Analysis of the $Y(4140)$ with QCD sum rules

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Abstract

In this article, we assume that there exists a scalar $D_s^*\bar{D}_s^*$ molecular state in the $J/\psi\phi$ invariant mass distribution, and study its mass using the QCD sum rules. The predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. The value of the mass is about $M_{D_s^*\bar{D}_s^*} = (4.43 \pm 0.16)$ GeV, which is inconsistent with the experimental data. The $D_s^*\bar{D}_s^*$ is probably a virtual state and not related to the meson $Y(4140)$. Other possibility, such as a hybrid charmonium is not excluded.

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1 Introduction

Recently the CDF Collaboration observed a narrow structure ($Y(4140)$) near the $J/\psi\phi$ threshold with statistical significance in excess of 3.8 standard deviations in exclusive $B^+ \rightarrow J/\psi\phi K^+$ decays produced in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV [1]. The mass and width of the structure are measured to be $4143.0 \pm 1.2$ MeV and $11.7^{+8.3}_{-5.0} \pm 3.7$ MeV, respectively. The meson $Y(4140)$ is very similar to the charmonium-like state $Y(3930)$ near the $J/\psi\omega$ threshold [2, 3]. The mass and width of the $Y(3930)$ are $3914.6^{+3.8}_{-3.4} \pm 2.0$ MeV and $34^{+12}_{-8} \pm 5$ MeV, respectively [3].

In Ref.[4], Liu et al study the narrow structure $Y(4140)$ with the meson-exchange model, and draw the conclusion that the $Y(4140)$ is probably a $D_s^*\bar{D}_s^*$ molecular state with $J^{PC} = 0^{++}$ or $2^{++}$ while the $Y(3930)$ is its $D^*\bar{D}^*$ molecular partner. In Ref.[5], Mahajan argues that it is likely to be a $D_s^*\bar{D}_s^*$ molecular state or an exotic ($J^{PC} = 1^{-+}$) hybrid charmonium.

The mass is a fundamental parameter in describing a hadron, in order to identify the $Y(4140)$ as a scalar molecular state, we must prove that its mass lies in the region $(4.1 - 4.2)$ GeV. In this article, we assume that there exists a scalar $D_s^*\bar{D}_s^*$ molecular state in the $J/\psi\phi$ invariant mass distribution, and study its mass with the QCD sum rules [6, 7].

In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [6, 7].

The article is arranged as follows: we derive the QCD sum rules for the mass of the $Y(4140)$ in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

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2 QCD sum rules for the molecular state $Y(4140)$

In the following, we write down the two-point correlation function $\Pi(p)$ in the QCD sum rules,

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | T \left\{ J(x) J^\dagger(0) \right\} | 0 \rangle,$$  

where the scalar current $J(x)$ is defined by

$$J(x) = \bar{c}(x) \gamma_\mu s(x) \bar{s}(x) \gamma^\mu c(x),$$  

we choose the scalar current $J(x)$ to interpolate the molecular state $Y(4140)$.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J(x)$ into the correlation function $\Pi(p)$ to obtain the hadronic representation [6, 7]. After isolating the ground state contribution from the pole term of the $Y(4140)$, we get the following result,

$$\Pi(p) = \frac{\lambda_Y^2}{M_Y^2 - p^2} + \cdots,$$  

where the pole residue (or coupling) $\lambda_Y$ is defined by

$$\lambda_Y = \langle 0 | J(0) | Y(p) \rangle.$$

In the following, we briefly outline the operator product expansion for the correlation function $\Pi(p)$ in perturbative QCD. The calculations are performed at the large space-like momentum region $p^2 \ll 0$. We write down the “full” propagators $S_{ij}(x)$ and $C_{ij}(x)$ of a massive quark in the presence of the vacuum condensates firstly [7],

$$S_{ij}(x) = \frac{i \delta_{ij} \not{x}}{2\pi^2 x^4} \left( \frac{\delta_{ij} m_s}{4\pi^2 x^2} + \frac{\delta_{ij}}{12} \langle \bar{s}s \rangle + \frac{i \delta_{ij}}{48} m_s \langle \bar{s}s \rangle \not{x} - \frac{\delta_{ij} x^2}{192} (\bar{s}g_s \sigma G s) \right) + \frac{i \delta_{ij} x^2}{1152} m_s (\bar{s}g_s \sigma G s) \not{x} - \frac{i}{32\pi^2 x^2} G_{\mu\nu}^{ij}(x) \sigma^{\mu\nu} \not{x} + \cdots,$$

$$C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \frac{\delta_{ij}}{k - m_c} - \frac{g_s G_{\alpha\beta}^\dagger}{4} \sigma_{\alpha\beta} (k + m_c) + (\bar{k} + m_c) \sigma_{\alpha\beta} \right\} \left( \frac{\alpha_s G}{\pi} \right) \delta_{ij} m_c (k^2 - m_c^2) \not{x} + \cdots \right\),$$

where $\langle \bar{s}g_s \sigma G s \rangle = (\bar{s}g_s \sigma_{\alpha\beta} G^{\alpha\beta} s)$ and $\langle \alpha_s G \rangle = (\alpha_s G)\frac{\alpha_s G}{\pi}$, then contract the quark fields in the correlation function $\Pi(p)$ with Wick theorem, and obtain the result:

$$\Pi(p) = \int d^4x e^{ipx} Tr \left[ \gamma_\mu S_{ij}(x) \gamma_\alpha C_{ji}(-x) \right] Tr \left[ \gamma^\mu C_{mn}(x) \gamma^\alpha S_{nm}(-x) \right],$$

where the $i, j, m$ and $n$ are color indexes.

Substitute the full $s$ and $c$ quark propagators into the correlation function $\Pi(p)$ and complete the integral in the coordinate space, then integrate over the variables in the momentum space, we can obtain the correlation function $\Pi(p)$ at the level of the quark-gluon degrees of freedom.
We carry out the operator product expansion to the vacuum condensates adding up to dimension-10 and take the assumption of vacuum saturation for the high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, and factorization works well in large $N_c$ limit. In calculation, we observe that the contributions from the gluon condensate are suppressed by large denominators and would not play any significant roles [8, 9, 10, 11].

Once analytical results are obtained, then we can take the quark-hadron duality and perform Borel transform with respect to the variable $P^2 = -q^2$, finally we obtain the following sum rule:

$$\lambda_Y^2 e^{-\frac{M_Y^2}{M^2}} = \int_{4(m_c + m_s)^2}^{s_0} ds \rho(s) e^{-\frac{M_Y^2}{M^2}}, \tag{8}$$

where

$$\rho(s) = \rho_0(s) + \rho_{(\bar{s}s)}(s) + \left[ \rho^A_{(GG)}(s) + \rho^B_{(GG)}(s) \right] \left( \frac{\alpha_s GG}{\pi} \right) + \rho_{(\bar{s}s)^2}(s), \tag{9}$$

the lengthy expressions of the spectral densities $\rho_0(s)$, $\rho_{(\bar{s}s)}(s)$, $\rho^A_{(GG)}(s)$, $\rho^B_{(GG)}(s)$ and $\rho_{(\bar{s}s)^2}(s)$ are presented in the appendix.

Differentiating the Eq.(8) with respect to $\frac{1}{M^2}$, then eliminate the pole residue $\lambda_Y$, we can obtain a sum rule for the mass of the $Y(4140)$,

$$M_Y^2 = \frac{\int_{4(m_c + m_s)^2}^{s_0} ds \frac{d}{d(1/M^2)} \rho(s) e^{-\frac{M_Y^2}{M^2}}}{\int_{4(m_c + m_s)^2}^{s_0} ds \rho(s) e^{-\frac{M_Y^2}{M^2}}} \tag{10}.$$

### 3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle$, $\langle \bar{s}g_s Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.2) \text{GeV}^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{GeV})^4$, $m_s = (0.14 \pm 0.01) \text{GeV}$ and $m_c = (1.35 \pm 0.10) \text{GeV}$ at the energy scale $\mu = 1 \text{GeV}$ [6, 7, 12].

In the conventional QCD sum rules [6, 7], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $M^2$ and threshold parameter $s_0$.

In Fig.1, we plot the contribution from the pole term with variation of the threshold parameter $s_0$. From the figure, we can see that the value $s_0 \leq 20 \text{GeV}^2$ is too small to satisfy the pole dominance condition.

In Fig.2, we plot the contributions from different terms in the operator product expansion. The contribution from the term $\langle \frac{\alpha_s GG}{\pi} \rangle$ is very small, the contributions from the terms involving the gluon condensates are less than (or equal) 10% at the values $M^2 \geq 2.3 \text{GeV}^2$ and $s_0 \geq 22 \text{GeV}^2$, the gluon condensate plays a minor important role. The vacuum condensates of the highest dimension $\langle \bar{s}s \rangle \langle \bar{s}g_s Gs \rangle + \langle \bar{s}g_s Gs \rangle^2$ serve as a criterion for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. At the values $M^2_{\text{min}} \geq 2.6 \text{GeV}^2$ and $s_0 \geq 23 \text{GeV}^2$, their contributions are less than 15%. The contribution from the vacuum condensate of high dimension $\langle \bar{s}s \rangle^2$ varies with the Borel
parameter $M^2$ remarkably and serves as another criterion for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. At the value $s_0 \geq 23 \text{GeV}^2$ and $M^2 \geq 2.4 \text{GeV}^2$, its contribution is less than (or equal) 20%. In the region $M^2 \geq 2.6 \text{GeV}^2$, the leading contribution comes from the perturbative term, while the next-to-leading contributions come from the terms $\langle \bar{s}s \rangle + \langle \bar{s}g_\sigma G_s \rangle$. The operator product expansion is convergent at the values $M^2_{\text{min}} \geq 2.6 \text{GeV}^2$ and $s_0 \geq 23 \text{GeV}^2$. For the central values of the input parameters, the contribution from the pole term is larger than 49% at the values $M^2_{\text{max}} \leq 3.0 \text{GeV}^2$ and $s_0 \geq 23 \text{GeV}^2$.

In this article, the threshold parameter and the Borel parameter are taken as $s_0 = (24 \pm 1) \text{GeV}^2$ and $M^2 = (2.6 - 3.0) \text{GeV}^2$ respectively, the contribution from the pole term is about $(49 - 72)$% for the central values of the other input parameters, the two criteria of the QCD sum rules are full filled \cite{6,7}. One may expect to take smaller Borel parameter and threshold parameter to satisfy the two criteria of the QCD sum rules marginally, however, it is not feasible. The contributions from the different terms in the operator product expansion change quickly with variation of the Borel parameter $M^2$ at the value $M^2 \leq 2.6 \text{GeV}^2$ (see Fig.2) and will not result in a stable sum rule for the mass (see Fig.3).

Taking into account all uncertainties of the input parameters, finally we obtain the values of the mass and pole residue of the $Y$, which are shown in Figs.4-5,

\begin{align}
M_Y & = (4.43 \pm 0.16) \text{ GeV}, \\
\lambda_Y & = (5.46 \pm 1.21) \times 10^{-2} \text{GeV}^5. \quad (11)
\end{align}

The central value $M_Y = 4.43 \text{ GeV}$ is about 200 MeV above the $D^*_s \bar{D}^*_s$ threshold, the $D^*_s \bar{D}^*_s$ is probably a virtual state and not related to the meson $Y(4140)$. Other possibility, such as a hybrid charmonium is not excluded. We can explore the hidden charm two-body

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{pole_contribution.png}
\caption{The contribution from the pole term with variation of the Borel parameter $M^2$. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\tau$, $\xi$ and $\rho$ correspond to the threshold parameters $s_0 = 19 \text{GeV}^2$, $20 \text{GeV}^2$, $21 \text{GeV}^2$, $22 \text{GeV}^2$, $23 \text{GeV}^2$, $24 \text{GeV}^2$ and $25 \text{GeV}^2$, respectively.}
\end{figure}
Figure 2: The contributions from the different terms with variation of the Borel parameter $M^2$ in the operator product expansion. The $A$, $B$, $C$, $D$, $E$ and $F$ correspond to the contributions from the perturbative term, $\langle \bar{s}s \rangle + \langle s \sigma G s \rangle$ term, $\langle \frac{\omega_{GG}}{\pi} \rangle$ term, $\langle \omega_{GG} \rangle + \langle \frac{\omega_{GG}}{\pi} \rangle \left[ \langle \bar{s}s \rangle + \langle s \sigma G s \rangle + \langle \bar{s}s \rangle^2 \right]$ term, $\langle \bar{s}s \rangle^2$ term and $\langle \bar{s}s \rangle \langle s \sigma G s \rangle + \langle s \sigma G s \rangle^2$ term, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\tau$ and $\rho$ correspond to the threshold parameters $s_0 = 20 \text{ GeV}^2$, 21 GeV$^2$, 22 GeV$^2$, 23 GeV$^2$, 24 GeV$^2$ and 25 GeV$^2$, respectively. Here we take the central values of the input parameters.
decay $J/\psi\phi$ and the open charm two-body decays $D_s D_s, D_s D_s^*$ to make further studies. More experimental data are still needed.

From Eq.(11), we can see that the uncertainty of the mass $M_Y$ is rather small (about 3.6%) while the uncertainty of the pole residue $\lambda_Y$ is rather large (about 22.2%). The uncertainties of the input parameters $((\bar{q}q), \langle \bar{s}s \rangle, \langle \bar{s}g_s\sigma Gs \rangle, m_s$ and $m_c)$ vary in the range ($7 - 25$)%, so the uncertainty of the pole residue $\lambda_Y$ is reasonable. We obtain the value of the mass $M_Y$ through a fraction (see Eq.(10)), the uncertainties in the numerator and denominator which origin from a given input parameter (for example, $\langle \bar{s}s \rangle, \langle \bar{s}g_s\sigma Gs \rangle$) cancel out with each other. It is not unexpected that the net uncertainty is smaller than the uncertainties of the input parameters.

At the energy scale $\mu = 1$ GeV, $\frac{\alpha_s}{\pi} \approx 0.19$ [13], if the perturbative $\mathcal{O}(\alpha_s)$ corrections to the perturbative term are companied with large numerical factors, $1 + \xi(s, m_c)\frac{\alpha_s}{\pi}$, for example, $\xi(s, m_c) > \frac{\alpha_s}{\pi} \approx 5$, the contributions may be large. We can make a crude estimation by multiplying the perturbative term with a numerical factor, say $1 + \xi(s, m_c)\frac{\alpha_s}{\pi} = 2$, the mass $M_Y$ decreases slightly, about $20$ MeV, the pole residue $\lambda_Y$ increases remarkably. The main contribution comes from the perturbative term, the large corrections in the numerator and denominator cancel out with each other (see Eq.(10)). In fact, the $\xi(s, m_c)$ are complicated functions of the energy $s$ and the mass $m_c$, such a crude estimation maybe underestimate the $\mathcal{O}(\alpha_s)$ corrections, the uncertainties originate from the $\mathcal{O}(\alpha_s)$ corrections maybe larger.

In this article, we also neglect the contributions from the perturbative corrections $\mathcal{O}(\alpha_s^2)$. Those perturbative corrections can be taken into account in the leading logarithmic approximations through anomalous dimension factors. After the Borel transform, the effects of those corrections are to multiply each term on the operator product expansion side by the factor,

$$\left[\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right]^{2\Gamma_J - \Gamma_{\mathcal{O}_n}},$$

where the $\Gamma_J$ is the anomalous dimension of the scalar interpolating current $J(x)$, the $\Gamma_{\mathcal{O}_n}$ is the anomalous dimension of the local operator $\mathcal{O}_n(0)$ in the operator product expansion,

$$T\left\{J(x)J^\dagger(0)\right\} = C_n(x)\mathcal{O}_n(0),$$

here the $C_n(x)$ is the corresponding Wilson coefficient.

We carry out the operator product expansion at a special energy scale, say $\mu = 1$ GeV, and can not smear the scale dependence by evolving the operator product expansion side to the energy scale $M$ through Eq.(12) as the anomalous dimension of the scalar current $J(x)$ is unknown. Furthermore, the anomalous dimensions of the high dimensional local operators have not been calculated yet, and their values are poorly known. In this article, we set the factor $\left[\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right]^{2\Gamma_J - \Gamma_{\mathcal{O}_n}} \approx 1$, such an approximation maybe result in some scale dependence and weaken the prediction ability; further studies are stilled needed.

In the QCD sum rules, the high dimension vacuum condensates are always factorized to lower condensates with vacuum saturation, factorization works well in large $N_c$ limit. In the real world, $N_c = 3$, there are deviations from the factorable formula, we introduce a factor $\kappa$ to parameterize the deviations,

$$\langle \bar{s}s \rangle^2, \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle, \langle \bar{s}g_s\sigma Gs \rangle^2 \rightarrow \kappa \langle \bar{s}s \rangle^2, \kappa \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle, \kappa \langle \bar{s}g_s\sigma Gs \rangle^2. \quad (14)$$
In Fig.6, we show the mass $M_Y$ with variation of the parameter $\kappa$ at the interval $\kappa = 0 - 2$. From the figure, we can see that the value of the $M_Y$ changes quickly at the region $M^2 \leq 2.6$ GeV$^2$, and increase with the $\kappa$ monotonously at the region $M^2 \leq 3.2$ GeV$^2$. At the interval $M^2 = (2.6 - 3.0)$ GeV$^2$, the value $\kappa = 1 \pm 1$ leads to an uncertainty about 50 MeV, which is too small to smear the discrepancy between the present prediction and the experimental data. In the limit $\kappa = 0$, which corresponds to neglecting the vacuum condensates $\langle \bar{s}s \rangle^2$, $\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle$ and $\langle \bar{s}g_s \sigma Gs \rangle^2$, we obtain the smallest value. It is not unexpected. From Fig.2E-2F, we can see that there are cancelations among the vacuum condensates $\langle \bar{s}s \rangle^2$, $\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle$ and $\langle \bar{s}g_s \sigma Gs \rangle^2$, the net contribution is rather small. If we assume the $\kappa$ has the typical uncertainty of the QCD sum rules, say about 30%, the correction is rather mild, we can neglect the uncertainty safely and take $\kappa = 1$, i.e. the factorization works well. In the QCD sum rules for the masses of the $\rho$ meson and nucleon, $\kappa \geq 1$ [14]. If the same value holds for the tetraquark states, the deviation from the factorable formula means even larger discrepancy between the present prediction and the experimental data.

The $c$-quark mass appearing in the perturbative terms (see e.g. Eq.(17)) is usually taken to be the pole mass in the QCD sum rules, while the choice of the $m_c$ in the leading-order coefficients of the higher-dimensional terms is arbitrary [15]. The $\tilde{MS}$ mass $m_c(m_c^2)$ relates with the pole mass $\hat{m}$ through the relation

$$m_c(m_c^2) = \hat{m} \left[ 1 + \frac{C_F \alpha_s(m_c^2)}{\pi} + (K - 2C_F) \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right]^{-1}, \quad (15)$$

where $K$ depends on the flavor number $n_f$. In this article, we take the approximation $m_c \approx \hat{m}$ without the $\alpha_s$ corrections for consistency. The value listed in the Particle Data Group is $m_c(m_c^2) = 1.27^{+0.07}_{-0.11}$ GeV [16], it is reasonable to take the value $m_c = m_c(1 \text{ GeV}^2) = (1.35 \pm 0.10)$ GeV. In Fig.4, we also present the result with smaller value $m_c = 1.3$ GeV, which can move down the central value about 0.06 GeV. The central value $M_Y = 4.37$ GeV is still larger than the $D_s^0\bar{D}_s^0$ threshold about 150 MeV.

The QCD sum rules is just a QCD-inspired model, we calculate the ground state mass by imposing the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. In fact, we can take smaller threshold parameter $s_0$ and larger Borel parameter $M^2$ to reproduce the experimental value by releasing the pole dominance condition.

We usually consult the experimental data in choosing the Borel parameter $M^2$ and the threshold parameter $s_0$. The present experimental knowledge about the phenomenological hadronic spectral densities of the multiquark states (irrespective of the molecule type and the diquark-antidiquark type) is rather vague, even existence of the multiquark states is not confirmed with confidence. The nonet scalar mesons below 1 GeV (the $f_0(980)$ and $a_0(980)$ especially) are good candidates for the tetraquark states. However, they can’t satisfy the two criteria of the QCD sum rules, and result in a reasonable Borel window. If the perturbative terms have the main contribution (in the conventional QCD sum rules, the perturbative terms always have the main contribution), we can approximate the spectral density with the perturbative term [11], then take the pole dominance condition, and obtain the approximate relation,

$$\frac{s_0}{M^2} \geq 4.7. \quad (16)$$
Figure 3: The mass with variation of the Borel parameter $M^2$. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\tau$, $\xi$ and $\rho$ correspond to the threshold parameters $s_0 = 19\text{ GeV}^2$, 20\text{ GeV}^2, 21\text{ GeV}^2$, 22\text{ GeV}^2, 23\text{ GeV}^2, 24\text{ GeV}^2$ and 25\text{ GeV}^2$, respectively.

If the Borel parameter has the typical value $M^2 = 1\text{ GeV}^2$, then $s_0 \geq 4.7\text{ GeV}^2$, the threshold parameter is too large for the light tetraquark state candidates $f_0(980)$, $a_0(980)$, etc.

Once the main Fock states of the nonet scalar mesons below 1\text{ GeV}^2 are proved to be tetraquark states, we can draw the conclusion that the QCD sum rules are not applicable for the light tetraquark states. We can either reject the QCD sum rules for the multiquark states or release one of the two criteria (pole dominance and convergence of the operator product expansion) [8], for example, we can cut the threshold parameters for the $f_0(980)$ and $a_0(980)$ slightly larger than 1\text{ GeV}^2 by hand.

4 Conclusion

In this article, we assume that there exists a scalar $D^*_s\bar{D}^*_s$ molecular state in the $J/\psi\phi$ invariant mass distribution, and study its mass using the QCD sum rules. Our predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. The numerical result indicates that the mass is about $M_Y = (4.43 \pm 0.16)\text{ GeV}$, which is inconsistent with the experimental data. The $D^*_s\bar{D}^*_s$ is probably a virtual state and not related to the meson $Y(4140)$. Other possibility, such as a hybrid charmonium is not excluded; more experimental data are still needed to identify it.
Figure 4: The mass with variation of the Borel parameter $M^2$.

Figure 5: The pole residue with variation of the Borel parameter $M^2$. 
Figure 6: The mass with variation of the parameters $\kappa$ and $M^2$, other parameters are taken to be the central values.

Appendix

The spectral densities at the level of the quark-gluon degrees of freedom:

$$\rho_0(s) = \frac{3}{1024\pi^6} \int_{\alpha_i}^{\infty} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^3 (s - \tilde{m}_c^2)^2 (7s^2 - 6s\tilde{m}_c^2 + \tilde{m}_c^4)$$

$$+ \frac{3}{1024\pi^6} \int_{\alpha_i}^{\infty} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2 (s - \tilde{m}_c^2)^3 (3s - \tilde{m}_c^2)$$

$$+ \frac{3m_s m_c}{512\pi^6} \int_{\alpha_i}^{\infty} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) (1 - \alpha - \beta)^2 (s - \tilde{m}_c^2)^2 (5s - 2\tilde{m}_c^2), \quad (17)$$
\[
\rho_{(ss)}(s) = \frac{3m_s\langle \bar{ss} \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1-\alpha-\beta) (10s^2 - 12sm_c^2 + 3m_c^4)
\]
\[
+ \frac{3m_s\langle \bar{ss} \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (s - m_c^2)(2s - m_c^2)
\]
\[
- \frac{m_s\langle \bar{sg}\sigma G_s \rangle}{64\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta [6(2s - m_c^2) + s^2 \delta(s - m_c^2)]
\]
\[
- \frac{3m_c\langle \bar{ss} \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1-\alpha-\beta)(s - m_c^2)(2s - m_c^2)
\]
\[
+ \frac{3m_c\langle \bar{sg}\sigma G_s \rangle}{128\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s + \beta)(3s - 2m_c^2)
\]
\[
- \frac{3m_s m_c^2\langle \bar{ss} \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s - m_c^2)
\]
\[
- \frac{m_s\langle \bar{sg}\sigma G_s \rangle}{64\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha (1-\alpha)(3s - 2m_c^2)
\]
\[
+ \frac{3m_s m_c^2\langle \bar{sg}\sigma G_s \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha ,
\]

(18)

\[
\rho_{(ss)^2}(s) = \frac{m_s^2\langle \bar{ss} \rangle^2}{4\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha + \frac{m_s^2\langle \bar{sg}\sigma G_s \rangle^2}{64\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha m_c^4 \delta(s - m_c^2)
\]
\[
- \frac{m_c^2\langle \bar{ss} \rangle\langle \bar{sg}\sigma G_s \rangle}{8\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{m_c^2}{M^2} \right] \delta(s - m_c^2)
\]
\[
- \frac{m_s m_c\langle \bar{ss} \rangle^2}{16\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 2 + s \delta(s - m_c^2) \right]
\]
\[
+ \frac{5m_s m_c\langle \bar{ss} \rangle\langle \bar{sg}\sigma G_s \rangle}{96\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{m_c^2}{M^2} + \frac{m_c^4}{2M^4} \right] \delta(s - m_c^2),
\]

(19)
\[
\rho_{(GG)}^A(s) = -\frac{m_c^2}{256\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1-\alpha-\beta)^3 \left[ 2s - \bar{m}^2_c + \frac{s^2}{6} \delta(s - \bar{m}^2_c) \right] \\
+ \frac{3m_c m_e - m_e^2}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1-\alpha-\beta)^2 (3s - 2\bar{m}^2_c) \\
- \frac{m_c^3}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)(1-\alpha-\beta)^2 [2 + s\delta(s - \bar{m}^2_c)] \\
- \frac{1}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(1-\alpha-\beta)^2 (10s^2 - 12s\bar{m}^2_c + 3\bar{m}^4) \\
+ \frac{1}{256\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(1-\alpha-\beta)(s - \bar{m}^2_c)(2s - \bar{m}^2_c) \\
- \frac{3m_c m_e}{128\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1-\alpha-\beta)(3s - 2\bar{m}^2_c) \\
- \frac{m_c^2 (ss)}{96\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1-\alpha-\beta) \\
\left[ 1 + \frac{\bar{m}^2_c}{M^2} + \frac{\bar{m}^4_c}{2M^4} \right] \delta(s - \bar{m}^2_c) \\
- \frac{m_c^2 (ss)}{192\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \left[ 1 + \frac{\bar{m}^2_c}{M^2} \right] \delta(s - \bar{m}^2_c) \\
+ \frac{m_c^4 (ss)}{1152\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \bar{m}^4_c \delta(s - \bar{m}^2_c) \\
+ \frac{m_c^4 (ss)}{48\pi^2 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \delta(s - \bar{m}^2_c) \\
+ \frac{m_c (ss)}{192\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)(1-\alpha-\beta) \\
\left[ 1 + \frac{\bar{m}^2_c}{M^2} \right] \delta(s - \bar{m}^2_c) \\
- \frac{m_c^3 (sgs)}{768\pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)\bar{m}^2_c \delta(s - \bar{m}^2_c) \\
- \frac{m_c (ss)}{64\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1-\alpha-\beta) [2 + s\delta(s - \bar{m}^2_c)] \\
+ \frac{m_c (sgs)}{256\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \left[ 1 + \frac{\bar{m}^2_c}{M^2} \right] \delta(s - \bar{m}^2_c) \\
- \frac{m_c^2 (ss)}{16\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) \delta(s - \bar{m}^2_c) \\
- \frac{m_c (ss)}{64\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) \left[ 1 + \frac{2\bar{m}^2_c}{3} \delta(s - \bar{m}^2_c) + \frac{\bar{m}^4_c}{6M^2} \delta(s - \bar{m}^2_c) \right] \\
+ \frac{m_c (ss)}{32\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 2 + s\delta(s - \bar{m}^2_c) \right], \quad \text{(20)}
\]
\[ \rho^B_{\langle GG \rangle}(s) = -\frac{m_c^4 \langle ss \rangle^2}{72 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \delta(s - \tilde{m}_c^2) \\
- \frac{m_s m_c^3 \langle gs \sigma Gs \rangle}{192 \pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \delta(s - \tilde{m}_c^2) \\
+ \frac{m_s m_c^2 \langle gs \sigma Gs \rangle}{1152 \pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1 - \alpha}{\alpha^2} + \frac{\alpha}{(1-\alpha)^2} \right] \tilde{m}_c \delta(s - \tilde{m}_c^2) \\
- \frac{m_s m_c^3 \langle ss \rangle^2}{288 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \left[ 1 - \frac{\tilde{m}_c^2}{M^2} \right] \delta(s - \tilde{m}_c^2) \\
+ \frac{m_c^2 \langle ss \rangle^2}{24 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2} \right] \delta(s - \tilde{m}_c^2) \\
+ \frac{m_s m_c \langle ss \rangle}{64 \pi^2 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{\alpha}{\alpha^2} + \frac{1}{(1-\alpha)^2} \right] \delta(s - \tilde{m}_c^2) \\
- \frac{m_s m_c^2 \langle ss \rangle^2}{96 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1 - \alpha}{\alpha^2} + \frac{\alpha}{(1-\alpha)^2} \right] \tilde{m}_c \delta(s - \tilde{m}_c^2) \\
+ \frac{m_s \langle ss \rangle}{384 \pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 2 + s \delta(s - \tilde{m}_c^2) \right] \\
- \frac{m_c \langle gs \sigma Gs \rangle}{128 \pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{\tilde{m}_c^2}{M^2} \right] \delta(s - \tilde{m}_c^2), \tag{21} \]

where \( \alpha_f = \frac{1 + \sqrt{1 - 4m_c^2 s}}{2} \), \( \alpha_i = \frac{1 - \sqrt{1 - 4m_c^2 s}}{2} \), \( \beta_i = \frac{\alpha m_c^2}{\alpha s - m_c^2} \), \( \tilde{m}_c^2 = \frac{(\alpha + \beta) m_c^2}{\alpha \beta} \), \( \tilde{m}_c = \frac{m_c^2}{\alpha (1-\alpha)} \).

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