THE SHAPLEY SUPERCLUSTER. III. COLLAPSE DYNAMICS AND MASS OF THE CENTRAL CONCENTRATION

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ABSTRACT

We present the first application of a spherical collapse model to a supercluster of galaxies. Positions and redshifts of ~3000 galaxies in the Shapley supercluster (SSC) are used to define velocity caustics that limit the gravitationally collapsing structure in its central part. This is found to extend at least to 8 h⁻¹ Mpc of the central cluster, A3558, enclosing 11 ACO clusters. Infall velocities reach ~2000 km s⁻¹. Dynamical models of the collapsing region are used to estimate its mass profile. An upper bound on the mass, based on a pure spherical infall model, gives \( M(<8 \ h^{-1} \ \text{Mpc}) \leq 1.3 \times 10^{16} \ h^{-1} \ M_\odot \) for an Einstein–de Sitter (critical) universe and \( M(<8 \ h^{-1} \ \text{Mpc}) \leq 8.5 \times 10^{15} \ h^{-1} \ M_\odot \) for an empty universe. The Diaferio & Geller model, based on estimating the escape velocity, gives a significantly lower value, \( M(<8 \ h^{-1} \ \text{Mpc}) \approx 2.1 \times 10^{15} \ h^{-1} \ M_\odot \), very similar to the mass Geller et al. found around the Coma cluster by the same method and comparable to or slightly lower than the dynamical mass in the virialized regions of clusters enclosed in the same region of the SSC. In both models, the overdensity in this region is substantial, but it is far from the value required to account for the peculiar motion of the Local Group with respect to the cosmic microwave background.

Key words: cosmology: observations — dark matter — galaxies: clusters: individual (A3558) — large-scale structure of universe — methods: analytical — methods: statistical

1. INTRODUCTION

This is the third in a series of papers analyzing the structure and physical parameters of the Shapley supercluster (SSC), based on galaxy redshifts. The first (Quintana et al. 1995, hereafter Paper I) presents and gives an initial analysis of the results of spectroscopic observations of the central region. The second (Quintana, Carrasco, & Reisenegger 2000, hereafter Paper II) presents a much extended sample of galaxy redshifts and gives a qualitative discussion of the SSC’s morphology. Here we use dynamical collapse models applied to this sample in order to obtain the mass of the central region of the SSC. An upcoming paper (Carrasco, Quintana, & Reisenegger 2000, hereafter Paper IV) will analyze the individual clusters of galaxies contained in the sample to obtain their physical parameters (velocity dispersion, size, mass), search for substructures within the clusters, and determine the total mass contained within the virialized regions of clusters in the whole Shapley area.

The Shapley concentration (Shapley 1930) is the richest supercluster in the local universe (Zucca et al. 1993; Einasto et al. 1997; but see also Batuski et al. 1999). This makes its study important for three main reasons. First, its high density of mass and of clusters of galaxies provides an extreme environment in which to study galaxy and cluster evolution. Second, its existence and the fact that it is the richest supercluster in a given volume constrain theories of structure formation and particularly the cosmological parameters and power spectrum in the standard model of hierarchical structure formation by gravitational instability (e.g., Ettori, Fabian, & White 1997; Bardelli et al. 2000). Finally, it is located near the apex of the motion of the Local Group with respect to the cosmic microwave background. Thus it is intriguing whether the SSC’s gravitational pull may contribute significantly to this motion, although most mass estimates (e.g., Raychaudhury 1989; Raychaudhury et al. 1991; Paper I; Ettori et al. 1997; Bardelli et al. 2000) make a contribution beyond a 10% level very unlikely.

Galaxy counts in redshift space (Bardelli et al. 2000) suggest that most of the supercluster has a density several times the cosmic average, while the two complexes within ~5 h⁻¹ Mpc of clusters A3558 and A3528 have overdensities ~50 and ~20 times, respectively. These regions are therefore far outside the “linear regime” of small density perturbations but still far from being virialized after full gravitational collapse. The same conclusions are easily reached by even a casual glance at the redshift structure presented in Paper II. The density of these complexes indicates that they should be presently collapsing (e.g., Bardelli et al. 2000), and in the present paper we study this hypothesis for the main complex, around A3558, where substantially more data with better areal coverage are available (see Paper II).

It should be pointed out that within each of these complexes we expect very large peculiar velocities, which dominate by far over their Hubble expansion. In this case, redshift differences among objects within each complex can give information about its dynamics, which we will analyze below, but essentially no information about relative positions along the line of sight (except, perhaps, nontrivial information within a given dynamical model). While this is generally acknowledged to be true within clusters of galaxies, where galaxy motions have been randomized by the collapse, it is sometimes overlooked on somewhat larger, but still nonlinear scales. For instance, Ettori et al. (1997) calculate three-dimensional distances between clusters in the Shapley region on the basis of their angular separations and redshifts, and they conclude, because of moderate dif-
ferences in redshift, that the group SC 1327-312 and the cluster A3562, at projected distances $\sim 1$ and $3$ $h^{-1}$ Mpc from the central cluster A3558, are between 5 and $10$ $h^{-1}$ Mpc from it in three-dimensional space. However, there is evidence for interactions between these clusters and groups (Venturi et al. 1999), suggesting true distances much closer to the projected distances. The discrepancy is naturally explained by quite modest peculiar velocities of several hundreds of kilometers per second, easily caused by the large mass concentration.

In the present paper, we analyze the region around A3558 in terms of an idealized, spherical collapse model, which is used both in its pristine, but undoubtedly oversimplified, original form (Regös & Geller 1989), seen from a slightly different point of view, and in its less appealing, but possibly more accurate, modern fine-tuning calibrated by simulations (Diaferio & Geller 1997; Diaferio 1999). Section 2 explains the models, argues for the presence of velocity caustics, and gives the equations relating the caustics position to the mass distribution. (Some mathematical remarks regarding this relation are given in the Appendix.) In § 3 we present the data, argue that velocity caustics are indeed present, and explain how we locate their position quantitatively. Section 4 presents and discusses our results, and § 5 contains our main conclusions.

2. THE MODELS

2.1. Pure Spherical Collapse

In this approach, we consider a spherical structure in which matter at any radius $r$ moves radially, with its acceleration determined by the enclosed mass $M(r)$. At a given time $t_1$ of observation, the infall velocity $u(r)$ (traced by galaxies participating in the mass inflow) can give direct information about the mass profile (Kaiser 1987; Regös & Geller 1989). Of course, $u(r)$ is not directly observable. Instead, for each galaxy we only observe its position on the sky, which translates into a projected distance from the assumed center of the structure, $r_p < r$, and its redshift, which can be translated into a line-of-sight velocity $v$ with respect to the same center. The “fundamental” variables $r$ and $u$ and the “observed” variables $r_p$ and $v$ (with the line-of-sight velocity of the structure’s center already subtracted) are related by

$$v = \pm \left[ 1 - \left( \frac{r_p}{r} \right)^2 \right]^{1/2} u(r).$$

Contrary to the Hubble flow observed on large scales, $v$ is negative (approaching) for the more distant galaxies on the back side of each shell, and positive (receding) for the closer galaxies on the front side. At any given projected distance $r_p$, one observes galaxies at many different true distances from the center. The infall velocity $u(r)$ decreases at large enough distances $r$, reaching zero at a finite (“turnaround”) radius $r_t$, and matching onto the Hubble flow, $u(r) = -Hr$, for $r \gg r_t$. The projection factor increases from zero at $r = r_t$ (galaxies moving perpendicularly to the line of sight), asymptotically approaching unity for $r \gg r_t$. Therefore, there will be some maximum projected velocity

$$\mathcal{A}(r_p) \equiv \max_{r \approx r_p} |v(r_p, r)| = \max_{r \approx r_p} \left[ 1 - \left( \frac{r_p}{r} \right)^2 \right]^{1/2} u(r),$$

which is a monotonically decreasing function of $r_p$ (see Appendix), giving rise to caustics in the $(r_p, v)$ diagram with the characteristic “trumpet shape” described by Kaiser (1987) and by Regös & Geller (1989).

In order to obtain the infall velocity $u(r)$ from the galaxy redshifts, we first identify the caustics amplitude $\mathcal{A}(r_p)$ from the $(r_p, v)$ diagram by the procedure outlined in § 3. Given this relation, we can invert equation (2) to obtain

$$u(r) \leq u_p(r) \equiv \min_{r_p < r} \frac{\mathcal{A}(r_p)}{1 - (r_p/r)^2} \left[ 1 - (r_p/r)^2 \right]^{1/2}.$$  

This is an inequality rather than an equality, because for an arbitrary shape of $u(r)$ it is not guaranteed that the shells at every $r$ will correspond to a maximum amplitude for some $r_p$ (see Appendix for a detailed mathematical discussion).

If the mass density decreases outward, then the collapse occurs from the inside out, with innermost mass shells first reaching turnaround and recontraction, and outer shells following in succession. Any given shell will enclose the same mass $M$ at all times until it starts encountering matter that has already passed through the center of the structure and is again moving outward. The latter is only expected to happen in the very central, nearly virialized, part of the supercluster. Elsewhere, the dynamics of any given shell is described by the well-known parameterized solution

$$r = A(1 - \cos \eta); \quad t = B(\eta - \sin \eta); \quad A^3 = GM^2$$

(e.g., Peebles 1993, 484). Here, $A$ and $B$ are constants for any given shell (related to each other by the enclosed mass $M$), and $\eta$ labels the “phase” of the shell’s evolution (initial “explosion” at $\eta = 0$, maximum radius or turnaround at $\eta = \pi$, collapse at $\eta = 2\pi$). As we are observing many shells at one given cosmic time $t_1$ (measured from the big bang, at which all shells started expanding, to the moment at which the structure emitted the light currently being observed), for each shell we can write

$$t_1 \frac{\dot{r}}{r} = -H_0 t_1 \frac{u}{H_0 r} = \sin(\eta - \sin \eta),$$

where $H_0$ is the current value of the Hubble parameter.

We can determine an upper bound on $u(H_0 r)$, and therefore on $u/(H_0 r)$, by the procedure described above (note that $r$ itself depends on the uncertainty in the cosmic distance scale). The combination $H_0 t_1$ is a dimensionless constant, dependent on the cosmological model (identified by dimensionless constants such as the density parameters $\Omega$). At the redshift of the SSC ($z \approx 0.048$), it is likely to lie in the range $0.62 \leq H_0 t_1 \leq 0.95$, with the lower limit corresponding to an Einstein–de Sitter universe (with critical matter density, $\Omega_m = 1$, and no other ingredients), and the upper limit corresponding to an empty universe ($\Omega_m = 0 = \Omega_\Lambda$) or a flat, low-density, $\Lambda$-dominated universe ($\Omega_m = 1 = \Omega_\Lambda \approx 0.27$; e.g., Peebles 1993, 314–317). For an assumed value of this parameter, the left-hand side becomes fully determined, and the equation can be solved for the value of $\eta$ for each shell. The equations can also be combined to yield

$$H_0 M(r) = \frac{(H_0 r)^3}{G(H_0 t_1)^2} \left( \frac{\eta - \sin \eta}{\cos \eta} \right),$$

the mass enclosed within the shell (of current radius $r$).

For two reasons, a mass estimate obtained by applying this model to real data should be regarded as an upper bound on the true mass. First, the model gives $u_p(r)$, an upper bound
on \(u(r)\), and this upper bound is used to derive the mass. Second, the caustics amplitude \(\mathcal{A}(r)\) is amplified through random motions due to substructure within the infalling matter (Diaferio & Geller 1997). Keeping this in mind, we will use the model to put a bound on the mass of the collapsing region around A3558.

### 2.2. Diaferio’s Prescription

On the other hand, Diaferio & Geller (1997; see also Diaferio 1999) have shown that the mass profile of structures forming in numerical simulations can be recovered to good precision from the formula

\[
M(r) = \frac{\mathcal{F}}{G} \int_0^r \mathcal{A}^2(r) \, dr. \tag{7}
\]

There is no rigorous derivation for this result, although it can be justified heuristically by assuming that \(\mathcal{A}\) reflects the escape velocity at different radii, i.e., that all galaxies within the caustics are gravitationally bound to the structure. One has to assume further that the radial density profile lies between \(\rho \propto r^{-3}\) and \(\rho \propto r^{-2}\) (Diaferio & Geller 1997; Diaferio 1999), as in the outskirts of simulated clusters of galaxies (e.g., Navarro, Frenk, & White 1997). This mass estimate is independent of the parameter \(H_0 \Omega\), since no dynamical evolution is involved. It has already been applied to the Coma cluster (Geller, Diaferio, & Kurtz 1999).

### 3. Identifying the Structure

The \((r_\perp, v)\) diagram for the galaxies with available redshifts in the Shapley region (see Paper II), shown in Figure 1 for an adopted center at the main cluster A3558 \((x_c = 13^h27^m56^s9; \delta_c = -31^\circ29'44'', c_z = 14,300 \text{ km s}^{-1}\) indeed shows the predicted trumpet shape, with a maximum half-width \(\mathcal{A}(r_\perp) \approx 2000 \text{ km s}^{-1}\) and extending cleanly out to \(r_\perp \approx 8 \text{ h}^{-1} \text{ Mpc}\), and less cleanly to perhaps \(14 \text{ h}^{-1} \text{ Mpc}\) from the center.\(^3\) Thus, there is indeed a coherent structure ("extended core"), enclosing at least (i.e., within \(8 \text{ h}^{-1} \text{ Mpc}\)) 11 Abell clusters (Abell, Corwin, & Olowin 1989) and three rich groups not included in the Abell catalog (see Table I). All of these have relative velocities with respect to A3558 smaller than \(1200 \text{ km s}^{-1}\), with a median \(|v|_{\text{med}} \approx 200 \text{ km s}^{-1}\). Given its shape, this extremely cluster-rich structure is most likely due to gravitational collapse.

Of course, the morphology of the SSC or even of its extended core does not support the assumption of spherical symmetry. The inner core (formed by A3558, A3556, SC 1327-312, and SC 1329-314) is clearly elongated, and there is evidence that both the galaxies and the clusters in the region of interest tend to form a planar structure (Paper I; Bardelli et al. 2000; Paper II). This is supported by the elementary fact that (within the extended core and a fair distance beyond it) all the clusters in the north (A3559, A3557a, A3555, farther away A1736b) have lower redshifts than A3558, while those in the south (A3560, A3554, others at larger distances) and west (particularly those belonging to the Northwest Filament, which extends toward A3528 and is described in Paper II) have higher redshifts. It is also hinted at by the slight upturn of the structure in Figure 1 at \(\theta > 3.5\) \((r_\perp > 8 \text{ h}^{-1} \text{ Mpc})\), which is due to the concentrations of clusters to the southwest and southeast of the central concentration. (The downward-pointing arm at \(\theta \approx 4\) is due to the clusters A1736a, b and A3571.) However, (1) the extreme simplicity of this symmetry assumption compared with any possible improvement to it,

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**Table 1**

| Name         | \(r_\perp\) \((\text{h}^{-1} \text{ Mpc})\) | \(v\) \((\text{km s}^{-1})\) | \(\sum M_{500}\) \((10^{14} \text{ h}^{-1} \text{ M}_\odot)\) | Source for \(M_{500}\) |
|--------------|------------------------------------------|-------------------------------|------------------------------------------|--------------------------|
| A3558        | 0.00                                     | 0                             | 6.09                                    | 6.09                     | 1                        |
| SC 1327-312  | 0.94                                     | +700                          | 2.14                                    | 8.23                     | 1                        |
| A3556        | 1.92                                     | −16                            | 1.82                                    | 10.05                    | 1                        |
| SC 1329-314  | 1.93                                     | −956                           | 1.03                                    | 11.08                    | 1                        |
| A3562        | 2.94                                     | −12                            | 3.06                                    | 14.14                    | 1                        |
| A3560        | 4.36                                     | +144                           | 2.72                                    | 16.86                    | 1                        |
| A3552        | 4.48                                     | +1153                          | 0.36                                    | 17.22                    | 2                        |
| A3559        | 4.65                                     | −376                           | 0.83                                    | 18.05                    | 1                        |
| A3554        | 6.13                                     | +99                            | 1.26                                    | 19.31                    | 2                        |
| A3557a       | 6.18                                     | −114                           | 0.47                                    | 19.78                    | 2                        |
| A3555        | 7.07                                     | −419                           | 0.10                                    | 19.88                    | 2                        |
| SC 1342-302  | 7.77                                     | +119                           | 0.20                                    | 20.08                    | 2                        |
| A724S        | 7.86                                     | +367                           | 0.88                                    | 20.96                    | 2                        |
| A726S        | 7.89                                     | +206                           | 0.59                                    | 21.55                    | 2                        |

**Note.**—Columns: (1) cluster identification, (2) projected distance from A3558, (3) average line-of-sight velocity, relative to A3558, (4) adopted cluster mass within a radius enclosing an average density 500 times the critical density, from X-ray or optical observations, (5) cumulative sum of cluster masses, (6) source of the cluster mass estimate: 1. X-ray deprojection analysis of Ettori et al. 1997; 2. preliminary mass estimate from optical determination to be refined in Paper IV. (The entries in cols. [2] and [3] were calculated from the data given in Paper II.)

\(^3\) At the redshift of A3558, \(z_c = 0.048\), assumed to be of purely cosmological origin, relative line-of-sight velocities are given by \(v = c(z - z_c)/(1 + z_c) = 0.955c(z - z_c)\) (Harrison & Noonan 1979). In an Einstein–de Sitter Universe \((q_0 = 1/2)\), physical distances projected on the sky are given by \(r_r = 2c\theta/[1 - (1 + z_c)^{-1/2}]/\{H_0(1 + z_c)\}\) (e.g., Peebles 1993), where \(\theta\) is the angular distance in radians. Again at the redshift of A3558, \(1\) corresponds to \(2.3 \text{ h}^{-1} \text{ Mpc}\). The latter result is fairly insensitive to the choice of cosmological model, therefore we adopt it in general.
Diaferio (1999) general approach in first smoothing the data, i.e., obtaining a smooth estimate \( f(r_{\perp}, v) \) for the density of observed galaxies on the \((r_{\perp}, v)\) plane, and then applying a cut at some density contour which is taken to correspond to the caustics. The details of how each of these steps is carried out differ slightly from Diaferio’s approach, and are discussed in the rest of this section.

3.1. Density Estimation in the \((r_{\perp}, v)\) Diagram

For a global analysis of the central (collapsing) region of the SSC, we need to obtain a smooth estimate \( f(r_{\perp}, v) \) of the density of galaxies in the \((r_{\perp}, v)\) diagram. Density estimation has been discussed by many authors, such as Silverman (1986), and in the astronomical context Pisani (1993; 1996) and Merritt & Tremblay (1994).

Diaferio (1999) applied density estimation to the particular problem of interest here. For \( N \) data points (galaxies) with coordinates \((r_{\perp i}, v_i)\), he adopts the estimate

\[
 f(r_{\perp}, v) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h^*_i h^*_v} K\left( \frac{r_{\perp} - r^*_{\perp i}}{h^*_r}, \frac{v - v^*_i}{h^*_v} \right),
\]

where

\[
 K(\vec{t}) = \begin{cases} 
 4\pi^{-1}(1 - |\vec{t}|^2)^{3} & \text{if } |\vec{t}| < 1 , \\
 0 , & \text{otherwise ,} 
\end{cases}
\]

is a smooth, but centrally peaked, kernel function. The ratio of smoothing lengths, \( q = h^*_v/h^*_r \), is fixed, approximately equal to the ratio of observational uncertainties \( q = 50 \text{ km s}^{-1}/0.02 \text{ h}^{-1} \text{ Mpc} = 25 H_0 \), and the individual values of, say, \( h^*_i \), are chosen by an adaptive algorithm.

Any choice of smoothing lengths is a compromise between keeping as much structure as possible (favoring small smoothing lengths) while eliminating as much noise as possible (favoring large values). The ideal compromise, though quite subjective in any case, depends on the density of data points, which generally varies over the volume being studied, motivating the choice of a different smoothing length for each data point. For our particular application, the density of points does not vary enormously over the area of interest, and we are only interested in the overall envelope of the structure, not in fine details. Therefore, we consider the additional computational effort of adaptive smoothing with different local smoothing lengths unjustified.

We apply fixed, overall smoothing lengths \( h_r = 1 \text{ h}^{-1} \text{ Mpc}, h_v = 500 \text{ km s}^{-1} \) (giving \( q = 5 H_0 \)), chosen by eye to preserve the overall shape while minimizing the noise, and each corresponding to about one-eighth of the total extension of the structure studied. Changing either of the two lengths by a factor of 2 either way does not substantially change our results. We note also that the chosen lengths are much larger than the respective uncertainties in the data, which therefore become irrelevant in determining the detected structures.

One problem with the smoothing kernel given above is that the data have a natural cutoff at \( r_{\perp} = 0 \), where they go abruptly from a fairly high (near maximum) density (at \( r_{\perp} > 0 \)) to zero (at \( r_{\perp} < 0 \)). When applied to points of small (positive) \( r_{\perp} \), the smoothing kernel extends to negative values (where there are no data points), producing a decrease in \( f(r_{\perp}, v) \) when approaching \( r_{\perp} = 0 \) from above. This causes isodensity contours to narrow as \( r_{\perp} \to 0^+ \), as seen, e.g., in Figure 1 of Geller et al. (1999), in Figures 4 and 5 of Diaferio (1999), and in Figure 2a of the present paper.

This can be cured, e.g., by making a mirror image of the data at \( r_{\perp} < 0 \) and letting the smoothing kernel integrate over both the real data and their image (Fig. 2b). Aside from correcting for the “misbehavior” at \( r_{\perp} = 0 \), this procedure gives results very similar to that of Diaferio (1999).

A more rigorous approach is suggested by Merritt & Tremblay (1994), who deal with density estimation in circularly symmetric structures. They focus on the surface...
density \( \Sigma(r_\perp) \) (number per unit area) rather than the radial density \( N(r_\perp) = 2\pi r_\perp \Sigma(r_\perp) \) (number per unit radial coordinate), and estimate \( \Sigma(r_\perp) \) with a circularly averaged kernel. In our case (with one additional coordinate \( v \), which is not affected by this problem), we can define a number density per unit (projected) area per unit line-of-sight velocity as \( f(r_\perp, v) \) and estimate it through a kernel which is a product of a standard, one-dimensional kernel for \( v \) and a circularly averaged kernel for \( r_\perp \). In particular, Figure 2c shows the results of applying to our data a one-dimensional quadratic (Epanechnikov) kernel for \( v \), and an annularly averaged, two-dimensional quadratic kernel for \( r_\perp \). (See Merritt & Tremblay 1994, eqs. [9a] and [28a] for explicit formulae.) The density \( \Sigma \) obtained from this procedure is well behaved in all respects, decreasing from outward. Indeed, it decreases so quickly that the density contours tend to close at fairly small radii, contrary to the visual impression from the data. Therefore, these contours are unlikely to be realistic representations of the velocity caustics.

A final alternative (with the added virtue of reducing biases due to nonuniform spatial sampling) is to normalize \( f(r_\perp, v) \) at each given \( r_\perp \) with respect to the value at \( v = 0 \), i.e., take a density estimate

\[
\tilde{f}(r_\perp, v) = \frac{f(r_\perp, v)}{f(r_\perp, 0)},
\]

with the “original” \( f(r_\perp, v) \) determined by any of the other methods (the case shown in Fig. 2d is based on Diaferio’s estimator). This estimator gives results very similar to the first two.

Overall, we consider that the second procedure (the “mirror image” density estimate) is the one that most closely represents the visual appearance of the data, while at the same time having mathematically desirable properties (smooth density contours slowly narrowing with increasing \( r_\perp \)), and being close enough to Diaferio’s to permit a direct comparison of results. Therefore, we use the mirror image density estimation for the analysis that follows. However, we stress that it is a very arbitrary choice, and that other choices may give quite different final results. However, discarding the very different result based on the “surface density” scheme, the other procedures discussed above give masses that, at any given radius, differ by less than 10% from the one obtained by the mirror image procedure.

3.2. Finding the Caustics

Given the estimated density \( f(r_\perp, v) \), we now turn to finding the caustics which separate the collapsing structure from the galaxies in the foreground and background. We are again inspired by Diaferio (1999), who uses a fixed density cutoff, \( f(r_\perp, v) = \kappa \), with the value of \( \kappa \) fixed by virial arguments applied to the most central region. In principle, taking a fixed value is somewhat arbitrary. By plotting in \( (r_\perp, v) \) space, we are summing galaxies over annuli, and therefore a uniform background of galaxies would result in a density increasing \( \propto r_\perp \), and therefore a fixed cutoff might include more and more of the background as \( r_\perp \) increases. The problem is worsened by the nonuniformly sampled

![Figure 2](image-url)

Fig. 2.—Central part of the diagram in Fig. 1, with line-of-sight velocities expressed with respect to A3558, with the projected radius \( r_\perp \) as the abscissa, and with an isodensity contour superposed. Each panel corresponds to a different density estimation scheme. In panel (a), the density estimate is a standard two-dimensional (fixed) kernel smoothing; in (b) it is modified by considering a mirror image of the data to the left-hand side of the vertical axis; in (c) the estimate is the circularly averaged surface-density estimate based on Merritt & Tremblay (1994); in (d) the estimate is the same as in (c), normalized to the values on the horizontal axis. More details on each of these are given in § 3.1, and the choice of isodensity contours is discussed in § 3.2.
data in our particular case. Therefore, a more natural and in principle better way to distinguish the structure from the background might be to fix on maxima of \( \frac{\partial f}{\partial v} \) for each \( r_p \).

However, taking and maximizing a derivative of the numerically determined function is much noisier than just imposing a fixed cutoff. Therefore, we follow Diaferio in adopting the latter approach.

In order to choose the value of the cutoff, Diaferio (1999) uses the virial theorem to relate the escape velocity (in his interpretation represented by the velocity amplitude \( \mathcal{A} \)) and the velocity dispersion \( \sigma \) of the central cluster, therefore writing \( \langle \mathcal{A}^2 \rangle_{\kappa, R} = 4 \sigma^2 \), where the average is a galaxy number-weighted average over the region enclosed by the virial radius \( R \) of the central cluster, for a given density cutoff \( \kappa \). For A3558, the velocity dispersion is a relatively robust number \( (\sigma \approx 928 \text{ km s}^{-1}) \), not sensitive to the radius of the sphere to be averaged over, and in the central region \( \mathcal{A} \) (determined with any reasonable cutoff) is also fairly radius independent. Therefore, the dependence on the (poorly determined) virial radius is weak, and we arbitrarily choose it as \( R = 1 \text{ h}^{-1} \text{ Mpc} \), and do a straight radial average (not number weighted) to calculate \( \langle \mathcal{A}^2 \rangle_{\kappa, R} \) and determine \( \kappa \) by Diaferio’s condition.

We tested the validity of Diaferio’s condition by the following procedure. Figure 3 shows the area of the \((r_p, v)\) diagram enclosed by contour levels with different \( \kappa \). We clearly distinguish three regimes, as follows.

1. At very low densities, the enclosed area is most of the diagram, therefore enclosing much of the background, not belonging to the structure. The area rapidly decreases as the threshold density is increased.

2. At intermediate densities, the decreasing curve becomes much flatter \( (dA/dr \approx \text{constant}) \), and we interpret this as having most of the background excluded and probing progressively denser parts of the structure. This is confirmed by watching the contour plots, which indeed trace the boundaries of the structure, and become progressively tighter.

3. Finally, at high values of \( \kappa \), only a few isolated peaks in the structure are left enclosed, and these finally disappear when \( \kappa \) reaches the maximum density present.

This analysis suggests choosing the threshold at the transition between regimes 1 and 2. This falls close to the threshold value chosen by Diaferio’s (1999) condition as outlined above, marked by the vertical line. This strengthens the argument for Diaferio’s choice of cutoff, already used to choose the contours in Figure 2, and adopted hereafter.

The chosen density contour of course gives two values of \( v \) (one positive, \( v_u \), and one negative, \( v_l \)) for each value of \( r_p \).

In the SSC, it turns out that the upper contour is much “cleaner” (separating a dense region from a nearly empty one), therefore we simply adopt \( \mathcal{A}(r_p) = v_u(r_p) \) rather than Diaferio’s prescription \( \mathcal{A}(r_p) = \min \{ |v_u(r_p)|, |v_l(r_p)| \} \).

4. RESULTS AND DISCUSSION

Figure 4 shows the enclosed mass as a function of radius, \( M(r) \), as determined by the two methods discussed in § 2, together with a third determination, namely the cumulative mass of the clusters enclosed in the given radius, given in Table 1. Note that the mass estimates \( M_{500} \), taken from Ettori et al. (1997) for the most important clusters, are masses within a radius enclosing an average density 500 times the critical density \( \rho_c \). This is substantially higher than the standard “virialization density” of \( \sim 200 \rho_c \), and therefore gives a conservative lower limit to the total virialized mass, which may be increased by a factor \( \sim (500/200)^{1/2} \approx 1.58 \) for a more realistic estimate.

Several comments are in order, as follows.

1. As discussed above, the pure spherical infall model is highly idealized and, even if correct, can only give an upper bound on the mass within any given radius. Therefore, the upper (dot-dashed) curve, corresponding to pure spherical collapse in a cosmological model with \( H_0 t_s = 0.62 \) should be regarded as a fairly robust upper limit to the mass within any given radius.

2. The model of Diaferio & Geller (1997) has been calibrated against simulations. Applied to the infall regions around clusters of galaxies, it should in principle give the
correct mass to within about 25% (Geller et al. 1999). However, it assumes a density profile decreasing at least as fast as \( \rho(r) \propto r^{-2} \), which may not apply to the very noisy region around A3558, which contains a number of other, fairly rich clusters. It appears surprising that the mass profile it gives for this region is quite similar (both in shape and in amplitude) to that obtained by Geller et al. (1999), considering that Coma is a quite massive cluster (as massive or perhaps even more massive than A3558) but is not surrounded by any other massive clusters.

3. The mass estimate based on individual cluster masses is uncertain for two reasons. First, of course it does not consider the mass in the nonvirialized outskirts of the clusters or not associated with clusters at all, and therefore it would be expected to underestimate the total mass. On the other hand, in the absence of information on the three-dimensional distance \( r \) of each cluster to A3558, and given that the velocity does not give reliable distance information within the collapsing structure, each cluster was put at its projected radius \( r_1 \leq r \), and therefore contributes to the enclosed mass already at radii smaller than its true position. Therefore, the mass in virialized clusters within any given radius \( M(r) \) is overestimated by the projection into radius \( r \) of clusters actually at larger radii.

Given these caveats, there seems to be fair agreement among the different mass determinations, and it seems safe to say that the mass enclosed by radius \( r = 8 h^{-1} \) Mpc lies between \( 2 \times 10^{15} \) and \( 1.3 \times 10^{16} h^{-1} M_\odot \). It is interesting, nevertheless, that Diaferio’s method gives results that differ so little from the lower limit to the virialized mass in clusters. Therefore, if Diaferio’s method is applicable to the SSC, then either there would be very little mass outside the inner, virialized parts of clusters of galaxies in this region or the cluster mass estimates would have to be systematically high.

For comparison, Ettori et al. (1997) used three different mass estimates, namely: (1) the sum of the gravitational masses of clusters as obtained from their X-ray emission profile, \( M_{\text{grav}} \), (2) the total mass expected to be associated with the baryons observed in clusters, \( M_{\text{PN}} \), and (3) the mass obtained from applying the virial theorem to the enclosed clusters used as test particles, \( M_{\text{vir}} \). They applied these methods to four progressively larger structures, each enclosing the previous one. The one most similar to our \( 8 h^{-1} \) Mpc sphere appears to be the second, enclosing 12 clusters and with a nominal three-dimensional radius of \( 13.9 h^{-1} \) Mpc, obtained by treating redshift as a third coordinate, which, we have argued, gives an overestimated depth in the collapsing region. For this region, they find \( M_{\text{grav}} = 2.15 \), \( M_{\text{PN}} = 5.2 \Omega_m \), and \( M_{\text{vir}} = 1.75 \), all in units of \( 10^{15} h^{-1} M_\odot \). The first two are likely underestimates (as they consider only the matter in the virialized regions of clusters observed in X-rays), and the third is completely uncertain, given the uncertain distances along the line of sight and the fact that the SSC is not virialized (but see Small et al. 1998 for a modern, more careful application of the virial theorem to the Corona Borealis supercluster). Thus, it is reasonable that we find a somewhat higher mass for the (optically observed) clusters and a possibly much higher total dynamical mass, as suggested by the spherical collapse model.

The average enclosed density (see Fig. 5) drops from a value 400–500 times the critical density within \( 1 h^{-1} \) Mpc (consistent with the presence of a massive, already collapsed and virialized cluster) to a value still several times critical (~3.7 to 23 times, depending on the model) within our outermost radius, \( 8 h^{-1} \) Mpc. From galaxy counts in redshift space, Bardelli et al. (2000) find an overdensity \( N/N = 11.3 \pm 0.4 \) within a region of equivalent radius \( 10.1 h^{-1} \) Mpc. Assuming that galaxies trace mass, this might in principle allow us to determine the universal matter density parameter \( \Omega_m = \rho/\rho_{\text{crit}} = [\rho(r)/\rho_{\text{crit}}]/[N(r)/N] \). In practice, however, the uncertainty in this estimate is still much too large to put a useful constraint on \( \Omega_m \).

Figure 6 shows that, in the spherical collapse model, the whole structure within \( 8 h^{-1} \) Mpc has already been contracting for more than one-third of its lifetime, and the inner regions are in the final stages of collapse, consistent with the presence of a massive cluster. Even if there were no additional mass beyond \( 8 h^{-1} \) Mpc, the current turnaround radius would be at \( \sim 14 h^{-1} \) Mpc, and the bound region (to collapse eventually) would extend to \( \sim 20 h^{-1} \) Mpc, enclosing essentially the whole supercluster, including the strong concentration around A3528, A3530, and A3532. However, the much lower enclosed densities in the Diaferio & Geller...
model would imply that the 8 h\(^{-1}\) region around A3558 is (at best) only now reaching turnaround and has another Hubble time to go for final collapse.

As discussed in Paper I, the mass required at the distance of the SSC to produce the observed motion of the Local Group with respect to the cosmic microwave background is given by

\[ M_{\text{dipole}} \approx 4.5 \times 10^{17} \Omega_m^{0.4} \ h^{-1} \ M_\odot. \]

The mass within 8 h\(^{-1}\) Mpc can therefore produce at most 3\(\Omega_m^{0.4}\)% of the observed Local Group motion, which makes it unlikely that even the whole SSC would dominate its gravitational acceleration.

Unfortunately, not much can be said about the regions beyond a radius of \(\sim 8\ h^{-1}\) Mpc from the center. It seems safe to assert, though, that the collapsing region does not extend far beyond, say, 14 h\(^{-1}\) Mpc. This gives an upper bound on the average density enclosed in larger radii, \(\bar{\rho} < 3\pi/(32Gt_1^2)\). Thus, in order to produce the peculiar velocity of the Local Group, one would need a region of radius

\[ r > 55 \ h^{-1} \Omega_m^{0.13}(H_0 t_1)^{2/3} \ Mpc, \]

which, for the range of cosmological values considered before, corresponds to a lower limit \(\sim 40\ h^{-1}\) Mpc. Therefore, in the unlikely case that the whole SSC (characterized in Paper II) were on the verge of gravitational collapse, it would be able to produce on its own the observed peculiar velocity of the Local Group. This statement ignores, of course, that the apex of the Local Motion does not point exactly at the SSC, and therefore some additional contribution is necessary in any case.

5. CONCLUSIONS

We have presented the (to our knowledge) first application of a plausible dynamical model to a supercluster of galaxies containing a substantial number of clusters. The central 8 h\(^{-1}\) Mpc region of the Shapley supercluster (and probably a much more extended region surrounding it) is argued to be currently collapsing under the effect of its own gravity. Its mass, although uncertain owing to idealizations in the model, indicates a large enhancement over the average density of the universe, although still far from that required to produce the Local Group’s observed motion with respect to the cosmic microwave background.

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APPENDIX A

GEOMETRIC INTERPRETATION AND MATHEMATICAL PROPERTIES OF THE LINE-OF-SIGHT VELOCITY AMPLITUDE \(\mathcal{A}_\beta(r)\) AND THE INFALL VELOCITY \(u(r)\)

In order to derive and understand intuitively the properties of the observed velocity amplitude \(\mathcal{A}_\beta(r)\) and the infall velocity \(u(r)\) producing it in the spherical model, it is convenient to define new variables \(P = r_1^2\), \(R = r^2\), \(V = \mathcal{A}_\beta^2\), and \(U = u^2\). Then

\[ V(P) = \max_{R > P} F(P, R), \quad \text{with} \quad F(P, R) \equiv \left(1 - \frac{P}{R}\right) U(R). \]  \(\text{(A1)}\)

(We consider only the interval in which \(u(r) = -\dot{r} \geq 0\), corresponding to infall.)

Note that, seen as a function of \(P\) for given \(R\), \(F(P, R)\) is a straight line intersecting the horizontal axis at \(P = R\) and the vertical axis at \(F(0, R) = U(R)\). Therefore, the function \(U(R)\) defines a set of straight lines whose upper envelope gives the function \(V(P)\) (see Fig. 7). The relation between the functions \(U(R)\) and \(V(P)\) is very similar to the Legendre transformation (e.g., Courant and Hilbert 1989, § I.6) and shares many of its properties.

Since \(U(R) \geq 0\) for all \(R\), \(V(P)\) is positive \((V \geq 0)\), monotonically decreasing \((V' \leq 0)\), and convex \((V'' \geq 0)\). Figure 8 shows the function \(V(P)\) obtained from the data in the way described in this paper. It is clear that it does not strictly satisfy the conditions of monotonicity and convexity, indicating that, as expected, the pure spherical infall model does not exactly represent the data.

We can establish a relation \(R(P)\) in the sense that, for any given \(P = P_0, R_0 \equiv R(P_0)\) is (are) the value(s) of \(R\) for which the maximum of \(F(P_0, R)\) occurs, so that

\[ V(P_0) = F(P_0, R_0) = \left(1 - \frac{P_0}{R_0}\right) U(R_0) \]  \(\text{(A2)}\)

(see Fig. 7). In addition, the linear function \(F(P, R_0)\) is the tangent to \(V(P)\) at \(P_0\), so

\[ V'(P_0) = \frac{\partial F}{\partial P}(P_0, R_0) = -\frac{U(R_0)}{R_0}. \]  \(\text{(A3)}\)

\(R(P)\) is a strictly increasing function, but it is not necessarily continuous. For example, it can happen that, for some point \(P_0\), the maximum occurs at the intersection of two straight lines labeled by \(R = R_1\) and \(R = R_2 > R_1\) (see Fig. 9), with the
Fig. 7.—Schematic illustration of the transformation relating $V(P) = \omega(r^2)$ to $U(R) = u^2(r^2)$. See Appendix for an explanation.

Fig. 8.—The function $V(P) = \omega(r^2)$ determined from the data.

Fig. 9.—Schematic illustration of the transformation relating $V(P) = \omega'(r^2)$ to $U(R) = u^2(r^2)$ when there is a discontinuity in the relation $R(P)$. See Appendix for an explanation.
lines corresponding to all other values of $R$ lying below it, i.e.,

$$V(P_0) = \left(1 - \frac{P_0}{R_1}\right) U(R_1) = \left(1 - \frac{P_0}{R_2}\right) U(R_2) \geq \left(1 - \frac{P_0}{R}\right) U(R) \quad \forall R \in (R_1, R_2).$$  \hfill (A4)

The values of $U(R)$ in the open interval $(R_1, R_2)$ do not affect the function $V(P)$ and therefore cannot be recovered from it. In general, it can only be said that

$$U(R) \leq U_{\delta}(R) \equiv \min_{P < R} \frac{V(P)}{1 - P/R}. \hfill (A5)$$

This is an equality for those $R$ which correspond to a maximum $F(P, R)$ for some $P$, and a strict inequality in all other cases. The latter can in principle be diagnosed by realizing that, in the case discussed above,

$$\lim_{P \to P_0} V(P) = - \frac{U(R_1)}{R_1} < \lim_{P \to P_0} V(P) = - \frac{U(R_2)}{R_2},$$  \hfill (A6)

so both $P(R)$ and $V(P)$ are discontinuous at $P = P_0$. In practice, with noisy data, a discontinuity in $V(P)$ is difficult to detect, and the inequality, equation (A5), has to be used as such. It is interesting to note that, taking the reciprocal value of all variables, one can write

$$U^{-1}(R^{-1}) \geq U_{\delta}^{-1}(R^{-1}) = \max_{P^{-1} > 0} \left(1 - \frac{R^{-1}}{P^{-1}}\right) V^{-1}(P^{-1}).$$  \hfill (A7)

This has the same form as equation (A1). We can conclude that $U^{-1}_{\delta}(R^{-1})$ has the same properties as $V(P)$, being positive, monotonically decreasing, and convex; in fact, it is the convex and decreasing lower envelope of $U^{-1}(R^{-1})$. As long as the latter is itself convex and decreasing, then $U_{\delta}(R) = U(R)$, while in general $U_{\delta}(R) \geq U(R)$. Figure 10 shows $U^{-1}_{\delta}(R^{-1})$ as obtained from the data. Its curved parts (here absent) and “corners” are expected (within the pure spherical collapse model) to correctly estimate $U^{-1}(R^{-1})$, while the straight segments are lower bounds.

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