Special aspects in interference of in-phase and anti-phase waves with unequal phase velocities in coupled lines under pulse impact

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Abstract. We considered the propagation of a short pulse in the picosecond range in coupled lines with a ratio of phase velocities of anti-phase and in-phase waves of 3:1. We showed experimentally a special aspect in the interference of in-phase and anti-phase waves in such structures, which leads to the input pulse separation between three ports without a significant energy loss associated with the reflection from the input. The wave interference was found to lead to a change in the directional properties of the structures under consideration and a separation of the pulse spectral components between the ports.

1. Introduction

The concept of in-phase and anti-phase waves was introduced in the theory of coupled lines (CL) when solving telegraph equations [1-3]. In this and other works, the phase velocities of in-phase \( v_e \) and anti-phase \( v_0 \) waves were considered as equal and the process of wave propagation in lines was considered as a result of the wave interference with the same propagation constants. However, with the development of the theory and practice of coupled strip lines with dielectric filling inhomogeneous in cross section, the effect of inequality between \( v_e \) and \( v_0 \) on the frequency characteristics of the devices based on CL became obvious. Works [4-8] considered the physical regularities of wave interference in coupled stripline structures when \( v_e \neq v_0 \), as well as qualitative and quantitative changes in the frequency characteristics of the devices based on CL and multiply connected stripline structures. It was theoretically shown for the first time in 1969, in [4], that some all-pass schemes acquire filtering properties when \( v_e \) and \( v_0 \) are not equal. It was confirmed experimentally in [8]. The practical application of CL with inhomogeneous dielectric filling stimulated the search for and creation of new varieties of CL designs, the creation of which was aimed to achieve two contradictory goals. The first of them was aimed to bring \( v_e \) and \( v_0 \) closer together as much as possible in order to avoid the wave interference leading to resonance phenomena [9, 10]. The second one was to determine the degree of inequality between \( v_e \) and \( v_0 \) to solve the problems of creating the best frequency-selective characteristics for the devices based on CL [11, 12], creating devices for protecting equipment from short pulses [13, 14], etc.

The phase velocities \( v_e \) and \( v_0 \) are usually determined through the effective relative dielectric permeabilities for in-phase excitation \( \varepsilon_{r_{\text{eff}}} \) and anti-phase excitation \( \varepsilon_{r_{\text{eff}}} \) of coupled lines [5, 6]:
\[ v_{e,0} = c/\sqrt{\epsilon_{r e f f e,0}} , \]  

where \( c \) is the speed of light.

If CLs are equal, then \( \epsilon_{r e f f e,0} \) can be determined through the linear parameters of the coupled lines [15]:

\[ \epsilon_{r e f f e} = cL_{11} + L_{11}C_{11} - |C_{12}|, \quad \epsilon_{r e f f o} = cL_{11} - L_{11}C_{11} + |C_{12}|, \]

where \( C_{11}, C_{12} \) are the elements of the per-unit capacitance matrix, \( L_{11}, L_{12} \) are the elements of the per-unit inductance matrix.

The study of the dependence of the coupled lines parameters on the ratio \( v_e/v_0 \geq 3 \) led to the creation of CL designs, in which the ratio of phase velocities is \( v_e/v_0 \geq 3 \) [16-18]. It was shown that when \( v_e/v_0 \geq 3 \), the interference of in-phase and anti-phase waves leads to a qualitative change in the frequency dependence of the CL segment parameters. The practical application of such CL was found in the creation of a transdirectional coupler (TrDC) [16] on the CL. The coupler is implemented on a structure with a vertical insert and a high dielectric permeability, that enables to provide a threefold ratio of phase velocities \( v_e \) and \( v_o \), and acceptable impedance matching. A computer model of such a structure, which was based on the method of numerical conformal transformations [19], was described in the following publications. In [17, 18], the characteristics of CL with the ratio of \( v_e/v_0 \geq 3 \) were studied in the frequency domain.

This work considers the propagation of an ultra-wideband pulse with picosecond range in coupled lines when \( v_e/v_0 \geq 3 \). The pulse was applied to the input port of the first line. We carried out an experimental measurement of the pulsed signals at the input/output of the other three ports and the signal reflected from the input port of the device. We calculated the spectra of the received impulse responses and analysed the physical features of the interference of in-phase and anti-phase waves in CL under pulse impact.

2. Design and measurement scheme

The phase velocity ratio of in-phase and anti-phase waves \( v_e/v_o \) depends on the coupled lines design. It is achieved in works [16-19] by vertical and horizontal positioning of substrates 1 and 4 made of different dielectrics with relative dielectric permeabilities \( \epsilon_1 \) and \( \epsilon_2 \), and conductors 2 and 3 with width of \( w_1 \) and \( w_2 \) (figure 1). This figure shows the structural design modification of the coupled strip lines, which enables to vary \( v_e/v_o \) over wide range. Figure 2 illustrates the CL cross section. Coupled strips with a width of \( w_1 = 2 \) mm are put on the substrate placed vertically to the grounded base. A gap between the bottom end of the substrate and the base is \( D = 0.5 \) mm. Relative dielectric permeability of the substrate is \( \epsilon_1 = 16.0 \). Material for the substrate is Flan-16. Other dimensions of the SPL cross section: \( h_1 = 1.0 \) mm; \( h_2 = 0.5 \) mm. The design has coaxial-to-stripline connectors. The length of segment of the coupled lines is \( l = 0.1 \) m. Experimental studies were carried out on a stand, the scheme of which is shown in figure 3.

The unit scheme consists of a pulse generator, which is a combination of a Geozondas GZ1105DLP2 reference generator and a GZ1117DN-35 pulse shaper, a Picosecond 5372 divider (splitter). The pulse generator is connected with divider’s input that has 14 dB decoupler with output 1. The pulse arriving at the divider’s output 2 is attenuated by 2 dB. This pulse is fed through a piece of coaxial cable to port 1 of the device under study. The DSA 8300 type oscilloscope has two inputs, which enables to observe two pulse signals from divider’s output 2 and from one of three ports 2, 3, 4. In this case, the oscilloscope is synchronized from the pulse generator.
Figure 1. Coupled strip lines design: 1 – vertically placed substrate and strips put on it 2; 3 – strips horizontally placed on the substrate 4; 5 – grounded base with a gap under the coupled region between the strips 3.

Figure 2. Cross section of the coupled strip lines with a wide-ranging phase velocity ratio of in-phase and anti-phase waves when changing the dimensions and dielectric constants of vertically and horizontally located substrates and conductors.

Figure 3. Scheme for studying the pulse characteristics of the segment of the coupled strip lines.

3. Experimental results
The pulse signals measuring at the same port loads of $1 - 4 Z_{\text{load}1}, ..., Z_{\text{load}4} = 50$ Ohms was carried out as a part of experimental studying. The problem was to identify the special aspects of the input pulse propagation along the coupled lines of the device. Figure 4 illustrates recording a pulse applied to port 1 from divider's output 2 with a duration of 40 ps and an amplitude of $-0.7$ V. The voltage-time relationship at the output of port 3 shows that the input pulse, when transmitted to port 3, was split into two pulses with amplitudes of $-0.26$ V and $-0.22$ V. The group delay of these pulses relative to the input pulse is $\tau_e = 346$ ps and $\tau_0 = 996$ ps, respectively. Pulse signals on ports 2 and 4 are shown in figure 5 and figure 6. In this case, we observe a more complex picture of the input pulse transmission in the form of a pulses sequence with a shape close to the shape of the input pulse, and pulse surges of a more complex shape and with a greater delay. The amplitude of the first less-delayed pulse transmitted to port 2 is $-0.30$ V, and the first pulse transmitted to port 4 is $-0.27$ V.

The input pulse reflection from port 1 is an important characteristic for the analysis of the mechanism for the input pulse propagation along the coupled lines of the transdirectional coupler. Therefore, the reflected pulse was measured. For this purpose, a segment of coaxial cable was connected between the divider and the device under study. As a result, the input pulse and the reflected pulse were separated by the time of their arrival at the oscilloscope input.
Figure 4. Pulse signals at the input (port 1) and at port 3 when ports 2 and 4 are loaded with 50 Ohms.

Figure 5. Pulse signals at input (port 1) and at port 2.

Figure 6. Pulse signals at input (port 1) and at port 4.

Figure 7 illustrates a comparison of the input pulse and the reflected pulse. The maximum amplitude of the reflected pulse is 0.1 V. It enables to qualitatively evaluate the reflection coefficient modulus of a transdirectional coupler on exposure to an ultra-wideband pulse as a ratio of amplitudes $|\Gamma| = 0,1/0,7 = 0,14$.

It is known [17] that the frequency dependence of the input reflection coefficient of CL segment in a wide frequency range is characterized by periodic poles. Therefore, a significant reflected pulse was expected when studying the device. But the received reflected signal shown in figure 7 did not confirm our assumption. To explain the observed effect, we made an analysis of the pulsed signal spectra at the device input and at ports 2 - 4 (figures 4, 5 and 6).

Figure 7. Comparison of the input pulse and reflected pulse.
Figure 8 shows the envelope curves of the pulsed signal spectra at the first line input (port 1), at its output (port 3) and the reflected signal from the input of the device under study.

As it is shown in figure 8, the actuating signal (port 1) has a continuous spectrum, the signal at port 3 has a grating spectrum. The presence of relatively small reflections of harmonic components from the input (shown in black) does not cause a significant decrease in the transmission coefficient of the harmonics transmitted to port 3 in the frequency range from 0.1 GHz to 8 GHz, since the reflection coefficient is low.

Because of the presence of a grating transmission spectrum and the absence of poles of total reflection of the harmonic components of an ultra-wideband signal, the question arises about the direction of transmission of harmonics that have not passed to port 3 to other ports. Envelope curves of the signal spectra at ports 2 and 4 were calculated (figure 9).

Analysis of the spectral characteristics (figure 9) shows that harmonic components that have not passed to port 3, with almost the same amplitudes up to 8 GHz, entered ports 2 and 4.

**Figure 8.** Envelope curves of the pulsed signal spectra at the first line input of the device (port 1), at its output (port 3) and the signal reflected from the input.

**Figure 9.** Envelope curves of the pulsed signal spectra at the input of the device (port 1) and at ports 2 and 4.

Analysis of the spectral characteristics (figure 9) shows that harmonic components that have not passed to port 3, with almost the same amplitudes up to 8 GHz, entered ports 2 and 4.

**4. Wave analysis in coupled lines**

The analysis of waves propagating in coupled strip lines with unequal phase velocities of in-phase and anti-phase modes is considered in [4-8]. The initial data are the matrices of linear capacitances $C$ and inductances $L$ determined from [19]:

$$C = \begin{bmatrix} 298 & -272 \\ -272 & 298 \end{bmatrix} \times 10^{-12} F/m, L = \begin{bmatrix} 0.322 & 0.144 \\ 0.144 & 0.322 \end{bmatrix} \times 10^{-6} H/m$$  (3)
We considered a segment of coupled lines as an eight-pole, to the input port of which, at number 1, an $E_1(f)$ with the amplitude and phase spectral composition of the input pulse is supplied (figure 4, figure 9, blue). Using the known relationship of voltages and currents at the input and output of coupled lines in terms of [7], we find the absolute values of voltages and currents at points $x = 0$ and $x = l$ (figure 3) in the form of matrices

$$\begin{bmatrix} U(0) \\ I(0) \end{bmatrix} \text{ and } \begin{bmatrix} U(l) \\ I(l) \end{bmatrix},$$

(4)

where $U(0), I(0)$ are voltages and currents at the device input, and $U(l), I(l)$ are voltages and currents at the output of coupled lines.

We obtained a relationship between the amplitudes of in-phase and anti-phase modes of incident waves $A_e, A_o$ and reflected waves $D_e, D_o$ in the first line [7]:

$$\begin{bmatrix} A_e \\ A_o \\ D_e \\ D_o \end{bmatrix} = [Am]^{-1} \begin{bmatrix} U_1(0) \\ U_2(0) \\ I_1(0) \\ I_2(0) \end{bmatrix},$$

(5)

where $[Am]$ is the normalized amplitude matrix. It is determined as follows:

$$[Am] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ k_e & k_o & k_e & k_o \\ Y_{1e} & Y_{1o} & -Y_{1e} & -Y_{1o} \\ Y_{2e} & Y_{2o} & -Y_{1e} & -Y_{1o} \end{bmatrix},$$

(6)

The matrix $[Am]$ coefficients included in the expression are calculated according to [7]. Figure 10 shows the dependence of the amplitudes $A_e, A_o, D_e, D_o$ on the frequency of the harmonic components of the actuating impulse.

![Figure 10](image)

**Figure 10.** Frequency dependence of amplitudes of in-phase and anti-phase wave components in the first line.

Figure 10 shows that the incident components of the in-phase and anti-phase waves dominate in the first line. The reflected components have a smaller amplitude, and the oscillation period of the in-phase component amplitude is three times less than the oscillation period of the anti-phase component. The obtained values $A_e, A_o, D_e, D_o$ enable to construct a complete picture of waves propagating in coupled strip lines and to reveal the special aspects of their interference in connection with a significant phase
velocity inequality of the in-phase and anti-phase components. We write down expressions for finding voltages and currents in coupled strip lines, which are essentially a solution to a system of telegraph equations with known boundary conditions at the ends of the lines:

\[
\begin{align*}
U1(x) &= A_e \cdot \exp(-\gamma_e x) + A_o \cdot \exp(-\gamma_o x) + D_e \cdot \exp(\gamma_e x) + D_o \cdot \exp(\gamma_o x); \\
U2(x) &= A_e k_e \cdot \exp(-\gamma_e x) + A_o k_o \cdot \exp(-\gamma_o x) + D_e k_e \cdot \exp(\gamma_e x) + D_o k_o \cdot \exp(\gamma_o x); \\
I1(x) &= A_e Y_1 e \cdot \exp(-\gamma_e x) + A_o Y_1 o \cdot \exp(-\gamma_o x) + D_e Y_1 e \cdot \exp(\gamma_e x) + D_o Y_1 o \cdot \exp(\gamma_o x); \\
I2(x) &= A_e Y_2 e \cdot \exp(-\gamma_e x) + A_o Y_2 o \cdot \exp(-\gamma_o x) + D_e Y_2 e \cdot \exp(\gamma_e x) + D_o Y_2 o \cdot \exp(\gamma_o x),
\end{align*}
\]

where \(\gamma_e, \gamma_o\) are propagation constants for in-phase and anti-phase waves;

\[
k_e = \frac{\gamma_e^2 - a_{1,1}}{a_{1,2}}, \quad k_o = \frac{\gamma_o^2 - a_{1,1}}{a_{1,2}}, \quad Y_{1e} = \frac{Y_{1,1} + k_e Y_{1,2}}{\gamma_e}, \quad Y_{2e} = \frac{Y_{1,2} + k_e Y_{2,2}}{\gamma_e}, \quad Y_{1o} = \frac{Y_{1,1} + k_o Y_{1,2}}{\gamma_o}, \quad Y_{2o} = \frac{Y_{1,2} + k_o Y_{2,2}}{\gamma_o},
\]

\(a_{1,1}, a_{1,2}\) are the elements of matrix \(\alpha = ZY\), where \(Z = R + i\omega L\) is the impedance matrix, \(Y = G + i\omega C\) is the conductance matrix, written as \(R, L, G, C\) – matrices of per-unit parameters, respectively, active resistances, inductances, conductivities and capacitances.

We calculated the response of coupled lines to the action of a pulse using formulas (2)-(4) and wave propagation along the coordinate \(x\), described as a result of the interference of in-phase and anti-phase incident and inverse waves by expressions (7). The response was determined as a Fourier series

\[
u(t) = \sum_{n=1}^{N} |U_n| \cos(2\pi nf t + \varphi_n),
\]

where \(f\) is the frequency of the fundamental harmonic of the actuating pulse signal; \(|U_n|, \varphi_n\) is the module and phase of the response at harmonic number \(n\) of the corresponding port.

Figure 11 shows the results of calculating the dependence of voltage in port 2 \(u2(t)\) on time, calculated in two ways when determining the frequency dependence of currents and voltages – on the basis of matrix relations (4) (blue) and by superposition of in-phase and anti-phase components of waves (7) (red).

Figure 11 shows a good match of signals calculated in different ways. That is ground to assert that the amplitudes of in-phase and anti-phase incident and inverse waves are determined correctly (formula (6), figure 10). The inequality of the phase velocities of the in-phase and anti-phase waves leads to a change in both the phase-frequency and amplitude-frequency dependences of the signals in ports 2-4. The frequency separation of the original signal harmonics was shown in figure 10, figure 11. Phase-
frequency distortion introduced into the original actuating signal at ports 2 and 4, calculated and obtained experimentally, is shown in figure 12, figure 13.

Comparison of the frequency dependence of the introduced phase shift shown in Figure 12 and Figure 13 shows their significant difference. It lies in the fact that the voltage phase shift in port 2 is formed as a result of the interference of waves with different phase velocities. Therefore, the derivative of phase with respect to frequency in the transmission edge vicinity, marked with dashed lines, changes its sign. This indicates a different effect of inverse waves with different phase velocities, despite their small amplitudes. When transmitting a signal to port 4, the phase-frequency characteristic approaches the form characteristic of a single line segment, since when the waves are added, the incident waves of the in-phase and anti-phase modes dominate in an amplitude ratio of about 2:1, and the contribution of the inverse waves decreases to the end of the line.

![Figure 12](image1.png)

**Figure 12.** Frequency dependence of the phase shift introduced in the signal when it is transmitted from port 1 to port 2: experimental data is in blue, calculated data – in red.

![Figure 13](image2.png)

**Figure 13.** Frequency dependence of the phase shift introduced in the signal when it is transmitted from port 1 to port 4: experimental data is in blue, calculated data – in red.

5. **Discussions of the results**

The measurements and calculations have shown the specific features of the propagation of an ultra-wideband pulse in coupled strip structures with a strong imbalance of the electromagnetic coupling between the lines. The experiment has shown that a short pulse with a continuous spectrum up to 30 GHz applied to port 1 is reflected from it with a relatively low (less than 0.2) reflection coefficient. In such a case, there occurs a power dividing of the harmonic components of the pulse signal between ports 2, 3 and 4. Harmonic components that have not passed to port 3 are transmitted with approximately the same amplitudes to ports 2 and 4, which are the beginning and end of the second strip conductor. The analysis of the phase relationships of the harmonic components in ports 2 and 4, which arrived with approximately the same transmission attenuation, shows a phase difference of 90 degrees in the frequency range of 0.62-0.83 GHz with periodic repetition in the range of 3.54-3.75 GHz, etc., and 180 degrees in the range of 1.04-1.14 GHz, 4.06-4.16 GHz. The noted special aspects of the short pulse
propagation in coupled lines are a combination of the properties of directional power dividing and simultaneously directed filtering the harmonic components of the signal through three ports.

6. Conclusion
Thus, we have shown the possibility for propagation of picosecond ranged ultra-wideband pulses without significant energy loss for reflection from the input in the segments of coupled strip lines with a phase velocity ratio of in-phase and anti-phase waves of 3:1. It was found experimentally and as a result of the analysis that in this case, the spectral components of the pulse are separated between the ports. It is concluded that the interference of in-phase and anti-phase waves occurs along the length of the coupled lines with different delays of the propagating modes. The result is a combination of directional division characteristic and frequency selection (filtering), which can be used to generate complex pulse signals with spectral components at ports with equal amplitudes and orthogonal or opposite in phase. The results obtained in this article complement the understanding of the short pulse splitting effect in modal filters based on coupled multiwire lines [13, 14].

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