MINIMAL SPATIO-TEMPORAL EXTENT OF EVENTS, NEUTRINOS, AND THE COSMOLOGICAL CONSTANT PROBLEM

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Chryssomalakos and Okon, through a uniqueness analysis, have strengthened the Vilela Mendes suggestion that the immunity to infinitesimal perturbations in the structure constants of a physically-relevant Lie algebra should be raised to the status of a physical principle. Since the Poincaré-Heisenberg algebra does not carry the indicated immunity it is suggested that the Lie algebra for the interface of the gravitational and quantum realms (IGQR) is its stabilized form. It carries three additional parameters: a length scale pertaining to the Planck/unification scale, a second length scale associated with cosmos, and a new dimensionless constant. Here, I show that the adoption of the stabilized Poincaré-Heisenberg algebra (SPHA) for the IGQR has the immediate implication that ‘point particle’ ceases to be a viable physical notion. It must be replaced by objects which carry a well-defined, representation space dependent, minimal spatio-temporal extent. The ensuing implications have the potential, without spoiling any of the successes of the standard model of particle physics, to resolve the cosmological constant problem while concurrently offering a first-principle hint as to why there exists a coincidence between cosmic vacuum energy density and neutrino masses. The main theses which the essay presents is the following: an extension of the present-day physics to a framework which respects SPHA should be seen as the most natural and systematic path towards gaining a deeper understanding of outstanding questions, if not providing answers to them.

Keywords: Stabilized Poincaré-Heisenberg algebra, generalized uncertainty relations, cosmological constant.

1. Two problems and a coincidence

Poincaré and Heisenberg algebras, supplemented by the principle of local gauge invariance, play a pivotal and defining role in the formulation and foundations of modern physics. If I do not invoke equivalence principle directly it is because, following Steven Weinberg, I take the view that Poincaré spacetime symmetries, in conjunction with Heisenberg algebra, not only define the notion of point particle but also suggest the equality of the inertial and gravitational masses.¹–³

¹This essay received an “honorable mention” in the 2005 Essay Competition of the Gravity Research Foundation.
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Seen in this light, irrespective of one’s preference for a candidate theory of quantum gravity, or, the theory of everything, a general question that may be asked is as to what extent the Poincaré and Heisenberg algebras remain undeformed at the interface of gravitational and quantum realms (IGQR). Further, as a secondary question, as to what extent the notion of point particle becomes untenable in any such deformation.

The answer to these questions has a direct impact on the conceptual and internally consistent formulation of any quantum theory of gravity. For instance, a consistent Lorentz-covariant quantum theory of strings requires not only gravity but it carries the advantage that short distance divergences of the field theory no longer exist. While this is far from a trivial justification in favor of abandoning the notion of point particle, it would be desirable to provide a deeper first-principle reason for doing so. In addition, it may have direct implication on how theories of extended objects are formulated, or how other quantum gravity programs are implemented.

The setting thus presented cannot but leave the reader with the expectation that perhaps an argument is now to be presented that Poincaré and Heisenberg algebras must suffer a deformation at IGQR. Indeed that is the case. But, as I hope to argue below, the deformation arises not as an ad hoc suggestion but is based on a principle that requires Lie algebraic stability as a minimal requirement for any physically-viable algebra.

Concurrently with this circumstance there exists yet another problem which challenges the underlying Poincaré-Heisenberg algebraic structure of the standard quantum field theory. The latter predicts that each quantum field carries a non-vanishing zero point energy and that for the standard model fields these quantum fluctuations do not cancel. This result is deeply intertwined with the underlying Poincaré-Heisenberg algebra.

To make the statement of the problem explicit, recall and consider an effective field theory that takes into account only degrees of freedom with energies below about 100 GeV, with all higher energy radiative corrections buried in corrections to various parameters in the effective Lagrangian. In this effective field theory, the all-pervading cosmic vacuum energy density, that serves to explain the recently inferred cosmic acceleration, may be symbolically written as

\[ \rho_{\text{vac}} = \frac{1}{2} \sum \hbar \omega = \frac{c^4 \Lambda}{8 \pi G} \]  

where \( \Lambda \) is the cosmological constant and the sum symbolizes, (a) contribution of all zero point energies in the fields of the effective field theory (with due regard for the sign for fermionic and bosonic fields), and (b) it is cut off at particle energies equal to roughly 100 GeV. In units with \( h = c = 1 \), we have

\[ \frac{1}{2} \sum \hbar \omega \approx (100 \text{ GeV})^4. \]

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\(^a\)See, e.g., Refs. 4–21 for a wide spectrum of views and discussions.
However, observations do not allow $\rho_{\text{vac}}$ to be significantly greater than the critical density $10^{-48}$ GeV$^4$; or, equivalently $(10^{-3}$ eV$)^4$. This mismatch of fifty-six orders of magnitude (in these units) is called the cosmological constant problem and may be interpreted as an outstanding failure of the Poincaré-Heisenberg Lie algebraic structure of the standard model of particle physics. It goes without saying that any well-motivated deformation of the Poincaré-Heisenberg algebra may have a direct impact on the cosmological constant problem.

This situation is made even more intriguing by the fact that there exists a coincidence between the observed cosmic vacuum energy density $\rho_{\text{vac}}$ and the neutrino masses.$^{26,27}$

1.1. Yet, why is the standard model so successful?

That the discordance manifests itself so dramatically in an otherwise successful model of particle physics thus becomes a problem in itself. Within the context of this essay the answer resides in the fact that the Lie algebraic stability leaves the Lorentz sector intact, while introducing modifications via length scales which are either unaccessible at low energies, or carry cosmological dimensions.

2. Introducing the principle of Lie-algebraic stability

First, in all successful physical theories Lie algebras have played a pivotal role in defining the fundamental notion of particle and its evolution. Secondly, as emphasized by Chryssomalakos,$^{28}$ Lie algebras naturally divide themselves in two classes. Those which are stable and those which are unstable. Under infinitesimal perturbations in their structure constants, the former are isomorphic to all Lie algebras in their vicinity, while the latter are not.

Following Vilela Mendes$^{29}$ and Chryssomalakos,$^{28}$ I here suggest that the immunity to infinitesimal perturbations in the structure constants of a physically-relevant Lie algebra should be raised to the status of a principle.$^b$

The reason for this suggestion lies in the fact that, in retrospect, the quantum and relativistic revolutions can be seen as to have been born from a unifying Lie-algebraic stability theme. That is, quantum and relativistic frameworks correspond to the Lie-algebraic stabilization of the algebras underlying the classical mechanics and Galilean relativity. By promoting the Lie algebraic stability to a principle, side by side, say, the principle of local gauge covariance, one hopes that a bewildering set of possibilities that a theorist encounters can be further narrowed. I hasten to add that this is not an abstract idea, but a paradigm which relies on physical and mathematical robustness of the underlying algebraic structures. That only such robust frameworks have a chance of describing physical reality follows if one one wishes to avoid various fine tuning problems which can ultimately, and often, be

$^b$Furthermore, this should be considered as a minimal algebraic requirement, and not as the most general one.
traced back to stability versus instability of the underlying Lie algebras. Stability, I then conjecture, equates to absence of a fine tuning in a physical theory.

3. Stabilized Poincaré-Heisenberg algebra

In the standard general relativistic and quantum framework, a freely falling frame at the IGQR carries the Poincaré-Heisenberg algebra. Within this framework while the position and momentum derive their operational meaning from the fundamental commutator \([x, p_x] = i\hbar, \ldots\), the vanishing commutators for the operators associated with position allow an uncertainty-free \((\Delta x \Delta y = 0 \text{ etc.})\) specification of an event. This underlies the operational framework for the notion of a point particle.

At the same time, as is apparent from the work of Wigner, see, e.g., Ref. 2, the Poincaré spacetime symmetries provide kinematical wave equations which describe the world lines of these point particles. The quantum aspect is then implemented by

— Using the kinematical wave equation to define a Lagrangian density,
— Interpreting the functions on which the wave operators act as field operators, and by imposing Heisenberg’s fundamental commutators/anticommutators for bosonic/fermionic fields and the Lagrangian-density implied canonical momenta, and
— Introducing interactions by invoking form covariance of the Lagrangian density under a suitable spacetime-dependent phase transformations of the involved fields. The simplest of these being a local, i.e. spacetime-dependent, \(U(1)\) transformation which introduces a massless vector field and results in quantum electrodynamics.

At the classical level the effects of a background gravitational field are then incorporated by demanding form covariance of these wave equations under general co-ordinate transformations. In making these transformations the flat spacetime metric is replaced by appropriate metric compatible with energy-momentum density associated with the gravitational background.\(^{6}\) If one now studies a weak-field and non-relativistic limit of these wave equations (with a background gravitational field), and finally invokes Ehrenfest limit, then one verifies that inertial and gravitational masses indeed cancel out from equations of motion. Otherwise, the mass-dependence of test particle survives in the wave equations and results in either gravitationally-induced Bohm-Aharonov like effects, or gravitational redshift of flavor oscillation clocks for neutrinos.\(^{30–34}\)

While these latter effects may be seen as an implication of the equivalence principle at IGQR beyond its original “\(m_i = m_g\)” formulation, the historically-assumed mass-independence of the equations of motion no longer survives. This circumstance, for the case of neutron interferometry, was verified in the classic 1975 experiment of

\(^{6}\)This is where Einstein’s equations come into play.
While the just summarized observations speak of the great strength quantum and relativistic frameworks embody at IGQR, troubles arise not only at well-known attempts to quantize gravity but also in the following two facts: (a) When gedanken experiments incorporate gravitational effects into position measurements the operators associated with the latter cease to commute; and, (b) The Poincaré-Heisenberg algebra at IGQR induces irremovable and intrinsic zero-point energy in freely falling frames. This implies an intrinsic element of curvature, i.e., gravity, in freely falling frames. These observations become even more intriguing when, in the absence of gravity, Sivasubramanian et al. arrive at non-commutative geometry for position measurements of polarized photons.

Within the framework of the paradigm proposed in this essay, the above remarks suggest to question validity of the Poincaré-Heisenberg algebra at IGQR. Specifically, the paradigm of Lie algebraic stability suggests that the problem of constructing a theory of quantum gravity may lie in the fact that Poincaré and Heisenberg algebras cease to be adequate enough at IGQR. Recent physics literature contains numerous efforts to attend to this suspicion. However, with the exception of the 1994 work of Vilela Mendes (and a few important works cited therein) essentially all the attempts fail to arise from some deeper universal principle. The way Vilela Mendes avoids the ad hoc element in his proposal is to discover, and point out, that

— Conceptually, the quantum and relativistic revolutions of the twentieth century can be viewed as Lie-algebraic stabilization of the algebras underlying the classical mechanics and Galilean relativity. Modulo minor technical remarks, the Poincaré and Heisenberg algebras, separately, are endowed with Lie algebraic stability. It was first realized by Faddeev.

— The combined Poincaré-Heisenberg algebra lacks Lie algebraic stability.

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4It is noteworthy that in 1997 the pioneering Werner group published a discrepancy between the theoretically predicted and experimentally measured values of the gravitationally-induced phase shift in neutron interferometry at 1 part in 10^3 level. This discrepancy, to the best of my knowledge, remains unexplained. The fact that no similar signal is seen for the violation of equivalence principle in atomic interferometry by the Stanford group of Chu, raises the possibility that — despite the unexpectedly large violation of the equivalence principle — this may be a quantum gravity effect which manifests itself only for polarized particles. Such a possibility naturally occurs in the proposal of Corichi and Sudarsky. If the Corichi and Sudarsky’s phenomenological proposal is indeed at the origin of the unexpected discrepancy (provided one is able to reconcile the unexpectedly large violation of the equivalence principle), then apart from offering a possibility for new laboratory experiments in quantum gravity, it may have important physical consequences for neutron stars.

5This assertion is as valid for the measurement of different positional components of the same event, as for position measurements of two different events.

6Even if one momentarily does not worry about the associated problem of cosmological constant.

7Rephrasing the earlier-noted definition: From a physicist’s point of view a Lie algebra is considered stable (or, rigid) if infinitesimal perturbations in its structure constants results in isomorphic algebras. See, e.g., Ref. 28.
Having done that, Vilela Mendes then proceeded to present a stabilized form of the Poincaré-Heisenberg algebra. The uniqueness of the Vilela Mendes’ proposal, with additional elements and insights, was demonstrated in the Winter of 2004 by Chryssomalakos and Okon. This circumstance raised the Lie-algebraic stability from a suggestion to a new testable principle.

The stabilized Poincaré-Heisenberg algebra (SPHA) reads:

\[
\begin{align*}
\{ J_{\mu\nu}, J_{\rho\sigma} \} &= i \left( \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\nu\sigma} J_{\mu\rho} \right), \\
\{ J_{\mu\nu}, P_\lambda \} &= i \left( \eta_{\nu\lambda} P_\mu - \eta_{\mu\lambda} P_\nu \right), \\
\{ J_{\mu\nu}, X_\lambda \} &= i \left( \eta_{\nu\lambda} X_\mu - \eta_{\mu\lambda} X_\nu \right), \\
\{ P_\mu, P_\nu \} &= i \left( \frac{\hbar^2}{c^2} \right) J_{\mu\nu}, \\
\{ X_\mu, X_\nu \} &= i \ell_P^2 J_{\mu\nu}, \\
\{ P_\mu, X_\nu \} &= i \hbar \left( \eta_{\mu\nu} F + \beta J_{\mu\nu} \right), \\
\{ P_\mu, F \} &= i \left( \frac{\hbar}{\ell_P^2} \right) X_\mu - \beta P_\mu, \\
\{ X_\mu, F \} &= i \left( \beta X_\mu - \left( \frac{\ell_P^2}{\hbar} \right) P_\mu \right), \\
\{ J_{\mu\nu}, F \} &= 0.
\end{align*}
\]

Here, \( J_{\mu\nu} \) are generators of rotation \( \mathbf{J} \) and boosts \( \mathbf{K} \) (\( J_{ij} = -J_{ji} = \epsilon_{ijk} J_k \) and \( J_{0i} = -J_{i0} = -K_i \); Latin indices run over 1, 2, 3). \( P_\mu \) are generators of spacetime translations, while \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \). The are several novel features in the stabilized algebra. First is the existence of two new length scales. One of these may be identified with the gravitational unification scale \( \ell_U \) defined as \( \gamma \ell_P \). Here \( \ell_P \) is defined as \( \hbar / (m_P c) = \sqrt{\hbar G / c^3} \), while \( \gamma \) may lie anywhere in the range \( 10^{-17} \leq \gamma \leq 1 \), and \( m_P \) is the Planck mass. The other length scale can be taken as \( \ell_C = \sqrt{c^3 / 8\pi G \rho_{\text{vac}}} \), with \( \Lambda \) being the cosmological constant, and \( \rho_{\text{vac}} \) is the vacuum energy density presumably arising from the (modified) zero-point energies of various field. In the process, the underlying algebra unifies the extreme microscopic (i.e., Planck/Unification realm) and the extreme macroscopic (i.e., cosmological scale). For reasons underlying these specifications, see Refs. 3–29. Second, there exists a new dimensionless constant \( \beta \neq 0 \in \mathbb{R} \). Third, existence of \( F \) which...
ceases to be central.

For the purposes of this essay, and to hint at the conceptual and predictive strength of the principle of Lie algebraic stability, I now discuss two issues. The first one concerns the notion of point particle (and associated questions and observations), while the second is a preliminary study on the implications for the cosmological constant problem.

4. Inevitability of abandoning the notion of point particle, and related observations

The fact that the Heisenberg’s fundamental commutator (8) undergoes non-trivial modifications with \(\mathcal{F}\) ceasing to be central, and \(\beta \neq 0\), has the following immediately identifiable consequence: the position-momentum Heisenberg uncertainty relations get modified. For example,

\[
\Delta x \Delta p_x \geq \frac{\hbar}{2} |\langle \mathcal{F} \rangle|,
\]

(12)

while \(\Delta x \Delta p_y\) no longer vanishes, but instead is given by

\[
\Delta x \Delta p_y \geq \frac{\beta \hbar}{2} |\langle J_z \rangle|.
\]

(13)

That is, \(\Delta x \Delta p_x\) is sensitive to \(\mathcal{F}\); while sensitivity to \(\beta\) is carried in \(\Delta x \Delta p_y\).

Furthermore, in the usual notation, one has the following representative expression for the product of uncertainties in position measurements:

\[
\Delta x \Delta y \geq \frac{\ell_U^2}{2c} |\langle J_z \rangle|,
\]

(14)

with

\[
\Delta p_x \Delta p_y \geq \frac{\hbar^2}{2\ell_U^2} |\langle J_z \rangle|.
\]

(15)

complementing equation (14) for momentum measurements. The expectation value, denoted by \(\langle \ldots \rangle\) in the above expressions, is with respect states that arise in a (yet to be fully formulated) quantum field theory based on Lie algebra for \(\text{IGQR}\), i.e., the \(\text{SPHA}\).

Above modified uncertainty relations are to be further supplemented by relations of the form

\[
\Delta x \Delta t \geq \frac{\ell_U^2}{2c} |\langle K_x \rangle|.
\]

(16)

For \(e^\pm\) it takes the form

\[
\Delta x \Delta t_{|e^z} \geq \frac{\ell_U^2}{4c} \left| \left( \begin{array}{cc} -i\sigma_x & 0 \\ 0 & i\sigma_x \end{array} \right) \right|.
\]

(17)

\(^1\)In equations (17) below the factor of \(i\) is left to remind the reader that for \((1/2,0)\) Weyl spinor the boost generator is \(-i\sigma/2\), while for a \((0,1/2)\) Weyl spinor the boost generator is \(+i\sigma/2\).
For the standard-model $\nu_e$ and $\overline{\nu}_e$, the counterparts are
\[
\Delta x \Delta t|_{\nu_e} \geq \frac{\ell_U^2}{4c} \left| \left( -i\sigma_x \ 0 \ 0 \right) \right|, \quad \Delta x \Delta t|_{\overline{\nu}_e} \geq \frac{\ell_U^2}{4c} \left| \left( 0 \ 0 \ i\sigma_x \right) \right|. \tag{18}
\]

For a massive vector particle $B^\mu$, using the results given in Ref. 46 I obtain\(^k\)
\[
\Delta x \Delta t|_{B^\nu} \geq \frac{\ell_U^2}{2c} \left| \left( -i\sigma_x \ 0 \ 0 \right) \right|. \tag{19}
\]

Reader’s attention is drawn to different numerical factors in the right hand sides of Eqs. (17)-(19), and that states that appear in (…) correspond to the indicated particles. The species dependence of these relations is reminiscent of the results found in Ref. 42 (where it is apparent, though not explicitly stated, that deciphered granularity of the spacetime is probe dependent).

For comparison, Eqs. (13)-(15) for $e^\pm$ take the form
\[
\Delta x \Delta p_y|_{e^\pm} \geq \frac{\beta \hbar}{4} \left| \left( \sigma_z \ 0 \ 0 \right) \right| \tag{20}
\]
\[
\Delta x \Delta y|_{e^\pm} \geq \frac{\ell_U^2}{4} \left| \left( \sigma_z \ 0 \ 0 \right) \right| \tag{21}
\]
\[
\Delta p_x \Delta p_y|_{e^\pm} \geq \frac{\hbar^2}{4\ell_C^2} \left| \left( \sigma_z \ 0 \ 0 \right) \right|. \tag{22}
\]

For $\nu_e$ and $\overline{\nu}_e$ of the standard model, Eqs. (13)-(15) have the following explicit form
\[
\Delta x \Delta p_y|_{\nu_e} \geq \frac{\beta \hbar}{4} \left| \left( \sigma_z \ 0 \ 0 \right) \right|, \quad \Delta x \Delta p_y|_{\overline{\nu}_e} \geq \frac{\beta \hbar}{4} \left| \left( 0 \ 0 \ \sigma_z \right) \right| \tag{23}
\]
\[
\Delta x \Delta y|_{\nu_e} \geq \frac{\ell_U^2}{4} \left| \left( \sigma_z \ 0 \ 0 \right) \right|, \quad \Delta x \Delta y|_{\overline{\nu}_e} \geq \frac{\ell_U^2}{4} \left| \left( 0 \ 0 \ \sigma_z \right) \right| \tag{24}
\]
\[
\Delta p_x \Delta p_y|_{\nu_e} \geq \frac{\hbar^2}{4\ell_C^2} \left| \left( \sigma_z \ 0 \ 0 \right) \right|, \quad \Delta p_x \Delta p_y|_{\overline{\nu}_e} \geq \frac{\hbar^2}{4\ell_C^2} \left| \left( 0 \ 0 \ \sigma_z \right) \right|. \tag{25}
\]

Whereas for massive vector particles, the counterpart of these is obtained from Eqs. (13)-(15) by the replacement
\[
J_z|_{B^\nu} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & -i \sigma_x & 0 \\ 0 & i & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{26}
\]

The spatio-temporal extent, besides representation space, depends on $\ell_U^2$; the uncertainty products such as $\Delta p_x \Delta p_y$ depend on $\hbar^2 \ell_C^{-2}$. As a reminder, $\Delta x \Delta p_x$ is sensitive to $F$; while sensitivity to $\beta$ is carried in $\Delta x \Delta p_y$.

\(^k\)The needed from of $K_p$ is obtained from “$K_p$” given in Eq. (4) of the indicated reference, and then evaluating $S^{-1}K_pS^{-1}$ (where $S$ is given by Eq. (18) of the said reference).
Referring to Eqs. (3) and (4), note is to be taken that since the Lorentz sector remains intact the $J^2$ and $J_z$ still commute (while $J_x$ does not commute with $J_x$ and $J_y$). This allows to choose states with well-defined $J^2$ and $J_z$. If we tentatively identify $J^2$ with the standard model fermions and bosons, then its eigenvalues, with exception of Higgs, are non-zero. That is, all matter and gauge field (with exception of Higgs) cannot be identified as point particles. Their position measurements carry a fundamental and irreducible uncertainty. If in Eq. (14), $\Delta y$ is taken as zero these particles acquire the interpretation of string-like objects. Or, if $\Delta y \approx \ell_U$, then one obtains the interpretation of a membrane-like entity. Yet, for physical states for which $\langle J_z \rangle$ vanishes, the point-like interpretation holds. The fundamental spatial extension is bounded from below by $(\ell_U^2/2)|\langle J_z \rangle|$; and it vanishes for a small subset of states for which $\langle J_z \rangle$ is zero.

As such point particle ceases to be a viable notion in IGQR. Furthermore, a concrete modification is suggested for the algebra underlying freely falling frames. It consist of replacing the Poincaré and Heisenberg algebras by the Lie stabilized Poincaré-Heisenberg algebra, SPHA. The latter governs and defines the evolution of the emergent extended objects.

The following questions and observations immediately arise and may be worthy of systematic exploration:

1. **Modification to wave-particle duality** — Since now $p_x \neq \frac{\hbar \partial}{\partial x}$, the considerations found in Refs. 47 and 48 suggest a fundamental modification to the wave-particle duality. That is, de Broglie relation $\lambda_{DB} = \frac{\hbar}{p}$ is no longer viable and must suffer a well-defined modification. In particular, I expect it to have same qualitative behavior as found in Ref. 48. That is, the modified $\lambda_{DB}$ saturates to $\ell_U$ as $p \to \infty$. In fact, given that spatial co-ordinates of an event no longer commute, and that point particle is no longer a viable notion, suggests that an event carrying momentum $p$ is characterized by a set of wavelengths.

2. **Lack of primitiveness of $X_\mu$** — What physical interpretation is to be associated with the non-commutative $X_\mu$. It’s interpretation as space-time coordinates lacks the required primitive nature for Lie algebra generators, as noted by Chryssomalakos and Okon.

3. **Configuration-space wave equations** — If one assumes that the boost parameter, in the notation of Ref. 3, remains unchanged\(^1\)

$$\cosh \varphi = \frac{E}{m}, \quad \sinh \varphi = \frac{p}{m}, \quad \hat{\varphi} = \frac{P}{p}$$

then, the momentum-space wave equations for the SPHA remain intact, but their configuration-space form depend on (a) resolution of the question just

\(^1\)It is not obvious that such an assumption is valid. For one things, the notion of inertial frames now requires a careful examination and spacetime now carries non-commutative elements. Yet, such an assumption is consistent with dispersion relation $E^2 = p^2 + m^2$. But, I see no reason that the dispersion relation itself should not suffer a modification.
enumerated, and (b) the precise conceptual understanding and form of $P_\mu$ as a differential operator which solves the SPHA.

(4) **Lagrangian density, and quantization rules** — The answer to the above question guides to write down the quantum field operator, to obtain the Lagrangian density (note it immediately follows once “configuration space” wave equation is known), and to define $J_{\mu\nu}$ and $P_\mu$, and perhaps $X_\mu$ and $F$, in terms of the field operator. The quantization rules may then be obtained by demanding that the resulting objects satisfy the SPHA.

(5) **Discrete symmetries** — How are the notions of charge conjugation, parity, and time reversal defined. Is the theory symmetric under modified form of these symmetries? Answer to this question, e.g., carries relevance to the observed matter-antimatter asymmetry in the universe.

(6) **S-matrix** — What is the associated S-matrix structure?

(7) **Equivalence principle** — Do the notions of inertial and gravitational masses undergo any change? This question can be examined by concurrently studying the non-relativistic and Ehrenfest limit of the “configuration space” wave equations and by repeating the 1964 analysis of Ref. 1 in the new context.

5. **Impact of SPHA on the cosmological constant problem**

The cosmological constant problem and the zero point energy for bosonic and fermionic field are directly related, and rest on Poincaré and Heisenberg algebras. In order to define the impact of the Lie algebraic stabilization on the cosmological constant problem it is first helpful to add a few brief comments. These complement the discussion given in the opening section. The remarks are then followed by the subject matter of the impact of Lie algebraic stabilization of the Poincaré-Heisenberg algebra on the cosmological constant problem.

**Few brief remarks on the standard zero point energy** — Within the context of Heisenberg algebra, a heuristic understanding of the zero point energy is gained by considering a one-dimensional non-relativistic harmonic oscillator. In the standard notation, it is characterized by the Hamiltonian: $H = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The zero point energy of $\frac{1}{2}\hbar\omega$ arises directly when one determines the eigenspectrum of $H$ with $x$ and $p_x$ satisfying the fundamental Heisenberg commutator $[x, p_x] = i\hbar$. It corresponds to the energy of the ground state. In obtaining this result, the spacetime is assumed to be commutative. In transition from quantum mechanics of point particles, to a relativistic quantum field describing point particles, instead of requiring $[x, p_x] = i\hbar$ (with commutative spacetime), one now imposes the same relations, with right hand side now being a Dirac delta function, $i\hbar\delta(x - x')$, or zero (for field-field, and momentum-momentum, commutators), and $x \rightarrow \psi(x)$, the field, while $p_x \rightarrow \pi(x)$, with the latter representing the canonically conjugate momentum associated with $\psi(x)$; and further replacing the commutator by anticommutator if the field is fermionic. For the standard model fields, each of the the fermionic fields, as is well known, is found to carry a zero point energy of $-\frac{1}{2}\hbar\omega$, while each of the
bosonic fields carries $+\frac{1}{2}\hbar\omega$, for each mode of angular frequency $\omega$. The cosmological constant problem arises because for the standard model fields the bosonic and fermionic contributions do not cancel, and because these contributions when summed over all accessible energies, up to a cut off, give a result which violently disagrees with observational data.\footnote{The numerical aspect of the disagreement was made specific in the opening section of this essay.}

5.1. Zero point energy with Lie-algebraically SPHA: Naive arguments

Naively, one may begin with $H$, considered above, for the simple harmonic oscillator. Such an exercise, with $\beta = 0$, has been undertaken by Vilela Mendes.\footnote{The indicated domain of validity in Eq. (29), corresponds to Vilela Mendes assumption $\hbar^2 / 4\ell_U^2 m^2 \omega^2 \gg 1$.} The result of Vilela Mendes can be put in a closed form for the ground state if one sets $n = 0$ in Eq. 43 of Ref. 49, identify a dimensionless parameter $\zeta$, notice a pattern in the leading order terms, and then sum the indicated series. This set of steps results in a closed form expression for the modified zero point energy, and reads\footnote{The indicated domain of validity in Eq. (29), corresponds to Vilela Mendes assumption $\hbar^2 / 4\ell_U^2 m^2 \omega^2 \gg 1$.}

$$E_0 = \frac{1}{2} \left(1 - \frac{\zeta^2}{1 - \zeta^2}\right) \hbar \omega, \quad \text{for} \quad \zeta \ll \frac{1}{2\sqrt{2} \left(\frac{\ell_U}{\hbar mc}\right)}$$

where the dimensionless parameter $\zeta$ is defined as

$$\zeta \equiv \frac{1}{2} \left(\frac{\ell_U}{\hbar mc}\right) \sqrt{\frac{\hbar\omega}{mc^2}} = \frac{1}{2} \left(\frac{m}{m_U}\right) \sqrt{\frac{\hbar\omega}{mc^2}}.$$  \hspace{1cm} (30)

Here, $m$ represents the mass of the bosonic oscillator, $m_U$ corresponds to cut off mass scale for the effective theory, and $\omega$ is the angular frequency of oscillation.

For $\zeta \ll 1$, i.e. at ‘low’ angular frequencies, the zero point energy $E_0$ remains close to $\frac{1}{2}\hbar\omega$. Whereas as $\zeta$ approaches the unification scale, the $E_0$ vanishes at $\zeta = 1/\sqrt{2} \equiv \zeta_c$, while concurrently one enters the parameter space where the validity criterion in Eq. (29) is crossed. In terms of the $\zeta_c$, the domain of validity for Eq. (29) translates to

$$\left(\frac{m}{m_U}\right)^2 \sqrt{\frac{\hbar\omega}{mc^2}} \ll \zeta_c$$

(31)

For $m \ll m_U$, the domain of validity extends to $\hbar\omega \sim mc^2$. For $m \sim m_U$, the domain of validity is severely restricted to $\hbar\omega \ll mc^2$.

The result (29) is in sharp contrast to the unstable form of Poincaré-Heisenberg where high angular frequencies result in increasingly higher contributions to the zero point energy.

It is quite clear that the preliminary considerations presented here point towards dramatic softening, if not the complete resolution, of the cosmological constant
problem. A stronger claim cannot be made because such an analysis is too naive. For one thing, there is no reason to believe that the form of $H$ remains valid when the underlying spacetime is no longer commutative. More importantly, one has no unique guiding principle to define as to what one means by a simple harmonic oscillator for the latter circumstance. To bypass this problem, it is advisable that heuristic argument given here serve only as a motivation to look at the spectrum of free bosonic and fermionic fields as they exist for (yet to be developed) quantum fields based on SPHA.

If the naive result contained in Eq. (29) — is essentially confirmed by a rigorous analysis of the quantum fields based on SPHA, and if it — captures the essence of the exact result

then for heavy particles, $\zeta$ approaches $\zeta_c$ faster than that compared with light particles. Therefore, the dominant contribution comes to the cosmological constant from the lightest particles in the standard model, i.e., the neutrinos of the fermionic sector and the photons of the bosonic sector. Furthermore, for massive particles this contribution comes not from the angular frequencies $\omega \sim mc^2/h$, but from the lower spectrum of angular frequencies. It may underlie the observation that the vacuum energy density associated with $\Lambda$ is of the same order as that of neutrinos. Specifically, the coincidence between $\rho_{\text{vac}} \approx (10^{-3} \text{ eV})^4$ and neutrino masses as suggested by the atmospheric and solar neutrino data, see e.g. Refs. 26 and 27, acquires a plausible first-principle explanation in the just outlined scenario.

6. To sum up

Within the framework of the standard model of particle physics, the cosmological constant problem poses a dramatic discordance between reality and prediction. The SPHA, summarized in Eqs. (3)-(11), offers the next logical step towards extension of the standard model in IGQR without spoiling any of its grand successes. In the process it offers a well-defined departure from the notion of point particle where an event carries a minimal spatio-temporal extent. The latter depends on the representation space to which the event belongs.

At this early stage it is difficult to assert with any confidence if SPHA is indeed the next approximation to the physically-realized algebraic structure over which to extend the standard model of particle physics (and which incorporates gravity in its quantum nature). However, from a theoretical point of view, an extension of the present-day physics to a framework which respects SPHA should be seen as the most natural and systematic path towards gaining a deeper understanding of outstanding questions, if not providing answers to them.
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