Ya. B. Zeldovich and foundation of the accretion theory

Nikolay I. Shakura1,*

Sternberg Astronomical Institute, Moscow M.V. Lomonosov State University, 13 Universitetskij pr., Moscow 119992, Russia; Kazan Federal University, Kremlevskaya 18, 420008 Kazan, Russia

This short review is dedicated to academician Yakov Borisovich Zeldovich, the science of his epoch and the creation of modern accretion theory.

1. INTRODUCTION

Yakov Borisovich Zeldovich was born on March 8, 1914, in Minsk. His father Boris Naumovich Zeldovich was a lawyer, a member of a College of Lawyers. His mother Anna Petrovna Zeldovich (Kiveliovich) was a translator, a member of the Union of Writers.

My first acquaintance with Ya. B. Zeldovich began with buying of his book “Further mathematics for beginners”. In the middle school my math teacher Alfred Viktorovich Baranovskiy explained us a method of finding extremum of parabola using Vieta’s formulas. And he told us by the way that using of the methods of further mathematics allow to make this process easier and more elegant. He did not tell any details and I was intrigued. And suddenly in a book store of Bobruisk city (no far from my town) I saw that Ya. B. Zeldovich’s math book. I bought it hoping to find out the method of finding extrema of parabola. But few days later I went to pass entrance exams to Faculty of Physics of the Moscow University.

In 1963 I entered Astronomical Division of the Lomonosov Moscow State University (MSU). My decision to study astronomy was inspired by another book that somehow found its way to Belarus of my childhood: “Essays About The Universe” written by Boris Aleksandrovich Vorontsov-Velyaminov, a professor of astronomy at MSU. Later I attended his lectures as MSU student and was examined by him — something I had never even dreamed about it two or three years ago when I studied his textbook “Astronomy” in school.
In my first three years at the MSU, Ya. B. Zeldovich was nowhere in my surroundings, nor did I ever remember the book I had bought in Bobruisk. This book was not included in the list of recommended texts — by no means because it was bad, but because the academician addressed it to beginning engineers and technicians as a self-education book on mathematics. Academician Ya. B. Zeldovich is known as an author of more than ten high-quality books on a wide range of topics in the sciences.

For the first time I met Jakov Borisovich when the Dean’s office of Faculty of Physics organized a meeting of students with the editorial board of the famous Soviet scientific journal Physics Uspechhi. It was the third year of my studying in MSU. The Large Physics Auditorium being the venue. I was, of course, very impressed with the brilliance of Eduard Vladimirovich Shpolsky, the Editor-in-Chief. Yakov Borisovich Zeldovich was silent all the time, sitting with his densely hairy hands crossed under his chin and being deep in his own minds. Later on, already as his co-worker, I found out that he had never been a full-time undergraduate student at any higher education institution. Having graduated from a ten-year school in Leningrad in 1930, he started to work as a laboratory technician at the Institute of Mechanical Processing of Minerals for some time and then at the Institute of Chemical Physics (ICP) where, at the age of twenty (!), he began his postgraduate research under supervising of Nikolay Nikolayevich Semyonov and where it took him a fantastically short time to get to the very top of the academic career ladder.

In late 1930-s Soviet Journal of Experimental and Theoretical Physics published several papers about uranium fission by Ya. B. Zeldovich and Yuliy Borisovich Khariton [1–3]. In the same time two papers about a division of uranium nuclei by free neutrons were published: experimental paper by O. Hahn and E. Strassmann [4], and theoretical paper by L. Meitner and O. R. Frisch [5]. These papers marked the begging of the new era of civilisation development: the creation of powerful nuclear weapons. For the long time (1948–1965) Ya. B. Zeldovich was one of the leading member of the scientific team of developing Soviet atomic project. When he was asked about that time he drew a big black square — a “Black square” by Zeldovich.

In the September of 1966, we, the fourth-year astronomy undergraduates of the Faculty of Physics, found ourselves enrolled in a new lecture course, “The structure and evolution of stars”, to be taught by Ya. B. Zeldovich. Namely, at these lectures my first personal contact with the academician occurred. The lectures were held on Fridays, and on Thursdays Ya. B. (some of other academicians called Yakov Borisovich so) led a joint astrophysical seminar (JAS) at the Sternberg Astronomical Institute (SAI MSU), both for full-fledged scientists and for young higher education graduates. Undergraduates did not have this seminar in their schedule and could only drop in on when possible. When Ya. B. finished his first lecture, he asked if there were in the
audience students wishing to receive topics for their course theses and could they please stay on for a while. I was among those few who did. When my turn came, he asked me whether I had been at the JAS session the previous day and whether I had heard the talk on the (then mysterious) sources of cosmic X-ray radiation, and when I twice said yes, he made a suggestion. He offered me to calculate the structure and spectrum of the radiation of a strong shock wave that arises near the surface of a neutron star due to gas falling onto the star.

2. BEGINNINGS OF X-RAY ASTRONOMY

In 1962, a group of American scientists led by Prof. Riccardo Giacconi discovered first X-ray sources. Before that, astronomers had known the only one X-ray source of extraterrestrial origin, namely, the solar corona. The coronal gas heated to million degrees due to some unknown mechanisms.
produces X-ray emission. The luminosity of the solar corona in X-rays is approximately one millionth of the optical luminosity of the Sun ($4 \times 10^{33}$ erg/s). So it was natural to assume that other stars are surrounded by hot coronas too. However, simple calculations showed that detectors available at that time could not reveal coronas even around the nearest stars on distances of several parsecs.

Nevertheless, astronomers tried to detect X-ray radiation from the Moon! The Moon has no atmosphere, but likely some radiation can be produced as a luminescence of the Moon’s surface being attacked by the solar X-rays and charged particles. Thus, precisely at midnight of June 18, 1962, when the full Moon was shining, the “Aerobee” rocket was launched. It reached a height of 225 km, its flight continued for 350 s and was quite successful: two of the three Geiger counters, with large surface and good sensitivity in the range $1.5 - 6$ keV, were operating during the flight time. In this energy range, the Earth’s atmosphere is totally opaque. Suddenly, instead of X-ray radiation from the Moon, a bright and before unknown source was discovered, which was far beyond the Solar system in the direction of the Scorpion constellation. It was named Sco X-1.

In the following years, new rocket flights brought more and more discoveries of new X-ray sources. Gradually, a sky map covered with X-ray sources of different nature was created. First sources got their names according to directions of their location (Cyg X-1, Cyg X-2, Her X-1, Cen X-3, and so on). Later it was revealed that their X-ray luminosities were thousands or even tens of thousand times stronger than the solar luminosity in the optical range. The epoch of X-ray astronomy, the epoch of stunning discoveries in the Universe began.

According to own Ya. B. Zeldovich’s simple estimations, the shock wave originating when the gas surrounding a neutron star falls onto its surface should produce radiation primarily in the X-ray range. My goal was to carry out the full calculation and investigate the process in detail. The main difficulty was connected with the following circumstance: the free-path length of a falling particle near the neutron star’s surface is much greater (tens of times) than the characteristic timescale of interaction between the radiation and matter. Usually it is not necessary to calculate the structure of the shock wave: it is sufficient to specify a change in density, pressure, temperature, and other physical parameters depending on the velocity and the adiabatic index of the falling gas. In my problem, the density, temperature and other parameters depended on energy release in the braking zone. Moreover, plasma oscillations may arise in this zone. To describe them, a consideration of kinetic plasma equations is required instead of ordinary hydrodynamics. Finally, it was shown that shock wave emission spectra from accreting neutron stars could explain observational data obtained with the launched equipment.

This investigation was outlined in the article written by Ya. B. and me, the article was submitted to “Soviet Astronomy” in 1968 and published in 1969 [6]. Meanwhile, in 1967 first radio pulsars were discovered which turned to be a strongly magnetized rotating neutron stars [7]. This discovery starts new epoch of radio astronomy: the epoch of study of various radio pulsars. But here I will discuss accreting X-ray pulsars!

As an undergraduate, I attended lectures on general astrophysics by Dmitrii Yakovlevich Martynov, then a Director of SAI, who paid special attention to close binary stellar systems with the matter flowing from one star to another. Due to the relative orbital motion of the two stars, this process results in an origin of a disk-shape envelope around one of these stars. It seemed natural to consider a binary stellar system where one of the components is a neutron star or even a black hole.

Thus, imagine a binary stellar system consisting of a normal star and a black hole. The size of the
ordinary star in this system is limited by that of the so-called Roche lobe. The normal star can increase in size as stellar evolution proceeds, and after the Roche lobe is filled, the matter starts flowing from its surface to the region of gravitational attraction of the black hole (left panel of Fig. 13). Because of the relative orbital motion of the binary components, the matter does not fall directly onto the black hole but forms a differentially rotating disk-like envelope around it. Due to layer-to-layer friction, the matter that accumulates in the disk strongly heats up and starts glowing. In its rapid revolution, the matter in the disk slowly moves radially toward (or accretes upon) the black hole, losing its angular momentum in the process. The glowing of the disk is caused by the matter accretion releasing its gravitational energy. Indeed, the inner parts of the disk closest to the black hole become hot enough to emit in the X-ray range.

In a more complex accretion disk formation scenario, the optical companion does not fill its Roche lobe and flows outward in all directions via stellar wind. In this case, a head-on shock is naturally expected to form in the region where the stellar wind stream is under the gravitational influence of the black hole. After the passage of the shock wave, the matter in the gravitational capture region of the black hole starts falling onto it – but not strictly radially! Due to its rotational motion, the falling matter has a specific angular momentum, which is somewhat greater than the specific orbital angular momentum of the hole. In the case of the falling with conserved angular momentum the matter acquires the orbital motion at some distance from the black hole (right panel of Fig. 13). And then, well, disk type accretion again!

Replacing the black hole by a strongly magnetized neutron star in the binary system has a consequence that the stellar magnetic field starts destroying the accretion disk at a distance of about a hundred star radii. The accreting matter then starts rapidly falling along the magnetic force lines, encountering the surface of the neutron star in the vicinity of the magnetic poles. Because magnetic and geographic poles are usually far apart, the rotation of the neutron star causes it to be observed as an accretion pulsar.

The presence of accreting black holes and neutron stars in binary stellar systems was first detected in the early 1970s by an experiment aboard the US purpose-built satellite Uhuru. The satellite was launched into a circular orbit about 500 km high from the Italian marine platform San Marco off the coast of Kenya. The satellite got name Uhuru (which is Swahili for freedom) due to the fact that the launch date, 12 December 1970, was Kenya’s Independence Day. Recognition is not always quick to come in science, and it was only in 2002 that the mission project leader Riccardo Giacconi, a US astrophysicist of Italian origin, was awarded the Nobel Prize in Physics for this pioneering work. A lot of discovered by Uhuru X-ray sources are turned to be accreting black holes and neutron stars.

Virtually simultaneously with the discovery of accreting black holes and neutrino stars in binary stellar systems, the foundation of the theory of disk accretion on gravitating centers was laid by me under the guidance of Ya. B. The paper was submitted to “Soviet Astronomy” in June 1971 and published in November 1972 [9]. Remarkable that the first publication with the results of Uhuru was received by editors of Astrophysical Journal in 17th May 1971 and was published on July 1971 [10].

The bulk of the work was done together with Rashid Sunyaev. In another of our joint efforts, the so-called standard model of disk accretion was developed and worked out in detail, which was presented [11] at the 55th Symposium of the International Astronomical Union held in May 1972 in Madrid. The symposium covered not only data from Uhuru but also the first theoretical results on modeling the compact X-ray sources detected in binary stellar systems, i.e., accreting black holes and neutron stars. Our joint work was presented by Jim Pringle (UK) – Rashid and I were then “nevyezdnye” (not free to go abroad) – and was in fact an introduction to a large scale paper [12] which was later published in 1973 in the high-profile European journal Astronomy and Astrophysics, and on the basis of which Igor Dmitrievich Novikov and Kip Thorne were able to calculate accurately the relativistic corrections to radiation due to general relativity effects near black holes [13].
The results UHURU produced during its three years of operation were spectacular. Not only a large number (339) of newly discovered X-ray sources were catalogued, but UHURU also provided guidance for other space observatories. Currently, space X-ray sources of various natures (not necessarily accreting relativistic stars in binary stellar systems!) number in the hundreds of thousands.

Rashid’s and my pioneering paper [12] gained wide popularity; its number of citations as of July 2018, i.e., forty five years on, exceeding 8500.

The study of accretion disks has led to the discovery in the cores of active galaxies and in quasars of supermassive black holes with masses ranging from a few dozen to a few hundred millions solar masses. The first theoretical results on the glowing of accretion disks around supermassive black holes were published by David Lynden-Bell (UK) in 1969 in Nature [14].

In the last 45 years a great advance has been made in observations, as well as in the theory of accretion disks. Now accretion disks is a specific extensive branch of astrophysics, important to the same degree as the structure and evolution of stars, interstellar medium, etc. The basis for the modern theory of accretion disks is: a) cosmic (magneto)hydrodynamics, b) radiative transfer, c) general relativity effects in the vicinity of compact stars.

But what about Moon? “Did R. Giacconi and his colleagues miss their target?”, you can ask me. Lunar X-ray light was discovered later by satellite ROSAT, that contains more perfect X-ray telescope with grazing incidence mirrors.

More detailed historical review was published in the year of Ya. B.’s 100 anniversary in “Physics Us-
3. STANDARD MODEL OF DISK ACCRETION

Many excellent books and proceedings on accretion disk theory have been published so far. I especially want to mention two of them. Namely, “Accretion Power in Astrophysics” by Juhan Frank, Andrew King and Derek Raine [16] and “Black Hole Accretion Disks. Towards a New Paradigm” by Shoji Kato, Jun Fukue and Sin Mineshige [17]. Below the theory of axisymmetric geometry thin and optically thick accretion disks (a standard Shakura–Sunyaev $\alpha$-model) will be briefly outlined.

3.1. Main equations

The height-integrated continuity equation for the disk layer on some radius $r$:

$$\frac{\partial \Sigma_0}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (\Sigma_0 v_r r) ,$$

where $v_r$ is the radial velocity of the accreting matter, $\Sigma_0 = \int_{-z_0}^{z_0} \rho \, dz$ is surface density of accretion disk, $\rho$ is the accreting matter density, $z_0$ is the semi-height of the disk and $r$ and $t$ are coordinate along disk axis and time correspondingly.

The first Euler’s equation:

$$v_r \frac{\partial v_r}{\partial r} - \omega^2 r = -\frac{GM}{r^2} - \frac{1}{\rho} \frac{\partial P}{\partial r} + \ldots ,$$

where $\omega$ is angular velocity of accreting matter, $M$ is the mass of central object (i.e., black hole), $P$ is the accreting matter pressure and ellipsis denotes other terms, such as viscosity, small compared to main terms. For geometrically thin accretion disks Keplerian rotation law takes place:

$$\omega = \frac{v_r}{r} = \sqrt{\frac{GM}{r^3}} .$$

The second Euler’s equation (the equation of angular momentum):

$$\Sigma_0 v_r \frac{\partial (\omega r^2)}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} (W_{r\phi} r^2) ,$$

where $W_{r\phi} = \int_{-z_0}^{z_0} w_{r\phi} \, dz$ is the height-integrated viscous shear stress.

The equation of hydrostatic equilibrium along $z$-coordinate (see Fig. 20):

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{GM}{r^3} z .$$

3.2. The equation of the energy balance

$$\Sigma_0 T v_r \frac{\partial S}{\partial r} = Q^+ - Q^- ,$$

where $T$ is the typical (along $z$-coordinate) accreting matter temperature, $S$ is the entropy per unit mass, $Q^+$ and $Q^-$ are the vertical-integrated rate of thermal energy release by viscous shear and rate of thermal energy losses by radiation. Left-hand side of the equation describes advection energy transfer, for geometrically thin disks this value is small and $Q^+ = Q^-$. 

3.3. Viscosity

The main challenge of solving the disk accretion problem is to account for viscous forces. Usually, viscosity of ionized plasma is very low. That is why it is necessary to assume the presence of developed turbulence or magnetic fields in the accretion disk. As the first step, let us introduce turbulence viscosity:

$$w_{r\phi}^t = \nu_t \rho \frac{\partial \omega}{\partial r} = \rho < v_t l_t > r \frac{\partial \omega}{\partial r} ,$$

Figure 19. Professor Joachim Trumper, the PI of ROSAT and my close friend in Germany.

pekhi” [15].

Figure 20. $z$-component of gravitational acceleration.
where \( \nu_t \) is turbulent kinematic viscosity, \( v_t \) and \( l_t \) are turbulent speed and length.

Using empirical Prandtl’s law \( \nu_t = \ell_t r |\partial \omega / \partial r| \) for Keplerian disk we can rewrite:

\[
\nu_t = -\rho v_t^2 = -\alpha \rho v_s^2 ,
\]

where \( v_s \) is the sound velocity of the accreting matter. Here we introduce the \( \alpha \)-parameter:

\[
\alpha = \left( \frac{M}{v_t} \right)^2 = \left( \frac{v_t}{v_s} \right)^2 ,
\]

where \( M \) is the turbulent Mach number.

### 3.4. Energy generation and transfer

The energy release occurs due to the differential rotation of the viscous turbulent disk, the energy release rate per unit volume is

\[
\epsilon = r w_{r \varphi} \frac{\partial \omega}{\partial r} = -\frac{3}{2} \omega w_{r \varphi} .
\]

The vertical gradient of the radiation flux \( q \) is proportional to the energy release rate:

\[
\frac{\partial q}{\partial z} = \epsilon .
\]

The equation transfer equation in the diffusion approximation:

\[
\frac{c}{3 \kappa \rho} \frac{\partial (a_r T^4)}{\partial z} = -q ,
\]

where \( \kappa \) is the opacity coefficient and \( a_r \) is the radiation constant.

### 3.5. Vertical and radial disk structure

Equations (5), (10) and (11) with the continuity equation in the form of \( \partial \sigma / \partial z = \rho \) form the system of ordinary differential equations, its solution for right boundary conditions provides vertical disk structure. This problem is like the problem of calculation of stellar structure, and can be solved numerically, see, i.e., [18, 19].

In the case of stationary accretion disk, when accretion rate \( \dot{M} = 2 \pi \Sigma_0 v_r r \) is constant, we can integrate Eq. (4) and obtain radial dependence of viscous shear:

\[
W_{r \varphi} = \frac{\dot{M} \omega}{2 \pi} \left( 1 - \sqrt{\frac{R_{in}}{r}} \right) ,
\]

where \( R_{in} \) is the inner disk radius, that equals innermost stable circular orbit for the case of accretion on the black hole. Thus, radial structure of accretion disk can be found as a solution of algebraic equations [12]. Using the vertical structure solution analytical relations for radial distribution of various parameters of accretion disk can be obtained, see, i.e., [20, 21].

In the most simple case when radiation from the disk surface is black-body the radiation flux is

\[
Q = \frac{1}{2} \frac{\partial \omega}{\partial r} W_{r \varphi} = \frac{3}{8 \pi} \frac{GM M}{r^3} \left( 1 - \sqrt{\frac{R_{in}}{r}} \right)
\]

\[
= \sigma_{SB} T_{eff}^4 ,
\]

where \( \sigma_{SB} \) is the Stefan–Boltzmann constant.

The spectral radiative flux in a unit solid angle from a flat accretion disk at distance \( d \) from the disk is equal to

\[
F_{\nu} = \frac{2 \pi}{d^2} \cos i \int_{R_{in}}^{R_{out}} I_{\nu} r dr ,
\]

where \( \nu \) is radiation frequency, \( i \) is the inclination of the disk to the line of sight, \( R_{out} \) is the outer disk radius and \( I_{\nu}(r) \) is the intensity of radiation from the disk surface.

With the assumption of black-body radiation Eq. (15) gives spectral disk flux shown in Fig. 21. If the case of electron scattering spectrum has more complicated shape, see [22] for details.

### 3.6. Non-stationary disk accretion

Substituting Eq. (4) to Eq. (1) gives us equation of viscous disk evolution. It is convenient to introduce a variable \( F = -W_{r \varphi} \), henceforth \( 2 \pi F \) means the total momentum of viscous forces acting between the
adjacent layers, and an independent variable $h = \omega r^2$ (Keplerian angular momentum per unit mass). Then we have:

$$\frac{M(r, t)}{2\pi} = -\frac{\partial F}{\partial h}$$

and

$$\frac{\partial \Sigma_0}{\partial t} = \frac{1}{2} \left( \frac{GM}{h^3} \right)^2 \frac{\partial^2 F}{\partial h^2}.$$  

A relation between $\Sigma_0$ and $F$ depends on the vertical structure, i.e. on opacity law, value of $\alpha$, etc. This equation is a non-linear partial differential equation of diffusion that in the general case can be solved only numerically.

For the case of power-law opacity $\kappa(\rho, T) = \kappa_0 \rho^a / T^b$:

$$\Sigma_0 = \frac{(GM)^2 F^{1-m}}{2(1-m)Dh^{3-n}},$$

where $m$ and $n$ are dimensionless coefficients that depend on $a$ and $b$, and $D$ is dimension coefficient [20]. For this relation between $\Sigma_0$ and $F$ the equation of viscous evolution (16) has the following form

$$\frac{\partial F}{\partial t} = D \frac{F^m}{h^n} \frac{\partial^2 F}{\partial h^2}.$$  

There are a number of self-similar solution of this equation, see [20, 23–25]. For the case of $m = 0$ Green functions of this problem were found in [26, 27].

The theory of non-stationary accretion applies successfully to X-ray outbursts of accretion disks in close binary systems with black holes or neutron stars. Often such outbursts cannot be described in the terms of self-similar solutions of the viscous equation (16). On the other hand numerical solution of this equation provides possibility to take into account effects of disk self-irradiation and hydrogen recombination. The examples of such numerical modeling are shown in Fig. 22 and 23. These simulations showed that $\alpha$-parameter is not small, it is about 0.7.

In the early 80-s Ya. B. analysed the results of Sir Taylor’s laboratory experiment [28] and explained generation of turbulence in it [29]. Several words about this experiment. Two coaxial cylinders were taken, the gap between them was filled with some liquid. The bigger cylinder was rotated with acceleration, the smaller one was stable. At some moment the liquid became turbulent. It was found that the bigger gap between cylinders required larger rotation velocity to arise turbulence. This relation was connected with critical Reynold’s number. Ya. B. Zeldovich suggested the theoretical basement which explained these empirical data obtained in Taylor’s experiment. Nevertheless, Ya. B. pointed out that this result cannot be applied to the disk accretion.

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**Figure 22.** X-ray light curve of 1975 outburst of A0620–00 [30]. Dots are observations, line is numerical model of accretion disk evolution.

**Figure 23.** Temporal evolution of accretion rate on the black hole in 2002 outburst of 4U1543–47 [31].

The modern astrophysics is largely impacted by Zeldovich’s papers on the interaction of matter and radiation in the Universe and on formation of large-scale structure of the Universe. Yakov Borisovich Zeldovich died on the 2nd of December 1987.

I thank Konstantin Malanchev and Natalia Shakura for preparation of the manuscript.

**Figure 24.** The critical Reynolds number dependence on the gap’s width $t/R_1$. Taylor’s experimental results and the exponential approximation by Ya. B. are shown [29].
4. INSTEAD OF EPILOGUE

From the resolution of the Presidium of the Russian Academy of Sciences of 11 Feb. 2014

- Establish the Academician Ya. B. Zeldovich gold medal, to be awarded by the Russian Academy of Sciences for distinguished works in physics and astrophysics.
- Name a street in Moscow after Ya. B. Zeldovich.
- Set the commemorative plaques in memory of the Academician Ya. B. Zeldovich on the buildings of:
  - the M. V. Keldysh Institute of Applied Mathematics of RAS
  - the Space Research Institute of RAS
  - the N. N. Semyonov Institute of Chemical Physics of RAS

Thank you for the accretion!

Thank you for the attention!

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