GENERAL RELATIVISTIC
SINGULARITY-FREE COSMOLOGICAL
MODEL

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Abstract

We "explain", using a Classical approach, how the Universe was created out of "nothing", i.e.,
with no input of initial energy nor mass. The inflationary phase, with exponential expansion,
is accounted for, automatically, by our equation of state for the very early Universe. This is a
Universe with no-initial infinite singularity of energy density.
1 INTRODUCTION

Berman and Trevisan (2001), suggested that with a "correct" equation of state, the Universe, as depicted with Robertson-Walker's metric, and the Einstein's field equations, appeared to have been created out-of-nothing. In the present paper, we derive an equation of state for the very early Universe, which consequence is a singularity-free exponential big-bang, such that inflation may be, in fact, the primordial phase of the Universe.

The science of cosmology (Weinberg, 1972; 2008) has progressed up to a point where it is possible to make a valid picture of the first moment of the Universe. It is widely accepted that the creation of the Universe should be attributed to Quantum fluctuations of the vacuum. As a consistent Quantum gravity theory is not yet available, cosmologists employ General Relativity, or other classical theories, in order to study the evolution of the Universe after Planck's time \( t \approx 10^{-43} s \).

The present author will try to offer now a Classical picture covering the creation of the Universe (Berman, 2007, 2007a, 2008). The only thing that we need to admit, is that Einstein's field equations yield the average values for the quantities that, in the Quantum Universe, when \( t < 10^{-43} s \), fluctuate quantum-mechanically around those average values, somehow like the Path Integral theory of Feynman (1965) admits paths that fluctuate around the average trajectories given by the Classical theory. Throughout this paper, when we refer to the alias Quantum Universe, all quantities should be understood as given by the most probable values of those quantities, even if this is not explicitly stated.

Along a rather similar situation, Kiefer, Polarski and Starobinsky (1998) have shown that although according to the inflationary scenario for the very early Universe, all inhomogeneities in the Universe are of genuine quantum origin, clearly no specific quantum mechanics properties are observed when looking at this inhomogeneities and measuring them. It looks like we can get along without a Quantum gravity theory, and still "explain" the origin of the Universe.
It was shown by Cooperstock and Israelit (1995) that the total energy of the Robertson-Walker’s Universe is zero. The total energy is the sum of the positive matter energy plus the negative energy of the gravitational field, and it is zero. Thus, in the very creation moment there was no supply of energy to the Universe, because it was not needed (see Feynman, 1962-63). We may look at the subject like follows: in the very beginning there was nothing, which we identify more or less philosophically, by assuming zero-total initial energy. From conservation of energy, we may still consider that the Universe is a zero-total energy entity.

In Section 2, we show that the Robertson-Walker’s metric is zero-total energy valued. In Section 3, we introduce a solution for the very early Universe, which is in fact an inflationary phase. In Section 4, we discuss the singularity-free properties available with the present model. Section 5 concludes the paper.

2 ZERO-TOTAL ENERGY OF THE UNIVERSE

Consider Minkowski’s metric,
\[ ds^2 = dt^2 - [dx^2 + dy^2 + dz^2] \quad (2.1) \]
This is an empty Universe, except for test particles. We agree that its total energy is zero (Weinberg, 1972).

Now consider the expanding flat metric:
\[ ds^2 = dt^2 - R^2(t) [dx^2 + dy^2 + dz^2] \quad (2.2) \]
Here, \( R(t) \) is the scale-factor. At any particular instant of time, \( t = t_0 \), we may define new variables, by the reparametrization,
\[ dx'^2 \equiv R^2(t_0) dx^2 \quad (2.3) \]
\[ dy'^2 \equiv R^2(t_0) dy^2 \quad (2.4) \]
\[ dz'^2 \equiv R^2(t_0) dz^2 \quad (2.5) \]
\[ dt'^2 \equiv dt^2 \]

Then,
\[ ds^2 = dt'^2 - [dx'^2 + dy'^2 + dz'^2] \quad (2.6) \]
The energy of this Universe is the same as Minkowski’s one, namely, \( E = 0 \). We remember that in energy calculations, the instant of time is fixed.

Consider now the metric:

\[
d s^2 = dt^2 - \frac{R^2(t)}{[1 + kr^2/4]^2} [dx^2 + dy^2 + dz^2] \tag{2.7}
\]

Here, \( k = 0 \) yields the flat case, already studied. When \( k = \pm 1 \), we have finite closed or infinite open Universes.

We want to calculate its energy. We are allowed to choose the way into making the calculation, so we choose a fixed value \( \bar{r} \) of the radial coordinate, for which we reparametrize the metric:

\[
d x^{i2} \equiv \frac{R^2(t_0)dx^{i2}}{[1 + kr^2/4]^2} \tag{2.8}
\]

\((i = 1, 2, 3)\)

For this value of \( r = \bar{r} \), the reparametrized metric has zero energy value, by the same token as above. Now we sum for all other values of \( r \), obtaining an infinite sum of zeros, which yields a total energy of zero value.

### 3 Graceful Entrance into Inflation

We suppose that the Universe obeyed Einstein’s field equations, and Robertson-Walker’s metric. As the field equations yield an expansion, the most probable value of \( R \) (the scale factor) increases, beginning with zero value.

Let us consider Robertson-Walker’s metric:

\[
d s^2 = dt^2 - \frac{R^2}{[1 + kr^2/4]} [dr^2 + r^2 d \theta^2 + r^2 \sin^2 \theta d \phi^2] , \tag{3.1}
\]

where \( k \) represents the tricurvature (0, +1, −1).

We can check that, Einstein’s field equations read, for the above metric, and a perfect fluid energy-tensor:

\[
\kappa \rho = 3H^2 + 3kR^{-2} + \Lambda , \tag{3.2}
\]

\[
\kappa p = -2R^{-1} \ddot{R} - H^2 - kR^{-2} - \Lambda , \tag{3.3}
\]
where $\rho, \kappa, \Lambda, p, and H$, stand respectively for the energy density, Einstein’s gravitational constant, cosmological constant, cosmic pressure, and Hubble’s parameter ($H = \dot{R}/R$).

On solving (3.2) and (3.3), we employ Berman and Trevisan’s solution (Berman and Trevisan, 2001),

$$R(t) = \exp(\beta t) - \exp(-\beta t),$$

(3.4)

where $\beta$ is a constant to be determined. The reason for introducing such metric will be seen later; it obeys the following good conditions:

(a) $\lim_{t \to 0} R(t) = 0$ ;
and,

(b) $\lim_{t \to \infty} R(t) = \text{exponential scale-factor (inflation)}$ .

We remember that Berman and Trevisan obtained (3.4), by imposing an equation of state of the type,

$$p = -\frac{1}{3} \rho,$$

(3.5)

but here, we leave the equation of state non-determined, for the time-being.

From the given solution, we obtain now, the expression for Hubble’s parameter,

$$H \equiv \frac{\dot{R}}{R} = \beta \left[ \frac{1 + e^{-2\beta t}}{1 - e^{-2\beta t}} \right].$$

(3.6)

If necessary, we may also remember that,

$$\ddot{R} = \beta^2 R .$$

(3.7)

We consider now the case $k = 0$, for a flat Universe. The field equations become,

$$\kappa \rho = 3\beta^2 \left[ \frac{1 + e^{-2\beta t}}{1 - e^{-2\beta t}} \right]^2 + \Lambda ,$$

(3.8)

$$\kappa p = -2\beta^2 - \beta^2 \left[ \frac{1 + e^{-2\beta t}}{1 - e^{-2\beta t}} \right]^2 - \Lambda .$$

(3.9)

We now can find the necessary equation of state: we sum (3.8) and (3.9) and obtain,
\[ p = -\rho + \frac{2\beta^2}{\kappa} \left\{ \left[ \frac{1+ e^{-2\beta t}}{1-e^{-2\beta t}} \right]^2 - 1 \right\} . \] (3.10)

We may also write,

\[ p = -\frac{1}{3}\rho - \frac{2}{\kappa} \left( \beta^2 + \frac{1}{3}\Lambda \right) . \] (3.11)

We have possibly negative cosmic pressures, but always larger than \(-\rho\) and smaller than \(-\frac{1}{3}\rho\).

For larger values of \(t\), as we are in face of rapid expansion, the negative exponential can be neglected from our solution for the scale-factor. This solution shows a graceful entrance into inflationary epoch (Guth, 1981).

We shall see bellow, that, the resulting solution that makes \(R(0) = 0\) at \(t = 0\), which does avoid the problems associated with infinite energy densities at the initial instant of the Universe, results in that Einstein’s field equations, may apply even when Quantum phenomena come to play a vital rôle.

Berman(1991; 1991a) worked the full field equations for lambda variable, for a general perfect gas equation of state, and a perfect fluid energy-tensor.

Einstein’s theory need not be substituted by Quantum Gravity theories in dealing with Planck’s Universe, because we are taking the care of the understanding that the Classical scale-factor, is representing an average value, over the uncertain quantum paths.

We have, so far, derived the equation of state that might apply for the very early Universe, and have shown that it leads towards a graceful entrance into inflationary epoch. For large \(t\), we obtain from (3.8), (3.9) and (3.10) the following limiting equation of state,

\[ p \simeq -\rho = -[3\beta^2 + \Lambda] . \] (3.12)

This is the usual inflationary result.

4 SINGULARITY-FREE UNIVERSE

Now, we can check the creation instant, where \(t \to 0\). Calling \(M\) the mass of the Universe, we have, for a tri-dimensional volume \(V\), that,
\[
\lim_{t \to 0} R(t) = 0, \quad (4.1)
\]

and,
\[
\lim_{t \to 0} M = \lim_{t \to 0} \rho V = \frac{4}{3} \pi \lim_{t \to 0} \rho R^3 = \lim_{t \to 0} \left(1 - e^{-2\beta t}\right)^2 = 0 \quad . \quad (4.2)
\]

We find that the scale factor increases from its zero value when \( t = 0 \) and \( M = 0 \) while the total energy input was also zero. (Feynman, 1962-63) This is creation out of nothing. Nevertheless, we presuppose that Einstein’s equations are valid along with the Robertson-Walker’s metric for the Universe since \( t = 0 \) onwards. In the Quantum Universe, of course, Planck’s constant \( h \) would have a specific rôle.

It must be remarked that a non-null lambda constant value, and, then, also of dark energy, in order to get a negative value for the cosmic pressure, points to an accelerating present Universe. The origin of the lambda concept should belong to the Quantum Gravity chapter of particle physics in the very early Universe but Berman (2008) has hinted that the cosmological term may have a Classical origin (as a centrifugal acceleration in a rotating evolutionary Universe – see Berman, 2008a).

We have seen that the Universe begins from a state of no-mass and no-energy. But we still need to think about the energy density in the origin of time. I suggest that we employ a time-varying cosmological ”constant” term, given by,
\[
\Lambda = \Lambda_0 - 3H^2 f(t), \quad (4.3)
\]

where \( \Lambda_0 \) is a constant, \( H \) is given by (3.6) as it should be, and the new function \( f(t) \) must obey the condition,
\[
\lim_{t \to 0} f(t) = 1 \quad . \quad (4.4)
\]

In this case, we shall have a finite energy density in the origin of time, given by \( \frac{\Lambda_0}{k} \). Now we have removed the initial singularity, while keeping intact the rest of the previous model. The analysis of varying \( \Lambda \), has been authoritatively clarified in a paper by Overduin and Cooperstock (1998).

5 CONCLUSIONS
The above original model, seems not to have been considered by any author. We obtained, from Robertson-Walker’s metric, a scale-factor that begins from zero-value at the initial time, while we showed that the initial energy and mass are equally zero. The energy density at the origin has a fixed finite value, if we introduce a convenient time-varying cosmological term. The exact form of the function $f(t)$, is left for future analyses, except for the condition (4.4).

As Raychaudhuri et al. (1992) have commented, the singularity problem is currently considered the most outstanding problem of Theoretical Cosmology. The usual Friedman expanding Universe was endowed with the $R = 0$ problem, at a finite time in the past. Richard Tolman and Arthur Eddington had thought that the singularity was a mere consequence of an oversimplification, i.e., the isotropy and homogeneity assumptions, but that in more realistic models, it would disappear. The use of Raychaudhuri’s equation (Raychaudhuri, 1955; 1979) has clarified the establishment of singularity theorems. The infinite values of physical variables at some points, may be over-thrown if we ”cut-out” from the model the relevant portion of spacetime where the variables blow-up, and the truncated region may be presented as a ”regular” spacetime. But, of course, we found a better solution in this paper.

Our present model is both appropriate and convenient.

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