Measurements of Angular Distributions of Muons From Upsilon Meson Decays in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

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The angular distributions of muons from $\Upsilon(1S,2S,3S) \rightarrow J/\psi\mu^+\mu^-$ decays are measured using data from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV corresponding to an integrated luminosity of $6.7$ fb$^{-1}$ and collected with the CDF II detector at the Fermilab Tevatron. This analysis is the first to report the full angular distributions as functions of transverse momentum $p_T$ for $\Upsilon$ mesons in both the Collins–Soper and $s$-channel helicity frames. This is also the first measurement of spin alignment of $\Upsilon(3S)$ mesons. Within the kinematic range of $\Upsilon$ rapidity $|y| < 0.6$ and $p_T$ up to 40 GeV/c, the angular distributions are found to be nearly isotropic.

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Heavy quarkonium production in hadron collisions provides critical tests of quantum chromodynamics (QCD) because it involves both short-distance and long-distance contributions. The cross sections for direct charmonium and $\Upsilon$ production measured at the Tevatron in $p\bar{p}$ collisions [1] greatly exceeded the predictions from leading-order “color singlet” models [2]. Further measurements of spin alignment in $J/\psi\mu^+\mu^-$ and $\Upsilon$ production [4] proved to be in dramatic disagreement with predictions of “color octet” models that were developed to explain the cross section results [3]. While spin alignment can provide very sensitive tests for QCD models, the measured elements of the spin density matrix for the spin-1 $\Upsilon$ states depend critically on the choice of coordinate frame.

Recently, it has been pointed out that improved experimental measurements are needed to clarify this situation [6]. In general, the angular distribution of the $\mu^+$ in the rest frame of an $\Upsilon \rightarrow \mu^+\mu^-$ decay can be written...
\[ \frac{dN}{d\Omega} \sim 1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos 2\varphi + \lambda_\theta, \sin 2\theta \cos \varphi. \]

in which \( \theta \) is the polar angle measured with respect to a quantization axis, and \( \varphi \) is the azimuthal angle measured with respect to the production plane containing the \( \Upsilon \) and the beam axis. The coefficients are directly related to the elements of the spin-density matrix for the ensemble of \( \Upsilon \) states observed \[8\]. Previous studies at hadron colliders have measured only \( \lambda_\theta \) in the \( s \)-channel helicity frame, where the quantization axis coincides with the direction of the \( \Upsilon \) momentum, and cannot be transformed into different coordinate frames without additional information. This precludes model-independent comparisons of results obtained in different coordinate frames or experimental environments. Improving on this experimental situation requires not only measuring all three coefficients, but also carrying out these measurements in multiple coordinate frames. This would allow a comparison of quantities such as \( \lambda = (\lambda_\theta + 3\lambda_\phi)/(1 - \lambda_\phi) \), which should have the same value in different coordinate frames \[8\]. Such a test would provide an important demonstration that observations have not been seriously biased due to poor determination of experimental acceptance or subtraction of highly non-isotropic backgrounds.

In this Letter, we report on the first analysis of angular distributions of muons from \( \Upsilon(1S, 2S, 3S) \rightarrow \mu^+\mu^- \) decays produced in \( p\bar{p} \) collisions carried out using this formalism: the distributions are quantified in both the \( s \)-channel helicity frame and in the Collins–Soper frame, which approximates, on average, the direction of the velocity of the colliding beams. It is also the first analysis to provide information on the angular distributions of muons in decays of the \( \Upsilon(3S) \) state which is more likely to be produced directly, rather than as a decay product of higher mass quarkonium states.

The \( \Upsilon \rightarrow \mu^+\mu^- \) decays were collected using the CDF II detector, which reconstructs charged-particle tracks and measures their momenta using a six-layer silicon strip detector \[10\] and a large-volume drift chamber \[11\], both with approximate cylindrical geometry and positioned in a 1.4 T solenoidal magnetic field. The tracking detectors are surrounded by calorimeters and three separate muon detector subsystems. The CMU system \[12\] consists of four layers of drift tubes that are located outside the hadron calorimeter and cover the central range of pseudorapidity \( |\eta| < 0.6 \). The CMP muon system \[13\] comprises more layers of drift tubes and scintillator placed behind additional steel absorber material and covers approximately \( |\eta| < 0.4 \). Extended muon coverage in the forward region \( 0.6 < |\eta| < 1 \) is provided by the CMX sub-detector which is also constructed from scintillator and drift tubes.

A three-level online event selection system (trigger) is used to identify events that contain oppositely charged dimuon candidates. The level-1 trigger requires two tracks with \( p_T > 1.5 \text{ GeV}/c \) to be identified in the tracking chamber and to be geometrically correlated with activity in the CMU or CMP muon systems \[14\]. The level-2 trigger requires the muons to have opposite charge, and it requires that one of the muons have \( p_T > 3 \text{ GeV}/c \) and that it be detected in both the CMU and CMP systems. Events satisfying the level-2 trigger are passed to the level-3 trigger system, which employs a version of the full event reconstruction software optimized for speed. The event selection used in the level-3 trigger requires the presence of two oppositely charged muon candidates with invariant mass in the range \( 8 < m(\mu^+\mu^-) < 12 \text{ GeV}/c^2 \). It requires one to be reconstructed in both CMU and CMP systems with \( p_T > 4 \text{ GeV}/c \) and the other to be reconstructed in either the CMU or CMX detectors with \( p_T > 3 \text{ GeV}/c \). In this Letter, we refer to the trigger scenario that selects two central muons as CC, and the scenario that selects one central and one forward muon as CF. From 6.7 fb\(^{-1}\) of integrated luminosity, the combination of these triggers provides an event sample containing approximately 550,000 \( \Upsilon(1S) \), 150,000 \( \Upsilon(2S) \), and 76,000 \( \Upsilon(3S) \) decays.

The criteria used to select dimuon candidates closely follow those previously used in Ref. \[15\]. Muon candidates are reconstructed from tracks in the drift chamber that extrapolate to a track segment reconstructed in at least one of the muon detector systems. Geometric restrictions are imposed on muon candidates to ensure that they are contained in regions of the detector with well-measured trigger and track reconstruction efficiencies. Efficiencies for the level-1 trigger and for these selection criteria are measured using the unbiased track in \( J/\psi \rightarrow \mu^+\mu^- \) decays that were recorded using a single-muon trigger. This analysis also makes use of information from the CMP muon system in the level-2 trigger. The efficiency for selecting such muons is measured using samples of \( J/\psi \rightarrow \mu^+\mu^- \) decays obtained using triggers that required information from only CMU or CMX.

The angular distribution analysis is performed separately in each of the 12 ranges of dimuon mass shown in Fig. \[1\]. The angular distributions of \( \Upsilon \) decays are analyzed in eight ranges of \( p_T(\Upsilon) \) from 0 to 40 GeV\(/c\) and are restricted to the central region of rapidity \( |y(\Upsilon)| < 0.6 \). For a given range of transverse momenta, the sample of dimuon candidates is divided into two subsamples according to whether one of the muons is reconstructed precisely using measurements from the silicon detector and its extrapolated trajectory misses the beam axis by a distance \( |d_0| > 150 \mu\text{m} \). Events with at least one muon satisfying this requirement are referred to as the “displaced” sample since they are consistent with the presence of a long-lived parent particle, which is a characteristic feature of the dimuon background arising from semileptonic decays of heavy quarks. These criteria do not bias the angular distribution and, since the displaced...
sample contains almost no $\Upsilon$ signal, it provides a good description of the dimuon background that remains in the complementary "prompt" sample. We verify that this is the case by comparing the angular distributions of prompt and displaced samples projected onto the $\cos \theta$ and $\varphi$ axes for mass ranges in the sidebands of the $\Upsilon$ signals. The level of agreement is quantified by computing the Kolmogorov–Smirnov statistic for each distribution.

The prompt sample contains most of the $\Upsilon \to \mu^+ \mu^-$ signal, while a small fraction is retained in the displaced sample due to the 30 $\mu$m resolution of the $d_0$ measurement (see Fig. 1). The fraction $f_p$ that is present in the prompt sample is measured using a simultaneous binned likelihood fit to the dimuon mass distributions of both prompt and displaced samples. The $\Upsilon$ signals are described by Gaussian functions with common widths, their mass splittings constrained to the known values [16], and their yields scaled by $f_p$ and $1 - f_p$ in the prompt and displaced samples, respectively. The value of $f_p$ ranges from 96–99% depending on the $\Upsilon$ $p_T$ and whether the candidate was recorded with the CC or CF trigger scenario.

A second fit is performed to measure the mass-dependent ratio $r(m)$ of the prompt and displaced mass distributions. This is similar to the first fit, but uses only the sidebands in the prompt sample, $m(\mu^+\mu^-) < 9 \text{ GeV}/c^2$ or $m(\mu^+\mu^-) > 10.5 \text{ GeV}/c^2$, to avoid the need to model the $\Upsilon(nS)$ line shapes. The mass distribution of the background in the displaced sample is parametrized by a gamma function at low-$p_T$ and by an exponential function at higher $p_T$. The mass distribution of background in the prompt sample is accurately described by the mass distribution in the displaced sample multiplied by a scale factor that varies linearly with mass, $r(m) = a + bm$, with the coefficients $a$ and $b$ determined from the fit. The value of the function $r(m_{\Upsilon(nS)})$, evaluated at the $\Upsilon(1S)$ mass, varies between 1.8–3.9 over the range of dimuon $p_T$ considered. In the subsequent analysis of angular distributions, the value of this function and its uncertainty, both evaluated at the center of each mass range containing the $\Upsilon$ signals, are used to impose a Gaussian constraint on the background yield in the prompt sample.

The displaced sample provides a good description of the angular distribution of background in the prompt sample. This observation is consistent with all background arising from semileptonic decays of heavy quarks, or any small non-heavy flavor background component having the same angular distribution in prompt and displaced samples. We observe good agreement between the angular distributions in the prompt and displaced background, outside the $\Upsilon$ signal mass regions, even though the angular distributions change rapidly with dimuon $p_T$. A typical example illustrating this comparison is shown in Fig. 2. We then proceed to use the displaced sample to constrain the angular distribution of the background when analyzing the angular distribution of muons from $\Upsilon$ decays.

The prompt sample provides a good description of the dimuon background that remains in the complementary “prompt” sample. We verify that this is the case by comparing the angular distributions of prompt and displaced samples projected onto the $\cos \theta$ and $\varphi$ axes for mass ranges in the sidebands of the $\Upsilon$ signals. The level of agreement is quantified by computing the Kolmogorov–Smirnov statistic for each distribution, which suggests that any differences in their observed shapes are consistent with statistical fluctuations.

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In mass ranges containing the $\Upsilon$ signals, we perform a third simultaneous fit, in both the prompt and displaced samples, to the distributions of angles ($\cos \theta$, $\varphi$) collected.
in 20 \times 36 discrete intervals. Separate fits are performed for the two cases in which the angles represent the direction of the positive muon with respect to the axes of the s-channel helicity frame or those of the Collins–Soper frame. For both frames, the angular distributions are described by linear combinations of probability density functions for signal and background components. We factor these functions into an underlying angular distribution, with that of the \( \Upsilon \) signal parametrized using Eq. (1), and an acceptance function that accounts for the geometry of the muon detectors and the kinematic restrictions imposed by the trigger. The parameters in the underlying angular distributions for the \( \Upsilon \) signal and for the background are determined using a simultaneous, binned likelihood fit in which the expected numbers of events in each discrete angular interval are expressed as

\[
\frac{dN_p}{d\Omega_{ij}} \sim N_T f_p A_T (\cos \theta_i, \phi_j) \cdot w_T (\cos \theta_i, \phi_j; \vec{\lambda}_T) + \\
N_d s_p A_b (\cos \theta_i, \phi_j) \cdot w_b (\cos \theta_i, \phi_j; \vec{\lambda}_b),
\]

\[
\frac{dN_d}{d\Omega_{ij}} \sim N_T (1 - f_p) A_T (\cos \theta_i, \phi_j) \cdot w_T (\cos \theta_i, \phi_j; \vec{\lambda}_T) + \\
N_d A_b (\cos \theta_i, \phi_j) \cdot w_b (\cos \theta_i, \phi_j; \vec{\lambda}_b).
\]

In these expressions, \( N_T \) and \( N_d \) are the \( \Upsilon \) and displaced background event yields, \( f_p \) is the fraction of the \( \Upsilon \) signal in the prompt sample, and \( s_p \) is the ratio of the background yields in the prompt and displaced samples, which is Gaussian-constrained to \( r(m) \). The acceptance for signal \( A_T \) and background \( A_b \), which are calculated separately, are described below. The modeling of the background angular distribution is improved by imposing the additional kinematic restriction \( |p_T (\mu^+) - p_T (\mu^-)| < (p_T (\mu^+ \mu^-) - 0.5 \text{ GeV/c}) \), which removes back-to-back muons that have large values of \( \cos \theta \) in the s-channel helicity frame. This has a negligible effect on the \( \Upsilon(nS) \) acceptance for \( p_T (\Upsilon) > 6 \text{ GeV/c} \). The angular distributions for \( \Upsilon \) signal \( w_T \) and background \( w_b \) are described by sets of parameters \( \vec{\lambda}_T \) and \( \vec{\lambda}_b \). For the signal, \( w_T \) has the form of Eq. (1), whereas \( w_b (\cos \theta, \phi; \vec{\lambda}_b) \) has, in addition to the terms in Eq. (1), an empirical \( \lambda_4 \cos^4 \theta \) term that parametrizes the background shape more accurately in some ranges of \( p_T \) and invariant mass. The free parameters in the fit are \( N_T, N_d, s_p, \vec{\lambda}_T \), and \( \vec{\lambda}_b \), while \( f_p \) is fixed to the value determined previously from the first fit to the dimuon mass distributions.

The detector acceptance is calculated using a Monte Carlo (MC) simulation in which dimuon events are generated with isotropic distributions of decay angles are processed using the standard CDF II detector simulation and event reconstruction programs. Separate samples are simulated at fixed masses to calculate the acceptance for the three \( \Upsilon(nS) \) signals, while the acceptance for the dimuon background is calculated using a continuum of dimuon invariant masses ranging from 8 to 12 GeV/c^2.

Figure 3 shows distributions for the data and the best fit model in one of the ranges of dimuon \( p_T \) in the mass range containing the \( \Upsilon(1S) \) signal. The shapes of the projected distributions are primarily determined by the acceptance since the resolution with which \( \cos \theta \) and \( \phi \) are measured is much smaller than the binning chosen for the fit. However, the agreement between the data and the model in regions where the acceptance changes rapidly, demonstrates that resolution effects are modeled accurately in the detector simulation. No significant discrepancy between the data and the fit model is observed in \( \chi^2 \) tests applied to one dimensional projected distributions over the analyzed range of dimuon mass and \( p_T \).

Systematic uncertainties on the parameters \( \lambda_0, \lambda_\varphi \) and \( \lambda_{0\varphi} \) due to the limited precision with which the trigger and reconstruction efficiencies are determined are evaluated by repeating the analysis with acceptances recalculated with all efficiencies simultaneously varied by \( \pm 1 \sigma \). The resulting change in the fitted parameters provides a conservative estimate of the sensitivity to the measured acceptance. Because the measured parameters depend on the estimated background in the prompt sample, an alternate, quadratic parametrization of the function \( r(m) \) was also investigated. The resulting small variations in the fitted angular distribution parameters are treated as an additional systematic uncertainty. Finally, the contribution of the uncertainty in the fitted parameters due to the finite MC sample size used to calculate the acceptance was estimated using ensembles of MC simulations with the same size used in the analysis of
the data. The uncertainties due to finite MC sample size and the determination of efficiencies are at most 30% of the size of the statistical uncertainty for the three \( \Upsilon(nS) \) states, while the uncertainty due to the treatment of the prompt scale factor function is no more than 20% of the statistical uncertainty.

Figure 4 shows the rotational invariant \( \hat{\lambda} \) which is calculated from the measured values of \( \lambda_0 \) and \( \lambda_0 \) in each \( p_T \) range for both the Collins–Soper and \( s \)-channel helicity frames. Uncertainties in \( \hat{\lambda} \) measured in the two coordinate frames are highly correlated. Monte Carlo simulations are used to calculate the expected sizes of differences between the two values of \( \hat{\lambda} \) and in most cases, the observed deviations are found to be consistent with purely statistical fluctuations. A systematic uncertainty derived from the difference between \( \hat{\lambda} \) measured in the two coordinate frames is only significant for the lowest three \( p_T \) ranges of the \( \Upsilon(3S) \). In the lowest \( p_T \) range, the values of \( \hat{\lambda} \) measured for the \( \Upsilon(3S) \) differ by 2.4 \( \sigma \), without accounting for systematic uncertainties, and is the only case where the angular distribution is observed to be significantly non-isotropic. However, we can find no evidence to suggest that this is due to a bias or systematic effect since we do not see a similar trend for the \( 1S \) and \( 2S \) states and can find no anomalous behavior in any of the underlying distributions.

The values of \( \hat{\lambda} \approx 0 \) suggest that the decays of all three \( \Upsilon(nS) \) resonances are consistent with an unpolarized mixture of states. Table II lists the values of \( \lambda_0 \) measured in the \( s \)-channel helicity frame for the \( \Upsilon(1S) \), \( \Upsilon(2S) \), and \( \Upsilon(3S) \) states, with the systematic uncertainties described above added in quadrature [17]. Figure 5 shows a comparison of the \( \lambda_0 \) parameter, measured for the \( \Upsilon(1S) \) state in the \( s \)-channel helicity frame, with previous measurements. The current result is found to be statistically consistent with the previous measurement from CDF [4], which was made for \( |y| < 0.4 \) at \( \sqrt{s} = 1.8 \) TeV rather than \( |y| < 0.6 \) and \( \sqrt{s} = 1.96 \) TeV. Restricting the current measurement to \( |y| < 0.4 \) does not change the results appreciably. The current \( \Upsilon(1S) \) result is inconsistent with the previous measurement from the D0 experiment [13] at the level of 4.5\( \sigma \).

In conclusion, we have measured the angular distributions of muons from \( \Upsilon(1S) \), \( \Upsilon(2S) \), and \( \Upsilon(3S) \) decays with \( |y| < 0.6 \) and in several ranges of transverse momentum up to 40 GeV/c. We find that the decay-angle distributions of all three \( \Upsilon(nS) \) states are nearly isotropic, as suggested by previous measurements [4] in the case of the \( \Upsilon(1S) \). This is the first measurement to simultaneously determine the three parameters needed to fully quantify the angular distribution of \( \Upsilon(nS) \rightarrow \mu^+\mu^- \) decays. This is also the first analysis to present information on the angular distribution of \( \Upsilon(3S) \) mesons produced in high energy \( p\bar{p} \) collisions.

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| $p_T$ (GeV/c) | $\lambda_\theta (\Upsilon(1S))$ | $\lambda_\theta (\Upsilon(2S))$ | $\lambda_\theta (\Upsilon(3S))$ |
|-------------|-------------------------------|-------------------------------|-------------------------------|
| 0–2         | $0.01 \pm 0.10 \pm 0.05$     | $-0.07 \pm 0.20 \pm 0.05$    | $0.32 \pm 0.36 \pm 0.12$    |
| 2–4         | $-0.08 \pm 0.04 \pm 0.03$    | $-0.29 \pm 0.10 \pm 0.06$    | $-0.19 \pm 0.17 \pm 0.08$   |
| 4–6         | $-0.11 \pm 0.04 \pm 0.03$    | $-0.11 \pm 0.10 \pm 0.05$    | $-0.01 \pm 0.19 \pm 0.09$   |
| 6–8         | $-0.12 \pm 0.06 \pm 0.04$    | $-0.09 \pm 0.12 \pm 0.07$    | $-0.48 \pm 0.14 \pm 0.13$   |
| 8–12        | $-0.15 \pm 0.06 \pm 0.02$    | $-0.18 \pm 0.11 \pm 0.03$    | $0.07 \pm 0.18 \pm 0.04$    |
| 12–17       | $-0.13 \pm 0.08 \pm 0.04$    | $-0.09 \pm 0.15 \pm 0.08$    | $0.14 \pm 0.22 \pm 0.07$    |
| 17–23       | $-0.23 \pm 0.13 \pm 0.08$    | $0.06 \pm 0.25 \pm 0.12$     | $0.14^{+0.36}_{-0.33} \pm 0.12$ |
| 23–40       | $-0.21 \pm 0.24 \pm 0.11$    | $0.00^{+0.39}_{-0.33} \pm 0.16$ | $0.14^{+0.50}_{-0.42} \pm 0.13$ |

TABLE I: Values of $\lambda_\theta$, measured in the $s$–channel helicity frame for each range of $p_T(\Upsilon)$. The first uncertainty is statistical while the second uncertainty is systematic and is described in the text.

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