Quark Model From Lattice QCD

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Abstract

We study the valence approximation in lattice QCD of hadrons where the cloud quarks and antiquarks are deleted by truncating the backward time propagation (Z graphs) in the connected insertions. Whereas, the sea quarks are eliminated via the quenched approximation and in the disconnected insertions. It is shown that the ratios of isovector to isoscalar matrix elements in the nucleon reproduce the SU(6) quark model predictions in a lattice QCD calculation. We also discuss how the hadron masses are affected.

1 Introduction

In addition to its classification scheme, the quark model is, by and large, quite successful in delineating the spectrum and structure of mesons and baryons. One often wonders what the nature of the approximation is, especially in view of the advent of quantum chromodynamics (QCD). In order to answer this question, we need to understand first where the quark model is successful and where it fails.

To begin with, we need to define what we mean by the quark model. We consider the simplest approach which includes the following ingredients:

• The Fock space is restricted to the valence quarks only.

• These valence quarks, be them the dressed constituent quarks or the bare quarks, are confined in a potential or a bag. To this zeroth order, the hadron

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wavefunctions involving u, d, and s quarks are classified by the totally symmetric wavefunctions in the flavor-spin and orbital space according to the \( SU(6) \times O(3) \) and totally antisymmetric/symmetric in the color space for the baryons/mesons.

- The degeneracy within the the multiplets are lifted by the different quark masses and the residual interaction between the quarks which is weak compared to the confining potential. The one-gluon exchange potential is usually taken as this residual interaction to describe the hyper-fine and fine splittings of the hadron masses.

Given what we mean by the quark model, it is easier to understand where the quark model succeeds and fails. It is successful in describing hadron masses, relations of coupling and decay constants, magnetic moments, Okubo-Zweig rule, etc. It is worthwhile noting that all these are based on the valence picture aided with \( SU(6) \times O(3) \) group for its color-spin and space group. On the other hand, it fails to account for the U(1) anomaly (the \( \eta' \) mass), the proton spin crisis and the \( \pi N \sigma \) term. All these problems involve large contribution from disconnected insertions involving sea-quarks \[1\]. It is natural not to expect the valence quark model to work. There are other places where the valence quark model does not work well. These include \( \pi \pi \), \( \pi N \) scatterings, current algebra relations, and the form factors of the nucleon which are better described by meson effective theories with chiral symmetry taken into account. For example, the \( \pi \pi \) scattering is well described in the chiral perturbation theory, the \( \pi N \) scattering and the nucleon electromagnetic, axial, and pseudoscalar form factors (especially the neutron charge radius), Goldberg-Treiman relation are all quite well given in the skyrmion approach \[2\]. One common theme of these models is the chiral symmetry which involves meson cloud and hence the higher Fock space beyond the valence.

## 2 Valence Approximation

It is then clear that there are three ingredients in the classification the quarks, i.e. the valence, the cloud, and the sea quarks. The question is how one defines them unambiguously and in a model independent way in QCD. It has been shown recently \[3\] that in evaluating the hadronic tensor in the deep inelastic scattering, the three topological distinct contractions of quark fields lead to the three quark-line skeleton diagrams. The self-contraction of the current leading to a quark loop is separated from the quark lines joining the nucleon interpolating fields. This disconnected insertion (D.I.) refers to the quark lines which are of courses connected by the gluon lines. This D.I. defines the sea-parton. One class of the connected insertion (C.I.) involves an anti-quark propagating backwards in time between the currents and is defined as the cloud anti-quark. Another class of the C.I. involves a quark propagating forward
in time between the currents and is defined to be the sum of the valence and cloud quarks. Thus, in the parton model, the antiquark distribution should be written as

$$\bar{q}(x) = \bar{q}_c(x) + \bar{q}_s(x).$$  \hspace{1cm} (1)$$

to denote their respective origins for each flavor i. Similarly, the quark distribution is written as

$$q^i(x) = q^i_V(x) + q^i_c(x) + q^i_s(x)$$  \hspace{1cm} (2)

Since $$q^i_s(x) = q^i_s(x)$$, we define $$q^i_c(x) = q^i_c(x)$$ so that $$q^i_V(x)$$ will be responsible for the baryon number, i.e. $$\int u_V(x) dx = \int [u(x) - \bar{u}(x)] dx = 2$$ and $$\int d_V(x) dx = \int [d(x) - \bar{d}(x)] = 1$$ for the proton.

We can reveal the role of these quarks in the nucleon matrix elements which involve the three-point function with one current. The D.I. in the three-point function involves the sea-quark contribution to the m.e. It has been shown that the this diagram has indeed large contributions for the flavor-singlet scalar and axial charges so that the discrepancy between the valence quark model and the experiment in the $$\pi N\sigma$$ term and the flavor-singlet $$g_A$$ can be understood. Thus we conclude that in order to simulate the valence quark model, the first step is to eliminate the quark loops. This can be done in the well-known quenched approximation by setting the fermion determinant to a constant.

In order to reveal the effect of the cloud degree of freedom, we have calculated the ratios of the isoscalar to isovector axial and scalar charges in a quenched lattice calculation. The ratio of the isoscalar (the C.I. part) to isovector axial charge can be written as

$$R_A = \frac{\langle p|\bar{u}\gamma_3\gamma_5 u + \bar{d}\gamma_3\gamma_5 d|p\rangle}{\langle p|\bar{u}\gamma_3\gamma_5 u - \bar{d}\gamma_3\gamma_5 d|p\rangle} \bigg| C.I. \bigg. = \frac{g_A^1}{g_A^3} \bigg| C.I. \bigg. = \frac{\int dx [\Delta u(x) + \Delta d(x)]]}{\int dx [\Delta u(x) - \Delta d(x)]} \bigg| C.I.$$.  \hspace{1cm} (3)$$

where $$\Delta u(\Delta d)$$ is the polarized parton distribution of the u(d) quark and antiquark in the C.I. For the non-relativistic case, $$g_A^3$$ is 5/3 and $$g_A^1$$ for the C.I. is 1 Thus, the ratio $$R_A$$ should be 3/5. Our lattice results based on quenched $$16^3 \times 24$$ lattices with $$\beta = 6$$ for the Wilson $$\kappa$$ ranging between 0.154 to 0.105 which correspond to strange and twice the charm masses are plotted in Fig. 1 as a function of the quark mass $$ma = \ln (4\kappa_c/\kappa - 3)$$. We indeed find this ratio for the heavy quarks (i.e. $$\kappa \geq 0.133$$ or $$ma \geq 0.4$$ in Fig.1). This is to be expected because the cloud antiquarks which involves Z-graphs are suppressed for non-relativistic quarks by $$O(p/m_q)$$. Interestingly, the ratio dips under 3/5 for light quarks. We interpret this as due to the cloud quark and antiquark, since in the relativistic valence quark models (i.e. no cloud nor sea quarks) the ratio remains to be 3/5. To verify that this is indeed caused by the cloud antiquarks from the backward time propagation, we perform the following approximation. In the Wilson lattice action, the backward time hopping is prescribed by the
term $-\kappa(1 - \gamma_4)U_4(x)\delta_{x,y-a_4}$. We shall amputate this term from the quark matrix in our calculation of the quark propagators. As a result, the quarks are limited to propagating forward in time and there will be no Z-graph and hence no cloud quarks and antiquarks. The Fock space is limited to 3 valence quarks. Thus we shall refer to this as the \textit{valence approximation} and we believe it simulates what the naive quark model is supposed to describe by design. After making this valence approximation for the light quarks with $\kappa = 0.148, 0.152$, and 0.154 (The quark mass turns out to differ from before only at the perturbative one-loop order, i.e. $O(\alpha_s)$, which is very small. we find that the ratio $R_A$ becomes $3/5$ with errors less than the size of the circles in Fig. 1. Since the valence quark model prediction of $R_A$ is well reproduced by the valence approximation, we believe this proves our point that the deviation of $R_A$ from $3/5$ in Fig. 1 is caused by the backward time propagation, i.e. the cloud quarks and antiquarks.

Similar situation happens in the scalar matrix elements. In the parton model description of the forward m.e., the ratio of the isovector to isoscalar scalar charge of the proton for the C.I. is then approximated according to eqs. (1) and (2) as

\[
R_S = \frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \quad C.I. = \frac{1 + 2 \int dx[\bar{u}_c(x) - \bar{d}_c(x)]}{3 + 2 \int dx[\bar{u}_c(x) + \bar{d}_c(x)]}
\]

Since the quark/antiquark number is positive definite, we expect this ratio to be $\leq 1/3$. For heavy quarks where the cloud antiquarks are suppressed, the ratio is indeed $1/3$ (see Fig. 2). For quarks lighter than $\kappa = 0.140$, we find that the ratio is in fact less than $1/3$. The lattice results of the valence approximation for the light quarks, shown as the circles in Fig. 2, turn out to be $1/3$. This shows that the deviation of $R_S$ from $1/3$ is caused by the cloud quarks and antiquarks. With these findings, we obtain an upper-bound for the violation of GSR [3], i.e. $n_{\bar{u}} - n_{\bar{d}} \leq -0.12 \pm 0.05$. This clearly shows that $n_{\bar{u}} - n_{\bar{d}}$ is negative and is quite consistent with the experimental result $\int dx[\bar{u}^p(x) - \bar{d}^p(x)] = -0.14 \pm 0.024$.

3 Hadron Spectroscopy

To further explore the consequences of the valence approximation, we calculate the baryon masses. Plotted in fig. 3 are masses of $\Delta, N, \rho$, and $\pi$ as a function of the quark mass $m_a$ on our lattice with quenched approximation. We see that the hyperfine splittings between the $\Delta$ and $N$, and the $\rho$ and $\pi$ grow when the quark mass approaches the chiral limit as expected. However, it is surprising to learn that in the valence approximation, the $\Delta$ and $N$ become degenerate within errors, so do the $\rho$ and $\pi$ as shown in Fig. 4. Since the one-gluon exchange is not switched off in the valence approximation, the hyperfine splitting is probably not due to the one-gluon exchange potential as commonly believed. Since this is a direct consequence of eliminating the
cloud quark/antiquark degree of freedom, one can speculate that it has something to do with the cloud. It seems that a chiral soliton like the skyrmion might delineate a more accurate dynamical picture than the one-gluon exchange spin-spin interaction.

To conclude, we find that the valence approximation in QCD reproduces the SU(6) results of the valence quark model better than we anticipated. Especially in hadron masses, the results seem to indicate that there are no hyper-fine splittings, modulo the uncertainty due to the statistical and systematic errors.

References

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[2] For example, Chiral Solitons, ed. K.F. Liu (World Scientific, 1987).
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Figure Captions:
Fig. 1 The ratio $R_A$ of eq. (3) as a function of the quark mass $m a = ln(4k_c/k - 3)$.
Fig. 2 The ratio $R_S$ of eq. (4) as a function of the quark mass $m a$.
Fig. 3 Masses of $\Delta$, $N$, $\rho$, and $\pi$ (in lattice units) as a function of the quark mass $m a$ in the quenched approximation.
Fig. 4 The same as in Fig. 3 with the valence approximation.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9411067v1
