Additive-multiplicative fixed-pattern noise model of an IR photodetector, taking into account the exposure time

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Abstract. The article discusses the additive-multiplicative fixed-pattern noise model of the infrared photodetector and a new way of its compensation, invariant to a change in exposure time. Examples of images showing the effectiveness of a new method for compensating fixed-pattern noise when the exposure time of a photodetector is changed are given.

1. Introduction

Fixed-pattern noise (FPN) is a deterministic interference for a specific photodetector, and this interference has two components: additive and multiplicative. The additive component is due to the uneven thermal generation of charge carriers in the elements of the matrix photodetector. The multiplicative component is due to the heterogeneity of the sensitivity of the elements of the matrix photodetector. A mathematical additive-multiplicative model describing FPN has the form of a linear equation with constant coefficients:

\[ y = kx + b, \]

where \( y \) is the output brightness values of the elements of a matrix photodetector with FPN, \( b \) is the additive component of the FPN, which characterizes for each element the non-uniformity of thermal generation, \( k \) is the multiplicative component of FPN [1–6].

In this case, \( x \) is the initial brightness values of the elements of the matrix photodetector without FPN, and \( k \) are the coefficients characterizing the non-uniformity of sensitivity for each element.

FPN is usually fought by the compensation method, which consists in subtracting the pre-stored values of the additive component \( b \) and multiplying (or dividing) by the previously calculated compensation factors \( k \) for the multiplicative component [1–6].

This method provides a preliminary calibration of the matrix photodetector of the visible range of spectrum, providing alternate overlap of the radiation flux and uniform irradiation of the photodetector. The resulting brightness values of the image frame elements \( y_1 \) for the blocked radiation flux and \( y_2 \) for a uniformly irradiated photodetector are memorized. Then the coefficients \( k \), which characterize the relative sensitivity of the photodetector elements according to the formula \( k = y_2/m_2 \), where \( m_2 \) is the average image brightness \( y_2 \), are calculated. Under informative irradiation of the photodetector, FPN is compensated in the resulting digital brightness values \( y \) of the frame elements, forming the output brightness values \( x \) of the frame elements by the inverse expression of the FPN mathematical model:

\[ X = (y - b) / k, \text{ where } b = y_1. \]

To compensate for the FPN of the matrix infrared photodetector, the method is somewhat modified [1–6] and involves finding the coefficients \( k \) from the system of equations with two unknowns:

\[ y_1 = kx_1 + b, \ y_2 = kx_2 + b. \]
At the same time, \( x_1 = m_1 \) and \( x_2 = m_2 \) are the initial values of the brightness of the elements of a matrix photodetector without FPN with a uniform low and high irradiance of the elements of the photodetector matrix, corresponding to the average brightness values \( m_1 \) and \( m_2 \) of the images \( y_1 \) and \( y_2 \). This modification of the method is called the two-point correction.

From the system of equations considered above, the following value follows:
\[
k = \frac{(y_2 - y_1)}{(m_2 - m_1)}
\]
\[
b = \frac{(y_1 x_2 - y_2 x_1)}{(x_2 - x_1)}.
\]

If \( x_1 \rightarrow 0 \), then \( k \approx \frac{y_2}{x_2}, \ b \approx y_1 \). However, as computer modeling shows, the approximate calculation of the \( b \) coefficient has almost no effect on the accuracy of the two-point correction, and the approximate calculation of the coefficient \( k \) worsens it significantly (figure 1).

![Figure 1](image)

Figure 1. Graphs of changes in standard deviation of residual FPN for two-point correction – a graph (row 1), with an approximate calculation of the \( b \) coefficient – (row 2, coincides with row 1), with an approximate calculation of the \( k \) coefficient – a graph (row 3)

It should be noted that computer modeling shows the limitations in the application of the methods discussed above with increasing dynamic range of a signal change [7, 8].

A common disadvantage of the known compensation methods is the low accuracy of the compensation of the FPN of the photodetector with an exposure time (accumulation) different from the exposure time used in the calibration process. Low accuracy is due to the fact that, for example, with an increase in the exposure time \( t \) of the photodetector during its informative irradiation within the working exposure range \( t_{\text{min}} \leq t \leq t_{\text{max}} \), relative to the minimum exposure time \( t_{\text{min}} \) used in the calibration process, an error occurs in compensating for FPN. The error of the FPN compensation, in turn, is explained by the fact that the additive component of the FPN is directly proportional to the accumulation time. In the image, the error of compensation manifests itself in the form of interference – a granular structure. Wherein, the longer the set accumulation time \( t \), the greater the compensation error and the more noticeable the disturbance.

2. Results and discussion

Figure 2 and figure 3 show the experimentally removed dependences of the mathematical expectation of a signal on the temperature of a radiation source at a fixed exposure and the dependence of the mathematical expectation of a signal on the magnitude of the exposure of a photodetector at a fixed temperature. Unlike the dependence shown in figure 2, the dependence (figure 3) is linear.

The presence of a linear dependence of the average brightness of the image \( m \) on the exposure time \( t \) in the interval \( t_{\text{min}} \leq t \leq t_{\text{max}} \) makes it possible to simply take into account the error of compensation of the FPN when changing the exposure, since the magnitude of the additive component of the FPN has a similar nature depending on the exposure time.

Below is a method for compensating the FPN of a photodetector [9], invariant for a set time \( t \) of exposure of a photodetector within \( t_{\text{min}} \leq t \leq t_{\text{max}} \).
Figure 2. Dependences of the average value of the signal on the temperature of the radiation source at a fixed exposure of the photodetector.

Figure 3. Dependences of the average value of the signal on the exposure value of the photodetector at a fixed temperature of the radiation source.
The essence of the method lies in the fact that the calculated brightness values of the additional correction frame of the image \( dx \) corresponding to the compensation error for the exposure time \( t \). With informative flare, the \( dx \) values are subtracted from the brightness values of the corresponding elements of the output frame, which are generated from the coefficients \( k \) calculated at the calibration stage with the exposure time \( t_{min} \).

Calculations of \( dx \) are made based on the linear dependence of the average brightness of the image \( m \) on the exposure time \( t \) on the interval \( t_{min} \leq t \leq t_{max} \) corresponding to the average brightness values \( m_1 \) and \( m_{max} \). In other words, by the known brightness values \( x_1 \) and \( x_2 \), the brightness values of the elements of the additional correction image frame \( dx \) for the exposure time \( t \) are interpolated.

In practice, it is advisable to use the following sequence of operations carried out at the stage of calibration of the photodetector. In addition to the images \( y_1 \) and \( y_2 \), the image \( y_{max} \) is formed, obtained with a low level of uniform irradiation of the photodetector with an exposure \( t_{max} \).

The initial digital brightness values of the image elements \( y_1, y_2, y_{max} \), as well as the values \( t_{min} \) and \( t_{max} \) entered into the computer are processed to calculate the average values \( m_1, m_2 \) and \( m_{max} \) of the brightness of elements in the image frames \( y_1, y_2 \) and \( y_{max} \).

Next, images \( x_1 \) and \( x_2 \) are formed, containing residual FPN at exposures \( t_{min} \) and \( t_{max} \) after compensation of the image \( y_2 \). To do this, the transformations are carried out according to the formulas:

\[
x_1 = (y_2 - y_1)k + m_1, \quad x_2 = (y_{max} - y_1)k + m_{max}.
\]

Then the values of the \( a \) coefficients are calculated by the formula:

\[
a = \frac{(t - t_{min})}{(t_{max} - t_{min})}.
\]

The brightness values of the additional correction frame of the image \( dx \) corresponding to the compensation error for the exposure time \( t \) correspond to the expression, as the weighted sum \( x_2 \) and \( x_1 \):

\[
dx = ax_2 + (1 - a)x_1.
\]

Under informative irradiation with exposure time \( t \), an image is entered into the computer with the brightness values of the elements \( y \), which are processed to form the output digital brightness values of the image elements \( X \) in accordance with the expressions:

\[
x = (y - y_1)k + m_1, \quad X = x - dx + m_0, \quad \text{if} \ t \neq t_{min}, \quad \text{and} \quad X = x, \quad \text{if} \ t = t_{min}, \quad \text{where} \ m_0 \text{ is the average brightness} \ dx.
\]

**Figure 4.** The original, containing FPN (left), and the resulting images and waveforms of the line, obtained without taking into account the exposure time (center) and in a new way taking into account the exposure time (right) with an exposure time \( t_{min} = 3\text{ms} \) during the calibration of the photodetector and exposure time \( t = 6\text{ms} \) in the process of informative irradiation.
Figure 4 shows the source, containing the FPN (top), and the resulting images and waveforms of the line obtained without taking into account the exposure time (center) and the new method taking into account the exposure time $t_{\text{min}}=3\text{ms}$ during the calibration of the photodetector and exposure time $t=6\text{ms}$ in the process of informative irradiation. In the image shown in the center, the disturbance in the form of a granular structure is visible. In the image obtained by the new method (right), the grain is significantly reduced. Under the images, the corresponding waveforms of the rows with the same numbers, illustrating a significant reduction in interference with the implementation of the new method compared to the method that does not take into account the change in exposure time, are presented.

3. Conclusion
The linear dependence of the average brightness of the image on the exposure time $t$ makes it possible to take into account the change in FPN in its additive multiplicative model.

For this, in addition to the $y_1$ and $y_2$ images, it is sufficient to form an additional reference image with the exposure $t_{\text{max}}$ at the calibration stage with a low level of uniform irradiation of the photodetector.

The brightness values of the additional correction frame of the image $dx$ corresponding to the compensation error for the exposure time $t$ are determined by the weighted sum of the images $x_1$ and $x_2$ containing the residual FPN at the exposures $t_{\text{min}}$ and $t_{\text{max}}$.

References
[1] Bychkov B N, Kuznetsov N N and Timofeev B S 1981 Digital correction of the TV signal sensors *Technique of film and television* 1 46–52
[2] Milton A 1985 Influence of nonuniformity to infrared focal plane array performance Opt. Eng. N 24, 855–862
[3] Chromov L I, Lebedev N V, Tsitsulin A K and Kulikov AN 1986 Solid television. Television systems with variable parameters on CCD and microprocessors Moscow *Radio and communication* 16–23
[4] Scribner D, Kruer M and Gridley C 1987 Physical limitations to nonuniformity correction in focal plane arrays *Proc. SPIE* Vol 865 185–201
[5] Scribner D A, Kruer M R, Sarkady K and Gridley J C 1989 Spatial noise in staring IR focal plane arrays *Proc. SPIE* Vol 930 56–63
[6] Bronds D S and Kharitonov E N 2008 Correction of geometric noise MFPU using the least squares approximation of the transfer characteristics of the matrix by the T–order polynomial using the least squares method *Journal of Radio Electronics* 11 1–29
[7] Kornyshev N P, Lyubimov M D and Senin A S 2017 Evaluation of the results of the correction of geometric noise in a highly sensitive matrix IR photodetector *Issues of radio electronics series. Television equipment* 4 3–8
[8] Kornyshev N P, Lyubimov M D and Senin A S 2017 Quantitative and qualitative assessment of the geometric noise of the matrix photodetectors *Bulletin of NovSU. Ser.: Technical science.* 7 (105) 23–26
[9] Senin A S, Lukin K G, Kornyshev N P, Stepanov Y V and Golovkin S 2019 NThe method of compensation of the geometric noise of the matrix photodetector Patent RU 2679547