Does phantom energy produce black hole?

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Abstract

We have found an exact solution of spherically symmetrical Einstein equations describing a black hole with a special type phantom energy source. It is surprising to note that our solution is analogous to Reissner-Nordström black hole.

Recent astrophysical observations have confirmed that the Universe at present is expanding with an acceleration. It is proposed that this unexpected cosmological behavior is caused by a hypothetical dark energy with a positive energy density and a negative pressure. The matter with the property, energy density $\rho > 0$ but pressure $p < 0$ has been denoted 'phantom energy'. Several authors have recently discussed accelerating phase of the Universe by using phantom energy as source [1-8]. Since traversable Wormholes require so called exotic matter with a negative pressure $p < 0$, some authors have recently investigated the physical properties and characteristic of traversable Wormholes by taking phantom energy as source [9-13]. In this article, we present a black hole solution with special type phantom energy as a source. By choosing the parameters adequately, the solution coincides with Reissner-Nordström black hole solution. Before discussing this surprising result, we proceed to show the solution.

We look for static spherically symmetric solution with the line element

$$ds^2 = -e^{2f(r)}dt^2 + \frac{1}{[1 - \frac{b(r)}{r}]}dr^2 + r^2d\Omega^2$$

(1)
Because of spherical symmetry the only non zero components of stress energy tensor are \( T_{0}^{0} = -\rho(r) \), \( T_{1}^{1} = p(r) \) and \( T_{2}^{2} = T_{3}^{3} = p_{tr}(r) \) where \( \rho \) is the energy density, \( p \) is the radial pressure and \( p_{tr} \) is the transverse pressure. Using the Einstein field equations \( G_{\mu\nu} = 8\pi T_{\mu\nu} \), in orthonormal reference frame (with \( c = G = 1 \)), we obtain the following stress energy scenario,

\[
\rho(r) = \frac{b'}{8\pi r^2} \tag{2}
\]

\[
p(r) = \frac{1}{8\pi} \left[-\frac{b}{r^3} + 2\frac{f'}{r}(1 - \frac{b}{r})\right] \tag{3}
\]

\[
p_{tr}(r) = \frac{1}{8\pi} (1 - \frac{b}{r})[f'' - \frac{(b'r - b)}{2r(r - b)}f' + f'^2 + \frac{f'}{r} - \frac{(b'r - b)}{2r^2(r - b)}] \tag{4}
\]

Using the conservation of stress energy tensor \( T^{\mu\nu}_{\nu} = 0 \), we can obtain the following equation

\[
p' + f'\rho + (f' + \frac{2}{r})p - \frac{2}{r}p_{tr} = 0 \tag{5}
\]

From now on, we assume that our source is characterized by the special type phantom energy with equation of state that contains a radial pressure

\[
p = -\rho \tag{6}
\]

we suppose also that pressures are isotropic and

\[
p_{tr} = \rho \tag{7}
\]

Since only two equations of the system (2) - (4) are independent, it is convenient to represent them as follows:

\[
b' = 8\pi r^2 \rho(r) \tag{8}
\]

\[
f' = \frac{(8\pi r^3 + b)}{2r(r - b)} \tag{9}
\]

One can find the solution of \( \rho \) from (5) by using (6) and (7) as

\[
\rho(r) = \frac{\rho_0}{r^4} \tag{10}
\]

where \( \rho_0 \) is an integration constant.
Plugging (10) in (8) and (9), one can get the following solutions of $b$ and $f$ as

$$b = A - \frac{8\pi \rho_0}{r} \quad (11)$$

$$2f = \ln f_0 \left[ \frac{r^2 - Ar + 8\pi \rho_0}{r^2} \right] \quad (12)$$

where $f_0$ and $A$ are integration constants.

Rescaling the time coordinate appropriately, the line element becomes,

$$ds^2 = -[1 - \frac{A}{r} + \frac{8\pi \rho_0}{r^2}] dt^2 + \frac{1}{[1 - \frac{A}{r} + \frac{8\pi \rho_0}{r^2}]} dr^2 + r^2 d\Omega^2_2 \quad (13)$$

The structure of this solution is similar to the Reissner-Nordström black hole solution.

In the absence of the source i.e. when $\rho_0$ is zero, then the metric (13) becomes Schwarzschild metric and comparing with Schwarzschild metric, the constant $A$ can be chosen to be $2M$, $M$ is the mass of the black hole.

**Properties of the solution:**

For $A > \sqrt{32\pi \rho_0}$, there are two zeros of $1 - \frac{A}{r} + \frac{8\pi \rho_0}{r^2}$ at $r = r_\pm$ where

$$r_\pm = \frac{A \pm \sqrt{A^2 - 32\pi \rho_0}}{2}$$

which correspond to two horizons.

The Kretschmann scalar

$$K = R_{abcd} R^{abcd} = \frac{4}{r^6} [(A - 32\pi \rho_0)^2 + (A - 16\pi \rho_0)^2 + (A - 8\pi \rho_0)^2]$$

is finite at $r_\pm$ and is divergent at $r = 0$, indicating that $r_+$ and $r_-$ are regular horizons and the singularity locates at $r = 0$.

For $A = \sqrt{32\pi \rho_0}$, the black hole has only one event horizon located at $r = \frac{A}{2}$. Thus two horizons $r_+$ and $r_-$ match to form a regular event horizon while $r = 0$ is still a singularity.

We also see that if $A < \sqrt{32\pi \rho_0}$, the solution does not describe a black hole at all, but, rather a naked singularity.
Now we find entropy $S$ and Hawking temperature $T_H$ of the black hole following Hawking’s remarkable discovery - the laws of black hole thermodynamics [14].

$$S = \frac{1}{4} (\text{area}) = \frac{\pi}{4} [A + \sqrt{(A^2 - 32\pi\rho_0)}]^2$$

$$T_H = \frac{1}{4\pi \sqrt{-gt_{rr}}} \frac{d}{dr} (-gt_{tt})|_{\text{horizon}} = \frac{1}{\pi} \left[ \frac{\sqrt{(A^2 - 32\pi\rho_0)}}{A + \sqrt{(A^2 - 32\pi\rho_0)}} \right]^2$$

In the limiting case i.e. when $A = \sqrt{32\pi\rho_0}$, the black hole exhibits a non zero entropy $S_0 = \frac{\pi A}{4}$, at zero temperature. One may consider it as a result of a dual symmetric that generates degenerate ground states of black hole.

In conclusion, we give a black hole solution by taking special type phantom energy as source. The structure and thermodynamic properties of this black hole is similar to Reissner-Nordström black hole. Three possible questions could there be to our model.

[1] Is any spherically charged distribution of matter has the same notion of special type phantom energy and obeys equation of state : $p = -\rho, p_{tr} = \rho$ ?

[2] It is argued that apart from the null energy condition violation, phantom energy possesses a strange property namely, phantom energy mediates a long range repulsive force [8]. So, how it is possible to a source ( distribution of matter ) which produces repulsive force to form a black hole ?

[3] Comparing the metric (13) with Reissner-Nordström black hole metric, one can find $8\pi\rho_0 = e^2$ i.e. $\rho = \frac{e^2}{8\pi r^4}$ [ where $e$ is the charge of the matter and $\rho$ is the matter density of the phantom energy source ]. So, is it possible to relate gravity with electromagnetic field?

The answer of these questions is under current consideration and we hope to report this elsewhere.
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