Regression mathematical models of the number of small mammals in the Kaluga region of the Russian Federation

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Abstract. An analytical review of mathematical model of the dynamics of the population of small mammals, feeders for ixodic ticks, is presented. Rodents are an essential link in the circulation of agents of numerous zoonoses, as a result, a constant natural nidality within a specific area is maintained. Small mammals also serve as host feeders for a number of hematophagous arthropods, which are natural reservoirs of causative agents of dangerous infections and invasions. Muroid rodents are the most numerous mammals that inhabit the territory of the Russian Federation. A combination of natural disease foci often occurs as a result of territorial convergence and the existence of common vectors and carriers in the same area. Tularemia can be combined with plague, pseudotuberculosis, listeriosis, leptospirosis, pasteurellosis, etc. Small mammals are also involved in the circulation of encephalitis virus, viral hemorrhagic fever, orthohantavirus, Lyme disease, toxoplasmosis, leishmaniasis, babesiosis, anaplasmosis, and many other diseases affecting animals and humans. All the above mentioned diseases require continuous monitoring of the population dynamics of small mammals within a specific natural and climatic area under study. Mathematical models of the dynamics of population of small mammals make it possible to effectively forecast outbreaks of natural focal diseases without significant financial investments. An accurate forecast will provide an opportunity to be prepared and will enable a prompt response to the epizootological situation. The purpose of the study is to develop mathematical models of the population of small mammals in the Kaluga region acting as ixodid tick host feeders, therefore making it possible to estimate the probability of outbreaks of vector-borne zoonoses. The obtained mathematical models enable the risk quantification of outbreaks of vector-borne zoonoses without incurring costs associated with field study and allow to implement competent, timely and effective preventive measures.

1. Relevance
Small mammals are found in all climatic zones due to their ability to adapt to changing environmental conditions [1].

Rodents form an important link in the circulation of pathogens of many zoonanthropic diseases, which result into constant natural foci on a certain territory.
Moreover, small mammals serve as feeders, or hosts, for many blood-sucking arthropods, which are directly reservoirs of pathogens [2; 3].

Chipmunks, squirrels, hares (both roe and squirrel) and other species of small rodents, along with birds (some forest species, black grouse, blackbird) are the prey for both preimaginal phases and adults of ixodid ticks [4].

Complex methods are needed to calculate the number of mouse-like rodents, to study field material for the most significant nosological forms of natural focal infections which are typical for specific territories. An annual monitoring of natural focal infections is required, which is laborious, requires the significant financial investments and human resources.

The presence of mathematical models of the dynamics of the population of small mammals makes it possible to effectively forecast outbreaks of natural focal diseases without significant financial investments [5].

2. Materials and research methods

The analytical mathematical models demonstrating the dynamics of the population of small mammals depending on the average annual temperature and average annual rainfall are presented in the research. During the the decade (from 2009 to 2019 inclusively), a multivariate experiment of type 2 was conducted in the field according to the standard method described in V.V. Kalmykov's works [6].

The object of the study were the small mammals found in the Kaluga region, like gray vole, red vole (Myodes glareolus), field mouse (Apodemus agrarius), small forest mouse (Apodemus uralensis), gray rat (Rattus norvegicus), house mouse (Mus musculus).

The carried out field studies of small mammals of all areas of the Kaluga Region over the decade from 2009 to 2019 were included in the current research as well.

3. Research results and their discussion

The mathematical models of the number of small mammalian populations are obtained depending on three factors: average monthly temperature, average rainfall and average atmospheric pressure during the year in the Kaluga Region.

A total amount of 3943 small mammals were collected over the 10 years of the observation. Graphically, the species composition of the small mammals is presented in figures 1.

![The number of captured small mammals in 2009-2019](image)

**Figure 1.** The number of captured small mammals in 2009-2019 at stationary observation points.
To obtain mathematical models, a complete factorial experiment was conducted on the collected statistical data. The values of the levels of factors are presented in Table 1.

### Table 1. Factor variation ranges.

|     | –1          | 0          | +1          |
|-----|-------------|------------|-------------|
| X1  | +4.57°C     | +6.55°C    | +7.57°C     |
| X2  | 31.6 mm     | 49.5 mm    | 64.14 mm    |
| X3  | 741.0 mm Hg | 745.5 mm Hg| 750.0 mm Hg |

X1 is average monthly temperature (°C), X2 is average monthly relative humidity (Q, %), X3 refers average atmospheric pressure (P mm Hg).

The matrix of the experimental design is presented in Table 2.

### Table 2. The matrix of the experiment with three factors.

| № | experience | X0 | X1 | X2 | X3 | X1X2 | X1X3 | X2X3 | X1X2X3 | Y1 | Y2 | Y3 | S² |
|---|------------|----|----|----|----|------|------|------|--------|----|----|----|----|
| 1 | +          | +  | +  | +  | +  | +    | +    | +    | +      | 230| 218| 242| 144|
| 2 | +          | –  | +  | +  | –  | –    | –    | +    | –      | 265| 251| 279| 196|
| 3 | +          | +  | –  | +  | –  | +    | –    | –    | –      | 322| 307| 337| 225|
| 4 | +          | –  | –  | +  | +  | –    | –    | +    | –      | 292| 303| 281| 121|
| 5 | +          | +  | +  | –  | +  | –    | –    | –    | –      | 290| 274| 306| 256|
| 6 | +          | +  | –  | –  | –  | +    | –    | +    | +      | 180| 172| 188| 64 |
| 7 | +          | +  | –  | –  | –  | –    | +    | +    | +      | 383| 393| 373| 100|
| 8 | +          | –  | –  | –  | –  | –    | +    | –    | –      | 116| 112| 120| 16 |

\[ Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{23} X_2 X_3 + b_{123} X_1 X_2 X_3 \]

To exclude the influence of systematic errors caused by external conditions, it is recommended that experiments with a given experimental design be conducted randomly in time. When organizing the experiment, the need to evaluate the variance of the experiments was taken into account. For this, the experiments were duplicated, i.e. determination of the number of individuals of small mammals took place in all areas of the region.

To obtain a mathematical model in the form of a regression equation on a normalized scale, the coefficients were determined:

\[ b_0 = +259.75; b_1 = +46.50; b_2 = –18.50; b_3 = + 17.50; b_{12} = –27.75; b_{23} = –11.25; b_{123} = + 11.50; \]

Thus, the mathematical model takes the following form:

\[ Y = 259.75 + 46.50 X_1 –18.50 X_2 + 17.50 X_3 –27.75 X_1 X_2 –11.25 X_1 X_3 –11.50 X_2 X_3 + 11.50 X_1 X_2 X_3 \]

The average monthly temperature influence the dynamics of the population of small mammals to the greatest extent. The degree of its influence is 2.5 times stronger than the influence of the average monthly rainfall and 2.6 times stronger than the influence of average atmospheric pressure. The “+” sign indicates that the higher the temperature is, the larger the number of small mammals ips. The “-” sign with a monthly average rainfall coefficient indicates a slight decrease in the number of small mammals followed by the increase in rainfall. A graphical representation of the friction coefficients characterizing the influence of factors on the response is shown in figure 1.

The pair effect of the interaction of the average temperature and the average monthly rainfall also reduces the number of small mammals and is 1.67 times weaker than the influence of the average temperature separately and 1.5 times stronger than the influence of the average rainfall separately.
The peculiarity of this model refers to the interaction of two factors having the greatest effect on the population of small mammals, which are the average temperature and the average atmospheric pressure. The strength of the two factors interaction's influence is more significant than the influence of each of the three studied factors separately. In particular, it is stronger than the degree of influence of the average temperature factor by 1.02 times, the force of influence of one amount of precipitation by 2.58 times, than the force of influence of the average atmospheric pressure by 2.72 times.

The paired effects of the atmospheric pressure and humidity are comparable in strength to the separate atmospheric pressure. Figure 2 shows a graphical representation of the adhesion coefficients characterizing the effect of factors on the response.

![Figure 2. Bar diagram of adhesion coefficients.](image)

To obtain a calculated mathematical model, it is necessary to bring it to the natural scale. For this, factors X1, X2, X3 should be replaced with the average monthly temperature (t °C), average monthly rainfall (S, mm), average atmospheric pressure (P mm Hg), according to the following formulas:

\[
X_1 = \frac{t - 6.55}{1.5},
\]
\[
X_2 = \frac{S - 49.5}{14.77},
\]
\[
X_3 = \frac{P - 745.5}{4.5},
\]

The calculated mathematical model after transformations to calculate the number of small mammals will take the form:

\[
N = 96,002P + 696,388S + 9623,358t - 0.925PS - 12,784Pt - 87,245St + 0.11535PSt - 71856,560
\]

4. Conclusion

The presented mathematical model can be used as an instrument to calculate the population of small mammals without resorting to expensive field experiments, which may be useful in predicting the population size of the small mammals in the Kaluga region and other regions with similar climatic conditions.

The advantage of the applied modeling allows taking into account the impact on the observed object of a combination of all factors and their interaction effects.
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