Role of neutrons to protons ratio in determining the symmetry energy at sub and supra-saturation densities

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Abstract. The symmetry energy at sub and supra-saturation densities has a great importance in understanding the exact nature of asymmetric nuclear matter as well as neutron stars. We will implement neutrons to protons ratios from different kind of fragments to understand the symmetry energy behavior by using the Isospin Quantum Molecular Dynamics (IQMD) model. For the sub-saturation behavior of symmetry energy, we will compare the single and double ratio of neutrons to protons with the experimental data of NSCL collaboration. Along with experimental data, double ratio results of IQMD will be compared with the different theoretical results of BUU97, IBUU04, BNV and ImQMD models. For the supra-saturation behavior of symmetry energy, we will present the incident energy and isospin asymmetry dependence of double neutrons to protons ratio from different kind of fragments (free, light charged particles and intermediate mass fragments) by using the soft and stiff symmetry energies.

At sub-saturation densities, the soft symmetry energy is favored by our calculations, while at supra-saturation densities, the neutrons to protons ratio from free nucleons is found to be most sensitive compared to all other kind of fragments.

1. Introduction
In the present era, the symmetry energy is considered to be a prominent candidate towards the isospin dependence of nuclear equation of state (NEOS) in intermediate energy heavy-ion collisions. In the past years, many studies were performed for the density dependence of symmetry energy at sub-saturation densities by using isotopic scaling [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13], isobaric ratio [14], single and double ratios [15, 16, 17, 18, 19, 20, 21, 22], isospin diffusion [23, 24], isospin distillation/fractionation [23], and isospin migration/drift [23, 25, 26, 27] etc. Apart from these, transverse and elliptic flow of neutrons and protons are also considered as a good candidate to emphasize on the importance of density dependence of symmetry energy [28, 29, 30, 31, 32, 33]. Even with the help of these studies, the exact determination of symmetry energy is still under the way.

Since from the past study, we are interested in the detailed analysis of neutrons-protons single and double ratio. The single ratio study in heavy-ion collisions have already been done by different experimental and theoretical groups [15, 16, 21]. In the experiments, near Fermi energy, Hilscher et al. [16] found that single ratio of pre-equilibrium nucleons is consistently higher than that of projectile-target system and it can not be explained by the Coulomb effects alone. Another experimental observation is the ratio of free neutrons and protons from two
isotopic systems at 26 MeV/nucleon. A lot of interesting observations were made from the data. Schroder et al. [34] also systematically studied the spectra of pre-equilibrium neutrons and protons in both isospin symmetric and asymmetric systems. Recently, at NSCL/MSU Famiano et al. [15] measured the single and double ratios of free neutrons to protons for $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn at 50 MeV/nucleon. The results of double ratio of the above data have also been reproduced by different theoretical models, such as BUU97 [17], IBUU04 [18], BNV [19] and ImQMD [20]. Even, there are a lot of uncertainties in the determination of symmetry energy in term of different parameters like cross-section, symmetry energy coefficient, impact parameter and method of clusterization etc.

From the literature, it is concluded that no comparison of single N/Z ratio of free nucleons (FN) as well as intermediate mass fragments (IMFs) is available. Moreover, no one have tried to compare the double ratio findings for experiments and theories at one place to see which one is the most appropriate form of the symmetry energy.

Apart from this, the present status of supra-saturation density dependence of the symmetry energy is quite uncertain and interesting. The high-density behavior of the symmetry energy in the literature is studied by using two important parameters: one is the yield ratio parameter and second is the flow parameter. The yield ratio parameter has been studied in term of single and double ratios of neutrons to protons [35, 36, 37, 38], single and double ratios of $\pi^-/\pi^+$ [39, 40, 41, 42, 43, 44], the $\Sigma^-/\Sigma^+$ ratio [40], the $K^-/K^+$ ratio [41], and isospin fractionation [37], while, the flow parameter has been studied in terms of relative and differential flows (single and double ratios) of neutrons to protons or $^3$H to $^3$He [45], and in terms of the ratio [28] or difference [29] of neutron-to-proton elliptic flow. Before using the $^3$H and $^3$He particle yield and flow ratios for the density dependence of the symmetry energy at high incident energies, one must check the production of these particles in the supra-saturation density region, which is obtained during the highly compressed stage only. However, the production of neutrons and protons occurs in large amounts and can explain the high density dependence of symmetry energy with great accuracy. In brief, to check the sensitivity of neutrons to protons ratio from the different kind of fragments towards the high density behavior of the symmetry energy is utmost important.

Finally, the objectives of the manuscript are as as follow:

* To compare the single N/Z ratio of free nucleons as well as IMFs with the NSCL experimental findings.
* To compare the double ratio results of IQMD with the NSCL experimental as well as different theoretical findings.
* To check the sensitivity of different fragments N/Z ratio toward the high density behavior of symmetry energy.

The brief description of the model is presented in Sec. 2, followed by the results and discussion in Sec. 3. The results are concluded in Sec. 4.

### 2. Isospin Quantum Dynamical (IQMD) Model

In the IQMD model [46, 47, 48], nucleons are represented by wave packets, just as in the QMD model of Aichelin [49, 50, 51, 52]. These wave packets of the target and projectile interact via the full Skyrme potential energy, which is represented by $U$ and is given as:

$$U = U_\rho + U_{Coul}.$$  

Here $U_{Coul}$ is the Coulomb energy and $U_\rho$ originates from the density dependence of the nucleon optical potential, and is given as

$$U_\rho = \frac{\alpha r^2}{2 \rho_0} + \frac{\beta}{\gamma + 1} \frac{\rho^{\gamma+1}}{\rho_0} + E_{Sym}^\rho(\rho) \delta^2.$$  

The first two of the three parameters of Eq. 2 ($\alpha$ and $\beta$) are determined by demanding that, at normal nuclear matter densities, the binding energy should be equal to 16 MeV and the total energy should have a minimum at $\rho_0$. The third parameter $\gamma$ is usually treated as a free parameter. Its value is given in terms of the compressibility:

$$\kappa = 9\rho_0^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E_A}{A} \right).$$

(3)

The different values of compressibility give rise to soft and hard equations of state. The soft equation of state is employed in the present study with the parameters $\alpha = -356$ MeV, $\beta = 303$ MeV, and $\gamma = 7/6$, corresponding to an isoscalar compressibility of $\kappa = 200$ MeV.

In the third term $E_{Sym}^{pot}$ is the potential part of the symmetry energy, which is adjusted on the basis of calculations from the microscopic or phenomenological many-body theory, having the form

$$E_{Sym}^{pot} = \frac{C_{s,p}}{2} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i}.$$  

(4)

Here $C_{s,p} = 35.19$ MeV, parameterized on the basis of the experimental value of the symmetry energy, is known as the symmetry potential energy coefficient. On the basis of the $\gamma_i$ value, symmetry energy is divided into two types with $\gamma_i = 0.5$ and $\gamma_i = 1.5$, corresponding to soft and stiff symmetry energies, respectively.

The total symmetry energy per nucleon employed in the simulation is the sum of the kinetic and potential terms and is given as

$$E_{Sym}(\rho) = \frac{C_{s,k}}{2} \left( \frac{\rho}{\rho_0} \right)^{2/3} + E_{Sym}^{pot},$$

(5)

where $C_{s,k} = \frac{\hbar^2}{3m} \left( \frac{3\pi^2\rho}{2} \right)^{2/3} \approx 25$ MeV is known as the symmetry kinetic energy coefficient. The kinetic symmetry energy originates from the Fermi-Dirac distribution.

Finally, we get a density and isospin-single particle potential in nuclear matter as follows:

$$V_{\tau}(\rho, \delta) = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^{\gamma} + E_{Sym}^{pot}(\rho)\delta^2 + \frac{\partial E_{Sym}^{pot}(\rho)}{\partial \rho} \rho \delta^2 + E_{Sym}^{pot}(\rho)\rho \frac{\partial \delta^2}{\partial \rho_{\tau,\tau'}}.$$  

(6)

Here $\tau \neq \tau'$, $\frac{\partial \delta^2}{\partial \rho_{\tau \tau'}} = \frac{4\delta p_{\tau}}{\rho^2}$, and $\frac{\partial \delta^2}{\partial p_{\tau}} = -\frac{4\delta p_{\tau}}{\rho^2}$. The potential also depends on the momentum-dependent interactions, which are optional in the IQMD model.

Note that the $\gamma$ used in the determination of the equation of state and $\gamma_i$ used in the determination of symmetry energy are different parameters. The interesting feature of symmetry energy is that its value increases with decreasing $\gamma_i$ at sub-saturation densities, while the opposite is true at supra-saturation densities. In other words, soft symmetry energy is more pronounced at sub-saturation densities, while stiff symmetry energy is more pronounced at supra-saturation densities.

In the calculations, we use the isospin and energy dependent cross section in the collision term and the Pauli blocking effects as in the QMD model [49, 50]. The cluster yields are calculated by means of the coalescence model, in which particles with relative momentum smaller than $P_{Fermi}$ and relative distance smaller than $R_0$ are coalesced into a one cluster. The value of $R_0$ and $P_{Fermi}$ for the present work are 3.5 fm and 268 MeV/c, respectively.
3. Results and Discussion

The neutrons-to-protons ratio is among the first observables that was proposed as a possible sensitive probe for symmetry energy prediction at sub-saturation densities [17, 20]. In this article, the sensitivities of free nucleons, light charged particles (LCPs, having charge number between 1 and 2), and intermediate mass fragments (IMFs, having charge number between 3 and $Z_{Total}/6$) toward the sub and supra- saturation density behavior of symmetry energy are checked by researching the results of kinetic energy, incident energy and isospin asymmetry dependences of single and double neutron-to-proton ratios.

To perform the study, thousand of events are simulated for the isotopes of Sn, namely $^{112}$Sn + $^{112}$Sn, $^{124}$Sn + $^{124}$Sn and $^{132}$Sn + $^{132}$Sn between incident energies of 50 and 600 MeV/nucleon at semi-central geometry by using the soft and stiff symmetry energies of $\gamma_i = 0.5$ and 1.5, respectively. The single ratio is just the ratio of neutrons to protons and is represented in the study by $R(N/Z)$ or $R_{N/Z}$, while double ratio is the ratio of the single ratios of any two isotopes of Sn. In order to study the systematics of the isospin effects, the single ratio of the isotope with a greater number of neutrons is always mentioned in the numerator when the double ratio is calculated. Mathematically, the double ratio is represented by $DR(N/Z)$ or $DR_{N/Z}$ and is given as

$$DR(N/Z) \text{ or } DR_{N/Z} = \frac{R(N/Z)^{\text{neutron rich}}}{R(N/Z)^{\text{neutron weak}}}.$$  \hspace{1cm} (7)

**Figure 1.** (Color online) The comparison of neutrons to protons ratio from free nucleons, for the systems $^{112}$Sn + $^{112}$Sn (top) and $^{124}$Sn + $^{124}$Sn (bottom) at $E = 50$ MeV/nucleon and impact parameter $b \leq 5$, with the experimental data of MSU/NSCL collaborations [15].

**Figure 2.** The comparison of neutrons to protons ratio with in IMFs, for the system $^{112}$Sn+$^{112}$Sn at $E = 50$ MeV/nucleon with the experimental data of MSU collaborations. The black stars are the experimental data, while, solid circle with line represents the IQMD calculations with $\gamma_i=0.5$. 


3.1. Singe N/Z ratio of free Nucleons and IMFs
In the Fig. 1, we have compared the results of single ratio of neutrons to protons from free nucleons for neutron-weak system $^{112}$Sn + $^{112}$Sn (in top panel) and for neutron-rich system $^{124}$Sn + $^{124}$Sn (in bottom panel) at 50 MeV/nucleon with the experimental data [15, 22]. The conclusion from the figure is that (1) Experimentally, more $R_{N/Z}$ is observed for more neutron-rich system, which is also predicted by theoretical predictions. This is because, in the more neutron-rich system, the symmetry energy is more repulsive (attractive) for neutrons(protons) and hence more neutrons are produced. (2) Furthermore, the $R_{N/Z}$ decreases with increase in kinetic energy. This is due to the coulomb repulsion, which shifts the protons from low to high kinetic energy. (3) $R_{N/Z}$ shows increment at very high kinetic energy, especially for $^{124}$Sn + $^{124}$Sn. At sufficient high kinetic energy, the symmetry energy dominates over the coulomb interactions, which will lead to increase in the production of neutrons and hence $R_{N/Z}$ ratio is increased. The similar behavior and findings are also reported in Ref.[23]. The study indicates that the theoretical results are consistent with the experimental one.

The results are in good agreement with the soft symmetry energy except at very low and very high kinetic energy. The difference between soft and stiff symmetry energy results for the neutron-weak system is almost comparable to the error bar, while, for neutron-rich system, the difference has a great importance over the error bar. In other words, the error bar of the theoretical results with the soft symmetry energy covers the error bar of the experimental data for both the systems under consideration. The difference at high kinetic energy between theoretical and experimental results for neutron-rich system is due to the large uncertainty in the measurement of the $R_{N/Z}$.

To strengthen the findings of Fig.1, in Fig.2, we displayed the results for the kinetic energy dependence of the single neutrons to protons ratio with in IMFs. The theoretical result are shown only with the soft symmetry energy with $\gamma_i = 0.5$. The kinetic energy scale is of small range for the IMFs compared to the free nucleons. This is due to the slow movement of the heavier fragments compared to the free one. However, just like the single N/Z ratio in free nucleons, the single N/Z ratio of IMFs also obey the soft symmetry energy at sub-saturation densities. By using the single ratio observable, one can reach at a partial conclusion that the asymmetric nuclear matter favors the soft symmetry energy at sub-saturation densities, which is also consistent with the other findings in the literature [17, 18, 19, 20, 21, 25, 28, 29].

3.2. Double N/Z ratio of free nucleons
The problem in the results of the single ratio is that along with the symmetry energy, they also include the effects of coulomb interactions. In order to cancel the effect of coulomb interactions, we will opt the double ratio parameter in Fig.3. The double ratio has been studied many times in last couple of years by different groups with the help of the BUU97, IBUU04, BNV and ImQMD models and compared with the experimental results. Even so, we are still far away from the exact conclusion about the symmetry energy form. We have, along with all the possible results in the literature, compared the double ratio with the IQMD model in Fig. 3. Let us start with very first comparison of the BUU97 [17]. The results were very close to the experimental one, but, the reaction conditions were different. Firstly, in the BUU97 calculations, the incident energy was 40 MeV/nucleon, not 50 MeV/nucleon, just like the experimental one. Secondly, data set is only for the transverse emission, while in the BUU97 calculations, the nucleons used are emitted in all the directions. Move one step ahead to the IBUU04 results [18], where the symmetry energy is introduced with the help of momentum dependent interactions, the results are very far from the experimental data. This happened because of not detailed consideration of effective mass in the determination of symmetry energy from the momentum dependent interactions at that time. This concept when implemented later on, is found to explain the directed flow and pion ratio nicely[23]. The BNV model calculations with stiff symmetry energy (not shown here)


Figure 3. (Color online) The comparison of free neutrons to protons double ratio at $E = 50$ MeV/nucleon and $b \leq 5$ with the MSU/NSCL data, BUU97, IBUU04, BNV, and ImQMD simulations.

Figure 4. (Color online) Time evolution of different fragments at semi-central geometry for $^{132}$Sn + $^{132}$Sn using the soft symmetry energy. The solid, dash, dash dotted, dash dot dot and short dash represents the incident energy 50, 100, 200, 400, and 600, respectively. The vertical line represents our time limit before which the system can be in the supra-saturation region.

were close to experimental findings [19] compared to soft one. By looking carefully, one can find that the soft symmetry energy is more soft and stiff symmetry energy is less stiff in the BNV compared to the IQMD and ImQMD models. In other words, the stiff symmetry energy from the BNV model and soft symmetry energy from the IQMD and ImQMD models lies between the stiff symmetry energy from the IQMD and ImQMD models and the soft symmetry energy from the BNV calculations. It means that the data is favoring almost the same symmetry energy from both models. The most closeness between the data and the calculation is observed by the ImQMD model in 2009 [20]. They found that the results with $\gamma_i = 0.75$ are best fit with the experimental data for impact parameter $b \leq 2$ fm. In the present study, we have performed simulations for $b \leq 5$ fm and for the angular cuts, as mentioned in the experiments, with the soft and stiff symmetry energy and displayed the theoretical results over the whole range of the kinetic energy. The Fig. 3 clearly indicates that our results with soft symmetry energy are very close to experimental data.

3.3. High density behavior by using $N/Z$ ratio

To check the sensitivities of $N/Z$ ratio towards the high density behavior of symmetry energy using different fragments, in Fig. 4 we display time evolution of free nucleons (top), LCPs
(middle), and IMFs (bottom) at semi central geometry for incident energies ranging from 50 to 600 MeV/nucleon. The behavior for all kinds of fragments is consistent with the results in the literature [53, 54]. The production of free nucleons increases with incident energy, and LCPs production decreases after 400 MeV/nucleon. In Ref. [48], LCPs production is correlated with the nuclear stopping and is also found to have a maximum at 400 MeV/nucleon. IMFs production is found to decrease after 100 MeV/nucleon. This is due to the different origin of the production of IMFs as compared to free and LCPs. For more details about the incident energy dependence of IMFs, see Ref. [54, 55, 56].

Our main task is to check the sensitivities of the fragments in the high-density region. For this, we apply the limit that at least one particle/fragment must be produced before the time 20 fm/c, because, in an average, after that time the density becomes lower than normal nuclear matter density for all the incident energies under consideration. The free nucleons are highly sensitive at all the energies. This is not true for LCPs and IMFs. LCPs are produced in this region only after the incident energy reaches 200 MeV/nucleon. In contrast, no IMFs are produced in the supra-saturation density region. This means that IMFs are not so sensitive to the high-density dependence of symmetry energy; however, they can be used at sub-saturation and saturation densities [20]. Here we conclude that the neutrons-to-protons ratio from free nucleons as well as LCPs can act as a probe of the high-density behavior of the symmetry energy.

Figure 5. (Color online) Excitation function of the different isotopes \(D\bar{R}(N/Z)\) ratio for different fragments. The vertical line represent the energy above which \(D\bar{R}(N/Z)\) becomes less insensitive. Solid and open circles are for soft and stiff symmetry energies. The solid, dashed, and dot-dashed line corresponds to \(D\bar{R}\) from \(^{132}\text{Sn}\) to \(^{124}\text{Sn}\), \(^{124}\text{Sn}\) to \(^{112}\text{Sn}\), and \(^{132}\text{Sn}\) to \(^{112}\text{Sn}\), respectively.

Figure 6. Isospin asymmetry dependence of the double neutron-to-proton ratio from free nucleons at different incident energies. The different symbols have the same meaning as in Fig. 5.

In order to cancel the Coulomb effects and to see the effect of symmetry energy, we have
shown in Fig. 5, the incident energy dependence of the double ratio from different isotopes of Sn with different combinations, namely, $^{132}\text{Sn} + ^{132}\text{Sn}$ and $^{124}\text{Sn} + ^{124}\text{Sn}$, $^{124}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$, $^{132}\text{Sn} + ^{132}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$, having differences of 8, 12, and 20 neutrons and the same number of protons. The upper and lower panels are for free nucleons and LCPs. The double ratio is found to increase with the difference of the number of neutrons in the different combinations. It is similar to the results obtained at sub-saturation densities by different models [17, 20]. The double ratio is decreasing with increase in incident energy. The decrease in the double ratio with increasing incident energy is not due to the Coulomb effect, but is due to the pion effect, which remains active in the double ratio and becomes more and more dominant with increasing incident energy. The detail is discussed later. In contrast, this effect is valid only for the double ratio from the free nucleons and not from the LCPs. The double ratio from LCPs is found to be constant above 200 MeV/nucleon. This indicates that the effect of the symmetry energy for the ratio from LCPs can be analyzed only near sub-saturation densities close to $1.1\rho_0$. The double ratio from the symmetry energy becomes independent of the incident energy after 200 MeV/nucleon. This type of dependence for the single $\pi^-/\pi^+$ ratio can be observed above 1 GeV/nucleon [39]. This indicates that LCPs production is also not a sensitive probe for investigating the high-density behavior of the symmetry energy. The only possible probe from the fragments is the double ratio of neutrons to protons from free nucleons. Another possible probe is the $\pi^-/\pi^+$ ratio, which recently was compared with the experimental data of the FOPI by the IBUU04 and ImQMD calculations [42, 43].

In order to strengthen our conclusion, in Fig. 6, we display the isospin asymmetry dependence of the double ratio from free nucleons at different incident energies. All the curves are fitted with a power law of the form $y = ax^\tau$, where $y$ is the double ratio from free nucleons and $x$ is the double ratio of the systems. The power-law exponent $\tau$ is found to vary drastically with the symmetry energy. At low incident energy, the main constituents of fragmentation process are neutrons, protons and fragments constructed from them. Due to the dominating nature of symmetry potential within mean field, the DR(N/Z) as well as symmetry energy contributions are the maximum at low incident energy. With increase in incident energy, primary nucleon-nucleon collisions increases and isospin effects due to symmetry energy starts decreasing and hence there is decrease in DR(N/Z) as well as effect of symmetry energy on it. At sufficient high

Figure 7. The $Z_{\text{bound}}$ dependence of multiplicity of neutrons and protons from the projectile spectator fragmentation of $^{124}\text{Sn}$ at $E = 600$ MeV/nucleon. In the L.H.S., the theoretical results are compared with the experimental findings of ALADIN 2000 collaborations[57].
incident energy, where the primary nucleon-nucleon collisions saturates, the contribution of pions from secondary chance nucleon-nucleon collisions starts taking place. If a first chance nucleon-nucleon collision converts a neutron to the proton by producing a $\pi^-$, then the subsequent collisions of the energetic protons can convert them back to neutrons by producing a $\pi^+$. Therefore, at sufficient high incident energy, the contribution of neutrons and protons becomes stable and the $\text{DR}(N/Z)$ of free nucleons approaches to the $\text{DR}(N/Z)$ of the system and hence the effect of symmetry energy decreases to minimal level, which can be seen more clearly in pions production. It indicates that the pion production effect is very important at high incident energy and is equally useful for understanding the high-density behavior of the symmetry energy [42, 43]. The difference in the double ratio obtained with the soft and stiff symmetry energies here is also found to increase from the neutron-poor to the neutron-rich system, just like the single pion ratio in the literature [42, 43].

### 3.4. Symmetry energy from projectile spectator fragmentation

Recently, we have performed the analysis for the multiplicity of neutrons from the projectile spectator fragmentation for $^{124}\text{Sn} + \text{nat}\text{Sn}$ at 600 MeV/nucleon over the whole range of $Z_{\text{bound}}$ and the results are compared with the experimental findings of ALADIN 2000 collaborations [57]. The results are displayed in Fig.7. Just like the experiments, the rise and fall behavior for the $Z_{\text{bound}}$ dependence of $M(n)$ as well as $M(p)$ is observed. The decrease in the multiplicity at large $Z_{\text{bound}}$ is due to the involvement of the free particles in the bound fragments. $M(n)$ is found to be more sensitive toward the symmetry energy compared to $M(p)$. This is due to only symmetry energy effects in the production of neutrons, on the contrary, symmetry energy and coulomb effects are involved in the production of protons. The comparison with the experimental data of ALADIN 2000 collaboration data once again favors the soft symmetry energy.

### 4. Conclusions

In conclusion, we have studied the sub-and supra-saturation behavior of symmetry energy in intermediate energy heavy-ion collisions. At sub-saturation densities, the comparison of the theoretical results of single and double ratios with the experimental data emphasizes on the softness of the symmetry energy at sub-saturation densities. The double ratio is a relative good candidate for density dependence of symmetry energy at sub-saturation densities because of the cancel of Coulomb effect between two systems.

At supra-saturation densities, the double neutron-to-proton ratio from free nucleons is highly sensitive to the symmetry energy, incident energy, and isospin asymmetry of the system. However, the sensitivity of the neutron-to-proton double ratio from LCPs to the nuclear symmetry energy is almost beam-energy independent above 200 MeV/nucleon. The same trend is observed for the single $\pi^-/\pi^+$ ratio above 1 GeV/nucleon. In simple words, just like the $\pi^-/\pi^+$ ratio, the neutron-to-proton double ratio from free nucleons can act as a useful probe to constrain the high-density behavior of symmetry energy. Experiments are planned at MSU, GSI, RIKEN, and FRIB to determine the high-density behavior of symmetry energy by using the neutron-to-proton ratio.

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### References

[1] Xu H S et. al. 2000 Phys. Rev. Lett. 85 716.
[2] Tsang M B et al. 2001 Phys. Rev. Lett. 86 5023.
[3] Botvina A S, Lozhkin O V and Trautmann W 2002 Phys. Rev. C 65 044610.
[4] Fevre A Le et al. 2005 Phys. Rev. Lett. 94 162701.
[5] Colonna M and Tsang M B 2006 Eur. Phys. J. A 30 165.
[6] Natowitz J B et al. 2010 Phys. Rev. Lett. 104 202501.
[7] Kowalski et al. 2007 Phys. Rev. C 75 014601.
[8] Chen Z et al. 2010 Phys. Rev. C 81 064613.
[9] Shetty D V, Yennello S J and Souliotis G A 2007 Phys. Rev. C 75 034602.
[10] Souliotis G A et al. 2007 Phys. Rev. C 75 011601.
[11] Li B A et al. 2004 Phys. Rev. C 69 064610.
[12] Li B A et al. 2005 Phys. Rev. C 72 064603.
[13] Tian W D et al. 2007 Phys. Rev. C 76 024607.
[14] Ma C W, Wang F, Ma Y G and Jin C 2011 Phys. Rev. C 83 064620.
[15] Famiano M A et al. 2006 Phys. Rev. Lett. 97 052701.
[16] Hilscher D et al. 1987 Phys. Rev. C 36 208.
[17] Li B A, Ko C M and Ren Z Z 1997 Phys. Rev. Lett. 78 1644.
[18] Li B A, Chen L W, Yong G C and Zuo W 2006 Phys. Lett. B 634 378.
[19] Wolter H H et al. 2007 arXiv:0712.2187v1.
[20] Zhang Y et al. 2009 Phys. Rev. Lett. 102 122701.
[21] Tsang M B et al. 2011 Prog. Part. Nucl. Phys. 66 400.
[22] Kumar S, Ma Y G, Zhang G Q and Zhou C L 2011 Phys. Rev. C 84 044620.
[23] Li B A, Chen L W and Ko C M 2008 Phys. Rep. 464 113.
[24] Tsang M B et al. 2004 Phys. Rev. Lett. 92 062701.
[25] Toro M D et al. 2008 Int. J. Mod. Phys. E 17 1799.
[26] Baran V, Colonna M, Toro M D, Greco V, Phabe M Z and Wolter H H 2002 Nucl. Phys. A 703 603.
[27] Lombardo I et al. 2010 Phys. Rev. C 82 014608.
[28] Russotto P et al. 2011 Phys. Lett. B 697 471.
[29] Cozma M D 2011 Phys. Lett. B 700 139.
[30] Gautam S, Sood A D, Puri R K and Aichelin J 2011 Phys. Rev. C 83 034606.
[31] Gautam S et al. 2010 Phys. Rev. C 82 014604.
[32] Gautam S et al. 2011 Phys. Rev. C 83 014603.
[33] Gautam S et al. 2012 Phys. Rev. C 85 067601.
[34] Li B A and Udo Schröder W 2001 Isospin Physics in Heavy-Ion Collisions at Intermediate Energies (New York: Nova Science Publishers) Chapter 3.
[35] Li B A, Das C B, Gupta S D and Gale C 2004 Phys. Rev. C 69 011603 (R).
[36] Li B A 2005 Phys. Rev. C 71 044604.
[37] Li B A, Chen L W, Ma H R, Xu J and Yong G C 2007 Phys. Rev. C 76 051601 (R).
[38] Kumar S, Ma Y G, Zhang G Q and Zhou C L 2012 Phys. Rev. C 85 024620.
[39] Li B A 2003 Phys. Rev. C 67 017601.
[40] Li Q, Li Z, Zhao E and Gupta R K 2005 Phys. Rev. C 71 054907.
[41] Wolter H H, Prassa V, Lalazissis G, Gaitanos T, Ferini G, Toro M D and Greco V 2009 Prog. Part. Nucl. Phys. 62 402.
[42] Xiao Z G, Li B A, Chen L W, Yong G C and Zhang M 2009 Phys. Rev. Lett. 102 062502.
[43] Feng Z Q and Jin G M 2010 Phys. Lett. B 683 140.
[44] Guo Y, Zhang L, Zhang H F, Chen X M and Yong C G 2011 Phys. Rev. C 83, 047602.
[45] Yong C G, Li B A and Chen L W 2011 Int. J. Mod. Phys. E 19 1647.
[46] Hartnack C, Puri R K, Aichelin J, Konopka J, Bass S A, Stoecker H, and Greiner W 1998 Eur. Phys. J. A 1 151.
[47] Hartnack C, Oeschler H, Leifels Y, Bratkovskaya E L and Aichelin J 2012 Phys. Rept. 510 119.
[48] Kumar S and Kumar S 2011 cent. Eur. J. of Phys. 9 986.
[49] Aichelin J 1991 Phys. Rep. 202 233.
[50] Lehmann E et al. 1993 Prog. Part. Nucl. Phys. 30 219.
[51] Puri R K and Aichelin J 2000 J. comp. Phys. 162 245.
[52] Puri R K, Hartnack C and Aichelin J 1996 Phys. Rev. C 54 R28.
[53] Kumar S, Kumar S and Puri R K 2008 Phys. Rev. C 78 064602.
[54] Kumar S and Kumar S 2010 Pramana J. of Phys. 74 731.
[55] Verveni Y K and Puri R K 2009 Eur. Phys. Lett. 85 62001.
[56] Verveni Y K et al. 2010 J. Phys. G: Nucl. and Part. 37 015105.
[57] Trautmann W et al. 2011 PoS BORMIO2011 018.