Numerical and experimental study of a weakly conducting fluid motion, driven by a strong inhomogeneous electric field

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Abstract. This work presents the results of numerical simulation and experimental studies of a weakly conducting fluid motion in a system of two electrodes “wire above the plane”.

1. Introduction

Deviations from the Ohm’s law for slightly ionized solid media in pre-breakdown uniform electric fields were experimentally discovered by Poole about 100 years ago [1]. In a weakly conductive liquid media the same experimental effect was obtained by M. Wien about 10 years later [2].

Theoretically this exponential effect for considerable number of media was explained by Frenkel for solid dielectrics and by Onsager [2] for liquid weak electrolytes and for weakly conductive liquid dielectrics. The space charge and electro-hydrodynamic (EHD) flows have been observed in these dielectrics at the pre-breakdown conditions [3].

The space charge formation, according to [4], occurs in the pre-breakdown fields until all the EHD characteristics become steady. Wherein steady conduction may be unipolar (corona discharge type) as well as quasi-neutral (plasmas or electrolyte type). The last was considered early in [4] and in the present work.

The pre-breakdown current-voltage theoretical and experimental characteristics of considerable media in non-uniform electric fields are described by us in [5]. Purpose of present work is researches of the electro-hydrodynamic flows, caused by these high non-uniform electric field. These intense flows are observed in transformer oil type liquids [6, 7] with complex molecular structure [8]. The hydrodynamic transfer of high voltage space charge, appeared in considerable liquids, is described in [9]. In the review [7] the surface high voltage electrode effects influence on considerable pre-breakdown electro-hydrodynamic flows is researched. This influence must be taken into account at plan high voltage electrodes. When applied high voltage field is non-uniform the volume effects influence on considerable flows is dominated.
2. Theoretical model of electro-hydrodynamic processes in the liquid insulator

The following inequalities are valid for bulk charge concentrations $n_\pm$, impurities concentrations $n_p$ and neutrals $n_a$ concentrations in a low-conducting medium:

$$
\begin{align*}
\begin{cases}
n_\pm & \ll n_a, \\
n_p & \ll n_a.
\end{cases}
\end{align*}
$$

(1)

The above relations can be considered as a condition for a weak ionization (dissociation) or low conductivity in the considered medium. The rates of volume ionization (or dissociation) and recombination are supposed to be known as the thermodynamics functions of the above mentioned concentrations, temperature and field intensity $|E|$. They can be represented in the following form

$$
W_I = W_I(n_a, n_p, T, |E|) = W_I(n_a, n_p, T, 0) r(|E|).
$$

(2)

Here $T$ is the absolute temperature. $|E|$ is the intensity of the electric field. The rate of the ions recombination is

$$
\begin{align*}
W_r &= K_r n_+ n_-, \\
K_r &= (b_+ + b_-) |e| Z \frac{1}{\varepsilon\varepsilon_0}.
\end{align*}
$$

(3)

Here $W_I$ denotes the rate of ionization (dissociation), $W_r$ is the rate of ion recombination, $b_+$ and $b_-$ are the mobilities of the ions. The expression for $K_r$ (recombination constant) was obtained by Langevin in 1903 and by Onsager in 1934 for particular case of weak electrolytes. This is the two-particle ion-ion recombination, when the energy excess is absorbed by a medium.

Onsager had shown weak influence of sub-breakdown electrical fields on $K_r$ when direct and reverse processes are ionization and recombination correspondingly. But in the case when these processes are dissociation and reverse dissociation the influence of $E$ on $K_r$ can not be neglected. Besides weak electrolytes, the Langevin’s formula is valid for dense gases with chemical reactions, where ionic conductivity is much greater than electronic one. Corresponding conductivity occurs due to various processes. Among them are the neutral molecules (with ionic chemical bonds) dissociation and the neutral molecules (with covalent chemical bonds) ionization. The latter goes through the electron ionization from some neutral particle with relatively low ionization potential and attaching it to another with relatively high electron affinity.

For the ion diffusion coefficient we have used more known Einstein-Nernst relation:

$$
ZD_\pm = \frac{k_B T b_\pm}{|e|},
$$

(4)

where $k_B$ is the Boltzmann constant, $T$ is supposed to be a constant and equals to 300 $K$, because the Joule heat is small for the pre-breakdown phenomena.

Function $f(|E|)$ describes the dependence of the ionization (dissociation) rate on the electric field intensity. The expression, describing it, was obtained by Frenkel for the solid dielectric and generalized by Ostroumov in [3] for the case when $Z > 1$ with ion chemical bonds in molecules.

It was used in the present research, and has the form

$$
f(E) = \exp \left(\frac{\beta |E|}{\sqrt{\varepsilon\varepsilon_0 k_B T}} \right),
\quad \beta = \frac{|Z e|^{3/2}}{\sqrt{\pi \varepsilon\varepsilon_0 k_B T}}.
$$

(5)

This function $f$ of the high field shock ionization rate is not well known for gases.

The equations, describing creation and annihilation of the space charge, high voltage conductivity and the electric field distribution can be written as [4]

$$
\begin{align*}
\begin{cases}
\frac{\partial q}{\partial t} + (\vec{V}, \nabla q) - \frac{k_B T b}{|Ze|} \Delta q + (\vec{E}, \nabla \sigma) = -\frac{q \sigma}{\varepsilon \varepsilon_0}, \\
\frac{\partial \sigma}{\partial t} + (\vec{V}, \nabla \sigma) + b^2 (\vec{E}, \nabla q) - \frac{k_B T b \Delta \sigma}{|Ze|} \frac{\sigma}{Z} - \frac{\sigma^2}{\varepsilon \varepsilon_0} \exp \left(\frac{\beta |E|}{\sqrt{\varepsilon\varepsilon_0}} \right) + \frac{\sigma^2}{\varepsilon \varepsilon_0} = 0,
\end{cases}
\end{align*}
$$

(6)
here $b$ is the mobility of the neutrals, $\phi$ is the potential. And in this case $b_+ = b_- = b$ (this supposition is more correct for liquids than for gases).

The well-known hydrodynamics equations should be added to equations (6) to construct the closed system of equations. The first of them is the continuity equation for incompressible media:

$$\text{div} \vec{V} = 0.$$  \hspace{1cm} (7)

We used the equation (7) to obtain electrodynamic equations (6). For our purpose it is enough to consider the law of momentum conservation without considering the law of energy conservation. The momentum conservation equation can be written as

$$\begin{cases}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + qE_x, \\
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + qE_y,
\end{cases}$$  \hspace{1cm} (8)

The initial conditions for the system of equations (6)–(8) are

$$q(t = 0) = V(t = 0) = 0, \quad \sigma(t = 0) = \sigma_0.$$  \hspace{1cm} (9)

The boundary conditions. Condition of adhesion ($u = 0, v = 0$) are observed on the surface of the wire-electrode ($\sqrt{x^2 + y^2} = r_0$) and the counter-electrode. On the other boundaries there are free outflow conditions and $q = 0$.

The other parameters are represented as:

$Z = 2$ (according to Professor V G Arakelyan),

$\varepsilon = 2.5$,

$T = 300$ K,

$\rho = 880$ kg/m$^3$,

$\nu = 0.2 - 22.5$ cSt (additional notes about viscosity will be presented in the text further),

$\sigma_0 = 10^{-13}$ Ohm$^{-1}$cm$^{-1}$.

Additional description and formula you can find in [10].

3. Numerical calculations

Numerical calculations of the isothermal two-dimensional system of equations (7)–(9) were carried out for the “wire above the plane” configuration of electrodes mentioned above. The need to determine the maximum speed of the fluid is important task due to the cooling problem of low-power non-pressure 10 kV transformers. We believe that the voltage inside such a transformer is proportional to the generated one and can reach value of 500 V and higher.

The method applied and solving system of equations was described in detail earlier [11]. In short, both explicit and implicit versions of used MacCormack method allow one to solve hyperbolic and parabolic equations for a positive time step, while exhibiting good dissipative and dispersive boundary properties. The ease of application of the McCormack method, high efficiency and accuracy of the solution are very important properties of this numerical method. Calculations, mentioned in this paper, were carried out for the wire with square cross-section. This was done because the transverse dimension of the wire is much smaller than the linear dimensions of the calculation region. This region has to be not less than the distance from the wire to the flat electrode, and, in its turn, should be quite large to minimize the influence of boundary conditions on the calculation. As a compromise on the speed of calculation and flow details outside the region of wire electrode such a grid was chosen, in which the size of each side of the wire electrode equals to 8 computational cells.

When conducting full-scale experiments and in reality the cross section of wire electrode is a circle. There is a question: how reliable are the simulation results with the square-shape electrode and how much are they differ from the simulation results with a round-shape electrode?
Figure 1. Distribution of the axial velocity between the electrodes in the electrode system “wire above the plane” in the axisymmetric electro-hydrodynamic flow of a weakly conducting liquid. Voltage between the electrodes equals to 500 V. Inter-electrode distance equals to 2 cm. Different configuration of the wire cross-section and various fineness of the calculation grid are presented.

To determine this, four calculations were carried out: two on a coarse grid, with a wire of square and “round” (as far as possible) cross-sections, and two on a finer grid with a similar sectional view. The calculations were carried out until the flow was completely established.

Coarse grid parameters: $100 \times 100$ cells, cell size equals to 0.0005 m or 0.5 mm. Parameters of the finer grid: $500 \times 500$ cells, cell size equals to 0.0001 m or 0.1 mm.

Figure 1 shows the axial velocities of fluid motion along the segment, connecting the electrode-wire and the electrode-plane. The distance between two electrodes equals to 20 mm. Viscosity of oil equals to 0.0198 Pas – real viscosity of GK oil at 20°C. It can be seen that the result has not changed qualitatively, but the shape of the electrode affects the quantitative result quite strongly. The maximum flow speed on the straight line connecting the electrodes increases with increasing fineness of the grid. In addition, when calculating on a fine grid in the region of a square-section electrode, the reverse flow is noticeable at a distance of 18 to 20 mm from the flat electrode (shown by the arrow in figures 1 and 2 a). Besides, the maximum of fluid velocity is greater if the shape of the wire cross-section is close to rounded. On the figures 2 a) and b) you can see differences between square and round electrodes in velocity field. The streamlines are presented only on the left side of the figure so that the velocity field on the right side can be seen more clearly.

4. Experiment

The experimental setup is shown on a figure 3. In a container (1) filled with transformer oil (2), two electrodes are placed at a distance of $l = 20$ mm from each other: a copper plate with a width of $D = 120$ mm (3) and a copper wire (4) with a diameter of $d = 2$ mm ($d \ll D$). In order to visualize the flow arising under the action of an electric field in the volume between the electrodes, the oil was seeded with aluminum powder with a characteristic particle size in it $d_p = 10 \div 50 \mu$m. Illumination of particles suspended in the oil was carried out by the “laser knife” method, which allows to isolate a thin layer of oil in the required plane and to ensure the possibility of obtaining a high-quality image of particles that are suspended in this layer.
Figure 2. Streamlines and magnitude of the velocity for a weakly conducting liquid in an electric field. The cross section of the wire-electrode is square (a) and round (b). Voltage between the electrodes equals to 500 V. The calculation grid has a size of 500 × 500 cells.

Figure 3. Schematics of the experimental setup.

A “laser knife” with a thickness of Δ = 1 mm was formed using an optical system from a laser beam with a wavelength \( \lambda = 527 \text{ nm} \) generated by a pulsed laser (5) with a frequency \( \nu = 15 \text{ kHz} \). The optical system also includes a telescope and a lens to open the beam into the plane. Video recording of the movement of aluminum particles was performed using a Canon EOS 560D camera with a frame rate of 50 Hz on a scale of \( M = 1 : 2 \) (with a resolution of 0.0078 mm/pixel). Observation and recording is done from above.

We have not seen any movement when the applied voltage between two electrodes was 500 V in experiment. Significant visual movement began when we applied voltage 3000 V and acetone was added to the oil in a quantity of 5% by weight. At the moment we have not figured out what it is connected with: with decrease in oil viscosity or with increase in the formed space charge. Figure 4 a) shows the axial velocity of fluid motion between the electrode-wire and the electrode-plane obtained in experiment with the voltage between the electrodes equals to
Figure 4. a) Distribution of the axial velocity between the electrodes in the electrode system “wire above the plane” in the axisymmetric electro-hydrodynamic flow of a weakly conducting liquid. Voltage between the electrodes equals to 3000 V. b) Streamlines and magnitude of the velocity under the round electrode in numerical calculation. Voltage between the electrodes equals to 3000 V. The calculation grid has a size of 500 × 500 cells.

Figure 5. Vortexes in oil during experiment a) and numerical calculation b).

3000 V. Suggesting that with the introduction of acetone the viscosity of the oil decreased, we carried out a calculation on the fine grid (500 × 500 cells) with the viscosity of the oil equals to 0.00016 Pas (this is the real viscosity of the transformer oil in a work regime). The axial velocity from the numerical calculation you can see on the same figure. The convex and concavity parts of the red line on the right-hand side of the figure are associated with the reverse flow from below of the electrode, which can be seen in figure 4 b). It can be seen that the maximum axial velocities are approximately equal to each other and the addition of acetone may really reduce the viscosity.

On a figure 5 a) you can see tracks in streams of this mixture forming two vortexes on either sides of the line connecting two electrodes. This picture was obtained with exposition of 1
second. In calculations carried out earlier (figure 2) with the free flow boundary conditions we have not seen such a vortexes. If use the adhesion conditions not only on the flat electrode, but also on the side walls of the computational domain, you can observe these two vortexes (5 b). The streamlines have a differences in their pattern on the left and right sides of the figure 5 b). This was done so to see the reverse flown under the electrode more clearly. The stream under the electrode is symmetrical indeed. So the formation of this vortexes is connected with presence of two side walls of container with oil.

It is worth to note that during the experiments, breakdowns between the electrodes or other parts of the installation have not been observed.

5. Conclusion
1) This work shows how fineness of the calculation grid and the cross-section of the wire electrode influence on the weakly conducting liquid flow in the inter-electrode space. Flow structures and velocity distribution are obtained both for square and round shape of the wire electrode. The maximum axial velocity between the electrodes increases if the wire with the round cross-section is considered. Besides that, it is shown that this maximum axial velocity increases while using finer calculation grid.

2) Significant visual movement of oil begins with the addition of acetone into the oil. This can be connected with the increasing of its conductivity or the decreasing of oil viscosity. Assuming the last, the velocity distributions between the two electrodes were obtained from experiment and calculation. It turned out that the maximum velocities are approximately equal to each other.

3) Formation of two vortexes between the electrodes is connected with the presence of side walls.

Acknowledgements
This work was supported by the Russian Foundation for Basic Research (RFBR) (project no. 18-08-00136).

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