A NEW CONSTRAINTS SEPARATION FOR THE ORIGINAL D=10

MASSLESS SUPERPARTICLE

D.Dalmazi*†

UNESP - Campus de Guaratinguetá - DFQ -, CP 205, Guaratinguetá - S.P., Brazil.

Abstract

We study the problem of covariant separation between first and second class constraints for the $D=10$ Brink-Schwarz superparticle. Opposite to the supersymmetric light-cone frame separation, we show here that there is a Lorentz covariant way to identify the second class constraints such that, however, supersymmetry is broken. Consequences for the $D=10$ superstring are briefly discussed.

* Partially supported by CNPq
† E-mail: dalmazi@grt000.uesp.ansp.br

hep-th/9401011
1. - Introduction

There are basically two different formalisms for treating supersymmetric critical \((D = 10)\) strings, namely, the Neveu-Schwarz-Ramond (NSR) formulation and the Green-Schwarz (GS) approach (see [1] for a review). The GS superstring has the advantage of being explicitly spacetime supersymmetric as opposite to the NSR string, also called spinning string, where spacetime supersymmetry is only achieved after the GSO projection.

For the calculation of scattering amplitudes the manifest supersymmetry of the GS approach represents, in principle, a great technical advantage since some divergences which appear in the NSR formulation before the GSO projection do not show up in the GS approach. In practice, however, due to the lack of a Lorentz covariant quantization of the GS superstring its advantages are withdrawn by the difficulties which appear in the light-cone [2] (or semi-light-cone [3]) gauges where this theory is usually quantized.

One of the difficulties of the noncovariant GS approach is that Lorentz covariance of the amplitudes can only be proven indirectly by relating the GS-superstring in the light-cone gauge with the light-cone NSR string which is on its turn equivalent [4] to the covariant NSR string. It is thus desirable to have an explicitly Lorentz covariant quantization of the GS-superstring which, however, has become a formidable task. In order to solve this problem we need a better understanding of a local fermionic symmetry of the GS-superstring which was discovered by Siegel [5] and is called \(\kappa\)-invariance. Fortunately, this symmetry also appears in the infinite tension limit of the GS superstring, namely, the ten-dimensional massless superparticle [6].

In fact, all the problems which prevent a covariant quantization of the GS superstring are also present in the simpler case of the superparticle. From the lagrangean point of view we have the following problems; on one hand the algebra of gauge transformations only closes on shell and the gauge generators are not linearly independent. On the other hand, there is, apparently, no consistent Lorentz covariant gauge condition to fix the \(\kappa\)-invariance. The first two problems do not permit the use of the Faddeev-Popov procedure to fix the gauge, but they can be circumvented [7] by applying the Batalin-Vilkovisky [8] formalism according to which it is necessary to introduce an infinite set of ghosts of alternating
statistics [7]. The problem of choosing a consistent Lorentz covariant gauge condition is more dramatic and it has not been solved yet. In the hamiltonian formalism the problems appear when we try to identify the second class constraints among the full set of constraints (see, e.g., ref.[9]). There seems to be no Lorentz covariant way of identifying the second class constraints. Besides, the first class constraints can only be covariantly written in a redundant (linearly dependent) way. In constructing the BRST charge this redundancy leads to difficulties, but for a Dirac quantization it does not represent a problem.

The apparently non-existence of both a Lorentz covariant gauge choice and a Lorentz covariant identification of the second class constraints has led many authors to suggest new formulations for the $D = 10$ massless superparticle and the $D = 10$ superstring, see e.g. [9,10-18] and [19,20] respectively. In [9,10-13] the basic idea was to extend the original superspace by introducing pure gauge variables while, the most promising approach seems to be the one initiated in [15] and further pursued in [18-20] where one tries to convert part of the $\kappa$-invariance in worldline (worldsheet) supersymmetry by the use of twistors and supertwistors variables, thus making a direct connection between the NSR and the GS formulations.

In this paper we come back to the original superparticle [6] and analyse it from the hamiltonian point of view. Since we are only interested in a Lorentz covariant Dirac quantization we just have to face the constraints separation problem. Without fixing any gauge we show that besides the light-cone frame constraints separation [9,21] there is a Lorentz covariant separation which works as follows, first of all we show that a light-like vector $n^\mu$ satisfying some conditions is all we need (see also [9]) for a consistent Lorentz covariant identification of the second class constraints. Opposite to [9] we do not introduce the vector $n^\mu$ by hand but construct it (see formula (3.1)) out of the canonical pair $(x^\mu, p^\mu)$. Although the second class constraints are identified in a redundant way we are able to eliminate them by a kind of Dirac bracket. By using such bracket we check that the Lorentz algebra closes classically although supersymmetry and translation invariance are lost. We finally show that for the $D = 10$ superstring the situation is different and it is possible, in principle, to separate the constraints keeping both supersymmetry and
Lorentz invariance.

2. - Global symmetries and constraints structure.

The action for the massless superparticle [6] in ten dimensions can be written in the first order formalism as:

\[ S = \int d\tau (p \cdot \dot{x} - i \theta \tilde{p} \dot{\theta} - \frac{e p^2}{2}) \]  \hspace{1cm} (2.1)

where \( \tilde{p}_{\alpha\beta} = p_\mu (\tilde{\sigma}^\mu)_{\alpha\beta} \) \((\alpha, \beta = 1, 2, \ldots, 16; \mu = 0, 1, \ldots, 9)\) and \( \theta^\alpha \) is a right handed Majorana-Weyl spinor in ten dimensions, which has 16 real components. The real and symmetric matrices \((\tilde{\sigma}^\mu)_{\alpha\beta}\) and \((\sigma^\mu)_{\alpha\beta}\) satisfy

\[
\begin{align*}
(\sigma^\mu \tilde{\sigma}^\nu + \sigma^\nu \tilde{\sigma}^\mu)_{\beta}^\alpha &= (\tilde{\sigma}^\mu \sigma^\nu + \tilde{\sigma}^\nu \sigma^\mu)_{\alpha}^\beta = 2 \eta^{\mu\nu} \delta_{\beta}^\alpha, \hspace{1cm} (2.2a) \\
Tr(\tilde{\sigma}^\mu \sigma^\nu) &= Tr(\sigma^\mu \tilde{\sigma}^\nu) = 16 \eta^{\mu\nu}, \hspace{1cm} (2.2b) \\
(\sigma^\mu)_{(\alpha\beta}(\sigma_\mu)_{\gamma\delta)} &= 0 = (\tilde{\sigma}^\mu)_{(\alpha\beta}(\tilde{\sigma}_\mu)_{\gamma\delta)} \hspace{1cm} (2.2c)
\end{align*}
\]

where \( \eta^{\mu\nu} = (-, +, \ldots, +) \) stands for the spacetime metric.

The global invariances of the action (2.1) consist of superpoincaré transformations and dilatations which we call superweyl transformations. Infinitesimally we have,

\[
\begin{align*}
\delta x^\mu &= -2 \omega^{\mu\nu} x^\nu + a^\mu - i e \tilde{\sigma}^\mu \theta + \lambda x^\mu, \\
\delta p^\mu &= -2 \omega^{\mu\nu} p^\nu - \lambda p^\mu, \\
\delta \theta^\alpha &= \frac{1}{4} \omega^{\mu\nu} (\tilde{\sigma}_{\mu\nu} \theta)^\alpha + \epsilon^\alpha + \lambda \theta^\alpha, \\
\delta e &= 2 \lambda e \hspace{1cm} (2.3)
\end{align*}
\]

where \( \tilde{\sigma}_{\mu\nu} = \sigma_\mu \tilde{\sigma}_\nu - \sigma_\nu \tilde{\sigma}_\mu \) \((\sigma_{\mu\nu} = \tilde{\sigma}_\mu \sigma_\nu - \tilde{\sigma}_\nu \sigma_\mu)\).

Using the basic Poisson brackets\(^1\) \( \{x^\mu, p_\nu\} = \delta^\mu_\nu; \{e, \Pi_\epsilon\} = 1 \) and \( \{\theta^\alpha, \Pi_\beta\} = \delta^\alpha_\beta \), where \( \Pi_\beta = \frac{\partial e}{\partial \theta^\beta} \), we can check that the superweyl transformations (2.3) can be generated by the following quantities:

\[
J_{\mu\nu} = x_\nu p_\mu - x_\mu p_\nu + \frac{1}{4} \Pi \tilde{\sigma}_{\mu\nu} \theta \hspace{1cm} (2.4a)
\]

\(^1\) We use the same symbol for Poisson brackets and Poisson anti-brackets which are henceforth called generically Poisson brackets.
\[
Q_\alpha = \Pi_\alpha - i(\bar{\psi}\theta)_\alpha , \quad (2.4b)
\]
\[
P_\mu = p_\mu , \quad (2.4c)
\]
\[
D = x \cdot p + \frac{\Pi \theta}{2} + 2 e \Pi_e , \quad (2.4d)
\]
which satisfy the superweyl algebra below:

\[
\{ J_{\mu\nu}, J_{\alpha\beta} \} = \delta_{\mu\beta} J_{\nu\alpha} - \delta_{\mu\alpha} J_{\nu\beta} + \delta_{\nu\alpha} J_{\mu\beta} - \delta_{\nu\beta} J_{\mu\alpha} , \quad (2.5a)
\]
\[
\{ J_{\mu\nu}, P_\alpha \} = \delta_{\nu\alpha} P_\mu - \delta_{\mu\alpha} P_\nu , \quad (2.5b)
\]
\[
\{ Q_\alpha, Q_\beta \} = -2 i (\bar{\psi})_{\alpha\beta} ; \quad \{ J_\mu, Q_\alpha \} = -\frac{(\sigma_{\mu\nu} Q)_\alpha}{4} , \quad (2.5c)
\]
\[
\{ P_\mu, D \} = -P_\mu ; \quad \{ Q_\alpha, D \} = -\frac{1}{2} Q_\alpha , \quad (2.5d)
\]

with the remaining brackets vanishing.

The superparticle (2.1) contains two bosonic first class constraints:

\[
\varphi_1 : \quad p^2 \approx 0 , \quad (2.6)
\]
\[
\varphi_2 : \quad \Pi_e \approx 0 , \quad (2.7)
\]

and the following 16 fermionic constraints.

\[
d_\alpha = \Pi_\alpha + i(\bar{\psi})_\alpha = 0 . \quad (2.8)
\]

The only nonvanishing brackets among the constraints are the following ones

\[
\{ d_\alpha, d_\beta \} = 2 i (\bar{\psi})_{\alpha\beta} . \quad (2.9)
\]

Due to (2.6) and the relation (2.2a) we get

\[
\{ d_\alpha, (\bar{\psi}d)^\beta \} \approx 0 . \quad (2.10)
\]

Since \( \{ p^2, (\bar{\psi}d)^\beta \} = 0 = \{ \Pi_e, (\bar{\psi}d)^\beta \} \) we conclude that the combination \( \bar{\psi}d \), which is weakly equivalent to \( \bar{\psi}\Pi \), corresponds to first class constraints. Indeed, the constraints \( \bar{\psi}\Pi \) generate the local \( \kappa \)-invariance [5] of (2.1), which reads

\[
\delta_{\kappa} \theta = \bar{\psi} \kappa(\tau) , \quad (2.11)
\]
\[
\delta_{\kappa} x^\mu = -i \theta \bar{\psi} \sigma^\mu \kappa(\tau) ,
\]
\[
\delta_{\kappa} e = -2 i \theta \kappa(\tau) ,
\]
with \( \kappa_\alpha \) a left-handed Majorana-Weyl spinor.

In order to understand the meaning of the remaining fermionic constraints we suppose the existence of a vector \( n^\mu \) satisfying

\[
p \cdot n \neq 0 \quad ,
\]

such that by use of (2.2a) we can write:

\[
d = \mathcal{P}_+ d + \mathcal{P}_- d \quad ,
\]

where \( \mathcal{P}_+ = (2n \cdot p)^{-1} \not p p \) and \( \mathcal{P}_- = (2n \cdot p)^{-1} \not p \not p \). By further assuming that \( n^\mu \) is strongly light-like (\( n^2 = 0 \)) we show, using also (2.2b), that

\[
(\mathcal{P}_\pm)^2 = \mathcal{P}_\pm \quad ; \quad \mathcal{P}_+ \mathcal{P}_- = 0 \quad ,
\]

\[
Tr (\mathcal{P}_+) = Tr (\mathcal{P}_-) = 8
\]

Therefore \( \mathcal{P}_\pm \) are projection operators (strongly) and the 16 constraints \( d_\alpha = 0 \) are equivalent to the 8 first class constraints \( \not p d = 0 \) and the 8 remaining constraints \( \not \not p d = 0 \).

If the light-like vector \( n^\mu \) satisfies

\[
\{ n^\mu , n^\nu \} = 0 \quad ,
\]

\[
\{ n^\mu , d_\alpha \} = 0 \quad ,
\]

then, we can prove that \( \not \not p d \) represent 8 second class constraints obeying:

\[
\{ (\not \not p d)^\alpha , (\not \not p d)^\beta \} = 4 i (n \cdot p) (\not \not p)^{\alpha \beta} \quad .
\]

So we come to the correct counting of independent constraints of the \( D = 10 \) massless superparticle, 8 first class + 8 second class, which was deduced long ago [22] in the light-cone frame where anyone of the light-cone directions \( n^\mu_\pm = (\mp 1, 0, \cdots, 0, 1) \) satisfies (2.15), (2.16) and \( p^\mu \) can be assumed to obey (2.12). In the next section we show that there is another couple of light-like vectors \( n^\mu_\pm \) which leads to a Lorentz covariant separation still with the correct counting of constraints.
3. - Lorentz covariant constraints separation

We can always take appropriate nonlinear combinations of two spacetime vectors to construct a couple of light-like vectors, for instance, taking \( x^\mu \) and \( p^\mu \) we have

\[
n^\mu_{\pm} = x^2 p^\mu - [x \cdot p \pm ((x \cdot p)^2 - x^2 p^2)^{\frac{1}{2}}] x^\mu \ ,
\]

which identically satisfy \( n^2_{\pm} = 0 \). In order that \( n^\mu_{\pm} \) be real we have to impose the following scale invariant restriction on the phase space:

\[
f \equiv (x \cdot p)^2 - x^2 p^2 > 0 \ .
\]

The above restriction can only be violated by some noncausal configurations inside the region where both \( p^\mu \) and \( x^\mu \) are space-like, i.e., \( x^2 > 0 \) and \( p^2 > 0 \). Notice also that (3.2) will be automatically satisfied on the physical states \( (p^2)_{\text{phys}} = 0 \).

Actually, the restriction (3.2) is too strong, since we would have real \( n^\mu_{\pm} \) also for \( f = 0 \), but if we intend to use \( n^\mu_{+} \) or \( n^\mu_{-} \) to identify the second class constraints we must have from (2.12) that \( p \cdot n_{\pm} = -f^{\frac{1}{2}} (f^{\frac{1}{2}} \pm x \cdot p) \neq 0 \) which means that \( f = 0 \) must be discarded and we have to suppose either \( x \cdot p > 0 \) or \( x \cdot p < 0 \) if we choose \( n^\mu_{+} \) or \( n^\mu_{-} \) respectively. Henceforth we choose \( n^\mu_{+} \) and assume the scale invariant restriction

\[
x \cdot p > 0 \ ,
\]

which plays the role of the \( p^+ > 0 \) condition of the light-cone frame.

The restrictions (3.2) and (3.3) guarantee that the decomposition (2.13) holds and the 8 constraints\(^{22}\) \( \eta^d \) are real such that we have the correct counting of constraints. It is easy to check, quite surprisingly, that \( n^\mu \) satisfies (2.15). The condition (2.16), however is not obeyed and instead of (2.17) we get

\[
\{(\eta^d)^\alpha, (\eta^d)^\beta\} = 4i(\eta)^{\alpha\beta} \left( n \cdot p - \frac{\Pi \theta}{2} \right) + \text{terms proportional to } \eta^d \ ,
\]

\(^{22}\) From now on we drop the subscript '+' of \( n^\mu_{+} \).
where we have used the identity

\[(d \gamma \sigma^\mu \theta) \sigma^\alpha \sigma^\beta = (\sigma^\mu d)^\alpha (\sigma^\mu \bar{\theta})^\beta = 2 d \theta (\bar{\gamma})^\alpha \beta , \tag{3.5}\]

which is a direct consequence of (2.2c). Taking into account that \( \gamma d \) has weakly vanishing Poisson brackets with the first class constraints \( \varphi_1, \varphi_2, \gamma d \) and assuming a third scale invariant restriction on the phase space:

\[n \cdot p - \frac{\Pi \theta}{2} \neq 0 , \tag{3.6}\]

which is also automatically satisfied on the physical states (assuming the restrictions (3.2) and (3.3)) where \( \Pi \theta = d \theta = 0 \) we conclude that \( \gamma d \) really represent 8 independent second class constraints which gives the correct counting of constraints once again.

Now we should notice that we cannot define the usual Dirac brackets to eliminate \( \gamma d \) since the right-handed side of (3.4) has no inverse due to the redundancy of the 16 linearly dependent constraints \( \gamma d \). In such cases there is, \textit{a priori} , no unique definition of the bracket (see [23]) used to eliminate the second class constraints, but by requiring that no further restrictions on the phase space than (3.2),(3.3) and (3.6) should be imposed we have just one possible definition, namely,

\[\{A, B\}^* = \{A, B\} - \{A, (\gamma d)^\alpha\} \frac{(\bar{\gamma})^\alpha \beta}{g} \{(\gamma d)^\beta, B\} , \tag{3.7}\]

where \( g = 4 i (n \cdot p) (2 n \cdot p - \Pi \theta) \). The Lorentz covariant definition (3.1) of \( n^\mu \) assures the classical closure of the Lorentz algebra for the constraints separation above. This closure can be confirmed by an explicit computation of the algebra of the superweyl generators \( G_a \) (given in 2.4) in terms of the bracket (3.7). By using the new brackets of the superspace variables given in the appendix the reader can check that \( \{G_a, G_b\}^* = \{G_a, G_b\} \) for most of the brackets (2.5) except for the following ones

\[\{P^\mu, P^\nu\}^* = \frac{(x \cdot p + f^2)}{g} d \sigma_\mu \bar{\sigma}_\nu d , \tag{3.8}\]

\[\{Q_\alpha, Q_\beta\}^* = \{Q_\alpha, Q_\beta\} - (\bar{\sigma}_\nu \theta)_\alpha (\sigma_\mu \theta)_\beta \{P^\nu, P^\mu\}^* \tag{3.9}\]

\[\{Q_\alpha, P_\mu\}^* = - (\bar{\sigma}^\nu \theta)_\alpha \{P_\nu, P_\mu\}^* \tag{3.10}\]
Therefore, as in the light-cone frame separation, the superwyel algebra will close only on the physical states $\{G_a, G_b\}^\ast|\text{phys} >= \{G_a, G_b\}|\text{phys} >$ where $d|\text{phys} >= 0$, but opposite to the light-cone case we are able to keep Lorentz covariance ($\{J_{\mu\nu}, J_{\alpha\beta}\}^\ast = \{J_{\mu\nu}, J_{\alpha\beta}\}$) loosing translation invariance and supersymmetry. Our results lead to the conjecture that we have to choose between Lorentz and supersymmetry invariance for the massless superparticle since we have not found any other self-consistent way to identify the second class constraints.

At this point it is important to comment upon the deep changes in the canonical structure of the phase space (see appendix) caused by the elimination of the constraints $\not\!d$ with $n^\mu$ given in (3.1). Of particular interest is the non-commutative nature of the variables $x^\mu$ (see (A.1)). It is remarkable that even on the physical states ($d = 0$) the coordinates $x^\mu$ have nonvanishing brackets among themselves, which will lead, at the quantum level, to noncommuting position operators. Such non-commutativity seems to be a consequence (see remarks in [24]) of having only positive energy states ($E > 0$) in the theory, which is certainly true, in our case, on the physical states where supersymmetry is unbroken. It should be remarked that the results found here and in the light-cone gauge [6] are in agreement with the non-existence of Lorentz covariantly defined commuting position operators for positive energy particles with nonzero spin (nonlocalization problem) which was established long ago [26,27,28].

4. - The superstring case and final remarks

Now we show how the ideas of the last sections can be generalized for the superstring case.

The D=10 GS superstring contains also 16 fermionic constraints

$$D_\alpha = \Pi_\alpha + i (\bar{\Pi}_\alpha \theta)_\alpha - i (\bar{\theta})_\alpha = 0 \quad ,$$

(4.1)

$\not^3$ Notice that although, our results are classical as opposed to the quantum nature of the nonlocalization problem the fact that $\{x^\mu, x^\nu\}^\ast \neq 0$ will clearly survive the quantization.

$\not^4$ The subscript $\sigma$ represents a coordinate of a point of the string and is not to be confused with the matrices $\sigma_\alpha$. 

9
where

\[ \Pi_\sigma^\mu = \partial_\sigma x^\mu + \partial_\sigma \theta \tilde{\sigma}^\mu \theta \],

\[ \Pi_\alpha = \frac{\partial^i \mathcal{L}_{GS}}{\partial \theta^\alpha (\tau, \sigma)} ; p^\mu = \frac{\partial \mathcal{L}_{GS}}{\partial x^\mu (\tau, \sigma)} \] (4.2)

with \( \mathcal{L}_{GS} \) standing for the Green-Schwarz superstring lagrangean (see [1]). The constraints (4.1) have equal time Poisson brackets analogous to (2.9):

\[ \{ D_\alpha (\sigma), D_\beta (\tilde{\sigma}) \} = 2i \delta (\sigma - \tilde{\sigma}) (p_s)_{\alpha \beta} \] . (4.3)

where \( p_s^\mu = -p^\mu + 2 \Pi_\sigma^\mu - \partial_\sigma x^\mu \).

As in the case of the superparticle, due to the fact that \( p_s^2 \approx 0 \), the constraints \( D_\alpha \) split into 8 first class and 8 second class. Thus, once again we have a decomposition like (2.13) where \( \hat{p}_s D \) represent 8 first class constraints while \( \hat{\mathcal{D}}D \) stand for the remaining 8 second class constraints with \( n^\mu \) a strongly light-like vector such that \( p_s \cdot n \neq 0 \). Now comes an important difference to the superparticle’s case, i.e., we have for the superstring other possibilities for building \( n^\mu \) than the Lorentz non-covariant light-cone directions and the non-supersymmetric choice (3.1). Indeed, instead of (3.1) we can define for the superstring, for instance, the light-like vectors

\[ n^\mu_{\pm} = \Pi_\sigma^\mu p_s^\mu - [\Pi_\sigma \cdot p_s \pm ((\Pi_\sigma \cdot p_s)^2 - p_s^2 \Pi_\sigma^2)] \Pi_\sigma^\mu . \] (4.4)

Due to the supersymmetric properties of \( \Pi_\sigma^\mu \) and \( p_s \) (\( \{ \Pi_\sigma^\mu, Q_\alpha \} = 0 = \{ p_s, Q_\alpha \} \)), contrary to (3.1), the definition (4.4) leads to a supersymmetric constraints separation. This and other possibilities are now being investigated. Actually, it is possible to show that the separation based on (4.4) corresponds exactly to the proposal of [25], so we partially unravel the quite mysterious suggestion of that reference.

Finally, it should be emphasized that the considerations made in this work are pure classical and for the separations (3.1) and (4.4) the Dirac brackets are rather involved (see appendix) which means that a correct treatment of the very likely normal ordering problems will be of fundamental importance.
Acknowledgements

I am grateful to Prof. V.O. Rivelles for discussions and a careful reading of the manuscript. I also thank a discussion with Prof. H.P. Nicola on the projection operators $P_\pm$. Conversations with H. Boschi-Filho and A. de Souza-Dutra are also acknowledged.

Finally I thank the financial support of DAAD/CAPES (Sandwich Fellowship) during my stay at the II. Institut für Theoretische Physik (Universität Hamburg) where this work was initiated.

Appendix

New brackets for the phase space variables:

$$\{x^\mu, x^\nu\}^* = -2 \frac{p \cdot n}{g} (\theta \bar{\sigma}^\mu \psi \bar{\sigma}^\nu \theta) + \frac{x^2}{g} (d\sigma^{[\mu} \bar{\psi} \bar{\sigma}^{\nu]} \theta)$$

$$+ \frac{x^4}{g} d\sigma^\mu \bar{\psi} \bar{\sigma}^\nu d$$ \hspace{1cm} (A.1)

$$\{p^\mu, p^\nu\}^* = \left(\frac{x \cdot p + f \frac{x}{2}}{g}\right)^2 (d\sigma^{\mu} \bar{\psi} \bar{\sigma}^\nu d)$$ \hspace{1cm} (A.2)

$$\{x^\mu, p_\nu\}^* = \delta^\mu_\nu +$$

$$\frac{x \cdot p + f \frac{x}{2}}{g} (-i d\sigma_\nu \bar{\psi} \bar{\sigma}^\mu \theta + x^2 d\sigma^{\mu} \bar{\psi} \bar{\sigma}^\nu d) .$$ \hspace{1cm} (A.3)

$$\{\theta^\alpha, \theta^\beta\}^* = -\frac{2 n \cdot p}{g} (\bar{\psi})^{\alpha\beta} .$$ \hspace{1cm} (A.4)

$$\{\Pi^\alpha, \Pi^\beta\}^* = \frac{2 n \cdot p}{g} (\bar{\psi} \bar{\pi} \bar{\psi})_{\alpha\beta} .$$ \hspace{1cm} (A.5)

$$\{\theta^\alpha, \Pi^\beta\}^* = \delta^\alpha_\beta - \frac{2 i (n \cdot p)}{g} (\bar{\psi} \bar{\pi} \bar{\psi})^\alpha_\beta .$$ \hspace{1cm} (A.6)

$$\{\theta^\alpha, x^\mu\}^* = \frac{2 i (n \cdot p)}{g} (\bar{\psi} \bar{\sigma}^\mu \theta)^\alpha + \frac{x^2}{g} (\bar{\psi} \bar{\pi} \bar{\sigma}^\mu d)^\alpha .$$ \hspace{1cm} (A.7)

$$\{\theta^\alpha, p^\mu\}^* = \frac{(x \cdot p + f \frac{x}{2})}{g} (\bar{\psi} \bar{\pi} \bar{\sigma}^\mu d)^\alpha .$$ \hspace{1cm} (A.8)

$$\{\Pi^\alpha, x^\mu\}^* = \frac{i x^2}{2} (\bar{\psi} \bar{\pi} \bar{\sigma}^\mu d)_\alpha - \frac{2 n \cdot p}{g} (\bar{\psi} \bar{\pi} \bar{\sigma}^\mu \theta)_\alpha .$$ \hspace{1cm} (A.9)

$$\{\Pi^\alpha, p^\mu\}^* = i \frac{x \cdot p + f \frac{x}{2}}{g} (\bar{\psi} \bar{\pi} \bar{\sigma}^\mu d)_\alpha .$$ \hspace{1cm} (A.10)

where $f = (x \cdot p)^2 - x^2 p^2$ and $g = 4 i n \cdot p (2 n \cdot p - \Pi \theta)$.
References

[1] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory” (Cambridge U.P., Cambridge 1987).
[2] M. B. Green and J. H. Schwarz , Nucl.Phys B243 (1984) 475.
[3] S. Carlip , Nucl. Phys. B284 (1987) 365.
[4] K. Aoki, E. D’Hoker and D. H. Phong, Nucl. Phys. B342 (1990) 149.
[5] W. Siegel Phys. Lett. B 128 (1983) 397.
[6] L. Brink and J.H. Schwarz, Phys. Lett. b100 B (1981) 310.
[7] R. E. Kallosh, Phys. Lett. B195 (1987) 369.
R. E. Kallosh and A. Morozov, Int. J. of Mod. Phys. A3 (1988) 1943.
[8] I.A. Batalin and G.A. Vilkovisky, Phys.Rev. D28 (1983) 2567 ; Nucl.Phys. B234 (1984) 106.
[9] L. Brink, M. Henneaux, C.Teitelboim, Nucl.Phys. B293 (1987) 505.
[10] H. Aratyn, Phys. Rev. D39 (1989) 503.
[11] M. B. Green and C. M. Hull, Mod. Phys. Lett. A5 (1990) 1399.
[12] R. Kallosh, Phys. Lett. B251 (1990) 134.
[13] A. Mikovic, M. Rocek, W. Siegel, P. van Nieuwenhuizen, J. Yamron and A. E. van de Ven, Phys. Lett. B235 (1990) 106.
F. Essler, M. Hatsuda, E. Laenen, W. Siegel, J. P. Yamron, T. Kimura and A. Mikovic, Nucl. Phys. B364 (1991) 67.
[14] E. Nissimov and S. Pacheva and S. Solomon, Phys. Lett. B189 (1987) 57.
[15] D. P. Sorokin, V. I. Tkach, D. V. Volkov and A. A. Zheltukhin, Phys. Lett. B216 (1989) 302.
[16] N. Berkovits, Nucl. Phys. B350 193(1991).
[17] Y. Eisenberg, Phys. Lett. B276 325(1992).
[18] A. Galperin and E. Sokatchev , Phys. Rev. D46 714(1992).
[19] N. Berkovits, Nucl. Phys. B379 96(1992).
[20] F. Delduc, A. Galperin, P. Howe and E. Sokatchev, “A twistor formulation of the
heterotic D=10 superstring with manifest (8,0) worldsheet supersymmetry”, Bonn-
preprint HE-92-19 (July-1992).

[21] J. M. Evans, Class. and Quantum Grav. 7 699(1990).

[22] I. Bengtsson and M. Cederwall, Göteborg preprint 84-21 (1984) unpublished.

[23] A. Dresse, J. Fisch, M. Henneaux and C. Schomblond, Phys. Lett. B210 141(1988).

[24] I. Bengtsson and M. Cederwall, Nucl. Phys. B302 (1988) 81.

I. Bengtsson, M. Cederwall and N. Linden, Phys. Lett B203 (1988) 96.

[25] J. M. Evans, Phys.Lett. B233 (1989) 307 .

[26] M. H. I. Pryce, Proc. R. Soc. A195 (1948) 62.

[27] T. D. Newton and E. Wigner, Rev. Mod. Phys. 21 (1949) 400.

[28] D. J. Almond, Ann. Inst. Poincarè A19 (1973) 105 .