Diffractive electroproduction of 
\( \rho \) meson excitations

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Abstract

We use perturbative QCD and parton-hadron duality to estimate the cross sections for the diffractive electroproduction of \( \rho' \)\(^{(1^-)} \) and \( \rho \)\(^{(3^-)} \) resonances which occur in the 1.3–1.8 GeV mass interval. We present the cross sections and the ratios \( \sigma_L/\sigma_T \) as a function of \( Q^2 \). We compare the predictions with those for the diffractive electroproduction of \( \rho \) mesons. We show how such diffractive electroproduction measurements at HERA can probe features of the perturbative QCD ‘Pomeron’.

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The experiments [1] at HERA are measuring the diffractive electroproduction processes $\gamma^* p \rightarrow (2\pi)p$ and $\gamma^* p \rightarrow (4\pi)p$ as a function of invariant mass $M$ of the pionic system, for different intervals of $Q^2$, the virtuality of the photon, and of $W$, the centre-of-mass energy of the $\gamma^* p$ system. As these are quasi-elastic processes we would expect that at high energy the pionic system will dominantly have spin-parity $J^P = 1^-$. Indeed a strong $\rho$ meson resonant peak is observed [1]. In a previous paper [2] we presented a QCD model which reproduced the observed $\rho$ electroproduction cross section and, in particular, described the $Q^2$ behaviour of the cross section ratio $\sigma_L/\sigma_T$ for $\rho$ meson production in longitudinally and transversely polarised states. Here we study the rate of $\rho'(1^-)$ resonance production in the higher mass interval, 1.3–1.8 GeV. Moreover we are also able to estimate the cross section for the diffractive production of the $\rho(3^-)$ resonance at HERA, the $\rho$ orbital excitation which also occurs in the above mass interval. Given sufficient data, the comparison of $\rho'(1^-)$ and $\rho(3^-)$ production would provide a unique opportunity to study how the QCD ‘Pomeron’ distorts the initial state.

![Diagram](image)

**Figure 1:** Diffractive open $q\bar{q}$ production in high energy $\gamma^* p$ collisions. The transverse momenta of the outgoing quarks are $\pm k_T$, and those of the exchanged gluons are $\pm \ell_T$.

A perturbative QCD description of diffractive $\rho$ electroproduction, $\gamma^* p \rightarrow \rho p$, was presented in [2]. The cross sections $\sigma_L$ and $\sigma_T$ in the HERA energy region were calculated in terms of the known gluon distribution of the proton. The approach was based on the ‘open’ production of light quark-antiquark pairs and parton-hadron duality, see Fig. 1. That is the diffractive dissociation $\gamma^* \rightarrow q\bar{q}$ was calculated in a $q\bar{q}$ invariant mass interval around the $\rho$ resonance for high values of $Q^2$ and high $\gamma^* p$ c.m. energy, $W^2 \gg Q^2$, corresponding to the small $x$ regime. In the $\rho$ meson mass interval phase space restrictions force the $q\bar{q}$ pair to dominantly hadronize into two (or three) pions. The projections of the $q\bar{q}$ system into the $I^G = 1^+$ and $0^-$ channels, corresponding to $2\pi$ and $3\pi$ production respectively, are in the ratio 9:1. Therefore to obtain the prediction for $\rho$ diffractive production we multiply the $q\bar{q}$ rate by 0.9.
In order to generalize the approach to the diffractive production of higher mass $\rho$ states we summarize the relevant formulae. The cross sections for the production of a $q\bar{q}$ system of mass $M$ by a photon polarised along, and transverse to, the $\gamma^*$ direction are respectively given by

\[
\frac{d\sigma_L}{dM^2} = \frac{4\pi^2 e^2_\alpha}{3b} \frac{Q^2}{(Q^2 + M^2)^2} \frac{1}{8} \int_{-1}^{1} d\cos \theta \left| d'_{10}(\theta) \right|^2 |I_L|^2,
\]

(1)

\[
\frac{d\sigma_T}{dM^2} = \frac{4\pi^2 e^2_\alpha}{3b} \frac{M^2}{(Q^2 + M^2)^2} \frac{1}{4} \int_{-1}^{1} d\cos \theta \left( |d'_{11}(\theta)|^2 + |d'_{1-1}(\theta)|^2 \right) |I_T|^2,
\]

(2)

where $d'_{\lambda \mu}(\theta)$ are the usual rotation matrices. The outgoing quark has the polar angle $\theta$ in the $q\bar{q}$ rest frame with respect to the direction of the incoming proton; that is the transverse momentum of the outgoing $q$ is $k_T = \frac{1}{2} M \sin \theta$. The cross sections have been integrated over $t$ assuming an $\exp(-b|t|)$ behaviour. We take the observed slope $b = 5.5$ GeV$^{-2}$, consistent with the average experimental value found in high energy $\rho$ electroproduction for $Q^2 \gtrsim 10$ GeV$^2$.

The quantities $I_{L,T}$ in (1) and (2) are integrations over the transverse momenta $\pm \ell_T$ of the exchanged gluons, see Fig. 1,

\[
I_L(\theta) = K^2 \int \frac{d\ell_T^2}{\ell_T^2} \alpha_S(\ell_T^2) f(x, \ell_T^2) \left( \frac{1}{K^2} - \frac{1}{K_T^2} \right),
\]

(3)

\[
I_T(\theta) = \frac{K^2}{2} \int \frac{d\ell_T^2}{\ell_T^2} \alpha_S(\ell_T^2) f(x, \ell_T^2) \left( \frac{1}{K^2} - \frac{1}{2k_T^2} + \frac{K^2 - 2k_T^2 + \ell_T^2}{2k_T^2 K_T^2} \right),
\]

(4)

where

\[
K^2 = k_T^2 \left( 1 + Q^2/M^2 \right) = \frac{1}{4} (Q^2 + M^2) \sin^2 \theta,
\]

(5)

\[
K_T^2 = \sqrt{(K^2 + \ell_T^2)^2 - 4k_T^2 \ell_T^2},
\]

(6)

and where $f(x, \ell_T^2)$ is the gluon distribution of the proton, unintegrated over $\ell_T^2$, evaluated at small $x$ given by $x = (Q^2 + M^2)/W^2$. If we were to assume that the main contributions to $I_{L,T}$ come from the domain $\ell_T^2 < K^2$ then we would have the leading $\ln K^2$ approximation

\[
\frac{d\sigma_{LLA}}{dM^2} = \frac{d\sigma_{TLA}}{dM^2} = \frac{\alpha_S(K^2)}{K^2} \int K^2 \frac{d\ell_T^2}{\ell_T^2} f(x, \ell_T^2) = \frac{\alpha_S(K^2)}{K^2} x g(x, K^2)
\]

(7)

where $g$, the conventional (integrated) gluon distribution is sampled at the scale given in (3). Here we do not use this approximation, but perform the $d\ell_T^2$ integrations explicitly. To be precise, for $\ell_T^2 > \ell_0^2$ we evaluate the integrals using

\[
f(x, \ell_T^2) = \frac{\partial (xg(x, \ell_T^2))}{\partial \ln \ell_T^2}
\]

(8)

where the gluon distribution $g$ is taken from a recent set of partons, whereas for the contribution from $\ell_T^2 < \ell_0^2$ we assume that the gluon vanishes linearly with $\ell_T^2$

\[
\alpha_S(\ell_T^2) g(x, \ell_T^2) = \frac{\ell_T^2}{\ell_0^2} \alpha_S(\ell_0^2) g(x, \ell_0^2).
\]

(9)
This is equivalent to the physical hypothesis that the gluon-proton cross, \( \sigma(gp) \sim \alpha_S g(x, \ell_T^2)/\ell_T^2 \), should tend to a constant (modulo logarithmic behaviour) at small values of \( \ell_T^2 \) and \( x \). We choose \( \ell_0^2 = 1.75 \) GeV\(^2\) and test the sensitivity of the prediction to variation of \( \ell_0^2 \) about this value.

Since the perturbative QCD ‘hard’ pomeron distorts the initial \( \gamma^* \rightarrow q\bar{q} \) wave function, the diffractively produced \( q\bar{q} \) pair is not necessarily in a pure \( J^P = 1^- \) state. In fact the \( (q\bar{q}) \)-proton interaction is proportional to the square of the separation, \( \Delta\rho_T \), of the quarks in the transverse plane — an example of the effect of colour transparency. The incoming \( q\bar{q} \) wave function is distorted by this \( \Delta\rho_T \)-dependent amplitude so that the produced system contains a superposition of the \( J^P = 1^-, 3^- \) \( q\bar{q} \) states which are accessible by Pomeron exchange. 

Thus the angular momentum \( J \) of the \( q\bar{q} \) state may be changed, although for forward scattering the \( s \) channel helicity is still conserved. The distortion enters the cross section formula \((4) \) and \((2) \) through the \( \theta \) dependence of \( I_{L,T} \); the integrals which arise from the \( q\bar{q} \) interaction with the proton. The \( q\bar{q} \) production amplitudes which enter \((4) \) and \((2) \) may therefore be decomposed in the form

\[
d_{1\lambda}(\theta) \ I_i(\theta) = \sum_J c_i^J(\lambda) \ d_{1\lambda}^J(\theta), \tag{10}\]

where \( i = L \) or \( T \), so that the probability amplitude for spin \( J \) production is proportional to the coefficient

\[
c_i^J(\lambda) = \frac{2J + 1}{2} \int d\cos\theta \ \left[ d_{1\lambda}^J(\theta) \ I_i(\theta) \right] \ d_{1\lambda}^J(\theta) \tag{11}\]

and so spin \( J \) production in longitudinally and transversely polarised states is proportional to \( |c_i^J(0)|^2 \) and \( |c_i^J(1)|^2 + |c_i^J(-1)|^2 \) respectively. Clearly if the \( I_i(\theta) \) were independent of \( \theta \) then \( J^P = 3^- \) \( q\bar{q} \) production would be zero, and pure \( J^P = 1^- \) production would occur. The integrals are infrared convergent not only for \( \sigma_L \), but also for \( \sigma_T \) — we quantify the infrared sensitivity of the predictions below.

In ref. \( [2] \) parton-hadron duality was invoked to estimate diffractive \( \rho \) electroproduction. The \( q\bar{q} \) production cross section, projected into the \( 1^- \) channel, was integrated over the mass interval \( 0.6 < M < 1.05 \) GeV. We summed over \( u\pi \) and \( d\bar{d} \) production and multiplied by 0.9 to allow for \( \omega \) production. We found good agreement with the measurements of \( \gamma^* p \rightarrow pp \) obtained at HERA \([3]\). Of course the absolute normalisation of the cross section depends on the choice of mass interval and also on the scale of \( \alpha_s \) used to estimate the \( K \) factor enhancement coming from higher order contributions (which is evaluated as described in refs. \([4, 2]\)). Thus it is the \( Q^2 \) behaviour of the total cross section and, more particularly, of the ratio \( \sigma_L/\sigma_T \) which offer tests of the model. To the best of our knowledge, this is the first time that the \( Q^2 \) behaviour of the ratio \( \sigma_L/\sigma_T \) has been satisfactorily described.

Now measurements of diffractive electroproduction into higher mass pionic systems are becoming available, for instance \( \gamma^* \rightarrow 4\pi \) has been observed in the mass range \( 1.4 < M < 2 \) GeV \([5]\). Indeed there is evidence of a broad peak for \( M \sim 1.6 \) GeV in the \( 4\pi \) channel, which may be attributed to a combination of the \( \rho'(1450, 1700) \) and \( \rho(1690) \) resonances with \( J^P = 1^- \) and \( 3^- \) respectively. To provide estimates of the QCD expectations for the diffractive production of
these resonances we again invoke parton-hadron duality. In this case we integrate the $J = 1$ and $J = 3$ projections over the mass interval $1.3 < M < 1.8$ GeV. Again we multiply by 0.9 to project onto the $I = 1$ channel. The results are shown in Figs. 2 and 3 by the curves labelled $\rho'(1^-)$ and $\rho(3^-)$ respectively. We also show the results for $\rho$ meson production for comparison. The gluon of the MRS(R2) set of partons is used.

![Graph showing QCD predictions for total cross section](image)

Figure 2: The QCD predictions for the total cross section for diffractive $\rho(1^-, 770$ MeV), $\rho'(1^-)$ and $\rho(3^-)$ electroproduction as a function of the photon virtuality $Q^2$, where the gluon distribution $g(x, l_T^2)$ is evaluated at $x = (Q^2 + M^2)/W^2$ with $W = 83.6$ GeV.

How dependent are the predictions on the treatment of the infrared regions of low transverse momenta of the exchanged gluons $\pm l_T$ and of the produced quarks $\pm k_T$? First we note that the results are hardly sensitive to variation of the value of $l_T^2$ below which we assume that the gluon distribution vanishes linearly in $l_T^2$, see (9). For convenience we take $l_T^2 = 1.75$ GeV$^2$, but essentially no change occurs for different choices of $l_T^2$ about this value.

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2 An alternative approach for $\rho'$ production, which is based on the colour dipole model, can be found in [6].
We now come to the integration over the quark transverse momentum \( k_T \), which in \((11)\) has been transformed into an integration over \( \cos \theta \). In principle, in pure perturbative QCD we should integrate essentially down to \( k_T = 0 \). However, confinement eliminates the large distance contribution and hence we impose a cut-off \( K_0 \sim 1 \text{ fm}^{-1} \approx 0.2 \text{ GeV} \). The results shown in Figs. 2 and 3 are obtained with \( K_0 = 0.2 \text{ GeV} \). That is we cut-off the low sin \( \theta \) contributions from the projection integrals given in eq. \((11)\). The sensitivity to the choice of \( K_0 \) is displayed in Table 1, which shows the diffractive electroproduction cross sections obtained by taking first \( K_0 = 0.1 \text{ GeV} \) and, second, \( K_0 = 0.3 \text{ GeV} \). It is evident from Fig. 3 that the sensitivity to the cut-off becomes less for large \( Q^2 \), since the infrared domain \( K^2 < K_0^2 \) corresponds to an increasingly smaller part of the region of integration, see \((5)\).

\footnote{The essential distances in the process are controlled by \( K \), which is related to \( k_T \) by eq. \((3)\) \((4)\).}
is in the prediction of $\sigma_T$ for $3^-$ production. This is to be expected. For $\sigma_L$ the contribution from the infrared region of small $\sin\theta$ in Eq. (11) is suppressed since $d_{10}\sim \sin^2 \theta$ as $\sin \theta \to 0$ for both $J=1$ and 3. However, the suppression factor is absent in $\sigma_T$. Now the distortion $I_i(\theta)$ of the $q\bar{q}$ state behaves approximately as $(\sin^2 \theta)^{\gamma-1}$ where $\gamma$ is the anomalous dimension of the gluon, $g(x,K^2) \sim (K^2)^\gamma$. Thus for $\sigma_T$ the distortion is larger in the region of small $\sin \theta$.

We note that the projection onto the higher $J$ state is more sensitive to this region, due to the behaviour of $d_{J,\pm 1}$ for small $\sin \theta$. A comparison of the predictions obtained with $K_0 = 0.1$ and 0.3 GeV shows just these trends. $\sigma_L$ is much more insensitive to variation of $K_0$ than is $\sigma_T$, and $1^-$ production is less sensitive than $3^-$ production.

| $K_0$ (MeV) | $\sigma_L$ (nb) | $\sigma_T$ (nb) |
|-------------|------------------|------------------|
| $\rho$      | 25.0 24.8        | 8.2 7.2          |
| $\rho'(1^-)$| 22.8 22.8        | 26.7 24.0        |
| $\rho(3^-)$ | 2.2 2.1          | 1.6 0.8          |

Table 1: The cross sections $\sigma_{L,T}(\gamma^* p \to \rho p)$ for diffractive electroproduction of longitudinally and transversely polarised $\rho$ resonances, for $Q^2 = 10$ GeV$^2$ for two different choices of the infrared cut-off $K_0$ of the variable $K$ defined by (5). The $\rho$ cross sections come from integrating $q\bar{q}$ production over the mass interval $0.6 < M < 1.05$ GeV, whereas $\rho'(1^-)$ and $\rho(3^-)$ correspond to integration over the interval $1.3 < M < 1.8$ GeV.

From Fig. 2 and Table 1 we see that $\rho'(1^-)$ diffractive electroproduction is predicted to occur at a comparable rate to $\rho$ production at the lower values of $Q^2$ shown, but decreases more slowly as $Q^2$ increases. Fig. 3 shows that the expected values of $R = \sigma_L/\sigma_T$ are smaller for the higher mass states than those for $\rho$ production. The main reason is the presence of the kinematic factor $Q^2/M^2$ in $R$. We also see that $3^-$ production is not insignificant. The $\rho(3^-)$ state is expected to occur at about 0.1 the rate of $\rho'$ production. The cross section $\sigma_L(3^-)$ is much better determined than $\sigma_T(3^-)$; the latter has about a factor of 2 uncertainty.

We conclude that if a sufficient number of diffractive electroproduction events are observed at HERA so as to be able to separate the $1^-$ and $3^-$ systems, then these processes will allow new insights of the ‘QCD’ pomeron. However, first we must consider the hadronization of the produced $q\bar{q}$ pair. Recall that in the $\rho(770)$ mass region, phase space restrictions force the $q\bar{q}$ pair to hadronize dominantly into two pions. In the higher mass region the situation is more complicated. From the branching ratios of the resonances listed in the PDG tables [7] we expect that the $q\bar{q}$ state hadronizes into the $4\pi$ channel some 70–80% of the time, with about 20% going into $2\pi$. Of course it is not easy to perform a partial wave analysis of the $4\pi$ channel to separate out the $3^-$ component. Fortunately the $\rho(1690)$ has healthy branching ratios into two hadron decay modes; for instance $\text{BR}(\rho^0 \to \omega \pi^0) = 16\%$ and $\text{BR}(\rho \to 2\pi) = 24\%$.

It would also be very interesting to observe the diffractive $\gamma^* \to q\bar{q}$ dissociation into the $2^+$ resonant state, that is $\gamma^* \to f_2(1270)$. Unfortunately the cross section is very small. There
are two main reasons for the suppression. First, the resonance isospin $I = 0$ gives rise to a 1:9 suppression in comparison to $I = 1$ $\rho$ electroproduction, due to the electric charges of the quarks. Second, the $\gamma^*(1^{-}-) \rightarrow f_2(2^{++})$ production process occurs via odderon, rather than pomeron, exchange. The odderon amplitude is mainly real and in perturbative QCD is given by 3 gluon exchange, to leading order. Numerical estimates indicate that $|A(\text{odderon})|/|A(\text{P})| \lesssim 0.03$, which agrees well with the limits obtained from the ratio of the real to the imaginary part of the forward scattering amplitude measured in $pp$ and $p\bar{p}$ elastic scattering. So we expect a suppression of $2^+ \bar{q}q$ production of at least $\frac{1}{9}(0.03)^2 \sim 10^{-4}$ in comparison to diffractive $\rho$ production. We thus estimate a $\gamma^* \rightarrow f_2$ cross section will be less than 1 pb at $Q^2 = 10$ GeV$^2$ which will make observation at HERA very difficult.

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