Neutral Trilinear Gauge Boson Couplings in Little Higgs Models

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We compute the one loop new physics effects to the CP even triple neutral gauge boson vertices $\gamma^* \gamma Z, \gamma^* Z Z, Z^* Z \gamma$ and $Z^* Z Z$ in the context of Little Higgs models. In addition, we re-examine the MSSM contribution at the chosen point of SPS1a’ and compare with the SM and Little Higgs models.

1 Introduction

The measurement of couplings among the electroweak gauge bosons can lead to understanding the non-abelian gauge structure of the Standard Model (SM) and confront the presence of the new physics above the weak scale. The forthcoming experiments at the LHC and proposed ILC offer the exciting prospect of probing physics beyond SM. The neutral gauge-boson couplings $Z \gamma \gamma, ZZ \gamma$ and $ZZ Z$ which can be studied in $Z\gamma$ and $ZZ$ pair production in $e^+ e^-$ and in hadron colliders through $e^+ e^- \rightarrow Z\gamma, ZZ$ and $q\bar{q} \rightarrow Z, ZZ$ respectively have been analyzed within the SM and MSSM [1, 2]. In this paper we study the CP conserving trilinear neutral gauge-boson couplings in little Higgs models and MSSM.

2 Neutral gauge-boson couplings

Bose-Einstein statistics render the three neutral gauge-boson couplings $\gamma \gamma Z, \gamma ZZ$ and $ZZ Z$ to vanish when all the three vector bosons are on shell. The most general CP conserving coupling of one off-shell boson $V \equiv Z/\gamma$ to a pair of on-shell $Z\gamma$ and $ZZ$ gauge bosons (all incoming) can be written as (see Ref. [2])

$$\Gamma_{VZ\gamma}(Q,p_1,p_2) = i \left[ \mathcal{H}^\gamma_V \epsilon^{\mu\beta\alpha\eta} p_{2\eta} + \frac{\mathcal{H}^\gamma_V}{M_Z^2} \left\{ \epsilon^{\mu\beta\rho\eta} p_{2\rho} Q^\alpha \right\} \right]$$

$$\Gamma_{VVZ}(Q,p_1,p_2) = i \left[ \mathcal{F}^V \epsilon^{\mu\alpha\beta\sigma} (p_1 - p_2)_{\eta} \right]$$

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where $\Gamma^{\mu\alpha\beta}_{\alpha\beta}(Q, p_1, p_2)$ represents the coupling of off-shell neutral gauge boson $V^\mu$ carrying momentum $Q$ with the bosons $V_1^\alpha$ and $V_2^\beta$ carrying momenta $p_1$ and $p_2$ respectively.

In the SM these couplings vanish at the tree level. These couplings can however, be generated at the loop level. These couplings can in general be complex quantities. However, they pick up imaginary contribution only when $Q^2$ crosses the threshold for fermion pair production (i.e. $Q^2 > 4m_f^2$) for timelike $Q^2$ or when $M_Z^2$ exceeds this threshold (i.e. $M_Z > 2m_f$) for spacelike $Q^2$. Further at the one loop level, the couplings $H_4^\gamma = H_4^Z = 0$.

The contribution of the fermionic triangle graphs to the trilinear vector boson couplings $F_5^\gamma, Z$ and $H_3^\gamma, Z$ can be expressed in terms of scalar Passarino-Veltman (PV) functions.

2.1 SM and MSSM Contribution
The contribution to the trilinear neutral gauge couplings in the SM arise from the three families of quarks and leptons. The anomaly cancellation ensures that all the couplings go to zero for $Q^2$ much larger than the fermion pair production threshold. It is obvious that of all the thresholds (at $Q^2 = 4M_f^2$), the largest contribution comes from the heaviest fermion loop.

The MSSM contribution to the trilinear neutral gauge couplings has been calculated in the references [1, 2]. We re-calculate the MSSM contribution in the light of the reference point SPS1a’ which is defined at a characteristic scale of 1 TeV with its origin in minimal super-gravity (mSUGRA) [9]. The root GUT scale mSUGRA parameters in this reference point SPS1a’ are the gaugino mass $M_1/2 = 250$ GeV, the universal scalar mass $M_0 = 70$ GeV, the trilinear coupling $A_0 = -300$ GeV, $\tan(\beta(\tilde{M})) = 10$ and $\text{sign}(\mu) = +1$. Extrapolating these parameters to $\tilde{M} = 1$ TeV generates the MSSM Lagrangian parameters. The relevant evolved MSSM parameters for our calculations are the Higgs mixing parameter $\mu = 396$ GeV and $M_2 = 193.2$ GeV.

2.2 Contribution in Little Higgs Models
Recently there has been a proposal to consider Higgs fields as pseudo Nambu-Goldstone bosons of a global symmetry [3, 4] which is spontaneously broken at some high scale. The realization of little Higgs mechanism discussed in the literature essentially fall into two classes namely the product gorup and simple group depending on the gauge symmetry. The Littlest Higgs model (LH) is a minimal model of the product group class which accomplishes this task to one loop order within a minimal matter content. SU(3) simple group model is a representative model of the second class.

In the Littlest Higgs model $[SU(2) \times U(1)]^2$ gauge symmetry is embedded in an $SU(5)$ global symmetry. The gauge symmetry is broken down to the SM $SU(2) \times U(1)$ gauge symmetry by a single vacuum condensate $f \approx 1$ TeV. The new fermionic degrees of freedom
in the Littlest Higgs model are in the heavy quark sector and consist of a pair of vector-like 
$SU(2)$-singlet quarks that couple to the top sector. The resultant top sector consists of a 
top quark $t$ and its heavy partner $T$ whose masses and couplings are given in terms of model 
dependent parameters by $m_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ and $M_T = \sqrt{\lambda_1^2 + \lambda_2^2} f = \frac{1}{\sqrt{X_L (1 - X_L)}} \frac{m_t}{v} f$. Here 
$X_L = \lambda_1^2 / (\lambda_1^2 + \lambda_2^2)$, $\lambda_1$ and $\lambda_2$ being the couplings that appear in the heavy quark sector of 
the interaction lagrangian. Extra Contribution to TNGBC’s comes from the triangle graph 
with only heavy top and also both the SM top $t$ and heavy top $T$ simultaneously present in 
the loop.

The second class of little Higgs models feature a simple group that contains an $SU(N) \times 
U(1)$ gauge symmetry that is broken down to $SU(2)_L \times U(1)$, giving rise to a set of TeV-scale 
gauge bosons. The two gauge couplings of $SU(N) \times U(1)$ are fixed in terms of two SM gauge 
couplings, leaving no free parameters in the gauge sector. Furthermore, due to enlarged 
$SU(N)$ gauge symmetry, all fermionic SM representations are extended to transform as fundamental or conjugate representations of $SU(N)$ giving rise to additional heavy fermions 
in all the three quark and lepton sectors. The simplest realization of this simple group class 
is the $SU(3)$ simple gauge model \[4\] with anomaly-free embedding of extra fermions. In this 
model extra fermions present are – heavy fermions associated with each $SU(2)_L$ doublet of 
the SM, new TeV-scale D and S quarks of charge $-1/3$, heavy third generation quark $T$ of 
charge $+2/3$ and electrically Neutral Heavy leptons $N_i$ of three generations. The masses of 
these heavy fermions are given in terms the parameters of the model. The extra one-loop 
Contribution to TNGBC’s comes from the triangle graph with mixed SM top $t$ and heavy 
top $T$, mixed SM and TeV range quarks of the first two generations and the mixed neutrino 
and TeV mass heavy neutrinos ($N_i$) of all the three generations in the triangle loop.

Constraints from electro-weak precision measurements require the breaking scale $f$ to 
be greater than 5 TeV in the Littlest Higgs Model and $f > 3.9$ TeV for $t_\beta = 3$ in the 
amanomaly free $SU(3)$-simple group \[5\]. Both these models suffer from severe constraints 
\[6\] \[7\] \[5\] from precision electro-weak measurements. Motivated by these considerations, an 
implementation of a discrete symmetry called T-parity is proposed. T-parity explicitly 
forbids any tree-level contribution from the heavy mass states to observables involving only 
the SM particles. It also forbids the interactions that impart vev to triplet Higgs, thereby 
generating the corrections to precision electro-weak observables only at the one loop level. 
In this little Higgs model with T-parity (LHT) \[8\] there are heavy T-odd partners of the 
SM gauge bosons and SM fermions called mirror fermions that couple vectorially to $Z$. In 
the top quark sector, the model incorporates two heavy T-even and T-odd top quarks in 
addition to the T even SM top quark, which are required for canceling the quadratically 
divergent contribution of the SM top quark to the Higgs mass. Further, because of $T$-parity

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conservation there is no coupling of $Z_\mu$ with a T-odd and a T-even fermion i.e. the coupling $Zf_+f_- = 0$. The T-even partner of the top quark $T_+$ however, has both axial and vector couplings with $Z_\mu$ and hence contributes to the triangle loop. In addition to the contribution from SM fermions in the loop, we now get additional contributions from heavy T-even partner and the top quark as well as contributions from the triangle loops with mixed contributions from $t$ and $T_+$ quarks.

| Ratio | $M_{T_+} \ (\text{GeV})$ | $\sqrt{Q^2}$ | $\mathcal{H}_1^T (10^{-4})$ | $\mathcal{H}_3^Z (10^{-4})$ | $\mathcal{F}_1^Z (10^{-4})$ | $\mathcal{F}_3^Z (10^{-4})$ |
|-------|--------------------------|-----------|----------------------------|----------------------------|----------------|----------------|
| 0.5   | 889.2                    | 2$m_t$    | $-93.40 - i 0.0140$        | $28.85 + 0 i$              | $-30.83 + 0 i$ | $-20.29 + 0 i$ |
|       |                          | $m_t + M_{T_+}$ | $4.068 - i 22.063$        | $-0.705 + i 7.341$        | $1.413 - i 7.612$ | $-1.001 - i 10.82$ |
|       |                          | $2M_{T_+}$ | $4.595 - i 10.66$          | $-1.499 + i 3.875$        | $1.441 - i 3.666$ | $1.574 - i 6.475$ |
| 1.0   | 711.4                    | 2$m_t$    | $-89.34 - i 0.0130$        | $25.49 + 0 i$              | $-27.65 + 0 i$  | $-18.06 + 0 i$  |
|       |                          | $m_t + M_{T_+}$ | $0.9154 - i 28.02$        | $5.388 + i 7.730$        | $-1.313 - i 9.229$ | $-8.534 - i 11.58$ |
|       |                          | $2M_{T_+}$ | $3.710 - i 14.24$          | $-0.3113 + i 6.901$       | $-0.0809 - i 5.481$ | $1.717 - i 10.66$ |
| 2.0   | 889.2                    | 2$m_t$    | $-81.80 - i 0.0094$        | $19.84 - i 0.0054$        | $22.00 + 0 i$    | $-14.31 + i 0.0002$ |
|       |                          | $m_t + M_{T_+}$ | $0.7583 - i 19.59$        | $14.88 + i 3.813$        | $-4.727 - i 7.226$ | $-15.09 - i 8.566$ |
|       |                          | $2M_{T_+}$ | $1.116 - i 9.099$          | $1.424 + i 9.069$        | $-2.866 - i 5.549$ | $-5.549 - i 13.21$ |
| 3.0   | 1185.6                   | 2$m_t$    | $-78.51 - i 0.0079$        | $17.58 - i 0.0068$        | $-19.40 + 0 i$   | $-12.69 + i 0.0002$ |
|       |                          | $m_t + M_{T_+}$ | $0.4505 - i 13.03$        | $18.70 + i 2.076$        | $-6.278 - i 6.100$ | $-16.71 - i 6.937$ |
|       |                          | $2M_{T_+}$ | $-0.623 - i 5.49$          | $1.759 + i 9.726$        | $-3.972 - i 5.508$ | $4.953 - i 13.55$ |
| 4.0   | 1511.7                   | 2$m_t$    | $-77.65 - i 0.0072$        | $16.60 - i 0.0074$        | $-18.16 + 0 i$   | $-11.95 + i 0.0002$ |
|       |                          | $m_t + M_{T_+}$ | $0.2019 - i 9.182$        | $20.18 + i 1.320$        | $-7.086 - i 5.541$ | $-17.0 - i 6.011$ |
|       |                          | $2M_{T_+}$ | $-1.712 - i 3.614$         | $1.824 + i 9.939$        | $-4.461 - i 5.508$ | $5.479 - i 13.48$ |

Table 1: Ratio $r = \lambda_1/\lambda_2$ in the LHT Model. $f = 500 \text{ GeV}$ and $m_t = 175 \text{ GeV}$.

3 Results and Conclusions
We calculate the one-loop contribution to the CP-conserving trilinear neutral gauge boson couplings in SM, MSSM and the two classes of Little Higgs Models for various parameters of the models. Certain features are common to all these graphs which we note here. All couplings vanish asymptotically for large $\sqrt{Q^2}$ compared to the highest fermion mass in the theory. The relative importance of the real and imaginary parts of the couplings is strongly energy dependent. Below the $2m_t$ threshold, the imaginary parts of all the couplings are negligible. At and above this threshold the imaginary parts become comparable or even dominant in comparison to the real parts. The $\sqrt{Q^2}$ variation of the real and imaginary parts of the couplings in all the four models is shown in Fig. 2. Values at some typical $\sqrt{Q^2}$
Table 2: The values of various couplings (written as complex numbers) at some typical $\sqrt{Q^2}$ (where peaks are expected) in the SU(3) simple model with anomaly free embedding. All values correspond to $\tan\beta = r = 3$, scale $f = 3$ TeV and $m_t = 175$ GeV. At these values of parameters, the mass of heavy top is $M_T = 1.8$ TeV and masses of all other heavy fermions have been taken to be $M_i = 3$ TeV.

| $\sqrt{Q^2}$ (in TeV) | $\mathcal{H}_3^Z$ $(10^{-4})$ | $\mathcal{H}_3^F$ $(10^{-4})$ | $\mathcal{F}_5^Z$ $(10^{-4})$ | $\mathcal{F}_5^F$ $(10^{-4})$ |
|------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $2m_t$                 | $-94.17 - i 0.0158$      | $29.53 + i 0$            | $-31.50 + i 0.0149$      | $-22.42 + i 0.0254$      |
| $m_t + M_T$            | $4.533 - i 9.136$        | $-1.487 + i 3.008$       | $1.757 - i 2.802$        | $-0.1062 - i 4.751$      |
| $2M_T$                 | $2.582 - i 3.448$        | $-2.417 - i 0.0712$      | $0.5535 - i 0.677$       | $0.9483 + i 0.9624$      |
| $M_U$                  | $3.146 - i 4.660$        | $-2.503 + i 2.036$       | $1.146 - i 1.152$        | $1.699 - i 2.449$        |
| $2M_U$                 | $1.372 - i 1.455$        | $0.1424 - i 1.523$       | $2.947 - i 0.191$        | $-3.151 + i 2.236$      |

are also given in the Tables 1 for different values of parameters of the model. However, there are certain features that are different in various models. In the MSSM, $\sqrt{Q^2} = 2m_{\chi^+}$, which is very near to the $2m_t$ SM peak resulting in the enhancement of the couplings at this point. This effect is more pronounced in the imaginary parts of the couplings. In the MSSM new peaks appear at $m_{\chi^+_1} + m_{\chi^+_2} \simeq 600$ GeV and $2m_{\chi^+_2} \simeq 800$ GeV which is more pronounced in the real parts of $\mathcal{F}_5^Z$ and $\mathcal{F}_5^F$. In SM and Little Higgs Model there is no such effect up to 1 TeV.

The effect of extra heavy fermions in the LHT Model is to decrease the threshold effects of the SM whereas the particles in MSSM enhance it.

The new threshold in the LHT at $\sqrt{Q^2} = m_t + M_{T^+}$ and in the MSSM as mentioned above are opposite to each other but the magnitudes are comparable. The anomaly free SU(3) simple Model does not show any appreciably different behaviour than the SM up to $\sqrt{Q^2} = 1$ TeV.

At higher $\sqrt{Q^2}$, the effect of new heavy fermions shows up but the threshold values are an order of magnitude lower than that at the $2m_t$ threshold. However, at these $\sqrt{Q^2}$, the SM contribution is negligible.

For higher ratios, a very interesting behaviour is shown by the couplings $\mathcal{H}_3^Z$ and $\mathcal{F}_5^Z$. Not only the imaginary part becomes appreciable at high $\sqrt{Q^2}$ but also the threshold values of the couplings at $\sqrt{Q^2} = m_t + M_{T^+}$ are higher than those at $\sqrt{Q^2} = 2m_t$ and are comparable to the SM value (See Figure 1 and Table 1).

Our analysis presented above allows us to confront and discriminate among various models considered here on the basis of these couplings.

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Figure 1: $\sqrt{Q^2}$-variation of real and imaginary parts of $H_3^Z$ and $F_5^Z$ in the range $0-3$ TeV in Little Higgs Model with T-parity for $r = 3$ and $f = 0.5$ TeV. With this choice of parameters the mass of T-even top, $M_{T^+_e} = 1.186$ TeV.
Figure 2: $\sqrt{Q^2}$-variation of the real and imaginary parts of the couplings in various models for model parameter values $f = 500$ GeV, $r = 1$ for LHT and $f = 3$ TeV, $t_\beta = 3$ and masses of all heavy fermions, $M_i = 2$ TeV for the SU(3) Model. MSSM parameters are as discussed in the text.