Facets of Chiral Perturbation Theory

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Motivation and overview

Goal

systematic and quantitative treatment of the Standard Model at low energies ($E < 1$ GeV)

- Effective Field Theory (EFT)
- Lattice Field Theory

main objectives

- understand physics of the SM at low energies
- look for evidence of new physics
$E < 1 \text{ GeV}$:
- strong-coupling regime of QCD \[\rightarrow\] not accessible in standard perturbation theory

key concept for EFT: approximate chiral symmetry of QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{tr}(G_{\mu\nu} G^{\mu\nu}) + \sum_{f=1}^{6} \bar{q}_f \left( i\gamma^\mu D_\mu - m_f \mathbb{1}_c \right) q_f$$

for $m_f = 0$: chiral components can be rotated separately

$\bar{q}_{fL} = \frac{1}{2} (1 - \gamma_5) q_f$, \quad $\bar{q}_{fR} = \frac{1}{2} (1 + \gamma_5) q_f$

\[\rightarrow\] chiral symmetry $SU(n_F)_L \times SU(n_F)_R \times U(1)_V$
\( m_f = 0 : \)
- very good approximation for \( n_F = 2 \) (\( u, d \))
- reasonable approximation for \( n_F = 3 \) (\( u, d, s \))

in contrast to isospin \( SU(2) \) or flavour \( SU(3) \):

**no sign of chiral symmetry in hadron spectrum**

many other arguments in favour of

\[
SU(n_F)_L \times SU(n_F)_R \times U(1)_V \quad \rightarrow \quad SU(n_F)_V \times U(1)_V
\]

**Goldstone** theorem:

\[ \exists n_F^2 - 1 \text{ massless (for } m_f = 0 \text{) Goldstone bosons} \]
Goldstone fields parametrize $SU(n_F)^L \times SU(n_F)^R / SU(n_F)^V$

| $n_F$ | $n_F^2 - 1$ | Goldstone bosons |
|------|-------------|------------------|
| 2    | 3           | $\pi$            |
| 3    | 8           | $\pi, K, \eta$   |

even in the real world ($m_q \neq 0$):

pseudo-scalar meson exchange dominates amplitudes at low energies

$\rightarrow$ construct EFT for pseudo-Goldstone bosons

nonlinear realization of chiral symmetry $\rightarrow$

effective Lagrangian necessarily nonpolynomial
consequence:

**EFT nonrenormalizable QFT: Chiral Perturbation Theory (CHPT)**

Weinberg, Gasser, Leutwyler,...

nevertheless: CHPT fully renormalized QFT
(to next-to-next-to-leading order)

basis for systematic low-energy expansion:

pseudo-Goldstone bosons decouple for vanishing momenta and masses

systematic approach for low-energy hadron physics

most advanced in meson sector (up to 2 loops)
also single-baryon sector and few-nucleon systems
electroweak interactions can be included
### Effective chiral Lagrangian (meson sector)

| $\mathcal{L}$ | loop order |
|---------------|------------|
| $\mathcal{L}_{\text{chiral order}}$ (# of LECs) | |
| $\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}^{\text{odd}}(0) + \mathcal{L}_{G_F p^2}^{\Delta S=1}(2) + \mathcal{L}_{G_8 e^2 p^0}^{\text{emweak}}(1)$ | $L = 0$ |
| $+ \mathcal{L}_{e^2 p^0}^{\text{em}}(1) + \mathcal{L}_{\text{kin}}^{\text{leptons}}(0)$ | $L \leq 1$ |
| $+ \mathcal{L}_{p^4}(10) + \mathcal{L}_{p^6}^{\text{odd}}(23) + \mathcal{L}_{G_8 p^4}^{\Delta S=1}(22) + \mathcal{L}_{G_8 e^2 p^2}^{\Delta S=1}(28)$ | $L \leq 2$ |
| $+ \mathcal{L}_{G_8 e^2 p^2}^{\text{emweak}}(14) + \mathcal{L}_{e^2 p^2}^{\text{em}}(13) + \mathcal{L}_{e^2 p^2}^{\text{leptons}}(5)$ | |
| $+ \mathcal{L}_{p^6}(90)$ | |

**LECs:** low-energy constants $\equiv$ coupling constants of CHPT

**in red:** Lagrangians relevant for nonleptonic $K$ decays
**Nonleptonic kaon decays**

**dominant decays:**  \( K \rightarrow 2\pi, 3\pi \)

LO  

NLO  

NLO + isospin violation + rad. corrs.  

\[ \rightarrow \text{LO couplings } G_8, G_{27} \text{ well known} \]
### Effective chiral Lagrangian (meson sector)

| $L_{\text{chiral order}}$ ($\#$ of LECs) | loop order |
|----------------------------------------|------------|
| $L_p^2(2) + L_{p}^{\text{odd}}(0) + L_{G_F}^{\Delta S=1}(2) + L_{G_8 e^2 p^0}(1)$ + $L_{e^2 p^0}(1) + L_{\text{kin}}^{\text{leptons}}(0)$ | $L = 0$ |
| $+ L_{p}^4(10) + L_{p}^{\text{odd}}(23) + L_{G_8 p^4}^{\Delta S=1}(22) + L_{G_8 e^2 p^2}^{\Delta S=1}(28)$ + $L_{e^2 p^2}(13) + L_{e^2 p^2}(5)$ | $L \leq 1$ |
| $+ L_{p^6}(90)$ | $L \leq 2$ |

**in red:** LO Lagrangian for nonleptonic $K$ decays

**in blue:** NLO Lagrangian
Nonleptonic kaon decays

dominant decays: $K \rightarrow 2\pi, 3\pi$

LO

NLO 

NLO + isospin violation + rad. corrs. Cirigliano, E., Neufeld, Pich Bijnens, Borg

$\rightarrow$ LO couplings $G_8, G_{27}$ well known

N.B.: all other nonleptonic transitions start at NLO = $O(G_Fp^4)$

problem: 22 (octet) + 28 (27-plet) LECs

Theorists’ favourite nonleptonic decays

$K_S \rightarrow \gamma\gamma$, $K_L \rightarrow \pi^0\gamma\gamma$ [, $K_S \rightarrow \pi^0\pi^0\gamma\gamma$]

no LECs at all at NLO!
Status at $O(G_F p^4)$

$K_S \rightarrow \gamma \gamma$

$K_L \rightarrow \pi^0 \gamma \gamma$

$K_S \rightarrow \pi^0 \pi^0 \gamma \gamma$

D’Ambrosio, Espriu; Goity

E., Pich, de Rafael; Cappiello, D’Ambrosio

Funck, Kambor

$O(G_F p^2)$ no contribution

$O(G_F p^4)$ LECs do not contribute $\rightarrow$ finite loop amplitude

pre-CHPT: $K_L \rightarrow \pi^0 \gamma \gamma$ vector-meson dominated

compare 2-photon spectra
Normalized decay distribution in $z = M_{\gamma\gamma}^2 / M_K^2$

E., Pich, de Rafael

leading order [$O(p^4)$]

pure vector resonance exchange

$[ a_V = -0.32$
$K_L \rightarrow \pi^0\gamma\gamma$ decay distribution in $m_{34} = M_{\gamma\gamma}$

NA48 (2002)

KTeV (2008)
higher-order corrections (starting at NNLO = $O(G_F p^6)$)

- rescattering (unitarity) corrections largely model independent
  Cappiello, D’Ambrosio, Miragliuolo; Cohen, E., Pich; Kambor, Holstein
  $K_S \to \gamma\gamma$ “trivial” in terms of $K \to 2\pi$ rate
  $K_L \to \pi^0\gamma\gamma$ more involved but straightforward

- resonance contributions
  Cohen, E., Pich; D’Ambrosio, Portolés; Buchalla, D’Ambrosio, Isidori
  $K_S \to \gamma\gamma$ small (vector mesons cannot contribute)
  $K_L \to \pi^0\gamma\gamma$ vector meson contribution model dependent
  good approximation: single parameter $a_V$
puzzling result of NA48 (2003): rate substantially bigger than $O(p^4)$ result

KLOE (2008):

$$B(K_S \rightarrow \gamma\gamma) = 2.26(12)(06) \times 10^{-6}$$

→ perfect agreement

not a good idea:

PDG averages NA48/03 and KLOE

$$B(K_S \rightarrow \gamma\gamma) = 2.63(17) \times 10^{-6}$$

??
Motivation

Nonleptonic K decays

Carbogenesis

Lattice QCD

Conclusions

for reasonable values of $a_V$:

pion loop dominates $2\gamma$-spectrum

rate more affected both by rescattering corrections and by $a_V$

⇒ excellent agreement between theory and experiment

$$B(K_L \rightarrow \pi^0 \gamma \gamma) \cdot 10^6 = \begin{cases} 1.27 \pm 0.04 \pm 0.01 & \text{NA48 (2002)} \\ 1.28 \pm 0.06 \pm 0.01 & \text{KTeV (2008)} \\ 1.273 \pm 0.033 & \text{PDG (2012)} \end{cases}$$

$$a_V = -0.43 \pm 0.06 \quad \text{PDG (2012)}$$

important consequence

CP-conserving contribution $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+ e^-$ negligible in comparison with CP-violating amplitudes
almost all carbon produced in stellar nucleosynthesis via

\textbf{triple-\(\alpha\) process}

\textbf{Hoyle (1954):} to explain observed carbon abundance

\[ \exists \text{ excited } 0^+ \text{ state of } ^{12}\text{C} \text{ near } ^8\text{Be-\(\alpha\)} \text{ threshold} \]

observed soon afterwards
properties of Hoyle state

\[ \epsilon = 379.47(18) \text{ keV} \] (above 3\(\alpha\) threshold)

\[ \Gamma_{\text{tot}} = 8.3(1.0) \text{ eV}, \quad \Gamma_{\gamma} = 3.7(5) \text{ meV} \]

triple-\(\alpha\) rate \(\sim \Gamma_{\gamma} \exp(-\epsilon/kT) \rightarrow\) mainly sensitive to \(\epsilon\)

example of anthropic principle?

Livio et al. (1989), Oberhummer et al. (2004)

\[ \Delta \epsilon \lesssim 100 \text{ keV} \] tolerable to explain abundance of \(^{12}\text{C}\), \(^{16}\text{O}\)

\(\rightarrow\) not exactly severe fine-tuning

however:

more interesting issue:

dependence of \(\epsilon\) on fundamental parameters of strong and electromagnetic interactions
one-parameter (p) nuclear cluster model

Oberhummer et al.

tolerances

\[ \Delta p/p \lesssim 0.5\% \quad \Delta F_{\text{Coulomb}}/F_{\text{Coulomb}} \lesssim 4\% \]

open question:

relation to fundamental parameters of QCD and QED?

chiral EFT of nuclear forces

Weinberg (1990), . . .

expansion of nuclear potential (2-,3-,4-nucleon forces)

successful approach for small nuclei \((A \lesssim 3)\)

more recent development

nuclear lattice simulations (Muller, Lee, Boraso, . . .)

lattice dofs: nucleons (not quarks!) and pions
Monte-Carlo techniques

\[ \rightarrow \text{energies of low-lying states of } ^{12}\text{C (in MeV)} \]

|     | \(0_1^+\) | \(2_1^+(E^+)\) | \(0_2^+\) |
|-----|-----------|----------------|-----------|
| LO  | -96(2)    | -94(2)         | -89(2)    |
| NLO | -77(3)    | -74(3)         | -72(3)    |
| NNLO| -92(3)    | -89(3)         | -85(3)    |
| Exp | -92.16    | -87.72         | -84.51    |

Epelbaum et al.

\(0_2^+\): Hoyle state

Method allows to study dependence on quark masses (via \(M_π^2 \sim (m_u + m_d)\))

fine-structure constant \(α_{em}\) (not \(α_{QCD}\))

**Final conclusion for tolerances**

\[ \Delta m_q/m_q \lesssim 3\% \quad \Delta α_{em}/α_{em} \lesssim 2.5\% \]

\[ \rightarrow \text{fine-tuning in } m_q, α_{em} \text{ much more severe than in } ε \]
Motivation
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Low-energy constants and lattice QCD

Motto (Laurent Lellouch, John F. Kennedy)
Ask not what CHPT can do for the lattice, but what the lattice can do for CHPT

- CHPT $\rightarrow$ lattice
  (chiral) extrapolation to physical quark (meson) masses
  still useful, but less needed than 5 years ago

- lattice $\rightarrow$ CHPT
  determination of LECs (\textit{FLAG}, \ldots)
  especially welcome for LECs multiplying quark mass terms
  advantage of lattice simulations compared to phenomenology:
  quark (and therefore meson) masses can be tuned
Motive

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Illustrative example:

Chiral $SU(3)$ Lagrangian (strong interactions)

$$
\mathcal{L}_{p^2}(2) = \frac{F_0^2}{4} \langle D_\mu U D^{\mu} U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle
$$

$$
\mathcal{L}_{p^4}(10) = \cdots + L_4 \langle D_\mu U D^{\mu} U^\dagger \rangle \langle \chi U^\dagger + \chi^\dagger U \rangle + \cdots
$$

$\langle \ldots \rangle$  flavour trace

$F_0 = \lim_{m_u,m_d,m_s \to 0} F_\pi$, $\chi = 2B_0 M_q$ ($B_0 \sim$ quark condensate)

$U = 1 +$ meson fields

Gauge-covariant derivative $D_\mu U$ (contains $A_\mu, W^{\pm}_\mu$)
Motivation

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\[ \mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}(10) = \frac{1}{4} \langle D_\mu U D^\mu U \rangle \left[ F_0^2 + 8L_4 \left( 2\hat{M}_K^2 + \hat{M}_\pi^2 \right) \right] + \ldots \]

\[ \hat{M}_P \quad \text{lowest-order meson mass} \]

\[ F_\pi^2 / (16M_K^2) = 2 \times 10^{-3} \quad \sim \quad \text{typical size of NLO LEC} \]

consequences

- strong anticorrelation between \( F_0 \) and \( L_4 \) in global fits

Bijnens, Jemos

[Graph: \( L_4(M_0) \times 10^3 \) vs \( F_0 \) (MeV)]

J. Bijnens, I. Jemos: Global fit for SU(3) LECs
Nucl. Phys. B854 (2012) 631
“convergence” of SU(3) CHPT depends (also) on value of $F_0$
but LO LEC $F_0$ less known than many higher-order LECs
rather wide spread in $F_0$ also from lattice studies →
FLAG (2011) does not perform an average
$L_4$ is large-$N_c$ suppressed → in Gasser, Leutwyler (1985)
set to zero (more precisely: $L_4^r(M_\eta) = 0 \pm 0.5 \times 10^{-3}$)
FLAG (2011): published lattice determinations (for $L_4^r(M_\rho)$)
comparison between $SU(2)$ and $SU(3)$

$$F = \lim_{m_u, m_d \rightarrow 0} F_\pi$$

$$F_0 = F - F^{-1} \left\{ (2M_K^2 - M_\pi^2) \left( 4L_4^r(\mu) + \frac{1}{64\pi^2} \log \mu^2 / M_K^2 \right) \right.$$ 
$$+ \frac{M_\pi^2}{64\pi^2} \right\} + O(p^6)$$

“paramagnetic” inequality (Descotes-Genon, Girlanda, Stern)

$$F_0 < F \quad \longrightarrow \quad L_4^r(M_\rho) > -0.4 \times 10^{-3}$$

FLAG (2011): $F = F_\pi / 1.073(15)$

$\longrightarrow$ linear relation between $F_0$ and $L_4$
suggestion: determine $F_0, L_4$ from $SU(3)$ lattice data for $F_\pi$

essential: CHPT to NNLO = $O(p^6)$ (Amoros, Bijnens, Talavera)

drawback: only available in numerical form

in addition: need some knowledge of $L_5$ and LECs of $O(p^6)$
(e.g.: from analysis of $F_K/F_\pi$)

with large-$N_c$ motivated approximation for 2-loop calculation

Blue: fitting $F_\pi$ with RBC/UKQCD data (2011)
Red: $F_0 (F, L_4' (M_\rho))$ with $F_\pi/F = 1.073$ (15) (FLAG 2011)

RBC/UKQCD data (2011)
$F_0 = (78.8 \pm 4.1)$ MeV
$L_4' (M_\rho) = (0.0 \pm 0.1) \cdot 10^{-3}$

however:
chiral extrapolation $\rightarrow$
$F_\pi \approx 86.4$ MeV !?
suggestion:

determine $F_0, L_4$ from $SU(3)$ lattice data for $F_\pi$

essential: CHPT to NNLO $= O(\rho^6)$ (Amoros, Bijnens, Talavera)

drawback: only available in numerical form

in addition: need some knowledge of $L_5$ and LECs of $O(\rho^6)$
(e.g.: from analysis of $F_K/F_\pi$)

with large-$N_c$ motivated approximation for 2-loop calculation

Blue: fitting $F_\pi$ with RBC/UKQCD data (2011)
Red: $F_0 \left( F, L_4 \left( M_\rho \right) \right)$ with $F_\pi/F = 1.073 (15)$ (FLAG 2011)

E., Masjuan, Neufeld

include in data sample:

$F_\pi = (92.2 \pm 1.0)$ MeV

$[F_\pi^{PDG} = (92.21 \pm 0.14)$ MeV]

reason for discrepancy?
Conclusions

main objectives

- understand physics of the SM at low energies
- look for evidence of new physics

objectives accomplished?

- we have gone some way in understanding the SM at low energies
- on the other hand: we have not found evidence for new physics
- but neither has the LHC!