Optimization design of two-layer Walker constellation for LEO navigation augmentation using a dynamic multi-objective differential evolutionary algorithm based on elite guidance

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Abstract
In recent years, low earth orbit navigation augmentation (LEO-NA) has attracted increasing attention and is expected to become a new addition to global navigation satellite systems (GNSSs). When solving complex constellation design problems, traditional optimization algorithms often fail to achieve satisfactory results and are sensitive to parameter settings. We propose a dynamic multi-objective differential evolutionary algorithm based on elite guidance (DMODE-EG). It can select the evolutionary strategy based on the evolutionary state reflected by elite individuals and dynamically modify evolution parameters. Moreover, to achieve more uniform global coverage, we construct a two-layer Walker constellation model for LEO-NA. Then, we use the DMODE-EG algorithm to solve the corresponding multi-objective optimization problem and obtain the optimal constellation parameters. With the augmentation of this LEO-NA constellation to the BeiDou-3 system, the average position dilution of precision (PDOP) values drop to 1.2–2.0 from 1.5–5.5, and the number of visible satellites increases from 8–10 to 13–18. By contrast, some realistic LEO constellations and constellations designed by other algorithms bring weaker improvements and cannot address the problem of high PDOP values in some regions. In addition, simulation results on standard test sets verify the excellent convergence and stability of the DMODE-EG algorithm.

Keywords LEO constellation · Navigation augmentation · Multi-objective optimization · Differential evolution algorithm

Abbreviations
BDS-3 Beidou-3 system
CR Crossover rate
DE Differential evolutionary
DMODE-EG Dynamic multi-objective differential evolutionary algorithm based on elite guidance
dMOPSO MOPSO based on decomposition
DOP Dilution of precision
DTLZ Benchmark MOP proposed by Deb, Thiele, Laumanns and Zitzler
GA Genetic algorithm
GD Generation distance
GEO Geostationary earth orbit
GNSS Global navigation satellite system
JADE J adaptive differential evolutionary
LEO Low earth orbit
LEO-NA Low earth orbit navigation augmentation
MEO Medium earth orbit
MODE Multi-objective differential evolutionary
MODE-RMO Multi-objective differential evolution with ranking-based mutation operator
MOP Multi-objective optimization problem
MOPSO Multi-objective particle swarm optimization
PDOP Position dilution of precision
PF Pareto front
PNT Positioning and timing
PPP Precise point positioning
RAAN Right ascension of ascending node
RM-MEDA Regularity model-based multi-objective estimation of distribution algorithm
SP Spacing
UF Unconstrained benchmark MOP
WFG Benchmark MOP proposed by walking fish group
ZDT Benchmark MOP proposed by Zitzler, Deb, and Thiele

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Introduction

The low earth orbit (LEO) satellites have attracted much attention in recent years. Though medium earth orbit (MEO) satellites achieve broader global coverage and establish inter-satellite links more easily, they suffer from problems such as susceptibility to interference and large signal attenuation. By contrast, the lower altitude enables LEO satellites to overcome these problems. Therefore, many companies have announced the launch and deployment of commercial LEO constellations to provide broadband communication services worldwide (Del Portillo et al. 2019). LEO satellites are not only of great significance for communication, but also have great potential value for navigation. The global navigation satellite system (GNSS) has been widely used in many fields such as navigation, positioning and timing (PNT) (Zidan et al. 2020). However, some application services are limited by shortcomings of GNSS, such as limited positioning accuracy, weak signal strength indoors and in remote areas, susceptibility to interference, and long convergence time of precise point positioning (PPP) (Borio 2010; Choy et al. 2017). The navigation augmentation of GNSS by LEO satellites is an effective way to ameliorate these problems. Studies have shown that LEO satellites can assist GNSS in significantly reducing the convergence time of PPP and improving the orbit determination precision (Ge et al. 2018; Zhu et al. 2004). In addition, LEO constellations can improve GNSS navigation performance through both information enhancement and signal enhancement. LEO navigation augmentation (LEO-NA) will be implemented in some LEO constellations, such as Iridium NEXT, Kepler and Hongyan. With a limited number of satellites, the constellation design becomes an important issue as it directly determines the performance and cost.

The objective of constellation design is to determine the configuration and orbital parameters. For multi-layer constellations, an additional determination of the number of satellites per layer is required when the total number of satellites is fixed. Early researchers used geometric resolution for constellation design. Walker (1971, 1984) analyzed the number of satellites required to achieve single and double coverage of the earth. Luders (1961) proposed a method to determine the minimum number of satellites required to provide continuous coverage of a latitudinally bounded zone. However, with the complexity of constellation configurations and mission objectives increasing, geometric analytical methods are no longer effective. Therefore, researchers began to transform the constellation design problem into a multi-objective optimization problem (MOP), and some optimization algorithms were adopted to solve this problem. Noer et al. (2020) designed the Indonesian regional navigation satellite system using the genetic algorithm (GA) with the objective of minimum mean geometric accuracy dilution. Kohani and Zong (2019) used GA to design a LEO triplet constellation (LEO) to provide regional coverage of the earth. Meng et al. (2009) studied a hybrid navigation constellation consisting of MEO and GEO satellites based on a multi-objective particle swarm algorithm. Wagner and Black (2020) used a multi-objective genetic algorithm to design the rideshare and heterogeneous constellation.

In fact, optimization algorithms have been used in various fields, including but not limited to constellation design. For example, Khishe et al. (2017, 2019) applied the improved biogeography-based optimization and the developed whale optimization algorithm to improve the performance of classification on the sonar datasets. Hu et al. (2021) used the chimp optimization algorithm to design an accurate DCNN model for detecting positive COVID-19 X-ray. Among these algorithms, the differential evolutionary (DE) (Storn and Price 1997) algorithm has excellent performance and is relatively simple to implement, requiring only two parameters to be set. Some traditional algorithms are significantly improved after replacing their original evolutionary operator with DE. Therefore, DE is widely used to solve both single-objective and multi-objective optimization problems. Reynoso-Meza et al. (2010) proposed a new multi-objective differential evolutionary (MODE) algorithm based on spherical pruning for the design of Laplace domain controllers. Chen et al. (2014) adopted the MODE algorithm for chemical process optimization. Liang et al. (2019) proposed a hybrid adaptive differential evolution algorithm to solve the flow shop scheduling problem.

In this study, we co-simulate the Beidou-3 system (BDS-3) with some realistic LEO constellations and find the potential to further improve global navigation performance. Therefore, we construct a two-layer Walker constellation model for LEO-NA. Then, after determining the decision variables and objective functions, we transform the design problem into a MOP. However, we find that traditional optimization algorithms often fail to achieve satisfactory results in solving this problem and are sensitive to parameter settings. Therefore, we propose a dynamic multi-objective differential evolutionary algorithm based on elite guidance (DMODE-EG), which is able to select the evolutionary strategy based on the evolutionary state reflected by elite individuals and dynamically adjust the parameters. Finally, we use the DMODE-EG algorithm to solve the MOP and thus obtain the optimal constellation parameters. For convenience, the background, some tables and figures are given in supplementary materials.
Multi-objective optimization for LEO-NA constellation

In order to transform a constellation design problem into a MOO, we first determine the optimization objectives based on the metrics that reflect the navigation performance and system costs. In addition, decision variables correspond to some configuration parameters of the LEO-NA constellation. Finally, the overall optimization model is described in detail.

Optimization objectives

First, since the positioning error is proportional to the dilution of precision (DOP), DOP is often used to measure the positioning accuracy of GNSS. DOP mainly depends on the geometric relationship between ground stations and visible satellites. And the position dilution of precision (PDOP) can reflect the 3D geometric relationship. Then, PDOP is used as the first optimization objective. Assuming that the weight coefficient matrix in the topocentric coordinate system is:

\[
H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}
\]

(1)

then PDOP can be calculated as:

\[
\text{PDOP} = \sqrt{h_{11}^2 + h_{22}^2 + h_{33}^2}
\]

(2)

where \(h_{11}, h_{22}, h_{33}\) are the elements in the diagonal of matrix \(H\).

Second, more visible satellites can generally shorten the PPP convergence time and reduce the impact of some satellites failures on user positioning. Therefore, the number of visible satellites is used as the second optimization objective. A satellite is considered visible if the elevation angle between the satellite and the ground station is higher than the elevation mask angle. The mathematical formula is shown as:

\[
\begin{align*}
\text{VN} &= \sum_{i=1}^{N} v_n_i \\
v_n_i &= \begin{cases} 
1, & \epsilon_i \geq \alpha \\
0, & \epsilon_i < \alpha 
\end{cases}
\end{align*}
\]

(3)

where \(N\) denotes the total number of satellites in the constellation, \(\epsilon_i\) denotes the elevation angle between the \(i\)-th satellite and the ground station, and \(\alpha\) denotes the elevation mask angle.

Third, since the orbital altitude of LEO satellites is lower than that of MEO and GEO satellites, the land signal power and anti-interference of the navigation signal are much stronger. In view of the effect of the atmospheric drag and the inner Van Allen radiation belts (Guan et al. 2020), the ideal range of the orbital altitude for the LEO-NA constellation is [900, 1500]. In addition, the increase in orbital altitude makes the satellites more expensive to launch. Then, the orbital altitude is used as the third optimization objective.

Decision variables

Considering that many commercial companies plan to construct the LEO constellations with 100–200 satellites in the near future, we choose the middle value of 150 as the total number of the two-layer Walker constellation. In addition, since the combination of the inclined Walker constellation and the polar orbit constellation enables more uniform global coverage, we construct a two-layer Walker constellation model for LEO-NA.

First, since the total number of satellites is fixed, we need to allocate the number of each layer \(N_1\) and \(N_2\). And the number of orbit planes \(P\) and the phase factor \(F\) should be determined. In addition, certain simplifications are made. True anomaly \(M\), argument of perigee \(\omega\) and eccentricity \(e\) of each layer are set to 0. Then, right ascension of ascending node (RAAN) \(\Omega\), inclination angle \(i\) and orbital altitude \(h\) of each layer need to be determined. For the uniform global coverage, the range of the orbital inclination is set to [0, 45] and [45, 90], respectively. The specific parameter ranges are shown in Table 1.

LEO-NA optimization model

First, it can be found that the decrease in PDOP and the increase in the number of visible satellites both indicate the improvement in navigation performance. Thus, they are used as the first and second objectives. In order to reduce the system cost, the third objective is to minimize the average orbital altitude of the two-layer walker constellation. In addition, considering the actual distance and the uniformity of distribution, we obtained 104 evenly distributed ground stations worldwide. The elevation mask angle is set to 10°. Observations are made at 5 min intervals throughout the day. Finally, the average

| Layer | Parameter | \(N\) | \(\Omega\) (°) | \(i\) (°) | \(h\) (km) | \(P\) | \(F\) |
|-------|-----------|-----|--------------|------|-------|-----|-----|
| 1     | \([1, 149]\) | \([0, 360]\) | \([0, 45]\) | \([900, 1500]\) | \{factors of \(N_1\)\} | \([0, P_1 - 1]\) |
| 2     | \(\cdots\) | \([0, 360]\) | \([45, 90]\) | \([900, 1500]\) | \{factors of \(N_2\)\} | \([0, P_2 - 1]\) |
observation of all stations is used as an evaluation metric of navigation performance.

In summary, the three objective functions are:

\[
\begin{align*}
    f_1(x) &= \min \frac{\sum_{i=1}^{T} \sum_{l=1}^{L} PDOP(x,l,t)}{T \cdot L} \\
    f_2(x) &= \max \frac{\sum_{i=1}^{T} VN(x,l,t)}{T \cdot L} \\
    f_3(x) &= \min \left( h_1 + h_2 \right) / 2
\end{align*}
\]

(4)

where \( x \) denotes a LEO-NA constellation, \( PDOP(x,l,t) \) denotes the PDOP of the \( l \)-th station at time \( t \), \( VN(x,l,t) \) denotes the number of satellites visible from the \( l \)-th station at time \( t \), \( T \) denotes the total number of observations, and \( L \) denotes the total number of ground stations. In addition to the decision variable constraints specified in Table 1, the constellation needs to satisfy a coverage constraint:

\[
COV(x,t) = 100% \quad (5)
\]

This constraint ensures that the satellites in designed LEO-NA constellations can be observed by all stations at any observation time.

**DMODE-EG algorithm**

Traditional evolutionary algorithms are sensitive to parameter settings and easy to fall into local optimum. To alleviate these problems, we propose the DMODE-EG algorithm, which can determine the evolutionary state of the population so as to dynamically adjust the evolutionary strategy. And the inverse parameter control method is used to correct the evolutionary parameters. For convenience, the background of MOP and DE is given in supplementary materials.

**Identification of evolutionary states**

Suppose \( x_i^t \) denotes the \( i \)-th individual in the \( t \)-th generation population. First, we adopt the fast non-dominated sorting (Deb et al. 2002) to obtain the frontier number of each individual \( FrN_i^t \) and the crowding distance of each individual \( CrD_i^t \). A small \( FrN_i^t \) indicates that the individual is dominated by few other individuals, and a large \( CrD_i^t \) implies a uniform distribution. The individual with a small binary group \( (FrN_i^t, CrD_i^t) \) is considered to have good performance. In addition, since \( FrN_i^t \) can reflect the dominance relationship between individuals, the evolutionary state of a population can be determined based on the distribution of frontier numbers of individuals.

In the early stages of evolution, there are fewer individuals with a frontier number of 1 due to the large variation between individuals. At this point, these individuals are simply identified as elite individuals. This stage is defined as the exploration stage. As the population evolves, the number of individuals with a frontier number of 1 keeps increasing. The population is considered to evolve to the exploitation stage when the number reaches a critical value. At this stage, both crowding distances and frontier numbers are required to identify elite individuals. The pseudo-code for this part is shown in Algorithm 1.

**Algorithm 1 Identification of evolutionary states**

| Input: \( x^i = \{ x_i^t \} \) |
| Output: \( FrN_i^t = \{ FrN_i^t \} \), \( CrD_i^t = \{ CrD_i^t \} \), state |
| 1: \( FrN_i^t = \text{Fast-Nondominated-Sort}(x^i) \); |
| 2: \( CrD_i^t = \text{Crowd-Distance}(x_i^t, FrN_i^t) \); |
| 3: \( eNum = 0 \); |
| 4: for \( i = 0; i < N; i++ \) do |
| 5: \( \text{if } FrN_i^t = 1 \text{ then} \) |
| 6: \( eNum = eNum + 1 \); |
| 7: end if |
| 8: end for |
| 9: \( \text{if } eNum < \beta \cdot N \text{ then} \) |
| 10: \( \text{state} = \text{exploration state} \); |
| 11: else |
| 12: \( \text{state} = \text{exploitation state} \); |
| 13: end if |

**Dynamic selection of evolutionary strategies**

In order to avoid falling into local optimal solutions, the evolutionary strategy in the exploration phase is to explore the solution space as much as possible to maintain the diversity of the population. Therefore, the DE/rand/1/bin operator is adopted to perform the variational operations as follows:

\[
v_i^t = x_{r_1}^t + F_i^t \cdot \left( x_{r_2}^t - x_{r_3}^t \right)
\]

(6)

where \( v_i^t \) denotes the mutation vector, \( F_i^t \) denotes the mutation factor, and \( r_1, r_2 \) and \( r_3 \) are the random numbers. At the same time, in order to make use of the guiding role of elite individuals, we rank individuals based on the binary group before the selection of the base vector and the difference vector. The selection probability is inversely proportional to the ordinal number. Then the best individual among the three is used as the base vector and the others as the difference vector.

The evolutionary strategy needs to be switched when the population evolves to the exploitation stage. The goal of this stage is to enhance the local mining ability and accelerate the convergence. Therefore, we introduce the DE/current-to-best/1 difference operator proposed in the single-objective
evolutionary algorithm J adaptive differential evolutionary (JADE) (Zhang and Sanderson 2009). The formula is expressed as

$$v_i^t = x_i^t + F_t^i \cdot (x_{pbest}^t - x_i^t) + F_t^i \cdot (x_{i_1}^t - x_{i_2}^t)$$  (7)

where $x_{pbest}^t$ is randomly selected from the top p% of individuals based on the single-objective function. It can be found that $x_{pbest}^t$ refers to the elite individual with good performance in the population. However, in multi-objective optimization, it is not feasible to compare individuals directly based on the objective function. Therefore, we need to modify the selection strategy for $x_{pbest}^t$. Since the binary group $(FrN_i^t, CrD_i^t)$ can evaluate individuals in multi-objective optimization, we randomly select $x_{pbest}^t$ from the top p% of individuals ranked according to $(FrN_i^t, CrD_i^t)$. In summary, the evolutionary strategy can be described as:

$$v_i^t = \begin{cases} 
  x_{i_1}^t + F_t^i \cdot (x_{i_2}^t - x_{i_1}^t), & \text{exploration state} \\
  x_{i_1}^t + F_t^i \cdot (x_{pbest}^t - x_{i_1}^t) + F_t^i \cdot (x_{i_1}^t - x_{i_2}^t), & \text{exploitation state} 
\end{cases}$$  (8)

The dynamic selection of evolutionary strategies balances the global exploration ability and the local exploitation ability.

**Dynamic modification of evolution parameters**

The performance of multi-objective evolutionary algorithms is greatly influenced by evolutionary parameters. It often takes much time to tune parameters because the optimal parameters are unknown in practice. Therefore, we adopt the reverse parameter control method (Leung et al. 2012) to dynamically adjust the evolutionary parameters during the evolutionary process.

First, the validity of evolutionary parameters should be determined. If an individual is dominated by the trial vector it generates, the parameter setting is considered successful. These parameters will be maintained into the next generation. Conversely, the parameter setting is considered unsuitable for the current population and these parameters need to be modified.

1. If the parameter $\theta$ has not been inverted before, the inverse operation is performed as:

$$\theta_i^{t+1} = a + b - \theta_i^t$$  (9)

where $a$ and $b$ are the lower and upper limits of the parameters respectively.

2. If the parameter $\theta$ has already been inverted, a random number within its range $[a, b]$ is selected. The pseudo-code for this section is shown in Algorithm 2.

**Algorithm 2** Modification of evolution parameters

```plaintext
Input: $\theta_i^t$
Output: $\theta_i^{t+1}$
1: if $u_i^t$ dominate $x_i^t$ then
2: $\theta_i^{t+1} = \theta_i^t$;
3: else
4: if $\theta_i^t$ is under opposition then
5: $\theta_i^{t+1} = a + b - \theta_i^t$;
6: else
7: $\theta_i^{t+1}$ is randomly chosen from $[a, b]$;
8: end if
9: end if
```

**Overall algorithm flow**

The above three sections explain the core ideas of the DMODE-EG algorithm. Next, we will present the overall process of this algorithm, as shown in Fig. 1. And the detailed steps will be given as follows.

1. **Step 1: Initialization**
   The initial population is randomly generated according to the value range of decision variables. Then the values of all objective functions for each individual is calculated. After that, the fast non-dominated sort is performed and the crowding distance of each individual is calculated. In addition, evolutionary parameters need to be initialized.

2. **Step 2: Differential evolution**
   The evolutionary state of the population needs to be identified. Then, the corresponding evolutionary strategy is selected to generate mutation individuals. After that, the crossover operation is performed to obtain the trial individuals. Finally, the objective function values of each individual are calculated.

3. **Step 3: Parameter modification**
   The validity of the parameter setting need to be judged based on the dominance relationship between the trial individual and the target individual. If the parameter setting is unsuccessful, the reverse parameter control method will be used to modify the parameters.

4. **Step 4: Select offspring**
Simulations and result analysis

To evaluate the performance of the DMODE-EG algorithm, we first compare it with other evolutionary algorithms on some standard test sets. Moreover, we analyze its sensitivity to parameters and verify the effect of reverse parameter control. Then, we apply these algorithms to obtain the optimal configuration parameters of LEO-NA constellations. Finally, we compare the navigation augmentation for BDS-3 by some realistic LEO constellations and these designed constellations.

Performance on standard sets of test functions

Since the ideal performance on test problems is a prerequisite for the DMODE-EG algorithm to solve real-world problems, we analyze its performance on some standard sets of test functions. The relevant results are presented in this section.

Simulation design

Standard test sets contain many single and multi-objective functions of different properties. In order to analyze the performance of DMODE-EG algorithm comprehensively, we conduct simulations on ZDT, DTLZ, WFG, and UF (Full name of test sets can be found in the "Acronymes" section), respectively. And the generation distance (GD) and spacing (SP) are used as the evaluation index of the performance. GD can measure the distance of the obtained Pareto front (PF) from the real PF. A small GD indicates that the obtained optimal set of solutions is close to the standard one. It can be calculated as:

$$GD = \sqrt{n \sum_{i=1}^{n} d_i^2}$$

where $n$ denotes the size of the Pareto solution set, $d_i$ denotes the Euclidean distance between the $i$-th solution obtained and the nearest solution in the real PF. In addition, SP can reflect the uniformity of distribution of the Pareto solution set. A smaller SP means better uniformity of distribution. It can be calculated as:

$$SP = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\bar{d} - d_i)^2}$$

where $\bar{d}$ is the average of all $d_i$.

In the simulations, the DMODE-EG algorithm is compared with multi-objective differential evolution with ranking-based mutation operator (MODE-RMO) algorithm,
multi-objective particle swarm optimization (MOPSO) algorithm, MOPSO based on decomposition (dMOPSO) algorithm and regularity model-based multi-objective estimation of distribution (RM-MEDA) algorithm. The initial crossover rate (CR) and mutation rate F are set to 0.2 and 0.5 respectively. The population size and the maximum number of evaluations are set to 100 and 30,000, respectively. To mitigate the randomness, all algorithms are run 100 times independently. The nonparametric test to justify significant difference is given in supplementary materials.

**Analysis of algorithm performance**

Regarding the two-objective optimization, Table S2 of the supplement shows that the mean of GD of the DMODE-EG algorithm is smaller than the other four algorithms on all test sets. Moreover, except for the slightly worse performance on UF7 than the MODE-RMO algorithm, the GD’s standard deviation of this algorithm is the lowest. In addition, in terms of the mean of SP, Table S3 of the supplement indicates that the DMODE-EG algorithm achieves the optimal performance on UF5, UF6 and UF7. However, the mean and standard deviation of GD is higher than some other algorithms on ZDT1, ZDT2 and ZDT3.

As for the three-objective optimization, the DMODE-EG algorithm also has excellent performance. Table S4 of the supplement shows that the mean and standard deviation of GD of the DMODE-EG algorithm are still smaller than other algorithms on all test sets. In terms of the mean of SP, Table S5 of the supplement indicates that it achieves the best performance on UF8 and WFG1, but performs worse than other algorithms on other test sets. In addition, on DTLZ2, UF8 and WFG1, its standard deviation of SP is the lowest.

In order to further analyze the stability of these algorithms, we record the success rate in achieving the specified accuracy of GD and the corresponding number of evaluations, as shown in Table S6 of the supplement. It can be found that the DMODE-EG algorithm achieves almost 100% success rate on all test sets. Moreover, except for the slightly worse performance on UF8 and DTLZ5 than the MOPSO algorithm, its number of evaluations is the lowest. It is concluded that the DMODE-EG algorithm has good stability.

**Analysis of algorithm sensitivity**

The DE algorithms are sensitive to CR and F. When CR increases, the proportion of mutation individuals in the trial individuals also rises. And F controls the diversity and convergence of populations. To analyze the effect of parameters on the performance, we tested the sensitivity of the DMODE-EG algorithm on DTLZ2. All parameters are set the same as the above simulations except for CR and F.

First, according to Table S7 of the supplement, the changes in CR and F have little effect on SP. With (0.2, 0.5) as the benchmark, a change of 0.2 in F increases the mean value of GD by 13.6%. Nevertheless, the effect on the DMODE-EG algorithm is limited, which is partly attributed to the inverse parameter control. In addition, a set of controlled tests are conducted with the parameters set to (0.6, 0.7). The results show that the reverse parameter control can largely attenuate the performance degradation caused by improper parameter settings.

Overall, the DMODE-EG algorithm has excellent convergence, though sometimes the distribution of solutions isn’t particularly good. It is confirmed by the fact that its mean GD is smaller than other algorithms on all standard test sets. Moreover, the nearly 100% success rate of seeking optimal solutions verifies its good stability. In addition, its ability to dynamically modify the evolutionary parameters is advantageous in solving practical problems.

**Performance on LEO-NA constellation design**

First, we adopt different evolutionary algorithms to obtain the optimal configuration parameters of the LEO-NA constellation. Then, these designed constellations and some realistic LEO constellations are used to augment BDS-3. The comparative results are presented in this section.

**Optimal parameters of LEO-NA constellation**

After determining the decision variables and the objective functions, the problem of designing the LEO-NA constellation is transformed into a multi-objective optimization problem. We adopt different evolutionary algorithms to obtain the corresponding Pareto optimal solution sets, as shown in Fig. 2. It is worth noting that the solutions obtained by the dMOPSO algorithm are distributed on a straight line. Since the dMOPSO algorithm substitutes entirely the dominance approach with decomposition, it sometimes fails to achieve the expected results when solving some MOPs with the complex PF. Therefore, the dMOPSO algorithm will be ignored in the following analysis. It can be found that other solutions are all concentrated in five parts of the solution space. Except for the DMODE-EG algorithm, the continuity of parts isn’t ideal. For the MOPSO algorithm and the RM-MEDA algorithm, the spacing between different parts is not uniform. By contrast, the solutions obtained by the DMODE-EG algorithm are uniformly distributed and continuous, which indicates that the algorithm can explore the solution space effectively and maintain the population diversity in solving the LEO-NA constellation design problem.

Since these algorithms have obtained the optimal solution sets, we need to determine the optimal solution. We first rank the individuals separately according to each objective and
then score each solution based on the ranking and objective weights. The specific formula is as follows:

\[ S_i = \sum_{j=1}^{3} \alpha_j \cdot \left( N - \text{rank}_j^i + 1 \right) \]  \hspace{1cm} (12)

where \( n \) denotes the score of the \( i \)-th individual, \( N \) denotes the size of population, \( \text{rank}_j^i \) denotes the ranking of the \( i \)-th individual based on the \( j \)-th objective. \( \alpha_j \) denotes the weight of the \( j \)-th objective and is calculated as:

\[ \alpha_j = \frac{f_j^{\max} - f_j^{\min}}{f_j^{\max}} \]  \hspace{1cm} (13)

where \( f_j^{\max} \) and \( f_j^{\min} \) denote the maximum and minimum value of the \( j \)-th objective, respectively. It is worth mentioning that in order to make the score restricted to 1 to 100, we use the normalized values of three weights in practice. Finally, the solution with the highest score is identified as the optimal solution. After calculation, the configuration parameters of the LEO-NA constellations obtained by four algorithms are denoted as C1, C2, C3 and C4, as shown in Table 2.

### Analysis of navigation augmentation of realistic LEO constellations

Some realistic LEO constellations represented by Iridium NEXT, Hongyan and Centispase were designed with the capability of navigation augmentation. Iridium NEXT and Hongyan are polar orbit constellations, while Centispase is a hybrid constellation consisting of inclined and polar orbits. For the convenience, we denote these three constellations as C5, C6 and C7. The specific parameters are shown in

| Parameters | C1 (DMODE-EG) | C2 (MODE-RMO) | C3 (MOPSO) | C4 (RM-MEDA) |
|------------|---------------|---------------|------------|--------------|
| Layer1 | Layer2 | Layer1 | Layer2 | Layer1 | Layer2 | Layer1 | Layer2 |
| \( N \) | 14 | 136 | 14 | 136 | 14 | 136 | 14 | 136 |
| \( \Omega \) (°) | 360.0 | 185.1 | 290.2 | 0.0 | 91.0 | 247.9 | 346.7 | 0 |
| \( i \) (°) | 33.8 | 70.8 | 22.5 | 72.8 | 18.0 | 53.4 | 28.0 | 71.5 |
| \( h \) (km) | 974.3 | 1500.0 | 900.0 | 1184.1 | 900.0 | 1334.5 | 912.8 | 1337.8 |
| \( P \) | 2 | 8 | 2 | 12 | 2 | 12 | 2 | 8 |
| \( F \) | 1 | 6 | 0 | 2 | 0 | 1 | 0 | 6 |
Table 3. Configuration parameters of the realistic LEO constellations

| Parameters | C5 (Iridium NEXT) | C6 (Hongyan) | C7 (Centispace) |
|------------|-------------------|--------------|-----------------|
| Layer 1    | 66                | 54           | 120             |
| Layer 2    | 6                 | 6            | 12              |
| i (°)      | 86.4              | 85.0         | 55.0            |
| h (km)     | 780               | 1070         | 975.0           |

Fig. 3. Distribution of PDOP values for different constellations. From left to right and top to bottom, each figure corresponds to BDS-3, BDS-3 + C5, BDS-3 + C6 and BDS-3 + C7.

Fig. 4. Cumulative probability of PDOP values for different constellations.

Fig. 5. Number of visible satellites at different latitudes for different constellations.

Fig. 6. Number of visible satellites at different longitudes for different constellations.

Table 3. Based on the observations from ground stations at the 5° interval, we evaluate their navigation augmentation performance for BDS-3. First, the distribution of PDOP is shown in Figs. 3, 4 and Fig. S3. PDOP values for BDS-3 are mainly below 3.5. It is worth noting that PDOP values are much higher in some areas than in others, which leads to a long trailing shown in Fig. 4. For example, PDOP values in 120°W and nearby are up to 5 or more. With the navigation augmentation, most PDOP values drop below 3 for BDS-3 + C5 and BDS-3 + C6, and below 2 for BDS-3 + C7. However,
although the problem of high PDOP values has been allevi-
ated, it remains unresolved.

In addition, we analyze the variation of the number with
latitude and longitude, as shown in Figs. 5 and 6. Due to
the augmentation of the realistic LEO constellations, there
has been some increase in the number of visible satellites
around the world. BDS-3 + C6 has a slightly higher number
of visible satellites than BDS-3 + C5 at different latitudes,
but almost the same number at different longitudes. Com-
pared to BDS-3 + C5 and BDS-3 + C6, BDS-3 + C7 has a
more even number of visible satellites at different latitudes
and longitudes. Since the number of satellites in the polar
orbit of Centispace is less than Iridium NEXT and Hong-
yan, BDS-3 + C7 has less visible satellites near the pole.
Overall, BDS-3 + C7 performs better than BDS-3 + C5 and
BDS-3 + C6 due to the advantage of orbital altitude and the
number of satellites.

Analysis of navigation augmentation of designed LEO
constellations

Based on the configuration parameters obtained by dif-
ferent evolutionary algorithms, we analyze the navigation
augmentation performance of these designed constella-
tions for BDS-3. First, the distribution of PDOP is shown
in Figs. 7, 8 and Fig. S4. It can be found that PDOP values
for BDS-3 + C1, BDS-3 + C2, and BDS-3 + C3 all decrease
mainly to below 2. In contrast, BDS-3 + C3 has the worst
PDOP distribution. Although the curve is similar to other
constellations when PDOP is below 1.8, it converges slowly
afterward. It is worth mentioning that only C1 obtained by
the DMODE-EG algorithm can assist BDS-3 in solving the
problem of high PDOP values in some regions.

Besides, the variation of the number of visible satellites
with latitude and longitude is shown in Figs. 9 and 10. It
can be found that BDS-3 + C3 has a high number of vis-
ible satellites at low and middle latitudes, but it performs
much worse than other constellations at high latitudes. By
contrast, BDS-3 + C1 performs particularly well at high
latitudes while also achieving good performance at mid and
low latitudes. Moreover, BDS-3 + C1 enables ground sta-
tions at different longitudes to observe more satellites than
other constellations. It can be concluded that BDS-3 + C1
outperforms other designed constellations in terms of both
PDOP and the number of visible satellites.

Fig. 7 Distribution of PDOP values for different constellations.
From left to right and top to bottom, each figure corresponds to
BDS-3 + C1, BDS-3 + C2, BDS-3 + C3 and BDS-3 + C4

Fig. 8 Cumulative probability of PDOP values for different constel-
lations

Fig. 9 Number of visible satellites at different latitudes for different
costellations
Compared to these realistic LEO constellations, C1 also performs better in augmenting the navigation performance of BDS-3. First, the smaller number of satellites makes C5 and C6 weaker than C1 and C7 in improving the navigation performance. Although C1 and C7 have the same number of satellites, there are significant differences in their configuration parameters. One is due to the different design approaches. The configuration parameters of C1 were directly obtained by the DMODE-EG algorithm. In contrast, C7 was originally designed to consist of only 120 satellites. Subsequently, 30 satellites in the higher orbit were added to achieve seamless global coverage. In addition, C1 is more focused on improving the navigation performance, while realistic LEO constellations are generally used not only for navigation augmentation, but also for communication services. And as commercial constellations, they need to meet certain performance requirements while controlling costs. Some trade-offs limit the improvements of C7 in navigation performance. First, the PDOP distribution of BDS-3 + C7 is worse than that of BDS-3 + C1. Not only the average PDOP value of BDS-3 + C7 is higher, but it cannot solve the problem of high PDOP values in some regions. In addition, BDS-3 + C7 has less visible satellites than BDS-3 + C1 at different longitudes and latitudes, with respective amount ranging from 1 to 6.

**Conclusions**

We propose the DMODE-EG algorithm, which determines the evolutionary state of a population based on the number of elite individuals, and thus dynamically adjusts the evolutionary strategy. Meanwhile, it can correct the evolutionary parameters based on the inverse parameter control method. The mean GD of the DMODE-EG algorithm on standard test sets is smaller than other algorithms, which indicates its good convergence. And the nearly 100% success rate of seeking optimal solutions shows that it has good stability. In addition, we construct a two-layer Walker constellation model for LEO-NA. Then, the constellation design problem is transformed into a MOP after the determination of decision variables and objective functions. By solving the MOP with the DMODE-EG algorithm, we obtain the configuration parameters of the optimal constellation C1. With the augmentation of C1, the navigation performance of BDS-3 is significantly improved. The average PDOP values drop to 1.2–2.0 from 1.5–5.5, and the number of visible satellites increases from 8–10 to 13–18. In contrast, the designed constellations obtained by other algorithms and some realistic constellations bring weaker improvements in navigation performance, and they cannot solve the problem of high PDOP values in some regions.

Overall, the DMODE-EG algorithm has the excellent performance, and the two-layer Walker constellation model for LEO-NA effectively improves the navigation performance of BDS-3. Nevertheless, when designing realistic constellations, some adjustments should be considered, e.g., the number of satellites should not be limited to 150. Launch and maintenance costs shouldn't simply be reflected by orbital altitude, but need to be calculated in detail. And some simplified orbital parameters, such as the argument of perigee, need to be controlled in practice. In addition, considering that some LEO constellations tend to have communication capabilities, it would also be valuable to consider some communication indexes when designing the LEO-NA constellations in the future.

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**Data availability** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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