Adaptive Optimal PMU Placement Based on Empirical Observability Gramian

Junjian Qi, Member, IEEE, Kai Sun, Senior Member, IEEE, and Wei Kang, Fellow, IEEE

Abstract—In this paper we compare four measures of the empirical observability Gramian that are used to quantify the observability of system states and to obtain the optimal PMU placement for power system dynamic state estimation. An adaptive PMU placement method is also proposed by making use of the advantages of these measures. The effectiveness of the proposed method is validated by performing dynamic state estimation on NPCC 48-machine system.

Index Terms—Adaptive, dynamic state estimation, empirical observability Gramian, observability, PMU placement.

I. INTRODUCTION

OPTIMAL PMU placement for power system dynamic state estimation [1], [2] has been formulated as an optimization problem that maximizes the determinant of the empirical observability Gramian [3]. As in [3], there are various measures of the empirical observability Gramian that can be used to quantify the observability. In this letter we compare the measures and further propose an adaptive PMU placement method by automatically choosing proper measures as the objective function to guarantee best observability.

Section II introduces the optimal PMU placement method in [3]. Section III compares four measures of the empirical observability Gramian. Section IV proposes and validates an adaptive PMU placement method by automatically choosing proper measures and compares the measures and further proposes an adaptive PMU placement for power system dynamic state estimation. An adaptive PMU placement method by automatically choosing proper measures as the objective function to guarantee best observability.

II. OPTIMAL PMU PLACEMENT BY USING EMPIRICAL OBSERVABILITY GRAMIAN

The discrete nonlinear system model obtained by discretizing the differential equations describing the dynamics of generators and the measurement functions is described as [3]

\[
\begin{align*}
    x_k &= f(x_{k-1}, u_{k-1}) \quad (1a) \\
    y_k &= h(x_k, u_k) \quad (1b)
\end{align*}
\]

where \( f \) and \( h \) are the state transition and output functions, \( x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^r \), and \( y_k \in \mathbb{R}^p \) are the state vector, input vector, and output vector at time step \( k \).

By defining the following sets

\[
T^n = \{T_1, \cdots, T_r; \ T_l \in \mathbb{R}^{n \times n}, \ T_l^T T_l = I_n, \ l = 1, \cdots, r\} \\
M = \{c_1, \cdots, c_s; \ c_m \in \mathbb{R}, \ c_m > 0, \ m = 1, \cdots, s\} \\
E^n = \{e_1, \cdots, e_n; \ \text{standard unit vectors in} \ \mathbb{R}^n\}
\]

where \( T^n \) defines \( r \) initial state perturbation directions, \( I_n \) is an \( n \times n \) identity matrix, \( M \) defines \( s \) perturbation sizes for each direction, and \( E^n \) defines the state to be perturbed, the empirical observability Gramian can be defined as [4]

\[
W = \sum_{l=1}^{r} \sum_{m=1}^{s} \frac{1}{r s c_m^2} \sum_{k=0}^{K} T_l \Psi_k^{lm} T_l^T \Delta t_k \quad (2)
\]

where \( \Psi_k^{lm} \in \mathbb{R}^{n \times n} \) is given by \( \Psi_k^{lm} = (y_k^{lm} - y^{lm,o})^T (y_k^{lm} - y^{lm,o}) \), \( y_k^{lm} \) is the output at time step \( k \) corresponding to initial condition \( x(0) = c_m T_l e_l + x_0 \), \( y^{lm,o} \) is the output for unperturbed initial state \( x_0 \), \( K \) is the number of points, and \( \Delta t_k \) is the time interval between two points.

Based on empirical observability Gramian the optimal PMU placement for dynamic state estimation can be formulated as

\[
\max_{z} F(W(z)) \\
\text{s.t.} \ \sum_{i=1}^{g} z_i = \bar{g} \quad (3)
\]

where \( F \) is a function (measure) of the Gramian \( W \), \( z \) is the vector of binary control variables determining PMU placement, \( g \) and \( \bar{g} \) are the number of generators and PMUs.

The objective function \( F \) can be the determinant (det) [3], the trace (Tr) [3], the smallest eigenvalue (\( \sigma_{\min} \)) [6], [7], or the opposite of the condition number (\( -\kappa \)) [7]. By solving (3) an optimal PMU placement can be obtained for each \( \bar{g} \) and each \( F \). More details on implementation can be found in [3].

III. COMPARING DIFFERENT MEASURES OF THE EMPIRICAL OBSERVABILITY GRAMIAN

Different measures of the observability Gramian reflect various aspects of observability. The determinant and the trace measure the overall observability in all directions in noise space. However, the trace cannot tell the existence of a zero eigenvalue and an unobservable system may still have a large trace. The smallest eigenvalue defines the worst scenario of observability while the condition number emphasizes the numerical stability in state estimation.

Dynamic state estimation is performed by using square-root unscented Kalman filter [8] for 50 times on NPCC 48-machine system [9] under optimal PMU placements for different \( F \) and random PMU placements to estimate the state trajectory on [0, 5s]. For each case a three-phase fault is applied at the from bus of one of the 50 branches with highest line flows. The generator and measurement model in Section III.C of [3] is used. Measurements are voltage phasors and current phasors from PMUs at generator terminal buses with a sampling rate of 60 frames/s. All the other settings are the same as [3].
Fig. 1 shows the average error of rotor angles $\bar{e}_t = \sqrt{\sum_{i=1}^{N} \sum_{t=1}^{T_s} (x_{t,i}^{\text{est}} - \bar{x}_{t,i}^{\text{true}})^2 / (g T_s)}$ where $x_{t,i}^{\text{est}}$ and $\bar{x}_{t,i}^{\text{true}}$ are estimated and true angle of the $i$th generator at time step $t$ and $T_s$ is the number of time steps. Fig. 2 shows the average number of convergent angles for which the differences between the estimated and true values in the last 1 second are less than 2% of the absolute value of true values. It is seen that: 1) optimal placements are better than random placements; 2) $\text{Tr}$ is not a good measure due to being unable to tell the existence of a zero eigenvalue; 3) $\det$ is a good measure if the number of PMUs is not too small and observability is not too weak; otherwise it has too small values; 4) $\sigma_{\min}$ and $-\kappa$ work better than $\det$ for small number of PMUs, in which case they have more reasonable values to indicate observability.

IV. ADAPTIVE PMU PLACEMENT METHOD

By making use of the advantages of different measures we propose the following adaptive PMU placement method. $z^*_{\text{det}}(\bar{g})$ if $\det(z_{\text{det}}(\bar{g})) \geq 1$

$z^*_{-\kappa}(\bar{g})$ if $\det(z_{\text{det}}^*(\bar{g})) < 1$ and $R_{-\kappa} \geq R_{\sigma_{\min}}$

$z^*_{\sigma_{\min}}(\bar{g})$ if $\det(z_{\text{det}}(\bar{g})) < 1$ and $R_{-\kappa} < R_{\sigma_{\min}}$

where $z^*(\bar{g})$ is the optimal placement of $\bar{g}$ PMUs, $z^*_{\text{det}}(\bar{g})$ is the optimal placement of $\bar{g}$ PMUs by using $F \in \{\det, \sigma_{\min}, -\kappa\}$ as the objective function in (3), $F'(W(z^*_{\text{det}}(\bar{g})))$ is written as $F'(z^*_{\text{det}}(\bar{g}))$ and is the function value of $F' \in \{\det, \sigma_{\min}, -\kappa\}$ for $W(z^*_{\text{det}}(\bar{g}))$ and $R_{-\kappa} = \kappa(z_{\text{det}}^*(\bar{g}))/\kappa(z^*_{-\kappa}(\bar{g}))$ and $R_{\sigma_{\min}} = \sigma_{\min}(z_{\text{det}}^*(\bar{g}))/\sigma_{\min}(z_{\text{det}}^*(\bar{g}))$ indicate the improvement of the condition number using $-\kappa$ objective function and that of the minimum eigenvalue using $\sigma_{\min}$ objective function compared with using $\det$ objective function.

Fig. 3 shows how the proposed method works. In our case, when the number of PMUs $\bar{g} \geq 10$, $\det(z_{\text{det}}^*) \geq 1$ (log $\det(z_{\text{det}}^*) \geq 0$) and thus the determinant is big enough to be used as objective function. When $\bar{g} < 10$, $\sigma_{\min}$ or $-\kappa$ is considered. Since the improvement for condition number is higher ($R_{-\kappa} > R_{\sigma_{\min}}$), $-\kappa$ is chosen as objective function. Fig. 4 shows the average numbers of convergent angles for adaptive optimal placements, which are much greater than those under random placements and are also better than those only using one measure as objective function.

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