On weak interactions as short-distance manifestations of gravity

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Abstract

We conjecture that weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi constant is related to the Newtonian constant of gravitation. In this framework one may understand the violations of fundamental symmetries by the weak interactions, in particular parity violations, as due to fluctuations of the spacetime geometry at a Planck scale coinciding with the Fermi scale. As a consequence, gravitational phenomena should play a more important role in the microworld, and experimental settings are suggested to test this hypothesis.

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Progress in Physics is sometime stimulated by critically revising already collected facts under a novel perspective, and the emergence of previously unnoticed connections. As a chief example, the realization that static measurements of the vacuum permittivity $\varepsilon_0$ and permeability $\mu_0$ constants are related to the speed of light $c$ - determined in a setting perceived as completely different - as $c = (\mu_0\varepsilon_0)^{-1/2}$, stimulated the development of a unified electromagnetic theory.

In this spirit, in this communication we discuss a somewhat similar situation with an attempt to relate weak and gravitational interactions, which naturally share various features. First, they are universal, acting on all fundamental particles discovered so far, with the only exception of the gluon which does not seem to interact weakly, although it would be extremely difficult to experimentally evidence possible $gW^\pm$ and $ggZ^0$ vertices. Second, they have peculiar behavior with respect to external symmetries: the gravitational interaction, in the Einsteinian view, is the manifestation of curved spacetime itself, and weak interactions manifest parity violations, i.e. their structure is also quite sensitive to the spacetime. Third, they have small interaction couplings, to be meant in a qualitative way that they can both be neglected when dealing with extremely short timescales. Finally, their coupling constants, the Newton gravitational constant $G_N$ and the Fermi constant $G_F$, have physical dimensions. These features are quite distinctive from the ones characteristic of electromagnetic and color interactions, which are instead selectively discriminating among particles based on the presence of electric and color charges, do preserve all external symmetries, and have dimensionless coupling constants large enough to determine the short-time dynamics of nearly all their bound states.

Therefore, a sensible question to ask is whether a joint analysis of weak and gravitational interactions is possible. Previous attempts have focused on embedding weak interactions in a broader structure of spacetime, such as the Einstein-Cartan theory in which contact interactions mediated by spin and torsion emerge naturally [1-4], or in introducing graviweak interactions allowing the gravitational interaction to break external symmetries [5]. Such a geometrical approach to weak interactions has been recently reconsidered in the framework of the so-called $f(R)$ gravity [6, 7], since the latter allows for a running coupling constant potentially capable to match the coupling constant present in the contact interaction of the Einstein-Cartan theory with the phenomenologically determined Fermi constant of charged weak interactions.
More specifically, we aim to provide further arguments in favor of a geometrization program of the weak interactions based upon simple dimensional and numerical arguments. The Fermi constant is

\[ G_F = 1.166364 \times 10^{-5} \text{GeV}^{-2} (\hbar c)^3 = 1.43583 \times 10^{-62} \text{ m}^5 \text{ kg s}^{-2}, \]  

(1)

where the last expression has been obtained by using MKSA units, and the dimensions of \( G_F \) are \([G_F] = L^5 M T^{-2}\). The Newton universal constant of gravitation is instead

\[ G_N = 6.67384 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \]  

(2)

with dimensions \([G_N] = L^3 M^{-1} T^{-2}\). The ratio between the two constants has therefore dimensions \([G_F/G_N] = L^2 M^2\), which is also the dimension of the square of the ratio of the two fundamental constants \( \hbar \) and \( c \). From the dimensional viewpoint, we can therefore write a relationship \( G_F = \xi (\hbar/c)^2 G_N \), where \( \xi \) is a dimensionless constant to be determined. By substituting the experimentally determined values of \( G_F, G_N, \hbar, \) and \( c \) we obtain \( \xi = 1.73867 \times 10^{33} \). We notice that \( G_N \) is measured in a range larger than the cm scale, while \( G_F \) is measured through decay or scattering processes occurring in the subnuclear world, i.e. at distances smaller than \( 10^{-15} \text{ m} \). It is therefore highly tempting to conjecture that the prefactor \( \xi \) would actually become of order unity if \( G_N \) were also measured at subnuclear distances. In other words, by reabsorbing \( \xi \) into the Newton’s constant, we may write instead (the presence of the \( \sqrt{2} \) factor will become clear in Eq. (4)):

\[ G_F = \frac{1}{\sqrt{2}} \left( \frac{\hbar}{c} \right)^2 \tilde{G}_N, \]  

(3)

provided that \( \tilde{G}_N = 2.45885 \times 10^{33} G_N = 1.641 \times 10^{23} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \) is a renormalized Newton’s constant as measurable at subnuclear distances. Equation (3) suggests an interpretation of weak interactions as corresponding to a specific class of short-distance gravitational phenomena occurring due to quantum relativistic fluctuations of spacetime, and disappearing in the classical, non relativistic limit (\( \hbar \to 0 \) and \( c^{-1} \to 0 \)). While on macroscopic lengthscales the effect of metric fluctuations is negligible, at small distances the foamy structure of spacetime [8–11] allows for large fluctuations of the geometry. This could possibly include also states with opposite parity and phenomena such as a local non-conservation of the electric charge, which then allows charge transport between two locations of spacetime, corresponding to the propagation of intermediate charged vector bosons \( W^\pm \) in the standard model.
This interpretation is also consistent with the fact that the Planck length, usually defined as \( \Lambda_P = \sqrt{\hbar G_N/c^3} = 1.6162 \times 10^{-35} \) m, is accordingly boosted by a factor \((\sqrt{2}\xi)^{1/2} = 4.95868 \times 10^{16}\) when replacing \(G_N\) with \(\tilde{G}_N\), leading to \(\tilde{\Lambda}_P = 8.0142 \times 10^{-19}\) m. The corresponding Planck energy is \(\tilde{E}_P = \hbar c/\tilde{\Lambda}_P = 3.9449 \times 10^{-8}\) J = 246.221 GeV. Therefore the Planck energy coincides with the vacuum expectation value \(v\) of the Higgs field in the electroweak model, as seen by inverting Eq. (3), thereby avoiding the hierarchy problem

\[
\tilde{E}_P = \left(\frac{\hbar c^5}{\tilde{G}_N}\right)^{1/2} = \left(\frac{(\hbar c)^3}{\sqrt{2}G_F}\right)^{1/2} = v.
\]

If all above has anything to do with reality, the Newtonian constant of gravitation should be running, reaching at the attometer scale a value about \(2.46 \times 10^{33}\) times larger than its value measured at the macroscopic level, i.e. 33 orders of magnitude over 13 orders of magnitude in lengthscale. We have no precedent of such a steep distance-dependence for other coupling constants, however our current phenomenological knowledge of gravity cannot rule out this possibility without further experimental scrutiny, since the upper bounds on gravitational or gravitational-like forces are rather loose. It is worth to point out that, with different motivations, the concept of strong gravity has appeared from time to time in the literature, especially in connection with the possibility that gravity plays a role in the confinement of quarks inside hadrons through black-hole analogies, although not within the framework of considering weak interactions as derivable from gravity at short lengthscale.

We envisage here at least three experimental scenarios in which this “Newton-Fermi conjecture,” centered around Eq. (3), could give rise to novel observable effects. Specifically:

(a) **High-precision measurements of the Newton gravitational constant in the micrometer to nanometer range**

Unless there is an abrupt dependence of \(G_N\) on distance in the form of a resonant phenomenon, we expect that its increase with decreasing distance will be distributed over many distance decades. This means that evidences for \(G_N\) as a running coupling constant could be achieved by comparing high-precision measurements at different distances, for instance in the cm, \(\mu m\), and nm range. This provides further motivations to ongoing efforts to measure gravitationally-related forces in the submillimeter range, especially as a byproduct of Casimir force studies, combining precision measurements and accurate theoretical predic-
(b) **Gravitationally bound states of hadrons and leptons in the subnanometer to femtometer range**

Bound states may be sensitive to an anomalous gravitational coupling. The Rydberg constant is known from hydrogen spectroscopy with a precision of 5 parts per trillion, so this leads to bounds at the scale of $10^{-10}$ m on $G_N(10^{-10}m) < 6 \times 10^{27} G_N < 2.4 \times 10^{-6} \tilde{G}_N$. With such a bound on $G_N$ in the nanometer range it seems very difficult to observe gravitationally bound dineutron states, for instance the Bohr radius should be of the order of 10 $\mu$m, with a ground state binding energy of 2.6 nK. By using reflecting surfaces as in [22], we expect that neutrons of energy large enough to approach the surface within hundreds of femtometers should feel the gravitational attraction of the atoms on the surface, resulting in a different energy spectrum with respect to the one already measured with ultracold neutrons [23]. Other suitable states are provided by exotic atoms. The spectroscopy of antiproton-nucleus atoms [24] was used to give an upper bound $G_N(10^{-13}m) < 1.3 \times 10^{28} G_N$ [25]. Muonic hydrogen should be also sensitive because of the small Bohr radius of $2.5 \times 10^{-13}$ m. The recently observed anomalous Lamb shift, so far interpreted as a measurement of the proton radius in disagreement with the CODATA value [26], could instead be due to different gravitational binding energy in the $2s_{1/2}$ and $2p_{1/2}$ states, requiring evaluation of the related contribution beyond perturbation theory.

(c) **Gravitationally bound states of heavy quarks**

Gravitation should contribute to the binding energy of the nuclei more substantially if coupled through $\tilde{G}_N$. It is however easy to check that for the value of $\tilde{G}_N$ assumed above this contribution is still negligible even with respect to the electrostatic correction, and below the accuracy achieved so far in the comparison between theoretical semiempirical mass formulae and binding energies as determined via mass spectrometry. We instead expect that bound states of heavy quarks will get a significant contribution from anomalous gravitational energy. An estimate for the gravitational contribution to the binding energy of heavy quarkonia of mass $m_q$, proportional to $\tilde{G}_N m_q^2$, is naturally obtained through its comparison to the corresponding electrostatic contribution, proportional to $e^2/4\pi\epsilon_0$ via fractional charges $2/3$ and $1/3$ for top and bottom quarks, respectively. The situation seems favourable.
in the case of $t\bar{t}$ pairs, since the larger mass leads to gravitational binding energy larger than the electromagnetic contribution by a factor of about 130 even taking into account the larger value of $\alpha_{\text{em}}$ due to its running at the top mass scale. However, in this case the intrinsically short lifetime of the top quark may prevent an easy signature, also considering the limits in energy resolution intrinsic in the initial states using hadron colliders such as Tevatron and LHC. Nevertheless, it is intriguing to investigate if recent anomalies in the $t\bar{t}$ production at the Tevatron [27, 28] could find an explanation in this setting. Next to this, $b\bar{b}$ bound states should have instead an anomalous gravitational contribution about 3 times smaller than the electromagnetic one. In principle, high resolution spectroscopy of $b\bar{b}$ bound states could identify such a further contribution to the binding energy using future electron-positron collider machines optimized for bottom quarks such as SuperB and SuperKEKB, although extraction of this anomalous term requires an accurate mastering of the dominant binding energy due to color interactions. From this perspective, the best scenario is an electron-positron storage ring producing bound states of $\mu^+\mu^-$ [29] or $\tau^+\tau^-$ [30, 31], allowing one to study their spectroscopy at high precision.

It may be worth to iterate that, as clearly emphasized in the conclusions of [7], unification schemes based on attempts to geometrize weak interactions differ conceptually from the usual one pursued in the latest decades based on gauging internal symmetries. In the latter case the general idea is to incorporate the standard model into broader algebraic structures which, apart from the elegant, predictive, falsifiable example of SU(5) GUT, in general have more free parameters and make predictions predominantly at higher, still unexplored, energies. In this geometrization program instead, the idea is to strive for an economic and falsifiable description of natural phenomena - without introducing further degrees of freedom or free parameters - in which the Fermi constant loses its fundamental character and is considered as an effective coupling constant emerging from the short-distance geometry of quantum vacuum. We believe that such a geometrical approach to unification is closer in spirit to the successful unification pursued by Maxwell for electric and magnetic phenomena, and allows for tests in the whole range of energies explored so far - as discussed above - without providing ample margins for arbitrarily tuning free parameters to accommodate possible unsuccessful predictions at the currently explored energy scale.

In conclusion, we conjecture that weak interactions should be considered as empirical evidences of quantum gravity at the Fermi scale. The Fermi constant plays the role of an
effective gravitational constant taking into account the presence of quantum fluctuations of the spacetime foam \[8-11\]. The existence of violations of external symmetries such as parity, and internal symmetries such as flavor, or local nonconservation of the electric charge as witnessed by the presence of charged intermediate vector bosons like \(W^{\pm}\), may be related in this framework to topology changes induced by quantum fluctuations through tunnelling among superselected spaces. This could also provide a geometrical interpretation of the universality of weak interactions à la Cabibbo, i.e., via CKM and PMNS unitary matrices for quarks and leptons, respectively. While at this stage we prefer to maintain an agnostic attitude towards the origin of the conjectured running of the Newtonian constant, we note that - besides extra-dimensions and moduli predicted by string theory - recently developed \(f(R)\)-gravity \[6\] and Higgsless models \[32\] may provide natural settings for this effect. A gravitational running coupling constant may be a concept less difficult to accept with respect to the existence of extra-dimensions since, unlike the latter for which we have collected null experimental evidence so far, we are certain about the reality of running coupling constants for all other fundamental interactions. Further implications and remaining challenging questions will be analysed in the future, in particular the role of neutral weak currents, the possible relationship between the spin-1 bosons \(W^{\pm}\) and the spin-2 graviton, and more in general the consistent incorporation within this unified scenario of all the experimental facts successfully corroborating the electroweak model.

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