Effect of Redshift Distributions of Fast Radio Bursts on Cosmological Constraints

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ABSTRACT

Nowadays, fast radio bursts (FRBs) have been a promising probe for astronomy and cosmology. However, it is not easy to identify the redshifts of FRBs to date. Thus, no sufficient actual FRBs with identified redshifts can be used to study cosmology currently. In the past years, one has to use the simulated FRBs with “known” redshifts instead. To simulate an FRB, one should randomly assign a redshift to it from a given redshift distribution. But the actual redshift distribution of FRBs is still unknown so far. In the present work, we study the effect of various redshift distributions on cosmological constraints. We find that different redshift distributions lead to different constraining abilities from the simulated FRBs. This result emphasizes the importance to find the actual redshift distribution of FRBs, and reminds us of the possible bias in the FRB simulations due to the redshift distributions.

PACS numbers: 98.80.Es, 98.70.Dk, 98.80.-k
I. INTRODUCTION

Currently, fast radio bursts (FRBs) have become a thriving field in astronomy and cosmology. Since their first discovery, an extragalactic/cosmological origin is strongly suggested to FRBs, due to the large dispersion measure (DM) of observed FRBs well in excess of the Galactic value. To date, the redshifts of several FRBs have been identified by the precise localizations of their host galaxies. For example, the redshift of the first known repeating FRB (namely FRB 121102) has been identified as \( z = 0.19273 \). Currently, FRB 190523 has the largest identified redshift \( z = 0.66 \). The 12 FRBs with identified redshifts as of November 2020 were summarized in e.g. [23]. Clearly, they are all at cosmological distances. Therefore, it is justified and well-motivated to study cosmology by using FRBs. We refer to e.g. [24–40] for some interesting works on the FRB cosmology.

As is well known, one of the key observational quantities of FRBs is the dispersion measure DM. The radio signals of different frequencies from FRB reach earth at different times, if there is a cold plasma along the path. According to e.g. [11], in the rest frame, an electromagnetic signal propagates through an ionized medium (plasma) with a velocity less than the speed of light in vacuum \( c \), and hence this signal of frequency \( \nu \gg \nu_p \) is delayed relative to a signal in vacuum by a time proportional to \( \nu^{-2} \) and the column density of the free electrons, where \( \nu_p \) is the plasma frequency. In practice, it is convenient to measure the time delay \( \Delta t \) in the observer frame between two signals of frequencies \( \nu_1 \) and \( \nu_2 \). Taking the redshift effect into account, this time delay is given by

\[
\Delta t = \frac{e^2}{2\pi m_e c^2} \left( \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \int \frac{n_e(z)}{1+z} dl = \frac{e^2}{2\pi m_e c^2} \left( \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) DM,
\]

where \( n_e(z) \) is the number density of free electrons in the medium (given in units of \( \text{cm}^{-3} \)) at redshift \( z \), \( m_e \) and \( e \) are the mass and charge of electron, respectively. Using Eq. (1), one can get the column density of the free electrons

\[
DM \equiv \int \frac{n_e(z)}{1+z} dl
\]

by measuring the time delay \( \Delta t \) between two signals of frequencies \( \nu_1 \) and \( \nu_2 \). It is worth noting that the distance \( dl \) along the path in DM records the expansion history of the universe. Therefore, DM plays a key role in the FRB cosmology.

Clearly, the observed DM of FRB can be separated into

\[
DM_{obs} = DM_{MW} + DM_{IGM} + DM_{HG},
\]

where \( DM_{MW} \), \( DM_{IGM} \), and \( DM_{HG} \) are the contributions from the Milky Way, the intergalactic medium (IGM), and the host galaxy (HG, including interstellar medium of HG and the near-source plasma), respectively. Since thousands of pulsars in the Milky Way and the Small/Large Magellanic Clouds were observed, one can reliably infer the density distribution of the free electrons in or nearby the Milky Way from the observed DMs of these pulsars. So, for a well-localized FRB, the corresponding \( DM_{MW} \) can be estimated with reasonable certainty by using the well-known tools NE2001 [44, 45] or YMW16 [46]. Thus, subtracting the “known” \( DM_{MW} \) from \( DM_{obs} \) in Eq. (2), it is convenient to introduce the extragalactic DM of an FRB as the observed quantity

\[
DM_E \equiv DM_{IGM} + DM_{HG}.
\]

The main contribution to DM of FRB comes from IGM. In fact, \( DM_{IGM} \) carries the key information about IGM and the cosmic expansion history. In principle, one can constrain cosmological models by using the observed DM of a large number of FRBs with identified redshifts.

Unfortunately, it is not so easy to identify the redshifts of FRBs to date. Since the first discovery of FRB [13, 14], the redshifts have been identified only for 12 extragalactic FRBs, as is summarized in e.g. [23]. Therefore, no sufficient actual FRBs with identified redshifts can be used to study cosmology currently. In the past years, one has to use the simulated FRBs with “known” redshifts instead. The devil is in the details. To simulate an FRB, one should randomly assign a redshift to it from a given redshift distribution. However, the actual redshift distribution of FRBs is still unknown, because there are no sufficient actual FRBs with identified redshifts to date. Although various redshift distributions have been extensively considered in the literature, most of them are artificial in some sense, or directly borrowed from other objects such as Gamma-ray bursts (GRBs) or star-formation history (SFH). In the present work, we are interested to see whether or not various redshift distributions used to simulate FRBs can affect cosmological constraints considerably.
The rest of this paper is organized as follows. In Sec. II, we introduce various redshift distributions for FRBs considered extensively in the literature. In addition, we also propose two new redshift distributions inferred from the actual FRBs data to date, which are fairly different from the existing ones in the literature. In Sec. III, we briefly describe the key points to simulate FRBs. In Sec. IV, we constrain various cosmological models by using these simulated FRBs, and try to see the effect of redshift distributions on cosmological constraints. In Sec. V some brief concluding remarks are given.

II. VARIOUS REDSHIFT DISTRIBUTIONS FOR FRBS

A. New redshift distributions

In the literature, various redshift distributions for FRBs have been extensively considered. To our best knowledge, (almost) all of them are not inferred from the actual FRBs data. So, let us try it at first. Note that an online catalogue of the observed FRBs can be found in FRBCAT [47], which summarizes almost all observational aspects concerning the published FRBs. As of January 2021, FRBCAT catalogue contains 129 observed FRBs. Of course, most of them have no identified redshifts. However, one can roughly infer the redshift from the observed DM of FRB, following the methodology described in e.g. Sec. 2.2 of [48]. Since this is an inferred redshift, we do not require a high precision, and hence we can slightly simplify the methodology of [48]. For an observed FRB, its DM$_{obs}$ can be separated into three components as in Eq. (2). One can directly read its DM$_{MW}$ from FRBCAT [47], which is estimated by using NE2001 [44, 45] or YMW16 [46]. On the other hand, one can assume DM$_{HG} = 50/(1+z)$ pc cm$^{-3}$ following e.g. [48, 49].

The mean DM$_{IGM}$ can be estimated by DM$_{IGM} = 3cH_0\Omega_b/8\pi G m_p \int_0^\infty (f_{IGM}(\tilde{z})f_c(\tilde{z})(1+z)/E(\tilde{z})) \, d\tilde{z}$ (see e.g. [24, 25, 35–38, 48]), in which one can assume the simplest flat $\Lambda$CDM cosmology to estimate $E(z) = (\Omega_m (1+z)^3 + (1-\Omega_m))^{1/2}$ and adopt the values of $\Omega_m$, $\Omega_b$, $H_0$ from Planck 2018 results [50], while $f_{IGM} = 0.83$ and $f_c = 7/8$ as in e.g. [25, 26, 28, 35]. So, the right hand side of Eq. (2) becomes an explicit function of redshift $z$. For an observed FRB, one can infer its redshift $z$ by numerically solving Eq. (2) with the observational value of DM$_{obs}$. Of course, we stress that it is just a roughly inferred redshift only for reference. Following this methodology, now we have 129 actual FRBs with inferred redshifts. To get a reasonable redshift distribution, we need an anchor. Very recently, FRB 200428 in our Milky Way was observed (see e.g. [51, 54]). So, we also take this FRB at $z = 0$ into account. We fit these 130 actual FRBs with the fitter Python package [55], which can find the most probable distribution(s) for a given data sample by using 80 distributions in SciPy [56]. Finally, we obtain the best redshift distribution for these 130 actual FRBs, namely Burr distribution [57] (note that Burr Type III distribution is called Burr distribution for short in SciPy). The standardized Burr distribution is given by [57]

$$f_{\text{Burr}}(x, b, k) = \frac{bkx^{b-1}}{(1+x^k)^{k+1}},$$

where $x \geq 0$, $b > 0$ and $k > 0$. One can shift and/or scale this distribution by using the shift and scale parameters ($\ell$ and $s$), namely [57]

$$P_{\text{Burr}}(z, b, \ell, s) = f_{\text{Burr}}((z-\ell)/s, b, k)/s.$$  

The best parameters for the 130 actual FRBs mentioned above are $b = 2.8733$, $k = 0.4568$, $\ell = -0.0043$ and $s = 0.7357$. We present this best Burr distribution in the left panel of Fig. 1.

As mentioned above, there are 12 extragalactic FRBs with identified redshifts to date, as is summarized in e.g. Table 2 of [23]. Eleven of them are also compiled in the above 129 FRBs catalogue, while FRB 200430 is not. We replace the inferred redshifts of these 11 FRBs by the actually identified ones, and also take FRB 200430 into account. Similarly, we fit these 131 FRBs with the fitter Python package [55], and then obtain the best redshift distribution, namely Burr Type XII distribution (Burr12) [58]. The standardized Burr Type XII distribution is given by [58]

$$f_{\text{Burr12}}(x, b, k) = \frac{bkx^{b-1}}{(1+x^k)^{k+1}},$$
where \( x \geq 0, b > 0 \) and \( k > 0 \). One can shift and/or scale this distribution by using the shift and scale parameters \( (\ell, s) \), namely

\[
P_{\text{Burr12}}(z, b, k, \ell, s) = f_{\text{Burr12}}((z - \ell)/s, b, k)/s.
\]

The best parameters for the 131 actual FRBs mentioned above are \( b = 1.4653, k = 3.9060, \ell = -0.0064 \) and \( s = 1.3963 \). We present this best Burr Type XII distribution in the middle panel of Fig. 1.

We compare the best Burr and Burr12 distributions obtained above in the right panel of Fig. 1. It is easy to see that they are fairly close in fact. We stress that these two new redshift distributions might be not the actual one of FRBs, since the inferred redshifts are rough, while the 129 FRBs from FRBCAT might not represent the actual ones because most of them are at low redshifts < 1 (and it is possible that many FRBs at high redshifts are absent due to the selection effect). So, these two new redshift distributions for FRBs (Burr and Burr12) are just for reference and inspiration.

B. Existing redshift distributions

In fact, many existing redshift distributions for FRBs have been extensively considered in the literature. Note that they do not come from the actual FRBs. Since the actual redshift distribution of FRBs is still unknown to date, one might borrow the ones of other objects. For example, in e.g. [25–27, 34, 35, 38], one can argue that FRBs are similar/related to Gamma-ray bursts (GRBs), and hence assume that the redshift distribution of FRBs takes the one of GRBs \([59]\) (termed “PzGRB”), namely

\[
P_{\text{GRB}}(z) \propto z e^{-z},
\]

which is a special case of Erlang distribution \([60]\). PzGRB was used extensively in the literature.

In e.g. \([61]\), two redshift distributions for FRBs were proposed. The first one (termed “Pzconst”) assumes that FRBs have a constant comoving number density, and the corresponding redshift distribution function is given by \([61]\)

\[
P_{\text{const}}(z) \propto d_C^2(z)/(1 + z) H(z) \exp\left(-\frac{d_L^2(z)}{2d_L^2(z_{\text{cut}})}\right),
\]

where \( H(z) \) is the Hubble parameter, \( d_C(z) = d_L(z)/(1 + z) = c \int_0^z d\tilde{z}/H(\tilde{z}) \) is the comoving distance, \( d_L(z) \) is the luminosity distance. Gaussian cutoff at \( z_{\text{cut}} \) is introduced to represent an instrumental signal-to-noise threshold. The second one (termed “PzSFH”) assumes that FRBs follow the star-formation history (SFH) \([62]\), whose density is given by

\[
\dot{\rho}_*(z) = \frac{(b_1 + b_2 z) h}{1 + (z/b_3)^{\alpha}}. 
\]
with \( b_1 = 0.0170, b_2 = 0.13, b_3 = 3.3, b_4 = 5.3 \) and \( h = 0.7 \). In this case, the SFH-based redshift distribution function reads \[ P_{SFH}(z) \propto \dot{\rho}_* (z) \frac{d^2L(z)}{(1+z)H(z)} \exp \left( -\frac{d_L^2(z)}{2d_L^2(z_{cut})} \right). \] (11)

In the literature, the cutoff \( z_{cut} \) has been set to various values. In \cite{61} and e.g. \cite{36,40}, \( z_{cut} = 0.5 \) was adopted. On the other hand, \( z_{cut} = 1.0 \) was considered in e.g. \cite{31,37}. We call the corresponding redshift distributions “ \( P_{zconst}(0.5) \)”, “ \( P_{zconst}(1.0) \)”, “ \( P_{zSFH}(0.5) \)”, “ \( P_{zSFH}(1.0) \)”, respectively.

Another type of redshift distribution for FRBs was proposed in e.g. \cite{65}. One might argue that the distribution of FRBs closely trace the cosmic star-formation rate (SFR) for young stellar FRB progenitors. In e.g. \cite{66}, the cosmic SFR function is given by \[ \psi(z) = 0.015 \frac{(1+z)^{2.7}}{1+((1+z)/2.9)^{5.6}} \frac{M_\odot}{yr Mpc^{-3}}. \] (12)

The appropriately weighted redshift distribution is obtained by considering the quantity \[ \zeta_{SFR} = \frac{\int_{z_{cut}}^{z_{max}} \psi(z) d\tilde{z}}{\int_0^{z_{max}} \psi(z) dz}. \] (13)

and drawing it as a uniform random number between 0 and 1. Since the right hand side of Eq. (13) is an explicit function of redshift \( z \), for any uniform random number \( 0 \leq \zeta_{SFR} \leq 1 \), one can obtain the corresponding redshift \( z \) by numerically solving Eq. (13). So, the SFR-based redshift distribution (termed “ \( P_{zSFR} \)” can be generated for FRBs. In principle, \( z_{max} \) can be set to any value, and then \( P_{zSFR} \) generates random redshifts in the range of \( 0 \leq z \leq z_{max} \).

Naively, since there is no guideline for the redshift distribution of FRBs to date, it is also reasonable to just consider a uniform distribution. One can uniformly assign a random redshift \( z \) from 0 to \( z_{max} \). In the present work, we also take this uniform redshift distribution into account.

In the left panel of Fig. 2, we summarize the 9 redshift distributions for FRBs, which are all normalized. Notice that the distributions \( P_{zconst} \) and \( P_{zSFH} \) are plotted just for demonstration by assuming the simplest flat \( \Lambda \)CDM cosmology with \( \Omega_m = 0.3153 \) taken from Planck 2018 results \cite{50}. We stress that there are other types of redshift distributions for FRBs in the literature. We do not try to consider all redshift distributions for FRBs in a limited work. With these 9 redshift distributions, we simulate FRBs and then try to see the effect of redshift distributions for FRBs on cosmological constraints.

III. SIMULATING FRBS

Here, we briefly describe the key points to simulate FRBs. As mentioned in Sec. I, we consider the extragalactic DM defined in Eq. (5) as the observed quantity. The main contribution comes from IGM.
FIG. 3: Panels (a) to (i) correspond to the redshift distributions Burr, Burr12, PzGRB, Pzconst(0.5), Pzconst(1.0), PzSFR, PzSFH(0.5), PzSFH(1.0), and Uniform, respectively. In each panel, the marginalized 1σ constraints on the cosmological parameter $\Omega_m$ of the flat $\Lambda$CDM model for 100 simulations are presented. In each simulation, $N_{\text{FRB}} = 1000$ FRBs are generated by using the flat $\Lambda$CDM model with the preset parameter $\Omega_m = 0.3153$ (indicated by the magenta dashed lines). The blue means with error bars (the chocolate means with error bars) indicate that the preset $\Omega_m = 0.3153$ is consistent (inconsistent) with the simulated FRBs within 1σ region, respectively. $n$ and $100-n$ are the numbers of blue and chocolate means with error bars, respectively. $\langle \sigma \rangle$ is the mean of the uncertainties of 100 constraints on the cosmological parameter $\Omega_m$. See Sec. IV for details.

As is shown in e.g. [24, 25, 35–38], the mean of DM$_{\text{IGM}}$ is given by

$$\langle \text{DM}_{\text{IGM}} \rangle = \frac{3\varepsilon H_0 \Omega_b}{8\pi G m_p} \int_0^\zeta \frac{f_{\text{IGM}}(\zeta) f_e(\zeta) (1 + \bar{\zeta}) d\bar{\zeta}}{E(\zeta)},$$

(14)

where $\Omega_b$ is the present fractional density of baryons, $m_p$ is the mass of proton, $H_0$ is the Hubble constant, $E \equiv H/H_0$ is the dimensionless Hubble parameter. $f_{\text{IGM}}$ is the fraction of baryon mass in IGM, which is a function of redshift $z$ in principle [36–38]. Following e.g. [24, 26, 28, 35], here we adopt a constant $f_{\text{IGM}} = 0.83$ (see e.g. [62, 68] and [24]). The ionized electron number fraction per baryon is

$$f_e(z) \equiv Y_H \chi_{e,H}(z) + \frac{1}{2} Y_{He} \chi_{e,He}(z),$$

(15)

in which hydrogen (H) and helium (He) mass fractions are $Y_H = (3/4) y_1$ and $Y_{He} = (1/4) y_2$, where $y_1 \sim 1$ and $y_2 \sim 4 - 3 y_1 \sim 1$ are the hydrogen and helium mass fractions normalized to the typical values $3/4$ and $1/4$, respectively. In principle, the ionization fractions $\chi_{e,H}(z)$ and $\chi_{e,He}(z)$ are both functions of redshift $z$. It is expected that intergalactic hydrogen and helium are fully ionized at redshifts $z \lesssim 6$.
and \( z \lesssim 3 \) [69, 70] (see also e.g. [71]), respectively. Thus, for FRBs at redshifts \( z \leq 3 \), they are both fully ionized, namely \( \chi_{e,H}(z) = \chi_{e,He}(z) = 1 \). So, \( f_e(z) \simeq 7/8 \) for \( z \leq 3 \).

Note that DM_{IGM} will deviate from the mean \( \langle DM_{IGM} \rangle \) if the plasma density fluctuations are taken into account [72] (see also e.g. [32, 42]). The uncertainty \( \sigma_{IGM} \) was studied in e.g. [72], where three models for halo gas profile of the ionized baryons were used. Following e.g. [36], we consider the simplest one, namely the top hat model, and the corresponding \( \sigma_{IGM} \) was given by the green dots in the bottom panel of Fig. 1 of [72]. It is easy to fit these 27 green dots with a simple power-law function [36]

\[
\sigma_{IGM}(z) = 173.8 z^{0.4} \text{ pc cm}^{-3}.
\]

In the right panel of Fig. 2 we reproduce these 27 green dots from [72], and also plot the power-law \( \sigma_{IGM}(z) \) given by Eq. (16). Obviously, they coincide with each other fairly well.

The contribution from the host galaxy of FRB, i.e. DM_{HG}, is poorly known. The observed DM_{HG} for an FRB at redshift \( z \) is given by (e.g. [25–28, 35–37])

\[
DM_{HG} = DM_{HG,loc}/(1 + z),
\]

where DM_{HG,loc} is the local DM of FRB host galaxy. Following e.g. [28, 33, 36], we reasonably assume that DM_{HG,loc} is independent of redshift \( z \).

We briefly describe the steps to generate the simulated FRBs with “known” redshifts. At first, we assign a random redshift \( z_i \) to the \( i \)-th simulated FRB from a given redshift distribution (one of the nine mentioned in Sec. II). In this step, the distributions Pzconst and PzSFH should use a given cosmology characterized by \( E(z) = H(z)/H_0 \) (which will be specified in Sec. IV) to calculate the comoving and
luminosity distances, while the other distributions should not. As mentioned above, both the intergalactic hydrogen and helium are fully ionized at \(z \leq 3\), and hence we choose to generate the FRB redshifts in the range of \(0 \leq z_i \leq 3\) (namely \(z_{\text{max}} = 3\)). The second step is to randomly assign \(\Delta M_{\text{IGM}, i}\) and its uncertainty \(\sigma_{\text{IGM}, i} = \sigma_{\text{IGM}}(z_i)\) to this simulated FRB from a Gaussian distribution,

\[
\Delta M_{\text{IGM}, i} = \mathcal{N}(\langle \Delta M_{\text{IGM}} \rangle(z_i), \sigma_{\text{IGM}}(z_i)).
\]  

(18)

Here, \(\langle \Delta M_{\text{IGM}} \rangle(z_i)\) in Eq. (14) is calculated by using a given cosmology characterized by \(E(z)\) (which will be specified in Sec. IV), and \(\sigma_{\text{IGM}}(z_i)\) is calculated by using Eq. (16). The third step is to assign \(\Delta M_{\text{HG}, i} = \Delta M_{\text{HG,loc}, i}/(1 + z_i)\) and its uncertainty \(\sigma_{\text{HG}, i} = \sigma_{\text{HG,loc}, i}/(1 + z_i)\) to this simulated FRB, according to Eq. (17) and following e.g. [25–28, 35–37]. Here, \(\Delta M_{\text{HG,loc}, i}\) can be randomly assigned from a Gaussian distribution with the mean \(\langle \Delta M_{\text{HG,loc}} \rangle\) and a fluctuation \(\sigma_{\text{HG,loc}}\) [25–28, 35–37], namely

\[
\Delta M_{\text{HG,loc}, i} = \mathcal{N}(\langle \Delta M_{\text{HG,loc}} \rangle, \sigma_{\text{HG,loc}}),
\]

(19)

while \(\sigma_{\text{HG,loc}, i} = \sigma_{\text{HG,loc}}\). In the literature, \(\Delta M_{\text{HG,loc}} = 50\) pc cm\(^{-3}\) is frequently used (see e.g. [22, 48, 49]). On the other hand, it was argued in [73] that the median of \(\Delta M_{\text{HG,loc}}\) is about \(30 \sim 70\) pc cm\(^{-3}\), while the uncertainty \(20\) pc cm\(^{-3}\) was frequently used in the literature (e.g. [22, 52–56, 58]). So, we adopt the fiducial values \(\langle \Delta M_{\text{HG,loc}} \rangle = 50\) pc cm\(^{-3}\) and \(\sigma_{\text{HG,loc}} = 20\) pc cm\(^{-3}\) in this work. Finally, the simulated \(M_E\) data and its uncertainty for the \(i\)-th simulated FRB are given by

\[
M_{E,i} = M_{\text{IGM}, i} + M_{\text{HG}, i}, \quad \text{and} \quad \sigma_{E,i} = (\sigma_{\text{IGM}, i}^2 + \sigma_{\text{HG}, i}^2)^{1/2}.
\]

(20)

One can repeat the above steps for \(N_{\text{FRB}}\) times to generate \(N_{\text{FRB}}\) simulated FRBs.
IV. COSMOLOGICAL CONSTRAINTS FROM THE SIMULATED FRBS

Now, we consider the constraints on various cosmological models from the simulated FRBs. For a specified cosmological model, its dimensionless Hubble parameter $E(z) = H(z)/H_0$ is given. So, one can calculate the theoretical extragalactic DM of an FRB by using

$$DM_{E}^{th}(z) = \langle DM_{IGM}(z) + (DM_{HG, loc})/(1+z) \rangle,$$

where $\langle DM_{IGM}(z) \rangle$ is given by Eq. (14), and the universal constant $\langle DM_{HG, loc} \rangle$ is a model parameter for HG. The model parameters can be constrained by performing a $\chi^2$ analysis, while

$$\chi^2 = \sum_i \frac{(DM_{E,i} - DM_{E}^{th}(z_i))^2}{\sigma_{E,i}^2}.$$

The lower-limit estimates for the number of FRB events are a few thousands per sky per day [3, 74]. Even conservatively, the all-sky burst rate floor derived from the pre-commissioning of CHIME/FRB is $3 \times 10^2$ events per day [75]. Several projects designed to detect and localize FRBs with arcsecond accuracy in real time are under construction or in commission, for example DSA-10 [76], DSA-2000 [77], MeerKAT [78], UTMOST-2D [79], and LOFAR [80]. It is expected that numerous FRBs with identified redshifts will be available in the future. Thus, $N_{FRB}$ can be large, for example $O(10^3)$ or even more.
In this work, we use the Markov Chain Monte Carlo (MCMC) code **CosmoMC** \[81] to this end. Since we are mainly interested in the effect of redshift distributions on cosmological constraints, to save the length of paper, we do not present the constraints on HG parameter \( \langle DM_{HG, loc} \rangle \) in the following, although they are also available in fact.

At first, we consider the simplest cosmological model, namely the flat \( \Lambda \)CDM model. In this case, the dimensionless Hubble parameter is given by (e.g. \[82, 83\])

\[
E(z) = (\Omega_m(1 + z)^3 + (1 - \Omega_m))^{1/2},
\]

where \( \Omega_m \) is the only free cosmological parameter. We simulate \( N_{FRB} \) FRBs with the preset cosmological parameter \( \Omega_m = 0.3153 \) taken from Planck 2018 results \[50]. Then, we constrain the flat \( \Lambda \)CDM model with these simulated FRBs. To avoid the statistical noise due to random fluctuations, one should repeat the constraints for a large number of simulations. However, it is fairly expensive to consider too many simulations since they consume a large amount of computation power and time. As a balance, we choose to consider 100 simulations, which is enough in fact.

In Fig. 3, the marginalized 1σ constraints on the cosmological parameter \( \Omega_m \) of the flat \( \Lambda \)CDM model for 100 simulations are presented. In each simulation, \( N_{FRB} = 1000 \) FRBs are generated. It is easy to see from Fig. 3 that the preset parameter \( \Omega_m = 0.3153 \) can be found within 1σ region in most of the 100 simulations (64 ∼ 77%), for all cases of the 9 redshift distributions introduced in Sec. II. This implies that the cosmological constraints from simulated FRBs are fairly reliable and robust. However, the uncertainties of the constraints are different. Using the naked eye, we find from Fig. 3 that the error bars of right panels (c), (f), (i) are shortest, the ones of bottom-left panels (d), (g) are longest, and the ones of other four panels (a), (b), (e), (h) are moderate. Quantitatively, \( \langle \sigma \rangle \) in each panel gives the mean of the uncertainties of 100 constraints. Using \( \langle \sigma \rangle \) in Fig. 3 we confirm that the cosmological constraints
from FRBs simulated with the redshift distributions (c) PzGRB, (f) PzSFR, (i) Uniform are tightest, the ones with the redshift distributions (d) Pzconst(0.5), (g) PzSFH(0.5) are loosest, and the ones with the redshift distributions (a) Burr, (b) Burr12, (e) Pzconst(1.0), (h) PzSFH(1.0) are moderate. Clearly, they are separated into three distinct groups.

In Figs. 4 and 5, the number of simulated FRBs increases to \( N_{\text{FRB}} = 5000 \) and 10000, respectively. Clearly, the cosmological constraints become tighter when the number of simulated FRBs increases, for all cases of the 9 redshift distributions. However, the insight about the constraining ability keeps unchanged. The FRBs simulated with the redshift distributions PzGRB/PzSFR/Uniform, Pzconst(0.5)/PzSFH(0.5), Burr/Burr12/Pzconst(1.0)/PzSFH(1.0) have strong, weak, moderate constraining abilities, respectively. These three groups of redshift distributions lead to different constraining abilities from the simulated FRBs. Using FRB simulations with different redshift distributions, one will make optimistic, pessimistic, or moderate predictions about the future of the FRB cosmology.

Let us turn to other cosmological models to see whether or not the above insight changes. The second is the flat \( \omega \)CDM model, in which the dimensionless Hubble parameter is given by (e.g. \cite{82, 83})

\[
E(z) = \left[ \frac{\Omega_m(1+z)^3 + (1 - \Omega_m)(1+z)^{3(1+w)}}{\Omega_m} \right]^{1/2},
\]

where \( \Omega_m \) and \( w \) are free cosmological parameters. We simulate \( N_{\text{FRB}} \) FRBs with the preset cosmological parameter \( \Omega_m = 0.3153 \) and \( w = -0.95 \). Then, we constrain the flat \( \omega \)CDM model with these simulated FRBs. In Figs. 6 and 7 the marginalized 1\( \sigma \) constraints on the cosmological parameters \( \Omega_m \) and \( w \) are
presented, respectively. Since the insight about the constraining ability keeps unchanged when the number of simulated FRBs varies, we only consider the case of $N_{\text{FRB}} = 5000$ for the flat $w$CDM model. Once again, it is easy to see from Figs. 6 and 7 that both the preset parameters $\Omega_m = 0.3153$ and $w = -0.95$ can be found within 1σ region in most of the 100 simulations, for all cases of the 9 redshift distributions introduced in Sec. II. This implies that the cosmological constraints from simulated FRBs are fairly reliable and robust. On the other hand, since there are two free cosmological parameters $\Omega_m$ and $w$ in the flat $w$CDM model while there is only one cosmological parameter $\Omega_m$ in the flat $\Lambda$CDM model, the constraints on $\Omega_m$ in the flat $w$CDM model (Fig. 6) are looser than the ones in the $\Lambda$CDM model (Fig. 4), as expected. From Figs. 6 and 7, one can find that the cosmological constraints on both $\Omega_m$ and $w$ from FRBs simulated with the redshift distributions $P_{z\text{GRB}}/P_{z\text{SFR}}/\text{Uniform}$, $P_{z\text{const}}(0.5)/P_{z\text{SFH}}(0.5)$, and $\text{Burr}/\text{Burr12}/P_{z\text{const}}(1.0)/P_{z\text{SFH}}(1.0)$ are tightest, loosest, and moderate, respectively. These three groups of redshift distributions lead to different constraining abilities from the simulated FRBs. This insight still holds in the case of flat $w$CDM model.

Finally, we consider the flat Chevallier-Polarski-Linder (CPL) model [84, 85], in which the equation-of-state parameter (EoS) of dark energy is parameterized as

$$ w = w_0 + w_a (1 - a) = w_0 + w_a \frac{z}{1+z}, \quad (25) $$

where $w_0$ and $w_a$ are constants. As is well known, the corresponding $E(z)$ is given by (e.g. [84, 85])

$$ E(z) = \left[ \frac{\Omega_m (1 + z)^3 + (1 - \Omega_m)(1 + z)^{3(1 + w_0 + w_a)}}{1 + z} \right]^{1/2}. \quad (26) $$

We simulate $N_{\text{FRB}}$ FRBs with the preset parameters $\Omega_m = 0.3153$, $w_0 = -0.95$ and $w_a = -0.3$. Then,
we constrain the flat CPL model with these simulated FRBs. Similarly, we only consider the case of $N_{\text{FRB}} = 5000$ for the flat CPL model. In Figs. 8–10, the marginalized 1σ constraints on the cosmological parameters $\Omega_m$, $w_0$, and $w_a$ are presented, respectively. Again, we find from Figs. 8–10 that the preset cosmological parameters $\Omega_m = 0.3153$, $w_0 = -0.95$ and $w_a = -0.3$ can be found within 1σ region in most of the 100 simulations, for almost all cases of the 9 redshift distributions introduced in Sec. II. There are 3 free cosmological parameters in this model, and hence the cosmological constraints will become worse than the flat ΛCDM and $w$CDM models which have 1 and 2 free cosmological parameters, respectively. This can be verified by comparing Figs. 8–10 with Figs. 4, 6 and 7.

Let us look at the uncertainties of cosmological constraints. In the case of $\Omega_m$ (Fig. 8), the same insight keeps unchanged as in the flat ΛCDM and $w$CDM models, namely the cosmological constraints on $\Omega_m$ from FRBs simulated with the redshift distributions PzGRB/PzSFR/Uniform, Pzconst(0.5)/PzSFH(0.5), and Burr/Burr12/Pzconst(1.0)/PzSFH(1.0) are tightest, loosest, and moderate, respectively. However, it is slightly changed in the cases of $w_0$ and $w_a$. The tightest, loosest, and moderate groups are changed to the distributions Burr/Burr12/PzGRB/PzSFR/Uniform, Pzconst(0.5)/PzSFH(0.5)/PzSFH(1.0), and Pzconst(1.0)/PzSFR in the case of $w_0$ (Fig. 9), respectively. On the other hand, the tightest, loosest, and moderate groups are changed to the distributions Burr/Burr12/PzGRB/PzSFR/Uniform, Pzconst(0.5)/PzSFH(0.5), and Pzconst(1.0)/PzSFH(1.0) in the case of $w_a$ (Fig. 10), respectively. This is mainly due to the correlation between the cosmological parameters $w_0$ and $w_a$. Nevertheless, it is still unchanged that different redshift distributions lead to different constraining abilities from the simulated FRBs. If one uses the unsuitable redshift distributions to simulate FRBs, rather than the actual one of FRBs (which is still unknown to date), overoptimistic or overpessimistic predictions about the future of the FRB cosmology might be made.

FIG. 10: The same as in Fig. 8 but the marginalized 1σ constraints are on the cosmological parameter $w_a$.  

![Cosmological parameter constraints](image_url)
V. CONCLUDING REMARKS

Nowadays, FRBs have been a promising probe for astronomy and cosmology. However, it is not easy to identify the redshifts of FRBs to date. Thus, no sufficient actual FRBs with identified redshifts can be used to study cosmology currently. In the past years, one has to use the simulated FRBs with “known” redshifts instead. To simulate an FRB, one should randomly assign a redshift to it from a given redshift distribution. But the actual redshift distribution of FRBs is still unknown so far. In the present work, we study the effect of various redshift distributions on cosmological constraints. We find that different redshift distributions lead to different constraining abilities from the simulated FRBs. This result emphasizes the importance to find the actual redshift distribution of FRBs, and reminds us of the possible bias in the FRB simulations due to the redshift distributions.

In this work, we have proposed two new redshift distributions, namely Burr and Burr12, from the actual FRBs to date. Although they might be not the actual one of FRBs (which is still unknown) due to the possible selection effect, we suggest that they could be used for reference and inspiration.

We stress that there are other types of redshift distributions for FRBs in the literature. We do not try to consider all redshift distributions for FRBs in a limited work. However, it is expected that our main conclusion will not change for the other redshift distributions unused in this work.

As mentioned above, the 9 redshift distributions can be separated into three distinct groups, namely PzGRB/PzSFR/Uniform, Pzconst(0.5)/PzSFH(0.5), and Burr/Burr12/Pzconst(1.0)/PzSFH(1.0), which lead to strong, weak, and moderate constraining abilities, respectively. In fact, we can find some clues from the left panel of Fig. 2. The normalized redshift distributions PzGRB/PzSFR/Uniform are commonly “short and wide”, and hence the simulated FRBs span almost the whole redshift range from 0 to 3. On the contrary, the normalized redshift distributions Pzconst(0.5)/PzSFH(0.5) are commonly “tall and thin” with a sharp peak nearby the low redshift 0.5, and hence the simulated FRBs concentrate in a narrow redshift range around the redshift 0.5 (in fact it is rare to have redshifts > 1). On the other hand, the normalized redshift distributions Pzconst(1.0)/PzSFH(1.0) are moderate, and hence the simulated FRBs span a fairly wide redshift range from 0 to ~2.2. Although the normalized redshift distributions Burr/Burr12 tilt to low redshifts < 1, they have not so small probability to generate redshifts in the range from 1 to 2. In this sense, Burr/Burr12 are similar to Pzconst(1.0)/PzSFH(1.0), so that they are also in the moderate group. These are clues found from the left panel of Fig. 2. Naively, we try to understand them as follows. For FRBs at high redshifts, DM_E is dominated by the contribution from IGM, namely DM_{IGM}, which carries the key information about IGM and the cosmic expansion history. In this case, the contribution from host galaxy, DM_{HG} = DM_{HG, loc} / (1 + z), becomes relatively small. So, it is expected that the constraints on the cosmic expansion history accordingly become tight. Thus, the redshift distributions having larger probability to generate redshifts > 2 or > 1 lead to stronger constraining abilities from the simulated FRBs.

Of course, we cannot say “So it is better to simulate FRBs with the moderate redshift distributions Pzconst(1.0)/PzSFH(1.0)” or something like that. In fact, most of the actual FRBs to date have low redshifts < 1 inferred from the observed DM, as is shown by the normalized histograms in Fig. 1. To this problem, the key is to find the actual redshift distribution of FRBs, which is still unknown to date. We hope that it will be available soon as the observed FRBs with identified redshifts accumulate in the future. We consider this is an important topic in the field of FRBs, especially for the FRB cosmology.

ACKNOWLEDGEMENTS

We are grateful to Hua-Kai Deng, Shu-Ling Li, Zhong-Xi Yu, Han-Yue Guo and Jing-Yi Jia for kind help and useful discussions. This work was supported in part by NSFC under Grants No. 11975046 and No. 11575022.

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