Effect of correlations on cumulants in heavy-ion collisions

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We study the effects of correlations on cumulants and their ratios of net-proton multiplicity distributions which have been measured for central (0%-5%) Au + Au collisions at the Relativistic Heavy Ion Collider (RHIC). This effect has been studied assuming individual proton and anti-proton distributions as a Poisson or Negative Binomial Distribution (NBD). In-spite of significantly correlated production due to baryon number, electric charge conservation and kinematical correlations of protons and anti-protons, the measured cumulants of the net-proton distribution follow the independent production model. In the present work we demonstrate how the introduction of the correlations will affect the cumulants and their ratios for the difference distributions. We have also demonstrated this study using the proton and anti-proton distributions obtained from the HIJING event generator.

PACS numbers: 25.75.Gz,12.38.Mh,21.65.Qr,25.75.-q,25.75.Nq

I. INTRODUCTION

In recent years, the Beam Energy Scan (BES) program at Brookhaven National Laboratory’s Relativistic Heavy Ion Collider (RHIC) has drawn much attention to map the quantum chromodynamics (QCD) phase diagram in terms of temperature (T) and baryon chemical potential (\(\mu_B\)) [1]. Lattice QCD calculations combined with other theoretical models suggest that there should be a critical point where the phase transition line of first order originating from high \(\mu_B\) ends [2, 3]. Experimentally the location of the critical point can be measured by scanning the \(T-\mu_B\) plane by varying the center of mass energies of the colliding ions.

The moments of the multiplicity distribution of conserved quantities are related to the correlation length (\(\xi\)) of the system and hence can be used to look for signals of a phase transition and critical point [4, 5]. The variance (\(\sigma^2\)) of these distributions is related to \(\xi^2\) as \(\sigma^2 \sim \xi^2\) [3]. The skewness (\(S\)) goes as \(\xi^{4.5}\) and the kurtosis (\(\kappa\)) is related as \(\kappa \sim \xi^{2.5}\). Also, these quantities have been used to extract the freeze-out parameters of the system. For example, higher moments of net-charge distributions are used to extract \(\mu_B\) and are found to be in good agreement with methods using particle ratios [10, 12]. Both the current experiments at RHIC (STAR and PHENIX), have reported their measurements of higher cumulants for net-charge [12, 13] and net-proton [14] multiplicity distributions at different collision energies. The net-proton results from the STAR measurements are reasonably described by assuming independent production of protons and anti-protons, indicating that there are no apparent correlations between the protons and anti-protons for the observable presented [13]. In the independent production (IP) model, the measured cumulants of protons and anti-protons are used to construct the cumulants of the net-proton distribution. If the individual proton and anti-proton distributions are assumed to be Poisson distributions, the resultant net-proton distribution will be a Skellam distribution [13]. Poisson distributions fall into the class of “integer valued \(\text{Levy processes}\) for which the cumulants of the distribution \(P(n_+ - n_-)\) of the difference of samples from positive (\(n_+\)) and negative (\(n_-\)) distributions \(P(n_+)\) and \(P(n_-)\), with cumulants \(C_n^+\) and \(C_n^-\), respectively, are

\[
C_n = C_n^+ + (-1)^n C_n^- \tag{1}
\]

so long as the distributions are not correlated [16, 17]. This result is the same as if the distributions \(P(n_+)\) and \(P(n_-)\) are statistically independent. A similar exercise has been carried out in Refs. [18, 19] using heavy-ion event generators such as HIJING and UrQMD assuming both the individual distributions to be a Poisson or a negative binomial distribution (NBD). Further, the moments of the distributions are related to the cumulants as: mean (\(M\)) = \(C_1\); \(\sigma^2 = C_2 - \langle (\delta N)^2 \rangle\); \(S = \frac{C_3}{C_2^{3/2}} = \frac{\langle (\delta N)^3 \rangle/\sigma^3}{\langle (\delta N)^2 \rangle/\sigma^2}\); \(\kappa = \frac{C_4}{C_2^{3/2}} = \frac{\langle (\delta N)^4 \rangle/\sigma^4 - 3}{\langle (\delta N)^2 \rangle/\sigma^2}\). Here, \(N\) is the multiplicity and \(\delta N = N - M\). Hence, the ratios of the cumulants are related to the moments as:

\[
M/\sigma^2 = C_1/C_2, \quad S\sigma = C_3/C_2, \quad \kappa \sigma^2 = C_4/C_2, \quad S\sigma^3/M = C_5/C_1.
\]

Recently, measured cumulants of net-charge distributions by the PHENIX experiment show that individual positively and negatively charged hadron multiplicity distributions can be described by NBD for energies from \(\sqrt{s_{NN}} = 7.7\) to 200 GeV in Au + Au collisions [12]. Since NBD also lies in the class of “integer valued \(\text{Levy processes}\)”, it also follows Eq. (1). Hence, cumulants calculated from the event-by-event (e-by-e) net-charge distributions agree with the cumulants obtained from individual positive and negative multiplicity distributions using Eq. (1).

In Refs. [20, 21], the individual cumulants are shown for different particle production mechanisms and it is
observed that within the STAR kinematical acceptance, models satisfy the independent production model with the e-by-e measured distributions. An ideal hadron resonance gas model in the grand canonical ensemble by construction treats the susceptibility of net-protons in a similar way as they are treated in the IP model as: 
\[
\chi_{p-\bar{p}}^{(n)} = \chi_p^{(n)} + (-1)^n \chi_{\bar{p}}^{(n)} \quad \text{where,} \quad \chi_{p-\bar{p}}^{(n)} \text{ is the } n^{th} \text{ order susceptibility for net-protons,} \quad \chi_p^{(n)} \text{ and } \chi_{\bar{p}}^{(n)} \text{ are the } n^{th} \text{ susceptibilities for protons and anti-protons, respectively.}
\]
The STAR collaboration has reported that the product susceptibility for net-protons is found to have values close to expectations based on independent proton and anti-proton production. However, it has been puzzling since then that in spite of significantly correlated production due to baryon number, electric charge conservation and kinematical correlations of proton and anti-protons, why the measured cumulants follow the independent production model. In the present work, we demonstrate the results of such cases by considering the Poisson and NBD distributions for the particle production. We compare the results of cumulants and their ratios by making an e-by-e distribution to those which are derived from Eq. after introducing the correlations.

The paper is organized as follows. In the following section, we discuss the method used to include the correlation in this study. In Section III, the results for the observable \(C_1/C_2, C_3/C_2, C_4/C_2\) and \(C_3/C_1\) as a function of the correlation coefficients are presented for Poisson and NBD distributions along with their cumulants. The correlation effect is also discussed using the HIJING event generator. Finally in Section IV, we summarize our findings and discuss the implications of this work to the current experimental measurements in high energy heavy-ion collisions.

II. METHOD

Suppose we have two independently produced distributions \(Y_1\) and \(Y_2\), from which one can construct the distribution of the difference \((Y_1 - Y_2)\) on an e-by-e basis. One can introduce the correlation between individual distributions by taking a third independently produced distribution \(Y_{12}\), such that \((Y_1 + Y_{12})\) and \((Y_2 + Y_{12})\) are new distributions which are correlated as they share a common distribution \(Y_{12}\). The difference of these two distributions \((Y_1 + Y_{12})\) and \((Y_2 + Y_{12})\) will be same as \((Y_1 - Y_2)\). Therefore, in spite of a correlation, it is possible that the difference distribution remains the same as if there is no correlation. Let us define two independently produced bivariate Poisson distributions \(X_1\) and \(X_2\) as the joint distribution of the random variables as is given in [16]

\[
X_1 = Y_1 + Y_{12} \quad \text{and} \quad X_2 = Y_2 + Y_{12} \quad \text{(2)}
\]
where \(Y_1\), \(Y_2\) and \(Y_{12}\) are mutually independent Poisson random variables with the means \(\lambda_1\), \(\lambda_2\) and \(\lambda_{12}\) respectively. It can be easily shown that \(X_1\) and \(X_2\) have Poisson distributions with means \(\lambda_1 + \lambda_{12}\) and \(\lambda_2 + \lambda_{12}\) respectively. In case of Poisson distributions the correlation coefficient between the bivariate distributions, \(\rho(X_1, X_2)\) is defined by:

\[
\rho(X_1, X_2) = \frac{\text{Cov}(X_1, X_2)}{\sqrt{\text{Var}(X_1)\text{Var}(X_2)}} \quad \text{(3)}
\]

Since the variance and the mean are the same for a Poisson distribution therefore,

\[
\rho(X_1, X_2) = \frac{\lambda_{12}}{\sqrt{(\lambda_1 + \lambda_{12})(\lambda_2 + \lambda_{12})}} \quad \text{(4)}
\]

This definition of a bivariate Poisson distribution automatically provides a method to generate correlated Poisson random variates such that \(X_1\) and \(X_2\) have Poisson distributions with the specified mean values and the correlation coefficient \((\rho) > 0\). The correlation coefficient is used as a measure of the degree of linear dependence between the two random variables \(X_1\) and \(X_2\). In the present work, we generate three independent Poisson distributions \(Y_1\), \(Y_2\), and \(Y_{12}\) on an e-by-e basis using a Monte-Carlo technique, with carefully chosen means \(\lambda_1\), \(\lambda_2\) and \(\lambda_{12}\), and obtain \((X_1, X_2)\) through the two operations shown in Eq. The mean values of the generated

FIG. 1: Correlated \(N_p\) and \(N_\bar{p}\) distributions for a correlation coefficient \((\rho)\) value of 0% in panel (a) and 80% in panel (b).
are uncorrelated \((\rho = 0)\), then the correlated distribution will have uniform circular distribution. The mean multiplicities used for \(N_p\) are 5.664 ± 0.0006 and 11.375 ± 0.003 for \(\sqrt{s_{NN}} = 200\) and 19.6 GeV, and the mean multiplicities for \(N_\bar{p}\) distributions are 4.116 ± 0.0005 and 1.15 ± 0.001, respectively as given in Refs. \[14, 24\]. We have repeated the same exercise assuming the individual \(N_p\) and \(N_\bar{p}\) distributions are given by NBDs. The negative binomial distribution function of an integer \(n\) can be defined as:

\[
P(n) = \frac{\Gamma(n + k)}{\Gamma(n + 1)\Gamma(k)} \left(\frac{n}{k}\right)^n \chi^{n+k} \tag{5}\]

where \(\langle n \rangle\) is the mean number of particles and \(k\) is an additional parameter. In the limiting case of \(k \to \infty\), the NBD reduces to a Poisson distribution. The sum of two NBDs is also a negative binomial distribution. Figure 2 shows the mean values of the correlated (third) distribution which has been added to the individual \(N_p\) and \(N_\bar{p}\) distributions as a function of the correlation coefficient for two different energies \(\sqrt{s_{NN}} = 19.6\) and 200 GeV. As we increase the mean value of the mixing distribution the correlation coefficient of the distributions also increase. However, there is very little energy dependence in \(\lambda_{12}\) as a function of the correlation coefficient.

III. RESULTS AND DISCUSSION

Experimentally, the measured higher moments of conserved quantities such as net-proton, net-charge and net-kaon are compared with baseline values, which are calculated by assuming the particle distributions as Poisson or negative binomial distributions. The Poisson statistics is a limiting case of NBD, where both the mean and variance of the distribution are the same. In the case of NBD the variance is larger than the mean of the distribution. In the following we will demonstrate the correlation effect on the higher moments and their ratios of net-multiplicities assuming individual positive and negative distributions as Poisson or NBD.

A. Poisson distribution

A statistically random expectation of the observable is described by Poisson distribution. The individual proton and anti-proton distributions are independently generated by using the measured mean values as given in Refs. \[14, 24\]. Both the distributions are independent if the correlation coefficient is zero. We generate a third Poisson distribution by taking the different mean values which correspond to different \(\rho\) as shown in Fig. 2. Event-by-event we add the third distribution \((N_{mix})\) to the independently generated \(N_p\) and \(N_\bar{p}\) distributions. One can construct a corresponding net-distribution \((N_{diff})\) by taking the difference of the correlated \((N_p + N_{mix})\) and \((N_\bar{p} + N_{mix})\) distributions. The \(N_{diff}\) distribution will be a Skellam distribution. In the present study, the cumulants of the net-distribution are calculated in two different ways. In first case cumulants of the net-distribution are calculated from the Skellam \(N_{diff} = (N_p - N_\bar{p})\) distribution which is built by taking the correlated \(N_p\) and \(N_\bar{p}\) distributions on an e-by-e basis. In the second case the cumulants are calculated assuming independent production of particles as given in Eq. 1. Figure 3 shows the comparison of the cumulants calculated from the e-by-e \(N_{diff}\) distributions and by assuming independent production as a function of the correlation coefficient for \(\sqrt{s_{NN}} = 19.6\) and 200 GeV. As the measured mean \(\langle C_1 \rangle\) of proton and anti-protons are used to construct the Poisson distribution of \(N_p\) and \(N_\bar{p}\), the \(C_1\) of the difference distribution calculated from the IP model and from e-by-e measured \(N_{diff}\) distribution agrees. The \(C_2\) and \(C_4\) values obtained from the e-by-e \(N_{diff}\) distributions are independent of \(\rho\) as \(N_{diff}\) is generated by taking the difference of \((N_p + N_{mix})\) and \((N_\bar{p} + N_{mix})\). However, there is strong dependence of \(C_2\) and \(C_4\) values as a function of \(\rho\) when obtained using the IP model. As one increase the correlation coefficient, the \(C_2\) and \(C_4\) values deviate from the uncorrelated values. It is observed that, if two distributions are correlated more than \(\sim 20\%\), the \(C_2\) and \(C_4\) do not follow the “integer valued Leaky processes”. The \(C_3\) values are independent of the correlation coefficient similar to the \(C_1\) of the net-proton distribution. Similar behavior is observed for both \(\sqrt{s_{NN}} = 19.6\) and 200 GeV. The statistical uncertainties for the various cumulants and their ratios are calculated using Delta theorem method \[25\]. As noted in previous section, the uncertainties on the mean values of the protons and anti-protons are very small which gives negligible effect (less than 0.3%) on the higher cumulants and their ratios. More discussion on the statistical uncertainties on higher cumulants also can be found in \[18\]. Figure 4 shows the ra-
tios of cumulants as a function the correlation coefficient for \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV. The cumulant ratios obtained from the e-by-e measured \( N_{diff} \) distributions are independent of \( \rho \) as \( N_{mix} \) distribution gets canceled-out while constructing the net-distribution. In case of independent production model, the correlation added to the individual \( N_{p} \) and \( N_{\bar{p}} \) distributions preserve. The \( C_1/C_2 \) and \( C_3/C_2 \) ratios show strong dependence of \( \rho \) calculated using Eq. 1. However, the \( C_4/C_2 \) and \( C_3/C_1 \) ratios are independent of \( \rho \) in both the cases. Like individual cumulant case, the \( C_1/C_2 \) and \( C_3/C_2 \) ratios for very small \( \rho \) starts deviating from the uncorrelated baseline ratios. Although there is larger correlation between the particles which are produced close to the critical end point (CEP) or phase transition, it will be difficult to observe in \( C_4/C_2 \) and \( C_3/C_1 \) ratios as these two ratios are independent for any degree of correlation. This indicates that, in heavy-ion collisions, even if the particles have small correlation, it can be seen in \( C_1/C_2 \) and \( C_3/C_2 \) ratios.

**B. Negative Binomial Distributions**

The particle multiplicity distribution in elementary \((e^+ + e^- \text{ or } p + p)\) as well as heavy-ion collisions are well described by negative binomial distribution.\(^{24,28}\) In the present study, the individual \( N_p \) and \( N_{\bar{p}} \) distributions are assumed to be negative binomial distributions, which are constructed by taking the measured \( C_1 \) and \( C_2 \) of the proton and anti-proton distributions at \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV as given in Ref.\(^{14,24}\). It is assumed that the individual \( N_p \) and \( N_{\bar{p}} \) distributions are produced independently. As it is known that sum of two NBDs are also negative binomial distribution. A third NBD distribution \( (N_{mix}) \) has been added to the individual \( N_p \) and \( N_{\bar{p}} \) distributions on an e-by-e basis so that the resulting proton and anti-proton distributions will be correlated. The mean values of the correlated distribution correspond to different correlation coefficient as shown in Fig. 2. Similar to Poisson distribution, the cumulants are calculated in two different ways as discussed before. Figure 4 shows the cumulants of the net-proton distributions calculated in both the methods as a function of the correlation coefficient for \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV. Similar to the case of Poisson distribution, the \( C_1 \) and \( C_3 \) of net-proton distributions are obtained from both the methods are independent of \( \rho \). Deviations from the uncorrelated baseline are observed for \( C_2 \) and \( C_4 \) of the net-proton distributions calculated in the IP model. As we increase the correlation coefficient the deviation of \( C_2 \) and \( C_4 \) increase from the baseline values. These two cumulants behave as uncorrelated until the correlation coefficient is less than \( \sim 20\% \). Figure 4 shows the ratios of the cumulants as a function of the correlation coefficient for \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV. The cumulant ratios calculated from the e-by-e \( N_{diff} \) distributions are independent of \( \rho \). Where as the \( C_1/C_2 \) and \( C_3/C_2 \) ratios

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**FIG. 3:** Cumulants of net-proton distributions obtained on an e-by-e basis and by assuming independent production of particles are shown as a function of the correlation coefficient \( \rho \) for \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV. The errors on the cumulants are statistical only and smaller than the symbol.

**FIG. 4:** Ratios of cumulants of net-proton distributions obtained on an e-by-e basis and by assuming the independent production of particles are shown as a function of the correlation coefficient \( \rho \) for \( \sqrt{s_{NN}} = 19.6 \) and 200 GeV. The errors on the cumulants are statistical only and smaller than the symbol.
of the net-proton distributions calculated from the individual \(N_p\) and \(N_{\bar{p}}\) distributions deviate from the uncorrelated baseline values. As one increases the correlation coefficient, the \(C_1/C_2\) and \(C_3/C_2\) ratios deviate from the uncorrelated values. The \(C_4/C_2\) and \(C_3/C_1\) ratios are found to be independent of the \(\rho\) for both the cases.

The \(C_4/C_2\) and \(C_3/C_1\) ratios from the Poisson distribution and NBD are independent of \(\rho\) which implies, although particles are strongly correlated in heavy-ion collisions, still the cumulant ratios of net-proton distribution can be explained by independent particle production model. On the other hand, \(C_1/C_2\) and \(C_3/C_2\) ratios calculated using the IP model are strongly dependent on the correlation coefficient. If the particles produced in heavy-ion collisions close to the CEP are highly correlated, that can be observed in the \(C_1/C_2\) and \(C_3/C_2\) ratios. However, in the present study we have simulated the correlation as an independent Poisson or NBD distribution. This correlation may not be the same as that from the QCD based arguments, about the sensitivities of the higher moments which are based upon the expected critical behavior of the correlation length \(k_0\). From the above study, we show that \(C_1/C_2\) and \(C_3/C_2\) ratios are more sensitive to the correlation coefficient.

C. Understanding the correlation effect with the HIJING model

It is observed in Ref. [14] that the experimental data on net-proton cumulant ratios have been well explained by the independent production model. It can be argued that at lower collision energies the cumulants of net-proton distributions are mostly dominated by the cumulants of the corresponding proton distribution as the number of anti-proton production is very small. The \(\bar{p}/p\) ratios are \(\sim 0.01\) and \(\sim 0.06\) at \(\sqrt{s_{NN}} = 9.2\) and 17.3 GeV respectively [31, 32]. However this argument will not hold for higher \(\sqrt{s_{NN}}\) as it is known from the measured \(\bar{p}/p\) ratio \(\sim 0.77\) at mid-rapidity in Au + Au collisions at \(\sqrt{s_{NN}} = 200\) GeV [32] that there is significant correlation between proton and anti-proton production. Here we have discussed this aspect using the Heavy Ion Jet interaction Generator (HIJING) model [33]. It is a perturbative QCD model, which produces minijet partons that later are transformed into string fragments that then fragment into hadrons. Figure 7 shows the cumulants of net-proton distributions calculated using both e-by-e measured \(N_{adj}\) and individual cumulants of protons and anti-protons using Eq. 1 for the most central (0%–5%) Au + Au collisions at different \(\sqrt{s_{NN}}\). The results shown for net-proton distributions are for the same acceptance as the experimental data [14]. The cumulants from both the methods are in agreement, although there

\[
\begin{align*}
\frac{C_1}{C_2} & \sim 0.01, \\
\frac{C_3}{C_2} & \sim 0.06, \\
\frac{C_4}{C_2} & \sim 0.01.
\end{align*}
\]
FIG. 7: Cumulants of net-proton multiplicity distributions obtained on an e-by-e basis and by assuming independent production of particles as a function of $\sqrt{s_{NN}}$ for (0%–5%) centralities in Au + Au collisions from the HIJING. The errors on the cumulants are statistical only and smaller than the symbol.

FIG. 8: Ratios of cumulants of net-proton multiplicity distributions obtained on an e-by-e basis and by assuming independent production of particles as a function of $\sqrt{s_{NN}}$ for (0%–5%) centralities in Au + Au collisions from the HIJING.

FIG. 9: Energy dependence of the correlation coefficient ($\rho$) calculated from $N_p$ and $N_{\bar{p}}$ distributions for (0%–5%) centrality in Au + Au collisions from the HIJING.

is small difference for $C_2$ and $C_4$ values as observed in Poisson and NBD cases. At higher collision energies, the $C_2$ and $C_4$ values calculated using the IP model are slightly higher than the cumulants obtained from the e-by-e $N_{diff}$ distribution. This indicates, the presence of more correlations between the protons and anti-protons at $\sqrt{s_{NN}} = 200$ GeV. Figure 8 shows the cumulant ratios as a function of $\sqrt{s_{NN}}$ obtained from the above cumulants in Au + Au collisions for (0%–5%) centrality. The cumulant ratios calculated in both the methods agree very well. As we have seen in the cumulant ratios for Poisson and NBD cases, if the correlation exists between the proton and anti-proton then it should show in $C_1/C_2$ and $C_3/C_2$ ratios. We have observed in Fig. 4 and Fig. 6 that the cumulant ratios calculated in both the methods agree even if the correlation coefficient $\sim 15\%$. This implies, the correlation between protons and anti-protons in the HIJING events are within that order. Figure 9 shows the energy dependence of the degree of correlation that exists in proton and anti-proton production calculated from the HIJING model. As discussed previously in this section, even if the correlation coefficient is $\sim 20\%$, the cumulants calculated in both the methods will agree, which has been observed in the case of the HIJING simulation. However, in all the studied cases (Poisson, NBD and HIJING), the $C_4/C_2$ and $C_3/C_1$ ratios calculated using both the methods agree. Hence, it is not surprising that the experimentally measured $C_3/C_2$ and $C_4/C_2$ ratios in Ref. [14] agree with the values calculated using the IP model. It may so happen that, after applying different kinematical cuts on the measurements within the experimental acceptance, the correlation coefficient values are reduced to less than $\sim 20\%$. Experimental data that can be explained by the IP model does not rule-out the existence of CEP. We have demonstrated that even if
particles are highly correlated the $C_4/C_2$ and $C_3/C_1$ ratios can be explained by the IP model. It is important to know how much correlation the protons and anti-protons should have so that one can claim to find the CEP. On the other hand, if experimentally measured particles have a correlation coefficient less than $\sim 20\%$, the independent production model can explain the experimental cumulant ratios such as $C_1/C_2$ and $C_3/C_2$.

**IV. SUMMARY**

In conclusion we have studied the effect of the correlations on the cumulants and their ratios assuming the experimentally measured proton and anti-proton distributions are described as a Poisson or NBD. The correlation is introduced in both the $N_p$ and $N_\bar{p}$ distributions. The cumulants and their ratios of net-proton distributions are calculated using e-by-e measured $N_{diff}$ distribution and from the independent production of $N_p$ and $N_\bar{p}$ distributions. We have demonstrated using Poisson and NBD distributions that, \textit{“integer valued \textit{Levy processes}”} i.e $C_n = C_n^+ + (-1)^n C_n^-$ for the net distribution is valid only if the correlation coefficient is less than $\sim 30\%-35\%$. The $C_4/C_2$ and $C_3/C_1$ ratios are independent of the correlation coefficient, where as $C_1/C_2$ and $C_3/C_2$ ratios are more sensitive to the correlation coefficient. The $C_4/C_2$ and $C_3/C_1$ ratios can be explained by the IP model, which was also observed in Ref. [14]. The agreement between experimental data and the independent production model is not a coincidence in which measurements have been carried out. We have discussed that, if the particles are highly correlated, one should look for $C_1/C_2$ and $C_3/C_2$ ratios as a function of collision energies, which will have larger deviations from the ratios obtained by uncorrelated baseline values. The cumulants and their ratios are calculated as a function of $\sqrt{N_{pp}}$ using the HIJING event generator. In the HIJING model the cumulants calculated using both the methods agree very well. Hence, experimentally measured cumulants will follow the independent production model calculations if the correlation coefficient is less than $\sim 20\%$. However, $C_4/C_2$ and $C_3/C_1$ values will follow the IP model for all the correlation coefficient values. The observation that the experimental data can be explained by the independent production of particles does not rule out the existence of the critical endpoint.

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