The angular momentum of dark halos: merger and accretion effects

Sébastien Peirani, Roya Mohayaee and José A. de Freitas Pacheco
Observatoire de la Côte d’Azur, B.P.4229, F-06304 Nice Cedex 4, France
emails: peirani@obs-nice.fr, roya@obs-nice.fr, pacheco@obs-nice.fr

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ABSTRACT
We present new results on the angular momentum evolution of dark matter halos. Halos, from N-body simulations, are classified according to their mass growth histories into two categories: the accretion category contains halos whose mass has varied continuously and smoothly, while the merger category contains halos which have undergone sudden and significant mass variations (greater than 1/3 of their initial mass per event). We find that the angular momentum grows in both cases, well into the nonlinear regime. For individual halos we observe strong correlation between the angular momentum variation and the mass variation. The rate of growth of both mass and angular momentum has a characteristic transition time at around \( z \approx 1.5 - 1.8 \), with an early fast phase followed by a late slow phase. Halos of the merger catalog acquire more angular momentum even when the scaling with mass is taken into account. The spin parameter has a different behavior for the two classes: there is a decrease with time for halos in the accretion catalog whereas a small increase is observed for the merger catalog. When the two catalogs are considered together, no significant variation of the spin parameter distribution with the redshift is obtained. We have also found that the spin parameter neither depends on the halo mass nor on the cosmological model. From our simple model developed for the formation of a disk galaxy similar to the Milky Way, we conclude that our own halo must have captured satellites in order to acquire the required angular momentum and to achieve most of the disk around \( z \approx 1.6 \). The distribution of the angular momentum indicates that at \( z \approx 1.6 \) only 22% of the halos have angular momentum of magnitude comparable to that of disk galaxies in the mass range \( 10^{10} - 5 \times 10^{11} M_\odot \), clearly insufficient to explain the present observed abundance of these objects.

Key words: dark matter, halos of galaxies, merger, accretion, angular momentum

1 INTRODUCTION
The origin of angular momentum remains a key factor in understanding the formation, evolution and particularly the morphological types of galaxies. In the gravitational instability paradigm of structure formation, angular momentum could arise from the tidal interaction of galaxies with their surroundings (Hoyle 1949; Peebles 1969; Doroshkevich 1970; White 1984). As a protogalaxy expands, its angular momentum grows linearly with time due to tidal interactions until it decouples from the background expansion, turns around and starts collapsing. After the turn around time the angular momentum essentially stops growing and becomes less sensitive to tidal torques (Peebles 1969), and hence this time marks the maximum angular momentum acquired by galaxies through this mechanism.

In a Universe dominated by dark matter, within the hierarchical scenario, galaxies are formed when baryonic gas falls into the gravitational potential well of dark matter halos. Therefore, many properties of galaxies, including their angular momentum, are expected to be closely related to those of their host halos. The acquisition of angular momentum by dark matter halos through tidal torques, can be formulated within the Lagrangian perturbation theory (White 1984; Cattaneo & Theuns 1996a, 1996b). At the first order, using Zel’dovich approximation, one can show that the angular momentum grows linearly with time. This result
has been confirmed by N-body simulations. However, the tidal torque theory (TTT) is an oversimplified explanation of the complex evolution of the angular momentum of dark halos. Real progenitors of dark matter halos are not isolated, as assumed in TTT, but are continuously growing by accretion and merger. In the nonlinear regime, no self-consistent theory, including mass accretion and merger history of halos yet exists. Most of the investigations rely on numerical simulations and or on semi-analytic modelings. The transfer of orbital angular momentum to spin during a merger event has been verified in simulations, where one observes a correlation between a sharp increase in the halo mass and sudden variations in the dimensionless spin parameter \( \lambda = J / E |^{1/2} / GM^{5/2} \) where \( J \) is the angular momentum, \( E \) is the total energy of the halo and \( M \) is its mass. The spin parameter is essentially the ratio of the angular momentum of an object to that required for rotational support. There also seems to be a general tendency for \( \lambda \) to decrease during periods of slow accretion (Vitvitska et al. 2002). Effects of mergers are clearly seen in the probability distribution of the spin parameter. Halos that have undergone major merger events at \( z \leq 2 \) have systematically larger spin parameters than those that have evolved only by accretion (Gardner 2001; Vitvitska et al. 2002; Maller, Dekel & Somerville 2002). Since on galactic scales ellipticals have lower angular momentum than spirals, these results could be troublesome if the former are merger remnants of the latter.

Thus, in spite of extensive works, many aspects of the evolution of the angular momentum in the nonlinear regime remain inconclusive. For instance, it remains unclear if the distribution of the spin parameter depends on the redshift or not (Lemson & Kauffmann 1999; Maller, Dekel & Somerville 2002). Concerning the variation of \( \lambda \) with the halo mass, the early investigations indicate a slight decrease with a mean slope \( d(\log < \lambda >)/d(\log M) = -0.17 \pm 0.07 \) (Barnes & Efstathiou 1987). Such a trend was also observed in more recent numerical simulations (Cole & Lacey 1996). Although this result seems to be compatible with the tidal torque theory, a small but opposite variation was obtained if halos were to acquire angular momentum by merging (Maller, Dekel & Somerville 2002). Recent tests of TTT by N-body simulations (Porciani, Dekel & Hoffman 2002a, 2002b) have shown that the values of the spin amplitudes predicted by TTT are in agreement with those derived from simulations if the linear growth stops somewhat earlier than maximum expansion. After shell crossing, some numerical studies suggest that, in general, the angular momentum decays (Barnes & Efstathiou 1987; Sugerman, Summers & Kamionkowski 2000), which is expected due to spin conservation if the system remains isolated. On the other hand, erratic variations are expected due to the transfer of orbital angular momentum to spin during merger events or due to accretion (Vitvitska et al. 2002).

In the present work, considering that models based on TTT do not agree in detail with the results of N-body simulations in the nonlinear regime, we investigate an alternative approach in which halos obtain most of their spin through the transfer of orbital angular momentum by continuous accretion and/or mergers. We identify the halos using FOF algorithm (Davis et al. 1985) and make two distinct catalogs: the ones which have never had a major merger event, and another catalog for halos which have undergone at least one merger episode, in which their mass increased by at least \( 1/3 \) of their initial value during the event. We show that even after shell crossing, the mean angular momentum of dark halos still grows, but with a rate different from that predicted by TTT. Individual halos have erratic variations of the angular momentum, which are strongly correlated with the mass accretion rate, producing on the average a growth of the angular momentum. We recognize two scaling regimes for the growth of the mean halo masses and angular momenta. The corresponding exponents differ for the accretion and merger samples. We find that, when no distinction between the two catalogs are made, i.e., when all the halos are considered together, no significant variation of the spin parameter distribution with the redshift is observed. However, when the halos are considered separately, the evolution of spin parameter of the accretion sample indicates a small but significant decrease with time, whereas for the merger catalog the spin parameter increases with time. We show that this behavior is a consequence of the different time scaling laws followed by the dynamical variables defining the spin parameter.

Finally, we also discuss the formation of disk galaxies. Past cosmological N-body/SPH simulations have shown that the scale length and angular momenta of disks (baryonic component) are about one order of magnitude smaller than the observed values. This problem is twofold: firstly, in N-body simulations baryons loose a significant fraction of their angular momentum, resulting in disks that are too small and secondly the angular momentum distributions reveal too much low angular momentum material. If the specific angular momentum distribution is conserved, which is usually assumed to be so for disk formation, the resulting disk shows a more centrally concentrated mass distribution making it difficult to explain the exponential mass profile of spirals (Navarro, Frenk & White 1995; Navarro & Steinmetz 1997, 2000; van den Bosch 2001; van den Bosch, Burkert & Swaters 2001). We discuss a simple collapse model to form a baryonic proto-disk. We will show how the halo parameters, in particular the specific angular momentum distribution, affect the disk properties like its mass distribution, spin parameter and how the angular momentum acquired during the growth phases of the halo influences the evolution of the disk. The plan of this paper is as follows: in Section 2, we review briefly the basic principles of TTT and alternative theories, in Section 3 we describe our N-body simulations and our halo-finding algorithms and we discuss the angular momentum evolution by accretion and merger events in Section 4. In Section 5 we develop our scenario describing the formation of a disk galaxy and in Section 6, we summarize our main results and conclusions.
2 ANGULAR MOMENTUM: DYNAMICAL DESCRIPTION

The global angular momentum $\mathbf{J}$ of a halo of $N$ particles is defined by

$$\mathbf{J} = \sum_{i=1}^{N} m_i \mathbf{r}_i \times \mathbf{v}_i,$$

(1)

where $\mathbf{r}_i$ and $\mathbf{v}_i$ are the position and velocity of the $i$th particles with respect to the halo centre of mass.

In the continuum limit, the angular momentum $\mathbf{J}$ of the matter contained at time $t$ in a volume $V$ of the Eulerian $x$-space becomes

$$\mathbf{J} = \int_{V(t)} \rho \, d^3 r \ (\mathbf{r} - \mathbf{r}) \times \mathbf{v},$$

$$= a(t)^5 \int_{V} \rho(x) \, d^3 x \ (x - \bar{x}) \times \dot{x},$$

$$= \bar{\rho} a(t)^5 \int_{V_0} d^3 q \ (\mathbf{q} - \bar{\mathbf{q}}) \times \frac{d\mathbf{S}}{dt},$$

(2)

where $a(t)$ is the scale factor, $x$ is the comoving Eulerian spatial coordinate, $\mathbf{r} = a(t)x$ is the physical distance, centre of mass coordinates are marked by tilde, $\rho$ is the matter density, $\bar{\rho}$ is the background density, $V_0$ is the Lagrangian volume and $\mathbf{S}$ is the displacement vector from the initial position $\mathbf{q}$ to the Eulerian position $x$ satisfying the mapping

$$\mathbf{x}(\mathbf{q}, t) = \mathbf{q} + \mathbf{S}(\mathbf{q}, t).$$

(3)

An analytic approximate expression for the angular momentum (2) can be found using the linear Lagrangian perturbation theory, i.e. the Zel’dovich approximation,

$$\mathbf{S}(\mathbf{q}, t) \approx S^{(1)}(\mathbf{q}, t) = D(t) \nabla \varphi(\mathbf{q}),$$

(4)

where $D(t)$ is the growing mode of the density perturbation: in the Einstein-de Sitter Universe $D(t) \approx a(t)$ and $\varphi(\mathbf{q})$ is the linear gravitational potential satisfying to first order in the density fluctuation, $\delta = (\rho - \bar{\rho})/\bar{\rho}$ the Poisson equation $\nabla^2 \varphi = \delta/\bar{\rho}$. Inserting (4) in the expression for the angular momentum (2), gives to first order

$$J_i^{(1)} = \bar{\rho} a(t)^5 D \int_{V_0} d^3 q \epsilon_{ijk} (q_j - \bar{q}_j) \frac{\partial \varphi}{\partial q_k}.$$  

(5)

Taylor expanding the gravitational potential around the centre of mass $\bar{\mathbf{q}}$, one obtains

$$J_i^{(1)} \approx a(t)^5 \bar{\rho} D \epsilon_{ijk} T_{kl} I_{lj}$$  

(6)

for the angular momentum in terms of the deformation tensor

$$T_{kl} = \left( \frac{\partial^2 \varphi}{\partial q_k \partial q_l} \right)_{\bar{\mathbf{q}}}$$  

(7)

and the quadrupole inertia tensor

$$I_{lj} = \int_{V_0} d^3 q (q_l - \bar{q}_l)(q_j - \bar{q}_j).$$  

(8)

Evidently, if the inertia and deformation tensors are aligned then the tidal torque is zero. Numerical simulations indeed show that these two tensors are mostly aligned and the contribution to the torque comes from statistical fluctuations in the misalignment (Porciani, Dekel & Hoffmann 2002a, 2002b). Thus, from expression (6), in the linear regime angular momentum increases linearly with time. The prediction of the linear theory were shown to be marginally affected by going to higher-order terms in Lagrangian perturbation theory (Catelan & Theuns 1996a).

If the Lagrangian volume is spherical, then the moment of inertia is $I_{lj} = 4\pi \delta_{lj} q_j^2 / 15$, where $\delta_{lj}$ is the Kronecker delta, and hence the angular momentum vanishes. This remains valid also at the second order in Lagrangian perturbation theory (Catelan & Theuns 1996b). However, the angular momentum of an Eulerian spherical volume does indeed grow only at second order (Peebles 1969). The discrepancy between these two results was shown to be due to the fact that the growth of the angular momentum of an Eulerian sphere at second order is not due to tidal torques but due to the convective transport of the angular momentum across the surface of the sphere (White 1984).

The subtlety in the generation of angular momentum comes from the fact that in an expanding universe the vortical modes decay and using Kelvin circulation theorem one can show that in a dissipationless system vorticity cannot be generated, however, angular momentum can indeed be generated (Peebles 1973). Such motion which is mainly a shearing flow and not a vortical flow, soon becomes complicated and vorticity can be generated. Furthermore, a minute amount of velocity dispersion in the initial flow would indeed lead to the generation of vorticity which would not show in cold dark matter simulations where the velocity dispersion is put equal to zero. A small vorticity can also be obtained in the averaged density-weighted streams after shell crossing (Pichon & Bernardeau 1999). Moreover, a collisionless system with a non-isotropic velocity dispersion can also generate vorticity (Ruzmaikin 1975).

Although TTT provides a good description of the growth of angular momentum in the linear regime, as compared with numerical simulation, it is clearly an oversimplified description of the dynamical process of spin acquisition. The angular momentum of halos in this model is overestimated by a factor of three compared to simulations, with a large scatter. Also the direction of the spin vector disagrees with numerical experiments, with errors typically of about 57° (Lee & Pen 2000). TTT predictions for the spin parameter are for all the matter which at $z = 0$ is found in a given halo and does not predict the spin of any particular halo progenitor. Here, we follow the evolution of the angular momentum of the most massive progenitor; a procedure more adequate for modeling galaxy formation (Vitvitska et al. 2002, Somerville & Primack 1999).

Alternative semi-analytic models for the acquisition of angular momentum by halos exist; for example the model in which the spin is built up randomly by mass accretion and merger events (Vitvitska et al. 2002, Maller, Dekel & Somerville 2002). The evolution of the angular momentum in this scenario is rather different from that expected from TTT, in which the halo spin grows linearly at very early times and essentially stops later.
The evolution of the spin parameter of the major halo progenitor in such an alternative picture, presents sharp increases during important merger episodes and a tendency to decline during periods of gradual accretion or capture of small lumps of matter. The analytical approach of the random walk theory predicts that the distribution of the spin parameter is a log-normal, with mean and dispersion that depend neither on the progenitor mass nor on the redshift.

3 THE SIMULATIONS

The N-body simulation analyzed uses adaptive particle-particle/particle-mesh (AP3M) code HYDRA (Couchman, Thomas & Pearce 1995). We ran a ΛCDM simulation with $h = 0.65$, $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$ and $\sigma_8 = 0.9$. The simulation was performed in a periodic box of side $30 h^{-1}$ Mpc with 256$^3$ particles (hence with mass resolution of $2.051 \times 10^6 \, M_\odot$). The simulation started at $z = 50$ and ended at the present time $z = 0$.

Halos catalogs at different redshifts were prepared by using a friends-of-friends (FOF) algorithm (Davis et al. 1985), with a linking length of $b = 0.15$ in units of the mean interparticle separation. Only structures with at least 50 particles were retained in the different catalogs, corresponding to a halo mass of about $1.03 \times 10^{13} \, M_\odot$. Halos with masses higher than $10^{13} \, M_\odot$ (about 49,000 particles) were excluded from our study, since they have many substructures and are likely to be representative of groups or clusters of galaxies. Once a structure is identified, the total energy of each particle is computed, with respect to the centre of mass, and those with positive energy are removed. The procedure is repeated with the new centre of mass, computed according to its usual definition, until no unbound particles are found. We comment that our halo-finding algorithm differs from those using the spherical virial radius around the most bound particle. Most of the halos characterized by this procedure are not virialized, having a virial ratio $2T/|W|$ around 1.4 - 1.8, at $z \sim 3$. As we shall discuss in a forthcoming paper (Peirani et al 2003), halos reach a state of relaxation only by now, depending strongly on their evolutionary history. The typical halo dimension was estimated calculating the gravitational radius, $r_g = GM^2/|W|$, where $W$ is the total gravitational energy of the system.

We have identified 8728 halos at $z = 0$ and out of this number we have prepared a sample of 780 halos which have never undergone a major merger event, and their mass have always increased by accretion or by capture of small lumps of matter. We also selected 561 halos which had at least one major merger episode in their history, corresponding to an increase of their masses at least by a factor of $1/3$ in the event. Since this limit, generally adopted in the literature is rather arbitrary, we have also examined how our results are modified if we decrease the above mass-fraction threshold to $1/6$. In this case, the accretion catalog is reduced to 607 halos, whereas the merger sample is increased to 734 halos. Here, we present our results mainly for the first set of the catalogs, with mass-fraction threshold of $1/3$, however our results are not significantly modified by the other set of catalogs with the mass-fraction threshold of $1/6$.

4 ANGULAR MOMENTUM EVOLUTION

4.1 Accretion effects

In order to study accretion effects, we have selected halos which up to the present time have not undergone any major merger event. This means that these halos have not captured any other objects with masses larger than one third (one sixth) of their own masses. These halos are mostly in filaments and their masses evolve only by a continuous accretion process or by the capture of small lumps of matter. The accreted matter comes from different directions and not only from the main axis, since filaments are much wider than the typical halo dimensions. The evolution of the selected halos satisfying these conditions has been followed from $z = 3.5$ until $z = 0$, using the following algorithm: since each particle in the simulation can be identified, it is possible to obtain the constitution of each halo at each time step. If more than 70% of the particles of a given halo are found in another halo at the subsequent time step, then we assume that both objects are the same. This scheme allows the tracing of the evolution of the most massive progenitor. In order to follow the halo evolution, the considered redshift interval ($3.5 \geq z \geq 0$) was divided into 28 steps from which positions and velocities of each particle were stored (specifically at: 0.0, 0.04, 0.09, 0.15, 0.20, 0.26, 0.33, 0.40, 0.48, 0.57, 0.67, 0.79, 0.92, 1.08, 1.26, 1.49, 1.6, 1.7, 1.77, 1.9, 2.0, 2.15, 2.3, 2.5, 2.68, 3.0, 3.2, 3.51).

The mass distribution of these 780 halos (or the 607 halos for the catalogs with the mass-fraction threshold of $1/6$) was calculated at different times and statistical parameters like the mean and the median of the logarithm of mass, as well as the root mean square deviation were computed. We emphasize that all of the scaling relations obtained in this paper use the median of the logarithm of the variable (which all vary over several decades) rather than the variable itself, since otherwise the statistical distribution would not have a fair representation. Both the mean and the median increase with time, indicating an average growth of about a factor of five in the considered time interval. As we shall see later and as it is expected, merging produces on the average much more massive halos.

In Fig. 1 we have plotted the variation of the median of the logarithm of the mass of the halos, as a function of time. The simulated data is well-fitted by the two power laws

$$M_{acc} \propto \begin{cases} z^{0.91} & \text{for } z > 1.8 \\ z^{0.58} & \text{for } z < 1.8 \end{cases}$$

where $M_{acc}$ is the median of the halo mass distribution at a given instant of time. If we consider the mass-fraction threshold of $1/6$ instead of $1/3$ to discriminate between the two catalogs, these results are not significantly modified: the transition between the two scaling
regimes still occurs near \( z \sim 1.8 \) and the exponent at higher redshifts becomes 0.89, while for \( z < 1.8 \) the exponent is 0.55.

It is worth mentioning that this behavior compares with the evolution of individual halos (Vitvitska et al. 2002), indicating a fast growth of the mass for \( z > 1.5 - 2.0 \), followed by a phase where the accretion rate is slower. The median mass of the distribution fits the relation \( M_{\text{acc}}(z) = M_{\text{acc}}(0)e^{-2z/(1+z_t)} \), where \( M_{\text{acc}}(0) \) is the present halo mass and \( z_t \) is a characteristic redshift at which the halo mass is a certain fraction of the present mass (Wechsler et al. 2002; Zhao et al. 2003). We found from our fit \( z_t = 4.0 \), indicating that at that redshift the median mass was about five times smaller than now.

As we have seen in Section 2, tidal torques are effective until the density perturbation attains the turnaround time, when they are expected to reach the maximum angular momentum. However, for lumps of matter that are continuously growing, the turnaround time is ill-defined since they do not simply follow the evolution of the spherical model. In particular, in the process of virialization, the inner mass shells cross the center and one would expect an important energy transfer from bulk to random motions, due to collective effects. In fact, this mechanism contributes to the relaxation of the halo towards a complete equilibrium state, which is however delayed by the continuous accretion of matter. Faced with this complex situation, some empirical approaches have been suggested in the literature to define the moment of the first shell crossing. Here, we take the time of maximum velocity dispersion to define such an instant (Sugerman, Summers & Kamionkowski 2000). It is worth mentioning that it was found that, on the average, the magnitude of the angular momentum, \( J \), still grows almost linearly \( (J \propto t^{0.85}) \) between turnaround and the first shell-crossing time, as defined above (Sugerman, Summers & Kamionkowski 2000).

We have estimated the latter for several halos of different masses, \( M \), and obtained that the first crossing time varies approximately with mass as

\[
 t_c = 0.112(M/10^{11})^{0.41}. \tag{10}
\]

In this equation the time is in units of \( H_0^{-1} \) and masses are in solar units. We stress that here the halo masses correspond to the instant of shell crossing. At \( z = 3.5 \), when we start to follow the evolution of the halos, the shell-crossing time of the median of the mass distribution corresponds to a redshift of about \( z \sim 6 \). Assuming roughly that the crossing time is twice the turnaround time (this is correct only in the spherical model), the latter is around \( z \sim 8 - 9 \). In Fig. 2 we show the estimated redshift of shell crossing by our procedure as a function of the halo mass. This result implies that the variations observed in the angular momentum of our simulated halos for \( z < 3.5 \) are probably not due to the tidal field of nearby lumps of matter, but are due to the transfer of angular momentum through accretion of mass.

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Variations of the angular momentum when halos are considered individually are rather erratic and are correlated with variations of the mass accretion rate. This behavior is illustrated in Fig 3, where the rate of variation of the modulus of the angular momentum is plotted in parallel with the mass accretion rate for two halos from our catalogs. Notice that positive and negative rates can be observed in both cases, but not always producing effects in the same sense, since they depend on the relative orientation of the halo spin with respect to the angular momentum vector of the accreted matter. This figure illustrates that accretion affects considerably the angular momentum history of the halos. It is worth mentioning that the FOF mechanism may introduce some artifacts like bridges between nearby halos, which may cause sudden variations in the mass accretion history. However our procedure always check if particles are physically bounded to the halo or not and in this way we reduce the possibility that spurious effects would influence our results.

In spite of the erratic variations of the spin when individual halos are considered, on the average, an increase of the angular momentum is observed. Fig. 4 shows the median value of angular momentum distri-
4.2 Merger effects

In order to compare the effects of merging on the angular momentum history of halos, we have selected 561 halos which have at least had one merger event in which the mass-fraction threshold is greater than 1/3. As before, to evaluate the consequences of this threshold on the results, another merger catalog including 734 halos was also studied, in which the threshold was lowered to 1/6.

In Fig. 1 we show how the median of the logarithm of mass evolves for these halos. We comment that as compared to halos which grow by accretion, the median grows faster, as expected, and we equally find two regimes and a characteristic redshift of $z \sim 1.5$, namely

$$M_{mer} \propto \begin{cases} t^{1.35} & \text{for } z > 1.5 \\ t^{0.81} & \text{for } z < 1.5 \end{cases}$$

(12)

Here again $M_{mer}$ corresponds to the median of the mass distribution at a given redshift. No significant modifications are noticed if the mass-fraction threshold is taken equal to 1/6 in order to classify the two catalogs: the exponent is 1.25 for $z > 1.5$ and decreases to 0.75 for $z < 1.5$.

In Fig. 4 we have plotted the median $(\log J)$ of the angular momentum distribution as a function of time, which can be compared with the evolution of the halos evolved only by accretion. Again, two regimes of growth are found with a transition at $z \sim 1.5$.

$$J_{mer} \propto \begin{cases} t^{2.70} & \text{for } z > 1.5 \\ t^{1.67} & \text{for } z < 1.5 \end{cases}$$

(13)

Notice the rapid rate of growth of the angular momentum due to merging for $z > 1.5$ and, in spite of increasing at a lower rate after the transition, the growth is still faster than the almost linear variation obtained for the accretion sample in the same evolutionary phase. No significant modifications are obtained changing the mass-fraction threshold to 1/6: the exponents become equal to 2.52 and 1.54 respectively before and after the characteristic redshift at $z \sim 1.5$. In the redshift interval $3.5 > z > 0$, the average angular momentum grows by a factor of 80, suggesting clearly that merging transfer angular momentum much more efficiently than accretion. However, a simple comparison as the one we have just made is not sufficient to explain this phenomenon. In fact, since the angular momentum scales with mass as $M^{5/3}$ (see below) and halos which have grown by merging are more massive, this effect must be taken into account when comparing both samples. In this case, it is more convenient to compare the distribution of the quantity $\log(J/M^{5/3})$. In Fig. 5 the distributions for the two samples at $z = 0$ are compared and one can see that merging events give a larger contribution to the final angular momentum of halos irrespective of the mass variations.

In the context of the present study, an interesting question, which we have also considered was the radial redistribution of the specific angular momentum during accretion or after a merger episode. In the accretion process, the specific angular momentum is distributed almost linearly with the distance to the centre ($j_{DM} \propto r$), and the amplitude at the periphery may increase or decrease with time, according to the relative orientations of the halo spin and that of the matter stream being...
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Figure 5. The distribution of the angular momentum, normalized by \( M^{5/3} \), for halos that have grown by merging (mass ratio 1/3) and those which never had a major merger event. The mean of the distribution is larger for halos which have undergone major merger events.

Captured. In the upper panel of Fig. 6, the radial distribution of the specific angular momentum is shown at three different redshifts, for a typical case where there is a small decrease of the maximum value, in spite of the increasing halo dimensions. However, notice that the distribution becomes more and more smooth, which is a general trend of the evolution when accretion dominates. The situation is rather different after an important merger event, and this is illustrated in the lower panel of Fig. 6. After the merger episode, the maximum specific angular momentum increases, which is a quite general situation. In the subsequent phases, the specific angular momentum is redistributed, decreasing in the central regions and increasing in the halo outskirts. We shall return to this problem in Section 5, when the formation of a disk galaxy will be discussed. However, to characterize well these effects, higher resolution simulations are needed.

4.3 The evolution of the spin parameter

Previous works have shown that the spin parameter \( \lambda \), obtained from simulations, has a log-normal distribution (Barnes & Efstathiou 1987; Cole & Lacey 1996; van den Bosch 1998; Ryden 1998),

\[
P(\lambda)d\lambda = \frac{1}{\sigma_\lambda \sqrt{2\pi}} \exp\left(-\frac{\ln^2(\lambda/\lambda_0)}{2\sigma^2}\right) \frac{d\lambda}{\lambda} \tag{14}\]

which seems to be a universal result, independent of the cosmological model. In Fig. 7 we show the distribution of the spin parameter at \( z = 0 \) for the two catalogs considered here. An inspection of this plot confirms again that halos which have undergone important merger episodes have, on the average, a larger spin parameter and a wider distribution than those evolved by accretion only. We will return to this point later in this section.

If the halos of both samples are considered together, the parameters defining the distribution at \( z = 0 \) are: \( \lambda_0 = 0.036 \) and \( \sigma_\lambda = 0.57 \), in agreement with previous

Figure 6. The distribution of the specific angular momentum for a typical halo of 12000 particles at \( z = 0 \) in the accretion case (upper panel) and of 30,000 particles at \( z = 0 \) in the merger case (lower panel). In the merger case, there is a general trend of transfer of angular momentum from the central regions to the periphery.

Figure 7. The distribution of the spin parameter for halos that grow by merger and those which grow by accretion at \( z = 0 \). The average spin parameter is higher for halos underwent merger events than for halos which have growth by accretion only.
Earlier investigations of the evolution of the spin parameter indicate that its distribution does not vary with redshift. This result seems to be valid both in the TTT and in the random walk capture model (Vitvitska et al. 2002). In Fig. 8 we plot statistical parameters of the $\lambda$ distribution, as the mean, median and $\lambda_\alpha$, as a function of the redshift, when the halos of each catalog separately and also both catalogs together are considered (1341 halos).

This figure demonstrates that, in spite of some erratic fluctuations, the variations in both $\lambda_0$ and the median are quite small, although a slight increase seems to be suggested by the evolution of the mean value. Taken at the face value, these results are compatible with no variation of $\lambda$ with $z$ or, at least compatible with a slight increase in the mean. However, the results are quite different if we analyze the evolution of the spin parameter for the two samples (accretion and merger) independently. The middle and the lower panels of Fig. 8 show respectively the evolution of the same statistical parameters for halos evolved only by accretion and those which have had at least one major merger event. We notice a clear and significant evolution in opposite senses: halos evolved by accretion have a decreasing spin parameter, whereas for halos evolved by merging $\lambda$ increases. Typical values for the rmsd of the mean value of the spin parameter $\lambda$ are 0.02.

How can this behavior be explained? The spin parameter depends on dynamical quantities since $\lambda \propto J\left| E \right|^{1/2}/M^{3/2}$. As we have seen above, both the angular momentum $J$ and the mass $M$ increase with time at different rates, according to the dominant mechanisms by which halos grow. But what about the total energy. In Fig. 9 we have plotted the median of the logarithm of the modulus of the total energy as a function of time. We notice that halos evolved by merging have a higher binding energy that also evolves faster than in the accretion case.

This question will be discussed in more detail by Peirani et al. (2003), when the dynamical relaxation of the halos will be examined. For the evolution of the magnitude of the energy, once again we observe two scaling regimes. For the accretion catalog, we have

$$|E|_{\text{acc}} \propto \begin{cases} t^{1.32} & \text{for } z > 1.8 \\ t^{0.66} & \text{for } z < 1.8 \end{cases} \quad (15)$$

and for the merger catalog we have

$$|E|_{\text{mer}} \propto \begin{cases} t^{1.03} & \text{for } z > 1.5 \\ t^{1.03} & \text{for } z < 1.5 \end{cases} \quad (16)$$

Next, we consider the evolution for $z < 1.8$, and put the above results together with the scaling relations in (9),(11), (12) and (13). In the accretion case we have $J \propto t^{1.02}$, $M \propto t^{0.56}$ and $|E| \propto t^{0.66}$, which imply $\lambda \propto t^{-0.05}$. The same scaling analysis for $z > 1.8$ gives $\lambda \propto t^{-0.12}$. Therefore the scaling regimes of the different dynamical variables lead to a decrease of the spin parameter. In the case of merging, for $z < 1.5$, $J \propto t^{1.07}$, $M \propto t^{0.81}$ and $|E| \propto t^{1.03}$, implying $\lambda \propto t^{0.16}$, or an increase of the spin parameter with time (for $z > 1.5$ we have $\lambda \propto t^{0.29}$), in agreement with the results of our simulations. Thus, in order to explain the evolution of the spin parameter, the scaling regimes of the three dynamical quantities $J$, $M$ and $E$ are needed to be considered.
A similar analysis may be performed in the case of the variation of the spin parameter with the halo mass. The total energy scales with mass as $|E| \propto M^n$ and the angular momentum as $J \propto M^{3/2}$. From a theoretical point of view, both exponents should be equal to 5/3, and consequently the spin parameter should not depend on the mass. From our simulations, we have calculated these exponents and have verified that: i) they practically do not vary with the redshift and are quite close to the expected theoretical value; ii) no variation is also detected when the results of both samples are compared, since for halos evolved by accretion the mean values are $\kappa = 1.647 \pm 0.006$ and $\gamma = 1.647 \pm 0.034$, while for halos having experienced merger events the mean values are $\kappa = 1.624 \pm 0.011$ and $\gamma = 1.672 \pm 0.023$. These results imply $\lambda \propto M^{-0.02}$, consistent with a very small dependence on the halo mass. Using simulations with lower mass resolution ($128^3$ particles), we have checked if the scale parameters $\kappa$ and $\gamma$ depend on the initial conditions, as claimed by Barnes & Efstathiou (1987) and no differences where noticed either using a ΛCDM or a CDM model.

5 FORMATION OF DISK GALAXIES

Mestel (1963) proposed that the distribution of matter in a collapsed disk galaxy could be determined, if each element of the proto-galaxy conserves its specific angular momentum. Previous, N-body/SPH simulations have showed that the scale length and specific angular momentum of simulated disks are one order of magnitude smaller than observed, the so-called “angular momentum catastrophe” (Navarro & Benz 1991; Navarro & White 1994). More recent simulations by van den Bosch et al. (2002) seem to confirm that both gas and dark matter have similar angular momentum distributions, but their spin vectors are, in general, misaligned.

In this section, in spite of not being the major goal of this work, we explore the formation of disk galaxies in connection with the results of our simulations on the angular momentum of halos. The bulge formation will not be considered here, since it may be considered as a separate system, once its collapse timescale is probably as short as that of ellipticals of comparable mass. Moreover, they host in general a supermassive black hole whose rôle in the bulge formation is not yet well established (see, for instance, Silk 2001). From a dynamical point of view, it has been suggested that if the globular cluster system and the bulge of our Galaxy have similar dynamics, then the bulge would have a specific angular momentum about a third that of the disk (Frenk & White 1980).

In the hierarchical scenario, galaxies are expected to be formed inside dark matter halos. As discussed above, halos may initially acquire angular momentum by tidal torques until they reach the turnaround phase. Then the tidal field decreases and the angular momentum increases as a consequence of accretion and merging episodes. In spite of the fact that our simulations include only dark matter, we would expect that mechanisms affecting the intrinsic angular momentum of such a component will also act on baryons. In this case, one would expect that the specific angular momentum of the gas (baryons) is proportional to that acquired by the dark matter, namely, $j_b = \beta j_{dm}$, where $\beta \sim 1$ is a parameter describing the efficiency of torques on the gas component.

Here a simple picture is developed, based on our simulations, to describe the resulting baryonic disk structure. We have chosen two examples to illustrate a possible evolutionary history of our own Galaxy. Both halos have masses at $z = 0$ comparable to that of our Galaxy, but have evolved differently. The first one has grown without having any major merger episode, while the second one captured some small satellites (with masses less than 1/3 of the mass of the main halo) between $2.0 > z > 1.0$. In Table 1 we give the main evolutionary characteristics of both examples.

The total angular momentum of the galactic (baryonic) disk is estimated to be $J_0 \approx 2.0 \times 10^{67} \text{kg.m}^2.\text{s}^{-1}$ (see, for instance, Fall & Efstathiou 1980). As we will see below, the baryonic to dark matter mass ratio in our Galaxy is taken to be $f_b \approx 0.034$ (which is obtained from tuning the model to roughly match the rotation curve of Milky Way). Thus if both components have the same specific angular momentum, the required total angular momentum of the halo must be (at least) $J_{dm} \approx 5.8 \times 10^{68} \text{kg.m}^2.\text{s}^{-1}$. Simple inspection of Table 1 shows that the first example attains the necessary angular momentum only by the present time, unable to form a disk 10-11 Gyr old. The second example, thanks to the capture of small halos, the required angular momentum is attained at $z \sim 1.6$, corresponding to an age of 10.4 Gyr, according to our adopted cosmological parameters. In what follows we consider the history of example 2 as representative of the halo in which the Milky Way is embedded.

The build up of the disk is gradual, following the growth of the halo mass and the increase of the angular momentum. If we suppose arbitrarily that the gas starts to settle down towards the equatorial plane, according to its specific angular momentum, at $z = 5$, the halo has a mass of about $3.6 \times 10^{11} M_\odot$ and a grav-
where and the circular rotation velocity at that point (see, for example, Bullock et al. (2001), who have concluded that the successively infalling shells. If a dynamical equilibrium situation is reached after the collapse, the dimension of the disk is fixed by the maximum value of the specific angular momentum. Supposing \( \beta = 1 \), the rotational equilibrium can be expressed as

\[
j_{dm}(l) = j_b(y) = y \sqrt{G M_{dm}(y)} + y \frac{\partial \phi_b(y)}{\partial y} \quad (18)
\]

where \( y \) is the distance to the spin axis in the disk after the collapse, \( M_{dm}(y) \) is the dark matter mass inside a radius \( y \), \( \phi_b(y) \) is the gravitational potential of baryons, supposed to be distributed in a thin disk and given by the equation

\[
\phi_b(y) = -2\pi G \int_0^\infty dk J_0(ky) \int_0^\infty dx \Sigma_b(x) J_0(kx) x \quad (19)
\]

where \( J_n(x) \) is the Bessel function of order \( n \) and \( \Sigma_b(x) \) is the projected baryon mass density of the disk.

In order to derived the resulting density distribution \( \Sigma_b(y) \), we have adopted the following procedure:

- a) from our simulations, we compute not only the specific angular momentum profile but also the total dark matter mass inside a given radius \( r \), the total projected mass inside a distance \( l \) from the spin axis or, equivalently the total projected mass with specific angular momentum less than \( j_{dm} \). The latter distribution at different redshifts is showed in Fig. 11 and, generically, is represented by \( M_{dm,p} = \phi(j_{dm}) \), where the extra subscript \( p \) means the projected mass.

- b) Using the condition of dynamical equilibrium, the baryon mass in the disk inside a distance \( y \) from the axis is

\[
M_{b,p}(y) = f_b(j_b(y)) \quad (20)
\]
and the derivative of this equation gives the projected baryon density

\[ \Sigma_b(y) = \frac{1}{2\pi y} \frac{dM_{b,p}(y)}{dy} \]  \hspace{1cm} (21)

c) since in the beginning the baryon distribution throughout the disk is not known, we start our calculations supposing that the gravitational field is essentially due to the dark matter. Then, an initial projected density is calculated and the corresponding gravitational potential (eq. 19), which is introduced in the dynamical equilibrium equation. The process is repeated until the required convergence precision is attained.

If we start at redshift \( z = 5 \), a quite well-known result is obtained. The halo does not have sufficient angular momentum and the resulting disk is quite small, with a radius of about 5 kpc. As the halo grows, its angular momentum increases and the outer layers acquire a higher specific angular momentum. If the gas follows the same trend a larger disk can be built up.

Fig. 11 shows the resulting projected density profile. The projected mass density decreases exponentially with a scale length of 3.4 kpc between 1 - 10 kpc. Beyond 12 kpc the density decreases more slowly but still exponentially, with a scale length of about 8.4 kpc. The present extension of the disk is approximately 40 kpc, where a density cutoff is obtained and its total mass (baryonic) is about \( 1.45 \times 10^{11} M_\odot \). The rotation curve imposes a strong constraint on the baryon fraction, which should be equal to \( \rho_b = 0.034 \), in order that the rotation velocity be about 230 km/s at a distance of 8-9 kpc from the spin axis (see Fig. 10).

The resulted mass density profile of the baryonic disk.

The angular momentum of dark halos: merger and accretion effects

Figure 11. The distribution of the projected dark matter mass with a maximum specific angular momentum \( J_{dm} \).

The distribution of the baryonic disk.

Figure 12. The resulted mass density profile of the baryonic disk.

Figure 13. Theoretical rotational velocity profile, including the contribution of baryonic and dark matter.

The required baryon fraction is a factor 4.5 lower than the mean cosmic value, but not incompatible with the ratio estimated for the Galaxy and the value adopted by Bullock et al. (2001) in their own calculations of the disk structure. Recent hydrodynamical calculations by van den Bosch, Abel & Hernquist (2003) considered the effects of preheating on galaxy formation. In particular, a fraction of the preheated gas may become unbounded, yielding a final baryon to dark matter ratio smaller than the mean cosmic value.

As we have mentioned above, if baryons and dark matter have the same specific angular momentum and distribution, we would expect that the spin parameter ratio between both components is \( \lambda_{dm}/\lambda_b \approx f_b \). Once that the disk structure was calculated, the total energy can be computed. For an exponential disk, \( E = -5.8 G_0 \Sigma_b \Lambda^{1/2} \), where \( \Sigma_b \) is the central projected mass density and \( \Lambda \) is the scale of length. Using the values derived from our model, we obtain \( |E| = 5.5 \times 10^{51} \) J and a spin parameter \( \lambda_b = 0.49 \). In this case, taking into account the derived baryon mass fraction, the expected halo spin parameter is \( \lambda_{dm} \approx 0.017 \). It is worth mentioning that the value derived from our simulations, when the halo (and the disk) reached about 90% of its final mass is \( \lambda_{dm} = 0.018 \), in agreement with our prediction.
6 CONCLUSIONS

We have studied the effects of accretion and merger on the dynamical evolution of dark matter halos, specifically on their angular momentum. The difference between accretion and merger could essentially be semantic, since the criterion for separating the two cases is somehow arbitrary. In this work, accretion should be understood as a "continuous and almost smooth variation" of the halo mass, while this variation in the merging case is "sudden and significant". When a halo mass changes by more than a 1/3 of its present value, then we consider the event a merger and otherwise accretion. In this manner, we have built and studied two separate merger and accretion catalogs. We have demonstrated that our results are only marginally modified if the mass-fraction threshold of 1/3 is reduced by a factor of two.

We found that in both cases (accretion and merger), there are two distinct phases of mass growth: an early and a fast phase, followed by a late phase of slow mass growth, e.g., in agreement with Zhao et al. (2003). The transition occurs at $z \sim 1.5 - 1.8$ (later than the value of $z \sim 3$ found by Zhao et al. 2003). The mass growth in both phases is well-represented by power laws. The exponents not only vary from one phase to another, but also change between the merger and the accretion catalogs. It is worth mentioning that in the hierarchical picture, if the power spectrum of the initial fluctuations is of the form $|\delta_i|^{2} \propto k^{n}$, in the linear regime the characteristic masses grow as $M \propto t^{4/(n+3)}$ (Peebles 1980). In this case, for a Harrison-Zel’dovich spectrum, the masses at the early phases grow almost linearly with time, an evolution close to that found from our simulations for halos in the merger catalog. A similar exponent was also found by Toth & Ostriker (1992), i.e., $M \propto t^{1.3}$, who considered the halo mass evolution in the hierarchical picture, using a standard CDM power spectrum.

The algorithm used in this work is adequate to follow the evolution of the more massive halo progenitor either in the accretion or merger case. Therefore, our procedure is more suitable to test the evolution of the angular momentum, for example according to the random walk capture scenario (Vivitska et al. 2002). It should be emphasized that unlike the present work, in that model no distinction between accretion and merger is made and essentially the effects of a cumulative capture of satellites by the progenitor is considered.

When considered individually, halos show erratic variations of the angular momentum strongly correlated (positively or negatively) with mass variations (Figs. 3). This correlation reinforces the fact that even after the first shell-crossing the angular momentum varies, and the reason is mainly the transfer of orbital angular momentum to the halo spin due to accretion and merger. Previously, a different evolution for individual halos in the simulations was found: in general the angular momentum decays after the first shell crossing (Sugerman, Summers & Kamionkowski 2000). It was then understood that such a trend is probably due to the redistribution of the angular momentum to bound particles outside the overdensity cut-off generated in the identification of the objects included in their catalog. Thus, unless all bound particles of a halo are identified, the apparent decay of the angular momentum at late times can arise as a numerical artifact.

The angular momentum distribution calculated at different redshifts indicates that the median and the mean value increase with time. Two regimes again are identified in both samples, with a transition at $z \sim 1.5 - 1.8$, consistent with the behavior observed in the mass growth. The two regimes of growth are again well represented by power laws (eq. 11) and are quite different from the linear variation expected from TTT. The present mass weighted distribution of the angular momentum (Fig. 5) and the distribution of the spin parameter (Fig. 7) indicate that halos which have undergone merger episodes gain more angular momentum, confirming some previous investigations (Gardner 2001).

The distinction between the two cases (accretion and merger) allows us to establish an important characteristic of the temporal behavior of the spin parameter. The median, the mean and $\lambda_{0}$ increase with time for halos which have undergone major merger episodes, while a decrease is obtained for the accretion catalog. We have shown that such a behavior can be understood taking into account the different scaling laws followed by the dynamical variables defining the spin parameter. When no distinction is made, i.e., all halos are considered together, the statistical parameters of the $\lambda$ distribution are practically independent of the redshift, apart from the mean which shows a slight increase with time. This result is consistent with studies in which no separation concerning the halo growth history is made (see, for instance, Lemson & Kauffmann 1999).

Concerning the possible dependence of the spin parameter on the halo mass (Barnes & Efstatiou 1987), we found that for $z < 3.5$ the angular momentum and total energy scale with mass approximately as $M^{5/3}$, i.e., the expected theoretical exponent. This results is valid both in $\Lambda$CDM and in CDM cosmologies. In this case, the spin parameter should be practically independent of the halo mass and of its previous growth history, since the same result was found for both catalogs.

We have also developed a simple model for disk formation based on considerations of the specific angular momentum distribution. The basic assumptions are the equality between the specific angular momentum of dark matter and baryons, and rotational equilibrium after the collapse. However, the distribution of the dark matter mass with the specific angular momentum was calculated here in a cylindrical symmetry, centered along the spin axis. Two possible progenitors of our own galactic halo were chosen based on their present mass. The first example concerns a halo evolved under accretion only, which is unable to acquire enough angular momentum to explain the observed value of the galactic disk. In the second example, the halo was taken from the merger catalog where it has captured a few satellites in the redshift interval $2 > z > 1$ and the required angular momentum is reached at $z \sim 1.6$. The disk is built up gradually, the inner 5 kpc is formed around $z \leq 5$, and the outskirts attain dimensions of about 30 kpc and 60% of the total mass at $z \sim 1.6$, corresponding to an age of about 10.4 Gyr. Only 17% of the total mass...
was accreted in the last 5 Gyr. This evolutionary path is probably not unique, since other halos with different histories can also form disks with similar characteristics. However, the general picture agrees with the work by Helmi, White & Springel (2003), who also consider that the galactic halo was formed gradually, with more than 60% of the mass already present at around 11 Gyr ago. The resulting projected mass density is exponential within the first 10 kpc, having a scale of length equal to 3.4 kpc (Fig.12). The spin parameter of the disk is \( \lambda_b \sim 0.49 \), in agreement with the relation \( \lambda_{\text{dm}}/\lambda_b \approx f_b \), expected if the specific angular momenta of baryons and dark matter are equal.

Finally, to conclude, it is important to notice that most disk galaxies have masses in the range \( 10^{10} - 5 \times 10^{11} M_\odot \). Considering that the angular momentum of baryonic and dark matter scales as \( M^{7/3} \), this translates into the fact that the halos associated to galaxies in that mass interval should be in the range \( 3 \times 10^{66} \) up to \( 2 \times 10^{69} \) kg m\(^2\) s\(^{-1}\). If disks have ages comparable to our own galactic disk, i.e., have been formed around \( z \sim 1.6 \), our simulations indicate that only 22% of the associated halos have acquired, at that redshift, the required angular momentum. If one neglects the eventual loss of spirals that have merged to form ellipticals, the estimated fraction is about a factor of three less than the present observed value. This is perhaps the "true" angular momentum problem!

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