Two Loop Finiteness of Higgs Mass and Potential in the Gauge-Higgs Unification

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The zero mode of an extra-dimensional component of gauge potentials serves as a 4D Higgs field in the gauge-Higgs unification. We examine QED on $M^4 \times S^1$ and determine the mass and potential of a 4D Higgs field (the $A_5$ component) at the two loop level with gauge invariant regularization. It is seen that the mass is free from divergences and independent of the renormalization scheme.

§1. Introduction

In the standard model of electroweak interactions the Higgs boson is vital to induce the electroweak symmetry breaking. It is one of the major goals in particle physics to discover the Higgs boson in the coming years. Its mass squared $m^2_H$, in general, acquires $O(\Lambda^2)$ radiative corrections where $\Lambda$ is a cutoff scale which is as large as $10^{16}$ GeV in grand unified theories. In order to have $m_H = O(100)$ GeV, unnatural fine-tuning of parameters of the theory is demanded.

The supersymmetry naturally solves this gauge hierarchy problem to push down $\Lambda$ to the TeV scale. It serves as a leading candidate for a model beyond the standard model, and is under intensive study. There are alternative scenarios to have a naturally light Higgs boson, among which is the gauge-Higgs unification. A 4D Higgs field is identified with a part of the extra-dimensional component of gauge potentials. When the extra-dimensional space is not simply connected, there appears a Wilson line phase $\theta_H$, an analogue of the Aharonov-Bohm phase in quantum mechanics. The 4D Higgs field is nothing but a field describing four-dimensional fluctuations of $\theta_H$. At the tree level the Higgs field appears massless, reflecting the nature of the Aharonov-Bohm phase. At the quantum level the effective potential for the Wilson line phase, $V_{\text{eff}}(\theta_H)$, is generated radiatively. It has been shown long ago that $V_{\text{eff}}(\theta_H)$ on $M^4 \times S^1$ and the Higgs mass $m_H$ are finite at the one loop level.\(^{3,5,6}\)

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This has significant relevance in the context of the gauge-Higgs unification. Although higher dimensional gauge theory is not renormalizable, the 4D Higgs mass can be predicted to be a finite value without suffering from the problem of divergences. As the Higgs field is associated with the nonlocal Wilson line phase, it is commonly said that the Higgs mass remains finite to all orders as no gauge-invariant local counter term can be written. It is not quite clear, however, whether this argument applies to non-renormalizable theories like the one under consideration. It is desirable to have explicit evaluation and confirm the finiteness of $m_H$ beyond one loop.

There have been significant advances in the gauge-Higgs unification in the electroweak theory in the last couple of years. In the early stage unification on orbifolds $M^4 \times (S^1/Z_2)$ and $M^4 \times (T^2/Z_2)$ was pursued with chiral fermions.\textsuperscript{10–24} It has been recognized that unification on warped spacetime such as the Randall-Sundrum spacetime works much better for having phenomenologically viable models.\textsuperscript{25–40} In all of such aspects as the Higgs mass, the Kaluza-Klein mass scale, the gauge self-couplings, and the Weinberg angle, the unification in the Randall-Sundrum spacetime gives natural consistent results. Suppression of the Higgs-gauge couplings and Yukawa couplings has been predicted, which can be tested at LHC.\textsuperscript{32,33} Furthermore it has been shown recently that the gauge-Higgs unification in the Randall-Sundrum spacetime is dual to the theory of holographic pseudo-Goldstone boson.\textsuperscript{34–38}

In view of these developments it is appropriate and necessary to strengthen and confirm the statement that the Higgs mass remains finite beyond one loop.\textsuperscript{47–50} Its finiteness has been investigated in the lattice simulation on orbifolds as well.\textsuperscript{51} The calculability of the $S$ and $T$ parameters in the electroweak theory has been discussed at the one loop level.\textsuperscript{52}

Evaluation of the Higgs mass at the two loop level is formidable in non-Abelian gauge theory. To get insight in the problem it is instructive to examine, as the first step, QED on $M^4 \times S^1$ in which the zero mode of the extra-dimensional component $A_5$ mimics the 4D Higgs boson. It is called a Higgs boson in the present paper.

To evaluate the Higgs mass $m_H$ at the two loop level, renormalization at the one loop level must be taken into account in due course. Two loop evaluation of the Higgs mass in QED on $M^4 \times S^1$ has been previously attempted by Maru and Yamashita,\textsuperscript{49} where the vacuum polarization tensors $\Pi^{MN}$ are evaluated near $\theta_H = 0$ for the zero-modes, without paying serious attention to the regularization. In this article we evaluate both the effective potential $V_{\text{eff}}(\theta_H)$ and the vacuum polarization tensors in renormalized perturbation theory in the dimensional regularization, maintaining the gauge invariance and the Ward-Takahashi identities. The computation is carried out with an arbitrary value of $\theta_H$ as a background. The effective potential $V_{\text{eff}}(\theta_H)$ is found to be minimized at $\theta_H = \pi$, and therefore the vacuum polarization tensors $\Pi^{MN}$ at $\theta_H = \pi$ become relevant for determining $m_H$.

The paper is organized as follows. In the next section renormalized perturbation theory for QED in $M^4 \times S^1$ is developed and renormalization conditions are given. In §3 the effective potential $V_{\text{eff}}(\theta_H)$ is evaluated at the two loop level. Relevant integral-sums are evaluated in Appendix A. In §4 the vacuum polarization tensors $\Pi^{MN}$ are determined at the one loop level. Details of the computation are given.
in Appendix B. With these results the Higgs mass is determined at the two loop level in §5. It is seen that the Higgs mass thus evaluated is independent of the renormalization scheme. Section 6 is devoted to a brief summary and discussion.

§2. QED in $M^4 \times S^1$

The model we analyze is QED defined in five-dimensional spacetime where the fifth dimension is a circle $S^1$ with a radius $R$. For the sake of simplicity we introduce only one fermion $\psi$ with a mass $m$. Both the gauge potential $A^M$ (the photon field) and $\psi$ are taken to be periodic. Renormalization, at least at the two loop level, is done with the standard renormalization procedure. Renormalized fields are defined by $A^{(0)}_M = Z_3^{1/2} A^M$ and $\psi^{(0)} = Z_2^{1/2} \psi$. The renormalized coupling constant is defined by $e^{(0)} = Z_1 Z_2^{-1} Z_3^{-1/2} e$. The renormalized mass of $\psi$ is given by $m^{(0)} = m + \delta m$. Here quantities with superscript (0) denote bare quantities. Renormalization conditions for $Z_1, Z_2, Z_3$ and $\delta m$ are specified below.

We develop renormalized perturbation theory around the non-vanishing Wilson line phase $\theta_H$. The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} (\partial_M A^M)^2 + \bar{\psi} (i\gamma^M D_M^c - m) \psi - eA_M \bar{\psi} \gamma^M \psi$$

$$- \frac{1}{4} \delta_3 F^{MN} F_{MN} + \bar{\psi} (\delta_2 i\gamma^M \partial_M - \delta_m) \psi - e\delta_1 A_M \bar{\psi} \gamma^M \psi .$$

Here $D_M^c = \partial_M - \delta_{MN5} i e A_5^c$ where $e A_5^c = \theta_H/2\pi R$ and the metric is $\eta_{MN} = \text{diag} (1, -1, -1, -1, -1)$. Counter terms are defined as $\delta_k = Z_k - 1$ and $\delta_m = Z_2 m^{(0)} - m$. We renormalize such that the Ward-Takahashi identity $Z_1 = Z_2$ is preserved so that

$$\theta_H = e \int_0^{2\pi R} dy A_5 = e^{(0)} \int_0^{2\pi R} dy A_5^{(0)} .$$

In other words the Wilson line phase is not renormalized.

On $M^4 \times S^1$ the fifth component of a momentum $p^5$ is discretized. With $\theta_H \equiv 2\pi a \neq 0$, $p^5 = (n - a)/R$ for $\psi$ and $p^5 = n/R$ for $A_M$ where $n$ is an integer. We denote a five-momentum by $\hat{p}^M = (\hat{p}^\mu, \hat{p}^5)$ ($\mu = 0 \sim 3$). Let us denote the sum of all 1-particle-irreducible diagrams for the fermion propagator by $\Sigma(\hat{p}; a, R)$, that for the photon propagator by $\Pi^{MN}(\hat{p}; a, R)$, and the amputated fermion vertex function by $\Gamma^M(\hat{p}, \hat{p}'; a, R)$. In the $R \to \infty$ limit these functions take 5D Lorentz covariant form, and are denoted with a superscript (0),

$$\Sigma(\hat{p}; a, R) = \Sigma^{(0)}(\hat{p}) + \Sigma^{(1)}(\hat{p}; a, R) ,$$

$$\Pi^{MN}(\hat{p}; a, R) = \Pi^{(0)MN}(\hat{p}) + \Pi^{(1)MN}(\hat{p}; a, R) ,$$

$$\Gamma^M(\hat{p}, \hat{p}'; a, R) = \Gamma^{(0)M}(\hat{p}, \hat{p}') + \Gamma^{(1)M}(\hat{p}, \hat{p}'; a, R) ,$$

where $\hat{p} = \hat{p}^M \gamma_M$ and $\Sigma^{(0)}(\hat{p}) = \lim_{R \to \infty} \Sigma(\hat{p}; a, R)$, etc. The vacuum polarization tensors $\Pi^{MN}(\hat{p}; a, R)$ can be expressed in terms of two invariant functions $\Pi(\hat{p}; a, R)$
and $F(\hat{p}; a, R)$. The current conservation $\hat{p}_M \Pi^{MN}(\hat{p}; a, R) = 0$ implies that

$$
\Pi^{\mu\nu} = (\eta^{\mu\nu} \hat{p}^2 - p^\mu p^\nu) \Pi - \frac{p^\mu p^\nu}{\hat{p}^2} \cdot (p^5)^2 F ,
$$

$$
\Pi^{55} = -p^2 (\Pi + F) ,
$$

$$
\Pi^{5\mu} = -p^5 (\Pi + F) ,
$$

(2.4)

where $p^5 = n/R$ ($n$: an integer). We remark that $F$ is finite and $\lim_{R \to \infty} F = 0$. The divergent contributions appear only for $\Pi^{(0)} = \lim_{R \to \infty} \Pi$, which can be cancelled by the counter term $\delta_3$ in (2.1). Also $\lim_{R \to \infty} \Pi^{MN} = (\eta^{MN} \hat{p}^2 - \hat{p}^M \hat{p}^N) \Pi^{(0)}(\hat{p}^2)$.

The full photon propagators $D^{MN}(\hat{p})$ are given by

$$
i D^{\mu\nu} = \frac{1}{\hat{p}^2(1 - \Pi)} \left( \eta^{\mu\nu} - \frac{p^\mu p^\nu}{\hat{p}^2} \right) + \frac{p^\mu p^\nu}{(\hat{p}^2)^2} \left( 1 - \frac{(p^5)^2}{\hat{p}^2(1 - \Pi - F)} \right) ,
$$

$$
i D^{\mu 5} = -\frac{p^\mu p^5}{(\hat{p}^2)^2} \frac{\Pi + F}{1 - \Pi - F} ,
$$

$$
i D^{55} = -\frac{p^2}{(\hat{p}^2)^2(1 - \Pi - F)} + \frac{(p^5)^2}{(\hat{p}^2)^2} .
$$

(2.5)

In particular, for the zero modes ($p^5 = 0$)

$$
D^{\mu\nu}|_{p^5=0} = \frac{-i}{\hat{p}^2(1 - \Pi)} \left( \eta^{\mu\nu} - \frac{p^\mu p^\nu}{\hat{p}^2} \Pi \right) ,
$$

$$
D^{\mu 5}|_{p^5=0} = 0 ,
$$

$$
D^{55}|_{p^5=0} = \frac{i}{\hat{p}^2(1 - \Pi - F)} ,
$$

(2.6)

where $\Pi$ and $F$ are evaluated at $p^5 = 0$. It will be found that $\Pi$ and $F$ are expanded in $p^2$ as $\Pi|_{p^5=0} = c_0 + \cdots$ and $F|_{p^5=0} = b_{-1}/p^2 + b_0 + \cdots$, respectively. Consequently the Higgs mass, or the mass of the zero mode of $A_5$, defined in the $p^2$ expansion in the inverse propagator is given by

$$
m_H^2 = \frac{b_{-1}}{1 - c_0 - b_0} .
$$

(2.7)

It slightly differs from the exact pole mass, but is convenient to relate to the effective potential $V_{\text{eff}}(\theta_H)$. The difference between the two is small in the weak coupling $\epsilon_4^2/4\pi \ll 1$ where $\epsilon_4$ is the four-dimensional gauge coupling $\epsilon_4^2 = e^2/2\pi R$.

There are two typical ways to impose renormalization conditions. The most convenient way is to impose them in the $R \to \infty$ limit, namely in $M^5$. In terms of $\Sigma^{(0)}$, $\Pi^{(0)}$ and $F^{(0)}$ in (2.3) and (2.4) and the five-dimensional fermion mass $m$ in $M^5$, renormalization constants $Z_j$ and $\delta m$ are fixed by

$$
\Sigma^{(0)}(\hat{p} = m) = 0 ,
$$
\[
\frac{d\Sigma^{(0)}}{d\hat{p}}(\hat{p} = m) = 0 , \\
\Pi^{(0)}(\hat{p}^2 = 0) = 0 , \\
\Gamma^{(0)M}(\hat{p} = \hat{p}') = \gamma^M .
\]

(2.8)

One can also adopt the mass-independent renormalization where \( m \) is set to be zero in the above equations (2.8). An alternative prescription is to impose the conditions on shell in \( M^4 \times S^1 \) for the 4D fermion with the lowest 4D mass (\( \equiv m^{4D}_{\text{phys}} \)). As is seen below, \( V^{\text{eff}}(\theta_H) \) has a global minimum at \( \theta_H = \pi \left( a = 1/2 \equiv a_0 \right) \). Hence the alternative renormalization conditions read

\[
S(\hat{p}) \bigg|_{p^5 = a_0 R^{-1}} = \frac{i}{\hat{p} - m - \Sigma} \bigg|_{p^5 = a_0 R^{-1}} \sim \frac{i}{\hat{p} + a_0 R^{-1} \gamma^5 - m}
\]

near \( p^2 = (m^{4D}_{\text{phys}})^2 = m^2 + \frac{a_0^2}{R^2} \),

\[
\Pi(p^2 = 0, p^5 = 0) = 0 , \\
\Gamma^\mu(p = p', p^5 = p'^5 = \frac{a_0}{R}) = \gamma^\mu .
\]

(2.9)

Renormalization with other reference values of \( a_0 \) is also possible. In all of these prescriptions the Ward-Takahashi identity \( Z_1 = Z_2 \) is preserved.

§3. Effective potential

The effective potential \( V^{\text{eff}} \) for \( \theta_H = 2\pi a \) is evaluated at the two loop level. We adopt the dimensional regularization method to maintain the gauge invariance. The evaluation is performed in \( M^d \times S^1 \), and the \( d \to 4 \) limit is taken at the end.

\( V^{\text{eff}}(a) \) in \( d + 1 \) dimensions at the one loop level is given by

\[
V^{\text{eff}}(a)^{1\text{loop}} = -\frac{f(d)}{2} \int \frac{d^dp_E}{(2\pi)^d 2\pi R} \sum_{n=-\infty}^{\infty} \ln \left( p_E^2 + \frac{(n-a)^2}{R^2} + m^2 \right)
\]

(3.1)

after Wick rotation. \( f(d) = 2^{[(d+1)/2]} \). In the \( m \to 0 \) limit it becomes

\[
V^{\text{eff}}(a)^{1\text{loop}} = \frac{f(d)\Gamma(\frac{d+1}{2})}{(2\pi^{3/2} R)^{d+1}} f_{d+1}(a) + \text{constant} ,
\]

(3.2)

where \( f_k(a) \) is defined in (A.3). The 4D effective potential is \( V^{4D}_{\text{eff}}(a) = V^{\text{eff}}(a)|_{d=4} \times 2\pi R \) so that

\[
V^{4D}_{\text{eff}}(a)^{1\text{loop}} = \frac{3}{16\pi^6 R^4} f_5(a) + \text{constant} ,
\]

(3.3)

where the constant is divergent, but is independent of \( a \).
Fig. 1. Diagrams contributing to the effective potential $V_{\text{eff}}(\theta_H)$. The second and third diagrams contain one loop counter term $\delta_2 \delta - \delta_5$ and $\delta_3 (\hat{P}_E \hat{Q}_E - \eta^{M N} \hat{P}_E^2)$, respectively.

The effective potential is minimized at $a = 1/2$ or $\theta_H = \pi$. The effective potential for the zero mode of $A_5$, or the Higgs field $\phi_H$, is given by $V_{\text{eff}}^{4D}(a)$ where $a$ is replaced by $e_4 R \phi_H$. Here the four-dimensional coupling is given by $e_4 = e/\sqrt{2\pi R}$. Hence the Higgs mass $m_H$ at the one loop level is given, for $m = 0$, by

$$m_H^2|_{1\text{loop}} = e_4^2 R^2 \frac{d^2}{d a^2} V_{\text{eff}}^{4D}(a)|_{a=\frac{1}{2}} = \frac{9 e_4^2 \zeta_R(3)}{16 \pi^4 R^2}, \quad (3.4)$$

where $\zeta_R(z)$ is Riemann’s zeta function.

At the two loop level the diagrams in Fig. 1 contribute to $V_{\text{eff}}(a)$. The contribution from the first diagram (a) is given by

$$-i V_{\text{eff}}(a)^{2\text{loop}(a)} = (-1)^{\frac{-i e}{2!}} \int \frac{d^d p}{(2\pi)^d} \frac{d^d q}{2\pi R} \sum_l \int \frac{d^d q}{(2\pi)^d} \frac{d^d q}{2\pi R} \sum_n$$

$$\times \frac{-i \eta_{M N}}{\hat{P} - \hat{Q}}^2 + i \epsilon \sum l \sum_n \frac{i}{\hat{P} - m + i \epsilon} \gamma^M \frac{i}{\hat{Q} - m + i \epsilon} \gamma^N ,$$

so that

$$V_{\text{eff}}(a)^{2\text{loop}(a)}$$

$$= -\frac{e^2}{2} \int \frac{d^d p_E}{(2\pi)^d} \frac{d^d q_E}{(2\pi)^d} \frac{1}{(2\pi R)^2} \sum_l \sum_n \frac{f(d)}{(d - 1)^2 (d + 1) m^2}$$

$$= -\frac{e^2 f(d)}{2} \left\{ \frac{2m^2}{(\hat{P}_E^2 + m^2)(\hat{Q}_E^2 + m^2)(\hat{P} - \hat{Q})^2} \right\} . \quad (3.6)$$

At this stage infinite sums over discrete momentum $P_E^2$ have to be evaluated. We summarize typical integral-sums in Appendix A. In terms of $G_j(a; m, d)$ defined there, $V_{\text{eff}}(a)^{2\text{loop}(a)}$ can be expressed as

$$V_{\text{eff}}(a)^{2\text{loop}(a)} = -\frac{e^2 f(d)}{2} \left\{ 2m^2 G_2(a; m, d) \right\} . \quad (3.6)$$
In the $mR \to 0$ limit (with $R$ kept fixed) it simplifies to

$$V_{\text{eff}}(a)^{2\text{loop}(a)} = \frac{e^2(d-1)f(d)}{4}\left\{G_1(a;0,d) - G_1(0;0,d)\right\}^2$$

(3.8)

up to an $a$-independent constant where $f_k(a)$ is defined in (A-3). Its contribution to the 4D effective potential is given by

$$V^{4\text{D}}_{\text{eff}}(a)^{2\text{loop}} = \frac{3e^4}{16\pi^4(2\pi R)^4} \left\{f_3(a) - f_3(0)\right\}^2.$$  

(3.9)

Contributions from one loop counter terms, namely from the second and third diagrams in Fig. 1, either vanish for $m = 0$ or are $a$-independent.

The effective potential at the two loop level is given by (3.3) and (3.9). The global minimum is located at $a = 1/2$. The two loop contribution to the effective potential is suppressed by an order of the fine structure constant, as seen in Eq. (3.9). Hence, even though the two loop contribution itself is minimized at $a = 0$, the effective potential is governed by the one loop contribution (3.3) as long as the coupling $e^2/4\pi$ is small. When one needs a very small value of $a$ for having a realistic model of the gauge-Higgs unification, the two loop contribution may play an important role and affect the location of the global minimum of the effective potential as discussed in Ref. 19).

The second derivative of $V^{4\text{D}}_{\text{eff}}(a)^{2\text{loop}}$ with respect to $a$ at the global minimum of $V^{4\text{D}}_{\text{eff}}(a)$ is related to the coefficient $b_{-1} = -\Pi^{55}_{[p^5=0, p^2=0]}$ introduced in §2, as easily confirmed by examining Feynman diagrams. Indeed,

$$b_{-1} = e^2 R^2 \left. \frac{d^2}{da^2} V^{4\text{D}}_{\text{eff}}(a) \right|_{a=\frac{1}{2}}.$$  

(3.10)

Hence to the two loop order we have

$$b_{-1} = \frac{9e^2\zeta_R(3)}{16\pi^4 R^2} - \frac{21e^4}{128\pi^6 R^2} \ln 2 \zeta_R(3).$$  

(3.11)

We would like to note that $e^2 R^2 (d^2 V^{4\text{D},2\text{loop}}_{\text{eff}}/da^2)$ vanishes at $a = 0$, which is in conformity with the result in Ref. 49).

§4. Vacuum polarization

The vacuum polarization tensors $\Pi^{MN}$ in QED on $M^4 \times S^1$ has been evaluated to the two loop order near $a = 0$ in Ref. 49). We need to determine $\Pi^{MN}$ at $a = 1/2$
which corresponds to the true vacuum. In order to determine the Higgs mass (2.7) to the two loop order, we need to find the coefficients $b_0$ and $c_0$, or $\Pi^{MN}$ to the one loop order as $b_{-1}$ has been already evaluated in (3.11). In this section $\Pi^{MN}(\hat{p}; a, R)$ is evaluated at the one loop level for an arbitrary value $a$. We note that $\Pi^{MN}$ in supersymmetric gauge theory on an orbifold $M^4 \times (T^2/Z_2)$ with a vanishing Wilson line phase has been evaluated at the one loop level.53)

The evaluation is straightforward. The contribution from a fermion loop is

$$\Pi^{MN}(\hat{p}) = \frac{ie^2}{(2\pi)^d 2\pi R} \sum_l \text{Tr} \gamma^M \frac{1}{\hat{q} - m + i\epsilon} \gamma^N \frac{1}{\hat{q} + \hat{p} - m + i\epsilon}.$$ \hspace{1cm} (4.1)

In the dimensional regularization scheme the gauge invariance is maintained so that $\Pi^{MN}$ satisfies the current conservation,

$$\hat{p}_M \Pi^{MN}(\hat{p}) = 0.$$ \hspace{1cm} (4.2)

The detailed evaluation of $\Pi^{MN}(\hat{p})$ is given in Appendix B, which is summarized in (B.5) and (B.6). To find the coefficients $b_0$ and $c_0$, we need $\Pi$ and $F$ at $p^5 = n/R = 0$. From (B.6) and (B.7) it follows that

$$\left( \begin{array}{c} \Pi \\ F \end{array} \right)_{p^5=0} = \left( \begin{array}{c} \Pi^{(0)}(p^2) \\ 0 \end{array} \right) - \frac{e^2 f(d)}{(4\pi)^{(d+1)/2}} \int_0^1 dx \int_0^\infty dt \frac{t^{(1-d)/2} e^{-t(m^2 - x(1-x)p^2)}}{2} \times \sum_{\ell \neq 0} e^{-\pi^2 R^2 \ell^2 / 4 \pi i a} \left( \frac{2x(1-x)}{p^2 - \ell^2} \right).$$ \hspace{1cm} (4.3)

The $\ell \neq 0$ terms give finite contributions. At $d = 4$ and $m = 0$

$$\Pi|_{p^5=0} = \frac{3e^2 R}{128} (-p^2)^{1/2} - \frac{e^2 f_1(a)}{6\pi^2} + \cdots,$$

$$F|_{p^5=0} = -\frac{3e^2 f_3(a)}{4\pi^4 R^2} \frac{1}{p^2} - \frac{e^2 f_1(a)}{12\pi^2} + \cdots.$$ \hspace{1cm} (4.4)

Note that the counter term $\delta_3 = \Pi^{(0)}(p^2 = 0)$ vanishes at $m = 0$. We expand the invariant functions around $p^2 = 0$ at the global minimum of the $V_{\text{eff}}(a)$, namely at $a = 1/2$,

$$\Pi|_{p^5=0, a=\frac{1}{2}} = c_0 + \cdots,$$

$$F|_{p^5=0, a=\frac{1}{2}} = \frac{b_{-1}}{p^2} + b_0 + \cdots.$$ \hspace{1cm} (4.5)
The coefficients at the one loop level are given by

\begin{align*}
    c_0 &= \frac{e_4^2 \ln 2}{6\pi^2}, \\
    b_0 &= \frac{e_4^2 \ln 2}{12\pi^2}, \\
    b_{-1} &= \frac{9e_4^2 \zeta_R(3)}{16\pi^4 R^2}.
\end{align*}

The coefficient $b_{-1}$ coincides with the result from the effective potential (3.4) or (3.11) as it should.

The gauge invariant mass for the zero mode of $A_5$ appears as a pole $1/p^2$ in $F$. We remark that there is similarity to the gauge invariant mass in the Schwinger model (QED in two dimensions) in which $\Pi$ develops a pole from a fermion loop.\(^{54}\)

It differs in the point that $F$ vanishes in the $R \to \infty$ limit, or in $M^2$, whereas the pole remains in the Schwinger model in $M^2$. The effective potential for $\theta_H$ in the Schwinger model on a circle has the same structure as in the current model.\(^{55}\) Its curvature at the minimum gives a mass for photons.

§5. The Higgs mass

The 4D effective action for the Higgs field $\phi_H$ takes the form

\[ \Gamma_{\text{eff}}[\phi_H] = \int d^4x \left\{ -V[\phi_H] + \frac{1}{2}Z[\phi_H] \partial_\mu \phi_H \partial^\mu \phi_H + \cdots \right\}. \tag{5.1} \]

The Higgs mass in this approach is

\[ m_H^2 = \left. \frac{1}{Z[\phi_H]} \frac{\partial^2 V[\phi_H]}{\partial \phi_H^2} \right|_{\phi_H^\text{min}}, \tag{5.2} \]

where $\phi_H^\text{min} = (2e_4 R)^{-1}$ is the location of the global minimum of $V[\phi_H]$. The effective potential $V[\phi_H]$ is given by $V_{\text{eff}}^4D(a)$ with $a = e_4 R \phi_H$. Its second derivative is related to $b_{-1}$ by (3.10). Similarly $Z[\phi_H^\text{min}]$ is related to $c_0$ and $b_0$ by $Z[\phi_H^\text{min}] = 1 - c_0 - b_0$. Hence the Higgs mass defined by (5.2) coincides with the mass defined by (2.7).

Inserting (3.11) and $c_0, b_0$ in (4.6) there, one finds that in the massless fermion limit $m = 0$

\[ m_H^2 = \frac{9e_4^2 \zeta_R(3)}{16\pi^4 R^2} \left\{ 1 - \frac{e_4^2 \ln 2}{24\pi^2} \right\}. \tag{5.3} \]

The coupling constant $e$ and the coefficients $b_{-1}, b_0, c_0$ depend on the renormalization scheme. However, the Higgs mass is a physical quantity so that it should not depend on the renormalization scheme employed. This can be confirmed from the results at the two loop level obtained above.

Let $e', b'_{-1}, b'_0, c'_0$ be the coupling constant and the coefficients in a second renormalization scheme. To be concrete, $(e, b_{-1}, b_0, c_0)$ are defined in the renormalization
in $M^5$ as employed in the preceding sections, whereas $(e', b_{-1}', b_0, c_0')$ are defined in the on-shell renormalization in $M^4 \times S^1$ at $a = 1/2$. For the sake of simplicity we suppose that $m = 0$. At the one loop level we write $\Pi(\hat{p}) = \Pi_r(\hat{p}) - \delta_3$ where $\Pi_r(\hat{p})$ is the contribution from a fermion loop. The counter terms are given by

$$
\delta_3 = \lim_{R \to \infty} \Pi_r(\hat{p}; a, R) |_{\hat{p}^2 = 0},
$$

$$
\delta'_3 = \Pi_r(p^2 = 0, p^5 = 0; a = \frac{1}{2}, R).
$$

The difference between the two, $\delta_3 - \delta'_3$, is finite. The coefficient $c_0$ is defined by the expansion of $\Pi(p^2, p^5 = 0) = c_0 + \cdots$ in $p^2$. Hence $c_0' - c_0 = \delta_3 - \delta'_3$. The Ward-Takahashi identity $Z_1 = Z_2$ implies $e^{(0)} = Z_3^{-1/2} e$. It follows that

$$
e^2 = \frac{Z_3'}{Z_3} e^2 \simeq \frac{e^2}{1 - c_0 + c_0'}.
$$

(5.5)

We observed that $F$ is finite at the one loop level in §4 and $V_{\text{eff}}(a)^{2\text{loop}(a)}$ itself is finite in the $mR \to 0$ limit in §3. We write $b_{-1} = e^2 b_{-1}^{(1)} + e^4 b_{-1}^{(2)} + \cdots$ and $b_0 = e^2 b_0^{(1)} + \cdots$. Then the finiteness implies that $b_{-1}^{(1)} = b_{-1}'^{(1)}$, $b_{-1}^{(2)} = b_{-1}'^{(2)}$, and $b_0^{(1)} = b_0'^{(1)}$. From these identities one finds that

$$
(m_H'^2)' = \frac{b_{-1}'}{1 - c_0' - b_0'} = e^2 \cdot \frac{b_{-1}'^{(1)} + e^2 b_{-1}'^{(2)}}{1 - c_0' - e^2 b_0'}
$$

$$
\simeq \frac{e^2}{1 - c_0 + c_0'} \cdot \frac{b_{-1}^{(1)} + e^2 b_{-1}^{(2)}}{1 - c_0' - e^2 b_0'}
$$

$$
\simeq \frac{b_{-1}}{1 - c_0 - b_0} = m_H^2.
$$

(5.6)

The Higgs mass is independent of the renormalization scheme to this order as it should be.

§6. Summary and discussion

In this paper we have determined the Higgs mass $m_H$ at the two loop level, or to $O(e^4)$, in the QED gauge-Higgs unification model on $M^4 \times S^1$. The mass is shown to be independent of the renormalization scheme. The evaluation of the vacuum polarization tensors, or equivalently $Z[\phi_H]$ in the effective action, at the one loop level is also required to find $m_H$ at the two loop level. The $\theta_H$-dependent part of the effective potential is found finite at the two loop level. Divergences in $\Pi^{MN}(\hat{p})$, which appear only in the $\Pi$ part, but not in the $F$ part, are absorbed by the counter term $\delta_3$. There is no need to introduce additional counter terms other than $\delta_1$, $\delta_2$, $\delta_3$ and $\delta_m$ at this level.

The fact that radiative corrections to the Higgs mass are finite and suppressed by a power of the four-dimensional gauge coupling constant with respect to the
Kaluza-Klein mass scale $m_{KK} = 1/R$ has an important implication in the gauge-Higgs unification. The Higgs mass is not an input parameter of the theory, but is definitively predicted in terms of other fundamental constants such as the gauge coupling and the size of the extra dimension. Its value is stable against higher order corrections. In other words the gauge-Higgs unification yields a naturally light Higgs boson in four dimensions. We remark that in the gauge-Higgs unification in flat space, however, the Higgs mass becomes small compared with the $W$ and $Z$ boson masses unless the Wilson line phase $\theta_H$ is sufficiently small.\(^{18-20,23}\) This problem can be naturally resolved in the gauge-Higgs unification in the warped space.\(^{28}\) Further the Higgs interactions with other fields and particles can be predicted as well.\(^{29,32,33,38,39}\)

Although we considered, for the sake of simplicity, the massless fermion limit ($mR \to 0$) to find $m_H$ in the present paper, the same features are expected to hold in the $m \neq 0$ case. Contributions of non-vanishing $\delta_1$, $\delta_2$ and $\delta_m$ have to be taken into account. In passing, we would like to point out that the massless fermion limit is well defined on $M^4 \times S^1$ and on $M^3 \times S^1$. As the effective potential is minimized at $a = 1/2$, the fermion propagator does not vanish at $\hat{p}^M = 0$ with $a = 1/2$.

Extension of our analysis to QED in $M^5 \times S^1$ is straightforward. To define a theory and determine a Higgs mass at the two loop level, one must include additional counter terms such as $(\partial_L F_{MN})^2$ in the original Lagrangian. Other than this the analysis remains intact with the substitution $d = 5$.

More important is the extension to the non-Abelian case and to higher order corrections in the viewpoint of the gauge-Higgs unification. Not only propagators of gauge fields have $\theta_H$ dependence, but also there appears a new interaction vertex proportional to $\theta_H$. It is curious to see how the large gauge invariance ($\theta_H \to \theta_H + 2\pi$) is maintained in perturbation theory.

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**Appendix A**

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**Integrals and Sums**

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We summarize useful formulas for the evaluation in §3. The first integral-sum is

$$G_1(a; m, d) = \int \frac{d^d p_E}{(2\pi)^d} \frac{1}{2\pi R} \sum_{\ell = -\infty}^{\infty} \frac{1}{\ell \frac{p_E^5}{m^2} + \frac{\ell - a}{R}} \quad (p_E^5 = \frac{\ell - a}{R})$$
\[= \int_0^\infty \frac{e^{-tm^2}}{(4\pi t)^{d/2}} \frac{1}{2\pi R} \sum_{\ell=-\infty}^\infty e^{-t(\ell-a)^2/R^2} \]

\[= \int_0^\infty \frac{e^{-tm^2}}{(4\pi t)^{(d+1)/2}} \sum_{\ell=-\infty}^\infty e^{-\pi^2 R^2 \ell^2/t} e^{-2\pi i \ell a} \]

\[= \frac{\Gamma\left(\frac{1-d}{2}\right)}{(4\pi)^{(d+1)/2}} m^{d-1} + \frac{m^{d-1}}{(2\pi)^d} \sum_{\ell=1}^\infty \frac{2 \cos(2\pi \ell a) K_{(d-1)/2}(2\pi \ell m R)}{(\ell m R)^{(d-1)/2}} . \] (A.1)

In the third equality the Poisson resummation formula has been employed. In the last expression \(K_\nu(z)\) is the modified Bessel function. For small \(mR\) one finds

\[G_1(a; m, d) = \frac{\Gamma\left(\frac{1-d}{2}\right)}{(4\pi)^{(d+1)/2}} m^{d-1} + \frac{2 \Gamma\left(\frac{d-1}{2}\right) f_{d-1}(a)}{(4\pi)^{(d+1)/2}(\pi R)^{d-1}} - \frac{2 \Gamma\left(\frac{d-3}{2}\right) f_{d-3}(a) m^2}{(4\pi)^{(d+1)/2}(\pi R)^{d-3}} + \cdots , \] (A.2)

where

\[f_k(a) = \sum_{\ell=1}^\infty \frac{\cos 2\pi \ell a}{\ell^k} . \] (A.3)

The second integral-sum is

\[G_2(a; m, d) = \int \frac{d^d p_E}{(2\pi)^d} \frac{d^d q_E}{(2\pi)^d} \frac{1}{(2\pi R)^2} \sum_{\ell=-\infty}^\infty \sum_{n=-\infty}^\infty \frac{1}{(\tilde{p}_E^2 + m^2) (\tilde{q}_E^2 + m^2) (\tilde{p}_E - \tilde{q}_E)^2} . \]

\[\left( \tilde{p}_E^5 = \frac{\ell - a}{R} , \tilde{q}_E^5 = \frac{n - a}{R} \right) \] (A.4)

Introducing Feynman parameters, exponentiating the denominator, integrating over \(p_E\) and \(q_E\), and making repeated use of the Poisson resummation formula, one finds

\[G_2(a; m, d) = \frac{1}{2} \int_\Omega dx dy \int_0^\infty \frac{t^2 e^{-t(x+y)m^2}}{(4\pi t)^{d+1} h(x, y)^{(d+1)/2}} \]

\[\times \sum_{\ell=-\infty}^\infty \sum_{n=-\infty}^\infty e^{-2\pi i (\ell+n) a} \exp \left\{ - \frac{\pi^2 R^2 S_{\ell n}(x, y)}{t h(x, y)} \right\} , \]

\[\Omega = \{(x, y); 0 \leq x, y, x + y \leq 1\} , \]

\[h(x, y) = (1-x)(1-y) - (1-x-y)^2 , \]

\[S_{\ell n}(x, y) = (1-x)\ell^2 + (1-y)n^2 + 2(1-x-y)\ell n \]  (A.5)

The \((\ell, n) = (0, 0)\) term gives contributions in \(M^5\), which is independent of \(a\). For small \(mR\) one finds

\[G_2(a; m, d) = \frac{\Gamma(2-d) m^{2(d-2)}}{2(4\pi)^{d+1}} \int_\Omega dx dy \frac{(x+y)^{d-2}}{h(x, y)^{(d+1)/2}} \]
We shift the integration variable $q$ and integrate over Wick-rotated $q$ to find
\[ \Pi \]
\[ \Pi \]
\[ \Pi \]
Evaluation of the vacuum polarization tensors proceeds as follows. Performing the trace in (4-1) and introducing a Feynman parameter, one obtains
\[ \Pi^{MN}(\bar{p}) = ie^2 f(d) \int_0^1 dx \int_0^\infty dt \frac{1}{2\pi R} \sum_\ell \frac{S^{MN}}{[\hat{q}^2 - m^2 + x(\bar{p}\hat{q} - 2\hat{q}\bar{p}) + i\epsilon]^2}, \]
\[ S^{MN} = 2\hat{q}^M\hat{q}^N + \hat{p}^M\hat{p}^N - \hat{q}(\hat{q} + \hat{p})\eta^{MN} + 2m^2\eta^{MN}. \] (B.1)
We shift the integration variable $q^\mu \rightarrow q^\mu + xp^\mu$, exponentiate the denominator, and integrate over Wick-rotated $q'_E$ to find
\[ \left( \begin{array}{c} \Pi^{\mu\nu} \\ \Pi^{55} \\ \Pi^{\mu5} \end{array} \right) = -\frac{e^2 f(d)}{(4\pi)^{d/2}} \int_0^1 dx \int_0^\infty dt \frac{t^{1-(d/2)}}{2\pi R} \sum_\ell \frac{1}{\ell} e^{-t[(q^5 +xp^5)^2 + m^2 - x(1-x)p^2]} \sum_\ell \frac{1}{\ell} \eta^{\mu\nu} - 2x(1-x)p^\mu p^\nu \]
\[ \frac{1}{2} \frac{d - 1}{t} x(1-x)p^2 + q^5(q^5 + p^5) + m^2 \]
\[ \frac{1}{2} \frac{d - 1}{t} x(1-x)p^2 + q^5(q^5 + p^5) + m^2 \]
\[ p^\mu \left\{ (1 - 2x)q^5 - xp^5 \right\}. \] (B.2)
Recalling $q^5 = (\ell - a)/R$ and $p^5 = n/R$, one employ the Poisson resummation formula to find
\[ \left( \begin{array}{c} \Pi^{\mu\nu} \\ \Pi^{55} \\ \Pi^{\mu5} \end{array} \right) = \frac{-e^2 f(d)}{(4\pi)^{(d+1)/2}} \int_0^1 dx \int_0^\infty dt \frac{t^{(1-d)/2}e^{-t(m^2 - x(1-x)p^2)}}{2\pi R} \sum_{\ell = -\infty}^{\infty} e^{-\pi^2 R^2\ell^2/t} e^{2\pi i\ell(a-xn)} \]
\[ \left( \begin{array}{c} \frac{d - 1}{2t} - \frac{\pi^2 R^2 \ell^2}{t^2} + (1 - 2x)\frac{i\pi n \ell}{t} + x(1-x)p^2 + m^2 \end{array} \right) \eta^{\mu\nu} \]
\[ -2x(1-x)p^\mu p^\nu \]
\[ \frac{d - 1}{2t} - \frac{\pi^2 R^2 \ell^2}{t^2} + (1 - 2x)\frac{i\pi n \ell}{t} - x(1-x)\left\{ (p^5)^2 + p^2 \right\} - m^2 \]
\[ p^\mu \left\{ (1 - 2x)\frac{i\pi R \ell}{t} - 2x(1-x)p^5 \right\}. \] (B.3)
The expression is simplified with identities

\[
\int_0^\infty \! \! dt \left\{ \frac{1 - d}{2t} - m^2 + x(1 - x)\hat{p}^2 + \frac{\pi^2 R^2 \ell^2}{t^2} \right\} t^{(1-d)/2} e^{-t(m^2 - x(1-x)\hat{p}^2)} e^{-\pi^2 R^2 \ell^2 / t} = \int_0^\infty \! \! dt \frac{\partial}{\partial t} \left\{ t^{(1-d)/2} e^{-t(m^2 - x(1-x)\hat{p}^2)} e^{-\pi^2 R^2 \ell^2 / t} \right\} = 0 \, ,
\]

\[
\int_0^1 \! \! dx \left\{ (1 - 2x)\hat{p}^2 - \frac{2\pi i \ell n}{t} \right\} e^{i x(1-x)\hat{p}^2 - 2\pi i \ell n x} = \int_0^1 \! \! dx \frac{1}{t} \frac{\partial}{\partial x} e^{i x(1-x)\hat{p}^2 - 2\pi i \ell n x} = 0 \, ,
\]

(B.4)

to

\[
\begin{pmatrix}
\Pi^{\mu\nu} \\
\Pi^{55} \\
\Pi^{\mu 5}
\end{pmatrix}
= \begin{pmatrix}
-e^2 f(d) \\
(4\pi)^{(d+1)/2} \\
(4\pi)^{d+1/2}
\end{pmatrix}
\int_0^1 \! \! dx \int_0^\infty \! \! dt \, t^{(1-d)/2} e^{-t(m^2 - x(1-x)\hat{p}^2)} \sum_{\ell = -\infty}^\infty e^{-\pi^2 R^2 \ell^2 / t} e^{2\pi i \ell (a-xn)}
\times \begin{Bmatrix}
2x(1-x)(\hat{p}^2 \eta^{MN} - \hat{p}^M \hat{p}^N) + (1 - 2x) \frac{i\pi \ell}{t} \left( \frac{n \eta^{\mu\nu}}{t} - \frac{2\pi^2 R^2 \ell^2}{t^2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right) \\
2x(1-x) + \frac{1}{\hat{p}^2}(1 - 2x)n \frac{i\pi \ell}{t} \\
\frac{p^2}{(\hat{p}^2)^2} \frac{2\pi^2 R^2 \ell^2}{t^2}
\end{Bmatrix} \, .
\]

(B.5)

Notice that only the \( \ell = 0 \) term survives in the \( R \to \infty \) limit. \( \Pi^{\mu\nu} \) and \( \Pi^{55} \) are even in \( \hat{p}^5 = n/R \), whereas \( \Pi^{\mu 5} \) is odd after the integration over \( x \).

As a consequence of the current conservation \( \Pi^{MN} \) can be expressed in terms of the two invariant functions \( \Pi \) and \( F \) in (2.4). Comparison of the two expressions for \( \Pi^{55} \) and \( \Pi^{\mu 5} \) in (B.5) shows that in the integrand \( -2\pi^2 R^2 \ell^2 / t^2 \) and \( (\hat{p}^2 / \hat{p}^5)(1 - 2x)R(i \pi \ell / t) \) give the same contribution in (B.5).

The invariant functions are given by

\[
\begin{pmatrix}
\Pi \\
F
\end{pmatrix}
= \begin{pmatrix}
-e^2 f(d) \\
(4\pi)^{(d+1)/2} \\
(4\pi)^{d+1/2}
\end{pmatrix}
\int_0^1 \! \! dx \int_0^\infty \! \! dt \, t^{(1-d)/2} e^{-t(m^2 - x(1-x)\hat{p}^2)}
\times \sum_{\ell = -\infty}^\infty e^{-\pi^2 R^2 \ell^2 / t} e^{2\pi i \ell (a-xn)} \left( 2x(1-x) + \frac{1}{\hat{p}^2}(1 - 2x)n \frac{i\pi \ell}{t} \right).
\]

(B.6)

In the \( R \to \infty \) limit, namely in \( M^{d+1} \), \( F \) vanishes and

\[
\Pi^{(0)}(\hat{p}^2) = \lim_{R \to \infty} \Pi
\]

\[
= -\frac{2e^2 f(d) \Gamma\left( \frac{3-d}{2} \right)}{(4\pi)^{(d+1)/2}} \int_0^1 \! \! dx (1-x) \left[ m^2 - x(1-x)\hat{p}^2 \right]^{(d-3)/2} \, .
\]

(B.7)
At $d = 4$ ($D = 5$), $\Pi^{(0)}(0)$ is removed by the counter term $\delta_3$ so that the renormalized $\Pi$ is $\Pi(\hat{p}) - \Pi^{(0)}(0)$. At $d = 5$ ($D = 6$), an additional counter term $(\partial_L F_{MN})^2$ is necessary to remove the divergence proportional to $\hat{p}^2$ in (B.7).

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