Self–Interacting Dark Matter from the Hidden Heterotic–String Sector

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Abstract

It has been suggested recently that self–interacting dark matter fits better the observational characteristics of galaxy dynamics. We propose that the self–interacting dark matter is composed from the glueballs of the hidden sector non–Abelian gauge group, while the hidden matter states exist in vector–like representation and decouple from the light spectrum. It is shown that these glueballs are semi–stable with the life–time larger than the present age of the Universe, if their mass is 1 GeV or less. The constraint on their abundance today suggests that the energy was stored in the hidden sector soon after inflation. This imposes an upper limit on the reheating temperature. We further study the naturalness of this scenario in the context of the free–fermionic string models and point out a class of such models where the self–interacting dark matter from the hidden sector is indeed plausible.
1 Introduction

Substantial experimental evidence indicates that most of the mass in the universe is invisible. The determination of the nature of this dark matter is one of the important challenges confronting modern physics. The Cold Dark Matter (CDM) class of cosmological models provides good description for a wide variety of observational results, ranging from the early universe, probed via the microwave background fluctuations to present day observations of galaxies and large scales structure. Flat cosmological models with a mixture of baryonic matter, cold matter and vacuum energy, can account for almost all observations on scales \( \geq 1 \) Mpc. More recently, improved observations and numerical simulations have enabled comparison of CDM models to observations on galactic scales of \( \sim \) few kpc **[1]**. These studies reveal that the collisionless CDM scenarios, which predict halo density profiles that are singular at the center, are in apparent contradiction with observations, which indicate uniform density cores. This conflict prompted Spergel and Steinhardt **[2]** to propose that the cold dark matter is self–interacting with a large scattering cross section but negligible annihilation or dissipation. The key feature of this proposal is that the mean free path of the self–interacting dark matter candidate should be \( 1 \text{ kpc} < \lambda_{\text{free path}} < 1 \text{ Mpc}. \) The effect of the self–interaction would smooth out the central profiles and suppress the number of satellite galaxies, hence improving the agreement with observations **[2]**.

In view of these developments, it is prudent to explore possible particle dark matter candidates that possess the required properties. One can imagine that to devise a particle with these desired characteristics is by no means too difficult. It is therefore essential to examine whether a particle with the coveted virtues, can be motivated in a larger context. This is for example the situation in the case of the very well motivated Cold Dark Matter candidates, like the neutralino and the axion, which are motivated, respectively, by supersymmetry and the strong CP problem.

In this paper we therefore study self–interacting dark matter which is motivated from string theory. One particular class of string motivated dark matter candidates are the strongly interacting states, the uniton and the sexton, which were proposed in ref. **[3]**. However, in ref. **[4]** it was shown that such states would accumulate in the center of the sun and the earth and would subsequently annihilate into energetic neutrinos at an unacceptable rate. Moreover, even though their self-interaction is sufficiently strong and the scattering cross-section is of the order of hadron-hadron scattering cross-section, their masses are expected to be quite large, which translates into a low number density and very large \( \lambda_{\text{free path}} \). What about other strongly interacting candidates, such as gluino LSP scenario, advocated in Refs. **[5, 6, 7]**? If the gluino is heavier than 1 GeV, this scenario again could be excluded from the indirect searches of the dark matter via the flux of energetic neutrinos. It seems, however, that if the mass of gluino is really low, i.e. \( \leq 1 \) GeV, it will not produce sufficiently energetic neutrinos, the indirect constraints do not apply, and, as was argued in Ref.
direct searches are also not sensitive to gluino–containing hadrons, as they will be considerably slowed down before reaching an underground detector.

Here we study a different type of self–interacting dark matter candidate which comes from the hidden strongly-interacting sector of the theory, reminiscent to what was considered in Ref. 8 twenty years ago. The existence of such a hidden sector is a generic consequence of the string theory. In particular we examine the case that the lightest hidden sector state is a stable glueball of a non–Abelian hidden gauge group, which arise from the hidden string sector, and interacts with the Standard Model states only via hidden sector heavy matter states. As is frequently the case in semi–realistic heterotic–string derived models, the hidden sector matter states are charged with respect to horizontal $U(1)$ symmetries which are broken near the string scale, and under which also the Standard Model states are charged. Therefore, the lightest hidden sector glueball states are strongly interacting among themselves and are very weakly interacting with the Standard Model states, where interactions are mediated by higher dimensional operators, which are suppressed by inverse powers of the string scale. Generically, these hidden sector glueballs are metastable, with strong dependence of the lifetime on the condensation scale.

In view of recent years progress in string theory, one must address the issue of what is the appropriate string scale to use. We pursue the minimalist approach which assumes the big desert scenario as suggested by the Standard Model multiplet structure and grand unification. The relevant framework is that of the heterotic–string and the string scale is of the order $10^{16}$–$10^{17}$ GeV. We further assume that the Standard Model gauge couplings as well as those of the hidden sector unify near the string scale, and are of the order extracted by the standard extrapolation of the MSSM gauge couplings. We then examine the constraints that are imposed on the possible non–Abelian hidden gauge groups by the results indicated by Spergel and Steinhardt and by the requirement that the lightest stable hidden state constitute the dark matter. We find that this set of assumptions rather tightly constrain the possible hidden sectors that can produce the desired characteristics. We examine the possible emergence of stable hidden states with these virtues from heterotic–theory. We show that a class of three generation free fermionic heterotic string models can in fact produce a hidden sector with the desired properties. The basic features that are needed, and which are reproduced in this class of string models, is a non–Abelian hidden gauge group with a small gauge content and that the hidden matter appears in vector–like representations and can decouple at a sufficiently high scale.

2 Hidden glueballs dark matter

Spergel and Steinhardt propose that the dark matter particles should have a mean free path, $1\text{kpc} \leq \lambda_{\text{free path}} \leq 1\text{Mpc}$. For a particle with mass $m_x$ this implies
an elastic scattering cross section of

\[ \sigma_{XX} = 8.1 \cdot 10^{-25} \text{cm}^2 \left( \frac{m_x}{\text{GeV}} \right) \left( \frac{\lambda}{1 \text{ Mpc}} \right)^{-1} \]  

(2.1)

Assuming that the dark matter particle scatter through strong interactions similar to hadronic scattering, the cross section is approximately equal to the geometric cross section, \( \sigma \approx 4\pi a^2 \), where \( a \) is the scattering length. Assuming \( a \approx 100 m^{-1} \) Spergel and Steinhardt obtain the estimate

\[ m_x = 4 \left( \frac{\lambda}{1 \text{ Mpc}} \right)^{1/3} f^{2/3} \text{GeV} \]  

(2.2)

Here we study the possibility that the self–interacting dark matter (SIDM) comes from the hidden non–Abelian sector of the theory. The effective low–energy gauge symmetry is therefore that of the Standard Model plus an hidden gauge group, which can be \( SU(2), SU(3) \) or another. However, as we elaborate in section \[ \Box \] inspired from the realistic heterotic–string models, we assume that all the hidden matter fields appear in vector–like representations, and can therefore decouple from the light spectrum at a higher scale. In section \[ \Box \] we will show that, assuming unification of the Standard Model as well as the hidden sector couplings, strongly constrains the allowed possibilities. If the additional hidden groups are not Higgsed, then at some scale \( \Lambda_h \), because of asymptotic freedom, the hidden sector gauge coupling, \( g_h \), becomes large and the hidden gauge group will be in the confining regime. Necessarily it will develop a mass gap in the spectrum \( \sim O(\Lambda_h) \) and the lowest glueball–like state will be stable. These particles will be strongly interacting among themselves and “almost” non–interacting with the Standard Model particles.

The lightest hidden sector state is non–interacting with the Standard Model states up to higher dimensional operators which are generated at the radiative level. These lowest order terms are of the form

\[ \mathcal{L}_{\text{eff}} = C_6 \frac{(H \dagger H)(G^a_{\mu\nu}G^a_{\mu\nu})}{M_S^2} + \sum_{\text{SM}} C_8^i \frac{(G^a_{\mu\nu}G^a_{\mu\nu})(F_{\mu\nu}F_{\mu\nu})}{M_S^4} + \sum_{\text{SM}} \tilde{C}_8^i \frac{(G^a_{\mu\nu}G^a_{\mu\nu})(F_{\mu\nu}\tilde{F}_{\mu\nu})}{M_S^4} + \ldots \]  

(2.3)

Here \( G^a_{\mu\nu} \) is the field strength of a non-Abelian gauge group from the hidden sector, \( F_{\mu\nu} \) represents the field strength of the SM gauge groups (\( U(1), SU(2), \) or \( SU(3) \)). The summation runs over the SM gauge groups and ellipses stands for other possible operators. \( C_6, C_8^i, \) and \( \tilde{C}_8^i \) are loop coefficients; \( M_S \) is a heavy mass scale, associated with the decoupling of the heavy vector–like matter fields, which transform under the hidden gauge group. These fields are also charged under horizontal \( U(1) \) symmetries, under which also the Standard Model fields are charged. Note that the scale \( M_S \) does not necessarily coincide with the string scale, although in the simplest scenarios that we consider we will assume that that is the case. We remark that \( C_6 \) may have an
additional suppression which depends on the Yukawa couplings. Eq. (2.3) is given in a non-supersymmetric form, and we note that the supersymmetric generalization is straightforward.

It is now possible to make a rough estimate of the lifetime of the hidden glueball, which decay is induced by operators (2.3). Let us assume that the lightest glueball is a scalar. Then, the operator \( G^a_{\mu\nu}G^a_{\mu\nu} \) can annihilate an exotic glueball (exoglueball) with the efficiency

\[
< 0 | G^a_{\mu\nu} G^a_{\mu\nu} | \text{exoglueball} > = f \Lambda_h^3 \phi_h,
\]  

(2.4)

where \( \phi \) is the wave–function of the glueball and \( f \) is some dimensionless coupling, presumably of the order one. Thus, at \( \Lambda_h \) and below the effective Lagrangian for the interaction of the exotic glueball with the Standard Model states becomes

\[
\mathcal{L} = C_6 f \frac{\Lambda_h^3}{M_S^2} \phi (H^\dagger H) + \sum C_8 f \frac{\Lambda_h^3}{M_S^4} \phi (F_{\mu\nu} F_{\mu\nu})
\]  

(2.5)

We first estimate the lifetime expected from the dimension eight operator, involving SM SU(3) fields. This induces hadronic decays of exotic glueballs, with the probability per unit time given by

\[
\Gamma = \frac{2m_\phi^3}{\pi} \left( C_8 f \frac{\Lambda_h^3}{M_S^4} \right)^2 \sim \Lambda_h \left( \frac{\Lambda_h}{M_S} \right)^8,
\]  

(2.6)

where we omitted (likely small) numerical coefficients and used the fact that \( m_\phi \sim \Lambda_h \). Taking \( M_S \approx 10^{16} \text{GeV} \) and \( \Gamma < 1/\tau_{\text{universe}} = 1/(10^{40} \text{yr}) \), we obtain the following condition

\[
\Lambda_h < 3 \cdot 10^9 \text{GeV}
\]  

(2.7)

Thus, for a reasonable \( \Lambda_h \) the decay rate is smaller than the inverse lifetime of the universe.

We next turn to estimate the lifetime expected from the dimension six operator. The decay is mediated by the virtual Higgs particle, decaying into all possible channels. Motivated by Spergel and Steinhardt’s proposal, we consider \( m_\phi \) in the ballpark of 1 GeV, which means that the decay width is saturated by hadronic channels and the effective Lagrangian can be written in the following form:

\[
\mathcal{L}_{\text{eff}} = C_6 f \frac{\Lambda_h^3}{M_{Higgs}^2 M_S^3} \frac{3\alpha_3}{8\pi} (F^a_{\mu\nu} F^a_{\mu\nu}),
\]  

(2.8)

where \( F^a_{\mu\nu} \) now is the gluon SU(3) field strength. Using same formulae as before, we get

\[
\Gamma_\phi = (C_6 f)^2 \frac{2m_\phi^3}{\pi} \left( \frac{3\alpha_3}{8\pi} \right)^2 \left( \frac{\Lambda_h^3}{M_S^2 M_{Higgs}^2} \right)^2 \sim 10^{-2} \frac{\Lambda_h^9}{M_S^4 M_{Higgs}^4}
\]  

(2.9)

The condition \( \Gamma \cdot \tau_{\text{universe}} < 1 \) yields, \( \Lambda_h \leq 10^5 \text{GeV} \), which is well above the range for \( m_x \), required by self–interacting dark matter scenario.
Thus we conclude, that if the requirement of strongly-interacting dark matter is satisfied, and the condensation scale $\Lambda_h$ is around 1 GeV or less, the lifetime of exotic glueballs is much larger than the present age of the Universe, and thus these glueballs might constitute (or partially contribute to) the dark matter. Now we turn to analyzing possible scenarios which could give a required cosmological abundance of these glueballs close to $\Omega_\phi h^2 \sim 1$.

3 Estimate of primordial abundance

We start this section by noting that there are no “natural” reasons for the mass density of exotic glueballs to be of the order of the required dark matter density. In this respect the situation is quite different from the neutralino LSP dark matter, where a correct abundance follows from the weak-scale annihilation cross section. In the case of exotic glueballs the low-energy annihilation cross section into the SM particles is extremely low, exactly for the same reasons that the decay width of an individual glueball turns out to be so small. When the annihilation cross section is so small, there is always a “danger” of overproducing these particles, so that they overclose the Universe.

It is possible to show on general grounds that the hidden sector cannot be in thermal equilibrium with observable matter. In a deconfining phase the energy density of the exotic gluons scales as $1/R^4$ with the size of the Universe $R$, and after the phase transition it scales as $1/R^3$. Thus, if at some point in the history of the Universe, the exotic gluons were in thermal equilibrium with normal matter, they carried comparable energy density and the confining phase transition should have occurred around the same $R_{eq}$ when the normal radiation–domination $\rightarrow$ matter–domination phase transition occurs. That is, around $T_{\text{normal matter}} \sim 5$ eV and long after nucleosynthesis. It implies that during the nucleosynthesis there were $2(N_2^2 - 1)$ additional massless degrees of freedom, associated with exotic gluons. For $SU(2)$ hidden gauge group we therefore get additional 6 degrees of freedom, seemingly in disagreement with the nucleosynthesis constraints. If the deconfining–confining phase transition occurred before nucleosynthesis, the energy density stored in exotic matter would have been much larger than observable matter density today. Therefore, the exotic gluons could not have been in thermal equilibrium with ordinary matter. Consequently, we have to come up with some mechanism responsible for their dilution and we assume that the exotic gluons were produced after inflation with $T_{\text{Reating}} < M_s$.

In that case, several possibilities are open. Some portion of an inflaton could decay directly into the exotic gluons, a straightforward possibility, which requires a fine–tuning of the corresponding coupling. Another, more interesting, case is when the coupling of the inflaton to the exotic sector is nil, and the decay occurs predominantly into the observable sector. Due to the existence of small but non-vanishing couplings between the two sectors, Eq. (2.3), the high-energy scattering of visible sector particles would lead to the creation of hidden sector particles with the effi-
ciency $C_n^2 T_R^{2n+1}/M_S^{2n}$, where $n$ is 2 or 4, depending on the operator, and $T_R$ is the temperature of visible sector matter, which defines the energy density of the visible sector matter.

$$\rho_R = \frac{\pi^2}{30} g_s T_R^4$$  \hspace{1cm} (3.1)

Let us denote the exotic gluon radiation density by $\rho_g$ and write down a system of cosmological equations which would determine the evolution of the hot Universe:

$$H^2 = \frac{8\pi}{3M_{Pl}^2} (\rho_R + \rho_g)$$  \hspace{1cm} (3.2)

$$\frac{d\rho_R}{dt} = -4H\rho_R - \alpha C_n^2 \frac{T_R^{2n+5}}{M_S^{2n}}$$  \hspace{1cm} (3.3)

$$\frac{d\rho_g}{dt} = -4H\rho_g + \alpha C_n^2 \frac{T_R^{2n+5}}{M_S^{2n}}$$  \hspace{1cm} (3.4)

where $\alpha$ is some dimensionless constant, with no parametric dependence on any of the relevant scales in the problem. These equation can be considerably simplified because $\rho_g$ gives a negligibly small contribution to the expansion rate of the hot universe, and away from dynamical equilibrium the dilution of $\rho_R$ due to the “leakage” into the hidden sector can be safely neglected:

$$H^2 = \frac{8\pi}{3M_{Pl}^2} \rho_R$$  \hspace{1cm} (3.5)

$$\frac{d\rho_R}{dt} = -4H\rho_R$$  \hspace{1cm} (3.6)

$$\frac{d\rho_g}{dt} = -4H\rho_g + \alpha C_n^2 \frac{T_R^{2n+5}}{M_S^{2n}}$$  \hspace{1cm} (3.7)

These equations reduce the problem to finding the energy density $\rho_g$ as the function of “external” temperature $T_R$, $\rho_g(t(T_R))$. Using the relation

$$t = \frac{1}{2} H^{-1} = \frac{1}{2} \frac{1}{\sqrt{\frac{\pi^2}{30} g_s M_{Pl}}} \frac{M_{Pl}}{T_R^2}$$  \hspace{1cm} (3.8)

we obtain a simple differential equation for $\rho_g(T_R)$:

$$\frac{d\rho_g}{dt} = -T_R^3 \frac{d\rho_g}{dT_R} \sqrt{\frac{\pi^2}{30} g_s} =$$

$$= -\frac{\pi^2}{30} g_s \frac{T_R^2}{M_{Pl}} \rho_g + \alpha C_n^2 \frac{T_R^{2n+5}}{M_S^{2n}}$$  \hspace{1cm} (3.9)

Integrating this equation explicitly, and using the initial condition $\rho_g(T_{RH}) = 0$, we arrive at

$$\rho_g(T_R) \sim \frac{\alpha C_n^2}{\sqrt{\frac{\pi^2}{30} g_s M_{Pl}}} \frac{M_{Pl}}{M_S^{2n}} \frac{1}{2n - 1} (T_{RH}^{2n-1} - T_R^{2n-1}) T_R^4$$  \hspace{1cm} (3.10)
More accurate treatment would require some knowledge about reheating, as most of $\rho_g$ created at $T_R$ close to $T_{\text{RH}}$. As one would naturally expect, soon after the reheating, the ratio of two energy densities becomes proportional to the dimensionless combination of the Plank scale, reheating temperature and $M_S$:

$$\frac{\rho_g}{\rho_R} \sim \frac{M_{\text{Pl}} T_{\text{RH}}^{2n-1}}{M_S^{2n}}. \quad (3.11)$$

After quick rescattering and thermalization, the exotic gluons will acquire their own temperature $T_g$, which will be lower than $T_R$, because we explicitly assume that $\frac{M_{\text{Pl}} T_{\text{RH}}^{2n-1}}{M_S^{2n}} << 1$. After that $T_g^4$ and $T_R^4$ both scale as $1/R^4$, until $T_g$ cools down to $\Lambda_h$, where the confining phase transition occurs. From that moment the energy density, stored in exotic glueballs, scales as $1/R^3$. After $T_R \approx 5\text{eV} \equiv T_{\text{EQ}}$ both the normal matter and dark matter scale as $1/R^3$. This leads to the following final estimate of the abundance,

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{baryon}}} \approx \frac{T_R(\text{at } T_g = \Lambda_h) T_{\text{RH}}^{2n-1} M_{\text{Pl}}}{T_{\text{EQ}} M_S^{2n}} = \frac{\Lambda_h}{T_{\text{EQ}}} \left(\frac{C_n T_{\text{RH}}^{2n-1} M_{\text{Pl}}}{M_S^{2n}}\right)^{3/4}, \quad (3.12)$$

which should be of order $O(10)$. This relation can viewed as the condition on the reheating temperature, because $\Lambda_h$ is more or less fixed by the requirement (2.2) and can be further related to $M_S$ via the renormalization group flow. Assuming $\Lambda_h \sim 1\text{GeV}$, $T_{\text{EQ}} \sim 5\text{eV}$, and $M_S \approx 10^{16}\text{GeV}$, we can get $T_{\text{RH}} \sim 10^{-4} M_S$ for the $n = 2$ maximal strength operator, and $T_{\text{RH}} \sim 10^{-2} M_S$ for $n = 4$. This range for the reheating temperature is quite reasonable. Scenarios with lower values of $T_{\text{RH}}$ would require a direct coupling between inflaton and the hidden sector to ensure sufficient abundance of exotic glueballs. On the other hand, scenarios with larger $T_{\text{RH}}$ in units of $M_S$ are excluded as in this case the energy density stored in exotic glueballs would overclose the universe.

### 4 Renormalization Group Analysis

In the previous section we showed that, assuming $\Lambda_h \sim 1\text{GeV}$, the exotic glueballs can account for the missing mass, and be in agreement with nucleosynthesis constraints, provided that the reheating temperature is of the order $O(10^{12-14}\text{GeV})$. In this section we study the evolution of the gauge couplings from the unification scale to the low scale. In this respect we examine which gauge groups, and under what conditions, can produce the exotic glueballs at a scale of the order $O(1)\text{GeV}$, as suggested by the cosmological data.

As we discussed in the introduction, in order to make connection with realistic superstring models we assume the framework of the Supersymmetric Standard Model unification, which indicates that the unification scale is of the order $10^{16-17}\text{GeV}$. 

7
Consistently with this assumption, we also assume the framework of the heterotic string, and consequently that the observable and hidden gauge couplings unify at the unification scale. With these assumptions, we extrapolate the hidden sector gauge coupling from the high to low scales. We take the magnitude of the hidden sector gauge coupling at the unification scale to be of the order that is expected by extrapolation of the Supersymmetric Standard Model couplings from the low scale to the unification scale. We divide our analysis into two parts. We first assume that all the vector–like matter states decouple at a scale which is identical with the $M_S$ scale. Therefore below the $M_S$ scale only the gauge sector contributes to the evolution of the gauge couplings. In the subsequent part we assume that the vector–like matter states decouple at an intermediate scale $M_h$, which is below $M_S$. In the evolution of the couplings we assume a supersymmetric spectrum. In the next section we will examine the plausibility of obtaining the required particle content from realistic superstring models.

Assuming that there are no intermediate scales between $M_S$ and $\Lambda_h$, the scale at which the hidden $SU(n)$ gauge group becomes strongly interacting is given by,

$$\Lambda_h = M_S \exp \left( \frac{2\pi}{(\frac{1}{2}N_f - 3N_C)} \left(1 - \frac{\alpha_0}{\alpha_0}\right) \right),$$

(4.1)

where $N_c = n$ is the number of colors, $N_f$ is the number of spin 1/2 fundamental multiplets, and $\alpha_0$ is the value of gauge coupling at the scale $M_S$. Assuming that all matter states decouple at $M_S$ gives $N_f = 0$. Then for the lowest possibility with $N_C = 2$, and taking and $M_S \approx 10^{16}$ GeV, $\Lambda_h \sim 1$ GeV, we obtain the required initial value of the gauge coupling, $\alpha_0 \approx 1/36$. This value of the coupling is 1.5 times smaller than conventional value $\alpha_0 \approx 1/24$, suggested by the unification of coupling constants from the observable sector. If we take $1/24$ for the value of the coupling, the transition to the strong coupling regime for the hidden sector will occur at $\Lambda_h \sim 3 \cdot 10^{15}$ GeV, which is, clearly, too high a scale to satisfy the strongly–interacting dark matter criterion.

However, although this scenario is appealing in its simplicity, a likely outcome suggested by realistic string models is that the hidden matter states decouple at an intermediate energy scale which is slightly or several orders of magnitude below the string scale. We will discuss the relevant string framework in the subsequent section. Here we continue with our qualitative analysis. We therefore assume that the matter vector–like states decouple at an intermediate energy scale and study the conditions for obtaining $\Lambda_h \approx 1$ GeV.

With this assumptions the one–loop Renormalization Group Equation (RGE) for the hidden group gauge coupling is given by

$$\frac{1}{\alpha_h(\mu)} = \frac{1}{\alpha_0} - \frac{1}{2\pi} \left(\frac{1}{2}N_f - 3N_C\right) \ln \frac{M_h}{M_S} - \frac{1}{2\pi} (-3N_C) \ln \frac{\Lambda_h}{M_h},$$

(4.2)
Writing $M_h = 10^6 \text{GeV}$, and taking $\Lambda_h = 1 \text{GeV}$; $\alpha_h(\Lambda_h) = 1$; $\alpha_0 = 1/24$, we obtain a relation between $N_f$, $N_c$ and $h$,

$$\frac{1}{2} N_f (16 - h) = 48 N_C - \frac{46 \pi}{\ln 10}$$

(4.3)

First we see that, since $0 < h < 16$ for any $N_C$ there will be solutions to this equation, which depend on the number of flavors. This is of course true, as for any $N_C$ we can add a number of flavor multiplets that slow the evolution of the gauge couplings. For larger $N_C$ we, of course, have to add more flavor multiplets. The more constraining framework then has to be sought in the context of the realistic superstring models.

For $SU(2)$ from eq. (4.3) we see that if the intermediate scale $M_h$ is just an order of magnitude below the MSSM unification scale, we need approximately 66 flavors in order for $\Lambda_h$ to be of order $1 \text{GeV}$, 16 flavors if $M_h \approx 10^{12} \text{GeV}$ and 8 flavors if $M_h \approx 10^8 \text{GeV}$. For $SU(3)$ we need 162 flavors for $M_h \approx 10^{15} \text{GeV}$, 40 flavors for $M_h \approx 10^{12} \text{GeV}$, and 20 flavors for $M_h \approx 10^8 \text{GeV}$. The number of needed flavors, of course, grows rapidly with increasing $N_c$. In the next section we examine the feasibility of obtaining the needed spectrum in realistic string models. However, we can already infer that the desirable gauge group should have the smallest number of colors, i.e. $SU(2)$.

## 5 String origins

In the previous section we showed that in order to get a hidden sector which becomes strongly interacting at the GeV scale requires that the hidden gauge group has a small gauge content and the existence of the hidden vector–like matter states at an intermediate energy scale. In this section we examine whether these needed characteristics can be obtained from realistic heterotic string models. From a purely field theoretic point of view we can of course construct a model with any number of colors and flavors, and a potential that will generate the required scales. The resulting model may be rather contrived, but not impossible to conceive. The more constraining framework can therefore be sought in the context of string theory. The class of string models that are of most interest in this respect are those that can potentially reproduce the observed physics of the standard particle model. It is then of great interest to examine if such models can produce a hidden sector with the characteristics that we discussed in the previous sections.

Examples of semi–realistic string models were constructed in the orbifold and free fermionic formulations. The most realistic models constructed to date are the free fermionic models which utilize the NAHE∗ set of boundary condition basis vectors [10, 11]. These constructions naturally give rise to three generation models with the

*NAHE=pretty in Hebrew. The NAHE set was first employed by Nanopoulos, Antoniadis, Hagelin and Ellis in the construction of the flipped $SU(5)$ heterotic–string model [9]. Its vital role in the realistic free fermionic models has been emphasized in ref. [10].
The realistic free fermionic models are defined in terms of a set of boundary condition basis vectors for all the world–sheet fermions, and the one–loop GSO amplitudes [13]. The physical massless states in the Hilbert space are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The basis is constructed in two stages. The first stage consists of the NAHE set [10, 11], which is a set of five boundary condition basis vectors, \{1, S, b_1, b_2, b_3\}. The gauge group after the NAHE set is \(SO(10) \times SO(6)^3 \times E_8\) with \(N = 1\) space–time supersymmetry. The space–time vector bosons that generate the gauge group arise from the Neveu–Schwarz (NS) sector and from the sector \(\zeta \equiv 1 + b_1 + b_2 + b_3\). The NS sector produces the generators of \(SO(10) \times SO(6)^3 \times SO(16)\). The sector \(\zeta\) produces the spinorial \(128\) of \(SO(16)\) and completes the hidden gauge group to \(E_8\). The sectors \(b_1, b_2\) and \(b_3\) produce 48 spinorial \(16\)'s of \(SO(10)\), sixteen from each sector \(b_1, b_2\) and \(b_3\).

The second stage of the basis construction consist of adding three additional basis vectors to the NAHE set typically denoted by \{\(\alpha, \beta, \gamma\}\}. Three additional vectors are needed to reduce the number of generations to three, one from each sector \(b_1, b_2\) and \(b_3\). At the same time the additional boundary condition basis vectors break the gauge symmetries of the NAHE set. The \(SO(10)\) symmetry is broken to one of its subgroups. The flavor \(SO(6)^3\) symmetries are broken to product of \(U(1)\)'s, and the hidden \(E_8\) is broken to one of its subgroups. In addition to the spin one and two multiplets, the Neveu–Schwarz (NS) sector produces three pairs of electroweak doublets, \{\(h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}\), three pairs of \(SO(10)\) singlets with \(U(1)\) charges, \{\(\Phi_{12}, \Phi_{23}, \Phi_{13}, \bar{\Phi}_{12}, \bar{\Phi}_{23}, \bar{\Phi}_{13}\)\}, and three singlets of the entire four dimensional gauge group, \{\(\xi_1, \xi_2, \xi_3\)\}. This generic structure is common to a large number of three generation models which differ in their detailed phenomenological characteristics. The analysis of the models proceeds by analyzing the cubic level and higher order terms in the superpotential and by imposing that the string vacuum preserves \(N = 1\) space–time supersymmetry. By studying specific models in this fashion, it was demonstrated that these models can potentially reproduce the fermion mass spectrum, and also produce models with solely the MSSM spectrum in the observably charged sector.

The fact that the free fermionic models produce models that look tantalizingly

\[\text{standard } SO(10) \text{ embedding of the Standard Model spectrum}^{\dagger}.\]

Furthermore, one of the generic features of semi–realistic string vacua is the existence of numerous massless states beyond the MSSM spectrum, some of which carry fractional electric charge and hence must decouple from the low energy spectrum. Recently, and for the first time since the advent of string phenomenology, it was demonstrated [12] in the FNY free fermionic model [13, 14], that free fermionic models can also produce models with solely the MSSM states in the light spectrum.

It is interesting to note that among the perturbative heterotic–string orbifold models the free fermionic models are the only ones which have yielded three generations with the canonical \(SO(10)\) embedding.

\[\text{standard } SO(10) \text{ embedding of the Standard Model spectrum}^{\dagger}.\]
realistic renders the search for possible signatures beyond the observed spectrum much more appealing. In this paper we focus on the possibility of the strongly interacting dark matter.

The basis vectors $\{\alpha, \beta, \gamma\}$, which break the observable gauge group, also break the hidden $E_8$ gauge group to one of its subgroups. This is a necessary consequence of the perturbative string consistency conditions, \textit{i.e.} of modular invariance. The resulting hidden gauge groups which arise depend on the specific models and are quite varied. We will comment more on this below. In addition to the generic spectrum from the NS sector and the sectors $b_1$, $b_2$ and $b_3$, outlined above, the models typically also contain additional massless states, which arise from the basis vectors which extend the NAHE set. The three sectors $b_j + 2\gamma$ produce hidden matter states that fall into the $\mathbf{16}$ representation of the $SO(16)$ subgroup of the hidden $E_8$, decomposed under the final $E_8$. These states are $SO(10)$ singlets but are charged under the horizontal $U(1)$ symmetries. In addition, vectors that are combinations of the NAHE set basis vectors and of the basis vectors $\{\alpha, \beta, \gamma\}$, produce additional massless sectors which break the $SO(10)$ symmetry explicitly. Some of these states are Standard Model singlets and can therefore also remain in the light spectrum.

The discussion above summarizes the general structure of the realistic free fermionic models. Our next task is then to make a survey of several models and to examine whether there exist, if any, models that can produce the characteristics desired for the self–interacting dark matter. As we discussed in section 4 to obtain $\Lambda_h \sim 1\text{GeV}$ we need a non–Abelian gauge group with small gauge content and matter spectrum at an intermediate mass scale. The prerequisite from the string model perspective is that the hidden $E_8$ gauge group is broken to a sufficiently small factor. Most favorably the hidden sector should contain an $SU(2)$ or $SU(3)$ gauge groups.

The second condition is that there should be a sufficient number of hidden matter multiplets to slow down the evolution of the gauge coupling.

The revamped flipped $SU(5)$ model of ref. \cite{9} produces a hidden sector with $SO(10) \times SO(6)$ gauge group. The model contains 5 multiplets in the 10 vectorial representation of the hidden $SO(10)$; 5 multiplets in the 6 vectorial representation of the hidden $SU(4)$ and 5 multiplets in the $(1, 4) \oplus (1, \bar{4})$ representations. For $SO(10)$ we find that even if we assume that all the matter states remain massless, the theory becomes strongly interacting at $\Lambda_h \sim 5 \cdot 10^{12}\text{GeV}$ (taking $M_S = 10^{16}\text{GeV}$). For $SU(4)$ we have that taking all the spectrum to remain massless gives $b_{SU(4)_h} = -2$, which may result in a sufficiently low scale for the hidden $SU(4)$. However, in the revamped flipped $SU(5)$ model the 4 + 4 states carry fractional electric charge and therefore must either decouple or the theory must confine at a much higher scale \cite{10}.

We next turn to the $SO(6) \times SO(4)$ model of ref. \cite{17}. In this model the hidden gauge group is $SU(8)$ and there are 5 multiplets in the 8 + 8 representations. This again yields $\Lambda_h \sim 5 \cdot 10^{12}\text{GeV}$ (taking $M_S \sim 10^{16}\text{GeV}$, even if we assume that all of the matter states remain massless.

Next we turn to the case of the string Standard–like Models \cite{13, 11, 18}. These
models represent the most interesting possibilities for the strongly interacting dark matter for the following reason. As we discussed above it is desirable to have a hidden gauge sector that contains small group factors, like $SU(2)$ and $SU(3)$. In the string standard–like models the observable gauge group is broken by two subsequent basis vectors. The modular invariance constraints then impose that similarly the hidden gauge group in these models has to be broken by the same two basis vectors. This means that in the string standard–like models small hidden group factors can indeed naturally arise. The same argument also applies to the left–right symmetric models of ref. [19]. On the other hand in the $SU(5) \times U(1)$ or $SO(6) \times SO(4)$ type models, the observable $SO(10)$ gauge group is broken by a single basis vector. Consequently, the hidden gauge group in these models contains larger group factors.

Turning then to the string standard–like models we find that indeed $SU(2)$ and $SU(3)$ hidden group factors frequently arise. For example, in the model of ref. [18] the hidden gauge group is $SU(5) \times SU(3) \times U(1)^2$. There exist 8 multiplets in the 3+3 representations, producing $b_{SU(3)} = -1$. Of those five carry fractional electric charge and therefore must decouple at a high scale. If we take the scale $M_I$ at which all the hidden matter fields decouple as in section 4 then requiring $\Lambda_h \sim 1\text{GeV}$ imposes $M_h \sim 10^6\text{GeV}$ for the intermediate energy scale. If on the other hand we assume that the electrically neutral states remain light down to $\Lambda_h$ and that the fractionally charged states decouple at the scale $M_I$ then we find that $M_I \sim 10^{11}\text{GeV}$, which seems more reasonable. All in all this qualitative analysis suggests that models with a hidden $SU(3)$ group factor may have enough flexibility to allow a small $\Lambda_h$.

Next we turn to the model of ref. [13]. The hidden gauge group in this model is $SU(3) \times SU(2) \times SU(2) \times U(1)^4$. Each hidden $SU(2)$ gauge group contains 10 multiplets in the fundamental representation, which are all electrically neutral. If we assume that all the matter states decouple at an intermediate scale $M_I$, then imposing $\lambda_h \sim 1\text{GeV}$ we obtain $M_h \sim 10^{11}\text{GeV}$. This model therefore demonstrates that hidden gauge group with a low confining scale and with matter which decouples at a much higher scale may indeed arise in realistic string models.

6 Discussion

The proposal of the self–interacting dark matter is a new interesting development which can help to reconcile computer–simulated features of galactic substructures with observations. Whether the self-interacting dark matter represents a considerable improvement over a more conventional cold dark matter scenario is, of course, an open question, which we are not trying to address here. Nevertheless, it is interesting to explore a possibility of having self–interacting dark matter within a particle physics context.

In this paper we argued that a naturally self–interacting dark matter can arise from the hidden sector gauge group. If this gauge group is not Higgsed, it will enter a strongly–interacting regime at some scale $\Lambda_h$, hadronize and develop a mass gap.
A lightest particle, with the mass presumably of the order $\Lambda_h$, will be stable and will have an elastic cross section roughly on the order of $\Lambda_h^{-2}$. If this scale $\Lambda_h$ is 1 GeV or smaller, these glueballs may fit into self–interacting dark matter criterion, put forward by Spergel and Steinhardt. The connection between the hidden and visible sectors may occur due to exotic heavy matter states, charged under the SM and hidden gauge groups, which decouple at a very high scale $M_S$, presumably comparable to unification scale. As a result, this connection is mediated by dimension 8 operators (dimension 6, if the SM Higgs has couplings to the exotic matter states). This induces a decay of exotic glueballs, suppressed by the eighth (fourth) power of the heavy scale. Consequently, if the condition on the self–interacting dark matter is satisfied, this suppression makes the life time of exotic glueballs to be much larger than the present age of the Universe.

The extreme smallness of the coupling between the visible and hidden sector at low energies poses certain difficulties in explaining the cosmological abundance of exotic glueballs, close to a required value. Indeed, since there is no ways of diluting the energy stored in the hidden sector through the decay into the SM particles, we conclude that the visible and hidden sectors have never been in thermal equilibrium. Consequently, we have to assume that the energy was stored in the hidden sector soon after inflation, either due to a direct coupling of the inflaton into exotic gluons or through the annihilation of the visible sector particles into exotic gluons. The efficiency of a latter process is governed again by the same effective operators, which connect the visible and hidden sectors. This process puts an upper limit on the reheating temperature, $T_{RH} \leq 10^{-4}M_S$ for dimension 6 operators and $T_{RH} \leq 10^{-2}M_S$ for dimension 8.

Choosing a specific gauge group, we can connect the two scales, $M_S$ and $\Lambda_h$. If we insist on the unification of couplings at $M_S$, the condensation scale $\Lambda_h$ is many orders of magnitude larger than desirable value of 1 GeV even for the minimal gauge group $SU(2)$. This problem can be cured only if one assumes a number of matter–like thresholds at some intermediate scale which would reduce the initial value of the coupling constant.

The cold dark matter candidates, like the axion and the neutralino, are well motivated by theoretical considerations. Does the stable hidden glueball have a similar appealing theoretical motivation? We believe that the answer is indeed affirmative. The existence of the hidden sector is a natural consequence of string theory, the only prevalent theory that at present offers a viable framework for quantum gravity. Realistic string models that reproduce the general structure of the Standard Model spectrum and have the potential of explaining its detailed features, also produce a hidden sector with the general characteristics that we assumed in this paper. Namely, a non–Abelian hidden gauge group with matter states in vector–like representations. It is then most intriguing that the models that come the closest to being fully realistic also produce the hidden gauge group with small gauge content, as is required if the hidden gauge group is to confine at the hadronic scale. Further exploration to reveal
whether other classes of string models \cite{20} can produce a hidden sector with similar characteristics are of enormous interest. Finally, at the turn of the new millennium it seems that a burning question in physics is: What is the universe made of? The answer to this question, whether in the direction advocated in this paper or otherwise, will have profound implications on our basic understanding of fundamental physics.

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