The black hole tunnel phenomenon

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The potentials of spin-weighted wave equations in various Kerr-Newman black holes are analyzed. They all form singular potential barriers at the event horizon. Applying the WKB approximation it is shown that no particle can tunnel out of the interior of a static black hole. However, photons inside a non-extremely rotating Kerr black hole may tunnel out into the outer space, whereas neutrinos, electrons, and gravitons may not. If the rotation is extremal, any particle may tunnel out, under restrictive conditions. It is unknown whether photons and gravitons may tunnel out if the black hole is charged and rotating.

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I. INTRODUCTION

In classical general relativity a causal particle inside the event horizon is inevitably pulled towards the center of the black hole, at least until it reaches the Cauchy horizon representing a barrier to predictability [1]. In particular, no classically relativistic particle, be it of positive or vanishing rest-mass, can escape from the black hole interior. However, Hawking [2] has shown that photons in effect can leave the black hole if a quantized photon field in the curved spacetime is supposed, cf. [3].

In general, quantization of spin-weighted waves in a curved spacetime yields the notion of spin particles in a gravitational field. As such, spin-weighted waves are the basis for semiclassical quantum gravity. Their properties and behavior in the outer space of Kerr black holes have been extensively studied [4, 5, 6, 7, 8], and could in part even be extended to charged Kerr-Newman black holes [9, 10, 11, 12]. By the symmetries of these spacetimes the equations turn out to be separable in special coordinate frames, thus being mathematically tractable to a certain extent.

In the framework of classical general relativity the examination of waves in the outer space of a black hole seemed sufficient, since the event horizon acts like a perfect semi-permeable membrane, letting in any form of energy and matter but allowing none to get out.

In the present paper we extend these considerations to the region beyond the event horizon. Topologically, this means nothing particular, since the spacetime curvature remains finite in this domain and the event horizon is nothing more than a coordinate singularity. As expected, the event horizon causes a singular repulsive potential in

\[ \frac{2M}{r} \]

the respective wave equations. However, more detailed examination shows the remarkable, and to our knowledge yet unmentioned, property of the potential barrier to be singular enough to prevent nearly all kinds of spin-weighted waves from tunneling through it — unless electromagnetic waves.

Our analysis below shows that in fact a photon inside an uncharged rotating black hole may tunnel through the event horizon to the outside region, but a graviton, a neutrino or an electron may not. If the black hole is non-rotating, no particle at all can tunnel out of it. In case of a rotating and electrically charged black hole it remains still unknown what kind of particle may tunnel out, it is only sure that electrons and neutrinos may not.

The present paper is organized as follows. In section II we introduce the basic notation and properties of Kerr-Newman spacetimes. In section III we analyze the equations for massless waves in Kerr geometry and present a proof that only photons of certain discrete frequencies may tunnel out of a rotating but non-extremal Kerr black hole. The case of extremal rotation is considered in section IV, the mathematically not completely tractable case of massless waves in a charged Reissner-Nordström spacetime in section V, and the Dirac equation in a general Kerr-Newman spacetime in section VI. Finally, in section VII we sum up the results and discuss them.

II. KERR-NEWMAN SPACETIMES

Suppose a Kerr-Newman black hole with the three real parameters \( M, a, \) and \( Q \). They are related to the mass \( M \) (in kg), the angular momentum \( J \) (in kg m\(^2\) s\(^{-1}\)) and the electrical charge \( q \) (in kg m\(^3\) s\(^{-1}\)) by the relations

\[ \begin{align*}
M &= \frac{e^2 M}{G}, \\
J &= acM, \\
q &= \frac{cQ}{\sqrt{G}}.
\end{align*} \]

Here \( G \) is the gravitational constant, and \( c \) the speed of light. The non-vanishing contravariant components \( g^{i\jmath} \)
of the metric tensor in Boyer-Lindquist coordinates \((x^0, \ldots, x^3) = (ct, r, \theta, \varphi)\) are then
\[
g^{tt} = \frac{(r^2 + a^2)^2 - a^2\sin^2 \theta}{c^2\rho^2\Delta}, \quad g^{rr} = \frac{\Delta}{\rho^2}, \quad g^{\theta\theta} = -\frac{1}{\rho^2}, \quad g^{\phi\phi} = \frac{\Delta - a^2\sin^2 \theta}{\rho^2\Delta \sin^2 \theta},
\]
\[(g^{t\phi} = g^{\phi t}), \text{ where}
\]
\[
\rho = r + ia \cos \theta, \quad \Delta = (r-r_+)(r-r_-),
\]
with the event horizon \(r_+\) and the Cauchy horizon \(r_-\) given by
\[
r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2}.
\]
Only the points of the set \(\rho = 0\) have infinite Riemann curvature and thus locate the curvature singularity of the spacetime. For \(a \neq 0\), it in fact forms a ring of radius \(|a|\) in the equatorial plane \(\theta = \pi/2\), cf. \[3\]. Because the cosmic censorship hypothesis forbids naked singularities, the event horizon \(r_+\) must necessarily exist. Thus the square root of \(\[3\] must be real, i.e.
\[
a^2 + Q^2 \leq M^2.
\]

### III. SPIN-WEIGHTED MASSLESS WAVES IN KERR GEOMETRY

Let be \(Q = 0\), and define \(s \in \{0, \pm \frac{1}{2}, \pm 1, \pm 2\}\) as the spin-weight. Then the sourcefree perturbation equations for scalar \((s = 0)\), two-component neutrino \((s = \pm \frac{1}{2})\), electromagnetic \((s = \pm 1)\), and gravitational fields \((s = \pm 2)\) are given by wave equations which by the the symmetries of the Kerr spacetime, viz. stationarity and axialsymmetry, admit the separable solutions
\[
\psi(t, r, \theta, \varphi) = R_s(r) S_s(\theta) e^{-i\omega t} e^{im\varphi}
\]
with the constants \(\omega \in (0, \infty)\) and \(m \in \mathbb{Z}\). Here \(R_s\) obeys the radial equation
\[
\Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{dR_s}{dr} \right) + \frac{(K^2 - 2is(r-M)K)}{\Delta} + 4is\omega r - \lambda R_s = 0,
\]
and \(S_s\) solves the angular equation
\[
\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dS_s}{d\theta} \right) + \left( (a\omega \cos \theta - s)^2 - (m + s \cos \theta)^2 \sin^2 \theta - s(s-1) + A \right) S_s = 0,
\]
with
\[
K = K(r) = (r^2 + a^2)\omega - am,
\]
and \(\lambda = A + a^2\omega^2 - 2am\omega\), cf. \[4\]. The constants \(\lambda, A \in \mathbb{R}\) are separation constants obtaining some discrete values depending on the boundary conditions of \(S_s\). (For details see \[3\], §2.1.) To be more explicit, \(\lambda = s\lambda_{0}^m = sE_{0}^m + a^2\omega^2 - s(s+1)\), where \(E_{0}^m = \frac{l(l+1) - 2am\omega a^2}{1(l(l+1))} + O((\omega a)^2)\), with \(l \geq \max(|s|, |m|)\) and \(l \in \mathbb{N}\) for integer spin \(s\), \(l\) half-odd integer for \(|s| = 1/2\), cf. \[4\] eq. (3.10).

**Theorem 1** Let be given the operator \(A(s) : C^2_0(\mathbb{R}, \mathbb{C}) \to C^2_0(\mathbb{R}, \mathbb{C})\),
\[
A(s) = \frac{d^2}{dr^2} + q_s
\]
with
\[
q_s = \frac{K^2 + (1 - s^2)(r-M)^2}{\Delta^2} + \frac{|s| - 1 - \lambda}{\Delta} + \frac{2is}{\Delta} \left( 2\omega r - \frac{(r-M)K}{\Delta} \right).
\]
With the functional transformation
\[
u_s = \Delta^{(s+1)/2} R_s
\]
the radial equation \([3]\) is equivalent to \(T_s P_s = 0\), with the differential operator
\[
T_s = \frac{d^2}{dr^2} + g_s \frac{d}{dr} + f_s,
\]
where
\[
f_s = \frac{K^2 - 2is(r-M)K + 4is\omega r - \lambda}{\Delta},
\]
and
\[
g_s = \frac{2(1 - |s|)(r-M)}{\Delta}.
\]
(cf. \[3\], eqs. (96), (97) in chapter 8). With
\[
\frac{1}{2} \frac{d g_s}{dr} \left( \frac{1}{\Delta} - \frac{1 - |s|}{\Delta} \right) \left( 1 - \frac{2(r-M)^2}{\Delta} \right)
\]
we see that \(q_s = f_s - \frac{1}{2} \frac{d g_s}{dr} \frac{1}{\Delta} \). Since \(d\Delta/dr = 2(r-M)\), we have \(f_s dr = (1 - |s|) \ln \Delta\). By elementary means (e.g. \[3\], p. 323) we deduce from this result that, by the transformation \(u_s = \Delta^{(1-|s|)/2} P_s\), the equation \(T_s P_s = 0\) is equivalent to \(A(s) u_s = 0\). \(\square\)
Lemma 2 Let \( u_s \) be a solution of the differential equation \( A_{(s)} u_s = 0 \). Then the asymptotic behavior of \( u_s \) is given by

\[
\begin{align*}
\lim_{r \to \infty} u_s &= a_{\text{in}} r^s e^{-i\omega r} + a_{\text{out}} r^{-s} e^{i\omega r}, \\
\lim_{r \to \infty} u_s &= b_{\text{in}} e^{-i k r} + b_{\text{out}} e^{i k r}
\end{align*}
\]  

with the complex amplitudes \( a_{\text{in}}, a_{\text{out}}, b_{\text{in}}, b_{\text{out}} \in \mathbb{C} \) and the complex exponent \( k \) given by (cf. figure 4)

\[
k^2 = \frac{[(M^2 + a^2) \omega - am]^2}{(M^2 - a^2)^2} + |s| - 1 - \lambda - 4i s \omega M}{M^2 - a^2}.
\]  

Proof. First we consider the behavior for \( r \to \infty \). In this case, \( q_s = q_{s,\infty} + O(r^{-2}) \) with \( q_{s,\infty} = \omega^2 - 2i s \omega / r \). Thus asymptotically \( A_{(s)} u_s \to u''_s + q_s u_s \), which is solved by the first limit (17) as is seen by direct computing.

On the other hand, we calculate by \( K(M) = (M^2 + a^2) \omega - am \) that \( q_s \to k^2 \) as \( r \to M \). Hence, \( A_{(s)} u_s \to u'' + k^2 u \), which is solved by the second limit in (17).

Note that the complex conjugate of \( A_{(s)} \) in (1) equals the radial operator \( A_{(-s)} \) of the waves of spin \(-s\), \( A_{(s)} = A_{(-s)} \). In particular, \( A_{(-s)}^* u_{-s} = A_{(-s)} u_{-s} = 0 \), and thus

\[
A_{(s)}^* u_{-s} = A_{(-s)} u_{-s} = 0.
\]  

The advantage of the transformation in theorem 3 is the fact that the Wronskian of two solutions of the differential equation \( A_{(s)} u_s = 0 \) is constant. In particular, the behavior of two independent solutions at the boundaries of the domain of definition determines the reflection and transmission coefficients of the potential \( V_s \), cf. (11). By the peeling theorem of Newman and Penrose (11), for a null field of spin weight \( s \) the ingoing and outgoing solutions differ in magnitude by the factor \( r^{2s} \). Accordingly, the solutions \( R_s \) show the asymptotic behavior

\[
R_s \propto \begin{cases} \frac{1}{r} e^{-i \omega r} & \text{ingoing waves} \\ \frac{1}{2s+1} r^{2s+1} e^{i \omega r} & \text{outgoing waves} \end{cases}
\]  

This is in accordance with the asymptotic behavior of \( u_s \), using the transformation (13) and \( \Delta = r^2 + O(r) \).

Theorem 3 The only massless particles that can tunnel out of a non-extremal Kerr black hole are photons with the discrete frequencies

\[
\omega_m = \frac{am}{r_s^2 + a^2}.
\]  

Their wave numbers \( k_m \) at \( r = M \) are given by

\[
k_m^2 = \frac{a^2 m^2 (M^2 - r_s^2)^2}{(M^2 - a^2)^2 (r_s^2 + a^2)^2} - \frac{\lambda + 4i s \omega_m M}{M^2 - a^2},
\]  

depending on the angular quantum number \( m \in \mathbb{Z} \). The amplitudes of a photon wave with the asymptotic behavior (14) is determined by the conservation law

\[
|a_{\text{out}}|^2 - |a_{\text{in}}|^2 = \frac{\text{Re} k_m}{\omega_m} (|b_{\text{out}}|^2 - |b_{\text{in}}|^2),
\]  

Proof. A Kerr black hole with mass \( M \) and rotation parameter \( a \) is non-extremal if and only if \( a^2 < M^2 \). For \( \omega = \omega_m \) as in (23) we compute straightforwardly the functions \( K_m = K \) as

\[
K_m = \omega_m (r - r_+) (r + r_+).
\]  

Inserting \( \omega_m \) into the expression for \( k \) in (18) yields

\[
k_m^2 = \left[ \frac{am (M^2 - r_s^2)}{(M^2 - a^2)(r_s^2 + a^2)} \right]^2 + \frac{|s| - 1 - \lambda - 4i s \omega_m M}{M^2 - a^2}.
\]  

By theorem 3 a massless particle of spin-weight \( s \) is given as a wave function \( u_s \) satisfying \( A_{(s)} u_s = 0 \). Since the WKB approximation is applicable if \( q_s \) is varying slowly enough,

\[
\frac{1}{\sqrt{q_s}} \frac{d^2}{dr^2} q_s \ll 1 \quad \text{and} \quad \frac{1}{\sqrt{q_s}} \frac{d}{dr} \sqrt{q_s} \ll 1
\]  

we may use it for a solution \( u \) in a region close enough to the event horizon at \( r = r_+ \) and far enough from the turning points \( q_s = 0 \), see [8] §8.2, say \( r_+ - \epsilon_+ < r < r_+ + \epsilon_+ \). In this regime, the transmission coefficient \( |T|^2 \) is given by

\[
|T|^2 = \exp \left[ -2 \int_{r_+ - \epsilon_+}^{r_+ + \epsilon_+} \text{Re} \sqrt{q_s} \frac{dr}{\sqrt{q_s}} \right]
\]  

The integral diverges if \( q_s \) has a pole at \( r = r_+ \) of order not smaller than 2, but it attains a finite value for a pole of order 1 since

\[
\int_{\epsilon_+}^{\epsilon_-} \frac{dr}{\sqrt{r}} \leq \int_{\epsilon_+}^{\epsilon_-} \frac{dr}{\sqrt{|r|}} = 2(\sqrt{\epsilon_+} + \sqrt{\epsilon_-}).
\]  

There are three terms in \( q_s \) which may result in a pole of order 2 at \( r = r_+ \), namely

\[
\Delta q_s = K^2 + (1 - s^2)(r - M)^2 + 2i s K (r - M) + O(\Delta).
\]
The third term, the imaginary part, vanishes (in the non-extremal case, where \( r_+ > M \)) if and only if \( K(r_+) = 0 \) or \( s = 0 \). The first two terms, the real part, vanish at \( r = r_+ \) if and only if \( K^2(r_+) = (s^2 - 1) (r - M)^2 \). Since \( q_s \) is never purely real and negative, a pole both in the real part as well as in the imaginary part of \( q_s \) is also a pole of \( \Re \sqrt{q_s} \), cf. figure [3]. Thus in (23) the real and the imaginary terms of \( q_s \), resulting in a pole of order 2 have to vanish simultaneously at \( r = r_+ \), i.e. \( K(r_+) = 0 \) and \( |s| = 1 \). This means that the integral \( \int_M^r \Re \sqrt{q_s} \dd r \) exists only for photons with frequencies \( \omega = \omega_m \). In particular, with (24) we obtain \( K_m/\Delta = \frac{m(r+r_+)}{(r-r_+)^2} \). This leads to the functions \( q_{sm} \)

\[
q_{sm} = \frac{\omega_m^2(r + r_+)^2}{(r-r_+)^2} - \frac{\lambda}{\Delta} + 2i \omega_m (r^2 + r_+ r + Mr_+) \frac{(r - r_+)(r - r_-)^2}{(r - r_+)(r - r_-)^2}.
\]

Now let \( u_s \) be a solution of \( A_{s}(u) = 0 \) with the asymptotic behavior (17). Then \( u_{-s} \) solves \( A_{-s}(u) = 0 \) and is therefore a wave of spin \(-s\), too. By \( A_{s}(u) = A_{s}(u) \) it follows that \( A_{s}(u_{-s}) = 0 \), i.e. \( u_{-s} \) is a wave with spin \( s \). Hence the Wronskian of the two solutions of \( A_{s}(u) = 0 \) is given asymptotically by

\[
\begin{align*}
[u_s, u_{-s}] & \to 2i(\omega_m |a_{out}|^2 - |a_{in}|^2), \\
[u_s, u_{-s}] & \to 2i \Re k (|b_{out}|^2 - |b_{in}|^2),
\end{align*}
\]

Since the Wronskian is constant for an equation of the form \( u'' + qu = 0 \), the assertion follows.

\( \square \)

IV. EXTREMAL ROTATION

An extremal Kerr black hole is given by the maximally possible angular momentum \( |a| = M \). In this case, we have \( r_+ = r_+ = M \), and the functions \( \Delta \) and \( K \) in (4) simplify to \( K = (r^2 + M^2) \omega - am, \Delta = (r - M)^2 \). According to (10) this yields

\[
q_s = \frac{K^2}{(r - M)^4} + \frac{|s|(1 - |s|) - \lambda}{(r - M)^2} + \frac{2is}{(r - M)^2} (\omega r - \frac{K}{(r - M)}) \frac{2i}{(r - M)^2} (\omega r - \frac{K}{(r - M)}) \frac{2i}{(r - M)^2} (\omega r - \frac{K}{(r - M)}).
\]

Now \( q_s \) has a pole of order 4 at \( r = M \). To apply a similar consideration as in the proof of theorem [3] we have to arrange the nominators to obtain a zero of order 3. For convenience, we again consider the imaginary and the real part of \( q_s \) separately. First we note that \( K(M) \) must vanish to avoid a pole of order 3, i.e. \( K(M) = 0 \), or equivalently

\[
\omega_m = \frac{m}{2a}, \quad (|a| = M).
\]

With this, \( K \) simplifies to \( K = (r - M)(r + M) \omega_m \), and therefore

\[
\text{Im} q_{sm} = \frac{2s \omega_m}{r - M},
\]
i.e., the imaginary part of \( q_s \) has a pole of order 1. Regarding the real part, we insert our derived expression for \( K \) to obtain

\[
\text{Re} q_{sm} = \frac{(r + M)^2 \omega_m^2 + |s|(1 - |s|) - \lambda}{(r - M)^2},
\]

At \( r = M \), the nominator vanishes if and only if \( \lambda = 2\omega_m^2 M + |s|(1 - |s|) \), or with (24),

\[
\lambda = \frac{m^2}{2} + |s|(1 - |s|).
\]

Therefore, massless spin waves only can tunnel out of an extremal Kerr black hole under a restrictive condition on the separation constant \( \lambda \). Since \( |s|(1 - |s|) \), is integer for integer spin \( s \), \( m \) has to be even to guarantee that \( m^2/2 \) and thus \( \lambda \) is an integer. (Remember that \( \lambda \) has to be integer for integer spin \( s \).) On the other hand, \( m \) also has to be even such that \( \lambda \) is a square of an odd-half integer for \( |s| = \frac{1}{2} \). (Note: \( |s|(1 - |s|) = \frac{1}{4} \), and \( \lambda = (2m^2 + 1)/4 \); since \( m^2 = 0 \) or 1 mod 4, \( 2m^2 + 1 = 1 \) or 3 mod 4; this is only a square number if \( 2m^2 + 1 = 1 \) mod 4, because any square number \( n = 0 \) or 1 mod 4.)

V. REISSNER-NORDSTROM BLACK HOLES

For \( a = 0 \) and \( 0 < Q^2 \leq M^2 \), the Kerr-Newman spacetime represents a static charged black hole, the Reissner-Nordstrøm black hole. A general master equation like Teukolsky’s equation governing massless spin waves in the Reissner-Nordstrøm geometry and leading to radial and angular equations like (3), (4) could not be derived to date. Nonetheless, the basic equations governing electromagnetic and gravitational waves (\( |s| = 1, 2 \)) can be transformed to \( T_{(s)} Y_{±i} = 0 \), where \( T_{(s)} = \frac{d^2}{dr^2} + g_s \frac{d}{dr} + f_s \) with

\[
f_s = \frac{r^4 \omega^2}{\Delta^2} + \frac{2i \omega r}{\Delta} - n^2 \Delta \left[ 1 + \frac{2 \bar{q}_j}{n^2 r} \right] \left[ 1 + \frac{2 \bar{q}_j}{n^2 r} \right],
\]

and

\[
g_s = \frac{2(r - M)}{\Delta} + \frac{d}{dr} \ln \left[ \left( r^6 \Delta^2 \right)^{2|s| - 3} \left( 1 + \frac{2 \bar{q}_j}{n^2 r} \right) \left( 1 + \frac{2 \bar{q}_j}{n^2 r} \right) \right].
\]

Here the separation constant \( n \) is given by \( n^2 = (l - 1)(l + 2) \), the indices \( i \) and \( j \) take the two values \( i, j = 1, 2 \), but \( i \neq j \), and the constants \( \bar{q}_j \) are given by

\[
\bar{q}_{1/2} = 3M + \sqrt{9M^2 + 4Qn^2},
\]

These relations can be readily deduced from equations (244) and (245) in [[3], §5 p. 244]. By the same procedure as in the proof of theorem [3] the spin-weighted wave equations can be transformed to \( A_{(s)} P_{2s} = 0 \) with \( A_{(s)} = d^2/4d^2 + q_s \) and \( q_s = f_s - g_s^2/4 - g_s^2/2 \). But since \( \Delta^2 q_s = r^4 \omega^2 + O(\Delta) \), \( q_s \) has a pole of order not smaller than 2 as long as \( \omega > 0 \). Hence for waves of spin-weight \( |s| = 1 \) or 2 no tunneling out of a Reissner-Nordstrøm black hole can occur.
VI. THE DIRAC EQUATION IN A KERR-NEUMANN SPACETIME

For a particle of mass $m$, and electrical charge $e$ let be given the constant $\mu_e = m_e c/(\sqrt{2} \hbar)$, such that $\sqrt{\pi}/\mu_e$ is the Compton wavelength. Then it is described by the Dirac equation in a general spacetime, cf. \[12\] eq. (16) (for $e = 0$). In the Kerr-Newman geometry it can be reduced to the complex radial differential equation $T(r)P_s = 0$, where $T(r) = \frac{d^2}{dr^2} + g_s \frac{d}{dr} + f_s$ with

$$f_s = \frac{K^2 - 2i\lambda(r-M)K}{\Delta^2} - \frac{\mu_e K}{(\lambda + 2i\mu_e r)\Delta} + \frac{2i(2\omega r + eQ - \mu_e^2 r^2 - \lambda^2)}{\Delta}$$

(37)

and

$$g_s = \frac{r-M}{\Delta} - \frac{2i\lambda}{\lambda + 2i\mu_e r}$$

(38)

(note that $P_s$ is related to the usual radial function $R_s$ by \[12\]), and the angular equation $\mathcal{L}S_s = 0$, with

$$\mathcal{L} = \frac{1}{\sin \theta \frac{d}{d\theta}} \left( \sin \theta \frac{d}{d\theta} + \frac{a\mu_e \sin \theta \frac{d}{d\theta}}{\lambda - 2asmu_e \cos \theta} \right) + (a\omega \cos \theta - s) + \frac{a\mu_e (a\omega \sin^2 \theta - m - s \cos \theta)}{\lambda - 2as\mu_e \cos \theta} - \frac{(m + s \cos \theta)^2}{\sin^2 \theta} - \frac{a^2 \mu_e^2 \cos^2 \theta - \lambda(s - 1) + A}{\Delta}.$$  

Here

$$K = K(r) = (r^2 + a^2) \omega - am + eQr,$$

(40)

and $\lambda^2 = A + a^2 \omega^2 - 2am \omega$. The constants $\lambda, A \in \mathbb{R}$ are separation constants obtaining discrete values depending on the boundary conditions of $S_s$. For details see \[10\], for the Kerr case ($Q = 0$) see \[8\], \[9\] §104 eqs. (123), (124).

To analyze the potential, a transformation analogous to the proof of theorem \[10\] has to be done, where now by $4s^2 - 1$,

$$\frac{dg_s}{dr} = \frac{1}{\Delta} \left( \frac{1 - 2(r-M)^2}{\Delta} - \frac{\mu_e^2}{(\lambda + 2i\mu_e r)^2} \right),$$

(41)

cf. \[10\]. This yields a complex potential $q_s$ differing from \[10\] only by terms depending on $\mu_e$. As long as $\lambda = -2is\mu_e r_+$, these terms are of order $O(\Delta^{-1})$, and the proof of theorem \[10\] in essence can be applied. Hence no tunneling of electrons out of a Kerr-Newman black hole can occur, unless for the case of extremal rotation.

If $\lambda = -2is\mu_e r_+$ the situation changes, since now the terms by which $q_s$ differs from the massless case, are of order $O(\Delta^{-2})$. First we notice that $\lambda + 2is\mu_e r = 2is\mu_e (r-r_+)$, Therefore, with $\Delta = (r-r_+)(r-r_-)$, $4s^2 = 1$, $4s^2 (M - r + r_+) = M - r_+$, we obtain

$$f_s = \frac{K^2 + iK(M-r_-)}{\Delta^2} + \frac{i(2\omega r + eQ)}{2s\Delta} - \frac{r + r_+ \mu_e}{r - r_- \Delta},$$

$$g_s = -\frac{M - r_+}{\Delta},$$

(42)

With these equations the terms deciding the occurrence of a pole of order 2 at the horizon $r = r_+$ are determined by

$$\Delta^2 q_s = K^2 + (M - r_-) \left( r + r_+ - \frac{5M}{4} \right) + 2iM(M - r_-) + O(\Delta).$$

(43)

To avoid a pole of order 2, the imaginary part has to vanish. This implies $K(r_+) = 0$ (even in the extremal case $M = a^2 + Q^2$, for then $\Delta = (r - M)^2$ and $M = r_-$), or

$$\omega_m = \frac{am - eQr_+}{r_+^2 + a^2}.$$

(44)

The real part $\Delta^2 \cdot \text{Re} q_s = (M - r_-)(r - \frac{5M}{4})$ then vanishes if and only if $r_+ = r_ = M$, which is equivalent to $M^2 = a^2 + Q^2$. This is the extremal case, and we see that then $K = \omega_m (r - M) (r + M)$, i.e. $q_s = f_s$ or

$$q_s = \frac{\omega_m^2 (r + M)^2}{(r - M)^2} + \frac{i(2\omega r + eQ)}{2s(r - M)^2} - \frac{r + M + 2i\mu_e}{r - M}.$$

(45)

Therefore, $q_s$ has a pole of order 2 at the event horizon, such that no tunneling may occur for $\lambda = -2is\mu_e r_+$.

VII. DISCUSSION

In the present paper we consider properties of spin-weighted waves in various Kerr-Newman spacetimes. We analyze the singular potential barriers of the respective wave equations emerging at the event horizon. It turns out that the potential barriers are repulsive enough to prevent spin waves with spin $|s| = 0, \frac{1}{2}, 1, 2$ from tunneling out of the black hole, but weak enough to admit the tunneling of photons in case of non-extremal rotation.

In particular, if $a$ denotes the angular momentum of a Kerr-Newman spacetime and $Q$ its electrical charge, then the following table shows the occurrence of tunneling (‘+’), its impossibility (‘-’) and the present ignorance (‘?’) with respect to the various kinds of spins $s$.

| spin $|s|$ | 0 | $\frac{1}{2}$ | 1 | 2 |
|---------|---|-------------|---|---|
| $a = 0$ | $-$ | $-$ | $-$ | $-$ |
| $0 < |a| < M$, $Q = 0$ | $-$ | $-$ | $+$ | $+$ |
| $|a| = M$, $Q = 0$ | $\times$ | $\times$ | $\times$ | $\times$ |
| $0 < |a|$, $|Q| < M$ | $-$ | $-$ | $-$ | $?$ |

(46)

The case $a \neq 0 \neq Q$, i.e. charged rotating Kerr-Newman black holes, still remains unknown. The crucial role that rotation plays in the known cases suggests that photons may tunnel out of any rotating black hole. A very strong hint supporting this conjecture is the analysis of one of the authors \[11\]. where any electromagnetic wave in the outer space of a rotating Kerr-Newman black hole can gain some energy from the hole. However, we still have no such hints for gravitons.
For the case of extremal rotation, $|a| = M$, any particle can tunnel out of the black hole. Because of the cosmic censorship hypothesis (1), $Q$ vanishes necessarily in this case. There is only the restriction on the separation constant $\lambda$ given by (33) to be an even integer. Whether such an integer exists at all depends on the Sturm-Liouville equation for $s$, (5) or (9), respectively.

Mathematically, in case of $Q = 0$ or $a = 0$ a necessary and sufficient condition for the occurrence of the tunnel phenomenon is the vanishing of $g_s$, the factor of the first derivative in the radial wave equations, and of the function $K$, cf. equations (15), (8) for massless waves of spin $s$, and (38), (40) for spin-$\frac{1}{2}$ waves of mass $m_v = \sqrt{2\hbar\mu_v}/c$. This condition yields a potential with an order-1 pole at the event horizon $r = r_+$, see figure 4. Remarkably, the real part of a potential with a pole of order one is attractive from the outer space of the black hole and repulsive from the interior, but the imaginary part tends to $+\infty$ from the outside and to $-\infty$ from the interior. A wave packet inside the black hole with nonvanishing outgoing amplitude $b_{\text{out}}$ then reaches the outer space with nonvanishing amplitude $a_{\text{out}}$; if, e.g., $b_{\text{out}}$ is known at $r = M$ and $b_{\text{in}} = 0$, the outside amplitude is given by (22) with $a_{\text{in}} = 0$.

Where does this all lead us to? Notably, the tunnel phenomenon studied in this paper is a semiclassical phenomenon. This implies that the black hole gravitational field has to be strong enough compared to the typical energy of the wave field such that it does not perturb the curvature essentially, i.e. such that it is a test field. The tunnel phenomenon seems to be related to another semiclassical phenomenon, the Hawking radiation. But whereas the Hawking radiation is a black hole thermodynamical effect occurring in particular for static black holes, the tunneling only exists in case of rotation. As such it resembles the classical superradiance effect, but this affects only, and all, particles with integer spin.

A complete theory of quantum gravity must include the tunnel phenomenon — and should explain it. Especially the puzzles concerning gravitons and photons in charged rotating black holes should be solved. To date, the most promising candidate for a theory of quantum gravity is M-theory. It has made strong progress in the last few years, not only with respect to black hole phenomena. One of the most remarkable aspects in this context certainly is the deduction of the notion of entropy of a black hole (9). However, to our knowledge the analysis of rotating black holes, be they uncharged or charged, in M-theory is still missing.

To conclude, rotation in general relativity over and over again reveals surprising phenomena, a fact which is not only demonstrated by (21), (22), or (23). Succeeding in integrating rotation in M-theory would give deep insight.

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