Charge asymmetry of pions in the process $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$

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Received: December 7, 2000

Abstract. The study of the charge asymmetry of produced particles allows to investigate the interference of different production mechanisms and to determine new features of the corresponding amplitudes. In the process $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$ the two–pion system is produced via two mechanisms: two–photon (C–even state) and bremsstrahlung (C–odd state) production. We study the charge asymmetry of pions in a differential in the pion momenta cross section originating from an interference between these two mechanisms. At low effective mass of dipions this asymmetry is directly related to the difference of $s^+$ and $p^-$ phases of elastic $\pi\pi$ scattering. At higher energies it can give new information about the dipion threshold the asymmetry is of the order of 1%. We show that with suitable cuts the signal to background ratio can be increased up to about 10%.

1 Introduction

The study of charge asymmetry in particle production can be used as an essential source of information about production amplitudes which is difficult to obtain otherwise. In this paper we discuss the charge asymmetry of pions produced in the reaction $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$. The pion pair (dipion) in this process is produced mainly via the two–photon mechanism (Fig. 1) or via bremsstrahlung (Fig. 2).

The first of them gives charge parity even (C–even) dipions, while the second mechanism leads to C–odd pion pairs. In the differential cross section the interference of these mechanisms results in terms which are antisymmetric under $\pi^+ \leftrightarrow \pi^-$ exchange. Certainly, these terms disappear in the total cross section or after a suitable averaging. Nevertheless, they are observable and could give new information about the production amplitudes which cannot be obtained unambiguously by other approaches. In particular, at low effective masses of dipions $W \sim 2m_\pi$ this interference is directly related to the difference of $s^+$ and $p^-$–phase shifts of the elastic $\pi\pi$ scattering. These phase shifts are of primary importance for low energy hadron physics (in particular, for the chiral dynamics which pretends to be the low energy QCD). At higher energies the interference can give new information about the $f_0(400–1200)$ meson (former $\sigma$), the $f_0(980)$ meson, etc. Their nature is now subject of wide discussions.

The opportunity using C–odd effects for such problems was firstly studied almost three decades ago in Ref. [3]. In that paper the case of the small total transverse momenta of the produced pion pair, $k_T^2 \ll m_\pi^2$, was considered. However, this region gives only a small fraction of the entire charge asymmetry discussed. In this paper we obtain formulae which allow to study the charge asymmetry in the main kinematical region, $k_T \lesssim W$, and discuss the main features of the effect and the background.

Fig. 1. Amplitude $\mathcal{M}_1$ for the two–photon production of pions. The $e^-$ and $e^+$ with initial 4–momenta (energies) $p_1$ ($E_1$) and $p_2$ ($E_2$) and final momenta (energies) $p'_1$ ($E'_1$) and $p'_2$ ($E'_2$) emit virtual photons with $q_i = p_i - p'_i$ ($\omega_i = E_i - E'_i$). These photons produce the C–even $\pi^+\pi^-$ system with total 4–momentum $k = p_+ + p_- = q_1 + q_2$ and effective mass $W = \sqrt{k^2}$, furthermore $s = (p_1 + p_2)^2 = 4E_1E_2$. 

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Recently the process $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ was considered in Ref. [1] for the case when the virtuality of one photon is large, $-q_1^2 \gg W^2$. The authors concentrate their efforts on a QCD analysis of the differential cross section in the $\gamma^*\gamma$ collision in that region (leading to a factorization of perturbative QCD subprocesses and a generalized two-meson distribution amplitude). The developed description of the process at $e^+e^-$ colliders also includes the interference of two-photon and bremsstrahlung production mechanisms. Naturally, the estimates for possible number of events of that work are considerably lower than those in the main region considered in the present paper. Note that the discussed charge asymmetry was observed at CLEO in the $e^+e^- \rightarrow e^+e^-\pi\pi$ reaction detecting additionally an electron scattered at large angle [3]. The effect of interest can be studied on $e^+e^-$ colliders with c.m. energies above 1 GeV. We show that the charge asymmetry is of the order of 1% and that the signal to background ratio can be considerably improved with suitable cuts. Let us emphasize that in the method of data preparation suggested in the present paper, the considered asymmetries can be obtained from the data independent on the uncertainty in calculating the background.

We consider an experimental set-up when only pion momenta $p_+$ and $p_-$ are measured (the so called no tag experiments). This set-up corresponds to the cross section $d\sigma/(d^3p_+d^3p_-)$ for the process $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$. Besides, our results are also valid for the single tag experiments in which additionally the scattered electron is recorded, and where an averaging is performed over the small unbalance of transverse momentum of the scattered electron and the dipion.

The outline of our paper is as follows: First we present a qualitative description of different contributions to the reaction $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$. In Sect. 2 the necessary variables are defined and the basic formulae are presented. The amplitude of the subprocess $\gamma^*\gamma \rightarrow \pi^+\pi^-$ is represented via helicity amplitudes in a model independent way. The charge asymmetry of pions is calculated in Sect. 3. The obtained result [see Eqs. (28),(29) and (31)] is given in a simple analytical form. To discuss the background problems more accurately, we present in Sect. 4 approximate formulae for the two-photon and bremsstrahlung production in the kinematical region which is essential for the charge asymmetry. In Sect. 5 we present an approximate description of the $\gamma^*\gamma \rightarrow \pi^+\pi^-$ subprocess entering the two-photon amplitude. To get an idea about the potentiality of future experiments we perform a numerical analysis in Sect. 6 restricting ourselves to the QED model (point–like pions) for the amplitudes which gives a reasonable description at $W < 1$ GeV. We present several important distributions and estimate the background. Additionally we study the charge asymmetry of muons in the process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ (Sect. 6). In the final Section we summarize our results. Details of the calculations are presented in the Appendices.

2 Qualitative description of different contributions

The main contribution to the cross section of the process can be written via the amplitudes $M_j$ shown in Figs. 2. It is a sum of $C$–even, $C$–odd and interference contributions:

$$d\sigma = d\sigma_{C=+1} + d\sigma_{C=-1} + d\sigma_{\text{interf}},$$

where

$$d\sigma_{C=+1} \propto |M_1|^2,$$

$$d\sigma_{C=-1} = d\sigma_2 + d\sigma_3 \propto |M_2|^2 + |M_3|^2,$$

$$d\sigma_{\text{interf}} = d\sigma_{12} + d\sigma_{13},$$

$$d\sigma_{12} \propto 2 \text{Re}(M_2^*M_1), \quad d\sigma_{13} \propto 2 \text{Re}(M_3^*M_1).$$

Let us discuss these contributions qualitatively. In the head–on collisions of the leptons the $z$–axis is chosen along the initial electron momentum.
The two–photon mechanism (Fig. 1) produces C–even dipions. It provides the main contribution to the total cross section. The corresponding part of the cross section \(d\sigma_{C^{\pm+}}\) can be expressed via the amplitudes \(M_{\pm}\) describing the collisions of virtual photons with helicities \(a\) and \(b\) (\(a, b = \pm 1, 0\)). Its dominant part is given by almost real photons which have virtualities \(q_1^2\) and \(q_2^2\) close to zero (small transfer momenta squared of electrons and positrons). The produced pairs are distributed almost uniformly over their total rapidity and peaked at small values of their total transverse momentum \(k_\perp\) (for details see review [9]). The corresponding part of the cross section is given by an almost real photon \(q_2\) and \(q_2\) due to opposite charge asymmetry of pions \(q_1\) and \(q_2\). It provides the main contribution to the total charge asymmetry of pions \(F\), for the two–photon and bremsstrahlung production. For example, taking into account the described features of the \(M_{\pm}\) distribution in the main regions and, therefore, do not give a logarithmic enhancement (contrary to the considered diagrams). As a result, the contributions of the \(s\)–channel (annihilation) diagrams and their interference with those of Figs. 1, 2, 3 are suppressed by a factor \(W^2/(sL)\) where \(L \sim 10 \div 15\) is a typical logarithm.

The bremsstrahlung mechanism (Fig. 2) produces pion pairs in \(C\)–odd state. Its contribution to the cross section \(d\sigma_{C^{\pm-}}\) was calculated in Ref. 9. It is proportional to \(|F_\pi(k^2)|^2\) where \(F_\pi\) is the pion form factor. The main contribution to \(d\sigma_{C^{\pm-}}\) is given by the region where the exchange photon with momentum \(q_2\) is almost real. In that domain the sum of energy and longitudinal momentum of the produced pair is close to that of the initial electron, while the transverse momentum of the pair \(k_\perp\) is very small.

However, in the kinematical region being essential for both two–photon production and interference, the \(k_\perp\) distribution of the pions is relatively wide. The interference between bremsstrahlung by an electron (amplitude \(M_2\)) and by a positron (\(M_3\)) is negligible small. Note that both contributions \(d\sigma_{C^{\pm+}}\) and \(d\sigma_{C^{\pm-}}\) are charge symmetric, they do not change under pion exchange \(\pi^+ \leftrightarrow \pi^-\).

The interference of \(C\)–even and \(C\)–odd contributions \(d\sigma_{\text{interf}} = d\sigma_{12} + d\sigma_{13}\) [see Eq. (3)] is antisymmetric under pion exchange \(\pi^+ \leftrightarrow \pi^-\) due to opposite charge parities of dipion states produced by two–photon and bremsstrahlung mechanisms. Therefore, this interference determine the charge asymmetry of pions, i.e.

\[
\frac{1}{2} \left[ d\sigma(p_+, p_-, \ldots) - d\sigma(p_-, p_+, \ldots) \right].
\]

To discuss that asymmetry, it is useful to introduce the operator of charge conjugation of pions \(\hat{C}_\pi\) by its action on an arbitrary function \(F(p_+, p_-)\):

\[
\hat{C}_\pi F(p_+, p_-) = F(p_-, p_+).
\]

In particular, we have

\[
\hat{C}_\pi d\sigma_{C^{\pm+}} = d\sigma_{C^{\pm-}}, \quad \hat{C}_\pi d\sigma_{\text{interf}} = -d\sigma_{\text{interf}}.
\]

Many features of this interference can be naturally explained taking into account the described features of the two–photon and bremsstrahlung production. For example, the main contribution to \(d\sigma_{12}\) is given by an almost real photon \(q_2\) (small transfer momentum squared of the positron). The produced pions fly mainly along the electron, \(k_\perp = p_1 + p_2 > 0\), and the dipion transverse momentum distribution is not peaked at small \(k_\perp\). Therefore the transverse momentum of the electron is not small, \(q_{1\perp} \approx k_\perp\). Similarly, the main contribution to \(d\sigma_{13}\) is given by the almost real photon \(q_1\) (small transfer momentum squared of the electron), whereas the transverse momentum of the positron is not small, \(q_{2\perp} \approx k_\perp\). The produced pions fly mainly along the positron, \(k_\perp < 0\).

Certainly, the sign of the observed effect is different for \(d\sigma_{12}\) (related to bremsstrahlung production of pions by an electron with negative charge) and for \(d\sigma_{13}\) (related to bremsstrahlung production of pions by a positron with positive charge). Therefore, some details of the asymmetry are different for \(e^+e^-\) and \(e^-e^-\) collisions.

The \(s\)–channel contributions. In the previous discussion we did not consider the additional set of diagrams which can be obtained from those in Figs. 1, 2 interchanging the outgoing electron by the incoming positron \((p_i' \leftrightarrow -p_2)\), etc. The contributions of those diagrams contain an additional factor \(1/s\) due to the photon propagator. Besides, the final electron and positron have a wide angular distribution in the main regions and, therefore, do not give a logarithmic enhancement (contrary to the considered diagrams). As a result, the contributions of the \(s\)–channel (annihilation) diagrams and their interference with those of Figs. 1, 2, 3 are suppressed by a factor \(W^2/(sL)\) where \(L \sim 10 \div 15\) is a typical logarithm.

3 Basic notations and general formulæ

The main notations are given in Figs. 1 and 2. As already mentioned, we consider the head–on collisions of electrons and positrons with the \(z\)–axis along the initial electron momentum. In this frame \(p_i = (E_i, 0, 0, \pm \sqrt{E_i^2 - m_e^2})\), \(i = 1, 2\). The virtual photon momenta for the two–photon production of Fig. 2 are \(q_i = p_i - p_i' = (\omega_i, q_{i\perp}, q_{i\parallel})\) with

\[
q_i^2 = 2q_i p_i - \frac{q_{i\perp}^2 + m_e^2 (\omega_i / E_i)^2}{1 - (\omega_i / E_i)} < 0.
\]

The 4–momenta of the produced pions are given by \(p_{\pm} = (\epsilon_{\pm}, p_{\pm\perp}, p_{\pm\parallel})\) with \(p^2_{\pm} = \mu^2\). Below we use the quantities

\[
x_{\pm} = \frac{\epsilon_{\pm} + p_{\pm\parallel}}{2E_1} = \frac{p_{\pm} p_2}{p_1 p_2}, \quad y_{\pm} = \frac{\epsilon_{\pm} - p_{\pm\parallel}}{2E_2} = \frac{p_{\pm} p_1}{p_2 p_1}.
\]

For ultra–relativistic pions flying along the initial electron or positron momentum, the quantity \(x_{\pm} (y_{\pm})\) is the fraction of energy transferred from the electron (positron) to \(\pi^\pm\). The variables \(x_{\pm} (y_{\pm})\) appear in the cross section \(d\sigma_{12}\) (\(d\sigma_{13}\)).

In what follows, we consider the symmetric and antisymmetric combinations of the pion momenta:

\[
k = p_+ - p_-, \quad \Delta = p_+ - p_-. \]

The pion charge conjugation operator leads in particular to

\[
\hat{C}_\pi \Delta^\mu = -\Delta^\mu.
\]

Therefore, the asymmetry effects are proportional to the components of the 4–vector \(\Delta\).
To describe that asymmetry we use the variables

\[ \xi = \frac{x_+ - x_-}{x}, \quad \eta = \frac{y_+ - y_-}{y}, \quad K_- = \frac{(p_2 - p_1) \Delta}{(p_2 + p_1) k}, \quad v = p_2 \perp - p_1 \perp = k \perp \Delta \perp. \]

The “transverse” variable \( v \) is a natural variable both for contributions \( d\sigma_{12} \) and \( d\sigma_{13} \). The “longitudinal” variable \( \xi \) naturally arises in describing the contribution \( d\sigma_{12} \) (whereas \( \eta \) – in describing \( d\sigma_{13} \)). The symmetric variable \( K_- \) is suitable to discuss the sum \( d\sigma_{12} + d\sigma_{13} \). Note that \( K_- \) is proportional to the difference of the longitudinal momenta of \( \pi^+ \) and \( \pi^- \) in the \( e^-e^+ \) center-of-mass system

\[ K_- = \xi \eta \approx y \quad \text{and} \quad K_- = -\eta \approx y. \]

The amplitude \( M_1 \) of the two–photon production is written via the amplitude \( M^{\mu\nu} \) of the subprocess \( \gamma^+ \gamma^- \to \pi^+ \pi^- \) as

\[ M_1 = \frac{4\pi\alpha}{q_1^2 q_2^2} (\bar{u}_1 \gamma_\mu u_1) (\bar{v}_2 \gamma_\nu v_2) M^{\mu\nu}, \quad \text{(9)} \]

where the bispinors \( u_1 \) (\( u_1' \)) and \( v_2 \) (\( v_2' \)) correspond to the initial (final) electron and positron, respectively.

Instead of the amplitude \( M^{\mu\nu} \) it is more convenient to use the helicity amplitudes \( M_{ab} \) which can be introduced via

\[ M_{\mu\nu} = \sum_{ab=\pm 1,0} (-1)^{a+b} e^{(a)(b)}_{\mu\nu} M_{ab}. \quad \text{(10)} \]

Here \( e^{(a)(b)}_{\mu\nu} \) is the polarization vector of the \( j \)-th virtual photon with helicity \( a = \pm 1, 0 \). A virtual photon is called transverse if its helicity is equal to \( \pm 1 \) and scalar (longitudinal) for zero helicity. Since the amplitude \( M^{\mu\nu} \) is \( C \)-even, the vectors of transverse and scalar polarization are \( C \)-odd and \( C \)-even, respectively (see Appendix [3] for details):

\[ \hat{C}_\pi M^{\mu\nu} = M^{\mu\nu}, \quad \hat{C}_\pi e^{(\pm 1)}_{\mu\nu} = -e^{(\pm 1)}_{\mu\nu}, \quad \hat{C}_\pi e^{(0)}_{\mu\nu} = e^{(0)}_{\mu\nu}, \quad \text{(11)} \]

we get

\[ \hat{C}_\pi M_{0\pm} = -M_{0\pm}, \quad \hat{C}_\pi M_{\pm 0} = M_{\pm 0}. \quad \text{(12)} \]

It is important that the amplitudes \( M_{ab} \) with scalar photons disappear near the photon mass shell

\[ M_{0\pm} \propto \sqrt{-q_1^2}, \quad M_{\pm 0} \propto \sqrt{-q_2^2}, \quad M_{00} \propto \sqrt{q_1^2 q_2^2} \quad \text{at} \quad q_{1,2}^2 \to 0. \quad \text{(13)} \]

So the amplitude \( M_1 \) of the two–photon production can be represented in the form

\[ M_1 = \frac{4\pi\alpha}{q_1^2 q_2^2} \sum_{ab=\pm 1,0} (-1)^{a+b} \times \left( \bar{u}_1 e^{(a)*}_{1} u_1 \right) \left( \bar{v}_2 e^{(b)*}_{2} v_2 \right) M_{ab}. \quad \text{(14)} \]

In a similar way the amplitude \( M_2 \) of the bremsstrahlung production by an electron can be written as

\[ M_2 = \frac{(4\pi\alpha)^2}{q_2^2 k^2} F_\pi(k^2) \times \sum_{c=\pm 1,0} (-1)^c \left( \bar{u}_1 \hat{C}^{(c)} u_1 \right) \left( \bar{v}_2 \hat{e}^{(c)*}_{2} v_2 \right), \quad \text{(15)} \]

where the quantity \( \bar{u}_1 \hat{C}^{(c)} u_1 \) corresponds to the amplitude of the virtual Compton scattering shown in Fig. [3]. \( F_\pi(k^2) \) is the pion form factor.

As a result, the interference of the amplitudes of \( M_1 \) and \( M_2 \) is given by the expression

\[ d\sigma_{12} = 2 \text{Re}(M_{12} M_{21}^{\ast}) \frac{d\Gamma}{2s}, \quad \text{(16)} \]

\[ = 2 \sum_{ab=\pm 1,0} \text{Re} \left( F_\pi M_{ab} \bar{e}^{b}_{2} C^{ac}_{1} \right) \frac{d\Gamma}{2s}, \quad \text{(17)} \]

the (non-normalized) density matrix of the second virtual photon is

\[ \rho_2^{bc} = \frac{(-1)^{b+c}}{2(-q_2^2)} \left( \hat{p}_2 - m_e \right) e^{(b)*}_{2} \left( \hat{p}_2 - m_e \right) e^{(c)}_{2}, \quad \text{(18)} \]

and

\[ C^{ac}_{1} = \frac{(-1)^a}{2} \text{Tr} \left( \hat{p}_1 + m_e \right) e^{(a)*}_{1} \left( \hat{p}_1 + m_e \right) e^{(c)*}_{1}. \quad \text{(19)} \]

4 The charge asymmetry

Let us remind the reader that the charge asymmetry is determined by the two contributions \( d\sigma_{12} \) and \( d\sigma_{13} \) arising from the interference of the two–photon diagram of Fig. [3] with bremsstrahlung diagrams of Figs. [3] [see Eq. (3)]. Calculating that asymmetry we limit ourselves to logarithmic accuracy (which is about 5% in our case).

First, we discuss the contribution \( d\sigma_{12} \). According to the qualitative description in Sect. [3] the main contribution to \( d\sigma_{12} \) is given by very small values of \( -q_2^2 \).
Therefore, the second virtual photon can be considered as almost real. Taking into account Eq. (13) we can use in all expressions
\[ q_2^2 = 0, \quad q_{1\perp} = 0, \quad q_{1,\perp} = k_{\perp}, \quad q_1^2 = -\frac{k_{\perp}^2}{1-x}, \]
(20)
except for the propagator of that second photon in the matrix element \( M_1 \) [Eqs. (10,11)]. This has the consequence that in that limit the amplitudes \( M_{ab} \) with \( b = 0 \) can be safely neglected [see Eq. (13)].

To obtain \( d\sigma_{12}/(d^3p_+d^3p_-) \), we transform the phase volume \( \{ \} \) to the form
\[ d\Gamma = \frac{d^2q_{1\perp}}{2(2\pi)^3 (E_1 - \omega_1)(E_2 - \omega_2)} \frac{d^3p_+ d^3p_-}{\epsilon_+ \epsilon_-}, \]
(21)
and integrate either over \( q_{2\perp} \) or \( q_2^2 \) and \( \varphi_2 \). The latter is the azimuthal angle of vector \( q_{2\perp} \). After integrating over \( \varphi_2 \) the non–diagonal elements of the \( \rho_{22}^2 \) matrix disappear and the final result contains \( \rho_{1\perp}^2 = \rho_{2\perp}^2 \) only (i.e. \( b = c = \pm 1 \))
\[
\int \rho_2^b \frac{d^2q_{2\perp}^b}{|q_{2\perp}^b|} d\varphi_2 = 2\pi \delta_{bc} \rho_{1\perp}^2 L_2,
\]
\[ L_2 = \int \frac{d^2q_{2\perp}^b}{|q_{2\perp}^b|} = \ln \frac{|q_{2\perp}^b|_{\text{max}}}{|q_{2\perp}^b|_{\text{min}}}. \]
\[ y_2 = \frac{2q_2 p_1}{s} = \frac{\omega_2}{E_2} = \frac{W^2(1-x) + k_2^2}{sx(1-x)}, \]
(22)
With the considered logarithmic accuracy the upper limit is determined by a scale at which the integrand (besides the photon propagator) starts to decrease significantly. For the pion pair production this leads to
\[ |q_{2\perp}^b|_{\text{max}} \sim \min \left\{ -\frac{k_2^2}{1-y_2}, m_{\rho}^2, W^2 \right\} \]
(23)
where \( m_{\rho} \) is the \( \rho \) meson mass which is the natural scale of the form factors. As a result, we have
\[ d\sigma_{12} = -\frac{\alpha^3}{32\pi^6} \frac{\rho_{1\perp}^2}{s^3 W^2 k_2^2} L_2 \Re(F^*_1T) d^3p_+ d^3p_- \]
\[ \frac{1}{\epsilon_+ \epsilon_-}, \]
\[ T = \sum_{ab} M_{ab} C_1^{ab}. \]
(24)
The calculation of trace (19) which determines \( C_1^{ab} \) is given in Appendix B.

To present the contribution \( \{ \} \) in a compact form, it is useful to introduce the auxiliary vector \( r_1 \) and the angle \( \phi \) between this vector and the vector \( k_{\perp} \) via
\[ r_{1\perp} = \frac{1}{2} \left( \Delta_{\perp} - \xi k_{\perp} \right) = \frac{x-p_{+\perp}}{x} - \frac{p_{-\perp}}{x}, \]
\[ r_{1\perp} k_{\perp} = |r_{1\perp}| |k_{\perp}| \cos \phi, \]
(25)
\[ \] The angle \( \phi \) is also the azimuthal angle between the vectors \( p_+ \) and \( -p_1 \) in the \( \gamma^* \gamma \) c.m.s.

and use the dimensionless quantities
\[ z_r = \frac{|r_{1\perp}|}{\mu}, \]
\[ z_k = \frac{|k_{\perp}|}{\mu}, \]
\[ d = 1 - x + \frac{(1 - \xi^2)z_k^2}{4(1 + z_r^2)}. \]
(26)
In these notations
\[ W^2 = 4\mu^2 \frac{1 + z_r^2}{1 - \xi^2}, \quad \rho_{1\perp}^2 = -\mu^2 \frac{z_k^2}{1 - x}, \]
\[ y_2 = \frac{4\mu^2}{s} \frac{(1 + z_r^2)d}{x(1-x)(1 - \xi^2)}, \]
\[ d^3p_+ d^3p_- \quad \frac{1}{\epsilon_+ \epsilon_-} = 4\mu^4 z_k dz_k z_r dz_r d\phi \frac{dx}{x} \frac{d\xi}{1 - \xi^2} \]
We obtain the following expression for the interference contribution \( d\sigma_{12} \):
\[ d\sigma_{12} = \left[ G_{++} \Re(F^*_1M_{++}) + G_{+-} \Re(F^*_1M_{+-}) + G_{0+} \Re(F^*_1M_{0+}) \right] d^3p_+ d^3p_- \]
\[ \frac{1}{\epsilon_+ \epsilon_-}, \]
\[ G_{ab} = \frac{\alpha^3}{8\pi^6} \rho_{1\perp} z_k L_2 \frac{g_{ab}}{s^3 W^2 x z_k d}, \]
\[ g_{ab} = \sum_{n=0}^{3} g_{n}^{ab} \cos (n\phi), \]
(28)
The nonzero coefficients \( g_{n}^{ab} \) are

\[ g_{0}^{++} = (2-x)\xi z_k, \]
\[ g_{1}^{++} = (2-x)z_r - \frac{2 - 2x + x^2}{1 - x} z_r d, \]
\[ g_{1}^{++} = - (2 - 2x + x^2)z_r, \]
\[ g_{2}^{++} = - (2 - x)\xi z_k, \]
\[ g_{3}^{++} = 2(d - 1 + x)z_r, \]
\[ g_{0}^{0+} = z_r (2-x) \sqrt{2(1-x)}, \]
\[ g_{1}^{0+} = -2 \xi z_k \sqrt{2(1-x)}, \]
\[ g_{2}^{0+} = - \frac{2(2-x)}{\sqrt{2(1-x)}} (d - 1 + x) z_r. \]
(29)
Let us briefly discuss that result.
We note that
\[ \frac{\rho_{1\perp}}{s^2} \approx \frac{1}{(s y)^2} = \frac{x(1-x)}{[W^2(1-x) + k_2^2]^2}. \]
Therefore, the effect under discussion does not decrease with growing \( s \), as one could imagine from a first look at Eq. (25).
The coefficient \( g^{0+} \) is of the same order as \( g^{++} \) and \( g^{+-} \). Near the mass shell the amplitude \( M_{0+} \propto \sqrt{-q_1^2} \sim \)
As [photon] $k_\perp$ [see Eqs. (13), (20)]. However, in the main region for the charge asymmetry the total transverse momentum of pion system $k_\perp$ is not small. Therefore, the contribution of the amplitude $M_{0+}$ to the interference is roughly of the same order of magnitude as the contributions of the other amplitudes.

The $\pi^+ \leftrightarrow \pi^-$ exchange is realized by $\xi \rightarrow -\xi$ and $\phi \rightarrow \pi - \phi$ replacements. As it was discussed above, the contribution $d\sigma_{12}/(d^3p_+d3p_-)$ is C–odd, i.e. it changes its sign under this exchange. Indeed, the coefficients $g^{\pi+}$ and $g^{\pi-}$ alter their signs while $q^{0+}$ remains unchanged. On the other hand, the amplitudes $M_{++}$ and $M_{+-}$ are unchanged (C–even) while $M_{0+}$ changes its sign (C–odd) [see Eq. (12)].

According to Eqs. (28,29) the contribution of amplitude $M_{+-}$ disappears after averaging over the azimuthal angle $\phi$ (this fact is explained in Appendix I):

$$
\langle d\sigma_{12} \rangle = -g^{\pi+} \text{Re} \left(F^*_\pi M_{++} - g^{\pi-} \text{Re} \left(F^*_\pi M_{0+}\right)\right) = -2(1-x) \times \left[\xi_\pm \text{Re} \left(F^*_\pi M_{++}\right) + z_\pm \sqrt{2(1-x)} \text{Re} \left(F^*_\pi M_{0+}\right)\right].
$$

For small transverse momentum of the produced pair $k_\perp \rightarrow 0$ our result for $\langle d\sigma_{12} \rangle$ coincides with that of Ref. [3] (see Appendix I). In this region the asymmetry in $\xi$ is negligible small compared with that in $v$.

We have obtained our equations in the dominant region of the effect $k_\perp^2 \sim -q_2^2 < W^2$. However, our sum $\sum g^{ab} \text{Re} \left(F^*_\pi M_{ab}\right)$ entering Eq. (28) coincides with the corresponding expression from Ref. [3] despite the fact that the latter was obtained in the quite different kinematical region (at $-q_2^2 > W^2$).

The term $d\sigma_{13}$ is obtained from the presented formulae using the substitution rules (see Appendix I for details)

$$
\langle d\sigma_{13} \rangle = \frac{\alpha^2}{32\pi^5 k_\perp^2} (T_A + T_B) \frac{d^3p_+ d^3p_-}{\varepsilon_+ \varepsilon_-}.
$$

Here $p_2^{bd}$ is defined in Eq. (15) and $g^{a0}_2$ is given by a similar expression with the evident changes of the other amplitudes.

The further calculations repeat those in Sect. I. At fixed value of $k_\perp$ there are two regions (A) and (B) where either $q_3^2 \approx 0$ or $q_1^2 \approx 0$. Those regions give the dominant contributions in logarithmic approximation

$$
\langle d\sigma_{C=+1} \rangle = \frac{\alpha^2}{32\pi^5 k_\perp^2} (T_A + T_B) \frac{d^3p_+ d^3p_-}{\varepsilon_+ \varepsilon_-}.
$$

In region (A) we can use Eqs. (29,30) which leads to

$$
T_A = \frac{p_2^{++}}{(s_2)^2} L_2 \sum_{n=0}^2 T_n \cos(n\phi).
$$

The coefficients $T_n$ are of the form

$$
T_0 = \frac{1}{2} \left(|M_{++}|^2 + |M_{+-}|^2\right) \left(1 - x + \frac{1}{2}x^2\right) + |M_{0+}|^2 (1 - x),
$$

$$
T_1 = -2 \text{Re} \left[(M_{+-}^\ast - M_{++}^\ast) M_{0+}\right] (2 - x) \sqrt{1 - x \over 2},
$$

$$
T_2 = -2 \text{Re} \left(M_{+-}^\ast M_{++}\right) (1 - x).
$$

The helicity amplitudes $M_{ab}$ have be taken in the limit

$$
M_{ab}(q_1, q_2, \Delta) \rightarrow (-1)^{\alpha+b} M_{ba}(q_1, q_2, \Delta).
$$

5 Two–photon and bremsstrahlung background

Let us present now the necessary formulae for the two– photon $d\sigma_1$ and bremsstrahlung $d\sigma_2$ contributions to the pion pair production which are background for the considered asymmetry.

The differential cross section of the two–photon contribution can be written via the helicity amplitudes $M_{ab}$ (10) as

$$
\langle d\sigma_{C=+1} \rangle = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} \sum_{abcd=\pm 1,0} M^\ast_{cd} M_{ab} g^{ac}_1 g^{bd}_2 \frac{d\Gamma}{2s}. \tag{32}
$$

For the bremsstrahlung contribution $d\sigma_{C=-1} = d\sigma_2 + d\sigma_3$ we use the results of Ref. I. For pions flying along electrons, the dominant contribution is given by the amplitude $M_2$ taken in the limit (20). In logarithmic approximation we have

$$
\langle d\sigma_{2} \rangle = \frac{|M_2|^2}{2s} \frac{d\Gamma}{2s} = \frac{\alpha^4}{8\pi^3} |F_x|^2 \times \frac{x^2 \rho^2_+}{W^2 (W^2 (1-x) + k_\perp^2 + m^2_\pi^2)} L T^- \frac{d^3p_+ d^3p_-}{\varepsilon_+ \varepsilon_-},
$$

$$
T^- = \left(1 - \frac{4m^2_\pi^2}{W^2}\right) \left[\frac{y^2 + \left(y_2 - \frac{W^2}{s}\right)^2}{s} - y^2 \eta^2 - (\eta y + y_2 x \xi)^2\right],
$$

$$
L = \ln \frac{s^2 (1-x)(1-y_2)}{m^2_\pi^2 [W^2 + s y_2 (1-x)]}. 
$$

In region (B) we have

$$
\langle d\sigma_{C=+1} \rangle = \frac{\alpha^2}{32\pi^5 k_\perp^2} (T_A + T_B) \frac{d^3p_+ d^3p_-}{\varepsilon_+ \varepsilon_-}.
$$

Here $p_2^{bd}$ is defined in Eq. (15) and $g^{a0}_2$ is given by a similar expression with the evident changes of the other amplitudes.
The denominator $|W^2(1-x) + k_1^2 + m_2^2x^2|^2$ shows that this contribution is dominated by small $k_1^2$ and large $x$ (i.e. $1-x \ll 1$).

For pions flying along positrons the corresponding contribution $d\sigma_3$ is obtained via $d\sigma_3 = d\sigma_2(x \leftrightarrow y, y_2 \leftrightarrow x_1, \xi \leftrightarrow \eta)$.

6 The $\gamma^*\gamma \to \pi^+\pi^-$ subprocess

For the pion pair production the strong interaction effects are of primary interest. Nevertheless, the pure Born QED model (point–like pions) gives a reasonable description of the squared two–photon amplitude at $W \lesssim 1$ GeV. On the contrary, the QED amplitude itself is real whereas the QED amplitude for the two–photon production with total helicity $0$. Since $xy = (W^2 + k_1^2)/s$, we can introduce the characteristic value

$$x_0 = \sqrt{\frac{W^2 + k_1^2}{s}}$$

and find that the total longitudinal momentum of the pion pair in the c.m.s. of the colliding electrons and positrons $k(p_2 - p_1)/\sqrt{s} \propto x - y$ is positive at $x > x_0$ and negative at $x < x_0$. Therefore, $|d\sigma_{12}| \gg |d\sigma_{13}|$ at $x \gg x_0$ and $|d\sigma_{13}| \gg |d\sigma_{12}|$ at $x \ll x_0$.

In the region $x \sim y \sim x_0$ both contributions are of the same order, $|d\sigma_{12}| \sim |d\sigma_{13}|$, but their distributions over the longitudinal variable $K_-$ and the transverse variable $v$ have different properties. Indeed, $K_-$ is antisymmetric whereas $v$ is symmetric under $p_1 \leftrightarrow p_2$ exchange. Having in mind relation (31), we conclude that

$$\frac{d\sigma_{13}}{dK_-} = \frac{d\sigma_{12}}{dK_-}(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2', q_1 \leftrightarrow q_2),$$

$$\frac{d\sigma_{13}}{dv} = -\frac{d\sigma_{12}}{dv}(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2', q_1 \leftrightarrow q_2).$$

To take into account the described properties and to summarize different contributions, it is natural to introduce the following quantities related to the charge asymmetry of pions.

$$\Delta \sigma_K = \int_D \epsilon(K_-)d\sigma, \quad \Delta \sigma_v = \int_D \epsilon(v)\epsilon(x-y)d\sigma. \quad (41)$$

In these definitions we denote by $D$ the kinematical region in phase space given by the detector array and suitably chosen cuts (certainly, it is necessary to test that this region is symmetric in $K_-$ and $v$).

The background for these effects is found integrating the two–photon and bremsstrahlung contributions over the same region $D$:

$$\Delta \sigma_B = \int_D (d\sigma_{C=+1} + d\sigma_{C=-1}). \quad (42)$$

7 Analysis of results

7.1 Studied quantities

As it was discussed before, the contribution $d\sigma_{12}$ dominates for pion pairs flying along the initial electron, i.e. at large values of $x$. On the other hand, $d\sigma_{13}$ dominates for dipions flying along the initial positron, i.e. at large values of $y$. Since $xy = (W^2 + k_1^2)/s$, we can introduce the characteristic value

$$x_0 = \sqrt{\frac{W^2 + k_1^2}{s}}$$

and find that the total longitudinal momentum of the pion pair in the c.m.s. of the colliding electrons and positrons $k(p_2 - p_1)/\sqrt{s} \propto x - y$ is positive at $x > x_0$ and negative at $x < x_0$. Therefore, $|d\sigma_{12}| \gg |d\sigma_{13}|$ at $x \gg x_0$ and $|d\sigma_{13}| \gg |d\sigma_{12}|$ at $x \ll x_0$.

In the region $x \sim y \sim x_0$ both contributions are of the same order, $|d\sigma_{12}| \sim |d\sigma_{13}|$, but their distributions over the longitudinal variable $K_-$ and the transverse variable $v$ have different properties. Indeed, $K_-$ is antisymmetric whereas $v$ is symmetric under $p_1 \leftrightarrow p_2$ exchange. Having in mind relation (31), we conclude that

$$\frac{d\sigma_{13}}{dK_-} = \frac{d\sigma_{12}}{dK_-}(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2', q_1 \leftrightarrow q_2),$$

$$\frac{d\sigma_{13}}{dv} = -\frac{d\sigma_{12}}{dv}(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2', q_1 \leftrightarrow q_2).$$

To take into account the described properties and to summarize different contributions, it is natural to introduce the following quantities related to the charge asymmetry of pions:

$$\Delta \sigma_K = \int_D \epsilon(K_-)d\sigma, \quad \Delta \sigma_v = \int_D \epsilon(v)\epsilon(x-y)d\sigma. \quad (41)$$

In these definitions we denote by $D$ the kinematical region in phase space given by the detector array and suitably chosen cuts (certainly, it is necessary to test that this region is symmetric in $K_-$ and $v$).

The background for these effects is found integrating the two–photon and bremsstrahlung contributions over the same region $D$:

$$\Delta \sigma_B = \int_D (d\sigma_{C=+1} + d\sigma_{C=-1}). \quad (42)$$

7.2 Numerical analysis

Below we consider the charge asymmetry effects within the QED model of point–like pions. First, we present in Table 1 the integrated pion charge charge asymmetry in the $e^+e^-$ c.m.s.
the variables $K_-$ and $v$ (signals $S$) and the background of two–photon and bremsstrahlung production (background $B$) at different c.m. energies $\sqrt{s}$.

Using the charge asymmetry one can study the two–photon amplitude and its phase shift as function of the effective mass of the $\pi^+\pi^-$ system $W = \sqrt{k^2}$. To see the potentiality of such a study, we present in Fig. 4 the distribution of the signal and the background over $W$.

![Figure 4](image)

**Fig. 4.** Contributions $\Delta \sigma_K$ and $\Delta \sigma_v$ (1) and background at $\sqrt{s} = 1$ and 10 GeV vs. $W$.

the background over $W$ for two collider energies. Both signal and background are concentrated near the threshold where the longitudinal asymmetry $|d\Delta \sigma_K/dW|$ is considerably larger than the transverse one $|d\Delta \sigma_v/dW|$. At $W > 400 \div 500$ MeV (depending on $\sqrt{s}$) the transverse asymmetry dominates over the longitudinal one. Nevertheless, having in mind the results for

Contribution of the signal and the background over $W$ for two collider energies. Both signal and background are concentrated near the threshold where the longitudinal asymmetry $|d\Delta \sigma_K/dW|$ is considerably larger than the transverse one $|d\Delta \sigma_v/dW|$. At $W > 400 \div 500$ MeV (depending on $\sqrt{s}$) the transverse asymmetry dominates over the longitudinal one. Nevertheless, having in mind the results for

### Table 1. Pion charge asymmetry signals and background at different c.m. energies

| $\sqrt{s}$, GeV | 1   | 4   | 10  | 200 |
|-----------------|-----|-----|-----|-----|
| $\Delta \sigma_K$, pb | -6.1 | -26 | -35 | -56 |
| $\Delta \sigma_v$, pb  | 3.3  | 17  | 27  | 51  |
| $\Delta \sigma_B$, pb  | 420  | 2900 | 6200 | 27000 |

We consider two typical intervals of effective mass values (over which we integrate):

1. $W = 300 \div 350$ MeV — near the threshold where QED is approximately valid,
2. $W = 475 \div 525$ MeV — far from the threshold and resonances where one hopes to describe the modules of the two–photon amplitudes reasonably within QED whereas the bremsstrahlung amplitude is enhanced compared to its QED value due to the $\rho$ meson resonance. For this region we expect that our numbers underestimate the effect.

The signal/background ratio (S/B) is introduced as

$$
\frac{S}{B} = \frac{|\Delta \sigma_S|}{\Delta \sigma_B}
$$

with $S = K_-, v$. (43)

Besides, it is useful to consider the statistical significance (SS) of the effect. This quantity is expressed via the number of events for the effect $N_S = \mathcal{L}|\Delta \sigma_S|$ and background $N_B = \mathcal{L}\Delta \sigma_B$ as

$$
SS = \frac{N_S}{\sqrt{N_B}}
$$

where $\mathcal{L}$ is the integrated luminosity of the collider. For the luminosity we use numbers proposed for the DAΦNE and PEP II colliders. We now demonstrate that the S/B and SS quantities can be considerably improved with suitable cuts on $k_\perp$ and $x$.

The variable $k_\perp^2$ describes both the transverse motion of the dipion and the virtuality of the photon. The charge asymmetry effect (28),(29) vanishes at small $|k_\perp|$, $\Delta \sigma_{12} \propto d|k_\perp|$. On the contrary, the two–photon contribution is singular at $|k_\perp| = 0$, see Eq. (36). In Fig. 4 we present signal and background integrated over $k_\perp > k_0$ (with $k_0$ being a cut–off from below). One observes that with increasing $k_0$ the background drops considerably faster than the signal. Therefore, some cut at small $k_\perp^2 = k_0^2$ is desirable, the best cut on the pair transverse momentum $k_0$ depends on $s$ and $W$.

The variable $x$ describes the dipion motion along the collision axis. The factor $W^2 d \equiv W^2 (1-x) + k_\perp^2$ in the denominator of Eq. (28) shows that in the interference the dipions tend to be concentrated at $x \sim 1$. On the contrary, in the two–photon production the $x$–distribution of the dipions is proportional to $1/x$. Therefore, some cut at not too low $x$ would be desirable. On the other hand, the bremsstrahlung contribution is concentrated near $x = 1$ more strongly than the charge asymmetry contribution. Moreover, the values of $x$ very close to 1 contribute only weakly to the charge asymmetry. Therefore, an additional cut at $x \sim 1$ is suggested. In Fig. 5 these features are demonstrated for signal and background contributions integrated over $x > x_0$.

To show the effect of both cuts we present in Table 2 some examples considering cuts in $k_\perp$ and two symmetrical regions in pion rapidity (contributions of pions flying along initial electron momentum $x_1 > x > x_2$ and initial positron momentum $x_1 > y > x_2$). For the DAΦNE collider we consider the region of small effective mass of
I.F. Ginzburg et al.: Charge asymmetry of pions in the process $e^-e^+ \rightarrow e^-e^+\pi^+\pi^-$

Table 2. Effect of cuts in $k_\perp$ and $x, y$

| $\sqrt{s}$ (GeV) | $k_\perp$ (MeV) | $W$ (MeV) | cuts | $\Delta\sigma_B$ (pb) | $\Delta\sigma_K$ (pb) | $S/B$ | SS
|-----------------|-----------------|--------|-----------------|-----------------|-----------------|-------|-------|
| 1               | 5               | 300    | no cuts         | 145             | -1.85           | 1.3   | 11    |
| DA\øNE         | $\div$         | $k_\perp > 100$ MeV, | 14.6          | -1.07           | 7.3             | 20    |
|                 | 350             | 0.4 < $x, y < 0.9$  |               |                 |                 |       |
| 10              | 30              | 475    | no cuts         | 433             | -3.13           | 0.72  | 8.2   |
| PEP-II          | $\div$         | $k_\perp > 150$ MeV, | 17.2          | -1.62           | 9.5             | 68    |
|                 | 525             | 0.3 < $x, y < 0.95$ |               |                 |                 |       |

Fig. 5. Contributions $\Delta\sigma_K$ and background at $\sqrt{s} = 1$ and 10 GeV for the interval $W = 300 \div 350$ MeV and $W = 475 \div 525$ MeV and $k_\perp > k_0$ vs. $k_0$.

Fig. 6. The contributions $\Delta\sigma_K$ and background at $\sqrt{s} = 1$ and 10 GeV for the intervals $W = 300 \div 350$ MeV and $W = 475 \div 525$ MeV and $x > x_{0}$ vs. $x_{0}$.

dipion $W = 300 \div 350$ MeV. The used cuts improve the $S/B$ by a factor about 5 and the SS – by a factor about 2. For the PEP-II collider we consider an intermediate mass region $W = 475 \div 525$ MeV. The used cuts improve both the $S/B$ and the SS by about one order of magnitude. It is natural to expect that the same type of improvement will take place at $W \sim 1$ GeV.

For the physical analysis of the results it is useful to consider the individual contributions of different helicity amplitudes $M_{\alpha\beta}$ to the charge asymmetry. The results for the longitudinal asymmetry $d\Delta\sigma_K/dW$ are shown in Fig. 5. In this distribution the amplitude $M_{++}$ is dominant whereas $M_{+-}$ contributes only weakly to the asymmetry (in accordance with the discussion at the end of Sect. 4). The last contribution can be even stronger suppressed, excluding pion pairs with small longitudinal momentum in the $e^+e^-$ c.m.s. additionally. Therefore, the distribution over $K_-$ allows us to obtain a clean information about the amplitude $M_{++}$.

In the transverse distribution $d\Delta\sigma_v/dW$ (Fig. 8) the contribution of the $M_{++}$ and $M_{+-}$ amplitudes are of the
same order, partially compensating each other. Therefore, the combined distributions over $K_-$ and $v$ can give complementary knowledge about individual contributions of the $M_{++}$ and $M_{+-}$ helicity amplitudes.

Finally let us notice, that in the $K_-$-distribution the contribution of the amplitude with one scalar photon $M_{0+}$ is not negligible.

### 7.3 Weighted cross sections

For future studies it is useful to look at the presented analysis from a more general point of view. The distributions of produced pions contain a charge asymmetric part. To extract it, we consider each event with the $C$-odd weight function $\epsilon(K_-)$ or $\epsilon(v)$. So, our asymmetry $\Delta\sigma_K$ and $\Delta\sigma_v$, defined in Eqs. (41) can be considered as "weighted cross sections" with these weights. Certainly, to extract the asymmetric part of the cross section, one can use also other $C$-odd weight functions. It seems to be attractive to explore more smooth weight functions, for example, given by factors $K_-$ and $v$ instead of $\epsilon(K_-)$ and $\epsilon(v)$, i.e. to introduce weighted cross sections

$$\Delta\sigma_K^C = \int_D K_- \, d\sigma, \quad \Delta\sigma_v^C = \int_D v \, \epsilon(x-y) \, d\sigma.$$  

They (or similar quantities) may be more suitable for a theoretical analysis, generalizations and data processing. In particular, the signs of small differences $p_{+z} - p_{-z}$ or $p_{+\perp} - p_{-\perp}$ cannot be reliably established from the data. However, the proposed weighted cross sections (45) are weakly sensitive to those small values.

The background for these "cross sections" is given by the total weighted cross section of the process in the same kinematical region,

$$\Delta\sigma_{BK}^C = \int_D |K_-| \, (d\sigma_{C=+1} + d\sigma_{C=-1}) ,$$

$$\Delta\sigma_{Bv}^C = \int_D |v| \, (d\sigma_{C=+1} + d\sigma_{C=-1}) .$$

### 8 Process $e^-e^+ \rightarrow e^-e^+\mu^+\mu^-$

The process $e^-e^+ \rightarrow e^-e^+\mu^+\mu^-$ can give an essential background while studying the the dipion production.
The charge asymmetry of muons in this process has been studied for the first time at small $k_\perp$ in Ref. [3]. The muon asymmetry without that limitation was obtained in Ref. [11] (in the same logarithmic accuracy as it is used here). We use these results as given in review [11] and transform them to a form convenient for analysis in notations (25), (26) (where $K$ and $L$ are used for the replacement of the same order of magnitude.

(iii) the longitudinal asymmetries for pions and muons are larger than that for pions (see Table 1); change is absent for pions. In summary, we consider just the same regions of production as a possible background for the pion production. Moreover, in some regions the longitudinal asymmetry of muons disappear contrary to that that of pions. In Table 3 we present the distributions similar to those given in Figs. 3—5 for pions. Having in mind the muon production as a possible background for the pion production, we consider just the same regions of $W$ as used for pions. It is interesting to note that at $\sqrt{s} = 10$ GeV the longitudinal asymmetry $d\Delta\sigma_K/dW$ (Fig. 4) changes its sign at $W \approx 500$ MeV. The position of this crossover depends on $\sqrt{s}$ (cf. the curve for $\sqrt{s} = 1$ GeV). Such a sign change is absent for pions.

9 Summary and conclusions

We have calculated the charge asymmetry contribution $d\sigma_{\text{int}}/(d^3p_+d^3p_-)$ for the process $e^-e^+ \rightarrow e^+e^-\pi^+\pi^-$. Our result summarized in Eqs. (25), (29), (31) is expressed via the helicity amplitudes $M_{\pi^+}$, $M_{\pi^-}$ and $M_{\pi^0}$ of the subprocess $\gamma^+\gamma \rightarrow \pi^+\pi^-$ and the pion form factor $F_\pi$ in analytical form. It can be used for experiments both with and without recording the scattered electrons (no tag experiments)

### Table 3. Muon pair charge asymmetry signals and background at different c.m. energies

| $\sqrt{s}$, GeV | 1  | 4  | 10 | 200 |
|-----------------|----|----|----|-----|
| $\Delta\sigma_K$, pb | -11 | 7.9 | 42 | 120 |
| $\Delta\sigma_{\nu}$, pb | 180 | 660 | 950 | 1700 |
| $\Delta\sigma_{\pi}$, pb | 7100 | $3.7 \times 10^4$ | $6.9 \times 10^4$ | $3.7 \times 10^5$ |

Given in Figs. 3—5 for pions. Having in mind the muon production as a possible background for the pion production, we consider just the same regions of $W$ as used for pions. It is interesting to note that at $\sqrt{s} = 10$ GeV the longitudinal asymmetry $d\Delta\sigma_K/dW$ (Fig. 4) changes its sign at $W \approx 500$ MeV. The position of this crossover depends on $\sqrt{s}$ (cf. the curve for $\sqrt{s} = 1$ GeV). Such a sign change is absent for pions.
The contribution Fig. 10. obtained in the QED model are typically around 1%. However, with the expected high luminosity of B– and φ– factories the statistical significance of the effect SS (13) is high enough (typically about 10) for the considered ideal case when all dipions are assumed to be recorded. The strongly different dependence of signal and background on the components of the total dipion momentum results in a large improvement of both S/B and SS (by about a factor 10 for $W \sim 0.5 \div 1$ GeV at the PEP-II) at suitable cuts in the total transverse momentum of pion pair $k_{\perp}$ and its total rapidity (see Table 3). With these estimates the perspectives of experiments look well.

The charge asymmetry in the resonance region will be considered elsewhere. Preliminary estimates indicate that the effect will be observable. Moreover, the quick change of the phase near the resonance results in a strong dependence of the asymmetry on $W$ in this region what could help to distinguish resonance models.

Finally note that the presented equations for the muon charge asymmetry (17) can be used to estimate the production of $c$ quarks in the process $e^+e^- \rightarrow e^+e^-c\bar{c}$ with production of open charm. The deviation from the QED result will be caused by violation of quark–hadron duality due to strong interactions. These deviations are expected to be large near the threshold where the $c\bar{c}$ resonance production is essential in both $p$– and $s$–waves. Simple QED calculations with an additional charge factor 8/9 give $\Delta \sigma_{\mu} \sim 0.27$ pb at $\sqrt{s} = 10$ GeV and about 4.1 pb at $\sqrt{s} = 200$ GeV (here we put $c$–quark mass of 1.75 GeV to take into account a threshold for the production of the open charm).

We are very grateful to A. Bondar, V. Fadin, I. Ivanov, E. Kuraev, V. Savinov, V. Serebryakov, W. Speth and V. Zhilich for useful discussions. This work was partially supported by grants RFBR 00-02-17592, 00-15-96691, “Universities of Russia” 015.0201.16, and Sankt-Petersburg Center of Higher education. V.G.S. acknowledges a support of Sächsisches Staatsministerium für Kunst und Wissenschaft, grant 4-7531.50-04-0361-00/25 and kind hospitality of NTZ of Leipzig university.

**A Kinematics of the process $\gamma^+\gamma^* \rightarrow \pi^+\pi^-$**

Due to 4–momentum conservation $q_1 + q_2 = p_+ + p_-$, the amplitude $M^{\mu\nu}$ of the process $\gamma^*\gamma^* \rightarrow \pi^+\pi^-$ depends only on three independent momenta $q_1, q_2, \Delta = p_+ - p_-$. We
use the Mandelstam variables
\[ W^2 = (q_1 + q_2)^2, \quad t = (q_1 - p_+)^2, \quad u = (q_1 - p_-)^2 \]
with the relations
\[ W^2 + t + u = 2q_1^2 + q_1^2, \quad t - u = -2q_1 \Delta = 2q_2 \Delta. \]

The polarization properties of the virtual photons are described by two polarization 4–vectors \( e_1^{(a)} \) and \( e_2^{(b)} \) with helicities \( a, b = \pm 1 \), \( 0 \). For scalar (longitudinal) photons \((a = b = 0)\) we use the following polarization vectors
\[ e_{1\mu}^{(0)} = \sqrt{-\frac{q_2}{X}} \left( q_{2\mu} - \frac{q_2 q_1}{q_1^2} q_{1\mu} \right), \]
\[ e_{2\mu}^{(0)} = \sqrt{-\frac{q_2}{X}} \left( q_{1\mu} - \frac{q_2 q_1}{q_1^2} q_{2\mu} \right), \quad (A.1) \]
The transverse photons \((a, b = \pm 1)\) can be described by two independent polarization vectors only. This choice can be realized by taking those vectors in the \( \gamma^*\gamma^* \) center–of–mass system in the form
\[ e_{1\mu}^{(\pm)} = e_{2\mu}^{(\mp)} = \pm \frac{1}{\sqrt{2}} (0, 1, \pm i, 0). \quad (A.2) \]

To construct the corresponding covariant expression, we introduce a metric tensor of a subspace which is orthogonal to the 4–vectors \( q_1 \) and \( q_2 \)
\[ R_{\mu\nu} = g_{\mu\nu} - \frac{1}{X} \left[ (q_1 q_2) (q_{1\mu} q_{2\nu} + q_{1\nu} q_{2\mu}) - q_2 q_2 q_{2\mu} q_{2\nu} - q_1 q_1 q_{1\mu} q_{1\nu} \right] \]
and define two 4–vectors in that subspace (both of them are antisymmetric under \( p_+ \leftrightarrow p_- \) exchange)
\[ r_{\mu} = \frac{1}{2} R^{\mu\nu} \Delta_{\nu}, \quad s_{\mu} = \varepsilon^{\mu\nu\alpha\beta} \Delta_{\nu} q_{1\alpha} q_{2\beta}, \quad r_{\mu} s_{\mu} = 0. \quad (A.3) \]
In the \( \gamma^*\gamma^* \) c.m.s. the tensor \( R_{\mu\nu} \) has only two nonzero components \( R_{xx} = R_{yy} = -1 \). Both 4–vectors \( r_{\mu} \) and \( s_{\mu} \) have only nonzero components in \( x \) and \( y \) directions and are perpendicular to each other\[6\]. Therefore, we can choose the \( x \)– and \( y \)–axes along these 4–vectors:
\[ e_{\mu}^{(x)} = \frac{r_{\mu}}{\sqrt{-r^2}}, \quad e_{\mu}^{(y)} = \frac{s_{\mu}}{\sqrt{-s^2}}. \quad (A.4) \]
In the \( \gamma^*\gamma^* \) c.m.s. \( e^{(x)} = (0, 1, 0, 0), e^{(y)} = (0, 0, 1, 0) \). As a result, the covariant expression for the vectors \((A.2)\) takes the form
\[ e_{1\mu}^{(\pm)} = e_{2\mu}^{(\mp)} = \pm \frac{1}{\sqrt{2}} \left( e_{\mu}^{(x)} \pm ie_{\mu}^{(y)} \right). \quad (A.5) \]
We define the helicity amplitudes for the discussed process as
\[ M_{ab} = e_{\mu}^{(a)} e_2^{(b)} M^{\mu\nu}, \]
where the inverse transformation is given in Eq. (10). Taking into account parity conservation, we obtain
\[ M_{++} = M_{--}, \quad M_{+-} = M_{-+}, \]
\[ M_{0+} = -M_{0-}, \quad M_{+0} = -M_{-0}. \quad (A.6) \]
Since under \( \pi^+ \leftrightarrow \pi^- \) exchange the \( C \)–even amplitude \( M^{\mu\nu} \) is symmetric whereas the 4–vectors \( e_{1,2}^{(\pm)} \) are antisymmetric [cf. Eq. (1)], the amplitudes \( M_{++}, M_{+-} \) and \( M_{00} \) are symmetric and the amplitudes \( M_{0+}, M_{+0} \) are antisymmetric under that exchange [see Eq. (12)].

Under photon exchange \((q_1 \leftrightarrow q_2)\) the polarization vectors have to be replaced by \( e_{1\mu}^{(0)} \rightarrow e_{2\mu}^{(0)}, e_{1\mu}^{(x)} \rightarrow e_{2\mu}^{(x)}, e_{1\mu}^{(y)} \rightarrow -e_{2\mu}^{(y)}, e_{1\mu}^{(\pm)} \rightarrow -e_{2\mu}^{(\mp)} \) what can be short-hand written as
\[ e_{1\mu}^{(a)} \rightarrow (-1)^a e_{2\mu}^{(a)}. \]
Taking into account \( M^{\mu\nu}(q_1, q_2, \Delta) = M^{\mu\nu}(q_2, q_1, \Delta) \), we obtain
\[ M_{ab}(q_1, q_2, \Delta) \rightarrow (-1)^{a+b} M_{ba}(q_1, q_2, \Delta). \quad (A.7) \]

It is useful to present these amplitudes for the pure QED case (point–like pions)
\[ M_{++++}^{QED} = 8 \rho - M_{++}^{QED}, \]
\[ M_{--}^{QED} = -8 \rho \frac{q_1^2 (W^2 - q_2^2)}{(u - \mu^2)(u - \mu^2)}, \quad (A.8) \]
\[ M_{0+}^{QED} = -2 \rho \frac{2q_1^2 q_2^2 (t - u)(W^2 + q_2^2 - q_1^2)}{(u - \mu^2)(u - \mu^2)}, \]
\[ M_{+0}^{QED} = 2 \rho \frac{2q_1^2 q_2^2 (t - u)(W^2 + q_1^2 - q_2^2)}{(u - \mu^2)(u - \mu^2)}. \]
with
\[ r^2 = \frac{W^2}{4X} \left[ (t - \mu^2)(u - \mu^2) - q_1^2 q_2^2 \right] + \mu^2 < 0. \quad (A.9) \]
In the \( \gamma^*\gamma^* \) c.m.s. the quantity \( (-r^2) \) is the squared transverse momentum of \( \pi^+ \) or \( \pi^- \).

**B Calculation of traces**

**B.1 Sudakov variables**

When calculating the trace [13], we neglect the electron mass \( m_e \) and decompose any 4–vector \( A \) into components in the plane of the 4–vectors \( p_1 \) and \( p_2 \) and in the plane orthogonal to them
\[ A = x A_p^1 + y A_p^2 + A \perp, \quad (B.1) \]
\[ x_A = \frac{2p_2 A}{s}, \quad y_A = \frac{2p_1 A}{s}, \quad A^2 = s x_A y_A + A \perp^2. \]
The normalization condition for the 4–vectors \( e \) in the decomposition of \((e^2 \pm x(0)e^\perp i)\) given in Eqs. (22) and (25). Besides, \( r = x_1 p_1 + y_1 p_2 + q_1 \) with \( x_1, y_1, x = x_+ + x_- \) and \( y = y_+ + y_- \) mentioned in Eqs. (4).

In the used logarithmic approximation [see Eq. (20)] the decomposition of \( q_2 \) and \( r \) defined in Eq. (A.3) is simplified

\[
q_2 = y_2 p_2, \quad r = y_1 p_2 + r_1, \quad r_1 = \frac{2}{x} r_\perp k_\perp
\]

with \( y_2 \) and \( r_\perp \) given in Eqs. (24) and (25). Besides,

\[
t - \mu^2 = -2q_2 p_- = -sy_2 x_-, \quad u - \mu^2 = -2q_2 p_+ = -sy_2 x_+,
\]

\[
t - u = 2q_2 \Delta = sy_2 x_-. \quad \text{(B.4)}
\]

Below we use the notations

\[
e^{(a)} = e_1^{(a)} = x^{(a)} p_1 + y^{(a)} p_2 + e_1^{(a)} \quad \text{(B.5)}
\]

with

\[
x^{(\pm)} = 0, \quad y^{(\pm)} = \frac{2e^{(\pm)} k_\perp}{sx}, \quad y_0^{(0)} = \frac{r_\perp k_\perp}{-q_1^2}, \quad e^{(a)} = \frac{k_\perp}{\sqrt{-q_1^2}}. \quad \text{(B.6)}
\]

The normalization condition for the 4–vectors \( e^{(a)} \) results in

\[
e_1^{(a)} e_1^{(b)} = \delta_{ab}, \quad a, b = \pm 1. \quad \text{(B.7)}
\]

The following expressions will be useful \( (i, j = x, y) \):

\[
\left( e_1^{(+)i} \right)_i \left( e_1^{(+i)} \right)_j + \left( e_1^{(-)i} \right)_i \left( e_1^{(-)i} \right)_j = \delta_{ij},
\]

\[
\left( e_1^{(+)i} \right)_j \left( e_1^{(-)i} \right)_i + \left( e_1^{(-)i} \right)_j \left( e_1^{(+)i} \right)_i = \delta_{ij} - 2e^{(x)}(x, e^{(x)}),
\]

\[
\left( e_1^{(+)i} \right)_i - \left( e_1^{(-)i} \right)_i = -\sqrt{2} e^{(x)}, \quad \text{(B.8)}
\]

where the vector \( e_1^{(x)} \) has the form [in accordance with definition (A.4)]

\[
e_1^{(x)} = \frac{r_\perp}{|r_\perp|}. \quad \text{(B.9)}
\]

B.2 Calculation of \( C_1^{ab} \)

Now we describe the calculation of trace (19) in the used approximation (20). First, we rewrite the trace \( C_1^{ab} \) with \( a = 0, \pm 1 \) and \( b = \pm 1 \) in the form of three terms

\[
C_1^{ab} = \frac{(-1)^{a+1}}{2} \times \quad \text{(B.8)}
\]

\[
\text{Tr} \left[ \hat{p}_1' \hat{e}^{(a)*} \hat{p}_1 \left( e^{(b)} \hat{p}_1 + \frac{q_2}{sy_2} \Delta - \hat{\Delta} \frac{\hat{p}_1^2 - \hat{q}_2}{sy_2(1-x)} e^{(b)} \right) \right]
\]

\[
= \frac{(-1)^{a+1}}{2sy_2(1-x)} \left[ N_1^{ab} + y_2 (N_3^{ab} + N_4^{ab}) \right] \quad \text{(B.9)}
\]

\[
N_1^{ab} = \text{Tr} \left[ \hat{p}_1' \hat{e}^{(a)*} \hat{p}_1 \left( (1-x)e^{(b)} \hat{p}_1 \Delta - \hat{\Delta} \hat{p}_1 e^{(b)} \right) \right],
\]

\[
N_2^{ab} = (1-x) \text{Tr} \left[ \hat{p}_1' \hat{e}^{(a)*} \hat{p}_1 \hat{p}_2 \Delta \right],
\]

\[
N_3^{ab} = \text{Tr} \left[ \hat{p}_1' \hat{e}^{(a)*} \hat{p}_1 \hat{\Delta} \hat{p}_2 \Delta \right].
\]

The polarization 4–vector \( e^{(a)} \) appears in \( C_1^{ab} \) in the combination \( \hat{e}^{(a)*} \hat{p}_1 \) only, therefore, the \( p_1 \) component of \( e^{(a)} \) does not contribute \( (\hat{p}_1^2 \hat{p}_1 \approx 0) \). Since \( x(\pm) = 0 \), such a component is also absent in \( e^{(b)} \). As a result, we can use all polarization 4–vectors in the form

\[
e^{(a)} = y^{(a)} p_2 + e^{(a)} \quad \text{(B.10)}
\]

Next, for \( N_1 \) we use the relations

\[
\hat{p}_1 e^{(b)} \hat{p}_1 = sy_2 (x, y \Delta) + \frac{-q_1^2}{s} (D \Delta e^{(a)} + y_\Delta (k_\perp e^{(a)} + y^{(a)} k_\perp) \Delta D \perp k_\perp).
\]

The polarization 4–vector \( e^{(a)} \) in the combination \( e^{(a)*} \hat{p}_1 \) only, therefore, the \( p_1 \) component of \( e^{(a)} \) does not contribute \( (\hat{p}_1^2 \hat{p}_1 \approx 0) \). Since \( x(\pm) = 0 \), such a component is also absent in \( e^{(b)} \). As a result, we can use all polarization 4–vectors in the form

\[
e^{(a)} = y^{(a)} p_2 + e^{(a)} \quad \text{(B.10)}
\]

The final result can be expressed in the form

\[
N_1^{ab} = -4s \left[ y_\Delta D_3 + \frac{-q_1^2}{s} D_2 + (1-x) y_\Delta + \frac{v}{s} \right] D_4
\]

where we have used the basic structures

\[
D_0 = e^{(a)*}_1 e^{(b)}_1, \quad D_1 = e^{(a)*}_1 (k_\perp) e^{(b)}_1 (\Delta \perp) k_\perp, \quad D_2 = e^{(a)*}_1 (\Delta \perp) e^{(b)}_1 k_\perp, \quad D_3 = e^{(a)*}_1 (k_\perp) e^{(b)}_1 k_\perp, \quad D_4 = sy^{(a)*}_1 (e^{(b)}_1 k_\perp), \quad D_5 = sy^{(a)*}_1 (e^{(b)}_1 \Delta \perp).
\]

Similarly we obtain

\[
N_2^{ab} = 2s(1-x) \left[ (v - q_1^2 x_\Delta) D_0 + D_1 - D_2 + x_\Delta D_4 + (1-x) D_5 \right],
\]

\[
N_3^{ab} = 2s [-v D_0 + D_1 + D_2 + (1-x) D_5]
\]
and the final expression for $C_{1}^{ab}$

$$C_{1}^{ab} = \frac{(-1)^{1+a}}{y_{2}(1-x)} \left[ -xy_{2}(v - \xi k_{z}^{2})D_{0} + (2 - x)y_{2}D_{1} + \right.$$  

$$+ \left( \frac{2q_{2}^{2}}{s} + xy_{2} \right) D_{2} - 2y_{2}D_{3} - $$

$$- \left( \frac{2v}{s} + 2(1 - x)y_{\Delta} - y_{2}x\xi(1-x) \right) D_{4} + $$

$$+ y_{2}(1-x)(2-x)D_{5} \right].$$

with

$$sxy_{\Delta} = 2x(p_{1}\Delta) = 2v - \xi(W^{2} + k_{z}^{2}).$$

### B.3 Calculation of $T = \sum_{ab} M_{ab}C_{1}^{ab}$

Using Eq. (B.6) we find for $T$ the expression

$$T = (C_{1}^{1+} + C_{1}^{-+})M_{++} + (C_{1}^{1-} - C_{1}^{+-})M_{+-} +$$

$$+ (C_{0}^{1+} - C_{0}^{-+})M_{0+}$$

(B.13)

Taking into account Eq. (B.7) the basic structures related to the amplitude $M_{++}$ are

$$D_{0} = 2, \quad D_{1} = D_{2} = v, \quad D_{3} = k_{z}^{2},$$

$$D_{4} = \frac{2}{x}D_{3}, \quad D_{5} = \frac{2}{x}D_{1}$$

which leads to the coefficient $G_{0}$

$$G_{0} = \frac{v}{x} \left[ 2(1-x)y_{2} - \frac{(2-x)y_{2}}{(1-x)s} \right] +$$

$$+ k_{z}^{2} \left[ y_{2}\xi - \frac{(2-x)y_{\Delta}}{x} \right].$$

(B.14)

Analogously we get for the basic structures related to $M_{+-}$ [taking into account Eq. (B.8)]

$$D_{0} = 0, \quad D_{1} = D_{2} = D_{1}^{(2)} = v - \frac{(v - \xi k_{z}^{2})(\Delta_{z}^{2} - \xi v)}{2r_{\perp}^{2}},$$

$$D_{3} = D_{3}^{(2)} = k_{z}^{2} - \frac{(v - \xi k_{z}^{2})^{2}}{2r_{\perp}^{2}},$$

$$D_{4} = \frac{2}{x}D_{3}^{(2)}, \quad D_{5} = \frac{2}{x}D_{1}^{(2)}$$

which gives the coefficient $G_{2}$:

$$G_{2} = \left[ \frac{2y_{2}}{x} \left( 1 - x + \frac{x^{2}}{2} \right) - \frac{k_{z}^{2}}{(1-x)s} \right] D_{1}^{(2)} +$$

$$+ \left[ (1-x)y_{2}\xi - \frac{(2-x)y_{\Delta}}{x} - \frac{2v}{xs} \right] D_{3}^{(2)}.$$  

Finally the coefficient $G_{1}$ is obtained from [using (B.3)]

$$D_{0} = D_{0}^{(1)} = \frac{v - \xi k_{z}^{2}}{\sqrt{-2q_{2}^{2}r_{\perp}^{2}}}$$

$$D_{1} = D_{1}^{(1)} = \frac{(\Delta_{z}^{2} - \xi v)k_{z}^{2}}{\sqrt{-2q_{2}^{2}r_{\perp}^{2}}}$$

$$D_{2} = vD_{0}^{(1)}, \quad D_{3} = k_{z}^{2}D_{1}^{(1)}$$

$$D_{4} = \frac{2-x}{x(1-x)}k_{z}^{2}D_{0}^{(1)}$$

$$D_{5} = \frac{2-x}{x(1-x)}D_{1}^{(1)}$$

as follows

$$G_{1} = \frac{-k_{z}^{2}}{x} \left[ y_{2}\xi - \frac{2y_{\Delta}}{x} - \frac{2v}{x(1-x)s} \right] D_{0}^{(1)} -$$

$$- \frac{2}{x}y_{2}D_{1}^{(1)}.$$  

(B.16)

The coefficients $G^{a}$ depend on the vectors $\Delta_{\perp}$, $k_{\perp}$ and $r_{\perp}$. Using the relation $\Delta_{\perp} = 2r_{\perp} + \xi k_{\perp}$ [see Eq. (B.3)] and introducing the angle $\phi$ between the vectors $r_{\perp}$ and $k_{\perp}$ we get after some algebra the compact expression for $T$ in the form of Eqs. (22, 23).

### C Some useful notes

#### C.1 Substitution rule for the contribution $d\sigma_{13}$

Let us briefly describe the necessary changes for $d\sigma_{13}$ in the case of electron–positron or electron–electron collisions

$$e^{-}(p_{1}) + e^{\pm}(p_{2}) \rightarrow e^{-}(p_{1}') + e^{\pm}(p_{2}') + \pi^{+}\pi^{-}.$$  

The contribution $d\sigma_{13}$ has the form

$$d\sigma_{13} = 2 \Re(M_{1}^{a}M_{1}^{b}) \frac{d\Gamma}{2s}$$

$$= -2 \left( \frac{4\pi\alpha}{q_{1}^{2}r_{1}^{2}k_{2}^{2}} \right)^{2} \sum_{abc=\pm1,0} \Re \left( F_{\pi M_{ab}}\varrho_{1}^{ac}C_{2}^{bc} \right) \frac{d\Gamma}{2s}$$

where $\varrho_{2}^{bc}$ and $C_{2}^{bc}$ are similar to $\varrho_{1}^{bc}$ and $C_{1}^{bc}$ in Eqs. (3), (9). $d\sigma_{13}$ can be obtained from $d\sigma_{12}$ under the exchange

$$p_{1} \leftrightarrow p_{2}, \quad p_{1}' \leftrightarrow p_{2}', \quad q_{1} \leftrightarrow q_{2}.$$  

It is not difficult to check that in that case

$$\varrho_{2}^{bc} \rightarrow (-1)^{b+c} \varrho_{1}^{bc}, \quad C_{1}^{ac} \rightarrow \mp (-1)^{a+c} C_{2}^{ac}.$$  

(C.1)

The sign $\mp$ in the last equation is in agreement with the transition from three $e^{-}$ vertices in $C_{1}^{ac}$ to three $e^{\pm}$ vertices in $C_{2}^{ac}$.

As a result, taking into account Eq. (A.7), we have

$$d\sigma_{13} = \mp d\sigma_{12}(p_{1} \leftrightarrow p_{2}, \quad p_{1}' \leftrightarrow p_{2}', \quad q_{1} \leftrightarrow q_{2})$$

where the sign “minus” corresponds to electron–positron collisions considered here and “plus” to electron–electron collisions.
C.2 Absence of amplitude $M_{+-}$ in $d\sigma_{12}$ averaging over angle $\phi$

The amplitude $M_{+-}$ enters the result (24) with coefficient $C_{1}^{+-} + C_{1}\equiv \frac{e}{x}$ [see Eq. (B.13)]. Let us consider $C_{1}^{+-}$ given by Eq. (B.11). Since

$$e^{(-)}* = e^{(+)} = -e$$

and

$$e_{q_{1}} = e_{q_{2}} = 0, \quad e_{p_{1}} = e_{p_{1}^{*}}, \quad e_{p_{2}^{*}} = 0,$$

we have

$$\hat{e}_{\hat{p}_{1}} \hat{e} = \hat{e}_{\hat{p}_{1}^{*}} \hat{e} = 2(e_{p_{1}}) \hat{e}.$$

Therefore, the $C_{1}^{+-}$ coefficient can be simplified to a trace of four Dirac matrices easily calculable

$$C_{1}^{+-} = -(e_{p_{1}}) \text{Tr} \left\{ \hat{e}_{\hat{p}_{1}}^{*} \hat{q}_{2} \hat{\Delta}_{\hat{p}_{1}^{*}} - \hat{e}_{\hat{p}_{1}} \hat{\Delta}_{\hat{p}_{1}^{*}} \right\}$$

with

$$f_{1} = 8x_{p_{1}} \Delta - 4(1 - x) q_{2} \Delta,$$

$$f_{2} = 4x_{p_{1}} k + 4(1 - x) q_{2} (p_{1} + p_{1}^{*}).$$

It is easy to check that only two scalar products depend on the azimuthal angle $\phi$:

$$2e_{p_{1}} = s_{y_{2}}^{(+)} = \frac{2}{x} e^{(\pm)} |k_{\perp}|\text{e}^{i\phi},$$

$$2x_{p_{1}} \Delta = 4 |r_{\perp}| |k_{\perp}| \text{cos} \phi - \xi \left( W^{2} - k_{\perp}^{2} \right).$$

As a result, the structure of $C_{1}^{+-}$ is

$$C_{1}^{+-} = (a + b \text{cos} \phi) e^{2i\phi} + c e^{i\phi}$$

which leads to

$$C_{1}^{+-} + C_{1}^{+-} = 2 \text{Re} C_{1}^{+-} = (b + 2c) \text{cos} \phi + 2a \text{cos} 2\phi + b \text{cos} 3\phi.$$

Therefore, this coefficient disappears after averaging over $\phi$

$$\langle C_{1}^{+-} + C_{1}^{+-} \rangle_{\phi} = 0. \quad (C.3)$$

C.3 Low $k_{\perp}$ limit for $d\sigma_{12}$

At small transverse momentum of produced the pion pair $k_{\perp}$ our result (24), (25) is simplified to

$$\varepsilon_{+} \frac{d\sigma_{12}}{d\hat{p}_{+} d\hat{p}_{-}} = -\frac{\alpha^{3}}{4\pi^{3}} \text{Re} \left\{ \frac{x}{W^{d}} \frac{k_{\perp} \Delta_{\perp}}{k_{\perp}^{2}} \times \left( 1 - y_{2} + \frac{1}{2} y_{2}^{2} \right) L_{2} \right\} \left( \langle 1 - x \rangle \text{Re} (F_{x}^{M_{++}}) - \langle 1 - x + \frac{1}{2} x^{2} \rangle \text{Re} (F_{x}^{M_{++}}) \right) \quad (C.4)$$

with $y_{2} = W^{2} / (sx)$. Analogously, in this limit the muon pair production [see Eq. (17)] takes the form

$$\frac{d\sigma_{12}}{d\hat{p}_{+} d\hat{p}_{-}} = 4 \left( \frac{\alpha^{3}}{\pi^{3}} \text{Re} \left\{ \frac{x}{W^{d}} \frac{k_{\perp} \Delta_{\perp}}{k_{\perp}^{2}} \times \left( 1 - y_{2} + \frac{1}{2} y_{2}^{2} \right) L_{2} \right\} \left( \langle 1 - x \rangle \text{Re} (F_{x}^{M_{++}}) - \langle 1 - x + \frac{1}{2} x^{2} \rangle \text{Re} (F_{x}^{M_{++}}) \right) \right).$$

Both distributions are proportional to the transverse variable $v = k_{\perp} \Delta_{\perp}$ and do not depend on the longitudinal variable $\xi$. These results coincide with those of Ref. [3].

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