INDIRECT SEARCH FOR SUPERSYMMETRY IN RARE B DECAYS

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**ABSTRACT**

QCD corrections to the gluino induced contribution to $b \to s\gamma$ are shown to be important in order to extract reliable bounds on the off-diagonal elements of the squark mass matrices.

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Indirect Search for Supersymmetry in Rare B Decays

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Abstract

QCD corrections to the gluino induced contribution to $b \to s \gamma$ are shown to be important in order to extract reliable bounds on the off-diagonal elements of the squark mass matrices.

Rare processes are an important tool for investigating new interactions. The standard model contributions are usually small and new physics may manifest itself clearly. In particular, rare decays provide guidelines for supersymmetry model building. The experimental observation of these Flavour Changing Neutral Currents (FCNCs), or the upper limits set on them yield stringent relations between the many parameters in the soft supersymmetry-breaking terms. The processes involving transitions between first and second generation quarks, namely FCNC processes in the $K$ system, are considered to be most efficient in shaping viable supersymmetric flavour models. Moreover, the tight experimental bounds on some flavour diagonal transitions such as the electric dipole moment of the electron and of the neutron, as well as $g - 2$, help constrain soft terms inducing chirality violations.

The severe experimental constraints on flavour violations have no direct explanation in the structure of the minimal supersymmetric standard model (MSSM). This is the essence of the well-known supersymmetric flavour problem. There exist several supersymmetric models (within the MSSM) with specific solutions to this problem. Most popular are the ones in which the dynamics of flavour sets in above the supersymmetry breaking scale and the flavour problem is killed by the mechanisms of communicating supersymmetry breaking to the experimentally accessible sector: In the constrained minimal supersymmetric standard model (mSUGRA) supergravity is the mediator between the supersymmetry breaking and the visible sector \cite{1}. In gauge-mediated supersymmetry breaking models (GMSBs) the communication between the two sectors is realized by gauge interactions \cite{2}. More recently the anomaly mediated supersymmetry breaking models (AMSBs) were proposed, in which the two sectors are linked by interactions suppressed by the Planck mass \cite{3}. Furthermore, there are other classes of models in which the flavour problem is solved by particular flavour symmetries.

Neutral flavour transitions involving third generation quarks, typically in the $B$ system, do not pose yet serious threats to these models. The rare decay $b \to s \gamma$ has already been detected, but the precision of the measurements is at the moment not very high \cite{4}. Nevertheless, it has already carved out some regions in the space of free parameters of most of the models in the above classes (see \cite{5} and references therein). In particular, it dangerously constrains several somewhat tuned realizations of these models \cite{6}. Once the experimental precision is increased, this decay will undoubtedly help selecting the viable regions of the parameter space in the above class of models and/or discriminate among these or other possible models. It is important to calculate the rate of this decay with theoretical uncertainties reduced as much as possible, and general enough for generic supersymmetric models. In the standard model, the rate for $b \to s \gamma$ is known up to next-to-leading order (NLL) in QCD \cite{7}. The NLL calculation reduces the large scale dependence present at the LL ($\pm 25\%$) to a mere percent uncertainty. This accuracy is however somewhat fortuitous as it is obtained through large accidental cancellations among different contributions to the NLL corrections \cite{8,9}. Indeed, the accuracy for the NLL calculation of the $b \to s \gamma$ rate in a two-Higgs model is substantially worse \cite{10}.

The calculation of this decay rate within supersymmetric models is still far from this level of sophistication. There are several contributions to the decay amplitude: Besides the $W^- - t$-quark and the $H^- - t$-quark contributions, there are also the chargino, gluino and neutralino contributions. All these contributions were calculated in \cite{10} within the mSUGRA model; their analytic expressions apply naturally also to the GMSB,
and AMSB models. The inclusion of QCD corrections needed for the calculation of the rate, was in [10] assumed to follow the SM pattern. A calculation taking into account solely the gluino contribution has been performed in [12] for a generic supersymmetric model, but no QCD corrections were included.

An interesting NLL analysis of $b \to s\gamma$ was recently performed in [13] in a specific class of models where the only source of flavour violation at the electroweak scale is that of the SM, encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This calculation, however, cannot be used in particular directions of the parameter space of the above listed models in which quantum effects induce a gluino contribution as large as the chargino or the SM contributions. Nor it can be used as a model-discriminator tool, able to constrain the potentially large sources of flavour violation typical of generic supersymmetric models.

Among these, the flavour non-diagonal vertex gluino-quark-squark induced by the flavour violating scalar mass term and trilinear terms is particularly interesting. This is generically assumed to induce the dominant contribution to quark flavour transitions, as this vertex is weighted by the strong coupling constant $g_s$. Therefore, it is often taken as the only contribution to these transitions and in particular to the $b \to s\gamma$ decay, when attempting to obtain order-of-magnitude upper bounds on flavour violating terms in the scalar potential. Once the constraints coming from the experimental measurements are imposed, however, the gluino contribution is reduced to values such that the SM and the other supersymmetric contributions cannot be neglected anymore. Any LL and NLL calculation of the $b \to s\gamma$ rate in generic supersymmetric models, therefore, should then include all possible contributions.

The gluino contribution, however, presents some peculiar features related to the implementation of the QCD corrections. In ref. [14] this contribution to the decay $b \to s\gamma$ is therefore investigated in great detail for supersymmetric models with generic soft terms. It is shown that the relevant operator basis of the SM effective Hamiltonian gets enlarged to contain magnetic and chromomagnetic operators with an extra factor of $\alpha_s$ and weighted by a quark mass $m_{su}$ or $m_{su}$ and also magnetic and chromomagnetic operators of lower dimensionality, as well as additional scalar and tensorial four-quark operators. A few results of our analysis in ref. [15] are given in the following, showing the effect of the LL QCD corrections on constraints on supersymmetric sources of flavour violation.

To understand the sources of flavour violation which may be present in supersymmetric models in addition to those enclosed in the CKM matrix, one has to consider the contributions to the squark mass matrices

$$\mathcal{M}_f^2 = \begin{pmatrix} m_{f,LL}^2 & m_{f,LR}^2 & m_{f,RR}^2 \\ m_{f,RL}^2 & F_{f,LL} + D_{f,LL} & F_{f,LR} + D_{f,LR} \\ m_{f,RR}^2 & F_{f,LR} + D_{f,LR} & F_{f,RR} + D_{f,RR} \end{pmatrix},$$

where $f$ stands for up- or down-type squarks. In the super CKM basis where the quark mass matrices are diagonal and the squarks are rotated in parallel to their superpartners, the $F$ terms from the superpotential and the $D$ terms turn out to be diagonal $3 \times 3$ submatrices of the $6 \times 6$ mass matrices $\mathcal{M}_q^2$. This is in general not true for the additional terms $D$, originating from the soft supersymmetric breaking potential. As a consequence, gluino contributions to the decay $b \to s\gamma$ are induced by the off-diagonal elements of the soft terms $m_{f,LL}^2$, $m_{f,RR}^2$, $m_{f,RL}^2$.

It is convenient to select one possible source of flavour violation in the squark sector at a time and assume that all the remaining ones are vanishing. Following ref. [11], all diagonal entries in $m_{d,LL}^2$, $m_{d,RR}^2$ and $m_{u,RR}^2$ are set to be equal and their common value is denoted by $m_q^2$. The branching ratio can then be studied as a function of

$$\delta_{LL,ij} = \frac{(m_{d,LL}^2)_{ij}}{m_q^2}, \quad \delta_{RR,ij} = \frac{(m_{d,RR}^2)_{ij}}{m_q^2},$$

$$\delta_{LR,ij} = \frac{(m_{d,LR}^2)_{ij}}{m_q^2}, \quad (i \neq j).$$

The remaining crucial parameter needed to determine the branching ratio is $x = m_{\tilde{g}}^2/m_q^2$, where $m_{\tilde{g}}$ is the gluino mass. In the following, we concentrate on the LL QCD corrections to the gluino contribution.

In Figs. 1 and 2 the solid lines show the QCD corrected branching ratio, when only $\delta_{LR,23}$ or $\delta_{LL,23}$ are non vanishing. The branching ratio is

![Figure 1](https://via.placeholder.com/150)

**Figure 1.** Gluino-induced branching ratio $BR(B \to X_s \gamma)$ as a function of $x = m_{\tilde{g}}^2/m_q^2$, obtained when only the source of flavour violation is $\delta_{LR,23}$ (see text).
plotted as a function of $x$, using $m_\tilde{q} = 500$ GeV. The dotted lines show the range of variation of the branching ratio, when the renormalization scale $\mu$ varies in the interval 2.4–9.6 GeV. Numerically, the scale uncertainty in $BR(B \to X_s \gamma)$ is about $\pm 25\%$. An extraction of bounds on the $\delta$ quantities more precise than just an order of magnitude, therefore, would require the inclusion of next-to-leading logarithmic QCD corrections. It should be noticed, however, that the inclusion of the LL QCD corrections has already removed the large ambiguity on the value to be assigned to the factor $\alpha_s(\mu)$ in the gluino-induced operators. Before adding QCD corrections, the scale in this factor can assume all values from $O(m_\tilde{q})$ to $O(m_W)$: the difference between $BR(B \to X_s \gamma)$ obtained when $\alpha_s(m_\tilde{q})$ or when $\alpha_s(m_W)$ is used, is of the same order as the LL QCD corrections. The corresponding values for $BR(B \to X_s \gamma)$ for the two extreme choices of $\mu$ are indicated in Figs. 1 and 2 by the dot-dashed lines ($\mu = m_W$) and the dashed lines ($\mu = m_\tilde{q}$). The choice $\mu = m_W$ gives values for the non-QCD corrected $BR(B \to X_s \gamma)$ relatively close to the band obtained when the LL QCD corrections are included, if only $\delta_{LL,23}$ is non-vanishing. Finding a corresponding value of $\mu$ that minimizes the QCD corrections in the case studied in Fig. 1, when only $\delta_{LR,23}$ is different from zero, depends strongly on the value of $x$. In the context of the full LL result, it is important to stress that the explicit $\alpha_s$ factor has to be evaluated - like the Wilson coefficients - at a scale $\mu = O(m_\tilde{q})$.

In spite of the large uncertainties which the branching ratio $BR(B \to X_s \gamma)$ still has at LL in QCD, it is possible to extract indications on the size that the $\delta$-quantities may maximally acquire without inducing conflicts with the experimental measurements (see [15]).

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