Particle Spectrum of the Supersymmetric Standard Model from the Massless Excitations of a Four Dimensional Superstring

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A superstring action is quantised with Neveu Schwarz(NS) and Ramond(R) boundary conditions. The zero mass states of the NS sector are classified as the vector gluons, W-mesons, $B\mu$-mesons and scalars containing Higgs. The fifteen zero mass fermions are obtained from the Ramond sector. A space time supersymmetric Hamiltonian of the Standard Model is presented without any conventional SUSY particles.

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There is an exciting possibility that the observed spectrum of elementary particles of the standard model is obtainable from a suitable $N=1, D=4$ superstring massless modes of excitations. If the supersymmetric group is $SO(6) \otimes SO(5)$, it can descend to $Z_3 \otimes SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ as a very special case [1]. Because the group $Z_3$ is enough to consider one of the three generations. $Z_3$ groups takes care of the other two generations. The number of fermionic modes as observed today in one generation is thirty due to the fifteen fermions. Similarly, including Higgs, the number of bosonic modes is twenty eight. If we include two noninteracting scalar modes, the bosonic degrees of freedom will match with those of the fermions. Therefore there could be a distinct supersymmetry scenario. With this aim in mind, we examine the case of a recently proposed new type of superstring in four dimensions [1] where this is realised.

A world sheet $(\sigma, \tau)$ supersymmetric action is written as [1]

$$S = -\frac{1}{2\pi} \int d^2 \sigma \left[ \partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}_{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} + i \bar{\phi}_{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right].$$

(1)

$X_\mu$ are the bosonic coordinates. $\psi_{\mu,j}, j=1,..,6$ and $\phi_{\mu,k}, k=7,..,11$ are the Lorentz Majorana spinors in the bosonic representation of $SO(3,1)$, with

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(2)

and

$$\rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$  

(3)

The upper index in the action refers to a row and lower index to a column. By counting the contributions of the fermions and bosons, the central charge of the action in equation (1) is 26 like Nambu-Goto string but unlike a normal superstring. This is, because, the action also contains the light cone ghosts

$$S_{l.c.} = \frac{i}{2\pi} \int d^2 \sigma \sum_{\mu=0,3} \left( \bar{\psi}_{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} - \bar{\phi}_{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right),$$

(4)

whose central charge adds up to eleven, like $\beta - \gamma$ superconformal ghost action. The action in equation (1) is invariant under the supersymmetric transformations,

$$\delta X^\mu = \bar{\epsilon} \left( e^j \psi^\mu_j - e^k \phi^\mu_k \right),$$

(5)

$$\delta \psi_{\mu,j} = -i \epsilon e^\rho \rho^\alpha \partial_\alpha X^\mu \epsilon, \text{ and}$$

(6)

$$\delta \phi_{\mu,k} = i e^k \rho^\alpha \partial_\alpha X^\mu \epsilon,$$

(7)

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with $\epsilon$ as the constant anticommuting spinor. $e^j$ and $e^k$ are the two unit component c-number row vectors with the property that $e^j e_j = \delta^j_0$ and $e^k e_k = \delta^k_0$ and it follows that $e^j e_j = 6$ and $e^k e_k = 5$. The commutator of two such transformations is a translation with coefficient $a^\alpha = 2i\epsilon_1 \rho^\alpha e_2$ provided $\psi^{\mu,j} = e^j \psi^\mu$ and $\phi^{\mu,k} = e^k \phi^\mu$ where

$$\Psi^\mu = e^j \psi^\mu_j - e^k \phi^\mu_k.$$  

(8)

and $\Psi^\mu$ is the superpartner of $X^\mu$ as can be checked. Writing out the 2-d local supersymmetric action of Brink, Di Vecchia, Howe, Deser and Zumino [2], we can write down the Noether current in the gravitinoless gauge,

$$J_\alpha = \frac{\pi}{2e} \frac{\delta S}{\delta X^\alpha} = \rho^\beta \rho_\alpha \bar{\psi} \bar{\psi} \partial X^\alpha = 0$$  

(9)

and in the simpler light cone co-ordinates

$$J_\pm = \partial_\pm X^\mu \Psi^\mu_\pm = 0,$$  

(10)

and the energy momentum tensor

$$T_\pm = \partial_\pm X^\mu \partial_\pm X^\mu + \frac{i}{2} \psi^{\mu,j}_\pm \psi_{\pm,\mu,j} - \frac{i}{2} \phi^{\mu,k}_\pm \phi_{\pm,\mu,k}.$$  

(11)

From action in equation (1), commutation and anticommutation of the fields can be found and the string is quantised using Neveu-Schwarz and Ramond [1] boundary conditions. The co-ordinate quanta, $\alpha^\mu_m$, satisfy

$$[\alpha^\mu_m, \alpha^\nu_n] = m \delta_{m,-n} \eta^\mu\nu.$$  

(12)

In the NS sector, the quanta of $\psi$ and $\phi$ are respectively $b^\mu_m$ and $b^{\mu'}_n$, with $r$ half integral and obey

$$\{b^\mu_m, b^{\nu'}_n\} = \eta^\mu\nu \delta_{\mu',\nu} \delta_{m,-n} \quad \text{and} \quad \{b^{\mu,k}_m, b^{\nu,k'}_n\} = -\eta^\mu\nu \delta^{k,k'} \delta_{m,-n}.$$  

(13)

Similarly the quanta $d$ and $d'$ of R sector satisfy (m, n integers), the nonvanishing anticommutations

$$\{d^\mu_m, d^{\nu'}_n\} = \eta^\mu\nu \delta_{\mu',\nu} \delta_{m,-n} \quad \text{and} \quad \{d^{\mu,k}_m, d^{\nu,k'}_n\} = -\eta^\mu\nu \delta^{k,k'} \delta_{m,-n}.$$  

(14)

The unusual negative signs are taken care of, by the phase factors,

$$b^{\mu}_{-r,j} = -b^{\mu'}_{r,j} \quad \text{and} \quad d^{\mu}_{-m,j} = -d^{\mu'}_{m,j}.$$  

(15)

It will be useful to note $b^{\mu}_{r,j} = e^j C^\mu_r$, $b^{\mu,k}_m = e^k C^\mu_r$ and $C^\mu_r = e^j b^{\mu}_{r,j} - e^k b^{\mu'}_{r,k}$. Similarly for the $d$, $d'$; $D^\mu_m = e^j d^{\mu}_{m,j} - e^k d^{\mu'}_{m,k}$.

Virasoro generators [8] (dot implies summation of all qualifying indices and : : for normal ordering)

$$NS \quad L_m = \frac{1}{2} \sum_{\pm = -\infty} \pm \alpha_{-n} \cdot \alpha_{m+n} : + \frac{1}{2} \sum_{r \in \mathbb{Z} + \frac{1}{2}} (r + \frac{1}{2} m) : (b_{-r} \cdot b_{m+r} - b'_{-r} \cdot b'_{m+r}) : \quad (16)$$

$$G_r = \frac{\sqrt{3}}{\pi} \int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ = \sum_{n = -\infty}^{\infty} \alpha_{-n} \cdot C_{n+r},$$  

$$R \quad L_m = \frac{1}{2} \sum_{\pm = -\infty} \pm \alpha_{-n} \cdot \alpha_{m+n} \cdot + \frac{1}{2} \sum_{n = -\infty}^{\infty} (n + \frac{1}{2} m) : (d_{-n} \cdot d_{m+n} - d'_{-n} \cdot d'_{m+n}) : ,$$  

(17)

$$F_r = \sum_{n = -\infty}^{\infty} \alpha_{-n} \cdot D_{n+r}.$$  

(18)

By direct calculation, it is easily checked that the generators satisfy super Virasoro algebra with anomaly term typical of central charge 26. To cancel these anomalies, we can use the known way of introducing the conformal ghosts $b$ and $c$.

The BRST charge operator should also include 11 pairs of commuting $(\beta^{(j,k)}, \gamma^{(j,k)})$ superconformal ghost system. Defining

$$\beta^{(j,k)}_{r(m)} = e^{(j,k)} \beta_{r(m)} \quad \text{and} \quad \gamma^{(j,k)}_{r(m)} = e^{(j,k)} \gamma_{r(m)},$$
such that \( \gamma_{r(m)} = e^j \gamma_{r(m),j} - e^k \gamma_{r(m),k} \) and \( \beta_{r(m)} = e^j \beta_{r(m),j} - e^k \beta_{r(m),k} \) with \( [\beta_{r(m)}, \gamma_{s(n)}] = -\delta_{r+s,m+n} \). Since the conformal dimension of \( \beta \) is \( \frac{3}{2} \) and of \( \gamma \) is \( -\frac{1}{2} \),

\[
[L_m, \gamma_{s(n)}] = \left( -\frac{3}{2} m - s(n) \right) \gamma_{m+s(n)} \quad \text{and} \quad [L_m, \beta_{r(n)}] = \left( \frac{1}{2} m - r(n) \right) \beta_{m+r(n)}.
\]

In NS sector, the BRST charge is

\[
Q = \sum L_m c_m - \frac{1}{2} \sum (m - n) : c_m c_{-n} b_{m+n} : - c_0 a + Q',
\]

and \( Q^2 = 0 \) if \( a=1 \) is proved directly. The nilpotency proves the unitarity. Details can be found in reference [1]. As can be shortly shown, the physical state conditions remove all the ghosts from the Fock space,

\[
(L_0 - 1)\phi > = 0,
\]

\[
(L_0 - 1)\phi > R = (F_0^2 - 1)*p > R = (F_0 + 1)*p > R = (F_0 - 1)*p > R = 0
\]

\[
L_m|p > = 0, \quad \text{for} \quad m > 0,
\]

\[
G_r|p > = 0 \quad \text{for} \quad r > 0,
\]

and

\[
F_m|p > R = 0 \quad \text{for} \quad m > 0.
\]

We shall assume the Gupta- Bleuler condition \( \alpha_{0,1}^0 |\phi > = 0 \). Since \( [\alpha_{0,1}^0, L_{m+j}]|\phi > = 0 \), we get \( \alpha_{0,m}^0 |\phi > = 0 \). Further \( [\alpha_{0,1}^0, G_{r+1}]|\phi > = 0 \) leads to \( C_{ij}^0 |\phi > = 0 \) and \( b^0_{r,j} |\phi > = e_j C_{ij}^0 |\phi > = 0 \). Similar analysis yields \( d^0_{r,k} |\phi > = 0 \) and \( d^0_{m,k} |\phi > = 0 \). There is no Lorentz metric longitudinal ghost in the Fock space.

The half integral values of mass spectrum (NS), \( \alpha M^2 = -1, 0, \frac{1}{2}, 1, \frac{3}{2} \) are projected by G.S.O method. To show that the tachyons should not contribute to the Hamiltonian, we construct the world sheet supersymmetric charge

\[
Q = \frac{i}{\pi} \int_0^\pi d\sigma \Psi^\mu \rho^a \partial_0 X_\mu
\]

\[
Q^\dagger = -\frac{i}{\pi} \int_0^\pi d\sigma \Psi^\mu \rho^\dagger \rho_0 \partial_0 X_\mu
\]

We that

\[
\sum_\alpha \{ Q^\dagger, Q_\alpha \} = 2H
\]

Thus

\[
\sum_\alpha |Q_\alpha|\phi_0 >= |^2 = 2 < \phi_0 |H|\phi_0 > \geq 0
\]

Obtaining a ghost free, anomaly free and unitary string, we calculate the zero mode string excitations leading to the Standard Model. This is easily understood as \( < 0 |(L_0 - 1)^{-1} |0 > \), the self energy is cancelled by \( - < 0 |(F_0 - 1)^{-1} (F_0 + 1)^{-1} |0 > \) of the fermion sector. The negative sign is due to the fermion loop. The \( B_\mu \), the hypercharge vector, is simply given from the state

\[
|\phi(p) >= \alpha_{0,1}^0 |0, p > B_\mu.
\]

\( L_0 \) condition [21] tells that \( B_\mu(p) \) is massless and \( L_1 \) condition gives the Lorentz condition \( p^\mu B_\mu = 0 \).

In NS sector, the objects like \( b^\mu_{-1,1}, b^\mu_{-1,2} \) are massless due to [21]. Dropping the suffix \(-\frac{1}{2}\), we construct the traceless tensor, \( (i,j) = 1,2,3 \), expressible in terms of vector fields \( A_{ij}^\mu \),

\[
G_{ij}^{\mu \nu} = \left( b^\mu_i b^\nu_j - b^\nu_i b^\mu_j - \frac{2}{3} \delta_{ij} b^\mu_k b^\nu_k \right)
\]

\[
= \partial^\nu A_{ij}^\mu - \partial^\mu A_{ij}^\nu + \left( A_{ij}^{\mu \nu} - A_{ij}^{\nu \mu} \right).
\]
The eight gluons are obtained by using the eight Gell-Mann \( \lambda_i \) matrices.

\[ V_\mu^i = (\lambda_i)_{ij} A_{ij}^\mu. \]  

(33)

Similarly for \((i,j) = (7,8)\), we get the \( W_\mu \)-mesons

\[ W_\mu^{ij} = b_i^{\mu} b_j^{\nu} - \delta_{ij} b_i^{\nu} b_i^{\mu} = \partial^\mu W_\mu^{ij} - \partial^\nu W_\nu^{ij} + \left( W_\mu^{il} W_\mu^{lj} - W_\mu^{il} W_\mu^{lj} \right), \]

(34)

(35)

with

\[ W_\mu^{ij} = (\tau)_{ij} W_\mu^{ij}, \]

(36)

where \( \tau_l, l=1,2,3 \) are the \( 2 \times 2 \) isospin matrices. Of the remaining \( b_\mu^j, j=4,5,6 \) and \( b_\mu^k, k=9,10,11 \), we can form six Lorentz scalars \( \epsilon_{ijkl} b_i^k b_j^\lambda \) and \( \epsilon_{ikm} b_k^\lambda b_m^\gamma \). Let us call them \( \phi_1, ..., \phi_6 \). The Higgs bosons are

\[ \phi_H = \left( \phi_1 + i\phi_2, \phi_3 + i\phi_4 \right), \]

(37)

leaving behind a scalar

\[ \varphi_s = \phi_5 + i\phi_6. \]

(38)

Let us consider the fermion sector. With \( \Gamma_\mu \) as Dirac matrices and \( u(p) \) being the fermion wave function, the states \( \phi_1, \phi_2, \phi_3, \phi_4 \) and \( \phi_5, \phi_6 \) with \( \Gamma_\mu d_{-1, j, \mu} |0, p > u(p) \) and \( \Gamma_\mu d_{-1, k, \mu} |0, p > u(p) \) are all massless states. Broadly \( j=1, ..., 6 \) are the colour sector and \( k=7, ..., 11 \) are the electroweak sector. The current generator condition gives \( \Gamma_\mu p_i u(p) = 0 \), the Dirac equation for each of them.

Recently one of us (BBD) has shown \[1\] that using Wilson loop technique, \( SO(6) \otimes SO(5) \) can directly descend to the standard model reducing the rank by one and without breaking supersymmetry. In fact, it was proven that \( SO(6) \otimes SO(5) \rightarrow Z_3 \otimes SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \) but we have only eleven Ramond Fermionic zero mass modes \( d_{-1, j}^i, d_{-1, k}^i \). We have to find a way to put these fermions into electroweak and colour groups of one generation.

Let us drop the Dirac indices, the suffix -1, and prefix primes and supply a factor 2 such that

\[ \{ d_i, d_j \} = 2 \delta_{ij} \]

(39)

Since \( SO(4) \) can come from either \( SO(6) \) or \( SO(5) \) \[1\]. We can choose any four of them and define new operators \( b \)'s as follows

\[ b_1 = \frac{1}{2}(d_1 + id_2), \quad b_1^* = \frac{1}{2}(d_1 - id_2), \quad b_2 = \frac{1}{2}(d_3 + id_4), \quad \text{and} \quad b_2^* = \frac{1}{2}(d_3 - id_4). \]

(40)

They satisfy the algebra \( (i,j) = 1,2 \),

\[ \{ b_i, b_j \} = 0, \quad \{ b_i^*, b_j^* \} = 0 \quad \text{and} \quad \{ b_i, b_j^* \} = 2 \delta_{ij}. \]

(41)

This is the \( U(2) \) group, identified here as the isospin group. We now construct a table giving the assignment of all fermions (Table.1).

Once the weak isospins are fixed, the hypercharge can be calculated from the charge also. There are fifteen fermions with 30 degrees of freedom matching the 30 bosonic degrees of freedom from above equations. We now proceed to construct the Hamiltonian which is supersymmetric and \( SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \) invariant following the ideas presented by one of us \[5\]. The vector fields denoted by \( V_{i, l}^\mu \), \( l=1, ..., 8 \), are gluon fields. \( l=9 \) is the \( U_Y(1) \) field and \( l=10, 11, 12 \) stand for \( W \)-mesons fields. We shall use the temporal gauge \[6\], where \( V_{i, l}^0 = 0 \). For each \( V_{i, l}^\mu \), there are electric field strength \( E_l^i \) and magnetic field strength \( B_l^i \). Following Nambu \[7\], the combination

\[ F_l^i = \frac{1}{\sqrt{2}} \left[ E_l^i + B_l^i \right], \]

(42)
The phase function for the colour is
\[ \psi(x) = \frac{1}{\sqrt{2}} \Gamma \cdot (d_i + id_j) [0 > \alpha], \]

and the isospin phase functions are
\[ U_Q(x) = \exp \left( \frac{ig}{2} \int_0^x \tau \cdot \mathbf{W}_i dy_i - \frac{i}{6} Y(x) \right), \]
\[ U(x) = \exp \left( \frac{ig}{2} \int_0^x \tau \cdot \mathbf{W}_i dy_i - \frac{i}{2} Y(x) \right), \]
\[ U_1(x) = \exp \left( -\frac{2i}{3} Y(x) \right), \]
\[ U_2(x) = \exp \left( \frac{i}{3} Y(x) \right), \]
and
\[ U_R(x) = \exp (iY(x)) \]

The ̂ψ will denote the fermions, \( l = 1, \ldots, 6 \) refer to the coloured quark doublets. The sum of the products
\[ \sum_{l=1}^6 F_{i}^{l\dagger} \cdot \psi^l = \sum_{l=1}^3 \begin{pmatrix} F_{i}^{l\dagger}, F_{i}^{l+3} \end{pmatrix} \begin{pmatrix} \psi^l \psi^{l+3} \end{pmatrix} = \sum_{l=1}^3 \begin{pmatrix} F_{i}^{l\dagger}, F_{i}^{l+3} \end{pmatrix} U_{Q} U_{C} \begin{pmatrix} u_{L}^l \end{pmatrix} \]

The singlet phased colour quarks are
\[ \psi^l = U_{1} U_{C} u_{L}^l, \quad l = 7, 8, 9 \] and
\[ \psi^l = U_{2} U_{C} d_{L}^l, \quad l = 10, 11, 12. \]

The Q=1, singlet \( \varphi_s \) and the \( e_R \) are phased as
\[ \phi_s = U_{R} \varphi_s, \quad \psi_R = U_{R} e_R. \]
The Higgs doublets $\phi$ and the lepton doublet $\psi^l$ are phased like

$$\phi = U \phi_H \quad \psi_L = U \left( \frac{\nu_e}{e^-} \right).$$

(56)

In writing down the supersymmetric charge $Q$, we shall use $2 \times 2$ matrices, $\sigma^\mu$, $\mu = 0, 1, 2, 3$; $\sigma^0 = I$ and $\sigma$'s are the three Pauli spin matrices,

$$Q = \int d^3x \left[ \sum_{l=1}^{12} \left( \sigma \cdot F^l(x) \right) + \sigma^\mu \partial_\mu \phi^l(x) \cdot \psi_L(x) + \sigma^\mu \partial_\mu \phi^l_3(x) \cdot \psi_L(x) + \mathcal{W} \psi^l \right].$$

(57)

where $\mathcal{W} = \sqrt{X} [\phi^l \phi - \nu^2]$ is the electroweak symmetry breaking term with $\sigma = \sqrt{2} \nu = 246$ GeV. We use the usual commutators and anticommutators at equal times and the $\sigma_i \sigma_j = \delta_{ij} + i \epsilon_{ijk} \sigma_k$ and obtain

$$\{ Q^l_\alpha, Q^l_\beta \} = (\sigma^\mu P_\mu)_{\alpha\beta}.$$

(58)

$P_0$ is the Hamiltonian $H$ and $P$ is the total momentum. We get the $SU(C) \otimes SU(L) \otimes U_Y(1)$ invariant Hamiltonian as

$$H = \int d^3x \left[ \frac{1}{2} \left( E^2 + B^2 \right) + i\psi^l(x) \sigma \cdot \nabla \phi^l(x) \right]
+ \pi^l_H(x) \pi^l_H(x) + (\nabla \phi^l) \cdot \nabla \phi^l + \psi^l(x) \sigma \cdot \nabla \psi^l(x) + \psi^l(x) \sigma \cdot \nabla \psi^l(x) + \mathcal{W} \psi^l \]
+ \int d^3x \left[ \pi^l_3(x) \pi^l_3(x) + (\nabla \phi^l_i) \cdot \nabla \phi^l_i \right].

(59)

and similarly lengthy expression for $P$ (see reference [3]). The last two terms involving $Y=2$ meson is absent in the standard model. But the Hamilton’s equation of motion for the extra meson, obtained from the total Hamiltonian, is

$$\left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \psi^l = 0.

(60)

and decouples from other standard model particles.

Supersymmetric transformations exhibiting the superpartners are

$$\delta \psi^l = \sigma \cdot F^l \epsilon + \delta_H \mathcal{W} \epsilon,$$

$$\delta F^l_i = -i \epsilon (\sigma \times \nabla)_i \psi^l,$$

$$\delta \psi^l_L = \sigma^\mu \partial_\mu \phi \epsilon,$$

$$\delta \phi = -i \epsilon \psi^l_L,$$

$$\delta \psi^l_R = \sigma^\mu \partial_\mu \phi^l_3 \epsilon,$$

$$\delta \phi^l_3 = -i \epsilon \psi_L^l.$$

(61-66)

The only particle, not in the standard model, $\phi^l_3$, is noninteracting and decoupled.

In summary, we wish to emphasise that the proposed superstring excitations of zero mass fall into the standard model particles in a very natural way. The bosons are themselves, the superpartners of the fermions and vice versa. It is not surprising that the exotic SUSYs are not observed in the electroweak scale and the present formulation provides a justification for their absence.

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