Electronic transport in an array of quasi-particles in the $\nu = 5/2$ non-abelian quantum Hall state

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The Moore-Read Pfaffian $\nu = 5/2$ quantum Hall state is a $p$-wave super-conductor of composite fermions. Small deviations from $\nu = 5/2$ result in the formation of an array of vortices within this super-conductor, each supporting a Majorana zero mode near its core. Here we consider how tunneling between these cores is reflected in the electronic response to an electric field of non-zero wave vector $\mathbf{q}$ and frequency $\omega$. We find a mechanism for dissipative transport at frequencies below the $\nu = 5/2$ gap, and calculate the $\mathbf{q}, \omega$ dependence of the dissipative conductivity. The contributions we find depend exponentially on $|\nu - 5/2|^{-1/2}$.

The $\nu = 5/2$ fractional quantum Hall state is expected to be characterized by quasi-particles obeying non-abelian statistics. There are strong indications that this state is well described by the Moore–Read Pfaffian wavefunction $|\text{1}\rangle$, which may be formulated within composite-fermion theory (each electron is bound to two flux quanta) as a $p$-wave superconductor of composite fermions (CFs) at zero magnetic field. Excitations in this superconductor are vortices carrying half a flux quantum and an electric charge of $e/4$, and fermions created in those by breaking pairs with an appropriate energy gap

$\Delta$. The Bogoliubov-de-Gennes (BdG) equation describing the fermionic excitations of a two-dimensional $p$-wave superconductor admits zero-energy solutions in the presence of well separated vortices, one solution near each vortex' core; These solutions are Majorana fermions $\gamma$, satisfying $\gamma^\dagger = \gamma$. As a consequence, the ground state is degenerate; For $2N$ well separated vortices, the ground state degeneracy is $2^{2N}$. The adiabatic interchange of two vortices induces a unitary transformation within the subspace of degenerate ground states. Two such transformations do not necessarily commute; Hence vortex excitations obey non-Abelian statistics. A related spin model showing similar non-abelian excitations was recently studied by Kitaev $|\text{4}\rangle$.

Experimental support to the Moore-Read theory is still needed. Relating the theory, and in particular the non-abelian nature of the quasi-particles, to measurable observables, is a major theoretical challenge. Interference experiments may be a venue towards that goal $|\text{5}\rangle$ $|\text{6}\rangle$ $|\text{7}\rangle$.

In this work we pursue a different method to probe the ground state degeneracy as well as some of the properties of the Majorana excitations, by considering the response of a quantum Hall system near filling factor $\nu = 5/2$ to an external electric field of wave-vector $\mathbf{q}$ and frequency $\omega$. In a fractional quantum Hall system at a filling factor $\nu = 5/2 \pm \epsilon$ ($\epsilon \ll 1$), the density deviation from $\nu = 5/2$ is accommodated by means of quasi-particles (vortices) whose density is $8\pi n$, where $n$ is the density of electrons. For a perfectly clean system, these quasi-particles form a lattice, and when their density is large enough, tunneling between their cores should be taken into account. The degeneracy of the ground state is partially removed by this tunneling, and a band is formed with a width of the order of the tunneling strength. The tunneling also breaks the particle-hole symmetry of the localized $\gamma_i$'s.

We study the electronic transport through that band for square and triangular lattices. We find that due to the existence of the band, there is a dissipative part to the conductivity below the $\nu = 5/2$ energy gap, with a unique $\mathbf{q}, \omega$ dependence. This contribution to the conductivity, which does not involve a motion of the vortices, depends exponentially on $|\epsilon|^{-1/2}$, due to its origin in tunneling. There is a qualitative difference between the two lattice types. The square lattice is described by an effective massless Dirac Hamiltonian, while the triangular one shows a gap of a fraction of the band width.

We calculate the dissipative part of the conductivity of the CFs using Kubo’s formula $|\text{11}\rangle$, and then map it to the electronic conductivity by a Chern-Simon transformation $|\text{12}\rangle$ $(\sigma^\dagger - \sigma)^{-1} = (\sigma^\dagger f - \sigma f)^{-1} + 2\hbar \omega \hat{\epsilon}$, (with $\hat{\epsilon}$ being the anti-symmetric tensor). For the square lattice, we find that the longitudinal and transverse CF conductivities are respectively

$$\Re (\sigma_{\omega,\|}^f, \sigma_{\omega,\perp}^f) = \frac{e^2}{h} \frac{\partial^2 (\eta_\triangledown)^2}{h} \left( \frac{\omega}{|\frac{1}{2} | \omega | } + \frac{3\eta_\triangledown^{1/2}}{|\omega|} \right)$$

where $\eta_\triangledown = \omega^2 - v_0^2 q^2$. Here $a$ is the lattice constant, $v_0$ is the velocity characterizing the Dirac spectrum, and $\theta$, to be defined below, is related to the tunneling strength.

For the triangular lattice, we find that the real part of the conductivity is

$$\Re \sigma_{\omega,\perp}^f = \frac{e^2}{h} \frac{\partial^2 (3a q)^2}{\hbar} \frac{\theta}{8} \left( \frac{|\omega|}{\sqrt{3} f} \right)$$

where $\eta_\Delta = \frac{\omega | \omega |}{\sqrt{3} f} - 2 - \frac{a^2 q^2}{2}$. As we explain below, the electronic conductivities are suppressed by a factor of $\omega^2$ relative to the CF conductivities.

There are four steps in the calculation leading to these response functions. First, we specify the Hamiltonian describing the array. This Hamiltonian turns out to be closely related to the Azbel-Hofstadter (A–H) Hamiltonian $|\text{13}\rangle$ $|\text{14}\rangle$, describing electrons on a tight binding lattice.
in a magnetic field. Second, we calculate the spectrum of the Hamiltonian. Third, we find how the system couples to gauge fields by expressing the density and current operators in terms of the Majorana operators $\gamma_i$ (with $i$ the vortex index); we also present a physical picture of this coupling. Finally, we calculate the response functions.

Based solely on the requirement of Hermiticity and on the relation $\gamma_i = \gamma_i^\dagger$, a lattice of well separated vortices is generally described by a tight binding Hamiltonian

$$H = it\sum_{ij} s_{ij} \gamma_i \gamma_j$$

(3)

where $\gamma_i$ are the Majorana operators satisfying $\{\gamma_i, \gamma_j\} = \delta_{ij}$, and where $i, j$ are nearest-neighbors lattice site indices. The tunneling strength $t$ is real and positive. The matrix $s_{ij} = \pm$ is anti-symmetric and indicates the sign of the tunneling along the bond $(i, j)$. While the freedom to redefine $\gamma_i \rightarrow -\gamma_i$ makes the elements $s_{ij}$ gauge dependent, the product of $s_{ij}$ over bonds creating a closed path is gauge independent. We now show that this product is determined by a non-trivial phase a Majorana fermion accumulates when encirling a plaquette, and give a simple formula for the effective flux per plaquette. This formula fixes the matrix $s_{ij}$ up to a choice of gauge.

In the absence of tunneling between vortex cores, the localized solution to a 2D $p$-wave BdG equation near a vortex embedded in a lattice of vortices is given by

$$\chi_i(r) = \begin{pmatrix} e^{-i\pi/4 + \frac{1}{2}\int_{P_i} \nabla \Phi_i(r) \cdot dl} g(r - R_i) \\ e^{i\pi/4 - \frac{1}{2}\int_{P_i} \nabla \Phi_i(r) \cdot dl} g(r - R_i) \end{pmatrix}$$

(4)

This is an approximate zero energy eigenstate of the first quantized 2D $p$-wave Hamiltonian $H_{BdG}$ (see [2,11]) of an order parameter $\Delta_0(r) \exp i\Omega(r; \{R_i\})$, where $r$ is the 2D-space coordinate and $\{R_i\}$ are the vortices’ positions and the phase $\Omega(r; \{R_i\})$ has the property of increasing by $2\pi$ around any closed path containing one vortex (clockwise). The phase appearing in the solution (4) is given by $\Phi_i(r; \{R_i\}) = \Omega(r; \{R_i\}) + \arg(r - R_i)$, where the first term originates from the order parameter and the second one originates from the $p_x + ip_y$ pairing, which induces a relative particle-hole angular momentum. The point $P_i$ is arbitrarily chosen close to the vortex core. The real wavefunction $g(r)$ is localized at the vortex core. The tunneling matrix elements for nearest neighbors are purely imaginary, and are given by $\pm it$ where $t = \frac{1}{2} \int_{P_i} \nabla \Omega \cdot dl + \frac{1}{2} \sum_{s=1}^\infty A_i$, where $A_i$ is the angle subtended by the path with respect to the $i$-th vortex, positive for clockwise traversal. The first term gives a $\pi$-winding for each vortex enclosed in the path. For each of these enclosed vortices, $A_i$ is given by minus the exterior angle, which can be written as $-(2\pi - I_i)$, where $I_i$ is the interior angle. For all other vortices $A_i = I_i$. We therefore get $\frac{1}{2} \int \nabla \Omega \cdot dl + \frac{1}{2} \sum_{s=1}^\infty A_i = \frac{1}{2} \sum_{s} I_i$, i.e. half the sum of interior angles of the polygon. This result is independent of whether the core of a vortex on the path is inside or outside of the polygon: if a path is deformed as to cross a core of a vortex, both the term related to the order parameter and the relevant angle $A_i/2$ acquire an extra $\pi$, and these two contributions cancel each other. For a general polygon of $n$ vortices we get a phase of $\pi n/2 - \pi$; Consequently, for a lattice whose plaquette is a polygon of $n$ vortices we get $n/4 - 1/2$ flux quanta per plaquette.

We note that for the A–H problem of tight-binding electrons on the same lattice with the same flux per plaquette, the Hamiltonian is

$$H^b = it \sum_{ij} s_{ij} c^\dagger_i c_j$$

(6)

The Hamiltonians (3) and (6) share the same Harper’s equation, their spectra are identical, but they differ considerably in the way they couple to gauge fields. Yet, there exist relations between their response functions.

Our determination of the effective flux in (3) singles out then a chain of A–H type problems, one for each value of $n$, where the flux per plaquette is determined by the geometry of the lattice. There is a qualitative difference related to this path, given by half the sum of the interior angles of the polygon. The origin of this phase is in an interplay between the phase of the order parameter and the $p$-wave pairing. First we calculate the tunneling matrix elements $<\chi_i|H_{BdG}|\chi_j> = t_{ij} \exp i\psi_{ij}$, where we use the tight-binding assumption to neglect the spatial dependence of the phase and explicitly set $r = (R_i + R_j)/2 \equiv C_{ij}$. For all bonds $t_{ij} = t$, while $\psi_{ij}$ is given by

$$\psi_{ij} = \frac{1}{2} \int_{P_i} \nabla \Omega(1) \cdot dl + \frac{1}{2} \int_{P_j} \nabla \arg(1 - R_1) \cdot dl$$

(5)

The first term depends only on the order parameter; it measures the change of the phase of the spinor due to vortices enclosed in the path. The second and third terms are the contributions to the phase due to the relative particle-hole angular momentum induced by the $p_x + ip_y$ pairing; they measure changes in the direction of the path. Considering $n$ tunneling events $t^n \exp i[\psi_{12} + \psi_{23} + \ldots + \psi_{n,1}]$, the total phase is given by $\int \frac{1}{2} \nabla \Omega(1) \cdot dl + \frac{1}{2} \sum_{i=1}^n A_i$, where $A_i$ is the angle subtended by the path with respect to the $i$-th vortex, positive for anti-clockwise traversal. The first term gives a $\pi$-winding for each vortex enclosed in the path. For each of these enclosed vortices, $A_i$ is given by minus the exterior angle, which can be written as $-(2\pi - I_i)$, where $I_i$ is the interior angle. For all other vortices $A_i = I_i$. We therefore get $\int \frac{1}{2} \nabla \Omega(1) \cdot dl + \frac{1}{2} \sum_{i=1}^n A_i = \frac{1}{2} \sum_{s} I_i$, i.e. half the sum of interior angles of the polygon. This result is independent of whether the core of a vortex on the path is inside or outside of the polygon: if a path is deformed as to cross a core of a vortex, both the term related to the order parameter and the relevant angle $A_i/2$ acquire an extra $\pi$, and these two contributions cancel each other. For a general polygon of $n$ vortices we get a phase of $\pi n/2 - \pi$; Consequently, for a lattice whose plaquette is a polygon of $n$ vortices we get $n/4 - 1/2$ flux quanta per plaquette.
between the triangular lattice, with an odd \( n \), and the square lattice, with an even \( n \); the former breaks time reversal symmetry in the effective A–H problem while the latter does not. The honeycomb lattice, for which \( n = 6 \), was considered in [4].

In the next step we calculate the spectrum and eigenvectors of the Hamiltonian \([H, \Gamma]\) [4]. After identifying the flux per plaquette, we choose a gauge which complies with it, commonly breaking translational symmetry. Translational symmetry is restored by choosing a unit cell which contains an integer multiple of the flux quantum. The sites of the unit cell are numbered \( z = 1, \ldots, 4 \). In this way, we divide our lattice into \( s \) sublattices. We aim at finding an operator \( \Gamma^\dagger \) satisfying the equation \([H, \Gamma^\dagger] = E\Gamma^\dagger \). We expand it in local site operators as \( \Gamma^\dagger = \sum_{i} \lambda_i \gamma_i \) ending with the following equation

\[
\hat{H}_{zz'} \lambda_{z'} = \sum_{z \in z'} s_{ij} e^{i k \cdot (R_j - R_i)} \lambda_{z'} = E \lambda_z
\]

where the site \( i \) is an arbitrarily chosen lattice site that belongs to the \( z \) sublattice. We denote the corresponding eigenvectors of \( \hat{H} \) by \( \lambda^{(\alpha)}(k) \). This results in the following operators

\[
\Gamma^{(\alpha)}_k = \sum_i \lambda^{(\alpha)}_i(k) \gamma_i = \sum_{z = 1}^s \lambda^{(\alpha)}_z(k) \sum_{i \in z} e^{i k \cdot R_i} \gamma_i
\]

which obey the usual fermionic anti-commutation relations \( \{ \Gamma^{(\alpha)}_k, \Gamma^{(\beta)}_{k'} \} = \delta_{\alpha\beta} \delta(k - k') \) and \( \{ \Gamma^{(\alpha)}_k, \Gamma^{(\beta)}_{k'} \} = 0 \) for positive energy modes. In terms of these operators the Hamiltonian is diagonal and is given by \( H = \sum_{k \alpha} E_{k\alpha} \Gamma^{(\alpha)}_k \Gamma^{(\alpha)}_k \).

For the square lattice the A–H Hamiltonian has half a quantum of flux per plaquette. We choose a gauge for which \( s_{ij} = + \) along columns and has alternating signs between adjacent rows. Having translational invariance in doubled lattice vectors, we may split the lattice sites into four sublattices, numbered \( z = 1, \ldots, 4 \). The Hamiltonian \( \hat{H} \) may be written in a \( 4 \times 4 \) matrix notation as

\[
\hat{H}_0 = 2t \sigma_x \otimes \tau_z \sin(akx) + 2t \sigma_x \otimes \tau_x \sin(aKy)
\]

In the limit \( |k| a \to 0 \) the Hamiltonian [10] has a doubly degenerate gapless isotropic Dirac spectrum \( \epsilon_{k\alpha} = \text{sgn}(\alpha) v_0|k| \), with \( \alpha = \pm 2, \pm 1, -1, -2 \) and the characteristic velocity \( v_0 = 2at \). The eigenvectors of Eq. (10) are given by

\[
\lambda^{(1)}_\pm = \lambda^{(2)}_\pm = \lambda^{(1)}_\mp = \frac{(i e^{i\theta_k}, e^{i\theta_k}, -i, 1)}{2}
\]

where \( e^{i\theta_k} = (k_x + ik_y)/|k| \).

For the triangular lattice the A–H Hamiltonian has a quarter of a flux quantum per plaquette. The Hamiltonian in the sublattice representation is given by

\[
\hat{H}_\triangle = 2t \sum_{i=1}^2 (\xi_i \sin(a_i \cdot \mathbf{k}) + \xi_3 \cos(a_3 \cdot \mathbf{k}))
\]

where \( \xi_1 = l \otimes \tau_x, \xi_2 = \sigma_y \otimes \tau_y, \xi_3 = \sigma_y \otimes \tau_z \), and where \( a_1 = (ax - \sqrt{3}ay)/2 \), \( a_2 = (ax + \sqrt{3}ay)/2 \), \( a_3 = ax \) are the three lattice directions. There is a doubly degenerate spectrum indexed again by \( \alpha \), and the spectrum is [12]

\[
\epsilon_{k\alpha} = \text{sgn}(\alpha) \epsilon_{\triangle, k},
\]

where \( \epsilon_{\triangle, k} = \sqrt{2t} \sqrt{3 + \cos(2ak_x) - 2 \cos(ak_y) \cos(\sqrt{3}ak_y)} \).

The spectrum is gapped, and there are two minima at \( k_0 = (\pm \pi/3a, 0) \), around which the spectrum is quadratic \( \epsilon_{k\alpha} + \epsilon_{k\alpha} \sim \text{sgn}(\alpha) \sqrt{3t}(1 + 2a^2 k^2) \). The eigenvectors of Eq. (12) are given by

\[
\lambda^{(1)}_k = \lambda^{(-2)*}_k = \frac{1}{N_k} (iB_{-k}, -iB_{-k}, 1, 1)
\]

\[
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\]

where \( N_k \) is a normalization factor and

\[
B_k = \frac{\epsilon_{\triangle}(k)/2t + \sin(a_1 \cdot \mathbf{k})}{\cos(a_3 \cdot \mathbf{k}) + i \sin(a_2 \cdot \mathbf{k})}
\]

The coupling of the Majorana states of the Hamiltonian [3] to an electric field is very different from that of the electrons in the A–H Hamiltonian [16], due to the particle–hole symmetry of the operators \( \gamma_i \). While each Majorana state [14] is electrically neutral, when tunneling between vortex cores is switched on, a non-zero density of charge appears between the vortices. Projected to the subspace of the Majorana states, the density operator may be written as \( \rho(\mathbf{r}) = \sum_{ij} \rho_{ij}(\mathbf{r}) \) where

\[
\rho_{ij}(\mathbf{r}) = i s_{ij} g(\mathbf{r} - \mathbf{R}_i) g(\mathbf{r} - \mathbf{R}_j) \gamma_i \gamma_j
\]

The operator \( i \gamma_i \gamma_j \) has two eigenvalues \( \pm 1 \) which describe the sign of the charge mostly sitting at the center of the bond; However, the operators \( \rho_{ij} \) do not commute if they share a common Majorana operator, and consequently one cannot specify the charge at all bonds simultaneously. Furthermore, two nearest-neighbour spinors \( \chi_i \) and \( \chi_j \) are exactly orthogonal \( \langle \chi_i | \chi_j \rangle = 0 \) due to a \( \pi \) phase difference between the overlap of the particles and that of the holes; however, they do support non-zero matrix elements of the charge operator \( \langle \chi_i | \sigma_i | \chi_j \rangle = i s_{ij} \theta \), where \( \theta = \int_g g(\mathbf{r} - \alpha \hat{x}) g(\mathbf{r}) \). Consequently, the excitation \( \Gamma^{(ij)} \) carries a charge of \( \theta |k| t \).

Next we identify the current operator. At \( q = 0 \), the current is found using the identity

\[
\mathbf{j}_{q=0} = i[H, \mathbf{d}] = -i \partial t \sum_{ijl} s_{ij} s_{jl} \left( \frac{\mathbf{R}_l - \mathbf{R}_j}{2} \right) \gamma_i \gamma_l
\]
where \( \mathbf{d} = \int \mathbf{r} \rho(\mathbf{r}) \) is the total dipole operator. The sum \( \sum_{ij} s_{ij} s_{jl} \neq 0 \) only for sites \( i \) and \( l \) separated by a doubled lattice vector. The \( q = 0 \) current may be transformed to \( k \)-space by inverting \([\textit{13}]\) and substituting it into \([\textit{13}]\). The \( q = 0 \) current is a conserved quantity. To see that, we examine the commutator of \( \mathbf{q} \) with the Hamiltonian

\[
[H, \mathbf{q} = 0] \propto \sum_{ijklm} s_{ij} s_{jl} s_{lm} \left( \frac{\mathbf{R}_i + \mathbf{R}_m}{2} - \frac{\mathbf{R}_j + \mathbf{R}_l}{2} \right) \gamma_i \gamma_m
\]

which is described by paths composed of three consecutive bonds connecting the vortices given by \( i, j, l, m \). All paths give zero contribution as they interfere destructively with paths formed by starting from one of the ends and reversing the order of steps to the other end.

At finite \( q \) we find the current by calculating \( \rho(\mathbf{q}) = \int e^{i \mathbf{q} \cdot \mathbf{r}} \rho(\mathbf{r}) \) and using charge conservation. The current operator is, in momentum space,

\[
\mathbf{j}(\mathbf{q}) = \sum_{k, \alpha, \beta} \tilde{e}_k \mathbf{v}_k \tilde{\gamma}^{\alpha \dagger} (\mathbf{k}) \Gamma^{\beta \dagger} (\mathbf{k}) \mathbf{1}_{\mathbf{k} - \mathbf{q}/2}
\]

where \( \tilde{e}_k = \lambda^{(\alpha) \dagger} (\mathbf{k}) \mathbf{1}_{\mathbf{k} - \mathbf{q}/2} \cdot \lambda^{(\beta)} (\mathbf{k}) \mathbf{1}_{\mathbf{k} - \mathbf{q}/2} \) are the density matrix elements of the associated A–H problem, and \( \mathbf{v}_k = e \mathbf{q} \tilde{e}_k / \xi \), \( \mathbf{v}_k = \partial_k \tilde{e}_k \) are the charge and velocity of the quasiparticle respectively. For comparison, the longitudinal component of the current in the associated A–H Hamiltonian \([\textit{6}]\) is

\[
\mathbf{j}^\parallel (\mathbf{q}) = \sum_{k, \alpha, \beta} J_{\alpha \beta}^\parallel (\mathbf{k}, \mathbf{q}) \tilde{\gamma}^{\alpha \dagger} (\mathbf{k} + \mathbf{q}/2) \tilde{\gamma}^{\beta \dagger} (\mathbf{k} - \mathbf{q}/2)
\]

where for the relevant transitions \( J_{1,-1}^\parallel = J_{2,-2}^\parallel = \frac{c_s^0}{2} (e^{i \theta_{\eta q/2}} - e^{-i \theta_{\eta q/2}}) \) for the square lattice and \( J_{2,-2}^\parallel = -J_{2,-1}^\parallel = \frac{2 c_s}{\sqrt{2}} (3 + 3 \sqrt{3} i) \) for the triangular lattice near the bottom of the band. Overall, all the matrix elements of the current operator in the Majorana problem \([\textit{19}]\) are smaller by a factor of \( qa \) relative to those of the A–H problem \([\textit{20}]\).

Having calculated the spectrum and identified the relevant operators, the response functions of the array of Majorana states is readily calculated employing the Kubo formula with the results given by Eqs. \([\textit{1}], [\textit{2}]\). This result affirms the existence of dissipative conductivity, hence the flow of in-phase current, even at frequencies below the \( \nu = 5/2 \) energy gap.

Two steps need to be taken to transform the composite fermions conductivities \([\textit{1}], [\textit{2}]\) into the measurable electronic conductivity. First, the imaginary part of the CF conductivity, \( i \rho_s e^2 / \omega \) (with \( \rho_s \) being the superfluid density of the CFs), originating from the superconductivity of the CF condensate, should be added to the calculated real part. Second, the Chern-Simon transformation should be used to transform the CF conductivity into the electronic one. In the limit of small \( \omega \), these two steps result in

\[
\text{Re} \sigma_{\parallel}(\mathbf{q}, \omega) = \left( \frac{\omega}{2 \hbar \rho_s} \right)^2 \text{Re} \sigma_{\perp}^c
\]

At finite temperature, assuming \( v_0 q \ll \omega \), the conductivity satisfies \( \sigma(T) = \sigma(T = 0) \text{sgn} \omega \text{tanh} (\hbar \omega / 8 k_B T) \).

Before closing, we note that the same methods may be used to find the response functions of the associated A–H problems. For the square lattice the conductivity is

\[
\text{Re} \sigma_{\parallel}(\mathbf{q}, \omega) = \frac{\omega}{8 \pi} \left( \frac{|\eta q/2|}{|\eta q/2|} \right)^{1/2} \text{Re} \sigma_{\perp}^c
\]

The value of the conductivity at the \( q = 0 \) limit is a universal \( e^2 / 8 h \) \([\textit{13}]\): the origin of the universality lies in an exact cancelation of the dependence on \( v_0 \) due to the linear density of states \( \propto \omega / v_0^2 \). The dependence of the conductivity on temperature is \( \sigma_{\parallel}(\omega) = \frac{1}{8 \pi} e^2 / 8 h \text{tanh} (\hbar \omega / 8 k_B T) \). For the triangular lattice the conductivity at the bottom of the band is again universal, \( \frac{e^2}{9 \hbar} \).

In summary, we calculated the electronic response of an array of immobile quasi-particles of the \( \nu = 5/2 \) state to an electric field of non-zero \( \mathbf{q}, \omega \), due to tunneling between the Majorana modes at their cores. We found a contribution to the dissipative conductivity, Eq. \([\textit{21}]\), that is of a unique \( \mathbf{q}, \omega \) dependence, and a strong exponential dependence on the deviation of \( \nu \) from 5/2. Our analysis neglected disorder, which will be discussed elsewhere.

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