Multifrequency synthesis algorithm based on the generalized maximum entropy method: application to 0954+658

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ABSTRACT

We propose the multifrequency synthesis (MFS) algorithm with the spectral correction of frequency-dependent source brightness distribution based on the maximum entropy method. In order to take into account the spectral terms of nth order in the Taylor expansion for the frequency-dependent brightness distribution, we use a generalized form of the maximum entropy method. This is suitable for the reconstruction of not only positive-definite functions, but also sign-variable functions. With the proposed algorithm, we aim to produce both an improved total intensity image and a two-dimensional spectral index distribution over the source. We also consider the problem of the frequency-dependent variation of the radio-core positions of self-absorbed active galactic nuclei, which should be taken into account in a correct MFS. The proposed MFS algorithm has first been tested on simulated data and then applied to the four-frequency synthesis imaging of the radio source 0954+658 using Very Large Baseline Array observational data obtained quasi-simultaneously at 5, 8, 15 and 22 GHz.

Key words: methods: data analysis – techniques: high angular resolution – techniques: image processing – techniques: interferometric – galaxies: jets – galaxies: nuclei.

1 INTRODUCTION

At present, very-long-baseline interferometry (VLBI) is the most powerful tool for studying morphological structures as well as the kinematic, polarization and spectral characteristics of active galactic nuclei (AGNs). It allows objects to be imaged with a very high angular resolution, reaching fractions of a milliarcsecond (mas). One of the topical problems of the VLBI mapping of AGNs is multifrequency image synthesis. Our interest in this method is mainly related to the peculiar geometry of the future high-orbit ground-space radio interferometer Radioastron (Kardashev 1997), which is expected to provide an ultra-high resolution (mas), but poor aperture filling (Bajkova 2005).

Multifrequency synthesis (MFS) in VLBI suggests mapping AGNs at several frequencies simultaneously in order to improve the instrument aperture filling. This is possible because the interferometer baselines are measured in wavelengths of the emission being received. The problem of MFS is complicated because of the frequency-dependent brightness of a radio source. Hence, to avoid undesirable artefacts in the reconstructed image, spectral correction should be made at the stage of its deconvolution.

Conway, Cornwell & Wilkinson (1990), Conway (1991), Sault & Wieringa (1994) and Sault & Conway (1999) have investigated the influence of spectral effects on images and they have developed methods for their correction. They have shown that if narrow frequency bands, up to ±12.5 per cent of the reference frequency, are used, the effects of the spectral dependence of the brightness of a radio source can usually be ignored for dynamic ranges of less than 1000:1. When the spectral errors are above the noise, they can be recognized and removed to ensure the required dynamical range of images. The spectral errors can usually be accounted for by parametrizing the image in terms of two unknowns at each pixel: the intensity at some reference frequency and the spectral index, if the spectral variation of the source emission is modelled by a power-law relationship. Thus, the use of MFS doubles the number of unknowns. However, in case of MFS we have nf more data of observations, where nf is the number of frequencies. As discussed by Conway (1991), if nf is greater than 2, the MFS image remains better constrained than the single-frequency image.

The algorithm of linear spectral correction based on the CLEAN method (Högbom 1974), called ‘double deconvolution’ (Conway et al. 1990), is the best-studied algorithm. In this algorithm, the ‘dirty’ image is first deconvolved with an ordinary ‘dirty’ beam, and the residual map is then deconvolved with the beam responsible for the first-order spectral term. An improvement of this method was proposed by Sault & Wieringa (1994), which consisted of a simultaneous reconstruction of the sought-for image and the map of the spectral term. The vector relaxation algorithm developed by Likhachev, Ladynin & Guirin (2006) can be considered as a...
An alternative deconvolution method, also actively used in radio astronomy, is the maximum entropy method (MEM). MEM was first proposed by Frieden (1972) and Ables (1974) for the reconstruction of images in optics and radio astronomy, respectively. Since then, the method has been developed in many studies (Wernecke & D’Addario 1976; Skilling & Bryan 1984; Cornwell & Evans 1985; Narayan & Nityananda 1986; Frieden & Bajkova 1994) and implemented in a number of software packages designed for image reconstruction (MEMSYS, AIPS, etc.). A comparative analysis of CLEAN and MEM in VLBI is given by Cornwell, Braun & Briggs (1999); essentially, these methods complement each other. In particular, CLEAN is preferred for reconstructing images of compact sources from relatively poor data, while MEM is more suitable for the imaging of extended sources from better-quality data.

A severe drawback of MEM compared to CLEAN, the bias of the solution (Cornwell et al. 1999), can easily be removed by generalizing the method to enable the reconstruction of sign-variable functions (Bajkova 1992; Frieden & Bajkova 1994). This generalized MEM also permits difference imaging, making it possible to substantially broaden the dynamic range of maps of sources, including both compact and extended, faint components (Bajkova 2007; Rastorgueva et al. 2011).

As shown by Bajkova (2008), applying Shannon’s MEM allows a tangible progress to be achieved in solving the problem of MFS, owing to the possibility of a simple allowance for the spectral terms of any order. This, in turn, allows the range of synthesized frequencies to be extended significantly. However, the MFS algorithms discussed here, based on both CLEAN and MEM deconvolution, can be directly applied only to those radio sources for which no frequency-dependent image shift is observed. Otherwise, as shown by Croke & Gabuzda (2008), an additional operation to align images at different frequencies should be performed to obtain the proper results in a multifrequency data analysis.

In this paper, our goal is to deduce our MFS algorithm based on MEM and to show the importance of the procedure for precorrecting the frequency-dependent image shift while implementing MFS.

The paper is structured as follows. We describe the frequency dependence of the intensity of a radio source in Sections 2. We derive the frequency-dependent constraints on the visibility function in Sections 3. We give the reconstruction method in Sections 4. Prior to describing the MFS algorithm, we consider the MEM and its generalized form in Sections 4.1 and 4.2, respectively. We deduce the proposed MFS algorithm with frequency correction in Section 4.3. We test our algorithm in a number of model experiments in Sections 5. We discuss the problem of aligning frequency-dependent images in order to properly construct the spectral index distribution in Sections 6. Finally, in Sections 7, we present the results of applying the MFS algorithm proposed to four-frequency Very Long Baseline Array (VLBA) data for the BL Lacertae object 0954+658.

### 2 FREQUENCY DEPENDENCE OF THE AGN RADIO BRIGHTNESS

The dependence of the intensity of a radio source on frequency $\nu$ in the model of synchrotron radiation is given by (Conway et al. 1990)

$$ I(\nu) = I(\nu_0) \left( \frac{\nu}{\nu_0} \right)^\alpha, $$

(1)

where $I(\nu_0)$ is the intensity of the radiation at the reference frequency $\nu_0$ and $\alpha$ is the spectral index. To simplify, hereafter we set $I_0 = I(\nu_0)$.

Retaining the first $Q$ terms in the Taylor expansion of equation (1) at point $\nu_0$, we can write the following approximate equality

$$ I(\nu) \approx I_0 + \sum_{q=1}^{Q-1} I_q \left( \frac{\nu - \nu_0}{\nu_0} \right)^q, $$

(2)

where

$$ I_q = \frac{Q!}{q!} \left[ (\alpha - 1) \cdots [\alpha - (q - 1)] \right]. $$

From equation (2), for each pixel $(k, l)$ of the source’s two-dimensional $(N \times N)$ brightness distribution, we have

$$ I(k, l) \approx I_0(k, l) + \sum_{q=1}^{Q-1} I_q(k, l) \left( \frac{\nu - \nu_0}{\nu_0} \right)^q, $$

(3)

where $k, l = 1, \ldots, N$.

Thus, the derived brightness distribution over the source (3) is the sum of the brightness distribution at the reference frequency $\nu_0$ and the spectral terms with the $q$th-order spectral map depending on the spectral index distribution over the source as follows:

$$ I_q(k, l) = \frac{Q!}{q!} \left[ (\alpha - 1) \cdots [\alpha - (q - 1)] \right]. $$

(4)

Of greatest interest is the first-order spectral map

$$ I_1(k, l) = \frac{I_0(k, l) \alpha (k, l) \cdots [\alpha (k, l) - (q - 1)]}{q!}, $$

because the spectral index distribution over the source can be estimated from this in the following way:

$$ \alpha(k, l) = \frac{I_1(k, l)}{I_0(k, l)}. $$

(5)

### 3 CONSTRAINTS ON THE VISIBILITY FUNCTION

The complex visibility function is the Fourier transform of the intensity distribution over the source, which satisfies the spectral dependence (1) at each pixel of the map $(k, l)$. Given the finite number of terms in the Taylor expansion (3), the constraints on the visibility function $V$ can be written as

$$ V_{u, v_0} = F \left[ I(k, l) \right] \times D_{u, v_0}, $$

$$ \approx \sum_{q=0}^{Q-1} F \left[ I_q(k, l) \left( \frac{\nu - \nu_0}{\nu_0} \right)^q \right] \times D_{u, v_0}. $$

(6)

Here, $F$ denotes the Fourier transform and $D$ represents the transfer function, which is the $\delta$-function of $u$ and $v$ for each measurement of the visibility function; different sets of $\delta$-functions correspond to different frequencies $\nu$, as suggested by the indices of $u$ and $v$.

Let us rewrite equation (6) for the real and imaginary parts of the visibility function $V_{u, v_0} = A_{u, v_0} + jB_{u, v_0}$ by taking into account the measurement errors as

$$ \sum_{q=0}^{Q-1} \sum_{k, l} I_q(k, l) a_{u, v_0}^{im} \left( \frac{\nu - \nu_0}{\nu_0} \right)^q + \eta_{u, v_0}^{re} = A_{u, v_0}, $$

(7)

$$ \sum_{q=0}^{Q-1} \sum_{k, l} I_q(k, l) b_{u, v_0}^{im} \left( \frac{\nu - \nu_0}{\nu_0} \right)^q + \eta_{u, v_0}^{im} = B_{u, v_0}. $$

(8)
Here, \(a_{kl}^m\) and \(b_{kl}^m\) are the constant coefficients (cosines and sines) that correspond to the Fourier transform, and \(\eta_{kl,im}^a\) and \(\eta_{kl,im}^b\) are the real and imaginary parts, respectively, of the instrumental additive noise, distributed normally with a zero mean and a known dispersion \(\sigma_{kl,iv}\).

### 4 METHOD

#### 4.1 Maximum entropy method

MEM is one of a large class of non-linear informational methods, based on the optimization of a functional, specified by some informational criterion for the quality of the solution, subject to the constraints that flow from the data. In our case, maximizing the Shannon entropy consists of finding the maximum of the functional

\[
E = - \int x(t) \ln[x(t)] \, dt,
\]

where \(x(t)\) is the desired distribution.

Because imaging in VLBI implies dealing with digital data, we present a discrete formulation of the optimization. Let a map of an object with a finite carrier be sampled in accordance with the Kotelnikov–Nyquist theorem (Oppenheim & Schafer 1999) and let it have a size of \(N \times N\) pixels. We denote the discrete measurements of the desired distribution by

\[
x_{kl}, \quad k, l = 1, \ldots, N - 1.
\]

We denote the known measurements of the two-dimensional Fourier spectrum of the object, which represent the visibility data, in accordance with the Van Cittert–Cernike theorem, as follows, separating the real, \(A_m\), and imaginary, \(B_m\), parts:

\[
V_m = A_m + j B_m, \quad m = 1, \ldots, M.
\]

Here, \(M\) is the number of known measurements and \(m\) is the number of the current measurement with coordinates \((\eta_m, \nu_m)\) in the uv-plane, not necessarily located at the nodes of the coordinate grid. This last circumstance means that there is no problem with pixelization of the data in the frequency domain. This represents a certain technical advantage of this method over other methods, and it appreciably enhances the accuracy of the reconstruction.

The practical MEM algorithm we have applied, taking into account the errors in the data (Bajkova 1993), implies the solution of the conditional optimization problem:

\[
\min \sum_k \sum_l x_{kl} \ln(x_{kl}) + \frac{1}{2} \sum_m \left(\frac{\eta_{kl,im}^a}{\sigma_{kl,im}^a}\right)^2 + \left(\frac{\eta_{kl,im}^b}{\sigma_{kl,im}^b}\right)^2,
\]

\[
\sum_k \sum_l x_{kl} A_{kl}^m + \eta_{kl,im}^a = A_m,
\]

\[
\sum_k \sum_l x_{kl} B_{kl}^m + \eta_{kl,im}^b = B_m,
\]

\[
x_{kl} \geq 0.
\]

As we can see from equation (10), the optimized functional has two parts: a Shannon entropy functional and a functional that is an estimate of the difference between the reconstructed spectrum and the measured data according to the \(\chi^2\) criterion. This latter functional can be considered an additional regulating, or stabilizing, term acting to provide a further regularization of the MEM solution. The influence of this additional term on the resolution of the reconstruction algorithm must be kept in mind.

Equations (11) and (12) represent linear constraints on the unknown images \(x_{kl}\) as well as noise terms \(\eta_{kl,im}^a\) and \(\eta_{kl,im}^b\). The non-negativity constraint (13) on the image can be omitted in this case, because of the nature of the entropy solution, which is purely positive. If the total flux of the source \(F_0\) is known, this automatically leads to the normalization of the solution:

\[
\sum_k \sum_l x_{kl} = F_0.
\]

The numerical algorithm for solution (10)–(12), treated as a non-linear optimization problem based on the method of Lagrange multipliers, is considered in detail in Bajkova (2007). Here, we present only the solution:

\[
x_{kl} = \exp \left[ - \sum_m \left( a_m a_{kl}^m + \beta_m b_{kl}^m \right) - 1 \right],
\]

\[
\eta_{kl,im}^a = \sigma_{kl,im}^a a_m, \quad \eta_{kl,im}^b = \sigma_{kl,im}^b \beta_m.
\]

This is expressed in terms of the Lagrange multipliers (dual variables) \(a_m\) and \(\beta_m\), through which the constraints (11) and (12), respectively, enter the Lagrange functional.

As we can see from equation (14), the standard MEM image is evidently positive. It can be shown that the MEM Hesse matrices are everywhere positive-definite, so that the entropy functional is convex and the solution is global. Various gradient methods can be used to search for the extrema of the corresponding dual functional. We use a coordinate-descent method.

#### 4.2 Generalized maximum entropy method

The generalized MEM (GMEM) was designed for the reconstruction of sign-variable and complex functions (Bajkova 1992; Frieden & Bajkova 1994; Bajkova 2007). For the GMEM, dealing with sign-variable real distributions, the Shannon-entropy functional has the form

\[
E = - \int \left[ x^+(t) \ln[x^+(t)] + x^-(t) \ln[x^-(t)] \right] \, dt,
\]

where \(x^+(t) \geq 0\) and \(x^-(t) \geq 0\) are the positive and negative components, respectively, of the sought-for image \(x(t)\) [i.e. the equation \(x(t)=x^+(t)-x^-(t)\) holds]. \(a \geq 0\) is a parameter responsible for the accuracy of the separation of the positive and negative components of the solution \(x(t)\), and it is therefore critical for the resulting image fidelity. As shown by Bajkova (2007), the solutions for \(x^+(t)\) and \(x^-(t)\) obtained with the Lagrange optimization method are connected by the expression

\[
x^+(t) \cdot x^-(t) = \exp(-2 - 2 \ln a) = K(a),
\]

which depends only on the parameter \(a\).

This parameter is responsible for dividing the positive and negative parts of the solution: the larger \(a\) allows the more accurate discrimination, because \(K(a) \rightarrow 0\) as \(a \rightarrow \infty\). However, the value of \(a\) is constrained by computational limitations. The main constraint comes from the \(\chi^2\) term in the optimized functional, which depends on the data errors. The larger a standard deviation is, the higher value of \(a\) can be set. If data are very accurate, a lower value of \(a\) is needed. In practice, \(a\) is chosen empirically. In our case, we had to compromise between data errors, which determine the resolution of the final MEM solution, and the need to divide the positive and negative parts of the solution as accurately as possible. It is fair to say that, given fixed errors in the data, a maximum possible chosen value of \(a\) provides us with the best possible resolution of the MEM solution. In this work, we used \(a = 10^6\).
4.3 GMEM-based MFS algorithm

In this case, the distributions \( I_q(k, l), q = 0, \ldots, Q - 1, k, l = 1, \ldots, N, \) and the measurement errors of the visibility function \( \eta_{a_{k,l}}^{re} \) and \( \eta_{a_{k,l}}^{im} \) are unknown. Note that although the brightness distribution over the source is described by a non-negative function, the spectral maps of arbitrary order (4) can generally take both positive and negative values because the spectral index distribution over the source is an alternating one.

By setting, in accordance with the approach described above,

\[
I_q(k, l) = I_q^+(k, l) - I_q^-(k, l), \quad q = 1, \ldots, Q - 1,
\]

we obtain the following entropic functional to be minimized:

\[
E = \sum_{k,l} I_q(k, l) \ln[a I_q(k, l)]
+ \sum_{q=1}^{Q-1} \left( \sum_{k,l} \left[ I_q^+(k, l) \ln[a I_q^+(k, l)] + I_q^-(k, l) \ln[a I_q^-(k, l)] \right] \right)
+ \frac{1}{2} \sum_{a_{k,l}} \left( \eta_{a_{k,l}}^{re} \right)^2 + \left( \eta_{a_{k,l}}^{im} \right)^2, \tag{15}
\]

where \( I_0(k, l) \geq 0, \quad I_q^+(k, l) \geq 0, \quad I_q^-(k, l) \geq 0. \)

The linear constraints (7) and (8) on the measured visibility function are rewritten accordingly:

\[
\sum_{k,l} I_q(k, l) a_{a_{k,l}} = a I_q(k, l) = \sum_{a_{k,l}} \left( \eta_{a_{k,l}}^{re} \right) \left( \frac{v - v_0}{v_0} \right)^q + \eta_{a_{k,l}}^{im},
\]

\[
\sum_{k,l} I_q(k, l) b_{a_{k,l}} = b I_q(k, l) = \sum_{a_{k,l}} \left( \eta_{a_{k,l}}^{re} \right) \left( \frac{v - v_0}{v_0} \right)^q + \eta_{a_{k,l}}^{im}, \tag{16}
\]

Minimizing the functional (15) with constraints (16) and (17) constitutes the essence of the MEM-based MFS algorithm, which seeks the solution for all unknown \( I_q(k, l), I_q^{+/-}(k, l), q = 1, \ldots, Q - 1, k, l = 1, \ldots, N \) and \( \eta_{a_{k,l}}^{re}, \eta_{a_{k,l}}^{im} \). A detailed algorithm for the numerical implementation of the proposed MFS method is given in Bajkova (2008).

5 TESTING THE METHOD: SIMULATION RESULTS

Here, we present the results of testing our MFS deconvolution algorithm on the example of four-frequency synthesis using simulated VLBI data at four frequencies of 5, 8, 15 and 22 GHz. As a reference frequency in the MFS algorithm, we adopted the central frequency equal to 13.6 GHz. A model source map at this frequency and a model spectral index distribution over the source are presented in Fig. 1 (left and right, respectively).

As can be seen, the source shows a structure on a mas angular scale, consisting of a bright compact core and a one-sided jet. Note that the model map also contains a number of weak small-scale details scattered around the main structure. Such a complication of the source structure was made in order to test the ultimate capabilities of the MFS algorithm. Note also that the structure of the model source was built similar to the structure of the radio source 0954+658, which is considered in Sections 7.

Fig. 2 shows the \( uv \)-plane coverages related to a model interferometer, consisting of 10 baselines of the VLBA array (BR–LA, BR–MK, BR–NL, BR–PT, BR–OV, BR–SC, BR–KP, BR–HN, BR–FD, LA–MK), and source with a declination \( \delta = 70^\circ \) for each of the ‘observation’ frequencies. Note that the choice of baselines was not principal for our task and was made in an arbitrary way. It was assumed that the duration of the observations is 9 h, and that visibility data are formed every 30 min. As can be seen, the \( uv \)-plane at different frequencies has the same topology but is scaled accordingly. As far as interferometer baselines are measured in wavelengths, \( u \) - and \( v \) -visibility coordinates are proportional to an observation frequency.

Model maps of the source at frequencies of 5, 8, 15 and 22 GHz are shown in Fig. 3 (left column). These maps were obtained from the model map shown in Fig. 1 (left), according to the model spectral index distribution (Fig. 1, right) and expression (1) for the spectral dependence. The parameters of the model source intensity
maps, such as total flux density ($S_{\text{tot}}$), peak flux density ($S_{\text{peak}}$) and entropy, are given in Table 1. Comparison of the maps and their parameters shows a not strong but still quite noticeable frequency dependence.

The complex visibility functions, computed in accordance with the Van Cittert–Cernike theorem, consists of 172 samples for each ‘observation’ frequency. Each visibility value was aggravated with an additive random error to form data with a typical signal-to-noise ratio (S/N) of 10⁴. It is necessary to emphasize that we do not consider here the self-calibration problems (Conway et al. 1990) and we concentrate only on removing spectral errors.

The main goals of the simulation fulfilled were: (i) to show efficiency of the MFS for improving intensity images in case of small-element interferometers with poor $uv$-plane filling; (ii) to illustrate the consequences of MFS without any spectral correction; (iii) to estimate the possibility of reconstruction of spectral index maps with a satisfactory quality.

We performed three tests. The first was single-frequency synthesis of the source images at 5, 8, 15 and 22 GHz. The second test was multifrequency (four-frequency) synthesis without any spectral correction. Finally, the third experiment was devoted directly to MFS with spectral correction, at the reference frequency of 13.6 GHz.

The single-frequency maps are presented in Fig. 3 (right column). Their parameters are given in Table 2. Analysis of the results obtained shows the following. The amount of data proves to be too small, and $uv$-plane filling too poor, to obtain images with sufficient quality. A comparison of Tables 1 and 2 allows us to judge the distortions of the reconstructed images. As a criterion of the reconstructed image quality, we chose the S/N listed in the last column in Table 2, which was calculated in the following way:

$$S/N = \frac{\sqrt{\sum_{k,l} I_{\text{mod}}^2(k,l)}}{\sqrt{\sum_{k,l} (I_{\text{mod}}(k,l) - I_{\text{rec}}(k,l))^2}}.$$ 

Here, $I_{\text{mod}}$ is the model map, $I_{\text{rec}}$ is the reconstructed map and $k, l = 1, \ldots, N$, where $N$ is the linear size of the map.

As seen from the single-frequency maps, at lower observation frequencies, the lower resolution is ensured. The lower-frequency maps show a larger-scale structure, while the higher-frequency maps show smaller-scale features. The highest accuracy of reconstruction is achieved at frequencies of 8 and 15 GHz ($S/N \sim 16$). At the lowest frequency, 5 GHz, and the highest frequency, 22 GHz, the obtained reconstruction quality was much worse ($S/N$ about 8).

The $uv$-plane corresponding to multifrequency (four-frequency) synthesis is presented in Fig. 4 (left). The multifrequency map obtained without any spectral corrections of the frequency-dependent source brightness distribution is shown in Fig. 4 (right). The parameters of the synthesized map are given in Table 3. Compared
Thus, in the source model with a sufficiently complicated extended structure, typical for AGNs at mas scales, with typical spectral index values, we have demonstrated the ability of the MEM-based MFS algorithm with spectral correction for both (i) improving intensity images and (ii) obtaining spectral index maps of high quality. We emphasize that the use of the MFS algorithm is especially effective in the case of small-element interferometers with poor uv-plane filling. We have also shown the consequences of ignoring the frequency dependence of the source brightness distribution in the MFS algorithm.

6 PROBLEM OF IMAGE ALIGNMENT

One of the most important sources of information about the physical conditions in the radio-emitting regions of AGNs is the spectral index distribution over the source. The core region is usually characterized by a large optical depth and an almost flat or inverted spectrum, while the jets are optically thin with respect to synchrotron radiation and have steeper spectra (Pushkarev et al. 2005; Croke & Gabuzda 2008; O'Sullivan & Gabuzda 2009).

The spectral index distribution over the source can be constructed using various methods. The traditional method suggests: (i) the formation of images at two separate frequencies, \( v_1 \) and \( v_2 \), with the solutions of the deconvolution problem (\( \text{CLEAN} \) or \( \text{MEM} \)) being convolved with the same ‘clean’ beam corresponding to the lower observation frequency; (ii) the calculation of the two-dimensional spectral index distribution over the source from equation (1). Obviously, this sequence of operations is legitimate only when the positions of the VLBI cores of sources (not to be confused with the physical core of the source, which is undetectable because of absorption effects) are frequency-independent.

The image reconstruction using the iterative self-calibration procedure is known (Thompson, Moran & Swenson 2001) to lead to the loss of information about the absolute position of the source on the sky. During the self-calibration phase, the centroid of the object is placed at the phase centre of the map with coordinates (0, 0). However, as most radio-loud AGNs are characterized by a dominant compact core (Kovalev et al. 2005; Lister & Homan 2005; Lee et al. 2008), the VLBI core of the source coincides with the peak radio brightness of the source in the overwhelming majority of cases.

Nevertheless, the standard theory of extragalactic radio sources (Blandford & Königl 1979) predicts a frequency-dependent VLBI core shift as a result of opacity effects in the source’s core region. Synchrotron self-absorption takes place in an ultra-compact region near the ‘central engine’ of the AGN, the mechanism of which is most efficient at low frequencies. As a result, the apparent origin of the jet manifests itself farther from the physical core along the jet axis at lower frequencies (Fig. 6). This theoretical prediction was confirmed by observations: the frequency-dependent shift in the core position was measured for several quasars by Lobanov (1998).

In the literature, this phenomenon is also actively debated from the point of view of the accuracy of astrometric measurements (Charlot & Hambly 2002; Bolotzky 2006; Kovalev et al. 2008a).

Thus, it follows that the multifrequency data analysis must be preceded by the alignment of images at different frequencies. This can be achieved in three ways: (i) performing VLBI observations of the objects under study together with reference sources; (ii) finding the parameters of the shift of one image relative to the other by aligning compact optically thin jet features, which are not subject to absorption effects to the same extent as in the source’s core (Paragi, Fejes & Frey 2000; Kovalev et al. 2008a); (iii) finding the shift...
parameters using a cross-correlation analysis (Croke & Gabuzda 2008). Being laborious from the point of view of performing observations and their subsequent reduction, the first method gives no significant advantage in determining the shift and its accuracy. Therefore, the second and/or third methods are used more often.

Recall that the alignment procedure implemented by shifting one image relative to the other is equivalent to the phase correction of the spectrum (or visibility function) of the image being shifted relative to the fixed one. The need to pre-correct the data for the source’s visibility function at different frequencies makes direct use of the MFS algorithm described above problematic, because the frequency dependence of the core shift is not known in advance. It can be determined by forming the images at each frequency and determining the corresponding shifts. As shown by Kovalev et al. (2008b), O’Sullivan & Gabuzda (2009) and Sokolovsky et al. (2011), the frequency dependence of the VLBI core position is well fitted by a hyperbolic dependence of the form \( r \propto v^{-1} \). Thus, our MFS procedure can be used after allowance for the shifts in the positions of the VLBI cores at different frequencies and their coordinates relative to the phase centre, and applying the corresponding frequency-dependent phase corrections to the visibility function.

### 7 REAL DATA PROCESSING: FOUR-FREQUENCY IMAGING FOR 0954+658

Here, we present the results of applying the developed multifrequency image synthesis algorithm to the real VLBI data of the extragalactic radio source 0954+658, a member of the complete sample of BL Lacertae objects (Kühr & Schmidt 1990). Note that 0954+658 is also a member of the 1FGL catalogue of \( \gamma \)-ray bright sources detected by the Large Area Telescope on-board the Fermi observatory and positionally associated with the \( \gamma \)-ray source 1FGL J1000.1+6539 (Abdo et al. 2010). This source of interest to us because it has a typical parsec-scale morphology that includes an optically thick VLBI core and a one-sided optically thin jet, which is expected to manifest itself in the spectral index distribution. General information on this source is given in Table 4.

The VLBA observations of 0954+658 were carried out in a ‘snapshot’ mode in 1997 April (1997.26) simultaneously at four frequencies: 5, 8, 15 and 22 GHz. The data were calibrated in the National Radio Astronomy Observatories (NRAO) AIPS package, using standard procedures. The images were formed within the framework of the Pulkovo ‘VLBImager’ software package, based on a self-calibration algorithm (Cornwell & Fomalont 1999) in combination with a GMEM-based deconvolution procedure.

The \( uv \)-plane coverages related to the observation frequencies of 5, 8, 15 and 22 GHz are shown in Fig. 7. The MEM-based single-frequency maps are shown in Fig. 8. Table 5 gives the parameters of these maps, obtained from the MEM solutions by convolution with ‘clean’ beams that determine the system’s resolution at each observation frequency.

Table 6 gives the parameters of the frequency-dependent image shift found by aligning the compact features of the optically thin jet. As expected, the direction of the shift coincides with the inner jet orientation. The \( uv \)-plane related to the multifrequency (four-frequency) data is shown in Fig. 9 (left).

First, we processed the multifrequency data, ignoring the dependence of the source brightness distribution on observation frequency. The image obtained is shown in Fig. 9 (right). Then, we applied our MFS algorithm with spectral correction to the multifrequency data, but we did not correct the data in accordance with the frequency-dependent VLBI core position shift found. The results of this MFS obtained at a central reference frequency of 13.6 GHz are shown in Fig. 10.

Finally, we applied our MFS algorithm with spectral correction to the observational data that were first corrected according to the alignment parameters (Table 6). Fig. 11 shows the image obtained

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### Table 4. General parameters of 0954+658.

| Other name | RA (J2000) | Dec. (J2000) | Opt. ID | Redshift | 1FGL |
|------------|------------|-------------|---------|----------|------|
| J0958+6533 | 9h 58m 47.2451s | +65°33′54.818″ | BL Lac | 0.367 | Y |
Multifrequency synthesis

Figure 8. Single-frequency maps of the source 0954+658 reconstructed from the data at 5 GHz (a), 8 GHz (b), 15 GHz (c) and 22 GHz (d). Relative right ascension and declination are given in mas along the horizontal and vertical axes, respectively. The lowest contour levels are given in Table 5 for each image, with the values of the succeeding levels doubled.

Table 5. Parameters of single-frequency maps for 0954+658.

| Frequency (GHz) | $S_{\text{tot}}$ (mJy) | $S_{\text{peak}}$ (mJy beam$^{-1}$) | Beam FWHM mas x mas, PA | Lowest contour level (per cent) |
|----------------|-------------------------|-----------------------------------|-------------------------|--------------------------------|
| 5.0            | 617                     | 360                               | 2.50 x 1.72, −17:3      | 0.20                           |
| 8.4            | 496                     | 311                               | 1.44 x 1.05, −16:2      | 0.10                           |
| 15.4           | 454                     | 219                               | 0.81 x 0.59, −16:0      | 0.25                           |
| 22.2           | 310                     | 166                               | 0.58 x 0.43, −21:6      | 0.70                           |

Table 6. Frequency-dependent VLBI core position shifting parameters with respect to the core position at 22 GHz.

| Frequency (GHz) | $\Delta r$ (mas) | PA   |
|----------------|------------------|------|
| 15.4           | 0.18             | −29:4|
| 8.4            | 0.46             | −27:1|
| 5.0            | 0.73             | −29:7|

at a reference frequency of 13.6 GHz and the corresponding two-dimensional spectral index distribution.

The intensity images shown in Figs 9–11 are obtained by convolving solutions for $I_0$ with an appropriate Gaussian beam. The spectral index images (Figs 10 and 11) are formed by dividing two solutions for $I_1$ and $I_0$ in accordance with equation (5), after convolving them with the same beam.

It is necessary to note that the main goal of this section is to demonstrate the necessity of taking into account the frequency-dependent image shift in order to correctly map the spectral index distribution.

Let us analyse the results obtained.

The VLBI structure of the radio source 0954+658 consists of an optically thick core (Fig. 11, right) and an optically thin jet, initially
extending in the north-west direction up to ~2 mas from the VLBI core and then turning to the west. As seen from single-frequency maps (Fig. 8), the apparent extent of the jet reaches about 12 and 2 mas at the lowest and the highest frequencies, respectively. The total flux density of the source varies from 0.6 up to 0.3 Jy at the lowest and the highest frequencies, respectively. We can see that the images obtained manifest large-scale structure features at lower observation frequencies and small-scale ones at higher frequencies.

Combining the data obtained at different observation frequencies allows us, in general, to reconstruct both large- and small-scale structures to a larger extent because of a better filled $uv$-plane. This has been proved, in particular, in Sections 5 in model experiments.

The intensity map (Fig. 9) synthesized directly from the linearly combined multifrequency data without any spectral corrections shows large source structure distortions and many artefacts. Multifrequency synthesis with spectral correction, fulfilled first without taking into account frequency-dependent VLBI core position shift, shows a slight improvement of the source intensity map, but it is still distorted, especially in the south-west direction (Fig. 10). However, the main consequence of image misalignment is an incorrect reconstruction of the spectral index distribution. In our case, we can see that the spectral index map obtained does not correspond to the physical meaning of an optically thick core and an optically thin jet – there are negative spectral indices in the core region and segments with positive spectral indices in the jet region. From Fig. 11, we realize that only allowance for the real shift found by aligning features of the optically thin jet yielded the proper result.

As seen, we managed to reconstruct a more extended jet structure (up to 8 mas along the RA axis) than in the case of using only the high-frequency data, but with the same high angular resolution. The spectral index map adequately reflects the physical characteristics of the regions of the optically thick compact VLBI core and the optically thin extended jet. We see a fairly regular structure with smooth transitions between segments of different intensities along the entire source. For comparison, one-dimensional slices of the spectral index distributions (see the left panels of Figs 10 and 11) obtained along the jet ridge line are shown in Fig. 12. From these, we can graphically evaluate the undesirable consequences of ignoring the frequency-dependent VLBI core position shift in the MFS algorithm.

**Figure 12.** Slices of the spectral index distributions along the jet ridge line beginning from the phase centre: (1) and (2) are related to the spectral index maps presented in Figs 10 and 11 respectively, and $l$ is the distance along the ridge line.

### 8 Conclusions

We developed and tested an efficient multifrequency image synthesis algorithm with the correction for the frequency dependence of the radio brightness of a source. The algorithm is based on the GMEM. It allows us to take into account the spectral terms of any order and to map both a total intensity image and spectral index distribution, which is of great importance in investigating the physical characteristics of AGNs.

The advantage of the proposed MFS algorithm is that the spectral terms of any order can be easily taken into account in the entropic functional being minimized. This allows for the spectral correction of images to be made, both in a wide frequency range and for large spectral indices.

We have shown how important the allowance for the frequency-dependent image shift is in applying the MFS algorithm. Our conclusions are based on the results of processing the multifrequency VLBA data for the BL Lac object 0954+658 with a fairly complex extended jet structure, which also manifests itself in the spectral index distribution over the source.

An analysis of the results obtained shows that MFS is an efficient method for improving the mapping quality. Low-frequency data allow the extended structure of a source to be reconstructed more completely, while high-frequency data allow a high spatial resolution to be achieved. It should also be emphasized that the spectral index distribution can also be mapped with a high quality.

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