Seismic vulnerability study of Soltaniyeh dome using nonlinear static and dynamic analyses

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Abstract
Soltaniyeh dome is the largest brick dome in the world and registered in UNESCO’s world heritage site that makes it one of the most important historical buildings in Iran. It suffered significant damages during its lifetime; therefore, preservation of that from the future destruction is vital. In this paper, the vulnerability of the structure was studied by FEM model which was verified by previous experimental and analytical studies. The extended FEM method (XFEM) was used to consider crack formation and propagation. Gravity, pushover and nonlinear dynamic analyses were conducted to study the behavior of structure under service and seismic load. The results indicate that the structure is adequate in the gravity loads but significant cracks are started from the opening of the second and third floors and propagate through the dome and its supports in the design earthquake (475-year return period). These results in addition to the drift diagram of structure suggest that the top of the structure is more vulnerable and needed to be rehabilitated.

Keywords Soltaniyeh dome · Seismic behavior · Vulnerability analysis · Crack analysis · Historical building

Introduction
Soltaniyeh dome is one of the most important historical buildings in Iran. It is the largest brick dome in the world and the seventh Iranian heritage registered in UNESCO’s world heritage sites (Clementi et al. 2016). The structure has been exposed to the destructive effect of environment for centuries and experiences numerous small–medium-size earthquakes during its lifetime. The structure was repaired for several times and numerous deep cracks still exist in the structure which affects its strength and stability. The experience of previous earthquakes such as Bam (2003) and Nepal (2015) demonstrated that the historical buildings are vulnerable and its damage causes significant cultural losses. Since the Soltaniyeh dome is located in a very active seismic region, the study of seismic vulnerability of this iconic structure is very important.

A few limited research on seismic vulnerability study of Soltaniyeh is conducted. Sanpaolesi and Kassai (1979) studies on the architect configuration of the dome (Sanpaolesi and Kassai 1979). The first study on the seismic vulnerability of the structure is conducted by Vasseghi et al. (2004a, 2004b, 2007), in which the vulnerability of structure under three earthquakes level which corresponds to earthquakes with 75-, 475- and 2500-year return period was studied without considering crack grow. In this study, the model was not reflecting the existing situation of the structure. All openings and the Torbat-Khane, which is an annex building to the main building and make it unsymmetrical, were not modeled. The result of the nonlinear static analysis showed that in the 75-year earthquake, some cracks may happen among other places, in the dome’s supports. In the case of the 475-year earthquake, the probability of cracking increases in the dome’s support, surrounding walls, second floor and interior walls of the structure. In the case of the 2500-year earthquake, tension and pressure stresses reach to the peak level in the main bearing systems of the structure such as foundation and internal walls. Therefore, the structure is expected to collapse in this level of earthquake. Ghaemi and Bakhshi (2013) conducted an experimental and analytical study on...
the structures. The ambient vibration test was employed to estimate three main modes of the structure and the structure was analyzed using nonlinear dynamic analysis at the 475-year earthquake level. Similar to previous studies, all parts of the structure were not modeled and crack propagation did not consider in the analysis. They concluded that the crack started at the location of the opening in the third story and continued to the dome.

By studying the existing literature on the analysis of historical buildings, it can be seen that the method of vulnerability analysis of these structures was evolved during the last couple of decades. A list of some available literature is shown in Table 1. As it can be seen, at first, most of the studies were conducted by static analysis (linear or nonlinear) under gravity or lateral loads. Recently, some studies were conducted by dynamic analysis with the limited number of records.

In this study, the seismic vulnerability of Soltaniyeh dome was studied using the more precise nonlinear FEM model by considering the crack grow using extended FEM (XFEM) under seven different earthquakes. In this study, it is tried to build the more detailed model of structure in which all parts of the structure were modeled. In addition, to consider the real behavior of materials, the extended FEM (SFEM) technique for modeling of crack propagation was used. For evaluation of structure, the nonlinear static and dynamic analyses were conducted and the capacity and vulnerability of structure were evaluated.

**Architecture of Soltaniyeh dome and existing damages**

Soltaniyeh dome shown in Fig. 1 is an octagonal building with each edge of 17 ms long and height of 48.5 m. The thickness of the building’s wall is about 4.7 m. It is built in a semi-square shape with 24 m long on each side. This structure is the best example of Ilkhanate architecture in the thirteenth century. There is a porch on each internal side so that eight porches are seen on ground floor (four large and four small), and there is another story above that overlooking the interior. Some believe that octagonal plan was chosen because of experienced architectural computations and central stability. The building is constructed in such a way that it appears as two stories from interior view and three-stories from exterior view. There is an annex building to the main building on the south side called Torbat-khane. There is a basement in that part which is composed of multiple spaces underneath. First floor, which contains numerous hallways, is set at level of 9.4 m above the ground and can be accessible via three staircases. Third floor (roof) is at a level of 27.8 meters above the ground where base of dome and octagonal piers of minarets locate. The dome is 19 m high from third floor. Soltaniyeh dome is symmetric to one axis and can be categorized as irregular structure due to its geometry.

Several factors, especially environmental factors, caused a lot of structural and architectural damage to dome during the past 700 years. It reaches to the certain point that the structure extensively geometrically deformed and suffered several severe structural damages. Figure 2 compares the situation of the building in 1910 and 2015. It can be seen that bricks in many parts of structures were fallen including dome, walls, vaults, minarets, and the Torbat-khane. During the reconstruction of the building in the 1960s, three reinforced concrete rings in the level of the dome’s support were constructed to reduce the radial outward forces in the button of the dome, which leads to the significant beneficial effect on the dome’s behavior under the gravity load.

| Analysis type          | Researchers/reference | Studied structure                      |
|------------------------|-----------------------|----------------------------------------|
| Static, gravity load   | Funicicello et al. (1995) | Roman Colosseum                        |
|                        | Guler and Celep (2003) | Hagia Sophia                           |
|                        | Alper (2004)           | Hagia Sophia                           |
|                        | Miltiadou-Fezans (2008) | Dafini Catholic temple                |
| Static, nonlinear      | Sinopoli and Di Pasquale (2007) | Colosseum (Theoretic approach) |
| Dynamic, nonlinear     | Betti and Vignoli (2008) | Italian Romanesque church             |
|                        | Betti and Vignoli (2011) | Santa Maria church                     |
|                        | Betti and Galano (2012) | Vicarious palace, Masonry building San Francesco church (Italy) |
|                        | Clementi et al. (2016) |                                         |
|                        | Ferraro et al. (2016)  | Historic center (City of L’ Aquila) (Italy) |
|                        | Haciefendiglu and Koç (2016) | Historic masonry bridges               |
|                        | Clementi et al. (2016) | Strategic building in catania (Italy)   |
Finite element modeling and material properties

In this research, the structure of Soltaniyeh dome was modeled by FEM method with high precision and greatest similarity to existing geometry which include all parts of the building such as minarets, entrance vaults, Torbat-khane, double skin dome, cellar, etc. (see Fig. 3). The building had symmetrical openings and arched entrance in its original form. Some of them have been filled with bricks and mortar after rehabilitation. The current condition of the building was considered in the modeling.

All parts were modeled by SOLID element (10-node quadratic tetrahedron type) and selected from nonlinear element family to increase the accuracy of results. To consider the cracking behavior of brick walls, the extended finite element (XFEM) method which proposed by Moes and et al. (1999) was used. This method was developed to incorporate the crack initiation and propagation in the materials. Recently, this method was successfully used in the modeling of the masonry buildings (Drosopoulos and Stavroulakis 2018; Changhai Zhai et al. 2017). In this study, the similar approach is used to model the creation and propagation of existing crack and new cracks in the structure. The maximum dimension of mesh element is taken as 30 cm but there are some parts which require less dimension. The mesh of FEM of the model is shown in Fig. 4.

The material property of bricks, given in Table 2, is taken from the previous studies such as Vasseghi (2004a, b; 2007), Ghaemi and Bakhshi (2013). In this table, the material property of concrete and steel used in modeling of the RC rings in the dome support is also given. Stress–strain diagram of the brick and concrete used in the Abaqus is illustrated in Figs. 5 and 6, respectively. Steel was defined as bilinear material.

Gravity and modal analyses

To investigate the strength of structure under its weights, gravity load is applied by considering a vertical acceleration of 9.81 m/s² to the whole model. The distribution of Von Mises yields criteria of the structure under gravity load shown in Fig. 7. Results indicate that maximum compressive stress is equal to 1.10 Mpa which occurs at main columns and walls of the first floor which is well below the ultimate stress of the bricks (3 Mpa). Despite using three RC rings in the dome’s support to reduce the radial displacement and tensile stress, a slight displacement in the radial direction and noncritical tensile stress is observed on the dome’s support. A significant stress concentration in the most of the openings in first and second floors was observed.

The modal analysis of building is conducted. The dominant mode shapes of the building in the main directions (X, Y, and Torsion) are shown in Fig. 8. The natural frequency and period of structure for these modes are given in Table 3. In this table, the results of previous numerical and experimental studies are given as well. It can be observed that there is a good similarity between the current and previous studies that endorse the accuracy of modeling.

Nonlinear static analysis (pushover) and formation of cracks

To evaluate the vulnerability of structure in this paper, first, a pushover analysis is conducted to determine the nonlinear behavior of the structure and identify the cracking pattern of structure. The lateral load that is compatible with the first mode shape of structure is used for the analysis. The base shear vs. top displacement of structure in X- and Y-directions of structures are shown in Fig. 9. It appears that both
diagrams are almost similar; therefore, similar conclusion in each direction can be made.

It can be seen from Fig. 9 that the first deviation from the linear line appearances in the displacement of 6 cm (region “A”) where the first crack appears. By continuing the analysis and increasing the load, the cracks will grow and propagate to different parts of structures. To understand the mechanism of the behavior of structure, the crack propagation and state of damage in different parts of the structure were divided into several stages which are shown in Fig. 9.

Description of damage in different parts of the structure in each state is defined in detail in Table 4. To describe the pattern of damages, in brief, cracks in the structure started from the third floor and support of dome and propagated to the second and first stories. In Fig. 10, the displacement profile of structure in each pushover stage is shown. It can be seen that the profile of deformation is almost linear in first–third stories and significantly increased in the fourth story (the dome). This indicates significant lower stiffness and strength of dome level which leads to more vulnerability.

Fig. 2 Comparison of condition of Soltaniyeh dome in 1910 and 2015. a 1910 and b 2015 (Vasseghi et al. 2007)
Fig. 3 Model of Soltaniyeh dome in ABAQUS software

Fig. 4 Meshing patterns of Soltaniyeh dome in Abaqus software

Table 2 Mechanical properties of brick, concrete and steel

| Property  | E     | Mass density | Poisson ratio | Compressive strength | Tensile strength |
|-----------|-------|--------------|---------------|----------------------|-----------------|
| Brick     | 250 MPa | 1600 kg/m³  | 0.21          | 3 MPa                | 170 kPa         |
| Concrete  | 20 GPa | 2450 kg/m³  | 0.2           | 20 Mpa               | 2 Mpa           |
| Steel     | 210 GPa | 7800 kg/m³  | 0.3           | 240 MPa              | 240 MPa         |
Compressive behavior

Tensile behavior

Fig. 5 The stress–strain diagrams of bricks (Ghaemi and Bakhshi 2013)

Fig. 6 The stress–strain diagrams of concrete ring

Fig. 7 Stress distribution of Soltaniyeh dome under gravity load
Fig. 8 The Main mode shapes of Soltaniyeh

Table 3 Comparison of characteristics of main modes of Soltaniyeh dome in the current and previous studies

| Mode number | Mode direction | This study | Ghaemi and Bakhshi (2013) analysis | Ghaemi and Bakhshi (2013) experiment | Vasseghi et al. (2007) |
|-------------|---------------|-----------|-----------------------------------|--------------------------------------|-----------------------|
|             |               | Frequency (HZ) | Period (sec) | Frequency (HZ) | Period (sec) | Frequency (HZ) | Period (sec) | Frequency (HZ) | Period (sec) |
| 1           | X             | 3.341      | 3.30         | 3.125      | 3.32        | 2.92         | 0.34         | 2.941      | 0.34         |
| 2           | Y             | 3.57       | 3.29         | 3.225      | 0.31        | 3.04         | 0.33         | 2.941      | 0.34         |
| 3           | Torsional     | 4.93       | 0.2          | 4.347      | 0.23        | 4.72         | 0.21         | 4.54       | 0.22         |

Fig. 9 Pushover curve of Soltaniyeh dome
Nonlinear dynamic analysis

To study the nonlinear dynamic behavior of structure and consider the uncertainty of earthquake, seven three-component time history records were chosen. The response of the structure under these records was obtained. The process of selection of the earthquake records and results are given in the following sections.

Record selection

The result of nonlinear dynamic analysis strongly depends on the type of selected record. In this study, following conditions were used for record selection.

- The effective length of record must exceed 3 times of structure’s period or 10 s.
- The earthquake magnitude must be greater than 6.5.
- Peak acceleration of original record must be greater than 0.2 g.

The list of selected earthquake records is given in Table 5. All records were then scaled to the PGA of 0.3 g as suggested by the design code. The response spectra of selected records are shown in Fig. 11. In this figure, the median of response spectrum of selected records is compared with the code’s response spectrum. As it can be observed in the figure, the mean of selected records is almost similar to the code’s spectrum in the interested range of period of first modes of structure which is less than 0.3 s.

Analyses and results

The nonlinear response of the structure under selected record was obtained by considering following conditions.

- Damping coefficient of the structure was considered as 5%.
- Dynamic implicit was considered for the solution of dynamic equations.
- All records of earthquakes applied to the base of structure in three directions X, Y, and Z.
- Analysis of crack propagation is conducted by XFEM method based on tensile and compressive behavior of materials.

Two results of analyses were obtained for studying the seismic behavior and vulnerability of the structure: (1) crack propagation and (2) response of structure.
Crack propagation

The patterns of crack growth of Soltaniyeh dome during the earthquake are studied. By investigation of crack pattern in all records, it can be seen that the most severe cracks occurred in Loma-Prieta and Imperial Valley earthquakes. In general, it is observed that the pattern in different records is almost the same and is similar to the static nonlinear analysis. This indicated that the frequency content of earthquake does not affect the damage mode of the structure. The typical gradual crack grows in the structure is shown in Fig. 12. It is observed that cracks were started from the opening in the third story and continued to progress to the dome and second and then the first stories.

The start and growth of cracks around the openings of the structure indicated that most of the tensile stresses happen in these parts such as entrance doors, windows, and vaults. At first, cracks start from most of the openings in second and third stories, and then propagate to the dome and first story. In the next phase, cracks are initiated and started in the openings and entrances of the first story. In this stage, intensity and depth of cracks are at the highest level in the second and third floor which is an indication of the more vulnerability of upper parts of the structure. Cracks in the dome are diagonal and started from the support of the dome, where the highest tension and compression stress in the structure occur and propagate through its middle parts. Shear cracks also are observed in top parts of ceiling and walls of Torbat-khane which connects that to the second floor of the main structure. Deep cracks are observed on the base of minarets which located on the roof of the third floor. This increases the possibility of falling of minarets and causing collateral damage on structure, especially on the roof of Torbat-khane.

Table 5  List of selected earthquake records

| Number | Record name   | Years | Station               | Mag  | Distance from fault (km) | Mechanism      | Vs30 (m/s) |
|--------|---------------|-------|-----------------------|------|--------------------------|----------------|------------|
| 1      | San Fernando | 1971  | Lake Hughes           | 6.61 | 19.30                    | Reverse        | 602.1      |
| 2      | Northridge    | 1994  | Topanga-Fire Station  | 6.69 | 15.82                    | Reverse        | 505.97     |
| 3      | Friuli, Italy | 1976  | Tolmezzo              | 6.5  | 12.69                    | Reverse        | 505.23     |
| 4      | Imperial Valley | 1979 | Parachute test site  | 6.53 | 13.34                    | Strike-slip    | 348.69     |
| 5      | Kobe          | 1995  | Nishi-Akashi          | 6.54 | 10.27                    | Strike-slip    | 316.64     |
| 6      | Landers       | 1992  | Joshua tree           | 7.28 | 16.83                    | Strike-slip    | 379.32     |
| 7      | Loma Prieta   | 1989  | WAHO                  | 6.93 | 12.16                    | Reverse Oblique| 388.33     |
Response of structure

To study the displacement of the structure in different earthquakes, 14 controlling points located in different parts of the structure were selected. The locations of these points are shown in Fig. 13. In Figs. 14, 15 the maximum displacement of structure in X and Y direction at controlling points are compared. In these diagrams, the displacement limit of cracking obtained from pushover analysis is shown as well. It can be observed from the figures that:

- The maximum displacement in all records is close. This is an indicator of similar behavior of structure in different earthquakes regardless of its frequency content.
- The displacement in all points are well above the cracking displacement which suggest that crack will happen in all parts of structure during the earthquake. Therefore, it can be concluded that the structure is vulnerable to the code’s earthquake level.
- The maximum displacement of dome is lower than the maximum capacity of structure (40 cm.) obtained from Fig. 9. This is an indicator of existing of reserve capacity in structure for design earthquake (475 return period).

The drift of different stories is shown in Fig. 16. It can be seen that lateral drift of stories 3 and 4 (dome and its supported) are more than other stories which are similar to the pushover analysis in Fig. 10. This is an indicator of lower stiffness and strength of these levels compared to others that lead to more cracks in these levels.
Conclusions

In this paper, the vulnerability of Soltaniyeh dome was evaluated using static and dynamic nonlinear analyses. In this study, the structure was modeled by FEM method with high precision and greatest similarity to existing geometry. Effect of cracks and its propagation on the behavior of structure was considered by the extended method of FEM (XFEM). To verify the model, the frequency of first three mode shapes of structure in the main directions was compared with the previous numerical and experimental studies. To study the seismic behavior of structure, the pushover and nonlinear dynamic analyses were conducted and distribution of cracks in the structure and maximum displacement of the structure were obtained. The results show that:

- The structure seems to be adequate in the gravity loads.
- In the case of earthquake, the cracks in the structure started from openings in second and third stories; then propagated to the dome and first story. Cracks in the dome are diagonal and started from the support of the dome, which has the highest tension and compression stress in the structure, and propagated through its middle parts. Shear cracks are also observed in top parts of ceiling and walls of Torbat-khane which connects that to the

Fig. 12 Typical gradual crack grow in the Soltaniyeh dome
second floor of main structure. Deep cracks are observed on the base of minarets which are located on the roof of third floor.

- The nonlinear analysis under seven carefully selected earthquakes shows that the state of structure is well above the crack initiating displacement and below the maximum capacity of structure. This indicates that although the structure experiences some damages in the design earthquake, it still has reserve capacity to withstand the earthquake.

- The most vulnerable parts of structure which need special attention in a rehabilitation project are: opening at second and third stories, dome and its support. It is suggested to employ a method which prevents crack from initiation and growth in these locations.
Fig. 15 Maximum displacement of Soltaniyeh dome control points. Y-Direction

Fig. 16 Maximum relative displacement of different stories of Soltaniyeh dome

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