On the effect of secondary protons on baryon and proton number cumulants in event-by-event analysis

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We investigate the effects of secondary (knockout) protons, which constitute about 20% of the observed protons at STAR, on the higher order cumulants of proton and baryon numbers measured by event-by-event analyses in relativistic heavy ion collisions. We argue that the contribution of this background effect on the cumulants is expressed by a simple formula, and that hence their effects can be removed in the experimental analysis. It is discussed that this background effect has non-negligible contribution to recently observed proton number cumulants at STAR, especially the third-order one, and that the removal of this effect is crucial to investigate the thermodynamical properties of the primordial hot medium appropriately.

I. INTRODUCTION

The beam energy scan (BES) program at the Relativistic Heavy Ion Collider (RHIC) has recently accumulated an abundance of experimental data on heavy ion collisions at various collision energies per nucleon, \( \sqrt{s_{NN}} \). Because hot media created by collisions with different \( \sqrt{s_{NN}} \) follow different trajectories on the temperature \( T \) and baryon chemical potential \( \mu_B \) plane during their time evolution, careful inspection of these data will enable us to understand the global phase structure of QCD at various collision energies per nucleon, \( \sqrt{s_{NN}} \). Therefore, the HRG model is used as the baseline from the non-hadronic and/or non-thermal feature of the primordial hot medium. In the HRG model, the probability distribution of the number of each baryon in a phase space is Poissonian (the net number distribution is thus given by the Skellam distribution) \([20]\). In the experimental results on the cumulants, they are often compared with the hadron resonance gas (HRG) model \([20]\). If the fluctuations are generated in a thermal medium described well by the free hadronic degrees of freedoms, the experimental results should well reproduce the HRG model. Therefore, the HRG model is used as the baseline from which the deviation of the cumulants encodes information on the non-hadronic and/or non-thermal feature of the primordial hot medium. In the HRG model, the probability distribution of the number of each baryon in a phase space is Poissonian (the net number distribution is thus given by the Skellam distribution) \([20]\). In the experimental results on proton number cumulants at STAR, small but statistically-significant deviations from this baseline are observed \([3]\). These deviations are expected to be due to non-trivial properties of the hot medium from the above argument, and theoretical and experimental verifications and investigations of these deviations are important for extracting the global nature of the QCD phase structure from these experimental data.

In experiments, various backgrounds affect measurements of observables. In the measurement of fluctuation observables in event-by-event analyses, background effects would make the distributions of the fluctuations approach Poissonian ones. The observation of the deviation of the distributions from the Poissonian ones thus
would be subject to such background effects. In order to see the signals of the non-hadronic property of the hot medium encoded as the deviation from the Poisson distribution, therefore, careful treatment on the background effect is necessary.

In the present study, we focus on the background effect of secondary (knockout) protons, which are protons knocked out from materials in the detector by reactions with particles created in the hot medium. We derive relations to remove this background effect from higher order cumulants, and argue that this background effect has a non-negligible contribution to the experimental measurement of the cumulants of net-proton and net-baryon numbers.

II. SECONDARY PROTONS

A. General properties

In the present study, we consider the effects of secondary protons on the cumulants of the net-proton and net-baryon numbers in relativistic heavy ion collisions. Secondary protons are protons which come from not fireballs created by heavy-ion collisions, but materials in the detector. Protons in the detector can be knocked out by reactions with particles from the fireballs, and enter the detector. A major part of the secondary protons can be identified and eliminated in the experimental analysis by introducing a cut for the distance between the reconstructed trajectory of the proton and the collision point, which is usually represented by \(d_{ca}\) in the literature [21]. Some of them, however, have reconstructed trajectories which pass near the collision point, and are misidentified as true protons created in the fireballs. The abundance of these misidentified protons for various transverse momentum (\(p_T\)) bins estimated by an event generator for STAR for 62 GeV AuAu collision is shown in Table I in Ref. [21]. With this table and the \(p_T\)-dependence of the proton yield, it is estimated that about 20% of protons detected in this experiment are the secondary ones for the typical \(p_T\) range used in the event-by-event analysis of proton number cumulants, 0.4 < \(p_T\) < 0.8 GeV. In the following, we denote the number of misidentified secondary protons in each event as \(N_{p}^{2nd}\). Notice that the anti-proton number is not affected by this background effect, because materials in the detector do not contain anti-particles.

The background effect by secondary protons is usually subtracted from experimental results on average values using estimates with event generators. Their effects on the second- and higher-order cumulants, however, are nontrivial, and to the best of the authors’ knowledge, this effect has not been taken care of in the existing experimental results on fluctuation observables. The purpose of the present study is to quantify their effects and make it possible to eliminate them in the experimental analyses.

In order to obtain such a relation, let us clarify several properties of secondary protons. First, we assert that the probability distribution of the number of secondary protons in each event, \(p_{2nd}(N_p^{2nd})\), is Poissonian, \(p_{2nd}(N_p^{2nd}) = \lambda^{N_p^{2nd}} e^{-\lambda/N_p^{2nd}}\), to a good approximation, with \(\lambda = \langle N_p^{2nd} \rangle\). To support this statement, we note that the probability \(r\) that a particle generated in the hot medium creates a misidentified secondary proton is small, \(r \ll 1\). In fact, whereas particles of order 10\(^3\) are created in each relativistic heavy ion collision, the value of \(\langle N_p^{2nd} \rangle\) is of order 10\(^2\) [21]. This shows that \(r\) is of order 10\(^{-2}\), which is significantly smaller than unity. Moreover, events where secondary protons are created are uncorrelated with each other, because the knockout reactions take place independently in the bulk detector. These two properties are sufficient to guarantee the Poissonian nature of \(p_{2nd}(N_p^{2nd})\). Whereas the total multiplicity for each event is not fixed even after fixing the collision geometry, and the probability to create a secondary proton depends on particle species, the Poissonian nature is not altered by these features. The Poisson distribution is specified by a single parameter \(\lambda = \langle N_p^{2nd} \rangle\), and all cumulants take a unique value,

\[
\langle (\delta N_p^{2nd})^n \rangle_c = \langle N_p^{2nd} \rangle,
\]

where \(\langle (\delta N_p^{2nd})^n \rangle_c\) is the \(n\)-th order cumulant of the misidentified secondary proton number.

Next, let us consider the correlation between \(N_p^{2nd}\) and the genuine proton number emitted from the hot medium, \(N_p^{col}\). Because of the creation mechanism of secondary protons, \(N_p^{2nd}\) for each event has a strong positive correlation with the total multiplicity \(N^{tot}\). Since the total multiplicity \(N^{tot}\), which is dominated by pions, is correlated with \(N_p^{col}\), in general, \(N_p^{2nd}\) and \(N_p^{col}\) are also correlated with each other. On the other hand, when collision events with a fixed \(N^{tot}\) are concerned, provided that \(r\) does not depend on particle species or momentum, the distribution of \(N_p^{2nd}\), which is Poissonian, is completely determined and hence has no correlation with \(N_p^{col}\) or any other particle numbers emitted from the hot medium. In experiments, the number of charged particles, \(N_{ch}\), is used as a proxy of \(N^{tot}\), and analyses are performed for collision events with finite ranges of \(N_{ch}\). For example, \(N_{ch}\) is used as the centrality selection at STAR, and central to peripheral collisions are usually classified centrality bins with the size of 5 or 10\% [17]. In the following, we assume that the cut of \(N_{ch}\) in the experimental analysis is sufficiently narrow to suppress correlations of \(N_p^{2nd}\) with \(N_p^{col}\), as well as with the numbers of anti-protons, baryons, and anti-baryons.

In the analysis of cumulants in STAR experiments, cumulants have been estimated by the weighted method [14]. In this method, cumulants are first determined with narrow ranges of \(N_{ch}\), and then they are averaged with a certain wider range of \(N_{ch}\). In this method, the correlation between \(N_p^{2nd}\) with \(N_p^{col}\) vanishes almost completely when the value of \(\langle N_p^{2nd} \rangle\) determined for each range of \(N_{ch}\) is used.
B. Proton number cumulants

Neglecting the correlation between $N^{\text{col}}_p$ and $N^{2\text{nd}}_p$, one can write the probability distribution of the total proton number detected by the detector, $P^{\text{exp}}_p = N^{\text{col}}_p + N^{2\text{nd}}_p$, as

$$P^{\text{exp}}_p(N^{\text{exp}}_p) = \sum_{N^{\text{col}}_p, N^{2\text{nd}}_p} \mathcal{P}_{\text{col}}(N^{\text{col}}_p)\mathcal{P}_{2\text{nd}}(N^{2\text{nd}}_p) \times \delta_{N^{\text{exp}}_p, N^{\text{col}}_p+N^{2\text{nd}}_p}. \quad (2)$$

Then the moment and cumulant generating functions of $N^{\text{exp}}_p$ are given by

$$G^{\text{exp}}_\theta(\theta) = \sum_N P^{\text{exp}}_N e^{N\theta} = G^{\text{col}}_\theta G^{2\text{nd}}_\theta(\theta), \quad (3)$$

and

$$K^{\exp}_\theta(\theta) = \log G^{\text{exp}}_\theta(\theta) = K^{\text{col}}_\theta(\theta) + K^{2\text{nd}}_\theta(\theta), \quad (4)$$

respectively, where the generating functions for $N^{\text{col}}_p$ and $N^{2\text{nd}}_p$ are defined, respectively, as

$$G^{\text{col}}_\theta = \sum_N \mathcal{P}_{\text{col}}(N)e^{N\theta} = \exp K^{\text{col}}_\theta(\theta), \quad (5)$$

$$G^{2\text{nd}}_\theta = \sum_N \mathcal{P}_{2\text{nd}}(N)e^{N\theta} = \exp K^{2\text{nd}}_\theta(\theta). \quad (6)$$

In Eq. (4), $K^{\exp}_\theta(\theta)$ is given by the sum of the two independent generating functions $K^{\text{col}}_\theta(\theta)$ and $K^{2\text{nd}}_\theta(\theta)$, owing to the independence of the two probabilities $\mathcal{P}_{\text{col}}(N)$ and $\mathcal{P}_{2\text{nd}}(N)$. Cumulants $\langle (\delta N)^{n} \rangle_c$ are defined by derivatives of the cumulant generating function as

$$\langle (\delta N)^{n} \rangle_c = \frac{\partial^n K(\theta)}{\partial \theta^n} \bigg|_{\theta=0}. \quad (7)$$

Cumulants up to fourth order satisfy $\langle \delta N \rangle_c = \langle N \rangle$, $\langle (\delta N)^2 \rangle_c = \langle (\delta N)^2 \rangle$, $\langle (\delta N)^3 \rangle_c = \langle (\delta N)^3 \rangle$, and $\langle (\delta N)^4 \rangle_c = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$. From Eq. (4), it is clear that the cumulant of total proton number is given by the sum of the two cumulants,

$$\langle (\delta N^{\text{exp}})^{n} \rangle_c = \langle (\delta N^{\text{col}})^{n} \rangle_c + \langle (\delta N^{2\text{nd}})^{n} \rangle_c$$

$$= \langle (\delta N^{\text{col}})^{n} \rangle_c + \langle (N^{2\text{nd}})^{n} \rangle_c, \quad (8)$$

where in the last equality we have used Eq. (1).

Eq. (8) relates the experimentally observed proton number cumulants, $\langle (\delta N^{\text{exp}})^{n} \rangle_c$, with the genuine proton number cumulants in the final state in heavy ion collisions, $\langle (\delta N^{\text{col}})^{n} \rangle_c$. With the value of $\langle N^{2\text{nd}} \rangle$ estimated in experiments, this relation enables one to eliminate the effect of secondary protons from higher-order cumulants.

C. Net-proton and baryon number cumulants

Next, we apply a similar argument to the cumulants of net-proton and net-baryon numbers, $N^{\text{col}}_{p-\bar{p}}$ and $N^{\text{col}}_{B-B}$, respectively. With the same argument addressed in Sec. II A, one can regard that the correlations between $N^{2\text{nd}}_{p}$ and the number of each baryon is well suppressed with a sufficiently narrow $N_{\text{ch}}$ bin. Assuming the absence of the correlation in a set of collision events analyzed, the probability distribution to observe $N^{\text{exp}}_{p}$ protons and $N_{\bar{p}}$ anti-protons in the experiment is written as

$$G^{\text{exp}}_\theta(N^{\text{exp}}_p, N_{\bar{p}}) = \sum_{N^{\text{col}}_p, N^{2\text{nd}}_p} G^{\text{col}}(N^{\text{col}}_p, N_{\bar{p}})\mathcal{P}_{2\text{nd}}(N^{2\text{nd}}_p) \times \delta_{N^{\text{exp}}_p, N^{\text{col}}_p+N^{2\text{nd}}_p}, \quad (9)$$

where $G^{\text{col}}(N^{\text{col}}_p, N_{\bar{p}})$ is the probability distribution function that the final state of each collision event has $N^{\text{col}}_p$ protons and $N_{\bar{p}}$ anti-protons in the phase space covered by the detector. Note that the anti-proton number $N_{\bar{p}}$ does not receive modification from secondary protons, and the distinction between “exp” and “col” is not needed. This relation immediately leads to

$$\langle (\delta N^{\text{exp}})^{n}_{p-\bar{p}} \rangle_c = \langle (\delta N^{\text{col}})^{n}_{p-\bar{p}} \rangle_c + \langle (\delta N^{2\text{nd}})^{n}_{p-\bar{p}} \rangle_c. \quad (10)$$

In Refs. [18, 19], formulas to obtain net-baryon number cumulants using experimentally-observed proton number fluctuations were derived. These formulas are based on the fact that the probability to observe a baryon coming from the hot medium as a proton or a neutron in the detector is even and uncorrelated for each baryon to a good approximation. This property leads to

$$G^{\text{col}}(N^{\text{col}}_p, N_{\bar{B}}) = \sum_{N^{\text{col}}_B, N^{2\text{nd}}_B} \mathcal{F}_{\text{col}}(N^{\text{B}}_B, N^{2\text{nd}}_B) \times B_{1/2}(N^{\text{p}}_B; N^{\text{B}}_B)B_{1/2}(N^{\text{\bar{B}}}_B; N^{\text{B}}_B), \quad (11)$$

where $\mathcal{F}_{\text{col}}(N^{\text{B}}_B, N^{2\text{nd}}_B)$ is the probability distribution to have $N^{\text{B}}_B$ baryons and $N^{2\text{nd}}_B$ anti-baryons in the phase space in the final state, with the binomial distribution function

$$B_{r}(N; M) = r^{N}(1-r)^{M-N} \frac{M!}{N!(M-N)!}. \quad (12)$$

Neglecting the correlation between (anti-)baryon number and $N^{2\text{nd}}$, one can substitute Eq. (11) into Eq. (9), obtaining

$$\langle (\delta N^{\text{exp}})^{n}_{B-B} \rangle_c = \langle (\delta N^{\text{col}})^{n}_{B-B} \rangle_c + 2\langle (\delta N^{2\text{nd}})^{n}_{B-B} \rangle_c, \quad (13)$$

where $\langle (\delta N^{\text{exp}})^{n}_{B-B} \rangle_c$ is the $n$-th order cumulant of the net-baryon number reconstructed from experimentally-observed proton number fluctuations using Eqs. (9)-(12) in Ref. [18], while $\langle (\delta N^{\text{col}})^{n}_{B-B} \rangle_c$ is the genuine net-baryon number cumulants in the final state.

By employing Eqs. (10) and (13), one can remove the effects of the secondary protons from the cumulants of net-proton and net-baryon numbers, respectively.
D. Effects on ratios of cumulants

Next, let us consider how the recent experimental results on the net-proton number cumulants are modified with the effect of secondary protons using Eq. [10].

Because each cumulant is an extensive quantity and proportional to the volume of the system, experimental results on proton number cumulants are usually discussed in terms of their ratios in order to cancel out the volume dependence. Eq. [10] shows that the effect of secondary protons modifies these ratios as

\[
R_{nm}^{\text{col}} = \frac{\langle (\delta N_{P-P}^{\text{col}})^n \rangle_c}{\langle (\delta N_{P-P}^{\text{exp}})^m \rangle_c} = \frac{\langle (\delta N_{P-P}^{\text{exp}})^n \rangle_c - \langle N_{2nd}^{\text{exp}} \rangle_c}{\langle (\delta N_{P-P}^{\text{exp}})^m \rangle_c - \langle N_{2nd}^{\text{exp}} \rangle_c}. \tag{14}
\]

This relation shows that when the ratio is unity, i.e. the Poissonian value, the effect of secondary protons does not alter the ratio. When the ratio \(R_{nm}^{\text{col}} \equiv \langle (\delta N_{P-P}^{\text{exp}})^n \rangle_c / \langle (\delta N_{P-P}^{\text{exp}})^m \rangle_c\) has a deviation from unity, however, removing the effect of secondary protons makes the difference large,

\[
|1 - R_{nm}^{\text{col}}| > |1 - R_{nm}^{\text{exp}}|. \tag{15}
\]

Substituting \(\langle N_{2nd}^{\text{exp}} \rangle \simeq 0.2 \langle N_{\text{exp}}^{\text{exp}} \rangle\) in Eq. [14], one has

\[
R_{nm}^{\text{col}} \simeq \frac{\langle (\delta N_{P-P}^{\text{exp}})^n \rangle_c - 0.2 \langle N_{\text{exp}}^{\text{exp}} \rangle}{\langle (\delta N_{P-P}^{\text{exp}})^m \rangle_c - 0.2 \langle N_{\text{exp}}^{\text{exp}} \rangle} = \frac{R_{nm}^{\text{exp}} - r}{1 - r} \tag{16}
\]

\[
= R_{nm}^{\text{exp}} - r(1 - R_{nm}^{\text{exp}}) + O(r^2), \tag{17}
\]

with \(r = 0.2 \langle N_{\text{exp}}^{\text{exp}} \rangle / \langle (\delta N_{P-P}^{\text{exp}})^m \rangle\). In the last line, \(r\) is regarded small and higher order terms in \(r\) are neglected; in the HRG model, \(r\) approaches 0.1 for large \(\sqrt{s_{NN}}\), while \(r \simeq 0.2\) for \(\sqrt{s_{NN}} \lesssim 20\) GeV for even \(m\).

At STAR experiment, \(R_{32}^{\text{exp}} \simeq 0.2\) at \(\sqrt{s_{NN}} \simeq 200\) GeV \[3\]. Substituting this value into Eq. [17], one has

\[
R_{32}^{\text{col}} \simeq R_{32}^{\text{exp}} - 0.1, \tag{18}
\]

with \(r \simeq 0.1\). The third order cumulant at large \(\sqrt{s_{NN}}\) thus will receive about 50% modification from the effect of secondary protons.

The values of \(R_{42}^{\text{exp}}\) at STAR, on the other hand, is slightly smaller than but not far from unity for all \(\sqrt{s_{NN}}\) analyzed in the BES program \[3\]; \(R_{42}^{\text{exp}} \simeq 0.8\) at \(\sqrt{s_{NN}} \simeq 20\) GeV is the smallest value. With this value of \(R_{42}^{\text{exp}}\), the difference between \(R_{32}^{\text{col}}\) and \(R_{32}^{\text{exp}}\) expected from Eq. [17] is a few percent and is not significant. While the correction is not large, we note that the important signal is in the difference from the Poissonian value, \(1 - R_{42}^{\text{col}}\). In terms of this difference, the effect of secondary proton gives rise to about 10% modification for \(R_{42}^{\text{exp}} \simeq 0.8\),

\[
1 - R_{42}^{\text{col}} \simeq 1.1(1 - R_{42}^{\text{exp}}), \tag{19}
\]

which would not be negligible in the discussion of non-hadronic nature of the primordial fireballs.

While we have discussed the effect of secondary protons on net-proton number cumulants in this subsection, when one wants to compare the experimental results with theoretical predictions, it is important to obtain the net-baryon number cumulants \[18, 19\]. When the baryon number cumulants are analyzed in experiments, the effect of secondary protons should be removed with Eq. [13].

III. DISCUSSION AND SUMMARY

While we have concentrated on the background effects by secondary protons on higher-order cumulants of the net-proton and net-baryon numbers in this study, similar arguments are also applied to other background effects. In particular, when a background effect can be regarded independent of the baryon numbers in the final state, the background effect can be removed in a way completely similar to the argument in the present study. One of such candidates is the effect of particle misidentification. On the other hand, the effects of efficiency and acceptance of the detector are approximately expressed as a binomial distribution \[18, 19, 22\], and different treatment is required \[19\].

In the present study, we considered the effects of secondary protons on higher order cumulants of the net-proton and net-baryon numbers measured by event-by-event analyses. We argued that the number of secondary protons in each event follows Poisson distribution, and is uncorrelated with the numbers of each baryon emitted from fireballs. Using these properties, effects of secondary protons on net-proton number cumulants were given in a simple form, Eq. [10]. This formula enables one to subtract the background effect from the experimental results on the cumulants. The formula for the net-baryon number cumulants was also given by Eq. [13].

It is found that the effect of secondary protons makes the ratios of the cumulants in experimental measurements closer to unity compared to the ratios with the number of the genuine protons coming from the hot medium. Such an effect will particularly modify the value of the third-order cumulant at large \(\sqrt{s_{NN}}\). Since the deviation of the ratio from unity carries information on the non-hadronic and non-thermal properties of the primordial fireballs, appropriate treatment on this effect is important to reveal the genuine thermodynamical nature of the primordial fireballs.
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