Measuring leptonic $CP$ Violation in Neutrinoless Double Beta Decay

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Abstract

We investigate under which circumstances one can show the existence of leptonic $CP$ violation with the help of a positive or negative signal in neutrinoless double beta decay. The possibilities of cancellations are investigated for special mass hierarchies and the different solar solutions. The possibility that the mixing angle connected with the solar neutrino problem is smaller or larger than $\pi/4$ is taken into account. The non–maximality of that angle in case of the LMA solution allows to make several useful statements. The four different $CP$ conserving possibilities are analyzed. It is implemented how precisely the oscillation parameters will be known after current and future experiments have taken data. The area in parameter space, in which $CP$ violation has to take place, is largest for the LOW solution and in general larger for the inverse mass scheme.

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1 Introduction

Evidence for lepton flavor violation has been collected in large amounts, courtesy of neutrino oscillation experiments [1, 2]. An explanation of the smallness of the implied neutrino masses is given by the see–saw mechanism [3], which introduces Majorana neutrinos and thus lepton number violation to the theory. In recent years, the search for this phenomenon has concentrated on neutrinoless double beta decay ($0^{\nu}\beta\beta$). The decay width of this process is proportional to the square of the so–called effective mass of the electron neutrino,

$$\langle m \rangle = \sum U_{ei}^2 m_i,$$

(1)

where $U$ is the leptonic Maki–Nakagawa–Sakata (MNS) mixing matrix [4]. Since $\langle m \rangle$ depends on the neutrino masses, the two mixing angles connected with solar and reactor experiments and two phases in $U$, any measurement or non–measurement of $0^{\nu}\beta\beta$ can in principle answer some of the open questions of neutrino physics, a topic which in the past has been addressed by a number of authors [5, 6, 7, 8]. For example, by combining oscillation experiments and $0^{\nu}\beta\beta$, one can investigate the solution of the solar neutrino problem, the mass scheme, the value of the smallest mass eigenstate or the presence of leptonic CP violation. In this note we shall concentrate on the latter point. For Majorana neutrinos there are three phases in $U$, two of which can in principle be measured through $0^{\nu}\beta\beta$. These two additional phases [3] are parameters of an extended Standard Model (SM) and are of interest e.g. regarding the stability of the neutrino mass matrix under radiative corrections [10] or in governing the magnitude of the baryon asymmetry of the universe via the leptogenesis mechanism [11]. Regarding the latter it has recently been shown in two quite different models [12, 13] that even for vanishing CP violation in oscillation experiments, there can still be a sufficient baryon asymmetry generated. The amount of CP violation found in neutrinoless double beta decay is then crucial in testing different leptogenesis models. In contrast to the quark sector, there are four CP conserving possibilities for Majorana neutrinos, all of which have different aspects. We shall discuss them in some detail, finding that in many cases they can be classified into two groups, sometimes even one single case can be identified. We decided to ignore the recently announced controversial indication for $0^{\nu}\beta\beta$ [14]. See [3, 15] for a criticism of the statistical methods used in that analysis and [16] for replies. We shall only quote the measurement of the life–time limit on the neutrinoless double beta decay of $^{76}$Ge, which is $1.5 \cdot 10^{25}$ y [17]. Using different calculations for the nuclear matrix elements (NME), a limit on the effective mass of

$$\langle m \rangle < (0.30 \ldots 0.97) \text{ eV}$$

(2)

can be set. See [18] for a discussion of the different calculations. As future limits are concerned, several proposals for new experiments exist, such as CUORE [19], EXO [20], MOON [21], Majorana [22], or GENIUS [23], see [24] for a recent overview. As possible landmark limits we will assume 0.01 and $2 \cdot 10^{-3}$ eV, where the latter corresponds to the 10t GENIUS project. The expected uncertainty of the result is estimated to be around 20 to 30 % [24].
The paper is organized as follows: in Section 2 the required formalism is briefly reviewed and in Section 3 the results are presented for some special mass hierarchies and then in Section 4 the general case is analyzed, including the current and future uncertainty in the knowledge of the oscillation parameters. Section 5 summarizes our conclusions.

2 Formalism

Since $\langle m \rangle$ is the absolute value of a sum of three complex numbers, it depends on two phases. The neutrino mixing matrix $U$ can be parameterized in a very convenient form, which treats these two phases as the two additional Majorana phases:

$$U = U_{\text{CKM}} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}),$$  \hspace{1cm} (3)

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. Within the parameterization (3), $\langle m \rangle$ depends on $\alpha$ and $\beta$. The third phase $\delta$ can be probed in oscillation experiments [25]. For $CP$ conservation, different relative signs of the masses $m_i$ are possible, corresponding to the intrinsic $CP$ parities of the neutrinos [26, 27]. Four situations are possible, with $m_i = |\eta_i| m_i$ one can write these cases as $(+++)$, $(+-−)$, $(-+-)$ and $(-+)$, where the $(±±±)$ correspond to the relative signs of the mass states. Special values of the phases correspond to these sign signatures [27],

$$(+++)$ $\eta_1 = \eta_2 = \eta_3 = 1$ $\leftrightarrow \alpha = \beta = \pi$

$$(+-−)$ $\eta_1 = -\eta_2 = -\eta_3 = 1$ $\leftrightarrow \alpha = \beta = \pi / 2$ \hspace{1cm} (4)

$$(-+-)$ $\eta_1 = -\eta_2 = \eta_3 = -1$ $\leftrightarrow \alpha = \beta / 2 = \pi / 2$

$$(-+)$ $\eta_1 = \eta_2 = -\eta_3 = -1$ $\leftrightarrow \alpha = 2\beta = \pi$

The neutrino mass itself is bounded by the result from the tritium spectrum [28],

$$m_0 = \sum |U_{ei}|^2 m_i < 2.2 \text{ eV}. \hspace{1cm} (5)$$

Next generation laboratory [29] as well as satellite [30] experiments will be able to probe neutrino masses down to $\sim 0.4$ eV. Finally, two possible mass schemes exist, the normal and inverse scheme can be written as follows:

**NORMAL**

$$m_3 = \sqrt{\Delta m^2_{\Delta} + m_1^2} \quad m_1 = \sqrt{\Delta m^2_{\Delta} + m_3^2}$$

$$m_2 = \sqrt{\Delta m^2_{\odot} + m_1^2} \quad m_2 = \sqrt{-\Delta m^2_{\odot} + m_1^2}.$$  \hspace{1cm} (6)

**INVERSE**

$$m_1 = 0 \ldots 2.2 \text{ eV} \quad m_3 = 0 \ldots 2.2 \text{ eV}$$
Regarding the mixing angles and the mass squared differences, the best-fit points of the atmospheric oscillation parameters are \[31\]
\[
\Delta m^2_A = 2.5 \cdot 10^{-3} \text{eV}^2, \quad \tan^2 \theta_2 = 1. \tag{7}
\]
The maximal $\Delta m^2_A$ is about $5 \cdot 10^{-3} \text{eV}^2$. As far as the solar solution is concerned, the Large Mixing Angle (LMA) solution is favored over the low $\Delta m^2_\odot$ (LOW) solution by the latest data, especially after the recent SNO results \[4\]. The other possibilities are the vacuum solution (VAC) and the Small Mixing Angle (SMA) solution, which are currently highly disfavored. We will therefore concentrate mainly on LMA and LOW. Typical best-fit points are \[32\]
\[
\Delta m^2_\odot = 4.5 \cdot 10^{-5} \text{eV}^2, \quad \tan^2 \theta_1 = 0.4 \quad \text{LMA}
\]
\[
\Delta m^2_\odot = 1.0 \cdot 10^{-7} \text{eV}^2, \quad \tan^2 \theta_1 = 0.7 \quad \text{LOW} \tag{8}
\]
\[
\Delta m^2_\odot = 4.6 \cdot 10^{-10} \text{eV}^2, \quad \tan^2 \theta_1 = 2.4 \quad \text{VAC}
\]
The allowed ranges at 95 % C.L. of $t^2_1 \equiv \tan^2 \theta_1$ go from about 0.2 to 0.8 for LMA and 0.5 to 1 for LOW. In fact, the upper value of $t^2_1$ for LMA differs in the different available analyses, which appeared after the new SNO results \[33\]. We shall take 0.8 as an illustrative example. In case of the VAC solution, one may assume that $t^2_1$ lies between 0.4 and 3, i.e. also on the “dark side” of the parameter space. One may however safely state that maximal mixing is disallowed for the LMA solution, which will later on allow us to make some useful statements. See \[34\] for a detailed analysis of the consequences of this fact. The mass scale $\Delta m^2_\odot$ is lower than about $3 \cdot 10^{-4} \text{eV}^2$ for LMA and $3 \cdot 10^{-7} \ (3 \cdot 10^{-11}) \text{eV}^2$ for LOW (VAC). The third angle $\theta_3$ is constrained to be \[35\]
\[
\tan^2 \theta_3 \equiv t^2_3 \lesssim 0.08. \tag{9}
\]
In the future, it will be possible to reduce the error of the atmospheric parameters $\Delta m^2_A$ and $\sin^2 2\theta_2$ to about 10% by the MINOS, ICARUS and OPERA experiments \[36\]. The SNO, KAMLAND and BOREXINO collaborations are able to reduce the errors on $\sin^2 2\theta_1$ to about 5% and $\Delta m^2_\odot$ will be known to a precision of a few % for LMA, about 10% for LOW and a few % for VAC \[37\]. Long baseline experiments can make precision measurements of the atmospheric parameters on the few % level \[38\], neutrino factories even at a 1 % level \[39\]. Regarding $\tan^2 \theta_3$, the experiments for the atmospheric scale as well as future reactor experiments \[40\] can probe values of few·$10^{-3}$, long baseline experiments values of few·$10^{-5}$.

3 \textit{CP} violation in special hierarchies
3.1 Hierarchical scheme

This scheme is realized for \( m_3 \gg m_2 \gg m_1 \). The form of \( \langle m \rangle \) neglecting \( m_1 \) but including terms proportional to \( s_3 \) reads

\[
\langle m \rangle \simeq \sqrt{\Delta m_{\odot}^2} c_3^4 s_1^4 + \Delta m_{\odot}^2 s_3^4 + 2 \sqrt{\Delta m_{\odot}^2 \Delta m_{\odot}^2} s_1^2 s_3^2 c_3 c_2 \phi ,
\]

where \( \phi = \alpha - \beta \). The maximal \( \langle m \rangle \) for \( t_3^2 = 0 \) is about \( 7.7 \cdot 10^{-3} \) eV for a maximal \( \Delta m_{\odot}^2 \) of about \( 3 \cdot 10^{-4} \) eV\(^2\) in case of the LMA solution. For the other solutions it is around or below \( 10^{-4} \) eV. For the maximally allowed \( t_3^2 = 0.08 \) the effective mass can be at most \( 1.3 \cdot 10^{-2} \) (5.5 \( \cdot \) \( 10^{-3} \), 5.2 \( \cdot \) \( 10^{-3} \)) for LMA (LOW, VAC). For LMA and small \( s_3 \) the first term in Eq. (10) dominates, for larger \( s_3 \) the third term can cancel the first two when \( \phi \simeq \pi/2 \). For \( \alpha - \beta = \pi/4 \) the solar and atmospheric contributions to \( \langle m \rangle \) are just summed up. From values of \( \Delta m_{\odot}^2 \) smaller than about \( 10^{-7} \) eV\(^2\) on, the second term dominates in Eq. (10), \( \langle m \rangle \) is then proportional to \( s_3^2 \) and the dependence on \( \phi \) practically vanishes. This can be seen in Fig. [I], where \( \langle m \rangle \) is shown as a function of \( \phi \) for different \( \Delta m_{\odot}^2 \), \( s_3^2 \) and \( t_1^2 \). From (10), one can infer the phase difference as

\[
\cos 2(\alpha - \beta) = \frac{\langle m \rangle^2 - \Delta m_{\odot}^2 s_4^4 - \Delta m_{\odot}^2 s_3^4}{2 \sqrt{\Delta m_{\odot}^2 \Delta m_{\odot}^2} s_1^2 s_3^2} .
\]

As well known, it will be very difficult to measure \( \langle m \rangle \) in the hierarchical scheme when LMA is not the solar solution, because in order to give an accessible \( \langle m \rangle \), \( s_3^2 \) has to be very close to its current limit. For the LMA solution, \( \langle m \rangle \) can lie above \( 2 \cdot 10^{-3} \) eV even for vanishing \( s_3 \), which however requires large \( t_1^2 \) and \( \Delta m_{\odot}^2 \). If the two phases conspire to fulfill \( \alpha \simeq \pi/2 + \beta \), then cancellation occurs and \( \langle m \rangle \) can vanish. This happens also for the (− − +) and (− + −) configurations, i.e. when the second and third mass eigenstates have opposite signs. The (+ + +) and (− − +) cases correspond to \( \phi = 0 \).

Therefore, in case of the LMA solution and if \( s_3^2 \) is sizable (i.e. larger than about 0.03) and \( \langle m \rangle \) lies below the GENIUS limit, then \( \phi \) is located around \( \pi/2 \). The (+ + +) and (− − +) configurations are then ruled out. Values of \( \langle m \rangle \) considerably larger than 0.001 eV would show that \( \phi \) is close to zero, which corresponds to the (+ + +) or (− − +) signatures. For solutions with lower \( \Delta m_{\odot}^2 \) and \( t_3^2 \simeq 0.03 \) no statements can be made because the predicted \( \langle m \rangle \) is below the GENIUS limit. In addition, there is practically no dependence on the phases, even for sizable \( t_3^2 \).

3.2 Inverse hierarchical scheme

This scheme is realized if in the inverse scheme it holds \( m_1 \simeq m_2 \gg m_3 \). Thus, neglecting \( m_3 s_3^2 \), the effective mass reads

\[
\langle m \rangle \simeq \sqrt{\Delta m_{\odot}^2} \sqrt{1 - 4 s_1^2 c_1^2 s_3^2} = \frac{\sqrt{\Delta m_{\odot}^2}}{1 + t_1^2} \sqrt{(1 + t_1^2)^2 - 4 t_1^2 s_3^2} \simeq \sqrt{\Delta m_{\odot}^2} c_\alpha ,
\]
where the latter approximation holds for $t^2_1 \simeq 1$. Note that it is not a function of $\Delta m^2_{\odot}$. It allows for complete cancellation only for $t^2_1 = 1$, i.e. in the LMA solution there should be a non–vanishing effective mass (larger than about $5 \cdot 10^{-3}$ eV), whereas LOW allows for complete cancellation. For the best–fit points given in the previous section the effective mass is predicted to be smaller than 0.07 eV, independent of the solar solution. Fig. 2 shows $\langle m \rangle$ as a function of $\alpha$ for different $t^2_1$. One finds indeed very few dependence on $s^2_2$, $\beta$ and $\Delta m^2_{\odot}$. It is seen that for non–maximal solar mixing, $\langle m \rangle$ can be probed regardless of the phase. From Eq. (12), $\alpha$ can be calculated for given $t^2_1$, $\Delta m^2_{A}$ and $\langle m \rangle$:

$$s^2_\alpha \simeq \frac{1}{4} t^2_1 (1 + t^2_1)^2 \left(1 - \frac{\langle m \rangle^2}{\Delta m^2_{\odot}}\right) \simeq 1 - \frac{\langle m \rangle^2}{\Delta m^2_{\odot}},$$

where the last approximation holds again for $t^2_1 \simeq 1$. If e.g. $\langle m \rangle = 0.04$ (0.03) eV, then one gets $s^2_\alpha \simeq 0.4$ (0.8) for the best–fit point of the LMA solution and $s^2_\alpha \simeq 0.2$ (0.6) for the LOW case. With $s^2_\alpha \leq 1$ it is possible to find the condition

$$\frac{\langle m \rangle}{\sqrt{\Delta m^2_{\odot}}} \geq \frac{1 - t^2_1}{1 + t^2_1},$$

under which the inverse hierarchical scheme is valid. The number on the right–hand side lies between 1/9 and 2/3 for the LMA solution, between zero and 1/3 for the LOW case and zero and 1/2 for VAC. For instance, if $t^2_1 = 0.4$ (0.7, 2.4), then $\langle m \rangle \gtrsim 0.02$ (0.009, 0.02) eV. In this scenario, the best–fit points of the LMA and VAC solar solutions do not make any difference since they yield identical results for $\langle m \rangle$.

Therefore, maximal solar mixing and $\langle m \rangle$ above 0.01 eV means that $\alpha$ is small or close to $\pi$, which corresponds also to the $(+++) \text{ and } (---)$ cases. A value of $\langle m \rangle$ below the GENIUS limit implies that $\alpha \simeq \pi/2$, which is also possible for the $(+- -)$ and $(---)$ signatures. However, for such a small $\langle m \rangle$ a solar mixing angle very close to $\pi/4$ is required, i.e. if the LMA solution and the inverse scheme are verified, but $\langle m \rangle$ lies below the GENIUS bound, the inverse hierarchical scheme is ruled out. For non–maximal mixing, the dependence on $\alpha$ becomes smaller. Values of $\langle m \rangle$ below 0.01 eV are only possible for $\alpha \simeq \pi/2$ and $\theta_1 \simeq \pi/4 \pm 0.1$. Since in the inverse hierarchy the phase $\beta$ is connected with the smallest mass state $m_3$ as well as with the small quantity $s^2_3$, a determination of this parameter is very questionable. However, two of the four $CP$ conserving possibilities may be ruled out. The $(+++)$ and $(-+)$ cases as well as the $(++-)$ and $(+-)$ configurations give the same values because of the smallness of $m_3 s^2_3$.

\footnote{In the following, we will use positive $\langle m \rangle$. For $t^2_1 > 1$ it is understood that the absolute value of the right–hand side of (14) is taken.}
3.3 Degenerate scheme

This scheme is realized for \( m_1^2 \simeq m_2^2 \simeq m_3^2 \equiv m_0^2 \gg \Delta m^2_{\odot} \). It is then useful to define an “averaged mass” \( \bar{m} = \langle m \rangle / m_0 \), which reads

\[
\bar{m} \equiv \frac{\langle m \rangle}{m_0} = c_3^2 \sqrt{c_1^2 + s_1^2 + t_1^2 + 2(s_1^2 t_3^2 c_{2(\alpha-\beta)} + c_1^2(s_2^2 c_{2\alpha} + t_3^2 c_{2\beta}))} .
\]

(15)

No dependence on the solar \( \Delta m^2 \) exists. The four \( CP \) conserving configurations can be written as

\[
\bar{m} = \begin{cases} 
1 & (++) \\
\frac{1}{1 + t_1^2}(1 - t_1^2 - 2s_3^2) & (--) \\
\frac{1}{1 + t_1^2}(1 - t_1^2(1 - 2s_3^2)) & (+-) \\
1 - 2s_3^2 & (-+) 
\end{cases} .
\]

(16)

Note that the \((+++)\) and \((-+-)\) cases have an averaged mass independent on the solar solution. For the \((+++)\) configuration, \( \langle m \rangle = m_0 \) and for the \((-+-)\) case \( \langle m \rangle = m_0(1 - 2s_3^2) \), which is identical to the result for \((+++)\) for vanishing \( \theta_3 \). In the same limit, as well as for maximal solar mixing, the \((-+-)\) and \((+-+)\) cases are identical. In general, the \((-+-)\) and \((+-+)\) cases are connected via \( \theta_1 \rightarrow \pi/2 - \theta_1 \). Therefore, e.g. \( t_1^2 = 0.5 \) in the \((-+-)\) case is identical to \( t_1^2 = 2.0 \) in the \((+-+)\) configuration. This is however only interesting for the VAC solution, since the other ones have \( t_1^2 \leq 1 \). If \( t_1^2 < 1 \), then the minimal \( \bar{m} \) occurs for the \((-+-)\) configuration, if \( t_1^2 > 1 \), then in the \((+-+)\) case.

Cancellation can only occur for the \((-+-)\) and \((+-+)\) signatures, which however requires that \( t_1^2 \) is very close to one. In fact, for the \((-+-)\) case only \( t_1^2 = 1 \) together with \( s_3^2 = 0 \) can give full cancellation, whereas for \((+-+)\) also close to maximal solar mixing with non-vanishing \( \theta_3 \) is sufficient for complete cancellation. The minimal \( \bar{m} \) for the \((-+-)\) signature is 0.84, in the LMA solution the range of \( \bar{m} \) is 0.022 to 0.67 (0.11 to 0.69) for the \((-+-)\) \(((+-+)\) case, respectively. In the LOW solution, \( \bar{m} \) ranges from zero to 1/3 (zero to 0.44) for \((-+-)\) \(((+-+)\) and the VAC solution predicts \( \bar{m} \) to lie between zero and 0.54 (zero and 1/2) for \((-+-)\) \(((+-+)\).

We give a few examples for possible statements: for \( t_1^2 < 1 \) the minimal \( \langle m \rangle \) is obtained for the \((-+-)\) signature, thus, if \( \bar{m} \ll 0.02 \), then the LMA solution is ruled out. For \( \bar{m} \gtrsim 0.7 (0.6) \), the \((+++)\) or \((-+-)\) case has to be realized for the LMA (LOW, VAC) solution. For \( \bar{m} \gtrsim 0.84 \) the \((-+-)\) and \((+-+)\) cases are ruled out.

In analogy to Eq. (12) one can in the degenerate scheme write an equation for \( \langle m \rangle \) when \( s_3^2 \) is neglected:

\[
\langle m \rangle \simeq m_0 \sqrt{1 - 4s_1^2 c_1^2 s_3^2} = \frac{m_0}{1 + t_1^2} \sqrt{(1 + t_1^2)^2 - 4t_1^2 s_3^2} \simeq m_0 c_\alpha ,
\]

(17)
from which one obtains a formula for the phase $\alpha$,

$$s_\alpha^2 \simeq \frac{1}{4t_1^2} (1 + t_1^2)^2 \left(1 - \tilde{m}^2\right) \simeq 1 - \tilde{m}^2,$$

(18)

where the last approximation holds again for $t_1^2 \simeq 1$. The corresponding equation (13) for the inverse hierarchy should be a more appropriate relation since there the small quantity $s_3^2$ is multiplied with the smallest mass. In the degenerate scheme it contributes together with $m_0$, which for sizable $s_3^2$ could be a non-negligible number. In general, an area in $\alpha-\beta$ space can be identified, when a limit or value of $m_0$ or $\langle m \rangle$ is known [41, 27]. The smaller $\tilde{m}$ is, i.e. the more cancellation occurs, the closer $\alpha$ is to $\pi/2$. This however is equivalent to the $(+--)$ and $(-+-)$ signatures. Since (17) allows cancellation only for $t_1^2 = 1$, a vanishing $\langle m \rangle$ in the LMA case for a neutrino mass of $m_0^2 \gg \Delta m^2_A$ would mean that $s_3^2$ is not zero. The consistency relation for the scenario in this section reads

$$\tilde{m} \geq \frac{1-t_1^2}{1+t_1^2}.$$

(19)

Violation of this condition implies sizable $s_3^2$, which, from (16), can be obtained for $t_1^2 < 1$ as

$$s_3^2 = \frac{1}{2} \left(1 - t_1^2 - \tilde{m}(1 + t_1^2)\right).$$

(20)

For $t_1^2 > 1$ the $(-+-)$ case gives the minimal $\langle m \rangle$, and $s_3^2$ is obtained in this case as

$$s_3^2 = \frac{1}{2t_1^2} \left(\tilde{m}(1 + t_1^2) - (1 - t_1^2)\right).$$

(21)

We finally comment on a small possibility to calculate the phase $\beta$ for the SMA solution. Since $s_1^2 \approx 0$, $\langle m \rangle$ does hardly depend on the phase $\alpha$. If in Eq. (15) $c_1 \simeq 1$, then one gets

$$s_\beta^2 \simeq \frac{1 - \tilde{m}^2}{4s_3^2 c_3^2}.$$

(22)

The condition under which this is possible can be obtained from $s_\beta^2 \leq 1$ and reads

$$\tilde{m} \gtrsim \sqrt{1 - 4s_3^2 c_3^2} \gtrsim 0.84.$$

(23)

When $\langle m \rangle$ is close to 0.4 eV then this situation seems unlikely, since $s_3^2$ has to be close to its current limit and in addition $m_0$ must be close to the lowest experimentally accessible value in order to probe $\beta$.

3.4 Partial hierarchical schemes

These schemes are realized when the smallest mass state is of order of $\sqrt{\Delta m^2_A}$, say, between 0.01 and 0.1 eV.
3.4.1 Normal scheme

In the “normal partial hierarchical scheme” one can define again an averaged mass \( \langle m \rangle / m_1 \). In this scenario it can be obtained from Eq. (13) with the replacement

\[
\frac{\langle m \rangle}{m_1} = \bar{m} \left( m_0 \to m_1, s_3^2 \to s_3^2, \sqrt{1 + \frac{\Delta m^2_3}{m_1^2}} \right).
\] (24)

Again, no dependence on the solar \( \Delta m^2 \) is present. Depending on the value of \( m_1 \), the dependence on \( t_3^2 \) is more or less strengthened through the presence of the square root

\[ w_n = \sqrt{1 + \frac{\Delta m^2_3}{m_1^2}}. \]

The maximal \( \langle m \rangle \) is given by \( \langle m \rangle \simeq m_1(1 + s_3^2 w_n) \lesssim 0.11 \text{ eV} \). For vanishing \( t_3^2 \) complete cancellation is again only possible for \( t_1^2 = 1 \), i.e. not for the LMA solution. The situation is then equivalent to the one for the inverse hierarchical scheme. For vanishing \( t_3^2 \) one can write an equation for \( \alpha \) in analogy to (13), with the replacement \( \Delta m^2_3 \to m_1^2 \).

For \( t_1^2 = 0.4 \), \( \langle m \rangle \) lies below 0.01 eV only for \( \alpha \simeq \pi/2 \) and \( m_1 \lesssim 0.02 \text{ eV} \). If also \( \beta \simeq \pi/2 \) and \( t_3^2 \gtrsim 0.02 \), then \( \langle m \rangle \) can lie below the GENIUS bound. With increasing \( t_1^2 \), the minimal value of \( m_1 \), for which this happens, is increasing. Values below \( 2 \cdot 10^{-3} \text{ eV} \) are only possible if \( m_1 \) and \( t_1^2 \) are small and \( t_2^2 \) is sizable or if the mixing is close to maximal and \( t_3^2 \) is small. When \( t_1^2 \gtrsim 1.5 \), similar statements hold, however, \( \beta \simeq 0 \) is now required in order to allow for large cancellations. Thus, for large \( t_1^2 < 1 \) \( (t_1^2 > 1) \), sizable \( t_2^2 \) and \( \langle m \rangle < 2 \cdot 10^{-3} \text{ eV} \), then \( m_1 \) has to be small, \( \alpha \) around \( \pi/2 \) and \( \beta \) around \( \pi/2 \) \( (\pi) \). This situation corresponds to the \((+ - -) \) \( ((- + +)) \) configuration. For close to maximal solar mixing and \( \langle m \rangle < 2 \cdot 10^{-3} \text{ eV} \), \( \alpha \) is around \( \pi/2 \). If \( t_3^2 \) is sizable, then in addition \( m_1 \) has to be small. When \( \langle m \rangle \) is around 0.01 eV, the phase \( \alpha \) has to be small, which corresponds to the \((+ - -) \) or \((- + +) \) signature.

3.4.2 Inverse scheme

In the “inverse partial hierarchical scheme” \( \langle m \rangle / m_3 \) can be obtained from Eq. (13) with the replacement

\[
\frac{\langle m \rangle}{m_3} = \bar{m} \left( m_0 \to m_3, c_1^2 \to c_1^2, \sqrt{1 + \frac{\Delta m^2_3}{m_3^2}}, s_1^2 \to s_1^2, \sqrt{1 + \frac{\Delta m^2_A}{m_3^2}} \right),
\] (25)

which has a slightly weaker dependence on \( t_3^2 \) than in the normal hierarchy, because the contributions of \( c_1^2 \) and \( s_1^2 \) are enhanced by the factor \( w_i = \sqrt{1 + \frac{\Delta m^2_A}{m_3^2}} \). The maximal \( \langle m \rangle \) is given by \( \langle m \rangle \simeq m_3(w_i + s_3^2) \lesssim 0.12 \text{ eV} \). For vanishing \( t_3^2 \) one can obtain \( \alpha \) via Eq. (13) and the replacement \( \Delta m^2_3 \to w_1^2 m_3^2 = m_3^2 + \Delta m^2_A \). For \( t_1^2 \lesssim 0.6 \) and \( t_3^2 \gtrsim 1.5 \), \( \langle m \rangle \) lies always above 0.01 eV. For closer to maximal (but not exactly maximal) mixing it can lie...
below 0.01 eV for large \( t_{3}^{2} \) and low \( m_{3} \) or only for small \( t_{3}^{2} \) as long as \( \alpha \approx \pi/2 \) and \( \beta \approx \pi/2 \) (0) for \( t_{1}^{2} < 1 \left( t_{2}^{2} > 1 \right) \). Thus, values below 0.01 eV are possible when the solar mixing is maximal and \( \alpha \approx \pi/2 \) (which is equivalent to the \((+−−)\) or \((−++\)) signature) or if for \( t_{1}^{2} < 1 \) the second phase \( \beta \) is around \( \pi/2 \), which is the \((+−−)\) configuration. For \( t_{2}^{2} > 1 \), \( \beta \) should be close to \( \pi \), which corresponds to the \((−+++\)) case.

Once we finished now the discussion of the special hierarchies, we can order them with respect to the maximal \( \langle m \rangle \) they predict:

\[
\text{degenerate} > \text{partial inverse} > \text{partial normal} > \text{inverse} > \text{normal} .
\] (26)

4 General case

We shall use the best–fit oscillation parameters as given in Eqs. (7,8) and assume the following uncertainties of the solar \( \Delta m_{2}^{2} \): 5 % for LMA, 10 % for LOW and 5 % for VAC. For \( \tan^{2} \theta_{1} \) and \( \Delta m_{3}^{2} \) we assume an uncertainty of 5 and 10 %, respectively. The effective mass is analyzed as a function of the smallest mass state for different \( t_{3}^{2} \). What results with these assumptions is an area in parameter space, which denotes the region between the maximal and minimal \( \langle m \rangle \). Unless otherwise stated, the area for the \((+++\)) case is so small that it appears as a line.

4.1 Normal scheme

For the normal hierarchy, the result is shown in Figs. 3 to 5. The structure of the “\( CP \)-violating” area is the less complicated the smaller \( t_{3}^{2} \) is. From \( t_{3}^{2} \) smaller than about \( 10^{-3} \) \((10^{-4})\) on, the \((−+−)\) and \((+−−)\) signatures become indistinguishable for the LMA (LOW, VAC) solution. Up to six separated areas exist, for \( t_{3}^{2} \approx 10^{-3} \) they merge into one, which area is smaller than the sum of the areas for sizable \( t_{3}^{2} \). For the LOW solution the area is significantly larger than for LMA and for VAC. In case of VAC the area is smallest.

In Fig. 6 the consequences of different uncertainties of the oscillation parameters are shown. We concentrate on the LMA solution, \( t_{3}^{2} = 0.01 \) and start with a typical allowed parameter space, which is \( \Delta m_{3}^{2} = (1.5\ldots4) \cdot 10^{-3} \) eV\(^2\), \( \Delta m_{\odot}^{2} = (1\ldots12) \cdot 10^{-5} \) eV\(^2\) and \( t_{1}^{2} = 0.2\ldots0.8 \), which is denoted as “everything”. Then it is allowed for uncertainties of the solar and atmospheric parameters around the best–fit values of Eqs. (7,8), which are indicated in the figure. The lower right plot is for exact measurements, which coincides with the situation analyzed in Fig. 6. Similar plots have been presented first in Fig. 7, where the \( \Delta m^{2} \) have been allowed to vary within their 90\% C.L. values and different \( t_{1}^{2} \) and \( t_{3}^{2} \) have been taken. The situation under study in the present paper is more accurate with respect to the expected future uncertainty of the oscillation parameters. Now an area for the \((+++\)) case can be identified. For an uncertainty of 5\% and 10\% for the solar and
atmospheric parameters (as used in Fig. 3), the area becomes again a line and consequently is not shown anymore. Currently, there is only a small $CP$–violating area, between the minimal ($- - +$) and the maximal ($- + -$) line, although it exists at large $\langle m \rangle$ and $m_1$. The area grows with decreasing uncertainty and takes the complicated form known from the previous figures when the solar parameters are known to a precision better than 10%. These additional areas appear however around or below the maximal GENIUS limit.

### 4.2 Inverse scheme

The plots with the $CP$–violating areas are shown in Figs. 7, 8 and 9 for the LMA, LOW and VAC solution. The situation is now much simpler, $CP$ violation occurs only between the minimal ($- - +$) and the maximal ($- + -$) line. There is also a small area between the minimal ($- + -$) and the maximal ($+ - -$) line, which disappears for $t_3^2 \lesssim 0.01$. The ($+ - -$) and ($- + -$) signatures become indistinguishable for $t_3^2 \lesssim 0.01$. With the chosen oscillation parameters there is no complete cancellation and therefore all expected $\langle m \rangle$ values lie above 0.01 (0.006) eV for the LMA and VAC (LOW) case. Again, the LOW case provides the largest $CP$–violating area, the LMA and VAC areas are of comparable size.

As for the normal scheme, large part of the areas cover a range of $\langle m \rangle$ that is larger than the expected 20% uncertainty of the experimental results. This is negligible with respect to the uncertainty stemming from the NME calculations. Consequently, these have to be overcome in order to make reasonable statements (not only) about the presence of $CP$ violation in $0\nu\beta\beta$.

### 5 Final remarks and summary

The presence of leptonic $CP$ violation especially in $0\nu\beta\beta$ can strengthen our believe in leptogenesis, the creation of a baryon asymmetry through out–of–equilibrium decays of heavy Majorana neutrinos. These heavy neutrinos are also responsible for the light neutrino masses through the see–saw mechanism, linking thus neutrino oscillations with leptogenesis. Baryon number and $CP$ violation are necessary conditions for creating a baryon asymmetry. Given that in most models the heavy Majorana neutrinos are too heavy ($\gtrsim 10^{10}$ GeV) to be produced at realistic collider energies, the demonstration of lepton number violation and leptonic $CP$ violation could be the only possibility to validate leptogenesis. A general feature of models presented in the literature is the dependence of the baryon asymmetry $Y_B$ on the Majorana phases $\alpha$ and $\beta$. For example, in the left–right symmetric model presented in [12], a sufficient $Y_B$ can be generated without the “Dirac phase” $\delta$. This has also been observed in the minimal $SO(10)$ model analyzed in [13]. The presence of $CP$ violation in $0\nu\beta\beta$ is required there to produce the correct amount of $Y_B$. This is why $CP$ violation in $0\nu\beta\beta$ plays an important role.
In the light of recent data we analyzed the presence of $CP$ violation in neutrinoless double beta decay. The observed non–maximality of the solar mixing in case of the LMA solution allowed to make some statements about possible cancellations. The four $CP$ conserving sign signatures can in many cases be grouped into two pairs, in some cases even one unique solution can be identified. In the hierarchical scheme the $(+++)\) and $(+- -)$ cases are equivalent because $\langle m \rangle$ depends on the difference of two phases. In the inverse hierarchical scheme only one phase can be probed, which leads to identical results for the $(+++)\) and $(- --)$ cases as well as for the $(+- -)$ and $(- + -)$ cases. Due to the large solar mixing and the smallness of $s_{3}^{2}$, these two pairs also exist for the degenerate and partial hierarchical schemes. Simple formulas for the Majorana phases and consistency relations for these hierarchies have been collected and the different situation for values of $t_{1}^{2}$ smaller or bigger than one has been commented on. The $CP$ violating areas including present and future uncertainties of the mixing parameters were identified. The LOW solution provides the best opportunity to establish the presence of leptonic $CP$ violation, since the relevant area in parameter space is largest in this case, regardless of the mass scheme. Obviously, in the inverse scheme, where for small neutrino masses the predicted $\langle m \rangle$ is considerably higher, the situation is better. However, the uncertainty stemming from the calculation of the nuclear matrix elements will remain the big drawback for this possibility.

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References

[1] SuperKamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 85, 3999 (2000); SuperKamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 86, 5651 (2001); SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001).

[2] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002) and Phys. Rev. Lett. 89, 011302 (2002).

[3] M. Gell–Mann, P. Ramond, and R. Slansky in Supergravity, p. 315, edited by F. Nieuwenhuizen and D. Friedman, North Holland, Amsterdam, 1979; T. Yanagida, Proc. of the Workshop on Unified Theories and the Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto, KEK, Japan 1979; R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[4] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theo. Phys. 28, 870 (1962).
[5] S.M. Bilenky et al., Phys. Rev. D 54, 1881 (1996), Phys. Rev. D 54, 4432 (1996), Phys. Lett. B 465, 193 (1999); V. Barger, K. Whisnant, Phys. Lett. B 456, 194 (1999); F. Vissani, JHEP 06, 022 (1999); M. Czakon, J. Gluza, and M. Zralek, Phys. Lett. B 465, 211 (1999); H.V. Klapdor–Kleingrothaus, H. Päs, and A.Yu. Smirnov, Phys. Rev. D 63, 073005 (2001); D. Falcone, F. Tramontano, Phys. Rev. D 64, 077302 (2001); S.M. Bilenky, S. Pascoli, and S.T. Petcov, Phys. Rev. D 64, 053008 (2001); P. Osland, G. Vigdel, Phys. Lett. B 520, 143 (2001); M. Czakon et al., Phys. Rev. D 65, 053008 (2002); H. Minakata, H. Sugiyama, Phys. Lett. B 526, 335 (2002); V. Barger et al., Phys. Lett. B 532, 15 (2002).

[6] F. Feruglio, A. Strumia, and F. Vissani, Nucl. Phys. B 637, 345 (2002).

[7] S. Pascoli, S.T. Petcov, and L. Wolfenstein, Phys. Lett. B 524, 319 (2002).

[8] S. Pascoli, S.T. Petcov, hep-ph/0111203.

[9] S.M. Bilenky, J. Hosek, and S.T. Petcov, Phys. Lett. B 94, 495 (1980); J. Schechter, J.W.F. Valle, Phys. Rev. D 22, 2227 (1980); Phys. Rev. D 23, 1666 (1981).

[10] N. Haba, Y. Matsui, and N. Okamura, Eur. Phys. J. C 17, 513 (2000).

[11] For recent reviews see, e.g. A. Pilaftis, Int. J. of Mod. Phys. A 14, 1811 (1999); W. Buchmüller, M. Plümacher, Int. J. of Mod. Phys. A 15, 5047 (2000).

[12] A.S. Joshipura, E.A. Paschos and W. Rodejohann, JHEP 08, 029 (2001); K.R.S. Balaji, W. Rodejohann, Phys. Rev. D 65, 093009 (2002).

[13] G. Branco et al., hep-ph/0202030.

[14] H.V. Klapdor–Kleingrothaus et al., Mod. Phys. Lett A 16, 2409 (2001).

[15] C.E. Aalseth et al., hep-ex/0202018.

[16] H.V. Klapdor–Kleingrothaus, hep-ph/0205228, H.L. Harney, hep-ph/0205293.

[17] H.V. Klapdor–Kleingrothaus et al., Eur. Phys. J. A 12, 147 (2001).

[18] A. Morales, Nucl. Phys. B (Proc. Suppl.) 77, 335 (1999); W.C. Haxton, G.J. Stephenson, Prog. Part. Nucl. Phys. 12, 409 (1984).

[19] S. Pirro et al., Nucl. Instrum. Methods A 444, 71 (2000).

[20] M. Danilov et al., Phys. Lett. B 480, 12 (2000).

[21] H. Ejiri et al., Phys. Rev. Lett. 85, 2917 (2000).

[22] C.E. Aalseth et al., hep-ex/0202020.

[23] H.V. Klapdor–Kleingrothaus et al., hep-ph/9910203.
[24] S.R. Elliot, P. Vogel, [hep-ph/0202264].

[25] M. Freund, P. Huber, and M. Lindner, Nucl. Phys. B 615, 331 (2001); Y. Farzan, A. Yu. Smirnov, Phys. Rev. D 65, 113001 (2002); P. Huber, M. Lindner, and W. Winter, [hep-ph/0204352] and references therein.

[26] L. Wolfenstein, Phys. Lett. B 107, 77 (1981); S.M. Bilenky, N.P. Nedelcheva, and S.T. Petcov, Nucl. Phys. B 247, 61 (1984).

[27] W. Rodejohann, Nucl. Phys. B 597, 110 (2001).

[28] Mainz Collaboration, C. Weinheimer et al., Nucl. Phys. B (Proc. Suppl.) 91, 273 (2001).

[29] KATRIN Collaboration, A. Osipowicz et al., [hep-ex/0109033].

[30] MAP project, [http://map.gsfc.gov], PLANCK project [http://astro.estec.esa.nl/SA-general/Projects/Planck].

[31] T. Toshito for the SuperKamiokande collaboration, [hep-ex/0105023].

[32] J.N. Bahcall, M.C. Gonzalez–Garcia, and C. Peña–Garay, JHEP 08, 014 (2001).

[33] A. Bandyopadhyay, S. Choubey, and S. Goswami, [hep-ph/0204173]; J.N. Bahcall, M.C. Gonzalez–Garcia, and C. Peña–Garay, [hep-ph/0204194]; V. Barger et al., Phys. Lett. B 537, 179 (2002); A. Bandyopadhyay et al., Phys. Lett. B 540, 14 (2002); P. Aliani et al., [hep-ph/0205053]; P.C. de Holanda, A. Yu. Smirnov, [hep-ph/0205241].

[34] S. Pascoli, S.T. Petcov, [hep-ph/0205022].

[35] S.M. Bilenky, S.T. Petcov, and D. Nicolo, Phys. Lett. B 538, 77 (2002).

[36] V. Barger et al., Phys. Rev. D 65, 053016 (2002).

[37] V. Barger, D. Marfatia, and B.P. Wood, Phys. Lett. B 498, 53 (2001); A. Strumia, F. Vissani, JHEP 11, 048 (2001).

[38] Y. Itow et al., [hep-ex/0106019].

[39] A. Bueno, M. Campanelli, and A. Rubbia, Nucl. Phys. B 589, 577 (2000); see also M. Freund, P. Huber, and M. Lindner in [25] and references therein.

[40] Yu. Kozlov, L. Mikaelyan, and V. Sinev, [hep-ph/0109277]; S. Schoenert, Nucl. Phys. Proc. Suppl. 110, 277 (2002).

[41] H. Minakata and O. Yasuda, Phys. Rev. D 56, 1692 (1997); T. Fukuyama, K. Matsuda, and N. Nishura, Phys. Rev. D 57, 5844 (1998); Mod. Phys. Lett A 13, 2279 (1998); R. Adhikari and G. Rajasekaran, Phys. Rev. D 61, 031301 (2000); K. Matsuda et al., Phys. Rev. D 64, 013001 (2001).
Figure 1: The effective mass $\langle m \rangle$ in the hierarchical scheme as a function of $\phi = \alpha - \beta$ for different $\tan^2 \theta_3$, $\tan^2 \theta_1$ and $\Delta m^2_{\odot}$. The smallest mass is $m_1 = 10^{-5} \text{ eV}$, $\Delta m^2_{\odot} = 2.5 \cdot 10^{-3} \text{ eV}^2$ and $\beta$ is chosen to be zero.
Figure 2: The effective mass $\langle m \rangle$ in the inverse hierarchical scheme as a function of $\alpha$ for different $t_1^2$. The smallest mass is $m_3 = 10^{-5}$ eV, $\Delta m_A^2 = 2.5 \cdot 10^{-3}$ eV$^2$, $\Delta m_\odot^2 = 4.5 \cdot 10^{-5}$ eV$^2$, $s_3^2 = 0.01$ and $\beta = 0$. 
Figure 3: The range of $\langle m \rangle$ in the normal scheme for the LMA solution and an uncertainty of the oscillation parameters as described in the text. The “$CP$ violating” area is indicated by the hatched area.
Figure 4: The range of $\langle m \rangle$ in the normal scheme for the LOW solution and an uncertainty of the oscillation parameters as described in the text. The “$CP$ violating” area is indicated by the hatched area.
Figure 5: The range of $\langle m \rangle$ in the normal scheme for the VAC solution and an uncertainty of the oscillation parameters as described in the text. The "CP violating" area is indicated by the hatched area.
Figure 6: The range of $\langle m \rangle$ in the normal scheme for the LMA solution, $\theta_3^2 = 0.01$ and different uncertainties of the oscillation parameters as described in the text. The “CP violating” area is indicated by the hatched area.
Figure 7: The range of $\langle m \rangle$ in the inverse scheme for the LMA solution and an uncertainty of the oscillation parameters as described in the text. The “$CP$ violating” area is indicated by the hatched area.
Figure 8: The range of $\langle m \rangle$ in the inverse scheme for the LOW solution and an uncertainty of the oscillation parameters as described in the text. The “$CP$ violating” area is indicated by the hatched area.
Figure 9: The range of $\langle m \rangle$ in the inverse scheme for the VAC solution and an uncertainty of the oscillation parameters as described in the text. The "CP violating" area is indicated by the hatched area.