Assessing the Privacy Cost in Event-Based Demand Response Management of Microgrids

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Abstract—Demand response (DR) management programs have emerged as a potential key enabling ingredient in the context of smart grid (SG). Nevertheless, the rising concerns over privacy issues raised by customers subscribed to these programs constitute a major threat towards their effective deployment and utilization. This has driven extensive research to resolve the hindrance confronted, resulting in a number of methods being proposed for preserving the privacy of participant customers. While these methods are capable to protect customer privacy, only limited attention has been paid to their computational efficiency and performance quality. Under the paradigm of differential privacy, this paper initiates a systematic empirical study on the trade-off between privacy and optimality in centralized DR systems for control and optimization of Microgrids (MGs). Aiming to reveal and quantify the factors governing this trade-off, we compare quality and privacy level of the solutions computed by the featured privacy preserving DR strategy based on Laplace mechanism, which maximizes the cumulative customer utility, subject to the overall net satisfiable apparent power generation on MG. The analytic results derived from the comparison are complemented with empirical findings, corroborated extensively by numerical simulations with up to thousands of customers. By contrasting these two metrics, this pilot study serves DR practitioners when considering the implications of deploying privacy preserving DR programs in practice as well as stimulates future research on exploring more efficient mechanisms with definite guarantees on optimality for maintaining privacy of customers on demand side.

Index Terms—Demand response, differential privacy, microgrids, privacy preserving energy management, randomized response.

I. INTRODUCTION

THE emerging environmental and economic concerns necessitate modernization of the aging power grid infrastructure into a more sustainable and Smart Grid (SG). Microgrids (MGs), seen as one of the key enabling contributors towards this transition, facilitate large scale deployment of renewable energy based Distributed Generation (DG) and incorporation of new load types such as plug-in hybrid electric vehicles (PHEVs). However, the volatile nature of the renewable energy sources along with increasing customer expectation for both power demand and quality make the energy management of MGs more intricate and critical.

Demand Response (DR) has been widely recognized as an essential mechanism for efficient energy procurement and optimization in MGs [1]. To achieve the desired system reliability, resilience and power quality, DR programs aim at establishing a mutually beneficial interaction framework for DR participants and aggregators where power generation drives the demand. This allows reducing not only the peak loads but also the cost of generation and transmission capacity expansion. Along with a number of benefits, DR has been proven to reduce price variations [2], lower electricity bills [3], enhance congestion management [4] and strengthen system security [5].

DR benefits are elevated with increasing customer participation, as the number of controllable loads grows. Thus, it is an important consideration for utility companies to attract more DR subscribers by offering various incentives. Concerns over customer privacy pose a significant hindrance towards this goal [6], [7]. DR programs with centralized control architecture, which are commonly implemented in practice [8], require information on customer loads and preferences as inputs to the optimization problem. The studies in [9], [10] have shown that this may limit effective customer participation since customers may be unwilling to reveal their private information. These factors highlight the significance of deriving efficient privacy preserving strategies for solving large-scale DR problems under centralized control scheme.

This paper adheres to the framework of differential privacy as a mean to quantify the extent of privacy of a customer participating in DR program. Originated from the research in [11] and defined by Dwork [12], differential privacy now serves as a traditional definition of privacy in computer science. The notion of differential privacy can be described as a guarantee that altering an individual record in the input set does not impact the distribution of the computation outcomes significantly. This provides a strong privacy guarantee. Under differential privacy framework, the probability of an adversary inferring customer’s data from the output of a private algorithm is as likely as obtaining the same information when the customer has no contribution in the output. Furthermore, this statement holds true independent of additional information the adversary may possess.

Several studies have addressed the issue of preserving privacy in DR programs. An efficient privacy preserving metering scheme is proposed in [13]. The privacy is achieved by adding a randomly generated number to the measurement sent from the smart meter to the power provider, making it a simple and low complexity approach. The solution meets the privacy
and utility requirements. In [14], the metering data is sent to the DR provider, while maintaining its integrity during all the processes. The privacy is preserved by separating the identity of the customer from the metering data. That is, the DR provider only has access to either the customer identity or the metering data at a time but never to both. In [15] an efficient privacy preserving DR scheme is proposed with adaptive key evolution. The power demand aggregation and response are performed securely and efficiently using the homomorphic encryption and the key evolution techniques. Moreover, the desired trade-off between the communication efficiency and security can be achieved by adaptively controlling the key evolution. The authors of [16] study a privacy preserving energy scheduling problem for MGs with high renewable energy penetration. The problem is formulated as a linear program with privacy constraints. A new method, called dual decomposition, has been developed to optimally solve this problem.

One of the downsides of differential privacy framework is the difficulty to find efficient algorithms [17] meeting the criteria of the framework. Initial studies in this field were focusing on understanding the extent to which finding solutions compatible with differential privacy is feasible, while paying little attention to the computational efficiency. As a consequence, many fundamental problems currently have private solutions, while lacking efficient algorithms. In [18], for instance, the authors show that any learnable concept class can be privately learnt with a polynomial increase in the number of samples, but likely in an exponential time. In [19] algorithms for private combinatorial optimization problems are studied. However, these are specialized algorithms for specific problems only. In general, it is unclear whether a computationally efficient algorithms can be designed for solving these problems.

Typically, in a DR management scheme, there is a single load-serving entity (LSE) or an operator of MG, who coordinates the decisions of DR participants. Various approaches for modeling the energy management in MGs have been proposed in the literature with different extents of consideration of characteristics and operating constraints. The setting studied in this paper is driven by the corresponding operating modes of an MG, namely grid-connected and islanded. There is a high probability that an MG once operating in isolated mode will be short of power. The considered DR scheme resembles such a scenario, where customers may suffer from a reduction of generation occasionally due to limited MG capacity. The employed MG model encompasses a hybrid mix of traditional and renewable energy (RE) supplies that could collectively have a variable (depending on the availability of RE and storage available) yet dispatchable capacity. Constrained by the net available apparent generation, LSE is required to make control decisions in real time as to maximize the total utility of satisfied customers without violating their privacy. To solve the resulting optimization problem privately, a randomized differentially private approach [17] is employed with a definite theoretical guarantee on the level of privacy. The proposed approach relies on the Laplace mechanism introduced in [12].

The findings featured in this study shed light on the cost imposed on the quality of solution that arises with incorporation of privacy preserving mechanism into DR optimization problem. The results observed through extensive analytical and empirical analysis indicate that optimality of the solutions produced by the proposed privacy preserving approach degrades severely as the number of customers increases as well as when increasing privacy level of an individual customer.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Towards introducing a comprehensive mathematical formulation of the proposed privacy preserving DR optimization problem, as a first and fundamental step, this section models the system and its components. The adopted DR model envisions a single LSE procuring the responses of customers’ demands over a decision horizon \( T \triangleq \{1, \ldots, m\} \). The decision horizon \( T \) is discretized into \( m \) equal periods with a duration corresponding to the required time resolution granularity at which DR management decisions are to be produced. At each time slot \( t \in T \), the net available generation capacity of MG is denoted by \( C_t \in \mathbb{R}_+ \).

A. Load Model

Consider a set of customers \( \mathcal{N} \triangleq \{1, \ldots, n\} \) for a DR management scheme run by LSE. A customer \( k \in \mathcal{N} \) is associated with a complex-valued power demand \( S_k \in \mathbb{C} \) required for operating certain electric appliances at particular time instant. Denote by \( \Re(S_k) \) the active power demand of customer \( k \), and by \( \Im(S_k) \) the reactive power demand. For clarity of presentation, this paper assumes (via a rotation of power demand vectors) that \( \Re(S_k) \geq 0 \) and \( \Im(S_k) \geq 0 \) for \( \forall k \in \mathcal{N} \). Also, it is assumed that MG load is comprised solely of controllable loads which are drawn for supplying DR customers’ demands.

The customers’ demands are categorized into two types according to their operation and energy consumption characteristics, elastic (divisible) and inelastic (indivisible). The demand of a customer possessing inelastic load can be either shed or fed completely. This models the electric appliances that can operate only under particular energy supply level (e.g., washing machine, vacuum cleaner). Different from the inelastic demands, an elastic load may be satisfied partially and adjusted to operate with different energy consumption levels (e.g., air conditioners, LED light bulbs).

B. Customer Preference Model

In a DR program where each subscribed customer is an independent decision maker, typically, the response to the incentives by the DR aggregator is modeled by an utility function. The response of a subscribed customer may vary depending on the particular time of a day (e.g. at peak and non-peak hours). Moreover, a response may vary from customer to customer based on the consumption profile (i.e., when considering residential and commercial customers).

For simplicity, the utility function is summarized by an utility value \( u_{k,t} \) associated with a customer \( k \in \mathcal{N} \) that
quantifies the extent of satisfaction obtained (or alternatively, the payment) by customer $k$ when own power demand is satisfied at time $t \in T$. In the case with inelastic demands, if $S_k$ is satisfied at time slot $t$, $u_{k,t}$ is the perceived utility for customer $k$, otherwise zero utility is perceived. As for a customer $k' \in N'$ with an elastic load, a portion $b \in [0,1]$ of the power demand $S_{k'}$ drawn from MG at time instant $t$ imparts an utility of $b \cdot u_{k',t}$.

C. Privacy Preserving DR

Recall that the privacy preserving DR scheme under study requires protecting customer sensitive information in the input data. In particular, this paper focuses on the case where utilities, which define the objective function of DR optimization problem, are the sensitive information to customers, while the remaining data is considered as public.

In the adopted DR program featuring a centralized control scheme each customer declares his reactive and active power demand to LSE upon arrival. In order to prevent customer utilities from being exposed to the DR aggregator or a potential eavesdropper the proposed strategy submits to LSE a perturbed utility value to obfuscate customer’s true valuation. Section III scopes a detailed explanation of this mechanism and provides provable guarantees on its privacy level in the context of differential privacy.

To effectively control DR management, low-latency communication infrastructure between the LSE and customers using separate power supply will be utilized in the proposed scheme. Such a low-latency communication infrastructure is enabled by the standards of smart grid communication protocols [20]. Additionally, it is to be assumed that LSE has full control over the on/off operations of its customers’ demands.

D. Optimization Problem

Now that the system model is established, the utility maximizing demand response problem (UMDR) can be formulated by the following quadratically constrained integer programming problem.

\[
\text{(UMDR)} \quad \max_{(x_k)_{k \in N}} \sum_{k \in N} u_{k,t} x_k \\
\text{subject to} \quad \sum_{k \in N} S_k x_k \leq C_t \quad (1) \\
x_k \in \{0, 1\} \quad \forall k \in N \quad (2)
\]

Here, $x_k$ is a binary decision variable that takes value 1 if and only if the $k$-th customer’s power demand $S_k$ is satisfied and 0 otherwise. The UMDR problem aims at maximizing the overall net utility of customers while maintaining the apparent power generation $C_t$ bound at time instant $t \in T$.

Evidently, UMDR is NP-HARD (as shown to be strongly NP-HARD [21]), since the 0-1 classical knapsack problem is its special case. Relaxing (discrete) binary decision variables ($x_k$) to continuous ones in UMDR problem, such that $x_k \in [0,1]$, yields a convex quadratic programming problem denoted by UMDR$_L$. This seemingly small change in problem formulation, in fact, alters the complexity of DR optimization problem notably, since the convex variant can be solved optimally in polynomial time, for e.g., by applying Interior Point methods. Aside from complexity, setting the decision variable ($x_k$) to be discrete or continuous alters the practical application aspects. Particularly, the continuous case corresponds to customers having only elastic power demands. As for the case with binary decision variables, DR participant demands comprising the customer set consist of inelastic loads.

Before the proposed privacy preserving mechanism is introduced, a performance metric for assessing the level of privacy of the computed solution of UMDR and UMDR$_L$ problems is provided in the subsequent subsection.

E. Differential Privacy

The framework of differential privacy represents information private to a customer as a set $D$ referred to as database. In the setting studied here $D = \{u_{k,t}\}_{k \in N}$ at time $t$ of arriving customers. The basic idea underlying this framework is to draw an association between privacy and impact of an individual customer in the database. The impact, that is, changes that occur in the database when altering or deleting a customer’s record is typified by the concept of neighboring databases. Define databases $D$ and $D'$ to be neighboring if they are identical except a single record. In other terms, shifting between neighboring databases affects the objective $\sum_{k \in N} u_{k,t} x_k$ of UMDR problem leaving constrain matrix unchanged.

The notion of differential privacy is formalized by the definition below.

**Definition 1** (Differential privacy [12]). A randomized function $M : D \to \mathbb{R}$ that maps databases to an output range is $(\epsilon, \delta)$-differentially private if for every pair of neighboring databases $D, D'$

\[
Pr[M(D) \in S] \leq e^\epsilon \cdot Pr[M(D') \in S] + \delta \quad \forall S \subseteq \mathbb{R}, \quad (3)
\]

where $\epsilon > 0$ and $\delta \in [0, 1)$.

Similarly, a function $M$ is called $\epsilon$-differentially private, if $\delta = 0$. The level of privacy is characterized by the constant $\epsilon$. The smaller the $\epsilon$ the higher is privacy level.

One of the salient properties and strengths of differential privacy framework is that provided privacy guarantees hold regardless of adversary’s strength or ancillary information possessed. In this context, devising computationally efficient algorithms with provable optimality guarantees is substantially difficult. In fact, it was shown in [17] that if one makes no assumptions on the sensitivity of the private data it is impossible to devise a non-trivial optimality guarantees under the paradigm of differential privacy. This necessitates the need for introducing the simplifying assumption stated hereunder that will be followed throughout this paper.
Assumption 1. There exist positive $u_{\text{max}}$ and $u_{\text{min}}$ known to LSE apriori such that for $\forall k \in N$

$$u_{\text{min}} \leq u_{k,t} \leq u_{\text{max}} \quad \forall t \in T.$$ 

III. PROPOSED STRATEGY

This section presents an efficient randomized mechanism to compute solutions of UMDR and UMDR_L problems with a precise theoretical guarantee on their accuracy under differential privacy. The method relies on a principle differential privacy technique which is explained in what follows.

To this end, let a function $f : D \to \mathbb{R}$ be $\triangle$-sensitive if

$$|f(D) - f(D')| \leq \triangle \quad \forall \; \text{neighboring} \; D, D'.$$  

Instead of releasing the true customer valuations, the proposed mechanism perturbs each customer’s utility $u_{k,t}$ independently by adding a noise drawn from the Laplace distribution. When applying Laplace mechanism to a $\triangle$-sensitive function $f(D)$ the output releases

$$f(D) + \mu,$$  

where $\mu$ is a member of Laplace distribution with parameter $\frac{\triangle}{\epsilon}$ with $\epsilon > 0$.

Theorem 1 (\cite{22}). The Laplace mechanism explained above is $\epsilon$-differentially private.

Define

$$\hat{u}_{k,t} \triangleq u_{k,t} + \text{Lap}(\frac{u_{\text{max}} \sqrt{8n \log (\frac{n}{\beta})}}{\epsilon})$$  

(6)

to be the perturbed utility of customer $k \in N$ at time $t \in T$. Then the private analog of UMDR_L problem (DR with elastic demands) is embodied by the following convex programming problem.

(UMPDR_L) \quad \max_{\hat{x}_k \in \mathbb{N}} \sum_{k \in \mathbb{N}} \hat{u}_{k,t}x_k

subject to

$$\sum_{k \in \mathbb{N}} S_kx_k \leq C_t$$  

(7)

$$x_k \in [0, 1] \quad \forall k \in N$$  

(8)

Solving UMPDR_L problem non-privately results in a private solution to the original one. Note that any feasible solution to UMPDR_L is also feasible for UMDR_L. The following theorem is defined.

Theorem 2 (\cite{17}). Let $\beta \in (0, 1)$ be given and $\hat{x}^* \triangleq (\hat{x}_{k}^*)_{k \in \mathbb{N}}$ be an optimal solution for UMPDR_L problem. Then, it is $(\epsilon, \delta)$-differentially private feasible solution to UMDR_L that with probability $1 - \beta$ satisfies the following additive optimality bound

$$\sum_{k \in \mathbb{N}} u_{k,t}x_k^* \geq \text{OPT}_L - \alpha$$

where $\alpha = \frac{4u_{\text{max}} \sqrt{8n \log (\frac{n}{\beta})}}{\epsilon}$ and $\text{OPT}_L$ is the objective value of an optimum solution of UMDR_L.

The proof of Theorem 2 is deferred to the appendix.

IV. EMPIRICAL EVALUATION

To complement the analytic result in Theorem 2, this section evaluates the proposed mechanism empirically under extensive scenarios. The privacy preserving DR scheme under study is applied on a simulated MG. The objective is to investigate and quantify the trade-off between optimality and privacy of the proposed approach under the framework of differential privacy. CPLEX optimizer is utilized to obtain the close-to-optimal solutions numerically for UMDR, UMDR_L problems and their differentially private analogs.

A. Simulation Setup and Settings

An MG with an overall generation capacity of 4MVA is simulated with over 1500 customers. Each customer has a specific power demand (including both active and reactive power) and a utility that is generated according to a probability preference model. The amount of generation on an MG is typically less than the amount of demand and thus, the customers may suffer from a reduction of generation capacity occasionally. Various types of loads are considered including residential and commercial customers. The output solutions of CPLEX optimizer when applied to UMDR, UMDR_L, UMPDR, UMPDR_L problems are denoted by $\text{OPT}$, $\text{OPT}_L$, $\text{OPT}_{DP}$, and $\text{OPT}_{LDP}$, respectively.

The following parameters were set in CPLEX optimizer: (1) the total time expended for solving the problem was 200 seconds, (2) absolute mixed integer programming (MIP) optimality gap (i.e., the threshold of the absolute gap between the lower and upper objective bound) was set to zero, and (3) infeasibility tolerance was set to $10^{-9}$. It is worth noting that there are no guarantees that given an integer programming problem the optimizer will return an optimal solution nor will it terminate in a reasonable time (i.e., within 200 seconds for each run). Whenever the optimizer exceeds the time limit, the current best solution is considered to be optimal.

The simulations were evaluated using 2 Quad core Intel Xeon CPU E5607 2.27 GHz processors with 12 GB of RAM. The simulation environment was implemented using Python programming language with Scipy library for scientific computation. Typically, the load power factor varies between 0.8 to 1 (to comply with IEEE standards) and thus the maximum phase angle $\theta$ between any pair of demands is restricted to be in the range of $[0, 360^\circ]$.

B. Case Studies

Various case studies are performed to evaluate the proposed mechanism by taking into account the correlation between customer load and utility considering various load types. The following are settings for the case studies in this paper.

(i) Utility-demand correlation:
In this paper, the case studies will be represented by the aforementioned acronyms. For example, the case study named QM stands for the one with mixed customers and quadratic utility-demand correlation.

a) **Quadratic utility (Q):** The utility of a customer is a quadratic function of the power demand in the form of

\[ u_{k,t}(|S_k|) = a \cdot |S_k|^2 + b \cdot |S_k| + c, \quad \forall t \in T \quad (9) \]

where \( a > 0, b, c \geq 0 \) are predetermined constants.

b) **Uncorrelated setting (U):** The utility of each customer is independent of the power demand and is generated randomly from \([0, |S_{\text{max}}(k)|]\), where \(|S_{\text{max}}(k)|\) depends on the customer type: if customer \( k \) is a commercial customer then \(|S_{\text{max}}(k)| = 0.8 \text{MVA}\), otherwise if residential \(|S_{\text{max}}(k)| = 13 \text{KVA}\).

(ii) **Customer types:**

a) **Residential (R) customers:** The customer set is comprised of residential customers having small power demands ranging from 1500VA to 13KVA.

b) **Mixed (M) customers:** The customer set is comprised of a mix of commercial and residential customers. Commercial customers have big power demands ranging from 300KVA up to 0.8MVA and constitute no more than 10\% of all customers chosen at random.

In this paper, the case studies will be represented by the aforementioned acronyms. For example, the case study named QM stands for the one with mixed customers and quadratic utility-demand correlation.

C. Simulation Results

In this subsection the employed privacy preserving mechanism is evaluated in terms of quality of solution. The optimal solutions computed by CPLEX optimizer for UMDR and UMDR\(_L\) problems, which correspond to non-private DR management, are considered to be the base case for the comparison. The method is applied to various case studies where each case study is analyzed considering changes in the set of customers. For each DR optimization problem, two different privacy levels are examined where the corresponding value of \( \epsilon \) is set to 0.1 and 1. As for additive privacy parameter \( \delta \), the value is fixed to 0.5 for all case studies. It is worthy to note that the smaller these values are, the higher is the privacy level guaranteed. As an example, the employed method with an input parameter of \((\epsilon = 0.1, \delta = 0.5)\) is applied to UMDR problem (i.e., this corresponds to solving UMPDR with perturbed customer utilities calculated according to Eqn. (6)) 30 times for each of the \( m \) number of customers (where \( m \) varies between 500 to 1500 in steps of hundred) for case study QR (i.e., quadratic, residential) considering random changes in demands and utilities of customers. Thus, the total number of experiments for each case study is 300. The observed results are summarized in Figures 1 and 2.

One of the important findings observed is a common trend appearing in all the case studies performed. As can be inferred from Figures 1 and 2 increasing \( \epsilon \) reduces the optimality gap. That is, the lower the level of privacy the higher the quality
of the solution. This observation is supported by Eqn. \ref{eqn:laplace_noise} since the Laplace noise added to each customer’s increases when lowering $\epsilon$. As illustrated in Fig. \ref{fig:laplace_noise}, on a MG with 500 customers the proposed privacy preserving mechanism attained nearly 0.4 approximation when $\epsilon = 0.1$, whereas when $\epsilon = 1$ the optimality gap is about 0.55 considering the case study QR.

Another important observation is that with increasing customer participation optimality of the proposed mechanism degrades drastically especially for UMPDR problem. As depicted in Fig. \ref{fig:optimality_degradation} when the number of customers is small $\text{OPT}^{\text{DP}}$ approaches $\text{OPT}$, while as the customer set cardinality grows $\text{OPT}^{\text{DP}}$ drifts far away. This highlights the necessity of devising efficient privacy preserving mechanisms, with a constant factor guarantee on the optimality gap, capable of solving large-scale DR management problems. For case study QR, which is depicted in Figure \ref{fig:case_study}, the optimality gap is deceasing as the number of customers goes up when considering up to 1000 customers. This is due to the fact that at this scale all customers demands are below the total capacity 4MVA and hence can be all satisfied in non-private DR (i.e., OPT). Whereas, in $\text{OPT}^{\text{DP}}$ some customers are discarded despite the sufficient generation capacity since the added Laplace noise to their utility turns it negative. As the number of customers grows up to 1000, the number of customers with positive utility increases and hence does the objective value. This phenomena to occur decreases.

An auxiliary yet interesting result is the impact scale of the studied mechanism on optimality for the case studies with inelastic versus elastic demands. Figures \ref{fig:inelastic} and \ref{fig:elastic} show that for case study QR incorporation of privacy has more significant degrading effect on the optimality for the scenarios with inelastic demand. In particular, the objective value for the case of elastic demands drops by approximately 80%, while for the case of inelastic demand the maximum drop of the objective doesn’t exceed 50%. Moreover, for UMDR$_L$ problem, the optimality gap is not affected by the number of customers. This is not the case with inelastic demands as illustrated in Figure \ref{fig:case_study} Also it is interesting to examine the impact of utility to demand correlation on the performance of the adopted mechanism. According to Figure \ref{fig:correlation}, in contrast to quadratic utilities, the impact of privacy integration is more moderate in the case of random utilities.

V. CONCLUSION

This paper studies the trade-off between privacy and optimality in centralized DR management of MGs. Under the framework of differential privacy, the impact on quality of solution posed by privacy constraints is quantified empirically through extensive numerical simulations. The observed results illustrate the striking effect posed on the optimality of produced solutions to DR optimization problem when considering increased privacy levels. According to the findings, the optimality gap approaches nearly 80% in some cases, which urges the need for efficient privacy preserving algorithms with constant theoretical guarantee on the worst case performance.

The two major factors identified in this study that define this trade-off are the elasticity of demand and the cardinality of the customer set.

APPENDIX

In the appendix, the full proof of Theorem \ref{thm:optimality_gap} is presented.

A. Proof of Theorem \ref{thm:optimality_gap}

Proof. Since $n$ independent samples are drawn from the Laplace distribution the $(\epsilon, \delta)$-differential privacy follows directly from Theorem \ref{thm:laplace_noise}.

Given that with probability at least $\frac{1 - \beta}{n}$, where $\beta \in [0, 1]$, the magnitude of a single draw from Laplace distribution with chosen scale parameter is upper bounded by $\frac{\alpha}{2} \triangleq \frac{2u_{\text{max}}}{\sqrt{8n \log(\frac{1}{\delta})}}$, then by the union bound this condition holds with probability $1 - \beta$ for all $n$ draws.

Denote by $x^*$ and $\hat{x}^*$ the optimal solutions of UMDR$_L$ and UMDPR$_L$ problems, respectively. Also, let $\text{OPT}_L$ be the corresponding total valuation of the optimal solution $x^*$ to UMDR$_L$. Note that $\hat{x}^*$ is also a feasible solution to UMDR$_L$. The additive optimality bound is proved by contradiction. Assume $\sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k < \text{OPT}_L - \alpha$. The noise added at time $t \in T$ to each customer’s utility $u_{k,t}$ is bounded by $\frac{\alpha}{2}$, thus if $\sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k < \text{OPT}_L - \alpha$ then

$$\sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k < \sum_{k \in \mathcal{N}} (u_{k,t} + \frac{\alpha}{2}) \hat{x}^*_k$$

$$< \sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k + \sum_{k \in \mathcal{N}} \frac{\alpha}{2} \hat{x}^*_k$$

$$< \text{OPT}_L - \alpha + \frac{\alpha}{2} \sum_{k \in \mathcal{N}} \hat{x}^*_k$$

$$< \text{OPT}_L - \frac{\alpha}{2}.$$  \hspace{1cm} (13)

The last inequality holds since it is assumed that optimal solution has $l_1$ norm of 1. Note that one may easily scale down the original perturbed problem UMPDR$_L$ so as the resulting scaled variant has a solution $\hat{x}^*$ with $l_1$ norm of 1. On the other hand, it follows from the property of the Laplace distribution that

$$\sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k \geq \sum_{k \in \mathcal{N}} (u_{k,t} - \frac{\alpha}{2}) \hat{x}^*_k$$

$$\geq \sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k - \frac{\alpha}{2}$$

$$\geq \text{OPT}_L - \frac{\alpha}{2}.$$ \hspace{1cm} (16)

Eqns. (16) and (13) together contradict the optimality of $\hat{x}^*$. This completes the proof of the theorem showing that $\sum_{k \in \mathcal{N}} u_{k,t} \hat{x}^*_k \geq \text{OPT}_L - \alpha$. \hfill $\square$

Theorem \ref{thm:composition}(\ref{thm:privacy_integration}). The composition of $n$ adaptively chosen $\epsilon'$-differentially private mechanisms is $(\epsilon, \delta)$-differentially private if for all $\delta \in (0, 1)$ the following holds

$$\epsilon' = \frac{\epsilon}{\sqrt{8n \log(\frac{1}{\delta})}}.$$
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