Turbulence production and turbulent pressure support in the intergalactic medium

L. Iapichino,1⋆ W. Schmidt,2 J. C. Niemeyer2 and J. Merklein3

1Zentrum für Astronomie der Universität Heidelberg, Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, D-69120 Heidelberg, Germany
2Institut für Astrophysik, Universität Göttingen, Friedrich-Hund-Platz 1, D-37077 Göttingen, Germany
3Abteilung Bioklimatologie, Universität Göttingen, Bäusgenweg 2, D-37077 Göttingen, Germany

Accepted 2011 February 14. Received 2011 February 14; in original form 2010 November 30

ABSTRACT
The injection and evolution of turbulence in the intergalactic medium is studied by means of mesh-based hydrodynamical simulations, including a subgrid-scale (SGS) model for small-scale unresolved turbulence. The simulations show that the production of turbulence has a different redshift dependence in the intracluster medium (ICM) and the warm-hot intergalactic medium (WHIM). We show that the turbulence in the ICM is produced chiefly by merger-induced shear flows, whereas the production in the WHIM is dominated by shock interactions. Secondly, the effect of dynamical pressure support on the gravitational contraction has been studied. This turbulent support is stronger in the WHIM gas at baryon overdensities $1 \lesssim \delta \lesssim 100$ and less relevant for the ICM. Although the relative mass fraction of the gas with large vorticity is considerable (52 per cent in the ICM), we find that for only about 10 per cent in mass this is dynamically relevant, namely not associated with an equally large thermal pressure support. According to this result, a significant non-thermal pressure support counteracting the gravitational contraction is a localized characteristic in the cosmic flow, rather than a widespread feature.

Key words: hydrodynamics – shock waves – turbulence – methods: numerical – large-scale structure of Universe.

1 INTRODUCTION
Often considered just a by-product of the virialization mechanism, the injection of the turbulence energy in the intergalactic medium (IGM) has been recognized as an interesting process in its own right. Put in a broader context, the information about non-thermal phenomena inside and outside galaxy clusters can complement the information derived from the hot, X-ray-emitting gas. Turbulence, in particular, is of utmost importance because of the apparent link to the acceleration mechanisms of cosmic rays (CRs) and to the cluster diffuse radio emission (Ferrari et al. 2008; Brunetti 2009; Brunetti et al. 2009; Cassano 2009; Cassano et al. 2010). Turbulent velocity fluctuations are also considered to be important for amplifying magnetic fields in the intracluster medium (ICM) (Subramanian, Shukurov & Haugen 2006) and in the IGM (Ryu et al. 2008).

To date, constraints on the turbulent velocity in clusters have been determined by measuring the resonant scattering suppression (Churazov et al. 2004; Werner et al. 2009) and by $\textit{XMM–Newton}$ spectroscopic observations of clusters with a compact core (Sanders et al. 2010; Sanders, Fabian & Smith 2011). Recently, the $\textit{Suzaku}$ satellite made deep X-ray observations out to the virial radius of several clusters possible (Bautz et al. 2009; George et al. 2009; Reiprich et al. 2009; Hoshino et al. 2010; Kawaharada et al. 2010), thanks to its low and stable particle background. In most of the published data, the observed clusters show departures from the hydrostatic equilibrium in their outskirts; in one case (Kawaharada et al. 2010), the non-thermal support is as large as about 50 per cent of the total pressure.1

As for the gas outside clusters, the strongest recent arguments in favour of a significant effect of small-scale turbulence have been derived from low-$z$ $\textit{OVI}$ observations compared to numerical simulations (Oppenheimer & Davé 2009). In this work, turbulence had to be added by hand as a post-process correction since a subgrid-scale (SGS) turbulence model was unavailable.

Hereas the theoretical and numerical exploration of the turbulence in the hot ICM of galaxy clusters has received a fair amount

1 By ‘non-thermal pressure’ one indicates generically pressure contributions from the turbulence, magnetic fields and non-thermal particles (CRs); in the following, the latter two contributions will not be addressed. Moreover, non-equipartition effects of the low-density plasma in the cluster outskirts could mimic a non-thermal contribution (see Wong & Sarazin 2009; Rudd & Nagai 2009, and references therein).
of attention (e.g. Dolag et al. 2005; Subramanian et al. 2006; Vazza et al. 2006; Iapichino & Niemeyer 2008; Scannapieco & Brüggen 2008; Brüggen & Scannapieco 2009; Maier et al. 2009; Vazza et al. 2009b), the turbulent state of the cluster outskirts and of the warm-hot intergalactic medium (WHIM), which is believed to contain a significant fraction of the baryons in the low-z universe (Cen & Ostriker 1999; Oppenheimer & Davé 2008), has not yet been addressed in a similarly systematic way. The properties of the turbulence in the cluster outskirts have been studied recently in simulations by Burns, Skillman & O’Shea (2010) who found that those regions are not in hydrostatic equilibrium and have a substantial turbulent pressure support. Based on a sample of 16 cluster simulations, Lau, Kravtsov & Nagai (2009) conclude that the support of turbulent motions increases towards the cluster periphery. Shaw et al. (2010) find that the previous results lead to a significant reduction in the Sunyaev–Zel’dovich power spectrum at angular scales of a few arcminutes. Zhu, Feng & Fang (2010) present an instructive analysis of the resolved vorticity field and the influence of the turbulent pressure in the cosmological simulation. Using the dynamical equation for the rate of change of the divergence (cf. Schmidt 2009b), they estimate the effect of the turbulent pressure on the gravitational contraction of the baryonic gas. In this way, the role of the gas turbulence in the clustering process is studied (see also Bonazzola et al. 1992).

There are several mechanisms that are potentially able to stir the baryons and inject turbulence in the fluid. In the framework of our simulations, we neglect the effects of galaxy motions in the ICM (Bregman & David 1989; Kim 2007; Parrish, Quataert & Sharma 2010; Ruszkowski & Oh 2011) and outflows from the active galactic nucleus (AGN) activity (Heinz et al. 2006; Sijacki & Springel 2006; Brüggen et al. 2009). Cluster mergers and curved shocks thus remain as the main stirring agents to be considered.

According to the hierarchical scenario for clustering, a halo accretes most of its mass by mergers. In particular, one can distinguish between an earlier phase of major mergers, where the ratio of the masses of the merging subclumps is close to unity, and a subsequent minor merger phase, when smaller subhaloes fall into the cluster gravitational well.

Both merger phases perturb the cluster medium and are thus related to the injection of the turbulence in the ICM. According to Subramanian et al. (2006), the turbulence produced in the major-merger phase has a large volume-filling factor, as expected from events which deeply stir and rearrange the cluster structure (Roettiger, Burns & Loken 1993; Roettiger, Loken & Burns 1997; Ricker & Sarazin 2001; Mitchell et al. 2009; Paul et al. 2011). In the case of minor mergers, as the study of idealized setups has shown, the shear at the boundary between the ICM and the accreting subcluster triggers the Kelvin–Helmholtz instability, locally injecting turbulence in the wake of the moving subclumps (Heinz et al. 2003; Iapichino et al. 2008; Maier et al. 2009).

Another stirring mechanism is linked to the baroclinic vorticity generations at curved shocks. As known from theory, vorticity \( \omega \) is produced where the pressure and density gradients are not parallel. Taking the curl of the Euler equation (Landau & Lifshitz 1959; Kang et al. 2007):

\[
\frac{\partial \omega}{\partial t} = \nabla \times (v \times \omega) - \frac{\nabla p \times \nabla \rho}{\rho^2},
\]

where the second term on the right-hand side is non-vanishing at the locations of non-planar shocks. In filaments and cluster outskirts, the unprocessed gas is accreted from the voids and accelerated towards the growing structures, where accretion shocks are formed. The injection of turbulence is therefore a by-product of the gas accretion at curved shocks.

The main difference in the two distinct mechanisms described above is in the driving: in merger events, turbulence is generated by shearing instabilities, whereas at shocks, the generation is driven by compressional modes. This difference is expected to affect the flow features in a quantifiable way that will be explored through our numerical simulations. Even for a fixed temperature, numerical studies of forced supersonic turbulence indicate significant differences in the distributions of density and velocity fluctuations, depending on the forcing (Federrath, Klessen & Schmidt 2008; Schmidt, Federrath & Klessen 2008; Schmidt et al. 2009; Federrath et al. 2010).

It is clear from the above that turbulence is generated on essentially all cosmologically-relevant scales. Moreover, the turbulent WHIM and ICM include a temperature range from \( 10^3 \) to \( 10^8 \) K and corresponding Mach numbers ranging from \( 10^{-2} \) to transonic values. Adding to the inherent difficulties in simulating turbulence even under more simplified (i.e. homogeneous and isotothermal) conditions, these complications make the exploration of large-scale structure turbulence challenging, to say the least. A convincing statistical analysis of the turbulence properties of the IGM on numerically resolved scales is infeasible with present codes and resources.

On the other hand, much can be learnt from looking at the magnitude of the production terms and keeping track of the amount of turbulent kinetic energy on unresolved scales by means of a SGS model. The central simplifying assumption here is that turbulence can be considered to be statistically isotropic on sufficiently small scales. Clearly, this is only a first approximation which cannot replace the information gained by increased resolution. However, extensive experience with large-eddy simulations (LES), as this technique is commonly referred to, shows that many properties of unresolved turbulence (most importantly, those related to transport and dissipation) can be captured with a certain degree of confidence (for further references, see Schmidt, Niemeyer & Hillebrandt 2006; Schmidt 2009a). Once a reliable model for small-scale turbulence in cosmological simulations is available, many of the questions listed above can already be addressed to some extent. This is the approach we will take in this work.

In a previous paper (Maier et al. 2009), we introduced a version of LES suitable for adaptive mesh refinement (AMR) called FEARLESS (Fluid mEchanics with Adaptively Refined Large Eddy Simulation). Its main properties will be summarized in Section 2. Here, we apply this model to a large-scale structure simulation with adiabatic gas dynamics for the first time. Considering only the SGS turbulence energy as a probe for the production of turbulence for the reasons explained above, we find that its evolution indeed differs significantly for the WHIM and ICM phases of the IGM. Although this result is limited by the approximate nature of the turbulence SGS model, we offer an interpretation in terms of different dominant production mechanisms in these phases and present supporting evidence in Section 3.

In Section 4, we will further elaborate on this approach. The turbulence SGS model allows us to predict the contribution of the turbulent pressure on the grid scale in addition to the effects caused by numerically resolved turbulence. Comparing the WHIM and the ICM, we find that the support of the gas against the gravitational contraction by turbulence is more pronounced at low densities in the WHIM than at higher densities in the WHIM and ICM, in which the support is mainly thermal. A further important finding which will be discussed is that the turbulence-supported gas has a fairly low mass and volume fraction.
We conclude with a summary of our results and suggestions for future directions in Section 5.

2 NUMERICAL TOOLS

This work is based on hydrodynamical simulations of the evolution of the cosmic large-scale structure, performed using the FEARLESS numerical technique (Maier et al. 2009) for simulating intermittent turbulent flows in clumped media. This tool has been implemented on the public release of the grid-based, AMR hybrid (N-body plus hydrodynamical) code ENZO (v1.0) (O’Shea et al. 2005).

2.1 Setup of the simulations

A flat Λ cold dark matter cosmology is assumed, with ΩΛ = 0.721, Ωm = 0.279, Ωb = 0.046, h = 0.7, σ8 = 0.817 and n = 0.96.

The computational box has a side of 100 Mpc h⁻¹ and is resolved with a root grid of 128³ cells and 128³ N-body particles. The mesh is refined with four additional AMR levels (refinement factor N = 2), leading to the effective spatial resolution of l_{α,4} = 48.8 kpc h⁻¹. The force resolution of the gravity solver is of the order of 2 × l_{α,4}. The AMR criteria are based on the baryon and dark matter overdensity, with overdensity factors f = 4 (Iapichino & Niemeyer 2008).

The initial redshift of the simulations is z = 60 and the initial conditions are produced with the Eisenstein & Hu (1999) transfer function. The evolution is then followed to z = 0. Additional physics such as cooling, feedback and transport processes are neglected. An ideal equation of state was used for the gas, with γ = 5/3.

2.2 Subgrid-scale model and the FEARLESS approach

FEARLESS combines AMR with a SGS model for the unresolved turbulence energy, which encompasses the production, diffusion and dissipation of kinetic energy on SGSs (see Schmidt et al. 2006). Details, numerical tests and applications to the physics of galaxy clusters are presented elsewhere (Maier et al. 2009) and here we recall the main features of this tool.

In Schmidt et al. (2006), it is shown how the governing equations of a compressible, viscous, self-gravitating fluid can be decomposed into a large-scale (resolved) and a small-scale (unresolved) part by exploiting the Germano (1992) filtering formalism, applied to density-weighted variables (Favre 1969). According to this formalism, once a filtering length is set, a variable f can be decomposed into a smoothed part f and a fluctuating part f', with (f) varying only on scales larger than the filter length-scale. A filtered quantity f is thus defined by

\[ \langle f \rangle = \langle f \rangle f \Rightarrow f' = \langle f \rangle f/\langle \rho \rangle. \]  

(2)

In the following, we assume that the filter length-scale is generically given by the grid scale l_α, that is, the size of the grid cells at any level of refinement.

By applying this formalism one can derive the filtered equations of the fluid dynamics (cf. Schmidt et al. 2006). For the sake of conceptual clarity, we do not include the cosmological expansion here and refer the reader to Maier et al. (2009) for a complete formulation in comoving coordinates. The resulting equations read

\[ \frac{\partial}{\partial t} \rho + \frac{\partial}{\partial r_j} \tilde{v}_j \rho = 0, \]  

(3)

\[ \frac{\partial}{\partial t} \rho \tilde{v}_i + \frac{\partial}{\partial r_j} \tilde{v}_j \rho \tilde{v}_i = -\frac{\partial}{\partial r_j} \langle \rho \rangle + \frac{\partial}{\partial r_j} (\sigma_{ij}) \]  

(4)

\[ \frac{\partial}{\partial t} (\rho \tilde{v}_i) + \frac{\partial}{\partial r_j} \tilde{v}_j (\rho \tilde{v}_i) = -\frac{\partial}{\partial r_j} \tilde{\rho} + \frac{\partial}{\partial r_j} \tilde{e} \]  

(5)

where ρ(r, t) is the baryon density, v_r(t, t) are the velocity components, e(r, t) is the total specific energy, p is the pressure, g is the gravitational acceleration and σ_{ij} is the viscous stress tensor.

In the above equations, the generalized moments of the arbitrary quantities f and g are given by

\[ \tilde{f} = \langle f \rangle - \langle f \rangle f . \]  

(6)

The filtering of energy leads to the definition of a total resolved energy \( e_{res} = e_{int} + \frac{1}{2} \tilde{e} \tilde{v} \), where \( e_{int} \) is the internal energy and the term \( \frac{1}{2} \tilde{e} \tilde{v} \) is the resolved kinetic energy. On the other hand, the filtered kinetic energy \( \tilde{e}_{kin} \) also includes an unresolved contribution, expressed by a second-order moment of the velocity field \( \tilde{v}(v_i, v_j) \), the turbulent stress tensor:

\[ \tilde{e}_{kin} = \frac{1}{2} \tilde{v} \tilde{v}_{ij} + \frac{1}{2} \tilde{e}(v_i, v_j)/\langle \rho \rangle. \]  

(7)

As in Germano (1992) we identify the trace of \( \tilde{e}(v_i, v_j)/\langle \rho \rangle \) with the square of the SGS turbulence velocity q, so that we define the SGS turbulence energy as

\[ e_t = \frac{1}{2} q^2 := \frac{1}{2} \tilde{e}(v_i, v_j)/\langle \rho \rangle, \]  

(8)

Since the trace of \( \tilde{e}(v_i, v_j) \) can be added to the thermal pressure \( p \) in the filtered momentum equation (4), this identity immediately implies that

\[ p_t = \frac{2}{3} \langle \rho \rangle e_t \]  

(9)

is the turbulent pressure associated with the turbulent velocity fluctuations on length-scales smaller than the grid scale \( l_\alpha \).

The governing equation of \( e_t \), as derived by the filtering of the equations of fluid dynamics, is

\[ \frac{\partial}{\partial t} \rho \tilde{e}_t + \frac{\partial}{\partial r_j} \tilde{v}_j \rho \tilde{e}_t = \mathcal{D} + \Sigma + \Gamma - \langle \rho \rangle (\lambda + \epsilon), \]  

(10)

where the terms on the right-hand side have to be explicitly defined as a function of large-scale filtered quantities and of \( e_t \). Their definitions (the so-called closures) represent the turbulence SGS model. The physical interpretation and the expressions of the terms in equation (10) are as follows:

(i) \( \mathcal{D} \) represents the diffusion of SGS turbulence energy. Its expression is based on the gradient–diffusion hypothesis (Sagaut 2006):

\[ \mathcal{D} = \frac{\partial}{\partial r_j} C_D \langle \rho \rangle l_\alpha q \frac{\partial}{\partial r_j} q, \]  

(11)

with \( C_D = 0.4 \), as inferred by numerical experiments (Schmidt et al. 2006), and \( l_\alpha \) is the cut-off scale.

(ii) \( \Sigma \) is the production term, that is, the term which accounts for the flux of kinetic energy from the resolved to the SGS component.
The production terms arises in equation (10) as the contraction of the turbulent stress tensor and the Jacobian of the resolved velocity:

$$\Sigma = -\hat{\varepsilon}(v_i, v_j) \frac{\partial v_i}{\partial x_j}$$

(12)

The turbulence stresses $\hat{\varepsilon}(v_i, v_j)$ are given by the commonly used eddy-viscosity closure. In the compressible formulation (Schmidt et al. 2006; Maier et al. 2009), this closure reads

$$\hat{\varepsilon}(v_i, v_j) = -2(\rho)C_l \Delta q S_{ij}^\ast + \frac{3}{2} \delta_{ij} \langle \rho \rangle q^3,$$

(13)

where $S_{ij}^\ast$ is the trace-free part of the rate-of-strain tensor

$$S_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

(14)

(iii) $\Gamma$ accounts for small-scale gravitational effects and is neglected in our implementation. As further simplifications in the model, the influence of the viscous stress tensor $(\tau_{ij})$ in equations (4) and (5) is neglected (which is well justified for high Reynolds numbers), and so is also the SGS transport of internal energy, given by the divergence of $\hat{\varepsilon}(v_i, \epsilon_{int})$ in equation (5).

(iv) $\epsilon$ is the term for SGS turbulence dissipation into internal energy, treated as an effect acting only at a SGS. This term is therefore added to the numerical dissipation of the code, but is modelled in a physically motivated way (see Maier et al. 2009 for its role in the physics of the ICM). Its closure follows Sarkar (1992):

$$\epsilon = C_\epsilon \frac{q^3}{\Delta^3} (1 + \alpha_1 M^2),$$

(15)

where $C_\epsilon = 0.5$ (Sagaut 2006), $\alpha_1 = 0.5$ and $M_1 = q/c_s$ is the turbulent Mach number, where $c_s$ is the speed of sound.

(v) $\lambda$ is the pressure dilatation term, which models the effect of unresolved pressure fluctuations and, effectively, is an exchange term between $\epsilon_i$ and $\epsilon_{int}$. Like for the previous term, a closure suggested by Sarkar (1992) is adopted (Maier et al. 2009).

An enlightening graphical representation of the SGS terms and their role in the energy budget is given by Iapichino et al. (2010, cf. Maier 2008).

The innovation of the FEARLESS approach is a consistent treatment of the interchange between the kinetic energy on resolved length-scales and the SGS turbulence energy for a cut-off length that varies in space and time. Apart from the energy flux through the turbulent cascade, this involves the increase in resolved kinetic energy at the cost of SGS turbulence energy if refined subgrids are inserted and, thus, the numerical cut-off length-scale decreases.

The interplay between the AMR of ENZO (based on the method of Berger & Colella 1989) and the turbulence SGS model in FEARLESS exploits an additional assumption of the Kolmogorov scaling of the turbulent energy (Kolmogorov 1941; Frisch 1995). Considering two AMR levels with spatial resolutions $\Delta_{i,j}$ and $\Delta_{k,j}$, it means that, at grid refinement (or derefinement), the SGS turbulence energies are statistically related by

$$\frac{\epsilon_{i,j}}{\epsilon_{k,j}} = \left( \frac{\Delta_{k,j}}{\Delta_{i,j}} \right)^{2/3}.$$

(16)

When a region is refined, a new ‘grid’ is created, that is, the refined grid patch is handled as a single AMR object. In this new fine grid, the values of the hydrodynamical variables are interpolated from the coarse grid, but the SGS turbulence energy is scaled according to equation (16), and the velocities are corrected such that the sum of resolved energy and turbulent energy remains conserved. An opposite procedure applies to grid derefinement, as described by Maier et al. (2009).

An important preliminary check concerns the range of applicability of the turbulence SGS model, which is devised to the study of, at most, moderately compressible flows. This limitation is incorporated in the code as a safeguarding mechanism for the value of the turbulent Mach number. In order to prevent numerical instabilities, a threshold is set at $M_{t,\max} = \sqrt{2}$, as motivated in Maier et al. (2009).

The issue of the applicability of the turbulence SGS model for different baryon temperatures is summarized in Fig. 1. Most of the cold gas ($T < 10^3$ K) lies in the plot at $M_t = \sqrt{2}$: for this baryon phase the low sound speed makes the gas motions very supersonic. The turbulence production is therefore unphysical for this gas, because the implemented SGS model is not suitable for its study and also because the cold and rarefied medium is poorly resolved by our refinement criteria. Anyway, the study of cold gas phase is physically not well posed in simulations without ultraviolet background heating.

In order to circumvent this shortcoming, we will limit our analysis to the gas with $T > 10^3$ K. The threshold on the turbulent Mach number does not affect the gas above this temperature, where $M_t$ is typically below unity. The SGS turbulent energy is thus not a dominant component in the energy budget, as already discussed in Maier et al. (2009).

3 EVOLUTION OF THE WHIM AND THE ICM

3.1 Baryon phases

The main results of this work make use of a distinction of the baryons in the two phases. This distinction is somewhat arbitrary and indeed the mass distribution function in Fig. 2 shows that the cosmic gas is characterized by a continuum of gas states in temperature and density. The problem is generally addressed in the literature by using a threshold based on gas temperature (e.g. Cen & Ostriker 1999).
Turbulence production and pressure in the IGM

and/or density (Skillman et al. 2008; Vazza, Brunetti & Gheller 2009a). Ideally, a physically motivated distinction should be done on a dynamical basis, between the gas belonging to virialized structures and that not belonging to virialized structures. This criterion would be computationally too demanding and is not used in our analysis.

In the following, we will make a distinction based on the baryon overdensity of the gas \( \delta = \rho/(\Omega_b \rho_c) \), where \( \rho_c = 3H_0^2/(8\pi G)(1+z)^2 \) is the critical density at redshift \( z \). The gas will be labelled as ‘WHIM’ if \( \delta < 10^3 \) and as ‘ICM’ if \( \delta > 10^3 \). As discussed at the end of Section 2.2, for both phases the additional constraint \( T > 10^8 \) K is imposed. The former baryon phase is mostly to be found in filaments and in outer halo atmospheres, whereas the latter is in the potential wells of groups and clusters (cf. Fig. 3). From the definition of the AMR criteria in our simulation, it follows that the ICM gas is always resolved at the highest AMR level \( l_A = 4 \), whereas the mass-weighted average refinement level for the WHIM is 2.6.

3.2 Turbulent energy

A first overview of the simulation data at \( z = 0 \) (Fig. 3) already shows a match of the locations where both the internal and the SGS turbulent energies are large. This indication is consistent with the close link occurring between the gravitational collapse, virialization and injection of turbulent energy during the cosmological structure formation. In this section, we use the SGS turbulent energy (measuring turbulent velocity fluctuation at the numerical cut-off scale) in relation to the internal energy (measuring the temperature) of the gas as a diagnostic of the properties of turbulence. Although turbulence is produced on length-scales larger than the grid cut-off scale, the local energy injection is imprinted on the SGS turbulent energy because of the energy transport through the turbulent cascade.

In Fig. 4, the temporal evolution of the mass-weighted average of the internal \( (e_{\text{int}}) \) and SGS turbulent \( (e_t) \) specific energies, both for the WHIM phase and for the ICM phase, is reported in more detail. From \( e_t \) one can derive the average SGS turbulent velocities for the WHIM and ICM: they are 59 and 76 km s\(^{-1}\), respectively, at \( z = 0 \). The corresponding average turbulent Mach numbers are 0.18 and 0.14.

The key feature to be noted in Fig. 4 is the different trend for the evolution of \( e_t \) with time: for the WHIM phase, \( e_t \) increases steadily to \( z = 0 \), whilst in the ICM it reaches a peak between \( z \sim 1.0 \) and 0.65 and then decreases. In the following, we speculate that this different evolution is related to the mechanisms of turbulence generation in the two baryon phases. More specifically, the time-evolution of \( e_t \) in the ICM and WHIM can be interpreted as turbulence production by mergers in the ICM and by shock interactions in the WHIM.

There is supporting evidence corroborating our hypothesis. The gas in the ICM belongs to collapsed structures, which experienced merger episodes during their evolution. These merger events are related to the stirring of the baryons and the subsequent injection of turbulence (e.g. Maier et al. 2009; Paul et al. 2011). It is therefore

Figure 2. Two-dimensional mass distribution function of the gas temperature as a function of the baryon overdensity, at \( z = 0 \). The mass is coded according to the colour bar on the right-hand side.

Figure 3. The panels show projections of a cube with side 20 Mpc \( h^{-1} \), extracted from the computational domain of the FEARLESS run, at \( z = 0 \). Baryon overdensity is shown in the left-hand panel, internal energy in the middle panel and the SGS turbulent energy in the right-hand panel, with density contours overlaid and the corresponding colour bar under the panels.
not surprising that the maximum of $e_t$ in the ICM is consistent with the formation time (defined by the major-merger phase) in the evolution of clusters in the mass range $10^{13} < M < 10^{14} \, M_{\odot}$, as inferred by analytical models of hierarchical clustering (Lacey & Cole 1993, Sheth & Tormen 2004, and fig. 4 of Giocoli et al. 2007).

At later times, the level of SGS turbulence decreases, although the local injection in minor mergers likely contributes to slow down its decay (cf. Subramanian et al. 2006).

As for the WHIM, the injection of turbulent energy at curved shocks is obviously related to the features of the gas accretion on filaments and haloes, that is, more specifically, on the amount of the kinetic energy processed by the external shocks. In fact, a qualitative similarity can be noted between the evolution of $e_t$ and the flux of kinetic energy through external shocks (fig. 10 of Miniati et al. 2000 or fig. 2 of Skillman et al. 2008). This hints towards a link between the SGS turbulence energy production in the WHIM and the gas accretion on shocks associated with growing structures.

Recently, Cavaliere, Lapi & Fusco-Femiano (2011) investigated thoroughly the injection of turbulence in cluster outskirts (at the accretion shocks) with analytical calculations, finding that the turbulent support increases from $z \sim 0.5$. In their analysis, this is due to the shock weakening, the decrease in accretion rates on clusters and the decrease in the gas infall speed at low redshift. The evolution of $e_t$ for the WHIM at redshift around 0.5 in Fig. 4 is in qualitative agreement with the model of Cavaliere et al. (2011).

In order to verify the resolution insensitivity of our results, we analysed the innermost part of the FEARLESS cluster simulation discussed in Maier et al. (2009). In that setup, a computational box with a side of 128 Mpc $h^{-1}$ is simulated with a root grid resolution of $128^3$ cells and $128^3$ N-body particles, but in a small cube with a side of 32 Mpc $h^{-1}$; an additional static grid is nested and seven AMR levels are allowed, such that the local root grid resolution is equivalent to $256^3$ for both the mesh and the N-body particles, and the effective spatial resolution is 7.8 kpc $h^{-1}$. This small volume is centred on a growing cluster and therefore is not representative of a random realization of the cosmological initial conditions, but interestingly in this region the time-evolution of $e_t$ and $e_{int}$ is equal to that shown in Fig. 4. Our results therefore look robust with respect to an increase in spatial and force resolution.

### 3.3 Compressive ratio

Our investigation, started from the analysis of the subgrid turbulent energy in Fig. 4, is furthermore supported by the structure of the velocity field at resolved scales. To this aim, we define the small-scale compressive ratio (Kida & Orszag 1990; Schmidt et al. 2009):

$$r_{cs} = \frac{\langle d^2 \rangle}{\langle d^2 \rangle + \langle \omega^2 \rangle},$$

where $\langle d^2 \rangle$ and $\langle \omega^2 \rangle$ are the averages of the squares of the divergence and the vorticity of the velocity field, respectively. This ratio quantifies the relative importance of compressional and solenoidal modes in a flow. As shown by Schmidt et al. (2009), the values of $r_{cs}$ tend to be higher for compressively driven turbulence.

The evolution of $r_{cs}$ for the two baryon phases is reported in Fig. 5. As expected from the previous considerations, the gas in the WHIM phase has a larger value of the compressive ratio throughout the simulation, with respect to the ICM phase, indicating a higher contribution from compressional modes in filaments and cluster peripheries. A more detailed analysis in the $T-\delta$ plane at $z = 0$ (Fig. 6) shows that the compressive ratio is low at high densities and temperatures (the ICM), while it is significantly higher elsewhere. The compressive ratio is particularly high also at the extrema of the overdensity distribution, resulting either from strong rarefactions or from compressions.

A similar conclusion can be drawn from the visual inspection of the projection in Fig. 7. $r_{cs}$ is generally lower in clusters, except for localized regions (e.g. in the cluster at the centre of the projected volume), likely to be associated with weak shocks in the ICM.

### 3.4 Energy comparison with the adiabatic run

The role of the turbulence SGS model in our large-scale structure simulations and the consistency of the energy budget in this framework is further investigated with a comparison between a simulation

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3 The kinetic energy flux in Miniati et al. (2000) is averaged over all Mach numbers, whereas in Skillman et al. (2008) it is sorted in Mach number, with results grossly comparable to the former study.
using FEARLESS and an adiabatic reference run. In these simulations, the flow is mostly subsonic and the energy content of the SGS turbulence is globally almost negligible (Maier et al. 2009). Quantitatively, it means that, for every baryon phase, the sum of kinetic and thermal energy should be approximately equal in the adiabatic and the FEARLESS run (in the latter case, the sum is extended to the SGS turbulent energy). In fact, good agreement (mostly within 2 percent) is found on the global gas properties of the WHIM and ICM (mass fractions, energy content and their time-evolution).

As a more sensitive diagnostic for the detailed energy budget in the ICM and WHIM, for the different phases, we study the evolution of the total energies

$$E_k = \sum_{i,\text{phase}} e_{k,i} \rho_i V_i,$$

where $e_{k,i}$ is the value of the specific energy $e_k$ in cell $i$ (‘$e_k$’ refers to $e_{k,i}$ or $e_k$; cf. Fig. 4) and $\rho_i V_i$ is the baryon mass in cell $i$. The sum is performed on the cells belonging to the same baryon phase, either the WHIM or the ICM.

Figure 6. Two-dimensional distribution function of the average compressive ratio $r_{cs}$ in the $T$–$\delta$ plane, at $z = 0$.

Figure 7. Same projection volume as the panels in Fig. 3, but the compressive ratio $r_{cs}$ is shown. Baryon density contours are overlaid.

Using data from a cosmological hydrodynamic simulation, Zhu et al. (2010) present an in-depth analysis of the vorticity and divergence fields in the IGM. The rationale behind their analysis is similar to ours, except that they infer turbulence properties from the derivative of the resolved velocity field. Moreover, they consider dynamical equations for the modulus of the vorticity and the divergence. Of particular importance is the rate of change in the divergence, which is generalized to a comoving coordinate system:

$$\frac{D}{Dt} d = \frac{1}{a} \left[ \frac{1}{2} |\omega|^2 - |S|^2 \right] - \frac{1}{p} \nabla^2 p + \frac{1}{p} \nabla p \cdot \nabla p$$

$$- \frac{1}{a^2} \left[ 4 \pi G (\rho + \rho_{dm}) - \frac{3 H^2}{2} \Omega_m \right] - H d,$$

where $a$ is the time-dependent cosmological scalefactor, $D/Dt = \partial/\partial t + a^{-1} v \cdot \nabla$ is the material derivative in comoving coordinates, $G$ is the gravitational constant, $\rho_{dm}$ is the local dark matter density, $\Omega_m$ is the cosmological mean density parameter of baryonic and dark matter and $H = a/\alpha$ is the Hubble parameter. This is the same as in Zhu et al. (2010) (their equation 3), with only the gravity terms (in the second line of equation 19) slightly rearranged, and note that, for the components of the rate of strain tensor, $2 S_{ij} \delta_j = |S|^2$.

4 THERMAL AND TURBULENT PRESSURE SUPPORT

In Fig. 8, we compare $E_i$ with $\Delta E_{int}$, the difference of $E_{int}$ between the adiabatic and FEARLESS simulations. In both phases, $E_{int}$ (not in the plot) is 40 times or more larger than $E_i$.

We observe that $E_i$ and $\Delta E_{int}$ are of the same order of magnitude during the simulation, both for the ICM and for the WHIM, indicating that the SGS turbulent energy acts as an energy buffer between the resolved and unresolved scales. In other words, the global decrease in $E_{int}$ in the FEARLESS run is partly balanced by $E_i$, so that the global energy budget is nearly unaffected.

Further physical interpretations of this energy budget are difficult, because of the high complexity of the flow in cosmological simulations; we refer the reader to Maier et al. (2009) for more tests in simplified setups. Another caveat, however, is that the turbulence SGS model is pushed to its limit in the WHIM because of the relatively high Mach numbers in the flow. For this reason, the result in the WHIM has to be confirmed with a SGS model that does not suffer from such constraints (see Schmidt & Federrath 2011).
The advantage of the filtering approach outlined in Section 2.2 is that we can easily include SGS terms, in particular, turbulent pressure terms. The SGS model described in Section 2.2 allows for a direct computation of the turbulent pressure that is associated with the grid scale: $p_t = \frac{1}{2} \rho \kappa_c$. Since the divergence equation is derived from the momentum equation, in which the turbulent pressure is simply added to the thermal pressure, it follows that the filtered version of the divergence equation is readily obtained from equation (19) by substituting $p$ with $p + p_t$ everywhere:

$$\frac{D}{Dt} \rho = \frac{1}{\rho} \left[ \frac{1}{2} (\omega^2 - |S|^2) - \frac{1}{\rho} \nabla^2 (p + p_t) + \frac{1}{\rho^2} \nabla \rho \cdot \nabla (p + p_t) \right]$$

and by considering filtered quantities (we dropped the hats for brevity). The trace-free part of the turbulence stress tensor is neglected in the above equation. The expression on the right-hand side specifies the net negative compression rate of a fluid parcel. The self-gravity term on the extreme right-hand side stems from the Poisson equation for the gravitational potential and tends to decrease the divergence.

To understand the meaning of the various terms in equation (20), we consider different limiting cases:

(i) **Incompressible limit.** The fluctuations of the density with respect to the mean density vanish and $d = 0$. Thus,

$$\frac{1}{2} (\omega^2 - |S|^2) = \frac{1}{\rho} \nabla^2 (p + p_t).$$

(ii) **Infinite resolution** ($l_\Delta \to 0$). In this case, $p_t$ vanishes and equation (19) is obtained.

(iii) **Global filtering** ($l_\Delta \sim L$, where $L$ is the integral length-scale for turbulence injection). The filter formalism (Germano 1992) encompasses the limit of a statistical theory. If flow structure is smoothed over the largest scales, that is, the size of galaxy clusters, then the velocity derivative becomes negligible and the effect of turbulence is entirely given by the turbulent pressure:

$$\frac{1}{\rho} \nabla^2 (p + p_t) + \frac{1}{\rho^2} \nabla \rho \cdot \nabla (p + p_t)$$

$$= \frac{1}{a} \left[ 4\pi G (\rho + \rho_{dm}) - \frac{3H^2}{2} \Omega_m \right].$$

Neglecting the effects of pressure gradients that are unaligned with the density gradients and comparing the limiting cases (ii) and (iii), we see that the term $\frac{1}{2} \rho (\omega^2 - |S|^2)$ in a fully resolved simulation is equivalent to $-\nabla^2 p_t$ if the flow is filtered on the largest scale of the system. In a LES, we have an intermediate case, where part of the effect of turbulence is captured by the vorticity and the rate of strain of the resolved flow, while the turbulent pressure at the grid scale accounts for numerically unresolved turbulence. If $\omega > |S|$, numerically resolved turbulence counteracts the gravitational contraction of the gas. The turbulent pressure of unresolved velocity fluctuations counteracts self-gravity if $\nabla^2 p_t < 0$. The relative contribution of $p_t$ depends on the grid scale.

It is important that, by its very definition, the turbulent pressure is a *scale-dependent* quantity (Schmidt & Federrath 2011). Zhu et al. (2010) investigate the scale-dependence of the turbulent pressure by integrating the spectrum of the kinetic energy density for all wavenumbers greater than a certain wavenumber (corresponding to a particular length-scale). Since the resulting turbulent pressure spectrum is rather flat, no clear distinction is made between the integral turbulent pressure of the resolved flow and the turbulent pressure of velocity fluctuations below the grid scale. The advantage of our approach is that we can investigate both resolved turbulence and SGS turbulence effects. In Fig. 9, we show the mass-weighted correlation diagrams of $\frac{1}{2} \rho (\omega^2 - |S|^2)$ and $-\nabla^2 p_t$. Both for the WHIM and for the ICM, these quantities are roughly correlated. This is expected, because SGS turbulence is produced by the interactions with turbulent velocity fluctuations on the smallest resolved length-scales, which are probed by $\omega$ and $|S|$. However, the non-local nature of the SGS turbulence energy (see equation 10) implies that there is no simple algebraic relationship between the resolved and unresolved turbulent pressures. This becomes

![Figure 9](https://academic.oup.com/mnras/article-abstract/414/3/2297/1040121/2297-2308)

**Figure 9.** Mass-weighted correlation diagrams of the ‘resolved turbulent pressure’, $\frac{1}{2} \rho (\omega^2 - |S|^2)$, and the turbulent pressure on SGSs, $-\nabla^2 p_t$, for the WHIM (a) and the ICM (b) at redshift $z = 0$. In both panels, the diagonal line marks the location of the equality $\frac{1}{2} \rho (\omega^2 - |S|^2) = -\nabla^2 p_t$. 

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manifest in the large scatter of the correlation diagrams. Consequently, the SGS model is essential for the computation of the support by the turbulent pressure, $-\nabla^2 p$. Compared to a given value of the resolved turbulent pressure (corresponding to a horizontal cut through the two-dimensional distribution), $-\nabla^2 p$ is typically an order of magnitude smaller. Locally, however, the contribution from SGS turbulence can become comparable to or even exceed the resolved contribution, with the exception of extremely strong turbulence intensity in the upper right-hand corner of the distribution $\left(\frac{1}{2} \rho |\omega|^2 - |S|^2 \right) \lesssim 10^{-29}$ and $10^{-30}$, in arbitrary units, for panels (a) and (b) in Fig. 9, respectively.

To analyse the effects of intense vorticity relative to the support of the gas due to its thermal pressure, Zhu et al. (2010) calculate the ratio of $\frac{1}{2} \rho |\omega|^2 - |S|^2$ to $-\nabla^2 p$.

\footnote{However, $\nabla^2 p$ appears with a positive sign in fig. 10 of Zhu et al. (2010). It is not clear whether this is an error in the labelling or an inconsistency of their calculation.}

According to the above discussion, the ratio $\frac{1}{2} \rho |\omega|^2 - |S|^2 \lesssim \nabla^2 p$ is a considerable fraction of the gas, in mass and entropy. Zhu et al. (2010) calculate the ratio of $\frac{1}{2} \rho |\omega|^2 - |S|^2$ to $-\nabla^2 p$.

\begin{equation}
\frac{1}{2} \rho |\omega|^2 - |S|^2 \lesssim \nabla^2 p, \quad \text{(23)}
\end{equation}

subject to the constraints
\begin{equation}
\frac{1}{2} \rho |\omega|^2 - |S|^2 > 0 \quad \text{and} \quad -\nabla^2 p > 0. \quad \text{(24)}
\end{equation}

The distribution in Fig. 10(a) suggests higher values of $r_{wp}$ for overdensities $\delta \sim 10$. To be able to discern this trend more clearly, in Fig. 10(b), the mass distribution function of $r_{wp}$ as a function of $\delta$ is shown. A bimodal mass distribution becomes apparent, where the low-density peak is a feature of the WHIM, and the high-density peak is related to the ICM. Besides fluctuations, the average of $r_{wp}$ decreases for increasing baryon overdensity. Clearly, the turbulent support (resolved plus unresolved) is important for the low-density gas in the WHIM ($1 \lesssim \delta \lesssim 100$). For this gas, the average $r_{wp}$ is larger than for the WHIM at higher overdensity and for the ICM. This fact can be linked with the energy evolution seen in Fig. 4: the driving of large-scale turbulence in the ICM at $z = 0$ has passed its maximum and is declining, while the turbulence production in the WHIM is just saturating. Moreover, at low redshift, $e_t$ is a larger fraction of $e_{\text{tot}}$ in the WHIM, rather than in the ICM (according to $M_t$, for the two phases, cf. Section 3.2). In the ICM, we observe a declining trend of $r_{wp}$ towards large overdensities, meaning that the support for the densest gas is, on the average, mainly thermal.

A more precise comparison to Zhu et al. (2010, their fig. 10) is not straightforward, because we use mass distribution functions in place of their scatter plot, and we include the contribution of SGS turbulence to the total pressure. Moreover, since the WHIM and the ICM are not clearly separated in their plot, the trend with the density is obscured. Anyway, the average values in Fig. 10 are qualitatively similar to those of Zhu et al. (2010), making us confident of the consistency of these two analyses.

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Figure 10. Panel (a): two-dimensional distribution function of the average pressure ratio $r_{wp}$, defined in equations (23) and (24), in the $T-\delta$ plane, at $z = 0$. Panel (b): two-dimensional mass distribution function of $r_{wp}$ as a function of baryon overdensity. The mass-weighted average is overlaid.
based on its vorticity; therefore, it is not surprising that it compares well with this study based on the turbulent pressure support.

As interesting as it may be, the information brought by \( F_{\varphi} \) has to be complemented: given the tight link between turbulence injection and thermalization, it is meaningful to consider the role of the turbulence only in combination with the thermal component of the energy budget. To this aim, in equation (23) we defined the ratio \( r_{\varphi} \) between the dynamical and thermal terms opposing gravitational collapse in the divergence equation (20). Conservatively, we will assume that for \( r_{\varphi} > 0.1 \) the role of the dynamical support in equation (20) becomes non-negligible. According to this threshold, let \( F_{\varphi, \text{WHIM}} \) be defined as the fraction of \( \Omega_{\text{WHIM}} \) where both relations of equation (24) are fulfilled and \( r_{\varphi} > 0.1 \). An analogous quantity is defined for the ICM and both are reported in Table 1, in mass and volume fractions at \( z = 0 \).

Table 1. Mass (second column) and volume fractions (third column) of the WHIM and ICM gas, selected according to the definitions (first column) introduced in Section 4, at \( z = 0 \).

| Mass fraction | Volume fraction |
|--------------|----------------|
| WHIM         |                |
| \( \Omega_{\text{WHIM}} \) | 0.315 \( \times 10^{-2} \) |
| \( F_{\varphi, \text{WHIM}} \) | 0.287 \( \times 10^{-2} \) |
| \( F_{\varphi, \text{WHIM}} \) | 7.74 \( \times 10^{-2} \) |
| ICM          |                |
| \( \Omega_{\text{ICM}} \) | 7.98 \( \times 10^{-2} \) |
| \( F_{\varphi, \text{ICM}} \) | 0.523 \( \times 10^{-5} \) |
| \( F_{\varphi, \text{ICM}} \) | 0.128 \( \times 10^{-1} \) |

Although the mass and volume fractions expressed by \( F_{\varphi} \) are relatively large, the turbulent support is non-negligible only for a smaller subset of these cells. \( F_{\varphi} \) for the ICM is of the order of 10 per cent of the global mass and volume, and somewhat smaller for the WHIM. This result suggests that a significant non-thermal pressure support, counteracting gravitational contraction, is a local behaviour of the cosmic flow, rather than a widespread feature.

5 CONCLUSIONS

In this work, we studied the evolution of the energy budget of the ICM and the WHIM in mesh-based hydrodynamical cosmological simulations. Besides the internal and the resolved kinetic energy, the fearless method combining AMR and LES allows us to also follow the evolution of the unresolved, SGS turbulent energy, defined according to the model described in Maier et al. (2009). Since an energy cascade sets in from the integral length-scale down to the (unresolved) Kolmogorov scale, the SGS turbulence contains information on turbulent injection and evolution at larger, resolved scales, with the advantage of being computed cell-wise and thus easily accessible without further post-processing the resolved gas velocity data.

A first result of this work is the indication of a production of turbulence with different properties in different baryon phases (Fig. 4). In the ICM, the SGS turbulent energy peaks approximately at the expected formation redshift of the haloes with mass \( 10^{13} - 10^{14} \ M_{\odot} \), indicating a turbulence-driving mechanism associated with merger events. Indeed, the compressive ratio of the ICM baryon phase is relatively low (Fig. 5), as expected in a flow where the driving mechanism is dominated by shearing motions.

In the WHIM phase, the SGS turbulent energy grows more smoothly with time, hinting towards a different production mechanism. It is straightforward to call for the role of shocks (in particular, external shocks) in this process, because they enclose the WHIM gas in filaments and outer cluster regions. Interestingly, the flux of kinetic energy through the external shocks, as simulated by Miniati et al. (2000) and Skillman et al. (2008), closely resembles the temporal evolution of \( e_{\varphi} \). A similar trend has also been predicted analytically in the cluster peripheries by Cavaliere et al. (2011). Further evidence is provided by the compressive ratio for the WHIM, which is larger than the values found for the ICM and thus indicates a flow dominated by a compressive driving mechanism. The energy content of SGS turbulence is larger for the ICM phase (Fig. 4), but the relative importance with respect to the internal energy is larger for the WHIM, as testified also from the slightly larger turbulent Mach number (Section 3.2).

Some cautionary remarks are needed for a correct interpretation of the results. First of all, this bimodality is not unambiguous. As known, weak shocks are ubiquitous in the ICM (Miniati et al. 2000; Ryu et al. 2003; Pfrommer et al. 2006; Skillman et al. 2008; Vazza et al. 2009a), thus adding a compressive contribution in that baryon phase, and similarly small clumps move also along the filaments (Dolag et al. 2006), contributing to solenoidal driving modes in the WHIM. Nevertheless, the mixing of compressive and solenoidal modes is dominated by the former for the WHIM and by the latter for the ICM. It would be interesting to explore if and how further stirring mechanisms like AGN outflows change the simulated scenario.

Zhu et al. (2010) investigate the effects of the turbulent pressure on the rate of change of the divergence. This quantity turned out to be important for the clustering of the baryonic gas in the gravitational wells of the dark matter. Following Iapichino et al. (2008) and Schmidt et al. (2009), we propose to utilize the negative growth rate of the divergence as a control variable for AMR in cosmological simulations.

The study of the divergence equation (20) has been extended with the SGS turbulence modelling, finding the following:

(i) In general, the contribution of unresolved pressure to equation (20) is not a dominant term, although locally it can become relevant (Fig. 9).

(ii) The turbulent support (defined by the first relation in equation 24) is largest for the WHIM gas in the overdensity range \( 1 \lesssim \delta \lesssim 100 \) and tends to be small for the ICM gas (\( \delta > 10^3 \)).

(iii) A non-negligible mass fraction of the WHIM (28.7 per cent) and the ICM (52.3 per cent) is characterized by a large vorticity, according to the criterion given by the first relation in equation (24). However, in most of this gas, the thermal support is not smaller than the dynamical one, so that the mass fractions of the gas where \( r_{\varphi} > 0.1 \) are only 7.7 and 12.8 per cent of the WHIM and ICM at \( z = 0 \), respectively.

For completeness, we note that at \( z = 0 \) in our simulation about 31 per cent of the gas is in the WHIM phase and 8 per cent in the ICM phase: most of the turbulence-supported gas is thus in the WHIM phase, although the ICM gas is more tightly related to observables. The emerging picture is that of turbulence as a low-redshift (mostly \( z < 0.5 \)) feature in the WHIM, while in the ICM the main stirring epoch is slightly earlier. The total mass fraction of baryons with \( T > 10^7 \) K where the first relation in equation (24) holds is 13.2 per cent, in good agreement with the previous investigation of Zhu et al. (2010).

The fact that \( F_{\varphi} = F_{\varphi, \text{WHIM}} \) means that the turbulence-supported gas is a substantial fraction of the cosmic baryons, but the flow
producing significant non-thermal support fills only small fractions of space. Although this result reflects a typical feature of turbulence and can be understood by considering that turbulence injection is a by-product of thermalization during the structure growth, this is apparently at odds with recent observational and theoretical claims of non-thermal pressure support, especially in the cluster outskirts (e.g. Lau et al. 2009; Kawaharada et al. 2010; Cavaliere et al. 2011). From the presented results, it seems that turbulence has a dynamical role only in a volume fraction of about 6 per cent of the gas with $T > 10^6$ K. One possibility is that the spatial resolution of our simulation is not adequate to model the flow close to the accretion shock of growing clusters, which plays an important role in injecting turbulence in the cluster outskirts (Cavaliere et al. 2011). However, we tested the results in Table 1 in the well-resolved, innermost region of the computational box presented in Maier et al. (2009), obtaining basically the same results (see also Section 3.2).

We observe that the diagnostics used here for the study of the turbulent support (pressure Laplacians) are different from other studies, based on ratios between turbulent and internal energies (or, equivalently, pressure ratios). Only in Lau et al. (2009), as far as we are aware, the analysis is complemented by the computation of pressure gradients, the very quantities that appear in the cluster mass estimate (Rasia et al. 2006). The discrepancy of the definitions could explain why turbulence ‘support’ (in the sense of the pressure ratio) is found large and widespread in many galaxy cluster simulations, although the dynamical role elucidated in this work is much less significant. One could guess that diagnostics like the energy or pressure ratios track the turbulent gas in a way similar to $F_{\text{iso}}$, rather than $F_{\text{iso}}^\text{corr}$.

This work should be considered an exploratory study to be complemented by future simulations, possibly including a more sophisticated treatment of additional physics (ionization background, radiative cooling, galactic winds). However, the physical processes discussed in this work are governed primarily by gravity; thus, the role of additional physics is not expected to change the scenario drastically (cf. Kang et al. 2007). There is also room for improvement in the SGS modelling of turbulence, focusing in particular on compression-dominated and inhomogeneous turbulence (Schmidt 2009a). As mentioned above, a useful complement to this study will be a similar analysis of turbulent pressure and turbulence support in simulations of single clusters. This is left for future work.

The study of turbulence in many branches of astrophysics has been making progress in recent years, both from the theoretical and from the observational viewpoint. The more one goes into the details of this field, the clearer it becomes that a simplistic way of approaching turbulence [like simply assuming the classical reference results by Kolmogorov (1941) for energy spectra] is wrong or incomplete in most cases. Turbulence driving already emerged as a key issue in the context of compressible turbulence in isothermal gas, relevant for the problem of star formation (Federrath et al. 2010). Kritsuk et al. (2010), on the other hand, argue that a universal scaling law should be observed at length-scales that are sufficiently small compared to the forcing scale. This idea is supported by recent findings of Schmidt & Federrath (2011). In this work, a SGS model for supersonic turbulence indicates a scaling exponent for the unresolved turbulent pressure that is independent of the forcing. Nevertheless, the global statistics of turbulent pressure varies with the forcing. The physical conditions of the gas and the turbulence in the cosmological large-scale structure are obviously different: here we are dealing mostly with subsonic or transonic flow. As this study shows, one needs the investigation of the role of turbulence forcing and turbulent support in this regime, and their applications to cosmological simulations, in order to better understand the gas dynamics in the IGM.

**ACKNOWLEDGMENTS**

The numerical simulations were carried out on SGI Altix 4700 HLRRB-II of the Leibniz Computing Centre in Garching (Germany). The enzo code is developed by the Laboratory for Computational Astrophysics at the University of California in San Diego. The data analysis was performed using the yt toolkit (Turk et al. 2011). Thanks to Carlo Giocoli for useful discussions.

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APPENDIX A: DIFFERENT DEFINITIONS OF THE BARYON PHASES

Throughout this work, the distinction between the gas belonging to clusters and that belonging to filaments has been based solely on baryon overdensity. We note here that the main results of this work do not depend on this definition by testing different characterizations of the baryons. In particular, we computed the evolution of the mass-weighted averages of the SGS turbulence energies of ‘clusters’ and ‘filaments’, according to criteria based on temperature (e.g. Cen & Ostriker 1999) and on the total overdensity (e.g. Vazza et al. 2009a).

The temperature ranges defining the WHIM and ICM are $10^{5} < T < 10^{7}$ K and $T > 10^{7}$ K, respectively. In Vazza et al. (2009a), the baryon phases are characterized by the total overdensity $\delta = (\rho + \rho_{\text{dm}})/\rho_{\text{cr}}$, where the suffix ‘dm’ indicates dark matter. In that work, filaments and sheets have overdensity $3 < \delta < 30$ and the clusters are defined by $\delta \geq 50$.

In Fig. A1, the evolution of $e_{i}$ for the two baryon phases and the different definitions are shown. The definitions are not completely equivalent and therefore the average values differ from each other, in some cases up to an order of magnitude, but in Fig. A1, they are scaled for the sake of clarity. The comparison shows that the trends in the time-evolution of $e_{i}$ presented and discussed in Section 3.2 are apparent also with other definitions of the baryon phases and are not caused by the particular choice of using the baryon overdensity.

![Figure A1. Time-evolution of the SGS turbulence energy for the phases labelled as ‘clusters’ and ‘filaments’ according to our definition based on baryon overdensity (solid lines) and on the criteria based on temperature (dashed lines) and the total overdensity (Vazza et al. 2009a, dotted lines). The single lines are scaled by arbitrary factors, in order to ease the visualization on the plot.](https://example.com/figureA1.png)