Collisions of Terrestrial Worlds: The Occurrence of Extreme Mid-infrared Excesses around Low-mass Field Stars

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Abstract

We present the results of an investigation into the occurrence and properties (stellar age and mass trends) of low-mass field stars exhibiting extreme mid-infrared (MIR) excesses ($L_{\text{IR}}/L_\ast \gtrsim 0.01$). Stars for the analysis were initially selected from the Motion Verified Red Stars (MoVeRS) catalog of photometric stars with Sloan Digital Sky Survey, 2MASS, and WISE photometry and significant proper motions. We identify 584 stars exhibiting extreme MIR excesses, selected based on an empirical relationship for main-sequence W1 − W3 colors. For a small subset of the sample, we show, using spectroscopic tracers of stellar age (Hα and Li i) and luminosity class, that the parent sample is most likely comprised of field dwarfs ($\gtrsim$1Gyr). We also develop the Low-mass Kinematics (LoKi) galactic model to estimate the completeness of the extreme MIR excess sample. Using Galactic height as a proxy for stellar age, the completeness-corrected analysis indicates a distinct age dependence for field stars exhibiting extreme MIR excesses. We also find a trend with stellar mass (using $r − z$ color as a proxy). Our findings are consistent with the detected extreme MIR excesses originating from dust created in a short-lived collisional cascade ($\lesssim$100,000 years) during a giant impact between two large planetisimals or terrestrial planets. These stars with extreme MIR excesses also provide support for planetary collisions being the dominant mechanism in creating the observed Kepler dichotomy (the need for more than a single mode, typically two, to explain the variety of planetary system architectures Kepler has observed), rather than different formation mechanisms.

Key words: circumstellar matter – infrared: stars – methods: statistical – planet–disk interactions – stars: late-type – stars: low-mass

Supporting material: FITS file

1. Introduction

The ability to study circumstellar environments around stars has greatly improved over the past decade, due in part to new technologies that provide higher sensitivity and greater resolution at infrared (IR) and radio wavelengths. Examples of facilities that have contributed to this advance include but are not limited to the Spitzer Space Telescope (Werner et al. 2004), the Atacama Large Millimeter Array, the Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010), and the Herschel Space Observatory (Pilbratt et al. 2010). In recent years, observations at these facilities have led to the discovery of stars exhibiting large amounts of excess mid-IR (MIR) flux ($L_{\text{IR}}/L_\ast \gtrsim 10^{-2}$), termed “extreme debris disks” (Meng et al. 2012, 2015) or “extreme IR excesses” (Balog et al. 2009). Typically found around stars with ages between 10 and 130 Myr (Meng et al. 2012, 2015), these systems are believed to have hosted collisions between terrestrial planets or large planetisimals (Meng et al. 2014).

The majority of stars exhibiting extreme MIR excesses have been found with ages coinciding with the final stages of terrestrial planet formation (10–200 Myr; Meng et al. 2015). However, until recently, there was one known system that did not fall into the same category, BD +20 307, a $\sim$1 Gyr old spectroscopic binary composed of two late-F-type stars (Weinberger 2008) exhibiting a significant MIR excess ($L_{\text{IR}}/L_\ast \approx 0.033$; Song et al. 2005; Weinberger et al. 2011). An in-depth study of the disk mineralogy for BD +20 307 found that the best explanation for the observed large MIR excess and low level of crystallinity was a giant impact between two large terrestrial bodies, similar to the Moon-forming event in our solar system (Weinberger et al. 2011). However, such collisions are expected to occur much earlier during planetary system formation (as stated above), and the lifetime for the observable collisional cascade is expected to be short ($\sim$100,000 years; Melis et al. 2010). It is also possible that the close binary nature of BD +20 307 may have played a role in this late-time collision. The potential for impacts between terrestrial bodies on timescales $\gtrsim$1 Gyr is particularly important for low-mass stars ($M_\ast \lesssim 0.8 M_\odot$), which are known to host multiple terrestrial planets (about three planets per star on average; Ballard & Johnson 2016), all orbiting closely to their host stars due to the proximity of the snow line ($\lesssim$0.3 au; Ogihara & Ida 2009).

Low-mass stars make ideal laboratories for studying the occurrence of extreme MIR excesses and investigating the hypothesis of planetary collisions as their origin. In addition to the observational evidence suggesting an abundance of close-in terrestrial planets surrounding them, low-mass stars are ubiquitous, making up more than 70% of the stellar population (Bochanski et al. 2010). Until recently, all of the aforementioned observed extreme MIR excesses have been found around solar-type (FGK-spectral type) stars. However, no explanation has been put forward to explain the dearth of low-mass stars exhibiting similar extreme MIR excesses. In particular, the relative frequency of low-mass stars to solar-type stars should make it more likely to find extreme MIR excesses around low-mass stars, barring any observational limitations.

Simulations of planet formation around Sun-like stars indicate that impacts are quite common during the period of terrestrial planet formation (Quintana et al. 2016). Quintana et al. (2016) noted that highly energetic giant impacts (similar
to the Moon-forming event) occur far more rarely than smaller collisions, but are a necessity to build a system analogous to our present-day solar system. One interesting finding by Quintana & Barclay (2016) is that by removing giant planets from their dynamical simulations, giant impacts can occur much later in the system’s evolution (100 Myr to a few Gyr versus 10–100 Myr). This may have strong implications for planetary systems around low-mass stars, which do not typically form giant planets (e.g., Johnson et al. 2010; Bonfils et al. 2013). Efforts are currently under way to extend these models to low-mass stars, but initial circumstellar disk conditions are not as well constrained observationally at the bottom of the main-sequence.

A number of studies have undertaken searches for low-mass stars exhibiting signs of disks and/or M(IR) excesses (e.g., Plavchan et al. 2005, 2009; Avenhaus et al. 2012; Wu et al. 2013). Plavchan et al. (2009) provided a theoretical framework for why primordial disks around low-mass stars could persist on longer timescales than those around higher mass stars, in spite of most observational evidence suggesting primordial disks are dispersed around low-mass stars in less than 100 Myr. For a low-mass star (M0), the timescales for dust removal by Poynting–Robertson drag and grain–grain collisions are $\sim 10$ times longer and 40% longer than for a higher mass star (G0), respectively (Plavchan et al. 2009). Primordial disks around low-mass stars have been observed to be longer lived than those around higher mass stars (e.g., Ribas et al. 2015), potentially because the timescales for Poynting–Robertson drag to remove grains from these systems are longer than for higher mass systems ($\sim 10$ times longer for an M0 star versus a G2 star; Plavchan et al. 2009). However, there are currently little to no observational data to support the hypothesis that primordial disks around low-mass stars survive past 10s of Myr, which indicates that the evolution of primordial disks around low-mass stars follows an evolution similar to that of primordial disks around solar-mass stars.

A search for low-mass stars exhibiting extreme MIR excesses was conducted by Theissen & West (2014, hereafter TW14). The initial sample was pulled from the Sloan Digital Sky Survey (SDSS; York et al. 2000) Data Release 7 (DR7; Abazajian et al. 2009) spectroscopic sample of M dwarfs (70,841 stars; West et al. 2011). TW14 discovered 168 low-mass field stars exhibiting large amounts of excess MIR flux, and they estimated a collision rate of $\sim 130$ collisions per star over its main-sequence lifetime. This rate is significantly higher than the rate estimated by Weinberger et al. (2011) for A–G type stars (0.2 impacts per star). The result of TW14 suggests that collisions may be more common among low-mass stars, possibly due to a longer timescale over which collisions can act, coupled with the extremely long main-sequence lifetimes of low-mass stars (Gyr; Laughlin et al. 1997) and/or the higher density of planets with small semimajor axes. One limitation of the TW14 study was the use of the SDSS DR7 spectroscopic sample, which was not produced in a systematic way, making estimates of completeness difficult. To further investigate the mechanism responsible for creating these observed extreme MIR excesses, a larger sample must be gathered, and methods to estimate the completeness of the sample must be developed.

Although many large spectroscopic samples exist for low-mass stars, such as the SDSS spectroscopic M dwarf sample (West et al. 2011), the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (Cui et al. 2012) Data Release 1 (DR1; Luo et al. 2015) M dwarf catalog (93,619 stars; Guo et al. 2015), and the Palomar/Michigan State University Nearby Star Spectroscopic Survey (~2400 stars; Reid et al. 1995; Hawley et al. 1996), these samples are dwarfed by the millions of photometric data products for low-mass stars that are currently available. Unfortunately, many photometric objects share similar colors and point-source-like morphologies with low-mass stars (e.g., giants, quasars, and distant luminous galaxies). One way of distinguishing dwarf stars from other similarly colored objects is through the use of proper motions. Distant objects will show little to no tangential motion on the sky, while nearby stars will show significant measurable motion in reference to background stars.

The largest catalog of low-mass stars with proper motions to date is the Motion Verified Red Stars catalog (MoVeRS, containing ~8.7 million stars; Theissen et al. 2016). MoVeRS was created using data from SDSS, the Two Micron All-Sky Survey (2MASS; Skrutskie et al. 2006), and the WISE (Wright et al. 2010). The Late-Type Extension to MoVeRS was recently released with additional very-low-mass objects later than M5 (LaTE-MoVeRS; Theissen et al. 2017). The MoVeRS catalog enables the search for extreme MIR excesses in a larger capacity than was previously available.

This paper performs a thorough investigation of the dependence on mass, spatial extent, and age of extreme MIR excesses around low-mass field stars. In Section 2 we describe the sample from which the stars are drawn. Section 3 briefly discusses the methods used in estimating stellar parameters (Section 3.1) and distances (Section 3.2), describes how we cull the sample of stars (Section 3.3), account for interstellar extinction (Section 3.3.3), distinguish extreme MIR excess candidates (Section 3.4), investigate the fidelity of the WISE measurements (Section 3.5), obtain spectroscopic observations for youth (Section 3.6), and the inherent biases in the sample (Section 3.9). Section 4 provides details about the Galactic model that we use to estimate the completeness of the sample, and discuss the completeness-corrected results. In Section 5 we investigate the non-significant MIR excess sample for trends with stellar age. In Section 6 we summarize the conclusions and provide a discussion of our results. Details regarding the methods for estimating stellar parameters, including the Markov chain Monte Carlo (MCMC) method for estimating $T_{\text{eff}}$ and log $g$ and the methods for estimating stellar size, are found in Appendix A. Details for building and using the LoKi galactic model to estimate the level of completeness are discussed in Appendix B.

### 2. Data: The MoVeRS Catalog

The occurrence rate for low-mass stars exhibiting extreme IR excesses was shown to be extremely low by TW14 (~0.4%). To build a larger sample of candidate stars with extreme IR excesses, a massive input catalog of bona fide low-mass stars is required. Although photometric catalogs exist for large numbers of low-mass stars (e.g., Bochanski et al. 2010), proper motions are a way to definitively separate dwarf stars from giants and extragalactic objects of similar photometric colors. Theissen et al. (2016) created the MoVeRS catalog, a photometric catalog of low-mass stars extracted from the SDSS, 2MASS, and WISE data sets, that were selected based on their significant proper motions. The MoVeRS catalog contains 8,735,004 stars, 8,534,902 of which have cross-
matches in the \textit{WISE} AllWISE catalog. Along with proper motions computed in Theissen et al. (2016), the current version of the MoVeRS catalog contains photometry from SDSS, 2MASS, and \textit{WISE}, where available, for each star.

To build the MoVeRS catalog, Theissen et al. (2016) initially selected stars based on their SDSS, 2MASS, and \textit{WISE} colors, tracing the stellar locus for stars with $16 < r < 22$ and $r - z \geq 0.5$. Stars were then selected based on a number of quality flags and proximity to neighboring objects. Proper motions for the remaining objects were computed using astrometric information from SDSS, 2MASS, and \textit{WISE}, which spans a $\sim$12 year time baseline. The precision of the catalog is estimated to be $\sim$10 mas yr$^{-1}$. Only stars with significant proper motions ($\mu_{\text{tot}} \geq 2\mu_{\text{ref}}$) were included in the final catalog, increasing the likelihood that the catalog contains nearby stars as opposed to other astrophysical objects.

To illustrate the effectiveness of removing giants using proper motions, we consider a giant star at the edge of the photometric selection criteria used for MoVeRS ($r = 16$). A giant star would be approximately 1000 times more luminous than its dwarf counterpart, which places a giant approximately 30 times farther away than a dwarf for a given magnitude. The median photometric distance for stars in the MoVeRS sample is 200 pc, placing a giant star at 6 kpc. The minimum required proper motion within MoVeRS is approximately 20 mas yr$^{-1}$. For a giant at a distance of 6 kpc, this translates into a tangential velocity of $570 \text{ km s}^{-1}$. Figure 6 within \textit{Gaia} Data Release 1 (DR1; Gaia Collaboration et al. 2016a) shows that red giants with such high tangential velocities (hypervelocity stars) are a negligible fraction of the entire population and are likely to be unbound from the Galaxy.

If we assume a similar proper motion distribution between giant stars and QSOs (both essentially non-moving on the sky) for motions measured with \textit{WISE}+SDSS+2MASS, we can use Figure 3 from Theissen et al. (2016) to make a statistical estimate of the contamination rate of giants. The average time-baseline of 12 years translates into a combined proper motion uncertainty of 10 mas yr$^{-1}$ for a non-moving population. This gives a point-source with a proper motion of 20 mas yr$^{-1}$ a 4.5% chance of being a giant. Combined with the relative fraction of all point sources that are giants (versus dwarfs) at the blue limit of the MoVeRS samples ($\sim$2%; Covey et al. 2008), this gives the likelihood of having an interloping giant with a proper motion of 20 mas yr$^{-1}$ lower than 0.1%. The vast majority of MoVeRS stars have proper motions that exceed 20 mas yr$^{-1}$, making the likelihood for contamination by giants significantly smaller than this. More information about the construction and properties of the MoVeRS catalog can be found in Theissen et al. (2016). The Late-Type Extension to MoVeRS was recently released and contains stars with spectral types later than M5 (LaTE-MoVeRS; Theissen et al. 2017).

Photometry from \textit{WISE}, taken in four MIR bands (W1, W2, W3, and W4) with effective wavelengths at 3.4, 4.6, 12, and 22 $\mu$m, respectively, is particularly crucial for finding extreme MIR excesses around K and M dwarfs because dust orbiting within the snow line, where terrestrial planets form, is warm ($\sim$300 K), with its thermal emission peaking in the MIR. The W3 band also samples the 10 $\mu$m silicate feature prominent in the types of disks expected to produce these extreme MIR excesses. The sensitivity of \textit{WISE}, particularly the W3 band ($\sim$730 $\mu$Jy at 12 $\mu$m; Wright et al. 2010), allows these extreme MIR excesses to be detected at much higher precision than previous all-sky MIR observatories (e.g., the \textit{Infrared Astronomical Satellite}, Neugebauer et al. 1984, and \textit{Akari}, Murakami et al. 2007).

3. Methods

3.1. Estimating Stellar Parameters

An important step in identifying and quantifying the significance of a MIR excess is measuring the deviation between the expected photospheric MIR values and the measured photometric values, which requires an estimate of the fundamental stellar parameters (e.g., $T_{\text{eff}}$). Additionally, estimates for stellar temperature ($T_{\text{eff}}$) and size ($R_*$) place constraints on dust temperature and orbital distance (Jura et al. 1998; Chen et al. 2005). Photospheric models for low-mass stars are limited in replicating the myriad of complex molecules found in low-mass stellar atmospheres owing to the low-temperature environments (Schmidt et al. 2016). Furthermore, the onset of potential clouds forming in the coolest stars provides further complications for modeling (Allard et al. 2013). However, these models are good at producing the overall expected stellar energy distributions (SEDs) and are effective for constraining many of the fundamental stellar parameters. TW14 estimated stellar parameters using a grid of BT-Settl models based on the PHOENIX code (Allard et al. 2012a, 2012b), which have taken into account many molecular opacities and cloud models.

TW14 compared synthetic photometry and spectra from models to data from SDSS, 2MASS, and \textit{WISE} to estimate goodness-of-fit. Because of the lack of spectra for the MoVeRS sample, we only considered synthetic photometry in deriving the goodness-of-fit. This process involved fitting synthetic photometry, derived using relative spectral response curves for SDSS (Doi et al. 2010), 2MASS (Cohen et al. 2003), and \textit{WISE} (Wright et al. 2010), to actual measurements from each photometric survey.

TW14 derived stellar parameters by computing reduced-$\chi^2$ values over the entire range of models, a method that is intractable computationally for the large number of stars in the MoVeRS catalog. To reduce the parameter space, we employed an MCMC technique to sample and build posterior probability distributions for each of the stars, which we used to estimate best-fit parameters and uncertainties (using the 50th percentile value, and the 16th and 84th percentile values, respectively). Details of the MCMC method are described in Appendix A.1. Using this process, we estimated $T_{\text{eff}}$ and log $g$ values for all 8.7 million sources in the MoVeRS catalog. We used the $T_{\text{eff}}$ values to derive a color–$T_{\text{eff}}$ relationship, also found in Appendix A.1. With distance estimates, the scaling values derived from this fitting procedure were used to estimate stellar size ($R_*$) from a radius–color relation (also found in Appendix A.1). The new MoVeRS catalog (MoVeRS 2.0), with the estimated stellar parameters, is available through SDSS CasJobs\footnote{http://skyserver.sdss.org/casjobs/} and VizieR.\footnote{http://vizier.u-strasbg.fr/}

3.2. Estimating Distances: Photometric Parallax

Distances to stars are important for estimating luminosities, radii, and many other stellar and kinematic parameters (see TW14 for details). For stars with resolved disks, distances can be used to convert angular sizes into absolute sizes. For
unresolved disks, stellar sizes can give approximate orbital distances for circumstellar dust and approximate dust masses. Few parallax measurements have been made for M dwarfs, relative to higher mass stars, due to their intrinsic faintness. The two largest astrometry databases, the General Catalog of Trigonometric Stellar Parallaxes, Fourth Edition (the Yale Parallax Catalog; van Altena et al. 1995), and the Hipparcos catalog (Perryman et al. 1997; van Leeuwen 2007), are both severely incomplete for M dwarfs and brown dwarfs (Dittmann et al. 2014). Although large parallax databases are incomplete for low-mass stars, two nearby stellar samples now have many parallax measurements: the REsearch Consortium On Nearby Stars (RECONS; Riedel et al. 2014; Winters et al. 2015) and MEarth (Nutzman & Charbonneau 2008). The RECONS sample includes parallaxes for over 1400 M dwarfs within 25 pc (Winters et al. 2015), and the MEarth sample includes over 1500 M dwarfs within 33 pc (Dittmann et al. 2014). There is very little overlap between the two samples since the RECONS survey began operating in the southern hemisphere, while MEarth started as a survey in the northern hemisphere, only recently adding telescopes to the southern hemisphere (Irwin et al. 2015). Additionally, a few studies have measured trigonometric parallaxes for substellar objects (e.g., Faherty et al. 2012; Manjavacas et al. 2013; Marocco et al. 2013; Marsh et al. 2013; Smart et al. 2013; Zapatero Osorio et al. 2014; Weinberger et al. 2016), but these studies are limited by small numbers.

Unfortunately, none of these trigonometric parallax surveys have data in SDSS passbands, which makes deriving a photometric relationship impossible without adding in additional errors from color transformations. The most commonly used photometric parallax relationship for low-mass stars with SDSS colors comes from Bochanski et al. (2010; hereafter B10). These relationships are derived from 86 low-mass stars with trigonometric parallax measurements from various sources (B10). The average uncertainty in these relationships is ~0.4 mag in absolute $r$-band magnitude ($M_r$), due in part to luminosity differences between stars of different metallicities (see Savcheva et al. 2014) and magnetic activity (see Bochanski et al. 2011). This uncertainty in absolute magnitude corresponds to distance uncertainties of ~20%. Efforts are under way to obtain SDSS magnitudes for many of the low-mass stars with parallax measurements in the samples listed above (C. Theissen et al. 2017, in preparation), however, to date, such measurements do not exist. For this purpose, we chose to use the B10 $r-z$ relationship to estimate distances for the entire MoVeRS sample. Using these distances, we also estimated stellar radii for the MoVeRS sample (see Appendix A.2). The new MoVeRS 2.0 catalog also includes our distances estimates.

### 3.3. Sample Selection for Stars with MIR Excesses

To compile a clean set of stars for our analysis, we used a number of selection criteria, most of which have been adapted from TW14. We applied the following selection criteria to the MoVeRS sample:

1. We selected stars that did not have a WISE extended source flag ($\text{EXT\_FLG} = 0$). This requirement ensured a point-source morphology through all WISE bands. This cut left 8,483,499 stars.
2. We selected stars that did not have a contamination or confusion flag in either $W1$, $W2$, or $W3$ ($\text{CC\_FLG} = 0$). This ensured clean photometry for those bands. This cut left 7,899,559 stars.
3. We selected stars with at least a signal-to-noise ratio (S/N) of 3 in $W1$, $W2$, and $W3$ ($\text{WXSNR}_{w=W1,W2,W3} \geq 3$). This cut left 185,121 stars.
4. We kept only the highest fidelity stars, retaining relatively bright stars satisfying Equation (12) of Theissen et al. (2016). This cut ensures stars have high-precision proper motion measurements and fall within the regime confirmed with independent checks to other proper motion catalogs. This cut left 145,526 stars.
5. Last, to minimize source confusion and reduce contamination due to dust extinction, we removed stars close to the Galactic plane ($|b| < 20^\circ$) and in the Orion region ($-30^\circ < b < 0^\circ$ and $190^\circ < l < 215^\circ$). This cut left 126,976 stars.

#### 3.3.1. WISE Sensitivity Limits

To directly address one of the limitations of the TW14 study, we constructed a uniform sample of stars. We broadly categorized the stars into three groups: (1) stars that are close enough that WISE can significantly detect their photospheres at 12 μm; (2) stars that are far enough away that their photospheres are undetectable at 12 μm, but for which an extreme MIR excess (on the order of those found in TW14) is significantly detectable by WISE; and (3) stars that are too far away to be detectable by WISE, even if they have an extreme MIR excess. We were only interested in stars that have measurable detections in W3. Below, we discuss the methods for building the “full” sample, stars that meet criterion (2), and the “clean” sample, stars that meet criterion (1), which is a subset of the “full” sample. We first discuss selecting stars exhibiting excess MIR flux (Section 3.4), and apply further criteria to select stars with extreme MIR excesses ($L_{IR}/L_* \geq 0.01$) in Section 3.4.1.

The W3 5μ point-source sensitivity limit is estimated to be 730 μJy (also the approximate 95% completeness limit; hereafter referred to as the W3 flux limit), based on external checks with Spitzer COSMOS data, which translates into a flux density of $1.89 \times 10^{-13}$ erg s$^{-1}$ cm$^{-2}$. Using the sample of 126,976 stars, we computed the expected photospheric W3 flux for each star by scaling the best-fit stellar model to the measured $z$-band flux. This gave us a measure of the expected W3 flux from the stellar photosphere for each star. The map of expected W3 stellar flux for a given $r-z$ color and $r$-band magnitude is shown in Figure 1. Figure 1 shows that a constant expected W3 flux is approximately linear in this color–magnitude space.

To quantify the relationship between $r$, $r-z$, and the expected W3 flux, we started at $r = 16$ and binned each 0.1 mag along the $r$-band axis, and binned each slice in 0.1 mag $r-z$ bins. We identified the $r$, $r-z$ value where the expected W3 flux dropped below $1.89 \times 10^{-13}$ erg s$^{-1}$ cm$^{-2}$ (the W3 flux limit). We repeated this process between 16 ≤ $r$ ≤ 22, and then fit a line to the $r$, $r-z$ values. Our linear fit is shown as a red dashed line in Figure 1 and is given by

$$r = 13.40 + 1.38(r-z).$$

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4 http://wise2.ipac.caltech.edu/docs/release/allwise/expsup/sec2_3a.html
Every star brighter than this limit should fall within the W3 flux limit, regardless of whether the star has a 12 μm excess. This gives us a very simple sample, free of a W3 sensitivity bias. Stars equal to or brighter than Equation (1) are referred to as the “clean” sample, which consists of 6129 stars.

Many of the stars in the TW14 sample had extremely high W3 excesses above the expected photospheric values, with the majority of observed 12 μm fluxes being 10 times greater than the expected photospheric values. Considering that we were looking for similarly high excesses, the volume of space over which we might effect a true W3 detection can be increased. To illustrate this point, Figure 1 shows the expected r, r − z limit at which stars with 12 μm excesses 10 times their photospheric values would equal the W3 detection limit (dash−dotted line). However, to increase the detections (source counts) of stars with MIR excesses, we must also consider the larger sample of stars that reside outside the W3 bias-free limit, where a MIR excess could be detected (at larger distances, and hence larger volumes). This is illustrated in Figure 1, where we plot the estimated distance limits corresponding to different r, r − z values.

The WISE sensitivity limits are highly dependent on the source position on the sky because of different coverage depths and zodiacal foreground emission. Many of the stars fainter than the imposed limit can therefore yield true detections, but stricter criteria must be implemented in their selection. Sensitivity maps for the WISE bands have been created using a profile-fit photometry noise model. These sensitivity maps have been checked using 2MASS stars with spectral types earlier than F7 to estimate the sensitivity of the W3 band at different positions over the entire sky. The external comparison against 2MASS has shown that the W3 sensitivity map may slightly underestimate the sensitivity of the ALLWISE catalog, but provides a consistent model against which we can examine the measured W3 fluxes for significance as a function of position on the sky.

To select the highest fidelity stars outside the limits of the clean sample, we required that each source have a W3 flux ≥ the W3 flux limit for its position on the sky according to the noise model sensitivity map. This sample, termed the “full” sample, consists of the clean sample and an additional 19,354 stars, for a total count of 25,483 stars.

### 3.3.2. Visual Inspection

To retain the highest quality detections, we performed visual inspection for each of the stars. The W3 band is especially susceptible to background and nearby contaminants because of its large point spread-function (PSF; ∼6′.5). Visual inspection removed stars superimposed on galaxies or blended with other nearby stars, which could cause the elevated MIR fluxes. Visual inspection also removed stars close to nearby bright objects that could produce additional MIR flux, or stars in areas of high IR cirrus. During visual inspection, we viewed SDSS and WISE archival images to ensure that the candidate objects were real MIR detections, a process similar to the procedure in TW14. Stars were assigned a QUALITY flag, with QUALITY = 1 indicating a star free of any contaminants and of the highest visual quality, and QUALITY = 2 indicating that the 12μm source is good but may be affected by nearby or background contamination, slightly offset between other WISE bands, or low contrast in W3. After visual inspection, we were left with 20,502 stars in the full sample and 5786 stars in the clean sample. The breakdown of the samples and quality flags is shown in Table 1. This provides a clean sample from which to select stars with excess MIR flux (Section 3.4) and account for interstellar extinction (Section 3.3.3).

### 3.3.3. Accounting for Interstellar Extinction

Owing to the distances to the stars in the sample (≥100 pc), interstellar extinction may affect the photometry. Since dust grains along a line of sight in the interstellar medium both extinct and redden an object’s SED, interstellar extinction increases the likelihood of a false MIR excess detection. For wavelengths longer than ∼5 μm, extinction effects should be negligible, with the exception of the 10 μm silicate feature (Gao et al. 2013). Although we expect extinction to minimally affect the SED fits for the sources in our sample, extinction must still be evaluated because of the requirement that stars reside at relatively high Galactic latitudes (|b| > 20°), especially since the W3-band samples the 10 μm silicate feature.

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3. http://wise2.ipac.caltech.edu/docs/release/allwise/expsup/sec2_3a.html

6. http://wise2.ipac.caltech.edu/docs/release/allwise/expsup/sec2_3a.html
Directly measuring extinction for a star is most accurately done with an optical spectrum that samples the “knee” of the extinction curve, and a comparison to an un-extincted template of the same spectral type (Jones et al. 2011). However, because optical spectra are unavailable for the vast majority of the MoVeRS sample, we employed a broader approach. SDSS provides estimates for the relative extinction, $A_V/A_0$ (the ratio of extinction in a given bandpass to extinction in the $V$ band), for each star and each band in the photometric catalog. These extinction values were estimated along the line of sight using the Schlegel et al. (1998) dust maps, created using galactic extinction measurements from the Cosmic Microwave Background Explorer (COBE; Boggess et al. 1992) and the Infrared Astronomical Satellite (IRAS; Neugebauer et al. 1984). These maps estimate the total extinction along a line of sight out of the Galaxy and may therefore overestimate the actual extinction values for stars closer than 1–2 kpc. Extinction effects may also occur as a result of circumstellar material, which is expected of the MIR excess candidates. However, the probability that an optically thick disk is seen directly edge-on is small assuming inclinations are random (Beatty & Seager 2010), although an edge-on view has the highest probability ($\sim$3.5% chance to view within $\pm2^\circ$ of edge-on). Therefore we may assume the disk to be optically thin at visible wavelengths (similar to Weinberger et al. 2011).

To estimate the extinction in the sample, we used the SDSS extinction estimates for the $riz$-bands ($A_r$, $A_i$, and $A_z$). The extinction values for the clean and full samples are shown in Figure 2. The vast majority of the sources have low extinction values (<0.1 mag), with median values for $A_r$, $A_i$, and $A_z$ of 0.08, 0.06, and 0.04 for the full sample, and 0.09, 0.07, and 0.05 for the clean sample, respectively. Therefore we do not expect extinction to affect the majority of our model fits from Appendix A.1. Furthermore, extinction tends to move stars parallel to our initial selection criteria (see Figure 1) and should minimally bias our selected sample (Section 3.3.1). For our full and clean samples, we corrected for extinction using the the SDSS estimates for $A_r$, $A_i$, and $A_z$, and the relative extinction values ($A_V/A_0$) for SDSS bandpasses from Schlegel et al. (1998), Table 6, to compute $A_V$ values. We then applied corrections to the $rizHK_s$ bandpasses using relative extinction measurements from the Asiago Database (Moro & Munari 2000; Fiorucci & Munari 2003) and an $R_V = 3.1$. Further details of this method can be found in Theissen & West (2014).

Rieke & Lebofsky (1985) found that the relative extinction at 10 $\mu$m due to the Galactic ISM extinction curve can be as large as the relative extinction in the K band. Davenport et al. (2014) used 1,052,793 main-sequence stars from SDSS DR8 (Alfaro et al. 2011) with $|b| > 10^\circ$ to measure the dust extinction curve relative to the $r$ band for the first three WISE bands. Davenport et al. (2014) derived $A_r/A_K = 0.60, 0.33,$ and 0.87 for $W1, W2,$ and $W3$, respectively. Another study by Xue et al. (2016) using GK-type giants from the SDSS Apache Point Observatory Galaxy Evolution Experiment (APOGEE; Eisenstein et al. 2011) spectroscopic survey found that the MIR relative extinction values were extremely sensitive to the NIR extinction, which is commonly expressed as a power law, $A_V \propto \lambda^{-\alpha}$. This power law also corresponds to the relative extinction between the $J$ and $K_s$ bands.

### Table 2

| SDSS DR8+ objID | R.A. (H:M:S) | Decl. (D:M:S) | Radial Velocity ±7 (km s⁻¹) | Spectral Type | Hα EW⁴ | Telescope | L |
|----------------|-------------|--------------|----------------------------|--------------|--------|-----------|---|
| 123766536903878268 | 10:17:40.54 | +28:58:51.62 | +39.5 | M1 | ... | SDSS | 21.34 |
| 1237651250974556408 | 15:47:54.70 | +52:48:57.52 | -32.5 | M1 | ... | SDSS | 13.77 |
| 1237657071156723794 | 01:27:51.44 | +00:16:33.17 | +6.2 | M2 | ... | SDSS | 21.98 |
| 1237659692480151822 | 15:16:10.43 | -01:42:37.24 | -48.4 | M2 | ... | SDSS | 16.95 |
| 1237677112537461409 | 09:32:04.26 | +14:08:26.51 | +39.0 | M3 | ... | SDSS | 92.45 |
| 1237662619722449089 | 15:38:25.49 | +32:28:44.59 | -10.0 | M4 | ... | SDSS | 36.11 |
| 1237667254011101278 | 11:30:25.02 | +29:14:16.37 | -23.5 | M5 | ... | SDSS | 59.30 |
| 1237659516736315205 | 15:48:31.45 | +42:53:07.14 | -21.1 | M6 | ... | SDSS | 179.04 |
| 123766512854591120 | 12:42:03.86 | +34:55:37.74 | -45.7 | M7 | ... | SDSS | 240.58 |
| 1237661068171346281 | 09:31:07.08 | +10:06:07.25 | +56.2 | M7 | 10.3 ± 0.9 | SDSS | 327.29 |
| 1237668331488084142 | 14:12:46.44 | +15:01:52.55 | -42.1 | M0 | ... | DCT | -1.97 |
| 1237651250974556408 | 15:47:54.70 | +52:48:57.52 | -8.4 | M2 | ... | DCT | 17.59 |
| 1237655749395022353 | 18:04:45.57 | +46:36:57.79 | -51.4 | M2 | ... | DCT | 41.55 |
| 1237672022649167591 | 22:41:17.31 | +33:40:21.14 | -43.6 | M2 | ... | DCT | 22.31 |
| 1237664852033142893 | 14:15:55.43 | +32:54:33.84 | +25.1 | M3 | ... | DCT | 11.19 |
| 123766250006461639 | 16:01:09.94 | +36:35:30.07 | +5.3 | M3 | ... | DCT | 38.02 |
| 1237657577797563146 | 17:45:18.61 | +57:53:59.65 | +4.3 | M4 | ... | DCT | 28.02 |
| 1237687834684959988 | 18:35:14.13 | +40:26:51.95 | -93.0 | Pe⁵ | ... | DCT | ... |
| 1237671941420483289 | 19:06:24.80 | +64:36:19.88 | -56.3 | M4 | ... | DCT | 40.04 |
| 1237656241415901294 | 21:58:10.54 | +11:42:01.70 | -122.0 | M4 | ... | DCT | 30.99 |
| 1237659330309456141 | 15:35:00.41 | +48:53:42.51 | -111.1 | M5 | ... | DCT | 51.76 |
| 1237654656522923838 | 16:17:07.09 | +45:52:14.97 | -70.0 | M5 | ... | DCT | 86.02 |
| 1237652943695095656 | 22:00:46.74 | +12:44:01.96 | -32.4 | M5 | 6.3 ± 0.5 | DCT | 76.04 |
| 1237652937790015940 | 20:53:41.55 | +08:35:14.57 | -26.7 | M6 | 3.5 ± 0.9 | DCT | 241.09 |
| 12376787920195637464 | 22:35:47.06 | +11:42:15.67 | -43.5 | M7 | 15.8 ± 1.8 | DCT | 103.27 |

**Notes.**

- Positive EW measurements indicate emission. Inconclusive measurements are not listed.
- This object shows peculiar spectral features. The TIO bands at ~7050 are indicative of a low-mass star. However, the numerous bumps in the spectrum may indicate a carbon dwarf.
i.e., $A_I/A_{K_S} = (\lambda_I/\lambda_{K_S})^{-\alpha}$. Rieke & Lebofsky (1985) measured $\alpha = 1.65$ using a small number of stars, but Xue et al. (2016) measured a slightly higher value of $\alpha = 1.79$. The value of $\alpha$ corresponding to the measurements from Davenport et al. (2014) is 1.25, significantly less steep than the value of other studies. Wang & Jiang (2014) studied the universality of the NIR extinction law using color-excess ratios of APOGEE M and K giants and found that the extinction law shows very little variation across different environments. We chose to adopt the relative extinction values from Xue et al. (2016), whose measurement of $\alpha$ is consistent with other measurements from the diffuse ISM (Martin & Whittet 1990), to correct for extinction in each WISE passband. Using the extinction-corrected photometry, we reran the full and clean samples through the stellar parameters pipeline (Section 3.1) to obtain new estimates for $T_{\text{eff}}$ and $R_\star$. For the remainder of this study we use the unreddened photometry.

3.4. Determining Infrared Excesses

TW14 explored two different methods to determine which stars showed high levels of excess IR fluxes over the expected photospheric values ("extreme" MIR excesses are evaluated in Section 3.4.1). The first method, and the method ultimately used by TW14, is a modified version of the empirical calibrations from Avenhaus et al. (2012), using main-sequence stars to determine the expected WISE colors as a function of $r-z$ color (denoted as $\sigma'$). Figure 3 shows the $r-z$ versus $W1 - W3$ distribution for the full and clean samples, along with the empirical calibration of TW14. Figure 4 shows the residual distribution with the TW14 empirical calibration (red line; Figure 3) subtracted. Although it is common to define stars with disks to be only those with highly significant deviations from the expected photospheric values in a binary fashion, we acknowledge that the distribution is continuous, and many of the stars with non-significant deviations may have true detections but lower disk masses or dust that is becoming optically thin. Although we used the more classical binary description of stars with an excess versus stars without an excess, we address this continuous distribution in Section 3.4.2.

Rather than making a blanket cut on stars with $\sigma' \geq 5$, as was done in TW14, we used the distributions from Figure 4 to evaluate the false-positive probabilities of the candidates. To obtain stars with a 99% probability of hosting a true MIR excess, we define the probability threshold (assuming normal distributions),

$$P_{\text{FP}}(\text{MIR Excess}) \times N_{\text{sample}} < 0.01,$$

where $P_{\text{FP}}(\text{MIR Excess})$ is the probability that the MIR excess is a false-positive, and $N_{\text{sample}}$ is the number of sources within the given sample. For the full sample, $P_{\text{FP}}(\text{MIR Excess}) < 4.88 \times 10^{-7}$, and for the clean sample $P_{\text{FP}}(\text{MIR Excess}) < 1.73 \times 10^{-6}$. Converting these false-positive probabilities into $\sigma'$ values for each sample, we define stars with true MIR excesses to have $\sigma' > 3.48$ for the full sample (4.90$\sigma$), and $\sigma' > 2.53$ for the clean sample (4.64$\sigma$). The two limits are shown in Figure 4 (red dotted line), and candidates that meet these thresholds are marked as red points in Figure 3.

Figure 4 indicates that the TW14 calibration appears to be shifted to slightly redder WISE colors than the bulk of the
stellar population. The peak of the distribution is shifted negative of zero, suggesting that either the TW14 calibration needs to be recalibrated, or that some effect, such as the metal content of the stellar ensemble, has shifted these values.

The second method takes the difference between the measured flux and the expected flux (estimated from a stellar photospheric model), weighted by the measurement uncertainty. This value is commonly abbreviated as

\[ \chi_{12} = \frac{F_{12 \mu m, measured} - F_{12 \mu m, model}}{\sigma_{F_{12 \mu m, measured}}} \]  

Using stellar parameters and scaling values from the MCMC method (Section 3.1), we computed the expected 12 \( \mu m \) flux densities for stars in both the full and clean samples. Next, we converted W3 magnitudes into flux densities using the WISE all-sky explanatory supplement\(^7\) (further details can be found in TW14). Figure 5 shows the distribution of \( \chi_{12} \) values for the full and clean samples. The majority of both samples are well represented by normal distributions with similar widths, although the full sample is shifted to slightly higher \( \chi_{12} \) values due to a distance bias that we discuss in Section 3.9.

Avenhaus et al. (2012) showed that the empirical method outlined above was able to detect the disk around AU Mic at 22 \( \mu m \), while methods involving SED fitting were unable to significantly detect the same disk using observational data at similar wavelengths (Liu et al. 2004; Plavchan et al. 2009; Simon et al. 2012). Presumably this indicates that \( \sigma' \) is a stronger discriminator of MIR excess significance. Although the SED fitting is important for estimating parameters that will allow us to then constrain disk parameters, we chose to select excess sources based solely on their \( \sigma' \) significance, similar to TW14.

Selecting stars with MIR excesses using the aforementioned criteria produced 609 stars in the full sample and 2 stars in the clean sample. The cumulative false-positive probabilities for our selected stars are 0.0386\% (\( \sim 0.24 \) stars) for the full sample.

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\(^7\)http://wise2.ipac.caltech.edu/docs/release/allsky/expsup/sec4_4h.html
3.4.1. Extreme MIR Excesses

Extreme MIR excesses arising from planetary collisions are expected to produce large amounts of dust and hence high fractional IR luminosities ($L_{IR}/L_*$ $\lesssim 10^{-2}$). The primary focus of this study are these extreme MIR excesses, but this requires knowledge about the total IR flux of the dust grains. For stars that have both W3 and W4 detections, we can fit a simple blackbody to the MIR flux, similar to what was done in TW14. We acknowledge that the disks we are interested in observing should emit a strong silicate features (e.g., Meng et al. 2014), which would make W3 a poor indicator of the underlying blackbody continuum of the dust. However, with no ability to discern the blackbody continuum from the silicate emission (e.g., a MIR spectrum), we use the approximation that W3 is dominated by the continuum radiation. Using the extreme MIR excess candidates that had a W4 detection with a S/N $> 2$, we fit a combined model comprised of the best-fit photospheric model found in Section 3.3.3 and a simple blackbody function. To determine the best-fit blackbody function, we used a least-squares minimization, fitting for $T_{dust}$ and the multiplicative scaling factor for the blackbody. For the least-squares fit, we used the best-fit photosphere model and fit the dust blackbody function to the W3 and W4 measurements, weighted by the measurement uncertainty. An example fit from this process is shown in Figure 6. For stars without a W4 detection, we assume that the peak SED flux is at W3, giving an estimate for $T_{dust}$ $\approx 317.4$ K (TW14).

To compute $L_{IR}/L_*$, we integrated the best-fit photospheric model to estimate $L_*$, and for $L_{IR}$, we subtracted the stellar model from the combined fit (stellar model plus best-fit blackbody), and integrated the residual flux to estimate $L_{IR}$, taking the ratio of the two values (similar to Patel et al. 2014, 2017). Keeping only the stars with $L_{IR}/L_*$ $\geq 10^{-2}$, we were left with 584 stars in the full sample and two stars in the clean sample, removing none of our stars. The reason for this probably is that our initial selection criteria required significant MIR excesses. We address “non-significant” MIR excesses in the following section and again in the discussion (Section 6).

3.4.2. Non-significant MIR Excesses

In studies of disks that are inferred from their MIR excesses, it is common to only select stars with significant excesses, which deviate from the expected photospheric value. However, the distribution of stars with or without excesses is continuous, with a very subtle area between what is considered to have an excess and what is not considered to have an excess. Many of the stars that are not included in the bona fide sample of stars with MIR excesses are indeed stars with excess MIR emission above their photosphoric values. For example, the region between the $2\sigma$ value and our cutoff limit (1.09 $< \sigma' < 3.48$; Figure 4) contains many stars with real excesses and may trace the end of a collisional cascade where the dust is becoming optically thin. The problem is that we cannot confidently identify individual stars that have excesses in this range, since some of the stars in the $1.09 < \sigma' < 3.48$ range are interlopers from the stellar distribution of $\sigma'$. Instead, we can statistically examine this population.

Using the $\sigma'$ distributions (Figure 4), we explored the number of excesses that exist within the non-significant excess region. We fit normal distributions to the core of the $\sigma'$ distributions to minimize effects from the long tail of excess sources (blue line; Figure 4). Next, we subtracted the best-fit normal distribution (scaled from the normalized distribution to the true distribution) interpolated at the mid-point of each bin from the distribution of $\sigma'$ values. The residual histograms are shown in Figure 7. The scatter within the $1\sigma$ range (and to a lesser extent the $2\sigma$ range) can be considered noise since the distribution is not perfectly normally distributed. However, the bumps at $\sigma'$ values greater than $2\sigma$ can be considered real since there is no corresponding scatter at similar negative $\sigma'$ values. These bumps represent real sources harboring MIR excesses.

To quantify the number of potentially missing stars with MIR excesses, we integrated the region between the $2\sigma$ limit (light gray region, $\sigma' = 1.09$ for the full sample and $\sigma' = 0.58$ for the clean sample; Figure 7) and the significant cutoff we imposed (red dotted line, $\sigma' = 3.48$ for the full sample and $\sigma' = 2.53$ for the clean sample; Figure 7). We estimate that ~1400 stars are excluded from the full sample and ~90 stars from the clean sample. However, this assumes that all missing stars are hosts to “extreme MIR excesses.” We computed fractional IR luminosities using the same method from the preceding section, finding that 5.6% of the non-excess stars in the full sample and 0.5% of the non-excess stars in the clean sample hosted extreme MIR excesses. This translates into ~80...
Residual histograms subtracting the best-fit normal distribution from the distribution of \( \sigma' \) values (see Figure 4). The red dotted line represents the cutoff for significance used in identifying MIR excess candidates (Section 3.4). The black dashed line shows the mean of the best-fit normal distribution (the approximate center of the distribution), and the black dashed line denotes a residual value of 0. The gray shaded areas show the 1 and 2\( \sigma \) regions about the mean. Within the 2\( \sigma \) regions, the positive/minus scatter is approximately equal and can be thought of as noise. In the positive region, at values of \( \sigma' \) higher than 2\( \sigma \), there are significant bumps out to the imposed cutoff limit (red dotted line), indicating a large portion of true MIR excesses within this significance region.

and \( \sim 1 \) star(s) missing from the full and the clean samples, respectively. Although we cannot definitively say which stars within this region actually harbor a true MIR excess, it is important to consider this missing population in the context of the frequency of low-mass field stars exhibiting MIR excesses. If we consider the clean sample (as the full sample has a number of inherent biases that we account for in Section 4), then accounting for the missing sources, we estimate the fraction of stars exhibiting a MIR excess is \( \sim 0.05\% \). We discuss this statistic further in Section 6.

3.5. Fidelity of Excesses: Cross-match to Spitzer

To examine the validity of the extreme MIR excess detections, we cross-matched the candidates with the Spitzer Enhanced Imaging Products catalog (this includes both IRAC and MIPS observations). We found ten candidates with Spitzer photometry matched to within 6\( ^\circ \). A search through the literature indicated that none of the Spitzer data for these sources have been published previously. Figure 8 shows the SEDs for these ten matching stars, demonstrating that the Spitzer photometry is consistent with the WISE photometry (for both W3 and W4 detections). All of these stars appear to have true MIR excesses. We are confident that the detected MIR excesses are true excesses originating from their host stars. However, younger populations of stars are expected to exhibit MIR excesses, therefore we must test for youth where available in the samples.

3.6. Spectroscopic Tracers of Youth

One strength of the TW14 sample over the MoVeRS sample is the availability of optical SDSS spectra for each star. This ensured that all objects were low-mass stars and enabled an investigation for youth. TW14 used age diagnostics such as H\( \alpha \) emission to determine that the stars in their sample were older field stars and not young pre-main-sequence stars, the latter of which we expect to host circumstellar disks (and therefore MIR excesses). To examine possible age diagnostics and confirm our selection of low-mass dwarfs for the sample, we identified ten SDSS spectroscopic targets within the extreme MIR excess sample, and received time on the Discovery Channel Telescope (DCT) to obtain optical spectra for 15 additional extreme MIR excess candidates. Unfortunately, none of the spectroscopic subsample overlapped with the stars with Spitzer data (Section 3.5).

TW14 used two age-dependent spectroscopic diagnostics: H\( \alpha \) (e.g., West et al. 2006, 2008) and Li \( \text{I} \) (e.g., Cargile et al. 2010). H\( \alpha \) emission (in addition to other Balmer transitions) is a strong indicator of accretion, resulting in large equivalent width (EW) measurements\(^8\) (EW \( \gtrsim \) 4 A; Barrado y Navascués & Martín 2003) and broad lines (10% widths \( \gtrsim 270 \text{ km s}^{-1} \); White & Basri 2003). Stars exhibiting H\( \alpha \) as a consequence of accretion are also young (<10 Myr) and are typically found in young associations rather than the field.

For older populations of stars \( (\gg 100 \text{ Myr}) \), H\( \alpha \) emission (and other Balmer transitions) is also tied to “magnetic activity,” as strong magnetic fields lead to chromospheric heating (West et al. 2015). West et al. (2008) demonstrated that the lifetime for magnetic activity (as traced through H\( \alpha \) emission) is mass dependent in the M-dwarf regime. For the highest mass M dwarfs, the lifetime for magnetic activity is 500 Myr–1 Gyr, increasing to \( \geq 8 \) Gyr for the lowest mass M dwarfs. This makes H\( \alpha \) emission a moderate age diagnostic for field stars when coupled with stellar mass or spectral type. A lack of detectable H\( \alpha \) emission in the earliest type stars in our sample would indicate a relatively old (>1–2 Gyr) field population. We used the same regions as TW14 to measure the EW of H\( \alpha \) and determine stars for which an EW measurement could or could not be made.

Lithium absorption is more strongly correlated with youth than H\( \alpha \) emission, but it is also mass dependent. Modeling results by Chabrier & Baraffe (1997) demonstrated that the initial lithium abundance will deplete by a factor of 10 in 10 Myr for a 0.7 \( M_\odot \) star (~M0), while a star with a mass of 0.08 \( M_\odot \) (~M8) will take \( \sim 100 \) Myr to deplete by the same factor. This makes Li \( \text{I} \) absorption a strong discriminator of youth.

Because it is difficult to measure the EW of Li \( \text{I} \) (primarily caused by the strong TiO features around Li \( \text{I} \) and typically low S/N), we applied a comparative technique, using SDSS template spectra (Bochanski et al. 2007b), similar to what was done by Cargile et al. (2010). The template spectra from

\(^8\) As is convention in studies of small stars, positive EW measurements indicate emission.
Bochanski et al. (2007b) were built from a composite of SDSS field-star spectra. Therefore they should indicate the baseline shape of the spectrum near the Li I feature for low-mass field stars devoid of Li I absorption. A comparison between the spectra and the Bochanski et al. (2007b) template spectra provides a means to detect Li I absorption without making a direct measurement of the EW. Further details of the method are described in TW14.

We discovered that ten of the extreme MIR excess candidates had been previously observed through one of the SDSS spectroscopic programs and had spectra available. Nine of these stars were included in TW14 because they were part of the

Figure 8. SEDs for all objects with Spitzer detections. For all sources, the WISE and Spitzer photometry agree well, with all stars appearing to have true MIR excesses.
SDSS DR7 spectroscopic sample of M dwarfs (West et al. 2011), and one of the stars was observed after the West et al. (2011) sample was compiled. All ten of these stars are classified as M dwarfs, confirming our selection of low-temperature dwarfs. The radial velocity (RV) corrected SDSS spectra are shown in Figure 9. Only one of these stars (an M7) showed significant Hα emission. The average activity lifetime of an M7 star is $\sim$8 Gyr (West et al. 2008). None of these stars had detectable amounts of lithium. Our Li I analysis sets a lower age limit of $>100$ Myr. The lack of Hα emission for stars earlier than M7 indicates a typical minimum age of $\sim$1 Gyr for the sample (West et al. 2008), indicative of an older field population.

To further assess the age for the sample of extreme MIR excess candidates, we obtained optical spectra with the DeVeny Spectrograph on the 4.3 m DCT for an additional 15 candidates with high-signal MIR excesses ($\sigma > 10$), shown in Figure 10. The spectra cover the range 5600–9000 Å at a resolution of $\lambda / \Delta \lambda \approx 2850$ (2.5 pixel). The candidates were selected based on location in the sky, and should represent a relatively unbiased subsample of the full sample.

Spectra were reduced using a modified version of the pyDIS Python package (Davenport et al. 2016), originally designed for use with the APO 3.5 m Dual Imaging Spectrograph (DIS). Stars were spectral typed using the PyHammer Python package (Kesseli et al. 2017). Although this is a small portion of the total sample, we expect a similar age distribution for the parent population.

The spectroscopic observations collected indicate that the DCT sample also consists of low-temperature stars, further confirming our sample selection. One of the stars (SDSS objID 1237668734684955989; 2MASS J18351414+4026520) has peculiar features. The TiO bands found at 7053 Å are consistent with a cool star, but other features are consistent with a carbon dwarf (dC; Green 2013), while some of the features are not. This object motivates further investigation to determine its true nature. From the full spectroscopic sample of 25 stars, we estimate a contamination rate of 4% for our entire sample due to objects that are not typical low-mass stars.

We observed that only three of the stars for which we have DCT spectra, all within the fully convective regime ($\gtrsim$M4), showed signs of Hα emission. Additionally, none of the stars had detectable amounts of Li I. This lack of Li I absorption is consistent with the stars having ages $\gtrsim$100 Myr estimated from $\sigma^2 > 1.0$.

Figure 8. (Continued.)
the SDSS spectra. Considering the stars without Hα emission, this indicates that the average age of the population is \( \gtrsim 1 \) Gyr (West et al. 2008), again consistent with the findings from the SDSS spectra. Based on the age limits from the two spectroscopic subsamples, we concluded (as did TW14) that the orbiting dust (inferred from the MIR excesses) was not primordial in nature, since the primordial disk is expected to be dispersed on timescales much shorter than the presumed ages.

### 3.6.1. Spectroscopic Estimates of Luminosity Classes

We also estimate the contamination rate of giants in our subsample of the MoVeRS catalog using the collected spectra.

![Scaled and RV-corrected SDSS spectra](image1)

**Figure 9.** Scaled and RV-corrected SDSS spectra. All SDSS spectra appear to be low-mass stars (M dwarfs), confirming our sample selection. The dotted red line indicates the wavelength corresponding to Hα. Only one of the objects has detectable Hα emission, and none of the objects show detectable amounts of Li I absorption. Spectral types using the PyHammer python package (Kesseli et al. 2017) are listed above each spectrum. The large feature commonly found at 5600 Å is an artifact caused by the SDSS spectrograph and is not a real feature (Silvestri et al. 2006; Morgan et al. 2012).

![Scaled and RV-corrected spectra from the DCT](image2)

**Figure 10.** Scaled and RV-corrected spectra from the DCT. The dotted red line indicates the wavelength corresponding to Hα. The top three stars have detectable amounts of Hα emission. Spectral types using the PyHammer python package (Kesseli et al. 2017) are listed above each spectrum. All spectra appear to be M-type stars. The eighth spectrum from the bottom has peculiar characteristics, partially consistent with a cool star and a carbon star (discussed in the text).
Table 3
Spectroscopic Indices

| Index Name | Band (Å) | Continuum (Å) |
|------------|----------|---------------|
| Na I (a)²  | 5868–5918| 6345–6355     |
| Ba t/Fe i/Mn i/Ti t² | 6470–6530 | 6410–6420     |
| CaH²       | 6814–6846| 7042–7046     |
| CaH³       | 6960–6990| 7042–7046     |
| TiO⁵       | 7126–7135| 7042–7046     |
| K t²       | 7669–7705| 7677–7691, 7802–7825 |
| Na I (b)²  | 8172–8197| 8170–8173, 8232–8235 |

Notes.
² Measured as an EW. Linear interpolation is made through the continuum ranges to estimate the continuum.
³ Measured as a band index by calculating the mean flux within each wavelength range and taking the ratio between the band mean flux to the continuum mean flux.

A thorough investigation into separating M-type stars based on luminosity class was undertaken by Mann et al. (2012), using a modified method similar to the method of Gilbert et al. (2006) for Kepler target stars. The spectroscopic features Mann et al. (2012) used for determining luminosity classes included: (1) the CaH2 (6814–6846 Å) and CaH3 (6960–6990 Å) indices (Reid et al. 1995); (2) the Na I doublet (8172–8197 Å; Schiavon et al. 1997); (3) the Ca II triplet (8484–8662 Å; Cenarro et al. 2001); (4) the mix of atomic lines (Ba II, Fe I, Mn I, and Ti I) at 6470–6530 Å (Torres-Dodgen & Weaver 1993); and (5) the K I (7669–7705 Å) and Na I lines identified in Mann et al. (2012). The Ca II triplet falls within a region prone to fringing at the red end of the DCT spectra, therefore we omitted measuring this feature. Most of the spectroscopic features above change with surface gravity and temperature, therefore we compare the above spectroscopic indices against the TiO5 index (Reid et al. 1995), which is sensitive to both metallicity and temperature (Woolf & Wallerstein 2006; Lépine et al. 2007), but relatively insensitive to surface gravity (e.g., Jao et al. 2008). All other aforementioned features were measured using the available SDSS and DCT spectra following the same prescription as outlined in Mann et al. (2012). Table 3 contains the information for the continuum region(s) and band region used to measure EWs and spectral indices.

To determine the expected EWs and spectral indices for low-mass dwarfs, we measured the same features for 38,722 stars from the West et al. (2011) spectroscopic sample of M dwarfs with good photometry (GOODPHOT = 1) and good proper motions (GOODPM = 1). Although some small amount of giant contamination within this sample is expected, it is estimated to be lower than 2%, and the use of good proper motions should further minimize giant contamination. We also obtained optical spectra for 154 giant stars from Flukas et al. (1994), Danks & Dennefeld (1994), Serote Roos et al. (1996), and SDSS. All giant spectra were sampled to the same resolution as our sample spectra before measuring spectroscopic indices to remove any potential bias.

To estimate the likelihood that each star in our sample is either a dwarf or a giant, we built 2D probability distributions for both the dwarf and giant comparison samples for each spectroscopic tracer using a Gaussian kernel density estimation using Silverman’s rule (Silverman 1986), as is shown in Figure 11. The likelihood that source i is a dwarf given spectroscopic index j is estimated by the log-likelihood,

$$L_{ij} = \log_{10} \left( \frac{P_{\text{dwarf}}}{P_{\text{giant}}} \right)$$

The likelihood given all indices that a source is a dwarf versus a giant is

$$\langle L_i \rangle = \frac{\sum_j w_j L_{i,j}}{\sum_j w_j},$$

where $w_j$ is a weighting factor for spectroscopic index j. Mann et al. (2012) found that setting weights to unity (allowing all spectroscopic tracers to be equally weighted) did not significantly alter results. We chose to equally weight all the measured spectroscopic indices, simplifying Equation (5) to $\langle L_i \rangle = \sum_j L_{i,j}$.

Each source was then either assigned to the category of dwarf star ($L_i > 2$), giant star ($L_i < -2$), or undetermined ($-2 < L_i < 2$), based on the 99% confidence that one training set was more likely to host the source. All but one of our sources has a high probability of being a dwarf versus a giant. The earliest type star in our sample has an inconclusive classification, primarily due to all spectroscopic indices for both training sets beginning to converge for the earliest type stars (highest values of TiO5). Given this object’s measured proper motion in multiple catalogs, this is most likely a dwarf star. The inclusion of this object in Gaia DR1 indicates that both a higher precision proper motion measurement and a trigonometric distance are forthcoming, which will definitely determine the luminosity class of this object. We did not attempt to ascribe a luminosity class to our peculiar object due to multiple non-similarities in its spectrum as compared to both our training sets. Based on our above analysis, we do not change our estimated contamination rate of $\sim 4\%$.

3.7. Disk Properties

We can further explore the properties of our extreme MIR excess systems by making some basic assumptions about the disk properties. Dust temperatures allow us to estimate both the orbital distance of the dust and the minimum dust mass. Using the dust grain temperature estimates (Section 3.4.1), we calculated the minimum orbital distance of the dust assuming the dust grains are in thermal equilibrium with the host star, given by

$$D_{\text{min}} = \frac{1}{2} \frac{T_*}{T_0} R_*,$$

where $T_*$ and $T_0$ are the stellar effective temperature and dust grain temperature, respectively, and $R_*$ is the stellar radius. When we assume a simple geometry for the orbiting dust, a dust mass ($M_d$) can be estimated. Similar to TW14, we assumed that the dust is in a thin shell, orbiting at a distance $D_{\text{min}}$ from the host star, with a particulate radius $a$ and density $\rho$, and a cross section equal to the physical cross section of a spherical grain. We take $\langle a \rangle = 0.5\,\mu m$ and $\rho = 2.5\,g\,cm^{-3}$, similar to TW14. The dust mass is then defined as

$$M_d \geq \frac{16}{3} \frac{L_{\text{IR}}}{\lambda(a) D_{\text{min}}},$$

where $L_{\text{IR}}$ contains the information for the continuum region used to measure EWs and spectral indices.
Further details regarding this process can be found in TW14. The orbital distances and dust masses for the extreme MIR excess candidates are shown in Figure 12. The majority of stars harbor dust within 1 au, with the peak of the distribution at a few tenths of an au, within the snow line for low-mass stars (∼0.3 au; Ogihara & Ida 2009). For the majority of our sample, which only have W3 measurements, the dust temperature was assumed to be 317.4 K, which predetermined the estimated orbital distance of the dust to be within the snow line. A colder disk (<317.4 K) would need to be even more massive to have a similar flux level at W3, making it more likely that we are observing a less massive hotter disk. Our dust mass estimates are comparable to those found in TW14, with the median value of $-10^5 \text{Moon}$. Obtaining MIR spectra of these stars with the next generation of telescopes will help to further characterize these dust populations (e.g., constrain mineralogy).

3.8. The Extreme MIR Excess Sample

The general characteristics of our sample of stars with extreme MIR excesses are similar to those from TW14. We show the $r-z$ color distribution, distance distribution, and Galactic spatial distribution of sources in Figure 13. The $r-z$ color distribution peaks at $r-z \approx 2$, which is equivalent to a dM4, which corresponds to the peak of the initial mass distribution ($M_* \approx 0.125 M_\odot$; Baraffe & Chabrier 1996; Chabrier 2003). The distance distribution peaks at approximately 200 pc, which is consistent with other low-mass stellar samples from SDSS (e.g., West et al. 2011).

The candidates are fairly spread out within the SDSS footprint. To test for clumping of objects, we ran a friends-of-friends algorithm to test for spatial groupings within 10 pc of one another (see TW14 for further details). We found ten pairs of stars within 10 pc of each other, with no other groupings larger than two stars. We tested each pair for similar 2D kinematics (they are moving together through the Galaxy) using Equation (6) from Dhital et al. (2010), given by

$$\frac{\Delta \mu_v}{\sigma_{\Delta \mu_v}} + \frac{\Delta \mu_s}{\sigma_{\Delta \mu_s}} \leq 2,$$
where $\Delta \mu_a$ and $\Delta \mu_b$ are the differences between the two proper motion components for each pair, and their uncertainties are the quadrature sum of each individual proper motion uncertainty. The lowest value for this metric among the pairs was 5, indicating that none of these pairs showed similar 2D kinematics. This indicates that these distances are more likely chance alignments than actual physical groupings. The catalog of candidates is available through the online journal, and the column descriptions are listed in Table 4.

3.9. Distance and Color (Temperature) Bias

Because SDSS is a magnitude-limited survey, our selection of stars suffers a distance bias that is dependent on stellar effective temperature. For each stellar temperature range, there will be a minimum and maximum distance over which a dwarf star can be observed because of the saturation and faintness limits of SDSS, respectively. To explore where this bias occurs, we examined the flux ratios ($F_{12\,\mu m,\text{measured}}/F_{12\,\mu m,\text{model}}$) as a function of $r - z$ color and distance (Figure 14). Figure 14 also shows the distance corresponding to the W3 flux limit (730 $\mu$Jy; see Section 3.3.1).

For the full sample, the spread in distances is typically larger than the limit corresponding to the distance at which the photospheric flux level would be detectable at the W3 flux limit (dashed line). This makes many of the stars in the full sample undetectable (at this flux limit) unless they have a MIR excess (assuming no line-of-sight dependence on sensitivity). Figure 14 further illustrates that we can only detect the bluest stars in W3 if they have an extreme MIR excess, since their distances are too large to detect their photospheres at the W3 flux limit. This is true for some of the redder sources as well, but we have the ability to observe many of their photospheres at 12 $\mu$m. As a consequence of the distance spread above the W3 flux limit distance in the full sample, there is a bias for which we must account.

The case is different for the clean sample, where the distance spread for all $r - z$ colors is closer than the distance corresponding to the W3 flux limit. The clean sample should therefore be free of a higher limit distance bias, unlike the full sample, but may suffer from a lower distance limit bias due to saturation. Moreover, the clean sample does not cover the same $r - z$ color range (a proxy for stellar temperature and mass) as the full sample, restricting its use for only mid- to late-spectral type low-mass stars. The distance bias is accounted for using a Galactic model.
### Table 4

**Extreme MIR Excess Candidates Catalog Schema**

| Column Number | Column Description | Units |
|---------------|-------------------|-------|
| 1             | SDSS Object ID    |       |
| 2             | SDSS R.A.         | deg   |
| 3             | SDSS decl.        | deg   |
| 4             | SDSS u-band PSF mag | mag   |
| 5             | SDSS u-band PSF mag error | mag |
| 6             | SDSS u-band extinction | mag |
| 7             | SDSS u-band unreddened PSF mag | mag |
| 8             | SDSS g-band PSF mag | mag |
| 9             | SDSS g-band PSF mag error | mag |
| 10            | SDSS g-band extinction | mag |
| 11            | SDSS g-band unreddened PSF mag | mag |
| 12            | SDSS r-band PSF mag | mag |
| 13            | SDSS r-band PSF mag error | mag |
| 14            | SDSS r-band extinction | mag |
| 15            | SDSS r-band unreddened PSF mag | mag |
| 16            | SDSS z-band PSF mag | mag |
| 17            | SDSS z-band PSF mag error | mag |
| 18            | SDSS z-band extinction | mag |
| 19            | SDSS z-band unreddened PSF mag | mag |
| 20            | SDSS z-band PSF mag | mag |
| 21            | SDSS z-band PSF mag error | mag |
| 22            | SDSS z-band extinction | mag |
| 23            | SDSS z-band unreddened PSF mag | mag |
| 24            | 2MASS J-band PSF mag | mag |
| 25            | 2MASS J-band PSF corr. mag unc. | mag |
| 26            | 2MASS J-band PSF total mag unc. | mag |
| 27            | 2MASS J-band SNR | ... |
| 28            | 2MASS J-band $\chi^2$ goodness-of-fit | ... |
| 29            | 2MASS J-band extinction | mag |
| 30            | 2MASS J-band unreddened PSF mag | mag |
| 31            | 2MASS H-band PSF mag | mag |
| 32            | 2MASS H-band PSF corr. mag unc. | mag |
| 33            | 2MASS H-band PSF total mag unc. | mag |
| 34            | 2MASS H-band SNR | ... |
| 35            | 2MASS H-band $\chi^2$ goodness-of-fit | ... |
| 36            | 2MASS H-band extinction | mag |
| 37            | 2MASS H-band unreddened PSF mag | mag |
| 38            | 2MASS K$_s$-band PSF mag | mag |
| 39            | 2MASS K$_s$-band PSF corr. mag unc. | mag |
| 40            | 2MASS K$_s$-band PSF total mag unc. | mag |
| 41            | 2MASS K$_s$-band SNR | ... |
| 42            | 2MASS K$_s$-band $\chi^2$ goodness-of-fit | ... |
| 43            | 2MASS K$_s$-band extinction | mag |
| 44            | 2MASS K$_s$-band unreddened PSF mag | mag |
| 45            | 2MASS photometric quality flag | ... |
| 46            | 2MASS read flag | ... |
| 47            | 2MASS blend flag | ... |
| 48            | 2MASS contamination and confusion flag | ... |
| 49            | 2MASS extended source flag | ... |
| 50            | WISE W1-band PSF mag | mag |
| 51            | WISE W1-band PSF mag unc. | mag |
| 52            | WISE W1-band SNR | ... |
| 53            | WISE W1-band $\chi^2$ goodness-of-fit | ... |
| 54            | WISE W1-band extinction | mag |
| 55            | WISE W1-band unreddened PSF mag | mag |
| 56            | WISE W2-band PSF mag | mag |
| 57            | WISE W2-band PSF mag unc. | mag |
| 58            | WISE W2-band SNR | ... |
| 59            | WISE W2-band $\chi^2$ goodness-of-fit | ... |
| 60            | WISE W2-band extinction | mag |
| 61            | WISE W2-band unreddened PSF mag | mag |
| 62            | WISE W3-band PSF mag | mag |

### Table 4 (Continued)

| Column Number | Column Description | Units |
|---------------|-------------------|-------|
| 63            | WISE W3-band PSF mag unc. | mag |
| 64            | WISE W3-band SNR | ... |
| 65            | WISE W3-band $\chi^2$ goodness-of-fit | ... |
| 66            | WISE W3-band extinction | mag |
| 67            | WISE W3-band unreddened PSF mag | mag |
| 68            | WISE W4-band PSF mag | mag |
| 69            | WISE W4-band PSF mag unc. | mag |
| 70            | WISE W4-band SNR | ... |
| 71            | WISE W4-band $\chi^2$ goodness-of-fit | ... |
| 72            | WISE W4-band extinction | mag |
| 73            | WISE W4-band unreddened PSF mag | mag |
| 74            | WISE contamination & confusion flag | ... |
| 75            | WISE extended source flag | ... |
| 76            | WISE variability flag | ... |
| 77            | WISE photometric quality flag | ... |
| 78            | Spitzer IRAC Ch1 PSF flux density | µJy |
| 79            | Spitzer IRAC Ch1 PSF flux density unc. | µJy |
| 80            | Spitzer IRAC Ch2 PSF flux density | µJy |
| 81            | Spitzer IRAC Ch2 PSF flux density unc. | µJy |
| 82            | Spitzer IRAC Ch3 PSF flux density | µJy |
| 83            | Spitzer IRAC Ch3 PSF flux density unc. | µJy |
| 84            | Spitzer IRAC Ch4 PSF flux density | µJy |
| 85            | Spitzer IRAC Ch4 PSF flux density unc. | µJy |
| 86            | Spitzer MIPS Ch1 PSF flux density | µJy |
| 87            | Spitzer MIPS Ch1 PSF flux density unc. | µJy |
| 88            | Proper motion in R.A. ($\mu_\alpha \cos \delta$) | mas yr$^{-1}$ |
| 89            | Proper motion in decl. | mas yr$^{-1}$ |
| 90            | Total error in R.A. proper motion | mas yr$^{-1}$ |
| 91            | Total error in decl. proper motion | mas yr$^{-1}$ |
| 92            | Full Sample Flag | ... |
| 93            | Clean Sample Flag | ... |
| 94            | Visual Quality Flag | ... |
| 95            | Photometric distance | pc |
| 96            | Distance from the Galactic plane | pc |
| 97            | $\alpha$ | ... |
| 98            | $T_{eh}$ estimate | K |
| 99            | Upper $T_{eh}$ limit | K |
| 100           | Lower $T_{eh}$ limit | K |
| 101           | Log g estimate | dex |
| 102           | Upper Log g limit | dex |
| 103           | Lower Log g limit | dex |
| 104           | $\chi^2_{12}$ | ... |
| 105           | $\chi^2_{22}$ | ... |
| 106           | $L_{IR}/L_*$ | ... |
| 107           | $D_{min}$ | au |
| 108           | $M_d$ | $M_{Moon}$ |
| 109           | $T_{sp}$ | K |
| 110           | $\sigma_{sp}$ | K |

**Note.**

* Defined in Section 3.4.

(This table is available in its entirety in FITS format.)

#### 4. LoKi Galactic Model: Estimating Stellar Counts and Proper Motions for Completeness

A major limitation of the extreme MIR excess study completed by TW14 was a nonuniform sample and the lack of a method to estimate completeness. To estimate the completeness of the current sample, we used a Galactic model to estimate...
how many stars were missing from the sample (e.g., within a local volume or along a line of sight). Galactic models have been used to simulate stellar densities (e.g., Jurić et al. 2008; van Vledder et al. 2016), kinematics (e.g., Ivezić et al. 2008; Dhillon et al. 2010, 2015; hereafter D10), or both (Robin et al. 2003; Sharma et al. 2011). Galactic models are typically comprised of three main components, the thin disk (cold component), the thick disk (warm component), and the halo. Each component is individually modeled in terms of its mixing fractions and kinematics. We created a model, dubbed the LoKi galactic model, to estimate the total number of stars we would expect to observe within a given volume, and their respective kinematics. The model incorporates a luminosity function (LF; Bochanski et al. 2010) to select stars in proportion to their abundance in the Galaxy, in addition to simulating their positions and kinematics. We ran 100 realizations of the model over the entire simulated Galactic plane, and over time, are dynamically heated away from the plane (e.g., West et al. 2006, 2008). This method of assigning ages to ensembles of stars based on absolute distance from the Galactic plane is commonly referred to as “Galactic stratigraphy” (West et al. 2015).

TW14 identified a weak trend of decreasing MIR excess fractions as a function of increasing stellar age. However, their sample was small and incomplete. To further investigate the findings of TW14, we computed MIR excess fractions using stars with extreme MIR excesses (584 stars in the full sample and two stars in the clean sample, Section 3.4.1; numerator value), and modeled stellar counts (denominator value) over the same volume as the SDSS observations, and with proper motions detectable by MoVeRS (dependent on stellar color and line-of-sight; see Appendix B). Figure 15 shows the methods involved in building and using LoKi are described in detail in Appendix B.

4.1. Extreme MIR Excess Fractions

Using the larger photometric sample from MoVeRS and the LoKi galactic model, we were able to extend the findings of TW14. Using LoKi, we were able to explore the occurrence of extreme MIR excesses as a function of color (a proxy for stellar mass) and Galactic height (a proxy for stellar age). This was done by simulating the total number of stars expected to be observed within the given volume observed by SDSS. These simulations provide stellar counts and Galactic height distributions, which we used to investigate the occurrence of extreme MIR excesses in low-mass stars.

TW14 compared the stars with MIR excesses to the entire W11 catalog to calculate the fraction of stars exhibiting an extreme MIR excess (\(\sim 0.4\%\) of field M dwarfs exhibit an extreme MIR excess), or the “extreme MIR excess fraction” (i.e., the ratio of the number of stars exhibiting an extreme MIR excess to the total number of stars). Using the same parent population selection criteria as TW14 (i.e., using all 390,006 stars with \(J \leq 17\)), we calculated a global extreme MIR excess fraction from the MoVeRS sample of \(\sim 0.1\%\). However, because MoVeRS is not a volume-complete catalog, these fractions are very likely overestimates and need to be corrected using a Galactic model. In addition, as described in Section 3.4.2, we exclude a number of potentially real extreme MIR excesses. Without the ability to determine which of these stars harbor true excesses, as they fall within the statistical scatter of the parent population, the results in this section should be taken as lower limits.

We used the LoKi galactic model to simulate the number of stars expected in the observed footprint (see Appendix B for details) and their distribution in the Galaxy. Using the model, we computed volume-complete fractions, i.e., estimated the denominator value for the number of stars for which we should have been able to detect an extreme MIR excess. We computed the global extreme MIR excess fraction from the model stellar counts using the mean value of the stellar counts across all 100 simulations, estimating an extreme MIR excess fraction of \(\sim 0.02\%\). The model-complete MIR excess fraction is an order of magnitude smaller than that found by TW14, but still orders of magnitude larger than the extreme MIR excess fraction estimated for A-G type stars by Weinberger et al. (2011; \(\sim 0.0007\%\)). We discuss this further in Section 6.

Galactic height is strongly correlated with stellar age for ensembles of stars. The reason is that stars are born close to the Galactic plane, and over time, are dynamically heated away from the plane (e.g., West et al. 2006, 2008). This method of assigning ages to ensembles of stars based on absolute distance from the Galactic plane is commonly referred to as “Galactic stratigraphy” (West et al. 2015).

Figure 14. Distance as a function of \(r - z\) color for the full (top) and clean (bottom) samples. Each bin is 0.1 mag \(\times\) 10 pc, and the color is the mean flux ratio (measurement/model) in the W3 band. The distances are compared to the estimated maximum distance corresponding to the W3 flux limit (see Section 3.3.1) used for the clean sample (red dashed line). For the full sample, there is an inherent bias due to the distances for the bluest stars in the sample, requiring stars to exhibit high MIR excesses to be detected in W3. The clean sample is located much closer (within the bias distance limit), and should not have any significant bias.

\(^{10}\) https://github.com/ctheissen/LoKi
model-corrected extreme MIR excess fractions as a function of absolute distance from the Galactic plane \((Z)\). Each bin has two points corresponding to the 1st and 99th percentile values across all model runs, with error bars representing the greatest and smallest binomial errors between the two percentiles. The fact that much of the sample is not at low Galactic latitudes should result in very few young stars. The estimated ages from Section 3.6 and the results from TW14 suggest that the vast majority of stars within SDSS at high Galactic latitudes are members of the field population \((\gg 100 \text{ Myr})\). Figure 15 shows a declining trend with Galactic height, with the majority of stars with extreme MIR excesses found within 100 pc of the Galactic plane. To assess the statistical significance of this trend, we performed a least-squares linear fit (of the form \(y = mx + b\)) to the average fraction for each bin, weighted by the average binomial uncertainty, finding a slope of \(m = (-6.836 \pm 1.468) \times 10^{-7} \text{pc}^{-1}\). This indicates that younger field populations are more likely to have extreme MIR excesses, and that stars are less likely to host extreme MIR excesses as they age (using “Galactic stratigraphy”; West et al. 2006, 2008). This also indicates that there is some typical age after which the mechanism responsible for creating extreme MIR excesses ceases to act.

TW14 did not attempt to examine a stellar mass dependence with MIR excess fractions. However, with the larger sample of extreme MIR excess candidates and the Galactic model, we were able to examine the MIR excess fractions as a function of \(r - z\) color (a proxy for stellar mass). Figure 16 shows the fraction of stars exhibiting an extreme MIR excess as a function of \(r - z\) color. Again, we fit a linear function to the trend and found a slope of \(m = (1.486 \pm 0.424) \times 10^{-4} \text{pc}^{-1}\), indicating an upward trend. There is a slight distance (and hence age) bias in Figure 16, as bluer stars tend to be at greater distances (older) than redder stars. This effect is due to SDSS observing primarily out of the plane of the Galaxy, which makes distance strongly correlated with vertical distance from the Galactic plane (e.g., see Bochanski et al. 2010). Furthermore, the vertical distribution of stars from the Galactic plane is strongly correlated with vertical distance (Ma et al. 2016), with older stellar populations found farther from the Galactic plane on average. Considering the upward trend with redder colors, this is consistent with Figure 15, as younger stellar populations tend to have larger extreme MIR excess fractions.

To minimize selection effects and explore the interplay among extreme MIR excess fractions, stellar age, and stellar mass, we examined extreme MIR excess fractions as a function of absolute distance from the Galactic plane binned in three \(r - z\) color regimes (Figure 17). The first bin \((0.5 \leq r - z < 2)\) potentially suffers from selection effects due to the inherently large distances to these objects, dictated by the saturation limit of SDSS (see Figure 14), placing the majority of observed stars farther away from the Galactic plane (76% with \(|Z| > 200 \text{ pc}\)). Although the model attempts to recover some fraction of these stars, we implemented the same magnitude and proper motion cuts on the model sample, therefore both the model and our sample will suffer from a similar selection effect. The intermediate-mass stars within the sample \((2 \leq r - z < 3.5)\) show a slight trend with \(|Z|\), and these bins are likely to be relatively free of the selection effects affecting the other mass bins. The lowest mass bin \((3.5 \leq r - z < 5)\) has very few sources and most likely does not sample a large enough volume to detect MIR excesses if excesses occur at similar rates across all stellar masses. The measured best-fit slopes for all three color bins from bluest to reddest are \(m = (-4.254 \pm 0.788) \times 10^{-7} \text{pc}^{-1}\), \(m = (-2.683 \pm 1.389) \times 10^{-6} \text{pc}^{-1}\), and \(m = (-3.358 \pm 16.809) \times 10^{-6} \text{pc}^{-1}\).

5. Non-significant MIR Excesses Revisited: A Further Investigation into Timescales

The strong trend of decreasing extreme MIR excess fraction with Galactic height indicates a trend with stellar age and
motivates further investigation. To explore if the overall distribution of non-significant excess sources changes as a function of age, we examined the $\sigma'$ distribution as a function of $|Z|$ for the full and clean samples, using stars with $2.0 \leq r - z < 3.5$ to minimize selection effects that are due to distance. Figure 18 shows how the distribution of $\sigma'$ changes as a function of $|Z|$.

To assess if there is a significant difference between the distributions in the full and clean samples, we investigated the skew of each sample distribution. The underlying hypothesis is that all samples come from a nearly Gaussian parent distribution, with the stars with excess skewing that parent population to more positive $\sigma'$ values. To statistically assess the skew of each distribution, we took 100,000 bootstrap samples of each distribution and measured the skew of the resulting distribution. We report the mean values along with the 68% (16th and 84th percentiles) and 95% (2.5th and 97.5th percentiles) confidence intervals in Table 5. The full sample shows a trend toward more excess sources (larger skewness) at farther distances away from the Galactic plane. This is most probably because we preferentially select stars with excesses at larger distances.

The clean sample should be devoid of selection effects associated with distance at the expense of a smaller spread in Galactic height. In Figure 18 we see a decrease in the number of high $\sigma'$ sources (MIR excess sources) at higher Galactic heights, which is also illustrated by the decreasing skew in Table 5, although the observed decrease is a tentative result. The decrease in skewness would be consistent with there being an age evolution in all of the stars with MIR excesses, not only stars exhibiting extreme MIR excesses.

6. Conclusions and Discussion

The large sample of low-mass stars contained within the MoVeRS catalog has allowed us to compile the largest sample of low-mass field stars exhibiting large MIR excesses to date (584 stars). We examined the dependence of MIR excess occurrence with stellar mass (using $r - z$ color as a proxy), and stellar age (using Galactic height as a proxy). The sample is divided into a

![Figure 17](image1.png)

**Figure 17.** Fraction of stars exhibiting an extreme MIR excess as a function of absolute distance from the Galactic plane in color bins. We see a declining trend in MIR excess fractions with Galactic height. The bluest bin suffers from a selection effect because the majority of these stars is located at relatively large distances (which is strongly correlated with distance from the Galactic plane), which is why we are missing many of the stars that actually reside close to the Galactic plane due to the saturation limits of SDSS. The reddest bin does not sample a large enough volume to detect a larger number of stars with MIR excesses if they occur at similar rates across the stellar mass regime.

![Figure 18](image2.png)

**Figure 18.** Normalized distributions of $\sigma'$ values as a function of $|Z|$ for stars with $2.0 \leq r - z < 3.5$. Dashed lines, dotted lines, and shaded regions are the same as Figure 7. The nearest bin (0–30 pc) has been omitted because of a bias from the SDSS saturation limit. The full sample shows a slight shift to higher $\sigma'$ values at larger Galactic heights. This is most likely due to a bias because fewer stars without MIR excesses are detectable in W3 at distances greater than 100 pc. The clean sample shows the longest tail for the 30–60 pc bin, indicating a possible dependence on age for the stars with lower MIR excesses.
### Table 5
Sample Skewness

| Sample | Distance Range | Skewnessa |
|--------|----------------|-----------|
| Full   | 30–60 pc       | 0.74±0.08|0.15 |
| Full   | 60–90 pc       | 0.85±0.07|0.13 |
| Full   | 90–120 pc      | 1.07±0.07|0.14 |
| Clean  | 30–60 pc       | 0.39±0.09|0.17 |
| Clean  | 60–90 pc       | 0.34±0.07|0.13 |
| Clean  | 90–110 pc      | 0.19±0.08|0.16 |

Note.

a Confidence intervals correspond to the 68% confidence and the 95% confidence (inside parenthesis).

“full” sample (584 stars), consisting of stars with high-fidelity high-sensitivity MIR excess detections, and a “clean” sample (two stars), which also contains high-fidelity high-sensitivity stars with excesses, but is magnitude (volume) limited.

To build the samples, we implemented cuts to ensure relatively bright sources with high S/N WISE observations. These stars were then visually inspected to reduce contaminants (e.g., crowded fields). The final samples, including both stars with and without excesses, were made up of 20,502 stars (full sample; 584 stars with extreme MIR excesses) and 5786 stars (clean sample; two stars with extreme MIR excesses). Stars with extreme MIR excesses were selected using modified empirical criteria from TW14. A cross-match to the Spitzer Enhanced Imaging Products catalog identified ten stars and verified the WISE MIR excesses. The full sample covers the range 0.5 \( \leq r - z < 5 \), covering all spectral subtypes within the M-dwarf regime \((0.1 M_\odot \leq M_* \leq 0.7 M_\odot)\). The clean sample is biased toward later spectral type stars \((2 \leq r - z < 5; \ 0.1 M_\odot \leq M_* \leq 0.35 M_\odot)\), and was chosen to minimize biases that are due to distance or magnitude and WISE sensitivity.

Spectroscopic observations of 25 stars in the sample taken by SDSS and using the DCT support the hypothesis that the sample is made up of field stars and confirms the selection of M dwarfs, although one star has characteristics similar to a carbon dwarf, indicating a contamination rate of \(~4\%)\. Many carbon stars are known to show evidence of circumstellar material (Green 2013), potentially making us more likely to select for them in this study, and indicating that the contamination rate for the MoVeRS catalog is likely much lower than \(~4\%). For the remainder of the stars with spectra, the vast majority lack H\(\alpha\) emission, consistent with an inactive older \((\geq 100 \text{ Myr})\), field population. Furthermore, none of the stars have measurable Li\(1\) absorption, which is expected for stars with ages \(< 100 \text{ Myr}\). Since the magnetic activity lifetimes of lower mass stars are one to several Gyr and none of the stars had detectable Li\(1\) absorption, the parent population most likely has an average age \(> 1 \text{ Gyr}\). The samples and their derived quantities are available in the electronic format of this manuscript.

Our primary finding is the strong correlation with the fraction of field stars exhibiting an extreme MIR excess as a function of absolute distance from the Galactic plane. Although the bins with higher mass stars suffer selection effects and are biased toward stars farther away from the Galactic plane (because of the brightness of these stars and the saturation limits of SDSS) and the lowest mass stars are biased toward extremely close distances and therefore small volumes, we find a significant decreasing trend for stars with MIR excesses at larger Galactic heights, specifically in the intermediate-mass stars, which are largely unbiased. These data strongly support an age dependency on the presence of extreme MIR excesses. We also find that MIR excesses have a correlation with \(r - z\) color, indicating a possible dependence with stellar mass.

Giants collisions between large planetimals or terrestrial planets are expected to create a collisional cascade that may last for \(~100,000\) years (Weinberger et al. 2011). If we assume a typical stellar age for the sample of 1 Gyr and a timescale over which a MIR excess can be detected of 0.1 Myr, then only 0.01% of the sample should show a detectable excess, which reduces to \(~0.5\) stars for the clean sample and is roughly consistent with our findings. This is assuming a volume-complete sample and the ability of the mechanism that creates MIR excesses to act at any time during the lifetime of the star. Limiting the timescale over which the mechanism can act (to shorter than 1 Gyr) or increasing the lifetime of the collisional products would increase the number of predicted stars that are observed to have an extreme MIR excess. Although we are unable to link a distinct timescale over which a collision may occur, our findings are consistent with a short lifetime for the collisional cascade to create enough dust for a significant MIR detection. Additionally, multiple collisions can extend the lifetime of the collisional products past 100,000 years.

Using the clean sample, which is relatively unbiased and complete, we reinvestigated the collision rate found in TW14. The estimated fraction of stars undergoing collisions is \((3.5 \pm 1.7) \times 10^{-4}\), an order of magnitude smaller than the TW14 value. However, when we consider the different selection criteria for the parent population (34%, from Section 3.4) and the more stringent criteria applied for a star to be included in the extreme MIR excess sample (16%, from Section 3.4), we find that the TW14 fraction of 0.4% is reduced to 0.02%, consistent with this study. This fraction is still two orders of magnitude larger than the number estimated \((\sim 7 \times 10^{-6})\) by Weinberger et al. (2011) for A–G spectral type stars. Our updated fraction gives us a collision rate of about nine impacts per star up to its current age. This value is consistent with the findings of TW14 that planetary collisions occur more frequently around low-mass stars.

Investigating the continuous distribution of stars with excess MIR flux versus simply the high-sensitivity sample, we estimate that there are potentially 80 stars with actual extreme MIR excesses excluded from our full sample, and 1 star excluded from the clean sample. Non-extreme MIR excesses may represent the more evolved state of the aforementioned collisional disks, at the end of the lifetime for a collisional cascade where the disk is becoming optically thin, or perhaps smaller collisions. The addition of these stars would imply that the estimated fraction of stars undergoing collisions is underestimated by a factor of \(~4\), which indicates that collisions may be even more frequent in low-mass stellar systems.

Planetary collisions have also been put forth to explain a dichotomy found in the Kepler data. Kepler has found a wealth of planetary systems around low-mass stars, both singly transiting systems and multi-transiting systems. Numerous studies have used ensemble statistics to reproduce Kepler multi-planet observations with success (Lissauer et al. 2011; Fang & Margot 2012; Tremaine & Dong 2012; Fabrycky et al. 2014). However, as noted by Lissauer et al. (2011), the
best-fitting models underpredict the number of observed singly transiting systems by a factor of ~2. Lissauer et al. (2011) postulate that a second population of systems with higher inclination dispersions and/or lower multiplicities may explain the dearth of singly transiting systems. This proposed dual population has become known as the “Kepler dichotomy.”

Recently, Ballard & Johnson (2016) simulated planetary systems with a range of mutual inclinations and multiplicities to replicate Kepler results for the M-dwarf population. Ballard & Johnson (2016) found that a high multiplicity ($N \approx 7$ planets per star) with a typical mutual inclination of $2^\circ$ could produce a planetary population in good agreement with the Kepler multi-planet yield, both with and without invoking a range of eccentricities. Ballard & Johnson (2016) accounted for the dearth of singly transiting systems by invoking a second population of planetary systems, either with a single planet, or with two to three planets and a large scatter in mutual inclination ($4^\circ$–$9^\circ$). The best mixture between these two populations was found to be $\sim 50\%$.

Ballard & Johnson (2016) discuss two possible explanations for the Kepler dichotomy, initial formation conditions and dynamical disruption. In the former of these scenarios, Johansen et al. (2012) posit that for the case of solar-mass stars, the formation, migration, or scattering of a giant planet could suppress planet formation. This is a scenario similar to the Grand Tack model (Walsh et al. 2011), which was put forward to explain the anomalously low mass of Mars in our own solar system. However, the lack of massive planets found orbiting most low-mass stars makes this an unlikely scenario. Moriai & Ballard (2016) used $N$-body simulations of late-stage planet formation to attempt to reproduce Kepler observations and found that two separate disk surface mass densities could reproduce the dichotomy. However, it is unclear if two distinct surface density profiles are observationally motivated.

Dynamical disruption as an explanation for the Kepler dichotomy has also been explored through the use of models. Simulations of tightly packed planetary systems (Pu & Wu 2015; Volk & Gladman 2015) have shown that coplanar high-multiple planetary systems are metastable and are disrupted on Gyr timescales. Furthermore, in systems that experience dynamical instability, the most likely outcome is two planets colliding once they are excited to crossing orbits (Pu & Wu 2015). Such collisions would likely result in massive amounts of orbiting dust and potentially in planets scattered to higher inclinations. Combined with the findings of Quintana & Barclay (2016), that suppression of giant planets can extend the timescale over which collisions can occur to Gyr, late-time occurring giant impacts are a plausible explanation for the Kepler dichotomy.

Our observed extreme MIR excesses support the hypothesis that the Kepler dichotomy arises from late-occurring ($\sim 1$ Gyr) giant impacts due to dynamical disruption. Planetary collisions between orbiting planets with small semimajor axes would produce the massive dust populations inferred from these extreme MIR excesses. The high frequency of these impacts (relative to higher mass stars) has strong implications on the habitability of terrestrial planets around low-mass stars. This analysis motivates the search for similar extreme MIR excesses in higher and lower mass stellar populations.

The upcoming Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2014) will be instrumental in testing the evolution versus formation hypothesis for the Kepler dichotomy through a larger sample of low-mass stars than Kepler observed. TESS, and to a lesser extent the Kepler two-wheel mission (K2), will sample a larger distribution in Galactic height and rotation periods (both tracers of stellar age) to further estimate the timescale over which planetary collisions occur. Additionally, the upcoming James Webb Space Telescope (JWST; Gardner et al. 2006) will allow us to constrain the mineralogy of the disks detected with WISE, which can distinguish disks formed through violent collisions versus disks made of differentiated bodies, such as asteroids.

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![Figure 19. Effective temperature as a function of $r-z$ color from the MCMC estimation. Each bin is 0.1 mag x 100 K. Typical errors are shown in the bottom right corner. We plot the best-fit sixth-order polynomial along with relationships from the Dartmouth Stellar Evolution Database (Potter et al. 2008; Feiden & Chaboyer 2013) and Mann et al. (2015). Most relationships fail to replicate the reddest or coolest end of the main-sequence.](image)

Figure 19. Effective temperature as a function of $r-z$ color from the MCMC estimation. Each bin is 0.1 mag x 100 K. Typical errors are shown in the bottom right corner. We plot the best-fit sixth-order polynomial along with relationships from the Dartmouth Stellar Evolution Database (Potter et al. 2008; Feiden & Chaboyer 2013) and Mann et al. (2015). Most relationships fail to replicate the reddest or coolest end of the main-sequence.

Appendix A

Estimating Stellar Parameters

A.1. Markov Chain Monte Carlo Method for Stellar Parameters

We calculated the parameters of the orbiting dust ($D_{\text{dust}}$ and $M_{\text{dust}}$) using our estimates of the fundamental stellar parameters ($T_{\text{eff}}$ and $R_*$). We estimated stellar parameters using the BT-Settl models with solar abundances from Caffau et al. (2011) and mixing lengths calibrated on 2D or 3D radiative hydrodynamic simulations (CIFIST2015; Freytag et al. 2010, 2012; Baraffe et al. 2015). These models span temperatures ranging between 1200 and 7000 K in steps of 100 K or 50 K, dependent on surface gravity, and log $g$ values between 2.5 and 5.5 in steps of 0.5 dex, with metallicities and alpha abundances set to solar values. Using a previous version of the CIFIST models, Mann et al. (2013) found that the deviation between temperatures based on model comparisons to optical spectra and those derived empirically was 57 K.

To produce the best model fits to stellar data requires probing parameter space to fit for $T_{\text{eff}}$, [M/H], log $g$, $\alpha$-abundance, and the normalizing factor in the form of the square of the ratio of the stellar radius over the distance (i.e., $R_*/L_*/d^2$). To reduce the parameter space for fitting models to the millions of stars in the MoVeRS sample, a few basic assumptions were made that should not overly bias our results. Metallicity was set to solar abundances, removing this parameter from the search space. To further reduce the complexity of the algorithm, the normalization factor was removed from the parameter space by scaling the model fluxes to the measured $z$-band values (a similar process was used in TW14 using the $K_s$ band), leaving only two parameters for which to solve ($T_{\text{eff}}$ and log $g$).

We used the emcee package (Foreman-Mackey et al. 2013), a Python implementation of the Goodman & Weare (2010) affine invariant sampler, to explore the remaining stellar parameter space. Since the BT-Settl models are not continuous across the parameter space, we interpolated between grid points using a nearest-neighbor method for model selection. For each step in the MCMC, the log-likelihood is given as

$$\ln \mathcal{L}(\mathbf{X}, \sigma) = -\frac{1}{2} \sum_{n=1}^{N} \left( \frac{\Theta_n - X_n}{\sigma_n} \right)^2 + \ln(2\pi\sigma_n^2)$$

where $\Theta$ is a vector of length $N$ containing the model predicted, scaled fluxes for a given set of stellar parameters ($T_{\text{eff}}$ and log $g$), $\mathbf{X}$ is a vector containing the observed fluxes, $\sigma$ is a vector containing the measurement errors for the observed fluxes, and the length $N$ pertains to the number of bands in which data were available. Uniform priors were chosen across the parameter space, and we assumed all the parameters were normally distributed.

Instead of collecting the entire posterior probability distributions for each of the stars, we calculated the 16th, 50th, and 84th percentiles of the distributions for both $T_{\text{eff}}$ and log $g$. We plot the 50th percentile values as a function of $r-z$ color in Figure 19. The $T_{\text{eff}}$ estimates follow the expected trend with $r-z$ color. The width of the distribution is very likely due to different metallicity classes (Mann et al. 2015; Schmidt et al. 2016). Using an $F$ test, we compared different order polynomial relationships and found the best-fit to the observed trend between $T_{\text{eff}}$ and $r-z$ was a 6th order polynomial,

$$T_{\text{eff}} = a + bX + cX^2 + dX^3 + eX^4 + fX^5 + gX^6,$$

where the coefficients are listed in Table 6. We find good agreement between our relationship and Mann et al. (2015), except for the extremes where the Mann et al. (2015) fits are not well constrained.

$$\ln \mathcal{L}(\mathbf{X}, \sigma) = -\frac{1}{2} \sum_{n=1}^{N} \left( \frac{\Theta_n - X_n}{\sigma_n} \right)^2 + \ln(2\pi\sigma_n^2)$$
for hotter temperatures, but the relationship (plot our best-fit sixth-order polynomial, only for the color range over which the B10 relationships are valid. The scatter at the red end is most likely an artifact of extrapolating the B10 photometric parallax relationship, which is not well constrained for the reddest stars.

The relationship between effective temperature and stellar radii using our polynomial equations is shown in Figure 21. The relationship follows similar trends to both the relationship by Mann et al. (2015) and Boyajian et al. (2012). The upturn in radii at cooler temperatures is an artifact of the B10 photometric parallax relationship, which is not well constrained for the reddest stars.

**Appendix B**

Low-mass Kinematics (LoKi) Galactic Model

**B.1. Stellar Density Profile**

We implemented a galactic model framework similar to that used in Dhital et al. (2010). In the model, the stellar density for each galactic component is given in terms of standard galactic coordinates. For the thin (cold component) and thick (warm component) disks, the stellar density profiles are given by

\[
\rho_{\text{thin}}(R, Z) = \rho(R_0, 0) \exp \left(-\frac{|Z|}{H_{\text{thin}}}ight) \times \exp \left(-\frac{|R - R_0|}{L_{\text{thin}}}ight),
\]

\[
\rho_{\text{thick}}(R, Z) = \rho(R_0, 0) \exp \left(-\frac{|Z|}{H_{\text{thick}}}ight) \times \exp \left(-\frac{|R - R_0|}{L_{\text{thick}}}ight),
\]

where \(H\) is the scale height above and below the plane, and \(L\) is the scale length within the plane. The halo stellar density is expressed as a biaxial power-law ellipsoid,

\[
\rho_{\text{halo}}(R, Z) = \rho(R_0, 0) \left(\frac{R_0}{\sqrt{R^2 + (Z/q)^2}}\right)^{r_{\text{halo}}},
\]

where \(q\) is the halo flattening parameter, and \(r_{\text{halo}}\) is the halo density gradient. In each of the above formulas, \(R\) is the Galactic radius, \(R_0\) is the Sun’s distance from the Galactic center (8.5 kpc), and \(Z\) is the Galactic height. To obtain the total stellar density at a specific radius and height in the Galaxy, all three density profiles weighted by the fraction of all stars in each component are summed,

\[
\rho(R, Z) = f_{\text{thin}} \cdot \rho_{\text{thin}}(R, Z) + f_{\text{thick}} \cdot \rho_{\text{thick}}(R, Z) + f_{\text{halo}} \cdot \rho_{\text{halo}}(R, Z),
\]

A.2. Estimating Stellar Radii

Stellar radii can be inferred using distance estimates (Section 3.2) and the scaling factor of the best-fit model to the measured photometry (see Section 3.1 and Cushing et al. 2008). Figure 20 shows the estimated stellar radii as a function of \(r - z\) color. We again fit a polynomial relationship between \(R_{\ast}\) and \(r - z\) color and find a sixth-order polynomial provides the best-fit (using an \(F\) test). Our polynomial relationship is shown in Figure 20 and described by an equation similar to Equation (10), with coefficients listed in Table 6.

The scatter in the data is most likely an artifact of extrapolating the B10 photometric parallax relationship, which is not well constrained for the reddest stars.

**Table 6**

| Polynomial Relationship Coefficients |
|--------------------------------------|
| \(Y\) | \(X\) | \(a\) | \(b\) | \(c\) | \(d\) | \(e\) | \(f\) | \(g\) | \(\sigma\) | \(\chi^2\) | Range |
|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|--------|------|
| \(T_{\text{eff}}\) (K) | \(r - z\) | 6691.90 | -6000.26 | 5135.52 | -2513.18 | 679.434 | -94.2185 | 5.18804 | 47.41 | 1.39 | 0.5 \(\leq r - z \leq 4.84\) |
| \(R_{\ast}(R_\odot)\) | \(r - z\) | 0.41895 | 1.3345 | -1.9848 | 1.1474 | -0.34214 | 0.052184 | -0.0032136 | 0.027 | 0.022 | 0.9 \(\leq r - z \leq 4.30\) |

Figure 20. \(R_{\ast}\) as a function of \(r - z\) color from the MCMC estimation. Each bin is 0.1 mag \(\times 0.01\) \(R_\odot\). Typical errors are shown in the top right corner. We plot our best-fit sixth-order polynomial, only for the color range over which the B10 relationships are valid. The scatter at the red end is most likely an artifact of extrapolating the B10 photometric parallax relationships to redder colors.

Figure 21. \(R_{\ast}\) as a function of \(T_{\text{eff}}\). The red line shows the relationship using the polynomial values from Table 6. Comparison with the Mann et al. (2015) relationship (cyan line) and Boyajian et al. (2012) relationship (yellow line) shows an offset of \(<0.05\) \(R_\odot\) for hotter temperatures, but the relationship converges with Mann et al. (2015) at cooler temperatures. The Mann et al. (2015) and Boyajian et al. (2012) relationships were calibrated using nearby stars, and thus the observed offset in the relationships for hotter stars could be due to SDSS sampling a less active and/or lower metallicity stellar population, or possible extinction effects. The upturn for the coolest stars is due to extrapolating the B10 polynomial to redder colors.

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The relationship between effective temperature and stellar radii using our polynomial equations is shown in Figure 21. The relationship follows similar trends to both the relationship by Mann et al. (2015) and Boyajian et al. (2012). The upturn in radii at cooler temperatures is an artifact of the B10 photometric parallax relationship, which is not well constrained for the reddest stars.

Appendix B

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\]

\[
\rho_{\text{thick}}(R, Z) = \rho(R_0, 0) \exp \left(-\frac{|Z|}{H_{\text{thick}}}ight) \times \exp \left(-\frac{|R - R_0|}{L_{\text{thick}}}ight),
\]

where \(H\) is the scale height above and below the plane, and \(L\) is the scale length within the plane. The halo stellar density is expressed as a biaxial power-law ellipsoid,

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\]

where \(q\) is the halo flattening parameter, and \(r_{\text{halo}}\) is the halo density gradient. In each of the above formulas, \(R\) is the Galactic radius, \(R_0\) is the Sun’s distance from the Galactic center (8.5 kpc), and \(Z\) is the Galactic height. To obtain the total stellar density at a specific radius and height in the Galaxy, all three density profiles weighted by the fraction of all stars in each component are summed,

\[
\rho(R, Z) = f_{\text{thin}} \cdot \rho_{\text{thin}}(R, Z) + f_{\text{thick}} \cdot \rho_{\text{thick}}(R, Z) + f_{\text{halo}} \cdot \rho_{\text{halo}}(R, Z),
\]
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Component & Parameter & Description & Value \\
\hline
Thin disk & $f_{\text{thick}}$ & Fraction & $1 - f_{\text{thick}} - f_{\text{halo}}$ \\
& $H_{\text{thick}}$ & Scale height & 300 pc \\
& $l_{\text{thick}}$ & Scale length & 3100 pc \\
\hline
Thick disk & $f_{\text{thick}}$ & Fraction & 0.04 \\
& $H_{\text{thick}}$ & Scale height & 2100 pc \\
& $l_{\text{thick}}$ & Scale length & 3700 pc \\
\hline
Halo & $f_{\text{halo}}$ & Fraction & 0.0025 \\
& $r_{\text{halo}}$ & Density gradient & 2.77 \\
& $q = c/a$ & Flattening parameter & 0.64 \\
\hline
\end{tabular}
\caption{Galactic Model Parameters}
\end{table}

Notes. The parameters were measured using M dwarfs for the disk (bias-corrected values; B10) and MS turn-off stars for the halo (Ivezić et al. 2008) in the SDSS footprint.

\begin{itemize}
\item[a] Evaluated in the solar neighborhood.
\item[b] Assuming a biaxial ellipsoid with axes $a$ and $c$.
\end{itemize}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$r - z$ & $M_r$ & $\rho(R_0, 0)$ & Distance \\
& & (stars pc$^{-3}$) & (pc) \\
\hline
[0.5, 1.0) & [6.52, 8.01) & [0.000287, 0.000289] & [390, 1100] \\
[1.0, 1.5) & [8.01, 9.59) & [0.000257, 0.000259] & [215, 780] \\
[1.5, 2.0) & [9.59, 11.18) & [0.000677, 0.000680] & [90, 520] \\
[2.0, 2.5) & [11.18, 12.74) & [0.01005, 0.01010] & [60, 345] \\
[2.5, 3.0) & [12.74, 14.19) & [0.00657, 0.00660] & [35, 240] \\
[3.0, 3.5) & [14.19, 15.46) & [0.000489, 0.000493] & [15, 165] \\
[3.5, 4.0) & [15.46, 16.50) & [0.00461, 0.00464] & [10, 125] \\
(4.0, 4.5) & [16.50, 17.50) & [0.00143, 0.00146] & [10, 105] \\
(4.5, 5.0) & [17.50, 18.50) & [0.000886, 0.000889] & [10, 105] \\
\hline
\end{tabular}
\caption{Galactic Model Input Ranges}
\end{table}

Notes. 

\begin{itemize}
\item[a] This color range falls outside the limits of the B10 $M_r(r-z)$ relationship.
\item[b] Values estimated from the 5 Gyr isochrone from Baraffe et al. (1998).
\end{itemize}

with $f_{\text{thin}} + f_{\text{thick}} + f_{\text{halo}} = 1$. The local stellar density scaled to the Galactic plane, $\rho(R_0, 0)$, was obtained by integrating the bias-corrected single-star LF from B10 for low-mass stars from SDSS. Table 7 contains the adopted disk parameters for the model.

\section*{B.2. Stellar Densities and Distance Ranges}

Perhaps the most fundamental parameter required in the model is the local stellar density. Many studies have measured the local stellar density $\rho(R_0, 0)$, scaled to the Galactic plane (Jurić et al. 2008; Bochanski et al. 2010; van Vledder et al. 2016). Stellar number densities are commonly estimated through LFs (e.g., Cruz et al. 2007; Bochanski et al. 2010). We used the low-mass LF from B10 since the MoVeRS catalog (and hence, the sample) is built from the same photometric criteria used to create the B10 LF. However, as stated above, the B10 photometric parallax relationships extend to absolute magnitudes fainter than the B10 LF, therefore care must be taken in obtaining stellar densities for the reddest stars.

The B10 LFs are given for both $M_r$ and $M_J$. $M_J$ is a commonly used metric for the LF function, but the B10 photometric parallax relationships map SDSS colors to $M_J$. Bochanski (2008) gives a relationship between $M_r$ and $M_J$ that extends two magnitudes fainter in $M_r$ than the B10 $M_J$ LF. The Bochanski (2008) relationship also reaches $M_J \approx 12$, which is also two magnitudes deeper than the B10 $M_J$ LF ($M_J \lesssim 10$).

Using the $M_J$ LF from Cruz et al. (2007), which begins where the B10 $M_J$ LF ends, we mapped fainter $M_J$ magnitudes to $M_J$ magnitudes (using the Bochanski 2008 relationship), and estimated stellar densities past the limits of the B10 LFs. The stellar densities are shown in Table 8.

The distance ranges are dictated by both the SDSS saturation limits and the maximum distance at which we would find an extreme MIR excess. For the lower distance limits, we binned the MoVeRS sample in 0.5 mag bins in $r - z$ color and used the minimum distance value in each bin for the lower limit. The upper distance limit corresponds to the maximum MIR excess value above the photospheric value, since we can see an extremely high excess out to a farther distance than a lower MIR excess. Figure 14 shows the distribution of MIR excess values above the photosphere, and we found that 95% of the excesses had values up to 12 times the photospheric value. Using Equation (1) scaled $\sim 2.7$ mag fainter (12 times greater than the expected photospheric flux), we derived new distance limits using the B10 photometric parallax relationships. The distance limits are shown in Table 8. Since the B10 $M_r$ photometric parallax relationship did not extend to the reddest $r - z$ values as the sample, we used the Baraffe et al. (1998) 5 Gyr relationship between $4 < r - z < 5$. The Baraffe et al. (1998) model photometric parallax relationship is consistent with other photometric parallax relationships (Hawley et al. 2002; West et al. 2005) to the reddest $r - z$ extent that it can be compared to empirical data (see B10 Figure 9).

\section*{B.3. Stellar Kinematics}

Stellar kinematics are much more difficult to constrain than stellar densities, in part because of the difficulty in obtaining 3D kinematics of stars. Many studies have measured the mean velocities of stars as a function of Galactic height and the velocity dispersions for the thin (cold component) and thick (warm component) disks, along with the halo (e.g., West et al. 2006; Bochanski et al. 2007a; Jurić et al. 2008; Pineda et al. 2017). An in-depth prescription of the kinematical model we used can be found in D10. Here we summarize the model...
where the values of $k$ and $n$ are defined in Table 9. For halo stars, we used velocity dispersion values from Bond et al. (2010), using the dispersion relations taken at the Galactic plane ($Z = 0$ pc). These velocity distributions can then be sampled to obtain expected galactic cylindrical $V_r$, $V_\phi$, and $V_Z$ velocity distributions for samples of stars at any location in the Galaxy. These $V_r$, $V_\phi$, and $V_Z$ velocities can be transformed into $U/V/W$ velocities, which can then be transformed into proper motions and radial velocities using the methods of Johnson & Soderblom (1987).

B.4. Model Comparisons: SDSS Source Counts

To assess the validity of the model, we compared stellar counts from the model against counts from SDSS for all objects with colors similar to those expected for low-mass stars. Specifically, we obtained source counts for $1^\circ \times 1^\circ$ size bins within the entire SDSS footprint and required the following criteria (taken from Bochanski et al. 2010):

1. Objects were PRIMARY sources within the PHOTOOBJALL table (MODE = 1),
2. Objects had point-source-like morphologies within the PHOTOOBJALL table (TYPE = 6),
3. $i < 22$,
4. $z < 21.2$,
5. $r - i > 0.3$,
6. $i - z > 0.2$, and
7. $16 < r < 22$.

To compare SDSS source counts to the model, we integrated the $B10 M/L$ LF to obtain a total stellar density. Next, we integrated the model in $1^\circ \times 1^\circ$ size bins out to a distance of 2 kpc, the estimated depth of the $B10 M/L$. A comparison between the stellar counts and SDSS source counts is shown in Figure 22. The model has better than 90% agreement with SDSS at high Galactic latitudes. The model produces more stars in regions at the edges of the SDSS stripes, where we expect SDSS to be incomplete. Close to the Galactic plane, SDSS has a much higher number of sources. This is most likely due to bluer sources that are reddened and pulled into the color selection criteria from the higher extinction environment. Considering that the input parameters for the model are based on SDSS data, it is not surprising that the model and SDSS source counts agree to such a high degree. Further comparisons must be made with independent observations to verify the model.

B.5. Model Comparison: RECONS Sample

The REsearch Consortium on Nearby Stars (RECONS; Jao et al. 2005; Henry et al. 2006) has been compiling a sample of the low-mass stars within ~25 pc in the southern hemisphere. The current realization of the RECONS samples was published by Winters et al. (2015) and contains 1748 systems with an M-dwarf primary and with parallax measurements (trigonometric or photometric). These stars also all have significant proper motions ($\mu > 180$ mas yr$^{-1}$) to remove possible giant stars. The completeness of this sample is unknown, but extrapolating results from the 5 pc sample, Winters et al. (2015) estimate their 25 pc sample to be between 48% and 77% volume complete.

We chose to simulate a 3600 deg$^2$ patch of sky away from the Galactic plane ($0^\circ \leq \alpha \leq 60^\circ$ and $-60^\circ \leq \delta \leq 0^\circ$). Since the RECONS sample has parallax measurements with a variety of precisions, we applied a 20% normal uncertainty to the simulated stars and kept stars within 25 pc. We ran 1000 realizations of the model over the volume listed above using

Figure 22. Ratio of SDSS source counts to stellar counts from the model. Each bin is $1^\circ \times 1^\circ$. The model produces similar numbers to SDSS at high Galactic latitudes (typically less than 10% difference). Close to the Galactic plane, SDSS source counts are much higher, probably due to reddened higher mass stars that fall into the color selection. The model has higher source counts near edge regions of the SDSS stripes.

and explain some of the important differences in our specific model.

For a given stellar population, the average stellar kinematics can be represented in Galactic cylindrical coordinates by the following equations:

$$
\langle V_r(Z) \rangle = 0, \\
\langle V_\phi(Z) \rangle = V_{\text{circ}} - V_a - f(Z), \\
\langle V_Z(Z) \rangle = 0,
$$

(15)

where $V_r$, $V_\phi$, and $V_Z$ are the velocities in the radial, circular, and perpendicular directions, respectively. $V_{\text{circ}}$ is the circular velocity, taken as 240 km s$^{-1}$ (McMillan 2011; Schönenrich 2012). The $V_a$ term is due to the interactions that stars undergo over their lifetimes, which cause circular orbits to become more eccentric and more inclined to the Galactic plane. These interactions cause the velocity component along the direction of Galactic rotation to lag the local standard of rest for older stellar populations, a phenomenon known as asymmetric drift. $V_a$ is approximately equal to 10 km s$^{-1}$ for low-mass stars in SDSS (D10). The last term for $V_\phi$ is a polynomial relationship between the average velocity and Galactic height, given by $f(Z) = a|Z| - b|Z|^2$ km s$^{-1}$, where $a = 0.013$ km s$^{-1}$ pc$^{-1}$ and $b = 1.56 \times 10^{-5}$ km s$^{-1}$ pc$^{-2}$ (taken from D10). This last term accounts for a mixture of thin- and thick-disk stars, with the ratio highly dependent on Galactic height.

For the velocity dispersions, we chose to explore different functional forms rather than a power law as was used in D10, which gives zero dispersion at the Galactic plane. Using results from the kinematic study of Pineda et al. (2017), we found that velocity dispersions grew approximately linearly with Galactic heights up to ~1 kpc in all three velocity components for both thin- and thick-disk stars. The Pineda et al. (2017) sample is an adequate representation of the candidate stars since they all fall within this Galactic height limit. The linear fits to the velocity dispersions take the form

$$
\sigma(Z) = k + n|Z|,
$$

(16)
the full density computed from integrating the B10 single-star r-band LF. Our results compared to the RECONS sample are shown in Figure 23. Both the model distributions of distances and proper motions follow the observed distributions up to the survey limits. If we use the model to estimate the incompleteness within the volume probed, we estimate the RECONS sample to be 74% complete using the 95th percentile values. The proper motion distribution indicates that the majority of missing stars have small proper motions.

B.6. Model Comparison: SUPERBLINK Sample

The SUPERBLINK survey (Lépine et al. 2002, 2003; Lépine & Shara 2005) is a proper motion and magnitude-limited survey. For the comparison, we used the bright M-dwarf subcatalog (Lépine & Gaidos 2011). This catalog has a magnitude limit of $J < 10$ and a proper motion limit of $\mu > 40 \text{ mas yr}^{-1}$. The completeness for stars in the northern hemisphere is estimated to be $\approx 90\%$.

To properly simulate this sample, we were required to simulate the magnitude limits in the form of distance limits and distance uncertainties. The $J < 10$ limit was implemented using the $J$-band LF from B10 and calculating the distance for each $M_J$ bin using a limiting magnitude of $J = 10$. We integrated out to a distance of 200 pc although 80% of the stars in the Lépine & Gaidos (2011) sample have distances $\leq 75$ pc. This larger simulated maximum distance was chosen because distances were convolved with uncertainties before implementing a distance cut of 65 pc (comparing only to the Lépine & Gaidos 2011 stars with $d \leq 65$ pc).

The quoted distance uncertainty in the photometric parallax relationship used in Lépine & Gaidos (2011) is between 20% and 50%. To determine the best uncertainty to fold into the distances, we ran small batches of simulations using different normally distributed uncertainties (between 20% and 50%), and comparing their distance distributions to SUPERBLINK. We found that 30% uncertainty gave the expected trends in the distance distributions.

Again, we simulated a 3600 deg$^2$ patch of sky away from the Galactic plane ($160^\circ \leq \alpha \leq 220^\circ$ and $0^\circ \leq \delta \leq 60^\circ$) and ran...
1000 realizations. Figure 24 shows the SUPERBLINK distributions and the model results, along with the 5th and 95th percentile confidence intervals. We can again estimate a level of completeness using the 95th percentile values, but caution should be taken as the uncertainties folded into the simulations may be different than the actual uncertainties within the SUPERBLINK survey. The estimated completeness level for the simulated volume is 65%, with the majority of missing stars at smaller proper motions below the survey limit. As is shown in Figure 24, the completeness of SUPERBLINK should be extremely high for the largest proper motion stars. However, toward the proper motion limit of SUPERBLINK, the completeness drops off. This is to be expected as smaller proper motions are more difficult to measure to high precision. Some of this incompleteness may be accounted for if measurement uncertainty tends to scatter stars toward higher proper motions. However, there still appears to be a large population of nearby stars with small proper motions that has gone relatively undetected because of the requirement of larger proper motions (similar to the comparison with the RECONS sample). The complete SUPERBLINK sample (without the \( J < 10 \) criterion) will likely resolve much of this incompleteness when some of the fainter stars with smaller proper motions are added to the sample.

The *Gaia* (Perryman et al. 2001; Gaia Collaboration et al. 2016b) collaboration recently published Data Release 1 (Gaia Collaboration et al. 2016a), which has a proper motion precision of \( \sim 1 \text{ mas yr}^{-1} \) for non-*Hipparcos* Tycho-2 stars (Lindegren et al. 2016). However, the final data release for *Gaia* is expected to have a precision better than 0.1 mas yr\(^{-1}\). *Gaia* should detect all of the nearby (\( \lesssim 60 \text{ pc} \)) earliest type-M dwarfs and lower mass objects at closer distances. However, *Gaia* will not be able to detect the lowest mass M dwarfs out to the distances SDSS, 2MASS, and WISE were able to observe them because of its relatively blue filter (Ivezić et al. 2012). The *Gaia* completeness for low-mass dwarfs has been investigated using the LaTE-MoVeRS sample (Theissen et al. 2017) and *Gaia* Data Release 1. Theissen et al. (2017) found that *Gaia* was \( \sim 70\% \) complete for low-mass dwarfs with \( i < 20 \) and less than 30% complete for dwarfs with \( i \gtrsim 20 \). Although *Gaia* will not be able to probe the entire volume that the MoVeRS sample covers, it will allow us to validate the model across the entire proper motion range and with much smaller simulated distance uncertainties for nearby (\( \lesssim 30 \text{ pc} \)) stars. *Gaia* will be especially critical in uncovering the potential population of nearby stars with small proper motions that have been primarily ignored and for resolving the true completeness of the SUPERBLINK sample.
B.7. Simulating a Galactic Volume within the SDSS Footprint

To properly estimate the level of completeness, we need to simulate the complete volume ($\alpha$, $\delta$, and $d$) from where the sample was extracted. However, because of the time-delay-integrate nature of SDSS, obtaining the exact outline of the imaging footprint in $\alpha$ and $\delta$ coordinates is extremely complicated. To further complicate matters, some pipelines fail processing by the photometric pipeline. This is primarily due to large or bright objects within the frame causing the photometric processing by the photometric pipeline to time out (Blanton et al. 2011). To quantify the number of bad fields within the SDSS footprint, we retrieved all the field IDs and number of extracted objects within the field from the Field table using CasJobs. Of the 938,046 fields in SDSS, 6,239 fields contain zero objects (~0.67%). The vast majority of bad fields (4271) are found in stripes within the Galactic plane ($|b| < 20^\circ$), which we excluded from the sample. Therefore, bad fields were not a concern for the simulated SDSS volume.

Rather than try to simulate the entire SDSS footprint, we chose to simulate large areas within the footprint. Figure 25 shows the fields imaged by SDSS and the selected areas within that footprint. The stripe nature of SDSS is clearly shown, with darker regions indicating heavier coverage. The regions we chose are listed in Table 10, with larger regions divided into smaller subregions for computational ease and parallelization.

B.8. Sampling with the Model to Estimate Completeness

The level of completeness was estimated by simulating stars in regions defined in the previous section. This was done for all stars within the volume, and separately in absolute magnitudes bins defined in Table 8. The following steps were completed for all simulated regions:

1. For parallelization, different $r-z$ color ranges (a proxy for stellar mass ranges) were simulated individually. For each $r-z$ color range in Table 8, we used the B10 color–magnitude relations to obtain the range of absolute magnitudes ($M_\gamma$).

2. Since the color ranges were continuous, but the B10 $M_\gamma$ LF is given in discrete bins, we chose to interpolate the $M_\gamma$ LF. Using the single-star LF from B10, we interpolated the $M_\gamma$ LF over the $r-z$ color range from the previous step. The B10 LFs are given as median values with asymmetric uncertainties. All three values (median and asymmetric uncertainties) were used to provide a range of possible stellar number densities for the model. Three interpolations were done, one for the median $M_\gamma$ value, one for the upper $M_\gamma$ limit, and one for the lower $M_\gamma$ limit. This step is illustrated in Figure 26.

3. A random LF value was drawn for a given $M_\gamma$ value. To do this, the absolute magnitude range (from above) was divided into 10,000 evenly spaced bins. For each bin, a random LF value was drawn from a triangular probability distribution defined by the median value at the apex, and the lower limit and upper limit values as the first and third vertex, respectively. The median, upper limit, and lower limit values were taken from the interpolated LF at the center of each absolute magnitude bin. An example of this step is shown in Figure 26.

4. The LF values from the previous step were then integrated over the absolute magnitude range (from step 1) to produce the local stellar density scaled to the plane, $\rho(R_\odot, 0)$.

5. Using the stellar density from the previous step, we integrated the density profile, Equation (14), along the LOS in 1 pc deep, discrete pyramidal “cells.” Each cell along the LOS was parameterized by the $\alpha$ and $\delta$ range, and the distance range (defined in Table 8). Multiplying the volume of the cell by the average stellar density within the cell gave us the total number of stars within each cell. Summing all the cells gave us the total number of stars along the LOS.

6. The next step was distributing stars randomly within the given volume. For the relatively small angular ranges, we assumed that the $\alpha$ and $\delta$ positions for the stars were uniformly random within the range. Distances are more complicated as the distribution of distances is dependent on the line of sight through the Galaxy. To build a representative distribution of distances along the given line of sight, we used the number of stars in each cell and the distance to the center of each pyramidal cell from the previous step. This distribution was transformed into an inverse cumulative distribution function, which was sampled from the following step (known as inverse transform sampling; Press et al. 1992).

7. Stars were then distributed in a 3D space within the defined volume using the rejection method (Press et al. 1992). This generated uniformly random $\alpha$ and $\delta$ coordinates and distances randomly chosen through inverse sampling of the distribution created in the previous step.

8. The 3D $\alpha$, $\delta$, and distances were converted into Galactic cylindrical coordinates ($R$, $T$, $Z$).

9. Each star was then given $V_R$, $V_T$, and $V_Z$ velocities dependent on the average $V_R$, $V_T$, and $V_Z$ and corresponding dispersion found at each star’s Galactic height, based on Equation (16). These velocities were subsequently converted into $UWV$ velocities.

10. $UWV$ velocities were converted into proper motion components and radial velocities following the inverse of the methods described in Johnson & Soderblom (1987). We disregard the radial velocities as they are not required for the completeness estimates.

11. Last, a variable proper motion cut was made based on the minimum proper motion within the MoVeRS sample for the volume and color range simulated. This ensured the simulations only included stars that had distances and

| Region ID | $\alpha$ Range (deg.) | $\delta$ Range (deg.) |
|-----------|----------------------|----------------------|
| 1         | [0, 28]              | [−6, 10]             |
| 2         | [0, 28]              | [10, 26]             |
| 3         | [130, 182]           | [0, 20]              |
| 4         | [130, 182]           | [20, 40]             |
| 5         | [130, 182]           | [40, 58]             |
| 6         | [182, 235]           | [0, 20]              |
| 7         | [182, 235]           | [20, 40]             |
| 8         | [182, 235]           | [40, 58]             |
| 9         | [330, 360]           | [−6, 10]             |
| 10        | [330, 360]           | [10, 26]             |
12 The previous steps were repeated 100 times to build distributions of counts to estimate the random uncertainty in the model.

The LoKi Galactic model is available to the community through GitHub.12

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12 https://github.com/ctheissen/LoKi
