P, T-Violating Electron-Nucleon Interactions in the R-Parity Violating Minimal Supersymmetric Standard Model

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(December 7, 1999)

Abstract

We show that the present experimental limits on electron-nucleon interactions that violate both parity and time reversal invariance provide new stringent bounds on the imaginary parts of some of the products of the R-parity violating coupling constants in the R-parity violating Minimal Supersymmetric Standard Model.

11.30.Er,12.60.Jv,32.10.Dk
I. INTRODUCTION

In the Minimal Supersymmetric Standard Model (MSSM) [4], unlike in the Standard Model (SM) [2], the conservation of lepton number ($L$) and of baryon number ($B$) is not automatic [3, 4]. In particular, the superpotential can contain renormalizable and gauge invariant $L-$ and $B-$violating terms. The general forms of these are [4]

$$W_L = \frac{1}{2} \lambda^c_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \mu_i L_i H_u,$$

$$W_B = \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c_k,$$

where $i, j, k = 1, 2, 3$ are family indices, and summations over $i, j, k$ are implied. In Eqs. (1) and (2) $L_i, Q_i$ are the $SU(2)$-doublet lepton and quark superfields, $E^c_i, U^c_i, D^c_i$ are the $SU(2)$-singlet charged lepton, and up- and down-type quark superfields; $H_u$ is the Higgs superfield which generates the masses of the up-type quarks. The constants $\lambda_{ijk}$ are antisymmetric under the interchange $i \leftrightarrow j$, and $\lambda''_{ijk}$ is antisymmetric under $j \leftrightarrow k$.

The couplings in $W_L$ and $W_B$ violate invariance under $R$-parity ($R = (-1)^{3B+L+2s}$, where $s$ is the spin of the particle; thus $R = +1$ for the particles of the SM, and $R = -1$ for their superpartners) [5]. If both the $\lambda'_{ijk}$-term and the $\lambda''_{ijk}$-term is present, some of the products would have to be extremely small (for example, $|\lambda'_{11k} \lambda''_{11k}| \lesssim 10^{-22}$ for $k = 2, 3$ and $m_{d_{3R}} = 100$ GeV) to prevent too rapid proton decay [6]. One way to deal with this problem is to postulate $R$-parity invariance. This would eliminate both $W_L$ and $W_B$ [4]. Another possibility is that $B$ is conserved, but the $L$-violating terms are present. This scenario is obtained by demanding invariance under “baryon parity” (under baryon parity $Q_i \rightarrow -Q_i$, $U^c_i \rightarrow -U^c_i$, $D^c_i \rightarrow -D^c_i$, and $L_i, E^c_i, H_u, H_d$ remain unchanged) [4, 7]. The model we shall consider in the following is the $R$-parity violating MSSM ($\tilde{\text{MSSM}}$), defined as the MSSM with $W_L$ included in the superpotential [8].

The presence of $R$-parity violating couplings has rich phenomenological implications. If $R$-parity isviolated, the production of single supersymmetric particles becomes possible, and the lightest supersymmetric particle is no longer stable. The main source of constraints on the $R$-parity violating coupling constants is experimental data on processes with the SM particles, to which the $R$-parity violating couplings can contribute through the exchange of singlet squarks or sleptons. The present status of these bounds is given in the reviews in Ref. [8]. Most of the upper limits on the individual coupling constants are of the order of $10^{-2} - 10^{-1}$ for squark and slepton masses of 100 GeV. There are also more stringent bounds, including some on products of two coupling constants, coming mainly from processes forbidden in the SM.

The $R$-parity violating coupling constants can be complex, and thus represent new sources of CP-violation. Stringent constraints come from the experimental values of $\epsilon$ and $\epsilon'/\epsilon$ [10]. The effects of CP-violating $R$-parity violating interactions have also been considered in semileptonic K-decays [11], B-decays [12], semi-inclusive decays of heavy quarks [13], leptonic collider processes [14], and in lepton-pair production in $\bar{p}p$ reactions [15].

In this paper we show that the present experimental limits on $P, T$-violating e-N interactions (electron-nucleon interactions that violate both parity and time reversal invariance)
set stringent bounds on the imaginary parts of some of the products $\lambda_{ijk}^* \chi_{lmm}$. In the next section we analyze the $P, T$-violating e-N interactions arising from the $R$-parity violating couplings. In Section III we derive the bounds on the imaginary parts of products of the coupling constants involved, and consider the constraints on them from other data. In Section IV we summarize our conclusions.

II. $P, T$-VIOLATING e-N INTERACTIONS IN THE $\mathcal{R}$MSSM

The general form of $P, T$-violating e-N interactions, including non-derivative couplings only, is given by [16]

$$H_{P,T} = \sum_{a=p,n} \frac{G_F}{\sqrt{2}} [C_{Sa} \bar{e} i \gamma_5 e a + C_{Pa} \bar{e} e a i \gamma_5 a$$

$$+ C_{Ta} \frac{1}{2} i \epsilon_{\alpha\beta\gamma\delta} e a^{\alpha\beta} e a^{\gamma\delta} a], \hspace{1cm} (3)$$

where $a = p, n$ ($p = $ proton, $n = $ neutron) and $C_{Sa}, C_{Pa}$ and $C_{Ta}$ are real constants [17].

Stringent limits on $C_{Sa}, C_{Pa}, C_{Ta}$ [18] follow from experimental results on the electric dipole moments of the $^{133}Cs$ [19], $^{205}T\ell$ [20], $^{129}Xe$ [21] and $^{199}Hg$ [22] atoms, and on the $P, T$-violating spin-flip parameter $\nu$ of the $T\ell F$ molecule [23]. The best of these are [16]

$$|0.4 C_{Sp} + 0.6 C_{Sa}| < 3.4 \times 10^{-7}, \hspace{1cm} (4)$$

$$|C_{Pn}| < 1.4 \times 10^{-5}, \hspace{1cm} (5)$$

$$|0.75 C_{Pp} + 0.25 C_{Pn}| < 3 \times 10^{-4}, \hspace{1cm} (6)$$

$$|C_{Tn}| < 4 \times 10^{-8}, \hspace{1cm} (7)$$

$$|0.75 C_{Tp} + 0.25 C_{Tn}| < 4.5 \times 10^{-7}, \hspace{1cm} (8)$$

The limit (4) has been deduced from the experimental bound on $d(T\ell)$; the limits (5) and (7) come from $d(Hg)$, and the limits (6) and (8) from $\nu(T\ell F)$.

In the SM the constants $C_{Sa}, C_{Pa}$ and $C_{Ta}$ are very small: the Kobayashi-Maskawa phase contributes at the level of $10^{-16}$, and the contribution of the $\theta$-term is of the order of $10^{-11} - 10^{-10}$ [24]. They can be much larger, however, in some extensions of the SM. The $P, T$-violating e-N interactions have been studied for general electron-quark interactions [25], in multi-Higgs models [24,26,27], models with leptoquarks [24,27,28], and in the $R$-parity conserving MSSM [29]. In multi-Higgs and leptoquark models they can be as strong as allowed by the present experimental limits. In the $R$-parity conserving MSSM the constants $C_{Sa}$ (which are the dominant ones [29]) are smaller by several orders of magnitude than the present limit on $C_{Sa}$ [Eq. (4)]. This is implied by the limit on the pertinent CP-violating phase, obtained from the limit on the $P, T$-violating nucleon-nucleon interactions set by the experimental limit on $d(Hg)$ [30].
We shall consider now the $P$, $T$-violating e-N interactions in the $\mathcal{R}$MSSM. Relative to the $R$-parity conserving MSSM, in this model there are additional contributions to $P$, $T$-violating e-N interactions, originating from the $\lambda_{ijk}$ and $\lambda'_{ijk}$ couplings in Eq. (1), which appear already at the tree level. The $\lambda_{ijk}$ and $\lambda'_{ijk}$ couplings in (1) in terms of the components of the superfields are given by \[31\]

\[
\mathcal{L} = \lambda_{ijk}\left[\bar{\nu}_{iL}\bar{e}_{kR}\nu_{jL} + \bar{e}_{jL}\bar{\nu}_{kR}\nu_{iL} + \bar{e}_{kR}\bar{\nu}_{jL}\nu_{iL}\right]
- \nu_{jL}\bar{e}_{kR}\nu_{iL} - \bar{e}_{iL}\bar{\nu}_{kR}\nu_{jL} - \bar{e}_{kR}\bar{\nu}_{jL}\nu_{iL}
+ \lambda'_{ijk}\left[\bar{\nu}_{iL}\bar{d}_{kR}\nu_{jL} + \bar{d}_{jL}\bar{\nu}_{kR}\nu_{iL} + \bar{d}_{kR}\bar{\nu}_{jL}\nu_{iL}\right]
- \bar{e}_{iL}\bar{d}_{kR}\nu_{jL} - \bar{u}_{jL}\bar{\nu}_{kR}\nu_{iL} - \bar{d}_{kR}\bar{\nu}_{jL}\nu_{iL} + H.c.
\]

In Eq. (9) only the $\lambda_{ijk}$ with $i < j$ are nonvanishing, since the relation $\lambda_{ijk} = -\lambda_{ijk}$ that holds for the coupling constants in the first term in Eq. (1) has already been used.

Contributions to electron-quark interactions can come from the combination of either two $\lambda'_{ijk}$ terms, or from a combination of a $\lambda_{ijk}$ and a $\lambda'_{ijk}$ term. It is easy to see that in the former case the electron-quark interaction has no $P$, $T$-violating component. The reason is that the only terms in (9) involving the electron field are $\bar{\nu}_{iL}\bar{d}_{kR}\nu_{jL}$ and $\bar{d}_{kR}\bar{\nu}_{jL}\nu_{iL}$, and therefore their contribution to the electron-quark interactions must be proportional to $|\lambda'_{ijk}|^2$, which is insensitive to CP-violating phases. $P$, $T$-violating contributions do arise however from combinations of a $\lambda_{ijk}$ and a $\lambda'_{ijk}$ term. We find that there are two such contributions for each down-type quark: one mediated by $\bar{\nu}_{iL}$, and one by $\bar{\nu}_{jL}$ (see Fig. 1). The corresponding effective Hamiltonian is given by

\[
H = \sum_{k=1,2,3} \sum_{j=2,3} \frac{\lambda^*_{ij1}\lambda'_{jkk}}{4m^2_{\nu_j}} (1 + \gamma_5) \bar{e}_k(1 - \gamma_5) d_k + H.c.
\]

The $P$, $T$-violating component of (10) is

\[
H_{P,T} = \sum_{k=1,2,3} \sum_{j=2,3} \frac{\text{Im} \left( \lambda^*_{ij1}\lambda'_{jkk} \right)}{2m^2_{\nu_j}} \left( \bar{e}_k\gamma_5\bar{d}_k - \bar{d}_k\gamma_5\bar{e}_k \right).
\]

It follows that the constants in the e-N interaction (3) are given by

\[
C_{Sa} = \sum_{k=1,2,3} \sum_{j=2,3} \frac{\text{Im} \left( \lambda^*_{ij1}\lambda'_{jkk} \right) \sqrt{2}}{2m^2_{\nu_j}} \frac{f^{(d_k)}}{G_F},
\]

\[
C_{Pa} = -\sum_{k=1,2,3} \sum_{j=2,3} \frac{\text{Im} \left( \lambda^*_{ij1}\lambda'_{jkk} \right) \sqrt{2}}{2m^2_{\nu_j}} \frac{g^{(d_k)}}{G_F},
\]

\[
C_{Ta} = 0,
\]

where $f^{(d_k)}$ and $g^{(d_k)}$ are defined by

\[
\langle a | d_k d_k | a \rangle = f^{(d_k)} \bar{u}_a u_a \quad (k = 1, 2, 3; a = p, n),
\]

\[
\langle a | d_k d_k | a \rangle = g^{(d_k)} \bar{u}_a u_a \quad (k = 1, 2, 3; a = p, n).
\]
\[ \langle a | \bar{d} i \gamma_5 d | a \rangle = g^{(d)}_a \bar{u} a i \gamma_5 u_a \quad (k = 1, 2, 3; \ a = p, n) \ . \]  \tag{16} 

An estimate of the matrix elements \( \langle p | \bar{d} d | p \rangle \) and \( \langle p | \bar{s} s | p \rangle \) can be obtained from the \( \sigma \)-term (deduced from pion-nucleon scattering data) and, assuming octet type \( SU(3) \) breaking, from baryon mass splitting \[32\]. These yield \[33\]

\[ f_p^{(d)} \simeq 2.8 \ , \tag{17} \]
\[ f_p^{(s)} \simeq 1.4 \ . \tag{18} \]

One obtains also \( f_p^{(u)} \simeq 3.5 \) for the form factor in the matrix element \( \langle p | \bar{u} u | p \rangle = f_p^{(u)} \bar{u} u_p \). Charge symmetry and isospin invariance imply, respectively, \( f_n^{(d)} = f_p^{(u)} \) and \( f_n^{(s)} = f_p^{(s)} \), so that

\[ f_n^{(d)} \simeq 3.5 \ , \tag{19} \]
\[ f_n^{(s)} \simeq 1.4 \ . \tag{20} \]

The matrix elements \( \langle a | \bar{d} i \gamma_5 d | a \rangle \) and \( \langle a | \bar{s} i \gamma_5 s | a \rangle \) \( (a = p, n) \) can be estimated using the relation (see Ref. \[34\])

\[ g_p^{(q)} = \frac{M_p}{m_q} (\Delta q' + \frac{\alpha_s}{2\pi} \Delta g) \quad (q = u, d, s) \ , \tag{21} \]

where \( M_p \) is the proton mass, \( \Delta q' \) is the form factor at zero momentum transfer in the proton matrix element of the axial vector current \( \langle p | \bar{u} \gamma_\lambda \gamma_5 q | p \rangle = \Delta q' \bar{u} \gamma_\lambda \gamma_5 q u_p \) and \( \Delta g \) is defined by

\[ \langle p | Tr G_{\mu\nu} \tilde{G}^{\mu\nu} | p \rangle = -2M_p \Delta g \bar{u} \gamma_5 u_p \ , \tag{22} \]

where \( G_{\mu\nu} \) is the gluon field intensity and \( \tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \). With \( \Delta u' = 0.82, \Delta d' = -0.44, \Delta s' = 0.11 \[33\], deduced from data on polarized nucleon structure functions, and the estimate \( (\alpha_s/2\pi) \Delta g \simeq -0.16 \[34\], we find

\[ g_p^{(d)} \simeq 61 \ , \tag{23} \]
\[ g_n^{(d)} = g_p^{(u)} \simeq 121 \ , \tag{24} \]
\[ g_n^{(s)} = g_p^{(s)} \simeq -1.5 \ . \tag{25} \]

In Eqs. (24) and (25) we used again charge symmetry and isospin invariance.

There are no estimates available for \( f_a^{(b)} \) and \( g_a^{(b)} \). We shall use \( f_a^{(b)}/f_a^{(c)} \simeq m_c/m_b \) and \( g_a^{(b)}/g_a^{(c)} \simeq m_c/m_b \) (as expected on the basis of the heavy quark expansion of the corresponding operators (see Ref. \[36\])) and \( f_a^{(c)} \simeq 0.04, g_a^{(c)} \simeq -0.1 \[36\]. Then

\[ f_a^{(b)} \simeq 10^{-2} \quad (a = p, n) \ , \tag{26} \]
\[ g_a^{(b)} \simeq 3 \times 10^{-2} \quad (a = p, n) \ . \tag{27} \]
III. BOUNDS ON THE COUPLING CONSTANTS

We are now ready to consider the limits on the quantities \( \text{Im}(\lambda_{121}' \lambda_{211}') \). In deriving these we shall assume for each such product that it is the only one which may have a significant size. This assumption precludes additional constraints to apply on a given \( \text{Im}(\lambda_{121}' \lambda_{jkk}') \), and the possibility of cancellations among the various contributions to \( C_{Sa} \) and \( C_{Pa} \).

A. \( \lambda_{121}' \lambda_{211}' \) and \( \lambda_{131}' \lambda_{311}' \)

The contribution of \( \lambda_{121}' \lambda_{211}' \) to the constants \( C_{Sa} \) and \( C_{Pa} \) is [see Eqs. (12) and (13)]

\[
C_{Sa} = \frac{\text{Im}(\lambda_{121}' \lambda_{211}')}{2m_{\nu_{\mu}}^2} \frac{\sqrt{2}}{G_F} f_a^{(d)} \quad (a = p, n) ,
\]

\[
C_{Pa} = \frac{\text{Im}(\lambda_{121}' \lambda_{211}')}{2m_{\nu_{\mu}}^2} \frac{\sqrt{2}}{G_F} g_a^{(d)} \quad (a = p, n) .
\]

We shall consider first the information from \( d(T\ell) \). We note that although the ratio \( g_a^{(d)}/f_a^{(d)} \) is large (~20 and ~40 for \( a = p \) and \( a = n \), respectively), the contribution of the \( C_{Pa} \)-interaction to \( d(T\ell) \) can be neglected, since it is supressed by about four orders of magnitude relative to the contribution from \( C_{Sa} \) (two orders of magnitude due to the absence of enhancement by factors of \( Z \) and \( N \), and a further two orders of magnitude due to the fact that the \( C_{Pa} \)-interaction arises only as a correction to the nonrelativistic approximation). Using for \( f_p^{(d)} \) and \( f_n^{(d)} \) the values (17) and (19), we obtain from the limit (4)

\[
|\text{Im}(\lambda_{121}' \lambda_{211}')| \lesssim 1.7 \times 10^{-8} (m_{\nu_{\mu}}/100 \text{ GeV})^2 .
\]  

With the value (24) for \( g_n^{(d)} \) about the same limit \( |\text{Im}(\lambda_{121}' \lambda_{211}')| \lesssim 1.9 \times 10^{-8} (m_{\nu_{\mu}}/100 \text{ GeV})^2 \) follows from \( d(Hg) \) [Eq. (5)]. In \( d(Hg) \) the contribution of \( C_{Pa} \) is smaller than the contribution of \( C_{Sa} \) only by an order of magnitude. The reason is that in diamagnetic atoms a \( C_{Sa} \)-interaction can contribute to the electric dipole moment only with the participation of the hyperfine interaction [34]. As a consequence, the limit on \( |\text{Im}(\lambda_{121}' \lambda_{211}')| \) from the \( C_{Sa} \)-contribution is weaker then the one implied by the \( C_{Pa} \)-contribution by about a factor of four.

For \( \lambda_{131}' \lambda_{311}' \) one obtains in the same way as for \( \lambda_{121}' \lambda_{211}' \)

\[
|\text{Im}(\lambda_{131}' \lambda_{311}')| \lesssim 1.7 \times 10^{-8} (m_{\nu_{\mu}}/100 \text{ GeV})^2 .
\]  

B. \( \lambda_{121}' \lambda_{222}' \) and \( \lambda_{131}' \lambda_{322}' \)

With the values (18) and (20) for \( f_p^{(s)} \) and \( f_n^{(s)} \) the limit (4) from \( d(T\ell) \) yields

\[
|\text{Im}(\lambda_{121}' \lambda_{222}')| \lesssim 4 \times 10^{-8} (m_{\nu_{\mu}}/100 \text{ GeV})^2 ,
\]

\[
|\text{Im}(\lambda_{131}' \lambda_{322}')| \lesssim 4 \times 10^{-8} (m_{\nu_{\mu}}/100 \text{ GeV})^2 .
\]

The limits from \( d(Hg) \) are in this case weaker than those in (32) and (33), since \( g_p^{(s)} \) and \( g_n^{(s)} \) are small.
C. \( \lambda^*_{121}\lambda'_{233} \) and \( \lambda^*_{131}\lambda'_{333} \)

The best limits come again from \( d(T\ell) \) [Eq. (4)]. Using the value (26) for \( f_p^{(b)} \) and \( f_n^{(b)} \) gives

\[
|\text{Im} (\lambda^*_{121}\lambda'_{233})| \lesssim 4 \times 10^{-6} (m_{\tilde{\nu}_i}/100 \text{ GeV})^2 ,
\]

\[
|\text{Im} (\lambda^*_{131}\lambda'_{333})| \lesssim 4 \times 10^{-6} (m_{\tilde{\nu}_i}/100 \text{ GeV})^2 .
\]

In obtaining the bounds (30) - (35) we treated the fields in the Lagrangian (9) as if they were mass-eigenstates, which is not the case in general [38]. While mixing is compulsory only among the left-handed up-type or down-type quarks and, in the light of new evidence, among the neutrinos, the weak eigenstates are not expected to be identical with the mass eigenstates for any of the fields involved. In the presence of mixing the bounds (30) - (35) hold for the quantities \( \text{Im} (\lambda^*_{ij1}\lambda'_{jjk}) \) multiplied by a factor, which is the product of the appropriate elements of the mixing matrices. If the mixings are hierarchical, this factor would not be far from unity.

The bounds (30) - (35) depend only on the constants \( f_{a}^{(d_k)} \). The uncertainties in the values of \( f_{a}^{(d)} \) and \( f_{a}^{(s)} \) come from the experimental value of the \( \sigma \)-term, the values of the quark masses, and from \( SU(3) \)-breaking effects. All these are not likely to affect the values (17) - (20) by more than a factor of 3 - 4. The theoretical uncertainties in \( f_{a}^{(b)} \) are, of course, difficult to assess.

We shall consider now the constraints on the quantities \( \text{Im} (\lambda^*_{ij1}\lambda'_{jjk}) \) in (30) - (35) from other data. The available bounds on the individual coupling constants \( \lambda_{ijk} \) and \( \lambda'_{ijk} \) [9] imply upper limits on the \( \text{Im} (\lambda^*_{ij1}\lambda'_{jjk}) \) in the range \( 2 \times 10^{-2} - 2 \times 10^{-3} \) [9]. The electric dipole moment of the electron (\( d_e \)), which is of relevance, receives contributions involving the same products of \( \lambda_{ijk} \) and \( \lambda'_{ijk} \) as \( C_{S\alpha} \) and \( C_{P\alpha} \) through the two-loop diagrams shown in Fig. 2 [39]. These give [10]

\[
d_e \simeq e \frac{16}{3} \frac{\alpha}{(4\pi)^3} \left( \frac{m_{d_k}}{m_{\tilde{\nu}_j}} \right) \left( \ln \frac{m_{d_k}^2}{m_{\tilde{\nu}_j}^2} \right)^2 \text{Im} (\lambda^*_{ij1}\lambda'_{jjk}) .
\]

For \( k = 1 \) Eq. (36) and the experimental limit \( |d_e| < 4 \times 10^{-27} \text{ ecm} \) [20] imply for \( m_{\tilde{\nu}_j} \) in the range 100 GeV to 1 TeV a limit on \( \text{Im} (\lambda^*_{ij1}\lambda'_{jj1}) \), which is weaker than the limits (30) and (31) by three orders of magnitude. For \( k = 2 \) and \( m_{\tilde{\nu}_j} \) in the same range, the limit from \( d_e \) is weaker than the limits (32) and (33) by two orders of magnitude. However, for \( k = 3 \) and \( m_{\tilde{\nu}_j} = 100 \text{ GeV} \) we obtain

\[
|\text{Im} (\lambda^*_{ij1}\lambda'_{jj3})| \lesssim 6 \times 10^{-7} .
\]

The limit (37) (and also the limits from \( d_e \) for \( m_{\tilde{\nu}_j} \) in the range \( m_{\tilde{\nu}_j} = 100 \text{ GeV} \) to 1 TeV) is more stringent than (34) and (35) by an order of magnitude.

The couplings involved in the \( P, T \)-violating e-N interactions give rise also, through two loop diagrams [39] similar to the one in Fig. 2, to electric and chromoelectric dipole moments for the down-type quarks (\( d_{(d_k)} \) and \( d_{(d_k)}^c \), respectively). The best limits on \( d_{(d_k)} \)
and $d^c_{(d_k)}$ come from the experimental limit on the electric dipole moment of the neutron and the $^{199}$Hg - atom, respectively [10]. The limits implied on the quantities Im($\lambda^*_i j_1 \lambda'_{jkk}$) are weaker than those from $d_e$, since all the $d_{(d_k)}$ and $d^c_{(d_k)}$ are proportional to the electron mass (rather than to the mass of $d_e$), and also because the limits on $d_{(d)}$, $d^c_{(d)}$, $d_{(s)}$, and $d^c_{(s)}$ are weaker than the experimental limit on $d_e$ by two or three orders of magnitude [16], and the limits on $d_{(b)}$ and $d^c_{(b)}$ most likely by even more.

IV. CONCLUSIONS

In this paper we pointed out that experimental limits on $P$, $T$-violating e-N interactions provide stringent constraints on the $R$-parity violating interactions in the $\mathbb{R}$MSSM. Unlike in the MSSM with $R$-parity conservation, in the $\mathbb{R}$MSSM $P$, $T$-violating e-N interactions arise already at the tree level. We found that there are two such contributions for each down-type quark, mediated by the $\tilde{\nu}_\mu L$ and the $\tilde{\nu}_\tau L$ [Eq. (11)]. From the experimental bounds on $P$, $T$-violating observables in atoms and molecules the best limits on the associated coupling constant products Im($\lambda^*_i j_1 \lambda'_{jkk}$) ($j = 2, 3$; $k = 1, 2, 3$) come from the limit on the electric dipole moment of the $^{205}T_\ell$ atom. Here $j$ labels the mediating sneutrino, and $k$ the participating down-type quark. For sneutrino masses of 100 GeV the upper limits on Im($\lambda^*_i j_1 \lambda'_{jkk}$) are of the order of $10^{-8}$ for $k = 1$ and $k = 2$, and about two orders of magnitude weaker for $k = 3$ [see Eqs. (30) - (35)]. For $k = 1$ and $k = 2$ these limits are the best available bounds on these products. For $k = 3$ the limit from $d_e$ is more stringent by an order of magnitude.

Note added in proof: After this paper was submitted to Physical Review, two papers (Refs. [41] and [42]) appeared on fermion electric dipole moments in the $\mathbb{R}$MSSM. These papers also find that in the $\mathbb{R}$MSSM one-loop contributions to the electric dipole moments from trilinear $R$-parity violating couplings can arise only with the participation of either neutrino Majorana mass terms, or $\tilde{\nu}_i - \tilde{\nu}_i^c$ mixing. In Ref. [41] a limit on the coupling constants is derived from a two-loop contribution to the electron electric dipole moment, which is identical to the limit from the same source in our Eq. (37).

ACKNOWLEDGMENTS

I would like to thank S. M. Barr, G. Bhattacharyya, and R. N. Mohapatra for useful conversations. This work was supported by the Department of Energy, under contract W-7405-ENG-36.
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\[\bar{\nu}_i - \bar{\nu}_c^*\]
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FIG. 1. Diagrams contributing to P,T-violating electron-quark interactions in the R/MSSM. Here $j = 2, 3$ ($\tilde{\nu}_2^L \equiv \tilde{\nu}_{\mu L}$, $\tilde{\nu}_3^L \equiv \tilde{\nu}_{\tau L}$), and $k = 1, 2, 3$ ($d_1 \equiv d$, $d_2 \equiv s$, $d_3 \equiv b$).

FIG. 2. Two loop diagrams contributing to the electron electric dipole moment in the R/MSSM. As in Fig. 1, $j = 2, 3$ and $k = 1, 2, 3$. 
