Bose-Einstein Correlations
A Research Program for the 21-st century *

R.M.Weiner
Physics Department, University of Marburg, Marburg, Germany †
and
Laboratoire de Physique Théorique et Hautes Energies
Université de Paris-Sud, Orsay, France

Abstract

It is argued that Bose-Einstein correlations constitute one of the most important and characteristic effects of strong interactions. The progress made in our understanding of this phenomenon is reviewed and a program for future research in this field is formulated.

1 The state of the art

1.1 Introduction

Although the study of Bose-Einstein correlations (BEC) is going on for more than 40 years and many interesting theoretical developments took place the experimental facts which we know at present are very few (cf. subsection 1.5) and it will take many decades until this situation will significantly improve. This is necessary not only because BEC per se are of high scientific interest (this will hopefully emerge also from the following) but primarily because in analogy to the fact that the Hanbury-Brown Twiss effect lead to a new chapter in optics, namely Quantum Optics, one could expect that BEC will lead to a new chapter in strong interaction physics.

What can be done to achieve this goal? The purpose of this talk is to try to answer this question by formulating a program which could become an entry ticket to this new field.

The talk will be divided into two parts: in the first and main part I will try to summarize the present status of the field. Due to space limitations this summary has to be very sketchy. In the second part the open and most important problems will be enumerated.

1.2 Why are BEC interesting and important?

Intensity interferometry and BEC in particular constitute at present the only experimental method for the determination of sizes and lifetimes of sources in particle and nuclear physics.

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†E.-mail address: weiner@mailer.uni-marburg.de
The measurement of these is essential for an understanding of the dynamics of strong interactions which are responsible for the existence and properties of atomic nuclei. Moreover a new state of matter, quark matter, in which the ultimate constituents move freely, is within the reach of present accelerators or those under construction. The confirmation that we have really seen this “promised” new state is intimately linked with the determination of its space-time properties. Furthermore certain consequences of the standard model which could not be tested directly should be seen in BEC and one of the most actual tests of this model related to the much hunted Higgs particle is influenced by this effect. Last but not least besides this “applicative” aspect of BEC, this effect has important implications for the foundations of quantum mechanics.

Because of these facts in the last years there has been a considerable surge of interest in hadron interferenceometry and in particular in BEC. At present there is no meeting on multiparticle production where numerous contributions to this subject are not presented and at least one meeting was dedicated almost entirely to this topic (cf. [1]). Due to its excellent programing and organization the present meeting is a further milestone in this evolution.

1.3 Relation between BEC and and quantum field theory

Loosely speaking Bose-Einstein correlations can be viewed as a consequence of the symmetry properties of the wave function with respect to permutation of two identical particles with integer spin and are thus intrinsic quantum phenomena. At a higher level, these symmetry properties of identical particles are expressed by the commutation relations of the creation and annihilation operators of particles in the second quantisation (quantum field theory-QFT). The QFT is the more general approach as it contains the possibility to deal with creation and annihilation of particles and certain correlation phenomena like the correlation between particles and antiparticles can be properly described only within this formalism. Furthermore, at high energies, because of the large number of particles produced, not all particles can be detected in a given reaction and therefore one usually measures only inclusive cross sections. For these reactions the wave function formalism is impracticable. Related to this is the fact that the second quantisation provides through the density matrix a transparent link between correlations and multiplicity distributions. This last topic has been in the center of interest of multiparticle dynamics for the last 20 years (we refer among other things to KNO scaling and intermittency). Last but not least one of the most important properties of systems made of identical bosons and which is responsible for the phenomenon of lasing is quantum statistical coherence. This feature is also not accessible to a theoretical treatment except in field theory.

1.4 HBT versus GGLP. Final state interactions.

The method of photon intensity interferometry was invented in the mid fifties by Hanbury-Brown and Twiss for the measurement of stellar dimensions and is sometimes called the HBT method. In 1959-1960 G.Goldhaber, S.Goldhaber, W.Lee and A.Pais discovered that identical charged pions produced in $\bar{p}-p$ annihilation are correlated (the GGLP effect). Both the HBT and the GGLP effects are based on BEC. In HBT we deal with correlations between
photons, i.e. particles which practically do not interact, while in GGLP we have hadrons which interact. This fact made some people wonder whether hadron interferometry is possible at all. Before going into a theoretical discussion of this question it is useful to recall some qualitative experimental facts which suggest that the above doubts are unjustified. These facts are: a) Positive correlations are seen between identical particles in all reactions like e-e, hadron-hadron, lepton-hadron, hadron-nucleus and nucleus-nucleus at all energies and are not seen between non-identical particles, except for resonance effects (the issue of quantum statistical particle-antiparticle correlations is discussed in section 1.8). b) BEC increase with decreasing difference of the momenta of the pair as expected in any theoretical treatment of BEC. c) Radii extracted via BEC from heavy ion reactions increase with mass number of the participating nuclei. It is these few facts which were alluded to above as being the basis of our confidence into the fact that in the GGLP effect we really see BEC.

1.5 BEC and the search for quark gluon plasma

The experimental proof of the existence of quark-gluon plasma as a new phase of matter is certainly one of the most challenging tasks of high energy physics. BEC play in this game a double role: a) To prove that QGP is indeed a (new) phase one must prove that its lifetime is significantly larger than the typical hadronic time scale ($10^{-23}$ sec). Hence a lifetime measurement is necessary. b) To prove that a phase transition took place one must prove that the energy density achieved in the reaction exceeds the critical energy density predicted by statistical QCD. This implies the measurement of the volume of the fireball. Both a) and b) can be performed only with the help of BEC. Unfortunately, most of the experimental results on BEC with relativistic heavy ions are still called “preliminary”.

1.6 BEC and the foundations of quantum mechanics

There are some aspects of principles of quantum mechanics involved in the study of BEC, which have been discovered more recently and which are related to the very concept of identity of particles. It is well known that the principle of identity of particles is part of the fundamental postulates of quantum mechanics and states essentially that elementary particles are indistinguishable. The question what means identical has not been raised until recently, because it had been considered that the answer to it was obvious. This situation has changed when it was discovered that there exists also a quantum statistical correlation between particles and antiparticles.[2]

Another fundamental property of BEC emerges from Bose statistics: for small differences in the momenta of the pair, identical bosons are in general bunched while fermions are antibunched. However in BEC for the particular case of squeezed states (cf. section 1.9) also antibunching is possible.

1.7 BEC, quantum statistics and the standard model

Coherence While for the determination of sizes and lifetimes via intensity interferometry, both bosons and fermions can be used, BEC have another potential field of applications of major interest for particle physics, namely the determination of the amount of coherence of sources. This constitutes also a test of the presence of classical fields (any classical field
is a coherent state). To realise the importance of this topic it is enough to mention that some of the most important developments in particle physics of the last 25 years including the “standard model”, are based on spontaneously broken symmetries which imply classical fields. However so far there is no direct experimental evidence for these fields. On the other hand it is well known from quantum optics that BEC depend on the amount of coherence in a very characteristic way (a completely coherent source like a laser above threshold leads to a constant correlation i.e. no bunching) and therefore one hopes to obtain information about coherence from boson interferometry.

The density matrix  This dependence of BEC on coherence is a particular case of the fact that in quantum theory any probability or cross section of an operator $O$ is an expectation value and thus depends on the state of the system which in general is described by the density matrix $\rho$

$$< O > = Tr(\rho O).$$

$\rho$ is in principle determined by the theory. For hadron multiproduction this theory is quantum chromodynamics and for processes involving multiphoton production this theory is quantum electrodynamics. However in both cases the use of the fundamental theory is impracticable because of the complexity of the many body problem. That is why one has to use phenomenological approaches. The experience gained in photonics has been instrumental in the analogous problem of hadron multiparticle production and amounts essentially to postulating the form of the density matrix in the coherent state representation $|\alpha>$ . Let us consider a statistical distribution $P\{ \alpha \}$ and expand the density matrix in $|\alpha>$ . Given the fact that the coherent states form an (over)complete set, this means that the resulting density matrix $\rho$ is quite general. Indeed we write then

$$\rho = \int D\alpha \ P\{ \alpha \} \ |\alpha><\alpha|$$

where the symbol $D\alpha$ denotes an integration over the space of functions $\alpha$, and the statistical weight $P\{ \alpha \}$ is normalized to unity.

The simplest and most common form of density matrix used is the Gaussian form in the $P$ representation both because of its mathematical convenience as well as because of the fact that it corresponds to the physically important case when the number of independent sources is large (central limit theorem). Two important phenomenological consequences follow from the Gaussian form of $\rho$.

(i) The maximum of the BEC function is quantitatively well defined, independent of the concrete form of the field correlator and of the geometry; thus e.g. for the second order BEC function $max C_2 = 2$.

(ii) The density matrix can be expressed in terms of only two moments of $P$ (cf.below). Most experimental data so far, with the exception of annihilation in rest (cf. and references quoted there), are consistent with (i). As to (ii) the situation is less clear. An approximate confirmation of (ii) has been obtained in [4] based on the data of [5]. While it appears that the gaussian ansatz is at least an acceptable approximation, given the importance of the form of the density matrix, more precise tests are very important. Furthermore, the consequences of small deviations from this form for the relationship between correlation...
functions of different order and for the relationship between moments of the multiplicity distributions of different order could be theoretically worked out without difficulties. This would make the form of the density matrix more accessible to experimental tests.

**The current formalism**  In particle physics rather than working with the fields it is often convenient and sufficient to use classical currents \( J \) and this will be done in the following. One can show that this effectively amounts to substitute in the above eqs. the symbol \( \alpha \) characterizing the coherent field by the symbol \( J \) characterizing the classical current.

The current formalism has two important advantages: 1. The corresponding field theoretical Klein Gordon equation can be solved exactly. 2. The space time picture of the process we are interested in can be introduced immediately. The current \( J(x) \) can generally be written as the sum of a chaotic and a coherent component,

\[
J(x) = J_{\text{chaotic}}(x) + J_{\text{coherent}}(x)
\]

with

\[
J_{\text{coherent}}(x) = \langle J(x) \rangle,
\]
\[
J_{\text{chaotic}}(x) = J(x) - \langle J(x) \rangle.
\]

By definition, \( \langle J_{\text{chaotic}}(x) \rangle = 0 \). The case \( \langle J(x) \rangle \neq 0 \) corresponds to **single particle coherence**. In the following we shall also deal with two-particle coherence (squeezed states).

A Gaussian current distribution is completely determined by specifying its first and second moments: the coherent component,

\[
I(x) \equiv \langle J(x) \rangle
\]

and the 2-current correlator,

\[
D(x, x') \equiv \langle J(x) J(x') \rangle - \langle J(x) \rangle \langle J(x') \rangle
\]

\[
= \langle J_{\text{chaotic}}(x) \rangle \langle J_{\text{chaotic}}(x') \rangle.
\]

Consider first the case of an infinitely extended source. The correlation of currents at two space-time points \( x \) and \( y \) is described by a primordial correlator,

\[
\langle J(x) J(y) \rangle_0 = C(x - y).
\]

The correlator \( C(x - y) \) reflects dynamical properties of the particle source, rather than its space-time geometry. Effects of the geometry of the source are taken into account by introducing the space-time distributions of the chaotic and of the coherent component, \( f_{\text{ch}}(x) \) and \( f_c(x) \), respectively. The expectation values of the currents, \( I(x) \) and \( D(x, x') \), take nonzero values only in space-time regions where \( f_c \) and \( f_{\text{ch}} \) are nonzero. Thus, one may write

\[
I(x) = f_c(x)
\]
\[
D(x, x') = f_{\text{ch}}(x) C(x - x') f_{\text{ch}}(x').
\]
A microscopic theory like lattice QCD may eventually provide us with the exact form of $C$. Phenomenologically one proceeds however by postulating the analytical form of the correlator and parametrising this form. General theoretical considerations allow then to determine the minimum number of parameters. The next step is to describe the space-time distribution of the source $f(x)$. Here again an analytical form has to be postulated and the number of independent parameters determined from general considerations.

The primordial correlator $C$ contains in general two length scales (correlation lengths) characterising the two space directions, e.g. the longitudinal and transverse direction for a non-expanding source or the boost and transverse direction for an expanding source, and one time scale (correlation time). Next we come to the space-time distributions of the source $f(x)$ for which we have to distinguish between the chaotic and coherent part $f_{ch}$ and $f_c$ respectively. For each of these parts we have again to postulate the analytical form and to determine the minimum number of independent parameters. One can show that here again at least three parameters are necessary both for expanding and non-expanding sources. Last but not least the chaoticity

$$p(k) = \frac{D(k,k)}{D(k,k) + |I(k)|^2}$$

has to be determined. One can show that within the above classical current formalism this can be done by specifying the value of $p$ at $k = 0$. This means that all overall there are at least 10 independent parameters which have to be determined phenomenologically in order to characterize completely BEC.

### 1.8 Particle-Antiparticle correlations

A surprising consequence of the classical current formalism, which however holds also for quantum currents is the existence of particle-antiparticle correlations and the fact that BEC for neutral pions are different from BEC for charged ones. The experimental detection of these new effects which are quite small but nevertheless very important from a principle point of view demands among other things much higher statistics at small momentum differences than that achieved so far. For details of the derivation of these effects we refer

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1. For mathematical simplicity usually Gaussian forms for $f$ and $C$ are chosen. Contrary to the case of the density matrix, where the Gaussian form has, because of the central limit theorem, deep theoretical significance, this is not true for the correlator $C$ or the geometrical function $f$.

2. A semiclassical approximation of the current formalism is the Wigner function formalism. It is useful when combined with full fledged hydrodynamics as it provides a link between correlations and the equation of state, a subject of high current interest in the investigation of hadronic and quark matter. This combination is apparently powerful enough to explain a multitude of heavy ion physics data, including single and double inclusive cross sections, particle yields etc. Heuristically it also has the advantage that it makes assumptions about the forms of the primordial correlator $C$ and of the geometry $f$ unnecessary as these follow from the solutions of the equations of hydrodynamics.

On the other hand in the heavy ion physics literature sometimes “short cuts” of the Wigner formalism are used in which hydrodynamics is replaced by assumptions about the form of the source function which are equivalent to the assumptions made above about $C$ and $f$. However, not only is this last approach less economical than the current formalism in the form presented above, (the number of independent parameters for a chaotic source alone is ten; these parameters are dependent on the average momentum of the pair and this dependence is not predicted by the “theory”), but it is also less general as it is based on a semiclassical approximation, valid only for small values of the difference of momenta $q$. 

the reader to \cite{2,4}. Qualitatively these effects can be understood if one realises that in the s-channel the particle-antiparticle interaction is different from the charged-charged interaction, because of the annihilation channel.

1.9 Squeezed states

At this point one should also mention that besides ordinary coherent states used as the basis of the representation, squeezed coherent states have been introduced, which are of major interest both from a theoretical point of view as well as because of their application potential. BEC are one of the most sensitive tools for the detection of these squeezed states. So far they have been seen only in optics mainly because in particle physics there was missing a definite prescription how to produce them. However very recently it has been pointed out that they may be produced in sudden transitions like annihilation processes or the explosive hadronisation of a quark gluon plasma \cite{3}.

1.10 BEC and multiplicity distributions.

One of the most characteristic properties of strong interactions is many particle production. This in itself explains from the beginning the need and usefulness of statistical methods. The pertinent physical observable is first of all the multiplicity distribution \( P(n) \) which is given by the diagonal matrix elements of the density matrix in the number representation

\[
P(n) = \langle n | \rho | n \rangle.
\]

One classifies correlations in strong interactions into long range (LRC) and short range (SRC). If one restricts oneself to identical bosons the bulk of SRC is due to BEC. However LRC also influence BEC; this was seen in hadronic reactions \cite{10} but in \( e^+ - e^- \) reactions and heavy ion reactions the situation is less clear.

As long as one considers \( P(n) \) in the entire phase space they are dominated by LRC, while if one restricts the phase space to small windows (e.g. in rapidity) SRC play the main part. The fact that SRC for identical particles are essentially due to BEC has led to the proposal \cite{11} to combine the investigation of multiplicity distributions with that of BEC and to look for coherence effects simultaneously in \( P(n) \) and BEC.

In 1986 Bialas and Peschanski \cite{13} suggested that the moments of \( P(n) \) in narrow windows might scale and show an intermittency pattern. Although this interpretation of the data available at the end of the eighties had been disputed quite soon when it was pointed out that conventional SRC analogous to the quantum statistical ones might be responsible for these observations \cite{14}, the link with BEC was not accepted definitively until it was proven experimentally that the so called scaling effects were strongly enhanced if only identical particles were analysed. Subsequently Bialas \cite{15} suggested that the source itself might be fractal; this would explain the above new experimental observations without spoiling the “intermittency” interpretation.

As demonstrated however in \cite{16} the apparent scaling behaviour can be explained by conventional BEC with a source of fixed size. Nevertheless a more definitive clarification of this issue awaits better resolution at small \( q \) where an end of the present “scaling” is predicted by the conventional BEC theory.
Another aspect of the relationship between BEC and multiplicity distributions is the fact that $P(n)$ depends not only on the width of the rapidity region but also on the position on the rapidity axis $y$: in the center $P(n)$ is broader than in the fragmentation region. Taking over the quantum statistical language as explained above this suggests that $P(n)$ is more chaotic in the center [17], [18]. If this is true it should be seen in BEC and indeed there are some experimental hints in this direction [19]. However much more experimental work and in particular a drastic improvement of statistics, as well as the theoretical investigation of alternative explanations of the presumed observations are necessary before more stringent conclusions can be drawn.

1.11 Experimental problems

From the facts mentioned above the reader may realize that there has been considerable progress in our understanding of the phenomenon of BEC. Unfortunately these theoretical developments have not been matched in full with comparable experimental progress although the number of experimental papers on this subject is quite impressive. Some of the most important reasons for this deficiency are of technical nature like insufficient statistics, lack of particle and track by track identification, and limitations of the phase space accessible to detectors. As a matter of fact many of the experiments on BEC performed so far are not dedicated experiments but rather byproducts of other experiments.

Another difficulty in the experimental investigation of BEC is related to the problem of normalisation of the correlation functions. Correlation functions of order $n$ are by definition ratios of multiple inclusive cross sections of order $n$ and the $n$ fold product of single inclusive cross sections. Because of the phase space limitations of most detectors used at present, the single inclusive cross sections cannot be measured directly. To circumvent this difficulty “substitutes” for the denominator made of the products of single inclusive cross sections are used. As any substitute they are not ideal and introduce biases in the experimental results. This is particularly evident in recent measurements of BEC in $e^+ - e^-$ reactions at the CERN LEP accelerator where the values for the incoherence factor $\lambda$ obtained by different methods of normalisation at different detectors differ by factors up to 2. Because of this situation serious doubts have been expressed about the usefulness of these measurements [20].

2 The program

Based on the considerations of the previous section I would like to propose the following research program in the field of BEC. Given the discrepancy between theoretical and experimental progress, this program is mostly of experimental character and will be labeled by (E). Wherever appropriate, theoretical open problems will also be mentioned and labeled by (T).

1. Determination of the form of the density matrix in the $P$ representation (possible deviations from the gaussian form), mostly from higher order BEC (E+T).

2. Determination of all independent parameters, in particular separation of correlation lengths (times) from geometrical scales and determination of chaoticity (E). Compa-
son with results to be obtained in the mean time by lattice QCD and other theoretical
developments (T).

3. Determination of the form of the correlator and of the form of the space time
distribution (E); comparison with future results from lattice QCD and other theoretical
approaches (T).

4. Comparison of BEC in e-e, hadron-hadron and heavy ion reactions, with particular
emphasis on 1, 2, and 3 (E+T).

5. Measurement of particle-antiparticle correlations (E).

6. Simultaneous determination of BEC and multiplicity distributions in the same phase
space region (E).

7. Search for squeezed states, both in BEC (through overbunching and antibunching)
and in multiplicity distributions (through oscillations in $P(n)$) (E).

8. Normalisation of BEC using the separately determined single inclusive cross sections
in the entire phase space (E).

9. Track by track detection and improved identification of particles (E).

10. Improvement of statistics especially at small $q$ by at least 2 orders of magnitude (E).

The entire program could be summarised in two words: **Quantum Hadronics**. This name
on the one hand reflects the analogy with quantum optics and on the other the fact that
hadrons are not photons. The implementation of this program may not take 100 years,
but it is certain that it will not be realized this century; therefore the title of this talk.
Once the above points and in particular 1-3 will be clarified, one might be able to proceed
to a reconstruction of an effective density matrix for multiparticle production in strong
interactions along the lines of [2], [1]. This could be then the birth certificate of the new
chapter of physics alluded to in the introduction.

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