Mathematical model specification of the automated control system subject based on the differential approach

A A Sidaras¹,² and S V Chentsov²

¹ Chair of Applied mathematics and computer safety, Siberian Federal University, 79 Svobodny pr., 660041 Krasnoyarsk, Russia
² Chair of Systems of automatic equipment, automated management and design, Siberian Federal University, 79 Svobodny pr., 660041 Krasnoyarsk, Russia

E-mail: asidaras@sfu-kras.ru

Abstract. This article is devoted to describing the process of mathematical model specification of an automated control system subject based on a differential approach with a help of statistical data of parameters that affect the reliability of the psychological testing results. The mathematical model of the subject is based on the fuzzy logic conclusion system, which with a greater adequacy allows to describe the subject in comparison with similar solutions, which are also based on the differential approach. There are description of the ideas and results of statistical analysis and research, the stages of the model specification process, explanations each of these stages, formulation the main provisions on the possibility of additional application of the developed mathematical model in the article.

1. The need for the mathematical model specification

In assessing the impact of the human factor on the reliability of automated control systems, as well as information systems, the such properties and characteristics of a person are taken into account, which, first of all, will affect the reliability of specific operations directly [1]. From the human factor point of view, an automated control system is reliable when the system’s operator fully meets the requirements set, that is, he is competent [2]. Otherwise, from the human factor point of view, the system is less reliable or completely unreliable. In any information system or automated control system, it is absolutely logical to designate competent staff member as a leader of specific process.

The person (operator) involved in the control processes of the automated system will be called the subject of the automated control system. Quite often, a simple set of classical requirements meets to the subject, such as: the presence of specialized education, the experience in professional activities, as well as its duration, the information of advanced training, etc. However, it also makes sense to consider the subject as a system that has features (unique personality traits) which are not taken into account in the set of classical requirements. The work of the subject in an automated control system is associated with the performing of a certain type of activity, such as monitoring system processes, monitoring process parameters, managing a group of operators, and operational decision-making [3]. Due to the unique personality traits of the subject, the performance of various types of activity will have different efficiencies. Therefore, when assessing the reliability of the automated control system taking into account the human factor, it is also necessary to take into account the unique properties of the of the subject personality. It is obvious that the highest efficiency of performing a certain type of activity is
observed for those subjects that meet the entire set of classical requirements for them, as well as for those subjects whose psychological predisposition match with the performed type of activity. The converse statement is also logical. The lowest efficiency of performing a certain type of activity is observed in those subjects that do not meet any of the classical requirements sets, as well as for those subjects whose psychological predisposition is opposite to the performed type of activity. The state of the automated control system, and, consequently, its reliability, depends on the efficiency of performing the activities by the subjects.

In previously published works aimed at improving the sustainability [4] and reliability [5] of organizational and technical systems, the subject \( s \in \{ s_1, s_2, \ldots, s_n \} \) was considered as a system of two characteristics: the psychological predisposition \( P = \{ p_1, p_2, p_3, p_4 \} \) to a certain type of activity \( D = \{ d_1, d_2, d_3, d_4 \} \) and the educational characteristic \( Q = \{ q_1, q_2, q_3, q_4 \} \). In these works, psychological predisposition was considered as a unique personality trait. This approach to the unique personality traits consideration has a number of advantages: Firstly, it uses a modified model of personality psychotype, based on CG Jung’s differential psychology [6], which allows to define a set of basic unique personality traits that determine a psychological predisposition to a certain type activity. Secondly, this approach is very simple for the formal description; indeed, the unique properties of the personality in the form of psychological predisposition are represented by just one set \( \{ P \} \).

Nevertheless, it is quite obvious that such approach to the consideration of unique personality traits has a significant drawback: it rather roughly describes the subject’s belonging to a certain psychotype, in other words, to a psychological predisposition \( P = \{ p_1, p_2, p_3, p_4 \} \). The subject may belong to only one of the psychological predispositions. This drawback is not a drawback of the mathematical model, because formally \( p_i \in [0; 1] \), where \( i \in \{1, 4\} \). The origins of this drawback relate to the drawbacks of psychological testing results processing, namely, that there is no complete check for psychotype plausibility. Partially, a plausibility checks represented as a lie scale [7], for example, in the charactererological questionnaire of Leonhard. But when checking the plausibility of the test results, such parameters as the test passing time, the plausibility of the dominant share value of the psychotype are not taken into account. These parameters must be taken into account, since they greatly influence the obtained result.

The specification the model of the automated control system subject based on the differential approach firstly begins with assessing the plausibility of psychological testing results, specification of the fuzzy logic conclusion system for combining various plausibility parameters into a single parameter of test results plausibility, as well as introducing additional parameters to clarify the subject’s unique personality characteristics.

2. Model of the automated system subject

The plausibility of the psychological test results shows the expression of a certain psychotype for a subject. Therefore, it is most convenient to express this value with a dimensionless quantity belonging to the section \([0; 1] \), where the smallest plausibility is zero, and the highest plausibility is equal to one. It is logical not to introduce new variables and not to modify existing ones, but to identify the value of the psychological testing results plausibility with the value of psychological predisposition, because each \( p_1 \in [0; 1] \), where \( p \in \{ p_1, p_2, p_3, p_4 \} \). At the same time, the following property is preserved for the elements: \( \sum_{i=1}^{4} p_i = 1 \). Indeed, if the plausibility value for a certain psychological test result exceeds the plausibility value for all other results, then it is logical to take the value of the psychological test result equal to the plausibility value obtained for it. Since each value refers to a separate psychotype (psychological predisposition), it eliminates confusion when interpreting the results.

The assessment of the psychological test results plausibility will consist of assessments of the individual parameters plausibility. As mentioned above, the time for passing psychological test and the value of the psychotype dominant share play an important role in assessing the plausibility. The time \( t \) of psychological testing can take on a wide range of values from seconds to many hours. In order to level this diversity for the subsequent convenience of analyzing the obtained data, we will not use the
number of subjects spending a certain amount of time to pass the test, but we will use the number of subjects that fell within certain time intervals $\Delta t$ during the test. For the analysis, a random sample was generated, in which random people aged from 16 to 49 years from various regions of Russia, such as the Krasnoyarsk Territory, Stavropol Territory, Tomsk Region, St. Petersburg and other regions, took part in the psychological testing. The sample size $N$ was 3367 people (subjects). The results of the time spend for psychological testing are presented in the histogram of frequencies shown in figure 1. The number of intervals in the histogram is chosen equal to $31 > [1.72 \sqrt{N}]$ [8] for easy dividing the intervals into equal 4-minute intervals.

From the analysis of the sample and figure 1, it can be seen that the distribution of psychological testing time is asymmetrically (shifted to the left), unimodal with a mode = 20 minutes and has almost no outliers. Outlier = 140 with a value of $t = 4$ minutes is explained by the fact that for not every test subject passed the test honestly, that is, he had no sincere intentions to determine his psychological predisposition to a certain type of activity. It was established that during the time $t \leq 4$ minutes, the subject was able to write down the answers to the proposed questions thoughtlessly so that the system records the fact of passing the test. Therefore, this outlier can be ignored. The remaining sample values and frequency histogram in figure 1 are consistent with the expected results.
Obviously, the largest number of subjects passed the test in the time interval from 16 to 20 minutes. It is also obvious that a large number of subjects spent a time in the range from 12 to 28 minutes to pass the test. Thus, we can conclude that the value of \( t \) is not clearly defined and can take quite wide values. To assess the plausibility of the time \( t \), it is convenient to present in the form of a linguistic variable with three terms: “low”, “medium” and “high”, where each term is described by the corresponding membership function. After mathematical transformations from the frequency histogram, we obtain the membership functions for the linguistic variable \( T \) of the following form (figure 2).

The value of the psychotype dominant share shows how the psychological predisposition is expressed is in relation to the other psychotypes (psychological predispositions). A random sample was generated for analysis, in which random people aged from 16 to 49 from various regions of Russia took part in psychological testing. The sample size \( N \) was 108 people (subjects). The results are shown in the histogram of frequencies, shown in figure 3. The number of intervals in the histogram is chosen equal to \( 13 > \lceil 1.72\sqrt{N} \rceil \).

It can be seen from the analysis of the sample and figure 3 that the distribution of the dominant share of psychological predisposition has a slight asymmetry, unimodal with a mode = 0.3 and has no outliers. Due to the specifics of psychological testing, the dominant share cannot have a value less than 0.2 or more than 0.5. Therefore, the variability of this characteristic has the corresponding boundaries observed in figure 3.

The largest number of subjects has a dominant share of the psychotype in the interval from 0.27 to 0.33 minutes. Also, the majority of subjects have a dominant share of the psychotype in the range from 0.27 to 0.39 minutes. Thus, we can conclude that the value of the psychotype dominant share is not
clearly defined and can take quite wide values. To assess the plausibility, the dominant share of the psychotype is conveniently to represent as a linguistic variable with three terms: “low”, “medium” and “high”, where each term is described by the corresponding membership function. After mathematical transformations from the frequency histogram, we obtain the membership functions for the linguistic variable $A$ of the following form (figure 4).

The description of test time variables and the dominant share was used in the form of linguistic variables. Thus, to obtain a final level of reliability of test results, it is naturally necessary to create a fuzzy logic conclusion system, where the input variables are $T$ and $A$ variables and the output defuzzificated variable is $R$. The membership functions for $R$ are shown in figure 5. The membership functions for the final plausibility level are formed on the basis of expert estimates.

![Figure 5. Membership functions of the linguistic variable R.](image)

To complete the description of a fuzzy logic conclusion system, it is necessary to compile a database of conclusion rules (table 1).

| Rule number | Rule |
|-------------|------|
| 1           | $\min(\mu_T^{\text{low}}, \mu_A^{\text{low}}) = \mu_R^{\text{very low}}$ |
| 2           | $\min(\mu_T^{\text{low}}, \mu_A^{\text{medium}}) = \mu_R^{\text{low}}$ |
| 3           | $\min(\mu_T^{\text{high}}, \mu_A^{\text{low}}) = \mu_R^{\text{low}}$ |
| 4           | $\min(\mu_T^{\text{low}}, \mu_A^{\text{high}}) = \mu_R^{\text{medium}}$ |
| 5           | $\min(\mu_T^{\text{high}}, \mu_A^{\text{medium}}) = \mu_R^{\text{medium}}$ |
| 6           | $\min(\mu_T^{\text{medium}}, \mu_A^{\text{low}}) = \mu_R^{\text{medium}}$ |
| 7           | $\min(\mu_T^{\text{medium}}, \mu_A^{\text{high}}) = \mu_R^{\text{high}}$ |
| 8           | $\min(\mu_T^{\text{high}}, \mu_A^{\text{high}}) = \mu_R^{\text{high}}$ |
| 9           | $\min(\mu_T^{\text{medium}}, \mu_A^{\text{high}}) = \mu_R^{\text{very high}}$ |

To expand the set of unique personality traits, we introduce additional variables that clarify the educational characteristics $Q = \{q_1, q_2, q_3, q_4\}$ of the subject. The educational characteristic $\{Q\}$ did not initially distinguish between subjects with different periods of experience, therefore, for subjects with different periods of experience, the educational characteristic is the same. It seems logical to introduce
a new variable that could reflect the period of experience, expressed, for example, in years. Let introduce such a variable. Let us establish that the period of experience is denoted by the variable $dur \in [0;45]$.

Now, taking into account the introduced changes in the set of the personality unique properties of the subject, when calculating the level of competence $K_m \{s, d\}$, it is also convenient to use the fuzzy inference system [9].

3. Conclusion
The main feature of the model of the automated system subject based on the differential approach is that now, when assessing the reliability of psychological testing results, statistically reasonable parameters are used, such as the test passing time and the value of the dominant share of psychotype. New additional variables were able to expand the set of unique properties of the subject's personality, and the fuzzy logic conclusion system allowed to work easily with the introduced heterogeneous parameters. This model allows to clarify the result of psychological testing, as well as to assess its plausibility. It is necessary to continue statistical studies of various test results in order to obtain more substantiated results and discover new patterns.

References
[1] Cummings M, Gao F and Thornburg K 2015 Boredom in the Workplace: A New Look at an Old Problem Human factors 58 279–300
[2] Krasnov I and Tsygankov N 2016 Algorithm for calculating the level of competence regarding the required human activity Scientific technical vestnik Povolzhia 2 127–32
[3] Kulik S 2015 Sequential analysis method for testing a human operator Journal of applied informatics 10 100–8
[4] Chentsov S, Krasnov I and Sidaras A 2017 Ensuring the sustainability of information systems with regard to the human factor Fundamental researches 11 140–144
[5] Sidaras A and Chentsov S 2018 IOP Conf. Ser.: Mater. Sci. Eng. 450 052010
[6] Sidaras A 2016 Human Resilience Information Management Model Proceedings of the scientific conference dedicated to the Year of Education in the Commonwealth of Independent States 1 pp 56–9
[7] Myagkov A 2006 Falsehood scales: The attempt of sociological reinterpretation Vestnik IGEU 1 1–8
[8] Bardasov S 2012 The optimal number of histogram intervals Physics and mathematics and information technology: problems and development trends: Proceedings of int. scientific-practical conf.– Novosibirsk: SibAK 1 1–8
[9] Sidaras A 2019 Model of a fuzzy inference system to calculate the level of competence of the subject Fundamental scientific research: theoretical and practical aspects Proceedings of the international scientific-practical conference 1 93–6