How diverse are physics instructors’ attitudes and approaches to teaching undergraduate level quantum mechanics?

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Abstract

Understanding instructors’ attitudes and approaches to teaching undergraduate-level quantum mechanics can be helpful in developing effective instructional tools to help students learn quantum mechanics. Here we discuss the findings from a survey in which 12 university faculty members reflected on various issues related to undergraduate-level quantum mechanics teaching and learning. Topics included faculty members’ thoughts on the goals of a college quantum mechanics course, general challenges in teaching the subject matter, students’ preparation for the course, views about foundational issues and the difficulty in teaching certain topics, reflection on their own learning of quantum mechanics when they were students versus how they teach it to their students and the extent to which they incorporate contemporary topics into their courses. The findings related to instructors’ attitudes and approaches discussed here can be useful in improving teaching and learning of quantum mechanics.

Keywords: quantum mechanics, instructor attitude, instructor approaches, physics education research

1. Introduction

Learning quantum mechanics (QM) is challenging [1–27]. Many systematic investigations have been conducted on student difficulties in learning undergraduate level quantum

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mechanics. In these investigations, many student difficulties have been identified [1–27]. In order to reduce these difficulties and help students develop a good grasp of quantum mechanics, research-based instructional tools and approaches [28–55] are being developed and evaluated.

Faculty attitudes and approaches to teaching quantum mechanics must be taken into account to make further progress in improving QM teaching and learning. Unlike other subjects such as electricity and magnetism, there is no widespread agreement on the essential topics to teach in the college undergraduate QM course for physics majors, the order in which those topics should be taught and the amount of time that should be spent on various topics. Anecdotal evidence suggests that some instructors start such a course with wave functions in position representation, e.g., in the context of the one-dimensional infinite square well, some start with spin and yet others begin with the postulates of quantum mechanics and Dirac notation. Moreover, physicists in general do not have a consensus on how to teach topics such as quantum measurement and interpretations of quantum mechanics. Some instructors may consider these topics to be confusing to teach to their students and avoid them while others may devote extensive time to teaching students such foundational issues. These differences can lead to differing opinions among instructors regarding the course goals, content and the appropriate pedagogical approach to be used in college quantum mechanics courses and can impact student learning of this challenging and exciting subject.

Here we discuss the findings from a survey about physics instructors’ attitudes and approaches to teaching college undergraduate quantum mechanics which was given to twelve faculty members in the US, six from the University of Pittsburgh (Pitt) and six from other institutions. The survey administered to the instructors consisted of 21 questions on various topics related to QM and covered a variety of topics, including instructor’s thoughts on the goals of the course, their thoughts on their students’ mathematical preparation for the course, course content and the order in which the content is presented, their teaching styles/methods, their students’ difficulties and their teaching challenges, instructor’s experiences with learning quantum mechanics as a student versus how they teach the subject to their students and their opinions on some of the foundational issues such as the many interpretations of QM. We also discussed the responses individually with five of the six instructors at Pitt which provided opportunity to ask them further follow up questions for clarification. The instructor responses indicate that there is more agreement among instructors on many of the issues than we had expected anecdotally. Still, important variations remained on many of the issues covered in the survey. Below, we discuss the major findings from the survey.

2. Goals of a quantum mechanics course

Many physics instructors noted that although quantum mechanics is challenging, these courses can be a great tool to prepare physics majors for career in physics. Moreover, one of the questions on the survey related to the goal of a QM course was the following:

Feynman once said that nobody understands quantum mechanics. If understanding of QM is so difficult for experts who have spent years learning it, it will be challenging for students who spend a mere semester or two trying to learn it. In your opinion, what is the objective/goal of an upper level (undergraduate) QM class?
Interestingly, a majority of instructors shared common opinion on this issue. According to them, one of the primary goals of an undergraduate QM course is to learn the formalism, and gain expertise in applying it to solve problems. Some instructors also suggested that one of the major course goals is to make students aware of the fundamental issues in QM that are very mystifying such as those related to measurement. For example, the following are some of the faculty responses in which they emphasised that they cared more about having their students learn the formalism in order to apply it correctly as opposed to worry about where it came from:

‘Make sure that they can use the formalism correctly, that is, without making too many mistakes. ‘Understanding’ may come later (or not).’
‘I think that the application of quantum mechanics to different situations is easy, I think it can be made easy for students. That is my goal. I want them to understand the rules (which are easy), and to understand how to apply the rules, which is straightforward. I think that Feynman was referring to ‘where do these crazy rules come from?’ I don’t get into these philosophical details. I just want the students to understand the rules and know how to apply them. Where the rules ‘come from’ they can worry about on their own time.’
‘would want the students to have had, at least at their peak of understanding, a glimpse of the internal consistency of the theory. And in achieving this, they get an important message that there was a process that led to the creation of QM theory. This wasn’t the product of some science fiction writer’s imagination.’

‘In my opinion, I think an upper-level course in QM should follow a lower-division introduction that covers much of the qualitative/descriptive aspects of QM. Then, the upper division course should repeat the qualitative while developing the formal theory.’

‘I think there are a number of goals. (1) …I think the formalism of QM is ‘easy’. I try to encourage students to realize that this is a great example of a formalism that gives the best predictions for experiments—even if you do not completely understand the formalism. (2) I think a lot of the QM ‘strangeness’ is actually not that strange. Quantization and uncertainty principles are not as hard as I think we make it. The part nobody understands is the measurement issue. I think one goal is to get students to realize where the real fundamental issues are.’

Individual discussions suggest that some instructors also felt that it is highly unlikely that students can gain a good understanding of QM in a one or two semester course because of the difficulties involved in learning a subject which is radically different from those that students had learned previously. For example, one of the instructors who felt that instructors teaching quantum mechanics should not get discouraged if their students did not learn quantum mechanics as well as they had expected emphasised: ‘Students needs many coats of paint to understand QM’.

In order to gain more information about the specific goals of a college QM course for physics majors, we further posed the following question: Are you satisfied if your students at the end of the QM course can do the following: know how to solve the time-independent Schroedinger equation for a variety of potential energies, normalise a wave function, and calculate the expectation values? If this is not satisfactory, what else should they have learned at the minimum?
This question triggered a variety of responses regarding important topics and concepts that QM instructors felt were essential for students to have learned in an undergraduate QM course. The following are some of their responses:

'\textit{They should know time development of wave function, time-dependent Schroedinger equation, uncertainty principle, Dirac notation, postulates, conceptual framework of quantum mechanics done in the standard way}'.

'\textit{They also need to understand how to compute the probability of measuring a particular value for any of the basic quantities: momentum, energy, angular momentum}'.

'\textit{They should know about angular momentum, especially spin, two-level systems, the harmonic oscillator... and, of course, measurement and the collapse of the wave function}'.

It appears that the instructors do not differ significantly in what they consider to be important for students to have learned at the minimum. Some of the other instructors were even more formal in their explanation of what they expected their students to have learned at the minimum. Their overall emphasis was again on learning the formalism and they specifically emphasised that students must learn about Hilbert space among other things in order for the instructors to feel satisfied with student learning in their QM course as in the following instructor responses:

'\textit{They need to have learned about Hilbert space and linear algebra tools, about algebraic methods like ladder operators for the harmonic oscillator and angular momentum, and about systems that cannot be described by wave functions, such as spin}'.

'\textit{They should definitely know about Hilbert space, Dirac notation, and be comfortable with SHO [simple harmonic oscillator], particle in a box, free particle and spin-1/2. They should be able to calculate almost anything for those systems}'.

### 3. Topics, their order, and math preparation

#### 3.1. Important topics and order of topics

The faculty members were also asked about the important topics that they cover in a one semester or first semester of a two semester upper-level undergraduate course for physics majors. A majority of them noted that the following topics were very important for a one semester course: interpretation of wave function, time-dependence of wave function, measurement, collapse of wave function, expectation value, time dependence of expectation value, uncertainty principle, postulates of quantum mechanics, Dirac notation, bound states and scattering states, spin (basics, Stern–Gerlach experiment, Larmor precession), angular momentum, harmonic oscillator and hydrogen atom. In addition to these topics, EPR paradox, Bell inequality and entanglement were also considered to be important by a few faculty members even for a one semester undergraduate course.

Survey responses regarding the order of topics suggest that many faculty members follow the order of the textbook they use although they sometimes supplement the main textbook with other material. Some instructors teaching upper-level undergraduate course noted that they liked to start with wave function in position representation (covering problems such as the infinite square well, harmonic oscillator, free particle etc) first before moving on to the
postulates of quantum mechanics and spin while others noted that they liked to start with spin or the postulates of quantum mechanics and Dirac notation. One faculty member who introduced the postulates of quantum mechanics at the beginning of his course explained his rationale as follows:

‘My rationale is that I very much like a ‘postulate based’ approach with an emphasis on the concept of eigen functions and eigen values. I like the students to get some kind of intuition for Hilbert space so that the ideas of a state as some discrete vector (like angular momentum) or continuous function (like psi of x) are coherent and just different expressions of the same basic idea. I really don’t like a ‘historical approach’, since that is not best pedagogically. After introducing them to basic concepts and framework, I like to give them a strong dose of all the wave stuff, since waves are important to physicists and waves are fun, whereas all the formal stuff is so dry (just linear algebra). Then after the wave stuff I get back into the formal stuff and go in more depth. The idea is that now the students might be more motivated to learn it because they have been introduced already to basic fun ideas like particle in box. Then I move into 3-d with simple separation of variables, but then more difficult angular momentum stuff. After doing angular momentum in great detail I do radial equation...first for free particle and then for hydrogen atom. I do angular momentum first, since then the complicated Schroedinger equation in spher- ical coordinates reduces to something very easy, and the separation variables naturally become angular momentum. Then details of hydrogen atom, and then spin comes after, since it’s just another angular momentum, but a little more abstract...’

We note that some of the instructors who started the course with wave function in position representation (with examples such as the infinite square well and simple harmonic oscillator potential energy well) noted that they preferred to start this way because it was less abstract than other approaches to introducing QM and students had learned some of the material in an earlier modern physics course. They felt that students usually take a course in the physics of waves (and differential equations) and are more comfortable with this approach than with linear algebra. However, other instructors who started with the postulates of QM and Dirac notation felt that the rules of QM should be laid out first before students are exposed to specific systems in which those rules are applied. Some instructors who preferred to introduce a two state system (such as spin-1/2) before an infinite-dimensional Hilbert space (e.g., particle in an infinite square well or simple harmonic oscillator potential energy well) felt that a two state system is a simple system in which the postulates and formalism of QM can be taught to students without the mathematical complication of an infinite dimensional vector space. Some instructors, who preferred to introduce spin-1/2 first, also felt that there could be confusion between QM and classical mechanics ideas if students learn the quantum postulates in the context of measurement of position and momentum. In particular, they felt that since position and momentum are deterministic variables in classical mechanics but probabilistic in QM (that is, position and momentum are not well defined in a given quantum state and measurement of position or momentum in a given quantum state collapses the state into an eigenstate of the corresponding observable and we measure the corresponding eigenvalue), students may confuse classical and quantum ideas (e.g., of position and momentum) which can result in difficulty in developing a good grasp of the QM formalism. On the other hand, they felt that learning the QM postulates and formalism in the context of
spin-1/2 will not produce similar difficulty since students do not encounter spin-1/2 systems in classical mechanics.

3.2. Mathematics preparation

In order to understand instructors’ views about how mathematically well-prepared their undergraduate students are for taking their course in QM, we posed the following question:

Rate on a scale of 1–5 (1 means not at all prepared and 5 means extremely well-prepared) how well-prepared students are to take QM in terms of their math preparation in the following mathematical topics

(a) Linear algebra
(b) Special functions
(c) Ordinary differential equations
(d) Partial differential equations

The responses are described in the pie charts in figure 1. The data suggest that the mathematical preparation according to most faculty members in each of these categories is either average (rating of 3) or below average (rating of 2 or 1). Surprisingly, a majority of them felt that students’ understanding of special functions is below average (2/3rd of the faculty gave a rating of 2) whereas for topics like linear algebra, partial differential equations and ordinary differential equations, roughly one-thirds to two-thirds of the faculty members
gave a rating of 3. Individual discussions with some of the faculty members indicates that they felt that improving students’ mathematical preparation of these topics can be beneficial for helping students learn QM because many students struggled with the mathematics. They noted that instead of focusing on QM, those students often got distracted by the requisite math (which was challenging for them) which severely compromised their learning.

4. Teaching and learning strategies/methods

The objectives for surveying issues related to instructors’ teaching and learning strategies were to learn from them about their styles, methods and approaches for learning and teaching QM, to identify commonalities between their teaching styles/methods, and to learn from them about the difficulties their students have in learning QM and how they help their students overcome those difficulties.

4.1. Instructors’ recollections of their own undergraduate QM courses

One of the factors that may influence instructors in shaping their teaching styles is how they learned when they were students and the strategies and methods used by their instructors in helping them learn when they learned QM as students. Since we were motivated to investigate how instructors learned QM, we posed the following questions: Do you remember any difference between your own learning of quantum mechanics versus other areas of physics? What unique strategies, if any, did you use to learn QM? The following are some of the responses from the instructors that suggest that some of them were lost or had no sense of what they were learning in their undergraduate quantum mechanics course:

‘I was completely lost when taking QM as an undergraduate. I could do the math, so I churned out answers while having no conceptual basis for what I was doing. So I guess I really learned QM when I was in grad school, talking to my classmates’.

‘Yes. Everything else that I learned was straightforward. QM made no sense until graduate school. Even though I got A’s on every test, I really had no clue how it held together. I teach it completely different from how I learned it’.

Some faculty members reminisced about the fact that their instructors’ approaches were not necessarily conducive to helping them learn. The following examples elucidate these types of thoughts from the instructors about their own learning of QM:

‘I had a sub-par teacher, so I just learned from the book. That worked well for me’.

‘I first learned a bit of quantum mechanics as a sophomore in a course on device physics in electrical engineering. I don’t recall needing any special strategies. As a junior I took the two-term undergraduate quantum course based on Merzbacher’s book, which is really at the graduate level. My strategy was to work very hard. The complexity of the mathematics made it difficult to focus on the concepts. The instructor was a person who liked doing calculations’.

‘I was not really exposed [by my instructor] to bra-ket notation and I think that might have helped me. I remember getting a bit too bogged down in functions and integrals. Perhaps that’s a bit inevitable’.
Other faculty members recalled how much they enjoyed learning quantum mechanics or that they actually approached the learning of quantum mechanics by developing special strategies such as focusing on how to calculate things instead of trying to understand what everything meant. The following recollections shed light on these kinds of responses:

‘I don’t think the learning itself was different. If anything, I enjoyed learning quantum more than anything else, because I found it fascinating, so it may actually have been easier than, say, classical mechanics or E&M [electricity and magnetism].’

‘For me, it really came in two stages. I focused first on executing the mechanics and then trying to understand what it ‘meant’.’

In summary, written responses and individual discussions with some of the instructors (about their reflection upon their own learning of QM) suggest that some of them struggled to make sense of QM when they were learning the subject and felt that they were not in control of the subject matter even though they were facile at the mathematical calculations and managed to get good grades in the course. Some of them explicitly mentioned that they found it difficult to focus on the concepts or that they felt that all they were doing in their quantum mechanics course was mathematics without a good explanation or justification for why it was reasonable to do so. Others worked very hard and relied on teaching themselves from the textbook. These responses from QM instructors about their own learning of QM are very revealing and should be taken into account by QM instructors and curriculum developers alike. The diversity of students’ prior preparation in the physics classrooms has increased tremendously over the years. If instructors’ reflections suggest that many of them were having great difficulty in making sense of their quantitative calculations in their QM courses, it is important to consider effective strategies to bridge the gap between the conceptual and quantitative aspects of quantum mechanics in order to help students develop a good grasp of QM.

4.2. Challenging topics and examples of common difficulties

We also investigated instructors’ views about some of the difficulties their students have in learning quantum mechanics by asking the following question: From your experience, can you tell us about some of the specific difficulties students have in learning QM? The following are some of the responses:

‘I find that QM more than any other subject has two distinct levels: (1) Apply the rules. (2) formulate your own mental picture of what is physically happening, i.e. have an understanding of the concepts. To me, the rules are probably the most straightforward of any physics discipline but the concepts are perhaps the hardest. This large discrepancy causes students difficulty’.

‘Meaning of the wave function, meaning of measurement, how to apply postulates to different situations’.

‘Students can have difficulties at all levels, some more with conceptual issues, some more with mathematical issues. It strongly depends on the individual.’

‘Time development of wave function is difficult, misconceptions related to uncertainty principle, collapse of wave function is difficult to integrate with the rest of the framework involving Schroedinger equation when no measurement is done, reconciling classical ideas with quantum ideas’.
There is lots of baggage from modern physics courses and the stories they learn from there.

In summary, written responses and individual discussions with some of the QM instructors suggest that despite the fact that they rated students’ mathematical preparation to be often inadequate; their major concern was students’ conceptual difficulties in learning quantum mechanics. Moreover, some of them felt that the difficulty may be exacerbated by inaccurate descriptions in modern physics courses (some concrete examples that the instructors provided are discussed later). In discussion with one instructor, he pointed out that the misconceptions in introductory classical mechanics are often unavoidable because people try to make sense of their everyday experiences and the laws of physics do not conform to their naïve notions about force, motion, energy etc. On the other hand, he felt that the misconceptions students display in the context of quantum mechanics in many situations are due to the fact that students over-generalise their classical mechanics knowledge to quantum situations in which it is not applicable and also because students transfer inaccurate descriptions of quantum mechanics from their earlier modern physics courses to QM courses (e.g., what they learned about the Bohr model of hydrogen atom in an earlier modern physics course). These types of comments and reflection from QM instructors are consistent with the framework we have developed for why the difficulties beginning students have in learning introductory classical mechanics are analogous to the difficulties students in quantum mechanics courses have in learning QM.

We also asked the instructors about the topics that they find challenging to teach. Several topics were mentioned including measurement, formalism, Dirac notation, spin and scattering theory. The following are some of the instructors’ responses:

'Spin is one of the most challenging topics because it is the first without any real classical analog. The other one is the raising/lowering operators in the harmonic oscillator. This is the first time they use a purely 'abstract' formalism'.

'I find all topics in QM are fairly challenging. Perhaps the mathematics is the 'easiest' in the sense that there are some nice specific rules to follow'.

'The introduction of the complete linear algebra formalism is relatively challenging because it requires a lot of traditional teaching style (the instructor at the board talking to the class) until the students are ready to deal with non-trivial problems in that language themselves'.

'Addition of angular momentum is always tricky. And of course the whole measurement process'.

'formalism, Dirac notation, anything that is not about solving TISE [Time-independent Schroedinger equation] and position space wave function'.

'Perhaps very tricky perturbation things involving spin-orbit calculations can get hard'.

Interestingly, some faculty members found mathematical aspects of some topics in QM to be most challenging to teach while others found the conceptual aspects most difficult to teach. Although the challenging topics pointed out by the instructors varied from mathematical to conceptual, they were similar in their abstractness. The curriculum developers should take these challenges in account in order to develop effective instructional tools and approaches that address these difficulties and help diverse groups of students learn QM better.
4.3. Student engagement and strategies for reducing common difficulties

Prior investigations suggest that in a QM course, students can benefit from instructional approaches that involve students actively in the learning process and get them excited about learning [e.g., see 28–55]. Therefore, we asked the QM instructors about their strategies to keep college students actively engaged in the learning process in the QM courses as in the following question: **Do you use any special learning methods to keep students engaged in the class? If so, explain what you do.** While some of the instructors follow a traditional approach which mainly involves lecturing, the following are responses from three faculty members who noted using physical examples, incorporating group problem solving and considering integrating single-photon experiments with the lecture class to engage students in the learning process and help them develop a good grasp of the abstract formalism of quantum mechanics:

‘I introduce every major concept with a physical example and outline an experiment we want to understand. For example, to start 3d spherical stuff I will pose the question: suppose we evaporated metal onto a dirty surface and it clustered up into little balls? What then would be the allowed energies of an electron in the ball? For SHO [simple harmonic oscillator] I pose the question: Suppose I drop an atom on a surface and it sits in a corrugated potential? What are the allowed vibrational energies? Where might the atom sit? I set the problem up from an experimental starting point before diving into the math.’

‘We typically have 4 h of in class time available. Of that, approximate 2.5 is used for ‘standard’ lecture, though a fairly interactive style lecture. The other 1.5 h is used in supervised group problem solving. Typically, 3 problems are assigned, each with a pre question. When necessary, the class is stopped and general comments are made to the class on the problems.’

‘Starting next year, we will actually have students performing experiments and discussing these experiments in detail. Specifically, we will be discussing single-photon experiments in a Mach–Zehnder Interferometer. I also have my students work a HW [homework] problem in front of the class—everyone must do one during the semester and they volunteer for it in advance.’

Prior research also suggests that students are not blank slates and even though students do not explicitly reason about quantum phenomena in their everyday experiences, students in QM courses often display common misconceptions that can interfere with their learning. These misconceptions are often due to over-generalisation of concepts learned in a classical context or in a similar context in a modern physics course. Therefore, another question we asked the instructors was about their teaching strategies as follows: **What strategies do you use to prevent misconceptions about the concepts taught in QM? Please describe if you use any specific strategies/methods to avoid misconceptions.** Following are some typical responses:

‘I try to relate to the few basic concepts of QM in as many different applications, examples, or formulations as possible’.

‘I like to find the simplest examples that display the richness of a set of principles without adding complexity to the problem. That’s why I like to stick with canonical QM Hamiltonians like free particle, SHO [simple harmonic oscillator] and particle in a box’.
'I do not know of any strategies or methods that really work. Just keep asking the same questions in homework problems, and explaining to them why (and when) they get them wrong.'
'I try to select pre-lecture quizzes and in class problems that highlight the major challenges to emphasize these [quantum challenges] to the students'.
'I use concept tests in class, and tutorials with demonstrative simulations for home-study.'

In summary, in response to this question, only some of the faculty mentioned providing accessible experiments on various topics, opportunity for group problem solving or individual students presenting problem solutions to their peers and instructor, using tutorials or concept tests (clicker questions to get immediate feedback from students and to communicate to the students the goals of the course with concrete examples) in classes. Other instructors were either skeptical about whether any novel strategies or methods would really work (or were needed) or they mentioned emphasising and spiraling back to the basic concepts of quantum mechanics in many different situations. Since only a few faculty members mentioned physics education research based methods to teach QM, more effort is needed to disseminate these materials developed with extensive research in how students learn QM or make them adaptable to an instructor’s teaching style.

4.4. Helping students make sense of the wave function

In order to gather information from the instructors about how they teach a particular topic, we asked them about how they help students make sense of the wave function [56]. Some faculty noted that they focus on the fact that the wave function is an abstract vector in an infinite dimensional vector space whereas others were more focused on its connection with measurement. The following are sample comments that shed light on the typical methods the faculty members noted they use to help students make sense of an abstract topic like the wave function:

'I tell them that Hilbert space is a porcupine. Infinite dimensional space is a porcupine with each spine perpendicular to the other. Each spine represents an eigenstate. A wave function is a vector pointing in an arbitrary direction in the porcupine'.

'For cases where the spatial part of the wave function is real, we do a lot of sketching, relating the curvature to E–V [energy versus potential energy]. I give them some hypothetical messy potential and they have to sketch a wave function that is at least plausibly a solution'.

'I haven’t thought about this per se. I guess the main thing is focusing on the connection to measurements. I focus much more on the probability of making measurements than on the expectation value'.

'I guess I try to give students enough experience with the wave function that they know what to do with it. Knowing what to do with it in some sense gives you a way to make sense of it’.

'I try to make them think of wave functions as really large vectors (meaning, literally, an array of numbers), which they are in practice, if you discretize and bound space appropriately (as you often have to do for numerical calculations)'.

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In summary, written responses and individual discussions suggest that some faculty members emphasised the state of the system as a vector in the Hilbert space while others focused on helping students learn to draw the wave function in position space and learn to use it to answer questions about measurements of physical observables and expectation values.

4.5. Reconciling classical mechanics and quantum mechanics

By the time students take QM, they already have well-developed classical intuition. But classical mechanics is a deterministic theory whereas quantum mechanics is probabilistic. Therefore, it is important that students understand the similarities and differences between the two theories. In order to learn from the faculty members about the approaches they use to show students the connections and differences between classical mechanics and QM, we posed the following questions: In your class, what types of connections do you make between quantum physics and classical mechanics (CM)? Also, how do you explain the differences between QM & CM? Please give specific examples. The following are some typical responses:

‘I do not emphasize the connections between QM and classical physics. I find this mostly distracting. I use classical mechanics as a ‘foil’ and emphasize the differences that QM has with CM. I think it’s more important to understand the differences rather than the similarities (since students already have a good CM intuition, but tend to have no QM intuition, and these intuitions are very very different in my opinion)’.

‘Perhaps the main emphasis in this regard is a discussion in terms of how the state of a system is characterized. In CM we tend to characterize the state of the system by the position and velocity and how this determines energy, etc. In QM, the wave function characterizes the state. From there, the goal is to predict measurements, and we discuss the differences in this process’.

‘I try to connect QM to CM as much as practical. Deriving Ehrenfest’s theorem is an example. I also try to connect the time-rate of change for the expectation value equation to the CM equation in terms of Poisson Brackets—even though our students tend “not” to see the Poisson Bracket equation in our CM class’.

‘I do not try very hard to relate QM and CM. I do cover the Heisenberg picture, and Ehrenfest’s theorem’.

‘One should at least talk about the classical limit and how to take it—why quantum mechanics is not seen in everyday behavior’

‘Classical mechanics is used to give physical meaning to operators like momentum and Hamilton operator. The specific form of the Hamilton operator for various problems is derived from classical mechanics.’

Responses suggest that the faculty members have varied opinions on the extent to which the connections and differences between classical mechanics and QM should be emphasised. Individual discussions also suggest that those who considered that it is important to make these connections thought that they will help students develop intuition.

Another question that the faculty members were asked was whether they agree or disagree that the semi-classical models such as the Bohr model of the hydrogen atom can be misleading for students’ learning of QM. Some of the faculty members had very strong opinions about Bohr model. The following are typical responses:

‘…I avoid it. At least that’s how I feel when teaching QM (although out in the real world I appreciate that semiclassical ideas can be useful, like
understanding how electrons move in solids), but I avoid semiclassical stuff when teaching QM.’

‘I do not teach the Bohr model at all in quantum courses. It has its place in elementary modern physics courses, but quantum students should realize that energies come from diagonalizing the Hamiltonian, and wave functions do not really describe classical trajectories.’

‘we do not teach it in the upper level course. I definitely think the only real value of the Bohr model is to show how physics works as an experimental science—it was a great starting point as it was able to explain certain measurements. But, once more measurements are made (especially related to angular momentum, which it gets wrong) the model fails, and you need a different model. I would say the real misleading thing is to call it semi-classical. There are semi-classical techniques that work in some situations. The Bohr model is just wrong.’

‘I would argue that one can pull a reasonable picture of electron ‘shells’ as ‘wave resonances’ from the model. I think this can actually help students understand what QM is trying to do. But I don’t teach this in the upper level course. I will make reference to it, however.’

‘I find the Bohr model hideous. I really wish modern physics classes could go beyond the Bohr model some day, so that we don’t have to unteach it later. So after I’ve introduced the 3D Schroedinger equation we have a discussion about all the obviously unphysical aspects of the Bohr model. That, I think, usually cures them.’ The same faculty member added ‘…I’m not against semi-classical models per se, but the two most common examples of them I find counterproductive. The Bohr model has an inaccurate picture of an electron with a definite position and momentum. It predicts the wrong angular momentum (though by good fortune gets the energy levels right). It gives entirely unphysical orbitals that consequently have no predictive chemistry capability. The representation of $J = L + S$ vector in the Zeeman effect. Just by drawing it, one is building a model of specified $J_x$, $J_y$, and $J_z$, which is nonsense. Picturing the uncertainty in $J_x$ and $J_y$ as ‘precession’ is wrong and misleading.’

Although most of the instructors pointed out that the Bohr model is not taught in the upper-level undergraduate quantum mechanics, their level of support for the model in general varied. Some of them felt that it has a place in a college-level modern physics course, others felt that it can be misleading and can lead to the students developing misconceptions that will be difficult to eliminate later on.

4.6. QM postulates and single measurements

The postulates of quantum mechanics are non-intuitive [57], but they provide a coherent conceptual framework for making connections between the state of the system, which is an abstract vector in a Hilbert space, and measurements of physical observables. Since postulates are central in unifying the quantum mechanical theory, one question on the survey asked faculty members how much time they spend teaching about the postulates of quantum mechanics and whether they teach them at the beginning, middle or at the end of the course. Although different instructors introduced the postulates at different times in their courses, most instructors noted that the postulates were important. The sentiment of this instructor was
shared by many others: ‘I do postulates on the very first day and keep harping on them over and over through the course. I remember that I was not taught the postulates as an undergrad, and so QM at first seemed to me as a big mish mash of unconnected ideas. I think that the postulates are crucial for tying it together.’

Another question on the survey asked about whether the instructors teach about single measurements versus expectation values as follows: **Do you teach students about single measurements and the collapse of the wave function or only discuss expectation values?** **Explain.** Most of the instructors noted that they discuss single measurements and the collapse of the wave function in addition to the expectation values. For example, one faculty member noted ‘I push hard the idea of single measurements, and how repeated measurements builds up a histogram, which can yield an expectation value. I try to go through this and use examples of measurements of every reasonable observable (momentum, position, angular momentum, energy, etc).’ Other responses such as the following had a similar theme: ‘Students need to learn about single measurements and the collapse of the wave function. You cannot do quantum mechanics without that postulate, especially nowadays, when experiments on single quantum systems are becoming more and more commonplace.’

### 4.7. Introduction of novel developments in QM

In the last few decades, major experimental and theoretical advances have been made that elucidate foundational issues in quantum mechanics. Therefore, faculty members were asked to suggest any novel developments in the field of QM that offered new insights in resolving some of the foundational issues and whether they would discuss them with college undergraduate students in QM courses. Quantum information/computing, entanglement, EPR paradox, Bell inequality and its experimental confirmation topped the list with the following sample responses:

‘Quantum information theory should be introduced more to QM. Ideas of qubits and manipulating them should be made more important. Ideas of partial measurement, projections, could be useful.’

‘Being able to do single-photon experiments with students is new—although the experiments themselves aren’t new. It seems like quantum optics is a place where some new experiments are revealing that QM continues to predict very strange things correctly. I am completely open to discussing new things with students but there’s not much room in the course as is.’

‘The Bell inequality experiments are certainly a major breakthrough that should be covered in a quantum course if there is enough time to do it properly (which unfortunately is often not the case).’

‘…entangled states. This gets at fundamental issues (Bell’s inequality) and touches on some exciting topics (quantum computing).’

‘One thing that I think helped me that I try to get into my classes was EPR and Bell’s theorem. The fact that it was experimentally verified that local realism is violated made quite an impression on me.’

In response to another question about an experiment in QM that demonstrates the weirdness of quantum theory, the experiments cited included observing interference when two-slit experiment is conducted with particles, e.g., electrons, emitted from the source one at a time, single photon interference in a Mach–Zehnder Interferometer, experiments to test Bell’s inequality, and optical absorption of a small structure which shows quantum confinement. One faculty member noted ‘…we do the plain old 2-slit interference, with a dim
light and a black chamber (using a photomultiplier tube to measure the intensity). They do some calculations to determine that photons are going through the 2-slit apparatus one at a time, so the interference pattern is created by single photons. With enough discussion, they usually leave wide-eyed.’

It is encouraging that quantum mechanics instructors are incorporating exciting topics that touch upon some of the foundational issues in their classes. Such topics are likely to increase students’ enthusiasm for pursuing further studies in QM.

4.8. Instructors’ views about the various interpretations of QM

There have been disagreements and debates among physicists regarding the interpretation of quantum mechanics from early times but the Copenhagen interpretation is the most widely accepted interpretation and commonly taught to students. Since some physicists have issues with this interpretation [58], we wanted to learn from the faculty members about their views on the advantages and disadvantages of using the standard interpretation for teaching QM. Therefore, we posed the following question: There are many interpretations of quantum theory. However, the ‘standard interpretation’ (Copenhagen interpretation) is commonly used for teaching. Can you tell us about the advantages and disadvantages of using the standard interpretation of quantum theory for teaching QM?

The following are some typical responses from QM instructors who participated in the survey:

‘I don’t worry about ‘interpretations’. I teach the standard postulates like they are handed down by God, and then try to teach students how to interpret and use them in practical situations.’

‘Advantages—it gives students something to grasp onto. Disadvantages—it’s unfamiliar and students will inevitably try to take the picture literally.’

‘The advantage is obviously that it is “the” true interpretation, or at least the only sensible one. The disadvantage is that, of course, it is hard for anybody to understand properly.’

‘I’m not an expert in QM interpretation. I like covering Bell’s inequality and the Aspect experiments as a way to place limitations on possible hidden variable theories. I don’t think that is incompatible with the Copenhagen interpretation. I don’t present much on measurement theory because I don’t really understand what, for example, cosmologists like Hartle are doing.’

‘I think the standard interpretation used in the textbook is the only one that is easy enough to teach to undergraduate students even though it is not elegant due to separation of ‘normal’ time evolution and time evolution during measurement.’

‘I think it’s fine to teach Copenhagen, as long as students know that many have issues with this interpretation. They should know when to think and when to shut up and calculate.’

Thus, written responses and discussions with some of the instructors suggest that they were comfortable with the interpretation of quantum mechanics commonly taught to students. On the other hand, others explicitly noted that they had not spent much time thinking about the alternative interpretations. Moreover, some of the instructors were skeptical about the alternative interpretations while others were more open to making an effort to learn them if they were given an opportunity.
5. Summary

It appears that the QM instructors surveyed often shared common opinions about the goals of an undergraduate QM course for physics majors, although important variations remained with differences of opinion on the strategies for engaging students, the order of topics, whether they included certain topics and how they incorporate selected topics in their courses. For example, QM instructors’ written responses and individual discussions with some of them suggest that many of them felt that a QM course is a good preparation for research so ensuring that students learned to use the formalism in the QM course should be a major goal. However, some instructors started the course with the postulates and formalism of QM including the use of Dirac notation and some felt that the spin-1/2 system was a good system to start with since it is a simple two-state system in which all of the quantum postulates and formalism can be taught without the complication of an infinite-dimensional vector space and without any confusion between position and momentum in classical mechanics (in which they are deterministic variables) and in quantum mechanics (in which they are probabilistic). However, others felt that starting with the postulates and spin-1/2 is too abstract and could be difficult for students to grasp since students were not exposed to these concepts earlier and they preferred to start with the position representation (e.g., the solutions of the time-independent Schrödinger equation with different potential energies such as the infinite square well, finite square well and simple harmonic oscillator potential energy well etc).

Furthermore, some instructors focused more on the conceptual aspects of QM while other instructors focused more on the mathematical aspects even though they all agreed that students should learn the quantum formalism by the end of the course. The survey also suggests that some of the topics such as Dirac notation, spin, formalism and measurements were generally considered especially challenging to teach by most of the instructors who participated in the survey. However, some instructors felt that the conceptual aspects of QM were more challenging to teach while others felt that the mathematical aspects of QM were more challenging to teach.

Moreover, some instructors liked to emphasise the connection between classical mechanics and quantum mechanics while others did not like doing so. Also, some were opposed to teaching simplified models, e.g., Bohr model or pictorial representation of angular momentum (commonly displayed in the textbooks) because they felt that the model could give students the incorrect impression that the different components of the angular momentum of a quantum entity can have well-defined values in a given quantum state whereas other instructors were more favourable to introducing QM using these simplified models. The latter group felt that since these models are simpler, they can be used to introduce certain aspects of QM even though they miss certain other aspects (which other instructors worried could mislead students and may cause interference when students are taught the correct quantum mechanical models).

Also, many of the instructors surveyed appeared to be comfortable using the standard Copenhagen interpretation to teach QM and some of them brushed aside the question of interpretation of QM noting they do not worry about such things and are focused on helping students learn how to apply the standard formalism similar to Mermin’s advice of ‘shut up and calculate’ [59].

One interesting finding of the survey is that the instructors considered students’ mathematical preparation for learning QM to be inadequate. This finding suggests that it may be useful to help students develop the required mathematical skills (either in another physics or mathematics course) before they take QM. We also found that a majority of the instructors
follow the order of topics in the textbook they use and a survey of 48 instructors found that
David Griffiths’ textbook on QM was the most commonly used textbook.

Many instructors surveyed mostly used lecture-based teaching only and some were even
skeptical that any other approach would help students learn QM. However, some of them
supplemented their lectures with experiments (e.g., double slit with single particles), group
problem solving, pre-lecture quizzes and concept tests to help students develop conceptual
understanding. Several instructors recalled that learning quantum mechanics was particularly
challenging for them when they were undergraduates and some noted that they do not teach it
the way they were taught to their own students because they did not find the approach used by
their instructors to be effective.

These findings should be taken into account in developing learning tools to improve
students’ understanding of QM.

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