Vibration Analysis of Atomising Discs

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Abstract. The centrifugal atomisation of metallic melts using a spinning disc is an important process for powder production and spray deposition. In the manufacturing process the high-temperature melt flows down to the surface of the atomising disc spinning at very high speed. It is observed that there is a hydraulic jump of the melt flow prior to atomisation. In this paper, the dynamic model of the atomising disc as a spinning Kirchhoff plate with this hydraulic jump is established. The flowing melt is modelled as moving mass and weight force in the radial direction. Using a Galerkin method, it is found that the vibration properties of the atomising disc vary with the disc clamping ratio. The amplitude of the vibration is largely raised when the clamping ratio is smaller than the critical jump radius ratio. It is also found that the disc vibration is non-stationary before becoming steady and the amplitude decreases with increasing disc speed.

1. Introduction
Centrifugal atomisation of metallic melts using a spinning disc is an efficient process for powder production and spray deposition. It was originally developed by the Pratt & Whitney Aircraft Group to produce Ni-based superalloy powders [1] and is widely used for manufacturing high quality powders and near-net-shape preforms of a range of advanced metallic materials [2-5]. In the manufacturing process, the metallic melt flows from a nozzle under gravity onto the centre of the atomising disc spinning at high speed and gives the disc an impact. It then spreads quickly to the disc edge due to the large centrifugal force and induces moving weight, friction, thermal stress and pressure to the disc. At the edge of the disc, the melt is thrown off the disc and disintegrated into a spray of droplets with vast heat transfer in subsequence, which solidify in room temperature to form spherical particles with a narrow size distribution. The interaction between the metallic melt and the atomising disc is very complex.

The flow behavior of melts on the surface of the spinning disc has attracted a lot of investigations [6-10]. Previous research has shown that there are three modes of flow behaviour [11, 12]. At low volumetric flow rate, the direct droplet formation is observed at the edge of the disc. With increasing volumetric flow rate, there is a transition from the direct droplet formation to ligament disintegration. Film disintegration emerges at the higher flow rate and the surface of the atomising disc is covered by a continuous film moving from the centre to the circumference of the disc. It is observed that there is a hydraulic jump phenomenon encountered in the moving film prior to the disintegration. Zhao and his coworkers simulated the height distribution of the continuous film which is axisymmetric based on Newtonian fluid theory [13]. The aforementioned studies focused on the flow behaviour, while investigations of the vibration of the atomising disc are rare in the open literatures although it will definitely influence the flow behaviour and subsequently affect the quality of the powder production.
The vibration of atomising discs was first studied by Ouyang [14, 15]. The atomising disc was modeled as a spinning Kirchhoff plate and the continuous film as a moving distributed mass. A numerical example was given by assuming the mode shape of the disc in the radial direction as polynomial functions. The hydraulic jump of the continuous film was not considered in that paper. In this paper, the vibration of spinning atomising disc subjected to the moving film is analysed. The dynamic model is established based on Kirchhoff plate theory and the metallic melt is treated as moving load considering the hydraulic jump. The mode shape of the disc in the radial direction is assumed as a combination of Bessel functions which is more accurate than polynomials and a Galerkin method is employed to solve this problem.

2. Dynamic Model

In centrifugal atomisation, the metallic melt descends to the center of atomising disc spinning at a high speed and is accelerated toward the edge of the disc prior to disintegration. The load on the atomising disc is complicated, involving the gravity of the moving film, fluid inertial force, friction, thermal stress and so on. To establish the dynamic model, several assumptions are made in this paper: (I) the distribution of the melt film is axisymmetric and does not vary in the circumferential direction; (II) the friction between the film and the disc can be ignored; (III) the temperature of the disc is constant and the thermal stress is negligible; (IV) the influence of the vibration of the disc on the film height is assumed to be negligible.

The equation of the motion of the disc subjected to an external distributed load \( p^* \) based on Kirchhoff plate [16] is

\[
\rho h \frac{\partial^4 w^*}{\partial t^2} + D \nabla^4 w^* - h \left( \frac{r}{\partial r} \right)^2 \left( r \frac{\partial w^*}{\partial r} - h \frac{\partial^2 w^*}{r^2 \partial \theta^2} \right) = p^*(r, \theta, t) \quad r^* \in [a, b]
\]  

(1)

where the biharmonic differential operator when written in the cylindrical coordinate system is

\[
\nabla^4 = \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right)^2
\]  

(2)

where \( \rho, h, D, v, a, b \) are the mass density, the thickness, the flexural rigidity of the plate, Poisson’s ratio, inner radius and outer radius. It must be noticed that this equation is established in the rotating frame fixed to the spinning disc at speed \( \Omega \). Dimensionless variables are defined by

\[
r = r^*/b, w = w^*/h, t = t/\sqrt{D/h}\rho b^4, \Omega = \Omega^* \sqrt{D/h}\rho b^4, \sigma_\theta = \sigma_\theta^* h b^2/D, \sigma_r = \sigma_r^* h b^2/D, p = p^* b^4/Dh
\]  

(3)

Under definition (3), Eq. (1) becomes

\[
\frac{\partial^2 w}{\partial t^2} + \nabla^4 w - \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) - \frac{\sigma_\theta}{r^2} \frac{\partial^2 w}{\partial \theta^2} = p(r, \theta, t)
\]  

(4)

The in-plane stresses \( \sigma_r \) and \( \sigma_\theta \) in Eq. (4) are [17]:

\[
\sigma_r = d_1 + \frac{d_2}{r^2} + d_3 r^2, \sigma_\theta = d_1 - \frac{d_2}{r^2} + d_4 r^2
\]  

(5)

where

\[
d_1 = \frac{1 + v}{8} \frac{(v-1) a_c^4 - (3 + v) \Omega^2}{(v-1) a_c^2 - (1 + v)}, d_2 = \frac{1 - v}{8} \frac{(v+1) a_c^4 - (3 + v) \Omega^2}{(v-1) a_c^2 - (1 + v)^2} a_c^2, d_3 = -\frac{3 + v}{8} \Omega^2, d_4 = -\frac{1 + 3v}{8} \Omega^2
\]  

(6)
where \( a_i = a/b \) is clamping ratio of the atomising disc.

In the region where the melt is present on the disc surface, the external distributed load has two sources: one is due to the fluid inertia force and the other due to the weight of the film

\[
p^* = \begin{cases} \rho_m h_m(r^*, t^*) \left( \frac{\partial^2 w^*}{\partial t^2} + g \right) & a < r^* \leq r_m^*(t^*) \\ 0 & r_m^*(t^*) < r^* \leq b \end{cases}
\]

where \( \rho_m^*, h_m^* \) are the mass density and the height distribution of the film. If the film acts as a Newtonian fluid with a constant viscosity which is continuous and axisymmetric, its height distribution can be described as follows \[13\]

\[
h_m^*(r^*, t^*) = \begin{cases} \frac{v_i}{n_i \Omega} \frac{r_0}{\sqrt{3(2k-1)}} & a < r^* \leq r_c^* \\ -\frac{1}{n_i} \frac{v_i}{\Omega} \ln \left( 1 - \frac{n_i Q}{\pi n_i \sqrt{v_i \Omega}} \frac{1}{r} \right) & r_c^* < r^* \leq b \end{cases}
\]

where \( v_i \) and \( Q \) are kinematic viscosity and volume flow rate of the melt, constants \( n_i, n_k \) and \( n_b \) are 0.702, 0.587 and 0.739 respectively. \( w_0 \) is the initial velocity of the melt just prior to the impingement, \( r_0 \) is radius of the melt stream just before it impinges on the disc, \( r_c^* \) is critical jump radius.

The radius of the melt at time \( t^* \) is

\[
r_m^*(r^*, t^*) = \begin{cases} \int_0^{r^*} u_m(r^*) \, dt^* & a < r_m^*(t^*) < b \\ b & b \leq r_m^*(t^*) \end{cases}
\]

where \( u_m(r^*) \) is the radial velocity of the moving film.

\[
u_m^*(r^*) = \begin{cases} \frac{3n_i w_0^* r^*}{v_i} e^{-\frac{\rho_m^* h_m^*}{n_i v_i}} & a < r^* \leq r_c^* \\ n_i \Omega^2 r^* e^{-\frac{n_i Q}{v_i} \left( 1 - e^{-\frac{\rho_m^* h_m^*}{v_i}} \right)} & r_c^* < r^* \leq b \end{cases}
\]

Under the definition of Eq. (3), the dimensionless form of Eq. (7) become

\[
p = \begin{cases} -\rho_m h_m(r, t) \frac{\partial^2 w}{\partial t^2} - \rho_m h_m(r, t) g b^4 \frac{1}{D} & a_c < r \leq r_m(t) \\ 0 & r_m(t) < r \leq 1 \end{cases}
\]

where \( \rho_m^*, h_m^* \) and \( r_m^* \) are dimensionless parameters defined as \( \rho_m^* = \frac{\rho_m}{\rho}, h_m^* = \frac{h_m}{h} \) and \( r_m^* = \frac{r_m}{b} \).

### 3. Solutions

The solution of Eq. (1) can be written as

\[
w(r, \theta, t) = \sum_{j=-\infty}^{\infty} W_j(r) e^{ij\theta} q_j(t)
\]
where \( i = \sqrt{-1} \). The modes of the free disc are:

\[
W_i(r) = a^T \psi(r)
\]

where

\[
a = \left[ a_1 \ a_2 \ a_3 \ \ldots \right]^T, \quad \psi(r) = \left[ \psi_1(r) \ \psi_2(r) \ \psi_3(r) \ \ldots \right]^T
\]

In Eq. (14) \( \Psi_i(r) = R_{nn}(r) \) and \( R_{nn}(r) \) is the modes of the static disc, which can be represented by

\[
R_{nn}(r) = [\alpha, \ \beta, \ \gamma, \ \delta][J_n(\omega r), \ Y_n(\omega r), \ I_n(\omega r), \ K_n(\omega r)]^T
\]

where \( J_n(\omega r) \) and \( Y_n(\omega r) \) are Bessel functions of the first and second kind of order \( n \), and \( I_n(\omega r) \) and \( K_n(\omega r) \) are modified Bessel functions of the first and second kind of order \( n \). The value of \( m \) represents the number of nodal circles while, \( n \) represents the number of nodal diameters. The ratios of the constants \( \alpha, \ \beta, \ \gamma \) and \( \delta \) and the natural frequency parameter \( \kappa \) are determined by the boundary conditions of the disc [18]. The natural frequency parameter \( \kappa \) is given by \( \kappa^2 = \lambda^2 \) where

\[
\lambda = \omega \sqrt{\frac{phb^2}{D}}
\]

in which \( \omega \) is the real natural frequencies of the static disc.

By using a Galerkin method, Eq. (4) becomes

\[
\sum_{j=0}^{\infty} \left[ \left[ W_j e^{i\theta} q_j + \nabla^2 W_j e^{i\theta} q_j + L(W_j e^{i\theta} q_j) \right] \delta(W_k e^{i\theta}) \right] d \theta
dr d \theta
\]

\[
= \int p(r, \theta, t) \delta(W_k e^{i\theta}) r dr d \theta
\]

where

\[
L(w, r, \theta) = -\frac{\partial}{\partial r} \left( r \sigma_r \frac{\partial w}{\partial r} \right) - \frac{\partial^2 w}{\partial \theta^2}
\]

Substituting Eq. (13) into Eq. (17) yields

\[
\sum_{j=0}^{\infty} \left[ \left[ W_j e^{i\theta} q_j + \nabla^2 W_j e^{i\theta} q_j + L(W_j e^{i\theta} q_j) \right] \delta(W_k e^{i\theta}) \right] d \theta
dr d \theta
\]

\[
= \int p(r, \theta, t) \delta(W_k) r dr d \theta \int e^{ik\theta} d \theta
\]

Because of orthogonality of modes

\[
\int_0^{2\pi} e^{i(k+j)\theta} d \theta = \begin{cases} 2\pi & k = -j, k = j = 0 \\ 0 & \text{else} \end{cases}
\]

one gets

\[
\sum_{j=0}^{\infty} a^T \left[ \left[ (\psi^T \psi^T) q_j + \omega^2 \psi^T q_j + L(\psi\psi^T q_j) \right] r dr \right] = \int p(r, \theta, t) \psi^T r dr d \theta
\]

Substitution of Eq. (11) into the above equation leads to
Since Eq. (22) contains time-dependent parameters, a fourth order Runge-Kutta algorithm is developed to obtain numerical solutions. The initial vertical displacement and velocity of the atomising disc are assumed to be zero.

4. Results and Discussion

The parameters of the atomising disc under study are: $\rho_m = 4100/7850$, $h = 0.005m$, $b=0.2m$, $v = 0.3$, $E = 2.1e11$Pa, $v_k = 1.263\times10^{-6}$m$^3$/s, $Q = 6.096\times10^5$ m$^3$/s.

As shown in Fig. 1, the vibration of the atomising disc consists of three stages: (I) The downward deflection with flutter-like vibration due to the initial descending weight of the melt; (II) The growing vibration due to the growing mass and weight as the melt film spreads towards the circumference of the disc; (III) The steady vibration after the melt flies off the edge of the disc. The time-dependent
Fourier transformation in Fig. 2 indicates that the frequency decreases initially due to the growing mass when the melt spreads out before reaching the edge of the disc. At the third stage, the melt flow becomes steady and the frequency then also becomes constant as expected. These results are consistent with those reported in [15]. The results shown in Fig. 1 are obtained by setting $a_c = 0.3$ and $\Omega = 2$.

![Graphs showing vibration properties](image)

Figure 3. The disc vibration at: (a) $\Omega = 4$; (b) $\Omega = 6$; (c) $\Omega = 8$; (d) $\Omega = 10$

The spinning speed of the atomising disc is an important parameter which induces centrifugal force causing the melt film to spread and subsequently determines the size of produced particles. Figure 3 shows the influence of the spinning speed of the atomising disc on the vibration where $a_c$ is equal to 0.3. As the spinning speed increases, the time length of the first vibration stage and the deflection of the disc due to the moving film weight decreases. The amplitude of the steady vibration is also reduced slightly with the increase of the spinning speed. These results, the evidence of the stiffening effect of centrifugal force, agree well with the findings of [14, 15].

Figure 4 shows the vibration of the atomising disc when the clamping ratio $a_c$ changes to 0.1, while $\Omega$ remains 2 and other parameters stay the same. It clearly indicates that the vibration properties become quite different compared with Fig. 1. The downward deflection of the disc is very small. Thus, it is difficult to separate the first vibration stage from the second vibration stage. After the growing vibration stage, the second stage, there is a sharply increasing vibration stage prior to the steady stage which does not exist in Fig. 1. In the steady stage, the vibration amplitude is much larger than the
previous one, nearly ten times greater, as shown in Fig. 4. The reasons for this situation are analysed as the following. When the clamping ratio equals 0.3, it is larger than the critical jump radius ratio which is 0.14 specifically. No hydraulic jump emerges in the area of the disc where the deflection and the vibration are induced by the moving film weight. As shown in Fig. 5, the film prior to hydraulic jump is much thinner than the film after hydraulic jump. Thus, the weight of the melt film spreading prior to hydraulic jump is very small. As a consequence, the downward deflection of the disc due to the initial descending weight of the melt is also very small. This leads to the result that the first vibration stage is difficult to tell. After this the melt film keeps spreading out and the vibration grows with increasing weight. Then hydraulic jump emerges. The height of the film suddenly rises up and makes the deflection of the disc sharply increase. After hydraulic jump, the film spreads out smoothly and flies off the atomising disc. The vibration of the atomising disc becomes steady subsequently.

5. Conclusion
The vibration of the atomising disc spinning at high speed is investigated in this paper. The atomising disc is modelled as a spinning annular Kirchhoff plate clamped at the inner radius and free at the outer radius. The melt flowing on the surface of the disc is treated as moving mass and growing weight. The hydraulic jump phenomenon of the melt film is considered in this paper. Using a Galerkin method to solve the equation of motion, it is found from the simulated example that the vibration properties of the atomising disc vary with the disc clamping ratio. If the clamping ratio is greater than the critical jump radius ratio, the disc motion consists of three vibration stages: a downward oscillatory deflection, growing vibration and finally steady vibration. While the clamping ratio is smaller than the critical jump radius ratio, the amplitude of the vibration is largely raised, nearly ten times greater than the previous one. Moreover, there is a sharply increasing vibration stage between the growing vibration stage and the steady vibration stage due to the hydraulic jump. It is also found that the disc vibration decreases with increasing disc speed.

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