The Sixth Catalogue of Fundamental Stars (FK6) and the Problem of Double Stars

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1. Introduction

The Sixth Catalogue of Fundamental Stars (FK6) combines the ground-based astrometric data of the basic fundamental stars, obtained over more than two centuries and summarized in the FK5 (Part I: Fricke et al. 1988), with the observations of the HIPPARCOS astrometry satellite (ESA, 1997). This combination provides the most accurate proper motions available at present. As we shall discuss below, the proper motions given in the FK6 are significantly better than both the proper motions provided by the FK5 or by HIPPARCOS alone.

There are two main reasons why the ground-based observations are able to improve the HIPPARCOS proper motions considerably: (1) The lower positional accuracy of the ground-based observations is compensated by a much longer period of observations, covering more than 200 years for most FK6 stars; (2) Due to undetected astrometric binaries, the HIPPARCOS proper motions, measured ‘instantaneously’ during a period of about 3 years only, can deviate significantly from the long-term proper motions, while the ground-based results of the FK5 provide already fairly ‘time-averaged’ data. Hence the ground-based results allow to identify and to correct, at least partially, ‘cosmic errors’ in the HIPPARCOS proper motions.

The very accurate proper motions of the FK6 can be used either astrophysically, e.g. for purposes of galactic kinematics, or astrometrically, e.g. for improving the accuracy of the prediction of stellar positions.

With increasing measuring accuracy, double (and multiple) stars cause increasingly larger problems in astrometry. This is not only true for objects whose binary nature is already known but also for undetected binaries. The uncomfortable consequence for the user is that the FK6 has to give a variety of astrometric solutions for the astrometric parameters (proper motion, position) of a star. The accuracy of and the choice between these solutions depend on the (often unknown) actual nature of a star (single or double) and on the model assumed for the motion of the star. The problems of this situation are discussed by Wielen
The method of ‘statistical astrometry’ is able to handle quantitatively the astrometric consequences of undetected binaries in stellar ensembles.

2. Truly Single Stars

For a truly single star, we should use the simplest model: the star moves linearly in time on a straight line in space. The combination of the FK5 data with the HIPPARCOS observations is then rather straight-forward.

Before combining the two catalogues, we have to reduce the FK5 to the system of the HIPPARCOS Catalogue. This is done successfully by the analytical method described by Bien et al. (1977). Consequently, the resulting FK6 is on the HIPPARCOS system (i.e. on the ICRS).

Let us call the position in one coordinate (e.g. in declination $\delta$) $x$, the corresponding proper motion $\mu$, the mean epoch (at which $x$ and $\mu$ are uncorrelated) $T$. We use the index $F$ for the FK5 (in the HIPPARCOS system), and $H$ for HIPPARCOS. From the two positions, $x_F(T_F)$ and $x_H(T_H)$, we can derive a third proper motion $\mu_0$ (Wielen 1988), in addition to $\mu_F$ and $\mu_H$:

$$\mu_0 = \frac{x_H(T_H) - x_F(T_F)}{T_H - T_F}.$$  

The mean error of $\mu_0$ is given by

$$\varepsilon_{\mu,0} = \left(\frac{1}{\varepsilon_{\mu,F,ind}^2} + \frac{1}{\varepsilon_{\mu,F,sys}^2} + \frac{1}{\varepsilon_{x,H}^2}\right)^{1/2}.$$  

$\varepsilon_{x,H}$ is the mean measuring error of $x_H(T_H)$, $\varepsilon_{x,F,ind}$ is the random (‘individual’) error of $x_F(T_F)$, and $\varepsilon_{x,F,sys}$ is the mean error of the systematic difference in position between the FK5 system and the HIPPARCOS system at the epoch $T_F$ and at the position (and magnitude) of the star under consideration. In other words, $\varepsilon_{x,F,sys}$ describes the local uncertainty with which the FK5 system of positions can be reduced to the HIPPARCOS system (It is not the amount of the systematic difference itself).

If we neglect for a moment the fact that the HIPPARCOS astrometric results for a star are correlated, we can use a rather direct method for combining the FK5 and the HIPPARCOS data: The resulting FK6 proper motion $\mu_{FK6}$ is the weighted average of the three proper motions $\mu_0, \mu_F, \mu_H$. The weights of the quantities are the inverse squares of their mean errors, $1/\varepsilon_{\mu,0}^2, 1/\varepsilon_{\mu,F}^2, 1/\varepsilon_{\mu,H}^2$. Into $\varepsilon_{\mu,F}$ we have to include the uncertainty of the reduction of the FK5 proper motion system to the HIPPARCOS system. The weight of $\mu_{FK6}$ is the sum of the weights of $\mu_0, \mu_F, \mu_H$. Similarly, the mean epoch $T_{FK6}$ of the star in the FK6 is the weighted average of $T_F$ and $T_H$, and the mean position $x_{FK6}(T_{FK6})$ is the weighted average of $x_F(T_F)$ and $x_H(T_H)$, using the weights of the positions in both cases. Again, the weight of $x_{FK6}$ is the sum of the weights of $x_F$ and $x_H$. This simple averaging method gives already quite accurate results which can be understood rather easily. The averaging method is described in more details in another paper (Wielen et al. 1998). In the actual FK6, we use a strict method, which takes also care of the correlations between the HIPPARCOS values of the astrometric parameters (including the parallax) of a star.

We call the resulting solution for the proper motion and position of a star in the FK6
the ‘single-star mode’, because it is strictly valid for truly single stars only. In Table 1, we present the typical error budget for the determination of an FK6 proper motion in this single-star mode. The mean errors given in Table 1 are root-mean-squared (rms) averages over the individual values of all the FK6 stars or, for comparison, the median of the individual values. They are valid for one component, but are averaged over \( \mu_{\alpha*} \) and \( \mu_{\delta} \).

From Table 1, we see the following: (1) The proper motion \( \mu_0 \) is typically more accurate than \( \mu_F \) and \( \mu_H \). (2) The final accuracy of the FK6 proper motions in the single-star mode is about two times better than both the accuracy of the proper motions in the FK5 or in the HIPPARCOS Catalogue. This significant gain in accuracy justifies the compilation of the FK6 quite well, even for truly single stars.

### 3. Apparently Single Stars and Hidden Binaries

In real life, we can never be completely confident that a star is truly single. At best, we can collect a sample of ‘apparently single stars’. For stars in such a sample, we have either no indication for a binary nature of the object at all, or the known binary nature should not affect the astrometric parameters. Hence we may include e.g. very close spectroscopic binaries, stars with very distant companions, double stars with known orbits for which the center-of-mass motion is given etc.

In other papers (Wielen 1995, Wielen et al. 1997), we have shown that the HIPPARCOS proper motions \( \mu_H \) in such a sample of ‘apparently single stars’ suffer from cosmic errors, due to undetected astrometric binaries. The basic fundamental stars show the effect of cosmic errors most clearly, because of the small measuring errors in their proper motions and positions. The mean cosmic error \( c_\mu \) in \( \mu_H \), corresponding to \( (\eta(0))^{1/2} \) in the terminology of Wielen (1997), was determined from a comparison of \( \mu_F \) and \( \mu_H \) in a sample of 1202 apparently single FK stars as \( c_\mu = 2.13 \) mas/year.
Table 2
Cosmic error $c_\mu$ in HIPPARCOS proper motions

| Quantity                        | Catalogue: | Sample of stars: | rms of: | FK5 | FK5 | FK4 | FK3 | GC | GC |
|--------------------------------|------------|------------------|---------|-----|-----|-----|-----|----|----|
| rms difference $\mu_F$ or $\mu_0 - \mu_H$ | FK5        | 1202 FK          | $\mu_F - \mu_H$ | 2.38 | 2.04 | 2.07 | 2.12 | 2.18 | 2.63 |
| measuring errors:              |            |                  | $\mu_0 - \mu_H$ |       |       |       |       |     |     |
| $\varepsilon_{\mu,H}$         |            |                  |         | 0.68 | 0.68 | 0.68 | 0.68 | 0.68 | 0.76 |
| $\varepsilon_{\mu,F}$ or $\varepsilon_{\mu,0}$ (total) |            |                  |         | 0.82 | 0.59 | 0.55 | 0.61 | 0.67 | 1.43 |
| remaining part = cosmic error $c_\mu$ |            |                  |         | 2.13 | 1.83 | 1.88 | 1.91 | 1.96 | 2.07 |
| mean epoch difference $T_{cat} - T_H$ [years] |            |                  |         | 42   | 42   | 75   | 88   | 89  | 91  |
| median of $m_V$ [mag]          |            |                  |         | 4.9  | 4.9  | 4.9  | 4.9  | 4.9 | 6.8 |
| median of parallax [mas]       |            |                  |         | 11.0 | 11.0 | 11.0 | 11.0 | 11.0 | 6.8 |

Note: The units of $\mu$, $\varepsilon$, and $c_\mu$ are mas/year.

To confirm this value of $c_\mu$, we have compared $\mu_H$ also with $\mu_0$ for the same sample of FK stars. The proper motion $\mu_0$ has been obtained from combinations of the HIPPARCOS observations with the following compilation catalogues: FK5 (Fricke et al. 1988), FK4 (Fricke et al. 1963), FK3 (Kopff 1937, 1938), and General Catalogue GC (Boss et al. 1937). The cosmic error $c_\mu$ is derived by subtracting the measuring errors $\varepsilon_{\mu,H}$ and $\varepsilon_{\mu,F}$ (total) or $\varepsilon_{\mu,0}$ (total) of $\mu_H$ and of $\mu_F$ or $\mu_0$ quadratically from the rms difference $\mu_F - \mu_H$ or $\mu_0 - \mu_H$. Table 2 shows that the resulting values of the cosmic error $c_\mu$ are in good agreement, both among the various catalogues and with the value of $c_\mu$ quoted above and derived from a comparison of $\mu_{FK5}$ with $\mu_H$. The advantage of using $\mu_0$, instead of the proper motion given in the catalogue itself, is the good accuracy of $\mu_0$ even for old compilation catalogues. The use of e.g. $\mu_{GC}$, with typical measuring errors of the order of 10 mas/year, would not allow a meaningful determination of $c_\mu \sim 2$ mas/year. It is especially remarkable that the much larger sample of about 12 000 well-measured and apparently single GC stars gives essentially the same typical value for $c$ as we have determined earlier from the FK stars.

We have finally adopted for the FK6 the overall value of 2.13 mas/year for $c_\mu$. In detail, we are using in the FK6 a cosmic error $c_\mu(r)$ which depends on the distance $r$ of the star from the Sun, determined empirically from the run of $c_\mu$ with $r$ among the 1202 FK stars. The distance $r$ is obtained usually from the HIPPARCOS parallax. For small and uncertain parallaxes, we use photometric distances if these are expected to be more accurate. We adopt also a cosmic error of 10 mas in the HIPPARCOS positions $x_H(T_H)$.

In the presence of cosmic errors in the HIPPARCOS data, we determine, in contrast to the single-star mode of the FK6, now a ‘long-term prediction’ in the FK6. For the long-term prediction, we modify the method of combining the FK5 data with the HIPPARCOS...
In Table 3, we give the error budget of the FK6 proper motions in the long-term prediction mode. If we compare the accuracy of these FK6 long-term proper motions with the accuracy of HIPPARCOS proper motions, now taking the cosmic error $c_{\mu}$ in $\mu_H$ into account, we see that the FK6 is more accurate than HIPPARCOS by a factor of more than 4. This is certainly a remarkable improvement.

The long-term proper motions of the FK6 should be preferred over the instantaneous HIPPARCOS proper motions both for galactic kinematics as well as for long-term predictions of stellar positions. As soon as the epoch of the predicted position of a star differs by more than about 10 years from $T_H = 1991.25$, the long-term prediction of the FK6 is more accurate and gives more reliable error estimates than the direct use of the HIPPARCOS Catalogue.

For epochs close to $T_H$, one may use either directly the instantaneous HIPPARCOS data or the ‘short-term prediction mode’ of the FK6. This short-term prediction is derived by a modification of the methods described in Section 2 in a way formally similar to that used for the long-term prediction. The only difference is that in the short-term prediction, the cosmic...
errors have to be added to the FK5 mean errors of $\mu_F$ and $x_F(T_F)$, instead of the addition to the HIPPARCOS ones.

4. Known Double Stars

The basic FK5 contains 1535 stars. For 302 of these stars (20 percent), special solutions (C, O, G, X, V) are given in the HIPPARCOS Catalogue, indicating their binary nature. Other binaries known from ground-based observations, especially visual binaries with magnitude differences of more than about 3, are treated by HIPPARCOS as single stars (standard solutions).

For most of the known double stars, the combination of FK5 and HIPPARCOS data is usually not straightforward. Many different situations require rather different operations. In most cases, individual ‘corrections’ have to be applied to the data, either to FK5 or to HIPPARCOS, before we can combine the data into the FK6. Even then, the results for these binaries are often rather uncertain and of doubtful significance. In this paper, we can indicate a few of these problems only.

The first problem to be solved for binaries is usually to find a common astrometric ‘reference point’ for the FK5 and HIPPARCOS which is also meaningful for the user of the FK6. Good reference points are, for example, the center-of-mass of a binary, or the positions of the components A and B of a double star with purely linear relative motions. These reference points can be combined like single stars.

Other reference points are less well-defined. This is especially true for the photo-center of A and B. The photo-center depends on the photometric system (V or Hp), and implicitly on the ratio between the orbital period and the time coverage of observations. For binaries with periods of the order of a few decades, the FK5 gives essentially the time-averaged position and motion of the photo-center, while HIPPARCOS provides the instantaneous photo-center. If we know the orbit and the individual magnitudes and colours of the binary components accurately, we can harmonize the data. But in most cases, we can do this in a statistical manner only.

For determining the most appropriate reference point for a given double star, it is very important to know its orbital period $P$. For 47 visual binaries in the basic FK5, we know $P$ rather accurately from a well-determined relative orbit. For the remaining majority of cases, we have obtained a statistical estimate of $P$ from the observed separation, parallax and photometry, using the mass-luminosity relation. In many cases, these estimated orbital periods indicate rather clearly whether or not the FK5 or HIPPARCOS provide ‘time-averaged’ or ‘instantaneous’ data.

In favourable cases, we can obtain very accurate ‘corrections’. For example, for many short-period astrometric binaries with small separations, the O solutions of HIPPARCOS provide the center-of-mass as the reference point. In these cases, the FK5 data are to be interpreted as valid for the time-averaged photo-center. Using the known orbital elements, especially the eccentricity, we can derive the constant shift between the mean photo-center and the center-of-mass. This shift can then be used to reduce the FK5 position to the center-of-mass.
For 95 stars (6 per cent) of the basic FK stars, the HIPPARCOS Catalogue provides non-linear solutions of type G. It can be shown that most of these stars are astrometric binaries with orbital periods of a few years. For the stars with G solutions, we neglect the HIPPARCOS proper motions completely, because they differ from the FK5 proper motions typically by about 10 mas/year. Hence we combine only the HIPPARCOS positions with the FK5 positions and proper motions. The resulting FK6 data for the stars with G solutions by HIPPARCOS refer then to the time-averaged photo-center of these binaries.

5. Astrometrically Excellent Stars

Being confronted with the often very nasty problems caused by double stars, one may be inclined to eliminate at least known double stars entirely from the FK6. This could be indeed a solution from the purely astrometric point-of-view. In high-precision astrometry, the accuracy of the astrometric data for double stars is often by an order of magnitude or more lower than for single stars. However, for many astrophysical problems one needs at least the proper motion of a double star, since many interesting objects (e.g. δ Cep) are binaries. In that sense, the FK6 aims at deriving the ‘best’ proper motions also for such known double stars, even if the accuracy is not as good as for single stars.

As a compromise between the conflicting demands of highest astrometric accuracy and the inclusion of double stars for completeness, we shall identify in the FK6 a subsample of ‘astrometrically excellent stars’ by special flags. Such an astrometrically excellent star should behave essentially like a well-measured single star.

The selection criteria for astrometrically excellent stars are: (1) No disturbing duplicity is known, neither from ground-based observations nor from HIPPARCOS measurements. (2) Good agreement between the three proper motions \( \mu_H, \mu_F, \) and \( \mu_0 \). This agreement indicates, at least statistically, that the cosmic errors in the HIPPARCOS proper motions for these stars are small. (3) Small measuring errors of the proper motions.

The number of astrometrically excellent stars depends strongly on how stringent the selection criteria are chosen. We think that many hundreds of FK6 stars are qualified as astrometrically excellent stars. Perhaps we will indicate different levels of excellence by different types of flags, similar to the numbers of ‘Michelin stars’ for indicating the excellence of a restaurant. Of course, further observations may show that the star is actually not qualified as being astrometrically excellent. In any case, the ‘excellence’ attributed to a star in the FK6 is significant only on the level of accuracy reached at present. Future space missions may be much more disturbed by the binary nature of the majority of stars.

6. Conclusions

The proper motions of the FK6, and hence the positions predicted from the FK6 data, are expected to be significantly more accurate than the data given in the HIPPARCOS Catalogue and in the FK5. For truly single stars, the accuracy is improved by a factor of about 2 by the FK6. For the ensemble of apparently single stars, in which the HIPPARCOS proper motions suffer from cosmic errors due to undetected astrometric binaries, the gain is even higher, by a factor of more than 4. Double stars pose special problems which are handled rather individually in the FK6. Among all the FK6 stars, a subset of ‘astrometrically excellent stars’ is identified for high-precision astrometry, avoiding as far as possible a disturbing
binary nature of the objects.

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