Stability of nonlinear 1D laser pulse solitons in a plasma

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Abstract

In a recent 1D numerical fluid simulation study [Phys. Plasmas 13, 032309 (2006)] it was found that an instability is associated with a special class of one dimensional nonlinear solutions for modulated light pulses coupled to electron plasma waves in a relativistic cold plasma model. It is shown here that the instability can be understood on the basis of the Stimulated Raman scattering (SRS) phenomenon and the occurrence of density bursts in the trailing edge of the modulated structures are a manifestation of an explosive instability arising from a nonlinear phase mixing mechanism.
I. INTRODUCTION

The interaction of intense laser pulses with a plasma has been a topic of research interest for decades. The recent advent of ultra high intensity (∼$10^{20}$ w/cm$^2$) lasers has however led to a strong resurgence of this field. The processes of major interest taking place during these interactions are self focusing, soliton formation, wake-field generation, magnetic field generation etc. Among those the possibility of coherent nonlinear traveling pulse soliton like solutions for such a system has attracted keen attention from physicists both from a fundamental research point of view for their possible applications in diverse areas such as particle and photon acceleration, fast ignition concept of laser fusion etc. A large number of investigations [1, 2, 3, 4, 5, 6, 7, 8, 9] have been carried out to study the existence and accessibility of such coherent nonlinear solutions. In general two classes of soliton solutions have received considerable attention. One having single peak in vector potential ($R$) as well as in the scalar potential ($\phi$) profiles while other having multiple peaks of vector potential trapped inside a single peak envelope of scalar potential. The single peak one exhibit a continuous spectrum whereas the one with multiple peaks in $R$ correspond to a discrete spectrum [7, 8]. For practical applications of these solitonic structures one needs to develop a proper understanding of their dynamical properties like how they propagate in homogeneous and inhomogeneous plasmas and how they behave if subjected to mutual collisions etc. To reveal the dynamical properties of these interesting solutions an attempt has been made recently in our earlier work [9], where we dynamically evolved these nonlinear solutions with the help of fluid simulations.

It was found there that the single peak solutions evolve stably in a homogeneous plasma. They also remain almost unchanged and display nice reflection and transmission properties during their course of propagation in an inhomogeneous plasma. They were even found to remain intact when subjected to mutual collisions. On the other hand the solutions with multiple peaks in vector potential profile inside a single peak envelope of scalar potential, were seen to become unstable after a few tens of plasma periods by shedding radiation from their trailing edge. Further, these pulses also exhibited sharp density bursts in their wake region. Such an unstable behavior was also noticed in Vlasov simulation [23] of these solutions.

To date the mechanism of this instability and origin of the density bursts remain unclear.
The present work is devoted to a delineation of the physical mechanism underlying this process. We first carry out a detailed study of the instability occurring within the pulse extent and demonstrate that the forward Raman scattering process is in fact responsible for the instability by comparing our simulation results with the known analytical values of the growth rates of the Raman forward and backward scattering instability. It is seen that the backward scattering growth rates estimated from the simulation do not match with the corresponding analytical growth rates. Moreover in the simulation the wavelengths generated due to scattering processes are larger for the excited electrostatic waves than those for the scattered electromagnetic waves which is a clear feature of forward Raman scattering. Also backward scattering process tends to affect the front edge of the pulse and even in broad pulse it is expected to saturate in the leading part of the pulse itself and thus doesn’t affect the main body of the pulse [15] which is in contrast to our observations. We further provide an understanding of the density bursts observed in the wake of the pulse by employing a simple model calculation based on the relativistic wave breaking phenomenon. In particular, we try to predict the approximate time between two consecutive bursts on the basis of the calculation of the mixing time for two coexisting relativistic plasma waves and then compare it with the corresponding time observed in our simulations. A detailed investigation in this regard is presented.

In the next section a 1D model for the interaction of a relativistically intense laser pulse with a cold collisionless plasma with fixed ion background is described. In the same section we also recapitulate the possible solutions briefly. Then in the following section, a detailed analysis of the instability is provided. Further in section - IV we provide an understanding of the density bursts that are observed in the wake of the moving multipeak solution on the basis of the relativistic wave breaking phenomenon. Finally in the last section we present the conclusions of the paper.

II. BASIC EQUATIONS AND STATIONARY SOLUTIONS

The basic equations are the relativistic set of fluid evolution equations for a cold plasma in one dimension together with the Maxwell equations for the electromagnetic wave. We consider spatial variations to exist only along $x$, the direction of propagation, and consider the ions to be stationary. The relevant set of fluid and field equations are then,
\[ \frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0. \]  

\[ \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma} \frac{\partial A_{\perp}^2}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma} \frac{\partial A_{\perp}^2}{\partial x} = 0. \]  

\[ \frac{\partial^2 \phi}{\partial x^2} = n - n_0(x) \]  

\[ \frac{\partial^2 \vec{A}_{\perp}}{\partial x^2} - \frac{\partial^2 \vec{A}_{\perp}}{\partial t^2} = \frac{n \vec{A}_{\perp}}{\gamma} \]  

where (1) is the electron continuity equation, (2) is the parallel electron momentum equation, (3) is the Poisson’s equation for the electrostatic potential \( \phi \), (4) is the wave equation for the vector potential \( \vec{A}_{\perp} \) and other notations are standard. The perpendicular electron momentum equation has been integrated exactly to obtain the conservation of the transverse canonical momenta (sum of particle and the field momenta) as \( u_{\perp} - \frac{\vec{A}_{\perp}}{\gamma} = 0 \) and used to eliminate \( u_{\perp} \) in the above equations. Here \( \gamma \) is the relativistic factor \( \gamma = \sqrt{1 + \frac{A_{\perp}^2}{1 - u^2}} \). 

In writing the above equations we have chosen to normalize the density by some appropriate density \( n_0 \). The length is normalized by the corresponding skin depth \( c/\omega_{pe0} \) (where \( \omega_{pe0} = \sqrt{4\pi n_0 e^2/m_e} \)) and time by the inverse of the plasma frequency \( \omega_{pe0}^{-1} \). The scalar and vector potentials are normalized by \( mc^2/e \). In Poisson’s equation \( n_0(x) \) corresponds to the background ion density normalized by \( n_{00} \).

The coupled set of nonlinear equations (1-4) permit a variety of coherent solutions. A class of one dimensional propagating solutions with modulated envelope structure of the above set have been obtained in the past by using the coordinate transformation \( \xi = x - \beta t \) and \( \tau = t \) (where \( \beta \) represents the group velocity of the structure). The vector potential is assumed to be circularly polarized and has a sinusoidal phase variation of the form \( \vec{A} = (a(\xi)/2)\{[\hat{y} + i\hat{z}] \exp(-i\lambda \tau) + c.c. \} \). The plasma oscillations associated with the envelope structure are assumed to have no dependence on \( \tau \). This is the so called electrostatic approximation which is valid if there is not a significant change in the plasma parameters within the pulse duration. The above transformations convert Eqs. (1-2) into ordinary differential equations which can be integrated to give \( n(\beta - u) = \beta \) and \( \gamma(1 - \beta u) - \phi = 1 \), where one assumes that at the boundaries \( u = 0, \phi = 0 \) and \( n = 1 \). One eliminates \( n \) to write Poisson’s equation.
Here prime(\') denotes derivative with respect to $\xi$. Writing $a(\xi) = R \exp(i\theta)$, the wave equation [see Eq.(4)] can be written as
\[
R'' + \frac{R}{1 - \beta^2} \left[ \frac{\lambda^2 - M^2}{R^4} - \frac{\beta}{\beta - u} \frac{1 - \beta u}{1 + \phi} \right] = 0 \tag{6}
\]
Here $M = R^2[(1 - \beta^2)\theta' - \lambda \beta]$ is a constant of integration and $R^2 = A_x^2 + A_z^2$. Eqs.(5,6) form a coupled set of second order differential equations in two fields $\phi$ and $R$ respectively. The longitudinal velocity $u$ appearing in the two equations can be expressed entirely in terms of $R$ and $\phi$ as
\[
u = \beta \left( (1 + R^2) - (1 + \phi)[(1 + \phi)^2 - (1 - \beta^2)(1 + R^2)]^{1/2} \right) \tag{7}
\]
Eqs.(5,6) have been solved by Kaw et al. [2] and others [8] for $M = 0$.

In the absence of any further simplifying assumptions, above equations [see Eqs.(5,6)] cannot be solved analytically. However, for the general case several varieties of numerical solutions have been obtained(for $M = 0$). A detailed characterization of some of these solutions on the basis of group speed $\beta$ and the frequency parameter $\lambda$ has been made in some of the earlier studies [2, 7, 8] where $\lambda$ is defined as $\omega(1 - \beta^2)$.

We reproduce the $\lambda - \beta$ spectrum and show the two varieties of the solutions with that in Fig.1. A continuum spectrum in the $\lambda - \beta$ plane has been observed only for those solutions which have a single peak of vector potential $A$ as well as electrostatic potential $\phi$. These solutions have a reasonably lower amplitude and satisfy $\phi < A$. On the other hand there are solutions which occur only for discrete values of $\lambda$ for a given $\beta$. These solutions differ from those with continuum spectrum as they have several multiple peaks of the vector potential $A$ trapped inside an envelope of $\phi$. The scalar normalized potential $\phi >> A$ for these solutions. The electron density in the central region is strongly evacuated and the light wave is trapped in this density cavity.

In our recent numerical work [9] it was found that the solutions corresponding to the continuous spectrum and having single peak in vector potential display robustness during their propagation in a homogeneous as well as in an inhomogeneous plasma while those admitting a discrete spectrum and having multiple peaks in the vector potential tend to develop an instability in their trailing edge even while propagating in a homogeneous background plasma.
In the next section we carry out a detailed investigation of this characteristic instability of
the multipeak solutions and compare our simulation results, in particular the growth rate
of the instability, with the corresponding analytical results for forward stimulated Raman
scattering (fSRS) instability.

III. CHARACTERISTIC INSTABILITY OF THE MULTIPEAK SOLUTIONS: A
DETAILED INVESTIGATION

As mentioned above, the structures with multiple peaks in $R$ exhibit an interesting in-
stability wherein the perturbed fields are ejected from the trailing edge of the solutions as
shown in Fig.2. In various subplots of the figure, profiles of vector potential as well as scalar
potential are shown at different time instants. It is obvious from the figure that the struc-
ture seems to evolve stably for few tens of plasma periods before it starts emitting from the
trailing edge.

In Fig.3 we show the growth of the perturbations at three different time instants, viz.
$t = 100, 110, \text{ and } 120$ electron plasma periods, measured as the difference between exact
and the numerically observed value of the scalar potential $\phi$ and the vector potential $R$
within the structure. The structure has been identified by simultaneously plotting the
equilibrium electron density curve translated by $\beta t$ (the cavitation in electron density
essentially provides the spatial extent of the structure). The pulse is moving towards right
with a group speed of $\beta = 0.8$. It can be observed from the figure that a small amplitude
perturbation starts at the front edge of the pulse. It suffers continuous amplification as it
trails behind towards the rear edge. Finally from the rear edge of the solutions, structures
get ejected and this process continues.

Let us first have some understanding of the mechanism for this instability on the
basis of the well known relativistic stimulated Raman scattering (SRS) phenomena. The
physical mechanism of SRS is simple and can be understood by realizing that an incident
electromagnetic radiation generates a scattered light wave due to the transverse currents
in the plasma medium. The nonlinear interaction of the scattered light wave with the
incident light pulse in turn produces an electrostatic plasma wave. The plasma wave can get
resonantly excited to a very large amplitude if appropriate frequency matching conditions
are satisfied. The instability can only get excited provided a threshold condition on the electron density is satisfied which arises from the condition $\omega \geq \omega_{pe}/\sqrt{(\gamma)}$. Here $\omega$ and $\omega_{pe}$ are the laser frequency and the electron plasma wave frequency respectively and $\gamma$ is the relativistic factor. This shows that the instability can only be excited provided the electron density satisfies the condition of $n_e \leq n_{th}$, where $n_{th} = \gamma \omega^2/4$ within a reasonable spatial extent to observe several e-foldings in the growth.

Before we provide a comparison for the analytical growth rates of forward and backward Raman scattering with the numerically observed growth rates, let us discuss the well established theoretical results for the two cases. The growth rate for the relativistic Raman forward scattering instability for which the scattered wave moves in the same direction as the incident light pulse is given by the following expression \[13, 14, 15, 16\],

$$\Gamma_{rfs} = \frac{1}{2\sqrt{2\omega}} \frac{A_0}{(1 + A_0^2/2)}$$  

(8)

On the other hand the growth rate of the backward Raman instability for which the scattered wave moves opposite to the incident pulse is given by two different expressions in two different regimes \[10, 14\]. When the condition $v_{osc}/c < (\omega_{pe}/\omega)^{1/2}$ which in relativistic case becomes $A_0^2/\gamma^{3/2} < \omega_{pe}/\omega$, is satisfied the growth rate for the backward Raman scattering instability is given by

$$\Gamma_{brs} = \frac{\sqrt{\omega}}{4} \frac{A_0}{(1 + A_0^2)^{5/8}}$$  

(9)

and when $A_0^2/\gamma^{3/2} > \omega_{pe}/\omega$ holds the expression for the growth rate for the backward Raman scattering instability reads

$$\Gamma_{brs} = \sqrt{3} \left(\frac{\omega}{16}\right)^{1/3} \frac{A_0^{2/3}}{(1 + A_0^2)^{1/2}}$$  

(10)

Here, $\omega$ is the frequency of the light pulse and $A_0$ is the maximum amplitude of the vector potential. We compare the analytical growth rates for both kinds of the Raman scattering instability with those evaluated from the results of the numerical simulations in Fig.4.

The growth rate from the observed data is calculated using the expression for the amplification factor for the parametric instability \[11\],

$$\alpha = \exp\left(\Gamma_{sim}L/(V_1V_2)^{1/2}\right)$$  

(11)
where $\Gamma$ is the growth rate of the parametric instability, $L$ is the length of the interaction region, and $V_1, V_2$ are the relative group speed of the daughter waves measured with respect to the pump wave. In our simulations we observe both the daughter waves almost standing together behind the pump which leads to $V_1 = V_2 = \beta$ and the expression for the amplification factor reduces to

$$\alpha = \exp(\Gamma_{\text{sim}} L / \beta)$$

(12)

The amplification factor is the ratio of the final to the initial perturbation amplitudes in scalar potential $\phi$.

Now for a comparison, numerical growth rates from the simulations and analytical growth rates for the two cases viz. the forward and backward Raman scattering instability are obtained for 8 different solutions as discussed above. These solutions differ from each other with respect to the number of light wave peaks associated with them, the peak vector potential amplitude and its frequency. These parameter details of various solutions have been presented in Table - I together with the analytical growth rates for the forward as well as for the backward SRS and the growth rates obtained from the simulations. The table also shows the value of the threshold density $n_{\text{th}}$ and the minimum electron density $n_{\text{min}}$ for the solutions. Note that for all the cases $n_{\text{min}}$ is less than $n_{\text{th}}$. so that the threshold criterion for the excitation of SRS is adequately satisfied. The upper subplot of Fig.4 shows a plot of $\Gamma \omega$ with $A_0$ for the theoretical growth rates for forward Raman scattering instability as well as numerical growth rates. It is clear from the figure that for all varieties of multipeak solutions, the growth rate of the observed instability agrees closely with the analytical value of the forward SRS. In the lower subplot of Fig.4 we compare the theoretical growth rates for backward Raman scattering instability with the numerically observed growth rates. In fact we plot $\Gamma / \sqrt{\omega}$ with $A_0$, corresponding to 8 multipeak solutions for the theoretical growth rates for backward Raman scattering instability as well as for the numerical growth rates. We observe that there is a clear mismatch in the values.

We note from the figure and from the table as well, that for forward SRS instability the theoretical values of the growth rates match well with the numerically observed values which is not the case for the backward SRS instability. It should also be noted
TABLE I: Comparison of instability’s growth rates observed in simulations with the theoretical values. Here $\beta, \omega, p, A_0, w, n_{min}, n_{th}, \Gamma_{frs}, \Gamma_{brs}$ and $\Gamma_{sim}$ respectively stand for group speed and frequency of the pulse, number of extrema and peak amplitude of the vector potential, transition width for growth, minimum electron density value in the cavity, threshold electron density for Raman scattering, theoretically estimated growth rates for forward and backward SRS and numerically observed growth rates of the instability.

| $\beta$ | $\omega$ | $p$ | $A_0$ | $w$ | $n_{min}$ | $n_{th}$ | $\Gamma_{frs}$ | $\Gamma_{brs}$ | $\Gamma_{sim}$ |
|---------|---------|-----|-------|-----|------------|----------|--------------|--------------|--------------|
| 0.9     | 2.14266 | 3   | 1.5343| 6   | 0.5636     | 1.6053   | 0.1163       | 0.6437       | 0.1547       |
| 0.8     | 1.4803  | 3   | 2.2522| 7   | 0.4762     | 1.1559   | 0.1521       | 0.5462       | 0.1597       |
| 0.8     | 1.4371  | 4   | 3.1362| 9   | 0.4539     | 1.4974   | 0.1304       | 0.5049       | 0.1752       |
| 0.8     | 1.403   | 5   | 4.046 | 12  | 0.4488     | 1.8501   | 0.111        | 0.4688       | 0.1532       |
| 0.6     | 0.93522 | 5   | 7.2710| 17  | 0.3756     | 2.0927   | 0.1002       | 0.3437       | 0.1346       |
| 0.4     | 0.6943  | 4   | 10.3772| 19  | 0.2859     | 1.9353   | 0.0964       | 0.2778       | 0.0609       |
| 0.5     | 0.67225 | 7   | 19.1134| 35  | 0.3334     | 3.410    | 0.05473      | 0.2249       | 0.0255       |
| 0.5     | 0.59288 | 9   | 31.4606| 50  | 0.3333     | 3.1188   | 0.037833     | 0.1828       | 0.00852      |

that the scattered light wave structures are found to remain trapped inside the plasma wave structures which trail behind with respect to the moving pulse and the scale length of the scattered light wave amplitude $dR$ is shorter than that of the perturbed scalar potential $d\phi$ (as is clear from Fig.3) which are clear signatures of forward Raman scattering instability. Also the backward Raman instability is known to get saturated in the leading edge of the pulse itself thus not affecting the main body of the pulse which is in contrast with our observations. Therefore we discard the possibility of the backward Raman scattering to be a potential candidate for the depletion of the pulse. It should be noted here that even though the growth rate of the backward Raman scattered instability is more than the forward SRS the solutions seem to exhibit only the forward SRS instability.

To provide an additional evidence of the forward Raman scattering instability we display in Fig.5 the frequency spectrum of one component of the vector potential ($A_y$) measured both in the laboratory frame as well as in the frame moving with the pulse. As evident from the left subplot of Fig.5, the spectrum measured in the laboratory frame
has peaks at pump wave frequency $[\omega_0 = \lambda/(1 - \beta^2)]$, plasma frequency($\omega_{pe}$), sideband frequencies ($\omega_0 - \omega_{pe}$ and $\omega_0 + \omega_{pe}$ ) and at another frequency $2\omega_0 - \omega_{pe}$ which might result from the interaction between the left sideband and the pump wave frequency. The spectrum also affirms the presence of forward Raman scattering as we obtain both the frequency sidebands. It also supports our conclusion discarding the possibility of the backward Raman scattering as in the backward Raman scattering the power in the up-shifted frequency band is usually negligible which is in contrast with the present frequency spectrum. In the right subplot of the same figure where the spectrum measured in the moving frame is shown, there is only one peak at a frequency $\omega_0 - k\beta$. The result is as per our expectations because while measuring the frequencies in the moving frame of the pulse the Doppler effects come into picture. The reason why we don’t observe other Doppler shifted frequencies is also quite simple since if we fix a position with respect to the head of the pulse of the observation point we can only observe the frequency with which the wave vector components are oscillating ($\omega_0$ in the present scenario).

We now try to understand the total absence of the forward SRS instability for the single peak variety of the modulated solutions. For all the single peak solutions the scalar electrostatic potential $\varphi$ is observed to be very weak in comparison to the vector potential $a$. The smaller value of $\varphi$ for these solutions is due to the fact that electron cavitation in these structures are much weaker in comparison to the multipeak solutions. Table II provides a detailed description of the parameters of this variety of solutions. Note that even though the forward SRS growth rate of the solutions obtained from the analytical expression may be finite, but in all cases the threshold condition on density for the excitation of forward SRS is not satisfied and the growth rate has no meaning. This is the primary reason for the robustness of these structures in fluid simulations. Moreover, in the small amplitude limit these solutions are similar to the exact nonlinear Schrodinger soliton solutions which are known to be stable.

In the next section, we present a model calculation based on phase mixing/wave breaking phenomenon to explain the appearance of bursts in the wake of the moving multiple peak solution.
TABLE II: Comparison of minimum electron density and threshold density for RFS for single peak solitons

| $\beta$ | $\omega$ | $A_0$ | $n_{min}$ | $n_{th}$ |
|---------|---------|------|-----------|---------|
| 0.1     | 0.9495  | 0.7350 | 0.9392    | 0.2797  |
| 0.2     | 0.9896  | 0.5209 | 0.9822    | 0.2720  |
| 0.3     | 1.0329  | 0.3509 | 0.9959    | 0.2827  |
| 0.4     | 1.0857  | 0.2002 | 0.9995    | 0.3005  |
| 0.5     | 1.1453  | 0.2585 | 0.9984    | 0.3387  |
| 0.8     | 1.0066  | 0.1158 | 0.9998    | 0.6966  |

IV. A MODEL CALCULATION FOR THE DENSITY BURSTS IN THE WAKE OF THE MULTIPEAK SOLUTION

As a result of forward Raman instability the small perturbations in the front end of the pulse are continuously amplified before being ejected from the rear end. We observe that following this process there appear density spikes in the wake of the moving pulse.

A plausible explanation for the appearance of density spikes lie in the basic nonlinear effect associated with a relativistically intense plasma oscillation, where due to relativistic variation of electron mass, the effective plasma frequency becomes a function of position. As a result different fluid elements which are at different locations in space oscillate at different frequencies leading to loss of coherence due to phase mixing. Ultimately there occurs a point in time when two adjacent oscillating fluid elements cross through each other and the initial coherent oscillation explosively breaks (wave breaking). This effect is very similar to the phase mixing of non-relativistic plasma oscillations as discussed by Dawson et. al. [18] for an inhomogeneous plasma, where the spatial dependence of plasma frequency arises due to background inhomogeneity.

In the present case however, instead of a plasma oscillation, a spectrum of intense plasma waves are excited in the wake of the multipeak solution. Based on the above physics, we present below a model calculation where we treat the evolution of two relativistically intense plasma waves whose wave numbers differ by an amount $\Delta k$. This is an extension of Infeld’s [20] calculation for a relativistically intense plasma oscillation. In order to represent the
scenario behind the moving pulse at a time when the plasma waves are ejected out, we take the initial density and velocity perturbations as

\[ \delta n_e(x, 0) \approx \Delta \cos\left( \frac{\Delta k}{2} x \right) \cos((k + \frac{\Delta k}{2})x) \]  

(13)

and

\[ v_e(x, 0) \approx \frac{\omega_{pe}}{k} \cos\left( \frac{\Delta k}{2} x \right) \cos((k + \frac{\Delta k}{2})x) \]  

(14)

We now study the evolution of these perturbations. In Lagrange coordinate, the relativistic equation of motion of a fluid element is given by

\[ \ddot{\xi} \left( 1 - \frac{\dot{\xi}^2}{c^2} \right)^{3/2} + \omega_{pe}^2 = 0 \]  

(15)

where \( x = x_0 + \xi(x_0, \tau) \), \( \xi \) being the displacement of a fluid element from its equilibrium position \( (x_0) \). Using (13) and (14) as initial condition, in the weakly relativistic limit \( (\omega_{pe}\Delta/k \ll 1) \) and \( (\Delta k/k \ll 1) \), solution of equation (15) may be written as

\[ \xi(x_l, \tau) \approx \frac{\Delta}{k} \cos\left( \frac{\Delta}{2} x_l \right) \cos(\tilde{\omega}_{pe}\tau - kx_l) \]  

(16)

where \( x_l = x_0 + \xi(x_0, 0) \) is a new Lagrange coordinate and

\[ \tilde{\omega}_{pe} \approx \omega_{pe} \left[ 1 - \frac{3}{16} \frac{\omega_{pe}^2 \Delta^2}{k^2 c^2} \cos^2\left( \frac{\Delta k}{2} x_l \right) \right] \]  

(17)

Dependence of \( \tilde{\omega}_{pe} \) on the initial position \( x_l \) is a clear indication of phase mixing. Using Poisson’s equation, the electron density \( n_e \) can be expressed in terms of \( \xi(x_l, \tau) \) as

\[ n_e(x_l, \tau) = \frac{n_0}{1 + \frac{\partial \xi}{\partial x_l} \left| _{\tau=0} \right.} \]  

(18)

Substituting the expression for \( \xi(x_l, \tau) \) in the above expression, the electron density in terms of new Lagrange coordinate \( (x_l, \tau) \) finally stands as

\[ n_e(x_l, \tau) \approx \frac{n_0 \{1 + \Delta \cos(\frac{\Delta k}{2} x_l) \cos kx_l\}}{1 + \Delta \cos(\frac{\Delta k}{2} x_l)[\cos kx_l + \cos(\tilde{\omega}_{pe}\tau - kx_l)\{\frac{\partial \tilde{\omega}_{pe}}{\partial x_l} - 1\}]} \]  

(19)

The presence of secular term in the denominator clearly shows that the electron density will eventually explode in a time scale \( \omega_{pe}\tau_{mix} \sim \left( \frac{3\omega_{pe}^2 \Delta^3}{16k^3 c^2} \right)^{-1} \Delta k \). This time scale depends on the level of density fluctuation \( \Delta \) and the spread “\( \Delta k \)” of the plasma waves. We would like to emphasize here, that in this case wave breaking happens at arbitrarily low amplitudes.
This is in contrast to the earlier works on wave breaking \cite{17, 19, 22}, which require the wave to reach a critical amplitude before it breaks.

We now apply the above expression for phase mixing time to estimate the time between consecutive density bursts and compare it with our numerical solution. We calculate the values of $k$ and $\Delta k$ from the $k-$spectrum of the density profile of a relaxed state in between two bursts. Fig. 7 shows the frequency spectrum of such a relaxed state (no bursts) at $t = 112.1\omega_{pe}^{-1}$. In this state the maximum amplitude of the plasma waves is $\Delta \sim 4.0$ and from their $k$-spectrum $k \sim 1.3\omega_{pe}/c$ and $\Delta k \sim 0.2\omega_{pe}/c$. This gives $\omega_{pe}\tau_{mix} \sim 1$. This implies that a burst should be observed at $\omega_{pe}t \sim 113.1$ and indeed such a burst is observed as shown in Fig. 6 where two consecutive density bursts occurring at times $t = 110.9\omega_{pe}^{-1}$ and $t = 113.5\omega_{pe}^{-1}$ are shown together with the in between relaxed state at $t = 112.1\omega_{pe}^{-1}$ (dotted curve). If we assume that the relaxation time for a density spike to be of the same order as the time required for the formation of a spike from a relaxed state, then the time between two consecutive bursts turns out to be $\sim 2\omega_{pe}^{-1}$ which is in close agreement with the temporal spacing observed between two consecutive bursts (Fig. 6). The size of density spike observed in our simulation is limited by the grid size $\Delta x$ as $\delta n/n_0 \sim 1/\Delta x$. In the limit $\Delta x \rightarrow 0$, $\delta n/n_0 \rightarrow \infty$ as is really the case with wave breaking of a plasma wave in a cold plasma.

V. SUMMARY

In the present paper we have investigated the instability responsible for the break up of the multihump solution and we identify it to be the forward Raman scattering instability. The growth rate obtained from the simulations is compared with the theoretically estimated growth rates for both the forward and the backward Raman scattering instabilities and found to match well with those for the forward SRS. We also present and explain the Fourier spectrum of the scattered electromagnetic fields which also supports our arguments about the forward Raman scattering. Furthermore, we provide an explanation for the density bursts observed in the wake of the moving multipeak solutions by means of a model calculation based on the relativistic wave breaking phenomenon.
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FIGURE CAPTIONS

FIG.1: The $\lambda - \beta$ spectrum (in subplot 'S') and two possible variety of solutions. The plot in subplot tagged with 'A' is for $\beta = 0.05, \lambda = 0.92$ and corresponds to the continuous spectrum in the subplot 'S'. The other subplot tagged with 'B' is for $\beta = 0.8, \lambda = 0.50518049$ and corresponds to the discrete spectrum, in particular the dotted line with 'stars'.

FIG.2: The scalar potential, vector potential and electron density profiles of a multipeak solution with $\beta = 0.8, \lambda = 0.50518049$ are shown at four different times viz. $t = 0, 80, 100, 120$ electron plasma periods. The development of the instability in the trailing edge as well as the consequent density bursts are evident in the two lower subplots.

FIG.3: The growth of the perturbation in the scalar potential amplitude $\phi$ (solid curve in subplots of the left column) and in the vector potential amplitude $R$ (solid curve in subplots of the right column) is shown respectively in left and right column of subplots at three different times viz. $t = 100, 110$ and $120$ electron plasma periods. It is clear that there appear smaller wavelengths (smaller $k$'s) in the scattered radiation than in the excited electrostatic oscillations which is a clear signature of forward Raman instability. Also shown is the density cavity associated with the original pulse translated with $\beta$ (dotted curve in all the subplots of this figure).

FIG.4: Comparison of the growth rates estimated from the simulation (triangles) with the analytical value of the growth rates (circles) of the relativistic forward Raman instability (upper subplot) and the backward Raman instability (lower subplot) for 8 different solutions. The simulation values match well with the analytical values for forward Raman scattering instability and they don’t match with the analytical values for backward Raman scattering instability.

FIG.5: The two subplot show the Fourier power spectrum in frequency. The left subplot correspond to the lab frame whereas the right one is for the pulse frame. In the lab frame five we get peaks at $\omega_0 - \omega_{pe}, \omega_{pe}, \omega_0, 2\omega_0 - \omega_{pe}$ and $\omega_0 + \omega_{pe}$. On the other hand in the pulse frame the spectrum comprises of a single peak at a Doppler shifted frequency $\omega_0 - k\beta$. 
FIG. 6: Two consecutive density bursts in the wake of the pulse at $t = 110.9$ and $113.5$ electron plasma periods together with the in between relaxed state at $t = 112.1$ electron plasma periods (dotted line).

FIG. 7: An expanded view of the k-spectrum of the relaxed density at $t = 112.1$ in between two consecutive density bursts shown in previous figure. We note that the dominant $k$ has the value $\approx 1.3$ and the full width at half maximum gives $\Delta k \approx 0.2$. 
\[ n(x_0 + \beta t, 0) \]
\[ P(\omega) = \begin{cases} 
1 & \omega = \omega_0 - \omega_{pe} \\
2 & \omega = \omega_{pe} \\
3 & \omega = \omega_0 \\
4 & \omega = 2\omega_0 - \omega_{pe} \\
5 & \omega = \omega_0 + \omega_{pe} 
\end{cases} \]

**Lab Frame**

**Pulse Frame**
