Inversion methods for the measurements of MHD-like density fluctuations by Heavy Ion Beam Diagnostic

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ABSTRACT: We report here on the recent developments in the deconvolution of the path integral effects for the study of MHD pressure-like fluctuations measured by Heavy Ion Beam Diagnostic. In particular, we develop improved methods to account for and remove the path integral effect on the determination of the ionization generation factors, including the double ionization of the primary beam. We test the method using the HIBD simulation code which computes the real beam trajectories and attenuations due to electron impact ionization for any selected synthetic profiles of plasma current, plasma potential, electron temperature and density. Simulations have shown the numerical method to be highly effective in ISTTOK within an overall accuracy of a few percent (< 3%). The method here presented can effectively reduce the path integral effects and may serve as the basis to develop improved retrieving techniques for plasma devices working even in higher density ranges. The method is applied to retrieve the time evolution and spatial structure of $m = 1$ and $m = 2$ modes. The 2D MHD mode-like structure is reconstructed by means of a spatial projection of all 1D measurements obtained during one full rotation of the mode.

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1 Introduction

The efficient operation of a fusion reactor requires the active control of several plasma parameters and their spatial profiles within challenging accuracies. For instance, the ITER diagnostic system [1] comprising more than 40 sophisticated diagnostic techniques dedicated to machine protection, safe operation, plasma profiles control and physics studies will be used to ultimately provide the measurements necessary to develop the physics basis for operation of a DEMO grade device. As the demands on experimental measurements’ resolution and accuracy increase in order to better understand plasma intrinsic physics processes and interplays the increasing size and power of the devices implies that the capability to diagnose the burning plasma becomes more limited due to the inherent constrains on access (e.g. shielding, complex interfacing of sub-systems, etc.) and on the operational requirements of fusion power plants (e.g. power loads, radiation levels, etc.). There is therefore a clear motivation nowadays to obtain the most complete plasma physics knowledge utilizing the existing small and medium size devices in view of their installed diagnostic capability and higher flexibility to conduct a broad range of experiments.

The Heavy Ion Beam Diagnostic (HIBD) is a powerful tool that can in principle provide the local measurement of several quantities of interest being these the electron density and temperature, the internal poloidal field and the plasma potential with spatial resolution of several mm and down to the micro second range temporal resolution. For instance, experiments in ISTTOK [2–5], TJ-II [6], T-10 [6] and in the LHD device [7] have all together demonstrated the capabilities of the...
HIBD in plasma density, potential fluctuations and mean potential measurements, supporting MHD and turbulence analysis including Alfvén Eigen modes (TJ-II) [8] and Geodesic Acoustic modes (T-10) [9] studies.

Owing to the HIBD nature the current intensity obtained in the raw signals has a contribution from the local primary beam ionization rate (at the sample volume) and from the beams’ path integration (primary and secondary beams) determined by the plasma density and temperature profiles. This effect has the consequence of potentially affect the retrieval of plasma density and temperature absolute profiles, however it will not affect the retrieval of plasma potential and poloidal magnetic field profiles since they do not depend on the features of those profiles. It has in general been accepted that the path integral effect is somehow negligible under some conditions in the retrieving of fluctuation characteristics during plasma density measurements. For ISTTOK our study shows that for analyzing and validating data from MHD activity we must remove the integral path effects. In fact, integral path effects can lead to the signature of phantom patterns in the cross correlation information used to obtain the mode spatial structure. The purpose of this paper is to study based on modelling, the effect of integral path effects in the measurements of density and temperature profiles in ISTTOK. We develop here a retrieving method to account for the path integral effects and thus reduce the presence of artifacts in the interpretation of the data either from absolute profile measurements either from fluctuations measurements.

In this paper we start by introducing the HIBD concept and introduce the calculation of beam attenuation. We then move on to review and complete the methodology for eliminating the path integral effects from the raw signals for time independent profiles of density and temperature. In the second half of the paper we treat the retrieving of local plasma parameters under the effect of MHD activity. Several examples are presented and comparisons between raw and processed signals are made. Finally we present a couple of examples in using the HIBD for retrieving the 2D profile of rotating MHD modes. The conclusions section will close the paper.

2 The Heavy Ion Beam Diagnostic: trajectories and attenuations

The HIBD is based on the injection of a single ionized beam into the plasma (e.g. Cs+, Hg+, Xe+, etc.). This single charged ion beam (called the primary beam) will experience ionizing collisions with the plasma electrons generating double charged ions (called secondary beams). Mainly due to the magnetic toroidal field the trajectory of all beams is curved in the poloidal plane (determined by their Larmor radius). The beam energy, which also plays a role on the final levels of ionization, is normally chosen based on the geometrical arrangement that allows for the primary beam to cross and exit the plasma region of interest and for the collection of the secondary beams (figure 1).

In theory, while traveling through the plasma the primary beam and the secondary beams will suffer additional attenuation effects due to ionization to higher states (3+ and higher). The plasma density and temperature absolute values together with the detector noise level dictate the set of ionizing reactions that need to be considered in practice. In this paper, all HIBD simulations include the attenuation processes leading to the single and double ionization of the primary beam and the single ionization of the secondary beams. The number of detector cells used is \( N = 18 \) disposed along the vertical direction (yy) and covering the whole plasma diameter with a typical spatial resolution of 1 cm.
Figure 1. Trajectories of a 22 keV primary Xe$^+$ beam and corresponding cloud of secondary beams as calculated by the HIBD modelling code for the tokamak ISTTOK ($B_T = 0.45$ T). The detector represented by the red line has $N$ cells in total each indexed by $L$.

2.1 Attenuation effects

As was mentioned above the ions travelling in the plasma will have a certain probability to undergo several reactions such as ionization to next state or higher states by electron impact (proton impact ionization is only manifested for $T_p > 2$ keV). For each beam path the fraction of ionization is determined by the effective ionization cross section (a function of the plasma temperature) and local plasma density. For the case of the ISTTOK tokamak ($a = 8.5$ cm, $R = 46$ cm, $B_T = 0.45$ T, $I_p = 4 - 5$ kA, $\langle n_e \rangle \sim 5.10^{18}$ m$^{-3}$ and $T_e(0)$ up to 150 eV) the relevant reactions contributing for the detected currents are the single and double ionization of the primary beam, and the single ionization of the secondary beams. The equation describing the contribution of these attenuation processes in the detect current of secondary beams, $I^{2+}_{j(det)}$ can be given by
In eq. (2.1) $I_{0}^{+}$ is the injected primary beam current, $n(s_{i})$ is the plasma electron density evaluated over the beam path ($i = 1$ for primary beam and $i = 2$ for secondary beam) and $\hat{\sigma}_{12}$, $\hat{\sigma}_{13}$ and $\hat{\sigma}_{23}$ are respectively the effective cross sections for the single and double ionization of the primary beam, and the single ionization of the secondary beam\footnote{The effective cross section is a function of plasma temperature and is obtained by integration of the impact ionization cross section over the relative velocity ($v_{b}$) of the beam to the electron Maxwellian velocity distribution (normalized to the beam velocity): 

$$\sigma_{q,q'}(T_{e},v_{b}) = \frac{\langle \sigma_{q,q'}(v) \cdot v \rangle}{v_{b}} = \frac{1}{v_{b}} \left( \frac{m_{e}}{2 \pi K_{T_{e}}} \right)^{\frac{3}{2}} \int_{0}^{\infty} v \cdot \sigma_{q,q'}(v) \exp \left\{ - \frac{m_{e}v^{2}}{2 K_{T_{e}}} \right\} d^{3}v$$

Where $v$ is the relative velocity, $v_{b}$ is the beam velocity, $v_{e}$ is the electron thermal velocity, $\sigma_{q,q'}$ is the cross section for the reaction $q$ to $q'$, $m_{e}$ is the electron mass and $K$ is the Boltzmann constant.}. The factor 2 accounts for the double charge of the detected secondary ions. The factor $A$ is the integral of attenuation of primary beam from the plasma entrance ($R_{i}$) to the ionization point ($r_{j}$). Likewise the factor $B$ accounts for the attenuation of the secondary beam from the ionization point ($r_{j}$) until the exit of the plasma ($R_{p}$). The generation factor for the production of secondaries at each sample volume is given by the local factor $n_{e}(r_{j})\hat{\sigma}_{12}(r_{j})$ times the size of the detector cell projected along the primary beam trajectory $dl_{(cell)}$.

The path integral effects included in eq. (2.1) have long been the subject of several studies aiming at the determination of the effect of attenuation in the interpretation of the experimental data, in particular for the measurements of the plasma density fluctuations [10–13]. The previous work done at ISTTOK [3] extended the diagnostic capability for the determination of the absolute values of the local generation factor $n\hat{\sigma}_{12}(r_{j})$ taking into account the attenuation of the primary beam and secondary beams due to single ionization. In ISTTOK the HIBD makes use of a multiple cell array detector covering the secondary beams generated by the whole primary beam trajectory. In such arrangement it is possible to make account for all relevant attenuation effects that allow retrieving the absolute value of the local generation factor $n\hat{\sigma}_{12}(r_{j})$ which is the actual parameter directly measured by the HIBD. The work here presented extends the treatment in [3] to include additionally the attenuation of primary beam by double ionization. In this model, an overall reconstruction of the $n\hat{\sigma}_{12}(r_{j})$ absolute values for ISTTOK is obtained within 3% instead of the 7%–10% previously achieved. This improvement is shown to be helpful when retrieving time traces of the fluctuations. It will also improve the accuracy in retrieving the absolute values of density and temperature in future developments. It follows that while retrieving the absolute values of $n\hat{\sigma}_{12}(r_{j})$ and its local fluctuation this method preserves the phase information between all sample volumes.
2.2 The retrieval of the absolute values of the generation factor $n\hat{\sigma}_{12}(r_j)$

As a first order step in developing the model let us ignore the double ionization of primary beam and the single ionization of secondary beams. We take the integrals in eq. (2.1) and replace them by sums of discrete quantities representing the experimental conditions (finite sample volumes). Within the validity of these approximations the primary beam attenuation can be accounted by simply discounting the secondary currents collected at the detector in an iterative manner. Defining the primary beam current available for the first ionization volume ($j = 1$) as $I^+_1 = I^+_0$, where $I^+_0$ is the primary beam injected current, we can write for the primary beam current at each sample volume ($j$)

$$I^+_j = I^+_0 - \sum_{L=0}^{j-1} \frac{1}{2} \int_{L}^{(\text{det})} I^+_j dl$$

(2.2)

where $L$ is the cell number in the detector ($L = 0$ defines the initial conditions). As by assumption $\hat{\sigma}_{13}$ and $\hat{\sigma}_{23}$ are neglected, we can use eq. 1 in order to calculate the local $n\hat{\sigma}_{12}(r_j)$ factor given by,

$$n_e \hat{\sigma}_{12}(r_j) = \frac{I^+_j}{2 \int_{j}^{(\text{det})} dl}$$

(2.3)

where all quantities are known. This rather simple model does not recover the absolute value of $n\hat{\sigma}_{12}(r_j)$ but it does recover its normalized shape with an excellent matching for a large range of density and temperature profiles (figure 2). Let us call this the relative method for further reference.

Figure 2. Representation of three different $n\hat{\sigma}_{12}(r_j)$ profiles as inputted in the simulation code (full line) and the retrieved local $n\hat{\sigma}_{12}(r_j)$ from the detector currents (circles).

This important result constitutes the basis to develop a deconvolution algorithm that takes into account the contributions of $\hat{\sigma}_{13}$ and $\hat{\sigma}_{23}$ in eq. (2.1). As perceived from that equation the attenuation of the secondary beams (due to single ionization) is expected to depend on their path length ($\int ds_2$) from ionization point to the exit of the plasma, and also on the shape of the $n\hat{\sigma}_{23}$ profile. Simulations comparing the relative shapes of $n\hat{\sigma}_{12}$ along the primary beam path with $n\hat{\sigma}_{23}$ along the secondary beam path for several ionization points are presented in figure 3. We can observe that these normalized profiles match very well along all plasma diameter. The reason for
this result is that the effective cross section for the reactions $I^+ \rightarrow I^2+$ and $I^2+ \rightarrow I^3+$ present similar relative profiles and also de fact that in particular the primary and the secondary beams describe quasi-radial trajectories having therefore geometrically equivalent paths.

Let us proceed and include now the attenuation of the secondary ions ($\hat{\sigma}_{23} \neq 0$ and $\hat{\sigma}_{13} = 0$). For the range of densities and temperatures in ISTTOK the factor $B$ in eq. (2.1) can be considered a second order effect on the collected secondary current at each detector cell. We therefore can expand the exponential dependence of the factor $B$ and write for the tertiary lost current, $2\rightarrow 3 I_j^{2+}$,

$$2\rightarrow 3 I_j^{2+} = I_j^{2+} - I_j^{2+} \exp \left\{ - \int_{r_j}^{R_P} n_e \hat{\sigma}_{23} ds \right\} \approx I_j^{2+} - I_j^{2+} \left( 1 - \left\{ \int_{r_j}^{R_P} n_e \hat{\sigma}_{23} ds \right\} \right) = I_j^{2+} \left\{ 1 - \left( \int_{r_j}^{R_P} n_e \hat{\sigma}_{23} ds \right) \right\}$$

(2.4)

where $I_j^{2+}$ is the initial current of each secondary beam (at ionization point $j$). We can define the fraction of the ’current lost’ by one secondary beam to the sum of total secondary beams ’lost current’ using the ratio $\mathcal{R}_{j}^{2\rightarrow 3}$,

$$\mathcal{R}_{j}^{2\rightarrow 3} \frac{2\rightarrow 3 I_j^{2+}}{2\rightarrow 3 I_{\text{total}}} = \frac{\left\{ I_j^{2+} \int_{r_j}^{R_P} n_e \hat{\sigma}_{23} dl \right\}}{\sum_{j=1}^{N} \left\{ I_j^{2+} \int_{r_j}^{R_P} n_e \hat{\sigma}_{23} dl \right\}}$$

(2.5)

As it stands eq. (2.5) does not contain experimental parameters and can only be evaluated by simulation. However, the results that have been obtained above for the relative method allow for replacing eq. (2.5) by equivalent quantities obtained from experimental values as long they present a similar spatial evolution. Namely, from simulations we found that the normalized distribution of the secondary currents at the detector (plotted as a function of plasma radius) follows the same radial distribution as the secondary currents ($I_j^{2+}$) calculated at the ionization point. Additionally, as depicted from figure 3, the profile of $n_e \hat{\sigma}_{23}$ is very similar to the profile of $n_e \hat{\sigma}_{12}$. Therefore,
an equivalent of eq. (2.5) using only experimental discrete values can be given by the following weighing function,

\[
\mathbf{R}_{j}^{2\rightarrow3} = \frac{I_{j}^{2\rightarrow3} \sum_{L=j}^{N} (n_{e} \sigma_{12})_{L} dl_{L}}{\sum_{j=1}^{N} \left\{ I_{j}^{2\rightarrow3} \sum_{L=j}^{N} (n_{e} \sigma_{12})_{L} dl_{L} \right\}}
\] (2.6)

The term \( \mathbf{R}_{j}^{2\rightarrow3} \) can now be determined based on experimental values and corresponds to the fraction of current lost by a secondary beam traveling from the ionization point \( j \) to the corresponding detector cell \( L \). The absolute value of the current lost by each secondary beam is obtained multiplying \( \mathbf{R}_{j}^{2\rightarrow3} \) by the total ‘lost current’ of all secondary beams, which we determine in the following paragraph.

In the full physics scenario now being considered (\( \sigma_{23} \neq 0 \) and \( \sigma_{13} \neq 0 \)) the difference between the injected and collected primary beam current after crossing the plasma corresponds to the current lost for the generation of secondary ions \( (I_{2}^{2\rightarrow3})_{\text{total}} \) and tertiary ions \( (1\rightarrow3)I_{3}^{2\rightarrow3})_{\text{total}} \), weighted by their charge factor. We write such result as,

\[
I_{0}^{+} - I_{\text{det}}^{+} = \frac{1}{2} (I_{2}^{2\rightarrow3})_{\text{total}} + \frac{1}{3} (1\rightarrow3)I_{3}^{2\rightarrow3})_{\text{total}}
\] (2.7)

In order to simplify the above equation we seek for the dependence of the ratio between the total current of tertiary ions \( (1\rightarrow3)I_{3}^{2\rightarrow3})_{\text{total}} \) and secondary ions \( I_{2}^{2\rightarrow3})_{\text{total}} \) generated by the primary beam. This ratio was evaluated for a range of temperatures and density profiles using the simulation code. As depicted in figure 4, we found that for a given central temperature this ratio does not depend strongly (within 10–15%) on the temperature profiles and as expected is independent of the density profile (because both currents, secondary and tertiary, come from the same primary beam path).

![Figure 4](image)

**Figure 4.** Representation of the fraction of the total tertiary current to the total secondary current lost by the primary beam

This result allows us to relate these two quantities by an average factor \( \mathbf{\bar{R}} \) (over the temperature profile) which depends roughly only on the plasma maximum temperature,

\[
\frac{(1\rightarrow3)I_{3}^{2\rightarrow3})_{\text{total}}}{I_{2}^{2\rightarrow3})_{\text{total}}} = \mathbf{\bar{R}}
\] (2.8)
And replacing eq. (2.7) into eq. (2.8) we get,

$$I_0^+ - I_{\det}^+ = \frac{1}{2} \left( I_{\text{total}}^{2+} + \frac{2}{3} \bar{\mathcal{M}}_{\text{total}}^{2+} \right)$$

(2.9)

This last equation contains only known factors and allows us to calculate the total current that the primary beam loses for the generation of secondaries, $I_{\text{total}}^{2+}$. Likewise, from eq. (2.8) we obtain the total current that the primary beam has lost for the generation of tertiaries, $1^{1-3}I_j^{3+}$. Having determined the value of $I_{\text{total}}^{2+}$, the ‘lost current’ for each secondary beam counted from the ionization point to the detector can be obtained from combining eq. (2.6) and eq. (2.9)

$$2^{1-3}I_j^{2+} = \mathcal{R}_j^{2-3}I_j^{2+}_{\text{total}}$$

(2.10)

### 2.3 Estimation of $1^{1-3}I_j^{3+}$

In order to obtain the absolute values of the generation factor $n\hat{\sigma}_{12}(r_j)$ from eq. (2.3) apart from knowing the current of the secondary beam at the ionization volume, we also need to know the local current of the primary beam. To that end, we estimate now the tertiary current generated at each ionization point along the primary beam, $1^{1-3}I_j^{3+}$. We use similarly to the previous case, a weighing function now given by,

$$\mathcal{R}_j^{1-3} = \frac{\sum_{j=1}^{N} 1^{1-3}I_j^{3+}}{\sum_{j=1}^{N} 1^{1-3}I_j^{3+}} = \frac{3I_j^+ n_e \hat{\sigma}_{13} dl}{3 \sum_{j=1}^{N} I_j^+ n_e \hat{\sigma}_{13} dl}$$

(2.11)

In eq. (2.11) the numerator represents the current of a single tertiary beam and the denominator is the sum of all individual tertiary beam currents. Since we do not know the value of $n\hat{\sigma}_{13}$ we can make use of the roughly similar shapes between $n\hat{\sigma}_{12}$ and $n\hat{\sigma}_{13}$ generation factors as calculated for a range of profiles (figure 5).

![Figure 5](image-url)

**Figure 5.** Representation of the $n\hat{\sigma}_{12}$ and $n\hat{\sigma}_{13}$ profiles along the primary beam path (converted to plasma radial position) for a particular pair of density and temperature profiles.
Replacing in eq. (2.11) the retrieved \( n\hat{\sigma}_{12} \) profile we can estimate the fraction of ‘current lost’ by the primary beam in the production of tertiary ions. For each ionization point \( j \) we obtain,

\[
\mathcal{R}^{1-3}_j = \frac{I_1^+ n\hat{\sigma}_{12} dl}{\sum_{j=1}^{n} I_1^+ n\hat{\sigma}_{12} dl} = \frac{I_j^{2+}}{\sum_{j=1}^{n} I_j^{2+}} \approx \frac{I_{j\text{det}}^{2+}}{\sum_{j=1}^{n} I_{j\text{det}}^{2+}} \tag{2.12}
\]

and the absolute value of tertiary current generated at each ionization point is given by,

\[
1^{-3}I_j^{3+} = \mathcal{R}^{-3}_j (1^{-3}I_{\text{total}}^{3+}) \tag{2.13}
\]

A point of remark at this stage is that the charge factors shall be used correctly. For instance in eq. (2.13) as it stands, \( 1^{-3}I_j^{3+} \) is the absolute value of the tertiary beam detectable current (if we would have place a detector to collect the tertiary beams this would be the measured current in one cell). But, in terms of evaluating the primary beam lost current one needs to multiply this value by \( 1/3 \) before subtracting to the primary beam current since the quantity to conserve is the total number of beam particles (not the charge). As we are now able to account for all currents lost by the primary beam at each sample volume and also the current lost by each secondary beam from the sample volume to the detector cell we can thus obtain the values at ionization volume and use eq. (2.3) to compute the absolute value of \( n\hat{\sigma}_{12}(r_j) \). Consequently, eq. (2.2) must be extended to include all attenuations processes at the ionization volume:

\[
I_j^{+} = I_0^+ - \sum_{L=0}^{j-1} \left[ \frac{1}{2} I_L^{2+} + \frac{1}{3} (1^{-3}I_L^{3+}) \right] \tag{2.14}
\]

where \( L \) as before represents the detector cell number (\( L = 1, 2, \ldots \)) and by convention we take the initial values of all quantities indexed to \( L = 0 \). The values of \( I_L^{2+} \) are given by eq. (2.10) and the values of \( 1^{-3}I_L^{3+} \) are given by eq. (2.13). In figure 6 are presented the results of applying this method in the retrieval of several different \( n\hat{\sigma}_{12}(r_j) \) synthetic inputted profiles. One can see that the results provide the retrieval of \( n\hat{\sigma}_{12}(r_j) \) absolute values with average deviations below 3% (in ISTTOK).

The method here developed requires the information about the plasma central temperature (or average temperature). Due to the lack of a direct temperature measurement in ISTTOK in practice we obtain the plasma temperature matching the detected secondary currents using the simulation code with a density profile normalized to the interferometer measurement and compare the average temperature values obtained by the code with Spitzer resistivity measurements. After such ‘calibration’ we use the Spitzer resistivity for tracking average temperature relative changes.

### 3 Implementation of the retrieving method

The process of utilizing the retrieving method here developed can be summarized in the following steps:

1. Determination of the relative \( n_e\hat{\sigma}_{12}(r_j) \) radial profile using eq. (2.2) and eq. (2.3).
2. Determination of the total current lost by the primary beam in the production of secondaries and tertiaries using eq. (2.9) (data from figure 4 is required).
Figure 6. Inputted and retrieved \( n_e \hat{\sigma}_{12} (r_j) \) absolute values for a range of plasma temperature and density profiles. In the left column the central values of temperature and density are indicated on the top of each figure. In the second column the corresponding values are: a) \( n_e = 5 \times 10^{18} \text{ m}^{-3} \); b) \( n_e = 1 \times 10^{19} \text{ m}^{-3} \); c) \( n_e = 1.5 \times 10^{19} \text{ m}^{-3} \).

3. Built the weighing functions \( \mathcal{R}_j^{2-3} \) and \( \mathcal{R}_j^{1-3} \) using respectively eq. (2.6) and eq. (2.12).

4. Use eq. (2.10) and eq. (2.13) to calculate respectively the secondary and tertiary current at each ionization sample volume (\( L \)).

5. Use eq. (2.14) to calculate the primary beam current at each ionization sample volume (this includes now the attenuation processes due to tertiaries)

6. Use eq. (2.3) to calculate the absolute \( n_e \hat{\sigma}_{12} (r_j) \) profile replacing the quantities evaluated at the detector by the ones evaluated at the ionization sample volume.
4 The diagnose of MHD modes using the HIBD

In the previous section we developed a model that allows retrieving the absolute value of the local generation factor \( n_e \delta_{12}(r_j) \) for plasma profiles at a particular time instant. The present section deals with the modelling and analysis of the HIBD as a diagnostic tool for MHD modes, including their time evolution. The methodology here described was implicitly used to help interpreting recent experimental results [5].

The simulation code of the HIBD has been extended to include synthetic profiles in temperature and density similar to those measured during MHD activity in particular including tearing mode like profiles. The computer model takes into account the real acquisition sampling rates (2 Msamples/s) and the plasma radial boundary according to the minimum detectable currents (2–3 nA) limiting thus the calculations to a maximum plasma radius of around \( r \sim 5.5 - 6.0 \text{ cm} \). A multiple cell detector composed of 12 cells of 0.5 cm height (about 1 cm footprint in the primary beam path) is used to collect the secondary ions, simulating more closely the actual experimental setup (figure 7).

![Figure 7](image.png)

**Figure 7.** Representation of the sample volumes (and corresponding detector cell number L) as determined by the cell size and beam width.

4.1 Modelling of \( m = 1 \) mode

We modelled an \( m = 1 \) mode located at \( r = 0.025 \text{ m} \) superimposed in a density parabolic profile and in a peak temperature profile with a fluctuation level of 30% (figure 8a). The rotation frequency is 67 kHz and the mode has a growing rate of 6.7 kHz. Instead of allow for saturation of the mode the amplitude was taken as a \( \sin^2 \) like function, simulating growing and damping. Note that these are inputted synthetic profiles, not linked to an equilibrium code. Using this model in the HIBD code we calculate the collected secondary beam currents. In figure 8b we present the resulting temporal evolution of the current of secondaries as detected by cell#3 (for example).

The signal of cell#3 shows a fast frequency oscillation (67 kHz) due to the rotation of the island and a slower frequency oscillation (6.7 kHz) due to modulation of the amplitude of the island. The collected signals by all detector cells are used to compute the \( n_e \delta_{12}(r_j) \) values for the twelve sampled volumes. In figure 9 we take one representative period of the mode amplitude modulation...
Figure 8. a) $m = 1$ density profile (the box dimension is $0.1 \times 0.1$ m); b) Time evolution of the secondary beam current collected at cell #3.

Figure 9. One period evolution of local $n_e(r_j)\hat{\sigma}_{12}(r_j)$ values: inputted (full line) and retrieved (broken line) for each labeled radius (on the side of the plot). Starting from the top each radial position follows sequentially from the cells in the detector (cell#1 to cell#12).

and compare the time evolution of the absolute values of the inputted $n_e(r_j)\hat{\sigma}_{12}(r_j)$ with the retrieved ones at the corresponding sampling radius ($r_j$). We can see that the agreement is almost exact and one can retrieve the absolute value and the phase of the oscillation for every cell. It is clear that only cells acquiring from the mode location present the oscillatory characteristic of the mode. In particular, cells corresponding to positions in the plasma center and plasma periphery do not present such relatively large oscillations. We can also note on the $\pi$ phase shift expected for this type of mode between cells located at same radius but in opposite $yy$ coordinate (above and below the mid-plane).
Figure 10. Time evolution of the local inputted $n_e \hat{\sigma}_{12}(r_j)$ values and secondary currents at the detector cells #5 - #7.

We would like to put in evidence the capability of this method in retrieving the correct phase of the signals arising from each sample volume. In figure 10 is depicted the inputted $n_e \hat{\sigma}_{12}(r_j)$ used in the present simulation and the signal collected at each corresponding detector cell (only for the three most central cells). We can observe that the cell currents are dominated by the oscillations with the same frequency as the $n_e \hat{\sigma}_{12}(r_j)$ evolution (due to $m = 1$ rotation). However, in cell#6 the phase difference between the cell current and the inputted $n_e \hat{\sigma}_{12}(r_j)$ evolution is $\pi$ while we expected to exhibit the same phase (zero phase difference). This phase mismatch is caused by the path integral effect of the primary beam. We shall point out that in practice and for the present simulated conditions this effect might not be readily noticed since the calculated fluctuation level
of current in cell#6 is within the experiment noise level. However, for relatively higher S/N ratio cases and depending on detector arrangement and cell size such artifacts can take place at detectable levels. We shall therefore investigate the consequences of such effects in the capability of the HIBD to retrieve MHD mode characteristics.

4.2 Modelling of $m = 1$ and $m = 2$ mode

One common situation observed in practice is the presence of two MHD modes during the same discharge. We shall model this more complex situation in order to determine if the $n_e \hat{\sigma}_{12}(r_j)$ profile can be recovered keeping the phase information. We are also interested to find out if the detector raw currents will present some phase mismatch when compared with the $n_e \hat{\sigma}_{12}(r_j)$ evolution. We introduce in the present simulation a horizontal outwards plasma column shift of 0.015 m (we keep circular plasma cross section). In general, ISTTOK plasmas present vertical and horizontal column oscillations (with longer characteristic times than the analysis windows here presented) but we expect that due to the geometry of the diagnostic the most challenging to analyze are the horizontal oscillations as they can introduce larger spatial distortions in the signals. For the sake of clarity we neglect mode growth rates and have simulated an $m = 1$ ($r = 0.025$ m; 80 kHz) and an $m = 2$ ($r = 0.045$ m; 120 kHz) modes with constant amplitude (figure 11).

![Figure 11.](image)

**Figure 11.** $m = 1$ and $m = 2$ density and temperature profile inputted in the simulation code (the box dimension is $0.1 \times 0.1$ m). Plasma column is shifted 1.5 cm in the horizontal axis (xx)

In figure 12, we can visualize the inputted global $n_e \hat{\sigma}_{12}(r_j)$ profile evolution, the evolution of the detector currents in time and the retrieved $n_e \hat{\sigma}_{12}(r_j)$. We can clearly see a distortion of the spatial characteristics of the detector currents when compared to the imputed $n_e \hat{\sigma}_{12}(r_j)$ profile. On the other hand the retrieved $n_e \hat{\sigma}_{12}(r_j)$ profile recovers relatively well the inputted one and therefore preserves more accurate information for analysis of phase relations.

4.3 Phase analysis

In figure 13, is depicted the phase relation between the signals from periphery of the plasma with all the other regions (cross correlations between cell #2 and all other cells) and from the center.
with all other regions (cross correlations between cell #6 and all other cells). Concerning the ‘center’ raw in this figure we can notice mainly the phase pattern of the inputted $n_{e} \hat{\sigma}_{12}(r_{j}) $ values for $m = 1$ covering the detector cells #4 to #9. For all other peripheral cells the cross correlation with the central cell #6 is practically zero. Also, due to the plasma column offset the mode pattern does not present exactly at $\pi$ phase shift between cells #4 and #9 (as expected from a spatially anti-symmetric $m = 1$ mode crossing the sample volume of these cells). For the same reason the $m = 2$ mode (located at plasma ‘periphery’) which should be symmetric for top (#2) and bottom cells (#11) presents a non-zero phase shift between these cells. One important point observed here is that plasma column horizontal shifts can have a strong signature in the phase correlation in the detector cells and if not taking into consideration can lead to misinterpretation of the mode.
Figure 13. Cross-correlation between peripheral ($r = 4.5$ cm) and central ($r = 0.6$ cm) cells calculated against all detector cells.

It is also noticeable that the cross correlation between peripheral cell #2 and all central cells is practically null. The most interesting observation from figure 13 is that the retrieved $n_e \hat{S}_{12}(r_j)$ presents a correlation map above 0.3 very similar to the inputted $n_e \hat{S}_{12}(r_j)$. On the other hand, the correlation map of the raw detector currents strongly shows correlation structures above 0.3 not present in the inputted $n_e \hat{S}_{12}(r_j)$. In fact, the currents' signals exhibit noticeable correlation artifacts in both plasma regions (with correlation levels of $\sim 0.9$) not present in the plasma. It also presents a distorted correlation pattern for the central cell #6 (auto-correlation) and #7. On the other hand, the retrieved $n_e \hat{S}_{12}(r_j)$ presents a softer signature of such artifacts (levels of $\sim 0.3$, almost at the noise level). As a practical rule, an artifact can be identified by a relative strong change in the levels of the cross-correlation maps between the current detector cells and the retrieved $n_e \hat{S}_{12}(r_j)$. These results put in evidence the need to de-convolute the detector current signals from integral path effects in order to more accurately interpret the phase relations between the signals.

When calculating cross correlations of pure signals (no noise) between different detector cells one shall take into account that relatively large or small amplitude signals can give an equally noticeable signature as long a coherent phase relation exists. However, in the presence of noise depending on the signal fluctuation’s level the contribution of low amplitude fluctuations to the correlations’ map fades away remaining only the highest correlated patterns. Namely, if 20% of white noise is added to the cell signals the inputted and retrieved $n_e \hat{S}_{12}(r_j)$ exhibit very alike pattern relationships while the currents cross correlations plot still shows the presence of artifacts.
4.3.1 The high density case

We have tested this model (beyond its validity assumptions) for a ten-fold higher density and temperature values, in ISTTOK geometry. Due to lack of space we cannot describe further details rather than presenting some general conclusions. We have included in the simulation the additional channels for the reactions $I^+ \rightarrow I_4^+$ and $I_2^+ \rightarrow I_4^+$. We have used the presented retrieving algorithm to regain in this case the relative values of $n_e \hat{\sigma}_{12}(r_j)$ (therefore not accounting for the contribution of the additional attenuation due to $I_4^+$ ions in the retrieval process). The main results of such test are that the method can still perform the deconvolution of path integral effects. The retrieved relative $n_e \hat{\sigma}_{12}(r_j)$ profile presents an identical signature to the inputted relative $n_e \hat{\sigma}_{12}(r_j)$ profile in frequency and in the spatial cross correlation plots (figure 14) thus allowing for a more realistic interpretation of the mode dynamics and spatial structure. On the other hand, the spectrogram of the raw detector currents evolution clearly showed the presence of phantom frequencies. As it is depicted in figure 14 the spatial cross correlations produced by the raw cell currents are populated with artefacts that scramble the correct interpretation of the mode structure.

![Figure 14](image)

**Figure 14.** Higher density and temperature case: cross-correlation between peripheral ($r = 4.5$ cm) and central ($r = 0.6$ cm) cells calculated against all detector cells. In spite of the retrieving method here developed does not include the $I^+ \rightarrow I_4^+$ and $I_2^+ \rightarrow I_4^+$ reactions it can still put in contrast the main structure of the mode ($m = 1$ at $r = 2.5$ cm and $m = 2$ at $r = 4.5$ cm). On the contrary the raw cell currents cannot provide any useful information about spatial structure.
5 Reconstruction of the mode spatial structure

For the particular case of ISTTOK because the primary beam crosses all plasma diameter we can reconstruct a 2D image of the mode spatial structure. The assumption is that the beam time of flight in the plasma (0.6 $\mu$s) is much faster than the fluctuations rate in the plasma. Therefore one can consider the measurements instantaneous at each sampling interval. Also the maximum spread of combined time of flight due to primary and secondary beam generated at all sample volumes is small (0.5 $\mu$s) when compared with the fluctuations rate. Thus, we can ignore the time delays between signals collected from different cells (or otherwise correct if they became important). In such conditions, each detector acquisition of the 12 sample volumes corresponds to a 1D photograph of the $n_e\hat{\sigma}_12(r_j)$ profile. Knowing the correct mode number (from Mirnov coils) and frequency (either from Mirnov coils or HIBD) one can take several snapshots of $n_e\hat{\sigma}_12(r_j)$ during a full rotation of the mode and project all snapshots in a 2D plane for one full mode rotation (providing the mode structure does not change much during one rotation). For a given mode rotation frequency, $F_M$, the corresponding number of synchronized snapshots is given by the ratio $N_F = F_S/F_M$ where $F_S$ is the sampling frequency. The angle between each snapshot is given by $\theta_S = 2\pi/N_F$. One can take equivalently a cylindrical system of coordinates where the mode is at rest and the primary beam is rotating at frequency $F_M$ in the opposite direction (figure 15).

At each particular angle $\Sigma\theta_S$ we can convert to rectangular coordinates by projecting the $x$ and $y$ coordinate of each of the twelve sample volumes and take the value of $n_e\hat{\sigma}_12(r_j)$ at that position:

$$n_e\hat{\sigma}_12(x_j, y_j)_{\Sigma\theta_S} \text{ with } x_j = r_j \sin(\Sigma\theta_S); y_j = \cos(\Sigma\theta_S)$$

The calculation stops when the cumulative angle $\Sigma\theta_S = 2\pi$. As the primary beam is not exactly a straight line aligned along the plasma diameter one can calculate the initial angle offset from the real position of each sampling volume (in relation to the center of the chamber) and increment each sample volume angular position following the procedure above. One additional point is that since the primary beam crosses the all plasma diameter only half a turn is required to scan over the all the plasma cross section. However, the sample volumes on the top radius are not exactly symmetric to the sample volumes on the bottom radius and when they encounter each other after half rotation.
Figure 16. Figure 16 – Reconstruction of the modes using the retrieved $n_e \hat{\sigma}_{12}(r_j)$ absolute values. For each mode the sampling frequency and the mode frequency are used to calculate the angle increment $\theta_s$. All radial coordinates are given in meter.

an interfacing misalignment is produced. Therefore one can opt by using for instance the top-half radius and build up the profile from a full plasma rotation (this method is twice slower than using the full diameter but gives a smoother spatial correlation). Applying this method to the retrieved
values of the cases herein studied provides the results presented in figure 16. The single \( \text{m} = 1 \) mode case provides a relatively sharp view of the mode structure. In the case where two modes coexist, the \( \text{m} = 1 \) in the core and the \( \text{m} = 2 \) at the periphery, the reconstruction method must take into account at turn each of the modes’ rotation frequencies. Each mode is clearly identifiable but now under a less sharp contour plot due to the lag of one into each other in the retrieving process. Because only one mode is synchronized at turn the non-synchronized mode presents a more compact or extended distribution depending on the rotation frequency difference between the two modes. This can lead to “spatial aliasing” and give the impression of the presence of other modes (like \( \text{m} = 3 \) for instance in the center of the figure). However, the retrieved shape shall only be validated for the chosen mode number and frequency.

6 Conclusion

A method for removing the main path integral effect of the HIBD for the tokamak ISTTOK was presented. The retrieving of the absolute values of the generation factor \( n_e \delta_{12}(r_j) \) proves to be very efficient with absolute errors below 3% (depending on plasma parameters). The test of such method was extended to the analysis of plasmas with MHD-like fluctuations. The plasma electron temperature and density profiles where modeled by shaping functions in space and in time in order to simulate MHD modes. The results showed that in the case of pure signals (no noise considered) the direct characterization of the plasma from the raw detected currents can, for some sample volumes, give wrong phase information (depending on mode amplitude, number, growing rate, plasma position, etc.). Alternatively, the retrieving method here utilized is more robust in retrieving the phase correlations associated with the MHD activity. The method has shown to be efficient also in the case of ten-times fold higher densities and temperatures than are usual in ISTTOK where relatively large path integral effects dominate the raw current signals collected at the detector. Using the retrieved \( n_e \delta_{12}(r_j) \) profile the 2D structure of the mode was retrieved based on a series of snapshots of the 1D profile during a full mode rotation.

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