Continuous mass-dependent analysis of the non–singlet DIS data

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Abstract

We consider the issue of an accurate description of the evolution of the non-singlet structure function moments $M_n(Q)$ near heavy quark threshold. To this aim we propose a simple modification of the standard massless $\overline{\text{MS}}$ scheme approach to the next-to-leading QCD analysis of DIS data. We apply it to the processing of the modern CCFR data for $x F_3$ structure function and extract the value of

$$\alpha_s(M_Z) \approx 0.108 \pm 0.004$$

We check also the consistency of light gluino hypothesis with CCFR data.
1 Introduction

An important means of verification of the validity of pQCD (that is, of perturbative QCD "improved" by the RG summation) is an analysis of deep inelastic scattering (DIS) data. To interpret these data within pQCD, one should pay credit to a number of subtle physical effects: contributions of high twists, nuclear effects, high–order (three-loop) corrections and the influence of thresholds of heavy particles. All the introduced corrections are roughly of the same order of magnitude.

This paper is devoted to the problem of influence of the thresholds of heavy quark (HQ) on the pQCD analysis of DIS data that includes, in particular, the evolution of the strong coupling constant $\bar{\alpha}_s(Q)$. Recent estimates performed in [1, 2, 3] have revealed a significant role of threshold effects in the $\bar{\alpha}_s(Q)$ evolution when the DIS data lie close to the position of "Euclidean–reflected" threshold of heavy particles. The corresponding corrections to $\alpha_s(M_Z)$ can reach several per cent, i.e., they are of the order of the three–loop and nuclear effects on $\alpha_s(M_Z)$.

A common algorithm for the renormalization–group (RG) resummation is based upon beta–function $\beta(\alpha_s)$ and anomalous dimensions $\gamma(\alpha_s)$ calculation and the RG differential equations integration performed within the \MS renormalization scheme. However, the widespread massless \MS scheme fails to describe the data near thresholds of heavy particles $– b, c$ quarks and, maybe, light superpartners [2].

An appropriate procedure for the inclusion of threshold effects into the $Q^2$–dependence of $\bar{\alpha}_s(Q)$ in the framework of the massless \MS scheme was proposed more than 10 years ago [5, 6]: transition from the region with a given number of flavors $f$ described by massless $\bar{\alpha}_s(Q; f)$ to the next one with $f+1$ ("transition across the $M_{f+1}$ threshold") is realized here with the use of the so–called "matching relation" for $\bar{\alpha}_s(Q)$ [6]. The latter may be considered as the continuity condition for $\bar{\alpha}_s(Q)$ on (every) HQ mass

$$\bar{\alpha}_s(Q = M_{f+1}; f) = \bar{\alpha}_s(Q = M_{f+1}; f + 1)$$

that provides an accurate $\bar{\alpha}_s(Q)$–evolution description for $Q$ values not close to the threshold region. The condition (1) is used up to the three–loop level; the other version of the matching can be found in [6].

One needs also one more element, the matching procedure for the evolution of the structure function moment $M_n(Q, m)$. The corresponding expressions for anomalous dimensions $\gamma(n; f)$ are well known in the \MS scheme for a fixed $f$ value (see, e.g., [6]) but until now there is no recipe for obtaining a continuous interpolation across the HQ threshold for the moment evolution.

In this paper, we are going to focus just on this aspect of the problem: how does the HQ threshold influence the evolution of the DIS structure function? We will examine only non-singlet processes of DIS so as to pass over the delicate problem of modification of the operator product expansion in DIS through introducing a new scale, the mass of a heavy parton (for discussion, see [6]).

To solve the problem we propose a rather simple modification of the massless \MS scheme to take into account thresholds in analyzing the moments of the DIS non-singlet structure...
function at the two-loop level.

To simplify the exposition, we shall take advantage of the explicit analytic mass-dependent RG–solution derived in [9] and [10] that is expressed directly in terms of $\bar{\alpha}_s(Q, m)$ and $M_n(Q, m)$ perturbation expansion coefficients. This allows us to avoid the use of RG–generators, that are $\beta$ and $\gamma$–functions. In the next section, we present smooth analytic expressions for the evolution of $\bar{\alpha}_s(Q, m)$ and $M_n(Q, m)$ at the 2–loop level based on this mass–dependent RG formalism. We shall omit all theoretical and technical details (they can be found in refs. [9, 10, 11]) and write only final results. In Sect. 3, we introduce the “spline-approximation” to describe the two-loop level continuous moment evolution, and present there another proof of the matching condition (1). In Sect. 4, we describe briefly a method of analysis of the DIS data. On its base we carry out the fit of fresh CCFR Collab. data, extract the parameter $\alpha_s$ and estimate the contribution of threshold effects. We discuss the consistency of MSSM light gluino existence with CCFR data by using the spline–type evolution of $M_n(Q, m)$ in Sect. 5.

Throughout the paper we use the notation: $a = \alpha_s/4\pi$, $(\bar{a} = \bar{\alpha}_s/4\pi)$; indices in brackets stand for the loop number, e.g., $\beta_{k(\ell)} = \beta_{\ell-1}$; instead of the structure function moments $M_n(Q, m)$, we consider only its “evolution part” $\mathcal{M}_n(Q, m)$ (i.e., moments of the distribution function)

$$M_n(Q, m) = C_n(a, Q, m) \cdot \mathcal{M}_n(Q, m),$$

where $C_n$ are moments of the coefficient function of a certain DIS process (see, e.g., Ref. [13]).

## 2 Mass-dependent RG solutions

In the massless case, the moment two-loop evolution is described by the expression

$$\mathcal{M}_n^{(2)}(Q) = \mathcal{M}_n(\mu) \left( \frac{a}{\bar{a}^{(2)}(Q)} \right)^{d_n} \exp \{ [a - \bar{a}^{(2)}(Q)] f_n \}$$

with the numerical coefficients

$$d_n = \frac{\gamma_0(n)}{\beta_0}; \quad f_n = \frac{\beta_0 \gamma_1(n) - \beta_1 \gamma_0(n)}{\beta_0^2};$$

$$\beta_0 = \beta_{(1)}(f) = 11 - \Delta \beta_{(1)} f; \quad \Delta \beta_{(1)} = 2/3; \quad \beta_1 = \beta_{(2)}(f) = 102 - \Delta \beta_{(2)} f; \quad \Delta \beta_{(2)} = 38/3.$$

In the mass-dependent case, one should use instead of Eq.(3), a bit more complicated expression [3] of the same structure

$$\mathcal{M}_n^{(2)}(Q) = \mathcal{M}_n(\mu) \left( \frac{a}{\bar{a}^{(2)}(Q, m)} \right)^{D_n(Q,...)} \exp \{ [a - \bar{a}^{(2)}(Q, m)] F_n(Q,...) \}$$

with the functional coefficients $D_n, F_n$

$$D_n(Q, m, \mu) = \frac{\Gamma_{(1)}(n, Q)}{A_{(1)}(Q, m, \mu)}; \quad F_n(Q,...) = \frac{\Gamma_{(n)}(n, m)}{A_{(n)}(Q, m, \mu)};$$

where $\Gamma_{(n)}(n, m)$ is a certain function of the loop number, that incorporates threshold effects.
\[ F_n(Q, m, \mu) = \frac{A_{(1)}(Q, m, \mu)\Gamma_{(2)}(n, Q) - A_{(2)}(Q, m, \mu)\Gamma_{(1)}(n, Q)}{[A_{(1)}(Q, m, \mu)]^2} \] (7)

and the two-loop running coupling \( \bar{a} \) taken in the form

\[
\frac{1}{\bar{a}^{(2)}(Q, m; a)} = \frac{1}{a} + A_{(1)}(Q, m, \mu) + \frac{A_{(2)}(Q, m, \mu)}{A_{(1)}(Q, m, \mu)} \ln[1 + aA_{(1)}(Q, m, \mu)] . \] (8)

In the non-singlet case of DIS,

\[
\Gamma_{(1)}(n, m, Q) = \Gamma_{(1)}(n, l) = \gamma_{(1)}(n)l, \ l = \ln \left( \frac{Q^2}{\mu^2} \right) ,
\]

and the HQ-mass-dependent \( A_{(\ell)} \), \( \Gamma_{(\ell=2)}(n, Q) \) appearing in Exp. (6-8) are just perturbation expansion coefficients:

\[
\bar{a}(Q, m, \mu; a)_{\text{pert}} = a - a^2 A_{(1)}(Q, m, \mu) + a^3 \left\{ \left[ A_{(1)}(Q, ... \right]^2 - A_{(2)}(Q, m, \mu) \right\} + \ldots ;
\]

\[
\frac{M_n(Q)}{M_n(Q_0)} \bigg|_{\text{pert}} = 1 + a\Gamma_{(1)}(n, l) + a^2 \left\{ \frac{\Gamma_{(1)}(n, l) \left( \Gamma_{(1)}(n, l) - A_{(1)}(Q, m, \mu) \right)}{2} + \Gamma_{(2)}(n, Q, m, \mu) \right\} + \ldots \] (9)

satisfying the normalization condition \( -A_{(\ell)}(Q = \mu) = \Gamma_{(\ell)}(n, Q = \mu) = 0 \).

These coefficients consist of the usual massless part (for \( f = 3 \)) and HQ-mass dependent contributions, e.g.,

\[
A_{(1)}(Q, m, \mu) = \left( \frac{1}{\bar{a}^{(1)}(Q, m, \mu; a)} - \frac{1}{a} \right) = \beta_{(1)}(3)l - \Delta \beta_{(1)} \sum_h \left[ I_1 \left( \frac{Q^2}{m_h^2} \right) - C \right] \] (10)

with summation over HQ’s: \( h \geq 4 \). Here \( I_1 \) is the one-loop fermion mass-dependent contribution, like the polarization operator [14] or the three–gluon vertex loop [15] subtracted at \( Q^2 = 0 \) and \( C \) being some subtraction scheme-dependent constant.

“Massive” RG solutions (5) and (8) possess several remarkable properties:

- they are built up only of “perturbative bricks”, i.e., loop-expansion coefficients \( A_{(\ell)} \), \( \Gamma_{(\ell)}(n) \) (taken just in the form they appear in the perturbative input) and “contain no memory” about the intermediate RG entities such as \( \beta \) and \( \gamma \) functions;
- in the massless case with pure logarithmic coefficients, \( A_{(k)} = \beta_{(k)}l, \ \Gamma_{(k)}(n) = \gamma_{(k)}(n)l, \) they precisely correspond to the usual massless expressions, like Eq. (3);
- being used in QCD, they smoothly interpolate across heavy quark threshold between massless solutions with different flavors numbers.
3 Smooth schemes and MS massless schemes

3.1 Smooth mass-dependent scheme

We have above considered general formulae to describe the $M_n(Q)$-evolution including the threshold effects. It is clear that the mass–dependent MOM schemes automatically provide the most natural smooth description of thresholds. To use them in the framework of leading order, one needs a mass-dependent expression for $I_1$ presented of Appendix A (for two different schemes). So, to perform the one-loop evolution analysis of moments, one should substitute Eq. (23) or (27) in Appendix A into Eq. (10) and then into Eq. (8) and Eq. (6), and use the approximation

$$M_n(Q) = M_n(\mu) \left( \frac{a}{\bar{a}(Q)} \right) D_n(Q, ...).$$

(11)

We shall perform the fit of CCFR data following this formula in the next section.

However, the MOM scheme meets tremendous calculational difficulties in the next-to-leading order of pQCD. Moreover, each of the expansion coefficients $A(i), \Gamma(i)$ (see Sec. 2) becomes gauge–dependent at the two-loop level, which is not convenient. These difficulties are absent in the widespread \( \overline{\text{MS}} \) scheme. One can go far in loop calculations here (see [12]), but the scheme is not sensitive to the thresholds at all. Below we suggest a practical compromise between these different possibilities – the “spline” scheme. This scheme possesses both the sensitivity to thresholds and simplicity of the \( \overline{\text{MS}} \) procedure. Nevertheless, the \( \overline{\text{MS}} \) scheme looks like a conventional standard for all DIS calculations now. Therefore, one should recalculate the results obtained in other schemes to the \( \overline{\text{MS}} \) scheme at an appropriate number $f$. We do not need recalculation for $\alpha_s^{\text{MS}^{\text{spl}}}(M_Z)$ because the spline and \( \overline{\text{MS}} \) (at $f = 5$) schemes evidently coincide at $\mu = M_Z$ by construction, i.e.,

$$\alpha_s^{\text{MS}^{\text{spl}}}(M_Z) = \alpha_s^{\text{MS}}(M_Z; f = 5).$$

3.2 \( \overline{\text{MS}} \) – vulgate scheme

Usually, to obtain the evolution law, one calculates numerical expansion coefficients of generators $\beta(\alpha_s), \gamma_n(\alpha_s)$... in \( \overline{\text{MS}} \) scheme and solves the massless RG equations. In the solution, with all integration constants being omitted, one arrives at the final procedure which we shall name the \( \overline{\text{MS}} \) –vulgate scheme.

The first recipe to include the threshold mass $M_f$ into the framework of the \( \overline{\text{MS}} \) evolution was formulated in Ref. [3] as the “matching condition”, Eq. (1), for the coupling constant. Now all measurements on a low scale $Q$ are usually interpreted in terms of the $\alpha_s(M_Z)$ – RG solution in a certain scheme, with an appropriate matching of different numbers of active flavors which evolve from the scale $Q$ to $M_Z$. The matching condition (1) leads to a simple rule of including next “active flavors” $h$ into the evolution law

$$A(\overline{\text{MS}})(i) = \beta(i)(3)l - \Delta \beta(i) \cdot hl \rightarrow A^{\overline{\text{MS}^{\text{spl}}}}(Q, ...) = \beta(i)(3)l - \Delta \beta(i)l^*;$$

(12)
\[ l^* = \sum_h \left[ \theta(Q^2 - (M_{h(i)})^2) \ln(Q/M_{h(i)})^2 - (Q \to \mu) \right] \]

Nevertheless, the threshold value \( M_{h(i)} \) does not follow from this procedure and is left uncertain.

Note, the spline-type (in terms of the \( l \)-variable) expression (12) has an evident analogy with the approximation for the mass-dependent MOM scheme formulae for \( A_{(i)}(Q, \mu) \) (\( \Gamma_{(i)}(n, Q, \mu) \)) with the structure \( A_{(i)}(Q, \mu) \approx \left( I_{(i)}(Q, m) - I_{(i)}(\mu, m) \right) \), see, e.g., Eq. [10]. The approximation being discussed needs an asymptotic form of the mass-dependent calculation for the elements \( I_{(i)}(z = Q^2/m^2) \), \( m \). Well below the threshold, for \( Q, \mu \ll M_{h(i)} \), one usually uses some effective \( \overline{\text{MS}} \) scheme, that does not take mass of a particle into account. Above the threshold, a new particle cannot be ignored, but when \( Q, \mu \gg M_{h(i)} \), it can approximately be treated as massless within some other \( \overline{\text{MS}} \) scheme. How should the couplings \( a_{1}(\mu) \) and \( a_{2}(Q) \) in these two MS schemes be related? The answer can be obtained by the three–step algorithm:

(i) recalculating from the \( \overline{\text{MS}} \) to MOM scheme at \( q = \mu \ll M \) to get \( a^{\text{MOM}}(\mu) \);
(ii) performing the RG evolution of \( a^{\text{MOM}} \) up to \( q = Q \gg M \) in the MOM scheme;
(iii) recalculating to the \( \overline{\text{MS}} \) scheme, including the mass contribution, at \( q = Q \).

The final result of these successive steps leads to the approximate (due to power corrections) equality \( M_{h} \approx m_{h} \) for the threshold at the two–loop level. We obtain just the same result as usually used for the matching condition mentioned above with \( M_{h} = m_{h} \).

Consequently, proceeding in this way, one must modify the perturbative expansion coefficients for \( M_{h} \), \( i.e., \), \( \Gamma_{(i)}(n, Q) \) in (9), in the same manner as the expansion coefficients of the coupling constant \( A_{(i)} = \beta_{l} l \to A_{(i)}^{\text{MS}} \). For this aim we recall the structure for \( \gamma_{(2)}(n; f) \) in the framework of the \( \overline{\text{MS}} \) scheme (for details see, \( e.g., \), [7], [13]):

\[ \gamma_{(2)}(n; h + 3) = \gamma_{(2)}(n; 3) - \Delta \beta_{l} \cdot h \cdot \Delta \gamma_{(2)}(n); \quad (14) \]

\[ \Delta \gamma_{(2)}(n) = \frac{16}{9} \left\{ 10S_{1}(n) - 6S_{2}(n) - \frac{3}{4} \cdot \frac{11n^{2} + 5n - 3}{n^{2}(n + 1)^{2}} \right\}; \quad S_{k}(n) = \sum_{j=1}^{n} \frac{1}{j^{k}}. \]
where the first term $\gamma_2(n;3)$ in the r.h.s. of Eq. (14) consists of the usual massless part and the parameter $h$ numbers here heavy flavors. It is known (see e.g. [16]) that $\gamma_2(n;f)$ contains the terms generated by the evolution of the coupling constant. These terms naturally appear in the calculation of two-loop diagrams for $\gamma_2(n;f)$, they are proportional to the coefficient $\beta_1(1)$ (see the second term in (14)). Therefore in the $\overline{\text{MS}}$ expression for $\Gamma_2$ there appears a term proportional to the one-loop coefficient $A_1$: 

$$\Gamma_2(n,Q;h+3) = \Gamma_2(n,Q;3) - \Delta \beta_1(h \cdot l) \cdot \Delta \gamma_2(n); \quad (15)$$

The $h$-dependent part of the $A_1$-term is singled out of Exp.(15) in the form of $\Delta \beta_1(h \cdot l)$. To obtain the continuous coefficient $\Gamma_{\text{MS, spl}}(n,Q,h+3)$, one should substitute $A_1 \rightarrow A_{\text{MS, spl}}$, i.e., $(h \cdot l) \rightarrow l^*$ into the second term of the r.h.s. of Eq. (15), according to the recipe (12):

$$\Gamma_2(n,Q;h+3) \rightarrow \Gamma_{\text{MS, spl}}(n,Q;h+3) = \Gamma_2(n,Q;3) - \Delta \beta_1 l^* \cdot \Delta \gamma_2(n). \quad (16)$$

Now we can get the complete evolution law by substituting (12) and (16) into formulae (8) and (7) and then into the general formula (5):

$$\frac{M_2(n)}{M_n(\mu)} = \left( \frac{a}{\tilde{a}_{\text{MS, spl}}(Q,M)} \right)^{D_n(Q,M)} \exp \left\{ \left[ a - \tilde{a}_{\text{MS, spl}}(l) \right] F_{\text{MS, spl}}(Q,M) \right\}. \quad (17)$$

Recent CCFR Collab. experimental data on DIS are processed by this method (taking also account of the one-loop coefficient function) in the next section.

4 The QCD fit of the $xF_3$ CCFR data

4.1 Method of QCD Analysis

In this section, we present the QCD analysis of the CCFR data [18]. They are the most precise data on the structure function $xF_3(x,Q^2)$. This structure function is pure nonsinglet and the results of analysis are independent of the assumption on the shape of gluons. To analyze the data, the method of reconstruction of the structure functions from their Mellin moments is used [19]. This method is based on the Jacobi - polynomial expansion of the structure functions.

Following the method [19, 20], we can write the structure function $xF_3$ in the form:

$$xF_3^{N_{\text{max}}}(x,Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_j^{\text{NS}}(Q^2), \quad (18)$$
where \( \Theta_{\alpha\beta}^n(x) \) is a set of Jacobi polynomials and \( c_j^{(n)}(\alpha, \beta) \) are coefficients of their power expansions:

\[
\Theta_{\alpha\beta}^n(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)x^j.
\] (19)

The quantities \( N_{\text{max}}, \alpha \) and \( \beta \) have to be chosen so as to achieve the fastest convergence of the series in the r.h.s. of Eq.(18) and to reconstruct \( x F_3(x, Q^2) \) with the accuracy required. Following the results of [19] we have fixed the parameters – \( \alpha = 0.12, \beta = 2.0 \) and \( N_{\text{max}} = 12 \). These numbers guarantee an accuracy better than \( 10^{-3} \).

Finally, we have to parameterize the structure function \( x F_3(x, Q^2) \) at some fixed value of \( Q^2 = Q_0^2 \). We choose \( x F_3(x, Q^2) \) in a little bit more general form as compared to [21], where the same data have been analyzed within QCD in terms of \( \Lambda_{\text{MS}}^{(4)} \) without thresholds effects:

\[
x F_3(x, Q_0^2) = A x^B (1 - x)^C (1 + \gamma x).
\] (20)

Here \( A, B, C, \gamma \) and \( \alpha_0 = \bar{\alpha}_s(Q_0) \) are free parameters to be determined by the fit.

To avoid the influence of higher–twist effects, we have used only the experimental points in the plane \( (x, Q^2) \) with \( 5 < Q^2 \leq 501 \, (\text{GeV}/c)^2 \) and \( 0.015 \leq x \leq 0.65 \). The effect of target–mass corrections in \( x F_3 \) is taken into account to order \( M^2/Q^2 \) [22].

### 4.2 Results of Fit and Discussion

Here we present the results of processing the CCFR data obtained in the framework of two different approaches:

First, we have used the massless \( f \)-fixed \( \overline{\text{MS}} \) - scheme approach based on the two-loop evolution formula (3). The corresponding results for \( \alpha_0(f) \) are collected in the up-part of Table 1;

the second approach is based on mass-dependent evolution Eq.(5), for this formula we adapt the “spline” approximation (17). These results for \( \alpha_s^{\text{spl}} \) are collected in the down-part of Table 1.

Both the parts of Table 1 include the results of the LO and NLO fit; they are completed with the results of evolution of \( \alpha_0(f) \) and \( \alpha_s^{\text{spl}} \) to the point \( Q = M_Z \) with appropriate matching (1) of different numbers of active flavors (the last column – \( \alpha_s(M_Z) \); see, for comparison with other estimations, recent review [24]). Repeating the fit procedure for different values of \( Q_0^2 = 5, 50, 500 \, \text{GeV}^2 \) we have obtained the experimental dependence of \( \alpha_0(f) \) on the momentum transfer. In all fits, only statistical errors are taken into account. Here we make some comments on the fit results.

- A significant decrease of \( \chi^2(\text{NLO}) \) in comparison with \( \chi^2(\text{LO}) \) for all variants of the fit in Table 1 demonstrates that second loop effects are important for the kinematical region under consideration.

- To demonstrate the sensitivity of \( \alpha_0(f) \) on the \( f \), the results of the fit both for LO and NLO are shown for different \( f \) in the up-part of Table 1. The largest difference
between the values of $\alpha_0(f)$ for different $f$ is about 5% in the case of $Q_0^2$ close to kinematical boundaries $Q_0^2 = 5$ and 500 GeV$^2$. This difference reduces to 2% variations for $\alpha_s(M_Z; f)$. There are opposite relations for $\alpha_0(f)$ for these two points: $\alpha_0(3) > \alpha_0(4) > \alpha_0(5)$ for $Q_0^2 = 5$ GeV$^2$ and $\alpha_0(3) < \alpha_0(4) < \alpha_0(5)$ for $Q_0^2 = 500$ GeV$^2$. Note, the effects mentioned above can be described by estimating $\Delta \alpha_s = \alpha_s(f + 1) - \alpha_s(f) \approx d\alpha_s(f)/df$:

$$\Delta \alpha_s \approx \langle -\frac{\partial_f M_n(a,f,Q_{exp})}{\partial_a M_n(a,f,Q_{exp})} \rangle$$

Here, the brackets $\langle \ldots \rangle$ denote the average over experimental values of $Q_{exp}$. At the one–loop level, this expression leads to the simple estimate

$$\frac{\Delta \alpha_s}{\alpha_s} \sim -\Delta \beta_{(1)} \alpha_s \langle (\ln\frac{Q_{exp}}{Q_0}) \rangle,$$

in qualitative agreement with the results in Table 1.

- The final results for $\alpha_s(M_Z)$ depend on the $Q_0^2$ choice. For the $f$-fixed scheme this dependence amounts to 2% (two thousandth of the absolute value) and for the spline scheme, it is within only 1% (one thousandth) of the value of $\alpha_s(M_Z)$. Of course, the value of $\alpha_s^{spl}(Q_0)$ lies between the corresponding values of $\alpha_s^{MS}(Q_0; f = 4)$ and $\alpha_s^{MS}(Q_0; f = 5)$. This suppression of the residual dependence on $Q_0^2$ for the spline scheme results gives an additional “phenomenological” hint for preferring this scheme over the $\overline{MS}$ $f$-fixed version.

- The results of the fit are rather stable to the mass variations. The 10% change of $M_c$ and $M_b$ yields less than 0.5% change for $\alpha_0$.

Comparing the up- and down-parts of Table 1, we arrive at two main conclusions:

1. The spline scheme is more preferable than the traditional massless scheme, to process the experimental data involving thresholds, the values of $\alpha_s^{spl}(M_Z, Q_0)$ are focused tightly. The average value $\alpha_s^{spl}(M_Z) = 0.108$ is considerably smaller than the average LEP result and about 1 s.d. lower than that the CCFR result for $\alpha_s(M_Z)$ obtained recently from the $Q^2$–evolution of $xF_3$ and $F_2$ for $Q^2 > 15$ GeV$^2$ ($\alpha_s(M_Z) = 0.111 \pm .004$).

2. The threshold effects reveal themselves as approximately a 1% correction to the value of $\alpha_s(M_Z)$. The final results for the average values of $\alpha_s(M_Z)$ in the fit presented in Table 1 look like:

$$\alpha_s^{spl}(M_Z) = 0.108 \pm 0.001 \text{ (theor)} \pm 0.003 \text{ (stat)}$$

$$\alpha_s^{MS}(M_Z; f = 4) = 0.107 \pm 0.002 \text{ (theor)} \pm 0.003 \text{ (stat)}.$$
As it was emphasized in Sect. 3.1, the most natural way to include the thresholds effects into the analysis of data is to use the MOM scheme in formula (5). We present the results of processing the same experimental data by the one-loop evolution formulae (11) in Table 2. Note that one cannot directly compare the results obtained by different schemes, the spline fit or the \(\overline{\text{MS}}\) fit in Table 1 and the MOM scheme fit in Table 2 (see Sect. 3.1). Indeed, we extract, generally speaking, different kinds of the coupling constants, \(\alpha_s^{\overline{\text{MS}}}, \alpha_s^{\text{spl}}, \alpha_s^{\text{MOM}}\). To connect these quantities, we need relations between the \(\overline{\text{MS}}\) and our version of MOM–scheme. Here we write the relation at the one–loop level on scale \(\mu\)

\[
\frac{1}{\alpha_s^{\overline{\text{MS}}} (\mu)} = \frac{1}{\alpha_s^{\text{MOM}} (\mu)} + Q(1) (\mu) + \ldots
\]

(21)

The coefficients \(Q(i)\) depend, in general, on the gauge parameters and on the choice of a particular MOM scheme. The expression for \(Q(1)\) was represented, e.g. in [17], with the constant \(D_1 = \frac{5}{3} \cdot \beta(1)(3) - (11/12)C_A\) in the Landau gauge:

\[
Q(1) (\mu^2/m_h^2,..) = \frac{1}{(4\pi)^4} \left( \sum_h \Delta\beta_i^h \cdot \left( I_i^h (\mu^2/m_h^2) - \ln(\mu^2/m_h^2) \right) + D_1 \right).
\]

(22)

So, the quantity \(\alpha_s^{\overline{\text{MS}}} (M_Z)\) in the second rows of Table 2 was obtained by successive substitutions \(I_1\) into Eq.(22) and than into Eq.(21).

5 CCFR data and the light gluino window

In Subsec. 5.2, we have obtained a comparatively small effect, one-two thousandth to \(\alpha_s (M_Z)\) for the threshold contribution at the fit. The order of the effect is determined by the reason that only one \(b\)-quark threshold “works” really in the region in question. If Nature would provide few thresholds in the experimental interval \(5 \text{ GeV}^2 \leq Q^2 \leq 501 \text{ GeV}^2\), then they combined influence in a fit becomes significant and the preference of the spline scheme should look evident. To demonstrate this here we took an attempt to reconcile the existence of light MSSM gluino (\(\tilde{g}\)) and the CCFR data. The possibility of the light gluino existence \((m_{\tilde{g}}\) is of the order \(m_b)\) was intensively discussed few years ago in the context of the discrepancy between low energy \(\alpha_s\) values and the LEP data at the \(M_Z\) peak. This discrepancy has a chance to be resolved by including the light gluino [25]. The Majorana gluino leads to large effects in the evolution \(- \Delta\beta_{(1)}^g = 2, \ \Delta\beta_{(2)}^g = 48\) in Eq.(4) and slows the running both the coupling constant, and the moments \(M_n(Q)\). This reinforcement of the contribution of a new \(\tilde{g}\) threshold must influence the fit parameters. It is clear that the standard \(\overline{\text{MS}}\) -scheme at a fixed \(f\) everywhere is not adequate to the situation. We have performed the fit of CCFR data for different values of the gluino mass \(m_{\tilde{g}}\), the results of the fit for \(\alpha_s\) at \(m_{\tilde{g}} = 3, 4 \text{ GeV}\) and \(m_{\tilde{g}} = 10 \text{ GeV}\) are presented in Table 3.

Rather a strong dependence of \(\alpha_s^{\text{spl}} (M_Z, Q_0)\) value on \(Q_0^2\) (three thousandth), as well as a slight growth of \(\chi^2\), do not provide confidence that the data are consistent with the gluino with mass \(m_{\tilde{g}} \leq 4 \text{ GeV}\) (see up–part of Table 3). The “non-renormgroup” dependence of
\( \alpha_s^{sp(\tilde{g})}(M_Z, Q_0) \) on \( Q_0^2 \) reduces less than 1.5 thousandth for \( m_{\tilde{g}} \geq 10 \) GeV. This variation is not too far from the variation of \( \alpha_s \) in Table 2, so the gluino with these masses may be considered as accessible for the data. This leads to the estimation for \( \alpha_s(M_Z) \):

\[
\text{at } m_{\tilde{g}} = 10 \text{ GeV}, \quad \alpha_s^{sp(\tilde{g})}(M_Z) = 0.119 \pm 0.002 \text{ (theor)} \pm 0.004 \text{ (stat)},
\]

that is close to the LEP data [24]. Nevertheless, if one considers \( m_{\tilde{g}} \) as a fit parameter and “releases” it, the best \( \chi^2 \) will be reached at \( m_{\tilde{g}} \) beyond the fitting region, i.e., \( m_{\tilde{g}} \geq 22 \text{ GeV} \).

6 Conclusion

We have devised here a new approach for describing the two-loop evolution of the moment \( \mathcal{M}_n(Q^2) \) of structure function of lepton-nucleon DIS involving the threshold effects of heavy particles. The approach employs an analytic quasi-exact two-loop RG solution [9, 10] within the framework of mass-dependent Bogoliubov Renormalization Group (see Refs. [26, 27]), and on the results of papers [2], [1].

We adapt here the spline approximation (17) to the above mentioned RG solution and obtain as a result the simple modification of the formulae of the standard massless \( \overline{\text{MS}} \) evolution. For the particular case of a coupling constant evolution this approximation effectively leads to the same result as the “matching condition”. Finally, this recipe provides a more realistic continuous description for the \( \mathcal{M}_n(Q^2) \)-evolution and looks rather simple from a practical point of view. We performed the processing of the modern CCFR data to extract the value of \( \alpha_s(Q) \) by three different ways:

(i) the traditional \( \overline{\text{MS}} \)-scheme at the fixed numbers of flavors \( f \);
(ii) the spline scheme with break point at mass \( M = m \);
(iii) the MOM scheme (in the leading approximation);

and compare results of fits.

The results for the \( \overline{\text{MS}} \) spline – scheme processing of the data are the most adequate to the situation both for the physical and practical points of view. The threshold contribution to the value of \( \alpha_s(M_z) \) consists of about 1%; the extracted value of \( \alpha_s^{sp}(M_Z) \) is equal to \( 0.108 \pm 0.004 \). We examine the possibility to reconcile the CCFR data and the MSSM light gluino. It is possible for gluinos with mass \( m_{\tilde{g}} \geq 10 \) GeV, but it seems that similar gluinos are less probable due to other constraints (see [25]).

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Note Added in Proof

After this work was completed and submitted to the hep-ph database [a], the CCFR collaboration announced in Ref. [b] that the corrections of the energy spectrum calibration of the neutrino beam in the CCFR experiment change the “old” CCFR data of Ref.[18] and lead to the essentially larger values of $\bar{\alpha}_s(M_Z)$:

\[
xF_3 : \bar{\alpha}_s(M_Z) = 0.118 \pm 0.0025(stat) \pm 0.0055(syst) \pm 0.004(theory)
\]

extracted at the NLO of perturbative QCD using the integro-differential approach based on the GLAP equation. It would be better to verify this result by other methods. We believe that our qualitative result on the order of the “threshold correction” to the value of $\bar{\alpha}_s(M_Z)$ could not be changed by any modification of the experimental data and would be the same $\sim +10^{-3}$.

Additional References

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b. CCFR/NuTeV Collab., D.Harris, talk at the XXVIII Int. Conf. on HEP, Warsaw, July 1996.
Here we list the mass-dependent one-loop expressions for $I_1(z)$ and the corresponding $\beta$ functions in the MOM scheme. First, we write the well-known exact expression for the fermion polarization operator ($z = \mu^2/m_f^2$)

$$I_1^q(z) = 2\sqrt{1 + 4/z} \left(1 - \frac{2}{z}\right) \ln \left(\sqrt{1 + z/4 + \sqrt{z/4}}\right) + \frac{4}{z} - \frac{5}{3}; \quad \text{(23)}$$

$$I_1^q(z \to \infty) = \ln z - \left(c_q = \frac{5}{3}\right) + O(1/z) \quad ; \quad I_1^q(z \to 0) = \frac{z}{5} + O(z^2). \quad \text{(24)}$$

and the $\beta$ function

$$\beta_{(1)} = \frac{11}{3} C_A - \frac{4}{3} T_R \sum_{i=1}^{f} (1 - h_i(z));$$

where $h_i(z) = h_i^q(z) = \frac{6}{z} - \frac{12}{z^{3/2}} \frac{1}{\sqrt{1 + z/4}} \ln \left(\sqrt{1 + z/4 + \sqrt{z/4}}\right) \quad \text{(25)}$

The three–gluon contribution $I_0^q(z)$ can be expressed in terms of dilogarithms $Li_2(z)$, the final formula for $I_0^q(z)$ is too cumbersome and we do not demonstrate it here. As it was predicted in [15], the result for the corresponding three–gluon contribution to the $\beta$-function, $h_i(z) = h_i^g(z)$ –function may be expressed in terms of dilogarithms, as well,

$$h_i^g(z) = \frac{18}{z} - \frac{36}{z^{3/2}} \frac{1}{\sqrt{1 + z/4}} \ln \left(\sqrt{1 + z/4 + \sqrt{z/4}}\right) -$$

$$-6 \left\{ \frac{1}{\sqrt{z}(1 + z/3)} \frac{1}{\sqrt{1 + z/4}} \ln \left(\sqrt{1 + z/4 + \sqrt{z/4}}\right) +$$

$$\frac{1}{\sqrt{3}z} \Im [Li_2(z_3^*) - Li_2(z_3) + Li_2(z_4^*) - Li_2(z_4)] \right\},$$

$$z_3 = -\left(1 + \frac{i}{\sqrt{3}}\right) \left(\frac{\sqrt{z}}{2\sqrt{1 + z/4 + i\sqrt{z/3}}}\right); \quad z_4 = \left(1 - \frac{i}{\sqrt{3}}\right) \left(\frac{\sqrt{z}}{2\sqrt{1 + z/4 - i\sqrt{z/3}}}\right).$$

The function $h_i^g(z)$ is described by the rational approximation [13]:

$$h_i^g(z) \to \tilde{h}_i^g(z) = \frac{(1.2z + 1)z}{(0.15z + 1)(0.4z + 1)}. \quad \text{(26)}$$

This approximation works well (better than 1% of accuracy) when $z > 1$, but when $z \approx 10^{-2} - 10^{-3}$, the accuracy is about 10%. One can restore the corresponding approximate expression for $I_0^g(z)$ by the elementary integration of $\tilde{h}^g(z)$

$$\tilde{I}_1^g(z) = A \cdot \ln(1 + z/z_1) + (1 - A) \cdot \ln(1 + z/z_2); \quad \text{(27)}$$

$$A = 21/10 \quad ; \quad z_1 = 20/3 \quad ; \quad z_2 = 5/2 \quad ;$$

$$\tilde{I}_1^g(z \to \infty) = \ln z - (c_g \approx 2.98) + O(1/z) \quad \text{; (28)}$$

$$\tilde{I}_1^g(z \to 0) = \frac{z}{5} + O(z^2).$$
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| Quark content | 1-loop | 2-loops |
|---------------|--------|---------|
|               | $\chi^2$ | $\alpha_0$ | $\alpha_s(M_Z)$ | $\chi^2$ | $\alpha_0$ | $\alpha_s(M_Z)$ |
| $MS$-scheme, $f$-fixed, $M_q = 0$ |
| $Q_0^2 = 5 GeV^2$ |
| uds           | 104.4  | 0.2629±0.019 | 0.1097 | 84.7 | 0.2327±0.014 | 0.1085 |
| udsc          | 103.5  | 0.2557±0.019 | 0.1133 | 85.3 | 0.2282±0.013 | 0.1076 |
| udscb         | 102.7  | 0.2498±0.016 | 0.1173 | 86.1 | 0.2238±0.013 | 0.1066 |
| $Q_0^2 = 50 GeV^2$ |
| uds           | 103.2  | 0.1850±0.009 | 0.1103 | 84.0 | 0.1648±0.007 | 0.1069 |
| udsc          | 102.6  | 0.1856±0.009 | 0.1139 | 84.7 | 0.1668±0.007 | 0.1077 |
| udscb         | 102.0  | 0.1860±0.009 | 0.1177 | 85.4 | 0.1688±0.008 | 0.1085 |
| $Q_0^2 = 500 GeV^2$ |
| uds           | 104.2  | 0.1421±0.0053 | 0.1105 | 87.3 | 0.1275±0.0035 | 0.1038 |
| udsc          | 103.6  | 0.1448±0.0056 | 0.1140 | 87.9 | 0.1311±0.0043 | 0.1061 |
| udscb         | 103.1  | 0.1474±0.0055 | 0.1177 | 88.5 | 0.1348±0.0046 | 0.1085 |
| $Q_0^2 = 50 GeV^2$ |
| uds+(c+b)     | 102.2  | 0.2526±0.017 | 0.1167 | 85.8 | 0.2262±0.012 | 0.1072 |
| $Q_0^2 = 500 GeV^2$ |
| uds+(c+b)     | 102.1  | 0.1856±0.009 | 0.1175 | 85.1 | 0.1682±0.007 | 0.1083 |

### Table 1

The results of the LO and NLO QCD fit in the $\overline{MS}$-scheme of the CCFR [18] $xF_3$ structure function data in a wide kinematical region: $0.015 \leq x \leq 0.65$ and $5 GeV^2 < Q^2 < 501 GeV^2$ ($N_{exp.} = 81$). The value of the coupling constant is determined for different numbers of the flavor (in up-part of Table) and for few values of the momentum transfer $\alpha_0 = \alpha(Q_0^2 = 5, \ 50, \ 500 \ GeV^2)$. We present in the first column the flavor content involved in QCD–evolution using in fit. The results of fit with the matching at the thresholds corresponding to $m_c = 1.3 GeV$ and $m_b = 5 GeV$ are presented in the down-part of the table. The value of $\alpha_s(M_Z)$ is calculated with a statistical error about $\pm 0.003$. 


\[ \alpha_s^{\text{MOM}}, \alpha_s^{\text{MOM} \to \overline{\text{MS}}}, \text{ one-loop} \]

| Quark content | \( \chi^2 \)   | \( \alpha_0 \) | \( \alpha_s(M_Z) \) |
|---------------|---------------|---------------|-------------------|
| \text{MOM-scheme, } Q_0^2 = 3 GeV^2 | 103.4 | 0.281 ± 0.018 | 0.115 ± 0.0031 |
| \text{MOM} \rightarrow \text{MS } Q_0^2 = 3 GeV^2 | (f=4) | 0.282 ± 0.022 | 0.106 |
| \text{MOM-scheme, } Q_0^2 = 10 GeV^2 | 103.1 | 0.230 ± 0.014 | 0.116 ± 0.0037 |
| \text{MOM} \rightarrow \text{MS } Q_0^2 = 10 GeV^2 | (f=4) | 0.188 ± 0.014 | 0.106 |
| \text{MOM-scheme, } Q_0^2 = 500 GeV^2 | 104.3 | 0.1455 ± 0.0057 | 0.116 ± 0.0035 |
| \text{MOM} \rightarrow \text{MS } Q_0^2 = 500 GeV^2 | (f=5) | 0.130 ± 0.0052 | 0.106 |

Table 2 In the first rows of the Table 2 the results of LO MOM-scheme fit for different values of \( Q_0^2 \) are represented. The results of its transformation from \( \alpha_s^{\text{MOM}} \) to \( \alpha_s^{\overline{\text{MS}}} (f = 4) \) or \( \alpha_s^{\overline{\text{MS}}} (f = 5) \) are located in the nexts rows.
CCFR data and the light gluino window

| Particle content | 2-loops | $\chi^2$ | $\alpha_0^{spl}(g)$ | $\alpha_s^{spl}(\bar{g})(M_Z)$ |
|------------------|---------|---------|---------------------|-----------------------------|
| Spline, $M_c = 1.3$ GeV, $M_b = 5$ GeV, $M_{\tilde{g}} = 3$ GeV | $Q_0^2 = 5 GeV^2$ | 88.4 | 0.2162±0.012 | 0.1230 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 5 GeV^2$ | 87.2 | 0.1744±0.008 | 0.1251 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 5 GeV^2$ | 87.8 | 0.2188±0.012 | 0.1221 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 5 GeV^2$ | 86.7 | 0.1739±0.008 | 0.1248 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 5 GeV^2$ | 85.8 | 0.2254±0.013 | 0.1185 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 5 GeV^2$ | 85.7 | 0.1688±0.007 | 0.1201 |
| $uds+(c+b+\tilde{g})$ | | | |
| $Q_0^2 = 500 GeV^2$ | 89.1 | 0.1392±0.005 | 0.1186 |

**Table 3** The results of fit with the matching at the thresholds corresponding to $m_c = 1.3 GeV$, $m_b = 5 GeV$ and $m_{\tilde{g}} = 3, 4, 10 GeV$ are presented at the Table 3. The value of $\alpha_s^{spl}(\bar{g})(M_Z)$ is calculated with the error about ±0.004.