Analytical discussion and sensitivity analysis of parameters of magnetohydrodynamic free convective flow in an inclined plate

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Abstract. A mathematical model of the magnetohydrodynamic free convective flow of a viscous incompressible fluid, which is based on a system of coupled steady-state nonlinear deferential equations, is discussed. A new approach of the homotopy perturbation method is employed to derive analytical expressions of the fluid velocity, fluid temperature, and species concentration. The efficiency and accuracy of the derived results are tested against highly accurate and widely used numerical methods. The obtained analytical expressions are employed to study the effects of the magnetic field, chemical reaction, and other relevant flow parameters on fluid velocity, fluid temperature, and species concentration. Sensitivity analysis of these parameters is also presented.

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1. Introduction

The basic idea of magnetohydrodynamic (MHD) oscillatory flow in a channel is that, if the heat source is connected to a heat sink by an oscillating fluid, then the convective motion implies sharp spikes in velocity leading to optimal heat transport over pure conduction [24]. Underground water and energy storage systems, plasma physics, petroleum industries, nuclear reactors, and crystal growth are just a few applications of MHD convection with heat transfer.

In recent years, the development of heat and mass transfer processes, due to the effects of external forces, has been intensively studied in science and engineering research.

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Examples of these studies include the effect of heat transfer on MHD oscillatory flow in an asymmetric wavy channel [24], the MHD convective flow through a porous medium in a horizontal channel [20], the effect of viscous dissipation on the MHD boundary layer flow [5], the MHD free convection and mass transfer flow with a heat generation [14], and the chemical and radiation effects on MHD heat and mass transfer flow along a moving vertical porous plate and a long a porous medium bounded by an inclined surface [22].

The effect of magnetic fields on the MHD boundary flow layers has been examined in several research. As examples, we mention the studies on the effect of induced magnetic and convective heating on MHD stagnation point flow [13], the heat transfer of unsteady MHD convective assuming that the semi-infinite plate is moving in a transverse magnetic field [8], and the effect of an electromagnetic field on natural convection in an inclined porous medium [6].

The mathematical model of the MHD free convective heat and mass transfer flow is represented by a system of nonlinear boundary differential equations for which no exact solution exists. Despite the fact that reliable numerical methods have been implemented to find approximate solutions, the need for reliable analytical solutions is necessary to investigate the effects of parameters variation on the governing system and hence improve the efficiency and control of the heat and mass transfer processes. Of the numerical methods that have been implemented by researchers, we mention a sixth-order Runge-Kutta coupled with a shooting method [29], an implicit finite difference method of Crank-Nicolson-type [5], a numerical integration scheme over the entire range of physical parameters [19], and a DuFort-Frankel finite difference method [21].

In the last two decades, researchers have carried out several theoretical investigations to study the effect of thermal radiation on magnetohydrodynamics flow and heat transfer. However, efficient and reliable analytical methods to find accurate approximate analytical solutions for the underlined nonlinear differential equations in unsteady MHD flow are still largely outnumbered by numerical simulations despite some remarkable techniques. For example, an analytical solution was obtained in terms of two-term harmonic and nonharmonic functions to discuss the MHD free convective flow through a porous medium past a vertical plate in the presence of heat absorption [27]. A Laplace transform technique was employed by Hussein et al. in a series of articles to investigate the MHD heat and mass transfer under various assumptions. Of these articles, we mention their studies on: The combined effects of Hall current and rotation on unsteady MHD free convective heat and mass transfer flow, the effect of thermal radiation on magneto-nanofluids free convective flow in the presence of an inclined magnetic field, and the effect of magnetic field, heat absorption and chemical reaction on fluid flow (see [11, 12, 26] and the references therein). Also a Laplace transform approach was used to study the effect of hall current on MHD natural convection heat and mass transfer of Casson fluid flow past a vertical plate with ramped wall temperature [25].

Other methods that are prone to deliver reliable analytical or semi-analytical solutions include, but not limited to, the variational iteration method [1, 17], the homotopy analysis method [16], and a Greens function based method [2–4, 15]. In this article, analytical expressions of the velocity, temperature, and concentration distributions are
derived using a modified simple approach of the homotopy perturbation method. A sensitivity analysis is presented to explain the effect of parameter variation on the velocity profile.

2. Mathematical formulation of the problem

Consider a steady laminar, two-dimensional free convective flow of chemically reacting and viscous incompressible and electrically conducting along an inclined nonconducting plate kept at uniform temperature $T_w'$. It is assumed that the $x'$-axis is along the plate, the $y'$-axis is normal to it, and the flux is uniformly in the $y'$ direction. Initially, the fluid and the plate are assumed to have the same temperature, and for $t' > 0$, the plate temperature is raised to $T_w'$, and the concentration level close to the plate is also raised to $C_w'$. The physical model and the coordinate system are illustrated in Figure 1.

The nonlinear equations of momentum, energy, and diffusion for steady state are, respectively, given by:

\begin{align}
\nu \frac{d^2 u'}{dy'^2} + g \beta (T_w' - T_\infty') \cos \alpha + g \beta' (C_w' - C_\infty') \cos \alpha - \frac{\sigma B_0^2 u'}{\rho} - \frac{v'}{K'} u' &= 0, \\
\kappa \frac{d^2 T'}{dy'^2} + \mu \left( \frac{du'}{dy'} \right)^2 &= 0, \\
D \frac{d^2 C'}{dy'^2} - K'(C_w' - C_\infty') &= 0.
\end{align}

The corresponding initial and boundary conditions are:

\begin{align}
u' &= 0, \quad T' = T_w', \quad C' = C_w' \quad \text{at} \quad y' = 0,
\end{align}
\[ u' \to 0, \ T' \to T'_{\infty}, \ C' \to C'_{\infty} \quad \text{as} \quad y' \to \infty, \quad (5) \]

where \( \nu = \frac{\mu}{\rho} \) (\( \mu \) is the viscosity and \( \rho \) is the constant density of the fluid), \( g \) is the acceleration due to gravity, \( \beta \) is the coefficient of thermal expansion, \( \beta' \) is the coefficient of concentration expansion, \( \alpha \) is the inclination angle from the vertical direction, \( \sigma \) is the electrical conductivity, \( B_0 \) is the magnetic induction and \( K' \) is the permeability of the porous medium, \( \kappa \) is the thermal conductivity, \( D \) is the molecular, and \( K_r' \) is the chemical reaction constant. Introduce the following dimensionless variables:

\[
\begin{align*}
  u' &= \left( \nu g(T_w' - T'_{\infty}) \right)^{1/3}, \quad \theta = \frac{T' - T'_{\infty}}{T_w' - T'_{\infty}}, \quad C = \frac{C' - C'_{\infty}}{C_w' - C'_{\infty}}, \quad y' = \left( \frac{g\beta(T_w' - T'_{\infty})}{\nu^2} \right)^{1/3}, \\
  N &= \frac{\beta'(C_w' - C'_{\infty})}{\beta(T_w' - T'_{\infty})}, \quad u_0 = \left( \nu g\beta(T_w' - T'_{\infty}) \right)^3, \quad Pr = \frac{\mu C_p}{\kappa}, \quad Ec = \frac{u_0^2}{C_p(T_w' - T'_{\infty})}, \\
  K &= \frac{K'\nu^2}{\nu_0^2}, \quad M = \frac{\nu\sigma B_0^2}{u_0^2\rho}, \quad Sc = \frac{\nu}{D}.
\end{align*}
\]  

(6)

where \( N, M, K, Ec, Pr, Sc \) and \( Kr \) denote the buoyancy ratio parameter, magnetic parameter (Hartmann number), permeability parameter, Eckert number, Prandtl number, Schmidt number, and chemical reaction parameter, respectively. Substituting the variables in Eq. (6) into Eqs. (1)-(3) lead to the dimensionless steady state nonlinear equations of momentum, energy and diffusion:

\[
\begin{align*}
  \frac{d^2 u}{dy^2} - \left( M + \frac{1}{K} \right) u + \theta \cos \alpha + NC \cos \alpha &= 0, \quad (7) \\
  \frac{1}{PrEc} \frac{d^2 \theta}{dy^2} + \left( \frac{du}{dy} \right)^2 &= 0, \quad (8) \\
  \frac{d^2 C}{dy^2} - Sc Kr C &= 0, \quad (9)
\end{align*}
\]

with the new dimensionless boundary conditions:

\[
\begin{align*}
  u &= 0, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0, \quad (10) \\
  u &= 0, \quad \theta = 0, \quad C = 0 \quad \text{as} \quad y \to \infty, \quad (11)
\end{align*}
\]

where \( u, \theta, \) and \( C \) are dimensionless velocity, temperature, and concentration of fluid, respectively.

### 3. Derivation of analytical expression

Combining classical perturbation with homotopy theory, He [9, 10] developed the homotopy perturbation method (HPM), where the requirement of small parameters is waved. Over the past two decades, HPM has been employed by many researchers to obtain approximate analytical solutions for many nonlinear engineering dynamical systems.
In this section, a modified homotopy perturbation method \cite{23, 28} is employed to obtain an analytical solution of the MHD free convective flow past an inclined plate under a steady state condition. The basic idea of the HPM is described in Appendix A.

Letting $A = M + \frac{1}{k}$ in Eqs. (7)-(8) gives

\[
\frac{d^2 u}{dy^2} - Au + \theta \cos \alpha + NC \cos \alpha = 0, \tag{12}
\]

\[
\frac{1}{Pr Ec} \frac{d^2 \theta}{dy^2} + \left( \frac{du}{dy} \right)^2 = 0, \tag{13}
\]

with boundary conditions

\[
u = 0, \theta = 1, \text{ at } y = 0, \tag{14}
\]

\[
u = 0, \theta = 0, \text{ as } y \to \infty, \tag{15}
\]

The exact solution of Eq. (9) is readily obtained

\[
C = e^{-y \sqrt{Sc Kr}}. \tag{16}
\]

Applying the homotopy as described in Eq. (A4) to Eqs. (12) and (13) gives

\[
(1 - p) \left( \frac{d^2 u}{dy^2} - Au \right) + p \left( \frac{d^2 u}{dy^2} - Au + \theta \cos \alpha + NC \cos \alpha \right) = 0, \tag{17}
\]

\[
(1 - p) \left( \frac{1}{Pr Ec} \frac{d^2 \theta}{dy^2} - \theta \right) + p \left( \frac{1}{Pr Ec} \frac{d^2 \theta}{dy^2} - \left( \frac{du}{dy} \right)^2 - \theta + \theta \right) = 0, \tag{18}
\]

for which the approximate series solutions are, respectively, given by

\[
u = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \cdots, \tag{19}
\]

\[
\theta = \theta_0 + p\theta_1 + p^2 \theta_2 + p^3 \theta_3 + \cdots. \tag{20}
\]

Direct substitution of Eqs. (19) and (20) into Eqs. (17) and (18) leads to

\[
(1 - p) \left( \frac{d^2 (u_0 + pu_1 + p^2 u_2 + \cdots)}{dy^2} - A(u_0 + pu_1 + p^2 u_2 + \cdots) \right) \]

\[
+ p \left( \frac{u_0 + pu_1 + p^2 u_2 + \cdots}{dy^2} - A(u_0 + pu_1 + p^2 u_2 + \cdots) \right) \]

\[
+ (\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots) \cos \alpha + NC \cos \alpha \right) = 0, \tag{21}
\]

\[
(1 - p) \left( \frac{1}{Pr Ec} \frac{d^2 (\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots)}{dy^2} - A(\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots) \right) \]

\[
+ p \left( \frac{1}{Pr Ec} \frac{d^2 (\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots)}{dy^2} - \left( \frac{d(u_0 + pu_1 + p^2 u_2 + \cdots)}{dy} \right)^2 \right) \]

\[
- (\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots) + (\theta_0 + p\theta_1 + p^2 \theta_2 + \cdots) = 0. \tag{22}
\]
Rearranging Eq. (21) according to the powers of $p$, gives the following set of equations:

\[
p^0: \frac{d^2 u_0}{dy^2} - Au_0 = 0, \quad (23)
\]

\[
p^1: \frac{d^2 u_1}{dy^2} - Au_1 + \theta_0 \cos \alpha + NC \cos \alpha = 0, \quad (24)
\]

and from Eq. (22), we obtain the system

\[
p^0: \frac{1}{Pr Ec} \frac{d^2 \theta_0}{dy^2} - \theta_0 = 0, \quad (25)
\]

\[
p^1: \frac{1}{Pr Ec} \frac{d^2 \theta_1}{dy^2} - \theta_1 + \left( \frac{du_0}{dy} \right)^2 + \theta_0 = 0, \quad (26)
\]

The sets of corresponding boundary conditions are, respectively

\[
\begin{align*}
    u_0 &= -1, \quad \theta_0 = 1 \quad \text{at} \quad y = 0, \quad (27) \\
    u_0 &= 0, \quad \theta_0 = 0 \quad \text{as} \quad y \to \infty, \quad (28)
\end{align*}
\]

and

\[
\begin{align*}
    u_1 &= 1, \quad \theta_1 = 0 \quad \text{at} \quad y = 0, \quad (29) \\
    u_1 &= 0, \quad \theta_1 = 0 \quad \text{as} \quad y \to \infty. \quad (30)
\end{align*}
\]

Substituting Eq. (16) into Eq. (24) and solving Eqs. (23)-(24) with boundary conditions (BC) (27)-(28) lead to

\[
\begin{align*}
    u_0 &= -e^{-y\sqrt{A}}, \\
    u_1 &= e^{-y\sqrt{A}} + \frac{\cos \alpha}{Ec Pr - A} \left( e^{-y\sqrt{A}} - e^{-y\sqrt{Ec Pr}} \right) + \frac{N \cos \alpha}{Sc Kr - A} \left( e^{-y\sqrt{A}} - e^{-y\sqrt{Sc Kr}} \right). \quad (32)
\end{align*}
\]

The sum of $u_0$ and $u_1$ gives the following two-term HPM semi-analytic formula for the velocity:

\[
\begin{align*}
    u(y) &= \frac{\cos \alpha}{Ec Pr - A} \left( e^{-y\sqrt{A}} - e^{-y\sqrt{Ec Pr}} \right) + \frac{N \cos \alpha}{Sc Kr - A} \left( e^{-y\sqrt{A}} - e^{-y\sqrt{Sc Kr}} \right). \quad (33)
\end{align*}
\]

Similarly, solving the system (25)-(26) with BCs (29)-(30) leads to

\[
\begin{align*}
    \theta_0 &= e^{-y\sqrt{Ec Pr}}, \quad (34)
\end{align*}
\]
\[
\theta_1 = \frac{AEcPr}{4A - EcPr} \left( e^{-y\sqrt{EcPr}} - e^{-2y\sqrt{A}} \right) + \frac{y\sqrt{EcPr}}{2} e^{-y\sqrt{EcPr}}.
\] (35)

The sum of \( \theta_0 \) and \( \theta_1 \) gives the following two-term HPM semi-analytic formula for the temperature:
\[
\theta(y) = \frac{AEcPr}{4A - EcPr} \left( e^{-y\sqrt{EcPr}} - e^{-2y\sqrt{A}} \right) + \left( \frac{y\sqrt{EcPr}}{2} + 1 \right) e^{-y\sqrt{EcPr}}.
\] (36)

4. Analytical expressions for the skin friction and Nusselt and Sherwood numbers

As a direct conclusion of Eqs. (33), (36) and (16), analytic expressions for the local Skin-friction coefficient \((C_f)\), the Nusselt number \((N_u)\), and the Sherwood number \((S_h)\), which are essential material parameters to analyze the rate of fluid velocity and temperature near to the plate [11], can be derived.

**Skin friction** From Eq. (16), an analytical expression of the dimensionless skin friction is given by
\[
C_f = \frac{\partial u}{\partial y} \bigg|_{y=0} = \frac{\cos \alpha}{EcPr - A} \left( \sqrt{EcPr} - \sqrt{A} \right) + \frac{N \cos \alpha}{ScKr - A} \left( \sqrt{ScKr} - \sqrt{A} \right).
\] (37)

**Nusselt number** From Eq. (36), an analytical expression of the dimensionless rate of heat transfer (Nusselt number) is given by
\[
N_u = \frac{\partial \theta}{\partial y} \bigg|_{y=0} = \frac{AEcPr}{4A - EcPr} \left( 2\sqrt{A} - \sqrt{EcPr} \right) - \frac{EcPr}{2}.
\] (38)

**Sherwood number** From Eq. (16), an analytical expression of the dimensionless rate of mass transfer (Sherwood number) takes the form
\[
S_h = \frac{\partial C}{\partial y} \bigg|_{y=0} = -\sqrt{ScKr}.
\] (39)

4.1. Results and discussion

In this section, we present numerical simulations to test the accuracy and reliability of the proposed method. The analytical expressions obtained in this paper will be compared to the highly accurate numerical solutions obtained by the MATLAB routine bvp4c, which is a finite-difference code that implements the three-stage Lobatto IIIA formula.

The analytical and numerical solutions were plotted on the same coordinates for a wide range of possible values of the underlined problem parameters. Figures 2, 3, 5 and 8-11 reveal that the derived analytical expressions for the velocity, temperature, and concentration are in strong agreement with numerical solutions.

Velocity profiles for different values of buoyancy ratio parameter \((N)\) are shown in Figure 2, where it is observed that an increase in \(N\) leads to an increase in velocity. The
effect of the magnetic parameter ($M$) on the velocity is exactly opposite to that of $N$ as illustrated in Figure 3. That is, an increase in $M$ leads to a decrease in the velocity or, in other words, any decrease in the fluid angle on the inclined plate leads to an increase in the flow of the velocity profile.

Eckert number ($Ec$), which characterizes the influence of self-heating of a fluid also affects the velocity profile. From Figure 4, we notice that increasing $Ec$ implies a decrease in velocity. A similar effect on the velocity profile is caused by Schmidt number ($Sc$), which is the ratio of the kinetic viscosity to the molecular diffusion coefficient. The inverse proportionality relation between $Sc$ and velocity is presented in Figure 5.

Prandtl number is a dimensionless quantity that puts the viscosity of a fluid in correlation with the thermal conductivity. Figure 6 shows how an increase in $Pr$ results in a decrease in the velocity profile. The chemical reaction parameter ($Kr$) has a similar affect on the velocity. That is, an increase in $Kr$ leads to a decrease in the velocity, as illustrated in Figure 7. In fact, Schmidt, Prandtl, and Eckert numbers are all inversely proportional to velocity, as seen in Figures 4-7.

The exponential analytical expression of the temperature (Eq. (30)) justifies its inverse proportionality relation with Eckert and Prandtl numbers as well as the magnetic parameter, as depicted in Figures 8-10. Figure 11 shows that the increase in temperature that resulted from increasing the permeability parameter is insignificant.

The effects of problem parameters are summarized in the sensitivity analysis chart depicted in Figure 12. In this figure, where the rate of change of momentum $u$ was computed (using the experimental values $N = 30, \alpha = 0.8, M = 1, k = 1, Ec = 1, Pr = 1, Sc = 0.6$ and $Kr = 2$), it is shown that $\alpha$ has the most impact (72.3%) on the rate of change of velocity followed by Schmidt number with 13.6% impact on the rate of change of velocity. For the same experimental values, the sensitivity analysis of problem parameters reveals that $Ec$ and $Pr$ have the most impact (45% each) on the rate of change of temperature distribution. As for the concentration, the exact solution of equation (9) explains the reason why Schmidt number $Sc$ and the permeability parameter $Kr$ have an identical impact on the concentration profile (50% each).

5. Conclusion

In this paper, a free convective and mass transfer magnetic field of a viscous incompressible fluid in an inclined plate was presented. A modified version of the homotopy perturbation method was employed to derive analytical expressions for the fluid velocity, temperature, and concentration of species. These analytical expressions were used to study the effects of the system parameters on temperature and velocity profiles. Analytical expressions for the Skin-friction and Nusselt and Sherwood numbers were also derived. The accuracy of the analytical solutions was confirmed by noticing a strong agreement with MATLAB-generated numerical simulations.
Appendix A. The homotopy perturbation method

Consider the nonlinear differential equation

\[ A(u) - f(r) = 0, \quad r \in \Omega, \]  

(A1)

with the boundary condition

\[ B\left(u, \frac{du}{dr}\right) = 0, \quad r \in \Gamma, \]  

(A2)

where \( A, B, f(r) \) and \( \Gamma \) are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain \( \Omega \), respectively. Expressing \( A(u) \) as the sum of linear (\( L \)) and nonlinear (\( N \)) parts, Eq. (A1) becomes
L(u) + N(u) - f(r) = 0. \tag{A3}

The homotopy technique begins by defining \( v(r,p) : \Omega \times [0,1] \rightarrow R \), such that

\[ H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{A4} \]

where \( p \in [0,1] \) is an embedding parameter and \( u_0 \) is an initial approximation of Eq. \( (A1) \) that satisfies boundary conditions \( (A2) \). Evidently, Eq.\( (A4) \) implies that

\[ H(v,0) = L(v) - L(u_0) = 0, \tag{A5} \]
\[ H(v,1) = A(v) - f(r) = 0. \tag{A6} \]

As \( p \) changes from 0 to 1, \( v(r,p) \) changes from \( u_0 \) to \( u_r \), a process known as a homotopy. The solution of Eq. \( (A4) \) may be expressed in terms of a power series in the form:

\[ v = v_0 + pv_1 + p^2v_2 + \cdots. \tag{A7} \]
With $p = 1$, an approximate solution to Eq. (A4) is given by:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots.$$  \hspace{1cm} \text{(A8)}

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