Quasi-Inclusive and Exclusive decays of $B$ to $\eta'$

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Abstract

We consider the effective Hamiltonian of four quark operators in the Standard Model in the exclusive and quasi-inclusive decays of the type $B \to K^{(*)}\eta'$, $B \to \eta'X_s$, where $X_s$ contains a single Kaon. Working in the factorization assumption we find

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that the four quark operators can account for the recently measured exclusive decays $B \to \eta'/(\eta)K$ and $B \to K\pi$ for appropriate choice of form factors but cannot explain the large quasi-inclusive rate.

1 Introduction

Recently CLEO has reported a branching ratio for the process $B^- \to \eta'K^-$ at $(7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5}$ [1]. Upper limits on related exclusive decay modes with $\eta$ or $\eta'$ with a $K^*$ mesons in the final states were also obtained. The quasi-inclusive process $B \to \eta'X_s$ for high momentum $\eta'$ was also measured with a high rate [2]

$$B \to \eta'X_s = (6.2 \pm 1.6 \pm 1.3) \times 10^{-4}(p_{\eta'} > 2.0\text{GeV}) \quad (1)$$

where $X_s$ stands for one Kaon and up to 4 pions with at most one $\pi^0$.

Several explanations for the large quasi-inclusive decay rate have been proposed both within the Standard Model and beyond [3, 4, 5, 6]. However, some of these analyses have underestimated the effect of the effective four quark operators to this process. For example, it has been assumed that this contribution is small based on the smallness of $V_{ub}$ [4]. However, the penguin contribution to this process cannot be neglected. A simple estimate based on the magnitudes of the CKM elements associated with the tree and the penguin diagrams clearly suggests that this process is dominated by penguin contributions if one considers only the four quark effective Hamiltonian. In this paper we calculate the contribution of the effective Hamiltonian of the four quark operators to the exclusive decays $B \to K(\ast)\eta'(\eta)$ and $B \to K\pi$ along with the quasi-inclusive decay of the $B \to \eta'X_s$.

In the sections which follow, we describe the effective Hamiltonian of four quark operators, our calculation of the exclusive and quasi-inclusive rates.
2 Effective Hamiltonian

In the Standard Model (SM) the amplitudes for hadronic $B$ decays of the type $b \rightarrow q\bar{f}f$ are generated by the following effective Hamiltonian \[7\]:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{fb}V_{f\bar{q}}^*(c_1O_{f1}^q + c_2O_{f2}^q) - \sum_{i=3}^{10}(V_{ub}V_{u\bar{q}}^*c_i^u + V_{cb}V_{c\bar{q}}^*c_i^c + V_{tb}V_{t\bar{q}}^*c_i^t)O_{f1}^i] + H.C. \quad (2)$$

where the superscript $u$, $c$, $t$ indicates the internal quark, $f$ can be $u$ or $c$ quark and $q$ can be either a $d$ or a $s$ quark depending on whether the decay is a $\Delta S = 0$ or $\Delta S = -1$ process.

The operators $O_{f1}^q$ are defined as

$$O_{f1}^q = \bar{q}_\alpha\gamma_\mu\gamma_5L\bar{f}_\beta\bar{f}_\gamma\gamma_\mu\gamma_5L\bar{b}_\alpha$$

$$O_{f2}^q = \bar{q}_\alpha\gamma_\mu\gamma_5L\bar{f}_\beta\bar{f}_\gamma\gamma_\mu\gamma_5L\bar{b}_\alpha$$

$$O_{3,5}^q = \bar{q}_\alpha\gamma_\mu\gamma_5L\bar{q}_\beta\gamma_\muLR\gamma_\mu\gamma_5L\bar{q}_\alpha$$

$$O_{7,9}^q = \frac{3}{2}\bar{q}_\alpha\gamma_\mu\gamma_5L\bar{b}_\beta\gamma_\muLR\gamma_\mu\gamma_5L\bar{q}_\alpha$$

where $R(L) = 1 \pm \gamma_5$, and $q'\gamma_\mu\gamma_5L\bar{q}_\beta\gamma_\muLR\gamma_\mu\gamma_5L\bar{q}_\alpha$ is summed over $u$, $d$, and $s$. $O_1$ are the tree level and QCD corrected operators. $O_{3-6}$ are the strong gluon induced penguin operators, and operators $O_{7-10}$ are due to $\gamma$ and $Z$ exchange (electroweak penguins), and “box” diagrams at loop level. The Wilson coefficients $c_i^q$ are defined at the scale $\mu \approx m_b$ and have been evaluated to next-to-leading order in QCD. The $c_i^q$ are the regularization scheme independent values obtained in Ref. \[8\]. We give the non-zero $c_i^q$ below for $m_t = 176$ GeV, $\alpha_s(m_Z) = 0.117$, and $\mu = m_b = 5$ GeV,

$$c_1 = -0.307, \quad c_2 = 1.147, \quad c_3 = 0.017, \quad c_4 = -0.037, \quad c_5 = 0.010, \quad c_6 = -0.045,$$

$$c_7 = -1.24 \times 10^{-5}, \quad c_8 = 3.77 \times 10^{-4}, \quad c_9 = -0.010, \quad c_{10} = 2.06 \times 10^{-3},$$

$$c_{3,5}^{u,c} = -c_{4,6}^{u,c}/N_c = P_{s}^{u,c}/N_c, \quad c_{7,9}^{u,c} = P_{e}^{u,c}, \quad c_{8,10}^{u,c} = 0 \quad (4)$$

where $N_c$ is the number of color. The leading contributions to $P_s^i$ are given by: $P_s^i = \left(\frac{\alpha_s}{3\pi}\right)c_2\left(\frac{10}{9} + G(m_s, \mu, q^2)\right)$ and $P_e^i = \left(\frac{\alpha_s}{9\pi}\right)(N_c c_1 + c_2)\left(\frac{10}{9} + G(m_e, \mu, q^2)\right)$. The function $G(m, \mu, q^2)$
is given by
\[ G(m, \mu, q^2) = 4 \int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)q^2}{\mu^2} \, dx \] (5)

All the above coefficients are obtained up to one loop order in electroweak interactions. The momentum \( q \) is the momentum carried by the virtual gluon in the penguin diagram. When \( q^2 > 4m^2 \), \( G(m, \mu, q^2) \) develops an imaginary part. In our calculation, we use \( m_u = 5 \) MeV, \( m_d = 7 \) MeV, \( m_s = 200 \) MeV, \( m_c = 1.35 \) GeV \[9, 10\] and use \( q^2 = m_b^2/2 \).

3 Matrix Elements for \( B \to K^{(*)}\eta' (\eta) \), \( B \to K\pi \) and \( B \to X_s\eta' \)

The effective Hamiltonian described in the previous section consists of operators with a current \( \times \) current structure. Pairs of such operators can be expressed in terms of color singlet and color octet structures which lead to color singlet and color octet matrix elements. We use the factorization approximation, where one separates out the currents in the operators by inserting the vacuum state and neglecting any QCD interactions between the two currents. The basis for this approximation is that, if the quark pair created by one of the currents carries large energy then it will not have significant QCD interactions. Factorization appears to describe nonleptonic B decays rather well\[11\]. To accommodate some deviation from this approximation one can treat \( N_c \), the number of colors that enter in the calculation of the matrix elements, as a free parameter though the value of \( N_c \sim 2 \) is suggested by experimental data on low multiplicity hadronic B decays. In this section we describe the calculation of matrix elements for the exclusive decays \( B \to \eta'(\eta)K^{(*)} \), \( B \to K\pi \) and the quasi-inclusive decay \( B \to \eta'X_s \).
The $\eta$ and $\eta'$ mesons are mixtures of singlet and octet states $\eta_1$ and $\eta_8$ of $SU(3)$.

\[
\begin{bmatrix}
\eta \\
\eta'
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\eta_8 \\
\eta_1
\end{bmatrix}
\]  

(6)

\[
\eta_8 = \frac{1}{\sqrt{6}} \left[ u\bar{u} + d\bar{d} - 2s\bar{s} \right]
\]  

(7)

\[
\eta_1 = \frac{1}{\sqrt{3}} \left[ u\bar{u} + d\bar{d} + s\bar{s} \right]
\]  

(8)

where the mixing angle $\theta$ lies between $-10^0$ and $-20^0$. We can express the decay constants $f^u_{\eta'}, f^d_{\eta'}, f^s_{\eta'}$ in terms of octet and singlet decay constants $f_1, f_8$

\[
f^u_{\eta'} = f^d_{\eta'} = \sqrt{\frac{1}{3} f_\pi \left[ \cos \theta \frac{f_1}{f_\pi} + \frac{\sin \theta}{\sqrt{2}} \frac{f_8}{f_\pi} \right]}
\]  

(9)

\[
f^s_{\eta'} = \sqrt{\frac{1}{3} f_\pi \left[ \cos \theta \frac{f_1}{f_\pi} - \sqrt{2} \sin \theta \frac{f_8}{f_\pi} \right]}
\]  

(10)

and similarly one has

\[
f^u_{\eta} = f^d_{\eta} = \sqrt{\frac{1}{3} f_\pi \left[ \frac{1}{\sqrt{2}} \cos \theta \frac{f_8}{f_\pi} - \sin \theta \frac{f_1}{f_\pi} \right]}
\]  

(11)

\[
f^s_{\eta} = -\sqrt{\frac{1}{3} f_\pi \left[ \sqrt{2} \cos \theta \frac{f_8}{f_\pi} + \sin \theta \frac{f_1}{f_\pi} \right]}
\]  

(12)

In the $SU(3)$ and Nonet symmetry limit, the relations $f_8 = f_\pi = 130$ MeV and $f_1 = f_\pi$ hold. However these symmetries are not exact and we will consider values for $f_1$ and $f_8$ away from this symmetry limit. For example the value of $f_K \sim 1.28 f_\pi$ indicates about a 30% $SU(3)$ breaking effect.

### 3.1 Exclusive Decay

In this section we calculate the rates for the two body noleptonic decay for $B \to \eta' (\eta) K$ and $B \to \pi K$.

To evaluate the rates for the exclusive decays we have to calculate matrix element of the type $< hK | H_{eff} | B >$ where $h$ is a $\pi$ or $\eta' (\eta)$ and $H_{eff}$, as already described in the previous
section, has the form.

\[
H_{\text{eff}} = V_u(c_1 O_1 + c_2 O_2) - \sum_{i=3}^{10} \left[ V_u c_i^u + V_c c_i^c + V_t c_i^t \right] O_i
\]  

(13)

with \( V_u = V_{us} V_{ub} \), \( V_c = V_{cs} V_{cb} \) and \( V_t = V_{ts} V_{tb} \).

As an input to our calculation we need the form factors defined through

\[
\langle K(p_K)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B(p_B)\rangle = \left( p_B + p_K \right)_\mu - \frac{m_B^2 - m_{K^*}^2}{q^2} q_\mu \right] F_1^K(q^2)
\]

\[
+ \frac{m_B^2 - m_{K^*}^2}{q^2} q_\mu F_0^K(q^2)
\]

(14)

with \( q = p_B - p_{K^*} \).

The matrix element \( \langle h|\bar{u}\gamma_\mu(1 - \gamma_5)b|B\rangle \) is similarly described in terms of form factors \( F_1^h(q^2) \) and \( F_0^h(q^2) \).

For the \( K^* \) modes we define

\[
\langle K^*(p'_K)|\bar{s}\gamma_\mu(1 - \gamma_5)u|0\rangle = m_{K^*} g_{K^*} \epsilon^{\lambda \mu}
\]

(15)

The form factors in this case are defined as

\[
\langle K^*(p')|J^\mu|B(p)\rangle = \frac{2V(k^2)}{m_B + m_{K^*}} \epsilon^{\mu\nu\lambda\beta} \epsilon^\nu p_\lambda p_\beta + i(m_B + m_{K^*}) A_1(k^2) \epsilon^{*\mu} - i \epsilon^{*\mu} \cdot k \frac{A_2(k^2)}{m_B + m_{K^*}}(p + p')^\mu
\]

\[
+ i \epsilon^{*\mu} \cdot k \frac{2m_{K^*}(A_0(k^2) - A_3(k^2))}{k^2} k^\mu
\]

(16)

with

\[
A_3(k^2) = \frac{(m_B + m_{K^*}) A_1 - (m_B - m_{K^*}) A_2}{2m_{K^*}}
\]

where \( k = p_B - p_{K^*} \).

3.1.1 \( B^\pm \to K^\pm \eta'(\eta) \)

The amplitude for \( B^- \to K^- \eta' \) can be written as

\[
M = \frac{G_F}{\sqrt{2}} \left[ V_u(a_1 r_1 Q_K + a_2 Q_{\eta'}) - \sum_{i=u,c,t} V_i \left\{ T_i^1 r_1 + T_i^2 r_2 Q_K + T_i^3 Q_{\eta'} \right\} \right]
\]

(17)
where
\[
\begin{align*}
T_1^i &= 2a_3^i - 2a_5^i - \frac{1}{2}a_7^i + \frac{1}{2}a_9^i \\
T_2^i &= a_3^i + a_4^i - a_5^i + (2a_6^i - a_8^i) \frac{m_{\eta'}^2}{2m_s(m_b - m_s)} + \frac{1}{2}a_7^i - \frac{1}{2}a_9^i - \frac{1}{2}a_{10}^i \\
T_3^i &= a_4^i + 2(a_6^i + a_8^i) \frac{m_K^2}{m_u + m_s m_b - m_u} + a_{10}^i
\end{align*}
\]
with
\[
\begin{align*}
a_1 &= c_1 + \frac{\alpha_s}{N_c}, \\
a_2 &= c_2 + \frac{\alpha_s}{N_c}, \\
a_1^j &= c_j^i + \frac{\alpha_s}{N_c}, \\
a_1^{j+1} &= c_j^{i+1} + \frac{\alpha_s}{N_c}, \\
r_1 &= \frac{f_{\phi}}{f_\pi}, \\
r_2 &= \frac{f_{\phi'}}{f_{\pi'}}
\end{align*}
\]
\[
Q_K = iF_0^K(m_{\eta'}^2)(m_B^2 - m_K^2)f_{\pi}, \\
Q_{\eta'} = iF_0^{\eta'}(m_K^2)(m_{B}^2 - m_{\eta'}^2)f_{K},
\]
\[V_i = V_u, V_c, V_t \text{ and } N_c \text{ is effective number of colors.}\]

In the above equations we have used the quark equations of motion to simplify certain matrix elements. The expression for the amplitude can also be used for \(B \rightarrow \eta K\) by making the necessary changes. It is also straightforward to write down the amplitudes for \(B \rightarrow K^*\eta'\) and \(B \rightarrow K^*\eta\) decays.

\[
M = \frac{G_F}{\sqrt{2}} \left[ V_u \left( a_1 f_{\eta'}^u A + a_2 m_K g_K B \right) - \sum_{i=u,c,t} V_i \left\{ (T_1^i f_{\eta'}^i + T_2^i f_{\eta'}^i) A + T_3^i m_K g_K B \right\} \right]
\]

where
\[
\begin{align*}
T_1^i &= 2a_3^i - 2a_5^i - \frac{1}{2}a_7^i + \frac{1}{2}a_9^i \\
T_2^i &= a_3^i + a_4^i - a_5^i + (2a_6^i - a_8^i) \frac{m_{\eta'}^2}{2m_s(m_b - m_s)} + \frac{1}{2}a_7^i - \frac{1}{2}a_9^i - \frac{1}{2}a_{10}^i \\
T_3^i &= a_4^i + a_{10}^i
\end{align*}
\]

with \(A = 2m_K A_0 \varepsilon^* \cdot p_B, \ B = 2\varepsilon^* \cdot p_B F_1^{\eta'}(m_{K^*}^2)\) and we will use \(g_{K^*} = 221\ \text{MeV}\). A similar expression for \(B \rightarrow \eta K^*\) can also be obtained.

3.1.2 \(B \rightarrow K\pi\)

We give below expressions for the various \(B \rightarrow K\pi\) decays
\[ M(B^− \to K\pi^−) = -\frac{G_F}{\sqrt{2}} V_1 \left[ a_4' - \frac{1}{2} a_{10}' + \frac{2m_K^2}{(m_s + m_d)(m_b - m_d)} (a_6' - \frac{1}{2} a_8') \right] Q_\pi \]

\[ Q_\pi = i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2) \]

\[ M(B^− \to K^+\pi^-) = -\frac{G_F}{\sqrt{2}} V_i \left[ a_4' - \frac{1}{2} a_{10}' \right] 2m_K \cdot f_K \cdot \epsilon^\ast \cdot p_B F_1^{B\to\pi}(m_K^2) \] (19)

\[ M(\bar{B}^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} \left[ V_a a_2 - V_i \left\{ a_4' + a_{10}' + \frac{2m_K^2}{(m_s + m_u)(m_b - m_u)} (a_6' + a_8') \right\} \right] Q_\pi \]

\[ Q_\pi = i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2) \]

\[ M(\bar{B}^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} \left[ V_a a_2 - V_i \left\{ a_4' + a_{10}' \right\} \right] 2m_K \cdot f_K \cdot \epsilon^\ast \cdot p_B F_1^{B\to\pi}(m_K^2) \] (20)

\[ M(B^− \to K^−\pi^0) = \frac{G_F}{\sqrt{2}} \left[ V_a \left\{ a_1 Q_K + a_2 Q_\pi \right\} - V_i \left\{ T_1^i Q_K + T_2^i Q_\pi \right\} \right] \]

\[ M(B^− \to K^+\pi^0) = \frac{G_F}{\sqrt{2}} \left[ V_a \left\{ a_1 Q_{K'} + a_2 Q_{\pi'} \right\} - V_i \left\{ T_1^i Q_{K'} + T_2^i Q_{\pi'} \right\} \right] \] (21)

where \( T_1^i = -\frac{3}{2} a_9' + \frac{3}{2} a_9 = T_1^i, T_2^i = a_4' + a_{10}' + \frac{2m_K^2}{(m_s + m_u)(m_b - m_u)} (a_6' + a_8') \), \( T_2^i = a_4' + a_{10}' \), \( Q_\pi = i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi^0}(m_K^2) \) and \( Q_K = i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2) \) \( Q_{\pi'} = 2m_K \cdot f_K \cdot \epsilon^\ast \cdot p_B F_1^{B\to\pi^0} \), and \( Q_{K'} = 2m_K \cdot \epsilon^\ast \cdot p_B A_0^{B\to\pi} \frac{F_8}{\sqrt{2}} \)

### 3.2 Quasi-inclusive Decay

The technique to handle quasi-inclusive decays have been described in detail in Ref. [12]. We represent the total amplitude as the sum of a “two body” and a “three body” piece. The “two body” amplitude is given by

\[ M_2 = i \frac{G_F}{\sqrt{2}} f^\ast_{\mu} p_{\mu} < X|\bar{s}\gamma^\mu(1 - \gamma_5)b|B > FL_u \]

\[-i \frac{G_F}{\sqrt{2}} < X|\bar{s}\gamma^\mu(1 - \gamma_5)b|B > \frac{m_\pi^2}{2m_s} r f^\ast_{\mu} FR_u \] (22)

where

\[ FL_u = V_u \left( c_1 + \frac{c_2}{N_c} \right) + A_3 \left\{ 2 + \left( 1 - \frac{1}{N_c} \right) r \right\} + A_4 \left\{ \frac{2}{N_c} + \left( \frac{1}{N_c} + 1 \right) r \right\} \]
\[ FR_u = -2 \left( \frac{A_5}{N_c} + A_6 \right) + \left( \frac{A_7}{N_c} + A_8 \right) \]

with \( A_i = -(c_i^u V_u + c_i^c V_c + c_i^t V_t) \) and \( r = \frac{f_u^{\eta'}}{f_u^{\eta}} \)

The “three body” piece has the form

\[ M_3 = < \eta | u \gamma^\mu (1 - \gamma_5) b | B > < X | \bar{s} \gamma_\mu (1 - \gamma_5) u | 0 > L_u \]
\[ + < \eta | \bar{u} (1 - \gamma_5) b | B > < X | \bar{s} (1 + \gamma_5) u | 0 > E_u \] (23)

with

\[ L_u = \left( \frac{c_1}{N_c} + c_2 \right) V_u + \left( \frac{A_3}{N_c} + A_4 \right) + \left( \frac{A_9}{N_c} + A_{10} \right) \]
\[ E_u = -2 \left( \frac{A_5}{N_c} + A_6 \right) - 2 \left( \frac{A_7}{N_c} + A_8 \right) \]

Details of the calculations of the branching fraction and decay distributions are given in Ref. [12].

### 4 Results and Discussion

For the exclusive decays to the \( \eta' \) the inputs to the calculation are the form factors, the values of \( f_u^{\eta'} f_u^{\eta} \) or alternately the values of \( f_1 \) and \( f_8 \), \( N_c \) and the mixing angle. We will try to fit the experimental number for \( B \to K \eta' \) by assuming \( f_1 \sim (1.0 - 1.5) f_\pi \) and \( f_8 \sim (1.0 - 1.5) f_\pi \).

We will take \( N_c = 2 \), the effective number of colors. We find that there are several solutions corresponding to different values for the set of parameters \( f_1, f_8, \theta \) that can reproduce the experimental data on \( B \to K \eta' \) and the upper limit of \( B \to K \eta \) if we use the form factors in Ref. [13].

\[ \text{For the CKM parameters we choose two sets } (\rho = 0.15 \text{ and } \eta = 0.33) \text{ and} \]

\[ \text{For the BSW model the form factor } F_0(0) \text{ for } B \to \eta \text{ and } B \to \eta' \text{ are approximately same and so we assume this equality for the form factors in Ref. [13] where only the } B \to \eta \text{ form factor is calculated.} \]
(\(\rho = -0.15\) and \(\eta = 0.33\)) \cite{15}. In Table 1 we give the rates for the decays involving a \(K^*(s)\) and \(\eta, \eta'\) and \(\pi\) in the final state for \(\theta = -17^0\) and \(N_c = 2\) for two sets of the CKM parameters with various values of \(f_1\) and \(f_8\). Phenomenological studies involving radiative decays of the \(\eta\) and \(\eta'\) indicate values for \(f_1 \sim 1.1f_\pi\) and \(f_8 \sim 1.3f_\pi\) \cite{16}. We show the results of using the form factors in Ref \cite{13} and Ref \cite{14} in the third and fourth column of Table 1. The entries in Table 1 correspond to the choice of CKM parameters \((\rho = 0.15, \eta = 0.33)\) and for \(\eta'\) in the final state we also include a second number in the parentheses corresponding to the second set of CKM parameters though the data seem to favor a positive value of \(\rho\). The branching ratio for \(B \to \eta K\) is suppressed by about a factor of 10 or more relative to \(B \to \eta'K\) while \(B \to \eta K^*\) is enhanced relative to \(B \to \eta'K^*\) by a small amount. This is in qualitative agreement with Ref \cite{17} but we find a smaller enhancement \(B \to \eta K^*\) relative to \(B \to \eta'K^*\). From the branching ratios in Table 1, it is not possible to rule out the four quark operator explanation for the large branching ratio in \(B^- \to \eta'K^-\).

For the quasi-inclusive decay we will use the same parameters as used in the exclusive decays with the second set of the CKM parameters as this set gives the larger rate between the two choices. We plot the decay distribution \(d\Gamma/dM_{rec}\) in Fig. 1 showing the contributions from the effective Hamiltonian of four quark operators. From Fig. 1 we find the contribution from the four quark operators to the branching ratio \(B \to \eta'X\) is around \(1.3 \times 10^{-4}\) for \(E_{\eta'} > 2.2\) which is the signal region. This is far too small to account for the signal observed at high \(X_s\) mass. This continues to be true even if when all parameters are allowed to vary over a reasonable range.

5 Conclusion

Summarizing, we have calculated the effects of the effective Hamiltonian of four quark operators to the exclusive and quasi-inclusive decays of the B meson to \(\eta'\). Our analysis
indicate that the exclusive data can be explained by the four quark operators without significant contribution from the mechanism $b \to s g^*, g^* \to \eta' gg$ or from the intrinsic charm content of the $\eta'$. The contribution to the quasi-inclusive rate from the four quark operator is not enough to account for the observed signal.

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7 Figure Caption

- **Fig. 1** This figure shows the decay distribution as a function of the recoil mass $M_{rec}$. 

12
| Process                  | Experimental BR \[1\] | Branching Ratio \[13\] (BR) | BR \[14\] | \(f_1, f_8\) |
|-------------------------|-----------------------|-------------------------------|-----------|--------------|
| \(B^- \to K^- \eta'\)   | \((7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5}\) | \(5.33 \times 10^{-5}\)     | \(1.05 \times 10^{-5}\) | \(1.0, 1.0\) |
|                         |                       | \(6.1 \times 10^{-5}\)      | \(1.32 \times 10^{-5}\) | \(1.1, 1.3\) |
|                         |                       | \((6.44, 7.83) \times 10^{-5}\) | \(1.45 \times 10^{-5}\) | \(1.1, 1.5\) |
| \(B^0 \to K^0 \eta'\)   | \((5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5}\) | \(4.97 \times 10^{-5}\)     | \(9.82 \times 10^{-6}\) | \(1.0, 1.0\) |
|                         |                       | \(5.69 \times 10^{-5}\)      | \(1.24 \times 10^{-5}\) | \(1.1, 1.3\) |
|                         |                       | \((6.01, 7.3) \times 10^{-5}\) | \(1.36 \times 10^{-5}\) | \(1.1, 1.5\) |
| \(B^- \to K^- \eta\)    | \(< 8.0 \times 10^{-6}\) | \(7.82 \times 10^{-6}\)     | \(2.3 \times 10^{-7}\) | \(1.0, 1.0\) |
|                         |                       | \(5.13 \times 10^{-6}\)      | \(3.87 \times 10^{-7}\) | \(1.1, 1.3\) |
|                         |                       | \((3.68, 7.3) \times 10^{-6}\) | \(7.5 \times 10^{-7}\) | \(1.1, 1.5\) |
| \(B^0 \to K^0 \eta\)    | \(< 8.0 \times 10^{-6}\) | \(7.22 \times 10^{-6}\)     | \(2.16 \times 10^{-7}\) | \(1.0, 1.0\) |
|                         |                       | \(4.72 \times 10^{-6}\)      | \(3.68 \times 10^{-7}\) | \(1.1, 1.3\) |
|                         |                       | \((3.39, 6.7) \times 10^{-6}\) | \(7.13 \times 10^{-7}\) | \(1.1, 1.5\) |
| \(B^- \to K^- \eta'\)   | \(< 2.4 \times 10^{-4}\) | \(1.18 \times 10^{-5}\)     | \(4.79 \times 10^{-7}\) | \(1.0, 1.0\) |
|                         |                       | \(1.15 \times 10^{-5}\)      | \(5.67 \times 10^{-7}\) | \(1.1, 1.3\) |
|                         |                       | \(1.14 \times 10^{-5}\)      | \(5.84 \times 10^{-7}\) | \(1.1, 1.5\) |
| \(B^- \to K^- \eta\)    | \(< 2.4 \times 10^{-4}\) | \(1.36 \times 10^{-5}\)     | \(9.43 \times 10^{-7}\) | \(1.0, 1.0\) |
|                         |                       | \(1.31 \times 10^{-5}\)      | \(7.38 \times 10^{-7}\) | \(1.1, 1.3\) |
|                         |                       | \(1.27 \times 10^{-5}\)      | \(6.20 \times 10^{-7}\) | \(1.1, 1.5\) |
| \(B^\pm \to K^\pi^\pm\) | \((2.3^{+1.1+0.2}_{-1.0-0.2} \pm 2.0) \times 10^{-5}\) | \(2.25 \times 10^{-5}\)     | \(8.87 \times 10^{-6}\) | \(\ldots\) |
| \(B^\pm \to K^*\pi^\pm\) | \(\ldots\)            | \(1.50 \times 10^{-5}\)     | \(5.92 \times 10^{-6}\) | \(\ldots\) |
| \(B^0(\bar{B}^0) \to K^\pi^\pm\) | \((1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1) \times 10^{-5}\) | \(2.21 \times 10^{-5}\)     | \(8.79 \times 10^{-6}\) | \(\ldots\) |
| \(B^0(\bar{B}^0) \to K^*\pi^\pm\) | \(\ldots\)            | \(1.21 \times 10^{-5}\)     | \(4.77 \times 10^{-6}\) | \(\ldots\) |
| \(B^\pm \to K^{\pm}\pi^0\) | \((1.6^{+0.6+0.3}_{-0.5-0.2} \pm 0.1) \times 10^{-4}\) | \(1.02 \times 10^{-5}\)     | \(4.25 \times 10^{-6}\) | \(\ldots\) |
| \(B^\pm \to K^{\pm}\pi^0\) | \(\ldots\)            | \(5 \times 10^{-6}\)        | \(2.3 \times 10^{-6}\) | \(\ldots\) |
Four Quark