Point contact spectroscopy of hopping transport: effects of a magnetic field

V. I. Kozub, A. A. Zyuzin, O. Entin-Wohlman, A. Aharony, Y. M. Galperin, and V. Vinokur

1 A. F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia
2 Argonne National Laboratory, 9700 S. Cass Av., Argonne, IL 60439, USA
3 Department of Physics and the Ilse Katz Center for Meso- and Nano-Scale Science and Technology, Ben Gurion University, Beer Sheva 84105, Israel
4 Department of Physics and Center for Advanced Materials and Nanotechnology, University of Oslo, PO Box 1048 Blindern, 0316 Oslo, Norway

(Dated: March 23, 2022)

The conductance of a point contact between two hopping insulators is expected to be dominated by the individual localized states in its vicinity. Here we study the additional effects due to an external magnetic field. Combined with the measured conductance, the measured magnetoresistance provides detailed information on these states (e.g., their localization length, the energy difference and the hopping distance between them). We also calculate the statistics of this magnetoresistance, which can be collected by changing the gate voltage in a single device. Since the conductance is dominated by the quantum interference of particular mesoscopic structures near the point contact, it is predicted to exhibit Aharonov-Bohm oscillations, which yield information on the geometry of these structures. These oscillations also depend on local spin accumulation and correlations, which can be modified by the external field. Finally, we also estimate the mesoscopic Hall voltage due to these structures.

PACS numbers: 72.20.Ee, 73.40.Lq, 73.63.Rt

I. INTRODUCTION

Studies of current-voltage curves of tiny contacts between two electrodes—point contact spectroscopy—are a powerful tool for the investigation of charge transport through various systems. It has provided numerous important results for normal metals and superconductors. Recently, two of us have studied such point contacts between two hopping insulators (HI). When the diameter of the contact, D, is smaller than the typical hopping distance in the bulk, r_h, it has been found that the resulting conductance is determined by individual hopping processes, and not by an average over such processes (as required for the bulk). Indeed, in this case the tunnelling trajectory for a “critical hop” passes through the small contact region, see Fig. 1, and, in general, it is not straight. As a result, a typical trajectory connecting the two sides of the point contact, which determines the conductance of the whole system, is longer than the typical hop in the bulk.

FIG. 1: Tunnelling paths near the point contact.

In this paper we show that the magnetoresistance of such a point contact provides further information about the properties of the individual localized states at its vicinity. The contact can also serve as a detector for the local spin polarization. We discuss several mesoscopic effects revealed by the application of the magnetic field: (a) the shrinkage of the wave functions, (b) Aharonov-Bohm oscillations, (c) spin accumulation and correlations near the contact, and (d) the Hall effect.

In addition to discussing such individual hops, we also discuss the statistics of these hops, when the experiment is repeated over many realizations of the sample or of the Fermi level within one sample. The statistical properties of the resistance and of the magnetoresistance of point contacts are found to reflect the spatial and energy distributions of the individual hopping states near the contact. The calculated distribution function of the magnetoresistances at small values follows a power law; at large values it decays as a stretched exponential. The Aharonov-Bohm oscillations associated with the contact allow the determination of the occupation numbers of the relevant states, as well as the strength of the impurity potential. Spin-dependent effects reveal spin correlations and the point contact can thus serve as a detector of the local spin accumulation or depletion. Finally, the mesoscopic Hall effect is sensitive to the correlation length in the bulk, since the percolation cluster shunts the effective “Hall generator” located near the contact. Thus, the point contact spectroscopy is a powerful tool for quantitative studies of the nature and of the parameters of individual localized states.

There are two reasons for the large difference between the conductance of the bulk and the conductance through a point contact. First, as shown in Ref. 2, the total length of a typical “critical hop” between the sites located...
near the contact in different half-planes (or half-spaces) is about twice larger than for hopping in the bulk. Correspondingly, the resistance associated with this “critical hop” is exponentially larger than that associated with a typical hop in the bulk. This leads to specific statistical properties of the contact resistance and magnetoresistance. The second reason is related to the different geometries of the percolation cluster in the bulk, when the point contact is present or absent.

Let us start with the second effect. The necessity to go via the contact yields an additional constraint on the choice of the hopping sites, compared with the choice for the bulk conductor. To estimate the expected increase of the resistance, we consider first the bulk resistance, \( R = R_0 e^\delta \) (this is the definition of \( \xi \)). For the case of Mott hopping, this resistance is determined by the effective resistor network percolation cluster, for which \( \xi \) exceeds its critical percolation threshold, \( \xi = (T_0/T)^{1/(d+1)} \) (\( \gg 1 \)) in \( d \) dimensions (here, \( T_0 \gg 1/|g|^2 \)), where \( g \) is the density of states at the Fermi energy, and \( a \) is the decay length of the localized states. Taking \( \xi \) within the interval
\[
\xi_c < \xi < \xi_c + \Delta \xi ,
\]
the typical correlation length of the percolation cluster can be estimated by
\[
\xi_c < \xi < \xi_c + \Delta \xi ,
\]
where \( r_h \approx a \xi_c/2 \) is the hopping length (which replaces the usual lattice constant for the percolation problem) and \( \nu \) is the percolation correlation length exponent (equal to 4/3 in two dimensions). For a bulk HI, \( \Delta \xi \approx 1 \) and the correlation length is \( \xi \approx r_h \). In the present case, which pertains to two dimensions, the actual percolating cluster can then be thought of as being built of a regular two dimensional structure, made of random blocks of size \( \xi \). The final resistance \( R \) is then obtained by averaging the log of the resistance over these blocks.

In the presence of the point contact, the percolating cluster must pass via the orifice, with a probability of the order of unity. A sufficient condition for this is
\[
\frac{r_h}{\xi} \ll \frac{\xi_c}{D},
\]
namely \( \Delta \xi \geq \xi_c(r_h/D)^{1/\nu} \). Otherwise, the orifice may be surrounded by a finite cluster, which is disconnected from the bulk. This increase in \( \xi \) implies an increase in the total resistance, compared to that of the bulk HI. Thus, the resistance of the system may be larger than \( R_0 e^{\delta (1+((r_h/D)^{1/\nu}))} \). Given Eq. (3), this may be as large as \( R_0 e^{2\delta \xi} \) but the temperature dependence of the resistance is still dominated by that of \( \xi_c \), which is similar to that of the bulk (although it is renormalized).

In this paper we concentrate on the case \( r_h \gg D \). In that case, we argue that the total resistance is dominated by that of the individual hop near the orifice. As we show below, this situation generates a different temperature dependence of the resistance, which allows one to discriminate between the two situations experimentally.

The outline of the paper is as follows: Section II discusses the magnetoresistance, and shows how measurements of both the resistance and the magnetoresistance allow the extraction of detailed information on the individual localized states which dominate the hopping. This section also discusses the distribution of the magnetoresistance. Section III discusses the Aharonov-Bohm oscillations, due to the quantum interference between close paths near the point contact. In Sec. IV we show that the amplitude of these oscillations depends on the site occupation and on the spin configuration of the relevant localized states, and argue that this dependence can be used to identify spin accumulation. Finally, Sec. V discusses the mesoscopic Hall effect due to the individual configuration near the point contact.

II. MAGNETORESISTANCE

From now on we restrict the discussion to two-dimensional devices, and model the point contact by an orifice in the thin but infinitely strong barrier dividing the two-dimensional plane into the two semi-infinite half-planes, see Fig. 1. Similar arguments also work in three dimensions. The orifice diameter, \( D \), is taken such that \( a \ll D \ll n^{-1/2} \) where \( n \) is the concentration of the hopping centers. We next concentrate on one critical hop, between two sites on the two sides of the orifice, with coordinates \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) (both measured from the bottom of the orifice). The assumption \( D \ll n^{-1/2} \) ensures that there will be only one such site on each side of the orifice. A relatively weak magnetic field \( \mathbf{H} \), such that \( \lambda = (\hbar/eH)^{1/2} \gg a \), is applied normally to the plane.

We assume that the thickness of the sample, \( t \), satisfies the conditions
\[
a \ll t \ll L .
\]

The inequality \( t \ll L \) ensures the two dimensional character of hopping, and the inequality \( t \gg a \) allows using the three dimensional asymptotic behavior of the localized wave function. Our main results also hold for \( D \gtrsim a \).

The wave function of a hydrogen-like isolated localized state in a magnetic field can be written as (see, e. g., Ref. [1]),
\[
\Psi(\mathbf{r}, \mathbf{r}_i) = \frac{1}{\sqrt{N}} e^{-A(r,r_i)} , \quad N = \int d^3 r e^{-2A(r,0)} ,
\]
\[
A(\mathbf{r}, \mathbf{r}_i) = \frac{|\mathbf{r} - \mathbf{r}_i|}{a} + \frac{|\mathbf{r} - \mathbf{r}_i|^3 a}{24 \lambda^4} - \frac{ie}{2\hbar c} [\mathbf{H} \times \mathbf{r}_i] \cdot \mathbf{r} .
\]

The wave function shrinkage, described by the factor \( e^{-r/a}/24 \lambda^4 \), originates from the centrifugal potential induced by the magnetic field.

We next discuss the overlap integral, \( V_{12} \), between the wave functions centered at \( \mathbf{r}_1 \) and at \( \mathbf{r}_2 \). As long as we do not consider interference between this hop and other hops, the last term in \( A(\mathbf{r}, \mathbf{r}_i) \) only represents a phase factor, which does not affect the magnitude of the overlap.
between the wave functions at sites 1 and 2. Ignoring these phase factors, one has
\[ V_{12} \sim V_0 e^{-(r_1 + r_2)/\alpha} e^{-a(r_1^2 + r_2^2)/24\lambda^4}, \quad V_0 \sim e^2/\kappa a, \] (6)
where \( \kappa \) is the dielectric constant. The magnetoconductance ratio \( G(H)/G(0) \) is then given by the expression
\[ G(H)/G(0) = e^{-a(r_1^2 + r_2^2)/12\lambda^4}. \] (7)

At low fields, it is reasonable to assume that the same pair of centers, at distances \( r_1 \) and \( r_2 \), dominate both the zero field resistance and the low field magnetoresistance. Interestingly, measurements of both the resistance and the magnetoresistance can yield information on the individual hop near the orifice. For the given pair of sites, the zero-field conductance is given as
\[ G_{12} = G_0 \exp[-2(r_1 + r_2)/\alpha - \Delta\varepsilon/T], \] (8)
where \( \Delta\varepsilon \) is the activation energy. This conductance is definitely distinct from the bulk variable range hopping one, which is given by the Mott law,
\[ G = G_0 e^{-\varepsilon_c}, \quad \varepsilon_c = (T_0/T)^{1/(d+1)}. \] (9)

In fact, Eq. (8) exhibits an Arrhenius activation temperature dependence. Observing such a temperature dependence can indeed confirm that the conductance is dominated by the individual hop near the orifice, and not by the whole percolation cluster, as discussed at the end of the previous section. Measuring this temperature dependence can then yield estimates for both \( \Delta\varepsilon \) and \( G_0 e^{-2(r_1 + r_2)/\alpha} \). Measuring the conductance of a similar sample, without the orifice, would yield estimates for \( T_0 \) and for \( G_0 \). Assuming that \( G_0 \) is roughly the same in both cases (the difference in \( G_0 \) only adds a logarithmic correction to the exponent), one can end up with an estimate of the combination \( \lambda = (r_1 + r_2)/a \).

As stated in Ref. 3, the magnetoconductance of a similar bulk system has the form
\[ \frac{G(H)}{G(0)} = \exp \left[ -t_1 \left( \frac{a}{\lambda} \right)^4 \left( \frac{T_0}{T} \right)^{d/(d+1)} \right], \] (10)
where \( t_1 \) is a numerical factor. Since we know \( T_0 \) from the conductance, and we know the \( H \)-dependence of \( \lambda \), we can use this information for estimating \( (a/\lambda) \). Measuring Eq. (10) can thus yield estimates for the combination \( B = (r_1/a)^3 + (r_2/a)^3 \). Combining these two equations, one finds \( r_1/r_2 = a/2 \pm \sqrt{B/(3A)} - A^2/12 \).

It should be noted that the choice of the two sites which dominate the conductance may vary with temperature. In fact, one may end up with different Arrhenius curves for different temperature ranges. The above method can thus yield details about the individual pairs of sites which dominate the hopping in each such range. In principle, this information can be used for building up the distribution of the hopping sites near the orifice in real space and in energy.

Interestingly, the magnetoresistance allows one to find whether the transport is dominated by the states close to the contact. For bulk measurements, the conductance yields the hopping length \( r_h \) and then \( \log[G(H)/G(0)] \propto r_h^3 \). In contrast, when the conductance is dominated by the contact then \( r_h \) is replaced by \( r_{12} = r_1 + r_2 \).

We next assume that for the most probable configuration one has \( r_1 \sim r_2 \sim \bar{r} \equiv (\pi n)^{-1/2} \), where \( n \) is an effective concentration of the hopping sites relevant for the conductance (In the cases of the Mott or the Efros-Shklovskii conductance, this concentration is temperature-dependent). With this assumption, one has \( \log[G(h)/G(0)] \propto r_1^3 + r_2^3 \sim 2^{3} \bar{r}^3 \approx 3 r_{12}^3/4 \), namely about 4 times smaller than it would be expected for the bulk magnetoresistance for the same hopping length.

We next discuss the statistics of the magnetoresistance, when the measurements are done on many realizations of similarly prepared samples. For this purpose, we introduce the relative magnetoresistance, \( \rho = 1 - G(H)/G(0) \). We also use the typical value for the exponent in Eq. (10), \( r_1 = r_2 = \bar{r} = (\pi n)^{-1/2} \). The characteristic magnetoresistance is then
\[ \bar{\rho}(H) \equiv a\bar{r}^3/6\lambda^4 \ll 1. \] (11)

Here we have taken into account that \( \lambda \gg \bar{r} \). Equation (11) then implies that
\[ \rho = \bar{\rho}(r_1^3 + r_2^3)/2\bar{r}^3, \] (12)
and the quantity \( \bar{\rho} \) can be treated as a typical value for the magnetoresistance. We next recall our assumption of low concentration, \( D \ll n^{-1/2} \), which implies that there is only one relevant scatterer on each side of the orifice. At low fields, the same two centers will dominate both the zero field resistance and the magnetoresistance.

Under this assumption, the two distances \( r_1 \) and \( r_2 \) are independent of each other, and each of them is characterized by a Poisson distribution, \( \mathcal{P}(r) = (2/\pi^2) e^{-r/(\pi \bar{r})} \). The distribution of the \( 1 \rightarrow 2 \) magnetoresistance then becomes (see Appendix for details)
\[ \mathcal{P}(\rho) = \int \mathcal{P}(r_1)dr_1 \int \mathcal{P}(r_2)dr_2 \delta[\rho - \bar{\rho}(r_1, r_2)]. \] (13)

This distribution then obeys the scaling form \( \mathcal{P}(\rho) = (1/\bar{\rho}) \mathcal{F}(\rho/\bar{\rho}) \), where
\[ \mathcal{F}(z) = \frac{4z^{1/3}}{9} \int_0^z d\xi \frac{e^{-z^{2/3}i\xi^{2/3} + (1-\xi)^{2/3}}}{\xi^{1/3}(1-\xi)^{1/3}}, \] (14)
see the plot in Fig. 2. This scaling function has the limiting behaviors \( \mathcal{F}(z) \propto \sqrt{z} \) at small \( z \), and \( \mathcal{F}(z) \propto e^{-z^{2/3}} \) for large \( z \). Thus at small enough \( r_1 \) the magnetoresistance has only a power law dependence on \( r_1 \), in contrast to the contact resistance itself. Needless to say, this limit represents very rare realizations.

At very strong magnetic fields, where the exponent \( a\bar{r}^3/6\lambda^4 \) becomes comparable to \( \varepsilon_c \), the contact resistance can be lowered by including hopping between a
pair of sites with site energies outside of the hopping energy band, since the concentration of the corresponding sites is larger and thus their typical spatial separation is smaller. This additional hopping reduces the magnetoresistance. Such “switching” between the different pairs with an increasing magnetic field is expected to lead to giant mesoscopic fluctuations of the magnetoresistance, which are similar to the fluctuations of the conductance, i.e., oscillations of the magnetoconductance due to interference between different tunnelling paths. We effect, which are similar to the fluctuations of the conductance for a point contact, yields a sample specific Aharonov-Bohm effect, $\psi_{ij}$.

The characteristic field for the Aharonov-Bohm effect, $H_A = \Phi_0/S$, is of the same order as the critical field for the positive magnetoresistance originating from the shrinkage of the wave functions. Thus the Aharonov-Bohm oscillations decay strongly for $H > H_A$. However, one can expect that an oscillation pattern will be observed in addition to the smooth increase of the conductance. Nevertheless, the typical ratio $V_{ij}/V_{32}$ becomes exponentially small, and the oscillations will not be visible. The probability for this restriction to hold is of the order of the ratio between the typical area and the typical area per impurity, $\bar{r}^2$, i.e., $\sqrt{a/\bar{r}}$. Since most experiments are done close to the metal-insulator transition, this probability need not be very low. Once this restriction holds, one can expect that an oscillation pattern will be observed in addition to the smooth increase of the conductance. This typical ratio $V_{ij}/V_{32}$ is of order $1/(1 + J^2)$, which is large for $J > J_c$.

For Coulomb scattering centers with $V_0 \sim \epsilon_0^2/\kappa a \gg 1$, we can follow the estimates of Ref. [7] and conclude that the quantity $J$ is of order 1 ("strong scattering"). In the opposite case of weak scattering, $V_0 \sim \epsilon_0^2/k\bar{r}$, $r$ being a distance from the scattering center, the dependence of the matrix element on the hopping distance becomes important. Since in this case $J \ll 1$, the amplitude of the Aharonov-Bohm oscillations is significantly less than that for the case of strong scattering. Correspondingly, the interference contribution decreases with the temperature due to the growth of the hopping length.

---

**III. AHARONOV-BOHM EFFECT**

The presence of a third localized state, at $r_3$ near the point contact, yields a sample specific Aharonov-Bohm effect, i.e., oscillations of the magnetoresistance due to interference between different tunnelling paths. We next evaluate the interference between the “direct” tunnelling trajectory, $1 \rightarrow 2$, which hits the orifice, and the “scattered” trajectory, that starts at the same initial center, 1, and reaches the final center, 2, after being scattered by the scatterer 3. A typical arrangement of the centers is shown in Fig. 3. The trajectories interfere on the left side of the orifice, and almost coincide in the opposite half-space. The interference triangle for the case shown in Fig. 3 is $1 \rightarrow 2 \rightarrow 3\rightarrow 1$, the interfering paths being $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$. We next follow the perturbative approach used by Raikh and Wessels. To leading order, the terms which involve site 3 contain denominators like $(\epsilon_3 - \epsilon_1)$ and $(\epsilon_3 - \epsilon_2)$. The energy $\epsilon_3$ is spread within a band with a width of order $\epsilon^2/\kappa a \gg 1$, where $\Delta$ is the width of the hopping band. Since the energies $\epsilon_1$ and $\epsilon_2$ belong to the hopping band, typically one has $|\epsilon_3| \gg |\epsilon_{1,2}|$. Therefore, we approximate these denominators by $\epsilon_3$. The contribution of the triangle to the conductance then becomes

$$G \propto \frac{|V_{12}(1 + J e^{i\epsilon_3})|^2}{(\epsilon_1 - \epsilon_2)^2} = \frac{|V_{12}|^2}{(\epsilon_1 - \epsilon_2)^2} (1 + J^2 + 2 J \cos \varphi).$$

Here $r_{ij}$ is the distance between the sites $i$ and $j$ along the tunnelling trajectory, $S$ is the area of the interference triangle $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$, and $\Phi_0 = 2\pi \hbar/e$ is the flux quantum.

The typical area of the triangle, $S$, can be estimated from the following considerations, cf. Ref. [8]. Consider the triangle $1 \rightarrow 3 \rightarrow 2$ in Fig. 3. For a symmetric configuration, the difference between the lengths $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$ is $2\bar{r}^2/R$ where $h$ is the height of the triangle while $R$ is the distance between 1 and 2. As in Ref. [8], we restrict ourselves to configurations where this difference does not exceed $a$, so that typically $h \sim (R a)^{1/2}$ and $S \sim \bar{r}^2 a^{1/2}$. Otherwise, the ratio $V_{13}V_{32}/V_{12} \sim V_0 e^{-(r_{13}+r_{32}+r_{21})/a}$ becomes exponentially small, and the oscillations will not be visible. The probability for this restriction to hold is of the order of the ratio between $S$ and the typical area per impurity, $\bar{r}^2$, i.e., $\sqrt{a/\bar{r}}$. Since most experiments are done close to the metal-insulator transition, this probability need not be very low. Once this restriction holds, we may encounter large values of $|J|$ in Eq. (15), even for $max(|V_{13}|, |V_{32}|, |V_{12}|) << 1$. The sign of $J$ can be arbitrary, depending on whether the energy of state 3 is above or below the Fermi energy.

The characteristic field for the Aharonov-Bohm effect, $H_A = \Phi_0/S \sim \Phi_0/(\bar{r}^2 a^{1/2})$, is of the same order as the critical field for the positive magnetoresistance originating from the shrinkage of the wave functions. Thus the Aharonov-Bohm oscillations decay strongly for $H > H_A$. However, one can expect that an oscillation pattern will be observed in addition to the smooth increase of the conductance.

For Coulomb scattering centers with $V_0 \sim \epsilon_0^2/\kappa a \gg 1$, we can follow the estimates of Ref. [8] and conclude that the quantity $J$ is of order 1 ("strong scattering"). In the opposite case of weak scattering, $V_0 \sim \epsilon_0^2/k\bar{r}$, $r$ being a distance from the scattering center, the dependence of the matrix element on the hopping distance becomes important. Since in this case $J \ll 1$, the amplitude of the Aharonov-Bohm oscillations is significantly less than that for the case of strong scattering.
IV. NON-EQUILIBRIUM SPIN ACCUMULATION

The Aharonov-Bohm effect in a point contact can be used to detect non-equilibrium spin accumulations in hopping systems. Following Ref. [3], we consider the situation where site 2 is empty and site 1 is occupied. If site 3 is occupied, the tunnelling processes can be considered as tunnelling of the hole from site 2 to site 1. The interference takes place only if the electron spins on sites 1 and 3 are parallel. Indeed, the final configurations for the paths 2 → 1 and 2 → 3 → 1 are the same only for parallel 1-3 spins. Consequently, the overlap integral for parallel 1-3 spins. Consequently, the overlap integral for this case is given by Eq. (15). Otherwise, the interference contribution is absent. When both sites 2 and 3 are empty (tunnelling of an electron from 1 to 2), the interference effect is also present.

From Eq. (15), we derive the modulation depth of the conductance oscillations as

$$M \equiv \frac{G_{\text{max}} - G_{\text{min}}}{G_{\text{max}} + G_{\text{min}}} = \frac{2|J|}{1 + J^2} \left[ n_{1\uparrow} n_{3\downarrow} + n_{1\downarrow} n_{3\uparrow} \right]$$

$$+ n_1 (1 - n_3) (1 - n_2) = \frac{|J|}{1 + J^2} (4s_1 s_3 n_3 - n_3 + 2). \quad (16)$$

Here $n_{i\sigma}$ is the occupation number of spin $\sigma$ at site $i$, $n_1 \equiv n_{1\uparrow} + n_{1\downarrow}$ is the total occupation of the site $i$ (which is limited to be 0 or 1, due to strong on-site interactions), while $s_i \equiv (n_{i\uparrow} - n_{i\downarrow})/2$ is its spin accumulation. In deriving the last equality in Eq. (16), we have assumed that $n_1 = 1, n_2 = 0$, corresponding to the case of low temperatures and to hopping from left to right. The first term in the brackets is sensitive to the site spins. Indeed, if there is no spin polarization, and if there are also no spin correlations, then on average $\langle s_1 s_3 \rangle = \langle s_1 \rangle \langle s_3 \rangle = 0$. If there is a non-equilibrium source of spin accumulation or depletion, then the modulation depth will change, signaling the related local polarization.

We next apply an external magnetic field, in equilibrium. Writing the Zeeman energy as $-g \mu_B H s$, where $\mu_B = e\hbar/(2mc)$ is the Bohr magneton and $g$ is the $g$-factor, we would have $\langle s_i \rangle = \frac{1}{2} \tanh(H/H_p)$, where $H_p = 2kT/(\mu_B g)$. If $H$ is in the plane of the triangle, then $\cos \varphi = 1$, and the magnetoresistance would become

$$\frac{\delta G}{G} = \frac{J}{1 + J^2} n_3 \tanh^2(H/H_p). \quad (17)$$

When the field is perpendicular to the plane, then the magnetoresistance has both the Aharonov-Bohm oscillations, at a typical scale $H_c$, and the above contributions, which saturate for $H \gg H_p$. The ratio of these scales is given by

$$\frac{H_p}{H_c} = \frac{kT}{\pi g(h^2/ma^2)} \left( \frac{\tilde{r}}{a} \right)^{3/2} \quad (18)$$

Since $h^2/ma^2$ is of the order of the Bohr energy, say 200K, and since experiments would be typically done at a few degrees K, this ratio would become of order unity only when $\tilde{r}/a \sim 30$, but then it would be difficult to observe any conductance. Thus, usually $H_p \ll H_c$, and the spins would be saturated in the amplitude of the Aharonov-Bohm oscillations.

Since the field $H_p$ is well-defined, studies of the magnetoresistance in relatively weak magnetic fields can help calibrating the device, i.e., finding $J$ and $n_3$. Indeed, if $n_3 = 1$, the system exhibits a positive magnetoresistance, which saturates at $H > H_p$. Thus one should look for structures in which the positive magnetoresistance mentioned above is observed (which means that $n_3 = 1$) and then obtain $J$ from the saturated value of this magnetoresistance according to the formulae given above. Here we may profit from the fact that the saturation field depends only on well-defined values of $T$ and $g$, and is thus easily specified.

Note that the spins can be aligned by a magnetic field in any direction, while the Aharonov-Bohm effect is sensitive only to magnetic fields perpendicular to the plane of the tunnelling. Thus, even in the case when $H_p \sim H_c$ one can separate spin effects by applying the magnetic field parallel to the tunnelling plane. The effect depends on the product $s_1 s_3$, and therefore it is independent of the sign of the spin polarization. This offers an alternative way for the observation of the spin accumulation which is generated by an ac electric field, as predicted in Ref. [7].

V. MESOSCOPIC HALL EFFECT

In this paper we concentrate on the situation when the hopping through the point contact is dominated by the vicinity of this contact. In this case, the bias voltage is completely concentrated on the “critical” pair of sites (1 and 2) near the orifice. As a result, we expect the Hall voltage also to be generated only by the single triangle of sites 1–3–O (Fig. 3), and not to be affected by any averaging procedure. This triangle then serves as an elemental “Hall generator”. In the presence of an external magnetic field, $H$, perpendicular to the sample, this triangle generates an additional contribution to the potential difference between the sites 1 and 3 when the charge transfer occurs between the sites 1 and O. Consequently, a Hall voltage, $V_H$, is generated between the (transverse) sample edges.

One can present the triad 1–2–3 as the equivalent circuit shown in the left panel of Fig. 4. Here $R_{ik} \equiv G_{ik}^{-1}$ are the Miller-Abrahams resistors found from Eq. (8). The magnetic field changes the hopping probabilities of the inter-site hopping. As shown in Ref. [8], the changes of the potentials on the sites can be taken into account by introducing effective electromotive forces (EMFs), $E_{ik}$, proportional to the current entering through the site 1 and leaving through the site 2.

These EMFs vanish when $H = 0$. When $H \neq 0$, they can be expressed through the changes of the hop-
FIG. 4: Equivalent circuit of a Hall generator. The resistances and electromotive forces are calculated from the Miller-Abrahams resistances between the sites and the additional voltage differences induced by the magnetic field.

The approximate expression for the voltage between the Hall leads is convenient to represent the EMF of the Hall generator, while \( R_H \) is its internal resistance. The quantity \( \mathcal{E}_H \) then represents the EMF of the Hall generator, while \( R_H \) is its internal resistance. The approximate expression for \( \mathcal{E}_H \) at \( |\varepsilon_i| \gtrsim T \) is

\[
\mathcal{E}_H = V_{12} \frac{\mathbf{H} \cdot \mathbf{S}}{\Phi_0} \frac{R_{31} R_{23} (R_{12} + R_{23})}{R_{12} (R_{12} + R_{23} + R_{31})^2 (1 - n_s) \beta},
\]

\[
\beta = \text{sign}(V_{12} V_{23} V_{31}) \frac{2 \pi T V_0}{e^2 R_0} \frac{e^{(r_{12} - r_{13} - r_{23})/a}}{e^{-\varepsilon_{12}/T} + e^{-\varepsilon_{23}/T}},
\]

\[
R_H = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}.
\]

Note that the variation of the hopping probability by the magnetic field is determined by two-phonon processes and consequently \( \beta \ll 1 \). The typical internal resistance, \( R_H \), of the elemental Hall generator is of the the order of the hopping resistor, \( R_h \).

Now let us estimate the voltage between the Hall leads, see Fig. 5. The generator is connected to the Hall leads by a branch of the percolation cluster. This chain is bypassed by the resistance between the Hall electrodes, which consists of \( W_H / \xi \) branches. Here \( W_H \) is the width of the Hall electrodes while \( \xi \) is the correlation length of the percolation cluster. This ratio must be large, otherwise the Hall voltage would not be measurable because it would be subjected to very strong fluctuations. Thus the measured Hall voltage is smaller than the Hall voltage of a single triad, by the factor \( \mathcal{L}/W_H \ll 1 \),

\[
\frac{V_H}{V} \approx \beta \frac{H S}{\Phi_0} \frac{\mathcal{L}}{W_H}.
\]

One can expect that the mesoscopic Hall effect will exceed the bulk contribution of the electrodes. Assuming that almost all the voltage \( V \) drops at the contact region and only its small part, \( \eta V \) drops at the leads, and using the estimate for the bulk contribution to the Hall voltage from Ref. 3, we obtain the order-of-magnitude estimate

\[
\frac{V_H}{V^{(\text{bulk})}} \approx \frac{\mathcal{L} \xi}{\eta W R_h} \approx \frac{\xi \mathcal{L}_H}{\eta W}.
\]

Here we have assumed that \( W \approx W_H \), while \( \mathcal{L}_H \gg \mathcal{L} \) is the correlation length of the network of Hall generators in the bulk. This ratio can be large due to the exponential smallness of \( \eta \); this factor can be independently estimated as the ratio of the resistance in the bulk and the contact resistance.

VI. SUMMARY

Let us finally summarize the procedure for extracting local parameters of the point contact and the adjacent states. Studying the Aharonov-Bohm oscillations, one can extract the occupation at site 3, \( n_3 \), and the quantity \( J_s \), Eq. (15), from the amplitude of these oscillations. Similar information, as shown above, can be obtained from the weak-field positive magnetoresistance resulting from the spin alignment. Since the presence of these effects ensures that \( n_3 = 1 \), one can also find the average \( \langle s_3 \rangle \), which signals the presence of non-equilibrium spin accumulation or depletion. Indeed, at equilibrium one has \( \langle s_3 \rangle = 0 \), while a complete spin alignment gives \( \langle s_3 \rangle = 1/4 \). In this way, the point contact can be “calibrated”.

To conclude, we have considered the influence of an external magnetic field on the mesoscopic conductance of a point contact between two hopping insulators. We have shown that the applied magnetic field allows the identification and measurement of spin effects. We discussed the mesoscopic Hall effect and showed that it provides important information about the hopping cluster correlation length. All of these effects provide information about hopping processes associated with a single hopping event, not distorted by an averaging procedure. Furthermore, the statistics of the hopping transport through a point contact can, in principle, provide information about the distribution of the hopping states near the contact. In gated structures such statistics can be studied by changing the gate voltage in a single device.
APPENDIX A: DERIVATION OF THE DISTRIBUTION FUNCTION

We have \( \rho = \rho_1 + \rho_2 \), where \( \rho_i = \bar{\rho} (r_i / \bar{r})^3 \). Then

\[
\mathcal{P}(\rho) = 4 \int d\rho_1 d\rho_2 \delta(\rho_1 + \rho_2 - \rho) \times \prod_{i=1,2} \int x_i \, dx_i \, e^{-x_i^3} \delta \left( \rho_i - \bar{\rho} x_i^3 \right)
\]

(A1)

Since \( \int x \, dx \, \delta (\rho - \bar{\rho} x^3) = (3\bar{\rho})^{-1} (\bar{\rho}/\rho)^{1/3} \), we readily obtain Eq. (14).

ACKNOWLEDGMENTS

This work was supported by the U. S. Department of Energy Office of Science through contract No.DE-AC02-06CH11357, by a Center of Excellence of the Israel Science Foundation (at BGU), and by a grant from the German Federal Ministry of Education and Research (BMBF) within the framework of the German-Israeli Project Cooperation (DIP) (at BGU).

1 See e.g. Yu. G. Naïdyuk and I. K. Yanson, *Point-Contact Spectroscopy*, Springer Series in Solid-State Sciences, Vol. 145 (Springer, New York, 2004), and references therein.

2 V. I. Kozub and A. A. Zuzin, Phys. Rev. B 69, 115306 (2004).

3 B. I. Shklovskii and A. L. Efros, *Electronic properties of doped semiconductors* (Springer, New York, 1984).

4 M. E. Raikh, and G. F. Wessels, Phys. Rev. B 47, 15609 (1993).

5 N. V. Agrinskaya, V. I. Kozub, R. Rentzch, M. D. Lea, and P. Fozoni, Zh. Eksp. Teor. Fiz. 111, 1477 (1997).

6 H. I. Zhao, B. Z. Spivak, M. P. Gelfand, and S. Feng, Phys. Rev. B 44, 10760 (1991).

7 O. Entin-Wohlman, A. Aharony, Y. M. Galperin, V. I. Kozub, and V. Vinokur, Phys. Rev. Lett. 95, 086603 (2005).

8 A. Miller and E. Abrahams, Phys. Rev. B 120, 745 (1960).

9 Y. M. Galperin, E. P. German, and V. G. Karpov, Zh. Eksp. Teor. Fiz. 99, 343 (1991) [Sov. Phys. JETP 72, 193 (1991)].

10 T. Holstein, Phys. Rev. 124, 1329 (1961).