Factors That Determine the Power-law Index of an Energy Distribution of Solar Flares

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Abstract

The power-law index of an occurrence frequency distribution of flares as a function of energy is one of the most important indicators to evaluate the contribution of small-scale flares to coronal heating. For a few decades, many studies tried to derive the power-law index using various instruments and methods. However, these results are various and the cause of this uncertainty is unknown due to the variety of observation conditions. Therefore, we investigated the dependence of the index on the solar activity, coronal features, released energy range, and active region properties such as magnetic flux, twist, and size. Our findings are (1) annual power-law index derived from time series of total solar irradiance (Sun-as-a-star observation analysis) has a negative correlation with sunspot number; (2) power-law index in active region is smaller than that of the quiet Sun and coronal holes; (3) power-law index is almost constant in the energy range of $10^{25} \lesssim E \lesssim 10^{30}$ erg; and (4) active regions that have more magnetic free energy density, unsigned magnetic flux, and shear angle tend to have smaller power-law indices. Based on the results and energy-scaling law of Petschek-type reconnection, we suggest that the power-law index of sunspot-scale events is smaller than that of granule-scale events. Moreover, we indicated that sunspot-scale events follow CSHKP flare model whereas granule-scale events follow Parker’s nanoflare model.

Unified Astronomy Thesaurus concepts: Solar active regions (1974); Solar corona (1483); Solar coronal heating (1989); Solar flares (1496); Solar physics (1476); Solar activity (1475)

1. Introduction

Understanding the mechanism of the coronal heating is one of the most important problems in solar physics. Two primary models are suggested to explain this mechanism: nanoflare heating and wave dissipation. In the former model, the corona is heated by nanoflares ($E \approx 10^{24}$ erg), which are associated with the magnetic reconnections between the coronal magnetic fields tangled by the random motion of the foot point (e.g., Parker 1983, 1988). These dissipations of magnetic stresses are referred to as direct current (DC) heating. In the latter model, the dissipation of the Alfvén waves from the surface, which are generated by convection, heats the corona (e.g., Goldstein 1978; Hollweg et al. 1982). This heating process is referred to as alternating current (AC) heating as well. We focused on the nanoflare (DC) heating model in this study. In recent studies, the term nanoflare means “an impulsive energy release on a small cross-field spatial scale without regard to physical mechanism” (Klimchuk 2015). This change of terminology is caused by the prospect that waves can produce nanoflares (Klimchuk 2006).

The occurrence frequency distribution of flares as a function of energy is known to follow a power-law as following equation:

$$\frac{dN}{dE} = AE^{-\alpha}$$  \hspace{1cm} (1)

where $N$, $E$, $A$, and $\alpha$ are the number of flares, released energy, power-law constant, and power-law index (slope), respectively (Hudson 1991). The total energy released by flares $P$ is derived as follows:

$$P = \int_{E_{\text{min}}}^{E_{\text{max}}} dN \frac{dE}{E} = \frac{A}{-\alpha + 2} (E_{\text{max}}^{-\alpha + 2} - E_{\text{min}}^{-\alpha + 2}).$$  \hspace{1cm} (2)

This equation indicates that the contribution of smaller flares in coronal heating becomes dominant when $\alpha > 2$. Therefore, for a few decades, many studies tried to derive the power-law index using various methods and instruments. Shimizu (1995) reported the power-law index to be 1.5–1.6 based on an active region (AR) study using the Soft X-ray Telescope (SXT; Tsuneta et al. 1991) on board Yohkoh Satellite (Ogawara et al. 1991). Parnell & Jupp (2000) and Aschwanden et al. (2000) derived the indices using the EUV telescope of the Transition Region And Coronal Explorer (TRACE: Handy et al. 1999). As a result, whereas Parnell & Jupp (2000) implied that the power-law index is $>2$, Aschwanden et al. (2000) reported the index is approximately 1.8. As well, some studies suggested that the power-law index is $>2$ (e.g., Benz & Krucker 2002) whereas others found that the index is $<2$ (e.g., Jess et al. 2019; Kawai & Imada 2021a). Table 1 represents the methods and results of these studies. Due to the variety of the instruments, observation dates, regions of interest, and energy ranges, the cause of such a difference is unidentified.

Uyanov et al. (2019) surveyed the difference of energy distribution of flares between the solar minimum and the rising phase of cycle 24 in the quiet Sun (QS). As a result, the power-law index at the solar minimum ($\alpha \approx 2.8$) is greater than that of the rising phase ($\alpha \approx 2.3$). But they used different instruments (TESS/CORONAS-PHOTON for the solar minimum and SDO/AIA for the rising phase), therefore, it is not clear whether this difference is actually due to solar activity. The energy distribution of stellar flares have also been investigated using various instruments for many years. Their results are also
variable ($1.6 \lesssim \alpha \lesssim 2.7$; e.g., Kashyap et al. 2002; Maehara et al. 2012; Shibayama et al. 2013; Wu et al. 2015). Wu et al. (2015) suggested that the stars that have a shorter rotation period have a larger power-law index from their investigation of G-type stars observed by the Kepler Mission (Koch et al. 2010). On the other hand, Aschwanden & Güdel (2021) reported that the power-law index does not have time variability and dependence on the stellar spectral types. Unlike stellar cases, as far as we know, there are no studies which tried to reveal the cause of uncertainty of power-law index for solar flares.

Similar to the observational studies, power-law indices estimated by numerical simulations are also varied. Kanella & Gudiksen (2018) performed three-dimensional magnetohydrodynamic (3D MHD) simulation of loop-like magnetic structure. The obtained a power-law index of identified joule heating events is approximately 1.41 in the energy range of $10^{20} \lesssim E \lesssim 10^{26}$ [erg]. Bingert & Peter (2013) also employed 3D MHD simulation of an observed AR. The obtained distribution is not a single power-law and they fitted it as a double power-law distribution. The power-law index is 1.2 and 2.5 in the energy range of $10^{21} \lesssim E \lesssim 10^{24}$ and $10^{24} \lesssim E \lesssim 10^{26}$ [erg], respectively. There are other studies that suggest that the occurrence distribution does not follow a single power-law. Kawai & Imada (2021b) derives the distribution of an AR using 1D hydrodynamic model and a genetic algorithm. As a result, the power-law index is greater than two in higher energy range ($10^{26} \lesssim E \lesssim 10^{27.5}$) but not in a lower energy range.

As described above, the derived power-law indices from previous studies are very different even though the index is critical to evaluate the nanoflare heating model. Therefore, the motivation of this study is to reveal what determines the power-law index of a flare occurrence frequency distribution. To reach this goal, we investigated the dependence of the index on the solar activity, coronal features, released energy range, and AR properties. We compare a temporal series of power-law index derived by Sun-as-a-star observation and that of sunspot number in Section 2.1. We provide the difference of power-law index in ARs, QS, coronal holes, and off- limb in Section 2.2. In Section 2.3, we show occurrence frequency distributions from nanoflares to the largest flares. We investigate the relationships between some AR parameters such as total unsigned flux and power-law index in Section 2.4. Finally, we discuss possible scenarios that can explain the revealed dependences in Section 3.

### 2. Methods and Results

We used observation data obtained from the Atmospheric Imaging Assembly (AIA: Lemen et al. 2012) on board the Solar Dynamics Observatory (SDO: Pesnell et al. 2012). This instrument takes full-Sun images of nine UV and EUV broadband channels. We chose six EUV channels (94, 131, 171, 193, 211, 335 Å), which is responsive to the emission of coronal plasma (Boerner et al. 2012). The pixel size and temporal resolution of each channel are 0.64 and 12 s, respectively. The observation data is available at JSOC. All AIA images used in this study for Sun-as-a-star observation are calibrated by the aia_prep routine provided in SolarSoftWare (SSW: Freeland & Handy 1998).

#### 2.1. Power-law Index versus Solar Activity

To derive the occurrence frequency distribution of larger flares, we performed a Sun-as-a-star observation using the AIA data for approximately 11 years. In Sun-as-a-star observation, the intensity in a snapshot is integrated and only the temporal variation is analyzed like stellar observations. The observation duration used for this analysis was from 2010 August 5 to 2020 December 31. We used time series of total intensity of each snapshot normalized by the exposure time for each channel. To reduce the artificial fluctuation, we only used data that meets QUALITY keyword is zero.

We detected significant intensity enhancements from the light curves for each year and each channel. First we detected all the enhancements from the light curve. The detection threshold was defined by the following equation:

$$\delta I > 3 \times I_{\text{avg}}$$

where, $\delta I$ and $I_{\text{avg}}$ represent changes in the light curve from the beginning to the peak of enhancement and 1-minute average before the beginning of the enhancement, respectively. We neglected enhancements of only a single snapshot to reduce the effect of noise on the analysis. Figure 1 shows an example of light curves and detection results. The blue and red ticks represent the beginnings and peaks of detected enhancements.

We estimated the flare released energy of each enhancement assuming it to be the change in thermal energy between the onset and peak. The thermal energy was estimated based on the Differential Emission Measure (DEM) calculated by the DeepEM code developed by Paul Write. DeepEM is a convolutional neural network trained to output the DEM when AIA six EUV channels are input. The training was performed using solutions of Cheung et al. (2015). We chose DeepEM for our analysis because other inversion methods (e.g., Aschwanden et al. 2013) will be computationally too heavy to calculate all the AIA data we use.

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8 http://jsoc.stanford.edu/

4 https://github.com/PaulIWright/DeepEM
where $k_B$, $V$, $T_k$, and $EM_k$ represent the Boltzmann’s constant, volume, temperature, and emission measure of temperature range $\Delta T_k$. We used the relationship $n_k = [(DEM(T_k) \Delta T_k)/V_k]^{1/2}$, and we assumed $V_k$ is constant ($=V$). The temperature is discretized in a logarithmic range of $log T = 5.5, 5.6, ..., 7.2$ equidistantly. Extensive benchmark tests have been calculated that corroborate the correctness of Equation (4) in Aschwanden et al. (2015a).

Because the DEM is derived from Sun-as-a-star observation, the event volume is assumed to be the entire coronal volume in the field of view. Therefore, we assumed the event volume as following the equation:

$$E_{th} = \sum_k E_{th,k} = \sum_k 3k_B n_k T_k V_k = 3k_B V^{1/2} \sum_k T_k [DEM(T_k) \Delta T_k]^{1/2}$$

(4)

where $k_B$, $V$, $T_k$, and $EM_k$ represent the Boltzmann’s constant, volume, temperature, and emission measure of temperature range $\Delta T_k$. We used the relationship $n_k = [(DEM(T_k) \Delta T_k)/V_k]^{1/2}$, and we assumed $V_k$ is constant ($=V$). The temperature is discretized in a logarithmic range of $log T = 5.5, 5.6, ..., 7.2$ equidistantly. Extensive benchmark tests have been calculated that corroborate the correctness of Equation (4) in Aschwanden et al. (2015a).

Because the DEM is derived from Sun-as-a-star observation, the event volume is assumed to be the entire coronal volume in the field of view. Therefore, we assumed the event volume as following the equation:

$$V = \frac{2}{3} \pi [(R_{AIA})^3 - R_{\odot}^3]$$

(5)

where $R_{AIA} \approx 897$ Mm and $R_{\odot} \approx 696$ Mm represent the half length of a side of the field of view of the AIA and the solar radius.

Each panel in Figure 2 shows an occurrence frequency distribution of flares detected by AIA 193 A as a function of energy for each year. The red solid line represents the power-law fitted line. The values in the legends are the power-law index with the standard deviation. The power-law distribution was well reproduced in the energy range of $10^{27} \lesssim E \lesssim 10^{30}$ erg. The fitted energy range is defined as following the following procedures:

1. Determine the width of energy range $\Delta E_{fit}$ to be fitted.
2. Derive the power-law fitted line in the energy range of $E_{min} \leq E \leq E_{min} + \Delta E_{fit}$, where $E_{min}$ represents minimum energy bin of the histogram.

3. Calculate the mean squared error (MSE) between the histogram and the fitted line.
4. Shift the fitting energy range by the step of the energy bin size.
5. Perform steps 2–4 until the energy range reaches the maximum flare energy.
6. Obtain the fitted line of the energy range where the MSE is the least and the power-law index is greater than one.

In this paper, we defined $\Delta E_{fit} = 2$ in a logarithmic scale. The standard deviation of the power-law index seems to be smaller than those of previous studies. Because we used a CNN to energy estimation, the error of derived energy cannot be calculated. Therefore, we only estimated the standard deviation due to the fitting.

We compared the power-law index of derived flare distribution in each year with the solar activity. The top panel of Figure 3 presents annual time series of the power-law index obtained from each AIA channel. The black line in the bottom panel indicates a time series of monthly averaged sunspot numbers. We used the monthly smoothed sunspot number of the Observed Solar Cycle Indices Data provided by Space Weather Prediction Center of NOAA. The red line indicates the annual moving average of it. This figure shows negative correlations between the sunspot number and the power-law indices derived from all channels. The correlation coefficients are described in the legend on the top panel.

2.2. Power-law Index versus Coronal Features

We derived the flare occurrence frequency distribution in each coronal feature including active regions (ARs), quiet Sun (QS), coronal holes (CHs), and off-limb. We used image series of AIA EUV channels from 08:00 UT to 09:00 UT on 2014 February 25, including several ARs and CHs on the disk. First, we calculated 4 × 4 macro-pixel DEM maps in each snapshot using the DeepEM code as well. With reference to Adithya et al. (2021), we divided each image into ARs, QS, CHs, and off-limb based on the obtained DEM maps with the following criteria:

1. ARs
   (a) Calculate the medians of peak temperature $T_p$ and emission measure $EM_p$ from the derived DEM in each snapshot.
   (b) Detect macro pixels which satisfy $T_p > 1.3 \times \text{Median}(T_p)$ and $EM_p > 1.3 \times \text{Median}(EM_p)$.
   (c) Apply morphological closing with 15 × 15 kernel to smooth the contours.
   (d) Find connected macro pixels which have an area greater than 256 macro pixels (4096 AIA pixels).

2. CHs
   (a) Detect macro pixels which satisfy $T_p < 0.8 \times \text{Median}(T_p)$ and $EM_p < 0.8 \times \text{Median}(EM_p)$.
   (b) Remove the macro pixels where the distance from the disk center is greater than $R_{\odot}$.
   (c) Apply morphological closing with 15 × 15 kernel to smooth the contours.
   (d) Find connected macro pixels which have an area greater than 256 macro pixels.

https://www.swpc.noaa.gov/products/solar-cycle-progression
3. QS
   (a) Find macro pixels where the distance from the disk center is less than $R_e$.
   (b) Find macro pixels other than ARs or CHs.

4. Off-limb
   (a) Find macro pixels where the distance from the disk center is greater than $R_e$.
   (b) Find macro pixels other than ARs.

Figure 4 shows an example of the macro-pixel maps of $T_p$ (left), $E_{M_p}$ (center), and segmentation results (right). Red, blue, green, and purple in the segmentation results indicate the QS, ARs, CHs, and off-limb, respectively.

We applied the macro-pixel method as used in Shimizu (1995) in order to not miss faint enhancements and reduce the computational cost. As for the segmentation, the size of the macro pixel is 4 × 4 in this study. We detected significant enhancements from the light curve in each macro pixel. The detection criterion is the same as that of the Sun-as-a-star observation. When rising phases of multiple enhancements overlapped at adjacent macro pixels, they are regarded as a single event. The segment of each event is assigned based on its center of gravity.

Thermal energy is estimated by Equation (4) as well. However, event volume $V$ is assumed to be $S^3/2$ where $S$ represents the event area.

Figure 5 presents energy distributions of flares detected by AIA 171 Å in the QS (top left), ARs (top right), CHs (bottom left), and off-limb (bottom right) in 2014 (solar maximum). The power-law fitting criterion is the same as that of the Sun-as-a-star observation. When rising phases of multiple enhancements overlapped at adjacent macro pixels, they are regarded as a single event. The segment of each event is assigned based on its center of gravity. Thermal energy is estimated by Equation (4) as well. However, event volume $V$ is assumed to be $S^3/2$ where $S$ represents the event area.

Figure 6 represents the power-law indices in each region and of the events detected in each channel. As a result, the power-law index of flares detected in ARs tends to be smaller than that of the QS, CHs, and off-limb. This result seems to be consistent with the negative correlation between the power-law index and sunspot number. In darker channels (94, 131, 335 Å), sometimes the derived distributions cannot be fitted by a single figure.
power-law index using our algorithm due to the lack of energy range of detected events.

2.3. Power-law Index versus Released Energy

Figure 7 shows concatenated energy distribution using the result of Sun-as-a-star observation in 2014 and macro-pixel AR observations. The values in the legend represent the power-law indices. The power-law index is almost consistent over the energy range of \(10^{24} \lesssim E \lesssim 10^{30}\) erg. Or, in higher energy range, the index becomes larger, which is clearly shown in the result of 193 and 171 Å channel. We will discuss this point in Section 3.

2.4. Power-law Index versus AR Properties

To survey the dependence of the power-law index on the AR properties, we used Spaceweather HMI Active Region Patch (Bobra et al. 2014, SHARP) data set. SHARP includes various space-weather parameters calculated from the photospheric vector magnetic field. We used data series of hmi.sharp_720s in this study. Table 2 represents SHARP parameters which compared with power-law indices in this study. Thousands of AR data are stored in the data set, however, it is difficult to investigate all the ARs due to the computational cost. Therefore, we selected 108 ARs of the largest area in available data for the analysis. We obtained AIA six EUV observation data for 20 minutes for each AR. The beginning of the observation is when the mean longitude of the AR reaches the solar meridian to reduce the effect of a center-to-limb variation. The coordinates of these images are calibrated using LAT_MIN, LON_MIN, LAT_MAX, and LON_MAX quantities in the SHARP data set and Stonyhurst heliographic coordinate of each AIA pixel calculated by Sunpy modules (Mumford et al. 2020). The detection criterion and the way of energy estimation are the same as the previous analysis.

Figure 8 shows an example of energy distributions of flares detected in SHARP AR by each AIA channel. The HARP number of this AR is 5541. The red line indicates power-law fitted line which is derived by the same procedure described in Section 2.1.
We investigated dependences of power-law index on 18 SHARP quantities listed in Table 2 for 108 regions. For example, Figure 9 shows the relationship between the power-law index and total unsigned flux (USFLUX) of the ARs in each AIA channel. Each tick corresponds to the SHARP AR. Black lines are fitted to data points using least squares. There are weak negative correlations between them in all channels.

Table 2
SHARP Parameter List Compared with Power-law Indices (see Bobra et al. 2014 for more Details)

| SHARP keyword | Description |
|---------------|-------------|
| USFLUX        | Total unsigned flux |
| MEANGAM       | Mean angle of field from radial |
| MEANGBT       | Horizontal gradient of total field |
| MEANGBZ       | Horizontal gradient of vertical field |
| MEANGBH       | Horizontal gradient of horizontal field |
| MEANJZD       | Vertical current density |
| TOTUSJZ       | Total unsigned current |
| MEANALP       | Twist parameter |
| MEANIZH       | Current helicity |
| TOTUSHL       | Total unsigned current helicity |
| ABSNIZH       | Absolute value of the net current helicity |
| SAVNCP        | Sum of the modulus of the net current per polarity |
| MEANPOT       | Proxy for mean photospheric excess magnetic energy density |
| TOTPOT        | Proxy for total photospheric magnetic free energy density |
| MEANSHR       | Shear angle |
| SHRGT45       | Fractional of area with shear greater than 45° |
| R_VALUE       | Unsigned flux R (Schrijver 2007) |
| SIZE_ACR      | Projected area of active pixels on image |

Fursyak (2018) revealed that the greater the mean vertical current density, the higher the flare index of the AR. On the other hand, MEANGAM, MEANPOT, MEANSHR, SHRGT45, and R_VALUE have relatively strong negative correlations. These parameters are also important indicators for the flare prediction study (e.g., Hazra et al. 2020; Yi et al. 2021). Roughly speaking, ARs having more magnetic free energy tend to have smaller power-law indices.

3. Discussion and Summary

We statistically investigated the dependences of power-law index of energy distribution of flares on the solar activity, coronal features, released energy, and AR parameters. Our findings are as follows:

1. Yearly power-law index derived by the Sun-as-a-star observation has negative correlation with sunspot number.
2. The power-law index of detected flares in ARs is smaller than that of QS and CHs.
3. The power-law index is almost constant in the energy range of $10^{25} \leq E \leq 10^{30}$ erg.
4. Active regions with more magnetic free energy density, unsigned magnetic flux, and shear angle tend to have smaller power-law indices.

From the first, second, and fourth results, the power-law index of flares detected in magnetically active (flare productive) regions becomes smaller than those of the QS and relatively calm ARs. These results almost explain the difference of the indices in previous studies described in Table 1 (QS studies tend to report greater power-law indices than those of AR studies.). The third result suggests that the energy-release process is consistent regardless of energy scale in this energy range. In Figure 7, it seems that the power-law index becomes larger in higher energy range especially in 193 and 171 Å.
channel. This gap is probably caused by the difference of analysis methods because whereas the macro-pixel method focused on AR cores as shown in Figure 4, Sun-as-a-star observations of these channels respond to emissions from coronal loops and QS. In addition, except for some ARs, almost all the power-law indices are greater than 2, which supports the nanoflare (DC) heating model.

Based on the above results, we describe two scenarios for the energy-release process. The first is that the “apparent” power-law index varies, but the “actual” power-law index is consistent, regardless of the region. The other is that the physical process of energy release is different between ARs and QS. According to the first scenario, for example, an overlap of enhancements along the line-of-sight direction appears to cause the underestimation of the power-law index (larger events dominate). The overlap appears to occur easily because there are multiple sub-structures in the resolved coronal loops (Warren et al. 2008; Viall & Klimchuk 2012) and they are probably smaller than a single AIA pixel (Tajfouze et al. 2016; Kawai & Imada 2021b). However, in this scenario, the power-law index that is derived by Sun-as-a-star observation should be smaller than that of the macro-pixel method because...
there are more overlapped events in Sun-as-a-star observation. Alternatively, a higher event-occurrence rate results in more missed enhancements owing to the observation limitation. Aschwanden & Dudok de Wit (2021) identified the positive correlation between the sunspot number and power-law index of the waiting-time distribution of flares. The occurrence rate of smaller events is higher than that of larger events. Therefore, a lack of observation cadence may result in the underestimation of the power-law index.

To investigate the latter scenario in detail, we analyzed the dependence of the power-law index on its fitting energy range using SHARP data. Figure 11 shows scatter plots of the index and central energy of the fitting. This figure presents that the hotter channels (94, 131, 211, and 335 Å) have negative correlations between the index and its fitting range. In addition, these plots seems to be divided into two groups with 1025 erg as the boundary, as clearly seen in 94 Å. This indicates that the physical process of heating may differ around this energy.

Using 1.5D MHD simulations, Antolin et al. (2008) suggested that a coronal loop that is heated by nanoflares has a smaller power-law index than that of wave heating. Therefore, our results imply that the ARs are mainly heated by Alfvén waves. Moreover, the ratio of the nanoflare and wave heating is dependent on the magnetic properties, even among ARs. However, in this scenario, when nanoflare heating is dominant, the power-law index becomes smaller than 2, which does not support the nanoflare heating model. And the positive correlation between the power-law index and the vertical current density seems to deny this scenario because high current density helps magnetic reconnections. In fact, mean vertical current density has positive correlation with flare index (Fursyak 2018).

Dissipated magnetic energy $E_{\text{diss}}$ in Petschek-type magnetic reconnection can be described as the following equation:

$$E_{\text{diss}} = \left( \frac{B_{\text{free}}}{8\pi} \right) L^3$$

(6)

where $B_{\text{free}}$ and $L$ represents free energy field strength and length scale, respectively (Aschwanden 2020). Figure 12 shows its relationship and dashed line indicates $E_{\text{diss}} = 10^{25}$ erg. GOES M- and X-class flares have length scale of $1.3 \times 10^9 \lesssim L \lesssim 2.4 \times 10^{10}$ cm and free energy field of $1.0 \times 10^2 \lesssim B_{\text{free}} \lesssim 1.2 \times 10^3$ G (Aschwanden 2020). And the length scale of distance between leading and following sunspot is $10^9 \lesssim L \lesssim 10^{10}$ cm (Muraközy et al. 2012). On the other hand, the length scale of granules is approximately $10^8$ cm or smaller.

We attempt to explain the difference of power-law index between two event groups from the perspective of self-organized criticality model. The energy power-law index can be described by the following equations:

$$\alpha = 1 + \frac{S - 1}{\gamma D + 2/\beta}$$

(7)

$$F \propto V^\gamma$$

(8)

$$V \propto L^D$$

(9)

$$L \propto T^{3/2}$$

(10)

where $S$, $D$, $F$, $V$, and $T$ represent Euclidean geometry, mean fractal dimension, observed flux, flare volume, and avalanche duration, respectively (Aschwanden & Güdel 2021). In solar and stellar flare cases, Aschwanden (2014) assumed $S = 3$, $D = 2$, $\gamma = 1$, $\beta = 1$ (classical diffusion), and the derived power-law index was approximately 1.5. However, this estimation seems to assume the three-dimensional flare model such as CSHKP model (Carmichael 1964; Sturrock 1966; Hirayama 1974; Kopp & Pneuman 1976). In Parker’s nanoflare model (Parker 1983, 1988), mean fractal dimension $D$ seems to be $\sim 1$ and the index becomes larger than that of CSHKP model if other parameters are the same.

Based on this point, we suggest that the heating model as Figure 13. The energy distribution of flares can be divided into two power-law distributions. The first one is the larger energy events with smaller power-law index, which might follow the CSHKP model with a magnetic length scale of sunspots. The other is the smaller energy events with larger index, which
follows Parker’s model with a magnetic length scale of granules.

The degradation of the instrument is one of the concerns on our analysis because we used satellite data for over 10 years. Therefore, we investigated the dependence of the power-law index on the observation date using ARs studied in Section 2.4. Figure 14 shows the relationship between the power-law index and observation date (days from 2010 May 1) in each AIA channel. As shown in this figure, there are no significant effects of the degradation on the power-law index analysis.

We assumed the flare released energy to be changed into thermal energy. However, this assumption is not strictly correct because of the cooling and distribution to the Doppler motion and nonthermal energies (Kawai & Imada 2021a). Also, nonequilibrium of ionization can also have affects on the detection and energy estimation in the case of impulsive heating (Orlando et al. 1999; Reale & Orlando 2008; Imada et al. 2011). In addition, to reduce the computational cost, we performed Sun-as-a-star observation to derive the distribution of larger flares. Since this method cannot spatially resolve the events, we had to fix the size of events at the whole coronal volume. The flare energy derived in Sun-as-a-star observation may be over- or underestimated due to this assumption because we assumed flare energy to be the difference in thermal energy. But our volume assumption may be reasonable because energy distributions in Figure 7 seem to be smoothly connected.

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References

Adithya, H. N., Kariyappa, R., Shinsuke, I., et al. 2021, SoPh, 296, 71
Antolin, P., Shibata, K., Kudoh, T., Shiota, D., & Brooks, D. 2008, ApJl, 688, 669
Aschwanden, M. J. 2014, ApJ, 782, 54
Aschwanden, M. J. 2020, ApJ, 903, 23
Aschwanden, M. J., Boerner, P., Caspi, A., et al. 2015a, SoPh, 290, 2733
Aschwanden, M. J., Boerner, P., Ryan, D., et al. 2015b, ApJ, 802, 53
Aschwanden, M. J., Boerner, P., Schrijver, C. J., & Malanushenko, A. 2013, SoPh, 283, 5
Aschwanden, M. J., & Dudok de Wit, T. 2021, ApJ, 912, 94
Aschwanden, M. J., & Güdel, M. 2021, ApJ, 910, 41
Aschwanden, M. J., Tarbell, T. D., Nightingale, R. W., et al. 2000, ApJ, 535, 1047
Benz, A. O., & Knucker, S. 2002, ApJ, 568, 413
Berghmans, D., Clette, F., & Moses, D. 1998, A&A, 336, 1039
Bingert, S., & Peter, H. 2013, A&A, 550, A30
Bobra, M. G., Sun, X., Hoeksema, J. T., et al. 2014, SoPh, 289, 3549
Boerner, P., Edwards, C., Lemen, J., et al. 2012, SoPh, 275, 41
Carmichael, H. 1964, in Proc. AAS-NASA Symp., The Physics of Solar Flares, ed. W. N. Hess, Vol. 50 (Washington, DC: NASA), 451
Cheng, M. C. M., Boerner, P., Schrijver, C. J., et al. 2015, ApJ, 807, 143
Freeland, S. L., & Handy, B. N. 1998, SoPh, 182, 497
Furesy, Y. A. 2018, GeoAr, 58, 1129
Goldstein, M. L. 1978, ApJ, 219, 700
Handy, B. N., Acton, L. W., Kankelborg, C. C., et al. 1999, SoPh, 187, 229
Hirayama, T. 1974, SoPh, 34, 323
Hollweg, J. V., Jackson, S., & Galloway, D. 1982, SoPh, 75, 35
Hudson, H. S. 1991, SoPh, 133, 357
Imada, S., Hara, H., Watanabe, T., et al. 2011, ApJ, 743, 57
Jess, D. B., Dillon, C. J., Kirk, M. S., et al. 2019, ApJ, 871, 133
Kanella, C., & Gudiksen, B. V. 2018, A&A, 617, A50
Kashyap, V. L., Drake, J. J., Güdel, M., & Audard, M. 2002, ApJ, 580, 1118
Kawai, T., & Imada, S. 2021a, ApJ, 918, 51
Kawai, T., & Imada, S. 2021b, ApJ, 906, 2
Klimchuk, J. A. 2006, SoPh, 234, 41
Klimchuk, J. A. 2015, RSPTA, 373, 20140256
Koch, D. G., Borucki, W. J., Basri, G., et al. 2010, ApJL, 713, L79
Kopp, R. A., & Pneuman, G. W. 1976, SoPh, 50, 85
Lemen, J. R., Title, A. M., Akin, D. J., et al. 2012, SoPh, 275, 17
Maehara, H., Shibayama, T., Notsu, S., et al. 2012, Natur, 485, 478
Mumford, S., Freij, N., Christe, S., et al. 2020, JOSS, 5, 1832
Murakózy, J., Baranyi, T., & Ludmány, A. 2012, CEAB, 36, 1
Ogawara, Y., Takano, T., Kato, T., et al. 1991, SoPh, 136, 1
Orlando, S., Bocchino, F., & Peres, G. 1999, A&A, 346, 1003
Parker, E. N. 1983, ApJ, 264, 635
Parker, E. N. 1988, ApJ, 330, 474
Parnell, C. E., & Jupp, P. E. 2000, ApJ, 529, 554
Pesnell, W. D., Thompson, B. J., & Chamberlin, P. C. 2012, SoPh, 275, 3
Reale, F., & Orlando, S. 2008, ApJ, 684, 715
Schrijver, C. J. 2007, ApJL, 655, L117
Shibayama, T., Maehara, H., Notsu, S., et al. 2013, ApJS, 209, 5
Shimizu, T. 1995, PASJ, 47, 251
Sturrock, P. A. 1966, Natur, 211, 695
Tajiriouze, E., Reale, F., Petralia, A., & Testa, P. 2016, ApJ, 816, 12
Tsuneta, S., Acton, L., Bruner, M., et al. 1991, SoPh, 136, 37
Ulyanov, A. S., Bogachev, S. A., Reva, A. A., Kirichenko, A. S., & Loboda, I. P. 2019, AstL, 45, 248
Viall, N. M., & Klimchuk, J. A. 2012, ApJ, 753, 35
Warren, H. P., Ugarte-Urra, I., Doschek, G. A., Brooks, D. H., & Williams, D. R. 2008, ApJL, 686, L131
Wu, C.-J., Ip, W.-H., & Huang, L.-C. 2015, ApJ, 798, 92
Yi, K., Moon, Y.-J., Lim, D., Park, E., & Lee, H. 2021, ApJ, 910, 8