HEAVY QUARK PRODUCTION IN $\gamma\gamma$ COLLISIONS

JIŘÍ CHÝLA

Center for Particle Physics, Institute of Physics, Academy of Science of the Czech Republic, Na Slovance 2, 18221 Prague 8, Czech Republic
E-mail: chyla@fzu.cz

Current theoretical framework for the calculation of heavy quark production in $\gamma\gamma$ collisions is reviewed. The importance of including direct photon contributions up to the order $\alpha^2\alpha_s^2$ and of the proper choice of renormalization and factorization scales in the evaluation of $\sigma(\gamma\gamma \rightarrow Q\bar{Q})$ is emphasized.

1 Introduction

Heavy quark production in $\gamma\gamma$ collisions has recently received increased theoretical attention \(^1\), motivated in part by new experimental data on $c\bar{c}$ and $b\bar{b}$ production from LEP2 (see \(^4\) for references). Although the data on $b\bar{b}$ production have sizable errors and the theoretical predictions suffer from uncertainties, the excess by a factor of about three of the data over theoretical predictions represents serious problem for perturbative QCD. Interestingly, similar excess has been observed also in $\gamma^*p$ and $p\bar{p}$ collisions.

In such a situation it appears timely to reanalyze the theoretical framework used for analyses of heavy quark production in $\gamma\gamma$ collisions \(^4\), paying particular attention to the renormalization and factorization scale dependence, as these represent the main source of the theoretical uncertainty.

2 Basic facts

The factorization scale dependence of parton distribution functions (PDF) of the photon is determined by the system of coupled inhomogeneous evolution equations for quark singlet and nonsinglet and gluon distribution functions

\[
\begin{align*}
\frac{d\Sigma(x, M)}{d\ln M^2} &= \delta_{\Sigma} k_{q} + P_{qq} \otimes \Sigma + P_{qG} \otimes G, \\
\frac{dG(x, M)}{d\ln M^2} &= k_{G} + P_{Gq} \otimes \Sigma + P_{GG} \otimes G, \\
\frac{dq_{NS}(x, M)}{d\ln M^2} &= \delta_{NS} k_{q} + P_{NS} \otimes q_{NS},
\end{align*}
\]

where, for $n_f$ massless quark flavors, $\Sigma(x, M) \equiv \sum_{i=1}^{n_f}(q_i(x, M) + \bar{q}_i(x, M))$, $\delta_{\Sigma} = 6n_f(\alpha_s^2)$ and $q_{NS}(x, M)$ and $\delta_{NS}$ can be defined in two different ways.
The splitting functions admit perturbative expansion

\[ k_q(x, M) = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_q^{(2)}(x) + \cdots \right], \]

\[ k_G(x, M) = \frac{\alpha}{2\pi} \left[ \frac{\alpha_s(M)}{2\pi} k_G^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_G^{(2)}(x) + \cdots \right], \]

\[ P_{ij}(x, M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 P_{ij}^{(1)}(x) + \cdots. \]

where \( k_q^{(0)}(x) = (x^2 + (1 - x)^2) \) and \( P_{ij}^{(0)}(x) \) are unique, whereas all higher order ones \( k_q^{(j)}, k_G^{(j)}, P_{ij}^{(j)}, j \geq 1 \) depend on the choice of the factorization scheme. The equations (1-3) can be recast into evolution equations for \( q_i(x, M), \pi_i(x, M) \) and \( G(x, M) \) with inhomogeneous splitting functions \( k_q^{(0)} = 3e^2 k_q^{(0)} \). The couplant \( \alpha_s \) depends on the renormalization scale \( \mu \) and satisfies the equation

\[ \frac{d\alpha_s(\mu)}{d\ln \mu^2} = \beta(\alpha_s(\mu)) = -\frac{\beta_0}{4\pi} \alpha_s^2(\mu) - \frac{\beta_1}{16\pi^2} \alpha_s^3(\mu) + \cdots, \]

where for \( n_f \) massless quarks \( \beta_0 = 11 - 2n_f/3 \) and \( \beta_1 = 102 - 38n_f/3 \).

General solution of the evolution equations (1-3) can be written as the sum of a particular solution of the full inhomogeneous equations and a general solution, called hadron-like (HAD), of the corresponding homogeneous ones. A subset of the former resulting from the resummation of contributions of diagrams describing multiple parton emissions off the primary QCD vertex \( \gamma \rightarrow q\bar{q} \) and vanishing at \( M = M_0 \), are called point-like (PL) solutions. Due to the arbitrariness in the choice of \( M_0 \) the separation

\[ D(x, M) = D^{PL}(x, M, M_0) + D^{HAD}(x, M, M_0). \]

is, however, ambiguous. Because of a different nature of the UV renormalization of the couplant \( \alpha_s(\mu) \) and the mass factorization involved in the definition of dressed PDF I will keep the renormalization and factorization scales \( \mu \) and \( M \) as independent free parameters.

In the “next-to–leading order” QCD approximation to \( \sigma(\gamma\gamma \rightarrow Q\bar{Q}) \) is defined by taking into account the first two terms in the expansion of direct, as well as single and double resolved photon contributions

\[ \sigma_{\text{dir}} = \sigma_{\text{dir}}^{(0)} + \sigma_{\text{dir}}^{(1)}(M, \mu)\alpha_s^2(\mu) + \sigma_{\text{dir}}^{(2)}(M, \mu)\alpha_s^3(\mu) + \cdots, \]

\[ \sigma_{\text{sr}} = \sigma_{\text{sr}}^{(1)}(M, \mu)\alpha_s(\mu) + \sigma_{\text{sr}}^{(2)}(M, \mu)\alpha_s^2(\mu) + \sigma_{\text{sr}}^{(3)}(M, \mu)\alpha_s^3(\mu) + \cdots, \]

\[ \sigma_{\text{dr}} = \sigma_{\text{dr}}^{(2)}(M, \mu)\alpha_s^2(\mu) + \sigma_{\text{dr}}^{(3)}(M, \mu)\alpha_s^3(\mu) + \cdots. \]

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Three light quarks were considered as intrinsic for $e^-$ and four in the case of $b\bar{b}$ production. Note that $\sigma^{(0)}_{\text{dir}} = \sigma_0 c^{(0)}_{Q}(s/m_Q^2)$, where $\sigma_0 \equiv 12\pi\alpha^2 c^2_{Q}/s$ and $c^{(0)}_{Q}(x)$ is a function of $s/m_Q^2$ only, comes from pure QED.

The direct, single resolved and double resolved contributions as considered in $\gamma \rightarrow q\bar{q}$ start and end in (9-11) at different powers of $\alpha_s$. In the conventional approach this is justified by claiming that PDF of the photon behave as $\alpha/\alpha_s$ and, consequently, all three expansions in fact start and end at the same powers $(\alpha_s)^0 = 1$ and $\alpha_s$, respectively. However, as argued in $\delta$, the logarithm $\ln M^2$ characterizing the large $M$ behaviour of PDF of the photon comes from integration over the transverse degree of freedom of the purely QED vertex $\gamma \rightarrow q\bar{q}$ and cannot therefore be interpreted as $1/\alpha_s(M)$. If QCD is switched off by sending, for fixed $M$ and $M_0$, $\Lambda \rightarrow 0$, the quark and gluon distribution functions of the photon approach the QED expressions

$$ q_i(x, M) \rightarrow q_i^{\text{QED}} \equiv \frac{\alpha}{2\pi} 3\pi^2 k_i^{(0)}(x) \ln \frac{M^2}{M_0^2}, \ G(x, M) \rightarrow G^{\text{QED}} = 0. \quad (12) $$

This is manifestly true for the point-like parts of quark and gluon distribution functions, whereas the vanishing of the hadron-like parts $q^{\text{HAD}}_i$ and $G^{\text{HAD}}$ as $\Lambda \rightarrow 0$ follows from the fact that vector mesons (or in general any hadronic structure) can develop from the photon only as a consequence of strong interactions between the $q\bar{q}$ pair coming from the primary, purely QED, vertex $\gamma \rightarrow q\bar{q}$. As QCD coupling vanishes, so must these hadronic parts. Let me emphasize that there is no obstacle to performing this limit, as by decreasing $\Lambda$ we get ever closer to the asymptotic freedom point $\alpha_s = 0$ and thus our perturbation expansions are progressively better behaved. In the limit of vanishing colour coupling we thus get the purely QED result $\sigma^{(0)}_{\text{dir}}$. Had the PDF of the photon really behaved as $\alpha/\alpha_s$, we would get finite contributions from the lowest order single and double resolved photon contributions even in the limit of switching QCD off, which is clearly untenable.

All calculations have been performed with fixed quark masses, i.u. define $m_Q$ in the on-shell scheme. In this convention $\sigma^{(1)}_{\text{dir}} = \sigma_0 c^{(1)}_{Q}(s/m_Q^2)$, where $c^{(1)}_{Q}(x)$ is again a function of $s/m_Q^2$ only.

### 3 Direct photon contribution

For proper treatment of $\sigma_{\text{dir}}$, the total cross section for $e^+e^-$ annihilations into hadrons provides a suitable guidance. For $n_f$ massless quarks we have

$$ \sigma_{\text{had}}(Q) = \sigma_{\text{had}}^{(0)} + \alpha_s(\mu)\sigma_{\text{had}}^{(1)}(Q) + \alpha_s^2(\mu)\sigma_{\text{had}}^{(2)}(Q/\mu) + \cdots = \sigma^{(0)}_{\text{had}}(1 + r(Q)), \quad (13) $$

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where $\sigma^{(0)}_{\text{had}}(Q) = (4\pi\alpha^2/Q^2) \sum_f e_f^2$ comes, similarly as $\sigma^{(0)}_{\text{dir}}$ in (9), from pure QED, whereas genuine QCD effects are contained in the quantity

$$r(Q) = \frac{\alpha_s(\mu)}{\pi} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} r_1(Q/\mu) + \cdots \right].$$

In the case of the quantity (13) nobody calls the lowest order term $\sigma^{(0)}_{\text{had}}(Q)$ the “LO” and the next one, i.e. $\sigma^{(0)}_{\text{had}}(Q)\alpha_s(\mu)/\pi$, the “NLO” QCD approximations, but these terms are reserved for genuine QCD effects described by $r(Q)$! To work in a well-defined renormalization scheme (RS) of $\alpha_s$ requires including in (14) at least the first two (nonzero) powers of $\alpha_s(\mu)$ because the dependence of the coefficients $r_k(\mu/Q)$ on the RS starts with $r_1$. The explicit $\mu$-dependence of $r_1(Q/\mu)$ cancels to the order $\alpha_s^2$ the implicit renormalization scale dependence of the leading order term $\alpha_s(\mu)/\pi$ in (13) and thus guarantees that the derivative of the sum of first two terms in (13) with respect to $\ln \mu$ behaves as $\alpha_s^3$. Because $\alpha_s(\mu)$ is a monotonous function of $\mu$ spanning the whole interval $(0, \infty)$, the inclusion of first two consecutive nonzero powers of $\alpha_s$ is also a prerequisite for the applicability of any of the scale fixing methods (see for detailed analysis of renormalization scale dependence of (14)). For purely perturbative quantities the association of the term “NLO QCD approximation” with a well-defined renormalization scheme is a generally accepted convention, worth retaining for physical quantities in any hard scattering process, like the direct photon contribution $\sigma_{\text{dir}}$ in (9).

Unfortunately, in [1, 2, 3] the QED contribution $\sigma^{(0)}_{\text{dir}}$ is considered as the LO and the sum of the first two terms in (9), which for fixed $m_Q$ has exactly the same form as the first two term in (13), as the NLO approximation. Consequently, this approximation cannot be associated to a well-defined renormalization scheme of $\alpha_s$ and therefore does not deserve the label “NLO” even if the NLO expression for $\alpha_s(\mu)$ is used therein. For QCD analysis of $\sigma_{\text{dir}}$ in a

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well-defined renormalization scheme of $\alpha_s(\mu)$ the incorporation of the third term in (9), proportional to $\alpha^2\alpha_s^2$, is indispensable.

At the order $\alpha^2\alpha_s^2$ diagrams with light quarks appear as well and we can thus distinguish three classes of direct photon contributions differing by the overall heavy quark charge factor $CF$:

**Class A:** $CF = e_1^Q$. Comes from diagrams, like those in Fig. 1a-g, in which both photons couple to heavy $Q\overline{Q}$ pairs. Despite the presence of mass singularities in contributions of individual diagrams coming from gluons and light quarks in the final state and from loops, at each order of $\alpha_s$ the sum of all contributions of this class to $\sigma_{\text{dir}}$ is finite.

**Class B:** $CF = e_2^Q$. Comes from diagrams, like that in Fig. 1h, in which one of the photons couples to a heavy $Q\overline{Q}$ and the other to a light $q\overline{q}$ pair. For massless light quarks this diagram has initial state mass singularity, which is removed by introducing the concept of the (light) quark and gluon distribution functions of the photon.

**Class C:** $CF = 1$. Comes from diagrams in which both photons couple to light $q\overline{q}$ pairs, as those in Fig. 1i. In this case the analogous subtraction procedure relates it to the single resolved contribution of the diagram in Fig. 2f and double resolved contribution of the diagram in Fig. 2h.

As the diagrams in Fig. 1c and 1l give the same final state $q\overline{q}Q\overline{Q}$, we have to consider their interference term as well, but it turns out that it does not contribute to the total cross section $\sigma(\gamma\gamma \rightarrow Q\overline{Q})$. Because of different charge factors, the classes A, B and C do not mix under renormalization of $\alpha_s$ and factorization of mass singularities. At the order $\alpha^2\alpha_s^2$ all three classes of contributions are needed for theoretical consistency. As argued above, class A is needed if the calculation of $\sigma_{\text{dir}}$ is to be performed in a well-defined RS.

### 4 Resolved photon contribution

The classes B and C of direct photon contributions are needed to render the sum of direct and resolved photon contributions factorization scale invariant. To see this, let us write the sum of first two terms in (10-11) explicitly in terms of PDF and parton level cross sections

$$
\sigma_{\text{res}}^{(12)}(M, \mu) \equiv 2\alpha_s(\mu) \int dx G(x, M) \left[ \sigma_{\gamma G}^{(1)}(x) + \alpha_s(\mu)\sigma_{\gamma G}^{(2)}(x, M, \mu) \right] + 4\alpha_s^2(\mu) \int dx \sum_i g_i(x, M) \sigma_{\gamma q_i}^{(2)}(x, M) + 
$$

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Figure 2. Examples of diagrams describing resolved and related direct photon contributions.

\[
2\alpha_s^3(\mu)\int dx dy \Sigma(x, M)G(y, M)\sigma_{GG}^{(3)}(xy, M) + \int dx dy \sum q_i(x, M)\Sigma_i(y, M)\left[\sigma_{qq}^{(2)}(xy) + \alpha_s(\mu)\sigma_{qg}^{(3)}(xy, M, \mu)\right] + \int dx y G(x, M)G(y, M)\left[\sigma_{GG}^{(2)}(xy) + \alpha_s(\mu)\sigma_{GG}^{(3)}(xy, M, \mu)\right],
\]

where \(\sum q_i\) runs over \(n_f\) quark flavors and the factors of two and four reflect the identity of beam particles and equality of contributions from quarks and antiquarks. Recalling the general form of the derivative \(d\sigma_{\text{res}}/d\ln M^2\)

\[
\frac{d\sigma_{\text{res}}}{d\ln M^2} = \int dx W_0(x, M) + \int dx \left[\sum q_i(x, M)W_{q_i}(x, M) + G(x, M)W_G(x, M)\right] + \int dx dy \left[G(x, M)G(y, M)W_{GG}(xy, M) + \sum q_i(x, M)\Sigma_i(y, M)W_{qG}(xy, M)\right.
\]

\[
\left. + \Sigma(x, M)G(y, M)W_{qG}(xy, M)\right],
\]

using (15) and denoting \(\alpha_s \equiv \alpha_s(\mu)\), \(f \equiv df/d\ln M^2\) we find

\[
W_0 = \frac{\alpha_s^2}{\pi}\left\{\frac{1}{2\pi}k_{G}^{(1)}(x)\sigma_{\gamma G}^{(1)}(x) + 6k_{q}^{(0)}(x)\sum e_i^2\sigma_{qq}^{(2)}(x, M)\right\} + \cdots
\]

\[
W_{q_i} = \frac{\alpha_s^2}{\pi}\left\{4\pi\sigma_{qG}^{(2)}(x) + \int dz \left[P_{Gq}^{(0)}(z)\sigma_{gG}^{(1)}(xz) + 3e_i^2\alpha k_{q}^{(0)}(z)\sigma_{qq}^{(2)}(xz)\right]\right\} + \cdots
\]

\[
W_G = \frac{\alpha_s^2}{\pi}\left\{2\pi\sigma_{\gamma G}^{(2)}(x) + \int dz P_{GG}^{(0)}(z)\sigma_{gG}^{(1)}(xz)\right\} + \epsilon^2\alpha k_{q}^{(0)}(z)\sigma_{qG}^{(2)}(xz) + \cdots
\]

\[
W_{GG} = \frac{\alpha_s^3}{\pi}\left\{\pi\sigma_{GG}^{(3)}(x) + \int dz P_{GG}^{(0)}(z)\sigma_{GG}^{(2)}(xz)\right\} + \cdots
\]

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\[ W_{qq} = \frac{\alpha_s^3}{\pi} \left\{ 2\pi \hat{\sigma}_{qq}^{(3)}(x) + 2\int dzP_{qq}^{(0)}(z)\sigma_{qq}^{(2)}(xz) \right\} + \cdots \]  

\[ W_{qG} = \frac{\alpha_s^3}{\pi} \left\{ 2\pi \hat{\sigma}_{qG}^{(3)}(x) + \int dz \left[ P_{qG}^{(0)}(z)\sigma_{qG}^{(2)}(xz) + P_{qG}^{(0)}(z)\sigma_{GG}^{(2)}(xz) \right] \right\} + \cdots \]  

All integrals in (18-22) go formally from 0 to 1, but threshold behaviour of cross sections \( \sigma_{ij}(xz) \) restricts the region to \( xz \geq 4m_q^2/S \). The factorization scale invariance of (15) requires that the variation of (15) with respect to \( \ln M^2 \) is of higher order in \( \alpha_s \) than the approximation (15) itself. There is no question that direct photon contributions of classes B and C are needed to make the sum direct and resolved photon contributions factorization scale independent. The difference between the conventional and my approaches to QCD analysis of \( \sigma(\gamma\gamma \to Q\overline{Q}) \) concerns the question when this happens, i.e. what is the order of the approximation (15) and, consequently, which terms on the r.h.s. of (16) must vanish.

In the conventional approach both \( q(M) \) and \( G(M) \) are claimed to be of order \( \alpha/\alpha_s \), and the approximation (15) thus of the order \( \alpha^2 \alpha_s \). This implies that only terms up to this order must vanish in (16). As the expressions in round brackets of (18) and (19) do, indeed, vanish, the expression (15) is claimed to be complete NLO approximation to \( \sigma_{\text{res}} \). The fact that the expression on the r.h.s. of (17) does not vanish is of no concern in this approach as it is manifestly of the order \( \alpha^2 \alpha_s \) and thus supposedly of higher order than (15) itself. This line of argumentation is, however, untenable on physical grounds as it requires that not only the quark but even the gluon distribution function \( G(M) \) should behave as \( \alpha/\alpha_s \), despite the fact that gluons appear in the photon only due to their radiation off the quarks and their contribution to physical observables must therefore vanish if this radiation is switched off!

If we discard the untenable claim that \( q, G \propto \alpha/\alpha_s \) and realize the obvious, namely that \( q(M) \propto \alpha \), we are lead to the conclusion that \( W_0 \) is of the same order \( \alpha^2 \alpha_s^2 \) as \( q_i W_{qi} \) and other integrands in (16) and must also vanish for theoretical consistency of the approximation (15). This, in turn, necessitates the inclusion of class B direct photon contributions of the order \( \alpha^2 \alpha_s^2 \), like those in Fig. 2b,g, which provide the \( M \)-dependent terms the derivative of which cancel \( W_0 \) in the expression for \( d(\sigma_{\text{dir}}(M) + \sigma_{\text{res}}(M))/d\ln M^2 \). Note that \( W_{qi} \) in (18) receives contributions from the derivatives of both single and double resolved photon diagrams, proportional to \( \sigma_{\gamma G} \) and \( \sigma_{\gamma q} \), respectively. This fact reflects the mixing of single and double resolved photon contributions, which starts just at the order \( \alpha^2 \alpha_s^2 \). Note also, that for theoretical consistency of the sum \( \sigma_{\text{dir}} + \sigma_{\text{res}} \) up to this order, only the lowest order double resolved photon contribution needs to be included.

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5 Phenomenological implications

Although the current calculations of $\sigma(\gamma\gamma \rightarrow Q\overline{Q})$ do not represent complete NLO QCD approximation, one may ask how important are the missing terms, discussed above, numerically. To help answer this question it would be useful if the data could be expressed in terms of energy dependence of $\sigma(\gamma\gamma \rightarrow Q\overline{Q})$. The point is that for $b\overline{b}$ production direct photon contribution dominates below about $\sqrt{s} \simeq 25$ GeV, whereas for $200 \gtrsim \sqrt{s} \gtrsim 50$ GeV single resolved photon contribution makes up most of $\sigma(\gamma\gamma \rightarrow b\overline{b})$. If the disagreement comes from the former region, we would be in real trouble. As, however, the LEP data integrate over the whole WW spectrum, to which the direct and single resolved photon channels contribute roughly equally, there is no way how to determine wherefrom comes the disagreement with the data.

As argued above, the approximation used in (1), (2), (3) for the evaluation of the direct contribution is merely of the LO nature. On the other hand, as the lowest order QCD correction $\sigma^{(1)}_{\text{dir}}\alpha_s$ makes up a small fraction of the purely QED contribution $\sigma^{(0)}_{\text{dir}}$, the eventual incorporation of the (so far unknown) direct photon term $\sigma^{(2)}_{\text{dir}}\alpha_s^2$, coming from class A direct photon diagrams, will unlikely enhance $\sigma_{\text{dir}}$ by more than further few tens of percent.

The situation is different for the resolved photon contributions. In (5) I will discuss numerical aspects of the factorization and renormalization scale dependence of the approximation (15) and show that proper choice of these scales is essential for understanding the data on $b\overline{b}$ production and may explain part of the observed discrepancy between the data and theoretical calculations.

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