Cosmological evolution of the interacting phantom (quintessence) model in loop quantum gravity

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Abstract. The dynamics of the interacting dark energy model in loop quantum cosmology (LQC) is studied in this paper. The dark energy has a constant equation of state $w_x$ and interacts with dark matter through a form $3cH(\rho_x+\rho_m)$. We find that for the quintessence model ($w_x > -1$) the cosmological evolution in LQC is the same as that in classical Einstein cosmology, whereas for phantom dark energy ($w_x < -1$), although there are the same critical points in LQC and classical Einstein cosmology, the loop quantum effect significantly reduces the parameter spacetime $(c, w_x)$ required by stability. If parameters $c$ and $w_x$ satisfy the conditions that the critical points are existing and stable, the universe will enter an era dominated by dark energy and dark matter with a constant energy ratio between them, and accelerate forever; otherwise it will enter an oscillatory regime. Comparing our results with the observations we find at $1\sigma$ confidence level that the universe will accelerate forever.

Keywords: dark matter, dark energy theory, quantum gravity phenomenology
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1. Introduction

Many cosmological observations show that our universe is undergoing an accelerating expansion and now mainly consists of two dark components: dark matter and dark energy. The dark matter is a clumpy component that traces the baryonic matter and accounts for about 23% of present total cosmic energy; the dark energy is an exotic energy with negative pressure and accounts for about 72% of total cosmic energy today. The simplest dark energy candidate is the cosmological constant [1]; however it suffers from two problems. One is the cosmological constant problem: why is the inferred value of the cosmological constant so tiny (120 orders of magnitude lower) compared to the typical vacuum energy values predicted by the quantum field theory? The other is the coincidence problem: why is the dark energy density comparable to the matter density right now? Therefore a dynamical scalar field: quintessence [2] is proposed as an alternative of dark energy, but it cannot explain the region of the equation of state less than $-1$, which is favored by observations [3]. Later Caldwell [4] proposed a phantom field to explain the present cosmic accelerating expansion. This field possesses a negative kinetic energy and so has a super-negative equation of state. In the Einstein gravity it is found that if the universe is dominated by the phantom energy, it will end with a big rip, i.e., a future singularity [5].

Many works have been done trying to avoid this singularity [6]. There are many other scalar field models, such as the quintom [7] and hessence [8]. However these scale field dark energy models still suffer from the coincidence problem. A possible alleviation for this problem is obtained by assuming the existence of an interaction between dark matter and dark energy [9].

Recent investigations have shown that there are some new nice features appearing in loop quantum cosmology (LQC; see [10, 11] for recent reviews), such as easier inflation [12] and correspondence between LQC and braneworld cosmology [13]. The LQC is the application in the cosmology context of the loop quantum gravity (LQG; see [14] for recent reviews), which is a theory trying to quantize gravity with a non-perturbative and background-independent method. Due to the loop quantum effect the standard Friedmann equation can be modified by adding a correction term [11], [15]=[17],

$$H^2 = \frac{1}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right),$$

(1)
where $H$ is the Hubble parameter, $8\pi G \equiv 1$, $\rho$ is the total cosmic energy density, $\rho_c \equiv \sqrt{3}/16\pi^2 \gamma^2 G^2 \hbar$ denotes the critical loop quantum density and $\gamma$ is the dimensionless Barbero–Immirzi parameter (it is suggested that $\gamma = 0.2375$ by the black hole thermodynamics in LQG [18]). Since this modified equation is correct under the condition that the quantum state is semiclassical, this condition is assumed to be satisfied forever in this paper. In addition we assume that quantum correlations do not build up during the long-term evolution of cosmology; otherwise there are additional correction terms from LQC which become important [19]. The correction term appearing in equation (1) essentially encodes the discrete quantum geometric nature of spacetime. When this correction term becomes dominant, the universe begins to bounce and then expands backwards. By studying the early universe inflation and the fate of future singularity in LQC, it is found that the big bang singularity, the big rip and other future singularities can be avoided [11, 15, 16, 20]. Recently Samart and Gumjudpai [20], Wei and Zhang [21] studied the dynamics of phantom, quintom and hessence dark energy models in LQC, and found that the results are different from that obtained in classical Einstein cosmology. In this paper we will investigate the evolution of our universe dominated by a scalar field in LQC, which has constant equation of state and interacts with dark matter, and then investigate whether there are some interesting features arising from the loop quantum gravity effect.

2. The interacting model

We consider a spatially flat universe in which there are only dark matter and dark energy with a constant equation of state $w_x$. Apparently $w_x > -1$ corresponds a quintessence model and $w_x < -1$ is a phantom case. In addition we assume that between the dark matter and dark energy there is an interaction, $\Gamma$. Thus the conservation equations for dark matter and dark energy can be expressed as

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = -\Gamma, \tag{2}$$

$$\dot{\rho}_m + 3H\rho_m = \Gamma, \tag{3}$$

where $\rho_x$ and $\rho_m$ correspond to the energy densities of dark energy and dark matter respectively, and a dot denotes the derivative with respect to cosmic time $t$. The interacting term $\Gamma$ is assumed to be $\Gamma = 3Hc(\rho_x + \rho_m)$, where $c$ is a coupling constant denoting the transfer strength. A positive $c$ corresponds to energy transferred from dark energy to dark matter and the other way around for a negative one. In this paper we constrain our discussion in the case of $c > 0$. This type of interaction, motivated by analogy with dissipation of cosmological fluids, has been introduced to solve the coincidence problem [9], and has been studied in the context of quintessence [22], the phantom [23] and the (generalized) Chaplygin gas model [24]. In addition the observational constraints for this type of interaction dark energy model have been studied in [25, 26].

In LQC, using the conservation equation of cosmic total energy $\dot{\rho} + 3H(\rho + p) = 0$, where $\rho = \rho_x + \rho_m$, one can easily obtain the effective modified Raychaudhuri equation

$$\dot{H} = -\frac{1}{2}(\rho + p) \left( 1 - 2\frac{\rho}{\rho_c} \right), \tag{4}$$

where $p$ is the total pressure ($p = w_x \rho_x$ in this paper).
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To analyze the dynamical system, we set
\[ N = \ln a, \quad u = \frac{\rho_x}{3H^2}, \quad v = \frac{\rho_m}{3H^2}, \quad (5) \]
where \( a_0 = 1 \) is assumed. Using equations (2)–(4), one can obtain
\[ u' = -3u(1 + w_x) - 3c(u + v) + 3u(u + w_xu + v) \left( -1 + \frac{2}{u + v} \right), \quad (6) \]
\[ v' = -3v + 3c(u + v) + 3u(u + w_xu + v) \left( -1 + \frac{2}{u + v} \right), \quad (7) \]
where the prime denotes a derivative with respect to \( N \). As discussed in [9, 23, 26] this interacting model can solve, or at least ameliorate, the coincidence problem in classical Einstein cosmological since in the dynamical system there is a late time attractor solution with a constant energy ratio between dark energy and dark matter. Therefore, regardless the initial conditions, the universe evolves to a final state characterized by a constant dark matter to dark energy ratio. Here we will discuss in LQC whether in the dynamics system of the interacting model there exists an attractor solution, and then study the cosmic evolutions within different conditions. In order to obtain the possible attractor for the system given by equations (6) and (7), we should firstly solve these equations with \( u' = 0 \) and \( v' = 0 \) to get the critical points:

Point A : \[ u_c = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4c}{w_x}}, \quad v_c = 1 - u_c. \quad (8) \]

Point B : \[ u_c = \frac{1}{2} - \frac{1}{2} \sqrt{1 + \frac{4c}{w_x}}, \quad v_c = 1 - u_c. \quad (9) \]

Both critical points correspond to the era dominated by dark matter and dark energy with a constant energy ratio between them and exist for \( c \leq \frac{-w_x}{4} \). Apparently these critical points are the same as that obtained in classical Einstein cosmology [26]. If the critical point is stable, it is a late time attractor; otherwise the solution is oscillatory. In order to investigate the stability of the critical point, we linearize the system near the critical point and arrive at

\[ \delta u' = - \left[ 3c + 3(-1 + 2u_c + v_c) + 3w_x + 6w_xu_c \left( 1 - \frac{u_c + 2v_c}{(u_c + v_c)^2} \right) \right] \delta u \]
\[ - \left[ 3(c + u_c) + \frac{6w_xu_c^2}{(u_c + v_c)^2} \right] \delta v, \quad (10) \]

\[ \delta v' = \left[ 3c + 3(1 + w_x)(-v_c + \frac{2v_c}{u_c + v_c}) - 6v_c(u_c + w_xu_c + v_c) \left( u_c + v_c \right)^2 \right] \delta u \]
\[ - \left[ 3 + 3c - 3(1 + w_x)u_c - 6v_c + \frac{6w_xu_c^2}{(u_c + v_c)^2} \right] \delta v. \quad (11) \]

Apparently there are two eigenvalues of the coefficient matrix of the above equations. If the real parts of two eigenvalues for a critical point are all negative, this critical point is stable and is an attractor; otherwise it is unstable and thus oscillatory. We find that the point B is always unstable; however the point A is an attractor if the equation of state for
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Figure 1. The stable regions of $(w_x, c)$ parameter space. In the region II, critical point A is a late time attractor in LQC. In Einstein cosmology critical point A is a late time attractor in the region I + II. III represents the region of the solution without physical meaning.

dark energy $w_x$ and the coupling factor $c$ satisfy the conditions $(1 + w_x)/w_x \leq c \leq -w_x/4$ and $w_x > -2$. Comparing our results with that obtained in Einstein cosmology where point A is stable only under the condition $c \leq -w_x/4$ [26], we find that for quintessence dark energy $w_x > -1$ the results in LQC are the same as that in Einstein cosmology if a positive $c$ is considered since $(1 + w_x)/w_x < 0$. However for phantom dark energy $w_x < -1$ the conclusions seem to be different: in region $c < (1 + w_x)/w_x$ or $w_x < -2$, the point A is stable in Einstein cosmology, but it is unstable in LQC, that is, the quantum correction effect will break the stability of point A if the interaction factor $c$ is smaller than $(1 + w_x)/w_x$ or the equation of state for the phantom is less than $-2$. In figure 1 we show the stability regions of $(c, w_x)$ parameter space. Regions I + II are allowed for Einstein cosmology; however in LQC only region II is allowed to obtain a stable solution.

Since in LQC the interacting quintessence model has the same dynamics as that in Einstein cosmology, hereafter we will only discuss the case of phantom $w_x < -1$. In figures 2 and 3, we plot the numerical results for $c$ and $w_x$ satisfying the conditions $(1 + w_x)/w_x \leq c \leq -w_x/4$ and $w_x > -2$. Figure 2 shows the evolutionary properties of the universe controlled by the interacting phantom energy with $w = -1.2, c = 0.2$ and different initial conditions. Apparently the trajectories converge to the same final state determined by parameters $c$ and $w_x$. Since in the final state $\Omega_x = u_c$ and $\Omega_m = 1 - u_c$, our universe will contain both dark matter and phantom energy, and the energy ratio between them approaches a constant. Figure 3 shows the evolutionary curve of the equation of state for total cosmic energy $w = p_x/(\rho_x + \rho_m)$ with $w_x = -1.2$ and $c = 0.2$; we find that in the final state the equation of state is a constant and $w > -1$, which means that the total energy density decreases with the cosmic expansion but the universe accelerates forever. Therefore if $(1 + w_x)/w_x \leq c \leq -w_x/4$ and $w_x > -2$, regardless of the initial conditions, the universe will enter a final state with a constant energy ratio between dark energy and dark matter and accelerate forever.

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In the following we will give the numerical results for the cases \( c < (1 + w_x)/w_x, \)
\( w_x < -2 \) and \( c > -w_x/4 \) according to the equations (2)–(4) with \( \rho_c = 1.5 \). The cases \( c < (1 + w_x)/w_x \) and \( w_x < -2 \) are allowed for the stable solution in classical Einstein cosmology but ruled out by the loop quantum effect. In figures 4 and 5 we plot the evolutionary curves of \( H(t) \) and \( \rho(t) \) for the case \( c < (1 + w_x)/w_x \) with \( w_x = -1.2 \) and \( c = 0.1 \). In figures 6 and 7 we give the results for the case \( w_x < -2 \) with \( w_x = -2.5 \) and \( c = 0.25 \). It is easy to find from these figures that at the beginning the phantom energy density increases with time, which then leads to the increase of total cosmic energy density. When the total energy density equals \( \rho_c/2 \), \( H \) takes the maximum value. When \( \rho \) reaches its maximum value \( \rho_{\text{max}} \sim \rho_c \), \( H = 0 \) and then the universe undergoes contraction until the bounce is reached. Therefore the universe will oscillate forever.
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Figure 4. The evolution of $H$ with time under the condition of $c < (1 + w_x)/w_x$. Parameters are set as $w = -1.2$, $c = 0.1$ and $\rho_c = 1.5$.

Figure 5. The evolution of cosmic energy density with time under the condition of $c < (1 + w_x)/w_x$. The solid, dashed and dotted curves correspond to $\rho_x + \rho_m$, $\rho_x$ and $\rho_m$ respectively. Parameters are set as $w = -1.2$, $c = 0.1$ and $\rho_c = 1.5$. Obviously $\rho_x$ triggers the recollapses, while $\rho_m$ triggers the bounces.

Figures 8 and 9 show the results for the case $c > -w_x/4$ with $w_x = -1.2$ and $c = 0.35$ which corresponds to the case where the critical points do not exist. Comparing these figures with figures 4–7, we find that, although the universe finally also enters an oscillating regime, the process is different from that obtained with the $c < (1 + w_x)/w_x$ or $w_x < -2$. It is found that in this case the energy densities of dark energy and dark matter have the same evolution with time, and the $H$ changes from positive to negative (or inverse) when $\rho = 0$ or $\rho \approx \rho_c$, while in the case $c < (1 + w_x)/w_x$ or $w_x < -2$, $H$ changes from positive to negative (or inverse) only at $\rho \approx \rho_c$. 

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3. Conclusion

In this paper the cosmological evolution with the interacting phantom or quintessence dark energy in loop quantum cosmology is studied. We consider the case of dark energy with a constant equation of state $w_x$ and the interaction term with the form $\Gamma = 3HC(\rho_x + \rho_m)$. It is found that in LQC the dynamics of the interacting quintessence model is the same as that obtained in classical Einstein cosmology, whereas for the interacting phantom model, the loop quantum effect reduces significantly the parameter space in which the attractor solution exists. In LQC we obtain that the critical point is existing and stable under the conditions $(1 + w_x)/w_x \leq c \leq -w_x/4$ and $w_x > -2$; however in classical Einstein
cosmology only the condition $c \leq -w_x/4$ is required. If the coupling parameter $c$ satisfies the stability and existence conditions for a stable tracking solution, our universe will enter an era dominated by both dark energy and dark matter with a constant energy ratio between them, and accelerate forever, although the total energy density decreases with cosmic expansion; otherwise the universe will enter an oscillatory regime.

Recently using the WMAP3 [27] data, Olivares et al [26] obtained that at $1\sigma$ confidence level $c \leq 0.0023$. More recently in [25], by combining the gold SNe Ia, BAO and CMB data, the authors found that at $1\sigma$ confidence level $-0.99 < w < -0.83$ and $c = 0.0057^{+0.0030}_{-0.0026}$, which show at $1\sigma$ confidence level that our universe will enter a final stable state and cannot oscillate. Letting $c = 0.0057$ we find in LQC that if $-1.006 \leq w < -1$ the universe with an interaction between dark matter and dark energy
will accelerate forever, whereas if $w < -1.006$ it will enter an oscillatory regime. Therefore it is clear that at $2\sigma$ confidence level the current observations seem to be unable to predict the late time evolution of our universe with the interacting dark energy in LQC.

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