Radiative transitions and the mixing parameters of the D meson

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Abstract. Spectroscopic parameters of heavy-light flavoured D meson are obtained within the framework of phenomenological quark-antiquark potential (Coulomb plus linear confinement) model using the Gaussian wave function. We incorporated $O(1/m)$ corrections to the potential energy term and relativistic corrections to the kinetic energy term of the hamiltonian. We obtain the radiative (electric and magnetic) transitions and the mixing parameters of the $\bar{D}$â–Š $\bar{D}$ oscillation. The results are compared with various experimental measurement as well as other theoretical predictions.

1. Introduction
The radiative transitions are important tools to determine the properties of heavy-light mesons. Using the radiative transitions one can probe the internal charge structure of hadrons, hence it is useful for determining the quantum numbers and hadronic structures of mesons. Particle anti-particle mixing has been of fundamental importance in testing the Standard Model and its proposed extensions [1–3].

2. Radiative transitions
The radiative transition amplitude is determined by the matrix element of the EM current between the initial quarkonium state $i$ and the final state $f$, i.e., $\langle f \mid j_{\mu \nu}^E \mid i \rangle$ [2,3].

2.1. E1 radiative transitions
The partial width for an E1 radiative transition between states is given by [4]

$$\Gamma_{(i\rightarrow f+\gamma)} = \frac{4\alpha}{3} \left( \frac{mQe_q - m_qeT}{mQ + m_q} \right)^2 \left( \frac{M_i^2-M_f^2}{M_i^2} \right)^3 \frac{E_f}{M_i} C_{fi}\delta_{SS'} \times |\langle f \mid r \mid i \rangle|^2$$ (1)

The overlap integral is

$$\langle f \mid r \mid i \rangle = \int dr R_{n_i}^{\prime}(r) R_{n_i}^{\prime}(r)$$ (2)
A relativized Hamiltonian which includes the Laguerre polynomials. The variational parameter here is the variational parameter, \( \mu \). S-wave states of different quantum numbers given by [9] are the quantum numbers of the bound state and \( L \) are the Laguerre polynomials. The variational parameter \( \mu \) was fitted using virial theorem by using a relativized Hamiltonian which includes \( \mathcal{O}(1/m) \) corrections to the potential energy term and relativistic corrections to the kinetic energy term. For details see ref [5].

### Table 1. Electric dipole (E1) transitions widths of \( D \) meson.

| Meson transition | This work | Other work (\( \Gamma \) in KeV) |
|------------------|-----------|---------------------|
| \( D(1^3P_2) \) | 410 14.17 | 17.00 51 61.2 |
| \( D_1(1P) \)   | 398 0.20  | 13.77 30.87 39.9 |
| \( D_1(1P) \)   | 498 25.50 | 30.20 21.71 16.1 |
| \( D_1(1P) \)   | 380 11.26 | 1.24 10.25 8.6 |
| \( D_1(1P) \)   | 481 0.36  | 2.82 39.59 66  |
| \( D(1^3P_0) \) | 322 6.86  | 7.23 17 30    |
| \( D(2^3S_1) \) | 187 1.89  | 1.59            |
| \( D(2^3P_1) \) | 200 0.02  |                 |
| \( D(2^3P_1) \) | 220 1.85  |                 |
| \( D(2^3P_0) \) | 281 1.29  | 1.52            |
| \( D(2^1S_0) \) | 131 1.18  | 1.85            |
| \( D(2^1S_0) \) | 152 0.03  | 2.69            |
| \( D(1^3D_3) \) | 152 2.53  | 70.0            |
| \( D_1(1D) \)   | 303 1.62  |                 |
| \( D_1(1D) \)   | 277 5.06  |                 |
| \( D(1^3D_1) \) | 278 0.60  |                 |
| \( D(1^3D_1) \) | 291 7.00  |                 |
| \( D(1^3D_1) \) | 310 2.89  |                 |
| \( D(1^3D_1) \) | 369 20.13 | 53.8            |
| \( D_1(1D) \)   | 316 16.13 |                 |
| \( D_1(1D) \)   | 334 6.20  |                 |
| \( D_1(1D) \)   | 290 4.27  |                 |
| \( D_1(1D) \)   | 309 15.17 | 66.0            |

We employ Gaussian wave function for the calculation of the overlap integrals. The form of the wave function is given by

\[
R_{nl}(r) = \mu^\frac{3}{2} \left( \frac{2(n-1)!}{\Gamma(n + l + 1/2)} \right)^{\frac{1}{2}} \mu r^l e^{-\mu^2 r^2/2} L_n^{l+1/2}(\mu^2 r^2),
\]

(3)

here \( \mu \) is the variational parameter, \( n, l \) are the quantum numbers of the bound state and \( L \) are the Laguerre polynomials. The variational parameter \( \mu \) was fitted using virial theorem by using a relativized Hamiltonian which includes \( \mathcal{O}(1/m) \) corrections to the potential energy term and relativistic corrections to the kinetic energy term. For details see ref [5].

#### 2.2. M1 radiative transitions

The rates for magnetic dipole transitions correspond to triplet-singlet between S-wave states of the same \( n \) quantum number as well as either triplet-singlet or singlet-triplet transitions between S-wave states of different \( n \) quantum numbers given by [9]

\[
\Gamma_{M1}(i \rightarrow f + \gamma) = \frac{16\alpha}{3} \left( \frac{m_q e Q - m_Q e q}{4 m_q m_Q} \right)^2 E_\gamma^3(2 J_f + 1) |\langle f | j_0(E_\gamma r/2) | i \rangle|^2
\]

(4)

Here \( L = 0 \) for S-waves and \( j_0(x) \) is the spherical Bessel function.

\[
\langle f | r | i \rangle = \int dr R_{nl_i}(r) j_0(E_\gamma r/2) R_{n_j l_j}(r)
\]

(5)
Our calculated radiative magnetic dipole transitions widths for states of $D$ meson are tabulated in Table 2 using the masses estimated by our model [5].

| Meson transition                  | This work | Others ($\Gamma$ in KeV) |
|-----------------------------------|-----------|--------------------------|
| $D(1^3S_1) \rightarrow D(1^1S_0)$ | 123       | 0.271 0.339 10.8 1.8     |
| $D(2^3S_1) \rightarrow D(2^1S_0)$ | 72        | 0.055 0.007              |
| $D(3^3S_1) \rightarrow D(3^1S_0)$ | 53        | 0.021 0.001              |
| $D(2^3S_1) \rightarrow D(1^1S_0)$ | 659       | 6.371 100                |
| $D(2^1S_0) \rightarrow B(1^3S_1)$ | 508       | 8.594                    |
| $D_1(1P) \rightarrow D(1^3P_0)$   | 102       | 2.340                    |
| $D'_1(1P) \rightarrow D(1^3P_0)$  | 14        | 0.020                    |
| $D(1^3P_2) \rightarrow D_1(1P)$   | 36        | 0.314                    |
| $D'_1(1P) \rightarrow D(1^3P_2)$  | 88        | 1.524                    |
| $D(1^3D_3) \rightarrow D_2(1D)$   | 15        | 0.040                    |
| $D(1^3D_3) \rightarrow D(1^3D_1)$ | 43        | 0.936                    |
| $D'_2(1D) \rightarrow D(1^3D_3)$  | 43        | 0.539                    |
| $D_2(1D) \rightarrow D_2(1D)$     | 28        | 0.150                    |

3. Mixing parameters

The neutral $D$ meson exhibit particle-antiparticles mixing, leading to oscillations between the light (L) and heavy (H) mass eigenstates [1]. With assuming CPT conservation throughout, in each system, the light (L) and heavy (H) mass eigenstates are,

$$
\langle (D)^{L,H} \rangle = \frac{1}{\sqrt{1 + |(q/p)_q|^2}} \langle (D)^{L} \pm (q/p)_q (D)^{H} \rangle
$$

have a mass difference $\Delta m_q = m_H - m_L > 0$, and a total decay width difference $\Delta \Gamma_q = \Gamma_L - \Gamma_H$. The time evolution of the neutral $D$ meson doublet is described by the Schrodinger equation [1]

$$
\frac{d}{dt} \begin{pmatrix} D_q \\ \bar{D}_q \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} D_q \\ \bar{D}_q \end{pmatrix} - i \frac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \begin{pmatrix} D_q \\ \bar{D}_q \end{pmatrix}
$$

where $M_{12}$ and $\Gamma_{12}$ are the off-diagonal elements of the mass and decay matrices.

The formula of predictions for the off-diagonal elements of the mass and the decay matrices are [1]

$$
M_{12} = -\frac{G_F^2 m_W^2 \eta_D m_D f_D^2}{12\pi^2} S_0 (m_b^2/m_W^2) (V_{us}^* V_{cs})^2
$$

$$
\Gamma_{12} = \frac{G_F^2 m_W^2 \eta_D' m_D f_D^2}{8\pi} [ (V_{us}^* V_{cs})^2 ]
$$

The known function $S_0(x_t)$ can be approximated very well by $0.784 x_t^{0.76}$ [10], and $V_{ij}$ are the elements of the CKM matrix. The parameters $\eta_D$ and $\eta_D'$ correspond to the gluonic corrections for respective meson.

In the absence of CP violation, the time -integrated mixing probability ($\chi_q$) is given by

$$
\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2 + 1)}, \quad \text{where,} \quad x_q = \Delta m_q \tau_{D_q}, \quad y_q = \frac{\Delta \Gamma_q \tau_{D_q}}{2}
$$
The mass difference $\Delta m$ is a measure of the frequency of the change from a $(D)^0$ into a $\bar{D}^0$ or vice versa. We have the time-integrated mixing rate for semi-leptonic decays as \cite{1}

$$R_M \simeq \frac{1}{2}(x_q^2 + y_q^2). \quad (11)$$

For the estimation of the mixing parameters $x_q$, $y_q$, $\chi_q$ and $R_M$, we use $\eta_D = 0.86$, $\eta_D' = 0.21$ and the gluonic correction to the oscillation is given by Ref. \cite{11,12}. The bag parameter $B_{D_q} = 1.34$ is taken from the lattice result of \cite{13}, while the pseudoscalar mass ($M_D$), and the pseudoscalar decay constant ($f_D$) of the charmed mesons are taken from our previous study in the article \cite{5}. The calculated mixing parameters $x_q$, $y_q$, $\chi_q$ and $R_M$ are tabulated in Table 3.

**Table 3.** Mixing parameters of the $D$ meson.

| This work | $\Delta m_q \times 10^{-15}$ | $x_q \times 10^{-3}$ | $y_q \times 10^{-3}$ | $\chi_q \times 10^{-5}$ | $R_M \times 10^{-3}$ |
|-----------|---------------------------|---------------------|---------------------|------------------------|---------------------|
| $f_p$     | 2.8                       | 2.96                | 17.2                | 0.15                   | 0.152               |
| $f_{p_{cor}}$ | 1.27                   | 1.322               | 7.67                | 3.03                   | 0.03027             |
| Ref. 14   | $1.6\pm2.3 \pm 1.2$      | $5.7\pm2.0\pm1.3 \pm 0.7$ |                      |                       |                     |
| Ref. 15   | $8.0\pm2.9$              | 3.32                |                     |                       |                     |
| Ref. 16   |                           |                     |                     | 0.0864$\pm0.0311$     |                     |
| Ref. 17   |                           |                     |                     | 0.13$\pm0.22\pm0.20$  |                     |
| Ref. 18   |                           |                     |                     | 0.04$\pm0.6+0.7$      |                     |
| Ref. 19   |                           |                     |                     | 0.02$\pm0.47\pm0.14$  |                     |
| Ref. 20   |                           |                     |                     | 1.6$\pm2.9\pm2.9$     |                     |

4. Conclusion

From comparison of our estimated radiative (E1 and M1 dipole) transitions widths with other theoretical estimations, we conclude that the various models have different predictions of E1 and M1 dipole transitions may be due to different parameters and treatments are used in the relativistic corrections in the model. We have compared our results of the mixing parameters to PDG \cite{1} as well as other theoretical results and they are in agreement with our results.

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