Lorentz violation effects on astrophysical propagation of very high energy photons

Lijing Shao and Bo-Qiang Ma

1School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871
2Center for High Energy Physics, Peking University, Beijing 100871

Lorentz violation (LV) is predicted by some quantum gravity (QG) candidates, wherein the canonical energy-momentum dispersion relation, \( E^2 = p^2 + m^2 \), is modified. Consequently, new phenomena beyond the standard model are predicted. Especially, the presence of LV highly affects the propagation of astrophysical photons with very high energies from distant galaxies. In this paper, we review the updating theoretical and experimental results on this topic. We classify the effects into three categories: (i) time lags between photons with different energies; (ii) a cutoff of photon flux above the threshold energy of photon decay, \( \gamma \to e^+ + e^- \); (iii) new patterns in the spectra of multi-TeV photons and EeV photons, due to the absorption of background lights. As we can see, the details of LV effects on astrophysical photons depend heavily on the “phase space” of LV parameters. From observational aspects, available and upcoming instruments can study these phenomena hopefully, and shed light onto LV issues and QG theories. The most recent progresses and constraints on the ultra-high energy cosmic rays (UHECRs) are also discussed.

PACS numbers: 11.30.Cp, 04.60.-m, 13.85.Tp, 98.70.Sa

I. INTRODUCTION

The unification of standard model and general relativity is one of the most intriguing and desirous goals of modern physics, and it has stimulated many theoretical ideas towards quantum gravity (QG). Some of them predict Lorentz violation (LV), where the Lorentz symmetry of space-time breaks down at high energies, and it introduces some tiny “LV relics” at lower energies. This probability has arisen in the space-time foam [1–6], loop gravity [7–9], vacuum condensate of tensor fields [10, 11], and the so-called doubly special relativity (DSR; also known as the deformed special relativity) [12, 13].

LV is an important issue in QG theories, mainly because it provides some available scenarios to validate or falsify theories through “windows on QG” [14, 15]. Generally, we know that the quantum-gravitational effects are relevant only when the energy approaches the Planck energy, \( E_{Pl} \equiv \sqrt{\hbar c^5/G} \equiv 1.22 \times 10^{19} \text{ GeV} \).\(^1\) This huge number is obviously out of reach in the foreseeable future. However, at lower energies, the “relic” effects can demonstrate themselves and produce a modified energy-momentum dispersion relation with extra terms suppressed by \( \xi_n(E/E_{Pl})^n \). Here the power \( n \) and the coefficient \( \xi_n \) both depend on the specific theoretical framework under consideration [14, 15, 17].

To see the manifestations of the Lorentz-violating physics, novel insights are needed, and fortunately, some have already been proposed practically to probe tiny modifications to conventional physics. Accurate measurements in the laboratories on the earth [18, 19], violating processes in astrophysics with energetic particles [2, 20, 34], and accumulated effects through cosmological amplification mechanisms [2, 3, 6, 35–44], are three basic considerations till now to reveal new physics arisen in the LV scenarios.

\(^*\)Electronic address: mabq@pku.edu.cn

\(^1\) There are also arguments on a new energy scale rather than the Planck energy as the quantum gravity scale, see e.g., Ref. [16] and references therein.
Here we try to review and discuss some candidate theories on LV, and then utilize the modified energy-momentum dispersion relation to search observational clues in the photon sector (sometimes jointly, the electron and positron sectors). High energy photons originated from astrophysics are extremely promising to probe or falsify LV theories in various aspects [14, 15, 17]. This paper is organized as follows. In Sec. III several selected underlying models are briefly reviewed and discussed with the most recent observational constraints. In Sec. III we focus on photons from astrophysical sources. These photons can reveal LV effects through their time-of-flight with energy-dependent velocities, and their interactions with low energy background photons, namely the extra-galactic background light (EBL) and/or the 2.7 K cosmic microwave background (CMB) radiations. Multi-TeV \( \gamma \)-rays may also be involved in self-decays into \( e^+e^- \) pairs with proper LV parameters. In Sec. IV the most updating LV constraints from the spectra of ultra-high energy cosmic rays (UHECRs) are reviewed and discussed. Sec. V summaries the paper with discussions on several current and upcoming instruments related to LV physics.

II. SOME SELECTED THEORETICAL MODELS

In a rough manner, LV theories can mainly be classified into two categories. One is effective field theory (EFT), which provides an excellent framework where tiny LV effects are introduced into the Lagrangian through LV operators — renormalizable ones with mass dimension 3 and/or 4, see, e.g., the standard model extension (SME) [10, 11], and the further extended non-renormalizable ones with mass dimension 5 and/or 6 [45–50]. The other category includes those QG theories that cannot be embedded into the EFT framework, such as the quantum space-time foam model [1–6] and DSR [12, 13].

In the EFT framework, all renormalizable operators, which guarantee the gauge symmetry preserved, are introduced by Colladay and Kostelecky [10, 11] perturbatively and elegantly. The LV terms can be characterized by dimensionless tiny coefficients of order \( \mathcal{O}(10^{-23}) \) or even smaller. Lately, Myers and Pospelov [45] proposed dimension-5 non-renormalizable LV terms into the EFT Lagrangian. The dimension-5 CPT-odd terms in quantum electrodynamics (QED) read as [45, 46],

\[
\mathcal{L}_{\text{LV},-}^{5d,\text{QED}} = \frac{1}{E_{\text{Pl}}} \xi^{(5)} u^{\mu} u_{\mu} F_{\mu \nu} (u \cdot \partial) F^{\mu \nu} + \frac{1}{E_{\text{Pl}}} \bar{\psi}(\delta^{(5)} P_L + \delta^{(5)} P_R)(u \cdot \gamma)(u \cdot D)^2 \psi, \tag{1}
\]

while the dimension-5 CPT-even terms appear as [45, 46]

\[
\mathcal{L}_{\text{LV},+}^{5d,\text{QED}} = -\frac{1}{E_{\text{Pl}}} \bar{\psi} (u \cdot D)^2 (\zeta^{(5)} P_L + \zeta^{(5)} P_R) \psi . \tag{2}
\]

In above Lagrangians, \( u \) is a fixed timelike four-vector indicating the preferred frame, which is usually chosen as the frame where the CMB radiation is isotropic; \( \tilde{F} \) is the dual of \( F \), defined as \( \tilde{F}^{\mu \nu} = \epsilon^{\mu \nu \rho \lambda} F_{\rho \lambda}/2 \); \( P_{R,L} = (1 \pm \gamma^5)/2 \) are projection operators, and \( D_\mu = \partial_\mu + ieA_\mu \) is the QED covariant derivative; \( \xi, \delta_{R,L}, \) and \( \zeta_{R,L} \) are dimensionless LV coefficients naturally expected to be of order \( \mathcal{O}(1) \). Lately, the dimension-6 CPT-even terms are also introduced, by Mattingly [46],

\[
\mathcal{L}_{\text{LV},+}^{6d,\text{QED}} = -\frac{1}{2E_{\text{Pl}}^2} \xi^{(6)} u^{\mu} u_{\mu} F_{\mu \nu} (u \cdot D)^2 F^{\mu \nu} - \frac{i}{E_{\text{Pl}}} \bar{\psi} (u \cdot D)^3 (u \cdot \gamma) (\zeta^{(6)} P_L + \zeta^{(6)} P_R) \psi - \frac{i}{E_{\text{Pl}}} \bar{\psi} (u \cdot D) \Box (u \cdot \gamma) (\tilde{\zeta}^{(6)} P_L + \tilde{\zeta}^{(6)} P_R) \psi . \tag{3}
\]

We can see that in the non-renormalizable EFT, the LV terms are spontaneously suppressed by \( (E/E_{\text{Pl}})^2 \) or \( (E/E_{\text{Pl}})^7 \), compared to the renormalizable LV terms with tiny coefficients of order \( \mathcal{O}(10^{-23}) \) put in by hand [10, 11, 51]. However,
there remains a “naturalness problem” in the non-renormalizable EFT — radiative corrections may generate “baby” renormalizable terms which subsequently become even more important than the original “parent” non-renormalizable operators. Luckily, supersymmetry can play a role to prevent or alleviate this problem.

From the above Lagrangians, we finally arrive at the modified energy-momentum dispersion relations for photons and fermions, respectively,

\[ \omega^2 = k^2 + \xi_{\pm} \frac{k^{n-2}}{E_{Pl}^{n-4}}, \quad (4) \]

\[ E^2 = p^2 + m^2 + \eta_{\pm} \frac{p^{n-2}}{E_{Pl}^{n-4}}, \quad (5) \]

where \( m \) is the rest mass of fermions, and \( \eta_{\pm} \) are combinations of \( \delta_{R,L} \) and/or \( \xi_{R,L} \), where their subscripts “±” denote two opposite helicities. In the EFT, \( \xi_{\pm} = (-)^n \cdot \xi_{\pm}, \) and \( \eta_{\pm} \cdot \{ e^+ \} = (-)^n \cdot \eta_{\pm} \cdot \{ e^- \} \). However, \( \eta_+ \) is not necessary to equal to \( \eta_- \) in general. For more details on non-renormalizable EFT calculations and related issues, see e.g., Refs. \[ 15, 46–49 \].

From aspects of observational limits, unfortunately, the dimension-5 operators seem unnatural, even with supersymmetry included and softly broken \[ 54, 55 \]. The linear energy dependence in EFT is largely unfavored \[ 14, 15, 17 \]. The most strict observations come from the vacuum birefringence and synchrotron radiations \[ 14, 15, 17 \]. So we would like to fix \( n = 6 \) in the following analysis, and impose \( \eta_+ = \eta_- \) for simplicity (denoted as \( \eta \) hereafter; Liberati and Maccione also treated EFT in the same way in Ref. \[ 15 \]). This choice preserves CPT symmetry and helicity symmetry, though they are not the essential ingredients generally.

There are also several models where the local description in terms of effective Lagrangian breaks down. For instance, Ellis et al. proposed a Liouville-inspired stringy analogue of space-time foam model \[ 1–6 \]. In this model, a gas of D-particles roams in the bulk space-time of a high-dimension cosmology, wherein our universe is represented as a D3-brane. Photons are represented by open strings. Their interaction with D-particles excites and stretches the strings, and then decays and emits photons. Thus “gravitational medium effects” modify the canonical dispersion relation of photons to include an LV term which depends linearly on the energies of photons, \( \omega^2 = k^2[1 + \xi(k/E_{Pl})] \), while the dispersion relation for charged particles remains untouched and no helicity dependence is predicted \[ 1, 9 \]. Hence, it avoids many tight constraints from the vacuum birefringence of photons, the electron/positron sector, and the UHECRs sector \[ 33, 34, 56 \]. Most recently, Ref. \[ 49 \] claimed that the space-time foam model is unable to explain simultaneously the time lags of TeV \( \gamma \)-rays, and the non-observation of high energy photons above EeV. However, Ellis, Mavromatos, and Nanopoulos argued that there are ways to avoid the constraints \[ 57 \], hence the issue is still in dispute. On the other hand, the time-of-flight of photons from distant \( \gamma \)-ray bursts (GRBs) and active galactic nuclei (AGNs) strikingly favors the linearly energy dependent LV corrections \[ 3, 4, 44 \]. Therefore, it still has strong motivations and advantages to study this linearly dependent LV theory.

The above discussed LV theories are far from complete. Especially, DSR \[ 12, 13 \] is a well-motivated candidate where two invariant scales are included, namely the velocity of low energy photons, i.e., the conventional light speed \( c \), and a length scale \( l_{Pl} \equiv (G \hbar/c^3) \sim 1.61 \times 10^{-35} \) m (or equivalently, an energy scale \( E_{Pl} \)). In the DSR, the energy-momentum conservation laws may also be modified \[ 13 \], thus brings further complications to the analysis of LV phenomena. In this paper, the canonical energy-momentum conservation is assumed.

For a phenomenologically uniform description, we use the following dispersion relations, for photons and fermions, respectively,

\[ \omega^2 = k^2 \left[ 1 + \xi_n \left( \frac{k}{E_{Pl}} \right)^n \right], \quad (6) \]

\[ E^2 = m^2 + p^2 \left[ 1 + \eta_n \left( \frac{p}{E_{Pl}} \right)^n \right], \quad (7) \]
where only $n = 1$ (the linear modification) and $n = 2$ (the quadratic modification) are currently observationally relevant. More specifically, we should keep in mind that the case with $\xi_1 \neq 0$, $\eta_1 = \eta_2 = \xi_2 = 0$ corresponds to the space-time foam model, while the case with $\xi_1 = \eta_1 = 0$, $\xi_2 \neq 0$, $\eta_2 \neq 0$ represents the dimension-6 EFT with CPT-even LV operators. In addition, the case with $\xi_1 = \eta_1 \neq 0$, $\xi_2 = \eta_2 = 0$ is also of great interests, for the reason that it can be a consequence of the geometrically small scale of space-time and the equivalence principle to all species of particles.

### III. Astrophysical Photons in Presence of LV

As the dispersion relations are modified, we can suspect extra LV impacts beyond the standard model. The tiny corrections can be magnified when the energy $E$ is large, and also maybe through cosmological amplification mechanisms. Hence when two magnification mechanisms are combined, the astrophysical photons with very high energies provide plentiful motivations to be regarded as an important source to investigate LV physics.

The dynamics of LV is poorly understood, hence we focus on the kinematics of photon propagation from distant galaxies towards us. The effect of LV on high energy photon propagation is threefold. Firstly, the velocity of photons, defined as $v = \partial \omega / \partial k$, is no longer a constant, according to Eq. (6). Consequently, photons with different energies spend slightly different time-of-flight to arrive at the earth. It was firstly recognized in Refs. [1, 2], and is discussed extensively by the scientific society with great passion recently on the Fermi, Magic, and HESS observations [39–42, 44]. Secondly, the modified dispersion relation may allow the strictly forbidden reactions in the Lorentz-symmetric theories to occur, such as photon decay, $\gamma \rightarrow e^- + e^+$. It results in a cutoff in the spectra of $\gamma$-rays above the decay threshold. Thirdly, high energy photons interact with low energy background photons (CMB and/or EBL) on the way of propagation, through the reaction $\gamma + \gamma_{\text{CMB/EBL}} \rightarrow e^+ + e^-$. LV effects can influence the threshold of this process, and lead to interesting phenomena [20–22, 24–26, 29].

We discuss the above three scenarios in the following. Worthy to stress that, since the Lorentz symmetry is broken in the LV theory, the transformation laws between two relatively moving frames are unknown, hence the laboratory system is the only proper frame to discuss relevant physics before we get access to proper transformation laws.

#### A. Lags from time-of-flight

Amelino-Camelia et al. [1, 2] first suggested using GRBs to test LV physics. Due to the large cosmological distance and the fine time structure of GRBs, tiny LV effects can be amplified into observable quantities. By taking into account cosmological expansion of the universe, the time lag induced by the LV modified dispersion relation between photons with high energies $E_h$, and those with low energies $E_l$, is [37],

$$\Delta t_{\text{LV}} = \frac{1 + n}{2H_0} \xi_n \left( \frac{E^n_l - E^n_h}{E^n_{\text{Pl}}} \right) \int_0^z \frac{(1 + z')^n d z'}{\sqrt{\Omega_\Lambda + \Omega_M (1 + z')^3}},$$

(8)

where $n = 1$ and $n = 2$ stand for linear and quadratic energy dependence; $H_0 \approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the Hubble expansion constant; $\Omega_\Lambda \approx 0.73$ is the vacuum energy density, and $\Omega_M \approx 0.27$ is the matter energy density in the current universe; $z$ is the redshift of the source.

Fermi LAT discovered that high energy photons have a tendency to arrive later relative to low energy ones [40, 41], which might present potential evidence for LV with $\xi_1 < 0$ for the space-time foam model or $\xi_2 < 0$ for the dimension-6 CPT-even EFT. However, the determination of time lag from observational data is highly nontrivial and affected by many facets, both artificially and instrumentally [41, 44]. The primary uncertainty comes from the unknown effects from source activities, mainly due to our imperfect knowledge of the radiation mechanism of GRBs. However, we can
separate the source effects if we can achieve a survey of GRBs at different redshifts [32, 36, 44]. The time lag induced by LV accumulates with propagation distance, as it is a gravitational medium effect. On the contrary, the intrinsic source induced lag is likely to be a distance-independent quantity, which can be approximated as a constant. The relation between the observed delay $\Delta t_{\text{obs}}$ and the intrinsic time lag $\Delta t_{\text{in}}$ is

$$\Delta t_{\text{obs}} = \Delta t_{\text{LV}} + \Delta t_{\text{in}}(1 + z).$$

(9)

In Ref. [44], we systematically studied this scenario of Fermi LAT GRBs, and got the quantum gravity scale $E_{\text{QG}} \sim 2 \times 10^{17}$ GeV for the linear energy dependence, which corresponds to $\xi_1 \sim -10^2$. Worthy to mention that, $\xi_n > 0$ represents superluminal photon propagation, while $\xi_n < 0$ corresponds to subluminal propagation.

The time-of-flight of high energy photons is the clearest test to search LV effects of the photon sector, hence provides the most convincing results. AGNs and pulsars are also utilized to study LV effects through time-of-flight analysis [14, 15, 16, 44]. The observational results that high energy photons arrive relatively later than low energy ones suggest a negative $\xi_n$, and current data largely favor linearly energy dependent LV effects [42, 44].

B. Photon decay: a cutoff in the $\gamma$-ray spectra

The alteration of the dispersion relations of photons and fermions might also lead to some peculiar reactions to occur, like the basic QED vertex $\gamma \rightarrow e^+ + e^-$, called photon decay, which is generally forbidden by the canonical energy-momentum conservation in the standard model.

Photon decay might be realistic in the framework of LV theories with proper LV parameters. We consider a very high energy photon of momentum $k$, which decays into an electron of momentum $xk$, $x \in [0,1]$, and a positron of momentum $(1-x)k$. Utilizing the “threshold theorem” [38], energy conservation leads to [22]

$$k \left[ 1 + \frac{\xi_n}{2} \left( \frac{k}{E_{\text{Pl}}} \right)^{tn} \right] = xk \left[ 1 + \frac{m^2}{2xk^2} + \frac{\eta_n}{2} \left( \frac{xk}{E_{\text{Pl}}} \right)^{n} \right] + \{x \leftrightarrow 1-x\},$$

(10)

where we only keep the leading corrections of Planck-scale terms, and the leading term in the series of $(m/k)^2$. The consistence of the truncation can be checked. After a few steps, the equation turns into [22]

$$\frac{m^2 E_{\text{Pl}}}{k^{n+2}} = x(1-x) \left[ \xi_n - \eta_n (1-x)^{n+1} + x^{n+1} \right].$$

(11)

To get the threshold to valid photon decay, is equivalent to minimize $k$ on the left-hand side, hence to maximize the right-hand side of Eq. (11) [22]. We consider three theoretical scenarios in the following.

(i) $\xi_1 \neq 0, \eta_1 = \eta_2 = \xi_2 = 0$

This is the LV parameter configuration for the space-time foam model [14, 15]. Under these circumstances, Eq. (11) reduces to $m^2 E_{\text{Pl}}/k^3 = [x(1-x)]\xi_1$. We can see clearly that $\xi_1 \rightarrow 0$ leads to $k \rightarrow +\infty$, which is the very case in the standard model where no photon decay is kinematically allowed. For $\xi_1 < 0$, there is no meaningful solution for the photon decay process. On the contrary, for $\xi_1 > 0$, any high energy photon with its momentum larger than $k_{\text{th}} = (4m^2 E_{\text{Pl}}/\xi_1)^{1/3} \approx 23.4 \xi_1^{-1/3}$ TeV would decay rapidly into an $e^+e^-$ pair [22], thus we are not supposed to observe photons with momentum larger than $23.4 \xi_1^{-1/3}$ TeV from astrophysical sources.

On the other hand, the observation of multi-TeV photons can cast a constraint on the LV parameter $\xi_1$ from the $\xi_1 > 0$ side [21]. If $\xi_1$ is of order $O(1)$, then the relevant energy lays within the currently observational capability. Actually, the observations of the 50 TeV and 80 TeV photons from the Crab Nebula [15] have already constrained $\xi_1$ to be less than $10^{-2}$ in this scenario. It should be noticed that, here the power dependence of $k_{\text{th}}$ on $\xi_1$, $-1/3$, differs from that of Ref. [21], $-1/2$, because there the energy dependence of LV in terms of the maximal attainable velocity $\xi_1$ is different from that of the space-time foam model. However, in the relevant energy range, the results are roughly the same.
FIG. 1: The “phase diagram” of photon decay in the dimension-6 CPT-even EFT with LV parameters $\xi_2$ and $\eta_2$ in arbitrary unit.

(ii) $\xi_1 = \eta_1 \neq 0, \xi_2 = \eta_2 = 0$

If the modification of the dispersion relation is due to pure Planck-scale geometry of space-time, and the equivalence principle applies to all species of particles, then the fermions (electrons and positrons) share the same LV parameters as the photons. And if the dispersion relation is modified to the first order, i.e., linearly energy dependent, then from Eq. (11), the energy-momentum conservation leads to

$$m^2E_{Pl}/k^4 = 2[x^2(1-x) \xi_1]_1.$$  

Thus again, for $\xi_1 = \eta_1 \leq 0$, no photon decay is allowed, and for $\xi_1 = \eta_1 > 0$, there exists a threshold for photons with momentum larger than

$$k_{th} = (8m^2E_{Pl}/\xi_1)^{1/3} \sim 29.4 \xi_1^{-1/3} \text{ TeV to decay.}$$

This is also within the energy range of observational practicability if the LV parameters are of order $O(1)$.

(iii) $\xi_1 = \eta_1 = 0, \xi_2 \neq 0, \eta_2 \neq 0$

In the above two cases, the threshold happens when $x = 1/2$, i.e., the electron and the positron in the final state have the same amount of momentum. However, this is not a general property in the LV photon decay phenomenon, as first pointed out by Jacobson, Liberati and Mattingly [22].

In the dimension-6 CPT-even EFT framework, LV terms of photons and fermions both depend quadratically on the energy. Now Eq. (11) is reduced to

$$m^2E_{Pl}^2/k^4 = [x(1-x)] \times [\xi_2 - \eta_2 ((1-x)^3 + x^3)],$$

where the right-hand side is quartic of $x$ if $\eta_2 \neq 0$, and the discussions to derive the threshold energy become more subtle. If $\eta_2 = 0$, then it is a similar case as the space-time foam model, where $\xi_2 \leq 0$ corresponds to no photon decay, while $\xi_2 > 0$ leads to a threshold at

$$k_{th} = (4m^2E_{Pl}^2/\xi_2)^{1/4} \sim 0.11 \xi_2^{-1/4} \text{ EeV.}$$

For the $\eta_2 \neq 0$ case, after introducing $z = (2x-1)^2, z \in [0, 1]$, the constraint becomes quadratic of $z$

$$m^2E_{Pl}^2/k^4 = \frac{3\eta_2}{16} \left( z - \frac{2\xi_2 + \eta_2}{3\eta_2} \right)^2 - \frac{1}{12\eta_2} (\xi_2 - \eta_2)^2. \quad (12)$$

The maximum of the right-hand side can happen at $z^a = 0$ (no asymmetry), $z^b = 1$ (the maximum asymmetry), or $z^c = (2\xi_2 + \eta_2)/3\eta_2$ (a mediate asymmetry), depending on the realistic values of LV parameters [22]. After a detailed calculation, we can get the “phase diagram” for photon decay in the dimension-6 CPT-even EFT, illustrated in Fig. 1 [22], where

$$k_{th}^a = \left( \frac{16m^2E_{Pl}^2}{4\xi_2 - \eta_2} \right)^{1/4} \approx 0.16 (4\xi_2 - \eta_2)^{-1/4} \text{ EeV}, \quad (13)$$
We can see that the physical process $\gamma \rightarrow e^+ + e^-$ heavily depends on the phase space of LV parameters. In the region where $z = z^0 = 0$ ($x = 1/2$ thereof), it appears the naturally expected configuration of final state, where the electron and the positron share the same amount of the initial momentum. However, in some other phase space, the asymmetric momentum configuration of final state may appear, namely that the electron and the positron gain different proportions of the initial momentum, satisfying $z = z^c = (2\xi_2 + \eta_2)/3\eta_2$ ($x = \pm \sqrt{(2\xi_2 + \eta_2)/3\eta_2}$). Besides, there is also a regime where no photon decays can ever happen.

From the above discussions, we can see that within different theories, there can be a cutoff in the $\gamma$-ray spectra above a threshold energy with LV parameters in some certain regions. The cutoff is predominant, instead of a suppression, because the time scale of the photon decay is quite short and the process takes place in no time compared to their time-of-flight. Thus, the photon decay feature is obvious in the photon spectra if it exists, and hence should be observed explicitly. The $n = 1$ case has the cutoff energy four orders higher than that of the $n = 2$ case, thus it is more challenging to practical observations. However, the observations concerning the quadratic LV modification, as well as the linear LV modification, are already emerging.

### C. Modifications from pair-production absorption: an enhancement in the $\gamma$-ray spectra

Another interesting phenomenon comes from the interaction of a high energy photon $\gamma$ with a low energy photon $\gamma_{CMB/EBL}$ through the pair-production reaction $\gamma + \gamma_{CMB/EBL} \rightarrow e^+ + e^-$. CMB is the relic radiation from the big bang which evolves in the expanding universe after the lights decouple with the matters. And EBL is the faint diffuse light of the sky, consisting of the historically accumulated flux of all extragalactic sources. It is relevant for the formation of stars and galaxies, and also the large-scale structure of the universe. Its wavelength varies from the radio band to the ultraviolet band. The observed spectra of high energy photons from distant galaxies are modified on the way of propagation, due to the attenuation by these low energy background “absorbers”.

The presence of LV will change the reaction threshold and modify the reaction patterns in an intriguing and physically meaningful manner. It can be extended to the ultra-high energy photons, as well as the UHECRs, without any further difficulties. This might be the most extensively investigated scenario among the LV-induced “windows on QG”. The LV effects depend on the energy of photons involved, and the magnitude and also the sign of the LV parameters, as well as the theoretical framework under considerations.

As we can see in the photon decay section, different parameter configurations lead to different LV physical phenomena. Since four particles are involved in the pair-production absorption, we naturally expect more subtle and complicated dependence on the phase space of LV parameters than the photon decay case. Jacobson et al. provided the most detailed discussions on this issue. Here we look into two possible scenarios which are the most relevant and well constrained from observational aspects: (i) the propagation of multi-TeV photons, (ii) the photo-meson production through the GZK mechanism and the subsequently pion-produced photons with energies above EeV.

We would work in the simple framework provided by Jacob and Piran, who studied the linearly LV suppressed scenarios in a clear manner. Assuming that the LV parameters of fermions vanish, then by utilizing the “threshold theorem”, we can arrive at the threshold of the low energy background lights to interact with an incidental photon with energy $\omega$.

\[
\epsilon_{th} = \frac{m^2}{\omega} - \frac{\xi_n}{4} \left( \frac{\omega}{E_{Pl}} \right)^n \omega.
\]

\[
\xi_n = 0 \quad \text{leads to the conventional case} \quad \epsilon_{th} = \frac{m^2}{\omega}, \quad \text{of course.}
\]

If the LV parameters of electrons and positrons are also...
introduced, the LV modified patterns are hardly further modified. It only introduces a numerical rescaling of order $O(1)$ for the second term in Eq. (15), by assuming the likely magnitude of the LV parameters of fermions compared to that of photons [26]. Thus we here ignore them for simplicity.

In the superluminal propagation case with $\xi_n > 0$, the photon decay process dominates, where it results in a cutoff, and no photons are observable above energy larger than the decay threshold. Thus we focus on the subluminal case with $\xi_n < 0$ here. It is easily seen that there is a minimum on the right hand side of Eq. (15). The minimum appears as the most crucial feature from the pair-production absorption within LV theories [26]. Firstly, it means that the background photons with energies lower than the minimum never interact with the high energy photons. Secondly, most importantly, LV predicts a “reemergence” of the high energy photon flux when the incidental photons have energies above the critical energy [26],

$$\omega_{cr} = \left[ \frac{4m^2 E_n^{en}}{(n + 1)\xi_n} \right]^{1/(n+2)}.$$

For the $n = 1$ case, $\omega_{cr} \simeq 18.5 (\xi_1)^{-1/3}$ TeV, while for the $n = 2$ case, $\omega_{cr} \simeq 84.8 (\xi_2)^{-1/4}$ PeV. The critical energies are of the same order as the threshold energies of photon decays, because the photon decay can be viewed as a high energy $\gamma$-ray photon interacting with a “low” energy one with infinite wavelength (therefore zero energy actually). Thus photon decay is a special case of the reaction $\gamma + \gamma_{CMB/EBL} \to e^+ + e^-$ with a vanishing $\gamma_{CMB/EBL}$.

Above the critical energy, with the energies of high energy photons increasing, the threshold of low energy photons to interact also increases. In the relevant energy range, the photon density of the background lights decreases with energy. Hence, there are less low energy photons to annihilate with high energy photons above the critical energy, therefore, a recovery of the high energy photon flux is expected in the LV theories above the critical point [26].

As for the photons with energies above EeV, Galaverni and Sigl considered the ultra-high energy photons from GZK-cutoff-induced pion decays [24, 25]. In their work, the LV parameters for the electron and the positron are also fully taken under considerations. For the non-LV case, these ultra-high energy photons would interact with the CMB photons and lead to a cutoff in the spectra (actually, a suppression here) above the pair-production threshold. And if LV-corrected terms exist in the dispersion relations, as shown above, the threshold will shift and there is a reemergence above some critical energy if $\xi_n > 0$. Hence, it is supposed to observe the “reemergent flux” through Auger, AGASA, and Yakutsk observations, according to numerical calculations [24, 25]. However, these observations only find upper limits for photon flux in the relevant energy range. The non-observation situation allows very tight constraints on LV parameters. Refs. [24, 25] reported that, because of the non-observation of photons above $10^{19}$ eV, $|\xi_1|$ is limited to be below $10^{-14}$, and $\xi_2 > -10^{-6}$. A recent work utilized the same scenario, intending to rule out the space-time foam model mentioned above, after taking into the energy loss of photons in the stochastic interactions with the D-branes [29].

### IV. CONSTRAINTS FROM RECENT UHECRS SPECTRA AND DISCUSSIONS

Due to the most violating processes and long baseline propagations in astrophysics, there has been accumulating significant constraints for LV parameters from the astrophysical society [56]. Aside from the above mentioned scenarios, other processes are also taken under considerations by many authors and have achieved good progresses on the LV issues, e.g., vacuum birefringence, vacuum Čerenkov radiation, helicity decay, photon splitting [14, 15, 17, 56].

The LV modifications are usually suppressed by the ratio of the particle energy to the Planck scale, hence the high energy particles have a merit to amplify these tiny effects. We here review some most recent limits from UHECRs, because they possess the highest energies we have ever observed, and can cast tight constraints on LV theories.

UHECRs are among the most studied phenomena of LV for historical and practical reasons. In a certain period of time, people doubted whether a “standard” GZK cutoff exists, and many models are proposed to account for the
“shift” or “disappearance” of the GZK cutoff. Among them, the LV explanation was a promising candidate.

Nowadays, since the GZK cutoff is found around the predicted point, it is used to constrain the LV parameters $^{27, 28, 31, 32, 33, 34, 43}$. Bi et al. got the difference of the maximal attainable velocity $^{51}$ of pions to photons, $\delta_\pi = -0.8^{+3.2}_{-0.5} \times 10^{-23}$ and $\delta_\pi = 0.0^{+1.0}_{-0.4} \times 10^{-23}$, by utilizing the HiRes monocular spectra and the Auger combined spectra, respectively, under the assumptions that the LV modifications for nucleons are negligible and that the dominant component of UHECRs is the proton $^{28}$. Stecker and Scully derived a best fit to the LV parameter of $3.0^{+1.5}_{-3.0} \times 10^{-23}$ from studies on the Auger spectra $^{43}$. The above two studies both refer to the conception of the center of mass system, which might not be a well-defined conception in presence of LV. Maccione et al. worked in the framework of EFT with dimension-5 and dimension-6 operators, and utilized the Auger data to establish two-sided bounds on the coefficients for the proton and the pion, whose LV scales are found to be well above the Planck energy $^{27}$.

Though various UHECRs spectra studies favor the standard GZK explanation, the LV scenarios are not ruled out thoroughly. Small derivations from Lorentz symmetry are still under serious considerations. However, the fact that the cutoff point together with the spectrum of UHECRs are consistent with the standard theoretical expectation favors the GZK explanation without LV. Some authors argued that, the compound of UHECRs is not well understood yet. Even for general knowledge, we believe that they are protons or heavy ions. Then UHECRs are composite entities instead of fundamental particles, and their underlying degrees of freedom are actually quarks and gluons. This situation may limit the constraints derived from UHECRs $^{23}$. Moreover, the GZK cutoff may also be due to the accelerating capability of the UHECRs host galaxies. Therefore, more works on UHECRs from observational and theoretical aspects are needed to address these related issues.

On the other hand, as mentioned, Refs. $^{24, 25}$ realized that UHECRs lead to photo-pion production when undergoing inelastic collisions with CMB photons, i.e., the process $\pi \rightarrow \gamma$ always accompanies with the GZK cutoff process $p + \gamma_{\text{CMB}} \rightarrow N + \pi$. After a research into the influence of LV terms in this scenario, they reached the conclusion that $|\xi_1|$ is limited to be below $10^{-14}$, and $\xi_2 > -10^{-6}$, in order to explain the non-observation of photons in the UHECRs flux above $10^{19}$ eV $^{24}$.

Ending this section, let us compare and discuss some LV scenarios. The lag caused by the time-of-flight $^{2, 39}$ is the cleanest test ever, and it introduces no interaction into the phenomenological analysis, and some primary studies are emerging with attempt to disentangle the unknown (thus controversial) source effects $^{35, 36, 44}$. Hence, it provides the most convincing evidence for LV effects on the photon sector. Time-of-flight analysis favors linear energy dependence within current observations. However, the results are contradictory with other constraints from kinematics of reactions, where the linearly corrected LV terms are constrained to be very unlikely $^{14, 15, 17, 56}$. Even the stringy space-time foam model, where no corrections for charged particles are predicted thus avoiding many constraints from charged species, is debatably unfavored by the non-observation of the ultra-high energy photon flux $^{24, 25, 29, 57}$. As for species other than photons, e.g., electrons, positrons, quarks, and gluons, they can enjoy their own individual LV parameters differing from those of photons in principle. In some theories, particles with different helicities can also possess unrelated LV parameters. Thus lots of LV parameters are involved totally, and detailed analysis is needed and fortunately is performing to constrain all these tiny LV terms $^{56}$. Since most reactions contain more than one species, combined analysis is severely in demand. As we can see from the photon decays in the dimension-6 CPT-even EFT, the LV physics generally depends highly, sometimes even nonlinearly, on the phase space of LV parameters. Many efforts are still focusing to combine constraints from different types of analysis to derive a robust result $^{14, 15, 17, 56}$. As for composite particles, the jump from fundamental degrees of freedom to the compounded entities, e.g., the protons and the heavy ions in the UHECRs, requires more careful arguments and treatments.
V. SUMMARY

Lorentz violation (LV) may arise from many quantum gravity (QG) models. And apart from its origin, it is apparently important for its own sake. Theoretically, LV physics can originate from stringy considerations, effective field theories (EFTs), doubly special relativity, loop gravity theories, and so on. Correspondingly, fruitful and intriguing phenomena emerge from LV corrections, and fortunately they are testable on several operating platforms from various aspects nowadays, e.g., the Fermi satellite and the Pierre Auger Collaboration. This issue becomes more and more attractive to the scientific society nowadays, because evidence and constraints from experiments and observations are appearing continuously and steadily reachable and reliable. The most promising standpoint of the testability comes from the modified energy-momentum dispersion relation, where extra LV terms are introduced through different theoretical scenarios, compared to the canonical energy-momentum relation, \( E^2 = p^2 + m^2 \).

In this paper, we extensively review the studies of the LV influence on the astrophysical photons with very high energies. They are classified into three categories: (i) the velocity of photons \( v = \partial \omega / \partial k \) now depends on the wavelength of the light, hence introduces a time lag between photons with different energies, which can be observed from \( \gamma \)-ray bursts (GRBs) and the flares of active galactic nuclei (AGNs); (ii) newfangled reactions that are strictly forbidden in the standard model due to the non-conservation of energy-momentum might happen within LV theories, and a cutoff of photon flux above the threshold energy of photon decay, \( \gamma \rightarrow e^+ + e^- \), is expected with proper LV parameters; (iii) LV also alters the threshold energy of normal reactions, hence the spectra of the very high energy photons can be renewed to provide physically intriguing phenomena, due to the modified patterns of the reaction \( \gamma + \gamma_{\text{CMB/EBL}} \rightarrow e^+ + e^- \), for multi-TeV photons and EeV photons through interacting with low energy background lights.

To search for the LV physics in above scenarios, there are several indispensable steps. Most importantly, more sensitive detectors should play a decisive role, since generally, the higher the energy, the more diluted the flux. Currently, the Auger Collaboration and the HESS Collaboration are playing significant importance in relevant fields. An “Auger North” array intending to behave better than the present southern hemisphere Auger array is also under consideration. An improved knowledge of the astrophysically intrinsic mechanisms is also essential in searching LV physics from astrophysical side, e.g., the details of the background lights, the un-attenuated spectra of sources, the acceleration and radiation mechanisms of UHECRs. Space-based satellites, Fermi and the incoming “Extreme Universe Space Observatory onboard Japanese Experiment Module” (JEM-EUSO) are promising missions to pinpoint astrophysical issues. As we can see, LV physics reveals itself at high energy bands more obviously, thus the techniques to detect higher energy cosmic rays, e.g., \( > 10^{20} \) eV, are also of great significance.

In summary, theoretically, we cannot predict the values of LV parameters in LV theories from the first principle most of the time though, fortunately, experimental and observational data are serving tight constraints from various aspects to falsify theories and/or to constrain the LV parameters. These observations have developed and are developing rapidly within new platforms nowadays. Until now, no explicit LV effects deviating from the special relativity are observed determinately, and many theoretical scenarios are ruling out. Meanwhile, new clues of LV physics are emerging rapidly from astrophysical observations and ground-based laboratory experiments. In the following dozens of years, the LV issue is expected to meet a more clear situation then.

Acknowledgments

We thank helpful conversations with Zhi Xiao. This work is partially supported by National Natural Science Foundation of China (Nos. 11005018, 10721063, 10975003, 11035003). It is also supported by Hui-Chun Chin and Tsung-Dao Lee Chinese Undergraduate Research Endowment (Chun-Tsung Endowment) at Peking University, and
by National Fund for Fostering Talents of Basic Science (Nos. J0630311, J0730316).
[40] A. A. Abdo et al. [Fermi LAT and Fermi GBM Collaborations], *Science* **323**, 1688 (2009).
[41] A. A. Abdo et al. [Fermi GBM/LAT Collaborations], *Nature* **462**, 331 (2009) [arXiv:0908.1832 [astro-ph.HE]].
[42] G. Amelino-Camelia and L. Smolin, *Phys. Rev. D* **80**, 084017 (2009) [arXiv:0906.3731 [astro-ph.HE]].
[43] F. W. Stecker and S. T. Scully, *New J. Phys.* **11**, 085003 (2009) [arXiv:0906.1735 [astro-ph.HE]].
[44] L. Shao, Z. Xiao and B. Q. Ma, *Astropart. Phys.* **33**, 312 (2010) [arXiv:0911.2276 [hep-ph]].
[45] R. C. Myers and M. Pospelov, *Phys. Rev. Lett.* **90**, 211601 (2003) [arXiv:hep-ph/0301124].
[46] D. Mattingly, [arXiv:0802.1561 [gr-qc]].
[47] V. A. Kostelecky and M. Mewes, *Phys. Rev. Lett.* **99**, 011601 (2007) [arXiv:astro-ph/0702379].
[48] V. A. Kostelecky and M. Mewes, *Astrophys. J. Lett.* **689**, L1 (2008) [arXiv:0809.2846 [astro-ph]].
[49] V. A. Kostelecky and M. Mewes, *Phys. Rev. D* **80**, 015020 (2009) [arXiv:0905.0031 [hep-ph]].
[50] J. Alfaro and L. F. Urrutia, *Phys. Rev. D* **81**, 025007 (2010) [arXiv:0912.3053 [hep-ph]].
[51] S. R. Coleman and S. L. Glashow, *Phys. Rev. D* **59**, 116008 (1999) [arXiv:hep-ph/9812418].
[52] J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, *Phys. Rev. Lett.* **93**, 191301 (2004) [arXiv:gr-qc/0403053].
[53] J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi and R. Soldati, *Int. J. Mod. Phys. A* **25**, 3271 (2010) [arXiv:0904.3557 [hep-th]].
[54] S. G. Nibbelink and M. Pospelov, *Phys. Rev. Lett.* **94**, 081601 (2005) [arXiv:hep-ph/0410217].
[55] P. A. Bolokhov, S. G. Nibbelink and M. Pospelov, *Phys. Rev. D* **72**, 015013 (2005) [arXiv:hep-ph/0505029].
[56] V. A. Kostelecky and N. Russell, [arXiv:0801.0287 [hep-ph]].
[57] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, [arXiv:1004.4167 [astro-ph.HE]].
[58] D. Mattingly, T. Jacobson and S. Liberati, *Phys. Rev. D* **67**, 124012 (2003) [arXiv:hep-ph/0211466].
[59] F. R. Klinkhamer and M. Schreck, *Phys. Rev. D* **78**, 085026 (2008) [arXiv:0809.3217 [hep-ph]].