IMPROVED UPPER BOUND OF THIRD ORDER HANKEL DETERMINANT FOR OZAKI CLOSE–TO–CONVEX FUNCTIONS

MILUTIN OBRADOVIĆ AND NIKOLA TUNESKI

Abstract. In this paper we improve the upper bound of the third order Hankel determinant for the class of Ozaki close-to-convex functions. The sharp bound is conjectured.

Mathematics subject classification (2020): 30C45, 30C50.

Keywords and phrases: Analytic, univalent, Hankel determinant, upper bound, Ozaki close-to-convex.

REFERENCES

[1] D. BANSAL, S. MAHARANA, J. K. PRAJAPAT, Third order Hankel determinant for certain univalent functions, J. Korean Math. Soc. 52, 6 (2015), 1139–1148.
[2] F. CARLSON, Sur les coefficients d’une fonction bornée dans le cercle unité, Ark. Mat. Astr. Fys. 27A (1) (1940), 8 pp.
[3] A. W. GOODMAN, Univalent functions. Vol. II., Marinier Publishing Co., Inc., Tampa, FL, 1983.
[4] U. GRENDZANDER, G. SZEGÖ, Toeplitz forms and their applications, California Monographs in Mathematical Sciences. University of California Press, Berkeley-Los Angeles, 1958.
[5] W. K. HAYMAN, On the second Hankel determinant of mean univalent functions, Proc. London Math. Soc. 3, 18 (1968), 77–94.
[6] A. JANTENG, S. A. HALIM, M. DARUS, Coefficient inequality for a function whose derivative has a positive real part, J. Inequal. Pure Appl. Math. 7, 2 (2006), Article 50, 5 pp.
[7] A. JANTENG, S. A. HALIM, M. DARUS, Hankel determinant for starlike and convex functions, Int. J. Math. Anal. (Ruse), 1, 13–16 (2007), 619–625.
[8] B. KOWALCZYK, A. LECKO, Y. J. SIM, The sharp bound of the Hankel determinant of the third kind for convex functions, Bull. Aust. Math. Soc. 97, 3 (2018), 435–445.
[9] L. LI AND S. PONNUSAMY, On the generalized Zalcman functional $\lambda a^n - a_{n-2}$ in the close-to-convex family, Proceedings of the American Mathematical Society 145 (2017), 833–846.
[10] L. LI, S. PONNUSAMY AND J. QIAO, Generalized Zalcman conjecture for convex functions of order alpha, Acta Mathematica Hungarica 150, 1 (2016), 234–246.
[11] M. OBRADOVIĆ, S. PONNUSAMY AND K.-J. WIRTHS, Coefficient characterizations and sections for some univalent functions, Siberian Mathematical Journal 54, 1 (2013), 679–696.
[12] M. OBRADOVIĆ, N. TUNESKI, Hankel determinant of second order for some classes of analytic functions, arXiv:1903.08069.
[13] M. OBRADOVIĆ, N. TUNESKI, New upper bounds of the third Hankel determinant for some classes of univalent functions, arXiv:1911.10770v2.
[14] M. OBRADOVIĆ, N. TUNESKI, P. ZAPRAWA, New bounds of the third Hankel determinant for classes of univalent functions with bounded turning, arXiv:2004.04960.
[15] M. OBRADOVIĆ, N. TUNESKI, Hankel determinants of second and third order for the class $\mathcal{S}$ of univalent functions, Mathematica Slovaca, accepted, arXiv:1912.06439.
[16] S. OZAKI, On the theory of multivalent functions. II, Sci. Rep. Tokyo Bunrika Daigaku. Sect. A. 4 (1941), 45–87.
[17] S. PONNUSAMY, S. K. SAHOO, AND H. YANAGIHARA, Radius of convexity of partial sums of functions in the close-to-convex family, Nonlinear Analysis 95 (2014), 219–228.
[18] D. V. PROKHOROV, J. SZYNAL, Inverse coefficients for $(\alpha, \beta)$-convex functions, Ann. Univ. Mariae Curie-Sklodowska Sect. A. 35, 1981 (1984), 125–143.
[19] K. Sakaguchi, *A property of convex functions and an application to criteria for univalence*, Bull. Nara Univ. Ed. Natur. Sci. **22**, 2 (1973), 1–5.

[20] R. Singh, S. Singh, *Some sufficient conditions for univalence and starlikeness*, Colloq. Math. **47**, 2 (1982), 309–314 (1983).

[21] D. K. Thomas, N. Tuneski, A. Vasudevarao, *Univalent Functions: A Primer*, De Gruyter Studies in Mathematics **69**, De Gruyter, Berlin, Boston, 2018.

[22] D. Vamshee Krishna, B. Venkateswarlu, T. RamReddy, *Third Hankel determinant for bounded turning functions of order alpha*, J. Nigerian Math. Soc. **34**, 2 (2015), 121–127.