Directional information coupling dynamics in complex climate system

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Abstract Climate change has been significantly disturbing the dynamics of different earth system components, such as land surface and ocean, as well as the interactive relationship between different components. Here we aim to investigate the sea surface temperature dynamics and its remote connection to the precipitation patterns. Previous efforts on the remote ocean-land remote coupling are mostly relied on linear based statistical inference framework, disregarding the nonlinearity of the earth system dynamics. Here we apply a new inference framework that fully adapts to nonlinear system to quantify the coupling strength between Atlantic Oceanic temperature signals (AMO index) and US precipitation patterns. We found that the linear based coupling patterns are significantly different from the nonlinear based coupling patterns, which provides important insights into the system nonlinearity. We also conduct uncertainty analysis to quantify the estimated coupling strength uncertainty and discuss the robustness of the climate coupling between AMO and US precipitation.

1. Introduction
Climate warming has been ongoing since the pre-industrial era, due to the vast amount of greenhouse gas emissions and the positive radiative forcing of the potent greenhouse gases, such as CO₂ and CH₄. However, the warming over the ocean and land surfaces is neither evenly distributed, nor independently developed. Intrinsic relationship between land surface and sea surface temperature warming provides important insights into how the coupled earth system climate evolve dynamically, and how remote oceanic-land temperature signals relate to one another. Here, we particularly focus on the relationship between the Atlantic Sea Surface Temperature (SST) and United States land surface temperature. For the oceanic SST, the Atlantic Multidecadal Oscillation (AMO) index is considered, which has been known as the North Atlantic sea surface temperatures (SSTs) oscillating between high and low temperatures with a period of 60–80 years over a multidecadal timescales [1][2]. The AMO index of North Atlantic SST was calculated by averaging the low-pass filtered annual mean SST Anomalies over the region 0–60 N, 75 W–7.5 W [3]. A number of studies have shown that AMO have strong impacts to global and regional climate changes. For example, the AMO is associated with droughts and rainfall frequency over the conterminous United States. It may have a crucial influence on the teleconnection between the El Niño Southern Oscillation (ENSO) and winter precipitation [4]. AMO has been proposed to play an important role in the decades of changes in summer rainfall in India and the Sahel in the 20th century and is critically relevant to Atlantic hurricane activity [1]. The opposite phase of AMO anomaly from 1950 to 2015 corresponds to the significant difference in activity of atmospheric action and regional zonal circulation and heat and moisture transfer from the Atlantic to the continent [5]. North Atlantic Ocean have been suggested to be a key driver of European
AMO is remarkably consistent with anomalies in surface air temperature (SAT), sea level pressure (SLP) and precipitation in European [6].

Robustly investigating the relationship between AMO and land surface temperature is challenging due to the complexity and nonlinearity of earth system dynamics. Previous efforts have applied many different statistical inference frameworks to tackle this question. For example, Yu et al. applied empirical-orthogonal-function (EOF) analyses to the seasonal warm anomalies for getting the principal components (PCs). Based on the Linear correlation analysis between the PCs and the AMO index, they find that AMO can make a significant contribution to seasonal warm anomalies across the contiguous U.S. in the three most recent decades through large-scale atmospheric circulation [7]. However, correlation coefficients are not complete to describe correlation between variables since it just measures the linear dependence. In this work, we aim to overcome the challenge of nonlinearity in studying the complex relationship between AMO and land surface temperature dynamics. We propose to use the information entropy, which to quantify the amount of uncertainty involved in the value of a random variable or the outcome of a random process [8]. Particularly, the mutual information measures the information shared between two processes, which account for nonlinear dependence. we suggest that it is more suitable to apply the mutual information to evaluate the non-linear coupling between the AMO index and the land temperature or precipitation, compared with linear correlation-based analysis.

2. Methodology

2.1. Data description

We obtain Sea Surface Temperature (SST) for the region that define AMO Index Data from the Climate Analysis Section, NCAR, Boulder, USA, Trenberth and Shea (2006) [9], accessed January 30, 2019. We use the detrended AMO that averaged over the region 0-80N from 1958 to 2013 [6]. For precipitation data, we use the monthly Global Precipitation Climatology Center (GPCC) product ranging from 1958 to 2013 (56 years) [10]. Below, Figure 1 shows the detrended monthly averaged AMO index, which covers a relative dominated cold phase from about 1959 to 1995. Figure 2 shows the United States precipitation annual mean and standard deviation. Precipitation rates are high over the western coastal area and the southeast part of US.
2.2. Linear statistical model

We first use Pearson correlation coefficient to investigate the coupling relationship between AMO and US precipitation at monthly time scale.

\[
\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}
\]  

(1)

where \(\text{cov}(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]\) is the covariance, \(E\) is the expectation. \(\mu_X\) is the mean of \(X\), \(\mu_Y\) is the mean of \(Y\). \(\sigma_X\) is the standard deviation of \(X\), \(\sigma_Y\) is the standard deviation of \(Y\). The Pearson correlation has a value of between +1 and -1, where 1 is positive correlation, 0 is no linear correlation, while -1 is negative correlation.

2.3. Non-linear statistical model

We also use the mutual information derived from Shannon information entropy theory [9] to quantify the relationship between AMO and US regional precipitation dynamics.

\[
H(X) = -\sum_x p(x) \log_2 p(x)
\]  

(2)

\[
H(X|Y) = -\sum_{x,y} p(x,y) \log_2 p(x|y).
\]  

(3)

\[
I(X;Y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}.
\]  

(4)

where Eqn. 2 define the Shannon entropy \(H\), in the units of bits as it use a logarithm of base. \(p(x)\) is the probability of the source process \(X\) to occur. The conditional entropy of two processes \(X\) and \(Y\) is given by Eqn. 3. Where \(p(x,y)\) is the joint distribution, \(p(x|y)\) is the conditional distribution of \(X\) given \(Y\). The mutual information of \(X\) relative to \(Y\) is given by Eqn. 4. The mutual information measures mutual dependence between two variables which contain linear and nonlinear dependence [9]. When the special case that absolute values of Pearson correlation coefficient equal to 1, which means \(X\) and \(Y\) have a perfect linear correlation. In this case, the mutual information of \(X\) relative to \(Y\) are given as: \(I(X;Y) = H(X) = H(Y)\).
2.4. Analysis

In this study, we utilize two different types of statistical tools: linear and Non-linear inferences. Given the AMO index as the X variable and the US precipitation data is the Y variable. First, we apply the Pearson correlation coefficient equation (Eqn. 1) to analyse the linear correlation between AMO index data and American land precipitation data. Second, we evaluate the non-linear mutual information by measuring mutual information of AMO index relative to US precipitation data is given by Eqn. 4. Accurate estimation of the marginal possibility distribution function (pdf) of X and Y is crucial to estimate mutual information. We have used Ordinary ranking (ORD) and Gaussian kernel estimation (GKE) to estimate possibility distribution function.

3. Results and Discussion

3.1. Probability density estimate for mutual information calculation

Probability density distribution (PDF) of AMO and precipitation time series are estimated by Ordinary ranking (ORD) and Gaussian kernel estimation (GKE) with different bin numbers. As shown in the Figure 3, PDF obtained by ORD is significantly affected (high sensitivity and uncertainty) by the change in bins value. By contrast, the PDF obtained with GKE is relatively stable as bins' values equal to 9, 10, 11, 12. Therefore, we conclude that GKE is a more stable and reliable PDF estimator for our analysis.

![Figure 3](image)

**Figure 3** The possibility distribution function of normal distributed random samples estimated by Gaussian kernel estimation (GKE) and Ordinary ranking (ORD). bin=9,10,11,12 for top left, top right, bottom left, bottom right panels.

3.2. Linear versus nonlinear relationship inference

Pearson Correlation coefficient is a measure of linear and non-directional relationship between two variables. But Mutual information is a measure of directional non-linear causality. Here we quantify the spatial patterns of Pearson correlation coefficient (Figure 4 left panel) and mutual information (Figure 4 right panel) between the AMO and precipitation dynamics over the United States. Figure 4 shows that the two spatial pattern are not consistent. In general, the linear dependency between AMO and US precipitation is weak ($r^2<0.2$), while the non-linear relationship between AMO and US precipitation is relatively stable and statistically significant across a large part of US (colored regions all pass statistical significance test). Comparing the spatial distribution, the western United States, especially the southwestern region, has a strong non-linear relationship, while very little linear
dependency. This indicates a very strong nonlinear coupling relationship between the remote AMO and precipitation over this region, which may not have been identified and considered in previous studies, due to the statistical inference method limitation. In northern and central United States, both the linear and nonlinear coupling relationships are prominent, which is consistent with what previous studies suggested that AMO had a significant role in determining the circulation and participation in this region. [11] One possible reason is that our data covers a significant long AMO cold phase.

Figure 4 Spatial pattern of the Pearson correlation coefficient and Mutual information (PDF=GKE, bin=14) between AMO and precipitation in the United States.

3.3. Climate change effects on ocean-land coupling
Here we divide the AMO time series data into two equal segments, each has 28-year: 1958-1985 and 1986-2013. Since the 20th century, the average temperature of the earth's climate system has risen rapidly. It will increase the amount of evaporation and cause more precipitation. We attempted to quantify the effect of temperature rise on the nonlinear coupling between the Atlantic and continental precipitation in the United States by comparing the first and second 28-year AMO and precipitation mutual information fluctuations. Figure 5 shows that the mutual information between the AMO and precipitation have not changed significantly. Which means that global warming does not have a detectable impact on this coupling strength between the two processes of interest.

Figure 5 Spatial pattern of the Mutual information between amo and precipitation in the United States from 1958-1985(left) and 1986-2013(right). (PDF=GKE, bin=14)

3.4. Uncertainty analysis
The mutual information between AMO and precipitation will be affected by some uncertain factors, such as our choice of Probability Density Function (PDF) and Bin value. We conduct uncertainty analysis to show in Figure 6, that the mutual information between AMO and precipitation are subject to uncertainties. Figure 6 top panels use Ordinary Ranking (ORD) as the PDF estimator, and the Bin numbers are set to 10, 12, and 14, respectively. As the bin number increases, the identified nonlinear coupling relationship become stronger. The bottom panel of Figure 6 selects GKE as the pdf. As the Bin value increase, the change in the mutual information is limited, reflecting the stability of identified spatial distribution of mutual information. Consistent with our analysis in Figure 3, GKE is more stable as a PDF, which helps us to obtain a more accurate assessment of the nonlinear coupling between AMO and precipitation. By comparing the spatial maps of the top and bottom panels, it can be seen that the mutual information are different and the mutual information value calculated by ORD is higher.

Figure 6 Spatial pattern of the Mutual information between AMO and precipitation in the United States.PDF=GKE or ORD; Bin=10 or 12 or 14.

4. Conclusions
In this study, we quantify the coupling relationship between Atlantic Ocean surface temperature (AMO) and the US regional precipitation, through linear and non-linear inferences. We find that the linear relationship between AMO and US precipitation are generally weak, while the nonlinear relationships are statistically significant and particularly strong over the southwestern US. Such a strong nonlinear coupling relationship in the southwestern US is complex and likely associated with other important climatic mode ,i.e., ENSO, PDO and SOI. Previous study suggested that strong ENSO teleconnections with participation in the western US appears associated with the negative PDO phase and out of SOI phase which characterize the interaction of tropical air and sea circulation. [12] To understand the nonlinear coupling relationship between the AMO index and the participation furtherly, there still need more studies to figure out physical processes and the associations between various climate modes. In
northern and central United States, previous studies suggested that positive precipitation anomalies have been observed in this region during the negative phase of the AMO. [11] Such result also indicated in our linear analysis probably since our time series data is dominated by a cold phase in this region. However, the nonlinear coupling are also statistically significant over northern and central US, which has not been identified in previous studies. Moreover, climate warming during the historical period is not strong enough to significantly alter such non-linear coupling relationship. Finally, our uncertainty analysis reveal that the non-linear coupling patterns are robust using GKE probability density function estimator, while less robust using ORD probability density function estimator, the latter is sensitive to the bin numbers and does not clearly show spatial distinctions in the coupling strength over the US regions.

5. References

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