Hints for decaying dark matter from $S_8$ measurements

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(Dated: August 25, 2020)

Recent weak lensing surveys have revealed that the direct measurement of the parameter combination $S_8 \equiv \sigma(\Omega_m/0.3)^{0.5}$ – measuring the amplitude of matter fluctuations on 8 h$^{-1}$Mpc scales – is $\sim 3\sigma$ discrepant with the value reconstructed from cosmic microwave background (CMB) data assuming the $\Lambda$CDM model. In this Letter, we show that it is possible to resolve the tension if dark matter (DM) decays with a lifetime of $\log_{10}(\Gamma^{-1}/\text{Gyr}) = 1.75_{-0.95}^{+1.4}$ into one massless and one massive product, and transfers a fraction $\epsilon \simeq 0.7^{+2.7}_{-0.6}\%$ of its rest mass energy to the massless component. The velocity-kick received by the massive daughter leads to a suppression of gravitational clustering below its free-streaming length, thereby reducing the $\sigma_8$ value as compared to that inferred from the standard $\Lambda$CDM model, in a similar fashion to massive neutrino and standard warm DM. Contrarily to the latter scenarios, the time-dependence of the power suppression and the free-streaming scale allows the 2-body decaying DM scenario to accommodate CMB, baryonic acoustic oscillation, growth factor and uncalibrated supernova Ia data. We briefly discuss implications for DM model building, galactic small-scale structure problems and the recent Xenon-1T excess. Future experiments measuring the growth factor to high accuracy at $0 \lesssim z \lesssim 1$ can further test this scenario.

Introduction—The standard $\Lambda$-cold dark matter (LCDM) cosmological model provides a remarkable fit to a wide variety of observables, such as big bang nucleosynthesis (BBN), the cosmic microwave background (CMB), large-scale structures (LSS), baryonic acoustic oscillations (BAO), and uncalibrated supernovae of type Ia (SNIa) (see e.g. [1, 2] for reviews). Nevertheless, tremendous experimental developments have revealed curious discrepancies between different probes. At the heart of this study is the growing tension between the cosmological and local determination of the amplitude of the matter fluctuations on 8 h$^{-1}$Mpc scales, typically described through the parameter combination $S_8 \equiv \sigma(\Omega_m/0.3)^{0.5}$. Within LCDM, the latest $S_8$ value inferred from a fit to CMB data [3] is $\sim 2 - 3\sigma$ higher than that measured by a host of weak lensing surveys such as CFHTLenS [4], HSC [5], DES [6] and KiDS+Viking [7]. In particular, the recent joint analysis of KIDS1000+BOSS+2dFLenS has yielded $S_8 = 0.766_{-0.014}^{+0.020}$ [8] in $\sim 3\sigma$ discrepancy with CMB from Planck. While an unknown systematic effect at the origin of this discrepancy is not excluded, the existence of several independent observations disfavoring the LCDM predictions strengthen the case for new physics.

In this Letter, we show that the $S_8$ tension can be resolved if DM experiences 2-body decays where the decay products are one massive warm DM (WDM) particle and one (massless) dark radiation (DR) component. We will refer to the full model as $\Lambda$DDM. We find that it requires DM to have a lifetime of $\log_{10}(\Gamma^{-1}/\text{Gyr}) = 1.75_{-0.95}^{+1.4}$ and transfers a fraction $\epsilon \simeq 0.7^{+2.7}_{-0.6}\%$ of its rest mass energy into the DR species. Interestingly, depending on the velocity-kick received by the massive daughter, this scenario could help resolving some of the sub-galactic scales issues in LCDM (e.g. Refs. [9, 17]) or could explain the recent Xenon-1T excess [18, 19].

Many authors have attempted to explain the $S_8$ tension through new properties of DM (see, e.g., [20, 34]). Scenarios where the DM decays only into DR have been discussed in this context, but have been shown to be at odds with the latest Planck CMB lensing and BAO data [35–39]. The extension of these studies to the case of a massive daughter was recently performed in Ref. [32], where it was suggested that this scenario could resolve the ‘Hubble tension’ – The $\sim 5\sigma$ discrepancy between the value of the current expansion rate of the universe inferred from Planck data [3] under LCDM, and that measured using the cosmic distance ladder [40–44]. However, a recent series of analysis has shown that a combination of BAO, uncalibrated SNIa data-sets [45] and Planck data [46] excludes this model. Yet, authors of Refs. [32, 45, 46] have limited their analyses to the ADDM background evolution, or used crude approximations to deal with linear perturbations of the massive daughter particles.

In this work, we perform the first thorough analysis of the ADDM including a realistic treatment of linear cosmological perturbations. Our careful treatment of the warm daughter perturbations allows us to pin down the space of parameters resolving the $S_8$-tension. Indeed, the warm component produced by decay leads to a suppression of the matter power spectrum at late times, similar to that of massive neutrinos or standard WDM. However, contrarily to the latter scenarios, the specific time-dependence of the power suppression imprinted by the decay allows to accommodate Planck, BAO, uncalibrated SNIa and S8 measurements, though we confirm that it cannot simultaneously resolve the Hubble tension. Finally, we briefly discuss implications of these results for DM model building, Xenon-1T and the ‘small-scale crisis of CDM’.
Cosmology of 2-body DCDM— Our framework is characterized by two additional free parameters with respect to ΛCDM: the DCDM lifetime, Γ⁻¹, and the fraction of DCDM rest mass energy converted into DR, defined as follows [47]:

$$\varepsilon = \frac{1}{2} \left(1 - \frac{m_{\text{wdm}}^2}{m_{\text{dcdm}}^2}\right),$$  \hspace{1cm} (1)

where $0 \leq \varepsilon \leq 1/2$. The lower limit corresponds to the standard CDM case, whereas $\varepsilon = 1/2$ corresponds to DM decaying solely into DR.

The general form of the background and linear evolution equations for the DCDM, WDM and DR components can be found in Ref. [35, 48]. The dynamics of linear perturbations in the DR component is greatly simplified by integrating its phase-space distribution (PSD) over all the momentum degrees of freedom [35] (after expanding it over Legendre polynomials). To compute the WDM dynamics one cannot apply the same strategy, since the integral over the phase space is not analytic. One should follow the evolution of the full time- and momentum-dependent PSD, which would require solving $O(10^8)$ linear differential equations for realistic cosmological analyses. We tackle this issue by devising a new fluid approximation for the WDM species (we demonstrate its accuracy in Ref. [39]), which allows to integrate out the dependency on momenta and reduces the hierarchy of equations to the first three multipoles. The novel approximation scheme, based on the treatment of massive neutrinos as a viscous fluid by Refs. [50, 51], is only valid on sub-Hubble scales, where high- and low-\(\ell\) modes are effectively decoupled. In the synchronous gauge comoving with the DCDM species (see Ref. [52] for all relevant definitions), it yields the following expressions for the WDM continuity, Euler and shear equations [48, 49]:

$$\dot{\delta}_{\text{wdm}} = -3\mathcal{H}(c_g^2 - 1)\delta_{\text{wdm}} - (1 + w) \left(\theta_{\text{wdm}} + \frac{\dot{a}}{a}\delta_{\text{wdm}}\right)$$
$$+ (1 - \varepsilon) \frac{a\Gamma \rho_{\text{wdm}}}{\rho_{\text{dcdm}}}(\delta_{\text{dcdm}} - \delta_{\text{wdm}}),$$  \hspace{1cm} (2)

$$\dot{\theta}_{\text{wdm}} = -\mathcal{H}(1 - 3c_g^2)\theta_{\text{wdm}} + \frac{c_g^2}{1 + w} k^2 \delta_{\text{wdm}} - k^2 \sigma_{\text{wdm}}$$
$$- (1 - \varepsilon) \frac{a\Gamma \rho_{\text{wdm}}}{\rho_{\text{dcdm}}} \theta_{\text{wdm}},$$  \hspace{1cm} (3)

$$\dot{\sigma}_{\text{wdm}} = -3 \left(\frac{1}{2} + \frac{\mathcal{H} \xi}{1 + w} + (1 - \varepsilon) \frac{a\Gamma \rho_{\text{wdm}}}{\rho_{\text{dcdm}}}(\frac{1 + c_g^2}{1 + w})\right) \sigma_{\text{wdm}}$$
$$+ \frac{8w\varepsilon^2}{1 + w} \left[\theta_{\text{wdm}} + \frac{\dot{a}}{a}\right] - \frac{2 c_g^2 \xi}{1 + w} \rho_{\text{wdm}} \frac{\delta_{\text{dcdm}}}{\rho_{\text{dcdm}}},$$  \hspace{1cm} (4)

where $\delta_{\text{wdm}}$, $\theta_{\text{wdm}}$ and $\sigma_{\text{wdm}}$ represent the WDM density, velocity divergence and shear perturbations respectively, $w \equiv \rho_{\text{wdm}}/\rho_{\text{dcdm}}$ is the WDM equation of state, $\xi \equiv 2/3 - c_g^2 - \rho_{\text{wdm}}/3\rho_{\text{dcdm}}$ and the ‘dot’ refers to derivative with respect to conformal time. The shear equation is valid in the relativistic limit $\varepsilon \simeq 0.5$, and for $\varepsilon \leq 0.49$ we find that simply setting $\sigma_{\text{wdm}} \to 0$ is accurate at the $O(1\%)$ level in the matter power spectrum. As in Ref. [50], these equations rely on the assumption that the sound speed in the synchronous gauge $c_{\text{syn}}$ is nearly equal to the adiabatic sound speed $c_g$, and the latter can be computed as follows

$$c_g^2 = w \left(5 - \frac{5\rho_{\text{dcdm}}}{\rho_{\text{wdm}}} - \frac{\rho_{\text{dcdm}}}{\rho_{\text{wdm}}} \frac{\dot{\rho}_{\text{wdm}}}{\rho_{\text{wdm}}}(1 - \varepsilon)\right)$$
$$\times \left[3(1 + w) - \frac{\rho_{\text{dcdm}}}{\rho_{\text{wdm}}}(1 - \varepsilon)\right]^{-1},$$  \hspace{1cm} (5)

where the pseudo-pressure $p_{\text{wdm}}$ is a higher momenta integral of the background WDM PSD, equivalent to the standard pressure in the relativistic limit [50].

![FIG. 1: Reconstructed 2D posterior distribution in the ADDM and in the $\nu$ΛCDM models.](image)

Resolving the $S_8$ tension with DCDM— The 2-body ADDM scenario under study is fully described by the following set of free parameters:

$$\{\Omega_b h^2, \ln(10^{10} A_s), n_s, \tau_{\text{reio}}, \Omega_\text{dcdm}^{\text{ini}}, H_0, \log_{10} \Gamma, \log_{10} \varepsilon\}.$$  \hspace{1cm} (6)

We implement the DDM equations in the publicly available numerical code CLASS [50, 51]. We use a shooting method that guarantees that the budget equation is satisfied. We solve the Boltzmann hierarchy presented in Ref. [35] using 300 momentum bins. At sub-Hubble scales ($k \tau > 25$), we switch to the fluid equations given by Eqs. (2)-4. We make use of the code MONTEPYTHON-v3 [53, 54] to perform a Monte Carlo Markov chain (MCMC) analysis with a Metropolis-Hasting algorithm, testing the ADDM model against the
The decrease in S_{\text{KiDS1000+BOSS+2dfLenS}} measurement. Moreover, $\sigma_8 \equiv \min (\nu\Lambda \text{CDM}) \simeq 0.8$ (95% C.L.), which yields $S_8 \simeq 0.77$ in the ΛDDM model, for the best-fit ΛDDM model (red lines) and a νΛCDM model yielding the same $S_8$ (blue lines). The gray band indicates the approximate range of comoving wavenumbers contributing to $\sigma_8$. Bottom panel – Same as above, but for the (lensed) CMB TT, EE and lensing power spectrum. In this case, the gray bands show Planck 1σ errors.

**FIG. 2:** Top panel – Residuals in the linear matter power spectrum $P(k)$ at redshifts $z = 0.3$, with respect to our baseline νΛCDM model, for the best-fit ADDM model (red lines) and a νΛCDM model yielding the same $S_8$ (blue lines). The gray band indicates the approximate range of comoving wavenumbers contributing to $\sigma_8$. Bottom panel – Same as above, but for the (lensed) CMB TT, EE and lensing power spectrum. In this case, the gray bands show Planck 1σ errors.

Best-fit cosmology and impact on CMB and LSS– A C+WDM scenario like this one is expected to produce a suppression in the linear matter power spectrum on intermediate and small scales, with a non-trivial shape. The cut-off scale, as in the case of massive neutrinos, is determined by the WDM free-streaming scale $k_{fs}$. At wavenumbers $k > k_{fs}$, pressure becomes important and structure formation is inhibited. To better understand the ADDM success in resolving the $S_8$ tension compared to the case of massive neutrinos, we now illustrate the impact of both models on the relevant cosmological observables for our study.

In the top panel of Fig. 2 we compare the residual differences in linear matter power spectrum $P(k)$ with respect to our baseline νΛCDM model (first column of Tab. 1), for both the best-fit ADDM scenario (fourth column of Tab. 1) and a νΛCDM model with three degenerate massive neutrinos of total mass $M_\nu = 0.23$ eV (we adjust $\omega_{cdm} = 0.116$ whereas all other parameters are fixed to the baseline νΛCDM model), which yields $\sigma_8 = 0.76$ and $\Omega_m = 0.31$, in agreement with weak lensing data. These scenarios feature two key differences: i) a distinct redshift evolution for the power suppression. In the ADDM scenario, it is less significant at higher redshifts, since the abundance of the WDM
daughter is smaller; ii) a time-evolving cut-off scale; in the ADDM model, \( k_{\text{Ddm}} = \sqrt{3/2} H(a)/c_g(a) \), while in the \( \nu \Lambda\text{CDM} \) it is obtained by evaluating \( k_{\text{Ddm}} \) at the redshift at which neutrinos become non-relativistic \[20\]. As a consequence, the CMB power spectra, well constrained by Planck, are vastly different. This is illustrated in Fig. 3 bottom panel, for both the best-fit ADDM scenario and the \( \nu \Lambda\text{CDM} \) model which yield the same \( S_8 \) value. The \( \nu \Lambda\text{CDM} \) predicts different early-integrated Sachs-Wolfe effects, as well as different amount of lensing, because of a significant power suppression at \( z \sim 2 - 3 \), where the CMB lensing kernel peaks \[68\]. On the other hand, the differences between \( \Lambda\text{CDM} \) and ADDM until \( z \sim 2 \) are very small, explaining why Planck cannot disentangle between both scenarios. Detecting the DDM through its impact on CMB power spectra will be challenging, although CMB lensing measurements accurate at the \( \sim 1\% \) level could help (e.g. with CMB-S4 \[68\]). Fortunately, the differences between the growth rate \( f\sigma_8 \) in ADDM scenario and \( \Lambda\text{CDM} \) (shown in Fig. 3) at \( 0 \lesssim z \lesssim 1 \), while below the sensitivity of current experiments measuring, could be measured by upcoming surveys such as Euclid \[69\], LSST \[70\], and DESI \[71\].

Some implications of the DCDM—Concrete realizations of ADDM scenarios as the ones considered in this work may arise (for instance in the context of the ‘super weakly interacting massive particle’ (superWIMP) class of exotic particle physics models \[72\]–\[74\], whose super weak couplings make them evade many observational constraints \[75\]–\[76\]). To connect with parameters of these models, we can estimate typical orders of magnitude for the decay rate, \( \Gamma \approx (50 \text{ Gyr})^{-1}(g_{\text{eff}}/(10^{-18} \text{ or } 10^{-16}))^2(m/(0.01 \text{ or } 0.02 \text{ GeV}))^{\times(\varepsilon/0.007)^2} \times 4 \), where \( g_{\text{eff}} \) is the coupling constant for the decay, \( m = m_{\text{dcdm}} \) is the mass of the decaying particle, and the model-dependent power index is 2 for particle of spins \( \leq 1 \), and 4 for e.g. gravitino production. The decaying particle must have properties similar to CDM candidates, which sets a lower mass bound of \( m \gtrsim 5 \text{ keV} \) from structure formation if it is thermally produced in the early universe \[77\]–\[81\], raising to the MeV mass scale depending on couplings to standard model particles \[82\] (irrelevant for thermal production in hidden sectors \[83\] ). If produced non-thermally, the Tremaine-Gunn limit \[84\] of \( m \gtrsim 1 \text{ keV} \) \[85\] applies only to fermions.

Interestingly, late decays of CDM to WDM are among the possible proposed cures to some observational discrepancies with CDM on small (sub-galactic) scales after structure formation (e.g. \[86\]–\[91\], and e.g. \[80\]–\[81\] for reviews on small-scale issues)—see also e.g. \[88\]–\[91\] for different views. A detailed inspection of the effective parameter space was performed in Ref. \[92\], which was later supplemented by dedicated cosmological simulations \[17\], showing that this scenario mostly affects galaxy satellite/subhalo properties. In particular, a daughter particle speed \( 3 \times 10^{-3} \lesssim v \lesssim 2 \times 10^{-3} \) may reduce the abundance of subhalos and their concentrations for a large range of lifetimes, up to \( \sim 100 \text{ Gyr} \). Greater speeds are disfavored from the existence of dwarf galaxies (see also \[86\]–\[88\]), unless \( (\Gamma)^{-1} \gtrsim 100 \text{ Gyr} \), for which ADDM does not depart from \( \Lambda\text{CDM} \) as far as structure formation is concerned (this also holds for very low speeds irrespective of \( \Gamma \)). Further constraining our favored range for ADDM models along those lines would require to predict the non-linear matter power spectrum, which goes beyond the scope of this paper. However, our best-fitting parameters do fall in ranges that could partly address small-scale CDM challenges.

From different perspectives, remarking that the higher tail of the daughter particle speed contour reaches \( v \approx \varepsilon \sim 0.05 \) may lead to the intriguing possibility that this scenario be also connected with the excess events in the electronic recoils recently reported by the Xenon-1t Collaboration \[18\]. If one resorts to elastic scattering of our daughter particle off electrons, measured recoils in the keV energy range translate into a lower bound on \( m_{\text{wdm}} \sim m \gtrsim 1 \text{ MeV} \) \[19\], and require a local interaction rate with electrons of \( n_{\text{wdm}}\sigma_e \sim 3.5 \times 10^{-44} \text{ cm}^{-1} \). The former constraint is consistent with both thermal and non-thermal production of the parent CDM particle, while it is not clear whether the latter can be achieved while preventing the production of WDM particles in the early universe. Another possibility is that the daughter particle is a dark photon that kinetically mixes with photons, which could therefore transmit its entire mass energy to electrons from photoelectric processes \[94\]. This instead completely sets \( m_{\text{wdm}} \sim m \sim 2 \text{ keV} \) and no longer relies on \( \varepsilon \), which can therefore only be achieved.
in non-thermal production scenarios.Interesting realizations of the latter are e.g. found in models with a broken $U(1)_{B-L}$ gauge symmetry, also proposed, incidentally, to solve small-scale CDM issues (e.g. [95-97]).

Conclusions—In this Letter we have studied the cosmological phenomenology of a scenario in which CDM decays into one WDM and one DR species. Through a comprehensive MCMC analysis including up-to-date data, we demonstrated that the ADDM scenario can solve the $S_8$ tension. The inferred $S_8$ value is in excellent agreement with its direct measurement from WL, due to the suppression in the gravitational clustering induced by WDM free-streaming in a similar fashion to massive neutrino or standard WDM cosmologies. However contrarily to the latter, the specific time dependence of the power suppression imprinted by the decay allows to accommodate simultaneously all the data considered in this work. On the other hand, we confirm that this model cannot resolve the Hubble tension. Finally, we briefly outlined implications and possible future developments of our study, in view of DM model building, the ΛCDM small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess. Future experiments measuring the growth rate of fluctuations at small-scale crisis and the recent Xenon-1T excess.
| Model | Parameter | $\omega_{cdm}$ or $\omega_{cdm}^0$ | $H_0/[km/s/Mpc]$ | $\ln(10^{10}A_s)$ | $n_s$ | $\tau_{reio}$ | $M_\gamma/eV$ | $\log_{10}(\varepsilon)$ | $\log_{10}(\Gamma/[/Gyr^{-1}])$ | $A_\text{in}$ | $S_8$ | $\chi^2_{\text{min}}$ |
|-------|-----------|-------------------------------|-----------------|-----------------|-------|-------------|------------|-----------------|-----------------|--------|--------|---------------|
| $\Omega$ | $\omega_b$ | $2.245(2.242) \pm 0.013$ | $2.251(2.253) \pm 0.013$ | $2.243(2.242) \pm 0.013$ | $2.246(2.240) \pm 0.014$ |
| | $H_0/[km/s/Mpc]$ | $67.55(67.76) \pm 0.46$ | $67.85(68.08) \pm 0.47$ | $67.72(67.68) \pm 0.41$ | $67.94(67.70) \pm 0.42$ |
| | $\ln(10^{10}A_s)$ | $3.052(3.045) \pm 0.014$ | $3.047(3.043) \pm 0.014$ | $3.051(3.046) \pm 0.014$ | $3.049(3.051) \pm 0.014$ |
| | $n_s$ | $0.9676(0.9663) \pm 0.0037$ | $0.9697(0.9683) \pm 0.0036$ | $0.9674(0.9673) \pm 0.0037$ | $0.9683(0.9673) \pm 0.0038$ |
| | $\tau_{reio}$ | $0.058(0.055) \pm 0.007$ | $0.0569(0.0549) \pm 0.007$ | $0.0576(0.0552) \pm 0.0079$ | $0.0573(0.0558) \pm 0.0081$ |
| | $M_\gamma/eV$ | $<0.1395$ | $<0.1611$ | $<-0.79(-2.90)$ | $-2.23(-2.16)\pm 0.66$ |
| | $\log_{10}(\varepsilon)$ | -- | -- | -- | -- |
| | $\log_{10}(\Gamma/[/Gyr^{-1}])$ | -- | -- | -- | -- |
| | $A_\text{in}$ | $0.3127(0.3104) \pm 0.0057$ | $0.3083(0.3061) \pm 0.0056$ | $0.3101(0.3112) \pm 0.0054$ | $0.3068(0.3098) \pm 0.0053$ |
| | $S_8$ | $0.824(0.824) \pm 0.011$ | $0.81(0.816) \pm 0.01$ | $0.821(0.826) \pm 0.011$ | $0.795(0.773) \pm 0.015$ |
| | $\chi^2_{\text{min}}$ | $2053.4$ | $2060.5$ | $2053.4$ | $2055.1$ |

**TABLE 1:** The mean (best-fit) $\pm 1\sigma$ errors of the cosmological parameters from our combined analysis, with and without a split-normal likelihood on $S_8$ from Ref. [8]. For each model and data-set, we also report the best-fit $\chi^2$.
