Bound to bounce: a coupled scalar-tachyon model for a smooth cyclic universe

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Abstract

We introduce a string-inspired model for a cyclic universe, utilizing the tachyon-scalar coupling as well as contribution from curvature in a closed universe. The universe undergoes a locked inflation stage after each bounce. Cycles of inflation, decelerating expansion, followed by contraction are made possible because of the negative contribution to effective energy density by the curvature term. No ghosts are ever generated at any point in the entire evolution of the universe. The minimum size of the universe is nonzero for generic initial values of the scalar field. The Null, Weak, and Dominant Energy Conditions are not violated at the bounce points, contrary to many bounce models previously proposed. And the Strong Energy Condition is satisfied in periods with tachyon matter domination.

KEYWORDS: String cosmology, Tachyon inflation, Cosmic Singularity
1 Introduction and Summary

The idea of modelling the exponential expansion of the universe using simple scalar fields has led to remarkable progress in the study of cosmology [1], crystalizing the earlier attempts [2]–notably by Zeldovish and by Starobinsky–and paving the way to later developments [3]. In particular, the mechanism for generating matter from quantum fluctuations of the scalar field [4] successfully endows the inflation scenario with a casual mechanism for generating matter density fluctuations that agrees spectacularly well with the current...
array of observations [5][6][7]. This success has been crowned the “inflation paradigm” [8]. Despite the success story, improved theoretical understanding, on the other hand, points to many shortcomings of the paradigm. The most obvious is the lack of a quantum theory of gravity in modelling physics of the early universe. This is in turn reflected by the fact that the effective theory using scalar field necessarily breaks down as one approaches the Planck scale, $M_p$, and cosmic singularities are inevitable [9]. And trans-Plankian effects will eventually show up in the late time physics [10]. See, however, [11] for a different school of thought.

In an attempt to address the Big Bang singularity, various alternative models are proposed. These models can be loosely divided into four categories. The first consists of models that implement T-duality to circumvent the Big-Bang singularity, such as Brandenberger-Vafa scenario [12], string/brane gas bouncing universe models [13] and pre-big-bang models [14]. Models with modifications to Einstein’s theory of General Relativity (GR) to make gravity asymptotically free at very high energy, the like of a bouncing universe using $f(R)$ gravity [15] belong to the second category. The third is comprised of D-brane collisions in extra dimension as the interpretation of the Big-bang, building on the idea of [16]. Examples are Ekpyrotic universe [17] and cyclic Ekpyrotic scenarios [18]. The fourth is standard GR based bouncing models in which the dynamic behavior of each component’s equation of state is applied to the bounces, such as the bouncing universe with quintom matter [19]. Models in this category received a lot of attention as one can actually write down a Lagrangian and solve for the dynamic equations, even though many of these attempts encountered ghosts at bounce points. The spectrum of the density perturbation has been studied [20]. There are also quite a lot of cosmological applications of brane-world theory [21] and tachyonic fields [22]. For a summary of this active area of research the readers are referred to a few nice reviews [23] and a new book on string cosmology [24].

In this paper, we introduce a model with competition between tachyon-scalar coupling and curvature. This closed universe takes sufficiently soft bounce $\ddot{a}_* < a_*^{-1}$ and the Null Energy Condition, Dominant Energy Condition and Weak Energy Condition will not be violated. Such properties are consistent with the results of a model-independent analysis in [25]. By studying the coupling between the open string tachyon and a scalar field, we find a locked inflation stage. No ghost is ever generated because we use positive curvature to realize bounces.

According to our model the evolution of the universe consists of an initial phase of inflation–locked inflation–due to the vacuum energy of the tachyon field. After this initial inflation phase, the scalar field is strongly redshifted. The tachyon field starts rolling down from the top of its potential, marking the beginning of a phase of rolling tachyon inflation. As the rolling tachyon inflation finishes the tachyon field condensates rapidly and behaves like normal matter, a phase in which the equation of state is zero. With a positive spatial curvature, the universe with tachyon matter domination undergoes decelerated expansion until it reaches a local maximum in radius, after which a collapse ensues. During the
contracting phase, the scalar field is strongly blue-shifted to large values. As the scalar field and tachyon field increases in amplitudes so is the strength of coupling between them, pulling the tachyon back to the top of its potential, in the same of chaotic inflation. This process can be understood as the reversal of the tachyon condensation. Thus the total vacuum energy of the tachyon field and the scalar field dominates again and turns the universe from contraction to exponential expansion. Our universe is a result of a chain of the bouncing and contraction, perhaps ad infinitum, processes.

In Section 2, we introduce the components of the model building – a) the scalar-tachyon coupling, b) two possible ways to exit the locked inflation, and c) the curvature bounces and turnarounds. Universe evolution is studied in detail in Section 3. We then analyze the moduli space and possible e-folds in Section 4. Section 5 contains the discussion of unresolved issues, e.g. parametric resonance. In Section 6 we conclude this paper and provide prospective investigations one may pursue.

2 The Brick and Mortar of Model Building

2.1 Preliminary for the open string tachyon in string theory

Before we expound our model, we are briefly introducing the open string tachyon condensation and the decay of the unstable non-BPS D-brane structure in Type IIB string theory [26]. Readers who are familiar with open string tachyon may skip to the section 2.2 directly, where the Scalar-Tachyon Coupling Model is built.

A tachyon was defined, in the past, as a particle that travelled faster than light, which implied the mass-square of tachyon was negative. Such “faster-than-light” feature rendered tachyon utterly useless until quantum field theory provides a much better understanding of the role of tachyon. In quantum field theory, the mass-square of a particle-like state is defined as $m^2 = \frac{d^2V(\psi)}{d\psi^2} |_{\psi=0}$, where $\psi$ is a scalar field, $V(\psi)$ is its potential and has an extremum at the origin. It is clear that for $V''(0) < 0$ the particle of such state has a negative mass-square, which is nothing but a tachyon mentioned above. In this context, the existence of tachyon has a reasonable physical interpretation: 1) For $V''(0) > 0$, it describes a particle with positive mass-square. And the extremum of the potential $V(\psi)$ at the origin is a minimum. With small displacement of $\psi$, $\psi$ oscillates around the origin. The system is stable. 2) For $V''(0) < 0$, it describes a tachyon, and the extremum of the potential $V(\psi)$ at the origin is a maximum. Therefore, with small perturbation, the $\psi$ will roll down from the origin and grow rapidly in time. The existence of a tachyon, therefore, signals the emergence of an instability of the quantum system.

One may argue that, for the case of quantum field theories, we can simply expand the potential around a new point in the field space where it has a minimum, then it will give a particle with positive mass-square and remove the tachyon from this system. Such
argument is quite reasonable and makes us think deeper. Let us recall the knowledge of Standard Model \cite{27}. At the moment electroweak symmetry $SU(2) \times U(1)$ breaking, the Higgs field is becoming tachyonic and rolling from origin of its field space, which is the minimum of Higgs potential before the electroweak symmetry breaking, to the true vacuum after the electroweak symmetry breaking. This observation unveils a encouraging fact that the emergence of tachyon does not necessarily imply a flaw in this theory. In contrary the study of the tachyonic behaviors of the field can offer us a better understanding of the dynamics of phase transition associated with symmetry breaking.

The story of the open string tachyon is similar to what we discussed above. In string theory, the spectrum of single particle states is obtained from quantizing the oscillatory modes of a single string and each state is labeled by quantum numbers such as energy $E$, momentum $p$, and winding numbers. Occasionally, one can find states with such relation $m^2 = E^2 - p^2 < 0$, indicating the existence of particles with negative mass-square, the so-called tachyon, in the spectrum of the theory. Such tachyonic state was first found in the 26 dimensional bosonic string theory \cite{28}. At that time, the presence of tachyon was considered as an evidence of self-inconsistency. Fortunately, by introducing the fermionic sectors, the 10 dimensional supersymmetric string theory (superstring theory) was then shown to be tachyon-free \cite{29}. This fact implied that the open string tachyon only appears in non-supersymmetric cases. Further investigations, however, revealed non-supersymmetric structures in various superstring theories, for example, non-BPS Dp-branes for odd/even p in type IIA/IIB string theory respectively, as well as the BPS Dp–anti-Dp brane pair for even/odd p in type IIA/IIB respectively \cite{30}. It turned out, after close scrutiny, these non-supersymmetric structures play a crucial role in understanding annihilation and production of D-branes (i.e. the descent relations among different dimensional D-branes) \cite{31} and in the context of cosmology \cite{32, 33}.

Physically the endpoints of open strings are attached to a soliton-like configuration known as D-brane or anti-D-brane \cite{34}. These open string states represent quantum excitations of the D-brane system. Tachyon in this case represents the instability of D-branes and/or anti-D-branes structures. Moreover, the presence of open string tachyon in the non-BPS D-branes and D-branes–anti-D-branes pair made researchers reconsider the role of tachyon in the superstrings. Using purely string theory techniques, Ashoke Sen and other researchers \cite{26} found that the dynamics of open string tachyon indeed depicted the decay process of these unstable non-BPS D-branes and BPS D–anti-D-branes pair. This discovery revived the open string tachyon in superstring theories. Similar to electroweak symmetry breaking in which the Higgs field becomes tachyonic, open string tachyons appear when supersymmetry is being broken. The open string tachyon condensation, henceforth, describes the phase transition after the breaking of supersymmetry. Building upon this modern knowledge of open string tachyons in various superstring theories, cosmological implications of tachyon condensation have subsequently been explored in a large number of works \cite{35}.
2.2 Constructing the Scalar-Tachyon Coupling Model

In this paper, we specialize in the type IIB string theory with a coincident BPS D3–anti-D3-branes pair. The effective action of open string tachyon of a coincident BPS D3–anti-D3-branes pair in type IIB string theory takes this form [32, 36]:

\[ S = \int d^4x V(T) \sqrt{1 + \partial_\mu T \partial^\mu T / M_s^4}, \quad V(T) = \frac{V_0}{\cosh T / \sqrt{2} M_s} \]  

(1)

where \( T \) is the tachyon field, \( V_0 \) is the tension of D3–anti-D3-branes pair and the metric is \((-++,+++)\). We also assume the tachyon field is spatially homogenous. The tachyon potential \( V(T) \) has a maximum at \( T = 0 \) and two minima at \( T \to \pm \infty \). It is easy to see that when \( T = 0 \) and \( \dot{T} = 0 \), the tachyon field \( T \) stays on the top of the potential and the system stays in an unstable false vacuum, which represents the unstable D3-anti-D3-branes pair has not started to annihilate. With very small perturbation, the tachyon will roll down from the top of its potential and the \( T \) and \( \dot{T} \) will grow exponentially, and the annihilation of the D3-anti-D3 branes pair begins. Since the velocity of tachyon field \( \dot{T} \) is suppressed by the factor \( \sqrt{1 - \frac{1}{M_s^2} \dot{T}^2} \), \( \dot{T} \) will approach \( M_s^2 \). When \( T \) is very large and \( \dot{T} \) approaches \( M_s^2 \), the D3-anti-D3 branes pair has annihilated. This process can be seen clearly from the equation of state of tachyon field,

\[ \omega_T \equiv -1 - \partial_\mu T \partial^\mu T / M_s^4 \]  

(2)

When the tachyon field stays on the top of its potential, the D3-anti-D3 branes pair is intact and behaves like cosmological constant. The energy density of such cosmological constant is the tension of D3-anti-D3 branes. As the tachyon field is rolling down from the top of its potential, \( T \) has large value and \( \dot{T} \) is very close to \( M_s^2 \). Thus \( \omega_T \) approaches 0, which means the D3-anti-D3 branes pair has annihilated and their energy released to a pressureless fluid, called the “tachyon matter.” For more details of this original process of tachyon condensation, please refer to [32, 35, 37].

On the other hand, in the D-brane inflation scenario by Dvali and Tye [16], the D3 and anti-D3 branes are separated by a distance \( y \) at the beginning, and the distance between them is much larger than \( M_s^{-1} \). There is an attractive force, between the D3-brane and anti-D3-brane due to the exchange of closed string between these two branes. This attractive potential takes this form [24]:

\[ V_\phi = \frac{1}{2} m^2 \phi^2 + V_0 - \frac{V_0^2}{4\pi^2 v\phi^4} \]  

(3)

where \( \phi \equiv \sqrt{V_0} y \).

Due to this attractive potential, the universe inflates when these two branes approach each other. In general, D-brane inflation ends with the total annihilation of the two branes, resulting in the condensation of the tachyon. Models of this kind are derived from
string cosmology, and have been widely studied (See [23] for extensive reviews.) since the pioneer work of Dvali and Tye [16, 24, 38, 39], Sen [32], and Gibbons [33]. In these models the annihilation process of these two branes serves as the exit of inflation.

We are however motivated to consider the combined effect of tachyon rolling and inflation. The key point for such unification is that the model should have such properties:

1. When two branes are separated at a large distance, the tachyon field should be very small. Because when the distance between these two branes are large, they are approximately supersymmetric and do not annihilate in this process;

2. Once these two branes are close enough, i.e. the distance between them is less than string length $M_s^{-1}$, and this holds for sufficiently long time, the annihilation should take place.

3. There are also possibilities that the tachyon can oscillate up the potential well due to quantum fluctuations. We model that by introduce a coupling with an auxiliary scalar, $\phi$.

The novelty of this attempt is that the coupling term brings about two notable phases of cosmic evolution, the locked inflation phase which make the large e-folding number for D-brane inflation period possible, and the bounce phase which let universe bounce back before reaching the Big-bang singularity. The initial value of $\phi$ is taken to be large, and the coupling term between $\phi$ and $T$ stabilizes the system in the false vacuum at $T = 0$ and $\dot{T} = 0$. In the subsequent inflation phase $\phi$ is strongly red-shifted until the coupling term fails to stabilize the system in the false vacuum. Tachyon rolling ensues and the D3–anti-D3 brane pair annihilates. On the other hand, in the contraction phase of the universe, $\phi$ field is strongly blue-shifted to a large value. The coupling term $\lambda \phi^2 T^2$ then pulls the tachyon towards the false vacuum $T = 0$ and $\dot{T} = 0$. In other words, D3–anti-D3-branes are pair produced by quantum fluctuations.

Based on the analysis above, we take the coupled scalar-tachyon action as

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16G} R - V(T) \sqrt{-\omega_T} - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \lambda \phi^2 T^2 \right]$$

(4)

where $\frac{1}{16G} R$ is Hilbert-Einstein term. The second and third terms on the right hand of Eq. 3 are absent in the scalar-tachyon coupling action due to two reasons:
1) The second term is the tension of D3–anti-D3 branes pair and in this scalar-tachyon coupling action it has been replaced by the tachyon’s vacuum energy $V_0$;
2) The third term, which receives the contribution from the massive mode exchange of closed string between these two branes, is suppressed for large $\phi$. So for large $\phi$, this term can be omitted. And for the small $\phi$, the interaction and annihilation of D3-anti-D3-branes pair are described by the tachyon condensation. Therefore, the effect of this term is already captured by our scalar-tachyon coupling action.
From the scalar-tachyon coupling model, we can obtain the equations of motion for the scale factor \( a \), tachyon field \( T \) and scalar field \( \phi \) (in a flat space-time):

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3M_p^2} \left( \frac{V(T)}{\sqrt{-\omega_T}} + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m_T^2 \phi^2 + \lambda \phi^2 T^2 \right) \tag{5}
\]

\[
\frac{\ddot{a}}{a} = \frac{8\pi}{3M_p^2} \left[ \frac{V(T)}{\sqrt{-\omega_T}} \left( 1 - \frac{3\dot{T}^2}{2M_s^4} \right) + \frac{1}{2} m_T^2 \phi^2 - \dot{\phi}^2 + \lambda \phi^2 T^2 \right] \tag{6}
\]

\[
\frac{1}{M_s^4} \ddot{T} + (1 - \frac{T^2}{M_s^4}) \left( 3H \frac{\dot{T}}{M_s^4} + \frac{V'(T)}{V(T)} + 2\lambda \phi^2 T \frac{\sqrt{1 - \dot{T}^2}}{V(T)} \right) = 0 \tag{7}
\]

and

\[
\ddot{\phi} + 3H \dot{\phi} + (m^2 + 2\lambda T^2) \phi = 0 \tag{8}
\]

A couple of technical remarks are warranted here: We model the D3-brane and anti-D3 brane to be able to pass each other when they collide, with a certain probability for annihilation, thus \( \phi \) can take either positive value or negative value in the process. Meanwhile, after annihilation, the D-brane and anti-D-brane are taken to be infinitely thin but still existing. Thus the physical degrees of freedom of \( \phi \) do not disappear in the classical sense, and the effective action of \( \phi \) is still valid after the tachyon condensation.

### 2.3 The locked inflation

During locked inflation, the two branes oscillate with a large amplitude, preventing the production of tachyon, i.e. \( \lambda \langle \phi^2 \rangle = \lambda V_0 y^2 \gg V_0/M_s^2 \) and \( \langle T^2 \rangle \ll M_s^2 \). The universe is dominated by \( V_0 \), the vacuum energy between the branes. Here \( \langle \rangle \) means time averaging to remove oscillatory features.

Since the tachyon field \( T \) is very small and scalar field \( \phi \) is relatively large, the scalar-tachyon coupling \( \lambda T^2 \phi^2 \) gives the tachyon field a positive effective mass square, \( m_T^2 \equiv 2\lambda \langle \phi^2 \rangle - \frac{V_0}{2M_s^2} \). So the tachyon field is kept from rolling down at \( \langle T^2 \rangle \ll M_s^2 \). The energy density of this system, in the metastable vacuum is:

\[
\rho_{fv} = V_0 + \frac{1}{2} m_T^2 \phi^2 + \frac{1}{2} \phi^2 \approx V_0 \tag{9}
\]

Therefore, Friedmann equation is simplified to be

\[
H^2 = \frac{8\pi V_0}{3M_p^2} \tag{10}
\]

and the equations of motion for the tachyon and scalar \( \phi \) become

\[
\frac{1}{M_s^4} \ddot{T} + \left( -\frac{1}{2M_s^4} + \frac{2\lambda \phi^2}{V_0} \right) T = 0 \tag{11}
\]
and
\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0. \] (12)
accordingly.

The universe expands exponentially from Eq. 10 driven by vacuum energy:
\[ a \propto e^{\sqrt{8\pi V_0/3M_p^2} t}. \] (13)
We will call this period \textit{locked inflation} as it has an approximately constant Hubble rate. During the locked inflation period, \( \phi \) keeps oscillating and gets redshifted with
\[ \phi \propto a^{-3/2} e^{i\sqrt{m^2 - \frac{9}{4} H^2 t}}. \] (14)
From the oscillation term we notice \( \phi \) may pass zero many times during oscillation. The physical interpretation is that a D3-brane and an anti-D3-brane can meet many times before total annihilation. This is because the maximum distance between the branes (i.e. the amplitude of oscillation) is large. When they meet, this leads to a large relative velocity between the branes so time is insufficient for annihilation. One the other hand, \( a^{-3/2} \) tells us that the scalar field get strongly redshifted. In string theory, it means the maximum distance between the branes are decreasing. When the maximum distance is small enough, the two branes will annihilate, putting an end to locked inflation.

In this way, the string-inspired locked inflation proposed here is a more realistic proposal than, for example, the New Old Inflation model of Dvali and Kachru[40]. We will also go further by investigating two different ending conditions of locked inflation.

The previously proposed ending of locked inflation is caused by the change of the sign of \( m_T^2 \), the effective mass square of tachyon, marking the beginning of the tachyon’s rolling down its potential hill. Given \( m_T^2 = 2\lambda \langle \phi^2 \rangle - V_0/2M_s^2 \), the critical \( \langle \phi^2 \rangle \) at the ending of locked inflation, \( \langle \phi_c^2 \rangle \), is derived from
\[ m_T^2 = 0 \Rightarrow \langle \phi^2 \rangle = \langle \phi_c^2 \rangle \equiv V_0/4\lambda M_s^2. \] (15)
Therefore when \( \phi^2 \) reaches an expectation value of \( \phi_c^2 \), it can no longer hold the tachyon at the top of its potential hill. And \( T \) starts to roll down, ending the locked inflation.

There exists another way to exit locked inflation. When \( \phi \) oscillates much slower than tachyon, in which case as \( \phi \) passes zero, tachyon will have sufficient chance/time to roll down. Therefore even if the effective mass square of \( \phi \) is lower than \( V_0/2M_s^2 \), locked inflation can still hold if the amplitude of \( \phi \) is large enough: the larger amplitude of \( \phi \), the shorter the time \( \phi \) it stays in the region \( \langle \phi^2 \rangle \leq \langle \phi_c^2 \rangle \). We’ll call this ending condition “mass ending” and denote it by
\[ m_\phi^2 \langle \phi^2 \rangle = \frac{V_0}{2M_s^2} \langle \phi_c^2 \rangle, \] (16)
where \( m_\phi \) is the effective mass of \( \phi \).

These two conditions are connected at \( m_\phi^2 = V_0/2M_s^2 \). When \( m_\phi^2 \) is larger, then the normal ending discussed earlier should be adopted, and a smaller value of \( m_\phi^2 \) calls for "mass ending." We can see from the mass ending condition that a lower \( m_\phi^2 \) makes \( \langle \phi^2 \rangle \) larger at the end of locked inflation. This implies an earlier ending of the locked inflation than previously thought of. Although the two endings have different conditions, they have to meet the same criterion – the sufficiency of time for brane annihilation.

### 2.4 The Curvature-led Bounce and Turnaround

We use the single component curvature as the seed of both bounce and turnaround. With the metric of closed universe

\[
ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right),
\]

(17)

the Friedmann equation becomes

\[
H^2 = \frac{8\pi}{3M_p^2} \rho - \frac{k}{a^2}
\]

(18)

\[
= \frac{8\pi}{3M_p^2} \left( \rho + \rho_k \right),
\]

where

\[
\rho_k \equiv -\frac{3M_p^2 k}{8\pi} \propto a^{-2}
\]

(19)

is the equivalent energy density due to curvature (curvature energy). \( \rho \) is the total energy density summed over all species present. The equivalent pressure of curvature (curvature pressure) is

\[
P_k \equiv \frac{M_p^2 k}{8\pi a^2} = -\frac{1}{3} \rho_k.
\]

(20)

Since curvature behaves like an energy density component with an equation of state \( \omega_k = -1/3 \), we are lead to consider the positive curvature as a negative component of total energy density with \( \rho_k \propto a^{-2} \).

We can then introduce a bounce point at the beginning of each expansion and a turnaround point at the end. These points are identified as \( \rho_k = \rho_{\text{others}} \) where \( \rho_{\text{others}} \) (i.e. \( \rho \) in Eq. 18) is the total contributions to energy density by all other components. At the locked inflation stage, the "curvature energy" undergoes a process of \( \rho_k \propto a^{-2} \), while other components (i.e. fields) have almost constant total energy density, because of the domination by the vacuum energy of \( T \). We can then trace back through time from locked inflation back to the bounce where \( a(t) \) reached its minimum at \( \rho_{\text{other}} = \rho_k \). Similarly, at a later stage of matter domination, \( \rho_{\text{matter}} \propto a^{-3} \) declines faster than curvature energy, so a turnaround is expected when the an equilibrium is attained at \( \rho_{\text{matter}} \sim \rho_k \).
3 Universe Evolution

When D-branes and anti-D-branes approach each other, tachyons are developed signaling the existence of an instability of the system. We can furthermore add an interaction term between the tachyon and the scalar field, \( \phi \), to model possible symmetry breaking effects involved in the process. So the total Lagrangian is

\[
\mathcal{L} = -V(T)\sqrt{-\omega_T} - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \lambda \phi^2 T^2.
\]

(21)

whereas the geometry of the D-brane worldvolume takes the form Eq. [17] By construction, \( m \) is the mass of \( \phi \) and \( \lambda \) is the strength of the interaction.

\[ V(T) \]

\[ V_0 \]

\[ T_1 \]

\[ T_0 \]

\[ T_2 \]

\[ T_3 \]

Figure 1: Timestamps of universe evolution presented in the potential of \( T \). The dashed curve is the potential during locked inflation, and the solid one is after tachyon is released. The universe starts minimum at \( T_0 \). Locked inflation ends at \( T_1 \). Rolling inflation ends at \( T_2 \). Tachyon first reaches its vev at \( T_3 \).

Observations reveal a homogenous university today, we, therefore, assume homogeneity for all classical field configurations in the following calculation, unless otherwise noted. The Friedmann equation in a curved space-time becomes

\[
H^2 = -\frac{k}{a^2} + \frac{8\pi}{3M_p^2} \left( \frac{V(T)}{\sqrt{-\omega_T}} + \frac{1}{2} m^2 + \lambda T^2 \right) \phi^2 + \frac{1}{2} \dot{\phi}^2,
\]

(22)

and the equations of motion for the tachyon and the scalar field are

\[
\frac{\ddot{T}}{M_s^4} - \omega_T \left( \frac{3H\dot{T}}{M_s^4} + \frac{V'(T) + 2\lambda \phi^2 T \sqrt{-\omega_T}}{V(T)} \right) = 0,
\]

(23)
\[ \ddot{\phi} + 3H\dot{\phi} + (m^2 + 2\lambda T^2)\phi = 0. \] (24)

We will use these equations to study the evolution of the universe, starting with the minimum \( a(t) = a_0(t) \).

Fig. 1 shows the expected stages in the figure of the potentials of \( T \), with timestamps \((0, 1, \ldots)\) in subscripts to indicate the time at which variables stand. We start the universe from its minimum, timestamped as 0. Tachyon is initially locked at zero by the oscillating \( \phi \). The bounce stage is short. The curvature term is inflated away, and therefore the universe goes on to a locked inflation stage during which the amplitude of \( \phi \) is also red-shifted. When it drops to too low, it can no longer provide enough effective mass to \( T \). When it happens, \( T \) starts to roll down. The bounce and locked inflation stages are indicated by \( 0 \rightarrow 1 \) in Fig. 1 with Point 1 being the end of locked inflation.

At the beginning of tachyon’s rolling, when \( \dot{T}^2 \ll M^4_s \), there is another short period of inflation, rolling inflation, denoted by \( 1 \rightarrow 2 \) in Fig. 1. During rolling inflation, \( \langle \dot{\phi}^2 \rangle \) is damped and \( T \) picks up a vacuum expectation value.

Toward the end of the rolling inflation, the velocity of \( T \) becomes large. Inflation ends because the tachyon is matter-like. As shown in Fig. 1, tachyon is accelerately to catch up with its vev at this stage, indicated by \( 2 \rightarrow 3 \).

Once \( T \) reaches its vev, it will oscillate and relax. Its kinetic energy is lost through expansion. It then follows that the vev of tachyon increases slowly before contraction happens. This happens after Point \( T_3 \) in Fig. 1.

Matter-like components have larger damping rates than the curvature term, so curvature gradually becomes important and eventually balances all positive energy contributions in the Friedmann equation. The universe turns around at this point to enter a phase of contraction. During contraction, \( \phi \) is boosted and it drags \( T \) back to zero. When the universe is again dominated by the vacuum energy of the tachyon, the positive energy behaves like a cosmological constant and stays constant for further contraction whereas the curvature energy increases at a rate of \( a^{-2} \). A bounce takes place when the curvature energy and vacuum energy of the tachyon reaches exact equality. Even though the bouncing process is seemingly like the reverse of an expansion, the universe enters another cycle of expansion.

### 3.1 Initial Minimum and Locked Inflation

We start following the universe evolution from the minimum of \( a(t) \) by choosing the initial condition. Tachyon is initially locked at its vev zero by oscillating \( \phi \). Curvature energy and vacuum energy of \( T \) co-dominate and cancel each other at minimum \( a \), but their equation of state difference still leads to an expansion. The early co-dominant expansion stage is short (with a time scale comparable to \( \sqrt{\frac{3M^2_p}{8\pi V_0}} \)) and negligible compared with
Table 1: Labels of the six cases for separate consideration

| Case | Normal Ending | Mass Ending |
|------|---------------|-------------|
| a    | 2\(\lambda \langle T^2_2 \rangle > m^2 > 2\lambda \langle T^2_0 \rangle\) | d |
| b    | \(m^2 < 2\lambda \langle T^2_1 \rangle\) | e |
| c    | 2\(\lambda \langle T^2_0 \rangle > m^2 > 2\lambda \langle T^2_1 \rangle\) | f |

locked inflation. Because the amplitudes of \(\phi\) and \(T\) depends on \(a(t)\), not \(H\), they are not affected by this stage either. However the co-dominant stage plays a role in red-shifting curvature away so we can proceed without its complexity and can apply flat metric in subsequent calculations.

The universe then goes on to the locked inflation, as indicated 0 \(\rightarrow\) 1 in Fig. [1] During locked inflation, the interaction term produces a large effective mass for \(T\), so tachyon is locked at zero. The scalar field \(\phi\) is fast rolling about its vev zero, with its amplitude decreasing due to the inflation. At this stage, the universe is dominated by the vacuum energy of \(T\), so the Hubble rate is constant as long as \(T\) is locked.

Although the expectation value of tachyon is zero, \(\langle T \rangle = 0\), \(T\) still has a nonzero fluctuation \(\langle T^2 \rangle \neq 0\). The back-reaction of it may be large by generating an additional effective mass square \(2\lambda \langle T^2 \rangle\) for \(\phi\). In cases where the dominant part of effective mass of \(\phi\) differs, the emergence of those two fields is also different. Therefore, separate consideration is needed for those cases, specified by where \(m^2/\lambda\) is inserted into the relation of \(\langle T^2_1 \rangle < \langle T^2_0 \rangle \sim M_s^2\). For the same reason, two ending conditions of locked inflation are also considered separately. In all, there are \(2 \times 3 = 6\) cases as labelled \(a \rightarrow f\) in Table 1.

First we consider case \(a\), in which \(2\lambda \langle T^2_2 \rangle > m^2 > 2\lambda \langle T^2_0 \rangle\) with normal ending. In this case, \(m^2 > 2\lambda \langle T^2 \rangle\) holds throughout the entire period of locked inflation. The constant mass gives \(\langle \phi^2 \rangle \propto a^{-3}\). Tachyon however has a varying mass \(\lambda \langle \phi^2 \rangle \propto a^{-3}\), so the solution of its equation of motion is \(\langle T^2 \rangle \propto a^{-3/2}\). Such relations implies that the normal ending of locked inflation occurs at:

\[
\langle \phi^2_1 \rangle = e^{-3N_L} \langle \phi^2_0 \rangle = \langle \phi^2_c \rangle, \tag{25}
\]

where \(\phi_c\) is defined in Eq. [15] So the expected e-folds of locked inflation for case \(a\) is

\[
N_L^{(a)} = \frac{1}{3} \ln \frac{4\lambda M_s^2 \langle \phi^2_0 \rangle}{V_0}. \tag{26}
\]

Also,

\[
\langle (T_1^{(a)})^2 \rangle = \langle (T_0^2) \rangle \sqrt{\frac{V_0}{4\lambda M_s^2 \langle \phi^2_0 \rangle}}. \tag{27}
\]

Similarly, we can analyze other cases of interest. For case \(b\), because \(2\lambda \langle T^2 \rangle\) dominates \(\phi\)'s mass, there are \(\langle \phi^2 \rangle \propto a^{-2}\) and also \(\langle T^2 \rangle \propto a^{-2}\). Case \(c\) has a transition point at \(2\lambda \langle T^2 \rangle = m^2\), before which we apply the analysis for case \(b\) and after that the analysis for
Table 2: Locked inflation of six separate cases

|   | \(N_L\) | \(\langle T_1^2 \rangle\) | \(\langle \phi_1^2 \rangle\) |
|---|---------|----------------------------|----------------------------|
| a | \(\frac{1}{3} \ln \frac{4\lambda M_s^2 \langle \phi_0^2 \rangle}{V_0}\) | \(\langle T_0^2 \rangle \sqrt{\frac{V_0}{4\lambda M_s^2 \langle \phi_0^2 \rangle}}\) | \(\frac{V_0}{4\lambda M_s^2}\) |
| b | \(\frac{1}{2} \ln \frac{4\lambda M_s^2 \langle \phi_0^2 \rangle}{V_0}\) | \(\frac{V_0}{4\lambda M_s^2 \langle \phi_0^2 \rangle}\) | \(\frac{V_0}{4\lambda M_s^2}\) |
| c | \(\frac{1}{6} \ln \frac{32\lambda^3 M_s^4 \langle \phi_0^2 \rangle^2 \langle T_0^2 \rangle}{m^2 V_0^2}\) | \(\frac{m^2 V_0 \langle T_0^2 \rangle}{8\lambda^2 M_s^2 \langle \phi_0^2 \rangle}\) | \(\frac{V_0}{4\lambda M_s^2}\) |
| d | \(\frac{1}{3} \ln \frac{8\lambda m^2 M_s^4 \langle \phi_0^2 \rangle}{V_0^2}\) | \(\langle T_0^2 \rangle \sqrt{\frac{V_0^2}{8\lambda m^2 M_s^4 \langle \phi_0^2 \rangle}}\) | \(\frac{V_0^2}{8\lambda m^2 M_s^4}\) |
| e | \(\frac{1}{4} \ln \frac{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}{V_0^2}\) | \(\langle V_0^2 \langle T_0^2 \rangle \rangle \sqrt{\frac{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}}\) | \(\frac{V_0^2}{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}\) |
| f | \(\frac{1}{6} \ln \frac{128\lambda^3 m^2 M_s^8 \langle \phi_0^2 \rangle^2 \langle T_0^2 \rangle}{V_0^4}\) | \(\langle V_0^2 \langle T_0^2 \rangle \rangle \sqrt{\frac{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}{16\lambda^2 M_s^4 \langle \phi_0^2 \rangle \langle T_0^2 \rangle}}\) | \(\frac{V_0^2}{8\lambda m^2 M_s^4}\) |

Case a applies. The two sections are then connected at the transition point to obtain the complete solution. Cases d, e, f are more or less similar to a, b, c, except that the ending condition should be replaced by \(m_0^2 \langle \phi_1^2 \rangle = \frac{V_0}{2M_s^2} \langle \phi_0^2 \rangle\). So we get the six scenarios as listed in Table 2.

The six cases then correspond to different regions in the moduli space, dictated by the required conditions in the corresponding cases. For example, a requires \(2\lambda \langle T_0^2 \rangle > m^2 > 2\lambda \langle T_0^2 \rangle\) as in Table 1. All such restrictions of moduli space are collected, analyzed and presented in Section 4. We can temporarily ignore these restrictions when we follow the evolution of the universe.

### 3.2 Rolling Inflation

When \(\phi\) can no longer lock \(T\) at its origin, \(T\) will start to roll down to either side of the potential hill. At the beginning when \(T\) is small and thus \(\hat{T}^2 \ll M_s^4\), the vev of tachyon still dominates the universe. So this stage (indicated by \(1 \rightarrow 2\) in Fig. 1) is an additional epoch of inflation which is usually much shorter than locked inflation. Although most two-field models neglect this stage, we consider it here because it causes a delay between the motion of \(T\) and its vev. Such delay determines the length of following stages, and therefore should not be neglected in our analysis.

To good approximation we can set the mass squared of \(T\) to \(-M_s^2/2\), whereas the effective mass from interaction is neglected. Generically this approximation is valid only when the number of e-folds of rolling inflation \(N_R > 1\). It is also applicable to our case
even if \( N_R < 1 \). The rolling inflation stage is then short and itself can thus be neglected for e-folding counting. The equation of motion of \( T \) then simplifies to

\[
\ddot{T} + 3H\dot{T} - \frac{1}{2} M_s^2 T = 0,
\]

(28)

with a solution of

\[
T = T_1 e^{\left(\frac{1}{\sqrt{2}} M_s - \frac{3}{2} H\right) (t-t_1)}.
\]

(29)

So the e-folds of rolling inflation is

\[
N_R = H(t_2 - t_1) = \frac{1}{\sqrt{2} H M_s} \ln \frac{T_2}{T_1},
\]

(30)

where we approximately choose the end of rolling inflation to be at \( T_2 = \sqrt{2} M_s \) for simplicity of future calculations. From this equation we can see it is difficult to get a large \( N_R \) because \( H \ll M_s \).

Tachyon would later oscillate about its vev after rolling inflation, so keeping track of how the tachyon vev moves is necessary. We define \( T_V \), the vev of \( T \) as

\[
\left. \frac{\partial \rho}{\partial T} \right|_{T=T_V} = 0.
\]

(31)

and

\[
T_V e^{\frac{T_V}{\sqrt{2} M_s}} = \frac{V_0}{2\sqrt{2} \lambda M_s \langle \phi^2 \rangle \sqrt{1 - T^2/M_s^4}}.
\]

(32)

It is important to note the \( \dot{T} \) in the above equation is the real time velocity of tachyon, not the velocity of \( T_V \). At point 2, the equation becomes

\[
T_{V2} e^{\frac{T_{V2}}{\sqrt{2} M_s}} = \frac{V_0}{2\sqrt{2} \lambda M_s \langle \phi_2^2 \rangle \sqrt{1 - T_{V2}^2/M_s^4}},
\]

(33)

in which at \( T_2 = \sqrt{2} M_s \) there is \( 1 - T_{V2}^2/M_s^4 \sim < 1 \).

To get \( T_{V2} \), we still need to construct the relation between \( \langle \phi_2^2 \rangle \) and \( \langle \phi_1^2 \rangle \). The evolution of \( \phi \) however varies under two different situations – \( m^2 < 2 \lambda T^2 \) through rolling inflation, and a transition to \( m^2 > 2 \lambda T^2 \) occurs during rolling inflation. Here we don’t consider the case with \( m^2 > 2 \lambda T_2^2 \), because then the scalar field is too massive. For demonstration, such huge mass would not give any distinctive effect either.

First for cases \( b,e \), there is always \( m^2 < 2 \lambda T^2 \). Therefore

\[
\langle \phi_2^{(b,e)2} \rangle = \langle \phi_1^2 \rangle e^{-3N_R} \sqrt{\frac{\langle T_1^2 \rangle}{T_2^2}} = \langle \phi_1^2 \rangle e^{-\left(\frac{3}{2} + \frac{M_s^2}{2H^2}\right)N_R}.
\]

(34)
The second possibility is that there exists a transition to $2\lambda T^2 > m^2$ at $T^2 = m^2/2\lambda$, for the cases of $a, c, d, f$. Suppose the transition happens at $T_\alpha$, i.e. $m^2 = 2\lambda T^2_\alpha$, combining the above two processes gives

$$
\langle \phi_2^{(a,c,d,f)} \rangle^2 = \langle \phi_1^2 \rangle \frac{m e^{-3N_R}}{\sqrt{2\lambda T^2}}.
$$

(35)

We can see from these results the relationship is actually $\langle \phi^2 \rangle \propto a^{-3} m^{-1}_\phi$, and the physics in it is quite clear. $\langle \phi^2 \rangle$ is proportional to $a^{-3}$ because $\phi$ is a fast rolling field, so its amplitude is dampened by universe expansion by $a^{-3}$. It is then inversely proportional to $m_\phi$ because the kinetic energy is transferred to more potential energy as $m_\phi$ gets smaller.

We can also get this result from the energy aspect of view. The potential energy of $\phi$ is proportional to $m^2_\phi$, however $\phi$ is a harmonic oscillator, so potential energy is only half of its total energy when averaged with time, from which we can infer its total energy is proportional to $m_\phi$. Its total energy can be represented as $m^2_\phi \langle \phi^2 \rangle \propto m_\phi$, and we get $\langle \phi^2 \rangle \propto m^{-1}_\phi$.

### 3.3 Accelerating Tachyon

The period indicated by $2 \rightarrow 3$ is an accelerating process for tachyon with its velocity $\dot{T} \sim M_s^2$. When $T$ catches up with $T_V$, acceleration stops, signalling the end of this stage. The length of this stage hinges on the number of e-foldings from rolling period, for reasons explained above.

If we restrict the moduli space inside $m^2 < 2\lambda M_s^2$, there is then $\langle \phi^2 \rangle \propto a^{-3} T^{-1}$. To calculate the speed of $T_V$, we differentiate Eq. 32 w.r.t. time and get

$$
\left( \frac{1}{T_V} + \frac{1}{\sqrt{2} M_s} \right) \dot{T}_V = 3H \left( 1 - \frac{\dot{T}^2}{M_s^4} \right) + \frac{T}{\sqrt{2} M_s} \left( 1 + \frac{\sqrt{2} M_s}{T} - \frac{T}{T_V} e^{(T-V)/\sqrt{2} M_s} \right).
$$

(36)

Neglecting $3H \left( 1 - \dot{T}^2/M_s^4 \right)$, we have the solution of

$$
\frac{T_V}{T} e^{\frac{T-V}{\sqrt{2} M_s}} + \frac{T}{\sqrt{2} M_s} = C,
$$

(37)

where $C$ is a constant of integration.

Choosing point 2 as $T_2 = \sqrt{2} M_s$ and point 3 as $T_V = T_3$, by which we mean at point 3 the tachyon field reaches its $vev$ and starts to decelerate, we apply the solution Eq. 37 to the start and end points of this stage and arrive at

$$
T_3 = T_{V3} = T_V e^{\frac{T_{V3}^2}{\sqrt{2} M_s}} - 1.
$$

(38)

The universe is matter-like during $2 \rightarrow 3$. At the beginning, tachyon dominates with equation of state $\omega_T \approx 0$. Later when $\lambda T^2 \langle \phi^2 \rangle$ comes to dominate, $T$ is already large.
and its variation is negligible so $\lambda T^2\langle\phi^2\rangle \propto Ta^{-3} \propto a^{-3}$. Since both components are matter-like, we can also compute the e-folds of this stage

$$N = \frac{2}{3} \ln \frac{3H_2T_3}{2M_s^2}. \tag{39}$$

With a bit of calculation, we can see $N = 2N_R + \ldots$ where $\ldots$ indicate other contributions. Such dependence on $N_R$ is exactly as expected. The longer rolling inflation lasts, the larger delay is between $T$ and $T_V$, and thus the more e-folds are required in order for $T$ to catch up with $T_V$.

### 3.4 Relaxation and Reheating

When $T$ reaches its vev $T_V$ for the first time, it starts to oscillate and relax at its vev, gradually losing its kinetic energy due to the expansion. We will now follow these properties in detail.

Define $x \equiv 1 - \dot{T}^2/M_s^4$. The equation of motion of $T$ can then be transformed to

$$\frac{\dot{x}}{x} = 2\dot{T} \left( \frac{3H\dot{T}}{M_s^4} - \frac{1}{\sqrt{2}M_s} + \frac{4\lambda\langle\phi^2\rangle Te^{T/\sqrt{2}M_s}}{V_0} \right). \tag{40}$$

Here we neglect $3H\dot{T}/M_s^4$ term because it is much smaller than the other two. Adopting Eq. 32 at $T = T_V$ and $\langle\phi^2\rangle \propto a^{-3}T^{-1}$ to eliminate $\langle\phi^2\rangle$, we come to the following equation

$$\frac{dx}{2x} = \frac{dT}{\sqrt{2}M_s} \left( \frac{a^3}{a_r^3} e^{\frac{T-T_r}{\sqrt{2}M_s}} \sqrt{\frac{x}{x_r}} - 1 \right), \tag{41}$$

where subscript $r$ represents the time when the last vev of $T$ was reached, or when $T$ reaches its vev next time. The solution is

$$\frac{T - T_r}{\sqrt{2}M_s} = \ln \sqrt{\frac{x_r}{x}} - \ln \left( 1 - \frac{T - T_r}{\sqrt{2}M_s} \right) + 3N_r, \tag{42}$$

where $N_r \equiv \ln a/a_r$.

From this we can calculate the amplitude of $T$. Because $\dot{T}^2 \rightarrow M_s^4$ at $T = T_V$ before $T$ has relaxed, we will suppose $x_r \ll 1$. $T$ has an asymmetric potential w.r.t. $T_r$, so its amplitude on the sides of $T > T_r$ and $T < T_r$ are different. Defining the amplitudes at $T > T_r$ and $T < T_r$ as $T_+$ and $T_-$ respectively, we now calculate them in turn.

First for $T_+$ when $T > T_r$, the solution is simplified to

$$T_+ = T_r + \sqrt{2}M_s(1 - e^{3N_r}\sqrt{x_r}) \approx T_r + \sqrt{2}M_s. \tag{43}$$

All the logarithms are neglected here. We can tell from this equation that tachyon is pushed back swiftly by the interaction term once it goes beyond $T_V$. 
Similarly,
\[ T_- = T_r + (\ln \sqrt{x_r} + 3N_r)\sqrt{2}M_s \approx T_r + \sqrt{2}M_s \ln \sqrt{x_r}. \] (44)

Because \( x_r \ll 1 \), \( T \) can go quite far away from \( T_r \), which is different from \( T_+ \). Therefore \( T \) would stay at \( T < T_r \) most of the time during its oscillation, and the period when \( T > T_r \) is negligible.

To get tachyon relaxed at \( T \approx T_V \), we need \(|T_- - T_r| \sim M_s\), which, in turn, requires an e-folding of
\[ N_r \sim \frac{T_{V2}}{3\sqrt{2}M_s} \sim N_R, \] (45)

which is very slow compared with the oscillation of \( T \). Therefore, at this stage tachyon enjoys a much lower rate of damping from expansion than that from reheating. In other words, if allowed, tachyon would decay into other particles swiftly.

The mechanism of reheating is not the main concern of this paper, but we would like to briefly discuss the issue here. When one takes into account the effect of parametric resonance, discussed in Section 5, reheating from tachyon however may become insufficient. In this case we expect \( \phi \) to give the major contribution to reheating. After tachyon has rolled down, the effective mass of \( \phi \) is significantly increased, so its decay becomes much more effective. The decay of \( \phi \) implies a closer distance between D3-anti-D3 branes facilitating tachyon condensation. A larger tachyon field then further amplifies \( \phi \)'s decay. All in all the positive feed-back mechanism makes the decay of \( \phi \) into matter fast and effective.

### 3.5 Turnaround, Contraction and Cycles

Now let us consider the next stage of the evolution. Curvature energy has a damping rate of \( a^{-2} \) smaller than \( a^{-3} \), that of total energy density. So curvature will gradually catch up and turnaround will take place at the equality of curvature energy and the total energy density of the universe.

The contraction phase is almost a reversed process of expansion\(^\text{1}\). When contraction drives temperature high enough, a reversal of reheating is believed to take place and, in particular, D3- and anti-D3 are pair produced at sufficiently high temperature. Please note “reversal” here doesn’t merely mean a reverse of reheating process. Instead, it only indicates the energy transfer from matter/radiation particles to fields \( \phi \) and \( T \), which leads to the domination fields instead of matter/radiation. It is supposed, for the present discussion, this process is in thermal equilibrium so the entropy does not either increase or decrease. Afterwards, the energy in \( \phi \) is further pumped up by contraction, and gradually it pulls tachyon back up its potential well to zero. At this point the universe becomes dominated by vacuum energy of the tachyon again. So the vev of \( T \) behaves like

\(^{1}\)One also has to take into account of entropy generation, in which case, the reversal is only partial.
cosmological constant and dominates the inflationary contraction that follows. Curvature energy falls behind before vacuum energy domination, but catches up very soon after vacuum energy takes over. Therefore, a second bounce is expected.

Since the second bounce point can be treated as a new local minimum, the universe would then expand again and become cyclic. However due to the monotonic increase of entropy, the universe has a growing maximal size cycle by cycle and therefore is not truly periodic. At bounce point we find a local minimal size of the universe when an equality between the vacuum energy of tachyon and the curvature energy is reached.

In this sense, this model is free from tran-Planckian problems due to its cyclic nature. It is also free from horizon and entropy problems because of an extended inflationary period in each cycle of the evolution of the universe. The Hagendorn soup phase from string theory in the very beginning can be used to endow a minimal size to the universe and hence solving the singularity problem. But as this model is proposed in a closed universe, 50 e-folds of inflation or higher is required in the current cycle to solve the flatness problem. The issue of e-folding will be further elaborated in the following section.

4 Analysis

4.1 Phases of Universe Evolution

Let’s first list the constraints of the model which we have to take into account when computing the e-folding in the next subsection. We will use subscript 0 to denote initial value at the bounce point. Each constraint in this section is given followed with explanations.

CONSTRAINT 1 (Vacuum Energy Domination)

\[
\frac{V_0}{2\lambda \langle T_0^2 \rangle \langle \phi_0^2 \rangle} > 1, \quad \text{for } b, c, e, f
\]

\[
\frac{V_0}{m^2 \langle \phi_0^2 \rangle} > 2, \quad \text{for } a, d
\]

The first constraint comes from our expectation of initial tachyon (or its vacuum energy) domination. Not only it is the requirement from locked inflation, it is also because the total equation of state should be smaller than $-1/3$, that of curvature. Otherwise, the universe would contract instead of expand. Depending on which term is larger in the mass square of $\phi$, two cases are considered separately to get Eq. 46.
CONSTRAINT 2 (Initial Locking)

\[ \langle \phi_0^2 \rangle > \langle \phi_c^2 \rangle, \quad \text{for } a, b, c \]
\[ m^2 \langle \phi_0^2 \rangle > \frac{V_0}{2 M_s^2} \langle \phi_c^2 \rangle, \quad \text{for } d \]
\[ 2 \lambda \langle T_0^2 \rangle \langle \phi_0^2 \rangle > \frac{V_0}{2 M_s^2} \langle \phi_c^2 \rangle, \quad \text{for } e, f. \]

We would like to have a period of locked inflation, and we would like it to make a major contribution to e-folding. This constraint is to ensure initial tachyon locking, by making \( \langle \phi_0^2 \rangle \) larger than the \( \langle \phi_c^2 \rangle \) at the end of locked inflation. We categorize cases \( a \) to \( f \) according to different ending conditions of locked inflation, the definitions and therefore the values of \( \langle \phi_1^2 \rangle \) are also different in different cases.

CONSTRAINT 3 (Fastroll Scalar Field)

\[ m^2 > \frac{6 \pi}{M_p^2} (V_0 + m^2 \langle \phi_0^2 \rangle), \quad \text{for } a, d \]
\[ 2 \lambda \langle T_0^2 \rangle > \frac{6 \pi}{M_p^2} (V_0 + 2 \lambda \langle T_0^2 \rangle \langle \phi_0^2 \rangle), \quad \text{for } b, c, e, f \]
\[ M_p^2 \langle T_0^2 \rangle > 12 \pi M_s^2 \langle \phi_0^2 \rangle, \quad \text{for } b \]
\[ m^2 M_p^4 \langle T_0^2 \rangle > 144 \pi^2 \lambda V_0 M_s^2 \langle \phi_0^2 \rangle, \quad \text{for } c \]
\[ M_p^4 \langle T_0^2 \rangle > 288 \pi^2 \lambda^2 M_s^4 \langle \phi_0^2 \rangle, \quad \text{for } e, f. \]

To get an oscillating \( \phi \) during locked inflation, we need \( \phi \) to violate the slow-roll condition. The effective mass of \( \phi \) thus yields \( m_\phi^2 > 9 H^2/4 \). Neglecting the smaller contribution to \( \phi \)'s mass, the derivation of above constraint is straightforward. The first two relations are for initial fast-roll property and the rest are for fast-roll near the end of locked inflation.
CONSTRAINT 4 (Proper Mass of Scalar Field)

\[
m^2 > 2\lambda\langle T_0^2 \rangle, \quad \text{for } a, d
\]

\[
m^2 < \frac{V_0\langle T_0^2 \rangle}{2M_s^2\langle \phi_0^2 \rangle}, \quad \text{for } b
\]

\[
2\lambda\langle T_0^2 \rangle > m^2 > \frac{V_0\langle T_0^2 \rangle}{2M_s^2\langle \phi_0^2 \rangle}, \quad \text{for } c
\]

\[
m^2 < \frac{V_0}{2M_s^2}\sqrt{\frac{\langle T_0^2 \rangle}{\langle \phi_0^2 \rangle}}, \quad \text{for } e
\]

\[
2\lambda\langle T_0^2 \rangle > m^2 > \frac{V_0}{2M_s^2}\sqrt{\frac{\langle T_0^2 \rangle}{\langle \phi_0^2 \rangle}}, \quad \text{for } f.
\]

(49)

These and the next constraints come from the defining characteristics of each case. The above constraints ensure the correct relationship between \(\lambda\langle T_0^2 \rangle\), \(\lambda\langle T_1^2 \rangle\) and \(m^2\). The next ones apply to \(m_\phi^2\) and \(V_0/2M_s^2\) at Point 1.

CONSTRAINT 5 (Ending Condition of Locked Inflation)

\[
2m^2M_s^2 > V_0, \quad \text{for } a, c
\]

\[
\langle T_0^2 \rangle > \langle \phi_0^2 \rangle, \quad \text{for } b
\]

\[
2m^2M_s^2 < V_0, \quad \text{for } d, f
\]

\[
\langle T_0^2 \rangle < \langle \phi_0^2 \rangle, \quad \text{for } e.
\]

(50)

We can see from this set of constraints \(\sqrt{V_0/2M_s^2}\) is a critical value of \(m\) which divides the moduli space into normal ending and mass ending for \(m\) that is not too small (\(m^2 > 2\lambda\langle T_1^2 \rangle\)). This is reasonable because as long as \(m\) is not too small, it would already be \(m\) dominating in the mass of \(\phi\), so the constraint only applies to \(m\). Moreover, after checking with other constraints we find that the case \(e\) may only exist under \(m^2 < V_0/2M_s^2\). So in the analysis of moduli space in the next section, we will discuss the large \(m\) and small \(m\) separately.

CONSTRAINT 6 (Relaxed Tachyon)

\[
\lambda m^2M_s^4M_p^2\langle \phi_0^2 \rangle^2 > 3\pi V_0^3, \quad \text{for } a, d
\]

\[
2\lambda^2 M_s^4 M_p^2 \langle \phi_0^2 \rangle^2 \langle T_0^2 \rangle > 3\pi V_0^3, \quad \text{for } b, c, e, f.
\]

(51)
This is the last constraint, coming from the number of e-folds. We expect the universe to be still expanding when $T = T_V$ at Point 3, so there would be reheating and baryon genesis for late time universe. According to the evolution of the ratio of positive energy density to curvature energy, we write the constraint as $2(N_L + N_R) > N$, based on that during inflation energy density stays constant, and afterwards the universe becomes matter-like.

### 4.2 Moduli Space and E-folds

The string mass scale, $M_s$, is usually chosen to be $10^{-3}M_p$, the scale at which we are also adopting here. $V_0$ is the scale of $M_s^3M_p$ from definition of tachyon in string theory. Therefore we have four free parameters of the system, $\lambda$, $m$, $\langle T_0^2 \rangle$ and $\langle \phi_0^2 \rangle$. Once we give them definite values in the moduli space, the evolution of the universe is fully determined. For demonstration, we choose

$$\alpha \equiv \ln \frac{4\lambda M_s^2 \langle \phi_0^2 \rangle}{V_0}, \quad (52)$$

$$\beta \equiv \ln \frac{\langle T_0^2 \rangle}{\langle \phi_0^2 \rangle} \quad (53)$$

Figure 2: The moduli space diagram for $m^2 > V_0/2M_s^2$, when $m^2 = 10^{-3}M_p^2$, $\langle \phi_0^2 \rangle = 10^{-8}M_p^2$. Brown, blue and red correspond to cases $a, b, c$ respectively.
as the axes of moduli space, and use $\langle \phi_0^2 \rangle$ and $m^2$ as the free parameters. Any combination of values of $\langle \phi_0^2 \rangle$ and $m^2$ provides a different moduli space of $\alpha$ and $\beta$.

Such a choice of axes and free parameters provides convenience in several aspects. $\alpha$ is actually $\ln(\langle \phi_0^2 \rangle/\langle \phi_c^2 \rangle)$ so it partly indicates the e-folds of locked inflation. Definition of $\beta$ simplifies the constraints, especially Constraint 4. $m$ acts as a free parameter and provides distinctive moduli spaces under different values, as discussed in Constraint 5.

Given the constraints in Section 4.1 we can plot the moduli spaces. Fig. 2 is the case when $m^2 > V_0/2M_s^2$, and Fig. 3 is for $m^2 < V_0/2M_s^2$. From these two figures, we can tell that if we want to increase the e-folds of locked inflation (characterized by $\alpha$), we need to decrease $\beta$ (i.e. $\langle T_0^2 \rangle$, since $\langle \phi_0^2 \rangle$ remains constant as a free parameter) to preserve tachyon domination (Constraint 1). Other constraints can be found in the figures in the same way.

We can then further demonstrate the number of e-folds within the moduli space. Fig. 4 gives the figure of remaining e-folds after $T_3 = T_{V3}$ before turnaround, which is defined as $2(N_L + N_R) - N$. And Fig. 5 shows the total number of e-folds of inflation $N_L + N_R$. It can be seen that in most cases, locked inflation indeed contributes much more to total e-folds than rolling inflation. On the other hand, decreasing $\beta$ does generate additional e-folds that can contribute a small portion on total e-folds in Fig. 5. The vertical contours in Fig. 4 reveals that the e-folds required by the catch-up process of $T$ with its vev $T_V$ cancels that from rolling inflation.

Figure 3: The moduli space diagram for $m^2 < V_0/2M_s^2$, when $m^2 = 10^{-4}M_p^2$, $\langle \phi_0^2 \rangle = 10^{-8}M_p^2$. Blue, brown, green and red correspond to $a, d, e, f$. 
Figure 4: The diagram of remaining e-folds when $m^2 = 10^{-5} M_p^2$, $\langle \phi_0^2 \rangle = 10^{-8} M_p^2$

Figure 5: The diagram of total e-folds of inflation when $m^2 = 10^{-5} M_p^2$, $\langle \phi_0^2 \rangle = 10^{-8} M_p^2$
If we want the e-folds of locked inflation to be $N_L = 50$, there would be $\alpha \approx 150$ and $\beta < \sim -150$. The large $\alpha$ would imply a strong coupling between $\phi$ and $T$, which means tachyon will not generate until branes become very close. Meanwhile, $\beta$ demands an initial $\langle T^2 \rangle$ frozen at zero. This is reasonable because the initial large distance between the branes prevents tachyon production, and the strong coupling further ensures that. Therefore, our model needs a strong coupling between tachyon and the distance between branes. Otherwise it is difficult to get an e-folding larger than 50.

5 Discussion of Parametric Resonance

In the above analysis, we have neglected, for simplicity, the effect of parametric resonance. Here we briefly discuss its effect in the locked inflation scenario.

Parametric resonance during locked inflation transfers energy from $\phi$ field to tachyon, pumping up $\langle T^2 \rangle$. In the language of particle physics, tachyon particles are generated because of the interaction and the oscillation of $\phi$. This process is usually called “pre-heating” in many papers (see [43] for a brief review). When back-reactions of $T$ on $\phi$ are negligible, such preheating produces the number density of $T$ that is exponentially increasing for all modes below a certain momentum. Such exponential increase of $T$ would certainly ruin our model if no precaution is taken.

There are usually three ways to deal with unwanted resonance, one of which works for our model. Let us first see why the other two do not. First if the effective mass of $T$ from $\phi$ is much smaller than $T$’s bare mass, the effect of parametric resonance is negligible. It is certainly inapplicable to our model because the number of e-folding of locked inflation requires $\phi$ contribute an effective mass that is much larger than $T$’s bare mass, $\sqrt{V_0/2M^2_s}$. The second point is that if $\phi$ is fast rolling but not too fast, i.e. in the range of $10H > \sim m_\phi > 3H/2$, the Hubble damping due to expansion is even larger and cancels parametric resonance effect. So this mechanism works well for our model during expansion but it must fail at the contraction phase, because contraction acts as a boosting effect. Altogether with parametric resonance, $T$’s amplitude grows even faster during contraction.

The method we are using is to allow the back-reaction of $T$ so that parametric resonance between $\phi$ and $T$ is in equilibrium. Therefore the energy transfer to and fro each other by parametric resonance should be exactly equal, i.e. the growing rate of $\phi$ is proportional to the effective mass of $T$ and vice versa. With these relations, we have $m_\phi \langle \phi^2 \rangle_e = m_T \langle T^2 \rangle_e$, where subscript $e$ means equilibrium and the effective masses are taken to be $m_\phi = m$, $m_T = \sqrt{\lambda \langle \phi^2 \rangle_e}$. Since we require many e-folds during locked inflation, $m_T \gg m_\phi$ and $\phi$ is initially large. We thus get $\langle T^2 \rangle_e/\langle \phi^2 \rangle_e \ll 1$ initially, which in turn implies that the parametric resonance effect is negligible at the beginning. As $\langle \phi^2 \rangle$ decreases due to inflation, this ratio will grow and become significant. It may become
quite large near the end of locked inflation, and thereafter the stage of rolling inflation would be shortened or even bypassed. Consequently, the kinetic-potential energy ratio at point 3 may decrease significantly. In such cases, the reheating efficiency from tachyons kinetic energy is lowered and may become insufficient. This calls for other mechanisms for sufficient reheating, such as by taking into consideration $\phi$'s decay.

6 Conclusion

In this paper, we have proposed a coupled scalar-tachyon model for a cyclic universe without initial singularities in a $k = 1$ FLRW background. A scalar-tachyon coupling term have been introduced into the action to make D-brane inflation phase come out naturally from tachyon condensation. Further investigation reveals that such scalar-tachyon coupling term plays a crucial role in cosmic evolution of universe, and distinguishes the coupled scalar-tachyon model from other traditional single tachyon field cosmology and/or D-brane inflation cosmology. According to our model, universe undergoes six main phases, the locked inflation phase, tachyon rolling phase, tachyon matter dominated phase, turnaround, contraction and the a distinctive bounce phase.

By utilizing curvature, bounces and turnarounds are realized because of the competition between curvature and tachyon-scalar contributions to energy density. The entire evolution of universe is studied analytically and our model is proven to be self-consistent. Multiple cases are considered separately through explicit calculation. Whenever possible we also generalize the results to broader moduli spaces for the parameters in the model. In this process, “normal ending” and “mass ending” as well as behaviors of the model under different values of $\phi$'s mass and interaction strength $\lambda$ are investigated in detail. After one cycle of evolution of the universe is presented, comparison with observations let us impose further constraints on the values of the parameters.

Further studies are underway. We are investigating the effects of parametric resonance as well as the spectrum of density perturbations. A Monte Carlo investigation of the probabilities of obtaining a universe with a given size and degree of flatness is warranted. We have introduced the salient features of the model in this paper and will leave the above mentioned issues to a future publication.

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