Quantum states of neutrons in the gravitational field and limits for non-Newtonian interaction in the range between 1 µm and 10 µm

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Abstract. Quantum states in the Earth’s gravitational field can be observed, when ultra-cold neutrons fall under gravity. In an experiment at the Institut Laue-Langevin in Grenoble, neutrons are reflected and trapped in a gravitational cavity above a horizontal mirror. The population of the ground state and the lowest states follows, step by step, the quantum mechanical prediction. An efficient neutron absorber removes the higher, unwanted states. The quantum states probe Newtonian gravity on the micrometer scale and we place limits for gravity-like forces in the range between 1 µm and 10 µm.

1 A quantum system

Quantum physics is a fascinating but subtle subject. The subtlety of the quantum system described here arises from the fact, that gravity appears very weak in our universe. Quantum theory and gravitation affect each other, and, when neutrons become ultra-cold, the fall experiment of Galileo Galilei shows quantum aspects of the subtle gravity force in that sense that neutrons do not fall continuously. We find them on different levels, when they come close to a reflecting mirror for neutrons [1]. Of course, such bound states with discrete energy levels are expected when the gravitational potential is larger than the energy of the particle. Here, the quantum states have pico-eV energy, a value that is smaller by many orders of magnitude compared with an electromagnetically bound electron in a hydrogen atom, opening the way to a new technique for gravity experiments, for neutron optics, neutron detection and measurements of fundamental properties.

A side-effect of this experiment is its sensitivity for gravity-like forces at length scales below 10 µm. In light of recent theoretical developments in higher dimensional field theory [2,3,4], gauge fields can mediate forces that are $10^{10}$ to $10^{12}$ times stronger than gravity at submillimeter distances, exactly in the interesting range of this experiment and might give a signal in an improved setup.

The idea of observing quantum effects occurring when ultracold neutrons are stored on a plane was discussed long ago by V.I. Lushikov and I.A. Frank [5].
An in some aspects similar experiment was discussed by H. Wallis et al. [6] in the context of trapping atoms in a gravitational cavity. Retroreflectors for atoms have used the electric dipole force in an evanescent light wave [7,8] or they are based on the gradient of the magnetic dipole interaction, which has the advantage of not requiring a laser [9]. A neutron mirror makes use of the strong interaction between nuclei and an ultracold neutron, resulting in an effective repulsive force: Neutrons propagate in condensed matter in a matter similar to the propagation of light but with a neutron refractive index less than unity. Thus, one considers the surface of matter as constituting a potential step of height $V$. Neutrons with transversal energy $E_\perp < V$ will be totally reflected. Ultra-cold neutrons (UCN) are neutrons that, in contrast to faster neutrons, are retro-reflected from surfaces at all angles of incidence. When the surface roughness of the mirror is small enough, the UCN reflection is specular.

Neutron mirrors are interesting because they can be used to store neutrons, to focus neutrons, or even to build a Fabry Perot interferometer for neutron de Broglie waves. UCN storage bottle experiments have improved our knowledge about the neutron lifetime significantly or, together with the Ramsey method of separated oscillating fields, they have been used for a search for an electric dipole moment of the neutron.

2 Limits for non-Newtonian interaction below 10 $\mu$m

Discussions about deviations from the inverse square law for gravity have become popular in the past few years [10]. A new effective interaction coexisting with gravity would modify the Newtonian potential. On the assumption that the form of the non-Newtonian potential is given by the Yukawa expression, for masses $m_i$ and $m_j$ and distance $r$ the modified Newtonian potential $V(r)$ is having the form

$$V(r) = -G \frac{m_i \cdot m_j}{r} (1 + \alpha \cdot e^{-r/\lambda})$$

where $\lambda$ is the Yukawa distance over which the corresponding force acts and $\alpha$ is a strength factor in comparison with Newtonian gravity. $G$ is the gravitational constant. For a neutron with mass $m_n$, the gravitational force of the mass $m_E$ of the entire earth with radius $R_E$ lead to a free fall acceleration

$$g = \frac{G m_E}{R_E^2} = \frac{4\pi \cdot G \rho R_E^3}{3R_E^2} = \frac{4}{3} \pi G \rho R_E.$$  \hspace{1cm} (2)

When a neutron approaches the mirror, the mass of this extended source might modify $g$, when strong non-Newtonian forces are present. For small neutron distances $z$ from the mirror, say several micrometers, we consider the mirror as an infinite half-space with mass density $\rho$. By replacing the source mass $m_i$ by $dm_i$ and integrating over $dm_i$, the Yukawa-like term $\lambda$ modified Newtonian potential $V'(r)$ is having the form

$$V'(z, \lambda) = 2\pi \cdot m_n \rho \alpha \lambda^2 G \cdot e^{-|z|/\lambda}.$$  \hspace{1cm} (3)
Fig. 1. Limits for non-Newtonian gravity: Strength $|\alpha|$ vs. Yukawa length scale $\lambda$

a) Experiments with neutrons place limits for $|\alpha|$ in the range $1 \mu m < \lambda < 10 \mu m$.
b) Constraints from previous experiments [11,12,13,14,15] are adapted from Ref. [14].

and the non-Newtonian acceleration $g'$ as a function of distance $z$ is given by

$$g'(z, \lambda) = 2\pi \cdot \rho \alpha \lambda G \cdot e^{-|z|/\lambda}$$  \hspace{1cm} (4)

As a consequence, the ratio is

$$\frac{g'(z, \lambda)}{g(z, \lambda)} = \frac{3}{2} \alpha \cdot \frac{\lambda}{R_E} \cdot e^{-|z|/\lambda}.$$  \hspace{1cm} (5)
For \( z = \lambda = 10 \mu m \) and \( \alpha = 1 \), the ratio \( g'/g \) is about \( 10^{-12} \). Fig. 1a shows the present status of an experimental search for gravity-like forces below 10 \( \mu m \). The results of a fit of potential Eq. 3 to the measured data (see Fig. 5 and Fig. 6) yields predictions for 90% confidence level exclusion bounds on \( \alpha \) and \( \lambda \). These limits from this neutron mirror experiment are shown in Fig. 1a. They are the best known in the range \( 1 \mu m < \lambda < 3 \mu m \) and exclude for the first time gravity-like short-ranged forces at 1 \( \mu m \) with strength \( \alpha > 10^{12} \). The limit for strength \( \alpha \) at 10 \( \mu m \) is \( 10^{11} \) (Fig. 1a). There is a theoretical uncertainty in our limits since they depend on the model for the neutron absorber, estimated to be about one order of magnitude. In future experiments, these limits will be strongly improved by an enhanced setup and improved statistics by new UCN sources as a Monte Carlo simulation shows. Previous constraints \[11,12,13,14,15\] on both \( \lambda \) and \( \alpha \) are shown in Fig. 1b.

The method with neutrons has some advantages. Electromagnetic interactions at micrometer distances, serious sources of systematic error in distance force measurements, are effectively suppressed. The neutron carries no electric charge and direct electrostatic forces are ruled out. Yet, it bears a tiny magnetic moment \( \mu_n \) of approximately \( 0.5 \cdot 10^{-3} \cdot \mu_B \). This magnetic moment can induce a magnetic mirror force onto a neutron that hovers close to the surface of a body. Further, a neutron moving with the velocity \( v \) sees an induced electrostatic force, that is essentially some kind of Lorentz force and thus an effect of the order of \( v/c \). Both effects can be evaluated to yield electrodynamic energy shifts. With permittivity \( \epsilon \) and permeability \( \mu \), the order of magnitude is

\[
\Delta E_E \sim \epsilon_0 \left( \frac{\epsilon - 1}{\epsilon} \right) \cdot \frac{\mu_n^2 \cdot \mu^2}{48\pi} \cdot \frac{v^2}{z^3} \sim 10^{-26} \cdot 10^{-12} \text{eV}
\]

\[
\Delta E_B \sim \frac{\mu - 1}{\mu} \cdot \frac{\mu_0}{16\pi} \cdot \frac{\mu_n^2}{z^3} \sim 10^{-13} \cdot 10^{-12} \text{eV}
\]

for \( v \sim 10 m/s \) and \( z \sim 10 \mu m \). Thus, these effects can completely be neglected, since they are by far subgravitational in strength.

Motivations for gravity experiments come from frameworks for solving the hierarchy problem in a way of bringing quantum gravity down to the TeV scale. In such frameworks the Standard Model fields are localized on a 3-brane in a higher dimensional space by the presence of new dimensions of submillimeter size [2]. At the expected sensitivity, a number of gravity-like phenomena emerge. For example, a hypothetical gauge field can naturally have miniscule gauge coupling \( g_4 \sim 10^{-16} \) for 1 TeV, independent of the number of extra dimensions [3]. If these gauge fields couple to a neutron with mass \( m_n \), the ratio of the repulsive force mediated by this gauge field to the gravitational attraction is [3]

\[
\frac{F_{\text{gauge}}}{F_{\text{grav}}} \sim \frac{g_4^2}{Gm_n^2} \sim 10^6 \left( \frac{g_4}{10^{-16}} \right)^2.
\]  

(6)

With \( g_4 = 10^{-16} \) as a lower bound, these gauge fields can result in repulsive forces of million or billion times stronger than gravity at micrometer distances, exactly in the range of interest.
3 The experiment at the Institut Laue-Langevin

3.1 From hot to ultracold

Neutrons are produced in a spallation source or a research reactor. At production, these neutrons are very hot; the energy is about 2 MeV corresponding to $10^{10}$ degrees Centigrade. On the other side of the scale, the gravity experiment uses neutrons having $10^{18}$ times less energy in the pico-eV range (see Tab. 1). In a first step, spallation or fission neutrons thermalize in a heavy water tank at a temperature of 300 K. The thermal fluxes are distributed in energy according to Maxwellian law. At the Institut Laue-Langevin (ILL), cold neutrons are obtained in a second moderator stage from a 25 K liquid deuterium cold moderator near the core of the 57 MW uranium reactor. These cold neutrons have a velocity spectrum in the milli-eV energy range. For particle physics, a new beamline with a flux of more than $10^{10}$ cm$^{-2}$s$^{-1}$ over a cross section of 6 cm x 20 cm is available.

Ultra-cold neutrons are taken from the low energy tail of the cold Maxwellian spectrum. They are guided vertically upwards by a neutron guide (Fig. 2). The curved guide, which absorbs neutrons above a threshold energy, acts as a low-velocity filter for neutrons. Neutrons with a velocity of up to 50 m/s arrive at a rotating nickel turbine. Colliding with the moving blades of the turbine, ultra-cold neutrons exit the turbine with a velocity of several meters per second. They are then guided to several experimental areas. The exit window of the guide for the gravity experiment has a rectangular shape with the dimensions of 100 mm x 10 mm. At the entrance of the experiment, a collimator absorber system cuts down on the neutrons to a adjustable transversal velocity corresponding to an energy in the pico-eV range.

| Table 1. from hot to ultracold: neutrons at the ILL |
|---------------------------------|
| fission | thermal | cold | ultracold | this experiment |
| neutrons | neutrons | neutrons | neutrons | experiment |
| Energy | 2 MeV | 25 meV | 3 meV | 100 neV | 1.4 peV |
| Temperature | $10^{10}$ K | 300 K | 40 K | 1 mK | - |
| Velocity | $10^7$ m/s | 2200 m/s | 800 m/s | 5 m/s | $v_{\perp} \sim 2$ cm/s |

3.2 The setup

Fig. 3 shows a schematic view of the setup: Neutrons pass through the mirror absorber system eventually detected by a $^3$He-counter. The experiment itself is mounted on a polished plane granite stone with a passive antivibration table underneath. This stone is leveled with piezo translators. Inclinometers together
with the piezo translators in a closed loop circuit guarantee leveling with an absolute precision better than 10 µrad. Either one solid block with dimensions 10 cm x 10 cm x 3 cm or two solid blocks with dimensions 10 cm x 6 cm x 3 cm composed of optical glass serve as mirrors for UCN neutron reflection. Small angle X-ray studies [16] determined the roughness of the surface to be \( \sigma = 2.2 \pm 0.2 \) nm and the associated lateral correlation length to be \( \zeta = 10 \pm 2 \) µm. The plane can thus be regarded as a pattern that varies in height with 2 nm on a scale of 10 µm. Since the de Broglie wavelength of the neutrons is in the range of 40nm to 100nm, the neutrons do see a surface that is essentially flat. A neutron-absorber is placed above the first mirror. The absorber consists of a rough glass plate coated with an Gd-Ti-Zr alloy by means of magnetron evaporation. The absorbing layer is 200nm thick. The surface of the absorber
was parallel to the surface of the mirror. The absorber roughness and correlation length was measured with an atomic force microscope to be $\sigma = 0.75 \, \mu \text{m}$ and $\zeta = 5 \, \mu \text{m}$ respectively. Neutrons that hit the absorber surface are either absorbed in the alloy or scattered out of the experiment at large angles. The efficiency of removing these fast unwanted neutrons is 93%. The collimation system in front of the mirror absorber system is adjusted in that way, that classical trajectories of neutrons entering the experiment have to hit the mirror surface at least two times. After the second mirror we placed a $^3\text{He}$ counter for neutron detection. More information about the setup can be found in [17].

4 Gravity and quantum mechanics work together

4.1 Theoretical description

The neutrons fall under gravity onto the mirror. The calculation of the energy eigenvalues of the vertical motion of the neutrons in the mirror-absorber system is a nice example of quantum mechanics. In fact, we have two theoretical descriptions for the transmission of neutrons. The first one is the well known WKB
method. Usually, the accuracy of WKB quantization is 20% for the ground state, whereas the accuracy increases for higher levels. A similar calculation of energy levels for the gravitational field with the WKB method can also be found in [6]. We can compare the WKB result with an exact analytical solution using Airy-functions. Taking the neutron-absorber into account, the agreement of the two methods is significantly better than 10%. We start calculations from the one-dimensional stationary Schrödinger equation,

\[-\frac{\hbar^2}{2m} \Delta \Psi + V(z) = E\Psi\]  

(7)

with wave function \(\Psi\) for energy \(E\) and the potential

\[V(z) = mgz \text{ for } z \geq 0, \]

\[V(z) = \infty \text{ for } z < 0,\]  

(8)

ignoring the absorber for now. \(m\) is the mass of the neutron and \(g\) is the acceleration in the earth’s gravitational field. The quantum mechanical treatment of reflecting neutron mirror, made from glass, is simple. The glass potential is essential real because of the small absorption cross section of glass and with \(V = 100 \text{ neV}\) large compared with transversal energy \(E_\perp\). Therefore, the potential \(V\) is set to infinity at \(z = 0\). The quantum mechanical description follows in part [6]. It is convenient to use a scaling factor

\[\zeta = \frac{z}{z_0} \text{ with } z_0 = \left(\frac{\hbar^2}{2m^2g}\right)^{1/3}.\]  

(9)

Solutions of Equ. 1 for \(\Psi\) are obtained with an Airy function

\[\Psi_n(\zeta) = Ai(\zeta - \zeta_n)\]  

(10)

The displacement \(\zeta_n\) of the n-th eigenvector has to coincide with the n-th zero of the Airy function \((Ai(-\zeta_n)=0)\) to fulfill the boundary condition \(\Psi_n(0)=0\) at the mirror. Eigenfunctions \((n>0)\) are

\[Ai(\zeta - \zeta_n)\]  

(11)

|                  | Neutron | ^4He | ^85Rb | ^133Cs | Electron |
|------------------|---------|------|-------|--------|----------|
| \(E_1 [\text{peV}]\) | 1.4     | 38.4 | 35.7  | 154    | 0.11     |
| \(E_2 [\text{peV}]\) | 2.5     | 42.1 | 34.7  | 138    | 0.20     |
| \(z_1 [\mu]\)     | 13.72   | 5.5  | 0.7   | 0.5    | 2061     |
| \(z_2 [\mu]\)     | 23.99   | 9.5  | 1.2   | 0.9    | 3604     |

Table 2. Eigenenergies and classical turning points for neutrons, atoms and electrons, a comparison
with corresponding eigenenergies

\[ E_n = mgz_n \]  

(12)

and

\[ z_n = z_0\left(\frac{3\pi}{2}(n - \frac{1}{4})^2\right)^{1/3} \]  

(13)

\( z_n \) corresponds to the turning point of a classical neutron trajectory with energy \( E_n \). For example, Energies of the lowest levels \( (n = 1, 2, 3, 4) \) are 1.44 peV, 2.53 peV, 3.42 peV and 4.21 peV. The corresponding classical turning points \( z_n \) are 13.7 \( \mu \)m, 24.1 \( \mu \)m, 32.5 \( \mu \)m and 40.1 \( \mu \)m (see Tab. 2).

\[ \rho = C\Psi^*\Psi \]  

(14)

is the neutron density and can be interpreted as the probability to detect a neutron at height \( z \) above the mirror, see Fig. 4. C is a constant.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{neutron_density.png}
\caption{Neutron density above the mirror for states \#1 to \#4}
\end{figure}

In principle it is possible to visualize this neutron density distribution. The distribution is measurable with a nuclear track neutron detector having at the moment a spatial resolution of about 3 to 4 \( \mu \)m [18]. The nuclear track detectors is made out of CR39 plastic coated with 5mg/cm\(^2\) \( ^{235} \)UF\(_4\). Nuclear fission converts a neutron into a detectable track on CR39. The tracks can be visualized with a standard optical microscope after chemical treatment. The typical diameter of such a track is around 1.5 \( \mu \)m with a length of about 10 \( \mu \)m. Competing reactions from \( \gamma \) rays or alpha particles have a smaller track signature and as a consequence, background from these reactions is practically zero. The automatic readout of the CR39 detector was done in the CHORUS group at C.E.R.N. [18].
The microscope MICOS2 is normally used to scan radiated emulsion plates in a search for neutrino oscillation. The rectangular stage of the microscope can be moved by step motors with a reproducibility of 1 µm. The focal length of the microscope is adapted to a CCD camera. The resolution in terms of one pixel is approximately 0.34 µm. An image analysis program detects the tracks on CR39. Having followed the tracks in depth of CR39, the impact point of the fission product on CR39 was found and the spatial resolution of the detector was significantly improved.

The population of the ground state and lowest state follows the quantum mechanical prediction. Higher, unwanted states are removed by the rough neutron absorber made up of an alloy of Ti, Zn and Gd.

Fig. 5. Data vs. classical expectation

4.2 Observation of quantum states

Signatures of quantum states in the gravitational field of the earth are observed in the following way: A $^3$He counter measures the total neutron transmission $T$, when neutrons are traversing the mirror absorber-system as described in section
2. The transmission is measured as a function of the absorber height $h$ and thus as a function of neutron energy since the height acts as a selector for the vertical energy component $E_\perp$, see Fig. 3). The solid data points, plotted in Fig. 5, show the measured number of transmitted neutrons for an absorber height $h$ from zero up to 160 $\mu$m. From the classical point of view, the transmission $T$ of neutrons is proportional to the phase space volume allowed by the absorber. It is governed by a power law $T \sim h^n$ and $n = 1.5$ The solid line in Fig. 5 shows this classical expectation.

Above an absorber height of about 60$\mu$m, the measured transmission is in agreement with the classical expectation but below 50 $\mu$m, a deviation is clearly visible. From quantum mechanics, we easily understand this behavior: Ideally, we expect a stepwise dependence of $T$ as a function of $h$. If $h$ is smaller than the spatial width of the lowest quantum state, then $T$ will be zero. When $h$ is equal to the spatial width of the lowest quantum state then $T$ will increase sharply. A further increase in $h$ should not increase $T$ as long as $h$ is smaller than the spatial width of the second quantum state. Then again, $T$ should increase stepwise. At sufficiently high slit width one approaches the classical dependence.

![Graph showing data and quantum expectation](image)

Fig. 6. Data and quantum expectation

and the stepwise increase is washed out. Fig. 6 shows details of the quantum regime below an absorber height of $h = 50 \mu$m. The data follow this expectation.
as described: No neutrons reach the detector below an absorber height of 15\(\mu m\) as explained before. Then above an absorber height of 15\(\mu m\), we expect the transmission of ground-state neutrons resulting in an increase in count rate. The expectation in this case is shown in a solid line and agrees nicely with the data. The \(\chi^2\) is 56 for 35 degrees of freedom for one fit parameter, the neutron flux \([20]\). The expectation for neutrons behaving as classical particles is shown in a dotted line. The classical expectation for neutron transmission is in clear disagreement with the data (open circle). Especially, no neutrons are transmitted for an absorber height between zero and fifteen micrometer.

5 Summary

In this experiment, gravitational bound quantum states have been seen for the first time. The experiment shows that, under certain conditions, neutrons do not follow the classical Galileian expectation when reflected from neutron mirrors. The measurement does well agree with a simple quantum mechanical description of quantum states in the earth’s gravitational field together with a mirror-absorber system. We conclude that the measurement is in agreement with a population of quantum mechanical modes. Further, the spectrometer operates on an energy scale of pico-eVs and suitably prepared mirrors can usefully be employed in measurements of fundamental constants or in a search for non-Newtonian gravity. The present data constrain Yukawa-like effects in the range between 1 \(\mu m\) and 10 \(\mu m\). The limit for strength \(\alpha\) at 10 \(\mu m\) is \(10^{11}\) and at 1 \(\mu m\) the limit is \(10^{12}\). This work has been funded in part by the German Federal Ministry (BMBF) under contract number 06 HD 854 I and by INTAS under contract number 99-705.

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