Dynamics of Rotating, Magnetized Neutron Stars

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Using a fully general relativistic implementation of ideal magnetohydrodynamics with no assumed symmetries in three spatial dimensions, the dynamics of magnetized, rigidly rotating neutron stars are studied. Beginning with fully consistent initial data constructed with Magstar, part of the Lorene project, we study the dynamics and stability of rotating, magnetized polytropic stars as models of neutron stars. Evolutions suggest that some of these rotating, magnetized stars may be minimally unstable occurring at the threshold of black hole formation.

Keywords: stellar evolution; MHD; neutron stars; magnetars

Introduction: Neutron stars are fascinating astrophysical objects melding incredibly strong gravity with large densities and pressures. Along with powerful magnetic fields, these compact stars form magnetars which are suspected as the engines behind anomalous X-ray pulsars (AXPs) and soft gamma ray repeaters (SGRs).1 There is a long history of numerical models of neutron stars, but here their evolution is studied using a code developed for, and applied to, a number of astrophysical problems.2–7 This code adopts a fully nonlinear scheme for the Einstein gravitational field equations which allows for the easy extraction of gravitational wave signatures. This gravity is coupled to a magnetohydrodynamic (MHD) component which uses high resolution shock capturing (HRSC) methods to evolve a magnetized fluid in a general relativistic, finite-difference scheme. Further details concerning the formulation and numerical methods are presented in Ref. 8.

A number of tests are presented in Ref. 8. Evolutions of stable, rotating stars demonstrate convergence with increasing resolution. That is, conservation of quantities such as baryon mass and the \(z\)-component of the angular momentum improves as one moves to better resolution. Likewise, violations of the constraints remain closer to zero with better resolution.

Results: Also discussed in Ref. 8 were evolutions of unstable stellar solutions. Such solutions either expand, oscillating about some stable star, or they instead collapse, eventually forming a black hole. Beginning with a particular unstable solution, unavoidable numerical error will ultimately drive it to one of these end states. However, it was found in Ref. 8 that one could tune the perturbation via some generalized parameter \(p\) (e.g. the amplitude of some perturbation to the initial pressure) such that for some critical value \(p^*\), if \(p > p^*\) then the solution collapsed and if \(p < p^*\) the solution expanded. This ability to tune to threshold suggests that the unstable solutions have just a single unstable mode, and therefore that the solutions represent Type I critical solutions.

One question remaining, among many, is whether initial data far from these
unstable solutions in some configuration space will, upon tuning, evolve towards these unstable solutions and find them at threshold. The work of Ref. 9 answers this question with respect to nonrotating, spherical stars in spherical symmetry by evolving stable TOV solutions perturbed gravitationally by a pulse of scalar field. For energetic enough scalar pulses, the stable star is sufficiently disturbed that it collapses to a black hole. Tuning this initial scalar pulse, they find longer and longer lived unstable stellar solutions.

Because this code already has the capability to evolve a massless, complex scalar field (as studied within the context of the dynamics of boson stars), it is straightforward to include a scalar pulse as initial data. In particular, the real component of the scalar field \( \phi(x, y, z, t) \) is defined as a Gaussian pulse, \( \phi(t = 0) = Ae^{(r-R_0)^2/\delta^2} \), in terms of a radial coordinate \( r \equiv \sqrt{\epsilon_x^2 x^2 + \epsilon_y^2 y^2 + z^2} \) with real constants \( \epsilon_x, \epsilon_y, R_0, \delta, \) and \( A \). The imaginary component remains zero throughout the evolution and we define the initial time derivative of the field such that \( \phi \) is approximately in-going. There are two significant differences with respect to the work of Ref. 9: (i) the Einstein field equations are not re-solved to account for the scalar pulse and (ii) the scalar pulse is located outside the star initially, but not very far from it. In particular, two searches are carried out, both choosing \( R_0 = 30 \) and \( \delta = 6 \), and using \( A \) as the generalized parameter \( p \) which is tuned to threshold. The first search uses a spherically symmetric pulse described by \( \epsilon_x = 1, \epsilon_y = 0 \), and the second uses \( \epsilon_x = 1.25, \epsilon_y = 0.75 \).

It is found that tuning a single parameter results in the evolution approaching an unstable solution which once again suggests that the unstable solution is itself a co-dimension one unstable solution. The time \( T_0 \) for which the evolution is near the unstable solution is expected to scale as \( T_0 = -\sigma \ln |p - p^*| + C \) for some constant \( C \) that depends on the particular initial-data family and some universal constant \( \sigma \), the inverse of the growth rate of the instability. As indicated by Fig. 1, the results are consistent with this expected scaling, given the relatively large errors here. These numerical errors appear to be related to effects from the boundary which significantly limit how close to criticality one can achieve. These two searches are both tuned to approximately one part in a million.

Another interesting aspect to the dynamics of magnetized stars concerns the behavior of the magnetic field. In particular, for the case in which the magnetic moment of the star is not aligned with the rotational axis, one can look at the structure of the dynamic magnetic field in terms of poloidal and toroidal components. In Fig. 2, the evolution of rotating star with a magnetic moment rotated away from the \( z \)-axis is shown. Because the initial magnetic field is confined to the interior of the star, problems that generally occur when the magnetic field is strong compared to the density are avoided.

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Fig. 1. Results of two searches using a scalar pulse to perturb the same, magnetic, rotating star. **Left:** Stellar properties as functions of time in units of the period of rotation for near-critical evolutions. **Right:** Survival time of evolutions for the two searches demonstrates the expected scaling with the distance from criticality.

Fig. 2. Evolution of a rotating solution (counter-clockwise) with magnetic moment inclined with respect to the z-axis of $\pi/4$ radians. Shown are the values of $|\vec{B}|^2$ on the $z = 0$ plane for roughly half a period (from left to right). The computational domain extends in each direction a factor of four times the distance shown.