for any space interferometer, the degradation in sensitivity actually begins at a lower frequency, namely $f = (2\pi\ell)^{-1}$. This lower onset frequency has been used consistently for both instruments, so that their relative sensitivity is correctly portrayed in Fig.(1).

Both the galactic background and the periodic signals of nearby sources are seen in Figure (1) to be within the detection limits of either interferometer system. If the source model is made more conservative, by assuming that only the most rapid rotators, with periods closest to the estimate of [8], possess active r-mode instabilities (e.g., $P < 35 \text{ s}$, or $f > 3.8 \times 10^{-2} \text{ Hz}$), then OMEGA could still detect a large number of sources, while LISA would be limited to only the nearest and brightest.
In summary, if the r-mode relativistic instability is active in rapidly rotating white dwarfs, then the gravitational waves emitted would be detectable by proposed space-based interferometer systems. The actual relevance of the r-mode instability to rapidly rotating white dwarfs could then be determined observationally. As these sources are higher frequency than most other sources considered for the space-based interferometers, shorter baseline instruments would better suited to their detection.

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FIGURES

FIG. 1. The instrument noise curves for LISA and OMEGA are compared to the estimated spectral amplitude for r-mode limited rapidly rotating white dwarfs. A bandwidth of $10^{-7}$ Hz is assumed, which allows the majority of stars be resolved into individual GW spectral lines. The four known rapid rotators, indicated by the labeled points, are, from left to right, DQ Her, V533 Her, AE Aqr, WZ Sge.
TABLE I. The properties of the four rapidly rotating DQ Her systems are given, along with the values of $h$ which follow by assuming their rotation rate is limited by the r-mode instability.

| System   | Spin period (s) | $\frac{M}{M_\odot}$ | $\frac{R}{0.01 R_\odot}$ | $\frac{M}{10^{-6} M_\odot/\text{yr}}$ | $r$(pc) | $h$          |
|----------|-----------------|----------------------|---------------------------|---------------------------------------|---------|---------------|
| WZ Sge   | 28              | 1.4                  | 0.2                       | 1.6                                   | 75      | $2.27 \times 10^{-23}$ |
| AE Aqr   | 33.1            | 1.25                 | 0.5                       | 2.0                                   | 90      | $2.76 \times 10^{-23}$ |
| V533 Her | 63.3            | 1.0                  | 0.8                       | 63.                                   | 1000    | $2.06 \times 10^{-23}$ |
| DQ Her   | 71.1            | 0.83                 | 1.0                       | 126.                                  | 420     | $7.38 \times 10^{-23}$ |
