Constrained Molecular Dynamics simulation of the Quark-Gluon Plasma

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Abstract

We calculate the Equation of State of a quark system interacting through a phenomenological potential: the Richardson’s potential, at finite baryon density and zero temperature. In particular we study three different cases with different quark masses ($u$ and $d$), and different assumptions for the potential at large distances. We solve molecular dynamics with a constraint due to Pauli blocking and find evidences of a phase transition from “nuclear” to “quark matter”, which is analyzed also through the behaviour of the $J/\Psi$ embedded in the quark system. We show that the $J/\Psi$ particle behaves as an order parameter.

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I. INTRODUCTION

The production of a new state of matter, the Quark-Gluon Plasma (QGP), is one of the open problems of modern physics. Theoretically Quantum Chromodynamics (QCD) predicts such a state, QGP, but it can be applied only to some limited cases such as quark matter at zero density and high temperatures. Experimentally such a system can be obtained through ultra-relativistic heavy ion collision (RHIC) at CERN and at Brookhaven [1]. QGP can be formed in the first stages of the collisions, and can be studied through produced secondary particles.

Some features of the quark matter can be revealed by studying the properties of hadrons in a dense medium. The particle $J/\Psi$ is a good candidate because the formation of the QGP might lead to its suppression [2].

In this work we propose a semiclassical model which has an EOS resembling the well known properties of nuclear matter and predictions to the QGP at zero temperature and finite baryon densities. We simulate the nuclear matter composed of nucleons (which are by themselves composite three-quark objects) and its dissolving into quark matter. In addition, for our system of colored quarks, we will show how the color screening is related to the lifetime of a particle $J/\Psi$ in the medium. In particular we will see that the lifetime of the $J/\Psi$ as function of density behaves as an order parameter. Having a model which simulates the QGP might be useful when dealing with finite and, possibly, out of equilibrium systems. Infact dynamics and finite size effects might wash completely or hide a phase transition. The goal of our microscopic simulations is to help find unambiguous signals of the occurrence of the phase transition. In this work we will show that indeed using some phenomenological potential and with suitable chosen quark masses we can obtain an EOS which has some features of nuclear matter and its transition to the QGP. Infact we stress that two systems having a similar EOS will behave the same. An important ingredient of our approach is a constraint to satisfy the Pauli principle. The approach dubbed Constraint Molecular Dynamics (CoMD) as been successfully applied to relativistic and non relativistic [3, 4] heavy ion collision and plasma physics as well [5].

The paper is organized as follows: in Sect.II we introduce the method, molecular dynamics with a constraint for fermions, CoMD. In Sect.III we apply the method to calculate the Equation of State, with an arbitrary cut-off in the potential. In Sect.IV we use a screened
linear potential and we calculate the EOS. In Sect.V there is a brief summary.

II. NUMERICAL METHOD

We use molecular dynamics with a constraint for a Fermi system of quarks with colors. The color degrees of freedom of quarks are taken into account through the Gell-Mann matrices and their dynamics is solved classically, in phase space, following the evolution of the distribution function. Starting from quarks degrees of freedom, some dynamical approaches have been proposed in [6, 7, 8] based on the Vlasov equation [9, 10], and/or molecular dynamics type approach. Of course, in such approaches it is important to get quark clusterization and the correct properties of nuclear matter (NM) at the ground state (gs) baryon density $\rho_0 \sim 0.15 \text{ fm}^{-3}$ [11]. However the property of ground state nuclear matter, together with the high density phenomena, is not sufficiently studied from the point of view of quarks degrees of freedom.

In our work, the quarks interact through the Richardson’s potential $V(r_i, r_j)$:

$$V(r_{ij}) = 3 \sum_{a=1}^{8} \frac{\lambda_a^i \lambda_a^j}{2} \left[ \frac{8\pi}{33 - 2n_f} \Lambda \left( \Lambda r_{ij} - \frac{f(\Lambda r_{ij})}{\Lambda r_{ij}} \right) \right], \quad (1)$$

and [12]

$$f(t) = 1 - 4 \int dq \frac{e^{-qt}}{q \left( \ln(q^2 - 1) \right)^2 + \pi^2}. \quad (2)$$

$\lambda^a$ are the Gell-Mann matrices. We fix the number of flavors $n_f = 2$ and the parameter $\Lambda = 0.25 \text{ GeV}, (\hbar, c = 1)$ unless otherwise stated. Here we assume the potential to be dependent on the relative coordinates only. The first term is the linear term, responsible of the confinement, the second term is the Coulomb term [13].

The exact (classical) one-body distribution function $f(r, p, t)$ satisfies the equation [10]:

$$\partial_t f + \frac{p}{E} \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = 0, \quad (3)$$

where $E = \sqrt{p^2 + m_q^2}$ is the energy, $m_q$ is the $(u, d)$ quark mass and $U = U(r) = \sum_j V(r, r_j)$. Numerically the equation (3) is solved by writing the one body distribution function for each particle $i$ through the delta function:

$$f_i(r_i, p_i, t) = \sum_{a=1}^{Q} \delta(r - r_a) \delta(p - p_a), \quad (4)$$
where $Q = q + \bar{q}$ is the total number of quarks ($q$) and antiquarks ($\bar{q}$) (in this work $\bar{q} = 0$).

Inserting this expression in the exact equation (3) gives the Hamilton’s equations:

$$\frac{dr_i}{dt} = \frac{p_i}{E_i},$$

(5)

$$\frac{dp_i}{dt} = -\nabla r_i U(r).$$

(6)

Hence we must solve these equations of motion for our system of quarks.

Initially we distribute randomly the quarks in a box of side $L$ in coordinate space and in a sphere of radius $p_f$ in momentum space. $p_f$ is the Fermi momentum estimated in a simple Fermi gas model by imposing that a cell in phase space of size $h = 2\pi$ can accommodate at most $g_q$ identical quarks of different spins, flavors and colors. $g_q = n_f \times n_c \times n_s$ is the degeneracy number, $n_c$ is the number of colors (three different colors are used: red, green and blue) hence $n_c = 3; n_s = 2$ is the number of spins [1].

A simple estimate gives the following relation between the density of quarks with colors, $\rho_{qc}$, and the Fermi momentum:

$$\rho_{qc} = \frac{3n_s}{6\pi^2}p_f^3$$

(7)

We generate many events and take the average over all events in each cell on the phase space. For each particle we calculate the occupation average, i.e. the probability that a cell in the phase space is occupied.

To describe the Fermionic nature of the system we impose that average occupation for each particle is less or equal to 1 ($\bar{f}_i \leq 1$).

At each time step we control the value of average distribution function and consequently we change the momenta of particles by multiplying them for a quantity $\xi$: $P_i = P_i \times \xi$. $\xi$ is greater or less than 1 if $\bar{f}_i$ is greater or less than 1 respectively; which is the constraint [4].

With this procedure the basic quantities describing the system like: total energy, average occupation and order parameters (they will be described in this section below) after a given time will reach stationary values. We can see this in Fig. 1 in a typical case with $\rho_B = 5.995\text{fm}^{-3}$. We have repeated the calculations, in the same conditions, but two different starting points, i.e. from quarks with colors randomly distributed, i.e. QGP (left) and from quarks condensed in clusters of three with different colors, i.e. nucleons (right), respectively. We can see that in both conditions, the system will reach the same saturation values, though at different times. We stress that this behaviour is independent of the density.
FIG. 1: Time evolution of energy for quark (top panel), average occupation (middle panel) and reduced order parameters (bottom panel), with different starting point: QGP (left panels) and nucleon (right panels).

In the middle panel we display the time dependence of the average occupation, it is greater than 1 when we initially distribute randomly the quarks in the box and later it becomes nearly to 1 at saturation.

We define an order parameter to check the order of a phase transition if any. It is defined through the Gell-Mann matrices as [3]:

\[
M_{c3} = \frac{1}{N} \sum_{i=1}^{N} \sum_{a=3,8} \lambda_j^a \lambda_k^a + \lambda_i^a \lambda_j^a + \lambda_i^a \lambda_k^a = M_{c2} + \frac{1}{N} \sum_{a=3,8} \lambda_j^a \lambda_k^a + \lambda_i^a \lambda_k^a, \tag{8}
\]

where \(j(i)\) and \(k(i)\) are the two quarks closest to the quark \(i\). \(M_{c2}\) is the reduced order parameter which gives the color of the particle \(j\) closest to a particular quark \(i\).

In Fig. 1 (bottom) we show the time evolution of \(M_{c2}\) and \(M_{c3}\) when the quarks are initially randomly distributed in the system (left) and when they are clustered in nucleons (right), at the same conditions as above. The saturation values are equal in both cases, but the initial ones are different, near to 0 for the first case and near to \(3/2\) for the second. These
values are typical for QGP and a system of nucleons respectively, as we will show later in this section.

To better understand the clusterization of colored quarks we also define an higher order parameter $M_{c4}$ related to the colors of the 4 closest quarks:

$$M_{c4} = \frac{1}{N} \sum_{i=1}^{N} \sum_{a=3,8} \lambda_j^a \lambda_k^a + \lambda_j^a \lambda_l^a + \lambda_j^a \lambda_k^a + \lambda_j^a \lambda_l^a$$

where $l(i)$ is the third quark closest to the particular quark $i$.

We normalize the order parameters in this way:

$$\tilde{M}_{c2} = \frac{2}{3} [M_{c2} + 1]$$

$$\tilde{M}_{c3} = \frac{2}{9} [M_{c3} + 3]$$

$$\tilde{M}_{c4} = \frac{2}{15} [M_{c4} + 6]$$

From the properties of the Gell-Mann matrices it is easy to derive the following results for the order parameters: if the three closest quarks have different colors then $\tilde{M}_{c2} = 1$ ($M_{c2} = 1/2$), $\tilde{M}_{c3} = 1$ ($M_{c2} = 3/2$) and $\tilde{M}_{c4} = 1$ ($M_{c2} = 3/2$), in fact the fourth quark will have the same color of one of the first three, in this case we have isolated white nucleons. This case is recovered in the calculation at small densities, where the system is locally invariant for rotation in color space.

If four closest quarks have the same color $\tilde{M}_{c2} = \tilde{M}_{c3} = \tilde{M}_{c4} = 0$ we have a condition that we call EXOTIC COLOR CLUSTERING. Also in this case the system is locally invariant for rotation in color space. The corresponding potential energy is very large and repulsive.

If the three closest quarks have two different colors, independently of the color of the two closest quarks, i.e. the color of closest particle to quark $i$ is randomly chosen, we have the Quark Gluon Plasma, hence: $\tilde{M}_{c2} = \tilde{M}_{c3} = \frac{2}{3}$. In this state $\tilde{M}_{c4}$ can assume three different values: $\frac{4}{5}$; $1$; $\frac{3}{5}$, according to the colors of the four closest quarks and number of pairs of different color. If we have two pair of quarks with the same color (ex: $rggr$) $\tilde{M}_{c4} = \frac{4}{5}$, if the quarks have three different colors and two of the first three have two different colors (ex: $rggb$) $\tilde{M}_{c4} = 1$, instead if three of the four closest quarks have the same color but the first three have two different colors (ex: $rggg$) $\tilde{M}_{c4} = \frac{2}{5}$. In the next sections we will better analyze these quantities in different conditions. We note that in the QGP case the system is globally invariant for rotation in color space.
To test for a signature of the various states of matter we studied the behaviour of a pair of quarks $c$ and $\bar{c}$ embedded in the system, with $m_c = m_{\bar{c}} = 1.37$ GeV. For each density we calculate the lifetime of the $J/\Psi$ particle, through her survival probability in the system. The $J/\Psi$ embedded in matter might split essentially for two reasons. The first is that the internal kinetic energy of the $c$, $\bar{c}$ is large compared to their mutual attraction (this is true in the case where the interaction is neglected-which we will discuss below). The second, most important reason is that other quarks interact with the initial bound pair eventually splitting it. Intuitively, it is clear that the splitting occurs faster at higher densities where the $c, \bar{c}$ pair interacts with many other lighter quarks. The survival probability is related to the total number of pairs $c$ and $\bar{c}$ that stay bound after they are inserted in our saturated system of $u$ and $d$ quarks. We have fitted the $J/\psi$ survival probability with the expression:

$$P_{\text{sur}}(t) = \exp\left[-(t - t_D)/\tau\right]$$

(13)

and

$$t_{\text{sur}} = t_D + \tau$$

(14)

where $t_D$ is the delay time of the $J/\psi$ before the probability exponentially decreases, and $t_{\text{sur}}$ is the lifetime of $J/\psi$ in the system, similar to fission [15]. A typical example of the fit is given in Fig. 2, where the dotted line is an example of the real distribution and the full line is obtained through Eq.(14).

![FIG. 2: Time evolution of surviving $J/\Psi$s.](image)

In our study of the equation of state of quark matter at various baryon densities we look for some evidencies of phase transition to QGP also through the properties of the meson...
$J/\Psi$ in the medium. This is important because we want to see if the $J/\Psi$ can tell us about the occurrence and the order of the phase transitions. In future works for finite systems we want to test if the properties of the $J/\Psi$ remain. Infact, in an infinite systems there is all the time for dissolving the $J/\Psi$, but in a rapidly expanding QGP this might not occur also because of the relatively large charm masses.

III. RESULTS WITH CUT-OFF

When quarks, objects of different colors, are embedded in a dense medium such as in nuclear matter, the potential becomes screened in a similar fashion as ions and electrons in condensed matter. This is the Debye screening \cite{1,10}.

The screening can be obtained through the use of a Debye radius screening in the interaction, as will be discussed in the next section. In this section, the screening is produced directly, through the interaction of colored quarks. But our system is not really an infinite system, like nuclear matter, and this screening is insufficient to screen the linear potential and avoid its divergence for $r \to \infty$, hence we introduce a cut-off for the potential. The cut-off is a free parameter, when quark distances are greater then the cut-off, the interaction is equal to zero. Of course we are aware that by using a cut-off in the linear term the confinement property of the quarks might be lost. Neverthless we will show that this prescription leads to interesting effects which might be used in finite system studies. Furthermore the cut-off is relatively large thus it takes a considerable energy to have isolated quarks.

In Fig. 3 we plot some quantities related to the case with small quark masses, $m_u = 5$ MeV, $m_d = 10$ MeV and a cut-off of 3 fm.

The energy per nucleon and corresponding energy density in units of the $\varepsilon_F$ (energy density for a Fermi gas \cite{11}) (top panels) versus baryon density divided by the normal density $\rho_0$, have a very irregular behaviour that we can explain through the order parameters (third panel).

For small densities the quarks are condensed in clusters of three different colors, the system is locally white (isolated white nucleon). The normalized order parameters: $\tilde{M}_{c2}$ (circles), $\tilde{M}_{c3}$ (squares) and $\tilde{M}_{c4}$ (rombs) are near to 1, hence the two closest particle to quark $i$ have different colors and consequently the third closest quark to $i$ has the same color of one of the first two or of $i$, $\tilde{M}_{c2} = \tilde{M}_{c3} = \tilde{M}_{c4} \simeq 1$. At higher densities, the
FIG. 3: Energy per nucleon (top panel), energy density (2nd to top), normalized order parameters (middle panel) and time survival of $J/\psi$ (bottom panel) versus density divided by the normal density $\rho_0$, for $m_u = 5$ MeV, $m_d = 10$ MeV and cut-off $= 3$ fm.

quarks are not in clusters but randomly distributed, $\tilde{M}_{c2} = \tilde{M}_{c3} \simeq \frac{2}{3}$ and $\tilde{M}_{c4} \simeq \frac{4}{5}$, and we have the QGP ($\rho_B/\rho_0 \sim 1.2$). But the system does not stay in this state, it prefers the exotic color clustering state, where at least the four closest quarks have the same color. At density about $1.4 \sim 2.4$ times the normal density $\tilde{M}_{c2} = \tilde{M}_{c3} = \tilde{M}_{c4} \simeq 0$. The system reaches this state through a first order phase transition [1] at about 1.3 times the normal nuclear matter density. In the figure relative to the energy per nucleon, the transition is signaled by a discontinuity at the same density. The other discontinuities at larger densities...
\( \left( \frac{\rho_B}{\rho_0} > 1.5 \right) \), are probably due to the clusterization of more than four quarks of the same color. The reason for the phase transition at such small density is due to the small quark masses and to the large cutoff radius. As we will show more in detail below we can change those values and change not only the density where the transition occurs but also the order of the transition, if any.

In the present conditions, the linear term becomes very large and positive, hence the attraction between different clusters of quarks with different color distant in space prevails respect to repulsion between charges of the same color in each cluster, this explains the large values of the energy and consequently of energy density. For instance, we might have the formation of three red quarks cluster and these are attracted by an analogous three green quarks cluster.

It is the linear term that produces this very irregular behaviour, in fact repeating the same calculations with the Coulomb term only, triangles in Fig. 3, we obtain a constant contribution, not only to the energy, but also to the order parameters. The Coulomb term produces a permanent clusterization among quarks which prevents them to reach the ideal QGP state. Infact the density dependence of the Coulomb term is similar to the Fermi energy term. The difference between the two terms depends on the \( \alpha_s \) value, where \( \alpha_s \) is the strong constant coupling defined in the potential through the \( \Lambda \) parameter. When the linear term is included it prevails respect to the Coulomb one and the system stays in exotic color clustering state, \( \tilde{M}_{c2} = \tilde{M}_{c3} = \tilde{M}_{c4} = 0 \).

In Fig. 3 (bottom) we plotted the lifetime of \( J/\Psi \) versus baryon density divided by \( \rho_0 \) (full line). When the density increases, the lifetime decreases because it is more probable that a particle of different flavour gets in between a \( c\bar{c} \) pair and breaks the bond, then the number of survival \( J/\Psi \) in the medium decreases fastly and we have small values of \( t_{\text{sur}} \).

\( t_{\text{sur}} \) behaves similarly to an order parameter, infact it has a jump just where we found the phase transition \( \left( \frac{\rho_B}{\rho_0} \sim 1.3 \right) \). Analyzing the particle \( J/\Psi \) in the medium turning off the interaction (squares in Fig. 3 bottom), gives a different behaviour, i.e. a monotonic decrease with density. The survival time in the medium is always larger than that with interaction, because the forces break more easily the bonds between particles\((c,\bar{c} \) quarks). After the jump we notice a saturation of the surviving probability, again similarly to the order parameter. We would like to note that even though there is not much similarity between the EOS obtained here and nuclear matter with its transition to the QGP, it was the first case we
studied and its features are quite general as we will see in the following. In fact it would suffice a simple scaling around the critical density to compare to the other systems. We notice also that even if the $J/\Psi$ is usually studied at zero density and finite temperatures we expect a similar behaviour to the one discussed here, with the Fermi motion playing the role of the temperature.

![Graph](image)

**FIG. 4:** Energy per nucleon (top panel), energy density (2nd to top panel), normalized order parameters (middle panel) and time survival of $J/\psi$ (bottom panel) versus density divided by the normal density $\rho_0$ for $m_u = m_d = 180$ MeV and a cut-off of 1.26 fm.

In order to study the sensitivity of the results to the input parameters, we have repeated the calculations with $m_u = 180$ MeV, $m_d = 180$ MeV and a cut-off equal 1.26 fm. The
quark masses are chosen to reproduce the energy per nucleon of nuclear matter at normal density \[11\]. In Fig. 4 where we plot the same quantities of Fig. 3 (symbols have the same meaning), we can see a behaviour more regular than previously. At very high densities (almost 45 times the normal density), in the figure relative to the energy per nucleon (top) we see a flex, probably indicating a second order phase transition, which becomes a change of slope in energy density (figure below). The normalized order parameters are always positive, i.e. it never happens that three equal quark color states are on average in the same region in \(r\) space. At low densities actually they never reach the value of 1 (nucleons). Which implies that our potential is insufficient to get a good clusterization, in fact we do not obtain the minimum in energy per nucleon indicating a condition of stability for the system.

Values of order parameters at high density are always larger than \(2/3\) for \(\tilde{M}_{c2}\) and \(\tilde{M}_{c3}\), and \(4/5\) (one of the possible values of \(\tilde{M}_{c4}\) to have QGP). This indicates a residual clusterization between quarks of different color, which we associate to the semiclassical counterpart of pairing. In fact a residual attractive force especially due to the Coulomb term, couples quarks of different color.

Also in this case we studied the behaviour of the \(J/\Psi\) in the medium and we obtained a regular behaviour for the lifetime, Fig. 4 (bottom), i.e. a fast decrease for small densities, and after about \(7\rho_0\) a slow decrease with some fluctuation around \(1 \sim 2 fm/c\). The lifetime of \(J/\Psi\) behaves again similarly to the order parameters. In conclusion, we can see how changing quark masses and cut-offs we do not have the exotic color clustering and a first order phase transition, but a probable second order phase transition to QGP.

It is clear that the cut-off value changes the transition point consequently. In Fig. 5 we plot the reduced order parameters \(\tilde{M}_{c2}, \tilde{M}_{c3}, \tilde{M}_{c4}\) versus cut-off at density \(\rho_B = \rho_0\) (left) and \(\rho_B = 0.3558 fm^{-3} \sim 2.3\rho_0\) (right) for \(m_u = 5\) MeV, \(m_d = 10\) MeV. Changing the cut-off values from 1.26 to 4 fm gives a phase transition from QGP to exotic color clustering state, at \(3.1 fm\) for smaller density and at \(2.7 fm\) for \(\rho_B = 0.3558 fm^{-3}\), while the system was in a nucleonic state for small cut-off values for both cases.

At \(\rho_B = 3.49 fm^{-3} \sim 23.26\rho_0\) we calculated the order parameters versus cut-off, Fig. 6, for \(m_u = m_d = 180\) MeV (left) and versus quark masses for a cut-off = 1.26 fm (right).

As the previous case with smaller quark masses Fig. 5, for small cut-off values we have typical values of a nucleonic state. When the cut-off increases we have a phase transition from QGP to exotic color clustering state, at a cut-off = \(2.3 fm\). If we analyze the reduced
order parameters versus quark masses we find an almost constant behaviour. Very large variations of quark masses correspond little variations of the $\tilde{M}_{c2}, \tilde{M}_{c3}, \tilde{M}_{c4}$ values, hence it is the cut-off mainly responsible of the phase transition at different densities.

This suggests to use a different cut-off value to change the point of transition. In Fig. 7 we plot the energy per nucleon, energy density, normalized order parameters and time survival of $J/\psi$, for $m_u = 5$ MeV, $m_d = 10$ MeV and cut-off $= 2\, fm$.

At $\rho_B = \rho_0$ the total energy per nucleon (top) has a value similar to the typical value of
nuclear matter but we do not have the minimum. Important physics is in fact missing in our approach to describe the system of quark at small densities, i.e. the nuclear part. At $\rho_B = 10\rho_0$ the energy per nucleon and energy density display some fluctuation near to the QGP state. In fact the value of the reduced order parameters are close to $2/3$ for $\tilde{M}_{c2}$, $\tilde{M}_{c3}$ and $4/5$ for $\tilde{M}_{c4}$. Increasing the density further gives a first order phase transition to exotic color clustering state at $\rho_B \sim 40\rho_0$, as signaled by the reduced order parameters $\tilde{M}_{c2} = \tilde{M}_{c3} = \tilde{M}_{c4} \simeq 0$ and the large increase of energy and energy density.
The $J/\Psi$ particle displays some fluctuation around $10\rho_0$ and a jump around $40\rho_0$, similar to the reduced order parameters.

This equation of state could be our initial condition to simulate a finite system and a collision between nuclei.

IV. DEBYE SCREENING

In this section, to have a good screening of the linear interaction we use a particular expression of the linear term obtained through the resolution of the Poisson equation in one dimension [16]:

$$\nabla^2 \phi_{Lin} = - \sum_i q_i \rho_{qi}$$

(15)

where $\rho_{qi}$ is the linear density obtained in the Thomas-Fermi approximation:

$$\rho_{qi} \approx \frac{E_{F0}}{(6\pi^2)^{1/3}} \left[ \left(1 - \frac{\phi_{Lin}}{E_{F0}}\right)^2 - \frac{m_q^2}{E_{F0}^2}\right]^{1/2}$$

(16)

$E_{F0}$ is the Fermi energy calculated at large distances, where the field $\phi_{Lin}(r) \to 0$ and $P_{F0} = \left(\frac{6\pi^2}{9}\rho_0\right)^{1/3}$, i.e. we require that all the $r$-dependence is contained in the field and the density reduces to the free one for large distances from a given quark. Hence:

$$\phi_{Lin} = \begin{cases} 
K_{\chi l} \exp(-\chi_l r_{ij}) & \rho_q \neq 0 \\
Kr_{ij} & \rho_q = 0 
\end{cases}$$

with

$$\chi_l^2 = \frac{4}{\sqrt{3}} K \left(\frac{g_q}{6\pi^2}\right)^{1/3} \sqrt{\frac{P_F^2 + m_q^2}{P_F}}$$

(17)

$\chi_l$ is the linear Debye inverse radius [16], that for large densities goes to a constant. Also the linear potential goes to a constant and not to zero like for a total screening, but for distance larger than the Debye radius the potential is screened. $K$ is the string tension defined through the $\Lambda$ parameter [14]. We stress that the confinement property is recovered in the zero density limit.

In Fig. 8 we plot some results with this potential and $m_u = m_d = 324$ MeV. Also in this case we choose the masses of quarks to reproduce the energy per nucleon of nuclear matter at normal density.
FIG. 8: Energy per nucleon (top panel), energy density (2nd to top panel), normalized order parameters (middle panel) and time survival of $J/\psi$ (bottom panel) versus density divided by the normal density $\rho_0$ for $m_u = m_d = 324$ MeV.

In the top figure relative to energy per nucleon we found a minimum for low densities. The energy density, for large densities, shows some fluctuations, which can be due to numerical fluctuations.

The order parameters are smooth functions of the density and also in this case they never reach the QGP values (dotted lines): $\widetilde{M}_{c2} = \widetilde{M}_{c3} = \frac{2}{3}$ and $\widetilde{M}_{c4} = \frac{4}{5}$ (or other possible values), i.e. a residual clusterization remains.

 Probably it is an effect of the instability of quark pairs of different colors that produces
the instability at large energy density. Also in this case the reduced order parameters are always positive and we never have exotic color clustering.

The calculated lifetime of $J/\Psi$ in the medium versus density is shown in Fig. 8 (bottom) and it behaves like the order parameters.

V. SUMMARY

In conclusion, in this work we have discussed a semiclassical molecular dynamics approach to infinite matter at finite baryon densities and zero temperature starting from a phenomenological potential that describes the interaction between quarks with color. Pauli blocking, necessary for Fermions at zero temperature, is enforced through a constraint to the average one body occupation function. Color degrees of freedom for quarks are responsible for Debye screening, even though we have adopted some prescription mainly for numerical reasons to screen the linear term at large distances. Depending on parameters for the quark masses and the potential, we obtain EOS which exhibit a first, second or a simple cross over to the QGP. We stress that these transitions are due to changes in the system symmetries. In fact we have a local invariance for rotation at low densities i.e. for nucleons. This means that we can rotate locally the color of the quarks with no change in the energy. The QGP displays a global invariance, i.e. we can change randomly the quarks colors anywhere in the system without changing the system properties. Exotic color clustering is also a local property, infact we can change the color randomly but only within a cluster of identical colors. It is the breaking of these symmetries which gives the phase transitions. A suitable physical observable for the phase transition could be the $J/\Psi$, infact we have shown that for infinite systems it behaves like an order parameter and it is also able to distinguish between a first order and a second order (or a cross-over) phase transition. Finite size and dynamical studies within the model proposed will reveal if such a property remains. Those studies could also give indications on the possibility that the phase transitions are washed by finite sizes effects. Also other indicators of a phase transitions such as intermittency can be easily
studied in the framework of our model.

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