Electromagnetic Lorenz Fields

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Abstract: Gauge transformations are potential transformations that leave only specific Maxwell fields invariant. To reveal more, I develop Lorenz field equations with full Maxwell form for nongauge, sans gauge function, transformations yielding mixed, superposed retarded and outgoing, potentials. The form invariant Lorenz condition is then a charge conservation equivalent. This allows me to define three transformation classes that screen for Lorenz relevance. The nongauge Lorentz conditions add polarization fields which support emergent, light-like rays that convey energy on charge conserving phase points. These localized rays escape discovery in modern Maxwell fields where the polarizations are suppressed by gauge transformations.

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1. Introduction

In 1867, during the time when J. C. Maxwell (1831-79) was publishing his electromagnetic theory, L. V. Lorenz (1829-91) published his theory equating light vibrations with electric currents [1]. This work is translated to modern vector notation and critiqued in [2]. Starting from Kirchhoff’s Ohm’s law expression, Lorenz uses scalar potential retardation to obtain an expression for the retarded vector potential. The retarded potentials satisfy inhomogeneous wave equations with sources and, when charge is conserved, the eponic Lorenz condition [3, pp. 268-9]. Because the retarded potentials are nonlocal, all their derivatives also must be inhomogeneous wave function solutions for correspondingly differentiated remote sources. Since the potentials satisfy wave equations they can be augmented with solutions for the homogeneous wave equation. These outgoing augmentations are local, their derivatives depend only on proximate values. Although no longer strictly retarded, the composite potentials still satisfy a wave equation with one and the same propagation speed, but the Lorenz condition is modified.

In his 1867 paper, Lorenz never mentions the magnetic field and, therefore, never develops electromagnetic field equations. In fact, until now, this extension has not been examined. However, by defining magnetic induction as the vector potential curl, I derive electromagnetic Lorenz field equations with the Maxwell form. This derivation allows Lorenz retardation ramifications to be fully explored. When derived from nonlocal potentials, the fields in these equations must be nonlocal also. The analytically desirable locality can be restored as a far field approximation for systems that satisfy the dipole approximation which imposes specific size constraints [4, p. 222]. For the augmented potentials, the Lorenz field equations incorporate fields for electric displacement and magnetic field strength by adding local polarizations. To help understand the effect Lorenz retardation can have on light theory, this development is presented formally in Appendix A. It shows that wave function potentials satisfying the Lorenz condition assure charge conservation. Thus, the form invariant Lorenz condition and charge conservation are equivalent.

2. Potentials transformation

In the early twentieth century, starting from electromagnetic Maxwell field equations, H. A. Lorentz (1853-1928) found a condition that caused the vector and scalar potentials to be wave functions with the retarded forms. This Lorentz condition is the same mathematical equation found earlier by Lorenz. Since the Lorenz condition is equivalent to charge conservation, Maxwell fields must always be derivable from wave function potentials.

Having started from the field equations, Lorentz noted that the electric field strength and magnetic induction are invariant when the potentials are subjected to transformation with an
arbitrary gauge function, $\chi$, that causes
\[
\begin{align*}
A &= A_1 - \nabla \chi, \\
\Omega &= \Omega_1 + \frac{1}{c} \frac{\partial \chi}{\partial t}.
\end{align*}
\] (1)

Gauge restraints on potentials are now widely accepted in theoretical physics [5] even without gauge function specification [6] which further restrains the allowed potentials [7]. Although nonlocality and susceptibility to gauge restraint causes potential reality to be questioned, arbitrary gauge restraint permissibility is unquestioned. However, charge conservation equivalence to the Lorenz condition exposes this deficiency and imposes a requirement for extraordinary justification on any transformation that alters the Lorenz condition.

When the original potentials are retarded wave functions Eq.(1) resolves to three Lorenz classes that need not have the gauge transformation form. First, the new potentials are not wave functions. Second, the new potentials are wave functions with altered charge and current values. Third, the new potentials are wave functions with unaltered charge and current values. In the first case the Lorentz condition will not apply and the potentials will not be retarded; whereas recent tests listed in [8–11] show longitudinal electric fields to propagate with finite speed. The last two cases will conserve charge if the Lorentz condition persists. This persistence will assure Lorenz field equations in which charge and current densities are deemed observables that produce physical potentials and fields yielding emergent light-like waves that conserve charge progressively as described in Appendix B with localization to ray forming phase points on which energy is conveyed as described in Appendix C. For gauge transformations, this requires the Lorenz gauge, $\Box a\chi = 0$, that suppresses local electric and magnetic polarizations and, thus, their dependent, emergent rays that have been fruitlessly sought in Maxwell fields [12].

3. Conclusion

The forgoing shows that if Lorenz had developed field equations from his retarded potentials classical electromagnetism as we know it today could have been provided with retarded potentials as a solid etiological foundation. We would fully appreciate the Lorenz condition equivalence to charge conservation, light-like ray emergence from retarded fields and energy conveyance by field phase points. Based on this analysis I propose that distinct terms “Lorenz condition” and “Lorentz condition” used in the Abstract be retained to designate two uses for one equation relating vector and scalar potentials: the Lorentz condition only representing a potential transformation; the Lorenz condition representing charge conservation that potential transformations leave form invariant, Lorenz covariance. With this, Lorenz fields support localized, light-like rays that escape discovery in modern Maxwell fields.

A. Lorenz Field Equations

The electromagnetic Lorenz potentials are given by the forms
\[
\Omega = \int \int \int \frac{\rho'}{R} dv',
\] (2)

and
\[
A = \frac{1}{a} \int \int \int \frac{J'}{R} dv'.
\] (3)

where $\rho'$ and $J'$ indicate that $\rho$ and $J$ depend on $t - \frac{r}{c}$ and $r'$ rather than $t$ and $r$ for $R = r - r'$. They satisfy the wave equations
\[
\Box a \Omega = -4\pi \rho
\] (4)
and
\[ \square_a A = -\frac{4\pi}{a} J \]  \hspace{1cm} (5)

and, when charge is conserved, the Lorenz condition
\[ -a \nabla \cdot A = \frac{\partial \Omega}{\partial t}. \]  \hspace{1cm} (6)

Here, the D’Alembertian,
\[ \square_a = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{a^2} \frac{\partial^2}{\partial t^2}. \]

When the potentials are augmented by solutions to the homogeneous wave equation,
\[ \square_a \Omega_0 = \square_a A_0 = 0, \]
the Lorenz condition takes the modified form
\[ -a \nabla \cdot (A + A_0) = \frac{\partial (\Omega + \Omega_0)}{\partial t}. \]  \hspace{1cm} (7)

The augmentations contain implicit scalar strengths since one or both could be set to zero. For
\[ E = -\nabla \Omega - \frac{1}{a} \frac{\partial A}{\partial t} \]  \hspace{1cm} (8)

and
\[ B = \nabla \times A, \]  \hspace{1cm} (9)
we have the additional fields
\[ D = E + P \]
\[ H = B - M \]
with
\[ P = -\nabla \Omega - \frac{1}{a} \frac{\partial A_0}{\partial t} \]
and
\[ M = -\nabla \times A_0. \]

In these definitions the augmentation strengths are passed to the polarizations which vanish when the augmentations are derived from a gauge function, i.e. when
\[ A_0 = -\nabla \chi \]
and
\[ \Omega_0 = \frac{1}{a} \frac{\partial \chi}{\partial t}. \]

These fields satisfy the equations:
\[ \nabla \cdot B = 0, \]  \hspace{1cm} (10a)
\[ \nabla \cdot D = 4\pi \rho, \]  \hspace{1cm} (10b)
\[ a \nabla \times E = -\frac{\partial B}{\partial t}, \]  \hspace{1cm} (10c)
\[ \nabla \times H = \frac{1}{a} \left[ \frac{\partial D}{\partial t} + 4\pi J \right]. \]  \hspace{1cm} (10d)

Together Eqs. (10b) and (10d) give the condition for charge conservation
\[ \nabla \cdot J + \frac{\partial \rho}{\partial t} = 0. \]  \hspace{1cm} (11)

This also follows from simply applying the D’Alembertian to the Eq. (7) Lorenz condition. When \( E \) and \( B \) in the field equations are replaced by \( P \) and \( M \) using the expressions for \( D \) and \( H \), from \( H \times (\nabla \times D) - D \times (\nabla \times H) = \nabla \times (D \times H) \) Eqs. (10c) and (10d) give for energy continuity
\[ \nabla \cdot (D \times H) = -\frac{1}{a} \left[ H \cdot \frac{\partial D}{\partial t} + D \cdot \frac{\partial H}{\partial t} \right] + \frac{4\pi}{a} D \cdot J. \]  \hspace{1cm} (12)

More directly,
\[ \nabla \cdot (E \times H) = -\frac{1}{a} \left[ H \cdot \frac{\partial B}{\partial t} + E \cdot \frac{\partial D}{\partial t} \right] + \frac{4\pi}{a} E \cdot J. \]  \hspace{1cm} (13)

These expressions are identical when the polarizations vanish, but when \( E \) and \( B \) vanish the latter vanishes and the former carries the entire flux. This coupling allows local polarization waves to emerge from a nonlocal electromagnetic field. The Hertz dipole radiation solution
discussed in Appendices B and C is the only extant example for which this emergence is actually exhibited.

For Lorenz fields the coupling normally provided by the constitutive equations, \( \mathbf{D} = \varepsilon \mathbf{E} \) and \( \mathbf{B} = \mu \mathbf{H} \), can not be invoked without further augmenting the potentials with proportional retarded potentials, because fields derived from the retarded potentials can not be proportional to those derived from the potential augmentations. So the wave function speed must be adjusted for dielectric constant \( \varepsilon \) and magnetic permeability \( \mu \) by measurement. Being dependent on retarded potentials, all fields satisfy wave equations with the same propagation speed. Augmented potential polarization waves satisfying the Lorenz condition are discussed in Appendix B.

**B. Polarization Waves**

As representative Lorenz potential augmentations consider

\[
\mathbf{A}_0 = (f, g, h)
\]

and

\[
\Omega_0 = \frac{k}{k} \mathbf{A}_0 = \frac{k_x f + k_y g + k_z h}{k},
\]

When \( f, g \) and \( h \) are functionally dependent only on phase factors \( \omega t - \mathbf{k} \cdot \mathbf{r} \) where \( \mathbf{k} = (k_x, k_y, k_z) \) and \( a^2 k^2 = \omega^2 \), these potentials satisfy the homogeneous wave equation. Potentials in matter free space have not been restrained previously by the Eq. (6) Lorenz condition as these are. With this restraint, the potentials describe waves that can be considered to propagate by progressively conserving charge. An example with Lissajous vector potential is presented in Appendix C. It shows that the polarizations will describe rays without the longitudinal fields that plague other formulations [12]. The rays are real space paths determined by three linearly independent potential vector component phases. With neither wave packet dispersion [13] nor quantum mechanical nonlocality, these rays provide localization that has long eluded discovery [12]. Localization to rays is relaxed when the component phases are linearly dependent.

For the Eq. (14) and (15) potentials, the electric and magnetic polarizations become

\[
P = \frac{1}{\omega} \frac{k}{k} \times (k \times \dot{\mathbf{A}}_0) = \frac{1}{\omega} \frac{k}{k} \left( \frac{k}{k} \mathbf{A}_0 \right) k - \frac{1}{\omega} \frac{k}{k} \mathbf{k} \dot{\mathbf{A}}_0,
\]

since \( \nabla \Omega_0 = -\frac{k}{\omega} \Omega_0 \), and

\[
\mathbf{M} = \frac{1}{\omega} (k \times \dot{\mathbf{A}}_0).
\]

When the vector potential components are periodic functions they can be considered to be plane waves. Unlike Lorenz fields defined by nonlocal potentials, the polarizations should be considered to be local point functions. They are orthogonal, \( \mathbf{P} \mathbf{M} = 0 \) and have no longitudinal components, \( \mathbf{k} \mathbf{P} = \mathbf{k} \mathbf{M} = 0 \); they carry a flux \( \mathbf{M} \times \mathbf{P} = \omega^{-2} (k \times \dot{\mathbf{A}}_0)^2 k \) equal to \( \mathbf{D} \times \mathbf{H} \) when \( \mathbf{E} = \mathbf{B} = 0 \) and travel indefinitely in a ray direction \( \mathbf{k} \) at speed \( a \) without driving sources.

In the 19th century second half, H. von Helmholtz (1821-94) attempted to reconcile competing electromagnetic theories [14]. To this end he developed a theory based on electrical and magnetic polarization and obtained wave equations for polarization propagation in a homogeneous medium. His wave equation for the electric polarization contained an undetermined constant that allowed the propagation speed for longitudinal waves to have any non-negative value. So only difficult, precision measurements to establish the parameter’s value could complete the theory. For isotropic electric polarization his theory gives \( c/\sqrt{\varepsilon \mu} \) for component propagation speed, where \( c \) is the vacuum light speed. The Lorenz electric polarization does propagate isotropically. But we have just seen that the Lorenz condition suppresses its longitudinal component, because Eq. (10b) gives \( \nabla \cdot \mathbf{P} = 0 \).
To satisfy the Eq. (6) Lorenz condition, we find the scalar potential must have the form originating from a charge moving in an x-y plane. When By the Gauss theorem, this represents an equality between the temporal energy change in a volume and the energy flux through its surface. For monochromatic waves, This problem is revealed by the sensible benchmarks for photon properties ranging over 10 length inversely related to monochromaticity departure [21]. This dependence on chromaticity means that the photon power density must approach zero as the coherence length becomes very large for monochromatic photons. So the compact, quantum particle photon concept is untenable in this limit. The concept is further threatened by confounding photon size with wavelength.

To promote his theory, Helmholtz proposed a prize competition to experimentally establish a relation between electromagnetism and dielectric polarization. His former student, H. Hertz (1857-94), later claimed the prize and went on to observe electromagnetic reflection and interference. Based on these observations Hertz concluded that polarization propagation is analogous to vacuum light [15, pp. 19 and 122-3] and [16]. He studied waves in a hall with effective dimensions 15x8.5x6. These waves emanated from a 1 cm spark gap with calculated 14 ns resonant half period. Powerful discharges were obtained by applying interrupted induction coil output across the gap between two 15 cm radius spheres in a 100 cm long dipole with 40 cm by 40 cm brass plates at its far ends. In explaining his results, Hertz applied Maxwell’s equations to the cyclic gap charging and neglected the periodic arc discharge [4,15,16]. Today, we should prefer an alternative description in which the arc powered by the collapsing scalar potential is the radiation cause [17–19], because the arc generates light and induces remote circuit arcing that Hertz used to analyze the wave fields. Apparently arcing at his remote circuits was stroboscopically sensitized by light from his primary arc, so he could use only primary spark gaps near 1 cm where light emission was sufficient to make observations. Thus, Hertz’s dipole radiation fields displayed in textbooks are unlikely to represent the fields he actually studied; but, if photons have finite dimensions, his and recent [20] observations suggest their internal structure may be susceptible to study. The discussion in [2] shows that observations documented in [20] may represent an Ampere law based, magnetoinductive internal energy structure with anti-phased electric and magnetic fields.

C. Polarization Flux Localization

For vacuum polarization waves Eq. (12) takes the form

$$\nabla \cdot (\mathbf{M} \times \mathbf{P}) = \frac{1}{a} \left[ \mathbf{M} \cdot \frac{\partial \mathbf{M}}{\partial t} + \mathbf{P} \cdot \frac{\partial \mathbf{P}}{\partial t} \right]. \quad (18)$$

By the Gauss theorem, this represents an equality between the temporal energy change in a volume and the energy flux through its surface. For monochromatic waves, $\mathbf{M} \times \mathbf{P}$ can be written using the Hertz analogy as $h \nu \mathbf{f}$ where $h \nu$ is photon energy and $\mathbf{f}$ is photon flux with coherence length inversely related to monochromaticity departure [21]. This dependence on chromaticity means that the photon power density must approach zero as the coherence length becomes very large for monochromatic photons. So the compact, quantum particle photon concept is untenable in this limit. The concept is further threatened by confounding photon size with wavelength. This problem is revealed by the sensible benchmarks for photon properties ranging over $10^{36}$ ev in Table 1 where the lowest energy will have a wavelength greater than the Earth orbit radius.

As mentioned in Appendix B, the Lorenz field polarization waves provide, at least, a classical solution to this conceptual impasse. To see how, consider the vector potential

$$\mathbf{A}_0 = (\sin \phi_1, \sin \phi_2, \sin \phi_3) \quad (19)$$

with Lissajous phases $\phi_1 = \omega_1 t - k_1 \cdot \mathbf{r}$, $\phi_2 = \omega_2 t - k_2 \cdot \mathbf{r}$ and $\phi_3 = \omega_3 t - k_3 \cdot \mathbf{r}$ where $k_1 = (k_{1x}, k_{1y}, 0)$, $k_2 = (k_{2x}, k_{2y}, 0)$ and $k_3 = (0, 0, k_{3z})$. This vector potential may be looked upon as originating from a charge moving in an x-y plane. When $a^2 k_i^2 = \omega_i^2$ for $I = 1, 2$ or 3, $\nabla \cdot \mathbf{A}_0 = 0$. To satisfy the Eq. (6) Lorenz condition, we find the scalar potential must have the form

$$\Omega_0 = \frac{k_{1x}}{k_1} \sin \phi_1 + \frac{k_{2y}}{k_2} \sin \phi_2 + \sin \phi_3 \quad (20)$$

which also satisfies $\nabla \cdot \Omega_0 = 0$. So these potentials can be taken to define the Appendix A Lorenz polarizations given by

$$\mathbf{P} = \left( \frac{k_{2y} k_{3z}}{k_2^2} \cos \phi_2 - \frac{k_{3z}^2}{k_1} \cos \phi_1, \frac{k_{1x} k_{3z}}{k_1} \cos \phi_1 - \frac{k_{3z}^2}{k_2^2} \cos \phi_2, 0 \right) \quad (21)$$
Table 1. Some Vacuum Photon Properties

| Energy | Frequency | Wavelength |
|--------|-----------|------------|
| 1Tev   | 0.241E27  | 12.4E-17   |
| 1Gev   | 0.241E24  | 12.4E-14   |
| 1MeV   | 0.241E21  | 12.4E-11   |
| 1keV   | 0.241E18  | 12.4E-8    |
| 1eV    | 0.241E15  | 12.4E-5    |
| 1meV   | 0.241E12  | 12.4E-2    |
| 1µeV [15] | 0.241E9  | 12.4E1     |
| 1neV   | 0.241E6   | 12.4E4     |
| 1peV   | 0.241E3   | 12.4E7     |
| 1feV   | 0.241     | 12.4E10    |
| 1aev [20] | 0.241E-3 | 12.4E13    |

and

\[
M = (0, 0, k_2 \cos \phi_2 - k_1 \cos \phi_1)
\] (22)

with the flux

\[
M \times P = (k_2 \cos \phi_2 - k_1 \cos \phi_1) \times \left( -\frac{k_1 k_2}{k_1^2} \cos \phi_1 + \frac{k_1^2}{k_2} \cos \phi_1 \cos \phi_2, \frac{k_2^2}{k_1} \cos \phi_2 - \frac{k_1^2}{k_2} \cos \phi_1, 0 \right).
\] (23)

Taking \(\phi_1 = \phi_2 = \phi\) gives the simple harmonic polarizations

\[
P = \cos \phi \left( -\frac{k_1 k_2}{k_2} - \frac{k_1^2}{k_1}, \frac{k_1 k_2}{k_1}, 0 \right)
\] (24)

and

\[
M = \cos \phi (0, 0, k_2x - k_1y)
\] (25)

with the flux

\[
M \times P = (k_2x - k_1y) \cos^2 \phi \left( -\frac{k_1 k_2}{k_1}, \frac{k_1^2}{k_2}, \frac{k_2^2}{k_1}, k_1y, 0 \right)
\] (26)

having bead-chain [22] squared magnitude

\[
|M \times P|^2 = (k_2x - k_1y)^2 \left\{ \frac{k_2^2}{k_1 k_2} - 2 \frac{k_2 k_1}{k_1 k_2} (k_2x k_1y + k_2y k_1x) + k_1^2 \right\} \cos^4 \phi.
\] (27)

When \(k_1 = k_2 = \frac{k}{\sqrt{2}} = (k_x, k_y, 0)/\sqrt{2}\), the polarizations further simplify to

\[
P = \frac{1}{\sqrt{2}} \frac{k_2 - k_1}{k} \cos \phi (k_y, -k_x, 0)
\] (28)
This reduces to imposing some phase points. These localized energy bearers need to be marshaled into coordinated groups by like waves, the more general case in which the line defined by Eqs. (31). In the limiting case for which now has the Eq. (23) form. To prevent ray dissipation by flux components normal to the phase plane defined by Eqs. (31) with like those in Eq. (31) with squared magnitude
\[ |\mathbf{M} \times \mathbf{P}|^2 = \frac{1}{4} (k_x - k_y)^4 \cos^4 \phi. \]

This flux is highly anisotropic with maximum values for \( k_x = -k_y \) and null values for \( k_x = k_y \).

When \( k_1 \neq k_2 \) the phase moves on the parametric line with coordinates
\[
X(t) = \left[ k_{2y}(\omega_1 t - \phi) - k_{1y}(\omega_2 t - \phi) \right] / (k_{1x}k_{2y} - k_{2x}k_{1y}),
\]
\[
Y(t) = \left[ -k_{2x}(\omega_1 t - \phi) + k_{1x}(\omega_2 t - \phi) \right] / (k_{1x}k_{2y} - k_{2x}k_{1y})
\]
and velocity
\[
\mathbf{V} = (k_{2y}\omega_1 - k_{1y}\omega_2, k_{1x}\omega_2 - k_{2x}\omega_1, 0) / (k_{1x}k_{2y} - k_{2x}k_{1y})
\]
with squared magnitude \( V^2 = 2\sigma^2 k_1 k_2 [k_{1x} k_{2x} - (k_{1x} k_{2x} + k_{1y} k_{2y})] / (k_{1x}k_{2y} - k_{2x}k_{1y})^2 \). If the flux vector were to have a component perpendicular to the phase velocity the ray would dissipate. So the vector product components must vanish or, for the Eqs. (26) and (32) flux and velocity,
\[
(k_{2y}\omega_1 - k_{1y}\omega_2)\left( \frac{k_{2x}k_{2y}}{k_2} - \frac{k_{1y}^2}{k_1} \right) - (k_{1x}\omega_2 - k_{2x}\omega_1)\left( -\frac{k_{1x}k_{1y}}{k_1} + \frac{k_{2x}^2}{k_2} \right) = 0.
\]
This reduces to
\[
(k_{1x} + k_{2x}) [k_{1x} k_{2y} - (k_{1x} k_{2x} + k_{1y} k_{2y})] = 0
\]
which gives two alternative conditions to be satisfied by the \( k_1 \) and \( k_2 \) components for nontrivial flux direction alignment with the phase velocity direction. The phase \( \phi \) is a parameter that gives simple harmonic polarizations. But \( \phi \) is not a wave phase factor, because it is confined to the line defined by Eqs. (31). In the limiting case for which \( k_1 = k_2 = k/\sqrt{2} = (k_x, k_y, 0) / \sqrt{2} \), this ray-like character can be taken to persist even though this limit also allows the conventional plane wave phase characterization for \( \phi \). Failure to recognize this distinction as a real, physical possibility has prevented these light-like rays from being extracted from electromagnetic fields to help describe ray-like behavior in geometrical optics and photography [23, 24].

Although the case for \( k_1 \neq k_2 \) would appear to have physical relevance in describing light-like waves, the more general case in which \( \phi_1 \neq \phi_2 \) may provide a particle description for light. In this case, the phase point \((\phi_1, \phi_2)\) moves on the parametric path defined by equations like those in Eq. (31) with \( \phi \) replaced by \( \phi_1 \) or \( \phi_2 \) where appropriate. Furthermore, the flux now has the Eq. (25) form. To prevent ray dissipation by flux components normal to the phase path, restraints must again be applied to the components for \( k_1 \) and \( k_2 \). But these relations will depend, in general, on the \( \phi_1 \) and \( \phi_2 \) values. This therefore means that the flux is carried by phase points. These localized energy bearers need to be marshaled into coordinated groups by imposing some \( \phi_1 \cdot \phi_2 \) relation, such as \( \phi_1 = \phi_2 = \phi \), to get classical fields.

The preceding component phase independence discussion simplicity can be expected to be masked in real physical systems. For Hertz’s dipole field the electric field exhibits radial, transverse wave emergence in the far field. At intermediate distances the waves have superluminal radial speed that approaches light speed in the far field and the waves change from longitudinal to transverse as the radial direction changes from dipole length to dipole equator alignment. Hertz confirmed these properties in the equatorial plane by observing interference between free air and straight wire waves [15, pp. 150-5].

When phase point motion and wave flux are not aligned a flux component normal to the phase point motion direction would force ray dissipation. This is analogous to saying that all
light rays are electromagnetic, but not all electromagnetic waves are light rays. This analogy is supported by synchrotron light emitted as rays by high speed electrons in a circular orbit. These rays are attributed to the radial acceleration not the periodic linear acceleration required to maintain the electron energy, but they have not been identified in the electric field [25–27]. However, they are observed to be electrically polarized in the electron orbit plane [28]. These findings are expected if the rays are phase directed polarization waves in the orbit plane. Although component phase independence is mathematically compelling, its demonstrated physical nonexistence would support treating polarizations as point functions with closely bound potentials as described in Appendix B above. Even so, component phase independence is a classical field hidden variable whose consequence is unintuitive. Whether electromagnetic energy transport localization by phase independence is consistent with quantum statistics will have to be examined separately. However, the concept provides an unexplored means to help understand such light generation problems as anisotropic emission from excited atom charge distribution transitions for laser efficiency improvement.