Higher-order soft gluon corrections in single top quark production at the LHC

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Abstract

I present a calculation of soft-gluon corrections to single top quark production in pp collisions at the LHC via the Standard Model partonic processes in the t and s channels and associated top quark and W boson production. Higher-order soft-gluon corrections through next-to-next-to-next-to-leading order (NNNLO) are calculated at next-to-leading logarithmic (NLL) accuracy. The soft-gluon corrections in the s channel and in tW production are large and dominant, while in the t channel they are not a good approximation of the complete QCD corrections.

1 Introduction

The production of top quarks in hadronic collisions can proceed via strong interaction processes involving top-antitop pair production, or via electroweak processes involving the production of a single top (or antitop) quark. The top quark was first discovered via tt production at the Fermilab Tevatron by the CDF and D0 Collaborations in 1995 [1, 2]. The observation of single top quark events has been more elusive but there has been recent evidence of such events by the D0 Collaboration [3] with a cross section consistent with theoretical expectations [4]. Single top quark production is an important production mode for further understanding of electroweak theory, measurement of the \( V_{tb} \) CKM matrix element, and the search for new physics (see e.g. [5, 6, 7, 8, 9]), so it is crucial to have accurate predictions for the cross sections.

The Large Hadron Collider (LHC) at CERN has good potential for observation of single top quark events. The production of single top quarks in proton-proton collisions can proceed through three distinct partonic processes. The t-channel processes (\( q\bar{b} \to q't \) and \( q\bar{b} \to q't \), Fig. 1a) involve the exchange of a space-like W boson, the s-channel processes (\( qq' \to b't \), Fig. 1b) proceed via the exchange of a time-like W boson, and associated tW production (\( bg \to tW^- \), Fig. 1c) involves the production of a top quark in association with a W boson. At the LHC the t-channel process is numerically the largest, the s-channel process is much smaller, and associated tW production is intermediate in magnitude.

The next-to-leading order (NLO) QCD corrections to single top quark production have been calculated in Refs. [10, 11, 12, 13, 14]. In this paper we calculate the contribution of soft-gluon corrections to the cross section at the LHC beyond NLO. These corrections are normally dominant near partonic threshold for the production of a specified final state [15, 16, 17] and they arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (i.e. low-energy) gluons. For the partonic process \( f_1 + f_2 \to t + X \) the kinematical invariants are \( s = (p_1 + p_2)^2 \), \( t = (p_1 - p_t)^2 \), \( u = (p_2 - p_t)^2 \)
Figure 1: Leading-order $t$-channel (a), $s$-channel (b), and associated $tW$ production (c) diagrams for single top quark production.

$$u = (p_2 - p_t)^2, \quad s_4 = s + t + u - m_t^2 - m_X^2.$$ Near threshold, i.e. when we have just enough energy to produce the final state, $s_4 \to 0$. The threshold corrections then take the form of logarithmic plus distributions, $[\ln \left( \frac{s_4/m_t^2}{s_4} \right)]_{\geq 1}$, where $l \leq 2n - 1$ for the $n$-th order QCD corrections. These threshold corrections exponentiate; the resummation is carried out in moment space and it follows from the refactorization of the cross section into hard, soft, and jet functions that describe, respectively, the hard scattering, noncollinear soft gluon emission, and collinear gluon emission from the partons in the scattering $[15, 16]$. To obtain a physical cross section, the moment-space result must be inverted to momentum space thus necessitating a prescription to handle the Landau singularity. To avoid prescription ambiguities $[17]$ we provide fixed-order expansions of the resummed cross section, as has been done for many other processes $[4, 8, 17, 18, 19, 20]$ (for a review see $[21]$).

In the $n$-th order corrections in the strong coupling, $\alpha_s$, the leading logarithms (LL) are those with $l = 2n - 1$ while the next-to-leading logarithms (NLL) are those with $l = 2n - 2$. We calculate NLO, NNLO, and NNNLO soft-gluon threshold corrections at NLL accuracy, i.e. at each order including both leading and next-to-leading logarithms. We denote these corrections as NLO-NLL, NNLO-NLL, and NNNLO-NLL, respectively. More details are given in Ref. $[4]$, where single top quark production at the Tevatron was discussed.

A calculation of threshold corrections is only meaningful if these corrections dominate the perturbative expansion, i.e. if they are a good approximation to the complete QCD corrections. This can be determined by comparing the NLO soft-gluon corrections with the complete NLO corrections. At Tevatron energies both top-antitop pair $[17, 18]$ and single top quark $[4]$ production satisfy this criterion. Since the LHC energy is much bigger it cannot be assumed that the same holds there. Moreover, since the kinematics and color flows are quite different for each channel, the validity of the threshold approximation must be examined for each channel seperately. Indeed, a study of the corrections shows that while the threshold approximation works well for the $s$ channel and for $tW$ production, it is not valid for the $t$ channel at the LHC. Therefore for the $t$ channel we only update the NLO cross section.

In Section 2 we present numerical results for single top quark production via the $t$ channel at the LHC at NLO. Results including NNLO-NLL and NNNLO-NLL soft gluon corrections are provided for the $s$ channel in Section 3 and for associated $tW$ production in Section 4. We note that in the $t$ and $s$ channels the cross section for single antitop production is different from that for single top production and we provide results for the antitop quark as...
well. We use standard electroweak parameters \[22\] throughout.

## 2 Single top quark production via the \( t \) channel at the LHC

We begin with the \( t \) channel. The dominant processes (with percentage contribution to the cross section) are \( ub \rightarrow dt \) (68.1\% of the leading order cross section) and \( \bar{d}b \rightarrow \bar{u}t \) (11.5\%). Additional processes involving only quarks are \( cb \rightarrow st \) (6.5\%) and Cabibbo-suppressed processes \( (ub \rightarrow st, cb \rightarrow dt, us \rightarrow dt, \text{and further suppressed processes}) \). Additional processes involving antiquarks and quarks are \( \bar{s}b \rightarrow \bar{c}t \) (8.3\%) and Cabibbo-suppressed processes \( (\bar{d}b \rightarrow \bar{c}t, \bar{s}b \rightarrow \bar{u}t, \bar{d}s \rightarrow \bar{u}t, \text{and further suppressed modes}) \).

A calculation of the NLO soft-gluon corrections shows that they are not a good approximation to the full NLO QCD corrections in the \( t \) channel. In fact they are large and negative while the exact NLO corrections \[13\] are small. Therefore for the \( t \) channel we only update the NLO results using the MRST 2004 parton distribution functions (pdf) \[23\] and we provide uncertainties for the cross section.

The NLO cross section for a top quark mass \( m_t = 175 \text{ GeV} \) is \( \sigma^{t-\text{channel}}_{\text{top}}(m_t = 175 \text{ GeV}) = 146 \pm 4 \pm 3 \text{ pb} \), where the first uncertainty is from variation of the factorization and renormalization scales, \( \mu_F \) and \( \mu_R \), between \( m_t/2 \) and \( 2m_t \), and the second uncertainty is from the pdf \[24\]. Adding these uncertainties in quadrature gives \( \sigma^{t-\text{channel}}_{\text{top}}(m_t = 175 \text{ GeV}) = 146 \pm 5 \text{ pb} \).

The NLO cross section using the most recent value for the top quark mass, \( m_t = 171.4 \pm 2.1 \text{ GeV} \) \[25\], is \( \sigma^{t-\text{channel}}_{\text{top}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 150 \pm 5 \pm 2.5 \pm 3 \text{ pb} \), where the first uncertainty is due to the scale dependence, the second is due to the mass \( \pm 2.1 \text{ GeV} \), and the third is due to the pdf. Adding these uncertainties in quadrature we find that \( \sigma^{t-\text{channel}}_{\text{top}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 150 \pm 6 \text{ pb} \).

The cross section for the production of an antitop quark at the LHC is different from that for a top quark. The corresponding results for the two choices of mass are \( \sigma^{t-\text{channel}}_{\text{antitop}}(m_t = 175 \text{ GeV}) = 89 \pm 3 \pm 2 \text{ pb} = 89 \pm 4 \text{ pb} \) and \( \sigma^{t-\text{channel}}_{\text{antitop}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 92 \pm 3 \pm 1.5 \pm 2 \text{ pb} = 92 \pm 4 \text{ pb} \).

## 3 Single top quark production via the \( s \) channel at the LHC

We continue with the \( s \) channel. The dominant process is \( u\bar{d} \rightarrow \bar{b}t \) (85.0\% of the leading-order cross section). Additional processes are \( cs \rightarrow \bar{b}t \) (10.2\%) and Cabibbo-suppressed processes \( (u\bar{s} \rightarrow \bar{b}t, c\bar{d} \rightarrow \bar{b}t, u\bar{d} \rightarrow st, \text{and further suppressed modes}) \).

In Table 1 we give results for the LO cross section and for the approximate NLO, NNLO, and NNNLO cross sections which include the threshold soft-gluon corrections at NLL accuracy. The soft-gluon corrections are relatively large for this channel; this is also true for the exact NLO corrections \[13\]. We also note that the approximate NLO cross section is only about 4\% larger than the exact NLO result, showing that the threshold corrections
Table 1: The leading-order and approximate higher-order cross sections for top quark production in the $s$ channel in pb for $pp$ collisions with $\sqrt{S} = 14$ TeV and $m_t = 170, 172, \text{ and } 175 \text{ GeV.}$ We use the MRST2004 NNLO pdf [23] and we set $\mu_F = \mu_R = m_t.$

| $m_t$ (GeV) | LO | NLO approx | NNLO approx | NNNLO approx |
|------------|----|------------|-------------|--------------|
| 170        | 5.40 | 7.53       | 8.08        | 8.33         |
| 172        | 5.18 | 7.23       | 7.76        | 8.00         |
| 175        | 4.87 | 6.79       | 7.29        | 7.52         |

Figure 2: Left: The cross section for single top production at the LHC in the $s$ channel. Here $\mu = \mu_F = \mu_R = m_t.$ Right: the $K$ factors.

are dominant and provide the bulk of the QCD corrections and, thus, that the threshold approximation works quite well.

As shown in Table 1, the NNNLO approximate cross section for $m_t = 175 \text{ GeV}$ is $7.52 \text{ pb.}$ After matching to the exact NLO cross section [13], we find that the matched cross section (i.e. exact NLO plus NNLO-NLL and NNNLO-NLL threshold corrections) is $\sigma_{\text{top}}^{s\text{-channel}}(m_t = 175 \text{ GeV}) = 7.23^{+0.53}_{-0.45} \pm 0.13 \text{ pb,}$ where the first uncertainty is due to the scale dependence and the second is due to the pdf. Adding these uncertainties in quadrature we find $\sigma_{\text{top}}^{s\text{-channel}}(m_t = 175 \text{ GeV}) = 7.23^{+0.55}_{-0.47} \text{ pb.}$

The NNNLO approximate cross section for $m_t = 171.4 \text{ GeV}$ is $8.10 \text{ pb.}$ If we match this to the exact NLO cross section, then we find $\sigma_{\text{top}}^{s\text{-channel}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 7.80^{+0.58+0.36}_{-0.48-0.33} \pm 0.14 \text{ pb,}$ where the first uncertainty is due to scale dependence, the second is due to the mass ($\pm 2.1 \text{ GeV,}$) and the third is the pdf uncertainty. Adding these uncertainties in quadrature we find that $\sigma_{\text{top}}^{s\text{-channel}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 7.80^{+0.70}_{-0.60} \text{ pb.}$

In Fig. 2 we plot the cross section for single top quark production in the $s$ channel at the LHC with $\sqrt{S} = 14 \text{ TeV}$ setting both the factorization and renormalization scales to $\mu = m_t.$ On the left-hand side we plot the LO and approximate higher-order cross sections as functions of the top quark mass. On the right-hand side we show the $K$ factors, defined as
the ratios of the higher-order cross sections to the LO cross section. The $K$ factors are quite large, thus showing that the corrections provide a big enhancement to the cross section, and are fairly constant over the top-quark mass range shown. As seen in the plot, the NLO-NLL corrections provide a 40% increase of the LO cross section, the NNLO-NLL corrections provide an additional 10%, and the NNNLO-NLL corrections a further 5%.

| $s$ channel antitop | LO   | NLO approx | NNLO approx | NNNLO approx |
|---------------------|------|------------|-------------|--------------|
| $m_t = 170$         | 3.44 | 4.22       | 4.14        | 4.09         |
| $m_t = 172$         | 3.30 | 4.05       | 3.96        | 3.92         |
| $m_t = 175$         | 3.08 | 3.78       | 3.71        | 3.67         |

Table 2: The leading-order and approximate higher-order cross sections for antitop quark production in the $s$ channel in pb for $pp$ collisions with $\sqrt{S} = 14$ TeV and $m_t = 170, 172$, and 175 GeV. We use the MRST2004 NNLO pdf [23] and we set $\mu_F = \mu_R = m_t$.

The cross section for single antitop quark production in the $s$ channel at the LHC is different from that for single top quark production. In Table 2 we give results for antitop quark production at the LHC. The approximate NLO cross section is only about 10% smaller than the exact NLO result [13], showing again that the threshold corrections provide the bulk of the QCD corrections. The soft-gluon corrections beyond NLO are small and negative.

As shown in Table 2, the NNNLO approximate cross section for $m_t = 175$ GeV is 3.67 pb. After matching this to the exact NLO cross section [13], we find $\sigma_{\text{antitop}}^{s-\text{channel}}(m_t = 175 \text{ GeV}) = 4.03^{+0.10}_{-0.12} \pm 0.10$ pb, where the first uncertainty is due to the scale dependence and the second is due to the pdf. Adding these uncertainties in quadrature we find $\sigma_{\text{antitop}}^{s-\text{channel}}(m_t = 175 \text{ GeV}) = 4.03^{+0.14}_{-0.16}$ pb.

The NNNLO approximate cross section for $m_t = 171.4$ GeV is 3.96 pb. If we match this to the exact NLO cross section, we find $\sigma_{\text{antitop}}^{s-\text{channel}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 4.35^{+0.11}_{-0.13} \pm 0.21$ pb, where the first uncertainty is due to scale dependence, the second is due to the mass ($\pm 2.1$ GeV), and the third is the pdf uncertainty. Adding these uncertainties in quadrature we find that $\sigma_{\text{antitop}}^{s-\text{channel}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 4.35 \pm 0.26$ pb.

In Fig. 3 we plot the cross section for single antitop quark production in the $s$ channel at the LHC with $\sqrt{S} = 14$ TeV setting both the factorization and renormalization scales to $\mu = m_t$. On the left-hand side we plot the LO and approximate higher-order cross sections versus the top quark mass. The $K$ factors are shown on the right. They are not as big as for single top production in this channel and in fact the NNLO-NLL and NNNLO-NLL corrections are small and negative. As seen in the plot, the NLO corrections provide a 23% increase of the LO cross section, the NNLO corrections provide an additional -3%, and the NNNLO corrections a further -1.5%.

4 Associated $tW$ production at the LHC

Associated $tW$ production proceeds via $bg \rightarrow tW^-$ (Cabibbo-suppressed contributions from $sg \rightarrow tW^-$ and $dg \rightarrow tW^-$ are negligible).
Single antitop at the LHC $s$-channel $\sqrt{S} = 14$ TeV $\mu = m_t$

| $m_t$ (GeV) | LO | NLO approx | NNLO approx | NNNLO approx |
|------------|----|------------|-------------|--------------|
| 170        | 32.1 | 44.3       | 48.6        | 51.0         |
| 172        | 31.2 | 43.1       | 47.4        | 49.6         |
| 175        | 29.9 | 41.3       | 45.2        | 47.3         |

Table 3: The leading-order and approximate higher-order cross sections for associated $tW$ production in pb for $pp$ collisions with $\sqrt{S} = 14$ TeV and $m_t = 170, 172$, and 175 GeV. We use the MRST2004 NNLO pdf [23] and we set $\mu_F = \mu_R = m_t$.

In Table 3 we give results for the LO and the higher-order cross sections including the threshold soft-gluon corrections at NLL accuracy at each order. The soft-gluon corrections are relatively large for this channel, even more than in the $s$ channel, and they provide the bulk of the QCD corrections.

As shown in Table 3, the NNNLO approximate cross section for $m_t = 175$ GeV is 47.3 pb. After matching to the exact NLO cross section [12] we find $\sigma_{tW}(m_t = 175$ GeV) $= 41.1 \pm 4.1 \pm 1.0$ pb, where the first uncertainty is due to the scale dependence and the second is due to the pdf. Adding these uncertainties in quadrature we find $\sigma_{tW}(m_t = 175$ GeV) $= 41.1 \pm 4.2$ pb.

The NNNLO approximate cross section for $m_t = 171.4$ GeV is 50.1 pb. If we match this to the exact NLO cross section then we find $\sigma_{tW}(m_t = 171.4\pm 2.1$ GeV) $= 43.5\pm 4.5\pm 1.5\pm 1.0$ pb, where the first uncertainty is due to scale dependence, the second is due to the mass ($\pm 2.1$ GeV), and the third is the pdf uncertainty. Adding these uncertainties in quadrature we find that $\sigma_{tW}(m_t = 171.4 \pm 2.1$ GeV) $= 43.5 \pm 4.8$ pb.

In Fig. 4 we plot the cross section for associated $tW$ production at the LHC with $\sqrt{S} = 14$ TeV setting the scales to $\mu = m_t$. On the left-hand side we plot the LO and approximate higher-order cross sections, and on the right-hand side we plot the $K$ factors. The latter are
Figure 4: Left: The cross section for associated $tW$ production at the LHC. Here $\mu = \mu_F = \mu_R = m_t$. Right: The $K$ factors.

quite large, showing that the soft-gluon corrections provide a big enhancement to the cross section, and fairly constant over the top-quark mass range shown. As seen from the plot, the NLO corrections provide a 38% increase of the LO cross section, the NNLO corrections provide an additional 14%, and the NNNLO corrections a further 7%.

Finally, we note that the cross section for the associated production of an antitop quark is identical to that for a top quark.

5 Conclusion

We have studied single top quark production at the LHC and have calculated the soft-gluon corrections to the cross section in three different production channels. The results differ a lot among channels. In the $t$ channel the soft-gluon corrections are not a good approximation of the full NLO result. The cross section for $t$-channel single top quark production at the LHC is $\sigma_{t\text{-channel}}^{t_{\text{top}}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 150 \pm 6 \text{ pb}$, where the uncertainty indicated includes the scale dependence, the uncertainty in the top quark mass, and the pdf uncertainty. For single antitop quark production the cross section is $\sigma_{s\text{-channel}}^{s_{\text{antitop}}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 92 \pm 4 \text{ pb}$.

In the $s$ channel, however, the NLO soft-gluon corrections are a good approximation of the full NLO result and they are significant. The threshold approximation works well and the higher-order NNLO and NNNLO soft-gluon contributions, at NLL accuracy, provide further enhancements. Our best estimate for the single top quark cross section in this channel at the LHC is $\sigma_{s\text{-channel}}^{s_{\text{top}}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 7.80^{+0.70}_{-0.60} \text{ pb}$. For the single antitop quark the cross section is $\sigma_{s\text{-channel}}^{s_{\text{antitop}}}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 4.35 \pm 0.26 \text{ pb}$.

The threshold soft-gluon corrections for associated $tW$ production are also large. Our best estimate for the single top quark cross section in this channel at the LHC is $\sigma^{tW}(m_t = 171.4 \pm 2.1 \text{ GeV}) = 43.5 \pm 4.8 \text{ pb}$. The cross section is the same for associated antitop...
production.

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