Gapless two-flavor color superconductor

Igor Shovkovy\textsuperscript{1,*} and Mei Huang\textsuperscript{1,2}

\textsuperscript{1}Institut f"ur Theoretische Physik, J.W. Goethe-Universit"at, D-60054 Frankfurt/Main, Germany
\textsuperscript{2}Physics Department, Tsinghua University, Beijing 100084, China

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A new, gapless two-flavor color superconducting phase that appears under conditions of local charge neutrality and $\beta$-equilibrium is revealed. In this phase, the symmetry of the ground state is the same as in the conventional two-flavor color superconductor. In the low-energy spectrum of this phase, however, there are only two gapped fermionic quasiparticles, and the other four quasiparticles are gapless. The origin and the basic properties of the gapless two-flavor color superconductor are discussed. This phase is a natural candidate for quark matter in cores of compact stars.

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Sufficiently cold and dense quark matter is a color superconductor [1,2]. The ground state is characterized by a condensate of Cooper pairs that are made of quarks with equal and opposite momenta. In this phase of matter, the SU(3)\textsubscript{c} color gauge group is broken (partially or completely) through the Anderson-Higgs mechanism. At asymptotic densities, this phenomenon was studied in detail from first principles in Refs. [3–5].

In general, quark matter at high baryon density has a rich phase structure, consisting of different normal and color-superconducting phases. Changing the baryon chemical potential, it should be possible to obtain phases in which either two or three of the lightest quark flavors participate in Cooper pairing.

At not very large densities, one gets quark matter made of only up and down quarks. If the Fermi momenta of different types of quarks are approximately the same, the up and down quarks will form Cooper pairs in the color-antitriplet, flavor-singlet, spin-zero channel [2–4]. The corresponding ground state of matter is the so-called two-flavor color superconductor (2SC). In this phase, the color gauge group is broken by the Anderson-Higgs mechanism down to SU(2)\textsubscript{c} subgroup. With the conventional choice of the condensate pointing in the “blue” direction, one finds that the condensate consists of red up ($u_r$) and green down ($d_g$), as well as green up ($u_g$) and red down ($d_r$) diquark Cooper pairs. The other two quarks ($u_b$ and $d_b$) do not participate in pairing. This is the conventional picture of the 2SC phase [2–4].

At sufficiently large densities, i.e., when the value of the chemical potential exceeds the constituent (medium modified) mass of the strange quark, quark matter should consist of all three quark flavors. This opens the possibility for Cooper pairing between the three lightest quarks (up, down and strange). Depending on the value of the strange quark mass, as well as other parameters in the theory, one might get the color-flavor locked phase (CFL) [6], or even the more exotic crystalline phase [7] of quark matter. Besides that, it is also possible that up and down quarks form 2SC matter, while the strange quarks do not participate in pairing. We call this latter phase 2SC+s. Still another possibility is that the strange quarks by themselves form a color superconducting condensate. This will be then a spin-1 condensate [4,8].

It is natural to expect that some color superconducting phases may exist in the interior of compact stars. The estimated central densities of such stars might be as large as 10$\rho_0$ (where $\rho_0 \approx 0.15$ fm$^{-3}$ is the saturation density), while their temperatures should be in the range of tens of keV. These values are very encouraging.

Matter in the bulk of compact stars should be neutral with respect to electric as well as color charges. Also, such matter should remain in $\beta$-equilibrium. Satisfying these requirements may impose nontrivial relations between the chemical potentials of different quarks. In turn, such relations could substantially influence the pairing dynamics between quarks, for instance, by suppressing some color superconducting phases and by favoring others.

It was argued in Ref. [9], that the 2SC+s phase becomes less favorable than the CFL phase if the charge neutrality condition is enforced (note that the strange quark mass was chosen too small to allow the appearance of a pure 2SC phase in Ref. [9]). Therefore, one might speculate that only CFL quark matter could exist inside compact stars. In such a case, the outside hadronic layer of the star would make a direct contact with the CFL quark core through a separating sharp interface [10,11]. The baryonic density and the energy density would have a jump (smoothed only over microscopic distances) at the interface. Of course, a mixed phase of hadronic and CFL matter is another possibility, but it does not seem to be favorable [11] (for other studies of the stars with CFL quark matter in their interior see Ref. [12]).

We should mention that the general conclusion of Ref. [9] was essentially confirmed in Ref. [13] where it was claimed that the 2SC+s could exist only in a narrow window (about 10 to 15 MeV wide) of baryonic chemical potential around the midpoint $\mu \equiv \mu_B/3 \approx 450$ MeV. For lower values of $\mu$, no neutral 2SC phase was found. In this Letter, however, we argue that a neutral 2SC phase does exist at lower values of $\mu$. It is a gapless rather than an ordinary 2SC phase. In particular, the conventional density relations between the pairing quarks, used in Ref.
\[ \tau_\mu = \mu - \frac{2}{3}\mu_e - \frac{1}{3}\mu_s, \quad (5) \]
\[ \mu_{db} = \mu + \frac{1}{3}\mu_e - \frac{2}{3}\mu_s. \quad (6) \]

The effective potential for quark matter at zero temperature and in \( \beta \)-equilibrium with electrons takes the form

\[ \Omega = \Omega_0 - \frac{\mu_e^4}{12\pi^2} + \frac{m^2}{4G_S} + \frac{\Delta^2}{4G_D} - \frac{\sum_a}{\int \frac{d^3p}{(2\pi)^3}|E_a|}, \quad (7) \]

where \( \Omega_0 \) is a constant added to make the pressure of the vacuum zero. This is the zero temperature limit of the potential derived in Ref. [14]. The sum in Eq. (7) runs over all (6 quark and 6 antiquark) quasiparticles. The dispersion relations and the degeneracy factors of the quasiparticles read

\[ E_{ab}^\pm = E(p) \pm \mu_{ab}, \quad [x1] \quad (8) \]
\[ E_{db}^\pm = E(p) \pm \mu_{eb}, \quad [x1] \quad (9) \]
\[ E_{\Delta z}^\pm = \sqrt{|E(p) \mp \mu|^2 + \Delta^2 \pm \delta\mu}, \quad [x2] \quad (10) \]

where the shorthand notations are \( E(p) = \sqrt{p^2 + m^2}, \)
\[ \bar{\mu} \equiv (\mu_{ur} + \mu_{dd})/2 = \mu - \mu_e/6 + \mu_s/3, \]
and \( \delta\mu \equiv (\mu_{dg} - \mu_{ur})/2 = \mu_e/2. \)

In this study we concentrate exclusively on the 2SC phase of quark matter. The numerical study shows that the constituent quark mass \( m \) is zero in this phase [14]. Then by making use of the dispersion relations, we derive

\[ \Omega = \Omega_0 - \frac{\mu_e^4}{12\pi^2} + \frac{\Delta^2}{4G_D} - \frac{\mu_{ab}^4}{12\pi^2} - \frac{\mu_{db}^4}{12\pi^2} \]
\[ - 2\int_0^\Lambda \frac{p^2 dp}{\pi^2} \left( \sqrt{(p + \bar{\mu})^2 + \Delta^2} + \sqrt{(p - \bar{\mu})^2 + \Delta^2} \right) \]
\[ - 2\theta(\delta\mu - \Delta) \int_{-\mu}^{\mu} \frac{p^2 dp}{\pi^2} \left( \delta\mu - \sqrt{(p - \bar{\mu})^2 + \Delta^2} \right), \quad (11) \]

where \( \mu^\pm \equiv \pm \sqrt{(\delta\mu)^2 - \Delta^2} \). Note that the physical thermodynamic potential that determines the pressure, \( \Omega_{\text{phys}} = -P \), is obtained from \( \Omega \) in Eq. (11) after substituting \( \mu_s, \mu_e \) and \( \Delta \) that solve the color and electrical charge neutrality conditions, as well as the gap equation, i.e.,

\[ n_s = \frac{\partial\Omega}{\partial\mu_s} = 0, \quad n_Q = \frac{\partial\Omega}{\partial\mu_e} = 0, \quad \text{and} \quad \frac{\partial\Omega}{\partial\Delta} = 0. \quad (12) \]

Let us assume that the solution to the color neutrality condition, \( \mu_s(\mu_e, \Delta) \), is known for a given value of \( \mu \). In practice, we construct this function using a numerical set of solutions. By substituting this solution into the potential in Eq. (11), one is left with a function \( \Omega \) that depends only on two parameters, \( \mu_e \) and \( \Delta \). In this two-dimensional parameter space, the solution to each of the extra two conditions in Eq. (12) is a one-dimensional line. Graphically, this is shown in Fig. 1. Since both
conditions should be satisfied simultaneously, one needs to find the intersection points of the corresponding lines.

First we discuss the neutrality condition, \( n_Q = 0 \). The corresponding line of solutions in the two-dimensional parameter space is a monotonic, single valued function. In Fig. 1, it is represented by the thick dash-dotted line. This line of solutions consists of two branches in two qualitatively different regions of the parameter space that are separated by the line \( \Delta = \mu_c/2 \). In the leading order approximation, this neutrality line is independent of the coupling constant \( G_D \). This should be obvious from the derivation of the first two conditions in Eq. (12).

![Fig. 1](image1.png)

**FIG. 1.** The graphical representation of the solutions to the gap equation for three different values of the diquark coupling constant (thick solid and dashed lines), and to the electrical neutrality condition (thick dash-dotted line). The intersection points represent the solutions to both equations. The thin solid line divides two qualitatively different regions, \( \Delta < \mu_c/2 \) and \( \Delta > \mu_c/2 \). The results are plotted for \( \mu = 400 \text{ MeV} \) and three values of diquark coupling constant \( G_D = \eta G_S \) with \( \eta = 0.5, \eta = 0.75, \) and \( \eta = 1.0 \).

The line of solutions to the gap equation is most interesting. It is made of two different branches in two regions, \( \Delta < \mu_c/2 \) and \( \Delta > \mu_c/2 \), see Fig. 1. The upper is the main branch that exists down to \( \mu_c = 0 \) (i.e., no mismatch in the Fermi surfaces of the up and down quarks). The lower branch appears only in a finite window of electric chemical potentials, and it merges with the upper one at a point on the line \( \Delta = \mu_c/2 \). In Fig. 1, we show three solutions to the gap equation at \( \mu = 400 \text{ MeV} \) in three regimes with different coupling constants, corresponding to \( \eta = 0.5, \eta = 0.75, \) and \( \eta = 1.0 \). [The results look the same at all values of \( \mu_c \).] The three curves have qualitatively the same shape, but differ by overall scale factors. The difference in the overall scale has an important consequence. At weak coupling (\( \eta \lesssim 0.7 \)), there is no neutral 2SC phase because there is no intersection of the solution to the gap equation with the neutrality line, see \( \eta = 0.5 \) solution in Fig. 1. At intermediate \((0.7 \lesssim \eta \lesssim 0.8)\) and strong \((\eta \gtrsim 0.8)\) coupling, on the other hand, there is an intersection with the neutrality line at a point on the lower and upper branches, respectively. In these last two cases, neutral 2SC phases exist.

If the electric chemical potential were a free parameter, one would find that increasing its value leads to a first order phase transition [17]. This is the result of the appearance of two competing local minima (at \( \Delta = 0 \) and \( \Delta \neq 0 \)) of the potential \( \Omega(\Delta, \mu_c = \text{const}) \) for a range of intermediate values of the parameter \( \mu_c \). Note, that the location of the local maximum of the potential that lies between the two local minima is determined by the lower “unstable” branch of the solutions to the gap equation in Fig. 1. The first order phase transition happens at about \( \mu_c^* = \sqrt{2} \Delta_0 \) where \( \Delta_0 \) is the value of the gap at \( \mu_c = 0 \) (this estimate is derived in the approximation of weak coupling [18]). From our discussion below, it will be clear that this first order phase transition is unphysical under the requirement of local neutrality of quark matter. This is because the transition typically happens between two types of matter with different, nonzero charge densities (e.g., positively charged color superconducting matter and negatively charged normal quark matter). It is worth mentioning, however, that this first order phase transition can get physical meaning in a globally neutral mixed phase of quark matter [19], and in some condensed matter systems where there is no analogue of the charge neutrality condition [20].

The three-dimensional view of the potential \( \Omega \) as a function of two parameters, \( \Delta \) and \( \mu_c \), is represented by a surface in Fig. 2. The results are plotted for \( \mu = 400 \text{ MeV} \) and for the diquark coupling with \( \eta = 0.75 \). In this study, we are interested exclusively in locally neutral quark matter (mixed phases are discussed in Ref. [19]), and therefore we consider the potential only along the neutrality line (represented by a black solid line in Fig. 2).

![Fig. 2](image2.png)

**FIG. 2.** The effective potential as a function of the diquark gap \( \Delta \) and electric chemical potential \( \mu_c \). The black solid line gives the potential along the electric neutrality line. The white solid line shows the line of solutions to the gap equation. The results are plotted for \( \mu = 400 \text{ MeV} \) and \( G_D = \eta G_S \) with \( \eta = 0.75 \).

The study shows that, for any coupling strength, such a potential of the neutral quark matter, \( \Omega(\Delta, \mu_c | n_Q = 0) \),
as a function of $\Delta$ has one local minimum which is also the global one. In the regime of intermediate strength of diquark coupling (e.g., $\eta = 0.75$), this minimum corresponds to a point on the lower branch of the solutions to the gap equation in Fig. 1 (i.e., $\Delta \approx 68$ MeV and $\mu_c \approx 148.4$ MeV). This may sound very surprising because the points on the lower branch correspond to local maxima of the potential calculated for a fixed value of $\mu_c$ with the potential defined along the neutrality line in Fig. 3. As we see, after enforcing the condition of charge neutrality, a local maximum of the potential has the global minimum at $\Delta = 0$. This is in agreement with the fact that the neutrality line does not intersect the corresponding line of nontrivial solutions (e.g., $\eta = 0.5$ case in Fig. 1). For intermediate values of the coupling ($0.7 \lesssim \eta \lesssim 0.8$), the extremum at $\Delta = 0$ is a local maximum, while a minimum appears at $\Delta \neq 0$, see solid curve in Fig. 3. In this case, the minimum corresponds to an intersection of the neutrality line with the lower branch of the solutions to the gap equation (e.g., $\eta = 0.75$ case in Fig. 1). Finally, in the strongly coupled regime ($\eta \gtrsim 0.8$), there is a nontrivial minimum which corresponds to an intersection of the neutrality line with the upper branch of the solutions (e.g., $\eta = 1.0$ case in Fig. 1). The phase transition controlled by the coupling constant $\eta$ is a second order phase transition.

![FIG. 3. The effective potential as a function of the diquark gap $\Delta$ calculated at a fixed value of the electric chemical potential $\mu_c = 148.4$ MeV (dashed line), and the effective potential defined along the neutrality line (solid line). The results are plotted for $\mu = 400$ MeV and $G_D = \eta G_S$ with $\eta = 0.75$.](image)

Here we argue that the regime of intermediate couplings describes a new, gapless phase of 2SC quark matter. This phase of superconducting quark matter possesses four gapless and only two gapped fermionic quasiparticles in its spectrum, see Eqs. (8)–(10). Recall that the ordinary 2SC phase (which, in fact, appears in the region $\Delta > \mu_c/2$ in the strong coupling regime) has only two gapless and four gapped quasiparticles. The number of gapless modes changes when one crosses the line $\Delta = \mu_c/2$. The quasiparticle dispersion relations, originating from the red and green quarks, as described by Eq. (10), are shown in Fig. 4. Each line corresponds to two degenerate quasiparticles. The dispersion relations of gapless blue quarks, given in Eqs. (8) and (9), are not shown in Fig. 4.

In ordinary 2SC matter (with a solution in the region $\Delta > \mu_c/2$), there are two doublets of gapped modes: one with the gap $\Delta - \mu_c/2$ and the other with the gap $\Delta + \mu_c/2$. As one approaches the boundary between the regions, $\Delta \to \mu_c/2$, the two quasiparticles of the first doublet with the smaller gap gradually become gapless. They also remain gapless in the phase with $\Delta < \mu_c/2$ (regime of intermediate coupling). This justifies the name “gapless 2SC phase”.

In the study of the CFL phase in Ref. [21], it was revealed that the number densities of the pairing quarks are equal. By applying the same arguments in the case of 2SC quark matter, one might expect to get similar relations for the number densities of red up and green down (as well as green up and red down) quarks. Moreover, these relations have been numerically confirmed for the 2SC phase of quark matter in the strongly coupled regime in Ref. [22]. We find, however, that the number densities of pairing quarks are not always equal. In particular, they are not equal in the gapless 2SC phase, i.e., $n_{ur} = n_{ug} \neq n_{dr} = n_{dg}$. This is directly related to the fact that the corresponding solution is found in the region $\Delta < \mu_c/2$, where two extra gapless modes appear.

![FIG. 4. Three types of dispersion relations of quasiparticles that originate from quarks of two (red and green) colors participating in Cooper pairing. The two solid (dashed) lines represent the dispersion relations in the regime with $\Delta > \mu_c/2$ ($\Delta < \mu_c/2$). The dash-dotted line corresponds to the special case with $\Delta = \mu_c/2$. Each line corresponds to two degenerate quasiparticles.](image)

In conclusion, the analysis of this Letter confirms that the charge neutrality condition can strongly affect color superconducting quark matter, favoring some phases and disfavoring others. Moreover, as we argued, by imposing the charge neutrality condition, one can also obtain
new stable phases of matter which could not exist otherwise. The gapless 2SC phase of quark matter is a beautiful example of such a new phase. The symmetry of its ground state is the same as that of the conventional 2SC phase. However, the energy spectrum of the fermionic quasiparticles in gapless 2SC matter is different: it has two additional gapless modes. We should note that this phase is very different from another example of gapless color superconductivity, discussed in Ref. [24], which is a metastable state of CFL quark matter. The gapless 2SC phase is a stable, neutral state of two-flavor quark matter.

In nature, gapless 2SC quark matter could exist in compact stars. Indeed, this phase is neutral with respect to electric and color charges and satisfies the $\beta$-equilibrium condition by construction. Of course, satisfying these requirements is necessary, but not sufficient. In order to decide whether the gapless 2SC phase is likely to appear inside stars, further detailed studies are needed.

We could speculate that the thermodynamic properties of the gapless 2SC phase should be closer to the properties of neutral normal quark matter rather than to those of strongly coupled 2SC matter. Qualitatively, this is expected from observing the relative shallowness of the effective potential in Fig. 3 defined along the neutrality line. It indicates that, for a given value of the baryon chemical potential $\mu$, the pressure difference of (neutral) normal and gapless 2SC quark matter is substantially smaller than the pressure difference of the normal quark and conventional 2SC phase, $\delta P = (\mu \Delta / \pi)^2$ [21,23].

Here we studied only the most general properties of the gapless 2SC phase of quark matter. By taking into account that this phase is a realistic candidate for matter inside compact star cores, it would be very interesting to study its other properties that could potentially affect some compact star observables. In particular, this includes magnetic properties, neutrino emissivities, and various transport properties of gapless 2SC matter.

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* On leave of absence from Bogolyubov Institute for Theoretical Physics, 03143, Kiev, Ukraine.

[1] B. C. Barrois, Nucl. Phys. B129, 390 (1977); S. C. Frautschi, in “Hadronic matter at extreme energy density”, edited by N. Cabibbo and L. Sertorio (Plenum Press, 1980); D. Bailin and A. Love, Phys. Rep. 107, 325 (1984).
[2] M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B 422, 247 (1998); R. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998).
[3] D.T. Son, Phys. Rev. D 59, 094019 (1999); T. Schafer and F. Wilczek, Phys. Rev. D 60, 114033 (1999); D.K. Hong, V.A. Miransky, I.A. Shovkovy, and L.C.R. Wijewardhana, Phys. Rev. D 61, 056001 (2000); S.D.H. Hsu and M. Schwetz, Nucl. Phys. B572, 211 (2000); W.E. Brown, J.T. Liu, and H.-C. Ren, Phys. Rev. D 61, 114012 (2000).
[4] R.D. Pisarski and D.H. Rischke, Phys. Rev. D 61, 051501 (2000).
[5] I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Lett. B 476, 189 (1999); T. Schafer, Nucl. Phys. B575, 269 (2000).
[6] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B537, 443 (1999);
[7] M. G. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D 63, 074016 (2001); J. A. Bowers, J. Kundu, K. Rajagopal and E. Shuster, Phys. Rev. D 64, 014024 (2001); R. Casalbuoni, R. Gatto, M. Mannarelli and G. Nardulli, Phys. Rev. D 66, 014006 (2002); I. Giannakis, J. T. Liu and H. C. Ren, Phys. Rev. D 66, 031501 (2002); J. A. Bowers and K. Rajagopal, Phys. Rev. D 66, 065002 (2002).
[8] T. Schafer, Phys. Rev. D 62, 094007 (2000); M. G. Alford, J. A. Bowers, J. M. Cheyne and G. A. Cowan, Phys. Rev. D 67, 054018 (2003); M. Buballa, J. Hosek and M. Oertel, hep-ph/0204275; A. Schmitt, Q. Wang and D. H. Rischke, nucl-th/0301090.
[9] M. Alford and K. Rajagopal, JHEP 0206, 031 (2002).
[10] M. G. Alford, K. Rajagopal, S. Reddy and F. Wilczek, Phys. Rev. D 64, 074017 (2001);
[11] M. Alford and S. Reddy, nucl-th/0211046.
[12] G. Lugones and J. E. Horvath, astro-ph/0211638; M. Baldo, M. Buballa, F. Burgio, F. Neumann, M. Oertel and H. J. Schulze, nucl-th/0212096; S. Banik and D. Bandyopadhyay, astro-ph/0212340.
[13] A.W. Steiner, S. Reddy and M. Prakash, Phys. Rev. D 66, 094007 (2002).
[14] M. Huang, P. F. Zhuang and W. Q. Chao, Phys. Rev. D 67, 065015 (2003).
[15] S. Rüster, Diploma thesis, J. W. Goethe-University, 2003.
[16] D. Blaschke, S. Fredriksson, H. Grigorian and A. M. Oettas, nucl-th/0301002.
[17] P. F. Bedaque, Nucl. Phys. A 697, 569 (2002); O. Kiriyama, S. Yasui and H. Toki, Int. J. Mod. Phys. E 10, 501 (2001).
[18] A. M. Clogston, Phys. Rev. Lett. 9, 266 (1962); B. S. Chandrasekhar, Appl. Phys. Lett. 1, 7 (1962).
[19] I. Shovkovy, M. Hanauske and M. Huang, hep-ph/0303027, Phys. Rev. D, in production.
[20] G. Sarma, J. Phys. Chem. Solids 24, 1029 (1963).
[21] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 86, 3492
[22] F. Neumann, M. Buballa, and M. Oertel, Nucl. Phys. A 714, 481 (2003).
[23] V. A. Miransky, I. A. Shovkovy and L. C. Wijewardhana, Phys. Lett. B 468, 270 (1999).
[24] M. G. Alford, J. Berges and K. Rajagopal, Phys. Rev. Lett. 84, 598 (2000).