A New Subaperture Imaging Algorithm for the High Resolution Multi-Receiver SAS Imagery

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Abstract. In this paper, we propose a subaperture imaging algorithm that can be used flexibly to divide the aperture. In the implementation of the algorithm, firstly full aperture data is divided into the subapertures in azimuth time domain according to the needs of motion compensation or computing ability. Then each subaperture separately is operated by range migration correction, range compression, phase compensation and so on. Then the processed subaperture data are spliced into full aperture data in accordance with the division sequence. Finally, the image of the whole scene is obtained by the azimuth matched filtering for the processed full aperture data. Compared to the traditional subaperture imaging algorithms, the algorithm in this paper has the following two characteristics: 1, The operation of the coherent and image splicing is not needed, and the calculation is reduced. 2, The full aperture can be divided into the subapertures.

1. Introduction
Synthetic aperture sonar (SAS) is an important equipment for offshore exploration and environmental surveillance[1]. SAS processing is based on a two-dimensional correlation of the backscattered signal with a space-variant reference function. The commonly used processing algorithms use a two-dimensional Fast Fourier Transform (FFT) or 1-D FFT to perform the correlation process efficiently and to obtain a well image. Since the azimuth modulation of the backscattered signal is dependent on range and follows the range migration line, an interpolation and a reference function update or phase correction must be carried out in order to achieve high quality imagery. In addition to this, the platform motion errors, such as velocity and/or squint angle variation, may require an update of the reference function in azimuth direction, which will increase the computation time considerably[2]. Basic subaperture processing consists of dividing the received SAS signal (in range or in azimuth) into subapertures. The data in each subaperture is correlated with the matched reference function in order to obtain the low-resolution images. By adding all the subapertures coherently, a high-resolution image is obtained. At present, the main subaperture imaging algorithms are OSA[3], RTS[4] and PSAP[5]. But these subaperture algorithms require a uniform partition of the aperture, which limits the flexibility they use in the motion compensation algorithm.

The new subaperture imaging algorithm proposed by this paper, is a multi-receiver imaging algorithm that is adapt to compensate the platform motion errors for multi-receiver SAS/SAR and parallel hardware structure. However, in this paper, the focus is on the imaging algorithm, not the motion compensation and not the parallel compute.
This letter is organized as follows. Section II analyses the subaperture imaging algorithm proposed in this paper. Section III verifies the validity of the proposed algorithm by computer simulation. Section IV concludes.

2. Derivation of the Subaperture Imaging Algorithm

The subaperture imaging algorithm proposed in this paper is shown in the thick-lined boxes in Fig.1. As shown in Fig.1, the algorithm includes five steps, namely multi-receiver data processing, reconstruction azimuth data, division aperture, reconstruction imagery and coherent addition.

![Fig 1. Functional block diagram of the subaperture imaging algorithm.](image)

2.1. Multi-receiver Data Processing

After demodulation to baseband, the received signal of the $i$th receiver comes from the point target $\mathbf{P}$, which is expressed as

$$
ss_i(\tau, t; r) = p \left( \tau - \frac{R_i(t; r)}{c} \right) \cdot \omega_\mathbf{P} (\tau) \cdot \exp \left\{ j\pi k \left( \tau - \frac{R_i(t; r)}{c} \right)^2 \right\} \cdot \exp \left\{ -\frac{j2\pi f_0}{c} R_i(t; r) \right\}
$$

where $\tau$ is the fast time, $t$ is the slow time, $p(\tau)$ is the pulse envelope, $\omega_\mathbf{p} (\tau)$ is the antenna weighting, $k$ is the FM rate, $c$ is the speed of sound, $f_0$ is the carrier frequency, $R_i(t; r)$ is the instantaneous slant range, $r$ is the closest range between SAS and the target. According to the reference[6], $R_i(t; r)$ is approximately formulated as follows:

$$
R_i(t; r) \approx 2\sqrt{r^2 + (v + \frac{v}{c} r + \frac{d_i}{2})^2 + \frac{1}{4r} \left( \frac{2v}{c} + d_i \right)^2}
$$

where $v$ is the platform velocity and $d_i$ is the baseline between the transmitter and the $i$th receiver. Compared with the phase of a monostatic and uniformly sampled signal, the second term in $R_i(t; r)$ should be compensated. This compensation is the multi-receiver data processing.
2.2. Reconstruction Azimuth Data

![Diagram of Displaced Phase Centre Antenna](image1)

The concept of Displaced Phase Centre Antenna (DPCA) is introduced to multi-receiver side-looking SAR [1]. Assume the size of sub-receiver $d_i = d_{i-1} - d_i$, and the platform velocity $v$ are fixed, the multi-receiver SAS can be taken to be a single receiver SAS, which has a specific PRF to fulfill the timing constraint for uniform sampling.

$$P_{RF} = \frac{2v}{I\Delta d}$$ (4)

where $I$ is the number of receivers as shown in Fig.2. For the nonuniform DPCA sampling case, the reference [2] provides a solution. After DPCA sampling, the azimuth data can be obtained by (5) and the sampling frequency is $2v/\Delta d$.

$$R(t; r) \approx 2\sqrt{r^2 + (vt + \frac{r}{c} + \frac{d_i}{2})^2}$$ (3)

$$ss(t, r; r) = p\left(\tau - \frac{R(t; r)}{c}\right) \cdot \omega(t) \cdot \exp\left(j\pi k \left(\tau - \frac{R(t; r)}{c}\right)^2\right) \cdot \exp\left(-\frac{2\pi f_d}{c} R(t; r)\right)$$ (5)

$$R(t; r) = 2\sqrt{r^2 + (vt + \frac{r}{c})^2} \approx 2r + \frac{v^2}{r} \left(t + \frac{r}{c}\right)^2$$ (6)

2.3. Division Aperture

Division aperture is performed in the azimuth time domain. As shown in Fig.3, the full aperture integration time $T$ is divided into $N$ parts, each part respectively is $T_i$, $T_{i+1}$, etc. $T_i$ can be determined by something, such as motion error, and $T_e$ is the integer times of $\Delta d/2v$. The $n$th subaperture signal can be expressed as:

$$ss_n(t, r; r) = \begin{cases} ss(t, r; r) \\ T_{n-1} \leq t \leq T_{n-1} + T_e \end{cases}$$ (7)
where \( T_{n-i} = T_i + \cdots + T_{n-i-1} \). When \( T_n \) is a constant, the full aperture is divided into \( N \) uniform subapertures. When \( T_n \) isn’t a constant, the full aperture is divided into \( N \) non-uniform subapertures. This can be combined with the platform’s motion error to perform motion compensation with the optimal number of subapertures, and in addition, it can also enable non-uniform distribution of computational resources.

2.4. Reconstruction Imagery

The subaperture data is transformed by Fourier transform in range, and the results are as follows:

\[
S_{Rn}(f_r,t,r) = W_r(f_r)\omega_n(t)\exp\left\{-j\frac{2\pi(f_0 + f_r)}{c}R(t,r)\right\}\exp\left\{-j\frac{\pi f_r^2}{k}\right\}
\]

where \( W_r(\cdot) \) is the spectrum of the transmitted pulse, \( f_r \) is the range frequency. When the time-bandwidth product is 10, 95% of the signal energy is included in the bandwidth. When the time-bandwidth product is 100, 98% of the signal energy is included in the bandwidth. Therefore, the premise of using the stationary phase principle [7] to obtain the spectrum of the chirp signal is that the time-bandwidth product is far greater than 1. The subaperture azimuth signal is most likely not satisfied when the time-bandwidth product is greater than 1. Therefore, the method of the stationary phase principle will no longer be valid. Here, the two-dimensional spectrum of subaperture is obtained according to the definition of Fourier transform:

\[
S_{Rn}(f_r,f_c,r) = W_r(f_r)W_c(f_c)A(f_c)\exp\left\{-j\frac{\pi f_r^2}{k}\right\}\exp\left\{j\frac{4\pi f_r(f_c + f_0)}{c}\right\}\exp\left\{-j\frac{\pi f_r^2}{2c(f_c + f_0)}\right\}
\]

In the case of narrow bandwidth, there is:

\[
A(f_c) = -\frac{j}{2\sqrt{2\pi f_0}} \left[ c(v_0) + c(v_2) \right]^2 + \left[ s(v_0) + s(v_2) \right]^2 \right\}^{1/2} \exp\left\{ j\arctan\frac{s(v_0) + s(v_2)}{c(v_0) + c(v_2)} \right\}
\]

where \( c(\cdot) \), \( s(\cdot) \) is the Fresnel integral. (9) is the accuracy the two-dimensional spectrum of subaperture. The difference between the two-dimensional spectrum of the subaperture and of the full aperture is that \( A(f_c) \) is changed with the azimuth frequency and the range frequency. As the time-bandwidth product of the full aperture signal is far greater than 1, the inverse Fourier transform of (9) can be applied using the stationary phase principle and the signal in the range Doppler domain is obtained by:

\[
S_{R}(r,f_c,r) = p(r)W_r(f_c)A(f_c)\exp\left\{j\frac{\pi f_c^2}{2\lambda^2}\right\}\exp\left\{j\frac{2\pi f_cr}{c}\right\}\exp\left\{-j\frac{4\pi r}{\lambda}\right\}\exp\left\{-j(k\left(r - \frac{2r'}{c} + \frac{\lambda^2f_c^2}{8c^2}\right))\right\}
\]

where \( \lambda \) is the carrier wavelength. (11) shows the subapertures and the full aperture signal have the same range migration. According to (9) and (11), the imaging algorithms can be applied into the subapertures signal, such as RD[8], CS[9], WK[10, 11]. Then the subaperture signal is transformed to the two-dimensional time domain, and the full aperture data is spliced in the order of subaperture division.
Finally, the azimuth matching filter is carried out to complete the azimuth pulse compression, and the full aperture image can be obtained.

3. Simulation Results
In order to verify the validity of the subaperture imaging algorithm in the paper, simulations are carried out in this section. The system parameters are listed in Table 1.

| Carrier frequency | Bandwidth | Pulse width | PRI | Antenna length (transmitter) | Antenna length (aperture) | Velocity | Aperture number |
|-------------------|-----------|-------------|-----|------------------------------|--------------------------|----------|----------------|
| 150kHz            | 20kHz     | 10ms        | 200ms | 0.08m                        | 0.04m                   | 2.5m/s   | 25             |

A simulation based on a flat earth model is presented. Five-point target are used in the simulation. These point targets are illuminated at the same time. The separation between adjacent point targets is 2m in azimuth direction and is 2m in range direction.

Fig. 4 shows the result of processing five simulated point. Table 2 summarizes the point target analysis. The simulation results in Fig. 4 and Table 2 show that the range resolution, the azimuth resolution, the sidelobe ratio, the integral sidelobe ratio and the target location all reach the theoretical value.

![Simulated SAS data processed](image_url)
Table 2. Image Quality Parameters.

| Subapertures | Azimuth resolution | Range resolution | Azimuth PSLR | Range PSLR |
|--------------|--------------------|------------------|--------------|-----------|
| 4            | 4.20 cm            | 3.26 cm          | -21.39dB     | -13.45dB  |
| 10           | 4.22 cm            | 3.30 cm          | -21.29dB     | -13.65dB  |

4. Conclusion

In the traditional subaperture algorithm, the azimuth matching filter is made for each subaperture. However, due to the small time-bandwidth product of the subaperture, there is a large Fresnel ripple in the azimuth spectrum, and the rectangular amplitude-frequency response is not satisfied any more. Therefore, the azimuth signal will have higher sidelobes and fake aims after matched filtering. The azimuth matching filter is performed for the full aperture signal in this paper, so it is very good to avoid this problem.

Acknowledgments

This work was financially supported by the National Natural Science Foundation of China under Grant 61671461.

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