The $H_0$ tension: $\Delta G_N$ vs. $\Delta N_{\text{eff}}$

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Abstract

We investigate whether the 4.4$\sigma$ tension on $H_0$ between SH$_0$ES 2019 and Planck 2018 can be alleviated by a variation of Newton’s constant $G_N$ between the early and the late Universe. This changes the expansion rate before recombination, similarly to the addition of $\Delta N_{\text{eff}}$ extra relativistic degrees of freedom. We implement a varying $G_N$ in a scalar-tensor theory of gravity, with a non-minimal coupling of the form $(M^2 + \beta \phi^2)R$. If the scalar $\phi$ starts in the radiation era at an initial value $\phi_I \approx 0.3 \, M_{\text{Pl}}$ and with $\beta \approx -0.8$, a dynamical transition occurs naturally around the epoch of matter-radiation equality and the field evolves towards zero at late times. As a consequence the $H_0$ tension between SH$_0$ES (2019) and Planck 2018+BAO decreases, as in $\Delta N_{\text{eff}}$ models. However, mostly due to late-time constraints from Post-Newtonian (PN) local gravity, the tension is reduced only to 3.5$\sigma$ level. When including also the SH$_0$ES data in the fit, the varying $G_N$ model has $H_0 = 69.2^{+0.93}_{-0.75}$ and an improvement of $\Delta \chi^2 = -3.6$ compared to $\Lambda$CDM, at the cost of 2 extra parameters. This corresponds to a decrease of $7^{+3}_{-2}$ percent in the value of $G_N$ from the radiation era to the present time. For comparison, we update the fit of the $\Delta N_{\text{eff}}$ model to the same dataset. We find that the $\Delta N_{\text{eff}}$ model performs better than the simplest varying $G_N$ scenario, with $H_0 = 70^{+0.93}_{-0.95}$ and $\Delta \chi^2 = -5.5$. The $\Lambda$CDM limit of the $\Delta N_{\text{eff}}$ model is disfavored at slightly more than 2$\sigma$, since $\Delta N_{\text{eff}} = 0.316^{+0.15}_{-0.15}$. 


1 Introduction

The expansion rate of the Universe is currently at the center of an observational tension. On the one hand, the present Hubble parameter $H_0$ can be determined by measuring distances of astronomical objects according to the distance ladder method. The most precise recent measurement of this kind has been performed by the SH$_0$ES team using Supernovae data and gives $H_0 = 74.03 \pm 1.42$ km/s/Mpc [1], with a calibration method based on Cepheids. On the other hand, $H_0$ is independently determined by Cosmic Microwave Background (CMB) data, once a given cosmological model is specified. The latest fit of the standard six-parameter $\Lambda$CDM model to CMB temperature, polarization and lensing power spectra measured by the Planck collaboration gives $H_0 = 67.27 \pm 0.60$ km/s/Mpc [2]. Therefore, the two measurements disagree at $4.4\sigma$. When combined with baryon acoustic oscillations (BAO) data, Planck finds $H_0 = 67.66 \pm 0.42$ km/s/Mpc [2], assuming the six-parameter $\Lambda$CDM. Other local measurements generally prefer values of $H_0$ which are higher than the CMB measurement (see [3] and [4] for a complete review), with the exception of [5], which finds $H_0 = 69.8 \pm 0.8(\pm 1.1\text{stat}) \pm 1.7(\pm 2.4\text{sys})$ km/sec/Mpc, midway in the range defined by the current Hubble tension.

At the time of writing, no satisfactory explanation of the discrepancy between the various measurements based on systematic errors has emerged. While we wait for the final observational verdict, it is interesting to ask whether a cosmological model different from $\Lambda$CDM can resolve the tension between empirical and cosmological model-dependent determinations of $H_0$. Different efforts in this direction can be broadly classified as late or early time attempts (see [6] for a comprehensive review): the former modify the cosmological history only much after recombination, whereas the latter feature changes before or around recombination.

The Planck and SH$_0$ES measurements of $H_0$ can be made to agree better by changing the expansion history after recombination, with a different time evolution of the angular diameter distance with respect to standard $\Lambda$CDM. However, BAO data is in conflict with this kind of solution, which makes early time models the most effective strategy to solve the $H_0$ tension, see e.g. [7]. Arguably, the simplest such attempt consists in adding one additional parameter to the base six-parameter $\Lambda$CDM, allowing for extra relativistic species ($\Delta N_{\text{eff}}$) beyond the Standard Model neutrinos, e.g. axions [8]. This modification, shortly the ‘$\Delta N_{\text{eff}}$ model’, alleviates the tension [2, 8], but does not solve it completely, mainly because the required value of $\Delta N_{\text{eff}}$ also affects the photon diffusion scale and thus spoils the fit to the CMB damping tail. We will nonetheless consider such a model using recent 2019 SH$_0$ES data vs. the old 2018 SH$_0$ES data, showing a rather significant shift of the fit. Moreover, the $\Delta N_{\text{eff}}$ model constitutes a useful benchmark to compare how well other theoretical proposals perform.

A better fit to Planck, BAO and SH$_0$ES data than that of the six-parameter $\Lambda$CDM model was shown to be provided by the addition of an extra early dark energy (EDE) component, which contributes $\sim 5\%$ of the total energy density of the Universe just before the epoch of matter-radiation equality [9–11], and then dilutes faster than radiation, in such a way as to minimize the effects on the photon diffusion scale. Field theory realizations of this idea [9, 10] employ a light scalar field, which is initially frozen in its potential due to Hubble friction. Once the Hubble rate drops to values comparable to the curvature of the potential, the field starts rolling and may or may not oscillate, depending on the properties of the potential and the initial field value. Simple power-law potentials fit the data only marginally better than the $\Delta N_{\text{eff}}$ model, with $\phi^4$ being the preferred potential [12] (in this case the first few oscillations provide a short epoch where $w \gtrsim 1/3$). To date, the best fit to
the CMB, BAO and SH0ES data sets (in this kind of models) is provided by a scalar field
which initially sits in the concave region of a \( \cos(\phi)^n \) potential, with \( n > 1 \) (\( n = 3 \) being the preferred power) [9,10]; see however [13] for a recent critical take on EDE models.

Despite the relative success in alleviating the Hubble tension, the above scenarios may still be considered unattractive compared to the simpler \( \Delta N_{\text{eff}} \) extension of \( \Lambda \text{CDM} \), for the following reasons: Firstly, none of them explains why the transition in the EDE component occurs around matter-radiation equality. Rather, the curvature of the potential is fixed ad-hoc to be of the order of the Hubble rate at the relevant epoch. Therefore, these scenarios suffer from a coincidence problem. Secondly, the EDE scenario of [9] requires a potential which, although periodic, does not match the standard potential for axion-like fields, and whose field theory origin may thus be considered uncertain. Furthermore, the latter model introduces four extra parameters with respect to the base six-parameter \( \Lambda \text{CDM} \) [10,11].

It is this unsatisfactory situation which we take as motivation for this work. We aim at finding a model alternative to \( \Delta N_{\text{eff}} \), with the smallest number of extra parameters and which does not suffer from a coincidence problem. Our approach stems from the realization that the background effect of dark relativistic species in the early Universe, e.g. at the epoch of Big Bang Nucleosynthesis (BBN), can be mimicked by a varying Planck mass in a Universe with the standard matter and radiation content, see e.g. [14]. That this is indeed the case can be easily understood by looking at Friedmann’s equation: on the one hand in the \( \Delta N_{\text{eff}} \) model the additional energy density in dark radiation (and in the other species, including dark matter, to keep the fit to CMB data) necessarily corresponds to an increase in the Hubble parameter; on the other hand the same increase can be obtained by keeping the standard radiation and matter content but taking the Planck mass to be smaller in the early Universe than it is today. The extent of the analogy between the latter and the former models as we approach the epoch of matter-radiation equality is less clear, in particular with respect to the Hubble tension. Indeed, as we will show, the shift in the the total energy density caused by a varying \( G_N \) can scale differently from radiation at this epoch. Furthermore, the behavior of cosmological perturbations is in general different in the two models. In this work, we would therefore like to assess whether a varying Newton’s constant can alleviate the Hubble tension, as an alternative scenario to the \( \Delta N_{\text{eff}} \) model.

In order to do so, we will consider one of the simplest and popular implementations of this idea in field theory. This is provided by a scalar field non-minimally and quadratically coupled to gravity, similar to the old Brans-Dicke proposal [15]. The model which we will focus on adds only two extra parameters to the \( \Lambda \text{CDM} \) model, which correspond to the minimum number of parameters needed to describe a time varying Newton’s constant: its initial value in the early Universe and the rate of its variation. Very interestingly, this field theory scenario presents some of the ingredients that characterize the aforementioned oscillating scalar field models, without suffering from the same coincidence problem. Indeed, an essential feature of the epoch of matter-radiation equality is a change in the evolution of the Hubble parameter. As a consequence of Einstein’s equations, the gravitational background field also changes at matter-radiation equality: in particular, the Ricci scalar goes from being approximately vanishing during radiation domination to a non-zero value during matter domination. Therefore, by coupling a scalar field to the Ricci scalar, a dynamical transition at the epoch of matter-radiation equality arises naturally. While this is equivalent to a model with canonical Einstein action but with the scalar field coupled directly to the matter Lagrangian, it is easier to understand its most interesting aspects in the so-called Jordan frame, where the scalar field has a time dependent mass which is proportional to the
If we set the scalar potential to zero, the field is essentially frozen during radiation domination, and becomes dynamical only close to the onset of matter domination. Depending on initial conditions and on the value of the (dimensionless) non-minimal coupling, the field can then roll or oscillate around matter-radiation equality. Intriguingly, its energy density then redshifts faster than radiation and makes the scenario promising from the point of view of the Hubble tension. By performing a fit to cosmological data, we confirm that a larger Hubble constant can indeed be accommodated in our setup. However, we find that the inclusion of late-time constraints on modifications to General Relativity limits the likeliness of this scenario with respect to the $\Delta N_{\text{eff}}$ and $\Lambda$CDM models.

Before moving to the body of the paper, let us mention previous work in similar directions. Constraints from the CMB on a field theory scenario which is similar to ours have been presented in [16], where the relation to the Hubble tension was also investigated, albeit a fit with SH0ES data was performed only for a specific choice of the non-minimal coupling, i.e. the conformal case. Similar ideas to ours have been also recently discussed in [17], in frameworks with more extra parameters than the model which we focus on here. We comment further on the relation of our work to [16] and [17] below. Finally, non-minimal couplings to alleviate the Hubble tension have also been considered in [18], which however makes use only of threshold effects on the scalar field, due to neutrinos becoming non-relativistic. Such threshold effects are instead subdominant in our scenario.

Our paper is organized as follows. In section 2 we present the non-minimally coupled scalar field model; in section 3 we present the results of the fits to cosmological data and in section 4 we draw our conclusions. We use natural units throughout the paper.

2 Model

In order to investigate the implications of a varying $G_N$ scenario for the Hubble tension, we will consider a simple scalar-tensor model which introduces only two new parameters with respect to $\Lambda$CDM. It is easy to see that this is the minimum number of extra parameters which is needed to capture variations of $G_N$: one parameter corresponds to the difference between the values of the Newton’s constant in the early Universe and today; a second parameter is needed to describe the rate of variation of $G_N$. This is in contrast with the $\Delta N_{\text{eff}}$ proposal to address the Hubble tension, which introduces only one extra parameter to $\Lambda$CDM.\(^1\) More complicated models, with more than two new parameters, can also be considered (see e.g. [17] for recent work in the context of the Hubble tension) and we will briefly comment on this possibility later on.

The simplest scenario is a modification of Einstein’s gravity, obtained by coupling a scalar field $\phi$ that sets the value of $G_N$ to the Ricci scalar $R$. The action of this non-minimally coupled scalar, in the so-called Jordan frame, is

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ M^2 f(\phi)R + \partial_\mu \phi \partial^\mu \phi + L_{\text{tot}} \right],$$  \hspace{1cm} (1)

where

$$f(\phi) \equiv 1 + \beta \frac{\phi^2}{M^2},$$  \hspace{1cm} (2)

\(^1\)However, again at least two parameters are needed to describe new species which behave as radiation at early times, with a change in the equation of state parameter at late times.
\( \beta \) is a dimensionless coupling constant assumed to be negative, \( M \) is a constant mass scale and \( L_{\text{tot}} \) represents the remaining contents of the Universe, including radiation, dark matter, baryons, neutrinos and a cosmological constant \( \Lambda \). We assume a negligible mass in the Jordan frame for \( \phi \). The same model can be presented in the so-called Einstein frame by means of a Weyl transformation of the metric tensor \( g^{E}_{\mu\nu} = f(\phi)g_{\mu\nu} \). Then the Ricci term in the action becomes canonical, but the field \( \phi \) couples directly to the matter Lagrangian. Physically measurable quantities are of course the same in the two frames.

The background equation of motion of the scalar in a flat FLRW metric in the Jordan frame is

\[
\ddot{\phi} + 3H \dot{\phi} - \beta R \phi = 0, \tag{3}
\]

where a dot denotes a derivative with respect to to cosmic time \( t \), \( H \equiv \dot{a}/a \) is the Hubble parameter and \( a(t) \) is the FLRW scale factor. The Ricci scalar can be expressed as a function of \( H \) as

\[
R = 6 \dot{H} + 12H^2. \tag{4}
\]

In the deep radiation era, \( H(t) \simeq 1/(2t) \), thus \( R \approx 0 \) and the field is frozen at some initial value \( \phi_I \), which can be expected to be of the same order of \( M \).\(^2\) By writing Friedmann’s equation as \( 3H^2M^2 = \rho_\phi + \rho_{\text{tot}} \), one finds

\[
\rho_\phi = \rho_1 + \rho_2 + \rho_3 = \frac{1}{2}\phi^2 - 6\beta H \phi \dot{\phi} - 3\beta H^2 \phi^2, \tag{5}
\]

which may be interpreted as an energy density of the homogeneous scalar field in this frame. Here \( \phi^2/2 \) is the kinetic energy of the scalar field, and the last term is a shift in the usual critical energy density \( 3H^2M^2 \), since in our case the effective Planck mass is field dependent: \( M^2(\phi) \equiv M^2 + \beta \phi^2 \). Since \( \beta < 0 \), the Planck mass is smaller (and thus gravity is stronger) in the early Universe than it is today. In order to have always a positive \( M^2 \) we will impose \( \phi_I^2 < M^2/\beta \).

As the Universe approaches the matter dominated era, \( R > 0 \), the scalar field acquires an effective mass squared of order \( \beta H^2 \) and starts rolling towards zero reaching a final value \( \phi_0 \), typically much smaller than \( \phi_I \). Therefore, the dynamics naturally features the “release” of an initially frozen scalar around the epoch of matter-radiation equality. This is in stark contrast to usual EDE models, where the time of transition from \( w = -1 \) to \( w \geq 1/3 \) has to be set in an ad-hoc manner to address the Hubble tension.

We will investigate the transition from the radiation dominated era to the matter dominated epoch numerically. However, one of the most interesting features of our setup can be understood analytically, by solving (3) in the matter dominated era. Setting \( a \propto t^{2/3} \), one straightforwardly finds

\[
\phi \propto t^{\pm \frac{1}{2} \sqrt{1 + \frac{16}{3} \beta^{-rac{1}{2}}}} \propto a^{\pm \frac{1}{2} \sqrt{1 + \frac{16}{3} \beta^{-rac{1}{2}}}}, \tag{6}
\]

Therefore, for \( \beta < -3/16 \) the field undergoes damped oscillations, while for \(-3/16 \leq \beta < 0\) the evolution towards zero is monotonic. Since \( H \sim a^{-3/2} \), it is straightforward to check that all terms in (5) scale as \( a^{-4.5} \) when \( \beta \leq -3/16 \), averaging over oscillations when they are present. Therefore, the energy density of the background scalar field is diluted faster.

\(^2\)We neglect threshold effects due to particles that become non-relativistic [19] and the conformal anomaly due to the running of coupling constants [20]. The first effect has been used in [18] to address the \( H_0 \) tension. Both effects are subdominant in our scenario.
than radiation once matter dominates. The scaling of (5) at early times, during radiation domination, is found by setting \( \phi \sim \text{const.} \), so that only the last term in (5) contributes and scales as \( \sim H^2 \sim a^{-4} \), i.e. as radiation.

As noted in [9] a new species that dilutes faster than radiation after equality might fit the CMB and SN data better than the \( \Lambda \)CDM model with the addition of dark relativistic species. In our setup this happens as long as \( \beta \leq -3/16 \).

However, the scalar-tensor model which we analyze in this paper is subject to two additional constraints. The first one applies to any model which predicts a variation of Newton’s constant from the early Universe to today, since a too large deviation would spoil the agreement with Big Bang Nucleosynthesis (BBN) data. Secondly, at late times the simple model (1) effectively introduces a fifth force, whose strength is severely constrained by local gravitational bounds on Post-Newtonian (PN) parameters. We discuss both constraints and their implications on the parameter space of the model (1) in detail in Sec. 3. Let us simply point out here that the latter constraint could be evaded in more complicated models, with more than two parameters, where for instance the scalar field Lagrangian contains higher derivative terms which effectively screen the fifth force (see e.g. [17] for a recent discussion).

A detailed analysis of the dynamics and impact of the non-minimally coupled scalar requires the implementation of the model (1) in a Boltzmann code. We have done so by modifying \texttt{hi-class}, a public code for scalar-tensor theories [21,22], based on the Boltzmann code \texttt{CLASS} [23]. In the notation of \texttt{hi-class}, the model (1) corresponds to setting the functions \( G_2 = -\Lambda + X \equiv -(\Lambda + \partial_\mu \phi \partial^\mu \phi) / 2 \) and \( G_4 = M_*^2 / 2 \), with units \( M = 1 \), and \( G_3 = G_5 = 0 \). The need to modify \texttt{hi-class} arises because the original code does not support oscillating scalar fields, i.e. can only deal with the monotonically rolling case and thus does not allow for a full exploration of the relevant parameter space. In particular, our modifications to the code concern the evolution of the scalar field perturbations \( \delta \phi(t, x) \), which are coupled to matter and metric perturbations and can thus crucially affect the CMB and the matter power spectra. The relevance of perturbations from the point of view of alleviating the Hubble tension, in particular for oscillating scalar fields, has been recently stressed in [9]. While the original \texttt{hi-class} employs the variable \( V_x \equiv -a \delta \phi(t, x) / \dot{\phi} \) for the equations of the perturbations, our modified code works directly with \( \delta \phi(t, x) \) and it is thus

![Figure 1: Evolution of the scalar \( \phi \) in units of \( M \) (solid blue) and the quantity \( \Delta G_N^\% \equiv 100 |1 - G_N / G_0^N| \) (dashed blue) as a function of the redshift \( z \). On the left: \( \beta = -0.1 \) and \( \phi_I = 0.46M \). On the right: \( \beta = -0.8 \) and \( \phi_I = 0.32 M \). The orange vertical line denotes the redshift of matter-radiation equality. In both figures, we have fixed all cosmological parameters, except for \( \phi_I \) and \( \beta \), to the best-fit values reported for our model in Table 1.](image-url)
Figure 2: Contribution to the background energy density due to the scalar field, according to (5), as a function of $z$, normalized to the total energy density at matter-radiation equality. We have used the values of cosmological parameters corresponding to the best-fit reported in Table 1. The orange vertical line denotes the redshift of matter-radiation equality. The dashed and dotted lines show different scalings of $\rho_\phi$ with $a$.

well-behaved at the turning points of the background field.

We plot in Fig. 1 the evolution of $\phi$ (absolute value) as well as of $|\Delta G_N| \equiv |1 - G_N/G_0^N|$ as a function of redshift for two illustrative examples with $\beta = -0.1$ and $\beta = -0.8$. The latter example corresponds to the best-fit values of parameters which we will present and discuss in the next section (see Table 1). For this choice of parameters, we also plot in Fig. 2 the total energy density of the scalar field, according to (5), as a function of redshift, normalized to the total energy at matter-radiation equality. Deep in the radiation era the field behaves as a fluid which tracks the radiation background, whereas after matter-radiation equality its energy density dilutes faster than radiation, with equation of state parameter $w \approx 1/2$.

In Fig. 3, we show the ratio of $\rho_\phi$ to the total energy density as a function of redshift. For completeness, we also show the different contributions to $\rho_\phi$: it is clear that at early times the term $3\beta H^2 \phi^2$ dominates over the other terms in (5), whereas around and after matter-radiation equality the other terms can be equally relevant. These figures are produced using our modified version of hi-class.

 Changing the parameters $\beta$ and $\phi_I$ leads to variations in the redshift at which the field is released as well as in its contribution to the total energy density. In order to potentially alleviate the Hubble tension, the release should occur slightly before matter-radiation equality, while the scalar field energy density should be $O(5 - 10)\%$ of the background density at matter-radiation equality. The task of determining exactly which values of parameters lead to the best fit to cosmological data can be performed with a Monte Carlo analysis, whose results we report in the next section.

3 Datasets and Results

The data sets that we consider include the latest SH0ES 2019 measurement of the present day Hubble rate: $H_0 = 74.03 \pm 1.42$ km/s/Mpc [1], Planck 2018 high-$\ell$ and low-$\ell$ TT, TE,
Figure 3: Ratio of the energy density due to the scalar field to the total energy density, as a function of $z$. We have used the values of cosmological parameters corresponding to the best-fit reported in Tab. 1. The orange vertical line denotes the redshift of matter-radiation equality. The dashed, dotted and dot-dashed lines correspond to the different contributions to $\rho_c$ in (5).

EE and lensing data [24]. We also include BAO measurements from 6dFGS at $z = 0.106$ [25], from the MGS galaxy sample of SDSS at $z = 0.15$ [26], and from the CMASS and LOWZ galaxy samples of BOSS DR12 at $z = 0.38, 0.51$, and 0.61 [27]. We perform the analysis using the public code Monte Python [28]. We model neutrinos, using the standard treatment of the Planck collaboration, as two massless and one massive species with $m_\nu = 0.06$ eV [49]. The treatment of nonlinear corrections to power spectra, using the Halofit code, has been included in CLASS/Monte Python only for a more refined comparison between $\Lambda$CDM and $\Delta N_{\text{eff}}$. We study our model by trading the parameters $\beta$ and $\phi_I$ for $\beta \phi_I^2$ and $\phi_I$, on which we apply flat priors.

Further relevant constraints on scalar-tensor models come from BBN and from Solar System tests of General Relativity. The latter in particular provide a stringent bound on the so-called Post-Newtonian (PN) parameter $\gamma_{PN}$ (see e.g. [29]), which in our setup is predicted to be

$$\gamma_{PN} - 1 \equiv -\frac{f'\phi_0^2}{f(\phi_0) + 2f'(\phi_0)^2} \approx -4\beta^2 \frac{\phi_0^2}{M^2},$$

where we have considered only the first non-trivial order in $\phi_0/M$ in the last expression. The most recent bound comes from the Cassini mission and is given by $\gamma_{PN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [30,31], which we have included as a Gaussian constraint in our likelihood. The variation of the Newton constant from BBN until today is also constrained. In our setup, at the BBN epoch we have $G_{N,\text{BBN}} = G_{N}^I = 1/(8\pi M_\star(\phi_I))$, whereas the effective gravitational constant measured today in Cavendish-like experiments is given by [32]:

$$G_{N}^0 = \frac{1}{8\pi f(\phi_0)} \left( \frac{2f(\phi_0) + 4M^2 f'(\phi_0)^2}{2f(\phi_0) + 3M^2 f'(\phi_0)^2} \right) \approx \frac{1}{8\pi M^2} \left( 1 - \beta \frac{\phi_0^2}{M^2} \right),$$

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at the lowest order in $\phi_0/M$. By definition, the reduced Planck mass is taken to be $M_{Pl} \equiv (8\pi G_0^N)^{-1/2}$. Neglecting $(\phi_0/M)^2$ corrections our model predicts simply

$$\frac{G_{\text{BBN}}^{N}}{G_{N}^{0}} \approx 1 - \beta \phi_0^2 .$$

(9)

To the best of our knowledge, the most conservative constraint from BBN is $G_{\text{BBN}}^{N}/G_{N}^{0} = 1.01^{+0.20}_{-0.16}$ [33, 34] at 68% c.l. As we will shortly see, the large uncertainty on this bound makes it irrelevant for our analysis, therefore we do not implement this constraint in our likelihoods.\(^3\)

We have performed two analyses of our varying $G_{N}$ model, shortly the $\Delta G_{N}$ model, comparing with the standard six-parameter $\Lambda$CDM model and the $\Delta N_{\text{eff}}$ model, with the following combinations of data sets: Planck+BAO+PN, and Planck+BAO+PN+ SH\(_0\)ES.

The fit to Planck+BAO+PN gives $H_0 = 68.64^{+0.7}_{-0.89}$ in the $\Delta G_{N}$ model,\(^4\) so that the tension with SH\(_0\)ES 2019 is reduced to the level of 3.5\(\sigma\).

Interpreting the tension as a statistical fluctuation we combined also all datasets with SH\(_0\)ES 2019. In Table 1 we show the comparison among the three models and the contributions to the total $\chi^2$ are shown in Table 2, for their best-fit values. More complete results are shown in Tables 3-4 and in Figs. 4-5. The $\Delta N_{\text{eff}}$ model gives the following results at 68% c.l. for Planck+BAO+PN+ SH\(_0\)ES:

$$H_0 = 70.05^{+0.97}_{-0.98} (70^{+0.93}_{-0.95}) ,$$

$$\Delta N_{\text{eff}} = 0.3333^{+0.15}_{-0.16} (0.3165^{+0.15}_{-0.15}) ,$$

$$\Delta \chi^2 = -4.8 (-4.5) ,$$

(10)

the parentheses indicate the results using *Halofit*. These results disfavor the pure $\Lambda$CDM value ($\Delta N_{\text{eff}} = 0$) at more than 2\(\sigma\), which was not the case in the fit of [2]. This difference is due to the use of SH\(_0\)ES 2019 data, instead of SH\(_0\)ES 2018. Using the Akaike Information Criterion [37, 38] one has $\Delta \text{AIC} \equiv \Delta \chi^2 - 2 \Delta p = -2.8 (-2.5)$ in favor of the $\Delta N_{\text{eff}}$ model, where $\Delta p$ (equal to 1 in this case) is the number of additional parameters, beyond the six-parameter $\Lambda$CDM. For these runs the Gelman-Rubin parameter $R - 1$ has reached less than $10^{-3}$.

Our scalar-tensor model instead improves only by $\Delta \chi^2 = -3.6$ compared to $\Lambda$CDM (without *Halofit*), and is thus penalized by $\Delta \text{AIC} \equiv \Delta \chi^2 - 2 \Delta p = +0.4$, because of having 2 extra parameters. A small contribution to the $\chi^2$ is added to $\Lambda$CDM when comparing with this dataset, because of the PN constraint, which amounts to an additional 0.83. For these runs the Gelman-Rubin parameter $R - 1$ has reached less than 0.012. The main reason why our $\Delta G_{N}$ model performs worse than $\Delta N_{\text{eff}}$ is the PN constraint. Indeed we have checked that ignoring such a constraint we have $\Delta \chi^2 \approx -6$, compared to $\Lambda$CDM, which is better than $\Delta N_{\text{eff}}$.

We also note that the $\Delta G_{N}$ model has a slightly higher $\sigma_8$ and slightly lower $\Omega_M$, which leads to a small increase in the tension with weak gravitational lensing of galaxies [39] and

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\(^3\)Very recently [35] has updated this constraint, claiming that $\Delta G_{N} = 0.02 \pm 0.06$ at 95% C.L. While we postpone the application of this analysis to future work, we notice that such a constraint would affect our parameter space. However, as can be appreciated in Fig. 5, the preferred value of $H_0$ should not be significantly reduced.

\(^4\)The Gelman-Rubin [36] parameter $R - 1$ reached less than 0.02, thus we considered our Monte Carlo chains to be well converged, according to the criterion [36] $R - 1 < 0.1$. 

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smaller value of with the EDE scenarios of [9] (recently reanalyzed from this perspective in [13]) we find a galaxy clustering data [40] with respect to constant. The term was not included in this paper was analyzed in [16], with the following differences: (1) a cosmological constant can be seen in Fig. 5. with the a varying scalar (thus the coupling is effectively linear for small coupling rather than quadratic) and \( \omega_c \) and \( \rho_c \) respectively. Despite achieving a lower \( \chi^2 \) than our setup, the model appears to be in some tension with late-time constraints on modification of General Relativity [17]. It differs from ours in that the scalar field is exponentially coupled to the Ricci scalar, and the Lagrangian features an extra term proportional to \( X \Box \phi \). Therefore, the model of [17] has one more extra parameter with respect to ours. Despite achieving a lower \( \chi^2 \) than our setup, the model appears to be in some tension with late-time constraints on modification of General Relativity [17].

4 Conclusions

We have studied a very simple modification of gravity, where Newton’s constant \( G_N \) depends on a non-minimally coupled scalar field that decreases in time during the cosmological evolution, from an initial value \( \phi_I \) to a final value \( \phi_0 \), which is almost zero at present time. This setup slightly alleviates the \( H_0 \) tension, to a level comparable of that obtained by extending the base \( \Lambda \)CDM model with \( \Delta N_{\text{eff}} \). However, due to constraints on the Post-Newtonian parameter \( \gamma_{PN} \), the improvement is slightly less effective than the \( \Delta N_{\text{eff}} \) model. In such a varying \( G_N \) model the tension between Planck2018 + BAO data and the SH0ES 2019 data is reduced to 3.5\( \sigma \) (instead of 4.4\( \sigma \) in the case of \( \Lambda \)CDM), while being consistent with Post-Newtonian constraints on the parameter \( \gamma_{PN} \).

Interpreting the tension as a rare statistical fluctuation, we performed a combined fit of all such data, Planck+BAO+PN+SH0ES. Comparing the best-fits with respect to the base \( \Lambda \)CDM, we have \( \Delta \chi^2 = -5.8 \) (-4.5 using Halofit) for the \( \Delta N_{\text{eff}} \) model, with 1 extra parameter, which corresponds to a \( \Delta \text{AIC} = -2.8 \) (-2.5 using Halofit), with the Akaike Information Criterion. The \( \Lambda \)CDM limit of the \( \Delta N_{\text{eff}} \) model is disfavored at slightly more than 2\( \sigma \), since we find \( \Delta N_{\text{eff}} = 0.316^{+0.15}_{-0.15} \). For the \( G_N \) model we have instead only \( \Delta \chi^2 = -3.6 \) with 2 extra parameters, which corresponds to \( \Delta \text{AIC} = +0.4 \).

The value of \( \phi_0/M \) today in the non-minimally coupled model is typically of order \( 10^{-3} \) in the range of parameters preferred by the data, thus it is possible to satisfy present bounds on \( \gamma_{PN} - 1 \lesssim 10^{-5} \), from eq. (7). However a deviation from zero from eq. (7) is expected, constituting thus an interesting prediction for proposed future experiments [41], such as Phobos Laser Ranging [42] that could go down to to \( 10^{-7}-10^{-8} \) levels in \( \gamma_{PN} - 1 \) and even

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\(^{5}\)In the Einstein frame such a potential would actually be rescaled by \( f^{-2}(\phi) \) and thus act as a cosmological constant.
to $10^{-9}$ with the LATOR [43,44] and BEACON experiments [45], or to $2 \times 10^{-8}$ with gravitational time delay measurements (GTDM) [46]. Present constraints on the coupling constant $\beta$ itself are irrelevant, since we only know that $\beta_0 \approx -2\beta > -4.5$ from Pulsars [29, 47].

Despite its worse agreement with cosmological data, we think that two aspects of our setup may be considered as an improvement over EDE solutions to the Hubble tension: first, our setup does not feature a coincidence problem on the onset of the scalar field dynamics; second, it does not rely on non-generic scalar field potentials with several parameters, in contrast to [9,10]. We thus believe that these advantages may serve as a starting point for more sophisticated implementations of a varying $G_N$ to alleviate the Hubble tension.

$$
\begin{array}{|c|c|c|c|}
\hline
\text{Parameter} & \Lambda CDM & \Delta G_N & \Delta N_{\text{eff}} \\
\hline
\omega_b & 2.252 (2.257) & 2.256 (2.26) & 2.272 (2.262) \\
\omega_c & 0.1184 (0.1178) & 0.119 (0.1186) & 0.1239 (0.123) \\
\tau_{\text{reio}} & 0.06035 (0.06091) & 0.05882 (0.05962) & 0.05997 (0.05711) \\
10^{+9}A_s & 2.12 (2.119) & 2.123 (2.122) & 2.145 (2.13) \\
\eta_s & 0.9687 (0.9705) & 0.9757 (0.9769) & 0.9788 (0.9756) \\
1 - \frac{G_{\text{BBN}}}{G_N} / \Delta N_{\text{eff}} & - & -0.07003 (-0.08052) & 0.3336 (0.2635) \\
\tilde{\phi} & - & 0.3146 (0.3161) & - \\
\sigma_8 & 0.8093 (0.8075) & 0.8433 (0.8434) & 0.8248 (0.8202) \\
H_0 [\text{km/s/Mpc}] & 68.18 (68.42) & 69.2 (69.08) & 70.05 (69.53) \\
\hline
\end{array}
$$

Table 1: Mean values and 68% confidence intervals for relevant cosmological parameters, obtained with the dataset “Planck 2018 + BAO + $SH_0ES$ 2019 + PN”. In parentheses, the best-fit values for each model. Cosmological parameters follow the standard notation throughout the paper, as in [2].

$$
\begin{array}{|c|c|c|c|}
\hline
\text{Dataset} & \Lambda CDM & \Delta G_N & \Delta N_{\text{eff}} \\
\hline
\text{Planck highl TTTEEE} & 2349.56 & 2350.73 & 2351.67 \\
\text{Planck lowl EE} & 397.59 & 396.99 & 396.53 \\
\text{Planck lowl TT} & 22.46 & 21.72 & 22.09 \\
\text{Planck lensing} & 9.95 & 9.40 & 9.41 \\
\text{SH0ES 2019} & 15.60 & 12.14 & 10.06 \\
\text{bao boss dr12} & 3.38 & 3.55 & 3.48 \\
\text{bao smalls 2014} & 1.92 & 2.31 & 1.74 \\
\text{PN} & 0.83 & 0.83 & 0.83 \\
\text{Total} & 2801.29 & 2797.69 & 2795.82 \\
\hline
\end{array}
$$

Table 2: Contributions to the total $\chi^2_{\text{eff}}$ for individual datasets, for the best-fits of $\Lambda CDM$, $\Delta G_N$ and $\Delta N_{\text{eff}}$ models.
Table 3: Constraints on parameters for our $\Delta G_N$ model, using Planck 2018 high-$\ell$ TT,TE,EE+low-$\ell$ EE+ low-$\ell$ TT+ lensing, BAO, SH0ES 2019 and PN constraints. Parameters are our sampled MCMC parameters with flat priors. In particular a prior range has been set for the extra parameters: $-0.95 < \beta \phi_I^2 < 0$ and $0 < \phi_I < 0.95$. We did not use Halofit here. $H_0$ is in km/s/Mpc and $\phi_I$ is in Planck units. We also show the best-fit Likelihood $\mathcal{L}_\text{min}$, the $\chi^2$ and the difference $\Delta \chi^2$ and the $\Delta \text{AIC}$, as defined in the text, compared to $\Lambda$CDM. In such a comparison the $\chi^2$ shown for the $\Lambda$CDM model in Table 4 must be incremented by the contribution of the PN constraint, which amounts to an additional 0.83.

| Param | best-fit | mean±σ | 95% lower | 95% upper |
|-------|----------|---------|------------|------------|
| $100 \omega_b$ | 2.26 | 2.256$^{+0.014}_{-0.015}$ | 2.227 | 2.286 |
| $\omega_c$ | 0.1186 | 0.119$^{+0.001}_{-0.0011}$ | 0.1169 | 0.1212 |
| $H_0$ | 69.08 | 69.2$^{+0.62}_{-0.75}$ | 67.85 | 70.56 |
| $10^{+9}A_s$ | 2.122 | 2.123$^{+0.031}_{-0.035}$ | 2.058 | 2.188 |
| $n_s$ | 0.9769 | 0.9757$^{+0.0053}_{-0.0062}$ | 0.9646 | 0.9873 |
| $\tau_{\text{reio}}$ | 0.05962 | 0.05882$^{+0.00974}_{-0.0083}$ | 0.04342 | 0.07502 |
| $1 - G_{\text{BBN}}/G_0$ | -0.08052 | -0.07003$^{+0.058}_{-0.03}$ | -0.1528 | -5.08$e - 06$ |
| $\phi_I$ | 0.3161 | 0.3146$^{+0.083}_{-0.05}$ | 0.1497 | 0.4443 |
| $\gamma_{\text{PN}} - 1$ | $-2.171e - 10$ | $-1.143e - 06^{+1.1e - 06}_{-8.7e - 06}$ | $-1.81e - 05$ | $-6.722e - 17$ |
| $S_8$ | 0.8375 | 0.8373$^{+0.016}_{-0.022}$ | 0.8018 | 0.8766 |
| $\Omega_M$ | 0.2958 | 0.2958$^{+0.0065}_{-0.0065}$ | 0.2828 | 0.3085 |
| $\sigma_8$ | 0.8434 | 0.8433$^{+0.015}_{-0.024}$ | 0.8068 | 0.8854 |

$\Delta G_N$ model, $- \ln \mathcal{L}_\text{min} = 1398.85$, minimum $\chi^2 = 2797.7$, $\Delta \chi^2 = -3.6$, $\Delta \text{AIC} = +0.4$. 

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the best-fit Likelihood MCMC parameters with flat priors. We did not use TT, TE, EE+low-\ell EE+ low-\ell TT + lensing, BAO, SH0ES 2019. Parameters are our sampled MCMC parameters with flat priors. We did not use Halofit here. \(H_0\) is in km/s/Mpc. We also show the best-fit Likelihood \(\mathcal{L}_{\text{min}}\), the \(\chi^2\) and the difference \(\Delta\chi^2\) and the \(\Delta\text{AIC}\), as defined in the text, compared to \(\Lambda\text{CDM}\).

\[
\Delta N_{\text{eff}} \text{ model, } -\ln \mathcal{L}_{\text{min}} = 1397.49, \text{ minimum } \chi^2 = 2794.98, \Delta \chi^2 = -4.8, \Delta \text{AIC} = -2.8
\]

| Param   | best-fit | mean±\(\sigma\) | 95% lower | 95% upper |
|---------|----------|-----------------|-----------|-----------|
| \(100 \omega_b\) | 2.262 | 2.272±0.016\(+0.027\) | 2.239 | 2.306 |
| \(\omega_c\) | 0.123 | 0.1239±0.0028\(+0.0027\) | 0.1185 | 0.1294 |
| \(H_0\) | 69.53 | 70.05±0.97\(+0.97\) | 68.08 | 71.95 |
| \(10^{+9}A_s\) | 2.13 | 2.145±0.033\(+0.037\) | 2.075 | 2.217 |
| \(n_s\) | 0.9756 | 0.9788±0.006\(+0.0061\) | 0.9667 | 0.991 |
| \(\tau_{\text{reio}}\) | 0.05711 | 0.05997±0.0073\(+0.0083\) | 0.04453 | 0.07602 |
| \(\Delta N_{\text{eff}}\) | 0.2635 | 0.3336±0.15\(+0.16\) | 0.02888 | 0.6459 |
| \(\Omega_M\) | 0.3012 | 0.2989±0.0058\(+0.0059\) | 0.2873 | 0.3105 |
| \(S_8\) | 0.8219 | 0.8232±0.012\(+0.012\) | 0.7996 | 0.8464 |
| \(\sigma_S\) | 0.8202 | 0.8248±0.0095\(+0.0098\) | 0.8054 | 0.8443 |

\(\Lambda\text{CDM} \text{ model, } -\ln \mathcal{L}_{\text{min}} = 1400.23, \text{ minimum } \chi^2 = 2800.46\)

Table 4: Constraints on parameters for the \(\Delta N_{\text{eff}}\) and \(\Lambda\text{CDM}\) models, using Planck 2018 high-\(-\ell\) TT,TE,EE+low-\(\ell\) EE+ low-\(\ell\) TT + lensing, BAO, SH0ES 2019. Parameters are our sampled MCMC parameters with flat priors. We did not use Halofit here. \(H_0\) is in km/s/Mpc. We also show the best-fit Likelihood \(\mathcal{L}_{\text{min}}\), the \(\chi^2\) and the difference \(\Delta\chi^2\) and the \(\Delta\text{AIC}\), as defined in the text, compared to \(\Lambda\text{CDM}\.\)
Figure 4: Constraints on parameters for our $\Delta G_N$ model vs. the $\Delta N_{\text{eff}}$ model and the base $\Lambda$CDM model, using Planck 2018 high-$\ell$ TT,TE,EE+low-$\ell$ EE+ low-$\ell$ TT+lensing, BAO, SH0ES 2019 and PN constraints. Parameters are our sampled MCMC parameters with flat priors. In particular a prior range has been set for the extra parameters: $-0.95 < \beta \phi_I^2 < 0$ and $0 < \phi_I < 0.95$. Here $H_0$ is in km/s/Mpc. Contours contain 68% and 95% of the probability.
Figure 5: Above: constraints on parameters for our $\Delta G_N$ model, using Planck 2018 high-$\ell$ TT,TE,EE+ low-$\ell$ EE + low-$\ell$ TT+lensing, BAO, SH0ES 2019 and PN constraints. Parameters are our sampled MCMC parameters with flat priors. In particular a prior range has been set for the extra parameters: $-0.95 < \beta \phi I < 0$ and $0 < \phi I < 0.95$. Here $H_0$ is in km/s/Mpc and $\phi_I$ is in Planck units. Contours contain 68% and 95% of the probability.

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