Almost maximally broken permutation symmetry for neutrino mass matrix

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Abstract

Assuming three light neutrinos are Majorana particles, we propose mass matrix ansatz for the charged leptons and Majorana neutrinos with family symmetry $S_3$ broken into $S_1$ and $S_2$, respectively. Each matrix has three parameters, which are fixed by measured charged lepton masses, differences of squared neutrino masses relevant to the solar and the atmospheric neutrino puzzles, and the masses of three light Majorana neutrinos as a candidate for hot dark matter with $\sum |m_\nu| \sim 6$ eV. The resulting neutrino mixing is compatible with the data for the current upper limit, $(m_\nu)_{th} < 0.8$ eV, of neutrino-less double beta decay experiments, and the current data for various types of neutrino oscillation experiments. One solution of our model predicts that $\nu_\mu \rightarrow \nu_\tau$ oscillation probability is about $< 0.008$ with $\Delta m^2 \sim 10^{-2}$ eV$^2$, which may not be accessible at CHORUS and other ongoing experiments.
One of the important problems in particle physics is to understand the origin of fermion masses and flavor mixing and their hierarchical patterns. There are several different approaches to this problem, such as chiral flavor symmetries, texture zeros, mass matrix ansatz, determination of the top quark mass using the arguments based on infrared stable quasi-fixed point or the multiple point criticality principle, and so on [1].

Recently, Fritzsch and his collaborators [2] suggested quark mass matrix ansatz based on flavor democracy and its suitable breaking. This idea of flavor democracy is by no means new, and noticed by several authors before [3]. Their work on the quark sectors [2] yields the weak CP phase to be maximal (\( \alpha \approx 90^\circ \)) [4] in the mass matrix in order for the CKM matrix elements to be consistent with the current observations [5]. In view of the undergoing B-factory projects, it is quite interesting to see if this prediction of maximal CP-violation is really realized in the CP asymmetry in the B decays. These authors also considered the lepton mixing matrix without flavor democracy in the neutrino sector, and got some interesting predictions [6], which is essentially similar to the maximal mixing scenario.

In this letter, we extend Fritzsch’s approach to the lepton sector with a suitable modification which is different from Ref. [6]. As discussed below, the flavor democracy predicts one heavy fermion versus two massless fermions. This picture nicely fits with the charged fermion sectors, but not with the neutrino sector. If one requires three neutrinos to be a hot dark matter component of the universe, and solve the solar and the atmospheric neutrino puzzles in terms of neutrino oscillations, one has to invoke three almost degenerate neutrinos, instead of a large hierarchical structure in \( m_\nu \) \((i = 1, 2, 3)\). Therefore, we relax ‘the condition of flavor democracy’ in the Yukawa couplings and require the Yukawa matrices to possess ‘the permutation symmetry in the family indices.’ Then, the flavor democratic structure turns out to be a special case of the structure with the permutation symmetry.

We first recapitulate the idea by Fritzsch et al. in the context of the charged lepton mass matrix \( (M_l) \). Then, we introduce a mass matrix ansatz for three light neutrinos \( (M_\nu) \) invoking the permutation symmetry among three family indices \( (S_3) \), and its breaking to \( S_2 \) or \( S_1 \). Each lepton matrix depends on three independent real parameters, if we assume there is no particular relations among lepton masses. The form of the lepton mass matrix with three independent real parameters may be completely arbitrary in principle, as long as they correctly reproduce three lepton masses. In order to reduce this arbitrariness in the form of the mass matrix ansatz, we assume that the mass matrix ansatz respect the permutation symmetry among three families \((S_3)\) in the zeroth order approximation, and then this \( S_3 \) symmetry is subsequently broken to a smaller group \( S_2 \) or \( S_1 \). This requirement of permutation symmetry reduces the arbitrariness of our approach based on specific ansatz for lepton masses. Thus, there are six real parameters in total in the lepton mass matrices, and there is no CP violation in the leptonic sector in our model. These six parameters are fit to (i) the charged lepton masses, (ii) \( \Delta m^2 \) relevant to the solar and the atmospheric neutrino puzzles, and finally (iii) \( \sum_{i=1}^{3} |m_\nu| \sim 6 \) eV (in order to solve the dark matter problem) [7]. Diagonalizing the mass matrices yields a neutrino mixing matrix \( (V) \). By studying the resulting predictions to the various neutrino oscillation probabilities, and the neutrino-less double beta decay, we are led to make an ansatz for the neutrino mass matrix which fits every known constraint. It also predicts that \( P(\nu_\mu \rightarrow \nu_\tau) \sim 0.008 \), which is beyond the reach of the CHORUS and other similar experiments.

When one considers massive neutrinos, one could think of variety of models with massive
neutrinos including see-saw mechanism and GUT. In this work, however, we simply assume that the standard model (both particle contents and the symmetry group structure) is valid up to some scale $\Lambda \ll M_Z$. Then there is a dimension-5 operator $h_{ij}l_il_jH^2/\Lambda$, which generates the Majorana masses for three neutrinos $m_\nu \sim h\nu^2/\Lambda$ after the electroweak symmetry breaking [8]. If $\Lambda \sim M_{\text{Planck}}$, the neutrino masses are an order of $10^{-6}$ eV for $h \sim O(1)$, whereas $m_\nu \sim O(1)$ eV for $\Lambda \sim 10^{12}$ GeV (which is nothing but the usual intermediate scale in many beyond standard models).

2. For the charged lepton sector, we adopt modified Fritzsch’s form as in Ref. [2]:

$$M_l^\text{real} = c_l \begin{pmatrix} 0 & dl & 0 \\ dl & \frac{2}{3} \epsilon_l & -\frac{\sqrt{2}}{3} \epsilon_l \\ 0 & -\frac{\sqrt{2}}{3} \epsilon_l & 3 + \frac{1}{3} \epsilon_l \end{pmatrix},$$

(1)
n in the hierarchical basis where $c_l \simeq m_\tau/3$. We assume that $d_l \ll \epsilon_l \ll 1$. Then, one can treat the $d_l$ terms by a small perturbation, and solve $d_l = 0$ case first. The corresponding eigenvalues are

$$\lambda_0 = 0,$$

$$\lambda_{\mp} = \frac{1}{2} \left[ (3 + \epsilon_l) \mp 3 \sqrt{1 - \frac{2}{9} \epsilon_l + \frac{1}{9} \epsilon_l^2} \right],$$

(2)

with the eigenvectors

$$|\lambda_0\rangle = (1, 0, 0),$$

$$|\lambda_-\rangle = (0, \cos \theta_l, \sin \theta_l),$$

$$|\lambda_+\rangle = (0, -\sin \theta_l, \cos \theta_l),$$

(3)

where

$$\sin^2 \theta_l = \frac{2\epsilon_l^2}{2\epsilon_l^2 + (9 + \epsilon_l - 3\lambda_-)^2},$$

$$\cos \theta_l = \frac{(9 + \epsilon_l - 3\lambda_-)}{\sqrt{2\epsilon_l}} \sin \theta_l.$$  

(4)

At this stage, $m_e = 0$ MeV. To the leading order in $d$, we can identify $\lambda_{\mp}$ as $m_{\mu,\tau}$. Doing so, one gets

$$\epsilon_l = 0.287, \quad \text{and} \quad \theta_l = 2.66^\circ.$$  

(5)

For electron mass (0.511 MeV), a nonzero value of $d_l$ is needed, $d_l = 0.0128$ ($d_l^2 \approx 1.65 \times 10^{-4}$). $M_l$ can be approximately diagonalized by a unitary matrix

$$U_l = \begin{pmatrix} 1 \\ -d_l \left( \frac{\cos^2 \theta_l}{\lambda_-} + \frac{\sin^2 \theta_l}{\lambda_+} \right) \\ d_l \sin \theta_l \cos \theta_l \left( \frac{1}{\lambda_+} - \frac{1}{\lambda_-} \right) \end{pmatrix} \begin{pmatrix} \frac{d_l \cos \theta_l}{\lambda_-} & -d_l \sin \theta_l \\ \frac{d_l \cos \theta_l}{\lambda_-} & -d_l \sin \theta_l \\ \frac{d_l \cos \theta_l}{\lambda_-} & -d_l \sin \theta_l \end{pmatrix}. $$

(6)
Notice that the angle $\theta_l$ is completely fixed by the measured charged lepton masses.

3. Unlike the charged lepton sector, three neutrinos do not seem to have a hierarchy in their masses. In fact, the solar neutrino puzzle can be explained through the MSW mechanism if $\Delta m^2_{\text{solar}} \approx 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_{\text{solar}} \approx 8 \times 10^{-3}$ (small angle case), or $\sin^2 \theta_{\text{solar}} \approx 0.7$ (large angle case), and through the just-so vacuum oscillations if $\Delta m^2_{\text{solar}} \approx 10^{-10} \text{ eV}^2$. The atmospheric neutrino problem can be accommodated if $\Delta m^2_{\text{atmos}} \approx 10^{-2} \text{ eV}^2$ and $\sin^2 \theta_{\text{atmos}} \approx 0.5$. If light massive neutrinos provide the hot dark matter of the universe, one has to require

$$\sum_{i=1,2,3} |m_{\nu_i}| \sim 6 \text{ eV.}$$

All these data indicate that all three neutrinos may be almost degenerate in their masses, with $m_\nu \sim$ a few eV, rather than $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$, as usually assumed in the three neutrino mixing scenarios.

Since the flavor democratic neutrino mass matrix leads to a large hierarchy, one necessarily has to modify the symmetry relevant to the neutrino mass matrix. Here, we propose to consider the permutation symmetry among three family indices rather than the flavor democracy. Then, the lowest order neutrino mass matrix would look like (in the symmetry basis)

$$\tilde{M}^{(0)}_{\nu} = c_\nu \begin{pmatrix} 1 & r & r \\ r & 1 & r \\ r & r & 1 \end{pmatrix}.$$  

(8)

We have assumed that three light neutrinos are Majorana particles, so that the neutrino mass matrix is real symmetric $3 \times 3$ matrix. One recovers the flavor democratic case for $r = 1$. In the hierarchical basis, the above form becomes

$$M^{(0)}_{\nu} = c_\nu \begin{pmatrix} 1 - r & 0 & 0 \\ 0 & 1 - r & 0 \\ 0 & 0 & 1 + 2r \end{pmatrix}.$$  

(9)

Either for small $r(\sim 0)$ or for $r \sim -2$, all the three neutrinos are almost degenerate. For any $r$, two neutrinos are always degenerate, so that there is only one $\Delta m^2$ available.

In order to have two $\Delta m^2$ scales from our mass matrices, one has to lift the degeneracy for the first two neutrinos further. At this stage, there would be two simple ways to break this degeneracy:

1. The recent LSND data, if confirmed, indicates $\Delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2$ and $\sin^2 \theta_{\text{LSND}} \sim 10^{-3}$. Since the conclusions of two different analyses do not agree each other, we do not consider the possibility alluded by the LSND data in this work. See, however, Ref. for a discussion when the LSND data is included.

2. Note that the sign of the fermion mass is physically meaningless.
Case I:  
\[ M_{\nu}^{(I)} = c_{\nu} \begin{pmatrix} 1 - r & \epsilon_{\nu} & 0 \\ \epsilon_{\nu} & 1 - r & 0 \\ 0 & 0 & 1 + 2r \end{pmatrix} \]  
(10)

and

Case II:  
\[ M_{\nu}^{(II)} = c_{\nu} \begin{pmatrix} 1 - r & 0 & 0 \\ 0 & 1 - r & \epsilon_{\nu} \\ 0 & \epsilon_{\nu} & 1 + 2r \end{pmatrix} \]  
(11)

Other possibilities are equivalent to the above two by changing the labels \( i = 1, 2, 3 \) in the mass eigenstates of neutrinos.

4. Now, we analyze the above two ansatzs for the neutrino mass matrices, and study the consequences in the neutrino mixing.

Case I:
In this case, the neutrino mass matrix is diagonalized by

\[ U_{\nu}^{I} = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]  
(12)

with the eigenvalues

\[ m_{\nu_i}/c_{\nu} = (1 - r \mp \epsilon), (1 + 2r). \]  
(13)

Note that we have shown only one possibility for labeling three neutrino mass eigenstates. Furthermore, Eq. (12) is independent of neutrino mass eigenvalues, or equivalently, on \( c_{\nu}, \epsilon_{\nu} \) and \( r \). This is because the mass matrix \( M_{\nu}^{(I)} \) given in Eq. (10) still has a residual \( S_2 \) symmetry acting upon the first and the second family indices.

Combining with the \( U_{l} \) given in (6), one gets the neutrino mixing matrix, \( V^{I} \equiv U_{l}^{\dagger} U_{\nu}^{I} \). First of all, the mixing matrix is independent of neutrino masses, although it depends on the charged lepton masses. One can solve for \( c_{\nu}, r \) and \( \epsilon_{\nu} \) by requiring three conditions, \( \Delta m_{solar}^{2} \approx 10^{-10} \text{ eV}^2, \Delta m_{atmos}^{2} \approx 0.72 \times 10^{-2} \text{ eV}^2 \) and Eq. (7) \( ^{3} \). Then, we check if the solution satisfies the constraint from the neutrino-less double \( \beta \)-decay, and other data from neutrino oscillation experiments.

We find that there are three sets of parameters, \((c_{\nu}, r, \epsilon_{\nu})\):

\[ \begin{aligned} (1.0, \pm 0.667, \mp 2.997) \\ (r, c_{\nu}, \epsilon_{\nu}) = (1.001, \pm 0.666, \mp 3.004) \\ (0.999, \pm 0.667, \mp 2.999) \end{aligned} \]  
(14)

\(^{3}\)In this work, we solve the solar neutrino problem in terms of vacuum oscillations. The results would remain the same even if we invoke the MSW mechanism.
Since we are considering Majorana neutrinos, there is an additional constraint from non-observation of neutrino-less double $\beta$-decays \cite{12}:

$$\langle m_{\nu_e} \rangle \equiv | \sum_{i=1}^{3} V_{ei}^2 m_i | < 0.7 \text{ eV}. \quad (15)$$

This condition is not easy to satisfy in typical models with massive neutrinos when one tries to solve the hot dark matter problem in terms of light neutrinos with masses in a few eV range, as discussed in Ref. \cite{13}.

The values for $\langle m_{\nu_e} \rangle$ corresponding to Eq. (14) are

$$\langle m_{\nu_e} \rangle = 0.2771 \text{ eV}, \ 0.2763 \text{ eV} \ \text{and} \ 0.2781 \text{ eV}, \quad (16)$$

respectively. All of these solutions are well below the current upper limit given in Eq. (15). The expected atmospheric neutrino data $R$’s for different $L/E$ from the atmospheric are given in Table 1 along with the current data from KAMIOKANDE, IMB, FREJUS, NUSEX and SOUDAN. From Table 1, we find that the lepton mass matrix ansatz Eqs. (11) and (14) reproduce all the known data on the neutrino oscillation experiments.

Further test of our ansatz is provided with the long baseline experiments searching for $\nu_\mu \rightarrow \nu_\tau$ oscillation in the range of $\Delta m^2_\nu \simeq 10^{-2} \text{ eV}^2$. Our prediction is that

$$P(\nu_\mu \rightarrow \nu_\tau) \sim 7.6 \times 10^{-3},$$

with $\Delta m^2 = 0.72 \times 10^{-2} \text{ eV}^2$. This is still well below the current upper limit as well as the planned search for the $\nu_\mu \rightarrow \nu_\tau$ oscillations at CHORUS, NOMAD, ICARUS, etc.

\textit{Case II}:

In this case, the neutrino mass matrix is diagonalized by

$$U_{\nu}^{II} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_\nu & -\sin \theta_\nu \\ 0 & \sin \theta_\nu & \cos \theta_\nu \end{pmatrix}, \quad (18)$$

with the mass eigenvalues being

$$m_i/c_\nu = 1 - r, \quad \frac{1}{2} \left[ (2 + r) \pm \sqrt{9r^2 + 4\epsilon_\nu^2} \right]. \quad (19)$$

The mixing angle $\theta_\nu$ is determined by

$$\sin^2 \theta_\nu = \frac{2\epsilon_\nu^2}{(9r^2 + 4\epsilon_\nu^2) - 3r \sqrt{9r^2 + 4\epsilon_\nu^2}}, \quad (20)$$

with

$$\cos \theta_\nu = \left( \frac{\sqrt{9r^2 + 4\epsilon_\nu^2} - 3r}{2\epsilon_\nu} \right) \sin \theta_\nu. \quad (21)$$
Unlike the Case I, we now have the neutrino mixing matrix which does depend on the neutrino masses in a nontrivial way.

For a given set of $m_i^2$, one can get $\theta_\nu$. Scanning the parameter space in this case as before, we find no solution. Typically, $|V_{1e}| \sim 1$ and $|V_{2e}|, |V_{3e}| \ll 1$, we have too large $\langle m_\nu \rangle \simeq 2$ eV to be compared to (15). Thus, the neutrino mass ansatz Eq. (11) along with the charged lepton mass ansatz Eq. (1) does not fit the neutrino data.

5. In conclusion, we investigated the lepton mass matrices with the minimal number of parameters, three in each of the charged lepton and Majorana neutrino mass matrices ($M_l$ and $M_\nu$), with a permutation symmetry among three generations ($S_3$) and its suitable breaking into $S_1$ and $S_2$, respectively. We find the ansatz (1) and (10) lead to a lepton mixing matrix which is consistent with the current data on various types of neutrino oscillation experiments. Three light Majorana neutrinos can serve as the hot dark matter, with $\Sigma |m_\nu| \sim 6$ eV. The resulting $\nu_\tau \leftrightarrow \nu_\mu$ probability is in the range of 0.008 with $\Delta m^2 \simeq 0.7 \times 10^{-2}$ eV$^2$, which still lies beyond the scope of the planned CHORUS and other experiments searching for $\nu_\tau \leftrightarrow \nu_\mu$ oscillation. Furthermore, three neutrinos being almost degenerate, we expect that the lepton family number breaking will be very small.

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TABLE I. The atmospheric neutrino data $R$ for various $L/E$ along with our predictions for $\Delta m_{21}^2 = 0.72 \times 10^{-2} \text{eV}^2$, $\Delta m_{32}^2 = \times 10^{-10} \text{eV}^2$. We show the $r = (\mu/e)_{\text{incident}}$ values for each data point also.

| Experiments | $r$ | $L/E$ (km/GeV) | Measured   | Prediction |
|-------------|-----|----------------|------------|------------|
| KAMIOKA     | 4.5/1 | 5              | $1.27^{+0.61}_{-0.38}$ | 0.99       |
| (Multi-GeV) | 3.2/1 | 10             | $0.63^{+0.16}_{-0.16}$ | 0.97       |
|             | 2.2/1 | 100            | $0.51^{+0.15}_{-0.12}$ | 0.41       |
|             | 3.2/1 | 1000           | $0.46^{+0.18}_{-0.12}$ | 0.31       |
|             | 4.5/1 | 2000           | $0.28^{+0.10}_{-0.07}$ | 0.22       |
| KAMIOKA     | 2.5/1 | 80             | $0.59 \pm 0.10$        | 0.50       |
| (Sub-GeV)   | 2.1/1 | 12800         | $0.62 \pm 0.10$        | 0.48       |
| IMB         | 2.1/1 | 1000          | $0.54 \pm 0.13$        | 0.47       |
| FREJUS      | 2.1/1 | 500           | $0.87 \pm 0.18$        | 0.47       |
| NUSEX       | 2.1/1 | 500           | $0.99 \pm 0.32$        | 0.47       |
| SOUDAN      | 2.1/1 | 1000          | $0.69 \pm 0.21$        | 0.47       |