Tensions between measurements of the Hubble constant from the early and late Universe

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Abstract. Hubble constant ($H_0$) is one of the most important parameters in cosmology. There are mainly two ways to determine the value of Hubble constant, which measure the properties of early universe and the late universe, namely cosmic microwave background radiation (CMB) and Type Ia supernovae (SNe Ia). Those who had used these two methods to measure the Hubble constant won the Nobel Prize in Physics respectively, in 1978 and 2011. This article introduces the principle of accelerating universe and the methods to measure the Hubble constant. We analyze each method and discuss the uncertainties of them. In addition, we investigate possible reasons for Hubble constant discrepancy based on previous studies. We discuss about the conclusion and prospects of Hubble constant measurement.

1. Introduction

Hubble constant represents the expansion rate of the universe, which is one of the most important parameters in cosmology. For example, the age of the universe can be determined by the reciprocal of the Hubble constant [1]. It is also useful in galaxy evolution. In addition, the cosmological parameters, like dimensionless density parameters $\Omega_M$ and $\Omega_\Lambda$ are also related to Hubble constant, which helps to determine the ultimate fate of the universe, that is, whether it will continue to expand, or it will begin to collapse at a certain time. Therefore, Hubble constant is a fundamental parameter in astronomy and cosmology [2].

In 1912, an American astronomer named Vesto Slipher, measured the Doppler shift of the light that emitted from the galaxies. Among the involved 41 galaxies, the spectra of 36 of them have been found redshifted, but the rest were blueshifted. However, it has been proven that if the universe is moving in disorder, the number of blue-shifts should be the same as the number of red-shifts. Since most of them are redshifted, which indicated that the expansion of the universe is accelerating. This was also the first evidence to prove that the universe is expanding [3].

In 1929, Edwin Powell Hubble, the first man to determine the value of Hubble constant, got the value of around 500 km s\textsuperscript{-1} Mpc\textsuperscript{-1} [4]. From 1980, the value of the Hubble constant determined by astronomers basically varied between 50 and 100 (km s\textsuperscript{-1} Mpc\textsuperscript{-1}).

There are several methods of measuring the Hubble constant, and the most commonly used one is the distance-ladder method [5]. We utilize the Period-luminosity (P-L) relation of Cepheid and Hubble Space Telescope (HST) to calculate the distance of nearby galaxies to calibrate. Then we use the standard candles of Type Ia supernova (SNe Ia) to measure galaxies far away from the Earth in order to measure the Hubble constant.

In 1998, Saul Perlmutter, Brian Schmidt, and Adam G. Riess applied this method and discovered that the expansion of universe is accelerating, hence won the Nobel Prize in Physics in 2011 [6,7]. In 2019, the value of Hubble constant determined by Riess et al. 2019, was 74.03±1.42 km s\textsuperscript{-1} Mpc\textsuperscript{-1} [8].
Another method is to constrain the Hubble constant with cosmic microwave background radiation (CMB) [9]. Arno Allan Penzias and Robert Woodrow Wilson won the 1978 Nobel Prize in Physics for discovering the CMB. In recent years, the values of the Hubble constant measured with CMB were $H_0 = 67.4\pm0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [10] and $H_0 = 67.8\pm1.3$ [11].

Apart from those two methods mentioned above, gravitational lensing is also used to measure the Hubble constant [12,13], and the values got in this way are $H_0 = 71.9\pm2.7 \text{ (3.8\%)}$ [13], $H_0 = 74.5^{+5.4}_{-6.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$ [15]. In addition to that, Baryon acoustic oscillation (BAO) also provides a way to determine the Hubble constant [16].

In recent studies, the values of the Hubble constant obtained have become more precise. However, there is a discrepancy of the Hubble constant between early universe and late universe, and the difference between them is larger than three standard deviations [11].

2. The accelerating expansion of the universe

2.1 Hubble constant

In 1929, Edwin Powell Hubble [4] showed that the radial velocity of a galaxy is directly proportional to its distance from the earth, that is, the further away a galaxy is, the faster it is moving, and a formula is thus derived.

$$v = H_0 d,$$

where $H_0$ stands for Hubble constant, $d$ is the distance from the earth, and $v$ is the radial velocity. Hubble constant provides information about the age of the universe, and since the velocity ($v$) is equal to the ratio of distance ($d$) and the time ($t$)

$$v = \frac{d}{t},$$

Then the time is equal to the distance divided by velocity,

$$t = \frac{d}{v}.$$  

The reciprocal of $H_0$ is equal to the distance divided by the velocity,

$$\frac{1}{H_0} = \frac{d}{v}.$$  

So the reciprocal of $H_0$ is equal to the age of the universe,

$$\frac{1}{H_0} = t.$$  

Since the Hubble constant obtained in the 1920s was $500 \text{ km s}^{-1} \text{ Mpc}^{-1}$, it suggests the age of the Universe to be 2 billion years. However, at that time scientists believed that the age of the Earth was 3 billion years, they wanted to know why the age of the Universe calculated would be younger than that of the Earth. Therefore, they began to re-measure the Hubble constant.

The density of the universe can also be inferred from the Hubble constant. If the density of the universe is relatively small, then the gravitational force between galaxies will be generally week, which means that the universe will continue to expand. However, if the matter in the universe has more than a critical density, then the gravitational force will succeed in causing the universe to collapse. Currently, cosmologists believe that the three destinies of the universe are the flat Universe, the closed Universe, and the open Universe [2].

The Hubble Space Telescope (HST) launched in 1990 by National Aeronautics and Space Administration (NASA) helps to measure the Hubble constant. It provides possibility to use the distance ladder method to measure the Hubble constant [18]. For example, Riess et al. 2019 uses HST to measure the Hubble constant, $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which reduces the uncertainties to 4.8% [8].

2.2 Redshift

Redshift ($z$) is a kind of phenomenon due to the relative motion of other galaxies with respect to ours in an effect called the Doppler effect: an observer receiving waves emitted from a moving body observes that the wavelength of the waves has been altered to a new one, that is, when an object is moving away, the wavelength of the observed wave appears to be longer than expected, and all the features present
will be shifted towards the red end of the spectrum. In contrast, if the object is approaching to the observer, the wavelength of the observed wave emitted by the object will decrease and its frequency will increase, therefore it will shift towards the blue end of the spectrum. As astronomers have discovered that the spectra emitted from distant galaxies have redshifts, this demonstrates that the galaxy is moving away from the earth,

\[ z \equiv \frac{v_e}{v_0} - 1 = \frac{\lambda_0}{\lambda_e} - 1, \]  

(6)

\( z \) stands for redshift, \( V_0 \) stands for observed frequency, \( \lambda_0 \) stands for observed wavelength, \( V_e \) stands for emitted frequency, \( \lambda_e \) stands for emitted wavelength [1].

The relationship of redshift and radial velocity can be expressed as

\[ 1 + z = \frac{1 + v/c}{\sqrt{1 - v/c}}, \]  

(7)

where \( v \) stands for radial velocity, \( c \) stands for speed of light.

When \( v/c \) is small, as well as when the distance is small, redshift can be estimated as

\[ z \approx \frac{v}{c} = \frac{d}{D_H}, \]  

(8)

where \( D_H \) stands for the Hubble distance, which equals to the speed of light divided by the Hubble constant.

2.3 Distance measurement

In modern astronomy, there are many techniques to measure distance. One method is the trigonometric parallax. As the earth moves around the sun, it will be exactly on the opposite side of its orbit in six months. Therefore, we can see that the position of the star will be offset, and the parallax angle can be calculated. As we also know the size of the Earth’s orbit, geometry allows calculations of the distance to the star. However, since the farther the object is away from the earth, the smaller the parallax angle will be, and owing to the limitations of the accuracy of detectors, this method is only suitable for objects within a few hundred light-years away from us [19].

For stars far away from the earth, we can utilize another technique to measure. As we known, the brightness emitted by the star is related to its size and temperature, we can compare the brightness of a star observed with another star of known brightness. Therefore, we can deduce the distance of the star from us. For example, according to the Period-luminosity (P-L) relation of Cepheid, the longer the period (the period during which the brightness of a star changes from dim to bright) of a star, the greater its brightness will be,

\[ D_L \equiv \sqrt{\frac{L}{4\pi S}}, \]  

(9)

where \( D_L \) represents luminosity distance, \( L \) represents luminosity, and \( S \) represents flux [1].

\[ D_L = (1+z) D_M = (1+z)^2 D_A, \]  

(10)

where \( D_M \) is the comoving distance, which is the distance between two object that have the same distance from earth or the same redshift, but in different position of the universe. And \( D_A \) represents angular diameter distance [19, 20]. Finally, we can get the formula for distance modulus (DM) [1].

\[ DM \equiv 5\log\left(\frac{D_L}{10pc}\right). \]  

(11)

3. Measurement of Hubble constant

3.1 Supernova
A final destiny of stars is called a supernova (SN). A SN is a star that emits energy and materials around when it explodes. And its brightness may be brighter than an entire galaxy. This process usually lasts from a few weeks to several months, until it disappears [21]. It is also the main source of the heavy elements in the universe 22].

The hydrogen is converted into helium until it has been fused to become mostly iron. At this time, the temperature drops rapidly. As the electron degeneracy pressure is less than the pressure of gravity, and it will collapse inward, forming a shock wave [23].

There are two main types of supernovae (SNe), Type Ia supernovae (SNe Ia) and core-collapse supernovae (CC SNe). Furthermore, CC SNe includes Ib, Ic and Type II SNe.

SNe Ia are formed by white dwarfs’ explosions. A white dwarf accretes mass from a red giant or main sequence stars will explode until it reaches the Chandrasekhar limit, that is, 1.44 solar masses.

CC SNe may be formed when the mass of the progenitor star is eight time the mass of the sun. Its remnant forms neutron star if $8M_\odot < M < 20M_\odot$. Otherwise, it forms black hole if $20M_\odot < M < 50M_\odot$. In addition, if the mass of a progenitor star is greater than 50 solar masses, a black hole can be formed directly without a SN explosion [24].

Another type of SNe is Superluminous supernovas (SLSNe), which is defined as a SN with a peak brightness of less than 19.8 mag 25]. SLSNe can be categorized as two types, one is hydrogen poor (SLSNe-I) and hydrogen rich (SLSNe-II). In recent years, most of the SLSNe discovered are SLSNe-I. But in fact, our definition of SLSNe is still unclear. This is because this kind of SN is very difficult to find with only about 100 SLSNe have been discovered. As more SLSNe to be detected, we can further improve our SLSNe definition [25].

Most of the time, astronomers generally use SNe Ia as standard candles to measure cosmological parameters [6, 7]. As the maximum brightness of a SNe Ia explosion is linearly related to the rate of change of its brightness, that is, the Phillips relation [26].

With the Phillips relation, we can measure redshift and distance. However, it needs to be corrected in order to measure the Hubble constant. There are several ways to correct the peak magnitudes of SN Ia to make it a more standardized candle. 1. $\Delta m_{15}$ method [26], 2. MLCS method [27], 3. $\Delta C_{12}$ method [28], 4. MLCS2k2 method [29], 5. SALT method (SALT2) [30]. The last two methods (MLCS2k2, SALT2) are more commonly used by cosmologist’s methods among the five methods [21].

The values of the Hubble constant measured by the SNe Ia method are basically between 70 to 75 km s$^{-1}$ Mpc$^{-1}$, that is, $H_0=74.03\pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ [8].

3.2 Cosmic microwave background radiation
In 1964, two American scientists (Arno Allan Penzias, Robert Woodrow Wilson) tried to improve the satellite communication equipment, but found that there was a period of noise that could not be eliminated. Later they discovered that the wave band of this noise belonged to microwaves, and the temperature is about 3K. This is the first detection of the cosmic microwave background radiation [31]. In recent years, scientists found that the temperature of CMB is $2.72548\pm 0.00057$K [32].

This radiation is emitted when the universe was about 380,000 years old, and is an isotropic microwave [33]. At that time, stars and galaxies had not yet been formed, and the universe was full of plasma and thermal radiation. It was not until the temperature of the universe began to drop and atoms no longer absorbed thermal radiation, and the remaining radiation was explored through the radiation telescope 34]. European Space Agency (ESU) launched the Planck telescope in 2009 to detect CMB.

The values of the Hubble constant measured by the cosmic microwave background radiation method are basically less than 70 km s$^{-1}$ Mpc$^{-1}$ [10, 35, 36].

3.3 Gravitational lensing
In general, we cannot see the light behind an object. However, when light passes through a massive cosmic object, such as galaxy cluster, the nearby gravitational field is strong enough to bend the light passing through it. We will see multiple images formed, and a strong lens can even form Einstein rings [15].
Because the mass distribution of a cosmic object may be not uniformly distributed, the light travel path is not the same, hence the arrival time is different. As a result, there will be a time delay between different images. We can detect the corresponding distance and time, and estimate of the mass distribution of the lens galaxy, therefore the Hubble constant can be determined [12, 13].

There are two leading research groups, H0LiCOW and SHARP corporation to analyze six separate gravitational lensing systems. The uncertainties of the value of Hubble constant obtained is 4.9-9.1% [13, 37-41].

Birrer et al. 2020 combined data from 7 gravitational lenses and the value of Hubble constant they obtained is $74.5^{+5.6}_{-4.1}$ km s$^{-1}$ Mpc$^{-1}$.

3.3 Baryon Acoustic Oscillation
Baryon acoustic oscillations (BAO) can not only be detected from the cosmic microwave background radiation, but can also be observed by studying the distribution of galaxies in the universe [16]. In the early universe the temperature and density were very high, full of plasma, and baryons is one of the ions in it. Although the plasma is uniformly distributed in the universe, there will still be places with lower density and places with higher density. This is because the gas cloud will collapse due to gravity. The pressure of radiation pushes out the gas cloud. This baryon oscillation and density fluctuations are similar to the propagation of waves and form baryon acoustic oscillations $^{ab}$.

Andreu Font-Ribera et al. 2014 used Baryon Oscillation Spectroscopic Survey (BOSS) to model a distribution of hydrogen in the universe. As light passes through hydrogen, the electrons in the hydrogen will absorb a certain energy to reach a higher energy level, which will cause the spectral lines of the spectrum to be absorbed at a specific wavelength. By comparing the absorption spectra, the redshift can be measured [42]. As a result, we can determine the Hubble constant with reshifts and distances.

The value of the Hubble constant measured by BAO is $H_0=67.4\pm1.1$ km s$^{-1}$ Mpc$^{-1}$ [16].

4. Hubble tension
With the state-of-the-art technology, more detectors are launched, such as HST, Planck telescope, which makes the constraint on the Hubble constant more accurate with smaller error bars. However, we found that different measurements of the Hubble constant show significantly different values, and the discrepancy is more than 3 to 5 $\sigma$ [17]. This is known as the Hubble tension. The value of the Hubble constant measured in the late universe (SNe Ia, gravitational lensing) is around 70 to 75 km s$^{-1}$ Mpc$^{-1}$ [8, 43, 44], while the values determined in the early universe (CMB, BAO) are basically around 65 to 70 km s$^{-1}$ Mpc$^{-1}$ [10, 36, 45].

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\(^a\)http://www.sdss3.org/surveys/boss.php

\(^b\)https://qosmology.org/what-izaihs-baryon-acoustic-oscillations/
Fig. 1: the Hubble constant measured by different methods. The $H_0$ values measured through CMB and BAO are similar, while the values measured through SNe and Gravitational lensing are also consistent with each other. The $H_0$ measured by the SNe Ia method [8, 43, 46, 47], SNe II [48], gravitational lensing [14, 15, 44], CMB [10, 49], BAO [16], and gravitational waves [50], are marked in red, yellow, green, blue, grey, and orange, respectively.

For SNe Ia, its advantages are its large number (approximately 100 million SNe explosions during the lifetime of the Milky Way [51]), and a few parameters used to calculate the Hubble constant. But the disadvantages also exist. For example, since most SNe are located in start-forming galaxies, the light emitted by their explosions will be affected by its host galaxies, such as dust. Additional corrections are needed to constrain the absolute, which means there will be uncertainties in measuring distance [43]. In addition, Martinelli and Tutusaus [52] believe that the brightness of a SN is dependent on the redshift and its environment. According to the star formation rate (SFR) model, the light emitted by SNe in the relatively early universe is dimmer than the light emitted by the relatively late universe [53-56]. This indicates that there are uncertainties in the measurement of $H_0$ with SNe by the standard candles.

The source of uncertainties may come from the parameter used in the process. A slight change in the value of any parameter may result in errors in $H_0$. For the CMB, six parameters in $\Lambda$CDM are used in the process of measuring the Hubble constant along with it. The Hubble constant calculated by the Planck team has an uncertainty of only 0.74% [10]. In contrast, the SNe Ia uses two to three parameters, but the value obtained by Riess et al. 2019 has an error of 1.9% [8]. In term of gravitational lensing, the $H_0$ measured by each independent gravitational lensing system is different, which is strongly dependent on the lensing systems [15].

5. Conclusion

With the emergence of more and more methods, such as gravitational waves (GW) and type II supernovae (SNe II), these methods may reach agreement with the $H_0$ determined by different methods in the future. However, the differences between values of the $H_0$ may also become larger. Recently, the T de Jaeger team found that $H_0=75.8^{+5.2}_{-4.5}$ km s$^{-1}$ Mpc$^{-1}$ [53] by the property of SNe II, as it shows a similar linear relation between peak and plateau brightness [57]. This method is consistent with the value constrained with SNe Ia.

Furthermore, in 2017, Laser Interferometer Gravitational-wave Observatory (LIGO) [58] observed the gravitational wave (GW170817) emitted by the merger of two neutron stars. They determined that $H_0=70^{+12}_{-8}$ km s$^{-1}$ Mpc$^{-1}$ [50]. The value measured by GW method is consistent with both CMB and SNe Ia. Gravitational wave method is a new method. Neither will it distance ladder, nor will it contain any parameters during the measurement. Unfortunately, the rate of gravitational waves is very low, and it is difficult to be observed. Now its error bar is very large owing to the uncertainty of the sample, such as,
the distance measurement. As more gravitational waves can be detected in the future, the value of the Hubble constant will be more precise.

Many studies show that the difference between the values obtained in the early and late universes is because of the new physics. Many new theories have been proposed for this, for instance, dark matter decay, dark matter-radiation, neutrino-neutrino interactions or unknown field between the late and early universe [59-64]. If we can find evidence to prove these theories, then it may break down the discrepancy between SNe Ia method and the CMB method, which will thus solve the Hubble tension.

There is another opinion on the Hubble tension is that maybe the difference does not exist. Recently, Wendy Freedman found that through the latest observations, the difference between the values measured by the two methods (CMB, SNe) has become smaller, which means that we do not need to adjust the current parameters of the universe [65].

Not only do new theories and new methods will be utilized to tackle the Hubble tension, but also more detectors with higher resolution will be launched in the future to solve the problem. GAIA, which was launched by ESU in 2013, already brought back many useful detection results. James Webb Space Telescope (JWST), the Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT) will also be launched in the near future. These detectors will also collect a lot of meaningful data in the future, which will make the value of the Hubble constant more accurate, and uncertainties can be minimized.

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