Analysis and Prediction of COVID-19 Outbreak by the Numerical Modelling

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Abstract

Pandemic COVID-19 is a contagious disease affecting more than 200 countries, territories and regions. Recently, Iraq is one of the countries that has immensely suffered with this outbreak. The Kurdistan Region of Iraq (KRI) is also prone to the disease. Until now more than 23,000 confirmed cases have been recorded in the region. Since the onset of the COVID-19 in Wuhan, based on epidemiological modelling, researchers have used various models to predict the future of the epidemic and the time of peak, yielding a diverse number in different countries. This study aims to estimate the basic reproductive number ($R_0$) for COVID-19 in KRI, using the standard SIR (Susceptible-Infected-Removed) epidemic model. A system of nonlinear differential equations is formulated and solved numerically by the 4th order Runge-Kutta method. Reproductive numbers $R_0$ have been estimated by this method of fitting the curves between the actual daily data and numerical solution by applying the least square method. For the analysis, data were taken for the duration of 165 days from 1st of March to 12th August in a population of 5.2 million. It has been concluded that $R_0$ is fluctuating during the outbreak with an average of 1.33, predicting that infected cases will reach their maximum value of around 540,000 on 5th of November 2020. Then the spread of the disease will die out since the number of susceptible will decrease to about 3.2 million. While the number of removed individuals will reach approximately to 1.5 million.

Keywords: COVID-19, Numerical Model, 4th order Runge-Kutta, SIR, Reproductive Number

Introduction
The coronavirus disease 2019 (COVID-19) is a contagious disease which can be transmitted through droplets, aerosols, and direct contact (1-3). They are caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), a novel coronavirus, which emerged in Wuhan during early December, 2019 (4, 5). The symptoms of infection include, fever, cough, myalgia, fatigue, sputum production, headache, haemoptysis, diarrhoea, dyspnoea and lymphopenia (6). In more severe cases, COVID-19 can cause pneumonia and even death (7). The incubation period is believed to extend to 14 days with a median time of approximately 5.2 days (8).

The COVID-19 has spread rapidly throughout the world as it has been found to have higher levels of transmissibility and pandemic risk than the SARS-CoV (9). It has rapidly resulted in over 20 million confirmed cases and more 750,000 deaths worldwide in less than 9 months (7). Due to the sharp escalating level of spread on March 11, WHO declared the COVID-19 outbreak as a global pandemic (10).

The epidemiological prediction from mathematical modelling plays a key role to understand the pathway of the epidemic and propose effective strategies in controlling the disease (11).

SIR is a commonly used model for transmission of the disease from human to human (12). SIR is considered to be one of the most reliable simple tools which consists of three compartments; susceptible, infected and removed (11).

The basic reproduction number ($R_0$) is a measure to evaluate the transmissibility of the virus in a particular population. Sustainability of the transmission of the disease depends on $R_0$ value, when $R_0 > 1$ the disease is most likely developed to secondary cases. However, if $R_0 \leq 1$ it indicates that the secondary cases in the outbreak are declining (13).

SIR model has been used in several studies to analyse the spread of COVID-19 (14-16). Since the evaluation of COVID-19 by modelling in KRI has not been conducted to predict the behaviour of the disease spreading. So, in this study a numerical model for predicting the outbreak has been used to estimate how the quarantine, easy restriction and population mixing have an effect on outbreak progression. Accordingly, the model paves the way for the authority to manage measures’ related policy which can control and eradicate the infection.

**Materials and Methods**

In this study a coupled system of nonlinear differential equations derived by Kermack and McKendrick in 1927 (17) has been used. The system consists of three differential equations based on the classification which divide the population into three compartments Susceptible ($S$), Infected ($I$) and Removed ($R$) which is known as SIR model as shown in Figure 1. The interaction between the categories are controlled by transmission rate ($\beta$) and recovery rate ($\gamma$).
Figure 1: Schematic diagram of SIR Model; S, susceptible; I, infected; R, removed; $\beta$, transmission rate and $\gamma$, recovery rate.

The independent variable is time ($t$) measured in days and the dependent variables are ($S$, $I$ and $R$). We consider three related sets of dependent variables. Equations (1, 2 and 3) are the rate of change of susceptible population, the rate of change of infected population and the rate of change of removed population, respectively.

$$\frac{dS}{dt} = -\beta S(t)I(t) \quad (1)$$
$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) \quad (2)$$
$$\frac{dR}{dt} = \gamma I(t) \quad (3)$$

$$R_0 = \frac{\beta}{\gamma} \quad (4)$$

Equation (4), represents the reproductive number ($R_0$), which is the key point to identify the epidemic occurrence and disease severity.

**Assumptions**

For simplifying the analysis the following assumptions are made (18):

1- The total population number ($N$) is constant. Ignoring births, immigration and natural death. Accordingly, the rate of change of ($N$) will be zero. This will lead to:

$$N = S + I + R \quad (5)$$
$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \quad (6)$$

2- The population is homogeneously distributed.

3- Recovered individuals become immune (and hence remain in the removed compartment).

4- The removed compartment also includes the individuals who died from the disease.

**Runge-Kutta 4\textsuperscript{th} order method**
The 4\textsuperscript{th} order Runge-Kutta method for a system of differential equations has the following forms (19):

\begin{align}
S_{t+1} &= S_t + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)h \\
I_{t+1} &= I_t + \frac{1}{6} (G_1 + 2G_2 + 2G_3 + G_4)h \\
R_{t+1} &= R_t + \frac{1}{6} (L_1 + 2L_2 + 2L_3 + L_4)h
\end{align}

Where:

\begin{align}
K_1 &= f(t_i, S_i) \\
K_2 &= f\left(t_i + \frac{1}{2}h, S_i + \frac{1}{2}K_1h\right) \\
K_3 &= f\left(t_i + \frac{1}{2}h, S_i + \frac{1}{2}K_2h\right) \\
K_4 &= f(t_i + h, S_i + K_3h) \\
G_1 &= f(t_i, I_i) \\
G_2 &= f\left(t_i + \frac{1}{2}h, I_i + \frac{1}{2}G_1h\right) \\
G_3 &= f\left(t_i + \frac{1}{2}h, I_i + \frac{1}{2}G_2h\right) \\
G_4 &= f(t_i + h, I_i + G_3h) \\
L_1 &= f(t_i, R_i) \\
L_2 &= f\left(t_i + \frac{1}{2}h, R_i + \frac{1}{2}L_1h\right) \\
L_3 &= f\left(t_i + \frac{1}{2}h, R_i + \frac{1}{2}L_2h\right) \\
L_4 &= f(t_i + h, R_i + L_3h)
\end{align}

In order to solve the system of equation (7) to (9) by applying the 4\textsuperscript{th} order Runge-Kutta method, initial conditions are required. The data obtained from the KRI official website (20) from (1/3/2020 to 12/8/2020). Taking the total population of KRI as $N=5,200,000$, $S=S(0)$, $I=I(0)=4$, $R(0)=0$.

The model mainly depends on two parameters ($\beta$) the transmission rate, and ($\gamma$), the recovery rate. Estimating the values of parameters is often a complicated task. Inaccurate parameters for estimating the model is less useful as a predictive tool, although it may still be possible to describe the general behaviour (17). There are different approaches for estimating the parameters; the first one is to estimate parameters directly based on some experience, since covid-19 is a new disease, so it is difficult to adapt this approach. The second is to compare and fit actual data with numerical solution by method trial and error with the help of the least
square method which has been adapted in this study. The mentioned process of estimating the parameters is called calibrating the model (17).

Results

The result in Figure 2 shows the infected individuals that were announced by the KRI the ministry of health for the period between 1st of March to 12th of August 2020 (20). The number of confirmed cases started with 4 cases in the beginning of the outbreak in the region. In April, the confirmed cases declined to its minimum value. While, at the start of May the cases were increased steadily followed by a rocketed increase in June. Also, the number of infected cases has increased from the first of August until preparing this article.

Figure 2: COVID-19 infected cases. X axis: Time (days), Y axis number infected cases. Monthly infected cases (A-F), March (A), April (B), May (C) , June (D), July (E), August (F), total infected cases (G). Actual monthly infected cases resembling the black line and red line indicate the numerical results in KRI.
Figure 3 represents $R_0$ for each month as well as the whole period of the outbreak. April experienced the lowest value (0.063). May and June recorded the maximum values of (2.053) and (1.959) respectively. By using the overall data for the average $R_0$ value estimated as 1.327.

![Figure 3: The basic reproduction number ($R_0$) during the outbreak in KRI from the beginning of March until 12th of August.](image)

The general SIR model illustrated in figure 4 shows the predicted infected cases will reach their maximum value of around 540,000 on 5th of November 2020. Then, the spread of the disease will die out since the number of the susceptible will decrease to about 3.2 million and the number of removed individuals will reach approximately to 1.5 million.

![Figure 4: SIR model of prediction for COVID-19 outbreak in KRI (by using the mean of $R_0 = 1.327$). X axis: Time (days), Y axis: population number. Blue line (S) resembles susceptible population, red line (I) represents infected cases, and green line (R) indicates removed population.](image)

**Discussion**
The COVID-19 is spreading with astounding speed and it has severe consequences on the health, economic and social aspects. In the absence of population immunity, effective medicine and vaccines the spread of COVID-19 is still expanding exponentially in many countries. Numerical analysis will provide an early warning to the authority regarding the infected population and predicting the time frame for reaching the peak value. Also, it assists authorities to deal with measures amendment in a way fitting the dynamics of the spread of the disease (21, 22).

Numerical modelling has been revealed as a powerful tool to analyse disease behaviour and provide significant information for the authority to take any necessary actions when needed. From obtained results, the actual cases were compatible with the numerical analysis due to the well-fitting characteristics of the used method (15).

In general, most models follow exponential growth. Surprisingly, due to implementation of several restriction measures the trend of numerical infected cases in April did not follow the exponential trend. Also, in July due to high recovery rate the curve remained nearly in a steady state. Similarly, Maier and Brockmann found exponential growth decrease by imposing restriction measures (23) which may lead to proportional increase of the recovery cases.

Here we estimated the \( R_0 \) in KRI from the beginning of COVID-19 attack and assessed the SIR model to predict the afertime of the region with COVID-19 disease. Data released by the ministry of health were used as a source for numerical modelling in this study. Based on, the estimated \( R_0 \) values during the time period of six months from March to August fluctuated between 0.063 and 2.053. As a consequence of strict restrictions which were implemented by KRI authority in April to contain the outbreak, the lowest estimated value of \( R_0 \) was recorded. While, the sudden and unplanned lifting of measures in May caused a dramatic increase in confirmed cases of COVID-19 (24) which resulted in a highest \( R_0 \). Studies also revealed that travel bans, border closing, lockdown and social distancing are the most effective containment measures to enhance global readiness needed in response to COVID-19 (25). The average of \( R_0 \) for the time period was around 1.33 which is approximately fit with WHO’s declared lower value (1.4-2.5) (26). However, other studies estimated the maximum \( R_0 \) value to be around 6.7 with a similar minimum value to our study (27). \( R_0 \) values strongly correlated with COVID-19 restriction measures implementation, which can be varied among different populations which reflected on \( R_0 \) values (28).

According to our data analysis KRI hasn't reached the peak yet, it is estimated to reach the peak by the 5\(^{th}\) of November 2020. However, the spread of the virus will gradually decrease since the recovered population will increase and the number of susceptible individuals will decline. On the 26\(^{th}\) of November 2020, the number of susceptible individuals and recovered individuals will reach the same value.

**Conclusion**

By using the SIR model, we concluded that KRI has not reached the peak value yet. Our data analysis forecasted the peak by the beginning of November and it will reach around a half million. Accordingly, the model can predict susceptible populations by the time when the vaccine is available. The accuracy of the model depends on the quantity of availability of confirmed data. Therefore, it is important to increase the number of the diagnosis tests for the
non-hospitalized population in the region and use clinical manifestation-based technology to record as other sources of data.

**Author contribution**

DAM has performed data analysis, KMA, HMT and HMR contributed to write and prepare the manuscript.

**Conflict of interest**

No.

**References**

1. N. van Doremalen, T. Bushmaker, D. H. Morris, M. G. Holbrook, A. Gamble, B. N. Williamson, A. Tamin, J. L. Harcourt, N. J. Thornburg, S. I. Gerber, J. O. Lloyd-Smith, E. de Wit and V. J. Munster: Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1. *N Engl J Med*, 382(16), 1564-1567 (2020) doi:10.1056/NEJMc2004973

2. R. Xu, B. Cui, X. Duan, P. Zhang, X. Zhou and Q. Yuan: Saliva: potential diagnostic value and transmission of 2019-ncov. *Int J Oral Sci*, 12(1), 11 (2020) doi:10.1038/s41368-020-0080-z

3. L. Morawska and J. Cao: Airborne transmission of SARS-CoV-2: The world should face the reality. *Environ Int*, 139, 105730 (2020) doi:10.1016/j.envint.2020.105730

4. Q. Li, X. Guan, P. Wu, X. Wang, L. Zhou, Y. Tong, R. Ren, K. S. M. Leung, E. H. Y. Lau, J. Y. Wong, X. Xing, N. Xiang, Y. Wu, C. Li, Q. Chen, D. Li, T. Liu, J. Zhao, M. Liu, W. Tu, C. Chen, L. Jin, R. Yang, Q. Wang, S. Zhou, R. Wang, H. Liu, Y. Luo, Y. Liu, G. Shao, H. Li, Z. Tao, Y. Yang, Z. Deng, B. Liu, Z. Ma, Y. Zhang, G. Shi, T. T. Y. Lam, J. T. Wu, G. F. Gao, B. J. Cowling, B. Yang, G. M. Leung and Z. Feng: Early Transmission Dynamics in Wuhan, China, of Novel Coronavirus-Infected Pneumonia. *N Engl J Med*, 382(13), 1199-1207 (2020) doi:10.1056/NEJMoa2001316

5. N. Zhu, D. Zhang, W. Wang, X. Li, B. Yang, J. Song, X. Zhao, B. Huang, W. Shi, R. Lu, P. Niu, F. Zhan, X. Ma, D. Wang, W. Xu, G. Wu, G. F. Gao and W. Tan: A Novel Coronavirus from Patients with Pneumonia in China, 2019. *N Engl J Med*, 382(8), 727-733 (2020) doi:10.1056/NEJMo2001017

6. C. Huang, Y. Wang, X. Li, L. Ren, J. Zhao, Y. Hu, L. Zhang, G. Fan, J. Xu, X. Gu, Z. Cheng, T. Yu, J. Xia, Y. Wei, W. Wu, X. Xie, W. Yin, H. Li, M. Liu, Y. Xiao, H. Gao, L. Guo, J. Xie, G. Wang, R. Jiang, Z. Gao, Q. Jin, J. Wang and B. Cao: Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. *The Lancet*, 395(10223), 497-506 (2020) doi:10.1016/S0140-6736(20)30183-5

7. who.int: Coronavirus disease (COVID-19) pandemic. In: World Health Organization (2020)

8. Q. Li, X. Guan, P. Wu, X. Wang, L. Zhou, Y. Tong, R. Ren, K. S. M. Leung, E. H. Y. Lau, J. Y. Wong, X. Xing, N. Xiang, Y. Wu, C. Li, Q. Chen, D. Li, T. Liu, J. Zhao, M. Liu, W. Tu, C. Chen, L. Jin, R. Yang, Q. Wang, S. Zhou, R. Wang, H. Liu, Y. Luo, Y. Liu, G. Shao, H. Li, Z. Tao, Y. Yang, Z. Deng, B. Liu, Z. Ma, Y. Zhang, G. Shi, T. T. Y. Lam, J. T. Wu, G. F. Gao, B. J. Cowling, B. Yang, G. M. Leung and Z. Feng: Early Transmission Dynamics in Wuhan, China, of Novel Coronavirus–Infected Pneumonia. *New England Journal of Medicine*, 382(13), 1199-1207 (2020) doi:10.1056/NEJMo2001316

9. T. Liu, J. Hu, J. Xiao, G. He, M. Kang, Z. Rong, L. Lin, H. Zhong, Q. Huang, A. Deng, W. Zeng, X. Tan, S. Zeng, Z. Zhu, J. Li, D. Gong, D. Wan, S. Chen, L. Guo, Y. Li, L. Sun, W. Liang, T. Song, J. He and W. Ma: Time-varying transmission dynamics of Novel Coronavirus Pneumonia in China. *bioRxiv*, 2020.01.25.919787 (2020) doi:10.1101/2020.01.25.919787

10. who.int: WHO Director-General’s opening remarks at the media briefing on COVID-19 - 11 March 2020. In: World Health Organization, (2020)

11. G. Giordano, F. Blanchini, R. Bruno, P. Colaneri, A. Di Filippo, A. Di Matteo and M. Colaneri: Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy. *Nature Medicine*, 26(6), 855-860 (2020) doi:10.1038/s41591-020-0883-7
12. W. O. Kermack and A. G. McKendrick: A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character*, 115(772), 700-721 (1927)

13. E. Shim, A. Tariq, W. Choi, Y. Lee and G. Chowell: Transmission potential and severity of COVID-19 in South Korea. *International Journal of Infectious Diseases*, 93, 339-344 (2020) doi:10.1016/j.ijid.2020.03.031

14. S. Zhao and H. Chen: Modeling the epidemic dynamics and control of COVID-19 outbreak in China. *Quantitative biology (Beijing, China)*, 1-9 (2020) doi:10.1007/s40484-020-0199-0

15. leuwathe.com: COVID-19 dynamics with SIR model. In: Kai Sasaki, (2020)

16. T. Hasegawa and K. Nemoto: Efficiency of prompt quarantine measures on a susceptible-infected-removed model in networks. *Physical Review E*, 96(2), 022311 (2017) doi:10.1103/PhysRevE.96.022311

17. B. Barnes and G. R. Fulford: Mathematical Modelling with Case Studies: Using Maple and MATLAB. CRC Press, (2014)

18. F. B. Hamzah, C. Lau, H. Nazri, D. Ligot, G. Lee and C. Tan: CoronaTracker: worldwide COVID-19 outbreak data analysis and prediction. *Bull World Health Organ*, 1, 32 (2020)

19. S. C. Chapra and R. P. Canale: Numerical methods for engineers. Boston: McGraw-Hill Higher Education, (2010)

20. gov.krd: Coronavirus (COVID-19). In: Kurdistan Regional Government (2020)

21. L. López and X. Rodó: The end of social confinement and COVID-19 re-emergence risk. *Nature Human Behaviour*, 4(7), 746-755 (2020) doi:10.1038/s41562-020-0908-8

22. M. S. Cohen and L. Corey: Combination prevention for COVID-19. *Science*, 368(6491), 551-551 (2020) doi:10.1126/science.abc5798

23. B. F. Maier and D. Brockmann: Effective containment explains subexponential growth in recent confirmed COVID-19 cases in China. *Science*, 368(6492), 742-746 (2020) doi:10.1126/science.abb4557

24. K. M. Ali, H. M. Tawfeeq and H. M. Rostam: COVID-19 Second Spike as an Aftermath of the Sudden Restrictions Ease: Kurdistan Region of Iraq as an Example. *Passer Journal*, 57-62 (2020) doi:10.24271/psr.12

25. L. O. Gostin and J. G. Hodge, Jr: US Emergency Legal Responses to Novel Coronavirus: Balancing Public Health and Civil Liberties. *JAMA*, 323(12), 1131-1132 (2020) doi:10.1001/jama.2020.2025

26. who.int: Statement on the meeting of the International Health Regulations (2005) Emergency Committee regarding the outbreak of novel coronavirus (2019-nCoV). In: World Health Organization, (2020)

27. Y. Liu, A. A. Gayle, A. Wilder-Smith and J. Rocklöv: The reproductive number of COVID-19 is higher compared to SARS coronavirus. *Journal of Travel Medicine*, 27(2) (2020) doi:10.1093/jtm/taaa021

28. K. Linka, M. Peirlinck and E. Kuhl: The reproduction number of COVID-19 and its correlation with public health interventions. *Computational Mechanics* (2020) doi:10.1007/s00466-020-01880-8
Figure 1: Schematic diagram of SIR Model; S, susceptible; I, infected; R, removed; β, transmission rate and γ, recovery rate.

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