Abstract—In this correspondence, the comprehensive problem of joint power, rate, and subcarrier allocation have been investigated for enhancing the spectral efficiency of multi-user orthogonal frequency-division multiple access (OFDMA) cognitive radio (CR) networks subject to satisfying total average transmission power and aggregate interference constraints. We propose novel optimal radio resource allocation (RRA) algorithms under different scenarios with deterministic and probabilistic interference violation limits based on perfect and imperfect availability of cross-link channel state information (CSI). In particular, we propose a probabilistic approach to mitigate the total imposed interference on the primary service under imperfect cross-link CSI. A close form mathematical formulation of the cumulative density function (cdf) for the received signal-to-interference-plus-noise ratio (SINR) is formulated to evaluate the resultant average spectral efficiency (ASE). Dual decomposition is utilized to obtain sub-optimal solutions for the non-convex optimization problems. Through simulation results, we investigate the achievable performance and the impact of parameters uncertainty on the overall system performance. Furthermore, we present that the developed RRA algorithms can considerably improve the cognitive performance whilst abiding to the imposed power constraints. In particular, the performance under imperfect cross-link CSI knowledge for the proposed ‘probabilistic case’ is compared over the conventional scenarios to show the potential gain in employing this scheme.

I. INTRODUCTION

In recent years, a significant effort has been made towards improving the spectral efficiency of cellular networks in order to meet the growing demand and sophistication of wireless applications. Several technologies, such as small-cell (SC) solution, Full Duplex (FD) [1], cognitive radio (CR) networks and massive multiple-input multiple-output (MIMO), [2] - each with respective advantages and challenges - are promising candidates in this direction [3]. CR is outlined as an smart radio network that has the ability to sense the primary service behaviour and surrounding environment and adjust its spectrum usage and parameters based on the observed information [4]. According to Ofcom flexible spectrum-sharing for supporting CR and spectrum co-existence is a priority issue to overcome the current capacity crunch.

Orthogonal frequency-division multiplexing (OFDM) has emerged as a prominent technology for new generation of wireless communication systems and adopted in many modern wireless technologies. For OFDM-based multi-user applications, multiple-access can be accommodated through orthogonal frequency-division multiple-access (OFDMA) technique. In OFDMA systems, different subcarriers may be assigned to different users in order to exploit the random variations of users across each subcarrier. OFDMA is considered as an effective technology for CR networks as a result of the inherent advantages with regards to adaptability and flexibility in allocating radio resources in a shared-spectrum environments.

Radio resource Management (RRM) plays an important role in optimizing the spectral efficiency of conventional OFDMA systems [5] [6] [7]. Adaptive RRM is a prominent field for research in the context of multi-user CR networks with the aim of obtaining a balance between the CR performance and reducing the induced interference on the primary users. Optimal and suboptimal RRM policies are studied in [8], where the aggregate throughput of the CR system is maximized under a primary receiver (PRx) interference limit. Dynamics and adaptive MAC retry-limit aware adaptation is proposed in [9]. A queue-aware RRA algorithm is proposed in [10], to maximize the fairness in OFDMA-based CR networks subject to a total power constraint at the base station. A Lagrangian relaxation algorithm is adopted in [11] to probabilistically allocate resources based on the availability of the primary frequency band via spectrum sensing.

Most of the RRA algorithms for CR networks in the literature assume perfect channel state information (CSI) between the cognitive transmitter (CTx) and PRx, and few have considered imperfect cross-link CSI. However, due to technical reasons such as estimation errors and wireless channel delay, obtaining perfect cross-link CSI is difficult in practical scenarios. In [12] and [13], the ergodic capacity is derived over fading channels with imperfect cross-link knowledge, however, the analysis is carried out for a single cognitive user (CU). Furthermore, due to noisy cross-link information, it is unrealistic to assume that the secondary network strictly satisfies a deterministic interference constraint. The authors in [14] propose a RRA algorithm for maximizing instantaneous rate in downlink OFDMA CR systems subject to satisfying a collision probability constraint. However, [14] only considers the individual impact of probabilistic interference constraint per subcarrier.

To the best of authors’ knowledge, enhancing the average spectral efficiency of multi-user OFDMA-based CR systems has not been addressed in the literature. In this work, by
exploiting the advantages of channel adaptation techniques, we propose novel joint power, subcarrier, and rate allocation algorithms for enhancing the average spectral efficiency of downlink multi-user adaptive M-ary quadrature amplitude modulation (MQAM)/OFDMA CR systems. Given the received power restrictions on the CTx in order to satisfy the primary network interference limit and the cognitive network power constraint, the CTx transmit power is a function of the cognitive-cognitive direct-link and cognitive-primary cross-link fading states. We develop a closed-form expression for the cumulative distribution function (cdf) of the CR’s received signal-to-interference-plus-noise ratio (SINR) to evaluate the average spectral efficiency of the adaptive MQAM/OFDMA CR system.

The organization of this paper is as follows: Section II presents the network model and operation assumptions. In Section III, the resource allocation problem for enhancing average spectral efficiency of the adaptive multi-user MQAM/OFDMA under perfect cross-link CSI subject to power and deterministic interference constraints is developed. Section IV investigates the performance under a collision probability constraint with imperfect cross-link CSI and proposes a deterministic formulation of the probabilistic aggregate cross-link interference. Illustrative numerical results for various scenarios under consideration are provided in Section V. Finally, concluding remarks are presented in Section VI.

II. SYSTEM MODEL AND PRELIMINARIES

In this section, the multi-user OFDMA CR network model and operation assumptions are introduced. Further, interference management schemes and spectral efficiency of the adaptive MQAM/OFDMA system under consideration are studied.

A. Network Architecture and Wireless Channel

We consider an underlay shared-spectrum environment, as shown in Fig. 1 where a cognitive network with a single CTx and $n \in \{1, ..., N\}$ cognitive receivers (CRxs) coexist with a primary network with a primary transmitter (PTx) and $m \in \{1, ..., M\}$ PRxs. The cognitive network can access a spectrum licensed to the primary network with a total bandwidth of $B$ which is divided into $K$ non-overlapping sub-channels subject to not violating the imposed interference constraint set by a regulatory authority. The sub-channel bandwidth is assumed to be much smaller than the coherence bandwidth of the wireless channel, thus, each subcarrier experiences frequency-flat fading. Let $H^{sp}_{n,k}(t)$, $H^{ps}_{n,k}(t)$, and $H^{ss}_{n,k}(t)$, at time $t$, denote the channel amplitude gains over sub-channel $k$ from the CTx to $n^{th}$ CRx, PTx to the $n^{th}$ CRx, and CTx to $m^{th}$ PRx respectively. The channel power gains $|H^{sp}_{n,k}(t)|^2$, $|H^{ps}_{n,k}(t)|^2$, and $|H^{ss}_{n,k}(t)|^2$ are assumed to be ergodic and stationary with continuous probability density functions (pdfs) $f(|H^{sp}_{n,k}(t)|^2)(\cdot)$, $f(|H^{ps}_{n,k}(t)|^2)(\cdot)$, and $f(|H^{ss}_{n,k}(t)|^2)(\cdot)$, respectively. In addition, the estimated instantaneous values and distribution information of the secondary-secondary channel power gains is assumed to be available at the CTx [13]. In this work, we consider different cases with perfect and noisy cross-link knowledge between CTxs and PRxs.

Each sub-channel is assigned exclusively to at most one CRx at any given time, hence, there is no mutual interference between different cognitive users [15]. It should also be noted that by utilizing an appropriate cyclic prefix, the inter-symbol-interference (ICI) can be ignored [16]. The received SINR of cognitive user $n$ over sub-channel $k$ at time instant $t$ is

$$\gamma_{n,k}(t) = \frac{P_{n,k}|H^{ss}_{n,k}(t)|^2}{\sigma^2_n + \sigma^2_{ps}} \tag{1}$$

where $P_{n,k}$ is a fixed transmit power allocated to cognitive user $n$ over sub-channel $k$, $\sigma^2_n$ is the noise power, and $\sigma^2_{ps}$ is the received power from the primary network. Without loss of generality, $\sigma^2_n$ and $\sigma^2_{ps}$ are assumed to be the same across all users and sub-channels [17, 18]. Assuming stationariness of the channel gain, for the sake of brevity, we henceforth omit the time reference $t$.

Due to the impact of several factors, such as channel estimation error, feedback delay, and mobility, perfect cross-link information is not available [19]. With noisy cross-link CTxs to PRxs knowledge, we model the inherent uncertainty in channel estimation in the following form

$$H^{sp}_{m,k} = H^{sp}_{m,k} + \Delta H^{sp}_{m,k} \tag{2}$$

where over subcarrier $k$, $H^{sp}_{m,k}$ is the actual cross-link gain, $\hat{H}^{sp}_{m,k}$ is the channel estimation considered to be known, and $\Delta H^{sp}_{m,k}$ denotes the estimation error. $H^{sp}_{m,k}$, $\hat{H}^{sp}_{m,k}$, and $\Delta H^{sp}_{m,k}$ are assumed to be zero-mean complex Gaussian random variables with respective variances $\delta^2 H^{sp}_{m,k}$, $\delta^2 \hat{H}^{sp}_{m,k}$, and $\delta^2 \Delta H^{sp}_{m,k}$ [12, 20]. For robust receiver design, we consider the estimation $\hat{H}^{sp}_{m,k}$ and error $\Delta H^{sp}_{m,k}$ to be statistically correlated random variables with a correlation factor $\rho = \sqrt{\delta^2 H^{sp}_{m,k}}/\sqrt{\delta^2 \hat{H}^{sp}_{m,k} + \delta^2 \Delta H^{sp}_{m,k}}$, where $0 \leq \rho \leq 1$.

B. Interference Management

In a shared-spectrum environment, and particularly for delay-sensitive services, the licensed users’ quality of service (QoS) is highly dependent on the instantaneous received SIRs of cognitive users. In order to protect the licensed spectrum from harmful interference we impose a deterministic peak total interference constraint between CTxs and primary users

$$\sum_{n=1}^{N} \sum_{k=1}^{K} \varphi_{n,k}(\gamma_{n,k})P_{n,k}(\gamma_{n,k})|H^{sp}_{m,k}|^2 \leq I_{th}, \forall m \in \{1, ..., M\} \tag{3}$$

where $\varphi_{n,k}(\gamma_{n,k})$ is the time-sharing factor (subcarrier allocation policy), $P_{n,k}(\gamma_{n,k})$ is the allocated transmit power, and $I_{th}$ denotes the maximum tolerable interference threshold.

However, as a consequence of uncertainties about the shared-spectrum environment and primary service operation, it is unrealistic to assume that the CTx always satisfies the deterministic peak total interference constraint. In practical
scenarios, probability of violating the interference is confined
to a certain value that satisfies the minimum QoS require-
ments of primary users. Probabilistic interference constraint is
particularly critical for robust interference management given
noisy cross-link knowledge. To improve the overall system
performance and to mitigate the impact of channel estimation
efficiency of the adaptive MQAM/OFDMA CR network. In
this section, we solve the resource allocation problem
with perfect cross-link knowledge and deterministic interference
constraint.

A. Problem Formulation
Mathematically, the optimization problem can be stated as
follows.

Problem $\mathcal{O}_1$:

\[
\max_{\varphi_{n,k}, P_{n,k}} \sum_{n=1}^{N} \sum_{k=1}^{K} E_{\gamma_{n,k}} \left\{ \log_2 (M_{n,k}(\gamma_{n,k})) \varphi_{n,k}(\gamma_{n,k}) \right\}
\]
\[
\text{s. t.: } \sum_{n=1}^{N} \sum_{k=1}^{K} E_{\gamma_{n,k}} \left\{ \varphi_{n,k}(\gamma_{n,k}) P_{n,k}(\gamma_{n,k}) \right\} \leq P_t
\]
\[
\sum_{n=1}^{N} \sum_{k=1}^{K} \varphi_{n,k}(\gamma_{n,k}) P_{n,k}(\gamma_{n,k}) |H_{m_{n,k}}^{sp}|^2 \leq I_{th}^m, \forall m \in \{1, ..., M\}
\]
\[
\sum_{n=1}^{N} \varphi_{n,k}(\gamma_{n,k}) = 1, \forall k \in \{1, ..., K\}
\]
\[
\varphi_{n,k}(\gamma_{n,k}) \in \{0, 1\}, \forall n \in \{1, ..., N\}, \forall k \in \{1, ..., K\}
\]
\[
\xi_{n,k}(\gamma_{n,k}) \leq \xi, \forall n \in \{1, ..., N\}, \forall k \in \{1, ..., K\}
\]
where $\xi$ denotes the common BER-target.

In the adaptive multi-user MQAM/OFDMA \cite{22, 24} CR
system under consideration, different transmit power and con-
stellation sizes are allocated to different users and subcarri-
ers. Using the upper-bound expression for the Gaussian Q-
function, i.e., $Q(x) \leq (1/2) \exp(-x^2/2)$, the instantaneous

Employing square MQAM with Gray-coded bit mapping,
the approximate instantaneous bit-error-rate (BER) expression
for user $n$ over subcarrier $k$ is given by \cite{16, 22},

\[
\xi_{n,k}(\gamma_{n,k}) = \frac{4}{\log_2(M_{n,k}(\gamma_{n,k}))} \left( 1 - \frac{1}{\sqrt{M_{n,k}(\gamma_{n,k})}} \right) \times Q\left( \sqrt{\frac{3\gamma_{n,k}}{M_{n,k}(\gamma_{n,k}) - 1}} \right)
\]

(6)

where $M_{n,k}(\gamma_{n,k})$ denotes the constellation size vector of
MQAM when each element is a function of the instantaneous
received SINR of the cognitive user $n$ over subcarrier $k$, and
$Q(\cdot)$ represents the Gaussian Q-function.

The average spectral efficiency of the adaptive multi-user
MQAM/OFDMA system per subcarrier per user over the fading channel is defined as

\[
ASE = \sum_{n=1}^{N} \sum_{k=1}^{K} E_{\gamma_{n,k}} \left\{ \log_2 (M_{n,k}(\gamma_{n,k}) \varphi_{n,k}(\gamma_{n,k})) \right\}.
\]

(7)

In order to evaluate the $ASE$, the distribution of the received
SINR, a function of secondary-secondary and secondary-
primary channels, must be developed.

III. DETERMINISTIC INTERFERENCE CONSTRAINT WITH
PERFECT CROSS-LINK CSI

In this section, we solve the resource allocation problem
with perfect cross-link knowledge and deterministic interference
constraint.

C. Spectral Efficiency

The focus of this work is mainly on optimal power, rate,
and subcarrier allocation for enhancing the average spectral
}
BER for user $n$ over subcarrier $k$, subject to an instantaneous constraint $\xi_{n,k}(\gamma_{n,k}) = \xi$ can be expressed as
\[
\zeta_{n,k}^h(\gamma_{n,k}) \leq 0.3 \exp\left(-\frac{1.5\gamma_{n,k}}{M_{n,k}(\gamma_{n,k}) - 1 \min\left(\frac{P_n}{K \cdot N_m^s}\right)}\right),
\]
(9)
where $N_m^s = \sum_{k=1}^K |H_{m,k}^{sp}|^2$. With further manipulation, for a BER-target $\xi$, the maximum constellation size for user $n$ over subcarrier $k$ is obtained as
\[
M_{n,k}(\gamma_{n,k}) = 1 + \frac{\zeta_{n,k}^h P_n(\gamma_{n,k})}{\min\left(\frac{P_n}{K \cdot N_m^s}\right)},
\]
(10)
where
\[
\zeta = -1.5 \ln(\xi/0.3).
\]
(11)
According to the constraints \[(8b)\] and \[(8c)\] in the optimization problem $\Theta_1$, the joint cumulative density function (cdf) of $\gamma_{n,k}$ can be written as:
\[
F_{\gamma_{n,k}}(\Gamma) = \mathcal{P}\left(\frac{P_t}{K(\sigma_n^2 + 2\sigma_{ps}^2)} \leq \Gamma, \int_{0}^{\infty}\frac{I_{th}^{m}|H_{n,k}^{ss}|^2}{N_m^s(\sigma_n^2 + 2\sigma_{ps}^2)} \leq \Gamma\right).
\]
(12)
The probability expression in \[(12)\] can be further simplified by considering the cases $\frac{P_t}{K(\sigma_n^2 + 2\sigma_{ps}^2)} \leq I_{th}^{m}|H_{n,k}^{ss}|^2/\sigma_n^2(\sigma_n^2 + 2\sigma_{ps}^2)$ and conditioning on $N_m^s$
\[
1 - \mathcal{P}\left(\frac{P_t}{K(\sigma_n^2 + 2\sigma_{ps}^2)} \leq \Gamma, \int_{0}^{\infty}\frac{I_{th}^{m}|H_{n,k}^{ss}|^2}{N_m^s(\sigma_n^2 + 2\sigma_{ps}^2)} \leq \Gamma\right) = 1 - \begin{cases} \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{K(\sigma_n^2 + 2\sigma_{ps}^2)}{P_t}\right), & N_m^s = \frac{I_{th}^{m}K}{P_t}, \\ \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{N_m^s\Gamma(\sigma_n^2 + 2\sigma_{ps}^2)}{I_{th}^{m}K}\right), & N_m^s > \frac{I_{th}^{m}K}{P_t}. \end{cases}
\]
(13)

**Lemma 1:** For large values of $K$, given complex Gaussian random variables $H_{m,k}^{sp}$ with means $\mu_{H_{m,k}^{sp}}$ and equal variance $\delta_{H_{m,k}^{sp}}^2$ for all $k \in \{1, \ldots, K\}$, the non-central Chi-square random variable $N_m^s = \sum_{k=1}^K |H_{m,k}^{sp}|^2$ can be approximated as a Gaussian random variable with respective mean and variance $\mu_{N_m^s} = \delta_{H_{m,k}^{sp}}^2 \left[2K + \mu^\prime\right]$ and $\delta_{N_m^s}^2 = \delta_{H_{m,k}^{sp}}^4 \left[4K + 4\mu^\prime\right]$, where $\mu^\prime = \sum_{k=1}^K \frac{\mu_{H_{m,k}^{sp}}^2}{\delta_{H_{m,k}^{sp}}^2}$.

**Proof:** We can write $H_{m,k}^{sp} = \delta_{H_{m,k}^{sp}} G_{m,k}^{sp}$, where $G_{m,k}^{sp} \sim CN\left(\frac{\mu_{H_{m,k}^{sp}}}{\delta_{H_{m,k}^{sp}}^2}, 1\right)$. Assuming equal variance for random variables $H_{m,k}^{sp}$, $\sum_{k=1}^K |G_{m,k}^{sp}|^2$ is a non-central Chi-Square random variable with $2K$ degrees of freedom and non-centrality parameter $\mu^\prime = \sum_{k=1}^K \frac{\mu_{H_{m,k}^{sp}}^2}{\delta_{H_{m,k}^{sp}}^2}$. For large values of $K$, central limit theorem (CLT) can be invoked to show that the non-central Chi-Square random variable $\sum_{k=1}^K |G_{m,k}^{sp}|^2$ can be approximated as a Gaussian random variable as follows
\[
\sum_{k=1}^K |G_{m,k}^{sp}|^2 \sim N\left(2K + \mu^\prime, 4K + 4\mu^\prime\right).
\]
(14)
Hence, $N_m^s = \sum_{k=1}^K |H_{m,k}^{sp}|^2$ can be approximated by
\[
N_m^s = \sum_{k=1}^K |H_{m,k}^{sp}|^2 \sim N\left(\mu_{N_m^s}, \delta_{N_m^s}^2\right)
\]
(15)
where $\mu_{N_m^s} = \delta_{H_{m,k}^{sp}}^2 \left[2K + \mu^\prime\right]$ and $\delta_{N_m^s}^2 = \delta_{H_{m,k}^{sp}}^4 \left[4K + 4\mu^\prime\right]$. Denoting the pdf of $N_m^s$ with $f_{N_m^s}(\cdot)$, and the cdfs of $|H_{m,k}^{ss}|^2$ and $N_m^s$ with $F_{|H_{m,k}^{ss}|^2}(\cdot)$ and $F_{N_m^s}(\cdot)$, respectively, we obtain the cdf of $\gamma_{n,k}$ as
\[
F_{\gamma_{n,k}}(\Gamma) = 1 - A - B,
\]
(16)
where
\[
A = \int_0^{I_{th}^{m}K/P_t} \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{K(\sigma_n^2 + 2\sigma_{ps}^2)}{P_t}\right) f_{N_m^s}(N_m^s) dN_m^s
\]
\[
= \int_0^{I_{th}^{m}K/P_t} \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{K(\sigma_n^2 + 2\sigma_{ps}^2)}{P_t}\right) f_{N_m^s}(N_m^s) dN_m^s
\]
\[
= \int_0^{I_{th}^{m}K/P_t} \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{K(\sigma_n^2 + 2\sigma_{ps}^2)}{P_t}\right) F_{N_m^s}(N_m^s) dN_m^s
\]
and
\[
B = \int_{I_{th}^{m}K/P_t}^{\infty} \mathcal{P}\left(|H_{n,k}^{ss}|^2 > \frac{N_m^s \Gamma(\sigma_n^2 + 2\sigma_{ps}^2)}{I_{th}^{m}K}\right) f_{N_m^s}(N_m^s) dN_m^s.
\]
(17)
(18)
Recall that the cdf of a Normally-distributed random variable $X$ with mean $\mu$ and standard deviation $\sigma$ is given by $F_X(x) = \frac{1}{2} \left[1 + erf\left(\frac{x - \mu}{2\sigma}\right)\right]$, and the cdf of an Exponentially-distributed random variable $Y$ is computed by $F_Y(y) = 1 - e^{-y/\mu}$, where $\mu$ is the mean. Suppose that $|H_{m,k}^{ss}|^2$ follows an exponential distribution with mean $\mu_{|H_{m,k}^{ss}|^2}$, hence, the integrals in \[(17)\] and \[(18)\] can be simplified. Finally, a closed-form expression for the pdf of $\gamma_{n,k}$ is obtained in \[(19)\].

**B. Obtaining the Solutions**

It can be observed that the optimization problem, $\Theta_1$, is convex with respect to the transmit power $P_{n,k}(\gamma_{n,k})$. However, it is non-convex with respect to $\varphi_{n,k}(\gamma_{n,k})$ as the time-sharing factor only takes binary values. To obtain a sub-optimal solution for problem $\Theta_1$, we employ the Lagrangian dual decomposition algorithm. By applying dual decomposition, the non-convex optimization problem, $\Theta_1$, is decomposed into independent sub-problems each corresponding to a given cognitive user.
\[ f_{\gamma_{n,k}}(\Gamma) \approx K \left( \sigma_n^2 + \sigma_{ps}^2 \right) \exp \left( -\frac{KT(\sigma_n^2 + \sigma_{ps}^2)}{P_{t,\text{HSS}_{n,k}}^2} \right) \left( \text{erf} \left( \frac{I_{n,k}^\mu K}{\sqrt{2\delta_{N_{\text{HSS}}}} H_{\text{HSS}_{n,k}}^\mu} \right) + 1 \right) \]

\[ = \frac{2P_t |\mu| H_{\text{HSS}_{n,k}}^\mu|^2}{\sqrt{2\pi I_{n,k}^\mu |\mu| H_{\text{HSS}_{n,k}}^\mu|^2}} \left( \sigma_n^2 + \sigma_{ps}^2 \right) \delta_{N_{\text{HSS}}^\mu} \exp \left( -\frac{I_{n,k}^\mu K \mu_{\text{HSS}_{n,k}}^\mu |\mu| H_{\text{HSS}_{n,k}}^\mu|^2}{2P_t |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right) \]

\[ \times \exp \left( \frac{0.5(\sigma_n^2 + \sigma_{ps}^2)(I_{n,k}^\mu \mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2 - (\sigma_n^2 + \sigma_{ps}^2) \delta_{N_{\text{HSS}}^\mu} \Gamma)}{2I_{n,k}^\mu \mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right) \]

\[ \times \left( \text{erf} \left( \frac{I_{n,k}^\mu K - \mu_{N_{\text{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu}{\sqrt{2\delta_{N_{\text{HSS}}}} I_{n,k}^\mu |\mu| H_{\text{HSS}_{n,k}}^\mu} \right) \right) \]

\[ - \frac{I_{n,k}^\mu |\mu| H_{\text{HSS}_{n,k}}^\mu|^2}{I_{n,k}^\mu |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \]

(19)

Applying the KKT conditions yields the optimal power allocation policy for Lagrangian multipliers \( \mu \) and \( \eta(\gamma_{n,k}) \)

\[ P_{n,k}^*(\gamma_{n,k}) = \left[ \text{ln}(2) (\mu f_{\gamma_{n,k}}(\gamma_{n,k}) + \eta(\gamma_{n,k}) |H_{m,k}^{ps}|^2) - \min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right) \right]^{+} \]

(20)

The optimal subcarrier allocation problem is formulated as:

\[ n^* = \text{argmax} \left( \Lambda(\gamma_{n,k}) \right), \forall n \in \{1, \ldots, N\}, \forall k \in \{1, \ldots, K\} \]

(21)

where \( n^* \) is the optimal CRx index, and

\[ \Lambda(\gamma_{n,k}) = \left( \text{ln}(2) \int 1 + \frac{\zeta_{\gamma_{n,k},n,k} P_{n,k}^*(\gamma_{n,k})}{\min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right)} \right) \]

\[ + \left( \text{ln}(2) \int 1 + \frac{\zeta_{\gamma_{n,k},n,k} P_{n,k}^*(\gamma_{n,k})}{\min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right)} \right) \]

\[ - \frac{\text{ln}(2)}{\text{ln}(2)} \int \frac{\zeta_{\gamma_{n,k},n,k} P_{n,k}^*(\gamma_{n,k})}{\min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right)} \]

(22)

where \([x]^+ \triangleq \max\{x, 0\}\). The solution in (20) can be considered as a multi-level water-filling algorithm where each subcarrier has a distinct water-level for a given user. Note that the water levels determine the potential optimum amount of power that may be allocated to \( n^* \) CRx over subcarrier \( k \).

The optimal subcarrier allocation policy is therefore achieved by assigning the \( k^{th} \) subcarrier to the user with the highest value of \( \Lambda(\gamma_{n,k}) \) for all corresponding values of the \( \gamma_{n,k} \). To ensure optimality, \( \lambda_k(\gamma_{n,k}) \) should be between first and second maxima of \( \Lambda(\gamma_{n,k}) \). If there are multiple equal maxima, the time-slot can be identically shared among the respective users.

The subgradient method has been widely used for solving Lagrangian relaxation problems. The master problem sets the user resource allocation prices, and in order to update the dual variables, in every iteration of the subgradient method, the algorithm repeatedly finds the maximizing assignment for the sub-problems individually. For any optimal pair of \( (\gamma_{n,k}^*, \gamma_{n,k}^*) \), the dual variables of the problem are updated using subgradient iterations.

The potential optimum continuous-rate adaptive constellation size vector for user \( n \) over subcarrier \( k \) is written as

\[ M_{n,k}^*(\gamma_{n,k}) = \max \left( \frac{1}{\text{ln}(2)} \int \frac{\zeta_{\gamma_{n,k},n,k} P_{n,k}^*(\gamma_{n,k})}{\min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right)} \right), \]

(23)

Note that the aforementioned expression serves as an upper-bound for practical scenarios where only discrete-valued constellation sizes are applicable. Nevertheless, the real-valued \( M_{n,k}^*(\gamma_{n,k}) \) in (23) may be truncated to the nearest integer.

The corresponding maximum average spectral efficiency of the adaptive MQAM/OFDMA \([25]\) system is thus derived below

\[ ASE^* = \sum_{n=1}^{N} \sum_{k=1}^{K} E_{\gamma_{n,k}} \left\{ \text{log}_2 \left[ \max \left( 1, \frac{\zeta_{\gamma_{n,k},n,k} P_{n,k}^*(\gamma_{n,k})}{\min \left( \frac{P_t}{\text{ln}(2)} \frac{I_{n,k}^\mu K}{\mu_{\text{N_{HSS}}^\mu} |\mu| H_{\text{HSS}_{n,k}}^\mu|^2} \right)} \right) \right] \psi_{\gamma_{n,k},n,k} \right\} \]

(24)

According to (23), no transmission takes place, i.e., \( M_{n,k}^*(\gamma_{n,k}) = 1 \), when \( P_{n,k}^*(\gamma_{n,k}) = 0 \). Consequently, the optimized cut-off threshold, dictated by the channel quality, power constraint, and interference constraint, is given by:

\[ \gamma_{th} = \text{ln}(2)(\mu + \eta(\gamma_{n,k}))|H_{\text{HSS}_{n,k}}^\mu|^2 \]

\( \xi \)
IV. PROBABILISTIC INTERFERENCE CONSTRAINT

In a practical spectrum-sharing system, the tolerable collision level is confined by a maximum collision probability allowed by the licensed network. The tolerable collision level is highly dependent on the primary service type. For example, in case of real-time video streaming, a high collision probability is not desirable, however, delay-insensitive services can tolerate higher packet loss rates. In this section, we consider an underlay spectrum-sharing scenario where the primary users can tolerate a maximum collision probability \( \varepsilon_m, \forall m \in \{1, ..., M\} \). We derive the optimal power, rate, and subcarrier allocation algorithms for the multi-user OFDMA system.

According to \[28\], since the non-centrality parameter is small in previous sections, where we employ the Lagrangian dual optimization method as in the following (25b).

\[
\max_{\varphi_{n,k}} P_n, k \sum_{n=1}^{N} \sum_{k=1}^{K} E_{\hat{H}_{m,k}^{sp}} \left\{ \log_2 (M_n, k (\gamma_{n,k})) \varphi_{n,k} (\gamma_{n,k}) \right\} \\
\text{s. t.:} \text{ constraints in (8b), (8d), (8e), and (8f),}
\]

\[
\mathcal{P} \left( \sum_{n=1}^{N} \sum_{k=1}^{K} \varphi_{n,k} (\gamma_{n,k}) P_n, k (\gamma_{n,k}) |\hat{H}_{m,k}^{sp}|^2 > I_{th}^m \right) \leq \varepsilon_m, \forall m \in \{1, ..., M\}.
\]

We proceed by deriving \textit{a posteriori} distribution of the actual cross-link given the estimated channel gains.

**Proposition 1:** The posterior distribution of the actual channel \( \hat{H}_{m,k}^{sp} \) given the estimation \( 
\hat{H}_{m,k}^{sp} \) is a complex Gaussian random variable with respective mean and variance of

\[
\mu_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}} = E_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}} (\hat{H}_{m,k}^{sp}) = E_{\hat{H}_{m,k}^{sp}} (\hat{H}_{m,k}^{sp}) + \Delta H_{m,k}^{sp}(\hat{H}_{m,k}^{sp})
\]

\[
+ E_{\Delta H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}} (\Delta H_{m,k}^{sp} |\hat{H}_{m,k}^{sp}) = (1 + \rho^2)\hat{H}_{m,k}^{sp}
\]

and

\[
\delta_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}}^2 = \text{var}(\hat{H}_{m,k}^{sp}) + \Delta H_{m,k}^{sp}(\hat{H}_{m,k}^{sp})
\]

\[
= \text{var}(\hat{H}_{m,k}^{sp}) + \text{var}(\Delta H_{m,k}^{sp}) \hat{H}_{m,k}^{sp}
+ 2\text{cov}(\Delta H_{m,k}^{sp}, \hat{H}_{m,k}^{sp}) \hat{H}_{m,k}^{sp}
= (1 - \rho^2)\delta_{\hat{H}_{m,k}^{sp}}^2.
\]

Assuming equal variance \( \delta_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}}^2 \) across all users and subcarriers, the collision probability constraint in (25b) can be expressed as

\[
\mathcal{P} \left( \delta_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}}^2 \sum_{n=1}^{N} \varphi_{n,k} (\gamma_{n,k}) P_n, k (\gamma_{n,k}) |\Xi_m[k]|^2 > I_{th}^m \right) \leq \varepsilon_m
\]

where \( \Xi_m[k] \) is a complex Gaussian random variable with variance of one and mean of

\[
\mu_{\Xi_m[k]} = \frac{\mu_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}}^2}{\delta_{H_{m,k}^{sp}|\hat{H}_{m,k}^{sp}}^2}.
\]

It should be noted that in contrast to the sum of equally-weighted Chi-Square random variables in Lemma 1, \[28\] includes a sum of non-equally-weighted Chi-Square random variables. In general, obtaining the exact distribution of the linear combination of weighted Chi-Square random variables is rather complex. Although several approximations have been proposed in the literature, e.g., \[26\] \[27\] \[28\], most are not easy to implement. In this work, similar to \[29\] we use the following approximation:

\[
\delta' = \sum_{k=1}^{K} \mu_{\Xi_m[k]} + D = 2K
\]

\[
\Theta_m = \sum_{k=1}^{K} \beta_m (2 + \mu_{\Xi_m[k]})
\]

(29)

(28)

(27)

(30)

(31)

(32)

(26)

(25b)

(33)

(34)

(35)

(36)

(37)
Therefore, by solving the Lagrangian optimization problem the following optimal power allocation solution can be obtained for user $n$ over subcarrier $k$

$$P_{n,k}^*(\gamma_{n,k}) = \left[ \frac{f_{\gamma_{n,k}}(\gamma_{n,k})}{\ln(2)\left(\mu f_{\gamma_{n,k}}(\gamma_{n,k}) + \eta(\gamma_{n,k})\alpha_k\right)} - \min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right) \right]^+$$

(38)

where $\hat{N}_{sp}$ in the ‘probabilistic case’ is derived in Appendix A, Section C. The optimal subcarrier allocation policy is the solution to the following problem

$$n^* = \arg\max\left(\Lambda(\gamma_{n,k})\right), \forall n \in \{1, ..., N\}, \forall k \in \{1, ..., K\}$$

(39)

where $n^*$ is the optimal CRx index, and

$$\Lambda(\gamma_{n,k}) = \frac{\zeta_{n,k}P_{n,k}^*(\gamma_{n,k})f_{\gamma_{n,k}}(\gamma_{n,k})}{\ln(2)\left(1 + \frac{\zeta_{n,k}P_{n,k}^*(\gamma_{n,k})f_{\gamma_{n,k}}(\gamma_{n,k})}{\min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right)}\right)}$$

$$+ \min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right)\ln(2)\left(1 + \frac{\zeta_{n,k}P_{n,k}^*(\gamma_{n,k})f_{\gamma_{n,k}}(\gamma_{n,k})}{\min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right)}\right).$$

(40)

By employing the sub-gradient method, optimal expressions are derived for the constellation size and hence spectral efficiency under collision probability constraint and imperfect cross-link CSI:

$$M_{n,k}^*(\gamma_{n,k}) = \max\left(1, \frac{\zeta_{n,k}P_{n,k}^*(\gamma_{n,k})f_{\gamma_{n,k}}(\gamma_{n,k})}{\ln(2)\min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right)\left(\mu f_{\gamma_{n,k}}(\gamma_{n,k}) + \eta(\gamma_{n,k})\alpha_k\right)}\right),$$

(41)

$$ASE_n^* = \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{h_{n,k}} \left\{ \log_2\left[ \max\left(1, \frac{\zeta_{n,k}P_{n,k}^*(\gamma_{n,k})f_{\gamma_{n,k}}(\gamma_{n,k})}{\ln(2)\min\left(\frac{P_t}{K \cdot \frac{I_{th}}{N_{sp}}}\right)\left(\mu f_{\gamma_{n,k}}(\gamma_{n,k}) + \eta(\gamma_{n,k})\alpha_k\right)}\right)\right] \right\}.$$
I power setting enhances the achievable ASE in high users’ QoS. Further, imposing a higher maximum peak average at the cost of increased probability of violating the primary that the improved performance by increasing P th becomes the dominant power constraint. Note that the improved performance by increasing Ith limits the cognitive users’ transmit power. The improved performance however approaches a plateau in the high Ith region as the Pt threshold becomes the dominant power constraint. Note that the improved performance by increasing Ith is obtained at the cost of increased probability of violating the primary users’ QoS. Further, imposing a higher maximum peak average power setting enhances the achievable ASE in high Ith region.

Fig. 3: Optimal and dual values versus the number of iterations using the sub-gradient method. Results for the case with deterministic interference constraint and perfect cross-link CSI knowledge. System parameters are: \( K = 64, P_t = 30 \) Watts, \( I_{th} = 10 \) Watts, \( \xi = 10^{-2} \).

Fig. 4: ASE performance versus the tolerable interference power threshold level with different values of \( P_t \) and \( K \). Results for the case with deterministic interference constraint and perfect cross-link CSI knowledge. System parameters are: \( I_{th} = 10 \) Watts, \( \xi = 10^{-2} \).

Fig. 5: ASE performance using the proposed RRA algorithm versus \( I_{th} \) constraint for different BER-target values. Results correspond to the case with deterministic interference constraint and perfect cross-link CSI. System parameters are: \( K = 64, P_t = 30 \) Watts.

Fig. 6: Achievable ASE with imperfect cross-link CSI and ‘probabilistic case’ of estimation error against \( \epsilon \) with \( I_{th} \). System parameters are: \( K = 64, P_t = 40 \) Watts, \( \xi = 10^{-3}, \rho = 0.5, \delta_{\hat{\eta}^{k}_{\epsilon}} = 1 \).

\(-\) \( P_t \), for the particular values taken in this example, achieve the same ASE over small \( I_{th} \) settings. Moreover, increasing the number of subcarriers results in higher attainable performance.

Achievable ASE performance under different maximum tolerable interference thresholds for respective values of BER-target with perfect cross-link CSI availability is shown in Fig. 5. It can be seen that the system performance is improved under less stringent QoS constraints. For example, a 26.9% gain in ASE performance is achieved by imposing \( \xi = 10^{-2} \) in comparison to \( \xi = 10^{-3} \). However, the gap in performance becomes less significant for lower BER-target regimes.

Apart from the effect of \( \rho \) on the performance, higher values of pr increase the robustness of the interference management scheme but come at the cost of lower achievable spectral efficiencies. The results indicate that the improved ASE performance by decreasing pr in the lower half region.
(i.e., \( pr \leq 0.5 \)) is not significant yet it may cause critical interference to the primary service operation. For example, given \( p = 0.5 \), varying the value of \( pr \) from 0.5 to 0.1 results in a 40% increase in the probability of error bound violation but only provides an effective gain of 2.3% in the cognitive system performance.

The achievable performance with imperfect cross-channel information and ‘probabilistic case’ of estimation error versus the collision probability \( \epsilon \) with respective \( I_{th} \) values is illustrated in Fig. 6. Increasing the maximum probability of violating the interference constraint significantly improves the spectral efficiency of the cognitive network. The tradeoff is however the degradation of the primary service operation which is deemed highly undesirable in practical scenarios.

VI. CONCLUSIONS

In this paper, we have studied the spectral efficiency performance of adaptive MQAM/OFDMA underlay CR networks with certain/uncertain interfering channel information. We derived novel RRA algorithms to enhance the overall cognitive system performance subject to satisfying total average power and peak aggregate interference constraints. The proposed framework considers both cases of perfect and imperfect crosslink CSI knowledge at the cognitive transmitter. To compute the average spectral efficiency, we developed mathematical close form for the distribution of the received SINR for given users over different sub-channels in the respective cases under consideration. Through simulation results we studied the achievable performance of the cognitive system using our proposed RRA algorithms. By adapting the power, rate, and subcarrier allocation policies to the time-varying secondary-secondary fading channels and secondary-primary interfering channels, a significant gain in the spectral efficiency performance of the cognitive system can be realized, whilst controlling the interference on the primary service receivers. Furthermore, the impact of parameters uncertainty on overall system performance was investigated. The proposed ‘probabilistic case’ in this paper, which was derived as a low complexity deterministic constraint, provided an optimal trade-off between the achievable performance of the cognitive network and preserving the QoS of the primary users.

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