Effect of Zero Modes on the Bound-State Spectrum in Light-Cone Quantisation

ANNETTE S. MÜLLER
Sektion Physik, Ludwig–Maximilians–Universität München
D–80333 München

ALEXANDER C. KALLONIATIS
Institut für Theoretische Physik III, Universität Erlangen–Nürnberg
D–91058 Erlangen

HANS–CHRISTIAN PAULI
Max–Planck–Institut für Kernphysik
D–69029 Heidelberg

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Abstract

We study the role of bosonic zero modes in light-cone quantisation on the invariant mass spectrum for the simplified setting of two-dimensional SU(2) Yang-Mills theory coupled to massive scalar adjoint matter. Specifically, we use discretised light-cone quantisation where the momentum modes become discrete. Two types of zero momentum mode appear – constrained and dynamical zero modes. In fact only the latter type of modes turn out to mix with the Fock vacuum. Omission of the constrained modes leads to the dynamical zero modes being controlled by an infinite square-well potential. We find that taking into account the wavefunctions for these modes in the computation of the full bound state spectrum of the two dimensional theory leads to 21% shifts in the masses of the lowest lying states.
1 Introduction

It has always been clear that a ‘solution’ of quantum chromodynamics is constituted not just by numerical reproduction of the hadron spectrum to a certain accuracy, but also an accompanying intuitive picture of what are the essential degrees of freedom involved in the building up of hadrons. In other words, what are the ‘quasiparticles’? On the one hand, these quasiparticles must have something to do with the constituent quarks which are useful for, at the very least, classifying the hadron spectrum. On the other hand, they should emerge from some systematic treatment of the QCD Lagrangian. Finding adequate field theoretic and nonperturbative formalisms in which this can take place has been an ongoing problem for many years.

One must recognize that lattice gauge theory has taken some major steps in this direction in recent years: it has gone beyond mere ‘simulation’ of the hadronic spectrum (see for example, [1]) to quite refined calculations attempting to focus on specific ‘physical mechanisms’ related, for example, to confinement or symmetry breaking (as in, say, [2]). However, the focus on the putative ‘essential’ degrees of freedom is something that has to be imposed by the theoretician on the calculation after generation of a large number of arbitrary field configurations, rather than emerging naturally within the lattice gauge formalism. The complementary formalism of light-cone, or more properly, Dirac front-form [3], dynamics offers a framework in which the ‘quasiparticles’ naturally are central from the beginning [4, 5, 6, 7]. The reason for this is, by now well-known and even somewhat clichéd if not even completely correct, the so-called simplicity of the light-cone vacuum [4]. The essence of this simplicity is that the light-cone momentum operator is positive-definite in the absence of zero momentum modes and thus no states built out of Fock operators on the Fock vacuum can mix with this vacuum. In that case then, we have the ideal candidate for the vacuum on top of which quasiparticles – constituent quarks – can be built as elementary excitations.

However zero momentum modes of the basic fields in the Lagrangian are in principle present and cannot be ad hoc thrown away [8]. An open question which we will address in this paper is: can they have in fact any effect on the mass spectrum of a theory in the continuum limit?

The ‘zero mode problem’ of light-cone quantisation (see, for example, [1, 10]) is the problem of how to now introduce the zero momentum modes without simultaneously destroying the above pleasant picture. The formalism of discretised light-cone quantisation (DLCQ) developed originally in [3] ironically in the absence of zero modes, compactifies the light-cone ‘spatial’ direction \( x^- = (x^0 - x^1)/\sqrt{2} \) leading to discrete longitudinal momenta. It thus offers an elegant framework for studying this problem because the zero modes can be cleanly separated out. Despite the eminently unphysical nature of imposing periodic boundary conditions on points on a null-plane [11] – and the concomitant loss of some physics which can take place [12] – this approach has enabled some progress within various gauge theories (for a review, see [13]) and is finding application now in string theories and M-theory [14–15].
In DLCQ, some zero modes indeed appear in the problem as genuine degrees of freedom, with conjugate momenta \[16\]. These modes can indeed mix with the vacuum but because they do not have any transverse momentum dependence (see, for example, \[17\]) they do not obscure the quasiparticle interpretation of the field Fock modes. But they can carry other internal quantum numbers and thus are not so undesirable: by mixing between these modes and the Fock vacuum, the true vacuum can acquire nontrivial quantum numbers thus breaking a symmetry spontaneously. Moreover, this can happen without destroying the physical picture of quarks as quasiparticles. Other zero modes in DLCQ however have no conjugate momentum and are constrained \[8, 18\]. They are – in other words – dependent on the true dynamical degrees of freedom, including, in non-Abelian gauge theories, the dynamical zero modes. This dependence is given through constraint equations arising from the equations of motion which in general are highly nonlinear. Thus, though in DLCQ we have managed to preserve the simple physical picture sought at the outset, we have arrived at a mathematically difficult problem: solving these constraint equations. Pioneering work in this direction was done for two-dimensional scalar theory in \[19\].

The degree of difficulty of this problem in a gauge theory has warranted some of us studying its analogue in the simpler setting of two-dimensional SU(2) Yang-Mills theory coupled to scalar adjoint matter fields – originally studied by \[20\]. In this model, the problem remains significantly complicated. But the following insights have, to date, been attained through the works \[21, 22, 23\]. The dynamical zero modes are here — as is expected in fact for 3 + 1 dimensional gauge theories as well – strictly time dependent fields which are better expressed by Schrödinger wavefunctions rather than a Fock expansion. These wavefunctions are eigenfunctions of the zero mode projected Hamiltonian, the essential dynamics expressed in the effective potential for these variables. Neglecting the constrained zero mode contributions, the effective potential turns out to be an infinite square-well whose wavefunctions can be exactly determined \[21\]. Solving the constrained zero mode within certain approximations appears to generate a centrifugal barrier but itself of a height insufficient to significantly cause mixing of symmetric and antisymmetric states \[23\]. It is not yet clear if the small barrier height is a consequence of the severity of the approximations used in \[23\]. In any case, a square well potential is at least a good starting point for examining the more interesting question: if zero modes can ever in practice affect the invariant mass spectrum of bound states. This we carry out in this letter.

In the absence of zero modes the spectrum for two-dimensional gauge theory with scalar adjoint matter was computed in \[24\]. A similar spectrum is obtained by “freezing” the dynamical zero mode, as shown in \[24\]. The specific question we are interested in answering is whether inclusion of zero modes can lead to shifts in the invariant masses of the low energy spectrum computed in these two works. In the present paper, we provide numerical evidence that in fact this is the case. We show that, due to the dynamical zero mode, the lowest energy level in the
SU(2) theory is shifted upwards by twenty one percent after extrapolation to the continuum.

In the following we briefly review the formalism for two-dimensional, SU(2) Yang-Mills theory coupled to scalar matter. We then derive the invariant mass eigenvalue problem in the presence of the zero mode with a two particle truncation on the Hilbert space followed by the results for the low energy spectrum from the numerical solution of this problem. We summarise our result finally and comment on its possible significance.

2 Review of Two-Dimensional Model

The formalism of the model we shall consider has been extensively reviewed in the two previous papers \[21, 23\]. Because we wish to write down the Hamiltonian in as short a space as possible we shall here omit much of the detail of the quantisation and formalism. Our light-cone conventions will be those of \[25\]:

\[ x^\pm \equiv (x^0 \pm x^1)/\sqrt{2}. \]

The theory we consider is \((1+1)\) dimensional non-Abelian gauge theory covariantly coupled to massive scalar adjoint matter

\[ \mathcal{L} = \text{Tr} \left( -\frac{1}{2} F^{\alpha \beta} F_{\alpha \beta} + D^\alpha \Phi D_\alpha \Phi - \mu_0^2 \Phi^2 \right). \]  

This theory can be regarded as pure SU(2) Yang-Mills in \(2+1\) dimensions dimensionally reduced to \(1+1\). The field strength tensor and covariant derivative \(D_\alpha\) are respectively defined by

\[ D_\alpha = \partial_\alpha + ig[A_\alpha, \cdot] \] and \(F^{\alpha \beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha + ig[A^\alpha, A^\beta]\). As in \[21\] we represent field matrices in a colour helicity basis \(\Phi = \tau^3 \phi^3 + \tau^+ \phi^+ + \tau^- \phi^-\) with \(\tau^a = \sigma^a/2\). We use the light-cone Coulomb gauge \(\partial^- A^+ = 0\), which preserves the zero mode of \(A^+\). Then a single rotation in colour space suffices to diagonalise the SU(2) colour matrix \(A^+_3 = A^+_3 \tau_3\). The quantum mode \(A^+_3\) has a conjugate momentum \(p \equiv \delta L/\delta (A^+_3) = 2L\partial_+ A^+_3\) and satisfies the commutation relation \([A^+_3, p] = [A^+_3, 2L\partial_+) = i\). In the following it will be useful to invoke the dimensionless combination

\[ z \equiv \frac{gA^+_3 L}{\pi}. \]  

There are additional global symmetries which can be seen in terms of this mode. First, Gribov copies \[24, 27\] correspond to shifts \(z \rightarrow z + 2n, n \in Z\). Shifts \(z \rightarrow z + (2n + 1), n \in Z\) are ‘copies’ generated by the group of centre conjugations of SU(2). The finite interval \(0 < z < 1\) is called the fundamental modular domain, see for example \[28\]. Switching to the variable \(\zeta = (z - \frac{1}{2})\) enables realisation of the symmetry \(z \rightarrow -z\) and the composite symmetry \(z \rightarrow (1 - z)\) in a convenient way. After restriction to the fundamental domain, one is left with symmetry under reflection about \(z = \frac{1}{2}\) or \(\zeta = 0\). We shall work in Schrödinger representation for this quantum mechanical degree of freedom,

\[ \Psi_\tau[\zeta] = \langle \zeta | r \rangle. \]  

It is the effect of these wavefunctions in the computation of the full spectrum of the theory that we shall be concerned with.
The diagonal component of the hermitian scalar field $\Phi$ is $\varphi_3$. This is the field in which the constrained zero mode appears, studied in [23]. As mentioned in the introduction, we omit the consequences of the constrained mode on the spectrum, and hence drop it in the following. Then, the following field expansion can be given for this field

$$\varphi_3(x^-) = \frac{1}{\sqrt{4\pi}} \sum_{l=1}^{\infty} \left( a_l w_l e^{-il\pi x^-} + a_l^\dagger w_l e^{+il\pi x^-} \right)$$

where $w_l = 1/\sqrt{l}$ and the canonical commutation relations leading to Fock commutators $[a_l, a_{l'}^\dagger] = \delta_{l,l'} \ (l, l' > 0)$. Sometimes it will be convenient to write the Kronecker $\delta_{l,l'}$ as $\delta_{l'}^l$.

The off-diagonal components of $\Phi$ are complex fields with $\varphi_+ (x^-) = \varphi_+^\dagger (x^-)$. The momentum conjugate to $\varphi_-$ is $\pi_- = (\partial_- + igv)\varphi_+$. The other conjugate pair is obtained simply by hermitian conjugation. As in [21, 23] the field expansion can be made over half-integer momenta

$$\varphi_-(x) = \frac{e^{+im_0 \pi x^-}}{\sqrt{4\pi}} \sum_{m=\frac{1}{2}}^{\infty} \left( b_m u_m e^{-im\pi x^-} + d_m^\dagger v_m e^{+im\pi x^-} \right)$$

where $u_m(z) = 1/\sqrt{m + \zeta}$ and $v_m(z) = 1/\sqrt{m - \zeta}$. The objects $m_0$ and $\zeta$ are functions of $z$, defined by $m_0(z) = (\text{integer part of } z) - \frac{1}{2}$, $\zeta(z) = z - m_0(z)$. They satisfy the relations $m_0(z+1) = m_0(z) + 1$, $m_0(-z) = -m_0(z)$, $\zeta(z + 1) = \zeta(z)$, and $\zeta(-z) = -\zeta(z)$. The domain interval is now $-\frac{1}{2} < \zeta(z) < \frac{1}{2}$ for all values of $z$. For the fundamental domain $m_0 = -\frac{1}{2}$, but the specific choice no longer matters. The Fock modes then obey bosonic commutation relations $[b_n, b_{n'}^\dagger] = [d_n, d_{n'}^\dagger] = \delta_n^{n'}$ and all others zero. Finally one notes that a large gauge transformation $z \to z + 1$ produces only $m_0 \to m_0 + 1$ and thus only a change of the overall phase in Eq.(4). Most importantly the Fock vacuum defined with respect to $b_m$ and $d_m$ is invariant under these transformations.

The fields $A^-$, as usual in this type of gauge, are redundant variables and are solved by implementing Gauss’ law strongly. We do not give these equations explicitly here. What is important is that the zero mode colour diagonal component of the Gauss law must be imposed as a condition on physical states. After inserting the above field expansions this amounts to satisfying

$$\sum_{m=\frac{1}{2}}^{\infty} \left( b_m^\dagger b_m - d_m^\dagger d_m \right) |\text{phys}\rangle = 0 .$$

Thus physical states have equal numbers of “b” and “d” particles.

We can thus describe the complete Hilbert space in terms of states with the non-separable structure:

$$|\Psi_i\rangle = \sum_{r=0}^{\infty} \sum_{\nu=1}^{N} C_{r,\nu}^{(i)} \Psi_i |\nu\rangle$$

(7)
with $C_{r', \nu}^{(i)} \neq c_r^{(i)} c_{\nu}^{(i)}$, $\Psi_r$ the Schrödinger wave function for the zero mode and $|\nu\rangle$ a Fock space of an arbitrary number of $a$ modes but equal number of $b$ and $d$ modes.

The light-cone energy, $P^-$, and momentum, $P^+$, operators are obtained, as usual, from the energy-momentum tensor. The momentum operator turns out to be diagonal, $P^+ = \hat{K}$ with $\hat{K}$ the ‘harmonic resolution’,

$$\hat{K} = \sum_{n=1}^{\infty} a_n^\dagger a_n n + \sum_{m=\frac{1}{2}}^{\infty} ((m + \zeta) b_m^\dagger b_m + (m - \zeta) d_m^\dagger d_m).$$ (8)

The Hamiltonian $P^-$ is fully interacting and rather complicated. For the present it suffices to give its form schematically. In terms of the dimensionless operator $\hat{H}$ defined by $P^- = \frac{L}{\pi} \hat{H}$, we have

$$\hat{H} = -2\hat{g}^2 \frac{1}{\cos^2(\pi \zeta)} \frac{d}{d\zeta} \cos^2(\pi \zeta) \frac{d}{d\zeta} + H_F$$ (9)

where the first term is the kinetic term of the zero mode and the second term contains the Fock operator structure, as well as dependence on the zero mode $\zeta$ and the constrained zero mode. As said, the constrained mode we omit.

Our task is to solve for the low energy mass spectrum of this system by solving the eigenvalue problem

$$M^2 |\Psi\rangle = 2K \hat{H} |\Psi\rangle = 2K_i H_i |\Psi\rangle.$$ (10)

The eigenvalue $K$ of the harmonic resolution $\hat{K}$ is related to the size of the matrix system while $|\Psi\rangle$ are superpositions of states in the Hilbert space described above. The continuum limit is then taken by $K \to \infty$.

In the absence of the zero mode $\zeta$ this problem has been solved numerically now many times. We wish now to incorporate the zero mode into these calculations. The significant observation we make use of is that the vacuum of the theory, while no longer just the Fock vacuum, must still be a state of zero longitudinal momentum $K$ and can be picked out of the general Hilbert space state given above. Indeed it turns out to be the tensor product state $|\Omega\rangle \equiv \Psi_0[\zeta] \otimes |0\rangle$ (see also, [24]). The wavefunction $\Psi_0[\zeta]$ is an eigenstate of the quantum mechanical problem defined by the Hamiltonian

$$H_0 = -2\hat{g}^2 \frac{1}{\cos^2(\pi \zeta)} \frac{d}{d\zeta} \cos^2(\pi \zeta) \frac{d}{d\zeta} + \langle 0 | H_F | 0 \rangle.$$ (11)

The Fock vacuum expectation value of the Hamiltonian thus generates a potential governing the dynamics of the zero mode. As everywhere in this study, the constrained mode terms here are dropped. Formally $\langle 0 | H_F | 0 \rangle$ contains UV divergences which can be absorbed by a mass renormalisation

$$\frac{\mu^2}{4} = \frac{\mu_0^2}{4} + 2\hat{g}^2 \sum_{m=\frac{1}{2}}^{\Lambda} \frac{1}{m}$$ (12)
with $\Lambda$ a half-integer valued large momentum cutoff. The same renormalisation renders the Hamiltonian finite in the Fock sector, in the absence of the constrained zero mode. As shown in [21, 23] the renormalised potential is just an infinite square-well, leading to eigenmodes
\[
\sqrt{2} \sin[\pi (r + 1)(\zeta + \frac{1}{2})] \text{ with } r = 0, 1, 2, 3, \ldots. \]
As discussed in [16, 21], in the continuum limit the higher modes $r \neq 0$ become infinite in energy so that the ground state is actually the only relevant wavefunction. We are left with
\[
\Psi_0[\zeta] = \sqrt{2} \sin[\pi (\zeta + \frac{1}{2})] \quad (13)
\]
as the only relevant mode. We have thus, in an approximation that omits the constrained field contributions [23], solved the vacuum sector of the theory.

3 Eigenvalue Problem in Presence of Zero Mode

Formally, the problem we are confronted with now is the diagonalization of the following matrix
\[
2 \hat{K} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\zeta \Psi_0^\dagger(\zeta) \langle K; i | H_F | j; K \rangle \Psi_0(\zeta). \quad (14)
\]
Since we are interested in the low energy states at large harmonic resolution $K$, we can truncate the Fock space to two particle states which certainly in the absence of zero modes is a good approximation [20]. The same should hold also in the presence of the zero mode now as the mode does not change fundamentally the partonic structure of the theory expressed on the light-front.

As in [24], the two-particle sector contains the lowest state of neutral bound scalar field pairs. We are able to examine two classes of orthogonal states
\[
|aa\rangle_n \equiv a^\dagger_n a^\dagger_{-K-n}|0\rangle \quad n = 1, \ldots, \left\lfloor \frac{K}{2} \right\rfloor \quad \text{and}
\]
\[
|bd\rangle_m \equiv b^\dagger_m d^\dagger_{-K-m}|0\rangle \quad m = 1, \ldots, K - \frac{1}{2}. \quad (15)
\]
The notation $\left\lfloor \frac{K}{2} \right\rfloor$ means the highest integer which is smaller than or equal to the numeric value $\frac{K}{2}$. It is necessary in order to prevent double counting due to the identification $|aa\rangle_n = |aa\rangle_{K-n}$ because of the commutation relations, for all $n = 1, \ldots, K - 1$.

In the two particle sector we have then linear combinations of the $(\frac{K}{2} + K)$-basic states
\[
|\Psi_i\rangle = \sum_{n=1}^{\left\lfloor \frac{K}{2} \right\rfloor} C_n^{(i)} \Psi_0(\zeta) a^\dagger_n a^\dagger_{-K-n}|0\rangle + \sum_{n=1}^{K-\frac{1}{2}} C_m^{(i)} \Psi_0(\zeta) b^\dagger_m d^\dagger_{-K-m}|0\rangle. \quad (16)
\]
These states satisfy Gauss’ law and are eigenstates of the harmonic resolution operator, $\hat{K}|aa\rangle_n = K|aa\rangle_n$ and $\hat{K}|bd\rangle_m = K|bd\rangle_m$ for each $m, n$. 

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The functions $I_n, J_n, S_{aa}$ and $S_{bd}$ have been calculated in [30] and take the form

$$I_n(\zeta) = \mu^2 + \frac{g^2}{16\pi} \left( 16 + 16n \sum_{k=\frac{1}{2}}^{n-\frac{1}{2}} \frac{k^2 + \zeta^2}{(k^2 - \zeta^2)^2} + \zeta^2 \sum_{m=\frac{1}{2}}^{\infty} \frac{4}{m(m^2 - \zeta^2)} \right),$$

$$J_m(\zeta) = \mu^2 + \frac{g^2}{16\pi} \left( 16 + 8(m + \zeta) \left( \sum_{k=\frac{1}{2}}^{m-\frac{1}{2}} \frac{k^2 + \zeta^2}{(k^2 - \zeta^2)^2} + \sum_{k=1}^{m-\frac{1}{2}} \frac{1}{k^2} \right) + 4\frac{m + \zeta}{(m - \zeta)^2} \right) + \zeta^2 \sum_{n=\frac{1}{2}}^{\infty} \frac{2}{n(n^2 - \zeta^2)} + \zeta(m + \zeta) \sum_{k=m+1}^{\infty} \frac{16k}{(k^2 - \zeta^2)^2}$$

and

$$S_{aa}(n; \zeta) = \left( \sqrt{\frac{n_1}{n_3 + \zeta}} + \sqrt{\frac{n_3 + \zeta}{n_1}} \right) \left( \sqrt{\frac{n_2}{n_4 - \zeta}} - \sqrt{\frac{n_4 - \zeta}{n_2}} \right) \frac{(-2\bar{g}^2)}{(n_1 - n_3 + \zeta)^2},$$

$$S_{bd}(n; \zeta) = \left( \sqrt{\frac{n_1 + \zeta}{n_3 + \zeta}} + \sqrt{\frac{n_3 + \zeta}{n_1 + \zeta}} \right) \left( \sqrt{\frac{n_2 - \zeta}{n_4 - \zeta}} - \sqrt{\frac{n_4 - \zeta}{n_2 - \zeta}} \right) \frac{(-2\bar{g}^2)}{(n_1 - n_3)^2}$$

$$+ \left( \sqrt{\frac{n_2 - \zeta}{n_1 + \zeta}} - \sqrt{\frac{n_1 + \zeta}{n_2 - \zeta}} \right) \left( \frac{n_3 + \zeta}{n_4 - \zeta} - \frac{n_4 - \zeta}{n_3 + \zeta} \right) \frac{2\bar{g}^2}{(n_1 + n_2)^2}. \tag{24}$$

In these terms then, the matrix problem in the two particle sector can be written

$$2 \hat{K} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\zeta \left( n'\langle aa|\hat{H}|aa\rangle_n \ n'\langle aa|\hat{H}|bd\rangle_m \right) \sin^2[\pi(\zeta + \frac{1}{2})]. \tag{25}$$

In the ideal case, one could attempt to analytically carry out the $\zeta$ integrations in the expression of Eq. (21) and then extract, in the continuum limit, a modified bound state 't Hooft equation [31]. This has been, thus far, beyond our means. We are content in the following with a numerical analysis of the spectrum.
Figure 1: The square of the invariant mass $M_i^2$ in the two particle sector as a function of the harmonic resolution $K$. On the left hand side the spectrum with zero mode omitted and on the right the spectrum with the zero mode included.

4 Numerical Results

We diagonalize the matrix Eq.(25) numerically and compare the obtained spectra with the spectra calculated without zero modes in [24]. We stress that the spectrum without the zero modes has been computed again here independently of [24], and thus is also a control on our computation with the zero modes. To make contact with the original non-dimensionally reduced pure glue theory, we take the renormalised mass to be zero and normalise the coupling constant $\hat{g}^2 = 1$.

The result for the two spectra is shown in Fig. 1. We show the first eleven eigenvalues for values of the harmonic resolution between $K = 11$ and $K = 281$. There are certainly observable differences in the two spectra, though qualitatively they are both characterised by discrete levels whose spacing decreases with $M^2$. Let us focus now on the lowest state whose invariant mass is most accurately described in the two particle truncation. We are thus able to increase the harmonic resolution to quite high values for the purpose of making a reliable extrapolation to the continuum.

In Fig. 2 the ground state mass is shown as a function of $\frac{1}{K}$ for the two cases with and without the dynamical zero mode. We have gone up to $K = 1500$. The mass in both cases is seen to converge to a finite value.

We examine next the differences in the two values of the invariant mass-squared as a function
The ground state invariant mass squared including (curve above) and omitting (curve below) the dynamical zero mode. The invariant mass squared is shown as a function of $\frac{1}{K}$.

of $K$. This is shown in Fig. 3. We numerically extrapolate to the continuum limit by fitting the mass difference $\Delta M \equiv M_{\text{without}} - M_{\text{with}}$ to a power series in $\frac{1}{\ln K}$. The best fit turns out to be

$$\Delta M(K) = 1.47609 - \frac{3.36568}{\ln K} + \frac{5.20769}{(\ln K)^3} - \frac{3.34316}{(\ln K)^5}$$

and gives the continuous curve in Fig. 3. To see how significant the shift is against the mass excluding the zero mode, $\Delta M/M_{\text{without}}$, we also need the extrapolation

$$M_{\text{without}}(K) = 7.03883 - \frac{6.2887}{(\ln K)} + \frac{9.71745}{(\ln K)^3} - \frac{6.5194}{(\ln K)^5}.$$  

We thus obtain for $K \to \infty$

$$\frac{M_{\text{without}} - M_{\text{with}}}{M_{\text{without}}} = 0.20971.$$  

The extrapolation shows that the dynamical zero mode leads to a 21 percent shift upwards in the invariant mass of the lowest state with respect to the mass computed with zero modes suppressed.

5 Summary and Outlook

In this work the influence of the dynamical zero mode on the invariant mass spectrum of two-dimensional SU(2) Yang-Mills theory has been studied. Omission of the constrained zero mode
Figure 3: The difference $\Delta M = M_{\text{without}} - M_{\text{with}}$ of the lowest eigenvalue without and with the dynamical zero mode. The dots represent the value obtained numerically. The curve is the fit.

leads to the potential for the zero mode being an infinite square well. Taking the lowest eigenfunction for this quantum mechanical problem, we have been able to diagonalise the mass-squared operator in the two particle sector. In this truncation we expect the lowest state, if not the lowest few states, to be accurately computable. Extrapolating numerically to the continuum limit, we found a reduction in the mass of the lowest state by 21% due to the dynamics of the zero mode.

It may seem surprising that a single discrete momentum mode can have non-vanishing consequences in the continuum limit. However, it should be borne in mind that this zero momentum mode is nonetheless an infinite number of harmonic oscillator modes describing the lowest square-well eigenmode. Precisely how the Schrödinger wavefunction has conspired to cause this result is not immediately clear, though the obvious place is in the fact that the continuum bound-state ('t Hooft-like) equation becomes precisely sensitive to small momentum in the terms where a principal value prescription is usually invoked [31, 32, 33]. The most straightforward explanation of our numerical observation is then that the principal value prescription, once the zero mode wavefunctions are integrated over, undergoes modification.

Our result encourages further exploration within the present two dimensional Yang-Mills model now in the presence of the contributions of the constrained zero mode. As mentioned in the introduction, in [23] it was seen that this mode introduces a centrifugal barrier in the potential for the dynamical zero mode. Thus the appropriate quantum mechanical eigenfunctions are no longer those of the infinite square well, but rather the double-well oscillator. Even though the
size of the barrier was found to be small in [23], in light of the results of our work, it would not
be unreasonable to suspect further modifications in the final spectrum, particularly in the mass
of the lowest state of the spectrum. Within this model then, the shift in this lowest mass could
be determined as a function of the potential barrier height. At sufficient height, the vacuum
would break centre symmetry spontaneously [23] due to mixing of symmetric and antisymmetric
states in the double well. It would be interesting to determine the mass of the lowest state in the
spectrum – still a two-particle bound-state – at this critical value of the barrier height.

In a theory which includes fermions, alongside the features studied in [14], there will un-
doubtedly be an interplay between the specific ‘gluonic’ structures we have dealt with here and
states built out of quark Fock operators – namely mesons and baryons. Could this be a relevant
mechanism on the light-cone in a realistic gauge theory for giving mass to hadronic states that
would otherwise be thought of as massless? It is still far too early to tell.

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