Off-the-energy-shell \(pp\) scattering in the exclusive proton knockout \(^{12}\text{C}(p, 2p)\) reaction at 392 MeV

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(Dated: March 14, 2008)

The triple differential cross section (TDX) for the \(^{12}\text{C}(p, 2p)\) proton knockout reaction from the 1p3/2 single-particle state measured at 392 MeV is investigated by nonrelativistic distorted wave impulse approximation (DWIA) with accurate treatment of the kinematics of the colliding two protons and effects of in-medium modification to the matrix elements of the proton-proton (pp) effective interaction. Some simplifying approximations made in previous studies with DWIA are examined. The off-the-energy-shell matrix elements of the pp effective interaction are shown to play an essential role in describing the asymmetric two peaks of the measured TDX corresponding to the kinematics in which the momentum transfer is fixed.

PACS numbers: 24.10.Eq, 24.50.+g, 25.40.-h, 24.80.+y

Nucleon knockout reaction is a powerful tool to extract information on single-particle states of nuclei. Proton induced knockout reaction such as \((p, 2p)\) reaction at intermediate energies \(^1\)\(^2\)\(^3\) has been studied intensively because of its significantly large cross section compared to the electron induced \((e, e'p)\) one. A compensation for the large cross section is that the analyzing power \(A_y\) in particular, cannot be explained by standard distorted wave impulse approximation (DWIA) calculations \(^4\)\(^5\). Recently, this discrepancy has been attributed to effects of in-medium modification to the proton-proton (pp) effective interaction; the reduction of masses of mesons in nuclear medium \(^6\) was shown to improve the agreement between experimental data of \(A_y\) and results of theoretical calculation within the framework of relativistic impulse approximation (RIA) \(^7\)\(^8\)\(^9\)\(^10\). The reduction of meson masses, however, brought about a serious problem on other spin observables such as the spin transfer coefficients \(D_i\) \(^7\).

On the other hand, effects of in-medium modification to the matrix elements of the effective pp interaction, henceforth called in-medium effects, on observables of the \((p, 2p)\) reaction within the nonrelativistic framework, have not been investigated well. In particular, the off-the-energy-shell (off-shell) property of the \(pp\) scattering in nuclear medium was not treated properly in the foregoing studies with nonrelativistic DWIA \(^4\)\(^5\). Moreover, effects of refraction, i.e., change in the kinematics of the incoming and outgoing protons caused by distortion of the target nucleus, were also neglected. Since the refractive effect was shown to be essential to reproduce \(A_y\) data of inclusive \((p, p'x)\) reactions \(^1\)\(^1\), it is expected to be significant in the calculation of spin observables of also \((p, 2p)\) reactions. Careful analysis of the \((p, 2p)\) reactions with nonrelativistic DWIA taking account of the above-mentioned effects is necessary to draw a conclusion on the comparison between nonrelativistic calculation and experimental data of \((p, 2p)\) reactions.

As the first step to such theoretical analysis of \((p, 2p)\) reactions, in this Rapid Communication we analyze the triple differential cross sections (TDX’s) of the \(^{12}\text{C}(p, 2p)\) reaction at 392 MeV corresponding to the knockout of proton in the 1p3/2 single-particle state in \(^{12}\text{C}\). Our main purpose of the present study is to show that the off-shell \(pp\) scattering is essential to reproduce the experimental data of the TDX for specific kinematics in which the momentum transfer is fixed.

We start with the usual DWIA \(T\) matrix of the following form:

\[
T_{m'_1m'_2m_1M_j} = \int \int dr_1 dr_2 d\xi_1 d\xi_2 \chi_{t_1; k'_1, m'_1}^{-(-)*}(r_1, \xi_1) \times \chi_{t_2; k'_2, m'_2}^{-(+)}(r_2, \xi_2) \tau(s; \rho(R), K_{NN}) \times \bar{\phi}_{1p} \phi_{2p_M}^{-}(r_2, \xi_2),
\]

where \(\chi_{t_1} (\chi_{t_2})\) is the distorted wave for the outgoing proton 1 (proton 2) relative to the residual nucleus \(^{11}\text{B}; m_1 (m_2)\) and \(k'_1 (k'_2)\) are, respectively, the \(z\)-component of the spin and the asymptotic momentum in \(\hbar\) unit of the observed proton 1 (proton 2). The coordinate of the proton \(i (i = 1, 2)\) relative to the residual nucleus \(^{11}\text{B}\) is denoted by \(r_i\), and \(\xi_i\) is the corresponding internal coordinate. The distorted wave between the incident proton and \(^{12}\text{C}\) target is denoted by \(\chi_1\) with the asymptotic momentum \(k_1\) and the \(z\)-component \(m_1\) of the spin of the proton in the asymptotic region. The relative coordinate \(r\) for \(\chi_1\) is given by \(r_1 - r_2/A\) with the mass number \(A\) of the target nucleus. Furthermore, \(\bar{\phi}_{1p} \phi_{2p_M}^{-}\) is the single-particle wave function of the target proton in the 1p3/2 state and \(M_j\) is the \(z\)-component of the total angular momentum \(j\) that is 3/2 in the present case. \(\tau\) is the pp effective interaction in coordinate representation with \(s = r_1 - r_2\) and \(K_{NN}\) the scattering energy of the two proton system explained below in more detail. Note that \(\tau\) contains the antisymmetrization operator for the colliding two protons. We use in the present study the half-off-shell \(g\) matrix developed by the Melbourne group \(^1\)\(^2\) based on Bonn-B nucleon-nucleon \((N,N)\) potential \(^3\), the Melbourne \(g\) matrix, as for \(\tau\), which depends on also the nuclear density \(\rho\); we adopt \(\rho\) at \(\vec{R} = (r_1 + r_2)/2\) on the basis of local density approximation. The \(T\)-matrix elements of Eq. (1) are evaluated in the center of mass (c.m.) frame of the total reaction system with the relativistic kinematics. The

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TDX is given by

$$\frac{d^3\sigma}{dE_1d\Omega_1d\Omega_2} = \frac{JF_{\text{kin}}S}{2(2j+1)} \sum_{m_1'm_2'm_1M_j} |T_{m_1'm_2'm_1M_j}|^2,$$  \hspace{1cm} (2)

where $\Omega_1^j$ ($\Omega_2^j$) is the solid angle for the emitted proton 1 (proton 2) and $E_1^j$ is the emission energy of the proton 1; the superscript L denotes the laboratory frame. $J$ is the Jacobian for the transformation from the c.m. frame to the laboratory, $F_{\text{kin}}$ is the kinetic factor and $S$ is the spectroscopic factor that agrees with $2j+1 = 4$ if the single-particle description of the target nucleus is fully accurate.

We in the present study neglect the spin dependent part of the disturbing potentials for the $\chi$, which is known to give no significant effects on unpolarized cross sections. We then have

$$\chi_1^{-}(\eta R) \approx \chi_1^{(+)}(\eta R) \exp[ik_2^c(R) \cdot s],$$

where $\chi$ are the spatial parts of the $\tilde{\chi}$ and $\eta$ is the spin 1/2 wave function. The spin dependence of $\phi_{1p\tilde{\chi},m}$ is taken into account with

$$\phi_{1p\tilde{\chi},m}(r_2,\xi_2) = \sum_{m_2} \left(1 m \frac{1}{2} m_2 \frac{3}{2} M_j\right) \phi_{1p\tilde{\chi},m}(r_2) \eta_{m_2}(\xi_2),$$

where $\phi_{1p\tilde{\chi},m}$ is the spatial part of $\phi_{1p\tilde{\chi},m}$ with the z-component of the orbital angular momentum.

The relative coordinates concerned with the $\chi$ are expressed with $R$ and $s$ as follows:

$$r_1 = R + s/2,$n$$
$$r_2 = R - s/2,$n$$
$$r = \frac{A - 1}{A} R + \frac{A + 1}{2A} s \equiv \eta R + \zeta s.$$

We now make the local semi-classical approximation (LSCA) \cite{4} to the $\chi$, i.e.,

$$\chi(u + v) \approx \chi(u) \exp[ik(u) \cdot v].$$

(4)

The magnitude of the local momentum $k(u)$ is evaluated by the real part of the complex local momentum that satisfies the local energy conservation law at $u$. The direction of $k(u)$ is taken to be parallel with the flux of $\chi$ at $u$. The LSCA is shown to be valid for $|v|$ smaller than about 1.5 fm \cite{15}. The LSCA is applied to each distorted wave in the $T$ matrix of Eq. (1) as

$$\chi_1^{(+)}(\eta R + \zeta s) \approx \chi_1^{(+)}(\eta R) \exp[ik_1(\eta R) \cdot \zeta s].$$

Note that this approximation to the $\chi$ is valid for $s$ less than about 3 fm. Since the integrand of Eq. (1) is appreciable for small $s$ less than the range of $\tau$, which is about 1.5 fm, one can evaluate with high accuracy the $T$ matrix making use of Eqs. (5)–(7). One thus obtains

$$T_{m_1'm_2'm_1M_j} \approx \int dR \chi_1^{(-)}(\eta R) \chi_2^{(+)}(\eta R) \chi_{1',2'}^{(+)}(\eta R) \cdot s \times \sum_{m_2} \left(1 m \frac{1}{2} m_2 \frac{3}{2} M_j\right) \int d\xi_1 d\xi_2 \eta_{m_2}(\xi_2) \times \exp \left[-i \frac{k_2^c(R) - k_2^c(R)}{2} \cdot s\right] \tau(s; \rho(R), K_{NN}) \times \exp[i\kappa_1(\eta R) \cdot \zeta s] \eta_{m_1}(\xi_1) \eta_{m_2}(\xi_2) \phi_{1p\tilde{\chi},m}(R - s/2).$$

(8)

The spatial part of the proton single-particle wave function $\phi_{1p\tilde{\chi},m}$ can be written with its Fourier transform $f$ as

$$\phi_{1p\tilde{\chi},m}(R - s/2) = \frac{1}{(2\pi)^3} \int dk_2^A f_{1p\tilde{\chi},m}(k_2^A) \times \exp[ik_2^A \cdot (R - s/2)],$$

(9)

where $k_2^A$ can be interpreted as the relative momentum between the target proton and the residual nucleus in the c.m. frame of the target nucleus $^{12}$C. Inserting Eq. (9) into Eq. (8) one obtains

$$T_{m_1'm_2'm_1M_j} \approx \frac{1}{(2\pi)^3} \int dR \chi_1^{(-)}(\eta R) \chi_2^{(+)}(\eta R) \phi_{1p\tilde{\chi},m}(R - s/2) \times \sum_{m_2} \left(1 m \frac{1}{2} m_2 \frac{3}{2} M_j\right) \int dk_2^A f_{1p\tilde{\chi},m}(k_2^A) \times \exp[ik_2^A \cdot R M_{m_1'm_2'm_1m_2}(\kappa', \kappa)] \times \exp[-i\kappa_1(\eta R) \cdot \zeta s] \eta_{m_1}(\xi_1) \eta_{m_2}(\xi_2)$$

(10)

where the matrix elements $M$ of $\tau$ are defined by

$$M_{m_1'm_2'm_1m_2}(\kappa', \kappa) \equiv \int d\xi_1 d\xi_2 \eta_{m_1}(\xi_1) \eta_{m_1}(\xi_2) \times e^{-i\kappa_1(\eta R) \cdot \zeta s} \tau(s; \rho(R), K_{NN}) \times e^{i\kappa_1(\eta R) \cdot \zeta s}$$

(11)

with

$$\kappa' = \frac{k_1(\eta R) - k_2^A}{2},$$

$$\kappa = \frac{2\zeta k_1(\eta R) - k_2^A}{2}.\hspace{1cm} (12)$$

One immediately sees $\kappa'$ is the relative momentum between the two protons in the final state of the transition by $\tau$ at $R$ in the c.m. frame of the $p+^{12}$C system. For the initial state, since the motion of the target proton can be described well with nonrelativistic kinematics, one may approximate the relative momentum $\kappa$ as

$$\kappa \approx \frac{k_1(\eta R) - k_2 + \{k_1(\eta R) - k_1\}/A}{2} \approx \frac{k_1(\eta R) - k_2}{2},$$

(13)
where \( k_2 \equiv k_2^A - k_1/A \) is to be interpreted as the momentum of the target proton in the initial state in the \( p+^{12}\text{C} \) c.m. frame.

Thus, for given \( R \) and \( k_2^A \) in Eq. (10), the kinematics of the colliding two protons corresponding to the initial and the final states of the transition by \( \tau \) is uniquely determined. Specification of the momentum of each proton in the initial state is necessary since the scattering energy \( K_{NN} \) for \( \tau \), which we use in the present study, is defined by the kinetic energy of the incoming proton in the rest frame of the target proton [12].

Note that the momentum of the incoming proton to be used in the calculation of \( M \) is not the asymptotic momentum \( k_1 \) but the local one \( k_1(\eta R) \). Similarly, the local momenta \( k_1'(R) \) and \( k_2'(R) \) dictate the final states of the two protons of the transition by \( \tau \). This change in the magnitudes and the directions of the momenta of the incoming and outgoing protons is referred to as the refractive effect of distortion, which was shown to play an essential role in the accurate description of \((p,p'\pi)\) reactions [11]. Another remark on \( M \) of Eq. (11) is that \( \kappa' \neq \kappa \), hence, the off-shell matrix elements are necessary.

In foregoing studies with nonrelativistic DWIA [4, 5], simplification of \( M \) with no theoretical foundation was made as follows. First, the total local momentum of the two protons was assumed to be conserved in the transition by \( \tau \), namely, \( \kappa \) of Eq. (13) was replaced by

\[
\kappa^{\text{mc}} = \frac{2 k_1(\eta R) - k_1'(R) - k_2'(R)}{2}.
\]

This assumption resulted in the following form of the \( T \) matrix:

\[
T_{m_1' m_2' m_1 m_2}^{\text{mc}} \equiv \int dR \chi^{(-)\ast}_{1; 1} (R) \chi^{(-)\ast}_{1; 1} (R) \chi^{(+)}_{1; 1} (\eta R) \\
\times \sum_{m_1, m_2} (1 m_1 1 2 m_2 v_2 J_2) \phi_{1 p'} \frac{1}{2} m_{1} m_{2} (R) \\
\times M_{m_1' m_2' m_1 m_2} (\kappa', \kappa^{\text{mc}}). \tag{15}
\]

Second, the above-mentioned refractive effect of distortion was neglected, i.e.,

\[
\kappa' \rightarrow \kappa'_{\text{as}} = (k_1' - k_2')/2,
\]

\[
\kappa^{\text{mc}} \rightarrow \kappa^{\text{mc}}_{\text{as}} = (2 k_1 - k_1' - k_2')/2. \tag{16}
\]

The third simplification is the so-called on-shell approximation to \( M \), in which \( \kappa' \) is changed to \( \kappa \) with the direction of \( \kappa' \) fixed. Consequently, the following simplified \( T \) matrix was used in previous studies:

\[
T_{m_1' m_2' m_1 m_2}^{\text{prev}} \equiv \int dR \chi^{(-)\ast}_{1; 1} (R) \chi^{(-)\ast}_{1; 1} (R) \chi^{(+)}_{1; 1} (\eta R) \\
\times \sum_{m_1, m_2} (1 m_1 1 2 m_2 v_2 J_2) \phi_{1 p'} \frac{1}{2} m_{1} m_{2} (R) \\
\times M_{m_1' m_2' m_1 m_2} \left( \frac{\kappa^{\text{mc}}_{\text{as}}}{\kappa_{\text{as}}}, \kappa^{\text{mc}}_{\text{as}} \right). \tag{17}
\]

In some cases \( NN \) matrix instead of \( g \) was adopted, neglecting in-medium effects concerned with the Pauli principle in the evaluation of \( NN \) interactions from bare \( NN \) forces. This approximation can be simulated in the present calculation with the Melbourne \( g \) matrix by taking its limit in the free space, i.e., \( \tau \) with \( \rho = 0 \).

As one sees from Eq. (15), assumption of the local momentum conservation makes numerical calculation much simple. We thus first see in Fig. 1 the accuracy of this assumption. The solid line represents the TDX calculated with \( T \) of Eq. (10) putting \( S = 4 \). The energy and the emission angle of the proton 1 are fixed at \( 250 \) MeV and \( 32.5^\circ \), respectively, while the emission angle of the proton 2 is varied. The azimuthal angle of the detector for the proton 1 (proton 2) is \( 0^\circ \) (\( 180^\circ \)) in the Madison convention. We refer to this experimental condition as \emph{kinematics 1} following Ref. [16]. The horizontal axis of Fig. 1 is the recoil momentum \( p_R \) of \( ^{11}\text{B} \) that is to be interpreted as the momentum of the struck target proton in the initial state; the positive (negative) sign of \( p_R \) means that the projection of \( p_R \) onto the \( z \)-axis (the incident beam direction) is negative (positive). Also shown by the dotted line is the same as the solid one but assuming the local momentum conservation, i.e., using \( T^{\text{mc}} \) of Eq. (15). In both calculations the \( p-^{12}\text{C} \) optical potential based on the Dirac phenomenology [17] is adopted to generate the distorted waves and the proton single-particle wave function in the 1p3/2 state in \( ^{12}\text{C} \) is calculated with the global potential of Bohr and Mottelson [18]. We will discuss the dependence of the TDX on these potentials below. The two calculations shown in Fig. 1 agree well with each other, from which one can conclude that the local momentum conservation is practically fulfilled in the calculation of the TDX. Therefore, we henceforth show only the results of TDX assuming this conservation.

In Fig. 2 we show the result of the TDX with the \( T \) matrix of Eq. (15) compared with the experimental data [16]; the theoretical result is averaged over the the emission momentum \( k_1' \) of the proton 1 and the polar and azimuthal angles of the two

![Fig. 1: The TDX for the 1p3/2 proton knockout \((p, 2p)\) reaction from \(^{12}\text{C} \) at 392 MeV corresponding to \( \text{kinematics 1} \), as a function of the recoil momentum. The solid line shows the theoretical result including full kinematics, while the dotted line shows the result assuming the local momentum conservation. In both calculations the spectroscopic factor is set to be 4.](image-url)
follows. If the plane wave approximation is made to the TDX shown by the dotted line in Fig. 2, which can be explained as caused by the difference between the two results in Fig. 2; re-

found by detailed analysis that the on-shell approximation to the matrix elements of $\tau$ and $S = 1.84$. The experimental data are taken from Ref. [16].

The dotted line in Fig. 2 shows the result with the simplified matrix of Eq. (17), neglecting the in-medium effects due to the $pp$ interaction to the TDX is only about 20%, the change in $Q$ corresponding to the left (right) peak of the TDX is 3.0 (4.0) fm$^{-1}$. One sees from Fig. 3 that $d\sigma_{ppC}/d\Omega$ for the right peak is twice as large as that for the left peak. Thus, the $Q$ dependence of $d\sigma_{ppC}/d\Omega$ gives the asymmetry of the TDX. Note that if the on-shell $pp$ scattering is assumed, $Q$ is uniquely determined by the fixed $q$ as $Q = 3.1$ fm$^{-1}$. Although the contribution of the central $pp$ interaction to the TDX is only about 20%, the change in $d\sigma_{ppC}/d\Omega$ due to the off-shell property shown in Fig. 3 is clearly seen in Fig. 2.

There are other three sets of TDX data for the proton knock-off ($p, 2p$) reaction from the 1p3/2 orbit in $^{12}$C, corresponding to different kinematical conditions [16]. We show in Fig. 4 the results of the TDX just in the same way as in Fig. 2. In the kinematics adopted, which we refer to as kinematics $\mathfrak{f}$, the

Eq. (17), one finds

$$\frac{d^3\sigma}{dE_{1}d\Omega_{1}d\Omega_{2}} \propto \left| \tilde{f}_{1p_{\frac{3}{2}},m}(p_R) \right|^2 \frac{d\sigma_{pp}^{on}}{d\Omega},$$

where $d\sigma_{pp}^{on}/d\Omega$ is the on-shell $pp$ elastic cross section dictated by the momentum transfer $q = k_i - k_f$. Since the TDX concerned here is measured under the kinematical condition so that $q$ is fixed at 2.5 fm$^{-1}$, the TDX calculated with Eq. (18) has the same shape as $\left| \tilde{f}_{1p_{\frac{3}{2}},m}(p_R) \right|^2$, which is an even function of $p_R$. Although effects of distortion slightly affect the shape of the TDX, they never provide it with the striking asymmetry. Therefore, it can be said that the asymmetry of the two peaks of the TDX shown in Fig. 2 is evidence of the off-shell $pp$ scattering taken place in the ($p, 2p$) process.

If one considers in Eq. (18) the off-shell property of the $pp$ cross section, it depends also on $Q \equiv k_i + k_f$. In Fig. 3 we show the off-shell cross section of the $pp$ scattering due to the central part of the Melbourne $g$ matrix in the limit of the free space, i.e., at zero Fermi momentum, is adopted. The momentum transfer $q$ is set to be 2.5 fm$^{-1}$.

detectors, i.e., $\theta^i_1, \phi^i_1, \theta^i_2$ and $\phi^i_2$. The widths for these quantities corresponding to the experiment concerned are shown in Table I. This smearing procedure is done for all the results of TDX shown below. The spectroscopic factor $S$ in Eq. (2) is determined by the $\chi^2$ fit of the calculated TDX to the experimental data. We obtain $S = 1.71$ that agrees very well with the value determined by the previous $(e, e'p)$ experiment, i.e., $S = 1.72 \pm 0.11$ [19]. The change of the distorting potential and the proton single-particle potential adopted is found to have no effects on the shape of the resulting TDX but bring about change in $S$ of about 10%. This uncertainty is, however, of the same order as of the $(e, e'p)$ experimental value of $S$. The dotted line in Fig. 2 shows the result with the simplified $T$ matrix of Eq. (17), neglecting the in-medium effects due to the Pauli principle; the resulting $S$ is 1.84.

One clearly sees that the solid line reproduces the experimental data [16] very well in contrast to the dotted line. It is found by detailed analysis that the on-shell approximation to $M$ causes the difference between the two results in Fig. 2; re-

fractive effects of distortion and in-medium effects concerned with the Pauli principle turn out to be negligible in the present analysis of the TDX. Thus, it is concluded that the use of the off-shell matrix elements of $\tau$ is necessary to reproduce the experimental data. In fact, the on-shell approximation to $M$ inevitably results in the symmetric two peaks of the TDX as shown by the dotted line in Fig. 2, which can be explained as follows. If the plane wave approximation is made to the $\chi$ in

| $\Delta k_1^i$ | $\Delta \theta_1^i$ | $\Delta \theta_2^i$ | $\Delta \phi_1^i$ | $\Delta \phi_2^i$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| 0.022 $\times$ $k_1^i$ | 20 [mrad] | 30 [mrad] | 50 [mrad] | 45 [mrad] |

FIG. 2: Effects of the off-shell $pp$ scattering on the TDX. The solid line is the same as the dotted line in Fig. 1 but with $S = 1.71$. The dotted line is the result with the on-shell approximation to the matrix elements of $\tau$ and $S = 1.84$. The experimental data are taken from Ref. [16].

FIG. 3: The central part of the off-shell $pp$ cross section as a function of the exchanged momentum transfer $Q$; the Melbourne $g$ matrix in the limit of the free space, i.e., at zero Fermi momentum, is adopted. The momentum transfer $q$ is set to be 2.5 fm$^{-1}$.

TABLE I: The widths for $k_1^i, \theta_1^i, \phi_1^i, \theta_2^i$ and $\phi_2^i$ used to average the theoretical TDX. See the text for details.
energy of the proton 1 is chosen to be 250 MeV and the angle between the two detectors are almost fixed around 83°, i.e., 79°–86°. As discussed in Ref. [16], kinematics 3 was designed to minimize the off-shell property of the pp scattering in the target nucleus. This is indeed confirmed by the negligibly small difference between the two theoretical results shown in Fig. 4; both results reproduce the experimental data [16] well. The value of $S$ corresponding to the solid (dotted) line is 1.64 (1.78). Other features of the DWIA calculation, i.e., effects of refraction and in-medium modification to $\tau$ due to the Pauli principle, as well as the accuracy of the assumption of the local momentum conservation, are found to be the same as in the case of kinematics 1.

In summary, the triple differential cross sections (TDX's) for the 1p3/2 proton knockout ($p, 2p$) reaction from $^{12}$C at 392 MeV are analyzed with nonrelativistic distorted wave impulse approximation (DWIA) explicitly taking account of effects of distortion, the refractive effect in particular, and those of in-medium modification to the matrix elements $M$ of the proton-proton ($pp$) effective interaction $\tau$. We show by numerical calculation that the conservation of the total momenta of the colliding two protons is practically fulfilled in the description of the TDX's of the ($p, 2p$) reaction concerned. In-medium modification to $\tau$ corresponding to the Pauli principle and the refractive effect of distortion on the kinematics of the incoming and outgoing protons are both found to have no importance in the present analysis of the TDX's, which justifies previous nonrelativistic DWIA calculations in part. However, explicit treatment of the off-shell matrix elements of $\tau$ is necessary to reproduce the experimental data of the TDX with the kinematics in which the momentum transfer is fixed, i.e., kinematics I. Indeed, the asymmetry of the two peaks of the TDX is reproduced only when the off-shell matrix elements of $\tau$ are used, hence, this asymmetry is inferred to be evidence of the off-shell $pp$ scattering in the ($p, 2p$) process. In the analysis of the TDX with different kinematics in which the off-shell effects are expected to be small, i.e., kinematics 3, both calculations with and without the on-shell approximation to $M$ reproduce the experimental data well. Thus, the successful description of the TDX's with accurate nonrelativistic DWIA is opening the door to profound understanding of the reaction mechanism of ($p, 2p$) reactions. Analysis of the spin observables, the analyzing power $A_\tau$ in particular, with the present DWBA calculation, after inclusion of the spin-orbit part of the distorting potential as in Ref. [11], will be very interesting and important.

**Acknowledgments**

The authors would like to thank T. Noro and M. Kawai for valuable discussions. This work has been supported in part by the Grants-in-Aid for Scientific Research (Grant No. 17740148) of the Ministry of Education, Science, Sports, and Culture of Japan.

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