Spin content of constituent quarks
and one-spin asymmetries in inclusive processes

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Abstract

We consider mechanism for one-spin asymmetries observed in inclusive hadron production. The main role belongs to the orbital angular momentum of the quark-antiquark cloud in the internal structure of constituent quarks. We argue that the origin of the asymmetries in pion production is a result of retaining of this internal angular orbital momentum by the perturbative phase of QCD under transition from the non-perturbative phase. The non-perturbative hadron structure is based on the results of chiral quark models.

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Introduction

Significant one–spin asymmetries were observed in elastic scattering, hyperon and pion production in different reactions [1]. A number of models were proposed for qualitative and quantitative description of the corresponding data. Despite these models provide qualitative explanation of the experimental data and allow in several cases to get quantitative agreement with experiment it should be noted that there is no single model capable to describe all of the existing data. Also new data [2] often pose new problems for these models.
In the recent time interest in studying of spin phenomena was shifted to measurements of spin dependent structure functions and to extensive interpretations of the data. The substantial experimental information was obtained by the present time. As it is considered now, about one third of the proton spin is due to quark spins \([3, 4]\). It is interesting that calculations of \(\eta'\) couplings to vector mesons also predicted that quarks carry about one third of spin of vector mesons \([5]\). These results could be interpreted as that a substantial part of hadron spin would be due to orbital angular momentum of quarks.

The challenging problem is to relate the spin structure of nucleons studied in deep–inelastic scattering with the one–spin asymmetries measured in hadron processes.

In this paper we consider a possible origin of asymmetry in pion production under collision of polarized proton beam with unpolarized proton target. The experimental data \([6]\) for such processes were obtained at rather high energy where one could rely on perturbative QCD at high \(p_\perp\)'s.

We will use the scheme which incorporates perturbative and non–perturbative phases of QCD. We will argue that the orbital angular momentum of partons inside constituent quarks retained by the perturbative phase of QCD under transition from non-perturbative phase leads to significant asymmetries in hadron production with polarized beam.

In the nonperturbative regime QCD should provide the two important phenomena: confinement and spontaneous breaking of chiral symmetry. The scales relevant to these phenomena are characterized by the parameters \(\Lambda_{QCD}\) and \(\Lambda_\chi\), respectively. The values of these parameters are \(\Lambda_{QCD} = 100–300\) MeV and \(\Lambda_\chi \simeq 4\pi f_\pi \simeq 1\) GeV, where \(f_\pi\) is the pion decay constant \([6]\). Chiral \(SU(3)_L \times SU(3)_R\) symmetry is spontaneously broken in the range between these two scales. The chiral symmetry breaking results in particular in generation of quark masses and in appearance of quark condensates. Quark masses are comparable with the hadron mass scale. Therefore hadron is often represented as a loosely bounded system of the constituent quarks. These observations on the hadron structure lead to the understanding of several regularities observed in hadron interactions at large distances. Such a picture also provides reasonable values for the static characteristics of hadrons, for instance, their magnetic moments. Constituent quarks in this approach are extended objects. They are described by their size and quark matter distribution.
The general form of the effective lagrangian relevant for description of the non-perturbative phase of QCD \[ L_{QCD} \rightarrow L_{\text{eff}} \] includes three terms

\[ L_{\text{eff}} = L_\chi + L_I + L_C. \]

Here \( L_\chi \) is responsible for the spontaneous chiral symmetry breaking and turns on first. To account for the constituent quark interaction and confinement the terms \( L_I \) and \( L_C \) are introduced. The \( L_I \) and \( L_C \) do not affect the internal structure of constituent quarks.

However, the structure of hadron depends on the scale of the process and is different for different values of \( Q^2 \). Processes with large \( Q^2 \) can resolve partonic structure of constituent quarks and are described by perturbative QCD. Perturbative QCD provides well established calculation methods based on the use of the \( L_{QCD} \).

1 Structure and spin content of constituent quarks

In this section we specify the hadron structure and the spin structure of constituent quarks with account for results obtained in deep-inelastic scattering experiments.

In the framework of non-perturbative approach we consider a hadron as consisting from the valence constituent quarks located at the central part of a hadron and quark condensate surrounding this core. Experimental and theoretical arguments in favor of such picture were given in [9, 10]. The term \( L_\chi \) provides masses for quarks and leads to appearance of the quark condensate. We consider as a particular form of \( L_\chi \) the Nambu–Jona-Lasinio (NJL) model [11] with 6-quark interaction, i.e. we refer to the version of this model which takes into account the \( U(1)_A \)–symmetry breaking term [12]:

\[
L_\chi = \bar{\psi}(i\gamma \cdot \partial - \hat{m})\psi + \sum_{a=0}^{8} \frac{1}{2} G \left[ (\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2 \right] + K[\det \bar{\psi}_i(1 - \gamma_5)\psi_j + \det \bar{\psi}_i(1 + \gamma_5)\psi_j],
\]  

(1)

where quark field \( \psi \) has three colors (\( N_c = 3 \)) and three flavors (\( N_f = 3 \)) and matrix \( \hat{m} = \text{diag}(m_u, m_d, m_s) \) is composed from the current quark masses.
Eq. (1) may be considered as a minimal effective lagrangian which reflects some of the basic properties of nonperturbative QCD. The last term in Eq. (1) obeys the chiral $SU(3)_L \times SU(3)_R$ invariance, but it breaks the unwanted $U(1)_A$ symmetry. The four–fermion lagrangian of the NJL model reveals this symmetry in the $N_f \geq 3$ case. The first two terms represent the well–known NJL lagrangian. These terms ensure the dynamical breaking of the $SU(3)_L \times SU(3)_R$ chiral symmetry when the coupling constant $G$ is large enough. It has been shown that the chiral symmetry is broken dynamically and quark acquires a mass when the coupling constant $G$ is beyond its critical value. The lagrangian (1) in addition to the 4–fermion interaction of the original NJL model includes 6–fermion $U(1)_A$–breaking term. The constituent quark masses have been calculated in [12]:

$$m_U = m_u - 2G \langle 0 | \bar{u}u | 0 \rangle - 2K \langle 0 | \bar{d}d | 0 \rangle \langle 0 | \bar{s}s | 0 \rangle$$

In this approach massive quarks appear as quasiparticles, i.e. as current quarks and the surrounding clouds of quark–antiquark pairs which consist of a mixture of quarks of different flavors. Therefore besides its mass, quark acquires an internal structure and a finite size. Quark radii are determined by the radii of the clouds surrounding it. We assume that the strong interaction radius of quark $Q$ is determined by its Compton wavelength:

$$r_Q = \frac{\xi}{m_Q},$$

where constant $\xi$ is universal for different flavors. Quark formfactor $F_Q(q)$ is taken in the dipole form, viz

$$F_Q(q) \simeq \left(1 + \frac{\xi^2 q^2 / m_Q^2}{Q^2}\right)^{-2}$$

and the corresponding quark matter distribution $d_Q(b)$ is of the form [14]:

$$d_Q(b) \propto \exp(-m_Q b / \xi).$$

Quantum numbers of the constituent quarks are the same as the quantum numbers of current quarks due to conservation of the corresponding currents in QCD. The only exception is the flavor–singlet, axial–vector current, its $Q^2$–dependence is due to axial anomaly which arises under quantization. Axial anomaly gives contribution in the spin content of the constituent quark as it was discussed in [4, 13, 14, 15]. In particular, it was demonstrated that
constituent quark picture of a hadron with account for anomaly contribution is consistent with the results for the proton spin structure function $g_1(x)$ obtained in deep-inelastic scattering.

It is useful to note that in addition to $u$ and $d$ quarks constituent quark ($U$, for example) contains pairs of strange quarks, and the ratio of scalar density matrix elements

$$2\langle U|\bar{ss}|U\rangle/\langle U|\bar{uu} + \bar{dd}|U\rangle$$

is about 0.15 in the NJL model with axial $U(1)$ breaking [16]. It should be noted, however, that the following inequalities are valid for different mechanisms

$$\langle U|\bar{uu}|U\rangle \gg \langle U|\bar{dd}|U\rangle, \langle U|\bar{ss}|U\rangle.$$

The picture of hadron consisting from constituent quarks can be applicable at moderate momentum transfers, while interactions at high momentum transfers would resolve internal structure of constituent quarks and they are to be represented as clusters of non-interacting partons in this kinematical region.

In the framework of the NJL model transition to partonic picture is related to the need of a momentum cutoff $\Lambda = \Lambda_\chi \simeq 1$ GeV. We adopt the point that the need for such cutoff is an effective implementation of the short distance behavior in QCD [12].

Thus, loosely speaking we should consider three different regions for hadron structure depending on the typical scale of interaction: interactions with small momentum transfers ($Q < \Lambda_{QCD}$) do not resolve internal structure of hadrons, interactions with medium momentum transfers ($\Lambda_{QCD} < Q < \Lambda_\chi$) see hadrons as consisting from constituent quarks and interactions with high momentum transfers ($Q > \Lambda_\chi$) resolve the partonic structure of constituent quarks. Of course, this separation is approximate and real picture is definitely more complicated.

In the framework of the NJL model the partonic structure of constituent quarks was defined in [12]. The parton content $\omega_{q/Q}(x)$ of constituent quark $Q$ as it was shown is determined by the imaginary part of the virtual antiquark–quark scattering amplitudes $t_{1q/Q}(s, \mu^2)$ and $t_{2q/Q}(s, \mu^2)$ and can be written as follows:

$$\omega_{q/Q}(x) = \pi^2 \left[ \int_{\text{sh}}^\infty \frac{ds}{(2\pi)^3} \int_{-\infty}^{\mu \text{max}^2} d\mu^2 \text{Im}[t_{1q/Q}(s, \mu^2) + xt_{2q/Q}(s, \mu^2)] \right].$$
where $s$ is the squared center of mass energy for the quark–antiquark scattering and $\mu^2$ is the virtual quark squared mass; $\omega_{q/Q}(x)$ is the distribution of flavor $q$ quarks in the constituent quark $Q$.

Now we will address a complicated problem of the spin structure of constituent quark. The measurements of spin–dependent structure function $g_1(x)$ triggered the discussion of the role of axial anomaly in the nucleon spin. In the framework of perturbative theory it was argued that axial anomaly in QCD effectively reduces the total spin carried by quarks [17].

On the other hand the contribution of axial anomaly could have a non-perturbative origin. For example, in the NJL–model the 6-quark fermion operator in Eq. (1) simulates the effect of gluon operator

$$\frac{\alpha_s}{2\pi} G_{\mu\nu} \tilde{G}^\mu_\alpha^\nu,$$

where $G_{\mu\nu}$ is the gluon field tensor in QCD. Account for axial anomaly in the framework of chiral quark models results in compensation of the valence quark helicity by helicities of quarks from the cloud in the structure of constituent quark. The specific non-perturbative mechanism of such compensation is different in different approaches [12, 13, 14, 15], e.g. in [14] the modification of the axial U(1) charge of constituent quark is generated by the interaction of current quarks with flavor singlet field $\phi^0$. Forward matrix elements of the currents $A^j_{\mu5}$ between the constituent quarks $Q^i$ are written then as follows

$$\langle Q^i|A^j_{\mu5}|Q^i \rangle = \frac{1}{2}(\delta^i_j - \frac{2}{3}c)s^i_{\mu}, \quad (8)$$

where $s^i_{\mu}$ is the spin vector and the constant $c$ determines the derivative coupling between quarks and field $\phi^0$. Eq. (8) shows that constituent quark of any flavor contains a sea of polarized current quarks of the all other flavors. The case of $c = 1/2$ corresponds to complete compensation of current quark spins. The similar picture was developed by Fritzsch [13]. On this ground we can conclude that significant part of the spin of constituent quark should be associated with the orbital angular momentum of quarks inside this constituent quark, i.e. the cloud quarks should rotate coherently inside constituent quark. We consider effective lagrangian approach where gluon degrees of freedom are overintegrated and therefore we are not going to discuss subtle questions on the principal possibility of separation between the orbital angular momentum and gluon contribution in QCD (cf. [18]).
The important question concerns the origin of this orbital angular momentum. It is useful to address an analogy between hadron physics and superconductivity, in particular, anisotropic generalization of the theory of superconductivity which seems to match well with the above picture for constituent quark. Indeed, it was shown [19, 20] that pairing correlations have axial symmetry around the anisotropy direction \( \hat{l} \) which acts as the local \( z \) axis. Because of this anisotropy there are particle currents induced by pairing correlations. The corresponding calculations [19] indicate that particle at the origin is surrounded by a cloud of correlated particles that rotate around it with the axis of rotation \( \hat{l} \). The value of intrinsic orbital angular momentum \( L_0 \) is determined by the density of particles \( \rho \), the gap amplitude \( \Delta_0 \) and by the Fermi energy \( E_f \):

\[
L_0 = \frac{1}{2} (\rho - C_0) \simeq \rho \left( \frac{\Delta_0}{E_f} \right)^2
\]

It is clear that there is a direct analogy between the above picture and that of constituent quark. Axis of anisotropy \( \hat{l} \) is determined by the polarization vector of valence quark located at the origin of constituent quark. The orbital angular momentum \( \vec{L} \) lies along \( \hat{l} \) and its value is proportional to quark density.

Thus, the spin of constituent quarks \( J_{zU} \) is determined by the following sum

\[
J_{zU} = 1/2 = J_{zu} + J_{z(q\bar{q})} + \langle L_{z(q\bar{q})} \rangle.
\]  \( (9) \)

The value of the orbital momentum contribution into the spin of constituent quark can be estimated with account for new experimental results from deep-inelastic scattering [1] indicating that quarks carry one third of proton spin, i.e.

\[
(\Delta u + \Delta d + \Delta s)_p \simeq 1/3,
\]

and taking into account the relation between contributions of current quarks into proton spin and corresponding contributions of current quarks into the spin of constituent quarks and contributions of constituent quarks into the proton spin

\[
(\Delta u + \Delta d + \Delta s)_p = (\Delta U + \Delta D)(\Delta u + \Delta d + \Delta s)_U.
\]  \( (10) \)
Indeed, if we adopt $SU(6)$ model ($\Delta U + \Delta D = 1$) then we should conclude that

$$J_{zu} + J_{z\{\bar{q}q\}} \simeq 1/6$$

and from Eq. (3)

$$\langle L_{z\{\bar{q}q\}} \rangle \simeq 1/3,$$

i. e. about 2/3 of the $U$-quark spin is due to the orbital angular momenta of $u$, $d$ and $s$ quarks inside $U$-quark. Index $z$ will be dropped henceforth.

We argue that the existence of this orbital angular momentum, i.e. orbital motion of quark matter inside constituent quark, is the origin of the observed asymmetries in inclusive production at moderate and high transverse momenta. Indeed, since the constituent quark has small size, asymmetry associated with internal structure of this quark will be significant at $p_\perp > \Lambda_\chi \approx 1$ GeV where interactions at small distances give noticeable contribution.

The orbital motion of current quarks means that they have intrinsic transverse momenta. Estimation of the mean value of this momenta from the relation

$$\langle L_{\{\bar{q}q\}} \rangle = r_Q \langle k_\perp \rangle$$

with $\langle L_{\{\bar{q}q\}} \rangle \simeq 1/3$ and $r_Q = 1/3 - 1/6$ fm provides the values of 200–400 MeV which are in agreement with experimental values. Note, that these estimations correspond to $\xi \simeq 1/3$ in Eq. (3).

It should be noted that at high $p_\perp$ we will have a parton picture for constituent quark as a cluster of non-interacting quarks which however should naturally preserve their orbital momenta of the preceding non-perturbative phase of QCD, i.e. the orbital angular momentum will be retained in perturbative phase of QCD.

2 Model of hadron production and one-spin asymmetries

Consider now mechanism of hadron production based on the above picture of hadron structure. We will study the hadron processes of the type

$$h_1^\uparrow + h_2 \rightarrow h_3 + X$$
with polarized beam or target.

The picture of hadron consisting from constituent quarks embedded into quark condensate implies that overlapping and interaction of peripheral clouds occur at the first stage of hadron interaction. As a result massive virtual quarks appear in the overlapping region and some effective field is generated. Constituent quarks located in the central part of hadron are supposed to scatter in a quasi-independent way by the effective field. In the above picture generation of the effective field is related with the term $\mathcal{L}$ and formation of the final hadrons should be described by the term $\mathcal{L}_C$ in the Lagrangian $\mathcal{L}$.

Inclusive production of hadron $h_3$ results from recombination of the constituent quark (low $p_\perp$'s, soft interactions) and from the excitation of this constituent quark, its decay and subsequent fragmentation in the hadron $h_3$. The latter process is determined by the distances smaller than constituent quark radius and is associated therefore with hard interactions (high $p_\perp$'s). Thus, we adopt the two-component picture of hadron production which incorporates the non-perturbative and perturbative QCD phases.

Now we write down explicit formulas for corresponding inclusive cross-sections. The following expressions were obtained in \[1\] and take into account unitarity in the direct channel of reaction. They have the form

$$\frac{d\sigma^{\uparrow,\downarrow}}{d\xi} = 8\pi \int_0^\infty bdb \frac{I^{\uparrow,\downarrow}(s, b, \xi)}{|1 - iU(s, b)|^2},$$

(11)

where $b$ is the impact parameter. Here function $U(s, b)$ is the generalized reaction matrix (helicity non-flip one) which is determined by dynamics of the elastic reaction

$$h_1 + h_2 \rightarrow h_1 + h_2.$$

The elastic scattering amplitude $F$ (at the moment we consider spinless case for simplicity) is related to the function $U$ by the equation:

$$F = U + iUDF,$$

(12)

which we write here in the operator form. This equation allows one to obey unitarity provided inequality $\text{Im} \ U(s, b) \geq 0$ is fulfilled.

In accordance with the quasi-independence of valence quarks we represent
the basic dynamical quantity in the form of product [10]:

\[
U(s, b) = \prod_{i=1}^{N} \langle f_{Q_i}(s, b) \rangle
\]  

(13)

in the impact parameter representation, \( N = n_{h_1} + n_{h_2} \) is the total number of constituent quarks in the initial hadrons. Factors \( \langle f_{Q}(s, b) \rangle \) correspond to the individual quark scattering amplitude smeared over transverse position of \( Q \) inside hadron \( h_1 \) and over fraction of longitudinal momentum of the initial hadron carried by quark \( Q \).

The functions \( I^{\uparrow, \downarrow}(s, b, \xi) \) are related to the functions \( U_n(s, b, \xi, \{\xi_{n-1}\}) \) which are the multiparticle analogs of the \( U(s, b) \) and are determined by dynamics of the processes

\( h_1^{\uparrow, \downarrow} + h_2 \rightarrow h_3 + X_{n-1} \).

The kinematical variables \( \xi \) (\( x \) and \( p_{\perp} \), for example) describe the state of the produced particle \( h_3 \) and the set of variables \( \{\xi_{n-1}\} \) describe the system \( X_{n-1} \) of \( n-1 \) particles. Arrows \( \uparrow \) and \( \downarrow \) denote corresponding direction of transverse spin of the polarized initial particle.

Expressions for the functions \( I^{\uparrow, \downarrow} \) are the following [21]:

\[
I^{\uparrow, \downarrow}(s, b, \xi) = \sum_{n \geq 3, \lambda_2, \lambda_{X_n}} n \int d\Gamma_n^\prime |U_n^{\uparrow, \downarrow}(s, b, \xi, \{\xi_{n-1}\})|^2, 
\]  

(14)

where \( X_n = h_3 + X_{n-1} \). In the above formulas \( d\Gamma_n^\prime \) is the element of \( n-1 \)-particle phase space volume.

We introduce the two functions \( I_+ \) and \( I_- \):

\[
I_{\pm}(s, b, \xi) = I^{\uparrow}(s, b, \xi) \pm I^{\downarrow}(s, b, \xi),
\]  

(15)

where \( I_+(s, b, \xi) \) corresponds to unpolarized case. The following sum rule takes place for the function \( I_+(s, b, \xi) \):

\[
\int I_+(s, b, \xi) d\xi = \bar{n}(s, b) \text{Im}U(s, b),
\]  

(16)

where \( \bar{n}(s, b) \) is the mean multiplicity of secondary particles in the impact parameter representation.
Asymmetry $A_N$ defined as the ratio

$$A_N = \left\{ \frac{d\sigma^\uparrow}{d\xi} - \frac{d\sigma^\downarrow}{d\xi} \right\} / \left\{ \frac{d\sigma^\uparrow}{d\xi} + \frac{d\sigma^\downarrow}{d\xi} \right\}$$

can be expressed in terms of the functions $I_\pm$ and $U$:

$$A_N = \frac{\int_0^\infty b db I_-(s, b, \xi) / |1 - iU(s, b)|^2}{\int_0^\infty b db I_+(s, b, \xi) / |1 - iU(s, b)|^2}. \quad (17)$$

Using relations between transversely polarized states $|\uparrow, \downarrow\rangle$ and helicity states $|\pm\rangle$, viz

$$|\uparrow, \downarrow\rangle = (|+\rangle \pm |-\rangle)/\sqrt{2} \quad (18)$$
on one can write down expressions for $I_+$ and $I_-$ through the helicity functions $U_{(\lambda_i)}$:

$$I_+(s, b, \xi) = \sum_{n,\lambda_1,\lambda_2,\lambda_X} n \int d\Gamma_1 U_{n,\lambda_1,\lambda_2,\lambda_X} (s, b, \xi, \{\xi_{n-1}\})^2,$$

$$I_-(s, b, \xi) = 2 \sum_{n,\lambda_2,\lambda_X} n \int d\Gamma_1 \text{Im}[U_{n,\lambda_1,\lambda_2,\lambda_X} (s, b, \xi, \{\xi_{n-1}\}) \times U_{n,\lambda_1,\lambda_2,\lambda_X}^*(s, b, \xi, \{\xi_{n-1}\})]. \quad (19)$$

Taking into account Eq. (16), quasi-independence of the constituent quarks and assumption on hadron production as a result of interaction of the corresponding constituent quark with the effective field we adopt the following expressions for the functions $I_+(s, b, \xi)$ and $I_-(s, b, \xi)$:

$$I_\pm(s, b, \xi) = \bar{n}(s, b) \text{Im} \prod_{i=1}^{N-1} \langle f_{Q_i}(s, b) \langle \phi_{h_3/\bar{Q}}^\pm (s, b, \xi) \rangle \rangle,$$

where quark $\bar{Q}$ is the leading quark in the process of $h_3$ production, for example, $\bar{Q} = U$ for $h_3 = \pi^+$ and $\bar{Q} = D$ for $h_3 = \pi^-$. The functions $\langle \phi_{h_3/\bar{Q}}^\pm (s, b, \xi) \rangle$ describe the $h_3$ production as a result of interaction of the constituent quark $\bar{Q}$ with the effective field.

The central point of the model is a connection of the one-spin asymmetries in inclusive production with the orbital angular momentum of current quarks inside the constituent quark. This orbital momentum will affect the hadron
production only at small distances where internal structure of constituent quark could be probed. Thus, the function \( \langle \varphi_{h/\bar{Q}}^{-}(s, b, \xi) \rangle \) will be sensitive to interactions at small distances only, i.e. it will be determined by the hard processes which can be described in the framework of perturbative QCD, but with account for the internal orbital momentum of partons. This function can be written as the convolution integral:

\[
\langle \varphi_{h/\bar{Q}}^{-} \rangle = \langle \varphi_{\tilde{q}/\bar{Q}}^{-} \rangle \otimes D_{h/\bar{q}},
\]

where \( D_{h/\bar{q}} \) is the fragmentation function which is supposed to be spin-independent. Owing to inequalities (7) the leading contribution is given by the quark \( \tilde{q} \) of the same flavor as \( \bar{Q} \). Due to isospin invariance \( D_{\pi^+/u} = D_{\pi^-/d} \).

Fragmentation functions are almost completely unknown quantities in QCD. We consider these functions to be spin independent and the asymmetries are related here to the internal structure of constituent quarks. Note that the fragmentation functions might have a non-trivial spin dependence as it was discussed in [23].

Spin-independent function \( \langle \varphi_{h/\bar{Q}}^{+} \rangle \) gets contribution both from soft processes, where constituent quark interacts with the effective field as a whole (hadron \( h_3 \) arises in this case as a result of recombination of \( \bar{Q} \) with the virtual quarks) and from hard interactions associated with partonic structure of the constituent quark. Respectively, the function \( \langle \varphi_{h/\bar{Q}}^{+} \rangle \) contains the two terms, viz

\[
\langle \varphi_{h/\bar{Q}}^{+} \rangle = \langle \varphi_{\tilde{q}/\bar{Q}}^{+} \rangle \otimes D_{h/\bar{q}} + \langle \varphi_{\tilde{q}/\bar{Q}}^{+} \rangle \otimes D_{h/\bar{q}},
\]

where first term corresponds to soft and second one – to hard, spin-independent interactions. Since the second term in this formula and the function \( \langle \varphi_{h/\bar{Q}}^{-} \rangle \) are determined by the internal structure of constituent quark, then we can assume that their \( x \)-dependence is determined by the structure function of constituent quark \( \omega_{\tilde{q}/\bar{Q}}(x) \):

\[
\langle \varphi_{\tilde{q}/\bar{Q}}^{\pm} \rangle \otimes D_{h/\bar{q}} \propto \omega_{\tilde{q}/\bar{Q}}(x)
\]

while

\[
\langle \varphi_{\bar{Q}}^{\pm} \rangle \otimes D_{h/\bar{Q}} \propto \omega_{\bar{Q}/h_1}(x),
\]

where \( \omega_{\bar{Q}/h_1}(x) \) is the \( x \)-distribution of the constituent quark \( \bar{Q} \) in the hadron \( h_1 \).
It is an important question at this point what is the effect of nonzero orbital momentum of quarks and consequently their internal transverse momenta \( \langle k_\perp \rangle \) inside the constituent quark. It leads to a certain shift of transverse momenta and on the basis of Fourier transformation we suppose that this effect results in the phase factor \( \exp \left[ i \langle k_\perp \tilde{q}/\tilde{Q} \rangle r_{\tilde{Q}} \right] \) since the \( b \)-dependence of the functions \( \langle \varphi^+_{\tilde{q}/\tilde{Q}} \rangle \otimes D_{h_3/\tilde{q}} \) is determined by formfactor of the constituent quark \( \tilde{Q} \) (cf. Eqs. (4), (5)). On this ground we adopt the following relation between the spin-dependent function \( \langle \varphi^-_{\tilde{q}/\tilde{Q}} \rangle \) and the spin-independent function \( \langle \varphi^+_{\tilde{q}/\tilde{Q}} \rangle \):

\[
\langle \varphi^-_{\tilde{q}/\tilde{Q}} \rangle \otimes D_{h_3/\tilde{q}} \simeq \exp \left[ i \langle k_\perp \tilde{q}/\tilde{Q} \rangle r_{\tilde{Q}} \right] \langle \varphi^+_{\tilde{q}/\tilde{Q}} \rangle \otimes D_{h_3/\tilde{q}}.
\] (25)

Taking into account that the orbital angular momentum of \( \tilde{q} \) quarks in the constituent quark \( \tilde{Q} \) is proportional to its polarization we can rewrite exponential factor of Eq. (25) in the form

\[
\exp \left[ i \langle k_\perp \tilde{q}/\tilde{Q} \rangle r_{\tilde{Q}} \right] = \exp \left[ i \langle L_{\tilde{q}/\tilde{Q}} \rangle \right] \simeq \exp \left[ i P_{\tilde{Q}} \langle L_{\{\tilde{q}\bar{q}\}} \rangle \right],
\]

where \( \langle k_\perp \tilde{q}/\tilde{Q} \rangle \) and \( \langle L_{\tilde{q}/\tilde{Q}} \rangle \) are the mean transverse momenta and orbital momenta respectively of quark \( \tilde{q} \) inside quark \( \tilde{Q} \). The sign and value of the latter are determined by the polarization \( P_{\tilde{Q}} \) of the constituent quark \( \tilde{Q} \) inside the hadron \( h_1 \) and the mean orbital momenta of cloud quarks \( \langle L_{\{\tilde{q}\bar{q}\}} \rangle \).

Taking into account the above relations, we represent the asymmetry \( A_N \) in the form:

\[
A_N(s, x, p_\perp) = \frac{\sin[P_{\tilde{Q}} \langle L_{\{\tilde{q}\bar{q}\}} \rangle] \omega_{\tilde{q}/\tilde{Q}}(x) \phi_{\text{hard}}(s, p_\perp)}{\omega_{\tilde{q}/h_1}(x) \phi_{\text{soft}}(s, p_\perp) + \omega_{\tilde{q}/\tilde{Q}}(x) \phi_{\text{hard}}(s, p_\perp)},
\] (26)

where the function \( \phi_{\text{hard}} \) is determined by the interactions at small distances and reflects the structure of constituent quarks while \( \phi_{\text{soft}} \) is associated with the soft interactions and determined by a non-perturbative structure of hadron consisting from its constituent quarks. The explicit forms of these functions are determined by the integrals entering Eq. (17). We can rewrite Eq. (26) in more general form

\[
A_N(s, x, p_\perp) = \sin[P_{\tilde{Q}} \langle L_{\{\tilde{q}\bar{q}\}} \rangle] \frac{d\sigma_{\text{hard}}}{d\xi} / \left\{ \frac{d\sigma_{\text{soft}}}{d\xi} + \frac{d\sigma_{\text{hard}}}{d\xi} \right\},
\] (27)

which is appropriate for numerical analysis.
3 Numerical analysis

Prior to discussions of the experimental data it should be noted that the described mechanism is to be expected to work at high enough energies and transverse momenta when the structure of constituent quark can be resolved.

Explicit form of $A_N$ is determined by the specific parameterizations of quark and hadron formfactors, corresponding structure functions, mean multiplicity and several other distributions. These parameterizations imply rather large freedom and will unfortunately obscure the main features of the proposed mechanism.

Therefore, as a first step to numerical analysis of the data, it seems reasonable to use phenomenological parameterizations of inclusive cross-sections which can be matched with the above model as an input to obtain asymmetry $A_N$. Indeed, for such purposes we should consider a two-component parameterization of inclusive cross-sections which includes soft and hard contributions

$$\frac{d\sigma}{d\xi} = \frac{d\sigma_{soft}}{d\xi} + \frac{d\sigma_{hard}}{d\xi}.$$  

The parameterization of such type was used under analysis of the experimental data for cross-sections of the processes:

$$p + p \rightarrow \pi^\pm + X$$

at different energies [24]. Due to this we consider asymmetries $A_N$ in the processes with polarized initial proton:

$$p^\uparrow + p \rightarrow \pi^\pm + X.$$  

Asymmetry for the process

$$p^\uparrow + p \rightarrow \pi^0 + X$$

will be obtained using the following relation valid in a parton model:

$$A_N(\pi^0) = \frac{A_N(\pi^+) \frac{d\sigma}{d\xi}(\pi^+) + A_N(\pi^-) \frac{d\sigma}{d\xi}(\pi^-)}{\frac{d\sigma}{d\xi}(\pi^+) + \frac{d\sigma}{d\xi}(\pi^-)}.$$  

Eq. (28) follows also from the isospin relations for one-particle inclusive productions [22]. Now to get values of asymmetries we should fix $P_Q$ and
\( \langle L_{(q\bar{q})} \rangle \) for \( U \)-quarks (\( \pi^+ \)-production) and for \( D \)-quarks (\( \pi^- \)-production). For polarization of the constituent quarks we use \( SU(6) \) values \( P_U = \frac{2}{3} \) and \( P_D = -\frac{1}{3} \). Orbital angular momentum was estimated in sec. 2, therefore we take \( \langle L_{(q\bar{q})} \rangle = \frac{1}{3} \). The explicit form for \( d\sigma/d\xi \) and values of the parameters therein we borrowed from [24]. It has typical two-component behavior:

\[
\frac{d\sigma}{d\xi} = A \exp \left[ -B \sqrt{p_{\perp}^2 + m_0^2} \right] / (1 + e^{D(x_{\perp} - x_0)}) + C(1 - x_{\perp})^m (p_{\perp}^4 + M^4)^{-n/4},
\]

where \( x \simeq 0 \) and \( A \) and \( m_0 \) both have a weak energy dependence. The first term in Eq. (29) has typical form of soft contribution and will be identified with \( d\sigma_{\text{soft}}/d\xi \) and the second one, decreasing as a power of \( p_{\perp} \), is typical for hard contribution and is to be identified with \( d\sigma_{\text{hard}}/d\xi \). Thus, we have all parameters fixed and can evaluate now the asymmetries \( A_N \) at different energies. At high energies the experimental data for the process \( p_\uparrow + p \rightarrow \pi^0 + X \) are available at \( P_L = 200 \text{ GeV/c} \). Comparison of the calculations for \( A_N \) with the data and predictions for asymmetries in the processes of \( \pi^\pm \) and \( \pi^0 \) production at this energy as well as at \( P_L = 70, 800 \text{ GeV/c} \) and \( \sqrt{s} = 500, 2000 \text{ GeV} \) are given in Figs. 1–4. \( A_N \) has a weak energy dependence and gets significant values starting from \( p_{\perp} \simeq 1 \text{ GeV/c} \). As it is seen from Fig. 3 asymmetry \( A_N \) for the process \( p_\uparrow + p \rightarrow \pi^0 + X \) predicted by the model is systematically higher that the experimental data. This fact could confirm conclusion that hadron wave function deviates from \( SU(6) \) model as it was claimed in [27]. To check this statement we calculated the above asymmetries with \( P_U = -P_D = P_p \). The results are presented in Figs. 5-8. As it is clearly seen the agreement with the experimental data is better than for the case of \( SU(6) \) model. It is also evident that asymmetries in the production of charged pions are significantly higher than under the neutral pion production. This indicate that studies of the charged pion production would reveal significant asymmetries which are diluted in the case of neutral pion production. The corresponding values will allow to get conclusion on the mean orbital angular momenta of quark matter inside the constituent quarks.
4 Summary and discussion

First, we would like to summarize the main points of the considered model:

- asymmetry reflects internal structure of the constituent quarks and is proportional to the orbital angular momentum of current quarks inside the constituent quark;
- sign of asymmetry and its value are proportional to polarization of the constituent quark inside the polarized initial hadron, in the simplest case this polarization is determined by the $SU(6)$-symmetry.

We have not considered here quantitative description of the $x$–dependence of $A_N$, first of all, because it includes rather large freedom under the choice of explicit parameterization. Indeed, the realistic $x$–dependencies of the functions $\langle \varphi_{\tilde{q}/\tilde{Q}}^\pm \rangle \otimes D_{h_3/\tilde{q}}$ and $\langle \varphi_{\tilde{Q}}^\pm \rangle \otimes D_{h_3/\tilde{Q}}$ are definitely more complicated than those indicated in Eqs. (23) and (24). We should consider corresponding convolution integrals of the structure functions (of constituent quarks, current quarks inside constituent quarks) and fragmentation functions. Eqs. (23) and (24) show only characteristic parts of these dependencies. The realistic choice of the corresponding parameterizations of the structure and fragmentation functions as well as choice of the $x$–dependence of constituent quark polarization would allow to get description of the $x$–dependence of asymmetries.

The model predicts significant one-spin asymmetries at high $p_\perp$ values. At first sight it looks like contradiction since the model itself was inspired by QCD where we should expect helicity conservation in hard region due to the chiral $SU(3)_L \times SU(3)_R$ symmetry. Indeed, asymmetry $A_N$ results from interference between the two helicity functions $\U_{+\lambda_2,\lambda_n}$ and $\U_{-\lambda_2,\lambda_n}$ and therefore from the helicity conservation rule for exclusive processes \[ \lambda_1 + \lambda_2 = \lambda_n \]

we have to expect $A_N = 0$ at high $p_\perp$’s. However, the helicity conservation rule at hadron level is not a direct consequence of the chiral symmetry of the perturbative phase of QCD. In addition to helicity conservation at quark level it assumes that only S-state of quarks contributes to a hadron wave function. This statement was disputed in the work of Ralston and Pire \[ \text{26} \] where it was
demonstrated that helicity may not be conserved at hadron level. The origin of such effect is due to a nonzero orbital angular momentum component of the hadron wave function.

In our model the orbital angular momentum plays a role in the wave function of constituent quarks. The two helicity functions $U_{\pm \lambda_2, \lambda_n}$ in the impact parameter representation gain different phase factors due to internal transverse momentum of partons related with their coherent rotation inside constituent quark. It results in interference between these two functions and leads to significant values of $A_N$. This mechanism is not at all at variance with QCD.

Number of recent papers demonstrated that large asymmetries observed in inclusive processes do not contradict to QCD. Different mechanisms were proposed as a source of the asymmetries: higher twist effects [28], correlation of $k_\perp$ and spin in structure [29] and fragmentation [23, 27] functions, rotation of valence quarks inside a hadron [30]. Significant role in the above references belongs to orbital angular momentum of the constituents inside a hadron. It is worth to note here that this idea could be traced back to the model of rotating hadronic matter proposed by Chou and Yang [32].

As it was already argued the main role belongs to the orbital angular momentum of current quarks inside the constituent quark while constituent quarks themselves have very slow (if at all) orbital motion and may be described approximately by $S$-state of the hadron wave function. The observed $p_\perp$-behavior of asymmetries in inclusive processes seems to confirm such conclusions. The significant asymmetries appear to show up beyond $p_\perp > 1$ GeV/c, i.e. the scale where internal structure of a constituent quark can be probed.

The proposed mechanism, in principle, is appropriate for description of hyperon polarization, in particular, its $p_\perp$-dependence. We can assume that constituent quark $\tilde{Q}$ gets polarization due to multiple scattering in effective field by analogy with mechanism proposed in [31]. Then polarization of $\Lambda$-hyperon will be proportional to constituent quark polarization and polarization of $s$-quark inside constituent quark $\tilde{Q}$. The latter one can be related to the significant $s$-quark polarization measured in deep-inelastic scattering. Eq. (6) also indicates that constituent quarks have significant strangeness content. The $p_\perp$-dependence of polarization $P_\Lambda$ will be related to the specific behavior of soft and hard contributions to inclusive cross-section of $\Lambda$-production, but the general trends are expected to be the same
as in $p_{\perp}$-behavior of asymmetry in $\pi^-$-production. We should also expect that behavior of asymmetry $A_N$ in $\Lambda$-production will be similar to the corresponding behavior of polarization $P_{\Lambda}$ and this fact seems find confirmation in the data at $P_L = 200$ GeV/c [33].

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Figure captions

Fig. 1. Asymmetry $A_N$ ($SU(6)$ model) in the process $p_\uparrow + p \to \pi^+ + X$ (positive values) and in the process $p_\uparrow + p \to \pi^- + X$ (negative values) at $P_L = 70$ GeV/c (dashed curve), $P_L = 200$ GeV/c (solid curve) and at $P_L = 800$ GeV/c (dashed-dotted curve).

Fig. 2. Asymmetry $A_N$ ($SU(6)$ model) in the process $p_\uparrow + p \to \pi^0 + X$ at $P_L = 70$ GeV/c (dashed curve), $P_L = 200$ GeV/c (solid curve) and at $P_L = 800$ GeV/c (dashed-dotted curve). Experimental data from [6].

Fig. 3. Asymmetry $A_N$ ($SU(6)$ model) in the process $p_\uparrow + p \to \pi^+ + X$ (positive values) and in the process $p_\uparrow + p \to \pi^- + X$ (negative values) at $\sqrt{s} = 500$ GeV (dashed curve) and at $\sqrt{s} = 2000$ GeV (dashed-dotted curve).

Fig. 4. Asymmetry $A_N$ ($SU(6)$ model) in the process $p_\uparrow + p \to \pi^0 + X$ at $\sqrt{s} = 500$ GeV (dashed curve) and at $\sqrt{s} = 2000$ GeV (dashed-dotted curve).

Fig. 5. Asymmetry $A_N$ for the case $\mathcal{P}_U = -\mathcal{P}_D = \mathcal{P}_p$ in the process $p_\uparrow + p \to \pi^+ + X$ (positive values) and in the process $p_\uparrow + p \to \pi^- + X$ (negative values) at $P_L = 70$ GeV/c (dashed curve), $P_L = 200$ GeV/c (solid curve) and at $P_L = 800$ GeV/c (dashed-dotted curve).

Fig. 6. Asymmetry $A_N$ for the case $\mathcal{P}_U = -\mathcal{P}_D = \mathcal{P}_p$ in the process $p_\uparrow + p \to \pi^0 + X$ at $P_L = 70$ GeV/c (dashed curve), $P_L = 200$ GeV/c (solid curve) and at $P_L = 800$ GeV/c (dashed-dotted curve). Experimental data from [3].

Fig. 7. Asymmetry $A_N$ for the case $\mathcal{P}_U = -\mathcal{P}_D = \mathcal{P}_p$ in the process $p_\uparrow + p \to \pi^+ + X$ (positive values) and in the process $p_\uparrow + p \to \pi^- + X$ (negative values) at $\sqrt{s} = 500$ GeV (dashed curve) and at $\sqrt{s} = 2000$ GeV (dashed-dotted curve).

Fig. 8. Asymmetry $A_N$ for the case $\mathcal{P}_U = -\mathcal{P}_D = \mathcal{P}_p$ in the process $p_\uparrow + p \to \pi^0 + X$ at $\sqrt{s} = 500$ GeV (dashed curve) and at $\sqrt{s} = 2000$ GeV (dashed-dotted curve).