Modeling of the physical selenocentric surface using modern satellite observations and harmonic analysis methods

A O Andreev¹, N Y Demina¹², Y A Nefedyev¹², S A Demin², A A Zagidullin¹
¹Kazan Federal University, Engelhardt Astronomical observatory, Zelenodolski region, st. Observatory, 422526 Russia
²Kazan Federal University, Institute of Physics, Kazan, 420008 Russia

E-mail: alexey-andreev93@mail.ru

Abstract. On the basis of satellite observations taken during “Apollo”, “Clementine”, “Kaguya”, “LRO”, “GRAIL”, and “SMART-1” space missions a model of lunar physical surface of 18th order of expansion into a series of harmonic coefficients was constructed. In order to expand the data on relief into spherical functions a step-by-step regression was applied. All the constructed models included only significant elements. The regression analysis of the models of lunar relief for various expansion orders was carried out; as a result, it was determined that increase in order of expansion did not play a prominent role from a certain stage. The influence of model’s overdetermined structure on values of the parameters determined was also investigated.

1. Introduction

The modern experimental studies of external and internal characteristics of the Moon are related to the plans of its exploration in the near future. During the last two decades, the Moon has been the object of comprehensive investigation, as evidenced a number of space experiments such as “Lunar Laser Ranging” (LLR) (1969–2012), “Lunar Prospector” (1998–1999) [1], and “Clementine” [2] space missions. A few space programs aimed at thorough study of the Earth’s natural satellite were launched at the beginning of 21st century. It is necessary to note such space projects as “SMART–1” (European satellite, 2003–2006), “Kaguya” (Japanese satellite, 2007–2009) [3], “Chang–1” (Chinese spacecraft, 2007–2009), “Chang–2” (Chinese spacecraft, 2010) [4], “Chandrayaan–1” (Indian satellite, 2008–2009), “Chandrayaan–2” (Indian satellite, 2013) [1], “LRO–LCROSS” (U.S. satellites, 2009–2012) [1], “GRAIL” (The Gravity Recovery and Interior Laboratory, 2011, program on study gravitational field and internal structure of the Moon as well as modelling its heat history) [5]. The program included construction of selenographic mapping system, study of the Moon’s internal structure, investigation of the physical libration of the Moon subtle effects in rotational motion, use of intersatellite tracking in order to analyze lunar gravitational field.

The use of physics of complex systems methods and regression analysis allow studying and simulating multi-parametric celestial objects, which is of particular importance in terms of solution of planetary astrophysics problems. A number of modern space experiments are aimed at studying the Moon [6–10]. This fully applies to the Russian missions “Luna–25, –26, –27, –28” (2017–2020+) which imply obtaining a large amount of information on internal structure, inertial coordinate system, gravitational field, and dynamical figure of the Moon. It should be noted that the modern digital models of the lunar physical surface do not have the precise coordinate and time support. This occurs due to the fact that during the processing of topographic satellite observations of selenocentric data the reference surface relies on a satellite orbit’s coordinate system rather than the dynamic selenocentric coordinate system [11]. Nevertheless, based on the amount of satellite observational data obtained during space studies, expanding of astrophysical parameters in a series of spherical functions, and multi-parametric harmonic analysis it is possible to construct the dynamic selenocentric digital model of the lunar physical surface. In case of the Earth, the solution may be derived using the large amount of both space and ground-based observations. However, for other celestial bodies such capabilities are significantly limited, since there are no observations taken directly from their surfaces. The greatest amount of space information is collected for the Moon. And there is a great advantage in the presence
of observations taken on the Earth, which allows space selenographic observations binding to the inertial coordinate system.

2. The problem of the Moon macrofigure model construction
Currently, the entire data on lunar topography falls into two categories. On the one hand, the data obtained by the lunar sphere laser scanning from satellite describes the lunar relief well, but does not provide the accurate coordinates of the reference objects on the Moon. The other type of data provides the accurate coordinates of the reference objects by their direct binding to the stars, but does not describe the lunar relief with the sufficient accuracy. Besides, all these systems have different reference systems and orientation of coordinate axes. On the other hand, the data obtained during the space missions refer to quasi-dynamic coordinate system in which the reference center is the lunar center of mass, but the coordinate axes do not coincide with the Moon’s axes of inertia. The majority of modern selenodetic catalogues also refer to quasi-dynamic coordinate system, since neither their reference centers coincide with the Moon’s center of mass, nor the coordinate axes coincide with the lunar axes of inertia. Currently, there is no dynamic selenocentric coordinate system obtained on the basis of space observations covering sufficient part of the lunar surface. Besides, despite the high accuracy of determining the lunar physical relief by space missions, the reference surface of this relief presents a vague figure. Thus, it cannot be said that satellite topographic maps are models of full value with their certain reference surface of topographic data. To achieve the objectives of the present work the method of harmonic analysis based on expanding topographic data in spherical functions was used.

3. Selenocentric macrofigure construction method
In order to investigate reliability of topographic data of the reference surface one may:
1) Directly compare macroreliefs of certain lunar topographic maps;
2) compare coordinates of the Moon’s center of mass position in relation to the lunar center of figure obtained from various sources of topographic data.

When conducting these investigations, it is implied that all the sources of hypsometric data involved in calculation share equally accurate. In case of selenographic catalogues, there is high accuracy of the presented coordinates of reference objects on the lunar surface and there is low accuracy at relief description, but in case of satellite altimetry, there is sufficient accuracy at physical relief description and low accuracy of observational data binding to the selenocentric coordinate system. As for the problem of determining the position of the lunar center of mass in relation to the lunar center of figure, this is of great importance for both studying the Moon’s origin, its structure, evolution, and precise solving the near-Moon navigational problems. This work was performed in [11].

In the present paper, the data from “Apollo”, “LRO”, “Kaguya”, “Clementine”, “SMART–1”, and “GRAIL” space missions was used. The lunar megarelief model was constructed by expanding variations of the lunar surface points vectors in series of spherical functions. When expanding selenographic observational data in spherical functions the step-by-step regression method was used. The software developed for this purpose included only basic harmonic elements.

In order to achieve effective expansion order the model of the lunar physical surface was analyzed using the regression method. It was found that from the certain stage further increase in order did not play a significant role in the expansion. The problem of determining the most practical order of the harmonic expansion was also considered, and it was established that the 18th order of expansion fully satisfied the assigned task. As a model describing relief behavior the expansion of altitude function \( R(l, \phi) \) in spherical harmonics as regression model was used:

\[
R(l, \phi) = F(\tilde{C}_{nm}, \tilde{S}_{nm}, \tilde{P}_{nm}) + \varepsilon, \\
\]

where
\( l, \phi \) – latitude, longitude – known coordinates of lunar objects;
\( \tilde{C}_{nm}, \tilde{S}_{nm} \) – normalized harmonic amplitudes;
\( \bar{P}_{nm} \) – normalized associated Legendre functions;
\( \varepsilon \) – random regression error.

Solution of the over determined system (1) for various sources of hypsometric information was obtained within regression modelling approach which stipulated use of a number of quality statistics including external measures, meeting basic conditions of Ordinary least square diagnostics, adaptation in case of their violation in addition to usual stages (model (1) postulation and amplitudes \( \bar{c}_{nm}, \bar{s}_{nm} \) estimation). As computational schemes of Ordinary least square the Gauss-Jordan and Householder schemes were used. Direct use of model (1) for certain areas of the sphere (hemisphere etc.) might be complicated due to multicollinearity of the expansion coefficients; this is why in the present work the method of estimating model (1) amplitudes by preliminary expansion of a segment up to the full sphere which allows complete eliminating the effect of multicollinearity was used. Then, noise harmonics were removed by step-by-step regression.

4. Analysis of the Moon macrofigure construction results
The modern methods of constructing the Moon macrofigure models were analyzed. As a result, it was concluded that despite the high accuracy of presenting the physical relief obtained from space missions data, construction of lunar macrofigure models remained an unsolved problem due to the difficulties with determining coordinate grid’s dimension and inaccuracies of the lunar physical surface coordinates counting. The main method used for the study of the lunar macrorelief is numerical and analytical one, which involves expansion of selenographic data in harmonic series of spherical functions and application of regression analysis, described above. The model of lunar macrofigure was constructed on the basis of observations taken during “Apollo”, “LRO”, “Kaguya”, “Clementine”, “SMART–1”, and “GRAIL” space missions. The new data on global macrorelief of the lunar surface was obtained by multi-parametric harmonic analysis of radius-vectors (absolute heights) of 282 215 selenocentric points (Global Lunar Model – “GLM”). As a result, on the basis of this data a model of 18th order of expansion in spherical functions was developed. The section of global model at selenographic longitude of \( \lambda = 20^0 \) is shown in figure 1 (line 1). For comparison, the sections for the macrofigure obtained by missions “Clementine” (line 2) and “Kaguya” (line 3) at selenographic longitude of \( \lambda = 20^0 \) are presented as well.

![Figure 1. Comparison of lunar relief sections based on “GLM” (line 1), “Clementine” (line 2), and “Kaguya” (line 3) at selenographic longitude \( \lambda = 20^0 \). X-axis corresponds to height (km), Y-axis corresponds to latitude (degrees).](image-url)
The results of analysis of section at longitude $\lambda = 20^\circ$ (figure 1):

1) The mean level of heights in the southern hemisphere is higher than in the northern one (for about 0.5–1.0 km);
2) correlation coefficients for hypsometric curves “GLM”–“Clementine”, “GLM”–“Kaguya”, are 0.65 and –0.81 respectively;
3) standard deviation $\sigma$ is determined by formula

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2},$$

where $n$ is the number of measurements of 2 curves deviations, for “GLM”–“Clementine” deviation is 0.9 km, for “GLM”–“Kaguya” deviation is 1.7 km.

In figure 2 the 3D–model of the near side of the Moon of the mean radius of $1\,736.34 \pm 1.20$ km with the asperity of 0.9–1.5 km in the central part is presented. North of the parallel of $10^\circ$ radius-vectors are being gradually reduced and at latitudinal zone of $30^\circ - 70^\circ$ become, on average, 0.9 km less than lunar mean radius.

![Figure 2. Selenocentric model of the near side of the Moon.](image)

5. **Summary and conclusions**

In this paper, the model of the Moon’s physical surface was analyzed by regression method in order to determine the effective order of expansion. It was established that increase in the order of expansion from the certain stage did not play a significant role. Thus, the aim of this work was to find the most practical order of harmonic expansion of the lunar physical surface. It was found that the 18th order of expansion fully satisfied the assigned task. The optimal structure for the constructed model was analyzed as well as the impact of the obtained structure’s over determination on the parameters calculated. As a result, the model of the lunar physical surface for the 18th order of expansion was developed.
The main conclusions are as follows:

1) Analysis of dynamic and geometrical figures of the Moon showed that the most important problems were construction of selenocentric dynamic reference networks, development of lunar topocentric and gravimetric models, establishment of mutual position of lunar center of mass and center of figure, setting selenographic coordinates reference system for navigational orientation, and reference surface determination for lunar surface mapping [12].

2) The modern topographic models built on space observations have uncertain reference systems, and thus, inaccurate coordinates of the objects presented in them. Without reference catalogue of lunar objects best covering the area studied, the problem of space navigational networks thickening and widening is impracticable [13].

3) On the basis of the conducted analytical data analysis one may conclude the modern multiple processing of various sets of space data is necessary, since the processing methods and approaches, according to which the reduction of global selenodetic reference network is usually performed [14], keep improving.

4) The methods and data obtained in the present study are going to allow developing lunar coordinate systems that will serve as a basis for selenographic data reduction and provide navigational support for the Moon’s exploration and other space projects [15–20].

Acknowledgments
The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University. This work was supported by the Russian Foundation for Basic Research, project nos. 16-32-60071 mol_a_dk (N.D.), 18-02-01098 a, 18-32-00895 mol_a.

References
[1] Kirk R L, Archinal B A, Gaddis L R and Rosiek M R 2007 Cartography for Lunar exploration: current status and planned missions Proceedings of the 23rd International Cartographic Conference (International Cartographic Association) pp 1–12
[2] Smith D E, Zuber M T, Neumann G A and Lemoine F G 1997 J. Geophys. Res.-Planets 102 1591
[3] Noda H, Araki H, Goossens S, et al. 2008 Geophys. Res. Lett. 35 L24203
[4] Li C L, Ren X, Liu J J, et al. 2010 Science China Earth Sciences 53 1582
[5] Sharifi M A and Seif M R 2017 Journal of the Earth and Space Physics 43 489
[6] Robinson M S, Brylows M S, Tschimmel M, et al. 2010 Space science reviews 150 81
[7] Ping J S, Huang Q, Yan J G, et al. 2009 Science in China Series G: Physics, Mechanics and Astronomy 52 1105
[8] Zuber M T, Smith D E, Lemoine F G and Neumann G A 1994 Science 266 1839
[9] Spudis P D, Bussey D B J, Baloga S M, et al. 2010 Geophys. Res. Lett. 37
[10] Araki H, Tazawa S, Noda H, et al. 2009 Science 323 897
[11] Nefedev Y A, Valeev S G, Mikeev R R, Andreev A O and Varaksina N Y 2012 Advances in Space Research 50 1564
[12] Varaksina N Y, Nefedev Y A, Churkin K O, Zabbarova R R and Demin S A 2015 Journal of Physics: Conference Series 661 012014
[13] Rizvanov N G, Nefed’ev Yu A and Kibardin M I 2007 Solar System Research 41 140
[14] Petrova N, Zagidullin A, Nefedev Y, Kosulin V and Andreev A 2017 Advances in Space Research 60 2303
[15] Andreev A O, Demina N Y, Demin S A, Nefedev Y A and Churkin K O 2016 Nonlinear Phenomena in Complex Systems 19 271
[16] Nefedev Yu A and Rizvanov N G 2002 Astronomische Nachrichten 323 135
[17] Nefedyev Y, Andreev A, Demina N, Demin S and Andreeva Z 2017 The method of moonquakes selenophysical parameters analysis *International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM* vol 17 pp 961–966

[18] Nefedjev Yu A and Rizvanov N G 2005 *Astronomy and Astrophysics* **444** 625

[19] Demina N, Nefedyev Y, Churkin K, Demin S and Andreev A 2017 Analysis of 430322 lunar occultations *International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM* vol 17 pp 885–890

[20] Nefedyev Y A, Bezmenov V M, Demin S A, Andreev A O and Demina N Y 2016 *Nonlinear Phenomena in Complex Systems* **19** 102