Influence of pre-stressing over parameters of diagram of static-dynamic deformation of RC elements

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Abstract. The stress-strain state in RC elements of statically indeterminate structural systems under sudden restructuring is considered in this article. The solution of the problem on determining the parameters of a static-dynamic deformation diagram of prestressed RC elements was obtained on the energy basis excluding the structure dynamics instrument the problem. The deformation dependences for the pre-stressed flexural RC element are determined using the diagram method presented in the research of. For the possibility of obtaining an analytical expression of the desired moment convenient for practical application of the construction of the “moment-curvature” diagram for the pre-stressed element is performed using the simplest deformation dependence. The calculation analysis of the deformation of such a structural systems was performed using the level of primary and secondary design models.

1. Introduction

In many countries it was carried out a row of theoretical and experimental studies of RC structures under special accidental impacts to solve the problem of protecting buildings and structures against progressive collapse [1-13]. Up to the present moment non-stressed structures were considered in these research. However, the problem devoted to the restructuring of nonlinearly deformed prestressed RC structural systems were firstly in [12]. Following these studies the solution to the problem of determining of static-dynamic deformation parameters of prestressed RC elements of frame structural systems under sudden restructuring is provided in this article. The solution of the problem is carried out by the diagram method [14] on an energy basis excluding the structure dynamics instrument. The parameters of the static-dynamic deformation diagram of the prestressed RC element are determined using two and three-line «moment-curvature» deformation curves.

2. Models and methods

Let's consider an in-situ RC braced frame structural system in the form of a two-span multistorey frame, the beam of which are prestressed by the installation of prestressed reinforcement in two levels along the section height (Fig. 1). The calculational analysis of the deformation of such a structural system was performed using the level of primary and secondary design models. in figure 2.
Figure 1. The reinforcement model of the pre-stressed frame structure (a); cross section 1-1 (b).

Figure 2. Primary frame design model (a); secondary frame design model (b).

According to the constructed primary design model in figure 2 (a) of the frame (before one of the bearing elements is removed), the stress-strain state is determined in the elements of the structural system during operation loads. One of the vertical bearing elements is removed in the primary design
model, for example, the middle column along axis 1. The secondary design model is built without this element in figure 2(b) to check resistance of the structural system against progressive collapse in a quasistatic formulation. In this case, the loads in the secondary design model are accepted the same as in the primary design model. The instantaneous structural failure of the bearing element is modeled by the force calculated in this element according to the primary calculation model, and it is applied with the opposite sign in the secondary calculation model. The result of the calculation in the secondary design model are moments, curvatures, stresses and deformation in the elements of the structural system. These calculation results for one of the most stressed section of the frame in section 1-1, in figure 1 can be presented in the form of a «moment-curvature» diagram in figure 3.

![Diagram of static-dynamic deformation of section 1-1 of prestressed RC element of the 5 times the statically indeterminate structural system (a).](image)

The diagram parts two linear sections of static deformation of the section: parts 0-1 - before the cracking and parts 1-2 - after the cracking. Parts 3-2' describes the dynamic deformation of the considered section when the mentioned column is removed from the structural system along the axis 1.

The task is that, using the values of the static moment $M^s_n$ obtained according to the primary design model and the values of the moment $M^s_{n-1}$, obtained in the same section, using the secondary design model (when the middle column is removed), determine the moment values in the system $n-1$, taking into account the dynamic load of the system caused by the removal of the column structure.

The solution of this problem is performed on an energy basis by the quasistatic method [1,2], excluding the structural dynamics instrument. We perform using the simplest deformation dependence of the «moment-curvature» diagram for the prestressed element proposed in [15,16] to obtain the analytical expression of the desired moment convenient for practical application of the construction. The "moment-curvature $(M - 1/r)^n$" diagram for an arbitrary section of the RC under static loading of the frame for the level of operating load is described by section 0-1-3 in figure 3. The first section 0-1
corresponds to the deformation of the element with the initial stiffness $B_0 = \tan \alpha_0$ without cracks, the second section 1-3 corresponds to the deformation of the RC element after the formation of cracks when it takes stiffness $B_1 = \tan \alpha_1$. In the diagram, the value of the segment $M_1$, cut off from the axis of moments by the continuation of the linear dependence "$M - 1/r\)" depends on the level of crack resistance of the prestressed element.

Following the work [15], the values $B_i$ and $M_i$ are determined on the basis of the hypothesis that the dependence diagrams "$M - 1/r\)" at different values $(1/r)_{tot}$ can be taken parallel for flexible prestressed reinforced concrete elements, as well as eccentrically compressed elements. Therefore the value $B_1$ is taken independently of compressive force $(N_{tot} = 0)$. Then we can write for the curvature in the second section of deformation:

$$\frac{1}{r} = \frac{1}{B_1} (M - M_1) \tag{1}$$

Pass to the system of forces taken in the calculation of curvature in the traditional form [16, 17].

$$\frac{1}{r} = \frac{M_s}{h_y^2} \left[ \frac{\psi_s}{E_s A_y + E_s A_s} + \frac{\psi_s}{(\phi_f + \zeta)h_y E_s \nu} \right] - \frac{N_{tot}}{h_y \left( E_s A_y + E_s A_s \right)} \tag{2}$$

where $M_s$ is the moment from all the forces and prestressing relatively to the center of gravity of the cross-sectional area of the reinforcement $S$; $N_{tot}$ is the resultant of the axial force $N$ and the prestressing $P$.

We write the moment $M_i$ by analogy with the moment of crack resistance $M_{cric}$ in the following form:

$$M_i = \phi_2 b h^2 R_{y,sec} + \phi_4 N_{tot} \cdot (y_{sec} + r) \tag{3}$$

where $y_{sec}$ is the distance from the center of gravity of the reduced section of prestressed element to the center of gravity of the section of the reinforcement $S$; $r$ is the distance from the same center of gravity to the core point.

At such $M_i$ representation, the general formula for curvature application area of which is the same as in formula (2):

$$\frac{1}{r} = \frac{M_s - \phi_2 b h^2 R_{y,sec} + \phi_4 N_{tot} \cdot (y_{sec} + r)}{\phi_4 E_s A_y h_y^2} \tag{4}$$

The coefficients $\phi_1$ and $\phi_2$ in formulas (3) and (4) depend on the reinforcement and the shape of the cross section of the element and they are calculated by the formulas of [15]. The coefficient $\phi_3$ is found from the condition of equality of the curvatures found by formulas (2) and (4). In this case, it is assumed that the dependence "$M - 1/r\)" is described by a straight line passing through the points with coordinates $M = M_{tot}$, $1/r = 1/r_{tot} -$ is the level of the regulatory load and $M = M_{cric}$, $1/r = 1/r_{cric} -$ is formation of cracks in the stretched zone in figure 3.

When we calculated $M_i$ value and stiffness $B_i$ of cross section of prestressed element at the segment 1-2 with crack, let us determine parametres of dynamic segment 3-2' of the deformation in figure 3.

As mentioned earlier the value of the moment in the cross section 1-1 of the original-time statically indeterminate structure of the frame for a given operating load $M''_i$ is considered according to the primary design model. The value of the moment in the same section in a frame with a removed
column \( M^d_{n-1} \) is calculated according to the secondary calculation model under the assumption that the structural transformation of the primary \( n \)-times the statically indeterminate system into \( n-1 \)-times the static indeterminate system occurs slowly without its dynamic loading.

The value of the dynamic moment \( M^d_{n-1} \) in this section can be determined on an energy basis. During the dynamic deformation of the cross section of a RC element the condition of constancy of the total energy in accordance with the deformation diagram (see Figure 3) is written in the form:

\[
\Phi(1/r^d_{n-1}) - \Phi(1/r^d_{n-1}) = M^s_{n-1} \cdot (1/r^d_{n-1} - 1/r^d_{n-1})
\]  

(5)

We reveal the terms of the energy relation (5) for the adopted strain diagram in the following form:

\[
\Phi(1/r) = \frac{1}{2} \int (M_1 + (1/r)B_1)d(1/r)
\]  

(6)

and substituting them into equation (5), we get:

\[
\frac{B_1}{2} \cdot (1/r^d_{n-1})^2 + M_1 \cdot (1/r^d_{n-1}) - \left[ \frac{B_1}{2} \cdot (1/r^d_{n-1})^2 + M_1 \cdot (1/r^d_{n-1}) \right] = \frac{1}{B_1} \cdot \frac{M^s_{n-1} - M_1 + \sqrt{(M^s_{n-1} - M_1)^2 + B_1^2 \cdot (1/r^d_{n-1})^2 - 2B_1 \cdot (1/r^d_{n-1}) \cdot (M^s_{n-1} - M_1)}}
\]  

(7)

We obtain the following expression solving a quadratic equation (7) with respect to \( 1/r^d_{n-1} \):

\[
1/r^d_{n-1} = \frac{M^s_{n-1} - M_1 + \sqrt{(M^s_{n-1} - M_1)^2 + B_1^2 \cdot (1/r^d_{n-1})^2 - 2B_1 \cdot (1/r^d_{n-1}) \cdot (M^s_{n-1} - M_1)}}{B_1}
\]  

(8)

The value of the limit dynamic moment \( M^d_{n-1} \), which can be perceived by the section, is related to the dynamic strength of concrete and reinforcement and depends on the time of the dynamic addition loading in the section of the prestressed concrete element under consideration. The problem of the dynamic strength of concrete and reinforced concrete under a single high-speed loading (impact) was considered in a number of well-known publications, for example [2,17].

3. Results and discussion

Quantitative analysis of the parameters of the static-dynamic deformation diagram of a prestressed RC element in a statically indeterminate frame structural system is made to cross-section \( I-I \) the frame model presented in Figure 1. The frame is made of concrete of compressive strength class C40. The prestressed reinforcement was adopted by class A500, with a diameter of 8 mm in order to protect the frame from progressive collapse in case of a sudden redistribution of power flows. The reinforcement was installed symmetrically in the upper and lower zones of the cross-section of the beam \( A_{sp} = A_{sp} \).

The geometrical sizes of the prestressed beam are shown in Figure 1.

The first loss of prestress is 163 MPa, the second loss 40 MPa, controlled prestress with all losses \( \sigma_{p3} = 489.5MPa \) \( \gamma = 0.9 \) at the assigned initial stress.

According to the results of calculations for the considered section, the following values of the moments are obtained: \( M_{crv} = 2.36kNm \) the corresponding curvature is \( (1/r)_{crv} = 4.2 \cdot 10^{-5}(1/mm) \). The limiting value of the moment under operating load during static deformation of the considered section is \( M_{ult} = 3.09kNm \), the corresponding curvature was \( (1/r)_{ult} = 5.8 \cdot 10^{-5}(1/mm) \).

The value of the moment in the original \( n \)-times static-indefinable frame design, for a given operating load is \( M^s_{u1} = 2.50kNm \), the corresponding curvature is \( (1/r)^s_{u1} = 4.4 \cdot 10^{-5}(1/mm) \). The value of the moment in the same section in the frame with the column removed is \( M^s_{u1} = 9.32kNm \), the corresponding curvature is \( (1/r)^s_{u1} = 18.4 \cdot 10^{-5}(1/mm) \). The calculated value of the dynamic
moment is $M^v_{n1} = 15.57kNm$, the corresponding curvature is $(1/r)^v_{n1} = 27.3 \cdot 10^{-5}(1/mm)$. The value of the limiting curvature, calculated for a special limiting state from the values of the ultimate deformations of concrete and reinforcement [15] is $(1/r)^v_s = 47.5 \cdot 10^{-5}(1/mm)$.

4. Conclusions

The proposed analytical dependences and the static-dynamic deformation diagram of the section of a prestressed RC element make it possible to determine the increments of dynamic forces and curvatures in arbitrary sections of a statically indeterminate RC frame when one of the columns is suddenly removed from it.

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