An Example of Controlling the Obtained Characteristics of Technological Stress Distributions in a Layer-By-Layer Manufactured Product, Based on a Model of Additive Process Mechanics

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Abstract. Technological processes of layer-by-layer manufacturing of cylindrical products by the method of power winding of a thin tape on a sufficiently rigid circular base are considered. A characteristic factor for the processes under consideration is the creation of initial stresses in the elementary layers of additional material, which are sequentially included in the composition of the being formed product. It is assumed that the applied winding technology allows us to arbitrarily influence these stresses. This makes it possible to control the technological stresses developing in the resulting product. This paper is devoted to the formulation and solution of one of the problems of such control using current approaches of additive process mechanics. The simulation is performed in a quasi-static approximation within the limits of small elastic strains. A non-classical mechanical model of the process under study is proposed, and a mathematical formulation of the corresponding initial boundary value problem and its closed analytical solution are given. Based on the constructed solution, the regularities of the influence of the initial stress state of the attached material on the nature of the development of the contact pressure on the used base from the side of the material layer formed on it are studied. The problem of providing the required law of change in the value of this pressure in the process under consideration by setting the appropriate program for varying the initial stresses is formulated and solved. Corollaries of an applied nature are made.

1. Introduction

In a number of technological processes for manufacturing shell elements of structures, details of machines and mechanisms, their formation is carried out by layer-by-layer joining of the material to the surface of a pre-made base — a mold, mandrel, or substrate. Usually, due to the physical and chemical manifestations taking place in this process and also mechanical influences on the being formed product, stress fields and, consequently, deformations arise and begin to develop in the latter. The appearance and evolution of stresses in a solid during its formation depend not least on the initial stress-strain state of individual material layers being included in the composition of the solid. This means that by varying this initial state in some way, we can influence the current and resulting stresses.
in the manufactured product, that is, control these stresses. This paper is devoted to the formulation and solution of one of the problems on such control. It is considered an example of the technological process of manufacturing a product in the form of a cylindrical layer by slow power winding of a thin elastic tape on a circular base having a stiffness significantly higher than the stiffness of the material obtained on it during the winding process. When modeling this process we suppose the base absolutely rigid. The formed layer is considered to be homogeneous isotropic and so extended in the axial direction that its deformation can be considered plane. Study is carried out for quasi-static processes of deformation of the formed layer under the action of initial stresses in the attached material caused by the tension of the wound tape.

It is quite obvious that in the products obtained by winding, significant compressive circumferential stresses must arise and develop, which, as a result, cause radial compression of the base used. This compression can play a very positive role, restraining the working pressure that will be created inside the (hollow) cylindrical base during further operation of the finished product, and thereby increasing the functional characteristics of this product. We can talk here, for example, about products like pressure vessels or barrels of firearms. However, the same radial compression can also be a very negative factor, provoking, for example, a loss of stability of the base used already in the process of winding the material on it. Among other things, correct forecasting and evaluation of the possibilities for controlling the parameters of the mechanical interaction of the base used and the additional layer of material formed on it are of fundamental importance for the correct formulation and solution of different actual wear-contact problems [1, 2]. The present work aims to investigate the regularities of the influence of wound tape initial tension on the character of development of the radial compression of the base from the material layer being gradually formed on it, as well as to build a program of change in this pretension during winding to ensure the required law of change of compression in the process.

2. Specifics of Mechanical Stating the Problem Under Study

For the correct solution of the above-described problem, it is not enough to remain within the framework of classical solid mechanics which considers deformation of solids of a constant material composition. In our case, we are dealing with a gradually formed solid deforming already in the process of its formation. Such solids are commonly called accreted, or growing [3]. As is known [4], the main distinguishing feature of the deformation process of any accreted body is the absence of a completely stress-free configuration, in its traditional meaning for continuum mechanics. This feature makes the standard representation in terms of displacements of the strain state of an accreted body impossible and requires the development of other approaches [5, 6]. The latter is the subject of the works of the Russian scientific school in the field of mechanics of accreted solids, founded by Professor A.V. Manzhirov (see, for example, [7-11]), to which the author of this paper belongs.

In the simulated process, the particles of additional material continue to move as part of a continuous continuum after joining the solid being accreted. Consequently, at each moment of time, a sufficiently smooth velocity field of the solid deformation motion is determined in the entire domain instantly occupied by this continuum. Therefore, we can formulate the problem on deforming an accreted solid in terms of the rates of change in the characteristics of its stress-strain state, considered in a time-variable domain in physical space. Since the thickness of the elementary layers of material being attached in the considered winding process is considered to be quite small, it is permissible to model the winding as a process of continuous axisymmetric accretion of a cylindrical solid along its external side surface. The assumption that the resulting strains are small allows us to assume that the law of change over time of the external radius \( r \) of this solid is set. It is obvious that in the absence of rheological manifestations in the mechanical behavior of the material used, “time” can be understood as any parameter that monotonically changes over physical time, for example, the value of the external radius \( r \) itself.

The additional material attached to the solid being accreted has a certain initial stress state. This state, in our case, is caused by pre-tension of the wound tape, that is, the attached elementary layers of
the material are subjected to some stretching in the circumferential direction at the time of attachment. We assume that the value of this stretch does not have time to change within one turn of the tape, and therefore in our model of continuous growth there is a uniform stretching of each newly attached elementary material layer of radius $x$ by some circumferential stress $\sigma_{\phi}^0(x)$. Based on the real conditions of the creation of this stress, in our model it is possible to take

$$\sigma_{\phi}^0(x) = \frac{P(x)}{h(x)}$$

where $h(x)$ and $P(x)$ are the thickness (size in the radial direction) of the wound tape averaged over one turn and its tension force per unit of its width (size in the axial direction) at the moment of the winding process when the external radius of the entire layer having been already formed on the base is equal to $x$ (we will briefly call this moment “the moment $x$”). In the axial direction, the tape does not undergo stretching, so the axial stress at the external surface of the solid under consideration is zero. If the wound tape is not being pressed extra in radial direction against the cylindrical solid being formed during the winding process, then the radial stress on the external surface of this solid will also be zero. However, the latter does not in any way mean that radial stresses will not develop at the same points already inside the solid during its further accretion (see Introduction).

It should be noted that in the technological processes of winding, as a rule, a rotation of the formed layer together with the base takes place. In this case, the being formed layer will be exposed to inertial mass forces, and this effect, as shown in [12] on the example of centrifugal inertia forces, can significantly affect the process of deformation of this layer or even prevail over the mechanical effects on it from the superimposed additional material caused by the initial stresses created in this material. Since we focus in this paper on the latter effects, we will consider the rotation of the being formed layer together with the base to be slow and smooth enough to be able to ignore the forces of inertia of the rotational motion, both centrifugal and tangential.

3. Mathematical Model of the Investigated Mechanical Process

3.1. Characteristics of the Being Formed Layer Stress-Strain State

We will denote by $\rho$ the polar radius, that is, the distance from the considered point of the being formed cylindrical solid to its axis, by $\varphi$ — the polar angle measured in the cross-section of this solid, by $z$ — the axial (longitudinal) coordinate, and by $\mathbf{e}_{\rho}(\varphi)$, $\mathbf{e}_{\varphi}(\varphi)$, $\mathbf{e}_{z}$ — the oris that set the radial, circumferential and axial directions at the considered point, respectively.

In the constructed model, due to plane strain, axial symmetry, and the absence of factors that cause twisting of the being formed cylindrical layer, the velocity field of its deformation movement at the moment $x$ has the form:

$$V(\rho, \varphi, x) = V(\rho, x)\mathbf{e}_{\rho}(\varphi)$$

This field of velocities corresponds to the strain rate tensor

$$D(\rho, \varphi, x) = V'(\rho, x)\mathbf{e}_{\rho}(\varphi)\mathbf{e}_{\varphi}(\varphi) + \frac{V(\rho, x)}{\rho} \mathbf{e}_{\varphi}(\varphi)\mathbf{e}_{\varphi}(\varphi);$$

(1)
here and everywhere else we denote \( (\cdot)^{\prime} = \frac{\partial (\cdot)}{\partial \rho} \). The constitutive equation for the considered additively formed homogeneous isotropic elastic solid must be formulated in terms of tensor (1) and the stress rate tensor \( \mathbf{T} \):

\[
\mathbf{T} / G = 2\mathbf{D} + (\kappa - 1)\mathbf{I} \text{tr} \mathbf{D}
\]

(2)

where \( \mathbf{T} \) is the stress tensor, \( \mathbf{I} \) is the unit tensor of the second rank, \( \kappa = (1 - 2\nu)^{-1} \), \( \nu \) is the Poisson’s ratio, and \( G \) is the rigidity modulus; here and everywhere below we denote \( (\cdot) = \frac{\partial (\cdot)}{\partial x} \).

From (2) and (1) we find that

\[
\mathbf{T}(\rho, \varphi, x) = \hat{\sigma}_\rho(\rho, x) \mathbf{e}_\rho(\varphi) + \hat{\sigma}_\varphi(\rho, x) \mathbf{e}_\varphi(\varphi) + \hat{\sigma}_z(\rho, x) \mathbf{e}_z(\varphi)
\]

(3)

Where

\[
\hat{\sigma}_\rho / G = (\kappa + 1)V' + (\kappa - 1)V / \rho, \quad \hat{\sigma}_\varphi / G = (\kappa + 1)V / \rho + (\kappa - 1)V', \quad \hat{\sigma}_z = \nu(\hat{\sigma}_\rho + \hat{\sigma}_\varphi).
\]

(4)

It follows from (3) that

\[
\mathbf{T}(\rho, \varphi, x) = \sigma_\rho(\rho, x) \mathbf{e}_\rho(\varphi) + \sigma_\varphi(\rho, x) \mathbf{e}_\varphi(\varphi) + \sigma_z(\rho, x) \mathbf{e}_z(\varphi)
\]

Where

\[
\sigma_{\rho,\varphi,z}(\rho, x) = \sigma_{\rho,\varphi,z}(\rho, \rho) + \int_{\rho}^{x} \hat{\sigma}_{\rho,\varphi,z}(\rho, \zeta) d\zeta.
\]

(5)

3.2. Differential Equation in the Variable Domain Occupied by the Considered Accreted Solid, and Its General Solution

If \( x_0 \) is the radius of the base surface on which the material is wound, then at each moment \( x \) in the region of space \( x_0 < \rho < x \) occupied by the entire part of the having been currently formed cylindrical solid under consideration, the velocity analogue of the standard equilibrium equation will be valid:

\[
\nabla \cdot \mathbf{T} = 0.
\]

By adding representations (3), (4) to this equation, we can obtain the following equation for the equilibrium of the solid under consideration in terms of the velocities of its particles:

\[
V'' + V' / \rho - V / \rho^2 = 0.
\]

The general solution of this ordinary differential equation for a variable \( \rho \) with a parameter \( x \) relative to the desired function \( V(\rho, x) \) has the form:

\[
V(\rho, x) = C_1(x)\rho + C_2(x) / \rho
\]

(6)
where the unknown functions \( C_1(x) \) and \( C_2(x) \) are to be determined from the boundary conditions that must be set on the inner \( \rho = x_0 \) and outer \( \rho = x \) surfaces of the accreted layer in accordance with the above provisions.

3.3. Satisfaction of Boundary Conditions and Construction of Stress Evolution in the Layer Being Formed

On the inner surface of the layer under consideration, it is necessary to impose a condition of its complete adhesion to a rigid base:

\[ V(x_0, x) = 0. \]

Satisfying this condition, we find the connection \( C_2(x) = -x_0^2 C_1(x) \).

On the outer layer surface to which the additional material is attached, the values of all components of the stress tensor \( T \) are to be set in accordance with what is said in the Section “Specifics of Mechanical Stating...”:

\[ \sigma_\rho(x, x) = 0, \quad \sigma_\phi(x, x) = \sigma_\phi^0(x), \quad \sigma_z(x, x) = 0. \] (7)

However, the function \( V(\rho, x) \) can only restore the rate of change of stresses at each point of the solid, but not the stresses themselves. Therefore, from conditions (7) it is necessary to obtain a boundary condition of the velocity type. This can be done by calculating the full time-derivative of relation (7) for the component \( \sigma_\rho \) using the standard equilibrium equation. Omitting the corresponding conclusion, we give here only the final condition:

\[ \dot{\sigma}_\rho(x, x) = -\sigma_\phi^0(x)/x. \] (8)

Relation (8), as we can see, determines the starting rate of change in the radial stress at the newly joined points of the accreted solid under consideration, in other words, the rate at which the initially zero radial stress will begin to change at these points as soon as they become internal points of this solid. Satisfying condition (8) with the use of the first relation (4), we find

\[ C(x) = \frac{-\sigma_\phi^0(x)}{2Gx} \left( \kappa + \frac{x_0^2}{x^2} \right). \]

As a result, we have from (4) and (6) the following expressions for the rates of stress change:

\[ \dot{\sigma}_\phi(\rho, x) = \left( \kappa \pm \frac{x_0^2}{\rho^2} \right) \sigma_\phi^0(x)/x. \]

Hence, based on (5) and (7), we find

\[ \sigma_\rho(\rho, x) = -\left( \kappa + \frac{x_0^2}{\rho^2} \right) \int_0\sigma_\phi^0(\xi) d\xi, \quad \sigma_\phi(\rho, x) = \sigma_\phi^0(\rho) - \left( \kappa - \frac{x_0^2}{\rho^2} \right) \int_0\sigma_\phi^0(\xi) d\xi. \] (9)
4. Solving the Problem of Contact Pressure Control During the Winding Process

Using the mathematical model developed in this paper, we can predict the evolution of the contact pressure \( p(x) \) of the wound layer on the base surface during the winding process. From formula (9) for \( \sigma_p \) when \( \rho = x_0 \) we have:

\[
p(x) = -\sigma_p(x_0, x) = (\kappa + 1) \int_{x_0}^{x} \frac{\sigma_p^0(\xi) \xi d\xi}{\kappa \xi^2 + x_0^2}.
\]  

(10)

If it is necessary to provide the necessary law for changing the contact pressure on the base during the winding process by setting the appropriate program for changing the initial stresses \( \sigma_p^0(x) \), then this program can be found from (10) by differentiating by the time parameter \( x \):

\[
\sigma_p^0(x) = \frac{\kappa x^2 + x_0^2}{(\kappa + 1)x} \dot{p}(x).
\]

(11)

The dependence (11) shows, in particular, that when accreting with non-zero initial stresses, the contact pressure on the base cannot remain constant. And since, as can be seen from the definition of the material constant \( \kappa \) (see above), we have \( \kappa > 0 \) at any \( \nu < 1/2 \), it is impossible to achieve a decreasing contact pressure of this layer on the base over time when winding a layer with a positive tension force applied to the wound tape.

5. Summary

A model of the process of winding a material on the outer side surface of a circular cylindrical base is proposed. The simulation is carried out within the framework of additive process mechanics. The corresponding initial-boundary value problem is formulated. Its closed analytical solution is constructed. The initial state of the additional material attached to the being formed product is determined by the programs for changing the thickness and the force of tension of the wound elementary layers during the winding process. This state determines the stress-strain state of the entire manufactured solid. Based on the analytical dependences obtained in the work, the problem of controlling the technological pressure on the base from the side of the material layer produced on it by winding due to the proper variation of the initial stresses in the attached material is solved. Practical corollaries are made.

6. References

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