High $p_T$ in Nuclear Collisions at the SPS, RHIC, and LHC

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We review recent progress in the study of medium-induced modification of jet fragmentation in high energy nuclear collisions at the SPS and RHIC and present an outlook on jet physics at the LHC.

1. Introduction

Partonic energy loss is a potentially sensitive tomographic probe of matter produced in high energy nuclear collisions, generating observable modification of the fragmentation patterns of jets (“jet quenching”). Measurements at RHIC have revealed large medium-induced suppression at high transverse momentum (high $p_T$) of both the inclusive hadron yields [1, 2] and of back-to-back hadron pairs [3]. The principal energy loss mechanism underlying these effects is commonly thought to be medium-induced gluon bremsstrahlung, which is expected to dominate collisional (elastic) energy loss for very energetic partons [4].

The effects of medium-induced radiation have been calculated in various frameworks: multiple soft scatterings (BDMPS [5]), few hard scatterings (GLV [6]), twist expansion (Wang and Guo [7]), and light-cone path integral approach (Zakharov [8]). In the case of multiple soft scattering the medium is characterized by a single transport coefficient $\hat{q} = \mu^2/\lambda$, where $\mu$ is the average momentum kick of a gluon interacting in the medium and $\lambda$ is its mean free path. The gluon radiation spectrum is suppressed relative to the Bethe-Heitler spectrum due to coherence effects, leading to medium-induced radiated energy $\Delta E_{\text{medium}} \sim \alpha_s \hat{q} L^2$ [9, 10]. Longitudinal expansion of the medium reduces the length dependence to $\Delta E_{\text{medium}} \sim L$ while finite partonic energy truncates the medium-induced radiation spectrum, resulting in an energy-dependent energy loss. Although the induced radiation spectrum differs in detail between the BDMPS and GLV approaches, the total energy loss is similar for comparable medium properties [10]. For conditions relevant to RHIC collisions the energy is lost in both cases to a moderate number ($\sim 3$) of radiated gluons having moderate energy ($\sim 0.1 – 1$ GeV) [10].

This picture is conceptually appealing and pQCD-based calculations incorporating medium-induced bremsstrahlung reproduce much of the published data on high $p_T$ hadron production in nuclear collisions. Nevertheless, it is important to ask to what extent the data require this description to be the correct one. Are its detailed predictions, such as...
the expected $L^2$ dependence of $\Delta E$, observed? Alternatively, does collisional energy loss play a significant role in the finite kinematic regime of RHIC [11]?

We review recent measurements of partonic energy loss in hot matter, emphasizing results shown at this conference, and compare their systematic behavior to theoretical expectations. The variables at our disposal are energy, centralities and $p_T$ dependence of high-$p_T$ hadron production and correlations. We concentrate on the highest available $p_T$ at mid-rapidity.

2. SPS

A long-standing puzzle concerns the magnitude of partonic energy loss in nuclear collisions at the SPS. WA98 data [12] indicated a large enhancement of inclusive $\pi^0$ production at $p_T \sim 3 - 4$ GeV/c in central Pb+Pb relative to p+p collisions when normalized per binary collision ($R_{AA}(p_T)$), although a suppression was observed in central relative to peripheral nuclear collisions ($R_{CP}(p_T)$, also normalized per binary collision) by the same experiment.

The p+p spectrum used as a reference for $R_{AA}(p_T)$ at the SPS was not directly measured but was based on an extrapolation from measurements at higher $\sqrt{s}$. Some of the datasets used in this extrapolation are only marginally consistent, leading to large uncertainties in the p+p reference spectrum. A recent re-evaluation of the extrapolation by d’Enterria [15] found a reduced $R_{AA}(p_T)$, more consistent with $R_{CP}(p_T)$. The medium density inferred from the new $R_{AA}(p_T)$ values is now also consistent with expectations from Bjorken energy density estimates [13, 14].

New measurements of high-$p_T$ hadron suppression at the SPS were presented at this conference. Fig. 1 shows $R_{CP}(p_T)$ for $K^0_S$ from NA57 [16] (left panel) and charged pions and protons from NA49 [17] (right panel) compared to radiative energy loss calculations.
Figure 2. Inclusive high $p_T$ hadron suppression at RHIC. Left: $R_{AA}(p_T)$ for $\pi^0$ and charged hadrons in central Au+Au collisions from PHENIX [25]. The curves show suppression calculated in the single hard scattering approach [13]. Right: $N_{part}$ dependence of $R_{AA}(p_T)$ at $p_T > 6$ GeV for charged hadrons from Cu+Cu and Au+Au collisions measured by STAR [26], compared to calculations in the multiple soft scattering approach [27].

A marked Cronin effect is expected (“no E loss” on left panel) due to the steep $p_T$ spectrum. Introduction of energy loss in a medium, with gluon density scaling as $dN_{ch}/d\eta$ [14], results in good agreement between calculation and data.

These new data solve the high-$p_T$ suppression puzzle at the SPS: the medium densities inferred from bulk multiplicity and high-$p_T$ inclusive hadron measurements are consistent. However, significant theoretical uncertainties remain due to a potentially large Cronin effect at these lower energies.

3. RHIC: inclusive yields

Rich and initially unexpected phenomenology has emerged in the “intermediate $p_T$” region ($p_T \sim 2 - 5$ GeV/c) of nuclear collisions at RHIC. The enhancement of baryon relative to meson yields [18,19], the approximate scaling of elliptic flow $v_2$ with the number of constituent quarks [20], and correlation measurements in this region [21] suggest an interplay between hadronization of hard partons and the bulk medium, as discussed extensively elsewhere at the conference [20,22]. Here we will concentrate on the region $p_T \gtrsim 6$ GeV, i.e. above the region of the anomalous enhancement of the baryon/meson ratio [18,19], to avoid these complex hadronization phenomena that may obscure the modification of fragmentation due to partonic energy loss.

Fig. 2 left panel, shows the most recent measurement of $\pi^0$ $R_{AA}(p_T)$ from the high-statistics 2004 Au+Au run at RHIC [25]. The kinematic reach now extends to $p_T \sim 20$ GeV/c, well beyond the intermediate $p_T$ region. The observed $p_T$-independence of the suppression over this broad range is well described by partonic energy loss models in both the few hard scattering [13] (dashed and solid line) and multiple soft scattering [23,24] (not shown) limits, requiring medium densities $dN_g/dy \sim 1000 - 1200$ and $\hat{q} \sim 5 - 15$. 

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GeV$^2$/fm respectively. The transport coefficient $\dot{q}$ and gluon density $dN_g/dy$ in these models are time-averaged quantities. The connection between these averaged quantities and the gluon or energy density of the medium at early time is discussed elsewhere \cite{23, 24}.

The path length dependence of hadron suppression is a key test of its underlying mechanism. During the 2005 RHIC run, large samples of Cu+Cu collisions at $\sqrt{s_{NN}}=200$ GeV were collected to measure high-$p_T$ particle production in this smaller system with similar statistical reach as the long 2004 Au+Au run. First analyses of a subset of the total Cu+Cu dataset were presented at this conference. Fig. 2, right panel, shows the centrality dependence of $R_{AA}(p_T)$ for $p_T > 6$ GeV charged hadrons, from Cu+Cu collisions (squares), compared to existing results from Au+Au collisions (circles) \cite{26}. The most significant comparison is to central Cu+Cu, where the Glauber calculation uncertainties are smaller than for the semi-peripheral Au+Au collision data with similar $N_{part} \sim 100$. The data indicate that the inclusive hadron suppression is a function solely of $N_{part}$, with no observable sensitivity to the shape of the reaction zone. The lines indicate phenomenological fits to characterize the $N_{part}$ dependence of $R_{AA}(p_T)$. The data prefer a decrease with $N_{part}^{1/3}$, although the more commonly expected $N_{part}^{2/3}$ scaling is not strongly excluded. In general, the observed scaling behavior results from the combined effects of the spectrum shape, the collision geometry, and the path-length dependent energy-loss distribution. The gray bands indicate the results of a full calculation incorporating these effects \cite{27}, which reproduces the common suppression in Cu+Cu and Au+Au at the same $N_{part}$ but gives slightly larger suppression at low $N_{part}$ than observed in the data.

4. RHIC: correlations

Sensitivity of inclusive hadron suppression measurements to the properties of the medium is limited due to a surface bias: for a dense system, the observed hadrons are preferentially the fragments of partons produced near the surface and headed outwards, which suffer less than average energy loss\cite{28, 29}. The observed $R_{AA}(p_T) \sim 0.2$ in central Au+Au collisions is reproduced by calculations with a broad range of $\dot{q} \sim 5 - 15$ GeV$^2$/fm \cite{23}, suggesting that its value is essentially geometric in origin. A more detailed view of the medium is obtained from back-to-back correlations of hadron pairs, where the surface bias can be reduced by increasing the $p_T$-threshold for associated particles on the away-side. This leads to counterbalancing biases from the requirements on the trigger and recoil, providing a new and more sensitive probe of the medium.

It has been known for some time that the back-to-back high $p_T$ di-hadron yield is strongly suppressed in the most central Au+Au collisions \cite{3}. However, the kinematic cuts of this first measurement ($p_T^{\text{trig}} > 4$ GeV/c for the trigger and $p_T^{\text{assoc}} > 2$ GeV/c for the associated hadrons) were relatively low, giving rise to a large combinatorial background. Due to the large background and the strong suppression, a differential measurement of the recoil yield was not possible and only an upper limit could be established. Lowering $p_T^{\text{assoc}}$ reveals an excess and broadening of the recoil yield \cite{29}, to the extent that the correlation structure is compatible with expectations from simple momentum conservation without additional dynamic correlations. For these lower $p_T^{\text{assoc}}$, however, the backgrounds are yet larger and the uncertainties in the background yield and the flow modulation of the
Figure 3. Per-trigger correlated yield for d+Au and Au+Au collisions in two different centrality ranges, with $p_T^{\text{trig}} > 8$ GeV. The different rows show the correlation for different ranges in $p_T$ of the associated particle [32].

background make a quantitative measurement of the excess difficult. Qualitatively, these observation are compatible with a picture in which strong partonic energy loss in the core of the reaction volume softens the fragmentation. The response of the medium to the energy loss then becomes a key issue, which was discussed extensively elsewhere at this conference [21, 30, 31].

New analyses from the long 200 GeV Au+Au run at RHIC now provide much greater $p_T$ reach for correlation studies, for the first time extending beyond the intermediate $p_T$ region. Fig. 3 shows azimuthal correlations of charged hadrons for $p_T^{\text{trig}} > 8$ GeV/c and varying threshold on $p_T^{\text{assoc}}$ [32]. It is seen from the figure that the combinatorial background is negligible for the highest cuts and a clear back-to-back correlation signal emerges, for the first time enabling a quantitative differential measurement of partonic energy loss.

This is further explored in Fig. 4 [32], which shows distributions of the near- (left panel) and away-side (right panel) associated hadrons as a function of di-hadron fragmentation variable $z_T = p_T^{\text{assoc}}/p_T^{\text{trig}}$ [33], for $8 < p_T^{\text{trig}} < 15$ GeV trigger hadrons in d+Au and semi-central and central Au+Au collisions. The lines in the right-hand panel indicate an exponential fit to the d+Au data (solid line) which is scaled by 0.54 and 0.25 to match the semi-central and central Au+Au data respectively (dashed lines).

On the near side, no significant variation of the yield or the fragmentation distribution is seen between the systems. Calculations in Ref. [34] predict a strong enhancement of the associated yield in central collisions due to large energy loss and corresponding trigger bias effects, which is not observed in the data.

In contrast to the centrality invariance of the near-side correlation, a strong suppression
of the away-side yield is found in central collisions at a level (\(\sim 25\%\)) that is numerically similar to \(R_{AA}(p_{T})\). This away-side suppression is however not accompanied by observable modification to either the longitudinal fragmentation distribution (lines in Fig. 4) or the azimuthal distribution (see Fig. 3 and [32]).

Suppression that is independent of \(z_T\) for \(z_T > 0.4\) is in qualitative agreement with calculations by Wang [33], though quantitatively the measured suppression is stronger than predicted. Strong suppression could be accompanied by broadening of the recoil distribution in the event-averaged two particle correlation function, due either to large-angle emission of fragments from radiated gluons or to medium-induced acoplanarity of the dijets. In the multiple soft scattering approach the relation between energy loss \(dE/dx\) and induced acoplanarity has the general form [35]

\[
-dE/dx = \frac{\alpha_s N_c}{8} \cdot \langle p_{T}^2 \rangle_{jet},
\]

where \(\langle p_{T}^2 \rangle_{jet}\) refers to the momentum transverse to the initial direction of the hard parton that is acquired from interactions in the medium. Detailed calculations of the broadening using this approach are not yet available. A calculation for large energy loss in the GLV framework [36] predicts that induced radiation will dominate the recoil hadron distribution up to high \(p_{T} \sim 10\) GeV/c, leading to strong azimuthal broadening. In contrast, Fig. 3 shows no significant angular broadening in this kinematic range (see also [32]).

Current model calculations invoke independent gluon emission to calculate integrated energy loss [24] and may not incorporate both radiative and elastic energy loss in a con-
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sistent way. An improved theoretical framework is likely required to calculate accurately
the induced acoplanarity. On the experimental side, higher-order multi-particle corre-
lations may be able to distinguish broadening due to additional radiation and
induced acoplanarity of the jets, which is not possible with event-averaged two-particle
correlations.

Significant suppression of the recoil yield without measurable changes in the longitudinal
and azimuthal correlations would arise if this measurement is sensitive primarily to
hadronic fragments of partons that had little or no interaction with the medium. This
bias arises naturally in radiative energy loss calculations, which predict finite probability
to emit zero gluons in a medium of finite length [10], with emission of a moderate energy
gluon leading to sizeable suppression in the relatively high $p_T$ region of this analysis. A
quantitative evaluation of these effects, including a realistic nuclear geometry and path
length dependent energy loss distributions, is given in Ref. [27], mainly in the context of
single-hadron suppression. These calculations indicate that the probability for both the
trigger and recoil jet to have little interaction with the medium is small, and a contribu-
tion is expected from partons that experience significant interaction with the medium.
Refinement of these and other calculations within the constraints of the new data pre-
sent at this conference will help elucidate the mechanisms underlying jet quenching and
the properties of the medium. A tantalizing prospect from these jet quenching studies
is an experimentally measured upper bound on $\hat{q}$, which could constrain the number of
underlying degrees of freedom of the medium [38].

5. Jet Physics at the LHC

The CERN LHC is currently scheduled to commission p+p collisions in 2007, with
the first Pb+Pb run at $\sqrt{s_{NN}}=5.5$ TeV in 2008 [39]. The factor 30 increase in collision
energy relative to RHIC generates a huge increase in kinematic and statistical reach for
hard probes. Figure 5 shows the yield for various observables relevant to jet quenching
studies expected from one LHC year of Pb+Pb running ($10^6$ seconds) at nominal
luminosity. Simple binary collision scaling ($\propto A^2$) from calculated p+p cross-sections has
been applied, with no nuclear effects taken into account.

There will be statistically robust yields for jets well above $E_T \sim 200$ GeV, providing
logarithmically large energy variation over which to study jet quenching effects. The huge
statistics will enable the study of rare, perturbatively calculable fragmentation channels
such as very hard hadron pairs with small angular separation, whose distributions may
be modified by medium effects. The large yield of high energy jets raises the possibility
that multi-particle or calorimetric (quasi-)full jet reconstruction can be used even in the
presence of large backgrounds in central nuclear collisions. This potentially recovers the
energy radiated in gluons, allowing relatively unbiased reconstruction of the jet energy
and enabling complete characterization of jet quenching without the complications of
strong trigger and geometric biases that are present in leading hadron and di-hadron
measurements as performed at RHIC.

Jet reconstruction with good energy resolution is not straightforward in central nuclear
collisions even at the LHC, however. For jets with $E_T \sim 50 - 100$ GeV, $\sim 80\%$ of the
charged track energy is contained in a cone of phase space radius $R = \sqrt{\delta \eta^2 + \delta \phi^2} \sim 0.2$
Figure 5. Hard process rates at the LHC [40, 46]. Solid and dashed lines for inclusive jets indicate rates vs $E_T$ integrated over phase space cone of radius R=0.4 and 0.7.

[41], while an area of this size in a central Pb+Pb collision at 5.5 TeV may contain $\sim 75$ GeV [42] of uncorrelated energy from soft particle production. Simulation studies of jet reconstruction using unmodified fragmentation show that relatively small cone radii $R \sim 0.3-0.4$ provide optimum energy resolution [43], though the models of both jet signal (PYTHIA) and background (HIJING) in these studies do not capture the physics of jet softening and broadening due to medium effects. Full understanding of jet reconstruction capabilities and optimization of jet observables at the LHC must await data.

A recent calculation incorporating medium effects in the MLLA parton shower framework [44] predicts an enhancement in multiplicity due to jet quenching up to hadron $p_T \sim 5-6$ GeV/c for jets with $E_T \sim 100-200$ GeV. This excess should be measurable above background, enabling a detailed characterization of the modification of the fragmentation function. Measurement of high energy b-tagged and c-tagged jets can be contrasted to light hadron-led jets, which arise dominantly from gluons, to exploit the different color charge coupling of gluons and quarks to the medium (factor 9/4). A broad range of multi-hadron correlations will be accessible which interpolate between leading hadron studies and full jet reconstruction.

The golden channel for jet quenching is the coincidence measurement of a jet recoiling from a gauge boson ($\gamma$ or Z). The boson does not interact with the medium and therefore provides a clean calibration of the momentum transfer in the interaction [45], enabling
measurement of the true fragmentation of the recoiling jet. While this strategy remains attractive, the measurements are challenging. Fig. 5 shows that $\gamma+$jet rate is statistically robust in Pb+Pb only for $p_T < 40$ GeV/c. The $\gamma/\pi^0$ ratio (the key experimental parameter) exceeds 10% only for $p_T > 50$ GeV/c in p+p, though $\pi^0$ suppression may lower that bound to 20 GeV/c in central Pb+Pb \cite{46}. Most significantly, QCD fragmentation photons may dominate the prompt photon yield up to 50 GeV or higher \cite{47}, complicating the interpretation of the photon as a non-interacting messenger from the hard vertex. The $Z+$jet channel is background-free but suffers from small cross section (see Fig. 5) and will be statistically marginal at nominal Pb+Pb luminosity.

6. Summary

The SPS and RHIC data presented at this conference significantly extend previous jet quenching studies, in some cases providing qualitatively new insights. The data are in broad agreement with expectations based on the dominant paradigm of radiative energy loss, but experimental tests of this correspondence are not yet definitive.

A characteristic feature of radiative energy loss is the quadratic path length dependence that arises from coherence effects. The new Cu+Cu data, combined with existing Au+Au measurements, will provide the most precise tests of this scaling. First hadron suppression measurements in Cu+Cu are in rough agreement with radiative energy loss models, but theoretical uncertainties now dominate this comparison and further progress will come from the theory side.

New di-hadron correlation studies reveal a well-defined recoil peak at high $p_T$, enabling the first differential measurements of jet suppression in the medium. Many existing calculations miss essential features of these data, perhaps due to approximations in the treatment of nuclear geometry or energy loss. Comparison to the most complete calculation available \cite{37} gives a new constraint $\hat{q} \sim 5 - 7$ GeV$^2$/fm (using the non-reweighted version of this calculation), though the uncertainty on this number is difficult to estimate. Heavy flavor suppression, measured via non-photonic electrons, may be larger than expected from radiative energy loss alone \cite{48}, raising the question of significant collisional (elastic) energy loss.

The experimental study of jet quenching has reached a new level of detail and precision, and interpretation of the striking effects that have been observed is currently limited by theoretical uncertainties. Heavy ion collisions at the LHC will open up a huge new kinematic regime for jet quenching studies, with qualitatively new observables available. While the LHC regime may provide better grounds for quantitative theoretical predictions, a complete picture of jet interactions with dense QCD matter must describe all of the phenomena we observe, both at RHIC and at the LHC.

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