Inhomogeneous whistler turbulence in space plasmas

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ABSTRACT

A nonlinear two dimensional fluid model of whistler turbulence is developed that nonlinearly couples wave magnetic field with electron density perturbations. This coupling leads essentially to finite compressibility effects in whistler turbulence model. Interestingly it is found from our simulations that despite strong compressibility effects, the density fluctuations couple only weakly to the wave magnetic field fluctuations. In a characteristic regime where large scale whistlers are predominant, the weakly coupled density fluctuations do not modify inertial range energy cascade processes. Consequently, the turbulent energy is dominated by the large scale (compared to electron inertial length) eddies and it follows a Kolmogorov-like $k^{-7/3}$ spectrum, where $k$ is a characteristic wavenumber. The weak coupling of the density fluctuations is explained on the basis of a whistler wave parameter that quantifies the contribution of density perturbations in the wave magnetic field.

Key words: (magnetohydrodynamics) MHD, (Sun:) solar wind, Sun: magnetic fields, ISM: magnetic fields

1 INTRODUCTION

Whistler wave regime is ubiquitously present in many space, astrophysical and laboratory plasmas. For instance, in the magnetospheric plasma, electrons can be accelerated by whistler-mode and compressional ULF (fast mode waves) turbulences near the Earth’s synchronous orbit (Li et al, 2005). The whistler-mode turbulence can accelerate substorm injection electrons with several hundreds of keV through wave-particle gyroresonant interaction and hence may play an important role in the electron acceleration during substorms (Li et al, 2005). Vetoulis & Drake (1999) describe whistler turbulence at the magnetopause. Whistler mode turbulence can be triggered by electron beams in earth’s bow shock (Tokar et al, 1984). In the solar wind plasma, observations have identified a spectral break in the solar wind magnetic field spectrum (Goldstein et al 1995, Leamon et al 1999). The mechanism leading to the spectral break has been unclear and thought to be either mediated by the kinetic Alfvén waves (Hasegawa 1976), or by electromagnetic ion-cyclotron-Alfvén waves (Wu & Yoon, 2007), or whistler cascade regime (Gary et al, 2008), or by a class of fluctuations that can be dealt within the framework of the Hall magnetohydrodynamic plasma model (Alexandrova et al 2007, 2008; Shaikh & Shukla 2008, 2009). Stawicki et al (2001) argue that Alfvén fluctuations are suppressed by proton cyclotron damping at intermediate wavenumbers so the observed power spectra are likely to consist of weakly damped magnetosonic and/or whistler waves which are dispersive unlike Alfvén waves. Moreover, turbulent fluctuations corresponding to the high frequency and $k\rho_i \gg 1$ regime (where $k$ is wavenumber, and $\rho_i$ is ion gyroradius) lead to a decoupling of electron motion from that of ion such that the latter becomes unmagnetized and can be treated as an immobile neutralizing background fluid. While whistler waves typically survive in the higher frequency (and the corresponding smaller length scales) part of the solar wind plasma spectrum, their role in influencing the inertial range turbulent spectral cascades is still debated (Biskamp et al, 1996; Shaikh & Zank, 2003, 2005; Shaikh & Shukla, 2009, 2008).

Biskamp et al (1999) performed two and three dimensional simulations of incompressible (density perturbations are ignored) electron MHD model to demonstrate that the energy spectrum follows a $k^{-5/3}$ law for $k\rho_e > 1$ and $k^{-7/3}$ for $k\rho_e < 1$. They further reported that the 3D spectral properties are similar to those in 2D. This was lately confirmed by Shaikh (2009) using 3D simulations. Cho & Lazarian (2004) performed 3D simulations of incompressible electron MHD model to study anisotropic scaling that relates the parallel and perpendicular wavenumbers through $k_i \sim k_\perp^{1/3}$. In a much detailed work, Cho & Lazarian (2009) examined the anisotropy in the electron MHD and showed that the high-order statistics in electron MHD admit a scaling that is similar to the She-Leveque scaling for incompressible hydrodynamic turbulence.

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While there exists considerable literature describing the anisotropic cascades in whistler turbulence, the role of whistler wave in the spectral transfer of energy is still debated (Biskamp et al, 1996; Shaikh & Zank, 2003, 2005; Shaikh & Shukla, 2009, 2008). For instance, the Kolmogorov like dimensional arguments indicate that propagation of whistlers in the presence of a mean or an external constant magnetic field may change the spectral index of the inertial range turbulent fluctuations from $k^{-7/3}$ to $k^{-2}$ (Biskamp et al, 1996). By contrast, the numerical simulations (Biskamp et al, 1996; Shaikh & Zank, 2003, 2005) suggest that whistler waves do not influence the spectral migration of turbulent energy in the inertial range despite strong wave activity and that the turbulent spectra corresponding to the electron fluid fluctuations in whistler turbulence continue to exhibit a Kolmogorov-like $k^{-7/3}$ spectrum.

What is not clear from these work is the qualitative role of whistler and the corresponding mode coupling interactions that mediate the inertial range turbulent spectra. Furthermore, much of the work described above (Biskamp et al, 1996, 1999; Shaikh & Zank, 2003, 2005; Shaikh & Shukla, 2009, 2008; Cho & Lazarian, 2004, 2009) ignore the effect of density perturbations on the whistler mode turbulence. It is unclear if density fluctuations in the electron fluid modify the turbulent cascade properties. Since density fluctuations are critically important in many space and laboratory plasma phenomena, their role in whistler turbulence needs to be investigated.

The central object of this paper is to explore the nonlinear turbulent processes mediated by whistler waves in the presence of density perturbations. We will investigate nonlinear turbulent fluctuations, based on nonlinear fluid simulations, in $\omega > \omega_{ci}$ regime where correlation length scales of turbulence are comparable to the electron inertial length scales. Understanding of whistler turbulence in the presence of density fluctuations is important in the context of solar wind plasma (Krafft & Volokitin, 2003; Saito et al, 2003; Stavicki et al, 2001; Gary et al, 2008; Ng et al, 2003; Vocks et al, 2005; Salem et al, 2007; Bhattacharjee et al, 1998), magnetic reconnection in the Earth's magnetosphere (Wei et al, 2007) to interstellar medium (Burman, 1975) and astrophysical plasmas (Roth, 2007) where characteristic fluctuations can typically be of several astronomical units. These are only a few of the numerous other studies. For more literature, the readers can refer to the simulation work by Biskamp et al (1996) and others including Shukla (1978), Shukla et al (2001), Galtier (2008), Urrutia et al (2008), Saito et al (2008), Bengt & Shukla (2008), Shaikh (2009), Cho & Lazarian (2004, 2009) and numerous references therein.

This paper is organized as follows. Section 2 describes nonlinear whistler wave model for finite density perturbation. The finite density perturbations are described in terms of plasma beta (ratio of magnetic energy and electron fluid pressure). Section 3 deals with nonlinear fluid simulations. Energy spectra are discussed in Section 4 and section 5 describes the effect of the density fluctuations in whistler turbulence. In section 6, we discuss spectral anisotropy in inhomogeneous whistler turbulence. Section 7 concludes our work.

2 WHISTLER WAVE MODEL

Fluctuations in a magnetized plasma excite whistler modes when they propagate along a mean or background magnetic field with characteristic frequency $\omega > \omega_{ci}$, and the length scales are $c/\omega_{pe} < \ell < c/\omega_{pe}$, where $\omega_{pe}$ is the plasma ion and electron frequencies. In such a high frequency regime, the ions do not have time to respond to the electron motions. Hence the electron dynamics plays a critical role in determining the nonlinear interactions while the ions merely provide a stationary neutralizing background against fast moving electrons and behave as scattering centers. The whistler turbulence can be described by the electron magnetohydrodynamics (EMHD) model of plasma (Kingsep et al, 1990) that deals with the single fluid description of quasi neutral plasma. The EMHD model has been discussed in considerable detail in earlier work (Kingsep et al, 1990; Biskamp et al, 1996, 1999; Dastgeer et al, 2000a,b; Shaikh & Zank, 2003, 2005; Cho & Lazarian, 2004, 2009). In whistler modes, the currents carried by the electron fluid are important, and we therefore write down only those equations which are pertinent to electron motion. These are electron fluid momentum, electric field, currents, and electron continuity equations,

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e V_e) = -n_e e \nabla \cdot \mathbf{J} - \frac{n_e e}{c} \mathbf{V}_e \times \mathbf{B}$$

$$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{E} \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = 0$$

Here $\mathbf{V} \cdot \mathbf{P} = \mathbf{P} + \nabla \cdot \mathbf{P}$, the sum of pressure and stress tensors. The pressure becomes highly anisotropic in presence of a strong background magnetic field, especially in the low beta solar wind plasma. Hence the total pressure consists of the isotropic ($\nabla \cdot \mathbf{P}$) and the anisotropic ($\nabla \cdot \mathbf{P}$) parts. The remaining equations are $B = \nabla \times \mathbf{A}$, $\mathbf{J} = -e n_e \mathbf{V}_e$, $\nabla \cdot \mathbf{B} = 0$. Here $n_e, V_e$ are the electron mass, density and fluid velocity respectively. $\mathbf{E}, \mathbf{B}$ respectively represent electric and magnetic fields and $\phi, \mathbf{A}$ are electrostatic and electromagnetic potentials. The remaining variables and constants are, the collisional dissipation $\eta$, the current due to electrons flow $\mathbf{J}$, and the velocity of light $c$. The displacement current in Ampere’s law Eq. (3) is ignored because $\omega_{pe}/\omega_{pe} < 1$. The plasma is assumed to be quasineutral, hence $n_e \approx n_i \approx n_m$. Density fluctuations are considered to be incompressible at the leading order, but a first order compressibility is included to describe finite electron plasma beta ($\beta$ is the ratio of plasma pressure and magnetic field energy) effects. We adopt the approach described by Abdalla et al (2001) to include the finite density perturbation effect in the whistler wave model that couples density with pressure perturbations. The density field is determined from Poisson’s equation and momentum equation as follows,

$$-4\pi n_e e = \nabla \cdot \mathbf{E} = -\frac{1}{c} \nabla \cdot (n_e \mathbf{V}_e \times \mathbf{B}) - \nabla \cdot \left( \frac{\nabla P}{n_e} \right)$$

where electron inertia and pressure contribution is negligibly small. Here perturbed density ($n_e$) is small compared
to the mean background density ($n_0$) such that $n_e/n_0 < 1$. Furthermore, lengthscale on which density is varying ($\chi_n$) is small compared to the characteristic length ($l$) i.e $\chi_n < 1$. The leading order electron fluid velocity can then be associated with the rotational magnetic field through

$$V_e = -\frac{e}{4\pi n e} \nabla \times B.$$  

(6)

Substitution of Eq. (6) into Eq. (5) with further simplification gives

$$n_e \approx \left(\frac{d_e \omega_{ec}}{\omega_{pe}}\right)^2 \hat{z} \cdot \nabla \frac{\nabla^2 B}{\nabla^2 B_0} + \nabla^2 \beta,$$

(7)

where $\hat{z}$ is the direction of the background magnetic field and $\beta = 4\gamma p/B_0^2$. The above relation is further consistent with $n_e/n_0 < 1$ since $\omega_{ee}/\omega_{pe} < 1$ and $B/B_0 < 1$ in the low beta whistler mode turbulence. This inequality may however not hold for the $kd \gg 1$ modes. In any event, the latter is inaccessible by fluid theory as kinetic effects begin to play a crucial role for the smallest scales. We nonetheless restrict ourselves to $kd > 1$ only where the characteristic length scales are marginally smaller than electron inertial length scales. On taking the curl of Eq. (1) and, after slight rearrangement of the terms, we obtain

$$\frac{\partial \Omega'}{\partial t} + \nabla \times (V_e \times \Omega') = \nabla \times \left(\left(\frac{\nabla \cdot P}{mn_e}\right) - \mu \nabla \times V_e\right),$$

(8)

where $\Omega' = \nabla \times \xi = d_e^2 \nabla^2 B - \xi$.

It can be seen from Eq. (8) that in the ideal whistler mode turbulence (i.e. neglecting the term associated with the damping $\mu$), the Curl of generalized electron momenta is frozen in the electron fluid velocity. This feature is strikingly similar to Alfvénic turbulence where the magnetic field is frozen in the ideal two fluid plasma (Biskamp, 2003). Using electron continuity equation Eq. (4) in combination with Eq. (8), we obtain

$$\left(\frac{\partial}{\partial t} + V_e \cdot \nabla\right) \frac{\Omega'}{n_e} = \left(\frac{\Omega'}{n_e} \cdot \nabla\right) V_e + \nabla \times \left(\frac{\nabla \cdot P}{mn_e}\right)$$

+ $\mu \nabla \times V_e,$

(9)

We next introduce normalized generalized vorticity $\Omega$ as follows.

$$\Omega = \frac{\Omega}{n_e \omega_{ec}} = \frac{B}{B_0} - d_e^2 \nabla^2 \frac{B}{B_0} - \left(\frac{n_e}{n_0} - 1\right) \frac{B}{B_0}.$$  

On substituting $\Omega'$ into Eq. (8) and using appropriate vector identities, we obtain the three-dimensional normalized equation of EMHD describing the evolution of the magnetic field fluctuations in whistler wave.

$$\frac{\partial \Omega}{\partial t} + V_e \cdot \nabla \Omega - \Omega \cdot \nabla V_e = \nabla \times \left(\frac{\nabla \cdot P}{mn_e}\right) + \mu d_e^2 \nabla^2 B.$$  

(10)

The length scales in Eq. (10) are normalized by the electron skin depth $d_e = e/\omega_{pe}$ i.e. the electron inertial length scale, the magnetic field by a typical amplitude $B_0$, and time by the corresponding electron gyro-frequency. In Eq. (10), the diffusion operator on the right hand side is raised to 2n. Here $n$ is an integer and can take $n = 1, 2, 3, \cdots$. The case $n = 1$ stands for normal diffusion, while $n = 2, 3, \cdots$ corresponds to hyper- and other higher order diffusion terms. Eq. (10) alongwith Eq. (7) form a complete set of three dimensionl compressible whistler wave model. For simulation purposes, we use two dimensional (2D) model by ignoring the variation in the z=direction and transforming the magnetic field as follows

$$B(x,y,t) = \hat{z} \times \nabla \psi(x,y,t) + \phi(x,y,t) \hat{z}.$$  

Such representation preserves $\nabla \cdot B = 0$ in 2D. Here $\psi$ and $\phi$ are respectively orthogonal and longitudinal flux functions of the perturbed magnetic field $B$. This representation transforms Eq. (10) into its parallel and perpendicular components (of the magnetic field) as follows Abdalla et al (2001),

$$\frac{\partial}{\partial t} \left(\psi - d_e^2 \nabla \psi\right) = \frac{\partial}{\partial t} + \left[\nabla \psi, \nabla \phi\right] - \left[\nabla^2 \psi, \nabla \phi\right] + \left[\beta, \nabla \phi\right],$$

(11)

$$\frac{\partial}{\partial t} \left(\phi - d_e^2 \nabla \phi\right) = -\left[\phi, \nabla \psi - d_e^2 \nabla^2 \psi\right] - \left[\beta, \nabla^2 \phi\right],$$

(12)

$$\frac{\partial \beta}{\partial t} = -\left[\phi, \beta\right],$$

(13)

$$n = \lambda^2 \nabla^2 (\phi + \beta),$$

(14)

where $\lambda = d_e \omega_{ec}/\omega_{pe}$.

The linearization of Eq. (10) about a constant magnetic field $B = B_0 \hat{z} + \xi$, where $B_0$ and $\xi$ are respectively constant and wave magnetic fields, yields the following equation,

$$\omega_k (1 + d_e^2 k^2) \hat{B} + \frac{4\pi n e}{CB_0} i k^2 \hat{k} \times \hat{B} = 0.$$  

(15)

On eliminating the wave perturbed magnetic field from the above relation, one obtains the following dispersion relation,

$$\omega_k = \omega_{ce} \frac{d_e^2 k^2}{1 + d_e^2 k^2},$$

(16)

where $\omega_{ce} = eB_0/m_e c, k^2 = k_x^2 + k_z^2$ and $k_z = k_B B_0$. The use of Eq. (16) in Eq. (6) leads to the following relation between the wave magnetic field and the velocity field,

$$B = \pm \frac{i}{k} \hat{k} \times \hat{B}.$$  

(17)

The rhs of Eq. (17), in combination with Eq. (6), corresponds essentially to the whistler wave perturbed velocity field. This equation indicates that whistler waves consist of transverse fluctuations in the magnetized space plasma and they are produced essentially by rotational magnetic field that leads essentially to the velocity field fluctuations. On replacing the rhs in Eq. (17) with the perturbed velocity field, it can be shown that the whistler modes obey equipartition between the magnetic and velocity field components as $k^2 |B|^2 \approx |V|^2$. The whistler wave activity can thus be quantified by how closely the characteristic modes obey the turbulent equipartition relation. In 2 and 3D cases, we have estimated this relationship respectively in Shaikh & Zank (2005) and Shaikh (2009) in the incompressible limit. Particularly interesting is the 2D case where Eq. (17) exhibits a linear relation between $\psi_k$ and $\phi_k$ as $k|\psi_k| = |\phi_k|$ (Shaikh & Zank 2005). It is further evident from Eq. (16) that there exists an intrinsic length scale $d_e$, the electron inertial skin depth, which divides the entire turbulent spectrum into two regions; namely short scale ($kd_e > 1$) and long scale ($kd_e < 1$) regimes. In the regime $kd_e < 1$, the linear frequency of whistlers is $\omega_k \sim k_B k$ and the waves are dispersive. Conversely, dispersion is weak in the other regime.
Figure 1.

Mode structures are shown in the low beta whistler turbulence simulations at a time when turbulence is fully saturated and is in the steady state. The effect of the background magnetic field is evident from these structures that are elongated along the $x$ direction. The simulation parameters are: Box size is $L_x \times L_y = 2\pi \times 2\pi$, numerical resolution is $N_x \times N_y = 512 \times 512$, electron skin depth is $d_e = 0.015 - 1.0$, magnitude of constant magnetic field is $B_0 = 0.5$, dissipation $\mu = 10^{-4}$, time step $dt = 10^{-4}$.

$k d_e > 1$ since $\omega_k \sim k_\parallel/k$ and hence the whistler wave packets interact more like the eddies of hydrodynamical fluids.

3 NONLINEAR WHISTLER TURBULENCE

We develop a two dimensional nonlinear fluid code to numerically simulate Eqs. (11) to (14) that describe low plasma beta whistler turbulence. The spatial discretization employs a pseudospectral algorithm (Gottlieb et al, 1977; Shaikh & Zank, 2006, 2007) based on a Fourier harmonic expansion of the bases for physical variables (i.e. the magnetic field, velocity), whereas the temporal integration uses a Runge Kutta (RK) 4th order method. The boundary conditions are periodic along the $x$ and $y$ directions in the local rectangular region of the solar wind plasma. The turbulent fluctuations are initialized by using a uniform isotropic random spectral distribution of Fourier modes concentrated in a smaller band of lower wavenumbers. While spectral amplitudes of the fluctuations are random for each Fourier coefficient, it follows a certain initial spectral distribution proportional to $k^{-\alpha}$, where $\alpha$ is an initial spectral index. The spectral distribution set up in this manner initializes random scale turbulent fluctuations. We note that a constant magnetic field is included along the $z$ direction (i.e. $\mathbf{B}_0 = B_0 \mathbf{\hat{z}}$) to accommodate the large scale (or the background solar wind) magnetic field.

The evolution of whistler fluid fluctuations are governed by the nonlinear mode coupling interaction processes. In the presence of a constant background magnetic field, turbulent fluctuations not only couple nonlinearly with each other, but they also propagate along the direction of the background magnetic field as small scale whistler wave packets. The interaction of whistler waves with turbulent fluctuations complicates the dynamical evolution. Additionally, by virtue of nonlinear interactions the larger eddies transfer their energy to smaller ones through a forward cascade. According to Kolmogorov (1941), the cascades of spectral energy occur purely amongst the neighboring Fourier modes (i.e. local interaction) until the energy in the smallest turbulent eddies is finally dissipated gradually due to the finite dissipation. This leads to a damping of small scale motions.
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Whistler turbulence in the large scale $kd_e < 1$ regime exhibits a Kolmogorov-like inertial range power spectrum close to $k^{-7/3}$.

By contrast, the large-scales and the inertial range turbulent fluctuations remain unaffected by direct dissipation of the smaller scales. Since there is no mechanism that drives turbulence at the larger scales in our model, the large-scale energy simply migrates towards the smaller scales by virtue of nonlinear cascades in the inertial range and is dissipated at the smallest turbulent length-scales. A snap shot of fluctuations in density, magnetic and pressure fields is shown in Fig. (1). Consistent with 2D turbulence, a dual cascade phenomenon is observed Kraichnan (1965). In this process, the perpendicular component ($\phi$) of the magnetic field cascades predominantly towards the smaller scales whereas the parallel component ($\psi$) exhibits large scales in its spectrum (see Fig 1a & 1b). By contrast, density and pressure fields comprise of smaller scales (Fig 1c & 1d) as they follow a forward cascade. Since the spectrum of vorticity fields ($\nabla^2 \phi$ and $\nabla^2 \psi$) is dominated by the large k modes, they contain smaller scales (Fig 1e & 1f).

4 ENERGY SPECTRA IN WHISTLER TURBULENCE

The spectral transfer of turbulent energy in the inertial range is determined by neighboring Fourier modes in whistler turbulence. We find from our simulations that the mode coupling interaction follows a Kolmogorov phenomenology (Kolmogorov, 1941; Iroshnikov, 1963; Kraichnan, 1965) that leads to Kolmogorov-like energy spectra. It is evident from Fig. (2) that whistler turbulence in the $kd_e < 1$ regime exhibits a Kolmogorov-like $k^{-7/3}$ spectrum. This inertial range turbulent spectrum, in the context of low beta whistlers, is further consistent with previous 2D work (Biskamp et al, 1996; Dastgeer et al, 2000a,b). Surprisingly, the density fluctuations do not modify the energy spectrum in the $kd_e < 1$ regime. It turns out from the whistler wave dispersion relation that the wave effects dominate in the large scale, i.e. $kd_e < 1$, regime where the inertial range turbulent spectrum depicts a Kolmogorov-like $k^{-7/3}$ spectrum in our simulations. Our previous work, on the other hand, showed that the turbulent fluctuations in the smaller scale ($kd_e > 1$) regime behave like non magnetic eddies of hydrodynamic fluid and yield a $k^{-5/3}$ spectrum (Shaikh 2009). The wave effect is weak, or negligibly small, in the latter. The observed whistler turbulence spectra in the $kd_e < 1$ regime in Fig. (2) can be followed from the Kolmogorov-like arguments (Kolmogorov, 1941; Iroshnikov, 1963; Kraichnan, 1965) that describe the inertial range spectral cascades. We elaborate on these arguments to explain our simulation results of Fig. (2) as follows.

In the low plasma-$\beta$ regime, the whistler turbulence model described by the set of Eqs. (11) to (14) admits the following energy conservation law (Abdalla et al, 2001).

$$E = \frac{1}{2} \int dxdy \left[ \dot\phi^2 + (\nabla \psi)^2 + d_e^2 (\nabla \phi)^2 + d_e^2 (\nabla^2 \psi)^2 + d_e \frac{\omega_e}{\omega_{pc}} \left( \beta^2 + (\nabla \phi)^2 \right) \right].$$  

(18)

It is noted that the contribution due to the pressure and density fluctuations does not modify the energy conservation relation in the limit $\omega_{te}/\omega_{pe} < 1$, $B/B_0 < 1$ and $\beta < 1$.

The $kd_e < 1$ regime comprises the dispersive whistler waves and the total energy is dominated by first two terms in Eq. (18) such that $E \simeq \dot\phi^2 + (\nabla \psi)^2$. Turbulent equipartition between the velocity and magnetic field fluctuations yields $\dot\phi \simeq k \psi$ (Biskamp et al, 1996; Dastgeer et al, 2000a,b). Owing thus to the turbulent equipartition, the total energy can be given as $E \simeq \dot\phi^2$ in the $kd_e < 1$ regime. The group velocity of whistler waves in the $kd_e < 1$ regime is $v_g \sim k/\omega$ $k/\omega \sim k \sim \ell^{-1}$. Assuming that the nonlinear transfer of energy in the inertial range is governed by the eddy interactions whose velocity is $V_e \simeq \tilde{e} \times \nabla \phi$. Applying Kolmogorov-like dimensional arguments (Kolmogorov, 1941), we obtain the $k$th Fourier component of the electron fluid velocity as $v_k \sim k \phi_k$. The convective time scales on which the eddies transfer energy in the inertial range can be estimated as $\tau_{nl} \sim 1/(ku_k) \sim 1/(k^2 \phi_k)$. The nonlinear energy cascade rates ($\varepsilon$) are computed as $\varepsilon \sim E_k/\tau_{nl} \sim k^2 \phi_k^3$. 

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On using the Kolmogorov phenomenology that the spectral transfer is local and depends only on the energy dissipation rates and modes (Kolmogorov, 1941; Iroshnikov, 1963), the energy spectrum can be given by $E_k \sim \varepsilon^2 k^5$. Upon substituting the energy dissipation rates, we estimate the spectral energy as $E_k \sim \varepsilon^2 k^{-7/3}$.

Thus, the energy spectrum $k^{-7/3}$ derived on the basis of Kolmogorv-like arguments (Kolmogorov, 1941) is further consistent with our simulations in Fig. (2).

5 EFFECT OF DENSITY FLUCTUATIONS

We find from our simulations that the contribution due to the pressure and density fluctuations does not modify the energy conservation relation in whistler turbulence. Hence the density fluctuations do not influence the whistler wave cascades in the inertial range. This result appears to be a bit surprising at first, but our understanding is that the density fluctuations are only weakly coupled with the wave magnetic field and hence they are slaved to the magnetic field. During the evolution, density fluctuations simply follow the magnetic field and exhibit a Kolmogorov-like energy spectrum. The density spectrum from our simulations is shown in Fig. (3) for the $kd_e < 1$ regime.

To quantitatively demonstrate that the density fluctuations couple only weakly with the whistler wave magnetic field, we develop a novel diagnostic to determine the contribution of density perturbation in the whistler waves. This originates essentially from the whistler wave relationship that relates the poloidal $\psi$ and axial $\phi$ components of the magnetic field flux functions and is described by the expression $k|\psi_k| = |\phi_k|$ (Dastgeer et al, 2000a,b; Shaikh & Zank, 2005). It should be further noted that this relation is valid for the incompressible whistler mode fluctuations. The effect of compressibility due to the density perturbations however enters through Eqs. (13) & (14). The whistler relationship is thus modified to include the density fluctuations and it reads as

$$k^2 |\psi_k|^2 = \frac{n_k}{\lambda^2 k^2} - \beta_k^2,$$

(19)

where the left hand side corresponds to the energy associated with the wave field, whereas the right hand side of the expression describes the energy associated with the density field. Based on the modified whistler relationship, we develop a parameter called as whistler parameter in the following.

$$\chi(t) = \frac{\sum_k k^2 |\psi_k|^2 - \frac{n_k}{\lambda^2 k^2} - \beta_k^2}{\sum_k k^2 |\psi_k|^2 + \frac{n_k}{\lambda^2 k^2} - \beta_k^2}.$$

(20)

The whistler parameter is a dimensionless quantity that determines the contribution of density fluctuations in the whistler waves over the entire 2D turbulent spectrum. The physical picture emerging from this parameter can be described as follows; During the evolution of whistler waves and the corresponding density field, if there is a significant amount of energy being transferred in the density fluctuations (i.e. comparable to the whistler wave energy), then the difference in the magnitude of the wave energy $k^2 |\psi_k|^2$ and density fluctuations $|n_k/\lambda^2 k^2 - \beta_k^2|$ will be minimal.

Therefore a normalized contribution, obtained by dividing the difference by the total energy in the wave and the density field i.e. $\sum_k \sum_{\psi_k} (k^2 |\psi_k|^2 + |n_k/\lambda^2 k^2 - \beta_k|^2)$, will be much smaller than unity. This essentially corresponds to a state where $\chi \ll 1$. Such a criterion further characterizes a state in which the whistler wave magnetic field is greatly influenced or contaminated by the density fluctuations. By contrast, the limit $\chi \rightarrow 1$ corresponds to a state in which density fluctuations contribute only weakly or negligibly small in the wave magnetic field.

We follow the evolution of $\chi$ in our simulations. The result is plotted in Fig. (4). It is evident from Fig. (4) that $\chi$ is increasing progressively and it is approaching unity i.e. $\chi \rightarrow 1$. This means that energy associated with the wave field dominates over the energy in the density fluctuations. A decreasing trend of $\chi$, on the other hand, would have corresponded to state in which the energy in the density field was dominated over the wave. Since our simulations indiate that $\chi \rightarrow 1$, we believe that the effect of density perturbations is rather weak on the evolution of whistler waves and hence energy cascade rates are not altered. It is because of this reason that we find a Kolmogorov-like $k^{-7/3}$ energy spectrum in our simulations, a result similar to the one obtained previously by Biskamp et al (1996, 1999); Shaikh & Zank (2005); Cho & Lazarian (2004).

Although Kolmogorov-like $k^{-7/3}$ energy spectrum has been reported previously (Biskamp et al, 1996, 1999; Shaikh & Zank, 2005; Cho & Lazarian, 2004), they ignored the effect of compressibility due to the density field in whistler turbulence. Our work described here is therefore different in a sense that we have included the density fluctuations in whistler turbulence and found that they do not influence the energy cascade processes in the inertial range spectrum. The inertial range turbulent spectrum in the $kd_e < 1$ regime thus exhibits a Kolmogorov-like $k^{-7/3}$ power law. It thus follows from Figs (3) & (4) that the density field is simply slaved to the magnetic field and leads to Kolmogorov-like spectrum in
the wave-dominated $kd_e < 1$ regime. Moreover, it should be noted from Fig. (4) that the evolution of whistler parameter is dominated entirely by the wave energy (over the density field) right from the outset. A primary reason of such dominance is ascribed to the form of Eq. (14) that relates density to the small scale density and potential fluctuations. This essentially means that the density field is dominated by the characteristic small scale fluctuations. Such small scale fluctuations tend to possess a weaker tendency to influence the evolution of whistler parameter ($kd_e < 1$ regime) where the large scale whistler waves govern the entire nonlinear physics. This description further leads to another plausible explanation with regard to the time scales. The large scale energy containing whistler modes evolve on slower time scales [c.f. Eq. (16)]. By contrast, the small scale density fluctuations evolve on faster timescales. Owing to this temporal disparity associated with the two, the density field does not spend enough time with the wave field to influence its dynamical evolution. Hence the former is substantially incapable of coupling with the wave field. This leads to the negligibly small contribution of the density fluctuations in the wave field. The inertial range cascade is therefore governed predominantly by the whistler mode interactions.

6 ANISOTROPIC COMPRESSIBLE WHISTLER TURBULENCE

We next quantify the degree of anisotropy mediated by the presence of large scale magnetic in the nonlinear 2D whistler turbulence. In 2D turbulence, the anisotropy in the $k_x - k_y$ plane is associated with the preferential transfer of spectral energy that empowers either of the $k_x$ and $k_y$ modes. The background magnetic field is considered along the $x$ direction in our simulations. We therefore expect asymmetry in the evolution of the $k_x$ and $k_y$ modes. The anisotropy in the initial isotropic turbulent spectrum is triggered essentially by the background large scale magnetic field that regulate turbulent fluctuations to nonlinearly migrate the spectral energy in a particular direction. To measure the degree of anisotropic cascades, we employ the following diagnostics to monitor the evolution of $k_\parallel$ mode in time. The $k_\parallel$ mode is determined by averaging over the entire turbulent spectrum that is weighted by $k_x$ which is aligned in the direction of the background magnetic field.

$$k_\parallel(t) = \frac{\sum_k |k_xQ(k,t)|^2}{\sum_k |Q(k,t)|^2}$$

(21)

Here $Q$ represents any of $\phi$, $\psi$, $\nabla^2\psi$ and $\nabla^2\phi$. Similarly, the evolution of $k_\perp$ mode across the is background magnetic field determined by the following relation.

$$k_\perp(t) = \frac{\sum_k |k_yQ(k,t)|^2}{\sum_k |Q(k,t)|^2}$$

(22)

It is clear from these expressions that the $k_x$ and $k_y$ modes exhibit isotropy when $k_x \approx k_y$. Any deviation from this equality leads to a spectral anisotropy. We follow the evolution of $k_1$ and $k_2$ modes in our simulations for long enough time. Our simulation results describing the evolution of $k_\parallel$ and $k_\perp$ modes are shown in Fig. (5). It is evident from Fig. (5) that the initial isotropic modes $k_x \approx k_y$ gradually evolve towards an highly anisotropic state in that spectral transfer preferentially occurs in the $k_\perp$ mode, while the same is suppressed in $k_\parallel$ mode. Consequently, the spectral transfer in $k_\perp$ mode dominates the evolution and the mode structures show elongated structures along the $x$-direction, see Fig. (1).

The evolution of $k_1$ and $k_2$ shows a significant disparity in the two modes by virtue of an external magnetic field. The evolution of $k_\perp$ and $k_\parallel$ is clearly different as the spectral cascade in the parallel wavenumbers is dramatically suppressed. The suppression is caused by the excitation of whistler waves, which act to weaken spectral transfer along the direction of propagation. This can be understood as follows; We assume that the spectral transfer, essentially mediated by propagating whistler waves in wavenumber space, can be described by a three wave interaction mechanism, for which the frequency and wavenumber resonance criteria are, respectively, expressed by

$$\pm \omega_3 = \omega_1 - \omega_2,$$

and

$$k_3 = k_1 + k_2.$$

The resonance conditions indicate that two whistler waves ($\omega_1, k_1$) and ($\omega_2, k_2$) mutually interact and give rise to the third wave ($\omega_3, k_3$). Such conditions could, in principle, hold for a set of infinite waves as the indices ‘1’ and ‘2’ are merely dummy indices. With the help of dispersion relation (say, in the $kd_e < 1$ regime) and using the wavenumber $k_3 = k_1 + k_2$, we can obtain

$$\frac{k_{1y}}{k_{2y}} = \frac{k_2 + k_3}{k_1 - k_3}.$$

Let us now suppose the Kolmogorov turbulence hypothesis holds for EMHD nonlinear interactions as well, viz, that spectral transfer in the wavenumber space is local and occurs...
efficiently amongst the most adjacent Fourier modes, i.e. for which $|k - k_1| \approx |k_1|$. It, then, implies

$$k_{2y} < k_{1y}$$

(23)

since $k_1 \approx k_2 \approx k_3$, thereby indicating that there is a very little cascading along the $\hat{y}$-direction i.e. the magnetic field direction. Thus, the parallel wavenumbers ($k_\parallel$) appear to be suppressed and the spectral cascade mainly occurs in the perpendicular wavenumbers ($k_\perp$). This, we suggest, explains the wavenumber disparity ($\langle k_\perp \rangle \neq \langle k_\parallel \rangle$) observed in our simulations [see Fig. (5)].

7 DISCUSSION AND CONCLUSIONS

In summary, we present the results of freely decaying whistler turbulence calculated from an EMHD computer model. A major emphasis is to understand the effect of density fluctuations on the inertial range cascades in the whistler wave dominated $kd_e < 1$ regime. In this regime, large scale whistler waves govern the energy cascade processes. Interestingly we find that despite strong density perturbations, their effect on the cascade dynamics is non-sequential. Hence they couple weakly with the large scale waves in the $kd_e < 1$ regime. We find from our simulations that the density fluctuations do not influence the inertial range turbulent spectra. Consequently, the turbulent fluctuations in the inertial range are described by Kolmogorov-like phenomenology. Thus consistent with the Kolmogorov-like dimensional argument, we find that turbulent spectra in the $kd_e < 1$ regime is described by $k^{-7/3}$. Our results are important particularly in understanding turbulent cascade corresponding to the high frequency ($\omega > \omega_e$) solar wind, space and astrophysical plasmas where characteristic fluctuations are comparable to the electron inertial skin depth. Our work might also be useful in describing why many space plasmas are described by the Kolmogorov-like energy spectra despite the presence of strong density fluctuations and/or compressible effects. We find that density fluctuations exist on smaller scales and have typically higher frequency associated with their evolution. Owing to these disparate length and time scales, they do not modify the large scale wave-dominated turbulent processes. It is this large scale dynamics in the $kd_e < 1$ regime that leads to the Kolmogorov-like $k^{-7/3}$ spectrum in our simulations.

Note that our simulations of inhomogeneous whistler turbulence pertains to decaying turbulence only. In principle, turbulence can be driven. It remains to be seen whether the energy cascade processes in the driven whistler turbulence are altered by the density fluctuations.

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