Symmetry breaking in the prisoner’s dilemma on two-layer dynamic multiplex networks

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Abstract

Understanding the role of network structure in the evolution of cooperation is a key research goal at the intersection between physics and biology. Recent studies have particularly focused on multiplex networks given that multiple social domains are interrelated and cannot be represented by single-layer networks. However, the role of network multiplexity is not fully understood when combined with another important network characteristic: network dynamics. In the present study, we investigated evolutionary prisoner’s dilemma games played on dynamic two-layer multiplex networks in which the payoff combined across the two layers determined strategy evolution. In addition, we introduced network dynamics where agents can sever links with defecting neighbors and construct new links. Our simulation showed that link updating enhances cooperation but the resultant states are far from those of full cooperation. This modest enhancement in cooperation was related to symmetry breaking whereby the cooperation frequency in one layer disproportionately increased while that in the other layer remained the same or even diminished. However, this broken symmetry disappeared with sufficiently fast link updating. Our results show that the introduction of network dynamics enhances cooperation in the prisoner’s dilemma as previously reported, but this enhancement is accompanied by significant asymmetry once network multiplexity is considered.

Introduction

The origin of cooperation is an intriguing research topic at the intersection between physical and biological sciences [1–4]; it has been analyzed using a mathematical framework known as evolutionary game theory. The core dilemma in the study of cooperation is the discrepancy between myopic rationality and social efficiency. Specifically, non-cooperators can avoid the costs of cooperation while enjoying the benefits of others’ cooperation; thus, non-cooperation is advantageous to individual interests. Consequently, this free-riding leads to the prevalence of non-cooperation and lower social efficiency.

Network reciprocity, where network structure supports the maintenance of cooperation, offers one potential solution to this core dilemma. On the one hand, cooperators cannot survive in well-mixed populations because defectors can achieve larger payoffs on average by avoiding the cost of cooperation. On the other hand, a limited number of neighbors interacting in a network facilitate the formation of cooperative clusters, which enable cooperators to achieve higher fitness from the benefits of mutual cooperation. Since the seminal work of Nowak and May [5], researchers have examined the effects of various network characteristics, including degree heterogeneity [6, 7], average degree [8, 9], and assortativity [10], on cooperation. One study has also clarified the relationship between network reciprocity and a fundamental concept in evolutionary biology, namely inclusive fitness [9].

In the present study, we focus on the multiplexity and dynamics of networks in relation to cooperation. Multilayer networks that are not limited to multiplex networks are a key research focus in network science [11]. Understanding these types of network is crucial because multiple types of (social) activity are interrelated and should therefore be represented by networks with multiple layers. A seminal study showed that failure in one layer (e.g. in power
networks) can lead to severe fragmentation in multiple layers (e.g., in Internet networks as well as power networks) [12]. In addition, multiplex networks show novel epidemic spreading patterns [13] and contribute to robust diversity in culture formation models [14].

Multilayer networks are also vital to studying the evolution of cooperation. One widely examined interdependency is payoff coupling whereby individuals’ performances depend on the game payoff from multiple network layers [15–20] and the existence of an optimal interdependency level is indicated [21,22]. More complex situations, in which the layers differed in the games conducted [23–26] or topological characters [27], have also been studied. In addition, network layers can be coupled by other factors such as information about strategy frequency [28], imitation probability [29], reputation [30], social pressure [31], or the selection of imitation partners [32]. Furthermore, several studies have demonstrated the coevolution of cooperation and interdependency among network layers [33–37].

Multilayer networks have been found to support cooperation through incoherent behavior whereby individuals adopt different strategies in different network layers; some studies suggest that cooperation enhancement can be attributed to this incoherent behavior rather than the consistent adoption of cooperation in multiple layers of the network [38–59]. Moreover, such disparity over networks appears not only at the individual level but also at the macroscopic level; interdependent networks show symmetry breaking by which the overall cooperation levels in each layer diverge. Thus, although the same rules may be applied across the network, the extent of cooperation enhancement can differ across layers [40–42].

Network dynamics is another important realistic network characteristic [43] that researchers have investigated in relation to cooperation [44]. In evolutionary games, network dynamics imply that the existence or duration of links between individuals depends on the individuals’ attributes including their strategy. Both theoretical [45–60] and experimental [67,69] studies have demonstrated that network dynamics strongly enhance cooperation; however, overly fast link dynamics have been shown to hinder cooperation [52,70]. Researchers have also indicated that network dynamics are important to other types of cooperation-related phenomena such as fairness [71,73]. Recently, the role of network dynamics was elucidated further: sufficiently fast link updating in a network was shown to result in full cooperation in two-layer multiplex networks [74].

Although independent lines of research on multiplex networks and network dynamics have provided valuable insights into the role of networks in the evolution of cooperation, their coupled effects have yet to be fully examined (a notable exception is Ref. [74]). To remedy this situation, here we examined the prisoner’s dilemma game played on dynamic multiplex networks. Specifically, agents were located on networks with two layers (duplex networks) and played the prisoner’s dilemma game with their neighbors. The two layers were mutually related because the payoff that accumulated over both layers determined the evolution of strategies. Links in each layer could be modified depending on the strategy adopted by agents.

The results of our simulation showed that introducing link updating to the network increased cooperation but that the end result was far from full cooperation. The modest enhancement in cooperation is related to symmetry breaking whereby cooperation frequencies in one layer increase while those in another layer decrease or remain constant. However, we found that this broken symmetry disappeared once the speed of link updating became overly fast. In summary, our model shows that network dynamics support cooperation but are accompanied by nontrivial asymmetry once network multiplexity is considered.

**Simulation model**

We consider a duplex network in which $N$ agents occupy each node in each layer. Each agent will participate in the prisoner’s dilemma game with her direct neighbors. In each layer, agents adopt one of two strategies: cooperation ($C$) or defection ($D$). Agent $i$’s strategy in layer $l$ is denoted by $s^{(l)}_i$. Through the interaction with agent $j$ on layer $l$, agent $i$ acquires payoff $\pi^{(l)}_{s_is_j}$. In the prisoner’s dilemma, the order of the four payoff values is $\pi_{DC} > \pi_{CC} > \pi_{DD} > \pi_{CD}$. Because defection results in a larger payoff regardless of a partner’s decision, less profitable mutual defection tends to be realized. Following convention, we set the value of $\pi_{CC}$ ($\pi_{DD}$) to 1 (0) and controlled the harshness of social dilemmas through the values of $\pi_{DC}$ and $\pi_{CD}$.

In their initial states, agents are located on symmetrical Erdős-Rényi networks in which the two layers share the same set of agents and edges. Links between each pair of nodes are generated with a probability of $p$. Agents’ strategies in each layer are randomly assigned. Two types of
events modify this system: strategy updating and link updating. In each round, one of these two events occurs; link updating occurs with a probability of $w$ whereas strategy updating occurs with a probability of $1 - w$. This parameter controls the speed of the network dynamics relative to strategy evolution.

During strategy updating, agents may imitate their neighbor’s strategy in a specific layer. In such an event, one layer ($\lambda$) and one link in that layer ($e_{i,j}^{(\lambda)}$) are randomly selected; one of the connected agents is randomly selected to become a focal agent, while the other becomes a role agent [52,75]. In the explanation provided here, agent $i$ becomes a focal agent while agent $j$ becomes a role agent.

In this study, we adopt simple network interdependency and assume that agents’ payoffs are determined by the interactions with all of their neighbors on both layers [38]. Formally, the focal agent’s payoff is determined as follows:

$$\Pi_i = \sum_{l \in \{1,2\}} \sum_{k \in N(i)} \pi_{s_i^{(l)} s_k^{(l)}} / (z_i^{(1)} + z_i^{(2)}),$$

where $N(i)$ is the set of the focal agent’s neighbors in layer $l$. The accumulated payoff is regularized by the sum of the agent’s degree in two layers ($z_i^{(1)} + z_i^{(2)}$). The role agent earns her payoff, $\Pi_j$, in the same manner. The focal agent may imitate the role agent’s strategy on layer $\lambda$ when the role agent acquired larger payoff; the imitation probability is given by Wu et al. [76] as follows:

$$P(s_i^{(\lambda)} \leftarrow s_j^{(\lambda)}) = \max \left\{ (\Pi_j - \Pi_i) / (\pi_{DC} - \pi_{CD}), 0 \right\}.$$  

Here, payoff accumulation introduces interdependency between two layers but strategy transmission occurs only on the selected layer.

Strategy mutation occurs with a small probability ($\mu$) in strategy updating. In mutations, agents ignore the results of the payoff-based imitation described above and instead adopt one strategy randomly. Mutation causes small perturbations in the system and prevents spurious frozen states. These small perturbations are known to have significant impacts on various types of model [77].

In link updating, cooperative agents may sever a link with a defecting neighbor and form a new link. First, one link, $e_{i,j}^{(\lambda)}$, is selected in the same manner as in strategy updating, and a focal agent ($i$ in this example) is also selected. Agent $i$ severs the link with $j$ when $s_i^{(\lambda)} = C$ and $s_j^{(\lambda)} = D$, and then rewires that link to a randomly selected agent; nothing occurs in other combinations of the agents’ strategy [52]. Because accepting a link with a defecting agent does not improve the average payoff, defecting agents cannot rewire their links.

We conducted Monte Carlo simulations to examine this model. In these simulations, the relaxation process continued $4N \times 10^4 - 2N \times 10^5$ periods; subsequently, the sampling process continued $2N \times 10^4 - 5N \times 10^5$ periods. In order to enhance statistical accuracy, we conducted at least ten simulation runs for each combination of parameters. We recorded the mean cooperation frequencies of each layer ($\rho_C^{(1)}$ and $\rho_C^{(2)}$) in order to report simulation results. In addition, we recorded the absolute difference of cooperation frequencies across two layers at each period and calculated the average of these values ($\rho_C^{\lambda}$).

**Results and Discussions**

The average cooperation frequencies in two layers ($\rho_C = (\rho_C^{(1)} + \rho_C^{(2)}) / 2$) are given as a function of the frequency of link updating ($w$) to evaluate the overall cooperation level (Figure 1). With many of the payoff value combinations, the introduction of network dynamics has positive impacts on cooperation. Where $w$ is very large, the system escapes from almost non-cooperative states. The only exception to this positive impact of link updating is observed when payoff values are advantageous for cooperators ($\pi_{CD} = -0.05$ and $\pi_{DC} = 1.02$).

Although network dynamics increase cooperation up to
a point, further increases in \( w \) result in decreasing cooperation levels, which has also been observed in one-layer dynamic networks \([52]\). Furthermore, cooperation frequency in the network is about 0.5 even with optimal values of \( w \). This result contrasts with another model of games run on dynamic multiplex networks in which full cooperation was achieved with sufficiently fast link updating \([74]\).

Symmetry breaking across two layers is related to the modest cooperation enhancement shown in Figure 1. Figure 2 shows the absolute difference in the cooperation frequencies in two layers \((\Delta \rho^C)\), which helps to assess if the system shows symmetric behavior. The parameter values in Figure 2 are the same as those in Figure 1. Two layers show similar cooperation frequencies with fixed edges \((w = 0)\) and this pattern holds as long as the values of \( w \) remain small. Fixed networks also show asymmetric results with some parameter values. Previous studies have also reported symmetry breaking on static networks \([40, 42]\). However, as Figure 2 shows, this result occurs only with a limited combination of parameter values and the effects are small compared to those observed with link updating.

With moderate values of \( w \), the evolutionary process leads to clear symmetry breaking across two layers. In comparison to Figure 1, Figure 2 shows that this broken symmetry is accompanied by an increase in \( \rho_C \), which demonstrates that cooperation enhancement is uneven across two layers. Because the cooperation frequencies in only one layer increase, the overall enhancement of cooperation is modest. Further increases in \( w \), however, result in restored symmetry that corresponds to the diminishing cooperation frequencies shown in Figure 1.

We also examined the cooperation level in two layers separately to assess the evolutionary outcomes. Here \( \rho_{C}^{(\text{max})} \) and \( \rho_{C}^{(\text{min})} \) denote the average frequency of cooperators in a layer that showed higher and lower cooperation frequencies at each period, respectively. Figure 3 shows that one layer disproportionately enjoys the benefit of network dynamics. Specifically, panel (a) shows that network dynamics enhance cooperation in one layer at the expense of diminishing cooperation levels in the other layer. This result suggests that network dynamics destabilize the consistent selection of cooperation across two layers. Additionally, panel (b) shows that the introduction of link updating leads to cooperation enhancement in only one layer: cooperation frequencies in the other layer remain almost zero.

Broken symmetry is observed in the model with a wide range of payoff values. In Figure 4, the value of \( w \) was set to 0.5 and the values of two payoff parameters, \( \pi_{CD} \) and \( \pi_{DC} \), were varied. The upper panel shows that \( \rho_C \) is about 0.5 with large \( \pi_{CD} \) and small \( \pi_{DC} \), whereas small values of \( \rho_C \) are observed in the opposite scenario. Between these two scenarios, moderate values of \( \rho_C \) are observed, and this region corresponds to large \( \rho_{C}^{\text{max}} \), as shown in the lower panel.

To investigate how the system reaches an asymmetric state, the time evolution of strategy frequencies was assessed and is shown in Figure 5. The values of \( \rho_{CC} \), \( \rho_{CD} \), and \( \rho_{DD} \) show the frequencies of agents who adopt cooperation on both, either, or neither layers, respectively.
Without link updating ($w = 0$), the characteristic pattern observed in evolutionary games on networks is also observed in our model (panel (a)). Cooperation frequency increases due to the formation of cooperative clusters after the initial enduring periods [78]. This pattern is observed with the values of $\rho_{CC}$ and $\rho_{CD}$, which indicate that both layers enjoy the benefit of network reciprocity.

As shown in Figure 5, the adoption of cooperation in both layers cannot produce stable outcomes with link updating (panel (b)); the value of $\rho_{CC}$ decreases after reaching its peak. This pattern can be attributed to agents achieving a large average payoff if they achieve mutual cooperation in one layer. Although defection leads to the loss of links with cooperators, interactions in a cooperative layer mainly determine the average payoff as long as degree in that layer is sufficiently larger than degree in a low-cooperation layer. In a previous study that demonstrated full cooperation with fast link updating [74], activated agents chose one neighbor from two layers and payoff was determined by two game interactions; thus, two layers were counted equally in payoff calculation. We surmise that this difference in the payoff-collecting mechanism contributes to the different evolutionary outcomes observed.

The instability of cooperation in both layers can also be investigated using simulation runs with different initial cooperation frequencies. In panel (c) where the initial frequency of cooperation in each layer is 0.9, the frequency of agents who select cooperation on both layers diminishes over time. This pattern is observed in our model (panel (a)). Cooperation frequency increases due to the formation of cooperative clusters after the initial enduring periods [78]. This pattern is observed in evolutionary games on networks is also observed.

To further understand the time evolution of the system, figure 6 shows the cooperation level of the two layers in one simulation run. One layer shows stably higher cooperation levels with smaller $w$ but larger $w$ leads to alternation of cooperative layers. Other parameters: $N = 1000, p = 0.01, \mu = 10^{-4}, \pi_{CD} = -0.2$, and $\pi_{DC} = 1.02$. In contrast, fast link updating ($w = 0.55$) leads to alternation of cooperative layers that betokens the restored symmetry with further large $w$.

To investigate how the system reaches this asymmet-
Fig. 7 Frequency of cooperators as a function of degree in the layer that achieves a higher cooperation level at $t = 15000N$. Panel (a) shows the frequencies of cooperators in the high-cooperation layer whereas panel (b) shows the same in the low-cooperation layer. Degree in the high-cooperation layer and cooperation frequency in the low-cooperation layer show U-shaped relationships. Other parameters: $N = 1000$, $p = 0.01$, $w = 0.1$, $\mu = 10^{-4}$, $\pi_{CD} = -0.2$, and $\pi_{DC} = 1.02$.

In contrast, we observed U-shaped relationships between the cooperation frequency in the low-cooperation layer and the degree in the high-cooperation layer (panel (a)). Therefore, cooperators can apparently survive in the low-cooperation layer in two scenarios. First, free-riders who gain a large payoff in the high-cooperation layer can continue to choose cooperation in the low-cooperation layer. This explains the higher cooperation frequency observed among agents whose degree in the high-cooperation layer was small (a small degree suggests that the agents chose defection). Second, agents with a large degree in the high-cooperation layer can remain cooperative because interactions in the low-cooperation layer have a small impact on payoff. Although the cooperation frequency of the groups with the largest degree is noisy due to the small number of observations, these scenarios help to explain the observed relationships. This pattern holds until the cooperation level on the low-cooperation layer approaches zero.

Finally, we examined the robustness of the results using a larger ($N = 20000$) or smaller ($N = 250$) network size. Figure 8 shows $\rho^A_C$ as a function of $w$ in the same manner as Figure 2. As already shown in our network with $N = 1000$, we confirmed that the system shows symmetry breaking with moderate values of $w$ in different sized networks.

**Conclusion**

Here, we evaluated the evolutionary prisoner’s dilemma game played on a dynamic duplex network. The introduction of network dynamics led to enhanced cooperation but the resultant states were far from those of full cooperation. The pattern we observed was related to broken symmetry, whereby link updating led to enhanced cooperation in one layer while cooperation frequencies in another layer remained the same or deteriorated. This state was maintained as long as the frequency of link updating was not overly high. The robust findings of previous studies have demonstrated that network dynamics facilitate cooperation [44]. Our results show that the ramifications of link updating become more nuanced once network multi-
plexity is considered.
Lastly, we consider the potential future extensions of this study. Our simulation results suggest that individuals can show incoherent behavior across multiple social domains: they may choose cooperation in one domain but choose a defecting option in another domain. Future studies could therefore examine under which conditions individuals tend to show coherent behavior in multiple social domains. For example, the present study evaluated average payoff but additional studies might also consider accumulated payoff and a combination of the two methods [79]. Furthermore, although our study relies on imitation-based evolution (which to date has been widely adopted), other studies have indicated that the strategy-updating rule has a significant role in the evolution of cooperation [80]. Studies in these areas may further contribute to our understanding of the conditions under which network multiplexity and network dynamics affect the evolution of cooperation.

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