'Hidden' symmetry of linearized gravity in de Sitter space

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We demonstrate that the linearized Einstein gravity in de Sitter (dS) spacetime besides the evident symmetries also possesses the additional symmetry \(h_{\mu\nu} \rightarrow h_{\mu\nu} + \varepsilon_{\mu\nu}\chi\), where \(\varepsilon_{\mu\nu}\) is a spin-two projector tensor and \(\chi\) is an arbitrary constant. We argue that this hitherto 'hidden' property of the existing physics is indeed at the origin of the long-standing puzzle of ‘dS breaking’ in linearized quantum gravity.

**Introduction.** Recent three decades have witnessed a proliferation of conflicting approaches to dS quantum gravity, because of the long-standing puzzle of dS symmetry breaking for quantum field theory of gravity in dS spacetime (see \(^1\) and references therein). Together with the quantization of the dS minimally coupled scalar field, they set up a doublet of cases where dS quantum field theory would lead to such difficulties. The case of the minimally coupled scalar field was analyzed by Allen and Folacci \(^2\) and their results seemed definitive: dS invariance was broken and infrared divergences were present. The case of the graviton field, however, is more complicated among other things due to the presence of the local invariance, contrary to the scalar case, and it is still a source of contention in the literature: while the mathematical physics community maintain that there is no physical breaking of dS invariance, the particle physics community argue that gravitons inherit the dS breaking long recognized for the minimally coupled scalar field (see \(^3\) and references therein). It is therefore pertinent to develop theoretical benchmarks that will allow us to select the physically relevant approaches, and to subsequently use observational constraints in order to single out the few candidates that are actually the viable ones.

From the very beginning of this scientific dispute, a firm reasoning in favor of covariant quantization of the graviton field in the natural dS vacuum state (the Bunch-Davies vacuum) was put forward in Ref. \(^4\), and during recent three decades, it has been dynamically subject to scrutiny in a number of works (see, for instance, \(^5\)–\(^17\)). Let us make the idea lying behind of this reasoning explicit, using the so-called conformal (global) coordinates,

\[ x = (x^0 = H^{-1} \tan \rho, (H \cos \rho)^{-1} u), \rho \in ]\pi/2, \frac{\pi}{2}[, u \in S^3. \]

Basically, in Refs. \(^5\)–\(^17\), the physical graviton modes are respected as the transverse-traceless second-rank symmetric tensor spherical harmonics on the three-spheres, symbolized here by \(h_{\mu\nu}^{(Lim)}(\rho, u)\), with \(h_{00} = 0, L = 2, 3, ..., 0 \leq l \leq L\) and \(-1 \leq m \leq l\). It is indeed argued that each irreducible component of the representation of the dS group \(SO_0(1,4)\) given by linearized gravity (i.e., \(\Pi^+_{1,2} \oplus \Pi^-_{1,2}\) in the Dixmier’s notation\(^18\)) is formed by this set of solutions \(^4\). Note that, because the normalization factor breaks down at \(L = 0\) corresponding to the graviton zero-frequency mode, this mode is not considered in the set of solutions based upon which the Fock space and the vacuum state are constructed. It is shown that the value \(L = 1\) is not allowed either. This argument seems to entail making sense of the Bunch-Davies vacuum as the unique possibility for a dS-invariant dynamical gravitons state, and that implies somehow avoiding the appearance of the infrared divergences and the associated symmetry breaking in the theory.

In this Letter, we show that, although the above reasoning provides an appealing picture of dS quantum gravity, it tends to break down, whose root is a hitherto ‘hidden’ property of the existing physics. Indeed, we demonstrate that the linearized Einstein gravity in dS spacetime besides the evident symmetries also possesses an additional ‘hidden’ symmetry reminiscent of the shift symmetry of the dS minimally coupled scalar field. This ‘hidden’ symmetry of the classical theory reveals that it is not possible to construct a dS-invariant state for the graviton field without proper quantization of its zero-frequency mode violating the picture of dS quantum gravity in the Bunch-Davies vacuum. In fact, this mode has positive norm, but under the dS group action produces all the negative frequency solutions (with respect to the conformal time) of the graviton field equation.

**Covariant formulation.** We start our discussion by reminding the standard procedure to study linear perturbations of Einstein gravity around the dS metric. We denote the metric perturbation by a symmetric tensor \(h_{\mu\nu}\) propagating on a dS fixed background \(\tilde{g}_{\mu\nu}\), \(g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}\). This reparametrization invariance is translated at the linear level as the following gauge invariance

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\nabla_{(\mu} \xi_{\nu)}, \tag{1} \]
where $\zeta_\mu$ is an arbitrary vector field and $\nabla_\mu$ is the dS covariant derivative. In this framework, the linearized Einstein equation of motion in dS spacetime takes the form

$$\Box H + 2H^2)h_{\mu\nu} - (\Box H - H^2)\tilde{g}_{\mu\nu}h' - 2\nabla_\mu\nabla^\rho h_{\nu\rho} + \tilde{g}_{\mu\nu}\nabla^\rho h_{\lambda\rho} + \nabla_\mu\nabla_\nu h' = 0,$$

(2)

where $H$ is the Hubble constant, $\Box H = \tilde{g}_{\mu\nu}\Box^{\mu\nu}$ is the Laplace-Beltrami operator, and $h' = \tilde{g}^{\mu\nu}h_{\mu\nu}$ is the trace of $h_{\mu\nu}$ (all tensor indices are raised and lowered by the background metric). We will partially fix the gauge by using the Lorenz gauge condition, defined by $\tilde{g}^{\mu\nu}h_{\mu\nu} = e\nabla_\mu h'$, with $e = 1/2$.

**Isometric embedding.** For the sake of argument, it is useful to consider a more convenient set of coordinates, defined by the isometric embedding of the 3 + 1-dimensional dS space in the 4 + 1-dimensional Minkowski space ($\mathbb{M}_5$) as the ambient space,

$$M_H = \{ x \in \mathbb{M}_5; x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2}, \alpha, \beta = 0, 1, 2, 3, 4, \eta_{0\alpha} = \text{diag}(1, -1, -1, -1, -1),$$

with the ambient coordinates notations $x = (x^0, x^1, x^2, x^3, x^4)$. The dS metric would be the induced metric on the dS hyperboloid

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta|x^2 = -H^{-2} = \tilde{g}_{\mu\nu}dx^\mu dx^\nu,$$

where $\mu, \nu = 0, 1, 2, 3$ and $X^\mu$'s are the four local spacetime coordinates for the dS hyperboloid.

Utilizing the ambient space formalism is justified for two reasons. First, this embedding is a purely geometrical construction and hence has no effect on the dynamics, which always takes place in 3 + 1-dimensions. As a matter of fact, in the ambient space notations, a tensor field $K$ is a homogeneous function of the $\mathbb{M}_5$-variables $x^\alpha$, so that its degree of homogeneity is fixed $x_\alpha(\partial/\partial x_\alpha)K \equiv x \cdot \partial K = \tilde{g}_\alpha K$, with $\tilde{g} = 0$ and $\partial = 0$ to have correspondence between $\Box H \equiv \nabla_\alpha \nabla^\alpha$ on dS space and $\Box_\alpha = \Box^\alpha$ on $\mathbb{M}_5$. Moreover, to ensure that $K$ lies in the dS tangent space, it is constrained to obey the transversality condition $x \cdot K = 0$. Respecting the significance of the transversality property for dS fields, the transverse projection is defined by $(TK)_{\alpha_1...\alpha_q} = \theta^{\alpha_1}_\alpha...\theta^{\alpha_q}_\beta K_{\beta_1...\beta_q}$, with $\theta^{\alpha}_\beta = \eta^{\alpha\beta} + H^2x_\alpha x_\beta$, which guarantees the transversality in each index. $\theta^{\alpha}_\beta$ is indeed the only tensor which corresponds to the metric $\tilde{g}_{\mu\nu} = x^\mu_\alpha x^\nu_\beta \delta^{\alpha\beta}$, with $x^\mu_\alpha = \delta x^\mu/\delta x^\alpha$. Pursuing the same path, any intrinsic tensor field like the graviton field can be locally specified by its transverse counterpart in the ambient space notations, $h_{\mu\nu}(X) = x^\mu_\alpha x^\nu_\beta K_{\alpha\beta}(x(X))$. The covariant derivatives acting on a symmetric, second-rank tensor are also transformed according to $\nabla_\alpha \nabla_\beta h_{\gamma\rho} = \delta x^\mu_\alpha x^\nu_\beta T\partial_\beta T\partial_\beta K_{\gamma\rho}$, in which $\partial = \partial$ stands for transverse derivative in dS space.

Second, this way of describing the dS space constitutes the coordinate-independent approach, such that there is a close resemblance with the corresponding description on the Minkowski space and the link group theory is easily readable. More technically, in the ambient notations, the ten infinitesimal generators $L^{(r)}_{a\beta}$ of the dS group are simply defined by $L^{(r)}_{a\beta} = M_{a\beta} + S^{(r)}_{a\beta}$, where the orbital part is $M_{a\beta} = -i(x_\beta \partial_\beta - x_\beta \partial_\alpha)$ and the spinorial part $S^{(r)}_{a\beta}$ acts on the indices of rank-$r$ tensors as

$$S^{(r)}_{a\beta}K_{\alpha_1...\alpha_r} = -i \sum_{a=1}^r (\eta_{0a} K_{\alpha_1...\alpha_a} - \eta_{a\alpha} K_{\alpha_{a+1}...\alpha_r}) - (a = \beta).$$

The second-order Casimir operator of the dS group $Q_{0} = -\frac{1}{2}L^{(r)}_{a\beta}L^{(r)}_{\beta\alpha}$ is fixed on the carrier space of each dS unitary irreducible representation (UIR), and therefore, it can be used to classify the dS UIR's,

$$Q_{0} = (Q_{0}) \equiv \{ -p(p + 1) - (q + 1)(q - 2) \},$$

(3)

with $2p \in \mathbb{N}$ and $q \in \mathbb{C}$ (in the Dixmier's notation $\mathbb{18}$). The irreducible representations associated with our study are characterized by the dS discrete series representations $\Pi_{0}^{(r)}$, in which label 'q' has a spin meaning and '±' stands for the two types of helicity. For $q = q_{0}$, these representations correspond to the conformal massless cases. More precisely, the term 'massless' is used here in reference to the conformal invariance (propagating on the dS light cone). See a more detailed discussion in $\mathbb{20}$ $\mathbb{22}$.

In this sense, the dS massless spin-$2$ field is described by the UIR's $\Pi_{2}^{(2)}$, with $Q_{0} = \{Q_{2}\} = -6\tilde{g}$, and lies among the solutions of the dS-invariant equation $(Q_{2} + 6)K = 0$, supplemented with the divergencelessness condition $\partial \cdot K = 0$.

Taking all of the above into consideration, the ambient space counterpart of the linearized Einstein equations of motion $\mathbb{2}$ would be $\mathbb{22} \mathbb{24}$

$$(Q_{2} + 6)K + D_{2}\partial_2 \cdot K = 0,$$

(4)

which can be derived from the following action

$$S = \int d\sigma L_{i} = -\frac{1}{2x^{2}}K \cdot (Q_{2} + 6)K + \frac{1}{2}(\partial_2 \cdot K)^{2},$$

(5)

where $d\sigma$ is the volume element in dS space, $D_{2} \equiv H^{-2}S(\partial - H^{-2}x)$ in which $S$ is the symmetrizer operator $S_{\alpha\beta\omega\delta} = \theta_{\alpha\omega} \theta_{\beta\delta} + \theta_{\beta\omega} \theta_{\alpha\delta}$, $\partial_{2} \cdot K = \partial \cdot K - H^{2}x K' - \frac{1}{2}\partial K'$ in which $K'$ is the trace of $K_{\alpha\beta}$, and '·' is a shortened notation for total contraction. The Lagrangian density $\{3\}$ is invariant under the gauge transformation $K \rightarrow K + D_{2}\lambda$, where $\lambda$ is a vector field. This gauge transformation is exactly the ambient space counterpart of $\{1\}$. The Lorenz gauge condition in ambient space notations takes the form $\partial_{2} \cdot K = (b - 1/2)\partial K'$, with $b = 1/2$. Hence, the gauge fixing can be accomplished by adding the gauge-fixing term $\mathbb{L}_{g} = (1/2a)(\partial_{2} \cdot K)^{2}$ to $\{5\}$, where 'a' is an arbitrary constant. Finally, the field equation derived from $\mathbb{L} = \mathbb{L}_{i} + \mathbb{L}_{g}$ is

$$(Q_{2} + 6)K + cD_{2}\partial_2 \cdot K = 0,$$

(6)

where $c = (1 + a)/a$ is a gauge-fixing parameter.
Obviously, if one sets \( c = 1 \), the equation of motion becomes fully gauge invariant. For \( c \neq 1 \), the \( U(1) \)’s \( \Pi^2_{2,2} \), and hence the corresponding fields, are part of an indecomposable structure issued from the existence of gauge solutions. In the latter, the graviton tensor field \( K \) is traceless and, respectively, the gauge fields \( D_2 \lambda \) and the divergences \( \partial_2 \cdot K \) (the scalar states) have to obey \[ (1 - c)D_2(Q_1 + 6)\lambda = 0, \quad (1 - c)(Q_1 + 6)\partial_2 \cdot K = 0. \]

This means that both of them carry the same representation. More accurately, possessing the divergencelessness \((\partial \cdot \lambda = 0)\) and transversality \((x \cdot \lambda = 0)\) conditions, we have \((Q_1 + 6)\lambda = (Q_0 + 4)\lambda = 0\), which means that the space of gauge solutions is associated with the scalar case \( \Pi_{2,0} \). The same argument appears for \( \partial_2 \cdot K \). [The scalar representations \( \Pi_{0,0} \) in the discrete series are characterized by \( Q_0 = -(p - 1)(p + 2)\mathbb{I} \), with \( p = 1, 2, \ldots \) \[ 25 \].]

In summary, the indecomposable group representation structure of the full space of solutions to \((11)\) can be pictured as

\[
\begin{array}{ccc}
\Pi_{2,0} & \rightarrow & \Pi^+_{2,2} \oplus \Pi^-_{2,2} & \rightarrow & \Pi_{2,0}
\end{array}
\]

\[ \text{scalar states} \quad \text{physical states} \quad \text{gauge states} \]

The ‘zero-mode’ problem. In Ref. \[ 25 \], it has been shown that the general solution \( K_{\alpha\beta} \) to the field equation \((8)\) can be written as the resulting action of a second-order differential operator \((\text{the spin-two projector})\) on a minimally coupled scalar field \((\text{the structure function})\), \( K_{\alpha\beta} \equiv \mathcal{E}_{\alpha\beta} \phi \), so that

\[ K = K^{c=\frac{5}{2}} + \frac{2}{e - 1} D_2(Q_1 + 6)^{-1} \partial_2 \cdot K^{c=\frac{5}{2}}, \quad c \neq 1 \tag{8} \]

with

\[ K^{c=\frac{5}{2}} = \left( -2 \theta \partial \cdot \mathcal{Z} + sZ \mathcal{K} + \frac{1}{27} H^2 D_2 D_1 Z \cdot K \right) + \frac{1}{3} H^2 D_2 x \cdot Z K \left. \right| - \frac{1}{15} D_2 \partial_2 \cdot K^{c=\frac{5}{2}}, \tag{9} \]

and

\[ K = \left( \mathcal{Z}' - \frac{1}{2} D_1 (Z' \cdot \partial + 2 H^2 x \cdot Z') \right) \phi, \tag{10} \]

\[ Q_0 \phi = -H^{-2} \partial_1 \phi = 0, \tag{11} \]

in which \( Z \) and \( Z' \) are two constant five-vectors \((\mathcal{Z} \equiv TZ)\). Note: \((i)\) The gauge solutions are coupled to the term \( H^2 D_2 D_1 Z \cdot K \); on one hand, \( H^2 D_2 D_1 Z \cdot K \) is completely determined by its scalar content and does not carry any spin, \( L^0_{\alpha\beta} (H^2 D_2 D_1 Z \cdot K) = H^2 D_2 D_1 M_{\alpha\beta} Z \cdot K \); on the other hand, it satisfies \( D_2 (Q_1 + 6) H^2 D_1 Z \cdot K = 6 H^2 D_2 D_1 Z \cdot K \). \((ii)\) The second term in \[ 5 \] is responsible for the appearance of logarithmic divergences in the theory; therefore we set \( c = 2/5 \) to remove this term and to get the simplest indecomposable structure.

The minimally coupled scalar field, corresponding to the lowest case in the scalar discrete series representation \( \Pi_{1,0} \), can be identified by the so-called coordinate-independent dS plane waves as follows

\[ \phi(x) = (H x \cdot \xi)^\kappa, \quad \kappa = -p - 2 = -3, \tag{12} \]

where this 5-vector \( \xi \) lies on the null cone in \( M_5 \), \( \xi = (\xi^0, \xi) \in C = \{ \xi \in M_5; \xi^2 = 0 \} \). The solution \((12)\) is defined on connected open subsets of \( M_H \) such that \( x \cdot \xi \neq 0 \) (see \[ 26 \] for details).

Substituting \( \xi = ||\xi|| v \in M_4, v \in S^3 \), and \( ||\xi^0|| = ||\xi|| \), we obtain

\[ H x \cdot (\tan \rho) \xi^0 - \frac{1}{2 \cos \rho} u \cdot \xi = \frac{\xi^0 e^{i \varphi} (1 + z^2 - 2zt), \tag{13} \]

with \( z = i e^{-i\rho} \text{sgn} \xi^0 \) and \( t = u \cdot v \equiv \cos \varphi \). Then, considering the generating function for Gegenbauer polynomials, \((1 + z^2 - 2zt)^{-\lambda} = \sum_{n=0}^{\infty} z^n C_{\alpha}^\lambda (t), \) with \(|z| < 1\), we have the following expansion \((\lambda = -\kappa)\)

\[ (H x \cdot \xi)^\kappa = \left( \frac{\xi^0 e^{i \varphi}}{2i \cos \rho} \right) \sum_{n=0}^{\infty} z^n C_{\alpha}^\lambda (t), \quad \mathfrak{R} \kappa < \frac{1}{2}. \tag{13} \]

This formula is not valid in terms of functions since \(|z| = 1\). Nonetheless, the convergence is ensured if we give a negative imaginary part to the angle \( \rho \). Consequently, we extend ambient coordinates to the forward tube \([27, T^+ = \{ M_5 = i \mathcal{V}_5^+ \cap \bar{M}_H \}, \mathcal{V}_5^+ = \{ x \in M_5: x^2 \geq 0, x^0 > 0 \}. \]

Now, the dS plane waves \([13] \), using two expansion formulas involving Gegenbauer polynomials and normalized hyperspherical harmonics on \( S^3 \) and after putting \( \kappa = -3 \), can be written as \([28, 14] \)

\[ (H x \cdot \xi)^{-3} = 2 \pi^2 \sum_{L_m} \Phi_{Lm}^{-3}(\xi^0)^{-3}(\text{sgn} \xi^0)^L Y_{Lm}^* (v), \tag{14} \]

with

\[ \Phi_{Lm}^{-3}(x) = \frac{i^{L+3} e^{-i (L+3) \varphi}}{(2 \cos \rho)^{-3}} P^L_{\lambda} (-e^{-2i \varphi}) Y_{Lm} (u), \tag{15} \]

in which \( Y_{Lm} \) stands for the \( S^3 \) hyperspherical harmonics, with \((L, l, m) \in \mathbb{N} \times \mathbb{N} \times \mathbb{Z}, 0 \leq l \leq L \) and \(-l \leq m \leq l \), and

\[ P^L_{\lambda} (z^2) = \frac{\Gamma(L + 3)}{(L + 1) \Gamma(3)} 2 F_1 (L + 3, 2; L + 2; z^2). \]

The hyperspherical harmonics are linearly independent, therefore the functions \( \Phi_{Lm}^{-3} \) would be solutions to \([11] \) when adopting the appropriate separation of variables.

\[ ^1 \text{Note that, the case } \kappa = p - 1 = 0 \text{ is referred to as the trivial solution for the minimally coupled scalar field.} \]
With respect to the orthonormality of $Y_{Llm}$'s, then we have
\[
\Phi^{-3}_{Llm}(x) = \frac{(\text{sgn}(0))L}{2\pi^2(c^0)^{-3}} \int_{S^3} d\sigma(v)(Hx \cdot \xi)^{-3}Y_{Llm}(v). \tag{16}
\]

Note that, $\Phi^{-3}_{Llm}(x)$ are well defined, and are infinitely differentiable in the conformal coordinates $\rho, u \geq 0$. The above formulas present the 'spherical' modes in dS spacetime in terms of the dS plane waves, and allow us to bring the general solution $\Phi$ into the intrinsic form,
\[
h_{\mu\nu} \equiv \mathcal{E}_{\mu\nu}\Phi^{-3}_{Llm}(\rho, u) = (x^a_\mu x^b_\nu \mathcal{E}_{\alpha\beta})\Phi^{-3}_{Llm}(\rho, u). \tag{17}
\]

According to our choice of the global coordinate system, the dS-invariant inner product on the space of solutions (17) is defined by \(29\)
\[
\langle h_1, h_2 \rangle = \frac{1}{\rho^4} \int_{S^3, \rho = 0} \left((h_1)^{*} \cdot \partial_{\rho} h_2 \right. \left. + \frac{2}{\rho^2}((\partial_{\rho} x) \cdot (h_1)^{*}) \cdot (\partial_{\rho} h_2) - (1^* = 2)\right) d\sigma(u), \tag{18}
\]
where $h_1$ and $h_2$ are two arbitrary modes. Let us take a closer look at the behavior of this inner product. With respect to the following identity for hypergeometric functions \(30\)
\[
2F_1(a, b; c; z) = (1 - z)^{(c-a-b)}2F_1(c - a, c - b; c; z),
\]
one can find the alternative form of (15) as
\[
\Phi^{-3}_{Llm}(x) = i^{L+3}e^{-i\rho} \frac{\Gamma(L+3)}{(L+1)!\Gamma(3)} \
 \times 2F_1(-1, L; L + 2; -e^{-2i\rho})Y_{Llm}(u). \tag{19}
\]

Now, considering the behavior of the hypergeometric functions \(30\),
\[
2F_1(-1, L; L + 2; -e^{-2i\rho}) = 1 - \frac{L}{L + 2}e^{-2i\rho},
\]
it is clear that we face a degeneracy in the set of modes for $L \geq 0$; the mode associated with $L = 0$, i.e. $h^{(0,0,0)}_{\mu\nu}$, is orthogonal to the entire set of modes including itself. Obviously, because of this degeneracy, canonical quantization applied to the set of modes with $L \geq 0$ results in a non-covariant field (the zero-mode problem). This problem raises a natural question, which is decisive in evaluating different approaches to the long-standing puzzle of dS breaking in linearized quantum gravity: do we need to respect the space spanned by the $h^{(L_{\mu\nu}lm)}_{\mu\nu}$ with $L \geq 0$ as the complete set of modes or this is just a pure mathematical problem and we can drop the model $L = 0$ and the degeneracy associated with it, just like the procedure has been done in Refs. \(4\)\(,\)\(7\)?

Thanks to the mathematical structure presented thus far based on the ambient space formalism, one can easily observe that the theory, besides the spacetime symmetries generated by the Killing vectors and the gauge transformation \(1\), is also invariant under a 'hidden' gauge-like symmetry, i.e.
\[
h_{\mu\nu} \to h_{\mu\nu} + \mathcal{E}_{\mu\nu}\chi, \tag{20}
\]
is a solution to the field equation for any constant function $\chi$ as far as $h_{\mu\nu}$ is. This proves that the invariant null-norm subspace, generated by $L = 0$, can be interpreted as a space of 'gauge' states in the set of solutions. As a result, discarding the value $L = 0$ from the theory is equal to disregarding a part of the solutions to the field equation which leaves us with a non-complete set of solutions violating dS invariance of the theory. This argument explicitly casts serious doubts on the viability of the appealing picture of dS quantum gravity in the Bunch-Davies vacuum when it is evaluated in the transverse-traceless gauge, with $h_{\mu\nu} = 0$ and $L \geq 2$ \(3\)\(,\)\(17\), and shows that this canonical quantization scheme yields non-covariant results.

To restore full dS invariance of the theory, therefore, we need to delve more deeply into the $L = 0$ solutions. In this regard, by solving the field equation directly for $L = 0$, more accurately \(11\) (see \(31\)), we obtain two independent solutions as follows
\[
h_{\mu\nu}^{(0,0,0)} = \mathcal{E}_{\mu\nu} \frac{H}{2\pi}, \quad h_{\mu\nu}^{(0,0,0)} = \mathcal{E}_{\mu\nu} \left(-\frac{H}{2\pi}\rho + \frac{1}{2} \sin 2\rho\right),
\]

Both modes however have null norm, and the constant factors are selected to get $\langle h_{\mu\nu}^{(0,0,0)}, h_{\mu\nu}^{(0,0,0)} \rangle = 1$. Therefore, to circumvent the degeneracy problem, we define the 'true' normalized zero mode of the system as following combination of $h_{\mu\nu}^{(0,0,0)}$ and $h_{\mu\nu}^{(0,0,0)}$,
\[
h_{\mu\nu}^{(0,0,0)} = \mathcal{E}_{\mu\nu}\Phi^{-3}_{Llm}(\rho, u) = \mathcal{E}_{\mu\nu} \left(-\frac{H}{2\pi}\rho + \frac{1}{2} \sin 2\rho\right), \tag{21}
\]

with $\langle h_{\mu\nu}^{(0,0,0)}, h_{\mu\nu}^{(0,0,0)} \rangle = 1$. Taking into account this new mode \(21\) interestingly gives a complete set of strictly positive-norm modes $h_{\mu\nu}^{(L_{\mu\nu}lm)}$ for $L \geq 0$. The space spanned by these modes, however, is not dS invariant. For instance, we have
\[
(L_{03} + iL_{04})h_{\mu\nu}^{(0,0,0)} = \left(L_{03} + iL_{04}\right)\mathcal{E}_{\mu\nu}\Phi^{-3}_{0,0,0} \tag{22}
\]
+ $\mathcal{E}_{\mu\nu} \left((M_{03} + iM_{04})\Phi^{-3}_{0,0,0}\right)$. The first term is trivially invariant under the dS-group action. The dS invariance, however, is broken because of the second term. In fact, a direct computation gives\(3\)
\[
\Phi^{-3}_{0,0,0} = \frac{\sqrt{6}}{4} \left(h_{\mu\nu}^{(1,1,0)} + ih_{\mu\nu}^{(1,0,0)} + (1 + i)(h_{\mu\nu}^{(1,0,0)})^*\right).
\]

Note that, the gauge solutions $2\mathcal{E}_{(\mu\nu)}$, as already pointed out, are represented by the scalar discrete representations $\Pi_{2,0}$, while the gauge-like solutions $\mathcal{E}_{\mu\nu}\chi$ are obviously associated with the indecomposable representations \(7\).

See Ref. \(31\) for the action of $(M_{03} + iM_{04})$ on the scalar structure function $\Phi^{-3}_{0,0,0}$.
This result explicitly reveals that the graviton quantum field constructed through canonical quantization and the usual representation of the canonical commutation relations from the complete set of modes $h_{\nu L}(Llm)_{\mu}$ for $L \geq 0$ are not covariant.

To summarize, respecting the Bunch-Davies vacuum as the natural dS vacuum state, the above arguments demonstrate beyond doubt that dS breaking is universal and cannot be gauged away.

**Conclusion and outlook.** We have shown that in the case of linearized gravity in dS space, there exists an anomalous symmetry (the local symmetry of the classical theory, see (20), is absent at the quantum level), and therefore, one has to deal with the propagation of unphysical modes in the theory. In this sense, there is a deep analogy between this case and the quantization of the dS minimally coupled scalar field evaluated by Allen and Folacci in their seminal work [2], so that, for both cases, dS invariance is broken and infrared divergences are present. A possible way out would be to consider a quantum theory with a smaller symmetry group (spontaneous symmetry breaking). However, as dS symmetries are basic symmetries of field dynamics in dS space, it might also be worth developing our quantization scheme to a more general context which respects the dS symmetries. Motivated by this, our result highlights the need for structures that can be considered as a unified framework to treat gauge and gauge-like quantum field theories in dS spacetime.

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