Physica Scripta

PAPER

Muon \( (g - 2) \) in \( U(1)_{L\mu L\tau} \) scotogenic model extended with vector like fermion

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Keywords: beyond standard model physics, neutrino mass, scotogenesis, vector like fermion

Abstract

The latest results of anomalous muon magnetic moment at Fermilab show a discrepancy of 4.2 \( \sigma \) between the Standard Model (SM) prediction and experimental value. In this work, we revisit \( U(1)_{L\mu L\tau} \) symmetry with in the paradigm of scotogenic model which explains muon \( (g - 2) \) and neutrino mass generation, simultaneously. The mass of new gauge boson \( M_{Z'} \) generated after the spontaneous symmetry breaking of \( U(1)_{L\mu L\tau} \) is constrained, solely, in light of the current neutrino oscillation data to explain muon \( (g - 2) \). In particular, we have obtained two regions I and II, around 150 MeV and 500 MeV, respectively, in \( M_{Z'} - g_{\mu\tau} \) plane which explain the neutrino phenomenology. Region I is found to be consistent with muon neutrino trident \( (MNT) \) bound \( (g_{\mu\tau} < 10^{-3}) \) to explain muon \( (g - 2) \), however, region II violates it for mass range \( M_{Z'} > 300 \) MeV. We, then, extend the minimal gauged scotogenic model by a vector like lepton (VLL) triplet \( \psi_T \). The mixing of \( \psi_T \) with inert scalar doublet \( y \) leads to chirally enhanced positive contribution to muon anomalous magnetic moment independent of \( Z_{\mu\tau} \) mass. Furthermore, we have, also, investigated the implication of the model for \( 0\nu\beta\beta \) decay and \( CP \) violation. The non-observation of \( 0\nu\beta\beta \) decay down to the sensitivity of 0.01 eV shall refute the model. The model, in general, is found to be consistent with both \( CP \) conserving and \( CP \) violating solutions.

1. Introduction

The standard model (SM) of particle physics has been very successful in explaining the observed dynamics of fundamental particles and their interactions. The discovery of Higgs boson at large hadron collider (LHC) has affirmed our belief in the theory. Though there are few questions which still remain unanswered as of now. For example, observation of non-zero neutrino mass, matter-antimatter asymmetry, existence of dark matter (DM), to name a few, are some of the astounding unresolved issues in the SM and require new physics scenarios for their explanation [1] (and references therein). Recently, combining the previous result of Brookhaven National Laboratory (BNL) [2], the muon \( (g - 2) \) collaboration at Fermilab has reported a 4.2\( \sigma \) discrepancy between the experimental observation \( (a_{\mu}^{\text{exp}}) \) and SM prediction \( (a_{\mu}^{\text{SM}}) \) of muon anomalous magnetic moment \( a_{\mu} \) [3–15]. More precisely, they found \( \Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9} \) [16] which is a sign of new physics motivating theoretical initiatives to calculate various contributions to muon \( (g - 2) \) with high precision, in particular, the quantum chromodynamics (QCD) contribution [17].

Several new physics scenarios have been proposed which can cause muon \( (g - 2) \) to differ from the SM prediction. In supersymmetric models, particles like smuon, neutralino, chargino enter in the loop and can give additional contribution to muon \( (g - 2) \) [18, 19]. Also, there are plethora of models which attempt to explain DM and contributions to muon \( (g - 2) \). In some of these models DM particle participate in the loop diagram and contribute to muon \( (g - 2) \). There are another class of models where DM particle do not directly interact with the SM particles, alternatively, mediator particle interacts with both DM and SM particle(s). The mediator particle can give contribution to muon \( (g - 2) \) [20]. Furthermore, leptoquark models can, also, give
contribution to muon \((g - 2)\) through the diagrams involving leptoquarks and SM quarks [21]. The two Higgs doublet model (2HDM) and its variations have, also, been proposed for the explanation of muon \((g - 2)\) [22–28].

Apart from the above possibilities, another class of model extends the SM with anomaly free \(U(1)_{L_\mu - L_\tau}\) [29, 30] symmetry for the explanation of muon \((g - 2)\) [31–37]. After spontaneous breaking of \(U(1)_{L_\mu - L_\tau}\) symmetry, the new boson \(Z_\mu\) participate in the loop diagram to give additional contribution to muon \((g - 2)\). Further variation of \(U(1)_{L_\mu - L_\tau}\) model is the gauged extension with in the framework of scotogenic model. The gauged scotogenic model is promising variation which explain neutrino mass generation, dark matter and muon \((g - 2)\), simultaneously. These scenarios have been extensively explored in different variations e.g. see Refs. [38–42]. The neutrino masses are generated by usual scotogenic process at one loop level by requiring three right handed neutrinos and an inert doublet. It is known that, in this framework, the mass of \(Z_\mu\) (\(M_{Z_\mu}\)) has to be less than the bound \(O(10^{15})\) MeV coming from muon neutrino trident (MNT) process [43]. Although there are severe experimental constraints on new gauge boson mass from CCFR [44], WD cooling [44, 45], COHERENT [46, 47], BABAR [48], NA62 [49] and NA64 [50], but there still remains allowed parameter space in \(M_{Z_\mu}\) plane which can explain muon \((g - 2)\). There are few attempts to widen the mass range of \(Z_\mu\), explaining muon \((g - 2)\) while still being consistent with MNT and other experimental bounds [23, 51]. In general, the explanation of muon \(g - 2\) based on gauged scotogenic model assumes small mixing between \(Z_\mu\) and SM gauge boson \(Z\). Otherwise the new gauge boson may not explain the muon \((g - 2)\) due to constraints from \(Z\)-pole precision observable at LEP [52].

In the present work, we revisit the possible explanation of muon \((g - 2)\) in \(U(1)_{L_\mu - L_\tau}\) scotogenic model of the SM wherein the particle content has been enlarged with three right handed neutrino \((N_{k}, k = e, \mu, \tau)\), one scalar singlet \(S\) and one inert doublet \(\eta\). \(N_k\) and \(\eta\) are odd under unbroken \(Z_\mu\) symmetry making them suitable DM matter candidate while scalar singlet \(S\) is even. Unlike the earlier works, the mass range of new gauge boson \(M_{Z_\mu}\), explaining muon \((g - 2)\) has been constrained in light of the current neutrino oscillation data. Also, as discussed earlier, the explanation of muon \(g - 2\) based on gauged scotogenic model assumes small mixing between \(Z_\mu\) and SM gauge boson \(Z\). In this work, we implement an extension of the gauged scotogenic model with vector like lepton (VLL) triplet \(\psi_T\) wherein muon \((g - 2)\) can be explained independent of above mixing pattern and \(U(1)_{L_\mu - L_\tau}\) gauge boson mass. In fact, the mixing of \(\psi_T\) and inert doublet \(\eta\) of scotogenic model results in chirally enhanced positive contribution to muon \((g - 2)\).

The paper is organised as follows. In section 2, we present and discuss phenomenology of the minimal gauged scotogenic model. In section 3, we extend the minimal gauged scotogenic model by a vector like lepton triplet and show that it successfully explains muon \((g - 2)\) anomaly and neutrino oscillation data. Finally, in section 4, we summarize our conclusions.

### 2. Minimal Gauged Scotogenic Model

Scotogenic model proposed by E. Ma [53] is a promising framework to explain dark matter (DM) and non-zero neutrino mass, simultaneously. Within this model, the field content is enlarged with three right handed neutrinos and an inert doublet \(\eta\). In general, the contribution of inert doublet \(\eta\) to muon \((g - 2)\) is negative [39, 54, 55]. This motivates to study \(U(1)_{L_\mu - L_\tau}\) extensions of scotogenic model for possible explanation of muon \((g - 2)\) and neutrino phenomenology. The SM gauge group is extended by \(U(1)_{L_\mu - L_\tau}\) symmetry. The complete particle content of the model with corresponding charge assignments are given in table 1. Here, \(L_k (k = e, \mu, \tau)\) are the usual SM left handed lepton doublets, \(k_R (k = e, \mu, \tau)\) are SM right handed charged leptons, \(N_k (k = e, \mu, \tau)\) are right handed neutrino singlets, \(H\) is SM Higgs, \(\eta\) is inert doublet and \(S\) is scalar singlet field introduced to break \(U(1)_{L_\mu - L_\tau}\) symmetry to induce the mass of the new gauge boson \(Z_\mu\). All the beyond SM (BSM) fields are odd under \(Z_2\) except \(S\), the lightest of which can be DM candidate within the model.

The relevant terms in Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_N - \frac{1}{4} (Z_{\mu\tau})_{\mu\nu} Z_{\mu\tau}^{\mu\nu} - \frac{e}{2} (Z_{\mu\tau})_{\mu\nu} B^{\mu\nu},$$ (1)
where $\mathcal{L}_{\text{Scalar}}$ is the Lagrangian containing kinetic and potential terms of the scalar sector ($H, \eta, S$) of the model, $\mathcal{L}_N$ is the Lagrangian for neutrino sector. The fourth and fifth terms in equation (1) are, kinetic term for $Z_{\mu \tau}$ and mixing term of $Z_{\mu \tau}$-Z gauge bosons, respectively ($\epsilon$ is mixing parameter). The Lagrangian for scalar fields is given by

$$\mathcal{L}_{\text{Scalar}} = \left( \mathcal{D}_\mu H \right)^\dagger \left( \mathcal{D}_\mu H \right) + \left( \mathcal{D}_\mu \eta \right)^\dagger \left( \mathcal{D}_\mu \eta \right) + \left( \mathcal{D}_\mu S \right)^\dagger \left( \mathcal{D}_\mu S \right) - V(H, \eta, S),$$

where the covariant derivative $\mathcal{D}_\mu$ is given by

$$\mathcal{D}_\mu = \partial_\mu - ig_1 Z_\mu - ig_2 W_\mu - ig_\nu Y_{\nu \mu},$$

and the scalar potential $V(H, \eta, S)$ can be written as

$$V(H, \eta, S) = -\mu_H^2 (H^\dagger H) + \mu_\eta^2 (\eta^\dagger \eta) - \mu_S^2 (S^\dagger S) + \lambda_3 (H^\dagger H)^2 + \lambda_4 (\eta^\dagger \eta)^2 + \lambda_5 (H^\dagger \eta)^2 + \lambda_6 (S^\dagger S)^2 + \lambda_7 (H^\dagger H)(S^\dagger S) + \lambda_8 (\eta^\dagger \eta)(S^\dagger S) + H.\ldots$$

The SM gauge symmetry is broken by the neutral component of Higgs doublet $H$ while the $U(1)_{L_\mu - L_\tau}$ symmetry is broken by the non-zero vacuum expectation value (vev) of scalar singlet $S$ which results in a massive gauge boson $Z_{\mu \tau}$ with mass $M_{Z_{\mu \tau}} = g_{\mu \tau} v_S$ where $g_{\mu \tau}$ is $L_\mu - L_\tau$ gauge coupling. Also, we assume $\mu_\eta^2 > 0$ so that $\eta$ do not acquire any vev. After the symmetry breaking, neutral components of the scalar fields can be written as

$$H^0 = \frac{1}{\sqrt{2}} (v + h_1 + ih_2),$$
$$\eta^0 = \frac{1}{\sqrt{2}} (\eta_1 + i\eta_2),$$
$$S = \frac{1}{\sqrt{2}} (v_S + s_1 + is_2),$$

where $(S) = v_S/\sqrt{2}, (H) = (0, v/\sqrt{2})^T$ with $v = 246$ GeV.

In the basis $\{h_1, s_1, \eta_1, \eta_2\}$, mass matrix for the neutral scalars is

$$M_0^2 = \begin{pmatrix}
M_{11} & M_{12} & 0 & 0 \\
M_{12} & M_{22} & 0 & 0 \\
0 & 0 & M_{33} & M_{34} \\
0 & 0 & M_{34} & M_{44}
\end{pmatrix},$$

where

$$M_{11} = 2\lambda_3 v^2,$$
$$M_{22} = 2\lambda_S v_S^2,$$
$$M_{33} = \mu_\eta^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \text{Re}[\lambda_5] v^2 + \frac{1}{2} \lambda_8 v_S^2),$$
$$M_{34} = -\text{Im}[\lambda_5] v^2,$$
$$M_{44} = \mu_\eta^2 + \frac{1}{2} (\lambda_3 + \lambda_4 - \text{Re}[\lambda_5]) v^2 + \frac{1}{2} \lambda_8 v_S^2.$$

The mass matrix for the neutral scalar can be diagonalized by

$$\begin{pmatrix}
h_1 \\
\eta_1
\end{pmatrix} = \begin{pmatrix}
cos \alpha_1 & -\sin \alpha_1 \\
\sin \alpha_1 & \cos \alpha_1
\end{pmatrix} \begin{pmatrix}
h \\
\eta
\end{pmatrix},$$

and

$$\begin{pmatrix}
\eta_R^0 \\
\eta_I^0
\end{pmatrix} = \begin{pmatrix}
cos \alpha_2 & -\sin \alpha_2 \\
\sin \alpha_2 & \cos \alpha_2
\end{pmatrix} \begin{pmatrix}
\eta_R \\
\eta_I
\end{pmatrix},$$

where $\tan \alpha_{1,2} = \frac{\eta_{1,2}}{1 + \sqrt{1 + r_{1,2}^2}}$ with $\eta = \frac{v_S \lambda_5}{\lambda_4 v^2 - \lambda_8 v_S^2}$ and $r_2 = \frac{-\text{Im}[\lambda_5]}{\text{Re}[\lambda_5]} [56]$. Hence masses of scalars are

$$M_{2k}^2 = \lambda_k v^2 + \lambda_5 v_S^2 \pm (\lambda_3 v^2 + \lambda_8 v_S^2) \sqrt{1 + r_1^2},$$
while the masses of charged scalar $H^\pm$, pseudoscalars $\eta_i^0$ and $\eta_R^0$ are given by

\[ M_{H^\pm}^2 = \mu^2 + \frac{1}{2} (\lambda_5 v^2 + \lambda_6 v^3), \]
\[ M_{\eta_i}^2 = M_{H^\pm}^2 + \left( \frac{\lambda_4}{2} \pm |\lambda_3| \right) v^3, \]

where $+(-)$ sign is for $\eta_i^0 (\eta_R^0)$.

Furthermore, the Yukawa Lagrangian for the model is given by

\[ -\mathcal{L}_Y = y_{\ell e} \bar{\nu}_L H e + y_{\nu L} \bar{\nu}_L H \nu + y_{\nu R} \bar{\nu}_R H \nu + y_{\nu} (\bar{\nu}_L H \nu) \eta + y_{\nu} (\bar{\nu}_L H \nu) \eta^* N + y_{\nu} (\bar{\nu}_L H \nu) \eta^* N + \frac{1}{2} (\bar{\nu}_e N \eta + \bar{\nu}_\tau N^* \eta) \eta + \text{H.c.}, \]

where $\eta = i e^{-\delta} \eta^*$. Using equation (11), the charged lepton mass matrix $M_\ell$, Dirac Yukawa matrix $y_D$ and right handed neutrino mass matrix $M_R$ are given by

\[
M_\ell = \frac{1}{\sqrt{2}} \begin{pmatrix}
 y_{\ell e} & 0 & 0 \\
 0 & y_{\nu L} & 0 \\
 0 & 0 & y_{\nu R} 
\end{pmatrix}, \\
y_D = \begin{pmatrix}
y_{\ell e} \\
0 \\
0
\end{pmatrix},
M_R = \begin{pmatrix}
x & y \\
x & 0 \\
y & M_{\nu e} e^{\delta i}
\end{pmatrix},
\]

where $x \equiv y_{\ell e} v_s/\sqrt{2}$, $y \equiv y_{\nu L} v_s/\sqrt{2}$ and $\delta$ is the phase remaining after redefinition of the fields.

### 2.1. Neutrino Masses and muon (g − 2)

The neutrino masses are generated at one loop level (figure 1) resulting in the light neutrino mass matrix given by [53, 57]

\[
M_{\nu} = \sum_k \frac{y_{\nu_{ik}} y_{\nu_{ik}} M_k}{16\pi^2} \left[ \frac{M_{\eta_i}^2}{M_{\eta_i}^2 - M_k^2} \ln \frac{M_{\eta_i}^2}{M_k^2} - \frac{M_{\eta_i}^2}{M_k^2 - M_{\eta_i}^2} \ln \frac{M_k^2}{M_{\eta_i}^2} \right],
\]

where $M_k$ is the mass of $k^{th}$ right handed neutrino and $M_{\eta_i}, M_{\eta_R}$ are the masses of real and imaginary parts of inert doublet $\eta$. The Yukawa couplings appearing in equation (13) are derived from $y_D$ in the basis where $M_R$ is diagonal.

If the mass squared difference between $\eta_R^0$ and $\eta^0_i$ i.e. $M_{\eta_R}^2 - M_{\eta_i}^2 = \lambda_5 v^2 < < M^2$ where

\[ M^2 = \left( M_{\eta_R}^2 + M_{\eta_i}^2 \right)/2, \]

then above expression reduces to

\[
M_{\nu} = \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{y_{\nu_{ik}} y_{\nu_{ik}} M_k}{M^2 - M_k^2} \left[ 1 - \frac{M_k^2}{M^2 - M_k^2} \ln \frac{M^2}{M_k^2} \right].
\]

The effective low energy neutrino mass matrix $M_{\nu}$ obtained using equation (14) can be diagonalized to ascertain neutrino masses and mixing angles viz.,

\[ M_\nu = U M_{\nu} U^T, \]

where $U$ is unitary matrix and $M_{\nu} = \text{diag}(m_1, m_2, m_3)$, $m_i$ are neutrino mass eigenvalues. In term of the elements of the diagonalizing matrix

![Figure 1. The diagram responsible for neutrino mass generation in scotogenic model at one loop level.](image)
the neutrino mixing angles can be evaluated using

\[
\sin^2 \theta_{13} = |U_{e3}|^2, \quad \sin^2 \theta_{23} = \frac{|U_{e1}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}. \tag{17}
\]

**2.2. Numerical Analysis**

The mass of the gauge boson \( M_{Z_{\mu\tau}} \) is constrained by demanding the model to have consistent low energy phenomenology. The neutrino masses and mixing angles are obtained by diagonalising the neutrino mass matrix. In the numerical analysis, obtained values of neutrino mass squared differences and mixing angles \((\theta_{13}, \theta_{23}, \theta_{12})\) are compared with the experimental data \((3\sigma \text{ ranges})\) \cite{60} viz.,

\[
\begin{align*}
\sin^2 \theta_{13} &= (0.02034 - 0.02430), \\
\sin^2 \theta_{23} &= (0.407 - 0.620), \\
\sin^2 \theta_{12} &= (0.269 - 0.343), \\
\Delta m_{21}^2 &= (2.431 - 2.599) \times 10^{-3} \text{eV}^2, \\
\Delta m_{31}^2 &= (6.82 - 8.04) \times 10^{-3} \text{eV}^2,
\end{align*}
\]

(19)


to constrain the allowed parameter space. The free parameters in the neutrino mass matrix are varied randomly with in ranges given in table 2.

The allowed parameter space, in \( M_{Z_{\mu\tau}} \)-\( g_{\mu\tau} \) plane, which satisfies the low energy neutrino oscillation data and muon \((g-2)\) is shown in figure 3. It is evident from figure 3 that there exist two regions in \( g_{\mu\tau} - M_{Z_{\mu\tau}} \) plane, \( M_{Z_{\mu\tau}} \approx 150 \text{ MeV} \) and \( 500 \text{ MeV} \), for which neutrino phenomenology is satisfied within the model. The
experimental bounds from CCFR [43], WD cooling [44, 45], BABAR [48] and future sensitivities of experiments like NA62 [49] and NA64 [50] are, also, shown (dashed lines) in figure 3. The upper triangular region is excluded from cooling of white dwarf systems (WD) [44, 45]. It can be seen from figure 3 that region I (around 150 MeV) explains the muon (g − 2) while region II satisfies neutrino phenomenology but not muon (g − 2). Hence, the minimal U(1)_{U1,LL}-scotogenic model explains muon (g − 2) for \( M_{Z_{\mu\tau}} \approx 150 \text{MeV} \). In the next section, we propose a framework in which we can explain muon (g − 2) in region II i.e. in absence of contribution from \( Z_{\mu\tau} \). The BABAR constraint, shown in figure 3, excludes \( M_{Z_{\mu\tau}} \) in the range [0.212-1 GeV for coupling \( g_{\mu\tau} > 7 \times 10^{-4} \) [48]. It is evident from figure 3 that both the regions are outside the exclusion region from BABAR experiment.

**Benchmark Point:** In order to emphasise the viability of the model based on correct low energy neutrino phenomenology and experimental measurement of muon (g − 2), we have obtain the benchmark point of the numerical analysis for representative values of input parameters given in table 3. The values of neutrino mixing angles and mass-squared differences are

\[
\sin^2 \theta_{13} = 0.020, \quad \sin^2 \theta_{23} = 0.56, \quad \sin^2 \theta_{12} = 0.30,
\]

\[
\Delta m_{21}^2 = 2.4 \times 10^{-3} \text{eV}^2, \quad \Delta m_{31}^2 = 7.8 \times 10^{-5} \text{eV}^2.
\]

The value of \( \Delta a_{\mu}(Z_{\mu\tau}) = 2.78 \times 10^{-9} \) for \( M_{Z_{\mu\tau}} = 147 \text{MeV} \) and \( g_{\mu\tau} = 0.0008 \).
3. Extension of Minimal Gauged Scotogenic Model by a Vector like Lepton (VLL) triplet

In general, the explanation of muon $g-2$ based on gauged scotogenic model [38–40] assumes small mixing between $Z_{\text{\mu\tau}}$ and SM gauge boson $Z$. In absence of the $Z_{\text{\mu\tau}}$ contribution (large $M_{Z_{\text{\mu\tau}}}$ or large mixing paradigms), we extend the gauged scotogenic model by a VLL triplet $\psi_T$ to explain muon $(g-2)$

$$\psi_T = \left(\begin{array}{c} \psi_T^- \\ \frac{1}{\sqrt{2}} \\ \psi_T^+ \end{array}\right),$$

with charge assignments $(3, -1, 1)$ under $\text{SU}(2)_L \times U(1)_Y \times U(1)_{\text{\mu\tau}} - L$, symmetry and odd under $Z_2$.

The $L_{\text{\mu}} - L_{\tau}$ charge of $\psi_T$ allows it couples to muon only and shall not contribute in neutrino mass generation. The new terms in the Lagrangian (equation (1)) are

$$\mathcal{L}_{\psi_T} = \bar{\psi}_T i \gamma^\mu D_\mu \psi_T - \bar{\eta} \psi_T \beta L_{\mu} - M_{\psi} \bar{\psi}_T \psi_T + \text{H.c.},$$

where $M_{\psi} \bar{\psi}_T \psi_T$ is the bare mass term for $\psi_T$.

The $\text{SU}(2)_L$ triplet $\psi_T$ gives negative contribution to muon $(g-2)$ [52]. Also, as discussed in the previous section, the charged component of scalar doublet $\eta$ has a negative contribution to muon $(g-2)$. However, $\psi_T$ coupling with $\eta$ results in chirally enhanced positive contribution to $\Delta a_\mu$ through the second term of equation (21). The possible diagrams contributing to muon $(g-2)$ are shown in figure 4.

The contribution to muon magnetic moment is given by [52]:

$$\Delta a_\mu(\eta + \psi_T) = \frac{m_{\psi_T}^2 y_{\psi_T}^2}{32\pi^2 M_\eta} [5F_{\text{SS}}(M_{\psi_T}^2 / M_\eta^2) - 2F_{\text{SSF}}(M_{\psi_T}^2 / M_\eta^2)],$$

Figure 4. The diagrams responsible for positive contribution, from $\psi_T$ and $\eta$, to muon $(g-2)$ at one loop level.

Table 3. The values of parameters used to obtain benchmark point of the numerical analysis ($k = e, \mu, \tau$).

| Parameter | Value                      |
|-----------|----------------------------|
| $(M_{e\mu}, M_{\mu\tau})$ | (69.2, 66) GeV |
| $(x, y)$ | (54.8, 26.6) GeV |
| $\delta$ | 347.52° |
| $\gamma_{\psi_T}$ | (0.043, 0.048, 0.047) |
| $(M_{\psi_T}, M_{\psi})$ | (172.97, 172.98) GeV |
| $y_{\psi_T}$ | 184 GeV |
| $M_\eta$ | (7.96, 2.04, 6.77) × 10^7 GeV |
where $M_{\psi}$, $M_{\eta}$ are the masses of VLL triplet $\psi_T$ and inert scalar doublet $\eta$, $y_\psi$ is coupling constant,

$$F_{\text{FFS}}(t) = \frac{1}{6(t-1)^4}[t^3 - 6t^2 + 3t + 2 + 6t \ln t],$$

and

$$F_{\text{SSF}}(t) = \frac{1}{6(t-1)^4}[-2t^3 - 3t^2 + 6t - 1 + 6t^2 \ln t],$$ (23)

where $t = \frac{M^2}{M_\psi^2}$.

### 3.1. Numerical Analysis

In addition to the parameters in table 2, we randomly vary $M_{\psi}$ and $y_\psi$ in the ranges [100, 400] GeV and [1, 2], respectively. Following the procedure as described in section 2.2, we numerically diagonalize the low energy effective neutrino mass matrix equation (14) to obtain neutrino masses and mixing angles. For the sake of completeness, we have given some correlation plots depicting the allowed parameter space of the model. In figure 5(a) and 5(b) we have shown the correlation plots of $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, and $\sin^2 \theta_{13}$ with $\Sigma m_i$, respectively. It is evident that neutrino mixing angles are consistent with the latest neutrino oscillation data [60].

Also, information about the CP violation is encoded in CP rephasing invariants $J_{\text{CP}}$, $I_1$, and $I_2$. The Jarlskog CP invariant $J_{\text{CP}}$ is given by [63, 64]

$$J_{\text{CP}} = \text{Im}[U_{e1} U_{e2} U_{\nu e}^*],$$ (24)

while the other two CP invariants $I_1$, $I_2$ related to Majorana phases can be written as

$$I_1 = \text{Im}[U_{e1}^* U_{e2}], \quad I_2 = \text{Im}[U_{\nu e} U_{\nu e}^*].$$ (25)

Figure 6(a) shows the variation of Jarlskog CP invariant with sum of active neutrino masses $\Sigma m_i$, and figures 6(b) and 6(c) show the correlation of $I_1$ and $I_2$ with sum of active neutrino masses $\Sigma m_i$. The model is found to be consistent with both CP conserving and violating solutions.

Furthermore, there is a longstanding question in particle physics about the exact nature of neutrinos. Neutrinoless double beta decay ($0\nu \beta \beta$) process can shed light on whether neutrino is Dirac or Majorana particle. Other than the phase space factor, amplitude of this process is proportional to $U_{\nu e}$. The effective Majorana mass appearing in the $0\nu \beta \beta$ decay can be written as

$$m_{\text{ee}} \equiv M_{\nu 1} = \left| \sum_{i=1}^{3} U_{\nu e}^* m_i \right|. \quad (26)$$

Figure 7 shows the correlation of effective Majorana mass $m_{\text{ee}}$ with $\Sigma m_i$. Several $0\nu \beta \beta$ decay experiments with high sensitivities such as nEXO [65], NEXT [66, 67], KamLAND-Zen [68] and SuperNEMO [69] have bright prospect for its observation. The sensitivities of these experiments are, also, shown in figure 7 which have imperative implication for the model. For example, the non-observation of $0\nu \beta \beta$ decay down to the sensitivity of 0.01 eV will refute the model.

**Muon ($g-2$):**

It is well known that, in gauged $U(1)_{L_e - L_\mu}$ models, the vev of scalar which breaks this symmetry is chosen such that $M_{\nu_\mu}$ lies within muon neutrino trident (MNT) upper bound of 300 MeV to satisfy muon ($g-2$). It is
to be noted that though neutrino phenomenology can be satisfied for higher $M_{Z'}$ as discussed in Sec 2.2 but it does not provide a solution to $\Delta a_\mu$. The contribution to $a_\mu$ for this scenario is provided by coupling of $\psi_T$ with $\eta$ through equation (22). In figure 8 we have shown the region of parameter space contributing to anomalous magnetic moment of muon $\Delta a_\mu$. The horizontal lines depict the experimental allowed range of $\Delta a_\mu$. It is evident from figure 8 that the anomalous magnetic moment of muon is explained by $\psi_T$ for Yukawa coupling $y_\psi$ in the range $[1,2]$ which is within perturbative limit $y_\psi < \sqrt{4\pi}$ [70]. Furthermore, the large Yukawa couplings can create Landau pole below Planck scale. However, $y_\psi$ as large as 1.3 does not create Landau pole in case of fermion triplet [71].

**Benchmark Point:** For ready reference, we provide benchmark point showing the viability of the model. For input parameters as listed in table 4, the values of neutrino mixing angles and mass-squared differences are

\[
\sin^2 \theta_{13} = 0.021, \quad \sin^2 \theta_{23} = 0.55, \quad \sin^2 \theta_{12} = 0.33 \\
\Delta m_{21}^2 = 2.4 \times 10^{-3}\text{eV}^2, \quad \Delta m_{13}^2 = 7.0 \times 10^{-5}\text{eV}^2,
\]

**Figure 6.** The correlations of $CP$ rephasing invariants $J_{CP}$, $I_1$ and $I_2$ with sum of neutrino masses $\Sigma m_i$. The grey shaded region is disallowed by the cosmological bound on sum of neutrino masses [61,62].

**Figure 7.** The correlation of $\mu\mu$ with sum of active neutrino masses $\Sigma m_i$. The sensitivity reach of various $0\nu\beta\beta$ decay experiments are shown as the horizontal lines. The grey shaded region is disallowed by the cosmological bound on sum of neutrino masses [61,62].
and the contribution to muon anomalous magnetic moment \( \Delta a_\mu (\psi_T + \eta) \) is \( 2.24 \times 10^{-9} \) for \( M_{\psi} = 103 \text{ GeV} \), \( M_\eta = 102 \text{ GeV} \) and \( y_\psi = 1.07 \).

Finally, a comment about the possible DM in the model is in order. In this model, all the BSM fields except scalar singlet \( S \) are \( Z_2 \) odd and can be DM candidates. However, we have considered the real part of inert scalar doublet, \( R_0^I \), to be the lightest and, thus, is suitable DM candidate in the model.

4. Conclusions

In conclusion, muon \((g - 2)\) anomaly, non-zero neutrino mass, nature of neutrinos lacks an explanation within the SM. In this work, we revisit the gauged \( U(1)_{L_L - L_R} \) extension of the SM with in the framework of scotogenic model. The particle content of the model is extended by three right handed neutrinos, one inert scalar doublet and a SM gauge singlet scalar to implement conventional scotogenesis. \( U(1)_{L_L - L_R} \) symmetry is broken by the vev \( v_S \) resulting in massive gauge boson \( Z_{\psi_T} \). We have obtained the allowed parameter space of neutrino mixing parameters which is consistent with neutrino oscillation data and, simultaneously, explains muon \((g - 2)\). The mass of the gauge boson \( M_{Z_{\psi_T}} = g_{\psi_T} v_S \) is constrained by the neutrino phenomenology (through \( M_R \)). In fact, we obtain two distinct regions (around 150 MeV and 500 MeV) in \( M_{Z_{\psi_T}} - g_{\psi_T} \) plane which can explain the neutrino phenomenology. Region I is consistent with the MNT bound, however, region II violates it for mass range \( M_{Z_{\psi_T}} > 300 \) MeV. The explanation of muon \((g - 2)\) based on gauged scotogenic model assumes small mixing between \( Z_{\psi_T} \) and SM gauge boson \( Z \). In section 3, we propose an extension of the gauged scotogenic model with VLL triplet wherein muon \((g - 2)\) can be explained independent of above mixing pattern and \( U(1)_{L_L - L_R} \) gauge boson mass. In this case we have shown that, in light of the neutrino oscillation data, muon \((g - 2)\) can be explained via mixing between VLL triplet \( \psi_T \) and inert scalar doublet \( \eta \) in the mass range \((100-400) \) GeV. We have, also, given the benchmark points for both the scenarios emphasising the viability of the model. In addition, the implication of the model for \( 0\nu \beta \beta \) decay has, also, been studied. The non-observation of \( 0\nu \beta \beta \) decay down to the sensitivity of 0.01 eV shall refute the model. The model, in general, is consistent with both CP conserving and violating solutions.
Acknowledgments

S. Arora acknowledges the financial support provided by the Central University of Himachal Pradesh. M. K. acknowledges the financial support provided by Department of Science and Technology, Government of India vide Grant No. DST/INSPIRE Fellowship/2018/IF180327. The authors, also, acknowledge Department of Physics and Astronomical Science for providing necessary facility to carry out this work. B. C. Chauhan is also, thankful to the Inter University Centre for Astronomy and Astrophysics (IUCAA) for providing necessary facilities during the completion of this work.

Data availability statement

No new data were created or analysed in this study.

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References

[1] Kashav M and Verma S 2021 J. High Energy Phys. JHEP09(2021)100
[2] Bennett G W et al (Muon g − 2) 2006 Phys. Rev. D 73 072003
[3] Aoyama T et al 2020 Phys. Rept. 887 1–166
[4] Aoyama T, Hayakawa M, Kinoshita T and Nio M 2012 Phys. Rev. Lett. 109 111808
[5] Czarnecki A, Marciano W J and Vainshtein A 2003 Phys. Rev. D 67 073006
Czarnecki A, Marciano W J and Vainshtein A 2006 Phys. Rev. D 73 119901
[6] Gnendiger C, Stockinger D and Stockinger-Kim H 2013 Phys. Rev. D 88 053005
[7] Davier M, Hocker A, Malasue B and Zhang Z 2017 Eur. Phys. J. C 77 827
[8] Keshavarzi A, Nomura D and Teubner T 2018 Phys. Rev. D 97 114025
[9] Hoferichter M, Hoid B L and Kubs B 2019 J. High Energy Phys. JHEP08(2019)137
[10] Hoferichter M, Hoid B L, Kubs B, Leupold S and Schneider S P 2018 J. High Energy Phys. JHEP10(2018)141
[11] Colangelo G, Hagelstein F, Hoferichter M, Laub L and Stoffer P 2020 J. High Energy Phys. JHEP03(2020)101
[12] Blum T, Christ N, Hayakawa M, Izubuchi T, Jin L, Jung C and Lehner C 2020 Phys. Rev. Lett. 124 132002
[13] Colangelo G, Hoferichter M, Nystøl I, Passera M and Stoffer P 2014 Phys. Lett. B 735 90–1
[14] Kurz A, Liu T, Marquez P and Steinhauser M 2014 Phys. Lett. B 734 144–7
[15] Keshavarzi A, Nomura D and Teubner T 2020 Phys. Rev. D 101 014029
[16] Abi B et al (Muon g−2) 2021 Phys. Rev. Lett. 126 141801
[17] Borsanyi S et al 2021 Nature 593 51–5
[18] Martin S P and Wells J D 2001 Phys. Rev. D 64 033505
[19] Ali M I, Chakraborti M, Chattopadhyay U and Mukherjee S arXiv:2112.09867
[20] Pospelov M 2009 Phys. Rev. D 80 095002
[21] Biggio C and Bordone M 2015 J. High Energy Phys. JHEP02(2015)099
[22] Han T, Kang S K and Sayre J 2016 J. High Energy Phys. JHEP02(2016)097
[23] Chen C H, Chiang C W and Nomura T 2021 Phys. Rev. D 104 055011
[24] Arcadi G, de Jesus F A S, de Melo T and Villamizar Y S 2022 Nucl. Phys. B 982 115882
[25] DeY A, Lahiri J and Mukhopadhyaya B 2022 Phys. Rev. D 106 (5) 055023
[26] Delle Rose L, Khalil S and Moretti S 2021 Phys. Lett. B 816 136216
[27] Iguro S, Omura Y and Takeuchi M 2019 J. High Energy Phys. JHEP11(2019)130
[28] Chun E J and Mondal T 2020 J. High Energy Phys. JHEP11(2020)077
[29] He X G, Joshi G C, Lew H and Volkas R R 1991 Phys. Rev. D 43 22–4
[30] He X G, Joshi G C, Lew H and Volkas R R 1991 Phys. Rev. D 44 2118–32
[31] Borah D, Dasgupta A and Mahanta D 2021 Phys. Rev. D 104 075006
[32] Biswas A, Choubey S and Khan S 2016 J. High Energy Phys. JHEP09(2016)147
[33] Zhou S 2022 Chin. Phys. C 46 011001
[34] Guo W X, Xing Z Z and Zhou S 2007 Int. J. Mod. Phys. E 16 1–50
[35] Dev A arXiv:1710.02678
[36] Majumdar C, Patra S, Prithwita P, Senapati S and Yajnik A U 2020 J. High Energy Phys. JHEP09(2020)101
[37] Patra S, Rao S, Sahos N and Sahos N 2017 Nucl. Phys. B 917 317–36
[38] Borah D, Dutta M, Mahapatra S and Sahos N 2022 Phys. Rev. D 105 015029
[39] Jana S, Vishnu P K, Rodjeochari W and Saad S 2020 Phys. Rev. D 102 075003
[40] Baek S 2016 Phys. Lett. B 756 1–5
[41] Han Z L, Ding R, Lin S J and Zhu B 2019 Eur. Phys. J. C 79 1007
[42] Kang D W, Kim J and Okada H 2021 Phys. Lett. B 822 136666
[43] Altmannshofer W, Gori S, Pospelov M and Yavin I 2014 Phys. Rev. Lett. 113 091801
[44] Bauer M, Foldenauer P and Jarcckel J 2018 J. High Energy Phys. JHEP07(2018)094
[45] Kamada A, Kaneta K, Yanagi K and Yu H B 2018 J. High Energy Phys. JHEP06(2018)117
[46] Akinow D et al (COHERENT) 2017 Science 357 1123–6
[47] Akinow D et al (COHERENT) 2021 Phys. Rev. Lett. 126 012002
[48] Lees J P et al (BABAR) 2016 Phys. Rev. D 94 011102
[49] Krnjaic G, Marques-Tavares G, Redigolo D and Tobioka K 2020 Phys. Rev. Lett. 124 041802
[50] Gninenko S N, Krasnikov N V and Matveev V A 2015 Phys. Rev. D 91 095015
[51] Cheng Y, He X G and Sun J 2022 Phys. Lett. B 827 136989
[52] Freitas A, Lykken J, Kell S and Westhoff S 2014 J. High Energy Phys. JHEP03(2014)145
    Freitas A, Lykken J, Kell S and Westhoff S 2014 J. High Energy Phys. JHEP09(2014)155
[53] Ma E 2006 Phys. Rev. D 73 077301
[54] Queiroz F S and Shepherd W 2014 Phys. Rev. D 89 095024
[55] Calibbi L, Ziegler B and Zupan J 2018 J. High Energy Phys. JHEP07(2018)046
[56] Cabral-Rosetti L G, Gaitán R, Montes de Oca J H, Galicia R O and Garcés E A 2017 J. Phys. Conf. Ser. 912 012047
[57] Merle A and Platscher M 2015 J. High Energy Phys. JHEP11(2015)148
[58] Baek S and Ko P 2009 J. Cosmol. Astropart. Phys. JCAP10(2009)011
[59] Brodsky S J and Rafael E De 1968 Phys. Rev. 168 1620–2
[60] Esteban I, Gonzalez-Garcia M C, Maltoni M, Schwetz T and Zhou A 2020 J. High Energy Phys. JHEP09(2020)178
[61] Giusarma E, Gerbino M, Mena O, Vagnozzi S, Ho S and Freese K 2016 Phys. Rev. D 94 083522
[62] Aghanim N et al (Planck) 2020 Astron. Astrophys. 641 A6
    Aghanim N et al (Planck) 2021 Astron. Astrophys. 652 C4
[63] Krastev P I and Petcov S T 1988 Phys. Lett. B 205 84–92
[64] Jarlskog C 1985 Phys. Rev. Lett. 55 1039
[65] Licciardi [nEXO] C 2017 J. Phys. Conf. Ser. 888 012237
[66] Granena F et al (NEXT) arXiv:0907.4054
[67] Gomez-Cadenas J J et al (NEXT) 2014 Adv. High Energy Phys. 2014 907067
[68] Gando A et al (KamiLAND-Zen) 2016 Phys. Rev. Lett. 117 082503
[69] Barabash A S 2012 J. Phys. Conf. Ser. 375 042012
[70] Chakrabarty N, Ghosh D K, Mukhopadhyaya B and Saha I 2015 Phys. Rev. D 92 015002
[71] Kowalska K and Sessolo E M 2017 J. High Energy Phys. JHEP09(2017)112