Direction finding and beamforming using cylindrical array of dipole antennas in the presence of cylindrical scatterer/reflector including the mutual coupling effect

Sarah Poormohammad | Forouhar Farzaneh | Ali Banai

Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

Correspondence
Sarah Poormohammad, Department of Electrical Engineering, Sharif University of Technology, Tehran, 11155-4363, Iran.
Email: sara.poormohammad@gmail.com

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Abstract
Three-dimensional dipole arrays are normally used at the proximity of a reflective structure. Filamentary short-circuited dipoles are proposed to model the effect of the reflector structure. The computational burden is significantly reduced by using mutual impedance matrices. An analytical method of modelling the effect of the cylindrical reflector in a three-dimensional cylindrical geometry of dipole antenna arrays is introduced both for direction finding and beamforming applications in the presence of mutual coupling. The results of the implementation of the MUSIC (multiple signal classification) direction-finding algorithm show that the proposed model for accounting the cylindrical reflector, in the presence of a mutual coupling effect, has the required efficiency and reduced root-mean-square-error (RMSE). Monte-Carlo simulations for the beamforming algorithm are conducted to evaluate the signal-to-interference ratio (SIR) values in the presence of a cylindrical reflector and the results show that the model has the desired performance. The whole process of beamforming and pattern generation has been verified through high-frequency structural simulator (HFSS) simulation. The simplicity, adequate precision and low computational cost of this method can be appropriately used in the cylindrical reflector/scatterer in three-dimensional geometries, especially for base station applications.

1 | INTRODUCTION

The need for accurate direction-finding and beamforming processes arises in many applications such as cellular and wireless communications, radar, sonar and navigation systems. Over the past two decades different algorithms and arrays geometries have been proposed for direction finding and beamforming in smart antenna arrays [1,2]. For example, the simulations in [3,4] showed that the novel three-dimensional (3D) geometries had a better performance in comparison to the conventional geometries in terms of RMSE (root-mean-square-error) and SIR (signal-to-interference ratio). In realistic environments, the mutual coupling between elements and the presence of a nearby reflector or scatterers decreases the efficiency of beamforming and direction-finding algorithms. For better system performance, the mutual coupling effect needs to be compensated [5]. The mutual coupling effects were considered in Bartlett and (multiple signal classification) (MUSIC) direction-finding algorithms in [6,7]. The simulations in [7] showed that the proposed method modelled properly the mutual coupling effects in three-dimensional arrays and predicted correctly the RMSE in direction finding. The mutual coupling effects between dipole antenna arrays decreased the beamforming efficiency, especially in compact configurations. The sub-optimal beamforming for 3D cylindrical arrays of dipoles was proposed in [8] including mutual coupling effects. The method in [8] included analytical and low-computational cost beamforming in the presence of mutual coupling. The presence of a scatterer near the antenna array affects the precision of the direction-finding algorithm as well. The effects of a local scatterer on DOA (direction of arrival) estimation with MUSIC algorithm was represented in [9] in terms of angular spread and an approximate distribution for DOA was derived at. Another eigen-space method for...
computing the optimal weight vector in wide-angle spread circumstances due to scattered signal components was established in [10] for adaptive beamforming. In [11], the authors investigated the array calibration algorithm in beamforming to model the errors such as scattering by the edge of the ground plate and mutual coupling between elements. A self-calibration method in the presence of 3D scatterers of resonating size was presented in [12]. The proposed algorithm in [12] removed the effect of mutual coupling and 3D scatterers in a direction-finding algorithm. In [12] the scatterer was approximated as a sphere but the method had two drawbacks, firstly, it had limited application in cases where the time delay is important (due to its iterative method) and secondly, as the electrical size or number of scatterers increases the number of unknowns (which requires more antenna elements). In [13] the authors represented a similar iterative DOA estimation where the additive white Gaussian noise (AWGN) and 3D scatterers were simultaneously presented. The classical beamforming method in [13] was performed with the cubic, spherical and ellipsoidal scatterers based on electromagnetic simulations with iterative DOA estimation. The simulation in [13] showed that the presence of a scatterer and noise shifts the peak of the detected signal related to the direction of arrival. The near-field scattering can degrade the performance of adaptive beamforming algorithms. In [14] the presence of a local scatterer was modelled by a mismatch in the steering vector for a robust adaptive beamforming algorithm. The near-field scattered signal component was incorporated into the presumed steering vector for an adaptive beamforming algorithm in [15].

As mentioned above, a number of computational algorithms were proposed for direction finding and beamforming in the presence of mutual coupling and scattering effects. However, in all those methods there is a high computational burden while no three-dimensional array in the presence of a central reflector has been considered. A three-dimensional array of dipoles in the vicinity of a cylindrical reflector is considered by the authors. The cylindrical reflector was disintegrated into filamentary short-circuited dipole elements. Furthermore, the induced EMF method was used for computation of the mutual impedance matrix, which drastically reduces the computation time with respect to the conventional 3D electromagnetic simulators. The algorithms of [7,8] were conducted for accounting the mutual coupling with a novel formulation for the reflector’s effect. The effects of the central reflector were considered with an analytical method instead of using computationally time-consuming electromagnetic or iterative simulations.

In the following sections the model of a cylindrical reflector in the presence of mutual coupling effects is first presented. Section 3 a cylindrical dipole array at the vicinity of a cylindrical reflector is considered and by implementation of the MUSIC algorithm the direction-finding process is realised in the presence of mutual coupling. In Section 4 a sub-optimal beamforming method accounting the reflector and mutual coupling effect is proposed and examined for 3D array geometry. Finally, Section 5 draws the conclusions.

2 | MODEL OF A CYLINDRICAL REFLECTOR IN THE PRESENCE OF MUTUAL COUPLING EFFECTS

In this section a model is proposed for the cylindrical reflector at the centre of a cylindrical array of half-wave dipoles. For modeling the cylindrical reflector, one should have an appropriate formulation for the impedance matrix of the whole structure. For this purpose, the cylindrical array of six half-wave dipole antennas that are spaced equally by one wavelength distance and so the radius of cylindrical array is considered and is given by:

$$r_c = \frac{\lambda}{2\sin \frac{\pi}{6}} = \frac{\lambda}{6}$$  \hspace{1cm} (1)

As the distance between the reflector and the antenna array elements should be about $\frac{\lambda}{4}$, the radius of the cylindrical reflector should be $\frac{\lambda}{2}$. The height of the reflector is chosen to be $\frac{\lambda}{2}$. The cylindrical dipole array (with six elements) with conductive reflector at the centre is shown in Figure 1.

In this section for analytical formulations, we propose a novel model that replaces the cylindrical reflector with equally spaced filamentary short-circuited dipole reflectors. The number of reflectors for the sake of precision is chosen to be 18 and their height is equal to the height of the cylindrical reflector ($\frac{\lambda}{2}$). The geometry of cylindrical array with filamentary dipole reflectors is shown in Figure 2.

The excitation currents on the filamentary reflectors can be calculated from the antenna elements’ currents with analytical relations according to Equation (2).

$$Z_{11}I_1 + Z_{12}I_2 = V_1$$
$$Z_{21}I_1 + Z_{22}I_2 = V_2$$  \hspace{1cm} (2)

where the subscript 1 denotes the antenna array elements and subscript 2 denotes the reflector elements. Where $Z_{11}$ is the impedance matrix (of 6×6 dimension) of the dipole array, $Z_{22}$

![Figure 1](image-url) Six-element cylindrical dipole array at the vicinity of a cylindrical conductive reflector
is the impedance matrix of the reflector array (of 18×18 dimension) and $Z_{21}$ is the coupling impedance matrix between the dipole array and the reflector array (of 18×6 dimension) and $V$ and $I$ are the voltage and the current vectors, respectively. Note that for saving the computation time the authors have considered the mutual coupling only between the adjacent antenna elements, as such the $Z_{ii}$ matrices are mostly sparse. Considering the short-circuited filamentary reflector elements, the excitation current vector of filamentary reflectors is calculated by Equation (3).

$$
V_2 = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}
$$
$$
I_2 = -Z_{22}^{-1} Z_{21} I_1
$$

In order to test the proposed model, the authors compare the normalised radiation pattern of the cylindrical array (with uniform in-phase excitation) including the cylindrical reflector and the filamentary reflectors using our MATLAB (Matrix Laboratory) program and the HFSS (high-frequency structural simulator) simulations in Figure 3. For the sake of greater clarity, the radiation patterns of the arrays are compared with filamentary reflectors based on the numerically extracted $Z$ matrix using HFSS and those computed based on the $Z$ matrix computed by the induced EMF method, as shown in Figure 3. These simulations were conducted for the six-element cylindrical array using half-wave dipole antennas with a radius of $\frac{\lambda}{2}$ at a frequency of 1 GHz. The number of filamentary reflectors is considered to be 18 and the heights of the cylindrical reflector and filamentary reflectors are $\frac{\lambda}{2}$. The result of the radiation pattern (for uniform in-phase feed) for each configuration is shown at $\theta = 90^\circ$ (the x–y plane).

For the sake of comparison, in our simulations we have considered four cases. Firstly, a continuous reflector with antenna elements was considered and HFSS simulation was performed. Secondly, filamentary reflectors were considered and HFSS simulation was performed. Thirdly, the filamentary reflectors were considered with computation of the mutual impedance matrix using an induced EMF method within a MATLAB program. Fourthly, the filamentary reflectors were considered using the mutual impedance matrix obtained by HFSS within the MATLAB program. It is obvious that the first case is the most precise and the third case is the least time consuming.

As can be seen in Figure 3 there is good agreement between the results of HFSS simulations (cases 1 and 2) and the authors’ MATLAB program computations (cases 3 and 4). It is noteworthy that the difference between the case 1 (pure HFSS simulation result) and the case 3 (pure MATLAB computation) is the least. The results illustrated in Figure 3 indicate that the idea of replacing the cylindrical reflector with short-circuited filamentary reflectors can model the reflector’s effect properly. The excitation currents of filamentary reflectors are calculated by Equation (3) using the $Z$ matrix based on the EMF method. In the next sections the idea of using filamentary reflectors to model the cylindrical reflector in direction finding and beam-forming processes in the presence of mutual coupling effects for cylindrical geometry of dipole arrays is discussed.

### 3 DATA MODEL FOR THE MUSIC DIRECTION FINDING ALGORITHM IN THE PRESENCE OF THE REFLECTOR AND THE MUTUAL COUPLING EFFECTS

In this section a three-dimensional cylindrical array of 36 half-wave dipole antennas stacked in three-plane (12 elements in each plane) is considered. The distance between the planes of the array is considered to be $\frac{\lambda}{4}$ and the height of the centre reflector is set at $\frac{9\lambda}{4}$. The distance between the antenna elements in each plane is $\frac{\lambda}{2}$ and the radius of the cylindrical array is equal to $\frac{\lambda}{4 \sin (\frac{\lambda}{2})}$; consequently the radius of the reflector should be 0.72λ. The geometry of the three-dimensional cylindrical array including a cylindrical reflector is shown in Figure 4.
To model the cylindrical reflector at the centre of a three-layer cylindrical dipole array a three-layer array of filamentary reflectors is considered, as depicted in Figure 5.

Each plane consists of 36 filamentary reflectors with a height of 1.49λ and radius of $\frac{\lambda}{2}$ at a frequency of 1 GHz. As is clear in the MUSIC algorithm, the direction of the signal and interferences is determined by the maxima of the Pseudospectrum function. The Pseudospectrum function has been modified using the mutual impedance matrix [7] as described in Equation (4).

$$P_{\text{MUSIC}}(\theta, \varphi) = \frac{1}{|a^H(\theta, \varphi) (Z^{-1}_0)^H E_n E_n^H(Z^{-1}_0) a(\theta, \varphi)|}$$

where $a(\theta, \varphi)$ is the steering vector, $E_n$ is the eigen-vector of the noise subspace of the array correlation matrix and $Z_0$ is the normalised mutual impedance matrix.

The calculation of a normalised mutual impedance matrix is based on the induced EMF method [16]. The excitation currents of the short-circuited filamentary reflectors are related to the currents of antenna elements as shown here:

$$I_i = \left[ \begin{array}{c} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{array} \right] = \left[ \begin{array}{cccccc} Z_{44} & Z_{45} & Z_{46} \\ Z_{54} & Z_{55} & Z_{56} \\ Z_{64} & Z_{65} & Z_{66} \end{array} \right]^{-1} \cdot \left[ Z_{41} & Z_{42} & Z_{43} \\ Z_{51} & Z_{52} & Z_{53} \\ Z_{61} & Z_{62} & Z_{63} \right] \cdot \left[ I_1 \\ I_2 \\ I_3 \right]$$

Note that in Equation (5) every current is a vector itself and every matrix element $Z_{ij}$ is a matrix itself.

The subscripts 4, 5 and 6 refer to the filamentary reflector and the subscripts 1, 2 and 3 refer to the array elements themselves. In order to test the efficiency of the proposed model the authors use the three-dimensional geometry of Figure 5 for direction finding with the MUSIC algorithm including the mutual coupling effect. The sample azimuth and elevation angles of the signal and interferences are considered in Table 1.

The number of snapshots is 100 and the SNR value is chosen to be 10 dB. A two-dimensional MUSIC spatial spectrum as a function of azimuth and elevation angles for the geometry of Figure 5 is shown in Figure 6.

As shown in Figure 6, the proposed model estimates the direction of arrival of signal and interferences properly in the presence of a cylindrical reflector for a particular scenario. To further verify the efficiency of the method the authors performed a number of Monte-Carlo simulations. Through the Monte-Carlo simulations they estimated the RMSE of the related angle of arrivals. In this case the RMSE of arrival angles versus the number of snapshots and the signal-to-noise ratio (SNR) are presented. For calculation of the values of RMSE versus the number of snapshots 1000 random scenarios for azimuth and elevation angles of arrival with $\theta_i \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right]$ and $\varphi_i \in [0, 2\pi]$ for 10 simultaneous signal and interferences were considered. Simulations were conducted for fixed SNR = 10 dB and the values of RMSE were calculated for different numbers of snapshots between 20 and 200. Figure 7 shows the results of an RMSE evaluation versus the number of snapshots.

| Angle            | Azimuth | Elevation |
|------------------|---------|-----------|
| Signal           | 150     | 72        |
| Interference#1   | 35      | 64        |
| Interference#2   | 45      | 17        |
| Interference#3   | 70      | 57        |
| Interference#4   | 90      | 54        |
| Interference#5   | 115     | 42        |
| Interference#6   | 127     | 25        |
| Interference#7   | 100     | 72        |
| Interference#8   | 150     | 19        |
| Interference#9   | 17      | 65        |
snapshots for a cylindrical array of dipole antennas including the reflector and mutual coupling effect.

Now the simulations of RMSE versus the different values of SNR between 0 and 10 dB for a fixed number of 100 snapshots and for a thousand random scenarios of angle of arrivals can be conducted. The corresponding RMSE values versus the SNR are presented in Figure 8.

The results of Figures 7 and 8 show that the proposed model for accounting the cylindrical reflector in the presence of a mutual coupling effect has an acceptable RMSE in direction finding using the MUSIC algorithm.

The simplicity, satisfactory performances of the proposed model, analytical formulation for mutual coupling effects and low-computation time allow the real-time implementation of this model for different applications of direction finding, especially in base stations.

**FIGURE 6** The sample Pseudospectrum of the MUSIC algorithm for the cylindrical array including the reflector and the mutual coupling effects for the incident signal at $\theta = 72^\circ$ and $\phi = 150^\circ$ with nine interferers

**FIGURE 7** Direction-finding root-mean-square-error (RMSE) versus the number of snapshots for a cylindrical array of dipole antennas with a cylindrical reflector at fixed signal to noise ratio (SNR) = 10 dB

**FIGURE 8** Direction-finding root-mean-square-error (RMSE) versus the signal to noise ratio (SNR) for a cylindrical array of dipole antennas with a cylindrical reflector at a fixed number of snapshots = 100

**FIGURE 9** The resulting normalised radiation pattern of beamforming for a cylindrical dipole array in the presence of a cylindrical reflector for a sample single signal and nine interferences

**FIGURE 10** Normalised radiation pattern of beamforming for cylindrical dipole array in the presence of a cylindrical reflector for a sample single signal and nine interferences in dB. The deep nulls are well matched to the interferences
FIGURE 11  Comparison between 2D radiation patterns which have been made by the proposed method and those computed by high frequency structural simulator for five fixed elevation angles, versus the azimuth angle: (a) $\theta = 72^\circ$ plane; (b) $\theta = 19^\circ$ plane; (c) $\theta = 25^\circ$ plane; (d) $\theta = 42^\circ$ plane; and (e) $\theta = 64^\circ$ plane.
4 | SUB-OPTIMAL BEAMFORMING ALGORITHM IN THE PRESENCE OF THE MUTUAL COUPLING EFFECTS AND THE CYLINDRICAL REFLECTOR

The mutual coupling between antenna elements affects significantly the radiation patterns and the SIR values in beamforming processes. The sub-optimal beamforming algorithm was proposed and examined for a 3D cylindrical array of dipole antennas in the presence of mutual coupling effects in [8]. In this section a modified form of [8] is established for accounting for the presence of a cylindrical reflector in the beamforming process. The cylindrical reflector is modelled with three layers of filamentary reflectors with equal height, as depicted in Figure 5, and the excitation currents of the short-circuited filamentary reflectors are calculated from the currents of the excited antenna elements using Equation (5). To illustrate the efficiency of the proposed method, the beamforming process is conducted for the same scenario angle of arrivals including signal and interferences, as Table 1. The normalised radiation pattern constructed from the optimum weights as a function of azimuth and elevation angles is presented in Figure 9 and the direction of arrival of the desired signal is marked in the figure.

For better demonstrating the depth of nulls, the radiation pattern of Figure 9 is represented in Figure 10 in decibels and the directions of arrival for interferences are marked.

For further verification, the radiation pattern of the array in the presence of the cylindrical reflector is evaluated using HFSS software with optimum weights of antennas elements. The normalised radiation patterns constructed from beamforming optimum weights including mutual coupling and reflector effects for the fixed-elevation angle corresponding to a signal and five interferences are compared to the HFSS results in Figure 11a–e.

The radiation pattern of the array in the presence of the cylindrical reflector using the HFSS software with optimum weights is shown in Figure 12.

To evaluate the efficiency of the proposed method in the beamforming process, the Monte-Carlo simulations are conducted with 1000 random scenarios for azimuth and elevation angles of arrival with \( \theta_i \in \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \) and \( \varphi_i \in [0, 2\pi] \). The results of the mean value of SIR for cylindrical dipole array of Figure 5, including the cylindrical reflector, are compared to the same geometry without the reflector versus the number of interferences in Figure 13.

5 | CONCLUSIONS

Computing the effect of cylindrical reflectors or scatterers being normally at the proximity of three-dimensional cylindrical arrays, is important in precise evaluation of these array performances. A filamentary dipole array replacement has been proposed to model the performance of the reflector or a scatter. The mutual impedance matrices between dipole antennas were used as the main computational tool in these evaluations. Furthermore, only the couplings between a dipole and its proximity elements were considered as the corresponding impedance matrices became relatively sparse without compromising the validity of the computations. For the sake of verification, the whole array structure performance was compared to those obtained by HFSS simulations, both for uniform distribution and weighted distribution of the array excitations, which resulted in fairly good agreement between the corresponding results.

Through the authors’ evaluations it has been shown that the direction-finding precision and SIR values are relatively improved in the presence of a cylindrical reflector structure.

It is noteworthy that in each structure it is required to compute the mutual impedance/coupling matrices once and through the direction-finding/beamforming process the same matrices are used. It is clear that this method has a relatively lower computational burden than HFSS simulations. The method is useful for smart antenna applications in complex base stations.

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