CDCC calculations of elastic scattering for the systems $^6\text{Li} + ^{144}\text{Sm}$ and $^6\text{Li} + ^{208}\text{Pb}$. Effect of resonances of $^6\text{Li}$ on elastic scattering angular distributions

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Abstract. Calculations of elastic scattering angular distributions for reactions of the weakly bound projectile $^6\text{Li}$ with targets $^{144}\text{Sm}$ and $^{208}\text{Pb}$ at energies above the barrier, are performed with the Continuum Discretized Coupled-Channel method (CDCC). Ground, resonant and non-resonant continuum states of $^6\text{Li}$ are included up to some maximum energy $\epsilon_{\text{max}}$ for which convergence is achieved. In the three-body system, global interactions are used for the $\alpha - \text{target}$ and $d - \text{target}$ sub-systems. The effect of continuum resonant states of $^6\text{Li}$, i.e., $l = 2$, $j^+ = 3+, 2+$ and $1+$ on elastic scattering angular distributions is investigated by extracting these states from the continuum space. It is found that the calculated elastic scattering angular distributions are in good agreement with the measurements for most of the cases studied where consideration of couplings to continuum states is essential. It is also found that the resonance character of the continuum states is in some cases important to obtain agreement with the data.

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1. Introduction

Reaction mechanisms involving weakly bound nuclei, both stable and radioactive, have been a subject of strong research[1, 2, 3]. One of the most interesting topics, is the effect of breakup on elastic scattering and fusion reaction mechanisms. It is now known that couplings of the elastic channel to continuum breakup states of the weakly bound projectile, as well as, continuum-continuum couplings are essential to fit the experimental data. This is so since, couplings to and between continuum states give rise to a strong repulsive polarization potential that decreases the reaction absorption. On the other side, besides the non-capture breakup process, other nuclear mechanisms are present for reactions with weakly bound nuclei. For instance, sequential and direct complete fusion, incomplete fusion and nucleon transfer. Coupled channel calculations show that couplings between the incident elastic channel to fusion ones, give place to an attractive dynamical polarization potential $V_F$ that lowers the nominal Coulomb barrier $V_B$. So, the net effect of $V_F$ is to enhance fusion mainly for energies around the barrier. Similarly, couplings to breakup channels produce a repulsive dynamical polarization potential $V_{BU}$, that is particularly strong at energies below the barrier. In this energy range, $V_{BU}$ overcomes $V_F$, so that the net effect is twofold: a) An increase in the fusion barrier, that leads to a net
fusion suppression, and b) Most of the reaction cross section is due to breakup. Several CDCC calculations have been proposed to calculate the effect of breakup couplings on elastic and fusion cross sections of several weakly bound systems. For instance, the 2n-halo $^6$He on $^{59}$Co and $^{208}$Pb$[4, 5]$, $^6$Li with targets $^{28}$Si, $^{59}$Co, $^{58}$Ni, $^{144}$Sm and $^{208}$Pb$[4, 6, 7, 8, 9, 10, 11, 12, 13]$, also $^7$Li with $^{28}$Si$[14, 15]$ and $^{144}$Sm$[16]$, the n-halo $^{11}$Be with $^{208}$Pb$[17]$ and the p-halo $^8$B with $^{58}$Ni$[10, 18]$. Most of the CDCC calculations in these works, use either, microscopic density dependent double-folding interactions with normalization factors $N_R$, $N_I$ or Woods-Saxon potentials obtained from experimental systematic analyses. Except for a few cases, where contradictory results are found, e.g., $^8$B+$^{58}$Ni$[10, 18]$, most of the CDCC calculations show that couplings to continuum states of the weakly bound projectile have a strong effect and their consideration is very important to attain agreement with the experimental data. However, most of the studies for the weakly bound nucleus $^6$Li do not distinguish the effect that the resonance character of the continuum states $3+, 2+$ and $1+$ can have on elastic scattering. The purpose of the present work is to calculate this effect on reactions with the spherical targets $^{144}$Sm and $^{208}$Pb, at energies above the Coulomb barrier. The CDCC$[6, 19]$ method is used with global $d−target$ and $α−target$ interactions, which depend only on the target mass $A$ and incident collision energy $E_{lab}$. The global Woods-Saxon potentials, given in Ref.$[20]$ for the $d−target$ interaction $V_{dΤ}$, are obtained from a large analysis of experimental elastic and inelastic cross sections for reactions of deuterons with a large variety of target masses ($12 < A < 238$) and range of incident energies $E_{lab} < 183$ MeV. As a matter of fact, this interaction is an improvement of other widely used systematic parametrizations for shorter range of masses and energies$[21, 22, 23]$. It is the purpose of the calculations performed in this work, to test this improved parametrization of the $d−target$ interaction, when used to describe continuum states of the deuteron after the breakup of $^6$Li. The density dependent double-folding Sao Paulo potential (SPP)$[24]$ with adequate $α$-particle mass densities is used to describe the $α−target$ interaction $V_{αΤ}$. The non-local SPP has been widely used with success to describe elastic and inelastic scattering, as well as, nucleon transfer cross sections for a number of nuclear systems. An extensive systematics of nuclear densities has been performed by L.C. Chamon et al.$[25]$ to produce a parameter-free interaction. In our calculations, the $α−target$ interaction depends also on the target mass $A$ and the kinetic energy of the $α$-particle respect to the initial incident energy of $^6$Li. As for the $α−d$ cluster structure of $^6$Li, use is made of the Woods-Saxon parametrization given in Ref.$[26]$. Ground, and unbound continuum scattering states of $^6$Li are generated with this interaction. In order to calculate the effect of the resonances of $^6$Li, we remove the continuum states $l = 2, s = 1, π = (-)^l$, that is $j^p=3+, 2+$ and $1+$ from the continuum spectrum. This will be explained in section 3 where, the results are presented.

2. Basic CDCC formalism

In the CDCC formalism, continuum scattering states are replaced by a limited summation over bin-states. A bin-state $N_I$, $l = 1, ..., l_{max}$ is a set of discrete states, with a given angular momentum $l$, with parity $(-1)^l$, total angular momentum $j$, excitation energy $ε_{k,l}$ (respect to threshold energy), with discretization step $Δε_l$ up to a maximum energy $ε_{max}$. The $N_I$ bin-states are constructed in sequential intervals of energy, i.e., $ε_{i−1} < ε_k < ε_i$ with $i = 1, N_I$, where $ε_0 = 0$ and $ε_{N_I max} = ε_{max}$. Within the CDCC model, bin states $ϕ_{l,s,j}^{(i)}(r)$, $i = 1, N_I$ are square integrable and are constructed as a superposition of scattering wave functions within the interval $[k_{i−1}, k_i]$ with excitation energy corresponding to the mean value energy,

$$ε_i = \frac{ε_{k_i} + ε_{k_{i−1}}}{2},$$

(1)
where,
\[
\phi_{l,s,j}^{(i)}(r) = \frac{1}{\eta_i} \int_{k_{i-1}}^{k_i} w_i(k) \psi_{l,s,j}^{(scatt.)}(r,k) dk,
\]
where \( w_i(k) \) is a weight function that satisfies, \( \int_{k_{i-1}}^{k_i} |w_i(k)|^2 dk = \eta_i \). Resonance energies \( \epsilon_{i,res} \) and widths \( \Gamma_{i,res} \) of \( ^6\text{Li} \) correspond to the states \( j^n = 3+, 2+, 1+ \) with values \( \epsilon_3^+ = 0.716 \) MeV, \( \epsilon_3^+ = 0.204 \) MeV, \( \epsilon_2^+ = 2.84 \) MeV, \( \epsilon_3^+ = 1.7 \) MeV and \( \epsilon_1^+ = 4.18 \) MeV, \( \Gamma_1^+ = 1.5 \) MeV, respectively. Within the CDCC formalism, a set of coupled channel equations is obtained for the radial part of the relative motion wave function,
\[
\left[ T_R + U_{\beta\beta}(R) - E - \epsilon_\beta \right] u_{\beta}(R) = - \sum_{\beta'} U_{\beta\beta'}(R) u_{\beta'}(R),
\]
where angular coordinates have been integrated. Here, \( \beta = l, L, J \) are the quantum numbers of the ground and discrete states, \( \epsilon_\beta \) is the eigenvalue of the internal Hamiltonian of the nuclear system. \( U_{\beta\beta}(R) \) represents the diagonal matrix elements of the interactions and \( U_{\beta\beta}(R) \) non-diagonal elements. In particular, the diagonal matrix element \( U_{\beta\beta}(R) \) corresponds to the optical potential in the elastic channel. Inelastic excitations of the target are not explicitly considered in our calculations, while all continuum couplings are included. The deuteron-target potential \( V_{dT}(r) \) is that taken from Ref.[20].

Regarding the fragment-target interactions, the deuteron-target potential \( V_{dT}(r) \) is that taken from Ref.[20]. This potential is the result of a large systematic study of existing experimental data of elastic scattering angular distributions and non-elastic cross sections of incident deuteron on a wide range of target nuclei and energies. It is given in terms of Woods-Saxon shapes as,
\[
V(r) = -V_0 f_r(r) - iW f_V f_V(r) + 4iA_s W_s \frac{df_V}{dr} + \lambda_2 V_{so} + iW_{so} \frac{df_{so}}{dr} \mathbf{r} \cdot \mathbf{\sigma},
\]
where \( f_r(r) = 1 / [1 + \exp(-r/R_j)/a_j] \), \( R_j = r_j A^{1/3} \) with \( j = r, V, S, so. \) All potentials strengths, diffuseness and radii are given by the systematics, in terms of the target mass and deuteron energy \( E_d \). The non-local, double-folding density dependent interaction SPP has been used for the \( \alpha + target \) sub-system.
\[
V_{\alpha T}(r, E) = [N_R(E) + iN_I(E)] V_F(r) e^{-v(E)^2/c^2},
\]
r being the separation between the \( \alpha \)-particle and target and \( v(E) \) their relative velocity. The imaginary part is assumed with the same radial dependence as the real part. \( N_R(E) \) and \( N_I(E) \) are normalization coefficients that take into account polarization effects arising from direct reaction couplings and \( V_F(r) \) is the folding potential given by,
\[
V_F(r) = \int \rho_{A_1}(r_1) \rho_{A_2}(r_2) V_0 \delta[r - (r_1 - r_2)] dr_1 dr_2,
\]
with Fermi density distributions,
\[
\rho_i(r) = \frac{\rho_{0i}}{1 + \exp \frac{r - R_i}{a_i}}, i = A_1, A_2.
\]
Through an extensive systematic analysis of elastic scattering differential cross sections for several systems over a wide range of bombarding energies, global values have been determined for the zero-range potential strength \( V_0 = -456 \) MeV-fm\(^3\), the mass density diffuseness \( a_i = 0.56 \) fm and the radius parameter \( R_i = 1.31 A_i^{1/3} - 0.84 \) fm, \( i = A_1, A_2 \). Thus the SPP is a parameter-free interaction which depends only on the masses \( A_1, A_2 \) of the interacting nuclei and their collision energy \( E_{lab} \).
3. CDCC calculations of elastic scattering using global interactions

In this section CDCC calculations of elastic scattering angular distributions are presented for the projectile $^6\text{Li}$ with targets $^{144}\text{Sm}$ and $^{208}\text{Pb}$ for incident energies just above the corresponding Coulomb barriers. In our calculations, the code FRESO is used[27], where regarding the $V_{\alpha T}$ interaction, care is taken for the radial parameters $R_j$. This is so, since in the code FRESCO, the general relation $R_j = r_j(A_1^{1/3} + A_2^{1/3})$ is considered while a shorter $R_j = r_j A_1^{1/3}$ is used in Ref.[20], being $A_1 = 2$ and $A$ the target mass. To calculate the ground $\psi_{l=0}(E_{\text{thre}} = -1.47 \text{ MeV})$, resonant $l = 2, j^\pi = 3^+, 2^+, 1^+$ and non-resonant breakup discrete states of $^6\text{Li}$, we have used the parametrization for the interaction $V_{\alpha \text{d}}(r)$ given in Ref.[26]. The discretization is made as follows; the maximum angular momentum for the relative motion of the fragments $\alpha - d$ of $^6\text{Li}$ is $l_{\text{max}} = 3$. Larger values do not have any effect on the calculations. So, bin states are constructed for $l = 0$, $j^\pi = 1^+$ and $l = 1$, $j^\pi = 0^-, 1^-, 2^-$, with step $\Delta \varepsilon = 0.5 \text{ MeV}$ up to $\varepsilon_{\text{max}} = 6.8 \text{ MeV}$. Finer and variable steps are used for resonant states $l = 2, j^\pi = 3^+, 2^+, 1^+$, so as to obtain centroid excitation energies and widths close to the corresponding measured values. Table 1 shows the calculated values with the interaction $V_{\alpha \text{d}}$ of Ref.[26].

| Resonance Energies (MeV) for $^6\text{Li}$ | $E_{\text{cal}}$ | $\Gamma_{\text{cal}}$ | $E_{\text{exp}}$ | $\Gamma_{\text{exp}}$ |
|----------------------------------------|-----------------|-------------------|-----------------|-------------------|
| $3^+$                                  | 0.73            | 0.034             | 0.716           | 0.024             |
| $2^+$                                  | 3.09            | 1.3               | 2.84            | 1.7               |
| $1^+$                                  | 4.67            | 4.2               | 4.18            | 1.5               |

For bin states $l = 3, j^\pi = 4^-, 3^-, 2^-$, a larger step $\Delta \varepsilon = 1.0 \text{ MeV}$ is used. Convergence tests at $\varepsilon_{\text{max}} = 7.0, 7.5$ and $8.0 \text{ MeV}$ were done with no effect on elastic angular distributions. Similarly, larger steps $\Delta \varepsilon = 0.75$ and $1.0 \text{ MeV}$ were used with no appreciable effect on elastic scattering. Partial waves for the relative motion of $^6\text{Li}$ and target are considered up to $L_{\text{max}} = 600$ and nuclear and Coulomb potential multipoles are included up to $L_Q = 4$ for which convergence is achieved. As regards the normalization coefficients of the interaction $V_{\alpha \text{T}}$ of Eq.(5), we set these as $N_R = 1$ and $N_L = 1$. This is so, since for the nuclear systems and range of collision energies studied in this work, the $\alpha - \text{target}$ relative kinetic energy in the center of mass system is, in most cases, above the Coulomb barrier. Hence, polarization effects are negligible. The effect of resonances of $^6\text{Li}$ ($l = 2, j^\pi = 3^+, 2^+, 1^+$) on elastic scattering differential cross sections is studied by removing the narrow range of continuum bins around themselves. Figs.1 and 2 show the results for the elastic scattering angular distributions for the systems $^6\text{Li} + ^{144}\text{Sm}$ and $^6\text{Li} + ^{208}\text{Pb}$ for laboratory energies just above the barrier. As seen in Fig.1, for $^6\text{Li} + ^{144}\text{Sm}$ close agreement with the data is achieved when all couplings are considered (solid-lines). For the case when resonant continuum states are removed, the effect on elastic scattering is very small, except at the lowest energies (dashed-lines). In this case the calculations are slightly below those with resonance space included indicating a weak attractive polarization potential contribution from these states. At all energies, it is evident the importance of couplings to continuum states, since the calculations with only coupling to the elastic channel, that is, considering only the term $U_{\alpha d}$ in Eq.(3) is well below the data (dashed-lines). As for $^{208}\text{Pb}$, Fig.2 shows that except for the lowest energy the calculations with couplings to all continuum states are in agreement with the data. The effect of couplings to resonance states of $^6\text{Li}$ is in most cases small except at $E_{\text{lab}} = 39$ Mev at large angles.

In summary, CDCC calculations of the weakly bound projectile $^6\text{Li}$ with targets $^{144}\text{Sm}$ and $^{208}\text{Pb}$ have been performed with global fragment-target interactions dependent on the target mass and incident energy. In most cases, the results for the elastic scattering angular distributions at energies just above the barrier closely agree with the data when couplings to
continuum states of $^6\text{Li}$ are considered. Optical model calculations, without continuum couplings predict elastic scattering angular distributions well below the experimental values. This could mean that continuum couplings to breakup channels of $^6\text{Li}$ produce a net repulsive polarization potential that increases the Coulomb barrier. This fact, consequently leads to less absorption from the elastic channel. On the same footing, the effect of continuum resonant states of $^6\text{Li}$ on elastic scattering has been studied. The effect appears to be not too strong in most cases.

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Figure 2. Same as Fig.1 but for $^6\text{Li}^+^{208}\text{Pb}$. Data are taken from Refs.[29, 30, 31]

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