Unique contributions to the scalar bispectrum in ‘just enough inflation’

H. V. Ragavendra†, Debika Chowdhury‡ and L. Sriramkumar†

†Department of Physics, Indian Institute of Technology Madras, Chennai 600036, India
‡Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400005, India

A scalar field rolling down a potential with a large initial velocity results in inflation of a finite duration. Such a scenario suppresses the scalar power on large scales improving the fit to the cosmological data. We find that the scenario leads to a hitherto unexplored situation wherein the boundary terms dominate the contributions to the scalar bispectrum over the bulk terms. We show that the consistency relation governing the non-Gaussianity parameter $f_{\text{NL}}$ is violated on large scales and that the contributions at the initial time can substantially enhance the value of $f_{\text{NL}}$.

Suppressing the power on large scales: It is well known that a featureless and nearly scale invariant primordial spectrum, as is generated in slow roll models of inflation, is remarkably consistent with the observations of the anisotropies in the Cosmic Microwave Background (CMB) (for the most recent constraints from Planck, see Ref. [1]). However, intriguingly, it has been repeatedly noticed that suppressing the primordial scalar power on large scales roughly corresponding to the Hubble radius today improves the fit to the CMB data at the lower multipoles $2^\wedge 4$. There has been a constant effort to construct models of inflation that naturally result in lower power on the largest observable scales (see, for example, Refs. [3–14]).

In the standard slow roll models of inflation, the scalar fields are assumed to start on the inflationary attractor, and they evolve along the attractor until the end of inflation. A model that has drawn recent attention in the context of suppressing power on large scales involves a scalar field which begins its journey down the inflationary potential with the largest initial velocity possible (for the original discussion, see Ref. [8]; for recent discussions, see Refs. [15–19]). In fact, in such a situation, inflation begins only after about an e-fold or two, when the friction arising due to the expansion of the universe has reduced the velocity of the field adequately. Thereafter, the field rolls slowly down the inflationary potential and, as usual, inflation is terminated as the field approaches the bottom of the potential. Clearly, it is the large initial velocity of the field that results in inflation of a finite duration.

In slow roll inflation, the standard Bunch-Davies initial conditions are imposed on the perturbations when the modes are well inside the Hubble radius. Based on the constraints on the tensor-to-scalar ratio, it is possible to arrive at lower bounds on the required duration of inflation (when counted backwards from its end) if the largest observable scale today is to have emerged from sufficiently inside the Hubble radius. These arguments suggest that (under certain general conditions) inflation has to last for at least 60–65 e-folds in order for all observable scales to begin their evolution in the sub-Hubble domain (in this context, see Refs. [20–21]). In a scenario with kinetically dominated initial conditions, as we mentioned, inflation naturally lasts for a finite duration. If this duration is less than the above-mentioned number of e-folds, then a certain range of large scale modes of cosmological interest would never have been inside the Hubble radius. If we now choose to impose the Bunch-Davies initial conditions on the perturbations (irrespective of whether they are inside or outside the Hubble radius) when the scalar field rolls down with a large initial velocity, then one finds that the scalar power spectrum exhibits suppressed power on the largest scales. Interestingly, if the duration of inflation is chosen suitably, one finds that the power spectrum improves the fit to the CMB data at the lower multipoles $1^\wedge 2$.

Typically, features in the inflationary power spectra are generated due to deviations from slow roll and these departures also lead to larger levels of non-Gaussianities (see, for instance, Refs. [22–29]). In this work, we examine if the scalar bispectrum generated in the scenario with kinetically dominated initial conditions is consistent with the recent constraints from Planck on the scalar non-Gaussianity parameter $f_{\text{NL}}$ [30]. We numerically evaluate the scalar bispectrum in such a situation and show that, since the initial conditions on the perturbations are imposed at a finite early time, the contributions due to the boundary terms in the third order action governing the scalar bispectrum dominate over the contributions due to the bulk terms. This interesting situation does not seem to have been encountered earlier in the literature. In fact, we find that, for a high initial velocity, the contributions due to the boundary terms can enhance the value of the scalar non-Gaussianity parameter beyond the most recent constraints from Planck.

We shall set $\hbar = c = 1$ and $M_{\text{pl}} = (8 \pi G)^{-1/2}$. As usual, $a$ and $H$ shall denote the scale factor and the Hubble parameter associated with the Friedmann universe. Moreover, an overdot and overprime shall denote derivatives with respect to the cosmic and the conformal time coordinates, respectively. Further, while $N$ shall denote e-folds, $k$ shall represent the wavenumber of the modes.

Scalar power spectrum in ‘just enough inflation’: To illustrate the suppression of power on large scales that can arise in scenarios with kinetically dominated initial conditions, we shall consider two models of inflation driven by
the canonical scalar field $\phi$, viz. the quadratic potential $V(\phi) = m^2 \phi^2/2$ and the Starobinsky model described by the potential $V(\phi) = (\Lambda/8)[1 - e^{-\sqrt{7/3}(\phi/M_{Pl})}]^2$. We start the evolution of the background with a large initial velocity for the scalar field such that the first slow roll parameter $\epsilon_1 = \dot{\phi}^2/(2H^2M_{Pl}^2)$ is initially close to its maximum value, i.e. $\epsilon_{1i} \simeq 3$ \cite{13}. The initial value of the scalar field is chosen such that inflation lasts for about 60 e-folds. The expansion of the universe rapidly slows down the field and inflation sets in (i.e., $\epsilon_1$ becomes less than unity) after about an e-fold or so. As the velocity of the field reduces further, it soon settles down on the slow roll inflationary attractor with a small, nearly constant, velocity.

We shall numerically evolve the perturbations in such a background and calculate the resulting observable quantities of interest, viz. the scalar power and bispectra and the corresponding scalar non-Gaussianity parameter. Actually, for each model, we shall consider two situations wherein the perturbations are evolved from two different initial points in time, viz. from the onset of inflation (say, $N_i$, when $\epsilon_1 = 1$) and from the time (say, $N = 0$) when we begin the evolution of the background scalar field. Recall that the Bunch-Davies initial conditions are imposed on the perturbations when $k \gg \sqrt{z''/z}$, where $z = \sqrt{2} c_1 M_{Pl} a$. During slow roll inflation, $\sqrt{z''/z} \simeq \sqrt{2} a H$, and hence the above condition essentially corresponds to the modes being well inside the Hubble radius. Interestingly, we find that the equivalence $\sqrt{z''/z} \simeq a H$ proves to be roughly true even when the scalar field is rolling down the potential with a large initial velocity. Since we begin the evolution of the perturbations at a specific time, there naturally arises a finite initial value of the quantity $\sqrt{z''/z}$ (evaluated at $N = 0$ or $N_i$), which we shall refer to as $k_i$. This implies that, in the scenario of our interest, modes with $k < k_i$ would never be inside the Hubble radius. Despite this, if we were to impose the Bunch-Davies initial conditions on all the modes at the beginning of their evolution (i.e. at $N = 0$ or $N_i$), then one arrives at a scalar power spectrum with a sharp drop in amplitude for modes with $k \lesssim k_i$. In Fig. 1 we have plotted the scalar power spectra $P_s(k)$ arising in these two cases for suitable values of the parameters involved. Clearly, the spectra exhibit a distinct suppression of power over the modes that were never inside the Hubble radius (i.e. for $k < k_i$). The power spectra also contain oscillations (for modes with $k \simeq k_i$) before they turn nearly scale invariant at smaller scales. The two sets of spectra presented in the figure differ only in the nature of the transient oscillations with a higher initial velocity leading to oscillations of stronger amplitude and wider range.

**Evaluation of the scalar bispectrum:** The scalar bispectrum $G(k_1, k_2, k_3)$ — where $k_1$, $k_2$ and $k_3$ constitute a triangular configuration of wavevectors — is determined by the action describing the curvature perturbation at the third order (see, for example, Refs. \cite{31, 32}). This action, in turn, is arrived at from the original action governing the system of the gravitational and scalar fields. The third order action that is often used to calculate the scalar bispectrum contains six terms, which are arrived at after repeated integration by parts (in this context, see, for instance, Refs. \cite{24, 33}). In the case of standard slow roll inflation, it can be shown that, barring one term, the temporal boundary terms arising due to integration by parts do not contribute to the scalar bispectrum \cite{33}. (It can be easily shown that the spatial boundary terms do not contribute in any situation.) The boundary term that contributes (which we shall refer to as the seventh term) is often included as a term that arises due to a field redefinition \cite{26, 31, 33}. The remaining terms do not contribute in slow roll inflation for two reasons: the contributions from the extreme sub-Hubble domain are regulated by the introduction of a cut-off (which is nec-
necessary to choose the correct perturbative vacuum) and the late time contributions prove to be insignificant since the amplitude of the curvature perturbation freezes on super-Hubble scales \[ \text{(33)}. \]

In the scenario of our interest, while modes with \( k < k_i \) always remain on super-Hubble scales, modes with \( k > k_i \) begin in the sub-Hubble regime and eventually reach super-Hubble scales. As the amplitude of all these modes freeze in the super-Hubble regime, the contributions due to the boundary terms at late times turn out to be insignificant (apart from the seventh term usually taken into account through a field redefinition) as in the standard slow roll case. However, since the modes are evolved from a finite past, we find that we cannot ignore the contributions arising due to the boundary terms evaluated at the initial time (i.e. at \( N_i \) or at \( N = 0 \)). We numerically evaluate the contributions due to the six standard bulk terms and the seventh term often absorbed through a field redefinition. We also calculate the contributions due to all the boundary terms arising at the initial time. In Fig. 2 we have illustrated the various contributions to the scalar bispectrum in the equilateral limit. Interestingly, we find that over a range of modes near \( k_i \), the boundary terms turn out to be comparable to and even larger than the bulk terms. This is a rather novel result that does not seem to have been encountered earlier in the literature.

We had mentioned that the contributions to the scalar bispectrum from the sub-Hubble regime are regulated by the introduction of a cut-off \[ \text{(22, 28)}. \] As is usually done, we introduce a democratic (in the space of wavenumbers) cut-off of the form \( e^{-\kappa (k_1 + k_2 + k_3)/(3 a H)} \), where \( \kappa \) is a suitable cut-off parameter, when calculating the contributions due to both the bulk and the boundary terms. In slow roll inflation or in situations involving brief periods of fast roll sandwiched between epochs of slow roll, the value of the cut-off parameter \( \kappa \) is chosen depending on the depth inside the Hubble radius from which the integrals characterizing the bulk terms are carried out (in this context, see Ref. \[ \text{(28)}. \]) But, in the scenario of our interest, a range of modes (with \( k < k_i \)) are never inside the Hubble radius and another range (with \( k > k_i \)) do not spend an adequate amount of time in the sub-Hubble regime. Since the large scale modes (i.e. \( k < k_i \)) always remain on super-Hubble scales, the bispectrum evaluated over these range of modes is completely independent of the choice of the cut-off parameter \( \kappa \). However, we find that the results depend on the choice of \( \kappa \) for modes around \( k_i \) which do not spend an adequate amount of time in the sub-Hubble regime. As there exists no definitive procedure that can be adopted to circumvent this ambiguity, we make a judicious choice of \( \kappa \) for the remaining set of modes (i.e. for \( k > k_i \)) based on the natural demand that we are to recover the standard slow roll results at suitably small scales which emerge from sufficiently inside the Hubble radius (say, \( k > 10^2 k_i \)). We should mention here that we have set \( \kappa = 0.3 \) for all the modes in arriving at the results plotted in Fig. 2.

Amplitude and shape of \( f_{\text{NL}} \): With the scalar power and bispectra at hand, we can now evaluate the resulting non-Gaussianity parameter \( f_{\text{NL}} \). In Fig. 3 we have plotted \( f_{\text{NL}} \) in the equilateral limit for the two cases in each of the two models we have considered. While the quantity has a roughly constant value over wavenumbers \( k < k_i \), it exhibits oscillations around \( k_i \) before eventually settling down to the usual slow roll value for \( k \gg k_i \). Curiously, the constant value at large scales is higher in the case wherein the perturbations are evolved from the onset of inflation. However, over the oscillatory regime, the value of \( f_{\text{NL}} \) is larger in the case wherein the perturbations are evolved from a higher initial velocity of the background scalar field. Importantly, we should clarify that even the largest value of \( f_{\text{NL}} \) we encounter lies within the constraints (viz. \( f_{\text{NL}} = -26 \pm 47 \) for the equilateral shape) arrived at recently by Planck \[ \text{(30)}. \] Interestingly, under the same conditions, the amplitude of \( f_{\text{NL}} \) turns out to be significantly smaller in the Starobinsky model than in the quadratic potential.

Let us now turn to the behavior of the scalar non-Gaussianity parameter in the squeezed limit. In Fig. 4

![FIG. 2. The different contributions to the scalar bispectrum in the equilateral limit — the bulk contributions \( G_1(k) + G_2(k) \) (in red), \( G_2(k) \) (in blue), \( G_4(k) + G_7(k) \) (in green), \( G_5(k) + G_6(k) \) (in purple), and the boundary contributions \( G_8(k) \) (in cyan) and \( G_9(k) \) (in orange) — have been plotted in the scenario wherein the perturbations are evolved from the onset of inflation in the quadratic potential. Evidently, the contributions due to the boundary terms dominate the contributions due to the bulk terms over a range of modes. Moreover, note that the contributions due to the boundary terms prove to be considerably more significant around \( k_i \), before they die down rapidly on smaller scales. We find that the scalar bispectrum has roughly the same shape in all the models and cases we have considered.](image-url)
FIG. 3. The behavior of the non-Gaussianity parameter $f_{NL}(k)$ has been plotted in the equilateral limit for the quadratic potential (in the top two panels) and the Starobinsky model (in the bottom two panels) in the two cases evaluated from $N_i$ (in panels one and three, counted from the top) and $N = 0$ (in panels two and four). As expected, the non-Gaussianity parameter exhibits a burst of oscillations around $k_i$ before it settles down to the slow roll value at small scales. In a given model, $f_{NL}$ is considerably larger in the case wherein the bispectrum is calculated from $N = 0$ (plotted in panels two and four) than in the case wherein it is computed from the onset of inflation (plotted in panels one and three). Interestingly, $f_{NL}$ is significantly smaller in the Starobinsky model than in the quadratic potential.

we have plotted $f_{NL}$ in the squeezed limit as well as the consistency condition, viz. $f_{NL}^{CR} = (5/12) (n_s - 1)$, where $n_s$ is the scalar spectral index, again for both the models and in the two cases of our interest. Clearly, in the squeezed limit, the non-Gaussianity parameter has broadly the same shape as in the equilateral limit. It is roughly constant at small wavenumbers, which is followed by a burst of oscillations over the intermediate range, before its amplitude is restored to the standard slow roll value at larger wavenumbers. However, the strength of the oscillations in the squeezed limit proves to be considerably smaller than in the equilateral limit. Moreover, the consistency condition is violated for the large scale modes (for which the Bunch-Davies conditions are imposed in the super-Hubble domain), and it is eventually restored for small scale modes that emerge from sufficiently deep inside the Hubble radius.

**Conclusions:** The model of ‘just enough inflation’ is attractive for the reason that it leads to a suppression of power on large scales which provides a better fit to the CMB data than a nearly scale invariant spectrum produced in conventional slow roll inflation. It then becomes important to examine whether the non-Gaussianities generated in the model are consistent with the recent constraints from Planck [30].

In this work, we have numerically calculated the scalar bispectrum and the corresponding non-Gaussianity parameter $f_{NL}$ arising in this scenario for two models, viz. the quadratic potential and the Starobinsky model. Due to the fact that the initial conditions on the perturbations are imposed in the finite past when the scalar field is rolling rapidly down the inflationary potential, the model presents a novel and hitherto unexplored situation as far as the calculation of the scalar bispectrum is concerned. We find that, apart from the standard contributions due to the bulk terms in the third order action governing the curvature perturbation, there also arise contributions to the scalar bispectrum from the temporal boundary terms which are usually ignored. In fact, over a range of modes, we find that the contributions due to the boundary terms evaluated at the initial time (when the Bunch-Davies conditions are imposed on the perturbations) prove to be dominant when compared to the contributions due to the bulk terms. Moreover, we notice that the extent of the scalar non-Gaussianity generated depends on the velocity of the scalar field when the initial conditions are imposed on the perturbations, with the maximum value of $f_{NL}$ being larger when the velocity of the field is higher. Further, we find that, in the squeezed limit, the consistency condition governing the scalar bispectrum is violated for the large scale modes which are never inside the Hubble radius and exhibit a suppression in the power spectrum. Lastly and, importantly,
the amplitude of $f_{NL}$ generated in the quadratic potential and the Starobinsky model prove to be significantly different. These unique signatures of the scenario in the bispectrum can help in distinguishing it from other models that achieve similar suppression in scalar power and hence may seem degenerate in their performance against the existing CMB data at the level of the power spectrum. We are currently comparing models which lead to a suppressed amplitude on large scales in the scalar power spectrum and examining the possibility of being able to discriminate them through the bispectrum and its imprints on the CMB [34].

Acknowledgements: The authors wish to thank Dhiraj Hazra for discussions and comments on the manuscript. HVR would like to thank the Indian Institute of Technology Madras, Chennai, India, for financial support through half-time research assistantship. DC would like to thank the Tata Institute of Fundamental Research, Mumbai, India, for financial support. HVR and LS wish to acknowledge the use of the cluster computing facility at the Department of Physics, Indian Institute of Technology Madras, Chennai, India, where some of the numerical computations were carried out.

[1] Y. Akrami et al. (Planck), (2018), arXiv:1807.06211 [astro-ph.CO]
[2] S. L. Bridle, A. M. Lewis, J. Weller, and G. Efstathiou, Mon. Not. Roy. Astron. Soc. 342, L72 (2003)
[3] A. Shafieloo and T. Souradeep, Phys. Rev. D70, 043523 (2004)
[4] P. Hunt and S. Sarkar, Phys. Rev. D70, 103518 (2004)
[5] P. Hunt and S. Sarkar, Phys. Rev. D76, 123504 (2007)
[6] D. K. Hazra, A. Shafieloo, and G. F. Smoot, JCAP 1312, 033
[7] J. M. Cline, P. Crotty, and J. Lesgourgues, JCAP 0309, 010
[8] C. R. Contaldi, M. Peloso, L. Kolman, and A. D. Linde, JCAP 0307, 002
[9] B. A. Powell and W. H. Kinney, Phys. Rev. D76, 063512 (2007)
[10] G. Nicholson and C. R. Contaldi, JCAP 0801, 002
[11] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar, and T. Souradeep, JCAP 0901, 009
[12] R. K. Jain, P. Chingangbam, L. Sriramkumar, and T. Souradeep, Phys. Rev. D82, 023509 (2010)
[13] D. K. Hazra, A. Shafieloo, G. F. Smoot, and A. A. Starobinsky, Phys. Rev. Lett. 113, 071301 (2014)
[14] D. K. Hazra, A. Shafieloo, G. F. Smoot, and A. A. Starobinsky, JCAP 1408, 048
[15] E. Ramirez and D. J. Schwarz, Phys. Rev. D85, 103516 (2012)
[16] E. Ramirez, Phys. Rev. D85, 103517 (2012)
[17] W. J. Handley, S. D. Brechet, A. N. Lasenby, and M. P. Hobson, Phys. Rev. D89, 063505 (2014)
[18] L. T. Hergt, W. J. Handley, M. P. Hobson, and A. N. Lasenby, (2018), arXiv:1809.07185 [astro-ph.CO]
[19] L. T. Hergt, W. J. Handley, M. P. Hobson, and A. N. Lasenby, (2018), arXiv:1809.07337 [astro-ph.CO]
[20] S. Dodelson and L. Hui, Phys. Rev. Lett. 91, 131301 (2003)
[21] A. R. Liddle and S. M. Leach, Phys. Rev. D68, 103503 (2003)
[22] X. Chen, R. Easther, and E. A. Lim, JCAP 0706, 023
[23] R. Flauger and E. Pajer, JCAP 1101, 017
[24] P. Adshead, W. Hu, C. Dvorkin, and H. V. Peiris, Phys. Rev. D84, 043519 (2011)
[25] P. Adshead, C. Dvorkin, W. Hu, and E. A. Lim, Phys. Rev. D85, 023531 (2012)
[26] J. Martin and L. Sriramkumar, JCAP 1201, 008
[27] F. Arroja and M. Sasaki, JCAP 1208, 012
[28] D. K. Hazra, L. Sriramkumar, and J. Martin, JCAP 1305, 026
[29] S. Basu, D. J. Brooker, N. C. Tsamis, and R. P. Woodard, (2019), arXiv:1905.12140 [gr-qc]
[30] Y. Akrami et al. (Planck), (2019), arXiv:1905.05697 [astro-ph.CO]
[31] J. M. Maldacena, JHEP 05, 013
[32] D. Seery and J. E. Lidsey, JCAP 0506, 003
[33] F. Arroja and T. Tanaka, JCAP 1105, 005
[34] H. V. Ragavendra, D. Chowdhury, and L. Sriramkumar, manuscript in preparation.