Pores in Spherical Radiolarian Skeletons Directly Determined from Three-Dimensional Data

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We propose a method that uses three-dimensional data to directly determine the pores (holes) in the skeleton of a spherical radiolarian. Our goal is to automatically determine both the number and the distribution of the pores. We used a set of grid points on a spherical surface to approximate the skeletal structure, which was obtained from a micro X-ray CT scan. Next, we counted the number of pores by using an algorithm for counting clusters on a grid. Finally, we used Voronoi tessellation to determine the distribution of the pores. For noisy data, a smoothing filter was applied before the procedure. We applied our method to three real data sets, and the results showed that our method worked well.

Key words: Pore Number, Distribution of Pores, Voronoi Tessellation, Radiolarian

1. Introduction

Three-dimensional (3D) data processing has been applied in a number of scientific and industrial fields (Anderson, 2012). Two main technologies supporting this innovation are computer graphics and 3D printing. The former uses computers to facilitate the composition of 3D structures, and the latter facilitates the construction of 3D objects from 3D data. The combination of these technologies has allowed us to easily construct models that previously would have been extremely difficult to construct.

The 3D skeletal structure of a kind of radiolaria, Pantanellium, was determined by using 3D data processing technologies (Matsuoka et al., 2012). Radiolaria, single-celled marine plankton, are about 10−4 m in length, thus observing the details of their structure requires the use of a microscope. Matsuoka et al. (2012) reported the exact number of pores in a Pantanellium specimen and used a planar graph to represent the distribution of the pores. They used three steps to obtain this information: 1) use a micro X-ray CT scan to obtain the 3D structure of a specimen; 2) use a 3D printer to make a model; and 3) visually inspect these structures. Although it may seem that the third step could be done automatically by a computer, this is not yet possible.

The primary reason this cannot be done automatically is the absence of methods for counting and determining the distribution of the pores. The number of pores is one of the basic features used to describe the structure of a radiolarian skeleton, but no automatic counting procedure has been established. By visual inspection, we can only determine the number of pores to within an order of magnitude. The distribution of the pores is another basic feature. When these are determined by visual inspection, there is the possibility of inconsistencies due to either mistakes or subjectivity. One method that avoids this problem is to determine the relation of a given pore to its neighboring pores; however, it is necessary to first clarify the definition of a neighbor.

In this paper, we present a method that uses 3D data to directly determine the pores in a spherical radiolarian skeleton. This is a meaningful result because the method can be applied to other areas of science and technology. These include geophysical problems relating to the earth’s surface and optimal allocation problems for materials on a hemisphere. In the following sections, we describe this procedure and then use real 3D data to assess it.

2. Method

We used two different types of 3D images reconstructed from micro X-ray CT images; one was in Standard Triangle Language (STL) format and the other was in Tagged Image File Format (TIFF). The former consists of faces of triangles, and the latter is a collection of images of sections. Neither of them were real images from micro X-ray CT scans, but both were derived by reconstructing real images, and thus contain some errors.

We considered a collection of small volume elements, which were parts of a skeleton, as a collection of skeleton-forming points. For the STL data, we replaced each triangular face with its center, and we then considered the centers as a collection of the parts comprising the skeletal frame. For the TIFF data, after transforming the gradations to bi-
In the following, we demonstrate our procedure using an example of real STL data, which is shown in Fig. 1. This species, Mesozoic radiolarian *Pantanellium*, had long, large, opposing appendages (*polar spines*) with massive edges and double-layered spherical shells that contained about 30 pores. The inner and outer shells are called the medullary and cortical shells, respectively. The specimen presented in Fig. 1 lacks an inner or medullary shell.

To form a grid on a spherical surface, we used a geodesic grid (Sadouny et al., 1968), which is based on the equal triangulation of the faces of an icosahedron. We divided each edge of the triangular faces of a dodecahedron into $N$ equal segments and thus obtained a triangular grid consisting of $(10N^2 + 2)$ grid points. Most of the grid points have six neighbors, but the twelve grid points that correspond to the vertices of the original icosahedron each have only five neighbors. The origin of the Cartesian coordinates was placed at the center of all the grid points, and the distance of each of the grid points from the origin was normalized to one. We will denote the value of the $i$-th grid point by $x_i$ ($1 \leq i \leq 10N^2 + 2$). Figure 2 shows three examples of a geodesic grid ($N = 8, 16, 32$). The resolution of the system depends on $N$. We used the $N = 16$ geodesic grid (2,562 grid points) for the example in this section.

If necessary, prior to the main procedure, the original point data were screened; we will call this the cutoff procedure. Figure 3a shows an example of the result of the cutoff procedure.
procedure. This procedure is necessary in two cases: when there are unnecessary skeleton-forming points, such as the polar spines shown in Fig. 1, and when there are obvious errors, such as the padding of pores. The cutoff itself is arbitrary. For example, in the case of Fig. 1, we used only the skeleton-forming points that were between 0.28 and 0.45 units from the origin.

We counted the number of pores and determined their shapes as follows: 1) the skeleton-forming points were mapped to the grid points, 2) the number of pores was determined by counting the clusters on the grid, and 3) Voronoi tessellation was applied to the centers of the pores in order to determine the approximate structure of the skeleton.

First, we mapped the skeleton-forming points to the grid points. This procedure consisted of mapping the skeleton-forming points to the surface of the unit sphere and approximating each point by the nearest grid point. In order to map the skeleton-forming points to a spherical surface, we normalized their position vector with their length. Figure 3b shows the result of this first step. Next, we determined the nearest grid point for each normalized skeleton-forming point by selecting the grid point for which the inner product of that point and the normalized position vector was greatest. The values of the corresponding grid points were set to 0, and the values of the remaining grid points were set to 1. The small spheres in Fig. 3c show the result of the grid approximation; they correspond to the grid points set to 0. For clarity, the back of the hemisphere is hidden. It is obvious that there should be an appropriate number of grid points, $N$, for the 3D data set such that the resolution of the grid points does not exceed the resolution of the raw data. When there are too few grid points, some pores are padded. When
Fig. 6. Raw 3D data of *Pantanelium* B constructed from TIFF data. The skeleton-forming points are represented by small spheres.

Fig. 7. STL image of *Haliommilla*: (a) the entire shape, (b) the same specimen from a different viewpoint.

there are too many grid points, some skeleton edges have small pores because there are not enough skeleton-forming points to pad the grid points.

Next, we counted the number of pores. The concept of this procedure is the same as for counting the clusters on a grid. The algorithm we used comes from research on percolation (e.g., Stauffer and Aharony, 1994). Figure 4 shows the procedure schematically. The grid points corresponding to pores were changed from 1 to a different positive integer (Fig. 4a). After this procedure, we again changed the values of the grid points so that those that belonged to the same pore shared the same number. This procedure substitutes the value of a given nonzero grid point for the values of its nonzero neighbors. The grid point and its nonzero neighbors were set to the smallest value among them (Figs. 4b and 4c). This procedure was applied to all the nonzero grid points so that those that belonged to the same pore shared the same number. Next, we obtained the centers of the grid points that belonged to the same pore and normalized the location vectors of the centers with their distance from the origin of the sphere. The centers of the pores are shown as large spheres in Fig. 3c. The one-to-one correspondence between pores and large spheres indicates that this procedure worked well.

Finally, we carried out Voronoi tessellation on the sphere using the centers of the pores as generators. This tessellation automatically determined the appropriate neighbors of the generators. The algorithm we used for Voronoi tessellation on a spherical surface is from Okabe *et al.* (2000). The tessellation has three steps: 1) obtain the convex hull of the generators; 2) determine the equations of all the planes perpendicular to and containing the location vectors of the generators so that each plane corresponds to an appropriate generator; 3) obtain the intersecting lines of adjacent pairs of vertices of the convex hull.

Figure 5 shows the result of applying this method to the original STL images. Small red spheres denote the centers of the pores, and the blue frame denotes the frame structure approximated from the Voronoi tessellation. In order to combine the two images, we carried out only scaling and translation. Other kinds of transformation, such as rotation or Affine transformations, were not used. Because of the good correspondence between the two images, we concluded that we correctly extracted the structural features from this data. The estimated number of pores was 25. This value was consistent with the one obtained by visual inspection. The approximated polyhedron consisted of twelve pentagons and thirteen hexagons.

We also introduced a smoothing filter (e.g., see Gonzalez and Woods, 2010) to reduce the white noise in the data. When necessary, we applied this filter after approximating the grid points for the skeleton-forming points. The smoothing filter is defined as

$$x_i = \frac{1}{n_i} \left( x_i + \sum_j x_j \right), \quad (1)$$
where \( n_i \) is the number of neighbors of the \( i \)-th grid point, and the summation is carried out for the neighbors of the \( i \)-th grid point. This transformation is applied simultaneously to all the grid points. In this study, we repeated the filter three times. After this procedure, we discretized the values \( x_i \) to 0 or 1, according to the appropriate threshold value.

3. Results

In order to validate our proposed method, as discussed in the previous section, we prepared three 3D data sets obtained from micro X-ray CT scans. These were obtained from two different specimens of *Pantanelium* and one specimen of modern radiolaria *Haliommilla*. All of the specimens were about \( 10^{-4} \) m in length. The first was the STL data set of *Pantanelium*, shown in Fig. 1, and the results for this are shown in Fig. 5. We will refer this as *Pantanelium A*. The second was the TIFF data of another *Pantanelium*, shown in Fig. 6. The skeleton-forming points are represented by small spheres. This data set has noise due to the micro X-ray CT scanning process. Although some of this noise can be removed by converting from TIFF to STL, it cannot be removed completely, so we used this data to examine the robustness of our procedure. We refer to this data set as *Pantanelium B*. The third data were the STL data of *Haliommilla*, shown in Fig. 7. The images in Fig. 7 were also obtained from same STL data, but the viewpoint is different. In Fig. 7b, the back of the hemisphere was omitted for clarity. This specimen had many pores and a thin frame, compared with those of the two *Pantanelium* specimens, and it was impossible to count the pores by visual inspection. Furthermore, as shown in Fig. 7b, these data contained some errors around the center of the image. Without the use of the smoothing filter, neither of the last two data sets produced correct results. We use the word “correct” to describe the inherent properties of the specimen.

Figure 8 shows the results of applying the method to the original 3D image of *Pantanelium B*. We used a geodesic grid with \( N = 16 \) (2,562 grid points). We applied the smoothing filter because these data contain some noise. The red spheres indicate the centers of the pores, and the blue frame shows the result of the Voronoi tessellation of the centers of the pores. As shown in Fig. 8, the approximated polyhedron agreed with the original data. The estimated number of pores was 24. This value was also consistent with the one obtained by visual inspection.

For *Haliommilla* (Fig. 9), we did not obtain the correct number of pores because the one-to-one correspondence between pores and generators was broken in some regions, as shown in Fig. 9b. The images in Fig. 9 were taken using same viewpoint as those in Fig. 7. We used a geodesic grid with \( N = 128 \) (163,842 grid points) and applied the smoothing filter. The estimated number of pores was 262. As shown in Fig. 9a, determination of the pores was apparently successful in most regions. There was, however, a questionable region near the center of the image, as shown in Fig. 9b. Therefore, we conclude that, in this case, our procedure was insufficient for estimating the appropriate structure.
As shown in Figs. 8 and 9, we obtained not only the number of pores but also their pore frames. According to the results of *Patanellium* B (Fig. 8), the pore frame comprises twelve pentagons and twelve hexagons. The structures of both the *Patanellium* specimens considered in this study differed from that of the specimen reported by Matsuoka *et al.* (2012). We also obtained an approximated pore frame of *Haliommilla*, but it seemed to contain some errors due to the uncertainty in the number of pores.

For *Patanellium* B, we obtained results with and without the use of the smoothing filter, as shown in Fig. 10. These results were also obtained using a geodesic grid with $N = 16$. The red spheres correspond to the centers of the pores that were determined. In the case without filtering (Fig. 10a), some small pores were incorrectly indicated. These were due to the lack of skeleton-forming points to cover the area. On the other hand, when filtering was used (Fig. 10b), the small, incorrect pores disappeared and only large, correct pores were observed. We conclude that the smooth filter worked well in this case.

In order to examine the effect of the smoothing filter, we estimated the number of pores using different values of $N$, which changed the size the geodesic grid. The value of $N$ ranged from 8 to 256, with an interval of eight. Figure 11 shows the dependence of the results, for both filtered and unfiltered data, on the grid size; Figs. 11a and 11b show the results of *Patanellium* (A and B) and *Haliommilla*, respectively.

When $N$ was small, the estimated number of pores tended to be smaller than the correct number because the small number of grid points caused padding of the pores. On the other hand, when $N$ was large, the estimated number of pores tended to be larger than the correct number because the number of skeleton-forming points was insufficient for padding the grid points, as shown in Fig. 10a. As $N$ grew even larger, the estimated number of pores decreased with an increase in $N$ because the small, incorrect pores were connected.

The effects of the smoothing filter on the results for *Patanellium* A and B are shown in Fig. 11a. For filtered data from *Patanellium* A, the estimation of the number of pores was successful for values of $N$ ranging from 16 to 72. Without the filter, it was successful for values of $N$ ranging from 8 to 40. Thus, the smoothing filter increased the range of values of $N$ that gave good results. For filtered data from *Patanellium* B, the correct number of pores was obtained for $N = 16$ to 40; the correct value was not obtained for the unfiltered data.
In a case of *Haliommilla*, the estimated number of pores was not consistent. In the unfiltered case, it rapidly increased with an increase of \( N \), but in the filtered case, it only increased a small amount in the range \( N = 64 \) to 144. In that range, the determination of the number of pores failed only the particularly complex area. Therefore, we conclude that the estimate in this region is not correct but only a rough approximation.

4. Discussion

Our method works well for specimens with more massive skeletons, such as *Pantanellium*. Because the choice of \( N \) affects the accuracy, its selection is important. Various values of \( N \) should be tried, and then a value should be chosen such that the results are the same for various numbers of pores. The smoothing filter is effective not only for reducing noise but also for obtaining correct results.

When the skeleton was more delicate and contained many pores, our method did not obtain the correct number of pores. It did, however, provide a rough estimate in a range around the correct value. The main reasons for its failure were contamination of the data from various stages of the micro X-ray CT scanning process, insufficient resolution of the data, and the reconstruction process. It will be improved therefore by improvements in micro X-ray CT imaging.

In conclusion, our procedure works well for more massive structures, and for less massive structures, it can provide a range that restricts the estimated number of pores.

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