EVOLUTION OF SUPERMASSIVE BLACK HOLE BINARIES AND ACCELERATION OF JET PRECESSION IN GALACTIC NUCLEI

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Received 2007 January 7; accepted 2007 May 7

ABSTRACT

Supermassive black hole binaries (SMBHBs) are expected with the hierarchical galaxy formation model. Currently, physics processes dominating the evolution of a SMBHB are unclear. An interesting question is whether we could observationally determine the evolution of SMBHBs and give constraints on the physical processes. Jet precession has been observed in many active galactic nuclei (AGNs) and is generally attributed to disk precession. In this paper we calculate the time variation of jet precession and conclude that jet precession is accelerated in SMBHB systems but decelerated in others. The acceleration of jet precession, \( \frac{dP_{\text{pr}}}{dt} \), is related to the jet precession timescale, \( P_{\text{pr}} \), and the SMBHB evolution timescale, \( \tau_\nu \), as \( \frac{dP_{\text{pr}}}{dt} \simeq -\lambda(P_{\text{pr}}/\tau_\nu) \). Our calculations based on the models for jet precession and SMBHB evolution show that \( \frac{dP_{\text{pr}}}{dt} \) can be as high as about \(-1.0\), with a typical value of \(-0.2\), and can be easily detected. We discuss the differential jet precession for NGC 1275 that has been observed in the literature. If its observed rapid acceleration of jet precession is true, the jet precession is due to the orbital motion of an unbound SMBHB with a mass ratio of \( q \approx 0.76 \). When jet precesses from ancient bubbles to the currently active jets, the separation of the SMBHB decreases from about 1.46 kpc to 0.80 kpc, with an averaged decreasing velocity of \( \frac{da}{dt} \simeq 1.54 \times 10^6 \text{ cm s}^{-1} \) and an evolution timescale of \( \tau_\nu \approx 7.5 \times 10^7 \text{ yr} \). However, if we assume steady jet precession for many cycles, the observations imply a hard SMBHB with a mass ratio of \( q \approx 0.21 \) and a separation of \( a \approx 0.29 \text{ pc} \).

Subject headings: accretion, accretion disks — galaxies: formation — galaxies: individual (NGC 1275, 3C 84) — galaxies: interactions — galaxies: jets — gravitational waves

1. INTRODUCTION

In the hierarchical galaxy formation models of cold dark matter (CDM) cosmology, present-day galaxies are the products of successive mergers. Recent observations show that almost all galaxies harbor at their centers a supermassive black hole (SMBH) whose mass tightly correlates with both the mass and the velocity dispersion of the bulge (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Magorrian et al. 1998; Tremaine et al. 2002). During galaxy interactions and mergers, the gas at the galactic plane is driven toward the central SMBH, triggering the activity of active galaxies (Wilson & Colbert 1995) and accretion by black holes. SMBHs in galactic nuclei likely grow mainly through matter accretion. In this scenario, galaxy interactions and mergers are expected to trigger the formation of an unbound binary active galactic nucleus (AGN). In galaxy mergers, two galaxies and the SMBHs at their centers initially lose their orbital angular momentum due to galactic dynamic friction and form a bound supermassive black hole binary (SMBHB) at a separation of \( a_H \sim 10 \text{ pc} \), where the SMBHB binding energy becomes dominant. The dynamic friction is very efficient because of the trapping of stellar objects around each black hole, and the evolution timescale of SMBHBs is on the order of the local dynamic timescale, depending on the stellar velocity dispersions of the two galaxies. The evolution of a bound SMBHB is dominated by dynamic friction, but the evolution timescale depends on the inner surface brightness profiles of the galaxies. A SMBHB forms hard at a separation of \( a_H \sim 0.1 \sim 1 \text{ pc} \), when the loss of the orbital angular momentum is dominated by three-body interactions between the SMBHB and the stars passing by (Begelman et al. 1980; Quinlan 1996; Yu 2002). When the SMBHB becomes hard but orbital angular momentum loss because of gravitational wave radiation is unimportant, the SMBHB may stall at \( a \sim a_H \) on a timescale longer than the Hubble time. However, observations of nearby galaxies suggest that most SMBHBs should have passed through the hard phase and have coalesced quickly. This is the so-called final parsec problem (Merritt & Milosavljević 2005). To solve the problem, several processes with large uncertainties have been suggested in the literature (Merritt & Milosavljević 2005), and the hydrodynamic interaction with the gas disk may play a major role (Gould & Rix 2000; Liu et al. 2003; Liu 2004; Armitage & Natarajan 2005; Escala et al. 2004). An important question is whether we could detect a SMBHB at the center of a galaxy, determine its evolution, and give observational constraints on the formation and evolution of SMBHBs.

Although unbound AGN binary systems with separations of order kiloparsecs or larger have been imaged in interacting and merging galaxies (e.g., Komossa et al. 2003; Ballo et al. 2004; Rodriguez et al. 2006), no hard or bound SMBHBs have been directly detected. Close SMBHBs or binary coalescence have been introduced in explaining the observations of many AGNs; for example, periodic optical and radio outbursts (Sillanpää et al. 1988; Katz 1997; Liu et al. 1995, 1997, 2006; Liu & Wu 2002), periodic variation of the very long baseline interferometer (VLBI) jet position angle (Stirling et al. 2003; Sudou et al. 2003), the interception of jet formation in double-double radio galaxies (DDRGs; Liu et al. 2003), the X-shaped radio feature in winged radio sources (Liu 2004; Merritt & Ekers 2002), and the S- or Z-shaped morphological symmetry of radio jets (Begelman et al. 1980). A recent review of the observational evidence for SMBHBs was given by Komossa (2006). The periodic outbursts may be due to the periodic interaction of a SMBHB and a standard accretion disk or an advection-dominated accretion flow (ADAF), while the periodic variation of the VLBI jet position angle is due to binary orbital motion. However, when the disk mass inside the binary orbit is less than the mass of the secondary black hole, which is at a radius on the order of \( 10^3 \) times the Schwarzschild radius of the primary black hole, the interaction between the accretion disk and the
SMBHB will realign the inner accretion disk and the binary orbital plane (Ivanov et al. 1999; Liu 2004). When a SMBHB begins coalescing due to gravitational wave radiation, the interaction between the secondary black hole and the aligned accretion disk will remove the inner disk region and leave a truncated outer accretion disk, leading to the interruption of jet formation in DDRGs (Liu et al. 2003) and to the formation of delayed X-ray afterglow in a gravitational wave radiation burst (Milosavljević & Phinney 2005).

The S- or Z-shaped radio morphological symmetry has been observed for a high fraction of AGNs and was suggested as observational evidence of SMBHBs by Begelman et al. (1980). It is explained with jet precession of periods between approximately $10^3$ and $10^8$ yr (e.g., Gower et al. 1982; Hutchings et al. 1988; Dunn et al. 2006), due to geodetic precession of the spin axis of the primary rotating SMBH being misaligned with the binary total angular momentum (Begelman et al. 1980; Roos 1988), the orbital motion of the jet-ejecting black hole, the disk precession being tidally perturbed by the secondary black hole (Katz 1997; Dunn et al. 2006), or the precession of an inner warped disk due to the Bardeen-Petterson effect (e.g., Lu & Zhou 2005; Caproni et al. 2006). In all the models reviewed above, it is implicitly assumed that the precession of the jet orientation follows the precession of the spin axis of the emitting rotating black hole and the rotating axis of the inner region of the accretion disk. Depending on the driving energy resources. Because the small characteristic size of the jet production region (e.g., Meier et al. 2001) and the alignment of the rotating black hole and the inner region of the accretion disk due to the Bardeen-Petterson effect (Bardeen & Petterson 1975), it seems reasonable to assume that a jet would always orient along the rotating axis of both the black hole and the inner region of the accretion disk, irrespective of the driving mechanisms. Thus, in this paper we will make the same assumption that jets would, if present, precess with the rotating axis of the emitting central black hole and the accretion disk.

With this assumption, all the models in the literature can explain the observed jet precession, although the results depend on many parameters with very large uncertainties. One of the very important questions is how to tell which model is the right one and to determine its parameters. With the improvement of observational instruments, jet precession can be observed up to many cycles, which makes it possible to measure the precession timescale with very high accuracy. In this paper, we investigate the possibility of measuring the time derivative of the jet precession timescale. In the precession models, a rigid-body–like disk precession is assumed in the literature. Here we make the same assumption.

A circumbinary accretion disk could be warped by a massive SMBHB with a random orbital inclination angle relative to the accretion disk (Liu 2004; Ivanov et al. 1999). The interaction quickly realigns the inner warped disk region and finally also realigns the central rotating SMBH with the binary orbital plane, while the outer unperturbed disk region far from the binary orbit remains coplanar with the galactic gas plane. This scenario predicts the formation of X-shaped radio features in FR II radio galaxies and a random distribution of jet orientations with respect to the galactic plane (Liu 2004). A warped disk precesses, leading to jet precession. Therefore, before discussing the variations of the jet precession timescale, we calculate the precession of a warped circumbinary nonmassive disk in this paper. Although the detailed SMBHB models for jet precession are different, all of them predict a decrease of the precession timescale with binary evolution. As SMBHBs in galactic nuclei never get softer (Quinlan 1996), the secondary black hole always migrates toward the binary mass center, and jet precession is expected to be accelerated in the SMBHB models. After calculating the jet precession timescale and the SMBHB evolution timescale in different models, we show that the acceleration of jet precession could reach 20% or even higher, depending on the parameters of the SMBHB systems and the accretion disks. Differential measurement of jet precession can be used to distinguish the different precession models and determine the evolution timescale of SMBHBs in galactic nuclei. With the measurement of the acceleration of the precession, we could also determine the kinematic viscosity coefficient of the accretion disk and the binary parameters.

Following the different physical mechanisms driving the evolution of SMBHBs in galactic nuclei, in § 2 we start our calculations of the hardening rate of SMBHBs and the acceleration of jet precession by considering the regime in which stellar dynamic friction affects the merger. In § 3, we calculate the evolution of SMBHBs and the time variation of jet precession due to the interaction of a SMBHB with a massive circumbinary accretion disk, which is followed in § 4 by the calculations for the scenario in which the evolution of a SMBHB is dominated by the interaction between the SMBHB and a nonmassive circumbinary accretion disk. In § 5, we estimate the acceleration of jet precession due to the rapid evolution of SMBHBs dominated by gravitational wave radiation. As an example, in § 6 we discuss the differential observations of jet precession in a recently merged system, NGC 1275 (3C 84), and the implications for the SMBHB in the system. Our discussions and conclusions on the results are given in § 7.

2. HARDENING OF SMBHBs DUE TO GALACTIC DYNAMIC FRICTION

2.1. Unbound SMBHBs

Two SMBHS in merging galaxies are unbound until the gravitational force between the two SMBHs dominates the orbital motion, which occurs when the separation of the SMBHB, $a$, is (Yu 2002)

$$a > a_H = \frac{GM + m}{\sigma^2} \approx 1.12 \times 10^6 r_G (1 + q) \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^{-2},$$

(1)

where $\sigma$ is the one-dimensional velocity dispersion of the primary galaxy, $r_G = 2GM/c^2$ is the Schwarzschild radius of the primary SMBH of mass $M$, and $q = m/M$ is the mass ratio of the secondary SMBH (of mass $m$) to the primary SMBH. For $a > a_H$, the evolution of the SMBHB is dominated by galactic dynamic friction and the evolution timescale $\tau_a$ is approximately proportional to the separation $a$:

$$\tau_a = -\frac{a}{da/dt} \approx \tau_H \frac{a}{a_H},$$

(2)

where $\tau_H = -a_H/v_{dy}$ is the dynamic friction timescale at $a_H$. A minus sign is used in the definition of $\tau_a$ because a SMBHB at the center of a galaxy never gets softer (Quinlan 1996). The hardening rate of a SMBHB due to galactic dynamic friction $v_{dy}$ is approximately (e.g., Merritt 2000)

$$v_{dy} \approx -0.151 \left( \frac{\sigma_0^2}{\sigma^2} \right) \ln \Lambda.$$

(3)

Here $\ln \Lambda \approx 2$ is the Coulomb logarithm and $\sigma_0$ is the one-dimensional velocity dispersion of the smaller (secondary) galaxy.
Applying the empirical relation of the central black hole mass $M$ and the stellar velocity dispersion $\sigma$ of the host galaxy (Tremaine et al. 2002),

$$\log (M/M_\odot) = 8.13 + 4.02 \log (\sigma/200 \text{ km s}^{-1}),$$  (4)

to both the primary and secondary galaxies, we obtain

$$\tau_H \approx 8.71 \times 10^5 M_8^{1.02/0.02} q^{-3/4.02} (1 + q) \text{ yr},$$

and the SMBHB evolution timescale

$$\tau_a = - \frac{a}{v_{dy}} \approx 3.73 \times 10^7 M_8^{1.02/0.02} q^{-3/4.02} \frac{a}{10^3 r_G} \text{ yr},$$  (5)

where $q_{-1} = q/0.1$ and $M = M_8 \times 10^6 M_\odot$.

When $a > a_H$, any jets, if present, would precess because of the orbital motion of the emitting black hole with orbital period

$$P_{\text{orb}} = 2\pi \left[ \frac{a^3}{GM + m + M_e} \right]^{1/2}$$
$$\approx 3.93 \times 10^6 M_8 \left( \frac{a}{10^3 r_G} \right)^{3/2} \frac{5}{1 + q + M_e/M} \text{ yr},$$  (6)

where $M_e$ is the mass of the stellar objects inside the SMBHB orbit and $M_e > M + m$. Here a typical mass of $M_e \sim 5M$ is used, because we are interested in a SMBHB with $a \geq a_H$. This is the shortest precession period in a SMBHB system with a given binary separation $a$, and it has been introduced to explain the helical jet morphology on parsec scales and the periodic optical outbursts observed in some blazars (e.g., Villata & Raiteri 1999) and jet precession at kiloparsec scales or larger in some AGNs (e.g., Wirth et al. 1982). From equation (6), we can obtain the acceleration of jet precession in binary orbital motion due to the hardening of the SMBHB:

$$\frac{dP_{\text{orb}}}{dt} = - \frac{3}{2} \frac{P_{\text{orb}}}{\tau_a}. $$  (7)

From equations (6), (5), and (7), we get the acceleration of jet precession due to binary evolution:

$$\frac{dP_{\text{orb}}}{dt} \approx -0.16 \left( \frac{a}{10^3 r_G} \right)^{1/2} M_8^{1/4.02} q_{-1}^{-1/2} \left( \frac{5}{1 + q + M_e/M} \right)^{1/2}.$$

As the precession period is within the observable range of the jet precessions in the literature, equation (8) implies that one can measure the evolution of SMBHBs due to dynamic friction by detecting the acceleration of the jet precession.

### 2.2. Bound SMBHBs

When $a < a_H$, a SMBHB becomes bound, while a bound SMBHB becomes hard at a separation (Quinlan 1996)

$$a_b = \frac{GmM}{4\sigma^2(m + M)}$$
$$= 3.260 \times 10^4 r_G M_8^{-1/2.01} q_{-1}^{-1/4.02} \frac{1}{1 + q}.$$

The evolution of a SMBHB with separation $a_b < a \leq a_H$ is still dominated by galactic dynamic friction, but the hardening timescale is approximated with $\tau_{H} \propto a^{-5.5}$ (Yu 2002), where $\gamma$ is a fitting parameter in the Nuker law for the inner surface brightness profiles of galaxies,

$$I(r) = 2^\beta \gamma /r_{H} \left[ \frac{r}{r_{H}} \right]^{-\gamma} \left[ 1 + \left( \frac{r}{r_{H}} \right)^{\eta} \right]^{-\gamma},$$  (10)

and $\eta$, $\beta$, $I_H$, and $r_H$ are fitting parameters as well. The break radius $r_H$ is the point of maximum curvature in log-log coordinates, and $\gamma$ is the asymptotic logarithmic slope inside $r_H$. For core galaxies, $\gamma \leq 0.3$, while for power-law galaxies, $\gamma \geq 0.5$. Therefore, the evolution timescale of a bound SMBHB is approximately

$$\tau_a \approx \tau_H \left( \frac{a}{a_H} \right)^{-0.5}. $$  (11)

This equation gives

$$\tau_a \approx (1.75 \times 10^7) 13.03 - q_{-1}^{-3/4.02} (1 + q)^{1.02 - 0.03} \text{ yr},$$  (12)

for $a_b < a \leq a_H$. Equation (11) implies that the evolution timescale of SMBHBs decreases with binary separation for core galaxies but slowly increases for power-law galaxies.

If the orbital motion and the angular momentum are dominated by the total mass of the SMBHB, the jet may precess because of the geodetic precession of the spin axis of the primary black hole about the total angular momentum (Begelman et al. 1980), because of the orbital motion of the emitting black hole, and because of the accretion disk precession due to the tidal force of the inclined secondary SMBH outside the disk (Katz 1997). If the rotating primary SMBH is misaligned with the binary total angular momentum, the spin axis of the primary black hole undergoes geodetic precession about the total angular momentum with a period (Begelman et al. 1980; Roos 1988)

$$P_{\text{geo}} \approx 2.6 \times 10^7 M_8 q_{-1}^{-1} \left( \frac{a}{10^3 r_G} \right)^{5/2} \text{ yr.} $$  (13)

This equation gives an acceleration of jet precession due to the hardening of the SMBHB of

$$\frac{dP_{\text{geo}}}{dt} = - \frac{5}{2} \frac{P_{\text{geo}}}{\tau_a}. $$  (14)

Equations (13), (14), and (12) show that

$$\frac{dP_{\text{geo}}}{dt} \approx - (1.2 \times 10^3) 13.03 M_8^{1.02/2.01} q_{-1}^{-1.02/4.02} (1 + q)^{-1.5} \left( \frac{a}{10^3 r_G} \right)^{3-\gamma } $$

for $a_b < a \leq a_H$, implying that the acceleration of jet precession in the geodetic precession is too fast to be detected.

If the orbital plane of a SMBHB is inclined with respect to an accretion disk of radius $R_d$ inside the binary orbit, the disk
precesses like a rigid body due to the tidal force of the secondary with a precession period of (Katz 1997)

\[ P_{\text{td}} \simeq (5.1 \times 10^5 \text{ yr}) M_8^2 \left( \frac{a}{10^3 R_G} \right)^3 \left( \frac{R_d}{10^4 R_G} \right)^{-3/2} \left( 1 + q \right)^{1/2} \frac{1}{q - 1 \cos \theta}, \]

where \( \theta \) is the tilt angle between the disk plane and the binary orbital angular momentum. The disk size \( R_d \) could be the total radius extent of an accretion disk or the Bardeen-Petterson radius (Bardeen & Petterson 1975; Natarajan & Pringle 1998). When the SMBHB begins to harden, equation (16) suggests that the jet precession will be accelerated with

\[ \frac{dP_{\text{td}}}{dt} = -3 \frac{P_{\text{td}}}{\tau_{\text{td}}}. \]  

To obtain equation (17), we have assumed that the change of the disk radius \( R_d \) is insignificant, compared to the variation of the binary separation. From equations (16), (17), and (12), we find that the jet precession due to the binary-disk tidal interaction is accelerated with

\[ \frac{dP_{\text{td}}}{dt} = -(0.087)13/3 M_8^{4(01-\gamma)/2.01} \left( \frac{R_d}{10^4 R_G} \right)^{-3/2} (1 + q)^{-1} \frac{a}{10^5 R_G}, \]

where we have taken \( q \gg 1 \) and \( \gamma = 3 \) for power-law galaxies but moderate for core galaxies.

From equations (6), (7), and (12), we find that the jet precession period due to binary orbital motion will change with time as

\[ \frac{dP_{\text{orb}}}{dt} \simeq -7.5 \times 10^{-4} M_8^{2(3-\gamma)/2.01} q_{-1}^{3/4} \left( \frac{a}{10^5 R_G} \right)^{2-\gamma}, \]

where we have taken \( M_* \ll M + m \). Equation (19) shows that the change may be too small to be detectable.

3. EVOLUTION OF SMBHBs DUE TO INTERACTION WITH MASSIVE DISKS

When a SMBHB becomes hard at \( a \simeq a_b \), the evolution timescale \( \tau_a \) may be larger than the Hubble time, and the binary may stall if three-body interaction between the SMBHB and stellar objects dominates the binary evolution (Quinlan 1996; Yu 2002). For a SMBHB stalled at the hard radius \( a_b \), equations (13), (16), and (6), together with equation (9) give, respectively, the constant precession timescales

\[ P_{\text{geo}} \simeq 4.99 \times 10^4 M_8^{-0.89/4.02} q_{-1}^{1/2} (1 + q)^{-5/2} \text{ yr}, \]

\[ P_{\text{td}} \simeq 1.77 \times 10^4 M_8^{1.02/2.01} q_{-1}^{2} \left( 1 + q^{-5/2} \right) \left( \frac{R_d}{10^4 R_G} \right)^{-3/2} \frac{1}{q - 1 \cos \theta} \text{ yr}, \]

\[ P_{\text{orb}} \simeq 1.63 \times 10^4 M_8^{1.02/4.02} q_{-1}^{3/2} (1 + q) \text{ yr}. \]

However, gas disks exist in the central regions of AGNs, which should interact with the SMBHBs. In the AGN unification model, the size of an accretion disk around a central SMBH is on the order of \( 10^4 R_G \), whereas the broad emission-line region and the thick dust torus outside the accretion disk can be as large as \( \sim 10^5 R_G \sim 10 \) pc. Because the dust torus is geometrically thick and massive, the interaction between the secondary black hole and the dust torus is linear and the secondary cannot open a gap, probably leading to a rapid type I migration of the secondary toward the center of mass (e.g., Papaloizou & Terquem 2006). If the accretion disk is geometrically thin and is coplanar with the binary orbital plane, a secondary SMBH with mass ratio

\[ q > q_{\text{min}} = \frac{81 \pi}{8} \alpha \delta^2 \simeq 3.2 \times 10^{-4} \alpha_{-1} (\delta/0.01)^2 \]

will open a gap in the accretion disk and exchange angular momentum with disk gas via the nonlinear Lindblad resonant binary-disk interaction (Lin & Papaloizou 1986; Armitage & Natarajan 2002). Here the viscous parameter \( \alpha = 0.1 \alpha_{-1} \) is defined with the shear viscosity in the \( r-\phi \) plane (Shakura & Sunyaev 1973), \( \nu_1 = \alpha c H \), where \( H \) is the scale height of the unperturbed accretion disk and \( c_s \) is the sound speed, and \( \delta = H/r \) is the disk opening angle at radius \( r \). The migration of the secondary black hole is called type II migration. If the disk mass inside the binary orbit is larger than the mass of the secondary SMBH, the migration timescale of the secondary SMBH is the disk viscous timescale (Lin & Papaloizou 1986; Armitage & Natarajan 2002; Papaloizou & Terquem 2006). If the circumbinary disk is massive, the secondary SMBH also migrates on a disk viscous timescale even if the orbital plane and the accretion disk are misaligned (Ivanov et al. 1998). Because the mass ratio of a SMBHB formed in galaxy mergers within a Hubble time \( q > 10^{-3} > q_{\text{min}} \) (Yu 2002; Liu 2004), we consider only type II migration.

For type II migration, the secondary black hole migrates inward on a viscous timescale

\[ \tau_a \approx \tau_v \simeq -\frac{a}{v_{\nu}} \simeq \frac{2 a^2}{3 \nu_1}, \]

From equation (23) and \( \nu_1 = \alpha c H \), we have

\[ \tau_a \approx 1.66 \times 10^8 M_8^{1/2} \alpha_{{-1}}^{1/2} \delta_{-2}^{5/4} \left( \frac{a}{10^5 R_G} \right)^{5/4} \text{ yr}, \]

where \( \alpha_{-1} = \alpha/0.1 \) and \( \delta_{-2} = \delta/0.01 \). Here, for convenience, we have written

\[ \delta \equiv \delta_0 \left( \frac{r}{10^5 R_G} \right)^{1/4}, \]

where \( \delta_0 \) is the disk opening angle at \( r = 10^5 R_G \) and is weakly dependent on \( \alpha \), the central black hole mass \( M \), and the accretion rate \( \dot{m} \). For a standard \( \alpha \) disk, \( \delta \approx 1/8 \) if the disk is dominated by gas pressure and free-free absorption, while \( \lambda = 1/20 \) if the disk is dominated by gas pressure and electron scattering (Kato et al. 1998). To get equation (24), we have assumed that the disk is dominated by gas pressure and free-free absorption, with \( \lambda = 1/8 \), which would be valid for \( r \gtrsim 2.6 \times 10^4 R_G m^{2/3} (\text{Kato et al.} 1998) \), where the example is not specified. Eddington accretion rate \( \dot{M}_{\text{Edd}} = L_{\text{Edd}}/0.1 c^2 \) is related to the Eddington luminosity.

Because the disk is massive, the total angular momentum of the binary-disk system is dominated by the disk mass and there is no geodetic jet precession around the binary orbital angular momentum. However, an inner disk region that is misaligned with a rotating central SMBH may be warped and become aligned due
to the Bardeen-Petterson effect (Bardeen & Petterson 1975), and jets may precess because of the tidal interaction of a misaligned secondary black hole and a warped inner disk. From equations (16), (17), and (24), we have

\[ \frac{dP_{\text{td}}}{dt} \simeq -2.61 M_d \alpha_{-1} \delta_{-2}^2 \left( \frac{a}{10^4 r_G} \right)^{7/4} \times \left( \frac{r_{BP}}{50 r_G} \right)^{-3/2} (1 + q)^{1/2} q_{-1}^{-1} \cos^{-1} \theta, \]

where \( r_{BP} \), with a typical value of order \( \sim 50 r_G \), is the Bardeen-Petterson radius out to which the accretion disk flow is aligned with the black hole spin axis (Bardeen & Petterson 1975; Natarajan & Pringle 1998). Equation (26) implies that the acceleration of the jet precession is significant and can be detected very easily.

Jets will also precess because of the orbital motion of the emitting primary black hole in the case of a massive circumbinary accretion disk. However, the acceleration of jet precession due to binary orbital motion,

\[ \frac{dP_{\text{orb}}}{dt} = -\frac{3}{2} P_{\text{orb}} \tau_{G}, \]

\[ \simeq -2.5 \times 10^{-4} \alpha_{-1} \delta_{-2}^2 \left( \frac{a}{10^4 r_G} \right)^{1/4} (1 + q)^{-1/2}, \]

may be too small to be detectable.

When the inner disk region becomes aligned with the rotating black hole but is misaligned with the outer inclined accretion disk, the misaligned disk region and the spin axis of the central rotating black hole will precess with a precession timescale of (Natarajan & Pringle 1998)

\[ P_{\text{BP}} \simeq 1.51 \times 10^6 a_{8}^{11/16} \alpha_{-1}^{13/8} m_{-1}^{-7/8} M_8^{-1/16} \text{ yr}, \]

where \( a_c \) is the spin parameter of the primary SMBH. Equation (28) shows that the jet precession due to the Bardeen-Petterson effect is independent of the evolution of the SMBHB and does not change with time on the timescales in which we are interested.

4. EVOLUTION OF SMBHBs DUE TO INTERACTION WITH A NONMASSIVE DISK

When the secondary black hole migrates inward on a viscous timescale and reaches a critical radius \( r_m \), the disk mass inside the binary orbit will be equal to the mass of the secondary black hole. In a gas pressure– and electron-scattering–dominated \( \alpha \) disk, the unperturbed disk surface density is (Kato et al. 1998)

\[ \Sigma \simeq 2.4 \times 10^5 \alpha_{-1}^{-4/5} M_8^{1/5} m_{-1}^{-3/5} r_3^{-3/5} \text{ g cm}^{-2}, \]

where \( r_3 = r/10^3 r_G \). Note that we have used a different \( \alpha \) prescription. From equation (29), we can estimate the disk mass \( M_d \) inside a radius \( r \) to be

\[ M_d \simeq \frac{10}{r^3} \pi \Sigma r^2 \simeq 4.9 \times 10^5 \alpha_{-1}^{-4/5} M_8^{1/5} m_{-1}^{-3/5} r_3^{-3/5} \text{ M}_\odot, \]

When \( M_d = m \), from equation (30) we have

\[ r_m = 8.6 \times 10^3 q_{-1}^{4/7} m_{-1}^{-3/7} M_8^{-6/7} r_G. \]

When \( a < r_m \), the disk mass \( M_d \) inside the binary orbit is smaller than the mass of the secondary, and the migration of the secondary will be reduced (Syer & Clarke 1995; Ivanov et al. 1999). If the disk is gas pressure– and electron-scattering–dominated, the migration timescale is approximately (Ivanov et al. 1999)

\[ \tau_a \simeq 152 \left( \frac{15}{152} \right)^{519/16} \left( \frac{16}{11} \right)^{16/19} \left( \frac{M_d}{m} \right)^{-14/19} t_v, \]

where \( M_d = M t_v \) is the disk mass inside the binary orbit and \( t_v \) is the viscous timescale of the unperturbed accretion disk at \( r = a \). Taking \( t_v = (2/3)a^2/\nu \) and using equations (25) and (32), we have

\[ \tau_a \simeq 8.95 \times 10^6 m_{-1}^{-14/19} q_{-1}^{14/19} M_8^{5/19} \times \alpha_{-1}^{-5/19} \delta_{-2}^{-1/2} \left( \frac{a}{10^4 r_G} \right)^{19/19} \text{ yr}. \]

for a gas pressure– and electron-scattering–dominated disk with \( \lambda = 1/20 \).

If the rotating primary black hole is inclined to the binary total angular momentum, its spin axis will precess geodetically. From equations (13), (14), and (33), we obtain the acceleration of geodetic precession,

\[ \frac{dP_{\text{geo}}}{dt} \simeq -2.3 \times 10^{-2} m_{-1}^{-14/19} M_8^{14/19} \times q_{-1}^{-33/19} \alpha_{-1}^{5/19} \delta_{-2}^{-10/19} \left( \frac{a}{10^3 r_G} \right)^{81/38}. \]

When the disk mass inside the binary orbit is less than the mass of the secondary black hole, the secondary will warp, twist, and realign the inner accretion disk on a short timescale (Ivanov et al. 1999). However, the realignment will stop at the Bardeen-Petterson radius, \( r_{BP} \), and the disk region at \( r < r_{BP} \) will remain aligned with the rotating primary black hole and misaligned with the binary orbital plane (Liu 2004). Thus, the inner accretion disk has \( r < r_{BP} \), and so the jet orientation precesses due to the tidal interaction of the secondary black hole. Equations (16), (17), and (33) give the acceleration of the precession period due to binary-disk tidal interaction,

\[ \frac{dP_{\text{td}}}{dt} \simeq -4.8 \times 10^{-4} m_{-1}^{14/19} M_8^{33/19} q_{-1}^{-33/19} \alpha_{-1}^{5/19} \times \delta_{-2}^{-3/2} \left( \frac{a}{10^3 r_G} \right)^{50/19} \left( \frac{r_{BP}}{50 r_G} \right)^{32/3} \left( 1 + q \right)^{1/2} \cos \theta. \]

The time variation of the jet precession is very small.

When the disk mass \( M_d \) within the binary orbit is less than that of the secondary black hole, a SMBHB with an inclined orbital plane warps and realigns the inner disk region outside its orbit to a typical transitional radius \( r_{\text{td}} \) (Ivanov et al. 1999). The transitional radius \( r_{\text{td}} \) of the inner warped and the outer unperturbed disk regions depends on how warps dominate in the disk. Taking into account the internal hydrodynamics of the disk itself, Papaloizou & Pringle (1983) showed that for \( \alpha > \delta = H/r \), the warp transfers on a timescale of \( t_{\text{trans}} \approx 2\pi^2/3\nu^2 \), where \( \nu_2 \) is the vertical vorticity and relates to the shear viscosity \( \nu_1 \) in the \( r-\phi \) plane as \( \nu_2 \simeq \nu_1 f_0(2\alpha^{-2}/(1 + \alpha^2)) \), where \( f_0 = (1 + \alpha^{-2})/(1 + \alpha^2/4) \) (Kumar & Pringle 1985; Kumar 1990; Ogilvie 1999).
The quadrupole contribution of the secondary black hole to the gravitational potential causes the major axis of an elliptical orbit in the disk to precess with frequency (Ivanov et al. 1999)

$$\Omega_{ap} = \frac{3}{4} q \left( \frac{a}{r} \right)^2 \Omega_{K},$$

(36)

where $\Omega_K$ is the Keplerian angular velocity at $r$. The lines of nodes precess with frequency $\Omega_{ap} = -\Omega_{ap}$. Liu (2004) showed that for $f_0 = 1$, the transitional radius $r_{al}$ can be estimated by using $t_{ap} \approx \Omega_{ap}^{-1}$, which gives

$$r_{al} \approx \left( \frac{a}{f_0} \right)^{1/2} \delta^{-1} a,$$

(37)

where $r_{al} \leq r_m$. The precession period $P_{ap}$ of the aligned inner disk is determined by the precession of the lines of nodes at $r_{al}$:

$$P_{ap} \approx 2 \pi \Omega_{ap}^{-1} = \left( \frac{8\sqrt{2}}{3\pi} \right) 10^{21/2(1+i)} \frac{r G}{c} \frac{\alpha}{f_0 \delta_0} \left( \frac{a}{r G} \right)^{7/4(1+i)} \times \frac{q}{(3-4\lambda)/(4(1+i))} \left( \frac{a}{r G} \right)^{(3-4\lambda)/(2(1+i))}.$$  

(38)

For a gas pressure– and electron-scattering–dominated thin disk, $\lambda = 1/20$, and the precession period is

$$P_{ap} \approx \left( 2.52 \times 10^5 \right) M_S a^{2/3} \alpha^{-1/3} \delta^{-5/3} \delta^{-10/3} \left( \frac{a}{10^{3} r G} \right)^{4/3}.$$  

(39)

Because the warp transfer timescale $t_{wp} = (2\alpha/f_0)\tau_a$ is much smaller than the viscous timescale $t$, for a standard thin disk with $\alpha \ll 1$, an assumption of rigid body–like disk precession is reasonable. If the primary SMBH is rotating and is misaligned with the binary orbital plane, the warp transfers quickly inward and stalls at the Bardeen-Petterson radius $r_{BP}$ (Bardeen & Petterson 1975). The disk region within $r_{BP}$ and the spin axis of the rotating primary BH would also precess with a timescale $P_{BP}$.

From equation (38), the acceleration of the precession of a warped circumbinary disk is

$$\frac{d \ln P_{ap}}{dt} = \frac{d \ln M}{dt} - \frac{7}{2(1 + \lambda)} \frac{d \ln \delta_0}{dt} + \frac{3 - 4\lambda}{2(1 + \lambda)} \frac{d \ln a}{dt},$$

(40)

where we have assumed a constant mass ratio $q$ during the evolution of the SMBHB. In accretion disk theory, the opening angle $\delta_0$ depends on the accretion rate and the mass of the central black hole as

$$\delta_0 \propto m^\mu M^\zeta,$$

(41)

where $\mu > 0$ and $\zeta > 0$ (Kato et al. 1998). Substituting equation (41) into equation (40), we have

$$\frac{d P_{ap}}{d t} \approx \frac{7}{2(1 + \lambda)} \frac{M_S}{\tau_a} \frac{3 - 4\lambda}{2(1 + \lambda)} \frac{P_{ap}}{\tau_a},$$

(42)

where $\tau_a = -m/(dM/dt)$ is the variation timescale of the accretion rate, which is equivalent to the typical lifetime of the AGN and may be determined by the environment; for example, the supply of the gas from the galactic disk to the accretion disk and the interaction of the accretion disk and the stellar objects passing through the disk. To obtain equation (42), we have assumed that the mass growth of the primary SMBH is insignificant on the timescale in which we are interested, here, implying that the variation timescale of the black hole mass, $\tau_M = M/(dM/dt)$, is much longer than the binary hardening timescale $\tau_a$. Equation (42) suggests that the decrease of the accretion rate decelerates the jet precession, but the hardening of the SMBHB accelerates it. If a SMBHB is long-lived and it passes through the active phase of a galaxy, namely, $\tau_m \ll \tau_a$, equation (42) gives a deceleration rate of the jet precession of

$$\frac{d P_{ap}}{d t} \approx \frac{2}{3} \frac{P_{ap}}{\tau_m}$$

(43)

for a gas pressure– and electron-scattering–dominated standard thin disk with $\mu = 1/5$ and $\lambda = 1/20$. Equation (43) suggests that from the measurement of the jet precession and its deceleration rate, we can determine the disk evolution and the lifetime of an individual radio source. For a short-lived SMBHB with $\tau_a \ll \tau_m$, jet precession is accelerated as

$$\frac{d P_{ap}}{d t} \approx \frac{4}{3} \frac{P_{ap}}{\tau_a}$$

(44)

for a gas pressure– and electron-scattering–dominated thin disk.

From equations (44) and (33), we obtain the acceleration of the precession of a warped disk due to the reduced migration of the secondary black hole,

$$\frac{d P_{ap}}{d t} \approx -3.8 \times 10^{-2} \frac{6}{M_S} \frac{14}{q^{-1}} M_S \left( \frac{a}{10^{3} r G} \right)^{55/57} \times \frac{f_0}{(3-4\lambda)} \left( \frac{a}{10^{3} r G} \right)^{55/57}.$$  

(45)

At $a \sim 10^{3} r G$, the orbital period of a SMBHB is

$$P_{orb} \approx 8.8 \left( \frac{a}{10^{3} r G} \right)^{3/2} \frac{M_S}{(1 + q)^{1/2}} yr,$$

and the jet precession due to the binary orbital motion would be nearly constant at

$$\frac{d P_{orb}}{d t} \approx -3 \frac{P_{orb}}{2 \tau_a} \leq 10^{-5}.$$

5. RAPID EVOLUTION OF SMBHBs DUE TO GRAVITATIONAL WAVE RADIATION

When $a$ is on the order of $10^3 r_G$, the loss of the orbital angular momentum due to gravitational wave radiation becomes important (Armitage & Natarajan 2002), and the inspiraling velocity of the secondary black hole due to gravitational wave radiation is (Peters & Mathews 1963)

$$\dot{a}_{gw} = - \frac{64 G^3 M^3 q(1+q)}{5c^5 a^3} f = - \frac{8}{5} \left( \frac{r_G}{a} \right)^3 q(1+q)f c$$

$$\approx -4.8 \times 10^5 \left( \frac{a}{10^{3} r G} \right)^{-3} f q^{-1}(1+q) \text{ cm s}^{-1},$$

(46)

where $f$ is a function of the eccentricity $e$:

$$f = \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \left( 1 - e^2 \right)^{-7/2}.$$  

(47)
At large values of $a$, the evolutions of the SMBHB and the accretion disk are coupled, and the migration timescale of the secondary SMBH is given by equation (33). When $a$ is small, the loss of the binary orbital angular momentum is dominated by gravitational wave radiation, and the hardening timescale due to gravitational wave radiation is given by

$$\tau_a = \frac{a}{a_{gw}} \approx 1.95 \times 10^4 M_8 q_{-1}^{-2}(1+q)^{-1} f^{-1} \left( \frac{a}{10^2 r_G} \right)^4 \text{yr}.$$  

(48)

At a critical radius $a = a_{gw}$, the inspiraling timescale due to the gravitational wave radiation, $\tau_{gw}$, is approximately equal to the migration timescale of the secondary SMBH due to the interaction between the SMBHB and a nonmassive accretion disk, $\tau_a$. From equation (33) and the relation $\tau_{gw} = \tau_{gw}/a$, we obtain

$$a_{gw} \approx 4.27 \times 10^2 r_G m_{-1}^{14/69} M_8^{14/69} q_{-1}^{11/23} \times (1+q)^{10/3} \alpha_{-1}^{-5/6} a_{-2}^{-10/69} \frac{d}{10^2 r_G}.$$  

(49)

From equations (13), (14), and (48), we obtain the acceleration of the jet precession due to geodetic precession:

$$\frac{dP_{\text{geo}}}{dt} \approx 3.3 \times 10^{-2} (1+q) f \left( \frac{a}{10^2 r_G} \right)^{-3/2}.$$  

(50)

Equation (50) implies that the time variation of jet geodetic precession depends only on the binary separation and is independent of the parameters of the accretion disk and the SMBHB.

From equations (6), (7), (16), (17), and (48), the accelerations of the jet precession due to the tidal interaction between the binary and the disk and due to the orbital motion are, respectively,

$$\frac{dP_{\text{tid}}}{dt} \approx 2.2 \times 10^{-4} M_8 (1+q)^{3/2} f$$
$$\times \left( \frac{a}{10^2 r_G} \right)^{-1} \left( \frac{r}{50 r_G} \right)^{-3/2} \cos^{-1} \theta,$$  

(51)

$$\frac{dP_{\text{obb}}}{dt} \approx 2.1 \times 10^{-5} q_{-1}^{-1} (1+q)^{1/2} f \left( \frac{a}{10^2 r_G} \right)^{-5/2}.$$  

(52)

The accelerations are insignificant.

From equations (39), (44), and (48), we get the acceleration of the jet precession due to the precession of a warped circumbinary disk:

$$\frac{dP_{\text{wb}}}{dt} \approx -0.80 q_{-1}^{5/3} (1+q) f \alpha_{-1}^{5/3} a_{-2}^{-10/3} \left( \frac{a}{10^2 r_G} \right)^{-8/3}.$$  

(53)

6. DIFFERENTIAL JET PRECESSION AND SMBHBs IN NGC 1275

In previous sections, we have discussed the evolution of SMBHBs in different driving regimes and the corresponding acceleration of the jet precession. Equations (14), (17), (7), and (44) suggest that the acceleration of the jet precession due to SMBHB hardening can be written integrally as

$$\frac{dP_{\text{pr}}}{dt} \approx -\Lambda \frac{P_{\text{pr}}}{\tau_a},$$  

(54)

where $4/3 \leq \Lambda \leq 3$. Our calculations show that the acceleration of jet precession due to SMBHB evolution is significant and that it could be measured on the timescale of the jet precession period. If we measure the jet precession period $P_{\text{pr}}$ and compute the acceleration rate $\frac{dP_{\text{pr}}}{dt}$, we can determine the evolution timescale of a SMBHB in a galactic nucleus as

$$\tau_a \approx -\Lambda \frac{P_{\text{pr}}}{P_{\text{pr}}},$$  

(55)

Using these calculations, we discuss, as an example, the differential jet precession in the radio galaxy NGC 1275:

6.1. Jet Precession with Constant Timescale

The FR I radio galaxy NGC 1275 (3C 84) is a recently merged system at redshift $z = 0.01756$ (e.g., Holtzman et al. 1992), and the mass of its central SMBH is measured with the molecular gas hydrodynamic method to be $M = 3.4 \times 10^8 M_\odot$ (Wilman et al. 2005). The object has a bolometric luminosity of $L_{\text{bol}} \approx 1.07 \times 10^{44}$ ergs s$^{-1}$ (with $H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$ and $q_0 = 0.5$; Marchesini et al. 2004). With the measured black hole mass and the bolometric luminosity, we obtain the dimensionless accretion rate $\dot{m} \approx 2.4 \times 10^{-3} c_\odot^{-1}$. Dunn et al. (2006) imaged the S-shaped morphologies of jets and emission-line structure and differentially measured the jet precession timescale by identifying four components of different orientations in order of their formation: ancient bubbles, ghost bubbles, outer lobes, and inner jets. The observations of the precession angle $\Delta \phi$, the time difference $\Delta t$, the number of complete cycles $n$ between two components in succession, and the observed precession period $P_{\text{pr}}$ are taken from Dunn et al. (2006) and are summarized in Table 1. The number $n$ is the precession cycle number between two successive components and is estimated with $P_{\text{pr}} = 360^\circ \Delta t (\Delta \phi + n360^\circ)$. The observations suggest that the activity of NGC 1275 is intermittent and that the jet precession timescales are significantly different for different episodic activities. Intermittence of activity and significant differences of the jet precession periods for different activity episodes are also observed in the Seyfert 1.5 galaxy Mrk 6 (Kharb et al. 2006). For both NGC 1275 and Mrk 6, the observations show that the jet orientations at the beginning of each episodic activity are significantly different from those at the end of the last episodic activity, implying that the spin axis of the central black hole precesses even when the source is dormant or operating at a level of very weak activity.

Dunn et al. (2006) assumed that the jet precession in NGC 1275 remains steady for many cycles and that the observed differences in the precession timescale are due to the absence of different precession cycles when no bubble detaches. With this assumption, the significantly different precession timescales from the ancient bubbles through the ghost bubbles to the outer lobes are reconciled with one period, $P_{\text{pr}} \approx (3.31 \pm 0.46) \times 10^4$ yr, but the currently active jets still precess with a significantly shorter timescale, $P_{\text{pr}} \approx (2.72 \pm 0.54) \times 10^4$ yr (Dunn et al. 2006). Here the errors include the measurement error of the precession timescale, $\sigma_P/P_{\text{pr}} \approx 20\%$. The cycle number is given in Table 1. The jet precession from the ghost bubbles to the active jets is accelerated with $\langle dP_{\text{pr}}/dt \rangle \approx -0.19$, which implies that the jet precession is due to the SMBHB at the center of the galaxy. Equation (55) gives a model-independent evolution timescale of the SMBHB in NGC 1275 as $\tau_{\text{ob}} \approx (\Lambda/2) 3.62 \times 10^8$ yr. If we know the binary separation $a$, we can calculate a model-independent binary hardening rate or the migration velocity of the secondary,

$$v_{\text{ob}} \approx -54 \frac{a}{0.2 \text{ pc}} \left( \frac{\Lambda}{2} \right)^{-1} \text{cm s}^{-1},$$

where $4/3 \leq \Lambda \leq 3$. 

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Jets precessing with a constant timescale through several duty cycles of activity imply that the precession in NGC 1275 is independent of the accretion. All the models for jet precession that depend on the accretion disk are excluded, and the only reason-able scenarios for the jet precession from one bubble to another are geometric precession or binary orbital motion. If the precession is due to the orbital motion of a SMBHB, the observed period and equation (6) give

\[ a_{\text{orb}} \simeq 1.81 \times 10^7 r_G \left( \frac{1 + q + M_*/M}{5} \right)^{1/3} \simeq 5.9 \times 10^2 \text{ pc}, \]

which is much larger than the bound radius \( a_H \simeq 7.1 \times 10^5 r_H \) and implies an unbound SMBHB in NGC 1275. An unbound SMBHB is consistent with the observations that indicate a recent merger. However, equation (8) suggests a variation of the precession timescale from the ancient bubbles to the outer lobes for the south components of

\[ \frac{\Delta P_{\text{pr}}}{P_{\text{pr}}} \simeq \frac{dP_{\text{orb}}}{dt} \frac{\Delta t_{\text{ag}} + \Delta t_{\text{go}}}{2P_{\text{pr}}} \]

\[ \simeq -0.59 q_{-1}^{3/4} \left( \frac{5}{1 + q + M_*/M} \right)^{1/2}, \]

which, together with the assumption of steady precession, gives an upper limit of

\[ q \lesssim 2 \times 10^{-2} \left( \frac{1 + q + M_*/M}{5} \right)^{2/3}. \]

Equation (5) shows that to form such a binary in a minor merger, the dynamical friction timescale at \( a \sim 20 \text{ kpc} \) is

\[ \tau_d \gtrsim 1.4 \times 10^9 \left( \frac{1 + q + M_*/M}{10^3} \right)^{-1/2} \text{ yr}, \]

where \( M_0 \) is the stellar mass within the binary orbit with \( a \sim 20 \text{ kpc} \). A minor merger with \( q \lesssim 2 \times 10^{-2} \) is unlikely to be observable for such a long timescale. In this scenario, a steady precession from the ancient bubbles to the outer lobes is inconsistent with the acceleration of jet precession from the ghost bubbles to the present active jets.

The second possible precession scenario independent of the accretion rate is the geodetic precession of the primary SMBH. From equation (13), the constant precession timescale implies

\[ a_{\text{geo}} \simeq 6.7 \times 10^3 r_G q_{2/5}, \tag{57} \]

which is smaller than \( a_H \) for \( q \gtrsim 2.0 \times 10^{-2} \). When the source is operating at a level of very weak activity or is dormant, with an accretion rate much smaller than both its current accretion rate and the typical accretion rate for FR I radio galaxies, namely, \( \dot{m} \ll 10^{-3} \), the accretion disk cannot be a standard thin disk, but it can be a geometrically thick and optically thin advection-dominated accretion flow (ADAF). For such a binary-disk system, the total angular momentum is dominated by the orbital angular momentum and the jet precession is geodetic. For a ADAF-binary system, the jet precession other than the geodetic precession is the orbital motion with period \( P_{\text{orb}} \simeq 5 \times 10^7 \text{ yr} \). For an accretion disk with accretion rate \( \dot{m} = 2.4 \times 10^{-3} \), the inner region is an ADAF and the outer part of the disk is a standard thin disk. The transition radius between the two different accretion modes is (Meyer et al. 2000; Liu et al. 2002)

\[ r_{\text{tr}} \simeq 18.3 \dot{m}_{-3}^{-0.85} r_G \simeq 3.1 \times 10^3 r_G. \tag{58} \]

Equations (57) and (58) show that \( a_{\text{geo}} > a_H \) for \( q > 1.4 \times 10^{-2} \), but it is less than the transitional radius

\[ r_m \simeq 2.1 \times 10^4 r_G q_{-1}^{4/7} \alpha_{-1}^{4/7} \dot{m}_{-3}^{-3/7} \]

for

\[ q \gtrsim 2 \times 10^{-3} \alpha_{-1}^{20/11} \dot{m}_{-3}^{-15/11}, \]

where \( \dot{m}_{-3} = \dot{m}/10^{-3} \). The secondary SMBH migrates inward due to the interaction with the standard accretion disk on a timescale that is given by equation (33):

\[ \tau_d \simeq 3.88 \times 10^{8} q_{-1}^{84/95} \alpha_{-1}^{-5/19} \delta_{-2}^{-10/19}. \tag{59} \]
From equation (34), the time derivative of geodetic precession due to the migration of the secondary SMBH interacting with a circumbinary accretion disk is

$$\frac{dP_{\text{geo}}}{dt} \approx -0.21 q^{-8/9} \alpha_{-1}^{-5/19} \delta_{-2}^{10/19}. \quad (60)$$

The observed acceleration $\left\langle \frac{dP_{\text{pr}}}{dt} \right\rangle \approx -0.19$ and equation (60) suggest a binary mass ratio of

$$q \approx 0.11 \alpha_{-1}^{25/84} \delta_{-2}^{25/42},$$

and the secondary has a mass of

$$m \approx (3.8 \times 10^7 M_\odot) \alpha_{-1}^{25/84} \delta_{-2}^{25/42}.$$

The binary separation is approximately

$$a \approx 7.0 \times 10^3 r_G \alpha_{-1}^{5/24} \delta_{-2}^{21/24} \approx 0.23 \text{ pc},$$

and thus

$$a < a_h \approx 1.8 \times 10^4 r_G \alpha_{-1}^{25/84} \delta_{-2}^{25/42}.$$

The results are insensitive to the disk parameters.

However, the secondary black hole should warp the standard thin disk and the warped disk would precess, probably leading to the precession of the jet orientation on a timescale given by equations (39) and (57):

$$P_{\text{pr}} \approx 1.08 \times 10^7 q^{-6/5} \alpha_{-1}^{-5/3} \delta_{-2}^{-10/3} \text{ yr}. \quad (61)$$

If the precession timescale from the outer lobes to active jets is due to the precession of the warped disk, then equation (61) and the measured period $P_{\text{pr}} \approx 2.72 \times 10^7$ yr give

$$q = 0.21 \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}.\quad$$

The secondary has mass

$$m \approx (7.2 \times 10^7 M_\odot) \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9},$$

the binary separation is approximately

$$a \approx 9.1 \times 10^3 r_G \alpha_{-1}^{-5/9} f_0^{9/9} \delta_{-2}^{10/9} \approx 0.29 \text{ pc},$$

and thus

$$a < a_h \approx 3.1 \times 10^4 r_G \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}.$$

The secondary migrates inward because of the binary-disk interaction, leading to a time variation of the jet precession timescales of

$$\frac{dP_{\text{pr}}}{dt} \approx -1.4 \times 10^{-2} \alpha_{-1}^{-1.88} f_0^{-1.88} \delta_{-2}^{-3.75}, \quad (62)$$

$$\frac{dP_{\text{geo}}}{dt} \approx -0.11 \alpha_{-1}^{85/57} f_0^{-70/57} \delta_{-2}^{-110/57}. \quad (63)$$

Equations (62) and (63) imply that the time variations of jet precession cannot be detected because of the low observational accuracy. Therefore, the different precession timescales between the outer lobes and the active jets are most likely due to the different mechanisms for jet precession.

### 6.2. Rapid Acceleration of Jet Precession?

The argument for steady precession in NGC 1275 that is given by Dunn et al. (2006) is that if the precession were speeding up, the acceleration would be very rapid, and over the course of about 1.5 rotations, the precession timescale would change by around a factor of 10. However, our theoretical calculations suggest that rapid acceleration is possible and that there is no a priori requirement for a constant precession timescale. In this section, we discuss the implications of rapid acceleration of the jet precession.

We compute the precession timescale and the time derivatives for the case $n = 0$ in Table 1. From the averaged precession timescale from the ancient bubbles to the outer lobes, $\langle P_{\text{pr}} \rangle \approx 15.23 \times 10^7$ yr, and the averaged time derivative of the precession timescale from the ancient bubbles to the outer lobes, $\langle dP_{\text{pr}}/dt \rangle \approx -2.55$, we get a model-independent evolution timescale of the SMBHB of $\tau_{\text{pr}} \approx (\Lambda/1.5)^{9.86 \times 10^7}$ yr, which is about 3 times smaller than the timescale obtained under the assumption of a steady jet precession from the ancient bubbles through the ghost bubbles to the active jets. Meanwhile, from the averaged precession timescale from the ghost bubbles to the active jets, $\langle P_{\text{pr}} \rangle \approx 5.27 \times 10^7$ yr, and the averaged acceleration of the precession from the ghost bubbles to the active jets, $\langle dP_{\text{pr}}/dt \rangle \approx -1.30$, we compute a model-independent evolution timescale of the SMBHB of $\tau_{\text{pr}} \approx (\Lambda/1.5)^{6.08 \times 10^7}$ yr.

As the precession from the ancient bubbles to the active jets is continuous, without interruption when the activity of the object varies significantly, the possible mechanisms for jet precession are binary orbital motion and geodetic precession of the primary. If the precession is due to binary orbital motion, the averaged precession timescale from the ancient bubbles to the outer lobes, $\langle P_{\text{pr}} \rangle \approx 15.23 \times 10^7$ yr, and equation (6) give an averaged binary separation of

$$a_{\text{so}} \approx 5.01 \times 10^7 r_G \left(1 + q + M_*/M_\odot \right)^{1/3} \approx 1.46 \text{ kpc}.\quad$$

From the averaged time derivative of the precession timescale from the ancient bubbles through the ghost bubbles to the outer lobes, $\langle dP_{\text{pr}}/dt \rangle \approx -2.55$, and equation (8), we obtain a binary mass ratio of

$$q_{\text{so}} \approx 0.92 \left(1 + q + M_*/M_\odot \right)^{4.02/9}.$$

Meanwhile, the averaged precession timescale from the ghost bubbles to the active jets, $\langle P_{\text{pr}} \rangle \approx 5.27 \times 10^7$ yr, and equation (6) give an averaged binary separation of

$$a_{\text{sj}} \approx 2.47 \times 10^7 r_G \left(1 + q + M_*/M_\odot \right)^{1/3} \approx 0.80 \text{ kpc}.\quad$$

From the averaged time derivative of the precession timescale from the ghost bubbles through the outer lobes to the active jets, $\langle dP_{\text{pr}}/dt \rangle \approx -1.30$, and equation (8), we obtain a binary mass ratio of

$$q_{\text{sj}} \approx 0.60 \left(1 + q + M_*/M_\odot \right)^{4.02/9}.$$
The two averaged time derivatives give a consistent mass ratio and suggest a major merger with

$$\langle q \rangle \approx 0.76 \left( \frac{1 + q + M_*/M}{5} \right)^{4.02/9}.$$  

From galactic dynamics (Binney & Tremaine 1987), it is expected that the evolution timescale of a SMBHB due to dynamic friction is nearly proportional to the separation: $$\tau_a \propto a.$$ Our results give $$\tau_a/\tau_{ij} \approx 1.5$$ and $$d\tau_a/dq \approx 2.0,$$ which match the predictions very well and give an averaged dynamic friction velocity of

$$\left\langle \frac{da}{dt} \right\rangle \approx -1.54 \times 10^6 \left( \frac{1 + q + M_*/M}{5} \right)^{1/2} \text{ cm s}^{-1},$$  

where $$M_*$$ is the stellar mass inside the binary orbit at a separation of $$a \sim 1 \text{ kpc}.$$  

The alternative for jet precession independent of source activity and accretion is binary geodetic precession. From equation (13), to obtain the averaged precession timescale from the ancient bubbles to the outer lobes, we have a binary separation of $$a_{\text{geo}} \approx 1.24 \times 10^5 r_G q^{-1/5}\,$$ because

$$a_{\text{geo}} \approx a_h \approx 1.77 \times 10^4 r_G q^{-1}/(1 + q),$$  

the binary is hard and the secondary SMBHB may migrate inward when it interacts with a light standard disk, leading to the acceleration of jet precession as

$$\frac{dP_{\text{geo}}}{dt} \approx -0.410 m_{-3}^{4/19} q^{-1/2} \alpha_{-1}^{5/19} r_G^{-10/19}$$  

where we have used an accretion rate of $$\dot{m} \sim 10^{-3}$$ and equation (34). Equation (65) and the measured time derivative give

$$q \approx 1.3 \times 10^{-2} \dot{m}_{-3}^{5/6} q_{-1}^{25/84} r_G^{25/42}.$$  

Similarly, from observations of jet precession from the ghost bubbles to the active jets, we have $$a_{\text{geo}} \approx 8.13 \times 10^3 r_G q^{-1/2}$$ and

$$q \approx 0.97 \times 10^{-2} \dot{m}_{-3}^{5/6} q_{-1}^{25/84} r_G^{25/42}. $$  

Although the estimated mass ratios are consistent with each other within the uncertainties of the accretion rate, the hardening of a hard SMBHB depends on accretion, and the migration of the secondary should stop when the source becomes dormant and the accretion disk becomes an ADAF. Even if the migration could happen when the source is luminous and forms the bubbles, the timescale to form a hard binary with a mass ratio of $$q \sim 10^{-2}$$ is $$\tau \approx 2 \times 10^3 \text{ yr},$$ which is inconsistent with the scenario of a recent merger. Therefore, the scenario of geodetic precession for jet precession is less favorable.

### 7. DISCUSSION AND CONCLUSIONS

SMBHBs are expected by the hierarchical galaxy formation model and may have been observed in many AGNs. Jet precession, which has been observed in many AGNs, is one of the pieces of observational evidence. In this paper, we start our work with a discussion of different mechanisms for jet precession, including (1) geodetic precession of the spin axis of the central primary SMBH around the total angular momentum, (2) orbital motion of the SMBH that is ejecting plasma jets, (3) inner disk precession due to the tidal interaction of an inclined secondary black hole, (4) precession of a circumbinary disk warped by the SMBHB, and (5) disk precession due to the Bardeen-Petterson effect. The precession of a circumbinary disk warped by a SMBHB is discussed for the first time. We did not discuss the precession model due to disk instability (Pringle 1997), because it suggests a stochastic precession rather than a regular precession and is inconsistent with the observations of jet precession in most AGNs. Although the Bardeen-Petterson effect does not directly connect to the presence of a SMBHB, the origin of misalignment between the rotating central black hole and the accretion disk may be due to the interaction of an accretion disk and an inclined SMBHB. When the inner disk region becomes misaligned with the binary orbital plane due to the Bardeen-Petterson effect, the tidal interaction of the secondary SMBH to the warped inner disk also leads to jet precession.

In these scenarios for jet precession, the precession timescale ranges from being on the order of years to being much longer than 108 yr, depending on the parameters of the SMBHB and the accretion disk. However, the parameters are very difficult to determine, and the observations of the jet precession timescale cannot give restrictive constraints on the models and the parameters, as they are degenerate. Therefore, we proposed to observe one more quantity, the time variation of the jet precession timescale, in order to resolve the parameters. We calculated the time variation of the jet precession in different models and showed that jet precession is always accelerated in an evolving SMBHB system. The acceleration of the jet precession is related to the evolution timescale of the SMBHB with $$dP_{\text{pr}}/dt \approx -\Lambda P_{\text{pr}}/\tau_{\text{pr}},$$ which results from the fact that all SMBHB models for jet precession predict a relation of $$P_{\text{pr}} \propto a^\Lambda$$ with $$\Lambda > 0$$ and that a SMBHB in a galactic nucleus never gets softer. The parameter $$\Lambda$$ slightly depends on the model, with $$4/3 \leq \Lambda \leq 3.$$ Our investigations also show that jet precession due to the Bardeen-Petterson effect is decelerated with AGN evolution. Our results suggest that the sign of the time derivative of the precession timescale can be used to identify SMBHB models from others.

Our calculations show that the time variation of jet precession is proportional to the timescale ratio of jet precession and SMBHB evolution. We analytically estimated the evolution timescales of SMBHBs at different evolution stages and the time variation for jet precession in different models, based on our current knowledge of SMBHBs. Our calculations show that for an unbound SMBHB, the mechanism for jet precession is orbital motion, and the quick binary evolution due to galactic dynamic friction leads to an acceleration rate of around 20% or higher for the jet precession timescale. For a bound SMBHB system, jet precession could be due to geodetic precession of the rotating primary black hole, disk precession due to tidal interaction between a standard accretion disk and the secondary SMBH, or binary orbital motion. At this stage, the evolution timescale of the SMBHB depends on the inner surface brightness profile of the galaxy and is estimated with an asymptotic analytic relation of the binary hardening timescale and the separation given by Yu (2002). Although the estimate is very rough, our results suggest that the evolution timescale of SMBHBs is several orders of magnitude shorter than the geodetic precession timescale and longer than the binary orbital period. Thus, geodetic precession is not significant and the time variation of orbital motion is difficult to measure. However, if the jet precession is due to the tidal interaction of the secondary black hole and an inner misaligned accretion disk, the acceleration rate of the jet precession could be on the order of 10% or higher.

When a SMBHB becomes hard and stalls, jet precession may be steady for a timescale longer than the Hubble time. Because the migration of the secondary SMBH due to the interaction with
an ADAF is negligible (Narayan 2000; Liu 2004), a nearly steady jet precession may be possible if a SMBHB interacts with an ADAF or if the precession is due to the Bardeen-Petterson effect. The fundamental difference between the two scenarios is that the accretion disk is a geometrically thin standard disk in the latter but a geometrically thick ADAF in the former. The accretion mode depends on the relative accretion rate \( \dot{m} \), which one could infer by estimating the bolometric luminosity and the central black hole mass. However, it is most probable that a SMBHB interacts with a standard disk, either a massive one or a light one. We compute the time variation of jet precession due to the SMBHB–accretion disk interaction and show that in both cases the binary-disk interaction would lead to a significant acceleration of jet precession: the acceleration is significant for jet precession not only due to tidal interaction of the secondary SMBH and a massive disk, but also due to both geodetic precession and warped circumbinary disk precession in the case of a nonmassive disk, depending on the parameters of the binary system and the accretion disk.

When the evolution of a SMBHB is dominated by gravitational wave radiation, the binary separation is approximately hundreds of Schwarzschild radii or less. If a jet is ejected from the central black hole, it precesses because of the black hole geodetic precession, the tidal interaction of the binary and an inner misaligned disk, binary orbital motion, and the precession of a warped circumbinary disk. Our calculations show that the precession of a warped circumbinary disk is strongly accelerated due to the migration of the secondary black hole because of gravitational wave radiation. The acceleration of jet precession due to geodetic precession is also very significant for a SMBHB with a nonzero eccentricity.

When we calculate the jet precession timescale and its time variation, we have assumed that disk precession is rigid body–like and that the jet precession is directly related to it. Although almost all the disk precession models for jet precession in the literature have adopted the same assumption and have successfully explained the jet precession in some AGNs and microquasars (e.g., SS 433), this assumption needs more discussion. Whether the assumption of a rigid body–like precession is valid or not depends on the warp transfer in the disk. As we discussed in §4, the transportation of warps in a disk depends on the vertical shear viscosity, and the transfer timescale at the transition radius between the warped and unperturbed disk regions is on the same order of the precession timescale for both the Bardeen-Petterson effect and the warped circumbinary light disk (e.g., Natarajan & Pringle 1998; Liu 2004). Therefore, the rigid body–like approximation for disk precession is correct on the zeroth order of magnitude. As the jet precession and its time derivative depend on disk characteristics in a similar way, the relationship of the ratio of the precession timescale and its variation rate,

\[
\frac{P_{pr}}{P_{pr}} = \frac{\tau_p}{\tau_p} \left( \frac{\partial \tau_p}{\partial a} \right) a,
\]

and the SMBHB evolution timescale \( \tau_p \) would be expected to be insensitive to how warps transfer in the disk.

Following our theoretical investigation of the acceleration of jet precession, we discussed the implications of the differential observations of jet precession in NGC 1275 (3C 84), a recently merged radio galaxy. Its differential jet precession has been measured between four different components in order of their formation: ancient bubbles, ghost bubbles, outer lobes, and active jets. Between the formation of different components, the activity of the object becomes very weak or the source is dormant. The precession timescales are significantly decreased with time among the different components. Dunn et al. (2006) assumed a steady jet precession and the acceleration of jet precession just because of the absence of several cycles between adjacent components. However, even under this assumption, the acceleration of jet precession from the ghost bubbles to the active jets is still significant. Because the precession is steady when the source activity changes dramatically, the mechanism for the precession is independent of the accretion and thus most likely is either geodetic precession or orbital motion of the SMBHB. Under the assumption of steady jet precession, we discussed the two possible mechanisms. Our discussions suggest that if the precession is due to binary orbital motion, the SMBHB should have a mass ratio that is too small (\( q \approx 2 \times 10^{-2} \)), and the acceleration of jet precession from the ghost bubbles through the outer lobes to the active jets cannot be explained reasonably. Our results show that the steady precession from the ancient bubbles to the outer lobes is probably due to geodetic precession and that the jet precession from the outer lobes to the active jets may be due to the precession of a warped circumbinary light standard thin disk. In this scenario, the SMBHB formed in a major merger with a mass ratio of

\[
q = 0.21 \alpha^{-25/18} f_{25/18}^{25/0} \delta_{25}^{25/0},
\]

and the binary has a separation of

\[
a \approx 9.1 \times 10^3 f_G \alpha^{-5/9} f_5^{5/9} \delta_{10}^{10/9} \approx 0.29 \text{ pc}.
\]

The predicted acceleration of jet precession is about a few percent to 10% and may have not yet been observed because of low observational accuracy.

Since our theoretical investigations show that there is no a priori requirement for steady jet precession, we discussed the implications for the possibility that a continuous rapid acceleration of precession from the ancient bubbles to the active jets has indeed been observed. Our results show that in this case the mechanism for the jet precession is orbital motion and that the rapid acceleration of the jet precession is due to the rapid evolution of the SMBHB because of galactic dynamic friction. The calculations give a galactic dynamic friction evolution timescale of \( \tau_f \approx (6-9) \times 10^7 \text{ yr} \) and an averaged dynamic friction velocity of \( da/dt \approx -1.54 \times 10^6 \text{ cm s}^{-1} \). The SMBHB forms in a major galaxy merger with an averaged black hole mass ratio of \( q \approx 0.76 \) and has a separation of \( a \approx 0.8-1.46 \text{ kpc} \).

As our conclusions, we discussed the scenarios for jet precession in AGNs and calculated the time derivatives of the precession timescale. Our calculations show that jet precession is accelerated in SMBHB models but nearly steady in the Bardeen-Petterson effect scenario. We analytically computed the predicted acceleration of jet precession in the evolution of a SMBHB from unbound to gravitational wave–dominated stages and showed that the time variation is significant and can be detected easily. One can estimate the evolution timescale and mass ratio of a SMBHB and the parameters of an accretion disk in AGNs by measuring the central black hole mass, the accretion rate, the jet precession timescale, and its time derivative. If we have observations of jet precession acceleration of a sample of radio sources, we can test the hierarchical galaxy formation model and the galactic dynamics. We can also estimate the fraction of SMBHBs that can coalesce quickly and give rise to gravitational wave radiation bursts.
We are grateful to D. N. C. Lin, J. Magorrian, J. F. Lu, and X.-B. Wu for helpful discussions and comments. Many thanks are due to the referee for constructive comments, which helped us to improve the presentation of the paper significantly. This work is supported by the National Natural Science Foundation of China (grant 10573001).

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