Dynamic Model of a Non-equilibrium Chemical Composition Formation in the Shell of Single Neutron Stars

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Abstract—The process of a non-equilibrium chemical composition formation during cooling due to neutrino energy loss in the shells of hot, formed neutron stars is considered. A model constructed is to explain the presence of a large quantity of nuclear energy accumulated, which can maintain the X-ray luminosity of such compact objects for a long period of time. The study of the numerically obtained final chemical composition dependence on various parameters of the medium has been carried out.

Keywords: nuclear astrophysics, nuclear statistical equilibrium, non-equilibrium chemical composition

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1. INTRODUCTION

Neutron stars are the result of the massive star gravitational collapse at the end of its evolution. The densities of neutron stars are of the order of the atomic nucleus density \( 2 \times 10^{14} \text{ g cm}^{-3} \) in the inner parts and are \( 10^9 \text{ g cm}^{-3} \) on the outer shell boundary. Young formed neutron stars with temperatures \( T_9 > 10 \) (here and below \( T_9 = T/10^9 \text{ K} \)) cool rapidly, in the first seconds of life, due to neutrino energy loss.

The results of the chemical composition evolution in various systems are of interest to all astrophysics. The answer is well known for the slow stars evolution, when, as a result of various nuclear reactions, elements of the iron group are formed, which have the highest binding energy per nucleon \( \Delta E_{\text{Fe}} = 8.55 \text{ MeV} \), and up to heavier elements such as platinum and plumbum. The processes of nucleosynthesis were classified in the work [1], where authors figured out that elements heavier than iron are formed in the so-called \( s \)- and \( r \)-processes, the processes of slow and fast capture of neutrons by nuclei respectively, followed by beta decays [2, 3]. For the \( s \)-process, the relation between the capture probabilities of a neutron and an electron satisfies the inequality \( \lambda_n \ll \lambda_{\gamma} \), and its path runs along stable isotopes. For the \( r \)-process, the reverse inequality holds \( \lambda_n \gg \lambda_{\gamma} \) [4–6] and its path is far from the stability valley of chemical elements. No experimental nuclear data on strongly neutron-rich nuclei are available. For the nuclei production of the heaviest transuranic elements in the \( r \)-process, up to 150 free neutrons are required per seed nucleus with a mass number of \( A = 50–100 \). Astrophysical objects associated with the possible nucleosynthesis of \( r \)-elements are neutron stars. The formation of heavy elements in a neutron bunch was first considered in the work [7]. The conditions for \( r \)-process to occur are realized during the evolution of a close binary system of neutron stars of different masses [8], during the merging of neutron stars [9], and in a hot wind from the surface of neutron stars [10]. Note that strong magnetic fields change the properties and structure of nuclei [11], which, in turn, have impact on the nuclear reactions cross sections, for example, neutron capture rates \( (n, \gamma) \) [12]. Such strong magnetic fields can be present at supernovae explosions or neutron star mergers.

The aim of the current work is to carry out numerical calculations of the chemical composition evolution in the shell of the hot, formed neutron star that rapidly cools due to neutrino energy loss. This problem was first formulated and discussed in the work [13]. The calculations for the relaxation of the nuclear composition at some constant temperature and density values were made earlier in the papers [14, 15] that are in qualitative agreement to each other. Thermodynamic properties of matter at densities
\( \rho = 10^5 \text{--} 10^{10} \, \text{g cm}^{-3} \) and temperatures \( T_0 = 1 \text{--} 20 \) were studied in [16].

2. SCHEME OF A NON-EQUILIBRIUM LAYER FORMATION

Gradual and non-simultaneous closure of various nuclear reaction channels during cooling is crucial to the formation of the non-equilibrium layer [17]: first, a fusion of heavy charged nuclei and the protons capture by nuclei disappear (the probability of Coulomb barrier tunneling decreases with temperature drop [18]), then a photonuclear reactions vanish.

Initially, matter is at a high temperature and density: nuclei, protons and neutrons are non-degenerate \( kT > \varepsilon_{fe} \) (\( \varepsilon_{fe} \) is the Fermi energy), non-relativistic \( p_{fe} < m_{fe} c \) and are described by Boltzmann gas. Under these conditions, the substance is in equilibrium according to strong interaction reactions [19, 20]:

\[
\mu(A, Z) + \mu_i = \mu(A', Z').
\] (1)

Here we denote \( i = n, p \). Using the expression for the chemical potential \( \mu \) of the Boltzmann gas, one can obtain the Saha equation [21] that determines the concentration of each nucleus in the system as a function of free neutrons and protons concentrations in the form of:

\[
n(A, Z) = g(A, Z) \frac{n_{fe}^2 n_{np}^{A-Z}}{2^A} A^{3/2} \left[ \frac{2\pi m c^2}{m_k T} \right]^{3(A-1)/2} \exp \left[ \frac{Q(A, Z)}{kT} \right],
\] (2)

where \( Q(A, Z) = c^2[Zm_p + (A - Z)m_n - m(A, Z)] \) —a nuclear binding energy. Electrons are relativistic \( p_{fe} \gg m_{fe} c \) [22] and degenerate \( kT < \varepsilon_{fe} \), the degeneracy degree increases with cooling \( kT \ll \varepsilon_{fe} \). Cooling occurs mainly due to neutrino energy loss during the reactions:

\[
(A, Z) + e^+ \rightarrow (A, Z \pm 1) + v^+,
\]

\[
(A, Z \pm 1) \rightarrow (A, Z) + e^+ + v^-,
\] (3)

denoting \( v^\pm \) as antineutrino and neutrino, respectively. For \( kT \gtrsim m_c c^2 \), what corresponds to temperatures \( T_0 \gtrsim 6 \), it is necessary to take into account positrons in equilibrium. The condition for the annihilation reaction equilibrium of the pair \( e^- + e^+ \rightleftarrows 2\gamma \) for \( \mu_\gamma = 0 \) leads to the relation \( \mu_{e^-} = \mu_{e^+} = 0 \).

As a result of a temperature drop, the reactions with protons will freeze out and positrons will disappear. Only beta reactions with electrons as well as capture of neutrons and photonuclear reactions will remain at relatively low temperatures. Stable nuclei rapidly capture free neutrons (\( \tau_{\eta n} \ll \tau_{np} \) and \( \tau_{\eta n} \ll \tau_p \), \( \tau \) —characteristic reaction time), moving away from the valley of stability to the region of low binding energy values of neutrons in the nuclei, where direct reactions of neutron capture are balanced by photonuclear reactions (\( \tau_{\eta n} \sim \tau_{np} \)), resulting in formation of heavy nuclei with large atomic numbers \( A_j \) for each isotopic chain \( Z_j \). Electrons degenerate and occupy the lowest energy levels below Fermi surface, the Pauli exclusion principle restrict beta reactions: the nucleus can either decay at \( \varepsilon_{fe}^\beta = c^2[m(A, Z) - m_n - m_e] > \varepsilon_{fe} \), or capture electrons at \( \varepsilon_{fe} > Q_{\beta} = \varepsilon_{\beta} \) —a threshold Fermi energy for electron capture. As a result of the listed processes, nuclei accumulate in a non-equilibrium layer limited in \( Z \) and \( A \) [13].

In addition to the formation of a nuclear composition far from equilibrium [23, 24], disequilibrium also consists in a large excess of free neutrons. Free neutrons can neither decay (\( \varepsilon_{fe} > Q_{\beta n} = 0.78 \, \text{MeV} \)), nor join to neutron-rich nuclei with low neutron binding energies. As a result, the matter does not reach the minimum energy state. The number of allowed nuclei decreases with a temperature drop [13].

One can estimate the range of densities that makes a non-equilibrium layer formation possible. To evaluate from above, consider the evolution of matter with increasing density and at low temperature. For the threshold energy of electron capture, we approximately have \( \varepsilon_{fe}^\beta = Q_{\beta} = Q_n(A, Z - 1) \), where \( Q_n \) is the binding energy of the particle \( n \) in the nucleus. As the density build up, the Fermi energy of electrons \( \varepsilon_{fe} = \hbar c (3\pi n)^{1/3} \) increases and upon reaching \( \varepsilon_{fe} = \varepsilon_{\beta} \) nuclei capture electrons, decreasing their atomic charge numbers \( Z \) until they reach the neutron drip line \( Q_n = 0 \), at which \( \varepsilon_{\beta n} = Q_n = Q_{\beta n} \). The neutron drip line is illustrated in Fig. 1 based on the finite range droplet model [25] (see Section 3). At the moment the nucleus reaches \( Q_n = 0 \), subsequent electron captures will lead to the neutrons releasing from the nucleus [13]:

\[
(A, Z) + e^- \rightarrow (A - 1, Z - 1) + n + v^-.
\] (4)

The described process repeats until the nucleus reaches the maximum \( \varepsilon_{\beta} = Q_{\beta n}^{\text{max}} = 32 \, \text{MeV} \), which corresponds to carbon \( ^{22}\text{C} \). Figure 2 illustrates the proton binding energies in nuclei on the neutron drip line \( Q_n = 0 \). Proton binding energies in the used mass model are calculated as the following:

\[
Q_p(A, Z) = Q(A, Z) - Q(A - 1, Z - 1) - |Q_n(A - 1, Z - 1)|.
\] (5)
The correction for the negative neutron binding energies \( Q_n(A - 1, Z - 1) < 0 \) of the nuclei is taken into account. With further compression of the matter, beta reactions become non-equilibrium \( \varepsilon_{\beta < 0} > Q^{(\text{max})}_{\beta} \) that can lead to reheating to a temperature of \( T_0 \approx 5 \), at which nuclear statistical equilibrium is established. The maximum density at which matter consists of nuclei located on the neutron drip line \( Q_n = 0 \) is:

\[
\rho_{\text{max}} = \mu_z \times 10^6 \left( \frac{\varepsilon_{\beta}}{m_e c^2} \right)^3 \\
= \mu_z \times 10^6 \left( \frac{Q^{(\text{max})}_{\beta}}{m_e c^2} \right)^3 \approx 10^{12} \text{ g cm}^{-3},
\]

where \( \mu_z \geq 4 \) is the number of baryons per electron. The paper [26] demonstrates that when one take into account pycnonuclear fusion reactions with neutrons evaporation, the upper density limit of the non-equilibrium layer can reach \( \rho_{\text{max}} < 5 \times 10^{13} \text{ g cm}^{-3} \).

To evaluate from below, one can proceed as the following [13]. For \( Z > 10 \), the proton binding energies \( Q_{p0} \) at the nuclear drip line \( Q_n = 0 \) can be approximated by the linear function [27]:

\[
Q_{p0} = 29 - \frac{Z}{8} \text{ MeV},
\]

where expression \( A \approx 3.4Z \) is satisfied. We find an approximate dependence of the chemical composition on density:

\[
Z = 8[29 - 0.511(10^{-6} \rho/\mu_z)^{1/3}].
\]

According to the equation (8) one figure out that \( (A, Z) \) grows with decreasing density. At high values of \( (A, Z) \) nuclei become unstable with respect to fission and alpha decay reactions. As a result of spontaneous fission, the total number of nuclei increases continuously until all free neutrons are absorbed by them. According to the paper [27] for nuclei at the neutron drip line \( Q_n = 0 \), the evaluated half-life is:

\[
\log T_{1/2} = 157 - 0.93Z.
\]

We find out that for \( Z > 153 \) the half-life is more than \( 10^7 \) years. Using the expression (7) for \( Z = 153 \), we obtain that \( Q^{(\text{min})}_{p0} \approx 8 \text{ MeV} \). Similarly to the expression (6), we have the following:

\[
\rho_{\text{min}} = \mu_z \times 10^6 \left( \frac{Q^{(\text{min})}_{p0}}{m_e c^2} \right)^3 \sim 10^{10} \text{ g cm}^{-3}.
\]

We consider the densities \( \rho = 10^{11} \) and \( 10^{13} \text{ g cm}^{-3} \) in this paper.

3. NUCLEI PROPERTIES

Using the finite range droplet model [25] as a mass model, we obtain the neutron drip line (Fig. 1) and the proton binding energies at this boundary (Fig. 2).

Fig. 1. Neutron drip line based on the finite range droplet model [25] for \( Z \in [0; 83] \).
Within the framework of this model, the nucleus binding energy is described as:

\[ \Delta E(A, Z) = Z(m_p - m_n)c^2 + a_1A^{1/3}B_k + (A - Z)(m_p - m_n)c^2 + a_0A^0 + c_1\frac{Z^2}{A^{1/3}}B_3 + \left(-a_1 + J\delta^2 - \frac{1}{2}K\varepsilon^2 \right)A - c_2Z^2\frac{A^{1/3}}{B_1} + a_2B_1 + \frac{9J^2}{4Q}\left(\frac{B_1}{B_2}\right)A^{2/3} - c_4\frac{Z^3}{A^{1/3}} + W\left[1, A; Z = N \text{ and both odd}, 0, \text{ even} \right] \]

(11)

\[ - c_5Z^2\frac{B_wB_y}{B_1} - c_6(N - Z) + f_0\frac{Z^2}{A} - a_0Z^{2.39} \]

\[ \begin{cases} +\bar{\Delta}_p + \bar{\Delta}_n - \delta_{np}, & Z \text{ and } N \text{ odd,} \\ +\bar{\Delta}_p, & Z \text{ odd and } N \text{ even,} \\ +\bar{\Delta}_n, & Z \text{ even and } N \text{ odd,} \\ 0, & Z \text{ and } N \text{ even,} \end{cases} \]

where \( m_n = 1.660539 \times 10^{-24} \text{ g} \) is the atomic mass unit. \( B_i \) are the functions that take into account the finite range of nuclear forces, the shape of the nucleus, relative Coulomb energy, curvature, surface and volume energies; \( I = (A - 2Z)/A \) is the relative neutron excess; \( a_r, c_r, J, Q, L, C, \gamma, W, h, r_{max}, f_0 \) are the constants; \( \bar{\Delta}_p, \bar{\Delta}_n, \varepsilon, \delta, \text{ and } \delta_{np} \) are the functions that take into account the even–odd nuclei properties. A detailed description of each function can be found in the [25] paper.

4. NUMERICAL SIMULATION

A Fortran program was developed to simulate the cooling of hot matter. The explicit Runge–Kutta's fourth order method with automatic step selection was used. For numerical calculation, it is convenient to dimensionless the expressions involved. Transforming concentrations, the change in each isotope due to nuclear and beta reactions will be written as follows:

\[ \frac{dY(A, Z)}{dt} = -\lambda_{p_1}(A, Z)Y(A, Z) \]

\[ + \lambda_{p_2}(A, Z - 1)Y(A, Z - 1) - \lambda_{p_2}(A, Z)Y(A, Z) \]

\[ + \lambda_{p_3}(A, Z + 1)Y(A, Z + 1) - \lambda_{p_3}(A, Z)Y(A, Z) \]

\[ + \lambda_{p_4}(A, Z - 1)Y(A, Z - 1) - \lambda_{p_4}(A, Z)Y(A, Z) \]

(12)
we denote the reduced concentration as \( Y_i = n_i/\rho N_A \), \( N_A \) is Avogadro’s number. The two-particle reactions \( i^\gamma(j, \gamma^\nu)m^\nu \) rates \( \lambda_{ij}^\nu \), where \( j = n, p, \) and \( \eta \) and \( \nu \) take into account the excited states of the initial and final nuclei \( i \) and \( m \), respectively, are written in the form of \( \lambda_{ij}^\nu(A, Z) = Y_j \rho N_A \sigma_i^\nu \), where \( \sigma \) is the reaction cross section and \( \nu \) is the relative velocity of the interacting particles.

All isotopes in the range \( Z \in [0; 83] \) with nuclear data available are initially included in the system and distributed in accordance with (2) for the given initial \( \rho, T_0 \) and the ratio of the total number of protons to neutrons:

\[
R = \frac{n_p + \sum Z n(A, Z)}{n_n + \sum (A - Z) n(A, Z)}. \tag{13}
\]

The parameter \( Y_e \) is usually used in up-to-date literature as the number of electrons per baryon, which is related to the parameter \( R \) as \( Y_e = R/(R + 1) \).

At each time step with a temperature change \( T_0(t) \) and constant density \( \rho \), a new chemical potential of electrons and positrons is calculated \( \mu_{-e} \), hence new concentrations of electrons and positrons are known. In order to find them it is necessary to numerically solve the system of the following equations using Newton’s method:

\[
\begin{cases}
  n_{e^+} = \frac{1}{\pi^2} (kT/\hbar)^3 \int_0^\infty \frac{x^2 dx}{1 + \exp[(x^2 + \alpha^2 - \beta_\pm)]}, \\
  n_e = n_{e^+} + \sum_{A,Z} n(A, Z) Z,
\end{cases} \tag{14}
\]

with respect to implicitly given variables \( \beta_\pm = \mu_{e^-}/kT \) and \( n_e \). We denote some of the parameters as follows:

\[ \alpha = m_c^2/kT \] and \( x = cp/kT \). In the case when the electron gas is strongly degenerate and there are no positron, the case of low temperatures, the following relations \[18\] are used:

\[
\begin{cases}
  n_e = \sum_{A,Z} n(A, Z) Z, \\
  \mu_{e^-} \approx e_{fe} \left[ 1 - \frac{1}{3} \left( \frac{\pi kT}{e_{fe}} \right)^2 \right].
\end{cases} \tag{15}
\]

If the chemical potentials of electrons and positrons are known, one find the rates of beta reactions \( W \) and neutrino energy loss \( \Theta \) per nucleus \[28\] in reactions (3):

\[
W^{(a)} = \frac{g_z}{g_z} \ln 2 \left( \frac{kT}{F_{t1/2}^2} \right)^5 I_2, \tag{16}
\]

\[
\Theta^{(a)} = g_z \ln 2 \left( \frac{kT}{m_c^2} \right)^6 m_c^2 I_3, \tag{17}
\]

\[
\Theta^{(b)} = \frac{2}{F_{t1/2}^2} \sinh^{-1} \left( \frac{\Theta_{1/2}}{\nu} \right) \tag{18}
\]

the integrals \( I_k \) and \( I'_k \) are defined as:

\[
I_k = \int_0^\alpha \frac{x^2(x + \nu_0)^2 - \beta_\pm}{1 + \exp(x + \nu_0 - \beta_\pm)} dx, \tag{19}
\]

where we denote \( x_0 = e^2 [m(A, Z) - m(A, Z')]/kT \), and \( g_z \) are statistical weights. The constants \( F_{t1/2} \) are determined by the experimentally measured half-lives. \( F_{t1/2} \) values differ for various transitions to excited states of the same nuclei. All integral expressions in this work are calculated using the Gauss method and the similar ones, utilizing from 5 up to 11 points with the corresponding weights \[29, 30\]. In the case of strong degeneracy at low temperatures, the integral expressions (16) turn to \[28\]:

\[
W^{(a)} = \frac{g_z}{g_z} \ln 2 \left( \frac{\Theta_{1/2}}{\nu} \right) \tag{20}
\]

\[
\Theta^{(a)} = g_z \ln 2 \left( \frac{\Theta_{1/2}}{m_c^2} \right)^6 m_c^2 I_3, \tag{21}
\]

\[
\Theta^{(b)} = \frac{2}{F_{t1/2}^2} \sinh^{-1} \left( \frac{\Theta_{1/2}}{\nu} \right) \tag{22}
\]

which are calculated analytically \[28, 31\].

An approximate equation for the non-equilibrium layer cooling is used in the following form:

\[
\frac{2}{k} \sinh^{-1} \left( \frac{\Theta_{1/2}}{\nu} \right) \sum n(A, Z) = \sum \Theta(A, Z) n(A, Z). \tag{23}
\]

Only the volumetric energy loss of the non-equilibrium layer due to neutrino radiation are taken into account. Considering the heating of matter due to non-equilibrium beta processes, one can obtain a self-consistent case in which the rates of neutrino energy loss are compensated by non-equilibrium heating. Accurate accounting for non-equilibrium heating of matter is not considered in this work. It is assumed that non-equilibrium heating compensates for neu-
trino energy loss at a temperature of \( T_n = 0.1 \) and the matter is not cooled further.

At each time step with a constant density \( \rho \), beta reactions change the temperature \( T_n(t) \) and the ratio of the total number of protons to neutrons \( R(t) \). With a new \( T \) and \( R \), considering nuclear statistical equilibrium conditions (2), the implicit equations (2) and (14) are solved simultaneously, using Newton’s method. New nuclear composition, values \( \beta_\pm = \mu_c/kT \) and \( n_k \) are found. When the temperature drops to \( T_n \sim 3-5 \) and below, the equilibrium (2) is violated due to the Coulomb barrier. Equilibrium for protons and neutrons (2) is replaced with equilibrium for neutrons only. Similarly to the Saha expression (2), one derive equation for adjacent atomic mass numbers \( A \) in isotopic chain with fixed atomic charge numbers \( Z \) as function of free neutrons concentration:

\[
\frac{n(A, Z)}{n(A + k, Z)} = \left( \frac{A}{A + k} \right)^{3/2} \left( \frac{2}{n_k} \right)^k \left( \frac{m_e kT}{2\pi \hbar^2} \right)^{3k/2} \exp \left( \frac{Q(A, Z) - Q(A + k, Z)}{kT} \right).
\]

The criterion for terminating the calculation is that the concentrations of elements and most parameters reach constant values. It occurs in \( t \sim 2 \times 10^3 \) s for the problem considered.

Degenerate electrons make the key contribution to the pressure for the chosen conditions. We evaluate the pressure according to [32] in the following form:

\[
P \approx 1.2 \times 10^{15} \left( \frac{\rho}{\mu_c} \right)^{4/3} \text{ bar.}
\]

A pressure difference at the boundaries of the non-equilibrium layer between the densities \( \rho_{\text{max}} \) and \( \rho_{\text{min}} \) is:

\[
\Delta P = P_{\text{max}} - P_{\text{min}} \sim 10^{31} \text{ bar.}
\]

A mass evaluation of the non-equilibrium layer in the crust of the neutron star envelope is found using the forces balance equation that gives the mass value, considering the thin layer approximation, in the following form [13]:

\[
M_{\text{shell}} = \frac{4\pi R^4}{GM_{\text{NS}}} \Delta P = 0.1 \Delta P \sim 10^{30} \text{ g.}
\]

To evaluate the nuclear energy stored in the non-equilibrium layer relative to the equilibrium nuclear composition let us assume that \( \Delta E \sim 7 \times 10^{-3} m_p c^2 \) energy is released per nucleon [13] during the transition to equilibrium. For the total energy stored we get:

\[
E \sim \frac{M_{\text{shell}}}{m_p} \Delta E \sim 10^{48} \text{ erg.}
\]

The presence of such large nuclear energy reserves in the non-equilibrium layer is sufficient to maintain the X-ray luminosity at the level of \( L = 10^{36-37} \) erg/s for tens of thousands of years, and can also lead to powerful nuclear explosions accompanying starquakes of neutron stars [33–35]. Taking into account pycnonuclear reactions that increase the upper density boundary of the non-equilibrium layer by about an order of magnitude [26], as well as lower mass neutron stars with a thicker non-equilibrium layer [35], the nuclear energy reserve in the non-equilibrium layer increases by more than an order of magnitude.

As a result of various neutron star activity processes, such as starquakes, the matter of the non-equilibrium layer can be ejected from inner layers into the outer ones. Under conditions of low density, the electron degeneracy vanishes and the Pauli principle does not restrict beta reactions anymore, during the time scale of which the stored nuclear energy can be released in an explosive manner. The trigger of such an explosion can be considered in [36] paper a chain of nuclear reactions in the ejected part of the superheavy nuclei matter that leads to nuclear fission, followed by the explosion.

5. RESULTS AND DISCUSSION

Let us consider the results of a numerical calculation of the chemical composition evolution. The initial temperature for all calculating tracks is \( T_n = 10 \). For each density value \( \log \rho = 11,13 \), several values of the \( R \) parameter are selected. Only the most common elements with mass fractions \( x_j > 10^{-4} \) are presented on the graphs that display the initial and final isotope distributions. Figures 3 and 4 correspond to different trajectories of the parameter \( R \) at densities \( \log \rho = 11 \) and \( \log \rho = 13 \), respectively.

Figures 5 and 6 correspond to the Fermi energy trajectories \( \varepsilon_{fe} \) for different parameter \( R \) and density \( \rho \) values. Figures 7–12 and 13–18 represents the initial and final chemical compositions of the shell layer for different parameter \( R_n \) initial values at the densities \( \log \rho = 11 \) and \( \log \rho = 13 \), respectively. All possible isotopes \( Z \in [0;83] \) that are in the equilibrium described by the Saha equation (2) were chosen as seed nuclei. The density does not change during the calculations. The captions next to the labels in the figures mark the atomic mass numbers \( A \) of the isotopes.

As a result of calculations, a non-equilibrium composition of matter at a given density was obtained that is formed during cooling. The most prevalent nuclei have \( A \geq 3Z \) and are located next to the neutron drip line with low neutron binding energies \( Q_n = 0 \), and \( Q_{\beta \pi} = \varepsilon_{fe} \). The variety of resulting nuclei decreases with a temperature drop. The final composition of matter
depends on the relative neutrons number of the initial composition.

In the case of a low density $\log \rho = 11$, heavy nuclei are formed. The initial Fermi energies $\varepsilon_{fe}$ (Fig. 5) are less than the beta decays energies of nuclei at the neutron drip line. As the temperature drops, the nuclei capture neutrons and, passing to the $Q_n = 0$ boundary, undergo beta decay at $Q_\beta(A, Z) > \varepsilon_{fe}$, increasing their charge atomic numbers $Z$. The resulting nuclei also absorb neutrons, passing to the neutron drip line and decaying. The parameter $R$ value increases. The Fermi energy increases (Figs. 3, 5) until it equals $Q_\beta(A, Z)$.
Beta decays stop at this point. The final composition mainly depends on the initial parameter $R$ value and the ratio of the seed nuclei number to free neutrons number at the transition from Saha equilibrium described by equation (2) to quasi-equilibrium described by expression (20), when the total number of nuclei does not change anymore. The less parameter $R$ value ($R_0 = 0.05$ case, Figs. 7 and 8), the heavier nuclei are formed $A > 230$. The final chemical compositions for $R_0 = 0.20, 0.33$ (Figs. 10 and 12) do not
reach neutron drip line and match $A = 3Z$, what refers to $R = 0.5$ (Fig. 3). The reason is that there is insufficient amount of free neutrons remaining after the nuclear statistical equilibrium stage described by (2).

The lower free neutrons concentration $n_n$, the fewer nuclei close to the neutron drip line are formed (20).

In contrast, at high density $\log \rho = 13$ light elements are formed. The initial Fermi energy is
and only electron captures are available to nuclei in degenerate matter. As a result, the nuclei decrease their charge atomic number $Z$, passing to the neutron drip line $Q_n = 0$ and emitting neutrons. The parameter $R$ value decreases (Fig. 4) as well as $(A, Z)$ until the reducing Fermi energy of electrons equals the

Fig. 9. Initial chemical composition at $\log \rho = 11$, $T_0 = 10$, and $R_0 = 0.20$. Each isotope is marked by the atomic mass number $A$.

Final chemical composition, $\log \rho = 11$, $R_0 = 0.20$

Fig. 10. Chemical composition at the last calculated point $t = 200$ s for $\log \rho = 11$, $T_0 = 0.1$, and $R_0 = 0.20$. Each isotope is marked by the atomic mass number $A$. 

$\varepsilon_{\text{ee}} > Q_n^{(\text{max})}$ and only electron captures are available to nuclei in degenerate matter. As a result, the nuclei decrease their charge atomic number $Z$, passing to the neutron drip line $Q_n = 0$ and emitting neutrons. The parameter $R$ value decreases (Fig. 4) as well as $(A, Z)$ until the reducing Fermi energy of electrons equals the
threshold electron capture energies of light nuclei $\varepsilon_{fe} = \varepsilon_{\beta}$. As a result, the final nuclei distribution consists of free neutrons, helium $^8\text{He}$, beryllium $^{14}\text{Be}$, boron $^{19}\text{B}$, and carbon $^{22}\text{C}$ nuclei in different proportions that depend on the initial $R_0$ (Figs. 14, 16, and 18). In our calculations, we considered neutrons in equilibrium with nuclei according to 20. A drop in temperature below the established limit leads to the
disappearance of neutron photodetachment reactions. In this case, the resulting composition consists of only
one element $T_g = 0.1$, which lies on the boundary of
the existence of nuclei and $\varepsilon_{fe} \approx Q_{p0}$. Maximum value
in the used mass model corresponds to carbon $^{22}$C and
$Q_{p0}(22.6) = 32$ MeV.

6. CONCLUSION

The analysis of the calculation results reveals that the problem of the chemical composition evolution of
a hot neutron star crust at subnuclear densities, cooling due to neutrino energy loss, should be considered
taking into account the previous history of the system.
To obtain real initial conditions, it is necessary to have information on the dynamics of a collapsing supernova core and the parameters of a hot neutron star being born.

The properties of the non-equilibrium layer obtained within the framework of this model indicate the presence of a large reserve of neutron stars nuclear energy, capable of maintaining the X-ray luminosity.
for a long period of time. Energy stored in a non-equilibrium layer can be released during transient sources explosion, such as soft gamma-repeaters and anomalous X-ray pulsars [35], which is an alternative to the often considered model of explosions on neutron stars with super strong magnetic fields $\sim10^{15}$ G [37–39] called magnetars.

As a result of various stellar activity processes, such as starquakes, elements from the shell can be carried outward, leading to the appearance of explosive tran-
sient processes, as well as forming heavy elements, such as transuranium elements, that are part of the next generation objects, including the Earth.

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