GRAVITY WITH TORSION IN NON-CONSERVATIVE MAXWELL-LIKE GAUGE APPROACH

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Abstract

In this work we take into consideration a generalization of Gauge Theories based on the analysis of the structural characteristics of Maxwell theory, which can be considered as the prototype of such kind of theories (Maxwell-like). Such class of theories is based on few principles related to different orders of commutators between covariant derivatives. Their physical meaning is very simple, and lies in stating that the local transformations of a suitable substratum (the space-time or a particular phase space) and the imposed constraints define a “compensative mechanism” or the “interaction” we want to characterize. After a mathematical introduction, we apply this approach to a modified theory of gravity, in which the algebra of operators of covariant derivatives leads to an additional term in the equation of motion associated with the non-conservation of the energy-moment tensor. This offers the possibility to include, without ad hoc physical assumptions and directly from the formalism, new forms of coupling between matter and energy and the expression of the mixing between gravity and torsion.
1 Introduction

Symmetry and its physical implications on conservation principles have a long history in Physics that goes back to the majestic work by Emmy Noether [1]. The same importance, if not higher, is showed by the concepts of symmetry breaking and local gauge as the constructive principle to characterize interactions as a “compensation mechanism”. In particular, all that made possible a unified geometrical vision of fundamental interactions [2, 3]. It is in such context, at the crossroad of Theoretical Physics, Cybernetics, Category and Group Theory and Logical System Theory that the constructive approach here introduced has been developed in [4, 5], see also the recent [6]. We stress the mathematical aspects which make this approach a “theory to build geometric-based unified Theories”. The conceptual core of the procedure can be expressed in a five point nutshell:

- a) The description of a suitable substratum and its global and local properties on invariance;
- b) The field potentials are compensative fields defined by a gauge covariant derivative. They share the global invariance properties with the substratum;
- c) The calculation of the commutators of the covariant derivatives in (b) provides the relations between the field strength and the field potentials;
- d) The Jacobi identity applied to commutators provides the dynamic equations satisfied by the field strength and the field potentials;
- e) The commutator between the covariant derivatives (b) and the commutator (c) (triple Jacobian commutator) fixes the relations between field strength and field currents.

We re-analyse the modified theory of gravity in [4], in which the algebra of the covariant derivatives operators adds a term to the equations of motion like an eigenvalue equation of the double commutator of covariant derivatives. We think such theory gets a strong systemic value, because the GR syntax seems to regenerate itself from inside and to produce many schemes of classical coupling Raum - Zeit – Materie. This auto-poietic feature is a distinctive and propulsive of Theoretical Physics considered as a totality of structures that fixes the conditions of thinkability for its entities and “beables”. We also underline the conceptual meaning of gauging as cognitive compensation between known and unknown domains [7]. The proposed framework permits also a novel approach to the controversial issue of torsion in gravity.
2 The Maxwell Dream

As we know, Maxwell Theory has been the first model of gauge theory, even if the awareness of its geometric nature came quite later of its original formulation. In particular, Maxwell-like theories have an easy physical interpretation as “system theory”, i.e. theories where the dynamic equations are those with the characterization of their substratum. That is the case of General Relativity and - at a more speculative level - of the M-Theory dreams, where each interaction is a particular emergence of the world of brane [see for ex. 8]. In principle, a criticism can be raised to this approach. Leading historians such as Juergen Renn and Lee Smolin have suggested recently that Einstein and the rest of us have learned the wrong lessons from General Relativity because of the Einstein’s retrospective misinterpretation of his process of discovery. In particular, there exists a myth about “mathematical beauty” that should be scaled down [9, 10]. Also such authors as Ogievetsky and Polubarinov, decades ago, and Michael Deutsch et al., more recently, have shown that we can have the same kind of massive Yang-Mills Theory justifying the gauge approach by using “nuts-and-bolts” detailed reasoning from not-negotiable principles [for some recent contributes see for ex. 11, 12, 13].

A methodological rethinking about the “non-negotiable” principles and the “nuts-and-bolts” reasonings is what we need in order to understand what we call here the “Maxwellian Dream” [14]. The Lagrangian approach is the natural and historical extension of Newtonian Physics, based on the concept of particle and its conservation principles, and the unequaled model of “nuts-and-bolts” physics. With the advent of Quantum Mechanics (QM) and Quantum Field Theory (QFT) in particular - as well as the strong conceptual problems Unified Theories posed! - the Lagrangian-Newtonian scheme showed all its limitations. First of all, the “pointlike” particle concept itself showed to be extremely complicated, just like the attempts to define a particle “structure” [15, 16]. There are many recent experiments that seem to prove against the existence of permanent property-bearing objects [17]. We have to forget that the pointlike particle is at the root of the Re-normalization method, born to eliminate some embarrassing infinitives before becoming the key tool of the Effective Field Theories [18]. Actually, also the non-Abelian gauge theories end up coinciding with Super-Maxwell, thus the general schemes do not change so much. In our opinion, the greatly elegant work by Deutsch does not imply that the machineries of the standard model are better than the generalized gauge principles. Instead, it shows the possibility to preserve the global coherence of physics without considering problems about the particle’s structure. These problems are even stronger in the framework of quantum gravity [19, 20]. Thus, as the general tapestry is very indented and delicate, it is important to be careful for various reasons, which are not merely aesthetic. Hence, we think that it is interesting to explore also a generalized gauge approach, but without considering the framework of the non-linear generalizations of the QFT and without descriptions of the particle’s structure in the sense of “collective emergency”. For details on these issues, one can read the previously cited and precious review of Pessa [14]. For example,
a “generalized gauge” approach appears as the proper one to explore unified
theories by an “offsetting” method.

Historically, the Maxwell field seems to be the most “trustworthy”. We could
also take into consideration the “hydrodynamic” field, but it is less “genuine” and
difficult to handle. We know that we pass from Classical Mechanics to Maxwell
one by means of a potential vector which opens Classical Mechanics to something
new. Such “something” manifests itself as a characteristic non-commutativity,
a sort of logical scheme “openness”. In the case of the potential vector, there
comes out a new kind of derivative. The non-commutativity of this new operator
means that the new mechanic systems cannot be integrated and that invariants
are removed. Then, one must create a gauge field (the electromagnetic field)
which balances out the simple mechanic phenomena and introduces the new
field having new invariants and new equations. Concluding, on one hand, the
suggested method is not founded on the creation of new Lagrangians. On the
other hand, it not the opposite. In fact, the structure of the Lagrangias is
derived through a process of subsequent openings and compensations. The
Einstein methods can be considered as a third “middle way”. In fact, on one
hand one, one can use the Hilbert approach in deriving the field equations,
starting from the variation of the action, which is a general and non-negotiable
principle [21]. On the other hand, one cannot avoid the importance of the gauge
principle when operating in the linearized approximation or when discussing
the general covariance. In the case of the present paper, we stress that the argument
of the paper is the torsion in gravity, which is currently a controversial and
not completely understood issue. Thus, we think that any approach, which
could, in principle, help in a better understanding of torsion in the framework
of the gravitational theory should not be avoided, because it is also useful in
the framework of the Alternative Theories of Gravity [22].

We know that the Maxwell equations in the tensor form are

\[ \partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu, \]

\[ \partial^\alpha F^{\mu\nu} + \partial^\mu F^{\nu\alpha} + \partial^\nu F^{\alpha\mu} = 0, \]

where the controvant four-vector which combines electric current density
and electric charge density. \( J^\nu = (\epsilon p, J_x, J_y, J_z) \) is the four-current, the
electromagnetic tensor is \( F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \) with the four-potential \( A^\mu = (\Phi, A_x, A_y, A_z) \) containing the electric potential and vector potential. Using
the covariant derivative notation defined by

\[ D_\mu \equiv \partial_\mu - ieA_\mu \]

we have the classic relation

\[ [D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu = -ieF^{\mu\nu}. \]

Thus, we have

\[ F^{\mu\nu} = -F^{\nu\mu}. \]
We know the tensor form of the Maxwell equations. They are invariant for every affine transformation. In particular, the Maxwell equations are invariant for Lorenz transformations. We remember that we can obtain a general electromagnetic field from pure electrostatic field with a suitable Lorenz transformation. For more general transformations, the Maxwell equations are not invariant. Although rewriting the Maxwell equations in a way to be invariant for any transformations of the space-time this is usually achieved by coupling Maxwell theory to standard General Relativity, here we use a different approach. In order to write the extension of the Maxwell equations, we define the general commutators as in eq. (3) where \( F_{\mu\nu} \) is the general form for any field generated by the transformation. Using eq. (3), the equation

\[
[D_\mu, [D_\eta, D_\nu]] \psi + [D_\eta, [D_\nu, D_\mu]] \psi + [D_\nu, [D_\mu, D_\eta]] \psi = 0 \tag{5}
\]

can be rewritten as

\[
[D_\mu, F_{\eta\nu}] \psi + [D_\eta, F_{\mu\nu}] \psi + [D_\nu, F_{\mu\eta}] \psi = 0. \tag{6}
\]

We have also the equation

\[
[D_\gamma, [D_\alpha, D_\beta]] \psi = [D_\gamma, F_{\alpha\beta}] \psi = \chi J_{\gamma\alpha\beta} \psi \tag{7}
\]

In conclusion, the Maxwell scheme for a general gauge transformation is

\[
[D_\gamma, F_{\alpha\beta}] + [D_\alpha, F_{\beta\gamma}] + [D_\beta, F_{\gamma\alpha}] = 0 \tag{8}
\]

\[
[D_\gamma, F_{\alpha\beta}] \psi = \chi J_{\gamma\alpha\beta} \psi,
\]

where \( J_{\gamma\alpha\beta} \) are the currents of the particles that generate the gauge field \( F \). Such currents have the conservation rule

\[
J_{\alpha\beta\gamma} + J_{\beta\gamma\alpha} + J_{\gamma\alpha\beta} = 0. \tag{9}
\]

3 Maxwell-like Gauge Approach in Gravity

The development of a Maxwellian program for gravity has been done by Mignani-Pessa-Resconi and the recent developments about cosmological constant have drawn the attention of an increasing number of researchers to this theory [4, 5, 6]. Some proposals concern also application of the compensative method to the QM structure and to system theory [23, 24]. The coupling of gravity with a suitable substratum – a minimal model of quantum physical vacuum by a scalar field - implies the non conservation of the energy-momentum tensor and allows a simple way to introduce the cosmological constant. Let us label \( \Omega \) the affine transformation from general coordinates \( x^\mu \) in a given frame to general coordinates \( x'\nu \), that is [4]

\[
x^\nu \rightarrow y^\mu = g^\mu(\nu). \tag{10}
\]
By differentiating eq. (10) one gets the transformation law for contravariant vectors as

$$U^\mu = \frac{\partial y^\mu}{\partial x^\nu} V^\nu$$  \hspace{1cm} (11)$$

together with the inverse transformation

$$V^\nu = \frac{\partial x^\nu}{\partial y^\mu} U^\mu.$$  \hspace{1cm} (12)$$

Considering the ordinary derivative $\partial_\mu = \frac{\partial}{\partial x^\mu}$, the analysis in [4] permits to write down the covariant derivative as

$$D_\mu \equiv \partial_\mu + \left[ \frac{\partial x^\nu}{\partial y^\mu}, \partial_\mu \right] \frac{\partial y^\mu}{\partial x^\nu},$$  \hspace{1cm} (13)$$

which results to coincide with the ordinary covariant derivative of General Relativity, see [38] for a rigorous proof. Further, in [4] it has been shown that

$$[D_\mu, D_\nu] V_\xi = R^\alpha_{\beta\gamma\delta} V_\alpha$$  \hspace{1cm} (14)$$

where $R^\beta_{\beta\gamma\alpha}$ is the Riemann tensor. One also obtains [4]

$$F_{\gamma\alpha} = [D_\gamma, D_\alpha],$$  \hspace{1cm} (15)$$

and inserts the gravitational matter current as [4]

$$J_{\gamma\alpha\beta} = \left[ D_\gamma, [D_\alpha, D_\beta] \right]$$  \hspace{1cm} (16)$$

which is the gravitational analogous of eq. (7).

In this case, the Maxwell scheme gives us the set of equations [4]

$$[D_\gamma, R_{\alpha\beta}] + [D_\alpha, R_{\beta\gamma}] + [D_\beta, R_{\gamma\alpha}] = 0$$

$$[D_\gamma, R_{\alpha\beta}] = \chi J_{\alpha\beta\gamma}$$  \hspace{1cm} (17)$$

that together with eq. (14) give [4]

$$R^\beta_{\beta\gamma\delta} + R^\alpha_{\alpha\delta\beta} + R^\gamma_{\gamma\beta\delta} = 0$$

$$D_\gamma R^\alpha_{\beta\gamma\delta} + D_\beta R^\gamma_{\alpha\beta\gamma} + D_\delta R^\gamma_{\beta\alpha\gamma} = 0$$  \hspace{1cm} (18)$$

$$[D_\gamma, [D_\alpha, D_\beta]] V_\delta = \left( D_\gamma R^\mu_{\beta\alpha\delta} \right) V_\mu + R^\mu_{\gamma\alpha\beta} D_\mu V_\mu,$$

where the first equation is the first Bianchi identity, the second is the second Bianchi identity and the last is the basic equation for the Maxwell-like gauge approach in gravity [4]. The current can be given by the form

$$J_{\alpha\beta\gamma} = D_\alpha T_{\beta\gamma} - D_\beta T_{\alpha\gamma} - \frac{1}{2} (g_{\beta\gamma} D_\gamma T - g_{\alpha\gamma} D_\beta T)$$  \hspace{1cm} (19)$$

for which $D^7 J_{\alpha\beta\gamma} = 0$. The current can be given by the form

$$J_{\alpha\beta\gamma} = D_\alpha T_{\beta\gamma} - D_\beta T_{\alpha\gamma} - \frac{1}{2} (g_{\beta\gamma} D_\gamma T - g_{\alpha\gamma} D_\beta T)$$  \hspace{1cm} (19)$$
where $T_{\alpha\beta}$ is the energy-momentum tensor. The condition $D_\mu V_\mu = 0$ gives the gravitational field equation as

$$D^\beta R_{\alpha\beta} = \chi J^\beta$$

(20)

Using the expression of the current one gets

$$R_{\alpha\beta} = \chi \left( T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$$

(21)

that is the Einstein field equation.

## 4 Torsion in non-conservative Gravity

The most famous theory attempting to take into due account the presence of torsion in gravity is the so called Einstein–Cartan theory, also known as the Einstein–Cartan–Sciama–Kibble theory [26, 27, 29, 29]. This is a (classical) theory of gravitation similar to general relativity but with the important difference that it assumes the presence of a torsion tensor, which is the vanishing antisymmetric part of the the affine connection. Thus, the torsion results coupled to the intrinsic spin of matter in a similar way in which the curvature is coupled to the matter’s energy and momentum. This is because, in curved spacetime, the spin of matter needs torsion to not be null, but to work as a variable in the variational principle of stationary action. If one considers the metric and torsion tensors as independent variables, one finds the correct conservation law for the total (orbital plus spin) angular momentum due to the presence of the gravitational field. Historically, the theory was originally developed by Cartan, while additional contributions are due to Sciama and Kibble. Einstein worked on this theory in 1928 through a failed approach in which torsion should match the electromagnetic field tensor in the route for a unified field theory. At the end, this approach led Einstein to the different theory of teleparallelism [30]. In general, the Einstein–Cartan theory is considered viable and there are various researchers who still works on it, see for example [31].

Torsion in gravity is currently considered a controversial issue. For example, on one hand we find Hammond [32], who claims that torsion is required for a complete theory of gravity, and that without it, the equations of gravity violate fundamental laws. On the other hand, Kleinert claims that torsion can be moved into the curvature in a new kind of gauge transformation without changing the physical content of Einstein’s general relativity [33]. This should explain its invisibility in any gravitational experiment. This controversy has an important scientific value, as the contenders received the Fourth Award [32] and a Honorable Mention [33] respectively in the important Gravity Research Foundation Essay Competitions 2010 and 2011 for their works on torsion in gravity. To introduce the new wave equation for gravity we remember that in eq. (14) $D_\mu$ is the covariant derivative and the Riemann tensor is given by

$$R^\lambda_{\alpha\mu\nu} = \partial_\mu \Gamma^\lambda_{\nu\alpha} - \partial_\nu \Gamma^\lambda_{\mu\alpha} + \Gamma^\kappa_{\mu\kappa} \Gamma^\lambda_{\nu\alpha} - \Gamma^\kappa_{\nu\alpha} \Gamma^\lambda_{\mu\kappa}$$

(22)
With the double commutator we have the third of eqs. (18) as dynamic equation. Now, let us connect the commutator with the gravity current as

\[ \chi J_{\mu \alpha \beta} = (D_\mu R^\lambda_{\nu \alpha \beta}) V_\lambda + R^\lambda_{\mu \alpha \beta} (\nabla_\lambda V_\nu), \quad (23) \]

When \( \nabla_\mu V_\nu = 0 \) one can see through a bit of algebra that the Einstein field equation (21) is retrieved.

The original formulation of the general theory of relativity is assumed "torsion free". On the other hand, based on the increasing interest for the extended gravity theories, it is useful to see how torsion appears in a modern geometrical language. We define the torsion tensor as

\[ T^\lambda_{\mu \nu} \equiv \Gamma^\lambda_{\mu \nu} - \Gamma^\lambda_{\nu \mu} \quad (24) \]

Now, we show in an explicit way that it is possible to present the previous dynamical equation by a wave equation with particular source where the variables are symmetric and anti-symmetric Christoffel symbols including torsion in one geometric picture. We will define the new type of wave with an explicit computation of the commutator and of the double commutator. Thus, one gets

\[ D_\mu V_\alpha \equiv \frac{\partial V_\alpha}{\partial x_\mu} - \Gamma^\lambda_{\mu \alpha} V_\lambda. \quad (25) \]

A bit of algebra permits to compute the first commutator as

\[ F^\mu_{\nu, \alpha} = [D_\mu, D_\nu] V_\alpha = D_\mu D_\nu V_\alpha - D_\nu D_\mu V_\alpha = \]

\[ - (R^\lambda_{\alpha \mu \nu} V_\lambda + T^\lambda_{\mu \nu} D_\lambda V_\alpha). \quad (26) \]

In conclusion one gets

\[ -F^\mu_{\nu, \alpha} = [-D_\mu, D_\nu] V_\alpha = G^\mu_{\nu, \alpha} + \Omega^\mu_{\nu, \alpha} \quad (27) \]

with

\[ G^\mu_{\nu, \alpha} \equiv \left( \frac{\partial \Gamma^\lambda_{\nu \alpha}}{\partial x_\mu} - \frac{\partial \Gamma^\lambda_{\mu \alpha}}{\partial x_\nu} \right) V_\lambda \quad (28) \]

\[ \Omega^\mu_{\nu, \alpha} \equiv \left( \Gamma^\lambda_{\mu \xi} \Gamma^{\xi}_{\nu, \alpha} - \Gamma^\xi_{\mu, \alpha} \Gamma^{\lambda}_{\nu, \xi} \right) V_\lambda + \left( \Gamma^\lambda_{\mu, \nu} - \Gamma^\lambda_{\nu, \mu} \right) D_\lambda V_\alpha. \]

Now, let us consider the rescaling

\[ \Gamma^\lambda_{\mu, \nu} \rightarrow \Gamma^\lambda_{\mu, \nu} + \frac{\partial \chi}{\partial x_\mu} \quad (29) \]

One can easily show that \( G^\mu_{\nu, \alpha} \) is invariant under the rescaling (29). Now, we impose the Lorenz-like gauge condition as

\[ \frac{\partial \Gamma^\lambda_{\mu, \nu}}{\partial x_\mu} = 0. \quad (30) \]

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Some computations permits to obtain
\[ \frac{\partial G_{\mu\nu,\alpha}}{\partial x^\beta} + \frac{\partial G_{\beta\mu,\alpha}}{\partial x^\nu} + \frac{\partial G_{\nu\beta,\alpha}}{\partial x^\mu} = 0. \]  
(31)

The Lagrangian gravitational density is
\[ L \equiv F_{\mu\nu,\alpha}F^{\mu\nu,\alpha} = G_{\mu\nu,\alpha}G^{\mu\nu,\alpha} + \Omega_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} + 2G_{\mu\nu,\alpha}\Omega_{\mu\nu,\alpha} \]
(32)
where \( G_{\mu\nu,\alpha}G^{\mu\nu,\alpha} \) and \( \Omega_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} + 2G_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} \) are the Lagrangian density for the free gravitational field and the reaction field of the vacuum respectively.

The interaction term connects the gravitation field with the field of the vacuum.

With a bit of computing the dynamic equations for non conservative gravity can be obtained as
\[ [D_\beta, [D_\mu, D_\nu]] V^\alpha = J_{\mu\nu,\alpha\beta} \]
(33)

with
\[ D_\beta G_{\mu\nu,\alpha} = -J_{\mu\nu,\alpha\beta} - D_\beta\Omega_{\mu\nu,\alpha} - [D_\mu, D_\nu] D_\beta V^\alpha = J_{\mu\nu,\alpha\beta}. \]
(35)

Eq. (35) implies
\[ \frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} = \left( \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{\partial x^\beta \partial x^\mu} - \frac{\partial^2 \Gamma^\lambda_{\mu,\alpha}}{\partial x^\beta \partial x^\nu} \right) V^\lambda. \]
(36)

Setting \( \partial x_\beta = \partial x_\mu \), one gets
\[ \frac{\partial G_{\mu\nu,\alpha}}{\partial x_\mu} = \left( \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu} - \frac{\partial^2 \Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu \partial x_\mu} \right) V^\lambda. \]
(37)

The Lorentz-like gauge condition gives
\[ \frac{\partial^2 \Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu \partial x_\mu} = 0. \]
(38)

Then,
\[ \frac{\partial G_{\mu\nu,\alpha}}{\partial x_\mu} = \left( \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu} - \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{c^2 \partial^2 t} \right) V^\lambda = J_{\mu\nu}. \]
(39)

When the currents are equal to zero eq. (39) reduces to
\[ \left( \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu} - \frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{c^2 \partial^2 t} \right) V^\lambda = 0. \]
(40)
Hence, we find the interesting result that the derivative of the Christoffel connection $\Gamma^\lambda_{\nu,\alpha}$ has wave behavior.

Now, some algebra computations permits to write the "gravitational currents" as

$$J_{\mu\nu,\alpha\beta} = \frac{1}{2} R_{\mu\nu,\alpha\beta} \Rightarrow J_{\mu\nu,\alpha\beta} = R_{\mu\nu,\alpha\beta} - J_{\mu\nu,\alpha\beta}$$

where the second derivatives of the Riemann tensor represent a reaction of a virtual matter or medium (vacuum) and the second derivatives of $J$ are the ordinary currents for the ordinary matter represented by the energetic tensor. The non-linear reaction of the self-coherent system produces a current that justifies the complexity of the gravitational field and non-linear properties of the gravitational waves:

$$\left(\frac{\partial^2 \Gamma^\lambda_{\nu,\alpha}}{\partial^2 x} - \frac{\partial^2 \Gamma^\lambda_{\mu,\alpha}}{c^2 \partial^2 t}\right) V_\lambda = R_{\nu\alpha} - J_{\nu\alpha}$$

5 Conclusion remarks

We have shown that the non-conservative gravitational field is similar to a wave for 64 variables $\Gamma^\lambda_{\nu,\alpha}$ in a non linear material where we have complex non linear phenomena inside the virtual material that represents the vacuum. In previous equations, in the free field of the medium are present the Proca terms $\Gamma^\rho_\rho \Gamma^\lambda_\lambda$ the Chern-Simons terms $(\partial_\rho \Gamma^\rho_\lambda) \Gamma^\lambda_\lambda$ and the Maxwell-like terms $(\partial_\rho \Gamma^\rho_\mu) (\partial_\mu \Gamma^\lambda_\lambda)$. Hence, we have the mass terms, the topologic terms and the like electromagnetic field terms. We can model the gravitational wave with torsion as particle in a non linear medium which gives the mass of the particle, in a way that can be compared to usual SSB processes of the standard model.

Finally, we take the chance to stress that one gets a great intellectual and aesthetic pleasure by verifying the deep internal coherence between syntax and semantics in a physical theory. In the current approach, we have seen that the gauge philosophy of General Relativity can include torsion without “ad hoc” assumptions, differently from other theories where torsion is forcedly inserted, see the review for details. This issue improves our confidence on the power of symmetry in physics. One indeed argues that, differently from specific models, the most fundamental physical theories can be captured through some symmetry principle. This is the same direction of the formal unification between General Relativity and Quantum Mechanics. In his famous Essay, Kepler prophetically attributed the symmetries of the snowflake to its constituents. On one hand, maybe that only a future quantum theory of gravity could explain new behaviors of symmetry as "emergent". On the other hand, researchers can work having a full confidence on a strong complementarity between "the bricks of the Universe" and "symmetry". This is, perhaps, the richest and most intriguing speculative character of Einstein’s legacy.
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