Rational Competitive Analysis

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Abstract

Much work in computer science has adopted competitive analysis as a tool for decision-making under uncertainty. In this work we extend competitive analysis to the context of multi-agent systems. Unlike classical competitive analysis where the behavior of an agent’s environment is taken to be arbitrary, we consider the case where an agent’s environment consists of other agents. These agents will usually obey some (minimal) rationality constraints. This leads to the definition of rational competitive analysis. We introduce the concept of rational competitive analysis, and initiate the study of competitive analysis for multi-agent systems. We also discuss the application of rational competitive analysis to the context of bidding games, as well as to the classical one-way trading problem.

1 Introduction

Competitive analysis is a central tool for the design and analysis of algorithms and protocols for decision making under uncertainty [3]. It is a well studied and widely applicable approach that fits the framework of qualitative decision-making in AI (see e.g. [3, 4]). The competitive analysis approach attempts to minimize the ratio between the payoff an agent obtains and the

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payoff he could have obtained had he known the behavior of the environment. For example, consider the following trading problem (see [3], Chapter 14). An agent who holds $100 may wish to exchange them for British pounds. At each point in time, e.g. every minute in between 8AM and 4PM, an exchange ratio of dollars and pounds is announced. This ratio changes dynamically, and in an unpredicted manner. The agent would need to choose the time in which it will trade his $100 for pounds. Notice that if the agent would have known the sequence of exchange rates, \( e \), then he could have chosen a strategy \( o(e) \) that maximizes his payoff. If the agent chooses a strategy \( s \), then we can compute the ratio of the payoffs obtained by \( o(e) \) and \( s \). We can do similarly for every sequence of exchange rates \( e' \). Based on this, we can compute the highest (i.e. worst) ratio, over all possible sequences, that might be obtained when we compare optimal strategies to \( s \). This ratio is denoted by \( R(s) \). According to the competitive analysis approach, the agent will apply the competitive ratio decision criterion: he will choose a strategy \( s \) for which \( R(s) \) is minimal. This decision criterion may be quite helpful when we lack probabilistic assumptions about the environment. For example, assume that the minimal value of \( R(\cdot) \), which is obtained by some strategy \( s \), is 2. Then, by selecting \( s \), the agent guarantees himself a payoff which is at least half of the optimal payoff that he could have obtained had he known the actual environment behavior.

The competitive ratio has also an additive variant (also termed minimax regret [11]), where we replace the term ”ratio” by the term ”difference” in the definition of \( R \). So, in our example, if a strategy \( s \) that minimizes \( R(\cdot) \) obtains a competitive difference/regret of 20, then this implies that by performing \( s \) the agent gets a payoff (e.g. worth in British pounds) which is at most 20 lower than what he could have obtained had he known the behavior of the environment. In the sequel, we will use the additive version of the competitive ratio decision criterion.

Competitive analysis has been applied to a variety of classical problems in computer science, such as the k-server problem [14] and paging [4], as well as to more general algorithmic problems [2]. In all of these studies the environment that the agent acts in is non-strategic, and therefore does not assume to follow any ”rational” behavior. In this paper we extend the concept of competitive analysis to the context of multi-agent systems. In a multi-agent system the environment in which an agent takes his decision consists of other ”rational” agents. Following previous work on competitive
analysis, our approach is non-Bayesian and normative; we would like to find a decision rule for the agent that will rely as little as possible on assumptions about the behavior of his environment. Therefore, we adopt the requirement that agent $A$ should rule out a behavior $b_1$ of agent $B$ only if $b_1$ is dominated by another behavior, $b_2$, of that agent. Dominated behaviors are purely irrational in any decision making model. The agent will choose his behavior according to the competitive ratio decision rule. However, he should consider only rational behaviors of the other agents; a behavior of an agent will be considered irrational if and only if it is dominated by another behavior of it.

In section 2 we describe bidding games, a family of games that will serve us for the illustration of the basic concepts developed in this paper. Bidding games are representatives of k-price auctions, a central class of economic mechanisms [12]. In section 3 we present a competitive analysis of bidding games. In section 4 we introduce rational competitive analysis, a new tool for normative decision making, that generalizes competitive analysis to the context of rational environments, and apply it to bidding games. In section 5 we consider repeated (multi-stage) games. We present several results on the relationships between (rational) competitive analysis of repeated games and the competitive analysis of the particular (one-shot) games they consist of. Then, in section 6, we discuss and study variants and modifications of the one-way trading problem, using rational competitive analysis. In particular, we study the multi-agent one-way trading problem.

## 2 Bidding games

We start by recalling the general definition of a (strategic form) game.

**Definition 2.1** A game is a tuple $G = \langle N = \{1, 2, \ldots, n\}, S = \{S_i\}_{i=1}^{n}, \{U_i\}_{i=1}^{n} \rangle$, where $N$ is a set of $n \geq 2$ players, $S_i$ is the set of strategies available to player $i$, and $U_i : \prod_{j=1}^{n} S_j \rightarrow R$ is the utility function of player $i$.

In a game, each player selects a strategy from a set of available strategies. The tuple of strategies selected, one by each player, determines the payoff of each of the agents (as prescribed by the utility functions).

\[\text{Here and elsewhere we use the terms player and agent interchangeably.}\]
In a bidding game, a center attempts to obtain a service from a set of potential suppliers. Each such supplier has a certain cost associated with that service. This cost is taken to be an integer between $K - T$ and $K$, where $K$ and $T$ are w.l.o.g integers, $K > T > 0$. Each agent will offer his service and ask for a payment in the range in between $K - T$ and $K$. We will associate the request for payment of $K - i$ with the integer $i$, where $0 \leq i \leq T$. The center will choose as the service provider the supplier with the lowest asking price. There are various ways for determining the payment to that agent; in particular, the agent can be paid his asking price, the second lowest asking price, or the third lowest asking price. We assume that the costs for providing the service by each of the agents are common knowledge among them, although the center might not be familiar with these costs. Although this is quite natural for the above procurement problem, other assumptions can be treated similarly.

Our definition of bidding games will capture the above, by considering a fully isomorphic situation, namely: the auctioning of a good. The good is held by the center. Each agent has a valuation (i.e. maximal willingness to pay) for the good. Each agent needs to decide on his bid. The center will allocate the good to the agent with the highest bid (rather than to the agent with the lowest asking price, as in the isomorphic procurement problem).

Formally, we have:

**Definition 2.2** Given a set of $n$ players, and an integer $T >> 1$, a bidding game is determined by the tuple $B = (x_1, \ldots, x_n, k)$ where $x_i = \frac{1}{T}$ for some integer $0 \leq l_i \leq T$, and $1 \leq k \leq n$ is an integer. Player $i$’s strategy in $B$ is a decision about $b_i \in [0, T]$. Given a strategy profile $b = (b_1, b_2, \ldots, b_n)$ denote by $b_{[i]}$ the $i$-th order statistic of this tuple. Let $M(b)$ be the number of elements of $b$ that equals $b_{[1]}$. Then, $U_i(b) = \frac{1}{M(b)}(x_i - \frac{b_{[i]}}{k})$ if $b_i = b_{[1]}$, and $U_i(b) = 0$ otherwise.

In the above formalism, $x_i$ is the valuation of agent $i$ (that is normalized to the interval $[0,1]$), while $b_i$ denotes the bid made by agent $i$. The payment made by the winner is determined by the parameter $k$. If $k = 1$ we get the standard high-bid wins (or first-price) auction; if $k = 2$ then we get the famous Vickrey (second-price) auction, while if $k = 3$ we get the case of
third-price auctions.

For ease of presentation we will assume that \( 2 \leq l_i < T \) for every \( 1 \leq i \leq n \), that \( i \neq j \) implies \( l_i \neq l_j \), and that \( T \geq n \).

3 Competitive Analysis

In a game, agent \( i \) is facing an environment that consists of the other agents. The actions to be selected by these agents are not under the control of \( i \). Following the literature on competitive analysis, the competitive ratio decision rule may be used in order to choose an action for that agent.

**Definition 3.1** Given a game \( G \), and a strategy profile \( s = (s_1, s_2, \ldots, s_n) \in \Pi_{j=1}^n S_j \), the regret of player \( i \) is given by \( \text{Reg}_i(s_i, s_{-i}) = \max_{t \in S_i} U_i(s_1, \ldots, s_{i-1}, t, s_{i+1}, \ldots, s_n) - U_i(s) \). A strategy \( s \in S_i \) is a competitive strategy for agent \( i \) if \( s \in \arg\min_{t \in S_i} \max_{q \in S_{-i}} \text{Reg}_i(t, q) \), where \( S_{-i} \) denotes the possible strategy profiles of players in \( N \setminus \{i\} \).

Given the above definition we are interested in applying competitive analysis to bidding games. We now present three claims about competitive analysis of bidding games. These claims are associated with the competitive analysis of 1st, 2nd, and 3rd-price auctions, respectively.

**Claim 3.1** Given the bidding game \( B = (x_1, \ldots, x_n, 1) \), a competitive strategy for agent \( i \) yields a regret value of \( \frac{\alpha}{T} \), where \( \alpha \) equals the upper integer value of \( \frac{l_i - 1}{2} \).

Basic idea behind proof: Agent \( i \) can lose by submitting a bid that is higher than his valuation. On the other hand, by submitting a bid that is below \( l_i - 1 \) agent \( i \) might lose, since agent \( j \neq i \) might submit \( l_i - 1 \) as a winning bid. Since agents may submit the bid 0, agent \( i \) will minimize his regret by submitting a bid that equals (the upper integer value of) half of the difference between \( l_i - 1 \) and 0.

**Claim 3.2** Given the bidding game \( B = (x_1, \ldots, x_n, 2) \) a competitive strategy for agent \( i \) yields a zero regret.

Basic idea behind proof: Here the optimal strategy for an agent, regardless of what the others do, is to send his actual valuation as his bid; this is a well known property of the Vickrey auction \[13\]. As a result we get a regret of 0.

\[\text{Third-price auctions have been shown to have appealing properties in the context of Internet Auctions \[12\].}\]
Claim 3.3 Given the bidding game $B=(x_1, \ldots, x_n, 3)$, and assume w.l.o.g that $x_1 > x_2 > \cdots > x_n$, then agent $j$’s competitive strategy is to send the bid $\min(2l_j, T)$.

Basic idea behind proof: Given that agents may submit the bid 0, agent $j$ might reach a regret of $l_j T$ if he is not the winner. Submitting however a bid that is higher than $2l_j$ may also lead to a regret of $l_j T$, given that the agents may submit $2l_j$ as their bids. Combining these observations, we get that submitting the bid $\min(2l_j, T)$ is the competitive strategy.

4 Rational Competitive Analysis

Although competitive analysis is a most powerful concept from a non-Bayesian normative perspective, it may be quite restrictive when we consider decision-making in multi-agent systems. Following the spirit of competitive analysis for normative decision making, we refrain from using probabilistic assumptions and game-theoretic equilibrium analysis.\footnote{The debate about whether competitive ratio and non-Bayesian decision making are expressive or useful for normative or descriptive objectives is beyond the scope of this paper; see \cite{5} for sound and complete axiomatization of the competitive ratio decision criterion.} However, one can still improve on the use of competitive analysis by considering minimal rationality requirements.

Definition 4.1 Given a game $G = \langle N = \{1, 2, \ldots, n\}, \{S_i\}_{i=1}^n, \{U_i\}_{i=1}^n \rangle$, we say that a strategy $s_i \in S_i$ weakly dominates a strategy $s_i' \in S_i$ if $U_i(s_i, t) \geq U_i(s_i', t)$ for every strategy profile $t$ of the players in $N \setminus \{i\}$, and there exists such strategy profile $t'$ for which $U_i(s_i, t') > U_i(s_i', t')$. A strategy $s \in S_i$ will be called rational if there is no other strategy $\bar{s} \in S_i$ that weakly dominates it. Given a game $G$, the set of rational strategies for player $i$ will be denoted by $\text{Rat}(S_i)$.

In any reasonable model agents will choose only from the set of non-dominated strategies. Our idea is therefore to combine the powerful idea of competitive analysis and this minimal requirement of rationality, in order to re-introduce competitive analysis into the framework of multi-agent systems.

Definition 4.2 A strategy $s \in S_i$ is a rationally competitive strategy if $s \in \arg\min_{t \in S_i} \max_{q \in \text{Rat}(S_{-i})} \text{Reg}_i(t, q)$, where $\text{Rat}(S_{-i})$ denotes the possible
rational strategy profiles of players in $N \setminus \{i\}$, i.e. each player $j \in N \setminus \{i\}$ chooses its strategy from $\text{Rat}(S_j)$.

Basically, a rationally competitive strategy applies the competitive ratio decision criterion, while taking into account only rational activities of the environment. As the following claims illustrate, rational competitive analysis introduces an improved approach to normative decision making.

**Claim 4.1** Given the bidding game $B=(x_1, \ldots, x_n, 1)$, a rational competitive strategy for agent $i$ yields a regret of $\frac{\alpha}{T}$, where $\alpha$ equals the upper integer value of $\frac{\min(l_i, \max_{j \neq i} l_j) - 2}{2}$.

Basic idea behind proof: We observe that any strategy that tells agent $j$ to submit a bid which is greater than or equals to his valuation is dominated by the strategy of submitting his valuation minus 1. Given our assumptions about the possible valuations, all other strategies, excluding the strategy of submitting the bid 0, are not dominated. As a result, from the perspective of agent $i$, if his valuation is the highest one, he will minimize his regret if he will make a bid that is half of the distance between $\max_{j \neq i} l_j - 1$ and 1. If agent $i$’s valuation is not the highest one then he will minimize his regret (again, taking into account the assumptions on possible valuations) if he will make a bid in between $l_i - 1$ and 1.

Notice that rational competitive analysis allows us to improve upon the type of reasoning carried out in claim 3.1. Technically, in the case of a bidding game with $k = 1$, rationality implies that we need to take the minimum between $l_i$ and the highest other agents’ valuation in our analysis, rather than consider $l_i$ only.

**Claim 4.2** Given the bidding game $B=(x_1, \ldots, x_n, 2)$ a rational competitive strategy for agent $i$ yields a zero regret.

As we can see, unlike the major effect of the rationality assumption in the case of a first-price auction, there is no change in the analysis in the case of a second-price auction. In the case of a third-price auction, we see again the effect of the revised notion:

**Claim 4.3** Given the bidding game $B=(x_1, \ldots, x_n, 3)$, and assume w.l.o.g. that $x_1 > x_2 > \cdots > x_n$, then a rational competitive strategy for agent
j (j = 1, 2), is to submit the bid \( \min(2l_j - l_{[3]}, T) \), where \( l_{[3]} \) corresponds to the 3rd highest \( x_k \); a zero-regret rational competitive strategy for agent \( i, 3 \leq i \leq n \), is to submit \( l_i \).

Basic idea behind proof: First, observe that any strategy where the agent submits a bid that is below that agent’s valuation is dominated by the strategy that tells him to submit his actual valuation as his bid. As a result, for agents 3, 4, ..., \( n \) there is a 0 regret in submitting their actual valuations as their bids. Let us assume that agent \( i \) (where \( i \) is 1 or 2) submits a bid, then it can lose \( \frac{l_i - l_{[3]}}{T} \) if it turns out not to be the highest bidder (since agent \( j \) submits a higher bid). On the other hand, by submitting the bid \( b_i > l_i \) a loss of \( \frac{b_i - l_i}{T} \) may be caused, since (from the perspective of agent \( i \)) two other agents may submit the bid \( b_i \). This implies that the bid \( \min(2l_i - l_{[3]}, T) \) will minimize this agent’s regret.

As we can see, in the case of \( k = 3 \) as well, rational competitive analysis for bidding games leads to an improved normative approach to decision making. In particular, the competitive strategy of Claim 3.3 specifies a too high bid, and is not a rationally competitive strategy; as a result, it fails to serve in a multi-agent context.

5 Rational competitive analysis in repeated games: folk theorems

We first recall the notion of finitely repeated games [9].

Definition 5.1 Given an integer \( l > 0 \) and a game \( G = (N = \{1, 2, \ldots, n\}, S = \{S_i\}_{i=1}^n, \{U_i\}_{i=1}^n) \), a repeated game \( RG = (G, l) \) with respect to \( G \) is a game where \( G \) is repeatedly played \( l \) times. \( RG \) consists of the following strategies and utility functions: a strategy of agent \( i \) in \( RG \) determines the strategy of \( G \) to be taken by \( i \) in the \( k \)-th iteration of \( G \), as a function of the history of strategies of \( G \) selected by the others in iterations 1, 2, ..., \( k - 1 \). Given a tuple of strategies of \( RG \), one for each agent, the payoff for agent \( i \) is the sum of its payoffs along the \( l \) iterations. A sub-game of a repeated game \( RG \) is a repeated game that starts from iteration \( 1 \leq q \leq l \) of \( RG \) and consists of
Repeated games have been of much interest in the game-theory literature, due to the fact they enable to study agents’ actions as a function of past events and other agents’ actions. The study of repeated games is central to the understanding of basic issues in coordination and cooperation (e.g. [1]), as well as for the study of learning in games (e.g. [2]).

One of the central challenges for the study of repeated games is to establish general theorems (titled folk-theorems) that explain/recommend behavior in these (repeated) games by means of solution concepts for the games they consist of. In our case, it would be of interest to understand what will be a rationally competitive strategy in a repeated game, and try to relate it to the competitive analysis of the simple one-shot game that takes place at each iteration.

We now present a general result about competitive analysis in repeated games. For ease of presentation we will assume that $G$ is a two-player game, where all payoffs are distinct. We will also assume w.l.o.g that all payoffs are non-negative. Given a repeated game $(G, l)$, let us denote the highest payoff for agent $i$ in $G$ by $h_i(G)$, and the second highest payoff of agent $i$ in $G$ by $sh_i(G)$.

**Theorem 5.1.** Given a repeated game $(G, l)$ and assume that for each agent $i$ $h_i(G) \geq 2 \cdot sh_i(G)$, then a rationally competitive strategy for agent $i$ in the game $(G, l)$ is obtained by performing the competitive strategy of it in $G$ on iterations $1, 2, \ldots, l - 1$ and performing the rational competitive strategy of it in $G$ on the last iteration.

Basic idea behind proof: From the perspective of agent $i$, assuming we are at stage $k < l$, the selection of any strategy $s$ of $G$ by $j$ can be complemented to a non-dominated strategy of $j$; this non-dominated strategy will tell $j$ to choose the strategy associated with $h_j(G)$ in stages $k + 1, \ldots, l$. The reason that the resulting strategy is not dominated is that $j$ considers the strategy where $i$ will also choose in stages $k + 1, \ldots, l$ the strategy (of his) in $G$ that corresponds to $h_j(G)$, and does it only if in stage $k$ agent $j$ chooses $s$; in addition, according to this strategy $i$ will choose the strategy that

\[ l - q + 1 \text{ iterations. A (rationally) competitive strategy in } RG \text{ is a strategy that is a (rationally) competitive strategy at each of the sub-games of } RG. \]
corresponds to \(sh_j(G)\) is stage \(k\). This implies that agent \(i\) should consider at stages 1, 2, \ldots, \(l - 1\) all possible strategies of agent \(j\) in \(G\). In the last stage agent \(i\) is no longer subject to the above considerations and will choose the rationally competitive strategy of \(G\).

The above theorem shows a strong connection between competitive analysis in repeated games and competitive analysis in simple single-shot games. As it turns out, this connection can be further generalized to a much richer context:

**Definition 5.2** Let \(\bar{G} = (G_1, G_2, \ldots, G_m)\) be a sequence of games where \(N\) is the set of players in each of the games in the sequence, and game \(G_i\) is played in iteration \(i\). The strategy of agent \(t\) in \(\bar{G}\) determines its strategy in \(G_i\), \(1 \leq i \leq m\), as a function of the strategies of \(G_j\), \(1 \leq j < i\), selected by the other agents in previous iterations. Given a tuple of strategies of \(\bar{G}\), one for each agent, the payoff of agent \(i\) is taken as the sum of its payoffs in the \(m\) iterations.

**Theorem 5.2** Given a sequence of games \(\bar{G} = (G_1, G_2, \ldots, G_m)\) where \(N\) is the set of players in each of the games in the sequence, and game \(G_j\) is played in iteration \(j\), and assume that \(h_i(G_k) \geq 2 \cdot sh_i(G_l)\) for every \(1 \leq k, l \leq m\), and for every agent \(i\), then a rationally competitive strategy for agent \(i\) in the game \(\bar{G}\) is obtained by performing the competitive strategy of \(G_j\) in iterations 1, 2, \ldots, \(l - 1\) and performing the rational competitive strategy of \(G_m\) in the last iteration.

The above theorem can be generalized into a situation where \(n\) games from among the set of games \(\{G_1, G_2, \ldots, G_m\}\) are executed in some random order (with possible repetitions). Formally, this can be captured by the following definition and theorem:

**Definition 5.3** Given a set of games \(G = \{G_1, G_2, \ldots, G_m\}\), a random game with respect to \(G\), \(\bar{G}\), is a sequence of \(n\) games \((g_1, g_2, \ldots, g_n)\), where \(g_i \in G\) (\(1 \leq i \leq n\)) and \(N\) is the set of players in each of the games in the sequence. The game to be played in iteration \(i\), \(g_i\), is randomly selected from the set \(G\) independently of previous selections made. The strategy of agent \(t\) in \(\bar{G}\) determines its strategy in \(g_i\), \(1 \leq i \leq n\), as a function of the strategies of \(g_j\), \(1 \leq j < i\), selected by the other agents in previous
iterations. Given a tuple of strategies of $\bar{G}$, one for each agent, the payoff of agent $t$ is taken as the sum of its payoffs in the $n$ iterations. A sub-game of a random game $\bar{G}$ with respect to $G$, is a random game with respect to $G$ that starts from iteration $1 \leq j \leq n$ and consists of $n - j + 1$ iterations as above. A (rationally) competitive strategy in a random game is required to be a (rationally) competitive strategy at each sub-game of it.

**Theorem 5.3** Given a random game $\bar{G}$ with respect to $G = \{G_1, G_2, \ldots, G_m\}$, and assume that $h_i(G_k) \geq 2 \cdot sh_i(G_l)$ for every $1 \leq k, l \leq m$, and for every agent $i$, then a rationally competitive strategy for agent $i$ in the game $\bar{G}$ is obtained by performing the competitive strategy of game $g_i$ in iterations $1, 2, \ldots, n - 1$ and performing the rational competitive strategy of the game $g_n$ on the last iteration in the sequence.

## 6 One-way trading in multi-agent systems

In the previous section we have discussed competitive analysis for multi-agent systems in the framework of general repeated games and random games. In this section we look at a particular variant of repeated games that extends a well known and fundamental framework for competitive analysis – the one-way trading (see citations in chapter 14 of [3]).

One way to present the structure of one-way trading is as follows. An agent $a$ seeks buying $X$ units of a good or of a service. A supplier $A$ wishes to supply these units of good to $a$. The agents act in an environment that determines the actual payment for a unit of good in a non-deterministic way. For example, the payments might be specified in dollars, but since agent $A$ is a British company the actual payoffs it will obtain for providing the good will depend on the exchange ratio of the dollar and the British pound. Formally, the environment announces at each point in time, $1, 2, \ldots, t$, the payoff that will be obtained by agent $A$ for supplying a unit of good. The announcements are selected in an unpredicted non-deterministic manner from the interval $[m, M]$, where $M > m > 0$. For example, when $K$ is announced at point $i$, agent $A$ can supply the $X$ units of good and obtain a payoff of $X \cdot K$. Our assumption is that agent $A$ will obtain a zero payoff by not providing the units of good. The decision problem that agent $A$ faces is as follows: at each point he needs to decide whether he would like to supply the units of
good in the current rate. We assume that when agent \( A \) is willing to provide the service then he will provide and be paid for the whole quantity of goods requested by agent \( a \) (this property is termed one-way search).

The competitive analysis approach tells agent \( A \) in the above scenario to minimize his regret value. As it turns out, the competitive strategy in this case will tell the agent to accept the offer (i.e., supply the units of good) when the payoff reaches \( \frac{M - m}{2} \) in stage \( j \leq t - 1 \), and to accept the offer on stage \( t \) otherwise.

One-way trading is a typical setting for the use of competitive analysis. We now extend it to the case of several agents, where more than one agent may wish to supply the units of good requested by \( a \). We will first develop the multi-agent framework without considering the rationality assumption, and then will extend it to the case of rational competitive analysis.

### 6.1 Multi-agent one-way trading

For ease of exposition we consider the case of trading two agents (i.e., two suppliers who can provide the units of good requested by \( a \)): \( A_1 \) and \( A_2 \). The payment offers for the two agents are taken to be independent. For example, agent 1 may be a British company and agent 2 may be a Japanese company, and therefore the actual payment offers for them (from their perspective) will reflect the exchange ratio between the dollar and the British pound, and the exchange ratio between the dollar and the Japanese Yen, respectively. Formally, we have:

**Definition 6.1** Let \( M_1, M_2, m_1, m_2, t, X = 2K \) be positive integers, where \( M_1 > m_1 \) and \( M_2 > m_2 \), \( t \geq 3 \), and \( X \) is even. A multi-agent one way trading \( T = \langle N = \{1, 2\}, X, t, m_1, M_1, m_2, M_2 \rangle \) is a random game with the following players, strategies, and payoffs:

1. The players are 1 and 2.

2. There are \( t \) iterations. Each iteration \( i \) is associated with a pair of numbers \( (a_1, a_2) \) where \( m_1 \leq a_1 \leq M_1 \) and \( m_2 \leq a_2 \leq M_2 \). At each iteration each agent can “take” or “pass”. However, if an agent takes in iteration \( j \) then both agents can only “pass” in all iterations \( j \geq i \).
3. The payoff of each agent in iteration $i$ is 0 if it passes; if an agent performs "take" in iteration $i$ then its payoff will be $a_i X$ if the other agent passes and $a_i K$ if the other agent takes.

Intuitively, "take" means a decision of accepting the offer, while "pass" means rejecting it (at the given point). If both agents agree to "take" then each one of them will supply half of the units (and the payoff will be splitted among the agents). We now show what is the structure of the competitive strategy in a multi-agent one-way trading setting.

**Theorem 6.1** Given a multi-agent one way trading $T = \langle N = \{1, 2\}, X, t, m_1, M_1, m_2, M_2 \rangle$, a competitive strategy for agent $i$ is as follows:

1. For iterations $1 \leq j \leq t - 1$, take iff $a_i \geq \frac{2M_i + m_i}{4}$
2. If you arrive at iteration $t$ then take.

Basic idea behind proof: Consider iteration $j$, $1 \leq j \leq t - 1$, and consider the announcement $a_i = Y$. Then, by taking in round $j$, agent $i$ might suffer a regret of $2M_i K - 2Y K$ (notice that there is a regret when an agent takes only if the other does not take at that iteration). By not taking in stage $j$ agent $i$ might suffer a regret of $2Y K - m_i K$ (which is in fact $\max(K Y, 2Y K - m_i K)$). In order to minimize the regret agent $i$ will therefore have to take whenever $Y$ satisfies that $2M_i K - 2Y K = 2Y K - m_i K$, i.e. when $a_i = \frac{2M_i + m_i}{4}$. The fact that the regret is minimized in iteration $t$ by taking rather than passing is immediate.

### 6.2 Rational competitive analysis for multi-agent one way trading

We now show the result of applying rational competitive analysis to the context of multi-agent one way trading:

**Theorem 6.2** Given a multi-agent one way trading $T = \langle N = \{1, 2\}, X, t, m_1, M_1, m_2, M_2 \rangle$, a rational competitive strategy for agent $i$ is as follows:

1. For iterations $1 \leq j \leq t - 1$, if the other agent, $k$, is announced that $a_k = M_k$, then take.
2. For iterations $1 \leq j \leq t - 2$, if (1) does not hold then take iff $a_i \geq \frac{2M_i + m_i}{4}$.

3. If (1) does not hold, then in iteration $t - 1$ take iff $a_i \geq \frac{M_i + m_i}{4}$.

4. If you arrive at iteration $t$ then take.

Basic idea behind proof: Notice that if the other agent, $k$, is announced that $a_k = M_k$ then taking dominates any other strategy of it. In no other cases we can say that taking or passing in iterations $1, 2, \ldots, t - 1$ is dominated. Also, in stage $t$ passing is dominated by taking. As a result we will get that agent $i$ will minimize its regret by taking when $a_k = M_k$ or when it arrived in the last iteration. Assume that $a_i = Y$ in iteration $t - 1$, then the maximal regret we get by taking is $M_i K - 2K Y$, and by passing the maximal regret in this case is $2K Y - m_i K$ (which is in fact $\max(K Y, 2K Y - m_i K)$). This implies that the regret is minimized when $M_i K - 2Y K = 2K Y - m_i K$, i.e. when $Y = \frac{M_i + m_i}{4}$. The other case that refers to iterations $1, 2, \ldots, t - 2$ will be treated as in the case of (standard) competitive analysis.

7 Conclusion

Competitive analysis is a major tool in computer science, which has been used in a variety of contexts. In this paper we have introduced rational competitive analysis. Rational competitive analysis generalizes competitive analysis to the context of multi-agent systems. Moreover, we have shown its use in the context of bidding games and one-way trading, two problems of considerable importance, as well as in the context of general repeated games. Our approach adopts the non-Bayesian normative approach adopted in previous work, but modifies it to incorporate minimal rationality requirements. Such requirements are essential in multi-agent domains. Many of the previous studies in the context of competitive analysis can be naturally extended to multi-agent domains, and then rational competitive analysis can serve as a fundamental tool for the study of these extensions. We see the study of such extensions as a most attractive research topic, and hope that others will join us in addressing this challenge.
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