Gravity Cannot Cure Quantum Mechanics of its Malady of the Collapse of the Wavefunction

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Abstract

The speculation that gravity is the key to solving the quantum measurement problem has been alive for decades, without any convincing demonstration of a solution. One necessary factor in the relevant proposals is that the gravitational energy of mutual interaction, which scales quadratically with the mass, facilitates the spontaneous collapse of the wavefunctions in spatially separated superpositions. Relying on a simple physical input from electrodynamics, supported by robust first principle calculations, we show that the speculations connecting gravity and the hypothetical spontaneous collapse of the wavefunction are inconsistent and not tenable. The result suggests that the gravitational solution to the problem of the collapse of the wavefunction be put to rest.

Gravity is the most familiar of fundamental interactions, and its theoretical understanding has evolved and matured over 300 years. With the General Theory of Relativity, we have an encompassing theory, applicable from the laboratory to the largest scale imaginable. Yet, we are not sure about its unconditional applicability in the smallest of physical scales. The reasons are two-fold. Since gravity is relatively very weak, experimental indications are difficult come by from the physics of small masses and microscopic distances. More importantly, the dynamics at the atomic scales are dictated by the theory of quantum mechanics, which is known to have issues of compatibility with the theory of gravity.

Apart from the vigorous efforts to find the unified terrain of gravity and quantum mechanics, in a future theory of quantum gravity, there is another important research front that hopes to combine aspects from the two theories. This is to
invoke the macroscopic ‘classical’ theory of gravity to solve the long-standing vexing problems of quantum mechanics. The situation is somewhat paradoxical; on the one hand it is assumed that the classical theory of gravity is definitely incompatible with quantum mechanics, demanding fundamental modifications in the classical theory of General Relativity. But on the other hand, one is appealing to even the Newtonian features of the gravitational interaction to cure the age old maladies of quantum mechanics. The primary unsolved problem of the quantum theory is the quantum measurement problem, which can be traced to its ‘birth defect’ of the collapse of the wavefunction [1, 2]. The quantum measurement problem arises in the collapse of the entangled wavefunction of one microscopic and another macroscopic system. However, we do not even understand how a quantum state characterized by a simple superposition of two spatially separated wavefunctions reduces to one of the two, to faithfully represent the factually observed situation, in each trial of a quantum mechanical experiment. A related open issue is how is that we are able to prepare such superpositions for microscopic physical systems whereas it becomes progressively difficult as the mass and size of matter increase, eventually becoming impossible in the case of macroscopic objects.

The proposals that invoke the gravitational interaction as a universal ‘agent’ to facilitate the spontaneous collapse of a spatial superposition of wavefunctions have been discussed for several decades now [3, 4, 5, 6, 7]. The quantitatively detailed schemes are associated with the independent proposals from L. Diósi and R. Penrose, though often they are collectively called the Diósi-Penrose (D-P) models. The difference is in the exact theoretical basis for invoking exclusively gravity to induce the spontaneous collapse. While the Diósi mechanism is largely Newtonian, Penrose bases his reasoning on the Einsteinian association of gravity with the very properties of space and time where the quantum dynamics happens [7].

However, the quantitative details in both mechanisms are very similar. This is what is important for our analysis here, which eventually reliably rules out gravity as responsible for the collapse of the quantum superpositions of wavefunctions. The case under consideration is represented by the simple superposition like

\[ \Psi = a\psi(x - x_1) + b\psi(x - x_2) \] (1)

that represents the wavefunction of a single particle of mass \( m \). The functions \( \psi(x - x_1) \) and \( \psi(x - x_2) \) could be Gaussian functions centred on the coordinates \( x_1 \) and \( x_2 \), with their separation \( |x_1 - x_2| \) even larger than the nominal width of the functions. An observation will always result in either a detection centred at \( x_1 \) or at \( x_2 \), permanently collapsing the superposed wavefunction to either \( \psi(x - x_1) \) or \( \psi(x - x_2) \). The gravity-induced collapse mechanism invokes essentially the gravitational energy of a hypothetical mutual interaction, of matter distributions represented by the component wavefunctions, as an instability measure that can induce such collapse. The underlying logic is based on the literal or
In the models for the gravity-induced spontaneous collapse of the superpositions of quantum mechanical wavefunctions, as in the proposals by Diósi and Penrose, a spatial superposition is assumed to have an ontological counterpart in real space. Hence, the interaction energy of the two parts of the wavefunction is well defined. However, the hitherto overlooked physical aspect of much stronger interactions that directly influence the dynamics, like the electromagnetic interaction when the particle is charged, demolishes the entire scheme as a thoroughly nonviable and inconsistent proposition.

\[ \psi(x) = \frac{1}{\sqrt{2}} \psi_1(x-x_1) + \frac{1}{\sqrt{2}} \psi_1(x-x_2) \]

\[ \Delta E \leq \frac{Gm^2}{|x_1-x_2|} \]

Figure 1: In the models for the gravity-induced spontaneous collapse of the superpositions of quantum mechanical wavefunctions, as in the proposals by Diósi and Penrose, a spatial superposition is assumed to have an ontological counterpart in real space. Hence, the interaction energy of the two parts of the wavefunction is well defined. However, the hitherto overlooked physical aspect of much stronger interactions that directly influence the dynamics, like the electromagnetic interaction when the particle is charged, demolishes the entire scheme as a thoroughly nonviable and inconsistent proposition.

**ontological interpretation of a quantum superposition as spatially separated matter in real space, as schematically indicated in the figure**

We stress the caveat that such an interaction ‘between’ the wavefunctions of a superposition is purely hypothetical, requiring the (failed) identification of the wavefunctions with the ‘fuzzy’ matter distributions in real space, as was attempted in the very early days of quantum mechanics. In any case, this energy is identified by Penrose [5] to be the expression

\[ \Delta E = 4\pi G \int (\Phi_1 - \Phi_2) \left( \nabla^2 \Phi_1 - \nabla^2 \Phi_2 \right) d^3x \]  

(2)

where \( \Phi \) is the gravitational potential of each density distribution corresponding to the individual wavefunction in the superposition. This can be rewritten in terms of the mass density distributions, corresponding to the two configurations that are notionally implied by the superposition, as

\[ \Delta E = 4\pi G \int \int [\rho_1(x) - \rho_2(x)] \left( \frac{\rho_1(y) - \rho_2(y)}{|x-y|} \right) d^3x d^3y \]

(3)

The same expression is derived in the Diósi mechanism as well. This gravitational interaction energy of the difference between the mass distributions of each of the two quasi-localized stationary distribution is the sole decider of the average duration of stability of a superposition, in the D-P mechanism. In the argument advocating the role of gravity in state-vector reduction, Penrose viewed the macroscopic quantum superposition of two differing mass distributions as unstable (analogous to an unstable particle). Therefore, it was speculated that
such a superposition would decay, or collapse, into one of the many states of the superposition, in a characteristic ‘quantum time scale’ related to the gravitational self-energy $\Delta E$. Then, the average time for spontaneous collapse is $\tau = \hbar / \Delta E$. The gravitational interaction energy for quasi-localized configurations with separation $|x_1 - x_2| \approx a$ is $\Delta E \approx Gm^2 / a$.

We can now discuss the central significant result in this paper, that we reliably rule out any role of gravity or any other known interaction in the collapse of the superpositions of quantum wavefunctions. We will prove the simple and transparent result that any scheme in which the component wavefunctions that are spatially separated have an energy of self interaction is inconsistent and self-destructive.

Consider the commonly discussed D-P mechanism in which the superposition has two equal wavefunction components that are separated by a distance of the order of their sizes (figure 1). The gravitational energy even for the case of a particle of size 1 micron is very small, $E \approx Gm^2 / a < 10^{-19}$ eV, and it does not affect the observable quantum dynamics. Then, the time scale of the collapse is $\tau = \hbar / E \approx 10^5$ s. But this becomes less than 0.1 s for a particle 10 time larger, suggesting the plausibility of the D-P mechanism.

However, the D-P mechanism assumes that only gravity can be effective in the collapse of the wavefunction, excluding arbitrarily other fundamental interactions. This restriction is needed because gravity is the only known universal interaction, applicable to all quantum systems. Though it is not clarified how to deal with the other interactions, when they are present, this itself is not an irreparable deficiency of the D-P mechanism. If we take the Einsteinian space-time justification advocated by Penrose, then one can try to justify that gravity is the unique choice by virtue of its ability to alter the space-time itself. However, if the particle is charged, as the case in numerous experiments in the laboratory, the interaction energy of the electromagnetism is as real as the interaction energy of gravitation; it is unavoidable and no mechanism can have one type of energy without the other. While the electromagnetic energy is ineffective in the collapse of the wavefunction, by assumption, it is extremely important in the dynamics of the particle! None of the proponents for the gravity aided collapse ever considered this simple and natural possibility, and its destructive consequence to the D-P mechanism of the spontaneous collapse of the wavefunction. To see its crucial nature, let us calculate the electromagnetic energy in exactly the same situation as we considered earlier with only gravity. The expression is exactly the same as in the equation, with $G$ replaced by $1/4\pi\varepsilon_0$, and the mass distributions replaced by the charge distributions. At the localizable separation (figure 1), $|x_1 - x_2| \approx a$, the electromagnetic energy is $\Delta E_{em} \approx e^2 / 4\pi\varepsilon_0 a$. The ratio of the two contributions is

$$\frac{\Delta E_{em}}{\Delta E} \approx \frac{e^2}{4\pi\varepsilon_0 Gm^2}$$

which is independent of the details of the superposition. Since the quantities
∇^2 \Phi_i \) are nonzero, the mutual forces are also nonzero and large, unlike the gravitational forces. Also, the ratio of the mutual forces from the two interactions are independent of the details and evolution of the superposition. We can examine the routinely encountered situation of trapped ions of mass \( m \approx 10^{-23} \) kg, in a quantum superposition of localized stationary states separated by 0.1 microns [8]. While the mutual gravitational force is an utterly negligible \( 10^{-42} \) N, the mutual electromagnetic force is \( 10^{-14} \) N, an unbearably enormous force for an ion of the tiny mass of \( 10^{-23} \) kg! The resulting acceleration of \( 10^9 \) m/s^2 would catapult the ion out of the laboratory. It is obvious that proposals like the D-P mechanism that invoke the gravitational self interaction to induce the collapse of the wavefunctions in spatial superpositions are irretrievably ruled out by simple physical considerations.

Having decisively ruled out the D-P mechanism of the gravity-induced collapse of the quantum superpositions, we can now make some general remarks about other issues of principle in these models. One is the inconsistency of the theoretical scheme, which remains somewhat hidden as long as one considers only the superpositions involving only two-component wavefunctions, as the proponents of such spontaneous collapse mechanisms often do. But, when there are many components in the superposition, the very formalism implies a nonlinear increase in the self energy, violating the fundamental conservation constraints, which renders the proposal inconsistent. The problem arises because gravity can be invoked only to collapse the wavefunction, while the actual values of the probabilities of realization, through the Born’s rule, are not related at all to the gravitational physics. For definiteness, we consider a particle with the mass \( m \), and size \( a \), represented by the spherically distributed superposition with \( N \) component wavefunctions, with their locations centred at \( \vec{r}_i \),

\[
\Psi = \sum_{i=1}^{N} \frac{1}{\sqrt{N}} \psi_i (\vec{r} - \vec{r}_i) \tag{5}
\]

In the case of the two-component superposition, the instability energy \( \Delta E \) is scales as \( m^2 \), with no indication of the number \( N \) of the component wavefunctions. But, with the more elaborate wavefunction in the expression [5] the situation is very different. Following Penrose [7], we can calculate the gravitational energy that will trigger the collapse. For \( |r_i| \approx a \),

\[
\Delta E > G \left( N - 1 \right) \frac{N m^2}{2 |r_i|} + \frac{3N}{5} G m^2 / a \tag{6}
\]

This should be contrasted with the self energy when \( |r_i| \ll a \), which is just \( 3G m^2 / 5a \), which clearly proves the severe violation of the conservation of energy, inherent in the D-P mechanism. We have to rigorously keep in mind that there is only one material particle. The wavefunction, however complex and detailed, is but a representation of the physical state of the particle, and not the particle itself. Unfortunately, this crucial fact has been overlooked by the proponents of
the gravitational collapse models, and as a consequence, these self-destructing features were not noticed.

In summary, we have conclusively ruled out the widely discussed and actively researched mechanisms of the gravity-induced collapse of the spatial superposition of quantum wavefunctions, employing transparent first principle calculations and comparison with the factual empirical situation. Our result is entirely independent of the details of these collapse mechanisms, and it is obtained by the simple device of just considering other common aspects of real matter in dynamics, like the electric charge and its interaction energy. With this result, it is ever more clear that the solution of the quantum measurement problem and the problem of the resolution of spatial superpositions are to be sought in a deeper level of dynamics and quantum mechanics [9]. What seems conclusive is that gravity is not the cure for the primary malady of quantum mechanics.

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