Dynamics analysis in the design of turbomachinery using sensitivity coefficients

To cite this article: O Repetckii et al 2018 J. Phys.: Conf. Ser. 944 012096

View the article online for updates and enhancements.
Dynamics analysis in the design of turbomachinery using sensitivity coefficients

O Repetckii¹, I Ryzhikov²*, Tien Quyet Nguyen²

¹ Irkutsk State Agrarian University n.a. A.A. Ezhevsky, 664038 Irkutsk, Russia
² Irkutsk National Research Technical University, 664074 Irkutsk, Russia
* E-mail: rin111@list.ru

Abstract. Solving the optimization problems based on numerical models is an important step in the design of machinery and equipment. Special attention is given to the calculation gradient of the target and limitation functions behavior with the variation of design variables (known as sensitivity). Using the sensitivity function and varying the design variables the designer gets the opportunity to analyze various basic design variants quickly and efficiently without time-consuming calculations for each of them. This allows speeding up greatly the design process and reducing its time input. In this regard the numerical models development using algorithms of sensitivity calculation for the strength and vibration analysis of structures is an urgent task.

1. Introduction
Strength or vibration characteristics (displacement, stresses, frequencies) dependences on the geometric or mass variables parameters may act as the target function for aircraft structures. The rotor blade of a gas turbine engine can be specified as an example. Due to cantilever design the blade vibrates and the characteristics of this vibration are very sensitive to mass or geometric parameters changes. Using the sensitivity coefficients in blade natural frequencies analysis provide quality improvements of resonance detuning.

Many authors describe numerical methods of analysis of turbomachines rotors dynamics and strength [1-7]. Development of analysis methods using sensitivity algorithms is described in following publications [8-12].

2. Fundamentals
The statics equation in finite element method for constant speed of rotation and temperature is:
linear formulation:
\[ K \delta = f, \quad (1) \]
nonlinear formulation:
\[ (K + K_G + K_R) \delta = f, \quad (2) \]
where \( K \) is stiffness matrix, \( K_G \) is matrix of a geometrical stiffness, \( K_R \) is supplementary stiffness matrix arising from rotation, \( \delta \) is node displacements, \( f \) is vector of loadings.

The dynamic equation in finite element method in case of the free vibration without damping is [5]:
linear formulation:
\[ M \ddot{\delta} + K \delta = 0, \quad (3) \]

nonlinear formulation:
\[ M \ddot{\delta} + M_\varepsilon \dot{\delta} + (K + K_G + K_K) \delta = 0, \quad (4) \]

where \( M_\varepsilon \) is Coriolis matrix.

The solution of the equation (3) takes the form of harmonic oscillation:
\[ \delta = y \cos(\omega t - \beta), \quad (5) \]

substituting this expression in the equation (3), we obtain the following result:
\[ (\omega^2 M + K) y \cos(\omega t - \beta) = 0 \quad \text{or} \quad (K - \lambda M) y = 0, \quad (6) \]

where \( \omega \) is angular frequency, \( \lambda \) is eigenvalue.

This equation makes sense when the determinant is equals to zero:
linear formulation:
\[ \det(K - \lambda M) = 0, \quad (7) \]

nonlinear formulation:
\[ \det(K + K_G + K_K) - \lambda M = 0. \quad (8) \]

Solving the equation (8) in the linear formulation, we can define eigenvalues:
\[ \lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_N\}^T, \quad (9) \]

or modal displacements (mode shapes):
\[ y^{(i)} = \delta^{(i)} = [\delta_{i1}, \delta_{i2}, \ldots, \delta_{iN}]^T, \quad i = 1, 2, \ldots, N, \quad (10) \]

and vibration frequencies:
\[ f = \frac{1}{2\pi} \sqrt{\lambda} = \{f_1, f_2, \ldots, f_N\}^T. \quad (11) \]

Differentiating equations (1, 2) with respect to thickness, we obtain the equation of static displacements sensitivity:
linear formulation:
\[ K \frac{d\delta}{db} = \frac{df}{db} - \frac{dK}{db} \delta, \quad (12) \]

nonlinear formulation:
\[ (K_E + K_G(\delta) + K_K) \frac{d\delta}{db} = \frac{df}{db} - (\frac{dK_E}{db} + \frac{\partial K_G}{\partial \delta} \frac{d\delta}{db} + \frac{dK_K}{db}) \delta - \hat{K}_G(\frac{\partial \delta}{\partial \delta}) \delta. \quad (13) \]

To solve the equation (4) an iterative algorithm is used.

The sensitivity of the eigenvalues for free vibration without and with taking into account the static stress-strained state is respectively:
\[ \frac{d\lambda}{db} = y^T \left[ \frac{dK}{db} - \frac{\lambda M}{db} \right] y, \quad (14) \]
Taking into account that \( \lambda = (2\pi f)^2 \), the sensitivity of the free vibration frequency is:

\[
\frac{df}{db_i} = \frac{d\lambda}{db_i} \frac{df}{d\lambda} = \frac{1}{4\pi^2f} \frac{d\lambda}{db_i}.
\]  

(16)

The sensitivity of frequency to changes of structures thickness in the nodes is:

\[
\frac{dF}{db} = \sum \sum K_f(i, j).
\]  

(17)

A large absolute value of frequencies sensitivity means a greater intensity of frequency change while changing thickness. Positive or negative sensitivity means that the frequency decreases or increases (the trend) when changing the structure thickness. Two methods can be used to predict the trend and intensity of each frequency plate oscillation change. In the first method, the thickness change ratio for each individual node with the corresponding expected frequency changes is selected. In the second method given coefficient is chosen for each separate section.

Frequency change \( F_0 \) according to the frequencies sensitivity method is calculated by the formula:

\[
\Delta F = \sum \sum \Delta b(i, j)K_f(i, j), \quad i = 1, NCN, \quad j = 1, NE,
\]

(18)

where \( \Delta b(i, j) \) is thickness change value of node \( i \) in element \( j \).

Prediction of frequencies according to the method of sensitivity:

\[
F(b - \Delta b) = F_0 + \Delta F,
\]

(19)

where \( F \) is natural frequencies of original model\((F_1, F_2, F_3, F_4)\).

Analysis of frequencies resonant detuning possibilities means the calculation of frequency change range while changing design parameters. This is the basis for eliminating any possibility of blade resonance.

Sensitivity is positive when the function reaches a maximum value in the upper range of design parameters variations and, on the contrary, sensitivity is negative when the function reaches a maximum value in the lower range of design parameters variation. A large absolute sensitivity value means a greater degree of changes in the experimental parameters (e.g., frequency) when changing the design parameters (e.g., thickness).

The use of sensitivity coefficients allows achieving a more effective result and at the same time assessing the impact of selected option of thickness changes overall calculated frequencies range. The error estimation is sufficient for engineering calculations.

3. Numerical results

The analysis of compressor blade free vibration with the use of sensitivity coefficients was carried out. The parameters of the blade: length \( L = 0.186 \) m; chord \( X_k = 0.167 \) m; Young's modulus \( E = 2.16 \times 10^5 \) MPa; Poisson's ratio \( \mu = 0.3 \); density \( \rho = 7.85 \times 10^3 \) kg/m³; blade root radius \( R_0 = 0.236 \) m. The finite element blade model is shown in Figure 1. If the speed of blades rotation, gas stream pressure and temperature are constant, then it is possible to say that the aviation engine compressor is in a stationary mode (Figure 2). In our example, the rotational speed of the rotor is equal to 100 s⁻¹.
Figure 1. Finite element model of the rotor blade.

Figure 2. Resonance diagram of the rotor blade.

Frequencies sensitivity distribution for the nodes at coordinates \((x_0, y_0, z_0, b_0)\) is shown in Figure 3. Frequencies sensitivity distribution for the blade cross sections is shown in Figure 4.

Table 1. The effect of rotation on the natural frequency.

| Mode | \(n = 0 \text{s}^{-1}\) | \(n = 25 \text{s}^{-1}\) | \(n = 50 \text{s}^{-1}\) | \(n = 755 \text{s}^{-1}\) | \(n = 100 \text{s}^{-1}\) |
|------|----------------|----------------|----------------|----------------|----------------|
|      | \(F_0, \text{Hz}\) | \(F, \text{Hz}\) | \(\Delta F, \%\) | \(F, \text{Hz}\) | \(\Delta F, \%\) | \(F, \text{Hz}\) | \(\Delta F, \%\) | \(F, \text{Hz}\) | \(\Delta F, \%\) |
| 1    | 229.36          | 232.89        | 1.54           | 243.76         | 6.28           | 260.73         | 13.68          | 282.53         | 23.18          |
| 2    | 775.73          | 777.08        | 0.17           | 783.98         | 1.06           | 794.84         | 2.46           | 808.81         | 4.26           |
| 3    | 1082.62         | 1091.34       | 0.81           | 1091.34        | 0.81           | 1104.28        | 2.00           | 1122.70        | 3.70           |
| 4    | 1705.22         | 1703.34       | -0.11          | 1708.49        | 0.19           | 1716.69        | 0.67           | 1727.45        | 1.30           |

Figure 3. Sensitivity of the blade natural frequencies.
In addition, forecast accuracy of blade natural frequencies changes at a thickening of 10% along the entire surface of the blade or in certain zones was analyzed. The error of frequencies forecast by sensitivity coefficients is small in comparison with calculations results of the modified models, and it enables using sensitivity coefficients to assess the possible projects of the blade.

4. Conclusions
The mentioned results of the calculations demonstrate a sufficiently small error, that allows of applying the developed method for the analysis of natural frequencies changes using sensitivity coefficients for optimal design of the gas turbine engines rotor blades.

Acknowledgments
The authors grateful to Dr. Zainchkovsky K S and Dr. Nguyen Dinh Duong for their valuable comments and suggestions.

References
[1] Repetckii O V 1990 Automation of strength calculations of turbomachines (Irkutsk: publishing house) p103
[2] Repetckii O V 1990 Computer analysis of the dynamics and strength of machines (Irkutsk: Irkutsk State Technical University Publishing House) p 301
[3] Irretier H Repetckii O V 1998 Vibration and Life Estimation of Rotor Structure IFToMM Conf. on Rotor Dynamics (Darmstadt)
[4] Heiman B Gerdt V Popp K Repetckii O V 2010 Mechatronics: components, methods, examples (Novosibirsk: Publishing House of the SB RAS) p 602
[5] Repetckii O V and Buy Manh Cuong 2010 To a question of the choice of a numerical method of the analysis of stresses at an assessment of a multi-cycle fatigue of blades of transport turbomachines ISEA News 6 pp 153–158
[6] Elovenko D A Repetckii O V 2011 Analysis of thermophysical properties of heat insulation materials for new constructions of cylindrical walls in high-pressure autoclaves BSU News 6 pp 201–206
[7] Elovenko D A Repetckii O V 2011 Analysis of stress state of elastic half-plane loaded with constant pressure at limited intermediate sections with specified period by finite element method based on MSC Mars software BSU News 5 pp 171–175
[8] Repetckii O V Phan Van Tuan 2012 Construction of mathematical model for analysis of friction dampers influences on vibration of gas turbine engines blades BSU News 1 pp 200–205
[9] Repetckii O V Do Manh Tung 2012 Mathematical model operation and numerical analysis of vibrations of ideal cyclic and symmetric systems finite element method ISEA News 3 pp 149–153
[10] Repetckii O V Do Manh Tung 2014 Investigation of the characteristics of mistuned turbomachinery bladed discs vibration based on the reduced-order modeling by finite element method Bulletin SibSAU 1 (53) pp 60–66
[11] Repetkii O V Ryzhikov I N Nguyen Tien Quyet 2015 An approach to turbomachinery bladed disc durability estimation *Bulletin ISTU 5 (100)* pp 22-28

[12] Repetkii O V Ryzhikov I N Nguyen Tien Quyet 2016 Dynamics of gas turbine engines rotors taking into account non-linear effects *Vibroengineering PROCEEDIA*. 8 pp 361–365