ULTRA-LOW FREQUENCY GRAVITATIONAL RADIATION from MASSIVE BLACK HOLE BINARIES

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ABSTRACT

For massive black hole binaries produced in galactic mergers, we examine the possibility of inspiral induced by interaction with field stars. We model the evolution of such binaries for a range of galaxy core and binary parameters, using numerical results from the literature to compute the binary’s energy and angular momentum loss rates due to stellar encounters and including the effect of back-action on the field stars. We find that only a small fraction of binary systems can merge within a Hubble time via unassisted stellar dynamics. External perturbations may, however, cause efficient inspiral. Averaging over a population of central black holes and galaxy mergers, we compute the expected background of gravitational radiation with periods $P_w \sim 1 - 10y$. Comparison with sensitivities from millisecond pulsar timing suggests that the strongest sources may be detectable with modest improvements to present experiments.

Subject headings: black holes — galaxies:interactions — pulsars:timing — radiation mechanisms:gravitational

1. Introduction

The possibility that many galaxies contain a central black hole of $10^7 M_\odot$ or more, coupled with the ideas that elliptical galaxies probably formed from mergers (c.f. Barnes & Hernquist, 1992) and that all galaxies may have formed from multiple mergers of subunits, has led naturally to consideration of the fate of two or more massive black holes in a merger remnant. Simulations suggest that the cores of victim galaxies merge via violent relaxation in a few crossing times to form the remnant core (Barnes & Hernquist); with some possible exceptions (Governato et al. 1994), black holes in the victims then end up in the core of an elliptical or a spiral bulge. The holes sink towards the center of the stellar distribution on the dynamical friction time scale
of $\sim 10^6$ years until they become bound to one another (Begelman, Blandford & Rees 1980, hereafter BBR), forming a massive black hole binary (BHB). If the binary loses enough energy and angular momentum to the field stars, which is by no means certain, it will enter a regime where gravitational radiation alone can bring about inspiral and coalescence within the Hubble time.

During BHB inspiral, the gravity waves generated sweep through a range of frequencies. In the final merger of two $10^9M_\odot$ black holes, a ‘chirp’ of gravitational radiation with periods as small as $P_{GR} \sim 10^4s$ will be emitted. It has been suggested that this radiation might be detected through the Doppler tracking of spacecraft and by using space-based interferometers (Thorne & Braginsky 1976, Thorne 1992, Fukushige et al. 1992a). Unfortunately, the binary spends only few wave periods in the final chirp, so if inspiral is rare such events will be difficult to observe. In contrast, at binary frequencies $\sim 0.01 - 1 \mu$-Hertz, the BHB persists for many orbits. As first pointed out by Detweiler (1979), the strain amplitudes of the ultra-low frequency (ULF) gravity waves generated by sufficiently massive BHBs may be detected out to cosmological distances as perturbations in the pulse arrival times of quiet pulsars (Romani & Taylor 1983). With a new generation of timeable millisecond pulsars (Kaspi, Taylor & Ryba 1994, hereafter KTR; Camillo, Foster & Wolszczan 1994) allowing remarkable improvements in sensitivity, it seems appropriate to assess the probability of massive BHB detection.

In this paper we examine the fate of BHBs in a population of merger remnants, estimating the frequency of events and using these results to outline the required pulsar timing sensitivity needed to effect a detection or constrain the merger hypothesis. First we consider inspiral as the average result of repeated interactions with the surrounding stars (Mikkola & Valtonen, 1992, hereafter MV; Hills, 1993). The binary loses energy, shrinks and moves faster (hardens), with moderate increase in eccentricity. Numerical simulation (Makino et al. 1993, hereafter MFOE) seems to confirm this evolution in general, but warns of the importance of back-action of the BHB on the surrounding core, neglected in previous analytical investigations. Since the characteristics of the host core are crucial to the success of the inspiral, in section 2 we develop a numerical integration of the evolution of binding energy and angular momentum (hence eccentricity) during a merger event, which can be applied to a range of black hole masses and core properties. This approximate trajectory includes back-action on the core via loss cone depletion (BBR) and heating. In Section 2.3, we estimate $f_{SI}$, the fraction of mergers achieving a successful inspiral due to stellar scattering, by computing our evolutionary model for a range of core parameters and BHB properties. Additional mechanisms that may drive inspiral are also mentioned in Section 2.3. In section 3 we collect estimates of the number density of massive black hole cores and galaxy merger rates to estimate the number of merging sources visible as a function of gravity wave period and strain amplitude. In section 4 we compare these rates to present and anticipated pulsar timing sensitivities and conclude by commenting on the prospects for further constraining inspiral events.
2. Simulation of Inspiral

2.1. Binary Evolution Formulae

It was suggested by Fukushige et al. (1992b) that the evolution of a BHB can be computed from the classic Chandrasekhar formula (Chandrasekhar, 1943) for dynamical friction (DF) on a single object in a homogeneous background. Applying an approximate formula to both holes, they find a large growth in the eccentricity; this was our motivation to trace energy and angular momentum loss of a BHB. We use the results of MV, who track the net effect of an ensemble of three-body interactions between an equal-mass binary and an intruder from a background of light objects. They cast their results in the form \( \frac{\dot{a}}{a^2} = \frac{R_a}{v_\infty} \pi G \rho \), where \( a \) is the binary semi-major axis, \( \rho \) the density of background stars, and \( v_\infty \) the pre-encounter intruder speed. They find the numerical factor \( R_a \) to be well fit by \( R_a = -6.5/\left[1 + 2.44(v_\infty/W)^2\right] \), independent of binary eccentricity. Here \( W \) is the r.m.s. relative speed of the binary elements, i.e. the relative speed of a circular binary of equal energy.

We assume the galactic core to have a Plummer model velocity distribution, given by

\[
f(v) = \frac{16}{21\sqrt{3\pi^2}\sigma^2} \left(1 - \frac{v^2}{12\sigma^2}\right)^{7/2}.
\]  

Since the one-dimensional dispersion \( \sigma \) depends on position in a Plummer model, we use the central value \( \sigma_0 \). Using the fitting function, we calculate (as a function of \( \sigma_0 \)) \( \langle \sigma_0 R_a/v_\infty \rangle_{\sigma_0} \), the average over Plummer distributed \( v_\infty \). This gives

\[
\dot{E} = \frac{G M_1 M_2 \pi \rho}{2\sigma_0} \langle \sigma_0 R_a/v_\infty \rangle_{\sigma_0}
\]

for the evolution of the binary energy. In Fig. 1, we compare this to the energy loss rate of a circular binary due to DF, applying the Chandrasekhar formula (c.f. Binney & Tremaine, 1987) independently to each hole. In the \( W < \sigma_0 \) limit, the MV formula agrees by construction with the analytic calculation of Gould (1991); it thus differs from the DF result in the form of the Coulomb logarithm \( \Lambda \) (see Gould for a discussion). For the DF calculations of Fig. 1, we take the maximum impact parameter \( b_{\text{max}} \) contained in \( \Lambda \) to be a function of the binary period, as discussed in the next section. In the \( W \gtrsim \sigma_0 \) regime which proves most important in our calculation, we see in the figure that DF does not approximate well the more physically grounded and asymptotically sound three-body results.

For the eccentricity evolution, MV give results in the form

\[
\frac{dX}{d\ln a} = X^{n+1/2} - X \quad \text{where } X \equiv 1 - e^2, \quad \text{and } n = \frac{8 \left(\frac{v_\infty}{W}\right)^{1.5} - 0.3X}{1 + 16 \left(\frac{v_\infty}{W}\right)^{1.5}}
\]
For each $X$ and $W$ we again average over $v_\infty$ with distribution (1), and convert (3) to a time derivative, giving

$$
\dot{X} = -\frac{G^2 M_1 M_2 \pi \rho}{2 E \sigma_0} \left\langle \frac{\sigma_0 R_a}{v_\infty} \right\rangle_{\sigma_0} \left\langle X^{n+1/2} - X \right\rangle_{X,W/\sigma_0}
$$

(4)

In Fig. 1 we also compare the amount of eccentricity growth of the binary as a function of energy for DF and MV. Again, DF does not seem to be a suitable approximation. Since MV treat only the equal-mass binary, we impose a correction factor to force agreement with Gould’s analytic mass ratio scaling for $M_1 \neq M_2$. We use $M_1 \geq M_2$ throughout.

In following the orbit evolution, we also include the energy and angular momentum losses to gravitational radiation (GR): (Peters 1964)

$$
\langle \frac{dE}{dt} \rangle = -\frac{32 G^4 M_1^2 M_2^2 (M_1 + M_2)}{5 c^5 a^3 (1 - e^2)^{7/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right),
$$

(5)

$$
\langle \frac{dL}{dt} \rangle = -\frac{32 G^7/2 M_1^2 M_2^2 (M_1 + M_2)^{1/2}}{5 c^5 a^{7/2} (1 - e^2)^2} \left( 1 + \frac{7}{8} e^2 \right).
$$

(6)

Once these terms dominate, the BHB will circularize. When the time scale for binary evolution by GR alone is less than $10^{10}$ years, we call this the GR regime.
2.2. Properties of the Core

We use a Plummer model core of stars with mean mass $m_s$, core radius $r_c$ and isotropic central velocity dispersion $\sigma_0$. The core stars have total potential energy $W$ and kinetic energy $K$, satisfying the virial relation $2K + W = 0$. With

$$W(N_s, r_c) = -\frac{3\pi G(N_s m_s)^2}{32 r_c} - \frac{G N_s m_s (M_1 + M_2)}{r_c},$$

we use $K(N_s, \sigma_0) = (9\pi/32) N_s m_s \sigma_0^2$ and the virial relation to find

$$N_s = \frac{6\sigma_0^2 r_c}{G m_s} - \frac{32 M_1 + M_2}{3\pi m_s}. \tag{8}$$

The core mass is $M_c = N_s m_s$.

Only a subset of the core stars can effectively exchange energy with the binary. According to Heggie (1975), these are the stars whose interaction time with the binary is less than its orbital time scale. This criterion defines a maximum impact parameter $b_{\text{max}}$ (a function of the binary period) which we also used in the DF calculation for Fig. 1. At any distance from the center of the core, the condition on impact parameter selects stars whose velocities are within a cone aimed towards or away from the center, the so-called loss cone (Frank & Rees 1976). For our core, the fraction of stars with impact parameter at most $b_{\text{max}}$ is found by direct integration to be

$$\frac{N_{\text{avail}}}{N_s} = \frac{b_{\text{max}}^2}{b_{\text{max}}^2 + r_c^2}. \tag{9}$$

Since scattered stars enter a solid angle at a rate proportional to the solid angle size, stars can enter this loss cone no faster than the rate $N_{\text{avail}}/t_{\text{relax}}$. Thus $t_{\text{relax}}$ is also the loss cone repopulation time,

$$t_{\text{lc}} = t_{\text{relax}} = \left( \frac{N_s}{8 \ln \left( \frac{r_c \sigma_0}{G m_s} \right)} \right) \left( \frac{r_c}{\sigma_0} \right) \simeq 3.7 \times 10^{13} \text{ yr} \left( \frac{N_s}{10^{10}} \right) \left( \frac{r_c}{200 \text{ pc}} \right) \left( \frac{300 \text{ km s}^{-1}}{\sigma_0} \right) \tag{10}$$

(Binney & Tremaine 1987).

Once encounters with the binary deplete the loss cone (BBR), the binary evolution rate will be limited by the rate at which stars enter the loss cone, and the amount of energy each star can take from the binary. In the $W > \sigma_0$ limit, MV find the change in binary energy with each stellar encounter to be $\langle \delta E \rangle \sim 2E m_s/(M_1 + M_2)$. This is in good agreement with the results at several mass ratios of Roos (1988) and Hills (1983). Writing this as $\langle \delta E \rangle = (M_2/M_1) m_s v_{2c}^2$, where the circular velocity $v_{2c}$ of the lighter binary component is roughly the closest approach velocity of a star, we pass to the $W \leq \sigma_0$ regime by using $\sigma_0$ for $v_{2c}$. The energy the loss cone can take away without repopulation is thus $E_{\text{avail}} = N_{\text{avail}} \langle \delta E \rangle$, and from (10), energy enters the loss cone.
at a rate \((dE/dt)_{in} = E_{avail}/t_{relax}\). If each star which interacts with the binary is scattered into a random orbit, a fraction \(N_{avail}/N_s\) of stars will be returned to the loss cone. For a loss cone population in steady state, then, the limiting binary evolution rate is

\[
\left( \frac{dE_{bin}}{dt} \right)_{max} = \frac{N_s}{N_s - N_{avail}} \left( \frac{dE}{dt} \right)_{in} = \frac{b_{max}^2 + r_c^2}{r_c^2} \langle \delta E \rangle \frac{N_{avail}}{t_{relax}}
\]  

(11)

This rate is generally much smaller than that due to unrestrained dynamical friction. We approximate the situation by using the average core density in (8) and (9) until \(E_{avail}\) drops below the binding energy of the binary. From this point on the binary will affect the loss cone stellar density significantly, and we impose the steady-state evolution rate. Invariably this ‘loss cone catastrophe’ results in a binary evolution time scale \(E_{bin}/\dot{E}_{bin}\) much longer than the Hubble time. Similar but less specific treatments have been presented by BBR and Roos (1981).

We also treat eviction of stars from the core, found to be significant in the N-body work of Makino et al. (1993, hereafter MFOE). We assume all stars which interact with the binary after \(v_{2c}\) is larger than the Plummer central escape velocity \(v_{esc} = \sqrt{12\sigma_0}\) receive enough recoil velocity to be kicked out of the core. We calculate the resultant change in the core energies assuming rapid revirialization, readjusting \(r_c\) and \(\sigma_0\). The binary also heats the core before becoming hard, but this effect is small.

At the beginning of the simulation, the binary’s semi-major axis is the binding radius \(r_b\), defined such that the mass of stars within \(r_b\) is less than \(M_1\). Its center of mass is at the core center, and it has some initial eccentricity \(e_i\). We assume the core is not rotating after the merger, since N-body merger simulations show very small angular momentum in the remnant bulge (Barnes 1992), even when progenitor bulges rotate (Hernquist 1993). A simulation is terminated unsuccessfully if \(E_{bin}/\dot{E}_{bin}\) exceeds \(10^{10}\) years, or successfully if the GR regime is reached. To estimate the parameter space available for successful inspiral, we find \(\log(1 - e_i^2)\) for the marginally successful trajectory to one part in a thousand by using ten bisection steps on the initial value. Some sample trajectories are shown in Figure 2. If we ignore back-action and loss cone depletion the results are identical to those of MV.

Our simulations confirm the basic behavior described by MFOE. For a range of initial conditions, interaction with background stars can cause some increase in the binary eccentricity. If we choose core parameters and black hole masses to match those of MFOE, we find loss cone depletion to occur at binary radii similar to those at which they terminate their simulation, where they too note a slowdown in binary evolution which they attribute to a possible loss cone effect.

### 2.3. Success Rate and Alternative Merger Mechanisms

For all pairs of binary member masses drawn from the set \(\log M/M_\odot = \{10, 9.5, ..., 7.0, 6.5\}\) we calculated the average initial critical eccentricity required for successful inspiral in 80 galactic
cores, drawn from a smoothed distribution based on the data set of Lauer (1985). We chose the
distribution by noting that the binned data show
$N(M/10^9 M_\odot) = \exp(-M/22)$ (presumably
the tail of a Schechter function); matching this and the
$N(r_c)$ of the data set gives our core
distribution in the $r_c - \sigma_0$ plane. In drawing from the distribution we required that the stellar
mass of the host core be at least twice the total mass of the holes.

To estimate the fraction of successful inspirals $f_{SI}$ for any given binary mass, we assume
stochastically distributed initial eccentricities ($P(e) = 2e$), so the inspiral success probability
is $P(e > e_{\text{crit}}) = 1 - e_{\text{crit}}^2$. This choice of probability may be somewhat too low since N-body
simulations of collisions of bulge-disc-halo systems (Barnes 1992) show radial final infall of the
bulges; high initial binary eccentricity seems a likely consequence. Some critical eccentricities
for BHB (log $M_1/M_\odot$, log $M_2/M_\odot$) which will later prove most relevant for pulsar detection are:
(9.5, 9) $\rightarrow$ .985, (9, 9) $\rightarrow$ .987, and (9, 8) $\rightarrow$ .990. These yield $f_{SI}$ of 2.9%, 2.6% and 2.0%
respectively. Over the whole range of binary masses, the $f_{SI}$ are $\sim 1\% - 3\%$. From the dotted
curves in Fig. 1 we see that the dynamical friction formulae yield much more eccentricity growth,
hence earlier arrival in the GR regime as per equations (1) and (3), and larger success fraction.
We ran calculations with all the above features but using DF, finding $f_{SI}$ ranging from 1% to
50%. In the case of DF, it is possible to account for rotation of the stellar core, which was found
to be an important impediment to eccentricity growth in the simulations of MFOE. By giving all
angular momentum lost by the binary to the core, and applying the DF formulae to the motion of
the binary relative to the rotating background, we find much less eccentricity growth, resulting in
If even the modest eccentricity growth in the MV picture, as seen in Figure 2, is suppressed by spinup of the core, the MV $f_{SI}$ could be driven slightly lower.

Clearly stellar action on an isolated BHB in a galaxy core is of limited efficacy. However, the absence of many double active galactic nuclei (AGN’s) and of orbital variations in the broad line region, as well as short binary periods inferred from AGN jet precession (Roos 1988) suggest that inspiral beyond the loss cone limit does occur in many cases. Other means of binary inspiral have been proposed: gas may be ejected during nuclear activity, taking with it energy and angular momentum acquired from the binary; or it may be accreted by the larger hole, causing orbital contraction as $M_1 r_2$ remains constant to conserve angular momentum (BBR).

Roos (1988) has also pointed out that if merger events happen several times during the lifetime of a galaxy and if, in particular, mergers are related to the onset of accretion and nuclear activity in AGNs, then external perturbations due to incoming galactic masses should accelerate stellar scattering inspiral. Though a binary which fails to merge is left evolving on the relaxation (and loss cone filling) time scale, Roos estimates that by the time a new merging galaxy is within a few core radii of the remnant pair, the loss cone can be refilled quickly and repeatedly, driving the original binary to GR inspiral. His calculation of the flux of stars entering the loss cone in this tidal repopulation scheme suggests that the galactic orbit decays no faster than that of the binary. If the incoming galaxy also has a black hole, it seems likely to form a binary with the post-coalescence hole.

Accordingly, in the remainder of this paper we follow two hypotheses: that inspiral occurs with the low probabilities allowed before loss cone depletion by MV’s stellar encounter dissipation, and that repopulation ensures that inspiral always occurs. The latter picture assumes a galaxy formation model involving repeated mergers; these two cases certainly bracket the actual situation.

3. Merger Rates and Populations of Nuclear Black Holes

We wish to compute the rate of mergers observed from some redshift $z$. In an Einstein-de Sitter universe ($\Lambda = 0$, $\Omega = 1$) with the metric $ds^2 = dt^2 - a(t)^2 [d\chi^2 + \chi^2 d\Omega^2]$, let $F_m(z)$ be the number of merger events at redshift $z$ in the history of today’s bright ($L_\star$) galaxies, per dimensionless comoving volume per unit redshift. With this definition, $\int F_m(z) dz = N n_{gal} a_0^3$, where $n_{gal}$ is the current number density of such galaxies and $a_0$ the current scale factor; $N$ can range from $\sim 1/3$, if only single mergers occur to form elliptical galaxies, to $\sim 10$ in scenarios where all galaxies are formed from repeated merging of building blocks. At redshift $z$ we observe a dimensionless co-moving area $4\pi \chi^2(z)$, with $\chi(z) = \frac{2c}{a_0 H_0} \left( 1 - \frac{1}{\sqrt{1+z}} \right)$. During a period $T_z$ at redshift $z$, the comoving volume observed is $V_\chi = 4\pi \chi^2(c/a) T_z$. Thus the number of mergers per
unit observers’ time is

\[ \nu(z) \text{d}z = \frac{V_{\text{c}}F_m(z)\text{d}z}{T_z(a_0/a)} = 16\pi \left( \frac{c}{H_0} \right)^2 \frac{c}{a_0^3} \left( 1 - \frac{1}{\sqrt{1+z}} \right)^2 F_m(z) \text{d}z. \]  

(12)

Note that \( a_0 \) vanishes in any physical rate because of the normalization of \( F_m(z) \).

The form of \( F_m(z) \) is still quite uncertain. Burkey et al. (1994) find that the population of close galaxy pairs which seem certain to merge varies as \( \sim (1+z)^{3.5 \pm 5} \), in the interval \( 0 \leq z \leq 0.6 \). Colin et al. (1994) find that a galaxy density \( \sim (1+z)^{3.8} \) best fits the total galaxy count data in a model which accounts for photometric evolution. The 3.5 power law gives \( F_m(z) \sim (1+z)^2.5 \).

We normalize \( F_m(z) \) to the merger rate implied by the pair counts in Burkey et al. Applying this rate in the interval \( 0 \leq z \leq 1 \) implies 40 galaxies will have suffered a merging event; alternatively if this rate continues back to the epoch of high quasar activity, \( z \approx 3 \), then each bright galaxy will have experienced roughly 5 merger events. We find:

\[ \nu(z) \text{d}z = 7.6 \times 10^{-2} \text{yr}^{-1} h_{50}^{-2} \left( \frac{n_{\text{gal}}}{10^{-3} \text{Mpc}^{-3}} \right) (1+z)^{2.5} \left( 1 - \frac{1}{\sqrt{1+z}} \right)^2 \text{d}z, \]  

(13)

where \( H_0 = 50 h_{50} \text{km} \text{s}^{-1} / \text{Mpc} \).

As an alternative less dominated by early merging, we take merging rate per comoving volume constant in time, so that \( F_m(z) \sim (1+z)^{-5/2} \). If we assume that merging began at some \( z_m \) and continues to the present, resulting in \( N \) mergers per bright galaxy of number density \( n_{\text{gal}} \), we find

\[ \nu(z) \text{d}z = 0.55 \text{yr}^{-1} \left( \frac{N}{h_{50}^2} \right) \left( \frac{n_{\text{gal}}}{10^{-3} \text{Mpc}^{-3}} \right) \left( \frac{3}{2} \left( 1 + z_m \right)^{3/2} \right) \left( \frac{[1 - (1+z)^{-1/2}]^2}{(1+z)^{5/2}} \right) \text{d}z. \]  

(14)

If accretion onto massive central black holes is the source of AGN luminosity, the population of remnant holes can be estimated from models of AGN evolution (Cavaliere and Padovani, 1988; Small and Blandford, 1992). Recent HST detections of kinematic evidence for massive compact objects in nearby galaxy cores support this scenario. We adopt here Small and Blandford’s more conservative model IA, with a flat local Seyfert luminosity function. This gives the black hole number density spectrum

\[ N_R(M_{\text{BH}}) \text{d}M_{\text{BH}} = \begin{cases} 
7.0 \times 10^{-7} M_{\text{BH}}^{-1} \text{d}M_{\text{BH}} & \text{Mpc}^{-3} \quad \log M_{\text{BH}} < -1.6 \\
1.2 \times 10^{-5} M_{\text{BH}}^{-1.4} \text{d}M_{\text{BH}} & \text{Mpc}^{-3} \quad -1.6 < \log M_{\text{BH}} < 0.45 \\
1.1 \times 10^{-4} M_{\text{BH}}^{-3.5} \text{d}M_{\text{BH}} & \text{Mpc}^{-3} \quad 0.45 < \log M_{\text{BH}} 
\end{cases} \]  

(15)

Comparing the above to a Schecter luminosity function of bright galaxies

\[ N(L) \text{d}L = 2.5 \times 10^{-3} h_{50}^3 (L/L_*)^{-1.1} e^{-(L/L_*)} \text{d}(L/L_*) \text{Mpc}^{-3} \]  

(de Lapparent et al. 1989), we found that if all massive black holes are in bright galaxies, the integral of equation (15) implies that 21% of \( L > L_* \) galaxies contain a black hole with \( M > 10^{6.5} M_\odot \) today. Conservatively, we hold the co-moving number density of holes fixed: if there are \( N_m \) subunits at redshift \( z \) that will
become a bright galaxy, then the number of BHs per subunit is $0.21/N_m$. We note that the high inspiral rates ($\sim 1/3 - 10\text{yr}^{-1}$) estimated by Fukushige, et al. (1992a) are based on excessively optimistic assumption of an $M > 10^8M_\odot$ black hole in each of $\sim 10$ galaxy subcomponents forming an elliptical. In their picture there are 10 gravity wave chirps emitted for each bright elliptical seen today. As we shall see, such event rates are not consistent with bounds from pulsar timing.

4. Gravitational Wave Amplitude of BHB Inspiral

For a circular binary of orbital period $P_b$, reduced mass $\mu$ and total mass $M$, Thorne (1987) gives a characteristic strain amplitude averaged over direction and polarization of

$$h_c = 8(2/15)^{1/2}\mu(2\pi M/P_b)^{2/3}/r$$

for the emitted gravitational wave from a circular ($e = 0$) binary. In units convenient to the BHB problem this is

$$h_c = 1.8 \times 10^{-15}M_1M_2(M_1 + M_2)^{-1/3}r_{Gpc}^{-1}[g^{1/2}(e, n)/n]P_w^{-2/3}$$

where masses are in units of $10^9M_\odot$, distances are in kiloparsecs, $g(e, n)$ gives the fraction of the wave power in the $n^{th}$ harmonic as a combination of Bessel functions (Peters & Mathews 1963) and the observed gravitational wave period in years is $P_w = P_b(1 + z)/n$. Only the $n = 2$ harmonic is non-zero for a circular orbit.

Since the gravity wave flux scales as $h^2\omega^{-2}$, falling off with the luminosity distance as $d_L^{-2}$, one can write the gravity wave amplitude from a source at $z$ emitting waves that have period $P_w$ at redshift 0 in an Einstein-deSitter universe:

$$h_{-15}(z) = 0.29M_1M_2(M_1 + M_2)^{-1/3}h_{50}[g^{1/2}(e, n)/n]P_w^{-2/3}(1 + z)^{2/3}/[1 - (1 + z)^{-1/2}]$$

in dimensionless units of $10^{-15}$. The characteristic lifetime for the source at this period is

$$\tau_{GR} = 1.3 \times 10^4M_1^{-1}M_2^{-1}(M_1 + M_2)^{1/3}f(e)^{-1}[n P_w/2(1 + z)]^{8/3}y$$

where $f(e) = (1 + \frac{73}{21}e^2 + \frac{37}{90}e^4)/(1 - e^2)^{7/2}$, as in Eq. (9).

We wish to estimate, for a set of assumptions about the population, the expected number of BHBs detectable at a given strain sensitivity $h_{-15}$. Inverting equation (18) gives $z(h; M_1, M_2, P_w, e)$. We can combine this with the merger frequency rate, two independent draws from the black hole mass function (15), the successful inspiral fraction for these hole masses (averaged over cores that might contain such holes), and the lifetime of the resulting binary at the
indicated orbital period to get the number of visible gravity wave sources in a given amplitude range:

$$\frac{dN(h, M_1, M_2, P_w, e)}{dM_1 dM_2} = \nu(z) d\nu \frac{N_R(M_1) N_R(M_2)}{N_L^2} (f_{SI}(M_1, M_2)) \tau_{GR}(M_1, M_2, P_w, e, z) \frac{dh}{dz}. \tag{20}$$

Integration of this equation over $M_1$, $M_2$ and a range of $h$ gives the source population estimates shown in Figure 3a for waves of period $P_w = 10^7$. In practice, we find that even for inspiral driven by stellar encounters, the characteristic eccentricity of the binary decreases to $\lesssim 0.3$ by the time it reaches the range observable with pulsar timing, so restriction to circular orbits is reasonable for estimating fluxes. Depending on assumptions, we see that gravity waves should be detectable once a strain sensitivity of $\sim 10^{-15}$ is reached. The properties of the detected BH binaries can also be estimated by weighted sums over this population; for the case of $f_{SI} = 1$, and mergers proceeding as in equation (13), average characteristics of the detected binaries are shown in Figure 3b. It is also worth noting that with these parameters, we expect $\sim 1$ chirp per $10^4$ yr with an amplitude greater than $10^{-15}$ at periods $P_w \sim 10^4$ s, substantially lower than the rates quoted by Fukushige et al. (1992a).

Fig. 3.— Left – Source intensity distributions for several merger models. The filled dots give the distribution for tidally repopulated merging and open circles show numbers for evolution slowed by loss cone depletion, with the merger rate (13). Crosses show the population from mergers constant in $t$. Right – weighted observed source properties for $f_{SI} = 1$ mergers following rate (13). The binary mass ratio is $q$, and the GR inspiral time is $\tau$.

5. Conclusions and Observational Prospects
Millisecond pulsar timing has been used to place strong bounds on the energy density in a stochastic background of ultra-low frequency gravitational waves (KTR and references therein). For example, these studies have placed an energy density limit $\Omega_g < 2 \times 10^{-7} h_{50}^{-2}$ and have helped to rule out various exotic cosmologies, such as structure formation seeded by cosmic string loops. With a number of high-quality pulsars now being timed, it is worth considering whether astrophysical sources, such as BHBs, are within reach. In Figure 4 we show a simulated gravity wave spectrum computed from the amplitude number distributions (as in Figure 3a) for a range of wave periods. Two models are shown, the first with inspiral mediated by tidal repopulation ($f_{SI} = 1$), the second for evolution limited by loss cone depletion. Amplitude distributions were computed at each frequency; then the power was integrated over the low $h$ sources and Monte Carlo sampled from the rare bright sources. The corresponding amplitude is given in Figure 4, which thus shows the expected BHB ultra-low frequency gravity wave background.

Fig. 4.— Simulated ULF gravity wave backgrounds from $f_{SI} = 1$ mergers and loss cone limited stellar encounter (MV) mergers. Bright nearby sources rise above the background. Approximate sensitivities from anticipated pulsar timing experiments are shown as dashed lines (see text).

If one makes $N_{\text{obs}} \sim 20$ arrival time measurements per year over a period $T_{\text{obs}} \sim 10\text{y}$ with an accuracy of $\delta t \sim 1 \delta_{-6}\mu\text{s}$, then one can place a limit on the strain amplitude of a passing gravity
wave with period $P_w$ of roughly

$$h \sim \frac{\delta t}{P_w} (N_{\text{obs}} T_{\text{obs}})^{-1/2} \sim 2 \times 10^{-15} \delta_{-6}(N_{20} T_{10})^{-1/2} P_y^{-1}. \quad (21)$$

For this estimate to hold we must have $T_{\text{obs}} \gtrsim P_w$, so that fitting pulsar parameters does not significantly absorb any gravity wave signal (Blandford et al. 1984). This also assumes that the pulsar timing residuals are distributed as white noise. According to this estimate, intensive long-term timing programs can reach the sensitivity needed to detect the brightest BHB gravity wave sources at periods near 10y.

Present timing results (KTR) show that PSR1937+21 which has been monitored for over eight years shows significant unmodeled timing noise, presumably due to rotational variations intrinsic to the pulsar. PSR1855+09, on the other hand, shows random variations at its $\sim 0.8\mu$s arrival time accuracy with over seven years of timing; these variations are consistent with perturbations arising from instability of the best terrestrial clocks. PSR1855+09 residuals are constraining gravity wave sources at an amplitude of $h \approx 5 \times 10^{-15}$ (Fig. 4). Thus with the best present data, we estimate a chance $\sim 0.001$ of detecting a merging BHB. However, with a factor of $\sim 5$ increase in sensitivity (and a slightly increased experiment duration) we can anticipate detecting the brightest BHB gravity wave sources, for reasonable population assumptions. Whether this sensitivity increase can be effected is uncertain. In the last few years, timing programs have been initiated on several new pulsars (e.g. PSR J1713+0747, PSR J0437–4715) that provide sub-$\mu$s timing residuals. However, to take advantage of this precision it will be necessary to obtain improved atomic clock standards or time one pulsar against another. Finally, to reach interesting detection sensitivities we require the arrival time residuals to integrate down as white noise over the $\sim 10y$ periods. Clearly, intrinsic instabilities in PSR1937+21 prevent this; fortunately timing noise appears to correlate with period derivative and several of the new pulsars have period derivatives 10 - 20 times smaller than that of PSR 1937+21. It should be noted that the required single order of magnitude improvement represents a much better prospect than most other gravitational radiation search techniques!

In summary, we have developed a model of the inspiral of two massive black holes in a galaxy core driven by stellar encounters. We recover some of the behavior described by earlier workers, though the large eccentricity needed to circumvent the loss cone catastrophe in our calculations must be present \textit{ab initio}. Our simplified sum allows computation of this process in a range of cluster cores, and we see that simple stellar encounter dissipation is only effective in 1%-3% of all BHB-producing merger events. Nonetheless, it seems likely that other processes will ensure that inspiral occurs in the majority of cases. Turning to the rate of galaxy mergers, and the fraction of merging cores containing high mass black holes, we estimate the event frequency and typical gravity wave amplitude expected from a cosmological population of merging cores with central BHBs. Simulating the gravity wave spectrum produced by this population shows that, while no detection of ULF GR sources is expected to date, moderate improvements in present sensitivities will make detection of waves from binary sources with periods of $\sim 10yr$ possible \textit{via} timing observations of millisecond pulsars. Detection of correlated gravity wave signals in the arrival
times of several pulsars would, of course, constitute an exciting confirmation of this astrophysical class of gravity wave sources. However, even upper limits a factor $\sim 5$ lower than present bounds can constrain the population and merging behavior of massive BHBs throughout the universe.

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