Method for Estimating Outflow from the Bottom of Snowpack

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In order to evaluate the risk of slope disasters caused by melt water in the early spring, it is important to estimate the amount of melt water outflow from the bottom of snowpack on slopes. A model has been developed that enables estimation of the outflow from the bottom of snowpack on slopes using only four meteorological elements. This model estimates the outflow taking into account the effect of the aspect and inclination of a slope. A comparison of observed and estimated values confirmed that the outflow estimations correlated well with observed values.

**Keywords**: slope snowpack, outflow from the bottom of snowpack, slope model

1. Introduction

It is generally known that disasters such as slope collapse occur due to heavy rain. On the other hand, although few in number, slope collapse caused by melt water (snow melting disaster) can occur in snowy areas in the early spring.

Railway companies are introducing countermeasures such as track patrols in an attempt to prevent disasters using criteria such as rising temperatures in the early spring. It is considered that these countermeasures could be more effective and efficient if these patrols are dispatched based on decisions made with greater insight into mechanisms underlying this type of disaster. For this purpose, it is necessary to quantitatively evaluate the amount of melt water (and rainwater) outflow from the bottom of snowpack (hereinafter referred to ‘outflow water’) which is a contributing factor to this kind of disaster.

A model flat snowpack was constructed (hereinafter referred to as the ‘flat model’) with input values drawn solely from AMeDAS weather data (temperature, precipitation, sunshine hours, wind speed) that are generally available [1]. The flat model did not consider aspect and inclination of slopes. However, given the key influence of aspect and inclination of slopes, it was necessary to modify the flat model to make it applicable to a sloped snowpack considering these effects. This paper introduces the improved flat model adapted for modeling slopes.

2. Considerations required for applying the flat model to the slope

Heat which leads to melting snow is one of the main factors influenced by slope aspect and inclination, and among the factors that most affect heat is solar radiation. In addition, the speed at which water filters through the snowpack differs between snowpack on a flat surface and on a slope, because of the difference in stratigraphy. The flat model therefore also had to be adapted to consider these effects. This paper introduces the improved flat model adapted for modeling slopes.

2.1 Difference in solar radiation between a flat area and a slope

Observed solar radiation on the flat area and the slope (sunny weather in the middle of March) were compared (Fig. 2). Results showed a maximum solar radiation on the flat area of about 800 W/m², whereas it was about 1000 W/m² on the southeasterly facing slope and about 400 W/m² on the northwesterly facing slope. Results revealed that the daily maximum solar radiation differed significantly between the flat area and the slope. In addition, solar radiation peaked at around 12 o’clock on flat land, whereas it reached the maximum at 10 a.m. on the southeasterly slope and 2 p.m. on the northwesterly slope (c.f. Fig. 2: the dotted line represents the time when the maximum value was observed on each day). These observation results demonstrate that both
2.2 Difference in Infiltration speed of flat and slope snowpack

In order to clarify the time lag (time required for outflow to appear due to infiltration through the snowpack stratigraphy) between the flat area and the slope (the southeasterly slope), the time when the daily maximum snowmelt on the snow surface appears (estimated by the heat balance method), and the time when the daily maximum outflow water was reached (observed by the lysimeter method), were compared (Fig. 3). The target period involved snowpack made of granular snow with a snow depth of between 50 cm and 100 cm. Although there was one case where the outflow water peak appeared earlier than the surface snowmelt, on both flat ground and slope, the time lag between the daily maximum surface snowmelt volume and the daily maximum outflow water value varied only by about 1 to 2 hours, which means that there was no significant difference. Therefore, it is considered that there is no clear time lag between flat snowpack and sloping snowpack during the snow melting season when the snowpack is made of granular snow.

3. Improvements to the slope model

These observation results revealed the need to include in the flat model the influence of the slope to estimate solar radiation. This chapter outlines the flat model and then describes the improvements made to the slope model.

3.1 Overview of the flat model

The flat model is based on actual snow melting phenomena, and is composed of a surface model, a snow property model and an infiltration model [1]. This section explains the outline of the surface model and the infiltration model, which are closely related to the estimation of outflow from the bottom of snowpack during the snowmelt season.

3.1.1 The surface model

The surface model uses the amount of melt water on the snow surface (hereafter referred to as \( M_S \)) obtained through the heat balance method [3] (Fig. 4). The heat causing snowmelt \( Q_u \) (W/m²) at any given time is found with (1).

\[
Q_u = Q_s + Q_h + Q_c + Q_p + Q_l
\]  

(1)

where \( Q_s \) is the net radiation, \( Q_h \) is the sensible heat exchange, \( Q_c \) is the latent heat content of liquid precipitation, \( Q_p \) is the conductive heat. \( M_S \) is obtained by multiplying the value by time which is obtained by dividing \( Q_u \) (W/m²) derived from (1) by the latent heat of ice \( l = 0.334 \times 10^6 \text{J/kg} \) (equation (2)).

\[
M_S = \frac{Q_u}{l}
\]  

(2)

The flat model is a method of calculating \( M_S \) using only AMeDAS data which is made possible by formulating a part of the heat balance method. The calculation formulae for each heat quantity in the flat ground model are shown below.

The net radiation \( Q_s \) (W/m²) can be obtained from (3) as the summation of the shortwave radiation \( K_s \), the reflective shortwave radiation \( K_r \), the longwave radiation \( L_o \), and the outgoing longwave radiation \( L_s \) [4-6].

\[
Q_s = K_s + K_r + L_o + L_s
\]  

(3)

In this study, \( K_s \) is given by (4) [7] using the hours of sunshine \( n \) (min) and solar radiation at the upper end of the atmosphere \( S_0 \) (W/m²).

\[
K_d/S_0 = a_1 + a_2 n/60 + a_3 n^2/60
\]  

(4)

Factors \( a_1 = 0.20, a_2 = 0.80, a_3 = -0.21 \) were obtained by fitting coefficients based on the observed values of solar radiation at the Shiozawa Snow Testing Station. Further, \( K_s \) is given by (5) [3]. The albedo \( ref \) was obtained from a multiple regression analysis using the temperature \( T_s \) (°C), precipitation \( P \) (mm), solar radiation \( K_r \) (W/m²), and elapsed time since snowfall \( P_T \) (h) (equation (6)).

\[
K_s = (1-ref)K_d
\]  

(5)

\[
ref = f_1 T_s + f_2 P + f_3 K_r + f_4 P_T + f_5
\]  

(6)

Here, the coefficients \( f_1 \) to \( f_5 \) are coefficients obtained by
multiple regression analysis. For other elements, existing expressions were used.

As described above, by applying a formulated part of the heat balance equation in the flat model, it is possible to obtain the amount of heat to estimate snowmelt \( Q'_{\text{melt}} \) from only 4 meteorological elements.

### 3.2 Infiltration model

Since melt water infiltration is affected by the physical properties of stratigraphy, reproducing this phenomenon is very difficult. However, a recent study [8, 9] describes a method for expressing time lag \( (t) \) according to Darcy’s law (equation (7)). This research therefore also uses this technique.

\[
M'_h(t) = M'_h (t-1) \exp (-1/k_0) + M_s(t) + P_{\text{ref}} - \{M_s(t) + P_{\text{ref}} [t] \exp (-1/k_0) + M'_s(t) \} (7)
\]

Where \( M'_h(t) \) is the outflow water in the relevant time (time lag is taken into consideration), \( M'_h (t-1) \) is the outflow water 1 hour before the time \( (t) \) (time lag is taken into consideration), \( k_0 \) is the coefficient of storage which expresses the time lag, \( M_s(t) \) is the outflow water in the relevant time (time lag is not taken into consideration), and \( P_{\text{ref}}(t) \) is precipitation, respectively. \( M'_s(t) \) is the amount of snowmelt at the bottom of snowpack (snowmelt generated by ground warmth), assumed to be 0.075 mm/h [10] throughout this research.

It is known that time lag is dependent on snow depth or the structure of the snow layer. In this research, \( k_0 \) was given as a function of snow depth \( D_s \) based on the result of the weather/outflow observation in the Shiozawa Snow Testing Station (equation (8), Fig. 5). In addition, in reference to past research, it was considered that no time lag would occur if \( D_s \) was 0.5m or less.

\[
D_s > 0.5 \text{ m}, \quad k_0 = 1.0 \exp (1.6D_s) (8)
\]

### 4. Development of the slope model

In the slope model, the amount of heat leading to snowmelt \( Q'_{\text{melt}} / \text{(W/m}^2\text{)} \) in the snowpack on the slope is given by (9). Also, the net radiation amount \( Q'_{\text{n}} / \text{(W/m}^2\text{)} \) on the slope is given by (10).

\[
Q'_{\text{n}} = Q'_s + Q'_H + Q'_L + Q'_P
\]

\[
Q'_s = K'_s + L_j + L_s
\]

Where \( K'_s \) is the amount of solar radiation on the slope snowpack \( (\text{W/m}^2) \). The same estimation equations and infiltration models of other elements as in the flat ground model were used. The method for estimating \( K'_s \) is examined in the next section.

#### 4.1 The investigation of the surface of the snowpack on a slope

In the surface model of the slope snowpack, the slope solar radiation \( K'_s \) is found considering inclination and aspect of the slope using (11) devised in reference to past studies [11, 12].

\[
K'_s = K_{\text{dir}} + K_{\text{ref}} \quad (11)
\]

Where \( K_{\text{dir}} \) is the direct solar radiation on the slope, and \( K_{\text{ref}} \) is the solar radiation of the reflection on the snow surface \( (\text{W/m}^2) \). \( K_{\text{dir}} \) and \( K_{\text{ref}} \) are obtained with equations (12) to (14).

\[
K_{\text{dir}} = K_s \cdot \sin h' \quad (12)
\]

\[
\sin h' = \sin h \cdot \cos \theta + \cos h \cdot \cos (A - \alpha) \quad (13)
\]

\[
K_{\text{ref}} = K_s \cdot \text{ref} \cdot (1 - \cos \theta) / 2 \quad (14)
\]

Where \( h' \) is the solar altitude for the slope at any time and location, \( h \) is the solar altitude, \( A \) is the solar aspect angle, \( \theta \) is the slope angle of the slope, \( \alpha \) is the aspect angle of the slope, ref is the albedo of the flat snowpack, and \( K_s \) is the solar radiation on the flat (equation (4)).

In this analysis, in order to calculate the albedo \( \text{ref} \) necessary for obtaining \( K_{\text{ref}} \), the observed values from the flat snowpack were used. In order to test the equation (11),
the observed values of solar radiation on the slope per hour (2014 December - April 2015) were compared with the estimated values \( K' d \) from the slope model (Fig. 7). In addition, the hours of sunshine input to the slope model were taken from AMeDAS observation data from Tokamachi nearest to the Shiozawa Snow Testing Station, while for other elements, observed values from the station in the study were used. As a result, although there were slight variations, the coefficient of determination \( R^2 \) between the observed values and the estimated values was 0.8 or more for both slopes. This fact confirmed that the slope solar radiation can be estimated from AMeDAS data considering the inclination and aspect of the slope.

Next, the verification of (9) to find the heat leading to snowmelt on the surface of slope snowpack was carried out. For the heat required for snowmelt on the southeasterly slope, a comparison was made between snowmelt heat on the surface obtained through the heat balance method, the estimated value \( Q'M \) of the amount of heat for snowmelt on the surface obtained with the slope model using (9) and the estimated value \( Q'M \) from the flat model (Fig. 8).

Results demonstrated that although in the slope model, the daily maximum heat required for snowmelt on the surface of slope snowpack was slightly underestimated compared with the heat balance method, it was reproduced better than in the flat model.

4.2 Verification of the applicability of the slope model to the cold region

In order to verify the estimation accuracy of the outflow water using the slope model in cold regions, observed values in Hokkaido were compared with estimated values obtained through the slope model (Fig. 9). Outflow water observations were recorded by setting a lysimeter, similar to the one at the Shiozawa Snow Testing Station, on an artificial southeasterly slope (inclined 27°) in Hokkaido. The observation period was over the winter of 2014/15. Values for input to the model were obtained from the neighboring AMeDAS observation point. The comparison showed that the integrated values from March 1 to the day were the snowpack had completely melted was about 500 mm for both the observed values and the estimated values, which meant that the slope model accurately estimated the integrated value of the outflow from the bottom of snowpack during the snowmelt season. Next, temporal changes in the observed and estimated values were compared. It was found that the temporal change in observation values could be reproduced with high accuracy using the slope model. Furthermore, it was found that the root mean square error RMSE of estimated values with respect to observed values...
was 0.7 mm/h, and the determination coefficient $R^2$ between the two was 0.6, which shows a good correlation (Fig. 10).

5. Conclusion

A slope model was developed by preparing an estimation equation of the slope solar radiation with a focus on the difference in snow melting heat between flat and sloped surfaces which was incorporated into the flat model. Results confirmed that the outflow from the bottom of snowpack on slopes can be estimated by using widely available weather data.

Future studies will aim to resolve the current accuracy problems to develop a model for estimating outflow from the bottom of snowpack with greater precision, and wider scope of application.

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