Reconstruction of the Francis 99 main runner blade using a hybrid parametric approach.

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Abstract. The runner blade shape has been described as a set of discrete sections joining hub and shroud, resulting in such 3D free geometrical forms of considerable complexity that the search for an appropriate parametric approach for its reconstruction and optimization is still a matter of research field. For this reason, the main purpose of this study has been to reconstruct a real and twisted runner blade with a required accuracy through a methodology that uses several algorithms to combine sections and surfaces parametric approaches. Using the surface approach, the free-form shape of the 3D camber surface was defined by a multi-directional interpolation procedure. Using the section approach a Direct Profiling method was tested to fitting the hydrofoil section to the real one. For the blade reconstruction, ten profiles were stacked along the spanwise direction of the real blade to obtain by interpolation the pressure and suction sides using a lofting procedure, thus achieving an acceptable continuous and smooth surface definition. We assume that this methodology could be efficient to reconstruct a damaged and worn free-form blade, especially when the original surface geometry implies sections with different Thickness Distribution along the spanwise direction.

Key words: Turbomachinery, Francis turbine, runner blade, Thickness Distribution, Design Parameterization.

1. Introduction

In turbo-machinery, important efforts are concentrated on the parametric design of the rotor blades and the search of the optimal shape of flow passages remains as fundamental effort of the designer maintaining as an important research field. In the actual turbo-machinery design, the turbine blades can be seen geometrically as free form of considerable complexity resulting in blade shapes of twisted forms. Nowadays the coupling of aided computer tools (CAD, CFD, FEM, etc.) allow reaching greater upgrades in the geometry of the blades and increase the accessibility and comprehension to perform the design process by professionals with less experience.

Researchers classify blade parameterization in two main groups: those that use a sectional approach and those that use surface patches [1]. In regard to their potential for design optimization purposes, the sectional approaches provide a large number of shapes variations; however, they all present the disadvantage that an increase in the number of sections required to define the blade shape comes with an increase in the number of parameters involved. The use of surface patches seems effective for
reducing design parameters. Nevertheless, the existing formulations are defined analytically and for a limited range of applications.

Typically, the sectional approach consists in two phases: optimal design of 2D sections and then stacking them along the span-wise direction [2]. For the parameterization of the airfoil geometry, several methods have been proposed [3]. The airfoils are defined as a y-coordinate at prescribed chord wise locations. A second approach models the geometry as a linear combination of a basis airfoil and a set of perturbation functions. A third method use geometric parameters such as leading-edge radius, thickness and maximum curvature. A fourth method uses the control point of Non-Uniform Rational B-Splines (NURBS) curves to define the airfoils.

Concerning to the stacking optimization, usually the sections are arranged in radial direction along the stacking line. [2] parameterized the stacking line as a polynomial or free form curve. [4] represents the rotor blade shape by four blades profiles linearly interpolated in the spanwise direction. [5] stacked seven profiles and created a composite surface fitted to the stacked profiles. [6] assumed that the turbine works in a parallel flow having to construct a turbine blade from given planar profile curves (skeleton) in parallel planes.

Thus, the methodologies used have not permitted the stacking of an important amount of sections in order to better define the blade surfaces. In consequence, the search of an optimal flow passage by means of a flexible parametric approach still is fundamental for the designer and remains as a research field.

The main purpose of this study has been to reconstruct a real and twisted runner blade with a required accuracy through a methodology defined by [7] but that uses a new algorithm to combine the section and surface parametric approaches.

Using the surface approach, the free-form shape of the 3D camber surface was defined by a multi-directional interpolation procedure which created a grid conforming (same form or shape) to the specified boundaries of the real blade (hub and shroud profile and leading a trailing edge). As a result, the skeleton of the camber-surface inside the blade was defined in a bi-directional way; along the span and stream wise direction.

Using the section approach, the fitting of the hydrofoil section to the real one was tested through the Direct Profiling. The Bernstein polynomial of grade four was used to accurately represent the hydrofoil sections by control points through an automated optimization. Then, ten different sections were stacked in the spanwise direction. The interpolation of this number of sections created an increase in the number of stacked profiles. This action was fundamental in defining the blade shape through a lofting procedure for creating the 3D Pressure and Suction Sides of a twisted blade.

2. Methodology

As presented by [7], the original reconstruction methodology, shown in figure 1, consisted of three main steps: data extraction, blade reconstruction and numerical blade evaluation.

Figure 1. Blade reconstruction methodology.
However, this new methodology uses several real sections (red) stacked along spanwise direction instead of a main section to better reconstruct the thickness evolution of a twisted and reflexed blade.

3. Test case
In figure 2, the parts of the twisted and re-flexed free-form main blade is presented as a solid. This blade was provided as a digital file by NTNU-Norwegian University of Science and Technology under the Francis 99 workshop series, [8].

![Figure 2. Francis 99 runner main blade.](image)

It consists of a Pressure Side (PS) and a Suction Side (SS), and a Leading Edge (LE) where the water enters and a Trailing Edge (TE) where it exits, as well as an upper (hub) and lower (shroud) sections which are joined to the crown and band respectively.

4. Data extraction.
For the data extraction stage, discrete points that define the leading edge, trailing edge and Mean Camber Line (MCL) of the hub and shroud were extracted from the data file to construct the Blade Frame (BF), figure 3a. Through the Trans Finite Interpolation an algorithm (wit a mesh generating tool), the Camber Surface (CS) was obtained, figure 3b. The discretization of the CS allowed the generation of the Camber Skeleton (CSK), which was inserted into the real blade, figure 3c.

![Figure 3. Construction of the CSK.](image)

Due to the clustering and orthogonality of the mesh that defines the CSK, it was possible to stack ten cutting surfaces along the spanwise direction of the real blade, figure 4. The intersection along the spanwise direction between the cutting and the Pressure and Suction surfaces allowed extracting the discrete collected points, figure 5, with which the real hydrofoils were built.
The Thickness Distribution (TD) of the blade is used to offset points for the upper and for the lower section profile curves from de MCL and create the pressure and suction lines of the hydrofoil section. Figure 6 shows how the collected data, taken from each real hydrofoil, were related to the MCL to build the TD along the streamwise direction, i.e., from the LE to the TE. Moreover, figure (6) shows how the TD of each hydrofoil section evolves along the spanwise direction, i.e., from the crown to the band sections. In figure 6b, it is observed that the TD maintains the same evolution in the first 50% of the MCL; however, the main evolution changes are clearly visible after the maximum thickness, towards the TE of the blade.

[9] demonstrated that not only the Thickness Distribution of the blade TD, but also its Thickness Evolution (TE) can modify hydrodynamic and mechanically the runner performance. However, to define different sections at specific span positions will imply an increase in the number of parameters involved in the reconstruction process. For this reason, looking for improving its quality description but reducing the parametric dimensions, [7] stacked only one hydrofoil blade constructed with the mean TD but in a considerable number of span positions. In this new study, we will stack ten different hydrofoils sections along the spanwise of the real blade instead of only one to improve even more the numerical blade to the real one.

5. The blade reconstruction.
Due to the geometric complexity of the hydrofoils sections stacked along the spanwise direction of the
real blade, a four grade Bernstein polynomial was used to accurately represent them by adjusting its control points. According to [10], a Bernstein polynomial $P(x)$ of order $n$ approximates a function $f(x)$ along an interval using (1).

$$P(x) = \sum_{i=0}^{n} c_i B_i^n(x)$$  \hspace{1cm} (1)

Where the $B_i^n$ are Bernstein basis polynomials with respect to the variable $x$, and the $c_i$ are control points. (2) is the general form that can be used depending on $n+1$ control points:

$$P(x) = \sum_{i=0}^{n} f \left( i \frac{b-a}{n} + a \right) \frac{n!}{i! (n-i)!} \frac{(x-a)^i (b-x)^{n-i}}{(b-a)^n}$$  \hspace{1cm} (2)

Where $[a, b]$ represents the interpolation interval.

5.1 Hydrofoil sections

Using a new algorithm, it was possible to reconstruct the sections that define the surfaces of the blade. This algorithm was able to cut the curves of the PS and SS sides of each hydrofoil section, in three parts, this in order to be able to greatly reduce the complexity of the PS and SS curves. Subsequently, each new curve was adjusted and reproduced by a Bernstein polynomial of grade four with five parameters that defined five control points per curve. To define each section of the blade, 30 control points were used as is shown in figure 7.

![Figure 7](image)

**Figure 7.** Control points of the six curves used to define each section of the blade.

The shared control points at the ends of each curve were eliminated, resulting in a reduction to only three control points per curve resulting in 18 points per section and 180 to define the entire blade.

5.1.1 Pressure Side and Suction Side

The real hydrofoil sections, stacked along the blade, were fitting by the Adjusted Sections (AS). Several Interpolated Sections (IS) were obtained and stacked between the AS’s, (without adding more parameters). Figure 8 shows the AS and the IS which should define more exactly the geometry of the blade.
To generate continuous pressure and suction surfaces along the span of the blade, the interpolation algorithm can increase the IS to be stacked between the AS’s. Furthermore, the points that make up each section were kept constant, since according to [7], 214 points define precisely and uniformly each adjusted and interpolated section.

6. Shape error evaluation

In order to compare the adjustment of the reconstructed blade to the real one, (3) was used. This equation calculated the deviation \( d \) between the plane created by the collected points, shown in figure 5, and the grid points created by an orthogonal tensor product algorithm, [7], on the pressure and suction side \((x_i, y_i, \text{ and } z_i)\).

\[
d = \frac{|(ax_i + by_i + cz_i + d)|}{\sqrt{a^2 + b^2 + c^2}} \quad (3)
\]

Thus, from the deviation is obtained the mean and maximum shape error, which is presented in Table 1, in relation to the real runner diameter. It shows the number of interpolated sections that defines the blade shape and the grid points that defines each section, both used to reconstruct the blade. Using 19 sections, 10 AS and 9 IS, it was possible to improve by three orders of magnitude the maximum error established by [11], who considered, in an axial-radial blade, a maximum error of 5 mm, (0.1% of the turbine diameter) as valid to accept a blade reconstruction.

| Case | Blade Sections | Total Points To Define the Blade | Pressure Side Error (%) | Suction Side Error (%) |
|------|----------------|---------------------------------|------------------------|------------------------|
| Real | 10             | 2,140                           | -                      | -                      |
| 1    | 19             | 2,144                           | 5.07×10^4              | 3.57×10^3              |
| 2    | 37             | 4,066                           | 7.91×10^4              | 2.52×10^3              |
| 3    | 73             | 15,622                          | 1.43×10^4              | 1.24×10^3              |
| 4    | 145            | 31,030                          | 6.34×10^5              | 5.59×10^4              |
| 5    | 289            | 61,846                          | 3.17×10^5              | 5.07×10^4              |
| 6    | 577            | 123,478                         | 1.81×10^5              | 1.38×10^4              |
Figure 9 shows the volume change and convergence history of the reconstructed blade as the section number increases. The reconstructed blade suffers a relative volume increase of two orders of magnitude when the number of sections changes. If we use this parameter as a predictor of a better shape definition, this relative volume change could imply that the reconstructed blade has not been totally defined and its smoothing and continuous pressure and suction surfaces have not been reached yet.

Figure 9. Convergence history of the volume change and the maximum error as the interpolated sections are increased.

Figure 10 illustrates the distribution of the shape differences between the analytical and the actual surfaces, when different amount of interpolated sections were stacked. The maximum error reached in relation to the runner diameter is reduced as the interpolated sections are increased and it was found mainly at its reflexed and edges zones.

a) 19
b) 37
Figure 10. Maximum error distribution on the blade surface in (%) according to the stacked sections number.

Although the shape error convergence was unattainable, figure 9, the results suggest that at least 19 hydrofoil sections placed along the spanwise direction would be necessary to obtain an acceptable fitting to the real blade. However, it is necessary to reach the no volume change in order to assure a continuous and smooth blade.
7. Conclusion
This paper described the numerical process based on a hybrid parametric approach which was able to duplicate the real geometry of a real Francis runner blade. With this methodology has been possible to consider the evolution of the TD of a real blade, stacking an important number of hydrofoils sections along the spanwise direction. However, it is considered that some modifications should be taken into account to better represent different blade shapes.

Although a good fitting and an acceptable smoothness and continuity were reached for the reconstructed blade, it must be noticed that the real validation of this methodology should be related to the fluid and structural analysis to evaluate at what extent the blade performance could be affected or improved by the blade shape error.

Finally, since this method provided a simplified blade representation by creating only its camber-skeleton and the hydrofoil section, it would be possible to use it as Reverse Engineering Technology and recover the CAD model of old turbines blades, which could be of the interest for industrial purposes.

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