Macro- and Microscopic Self-Similarity in Neuro- and Psycho-Dynamics

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Abstract

The unique Hamiltonian description of neuro- and psycho-dynamics at the macroscopic, classical, inter-neuronal level of brain’s neural networks, and microscopic, quantum, intra-neuronal level of brain’s microtubules, is presented in the form of open Liouville equations. This implies the arrow of time in both neuro- and psycho-dynamic processes and shows the existence of the formal neuro-biological space-time self-similarity.

Keywords: Neuro- and psycho-dynamics, Brain microtubules, Hamiltonian and Liouville dynamics, Neuro-biological self-similarity

1 Introduction

Neuro- and psycho-dynamics have its physical behavior both on the macroscopic, classical, inter-neuronal level [1, 2], and on the microscopic, quantum, intra-neuronal level [3, 4, 5]. On the macroscopic level, various models of neural networks (NNs, for short) have been proposed as goal-oriented models of the specific neural functions, like for instance, function-approximation, pattern-recognition, classification, or control (see, for example [6]). In the physically-based, Hopfield-type models of NNs [7, 8] the information is stored as a content-addressable memory in which synaptic strengths are modified after the Hebbian rule (see [9]). Its retrieval is made when the network with the symmetric couplings works as the point-attractor with the fixed points. Analysis of both activation and learning dynamics of Hopfield-Hebbian NNs using the techniques of statistical mechanics [10], provides us with the most important information of storage capacity, role of noise and recall performance [11, 2].

Conversely, an indispensable role of quantum theory in the brain dynamics was emphasized in [12]. On the general microscopic intra-cellular level, energy transfer across the cells, without dissipation, had been first conjectured to occur in biological matter by [13]. The phenomenon conjectured by them was based on their 1D superconductivity model: in one dimensional electron systems with holes, the formation of solitonic structures due to electron-hole pairing results in the transfer of electric current without dissipation. In a similar manner, Fröhlich and Kremer conjectured that energy in biological matter could be transferred without dissipation, if appropriate solitonic structures are formed inside the cells. This idea has lead theorists to construct various models for the energy transfer across the cell, based on the formation of kink classical solutions [14, 15].
The interior of living cells is structurally and dynamically organized by cytoskeletons, i.e., networks of protein polymers. Of these structures, microtubules (MTs, for short) appear to be the most fundamental [16]. Their dynamics has been studied by a number of authors in connection with the mechanism responsible for dissipation-free energy transfer. Hameroff and his colleagues [17, 18, 19, 20] have conjectured another fundamental role for the MTs, namely being responsible for quantum computations in the human neurons. Penrose [21, 22, 23, 24] further argued that the latter is associated with certain aspects of quantum theory that are believed to occur in the cytoskeleton MTs, in particular quantum superposition and subsequent collapse of the wave function of coherent MT networks. These ideas have been elaborated by [25] and [26], based on the quantum–gravity language of [27], where MTs have been physically modelled as non–critical (SUSY) bosonic strings. It has been suggested that the neural MTs are the microsites for the emergence of stable, macroscopic quantum coherent states, identifiable with the preconscious states; stringy–quantum space-time effects trigger an organized collapse of the coherent states down to a specific or conscious state. More recently, the evidence for biological self-organization and pattern formation during embryogenesis was presented in [28].

In particular, MTs in the cytoskeletons of eukaryotic cells provide a wide range of micro–skeletal and micro–muscular functionalities. Some evidence has indicated that they can serve as a medium for intracellular signaling processing. For the inherent symmetry structures and the electric properties of tubulin dimers, the microtubule (MT) was treated as a 1D ferroelectric system in [29]. The nonlinear dynamics of the dimer electric dipoles was described by virtue of the double–well potential and the physical problem was further mapped onto the pseudo–spin system, taking into account the effect of the external electric field on the MT.

More precisely, MTs are polymers of tubulin subunits (dimers) arranged on a hexagonal lattice. Each tubulin dimer comprises two monomers, the α−tubulin and β−tubulin, and can be found in two states. In the first state a mobile negative charge is located into the α−tubulin monomer and in the second into the β−tubulin monomer. Each tubulin dimer is modelled as an electrical dipole coupled to its neighbors by electrostatic forces. The location of the mobile charge in each dimer depends on the location of the charges in the dimer’s neighborhood. Mechanical forces that act on the microtubule affect the distances between the dimers and alter the electrostatic potential. Changes in this potential affect the mobile negative charge location in each dimer and the charge distribution in the microtubule. The net effect is that mechanical forces affect the charge distribution in microtubules [30].

Various models of the mind have been based on the idea that neuron MTs can perform computation. From this point of view, information processing is the fundamental issue for understanding the brain mechanisms that produce consciousness. The cytoskeleton polymers could store and process information through their dynamic coupling mediated by mechanical energy. The problem of information transfer and storage in brain microtubules was analyzed in [31], considering them as a communication channel.

Therefore, we have two space–time biophysical scales of neuro– and psychodynamics: classical and quantum. Naturally the question arises: are these two scales somehow inter-related, is there a space–time self-similarity between them?

The purpose of the present paper is to prove the formal positive answer to the
self-similarity question. We try to describe neurodynamics on both physical levels by the unique form of a single equation, namely open Liouville equation: NN–dynamics using its classical form, and MT–dynamics using its quantum form in the Heisenberg picture. If this formulation is consistent, that would prove the existence of the formal neuro-biological space-time self-similarity.

2 Mathematics of Open Liouville Equation

2.1 Hamiltonian framework

Suppose that on the macroscopic NN–level we have a conservative Hamiltonian system acting in a 2ND symplectic phase space \( T^*Q = \{q^i(t), p_i(t)\}, i = 1 \ldots N \) (which is the cotangent bundle of the NN–configuration manifold \( Q = \{q^i\} \)), with a Hamiltonian function \( H = H(q^i, p_i, t) : T^*Q \times \mathbb{R} \rightarrow \mathbb{R} \) (see [32, 33, 34]). The conservative dynamics is defined by classical Hamilton’s canonical equations:

\[
\dot{q}^i = \partial_{p_i} H \quad \text{– contravariant velocity equation}, \\
\dot{p}_i = -\partial_{q^i} H \quad \text{– covariant force equation},
\]

(1)

(here and henceforth overdot denotes the total time derivative). Within the framework of the conservative Hamiltonian system (1) we can apply the formalism of classical Poisson brackets: for any two functions \( A = A(q^i, p_i, t) \) and \( B = B(q^i, p_i, t) \) their Poisson bracket is (using the summation convention) defined as [34, 3]

\[
[A, B] = (\partial_{q^i} A \partial_{p_i} B - \partial_{p_i} A \partial_{q^i} B).
\]

2.2 Conservative classical system

Any function \( A(q^i, p_i, t) \) is called a constant (or integral) of motion of the conservative system (1) if \( \dot{A} = \partial_t A + [A, H] = 0 \), which implies \( \partial_t A = -[A, H] \).

(2)

For example, if \( A = \rho(q^i, p_i, t) \) is a density function of ensemble phase–points (or, a probability density to see a state \( x(t) = (q^i(t), p_i(t)) \) of ensemble at a moment \( t \)), then equation

\[
\partial_t \rho = -[\rho, H]
\]

(3)

represents the Liouville theorem, which is usually derived from the continuity equation

\[
\partial_t \rho + \text{div}(\rho \dot{x}) = 0.
\]

2.3 Conservative quantum system

We perform the formal quantization of the conservative equation (3) in the Heisenberg picture: all variables become Hermitian operators (denoted by ‘\(^\wedge\)’), the symplectic phase space \( T^*Q = \{q^i, p_i\} \) becomes the Hilbert state space \( \mathcal{H} = \mathcal{H}_{q^i} \otimes \mathcal{H}_{\dot{p}_i} \) (where \( \mathcal{H}_{q^i} = \mathcal{H}_{q^1} \otimes \ldots \otimes \mathcal{H}_{q^N} \) and \( \mathcal{H}_{\dot{p}_i} = \mathcal{H}_{\dot{p}_1} \otimes \ldots \otimes \mathcal{H}_{\dot{p}_N} \)), the classical Poisson bracket \([,]\) becomes the quantum commutator \{,\} multiplied by \(-i/\hbar\) [36, 5]

\[
[,] \rightarrow -i\{,\} \quad (\hbar = 1 \text{ in normal units}).
\]

(4)
In this way the classical Liouville equation (3) becomes the quantum Liouville equation \[ \frac{\partial}{\partial t} \hat{\rho} = i\{\hat{\rho}, \hat{H}\}, \] (5)
where \( \hat{H} = \hat{H}(\hat{q}^i, \hat{p}_i, t) \) is the Hamiltonian evolution operator, while
\[
\hat{\rho} = \sum_a P(a) |\Psi_a><\Psi_a|,
\]
denotes the von Neumann density matrix operator, where each quantum state |\(\Psi_a\rangle\) occurs with probability \(P(a)\); \(\hat{\rho} = \hat{\rho}(\hat{q}^i, \hat{p}_i, t)\) is closely related to another von Neumann concept: entropy
\[
S = -\text{Tr}(\hat{\rho}\ln\hat{\rho}).
\]

2.4 Open classical system

We now move to the open (nonconservative) system: on the macroscopic NN–level the opening operation equals to the adding of a covariant vector of external (dissipative and/or motor) forces \(F_i = F_i(q^i, p_i, t)\) to (the right-hand-side of) the covariant Hamilton’s force equation, so that Hamilton’s equations obtain the open (dissipative and/or forced) form \[ \dot{q}^i = \partial_{p_i} H, \quad \dot{p}_i = -\partial_{q^i} H + F_i. \] (6)

In the framework of the open Hamiltonian system (6) dynamics of any function \(A(q^i, p_i, t)\) is defined by the open (dissipative and/or forced) evolution equation:
\[
\partial_t A = -[A, H] + F_i[A, q^i], \quad ([A, q^i] = -\partial_{q^i} A). \]
(7)

In particular, if \(A = \rho(q^i, p_i, t)\) represents the density function of ensemble phase–points then its dynamics is given by the open (dissipative and/or forced) Liouville equation \[ \partial_t \rho = -[\rho, H] + F_i[\rho, q^i]. \] (8)

Equation (8) represents the open classical model of our microscopic NN-dynamics.

2.5 Continuous neural network dynamics

The generalized NN–dynamics, including two special cases of graded response neurons (GRN) and coupled neural oscillators (CNO), can be presented in the form of a Langevin stochastic equation \[ \dot{\sigma}_i = f_i + \eta_i(t), \] (9)
where \(\sigma_i = \sigma_i(t)\) are the continual neuronal variables of \(i\)th neurons (representing either membrane action potentials in case of GRN, or oscillator phases in case of CNO); \(J_{ij}\) are individual synaptic weights; \(f_i = f_i(\sigma_i, J_{ij})\) are the deterministic forces (given, in GRN-case, by
\[
f_i = \sum_j J_{ij} \tanh[\gamma \sigma_j] - \sigma_i + \theta_i, \quad \text{with} \quad \gamma > 0
\]
and with the \(\theta_i\) representing injected currents, and in CNO–case, by
\[
f_i = \sum_j J_{ij} \sin(\sigma_j - \sigma_i) + \omega_i,
\]

where the \(\gamma\).
with $\omega_i$ representing the natural frequencies of the individual oscillators); the noise variables are given as

$$\eta_i(t) = \lim_{\Delta \to 0} \zeta_i(t) \sqrt{2T/\Delta},$$

where $\zeta_i(t)$ denote uncorrelated Gaussian distributed random forces and the parameter $T$ controls the amount of noise in the system, ranging from $T = 0$ (deterministic dynamics) to $T = \infty$ (completely random dynamics).

More convenient description of the neural random process (9) is provided by the Fokker-Planck equation describing the time evolution of the probability density $P(\sigma_i)$ [38, 3, 37]

$$\partial_t P(\sigma_i) = -\sum_i \partial_{\sigma_i} [f_i P(\sigma_i)] + T \sum_i \partial_{\sigma_i^2} P(\sigma_i).$$

Now, in the case of deterministic dynamics $T = 0$, equation (10) can be easily put into the form of the conservative Liouville equation (3), by making the substitutions:

$$P(\sigma_i) \to \rho, f_i = \partial_{\sigma_i}, \quad [\rho, H] = \text{div}(\rho \partial_{\sigma_i}),$$

where $H = H(\sigma_i, J_{ij})$. Further, we can formally identify the stochastic forces, i.e., the second-order noise-term $T \sum_i \partial_{\sigma_i^2} \rho$ with $F^i[\rho, \sigma_i]$, to get the open Liouville equation (8).

Therefore, on the NN–level deterministic dynamics corresponds to the conservative system (3). Inclusion of stochastic forces corresponds to the system opening (8), implying the macroscopic arrow of time.

2.6 Open quantum system

By formal quantization of equation (8), we obtain the quantum open Liouville equation [36, 5]

$$\partial_t \hat{\rho} = i\{\hat{\rho}, \hat{H}\} - i\hat{F}_i \{\hat{\rho}, \hat{q}^i\},$$

(11)

where $\hat{F}_i = \hat{F}_i(\hat{q}^i, \hat{p}_i, t)$ represents the covariant quantum operator of external friction forces in the Hilbert state space $\mathcal{H} = \mathcal{H}_{\hat{q}^i} \otimes \mathcal{H}_{\hat{p}_i}$.

Equation (11) represents the open quantum-friction model of our microscopic MT–dynamics.

2.7 Non–critical stringy MT–dynamics

In EMN–language of non-critical (SUSY) bosonic strings, our MT–dynamics equation (11) reads [36, 3, 5]

$$\partial_t \hat{\rho} = i\{\hat{\rho}, \hat{H}\} - i\hat{g}_{ij} \{\hat{\rho}, \hat{q}^i\} \hat{q}^j,$$

(12)

where the target-space density matrix $\hat{\rho}(\hat{q}^i, \hat{p}_i)$ is viewed as a function of coordinates $\hat{q}^i$ that parameterize the couplings of the generalized $\sigma$–models on the bosonic string world-sheet, and their conjugate momenta $\hat{p}_i$, while $\hat{g}_{ij} = \hat{g}_{ij}(\hat{q}^i)$ is the quantum operator of the positive definite metric in the space of couplings. Therefore, the covariant quantum operator of external friction forces is in EMN–formulation given as $\hat{F}_i(\hat{q}^i, \hat{q}^j) = \hat{g}_{ij} \hat{q}^j$. 
Equation (12) establishes the conditions under which a large-scale coherent state appearing in the MT-network, which can be considered responsible for loss-free energy transfer along the tubulins.

The system-independent properties of equation (12), are:

(i) Conservation of probability $P$

$$\partial_t P = \partial_t [\text{Tr}(\hat{\rho})] = 0.$$  \hspace{1cm} (13)

(ii) Conservation of energy $E$, on the average

$$\partial_t \langle \langle E \rangle \rangle \equiv \partial_t [\text{Tr}(\hat{\rho}E)] = 0.$$ \hspace{1cm} (14)

(iii) Monotonic increase in entropy

$$\partial_t S = \partial_t [-\text{Tr}(\hat{\rho} \ln \hat{\rho})] = (\hat{q}^i \hat{g}_{ij} \hat{q}^j) S \geq 0,$$ \hspace{1cm} (15)

due to the positive definiteness of the metric $\hat{g}_{ij}$, and thus automatically and naturally implying a microscopic arrow of time [27].

2.8 Equivalence of Neurodynamic forms

Both the macroscopic NN-equation (8) and the microscopic MT-equation (11) have the same open Liouville form, which implies the arrow of time [27, 5]. These demonstrates the existence of a formal neuro-biological space-time self-similarity.

3 Conclusion

We have described neuro- and psycho-dynamics of both NN and MT ensembles, belonging to completely different biophysical space-time scales, brain’s neural networks and brain’s microtubules, by the unique form of the open Liouville equation, which implies the arrow of time. In this way the existence of the formal neuro-biological space-time self-similarity has been proved.

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