1. INTRODUCTION

Gravitational lensing provides us with a powerful probe of the mass distributions of the universe. By comparing the lensing probabilities predicted by various cosmological models and the density profiles of lenses with observations, we are able to test the mass distribution of dark matter halos and, in particular, the inner density slope because the Jodrell-Bank VLA Astrometric Survey and the Cosmic Lens All-Sky Survey (CLASS; Browne & Meyers 2000; Helbig 2000; Browne et al. 2002; Myers et al. 2002) have provided us with the observed lensing probabilities at small image separations ($0.73 < \Delta \theta < 3\arcsec$).

It is well known that the cold dark matter (CDM) model has become the standard theory of cosmological structure formation. The ΛCDM variant of CDM with $\Omega_m = 1 - \Omega_\Lambda = 0.3$ appears to be in good agreement with the available data on large scales (Primack 2002). On smaller (subgalactic) scales, however, there seems to be various discrepancies. Issues that have arisen on smaller scales have prompted people to propose a wide variety of alternatives to the standard CDM model, such as warm dark matter (WDM) and self-interacting dark matter (SIDM). Now that problems arise from galaxy-size halos and observations, including the observations of CO rotation curves has recently been analyzed for low surface brightness galaxies, which suggests that the NFW profile is a universal, this model allows us to constrain the structure of the galactic centers using the strong lensing surveys.

In this Letter, we investigate the plausibility of the NFW+ bulge model by fitting the observational data of the JVAS/CLASS in a much more accurate way to improve our previous work (Chen 2003). Furthermore, we emphasize the importance of the flux density ratio $q$, of the two images produced by a central point mass in each galaxy for the predicted results.

2. LENSING EQUATION AND PROBABILITIES

We choose the most generally accepted values of the parameters for the ΛCDM cosmology, in which, with the usual symbols, the matter density parameter, vacuum energy density parameter, and Hubble constant are, respectively, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.75$. The NFW density profile is $\rho_{\text{NFW}} = \rho(r)/[r(r + r_s)^2]$, where $\rho$ and $r_s$ are constants. We can define the mass of a halo to be the mass within the virial radius of the halo $r_{\text{vir}}$: $M_{\text{vir}} = 4\pi \rho_{\text{vir}} f(c_s)$, where $f(c_s) = \ln(1 + c_s) - c_s/(1 + c_s)$; $c_s = r_{\text{vir}}/r_s = 9(1 + z)^{-1} (M/1.5 \times 10^{13} h^{-1} M_{\odot})^{-1}$ is the concentration parameter, for which we have used the fitting formula given by Bullock et al. (2001).
The surface mass density for a halo as a lens is

$$\Sigma(x) = M_{\text{eff}} \delta^2(x) + \Sigma_{\text{NFW}}(x),$$  \hspace{1cm} (1)

where $x = |x|$, $x = \xi r_\text{c}$ ($\xi$ is the position vector in the lens plane), $\delta^2(x)$ is the two-dimensional Dirac delta function, and $\Sigma_{\text{NFW}}(x)$ is the surface mass density for an NFW profile; $M_{\text{eff}}$ is a point mass ranging from 1 to 1000 times the mass $M_\bullet$ of a supermassive black hole (SMBH) inhabiting the center of each galaxy, and it is used to describe the contributions of galactic central SMBHs and galactic bulges to lensing probabilities (Merritt & Ferrarese 2001; Ferrarese 2002; Chen 2003). For a galaxy cluster, $M_{\text{eff}} = 0$. The lensing equation for this model is then

$$y = x - \mu_{\text{eff}} + g(x),$$ \hspace{1cm} (2)

where $y = |y|$, $\mu = yD_s^2/D_{\text{ls}}^2$ is the position vector in the source plane, in which $D_s$ and $D_{\text{ls}}$ are angular diameter distances from the observer to the source and to the lens, respectively. It should be pointed out that since the surface mass density is circularly symmetric, we can extend both $x$ and $y$ to their opposite values in equation (2) for convenience. The parameter $\mu_{\text{eff}} = 4\pi r_\text{c}^2/\Sigma_\text{ls}$ is independent of $x$, in which $\Sigma_\text{ls}$ is the critical surface mass density. The term $f_{\text{eff}} = 2.78 \times 10^{-4} f_\text{c}(y)M_{\text{ls}}^{0.5} M_{\text{ls}}^2/\Sigma_\text{ls}^2$, where $M_{15}$ is the reduced mass of an NFW halo defined as $M_{15} = M_{\text{DM}}/(10^{15} h^{-1} M_\odot)$, stands for the contribution of a point mass $M_{\text{ls}}$, and, of course, $f_{\text{eff}} = 0$ for cluster-size lenses. The function $g(x)$ stands for the contribution of the NFW halo, and it has an analytical expression originally given by Bartelmann (1996).

When the quasars at redshift $z_s = 1.5$ are lensed by foreground CDM halos of galaxies and clusters of galaxies, the lensing probability with image separations larger than $\Delta \theta$ and a flux density ratio less than $q_s$ is (Schneider, Ehlers, & Falco 1992)

$$P(\Delta \theta < q_s) = \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} n(M, z) \sigma(M, z) B(M, z) dM,$$

$$= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} n(M, z) \sigma(M, z) B(M, z) dM,$$

where $D_{\text{ls}}(z)$ is the proper distance from the observer to the lens located at redshift $z$. The physical number density $n(M, z)$ of virialized dark halos of masses between $M$ and $M + dM$ is related to the comoving number density $n(M, z)$ by $n(M, z) = n(M, z)(1 + z)^3$; the latter is originally given by Press & Schechter (1974), and the improved version is $n(M, z) dM = (\rho_0/M) f(M, z) dM$, where $\rho_0$ is the current mean mass density of the universe and $f = (0.301/M) f(\ln \Delta, \ln M) \times \exp[-\ln \Delta/1.68] + 0.64]^{3.89}$ is the mass function for which we use the expression given by Jenkins et al. (2001). In this expression, $\Delta = \delta_c(z)/\Delta(M)$, where $\delta_c(z)$ is the overdensity threshold for spherical collapse by redshift $z$ and $\Delta(M)$ is the rms of the present variance of the fluctuations in a sphere containing a mean mass $M$. The overdensity threshold is given by $\delta_c(z) = 1.68D(z)$ for the $\Lambda$CDM cosmology (Navarro, Frenk, & White 1997), where $D(z) = g[\Omega(z)/[g(\Omega_\Lambda)(1 + z)]$ is the linear growth function of the density perturbation (Carroll, Press, & Turner 1992), in which $g(x) = 0.5x(1/70 + 209x/140 - x^2/140 + x^3) - 1$ and $\Omega(z) = \Omega_\Lambda(1 + z)^3[1 - \Omega_\Lambda + \Omega_\Lambda(1 + z)]$. When we calculate the variance of the fluctuations $\Delta^2(M)$, we use the fitting formulae for the CDM power spectrum $P(k) = A k^2 f^2(k)$ given by Eisenstein & Hu (1999), where $A$ is the amplitude normalized to $\sigma_8 = 0.8 h^{-1}$ Mpc by observations. Note that the mass of an NFW halo is taken to be the virial mass $M_{\text{DM}} = 4\pi \rho_c r_{\text{vir}}^3/3$, where $\rho_c$ is the average density within $r_{\text{vir}}$.

The key step in working out the final results of lensing probabilities is how to calculate the lensing cross section $\sigma(M, z)$ in equation (3). Since we are interested in the lensing probabilities with image separations larger than a certain value $\Delta \theta$ (e.g., ranging from $0^\circ$ to $10^\circ$) and a flux density ratio less than $q_s$, the cross section should be defined under two conditions. The first condition is used to define the cross section of cluster-size NFW lenses, for which multiple images can be produced only if $|y| \leq y_{\text{cr}}$, where $y_{\text{cr}}$ is the maximum value of $y$ when $x < 0$, which is determined by $dy/dx = 0$ when $f_{\text{eff}} = 0$ in equation (2). On the other hand, because of the existence of the central point mass, theoretically, every galaxy-size lens will always produce two images. So the first condition fails in this case, and we need the second condition to define the cross section of galaxy-size lenses, which is the allowed upper limit of the flux density ratio of lensing images in any lensing survey experiments. So the flux density ratio $q_s$ for the two images is just the ratio of the corresponding absolute values of magnifications (Schneider et al. 1992; Wu 1996). $q_s = (\mu, |\mu|)$, where $\mu = \mu_1 \mu_2$, and $|\mu_1(x)\mu_2(x)| = (dy/dx)|_{y=0}$ and $\mu_1(x)\mu_2(x) = (f(x)\times dy/dx)|_{y=0}$. So $y_{\text{cr}}$ is determined by $|\mu_1(x)| = q_s$. The cross section for images with a separation greater than $\Delta \theta$ and a flux density ratio less than $q_s$ is (Schneider et al. 1992)

$$\sigma(M, z) = \pi r_{\text{eff}}^2 \delta^2(M - M_{\text{min}})$$

$$\times \begin{cases} y_{\text{cr}}^2, & \text{for } \Delta \theta < \delta \theta_{\text{cr}}, \\ y_{\text{cr}}^2 - y_{\text{cr}}^2, & \text{for } \delta \theta_{\text{cr}} < \Delta \theta < \delta \theta_{\text{rs}}, \\ 0, & \text{for } \Delta \theta \geq \delta \theta_{\text{rs}}. \end{cases} \hspace{1cm} (4)$$

where $\delta(x)$ is a step function and $M_{\text{min}}$ is the minimum mass of halos above which lenses can produce images with separations larger than $\Delta \theta$. From equation (2), an image separation for any $y$ can be expressed as $\Delta \theta(y) = r_{\text{ls}} \Delta(x) dM$, where $\Delta(x)$ is the image separation in the lens plane for a given $y$. So in equation (4), the source position $y_{\text{cr}}$, at which a lens images the separation $\Delta \theta$, is the reverse of this expression. And $\delta \theta_{\text{rs}} = \Delta(0)$ is the separation of the two images that are just on the Einstein ring; $\delta \theta_{\text{cr}} = \Delta y_{\text{cr}}$ is the upper limit of the separation above which the flux ratio of the two images will be greater than $q_s$. Note that since $M_{\text{DM}}(M_{15})$ is related to $\Delta \theta$ through $r_{\text{ls}} = (1.626/c_r) M_{15}/ \Omega_\Lambda^{0.5} + \Omega_\Lambda^{1/3}) h^{-1}$ Mpc (Li & Ostriker 2002), we can formally write $M_{\text{DM}} = M_{\text{DM}}(\Delta \theta(y))$, and we can determine $M_{\text{min}}$ for galaxy-size lenses by $M_{\text{min}} = M_{\text{DM}}(\Delta \theta(y_{\text{cr}}))$ and for cluster-size lenses by $M_{\text{min}} = M_{\text{DM}}(\Delta \theta(0))$. In the latter case, we have used the fact that the separation of the outermost images is insensitive to the value of $y$ in cluster-size NFW lenses.

To compare the predicted lensing probabilities with the combined data from the IVAS/CLASS, magnification bias must be considered. For the IVAS/CLASS sample, we use the result given by Li & Ostriker (2002): $B(M, z) = 2.22 A_{\text{eff}}(M, z)$, where $A_{\text{eff}}(M, z) = \Delta \theta(y = 0)/y_{\text{cr}}$.

One major uncertainty in the estimate of $P(\Delta \theta, q_s)$ by the NFW halo arises from the assignment of the concentration parameter $c_r$, to each halo of mass $M$. There exist several empirical fitting formulae or analytic models to fulfill the task. However, for a given halo mass and redshift, there is a scatter in the
Fig. 1.—Predicted lensing probability with image separations greater than $\Delta \theta$ and flux density ratios less than $q_i$ in $\Lambda$CDM cosmology. The cluster-size lens halos are modeled by the NFW profile, and galaxy-size lens halos by the NFW/H11001 bulge model. Instead of an SIS, we treat the bulge as a point mass; its value is so selected for each that the predicted lensing probability can match the results of the JVAS/CLASS represented by the histogram. In the left panel, the value of the concentration parameter is taken to be its median for any given halo mass and redshift. In the right panel, the scatter in $c_1$ is considered by averaging the probability with the well-known lognormal distribution. In both panels, $q_i = 0.95$, and the null result for lenses with $6' \leq \Delta \theta \leq 15'$ of the JVAS/CLASS is shown with the thick dashed horizontal line indicating the upper limit.

$c_1 = r_{ci}/r_i$, value that is consistent with a lognormal distribution with a standard deviation $\sigma_1 = \Delta(\log c) \approx 0.18$ (Jing 2000; Bullock et al. 2001). We take into account the scatter in $c_1$ by averaging the lensing probability with the lognormal distribution (the right panel of Fig. 1 and all the panels of Fig. 2). Another major uncertainty in predicting $P(>\Delta \theta, <q_i)$ arises from the considerable uncertainty regarding the value of the CDM power spectrum normalization parameter $\sigma_8$, so it would be useful to consider the effect of varying this parameter within its entire observational range, roughly $\sigma_8 = 0.7–1.1$ (see Fig. 2).

3. DISCUSSION AND CONCLUSIONS

The lensing probabilities predicted by equation (3) and calculated from the combined JVAS/CLASS data are compared in Figure 1. Since we are interested in the degeneracy between $q_i$ and $M_{\text{eff}}$ in matching the predicted results to observational data, we have calculated the lensing probabilities for four different values of $q_i$ and their corresponding values of $M_{\text{eff}}$, as indicated in Figure 1. We have assumed a “cooling mass” of $M_{\text{cool}} \approx 5 \times 10^{13} \, h^{-1} \, M_\odot$ above which the lenses are assigned the NFW profile and below which the lenses are assigned the NFW+point mass. The combined JVAS/CLASS is now completed, a subset of 8958 sources form a well-defined statistical sample containing 13 multiply imaged sources (lens systems) suitable for analysis of the lens statistics. The observed lensing probabilities can be easily calculated: $P_{\text{obs}}(>\Delta \theta) = N(>\Delta \theta)/8958$, where $N(>\Delta \theta)$ is the number of lenses with separation greater than $\Delta \theta$ in 13 lenses. $P_{\text{obs}}(>\Delta \theta)$ is plotted as a histogram in both panels of Figure 1.

First of all, as shown in Figure 1, when averaged over concentration parameter $c_1$ with the lognormal distribution (right panel), the probabilities are increased considerably at larger image separations and only slightly increased at smaller separations for all cases, and the “scatter” among the four cases is reduced. So the scatter in $c_1$ should be considered when one uses the NFW profile to constrain some related parameters.

It is also shown in Figure 1 that for low flux ratios ($q_i \leq 10$, as for the JVAS/CLASS), the NFW+single SMBH model produces far too few small separation lenses (the triple–dotted–dashed line in Fig. 1). This is confirmed by the fact that among the 22 confirmed lenses in the JVAS/CLASS, none have a fainter image very close to the center of the lens (Browne et al. 2002). In other words, an up-to-date strong gravitational lensing effect of a single SMBH has not been found.

A larger flux ratio requires a smaller fraction of the bulge mass as a point mass. It is interesting to note that when $q_i \approx 10^2$ and $M_{\text{eff}} \approx 30 M_\odot$ and when no scatter in $c_1$ is included (the dotted line in the left panel of Fig. 1), the predicted lensing probability fitted the JVAS/CLASS results quite well. As pointed out earlier, this conclusion can be equivalently obtained with the model of two populations of halos, the combination...
of SIS and NFW, when no constraints on the flux density ratio $q_j$ are taken into account (i.e., $q_j \sim \infty$; note that the flux density ratios with values as high as $q_j \sim 10^4$ and infinity have approximately the same effect on the predicted probabilities). However, since the flux density ratio for the well-defined JVAS/CLASS sample is limited to $q_j \leq 10$ (Chae et al. 2002), the above-mentioned good fit cannot be regarded as reflecting the real nature of CDM halos. In fact, to match the JVAS/CLASS results, for $q_j \leq 10$, about 20% of the bulge mass is required as a point mass (i.e., $M_{\bullet} \approx 200 M_\odot$, the solid lines in Fig. 1), while for $q_j \leq 10^4$, only 3% of bulge mass is needed. This difference is very important when one attempts to use the observational results of the JVAS/CLASS to constrain the density profile of galactic bulge and/or dark matter halos. It is well known that the flux density ratio will reduce the lensing cross section considerably. Without considering this, all the models would have overestimated the lensing probabilities.

We have noted that the solid lines in both panels of Figure 1 can only marginally fit the observations of the JVAS/CLASS to constrain the density profile of galactic bulge and/or dark matter halos. It is well known that the flux density ratio will reduce the lensing cross section considerably. Without considering this, all the models would have overestimated the lensing probabilities.

Although we believe that the reasonable match to the observations of the JVAS/CLASS has $q_j \leq 10$ and $M_{\bullet}/M_\odot = 200$ (the solid line in each panel of Fig. 1), it is helpful if we treat the four cases (the four upper lines matching the histogram) as a whole to constrain the sensitive parameter $\sigma_j$. We plot in each panel of Figure 2 the averaged lensing probability as a function of image separation $d\theta$. All the parameters are the same as those in the right panel of Figure 1, except for $\sigma_j$, for which five different values within the entire observational range (from 0.7 to 1.1, as explicitly indicated) have been investigated to see their effect on the predicted lensing probabilities. Clearly, the larger values of $\sigma_j$ will produce the higher probabilities. In the case of $\sigma_j = 0.7$, too few lenses are produced at small image separations, so a low value of $\sigma_j$ ($\leq 0.7$) is unlikely. The best fit is $\sigma_j \sim 1.0$. This result is very close to that obtained most recently by Bahcall & Bode (2002), in which $\sigma_j$ is determined from the abundance of massive clusters at redshifts $z = 0.5–0.8$. Our result is also in excellent agreement with that of Komatsu & Seljak (2002; $\sigma_j = 1.04 \pm 0.12$ at 95%), suggested by the excess cosmic microwave background fluctuations detected on small scales by the Cosmic Background Imager (Mason et al. 2002) and the Berkeley–Illinois–Maryland Association (Dawson et al. 2002) experiments.

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