An Application of Non-additive Measures and Corresponding Integrals in Tourism Management

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Abstract:
Non-additive measures and corresponding integrals originally have been introduced by Choquet in 1953 (1) and independently by Sugeno in 1974 (2) in order to extend the classical measure by replacing the additivity property to non-additive property. An important feature of non-additive measures and fuzzy integrals is that they can represent the importance of individual information sources and interactions among them. There are many applications of non-additive measures and fuzzy integrals such as image processing, multi-criteria decision making, information fusion, classification, and pattern recognition. This paper presents a mathematical model for discussing an application of non-additive measures and corresponding integrals in tourism management. First, the problem of tourism management is described for one of the tourism companies in Iraq. Then, fuzzy integrals (Sugeno integral, Choquet integral, and Shilkret integral) are applied with respect to non-additive measures to evaluate the grade of the gratification of the tourist of staying in a particular town for determining the best evaluation.

Key words: Choquet integral, Non-additive measures, Shilkret integral, Sugeno integral, Tourism management.

Introduction:
Non-additive measures (capacities (1), or fuzzy measures (2)) and fuzzy integral theory are an evolution of classical measure theory. Non-additive measures and fuzzy integral theory take into consideration the importance of criteria and interactions among them and have excellent potential for applications in different scientific fields. Non-additive measures and corresponding integrals have been studied and applied in diverse fields (see, e.g. (3-10)). This paper discusses an application of non-additive measures and corresponding integrals in Tourism management for the case study. The problem of tourism management that can be solved using the non-additive model mathematically (non-additive measures and corresponding integrals) is described. Then, fuzzy integrals (Sugeno integral, Choquet integral and Shilkret integral) are applied with respect to λ-non-additive measures to evaluate the grade of the gratification of the tourist of staying in a particular town for determining the best evaluation. This paper is organized as follows. Section 2 is devoted to describe non-additive measure. Section 3 applies non-additive measures and corresponding integrals in tourism management for the case study. Finally, the paper finishes with some conclusions.

Throughout the paper, the universal set $C=\{c_1,c_2,\ldots,c_n\}$ is the set of elements (criteria in our application) and $P(C)$ is the power set of $C$. $\lor, \land$, are applied which denoted to supremum and infimum respectively.

Non-additive Measures:
Non-additive measures are a generalization of classical measures by means of using non-additivity monotonicity instead of additivity property. The formal definition of the non-additive measure is given as follow.

Definition, (11): A non-additive measure on the set $C$ of elements is a set function $\mu: P(C) \rightarrow [0,1]$ satisfying the following axioms:
1- $\mu(\emptyset) = 0, \mu(C) = 1$ ,
2- $A \subset B \subset C$ implies $\mu(A) \leq \mu(B)$

In (12), Sugeno presented the concept of λ-non-additive measure which is a special case of...
non-additive measure. The following definition gives \( \lambda \)-non-additive measure.

**Definition, (13)(14):** Let \( C \) be a finite set. \( \lambda \)-nonadditive measure is a nonnegative set function \( \mu_A: P(C) \to [0,1] \) satisfying \( \mu_A(A \cup B) = \mu_A(A) + \mu_A(B) + \mu_A(A) \mu_B(B) \) for all \( A, B \in C \) whenever \( A \cap B = \emptyset \), where \( \lambda \) is a parameter, \( \lambda \in (-1, \infty) \).

The \( \mu_A(C) \) can be formulated as follows (15)

\[
\mu_A(C) = \mu_A^0(c_1, c_2, \ldots, c_n) + \lambda \mu_A^1(c_1) \mu_A^1(c_2) + \lambda^2 \mu_A^2(c_1, c_2) \mu_A^1(c_3) \mu_A^1(c_4) + \cdots + \lambda^n \mu_A^n(c_1, c_2, \ldots, c_n) = \frac{1}{\lambda} \prod_{i=1}^{n} (1 + \mu_A(c_i)) - 1 \]

The parameter \( \lambda \) can be calculated as stated in the following theorem.

**Theorem,(12):** Let \( C = \{c_1, c_2, \ldots, c_n\} \) when \( n \geq 2 \) and \( \mu_A \) be a \( \lambda \)-measure on \( (C), \) knowing \( \mu_A(c_1) = a_i \geq 0 \) and \( \mu_A(c) = b \geq a_i \), \( i = 1, 2, 3, \ldots, n \).

The value of \( \lambda \) can be uniquely determined by equation,

\[
b \lambda + 1 = \prod_{i=1}^{n} (1 + a_i \lambda) \]

It can be write as \( b \lambda + 1 = \prod_{i=1}^{n} (1 + b \mu_A(c_i)) \lambda \) since \( \mu_A(c) = b \), therefore above equation can be written as \( \lambda + 1 = \prod_{i=1}^{n} (1 + \mu_A(c_i)) \lambda \). \quad (2)

**Fuzzy Integrals:**

A special type of nonlinear integral is a fuzzy integral with respect to non-additive measure. The fuzzy integrals are appropriate tools to represent the weights of criteria with non-additive characteristics as the non-additive measure.

This section, briefly recalls the three most famous fuzzy integrals (i.e., the Sugeno, Choquet, and Shilkret integrals) on \([0,1]\)-valued function.

**Sugeno Integral:**

Sugeno gave the concept of fuzzy integral as nonlinear functions defined with respect to fuzzy measures as \( \lambda \)-fuzzy measure. The Sugeno integral (2) of a function \( f: X \to [0,1] \) with respect to \( \mu \) is defined by

\[
S_\mu(f) = \bigvee_{j=1}^{n} f(c_{s(j)}) \wedge \mu(A_{s(j)}) \quad (3)
\]

where \( f(c_{s(i)}) \) indicates that the indices have been permuted so that \( 0 \leq f(c_{s(1)}) \leq \cdots \leq f(c_{s(n)}) \leq 1 \), \( A_{s(i)} = \{c_{s(1)}, c_{s(2)}, \ldots, c_{s(n)}\} \), and \( A_{s(n)} = \emptyset \).

**Choquet Integral:**

Choquet integral is a generalization of the Lebesgue integral. That is, Choquet integral is defined with respect to a nonadditive measure. Choquet integral has algebraic properties which makes more suitable fuzzy integral for multicriteria decision making problems.

Let us suppose that \( \mu \) be a non-additive measure defined on \( C \) then Choquet integral of a function \( f:C \to [0,1] \) w.r.t a non-additive measure \( \mu \) is defined as (1)

\[
(c) \int f \, d\mu = \sum_{i=1}^{n} f(c_{s(i)}) \mu(A_{s(i)}) - \mu(A_{s(i+1)}) \quad (4)
\]

Where \( f(c_{s(i)}) \) indicates that the indices have been permuted so that \( 0 \leq f(c_{s(1)}) \leq f(c_{s(2)}) \leq \cdots \leq f(c_{s(n)}) \leq 1, A_{s(i)} = \{c_{s(1)}, \ldots, c_{s(n)}\}, A_{s(n+1)} = \emptyset \).

**Shilkret integral:**

Shilkret integral (16) originally has been defined for maxitive (\( v \)-additive) measures, but it is also defined for any fuzzy measure as follows.

Let \( \mu: C \to [0,1] \) be a non-additive measure defined on \( C \) then Shilkret integral of a function \( f:C \to [0,1] \) w.r.t a non-additive measure \( \mu \) is defined as (sh)

\[
\int f \, d\mu = \bigvee_{i=1}^{n} \mu(A_{s(i)}) \quad (5)
\]

where \( f(c_{s(i)}) \) indicates that the indices have been permuted so that \( 0 \leq f(c_{s(1)}) \leq f(c_{s(2)}) \leq \cdots \leq f(c_{s(n)}) \leq 1, A_{s(i)} = \{c_{s(1)}, \ldots, c_{s(n)}\}, A_{s(n+1)} = \emptyset \).

**An application of Non-additive Measures and Corresponding Integrals in Tourism Management**

This section will apply the non-additive model (non-additive measures and fuzzy integrals) on tourism management for one of the tourism companies in Iraq to evaluate the visitation more cities with greater satisfaction to the tourist of staying in a particular town. (i.e. to decide which the most acceptable town for the tourist to indwelling).

**Empirical Example:**

A case of tourism management is studied for one of the tourism companies in Iraq. The company of tourism is Al-Massal Company. Let \( C \) be a set of cities \( C = \{c_1, c_2, c_3, c_4, c_5\} \) that a travel in Iraq intends to visit these cities \( c_1, c_2, c_3, c_4, c_5 \) in Turkey. That is, Istanbul compatible to \( c_1 \), Ankara compatible to \( c_2 \), Bursa compatible to \( c_3 \), Antalya compatible to \( c_4 \) and Trabzon compatible to \( c_5 \). The grade of gratification (non-additive measure, fuzzy measure) of tourist to visiting the five mentioned cities is given in table 1. At first, we find the \( \lambda \) value using eq.(2)

\[
\prod_{i=1}^{n} (1 + \lambda \mu(c_i)) = 1 + \lambda
\]

\[*\] It is a company that organizes on several touristic trips to various countries of the world. It has officially obtained a work permit from the Iraqi Ministry of Tourism under the number 473. [www.almassal.com]
The measure of gratifications monotonically increasing i.e. (the more cities are visited) and is bounded i.e. (μ_3(∅) = 0 that is no visit implies no gratification and μ_3(C) = 1 means visiting all cities implies extreme gratification). Non-additive measure for some selected subsets A of the cities C is computed using eq(1), for example the subset \{c_1, c_2, c_3, c_5\} is given as

$$\mu_3(\{c_1, c_2, c_3, c_5\}) = \frac{1}{2} \left(\mu_3(c_1) + \mu_3(c_2) + \mu_3(c_3) + \mu_3(c_5)\right) - 1 = \frac{1}{2} \mu_3(c_1) + \frac{1}{2} \mu_3(c_2) + \frac{1}{2} \mu_3(c_3) + \frac{1}{2} \mu_3(c_5) - 1 = \frac{1}{2} \mu_3(\{c_3\}) + \frac{1}{2} \mu_3(\{c_4\}) - 1 = 0.99812$$

Similarly, for all subsets of C are shown in Table 1.

Table 1. The grade of gratifications for all subset of C

| Sets       | μ_3 | Sets       | μ_3 | Sets       | μ_3 |
|------------|-----|------------|-----|------------|-----|
| \{c_1\}   | 0.9 | \{c_2, c_4\} | 0.8036 | \{c_2, c_3, c_5\} | 0.9709004 |
| \{c_2\}   | 0.5 | \{c_2, c_3\} | 0.85042 | \{c_1, c_4, c_5\} | 0.94078 |
| \{c_3\}   | 0.8 | \{c_3, c_4\} | 0.92037 | \{c_3, c_5\} | 0.976944 |
| \{c_4\}   | 0.6 | \{c_3, c_5\} | 0.940672 | \{c_1, c_2, c_3, c_4\} | 0.99712 |
| \{c_5\}   | 0.7 | \{c_4, c_5\} | 0.880504 | \{c_1, c_2, c_3, c_5\} | 0.99812 |
| \{c_1, c_2\} | 0.95054 | \{c_1, c_2, c_3\} | 0.99102 | \{c_1, c_2, c_4, c_5\} | 0.99509 |
| \{c_1, c_3\} | 0.9808 | \{c_1, c_2, c_5\} | 0.9809003 | \{c_1, c_3, c_4, c_5\} | 0.99874 |
| \{c_1, c_4\} | 0.96064 | \{c_1, c_5\} | 0.9859 | \{c_2, c_5\} | 0.98905 |
| \{c_1, c_5\} | 0.970756 | \{c_1, c_4, c_5\} | 0.99305 | \{c_2, c_5\} | 0.96084 |
| \{c_2, c_3\} | 0.90048 | \{c_1, c_2, c_4, c_5\} | 0.995063 | \{c_1, c_2, c_4, c_5\} | 1 |

The value for the measure are expressed using the same terms as the grade of gratification, hence f(C) is comparable with μ(A) where A is a subset of C. Therefore, by using μ and f we can define a relation μ_A(f) = μ_A(\{c | f(c) ≥ f(c_i)\}). To apply three main fuzzy integrals (Sugeno, Choquet, and Shilkret), we shall use ascending permutation on f(c_i). That is

$$f(c_4) ≤ f(c_5) ≤ f(c_1) ≤ f(c_2) ≤ f(c_3) \quad \ldots \quad (7)$$

Table 2 gives measures for such accessibility from Erbil.

Table 2. Accessibility degrees from Erbil city

| Group or Collection | \{c_1\} | \{c_2\} | \{c_3\} | \{c_4\} | \{c_5\} |
|---------------------|---------|---------|---------|---------|---------|
| f                   | 0.6     | 0.7     | 0.5     | 0.3     | 0.8     |

The grade of gratification with reference to visiting definite towns will be joint with the grade of gratification of doing the tourist itself to the peculiar towns. This grade of gratification will be in attribution to the accessibility of such towns from the private location of the tourist. The grade of gratification will be expressed by a function f: C → [0,1]. Assume that the tourist sits at Erbil city, Trabzon town is the most agreeing town then Ankara city and so on. So that from these agreeing towns, we get the following permutation on (c).

$$f(c_4) ≤ f(c_5) ≤ f(c_1) ≤ f(c_2) ≤ f(c_3) \quad \ldots \quad (7)$$

Table 2 gives measures for such accessibility from Erbil.
Now by eq (3), we can get the Sugeno integral of $f(c)$ with respect to $\mu_{af}$ as follows:

$$\int f \, d\mu_{af} = \bigwedge\{\Lambda\{f(c_1), \mu_{af}(c_1)\}, \Lambda\{f(c_2), \mu_{af}(c_2)\}, \Lambda\{f(c_3), \mu_{af}(c_3)\}\}.$$ 

By eq (4), the Choquet integral of $f(c)$ with respect to a non-additive measure $\mu_{af}$ given as follows:

$$(c) \int f \, d\mu_{af} = f(c_1) \left( \mu_{af}(c_1) - \mu_{af}(c_2) \right) + f(c_2) \left( \mu_{af}(c_2) - \mu_{af}(c_3) \right) + f(c_3) \left( \mu_{af}(c_3) - \mu_{af}(c_4) \right),$$

Finally, we apply Shilkret integral given as follows:

$$(Sh) \int f \, d\mu_{af} = \bigvee\{f(c_1) \cdot \mu_{af}(c_1), f(c_2) \cdot \mu_{af}(c_2), f(c_3) \cdot \mu_{af}(c_3), f(c_4) \cdot \mu_{af}(c_4), f(c_5) \cdot \mu_{af}(c_5)\}.$$

The following an algorithm is the way to calculate the Sugeno, Choquet and Shilkret integral.

**Algorithm 2:** computation of fuzzy integral (Sugeno ,Choquet and Shilkret)

1. Input non additive measure $\mu_{af}$.
2. Input function $f$.
3. Sort $f$ ascending, sort $\mu_{af}$ descending.
4. $n=5, s=0$
5. for $i=1 : 5$
6. for $j=i : 5$
7. if $i < j$
8. calculate $r(i)=\min\{f(c_i), \mu_{af}(c_i)\}$
9. calculate $(s) \int f \, d\mu_{af} = \max(s)$
10. calculate $(c) \int f \, d\mu_{af} = \sum_{i=1}^{n} f(i)(\mu_{af}(i) - \mu_{af}(j+1))$
11. calculate $(Sh) \int f \, d\mu = \max(f(i), \mu_{af}(j))$
12. end for
13. end for

**Comparison of Results:**

To evaluate the grade of gratification of staying in a particular town (in this case, Erbil), we have used the Sugeno, Choquet and Shilkret integrals with respect to $\lambda$-non additive measures $\mu_{af}$. The results clearly show that Choquet integral is better than Sugeno and Shilkret integrals as a tool to determine the most suitable town for tourists to stay.

**Table 3. The grade of gratification of the tourist to visiting the five cities from Erbil city**

| Group or collection | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---------------------|-------|-------|-------|-------|-------|
| $\mu_{af}$          | 0.9859| 0.85042| 0.99812| 1     | 0.7   |

To apply Sugeno integral, ascending permutation on $g(c_i)$ is used. That is $g(c_2) \leq g(c_2) \leq g(c_1) \leq g(c_2) \leq g(c_4).$

| Group or collection | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_3$ |
|---------------------|-------|-------|-------|-------|-------|
| $\mu_{ag}$          | 0.99305| 0.99712| 0.92057| 0.6   | 1     |

To apply Sugeno integral, ascending permutation on $h(c_i)$ is used. That is $h(c_5) \leq h(c_3) \leq h(c_1) \leq h(c_5) \leq h(c_4).$

| Group or collection | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_3$ |
|---------------------|-------|-------|-------|-------|-------|
| $\mu_{an}$          | 0.99305| 0.99712| 0.92057| 0.6   | 1     |
The Sugeno integral of $h(c)$ with respect to non-additive measure $\mu_{\lambda h}$ is $SI(\mu_{\lambda h}) = 0.6$, Choquet integral is $Ch(\mu_{\lambda h}) = 0.61$ and the Shilkret integral $Sh(\mu_{\lambda h}) = 0.42$.

Now consider Basra city is the fourth alternatives to non additive measure for each cities (Erbil, Baghdad, Najaf, Basra).

Table 8. Display the accessibility grades from Basra city.

| Group or collection | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|--------------------|-------|-------|-------|-------|-------|
| $t$                | 0.41  | 0.32  | 0.45  | 0.6   | 0.1   |

The following (Fig.1). shows the results of Sugeno, Choquet, and Shilkret integral with respect to non additive measure for each cities (Erbil, Baghdad, Najaf, Basra).

Figure 1. Fuzzy integral (Sugeno, Choquet and Shilkret)

Experimental results for Sugeno, Choquet, and Shilkret integral with respect to a non-additive measure $\mu_{\lambda f}$ for Erbil city shown it has taken higher value than the same integrals with respect to $\mu_{\lambda d}$, $\mu_{\lambda h}$, $\mu_{\lambda t}$. Therefore, it can be deduced that the tourist will choose to stay in Erbil instead of staying in other cities.

Table 9. The grade of gratification of the tourist to visiting the five cities from Basra city.

| Group or collection | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|--------------------|-------|-------|-------|-------|-------|
| $\mu_{\lambda t}$ | 0.99305 | 0.99712 | 0.92057 | 0.6 | 1 |

To apply Sugeno integral, ascending permutation on $t(c_i)$ is used. That is $t(c_5) \leq t(c_2) \leq t(c_1) \leq t(c_3) \leq t(c_4)$.

The Sugeno integral of $t(c)$ with respect to non-additive measure $\mu_{\lambda h}$ is $SI(\mu_{\lambda h}) = 0.6$. Choquet integral is $Ch(\mu_{\lambda h}) = 0.53$ and the Shilkret integral $Sh(\mu_{\lambda h}) = 0.36$.

The following table compares the results of the Sugeno, Choquet and Shilkret integral with respect to non additive measures for each cities (Erbil, Baghdad, Najaf, Basra).

Table 10. Results of the Sugeno, Choquet and Shilkret integral

| Types of integrals | Result for $\mu_{\lambda f}$ (Erbil city) | Result for $\mu_{\lambda d}$ (Baghdad city) | Result for $\mu_{\lambda h}$ (Najaf city) | Result for $\mu_{\lambda t}$ (Basra city) |
|--------------------|------------------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|
| SI                 | 0.7                                      | 0.6                                      | 0.6                                    | 0.6                                    |
| Ch                 | 0.75                                     | 0.71                                     | 0.61                                   | 0.53                                   |
| Sh                 | 0.56                                     | 0.48                                     | 0.42                                   | 0.36                                   |

Conclusions:

This paper has established a simple mathematical model for representing an application of non-additive measures and corresponding fuzzy integrals in tourism management. Secondly, it has demonstrated how these fuzzy integrals (Sugeno, Choquet, Shilkret) with respect to $\lambda$- non additive measure can be applied to evaluate the gratification of the tourist of staying in a particular town. The results were compared to choose the town which has the best evaluation.

Conflicts of Interest: None.

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