The effect of aspect ratio on heat transfer in a square cavity filled with a porous medium

M A Theeb  
Mechanical Engineering Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq  
E-mail: maathe_a@yahoo.com, maathe@uomustansiriyah.edu.iq

Abstract. Two dimensional mathematical model of an enclosure filled with a porous medium and heated from the lower end with various values of the width to height ratio (aspect ratio) has been studied in the present article. A two dimensional free convection model has been solved numerically using finite difference technique to evaluate the streamlines, isotherm lines, and the average Nusselt number. The set values where; Rayleigh number of $10^6$, Prandtl number of 0.72, and aspect ratio ranging from 1:4 to 4:1. The model has been validated and the results showed that the aspect ratio has a clear effect on the average Nusselt number and that increasing the flow rate for aspect ratio form 1:4 to 4:1.

1. Introduction

The use of porous media is one of the necessary issues have obtained significant attention in the last years due to their extensive use in engineering applications. These include electronics cooling and improving the performance of heat exchangers, etc.

[1] Conducted a two-dimensional laminar flow model for studying the porous media at different wall thickness and concluded that the Nusslet number is directly proportional to Darcy Number and wall thickness. [2] Solved a non-dimensional unsteady model based on Darcy-Forchheimer model to investigate a square cavity with porous media, local heat source, and finite wall thickness. Rayleigh number and Darcy number were varied over ranges between $10^4$ and $10^6$ for the former and $10^{-5}$ to $10^{-3}$ for the latter. They concluded that Nusslet number increases with the increase in the time, and that an unstable thermal plume is obtained with increasing Rayleigh number. [3] Investigated a square enclosure enveloped by a solid wall with external boundary and constant temperature in the other side. The Boussinesq approximation was solved based on SIMPLER algorithm with a range of Rayleigh number ($10^3$ to $10^5$) and dimensionless conductivity (1 to 10), wall dimensionless width (0.15 to 0.5) and inclination degree (30° to 180°). The heat transfer is found to be directly affected by Rayleigh number and it has a maximum value at about 80° inclination angle. [4] Experimentally investigated the heat transfer over a porous media or randomly arranged packed beds. They have obtained the velocity fields by applying the technique of matched index of refraction with a Reynolds number of (700 to 1700). They found that increasing Reynolds number leads to increasing the peaks root mean temperature.

The aim of the present work is a numerical modeling of unsteady free convection modes for an enclosure with rectangular coordinates filled with a porous medium and with a different aspect ratio, Darcy number and its effect on temperature lines, stream function and the Nusselt number presented.
2. Problem description

The square enclosure of the present study is a two-dimensional square cavity filled with a porous medium as shown in fig (1). The vertical walls are adiabatically insulated \( \frac{\partial T}{\partial x} = 0 \). The lower surface with a constant temperature heat source \( T_h \) with a heating length equal to 20% of the total lower end and a constant temperature \( T_c \) at the upper cooled surface.

![Figure 1. geometry of the model.](image)

3. Mathematical Model

The domain of the current model is a two-dimension rectangular enclosure with a length of \( L \) and height of \( H \) filled with a porous media, as shown in Figure 1. The thermodynamic properties of the fluid are based on the Boussinesq approximation.

The two dimensional unsteady governing equations for the model of porous cavity using conservation of mass, momentum and energy are written as\[1][2][5][6][7]:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{\kappa} u \\
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) &= -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu}{\kappa} v + g \beta (T - T_0) \\
\frac{1}{\alpha} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}
\end{align*}
\]

Equation (1-4) can be written in other form with the absence of pressure. The discretized equations can be written in term of stream function \( \psi \) and the vorticity \( \omega \) as follows:

\[
\begin{align*}
\rho \left( \frac{\partial \psi}{\partial t} + u \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y} \right) &= \mu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \rho \beta g \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial y} \\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -\omega
\end{align*}
\]

Where the velocity components in the x-and y-direction and vorticity are:

\[
\begin{align*}
u &= \frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial \psi}{\partial x} - \frac{\partial u}{\partial y} \\
To put equations (5,6 and 7) in non-dimensional form, using the following relationships:
The mathematical model in dimensionless form are:

\[
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad \tau = \frac{t}{\tau_0}, \quad U = \frac{u}{V_0}, \quad \theta = \frac{T - T_o}{\Delta T}, \quad \Psi = \frac{\Psi_o}{\Psi_0}, \quad F_o = \frac{\alpha t_o}{H^2}, \quad Pr = \frac{\nu}{\alpha}
\]

\[
\Omega = \frac{\omega}{\omega_0}, \quad V_o = \sqrt{g\beta\Delta TH}, \quad \omega_0 = \frac{V_o}{H}, \quad \Psi_o = \frac{V_o H, Ra}{\nu \alpha} \frac{\beta(T_o - T_H)H^3}{\nu \alpha}, \quad Da = \frac{k}{l^2}
\]

The mathematical model is dimensionless form are:

\[
\frac{\partial \Omega}{\partial t} + U \frac{\partial \Omega}{\partial x} + V \frac{\partial \Omega}{\partial y} = \sqrt{Pr \frac{Ra}{\Omega}} \left( \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right) + \frac{\partial \theta}{\partial x} - \frac{1}{Da} \sqrt{Pr \frac{Ra}{\Omega}} \Omega
\]  \( (8) \)

\[
\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\Omega
\]  \( (9) \)

\[
\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{Pr \cdot Ra}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
\]  \( (10) \)

The average value of Nusselt number is the integral of temperature and can be written as:

\[
\overline{Nu} = \int_0^X \frac{\partial \theta}{\partial y} \, dX
\]  \( (11) \)

4. Validation
The present mathematical model was solved and validated with different experimental and numerical investigations\[1\][2][8] and a reasonable agreement was achieved as seen in Figure 2.

**Figure 2.** Stream lines and isotherm for Ra=10⁶,Da=10⁻⁵, τ=50, a- numerical results\[2][8], b- numerical results \[1\].

5. Results
The analyses in the present study are carried out with the dimensionless parameters Rayleigh number \( Ra = 10^6 \), \( Pr = 0.72 \), with the range of aspect ratio (A.R.) from 1:4 to 4:1, and the bottom side is uniformly heated with constant dimensionless temperature.

Figure 3 illustrate the effect of different values of A.R. on the streamlines for the flow in the enclosure. From the figure it can be seen that the flow is very weak for the aspect ratio less than 1 and the flow increased for aspect ratio 1:1,2:1 and 4:1 due to the increasing of the heat supplied to the lower end of the cavity.

Figure 4, shows the dimensionless temperature profiles for the square enclosure with different value of aspect ratio. The maximum dimensionless temperature equal to 1 at the lower end of the cavity and the temperature profile inside the enclosure affected by the value of aspect ratio. It can be considered from Figures 3a to 3c that the temperature lines appear to be horizontal due to the low value of flow rate of the internal fluid inside the cavity, while these lines begin to curve in the Figures 3d to 3g due to the growing flow rate and specifically in the quarter above of the heated section of the lower end of the enclosure.

Figure 5 indicates the effect aspect ratio on Nusselt number. It can be proven that the value of dimensionless Nusselt number increased significantly with AR due to the increasing of the convection heat transfer because of the flow rate increasing.
Figure 3. Stream lines with various aspect ratio.
Figure 4. Isotherm lines in enclosure with various aspect ratio.
6. Conclusions
The article investigates numerically the effect of changing the aspect ratio of rectangular coordinates enclosure filled with a porous medium on the heat transfer. The main conclusion of the study is that the aspect ratio affects the amount of the average Nusselt number and increased the flow rate for the aspect ratio form 1:1 to 4:1 where the conduction heat transfer is the dominant for the low aspect ratio while the effect of convection heat transfer appear for the values of 1:1 and more as it shown in the isotherms lines.

Appendix - Nomenclatures:
A.R.: Aspect ratio.
Da: Darcy number.
g: Gravitational acceleration.
H: enclosure height (m).
L: Width of enclosure(m).
Ra: Rayleigh number.
Pr: Prandtl number.
T: Temperature (K).
t :Time (sec).
x, y Space coordinates.
ρ: Density (kg/m³).
α: thermal diffusivity (m²/s).
β: Thermal expansion coefficient (1/k).
µ: Dynamic viscosity (Ns/m²).
ν: Kinematic viscosity (m²/s).
Ω: Vorticity (Dimensionless).
Ψ: Stream function (dimensionless).
Θ: Temperature (dimensionless).
τ: Time (dimensionless).

Figure 5. Effect of aspect ratio on the average Nu.
References

[1] Theeb M A and Fattah S A 2014 Numerical investigation of various thickness wall in square enclosure with a porous medium Al-Qadisiya J. Eng. Sci. 7 12–26

[2] Aleshkova I A and Sheremet M A 2010 Unsteady conjugate natural convection in a square enclosure filled with a porous medium Int. J. Heat Mass Transf. 53 5308–20

[3] Yedder R Ben and Bilgen E 2002 Laminar natural convection in inclined enclosures bounded by a solid wall Heat Mass Transf. 32 455–62

[4] T. Nguyen1, R. Muyshondt Y A H and N K A 2019 Physics of fluids Phys. Fluids 31

[5] Sathiyaamoorthy M, Basak T and Roy S 2011 Non-Darcy Buoyancy flow in a square cavity filled with porous medium for various temperature difference aspect aatios J. Porous Media 14 649–57

[6] Yan M, Lu C, Yang J, Xie Y, Luo J and Yu X 2019 Impact of Low- or high-permeability inclusion on free convection in a porous medium Geofluids 2019

[7] Calcagni B, Marsili F and Paroncini M 2005 Natural convective heat transfer in square enclosures heated from below Appl. Therm. Eng. 25 2522–31

[8] Tanmay Basak, S. Roy, T. Paul I P 2006 Natural convection in a square cavity filled with a porous medium: Effects of various thermal boundary conditions Int. J. Heat Mass Transf. 49 1430–1441