Inertia Evaluation for Power System Based on Quasi-Steady-State Data

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Abstract. With the integration of renewable energy sources, the power system equivalent inertia has dropped significantly. It is urgent to conduct the online evaluation of inertia. Current inertia evaluation methods are mostly based on the rate of change of frequency under large disturbances. In order to evaluate inertia under quasi-steady-state operation, an evaluation method is proposed based on system identification. The monitoring data under the quasi-steady-state operating conditions of the power system is acquired through the synchronous phasor measurement units (PMUs). And the PMU data including frequency and active power under small disturbances are used for the quantitative evaluation of inertia. The WSCC 9-bus system is established to verify the effectiveness of the proposed method.

1. Introduction
The inertia of the power system is expressed as the ability of the system to resist frequency changes caused by external disturbances. Inertia is an inherent property of the power system, which is generally expressed by the equivalent inertia constant [1]. In recent years, the proportion of renewables connected to the power system has been increasing [2]. Renewable energies are connected to the power system through power electronic equipment, and cannot provide inertia support to the system like traditional synchronous generators, which greatly reduces the equivalent inertia of power system, resulting in a short-term frequency drop under large disturbances [3]. On the other hand, as HVAC/DC transmission projects are put into operation one after another, the risk of large shortfall power disturbance accidents has increased [4]. Accurate evaluation of inertia is of great significance for analysing the dynamic frequency characteristics of power system.

The evaluation methods of inertia can be divided into two types. One is based on disturbance events, and the other is based on quasi-steady-state data. Among them, there are many researches on the evaluation methods based on disturbance events. The problem of power system equivalent inertia evaluation was studied earlier in [5]. They first performed polynomial fitting of the PMU measurement data, and calculated the system equivalent inertia based on the fitting coefficients. However, as the dynamic frequency behaviour of the power system becomes more and more complex, it is difficult to accurately fit the frequency dynamic trajectory. The system equivalent inertia was expressed as the ratio of the power imbalance and the frequency change rate at the moment of the event [6]. Based on this method, Ashton et al. used the frequency offset caused by the sudden disconnection of generators to estimate the equivalent inertia of the Great British power system [7]. Considering the spatial distribution...
characteristics of frequency, the node inertia was defined, and the inertia index could be extracted from the node frequency spectrum under small disturbances to visualize the power system inertia [8], [9]. Besides, the characteristic parameters of electromechanical oscillation could be extracted based on the swing equation to realize the online evaluation of the equivalent inertia of the power system [10], [11].

With the increasing number of power electronic devices in the power system, it is difficult to obtain the amount of power imbalance under an event [12]. As a result, it is difficult to accurately implement inertia evaluation based on disturbance events. And this method relies on external disturbances. In contrast, the inertia evaluation method based on quasi-steady-state data can reflect the characteristics of power system inertia, and has more research and application value. In the reference [13], authors build an autoregressive moving average model based on the frequency and active power deviation data measured by the PMU in the quasi-steady state, and the zero-pole equivalent gain of the transfer function of the reduced-order model is the equivalent inertia. The above method realizes the equivalent inertia evaluation under quasi-steady-state operating conditions, but there is still room for improvement in the evaluation accuracy under the scene of frequency fluctuations.

In this paper, the response characteristics of inertia of power system are described in detail, and the theoretical basis of equivalent inertia evaluation of power system is studied. Based on PMU measurement data and system identification theory, a method for estimating power system inertia under quasi-steady-state operating conditions is proposed.

The rest of the paper is organized as follows. The section II describes the inertia response mechanism of traditional synchronous generators and the concept of equivalent inertia of power system. The mathematical model of inertia estimation and the algorithm of inertia constant identification are illustrated in the section III. And then, the case study is presented in Section IV. Finally, the conclusion is drawn in Section V.

2. Theory of power system inertia

2.1. Inertia response mechanism of traditional synchronous generators

The inertia provided by the traditional synchronous generator to the power system is characterized by the rotational kinetic energy stored when the rotor rotates at the rated speed [14]. The calculation of inertia is as follows:

\[ E = HS = \frac{1}{2} J \omega_n^2 \]  \hspace{1cm} (1)

where \( E \), \( H \), \( S \), \( J \) and \( \omega_n \) are the rotor kinetic energy, inertia constant, capacity, moment of inertia and rated angular velocity of the generator, respectively.

The power imbalance of the system will cause the frequency change of each node of the system. Specific to each generator, the torque imbalance on the generator rotor caused by the electromagnetic power imbalance will cause the rotor speed to change. The response process of each state variable can be described by the rotor motion equation [15]:

\[
\begin{align*}
\frac{d\delta}{dt} &= (\omega - 1)\omega_n, \\
2H \frac{d\omega}{dt} &= T_m - T_e \approx P_m - P_e
\end{align*}
\]  \hspace{1cm} (2)

where \( t \) is the time; \( \omega_n \) is the system rated electrical angular velocity; \( H \) is the rotor inertia constant; \( \delta \) is the rotor power angle; \( \omega \) is the rotor electrical angular velocity; \( T_m \), \( T_e \), \( P_m \), \( P_e \) are the mechanical torque and electromagnetic torque of the rotor, respectively, Mechanical power and electromagnetic power. Except for \( t \), \( \omega_n \), and \( H \) which are well-known values, all the quantities in the formula are per unit values.
2.2. Power system equivalent inertia

With the increase in the penetration rate of wind and photovoltaic power in the system and the diversification of control methods, the inertia response after disturbance has become increasingly complex. At present, the main types of wind turbines are doubly-fed asynchronous turbines and permanent magnet direct-drive turbines. Wind power and photovoltaic units generally adopt maximum power point tracking (MPPT) strategy control, which cannot respond to power deviation by releasing or absorbing energy, so the power electronic interface of conventional control does not have inertia response characteristics [16].

In order to cope with the problem of the decrease of system inertia level caused by the high proportion of renewable energy integration, related researches use virtual inertia control to couple the electromagnetic power injected into the system with the mechanical power of the prime mover. The specific control strategies of virtual inertia include: droop control, virtual synchronous generator (VSG), swing equation simulation and other technologies [17]. The renewable energy unit under the control strategies of virtual inertia has the ability to provide inertia support. In addition, the static frequency characteristics of the load can resist frequency changes, and provide inertia response. In a low-inertia renewable energy power system, the support of load inertia cannot be ignored.

Therefore, it is necessary to clarify the concept of equivalent inertia of the renewable energy power system, so as to take into account the inertia provided by the traditional synchronous machine, the virtual inertia of the power electronic converter, and the load inertia. This paper expresses the equivalent inertia as:

$$E_{sys} = \sum_{i=1}^{N} H_{gen,i} S_{gen,i} + \sum_{j=1}^{M} H_{vir,j} S_{vir,j} + E_{load}$$

where $E_{sys}$ and $E_{load}$ are system equivalent inertia and load inertia, respectively; $H_{gen,i}$ and $H_{vir,j}$ are the inertia constants of synchronous generators and renewable energy units controlled by virtual inertia, respectively; $S_{gen,i}$ and $S_{vir,j}$ are the capacities of synchronous generators and renewable energy units controlled by virtual inertia, respectively.

The equivalent inertia constant of the system can be calculated from the above formula, which is used to evaluate the overall inertia level of the system as a whole or in a zone. The system is equivalent to a single generator model, and the system equivalent inertia time constant can be expressed as:

$$H_{sys} = \frac{E_{sys}}{S_{sys}} = \frac{f_n \Delta P}{2S_{sys} \frac{df}{dt}}$$

where $H_{sys}$ and $S_{sys}$ are the inertia constant and capacity of the system, respectively; $\Delta P$ and $f_n$ are the total active power deviation and rated frequency of the system; $f_c$ is the center frequency of the system, and the calculation method is as follows:

$$f_c = \sum_{i=1}^{N} \omega_i f_i$$

where $\omega_i$ is the weight, satisfies $\omega_i > 0$ and $\sum_{i=1}^{N} \omega_i = 1$; $N$ is the number of nodes in the system; $f_i$ is the measurement frequency of each node.

3. Inertia evaluation method under quasi-steady-state

3.1. Mathematical Model of Inertia Evaluation

The application of the Wide Area Measurement System (WAMS) realizes the simultaneous acquisition of transient response signals and noise-like quasi-steady-state signals [18]. Quasi-steady-state data mainly come from small disturbances such as load switch switching or unit output changes during
normal operation. The frequency and power data in the quasi-steady-state data collected by WAMS are the basis for inertia evaluation.

In order to realize the inertia evaluation at the system or zone level, it is necessary to set the system equivalent to a single-generator system and give a dynamic model containing the inertia constant. Considering the damping effect, the rotor motion equation can be written as follows:

$$\frac{df}{dt} = \frac{1}{2H} (P_m - P_e - D\Delta f)$$

(6)

where $P_m$ and $P_e$ are the mechanical power and electromagnetic power of the system's equivalent synchronous generator, respectively; $H$ and $D$ are the equivalent inertia constant and damping coefficient of the system, respectively. Except for $H$ and $D$, all the quantities in the formula are per unit values.

When operating under quasi-steady-state conditions, the power and frequency data show the characteristics of small fluctuations near the steady-state operating point, which can be written in incremental form:

$$\frac{d\Delta f}{dt} = \frac{1}{2H} (\Delta P_m - \Delta P_e - D\Delta f)$$

(7)

Under quasi-steady-state conditions, it can be assumed that the mechanical power input by the prime mover remains constant, that is, $\Delta P_m = 0$; Laplace transform equation (8) and write it as a transfer function:

$$G(s) = \frac{\Delta f(s)}{\Delta P_e(s)} = -\frac{1}{2Hs + D}$$

(8)

where $\Delta f$ is the rotor electrical frequency deviation of the equivalent synchronous generator, replaced by the system central frequency deviation; $\Delta P_e$ is the electromagnetic power deviation of the equivalent synchronous generator, replaced by the total power deviation of the system; $G(s)$ is the transfer function from $\Delta P_e$ to $\Delta f$, $s$ is the Laplace operator. Except for $H$ and $D$, all the quantities in the formula are standard unit values.

$\Delta P_e$ is the input of the system, and $\Delta f$ is the output of the system. The value of the inertia of the system can be estimated through parameter identification.

3.2. Inertial Constant Identification

First of all, pre-processing measures such as unitization, pre-filtering, and de-trending must be carried out on the monitored data. Based on the mathematical model obtained from the above analysis, it is necessary to select a suitable system identification algorithm to extract the inertia constant from the input and output data of the model.

A discrete state space model can be established [9], which is expressed as follows.

$$\begin{align*}
    x_{k+1} &= A x_k + B u_k + w_k, \\
    y_k &= C x_k + D u_k + v_k
\end{align*}$$

(9)

where state variable $x_k \in R^{n x}$, input variable $u_k \in R^{u x}$, output variable $y_k \in R^{y x}$; and $A \in R^{nx \times nx}$, $C \in R^{nx \times nx}$ and $D \in R^{nx \times ux}$ are the system matrices to be estimated; $w_k$ and $v_k$ represent process disturbance and measurement noise, respectively.

The Numerical algorithm for Subspace State Space System Identification (N4SID) algorithm is a commonly used algorithm for random subspace system identification. After transformations such as projection and Singular Value Decomposition (SVD) are performed on the state space model described by the formula, the parameter matrix is solved by the least square method, and the inertial constant can be extracted.
The matrix dimension of the N4SID algorithm based on the discrete state space model is relatively high. Once the inertia constant is estimated in a long time window, the calculation will take a long time. Therefore, this method is not suitable for online application. Autoregressive moving average with exogenous variable (ARMAX) models the collected quasi-steady-state discrete data, which is expressed as follows:

\[ A(q)y(t) = B(q)u(t) + C(q)w(t) \]  
\[ A(q) = 1 + a_1q^{-1} + \cdots + a_nq^{-n} \]  
\[ B(q) = b_1q^{-1} + \cdots + b_nq^{-n} \]  
\[ C(q) = 1 + c_1q^{-1} + \cdots + c_lq^{-l} \]  

where \( q^{-1} \) is the backward translation operator, \( q^{-1}u(t) = u(t-1) \), \( u(t) \) is the input at time \( t(t=1,2,\cdots,N) \), satisfy the static and continuous excitation conditions of sufficient order, \( y(t) \) is the output at time \( t(t=1,2,\cdots,N) \), \( w(t) \) is the noise, and the input \( \{u(t)\} \) are statistically independent. \( \theta_A = [a_1,\ldots,a_n]^T \) is the parameter vector of the autoregressive part (AR), \( \theta_B = [b_1,\ldots,b_n]^T \) is the parameter of the exogenous part (X), \( \theta_C = [c_1,\ldots,c_l]^T \) is the parameter vector of the moving average part (MA).

The first thing to be done is to identify the system structure to determine the order of each channel. It can be obtained based on prior information or offline structure identification methods. The parameter identification of the ARMAX model is then carried out. Use the observed data to identify the parameter vectors \( \theta_A \), \( \theta_B \) and \( \theta_C \) are the variance \( \sigma^2_w \) of the noise \( w(t) \).

For the channel to be identified, it can be written as follows:

\[ y(i) = -a_1y(i-1) - a_2y(i-2) - \cdots - b_1u(i-1) + b_2u(i-2) + \cdots + Ce(i) \]  

(14)

Extract the parameters to be identified, and get:

\[ \Phi = [-a_1,\ldots,-a_n,b_1,\ldots,b_n] \]  

(15)

The input and output data are respectively composed of output vector \( Y \) and matrix \( \Omega \):

\[ Y = [y(i) y(i+1) \cdots y(N)]^T \]  
\[ \Omega = \begin{bmatrix} y(i-1) & u(i-1) \\ y(i) & u(i) \\ \vdots & \vdots \\ y(N-1) & u(N-1) \end{bmatrix} \]  

(16)  
(17)

Then we can get the least squares estimate of \( \theta \)

\[ \hat{\theta} = (\Omega^T\Omega)^{-1}\Omega Y \]  

(18)

Then use the least square method to solve the model parameters, and then convert the difference equation into a transfer function to extract the inertia constant.

With the aid of the system identification toolbox of MATLAB, the above two mathematical models can be easily established and the inertial constants can be identified. The process of system inertia evaluation is shown in Figure 1.

4. Case study

This paper builds a WSCC 9-bus system to verify the effectiveness of the method proposed in this paper for power system inertia evaluation. The 9-bus system is shown in Figure 2. The rated capacity of each generator and the inertial time constant based on the system rated capacity of 100MW are shown in Table 1.
Begin
Input frequency and active power data
Data preprocessing
Form input and output matrix
Assign initial values to parameters
Solve parameter estimates
Whether the convergence conditions are met or not
Y
Update parameters
N
End

Figure 1. Flowchart of system inertia estimation

4.1. Inertia evaluation of synchronous generators
In order to simulate the load fluctuation under the quasi-steady state of the system, in the simulation software, the superposition of the sine function is used to simulate the random fluctuation of the load. Figure 3 shows the active power fluctuation of the No. 7 bus connected to the load simulated by the superposition of a sine function.

In order to verify the effectiveness of the proposed method, firstly, the inertia constant identification at the unit level is carried out, and the results are compared with the results of the inertial constant identification method under large disturbances. In order to realize online real-time tracking of unit inertia, the time window of inertia evaluation can be set to 10s, that is, the inertia time constant identification is performed every 10s of data collected.

Table 2 shows the results of the identification of unit inertia using the method introduced in this article and its comparison with the identification results of the method based on large disturbances. The selected method based on large disturbance is a more classic method based on polynomial fitting of frequency trajectory under power shortage disturbance. $H_i(s)$, $H_d(s)$, and $H_{idx}(s)$ in the table are the true value, the identification result of the large disturbance method and the identification result of this method respectively; $e_d$ and $e_{ide}$ are the error rate of the identification result of the large disturbance method and the identification result of this method, respectively. It can be clearly seen that the identification accuracy of the method proposed in this article is higher.

4.2. Inertia evaluation of power system
More practical value is to evaluate the inertia of a certain regional power system to determine whether the system inertia is within the safe operating range. In order to adapt to different types of load changes under the quasi-steady state conditions of the system, three types of load changes are set in simulation software: step sudden increase, ramp growth and random fluctuations simulated by sine function superposition. Figure 4. shows the fitting accuracy of the identification model established in this paper.
under the condition of random load fluctuations. The intercepted time window is 10s, and the fitting accuracy is close to 90%, which meets the requirements of engineering applications. Table 3 shows the system inertia identification results under different load changes. The results show that the method proposed in this paper can realize the assessment of the overall inertia level of the system while adapting to different load changes.

Table 1. Basic parameters of 9-bus system

| Synchronous generator | Rated power $S_r$/MW | Inertia constant $H$/s |
|-----------------------|-----------------------|------------------------|
| 1                     | 222.75                | 23.68                  |
| 2                     | 172.8                 | 6.4                    |
| 3                     | 115.2                 | 3.01                   |

Figure 4. Fitting accuracy under the condition of random load fluctuation

Table 2. Result of inertia of generators

| Generator | $H_r$/s | $H_d$/s | $e_d$ | $H_{adv}$/s | $e_{adv}$ |
|-----------|---------|---------|-------|-------------|-----------|
| 1         | 23.68   | 25.03   | 5.70% | 23.53       | 0.63%     |
| 2         | 6.4     | 5.48    | 14.38%| 6.34        | 0.94%     |
| 3         | 3.01    | 3.59    | 19.27%| 2.93        | 2.66%     |

Table 3. Result of System Inertia

| Type of load change               | $H_{adv}$/s |
|-----------------------------------|-------------|
| Step                              | 34.20       |
| Slope                             | 34.44       |
| Random fluctuations               | 39.87       |

5. Conclusion
This paper studies the inertia evaluation problem under the quasi-steady-state operation of the power system. The inertia response mechanism of the traditional synchronous generator is analysed, and the definition of the equivalent inertia of the power system is clarified. Based on the power and frequency data obtained by WAMS, a mathematical model for inertia evaluation is established. With the aid of system identification algorithm, a method evaluating power system inertia is proposed. The case study shows that the method in this paper can identify the unit inertia more accurately, and can evaluate the system inertia under the condition of various forms of load fluctuation.

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