Concluding Remarks

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Abstract. Some remarks are given on many body correlations from dilute to dense nuclear systems, putting stress on three subjects; cluster-gas-like states, dineutron correlation, and relation of mean-field dynamics and clustering dynamics.

1. Introduction
I here give some remarks on many body correlations from dilute to dense nuclear systems, from the viewpoint of a theorist who is long involved in the study of nuclear clustering. Nuclear clustering is important especially for dilute nuclear systems. We can easily enumerate several subjects of clustering in relation with dilute nuclear density. (i) Light nuclei; Lots of cluster states exist as excited states in light nuclei which have large surface region. (ii) Very dilute excited states in light nuclei; Recently cluster-gas-like states typified by alpha-condensation-like states have been discussed to be existent widely. (iii) Neutron-rich nuclei: Cluster structure with the neutron molecular orbit forms dilute nuclei and dineutron correlation is discussed in neutron skin, (iv) Heavy ion collisions: Multifragmentation in the expansion phase of the collision. (v) Crust of neutron star: Formation of clusters and possible alpha particle condensation.

Our study of alpha-condensation-like states in finite nuclei started on the excursion boat of the cluster conference at Rab, Croatia, in 1999. Peter Schuck asked me whether there is any possibility of alpha-condensation-like structure in finite nuclei. I recalled and mentioned 3-alpha cluster calculations [1, 2, 3] performed more than 30 years ago which concluded that the Hoyle state is a 3α gas-like state with very dilute density, about 1/3 of the ground state. Later we found [4] actually that the fully microscopic cluster wave functions of 3α RGM [2] and 3α GCM [3] are almost 100 % equivalent to the 3α THSR [5] wave function:

\[ |\langle \Phi(3\alpha THSRw.f.)|\Phi(3\alpha RGM/GCMw.f.)\rangle|^2 \approx 100\% \]  

(1)

The structure study of the Hoyle state has now developed into the proposal and study of the novel type of nuclear structure, the cluster-gas-like structure, in wide region of nuclear chart, which is typified by alpha-condensation-like states.

The subjects discussed in this conference are wide and therefore, as I mentioned before, I give below some remarks on the following three problems. They are (1) cluster-gas-like states, (2) dineutron correlation, and (3) relation of mean-field dynamics and clustering dynamics.
2. Cluster-gas-like states
We now know that the Hoyle state has a $3\alpha$-condensate-like structure, the lowest-energy state of cluster-gas-like states in $^{12}$C. It has long been discussed that the nucleus undergoes the liquid-gas phase transition, where the gas phase means the gas of nucleons. The excitation energy of the nucleon-gas state of mass-number $A$ nucleus is about $8A$ MeV and this high excitation energy makes the nucleon-gas state as the subject of nuclear matter and nuclear reaction rather than nuclear structure. On the other hand, the gas state of clusters is not so highly excited, and can be a discrete state accessible spectroscopically. This situation is shown in Fig. 1 for $^{12}$C.

![Figure 1](image1.png)

**Figure 1.** Excitation energies of $\alpha$-cluster gas state and nucleon gas state in the case of $^{12}$C.

If we can assume weak inter-cluster interaction, the lowest-energy state formed by $n$ clusters $C_1$, $C_2$, $\cdots$, $C_n$ is located near the breakup threshold of $C_1$, $C_2$, $\cdots$, $C_n$. The highest-energy breakup threshold is of course the full dissociation threshold into $A$ nucleons. The lowest-energy cluster structure of $C_1 + C_2 + \cdots + C_n$ can not be liquid-like, because a liquid-like compact cluster structure is equivalent to a mean-field-type structure. The lowest-energy cluster states are zero-temperature states. When the temperature is raised from zero, two types of excitation occur: one is the mean-field-type excitation of individual clusters with liquid nature and the other is the excitation of inter-cluster relative motions. For the understanding of the configuration of nuclear system at finite temperature, the investigation of the caloric relation is useful. Fig. 2 gives the constant-pressure caloric curve of $^{36}$Ar ($N = Z = 18$) calculated with AMD [6]. We see the liquid-gas phase transition occurs through the region of negative heat capacity. Fig. 3 shows fragment mass distribution for the ensembles along the $P = 0.05$ MeV/fm$^3$ line. The caloric curve comes down towards the zero temperature line (abscissa) when the pressure $P$ becomes smaller. The multiplicity distributions of clusters along a caloric curve looks consistent with the distribution of breakup thresholds into clusters.

![Figure 2](image2.png)

**Figure 2.** The constant pressure caloric curve for $^{36}$Ar obtained by AMD. The lines correspond to the pressure $P = 0.02$, 0.03, 0.05, 0.07, 0.10, 0.15, 0.20, 0.25, 0.30, and 0.40 MeV/fm$^3$ from the bottom. $E^*$ stands for the excitation energy and $A = 36$. The curves $E/A = T^2/(8\text{MeV})$ and $E/A = T^2/(13\text{MeV})$, and the line $E/A = (E^* + E_{\text{gas}})/A = (3/2)T$, are drawn for comparison.
3. Dineutron correlation

In order to study the halo phenomena of $^{11}\text{Li}$, Hansen and Jonson [7] introduced the dineutron picture for the two valence neutrons around $^9\text{Li}$ core. Bertsch and Esbensen [8] solved the three-body problem of $^9\text{Li} + n + n$ and found dineutron spatial correlation in the surface region in the two-neutron correlation density. They reported the necessity of the inclusion of high angular momentum of single particle states up to about 14, which is the reflection of spatial dineutron correlation. This point was shown also in the shell model study of Ref. [9] which reported rapid convergence of the calculation by the inclusion of the basis state with Jacobi coordinate. If $^{11}\text{Li}$ has '9Li + dineutron' structure, Coulomb breakup strength is approximately estimated by

$$\frac{dB(1E_1)}{dE_x} \propto \sqrt{S_c(E - S_c)^{3/2}} / E^4$$

which has the peak at $E(\text{peak}) = 1.6S_c$. If we use $S_c = S_{2n} = 0.3$ MeV, we have a peak at low energy $E(\text{peak}) \approx 0.5$ MeV, which agrees with the result of Ref. [8]. Unfortunately, the discrepancies among the experimental results of Coulomb breakup obtained at GSI, RIKEN, and MSU, made it difficult to pursue in more detail the validity of the di-neutron model. However, Nakamura and his group [10] remeasured the breakup reaction of $^{11}\text{Li}$ with improved experimental procedure at RIKEN, and obtained new results which differ from previous data from the three institutions. The peak position of the new data is now much closer to $E(\text{peak}) \approx 0.5$ MeV and the whole $B(1E_1)$ distribution due to new data is now in good agreement with the Bertsch-Esbensen result.

Peter Schuck and his coworkers discussed coexistence of BCS and BEC-like pair structures in nuclei for clarifying the two-neutron structure of $^{11}\text{Li}$ [11]. They used the solutions of the three-body model of core + $n + n$ which, in the case of $^{11}\text{Li}$, is almost the same as the Bertsch-Esbensen model. They analysed the $L = 0$ radial wave function $f_{L=0}(r, R)$ of the $S = 0$ component of $^{11}\text{Li}$ wave function, where $L$ and $S$ are orbital angular momentum and intrinsic spin of the system,respectively, and $r$ and $R$ are two-neutron relative coordinate and two–neutron center-of-mass coordinate measured from $^9\text{Li}$ core, respectively. As shown in Fig. 4, $f_{L=0}(r, R)$ as a function of $r$ has an oscillatory behavior for smaller $R$ where the nuclear density is normal, while it becomes a well localized single peak for larger $R$ the nuclear density is dilute. They argue that these features in $^{11}\text{Li}$ qualitatively correspond to the BCS and BEC-like structures of the pair wave function found in infinite nuclear matter. They also argue that, although the size of Cooper pair usually is very little influenced by pairing strength, in $^{11}\text{Li}$ it is strongly influenced by pairing strength, which stems from that fact that two neutrons occupy very weakly bound 0p and 1s orbits which spread out very far and make a halo structure.

In Ref. [11] the $^9\text{Li} - n$ potential $V_{\text{Li} - n}$ is deeper for positive parity than for negative parity.
Figure 4. The ground state two-particle wave functions, $r^2R^2|f_{L=0}(r, R)|^2$. The solid lines correspond to the two-particle wave functions of $^{11}\text{Li}$, while the dashed lines denote those of $^{16}\text{C}$.

It causes the narrowing of the energy gap between $0p$ and $1s$ orbits. This is necessary for reproducing the observation that $^{11}\text{Li}$ has large mixture of the $(1s)^2$ component besides the usual closed-shell component $(0p)^2$. We need to explain why positive-parity neutron orbit can be lowered than usual. Two kinds of Pauli-blocking effect have been proposed for this explanation. One is the effect of blocking of pairing excitation of $^9\text{Li}$ core which was originally proposed in Ref. [12] for explaining the parity inversion of the $^{11}\text{Be}$ ground state. The other is the effect of blocking of neutron-proton tensor excitation of $^9\text{Li}$ core [13] which coexists with the blocking effect of the two-neutron pairing excitation. Both kinds of Pauli blocking cause blocking for $0p$ orbit but no blocking for $1s$ orbit, which results in the narrowing of the energy gap between $0p$ and $1s$ orbits. Ref. [13] reports that these two kinds of Pauli-blocking effects contribute to the narrowing of the $0p - 1s$ energy gap with roughly the same amount of magnitude and the calculation well reproduces the experimental data which include the binding energy, matter radius, Coulomb breakup cross sections, and the $(1s)^2$ probability of the wave function [14]. It is to be noted that the approximate degeneracy of $0p$ and $1s$ orbits is favorable for the spatial correlation of dineuteron. We thus see that there are close relations among the three characteristics of $^{11}\text{Li}$, neutron halo, magicity breaking, and dineuteron.

4. Relation of mean-field dynamics and clustering dynamics

The investigation of the clustering correlation on the basis of the mean field is of course a fundamentally important way to study the relation between mean-field dynamics and clustering dynamics. Here, however, we would like to point out the fact that there exists a deep relation between mean-field nature and clustering nature of nuclei. This fact has been known for a long time but its fundamental importance has been emphasized only recently [15, 16, 17]. It is the duality of the ground-state wave function of the non-heavy nucleus that the wave function can be expressed in two ways, one in the mean-field-like type and the other in the cluster-model type. A typical example is the doubly-closed-shell wave function of $^{16}\text{O}$. We have the following relation;

$$
\det |(0s)^4(0p)^{12}| = c_L A \left[ R_{4L}(\vec{r}_{C-\alpha}, 3\nu) |Y_L(\vec{r}_{C-\alpha})\phi_L^{(12C)}|_{J=0}\phi(\alpha) \right] g(X_G, 16\nu),
$$

(2)
where \( L \) is arbitrary among \( L = 0, 2, \) and \( 4 \), \( R_{N,L}(r, \gamma) \) stands for the radial harmonic oscillator function with width parameter \( \gamma \) and \( N = 2n + L \) the number of oscillator quanta, \( g(r, \gamma) \propto \exp(-\gamma r^2) \), and \( \nu \) is the single-nucleon oscillator parameter. This duality is known as the Bayman-Bohr theorem [18].

The observed quantity which shows clearly the duality of the ground state wave function is the monopole transition strength between the ground state and excited clustering states in light nuclei. The observed transition strengths are all large and are of the order of single-nucleon strength [19]. In the description by the mean-field-type model, cluster states have (superpositions of) many-particle many-hole configuration(s), and thus it is very difficult to explain why almost all the observed monopole strengths are of single-nucleon strength. However, if we notice the duality of the ground state wave function, this observed fact that monopole transition strengths are all large and are of the order of single-nucleon strength is quite easy to explain.

We explain the above statement in the case of the monopole transition between the Hoyle state and the ground state of \(^{12}\text{C}\). The observed strength is \( M(E0; \text{g.s.} \leftrightarrow \text{Hoyle}) = 5.4 \text{ fm}^2 \). This is of the order of single-nucleon strength since the single-nucleon strength is estimated, by using schematic single-particle wave function of constant density, as \( M(E0) \sim \langle u_f | r^2 | u_i \rangle \sim (3/5)R^2 = 5.4 \text{ fm}^2 \), where the nuclear radius \( R = 3 \text{ fm} \) is used. The theoretical calculation of \( M(E0) \) value is performed as follows by using the duality of the ground state wave function which is represented by its dominant component given by the SU(3) shell model wave function, \(|(0s)^4(0p)^8, (\lambda, \mu) = (04)J = 0 \rangle \). The duality is expressed as

\[
| (0s)^4(0p)^8, (04)J = 0 \rangle = N_0 A \{ R_{4,0}(\xi_1, (8/3)\nu) R_{4,0}(\xi_2, 2\nu) \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3) \} g(X_G, 12\nu),
\]

where \( \xi_k \) are Jacobi coordinates of three \( \alpha \) clusters, \( \xi_1 = X_1 - (X_2 + X_3) / 2, \xi_2 = X_2 - X_3, \) with \( X_j \) standing for the center-of-mass coordinate of \( j \)-th \( \alpha \) cluster. The Hoyle state is described by the 3\( \alpha \) THSR [5] wave function

\[
\Phi(3\alpha \text{THSRw.f.}) = n_B A \{ \exp[-\frac{2}{B^2}(x_1^2 + x_2^2 + x_3^2)] \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3) \}, \quad x_k = X_k - X_G.
\]

From the forms of the wave functions of the ground and Hoyle states, we see that the monopole transition between these states is nothing but the monopole transition between two relative wave functions

\[
R_{4,0}(\xi_1, (8/3)\nu) R_{4,0}(\xi_2, 2\nu) \quad \longleftrightarrow \quad \exp[-\frac{2}{B^2}(x_1^2 + x_2^2 + x_3^2)].
\]

This fact means that only two degrees of freedom of inter-\( \alpha \) relative motion is relevant to the monopole transition between the ground and Hoyle states, while in the description by the mean-field-type model we have to treat much larger number of degrees of freedom of nucleonic motion. The relevance of only two degrees of freedom is the reason why the monopole transition between the ground and Hoyle states has the strength of the order of single-nucleon strength. This argument can be seen by the analytical expression of the monopole matrix element \( M(E0; \text{g.s.} \leftrightarrow \text{Hoyle}) \) given as [17]

\[
M(E0; \text{g.s.} \leftrightarrow \text{Hoyle}) = \sqrt{\frac{T}{6}} \sqrt{\frac{\langle F_4 \rangle}{\langle F_3 \rangle}} \sigma_5 \langle R_{40}(r, \nu) | r^2 | R_{60}(r, \nu) \rangle,
\]

where

\[
\langle F_n \rangle = \langle Q_n | A \{ Q_n \} \rangle, \quad Q_n = F_n(\xi_1, \xi_2) \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3),
\]

\[
F_n(\xi_1, \xi_2) = \frac{1}{4\pi} \sum_{n_1 + n_2 = n} \sqrt{(2n_1 + 1)!!(2n_2 + 1)!!} \frac{R_{2n_1,0}(\xi_1, (8/3)\nu) R_{2n_2,0}(\xi_2, 2\nu)}{2n_1!!2n_2!!}.
\]

\[
|\text{Hoyle}(0^+_2)\rangle = \sum_{n=5}^{\infty} \sigma_n \langle e_n A \{ Q_n \} \rangle, \quad ||e_n A \{ Q_n \}|| = 1.
\]
It is to be noted here that, when we rewrite the $E_0$ transition operator $O(E_0;^12C) = (1/2) \sum_{\alpha=1}^{12} (r_\alpha - X_G)^2$ as $O(E_0;\alpha_1) + O(E_0;\alpha_2) + O(E_0;\alpha_3) + (1/2)((8/3)\xi_1^2 + 2\xi_2^2)$, there is no contribution from $\alpha$-cluster degrees of freedom, $O(E_0;\alpha_1) \sim O(E_0;\alpha_3)$. The quantity $\langle F_n \rangle$ represents the effect of the antisymmetrization, but in the above analytical formula, it appears in the form of ratio, $\langle F_4 \rangle/\langle F_5 \rangle$, whose magnitude is close to unity. Thus the effect of antisymmetrization has only little influence on the $M(E_0)$ value. The quantity $\sigma_5$ is the amplitude of the $2\hbar\omega$ - jump component contained in the Hoyle state and its magnitude is around 0.25.

5. Summary
We have discussed various types of many body correlations from dilute to dense nuclear systems. The existence of various types of many body correlations shows the richness of the nuclear dynamics and system.

I close my talk by wishing Peter happy retirement and many more healthy and successful years.

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