The speed of gravitational waves for a single observation can be measured by the time delay among gravitational-wave detectors with Bayesian inference. Then multiple measurements can be combined to produce a more accurate result. From the near simultaneous detection of gravitational waves and gamma rays originating from GW170817/GRB 170817A, the speed of gravitational wave signal was found to be the same as the speed of the gamma rays to approximately one part in $10^{15}$. Here we present a different method of measuring the speed of gravitational waves, not based on an associated electromagnetic signal but instead by the measured transit time across a geographically separated network of detectors. While this method is far less precise, it provides an independent measurement of the speed of gravitational waves. For GW170817 a binary neutron star inspiral observed by Advanced LIGO and Advanced Virgo, by fixing sky localization of the source at the electromagnetic counterpart the speed of gravitational waves is constrained to 90% confidence interval (0.97$c$, 1.02$c$), where $c$ is the speed of light in a vacuum. By combing seven BBH events and the BNS event from the second observing run of Advanced LIGO and Advanced Virgo, the 90% confidence interval is narrowed down to (0.97$c$, 1.01$c$). The accurate measurement of the speed of gravitational waves allows us to test the general theory of relativity. We further interpret these results within the test framework provided by the gravitational Standard-Model Extension (SME). In doing so, we obtain simultaneous constraints on 4 of the 9 nonbirefringent, nondispersive coefficients for Lorentz violation in the gravity sector of the SME and place limits on the anisotropy of the speed of gravity.

I. INTRODUCTION

The first gravitational wave (GW) detection, GW150914 [1], was observed from a binary black hole (BBH) merger during the first observing run (O1) of Advanced LIGO [2] from September 12th, 2015 to January 19th, 2016. Later in O1, two BBH mergers GW151012 [3] and GW151226 [4] were also detected by the two Advanced LIGO detectors. The second observing run (O2) of the Advanced LIGO took place from November 30th, 2016 to August 25th, 2017. In O2 three BBH mergers GW170104 [5], GW170608 [6] and GW170823 [7] were detected by the two Advanced LIGO detectors. With the Advanced Virgo [8] detector joining in later O2, four more BBH mergers GW170729 [7], GW170809 [7], GW170814 [9], GW170818 [7] and one binary neutron star (BNS) inspiral GW170817 [10] were observed by the three-detector network [11].

General Relativity predicts that the speed of gravitational waves is equal to the speed of light in a vacuum. The GW seen by the Advanced LIGO and Advanced Virgo detectors can be used to test the theory of general relativity. The first measurement of the speed of gravitational waves using time delay among the GW detectors was suggested by Cornish et al [12]. By applying the Bayesian method the speed of gravitational waves is constrained to 90% confidence interval between 0.55$c$ and 1.42$c$ with GW150914, GW151226 and GW170104 [12].

Subsequent to Cornish et al [12], a more precise measurement of the speed of gravitational waves was facilitated by the measurement of the time delay between GW and electromagnetic observations of the same astrophysical source. On August 17, 2017, a binary neutron star inspiral GW170817 was observed by the Advanced LIGO and Advanced Virgo detectors, (1.74 $\pm$ 0.05)s later the Gamma-ray burst (GRB) was observed independently by Fermi Gamma-ray Laboratory. By using the lower bound of luminosity distance obtained from the GW signal, the time delay between the GW and GRB, and some astrophysical assumptions, the speed of gravitational waves ($v_g$) was constrained to $-3 \times 10^{-15} c < v_g - c < +7 \times 10^{-16} c$ [13].

In this paper, we employ an approach similar to that used in Ref. [12], to make a local measurement of the speed of gravity based on the difference in arrival time across a network of GW detectors for pure GW observations made during O2. We consider both measurements of the speed of gravitational waves from individual events and then demonstrate how the accuracy can be improved by combining measurements from multiple GW observations. In Sec. II, we discuss our methods and in Sec. III we present the speed of gravity results. Finally, in Sec. IV, we use a subset of the individual speed of gravity results to obtain constraints on local Lorentz violation in the context of the effective-field-theory-based test framework provided by the gravitational Standard-Model Extension (SME) [14–16], within which a number of recent

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ABSTRACT

We measure the speed of gravitational waves by combining multiple GW events. The speed is obtained by maximizing the likelihood for a single event. We then compute the posterior for the joint model using a Bayesian framework. Assuming the noise is stationary and Gaussian distributed, the likelihood for single event is: 

\[ p(\theta | d_1, d_2, ..., d_n) = \frac{1}{(2\pi)^{n/2} |\Sigma|^1/2} \exp\left(\frac{1}{2} (\theta - \mu)^T \Sigma^{-1} (\theta - \mu) \right) \] 

where \( \mu \) is the mean and \( \Sigma \) is the covariance matrix. The posterior for the joint model is obtained by maximizing the likelihood for all events, assuming that the noise is independent across detectors.

\[ p(\theta | d_1, d_2, ..., d_n) = \prod_{i=1}^{n} p(\theta | d_i) \] 

where \( p(\theta | d_i) \) is the posterior for a single event. The marginalized posterior is obtained by integrating over all parameters:

\[ p(\theta | d_1, d_2, ..., d_n) = \int \prod_{i=1}^{n} p(\theta | d_i) \, d\theta \] 

where \( \theta \) is a set of parameters in \( \bar{\theta} \). The marginalization is performed by using a Metropolis-Hastings algorithm. The speed of gravitational waves is obtained by maximizing the likelihood for a single event.
FIG. 1. Marginalized posterior distributions of \( v_g \) for GW170817. The solid line is obtained from the run with fixing \( \alpha \) and \( \delta \) at the electromagnetic counterpart, whereas the dashed line obtained from the run without fixing \( \alpha \) and \( \delta \).

FIG. 2. Posterior distributions of \( v_g \) for seven BBH events: GW170104, GW170608, GW170729, GW170809, GW170814, GW170818, GW170823 and combined posterior. The combined posterior is computed using Eq. 7.

In our analysis, we choose the IMRPhenomPv2 waveform [25] for all BBH events and TaylorF2 for the BNS event. IMRPhenomPv2 is a processing BBH waveform with inspiral, merger and ringdown. TaylorF2 [26–31] is a frequency domain post-Newtonian waveform model that includes tidal effects.

The first detection of binary neutron star inspiral GW170817 by Advanced LIGO and Advanced Virgo provides an accurate measurement for \( v_g \). The BNS event has a network signal to noise ratio(SNR) 33 [7] which is the highest in all GW events detected in the O1 and O2.

The sky localization is precisely constrained to an area of 16 \( \text{deg}^2 \). Those two aspects of GW170817 allow an accuracy \( v_g \) measurement to a \((0.95c, 1.06c)\) 90\% confidence interval. The later electromagnetic counterpart was discovered in the galaxy NGC4993 [32], which enable us to fix the right ascension(\( \alpha \)) and declination(\( \delta \)) at the electromagnetic counterpart during MCMC sampling. The later measurement shrinks the 90\% confidence interval of \( v_g \) to \((0.97c, 1.02c)\). The marginalized posteriors of \( v_g \) for GW170817 with and without fixing \( \alpha \) and \( \delta \) at the electromagnetic counterpart are shown in FIG. 1.

FIG. 2 shows the posterior distributions of \( v_g \) for all O2 BBH events and the combined posterior is obtained by using Eq. 7. Narrow sky localization and high SNR of a GW event can help to better constrain on \( v_g \). \( v_g \) for GW170809, GW170814 and GW170818 are well con-

FIG. 3. Posterior distributions of \( v_g \) for all events detected in O2: GW170104, GW170608, GW170729, GW170809, GW170814, GW170817, GW170818, GW170823 and combined posterior. For GW170817 \( \alpha \) and \( \delta \) are free parameters in the top plot, in the bottom plot \( \alpha \) and \( \delta \) are fixed at electromagnetic counterpart.
TABLE I. 90% confidence intervals of $v_g$ from individual events posteriors and combined posteriors. GW170817(fixed) obtain from the MCMC run with fixing $\alpha$ and $\delta$ at the electromagnetic counterpart and GW170817 treats $\alpha$ and $\delta$ as free parameters. Combined(BBH) obtained from the seven BBH. Combined(fixed) and Combined uses seven BBH and GW170817 with and without fixing $\alpha$ and $\delta$ respectively. Network SNR values are reported from the GstLAL search pipeline [7]. 90% confidence regions of the sky localization ($\Omega$) with fixing $\alpha$, $\delta$, $\gamma$ at $c$ are presented in GWTC-1[7] and without fixing $\alpha$ are computed from posteriors of $\alpha$ and $\delta$.

| Events          | 90% Confidence Intervals | Network SNR | $\Omega/\deg^2$ | GWTC-1 $\Omega/\deg^2$ |
|-----------------|--------------------------|-------------|------------------|------------------------|
| GW170104        | (0.34c, 3.27c)           | 13.0        | 923              | 5313                   |
| GW170608        | (0.91c, 1.38c)           | 14.9        | 396              | 1269                   |
| GW170729        | (1.56c, 5.83c)           | 10.8        | 1033             | 1287                   |
| GW170809        | (0.30c, 1.01c)           | 12.4        | 340              | 2252                   |
| GW170814        | (0.88c, 1.11c)           | 15.9        | 87               | 250                    |
| GW170817        | (0.96c, 1.06c)           | 33.0        | 16               | 53                     |
| GW170817(fixed) | (0.97c, 1.02c)           | 33.0        | 0                | 0                      |
| GW170818        | (0.59c, 1.21c)           | 11.3        | 39               | 168                    |
| GW170823        | (0.10c, 12.19c)          | 11.5        | 1651             | 6412                   |

| Events          | 90% Confidence Intervals | Network SNR | $\Omega/\deg^2$ |
|-----------------|--------------------------|-------------|------------------|
| Combined(BBH)   | (0.92c, 1.07c)           |             |                  |
| Combined        | (0.96c, 1.05c)           |             |                  |
| Combined(fixed) | (0.97c, 1.01c)           |             |                  |

The 9 nondispersive, nonbirefringent coefficients for Lorentz violation in the gravity sector of the SME cause modification of the group velocity of GWs. Using natural units and the assumption that the nongravitational sectors, including the photon sector, are Lorentz invariant, the modified group velocity can be written as follows [17]:

$$v_g = 1 + \frac{1}{2} \sum_{jm} (-1)^j Y_{jm}(\alpha, \delta) \pi_{jm}.$$

Here a basis of spherical harmonics $Y_{jm}$ in which $j \leq 2$ has been used to express the 9 Lorentz-violating degrees of freedom $\pi_{jm}$ present in this limit of the SME. While the sum on $m$ ranges from $\pm j$ in Eq. (8), the equivalent expansion over positive $m$:

$$v_g = 1 + \sum_{j} (-1)^j \left( \frac{1}{2} \pi_{j0} Y_{j0} + \sum_{m>0} \text{Re} \pi_{jm} \text{Re} Y_{jm} - \text{Im} \pi_{jm} \text{Im} Y_{jm} \right),$$

is conventionally chosen in expressing experimental sensitivities.

Restricting attention to the O2 events analyzed in this work, it is not possible to simultaneously constrain all 9 of the $\pi_{jm}$ coefficients for Lorentz violation as there are only 8 events. Further, some of these have significant uncertainty in both $v_g$ and sky position $\alpha, \delta$. Hence we explore a model formed by the $j \leq 1$ subspace of the full SME, using 4 of the most sensitive events and the following methods.

In the earlier sections of the paper, data from the multiple events were combined under the assumption of isotropic GW speeds to obtain a more sensitive measurement. Here we exploit a complementary advantage of the multiple observations in constraining direction-dependent speeds. To develop the methods, imagine that one had an exact measurement of $v_g$ as well as sky position for a GW event. Then Eq. (9) would form 1 equation with 4 unknowns (the 4 coefficients $\pi_{jm}$ in our model). Given 4 such events, assuming unique sky locations, the system of 4 equations that results could be solved for
FIG. 4. 90% confidence regions for the sky localizations of all GW events detected in O2. The solid contours are obtained from the posteriors where the speed of gravitational waves is fixed at the speed of light, the dashed contours shows the results for $v_g$ as a free parameter. Top: events detected by Advanced LIGO and Advanced Virgo (GW170729, GW170809, GW170814, GW170817, GW170818). Bottom: events detected by the two Advanced LIGO detectors (GW170104, GW170608, GW170823).

the 4 coefficients $\tilde{s}_{jm}$ forming a measurement of Lorentz violation. Of course in the present case of real experimental work we have a distribution for each of our 4 events rather than a signal value. We use this data by randomly drawing a sample from the distribution for each of the 4 events, solving for the corresponding values of the 4 coefficients $\tilde{s}_{jm}$, and repeating the process to build the $\tilde{s}_{jm}$ distribution.

Using GW170608, GW170814, GW170817, and GW170818 (lines 2, 5, 6, and 8 of Table I) we obtain the results shown in Fig. 5. In Fig. 6, we use the same events but with the fixed sky position as in line 7 of Table I. This generates a modest narrowing of the one sigma range for some coefficients. We also explored setting the
speed of gravity for the GW170817 event to that found in Ref. [13]. This results in negligible narrowing of confidence bands relative to those shown in Fig. 6.

Note that the measurements of the $s_{jk}$ shown in Figs. 5 and 6 are consistent with zero. Hence, we can interpret the one sigma range shown as upper and lower bounds on the values of the $s_{jk}$ coefficients, an exclusion of the simplest types of direction-dependent speeds. As with $v_g$, these limits are considerably weaker than some found in the literature [14]. However, they carry value in that they are obtained from significantly different methods than other tests, are the first effort to simultaneously constrain multiple $s_{jk}$ using speed of gravity measurements, and begin establishing methods for future higher-sensitivity tests.

As a final note, we point out that if the isotropic limit of the SME is considered such that $s_{00}$ is the only nonzero coefficient for Lorentz violation, then the combined $v_g$ results in the last line of Table I may be applied. Doing so yields $-0.2 < s_{00} < 0.07$.

V. CONCLUSIONS

While the association between GWs and gamma-rays observed with GW170817 and GRB 170817A have provided an extremely tight bound on the difference between the speed of gravitational waves and the speed of light, in this paper we have presented an independent method of directly measuring $v_g$, which, while less precise, is based solely on GW observations and so not reliant on multi-messenger observations. We continue to find measured values of $v_g$ consistent with the speed of light, as predicted by General Relativity, not just for GW170817 but also for other signals detected during the second observation run of Advance LIGO and Virgo. By combining these measurements and assuming isotropic propagation, we constrain the speed of gravitational waves to $(0.97c, 1.01c)$ which is within $3\%$ of the speed of light in a vacuum. We also obtain simultaneous constraints on nonbirefringent, nondispersive coefficients for Lorentz violation in the test framework of the SME. Though the constraints are not as strong as other methods, we simultaneously limit multiple coefficients using direct speed of gravity tests for the first time, directly constraining the possibility of an anisotropic speed of gravity. Other implications for deviations from general relativity arising from cosmological evolution were considered in Ref. [33].

There are some limitations of the approach used here that must be acknowledged. First, should the speed of gravitational waves differ from the speed of light in violation of the predictions of General Relativity then we would not necessarily expect other assumptions based on General Relativity predictions to necessarily hold. Among the assumptions that could affect us is the assumption that the gravitational waves only exist in two tensorial transverse polarizations. More general metric theories of gravity could allow for up to 6 independent po-
larizations, including in addition two longitudinal-vector polarizations and two scalar polarizations (one longitudinal and one transverse, thought these are indistinguishable in interferometric detectors). However our parameter estimation has continued to assume that only the tensor polarization states exists. Furthermore, we continue to assume that the gravitational waveforms are as predicted by general relativity. Nevertheless, our measurement of $v_g$ is mostly constrained by the measured times of arrival of the signal in the various detectors, so we believe it is reasonably robust.

In addition, as noted in Ref. [12] the searches that identify GW signals normally require a signal to be seen in two detectors and impose a time window. For example, for the LIGO Hanford Observatory and the LIGO Livingston Observatory, the searches required arrival times within 15 ms (while the light travel time between those detectors is 10 ms) [34]. This would seemingly create a selection bias against gravitational wave signals with $v_g < \frac{2}{3}c$. However (again as noted in Ref. [12]) a gravitational wave signal can also be identified in a single detector, and, for sufficiently loud signals, the presence of a signal in another detector at similar time would unlikely go unnoticed. For this reason we do not think there is a strong selection bias against slow moving gravitational waves.

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