Coupled dynamics of magnetizations in spin-Hall oscillators via spin current injection

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Abstract—An array of spin torque oscillators (STOs) for practical applications such as pattern recognition was recently proposed, where several STOs are connected by a common nonmagnet. In this structure, in addition to the electric and/or magnetic interactions proposed in previous works, the STOs are spontaneously coupled to each other through the nonmagnetic connector, due to the injection of spin current. Solving the Landau-Lifshitz-Gilbert equation numerically for such system consisting of three STOs driven by the spin Hall effect, it is found that both in-phase and antiphase synchronization of the STOs can be achieved by adjusting the current density and appropriate distance between the oscillators.

Index Terms—spintronics, spin Hall effect, spin torque oscillator, synchronization, Landau-Lifshitz-Gilbert equation

I. INTRODUCTION

An excitation of a mutually coupled motion of the magnetizations in nanostructured ferromagnets, such as synchronization between spin torque oscillators (STOs) [1-15], has attracted much attention from the viewpoints of both fundamental physics and practical applications such as phased arrays and brain-inspired computing [16,17]. The mechanism of the synchronization in the previous works was based on the electric and/or magnetic interactions among STOs, such as spin wave propagation, current injection, microwave application, and dipole interaction.

Spintronics devices have another possibility to excite a coupled dynamics of magnetizations by an injection of spin current. For example, the coupled motion of two ferromagnets in ferromagnetic resonance through spin pumping was studied previously [18-20]. Recently, we studied a synchronization of self-oscillations between STOs by the injection of spin current [21]. The system we considered was similar to an array of STOs proposed by Kudo and Morie for pattern recognition [17], where several STOs driven by the spin Hall effect [22-27] are connected by a common nonmagnetic electrode. Note that a self-oscillation in each STO is excited when a spin current is injected from a nonmagnetic heavy metal into the free layer of the STO. We noticed that the spin current simultaneously creates spin accumulation inside the free layer. When the free layers of the STOs are connected by a nonmagnet having a long spin diffusion length, another spin current flows in the connector, in accordance with the gradient of the spin accumulation. This additional spin current excites additional spin torques on the magnetizations, and leads to a coupled motion of the magnetizations. Considering two STOs, we showed that this type of coupling results in an antiphase synchronization of the magnetizations. The result indicates a possibility to excite a spontaneous synchronization between STOs without using electric and/or magnetic interactions. It is of interest accordingly to extend the system to a large number of STOs.

In this paper, theoretical investigation is given for the phase dynamics between three STOs driven by the spin Hall effect. It is found that two of three STOs show phase synchronizations, whereas the other STO shows an oscillation with a different frequency. An antiphase synchronization between two STOs is found for a relatively large-coupling case. For a relatively weak coupling case, on the other hand, the antiphase synchronization appears for a small current region, whereas the phase difference becomes an in-phase for a large current region.

II. SYSTEM DESCRIPTION

The basic idea of the coupling mechanism between N STOs is as follow, where N is the number of the oscillators. Each oscillator consists of a ferromagnet and a nonmagnetic heavy metal placed at the bottom. We use suffixes such as \( k, k' = 1, 2, \ldots, N \) to distinguish the ferromagnets \( F_k \). Electric current densities \( J_0 \) along the \( x \)-direction are applied to all of the bottom nonmagnet. The ferromagnet is placed onto the nonmagnet along the \( z \)-direction, as shown in Fig.
According to the experiment [28], we assume that the internal magnetic field $H_k$ of the ferromagnet $F_k$ consists of an in-plane anisotropy field $H_K$ along the $y$-direction and a demagnetization field $-4\pi M$ along the $z$-direction as

$$H_k = H_K m_{ky} e_y - 4\pi M m_{kz} e_z,$$  \hspace{1cm} (1)

where $m_k = (m_{kx}, m_{ky}, m_{kz})$ is the unit vector pointing in the magnetization direction of the $F_k$ layer. The magnetic energy density of the ferromagnet is given by $E_k = -M \int d m_k \cdot H_k = -(M H_K/2)m_{ky}^2 + 2\pi M^2 m_{kz}^2$. The energetically stable states correspond to $m_k = \pm e_y$.

The spin Hall effect in the bottom nonmagnet injects pure spin current having the spin polarization along the $y$-direction into the $F_k$ layer, and excites the spin torque

$$T_k = -\frac{\gamma h\theta_R J_0}{2eMd} m_k \times (e_y \times m_k),$$  \hspace{1cm} (2)

where $\gamma$, $M$, and $d$ are the gyromagnetic ratio, saturation magnetization, and thickness of the ferromagnet, respectively. An effective spin Hall angle, including the interface mixing conductance, is denoted as $\theta_R$ [13]. The spin torque given by Eq. (2) induces a self-oscillation of the magnetization $m_k$ around the $y$-direction [28]. We note that the pure spin current generated from the bottom nonmagnet simultaneously creates the spin accumulation in the $F_k$ layer, which obeys the diffusion equation [25,29] and is given by [30]

$$\delta \mu_F(z) = e\vartheta^* \lambda_F E_z m_{ky} \cosh \left(\frac{z - d}{\lambda_F}\right) m_k.$$  \hspace{1cm} (3)

Here, $\vartheta^* = \vartheta \{N \vartheta^* \tanh\left(d_S/(2\lambda_F)\right)\} / (1 - \beta^2 \sigma_F \vartheta N \sinh(d/\lambda_F))$ [21] depends on the conductivity $\sigma$ and spin-diffusion length $\lambda_F$, where we use the suffixes $F$ and $N$ to distinguish the quantities related to the ferromagnet and nonmagnet, respectively. The thickness of the bottom nonmagnet is $d_N$. The pure spin Hall angle in the nonmagnet and the spin polarization of the conductivity in the ferromagnet are $\vartheta$ and $\beta$, respectively. The quantity $\vartheta^*$ is related to the F/N interface resistance, whereas $g_N/S = h\vartheta_N/(2\sigma^2 \lambda_N)$ [30]. The origin of the $z$ axis locates at the F/N interface.

Now let us consider a coupling between the ferromagnets. We note that the spin accumulation given by Eq. (3) depends on the magnetization direction. Therefore, even when all the ferromagnets have the same magnetic properties and are under the effect of the same current densities, the spin accumulations in the ferromagnets are different when the magnetizations point to different directions. When the top surfaces of two ferromagnets, $F_k$ and $F_{k'}$, are connected by an additional nonmagnet $N'$, as shown in Fig. (1b), another spin current flows in the connector according to the gradient of the spin accumulation $\delta \mu_{F_k}$. When the spin-diffusion length of the connector is sufficiently longer than its dimensional length $L$, the spin current in the top connector flowing from the $F_k$ to $F_{k'}$ layer is given by

$$J^F_{F_k \to F_{k'}} \simeq \frac{h \sigma_{N'}}{2e^2 L} [\delta \mu_{F_k}(z = d) - \delta \mu_{F_{k'}}(z = d)].$$  \hspace{1cm} (4)

where $\sigma_{N'}$ is the conductivity of the top connector. The emission of the spin current given by Eq. (4) from the $F_k/N'$ interface results in an excitation of an additional spin torque acting on $m_k$ given by

$$T_{k \to k'} = -\frac{\gamma h\vartheta J_0}{2eMd} m_k \times (m_{k'} \times m_k),$$  \hspace{1cm} (5)

where we use Eqs. (3) and (4). We introduce $\vartheta$ as

$$\vartheta = \vartheta^* \frac{\sigma_N \lambda_F}{\sigma_L L}.$$  \hspace{1cm} (6)

Using Eqs. (2) and (5), the Landau-Lifshitz-Gilbert equation of the magnetization is given by

$$\frac{d m_k}{dt} = -\gamma m_k \times H_k + \alpha m_k \times \frac{d m_k}{dt} + T_k + \sum_{k' \neq k} T_{k \to k'},$$  \hspace{1cm} (7)

where $\alpha$ is the Gilbert damping constant. We should note that the coupling torque $T_{k \to k'}$ in Eq. (7) results in a coupled motion of the magnetizations because it depends on the magnetization directions $m_{k'}$ in the other ferromagnets. For a system consisting of two ($N = 2$) STOs, it was shown that this coupling torque leads to an antiphase synchronization of the magnetizations [21]. However, a coupled dynamics between STOs for $N \geq 3$ has not been investigated yet.

III. SYNCHRONIZATION OF THREE STOS

We study the coupled motion of the magnetizations by solving Eq. (7) numerically. It has been revealed in the field of nonlinear science that even the behavior of a small number of identical oscillators is rather complex [31]. For example, for three oscillators arranged in a ring coupled through electric interaction, three stable synchronous states are possible, depending on the coupling strength [32]. In our case, we note that the coupling strength $\vartheta$ depends on the distance $L$ between the ferromagnets. This fact means that the maximum number of the oscillators to connect all of them by the same coupling strength in two-dimensional space is three. Therefore, we consider the case of $N = 3$, and assume that each ferromagnet is located at the vertex of an equilateral triangle. Figure (2a) shows a possible alignment of the STOs.

The material parameters are derived from recent experiments on the spin Hall magnetoresistance in W/CoFeB metallic bilayer [33] and first-principles calculations [34] as $M = 1500$ emu/c.c., $H_K = 200$ Oe, $\gamma = 1.764 \times 10^7$ rad/(Oe s), $\alpha = 0.005$, $d = 2$ nm, and $\vartheta_R = 0.167$. The value of the coupling strength, $\vartheta$, for $L = 100$ nm was estimated to be $\vartheta = 0.027$ by assuming that $N'$ consists of Cu [21]. In this paper, we also study the case of a weak coupling, $\vartheta = 0.0027$. We note that, in the absence of the coupling, the self-oscillation in an STO is excited when the current density $J_0$ is in the range of $J_c < |J_0| < J^*$ [35], where

$$J_c = \frac{2\alpha eMd}{\hbar \vartheta_R} (H_K + 4\pi M),$$  \hspace{1cm} (8)

$$J^* = \frac{4\alpha eMd}{\pi \hbar \vartheta_R} \sqrt{4\pi M (H_K + 4\pi M)}.$$  \hspace{1cm} (9)

The critical current density $J_c$ is the minimum current density necessary to destabilize the magnetization staying near the easy axis and excites self-oscillation. On the other hand, $J^*$
two STOs for the strong coupling strength. The values $0$ and $33$ are antiphase-coupled. On the other hand, the magnetization in-phase and antiphase, respectively [13]. It shows that the current range of antiphase synchronization between two STOs appears for the initial state for $J \leq 25.0$ MA/cm$^2$, whereas they switch to the other stable state $m_k = -e_y$ for $J_0 > 25.3$ MA/cm$^2$. Also it should be noticed here that the current range of the self-oscillation is significantly suppressed by the coupling torque.

We also investigate a coupled motion of the magnetizations for a relatively weak coupling strength, $\tilde{\vartheta} = 0.0027$. Be reminded that the coupling strength can be adjusted by changing the distance between the STOs, as can be seen from Eq. (6). Figures 2(d) and 2(e) show examples of the magnetization oscillations for $J_0 = 26.0$ and 30.0 MA/cm$^2$, respectively. For $J_0 = 26.0$ MA/cm$^2$, two magnetizations show an antiphase synchronization, whereas the other magnetization oscillate with a different frequency. This behavior is similar to the result shown in Fig. 2(b). On the other hand, for $J_0 = 30.0$ MA/cm$^2$, two magnetizations show an in-phase synchronization, i.e., the magnetizations in $F_2$ and $F_3$ layers oscillate with the same phases. Figure 2(f) summarizes the current dependence of the phase difference between coupled STOs, indicating that the antiphase synchronization is stabilized in a relatively small current region whereas the in-phase synchronization appears in a relatively large current region.

An in-phase synchronization between oscillators is useful to enhance the emission power from devices such as microwave generator and magnetic sensor. On the other hand, an antiphase synchronization, or more generally, out-of-phase synchronization, can be used in practical devices such as phased array and pattern recognition [17,36-40]. Therefore, a precise control of the phases between STOs is of interest in applied physics. The result shown in Fig. 2(f) indicates a possibility to control the phase difference between the magnetizations in a spin Hall geometry by adjusting the current density and choosing an appropriate distance between the STOs.
IV. SUMMARY

In conclusion, theoretical investigation is carried out in a coupled motion of three magnetizations in a spin Hall geometry. The ferromagnets are coupled to each other through the injection of spin current by connecting the top surfaces of the free layers with nonmagnets having long spin diffusion lengths. For a relatively strong coupling case, two magnetizations showed an antiphase synchronization whereas the other magnetization oscillated with a different frequency. The current range of the self-oscillation was significantly suppressed compared with the case of free running. For a relatively weak coupling case, on the other hand, the phase difference between two STOs depends on the current magnitude. The antiphase synchronization appeared when the current was small, whereas an in-phase synchronization was found in the large current region. The results indicate a possibility to achieve a precise control of the phases in the STOs by adjusting the current density and choosing an appropriate distance between them.

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