Electric Quadrupole $E2$- Transitions of $^{170-174}Yb$ Isotopes

Mohd Kh. M. Abu El Sheikh\(^1\), Abdurahim A Okhunov\(^2\), Ph.N. Usmanov\(^3\), and Torla HJ Hassan\(^2\)

\(^1\) Department of Physics, Faculty of Science, University of Malaya, 50603, Kuala Lumpur, Malaysia
\(^2\) Department of Science in Engineering, International Islamic University Malaysia, 57528, P.O. 10, Kuala Lumpur, Malaysia
\(^3\) Institute of Nuclear Physics, Academy of Science of Uzbekistan, 100132, Tashkent, Uzbekistan

E-mail: abdurahimokhun@iium.edu.my, aaokhunov@gmail.com

Abstract. The non-adiabatic effects which is manifested in the electric properties of low-lying states of even-even deformed nuclei are studied. A simple phenomenological model which takes into account the Coriolis mixing of $K\pi = 0^+ + n$, $2^+ + n$ and $K\pi = 1^+ + \nu$ rotational bands.

The Calculations for isotopes $^{170-174}Yb$, are carried out. The reduced probability of electric quadrupole transitions from the states $0^+\nu$ and $2^+\nu$ - bands to the ground (gr) state band is calculated and non adiabatic effect is discussed. The ratio of $E2$– transitions $R_{1K}$ from $0^+_m$, $0^+_3$, $2^+_1$, and $2^+_2$ bands are calculated and compared with the experimental data.

1. Introduction

Despite the fact that the structure of deformed nuclei and nature of low excited levels have been substantially studies over more than four decades, this still occupies a central part of today’s research Refs. [1]-[3].

The isotopes of Yb is the deformed nuclei in rare-earth region and the spherical nuclei near $Z = 70$. For well deformed nuclei the states associated with ground state rotation produce regular sequences which dominate the yrast line. The nuclei $^{170,172,174}Yb$ have been well studied. In highspin spectroscopy studies of rare-earth nuclei, the isotopes of ytterbium have been a central focus for many experiments, and a large body of data has been established [4].

It is important to note that these are investigated in a number of ways such as radioactive decay of $^{170,172,174}Lu$, and different nuclear reactions. In these isotopes, many $1^+$ states and $K\pi = 0^+, 2^+$ bands have been observed.

Numerous conducted experiments on defining spectroscopic characteristics of low-lying exciting states, particular $K\pi = 1^+$ in deformed nuclei [5], have motivated the further theoretical investigations. In this case, investigations influence of $K\pi = 1^+$ states to the properties of low-lying levels is actual.

In the present paper analyzing properties of positive parity low-lying state in the transuranium nuclei, within the phenomenological model [6]-[8], which into account Coriolis mixture states of ground, $0^+_m$ ($m = 2, 3, ..$), $2^+\nu (\gamma \nu)$ and $K\pi = 1^+\nu$ rotational bands. The calculation are performed for the $^{170,172,174}Yb$ isotopes.
2. Electric Quadrupole E2– Transitions

We shall calculate the reduced probability for E2– transitions, using the wave function obtained by describing the energy of states. The expression for the reduced probability of E2– transitions between states \( I, K_i \) and ground state band and also intraband transitions of ground band within the framework our model \([9]\) as follows:

\[
B(E2; I_iK_i \rightarrow I_f01) = \left\{ \frac{5}{16\pi} e Q_0 \left[ \Psi_{I_01,01}^{I_i} \Psi_{01,K_i}^{I_i} C_{I_10;20}^{I_i} \sum_n \Psi_{I_n01}^{I_i} \Psi_{K_n,K_i}^{I_i} C_{I_i0;20}^{I_i} \right] \right. \\
+ \sqrt{2} \left[ \Psi_{I_01,01}^{I_i} \sum_n \frac{(-1)^{K_n} m_{K_n} \Psi_{01,K_i}^{I_i} \Psi_{I_00;2}^{I_i} C_{I_i0;20}^{I_i}}{\sqrt{1 + \delta_{K_n,0}}} \right] \right\}^2 \tag{1}
\]

Here \( m_{K_n} = \langle 0_1^+ | m(E2) | K_n^+ \rangle \) is matrix elements between intrinsic wave functions of ground \((0_1^+)\) and \( K_n^+ = 0_1^+, 2_2^+, 1_1^+ \) bands, whose values are defined from experimental data, \( Q_0 \) is intrinsic quadrupole moment of nucleus; and \( C_{I_i0;20}^{I_i} \) – Clebsch-Gordan coefficients.

In the adiabatic approximation, the following equations are valid for \( B(E2) \) factors from the \( I = 2 \) states to the \( K_n^+ \) rotational band:

\[
B^{adiabatic}(E2; 2K_n \rightarrow 00_1) = (2 - \delta_{K_n,0}) \left| m_{K_n} C_{20;2}^{00} \right|^2 \tag{2}
\]

which allows us to calculate the empirical values of the parameters \( m_{K_n} \) from the experimental data.

The \( K^\pi = 0_2^+, 0_3^+ \) and \( 2_2^+ \) bands are located close to each other in isotopes \( ^{170,172}Yb \), which leads to a strong mixing of states even \( I = 2 \). In this case, the adiabatic approximation \((2)\) becomes inapplicable to determine \( m_{K_n} \).

Therefore, to describing the experimental data for \( B(E2) \) in \( ^{170,172}Yb \) isotopes, the value of \( m_0 \) and \( m_2 \) parameters are varied slightly. The values of the parameters \( m_K \) which are used in calculating probability of E2– transitions are given in Table 1.

| \( A \) | \( m_{02} \) | \( m_{04} \) | \( m_{05} \) | \( m_{07} \) | \( m_{1r} \) | \( m_{21} \) | \( m_{22} \) | \( Q_0[10] \) |
|---|---|---|---|---|---|---|---|---|
| 170 | 2 | 24 | 3 | 8 | -5 | 19 | 780(4) |
| 172 | 10 | 1 | -6.9 | -8 | -5 | 15 | -8 | 791(4) |
| 174 | 8 | 1 | -6.9 | 8 | -10 \((m_{1r} = -1.7)\) | 15 | 8 | 782(4) |

The empirical values of parameters \( m_{K_n} \) have been defined by formula \((2)\), using the experimental data of the reduced probabilities of E2– transitions \( B(E2; 2K_n \rightarrow 00_1) \). In the case where the bandhead energy is close to each other the Coriolis mixing of state is manifested significantly even at low spins \( I = 2 \). For example, \( K^\pi = 0_2^+ \) and \( K^\pi = 0_3^+ \) bands in nucleus
$^{170}$Yb and also the $K^{-} = 0^{+}_{3}$ and $K^{+} = 2^{+}_{1}$ bands in nucleus $^{172}$Yb are close to each other. Therefore, for these bands the value of parameter $m_{K}$ is different from other bands (see Table 1). This mixture effect is more manifested in the electromagnetic than energy characteristics.

The magnitude and sign of the parameters $m_{11} = m_{1
u}$ were determined from the best agreement of the ratios $R_{IK} = B(E2; IK \rightarrow I + 10_{1})/B(E2; IK \rightarrow I - 10_{1})$ from odd states of the $K^{-} = 2^{+}_{1}$ and $1^{+}_{0}$ (states with the negative signature $\sigma = -1$).

The parameters $m_{0\nu}$ and $m_{2\nu}$ are determined, requiring the best agreement between calculated and experimental values of the ratios $B(E2)$ transitions from the $\beta_{-\nu}$ and $\gamma_{\ell}$ bands correspondingly, with positive signature $\sigma = +1$.

The comparison of the values of calculated reduced probabilities of $E2$– transitions with experimental data [11]–[14] are shown in Table 2.

| $A$ | $I, K_{i} \rightarrow I_{f}, K_{f}$ | Exp. | Thory | $I, K_{i} \rightarrow I_{f}, K_{f}$ | Exp. | Thory |
|-----|----------------------------------|------|-------|----------------------------------|------|-------|
| $^{170}$Yb | 221, 001 | 151(55)[11] | 90 | 001, 201 | 60(15)[11] | 43 |
| | → 201 | 269(60)[11] | 60 | 004, 201 | 567(118)[11] | 567 |
| | → 401 | 27(6)[11] | 10 | | | |
| $^{172}$Yb | 221, 001 | 75.6(63)[12]; 74.6(57)[14] | 82 | 004, 201 | 205(60)[12] | 100 |
| | → 201 | 121(12)[14] | 130 | 004, 201 | 14(1)[12]; 14(1)[14] | 13 |
| | → 401 | 7.3(6)[12]; 6.8(7)[14] | 8.6 | → 201 | 45(7)[12]; 52(8)[14] | 23 |
| | → 401 | 398(284)[12] | 15 | → 401 | 142(20)[12]; 140(20)[14] | 74 |
| | → 401 | 739(512)[12] | 81 | 004, 201 | 0.14(3)[12] | 1 |
| | → 401 | 152(11)[14] | 154 | 204, 201 | 0.4(1)[12]; 3.4(2)[14] | 3.6 |
| | → 401 | 79(6)[14] | 73 | → 201 | 0.6(4)[12]; 11.9(8)[14] | 3.0 |
| | → 401 | 20(4)[12]; 32(4)[14] | 23 | → 401 | 1.0(1)[14] | 1.2 |
| | → 201 | 31(2)[12]; 51(7)[14] | 38 | 004, 201 | >0.25[12] | 48 |
| | → 401 | 3.3(4)[14] | 2.2 | 204, 201 | 10(6)[12] | 12 |
| | → 401 | 54(7)[14] | 42 | 004, 201 | >0.27[12] | 64 |
| | → 401 | 22(3)[14] | 21 | 204, 201 | 19(9)[12] | 21 |
| | → 401 | | | → 201 | >0.18[12] | 23 |
| $^{174}$Yb | 002 → 201 | 81.6$^{+8.4}_{-20.5}$[13] | 64 | 221, 201 | 144(30)[13] | 133 |

In Table 3 provided theoretical values of the reduced matrix elements of $E2$– transitions for $^{172}$Yb, which are compared with experimental data and values taken by other models [15]–[17].

We note that our calculations were performed sequentially, i.e., first describes the energy state and then corresponding their wave functions are determined. Further, using these wave functions are calculated probabilities $E2$– transitions. From Table 4, one can see that, the results of calculation in the framework of our model for the majority cases provide a good agreement with experimental data.

To evaluate the degree of significant nonadiabaticity manifests in the values of reduced probabilities of $E2$– transition in Table 5 is given the comparison of theoretical ratios $R_{IK}^{\text{theory}} = I_{\gamma}(IK \rightarrow I_{0})/I_{\gamma}(IK \rightarrow I_{0})$ with experimental data [11]–[13], [10, 14, 18] and their adiabtic values $R_{IK}^{\text{adiabtic}}$, which defined by the formula:

$$R_{IK} = \frac{I_{\gamma}(IK \rightarrow I_{0})}{I_{\gamma}(IK \rightarrow I_{0})} \left( \frac{E_{\gamma}(IK \rightarrow I_{0})}{E_{\gamma}(IK \rightarrow I_{0})} \right)^{5}$$

(3)
Table 3. Reduced Matrix Elements of $E2^\gamma$ transitions in $^{172}$Yb, calculated within our model which comparison with experimental data [15] and are calculated using the rotational-vibrational model (RVM2) [16] and the IBA-1 model [17] ($eb$).

| $I,K_i \rightarrow I_f,K_f$ | Exp. | RVM2 | IBA-1 | Theory | $I,K_i \rightarrow I_f,K_f$ | Exp. | RVM2 | IBA-1 | Theory |
|--------------------------|------|------|-------|--------|--------------------------|------|------|-------|--------|
| 201 → 201                | -2.63_{-0.47}^{+0.28} | -2.92 | -2.92 | -2.93 | 201 → 001                | 2.45_{-0.15}^{+0.12} | 2.45 | 2.45 | 2.45 |
| 401 → 401                | -3.54_{-0.42}^{+0.12} | -3.73 | -3.69 | -3.74 | 401 → 201                | 3.76_{-0.19}^{+0.13} | 3.93 | 3.91 | 3.93 |
| 601 → 601                | -4.31_{-0.62}^{+0.23} | -4.43 | -4.33 | -4.46 | 601 → 401                | 5.34_{-0.27}^{+0.07} | 4.97 | 4.90 | 4.96 |
| 801 → 801                | -4.49_{-0.14}^{+0.43} | -5.05 | -4.83 | -5.08 | 801 → 601                | 5.90_{-0.30}^{+0.10} | 5.80 | 5.60 | 5.80 |
| 1001 → 1001              | -6.32_{-0.74}^{+0.32} | -5.60 | -5.22 | -5.63 | 1001 → 801               | 6.71_{-0.34}^{+0.13} | 6.54 | 6.29 | 6.54 |
| 1201 → 1201              | -6.15_{-0.44}^{+0.64} | -6.08 | -5.53 | -6.15 | 1201 → 2001              | 7.01_{-0.34}^{+0.13} | 7.19 | 6.79 | 7.20 |
| 1401 → 1401              | -2.20 | -0.01 | 0.203 | 0.20 | 2001 → 201               | 0.16 | 0.27 | 0.16 | 0.27 |
| 221 → 001                | 0.208_{-0.040}^{+0.101} | 0.21 | 0.20 | 0.203 | 002 → 201                | 0.166_{-0.015}^{+0.018} | 0.16 | 0.27 | 0.16 |
| → 201                    | 0.250_{-0.018}^{+0.016} | 0.25 | 0.31 | 0.255 | 202 → 001                | 0.090_{-0.040}^{+0.010} | 0.16 | 0.26 | 0.082 |
| → 401                    | 0.063_{-0.004}^{+0.009} | 0.062 | 0.10 | 0.066 | → 201                    | -0.162_{-0.071}^{+0.008} | 0.19 | -0.31 | 0.108 |
| 421 → 201                | 0.22_{-0.07}^{+0.07} | 0.20 | 0.13 | 0.11 | 401 → 401                | 0.27_{-0.08}^{+0.02} | 0.26 | 0.45 | 0.19 |
| → 401                    | 0.46_{-0.13}^{+0.12} | 0.38 | 0.45 | 0.27 | 402 → 401                | -0.27 | -0.13 | -0.13 | -0.13 |
| 321 → 201                | 0.32(11) | - | - | - | 0.328 | - | - | - | - |
| → 401                    | 0.235(6) | - | - | - | 0.226 | - | - | - | - |

$^a$) This matrix element was used to normalize the results of the model calculations.

where $I^\gamma (IK \rightarrow I_10_1)$ is intensity and $E^\gamma (IK \rightarrow I_10_1)$ is energy of $\gamma$ transition.
Table 4. The ratios of reduced probabilities of \(E2\)-transitions \(R_{1K} = B(E2; I_iK_i \rightarrow I_f0_1)/B(E2; I_iK_i \rightarrow I'_f0_1)\) for \(^{170}\text{Yb}\) isotope.

| \(A\) | \(I_iK_i\) | \(I_fK_f\) | Experiments | Theory | Alaga |
|---|---|---|---|---|---|
| \(^{170}\text{Yb}\) | \(22^+\) | \(20^+_1\) | \(00^+_1\) | 1.77(8)[11] | 1.86 | 1.43 |
| | | \(40^+_1\) | \(20^+_1\) | 0.098(11)[11] | 0.043 | 0.50 |
| | \(32^+_1\) | \(40^+_1\) | \(20^+_1\) | 0.78(4)[11] | 0.75 | 0.40 |
| | \(52^+_1\) | \(60^+_1\) | \(40^+_1\) | 1.39(46)[11] | 1.50 | 0.57 |
| | \(72^+_1\) | \(80^+_1\) | \(60^+_1\) | 1.27(24)[11] | 2.42 | 0.67 |
| | \(20^+_2\) | \(20^+_1\) | \(00^+_1\) | 1.94(52)[11] | 1.1 | 1.43 |
| \(40^+_2\) | \(40^+_1\) | \(20^+_1\) | | 1.82 | 0.91 |
| | \(60^+_2\) | \(60^+_1\) | \(40^+_1\) | 3.73(90)[11] | 1.67 | 0.81 |
| | \(80^+_2\) | \(80^+_1\) | \(60^+_1\) | 10.7(19)[11] | 2.45 | 0.77 |
| | \(100^+_2\) | \(100^+_1\) | \(80^+_1\) | 25.3(85)[11] | 29.2 | 0.74 |
| | \(120^+_2\) | \(120^+_1\) | \(100^+_1\) | 29.9(71)[11] | 2.0 | 0.73 |
| \(20^+_3\) | \(20^+_1\) | \(00^+_1\) | 1.81(11)[11] | 2.5 | 1.43 |
| | \(40^+_3\) | \(20^+_1\) | | 3.00(15)[11] | 2.3 | 1.80 |
| | | \(40^+_1\) | \(00^+_1\) | 5.43(18)[11] | 5.75 | 2.57 |
| \(20^+_4\) | \(40^+_1\) | \(00^+_1\) | 4.0(2)[11] | 3.9 | 2.57 |
| \(^{172}\text{Yb}\) | \(22^+\) | \(20^+_1\) | \(00^+_1\) | 1.62(12)[14] | 1.71(84)[12] | 1.59 | 1.43 |
| | \(40^+_1\) | \(20^+_1\) | | 0.056(5)[14] | 0.056(20)[12] | 0.066 | 0.072 |
| | \(32^+_1\) | \(40^+_1\) | \(20^+_1\) | 0.52(4)[14] | 0.56(3)[12] | 0.48 | 0.40 |
| | \(42^+_1\) | \(40^+_1\) | \(20^+_1\) | | 3.35(69)[12] | 6.0 | 2.94 |
| | \(22^+_2\) | \(20^+_1\) | \(00^+_1\) | 1.69(21)[14] | 1.55(30)[12] | 1.61 | 1.43 |
| | \(40^+_2\) | \(20^+_1\) | | 0.015(8)[14] | 0.064(38)[12] | 0.058 | 0.072 |
| | \(32^+_2\) | \(40^+_1\) | \(20^+_1\) | 0.163(56)[14] | 0.409(77)[12] | 0.504 | 0.40 |
| | \(42^+_2\) | \(40^+_1\) | \(20^+_1\) | | 4.11(17)[12] | 3.89 | 2.94 |
| | \(52^+_2\) | \(60^+_1\) | \(40^+_1\) | | < 1.10[12] | 0.82 | 0.57 |
| | \(20^+_3\) | \(20^+_1\) | \(00^+_1\) | 3.71(24)[14] | 2.88(36)[12] | 1.80 | 1.43 |
| | \(40^+_3\) | \(20^+_1\) | | 2.70(38)[14] | 2.61(11)[12] | 3.20 | 1.80 |
| \(40^+_4\) | \(40^+_1\) | \(20^+_1\) | | 6.78(1.36)[12] | 1.56 | 0.91 |
| \(^{174}\text{Yb}\) | \(22^+\) | \(20^+_1\) | \(00^+_1\) | > 0.49[10, 18] | 2.4(5)[13] | 1.59 | 1.43 |
| | \(40^+_1\) | \(20^+_1\) | | 0.167(75)[10, 18] | 0.256(92)[13] | 0.055 | 0.072 |
| | \(32^+_1\) | \(40^+_1\) | \(20^+_1\) | > 0.325[10, 18] | 0.67(13)[13] | 0.49 | 0.40 |
| | \(42^+_1\) | \(40^+_1\) | \(20^+_1\) | 4.83(5)[10, 18] | 4.77(63)[13] | 3.75 | 2.94 |
| | \(60^+_1\) | \(40^+_1\) | | < 0.14[10, 18] | | | |
| | \(20^+_2\) | \(20^+_1\) | \(00^+_1\) | > 8.8[10, 18] | > 9.7[13] | 2.63 | 1.43 |
| | \(40^+_2\) | \(20^+_1\) | | 3.45(5)[10, 18] | 2.9(2)[13] | 6.1 | 1.80 |
| | \(60^+_2\) | \(40^+_1\) | \(20^+_1\) | 7.82(26)[10, 18] | 11.8(2.5)[13] | 6.21 | 0.91 |
| | \(40^+_3\) | \(40^+_1\) | \(20^+_1\) | 1.50(22)[10, 18] | 0.59(7)[13] | 11.3 | 1.75 |
| | \(20^+_3\) | \(20^+_1\) | \(00^+_1\) | > 1.7[10, 18] | 1.99(27)[13] | 1.54 | 1.43 |
| | \(40^+_4\) | \(20^+_1\) | | 1.24(16)[10, 18] | 1.12(12)[13] | 2.11 | 1.80 |

From Table 4 we can see in \(^{170}\text{Yb}\), the experimental values of the ratios \(R_{10^+_2}\) for \(E2\)-transitions from the \(K^\pi = 0^+_2\) states is a differ from the adiabatic theory of a several dozen
times. This is associated with strong mixing of $K^\pi = 0^+_2$ and $K^\pi = 2^+_1$ bands. Then arises the question, why the value of rations $R_{I2_1}$ for the transitions from the $2^+_1$ bands is not so much different from the adiabatic theory compared $R_{I0_2}$? This can be explained by the fact that the matrix element $m_{21} = <0^+_1|\hat{m}(E2)|2^+_1>$ is 10 times greater than $m_{02} = <0^+_1|\hat{m}(E2)|0^+_2>$ (cm. Table 2)

One may see from these comparison that the mixing effect of states low-lying bands plays a crucial role which considerably demonstrates that $E2^-$ transitions even in low values of angular momentum $I$.

3. Conclusion

In the present work, non-adiabatic effects in energies and electric characteristics of excited states are studied within the phenomenological model which taking into account Coriolis mixing of all experimentally known rotational bands with $K^\pi < 3^+$.

The energy and structure of wave functions of excited states are calculated. And also the reduced probabilities of $E2^-$ transitions is calculated. The ratio of $E2^-$ transitions probability from $K^\pi = 0^+_m$ and $2^+_l$ bands are calculated and compared with experimental data which gives the satisfactory result.

If matrix elements of $E2^-$ transitions $m_K$ one of two strongly mixing bands $K$ is less than matrix element of $m_{K'}$ of $K'$ ($m_K < m_{K'}$), then the difference in the ratio $R_{IK}$ for the first band $K$ from Alaga rule is bigger than the difference in $R_{IK'}$ from Alaga rule. In other words, if $m_K < m_{K'}$, nonadiabaticity in the ratio $R_{IK}$ is stronger than that of $R_{IK'}$.

This work has been financial supported by the MOHE, Fundamental Research Grant Scheme FRGS13-074-0315, and also by the Committee for the Coordination of the Development of Science and Technology under the Cabinet of Ministers of the Republic of Uzbekistan, Fundamental Grant "OT-F2-75" (2017-2020).

References

[1] Bohr A and Mottelson B R 1997 Nuclear Structure 1,2 (Benjamin, New York)
[2] Soloviev V G 1974 Theory of difficult atomic nuclei (M. Nauk)
[3] Burke D G, Soloviev V G, Sushkov A V and Shirikova N Yu 1999 Nucle. Phys. Vol. A 656 287
[4] Firestone R B and Shirley V S 1996 Table of Isotopes (edited by Wiley, New York)
[5] Zilges A, Brentano P von, C.Wesselborg C and et al 1990 Nucl. Phys. A507 399
[6] Usmanov P N, Adam I, Salikhbaev U S and Solnyshkin A A 2010 Physics of Atomic Nuclei 73 (12) 1990-1996
[7] Okhunov A A, Usmanov Ph N and Hasan Abu Kassim 2014 J. Phys. G: Nucl. Part. Phys. 41 075102
[8] Okhunov A A, Sharrad F I, A. Anwer Al-Sammarraie and Khandaker M U 2015 Chinese Physics C 39(8) 084101
[9] Usmanov Ph N, I.N.Mikhailov IN 1997 Phys. Part. Nucl. Lett. 28 348
[10] Begzhanov R B, Belinkiy V M et al. 1989 The directory on the nuclear physics Vol.1,2 (Tashkent, FAN)
[11] Baglin M 2002 Nucl.Data Sheets 96, 611 [1996 Nucl.Data Sheets, 77(2) 125].
[12] Singh B 1995 Nucl.Data Sheets 75 199
[13] Browne E, Junde H 1999 Nucl.Data Sheets 87 15
[14] Reich C W, Greenwood R C, Lokken R A 1974 Nucl. Phys. A228 365
[15] Fahlander C, Varnestig B, Backlin F et al. 1992 Nucl. Phys. A147 157
[16] Faessler A, Greiner W, and R.K.Sheline R K 1965 Nucl. Phys. 70 33
[17] Arima A and Iachello F 1984 Adv. Nucl. Phys. 13 139
[18] Gasten R F, Von Brentano P, Kane W R 1973 Phys. Rev. C8 1035