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Two-stage DEA in banks: Terminological controversies and future directions

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Given the importance that two-stage Data Envelopment Analysis (DEA) models have attained in recent years, this paper presents a systematic review of the literature on the topic focusing on the banking industry. We discuss the two-stage terminology itself, which is not yet consolidated. We also discuss the current state-of-the-art and present opportunities, as well as challenges, for future studies. We analyse 59 papers, divided them into ten classes that cover various perspectives of two stage DEA studies, such as the economic context, geographic region of the banking units, methodological characteristics, and type of the models, either internal or external. Additionally, we investigate several controversial points regarding two-stage DEA models, such as the variable selection approach, the technique used in the second stage, and the possible impact of non-discretionary variables on efficiency. Results of the literature review indicate the lack of a uniform or universal terminology for two-stage DEA models in the baking industry. Moreover, the main objective of most papers involves extending or improving DEA models. Radial models, with variable returns of scale, and the intermediation approach are the most frequent configurations. Finally, we identify seven gaps in the literature for both internal and external two-stage DEA models and two specific gaps to external ones. Each gap is discussed in depth in the text and can be considered opportunities for future studies.

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1. Introduction

The banking sector is one of the most complex industries, and it is one of the main contributors to a country's wealth (Paradi, Rouatt, & Zhu, 2011, p. 99). Wang, Huang, Wu, and Liu (2014, p. 5) indicated that this sector plays an increasingly critical role in the development of the financial system. Given the relevance of these institutions, bank performance has been a matter of great interest for various stakeholders, such as regulators, customers, investors, and the general public (Fethi & Pasiouras, 2010), especially after the economic collapse of 2007–8 (LaPlante & Paradi, 2015). In the past, analysis of bank performance was done mainly through financial indices, which, according to Zhu (2000), are unsatisfactory measures of performance. With the advances in operational research techniques, this scenario changed with the emergence of two-stage DEA (DEA), which is currently one of the most popular techniques for analysing the efficiency of organizations (Wu, Yang, & Liang, 2006).

DEA consists of a non-parametric mathematical linear programming technique whose objective is to analyse a group of homogeneous production units known as Decision Making Units (DMUs) — which contain the same inputs and outputs — to identify the most efficient organizations and indicate the actions that inefficient ones must take to become efficient. It does not require specifications regarding the type of production frontier, which is constructed based on empirical observations. Little information is needed a priori to apply the model. The strengths of DEA include the following: it is effective in dealing with complex production processes (Schaffnit, Rosen, & Paradi, 1997, p. 270); it has the ability to work with inputs and outputs at different measurement scales (Svitalkova, 2014, p. 645); it has the ability to analyse each DMU individually, comparing them with other DMUs, with the optimization process performed for all DMUs in the sample (Řepková, 2014, p. 589); and it can identify inefficient DMUs, providing an indication of benchmarks (Angelooulos & Georgopoulos, 2017, p. 1172).

1 Put simply, this production frontier can be understood as the existing technology of DMUs in generating outputs from a set of inputs.
Despite their great popularity, traditional DEA models have been criticized for treating the production process like a black box, in which input variables are transformed in the DMU's production process, generating the output variables without an explicit modeling of how this transformation occurs (Färe & Grosskopf, 2000). Additionally, Paradi et al. (2011, p. 100) emphasized that most of the rejections by administrators of suggestions for improvements made by the DEA are due to the model not considering environmental factors outside the organization, which administrators have no control over. In other words, the environment in which the bank is inserted is not considered in the analysis. Often a bank is regarded as efficient simply because it is in a more favorable environment. Consequently, Jebali, Essid, and Khraief (2017, p. 994) emphasized that DEA indices, although consistent, are biased.

Seeking to improve the application of DEA, two-stage DEA models have been gaining prominence in the literature, precisely because they make it possible to overcome the aforementioned limitations. Emrouznejad and Yang (2017) analyzed the most popular keywords in DEA studies from 2015 and 2016 and found that the terminology two-stage DEA was the second-most popular keyword, while the banking sector was one of the most popular fields of study for the application of DEA. Thus, two-stage DEA is an emerging topic in the literature that, to the best of our knowledge, still lacks a systematic review.

Nevertheless, it is important to introduce a caveat regarding the two-stage DEA terminology. When searching for articles with this terminology, different models are identified, many of them with very different purposes. Although the distinction between these models can easily be made by reading the articles, often, this is not so evident when only reading the abstracts. These different models also make it difficult to clearly define exactly what the terminology two-stage DEA represents. In the study conducted by Emrouznejad and Yang (2017), it was not clear which type of two-stage DEA was becoming popular.

Hence, when searching for articles addressing two-stage DEA, there is a mixture of different models, although both are called two-stage DEA. When the production process is broken down into several subprocesses, these models are categorized here as internal two-stage DEA models; in turn, the approaches in which two analysis procedures are used, with DEA in the first stage and some other technique, either parametric or not, in the second stage, are called external two-stage DEA models. While the internal models enable the black box problem to be overcome, the external models enable a more complete analysis of DMUs.

In this context, although the two-stage DEA is a relevant research topic (Emrouznejad & Yang, 2017) and used in a large number of studies, results of our survey indicate that the terminology is still not well-established or a common ground. In addition, many studies in expert and intelligent systems have, as main objective, to advance the methodological perspective of the use of the two-stage DEA models, as in Izadikhah, Tavana, Di Caprio, and Santos-Arteaga (2018), Mohtashami and Ghiasvand (2020) and Örkcü, Özsoy, Örkcü, and Bal (2019). However, to the best of our knowledge, the studies did not synthesize and debate the problems raised here. Therefore, our survey of the literature contributes to the topic, by consolidating the state of the art of two-stage DEA models and by pointing out challenges and directions for future studies. We highlight that other review papers exploring expert and intelligent systems, such as Haixiang et al. (2017), Henrique, Sobreiro, and Kimura (2019), Zhang, Liu, Zhang, and Almpamidis (2017) and Zyoud and Fuchs-Hausch (2017), reflect the relevance of consolidating studies aiming at better understanding and mapping specific models and techniques. Therefore, our literature review seeks to carry out a critical and in-depth analysis of two-stage DEA. More specifically, we discuss the terminology on two-stage DEA models, and also provide researchers with gaps in the literature, which are opportunities for studies that further advance the current knowledge in the topic.

Accordingly, given the various studies described as two-stage, what exactly does the literature consider to be two-stage DEA models? When referring to two-stage models in banks, what has been most published regarding this? What is the most frequent technique used in the second stage of external models? How has this topic been discussed over the years? Such questions emerge when analyzing the publications on two-stage DEA models in banks. The contribution of the present study is that it proposes solutions to these questions. We believe that with this review, we can map the existing knowledge on this research theme and stimulate a debate on this emerging topic in the literature.

Regarding the applicability of two-stage DEA models, whether internal or external, this study contributes to the literature by fully exploring how they are applied in banks, identifying the most frequent scope, which can be understood as the approach used for the selection of the variables in the analysis of banks, which, in turn, will determine the model’s input and output variables. Furthermore, we will discuss which is the technique most used in the second stage of external models, the most used DEA model, the economic context and the continents of the most studied banking sectors, the type of study and its objectives, which authors produce the most research on the topic, and which publications are the most relevant.

The discussion regarding the research scope is of great importance because — as shown by Holod and Lewis (2011) — considering a variable as an input or an output significantly changes which DMUs are indicated as efficient by the model. Hence, considering that the main variable selection approaches in the literature analyze distinct bank functions and, therefore, assume different inputs and outputs, the importance of comparing studies that have used similar approaches becomes evident.

Another controversial topic in the literature that requires further discussion is the impact of exogenous variables on efficiency. The motivation to address this particular aspect of external two-stage DEA models is centered on the researchers’ recognition that environmental factors or exogenous variables can significantly influence the efficiency scores measured by DEA (Fried, Lovell, Schmidt, & Yaisawarng, 2002, p. 158). Despite the growing interest, the results in the literature regarding this impact have been quite ambiguous in that an environmental variable can have a positive impact on the bank in its role of financial intermediary but a negative impact on the function of offering services to clients, for example, which makes consolidation in the literature difficult. For this reason, as discussed previously, the analysis and comparison of the impact of an exogenous variable on efficiency should consider the scope of the study, that is, the approach used to select the variables. This review will make the results found in the literature regarding these impacts clearer by clarifying the approaches used in the studies and the respective effects of the variables on efficiency.

We emphasize that the goal of our manuscript is not to defend the two-stage DEA model or one technique over the other, but rather to present a discussion on the topic, focusing on bank efficiency studies. More specifically, we analyze diverse challenges for the use of two-stage DEA, including the terminology itself, and the statistical drawbacks, such as the separability problem. We argue that a systematic survey of the literature on this topic is urgent, since despite all the challenges, the number of studies has been quite large, as depicted in Emrouznejad, Parker, and Tavares (2008) and Emrouznejad and Yang (2017),

\( ^2 \) For more information, please see Holod and Lewis (2011).
Our work contribute to the discussion of two-stage DEA models, presenting the state of the art on the subject as well as identifying the challenges in studies using this technique, specially in the banking sector.

Finally, this study is the first conducted on the banking sector that adopts the systematic literature review method developed by Lage Junior and Godinho Filho (2010) and later disseminated by Mariano, Sobreiro, and Rebelatto (2015), Jabbour (2013), Silva, Tabak, Cajuereiro, and Dias (2017) and Henriques, Sobreiro, and Kimura (2018). As highlighted by Mariano et al. (2015) and Jabbour (2013), this method allows us to:

- Identify the main results of the studies analysed and relate them to emerging issues in the theme researched;
- Fully discuss and present the latest innovations regarding the key topics of the theme;
- Identify possible gaps and challenges for future research.

The article is structured as follows: a brief contextualization of two-stage DEA models is performed in Section 2; the research method is presented in Section 3; the classification and coding criteria for the analysed articles are described in Section 4; the results of the bibliometric analysis and coding are discussed in Section 5; and finally, the conclusions are provided in Section 6.

2. Brief summary of two-stage DEA model in banks

Despite the growing interest in two-stage DEA models, as highlighted in Emrouznejad and Yang (2017), several aspects remain ambiguous, including the terminology two-stage DEA model itself. The literature consists of two types of models that are completely different from each other, with distinct purposes, but that are both classified as two-stage DEA models. Given this, herein, we intend to briefly discuss the different approaches and techniques described as two-stage DEA models, categorizing them as either external two-stage DEA models or internal two-stage DEA models.

It is worth highlighting that both internal and external two-stage DEA models have emerged as a response to the limitations of conventional DEA models. In other words, Färe and Grosskopf (2000, p. 35) stated that variations in traditional DEA models seek to suit the application. Accordingly, regardless of the purpose, whether two-stage DEA models involve intermediate variables (internal) or the use of some technique after the application of DEA (external), the analysis will be closer to reality. The use of intermediate variables overcomes the black box problem, whereas the application of another technique after DEA enables a more complete analysis.

2.1. Internal two-stage DEA model

One of the main limitations of traditional DEA models is that they treat the production process like a black box, in which the input variables are transformed within this box to give the output variables. Although this is one of the advantages of DEA, i.e., revealing without needing to impose the structure of the transformation process (Färe & Grosskopf, 2000), in various applications, a more structured model is needed for better application. One great example of this situation is the banking sector. Because it is a highly complex sector (Schaffnit et al., 1997), an improved DEA model is ideal to make it possible to encompass this production process.

Thus, to overcome the black box problem, various researchers have sought to improve traditional DEA models to enable the analysis to be closer to reality. Internal two-stage DEA models represent such an effort — the two stages of the model refer to stages of the production process. Traditional models have only input and output variables, and based on the relationship between these variables, the DEA indicates which DMUs are efficient; in internal two-stage DEA models, the production process is divided into two subprocesses, in which the outputs of the first stage consist of the inputs of the second stage. Fig. 1 shows an example of the production process with intermediate variables. It is worth emphasizing that not necessarily all the outputs of the first stage will be the inputs of the second stage — some outputs may exit or some inputs may enter the system.

The first advance in this direction was made by Seiford and Zhu (1999) — the first study to apply internal two-stage DEA models in banks, which aimed to analyse the profitability and marketability of the 55 largest commercial banks in the United States. Accordingly, efficiency was measured in the first stage, considering profitability, with three inputs, i.e., number of employees, assets, and stockholders’ equity, and two outputs, i.e., profit and revenues. The variables profit and revenues — outputs of the first stage — are the input variables of the second stage — thus referred to as intermediate variables. The outputs of the second stage are market value, total return to investors, and earnings per share. In this stage, the bank’s efficiency in converting its profits and revenues into marketability was analysed.

Akther, Fukuyama, and Weber (2013) studied twenty-one Bangladeshi banks from 2005 to 2008, through a two-stage network Slacks-based inefficiency DEA model. The authors identified that the black box performance models had divergent results from the network DEA. Similarly, Fukuyama and Matousek (2011) showed that the precision and accuracy of DEA results are greater when using network models, compared to traditional DEA models. Kao (2017, p. 177) found that DMUs that had been indicated as efficient using traditional DEA models were not efficient using network models. The authors found that the efficiency of the productive process calculated by the black box models could be overestimated. This issue is more serious when more stages are involved. Lastly, Kao (2014) discusses cases where the overall productive system can be considered efficient, even if its sub-stages are not efficient. Likewise, the author found situations in which a DMU had efficiency rates below another DMU in its sub-stages, but presented superior efficiency scores when analysing from the black box perspective.

Despite the advance generated by the study by Seiford and Zhu (1999), the two-stage DEA model used by these authors — classified by Kao and Hwang (2010) as independent — can have problems related to the intermediate variables, given that by seeking maximization of the outputs in the first stage and minimization in the second, the same variables would be minimized and maximized. To solve this problem, researchers, such as Färe and Whittaker (1995), Färe and Grosskopf (1996b, 1996a), sought to include such intermediate variables in the DEA model itself, which led to the development of Network DEA (NDEA) models, later extended by Färe and Grosskopf (2000), Lewis et al. (2004), Kao and Hwang (2008), Kao (2009), Kao and Hwang (2011), Chen, Cook, and Zhu (2010) and Cook, Liang, and Zhu (2010), among others. In this regard, Fukuyama, Matousek, and Tzeremes (2020) argue that NDEA models can be divided in four main categories:

- Independent: Independent models investigate each stage of the productive process separately, without any relationship between stages;

Factors such as market value, earnings per share, and return to investors are part of the marketability, defined in the study of Seiford and Zhu (1999, p. 1271).
Connected: In Connected models, contrasting with Independent models, the interactions between the stages are taken into account in the calculation of the overall efficiency. Therefore, for a DMU to be overall efficient, it must necessarily be efficient in all stages considered;

Relational: Relational NDEA models, proposed by Kao (2009), consist of a combination of the two previous models. Relational models make it possible to measure the efficiency of each system and the overall efficiency. This category of models assumes an additive or multiplicative relationship between overall efficiency and the stage efficiencies;

Game theoretic: Game theoretic models assume each stage of the productive process as a player in a cooperative or a non-cooperative game.

Thus, the big difference between the network model and the models with independent intermediate variables is that the former includes all the stages of the process in its mathematical formulation; that is, the production process is divided into various sub-processes, with each sub-process being formulated mathematically in the model. Therefore, the network model enables the formulation of the intermediate variables, whereas the other, which consists of an application of basic DEA models at each stage, does not. Avkiran (2009) emphasized that NDEA has great potential for practical application and provides relevant information to managers.

Halkos, Tzeremes, and Kourtzidis (2014) point out that independent models have the burden of not consider connections between stages. However, independent models are less restrictive and generate the highest efficiency scores. Connected NDEA avoid conflicts between stages by considering the interactions between them, whereas Relational NDEA considers any mathematical relationship between the stages. Finally, game theoretic approach is more appropriate when the stages of the production process can be analysed as a game.

NDEA systems can have a serial or parallel structure (Halkos et al., 2014). Productive processes in a serial structure are connected in sequence. Each process uses potential exogenous inputs and outputs from the previous stage and produces potential exogenous outputs and intermediate variables to the next stage. In a parallel structure, the production processes operate simultaneously and independently. There are also NDEA systems in which the productive process is a mixture between the parallel and the serial structure.

In addition, network models can be static, dynamic or shared resources. Kao (2014) points out that static NDEA models analyse a single moment in time, whereas the dynamic NDEA consists of the repetition of the one-period system in subsequent periods, connected by carry overs. New improvements in dynamic NDEA can be seen in the study by Tone and Tsutsui (2014), which proposes an NDEA with dynamic slack-based measure and in the study by Kao (2013), which presented a relational dynamic NDEA.

It is important to highlight that each model is best suited to specific circumstances. Halkos et al. (2014) point out that less restrictive models can result in an overestimation of efficiency when more complex relationships between stages exist. In contrast, more restrictive models, when not necessary, can lead to underestimated efficiency scores. Nevertheless, despite the issues discussed earlier, the traditional DEA models are still a valid tool for analysing efficiency, when the productive system is simple. In the case of the banking sector, a highly complex system (Schaffnit et al., 1997), we recommend using NDEA, since it allows to incorporate the potential interrelationships among variables.

Finally, as a pitfall of the network approach, Chen, Cook, Kao, and Zhu (2014) indicate that in general there are two types of NDEA models: traditional multiplier-based DEA models, focused on DEA ratio efficiency, and envelopment-based NDEA models, focused on the production possibility set. Although for conventional DEA models, these two types of models are dual and equivalent, for NDEA models the duality and equivalence properties do not necessarily hold. The authors recommend that the envelopment-based NDEA model should be applied to determine the projection boundary for inefficient DMUs, whereas the multiplier-based NDEA model should be employed to measure the divisional efficiency. Chen et al. (2014) argue that these two types of NDEA follow different approaches and explore distinct efficiency concepts. The authors further indicate that many models currently used in production possibility set-based network DEA should be re-examined. More specifically, some studies using envelopment models failed to calculate divisional efficiencies. However, this result does not mean that it is impossible to calculate divisional efficiencies, but that more research is needed in order to extend the existing production possibility set-based network DEA and solve this issue.

2.2. External two-stage DEA model

The other branch of the literature refers to external two-stage DEA models, which consist of a second stage outside the production process. This is actually a procedure adopted by the researcher in which the efficiency indices are calculated in the first stage through DEA, and subsequently, these indices are used to power
some other technique, which may be some type of regression (e.g., Ordinary Least Squares or Bootstrapped Truncated Regression), an Analytical Hierarchy Process (AHP), or an Artificial Neural Network (ANN), among others, considering the various possibilities available to the researcher in the second stage. Fig. 2 shows the structure of the external two-stage DEA models. Thus, the main motivations for using external two-stage DEA models, as well as the respective technique in the second stage, include the following:

- Paradi et al. (2011, p. 100) emphasized, many of the rejections by managers of suggestions for improvements made by DEA are because traditional models do not consider that environmental factors, which are external to the organization, influence the results found in the model, and the administrators would have no control over such factors. Therefore, regression techniques are used in the second stage, in which the efficiency index calculated by the DEA model is the dependent variable and the exogenous variables are the independent ones;
- Given that DEA is very sensitive to the presence of outliers and statistical noise, ANNs can be used in the second stage for the purpose of finding data envelopes, which, instead of being based on outliers, are supported by the whole database (Wu et al., 2006). Additionally, the ANN allows the researcher to make predictions through training with the efficiency scores measured by the DEA; i.e., by being repeatedly exposed to the data, neural networks learn the relationship between the input and output variables of the DMUs (Athanassopoulos & Curram, 1996);
- Recognizing the importance of including qualitative indicators in the efficiency analysis, Azadeh, Ghaderi, Mirjalili, and Moghaddam (2011) used an external two-stage DEA model that integrated DEA with AHP, a multi-criteria decision technique developed by Saaty (1980) that allows modelling a complex problem in a hierarchical structure composed of different levels, with the top of the hierarchical structure representing the overall goal, while the lower levels consist of all possible alternatives (Sevkli, Koh, Zaim, Demirbag, & Tatoglu, 2007). With this, AHP reduces the complexity of the decision-making process to a series of simple comparisons and rankings.

For most of the studies analysed, it was clear whether the study involved an external two-stage DEA model or not. However, in other cases, this was not so obvious. There is a grey area that lacks a clear and accurate definition about when a study can be classified as an external two-stage DEA model. Exemplifying this situation, Wu et al. (2006) — who categorized their work as two-stage DEA — applied DEA to measure bank efficiency and, subsequently, used these efficiency scores to train an ANN. In turn, Mostafa (2009), despite basing his study on the study of Wu et al. (2006), did not categorize it as thus. After careful reading, we considered the study of Mostafa (2009) to be an external two-stage DEA model, considering that the scores of the DEA — which were measured in the first stage — were used in an ANN model in the second stage, as in Wu et al. (2006).

This difficulty in classifying the studies as involving models that are either external two-stage DEA or not highlights the importance of systematically analysing the theme — this work is an initial effort towards this, but restricted to the banking sector. If there was a clear definition for such models, there would be no difficulty in identifying which studies do and do not consist of an external two-stage DEA model.

It is also important to mention another ambiguous aspect of external two-stage DEA models, i.e., the impact that exogenous variables have on efficiency. Although analysing this effect is only one of various possible objectives in the second stage, this issue needs a more in-depth discussion, given that the literature presents quite controversial results, often without the necessary care when comparing results.

Authors have often used previous studies to support certain results found, although the studies in question use different approaches or measure different types of efficiency. It is known that variable selection strongly influences the results found. Holod and Lewis (2011) showed that in a study following the production approach, when keeping other variables constant, a larger number of deposits would lead to higher efficiency scores indicated by the DEA model, given that deposits would be a model output. By contrast, a researcher who had followed the intermediation approach and treated deposits as an input, keeping the other variables constant, would obtain higher efficiency scores when the bank had a fewer number of deposits.

This problem becomes even more interesting when considering two external stages. A study that followed the intermediation approach — that is, studied the bank’s role as a financial intermediary and analysed efficiency through the Banker, Charnes, and Cooper (BCC) model, which measures efficiency related only to administrative issues — and performed a regression in the second stage to analyse the impact that exogenous variables have on efficiency may encounter different results for this impact when compared to another study that adopted different criteria.

An example of this is to compare the situation above with another study that analysed banks in their function of offering services to customers, that is, a study that followed the production approach of Benston (1965) and measured efficiency through the Charnes, Cooper, and Rhodes (CCR) model, which measures technical efficiency, also referred to as overall efficiency. The effect of the exogenous variable in question may vary from one study to another simply because of the methodological differences adopted. It is not surprising, therefore, that the literature presents quite controversial results in this respect.

This review will guide this discussion by providing the results found by researchers who have addressed the two-stage DEA model in the banking sector, highlighting all the methodological aspects adopted by them, that is, the approach used to select the model’s variables, the type of efficiency analysed, the non-discretionary variables used and their respective impacts, considering the peculiarities of each study.

In addition to the problems mentioned above, external two-stage DEA models are sensitive to the problem of separability. Simar and Wilson (2007) found that traditional regression techniques in the second stage were not appropriate and proposed a bootstrap truncated regression model as an alternative, despite recognizing that this option could suffer from the same issue. As Daraio, Simar, and Wilson (2018) point out, if the condition of separability does not hold, the results of the second stage would present drawbacks and be difficult to analyse.

The issues discussed in Daraio et al. (2018) and Simar and Wilson (2011) are crucial for external two-stage DEA models. Such models are being extensively used in the literature, as Emrouznejad and Yang (2017) argue in their review, and may present statistical drawbacks as they fail to maintain the hypothesis of separability. In our review of the application of models with the terminologies Two-stage DEA in banks, we bring the discussion on separability aiming to present the state of the art. We also present strengths and weaknesses of the models, identifying gaps that signal opportunities for future studies. We argue that the tool developed in Daraio et al. (2018) to test separability, similarly to the tool indicated by Kneip, Simar, and Wilson (2016) to test constant versus variable returns to returns, can convey relevant information and should be considered in future research of external two-stage DEA models.

It is also important to reference a stream of research in the literature, which started with Daraio and Simar (2005, 2007), and
makes it possible to cope with separability problems. Daraio and Simar (2005) developed a conditional efficiency model that allows the estimation of the efficiency in the presence of environmental variables. These environmental variables will be neither inputs nor outputs in the production process. Examples of application of this approach in the banking sector are discussed, for instance, in Degl’Innocenti, Matousek, Sevic, and Tzeremes (2017), Degl’Innocenti, Grant, Sevic, and Tzeremes (2018), Kevork, Pange, Tzeremes, and Tzeremes (2017), Kevork, Kollias, Tzeremes, and Tzeremes (2017). Matousek and Tzeremes (2016) and Tzeremes (2017). Since conditional efficiency is not a two-stage DEA model, we did not include it in the survey of papers. However, we highlight that conditional efficiency models represent a relevant mechanism to deal with separability.

Table 1 presents a description of studies that used two-stage models — either internal or external — in the banking sector. It contains a brief review of each study identified, considering the literature review criteria discussed in Section 4. The studies were ordered from the oldest to the most current to show how topics related to two-stage DEA models in the banking sector have been discussed in the literature over time. The study by Luo (2003) — the oldest of the sample — was classified as number 1, Barth and Staat (2005) as number 2, and so on. Table 1 also lists the number of citations per article according to either Scopus or — if the article is not in this database — Web of Science up to August 2018.

3. Method

A review of the literature on DEA is nothing new. Emrouznejad et al. (2008), Emrouznejad and Yang (2017), Mariano et al. (2015), Sueyoshi, Yuan, and Goto (2017), Mardani, Zavadskas, Streimikiene, Jusoh, and Khoshnoudi (2017) and Liu, Lu, Lu, and Lin (2013b, 2013a) conducted literature reviews on DEA in various areas. However, to our knowledge, only Fethi and Pasiouras (2010) and Paradi and Zhu (2013) focused on the banking sector, and neither specifically reviewed two-stage DEA models in banks, a topic that has been gaining considerable prominence, as highlighted by Emrouznejad and Yang (2017).

According to Ferreira, Sobreiro, Kimura, and Barboza (2016, p. 7), the literature review is an important tool for gathering the results of previous studies on a certain theme, producing an in-depth analysis of the main studies. This method is particularly relevant for mapping the main topics studied and providing a complete view of the existing knowledge from the articles on the subject analysed, as well as for identifying the existence of possible gaps and opportunities for future studies. Accordingly, Jabbour (2013, p. 145) indicated that this technique identifies challenges for the development of future studies; that is, after identifying the characteristics of how the literature has been discussing a theme, it is possible to discover gaps and opportunities in topics that are not being discussed to the same degree as others. In addition to the previous observations, the review done here is important because, despite the existence of literature reviews regarding DEA in banks, namely, Fethi and Pasiouras (2010) and Paradi and Zhu (2013), neither have specifically analysed two-stage DEA models in banks.

Emrouznejad and Yang (2017) identified the most popular keywords in publications from 2015 and 2016 and found that in second place were keywords such as two-stage models and efficiency decomposition, which is one of the functionalities of these models, whereas in fourth place were words such as bootstrap and bootstrapping. Additionally, Emrouznejad and Yang (2017) found that the banking sector is the field of study with the second highest number of studies. Linking these two aspects, there is an emerging topic in DEA that, to the best of our knowledge, has not yet been systematically reviewed, i.e., two-stage DEA models in banks. Therefore, by reviewing this topic, this work contributes to the literature by presenting the state of the art on this topic and providing an agenda for future studies.

In spite of the popularity and the various years of research, questions that are quite frequent in studies on DEA in banks — such as what the orientation should be (input or output) and at what scope the bank should be analysed, which, in turn, will influence variable selection — still have no answers. In the particular case of two-stage DEA models, as discussed in Sec. 2, various other aspects require further discussion, for example, which type of two-stage DEA model is used most often and, in the case of external two-stage models, which technique is most popular in the second stage and what the impacts of non-discretionary variables on bank efficiency are. Perhaps the clearest aspect for researchers is that the CCR model should only be used when all companies are operating at the optimal scale level (Fethi & Pasiouras, 2010; Řepková, 2014; George Assaf, Barros, & Matousek, 2011); therefore, models that work with variable returns to scale have been prioritized in more recent studies (Fethi & Pasiouras, 2010, p. 191), but it is still not possible to state that one model is superior to another.

Considering the aspects without consensus in the literature, the present study makes a contribution by providing guidance to researchers in future studies, summarizing how the literature has...
Table 1
Brief description of objectives and results of each analysed study.

| N. | Study | Brief Summary | Number of Citations |
|----|-------|---------------|---------------------|
| 1  | Luo (2003) | By analysing three gaps in the literature, the authors examine the profitability and marketability efficiency of 245 US banks, as well as verify whether the bank location impacts its efficiency. The results suggest that banks' greatest source of inefficiency is marketability. The bank's location is unrelated to efficiency ratios, and overall technical efficiency can be used as a predictor of the likelihood of bank failure. | 134 |
| 2  | Barth and Staat (2005) | The study analyses the efficiency of 31 agencies from KölnBank in Germany. Verifying the impact of non-discretionary variables such as branch area, public transport, competition and others. The two-stage models were able to more accurately evaluate efficiency compared to one-stage models, and none of the environmental variables were statistically significant. | 9 |
| 3  | Wu et al. (2006) | The paper combines DEA with Neural Network (DEA-NN) to analyse the relative efficiency of branches of a Canadian bank. The results found by the proposed model are comparable to those of the traditional DEA models. The proposed model leads to a more robust boundary and identifies more efficient DMUs, but it is inferior in identifying benchmarks. | 162 |
| 4  | Pasiouras (2008) | The study evaluates bank efficiency in different countries (different contexts) by checking the impact of regulatory factors on efficiency. The results provide evidence for relevance of the three pillars of Basel's II Accord. Larger banks with lower funding loans showed better technical efficiency indices under all circumstances. Country-specific variables had a statistically significant impact on efficiency. | 105 |
| 5  | Mostafa (2009) | The paper analyses the efficiency of the largest Arab banks through the integration of DEA with neural networks (NN). NN have great potential to assess the relative efficiency of banks because of their flexibility and robustness. | 51 |
| 6  | Thoraneenitiyan and Avkiran (2009) | The authors investigate the relationship between cost and efficiency factors with bank efficiency in a sample of 110 banks in five Asian countries in the period 1997 to 2001. Bank restructuring does not necessarily increase banks' efficiency. Domestic M&A perform better on efficiency than foreign acquisition. | 30 |
| 7  | Staub et al. (2010) | The study estimates cost, allocative and technical efficiencies of Brazilian banks in the post-privatization period (2000–2007) through a three-stage model. Brazilian banks have a high degree of inefficiency compared to other countries. Stated owned banks were more efficient than private ones, and foreigners showed higher levels of cost inefficiency. Size is not an important variable that impacts efficiency. | 87 |
| 8  | Tsolas (2010) | The study measures the overall efficiency, comprised by the efficiency of profitability and effectiveness, of the 50 best branches of a Greek bank. Nineteen branches were efficient in profitability and effectiveness. Regarding overall efficiency, the main cause of inefficiency was profitability. The bank's performance can be largely improved by changing practices in branches identified as worst DMUs. | 20 |
| 9  | Azadeh et al. (2011) | The authors analyse and suggest strategies to optimize the productivity of workers from various branches of the Bank of Industry and Mining in Iran. Integrating AHP and DEA, the study verified that a large part of the inefficiency of the branches is due to low work quality level and high number of training hours. The proposed analysis technique leads to better results than others, exploring both qualitative and quantitative data. | 24 |
| 10 | Holod and Lewis (2011) | The authors propose a DEA model that considers the variable deposits as an intermediate variable. The results show that the decision to define deposits as input or output significantly affects the indexes and the efficiency ranking of traditional models and, for this reason, the method developed by the authors managed to avoid this dilemma. | 51 |
| 11 | Paradi et al. (2011) | The authors apply a two-stage DEA model to analyse the efficiency of 816 bank branches in order to reconcile the results indicated by this model with the opinions of the managers of these organizations. The efficiency indexes presented considerable variations among the different regions analysed. Branches in smaller markets were more efficient. Considering different approaches for analysing efficiency allowed finding results with greater consistency. | 111 |
| 12 | Shahroodi et al. (2011) | The study analyses the efficiency of 20 branches of Saderat Bank in Iran, pointing out which units are efficient and inefficient. Size is not an important variable that impacts efficiency. | 2 |
| 13 | Liu and Chen (2012) | The authors identify bank failures through a two-stage DEA model of worst practices, which makes it possible to work with negative outputs. The empirical analysis showed the applicability of the model to predict potential bank failures. The model predicted a number of potential banks to fail similar to what has been observed in Taiwan. | 2 |
| 14 | Magheryeh and Awartani (2012) | The study analyses the influence of reforms in the banking sector of six countries, whose objectives were to strengthen the financial and economic integration between these countries. These measures had a significant impact on the efficiency and homogenization of the banking sectors of the countries analysed. | 17 |
| 15 | Shyu and Chiang (2012) | The authors analyse the true managerial efficiency of 123 branches of a bank in Taiwan, through a three-stage model, adjusted for environmental variables and statistical noise. Traditional DEA models overestimated efficiency ratios. The main cause of branch inefficiency was the operated scale. Location did not show significant impacts on efficiency. Branches with greater scope of action and volume of deposits were more efficient. | 23 |
| 16 | Yang and Liu (2012) | The study integrates NDEA with Fuzzy to measure branch performance in Taiwan's banking industry. Most of the branches analysed had a better performance in the first stage of productivity. Interest cost is the largest factor in the first stage, while fund transfer income and interest income are key factors of the second stage. | 43 |
| 17 | Halkos and Tzeremes (2013) | The study examines the efficiency of 18 Greek banks in a period of Greece's fiscal crisis by checking how the banks' efficiency would react to possible Mergers and Acquisitions (M&A). The results suggest that, analysing the year before and the year after the crisis, M&A did not generate operational efficiency in the short term. M&A between efficient banks will not necessarily generate an efficient bank. | 32 |
| 18 | Kholousi (2013) | The paper measures the efficiency of 16 branches in Iran using an integrated DEA model with AHP. The location of the branches was a key factor of efficiency. The strengths of one branch can serve as benchmarking for the others. The use of AHP together with DEA provided more consistent results. | 1 |
| 19 | Lin and Chiu (2013) | The authors apply an integrated model for the measurement of bank efficiency in Taiwan through Independent Component Analysis (ICA) and the Network Slack-Based Measure (NSBM). Three dimensions of efficiency were analysed: production efficiency and profitability, efficiency, service efficiency and profitability efficiency. The results indicate that the proposed model was able to determine the main causes of bank inefficiency, presenting an excellent discriminatory feature. The study evaluates the risk management performance of Chinese banks in terms of their contribution to profitability through a three-stage NDEA model. The inclusion of the proxies for risk improved the efficiency measurement. | 69 |

(continued on next page)
| N. | Study | Brief Summary | Number of Citations |
|----|-------|---------------|---------------------|
| 21 | Özdemir (2013) | The study integrates DEA and Analytical Network Process (ANP) to evaluate the efficiency of commercial banks in Turkey, with the possibility of incorporating managerial preferences into the model. The proposed integration presented several advantages over traditional models, such as considering multiple performance measures. The weights of the model can be based on the preferences of the managers. | 1 |
| 22 | Xu (2013) | The paper verifies the impact of size and market power on the efficiency of 16 Chinese banks in the period from 2007 to 2011. The results found that size is a determinant of the efficiency of banks. A favourable economic environment (real GDP growth) also has a positive influence on efficiency. | 0 |
| 23 | Ebrahimnejad et al. (2014) | The authors propose a three-stage DEA model with two independent parallel stages, where the outputs of these stages serve as input to the third stage, with the presence of undesirable outputs. In a case study of 49 People’s Bank branches, the study corroborates the effectiveness and applicability of the model in bank efficiency studies. | 29 |
| 24 | Huang et al. (2014) | The study proposes a two-stage DEA Network Slack-Based Measures (NSBM) model with undesirable output aiming to open the black box of the production process. The proposed model has a better applicability than traditional models. All hypotheses suggested for efficiency determinants were confirmed for overall efficiency. On the other hand, the hypotheses could not be accepted when each stage were analysed individually. | 9 |
| 25 | Piot-Lepetit and Nzongang (2014) | The authors analyse the efficiency of Micro Financial Institutions (MFIs) both in the execution of financial tasks and in their role of coping with social problems, through, for instance, loans to poor people. In 46% of MFIs, there was no trade-off between the two dimensions analysed. Directives were given in order for MFIs to improve both their financial and social efficiencies. | 18 |
| 26 | Wang et al. (2014) | The study analyses the efficiency of the 16 largest Chinese banks in the period from 2003 to 2011, which corresponds to a reform in the Chinese banking sector. The authors consider deposits as intermediary variable and unrealized loans as undesirable output. The two-stage model was able to explain more appropriately the inefficiency of the banks than conventional DEA models. The efficiency of the banks increased during the period analysed because of the reform. State owned banks were more efficient before the reform, however difference to other banks decrease afterwards. | 76 |
| 27 | Wang et al. (2014) | The authors investigate the relationship between bank efficiency and intellectual capital in a sample of 16 US banks through a two-stage model. Profitability is included in the first stage and creation of value is included in the second. The authors found evidence that intellectual capital positively impacts efficiency. | 21 |
| 28 | Wanke and Barros (2014) | The study evaluates the 40 largest banks in Brazil regarding the optimization of costs and productive efficiency, establishing a connection between these two variables. Brazilian banks tend to be more efficient at translating administrative expenses and personnel expenses into shareholders’ equity and fixed assets than at managing physical and human resources. M&A, size and the fact that the bank is state owned are also variables that influence efficiency. | 48 |
| 29 | An et al. (2015) | The authors measure the efficiency of 16 Chinese banks in the period 2008 to 2012 through a two-stage DEA-SBM approach, in which the first stage was called a deposit generator and the second as a deposit user with the presence of undesirable output. The results indicate that efficiency has increased during these five years due to banks’ improvements in deposit creation. | 11 |
| 30 | Chao et al. (2015) | The study applies the Dynamic Network Slack-Based Measure Data Envelopment Analysis Model (DNSBM) to evaluate the performance of Taiwanese banks during the period 2005–2011. Using a three-stage model, the results indicate that banks have lost profitability since the 2008 crisis, while the creation of intellectual capital increased from 2008 to 2010. | 7 |
| 31 | Khalili-Damghani et al. (2015) | The paper evaluates the relative efficiency of customer services in 30 branches of an Iranian bank, through a hybrid model based on Multi-Criteria Satisfaction Analysis (MUSA) and NDEA. The proposed method was able to identify which branches were able to meet consumers’ expectations. | 0 |
| 32 | Fukuyama and Weber (2015) | The authors assess the dynamic efficiency and productivity of Japanese commercial banks, maximizing desirable outputs and minimizing undesirable outputs (Non Performing Loans). For a 3 year dynamic window, the inefficiency of Japanese banks ranged from 19.5% of average outputs and inputs in 2007–2009 to 21.5% of average outputs and inputs in 2008–2010. Banks could become more efficient by increasing the volume of deposits. | 17 |
| 33 | Kwon et al. (2015) | The authors combine two empirical data analysis techniques to evaluate and predict performance improvements for 181 US banks. The proposed model contributes, in an impactful way, to the managerial process of decision making. | 19 |
| 34 | Shavtari et al. (2015) | The authors measure the efficiency of Islamic Yemeni commercial banks, analysing the stability and efficiency of the sector. The study also checks for variables that may be affecting efficiency. The results suggest that the recent reforms adopted by the Yemeni government have failed to improve the sector since the efficiency scores were low. Islamic banks performed better than commercial banks. | 2 |
| 35 | Suflan (2015) | The study estimates the efficiency of Malaysian banks from 1999 to 2008, analysing the impact of several environmental variables such as liquidity, risk, size, profitability, capitalization level, macroeconomic conditions. Size, non-interest income, foreign control, and capitalization have a positive impact on productive efficiency. State owned banks were more inefficient. Credit risk and liquidity were not statistically significant. | 0 |
| 36 | Tsolas and Charles (2015) | The paper incorporates stochastic models in the DEA to analyse the efficiency of Greek banks in a period of crisis of the country incorporating variable related to risk. The model measures efficiency considering the possibility of stochastic variables in the DEA model. In addition, the model is able to control, from the efficiency indexes, the favourable operating conditions. | 21 |
| 37 | Wang and Lu (2015) | The study analyses the efficiency of banks in Taiwan by pointing out the marginal benefits of information technology (IT). In addition, considering the Basle III Accord, the impact of some proxies for risk on efficiency is measured. Most banks need to improve their returns to scale on IT inputs. The effect of risk proxies on efficiency was not universal in the study. | 0 |
| 38 | Nguyen et al. (2016) | The authors measure the cost efficiency of 32 Vietnamese banks in the period from 2000 to 2014, verifying the impact of two reforms in the banking sector, namely: partial acquisition by foreign banks and entry into the stock market. In addition, the study also analyses the impact of other environmental variables. Efficiency showed a slight upward trend in the period. Banks listed on the stock exchange or partially acquired by foreign capital presented better efficiency ratios. | 3 |
| 39 | Rayeni and Saljooghi (2016) | The authors investigate the relationship between efficiency and risk through a three-stage model in a case study with 14 branches. Risk causes banks to seek enhancement of their operations, thereby increasing their technical efficiency. Therefore, risk is positively related to efficiency. | 2 |
| N.  | Study                                                                 | Brief Summary                                                                                                                                                                                                                                                                                                                                 | Number of Citations |
|-----|----------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| 40  | Stewart et al. (2016)                                                | The study analyses the determinants of efficiency of Vietnamese banks from 1999 to 2009. The largest banks are more efficient than the medium and small banks, with the latter being the most inefficient. Profitability had a positive impact on efficiency, while the number of branches and number of years in operation had the opposite effect. As far as global efficiency is concerned, private banks are more efficient than state owned banks. | 15                  |
| 41  | Wanke et al. (2016)                                                  | The study estimates, through a two-stage model, the impact on virtual efficiency of M&A of Mozambique’s banks and also analyses the results taking into account whether the bank is state owned or has foreign control. The results indicate that control of the bank (state or foreign) affects efficiency and that mergers should occur between banks of different type of controls. M&A involving the analysed banks may lead, in most cases, to the situation of decreasing returns of scale. | 3                   |
| 42  | Wanke et al. (2016)                                                  | The study uses a new Fuzzy-DEA model to evaluate bank efficiency in Mozambique for the years 2003–2011. Several aspects explain bank efficiency in Mozambique as, for instance, labour price, capital price and deposits. The effect of the environmental variables was ambiguous, depending on the degree of uncertainty of the model. Banks should reduce the number of employees and make initiatives to leverage capital. | 5                   |
| 43  | Aggelopoulos and Georgopoulou (2017)                                | The authors analyse the efficiency of a bank’s branches in Greece during different periods of the economy, taking into account expansion followed by strong recessions. The study also verifies how efficiency has behaved over the years. Banks’ efficiency deteriorated at the beginning of the recession, and especially as it deepened. | 3                   |
| 44  | Alhassan and Tetteh (2017)                                           | The study examines the impact of exogenous variables on the efficiency of 26 Ghanaian banks in the period 2003 to 2011. A high level of inefficiency among Ghanaian banks is evident, mainly due to pure technical inefficiency. The size of the bank positively influences efficiency only to a certain degree, due to economies of scale. Market concentration, leverage, and loan loss provisions are other significant factors identified as determinants of efficiency. | 1                   |
| 45  | Azad et al. (2017)                                                   | The study evaluates and optimizes the productivity of employees from of the Bank of Industry and Mine in Iran by integrating DEA with AHP with quantitative and qualitative indicators. The results specify that the most inefficient branches are related to low work quality and high training hours. | 2                   |
| 46  | Farandey et al. (2017)                                               | The paper investigates the impact of exogenous variables on the efficiency of Islamic commercial banks in Indonesia from 2011 to 2014. The actual average efficiency of Islamic commercial banks in Indonesia is 91.82%. Assets and ROA had a positive impact, while the number of branches negatively affected the bank’s efficiency. | 1                   |
| 47  | Fukuyama and Matousek (2017)                                        | The authors extend the two-stage Network DEA (NDEA) by proposing a banking revenue function. In addition, the Nerlove model is also applied to identify bank inefficiencies. The results indicate that the Japanese regional banks did not reach the optimum point in their productive processes. The main cause of bank inefficiency is allocative efficiency. Capitalization and risk had a negative effect on efficiency. | 15                  |
| 48  | Gulati and Kumar (2017)                                             | The authors measure the operational and intermediation efficiency of 46 Indian banks through a two-stage Network DEA model, in addition to a bootstrapped truncated regression to verify the impact of variables on these indices. The overall efficiency of the sector needs improvement in the two stages analysed. Larger and private banks showed better results. | 2                   |
| 49  | Kamarudin et al. (2017)                                             | The paper analyses the determinants of productivity of Southern Asian Banks. National and foreign Islamic banks showed an improvement in Total Factor Productivity Change (TFPC). Among the exogenous variables analysed, capitalization, liquidity and world financial crisis had a significant influence on the level of productivity of banks. | 1                   |
| 50  | Kong et al. (2017)                                                   | The authors propose an extension of the two-stage DEA model developed by Chen et al. (2004), making it possible to work with negative data and undesirable outputs. Operational efficiency, calculated in the first stage, is statistically smaller than profitability efficiency, measured in the second stage. | 1                   |
| 51  | Shi et al. (2017)                                                    | The authors propose a model to estimate and decompose possible M&A gains from Chinese banks. The results show that banks can improve their operations, mainly in relation to technical efficiency, when engaging in M&A. In contrast, M&A have a negative impact on scale efficiency. | 3                   |
| 52  | Wanke et al. (2017)                                                  | The authors analyse the virtual efficiency of M&A of South African banks. In addition, the impact of contextual variables on the efficiency indices is tested. M&A tend to be beneficial to banks, increasing technical efficiency, especially in terms of production. M&A gains are larger when both banks are local. | 6                   |
| 53  | Chen et al. (2018)                                                   | The paper presents an innovative DEA model with SVM in the second stage in order to segregate efficiency groups. The study also analyses the effects of different context-related variables on efficiency indexes. For the sample of Chinese banks, efficiency is related to domestic origin and enlisting in the stock market. However, results show that performance of the Chinese banking sector is low. | 3                   |
| 54  | Du et al. (2018)                                                     | The authors analyse the impact of earning asset diversification on Chinese bank efficiency from 2006 to 2011. In addition, they proposed an innovation on the method by extending the bootstrap model of Simar and Wilson (2007) Chinese banks could improve their efficiency with an increase in the diversification of their asset portfolios. | 1                   |
| 55  | Fernandes et al. (2018)                                             | The study measures the efficiency of banks in peripheral countries in the Eurozone and examines the effects of determinants of risk on bank performance over 2007–2014. Results indicate that higher levels of liquidity and credit risk negatively influence efficiency, while capital and profit risk have a positive impact on banks’ performance. The crisis tends to amplify the effect of bank risk. | 0                   |
| 56  | Ouenniche and Carrales (2018)                                       | The authors investigate the efficiency of 109 UK banks in the period 1987–2015, through DEA with a regression feedback mechanism. Several types of DEA model were used as well as different orientations. The proposed model increased the discriminatory power of the DEA. The SIM presented more consistent results than RCC and CCR. | 0                   |
| 57  | Huang et al. (2018)                                                  | The study extends the NDEA to the Copula-Based Network SFA model, with application to US banks. The proposed model made it possible to overcome the convergence problem specifically when phenomena are subject to highly nonlinear simultaneous equations. The inefficiency of banks comes mainly from the first stage. | 1                   |
| 58  | Xu (2018)                                                           | The study measures the efficiency of Chinese commercial banks and assesses the impact of foreign capital participation on efficiency. Banks with foreign control tend to be more efficient, even if this share is owned by minority shareholders. In addition, efficiency is also influenced by macroeconomic factors. | 0                   |
| 59  | Zhou et al. (2018)                                                  | The authors develop a dynamic two-stage DEA-SBM model to identify the sources of inefficiency of Ghanaian banks. Banks’ efficiency ratios were considerably low. The biggest source of inefficiency is in the first stage, called the productivity stage. | 0                   |
been addressing these topics relevant to studies on two-stage DEA models in banks.

Briefly discussing the two reviews about DEA in banks mentioned earlier, Fethi and Pasiouras (2010, pp. 189–196) analysed 196 studies that applied operational research or artificial intelligence techniques in the banking sector. They searched the Scopus database using the following keywords: bank efficiency, bank and data envelopment analysis, bank performance, bank and neural networks, bank and artificial intelligence, and bank and operational research. The review period was from 1998 to 2009, and only articles in English were considered. Of the 196 articles, 151 used DEA and its variations to estimate several measures of banking efficiency and productivity growth. Therefore, DEA is the most used technique in the field of operational research. The articles analysed were published in 73 different journals, with 58% of the publications concentrated in 12 journals. The European Journal of Operational Research (EJOR) was ranked first, followed by the Journal of Banking and Finance and Applied Financial Economics, with 19, 15, and 13 publications. Regarding method-related questions, most of the studies focused on the measurement of technical efficiency, worked with variable returns to scale, used input orientation, and followed the intermediation approach to select variables.

In Section 3 of their work, Fethi and Pasiouras (2010, pp. 190–194) discussed the topics of interest in the studies analysed, in which it stands out their discussions about the determinants of efficiency. The non-discretionary variables that are studied typically included size, profitability, capitalization, and country-specific factors. Despite providing an enlightening discussion regarding this aspect, the authors neither identified the technique most used in the second stage for this purpose nor clearly specified what impact the reviewed studies found in relation to the non-discretionary variables on the different types of efficiency (technical efficiency (TE), pure technical efficiency (PTE), and scale efficiency (SE)). The other topics of interest included the relationship between stock returns and efficiency; bank ownership; corporate events, such as mergers and acquisitions, and efficiency; regulatory reforms or liberalizations and efficiency; a comparison of frontier techniques; and bank branch efficiency.

Paradi and Zhu (2013, pp. 61–70) reviewed 80 studies that applied DEA in bank branches, classified according to the following attributes: country or region, inputs, outputs, premise regarding the returns to scale, and the objective. With the exception of two studies, all of the others focused on branches in just one country. The five most researched countries were Canada, Greece, Portugal, the United States, and the United Kingdom, which accounted for 65% of the studies reviewed. The studies had two main focus: to develop more advanced DEA models (38%, or 30 articles) and to evaluate the efficiency and provide guidance for improvements (33%, or 26 articles). Of the 80 articles, 5 used deposits as an input, and 43 used it as an output; 47 followed the premise of constant returns to scale, 20 followed the premise of variable returns to scale, and 33% used both models.

Paradi and Zhu (2013, p. 69) concluded their review by stating that although DEA is a deterministic technique, its results are sensitive to the data used. Thus, generating statistical inferences and confidence intervals is of great relevance, as this enables the reliability and acceptance of the model to be demonstrated. The authors indicated that although several advances have already been made in this regard over the past 20 years, an opportunity for future studies in this area still remains. Therefore, in the case of bank branches, there is an opportunity for research that uses statistical techniques — such as the bootstrap technique of Simar and Wilson (2007) — together with DEA.

This review differs from others, such as Fethi and Pasiouras (2010) and Paradi and Zhu (2013), first, because it focuses on an emerging topic in the DEA literature, i.e., two-stage DEA models in banks (Emrouznejad & Yang, 2017). Fethi and Pasiouras (2010) discussed this aspect, but only briefly. Discussing how two-stage DEA models have been applied in banks and identifying the objectives of these studies, the results found, and even aspects related to the two-stage terminology itself are extremely important, and to the best of our knowledge, this has not yet been done. Second, the classifications and codings created here — proposed by Lage Junior and Godinho Filho (2010) and later disseminated by Mariano et al. (2015), Henriches et al. (2018) and Silva, Kimura, and Sobreiro (2017) — are unique in the area of banking efficiency. Finally, in relation to the review by Fethi and Pasiouras (2010), more than 8 years have passed since its publication, which, although not a long time, indicates the need for a new review that considers the context of two-stage models, given that these models have been gaining notoriety, especially in recent years, as shown by Emrouznejad and Yang (2017). Regarding the review of Paradi and Zhu (2013), they did not specifically discuss two-stage models and focused on studies of bank branches, not on the banks themselves.

Considering the aspects previously discussed, as well as the relevance that a literature review adds to the academic debate on a given theme, Lage Junior and Godinho Filho (2010, p. 1) presented five steps to be followed when conducting a review, later followed by Mariano et al. (2015), Jabbour (2013) and Henriches et al. (2018), as shown in Fig. 3.

Considering step 1, the first keyword used for the search was DEA Bank in the title and stage in the topic in the Web of Science, ScienceDirect, and Scopus databases. The reason for using only the word stage in the topic — which considers the title, abstract, and keywords — is that if the article happened to use DEA with more than one stage, the authors would possibly specify this. Therefore, there is no need to search for two-stage because these articles will already be found with the criterion adopted. Besides, studies that adopted a three-stage model could also be identified. Two searches were conducted, the first in June 2017 and the second in July 2018. Regarding the first search, 27 publications were found in Web of Science, which included 19 articles and 8 proceedings papers; 37 articles were found in ScienceDirect; and 27 publications were found in Scopus, including 22 articles, 2 articles in publication, 1 book chapter, and 2 conference papers.

Another search criterion used was Data Envelopment Analysis and bank in the title and stage in the topic, given that some articles could use the full nomenclature for DEA. Eleven documents were found in the Web of Science, including 6 articles and 5 proceedings papers; 13 articles in ScienceDirect; and 9 articles and 1 conference article in Scopus.

In accordance with Fethi and Pasiouras (2010, p. 190), it was decided to include only articles published in journals in the review. As many articles were identified more than once due to the different search criteria used, 77 articles were selected.

In step 2, a careful analysis was conducted to verify if the articles actually had a connection with the theme of the present study, i.e., two-stage DEA models in banks. This analysis is complex because of the absence of an accurate definition of what exactly characterizes these models, as discussed in Section 2. Of the 77 articles found, 47 had an appropriate relationship with the research theme. The second search — conducted in July 2018 in the same databases and considering the same keywords — found an additional 12 articles that had a relationship with the theme. Thus, the final sample was 59 articles.

4. Classification and coding

After the evaluation of the articles and considering step 3, an analytical framework was developed that contained ten classifica-
tions covering topics relevant to the literature on two-stage DEA models. Consequently, each article was classified and coded according to its characteristics and the results found. The classifications were composed of numbers and letters (A, B, D, E, and so on); thus, the coding consists of a combination of letters and numbers. This step is important in identifying the most studied topics and possible gaps in the studies in this area. First, we analyse all the articles jointly to present the general landscape of the literature on two-stage DEA models. Second, we segregate the external and internal two-stage DEA models to determine whether the gaps found are maintained.

Classification 1 addresses the type of two-stage DEA model adopted in the studies, which are coded as A (internal) or B (external) — the studies coded as A refer to those that analyse the production process in two or more stages by breaking this process down into subprocesses. The studies categorized as B are those that used another procedure in the second stage, outside the production process. The results of this classification will be important to understand exactly what is understood in the literature by the terminology two-stage, as well as to segregate these different types of models.

Classification 2 identifies the economic context of the country of the study in question — it has an A–C scale of coding possibilities. It should be noted that the C coding was restricted to theoretical studies or literature reviews that did not have a country as the focus of the study. According to Wanke and Barros (2014, p. 2337), much of the literature on bank efficiency focuses on the United States and Europe, neglecting countries with emerging economies. Additionally, when reviewing 80 studies on bank branch efficiency, Paradi and Zhu (2013, p. 64) identified a gap regarding studies considering more than one country in the analysis (only 2 of the 80 articles reviewed involved more than one country). Therefore, this classification enables determining whether the gaps found by Wanke and Barros (2014) and Paradi and Zhu (2013) also exist in the literature on two-stage DEA models in banks.

Classification 3 refers to the continent of the data analysed by the article in focus. The coding scale is composed of the letters A–F. The results of this classification will be important for identifying possible continents with few studies, thus indicating a gap in geographic perspective. Taking again the literature review of Paradi and Zhu (2013, p. 64), these authors determined that the bulk of research is concentrated in North America and Europe; thus, determining whether this also occurs with the literature on two-stage DEA models in banks is of great importance and will make it possible to direct future research to less studied continents.

Classification 4 analyses the articles in accordance with their research objectives, with the coding scale composed of the letters A–E. To construct this classification, the findings of Paradi and Zhu (2013, p. 61) were considered, which indicated that, in general, the main topics of the studies are as follows: changes in efficiency due to regulations, effect of exogenous variables on efficiency, measurement of efficiency with an indication of benchmarks, and

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**Fig. 3. Steps for the literature review.**

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Step 1: To conduct a comprehensive search of the articles published in relevant databases, using a set of pre-established keywords.

Step 2: To screen the articles by ascertaining their relationship with the researched topic.

Step 3: To develop criteria for classification of the analysed articles that can represent all dimensions of the object researched.

Step 4: To present the existing knowledge of the researched topic, indicating the main topics studied and the results found in the analysed articles.

Step 5: To analyse existing gaps, as well as the opportunities and challenges that may provide guidance for future research on the subject.
international comparison. As some of the studies analysed here were proposing new adaptations of two-stage models (e.g., the use of new techniques in the second stage, in the case of the external two-stage DEA model, or extensions in the mathematical formulations, in the case of internal models), coding was added for this situation.

Classification 5 identifies the level of research of the analysed works — the possibilities in the coding scale range from A to D. Exploratory research aims to develop, clarify, and modify concepts and ideas. In general, this type of research is the first step of a broader investigation on a given topic. In turn, the purpose of descriptive research is to describe the characteristics of a certain population or phenomenon or to establish relationships between variables. Explanatory research has as its central concern identifying the factors that determine or contribute to the occurrence of a particular phenomenon. Finally, predictive research seeks to predict future outcomes based on the analysed data. Through this classification, it will be possible to understand which type of research is predominant in this theme.

Classifications 6, 7, 8, 9, and 10 address aspects related to the DEA method. As discussed previously, despite the extensive application, there is still no unanimity regarding the basic aspects of a DEA study, for example, which orientation should be adopted (input or output), to select variables, and what technique should be used in the second stage.

It is known that models with constant returns to scale should be used only if all banks are operating efficiently in scale (Assaf et al., 2011), something that is unlikely to occur in practice. However, it is worth noting that one model is not necessarily superior to another given that they measure different phenomena — the CCR model measures the technical efficiency (TE) or overall efficiency, which is composed of purely technical efficiency (PTE) and scale efficiency (SE), while the BCC model analyses only the PTE, based solely on administrative capacities. In other words, PTE is equivalent to TE, disregarding the impact of the economies or diseconomies of scale.

Another frequent finding in relation to model orientation is that since banks generally do not have control over output levels, orientation towards the input is recommended (Schaffnit et al., 1997, p. 278). However, given the plurality of existing output variables, this may not always be true. Observing how the literature has been addressing this subject will be important in providing direction for future studies.

Accordingly, classification 6 refers to the DEA model used, with the following possible codings: radial DEA models, with the popular models of Charnes, Cooper, and Rhodes (1978) and Banker, Charnes, and Cooper (1984), known as CCR and BCC, and the non-radial DEA model of Slack-Based Measures (SBM) developed by Tone (2001). While radial models deal with proportional changes in inputs and outputs in order for a given DMU to become efficient, the non-radial models, which focus on slack, do not make this assumption (Tone, 2017). Fukuyama and Weber (2009) also highlight that both the radial and non-radial models may be biased when there are slacks in the restrictions that define the technology of the production process. A possible existence of slack in the input constraints indicates that a unit can be judged to be efficient even though it could reduce at least one input, maintaining the same output level (Fukuyama & Weber, 2002).

Classification 7 analyses the returns to scale considered in the studies, which can be constant or variable. It is worth noting that the articles could receive more than one code if they considered both constant and variable returns to scale. The use of the two types of return is necessary to calculate the different types of efficiency, namely, TE, SE, and PTE. In the radial models, the returns to scale take the acronym of the creators of the traditional DEA, the CCR and BCC models, whereas in the case of the non-radial models, although there was the possibility of working with constant or variable returns, this did not occur, given that they were not developed by the authors of the acronym. Articles that did not specify which return was adopted were coded as 7C.

Classification 8 deals with the orientation of the model, which can be as follows: input-oriented, in which, for an inefficient DMU to become efficient, it must keep its outputs constant and reduce its inputs; output-oriented, in which it is sought to increase the outputs while keeping the inputs constant; and non-oriented (often used in NDEA), the objective of which is to maximize the outputs while minimizing the inputs. As some authors do not clearly specify the orientation adopted, there is a coding for such studies (unidentified).

Classification 9 addresses the scope of the analysis, indicating which approach was used for variable selection, which, in turn, will determine which specific function of the bank is being analysed. According to Berger and Humphrey (1997, p. 197), the production approach — proposed by Benston (1965) and which considers the bank’s main objective as providing services to its clients — is more appropriate for bank branch studies, while the intermediation approach of Sealey and Lindley (1977) — which indicates the financial intermediator role as the primary function of the bank — is more appropriate for studies on the banks themselves. The profit approach — proposed by Drake, Hall, and Simper (2006) — analyses the bank as a producer of profit components, such as interest and fee income (outputs), generated through the use of inputs, such as operational expenses and the quality of the loan portfolio, i.e., loan components (Aggelopoulos & Georgopoulus, 2017). Studies that either followed less popular approaches in the literature or proposed a new approach were coded as Others (9D).

Another possible coding for classification 9 concerns studies that combined more than one approach, coded as 9E. For internal two-stage studies in banks, it is quite frequent for the authors to follow one approach in the first stage and another in the second. This is done mainly by combining the production approach with that of intermediation so that the researcher does not need to make a judgement call regarding the dilemma of deposits, as discussed by Holod and Lewis (2011) and Fukuyama et al. (2020). This coding, therefore, encompasses studies that have mixed approaches or have treated deposits as an intermediate variable.

Studies that did not follow any specific approach or adopted the same variables as previous studies in the literature were coded as 9F.

Classification 10 identifies the procedures adopted by the researcher that characterize the article as a two-stage DEA model. One of the possibilities in the second stage is to use the outputs of the first stage as inputs in the second, as in Seiford and Zhu (1999, pp. 1270–1288) — this procedure is known as the intermediate variable technique. In this case, the second stage refers to the production process of a specific bank and is often used to overcome the problem of the DEA treating the production process as a black box (Fukuyama & Weber, 2010). Another possibility is to use another procedure in the second stage — this technique is external to the production process. OLS regressions, censored models, such as Tobit, resampling techniques, such as bootstrap, and qualitative techniques, such as AHP, can be used in this stage.

Considering the observations of Simar and Wilson (2007, pp. 45–57) that using traditional regression techniques in the second stage would not be as appropriate for analysing the effect of the non-discretionary variables (because the DEA efficiency scores have a statistical bias and are highly correlated, requiring the bootstrap procedure to correct these problems), this classification allows the identification of the approach that is the most used in the second stage and if the researchers are following the observations in Simar and Wilson (2007, pp. 45–57). Additionally, analysing the techniques adopted in the second stage can be an
important step towards a more accurate understanding of the application and definition of the two-stage DEA model in banks.

It is important to highlight that, with the exception of classifications 4, 5, and 6, whose coding options are mutually exclusive, the articles could be coded with more than one code; therefore, the total number of articles in these categories could be more than 59. The classifications, as well as the coding possibilities discussed herein, are presented in Table 2.

5. Results of the literature analysis

To present the results in the most detailed manner, we performed a bibliometric analysis and codification. Bearing this procedure in mind, this section is divided into the two following subsections: bibliometric analysis and coding results. We believe that this will enable us to present the state of the art, opportunities, and challenges for future studies on two-stage DEA models in banks.

5.1. Bibliometric analysis

The first dimension presented is the bibliometric analysis. Table 3 shows that there is a decentralization of publications with regard to journals. In first place is the journal Expert System with Applications, the scope of which is the application of intelligent systems in businesses, governments, and universities, with nine publications, or 15.25% of the total; followed by the European Journal of Operational Research, with six publications (10.17%); Omega and Research in International Business and Finance, with three publications (5.08%); and the Journal of Banking and Finance, Annals of Operational Research, Economic Modelling, and International Journal of Productivity and Performance Management, with two publications each. The other journals, including Measurement, Socio-Economic Planning Sciences, Benchmarking, North American Journal of Economics and Finance, and Journal of Productivity Analysis, among others (30 in total), had only one publication, and accounted for 50.85% of the publications.

The first publication of the analysed sample was that of Luo (2003), followed by Barth and Staat (2005) and Pasiouras (2008). Between 2000 and 2010, there were few publications regarding two-stage DEA models in banks. The scenario began to change after 2010, with four publications in 2011 and 2014 and six in 2013 and 2014. In 2015, there were nine publications — the year with the second highest number of articles published, behind 2017, with ten publications. It is worth mentioning the publication in 2018, given that until July, the month when the article search was conducted, there were already seven publications. This analysis is important because it shows that, over time, the interest in two-stage DEA models in banks has grown considerably.

The year 2018 will possibly surpass the other years in terms of number of publications, and if there is no sudden change in the trend, the same will occur in 2019. The recent interest in two-stage DEA models specifically in the banking sector indicates an emerging topic in the literature, as already discussed in Section 3, which means that a great opportunity exists for researchers in future studies. It is worth noting that despite the growth in the number of publications at the global level, various opportunities for application still remain, given that there is a large plurality of non-discretionary variables to be analysed, as well as different models and approaches in DEA, in addition to different countries that lack research, for example, Latin America countries. Given this growth, reviewing how this model has been applied is of paramount importance for a better understanding of how two-stage DEA models have been applied in banks and to provide guidance for future studies.

| Table 2 | Codes used to analyse the articles. |
|---|---|
| Classification Meaning | Cryptography |
| 1 | Two-stage DEA | 1A - Internal. |
| 2 | Economic Context | 2A - Mature economy. |
| 3 | Geographical Region | 3B - South America. |
| 4 | Objective | 4A - To verify the change in efficiency taking into account reforms, e.g., liberalization and deregulation in the banking industry, changes in the market structure and changes in the economic environment. |
| 5 | Type of Research | 5A - Exploratory. |
| 6 | DEA Model | 6A - Radial. |
| 7 | Return of Scale | 7A - Constant. |
| 8 | Orientation | 8A - Input. |
| 9 | Approach | 9A - Intermediation. |
| 10 | Procedure Related to the Second Stage | 10A - Tobit. |

Regarding the relevance of the studies, the number of citations may be a good indicator (Mariano et al., 2015, p. 38). Table 1 shows that the most cited study (162 citations) was that of Wu et al. (2006), which combined DEA with ANN to analyse the efficiency of 142 branches of a Canadian bank, followed by the study of Luo (2003), with 134 citations, in which NDEA was used, with each stage independent of the other — profitability efficiency was analysed in the first stage and marketability efficiency in the second for 245 U.S. banks, similar to the work of Seiford and Zhu (1999).

The other three most cited articles were those of Paradi et al. (2011), Pasiouras (2008) and Staub, Silva Souza, and Tabak (2010), with 111, 105, and 87 citations, respectively. Paradi et al. (2011) analysed the efficiency of 816 branches of a Canadian bank through an external two-stage DEA model, in which the outputs of the second stage were the efficiency scores calculated in the first
stage, considering three different approaches. Pasiouras (2008) evaluated banking efficiency in 95 countries, verifying the impact of regulatory factors on efficiency. Staub et al. (2010) estimated the cost, allocative, and technical efficiency of Brazilian banks, analysing the impact of non-discretionary variables on efficiency.

Because the two-stage DEA model is a more recent variation of the DEA model, this number of citations has great potential to increase considerably over the next few years as more research is published. It is worth emphasizing that the number of citations was collected in August 2018 and may have increased since then.

Thirty-five studies were longitudinal, whereas 19 were cross-sectional. Thirteen studies analysed the efficiency of bank branches, whereas 46 considered the banks themselves. With very few exceptions, the vast majority involved 2 to 10 inputs and outputs at each stage, whereas the number of DMUs analysed varied considerably, from 16 in some cases up to 246, but always respecting the rule, discussed in Cooper, Seiford, and Tone (2006), of having three times more observations than the total number of variables.

5.2. Coding results

Considering now the second dimension of the results, the respective codings of each study are presented in Table 4. The gaps will be presented under the following abbreviations: $G_{12, x}$ for gaps that refer to both internal and external two-stage DEA models, $G_{12, x}$ for gaps referring only to internal two-stage DEA models, and $G_{12, x}$ for gaps referring only the external ones. First, to provide an overview of how the literature is categorized when referring to the two-stage terminology in banks, we will analyse the internal and external two-stage articles together, and second, we will segregate them to determine if the identified gaps are maintained and if there are new gaps to be indicated.

The first classification to be analysed addresses the type of two-stage DEA model adopted in the studies, with the following coding possibilities: A — internal two-stage DEA model, and B — external two-stage DEA model. There were 18 studies for the internal two-stage model, 29 for external two-stage, and 12 combining the internal and external models. These results are shown in Fig. 4 and indicate that when referring to the term two-stage in banks, models that use some technique after measuring the efficiency by DEA predominate. Interestingly, few studies used both the two-stage DEA model to overcome the black box problem (internal) and the model that allows a more complete analysis (external). Considering this aspect, the following gap emerges:

$G_{1}$: More studies could combine internal and external two-stage DEA models. These two types of two-stage models are becoming increasingly common in the literature and can complement each other to make the analysis even more realistic and complete. While internal two-stage DEA models overcome limitations related to the production process, the use of some technique in the second stage enables a more in-depth analysis.

Analysing the studies over time, it can be seen that combining the two types of two-stage DEA models has occurred in more recent years. The first study in the analysed sample that combined the two types of model was Xu (2013), followed by Huang, Chen, and Yin (2014) and Wang, Lu, and Liu (2014). In 2017 alone, four studies combined these models — of all the articles that applied internal and external two-stage DEA models, 33.33% were published in 2017.

Two-stage DEA models may be vulnerable to the problem of separability, as discussed in Simar and Wilson (2011) and Daraio et al. (2018). Considering this potential drawback, we suggested the tests presented in Daraio et al. (2018) to verify the separability condition. If the separability assumption is violated, conditional efficiency models could be used, following Degl’Innocenti et al. (2017, 2018), Kevork, Pange, et al. (2017), Kevork, Kollias, et al. (2017), Matousek and Tzeremes (2016) and Tzeremes (2015).

The second classification, which considers the economic context of the countries analysed, has the following coding possibilities: A — mature economy; B — non-mature economy; and C — not applicable, which corresponds to studies that had no empirical analysis. Twenty-three studies were conducted in countries considered to be mature economies or developed economies (i.e., they were coded with the letter A), whereas 33 studies were conducted in emerging non-mature economies (Fig. 5). Despite the predominance of studies in less developed economies, this difference is not very large, which indicates that the literature is not prioritizing one type of economy over another but rather analysing the banking sector in different economic contexts.

It is interesting to note that despite the predominance of undeveloped economic contexts in the sample of studies, this was not valid for the older articles, in which the analysis of the banking sector of mature economies prevailed. Upon analysing the coding of the ten oldest articles in the analysed sample, it could be seen that five considered mature economies, two considered both economic contexts, and only three dealt with non-mature economies. This indicates that — as discussed by Wanke and Barros (2014, p. 2337) — less developed countries were overlooked, something that has been reversed over time.

Comparing the publications on internal two-stage DEA models with external ones, it can be seen that there is a large difference in the economic contexts. If in the articles of the first type of model, there was a slight predominance of publications in developed economies (10 versus 8), when referring to the second type of model, 18 articles were focused on the banking sector in non-mature economies versus 9 in mature economies. Given this, the following question emerges:

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Table 3

| Analysed Criteria | Classification | Amount | Percentage (%) |
|-------------------|---------------|--------|----------------|
| Journal           | Expert System with Applications | 9      | 15.25          |
|                   | European Journal of Operational Research | 6      | 10.17          |
|                   | Omega         | 3      | 5.08           |
|                   | Research in International Business and Finance | 3      | 5.08           |
|                   | Annals of Operational Research | 2      | 3.39           |
|                   | Economic Modelling | 2      | 3.39           |
|                   | Int. Journal of Productivity and Performance Management | 2      | 3.39           |
|                   | Journal of banking and Finance | 2      | 3.39           |
|                   | Others        | 30     | 50.85          |
| Year              | 2003          | 1      | 1.69           |
|                   | 2005          | 1      | 1.69           |
|                   | 2006          | 1      | 1.69           |
|                   | 2008          | 1      | 1.69           |
|                   | 2009          | 2      | 3.39           |
|                   | 2010          | 2      | 3.39           |
|                   | 2011          | 4      | 6.78           |
|                   | 2012          | 4      | 6.78           |
|                   | 2013          | 6      | 10.17          |
|                   | 2014          | 6      | 10.17          |
|                   | 2015          | 9      | 15.25          |
|                   | 2016          | 5      | 8.47           |
|                   | 2017          | 10     | 16.95          |
|                   | 2018          | 7      | 11.86          |
When the two-stage DEA model used is the external type, why are researchers prioritizing less developed economic contexts, as opposed to studies that have used internal two-stage DEA models and the observations by Wanke and Barros (2014) that, generally, more developed countries are more frequently the focus of studies? One possible answer is that due to the instability of non-mature economies, environmental factors tend to exert a greater influence on efficiency — something that the researcher must consider. This hypothesis lacks testing but could be verified in future studies.

Only two studies (Thoraneenitiyan & Avkiran, 2009; Pasiouras, 2008) have been conducted considering these two contexts simultaneously — both involved external two-stage models. These two studies found that the difference in context between one country and another has a significant effect on efficiency. Therefore, more research is needed to analyse different economic contexts in the same study. It is worth highlighting that Paradi and Zhu (2013, p. 64) found the same gap when reviewing studies on bank branches, which indicates that this gap has existed for some time and has not been explored by researchers. One difficulty for this could be the limitation in obtaining data from more than one country, or the
difficulty of comparing banks in different countries when using DEA, a technique of relative efficiency; however, as some researchers — for example, Pasiouras (2008) and Thoraneenitiyan and Avkiran (2009) — have managed to overcome such limitations, others could also do the same by following these authors. Thus, the gap resulting from classification 2 is as follows:

\[ G_2: \text{Given that the economic context can have a significant effect on efficiency, more research that considers these different contexts — such as that by Pasiouras (2008) and Thoraneenitiyan and Avkiran (2009) — is needed.} \]

The geographical region of the countries evaluated is identified in classification 3. This classification, which aggregates information of the second classification, has the following coding options: A — North America, B — South America, C — Europe, D — Asia, E — other regions, and F — not applicable. Eight studies were conducted in North America, two in South America (Staub et al., 2010; Wanke & Barros, 2014), eight in Europe, 34 in Asia, six in other regions (Africa, Oceania, and Central America), and only one in more than one continent (Pasiouras, 2008), as shown in Fig. 6. When considering how articles studied the different geographic regions over time, as well as the type of two-stage DEA model used, no large variations were observed.

There was a large concentration of studies in the Asian continent — nine studies in China, seven in Taiwan, and five in Iran. Despite the predominance of publications that studied the banking sector of Asian countries, the United States was the focus of six studies. A similar situation occurred with Greece, with four articles. Both the USA and Greece accounted for virtually all the publications in their respective regions. Thus, the following gap was identified:

\[ G_3: \text{Why are researchers so focused on studying the Asian continent? In general, the other continents need more research, especially an analysis that encompasses more than one continent. Furthermore, various countries do not have any publications, for example, Latin American countries (excluding Brazil) and European countries (excluding Greece and Germany). Studies in other countries are also needed.} \]

Another relevant aspect is that only the study of Pasiouras (2008) was done in more than one continent, which indicates a clear need for more research that considers different continents. Although the gap discussed in classification 2 is related to this, the focus in that gap was to consider different economic contexts rather than different geographic regions. With this in mind, the following gap was identified:

\[ G_4: \text{Studies that consider different geographic regions are necessary so that international evidence can be found regarding the impacts that a given environmental variable has on efficiency, as the comparison between different studies in the literature has the complication that the authors can use different DEA models, as well as different variables as inputs or outputs, thus making it difficult to compare the results found. Consequently, a researcher maintaining the same model and the same variables in different continents would solve this problem and would also enable researchers to determine how the impact of these non-discretionary variables on efficiency would change with the continents considered, thus making an international comparison possible.} \]
Regarding classification 4, five coding possibilities were elaborated to categorize the objectives of the articles as follows: A — determine efficiency variation over time due to reforms, B — measure efficiency and indicate benchmarks, C — analyse the impact of non-discretionary variables on efficiency, D — propose an extension or a new DEA method/model, and E — international comparison. Fig. 7 shows the classifications of the studies: five were classified as 4A (i.e., they aimed to determine the impact of reforms and regulations on banking efficiency), twelve were classified as 4B (comprising the studies that measured efficiency and discussed benchmarks for improving inefficiencies), seventeen were classified as 4C (determination of the impact of non-discretionary variables on efficiency), twenty-three were classified as 4D (representing the studies that proposed new models or adaptations to two-stage DEA models with applications in the banking sector), and only two were classified as 4E, whose main objective was international comparison. Thus, a gap can be seen in this last coding; however, as this gap has already been discussed, no new gap was identified. It is worth mentioning that when analysing the objectives of the articles over time, interest in determining the impact of non-discretionary variables has increased.

The slight concentration of studies in the coding 4D was expected, given the predominance of publications in high impact journals, which, in turn, demands a certain degree of innovation from researchers, whether in changes to the mathematical formulations of the model or in the combination of new techniques with DEA in the second external stage. The coding with the second most published articles consists basically of one of the essentials of the external two-stage DEA models, i.e., determining the impact of exogenous variables on efficiency. Various internal factors of banks — size (Staub et al., 2010; Xu, 2013); state or private control (Staub et al., 2010; Wanke & Barros, 2014; Stewart, Matousek, & Nguyen, 2016; Sufian, 2015); foreign or domestic (Sufian, 2015; Wanke & Barros, 2014); dividend payment policies (Wanke, Barros, & Emrouznejad, 2016); capitalization (Sufian, 2015); profitability (Shawtari, Ariff, & Razak, 2015); intellectual capital (Wang et al., 2014); risk (Wang & Lu, 2015; Tsolas & Charles, 2015); macroeconomic factors (Xu, 2013), such as the country’s gross domestic product, inflation, and industry factors (Fukuyama & Matousek, 2017); and, finally, factors unique to each country (Pasiouras, 2008) — were considered. One topic of interest identified was the objective of assessing possible Mergers and Acquisitions (M&A) (Wanke, Azad, & Barros, 2016; Wanke & Barros, 2014; Wanke, Maredza, & Gupta, 2017; Wanke, Barros, Azad, & Constantino, 2016; Halkos & Tzeremes, 2013).

With the segregation of the types of two-stage DEA models, large variations in the results of the codings are expected, given that the internal and external models overcome the distinct limitations of traditional DEA models. Thus, for the internal models, only Rayeni and Saljooghi (2016) aimed to analyse the impact of exogenous variables on efficiency. These authors calculated three different models — one not including risk and two with this variable modelled in the NDEA. The predominant objective in this type of two-stage model was to propose extensions of the DEA models (10 publications), followed by the indication of benchmarks (6 publications).

Regarding the external models, the main objective was to analyse the impact of exogenous variables — 12 of the 29 studies had this objective. Eight studies proposed extensions to the DEA models, with the use of new techniques in the second stage or changes in the mathematical formulations of the model; four specifically analysed variations in efficiency due to banking freedom or deregulation; three sought to indicate benchmarks for improving efficiency; and the aim of two was international comparison.

As highlighted in Degl’Innocenti et al. (2018), the Global Financial Crisis (GFC) in 2007–2008 demonstrated the weaknesses of banking systems and the importance of understanding the mechanisms that improve bank performance. Considering the impact of financial institutions to the economy (Quenniche & Carrales, 2018; Paradi et al., 2011; Wang et al., 2014), and since banks are often the target of new regulations after fiscal crises (Halkos & Tzeremes, 2013; Thoraneenitiyan & Avkiran, 2009), we consider the following gap related to the external models, as internal models would not be appropriate for solving these situations:

\( Ge_i \): How banking efficiency is affected by economic crises and changes in the regulation of the sector? This question is even more relevant in the contemporary context, given the global economic recession triggered by the COVID-19. Some studies identify a relevant relationship between economy and efficiency in the banking sector. For instance, Fukuyama and Matousek (2011) and Fukuyama et al. (2020) identify an overall worsening in the efficiency of Turkish banks due to the economic environment. In Fukuyama and Matousek (2011), the
negative impact could be explained by a specific crisis in the country, whereas in Fukuyama et al. (2020) the explanation could be related to the GFC. Similarly, Degl’Innocenti, Kourtzidis, Sevic, and Tzeremes (2017) and Kevork, Kollias, et al. (2017) observe evidence of a degradation in productivity of European banks during the GFC. In addition, Degl’Innocenti et al. (2018) detect a positive nonlinear relationship among financial centres’ competitiveness, banks’ stability and innovation capacity levels. Future research could include further discussion regarding how financial crisis and changes in regulations affect the efficiency of banks.

Classification 5 discusses the level of research, with the following coding possibilities: A — exploratory, B — descriptive, C — explanatory, and D — predictive. One study was classified as exploratory, six as descriptive, forty-six as explanatory, and six as predictive (Fig. 8). Only the study of Luo (2003) was classified as exploratory, precisely because it was the first conducted in the group of studies analysed, thus providing guidance for future research. It is worth mentioning that the authors themselves also classified their studies as such. When separating the types of two-stage models, no large variation was perceived that would justify a segregated analysis.

The six studies classified as predictive were those of Wu et al. (2006), Mostafa (2009), Halkos and Tzeremes (2013), Kwon et al. (2015), Wanke et al. (2016) and Shi, Li, Emrouznejad, Xie, and Liang (2017), Halkos and Tzeremes (2013) and Wanke et al. (2016) sought to predict the efficiency behaviour of Greek and Mozambican banks, respectively, with possible M&As and, in the case of Wanke et al. (2016), with changes in the majority shareholder, for example, if a public bank were acquired by a private bank. Kwon et al. (2015), Wu et al. (2006) and Mostafa (2009) combined DEA with ANN techniques to develop a model to predict bank performance, while Shi et al. (2017) developed a production possibility set (PPS) for M&As. Considering the small number of predictive studies, the following gap was identified:

G5: As most of the articles in the literature on two-stage DEA are focused on explaining the efficiency scores found ex-post facto, there is a lack of studies seeking to predict efficiency behaviour in certain situations ex-ante facto, for example, in M&As, at times when the economy is heating or cooling, and how a new specific regulation would affect efficiency, among other possibilities. In short, there is a need for more predictive research.

It is worth noting that despite there being only one exploratory study (Luo, 2003), the need for further studies with this level of research was not suggested, as this type of research is the first stage of an investigation on a certain topic. It would be necessary for researchers to identify something that the literature has not yet discussed. Given the complexity of this, it was decided to not indicate gaps in this sense.

Classification 6 analyses the type of DEA model used and is coded with the following letters: A — radial and B — non-radial. Most of the articles (48) adopted radial models; whereas 11 articles involved non-radial models, using the SBM model. This indicates that in most of the studies, for a bank to become efficient, it must make proportional changes in its inputs or outputs, given that this is one of the characteristics of radial models. These results are shown in Fig. 9.

Regarding classification 7, which analyses the returns to scale adopted, models that involved variable returns to scale were the most used (present in 33 studies), with use as follows: individual; combined with constant return models to identify if the banks were presenting increasing, constant or decreasing returns to scale; or involving more complex models (e.g., network or fuzzy). Eight articles exclusively used the CRS model. The reason for the low utilization of the CRS model was precisely due to the aspect discussed by Assaf et al. (2011) — that the constant returns to scale model should only be used if all analysed DMUs are operating at the optimal level, which is very difficult to do in real terms. Fig. 10 shows the results for this classification.

The segregated analysis of internal and external models does not add new information to the discussion about classification 6 and classification 7. The predominance in both the internal and external models was of articles that adopted variable returns to scale, with the small difference that the CCR model was used...
proportionally more in the external models than the internal ones. However, this greater use of the CCR model is mainly due to the joint use with the BCC model, which makes it possible to calculate the SE. Thus, the results found in these classifications indicate that radial models account for the vast majority of DEA models used and confirm that researchers have sought to work with variable returns to scale models rather than constant returns to scale models, following the recommendation of Assaf et al. (2011). Because the methodological aspects of the DEA model are being addressed, no gap was identified in this classification.

Classification 8, which verifies the orientation of the DEA model in the analysed studies, had the following coding possibilities: A — input-oriented model, B — output-oriented model, C — unoriented, and D — not identified. Fourteen studies followed the input orientation; eleven followed the output orientation; two estimated the DEA model oriented at first to inputs and later oriented to outputs (Ouenniche & Carrales, 2018; Paradi et al., 2011); thirteen adopted an unoriented model, which sought both the minimization of inputs and the maximization of outputs; and nineteen studies did not specify the orientation adopted in their models. It is worth highlighting that the unoriented models were much more frequent in articles that used internal two-stage DEA models. These results are shown in Fig. 11.

The slight predominance of articles that adopted the input orientation rather than the output orientation reveals that researchers have generally followed the argument of Schaffnit et al. (1997, p. 278) that banks do not have control over their outputs and, therefore, input orientation is more appropriate. However, at
the time of the study by Schaffnit et al. (1997), the DEA models that simultaneously minimized inputs and maximized outputs were not yet popular. Thus, for the current context, the recommendation of these authors may not be as strong and relevant as at the time it was proposed.

Classification 9 deals with the approach used in the studies for selecting the model’s variables, which will define the scope of the analysis, with the following possible codings: A — intermediation approach, B — production approach, C — profit approach, D — other ways of selecting variables, E — combination of approaches, and F — not specified. Thirteen studies did not specify or did not use a specific approach (many of these studies only considered the variables used by other authors and opted to replicate them). Most of the studies (18) followed only the intermediation approach. Five used the production approach, five used the profit approach, and thirteen studies combined more than one approach (i.e., in the first stage, they analysed the bank’s efficiency in one function, and in the second stage, they analysed it in another). These results are shown in Fig. 12.

One important observation is that the suggestion of Berger and Humphrey (1997, p. 197) — that in studies dealing with branches, the production approach should be chosen, whereas in studies with banks, the intermediation approach should be chosen — is not being followed, given the small number of articles that have used the production approach.

Interestingly, despite the predominance of the use of the intermediation approach for variable selection, especially in studies that used external two-stage DEA models, when considering the studies that used internal two-stage DEA models, only Huang, Chen, and Lin (2018), Fukuyama and Matousek (2017) and Wang et al. (2014) used the intermediation approach. However, it is worth highlighting that most of these studies combined more than one approach (i.e., they were coded as 9E), seeking to treat deposits as intermediate or to analyse different bank functions. In this context, deposits was the most used variable as intermediate.

Another aspect that should be highlighted is the low number of studies that used more than one approach — only Paradi et al. (2011) used at least three approaches in different efficiency estimates to analyse how efficiency varied from one model to another. This is especially relevant in the case of the banking sector because, as discussed in Holod and Lewis (2011) regarding the deposits variable, the way a variable is treated will influence which banks will be indicated as efficient by the model. Additionally, there is no consensus in the literature as to the most appropriate approach to measure bank efficiency, as each of the approaches analyse the bank from a different perspective. Given the above, the following gaps emerge, in which G6 is a motivating gap for the use of internal two-stage DEA models, considering that only through the internal models is this issue resolved:

\[ G_6: \text{Given that DEA results are quite sensitive to the variables that will be part of the model, studies considering more than one approach are important to verify the behaviour of the results with different variables.} \]

\[ G_7: \text{Given the difficulty in dealing with the deposits variable (a judgement call must be made by the researcher), a new group of studies has been directing how to treat this variable (Holod \\& Lewis, 2011; Fukuyama et al., 2020), which is to consider it as an intermediate variable. Fukuyama et al. (2020) and Degl’Innocenti, Kourtzidis, Sevic, and Tzeremes (2017) argue that considering deposit as an intermediate variable provides a plausible solution to this dilemma, since this variable can be both input and output (Berger \\& Humphrey, 1997). Thus, the double role that the variable deposit can play would remain intact. Future research could take this aspect into account and conduct studies treating this variable as such.} \]

The last classification to be discussed — classification 10 — verified the procedures adopted by the researcher that characterize the article as a two-stage DEA model (either internal or external) and had the following coding possibilities: A — Tobit, B — AHP, C — bootstrapped truncated regression, D — OLS, E — Artificial Neural Networks, F — intermediate variables, and G — other techniques. The results in this classification showed that three studies used only Tobit in the second stage (Nguyen, Nghiem, Roca, \\& Sharma, 2016; Pasiouras, 2008; Farandy, Suwito, \\& Dabutar, 2017), two used only the AHP (Kholousi, 2013; Azadeh et al., 2011), ten used only bootstrapped truncated regressions, one used only an OLS regression (Shawtari et al., 2015), eighteen used only the
intermediate variable model, two used only ANN (Wu et al., 2006; Mostafa, 2009) and nine used only other techniques, such as stochastic simulations and the Monte Carlo algorithm (Tsolas & Charles, 2015), support vector machine (Wanke et al., 2016), beta regressions (Wanke et al., 2017; Xu, 2013), panel analysis (Shawtari et al., 2015), among others. Finally, in 14 studies, more than one technique was used in the second stage. The results of this coding are shown in Table 5.

Considering these results, it can be seen that the most frequent application of two-stage DEA models in banks is to use the outputs of the first stage as inputs of the second stage, that is, two-stage models concerned with the production process. Despite the predominance of external two-stage DEA models, as discussed in classification 1, there is a wide variety of possible techniques to apply in the second stage, while all the articles concerning two internal stages inevitably used intermediate variables. For this reason, this superiority is, to some extent, expected.

Considering only the articles on external models (either combined with internal models or not), there was a small dominance in the number of articles that adopted the bootstrapped truncated regression (15), followed by other techniques in the second stage (14). Examining only the studies whose objective was to determine the impact of non-discretionary variables on efficiency (classified as code 4C), of the 17 studies, 11 used bootstrapped truncated regression, which was the main technique for this purpose. Furthermore, following the trend — verified in classification 4 — of the growing interest of researchers in determining the impact of non-discretionary variables on efficiency over time, the bootstrapped truncated regression has become more popular in recent years.

As in categories 5, 6, and 7, no gap was identified in our classification, as the objective was to verify which technique was predominant in the second stage. However, this evidence helps future researches in the definition of which technique would be
To give this discussion direction, Table 6 presents information on the input and output variables, the type of efficiency analysed (PTE and TE), the variable selection approach used in the study, the exogenous variables used, and their impact on efficiency. Thus, the results obtained with respect to the effect of these environmental variables on efficiency can be analysed from the perspective of the approach used in each study, that is, in the different functions performed by the bank. It is worth emphasizing that Table 6 lists only the articles that analysed the effect of non-discretionary variables on efficiency.

Even when comparing the impact of non-discretionary variables in similar contexts, ambiguous results are observed. For example, Sufian (2015) — article number 35 in Table 6 — found a positive effect of capitalization on PTE, while Fukuyama and Matousek (2017) — article number 47 — found a negative impact. These two studies followed the intermediation approach and measured the same efficiency. The same occurred with the size variable, which, in the study of Alhassan and Tetteh (2017), negatively influenced both TE and PTE, while Stewart et al. (2016) identified a positive impact of size on TE, although these authors followed the same approach. Similar results were also verified in other studies.

Recognizing that it is not possible to accurately determine what the impact of non-discretionary variables on efficiency will be, in view of the ambiguous results found in the literature (even considering the function of the bank analysed and the type of efficiency), Table 6 can serve as a background for comparisons of future studies with those already in the literature.

### 6. Conclusions

Two-stage DEA models have been gaining prominence in research on efficiency in the banking sector because they overcome the limitations of traditional DEA models. Recognizing the existence of several controversial aspects in the literature, from the two-stage terminology itself to the application of these models in banks, this study analysed 59 articles related to two-stage DEA models in banks. All of these studies were found using the Scopus and Web of Science databases and Elsevier's ScienceDirect search engine.

This study followed the steps proposed by Lage Junior and Godinho Filho (2010) to review the literature. In this sense, ten classifications were created, ranging from the economic context and the geographic region to methodological aspects of the two-stage DEA models in banks, with several codification possibilities for each classification. We believe that with this study, which presented the existing knowledge, opportunities, and challenges for future studies, the state of the art in this emerging topic in the literature can be properly mapped.

Throughout this review, we highlighted the main characteristics of publications related to the term two-stage in banks. Although some gaps are common to both internal and external two-stage DEA models, we also showed the need to segregate these models to explore new gaps. The common terminology used for these two distinct types of models hinders a universal definition for two-stage DEA models in the banking sector. Based on the initial discussion herein, future studies can advance this work so that there is a clear terminological distinction between these models.

We find seven gaps in the literature, as highlighted in the discussion of the different classes or categories. The study identifies research opportunities related to: (i) the combination of internal and external two-stage DEA models, (ii) the analysis of how changes in regulations or market environment affect efficiency of banks, especially after the GFC in 2007–2008 and the COVID-19 pandemic in 2020, (iii) the analysis of efficiency on a more diverse list of different countries or continents, (iv) the investigation of banking efficiency not only in different geographic regions but also in different economic contexts, (v) the prediction of efficiency behaviour in certain situations ex-ante fact, (vi) the use of diverse approaches to select relevant variables to include in the model, and (vii) the set up of deposits as an intermediate variable. Each of
Table 6

Inputs, outputs, exogenous variables and their impacts on efficiency, by approach and efficiency.

| Number | Type of Efficiency | Scope of Analysis | Inputs 1st Stage | Intermediate Variables | Outputs 2nd Stage | Exogenous Variables | Impact on Efficiency |
|--------|--------------------|-------------------|-----------------|------------------------|-------------------|---------------------|----------------------|
| 37     | TE and PTE.        | Profitability (1st stage) and marketability (2nd stage). | Number of employees, fixed assets and information technology expenditure annual. | Deposits, liabilities and ATFD (Amount of trading by financial derivatives). | Operating diversification, branches and non performing loans recovered. | Two governance variables: Government Shareholdings (SOE), Financial Holding Subsidiary (FHS). Variables related to risk factors: Exchange Rate Volatility (ERV), Interest Volatility (INV). Long-term loan to capital (LCR). Variable related to Basel III Accord: Capital Adequacy Ratio (CAR). | TE: CAR, ERV, SOE and FHS have positive impact and LCR has negative impact. SE: CAR, SOE and FHS positive, ERV, IRV negative. PTE: CAR and FHS positive, ERV and IRV negative. |
| 24     | PTE.               | Deposits generation (1st stage) and loan generation (2nd stage). | Fixed assets, equity and personnel expenses. | Deposits and other raised funds. | Gross loans, other earning assets and an undesirable output of non-performing loans. |   |   |
| 11     | SBM - TE and PTE. | Intermediation approach, production approach and profit approach. | Production: nine personnel-related inputs. Intermediation (5): cash balances, fixed assets, other liabilities, net non performing loans, loan loss experience. Profit (6): personnel expenses, occupancy/computer expenses, loan losses, cross charges, other expenses and sundry expenses. | Do not apply. | Production: (9) segregated by the three main customers type: Retail: relationship, service, internal. Commercial: relationship, service, internal. Corporate: relationship, service, internal. Intermediation: (6) Wealth management, home-owner mortgages, consumer lending, commercial loans, commercial deposits, consumer deposits. Profit: (7) comissions, consumer deposits, consumer lending, wealth management, home mortgages, commercial deposits e commercial loans. | Regions, market size and scale. | Overall - Positive impact: risk, liquidity, shareholders behind and size. Negative impact: interest margin. Others variables were not statistically significant. Stage 1 - Positive impact: risk, liquidity, shareholders behind, size and GRM. Other variables were not statistically significant. Stage 2 - Positive impact: liquidity and assets. Negative impact: interest margin, Shareholders behind and GRM. Some regions of Canada showed higher efficiency values for each model analysed. Branches in Rural market had a better performance in profit and production than Small Urban and Major Urban branches. In the three efficiencies considered, increasing asset size results in a larger percentage of branches being classified as DRS. |
| 48     | Not identified.    | Intermediation (1st stage) and Profit (2nd stage). | Fixed assets, number of employees and loanable funds (Deposits and borrowings). | Advances and investment. | Interest income and non interest income. | Size, liquidity, profitability, risk, diversification, ownership, IC. | Intermediation efficiency: Size, liquidity and priority positively impacted, IC negatively impacted and the others variables were not statistically significant. Profit efficiency (operating): profitability and diversification had a positive impact, whereas the other variables were not significant. (continued on next page) |
### Table 6 (continued)

| Number | Type of Efficiency Analysis | Inputs 1°Stage | Intermediate Variables | Outputs 2° Stage | Exogenous Variables | Impact on Efficiency |
|--------|-------------------------------|----------------|------------------------|------------------|---------------------|----------------------|
| 50     | PTE. Intermediation and profit. | Operational expenses, loanable funds and capital stock. | Investment, performing loans and outputs that leaves the production system: service revenues and nonperforming loan. | Interest income and investment revenue. | Ratio of investments to loans and the ratio of nonperforming loans to performing loans. | No significance was found in the estimations assessed. |
| 49     | Malmquist. Intermediation approach. | Total deposits ($x_1$), total labour ($x_2$) and capital ($x_3$). | Do not apply. | Total loans ($y_1$) and total investments ($y_2$). | Bank specific (7): Size, credit risk, capitalization, market power, liquidity, management efficiency and dummy for domestic islamic bank. Macroeconomics: economic growth, inflation and world financial crisis. | TFPC — negative impact: capitalization. Liquidity and world financial crisis had positive impacts, however the relationship varies among models. |
| 28     | Not identified. Intermediation approach. | Number of branches and number of employees. | Administrative expenses and personnel expenses. | Equity and permanent assets. | Size, public, domestic, foreign and recent M&A. | Cost Efficiency: Size and recent M&A positively impacted, whereas the other variables were not significant. Productive efficiency: State owned positively impacted, recent M&A negatively impacted and the others variables were not significant. The efficiency assessed is the efficiency of each subprocess combined through the relational network model. HC, SC and RC positively impacted the efficiency. |
| 27     | Not identified. Intermediation approach. | $X_1$: total liability ratio; $X_2$: total equity ratio; $X_3$: unit cost of employee. | $Y_1$: profit ratio; $Y_2$: return on asset (ROA); $Y_3$: return on equity (ROE). | Book-to-market equity ratio ($B/M$) and Earnings to price ratio ($E/P$). | Intellectual capital, measured by three variables: human capital (HC), structural capital (SC) and relational capital (RC). | Intellectual capital, measured by three variables: human capital ($HC$), structural capital ($SC$) and relational capital ($RC$). |
| 34     | Not identified. Intermediation approach. | Capital, deposits and labour. | Do not apply. | Conventional banks: interest income, non-interest income and total loans. Islamic banks: financing income, non-interest income and total financing. | Three macroeconomic variables: Growth domestic product (GDP), inflation and concentration. Seven bank specific variables: Dummy for islamic banks, size, capitalization, profitability, credit risk, diversification and market power. | Three macroeconomic variables: Growth domestic product (GDP), inflation and concentration. Seven bank specific variables: Dummy for islamic banks, size, capitalization, profitability, credit risk, diversification and market power. |
| 17     | PTE. Intermediation approach. | Deposits, number of employees and fixed assets. | Do not apply. | Securities and loans. | Potential M&A. | During the crisis, the vast majority of potential M&A did not generate gains in efficiency. The efficiency assessed is the efficiency of each subprocess combined through the relational network model. HC, SC and RC positively impacted the efficiency. Market power, the fact that the bank is islamic, GDP, profitability and concentration showed positive impacts on efficiency. Size, capitalization and diversification had negative impacts and inflation and credit risk were not statistically significant. |
| 14     | PTE. Intermediation approach. | Interest expenses, operational expenses net of personnel expenses, personnel expenses and total deposits. | Do not apply. | Performing loans, other earning assets, interest revenue and non-interest revenue. | Influence of integration and coordination efforts on banking efficiency, and on convergence within the GCC countries. | Tests corroborate convergence in banking efficiency. Integration and harmonization measures had a significant effect on efficiency and on the degree of homogeneity in the GCC banking industry. |
| Number | Type of Efficiency | Scope of Analysis | Inputs 1st Stage | Intermediate Variables | Outputs 2nd Stage | Exogenous Variables | Impact on Efficiency |
|--------|-------------------|-------------------|------------------|------------------------|------------------|--------------------|----------------------|
| 35     | PTE.              | Intermediation    | Total deposits, capital and personnel expenses. | Do not apply. | Loans, investments and non-interest income. | Six bank specific variables: ratio of loan loss provisions to total loans (LLP/TL), ratio of non-interest income over total assets (NII/TA), ratio of non-interest expenses to total assets (NIE/TA), LOANS/TA, LN(TA), EQASS. Five external factors: LN(GDP), LN(INFL), LN(CR3), LN(Z-score), LN(MKTCAP/GDP). Bank ownership: (foreign, governmental, listed in the public stocks). | Positive impacts: Size (lnTA), capitalization (LN(EQASS)), diversification (ln (NII/TA)), GDP, CR3, Z-score (proxy for sector risk to default) and foreign. Negative impacts: LN (MKTCAP/GDP) (Proxy for financial market development), listed in the public stocks and governmental. Other variables were not statistically significant. |
| 38     | PTE.              | Intermediation    | Total funding, fixed assets and number of employees. | Do not apply. | Net profit and other earning assets. | Governance reform variables: foreign partial acquisition, public listing, short-term and long term partial foreign partial acquisition, short term and long term public listing. Control variables: time, state-owned banks, equity to total assets and GDP growth. | Public listing, time, state-owned banks, equity to total assets and GDP growth positively impacted efficiency. On the other hand, foreign partial acquisition negatively impacted efficiency. Other variables were not statistically significant. |
| 47     | PTE.              | Intermediation    | Number of employees and physical capital. | Deposits. | Performing loans, securities investments and a bad output: Nonperforming loans. | Capitalization, Net Interest Margin (NIM), risk, Industrial Index, bankrupt loans (BRL). | Capitalization, NIM, risk and BRL negatively impacted efficiency, whereas Industrial Index had a positive impact. The impacts analysed are not in the efficiency index but in the lacks of the inputs. Several variables had an impact on these lacks. |
| 6      | SBM-PTE.          | Intermediation    | Deposits, labour, capital and physical capital | Do not apply. | Loans adjusted to non-performing loans, investments and other earning assets, fee income and off-balance sheet items. | Variables related to restructuring measures: dummy variables for domestic bank mergers (MER), foreign bank entry (FOR), and state intervention (SI). Five country-specific factors: Market Concentration Index (MC), Interbank Interest Rate (INT), Intermediation Ratio (IR), per capita GDP (PCGDP), and IMF supports (IMFS); Control variable: Size. | The impacts analysed are not in the efficiency index but in the lacks of the inputs. Several variables had an impact on these lacks. |
| 7      | TE.               | Intermediation    | Labour, capital and purchased funds. | Do not apply. | Total loans net of provision loans, deposits and investments. | Allocative efficiency: NPL and equity negatively impacted; MS, the fact that the bank is domestic and state-owned positively impacted efficiency. Technical efficiency: MS had a positive impact; Cost efficiency: MS and state-owned positively impacted and MS of the previous year negatively impacted. Other variables were not statistically significant. | (continued on next page) |
| Number | Type of Efficiency | Scope of Analysis | Inputs 1st Stage | Intermediate Variables | Outputs 2nd Stage | Exogenous Variables | Impact on Efficiency |
|--------|--------------------|-------------------|------------------|------------------------|------------------|---------------------|----------------------|
| 46     | TE                 | Intermediation approach | Third-party funds, total assets, and labour costs. | Do not apply. | Financing and operating income. | Asset, the number of bank branches (BRANCHES), return on assets (ROA), capital adequacy ratio (CAR), and non-performing financing (NPF). Five bank specific variables: LOGTA which corresponds to the logarithm of bank's total assets and controls for bank's size; EQAS is the equity to assets ratio and controls for capital strength; LOANTA is bank's net loans to total assets ratio, and is a measure of loan activity; ROE is the pre-tax profit divided by equity; EXPTA is the non-interest expenses to assets ratio. 12 variables related to country-specific factors. | Negative impact: Asset and ROA. Positive impact: Branches. Others variables were not statistically significant. |
| 4     | TE and PTE.        | Intermediation approach. | Total deposits, total costs (interest expenses and non-interest expenses), and equity. | Do not apply. | Loans, other earning assets and non-interest income. | | PTE - Statistically significant impacts: Country-specific variables such as the protection of private property rights, market capitalization to GDP, bank claims to GDP, the number of branches and ATMs relative to the population, the presence of government-owned and foreign-owned banks and concentration. Positive impacts: Higher size and lower loan activity. Not significant: Capitalization, profitability and expenses relative to assets. TE: size, LLP and LOTA negatively impacted efficiency, whereas other variables were not statistically significant. PTE: size, LLP and LOTA negatively impacted efficiency, whereas other variables were not statistically significant. |
| 44    | TE and PTE.        | Intermediation approach. | Fixed assets, deposits and staff expenses. | Do not apply. | Investment, net loans and fees. | Size, bank asset concentration, leverage, loan loss provisions to Loans (LLP), ratio of loans to TA (LOTA) and ROA | |
| 40    | TE e PTE.          | Intermediation approach. | Number of employees, purchased funds and customer deposits. | Do not apply. | Costumer loans, other loans and securities. | ROA, COA, city, size, branches, age and the ratio of non-performing loans to customer loans. | TE: ROA, size and city positively impacted efficiency, whereas the number of branches and age (number of years the bank existed before 2009) negatively impacted. PTE: ROA had a positive impact, number of branches and age had a negative impact, whereas other variables were not statistically significant. |
| 22    | Not identified.    | Not identified.     | Employees, assets and net assets. | Deposits, loans, income and interest income. | Net interest income, net service income and profit. | Weight of shares held by the top 5 shareholders, the weight of shares held by the foreign strategic shareholders, the real GDP and the CPI. Two agency-specific variables: public transportation and automatic teller frequency. Other variables: potential customers and competitive environment. | The 3 market power proxies and CPI positively impacted efficiency, whereas other variables were not statistically significant. No significant impact was found. |
| 2     | PTE.               | Not identified.     | Personnel expenses, branch space, other expenses and risk index. | Do not apply. | Comissions, deposits and loans. | | |
| Number | Type of Efficiency | Scope of Analysis | Inputs 1st Stage | Intermediate Variables | Outputs 2nd Stage | Exogenous Variables | Impact on Efficiency |
|--------|-------------------|-------------------|------------------|------------------------|------------------|-------------------|---------------------|
| 15     | TE and PTE.       | Not identified.   | Number of operational staff, number of business personnel, branch office rent and operating expenses. | Do not apply.    | Net interest spread income and net fee income. | Two variables related to external economic environment: Real GDP growth and Consumer Price Index (CPI). Three agency-specific variables: branch floor area, years of operation, and loan amount. Two foreign capital participation proxies. Control variables: capital structure, real GDP, money supply growth rate, and bank loans’ weight in the total capital formation. Dummy for private or state-owned and the percentage of employees with a diploma. | The impact analysed is not in the efficiency index but in the slacks of the inputs. |
| 58     | Not identified.   | Operational efficiency and market efficiency (2nd stage). | Net asset, total asset and employees. | Deposits, loans and service income. | Net income, ROA and ROE. | Foreign ownership, government ownership, recent M&A, active dividend policy, and trend were not statistically significant. Price of deposits, price of labour, IFRS accounting principles, and market-share were significant, and the relationship (positive or negative) with efficiency depends on the reliability of input and output variables. Bank's location. | Market efficiency: Positively impacted: foreign ownership, money supply growth rate. Negatively impacted: real GDP. Other variables were not statistically significant. |
| 42     | Not identified.   | Production approach. | $X_1$: Total costs, $X_2$: employee costs. | Do not apply. | $Y_1$: total deposits, $Y_2$: income before tax, $Y_3$: total credit. | Control Variables (5): price of labour, price of capital, price of deposits, trend, market-share. Contextual Variables (5): foreign ownership, government ownership, M&A, IFRS accounting policy and Active dividend policy. | Foreign ownership, government ownership, recent M&A, active dividend policy, and trend were not statistically significant. Price of deposits, price of labour, IFRS accounting principles, and market-share were significant, and the relationship (positive or negative) with efficiency depends on the reliability of input and output variables. Bank's location. | No relevant impact of the location of the bank in the efficiency. |
| 1      | PTE.              | Production approach. | Number of employees, equity and total asset. | Profit and revenue. | Market value, earning per shares (EPS) and stock price. | Foreign ownership, government ownership, recent M&A and Same General Accepted Accounting Principles. | Foreign ownership, government ownership, recent M&A and Same General Accepted Accounting Principles. | Impact on the virtual efficiency of M&A: Foreign ownership, government ownership and same accounting principles positively impacted efficiency. Recent M&A was not statistically significant. Being national and listed in the stock market increase the likelihood of a bank being efficient, whereas (3) whether the bank is big or not; (4) Tier 1 ratio; (5) total capital ratio; and (6) relative interest expense on customer deposits decrease that likelihood. Gains from M&A are likely to be higher when the two banks are commercial and smaller and when banks are local. |
| 41     | PTE.              | Production approach. | 17 variables related to banking activity. | Do not apply. | 17 variables. | Foreign ownership, government ownership, recent M&A and Same General Accepted Accounting Principles. | Foreign ownership, government ownership, recent M&A and Same General Accepted Accounting Principles. |外国 ownership, government ownership, recent M&A and Same General Accepted Accounting Principles. |
| 53     | TE and PTE.       | Production approach. | (8): Reserves for Impaired loans, equity, impaired loans, operational cost, personnel expenses, number of employees, number of branches and depreciation. | Do not apply. | (8): Total assets, fixed assets, gross loans, total securities, total customer deposits, pre-tax profit, net interest income and total non-interest operating income. | (8): 1. Listed in stock market; 2. Foreign bank; 3. Big bank; 4. Tier 1 Ratio; 5. Total Capital Ratio; 6. Interest Expense on Customer Deposits/Average Customer Deposits; 7. National/Regional; and 8. Cost of deposits. | (8): 1. Listed in stock market; 2. Foreign bank; 3. Big bank; 4. Tier 1 Ratio; 5. Total Capital Ratio; and (6) relative interest expense on customer deposits decrease that likelihood. |
| 52     | TE                | Production approach (1st stage) and intermediation (2nd stage). | Employees, fixed assets and operational expenses. | Deposits and loans. | Interest income and non interest income. | Trend, Trend$^2$, commercial, local. | (continued on next page) |
these gaps can be considered a potential topic for future research on the subject.

It was found that the most frequent objective in the studies was to extend or improve DEA models, whereas the intermediation approach was the most used for variable selection, and the intermediate variables technique was the most popular in the second stage, in which the deposits variable was the most frequently adopted intermediate variable.

Despite operational research and expert and intelligent systems focus on extending or improving DEA models, several other aspects still need further in-depth analysis, as presented in the discussion of the gaps in the literature. In addition, we contribute to the literature by presenting the state-of-the-art on two-stage DEA models as well as by providing directions and gaps for further research. In this context, this systematic review reflects an effort to shed light at those points.

Considering the models that effectively adopted a procedure after measuring the efficiency scores via DEA (categorized as external two-stage DEA models), the application of a bootstrapped truncated regression was most common. Regarding the impact of non-discretionary variables on efficiency, even when comparing studies that analysed banks in similar functions, results remain ambiguous. It is important to highlight that these studies may be susceptible to the separability issue and that future research should carefully address this limitation of the method.

A limitation of this study is that it did not review all the articles that applied two-stage DEA models (internal or external) in banks. However, we believe that by analysing the 59 articles included, it was possible to present an overview of how the application of such models has occurred in banks, with an in-depth discussion on controversial issues. We hope that this study can assist in future applications and discussions on the theme.

CRediT authorship contribution statement

Iago Cotrim Henriques: Conceptualization, Methodology, Writing - original draft. Vinicius Amorim Sobreiro: Conceptualization, Methodology, Writing - review & editing, Supervision. Herbert Kimura: Writing - review & editing, Supervision. Enzo Barberio Mariano: Writing - review & editing, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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