Superheating fields of semi-infinite superconductors and layered superconductors in the diffusive limit: structural optimization based on the microscopic theory

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Abstract

The superheating field $H_{sh}$ of the Meissner state is thought to determine the theoretical field-limit of superconducting accelerator cavities. We investigate $H_{sh}$ of semi-infinite superconductors and layered structures in the diffusive limit using the well-established quasiclassical Green’s function formalism of the BCS theory. The coupled Maxwell–Usadel equations are self-consistently solved to obtain the spatial distributions of the magnetic field, screening current density, penetration depth, pair potential, and $H_{sh}$. For a semi-infinite superconductor in the diffusive limit, we obtain $H_{sh} = 0.795 H_{c0}$ at the temperature $T \to 0$. Here, $H_{c0}$ is the thermodynamic critical-field at the zero temperature. By laminating a superconducting film ($S$) with the thickness $d$ on a semi-infinite superconductor ($\Sigma$), we can engineer $H_{sh}(d)$ of the layered structure. When $d$ is the optimum thickness $d_m$, $H_{sh}$ can be larger than that of the simple semi-infinite superconductors made from the $S$ and $\Sigma$ materials: $H_{sh}(d_m) > \max \{H_{sh}^{S}, H_{sh}^{\Sigma}\}$. The present study addresses the calculation of $H_{sh}$ of the dirty heterostructure using the microscopic theory from beginning to end for the first time, which contributes to the microscopic understanding of the surface engineering for pushing up the accelerating gradient of superconducting cavities for particle accelerators.

Keywords: SRF, multilayer, non-linear Meissner effect, depairing current density, superheating field

(Some figures may appear in colour only in the online journal)

1. Introduction

The superconducting radio-frequency (SRF) resonant-cavity [1, 2] is the crucial component of modern particle accelerators, which efficiently imparts the electromagnetic energy to charged particles via the RF electric-field. The accelerating gradient $E_{acc}$, namely, the average electric field that charged particles see during transit, is proportional to the
amplitude $H_0$ of the RF magnetic-field at the inner surface of the cavity, e.g. $\mu_0H_0 = gE_{acc}$ and $g = 4.26 \, mT (MV \, m^{-1})^{-1}$ for the TESLA-shape cavity. Today, the best Nb cavities can reach $\mu_0H_0 \sim 200 \, mT$, which corresponds to $E_{acc} \simeq 40–50 \, MV \, m^{-1}$ [3–7]. The ultimate limit of $H_0$ is thought to be around $H_c$, irrespective of whether the cavity material is a type-I or a type-II superconductor. Here, $H_c$ is the thermodynamic critical field. This limitation comes from the fact that an SRF cavity is operated under the Meissner state, and the upper critical field $H_{c2}$ is irrelevant to SRF in contrast to some dc applications. Let us consider a semi-infinite superconductor in the Meissner state shown in figure 1(a) and suppose the penetration depth is given by $\lambda$. The external magnetic field $H_0 \sim H_c$ induces the screening current density $j_s \sim H_c/\lambda$ at the surface, which is close to the depairing current density $j_d$, the stability limit of the superfluid flow. Hence, $H_0$ cannot substantially exceed $H_c$ as long as a simple semi-infinite superconductor is used. The value of $H_0$ which makes the Meissner state absolutely unstable is the so-called superheating field $H_{sh}(\sim H_c)$.

In the Ginzburg–Landau (GL) regime, $H_{sh}$ of a semi-infinite superconductor has been thoroughly investigated [8–10]. The GL results are valid at a temperature $T$ close to the critical temperature $T_c$, while SRF cavities are operated at $T \ll T_c$ (e.g. $T/T_c \sim 0.1–0.2$ for Nb and Nb$_3$Sn cavities). Microscopic calculations of $H_{sh}$, which are valid at an arbitrary temperature $0 < T < T_c$, have been carried out for extreme type-II superconductors, including clean-limit superconductors [11–13], superconductors including homogeneous [14] and inhomogeneous impurities [15], and dirty-limit superconductors with and without Dynes subgap states [16]. Note that $H_{sh}$ for weak type-II superconductors (e.g. pure Nb) at $T \ll T_c$ has not yet been calculated. To estimate $H_{sh}$ of pure Nb at $T \ll T_c$, the GL result $H_{sh}(T) = 1.2H_c(T)$, which is valid at $T \simeq T_c$, is often used.

Besides the simple semi-infinite superconductor, the layered superconductor shown in figure 1(b) has also attracted much attention from SRF researchers because of its potential for increasing the ultimate field-limit. Gurevich [17] proposed the idea of multilayer coating, which introduces a higher-$H_c$ superconducting (S) layer formed on the top of the superconducting substrate ($\Sigma$). Here, the S layer is decoupled from $\Sigma$ by an insulator layer or a natural oxide layer. Using the London theory, it was later shown [18] that, when the penetration depth of the S layer is larger than that of the substrate $\Sigma$, a current counterflow induced by $\Sigma$ leads to a suppression of the current density in the S layer, resulting in an enhancement of the ultimate field-limit. The enhancement is maximized when the S layer has the optimum thickness $d = d_{opt} \sim \lambda_{sh}^{opt}$ [18]. It was also shown [19] that the similar consequences result from the GL calculations, which are valid at $T \simeq T_c$. However, we should use the microscopic theory for quantitative analyses at $T \ll T_c$. Fortunately, for a clean-limit s-wave superconductor at $T = 0$, analyses based on the microscopic theory are significantly simplified: the non-linear Meissner effect [15, 16, 48, 49] is negligible in this regime, and the London equation is valid even under a strong current density close to $j_d$. Combining the current distribution obtained from the London equation and $j_d$ for a clean-limit superconductor calculated from the microscopic theory, the more quantitative theory of the optimum multilayer is obtained [20]. The theory is extended to the case with a thick insulator layer [21]. The recent theoretical advances are reviewed in detail in [21]. Progress in experiments is summarized in [22] (see also progress in the last several years, e.g. [23–34]).

Also, Nb cavities processed by some materials-treatment methods (e.g. 120 °C baking, nitrogen infusion, etc) are known to have a thin dirty-layer (S) on the surface of the bulk Nb ($\Sigma$) [6, 7, 35, 36], which can be modeled by the geometry shown in figure 1(b). In fact, the calculations of the field-dependent non-linear surface resistance [37, 38] have shown that layered structures can mitigate the quality-factor ($Q$) degradation at high-fields (the so-called high-field $Q$-drop). Moreover, it was shown [21, 15] that the thin dirty-layer at the surface improves $H_{sh}$ by the same mechanism as that in the S-$\Sigma$ heterostructure: a current counterflow induced by bulk Nb leads to a suppression of the current density in the dirty-Nb layer at the surface, resulting in an enhancement of $H_{sh}$. These theoretical results are qualitatively consistent with experiments [6, 7, 39–41]. Other effects resulting from the materials treatment (e.g. effects on hydride precipitate [42, 43]) can also play significant roles in the performance improvements, which can be incorporated considering imperfect surface-structures such as proximity-coupled normal layer on the surface [37, 38, 44–47]. A thin normal-layer at the surface can increase and decreases $Q$ depending on its strength of the proximity effect [37, 38].

Despite the extensive studies, microscopically reliable calculations of $H_{sh}(d)$ are still limited. The existing theories of the layered heterostructure [20, 21] are correct when the non-linear Meissner effect is negligible, and $H_{sh}$ for both S and $\Sigma$ are known. For instance, the above conditions are satisfied when both S and $\Sigma$ are extreme type-II superconductors in the clean limit. However, for the case $\Sigma$ is pure Nb, the practically important material for particle accelerators, we do not know $H_{sh}^{opt}$ at $T \ll T_c$: remind $H_{sh}$ of a weak type-II superconductor.
at $T \ll T_c$ has not yet been known. Thus, [20, 21] use the extrapolation of the GL result or empirical values to estimate $H_{sh}^{(Na)}$. On the other hand, for both S and $\Sigma$ are disordered superconductors, we can calculate $H_{sh}$ of both the regions [14–16], even if we consider the case $\Sigma$ is Nb. However, in disordered materials, the non-linear Meissner effect ceases to be negligible. We need to self-consistently solve the microscopic theory of superconductivity combined with the Maxwell equations incorporating the current-induced pair-breaking effect and the resultant non-linear Meissner effect to evaluate $H_{sh}(d)$. It is nontrivial if the results in [20, 21] are valid in this case.

In the present paper, we focus on the latter case: the disordered heterostructure. We carry out the microscopic calculations of $H_{sh}(d)$ of the disordered heterostructure for the first time and identify the optimum thickness of the S layer. The paper is organized as follows. In section II, we briefly review the Eilenberger–Usadel–Larkin–Ovchinnikov formalism of the BCS theory in the diffusive limit and express physical quantities with the Matsubara Green’s functions. The solutions of the Usadel equation at $T = 0$ and the analytical expression of the depairing current density are also summarized. In section III, we consider a simple semi-infinite superconductor (see figure 1(a)). The coupled Maxwell–Usadel equations are self-consistently solved to obtain the spatial distributions of the magnetic field $H(x)$, the current density $j_s(x)$, pair potential $\Delta(x)$, and the penetration depth $\lambda(x)$. Then, the superheating field in the diffusive limit is derived. In section IV, we consider a layered superconductor (see figure 1(b)), self-consistently solve the coupled Maxwell–Usadel equations, and obtain the spatial distributions of $H(x)$, $j_s(x)$, $\Delta(x)$, and $\lambda(x)$. Then, we evaluate $H_{sh}$ as functions of the S-layer thickness $d$ for various material combinations and find the optimum thickness $d_{opt}$. Qualitative interpretation of the results are also discussed using an approximate formula of $H_{sh}(d)$. In section V, we discuss the implications of our results.

2. Theory

We consider the geometries shown in figure 1 and calculate the field and current distributions to evaluate the superheating field $H_{sh}$. We assume (a) the characteristic scale of current variation (London depth, $\lambda$) is much larger than the coherence length; (b) an insulator thickness $\delta$ between $S$ and $\Sigma$ is negligibly small compared with $\lambda$ but is adequately thick to suppress the Josephson coupling [50]. The condition (a) is always satisfied as long as we consider type-II superconductors in the diffusive limit. The condition (b) is also satisfied because $\lambda$ is typically a few hundreds of nm, while $\delta \simeq 1$ few mm is enough for suppressing the Josephson coupling. Hence, in the following, we neglect both the nonlocal effect and the insulator-thickness $\delta$.

2.1. Eilenberger–Usadel–Larkin–Ovchinnikov formalism

Let us briefly summarize the well-established Eilenberger–Usadel–Larkin–Ovchinnikov formalism of the BCS theory in the diffusive limit [47, 51–54]. The normal and anomalous quasiclassical Matsubara Green’s functions $G_{\omega_n} = \cos \theta$ and $F_{\omega_n} = \sin \theta$ and the pair potential $\Delta$ obey

$$\left( \Delta - \frac{s}{\sqrt{1 + \cot^2 \theta}} \right) \cot \theta = i \hbar \omega_n, \quad (1)$$

$$\ln \frac{T}{T_c} = \frac{2\pi k_B T}{\hbar D} \sum_{\omega_n > 0} \left( \frac{1}{\hbar \omega_n} \sin \theta + \frac{s}{\Delta} \right). \quad (2)$$

Here $s = (q/q_\xi)^2 \Delta_0$ is the superfluid-flow parameter, $\Delta_0 = \Delta(s,T)_{\hbar = 0}$ is the BCS pair potential for the zero-current state at $T = 0$, $\hbar q$ is the superfluid momentum, $q_\xi = \sqrt{2\Delta_0 / \hbar D}$ is the inverse of the coherence length, $D$ is the electron diffusivity, $\hbar \omega_n = 2\pi k_B T (n + 1/2)$ is the Matsubara frequency, $k_B T_c = \Delta_0 \exp(\gamma_E) / \pi \simeq \Delta_0 / 1.76$ is the BCS critical temperature, and $\gamma_E = 0.577$ is the Euler constant. The penetration depth $\lambda$ and the magnitude of supercurrent density $j_s$ are given by

$$\frac{\lambda_0^2}{\lambda^2(s,T)} = \frac{4\pi k_B T}{\Delta_0} \sum_{\omega_n > 0} \sin^2 \theta, \quad (3)$$

$$\frac{j_s(s,T)}{j_0} = \frac{\sin \lambda_0^2}{\Delta_0 \lambda^2(s,T)}. \quad (4)$$

Here $\lambda_0 = \lambda(0,0) = \sqrt{\hbar / \pi \mu_0 \Delta_0 \sigma_n}$ is the BCS penetration depth at $T = 0$, $\sigma_n = 2N_0 D e^2$ is the normal state conductivity, $N_0$ is the normal state density of states at the Fermi energy, $j_0 = H_{c0} / \lambda_0 = \sqrt{\pi \sigma_n D \Delta_0 q_\xi}$, and $H_{c0} = \sqrt{N_0 / \mu_0 \Delta_0}$ is the BCS thermodynamic critical field at $T = 0$.

In the geometries shown in figures 1(a) and (b), the magnetic field and the superfluid flow depend on the depth $x$ from the surface. These $x$ dependences are determined from the Maxwell equations, $j_s = -\partial \mathbf{E}/\partial t$ and $\mu_0 H = (q/2|e|)\partial q$, namely,

$$\frac{\partial^2 q}{\partial x^2} = \frac{q}{\lambda^2(s,T)}, \quad (5)$$

$$\frac{H}{H_{c0}} = \sqrt{\frac{\partial (q/q_\xi)}{\partial (x/\lambda_0)}}. \quad (6)$$

Suppose the magnetic field at the surface is given by $H_0$. Then, the boundary conditions can be written as

$$H(0) = H_0, \quad \lim_{x \to \infty} q(x) \to 0. \quad (7)$$

For the geometry shown in figure 1(b), we have the additional boundary conditions at the S-$\Sigma$ interface,

$$H(d_-) = H(d_+), \quad q(d_-) = q(d_+). \quad (8)$$

Here $d_\pm = \lim_{\delta \to 0} d \pm \delta / 2$; remind we assume $\delta$ is thick enough to suppress the Josephson coupling between $S$ and $\Sigma$ but negligible compared with $\lambda$. 

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2.2. Solutions and depairing current at $T \to 0$

For $T \to 0$, the Matsubara sum is replaced with an integral, and equations (1)–(4) reduce to the well-known formulas obtained by Maki many years ago [16, 55–57] (see also appendix A),

\[
\frac{\Delta(s,0)}{\Delta_0} = \exp\left[ -\frac{\pi s}{4\Delta(s,0)} \right],
\]

\[
\frac{\lambda^2}{\lambda^2(s,0)} = \frac{\Delta(s,0)}{\Delta_0} \left[ 1 - \frac{4s^2}{3\pi\Delta(s,0)} \right],
\]

\[
j_s(s,0) = \sqrt{\frac{\pi s}{A}} \frac{\Delta(s,0)}{\Delta_0} \left[ 1 - \frac{4s^2}{3\pi\Delta(s,0)} \right] \frac{H_0}{\lambda_0},
\]

for $0 \leq s \leq \Delta(s,0)$, namely, $0 \leq s \leq 0.456\Delta_0$ or $0 \leq |q/q_\xi| \leq 0.675$. Note that equations (9)–(11) are local and thus applicable to any geometry, e.g. nanowire, film, bulk, multilayer etc.; the effects of geometries are incorporated via $s = s(x)$, which will be discussed in sections 3 and 4. Shown in figure 2 are $\Delta$, $\lambda$, and $j_s$ at $T = 0$ as functions of $|q|$. While $\Delta$ and the superfluid density $\lambda^2/\lambda^2$ are monotonically decreasing functions, $j_s$ exhibits a non-monotonic behavior. At smaller $|q|$ regions, $j_s$ is proportional to $|q|$. As $|q|$ increases, $j_s$ becomes dominated by a rapid reduction of the superfluid density and ceases to increase. At a threshold value $(q_d)$, $j_s$ reaches the maximum value: the depairing current density $j_d$ (see the blob in figure 2(b)).

Using equations (9)–(11) and the condition $\partial j_s/\partial x = 0$, we have [55, 56, 58]

\[
j_d(0) = \sqrt{\frac{\pi s_d}{A}} \frac{\Delta_d}{\Delta_0} \left( 1 - \frac{4q_d^2}{3\pi} \frac{H_0}{\lambda_0} \right) = 0.595 \frac{H_0}{\lambda_0},
\]

where $q_d/\xi = \sqrt{x_d/\Delta_0} = 0.487$.

3. Semi-infinite superconductor

3.1. Spatial distributions of $H$, $j_s$, $\Delta$, and $\lambda$

For a simple semi-infinite superconductor shown in figure 1(a) and self-consistently solve the coupled Maxwell–Usadel equations at $T \to 0$ (equations (5)–(11)). Shown in figure 3(a) are the distributions of $H(x)$ and $j_s(x)$. For $H_0 < H_{0d}$ (blue), we have the solid and dashed curves that almost overlap, which can be understood as follows. Since the current density is so small that the non-linear Meissner effect is negligible, the London theory is applicable. Solving the London equation, we obtain $H(x)/H_0 = j_s(x)/J_{0d} = \exp(-x/\lambda_0)$, consistent with the numerical solutions. As $H_0$ increases (orange and red), however, the non-linear Meissner effect manifests itself, and the London theory is not applicable. In fact, the solid and dashed curves no longer overlap. Shown in figure 3(b) are the penetration depth $\lambda(x)$ and the pair potential $\Delta(x)$. The pair potential (penetration depth) is decreased (increased) at the surface due to the strong pair-breaking current and recovers at deeper regions where the current density is exponentially small.

3.2. Superheating field

For a simple semi-infinite superconductor in the diffusive limit, in which the current density is a monotonically decreasing function of $x$ (see figure 3), the superheating field $H_{sh}$ is given by $H_0$ which induces $j_s(x)|_{x=0} = j_d$. Then, we can derive a simple formula of $H_{sh}$. Integrating both the sides of equation (5) from $x = 0$ to $\infty$, we obtain

\[
\int_0^\infty \left( \frac{\beta^2}{\beta^2(s,0)} \right) \left( 1 - \frac{4s^2}{3\pi\Delta(s,0)} \right) \frac{H_0}{\lambda_0} dx = 0.595 \frac{H_0}{\lambda_0}.
\]
We can consider any materials combination, but here we focus on 4. Layered superconductors

The mean free path $\ell = \frac{\pi \rho d}{\lambda_0} / \lambda_0$, $\rho = \rho_n / \rho_h$.

Substituting the depairing value $s_d$ into $s(0)$, we obtain the formula

$$H_{ab}(T) = H_{c0} \sqrt{1 - \left(1 - \frac{\pi \rho d}{2}\right) e^{-\frac{s_d^2}{2}} - \frac{2}{3} \frac{s_d^2}{2}}$$

which is valid for an arbitrary $T$. Note that equation (17) reproduces the GL superheating field $H_{ab}(T) = (\sqrt{2}/3)H_c(T)$ at $T \approx T_c$ [16], consistent with the previous studies [8, 9].

For $T = 0$, substituting equations (10) and (15) into equation (17), we obtain $H_{ab}$, in the diffusive limit [16]

$$H_{ab}(0) = H_{c0} \sqrt{1 - \left(1 - \frac{\pi \rho d}{2}\right)} e^{-\frac{s_d^2}{2}} - \frac{2}{3} \frac{s_d^2}{2}$$

This is slightly smaller than that of an extreme type-II ($\lambda/\xi > 1$) superconductor in the clean limit [11, 12],

$$H_{ab}^{clean}(0) = 0.84H_{c0},$$

and consistent with the previous study on the effect of nonmagnetic impurities [14], in which $H_{ab}(\sim 0.8H_{c0})$ as a function of the mean free path $\ell$ takes its maximum value at $\ell = 5.32\xi_0 = \ell_s$ and decreases with $\ell$ for $\ell < \ell_s$.

### 4. Layered superconductors

Now we consider the layered heterostructure shown in figure 1(b). The model parameters are summarized in table 1: the S-layer thickness $d$ and the three ratios of materials parameters,

$$\Delta_0 = \Delta_0^{S}\left/\Delta_0^{S}\right., \quad r_\Delta = H_\Delta^{(S)} / H_\Delta^{(S)}, \quad r_\sigma = \sigma_n^{(S)} / \sigma_n^{(S)}.$$  

Here $\Delta_0 (i = S, \Sigma)$ is the pair-potential in the zero-current state at $T = 0$, $H_\Delta^{(S)}$ is the thermodynamic critical field at $T = 0$, and $\sigma_n^{(S)}$ is the normal-state conductivity. The other materials parameters can be expressed using these parameters; e.g. $D^{(S)} / D^{(S)} = r_\sigma r_\Delta^2 / r_\Delta$, $\lambda_0^{(S)} / \lambda_0^{(S)} = 1/\sqrt{\Delta_0^2}$, etc.

#### 4.1. Spatial distributions of $H, j_s, \Delta, \lambda$ in a layered superconductor

We can consider any materials combination, but here we focus on the simplest example that captures the striking feature of the layered structure: the surface-current-reduction effect. Let us assume that the S layer is made of the same material as the $\Sigma$ region but has a different concentration of nonmagnetic impurities, e.g. $\Sigma$ and $S$ are a bulk Nb and a ditter Nb-layer, respectively. We can fully solve the coupled Maxwell–Usadel equations for $T \rightarrow 0$ (equations (5)–(11)) in the same way as done for a semi-infinte superconductor in section 3.1. Shown in figure 4 are the distributions of $H(x), j_s(x), \Delta(x)$ and $\lambda(x)$ calculated for $d = 0.5\lambda_0^{(S)}$, $r_\Delta = r_\sigma = 1$, and $r_s = 0.25$. In the S layer, the magnetic field $H$ (dashed curves) slowly attenuates as $x$ increases, and the current density $j_s = -\partial_x H$ (solid curves) is significantly suppressed. As a result, $\Delta$ in the S region is less suppressed as compared with that in the $\Sigma$ region, even though it is the S layer which is directly exposed to the external magnetic field (see the solid curves in figure 4(b)). In the $\Sigma$ region, $H(x), j_s(x), \Delta(x)$, and $\lambda(x)$ are monotonic functions of $x$.

The non-monotonic decay of the current density is a common feature in the S-$\Sigma$ hetero-structure in which S has a different penetration depth from $\Sigma$ [15, 18–21]. When $\Sigma$ has a shorter (longer) penetration depth than S, the magnetic field in the S layer decays slower (more rapidly) than exponential, and the current density is suppressed (enhanced). Such reduction (enhancement) of the surface current results from a counterflow induced by the substrate $\Sigma$. When the penetration depths in S and $\Sigma$ are balanced, the magnetic field and the current density in the S layer exhibit the well-known exponential decay.

#### 4.2. Superheating field of a layered superconductor

Next, we evaluate $H_{ab}$ of the layered structure. Since the current density in the layered structure does not necessarily take its maximum value at $x = 0$ as seen in figure 4, the $H_{ab}$ formula given by equation (17) is not applicable. Instead, $H_{ab}$ is given by the surface magnetic-field which induces $j_s = j_d^{(S)}$ at $x = 0$ or $j_s = j_d^{(S)}$ at $x = d$. Shown as the solid curves in figure 5 are $H_{ab}$ as functions of $d$ calculated from the coupled Maxwell–Usadel equations for various materials combinations. We find $H_{ab}$ increases with $d$, takes its maximum value at $d = d_m \approx \lambda_0^{(S)}$, and decreases with $d$ at $d > d_m$. 

Table 1. Parameters of the layered structure.

| Parameter | Formula |
|-----------|---------|
| S layer thickness | $d$ |
| Pair-potential ratio | $r_\Delta = \Delta_0^{(S)} / \Delta_0^{(S)}$ |
| Critical-field ratio | $r_H = H_\Delta^{(S)} / H_\Delta^{(S)}$ |
| Normal-conductivity ratio | $r_\sigma = \sigma_n^{(S)} / \sigma_n^{(S)}$ |

Figure 4. Spatial distributions of (a) $H, j_s$, $\Delta$, and $\lambda$ in a layered superconductor calculated for $d = 0.5\lambda_0^{(S)}$, $r_\Delta = r_\sigma = 1$, and $r_\sigma = 0.25$. The penetration depth of the S layer in the zero-current state is given by $\lambda_0^{(S)} = \lambda_0^{(S)} / \sqrt{\Delta_0^2}$.
We use the values we obtained for the diffusive limit in good approximation of $\lambda_0^{(S)}$ respectively. Equations (21)–(23) are correct when both $S$ and $\Sigma$ are clean-limit superconductors at $T = 0$, but are not applicable to the diffusive limit.

In fact, if we substitute the clean-limit results $H_{sh}^{(i)} = 0.84H_{sh}^{(i)}$ into the formula, we obtain much larger $H_{sh}$ than figure 5, and equations (21)–(23) do not yield a good approximation of $H_{sh}$ of dirty heterostructure. To get a better approximation of $H_{sh}$ of dirty heterostructure, we use the values we obtained for the diffusive limit in section 3.2: $H_{sh}^{(S)} = 0.795H_{sh}^{(S)}$ and $H_{sh}^{(S)} = 0.795r_HH_{sh}^{(S)}$ (see equation (18)). The penetration depth of the S layer in the zero-current state is given by $\lambda_0^{(S)} = \lambda_0^{(S)} / \sqrt{\Delta r_\Sigma}$. Shown as the dashed gray curves in figure 5 are $H_{sh}^{(i)}(d)$ calculated from equation (21). The existence of the optimum thickness $d_{m}$ can be understood as follows [18–21]. Suppose $\lambda_0^{(S)} > \lambda_0^{(S)}$. Then, the counterflow induced by $\Sigma$ decreases the current density at the surface of S by a factor of $1/c_1$, and the maximum field that the S layer can withstand increases to $H_0 = c_1H_{sh}^{(S)}$. This enhancement is pronounced as $d$ decreases. On the other hand, the S layer attenuate the magnetic field down to $H_1 = H_0/\sigma_2$ at the S-$\Sigma$ interface, so the maximum field that $\Sigma$ can withstand is given by $H_0 = c_2H_{sh}^{(S)}$, which increases with $d$. The interplay between the reduction of the surface current and that of the shielding efficiency results in the optimum thickness $d_{m}$, at which the screening current densities in S and $\Sigma$ simultaneously reach $\mu_0H_{sh}^{(S)}$ and $H_{sh}^{(S)}$, respectively.

The disagreements between the full calculations (solid) and the approximate formula (dashed) result from the non-linear Meissner effect. The strong current density $\sim j_0$ increases the penetration depth from $\lambda_0$ to $\lambda(x, 0)/\lambda_{\Sigma}$, so that a larger $d$ becomes necessary to protect $\Sigma$ than expected from the London theory. As a result, the maximum in $H_{sh}^{(S)}(d)$ obtained from the full calculation is located at thicker regions.

Note $H_{sh}$ can decrease when S has a shorter penetration depth than $\Sigma$. In this case, the current density in the S layer is enhanced as mentioned in section 4.1 and can reach the depairing current density at rather small $H_0$. For instance, $(r_\Sigma, r_H, r_\Sigma) = (1, 1, 0.25)$, which yields $\lambda_0^{(S)} = \lambda_0^{(S)} / \sqrt{\Delta r_\Sigma} = 0.5\lambda_0^{(S)} < \lambda_0^{(S)}$, results in $H_{sh}^{(S)} = 0.46H_{sh}^{(S)}$ for $d = 0.05\lambda_0^{(S)}$.

5. Discussions

We have investigated a simple semi-infinite superconductor shown in figure 1(a) and a layered heterostructure shown in figure 1(b) in the diffusive limit. The coupled Maxwell–Usadel equations at $T \to 0$ have been self-consistently solved to obtain the spatial distributions of $H_\Sigma(x), j_\Sigma(x), \lambda(x)$, and $\Delta(x)$ for both the structures (see figures 3 and 4). The distributions of $H$ and $j_\Sigma$ obey the London theory for $j_\Sigma \ll j_0$, while the non-linear Meissner effect manifests itself for $j_\Sigma \sim j_0$, where the London theory is no longer valid. We have found the superheating field $H_{sh}$ of a semi-infinite superconductor in the diffusive limit is given by $H_{sh} = 0.795H_0$ at $T \to 0$; on the other hand, $H_{sh}$ of a layered structure depends on materials combinations and the thickness $d$ of the S layer, which can be maximized by tuning $d$ to the optimum thickness (see figure 5).

Our results can be tested by experiments. We can expect that the maximum operating field of an SRF cavity made from a bulk dirty-BCS-superconductor is given by its superheating field. Taking impurity-doped dirty Nb with $t \ll \xi$, for example, we have $\mu_0H_{sh} = 0.795 \times 200 \text{ mT} = 160 \text{ mT}$ at $T \to 0$. The maximum operating field can be improved by applying a layered structure onto the inner surface of a cavity. For instance, $H_{sh}$ of bulk dirty Nb is pushed up to $\mu_0H_{sh} = 1.04 \times 200 \text{ mT} = 210 \text{ mT}$ by laminating a thin dirtier-Nb-layer on the
surface (see figure 5(a)). Other materials combinations can also improve the field limit, e.g. $\mu_0H_{\text{th}} = 2.26 \times 200 \text{ mT} = 450 \text{ mT}$ for the Nb$_3$Sn-Nb structure (see figure 5(b)). These results can be tested using various techniques, e.g. high power RF pulse [59], RF characterization of samples [30, 60–62], third-harmonic voltage [25–27, 34, 63], magnetization measurements for ellipsoid samples [23], muon-spin-rotation technique [24, 64], etc. Note that the RF heating of the cavity wall due to quasiparticles [38, 44, 65, 66], vortices [67–76], and grain boundaries [77] can limit the achievable field. Also, it is important to minimize the inhomogeneity in the plane of the layer [13, 21]; topographic defects at the surface [78–86] can cause vortex penetration at a field much smaller than $H_{\text{th}}$; other types of materials defects (e.g. locally-suppressed superconductivity) can suppress $j_d$ and $H_{\text{th}}$ (see e.g. [44, 16]), limiting the achievable field.

The approximate formula (equation (21)), which was derived using the London equation [18], would be useful to know the optimum thickness $d_m$. For a clean-limit superconductor at $T = 0$, the non-linear Meissner effect is negligible. Hence, simply substituting the clean-limit result $H_{\text{th}}^{(i)} = 0.84H_{\text{th}}^{(0)}$ into equation (21), we obtain the microscopically valid theory [20]. On the other hand, for a dirty-limit superconductor, the clean-limit result $H_{\text{th}}^{(i)} = 0.84H_{\text{th}}^{(0)}$ cannot be applied to equation (21). Besides, since the non-linear Meissner effect is no longer negligible, it is nontrivial if the results obtained by substituting the dirty-limit result $H_{\text{th}}^{(i)} = 0.795H_{\text{th}}^{(0)}$ into equation (21) are valid. According to our results (see figure 5), however, the disagreements between the numerical solutions of the coupled Maxwell–Usadel and the approximate formula are $\sim$10%, and equation (21) would still be qualitatively useful to estimate $H_{\text{th}}(d)$ and $d_m$ even in the diffusive limit.

Besides, the above discussions provide an implication on surface processing of Nb cavities. As mentioned in section 1, some recipes (e.g. low-$T$ baking, N-infusion etc) result in an inhomogeneous impurity-distribution which decays within the penetration depth. The S-Σ structure offers a simple model of such an inhomogeneous material. Consider S as the dirtier region at the surface. Then, we find a current counterflow induced by bulk leads to a suppression of the current density at the surface, resulting in an enhancement of $H_{\text{th}}$ [21]. Using the results of the present study, we can guess $H_{\text{th}}$ of an impurity-doped dirty Nb can be pushed up to $\sim$200 mT by increasing impurity concentrations in the vicinity of the surface via some surface processing. To build a more sophisticated model, we may consider a depth-dependent impurity scattering rate instead of the S-Σ model. Ngampruetikom and Sauls [15] discussed such a model for an extreme type-II superconductor with a large GL parameter and showed that an inhomogeneous impurity-concentration pushes up $H_{\text{th}}$: consistent with the earlier prediction based on the S-Σ model of the baking and N-infusion [21]. The similar calculation for the dirty-limit materials is possible by extending the formulation in section 2.1.

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Appendix A. Derivations of equations (9) and (10)

We use $\Delta_0 = \Delta(0,0)$ as a unit of energy. The Mastubara Green’s function $u = \cot\theta$ for the current carrying state satisfies

$$
\left(1 - \frac{\zeta}{\sqrt{1 + u^2}}\right) u = \frac{\omega_0}{\Delta},
$$

(A1)

where $\zeta = s/\Delta(s,T)$. The self-consistency equation at $T \to 0$ reduces to

$$
0 = \int_0^\infty \frac{d\omega}{\Delta\sqrt{1 + u^2}} - \frac{1}{\sqrt{\omega^2 + 1}}
= \int_{u_0}^\infty du \left(1 - \frac{\zeta}{1 + u^2}\right)^{3/2}
\times \left(\frac{1}{\sqrt{1 + u^2}} - \frac{1}{\sqrt{\Delta^2 - [(1 - \zeta/\sqrt{1 + u^2})u]^2}}\right)
= - \ln \Delta - \sinh^{-1} u_0 - \zeta\left[\frac{\pi}{2} - \tan^{-1} u_0 - \frac{u_0}{1 + u_0^2}\right]
= - \ln \Delta - \sinh^{-1} u_0 - \zeta\left[\frac{\pi}{2} - \tan^{-1} u_0 - \frac{u_0}{1 + u_0^2}\right]
$$

(A2)

Here $u_0(\zeta)$ is defined by $(1 - \zeta/\sqrt{1 + u_0^2})u_0 = +0$. Equation (A2) is the formula of $\Delta_0$ valid for an arbitrary $s$. For $\zeta \leq 1$, we have $u_0 = 0$, and equation (A2) reduces to equation (9).

The superfluid density $\lambda^2_0/\lambda^2$ can be calculated from equation (3). At $T \to 0$, we find

$$
\lambda^2_0(2,0) = \frac{2}{\pi} \int_0^\infty \frac{d\omega}{1 + u^2}
= 2\Delta \int_{u_0}^\infty du \left(\frac{1}{1 + u^2} - \frac{\zeta}{1 + u^2} + \frac{\zeta u^2}{(1 + u^2)\pi}\right)
= \Delta \left[1 - \frac{2}{\pi} \tan^{-1} u_0 - \frac{4\zeta}{3\pi} \left\{1 - \frac{u_0 (3 + 2u_0^2)}{2(1 + u_0^2)^2}\right\}\right],
$$

(A3)

which is the formula of $\lambda$ valid for an arbitrary $s$. For $\zeta \leq 1$, we have $u_0 = 0$, and equation (A3) results in equation (10).

See also [16] for generalized formulas which incorporate the effects of a finite Dynes parameter.

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References

[1] Padamsee H 2017 50 years of success for SRF accelerators: a review Supercond. Sci. Technol. 30 053003

[2] Gurevich A 2017 Theory of RF superconductivity for resonant cavities Supercond. Sci. Technol. 30 034004

[3] Georg R L, Eremeev G V, Padamsee H and Shenelvin V D 2007 High gradient studies for ILC with single cell re-entrant shape and elliptical shape cavities made of fine-grain and large-grain niobium Proc. PAC2007 (Albuquerque, New Mexico, USA) (JACoW) p 2337

[4] Singer W et al 2013 Development of large grain cavities Phys. Rev. ST Accel. Beams 16 012003

[5] Kubo T, Ajima Y, Inoue H, Umemori K, Watanabe Y and Yamanaka M 2014 In-house production of a large-grain single-cell cavity at cavity fabrication facility and results of performance tests Proc. IPAC2014 (Dresden, Germany) (JACoW) p 2519

[6] Grassellino A et al 2017 Unprecedented quality factors at accelerating gradients up to 45 MV/m⁻¹ in niobium superconducting resonators via low temperature nitrogen infusion Supercond. Sci. Technol. 30 094004

[7] Dhakal P, Chetri S, Balachandran S, Lee P J and Ciocvati G 2018 Effect of low temperature baking in nitrogen on the performance of a niobium superconducting radio frequency cavity Phys. Rev. Accel. Beams 21 032001

[8] Kramer L 1968 Stability limits of the Meissner state and the mechanism of spontaneous vortex nucleation in superconductors Phys. Rev. 170 475

[9] Transtrum M K, Catelani G and Sethna J P 2011 Superheating field of superconductors within Ginzburg–Landau theory Phys. Rev. B 83 094505

[10] Liarte D B, Transtrum M K and Sethna J P 2016 Ginzburg–Landau theory of the superheating field anisotropy of layered superconductors Phys. Rev. B 94 144504

[11] Galaiako V P 1966 Stability limits of the superconducting state in a magnetic field for superconductors of the second kind Sov. Phys. JETF 23 475 (http://www.jetp.ac.ru/cgi-bin/e/index/e/23/3/p475?a=list)

[12] Catelani G and Sethna J P 2008 Temperature dependence of the superheating field for superconductors in the high-κ London limit Phys. Rev. B 78 224509

[13] Liarte D B, Posen S, Transtrum M K, Catelani G, Liepe M and Sethna J P 2017 Theoretical estimates of maximum fields in superconducting resonant radio frequency cavities: stability theory, disorder and laminates Supercond. Sci. Technol. 30 033002

[14] Pei-Jen Lin F and Gurevich A 2012 Effect of impurities on the superheating field of type-II superconductors Phys. Rev. B 85 054513

[15] Ngaumpruetikorn V and Sauls J A 2019 Effect of inhomogeneous surface disorder on the superheating field of superconducting RF cavities Phys. Rev. Res. 1 012015

[16] Kubo T 2020 Superfluid flow in disordered superconductors with Dynes pair-breaking scattering: depairing current, kinetic inductance and superheating field Phys. Rev. Res. 2 033203

[17] Gurevich A 2006 Enhancement of RF breakdown field of superconductors by multilayer coating Appl. Phys. Lett. 88 012511

[18] Kubo T, Iwashita Y and Saeki T 2014 Radio-frequency electromagnetic field and vortex penetration in multilayered superconductors Appl. Phys. Lett. 104 032603

[19] Posen S, Transtrum M K, Catelani G, Liepe M U and Sethna J P 2015 Shielding superconductors with thin films as applied to RF cavities for particle accelerators Phys. Rev. Appl. 4 044019

[20] Gurevich A 2015 Maximum screening fields of superconducting multilayer structures AIP Adv. 5 017112

[21] Kubo T 2017 Multilayer coating for higher accelerating fields in superconducting radio-frequency cavities: a review of theoretical aspects Supercond. Sci. Technol. 30 023001

[22] Valente-Feliciano A-M 2016 Superconducting RF materials other than bulk niobium: a review Supercond. Sci. Technol. 29 113002

[23] Tan T, Wolak M A, Xi X T, Tajima T and Civale L 2016 Magnesium diboride coated bulk niobium: a new approach to higher acceleration gradient Sci. Rep. 6 35879

[24] Junginger T et al 2018 Field of first magnetic flux entry and pinning strength of superconductors for RF application measured with muon spin rotation Phys. Rev. Accel. and Beams 21 032002

[25] Antoine C Z, Aburas M, Weiss F, Iwashita Y, Hayano H, Kato S, Kubo T and Saeki T 2019 Optimization of tailored multilayer superconductors for RF application and protection against premature vortex penetration Supercond. Sci. Technol. 32 085005

[26] Ito H et al 2019 Lower critical field measurement of NbN multilayer thin film superconductor at KEK Proc. SRF2019 (Dresden, Germany) (JACoW) p 632

[27] Katayama R et al 2019 Evaluation of the superconducting characteristics of multi-layer thin-film structures of NbN and SiO₂ on pure Nb substrate Proc. SRF2019 (Dresden, Germany) (JACoW) p 807

[28] Kubo T 2019 Optimum multilayer coating of superconducting particle accelerator cavities and effects of thickness dependent material properties of thin films Japan. J. Appl. Phys 58 088001

[29] Ito R, Nagata T, Hayano H, Katayama R, Kubo T, Saeki T, Iwashita Y and Ito H 2019 Nb₃Sn thin film coating method for superconducting multilayered structure Proc. SRF2019 (Dresden, Germany) (JACoW) p 628

[30] Oseroff T, Liepe M, Sun Z, Moekly B and Sowa M 2019 RF characterization of novel superconducting materials and multilayers Proc. SRF2019 (Dresden, Germany) (JACoW) p 950

[31] Thoeng E, Junginger T, Kolb P, Matheson B, Morris G, Muller N, Saminathan S, Baartman R and Laxdal R E 2019 Progress of TRIUMF beta-SRF facility for novel SRF materials Proc. SRF2019 (Dresden, Germany) (JACoW) p 964

[32] Turner D, Malyshov O B, Burt G, Junginger T, Gurran L, Dumbell K D, May A J, Pattalwar N and Pattalwar S M 2019 Characterization of flat multilayer thin film superconductors Proc. SRF2019 (Dresden, Germany) (JACoW) p 968

[33] Senevirathne I H, Ciocvati G and Delayen J R 2019 Measurement of the magnetic field penetration into superconducting thin films Proc. SRF2019 (Dresden, Germany) (JACoW) p 978

[34] Ito H, Hayano H, Kubo T and Saeki T 2020 Vortex penetration field measurement system based on third-harmonic method for superconducting RF materials Nucl. Instrum. Methods Phys. Res. A 955 163284

[35] Ciocvati G 2006 Improved oxygen diffusion model to explain the effect of low-temperature baking on high field losses in niobium superconducting cavities Appl. Phys. Lett. 89 022507

[36] Romanenko A, Grassellino A, Barkov F, Suter A, Salman Z and Prokscha T 2014 Strong Meissner screening change in superconducting radio frequency cavities due to mild baking Appl. Phys. Lett. 104 072601
[37] Gurevich A and Kubo T 2017 Surface impedance and optimum surface resistance of a superconductor with an imperfect surface Phys. Rev. B 96 184515
[38] Kubo T and Gurevich A 2019 Field-dependent nonlinear surface resistance and its optimization by surface nanostructuring in superconductors Phys. Rev. B 100 064522
[39] Charrier J P, Coadou B and Visentin B 1998 Improvements of superconducting cavity performances at high accelerating gradient Proc. EPAC1998 (Stockholm, Sweden) (JACoW) p 1885
[40] Lilje L et al 1999 Electropolishing and in-situ baking of 1.3 GHz niobium cavities Proc. SRF1999 (New Mexico, USA) (JACoW) p 74
[41] Saito K and Kneisel P 1999 Temperature dependence of the surface resistance of niobium at 1300 MHz Proc. SRF1999 (New Mexico, USA) (JACoW) p 277
[42] Birnbaum H K, Grossbeck M L and Amano M 1976 Hydride precipitation in Nb and some properties of NbH J. Less Common Met. 49 357
[43] Barkov F, Romanenko A and Grassellino A 2012 Direct observation of hydrides formation in cavity-grade niobium Phys. Rev. ST Accel. Beams 15 122901
[44] Kubo T 2020 Weak-field dissipative conductivity of a dirty superconductor with Dynes subgap states under a dc bias current up to the depairing current density Phys. Rev. Res. 2 013302
[45] Lechner E M, Oli B D, Makita J, Ciocvati G, Gurevich A, Ivarone M, Tunneling E and Photoelectron X-R 2020 Spectroscopy studies of the superconducting properties of nitrogen-doped niobium resonator cavities Phys. Rev. Appl. 13 044044
[46] Belzig W, Bruder C and Schön G 1996 Local density of states in a dirty normal metal connected to a superconductor Phys. Rev. B 54 9443
[47] Belzig W, Wilhelm F K, Bruder C, Schön G and Zaikin A D 1999 Quasiclassical Green’s function approach to mesoscopic superconductivity Superlattices Microstruct. 25 1251
[48] Xu D, Yip S K and Sauls J A 1995 Nonlinear Meissner effect in unconventional superconductors Phys. Rev. B 51 16233
[49] Groll N, Gurevich A and Chiorescu I 2010 Measurement of the nonlinear Meissner effect in superconducting Nb films using a resonant microwave cavity: a probe of unconventional pairing symmetries Phys. Rev. B 81 020504(R)
[50] Golubov A A, Kupriyanov M Y and Il’ichev E 2004 The current-phase relation in Josephson junctions Rev. Mod. Phys. 76 411
[51] Eilenberger G 1968 Transformation of Gorkov’s equation for type II superconductors into transport-like equations Z. Phys. 214 195
[52] Larkin A I and Ovchinnikov Y N 1969 Quasiclassical method in the theory of superconductivity Sov. Phys. JETP 28 1200 (http://www.jetp.ac.ru/cgi-bin/e/index/e/28/6/p1200?&la=eng)
[53] Usadel K D 1970 Generalized diffusion equation for superconducting alloys Phys. Rev. Lett. 25 507
[54] Kopnin N B 2001 Theory of Nonequilibrium Superconductivity (Oxford: Oxford University Press)
[55] Maki K 1963 On persistent currents in a superconducting alloy I Prog. Theor. Phys. 29 10
[56] Maki K 1963 On persistent currents in a superconducting alloy II Prog. Theor. Phys. 333 10
[57] Clem J R and Kogan V G 2012 Kinetic impedance and depairing in thin and narrow superconducting films Phys. Rev. B 86 174521
[58] Yu Kupriyanov M and Lukichev V F 1980 Temperature dependence of pair-breaking current in superconductors Sov. J. Low Temp. Phys. 6 210
[59] Posen S, Valles N and Liepe M 2015 Radio frequency magnetic field limits of Nb and Nb3Sn Phys. Rev. Lett. 115 047001
[60] Welander P B, Franzi M and Tantawi S 2015 Cryogenic RF characterization of superconducting materials at SLAC with hemispherical cavities Proc. SRF2015 (Whistler, BC, Canada) (JACoW) p 735
[61] Goudket P, Junginger T and Xiao B P 2017 Devices for SRF material characterization Supercond. Sci. Technol. 30 013001
[62] Oikawa H, Higashiguchi T and Hayano H 2019 Design of niobium-based mushroom-shaped cavity for critical magnetic field evaluation of superconducting multilayer thin films toward achieving higher accelerating gradient cavity Japan. J. Appl. Phys. 58 028001
[63] Lamura G, Aurino M, Andreone A and Villégier J-C 2009 First critical field measurements of superconducting films by third harmonic analysis J. Appl. Phys. 106 053903
[64] Keckert S et al 2019 Critical fields of Nb3Sn prepared for superconducting cavities Supercond. Sci. Technol. 32 075004
[65] Gurevich A 2014 Reduction of dissipative nonlinear conductivity of superconductors by static and microwave magnetic fields Phys. Rev. Lett. 113 087001
[66] Martinello M, Checchin M, Romanenko A, Grassellino A, Aderhold S, Chandrasekeran S K, Melnychuk O, Posen S and Sergatskov D A 2018 Field-enhanced superconductivity in high-frequency niobium accelerating cavities Phys. Rev. Lett. 121 224801
[67] Romanenko A, Grassellino A, Crawford A C, Sergatskov D A and Melnychuk O 2014 Ultra-high quality factors in superconducting niobium cavities in ambient magnetic fields up to 190 mG Appl. Phys. Lett. 105 234103
[68] Kubo T 2016 Flux trapping in superconducting accelerating cavities during cooling down with a spatial temperature gradient Prog. Theor. Exp. Phys. 2016 053001
[69] Huang S, Kubo T and Geng R L 2016 Dependence of trapped-flux-induced surface resistance of a large-grain Nb superconducting radio-frequency cavity on spatial temperature gradient during cooldown through Tc Phys. Rev. Accel. Beams 19 082001
[70] Posen S, Checchin M, Crawford A C, Grassellino A, Martinello M, Melnychuk O S, Romanenko A, Sergatskov D A and Trenikhina Y 2016 Efficient expulsion of magnetic flux in superconducting radiofrequency cavities for high Qo applications J. Appl. Phys. 119 213903
[71] Checchin M, Martinello M, Romanenko A, Grassellino A, Sergatskov D A, Posen S, Melnychuk O and Zasadzinski J F 2016 Quench-induced degradation of the quality factor in superconducting resonators Phys. Rev. Appl. 5 040419
[72] Liarte D B, Hall D, Koulafis P N, Miyazaki A, Senanian A, Liepe M and Sethna J P 2018 Vortex dynamics and losses due to pinning: dissipation from trapped magnetic flux in resonant superconducting radio-frequency cavities Phys. Rev. Appl. 10 054057
[73] Miyazaki A and DelSolaro W V 2019 Two different origins of the Q-slope problem in superconducting niobium film cavities for a heavy ion accelerator at CERN Phys. Rev. Accel. Beams 22 073101
[74] Orlov B et al 2019 High-frequency nonlinear response of superconducting cavity-grade Nb surfaces Phys. Rev. Appl. 11 064030
[75] Dhakal P, Ciocvati G and Gurevich A 2020 Flux expulsion in niobium superconducting radio-frequency cavities of
different purity and essential contributions to the flux sensitivity Phys. Rev. Accel. Beams 23 023102

[76] Pathirana W P M R and Gurevich A 2020 Nonlinear dynamics and dissipation of a curvilinear vortex driven by a strong time-dependent Meissner current Phys. Rev. B 101 064504

[77] Sheikhzada A and Gurevich A 2017 Dynamic transition of vortices into phase slips and generation of vortex-antivortex pairs in thin film Josephson junctions under dc and ac currents Phys. Rev. B 95 214507

[78] Iwashita Y, Tajima Y and Hayano H 2008 Development of high resolution camera for observations of superconducting cavities Phys. Rev. ST Accel. Beams 11 093501

[79] Ge M, Wu G, Burk D, Ozelis J, Harms E, Sergatskov D, Hicks D and Cooley L D 2011 Routine characterization of 3D profiles of SRF cavity defects using replica techniques Supercond. Sci. Technol. 24 035002

[80] Yamamoto Y, Hayano H, Kako E, Noguchi S, Shishido T and Watanabe K 2013 Achieving high gradient performance of 9-cell cavities at KEK for the international linear collider Nucl. Instrum. Methods Phys. Res. A 729 589

[81] Wenskat M 2019 First attempts in automated defect recognition in superconducting radio-frequency cavities J. Instrum. 14 06021

[82] Pudasaini U, Eremeev G, Reece C E, Tuggle J and Kelley M J 2020 Analysis of RF losses and material characterization of samples removed from a Nb$_3$Sn-coated superconducting RF cavity Supercond. Sci. Technol. 33 045012

[83] Knobloch J, Geng R L, Liepe M and Padamsee H 1999 High-field Q slope in superconducting cavities due to magnetic field enhancement at grain boundaries Proc. SRF 1999 (La Fonda Hotel, Santa Fe, New Mexico, USA) (JACoW) p 77

[84] Kubo T 2015 Field limit and nano-scale surface topography of superconducting radio-frequency cavity made of extreme type II superconductor Prog. Theor. Exp. Phys. 2015 063G01

[85] Kubo T 2015 Magnetic field enhancement at a pit on the surface of a superconducting accelerating cavity Prog. Theor. Exp. Phys. 2015 073G01

[86] Xu C, Reece C E and Kelley M J 2016 Simulation of nonlinear superconducting rf losses derived from characteristic topography of etched and electropolished niobium surfaces Phys. Rev. Accel. Beams 19 033501