Clash of Symmetries in a Brane World Picture*

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Abstract

If our (3+1) dimensional universe is a brane or domain wall embedded in a higher dimensional space, then a phenomenon that may be designated as "Clash of Symmetries" provides a new method of breaking continuous symmetries. The paper presents some non-trivial models containing the physical ideas.

1 Introductory remarks

Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection

Herman Weyl in Symmetry

We may hope that the remaining two decades of our century, which began under the sign of symmetry, physicists will be able to explain how great variety of non-symmetrical forms of the real world can arise out of the beautiful, symmetrical structure of the basic equations. At the beginning of the century, Einstein taught us to understand the meaning of symmetry; we have now to learn how to break it in the most symmetrical way so that his most cherished dream, that of building a unified theory of all interactions, may one day come true.

Luigi Radicati di Brozol

These two statements express two profound and basic aspects of physics today. Weyl, in his beautiful and elegant book, described as his swan song, elaborates on the role of symmetries in Nature, in art and architecture, and then shows how they

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are transformed in mathematical forms to express laws of physics. Radicatti, on the other hand, stresses the equally important fact of Nature, namely, the real world does not exist in the perceived perfect symmetric form, but more naturally in its "approximate" or "broken" forms. While we can build beautiful models of elementary particles and their interactions based on exact symmetries, real world defies them and remains provocative. Whether it be the Standard Model and beyond, SUSY, String theory or any other theory, symmetry breaking poses as a fundamental problem.

In what follows, I will briefly and qualitatively describe a new approach to symmetry breaking that has been the subject of study during the past few years [1]. What I am going to describe are some non-trivial, but still in the nature of toy models. We, think, however, they do indicate a new and fruitful approach to realistic models.

2 Clash of symmetries; basic idea

The conventional spontaneous symmetry breaking mechanism (SSB) in the standard model (SM) and its extensions assumes spatially homogeneous Higgs fields whose vacuum expectation values are determined by the minimization of a postulated Higgs potential. The non-vanishing constant values determine the masses of the gauge and matter fields through their couplings to the Higgs fields. The considerable arbitrariness involved in the choice of the potentials and the profusion of free parameters make this scheme unsatisfactory.

The non-commutative geometric framework of Alain Connes has provided a new and elegant scheme that elevates SSB to a new level of its understanding. This is accomplished by placing Higgs fields and gauge fields on a similar geometrical footing. This makes the models more predictive. However, this framework beyond its application to SM, requires departures from from the original rigorous approach and poses some problems that I cannot go into in this brief report.

There are, however, other types of solutions to Higgs fields that can serve as stable, static background fields: topological solitons, such as kinks, strings and monopoles. But their spatially non-homogeneous nature forbids them to be used as background fields in our 3+1 dimensional space-time universe, since they conflict with the strong evidence of large scale homogeneity. This objection, however, does not apply to brane-world models, since the non-trivial spatial dependence of the Higgs fields can be restricted to extra dimension co-ordinates only.

Consider, for instance, an extra dimension, coordinate $w$ and topologically stable Higgs field configurations $\phi_i(w)$, some of which have kink form with respect to $w$. Then the pattern of symmetry breaking becomes a function of $w$. Suppose that the 3+1 dimensional brane world is located at $w = 0$ with a set of localized physical fields confined to the brane at $w = 0$. If the fields are strictly confined to $w = 0$, then the unbroken symmetry is the stability group of $\phi_i(w = 0)$, say a subgroup $H(w = 0)$ of some internal symmetry group $G$. However, in quantal (or perhaps even in classical...
world), one would not expect the fields absolutely confined, in which case they couple, perhaps with reduced strength, to $\phi_i(w)$ states, where $w$ is different from $H(w = 0)$. If the stability group $\phi_i(0 < |w| < \epsilon), h(w \sim \epsilon)$ is different from $h(w = 0)$, then a rich effective symmetry breaking is possible on the brane.

This happens when isomorphic groups $H(|w| = \infty)$ left unbroken at $|w| = \infty$ can be differently embedded in the parent group $G$. The break down at finite $w$ is the intersection of the asymptotic stability groups

$$H(|w| < \infty) = H(-\infty) \cap H(+\infty) \equiv H_{\text{clash}}$$

Although $H(+\infty)$ and $H(-\infty)$ are isomorphic, their different embeddings leads to their intersection that is a smaller group.

Thus in general, if the vacuum manifold is disconnected and contains distinct vacuum states, the field can settle into different minimizing vacuum states in different spatial dimensions. Stable solitonic or kink-like domain wall configurations can exist if the vacuum manifold has the appropriate topology.

If our (3+1) dimensional universe is a brane or a domain wall, such a situation can arise, namely, a global $G_{\text{cts}} \otimes G_{\text{discrete}}$ is spontaneously broken to $H_{\text{cts}} \otimes H_{\text{discrete}}$, where $H_{\text{cts}}$ can be embedded in several different ways in the parent $G_{\text{cts}}$ and $H_{\text{discrete}} \subset G_{\text{discrete}}$.

As a consequence, a certain class of domain wall solutions connects two vacua that are invariant under differently embedded $H_{\text{cts}}$ subgroups and there is an enhanced symmetry breakdown to the intersection of the two subgroups on the brane or domain wall. In the brane limit, $H_{\text{cts}}$ prevails in the bulk, but smaller intersection symmetry on the brane itself.

One may call this phenomenon \textit{CLASH OF SYMMETRIES}.

### 3 A model with three higgs triplets

Consider a model with three Higgs triplets $\phi_{1,2,3}$ interacting through the potential

$$V = -\sum_{i=1}^{3}(\phi_i^\dagger \phi_i) + (\sum_{i=1}^{3} \phi_i^\dagger \phi_i)^2 + V_4$$

$$V_4 = \lambda/2 \sum_{i \neq j} (\phi_i^\dagger \phi_i \phi_j^\dagger \phi_j) + \sigma/2 \sum_{i \neq j} (\phi_i^\dagger \phi_j \phi_j^\dagger \phi_i)$$

The symmetry group of the potential is $G = G_{\text{cts}} \otimes G_{\text{discrete}}$, where $G_{\text{cts}} = SU(3) \otimes U(1)_1 \otimes U(1)_2 \otimes U(1)_3$ and $G_{\text{discrete}} = S_3$.

Minimization of the potential yields three degenerate vacua, leading to a manifold consisting of three disconnected pieces,

$$\text{vacuum}_i : (\phi_i^\dagger \phi_i) = 1/2, (\phi_2^\dagger \phi_2) = (\phi_3^\dagger \phi_3) = 0$$
A kink or one-dimensional domain wall configuration interpolates between the elements of disconnected vacua. A typical solution is shown in Fig. 1 (left panel).

A more interesting domain wall junction configuration is depicted Fig 1 (right panel). Three semi-infinite walls meet at a point, the origin or nexus, at angles of $2\pi/3$. Ignoring the superfluous $U(1)$’s, the clash of symmetries has the pattern:

$$H_{I\cap III} = U(2)_I \cap U(2)_{III} = U(1)_{III}$$

along wall III with corresponding results for $H_{II\cap III}$ and $H_{III\cap I}$.

Figures of global minima are shown in Figs. 2.

Figure 1: Left: Typical kink-type solution. Right: The three-star domain wall junction configuration.

Figure 2: $\phi_1, \phi_2, \phi_3$ components of the three-star configuration.
4 Domain wall solutions with Abelian gauge fields

The lagrangian consists of two complex scalar fields $\phi_{1,2}$ coupled to Abelian gauge fields $a_{1\mu}, a_{2\mu}$

$$\mathcal{L} = -(1/4)(\sum F_{\mu\nu}^i F_{\mu\nu}^i) + \sum (D_{\mu}\phi_i^\dagger) D_{\mu}\phi_i - V(\phi_1, \phi_2)$$

$$V(\phi_1, \phi_2) = \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 - v^2) + \lambda_2(\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2)$$

with the overall continuous symmetries $u(1) \otimes u(1)$ and the discrete symmetry $z_2$.

There are two distinct vacua. With appropriate boundary conditions for the scalar fields and the implied boundary conditions for the gauge fields, we find numerical solutions for the coupled equations. We obtain expected kink solutions for the scalar fields. The gauge fields diverge linearly on either side, but fall off exponentially on opposite sides (Fig. 3). The $U(1)$ symmetries are preserved in their respective vacua but broken elsewhere. The domain wall is sandwiched between domains of constant magnetic fields parallel to the wall. In the case of a domain wall with finite thickness, there will be magnetic fields parallel to the wall on either side associated with superconducting currents, as in the case of the superconducting string solution.

Thus, in addition to symmetry breaking on the brane, we find a new phenomenon such as the appearance of magnetic fields in the bulk.

![Figure 3: Plots of scalar and gauge fields for different sets of parameters](image)

5 A global $U(1) \otimes U(1)$ model with Randall-Sundrum-like gravity

The starting point is a model with two complex scalar fields in a five dimensional space-time with the action:

$$S = \int \left[ -\kappa \mathcal{R}/2 - \mathcal{L}(\phi_1, \phi_2) \right] \sqrt{-g} \ d^4x \ dw,$$
where
\[
\mathcal{L}(\phi_1, \phi_2) = g^{ab} \sum (D_a \phi_i)^* (D_b \phi_i) + V
\]
and the metric is given by
\[
ds^2 = dw^2 + e^{2f(w)} \eta^{\mu \nu} dx_\mu dx_\nu
\]
In addition to the $U(1) \otimes U(1)$ symmetry, there is a discrete symmetry $\phi_1 \leftrightarrow \phi_2$. The field equations that follow are
\[
2V(\phi_1, \phi_2) = -3\kappa (f'' + 4(f')^2)
\]
\[
\sum (\phi_i^* \phi_i) = -3\kappa f''
\]
\[
\phi_i'' + 4f' \phi_i' = (\delta V)/(\delta \phi_i),
\]
where 'prime ' denotes differentiation with respect to $w$.

We seek static solutions of the field equations with $w$ as the variable or alternately we attempt to find a potential that satisfies the equations. With the ansatz
\[
\phi_1 = (v/\sqrt{2}) \sqrt{(1 + \tanh \beta w)}
\]
\[
\phi_2 = (v/\sqrt{2}) \sqrt{(1 - \tanh \beta w)}
\]
we obtain an analytic solution
\[
e^{2f(w)} = (\cosh \beta w)^{-(\beta v^2)/(6\kappa)},
\]
which has the Randall-Sundrum limit
\[
e^{2f(w) \to e^{-(v^2 \beta)/(6\kappa)|w|}}
\]
provided,
\[
\beta \to \infty, v \to 0, v^2 \beta \to finite.
\]

The sextic potential that satisfies all the field equations and is both bounded from below and has the desired global minima:
\[
V = -\frac{\beta^2 v^4}{24\kappa} + \frac{\beta^2}{2v^2} (1 + \frac{v^2}{3\kappa}) \phi_1^2 \phi_2^2 + U + W,
\]
where
\[
U = -\frac{\beta^2}{v^2} \left( \frac{3}{2} + \frac{v^2}{3\kappa} \right) \phi_1^2 \phi_2^2 (\phi_1^2 + \phi_2^2 - v^2)
\]
and
\[
W = \zeta \frac{\beta^2}{4v^2} \left( \frac{3}{2} + \frac{v^2}{3\kappa} \right) (\phi_1^2 + \phi_2^2 - v^2)^2 (\eta + \frac{\phi_1^2 + \phi_2^2 - v^2}{v^2}),
\]
\( \zeta \) and \( \eta \) are parameters such that
\[
\zeta \geq 1; \quad \zeta \eta > 1.
\]

The equations are satisfied without \( W \), but then the potential is not bounded from below. We note that there is a negative cosmological constant in the bulk
\[
\lambda_5 = -\frac{\beta^2 \nu^4}{24\kappa}.
\]

In figure 4, contour plots of the potential for certain values of the parameters \( \zeta \) and \( \eta \) are shown to illustrate the qualitative features of the extrema of the potential.

![Contour plots](image)

Figure 4: Contours of the qualitative features of the extrema of the potential.

### 6 Summary and Conclusions

The model, although in the nature of a toy model has many realistic features. It has a solution featuring clash-of-symmetric Higgs kink configurations in a \( 4 + 1 \) Randall-Sundrum space-time. Gravity is localized to the dynamically generated brane, and the RS limit of the solution is well defined. For the chosen Higgs kink and metric configurations, the potential has to have a certain sextic form, whose properties we have studied in some depth. The symmetry breaking pattern varies as a function of the extra dimension coordinate \( w \), and displays the clash of symmetries phenomenon. At all points \(|w| < \infty\), both \( U(1) \) are broken with alternate \( U(1)'s \) restored as \( w \to \pm \infty \). The spontaneous breaking of the discrete symmetry guarantees topological stability for the matter-gravity induced brane.

This work sets the stage for incorporating gravity into more complicated models displaying the clash of symmetry idea. Our eventual aim is to construct a realistic brane-world model displaying clash of of symmetries to induce spontaneous symmetry breaking.
Acknowledgements

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References

[1] The work reported here is based on the following papers: A. Davidson, B.F. Toner, R.R. Volkas and K.C. Wali, Phys.Rev D65,125013; J.S. Rozowsky, R.R. Volkas and K.C. Wali, Physics Letters B 580 (2004)249-256; G. Dando, A. Davidson, D.P. George, R.R. Volkas and K.C. Wali, Phys.Rev D, 72, 045016 (2005) For details and references to related work, please see these papers.