A model with flavor-dependent gauged $U(1)_{B-L_1} \times U(1)_{B-L_2-L_3}$ symmetry

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Abstract: We propose a new model with flavor-dependent gauged $U(1)_{B-L_1} \times U(1)_{B-L_2-L_3}$ symmetry in addition to the flavor-blind symmetry in the Standard Model. The model contains three right-handed neutrinos to cancel gauge anomalies and several Higgs bosons to construct the measured fermion masses. We show the generic features of the model and explore its phenomenology. In particular, we discuss the current bounds on the extra gauge bosons from the K and B meson mixings as well as the LEP and LHC data, and focus on their contributions to the lepton flavor violating processes of $\ell_{i+1} \rightarrow \ell_i \gamma$ ($i = 1, 2$).

Keywords: flavor-dependent gauged Abelian symmetries, model building, numerical analysis of gauge boson masses and coupling

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1 Introduction

Two vector $U(1)$ gauge bosons often appear in grand unified theories (GUTs) such as $SO(10)$ gauged symmetry [1] when it spontaneously breaks down, when a flavor-blind gauged $U(1)_{B-L}$ can be naturally induced along with right-handed neutrinos. On the other hand, flavor-dependent $U(1)$ gauged symmetries are one of the promising scenarios to explain several anomalies beyond the Standard Model (SM), such as semi-leptonic decays involving $b \rightarrow s \ell \ell$, the muon anomalous magnetic moment, and so on [2].

In this paper, we propose a new model which contains two extra flavor-dependent gauge symmetries: $U(1)_{B-L_1} \times U(1)_{B-L_2-L_3}$, with the subscript numbers representing family indices besides the flavor-blind SM one. This type of the extension of the SM is, of course, difficult to embed into a larger group such as GUTs. But, due to the flavor dependence, there exist flavor changing processes via vector gauge bosons, resulting in slightly different signatures from typical gauged symmetries such as the flavor-blind $U(1)_{B-L}$ models.

This paper is organized as follows. In Section 2, we first construct our model by showing its field contents and their charge assignments, and then give the concrete renormalizable Lagrangian with scalar and vector gauge boson sectors. After that, we discuss the phenomenology, including the interaction terms, the bounds from the K and B meson mixings, the LEP [3] and LHC [4] experiments, and lepton flavor violations (LFVs). In Section 3, we perform numerical analysis. In Section 4, we extend our model to explain several anomalies indicated by current experiments. Finally, we conclude in Section 5 with some discussion.

2 Model setup and phenomenology

First of all, we impose two additional $U(1)_{B-L_1} \times U(1)_{B-L_2-L_3}$ gauge symmetries by including three right-handed neutral fermions $N_{B,L_{1,2,3}}$, with the subscripts representing the family indices. The field contents of the fermions and scalar bosons are given in Tables 1 and 2 respectively. Then, the anomaly cancellations among $U(1)_{B-L_1}$, $U(1)_{B-L_1}$, $U(1)_{B-L_2-L_3}$, $U(1)_{B-L_2-L_3}$.
Table 1. Field contents of fermions and their charge assignments under $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_{B-L_1}\times U(1)_{B-L_2-L_3}$, where the subscripts 1 and i=2,3 correspond to the family indices.

| Fermions | $Q_{L1}$ | $Q_{L2}$ | $u_{R1}$ | $u_{R2}$ | $d_{R1}$ | $d_{R2}$ | $L_{L1}$ | $L_{L2}$ | $e_{R1}$ | $e_{R2}$ | $N_{R1}$ | $N_{R2}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $SU(3)_C$ | 3        | 3        | 3        | 3        | 3        | 3        | 1        | 1        | 1        | 1        | 1        | 1        |
| $SU(2)_L$ | 2        | 2        | 1        | 1        | 1        | 1        | 2        | 2        | 1        | 1        | 1        | 1        |
| $U(1)_Y$  | $-\frac{1}{6}$ | $-\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | 0        | 0        |
| $U(1)_{B-L_1}$ | $0$ | $\frac{1}{6}$ | $0$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ |
| $U(1)_{B-L_2-L_3}$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $-1$ | $0$ | $-1$ | $0$ | $-1$ |

$U(1)_{B-L_1}^2 U(1)_Y$, $U(1)_{B-L_1} U(1)_Y^2$, $U(1)_{B-L_2-L_3} U(1)_Y$, $U(1)_{B-L_2-L_3}^2 U(1)_Y$, $U(1)_{B-L_1} U(1)_{B-L_2-L_3}$ and $U(1)_{B-L_1}^2 U(1)_{B-L_2-L_3}$ are the same as the typical single-flavor-independent gauged $U(1)_{B-L}$ symmetry, while those from $U(1)_{B-L_1} \times U(1)_{B-L_2-L_3}$ and $U(1)_{B-L_1}^2 U(1)_{B-L_2-L_3}$ are automatically cancelled because the two additional charge assignments are orthogonal to each other. In Table 2, $H_1$ is expected to be the SM Higgs.

Respectively, $\tilde{H} \equiv (i\sigma_2)H^*$, with $\sigma_2$ being the second Pauli matrix, and $i$ runs over 2 to 3.

Table 2. Field contents of scalar bosons and their charge assignments under $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_{B-L_1}\times U(1)_{B-L_2-L_3}$.

| Bosons | $H_1$ | $H_2$ | $\varphi_1$ | $\varphi_2$ |
|--------|-------|-------|-------------|-------------|
| $SU(3)_C$ | 1 | 1 | 1 | 1 |
| $SU(2)_L$ | 2 | 2 | 1 | 1 |
| $U(1)_Y$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 |
| $U(1)_{B-L_1}$ | 0 | $\frac{1}{2}$ | 0 | 0 |
| $U(1)_{B-L_2-L_3}$ | 0 | 0 | $\frac{1}{2}$ | 2 |

Scalar sector:

The scalar fields are parameterized as

$$H_i = \begin{bmatrix} \frac{w_i^+}{\sqrt{2}} \\ v_i + h_i + i z_i \end{bmatrix}, \quad \varphi_i = \frac{\varphi_i^+ + \varphi_i^- + i z_i}{\sqrt{2}}, \quad (i=1,2),$$

with all four CP-odd bosons $s_{1,2,\varphi_1,\varphi_2}$ massless, in which three of them are absorbed by vector gauged bosons while $H_2$ is another isospin doublet scalar boson, which plays a role in providing the mixings of the 1-2 and 1-3 components in the CKM matrix, as we will see below. Under these symmetries, the renormalizable Lagrangian for the quark and lepton sectors and scalar potential are given by:

$$-\mathcal{L} = y_u Q_L H \bar{H} u_R + y_d Q_L H \bar{H} d_R + y_e Q_L H \bar{H} e_R + y_{\ell} L_L H \bar{L} \ell_R + y_{\nu} L_L H \bar{L} \nu_R + \lambda H \bar{H} \bar{H} + \text{h.c.}$$

$$V = \frac{\mu_1^2}{2} |H_1|^2 + \frac{\mu_2^2}{2} |H_2|^2 + \lambda_1 |\varphi_1|^2 + \lambda_2 |\varphi_2|^2 + \lambda_3 |\varphi_1|^2 |\varphi_2|^2 + \lambda_4 |\varphi_1|^4 + \lambda_5 |\varphi_2|^4 + \lambda_6 |\varphi_1|^2 |\varphi_2|^2 + \lambda_7 |\varphi_1|^4 + \lambda_8 |\varphi_2|^4 + \lambda_9 |\varphi_1|^2 |\varphi_2|^2,$$
leading to $[h_1, h_2, \phi_{R_1}, \phi_{R_2}] = O_T^K [H_1, H_2, H_3, H_4]^t$. Here, we identify $H_3 \equiv h_{SM}$.

Fermion sector:

The SM Dirac fermions are diagonalized by unitary mixing matrices as $D_u = (U_{u,d,e})_R m_u (U_{u,d,e})_L$, and the active neutrinos are derived by an unitary mixing matrix as $D_{\nu} = U_{\nu}^t m_{\nu} U_{\nu}$, while the observed mixing matrices can be defined by $V_{\text{CKM}} \equiv U_{uL}^t U_{dL}$, and $V_{\text{MSN}} \equiv U_{uL}^t U_{eL}$, respectively [5]. However, we impose $U_{uL} = 1$ for simplicity. Hence, we reduce the formula to $V_{\text{CKM}} \equiv U_{uL}$. In the lepton sector, we classify the case of $V_{\text{MSN}} \approx U_{eL}$ or $V_{\text{MSN}} \approx U_{eL}$ below. Here, the neutrino mass matrix $m_{\nu}$ is induced via the canonical seesaw mechanism in Eq. (1).

2.1 Neutral gauge boson sector

$Z_{\text{SM}}$-$Z'$-$Z''$ mixing: Since $H_2$ and $\phi_{1,2}$ have nonzero $U(1)_{B-L}$ and $U(1)_{B-L_2-L_3}$ charges, there are mixings among $Z_{\text{SM}}$, $Z'$, and $Z''$. The resulting mass matrix in the basis of $(Z_{\text{SM}}, Z', Z'')$ is given by

$$
m_{Z_{\text{SM}}, Z', Z''}^2 = \begin{pmatrix}
g_2' v^2 & -\frac{1}{6} g_1' g_2 v^2 & \frac{1}{6} g_2' g_2 v^2 \\
\frac{1}{6} g_1' g_2 v^2 & \frac{1}{9} g_1^2 v^2 + 9 g_2^2 v^2 & -\frac{1}{9} g_1' g_2' v^2 - 9 g_2' v^2 \\
\frac{1}{6} g_2' g_2 v^2 & -\frac{1}{9} g_1' g_2' v^2 - 9 g_2' v^2 & \frac{1}{9} g_2^2 [v^2 + 9(4 v^2 + v' v^2)]
\end{pmatrix},
$$

(7)

where $g_1^2 \equiv g_1^2 + g_2^2$, $m_{Z_{\text{SM}}} = \sqrt{g_1^2 + g_2^2} \approx 91.18$ GeV, and $g_1$, $g_2$, $g_1'$, and $g_2'$ are the gauge couplings of $U(1)_Y$, $SU(2)_L$, $U(1)_{B-L_1}$, and $U(1)_{B-L_2-L_3}$, respectively. Here, we can identify the mass of $Z_1$ as the SM one, since we expect $v_2 < v_1 < v_1'$, in order to reproduce the SM fermion masses and the LEP measurement of $m_{Z_1} \approx m_{Z_{\text{SM}}}$. This approximation is in good agreement with the current experimental data, as the mass difference between $m_{Z_{\text{SM}}}$ and $m_{Z_1}$ should be less than $O(10^{-3})$ GeV.

The other part can be reduced to

$$
m_{Z', Z''}^2 \sim \begin{pmatrix}
g_2' v_1^2 & g_1' g_2' v_1^2 & g_2' v_2^2 \\
g_1' g_2' v_1^2 & g_2' v_2^2 & (4 v_1^2 + v_2^2) \\
g_2' v_2^2 & (4 v_1^2 + v_2^2) & (4 v_1^2 + v_2^2)
\end{pmatrix},
$$

(8)

which is diagonalized by the two-by-two mixing matrix $V_{Z_1}$ as $V_{Z_{1}} m_{Z', Z''} V_{Z_{1}}^t = \text{Diag}(m_{Z_1}^2, m_{Z_2}^2)$, with

$$
m_{Z_1}^2 = \frac{1}{2} \left[ g_2^2 (4 v_1^2 + v_2^2) + g_2' v_2^2 - \sqrt{g_1^2 (4 v_1^2 + v_2^2)^2 + g_2^2 v_2^2 + 2 g_2^2 g_2' v_2^2 (4 v_1^2 + v_2^2)} \right],
$$

(9)

$$
m_{Z_2}^2 = \frac{1}{2} \left[ g_2^2 (4 v_1^2 + v_2^2) + g_2' v_2^2 + \sqrt{g_1^2 (4 v_1^2 + v_2^2)^2 + g_2^2 v_2^2 + 2 g_2^2 g_2' v_2^2 (4 v_1^2 + v_2^2)} \right],
$$

(10)

$$
V_{Z_1} = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta
\end{bmatrix}, \quad s_\theta = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{g_1^2 (4 v_1^2 + v_2^2) - g_2^2 v_2^2}{m_{Z_1}^2 - m_{Z_2}^2}}.
$$

(11)

Note here that we have to satisfy the following condition:

$$
16 g_1^2 g_2^2 v_1^2 v_2^2 \leq \left[ g_1^2 v_1^2 + g_2^2 (4 v_1^2 + v_2^2) \right]^2,
$$

(12)

that arises from the need for the vector boson masses to be positive real.

Here, we evaluate the typical scale of $v_2$ that should be suppressed by the deviation of $m_{Z_1}$ from $m_{Z_{\text{SM}}}$, at the next leading order, $\delta m_{Z_1} \equiv |m_{Z_1} - m_{Z_{\text{SM}}}|$, approximately given by

$$
\delta m_{Z_1} \approx \frac{g_1' v_1^2}{12} \left( \frac{|g_1' \sqrt{1 + X} + g_2' \sqrt{1 + Y}|}{m_{Z_{\text{SM}}} - m_{Z_1}} \right)^2, \quad X = \frac{g_2^2 (4 v_1^2 + v_2^2) - g_2' v_2^2}{m_{Z_2}^2 - m_{Z_1}^2},
$$

(13)

where $\delta m_{Z_1}$ should satisfy $\delta m_{Z_1} \lesssim 2.1 \times 10^{-3}$ GeV from the electroweak precision test. As a result, we find, e.g., $v_2 \lesssim 19.5$ GeV for $v_1' \sim 10^5$ GeV and $g_1' \sim 10^{-3}$.

Interacting Lagrangian: The interactions in the kinetic term between the neutral vector bosons and quarks in terms of the mass eigenstates are given by:

$$
L_\phi = -\frac{1}{3} \left[ (g_1' c_\theta + g_2 s_\theta) d_{\mu} \phi_{Z_{1}^\prime} + (g_1' s_\theta + g_2 c_\theta) \sum_{i=e,t} \bar{u}_i \gamma^\mu u_i Z_{2i} \right] + \frac{1}{3} \left[ d_{\mu} \phi_{Z_{2}^\prime} + (g_1' c_\theta O_{1a} + g_2 s_\theta O_{1a'}) \sum_{i=e,t} \bar{d}_i \gamma^\mu (g_1' s_\theta O_{2a} + g_2 c_\theta O_{2a'}) d_i Z_{2i} \right],
$$

(15)

$$
O_{1a'} = V_{\text{CKM}} \text{diag}(1,0,0) V_{\text{CKM}}^t \approx \begin{bmatrix}
0.95 & -0.22 & 0.013 + 0.0032i \\
-0.22 & 0.0509 & -0.0030 - 0.00075i \\
0.013 - 0.0032i & -0.0030 + 0.00075i & 0.00019
\end{bmatrix},
$$

(16)
\[ O_{\text{ax}} = V_{\text{CKM}} \text{diag}(0,1,1) V_{\text{CKM}}^\dagger \approx \begin{bmatrix} 0.051 & 0.22 - 0.00014i & -0.0082 - 0.0033i \\ 0.22 + 0.00014i & 0.95 & -0.0030 - 0.00075i \\ -0.0082 + 0.0033i & -0.0030 + 0.00075i & 1.0 \end{bmatrix}, \] (17)

where we have used the central values for the CKM elements in \( V_{\text{CKM}} \) [6]. The interactions between the neutral vector bosons and charged-leptons depend on the parameterizations of \( V_{\text{MNS}} \) given by:

\[ V_{\text{MNS}} \approx U^\dagger_\ell: \quad \mathcal{L}_\ell^{(1)} = \left( g'_1 c_\theta + g'_2 s_\theta \right) \bar{\ell} \gamma^\mu \ell Z_\mu + \left( -g'_1 s_\theta + g'_2 c_\theta \right) \sum_{i=\mu, \tau} \bar{\ell}_i \gamma^\mu \ell_i Z'_\mu, \] (18)

\[ V_{\text{MNS}} \approx U_{\ell L}: \quad \mathcal{L}_\ell^{(2)} = \bar{\ell}_i \gamma^\mu \left( g'_1 c_\theta O_{\ell Z'} + g'_2 s_\theta O_{\ell Z''} \right) i_\mu \ell_i Z'_\mu + \bar{\ell}_i \gamma^\mu \left( -g'_1 s_\theta O_{\ell Z'} + g'_2 c_\theta O_{\ell Z''} \right) i_\mu \ell_i Z''_\mu, \] (19)

with \( i_{\ell, j} = (e, \mu, \tau) \), where \( O_{\ell Z', \ell Z''} \) are derived as

\[ O_{\ell Z'} = V_{\text{MNS}} \text{diag}(1,0,0) V_{\text{MNS}}^\dagger \approx \begin{bmatrix} 0.69 & -0.31 + 0.060i & 0.33 - 0.068i \\ -0.31 + 0.060i & 0.14 & -0.14 + 0.060i \\ 0.33 + 0.068i & -0.14 - 0.060i & 0.17 \end{bmatrix}, \] (20)

and

\[ O_{\ell Z''} = V_{\text{MNS}} \text{diag}(0,1,1) V_{\text{MNS}}^\dagger \approx \begin{bmatrix} 0.31 & 0.31 + 0.060i & 0.33 + 0.068i \\ 0.31 - 0.060i & 0.86 & 0.14 - 0.060i \\ 0.33 - 0.068i & 0.14 + 0.060i & 0.83 \end{bmatrix}. \] (21)

respectively, by taking the best fitted results in Ref. [6] for \( V_{\text{MNS}} \).

### 2.2 Phenomenology

Since \( Z_{1,2} \) interact with the SM fermions in a non-universal manner, as discussed before, the constraints are unlikely to be the same as those in the typical \( U(1)_{B-L} \) models. Here, we will examine the bounds on the extra gauge bosons from the K and B meson mixings as well as the LEP data, and discuss the lepton flavor violating processes of \( \ell_{i+1} \to \ell_i \gamma \) (\( i = 1 \) and 2).

1. \( M - \overline{M} \) meson mixings

The extra gauge bosons induce the neutral meson (\( M \))-antimeson (\( \overline{M} \)) mixings with \( M = (K^0, B_d, B_s) \), such as \( K^0 - \overline{K^0}, B_d - \overline{B_d}, \) and \( B_s - \overline{B_s} \), at the tree level. The formulas for the mass splittings are given by [7]

\[ \Delta m_M \approx \frac{\left| \left( g'_1 c_\theta O_{\ell Z} + g'_2 s_\theta O_{\ell Z''} \right) \right|^2}{m_{Z_1}^2} \left[ \frac{m_M}{m_Z^2} \right] \times \frac{5}{12} + \frac{1}{4} \left( \frac{m_M}{m_q + m_q'} \right)^2, \] (22)

for \( M = (K^0, B_d, B_s) \) with \( q q' = (d s, d b, d s) \), which should be less than the experimental values of \( (3.48 \times 10^{-2}, 3.33 \times 10^{-2}, 1.17) \times 10^{-11} \) GeV [6], where \( f_M = (156, 191, 200) \) MeV and \( m_M = (0.498, 5.280, 5.367) \) GeV.

2. Bounds on \( Z_{1,2} \) from LEP and LHC

From Eqs. (18) and (19), we obtain the effective Lagrangians as:

\[ V_{\text{MNS}} \approx U^\dagger_\ell: \quad \mathcal{L}_\ell^{(1)} = \frac{1}{2} \left( \frac{G^2}{m_{Z_1}^2} \right) (\bar{\ell} \gamma^\mu \ell) (\bar{\ell} \gamma^\mu \ell), \]

\[ V_{\text{MNS}} \approx U_{\ell L}: \quad \mathcal{L}_\ell^{(2)} = \sum_{i=1,2} \left[ \frac{1}{2} \left( \frac{V_{\ell_i}^\dagger V_{\ell_i}}{m_{Z_1}^2} \right) (\bar{\ell} \gamma^\mu \ell) (\bar{\ell} \gamma^\mu \ell) + \sum_{i=\mu, \tau} \frac{V_{\ell_i}^\dagger V_{\ell_i}}{m_{Z_1}^2} (\bar{\ell}_i \gamma^\mu \ell_i) (\bar{\ell}_i \gamma^\mu \ell_i) \right] + \sum_{q' = d, s, b} \left[ \frac{1}{2} \left( \frac{V_{q'q}^\dagger V_{q'q}}{m_{Z_1}^2} \right) (\bar{\ell} \gamma^\mu \ell) (\bar{q'} \gamma_q q') + \frac{V_{q'q}^\dagger V_{q'q}}{m_{Z_1}^2} (\bar{\ell} \gamma^\mu \ell) (\bar{u} \gamma_q q') \right], \] (23)

(24)
respectively, where \((V_{ij}^{(1)})_1 \equiv (g'_1 c_{0} O_{di(1)} x' + g'_2 s_{0} O_{di(2)} x')\),
\((V_{ij}^{(1)})_2 \equiv (-g'_1 s_{0} O_{di(1)} x' + g'_2 c_{0} O_{di(2)} x')\), \(G_1 \equiv g'_1 c_{0} + g'_2 s_{0}\),
and \(G_2 \equiv -g'_1 s_{0} + g'_2 c_{0}\). As a result, the bounds for \(Z'_{1,2}\)
from the measurements of \(e^+ e^- \to f f\) at LEP [3] and \(q\bar{q} \to e\bar{e}(\mu\bar{\mu})\) at LHC [4] are found to be

\[
V_{\text{MNS}} \approx U^{\dagger}_i \begin{pmatrix} m_{Z'_1}^2/(8\pi) & \leq V_{ij}^{(1)} G_i^2 \frac{(20.6 \text{TeV})^2}{8\pi} \\
\text{for LEP;} & \frac{(11.4 \text{TeV})^2}{12\pi} \leq \frac{m_{Z'_1}^2}{G_i (V_{ij}^{(1)})_{id}} \end{pmatrix},
\]

\[
\frac{(37 \text{TeV})^2}{12\pi} \leq \frac{m_{Z'_1}^2}{G_i (V_{ij}^{(2)})_{ad} G_2} \text{ for LHC,}
\]

and

\[
V_{\text{MNS}} \approx U_{eL} \begin{pmatrix} m_{Z'_1}^2/(8\pi) & \leq V_{ij}^{(1)} G_i^2 \frac{(26.0 \text{TeV})^2}{8\pi} \\
\text{for LEP;} & \frac{(18.9 \text{TeV})^2}{4\pi} \leq \frac{m_{Z'_1}^2}{(V_{ij}^{(1)})_{id}} \end{pmatrix},
\]

\[
\frac{(15.8 \text{TeV})^2}{4\pi} \leq \frac{m_{Z'_1}^2}{(V_{ij}^{(1)})_{id} G_i} \text{ for LEP,}
\]

\[
\frac{(11.4 \text{TeV})^2}{12\pi} \leq \frac{m_{Z'_1}^2}{(V_{ij}^{(2)})_{id} G_i} \text{ for LEP,}
\]

\[
\frac{(37 \text{TeV})^2}{12\pi} \leq \frac{m_{Z'_1}^2}{(V_{ij}^{(2)})_{ad} G_2} \text{ for LHC,}
\]

where \(f = e, \mu, \tau, d, u\). It is worth mentioning that these neutral gauge boson searches will be carried out by experiments such as the International Linear Collider (ILC) [8], and more stringent constraints should be obtained in the near future.

3. Lepton flavor violating processes

For \(V_{\text{MNS}} \approx U^{\dagger}_i\), one does not need to consider the lepton flavor violations from the \(Z'_{1,2}\) mediations, because the charged leptons are diagonal from the beginning. On the other hand, if \(V_{\text{MNS}} \approx U_{eL}\), the lepton flavor violating processes due to \(Z'_{1,2}\) can be induced. In this case, we get

\[
\text{BR}(\ell_{h}' \to \ell_{a} \gamma) \approx \frac{4\pi^2 \alpha_{em} C_{\mu} \left| V_{\ell_{h}'}^\dagger (V_{\ell_{a}'}^\dagger)_{ij} F_{ij} \left( \frac{m_{\mu}^2}{m_{Z'}^2} \right) \right|^2}{(4\pi)^4 G_F^2},
\]

\[
F_{ij}(r) = \int_0^1 2\pi x (1-x) F_{ij} r dr, (1-x)^2.
\]

where \(G_F\) and \(\alpha_{em}\) are the Fermi and fine structure constants, respectively, while \(C_{\mu} \approx 1\), \(C_{\tau} \approx 0.1784\) and \(C_{\tau} \approx 0.1736\). The current experimental limits are given by [9, 10]:

\[
\text{BR}(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13},
\]

\[
\text{BR}(\tau \to e\gamma) \lesssim 4.4 \times 10^{-8},
\]

\[
\text{BR}(\tau \to \mu\gamma) \lesssim 3.3 \times 10^{-8}.
\]

These constraints are imposed in the numerical analysis below.

3 Numerical analysis

In our numerical analysis, we explore the allowed gauge parameters of \(g'_{1,2}\) and \(m_{Z'_{1,2}}\) by taking \(s_0 = 0\) and \(s_0 = 1/\sqrt{2}\). We scan the parameter regions as follows:

\[
v_{\ell_{h}', \ell_{a}', m_{Z'_{1,2}}}, e_{\text{GeV}}, \frac{10}{5}, 1.
\]

\[
V_{\text{MNS}} \approx U^{\dagger}_i.
\]

In Fig. 1, we show the allowed parameter points in the planes of \(g'_{1,2}\) and \(m_{Z'_{1,2}}\) with the left-hand and right-hand plots showing \(s_0 = 0\) and \(1/\sqrt{2}\), respectively. The top left plot suggests a wide allowed range of values for \(g'_{1,2}\) for \(s_0 = 0\), whereas \(g'_{1}\) and \(g'_{2}\) should be degenerate for \(s_0 = 1/\sqrt{2}\). The bottom left plot indicates that any values with \(m_{Z'_{1}} \leq m_{Z'_{2}}\) are permitted for \(s_0 = 0\), whereas the allowed parameter spaces for both \(m_{Z'_{1}}\) and \(m_{Z'_{2}}\) should be narrow within 10 GeV \(\leq m_{Z'_{1,2}} \leq 10^6\) GeV for \(s_0 = 1/\sqrt{2}\). We get

\[
V_{\text{MNS}} \approx U_{eL}.
\]

In Fig. 2, similar to Fig. 1, we illustrate the corresponding results for the case of \(V_{\text{MNS}} \approx U_{eL}\) by including the plots for \(\text{BR}(\tau \to \mu\gamma) - \text{BR}(\mu \to e\gamma)\) at the bottom. The generic features for the first two plots from the top left in Fig. 2 are similar to those in Fig. 1. While \(g'_{2}\) is restricted to be \(g'_{2} \leq 0.2\), the allowed regions for \(g'_{1} - g'_{2}\) and \(m_{Z'_{1}} - m_{Z'_{2}}\) are more degenerate than the case of \(V_{\text{MNS}} \approx U^{\dagger}_i\) for \(s_0 = 1/\sqrt{2}\). For the lepton flavor violating processes at the bottom in Fig. 2, we see that \(\text{BR}(\mu \to e\gamma)\) reaches the current experimental bound in Eq. (31), which is clearly testable in the near future for both cases of \(s_0\). However, our result is much lower than the limit in

1) One can consider the anomalous magnetic moment because of the evading of the stringent constraint of the trident production via the \(Z'\) boson (flavor eigenstate) [12]. In our case, its value is of the order \(10^{-14}\), which is much smaller than the experimental value.

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Eq. (31). Note here that BR(τ → eγ) ≈ BR(τ → µγ).
We remark that the current bounds on the masses of the extra gauge bosons are around 3 TeV, from the LHC experiments [11], consistent with all the cases in our analyses with g′₁ = g₂ ≈ 0.72.

Finally, we also mention that the muon anomalous magnetic moment cannot be explained in our present model due to the constraint of the trident production via Z′ [12]. Moreover, the new contributions to the semi-leptonic decays of b → sℓ⁺ℓ⁻ from the Z′ mediations are negligibly small, so that our model sheds no light on the recent anomalies in B → K(ℓ⁺ℓ⁻)u⁻, unlike the models with an extra Z′ in the literature [2]. Thus, we minimally extend our model to explain these issues in the next section.

4 An extension

We now extend our model by introducing two extra vector-like fermions: Q′₃/₄ = (3, 2, 1/6, 1/2, −2/3) and L′₃/₄ = (1, 2, −1/2, 1/2, −2), along with a neutral inert complex scalar S = (1, 1, 0, −1/2, 1) under SU(3)C × SU(2)ᵢ × U(1)Y × U(1)B−L₁ × U(1)B−L₂ × L₃ [13], resulting in the following additional Lagrangian:

\[ \mathcal{L} = \int \bar{L}_L^i L'_R^i S + g_i \bar{Q}_L^i Q'R^i S + M_{Q'} Q' + M_L L' + h.c. \]  

where \( i = 2, 3 \). Here, we have assumed the mass eigenstates for the above down-quark and charged-lepton sectors in the SM, and \( f_3 < f_2 \). As a result, the \( b \rightarrow s \bar{c} c \) excess is negligible, which is consistent with the current experimental data, while \( \tau \rightarrow \mu \gamma \) at one-loop level is also suppressed to avoid the current experimental bound.

Note that \( S \) is a complex boson that is assured by the charge assignment under \( U(1)_{B−L₁} \times U(1)_{B−L₂−L₃} \), and its mass is denoted by \( m_S \).

Muon anomalous magnetic moment:

The muon anomalous magnetic moment is formulated by:

\[ \Delta a_\mu = \frac{|f_2|^2}{8\pi^2} \int_0^1 dx \frac{x^2(1-x)}{x(x-1)+r_L x+(1-x)r_S} \]  

(34)

where \( r_L \equiv (M_L/m_\mu)^2 \) and \( r_S \equiv (m_S/m_\mu)^2 \). The experimental deviation from the SM at 3.3σ C.L. is given by [14]

\[ \Delta a_\mu = (26.1 \pm 8.0(16.0)) \times 10^{-10} \]  

(35)

B → K⁺µ⁻ anomaly: The effective Hamiltonian for the \( b \rightarrow s \mu^+\mu^- \) transition is induced via the box diagram [15], given by

\[ \mathcal{H}_{\text{eff}}(b \rightarrow s \mu^+\mu^-) = \frac{g_2 g_4}{(4\pi)^2} F_{\text{box}}(m_S, m_{Q'}, M_L) \times (\gamma^\mu P_L b)(\bar{\mu}\gamma^\mu\bar{\mu})(\bar{\mu}\gamma^\mu\bar{\mu}) + h.c. \]

2) The LHC bounds are typically stronger than the LEP ones in case of a simple gauged \( U(1)_{B−L} \) model.
and the experimental result is given by

$$V \approx \frac{1}{\sqrt{2}}$$

where

$$C_{SM} \approx \frac{V_{th}V_{l}}{\sqrt{2}}$$

with $V_{th} \approx 0.9991$ and $V_{l} \approx -0.0403$ being the CKM matrix elements [6]. Here, we take $C_{0} = -C_{10}$, which is one of the promising relations to explain the anomaly [16], and the experimental result is given by

$$[-0.85, -0.50]$$

at $1\sigma$, 

$$[-1.22, -0.18]$$

at $2\sigma$,

where the best fit value is $-0.68$.

Neutral meson mixing: The neutral meson mixing gives

the bounds on $g_{s}$ and $M_{Q}$ at low energy, where our valid process is the $B_{s} \rightarrow B_{s}$ mixing in our case. Similar to the $B \rightarrow K^{*} \mu \bar{\mu}$ anomaly, the formula is derived by [7]:

$$\Delta m_{B_s} \approx (g_{s}g_{s}^{*})F_{B_{s}}(m_{S}, M_{Q}, M_{Q'}) \lesssim 1.17 \times 10^{-11} \times \frac{24\pi^{2}}{m_{B_s}f_{B_s}} \text{ GeV},$$

(39)

where the above parameters are found to be $f_{B_s} = 0.200$ GeV [7], and $m_{B_s} = 5.367$ GeV [6].

Dark matter candidate: We suppose that $S$ is a DM candidate. First, we assume that any annihilation modes coming from the Higgs potential are negligibly small. This is a reasonable assumption, because we can avoid the strong constraint coming from the spin independent scattering cross section reported by several direct DM detection experiments, such as LUX [17]. Second, we do not consider the modes through $Z_{i,2}$ coming from
The resulting relic density is found to be in the limit of massless final-state leptons and relative velocity; we randomly select the input parameters to obtain by expanding the cross section in powers of the correct relic density, and the neutral meson mixings as well as the LEP and LHC experiments. The allowed parameter spaces for the masses and couplings of $Z'_1,2$ have been given. Even though all the regions are within the current exclusion bounds ($\sim 3$ TeV) from the LHC [11], more stringent constraints or their discoveries will be found at ILC, with its sensitivity to the cut-off scale being around 50-100 TeV, which is stronger than the LEP constraints.

In addition, the possible effects on the flavor violating processes have been explored. Particularly, we have shown that the branching ratio of $\mu \rightarrow e\gamma$ for the case of $V_{\text{MNS}} \approx U_{\mu L}$ can be large, which is testable by future experiments.

Finally, we have discussed the possibility of explaining the muon $g-2$, $B\rightarrow K^{(*)}\mu\mu$, and dark matter candidate, by introducing vector-like fermions $Q',L'$ and an inert complex boson $S$ with appropriate charge assignments under $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_{B-L_1}\times U(1)_{B-L_2}$. We have also shown the allowed regions to satisfy all the anomalies and constraints, and

the kinetic term, since this is suppressed enough by the masses of $m_{\nu_1,2}$. We comment here that there are two resonant solutions around the points of $m_{\nu_1} = 2m_s$ and $m_{\nu_2} = 2m_s$. Subsequently, the dominant contribution to the thermal relic density comes from $f$ and $g$, and the cross section is approximately given by [18]

$$
(\sigma v_{\text{rel}}) \approx \frac{m_{\frac{2}{3}}^2}{16\pi} \left( \frac{|f|^4}{6(m_\phi^2 + M_{\phi}^2)^2} + \sum_{i=2,3} \frac{|g,g|^2}{(m_\phi^2 + M_{\phi}^2)^2} \right) v_{\text{rel}}^2 + O(v_{\text{rel}}^4),
$$

(40)

in the limit of massless final-state leptons and $m_{\nu_i}/M_{\phi} < 1$. Here, the approximate formula is obtained by expanding the cross section in powers of the relative velocity; $v_{\text{rel}}$. $\sigma v_{\text{rel}} \approx a_{\text{eff}} + b_{\text{eff}} v_{\text{rel}}^2$, where $a_{\text{eff}} = 0$. The resulting relic density is found to be

$$
\Omega h^2 \approx \frac{1.07 \times 10^9 x_f^2}{3\sqrt{g_*(x_f)M_{\text{Pl}}b_{\text{eff}}}},
$$

(41)

where the present relic density is $0.1199 \pm 0.0108$ [19], $g_*(x_f \approx 25) \approx 100$ counts the degrees of freedom for relativistic particles, and $M_{\text{Pl}} \approx 1.22 \times 10^{19}$ GeV is the Planck mass.

Numerical analysis:

We now perform a numerical analysis to satisfy the anomalies of the muon $g-2$, $B\rightarrow K^{(*)}\mu\mu$, the constraints of the correct relic density, and the neutral meson mixing, as discussed above. We randomly select the input parameters as follows:

$$
f_2 = [-1, \sqrt{4\pi}], \quad g_{2,3} = \pm [0.01, 0],
$$

$$
m_s = (10, 1000) \text{ TeV}, \quad (M_{Q'}, M_{L'}) = (1.2 m_s, 50000) \text{ GeV},
$$

(42)

where $1.2m_s$ is used to avoid the coannihilation processes among $Q',L'$ and $S$, for simplicity. We show the allowed regions in Fig. 3, where the left(right)-hand figure represents the $f_2(m_s) \rightarrow C_0$ plane, and the blue(red) points satisfy the muon $g-2$ in the range of $(26.1 \pm 8.0(16.0)) \times 10^{-10}$ in Eq. (35). The yellow(green) region denotes the experimentally allowed region $[-1.22(-0.85), -0.18(-0.50)]$ at $\sigma(2)$, where the black horizontal line inside the green region shows the best fit (BF).

5 Discussion and conclusions

We have proposed a new model with two flavor-dependent gauge symmetries: $U(1)_{B-L_1}$ and $U(1)_{B-L_2}$, in addition to the SM one, along with introducing three right-handed neutrinos to cancel gauge anomalies and several scalars to construct the measured fermion masses. We have examined the experimental bounds on the extra gauge bosons by considering the K and B meson mixings as well as the LEP and LHC experiments. The allowed parameter spaces for the masses and couplings of $Z'_1,2$ have been given. Even though all the regions are within the current exclusion bounds ($\sim 3$ TeV) from the LHC [11], more stringent constraints or their discoveries will be found at ILC, with its sensitivity to the cut-off scale being around 50-100 TeV, which is stronger than the LEP constraints.
found $0.5 \lesssim f_2 \lesssim \sqrt{\pi}$ for both blue and red points, and $10 \lesssim m_S \lesssim 170(90)$ GeV for red(blue) points. It is worth mentioning that the Z boson decay modes of $Z \to f_i f_j$ at one-loop level could restrict our parameter space, where $f_i$ represent all the SM fermions. It is expected that the sensitivities of these modes will further increase at future experiments, such as the CEPC [20], by several orders of magnitude.

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