Spin–motion entanglement and state diagnosis with squeezed oscillator wavepackets

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Mesoscopic superpositions of distinguishable coherent states provide an analogue of the ‘Schrödinger’s cat’ thought experiment. For mechanical oscillators these have primarily been realized using coherent wavepackets, for which the distinguishability arises as a result of the spatial separation of the superposed states. Here we demonstrate superpositions composed of squeezed wavepackets, which we generate by applying an internal-state-dependent force to a single trapped ion initialized in a squeezed vacuum state with nine decibel reduction in the quadrature variance. This allows us to characterize the initial squeezed wavepacket by monitoring the onset of spin–motion entanglement, and to verify the evolution of the number states of the oscillator as a function of the duration of the force. In both cases we observe clear differences between displacements aligned with the squeezed and anti-squeezed axes. We observe coherent revivals when inverting the state-dependent force after separating the wavepackets by more than 19 times the ground-state root mean squared extent, which corresponds to 56 times the root mean squared extent of the squeezed wavepacket along the displacement direction. Aside from their fundamental nature, these states may be useful for quantum metrology or quantum information processing with continuous variables.

The creation and study of non-classical states of spin systems coupled to a harmonic oscillator have provided fundamental insights into the nature of decoherence and the quantum–classical transition. These states and their control form the basis of experimental developments in quantum information processing and quantum metrology. Two of the most commonly considered states of the oscillator are squeezed states and superpositions of coherent states of opposite phase, which are commonly referred to as ‘Schrödinger’s cat’ (SC) states. Squeezed states involve a reduction of the fluctuations in one quadrature of the oscillator below the ground-state uncertainty, which has been used to increase sensitivity in interferometers. SC states provide a complementary sensitivity to environmental influences by separating the two parts of the state by a large distance in phase space. These states have been created in microwave and optical cavities, where they are typically not entangled with another system, and also with trapped ions, where all experiments performed have involved entanglement between the oscillator state and the internal electronic states of the ion. SC states have recently been used as sensitive detectors for photon scattering recoil events at the single-photon level.

Here we use state-dependent forces (SDFs) to create superpositions of distinct squeezed oscillator wavepackets that are entangled with a pseudo-spin encoded in the electronic states of a single trapped ion. We will refer to these states as squeezed wavepacket entangled states (SWESs) in the rest of the paper. By monitoring the spin evolution as the entanglement with the oscillator increases, we are able to observe the squeezed nature of the initial state directly. We obtain a complementary measurement of the initial state by extracting the number-state probability distribution of the displaced–squeezed states that make up the superposition. In both measurements we observe clear differences depending on the force direction. We show that the SWESs are coherent by reversing the effect of the SDF, resulting in recombination of the squeezed wavepackets, which we measure through the revival of the spin coherence.

The squeezed vacuum state is defined by the action of the squeezing operator $S(\zeta) = e^{\zeta \hat{a}^\dagger -\zeta^\dagger \hat{a}^2/2}$ on the motional ground state $|0\rangle$, where $\zeta = re^{i\phi}$, with $r$ and $\phi$ real parameters that define the magnitude and the direction of the squeezing in phase space. To prepare squeezed states of motion in which the variance of the squeezed quadrature is reduced by about 9 dB relative to the ground-state wavepacket we use reservoir engineering, in which a bichromatic light field is used to couple the ion’s motion to the spin states of the ion, which undergo continuous optical pumping. This dissipatively pumps the motional state of the ion into the desired squeezed state, which is the dark state of the dynamics. More details about the reservoir engineering can be found in ref. 18. This approach provides a robust basis for all experiments described below, typically requiring no recalibration over several hours of taking data. In the ideal case, the optical pumping used in the reservoir engineering results in the ion being pumped to $|\downarrow\rangle$. To create a SWES, we apply a SDF to this squeezed vacuum state by simultaneously driving the red $|\downarrow\rangle\langle n|\downarrow\rangle\langle n+1|$ and blue $|\downarrow\rangle\langle n+1|\langle n+1|$ on the spin–motion states. The resulting interaction Hamiltonian can be written in the Lamb–Dicke approximation (LDA) as

$$\hat{H}_D = \hbar \Omega \hat{a} \hat{a}^\dagger e^{-\hat{\theta}/2} + \hat{a} e^{\hat{\theta}/2}$$

where $\Omega$ is the strength of the SDF, $\phi_0$ is the relative phase of the two light fields, and $\theta$ is the phase difference between the two parts of the state by a large distance in phase space. This Hamiltonian results in displacement of the motional state in phase space by $\sin^2 \rho = \cos^2 \rho$, which gives in units of the root mean squared extent of the harmonic oscillator ground state. An ion prepared in $|\downarrow\rangle$ will be displaced by the same amount in the opposite direction. In the following equations we use $\pi$ in place of $\alpha$ for simplicity. Starting from the state $|\downarrow\rangle\langle \zeta|$, application of the SDF ideally results in the SWES

$$|\psi(\alpha)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + \zeta |\downarrow\rangle - \zeta |\downarrow\rangle - \zeta |\zeta\rangle - \zeta |\alpha\rangle)$$

where we use the notation $|\alpha, \zeta\rangle = \hat{D}(\alpha) S(\zeta) |0\rangle$ with the displacement operator $\hat{D}(\alpha) = e^{|\alpha|^2 - x^2 \hat{a}^\dagger \hat{a}}$. A projective measurement of the spin performed in the $\hat{\sigma}_z$ basis gives the probability of being in $|\downarrow\rangle$ as $P(|\downarrow\rangle) = (1 + X)/2$, where $X = (\zeta, \zeta - \alpha, \zeta - \alpha, -\alpha, -\alpha, \zeta, -\alpha, -\alpha)$. The probability of being in $|\uparrow\rangle$ gives the overlap between the two displaced motional states, which can be written as

$$X(\alpha, \zeta) = e^{-2|\alpha|^2} \cos(2\alpha^2 - 2\alpha \zeta) + \cos(-2\alpha \zeta)\cos(\Delta \phi)$$

where $\Delta \phi = \arg(\zeta) - \phi_0/2$. When $\Delta \phi = 0$, the SDF is aligned with the squeezed quadrature of the state, whereas for $\Delta \phi = \pi/2$, the SDF is
aligned with the anti-squeezed quadrature. At displacements for which X gives a measurable signal, monitoring the spin population as a function of the force duration τ for different choices of Δφ allows us to characterize the spatial variation of the initial squeezed wavepacket. For values of |x|^2 greater than the wavepacket variance along the direction of the force, the state in equation (2) is a distinct superposition of squeezed wavepackets that have overlap close to zero and are entangled with the internal state. For r = 0 (no squeezing) the state reduces to the familiar SC states that have been produced in previous work. For r > 0, the superposed states are the displaced-squeezed states.

The experiments use a single trapped ^{40}\text{Ca}^+ ion, which mechanically oscillates on its axial vibrational mode with a frequency close to ω_z/(2π) = 2.1 MHz. This mode is well resolved from all other modes. We encode a pseudo-spin system in the internal electronic states |↓⟩ ≡ |S_{3/2}, M_J = 1/2⟩ and |↑⟩ ≡ |D_{3/2}, M_J = 3/2⟩. All coherent manipulations, including the squeezed-state preparation and the SDF, make use of the quadrupole transition between these levels at 729 nm, with a Lamb–Dicke parameter of η ≈ 0.05 for the axial mode. This is small enough for the experiments to be well described using the LDA (a discussion of this approximation is given in the Methods).

We apply the SDF directly after the squeezed vacuum state has been prepared by reservoir engineering and the internal state has been prepared in |↓⟩ by optical pumping (in the ideal case, the ion is already in the correct state and this step has no effect). Figure 1 shows the results of measuring ⟨σ_z⟩ after applying displacements along the two principal axes of the squeezed state alongside the same measurement made using an ion prepared in the motional ground state. To extract relevant parameters regarding the SDF and the squeezing, we fit the data using P(↑) = (A + BX(α, ζ))/2, where the parameters A and B account for experimental imperfections such as shot-to-shot fluctuations in the magnetic field (Methods). Fitting the ground-state data with r fixed to zero allows us to extract Ω/(2π) = 13.25 ± 0.40 kHz (here and in the rest of the paper, all errors are given as s.e.m.). We then fix this when performing independent fits to the squeezed-state data for Δφ = 0 and Δφ = π/2. Each of these fits allows us to extract an estimate for the squeezing parameter r. For both the squeezed and anti-squeezed quadratures we obtain consistent values with a mean of r = 1.08 ± 0.03, which for a pure state would correspond to a 9.4 dB reduction in the variance of the squeezed quadrature. The inset of Fig. 1 shows the spin population as a function of the SDF phase φ_D with the SDF duration fixed to 20 μs. This is also fitted using the same equation described above, and we obtain r = 1.13 ± 0.03.

The loss of overlap between the two wavepackets indicates that a SWES has been created. To verify that these states are coherent superpositions, we recombine the wavepackets by applying a second ‘return’ SDF pulse for which the phase of both the red and blue sideband laser frequency components is shifted by π relative to the first. This reverses the direction of the force applied to the motional states for both the |+⟩ and |−⟩ spin states. In the ideal case a state displaced to ω(τ_1) by a first SDF pulse of duration τ_1 has a final displacement of δω = ω(τ_1) − ω(τ_2) after the return pulse of duration τ_2. For τ_1 = τ_2, δω = 0 and the measured probability of finding the spin state in |↑⟩ is 1. In the presence of decoherence and imperfect control, the probability with which the ion returns to the |↑⟩ state will be reduced. In Fig. 2 we show revivals in the spin coherence for the same initial squeezed vacuum state as was used for the data in Fig. 1. The data include a range of different τ_1. For the data for which the force was applied along the squeezed axis of the state (Δφ = 0), partial revival of the coherence is observed for SDF durations up to 250 μs. For τ_1 = 250 μs the maximum separation of the two distinct oscillator wavepackets is |Δx| > 19, which is 56 times the r.m.s. width of the squeezed wavepacket in phase space. The amplitude of revival of this state is similar to what we observe when applying the SDF to a ground-state cooled ion. The loss of coherence as a function of the displacement duration is consistent with the effects of magnetic-field-induced spin dephasing and motional heating. When the force is applied along the anti-squeezed quadrature (Δφ = π/2), we observe that the strength of the revival decays more rapidly than for displacements with Δφ = 0.

Simulations of the dynamics using a quantum Monte Carlo wavefunction approach including sampling over a magnetic field distribution indicate that this is caused by shot-to-shot fluctuations of the magnetic field (Methods).

We are also able to monitor the number-state distributions of the motional wavepackets as a function of the duration of the SDF. This provides a second measurement of the parameters of the SDF and the initial squeezed wavepacket, which has similarities with the homodyne measurement used in optics. To do this, we optically pump the spin state into |↓⟩ after applying the SDF. This procedure destroys the phase relationship between the two motional wavepackets, resulting in the mixed oscillator state \( \hat{ρ}_{\text{mixed}} = (|x, \zeta⟩⟨x, \zeta| + |−x, \zeta⟩⟨−x, \zeta|)/2 \) (we estimate that the photon recoil during optical pumping results in a decrease in the fidelity of our experimental state relative to \( \hat{ρ}_{\text{mixed}} \) by <3%, which would not be observable in our measurements). The two parts of this mixture have the same number-state distribution, which is that of a displaced-squeezed state. To extract this distribution, we drive Rabi oscillations on the blue-sideband transition and monitor the subsequent spin population in the \( \sigma_z \) basis. Figure 3 shows this evolution for SDF durations of τ = 30, 60 and 120 μs. For τ = 30 and 60 μs, the results from displacements applied parallel to the two principal axes of the squeezed state are shown (Δφ = 0 and π/2). We obtain the number-state probability distribution \( p(n) \) from the spin state population by fitting the data using a form \( P(↑) = bt + \frac{1}{2} \sum_{n} p(n) (1 + e^{-\gamma/n + i}) \), where \( t \) is the blue-sideband pulse duration, \( \Omega_{n,n+1} \) is the Rabi frequency for the transition between the ∣1⟩ and ∣n⟩ states, and \( \gamma \) is a phenomenological decay.
parameter\textsuperscript{25,26}. The parameter $b$ accounts for gradual pumping of population into the state $|\uparrow\rangle\langle 0|$ due to frequency noise on our laser\textsuperscript{19,27}. It is negligible when $p(0)$ is small. The resulting $p(n)$ are then fitted using the theoretical form for the displaced-squeezed states (Methods). The number-state distributions show a clear dependence on the phase of the force, which is also reflected in the spin population evolution. Figure 4 shows the Mandel $Q$ parameters of the experimentally obtained number-state distributions, defined as $Q = \langle (\Delta n)^2 \rangle / \langle n \rangle - 1$, in which $\langle (\Delta n)^2 \rangle$ and $\langle n \rangle$ are the variance and mean of $p(n)$, respectively\textsuperscript{28}. The solid lines are the theoretical curves given in ref. 19 for $r = 1.08$, and are in agreement with our experimental results. For displacements along the short axis of the squeezed state (Fig. 3), the collapse and revival behaviour of the time distribution, with the result that in the fitting $r$ and $\alpha$ are positively correlated; the errors stated do not take account of this. We think that this accounts for the apparent discrepancy between the values of $r$ and $\alpha$ obtained for $t = 60\,\mu$s. The dashed green line in the insets of d and f is the Poisson distribution for the same $\langle n \rangle$ as the created displaced-squeezed-state mixture, which is given by $\langle n \rangle = |a|^2 + \sinh r \langle n \rangle$ (ref. 19). Results are shown as means $\pm$ s.e.m.; the error bars were generated under the assumption that the dominant source of fluctuations was quantum projection noise.
evolution of $P(\{j\})$ is reminiscent of the Jaynes–Cummings Hamiltonian applied to a coherent state, but it has more oscillations before the ‘collapse’ for a state of the same $|n\rangle$. This is surprising because the statistics of the state is not sub-Poissonian. We attribute this to the fact that this distribution is more peaked than that of a coherent state with the same $|n\rangle$, which is obvious when the two distributions are plotted over one another (Fig. 3d, f). The increased variance of the squeezed state then arises from the extra populations at high $n$, which are too small to make a visible contribution to the Rabi oscillations. For the squeezing parameter in our experiments, sub-Poissonian statistics would be observed only for $|x| > 3$. For $r = 120 \mu s$ we obtain a consistent value of $r$ and $|x| = 4.6$ only in the case in which we include a fit parameter for scaling of the theoretical probability distribution, obtaining a fitted scaling of $0.81 \pm 0.10$ (Methods). The reconstruction of the number-state distribution is incomplete because we cannot extract populations with $n > 29$ as a result of frequency crowding in the $\sqrt{n+1}$ dependence of the Jaynes–Cummings dynamics. We therefore do not include these results in Fig. 4. Measurement techniques made in a squeezed-state basis could avoid this problem; however, these are beyond our current experimental capabilities for states of this size.

We have generated entangled supersposition states between the internal and motional states of a single trapped ion in which the superposed motional wavepackets are of a squeezed Gaussian form. These states present new possibilities both for metrology and for continuous variable quantum information. In an interferometer based on SC states separated by $|\Delta x|$ the interference contrast depends on the final overlap of the recombed wavepackets. Fluctuations in the frequency of the oscillator result in a reduced overlap, but this effect can be improved by a factor $\exp[-|\Delta x|^2(e^{-2r} - 1)/2]$ if the wavepackets are squeezed in the same direction as the state separation (Methods). In quantum information with continuous variables, the computational basis states are distinguishable because they are separated in phase space by $|\Delta x|$ and thus do not overlap. The decoherence times of such superpositions typically scale as $1/|\Delta x|^2$ (ref. 22). The use of states squeezed along the displacement direction reduces the required displacement for a given overlap by $e^r$, increasing the resulting coherence time by $e^r$, which is a factor of 9 in our experiments. We therefore expect these states to open up new possibilities for quantum-state engineering and control.
METHODS

Experimental details. The experiments make use of a segmented linear Paul trap with an ion–electron–distance of ∼185 μm. Motional heating rates from the ground state for a calcium ion in this trap have been measured to be 10 ± 1 quanta s⁻¹, and the coherence time for the number-state superposition |0⟩ + |1⟩/√2 has been measured to be 32 ± 3 ms.

The first step of each experimental run involves cooling all modes of motion of the ion close to the Doppler limit by using laser light at 397 and 866 nm. The laser beam used for coherent control of the two-level pseudo-spin system addresses the narrow-linewidth transition |1⟩ = [S₁/₂, M₁ = 1/₂] + |1⟩ = [D₁/₂, M₁ = 3/₂] at 729 nm. This transition is resolved by 200 MHz from all other internal state transitions in the applied magnetic field of 119.6 G. The SDFs and the reservoir engineering in our experiment require the application of a bichromatic light field. We generate both frequency components with the use of acousto-optic modulators (AOMs) starting from a single laser stabilized to an ultra-high-finesse optical cavity with a resulting linewidth of <600 Hz (at which point magnetic field fluctuations limit the qubit coherence). We apply pulses of 729 nm laser light with a double-pass AOM to which we apply a single radiofrequency tone, followed by a single-pass AOM to which two radiofrequency tones are applied. After this second AOM, both frequency components are coupled into the same single-mode fibre before delivery to the ion. The double-pass AOM is used to switch the light on and off. Optical pumping to |0⟩ is implemented using |1⟩ combined with two linearly polarized light fields at 854, 397 and 866 nm. The internal state of the ion is read out by state-dependent fluorescence using laser fields at 397 and 866 nm. The 729 nm laser beam enters the trap at 45° to the z axis of the trap, resulting in a Lamb–Dicke parameter of n ≈ 0.05 for the axial mode. For this Lamb–Dicke parameter, we have verified whether for displacements up to |x| = 9.75 the dynamics can be well described with the LDA. We simulate the wavepacket dynamics by using the interaction Hamiltonian with and without LDA. In the simulation we apply the SDF to an ion prepared in |⟩⟩. The interaction Hamiltonian for a single trapped ion coupled to a single-frequency laser field can be written as

\[ \mathcal{H}_I = \frac{\Omega_I}{2} \sigma_2 \sigma_1 \exp\left(\frac{i n a_0 (e^{-\alpha t} + \alpha e^{\alpha t})}{\sqrt{\nu}}\right) e^{i \phi_0} \mathcal{H}_c \]

where \( \Omega_I \) is the interaction strength, \( \sigma_2 \sigma_1 \) is because the frequency of the ion, \( \phi_0 \) is the phase of the laser, \( \alpha = \mu_0 \) is the detuning of the laser from the atomic transition, and \( \mathcal{H}_c \) is the Hamiltonian conjugate of the first term. In the laboratory, the application of the SDF involves simultaneously tuning both the blue-sideband and red-sideband transitions resonantly, resulting in the Hamiltonian \( \mathcal{H}_{\text{res}} = \mathcal{H}_{\text{rad}} + \mathcal{H}_b + \mathcal{H}_r \). Now the SDF is aligned along either the squeezing quadrature or the anti-squeezing quadrature. The revival in the spin coherence is not arising between the two superposed motional states. As a result, after the application of the second SDF pulse the residual displacement would be \( 4\alpha \) and the state overlap given by \( \langle \theta | \phi_0 \rangle \). For the data set of |x| = 4.6 (Fig. 3a, b) the direction parameter that gives us a value of 0.81 ± 0.1. We note that in this case 4% of the expected population lies above \( n = 29 \) but we are unable to extract these populations from our data.

The Mandel Q parameter is defined as

\[ Q = \frac{\langle (\Delta n)^2 \rangle - \langle n \rangle^2}{\langle n \rangle} \]

where \( \langle \Delta n \rangle \) and \( \langle \Delta n^2 \rangle \) are the mean and variance of the probability distribution. For a displaced-squeezed state these are given in ref. 19 as

\[ \langle (\Delta n)^2 \rangle = |\langle x | \mathcal{C} | \langle x \rangle \rangle|^2 + 2 \cosh^2 r \sinh^2 r \]

\[ \langle n \rangle = |\langle x | \mathcal{C} | \langle x \rangle \rangle|^2 \]

These forms were used to produce the curves given in Fig. 4.

Applications of SWESs. The SWES may offer new possibilities for sensitive measurements that are robust against certain types of noise. An example is illustrated in Extended Data Fig. 4, in which we compare an interferometry experiment involving the use of a SWES versus a more standard SC state based on coherent states. In both cases the superposed states have a separation of |2x| obtained using a SDF. For the SWES this force is aligned along the squeezed quadrature of the state. The interferometer is closed by inverting the initial SC state, resulting in a residual displacement that in the ideal case is zero. One form of noise involves shot-to-shot fluctuations in the oscillator frequency. On each run of the experiment, this would result in a small phase shift \( \Delta \theta \) arising between the two superposed motional states. As a result, after the application of the second SDF pulse the residual displacement would be \( 2x_0 = 2x \sin(\Delta \theta/2) \), which corresponds to the states being separated along the P axis in the rotating-frame phase space. The final state of the system would then be \( |\psi(2x_0)\rangle \) with a corresponding state overlap given by \( X(2x_0, \Delta) \). Therefore the contrast will be higher for the SWES (Extended Data Fig. 4a) than for the coherent SC state (Extended Data Fig. 4b) by a factor

\[ \exp(-2|x_0|^2 e^{-2 - 1}) \]

Although in our experiments other sources of noise dominate, in other systems such oscillator dephasing may be more significant.

30. Gerry, C. & Knight, P. Introductory Quantum Optics (Cambridge Univ. Press, 2005).
Extended Data Figure 1 | Quasi-probability distributions for displaced-squeezed states in phase space using LDA and non-LDA. a, c, e, The simulation results using LDA with different SDF durations. b, d, f, The results simulated using the full Hamiltonian.
Extended Data Figure 2 | Coherence of cat states with fixed magnetic field noise. The magnetic-field-induced energy-level shift of 1.5 kHz is used in this simulation. a, The duration of both SDF pulses is 60 μs. b, The duration of both SDF pulses is 120 μs. Dashed red and dash–dot green curves show the SDF aligned along the squeezed and anti-squeezed quadratures. The blue trace is for the SDF applied to a ground-state cooled ion.
Extended Data Figure 3 | Coherence of cat states with a magnetic field fluctuation distribution. With the assumption that the magnetic field exhibits a 50 Hz sinusoidal pattern with an amplitude of 2.2 mG, this plot shows the simulation results by taking an average over 100 samples on the field distribution. a, The duration of both SDF pulses is 60 µs. b, The duration of both SDF pulses is 120 µs. Definitions of the curve specification are the same as in Extended Data Fig. 2.
Extended Data Figure 4 | Possible application of using SWESs for interferometry. a, Use of squeezed-state wavepackets. b, Use of ground-state wavepackets. The first SDF pulse is used to create a spin–motion-entangled state. In the middle, a small phase shift $\Delta \theta$ is induced by shot-to-shot fluctuation in the oscillator frequency before the application of the second SDF pulse, which recombines the two distinct oscillator wavepackets.