Markovian Decision Modeling in Dam Projects - Niger Delta River Basin

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Abstract
This paper studied simulation modeling in Markovian Decision theory and its application in decision making as well as planning in water resources and environmental engineering. The research objectives deals with the multi-objective values of a River basin for its wide range of purposes such as Economic Efficiency, Regional Economic distribution, State Economic distribution, Social Well-being, and Environmental Quality control. In line with foregoing objectives, the researchers aim at achieving the following: (i) Measures the magnitude of the difference between alternative actions (ii) to present a framework for considering decision making under uncertainty. (iii) to evaluate the optimal policy or strategy or action that maximizes the expected benefit in the River Basin within the available limited resources and funds over the planning period of a course of action or alternatives. The Methodology applied involved Markovian decision model method for River basin. Data collection was based on technical literatures from books, journals, and newspapers, River Basin Engineering Development, Parastatals. The analysis and presentation of results were based on simulation of Markovian Models. Furthermore, Contingency association, Chi-square, Pearson Product Moment Correlation were carried out as interaction, reliability and validity tests. However, simulating the river basin variables using Markov chain Homogeneous analysis and policy iterations resulted to a decision policy of allocating resources to the river basin objectives based on a federal government budgetary appropriation of 100 billion Naira. In conclusion the model had policy decision made as follows: Economic Efficiency [64%], Regional Economic Distribution [9%], State Economic Distribution [19%], Social Well-Being [5%] and Environmental Control [3%] [see Figure 1 and 2]. The results indicate that Markov Chain can be successfully applied in optimum policy investment decision making in multi-objective water resources management.

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1.0 Introduction
Numerous major multiple-purpose reservoir systems have been constructed throughout the nation during the past several decades. Public needs and objectives and many factors affecting operation of these reservoirs change over time. Reservoir system operations are complex and often offer substantial increases in benefits for relatively small improvements in operating efficiency. Consequently, evaluation of refinements and modifications to the operations of existing reservoir systems is becoming an increasingly important activity. However, Reservoir operation for municipal and industrial water supply is based on meeting demands subject to institutional constraints related to project ownership.

However, against the foregoing the research work was initiated out of the concern of allocating budgetary resources to the various river basin purposes for functionality requirement as well as sustainability of the system arrangement.

Statement of Problem: The research will proffers’ solution on the allocation of resources to the multi-purpose dam projects such as Power generation, Navigation, water supply, Tourism, and Flood control, in the Niger Delta River Basin using Markov Modeling. In line with foregoing objectives, the research aim to achieve the following:

• To present selected empirical results of a study employing decision-making theory as a framework for considering decision making under uncertainty.
To evaluate the optimal policy or strategy or action that maximizes the expected yield of the River Basin purposes

**Area of Study:** The study area is the Niger delta river basin that lies between 6.83N and 6.75E; 5.38S and 5.37W. Niger delta basin development authority is a service-oriented organization that is positioned to meet the water requirements of stakeholders in the most satisfactory and cost effective manner, while ensuring good quality and sanitation and paying adequate attention to preservation of the ecosystem, using proven technology and a well-motivated force [NDBDA MISSION]. In terms of geographical coverage it serves Rivers state, Bayelsa and Delta states. The three states have an estimated population of 10.7 Billion people.

### 2.0 Methodology

Under this section, the researcher identified estimation methods of the two major parameters of river basin indicators and the Markova method of application as follows:

**Estimating Multipurpose Benefits**

There are six data categories that structure the multipurpose benefits framework. These categories are referred to herein as “uses”, and they represent a culmination of operations and services made possible due to existence of a reservoir. These uses are broadly classified to identify categories associated with a reservoir project, and serve as a foundation for assessing collective and inter-dependent relationships (Marisol Bonnet et al, 2015):

i. **Hydropower:** Operation and use of generating facilities and/or equipment for producing power by the sole source of water.

ii. **Flood Control:** Dams that facilitate the prevention and/or lessen the severity of flood damage to valuable resources within a flood basin.

iii. **Water Transport & Navigation:** The operation and control of locks to facilitate the transportation of goods via inland waterways.

iv. **Recreation & Tourism:** The use of water bodies (reservoirs or rivers) for physical and recreational activities (boating, swimming, etc.).

v. **Water Supply:** Public and private withdrawals of water used for consumption, municipal, and industrial needs.

vi. **Irrigation:** The withdrawal and use of water from reservoirs to meet the needs and requirement for crop and plant irrigation to sustain growth and production.

Based on the availability of both public and proprietary data, the following represent the methodologies used to compute the economic benefit of each multipurpose use.

**2.1 Markovian Simulation Method**

The method of Markov chain applied in this research work is homogeneous Markov chain one that does not evolve in time; that is, its transition probabilities are independent of the time step n. Then we have the “n-step” transition probabilities as stated below:

and we have

\[
P^{(n)}_{ij} = P(X_{n+1} = j | X_n = i)
\]

Equation 1

Now we can define a theorem: Chapman-Kolmogorov equation.

Theorem. Chapman-Kolmogorov equation.

\[
P^{(m)}_{ij} = \sum_{k \in \mathbb{Z}} P^{(r)}_{ik} P^{(m-r)}_{kj}
\]

\[\forall r \in \mathbb{N} \cup \{0\}\]

Proof.

\[
p_{ij} = P(X_m = j | X_0 = i) = \sum_{k \in \mathbb{Z}} P(X_m = j, X_r = k | X_0 = i)
\]

\[
= \sum_{k \in \mathbb{Z}} P(X_m = j, X_r = k, X_0 = i) P(X_r = k | X_0 = i)
\]

\[
= \sum_{k \in \mathbb{Z}} P(X_m = j, X_r = k) P(X_r = k | X_0 = i)
\]

\[
= \sum_{k \in \mathbb{Z}} P^{(r)}_{ik} P^{(m-r)}_{kj}
\]

Equation 2

We can write this as a matrix for convenience:
Corollary.

\[ \mathbf{P}^{(m)} = \left( (p_{ij}^{(m)}) \right) \]

Equation 3

Proof. Chapman-Kolmogorov in matrix form gives us

\[ \mathbf{P}^{(m)} = \mathbf{P}^{(r)} \mathbf{P}^{(m-r)} \quad \forall r \in \mathbb{N} \cup \{0\} \]

\[ \mathbf{P}^{(2)} = \mathbf{P} \times \mathbf{P} = \mathbf{P}^2 \]

\[ \mathbf{P}^{(3)} = \mathbf{P} \times \mathbf{P}^2 = \mathbf{P}^3 \]

\[ \mathbf{P}^{(m)} = \mathbf{P}^m, m \geq 2, \text{then} \]

\[ \mathbf{P}^{(m+1)} = \mathbf{P} \times \mathbf{P}^m = \mathbf{P}^{m+1} \]

Equation 4

Several definitions

A Markov Chain is completely determined by its transition probabilities and its initial distribution. An initial distribution is a probability distribution

\[ \{\pi_i = P(X_0 = i)|i \in \mathbb{Z}\} \]

such that

\[ \sum_i \pi_i = 1. \]

Equation 5

A distribution is stationary if it satisfies \( \pi = \pi \mathbf{P} \). The period of state \( i \) is defined as

\[ d_i = \gcd\{m \in \mathbb{Z} | \pi^{(m)}_{ii} > 0\} \]

Equation 7

that is, the \( \gcd \) of the numbers of steps that it can take to return to the state. If \( d_i = 1 \), the state is aperiodic – it can occur at non-regular intervals.

A state \( j \) is accessible from a state \( i \) if the system, when started in \( i \), has a nonzero probability of eventually transitioning to \( j \), or more formally if there exists some \( n \geq 0 \) such that

\[ P(\mathbf{X}_n = j | \mathbf{X}_0 = i) > 0. \]

We write this as \( (i \rightarrow j) \). We define the first-passage time (or “hitting time”) probabilities

\[ f_{ij}^{(m)} = P(X_m = j, X_k \neq j, 0 < k < m - 1 | X_0 = i), i, j \in \mathbb{Z}. \]

Equation 8

that is, the time step at which we first reach state \( j \). We denote the expected “return time” –

\[ \mu_{ij} = \sum m = 1 \rightarrow m f_{ij}^{(m)} \]

Equation 10

A state is recurrent if

\[ \sum_{m=1}^{\infty} f_{ij}^{(m)} = 1 \]

Equation 11

(and transient if the sum is greater than 1).

It is positive-recurrent if \( \mu_{ii} < \infty \). That is, we expect to return to the state in a finite number of time steps.

Fundamental Theorem of Markov Chains

Theorem. For any irreducible, aperiodic, positive-recurrent Markov chain \( \mathbf{P} \) there exists a unique stationary distribution \( \{\pi_j, j \in \mathbb{Z}\} \).

Proof. We know that for any \( m \),

\[ \sum_{i=0}^{m} P_{ij}^{(m)} \leq \sum_{i=0}^{\infty} P_{ij}^{(m)} \leq 1. \]

Equation 12

If we take the limit as \( m \rightarrow \infty \):

\[ \lim_{m \rightarrow \infty} \sum_{i=0}^{m} P_{ij}^{(m)} = \sum_{i=0}^{\infty} \pi_i \leq 1. \]

Equation 13

This implies that for any \( M \),
Now we can use Chapman-Kolmogorov:

$$p_{ij}^{(m+1)} = \sum_{k=0}^{\infty} p_{ik}^{(m)} p_{kj} \geq \sum_{l=0}^{M} p_{ik}^{(m)} p_{lj}$$

and take the limit again as $m, M \to \infty$

$$\pi_j \geq \sum_{k=0}^{\infty} \pi_k p_{kj}$$

For the purpose of this research work Homogeneous Markov Chain was adopted, which stated as follows:

The matrix $P$ is called a homogeneous transition or stochastic matrix because all the transition probabilities $P_{ij}$ are fixed and independent of time. The probability $P_{ij}$ must satisfy the conditions.

$$\sum_i P_{ij} = 1, \text{ for all } i$$

$$P_{ij} \geq 0 \text{ for all } i \text{ and } j$$

### 3.0 Data Estimation, Analysis and Optimization

Determination of benefits to purposes under various objectives in a multi-purpose/multi-objective Water Resources Project Planning:

At the onset of planning of multipurpose water resources project, it is necessary to declare the objectives against which efforts is being geared for their achievement, this serve as a criterion for measuring the projected end product of the planning process.

The main objective that can come into play in a multi-objective water resources development are (1) economic efficiency (economic optimization), (2) Regional economic redistribution, and (3) Social well-being. Any other objective can be incidental on the above three.

### 3.1 Application of Markov Theory in Multi-Purpose Multi-Objective Projects Optimization

Let’s consider Federal Government Allocation to Niger Delta River Basin where N100 billion is to be spent on a multi-purpose/multi-objective water resources development project. The purposes of interest are Navigation, Tourism, Flooding, Hydro-electric power generation and water supply. The objectives to be simultaneously achieved at optimum level are economic efficiency, regional economic redistribution, State Economic distribution, social well-being and Environmental quality.

The problem then becomes how to apportion the N100 billion development fund among the various purposes so as to optimize the objective even under the worst situation of conflict.

A benefit study of the five purposes under each of the five objectives was carried out. The results being the figures as shown in table 5.1. What we have by the table is basically a Matrix situation that satisfies the homogeneous Markov chain.
Table 1  Benefit to N100 Billion under various objectives [N X 10^9]

| State of Nature | Objectives                           | Economic efficiency allow[Billion Naira] | Regional economy | State economic distribution | Social well-being | Environment |
|-----------------|--------------------------------------|-----------------------------------------|------------------|----------------------------|--------------------|-------------|
| Purposes        |                                      |                                        |                  |                            |                    |             |
| Navigation      | 2                                    | 0.3                                     | 0.89             | 0.2                        | 0.1                |             |
| Tourism         | 30.5                                 | 0.68                                    | 0.75             | 0.8                        | 0.45               |             |
| Flooding        | 20.3                                 | 1                                       | 0.65             | 0.75                       | 0.35               |             |
| Hydropower      | 1.7                                  | 2                                       | 0.9              | 0.45                       | 0.59               |             |
| Water supply    | 1.4                                  | 0.75                                    | 0.8              | 0.35                       | 0.74               |             |

Table 5.1 above is in matrix form and is converted into homogeneous transition or stochastic matrix to satisfy Markov Chain process where the probability Pij must satisfy the conditions:

\[ \sum P_{ij} = 1, \text{ for all } i; \ P_{ij} \geq 0 \text{ for all } i \text{ and } j \]

Table 2: Represents Probability of “ij” in table 1

| State of Nature | Objectives                           | Economic efficiency allow[Billion Naira] | Regional economy | State economic distribution | Social wellbeing | Environment |
|-----------------|--------------------------------------|-----------------------------------------|------------------|----------------------------|--------------------|-------------|
| Purposes        |                                      |                                        |                  |                            |                    |             |
| Navigation      | 0.573066                             | 0.08595989                             | 0.25501433       | 0.0573066                  | 0.0286533         |             |
| Tourism         | 0.919228                             | 0.02049427                             | 0.02260398       | 0.0241109                  | 0.01356239        |             |
| Flooding        | 0.880694                             | 0.04338395                             | 0.02819957       | 0.032538                   | 0.01518438        |             |
| Hydropower      | 0.301418                             | 0.35460993                             | 0.15957447       | 0.0797872                  | 0.10460993        |             |
| Water supply    | 0.346535                             | 0.18564356                             | 0.1980198        | 0.0866337                  | 0.18316832        |             |

Converting Table 5.2 to a linear equation as following:
The above Matrix problem can be solved from the maximize point of view with the understanding that all purposes should be undertaken at positive level even under the worst circumstances or condition.

Let probability \( \pi_1 \) represent Navigation

Let probability \( \pi_2 \) represents Tourism

Let probability \( \pi_3 \) represents flooding

Let probability \( \pi_4 \) represents Hydropower

And Let probability \( \pi_5 \) represents Water supply

\[
P = \begin{bmatrix}
0.5730659 & 0.085959885 & 0.2550143 & 0.0573066 & 0.028653295 \\
0.91922845 & 0.020494274 & 0.022604 & 0.0241109 & 0.013562387 \\
0.88069414 & 0.043383948 & 0.0281996 & 0.032538 & 0.015184382 \\
0.30141844 & 0.354609929 & 0.1595745 & 0.0797872 & 0.104609929 \\
0.34653465 & 0.185643564 & 0.1980198 & 0.0866337 & 0.183168317 
\end{bmatrix}
\]

These Probabilities in the matrix were calculated by the formula:

\[
P_{ij} = \frac{N_{ij}}{\sum_{i=0}^{k} N_{ij}},
\]

Equation 17

Where \( N_{ij} \) is the number of observed transitions from state \( i \) to \( j \).
Then the Markov process equations can be stated as follows:
A stationary distribution of a Markov chain is a probability distribution that remains unchanged in the Markov chain as time progresses. Typically, it is represented as a row vector $\pi$ whose entries are probability summing to 1, and given transition matrix $P$, it satisfies

$$\pi P = \pi.$$ 

in other words, $\pi$ is invariant by the matrix $P$.
Ergodic Markov Chains have a unique stationary distribution, and absorbing Markov chains have stationary distribution with nonzero elements only in absorbing states. The stationary distribution gives information about the stability of a random process and in certain cases describes the limiting behavior of the Markov chain. Note that the limiting distribution does not depend on the number of population within the River Basin, that is why the researcher has chosen to work with a certain percentage of the population [1%]. Thus:

$$
\begin{align*}
0.573066\pi_1 + 0.08595989\pi_2 + 0.25501433\pi_3 + 0.0573066\pi_4 + 0.0286533\pi_5 &= \pi_1 \\
0.919228\pi_1 + 0.02049427\pi_2 + 0.02260398\pi_3 + 0.0241109\pi_4 + 0.01356239\pi_5 &= \pi_2 \\
0.080694\pi_1 + 0.04338395\pi_2 + 0.02819957\pi_3 + 0.032538\pi_4 + 0.01518438\pi_5 &= \pi_3 \\
0.080694\pi_1 + 0.04338395\pi_2 + 0.02819957\pi_3 + 0.032538\pi_4 + 0.01518438\pi_5 &= \pi_4 \\
0.346535\pi_1 + 0.18564356\pi_2 + 0.1980198\pi_3 + 0.0866337\pi_4 + 0.18316832\pi_5 &= \pi_5
\end{align*}
\]

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$$

4.0 Markov Chain Analysis
The equations having satisfied Markov homogeneous chain are analyzed by Markov steady state. There two methods for solving the infinite-stage problem. The first method calls for evaluating all possible stationary policies of the decision problem. This is equivalent to an exhaustive enumeration process and can be used only if the number of stationary policies is reasonably small. The second method, called policy iteration, is generally more effective because it determines the optimum policy iteratively. Conversely, the second method was adopted for this research work, using Microsoft Excel Power Matrix, developed by Charles E. Ebelings [2001] of University of Dayton. However, Table 5.3, 5.4 and 5.5 were all generated using Microsoft Excel Power Matrix.

### Table 3: Matrix-P, raised [Iterated] to the power 5

| State of Nature | Objectives |
|----------------|-------------|
| Purposes        | Economic efficiency allow [Billion Naira] | Regional economy | State economic distribution | Social wellbeing | Environment |
| Navigation      | 0.63895267  | 0.08956123  | 0.185548887 | 0.051933549 | 0.03400349 |
| Tourism         | 0.64035577  | 0.08946399  | 0.184386119 | 0.051819485 | 0.03397448 |
| Flooding        | 0.64023006  | 0.0894737   | 0.184487805 | 0.051829778 | 0.03397854 |
| Hydropower      | 0.63800871  | 0.0896538   | 0.186260611 | 0.052012138 | 0.0340647  |
| Water supply    | 0.63871545  | 0.08964735  | 0.185564309 | 0.051957671 | 0.03411515 |

### Table 4: Matrix- P, raised to the power 10

| State of Nature | Objectives |
|----------------|-------------|
| Purposes        | Economic efficiency allow [Billion Naira] | Regional economy | State economic distribution | Social wellbeing | Environment |
| Navigation      | 0.63925821  | 0.08954401  | 0.18528533 | 0.051908977 | 0.03400324 |
| Tourism         | 0.63925672  | 0.08954411  | 0.185286596 | 0.0519091 | 0.03400326 |
| Flooding        | 0.63925689  | 0.08954411  | 0.185286492 | 0.051909089 | 0.03400326 |
| Hydropower      | 0.63925922  | 0.08954395  | 0.18528454  | 0.051908903 | 0.03400323 |
| Water supply    | 0.63925833  | 0.08954401  | 0.185285226 | 0.05190897 | 0.03400325 |
Table 5: Matrix- P, raised to the power 50

| State of Nature | Objectives |
|-----------------|------------|
| Purposes        | Economic efficiency allow[Billion Naira] | Regional economy | State economic distribution | Social wellbeing | Environment |
| Navigation      | 0.63925797 | 0.08954403 | 0.185285613 | 0.051908996 | 0.03400324 |
| Tourism         | 0.63925791 | 0.08954402 | 0.185285613 | 0.0519099 | 0.03400324 |
| Flooding        | 0.63925791 | 0.08954403 | 0.185285598 | 0.051908996 | 0.03400325 |
| Hydropower      | 0.63925797 | 0.08954403 | 0.185285613 | 0.0519099 | 0.03400324 |
| Water supply    | 0.63925791 | 0.08954402 | 0.185285613 | 0.051908996 | 0.03400325 |

Looking at each column [1-5] of Table 4&5, it appears to be the same i.e. the iteration has reached a steady state and can no longer change; this can also be called optimum solution or values.

Table 6: River Basin Allocation

| Purposes | Alloc | Percentage Allocation | Allocation based on N100 Billion Naira |
|----------|-------|------------------------|----------------------------------------|
| Economic efficiency allow[Billion Naira] | 0.63925797 | 64% | N63.925797b |
| Regional economy | 0.08954403 | 9% | N8.954403b |
| State economic distribution | 0.185285613 | 19% | N18.5285613b |
| Social wellbeing | 0.051908996 | 5% | N5.1908996b |
| Environment | 0.03400324 | 3% | N3.400324b |

Table 7: Purposes verses Allocations

| Objective | Allocation |
|-----------|------------|
| Economic efficiency allow[Billion Naira] | 63.925797 |
| Regional economy | 8.954403 |
| State economic distribution | 18.5285613 |
| Social wellbeing | 5.1908996 |
| Environment | 3.400324 |
Figure 1: River Basin Purposes Allocation in percentage
Figure 2: River Basin Purposes Allocation in Billions of Naira

5.0 Contingency Coefficient And Its Associates

Chi-Square (X²) Contingency Test: The Chi-square test is a measure of relationships, association or independence. Introduced by Karl Pearson in 1990, the chi-square test is probably the best known and the most important of all non parametric method. It involves a measure of reliability by comparing observed frequency distribution failure mode with theoretical or expected distribution failure when that hypothesis is false. Non-parametric tests process the advantage of being fairly robust with respect to violations of assumptions having more power-efficiency (the power of a test relative to the sample size which permits one to compare the power of two different statistical tests. The power of a statistical test is then probability that the test will correctly reject the null hypothesis when that hypothesis is false) and sometimes providing more information about a phenomenon (i.e. interactions in the analysis of variance).

There are five basic conditions that must be met for Chi-square analysis to be validly applied. These are (a) the sample observations are independent of each other (b) sample data are drawn at random from the population (c) Sample data are expressed in original unites. (d) The sample should contain at least 50 observations. (e) There
should be not less than five observations in any one cell. (f) Not more than 20% of the expected frequency should be less than 5.

The $X^2$ can be used to treat data which are classified into nominal, non-ordered categories; it can also be employed with numerical data. The researcher may wish, however to analyze such data with more powerful parametric test. But for nominal data, few alternatives to $X^2$ analysis exist. The basic computation equation for $X^2$ is given below:

$$X^2 = \sum \frac{(O - E)^2}{E}$$

Equation 19

It should be noted that whenever $X^2$ is calculated from (1 by 2) or (2 by 2) cell tables (instances in which the degree of freedom is one) an adjustment known as Yates correction for continuity must be employed. To use this correction a value of 0.5 is subtracted from the absolute value (irrespective of algebraic sign) of the numerator contribution of each cell.

$$X^2 = \sum \frac{(O - E)^2 - 0.5}{E}$$

Equation 19

4 Contingency Coefficient, $C$ is given by

$$C = \sqrt{\frac{X^2}{N + X^2}}$$

Equation 20

Where $C$ = Contingency Coefficient  
$X^2$ = Chi-square 
$N$ = Grand total of subjects or cases

5 Correlation of Attributes

The degree to which one of the attributes depend upon is associated with or related to the other attribute is referred to as correlation of attributes. In the $k \times k$ Contingency the correlation of attributes

$$r = \sqrt{\frac{X^2}{N(X - 1)}}$$

Equation 21

For a 2 X 2 table the correlation attribute is called tetra choric.

5.1 Contingency and Reliability Test

Contingency and reliability in this paper is another alternative method of testing null hypothesis, the paper assesses the relationship and test the null hypothesis on:

“There is a relationship between the Watershed Purposes and Objectives”
Table 8: Observed Contingency Table

| State of Nature | Course of Action |
|-----------------|-----------------|
|                 | Irrigation      | Hydropower | Water supply | Recreation | Erosion Control |
|                 | 2               | 0.3        | 0.89         | 0.2        | 0.1             |
|                 | 30.5            | 0.68       | 0.75         | 0.8        | 0.45            |
|                 | 20.3            | 1          | 0.65         | 0.75       | 0.35            |
|                 | 1.7             | 2          | 0.9          | 0.45       | 0.59            |
|                 | 1.4             | 0.75       | 0.8          | 0.35       | 0.74            |

Step I: Calculation of the expected contingency table using the formula:

\[ \text{Cell}_{ij} = \frac{\text{I}^{\text{th}} \text{Row Total} \times \text{J}^{\text{th}} \text{Column Total}}{\text{Grand Total}} \]

Where I = is the i\(^{\text{th}}\) and J = is the j\(^{\text{th}}\) column

Step II: Computation of Chi-square using the formula:

\[ \chi^2 = \frac{\sum (O - E)^2}{E} \]
Table 10: Chi-square Table

| O | E     | O-E | (O-E)^2 | (O-E)^2/E |
|---|-------|-----|---------|-----------|
| 2 | 2.81110951 | -0.8111 | 0.65789864 | 0.234035222 |
| 0.3 | 0.237863112 | 0.062137 | 0.00386099 | 0.016231995 |
| 0.89 | 0.200649856 | 0.68935 | 0.47520362 | 2.368322763 |
| 0.2 | 0.12823487 | 0.071765 | 0.00515023 | 0.040162507 |
| 0.1 | 0.112142651 | -0.01214 | 0.00014744 | 0.00131479 |
| 30.5 | 26.72567723 | 3.774323 | 14.2455123 | 0.533027179 |
| 0.68 | 2.261403458 | -1.5814 | 2.5008369 | 1.105878249 |
| 0.75 | 1.907610951 | -1.15761 | 1.34006311 | 0.702482397 |
| 0.8 | 1.219149856 | -0.41915 | 0.1756866 | 0.14410583 |
| 0.45 | 1.066158501 | -0.61616 | 0.3796513 | 0.356092737 |
| 20.3 | 18.56621037 | 1.73379 | 3.00602647 | 0.161908457 |
| 1 | 1.570987032 | -0.57099 | 0.32602619 | 0.207529524 |
| 0.65 | 1.325208934 | -0.67521 | 0.4559071 | 0.344026585 |
| 0.75 | 0.84693804 | -0.09694 | 0.00939698 | 0.011095243 |
| 0.35 | 0.74065562 | -0.39066 | 0.15261181 | 0.206049626 |
| 1.7 | 4.542881844 | -2.84288 | 8.08197718 | 1.779041907 |
| 2 | 0.384397695 | 1.615602 | 2.61017081 | 6.790287368 |
| 0.9 | 0.324259366 | 0.575741 | 0.33147728 | 1.022259686 |
| 0.45 | 0.207233429 | 0.242767 | 0.05893561 | 0.284392378 |
| 0.59 | 0.181227666 | 0.408772 | 0.16709482 | 0.922016076 |
| 1.4 | 3.254121037 | -1.85412 | 3.43776482 | 1.05643422 |
| 0.75 | 0.275348703 | 0.474651 | 0.22529385 | 0.818212873 |
| 0.8 | 0.232270893 | 0.567729 | 0.32231634 | 1.387674253 |
| 0.35 | 0.148443804 | 0.201556 | 0.0406249 | 0.273671915 |
| 0.74 | 0.129815562 | 0.610184 | 0.37232505 | 2.868109794 |

Contingency coefficient, C is given by

\[
C = \sqrt{\frac{X^2}{N + X^2}}
\]

Where C = Contingency Coefficient
X² = Chi-square
N = Grand total of subjects or cases

\[
X^2 = 23.63436176
\]

\[
N = 69.4
\]

\[
C = 0.504, \text{ the maximum Contingency coefficient can go is 0.8.}
\]

Therefore \( C = 0.653/0.8 \)

\( C = 0.63 \)
Correlation of attributes $r$, is given as:

$$r = \sqrt[2]{\frac{X^2}{n(n-1)}}$$

$$r = \sqrt[2]{\frac{23.63436176}{69.4(5-1)}}$$

$$r = 0.292 = 0.3$$

6.0 Presentation of Results.
The Contingency of the raw data is $= 0.63$. The correlation of attributes of the raw data $= 0.3$. The $X^2$ value 23.63436176 is interpreted from the $X^2$ table of probability values at 0.10 level of significance. The degree of freedom necessary to intercept $X^2$ values are always determined from the frequency table by the number of rows minus one the number of columns minus one $(r-1)(c-1)$ i.e. $(5-1)(5-1) = 16$.

-Since the obtained $X^2$ value of 23.63436176 is less than the critical value of 32.000, therefore the Alternate Hypotheses is accepted. i.e. $X^2(23.63436176) < X^2_{0.10}(32.000)$. Therefore the Alternate Hypothesis is accepted, a clear indication that there is a relationship between the watershed purposes and the Objectives/Benefits.

-Therefore there is relationship between the state of the system (Dam Purposes) and the Dam Objectives.

-The Chi Square was not based on a fictitious data, in the case of Markov Decision Modeling in Niger Delta River Basin.

6.1 analysis of Variance (Anovar)
The Pearson Product Moment Correlation Coefficient often referred to as the Pearson R tests, is a statistical formula that measures the strength between variables and relationships. To determine how strong the relationship is between two variables, you need to find the coefficient value, which can range between -1.00 and 1.00. The computations are done as shown in Table 11 using equation 23 and results displayed graphically in Figure 3.

$$r = \frac{\sum XY}{\sqrt{\sum (X^2)(Y^2)}}$$

The analysis of variance in this research work can be done using the following methods:

(i) Let consider one of the river basin Objectives, at 1st Iteration and 50th Iteration. Using Pearson Product Moment Correlation Coefficient on the River basin Objective Initial benefits values and the 50th Iteration benefits values, $r = 0.9851$, this infer a perfect positive relation between Initial Values and iterative values. of the river basin objectives under Navigation [See Table 11).

The initial benefits Iteration and 50th Iteration benefits values were correlated using Pearson moment correlation coefficient formula and $r$ was determined as 0.9851 in table 11 and the graph represented.

| R.B. Objectives     | Initial iteration Benefits | 50th Iteration Benefits | $x$ | $y$ | $x^2$ | $y^2$ |
|---------------------|----------------------------|-------------------------|-----|-----|-------|-------|
| Economic Efficiency | 0.573066                   | 0.639258                | 0.323066 | 0.389258 | 0.125756 | 0.104372 |
| Regional Distribution| 0.08596                    | 0.089544                | -0.16404 | -0.16046 | 0.026321 | 0.026909 |
| State distribution  | 0.255014                   | 0.185286                | 0.005014 | -0.06471 | -0.00032 | 2.51E-05 |
| Social Well-being   | 0.057307                   | 0.051909                | -0.19269 | -0.19809 | 0.038171 | 0.037131 |
| Environment         | 0.028653                   | 0.034003                | 0       | 0     | 0.189924 | 0.168437 |

| $r$                  | 0.9851                     |

Table 11: Initial Benefits values and 50th Iterative benefits value
6.2 Model Validation
The initial iteration and 50th iteration were plotted, as in column 2 and 3 of Table 11 for the validation of the model. Therefore $R = 0.985$

Figure 3: Relationship between Initial and projected values of River Basin Objectives.

Graph Equation: $y = 1.099x - 0.019$

$R^2 = 0.971$
Therefore $R = 0.985393$

7.0 Conclusion and Recommendation
Based on the findings and conclusions reached on the study the following recommendations are made:
Niger delta has more water available; therefore it is recommended that Hydropower in this region should be considered and encouraged because of its immediate and long term benefits when compared to gas powered electric plants. Also clean environment should be embraced for a healthy land, water and air; and in turn increase the level of tourism as well as reduces flooding caused by environmental abuse.

7.1 Contribution to Knowledge
The study can provide an organized baseline for future work, mainly in obtaining superior estimates for institutional water use and planning by the aid of Markovian decision theory. However, the findings of the study can be vital input into the demand management process for long term sustainable water supply within Niger Delta River Basin and beyond.

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