Simulation of directional well trajectory using tension spline

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Abstract. In order to solve the problems in the directional well trajectory simulating, the tension spline function is used for calculating the hole deviation angle and azimuth angle of arbitrary point on the borehole axis of directional well and Simpson integral formula is also used for solving its spatial coordinates. By the method, the simulation software of well trajectory is written. The results of example calculated by the software show that the simulation precision of tension spline method is higher and more reliable than the cubic spline interpolation method and can meet the requirements of actual wellbore trajectory calculation. The method can solve stress fluctuation appearing in analyzing the mechanics of pipe string using the wellbore data calculated by the cubic spline interpolation. It can also reduce errors, improve the precision of pipe string mechanics analysis and meet the engineering needs.

1. Introduction

Since the wellbore parameters cannot be measured continuously during drilling, it can only be measured once every certain length, which makes it difficult to calculate the actual borehole parameters. In order to determine the well inclination angle and azimuth angle between two adjacent measuring points, the wellbore data is processed by the cylindrical helix model, the space arc model, the natural curve model, the constant tool face model and the spline curve model. The wellbore data are simulated by these methods. The smoothness of the curve at the measuring point is not always high, some methods cannot even guarantee the continuous of curve’s first derivative curve [1-3]. Although the cubic spline interpolation method can guarantee the smoothness of the wellbore curve, there will be extra inflection point. In the application of the spline curve model to deal with the wellbore data to calculate the stress of the column, the phenomenon of stress fluctuation occurs. The tension spline function can continuously calculate the inclination angle and azimuth angle on the axis of directional well, which avoids stress fluctuation in the subsequent processing and improves the accuracy of pipe string mechanics analysis.

2. Tension spline function interpolation method

The designed wellbore trajectory usually includes the following main data: sounding depth, borehole angle, azimuth angle, vertical depth, north-south displacement, east-west displacement, horizontal displacement and build-up slope. In wellbore trajectory data fitting, three variables of sounding, inclination and azimuth are adopted. The inclination angle and azimuth angle of the well are regarded as the tension spline function of the bathymetry [3-7].
Suppose there is a set of measured data on a well section \([l_0, l_n]\).  
Well depth: \(l_0, l_1, \ldots, l_{i-1}, l_i, \ldots, l_n\).
Well angle: \(\alpha_0, \alpha_1, \ldots, \alpha_{i-1}, \alpha_i, \ldots, \alpha_n\).
Azimuth: \(\varphi_0, \varphi_1, \ldots, \varphi_{i-1}, \varphi_i, \ldots, \varphi_n\).
Among them: \(l_0 < l_1 < \cdots < l_{i-1} < l_i < \cdots < l_n\).

The inclination angle and the azimuth angle are considered as a function of well depth. The wellbore tension spline function can be constructed according to the calculation method of tension spline function.

\[
\alpha^*(l) - \sigma^2 \alpha(l) = \left[ \alpha^*(l_i) - \sigma^2 \alpha_i \right] \frac{l_{i+1} - l}{h_i} + \left[ \alpha^*(l_i) - \sigma^2 \alpha_{i+1} \right] \frac{l_i - l}{h_i} 
\]

The solution of the differential equation is:

\[
\alpha(l) = \frac{1}{\sigma^2 \sinh(\sigma h_i)} \left\{ \alpha^*(l_i) \sinh(\sigma (l_{i+1} - l)) + \alpha^*(l_{i+1}) \sinh(\sigma (l_i - l)) \right\} 
\]

Then the first derivative of the equation is:

\[
\alpha'(l) = \frac{1}{\sigma^2 \sinh(\sigma h_i)} \left\{ -\alpha^*(l_i) \cosh(\sigma (l_{i+1} - l)) + \alpha^*(l_{i+1}) \cosh(\sigma (l_i - l)) \right\} 
\]

According to \(\alpha'(l_i^+) = \alpha'(l_i^-)\) and the boundary conditions: \(\alpha'(s_i) = \alpha_i'\), \(\alpha'(s_n) = \alpha_n'\), the following formula can be obtained:

\[
\begin{align*}
& a_i \frac{\alpha^*(l_{i-1})}{\sigma^2} + b_i \frac{\alpha^*(l_i)}{\sigma^2} + c_i \frac{\alpha^*(l_{i+1})}{\sigma^2} = d_i, \\
& b_i \frac{\alpha^*(l_{i+1})}{\sigma^2} + c_i \frac{\alpha^*(l_{i+1})}{\sigma^2} = d_i, \\
& a_n \frac{\alpha^*(l_{n-1})}{\sigma^2} + b_n \frac{\alpha^*(l_n)}{\sigma^2} = d_n,
\end{align*}
\]

Where,

\[
\begin{align*}
& a_i = \frac{1}{h_{i-1}} - \frac{\sigma}{\sinh(\sigma h_{i-1})}, \\
& b_i = \sigma \coth(\sigma h_{i-1}) - \frac{1}{h_{i-1}} + \sigma \coth(\sigma h_i) - \frac{1}{h_i}, \\
& c_i = \frac{1}{h_i} - \frac{\sigma}{\sinh(\sigma h_i)}, \\
& d_i = \frac{\alpha_i - \alpha_{i-1}}{h_{i-1}} - \frac{\alpha_{i+1} - \alpha_{i-1}}{h_{i-1}}, \\
& b_i = \sigma \coth(\sigma h_{i-1}) - \frac{1}{h_{i-1}}, \\
& c_i = \frac{1}{h_i} - \frac{\sigma}{\sinh(\sigma h_i)}, \\
& d_i = \frac{\alpha_i - \alpha_{i+1}}{h_{i+1}} - \frac{1}{h_i}.
\end{align*}
\]

The above equations are diagonally dominant tridiagonal equations, the pursuit method is appropriate to solve respectively. The solution \(\alpha''(l_i)/\sigma^2 (i=1, 2, 3, \ldots, n)\) can be substituted into the
$\alpha(l)$ expression. The well angle can be obtained in any position of $[l_0, l_n]$, the azimuths can be obtained at any depth of the well.

3. Calculation of borehole

The $Pxyz$ right-handed Cartesian coordinate system is established with the wellhead $P$ as the origin. The unit vectors along axes $x$ (north), $y$ (east) and $z$ (down) are denoted by $\hat{i}$, $\hat{j}$, $\hat{k}$ respectively, with the $l$ representing the arc coordinates. Through the figure 1, the geometrical position of any point $O(x, y, z)$ on the borehole axis in the three-dimensional space can be described by the vector: $\vec{r}_0(l) = x_0(l)\hat{i} + y_0(l)\hat{j} + z_0(l)\hat{k}$.

![Figure 1. The three dimensional borehole geometry.](image)

In the global coordinate system, the geometric coordinates of any point can be expressed as:

$$
\begin{align*}
x(l) &= x_0 + \int_0^l \sin \alpha(l) \cos \varphi(l) dl \\
y(l) &= y_0 + \int_0^l \sin \alpha(l) \sin \varphi(l) dl \\
z(l) &= z_0 + \int_0^l \cos \alpha(l) dl
\end{align*}
$$

From the Simpson integral formula, the coordinates of the well depth are:

$$
\begin{align*}
x_k &= x_{k-1} + \frac{l_k - l_{k-1}}{6} \left[ \sin \alpha(l_{k-1}) \cos \varphi(l_{k-1}) + 4 \sin \alpha(l_{k-0.5}) \cos \varphi(l_{k-0.5}) + \sin \alpha(l_k) \cos \varphi(l_k) \right] \\
y_k &= y_{k-1} + \frac{l_k - l_{k-1}}{6} \left[ \sin \alpha(l_{k-1}) \sin \varphi(l_{k-1}) + 4 \sin \alpha(l_{k-0.5}) \sin \varphi(l_{k-0.5}) + \sin \alpha(l_k) \sin \varphi(l_k) \right] \\
z_k &= z_{k-1} + \frac{l_k - l_{k-1}}{6} \left[ \cos \alpha(l_{k-1}) + 4 \cos \alpha(l_{k-0.5}) + \cos \alpha(l_k) \right]
\end{align*}
$$

Thus, the coordinate values of arbitrary points can be obtained. Wellbore trajectory threedimensional coordinate diagram method and the method of solving the wellbore curvature can refer to.

4. Example calculation

In order to verify the reliability and accuracy of the simulation of directional wellbore trajectory using tension spline and Simpson's integral formula, a simulation program was written in Visual Basic to
calculate the borehole spline curve for a directional well in Dagang. The wellbore spline curve was fitted fluctuation of the string stress was checked.

(1) The contrast of the wellbore curvature with the tension spline method and cubic spline method

The borehole trajectory data (well deviation angle, azimuth angle and curvature) is obtained by the wellbore trajectory fitting program of cubic spline method and tension spline method. Through the figure 2 (a) and figure 2 (b), it can be seen that the wellbore curvature curve obtained by the tension spline method is smoother than the wellbore curvature curve obtained by cubic spline method, which shows that the deviation angle of azimuth data obtained by tension spline method is small.

![Curvature by using tension spline method and cubic spline method](image)

**Figure 2.** Curvature by using tension spline method and cubic spline method.

(2) The stress fluctuation is calculated by tension string method

![Mises stress diagram with processed well track](image)

**Figure 3.** Mises stress diagram with processed well track.

From the comparison of (a) and (b) in figure 3, by using mechanical calculation of pipe string, mises stress curve of the obtained string obviously fluctuates. While the calculated mises stress curve of the tubing is smooth, which eliminates the phenomenon of stress fluctuation and improves the calculation precision.

(3) Results of interpolation calculation

The tension spline interpolation is used to calculate the deviation angle and azimuth angle of the original nine nodes. The results in table 1 show that the trajectory of the directional well is reliable and the accuracy is high. Therefore, it is appropriate to increase the measuring distance and reduce the number of measuring points, which will cut costs.
Table 1. Original tension sample interpolation calculation results.

| Depth(m) | Deviation angle | Azimuth angle |
|----------|----------------|--------------|
|          | Measured value (°) | Calculated value (°) | Relative error | Measured value (°) | Calculated value (°) | Relative error |
| 1504.78  | 11.36           | 11.52         | 0.0141         | 245.21           | 246.16         | 0.0039          |
| 1562.62  | 11.79           | 12.02         | 0.0195         | 244.53           | 245.04         | 0.0021          |
| 1620.09  | 12.90           | 12.63         | -0.0209        | 247.05           | 246.80         | 0.0010          |
| 1677.93  | 12.76           | 12.74         | -0.0016        | 249.74           | 249.47         | 0.0011          |
| 1735.51  | 12.76           | 13.21         | 0.0353         | 249.78           | 249.67         | -0.0004         |
| 1793.68  | 13.79           | 13.94         | 0.0109         | 247.49           | 248.25         | 0.0031          |
| 1846.75  | 14.48           | 14.29         | -0.0131        | 250.7            | 249.28         | -0.0057         |
| 1904.24  | 14.42           | 14.23         | 0.0118         | 250.79           | 251.19         | 0.0016          |
| 1953.54  | 14.58           | 14.24         | -0.0233        | 251.14           | 251.43         | 0.0012          |

5. Conclusions

(1) Tension spline function interpolates among arbitrary known points and obtains the inclination angle and azimuth angle of the well, hence it increases the measuring distance and reduces the number of measuring points, reducing the cost.

(2) Compared with the cubic spline function method, the tension spline function is more reliable and accurate to simulate wellbore trajectory of directional wells and can also be used to simulate the well trajectory of horizontal wells. If the interpolation data is used before the least squares method for smoothing the data processing, the simulation accuracy can greatly be improved.

(3) It is possible to eliminate the phenomenon of stress fluctuation, improve the accuracy of mechanical analysis and reduce the error, so as to meet the needs of engineering analysis.

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Entry name

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