Rate-Splitting Multiple Access for Quantized Multiuser MIMO Communications

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Abstract—This paper investigates the sum spectral efficiency maximization problem in downlink multiuser multiple-input multiple-output systems with low-resolution quantizers at an access point (AP) and users. We consider rate-splitting multiple access (RSMA) to enhance spectral efficiency by offering opportunities to boost achievable degree-of-freedom. Optimizing RSMA precoders, however, is highly challenging due to the minimum rate constraint when determining the common rate. The quantization errors coupled with the precoders make the problem more complicated. In this paper, we develop a novel RSMA precoding algorithm incorporating quantization errors for maximizing the sum spectral efficiency. To this end, we first obtain an approximate spectral efficiency in a smooth function. Subsequently, we derive the first-order optimality condition in the form of the nonlinear eigenvalue problem (NEP). We propose a computationally efficient algorithm to find the principal eigenvector of the NEP as a sub-optimal solution. We also extend the weighted minimum mean square error-based RSMA precoding to the considered quantization system. Simulation results validate the proposed methods. The key benefit of using RSMA over spatial division multiple access (SDMA) comes from the ability of the common stream to balance between the channel gain and quantization error in multiuser MIMO systems with different quantization resolutions.

Index Terms—Rate-splitting multiple access, low-resolution quantizers, spectral efficiency, precoding, nonlinear eigenvalue problem.

I. INTRODUCTION

NOWADAYS, 6G wireless communication has drawn significant attention beyond the era of 5G [1]. Accordingly, low-power yet high-speed wireless communications have become more indispensable; there are applications such as the internet-of-things (IoT) in which devices tend to be battery-limited and to have low computing capability while requiring high spectral efficiency [2]. We can alleviate the power consumption issue by utilizing low-power hardware, such as low-resolution analog-to-digital converter (ADC) and digital-to-analog converter (DAC), since the power consumption of the quantizers decreases exponentially according to the reduction of quantization bits [3]. Motivated by the power saving in using low-resolution quantizers, low-resolution quantization systems or mixed-resolution quantization systems where high- and low-resolution quantizers coexist have been widely studied [4], [5], [6].

Another key challenge needed to be addressed for the realization of 6G wireless communications is the severe inter-user interference due to the dramatic increase in the number of smart devices. In IoT communications, for example, the increase in the number of IoT devices and the high channel correlation among IoT devices [2] can incur a significant amount of inter-user interference, thereby leading to considerable performance degradation in spectral efficiency. In this regard, rate-splitting multiple access (RSMA) introduced in [7] for downlink multi-antenna wireless networks as a unified multiple access scheme can be an effective solution in overcoming the limits of the spectral efficiency gain in multiuser multiple-input-multiple-output (MU-MIMO) systems by reducing the inter-user interference [8], [9], [10], [11]. In this paper, we consider RSMA for downlink MU-MIMO systems with low-resolution quantizers and develop a novel and computationally efficient precoding method to maximize the spectral efficiency.

A. Prior Work

The information-theoretic performance limits when using low-resolution DACs and ADCs have been widely studied in the literature. The capacity of uniformly distributed quadrature phase shift keying (QPSK) was achieved in a quantized single-input single-output (SISO) additive white Gaussian noise (AWGN) channel [12]. In addition, the capacity of multiple-input single-output (MISO) fading channel with one-bit ADCs was derived in a closed form [13]. The analytical performance of low-resolution quantization systems was further studied via linear approximation of quantization process, namely, Bussgang decomposition [14] and additive quantization noise model (AQNM) [15]. In [14], the lower bound on the...
The achievable rate of the quantized MIMO channel was derived based on the Bussgang decomposition. In addition, in [15], optimal bandwidth and resolution of ADC for the SISO channel were analyzed employing the AQNM.

The existing conventional precoding methods for a perfect quantization system revealed highly limited spectral efficiency due to the non-negligible quantization error which was not taken into account [16], [17]. Accordingly, in [16], conventional precoding methods such as minimum mean square error (MMSE) or zero-forcing (ZF) for 3 to 4-bit DACs were proposed. In particular, the proposed precoders with 3 to 4-bit DACs achieved comparable performance with the high-resolution DAC system. The alternating direction method of multipliers (ADMM) was also used to solve the inter-user interference minimization problem in massive MU-MIMO systems with low-resolution DACs [18].

To design the precoder for downlink MU-MIMO systems with heterogeneous-resolution DACs and ADCs where the DACs (also ADCs) have different resolutions from each other, a generalized power iteration-based algorithm was proposed to maximize the energy efficiency [17]. Mixed DACs and ADCs architectures which are the special cases of the heterogeneous-resolution DACs and ADCs were further investigated, revealing the potentials in increasing the spectral efficiency compared to the case of homogeneous-resolution DACs and ADCs where the DACs (also ADCs) have the same resolutions [4], [19].

One-bit quantization systems have also been investigated to develop state-of-the-art channel estimation and detection techniques because of the implementation practicality and analytical tractability of one-bit quantizers [20], [21], [22], [23]. Channel estimation or detection techniques for a one-bit quantization system were developed by using a maximum likelihood detector [20] and Bussgang decomposition [21]. In addition, learning-based detection methods without the necessity of explicit channel estimation were proposed in [22] and [23]. Although the prior precoding methods for low-resolution quantization systems achieved high improvement in spectral efficiency, they are still limited to a conventional signaling method that may not be optimal in the system with severe inter-user interference. We also consider an advanced signaling method for low-resolution quantization systems to further improve spectral efficiency by managing the inter-user interference while reducing the power consumption at transceivers.

RSMA has emerged as a promising enabler in a wide range of applications [33], [34]. The performance analysis of RSMA in low-resolution quantization systems with regularized zero-forcing (RZF) was presented in [35]. In [36], a precoder design algorithm based on the ADMM of multi-antenna joint radar-communication (JRC) system in RSMA was proposed to solve the joint sum rate maximization problem considering low-resolution DACs. Furthermore, in [37], ADMM-based algorithm for energy-efficient dual-functional radar-communication in RSMA was proposed with low-resolution DACs and RF chain selection.

We remark that the prior works of RSMA successfully showed the benefits of RSMA and developed state-of-the-art RSMA transmission methods. However, a rigorous investigation on a precoding design problem for downlink RSMA considering the general number of DAC and ADC bits is still missing. Although in [36] and [37] the spectral and energy efficiencies of RSMA were studied for low-resolution DACs, the derived results are constrained to systems with homogeneous DACs at the transmitter and perfect ADCs at the receiver. In addition, the proposed methods in [36] and [37] are optimized for the joint radar and communications systems, which may lead to sub-optimal performance when considering communication only. More importantly, to compute the spectral efficiency in RSMA, it is necessary to consider the minimum rate of the common stream.
which is a non-smooth function. In most of the literature, such a non-smooth optimization problem induced by the minimum rate condition was often solved using the convex relaxation method based on the CVX toolbox. The existing optimization methods which are based on the CVX [36], [37] have high computational complexity. In this regard, developing a computationally efficient precoding method with improved performance is necessary to maximize the sum spectral efficiency for downlink RSMA systems with heterogeneous DACs and ADCs.

**B. Contributions**

In this paper, we propose a precoding optimization framework and study the effect of RSMA under coarse quantization systems. We summarize our contributions as follows:

- We consider an RSMA system in which a multi-antenna AP with low-resolution DACs of arbitrary resolutions transmits a common stream and private streams via linear precoding to single-antenna users with low-resolution ADCs of arbitrary resolutions. This starkly contrasts with [24], [36], and [37] where no quantization error or only DAC quantization error was considered with homogeneous DACs. In addition, we remark that our work further considers low-resolution ADCs as well as DACs for RSMA transmission. In other words, we consider not only heterogeneous DACs by generalizing the existing homogeneous DAC architecture but also heterogeneous ADCs for the advanced analysis of RSMA, which makes the problem more complicated. We then formulate the sum spectral efficiency maximization problem for the considered system. Since the common stream needs to be decodable by all RSMA users, the common rate becomes the minimum rate supportable by all users.

- We propose a novel and computationally efficient precoding method for solving the problem. To this end, we first utilize the LogSumExp approximation technique to convert the non-smooth minimum rate function into a tractable form. Since the minimum rate condition for the common stream was often handled by introducing an inequality constraint for each user’s common stream [24], [36], [37], such an approximation contributes to reducing algorithm complexity. Then, reformulating the sum spectral efficiency further, we establish the first-order optimality condition to identify stationary points. We interpret the derived condition as a nonlinear eigenvalue problem (NEP) [38] in which the eigenvalue and eigenvector correspond to the sum spectral efficiency and precoding vector, respectively. Hence, finding the leading eigenvector is equivalent to finding the best local optimal point. Based on the insight, we propose a quantized generalized power iteration for rate-splitting (Q-GPI-RS) algorithm by leveraging that a power iteration efficiently seeks the principal eigenvector. We also extend the WMMSE-based RSMA precoding method [24] to the considered system for comprehensiveness because its design principle that relies on the duality between the weighted sum rate maximization and WMMSE is different from Q-GPI-RS. This method is called quantization-aware WMMSE-based alternating optimization (Q-WMMSE-AO).

- Extensive numerical results show that Q-GPI-RS outperforms the conventional linear precoding method such as regularized ZF (RZF) in terms of spectral efficiency. Q-WMMSE-AO improves the spectral efficiency and shows higher robustness compared to the conventional WMMSE-AO approach [24], and Q-GPI-RS method further outperforms Q-WMMSE-AO in most environments. In addition, compared to spatial division multiple access (SDMA), RSMA offers a noticeable spectral efficiency gain in the medium to high signal-to-noise (SNR) regime where the performance is limited by the inter-user interference and quantization error. Therefore, the simulation results validate not only the effectiveness of the proposed methods, but also the benefit of RSMA over SDMA.

- Key findings from the numerical results are: (i) as shown analytically in [39], RSMA has been known to offer larger improvement of spectral efficiency with higher channel correlation, and we confirm such benefit of RSMA also in the presence of DAC and ADC quantization errors. (ii) Considering homogeneous DACs and ADCs, the spectral efficiency gain of RSMA increases with the resolutions due to the fact that quantization errors involved with the common stream cannot be canceled at the users. More importantly, we observe that the effect of ADC is more dominant than DAC in deciding the rate of the common stream. The findings indicate the importance of considering ADC quantization errors as well as DAC quantization errors in developing a transmission strategy. (iii) In the mixed-resolution DAC systems, antennas with low-resolution DACs tend to be turned off in the high SNR to reduce the quantization error for RSMA, which is not the case for SDMA. This phenomenon may lead to overloaded systems due to the insufficient number of active antennas, and thus RSMA can play a key role in maximizing the sum spectral efficiency by leveraging the common stream.

**Notation:** a is a scalar, a is a vector and A is a matrix. The superscripts $(\cdot)^{T}$, $(\cdot)^{H}$, and $(\cdot)^{-1}$ denote matrix transpose, Hermitian, and inversion, respectively. $E[\cdot]$ and $\text{tr}(\cdot)$ represent expectation operation and trace of a matrix, respectively. $I_K$ is the identity matrix of size $K \times K$. $0_N$ is the zero matrix of size $N \times N$ and $0_{N \times 1}$ is the zero vector of size $N \times 1$. $A = \text{blkdiag}(A_1,\ldots,A_n,\ldots,A_N)$ is a block diagonal matrix with block diagonal entries of $A_1,\ldots,A_N$.

**II. System Model**

We consider a downlink MU-MIMO system where an access point (AP) with $N$ antennas serves $K$ single-antenna users as shown in Fig. 1. The AP employs DACs with $b_{DAC,n}$-bit resolution where $b_{DAC,n}$ represents the number of quantization bits of the DAC pair at the $n$-th antenna. Each user employs an ADC of $b_{ADC,k}$-bit resolution where $b_{ADC,k}$
represents the number of quantization bits of the ADC pair at $k$-th user.

RSMA transmission is adopted in the considered system as shown in Fig. 1. In particular, we use the 1-layer rate-splitting (RS) architecture of RSMA [11], [24]. At first, we split an individual message $W_k$ into two parts, each of which is a common part $W_k^c$ and a private part $W_k^p$, respectively. Subsequently, we combine and jointly encode the common parts to generate one common message, i.e., $W^c = \{W_1^c, \ldots, W_K^c\}$. Here, the common message $W^c$ is encoded to be a common stream $s_c$ via a public codebook, so that it is aimed to be decoded at every user in the network. On the contrary, the private message $W^p_k$ is encoded to be a private stream $s_k$ via a private codebook, and thus, it is decodable only at the associated user. Once a user receives signal, it decodes the common stream $s_c$, eliminates it via SIC, and decodes the corresponding private stream $s_k$. Here we remark that once the common stream $s_c$ is decoded, user $k$ recovers its common message part $W_k^c$ from the common message $W^c$. We assume Gaussian signaling $s_c, s_k \sim CN(0,1)$, i.e., a complex Gaussian distribution with zero mean and unit variance.

The common stream and private streams are linearly precoded at the AP. The digital baseband signal $x \in \mathbb{C}^N$ is

$$ x = \sqrt{P}f_0s_c + \sqrt{P} \sum_{k=1}^K f_k s_k, \quad (1) $$

where $f_0 \in \mathbb{C}^N$ and $f_k \in \mathbb{C}^N$ are precoding vectors for the common and private streams, respectively, and $P$ is the maximum transmit power. Then, $x$ is quantized at the DACs. We adopt the AQNM method [40] that approximates the quantization process in linear form. We assume heterogeneous-resolution DACs and ADCs and thus, each antenna and each user have different quantization loss. After applying the AQNM, the quantized signal is represented as

$$ Q(x) \approx \varphi_q = \sqrt{P} \Phi_{\text{DAC}} f_0 s_c + \sqrt{P} \Phi_{\text{ADC}} \sum_{k=1}^K f_k s_k + \Phi_{\text{DAC}}, \quad (2) $$

where $Q(\cdot)$ is a scalar quantizer which applies for each real and imaginary part, $\Phi_{\text{DAC}} = \text{diag}(\alpha_{\text{DAC,1}}, \ldots, \alpha_{\text{DAC,N}}) \in \mathbb{C}^{N \times N}$ denotes a diagonal matrix of quantization loss, and $\Phi_{\text{ADC}} \in \mathbb{C}^N$ is a DAC quantization noise vector. The quantization loss of the $n$-th DAC $\alpha_{\text{DAC,n}} \in (0,1)$ is determined as $\alpha_{\text{DAC,n}} = 1 - \beta_{\text{DAC,n}}$, where $\beta_{\text{DAC,n}}$ is a normalized mean squared quantization error $\beta_{\text{DAC,n}} = \mathbb{E}[|x - Q_n(x)|^2]/\mathbb{E}[|x|^2]$.

If the value of $b_{\text{DAC,n}}$ is larger than 5, $\beta_{\text{DAC,n}}$ can be approximated as $\pi \sqrt{3}/2 - 2b_{\text{DAC,n}}$, since the AQNM assumes the scalar MMSE quantizer which provides an optimal centroid condition [42]. The quantization noise is not correlated with digital baseband signal $x$ and considered to follow $q_{\text{DAC}} \sim \mathbb{C}N(0, \sigma^2 I_N)$ which is the worst case in terms of spectral efficiency. Let $F = [f_0, f_1, \ldots, f_K] \in \mathbb{C}^{N \times (K+1)}$. The covariance matrix of $q_{\text{DAC}}$ is computed as [40]

$$ R_{q_{\text{DAC}}} = \Phi_{\text{DAC}} \Phi_{\text{DAC}} \text{diag} \left( \mathbb{E} \left[ xx^H \right] \right). \quad (3) $$

Then, the received analog baseband signal vector at users is

$$ y = H^H x + n, \quad (4) $$

where $H \in \mathbb{C}^{K \times N}$ is a downlink channel matrix between the AP and $K$ users, and $n = \mathbb{C}N(0_{K \times 1}, \sigma^2 I_K)$ is an AWGN with zero mean and variance of $\sigma^2$. We assume that both the AP and users have the perfect knowledge of channel state information (CSI) to focus on the effect of coarse quantization on the RSMA performance. At each user, the received analog signal $y_k$ is quantized by the ADCs of $b_{\text{ADC,k}}$ bits. Then, the received digital baseband signals are represented as [40]

$$ Q(y) \approx y_q = \Phi_{\text{ADC}} y + q_{\text{ADC}} = \sqrt{P} \Phi_{\text{ADC}} H^H \Phi_{\text{DAC}} f_0 s_c + \sqrt{P} \Phi_{\text{ADC}} H^H \Phi_{\text{DAC}} \sum_{k=1}^K f_k s_k + \Phi_{\text{ADC}} H^H \Phi_{\text{DAC}} + \Phi_{\text{ADC}} n + q_{\text{ADC}}, \quad (5) $$

where $\Phi_{\text{ADC}} = \text{diag}(\alpha_{\text{ADC,1}}, \ldots, \alpha_{\text{ADC,K}}) \in \mathbb{C}^{K \times K}$ denotes a diagonal matrix of quantization loss, and $q_{\text{ADC}} \in \mathbb{C}^K$ is a DAC quantization noise vector. The quantization loss of the $k$-th ADC $\alpha_{\text{ADC,k}}$ and $\beta_{\text{ADC,k}}$ is defined as likewise quantization loss of the DAC. The ADC quantization noise $q_{\text{ADC}}$ is not correlated with analog baseband signal $y$ and considered to follow $q_{\text{ADC}} \sim \mathbb{C}N(0_{K \times 1}, R_{q_{\text{ADC,DAC}}})$ which is the worst case in terms of spectral efficiency. The covariance is derived as

$$ R_{q_{\text{ADC}}} = \Phi_{\text{DAC}} \Phi_{\text{DAC}} \text{diag} \left( \mathbb{E} \left[ yy^H \right] \right). \quad (6) $$

Accordingly, the digital baseband signal at user $k$ is represented as

$$ y_{q,k} = \sqrt{P} \alpha_{\text{ADC,k}} \Phi_{\text{DAC}} f_0 \Phi_{\text{ADC}} f_0 s_c + \sqrt{P} \alpha_{\text{ADC,k}} \Phi_{\text{DAC}} f_0 f_k s_k + \Phi_{\text{ADC,k}} \sum_{i=1}^K \Phi_{\text{DAC}} f_i s_i + \alpha_{\text{ADC,k}} \Phi_{\text{DAC}} f_k s_k + q_{\text{ADC,k}}, \quad (7) $$

where $H_k$ is the $k$-th column of the channel matrix $H$. We remark that both the ADC and DAC quantization error terms, $\alpha_{\text{ADC,k}} H_k^H \Phi_{\text{DAC}}$ and $q_{\text{ADC,k}}$, in (7) involve the common stream as well as the private streams and corresponding precoders. Hence, it is expected that without properly designing the precoder for the common stream it is more difficult to accomplish the potential of RSMA in the considered low-resolution quantization systems than the systems with perfect quantization or low-resolution DACs.

1 Although ADC and DAC quantization on the channel sounding (along with other impairments) would lead to imperfect CSI knowledge, we shall leave the impact of such impairments for future studies.
A. Performance Metrics and Problem Formulation

Recall the decoding principle of RSMA [24]; each user decodes the common stream $s_c$ by treating the private streams as noise. After $s_c$ is successfully decoded, each user can cancel the common stream from the received signal $y$ by using SIC. In this regard, the combination of message split and SIC enables to partially decode interference and treat the remaining interference as noise, thereby increasing the sum spectral efficiency [7]. To successfully perform SIC, the common stream needs to be designed carefully in a way that is decodable to all RSMA users. Accordingly, the rate of the common stream is determined by the minimum of the spectral efficiencies of the common stream of all users. Then, the spectral efficiency of $s_c$ is defined as

$$ R_c = \min_{k \in K} \left\{ \log_2 \left( 1 + \frac{P \alpha_{ADC,k} |h_k^H \Phi_{\alphaDAC} f_0|^2}{IUI_k + QE_k + \alpha_{ADC,k} \sigma^2} \right) \right\} $$

$$ = \min_{k \in K} \{ R_{c,k} \}, $$

(8)

where

$$ IUI_k = P \alpha_{ADC,k} \sum_{i=1}^{K} |h_i^H \Phi_{\alphaDAC} f_i|^2, $$

$$ QE_k = \alpha_{ADC,k}^2 h_k^H R_{\alphaDAC \alphaDAC} h_k + r_{\alphaDAC,k \alphaDAC,k}. $$

(9)

After decoding and eliminating the common stream using SIC, the achievable spectral efficiency of the private stream $s_k$ of user $k$ is formulated as

$$ R_k = \log_2 \left( 1 + \frac{P \alpha_{ADC,k} |h_k^H \Phi_{\alphaDAC} f_i|^2}{IUI_k + QE_k + \alpha_{ADC,k} \sigma^2} \right), $$

(10)

where $IUI_k = P \alpha_{ADC,k} \sum_{i \neq k, i \in K} |h_i^H \Phi_{\alphaDAC} f_i|^2$.

We note that an individual value of $R_{c,k}$ does not affect the sum rate. Regarding the division between common and private streams, different division would lead to a different rate performance. Consequently, we remark that our primary goal is to maximize the sum rate of RSMA, wherein the rates of the common stream and the private stream are jointly identified as the preceding vectors are designed. This indicates that the derived precoding solution will optimize the rate division between the common stream and private stream to maximize the sum rate.

Remark 1 (SIC and error propagation): We note that when performing SIC users can only remove the quantized common stream, i.e., $\sqrt{P} \alpha_{ADC,k} h_k^H \Phi_{\alphaDAC} f_0 s_c$, not the quantization errors that come from the common stream. Under the AQNM, we consider that the additive quantization error follows a Gaussian distribution which is the worst case in terms of the spectral efficiency. Accordingly, the derived common rate is considered to be conservative. In this regard, we assume perfect decoding of the common stream if the derived rate of the common stream is less than or equal to the minimum of the common rates and thus, no SIC error propagation occurs.

We note that the user signals are quantized at the DACs and ADCs, and the quantization noise error terms appear in the rates of the common and private streams. To maximize the sum spectral efficiency, we formulate the problem as

$$ \begin{align*}
\text{maximize} & \quad R_c + \sum_{k=1}^{K} R_k = R_{\Sigma} \\
\text{subject to} & \quad \text{tr} \left( \mathbb{E} \left[ x_q x_q^H \right] \right) \leq P,
\end{align*} $$

(11)

(12)

where (12) is the transmit power constraint. Since the weight matrix of the quantization losses is intertwined with the precoder and channel matrix, and the RSMA transmission introduces the common rate with such a quantization loss matrix, we need to carefully handle our optimization problem for incorporating the quantization error. In the following section, we propose a novel computationally efficient precoding method to solve (11).

III. PRECODER OPTIMIZATION USING GENERALIZED POWER ITERATION

Since the direct solution of the problem in (11) is not available due to the inherent non-convexity and non-smoothness, we present key techniques to convert the problem to a tractable form and solve the reformulated problem.

A. Problem Reformulation

We first simplify the constraint in (12). The covariance matrix of $q_{\alphaDAC}$ in (3) is derived as

$$ R_{\alphaDAC \alphaDAC} = \Phi_{\alphaDAC} \Phi_{\alphaDAC} \text{diag} \left( P f_i^H f_i + P \sum_{i=1}^{K} f_i^H f_i \right) \Phi_{\alphaDAC}^H = \Phi_{\alphaDAC} \Phi_{\alphaDAC} \text{diag} \left( P F F^H \right). $$

(13)

(14)

Then, the power constraint in (12) is reformulated as

$$ \text{tr} \left( \mathbb{E} \left[ x_q x_q^H \right] \right) = \text{tr} \left( P \Phi_{\alphaDAC} \sum_{i=0}^{K} f_i^H f_i \Phi_{\alphaDAC}^H + R_{\alphaDAC \alphaDAC} \Phi_{\alphaDAC} \right) \approx \text{tr} \left( P \Phi_{\alphaDAC} FF^H \right) \leq P, $$

(15)

(16)

(17)

where (a) comes from (14) and $F = I_N - \Phi_{\alphaDAC}$. Finally, the constraint in (12) reduces to

$$ \text{tr} \left( \Phi_{\alphaDAC} FF^H \right) \leq 1. $$

(18)

Now, to address the challenges of the non-smoothness of the minimum operation and non-convexity of the spectral efficiency, we approximate the minimum rate of $R_{c,k}$ and further reformulate the problem into a tractable form. Let us define a positive constant $\tau > 0$. We utilize the LogSumExp technique to approximate the minimum function as [43]

$$ \min_{i=1,\ldots,N} \left\{ x_i \right\} \approx -\tau \ln \left( \sum_{i=1}^{N} \exp \left( -\frac{1}{\tau} x_i \right) \right), $$

(19)

where (19) becomes tight as $\tau \to 0$. Applying (19) to the common rate in (8), we have

$$ \min_{k \in K} \{ R_{c,k} \} \approx -\tau \ln \left( \sum_{k=1}^{K} \exp \left( -\frac{1}{\tau} R_{c,k} \right) \right). $$

(20)
We remark that once we obtain precoders using the approximation, we re-compute the achievable spectral efficiency by using the minimum operation without approximation to determine the actual common rate. Although the approximation in (20) converts the objective function in (11) to a smooth function, the non-convexity still resides in the problem which cannot be avoided, and the quantization errors which are the functions of the preceding vectors still make the problem more complicated to solve. To resolve the difficulty, we now re-express QE in (9) by reorganizing the DAC quantization error covariance-related term as

\[ h_k^H R_{\text{qDAC}} q_{\text{DAC}} h_k = h_k^H \Phi_{\alpha_{\text{DAC}}} \Phi_{\beta_{\text{DAC}}} \text{diag} \left( P \sum_{i=0}^{K} f_i f_i^H \right) h_k \]

\[ = P \sum_{i=0}^{K} f_i^H \Phi_{\alpha_{\text{DAC}}} \Phi_{\beta_{\text{DAC}}} \text{diag} \left( h_k^H h_k^H \right) f_i, \]

(21)

and the ADC quantization error covariance-related term as

\[ r_{\text{qADC}, \text{qADC}, k}^{\alpha_{\text{ADC}, k}/\text{ADC}, k} \]

\[ = h_k^H E \left[ x_n x_n^H \right] h_k + \sigma^2 \]

\[ = h_k^H \left( P \Phi_{\alpha_{\text{DAC}}} \sum_{i=0}^{K} f_i f_i^H \Phi_{\alpha_{\text{DAC}}} + R_{\text{qDAC}} q_{\text{DAC}} \right) h_k + \sigma^2 \]

(22)

\[ = P \sum_{i=0}^{K} f_i^H \Phi_{\alpha_{\text{DAC}}} h_k^H \Phi_{\alpha_{\text{DAC}}} + \Phi_{\alpha_{\text{DAC}}} \Phi_{\beta_{\text{DAC}}} \text{diag} \left( h_k^H h_k^H \right) f_i \]

\[ + \sigma^2, \]

(25)

where \( r_{\text{qADC}, \text{qADC}, k}^{\alpha_{\text{ADC}, k}/\text{ADC}, k} \) represents the \( k \)th diagonal entry of (6) and (a) follows from (22). Using (22) and (25), the SINRs of the common stream of user \( k \) is reorganized as (26), shown at the bottom of the next page. Similarly, the SINR of the private stream of user \( k \) is formulated in (27), shown at the bottom of the next page.

**Remark 2 (Effect of ADC and DAC quantization errors to RSMA):** To compare the effect of quantization errors from ADCs and DACs specifically to the use of the common stream, let us first assume that the quantization error comes only from ADCs. Then (26) becomes

\[ \gamma_{\text{ADC}} = \frac{\alpha_{\text{ADC}, k} | h_k^H f_i |^2}{\sum_{i=1}^{K} | h_k^H f_i |^2 + \left( 1 - \alpha_{\text{ADC}, k} \right) | h_k^H f_i |^2 + \sigma^2}, \]

(28)

Now, let us assume that the quantization error occurs only from DACs and they are homogeneous. Then (26) is represented as in (29), shown at the bottom of the next page, where \( 0 < \epsilon < 1 \) is a positive constant value less than one by assuming \( | f_i^H h_k^H |^2 > | f_0^H \text{diag}(h_k^H h_k^H) f_0 | \), since the precoding vector for the common stream \( f_0 \) should be constructively combined for the channels of the MSMA users, taking only the diagonal terms of \( (h_k^H h_k^H) \) deteriorates the design goal and reduces beamforming gains. Then, when comparing (28) and (29), the quantization error from \( s_c \) tends to be worse in (28) with the same resolution, thereby preventing the common stream more from using higher transmit power. Since there is no difference between the common and private SINRs in terms of the quantization errors, the same occurs for the SINRs of the private streams, i.e., the SINR of the private stream has the quantization error term associated with the common stream under ADC and DAC quantizations as \( (1 - \alpha_{\text{ADC}, k}) | h_k^H f_i |^2 \) and \( \epsilon (1 - \alpha_{\text{DAC}}) | h_k^H f_0 |^2 \), respectively. This indicates that allocating more power to the common stream gives more penalty to the private streams when using low-resolution ADCs than low-resolution DACs. In other words, the DAC quantizes the signals before the common precoder is constructively combined with the user channels, whereas the ADC quantizes the signals after the precoder is constructively combined with the channels, thereby providing larger quantization error. Therefore, it is concluded that using low-resolution ADCs tends to limit the use of the common stream than using low-resolution DACs for the same resolution. This will be confirmed in Section V.C.

**Remark 3 (Effect of ADC and DAC quantization errors to SMDA):** The behavior of the quantization error to the common stream in Remark 2 is due to the multicast nature of the common stream; \( f_0 \) needs to be constructively combined with all RSMA user channels, and thus, the significance of ADC and DAC quantization error is not the same to the use of private streams as analyzed as follows: if we consider SDMA, we have the SINRs of user \( k \) as

\[ \gamma_{\text{ADC}} = \frac{\alpha_{\text{ADC}, k} | h_k^H f_i |^2}{\sum_{i=1}^{K} | h_k^H f_i |^2 + \left( 1 - \alpha_{\text{ADC}, k} \right) | h_k^H f_i |^2 + \sigma^2}, \]

(29)

and (31), shown at the bottom of the next page. We note that when \( | f_i^H h_k^H |^2 = f_i^H \text{diag}(h_k^H h_k^H) f_i \), (30) and (31) are equal for the same ADC and DAC resolutions. For \( i = k \), we tend to have \( | f_i^H h_k^H |^2 > f_i^H \text{diag}(h_k^H h_k^H) f_i \), because \( f_k \) should be designed to be constructively combined with \( h_k \). For \( i \neq k \), however, we can assume \( | f_i^H h_k^H |^2 < f_i^H \text{diag}(h_k^H h_k^H) f_i \), since \( f_i \) is normally designed to nullify the interfering channel \( h_k \), but diagonalization of \( h_k^H h_k^H \) may break the goal. Hence, the DAC quantization error from the private streams may increase or decrease the SINR compared to the SINR under ADC quantization error. Therefore, no such claim in Remark 2 can be made for the use of private streams.

Now, we define the weighted precoding vector of user \( k \) as

\[ w_k = \Phi_{\beta_{\text{DAC}}}^{-1/2} f_k. \]

(32)

Let \( W = [w_0, w_1, \ldots, w_K] \). We vectorize the weighted precoding matrix \( W \) as \( \tilde{w} = \text{vec}(W) \). Here, we assume \( \text{tr}(W W^H) = 1 \) which indicates that the AP uses the maximum transmit power \( P \), which is optimal in terms of maximizing the spectral efficiency. Let \( G_k = (\Phi_{\beta_{\text{DAC}}}^{-1/2} h_k^H h_k^H \Phi_{\beta_{\text{DAC}}} + \Phi_{\beta_{\text{DAC}}} \text{diag}(h_k^H h_k^H)) \). Then using the SINRs in (26) and (27) with the vectorized weighted precoder \( \tilde{w} \), we represent \( R_{c,k} \) in a Rayleigh quotient form as

\[ R_{c,k} = \log_2 \left( \frac{\tilde{w}^H A_{c,k} \tilde{w}}{\tilde{w}^H B_{c,k} \tilde{w}} \right), \]

(33)
where
\[
\mathbf{A}_{c,k} = \text{blkdiag}(\mathbf{G}_k, \cdots, \mathbf{G}_K) + \mathbf{I}_{N(K + 1)} \frac{\sigma^2}{P},
\]
\[
\mathbf{B}_{c,k} = \mathbf{A}_{c,k} - \text{blkdiag}\left(\sigma_{\text{ADC},k} \mathbf{F}^{1/2}_{\text{DAC}}, \mathbf{h}_k^H \mathbf{h}_k^H \mathbf{F}^{1/2}_{\text{DAC}}, \mathbf{0}_N, \cdots, \mathbf{0}_N\right).
\]

Here, \(\mathbf{A}_{c,k}\) and \(\mathbf{B}_{c,k}\) are the diagonal matrices of size \(N(K + 1) \times N(K + 1)\). Similarly, we cast \(R_k\) into the Rayleigh quotient form as
\[
R_k = \log_2 \left( \frac{\mathbf{w}^H \mathbf{A}_k \mathbf{w}}{\mathbf{w}^H \mathbf{B}_k \mathbf{w}} \right),
\]
where
\[
\mathbf{A}_k = \text{blkdiag} \left( \mathbf{G}_k - \sigma_{\text{ADC},k} \mathbf{F}^{1/2}_{\text{DAC}} \mathbf{h}_k^H \mathbf{h}_k^H \mathbf{F}^{1/2}_{\text{DAC}}, \mathbf{G}_k, \cdots, \mathbf{G}_K \right) + \mathbf{I}_{N(K + 1)} \frac{\sigma^2}{P},
\]
\[
\mathbf{B}_k = \mathbf{A}_k - \text{blkdiag} \left( \mathbf{0}_N, \cdots, \sigma_{\text{ADC},k} \mathbf{F}^{1/2}_{\text{DAC}} \mathbf{h}_k^H \mathbf{h}_k^H \mathbf{F}^{1/2}_{\text{DAC}}, \mathbf{0}_N, \cdots, \mathbf{0}_N \right) \quad \text{(the (k + 1)th block)}.
\]

Finally, based on (20), (33), and (36), the optimization problem in (11) is reformulated as
\[
\begin{align*}
\text{maximize} \quad & \ln \left( \sum_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_{c,k} \mathbf{w}}{\mathbf{w}^H \mathbf{B}_{c,k} \mathbf{w}} \right)^{-\tau} \right) - \tau \\
+ & \frac{1}{\ln 2} \sum_{k=1}^{K} \ln \left( \frac{\mathbf{w}^H \mathbf{A}_k \mathbf{w}}{\mathbf{w}^H \mathbf{B}_k \mathbf{w}} \right) \quad \text{(38)} \\
\text{subject to} \quad & \|\mathbf{w}\| = 1. \quad \text{(39)}
\end{align*}
\]

Recall that the equality constraint in (39) comes from using the maximum transmit power. We remark that the problem in (38) is invariant to \(\mathbf{w}\) up to its scaling. Accordingly, we can effectively ignore the constraint in (39). The problem (38) is still non-convex with respect to the precoder \(\mathbf{w}\) and thus, we cannot find the global optimal solution. We note that (38) can be viewed as the variant of a Rayleigh quotient problem. Inspired by the similarity to the Rayleigh quotient problem, we first identify the first-order KKT condition of (38), then interpret it as the eigenvalue problem with respect to the precoding vector, and finally derive a local optimal solution by identifying a principal eigenvector of the problem.

### B. First-Order Optimality Condition

In this subsection, we derive the first-order optimality condition of (38) with respect to \(\mathbf{w}\).

**Lemma 1:** The first-order optimality condition of the optimization problem (38) is satisfied if the following holds:

\[
\mathbf{B}^{-1}_{\text{KKT}}(\mathbf{w}) \mathbf{A}_{\text{KKT}}(\mathbf{w}) = \lambda(\mathbf{w}) \mathbf{w},
\]

where (41) and (42), shown at the bottom of the next page,

**Proof:** with

\[
\lambda(\mathbf{w}) = \left\{ \frac{1}{K \ln 2} \sum_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_{c,k} \mathbf{w}}{\mathbf{w}^H \mathbf{B}_{c,k} \mathbf{w}} \right)^{-\frac{1}{\tau}} \right\} \left( \prod_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_k \mathbf{w}}{\mathbf{w}^H \mathbf{B}_k \mathbf{w}} \right)^{\frac{1}{\tau}} \right) \quad \text{(43)}
\]

\[
\lambda_{\text{num}} = \prod_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_k \mathbf{w}}{\mathbf{w}^H \mathbf{B}_k \mathbf{w}} \right),
\]

\[
\lambda_{\text{den}} = \left\{ \frac{1}{K \ln 2} \sum_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_{c,k} \mathbf{w}}{\mathbf{w}^H \mathbf{B}_{c,k} \mathbf{w}} \right)^{-\frac{1}{\tau}} \right\} \left( \prod_{k=1}^{K} \left( \frac{\mathbf{w}^H \mathbf{A}_k \mathbf{w}}{\mathbf{w}^H \mathbf{B}_k \mathbf{w}} \right)^{\frac{1}{\tau}} \right) \quad \text{(44)}
\]

See Appendix A.

*Lemma 1* states that the first-order optimality condition can be presented as a nonlinear eigenvalue problem for \(\mathbf{B}^{-1}_{\text{KKT}}(\mathbf{w}) \mathbf{A}_{\text{KKT}}(\mathbf{w})\). If the optimization problem (38) has multiple stationary points, it is possible to exist multiple \(\mathbf{w}\) satisfying (40). In addition, we consider that (40) is represented as a form of the eigenvector problem for the matrix \(\mathbf{B}^{-1}_{\text{KKT}}(\mathbf{w}) \mathbf{A}_{\text{KKT}}(\mathbf{w})\) which is a nonlinear function of the eigenvector \(\mathbf{w}\). In (40), the eigenvalue \(\lambda(\mathbf{w})\) indeed equivalent to the objective function of the problem (38). This means that if we find the leading eigenvector of (40), then
it is one of the stationary point and maximizes the objective function among them. Accordingly, we can obtain the best local optimal precoder, which is stated as follows:

**Proposition 1:** Denoting the leading eigenvector for the problem (40) to be \( \mathbf{w}^* \) and its corresponding eigenvalue to be \( \lambda^* \), i.e., \( \mathbf{B}_{\text{KKT}}^{-1}(\mathbf{w}^*) \mathbf{A}_{\text{KKT}}(\mathbf{w}^*) \mathbf{w}^* = \lambda^* \mathbf{w}^* \), the eigenvector \( \mathbf{w}^* \) is the stationary point that achieves the best local optimal solution of the problem (38).

Based on the observation, we propose a computationally efficient RSMA precoding algorithm that finds the best local optimal point to maximize the sum spectral efficiency.

**C. Quantized Generalized Power Iteration for Rate-Splitting**

To identify the leading eigenvector of (40), we adopt the generalized power iteration-based method [44]. Let \( \mathbf{w}_t \) be the vectorized weighted precoder at the \( t \)-th iteration. Then the matrices \( \mathbf{A}_{\text{KKT}}(\mathbf{w}_t) \) and \( \mathbf{B}_{\text{KKT}}(\mathbf{w}_t) \) are constructed using (41) and (42), and then \( \mathbf{w}_{t+1} \) is updated as

\[
\mathbf{w}_{t+1} = \frac{\mathbf{B}_{\text{KKT}}^{-1}(\mathbf{w}_t) \mathbf{A}_{\text{KKT}}(\mathbf{w}_t) \mathbf{w}_t}{\| \mathbf{B}_{\text{KKT}}^{-1}(\mathbf{w}_t) \mathbf{A}_{\text{KKT}}(\mathbf{w}_t) \mathbf{w}_t \|} \tag{45}
\]

The iteration is repeated until the algorithm satisfies the convergence criterion, i.e., \( \| \mathbf{w}_{t+1} - \mathbf{w}_t \| < \epsilon \) in which \( \epsilon > 0 \) denotes the threshold or reaches a maximum iteration count \( t_{\text{max}} \) which depends on a system requirement. Algorithm 1 summarizes the steps.

**Remark 4 (Algorithm Complexity):** The complexity of Q-GPI-RS depends on the calculation of \( \mathbf{B}_{\text{KKT}}^{-1}(\mathbf{w}) \). Since the matrix size is \( N(K+1) \times N(K+1) \), the inversion of \( \mathbf{B}_{\text{KKT}}(\mathbf{w}) \) typically requires the complexity order of \( O((K+1)^3 N^3) \). Noticing the block diagonal structure of \( \mathbf{B}_{\text{KKT}}(\mathbf{w}) \), only the complexity order of \( O((K+1)N^3) \) is required by computing the inversion of each \( N \times N \) sub-matrix separately since there are \( (K+1) \) sub-matrices.

**Algorithm 1 Quantized Generalized Power Iteration for Rate-Splitting (Q-GPI-RS)**

1. **initialize:** \( \mathbf{w}_0 \)
2. Set the iteration count \( t = 0 \).
3. while \( \| \mathbf{w}_{t+1} - \mathbf{w}_t \| > \epsilon \) \& \( t \leq t_{\text{max}} \) do
   4. Build matrix \( \mathbf{A}_{\text{KKT}}(\mathbf{w}_t) \) in (41).
   5. Build matrix \( \mathbf{B}_{\text{KKT}}(\mathbf{w}_t) \) in (42).
   6. Compute \( \mathbf{w}_{t+1} = \mathbf{B}_{\text{KKT}}^{-1}(\mathbf{w}_t) \mathbf{A}_{\text{KKT}}(\mathbf{w}_t) \mathbf{w}_t \).
   7. Normalize \( \mathbf{w}_{t+1} \) as
   
   \[
   t \leftarrow t + 1.
   \]
4. return \( \mathbf{w}_{t} \).

Let us denote \( T_{\text{GP}} \) as the number of outer iterations of Q-GPI-RS. Then, the total complexity order of Algorithm 1 is given as \( O(T_{\text{GP}}(K+1))^3 N^3) \). Similarly, including the number of iterations, the convex relaxation methods which are based on QCQP [24], [45] and CCCP [10] have the complexity orders of \( O(T_{\text{QCQP}}K^3.5N^{3.5}) \) and \( O(T_{\text{CCCP}}N^6K^{0.5}2^{-1.5}K) \) assuming the number of outer iterations are \( T_{\text{QCQP}} \) and \( T_{\text{CCCP}} \), respectively. In this regard, we note that Q-GPI-RS is still much more efficient in terms of the complexity compared to the other state-of-the-art precoding methods.

Prior to the numerical evaluation of the proposed method, we introduce an optimization approach based on direction extension of an existing state-of-the-art precoding approach for performance comparison in the next section.

**IV. EXTENSION OF EXISTING APPROACH:**

**WMMSE APPROACH**

Here, we briefly present the extension of the WMMSE-based alternating optimization (WMMSE-AO) approach [24] for precoding optimization in the considered system. To solve the non-convex problem of maximizing the sum spectral efficiency with respect to the precoder, [24] developed the WMMSE-AO method without considering the quantization error. Adopting the similar principle of the WMMSE-based algorithms in [24] and [36], we also solve our optimization problem. To this end, we first denote that \( \hat{s}_{c,k} \) and \( \hat{s}_k \) are estimated values for \( s_{c,k} \) and \( s_k \). Defining the scalar equalizers of the common and private streams for user \( k \) as \( g_{c,k} \) and \( g_k \), respectively, we compute the MSEs of the common stream \( \epsilon_{c,k} \) and private stream \( \epsilon_k \) received at user \( k \) as

\[
\epsilon_{c,k} = \mathbb{E}[(\hat{s}_{c,k} - s_c)^2] = \mathbb{E}[(\hat{s}_{c,k}g_{c,k} - s_c)^2] = |g_{c,k}|^2 T_{c,k} - 2\text{Re}\{\sqrt{P}g_{c,k}g_{c,k}^* h_k^H \Phi_{\text{DAC}} f_k\} + 1,
\]

\[
\epsilon_k = \mathbb{E}[(\hat{s}_k - s_k)^2] = \mathbb{E}[(\hat{s}_k - s_k)^2] = |g_k|^2 T_k - 2\text{Re}\{\sqrt{P}g_kg_k^* h_k^H \Phi_{\text{DAC}} f_k\} + 1. \tag{47}
\]

Here, \( T_{c,k} \) and \( T_k \) are defined as

\[
T_{c,k} = P\alpha_{\text{ADC},k}^2 \sum_{i=0}^{K} |h_i^H \Phi_{\text{DAC}} f_i|^2 + \alpha_{\text{ADC},k}^2 h_k^H R_{\text{DAC}q_{\text{ADC}}} h_k + r_{\text{DAC}q_{\text{ADC}},k} + \alpha_{\text{ADC},k}^2 \sigma^2, \tag{48}
\]

\[
T_k = P\alpha_{\text{ADC},k}^2 \sum_{i=1}^{K} |h_i^H \Phi_{\text{DAC}} f_i|^2 + \alpha_{\text{ADC},k}^2 h_k^H R_{\text{DAC}q_{\text{ADC}}} h_k + r_{\text{DAC}q_{\text{ADC}},k} + \alpha_{\text{ADC},k}^2 \sigma^2. \tag{49}
\]
Then, we achieve the minimum MSES when \( g_{c,k} = \sqrt{P_{\text{ADC,k}}} h_k^T \Phi_{\text{ADC,k}} h_k T_{c,k}^{-1} \) and \( g_{k} = \sqrt{P_{\text{ADC,k}}} h_k^T \Phi_{\text{ADC,k}} h_k T_{c,k}^{-1} \). Based on (48), the MMSE of the common stream is
\[
\epsilon_{c,k}^{\text{MMSE}} = T_{c,k}^{-1} (T_{c,k} - P_{\text{ADC,k}} \Phi_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_0)^2 \tag{50}
\]
and the MMSE of the private stream for user \( k \) is
\[
\epsilon_{k}^{\text{MMSE}} = T_{k}^{-1} (T_{k} - P_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_k)^2 \tag{51}
\]
Based on (46), the augmented WMSE of the common stream is defined by
\[
\xi_{c,k} = u_{c,k} \epsilon_{c,k} - \log_2(u_{c,k}) = \frac{P}{\kappa} \sum_{i=0}^{\kappa} f_i^T \left( \alpha_{\text{ADC,k}}^2 u_{c,k} |g_{c,k}|^2 \Phi_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_i \right)
- 2 \Re \left\{ \sqrt{P} u_{c,k} g_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_0 \right\}
+ \alpha_{\text{ADC,k}}^2 u_{c,k} |g_{c,k}|^2 r_{\text{ADC,k}}^T \Phi_{\text{ADC,k}} h_k
+ u_{c,k} |g_{c,k}|^2 r_{\text{ADC,k}}^T \Phi_{\text{ADC,k}} u_{c,k} |g_{c,k}|^2 \sigma^2
+ u_{c,k} - \log_2(u_{c,k}) \tag{52}
\]
Similarly, using (47), the augmented WMSE of the private stream for user \( k \) follows as
\[
\xi_{k} = u_{k} \epsilon_{k} - \log_2(u_{k}) = \frac{P}{\kappa} \sum_{i=1}^{\kappa} f_i^T \left( \alpha_{\text{ADC,k}}^2 u_{k} |g_{k}|^2 \Phi_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_i \right)
- 2 \Re \left\{ \sqrt{P} u_{k} g_{\text{ADC,k}} h_k^T \Phi_{\text{ADC,k}} f_k \right\}
+ \alpha_{\text{ADC,k}}^2 u_{k} |g_{k}|^2 r_{\text{ADC,k}}^T \Phi_{\text{ADC,k}} h_k + u_{k} |g_{k}|^2 r_{\text{ADC,k}}^T \Phi_{\text{ADC,k}} u_{k} |g_{k}|^2 \sigma^2
+ u_{k} - \log_2(u_{k}) \tag{53}
\]
We obtain the optimal weights to achieve the minimum of \( \xi_{c,k} \) and \( \xi_{k} \) as \( u_{c,k} = 1/\epsilon_{c,k}^{\text{MMSE}} \) and \( u_{k} = 1/\epsilon_{k}^{\text{MMSE}} \). Consequently, for given equalizers \( \xi_{c,k} \), \( \xi_{k} \), and weights \( u_{c,k} \), \( u_{k} \), the sum spectral efficiency maximization problem is solved by the following WMSE minimization problem:
\[
\begin{align*}
\text{minimize} & \quad \xi_{c} + \sum_{k=1}^{\kappa} \xi_{k} \tag{54} \\
\text{subject to} & \quad \text{tr} (\Phi_{\text{ADC,k}} F F^H) \leq 1, \tag{55} \\
& \quad \forall k \in K. \tag{56}
\end{align*}
\]
We remark that the problem (54) is the QCQP which can be solved by CVX. Accordingly, we compute the equalizers, weights, and precoders in the alternating manner as follows:
1) Update of equalizers and weights: we update the equalizers and weights by computing \( g_{c,k} \), \( g_{k} \), \( u_{c,k} = 1/\epsilon_{c,k}^{\text{MMSE}} \) and \( u_{k} = 1/\epsilon_{k}^{\text{MMSE}} \) for given precoding vectors.
2) Update of precoders and \( \xi_{c} \): then, the precoders and \( \xi_{c} \) can be derived by solving (54) via CVX for given equalizers and weights.
3) Repeat steps 1 and 2: the steps 1 and 2 are repeated until convergence.

V. NUMERICAL RESULTS
In this section, we compare the sum spectral efficiency of the proposed Q-GPI-RS and existing baseline methods. The channel vector \( h_k \) is computed based on its spatial covariance matrix \( R_k = \mathbb{E}[h_k h_k^H] \). We employ the one-ring channel model to generate \( R_k \) [46]; The channel covariance matrix \( R_k \) at the \( n \)-th antenna and \( m \)-th antenna is defined as \( [R_k]_{n,m} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \rho_{\phi} \psi(\phi) d\phi \), where \( \psi(\phi) = [\cos(\phi), \sin(\phi)] \), and \( r_k \) is the position vector of \( n \)-th antenna. Using the Karhunen-Loeve model, the channel vector \( h_k \) is decomposed as \( h_k = U_k \Lambda_k^1 \Lambda_k^2 \), where \( U_k \in \mathbb{C}^{N \times r_k} \) is eigenvectors of \( R_k \), \( \Lambda_k \in \mathbb{C}^{r_k \times r_k} \) is a diagonal matrix of eigenvalues of \( R_k \), each entry of \( \Lambda_k \) follows \( \mathcal{C}(0,1) \), \( \Lambda_k \in \mathbb{C}^{r_k \times r_k} \) is identically distributed, and \( r_k \) is the rank of \( R_k \). By using a block fading channel model, \( g_k \) is considered to be constant within one transmission block. The value of \( \theta_k \) is varied according to the user’s location. The AoD \( \theta_k \) follows IID uniform distribution between \( 0 \) and \( \pi \) as we consider that users are randomly located. In the case of correlated users, the differences of \( \theta_k \) for all users are randomly distributed within \( \pi/6 \). We set the simulation setting as \( \Delta_k = \pi/6, \epsilon = 0.01, \sigma^2 = 2 \).

For initialization, we set the precoding vector \( f_0 = h_k \), i.e., maximum ratio transmission (MRT). For the common stream, in particular, the average of channel vectors is adopted, i.e., \( f_0 = \frac{1}{N} \mathbb{E}[H] \). Then the stacked precoding vector \( \tilde{W}_0 \) is initialized based on such \( F \). Let us define an effective channel vector as \( h_{\text{eff}}^c = \Phi_{\text{ADC,k}} h_k \). Then, the comparing baseline methods are:
- **Q-GPI-RS (RSA):** The Q-GPI-RS algorithm is generalized power iteration for rate-splitting (Q-GPI-RS) inspired by a power iteration proposed in [32].
- **Q-WMMSE-AO (RSA):** The algorithm is introduced in Section IV. Q-WMMSE-AO approach is aligned with the ADMM-based algorithm in [36] when the radar pattern design is omitted and ADC quantization error is further incorporated.
- **WMMSE-AO (RSA):** The WMMSE alternating optimization (AO) approach is the state-of-the-art method for RSA precoding in [24].
- **WMMSE-CCCP (RSA):** The WMMSE concave-convex procedure (CCCP) algorithm is the advanced method for RSA precoding proposed in [10].
- **WMMSE (SDMA):** The WMMSE-based precoding method is one of the most well-known precoding method proposed in [29].
- **Q-GPI-SEM (SDMA):** The quantization-aware Q-GPI-based spectral efficiency maximization (Q-GPI-SEM) is proposed in the low-resolution ADC/DAC regime for SDMA [17].
- **Q-RZF (SDMA):** The conventional linear RZF precoder based on the effective channel \( h_{\text{eff}}^c \) such as quantization-aware RZF is evaluated for the SDMA.

We remark that except Q-RZF, WMMSE and Q-GPI-SEM are the state-of-the-art precoding methods for SDMA. In addition, we also evaluate the state-of-the-art benchmarks.
for RSMA such as GPI-RS, Q-WMMSE-AO, WMMSE-AO, and WMMSE-CCCP for comprehensive performance analysis of the proposed methods.

We define \( \text{SNR} = P/\sigma^2 \). Regarding the approximation parameter \( \tau \), we first find the value from the experimental results to the system configuration. Then we fix \( \tau \) with the obtained value for each evaluation point without online tuning of \( \tau \). The spectral efficiency in simulation results is computed based on (8) and (10) without the approximation of the minimum function.

A. Homogeneous Quantization Resolution

In Fig. 2, we consider randomly located users for \( N = 4 \) and \( K = 2 \) with \( b_{\text{DAC},n} = 4 \), \( \forall n \) and \( b_{\text{ADC},k} = 6 \), \( \forall k \). As shown in Fig. 2, Q-GPI-RS achieves the highest spectral efficiency. Since Q-WMMSE-AO cannot guarantee the best local optimal point, Q-WMMSE-AO reveals the performance limitation compared to Q-GPI-RS even though the quantization effect is taken into consideration. GPI-RS achieves similar performance to Q-GPI-RS in the low SNR regime. In the high SNR regime, however, the performance of GPI-RS shows increasing gap from Q-GPI-RS and becomes lower than that of Q-GPI-SEM because GPI-RS suffers from the quantization error. Similarly, WMMSE-AO and WMMSE-CCCP approaches also show performance degradation from Q-WMMSE-AO because they are designed without considering the quantization error. As a result, Q-RZF outperforms the WMMSE-based methods as the SNR increases. With properly designed RSMA precoding, the proposed Q-GPI-RS method shows the gain of RSMA by mitigating the interference compared to Q-GPI-SEM which is based on classical SDMA. Such an RSMA gain is also observed by comparing WMMSE-AO with WMMSE.

Therefore, it can be concluded by simulation results that the proposed algorithm Q-GPI-RS achieves the highest spectral efficiency compared to the benchmarks, and it is important to handle the quantization error for realizing the potential of RSMA.

In Fig. 3, we evaluate the spectral efficiency according to the user’s angular difference \( (\theta_k - \theta_{k'}) \) for \( N = 4 \), \( K = 2 \), \( b_{\text{DAC},n} = 4 \), \( \forall n \), and \( b_{\text{ADC},k} = 8 \), \( \forall k \). As shown in Fig. 3, the gap between Q-GPI-RS and the Q-GPI-SEM increases as the angular difference decreases. In other words, the gain of RSMA increases as user channel correlation increases, which was analytically shown for the perfect quantization system in [39]. This phenomenon occurs because, for highly correlated users, RSMA exploits the common stream to reduce the inter-user interference which is more severe when channel correlation is high. SDMA, however, suffers from high inter-user interference. User channels are typically correlated with small angular differences among the users in the indoor communications [47] which are often the environment of low-power applications such as IoT communications. Accordingly, we assume the correlated channels where the angular difference between users is less than \( \pi/6 \) in the rest of the simulations.

B. Heterogeneous Quantization Resolution

We analyze the spectral efficiency when DACs at the AP is configured with heterogeneous quantization bits. In Fig. 4, for \( N = 6 \), \( K = 4 \), and \( b_{\text{ADC},k} = 8 \), \( \forall k \) we consider that the number of DAC bits is set uniformly randomly from 2 to 8 bits. As shown in Fig. 4, Q-GPI-RS achieves the highest spectral efficiency than the other baseline methods with greater
improvement compared to the previous random channel case. In addition, the spectral efficiency of Q-WMMSE-AO is higher than that of WMMSE-AO by incorporating the quantization error. We also note that the performances of GPI-RS, WMMSE-AO, and WMMSE-CCCP decrease in the high SNR regime since they are developed without considering the quantization error. Comparing RSMA with SDMA, Q-GPI-RS reveals the gain of RSMA from Q-GPI-SEM since the proposed method has better capability in reducing the inter-user interference by employing the common stream as the SNR increases. The other RSMA algorithms, i.e., Q-WMMSE-AO and WMMSE-AO also provide higher spectral efficiency than WMMSE and Q-RZF because of the higher channel correlation.

In Fig. 5, we consider the mixed-resolution DAC case where a single DAC is an 8-bit DAC and the rest are 3-bit DACs for $N = 4$, $K = 2$, and $b_{ADC,k} = 8$, $\forall k$. As shown in Fig. 5(a), we note that Q-GPI-RS achieves the significant spectral efficiency improvement in medium-to-high SNR. In particular, the gain of RSMA of Q-GPI-RS compared to Q-GPI-SEM is achieved by assigning the high rate for the common stream as shown in Fig. 5(b). In addition, Fig. 5(c) illustrates that Q-GPI-RS concentrates the available transmit power on the antenna with the high-resolution DAC (8 bits in this case) to prevent the quantization error from significant increase as the SNR increases. This trend can be analyzed based on the SINR expressions: let us focus on the quantization error term from DACs in (26) and (27), $Q_{E\text{DAC}} = \sum_{i=0}^{K} |\Phi_{\text{DAC}}|^{2} \Phi_{\text{DAC}} \Phi_{\text{DAC}} \text{diag}(h_{i}h_{i}^{H})f_{i}$. Note that $Q_{E\text{DAC}}$ can be seen as the sum of quantization errors at each antenna $Q_{E\text{DAC},n}$, i.e., $Q_{E\text{DAC}} = \sum_{n=1}^{N} Q_{E\text{DAC},n}$, where $Q_{E\text{DAC},n} = |f_{i,n}|^{2} \alpha_{\text{DAC},n}(1 - \alpha_{\text{DAC},n}) \text{diag}(h_{i}h_{i}^{H})|_{n,n}$. Here, $[\cdot]|_{n,n}$ indicates the $n$th diagonal element of $[\cdot]$. Based on $Q_{E\text{DAC},n}$, it can be observed that the quantization error term from the $n$th DAC decreases as we increase the DAC resolution since the value of $\alpha_{\text{DAC},n}$ becomes approximately 1 for high-resolution DACs, which is natural and intuitive. Consequently, the preceding element $f_{i,n}, \forall i$, that corresponds to the high-resolution DAC can be assigned with large power with marginal increase of the quantization error and vice versa. In other words, we can manage the quantization error by allocating transmit power mostly on the antennas with high-resolution DACs in the heterogeneous DAC systems. This leads to reducing the number of active antennas, and thus, the system can become overloaded. In such a case, RSMA transmission plays its key role by leveraging the common stream to deal with the overloaded system. Q-GPI-SEM, on the other hand, allocates the transmit power to all antennas as shown in Fig. 5(c). Accordingly, although Q-GPI-SEM achieves the higher spectral efficiency than the other baselines, it still has performance limitation compared to Q-GPI-RS since Q-GPI-SEM cannot avoid the large increase of quantization errors with the SNR.

Since the mixed ADC/DAC architecture which is a special case of the heterogeneous ADC/DAC systems has shown to offer high potential in maximizing the spectral efficiency with low power consumption in the MIMO systems [5], we further evaluate the spectral efficiency versus the SNR for $N = 4$ AP antennas, $K = 2$ users, and $b_{ADC,k} = 8$ ADC bits, $\forall k$. We consider that one of the DACs is equipped with 8 bits and the rest are equipped with 3 bits.

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We consider the half of the DACs are equipped with 3 bits and the other half are equipped with 10 bits. In addition to the mixed DAC case, we plot the high-resolution DAC case for $N = 4$, $K = 4$, $b_{DAC,n} = 10$, $\forall n$, and $b_{ADC,k} = 10$, $\forall k$ for comparison. As shown in Fig. 6, the proposed Q-GPI-RS achieves the highest sum spectral efficiency in this environment. In particular, the gain of RSMA becomes larger as the SNR increases for both $N = 8$ and $N = 4$ cases. We note that the spectral efficiency of $N = 4$ case reduces its gap from that of $N = 8$ case as the SNR increases. This is due to the phenomenon that Q-GPI-RS with mixed-resolution DACs reduces the quantization error in the high SNR by allocating more power to antennas with high-resolution DACs. Accordingly, in the high SNR, the effective number of antennas becomes the number of antennas with high-resolution DACs, which makes $N = 8$ and $N = 4$ cases similar, while increasing the gap from the SDMA-based approaches. In the low-to-medium SNR, however, the gain of RSMA is small but there exists an antenna gain for $N = 8$ compared to $N = 4$ thanks to additional antennas with low-resolution DACs. Therefore, using the proposed RSMA precoding method with mixed-resolution DACs offers antenna gain in the low-to-medium SNR with the small cost of deploying the low-power hardware, and also provides the RSMA gain in the high SNR.

C. Effect of DAC and ADC Quantization to RSMA

Now, we analyze the effect of the number of DAC and ADC bits to verify the conjecture in Remark 2. We plot the spectral efficiency versus the number of DAC bits with $b_{ADC,k} = 10$, $\forall k$ in Fig. 7(a) and the number of ADC bits with $b_{DAC,n} = 10$, $\forall n$ in Fig. 7(b) for $N = 8$, $K = 4$, and SNR = 40 dB. In addition, Fig. 7(c) illustrates the power ratio between the common and private streams according to the number of quantization bits. As shown in Fig. 7, the proposed Q-GPI-RS achieves the highest performance over the considered number of bits. The gain of RSMA becomes larger as DAC and ADC resolutions increase because RSMA can allocate the higher rate to the common stream with the higher resolution, which corresponds to the observation in [35]. As shown in Fig. 7(c), it is observed that the portion
of the allocated power of the common stream increases as the resolution of quantizers increases. In particular, the allocated power on the common stream is more constrained by the number of ADC bits than that of DAC bits. Accordingly, the gain of RSMA is more sensitive to the ADC resolution than the DAC resolution, which confirms the analysis in Remark 2.

D. Convergence Analysis

In Fig. 8, we present the convergence result in terms of the iteration count for $N = 4$, $K = 2$, $b_{\text{ADC},k} = 8$, $\forall k$, and SNR $\in \{-10, 0, 10, 20\}$ dB considering that a single DAC is an 8-bit DAC and the rest are 3-bit DACs. Here, Q-GPI-RS converges within $T_{\text{GPI}} = 5$ iterations for all the considered SNR, which can be considered to be small. Therefore, the proposed algorithm can be more favorable to practical implementation than the other state-of-the-art precoding methods, offering much lower complexity as discussed in Remark 4.

E. Effect of Imperfect CSIT

Let us consider the estimated channel model. We consider time division duplex (TDD) systems and use the linear minimum mean square error approach to estimate the channel from uplink pilot sequences sent from the users [48]. Then, the estimated channel vector is formulated as

$$\hat{h}_k = h_k - e_k,$$

where $e_k$ is the channel estimation error vector. The error covariance matrix is computed as

$$E[\epsilon_k^H \epsilon_k] = \Psi_k = R_k - R_k \left( R_k + \frac{\sigma^2}{\tau_{ul}p_{ul}} \right)^{-1} R_k,$$

where $\tau_{ul}$ and $p_{ul}$ are uplink training length and transmit power, respectively. Under the imperfect CSIT assumption, we adopt the sample average approximation (SAA) technique [24] for the WMMSE-based algorithms. We use 1000 samples for the SAA technique. More detailed descriptions can be found in [24].

In Fig. 9, we compute the spectral efficiency when the channel estimation error exits for $N = 6$, $K = 4$, and $b_{\text{ADC},k} = 8$, $\forall k$. The number of DAC bits is set randomly from 2 to 8 bits. As shown in Fig. 9, the proposed Q-GPI-RS still achieves the highest performance for both cases. We also note that the spectral efficiencies of the WMMSE-based algorithms with SAA show smaller the gap from the proposed method when the channel estimation error is larger ($\tau_{ul}p_{ul} = 4$). This is because the WMMSE-based algorithms with SAA are developed by considering the impact of the channel estimation error. We can conclude that the proposed method can still achieve the higher performance than the other baselines with the imperfect channel estimation.

VI. CONCLUSION

In this paper, we proposed a promising precoding algorithm for downlink RSMA systems with low-resolution quantizers to maximize the sum spectral efficiency. Since the formulated problem is non-smooth, we used the LogSumExp technique to convert the problem into a tractable form. In addition, we further reformulated the non-convex problem to the product of the Rayleigh quotients for more tractability. We then derived
Lagrangian function is defined as by explicitly considering channel estimation error as well as promising to develop an optimal RSMA precoding algorithm for low-power transceiver architectures. For future work, it is both increasing the communication efficiency and designing Therefore, the proposed algorithm can provide benefits in improving the spectral efficiency, offering high adaptation and that the proposed RSMA precoding algorithm significantly with different DAC resolutions. Based on the observations, shows the high flexibility of RSMA in using antennas as a means of suppressing the quantization error, which transmits power for the antennas with low-resolution DACs. In particular, the gain of RSMA increased with the resolutions gain of RSMA exists in most DAC and ADC resolutions. To the baseline methods. It was also observed that the method achieved the highest sum spectral efficiency compared to the top of the page. The derivative of the Lagrangian function (62) and (63), shown at the top of the page. Rearranging (62), we derive \( A_{k,\text{KKT}} \bar{w}(\bar{w}) = \lambda(\bar{w})B_{k,\text{KKT}} \bar{w} \). Since \( B_{c,k} \) and \( B_k \) are Hermitian block diagonal matrices, \( B_{k,\text{KKT}} \) is invertible. Accordingly, we finally obtain the condition in (40). This completes the proof.

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The first-order optimality condition to find the stationary point. Interpreting the condition as the generalized eigenvalue problem, we developed a computationally efficient algorithm to find the best stationary point that maximizes the spectral efficiency. Simulation results demonstrated that the proposed method achieved the highest sum spectral efficiency compared to the baseline methods. It was also observed that the gain of RSMA exists in most DAC and ADC resolutions.

**APPENDIX A PROOF OF LEMMA 1**

From the reformulated optimization problem, the Lagrangian function is defined as

\[
L(w) = \ln \left( \frac{1}{K} \sum_{k=1}^{K} \exp \left( -\frac{1}{2} \log_2 \left( \frac{w^H A_{c,k} w}{w^H B_{c,k} w} \right) \right) \right) - \tau
+ \sum_{k=1}^{K} \ln \left( \frac{A_{k,\bar{w}}}{B_{k,\bar{w}}} \right). \tag{59}
\]

Then, we compute partial derivatives of the Lagrangian function (59) and find the stationarity condition by setting (59) to zero. Let us denote the first term and second term in (59) as \( L_1(\bar{w}) \) and \( L_2(\bar{w}) \), respectively. The partial derivative of \( L_1(\bar{w}) \) is represented as (60), shown at the top of the page. The derivative of the Lagrangian function \( L_2(\bar{w}) \) is derived as

\[
\frac{dL_2(\bar{w})}{d\bar{w}^H} = \frac{1}{\ln 2} \sum_{k=1}^{K} \left[ \frac{A_{k,\bar{w}}}{w^H A_{c,k} w} - \frac{B_{k,\bar{w}}}{w^H B_{c,k} w} \right]. \tag{61}
\]

Using (60) and (61), the first-order optimality condition is given as (62) and (63), shown at the top of the page. **REFERENCES**

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