Diagnostics of tension in long rods with the help of traveling bending waves

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Abstract. The article considers the possibility of measuring the tension of long thin structures by measuring the speed of traveling bending waves. Earlier, with the participation of the first two authors of the work, a theoretical consideration of the possibility of determining the stress of a continuous-welded railway track by measuring the velocity of bending waves was carried out. For verification, the velocity of propagation of bending waves was measured, presumably during the compression of a continuous-welded track. This is possible when the temperature of the rails is higher than the laying temperature of the continuous track. In the same consideration, it is taken into account that bending waves exist in an extended product of finite thickness without tension, and the results are presented for the case when the continuous track is in a tensioned state. This consideration takes into account the fact that in a long workpiece of finite thickness there are also bending waves without tension. These waves, unlike the dispersion-free waves of tension, possess dispersion. A method of measuring the speed of these waves is proposed.

1. Introduction

There are known methods to determine mechanical tensions based on measuring the velocities of longitudinal and transverse waves and turning the plane of their polarization [1-3]. A method for determining mechanical tensions based on the application of the acoustoelasticity effect [4] is also used. In particular, on the basis of this method, the engineering firm "Inkotes" developed a device named IN-5101A, the principle of which rests on the generation of the ultrasonic probing pulses and the recording of the speed of reflected elastic waves excited in the material of the controlled object.

As part of this work, the possibility of measuring the tension of long metal structures (smooth track rails, high-pressure vessels, pipelines) is considered by measuring the increase in the speed of the propagation of the bending waves of the audio range in the workpieces when they are stretched, excited by a calibrated impact.

Diagnostics of the tension force of thin strings using traveling bending waves is simple and clear. It is necessary to quickly create an initial excitation for a long stretched string in the form of the string’s local length deviation from the equilibrium state. It can be a quick impact or a quick plucking. The initial excitation, splitting in two, moves without changing the shape in both directions at a speed of

\[ C_T = \left( \frac{T}{\rho_{lin}} \right)^{1/2}, \]

as the traveling waves in a thin stretched string are dispersion-free (T is the force of
string tension per unit area, \( \rho_{\text{lin}} \) is the linear string density). Measuring the speed of \( C_0 \), we find a tension \( T \).

Diagnostics of tension in a stretched rod of finite thickness is complicated by the fact that bending waves in the thin rod also exist without tension. And these waves are dispersion waves, initial excitations are diffused. The additional tension of the rod changes the process of spreading bending waves along the rod. One needs to take into account this change in the pattern of the excitation spread and determine how to measure the tension \( T \) in the rod of finite thickness. In works [5, 6], we measured the velocity of flexural waves to diagnose the stress state of a continuous welded track, and the proposed article is a further development of the method.

2. **Theoretical consideration**

The dispersion ratio for the bending waves of the rod of finite thickness, according to Lamb [7], looks like this:

\[
\omega = C_T k + C_0 \frac{k \chi}{\sqrt{1 + k^2 \chi^2}},
\]

(1)

where \( C_0 = \frac{E}{\sqrt{\rho}} \) is the velocity of longitudinal waves in the media with Young’s modulus \( E \), \( \rho \) is the density of the media. The group velocity for thin waves is as follows:

\[
\nu_{gr} = \frac{\partial \omega}{\partial k} = C_T + C_0 \frac{2 k \chi + k^3 \chi^3}{\left(1 + k^2 \chi^2\right)^{\frac{3}{2}}},
\]

(2)

Here \( k \) is the wave number, and \( \chi \) is the rod cross-section radius of inertia. Besides, depending on the direction of the bending excitations relative to the plane, the value of \( \chi \) changes.

If we take the cross-section of the rod in the form of a rectangle, we can talk about two bending planes:

![Figure 1. Rod cross-section radii of inertia.](image)

Figure 1 depicts two directions of external impact (impacts) on the rod (1 – 1) and (2 – 2), which correspond to their own axes of rotation of bending excitations and their own inertia radii \( \chi_1 \) and \( \chi_2 \).
Further, let us keep in mind that, depending on the direction of external impact, it is necessary to use either \( \chi_1 \) or \( \chi_2 \) for the radius of inertia.

The phase velocity of the waves from (1) will be as follows:

\[
\nu_{\text{ph}} = C_T + C_0 \frac{k\chi}{\sqrt{1 + k^2 \chi^2}}.
\]  

(3)

From the comparison (3) and (2) it is clear that the group velocity at all \( k \) is more than the phase velocity. Besides, at \( k\chi >> 1 \) \( \nu_{\text{ph}} \rightarrow \nu_{\text{gr}} \). At \( k\chi << 1 \), but provided that \( 2C_0k\chi >> C_T \), the group velocity is maximally different from the phase velocity, twice exceeding it. And finally, when \( k\chi \) are so small that \( 2C_0k\chi < C_T \), the phase velocity tends to the group velocity, which tends to the value \( C_T \).

In these comparative characteristics, we implied that \( C_T << C_0 \).

The development of the bending excitation of the rod can be broken down into two stages:

1) the formation of the initial excitation under the influence of external impact;
2) the scattering of the initial excitation in the form of a wave process in both directions from the initial excitation.

Let us consider both stages. When considering the first stage, let us take into account that the external impact is an impact by the body of some mass \( M \) and size along the \( x \)-axis (along the rod) of the magnitude \( 2\Delta x_{\text{ip}} \).

![Figure 2. Formation of the initial excitation](image)

The area of initial excitation \( \Delta x \) is increasing with the velocity of \( \nu_{\text{gr}}(k_{\text{ph}}) \), where \( k_{\text{ph}} \) is the maximum wave vector of excitation at the time when it was of the size of \( \Delta x \). Let us accept that \( k_{\text{ph}} = \frac{2\pi}{\Delta x} \). With an increase of \( \Delta x \), the wave number decreases, the group velocity changes according to the formula (2). At the initial moment of the impact, \( k_{\text{phimp}} = \frac{2\pi}{\Delta x_{\text{imp}}} \), and \( k_{\text{phimp}}\chi >> 1 \). Then \( \nu_{\text{gr}}(k_{\text{phimp}}) \approx C_0 \). Over time, \( k_{\text{ph}}\chi \) decreases during the impact and, most certainly, even before the end
of the impact, the parameter of \( k_{\text{ph}} \chi \) will be much smaller than one. Let us denote the magnitude of \( k \chi \) at the end of the impact \( k_0 \chi << 1 \), and besides, \( k_0 = \frac{2 \pi}{\Delta x_0} \), where \( \Delta x_0 \) is the half-length of the initial bending excitation. The moment of the end of the impact can be identified with the moment when the body of mass \( M \) bounces away from the rod. Careful consideration of the formation of the initial bending excitation is a separate complex problem and is of only academic interest. For the purposes of diagnosis of tension \( T \), the details of the formation of the initial excitation are not important. A more powerful impact (with a greater mass \( M \) ) gives an increase of \( \Delta x_0 \).

Let us now consider the second stage. The magnitude of \( k_0 = \frac{2 \pi}{\Delta x_0} \) will be the maximum wave number for the initial excitation. Since the problem is linear, the composition by wave numbers at the stage of wave propagation evolution will not change in excitation. The edge of the wave excitation zone at the second stage will move with a velocity of:

\[
\nu_{gr0} = C_T + C_0 2k_0 \chi
\]

Let us assume that \( C_0 2k_0 \chi >> C_T \) and

\[
\nu_{gr0} \approx C_0 2k_0 \chi.
\]

The phase velocity of the wave excitation humps just behind the edge of the wave zone will be equal to \( \nu_{ph0} \approx C_0 k_0 \chi \). This effect can be tested experimentally (see the next section).

Sensors need to be positioned at points along the \( x \) further than \( \Delta x_0 \), although \( \Delta x_0 \) is not known in advance and needs to be specifically identified. The method of determining \( \Delta x_0 \) is in the next section.

Knowing the distance between the sensors \( \Delta_D \), by delaying the arrival of the wave zone boundary \( \Delta t_{delim} \) to the second sensor, we determine the group velocity:

\[
\nu_{gr0} = \frac{\Delta_D}{\Delta t_{delim}}.
\]

If there is no tension \( T \), then \( C_T = 0 \), and there is no clear second boundary of the wave packet, so the lower boundary of the wave packet moves from the side of \( \nu_{gr\min} \) which is equal to \( C_T \). If \( C_T \neq 0 \), then the speed \( C_T \) is determined by the delay \( \Delta t_{lbw} \) of the arrival of the lower boundary of the wave zone to the second sensor:

\[
C_T = \frac{\Delta_D}{\Delta t_{lbw}}.
\]

For a better measurement, it is necessary that \( C_T \) is not much smaller than \( \nu_{gr0} \). And to reduce \( \nu_{gr0} \), one needs to increase the power of the impact.
3. Experimental Method

It is necessary to identify $\Delta x_0$ for the given mass of the striker $M$. To do this, you need to have three sensors, placing them as shown in Figure 3.

![Figure 3. The location of measurement sensors.](image)

It is advisable that the sensor $D_2$ is immediately behind $\Delta x_0$, and the sensor $D_1$ is in front of $\Delta x_0$.

Then $\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1}$, the average speed of the front edge of the wave zone to the first sensor ($\Delta t_1$ is the time from the start of impact) $\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2}$ is the average velocity of the wave zone edge between the sensors $D_2$ and $D_1$. $\bar{v}_3 = \frac{\Delta x_3}{\Delta t_3} = v_{gr0}$. If $\Delta x_1 < \Delta x_0$, and $\Delta x_1 + \Delta x_2 > \Delta x_0$, then $\bar{v}_1 > \bar{v}_2 > \bar{v}_3 = v_{gr0}$.

By moving the sensor $D_1$ along $x$, we will find $\Delta x_1 = \Delta x_0$ such that $\bar{v}_1 > \bar{v}_2 = \bar{v}_3 = v_{gr0}$ is fulfilled.

Let us consider the experiment methodology according to the proof that the phase velocity of humps in the wave zone, just behind the front edge, is half the size of $v_{gr0}$.

Let us draw the supposed pattern of records from two sensors located at a distance of $\Delta x_D$.

If $\Delta x_D$ is large enough, then at the second sensor, due to the fact that the phase velocity is $v_{ph} < v_{gr0}$, a new hump (zero) is generated and together with other humps moves in the edge system deep into the wave zone.

Conditions of the generation of the first hump are:

$$\Delta t_{ph} - \Delta t_{gr} > T_{ph},$$

$$\Delta t_{ph} = \frac{\Delta x_D}{v_{ph}}, \quad \Delta t_{gr} = \frac{\Delta x_D}{v_{gr0}},$$

$T_{ph}$ is the time interval between the two humps in the image from a single sensor. Note that $\omega_0 = k_0 v_{ph} \rightarrow \frac{2\pi}{T_{ph}} = \frac{2\pi}{\Delta x_0 v_{ph}}$ and $T_{ph} = \frac{\Delta x_0}{\omega_0}$.

Then from (8) we obtain:

$$\Delta x_D > \Delta x_0 \frac{\beta}{\beta - 1},$$

where $\beta = \frac{v_{gr0}}{v_{ph}}$. At $\beta < 2$ magnitudes of $\frac{\beta}{\beta - 1} > 2$. 


If, on the contrary, we set $\Delta x_D$ in such a way that (9) is not fulfilled, that is, for example,

$$\Delta x_D < \Delta x_0$$

(10)

there will be no generation of additional humps of wave excitations, and then the humps on both sensors will be easily identifiable.

Figure 4. The supposed pattern of the observation of the signal on two sensors.

In this case, tracking the delay of $\Delta t_{ph}$ between the occurrence of the first hump at the second and the first sensor, we will obtain:

$$\nu_{ph} = \frac{\Delta x_D}{\Delta t_{ph}}$$

(11)

$$\nu_{gr0} = \frac{\Delta t_{ph}}{\Delta t_{gr}}$$

(12)

4. Conclusion

The performed consideration of the formation of a bending wave arising from the impact of an extended workpiece in a tensioned state allows one to continue measuring the velocity of bending waves in order to determine the stressed state of a continuous welded railway track. The above technique makes it possible to determine the tensile forces for long structures by increasing the velocity of bending waves.

References

[1] Gushcha O I, Guz A N, Lebedev V K, Makhort V G and Trotsenko V P 1975 Acoustic method of controlling stress in solid media Bull. of inventions 41

[2] Thompson R B and Alers G A 1978 US, Patent No. 40808836 Method of measuring stress in a material. Int.Cl. 73597 (G 01 B7/16).

[3] Gushcha O I and Makhort V G 1983 Acoustic method of controlling stress in solid media Bull. of inventions 36

[4] Nikitina N E 2005 Acoustic elasticity. Practical experience (Nizhny Novgorod: TALAM Publ.)

[5] Bardakov V M, Klimov N N, Dudakov S V, Lopatin M V, Muratov V I, Kutsenko S M and Filatov E V 2010 On the possibility of acoustic diagnostics of the stress state of a continuous welded track.
Problems and Prospects of Research, Design, Construction and Operation of Railways: Proceedings of the IV All-Russian Scientific and Practical Conference with International Participation. V.1 (Irkutsk: IrGUPS) pp. 341 - 354.

[6] Klimov N N, Zubkova D A, Kutsenko S M and Dudakov S V 2011 Automation of data processing of acoustic diagnostics of the stress state of a continuous welded track Pecsa Modern technologies. System analysis. Modeling 4(32) 209-214

[7] Lamb G 1960 The Dynamical Theory of Sound. State Publishing (House of Physics and Mathematics, FIZMATGYZ Publ)