Abstract

It is not possible to model semantics of natural language unless one can clearly establish the structure of contextual information and how "context" guides the construction of semantic identities of linguistic units. We claim that this cannot be done unless one can overcome the representational locality imposed by extensional formalisms. In this paper we present a formalism aiming at setting up a non-extensional framework allowing the expression of a notion of context. The very notion of identity is revisited and some forms of non-extensional identity are proposed. In the last section we suggest a representation of metaphor understanding that implements a form of interaction between global contextual information and local semantic identities.

1 Introduction

One of the most important problems in modeling semantics of Natural Language is undoubtfully the representational locality of the chosen formal features. Formal locality is apparently contradictory to empirical facts; the semantic identity of a linguistic unit—at whatever level it might be—is determined only within global structures. In formal constructions one generally recovers such global structures by more elementary ones, given by locally necessary and sufficient definitional means. Nevertheless, in Natural Language the primacy is rather to the global structures as they are necessary for the definition of the semantic character of the more elementary ones.

In interpretational semantics [Rastier, 1987] this last constitutes the major principle. The problem in such an approach is the construction of interpretation strategies. From a structuralistic point of view this turns out to be equivalent to the determination of a specific recurrence of semantic components giving isotopies. An isotopy is a form of logically coherent actualization of a possible co-occurrence of such components. Many different strategies may be operational and thus many interpretational schemata may be possible, corresponding to different senses of the same text. What precisely changes across such strategies is the semantic identity of the relevant linguistic entities insofar as different semic actualizations are selected.

Interpretational semantics as a general semantic theory is not yet formalized. The long-term aim of this paper is to provide a plausible approach towards a general formal model. Here we give a glimpse of the formalism by showing how to define a notion of semantic identity and a notion of context so that we shall be able to express semantic identity shifts as effects of interpretational strategies.

2 Formalism

The basic representational unit is called a "concept". Let $C$ be the set of all concepts and define $T_0$ such that:

$$T_0 = \bigcup_{i=1}^{n} C^i \text{; where every } C^i \text{ is given by}$$

$$C^i = \prod_{j=1}^{i} C_j \text{ and } C_j = C \text{ for every } 1 \leq j \leq i \tag{1}$$

The entities in $T_0$, of the form $\bar{e}_{(n)} = (c_{1}, c_{2}, \ldots, c_{n})$, are called "vector-concepts". $T_0$ is supposed to be extensional.

Let us also define the level $T_1$ ("theories" of level 1 in the sense of [Murphy and Medin, 1985]) that gives $T_0$ its structure.

$$T_1 = \{ t \mid t \text{ is a relation over } C'(1 \leq i \leq n) \} \tag{2}$$
The notion of extension of a theory provides the basic link between levels $T_0$ and $T_1$:

$$\text{ext}(t(n)_i) = \{ \bar{c} \mid \bar{c} \in T_0 \text{ and } \bar{c} \text{ satisfies } t(n)_i \}$$

Furthermore, we can introduce an indiscernibility relation $\text{=ext}$

$$t(n)_i \text{ =ext } t(n)_j \iff \text{ext}(t(n)_i) = \text{ext}(t(n)_j)$$

$T_1$ is non-extensional, and we suppose that if $E \subseteq T_0$ such that $\forall x, y \in E, x$ and $y$ are vector-concepts of the same arity, then there is a natural $n > 1$ such that $t_1, \ldots, t_n \in T_1$ such that $\forall i = 1, \ldots, n \text{ ext}(t_i) = E$.

Thus we have a relation between theories based on theories' extensions. On the other hand, the structure of the vector-concepts in the extension of a theory allows to consider it from a more "microscopic" point of view. This is done by the notion of projection of a theory.

Consider $t(n)_i$, an $n$-ary theory in $T_1$; we define its $j$-projection, $1 \leq j \leq n$ as a unary theory $T_1$, noted $t_{ij}$, such that:

$$\text{ext}(t_{ij}) = \{ c \mid \text{there is a } \bar{c} \in \text{ext } (t(n)_i) \text{ whose } j \text{-component is } c \}$$

Clearly theories can be related through their $j$-projections via a relation based on extensional equality:

$$t(n)_i \leq_{p-j} t(n)_k \iff \hat{t}_{ij} \leq \text{ext } \hat{t}_{kj}$$

Furthermore, we can extend this definition in order to build $k$-ary projections of a theory. There are several ways to build such projections. We shall discuss here only the non-adjacent $k$-ary projection of a theory $t(n)_i$, noted $P(k)(H)$, which is relevant for our aims; it is a matter of a $k$-ary theory $t(k)(H)$ such that:

$$\text{ext}(t(k)(H)) = \{ c_{g_1}, c_{g_2}, \ldots, c_{g_k} \mid \exists \bar{c} \in \text{ext } (t(n)_i) \text{ where } c_{g_1} \text{ is its } H_1 \text{-component, } c_{g_2} \text{ is its } H_2 \text{-component, } \ldots, c_{g_k} \text{ is its } H_k \text{-component} \}$$

where $H$ is an injection from $\{1, \ldots, k\}$ to $\{1, \ldots, n\}$.

Clearly there is an indiscernibility relation between theories through their $k$-ary projections: the ordered non-adjacent $k$-ary projective equality (noted $=P(k)^{\text{gen}}$):

$$t(n)_i =_{P(k)^{\text{gen}}} t(n)_p \iff \hat{t}_{(k)(H)} =_{\text{ext } \hat{t}_{(k)p}(H)}$$

However, one should notice that there may be several ways for two theories to be projectively indiscernible. On the other hand, one may even remove the constraint on the arity of theories being compared so that one can obtain a general projective equality between theories of different arity. For instance, one may define a general projective equality, noted $=_{P \text{ -gen}}$, based on the partial order $\leq_{P \text{ -gen}}$, which is given as follows: for $t(n)_i, t(m)_j \in T_1$, and $m \neq n$ naturals:

$$t(n)_i \leq_{P \text{ -gen}} t(m)_j \iff \hat{t}_{io} \leq \text{ext } \hat{t}_{jk}$$

These two indiscernibility relations are easily generalizable to $k$-ary projections. All the technical consequences and related notions are treated in [Zaldivar-Carrillo, 1995]. A special attention is given to the qualification of the notion of identity, as far as difference and identity are ubiquitous in every structuralistic analysis. The aim is to model the notion of identity which takes a large variety of local forms by virtue of the notion of projection. Moreover, logical relations are introduced and relativized to every local identity form.

Let now $T_2$ be a third level of theories defined as follows:

$$T_2 = \{ t \mid t \text{ expresses a n-ary relation over } T_1 \text{ or a binary relation over } T_1 \cup T_0 \text{ of the form } (\hat{c}, \hat{t_j}) \}$$

The motivation of $T_2$ is to "internalize" some notions such as the relation which expresses that concept $c_i$ belongs to the extension of theory $t_{(n)_j}$; such a theory is called a complex theory of level 2. All notions, and specifically those concerning the indiscernibility relations, introduced for theories of level 1, are generalizable at this level. We shall
not present them here. Nevertheless, one has to notice that all the notions based on the extension of a theory have to be reconsidered since level $T_1$ is non-extensional. Thus, two extensionally equal theories may not belong to the same extension of a theory of level 2. This remark allows us to introduce some further notions. First, the notion of saturation of a theory $t_2(n)_i$ (noted $s(t_2(n)_i)$); it corresponds to a theory $t_2(n)_j$ (of level 2) such that for every theory in the extension of $t_2(n)_j$ the whole class of $\equiv_{ext}$-equivalence of that theory belongs also to the extension of $t_2(n)_i$. Clearly, $t_2(n)_i \subseteq ext t_2(n)_j$. Secondly, a new indiscernibility relation between theories of level 2 may be defined, the virtual equality between two theories, noted $=_{virt}$:

$$t_2(n)_i =_{virt} t_2(n)_j \iff s(t_2(n)_i) =_{ext} s(t_2(n)_j)$$

Intuitively speaking, the virtual identity expresses the possibility to envisage two different theories as locally different selections of the same relational information (a saturated theory).

The rudiments of a non-extensional framework necessary to define the notion of semantic identity are now prompted. In order to formalize the very notion of semantic identity one has first to introduce the notion of the global intension of a concept (noted $int_g(c)$):

$$int_g(c) = df \left\{ t_{(m)k} \mid \exists t_{j,k}, 1 \leq j \leq m, \text{such that} \right\}$$

Thus, the global intension of a theory contains all the theories (of level 1 or 2) which, somehow, describe the concept, by expressing either one of the properties of the concept (as a unary theory) or a relation satisfied by the concept (a $n$-ary theory). Obviously, the global intension is still too broad to be operational. Indeed, in any "context" only a small part of the knowledge is necessary. This is captured by the notion of a semantic identity of a concept:

$$SI_k(c_i) \subset int_g(c_i)$$

Clearly, a concept possesses many semantic identities. On the other hand, we can introduce some intensional relations between concepts based on their semantic identity. For instance, we can define the local synonymy of two concepts (noted $s_{loc}$):

$$c_1 s_{loc} c_2 \iff SI_k(c_1) = SI_m(c_2)$$

Thus if two concepts have the same semantic identity in a given "context", we can consider them as locally indiscernible. One may at this level better understand our preoccupation about forms of non-extensional indiscernibility. Furthermore, one may define the virtual synonymy between two concepts which expresses the possibility to find a particular "context" in which the two concepts may be considered locally as semantically indiscernible:

$$c_1 =_{virt} c_2 \iff SI_k(c_1) \cap SI_m(c_2) \neq \emptyset$$

Let us now introduce a formal notion of context. As we have said, context may be considered as some global structure which constraints and guides the construction of more local representational structures. Thus we may define a context $K_i$ as a set of theories of level 2:

$$K_i = df \{ t_2 \mid t_2 \in T_2 \}; \quad K_i \subseteq T_2$$

Of course, we suppose that $K_i$ does not contain logically contradictory theories. Since context is represented as a structure of theories of level 2, we may formalize the notion of contextual significance for theories of level 1 and concepts. Let $K_i$ be a context; the set of theories of level 1 significant for the context $K_i$ (noted $T_{1i}$) is defined as follows:

$$T_{1i} = df \left\{ t_1 \mid \exists t_{2j}, t_1 \in ext(t_{2j}) \right\}$$

Consequently one may define the set of significant concepts in a context $K_i$ (noted $C^*_i$) as follows:

$$C^*_i = df \{ c \mid \forall t_1 \in SI_i(c), t_1 \in T_{1i} \}$$

Finally, let us introduce a function $\Phi$ that "puts the concept $c_1$ into a context" by returning, for a given context, all the pertinent concepts which have, somehow, a local semantic relation with $c_1$. More formally, let $c_1$ be a pertinent concept in $K_i$, then:

$$\Phi(c_1, K_i) = df \left\{ c_2 \mid c_2 \text{ is pertinent in } K_i \text{ and } SI_i(c_1) \cap SI_i(c_2) \neq \emptyset \right\}$$

$\Phi$ has an actual effect and expresses "top-down" constraints.

We have set up a non-extensional formal framework able to express a notion of context and a related notion of pertinence for theories of level 1 and concepts. Let us evaluate its descriptivity in a NL understanding example.
3 Example

Consider the metaphorical schema "argument is war" from [Lakoff and Johnson, 1980]. The way metaphors such as this one organize our everyday language is illustrated by the fact that the following expressions are commonly accepted: "Your claims are indefensible", "He attacked every weak point in my argument", "His criticism were right on target", "I demolished his argument", "I've never won an argument with him", "You disagree? Okay, shoot!", "He shot down all my arguments", to cite but a few. As Lakoff and Johnson notice, it is important to see that we do not only talk about arguments in terms of war, but we actually experience arguments like a real war with enemies, allies, battles, positions, strategies, etc.

In our conception, this means that there exist a concept "argument" which is somehow related with another concepts such as "opinion", "idea", "critique", "agreement", and there exist another concept "war" that is related to some other concepts like "battle", "weapon", "strategy", "victory", "defeat", "position", "attack", "destruction", "violence", "conquer". Understanding the metaphor "argument is war", means that one enhances the set of relations that describes "argument" with some relations taken from those describing "war"; thus one can see one's ideas as positions which may be defensible and someone's criticisms as attacks against these positions, and so on.

Thus, function $\Phi$, when applied to concept "argument" will return the set formed by concepts "opinion", "idea", "critique", "agreement"; this set is called the semantic field of concept "argument" and it is symbolized by $D_{\text{argument}}$.

The metaphoric mechanism can be formalized as a function that enhances the semantic identity and the semantic champ of the target concept with theories and concepts which belongs to the semantic identity and the semantic champ of the source concept. Such a function $\Phi$ may be defined as follows:

$$\Phi = \Phi(f_0, f_4, f_1)$$

where each $f_i$ is a function of the form:

$$f_0 : (c_1, c_2) \rightarrow c_1$$
$$f_4 : D_{c_1} \times D_{c_2} \rightarrow D'_{c_1}$$
$$f_{\Phi} : SI(c_1) \times SI(c_2) \rightarrow IS'_{c_1}$$

such that:

$$SI(c_1) = \Phi SI(c_1) \cup SI'(c_1)$$

where $SI'(c_1)$ is defined as follows:

$$SI'(c_1) = \Phi$$

In the simpler case $j = 1$ and $\Phi$ becomes the identity function. On the other hand $D'_{c_1}$ is the new semantic field after the application of function $\Phi$ for the new semantic identity of $c_1$. One can show that:

- $D'_{c_1}$ is conserved modulo $=_{\text{ext}}$ but this is not necessarily the case for the other forms of identity.
- It is possible that $c_2 \in D'_{c_1}$ but this is not generally the case.
- The semantic identities may be structured from more general to more specific ones and this may induce an order in the metaphoric mechanisms.
- $D'_{c_1}$ depends on the projective qualities of theories in $SI(c_2)$ and not in their extension, arity or number.

4 Concluding Remarks

Until now, the overall theory of interpretational semantics was rather set up on elementary set-theoretical foundations. But one requires non-extensional constructions insofar as extensionality is quite restrictive for semantic phenomena. Moreover, the kernel of the theory is based on two fundamental notions: that of relation between semantic components and that of semantic identity. The latter is drastically different from the usual extensional one as tightly dependent on contextual modifications; on the other hand, in an interpretational strategy, it takes a more "collective" form.

The formal framework presented in this paper is the first step towards a formalism able to capture some of the contextual features required to implement a system in this sense of doing some semantic interpretation of natural language. Future work will consist in building a system for computer-aided interpretation of literary texts.
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