Distribution network planning and inventory management in a multi-retailing supply chain

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Abstract. In this paper, we are aimed at discussing problems of distribution network planning and inventory management in a multiple-retailing supply chain. We study the distribution model and inventory management model, integrating both models to analyse the optimal decisions on maximum profit. The main methodology we used is literature study and model formulations. Through the research, we finally present the optimal distribution and inventory decision.

1. Introduction and Literature review
Distribution network planning is a widely studied problem in both transportation problems and supply chain management problems. Inventory management also plays a vital role in making decisions on maximum profit for enterprises. In this paper, we study distribution model and inventory model. Through the non-linear programming model, we can make distribution decisions in multi-retailing stores. In the non-linear programming model, the shipping quantity from warehouses to multiple stores is the basic decision variables. For inventory problems, we mainly use newsboy model to decide the optimal level. The final objective is aimed at obtaining the maximal profit for all multiple stores by optimizing the distribution network and inventory level of each product.

There is a great amount of research to study distribution and inventory problems in multiple stores. Optimal delivery network is able to benefit a lot in supply chain management, for example, low cost and quick response in supply chain (Chopra, 2003)[1]. Chopra (2003) addresses distribution network design, but Chopra concentrates on exploring the advantages and disadvantages of various distribution networks, and Chopra also come up with different factors that make influences on selecting a shipping network. Jayaraman and Ross (2003) believe transportation activity is a critical process in terms of improving supply chain performance[2]. In their research, distribution network design problems are discussed in a transportation system which include a central plant, multiple distribution centers, cross-docking locations and retail stores. Two models are formulated, with purpose of deciding the optimal distribution centers and best cross-docking locations, and determining the delivery amount between all shipping nodes, seperately. Another scholar Amiri (2006) comes up with a semblable problem of distribution
network design[3]. Nevertheless, the problem of cross-docking sites is not investigated by Amiri, but the problem of best sites of factories and warehouses is discussed. Compared with all kinds of papers, it can be found that the network design problem seems to be related with vehicle routing design for the supply chain management of multiple stores. In the research by Gaur and Fisher (2004), the problem of vehicle routing and delivery scheduling is discussed based on the case study of Albert Heijn supermarket in Holland[4]. This paper raise a question how to determine the weekly delivery schedule and vehicle routes from central distribution center to each retailing store at minimum cost and finally put forwards a solution by formulating a mathematical model. Shojaie and Hajghasem (2016) formulate a distribution model which is subject to a confined number of vehicles with a variety of capacities. The objective is to meet the shipping requirements with the minimal shipping cost[5].

The papers mentioned above do not consider inventory problem. In the research by Federgruen and Anily (1990), it explores the distribution network with one warehouse and many dispersed retail stores[6]. Federgruen and Anily (1990) build a mathematical model with an objective of minimizing the shipping cost and inventory cost. Daskin, Coullard and Shen(2003) investigate the problem of joint shipping and inventory problem with the assumption of single supplier and multiple retailers[7]. In consideration of the risk-pooling, some retail stores are viewed as distribution centers. They formulate a nonlinear integer-programming model so as to make it clear which retail stores will serve as the distribution centers and determine the inventory levels, and then restructure this model into a set-covering integer-programming model. The decision on distribution network and inventory is made by Garrido and Miranda (2004) [8]. They use continuous inventory review policy to calculate inventory levels, taking account of holding cost. Garrido and Miranda (2008) investigate a model which combine location, inventory and distribution problems[9]. The solution is based on Lagrangian relaxation. In another literature by Garrido and Miranda (2009), they also explore the problem of transportation network planning and stock management[10]. Two steps make up the optimization model: the first step optimizes the service level; on the foundation of the fixed service level in step 1, the second step seek for the distribution and inventory decisions. Taking holding cost, reorder point and safety stock into consideration, the inventory model is formulated in view of Economic Order Quantity. For Manzini and Gamberini (2009), they compare the minimal inventory cost and maximal customer service level, which is actually a cost trade-off. Then, they propose an optimization model considering the cost trade-off[11]. The safety stock levels are rationed in the shipping network planning. The above literature consider a problem of joint shipping and inventory.

2. Methodology and models

In our research, we are designed to formulate a model considering distribution and inventory decision in multiple stores. The quantitative model is based on a set of variables that vary within a specific range. We set maximal profit as the objective, formulating a non-linear programming model, subject to two constraints: supply capacity of warehouses and customer demand. Hence, it is essential to reckon consumer demand for all items in each store. In the first place, we presume that customer choice behavior is based on the maximal utility and we will build the demand model under multinomial logit model (MNL). Secondly, only customer purchasing behavior is considered, i.e. there is no substitution to occur. Specifically, once there is no consumer’s first preference in the shelf, the customer will leave the shop without any purchase (Maddah, Bish and Munroe, 2011)[12]. Next, we discuss inventory problem on the background of newsboy model. Using MNL choice to calculate the demand, we can followingly enumerate average demand and standard deviation. Then, we are able to obtain the inventory by combining critical ratio and checking standard Normal Distribution value. Because of the different unit delivery cost from different depots to different retailers, we generate demand for each item from all warehouses. Therefore, we make a non-linear programming formulation that focuses on solving the problem of joint distribution decisions and optimal inventory levels. All possible shipping connections are listed and the flowing quantity between each connection is the decision variables. Distribution result indicates the distribution quantity from each warehouse to each store. The optimal inventory depends on customer demand.
2.1. MNL model

On the foundation of Maddah & Bish (2007)[13], a parallel demand model is made in this section. Let N= {1, 2, 3, … n} denote the general set of items in a warehouse. Let Sm ⊆ N denote the set of items provided by store m. The price of items in a store can be denoted by P= {P1, P2, P3, ..., Ps}. The arrivals in a shop contributes to the demand for items in S. Assume that at most one item is picked by a consumer from S in a category, and thus the maximum surplus can be obtained. We can use Ut=αt-Pt+εt to indicate the consumer expected utility (surplus) for item i∈S, where αt represents the mean reservation price of item i and Pt is the retail price of item i in a shop. Generally speaking, buying or not mostly depends on the evaluation of the value of products. This assessment can be seen as reservation price, and we use εt, i ∈ S ∪ {0}, to express the Gumbel error variable with mean 0 and shape parameter 1 Maddah, Bish & Tarhini, 2014). If there is no-purchase behavior for a customer, let U0 = u0 + ε0 represent the utility. u0 denotes the mean utility for the no-purchase behavior.

We use qi(S, P) = Pr(Ui = max j∈S∪{0} Uj) to express the probability that a consumer purchases item i, and no-purchase probability can be indicated by q0(S, P) = 1 – Σj∈S qi(S, P). We use Gumbel distribution function and simplified functions (Mahajan & van Ryzin, 1999; Shmoys, Shen & Rusmevichientong, 2010; Tarhini, Bish & Maddah, 2014) are listed below[14-16].

\[
q_i(S, P) = \frac{e^{(\alpha_i - Pt)}}{\sum_{j \in S} e^{(\alpha_j - Pt)}}, i \in S, \tag{1}
\]

\[
q_0(S, P) = 1 - \sum_{i} e^{(\alpha_i - Pt)}
\]

In our choice model, there are two assumptions: (1) the retail price leads to the consumers’ purchasing decision, independently of the inventory level; (2) there is no-purchase behavior if consumers’ first choice of item (in S) is stocked out. We use λm to denote the mean arrivals at store m. λqi (S, P) is the expected demand for item i. It can be supposed that demand Xi follows Normal random variable with mean λqi (S, P) and standard deviation √λq(S, P) according to Poisson process (rate λ), when demand is generated from consumer arriving. The function X_i = λqi (S, P) + √λq(S, P)Z_i represents the demand for item i, where each Z_i follows standard Normal Distribution.

\[
q_0(S, P) = 1 - \sum_{i} e^{(\alpha_i - Pt)}
\]

As for cost, shortage cost, holding cost and salvage value for each item can be assumed to be zero. Procurement costs of items in a store can be expressed by C= {C4, C2, C3, ..., Cn}. According to newsvendor model, the underage cost of item i can be given by C^i = Pt - C_i, and overage cost of item i can be denoted by C^0 = C_i - S_i. Thus, critical ratio of item i is C^i = (Pt - C_i)/Pt. We can use yi(S, P) to indicate expected inventory level of item i and Π(S, P) represents the expected profit from S.

\[
y_i(S, P) = \lambda q_i (S, P) + \sqrt{\lambda q_i (S, P)} \Phi^{-1} \left( 1 - \frac{C_i}{P_t} \right) \tag{2}
\]

\[
\Pi(S, P) = \sum_{i \in S} \Pi_i (S, P)
\]

\[
= \sum_{i \in S} \left[ \lambda q_i (S, P) (P_i - C_i) - P_i \theta_i (P_i) \sqrt{\lambda q_i (S, P)} \right] \tag{3}
\]

\[
\theta_i (P_i) = \Phi \left( \Phi^{-1} \left( 1 - \frac{C_i}{P_t} \right) \right), C_i < P_t \tag{4}
\]

ϕ(.) refers to the density of the standard normal random variable and Φ(.) denotes the cumulative distribution function of the standard normal random variable. The expected profit of item i:
\[ \Pi_i(S, P) = \lambda q_i(S, P)(P_i - C_i) - P_i \theta_i(P_i) \sqrt{\lambda q_i(S, P)} \quad (5) \]

The maximal expected profit from S can be obtained by using equation (6) below. Next, we can enumerate the optimal inventory level by equation (2). The objective function maximal profit \( \Pi^* \) is expressed by equation (6)

\[ \Pi^* = \max_{S \in N} \max \{ \Gamma_i(S, P) \} = \Pi(S^*, P) \quad (6) \]

2.2. Distribution & inventory

We mainly study the problem of joint distribution decisions and optimal inventory in this section. We suppose there is the same price for an item in different stores, so does the cost. On the condition of fixed price, purchasing cost and unit shipping cost of each item, our model provides a solution to optimal shipping decision with maximal profit. Suppose there are multiple retail stores and different warehouses.

Let \( M = \{1, 2, 3 \ldots r\} \) denote the multiple stores.

Let \( W = \{1, 2, 3 \ldots d\} \) denote warehouses.

Let \( X^i_{wm} \) denote the quantity of item i distributed from warehouse w to store m.

Let \( C^i_w \) denote the purchasing cost of item i.

Let \( C^i_{wm} \) denote the unit shipping cost of item i from warehouse w to store m.

Let \( H^i_w \) denote the supply capacity of item i in each warehouse w.

Let \( D^i_m \) denote the demand for item i in each store m.

\[ N = \{1, 2, 3 \ldots n\}; \quad M = \{1, 2, 3 \ldots r\}; \quad W = \{1, 2, 3 \ldots d\}; \]

Max.

\[ \Pi = \sum_{i \in N} \sum_{m \in M} \sum_{w \in W} (P^i_m - C^i - C^i_{wm})X^i_{wm}, \quad (7) \]

S.t.

\[ \sum_{m \in M} X^i_{wm} \leq H^i_w, \quad \forall i \in N, w \in W, \quad (8) \]

\[ \sum_{w \in W} X^i_{wm} = D^i_m, \quad \forall i \in N, m \in M, \quad (9) \]

\[ X^i_{wm} \geq 0, \quad \forall i \in N, w \in W, m \in M, \]

The decision variable is flowing quantity of each distribution network connection \( X^i_{wm} \).

Equation (7) the objective function enables the total profit to maximize when products are transported from warehouses to retailers. In equation (8), it restricts outbound capacity of depots. In equation (9), the second constraint function enables the demand for items in each store to be meet.

According to demand model, \( D^i_m \) can be denoted by:

\[ D^i_m = \lambda \frac{e^{(\alpha_i - P_i)}}{\sum_{j \in S_m} e^{(\alpha_j - P_j)}} + \lambda \frac{e^{(\alpha_i - P_i)}}{\sum_{j \in S_m} e^{(\alpha_j - P_j)}} \Phi^{-1} \left( 1 - \frac{C^i + C^i_m}{P_i} \right) \quad (10) \]

As a consequence, the final non-linear model in multiple stores can be given as follows:

Max.
\[ \Pi = \sum_{i \in N} \sum_{m \in M} \sum_{w \in W} (p_i^m - c_i^m - c_{wm}^i) x_{wm}^i, \quad (11) \]

S.t.
\[ \sum_{m \in M} x_{wm}^i \leq h_w^i, \quad \forall i \in N, w \in W, \quad (12) \]
\[ \sum_{w \in W} x_{wm}^i = \frac{\lambda e^{(\alpha_i - p_i)}}{1 + \sum_{j \in S_m} e^{(\alpha_j - p_j)}} + \frac{\lambda e^{(\alpha_i - p_i)}}{1 + \sum_{j \in S_m} e^{(\alpha_j - p_j)}} \Phi^{-1}\left(1 - \frac{c_i^m + c_{wm}^i}{p_i}\right), \forall i \in N, \quad m \in M, \quad (13) \]

Through this model, we can obtain the flow quantity of each item in each store with maximal profit. The optimal inventory level of all products can be denoted by:
\[ \tau = \sum_{w \in W} x_{wm}^i \quad (14) \]

3. Conclusion
In this paper, we put forward a method to solve the problem of distribution and inventory decisions in multiple stores, which contributes to the management of distribution and inventory in a multi-retailing supply chain. We propose a solution to a specific distribution network planning and an optimal inventory decision. We succeed to combine two-field literature: distribution and inventory study.

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