Thermal properties of a rotating nucleus in a fluctuating mean field approach

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Abstract: The static path approximation to the path integral representation of partition function provides a natural microscopic basis to deal with thermal fluctuations around mean field configurations. Using this approach for one-dimensional cranking Hamiltonian with quadrupole-quadrupole interaction term we have studied a few properties like energy, level density, level density parameter\( (a) \) and moment of inertia as a function of temperature and spin for \( ^{64}\text{Zn} \) taking it as an illustrative example. We have also investigated the effects of variation in interaction strength on the level density and the parameter \( a \) as a function of temperature. The moment of inertia, \( I \) versus rotational frequency, \( \omega \) plot shows a sudden rise in the value of \( I \) due to rotation alignment of \( 0g_{9/2} \) orbitals at \( \omega \approx 1.0 \) MeV for a small temperature \( T \approx 0.5 \) MeV. At high \( T \approx 2.0 \) MeV about 40-45\% of each angular momentum is generated by alignment of \( 0g_{9/2} \) orbitals with an interesting result that at \( \omega \approx 1.0 \) MeV and spin \( J \sim 16 \) the moment of inertia has almost a constant, temperature independent value.

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1 Introduction

Heavy ion fusion reactions deposit energy into a nucleus in such a way that it is shared by various intrinsic and collective degrees of freedom. The excited nuclei thus produced may keep their energy long enough to reach internal statistical equilibrium. The equilibrated system takes a time of the order of $10^{-21} - 10^{-19}$ sec. to decay by particle emissions, depending on the excitation energy. During this time the intrinsic excitations are present as thermal excitations and they may be observed by the emission of photons. A quantitative interpretation of the $\gamma$-decay rates, and the nuclear structure information contained in, consequently depends on the nuclear level density. In addition to level density one also studies a few other interesting features at finite temperature and spin, e.g., shape transitions, collapse of proton and neutron pairing correlations, moment of inertia and rotational damping etc.

The problem of level density has been a subject of interest, theoretically as well as experimentally. Theoretically, it provides an important basis to test the validity of the approximations to many-body problem. The level density has been extensively studied semi-classically using Thomas-Fermi approach including quantal effects involving quite a few temperature dependent parameters. It has also been studied microscopically using finite temperature mean field theories, like, Hartree-Fock or Hartree-Fock Bogoliubov. The success of these mean field theories is based, to some extent, on the symmetry breaking. The symmetry breaking allows a considerable enhancement in variational Hilbert space and thereby includes various correlations appropriately. Ultimately these broken symmetries can be restored by using standard symmetry projection techniques. But each symmetry breaking introduces thermal and quantal fluctuations in the related degree of freedom, e.g. the nuclear orientation fluctuations caused by the breaking of rotational symmetry. Since, a nucleus is a finite system, these fluctuations play a vital role in understanding its dynamics. It has been shown explicitly by Egido that the quantal fluctuation at lower temperature dominates over the thermal fluctuation, but as temperature increases, thermal fluctuation grows faster whereas quantal fluctuation dies out. Recently, Alhassid and Bush have included
the effect of orientation fluctuation in Landau theory of phase transition and find that it is required to explain the observed angular distribution for giant dipole resonance. On the other hand, Goodman [6] explains the observed collectivity in a hot rotating $^{168}$Yb only when the thermal shape fluctuations are included in the mean field. However, the path integral representation of partition function [16] provides a natural framework to deal with the thermal as well as quantal fluctuations. The exact path integral representation of the partition function can be obtained by exploiting the Hubbard-Stratonovich transformation [17, 18]. This transformation introduces the path integration over some auxiliary field variables which are coupled to the one-body or the pairing density matrices and permits the linearization of two-body interaction with respect to these density matrices. One makes various approximations on these field variables to obtain mean field solution or a solution including quantal and thermal fluctuations around mean field. In static path approximation (SPA) [19, 20, 21] these fields are restricted to static paths (or static single-particle potential) which describes the motion of A-nucleons in a fluctuating mean field. As expected, it is found that the results obtained within SPA at high temperature are quite close to one obtained by exactly solvable models. Furthermore, when small amplitude quantal correction or RPA correlations [22, 23] are also included there is a remarkable improvement at low temperatures.

The static path approximation for nuclear partition function with quadrupole-quadrupole interaction Hamiltonian allows a nucleus to span entire collective space characterized by the deformation parameters $\beta$ and $\gamma$. So, to get some meaningful information, it is required to restore the rotational symmetry by using three-dimensional angular momentum projection at each point in the $\beta - \gamma$ plane [24]. This way of restoring symmetry needs the evaluation of five dimensional integrations, i.e, integration over three Euler angles and two deformation parameters, $\beta$ and $\gamma$. However, if one is not looking at quantities very sensitive to orientation fluctuations, then for a qualitative study cranking may be a reasonable proposition. Already in the previous work [25] we have seen that variation of moment of inertia as a function of rotational frequency at finite temperatures comes out quite encouraging. In the present work we
have used this formalism to constrain the average spin and studied the spin and temperature dependence of various quantities for $^{64}\text{Zn}$, including nuclear level density. Here we must mention that, as is well known, $\omega = 0$ does not really corresponds to $J = 0$ and only at high spins like $J \geq 8$ the angular momentum constraint implies a most probable spin value. Though this method does not affect the dimensionality of integrations, the dimension of Jacobian appearing in the expression for level density becomes four instead of three.

This paper is organized as follows: In the next section we present briefly the theoretical framework including basic expressions required for numerical computations, following mainly ref. \cite{19, 20}. Section 3 contains some numerical details and discussions of the results. Finally our main conclusions are presented in Section 4 along with a brief summary and prospects for subsequent calculations.

## 2 Theoretical framework

As already mentioned above, we follow the path integral representation of partition function within static path approximation (SPA) for quadrupole interaction as described by Lauritzen \textit{et al} \cite{19}. Only briefly we outline the formulation listing essential equations. The grand canonical partition function for a system rotating about intrinsic x-axis is

$$Z = Tr \ e^{-(\hat{H}_0 - \mu_p \hat{Z} - \mu_n \hat{N})/T}$$

where, $\omega$ is the rotational frequency and

$$\hat{H}_\omega = \hat{H} - \omega \hat{J}_x$$

$\hat{J}_x$ being the x-component of the angular momentum operator $\hat{J}$. Considering a quadrupole interaction Hamiltonian, written as

$$\hat{H} = \hat{H}_o - \frac{1}{2} \chi \hat{Q} \cdot \hat{Q}$$

where $H_o$ is the unperturbed spherical part and

$$\hat{Q}_\mu = r^2 Y_{2\mu}$$

4
is the quadrupole moment operator. The path integral representation of partition function in the SPA is given by

\[ Z(\mu_p, \mu_n, \omega, T) = Tr \hat{D} \]  

where,

\[ \hat{D} = 4\pi^2 \left( \frac{\alpha}{2\pi T} \right)^{5/2} \beta^4 d\beta \int_0^{\pi/3} | \sin 3\gamma | d\gamma e^{-\frac{\alpha^2}{\beta^2} e^{-\frac{(\hat{H}^\omega - \mu_{p} \hat{Z} - \mu_{n} \hat{N})}{T}}} \]  

is the static path statistical operator for quadrupole-quadrupole interaction Hamiltonian. The one-body operator \( \hat{H}^\omega = \sum_i h^\omega(i) \) is a Nilsson type deformed mean field Hamiltonian

\[ \hat{h}^\omega = h_o - \hbar \omega o \beta r^2 \left[ \cos \gamma Y_{20} + \frac{1}{\sqrt{2}} \sin \gamma (Y_{22} + Y_{2-2}) \right] - \omega \hat{J}_x \]

with \( h_o \) representing here the spherical basis space single-particle energies defined with respect to an appropriate inert core. The value of \( \hbar \omega_o = 41/A^{1/3} \) MeV and \( \alpha = (\hbar \omega_o)^2/\chi b^4 \) with \( \chi b^4 = 70 \) \( A^{-1.4} \) MeV as given by Baranger and Kumar [26].

The chemical potentials for proton and neutron are determined from the relations

\[ Z, N = T \frac{\partial}{\partial \mu_{p,n}} \ln Z(\mu_p, \mu_n, \omega, T) \]  

Similarly, the desired value of average angular momentum \( \sqrt{J(J+1)} \) is obtained by adjusting \( \omega \) such that

\[ \sqrt{J(J+1)} = < J_x > = T \frac{\partial}{\partial \omega} \ln Z(\mu_p, \mu_n, \omega, T) \]  

The energy as a function of temperature at a fixed number of particles and spin is given by

\[ E(T) = T^2 \frac{d}{dT} \ln Z + \mu_p Z + \mu_n N + \omega < J_x > \]  

and moment of inertia \( I \) is defined as

\[ I = \frac{< J_x >}{\omega_j} \]
where $\omega_J$ is such that the constraint Eq. (9) is satisfied. The nuclear level density is evaluated from the inverse Laplace transform of the partition function $Z$. For the fixed number of protons and neutrons, in the saddle point approximation, it is given by \[27\]

$$\rho(E, J) = \frac{e^S}{(2\pi)^2 D}$$

(12)

where,

$$S = \frac{(E - F)}{T}$$

(13)

is the entropy and the free-energy $F$ is given by

$$F = -T \ln Z + \mu_p Z + \mu_n N + \omega < J_x >$$

(14)

The quantity $D$ is the square root of Jacobian, i.e $\sqrt{J}$ with

$$J = \frac{\partial(E, Z, N, J)}{\partial(\beta, \alpha_p, \alpha_n, \lambda)}$$

(15)

where, $\beta = 1/T$, $\alpha_{p,n} = -\mu_{p,n}/T$ and $\lambda = -\omega/T$.

Finally, it may also be useful to calculate an effective level density parameter which is often used to connect the intrinsic excitation energy with a temperature

$$E^*(T) = a_{eff} T^2$$

(16)

That is $a_{eff} = (E(T) - E(T = 0))/T^2$. It is now obvious that the numerical value of $a_{eff}$ will strongly depend on the correct evaluation of the binding energy at T=0. On the otherhand SPA is not applicable in the T=0 limit. One way this ambiguity can easily be removed is by taking a derivative of $E^*(T)$ with respect to T, so that now (ignoring the dependence of $a_{eff}$ on T locally).

$$a_{eff} = \frac{1}{2T} \frac{dE}{dT} \quad (T > 0)$$

(17)

As it will be seen in the next section, we have also used another expression for $a_{eff}$ in terms of entropy to compute its values

$$a_{eff} = \frac{S}{2T}.$$ 

(18)
3 Numerical details and results

In this section we give some details of the numerical calculations performed, and present the main results for a nucleus $^{64}$Zn. We have studied the spin and temperature dependence of various quantities like, energy, level density parameter, level density and moment of inertia. In order to have a reasonable number of active valence particles we have chosen $Z=20$ and $N=20$ (i.e $^{40}$Ca) as an inert core. Thus we have 10 protons and 14 neutrons each in 30 orbitals spanning the model basis space up to $0g_{9/2}$. This model space would be very reasonable for zero temperature calculations even for high spin states. However, at high temperatures this would lead to truncation effects. We have tested that our results are reliable up to $T=2 - 2.5$ MeV. The spherical basis single-particle(sp) energies are -14.4, -10.2, -8.8, -8.3 and -4.4 (all in MeV) for the orbitals $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$ and $0g_{9/2}$, respectively. These values are precisely those given by Lauritzen and Bertsch [20]. The matrix elements of the sp Hamiltonian (7) can be easily calculated following Baranger and Kumar[26]. However, we should point out that the matrix elements of $r^2$ for the basis states beyond one major shell need to be reduced (renormalized) as is done in [26]. More realistic would be to use a radial function $f(r)$ like that employed in ref.[20], particularly when the basis space spans beyond one or two major shells.

Obviously as a first step of the calculations the deformed sp Hamiltonian (7) is diagonalised in the basis space at grid points in the $\beta - \gamma$ plane for a fixed value of $\omega$ so that a numerical integration( 12 point Gaussian in the $\beta$ space with $\beta_{max} = 0.5$ and 8 point Gaussian for $\gamma = 0 - 60^\circ$) in Eq. (3) can be performed to evaluate the partition function $Z$. One may compute the r.h.s. of Eq. (3) on a number of mesh points $\mu_p, \mu_n, T$ and $\omega$ so that later on a required value of $Z$ or its derivative at any $\mu_p, \mu_n, T$ and $\omega$ point could be computed numerically using multidimensional interpolation. However, we take the required differentiation $\partial/\partial\mu, \partial/\partial T$ or $\partial/\partial\omega$ directly inside the integration sign on the r.h.s. of Eq. (3) and then the ($\beta, \gamma$) integrations are performed for each quantity separately.
3.1 ENERGY

In Fig. 1 we have displayed the variation of energy (Eq. (10)) as a function of temperature for a few spin values J=0, 4, 8 and 16. Experimentally [28] for the yrast (T=0) J=2 and 4 states the excitation energies are $E_2 = 0.992$ MeV and $E_4 = 2.307$ MeV, respectively. But in the present calculation these come out quite compressed (our lowest T=0.3 MeV). Of course, at T $\leq$ 0.5 MeV the absence of pairing should be one of the main reasons for this compression. However, for T $\geq$ 1.0 MeV our results should be realistic. We also notice that $E_I - E_I(T = 0)$ is the largest for J=0 and this difference would have been even still larger particularly for the low spin states had the pairing correlations been included. The temperature dependent CHFB mean field calculations of Egido et al [29] for $^{164}$Er also show a similar behaviour.

At the first sight the plot of Fig. 1 in the $T \to 0$ limit appears like a rotational one. But actually it is in between rotational and vibrational, e.g. $E_6 - E_4 = 0.921$ MeV and $E_8 - E_6 = 1.42$ MeV. Besides the absence of pairing, the compression of the spectrum, particularly at high spins, is caused by the rotation alignment effects. At J=2 and T=0.5 MeV the occupation of $0g_{9/2}$ orbitals is zero. But at higher spins and/or higher temperatures $0g_{9/2}$ orbitals get partially occupied and help in easy generation of angular momenta through alignment at low energy expenses. More on this will be discussed later on.

Though in SPA there is integration over the deformation parameter $(\beta, \gamma)$ space, looking at the surface plots of the free energy like quantity defined as

$$f(\beta, \gamma) = \frac{\alpha \beta^2}{2} - \ln[\text{tr}\ exp(- (\hat{H}^{\omega} - \mu_p Z - \mu_n N)/T) + \mu_p Z + \mu_n N + \omega < J_x >]$$

may give some insight on the average shape evolution as a function of temperature and spin. We have chosen just three contour plots in the $\beta - \gamma$ space: Figs. 2a, b and c for T=0.5 MeV, J=0; T=0.5 MeV, J=8; T=2.0 MeV, J=16, respectively. The energy difference between the successive contour lines is 1.0 MeV and the numbers (energy in MeV) on a few of the lines give the idea of the free energy surface around the minimum value. The solid dots indicate the minimum point with equilibrium value of deformation parameters $(\beta_o, \gamma_o)$. Large spacings near the minimum free energy lines indicate the extent of shal-
lowness in energy and importance of shape fluctuations. Comparison of Figs. 2a and b shows that even at low temperature $T=0.5$ MeV and at $J=8$ the most probable (mean field or Hartree) shape has changed to oblate from prolate at $J=0$. At high spin $J=16$ and $T=2.0$ MeV also the shape is oblate (Fig. 2c), whereas at $J=0$ and $T=2.0$ MeV the mean field shape is spherical[20]. The value of $\beta_0$ in all the three cases is about 0.2. In Fig. 1 there is another curve (dashed) for $J=16$ evaluated at the most probable value of deformation parameter(see Fig. 2c) $\beta_0$ and $\gamma_0$. Though the difference between the two $J=16$ curve is not large ($\delta E \sim 500$ keV), it is not negligible particularly at higher temperatures. Even such small differences may be indicative of large fluctuation effects in other dynamical properties such as transition densities.

3.2 LEVEL DENSITY AND LEVEL DENSITY PARAMETER ’$a$’

At high temperatures the $\gamma$ decays are predominantly statistical in nature. Therefore, it is impractical to resolve these $\gamma$-rays to study the individual band structures. For a statistical analysis of these decay properties one needs excitation energy (temperature) and spin dependent level densities. Approximate spin dependent level density can be computed using the expression (12). The variation of $\ln \rho$ versus $T$ is shown in Fig. 3 for $J=0,4,8,16$ and 28. The $J=0$ curve is of course the $\omega=0$ no cranking result as in ref.[25]. Looking at $J=4,8$ and 16 curves there are two trends visible. One is that $\ln \rho$ is increasing with the increase of $T$ for all the $J$ values considered and the other is that at low temperature($T=1.0$ MeV) the value of $\ln \rho$ is lower for the smaller $J$ value and by $T=2.0$ MeV this trend gets reversed. Also at $T=2.5$ MeV the $J=16$ curve shows a tendency of saturation. The above mentioned reversal may be indicative of the limited basis space at high $T$ and high spin. Just to verify this argument we have repeated the calculation of $\ln \rho$ for $J=28$, as shown in Fig. 3 by a solid line with cross($\times$) marks. Slight flattening tendency of this curve for $T>2.0$ MeV once again may be indicative of the limited basis space.

It should be useful to study the spin and temperature dependence of the level density parameter $a$ which is often used to compute the level density.
employing semi-empirical or phenomenological expressions. Curves in Fig. 1 show a rather smooth variation of energy as a function of T. However, their slopes are changing, being very small for T < 1.0 MeV and almost a constant for T > 1.0 MeV. From Eq. (17) it is implied that a may be very small at low values of T. In Fig. 4 we have shown the inverse level density parameter K = A/a_{eff} as a function of temperature for some selected spins J=0, 4, 8 and 16. As in the absence of Fock term the slope of E* vs T curve is very small for T ≤ 0.5 MeV the corresponding K-curves in Fig. 4 show negative slopes. For T ≥ 1.0 MeV K increases with the increase of T for all spins. In our previous work [25] without the consideration of spin (ω = 0) change in K was slower; for T = 0.5 MeV to 1.5 MeV, increase in K is roughly by 8%. Here we must mention that presently the basis space is larger for neutrons as the inert core size is reduced to 40Ca whereas earlier [25] it was 48Ca. So, for example, at T = 1.0 MeV now K is smaller by about 10% and this is understood to be caused by the increase in collectivity, there being more number of active particles. In view of the present cranking calculations J = 8 and 16 cases should be taken more seriously and for these there is no flat (constant value of K) region. For T > 2 MeV the increase is even faster which may be an indication of the basis truncation effect.

As mentioned in Sec. 2 the quadrupole interaction strength χ (in MeV) = c/A^{1.4} with c=70 has been used in our computations. Without much justification we have taken it to be just the same as used by Baranger and Kumar [26] for the rare-earth nuclei. Then we have considered two more values of c=65 and 75 in order to investigate the effect of interaction on level density (only ω = 0 case). Of course, we know that higher the value of c interaction is more attractive and it should lead to lowering in energy and a more deformed system. In conformity with this we do get almost parallel running curves for c=65, 70 and 75 in the E-T plane. But its effect on the value of lnρ (Eq. (12) and K (Eq. (17)) does not seem to be a priori obvious. We have computed these quantities and enumerated them in Table 1 as a function of temperature with c=65, 70 and 75. First we consider the value of lnρ. At T = 1.9 MeV we notice that lnρ is practically independent of χ. At T < 1.9 MeV lnρ seems to be increasing with the increase in the value of χ, so much so that at T = 1.1
MeV \( \rho \) has increased by about 25% in going from \( c=65 \) to 75 (an increase of about 15%). Though actual numbers do not precisely support, we feel that at \( T \geq 2 \) MeV \( \ln \rho \) is not too sensitive to the interaction strength. At \( T=2.5 \) MeV we notice decrease in the value of \( \ln \rho \) with increase of \( \chi \) which may be a manifestation of limited basis space. The value of \( K \) seems to be slightly increasing with the increase of \( \chi \), that is the value of the level density parameter, \( a \) is decreasing with the increase of the value of \( \chi \). It does not look convincing but nonetheless may be true.

On the other hand the values of \( K_S \) in Table 1 indicate that using Eq. (18) \( a_s = S/2T \) is slightly higher for the large value of \( \chi \) at low temperatures and by \( T=2 \) MeV it becomes insensitive to about 15% increase in the value of \( \chi \). An important point to note here is that the magnitude of \( K_S \) is much smaller compared to the value of \( K \) and it is around the prevalent value in use in the semi-empirical calculations in the literature, that is about 8 to 10. In this sense the definition of the level density parameter as \( S/2T \) seems more reasonable as it is also consistent with the variation of \( \ln \rho \) presented in Table 1. Before proceeding to next section we may still note that the rate of increase in the value of \( K_S \) with \( T \) is rather large (not at all a constant) and is similar to that of \( K \).

Finally, we conclude this section with an important remark in support of our above investigation for the variation of \( \ln \rho \) with \( \chi \). In a very recent publication Alhassid and Bush [30] have studied the nuclear level density in SPA applied to an exactly solvable SU(2) model. Within this model study they do find that \( \ln \rho \) increases continuously with the increase in interaction strength at low excitation energy and at high excitation energy its value appears almost insensitive to the interaction strength (Fig. 7 in ref. [30]).

### 3.3 MOMENT OF INERTIA

According to Eq. (11) we define moment of inertia, \( I \) as a ratio of the angular momentum \( \sqrt{J(J+1)} \) to the cranking frequency \( \omega_J \). This way the moment of inertia is not a parameter solely describing the collective rotation of a nucleus. However, this makes it a more interesting parameter as its variation with \( \omega \) at
various temperatures can depict the effect of interplay between collective and
sp degrees of freedom, i.e. rotation alignment. Therefore, like a backbending plot\cite{31} we have shown in Fig. 5 the variation of $I$ as a function of $\omega$ at four
values of $T$ between 0.5 and 2.0 MeV. On each curve the points indicate the value of $J=2, 4, 6 \ldots$ such that the constraint relation (9) is exactly satisfied.
The horizontal dashed curve represents the value of rigid body moment of in-
ertia at $T=0$ and the deformation $\beta = 0.2$ and $\gamma = 0$. At a small temperature,
$T=0.5$ MeV plot shows a sudden rise of $I$ for $\omega > 0.5$ MeV implying the gen-
eration of spin by alignment of a high-$j$ sp orbital along the rotation axis. As
the Table 2 shows, the orbital $0g_{9/2}$ is unoccupied at $J=2$ and $T=0.5$ MeV.
But with a slight increase of $\omega$ the level $0g_{9/2}$ start getting partially occupied
and contribute to the generation of total angular momentum. So much so that
at $J=16$ the $0g_{9/2}$ orbitals, still with less than two particle in it, contribute
about 40%. Denoting the contribution of particles in $0g_{9/2}$ orbitals as aligned
angular momentum, $j_a$ and the rest as collective, $J_{coll}$ one can write

$$J = J_{coll} + j_a$$

In Table 2 we have listed the value of $j_a$, percentage contribution of $j_a$
to total angular momentum $< J_x >$ and the number of particles in $0g_{9/2}$ at two
temperatures $T=0.5$ and 2.0 MeV. Only a few values of angular momenta are
given, out of which $J=16$ is of a special interest. At $J=16$ the value of the
moment of inertia $I = < J_x > / \omega_J$ remains almost independent of tempera-
ture(see Fig. 5). Furthermore at $T=2.0$ MeV the occupation of $0g_{9/2}$ orbitals
has substantially increased compared to the $T=0.5$ case and keeps growing
with the increase of the value of $J$. However, the percentage contribution of $j_a$
at all the spins is almost the same. For $J < 16$ it is a temperature induced
alignment whereas for $J \geq 16$ it is the usual rotational alignment\cite{31} present
even at zero temperature. We may point out that with two protons and two
neutrons in $0g_{9/2}$ it can contribute maximum $j_a^{max} = 16$. As seen from Table 2
both protons and neutrons are contributing to the rotation aligned component
$j_a$ which may be termed as a coherent rotation alignment and this is stronger
at a higher temperature.
4 Summary and conclusions

Employing a one dimensional cranking Hamiltonian with quadrupole-quadrupole interaction term in the static path approximation we have studied the spin and temperature dependence of energy, level density, level density parameter and moment of inertia of $^{64}$Zn. The angular momentum is, of course, conserved only on the average according to the constraint given in Eq. (9). The level density parameter $a$ is calculated using two expressions: $dE/dT = 2aT$ and $S = 2aT$. The numbers obtained using latter relation seem to be more in agreement with the empirical values. Effect of variation of the interaction strength on the thermal properties is also investigated and it seems that at high temperature $T \geq 2.0$ MeV the dependence is rather weak. Within this investigation the variation of $a$ or $K = A/a$ as a function of $T$ again appears to support the $S = 2aT$ definition of $a$ so that its behaviour is consistent with that of the level density $\rho$.

The variation of moment of inertia as a function of spin as well as temperature is studied with a definition $I = \langle J_x \rangle / \omega_J$. At a given low temperature $T < 2.0$ MeV $I$ increases with the increase of $J$ and becomes almost a constant at $T = 2.0$ MeV. At $T < 1.0$ MeV and high spin $J \geq 16$ about 40% of the total spin value is generated by the alignment mechanism of $0g_{9/2}$ orbitals. On the other hand at $T = 2.0$ MeV the alignment of $0g_{9/2}$ orbitals contribute about 40% to all the angular momenta. We are planning to include $J$-dependence according to Eq. (8) of ref. [19] so that the validity of the present cranking results can be ascertained.
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Figure Captions

Figure 1: Energy versus temperature for $^{64}\text{Zn}$ at various spins, J=0, 4, 8 and 16. The dashed curve represents the energy for J=16 at different temperatures evaluated with most probable or equilibrium value of deformation parameters $\beta$ and $\gamma$.

Figure 2: Contour maps for free energy $f(\beta, \gamma)$ in $\beta - \gamma$ plane for $^{64}\text{Zn}$ at different spins and temperatures, (a) J=0 and T=0.5 MeV, (b) J=8 and T=0.5 MeV and (c) J=16 and T=2.0 MeV. Each contour represent a path in $\beta - \gamma$ plane for constant $f(\beta, \gamma)$ which differs by 1.0 MeV in magnitude for the adjacent contours. The solid dot indicates the point at which $f(\beta, \gamma)$ is minimum.

Figure 3: Logrithmic variation of nuclear level density $\rho$ for $^{64}\text{Zn}$ as a function of temperature at spins J = 0, 4, 8,16 and 28.

Figure 4: Spin and temperature dependence of inverse level density parameter $K = A/a_{eff}$ for $^{64}\text{Zn}$. The negative slope at lower spin and temperature indicates the lack of contribution due to pairing correlations and Fock energy.

Figure 5: Systematics of moment of inertia $I$ vs rotational frequency $\omega$ for $^{64}\text{Zn}$ at finite temperatures, T = 0.5 - 2.0 MeV. Points on these curves satisfy the angular momentum constraint (see Eq. (9)). The sudden rise in $I$ at $\omega \approx 1.0$ MeV for T=0.5 MeV shows a rotation alignment of $0g_{9/2}$ orbitals. The horizontal dashed line represent the rigid body moment of inertia $I_{rig}$ for $\beta = 0.2$ and $\gamma = 0$. 

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Table 1: The values of $\ln \rho$, $K$ and $K_S$ with no cranking at different temperature for interaction strength with $c_1$, $c_2$, $c_3$ corresponding to $c=65$, 70 and 75, respectively in $\chi = cA^{-1.4}$.

| $T$ (MeV) | $\ln \rho$ | $K$ (MeV) | $K_S$ (MeV) |
|-----------|-------------|------------|-------------|
|           | $c_1$ | $c_2$ | $c_3$ | $c_1$ | $c_3$ | $c_1$ | $c_3$ |
| 0.5       | 7.32  | 7.43  | 7.59  | 13.51 | 12.88 | 6.21  | 6.15  |
| 0.9       | 9.76  | 9.89  | 10.13 | 12.12 | 12.34 | 8.17  | 8.00  |
| 1.1       | 11.43 | 11.56 | 11.66 | 12.35 | 12.71 | 8.69  | 8.59  |
| 1.5       | 14.73 | 14.81 | 14.91 | 13.68 | 14.17 | 9.50  | 9.46  |
| 1.9       | 17.73 | 17.74 | 17.73 | 16.34 | 16.84 | 10.26 | 10.26 |
| 2.1       | 19.03 | 19.03 | 19.01 | 18.48 | 18.91 | 10.71 | 10.71 |
| 2.5       | 21.25 | 21.23 | 21.21 | 26.35 | 26.11 | 11.82 | 11.59 |
Table 2: The value of rotation aligned angular momentum $j_a$, percentage contribution of $j_a$ to total angular momentum $< J_x >$ and the number of particles in $0g_{9/2}$ at temperatures T= 0.5 and 2.0 MeV.

| $J(h)$ | T=0.5 MeV | | | T=2.0 MeV | | |
|---|---|---|---|---|---|---|
|  | $j_a$ | $\frac{J_x}{J}$ | $0g_{9/2}$ occupation($N_{g_{9/2}}$) | $j_a$ | $\frac{J_x}{J}$ | $0g_{9/2}$ occupation($N_{g_{9/2}}$) |
|  |  | Protons | Neutrons |  | Protons | Neutrons |
| 2 | 0.11 | 4.5 | 0.00 | 0.00 | 1.05 | 42.8 | 0.42 | 1.07 |
| 8 | 1.93 | 22.7 | 0.00 | 0.38 | 3.56 | 42.0 | 0.52 | 1.24 |
| 16 | 6.19 | 37.5 | 0.32 | 1.02 | 7.41 | 44.9 | 0.81 | 1.63 |
| 20 | 7.74 | 37.8 | 0.65 | 1.22 | 9.01 | 43.9 | 1.00 | 1.85 |
| 28 | 11.29 | 39.6 | 1.07 | 1.81 | 12.69 | 44.5 | 1.45 | 2.35 |