Measuring accretion-disk effects with gravitational waves from extreme mass ratio inspirals

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Gravitational wave observations of extreme mass-ratio inspirals (EMRIs) offer the opportunity to probe the environments of active galactic nuclei (AGN) through the torques that accretion disks induce on the binary. Within a Bayesian framework, we study how well such environmental effects can be measured using gravitational wave observations from the Laser Interferometer Space Antenna (LISA). We focus on the torque induced by planetary-type migration on quasi-circular inspirals, and use different prescriptions for geometrically-thin and radiatively-efficient disks. We find that LISA could detect migration for a wide range of disk viscosities and accretion rates, for both α and β disk prescriptions. For a typical EMRI with masses 50M⊙ + 106M⊙, we find that LISA could distinguish between migration in α and β disks and measure the torque amplitude with ∼ 20% relative precision. Provided an accurate torque model, we also show how to turn gravitational-wave measurements of the torque into constraints on the disk properties. Furthermore, if an electromagnetic counterpart is identified, the multimessenger observations of the AGN-EMRI system could yield direct measurements of the disk viscosity. Finally, we investigate the impact of neglecting environmental effects in the analysis of the gravitational-wave signal of our reference system, finding 3-sigma biases in the primary mass and spin. Our analysis can be easily generalized to any environmental effect, provided that the torque has a simple power law-like dependence on the orbital separation.

I. INTRODUCTION

Extreme mass-ratio inspirals (EMRIs) are a primary target of the future spaceborne gravitational-wave (GW) detector LISA [1]. A typical EMRI involves a stellar-mass compact object with mass O(10)M⊙ orbiting a massive black-hole (MBH) with mass O(109)M⊙. While emitting in the LISA band, the compact object will complete as many as hundreds of thousands of orbits around the MBH, making EMRIs ideal sources to test the nature of black holes (BHs), general relativity (GR), and the astrophysics of galactic nuclei [2,3].

Mergers involving MBHs commonly occur within gas-rich environments. Interactions with the gas can help binaries form and become more compact — including MBH binaries and EMRIs. Based solely on observations of actively accreting MBHs, it is expected that 1–10% of the EMRIs observed by LISA will reside in the accretion disk of active galactic nuclei (AGNs) [4]. Recently, Refs. 5 and 6 argued that there could be even more EMRIs taking place in AGNs than previously estimated, and that LISA could detect 10 – 104 EMRIs per year from dense accretion disks, compared to only 1 – 102 per year from relatively gas-free environments (see also Ref. [7]).

The presence of an accretion disk can considerably modify the orbital trajectory of an EMRI emitting in the LISA band. The modification comes in the form of torques, originating either from hydrodynamical effects such as “accretion winds” [8,9], or through purely gravitational torques1 such as dynamical friction [9,12], which can take the form of planetary-style migration if the density wakes produce torques through shocks [8,13,14]. Through these processes, the disk exchanges energy and angular momentum with the system.

It is expected that the parameters characterising EMRI signals will be measured with extreme precision by LISA, thanks to the large number of orbits that can be observed in the LISA band. For this reason, it is reasonable to believe that accretion-disk effects which are estimated to be detectable [12] can also be used to measure accretion-disk properties, such as the accretion rate and disk viscosity. This would open up the possibility of studying the properties of accretion disks through purely gravitational means.

In this work, we perform the first quantitative study of the measurability of accretion-disk effects using EMRI observations. In particular, we model migration in the disk with two distinct analytical prescriptions — the α [17] and β [18] — to account for expected uncertainties in the underlying disk physics. Our analysis uses state-of-the-art waveforms [19,22] and is performed within

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1 The direct pull from the accretion disk is negligible unless unrealistically large densities are considered [10].
a realistic framework for EMRI parameter estimation. For the first time, we perform a Bayesian analysis over the full parameter space of a circular-equatorial EMRI system. We show that LISA could detect and characterize migration in the EMRI GW signal thanks to an agnostic torque model.

We find that migration could be detected in both $\alpha$ and $\beta$ disks. Our results validate earlier studies [8] [14] within a fully Bayesian setting, and extend the discussion to the measurability of the disk parameters. Compared to Refs. [8] [14], we find that migration in an $\alpha$ disk can also be detected and characterized for realistic disk parameters. Furthermore, for our reference EMRI system, the GW observation alone can distinguish between disk prescriptions and constrain regions of the disk parameter space. Assuming a torque model, the GW measurement can be combined with electromagnetic observations to infer the viscosity of the host disk. Finally, we investigate the degree to which we expect key EMRI parameters to be biased if one ignores torques in EMRI parameter inference, finding that the primary’s mass and spin may be biased.

While our work focuses on migration in thin accretion disks, our torque parametrization could be used to study other effects from the environment or modifications of GR, provided that the induced loss of angular momentum or energy can be written as a simple power law of the orbital separation.

The paper is organized as follows. In Sec. II we present the models for the accretion disk structure and the migration torque used throughout our work, including a very general phenomenological torque model suited for GW parameter estimation. In Sec. III we describe the vacuum waveform model and how we modify it to account for an environmental torque. We also summarize our methods for parameter estimation and the properties of our reference EMRI. Finally, in Sec. IV we present the results: the projected constraints on the amplitude of environmental torques (Sec. IV A), a detailed study of an EMRI signal with a detectable disk torque (including a discussion of multimessenger implications, Sec. IV B), and a study of the bias induced on the EMRI parameters when environmental effects are present in the signal but ignored in the GW analysis (Sec. IV C). We discuss future prospects in Sec. V.

II. ACCRETION-DISK EFFECTS

We begin by describing the accretion-disk prescriptions, torque model and assumptions used in this work. We denote the primary and secondary masses with $M_1$ and $M_2$, respectively. The MBH accretes at a ratio $f_{\text{Edd}} \equiv M_1/M_{\text{Edd},1} = \epsilon M_1/L_{\text{Edd},1}$ of the Eddington rate $M_{\text{Edd},i} = 2.536 \times 10^{-8} M_i (\epsilon/0.1) \text{yr}^{-1}$. Here $\epsilon$ denotes the efficiency of conversion of mass-energy into luminosity in the disk. (The accretion efficiency and the Eddington ratio enter all disk quantities in the combination $\epsilon^{-1} f_{\text{Edd}}$.)

We work in units in which $G = c = 1$.

The astrophysics of accretion disks is notoriously uncertain. Sophisticated numerical simulations are often required to produce realistic disks [23], and to describe the rich phenomenology of the torques that disks generate [15]. However, a fully numerical hydrodynamical approach is intractable for this work’s scope, which requires models that are fast to generate. Thus, we decided to adopt analytical models for the disk prescriptions and torques. We would expect future analyses of real data to be conducted in conjunction with numerical simulations [15] [24].

We employ radiatively efficient, Newtonian, thin accretion-disk models, considering both the $\alpha$ and $\beta$-disk prescriptions for the standard Shakura-Sunyaev solutions for the inner (radiation-pressure dominated) region of the disk [17] [25]. For $\alpha$ disks the viscous stress is proportional to the total pressure $t_{r\phi} = \alpha (p_{\text{gas}} + p_{\text{rad}})$ [17], while for $\beta$ disks $t_{r\phi} = \beta p_{\text{gas}}$ [18]. There is a long-standing question regarding the stability and realism of these analytic solutions [23] [26] [29]. In particular, $\beta$ disks have raised concerns for being only loosely motivated by the absence of thermal instabilities, which appear in analytical solutions of $\alpha$ disks. Despite these instabilities, the $\alpha$ disk model is still considered a good approximation for realistic, turbulent accretion flows in the radiation-dominated regime (see however [23]). In both cases $\alpha$ is the viscosity, which parametrizes the complex (and uncertain) magnetohydrodynamic features of accretion disks. Viscosity in AGN disks is typically believed to be around $\alpha = 0.1$, and as low as $\alpha = 0.01$ [23] [30].

The disk surface densities and height for the $\alpha$ and $\beta$ profiles are given by [17] [18]

\[
\begin{align*}
\Sigma_\alpha [\text{kg/m}^2] & \approx 5.4 \times 10^3 \left( \frac{\alpha}{0.1} \right)^{-1} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{-1} \left( \frac{r}{10M_1} \right)^{3/2}, \\
\Sigma_\beta [\text{kg/m}^2] & \approx 2.1 \times 10^7 \left( \frac{\alpha}{0.1} \right)^{-4/5} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{3/5} \\
& \times \left( \frac{M_1}{10^9 M_\odot} \right)^{1/5} \left( \frac{r}{10M_1} \right)^{-3/5}, \\
H [M_\odot] & \approx 1.5 \left( \frac{f_{\text{Edd}}}{0.1} \right) \left( \frac{0.1}{\epsilon} \right) M_1. 
\end{align*}
\]

The disk prescriptions that we employ are valid for geometrically thin disks only,\(^2\) where $H \ll r$. The corresponding disk densities are obtained as $\rho = \Sigma / 2H$, and are inversely proportional to both the viscosity and the accretion rate. A detailed investigation of these models can be found in [32] and [8].

The presence of an accretion disk induces environmental torques that modify the trajectory of the compact object

\(^2\) When the thin disk condition is violated (e.g. for super-Eddington accretion), the disk is better described by a slim-disk solution [23] [31].
TABLE I: Parameters for the torque model (2.2) for the migration torques described in Sec. III. C is the overall torque amplitude, $n_r$ sets its dependence on the orbital separation, and $n_{f_{\text{Edd}}}, n_{M_1}$ that on the disk viscosity, accretion rate and primary mass.

| Parameter | Migration ($\alpha$) | Migration ($\beta$) |
|-----------|----------------------|---------------------|
| C         | $7.2 \times 10^{-10}$ | $8.1 \times 10^{-6}$ |
| $n_r$     | 8                    | 59/10               |
| $n_\alpha$| -1                   | -4/5                |
| $n_{f_{\text{Edd}}}$ | -3                     | -7/5               |
| $n_{M_1}$ | 1                    | 6/5                 |

orbiting the MBH. For EMRI sources in the LISA band, such torques are suppressed with respect to the effect of GW emission, but are potentially observable\textsuperscript{[10]}\textsuperscript{32}\textsuperscript{33}\textsuperscript{34}\textsuperscript{35}\textsuperscript{36}\textsuperscript{37}\textsuperscript{38}\textsuperscript{39}. Under these approximations, environmental torques can be modelled as:

$$\dot{L}_{\text{disk}} = A \left( \frac{r}{10M_1} \right)^{n_r} \dot{L}_{\text{GW}}^{(0)} \text{ with (2.2)}$$

$$A = C \left( \frac{\alpha}{0.1} \right)^{n_\alpha} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{n_{f_{\text{Edd}}}} \left( \frac{M_1}{10^6M_\odot} \right)^{n_{M_1}},$$

where $\dot{L}_{\text{GW}}^{(0)} = -\frac{32}{3} \frac{M_2}{M_1} \left( \frac{r}{M_1} \right)^{-7/2}$ is the leading order circular-orbit GW torque and $r$ is the distance between the compact object and the MBH (see also Appendix A). In this approximation, the torque is simply characterized by an amplitude $A$ and a radial slope $n_r$. The slope corresponds to a post-Newtonian (PN) order $-n_r\text{PN}$. Accretion-disk torques should wane within the inner edge of the disk, where the rapid plunge into the MBH ensures that gas densities are low. Modelling this phase is however not necessary: our waveform model also ignores the plunge of the EMRI system, because its duration is only a small fraction of the full inspiral\textsuperscript{3}.

Environmental effects in AGN disks may include the disk gravitational potential, the formation of density waves (dynamical friction or planet-like migration, depending on the scales involved), winds, and mass accretion on the primary or the secondary\textsuperscript{3}\textsuperscript{10}\textsuperscript{12}\textsuperscript{14}\textsuperscript{33}\textsuperscript{34}. In this work, we focus on migration since we found it to be the dominant one, and as also pointed out in Refs.\textsuperscript{3}\textsuperscript{8}\textsuperscript{10}\textsuperscript{12}\textsuperscript{14}.

In analogy with planet-planetary disk interactions\textsuperscript{36}\textsuperscript{37}, the EMRI secondary can undergo “type I” migration in AGN disks with large viscosity, if the disk density structure is unmodified, or “type II” migration, if a gap opens in the disk at the radius of the secondary’s orbit. We use the analytic formula for three-dimensional, isothermal disks in Ref.\textsuperscript{38}, which is valid for type-I migration. The torque parameters for both $\alpha$ and $\beta$ disks can be found in Table I. Note that the migration torque is proportional to the disk density and inversely proportional to the disk height\textsuperscript{2.1}, which means that it scales with inverse powers of the disk parameters $\alpha$ and $f_{\text{Edd}}$.

Due to the turbulent nature of AGN disks, migration can oscillate over time and average to both negative and positive values\textsuperscript{44}. Here we use the positive, constant overall torque amplitude predicted in Ref.\textsuperscript{38}, assuming that oscillations around the average migration torque can be modelled separately, see e.g. Ref.\textsuperscript{10}.

For type-II migration, previous work\textsuperscript{8}\textsuperscript{10} used the quasi-stationary approximation in Ref.\textsuperscript{39}. However, simulations have shown that the pile-up of gas modelled by Syer and Clarke does not happen generically\textsuperscript{40}\textsuperscript{42}. To avoid overestimating the torque in the type-II regime, we use the type-I formula whenever migration is active, as suggested by recent numerical simulations of migration in intermediate mass-ratio inspirals\textsuperscript{15}\textsuperscript{24}.

Migration can also be quenched at certain radii in the disk\textsuperscript{3}, a transition which could not be described by Eq. (2.2). We verified that the EMRI does not cross the quenching radius in the LISA band across the parameter space explored in this work.

### III. EMRI WAVEFORMS

The GW emission from EMRI systems will provide measurements of their parameters with unprecedented precision\textsuperscript{44}, but only if the waveforms are accurately modeled. EMRI waveform models rely on perturbative solutions in which the Einstein equations are expanded about the limit of small mass ratio $\eta = M_1/M_2 \ll 1$\textsuperscript{14}. In this limit, the orbital evolution of the compact object in vacuum is governed by the Kerr geodesic equations with a forcing term, called the gravitational self-force. The self-force takes into account the finite size and mass of the body and its backreaction on the background Kerr spacetime. As a result, the orbit of the compact object slowly decays into the massive black hole due to the emission of GWs. The presence of an accretion disk can further enhance the decay of the compact object’s orbit.

In this work we employ the FastEMRIWaveform\textsuperscript{3} few waveform model package\textsuperscript{19}\textsuperscript{22} and modify the trajectory evolution to take into account the presence of environmental effects. We model the evolution of Kerr circular-equatorial orbits at adiabatic order\textsuperscript{3} by interpolating the Teukolsky fluxes $L_{\text{GW}}$ using the Teukolsky package in the Black Hole Perturbation Toolkit\textsuperscript{45}. We model the disk-induced effects by writing the rate of change of angular momentum as $\dot{L} = L_{\text{GW}} + \dot{L}_{\text{disk}}$, the environmental contribution from Eq. (2.2). This implementation provides a fast and accurate adiabatic trajectory that can be fed into a waveform-generation formalism through the augmented analytic kludges\textsuperscript{46}.

\textsuperscript{3} Since the environmental effects considered in this work appear at negative PN orders, we do not expect post-adiabatic corrections to significantly affect our results.
We focus on circular and planar orbits (in the plane of the disk), as disk-induced density waves are expected to damp the EMRI inclination and eccentricity long before it enters the LISA band [5, 47]. We also conservatively assume prograde orbits, as retrograde orbits can suffer from even larger disk-induced torques [1, 51, 52]. Compact objects formed in the disk are expected to be on prograde orbits [53, 54], and prograde EMRIs can be seen to much greater distances, so we expect these to dominate detected LISA events [43]. However, our implementation is flexible enough to model generic orbits, once prescriptions for disk-induced effects become available for this scenario.

A circular equatorial EMRI waveform is described by the masses of the two bodies \(M_1, M_2\); the dimensionless spin parameter \(a\) of the primary; the initial phase and radius, \(\Phi_0\) and \(r_0\); the azimuthal and polar angles, \(\theta_K\) and \(\phi_K\), describing the orientation of the spin-angular momentum vector of the MBH; the polar and azimuthal sky location angles, \(\theta_S\) and \(\phi_S\), given in the solar system barycenter frame; and the luminosity distance \(d_L\). The presence of accretion effects introduces two additional parameters, \(A\) and \(n_r\). We will refer to \((\theta_S, \phi_S, \theta_K, \phi_K, d_L)\) as extrinsic parameters, and to \((M_1, M_2, a, r_0, \Phi_0)\) as intrinsic parameters.

The speed of generation of few waveforms allows us to perform Bayesian analyses with Markov Chain Monte Carlo (MCMC) methods. In this work, we use both emcee [55] and ptemcee [56], using a standard Gaussian likelihood \(\propto \exp\{-1/2 \left| s - h(\lambda) \right| \} \). Here we assume stationary Gaussian noise, we define the data stream \(s\), GW signal \(h(\lambda)\) with parameters \(\lambda\), and the inner product

\[ (a(t)|b(t)) = 4 \text{Re} \int_0^\infty \frac{\hat{a}^*(f)\hat{b}(f)}{S_n(f)} \, df \]  

(3.1)

weighted by the LISA Power Spectral Density (PSD), \(S_n(f)\) [57]. In all the studies reported we injected noise-free data streams, since this yields results averaged over noise realizations. We assume uniform priors in all parameters, restricted to a narrow range around the true parameters. All posteriors shown in this work have a support which is much tighter than the prior ranges.

For computational simplicity we focus on a reference EMRI with masses \(M_1 = 10^8 M_\odot\) and \(M_1 = 50 M_\odot\), and spin \(a = 0.9\). The relatively large secondary mass is motivated by the fact that black holes are expected to grow via accretion when originating in an AGN disk [7, 58]. The initial radius is fixed to \(r_0 = 15.482\) such that the compact object spirals into the MBH in 4 years. We fix the other parameters to randomly chosen values of \((\Phi_0, \theta_S, \phi_S, \theta_K, \phi_K) = (3.0, 0.54, 5.36, 1.73, 3.2)\) and a luminosity distance \(d_L = 1.456\) Gpc,\(^4\) chosen to give a signal-to-noise ratio SNR\(=\sqrt{|h|h|}=50\). A detailed investigation of how different configurations affect the detectability of environmental effects with EMRIs is beyond the scope of this work, but we stress that this choice of parameters is typical for observable EMRIs [43].

## IV. EMRI PARAMETER ESTIMATION WITH ENVIRONMENTAL TORQUES

There are three ways in which environmental effects could be relevant to GW observations of EMRIs:

1. If environmental effects are absent or too weak to affect the waveform, EMRI observations could be used to set an upper limit on the torque amplitudes. The same analysis can be used to forecast the detectability of a given effect, and it only requires knowledge of the torque’s radial dependence (Sec. IV A).

2. If environmental effects are strong enough and can be modelled with a simple power of the radius, their presence can be detected (Sec. IV B). Provided reliable torque models, we can use such detections to constrain some of the properties of the disk, especially in coordination with electromagnetic observations;

3. If we analyze GW data ignoring environmental effects and these are relatively strong, EMRI parameter estimation will be biased (Sec. IV C).

We discuss each of these scenarios in the next sections.

### A. Constraints on environmental torques

Assuming the disk models and migration torque from Secs. II we ask: when are environmental effects large enough to be detectable?

The dephasing \(\Delta \Phi\) between vacuum and matter-influenced waveforms is a commonly used metric for detectability, with \(\Delta \Phi \geq 1\) a reference threshold. However, a significant dephasing is a necessary, but not sufficient condition for detectability. Using this threshold as a proxy can lead to overestimating detectability, implying that it must be used as a qualitative, but not quantitative metric. To better assess the detectability of environmental effects, we perform a Bayesian parameter estimation over the EMRI parameters and the torque amplitude for a vacuum injection. This approach is similar to tests of GR with parametrized post-Einsteinian expansions of the phase [59]. We sample over the intrinsic EMRI parameters as well as the torque amplitude, fixing the slope \(n_r\) and the extrinsic parameters for computational efficiency. Sampling over the extrinsic parameters would not affect our results, because these are not strongly correlated with the amplitude \(A\) (see Sec. IV B).

\(^4\) This luminosity distance corresponds to a redshift \(z = 0.276\) for \(H_0 = 67.74\) km/s/Mpc and \(\Omega_M = 0.3075\).
The resulting posterior for $A$ has a variance that carries information about the detectability of the effect (slope) selected. Any prediction for the amplitude falling inside the 95% (symmetric) bound is 2σ-consistent with noise, and cannot be distinguished from an EMRI in vacuum.

We show the results obtained for our reference EMRI in Fig. 1 for the two slopes predicted for the migration torque by the two disk models (Table I). We find that the symmetric 95% bounds for $\beta$ and $\alpha$ disks are $A_{95\%} = \{3.5, 1.8\} \times 10^{-6}$. The constraint on $A$ become tighter as the slope increases, consistent with the fact that larger slopes correspond to more negative PN orders probed by the long inspiral.

We also indicate at what amplitude the torque induces a dephasing of 1 and $10^2$ radians. This shows torques may not be detected even for dephasings much larger than a radian, confirming that the previously adopted requirement $\Delta \Phi \gtrsim 1$ overestimates the detectability of environmental effects.

A more accurate estimate of detectability consists in requiring $A > A_{95\%}$. For migration in $\beta$ disks, we find that this implies

$$\left(\frac{\alpha}{0.1}\right)^{-4/5} \left(\frac{f_{\text{Edd}}}{0.1}\right)^{-7/5} \left(\frac{\epsilon}{0.1}\right)^{-3} > 2 \times 10^3 \quad (\beta \text{ disks}). \quad (4.1)$$

For typical parameters $\alpha = 0.1$, $f_{\text{Edd}} = 0.1$, and $\epsilon = 0.1$ the effect is detectable, as found in Ref. [4]. The dependence on the disk accretion rate and viscosity implies that lower values would lead to more observable effects.

Referring to the same typical parameters, Ref. [8] found that migration in the more realistic $\alpha$ disks could not be observed. In this case we find the condition

$$\left(\frac{\alpha}{0.1}\right)^{-1} \left(\frac{f_{\text{Edd}}}{0.1}\right) \left(\frac{\epsilon}{0.1}\right)^{-1} > 2 \times 10^3 \quad (\alpha \text{ disks}) \quad (4.2)$$

This implies that, while $\alpha = f_{\text{Edd}} = \epsilon = 0.1$ are still excluded, the effect could be detected in disks with lower accretion rates and/or viscosity. For instance, for values $\alpha \lesssim 0.05$ the effect is measurable for $f_{\text{Edd}} \lesssim 0.01$, $\epsilon = 0.1$. These values are within what is expected from global simulations [23] and X-ray observations of AGNs [60].

While here we have presented the bounds for the radial slopes predicted for the migration torque in two disk models, this analysis can be easily generalized to other effects in the same negative-PN regime. In Fig. 5 of Appendix B, we present the symmetric 95% bound on the torque amplitude as a function of the slope for $-4 \leq n_r \leq 10$. We find that for $n_r \geq 3$ the upper limit follows the relation

$$\log_{10} A_{95\%} = -4.63 - 0.14 n_r. \quad (4.3)$$

This can be readily used as an approximate bound for other effects.

### B. Detection of environmental torques

We now consider the more optimistic scenario in which disk effects are above the detection threshold. Can we measure and characterize environmental torques? Provided a reliable model, can we infer the parameters of the disk hosting the EMRI?

We again investigate this scenario by performing a Bayesian analysis for our reference EMRI. We limit our study to the more realistic $\alpha$ disks, injecting a signal with slope $n_r = 8$ and amplitude $A = 1.92 \times 10^{-5}$. Choosing $\alpha = 0.03$ as in Ref. [24], this amplitude corresponds to a value $f_{\text{Edd}} = 0.005$ (with $\epsilon = 0.1$). However, our agnostic procedure means the constraint applies to all combinations of $\alpha$, $f_{\text{Edd}}$, and $\epsilon$ resulting in the same amplitude through Eq. (2.2).

The full posterior can be found in the Appendix C in Fig. 6. As expected for typical EMRI observations, the intrinsic parameters are measured with $\sim 10^{-5}$ relative precision. The sky localization error, in particular, is $\Delta \Omega = 1.8 \text{ deg}^2$ [61, 62] and the relative luminosity distance error is 6%. 5 The comoving volume error for this source is $\approx 5 \times 10^{-5} \text{ Gpc}^3$, which means that this source would be promising for follow-up electromagnetic campaigns.

The marginalised posteriors of the environmental parameters $A$ and $n_r$ are shown in Fig. 2. The posterior of $A$ is inconsistent with $A = 0$ at more than 3σ, as expected

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5 We ignore errors on the luminosity distance due to lensing and peculiar velocity because they are an order of magnitude smaller [63].
with the intrinsic parameters are of the order \(\sim 0.6 - 0.7\), whereas with the sky localization and distance are of order \(\sim 0.2\) and 0.08, respectively. As expected, extrinsic parameters are not strongly correlated with the accretion parameters.

Our analysis shows that a sufficiently strong disk torque can be detected with a model-independent template, as long as the torque can be described as a power law of the radius. Provided that we have a trusted model for the torque slope and amplitude, the latter carries further information about the effect, which can be used in conjunction with measurements of the mass to extract the values of \(\alpha\) and \(f_{\text{Edd}}\) consistent with the observation. This is represented in the top panel of Fig. 3 as a derived distribution from the samples for \(A\) and \(M_1\) in Figs 2 and 6. These are related to \(\alpha\) and \(f_{\text{Edd}}\) through the second line of Eq. (2.2).

\[
\frac{A}{c} \left( \frac{M_1}{10^6 M_\odot} \right)^{-n_{M_1}} = \left( \frac{\alpha}{0.1} \right) \left( \frac{f_{\text{Edd}}}{0.1} \right)^{n/f_{\text{Edd}}}.
\]

We also show the undetectable region according to the criterion in Eq. (4.2) (black-shaded region). We find that purely GW observations can single out a narrow region of the parameter space \((\alpha, f_{\text{Edd}})\), constraining a relationship between \(\alpha\) and \(f_{\text{Edd}}\).

If the host AGN for this EMRI is identified in a follow-up campaign, electromagnetic observations across the spectrum could be used to determine the bolometric luminosity of the central engine. The bolometric luminosity, together with the GW measurement of the central BH mass and spin, could be used to determine the underlying efficiency and accretion rate of the AGN. Finally, this multimessenger determination of the accretion rate \(f_{\text{Edd}}\) could be used to extract the disk viscosity \(\alpha\) from the joint posterior provided by the GW analysis. We give a concrete example of this procedure for \(f_{\text{Edd}} = 0.01\) and \(\alpha = 0.00375\) consistent with amplitude \(A = 1.92 \times 10^{-5}\). We use the numerical fits provided in [6] to relate the (observable) bolometric luminosity to the intrinsic accretion rate and radiative efficiency. In the lower panel of Fig. 3 we show the constraints obtained on the viscosity when the AGN accretion rate is inferred with error \(\sigma_{f_{\text{Edd}}}\). This example showcases the potential of multimessenger observations of dirty EMRIs.

Electromagnetic detection and host-association will be somewhat challenging for our reference EMRI and an AGN with \(f_{\text{Edd}} = 0.01\). The bolometric luminosity of this AGN is \(L \approx 3 \times 10^{41} \text{ erg/s}\). Based on the quasar

\[6\] Note that we use different definitions for the accretion rate compared to Ref. [6].

\[7\] At the relatively low accretion rates considered here, the bolometric luminosity is proportional to the Eddington ratio \(f_{\text{Edd}}\). The BH mass and spin determined through the GW signal have negligible uncertainty, see Fig. 4. Therefore, the relative precision of a luminosity measurement translates directly into a relative precision on the Eddington ratio.
The inclination of the MBH spin) could be used \( \Delta \Omega = 0 \) with this luminosity and within our error volume is \( \sigma \) with uncertainty \( \sigma \) viscosity, assuming that the disk accretion rate is determined 

Near-Infrared Camera instrument on the James Webb Space Telescope (JWST) \(^{68}\). 

C. Biased parameter estimation from environmental effects

Finally, we consider the case in which the EMRI GW signal is analyzed ignoring environmental effects, and ask: how would ignoring environmental torques affect the inference of the EMRI parameters and our interpretation of the signal?

We investigate this aspect by analyzing the GW signals with environmental effects as injected in the previous section. This time, we use a waveform template that does not allow for environmental torques ("vacuum template").\(^8\) We run multiple "searches" where the MCMC walkers are allowed to explore a parameter space with priors extending up to 5% around the true value. Even though we use a naive, brute-force, search method and explore only a portion of the parameter space, our results qualitatively suggest how challenging EMRI search and inference studies could be when ignoring environmental effects.

\(^{8}\) Since this procedure is computationally expensive, we run this analysis only over the intrinsic EMRI parameters. We do not expect this choice to affect the conclusions.
environmental effects.

When using an incorrect template, we are not guaranteed to perfectly match the full signal. In fact, the migration torque considered here is strong enough that we cannot find any match for the full 4-year signal with a vacuum template. We are only able to find a good match to the signal with a vacuum template by considering a shorter portion of the signal.

As an example, Fig. 4 shows the posterior for primary mass and spin recovered with a vacuum template in the last 2 years of the EMRI inspiral. For reference, we also show the posterior distribution using the template matching the injection (“migration template”). We find that the vacuum-template posteriors are significantly biased, i.e. they are shifted 3-sigmas away from the true values. In particular, unaccounted (inward) migration leads to an overestimate of the mass and spin of the primary, as it increases the rate of inspiral.

The absolute size of these biases is small and would not adversely impact any conclusions about the astrophysics of the sources. However, if a similar bias occurred on a parameter that characterises a deviation from GR, it could shift the inferred value of that parameter away from zero and possibly lead to a false detection of a GR deviation. We expect to perform exquisitely sensitive tests of GR with EMRI observations 11, 43, but our analysis suggests it will be important to allow for additional environmental perturbations when carrying out these tests. Additionally, if environmental effects are ignored, the residuals between the template and the signal might affect parameter estimation of other sources 69.

V. DISCUSSION

In this paper, for the first time, we quantitatively study how to measure accretion-disk-induced torques with GW observations from EMRIs. The analysis we carried out assumes the binary is affected by migration in radiative-efficiency and geometrically-thin disks 8, 14, and it is based upon a realistic waveform-generation formalism for EMRI parameter estimation within a fully Bayesian framework.

We have investigated three different scenarios in which torques from accretion could affect EMRI parameter inference. In the first scenario we measure how well accretion effects are bounded to zero if they are absent and we find that the migration torque amplitude can be constrained with a precision of $\sim 5 \times 10^{-6}$. This allows us to infer when environmental effects are strong enough to be detectable. Interpreting the constraints as coming from migration torques, we confirm previous estimates in 14 that migration is observable assuming $\beta$ disk prescriptions, but we also point out that the same is true for a wide range of accretion-disk parameters with $\alpha$ disks. Our work assesses for the first time the detectability of an environmental effect, identifying the region of parameter space that can be realistically probed by GW observations.

In the second scenario, we analyze the GW signal of a typical migrating EMRI in an $\alpha$ disk. We find that we can distinguish between different disk prescriptions and constrain the amplitude of the environmental effects with $\sim 20\%$ relative error. Using the (agnostic) measurements of torque amplitude and mass, we infer 2D marginalized posteriors for the disk viscosity and accretion rate. Moreover, assuming a multimessenger measurement of the bolometric luminosity with 10\% precision, we show that the viscosity can be measured with 50\% precision.

In the third and final scenario, we investigate the size of biases in EMRI parameter estimates caused by ignoring a strong migration torque. Our proof-of-principle analysis shows that the size of the bias that one should expect from reasonable migration torques will not significantly affect the inference of the astrophysics of galactic nuclei, but these biases could have an adverse impact on tests of general-relativity with EMRIs 11.

Our work highlights the science potential of EMRI embedded in accretion disks and the need for accurate torque models. In this work, we used prescriptions for EMRI migration that are designed for planetary (type-I) migration in a 3D isothermal disk 38. This model has several limitations: for instance, it does not account for the fact that migration torques can change significantly close to the inner edge of the disk 70, where migration can halt altogether 71. In general, EMRI migration differs from planetary migration in that the secondary object inspirals rapidly due to GW emission 24. Targeted numerical simulations, although limited by the wide range of scales and timescales involved, will therefore be crucial to accurately model migration torques for GW observations. The first such simulations (in 2D) have shown promising results 15, 24. Similarly to what happened in planetary science, we could see in the next decade a progression from 2D to 3D simulations and the inclusion of more and more physics (radiation, magnetic fields, temperature and entropy gradients, thinner disks, etc.).

Another important takeaway point from this work concerns EMRI search and parameter inference strategies. Phenomenological models capable of capturing a host of environmental effects are likely to be needed in future analyses. Our work discusses a possible way of doing this. Our analysis should be considered as a proof-of-principle study of the impact of environmental effects on inference on EMRIs. The code used in this work will be available in the near future as extension of the few package 24.

While this work relies on a reference EMRI configuration, we expect the detectability of environmental effects to improve when the small compact object explores farther regions around the MBH. This is due to the negative PN order of the effects considered in this work, which become dominant over gravitational emission at low frequencies. For fixed inspiral length and primary mass, a larger secondary mass (due to accretion 7, 58 and/or hierarchical mergers 49, 72 in the disk) would not only have higher
SNR, but also be observable at larger radii.

While our study shows that accretion-disk properties can be resolved with EMRIs, it remains to be seen if the same holds true when considering competing torques, such as from dark-matter spikes [73–80], hierarchical triples [81], or modifications to GR [11, 82–84]. A detailed study of the distinguishability between these different effects will be the subject of future work.

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Appendix A: Additional dependence on the radius in analytic models

Our analysis relies on analytical torque models that are simple powers in the radius. However analytical models could be more complicated than this simple prescription. For instance, accretion-induced torques often carry over the factor \( F \equiv (1 - \sqrt{r_{in}/r})^{1/3} \) appearing in the description of the disk temperature and surface density \([32]\). In this work we have not included it both to maintain a more agnostic approach (potentially across environmental effects of different origins than accretion disks) and to avoid introducing too many parameters in the Monte Carlo analyses of Sec. IV. Our omission is a conservative choice, since \( F \) always increases the disk temperature and density in the inner region of the binary (perhaps unreasonably given the unavailability of gas there). Here we partly amend this omission by presenting how the analytical prescriptions of [8, 14] would change in the presence of this factor.

To reintroduce the F-factor in the expressions of the main body of the paper, the relevant quantities are the temperature in the central disk plane and surface density [32],

\[
T^4 = \frac{3\sum k_R}{4} T_{\text{eff}}^4, \quad \Sigma = \frac{M_1}{3\pi v}\beta^4, \quad F = \beta(1 - \beta)^{1/2} \left( \frac{F_{\text{disk}}}{M_1} \right)^{3/2} \left( \frac{M_1}{M_{\odot}} \right)^{1/5} \left( \frac{r_{10M_1}}{r_{10M_1}} \right)^{-3/5} F^{12/5},
\]

which carry further dependencies on the Boltzmann constant \( k_B \), proton mass \( m_p \), mean molecular weight \( \mu_0 = 0.615 \), and Stefan-Boltzmann constant \( \sigma \). We introduce \( H(r) = c_s M_1^{-1/2} r^{3/2} \) [32].

Notice finally that the \( \beta \) parameter is implicitly defined by

\[
\frac{\beta}{1 - \beta} p_{\text{gas}} = \frac{3\alpha k_B}{8\pi m_p^{1/2}} \left( \frac{\Sigma}{M_1 T^{7/2} r^{3/2}} \right) \approx \frac{p_{\text{gas}}}{p_{\text{prad}}} = \frac{3\alpha k_B}{8\pi m_p^{1/2}} \left( \frac{\Sigma}{M_1 T^{7/2} r^{3/2}} \right). \quad (A3)
\]

Solving this for \( \beta \ll 1 \) and \( \alpha \) disks (\( b=0 \)) yields \( \beta_\alpha \approx 1.14 r^{21/8} F^{-8} \), while solving it for \( \beta \) disks (\( b=1 \)) gives \( \beta_\beta \approx 1.11 r^{21/10} F^{-32/5} \). Inserting these values in (A1) and the definition of \( H \) then leads to the following modifications of the surface densities in Eq. (2.1),

\[
\begin{align*}
\Sigma_\alpha \left[ \frac{\text{kg}}{\text{m}^2} \right] &\approx 5.4 \times 10^3 \left( \frac{\alpha}{0.1} \right)^{-1} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{-1} \left( \frac{r_{10M_1}}{10M_1} \right)^{3/2} F^{-4}, \quad (A4) \\
\Sigma_\beta \left[ \frac{\text{kg}}{\text{m}^2} \right] &\approx 2.1 \times 10^7 \left( \frac{\alpha}{0.1} \right)^{-4/5} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{-3/5} \left( \frac{M_1}{10^6 M_\odot} \right)^{1/5} \left( \frac{r_{10M_1}}{10M_1} \right)^{-3/5} F^{12/5}, \quad (A5) \\
H[M_\odot] &\approx 1.5 \left( \frac{f_{\text{Edd}}}{0.1} \right)^{0.1} M_1 F^4, \quad (A6)
\end{align*}
\]

which in turn gives the following densities

\[
\begin{align*}
\rho_\alpha \left[ \frac{\text{kg}}{\text{m}^3} \right] &\approx 1.3 \times 10^{-6} \left( \frac{\alpha}{0.1} \right)^{-1} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{-2} \left( \frac{M_1}{10^6 M_\odot} \right)^{-1} \left( \frac{r_{10M_1}}{10M_1} \right)^{3/2} F^{-8}, \\
\rho_\beta \left[ \frac{\text{kg}}{\text{m}^3} \right] &\approx 4.7 \times 10^{-3} \left( \frac{\alpha}{0.1} \right)^{-4/5} \left( \frac{f_{\text{Edd}}}{0.1} \right)^{-2/5} \left( \frac{M_1}{10^6 M_\odot} \right)^{-4/5} \left( \frac{r_{10M_1}}{10M_1} \right)^{-3/5} F^{-8/5}. \quad (A7)
\end{align*}
\]

Carrying over the F factors through \( H, \Sigma \) and \( \rho \) in the expressions of the torques from [8] leads to the addition of a factor in the parametrization of Eq. (2.2),

\[
\dot{L}_{\text{disk}} = A \left( \frac{r}{10M_1} \right)^{n_F} F^{n_F} \dot{L}_{GW},
\]

where \( n_F = \{-12, -28/5\} \) for migration in \( \alpha \)-disks and \( \beta \)-disks. The other parameters in Table I remain unchanged.
Appendix B: Upper limit on the amplitude of effects with power law-like radial dependence

In our analysis we derived the constraints LISA could put on the amplitude of two disk-induced effects, which predict different torque powers \( n_r \). Other beyond-vacuum effects might also manifest with a specific power law-like dependence on the orbital separation. Here, we explore how the constraints change as a function of \( n_r \) for our reference EMRI source. We show the results in Fig. 5 in terms of the symmetric 95% bounds on the amplitude \( A_{95\%} \). We find that for \( n_r \gtrsim 3 \) the bound can be fitted with a straight line in log-space as follows,

\[
\log_{10} A_{95\%} = -4.63 - 0.14 n_r .
\]  

(B1)

Similar results are found in Fig. (8-9) of Ref. [88] and in Fig. (2) of Ref. [89], where the bounds are set on a different amplitude parameter. In future work, we plan to investigate how to map our parametrization to the parametrized post-Einsteinian expansions [59].

Appendix C: Full posterior probability with detectable accretion effect

In Sec. [IVB] we considered the case in which the effect of the environment is detectable in the GW signal. Here, we present the full posterior probability distribution for our reference EMRI. The sampler was run until chains were longer than \( 50\hat{\tau} \), where \( \hat{\tau} \) is the average autocorrelation time, determined across chains [90]. This criterion has been used for all the Bayesian analyses in this work. This the first appearance of the full posterior of a circular-equatorial EMRI in the literature.
FIG. 6: Full posterior for the reference EMRI injection described in Sec. IV.