A microscopic cluster-model study of $^3$He$^+$p scatterings

K. ARAI$^{1,*}$ S. AOYAMA$^{2,**}$, and Y. SUZUKI$^{3,***}$

$^1$Division of General Education, Nagaoka National College of Technology, 888 Nishikatakai, Nagaoka, Niigata, 940-8532, Japan,
$^2$Integrated Information Processing Center, Niigata University, Niigata 950-2181, Japan
$^3$Department of Physics, and Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

We calculate $^3$He$^+$p scattering phase shifts in two different microscopic cluster models, Model T and Model C, in order to show the effects of tensor force as well as $D$-wave components in the cluster wave function. Model T employs a realistic nucleon-nucleon potential and includes the $D$-wave, whereas Model C employs an effective potential in which the tensor-force effect is considered to be renormalized into the central force and includes only the $S$-wave for the cluster intrinsic motion. The $S$- and $P$-wave elastic scattering phase shifts are obtained in the $[^3\text{He}^+p]+(d+2p)$ coupled-channels calculation. In Model T, the $d+2p$ channel plays a significant role in producing the $P$-wave resonant phase shifts but hardly affects the $S$-wave non-resonant phase shifts. In Model C, however, the effect of the $d+2p$ channel is suppressed in both of the $S$- and $P$-wave phase shifts, suggesting that it is renormalized mostly as the $^4\text{He}(1/2^+)+p$ channel in the resonance region.

§1. Introduction

For studies of structure and reactions of light nuclei, a cluster model is known to be one of successful models$^1$). A microscopic cluster model like the resonating group method (RGM)$^{2,3}$ employs two ingredients. The first is to assume, mainly because of its simplicity, that the nucleus is composed of a few $s$-shell clusters such as $\alpha$, $^3\text{H}$, $^3\text{He}$, and $d$ and that the antisymmetry requirement on the total wave function is properly taken into account. The second is to employ an effective nucleon-nucleon ($N$-$N$) interaction, e.g., the Minnesota potential (MN)$^4)$. The intrinsic wave function of the $s$-shell cluster is usually approximated with the $0s$ harmonic-oscillator (h.o.) function whereas the cluster relative motion is solved accurately. Corresponding to the simplified cluster wave functions, only the central, $LS$, and Coulomb terms of the $N$-$N$ interaction is usually employed, and the effects of the tensor force and the short-range repulsion which are present in a realistic interaction are assumed to be renormalized in the central force of the effective interaction. Though there are some calculations available which employ another type of effective $N$-$N$ interactions including the tensor force, its contribution was considered only for the cluster relative motion but not for the cluster intrinsic motion.$^5$-$^7$)

In reality, it is well known that the ground state of $^4\text{He}$, for example, has a large admixture of the $D$-wave component due to the tensor force, amounting to

$^*)$ E-mail: arai@nagaoka-ct.ac.jp
$^{**)}$ E-mail: aoyama@cc.niigata-ct.ac.jp
$^{***)}$ E-mail: suzuki@nt.niigata-ct.ac.jp
\( P_d \approx 14\% \) for the AV8' potential.\textsuperscript{8)–10) It is discussed that this \( D \)-wave component plays an important role for the \( Q \)-moment of the ground state of \(^6\)Li\textsuperscript{6}\textsuperscript{11}) as well as for the \( \alpha+n \) \( P \)-wave phase shifts.\textsuperscript{12) The \( D \)-wave component with \((L, S) = (2, 3/2)\) in the ground state of \(^3\)He is 8.5\% for the AV8' potential and 7.0\% for the G3RS potential.\textsuperscript{10) In order to understand more deeply the structure and reactions of light nuclei, it is important to test the microscopic cluster model by taking into account both the tensor force of the \( N-N \) interaction and the \( D \)-wave components in the \( s \)-shell clusters.

The purpose of the present article is to focus on the effect of the tensor force in \(^3\)He+\( p \) \( S \)- and \( P \)-wave elastic scatterings by making comparative calculations in two different microscopic models, Model T and Model C. In Model T, a realistic force including the tensor force is employed and the \((L, S) = (2, 3/2)\) component of the \(^3\)He cluster as well as the \( D \)-wave component in the deuteron are taken into account. In Model C, however, an effective potential without the tensor term is employed and both the \(^3\)He and deuteron wave functions include only the \( S \)-wave components. Four \( P \)-wave broad resonances with spin and parity 2\(^−\), 1\(^−\), 0\(^−\), and 1\(^−\) are observed in the low incident energy region of 4–7 MeV in the \(^3\)He+\( p \) scattering,\textsuperscript{13) but no resonant behavior is observed in the two \( S \)-wave phase shifts with 0\(^+\) and 1\(^+\). The \(^3\)He+\( p \) scattering was previously investigated by various approaches with both realistic\textsuperscript{14),15) and effective interactions,\textsuperscript{5) Pfritzinger et al.} calculated the elastic \(^3\)He+\( p \) and \(^3\)H+\( n \) scatterings using the RGM with a realistic potential and discussed the phase shifts, analyzing powers and cross sections.\textsuperscript{15) In the present article, we will clarify both the effects of the \( D \)-wave components in the cluster wave functions and the mixing of the \( d+2p \) channel by comparing the results of Models T and C.

Firstly we calculate the phase shifts in Model T. In this calculation, the \( d+2p \) channel as well as other spin-parity states of \(^3\)He up to 5/2\(^±\) are included in order to take into account the breakup or distortion effect of \(^3\)He. These states of \(^3\)He other than the 1/2\(^+\) state are actually continuum states but they are approximated with discretized states in the present calculation. Secondly we repeat the phase shift calculation in Model C using the interaction which contains no tensor force.

The organization of this article is as follows. In the next section, Model T as well as Model C are briefly explained. In Sect.3, the phase shifts obtained by both models are presented. Calculations in the similar models are also performed for the ground state of \(^4\)He and \(^3\)H+\( p \) \( S \)-wave scattering phase shift. Summary is given in Sect. 4.

\section*{§2. Model}

In the present study, we have employed the microscopic cluster model as formulated by the RGM.\textsuperscript{2),3) In this method, all the nucleons are treated explicitly and they are assumed to be arranged in several clusters. The wave function consisting
of two clusters \((A+B)\) is given as

$$\Psi^{JM\pi}_{AB} = \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} A \left\{ \left[ \Phi^{A,i}_{I_A,\pi_A} \Phi^{B,j}_{I_B,\pi_B} I \chi^{A,i,B,j}_\ell (\rho) \right]_{JM} \right\}, \quad (2.1)$$

where \(\Phi^{A,i}_{I_A,\pi_A}\) and \(\Phi^{B,j}_{I_B,\pi_B}\) are the intrinsic wave functions of the clusters \(A\) and \(B\), and their spins, \(I_A\) and \(I_B\), are coupled to the channel spin \(I\) as indicated by the square bracket \([\cdot]\). The symbol \(N_A(N_B)\) stands for the number of the basis set for the cluster intrinsic wave function of the cluster \(A(B)\). The first state with \(i(j)=1\) is the ground state and the states with \(i \geq 2\) denote pseudostates. The ground states of \(^3\text{He}\) and \(d\) are bound but those of the \(2p\) and \(d(0^+)\) clusters are virtual states. Here \(2p\) stands for a di-proton cluster. The cluster relative motion function \(\chi^{A,i,B,j}_\ell (\rho)\) with the partial wave \(\ell\) is specified by the cluster relative distance coordinate \(\rho\). The total wave function \((2.1)\) is properly antisymmetrized as indicated by the intercluster antisymmetrizer \(\Lambda\). It contains no center-of-mass wave function, and has good total angular momentum \(JM\) and parity \(\pi\).

We take into account not only the \(^3\text{He}+p\) elastic channel but also the inelastic channels including the different spin-parity states of \(^3\text{He}\) and the rearrangement channel of \(d+2p\) as well. For all of the \(^3\text{He}\), \(d\), and \(2p\) clusters in each spin parity states, the pseudostates are taken into account in the present calculation. The pseudostates, when included in the phase-shift calculation, are expected to take account of the distortion of the clusters of the entrance channel.\(^{16,17}\) In the case of the coupled-channels calculation of \(\{A+B\}+\{A'+B'\}+\cdots\), the total wave function reads

$$\Psi^{JM\pi} = \Psi^{JM\pi}_{AB} + \Psi^{JM\pi}_{A'B'} + \cdots. \quad (2.2)$$

The intrinsic wave functions of \(^3\text{He}\) used in Eq. \((2.1)\) are given by three-body calculations of \(p+p+n\) as \((\alpha\) stands for \(^3\text{He}\))

$$\Phi^{\alpha,i}_{I_\alpha,M_\alpha,\pi_\alpha} = \sum_{\lambda_\alpha=1}^{N_\alpha} C^{\lambda_\alpha}_{\lambda_\alpha} A \left\{ \left[ \phi_{S_\alpha,T_\alpha} \left( \Gamma_{\ell_1}(\nu_1,\rho_1) \Gamma_{\ell_2}(\nu_2,\rho_2) \right) \right]_{I_\alpha,M_\alpha} \right\}. \quad (2.3)$$

The subscript \(\lambda_\alpha\) stands for a set of the labels \(\{S_\alpha, T_\alpha, L_\alpha, \ell_1, \ell_2, \nu_1, \nu_2\}\) and \(C^{\lambda_\alpha}_{\lambda_\alpha}\) is the coefficients of the \(i\)th eigenvalue obtained by diagonalizing the \(^3\text{He}\) cluster intrinsic Hamiltonian. The Gaussian basis function \(\Gamma_{\ell_i}(\nu_i,\rho_i)\) are given in Eqs. \((4)\) and \((5)\) of Ref. \(18)\) and \(\rho_1, \rho_2\) are the Jacobi coordinates in the \(p+p+n\) system with \(\ell_1, \ell_2\) denoting the corresponding orbital angular momenta. The function \(\phi_{S_\alpha,T_\alpha}\) is the spin and isospin part of \(^3\text{He}\) with \(S_\alpha\) and \(T_\alpha\) being the total spin and total isospin, respectively. The total angular momentum and parity \((I_{\alpha}^\alpha)\) of \(^3\text{He}\) is taken into account up to \(5/2^+\) with the restriction of \(\ell_1, \ell_2 \leq 2\) and \(S_\alpha=1/2\) or \(3/2\) and \(T_\alpha=1/2\). The wave function for the deuteron cluster, denoted as \(\beta\), has a form similar to Eq. \((2.3)\) as

$$\Phi^{\beta,j}_{I_\beta,M_\beta,\pi_\beta} = \sum_{\lambda_\beta=1}^{N_\beta} C^{\lambda_\beta}_{\lambda_\beta} A \left\{ \left[ \phi_{S_\beta,T_\beta} \Gamma_L(\nu_1,\rho_1) \right]_{I_\beta,M_\beta} \right\}. \quad (2.4)$$
The subscript $\lambda_\beta$ stands for a set of the labels \( \{ S_\beta, T_\beta, L, \nu_1 \} \). For the deuteron, \( I_\beta^\pi=1^+ \) with \( S_\beta=1, \ T_\beta=0, \ L=0 \) or 2. We also consider the \( pn \) cluster which have \( I_\beta^\pi=0^+ \) ( \( S_\beta=0, \ T_\beta=1, \ L=0 \) ). The \( 2p \) cluster are given similarly to Eq. (2.4) with \( I_\beta^\pi=0^+ \) ( \( S_\beta=0, \ T_\beta=1, \ L=0 \)). The spatial parts of the cluster wave functions are given in terms of a combination of Gaussian basis functions with different values of \( \nu_1 \).

The wave functions given in Eq. (2.1)−(2.4) are obtained by solving the respective \( A \)-nucleon Schrödinger equations with the Hamiltonian

\[
H = \sum_{i=1}^{A} T_i - T_{CM} + \sum_{i<j}^{A} V_{ij},
\]

where \( T_i \) is the kinetic energy of the \( i \)th nucleon, \( T_{CM} \) is the kinetic energy of the center-of-mass motion, and \( V_{ij} \) is the nucleon-nucleon interaction.

The cluster relative motion \( \chi_{A,i,B,j}^\pi(\rho) \) in Eq.(2.1) is solved with the microscopic \( R \)-matrix method (MRM),\(^{19}\) in which the configuration space for the relative motion between the clusters is divided into two regions, inner and outer, by a channel radius. The relative wave function in the inner region is approximated with a superposition of Gaussian basis functions \( \Gamma_\nu(\nu, \rho) \) with various range parameters \( \nu \).\(^{18}\) The same set of Gaussian basis functions is employed for all the channels. The range parameters are taken in the range of \( 0.1 \text{fm} < b (= 1/\sqrt{\nu}) < 15 \text{fm} \), and the number of \( \nu \) is 13 for Model T and 15 for Model C in the \( ^3\text{He}+p \) phase-shift calculation. The channel radius is chosen as 13.5fm for Model T and 15fm for Model C. In the case of \( ^3\text{H}+p \) scattering the number of basis set is 15 and the channel radius is taken as 15fm for both the models of T and C. In order to avoid the numerical instability in the MRM calculation, the range parameters in Eqs. (2.3) and (2.4) for the cluster intrinsic motion are taken in the range of \( b_i(= 1/\sqrt{\nu_i}) < 5 \text{fm} \). The relative wave function in the inner region is connected, at the channel radius, smoothly to the asymptotic form of the relative wave function which is expressed in terms of the Coulomb functions and the scattering \( S \)-matrix to be determined.

In Model T, we employ the G3RS potential which reproduces the \( N-N \) scattering data reasonably well.\(^{20}\) We use this 3-range Gaussian potential mainly because it saves a computer time compared to more recent potentials such as AV8' potential.\(^{8}\) The central, \( LS \) and tensor terms of the G3RS potential are included together with the Coulomb potential, but the \( L^2 \) and \( (LS)^2 \) terms which give a negligible contribution are omitted. The \( p+p+n \) three-body wave function for \( ^3\text{He} \) is obtained by taking into account the partial waves up to the \( D \)-wave for each Jacobi coordinate.

Model T takes into account the configurations of \( \{ ^3\text{He}(1/2^+, \ 3/2^+, \ 5/2^+) + p \} + \{ d(0^+,1^+) + 2p(0^+) \} \) and the partial wave \( \ell \) in Eq. (2.1) up to \( \ell=3 \). The \( 1/2^+ \) ground state of \( ^3\text{He} \) is approximated with fifteen Gaussian basis functions which are selected by the stochastic variational method (SVM).\(^{21,22}\) Table I lists the energy and root mean square (rms) radius with this limited number of basis set as well as the convergent result.\(^{10}\) This restriction for the basis dimension is necessary in order to make the \( ^3\text{He}+p \) calculation feasible. All of the other spin-parity states of \( ^3\text{He} \) are unbound and they are approximated with the wave functions of a bound-state
A microscopic cluster model study of $^3$He+p scatterings

Table I. Energies $E$ (MeV) and rms radii $r$ (fm) and probabilities of the $L=2$ component $P_D$ (%) of the $^3$He and the deuteron. ‘Limited’ denotes the results obtained with a limited number of basis set as explained in text, while ‘Full’ the converged results\(^{10}\) obtained with large enough basis set. The experimental energy of $^3$He is $-7.718$MeV.\(^{13}\)

| Wave function | G3RS | MN |
|---------------|------|----|
| $^3$He 0s h.o. | $E$ Limited | $E$ Full |
| $^3$He $p+p+n$ | $E$ Limited | $E$ Full |
| $^3$He $p$ | $r$ Limited | $r$ Full |
| $d$ $p+n$ | $P_D$ Limited | $P_D$ Full |

| Wave function | G3RS | MN |
|---------------|------|----|
| $^3$He 0s h.o. | $E$ Limited | $E$ Full |
| $^3$He $p+p+n$ | $E$ Limited | $E$ Full |
| $^3$He $p$ | $r$ Limited | $r$ Full |
| $d$ $p+n$ | $P_D$ Limited | $P_D$ Full |

The deuteron wave function in Model T is given by a superposition of three $S$-wave plus three $D$-wave Gaussian basis functions with $\nu_1$ in Eq. (2.4) being 2.40, 0.266, 0.0400 fm$^{-2}$ for the $S$-wave and 0.974, 0.328, 0.093 fm$^{-2}$ for the $D$-wave. Energy and rms radius of the deuteron with this limited basis set as well as those with a larger basis set are compared in Table I. Since the rms radius of the deuteron with the limited basis is smaller than the one with the full set, one might think that the contribution of the $d+2p$ channel is underestimated in the present $^3$He+p calculation. As will be discussed in Sect.3.3, these limited wave functions of $^3$He and the deuteron are found to be useful in accounting for the binding energy of the $0^+$ ground state of $^4$He within only 0.88MeV, compared with a more precise SVM calculation, which indicates that the pseudostates of $^3$He and $d$ taken into account in the present calculation make it possible to describe the distortion of the clusters in the $^3$He+p channel.

The three $S$-wave bases used for the deuteron wave function are also used to describe the unbound $0^+$ states in the $pn$ and $2p$ cluster systems by the bound state-type wave functions. Some possible effects of the three- and four-body channels of $d+p+p$ and $p+p+p+n$ are expected to be included in the the present $R$-matrix method through the $d(0^+, 1^+)+2p$ two-cluster channels. This sort of approximations was employed to discuss the three-body resonances in $^9$Be and $^{12}$C and gave the results consistent with those of the three-body complex scaling method.\(^{23}\)

In Model C, the MN potential\(^4\) with $u=0.98$ is employed as the effective $N-N$ interaction. This interaction can reproduce the $n-p$ triplet and $p-p$ singlet $S$-wave
In Model T, we employ the G3RS potential. The channel spin $I$ is 0 or 1 for the $1^-$ state, and 1 for the $0^-$ and $2^-$ states. The lines denote the results obtained including the following configurations: Solid $^3\text{He}(1/2^+) + p$; Dotted $^3\text{He}(1/2^\pm, 3/2^\pm, 5/2^\pm) + p$; Dash-dotted \{($^3\text{He}(1/2^+) + p$) + \{d(0^+, 1^+) + 2p(0^+)\}\}; Dash-double-dotted \{($^3\text{He}(1/2^\pm, 3/2^\pm, 5/2^\pm) + p$) + \{d(0^+, 1^+) + 2p(0^+)\}\}. The crosses denote the experimental data and the error bars of the data are omitted.

scattering lengths and effective ranges. With the spin-orbit term of Reichstein and Tang (set IV),\cite{Reichstein1958} this potential can reproduce low-energy $\alpha + n$ phase shifts for the $S$- and $P$-waves.\cite{Reichstein1959} Since the MN potential can fairly well reproduce the binding energies of $d$, $^3\text{H}$, and $^4\text{He}$ without a tensor term,\cite{Murray1977} any additional tensor term makes these $s$-shell clusters seriously overbound. Therefore, the MN potential without any tensor term is employed in Model C. Here the configurations of \{($^3\text{He}(1/2^+) + p$) + \{d(0^+, 1^+) + 2p(0^+)\}\} are included and the wave functions of $^3\text{He}$ and $d$ contain only $S$-wave components.

For the $^3\text{He}$ cluster, we employ two different wave functions. One is a superposition of the $(0s)^3$ h.o. functions with four different oscillator parameters, $\nu=1.234$, 0.548, 0.208, 0.0696 fm$^{-2}$. Second is the wave function obtained in the $p+p+n$ three-body calculation in which the partial wave for each Jacobi coordinate is restricted to the $S$-wave only. The wave function is a combination of fifteen Gaussians selected by the SVM. The deuteron wave function is given by four Gaussian basis set where the Gaussian parameters $\nu_1$ in Eq. (2.4) are 1.297, 0.552, 0.198, 0.040 fm$^{-2}$. The $0^+$ states of the $pn$ and $pp$ clusters are approximated with the bound-state type wave functions using the same basis set as used in the deuteron wave function. Energies and rms radii of $^3\text{He}$ and $d$ are listed in Table I. The partial waves for the cluster relative motion are taken up to $\ell=3$. 

Fig. 1. The $^3\text{He}+p$ $P$-wave elastic scattering phase shifts calculated by Model T (G3RS potential).
Fig. 2. The $^3$He+p S-wave elastic scattering phase shifts calculated by Model T (G3RS potential). The solid and dash-double-dotted lines denote the results with the $^3$He$(1/2^+)+p$ and $^3$He$(1/2^\pm, 3/2^\pm, 5/2^\pm)+p + \{d(0^+,1^+)+2p(0^+)\}$ configurations, respectively. The crosses denote the experimental data.\textsuperscript{26}

§3. Results

3.1. $^3$He+p elastic scattering in Model T

Figure 1 displays the $^3$He+p P-wave elastic scattering phase shifts in comparison with experiment.\textsuperscript{26} These are obtained in Model T with the inclusion of the D-wave for the $^3$He cluster. As mentioned in the Introduction, the P-wave scattering produces four negative-parity states with 2$^-$, 1$^-$($I=0, 1$), and 0$^-$, all of which correspond to the low-lying broad resonances of $^4$Li. The solid, dotted, dash-dotted, and dash-double-dotted lines of the figure denote the results obtained by including the configurations of $^3$He$(1/2^+)+p$, $^3$He$(1/2^\pm, 3/2^\pm, 5/2^\pm)+p$, $\{^3$He$(1/2^+)+p\}+\{d+2p\}$, and $\{^3$He$(1/2^\pm, 3/2^\pm, 5/2^\pm)+p\}+\{d+2p\}$, respectively, where the spin-parity of the d cluster includes both 0$^+$ and 1$^+$, and that of the 2$p$ cluster is 0$^+$. The calculation with all the configurations (dash-double-dotted lines) give results similar to those of the RGM calculation.\textsuperscript{15} Our result for the 0$^-$ state is in disagreement with the experimental data in the whole energy range. A further consideration for the model space or the nucleon-nucleon interaction such as three-body forces may be necessary in order to reproduce the experimental phase shifts.

Apparently, the single-channel calculation of $^3$He$(1/2^+)+p$ is quite insufficient for reproducing the P-wave phase shifts except for the 1$^-$($I=0$) case. The $d(1^+)+2p$ channel($I=1$) hardly changes the 1$^-$($I=0$) phase shift whereas it gives a significant contribution to the other P-wave phase shifts with $I=1$. The $d(0^+)+2p$ channel($I=0$) gives a minor contribution to the 1$^-$($I=0$) phase shift and hardly affects the 1$^-$($I=1$) phase shift. The contribution of the $d+2p$ channel is more important than that of the other spin-parity states of the $^3$He cluster. Thus we find that the $\{^3$He$(1/2^+)+p\}+\{d+2p\}$ calculation including the deuteron ($pn$) and diproton (2$p$) configurations is nearly sufficient to reproduce all the P-wave phase shifts. These results strongly indicate that the low-lying resonances of $^4$Li cannot
Fig. 3. The $^3\text{He}+p$ S-wave elastic scattering phase shifts calculated by Model C (MN potential).

The wave function of $^3\text{He}$ is given by four-range $(0s)^3$ h.o. functions. The solid and dashed lines denote the results of the $(^3\text{He}(1/2^+)+p)+\{d(0^+,1^+)+2p(0^+)\}$ configurations, respectively. The crosses denote the experimental data.\(^{26}\)

be described adequately in the single configuration of $^3\text{He}(1/2^+)+p$. This is a sharp contrast to the results in Model C as is discussed in the following subsection.

In contrast to the P-wave $^3\text{He}+p$ scattering phase shifts, the S-wave phase shifts with $0^+$ and $1^+$ gain negligible contributions from the channels other than the main $^3\text{He}(1/2^+)+p$ channel. Their phase shifts are shown in Fig. 2. A comparison of the S-wave and P-wave phase shifts clearly indicates that the $^3\text{He}+p$ interaction is attractive in the P-wave but repulsive in the S-wave. The calculation suggests that the attractive nature of the P-wave $^3\text{He}+p$ resonance cannot be taken into account fully in the single $^3\text{He}+p$ configuration but calls for more complex states or distorted configurations which couples with the elastic configuration.

3.2. $^3\text{He}+p$ elastic scattering in Model C

Figures 3 and 4 display the S- and P-wave elastic scattering phase shifts which are obtained with Model C using the MN potential. The $^3\text{He}$ cluster wave function is given by the $(0s)^3$ h.o. functions in this calculation. The results with the $^3\text{He}+p$ single configuration are shown by the solid lines, and those including additionally the $d+2p$ channel are shown by the dashed lines. In this calculation, the $d$ and $2p$ clusters have only the S-wave component. We see that the $d+2p$ channel gives a considerable contribution to not only the P-wave resonant phase shifts but also the S-wave non-resonant phase shifts, which is in sharp contrast to Model T case shown in Figs. 1 and 2.

The fact that the $d+2p$ channel is found to play an important role seems to suggest that the distortion of $^3\text{He}$ has to be taken into account. It should be noted, however, that the importance of the distortion of the clusters may depend on how accurately their wave functions are described. In order to examine this issue, we repeat the phase shift calculation by replacing the $^3\text{He}$ wave function from the simple $(0s)^3$ h.o. function with that of the $p+p+n$ three-body calculation as explained in Sect. 2. The P-wave phase shifts which result from this improved $^3\text{He}$ wave function are shown in Fig. 5. The single channel calculation (solid line) of $^3\text{He}+p$ gives only a
minor change on both the $S$- and $P$-wave phase shifts, compared to the corresponding case of Fig. 4. Now let us turn to the effect of including the $d+2p$ channel on the phase shifts (dashed line). In a sharp contrast to the case of Fig. 4, we see that the calculation using the improved $^3$He wave function leads to a significant suppression of the additional $d+2p$ channel. Especially, the contribution in the $1^-(I=0)$ and two $S$-wave phase shifts turns out to be negligibly small.

The different role of the $d+2p$ channel mentioned above can be explained as follows. The two configurations of $^3$He+$p$ and $d+2p$ are not orthogonal each other and have a significant overlap at the short distances of the cluster separation. The inclusion of the $d+2p$ channel plays a role of a distortion effect of the $^3$He cluster and modifies the $^3$He wave function indirectly. This additional $d+2p$ channel has a larger effect when the simple $(0s)^3$ h.o. function is used. Moreover this effect is noticeable at low incident energies because the $^3$He+$p$ threshold with the $(0s)^3$ h.o. function is predicted to be about 2.4 MeV too high compared to the one with the $p+p+n$ wave function, as seen in Table I. As a whole both of the $S$ and $P$-wave phase shifts are well reproduced by the single $^3$He($1/2^+$)+$p$ calculation if a realistic $^3$He wave function is used. To conclude, the effect of the cluster distortion strongly depends on whether or not the cluster intrinsic wave function is described appropriately according to the employed effective $N$-$N$ potential.

We have seen that the role of the $d+2p$ channel appears quite differently between Model T and Model C. In the former case using the realistic potential, the $d+2p$ channel plays a vital role particularly in the $P$-wave resonant phase shifts, responding to the complexity due to the tensor force. In the case of Model C using the effective potential, however, the situation is different. The potential is mainly central and
induces no complicated angular momentum couplings. Thus most of the dynamics are accounted for by the main configuration especially when the participating clusters are described realistically, and the effects of additional configurations are more or less suppressed.

A similar suppression by the improvement of the cluster wave function was noted in understanding the neutron-halo structure of $^6$He in the $\alpha+n+n$ cluster model. The issue there was the role of the additional $t+t$ channel. As was shown in Ref. 27), the use of the simple $(0s)^4$ h.o. function for the $\alpha$ particle led to the conclusion that the $t+t$ channel is really important to gain the binding energy of $^6$He, indicating the certain deviation from the three-body cluster picture. However, if the simple $(0s)^4$ h.o. wave function was replaced with the better one calculated in the $3N+N$ two-body model, the effect of the $t+t$ channel was reduced to a large extent, making it possible to maintain the dominant configuration of $\alpha+n+n$. This suggests that we must perform the multi-configuration calculation paying attention to the cluster intrinsic function so as not to overestimate the contribution by the other configurations such as the $d+2p$ channel in $^4$Li.

3.3. The ground state of $^4$He and $^3$H+p elastic scattering

In this subsection, we take up two problems relevant to the $0^+$ state of $^4$He, the ground state energy of $^4$He and the $^3$H+p $S$-wave scattering, in order to reinforce the arguments made in the preceding subsections. The analysis is performed in a scheme similar to the $^3$He+p calculation, namely using the configurations of $\{^3\text{He}(1/2^+)+n\}$, $\{^3\text{H}(1/2^+)+p\}$, $\{d(1^+)+d(1^+)\}$, $\{d(0^+)+d(0^+)\}$, and $\{2n(0^+)+2p(0^+)\}$. The intrin-
A microscopic cluster model study of $^3\text{He}+p$ scatterings

Fig. 6. The $^3\text{He}+p$ S-wave $0^+$ elastic scattering phase shift by Model T with the G3RS potential (the left panel) and by Model C with the MN potential (the right panel). The lines denote the results obtained including the following configurations: Solid $\{^3\text{He}+n\}+\{^3\text{H}+p\}$; Dotted $\{^3\text{He}+n\}+\{^3\text{H}+p\}+(d(1^+)+d(1^+))$; Dash-Dotted $\{^3\text{He}+n\}+\{d(1^+)+d(1^+)\}+(d(0^+)+d(0^+))+\{2n(0^+)+2p(0^+)\}$.

The binding energy of $^4\text{He}$ obtained in Model T is $-24.41$ MeV for the configuration of $\{^3\text{He}+n\}+\{^3\text{H}+p\}+(d+d)+(2n+2p)$, and $-22.16$ MeV for $\{^3\text{He}+n\}+\{^3\text{H}+p\}$, respectively. The contribution of the $d+d$ and $2n+2p$ channels to the energy gain is $2.25$ MeV. Here the $d(1^+)+d(1^+)$ channel gives the most important contribution ($2.1$ MeV). The content of this energy gain is brought about as follows: The kinetic energy gives the loss of $8.8$ MeV, while the central and tensor potentials give the gains of $4.4$ MeV and $6.9$ MeV, respectively. Thus we see that but for the tensor force, the $2N+2N$ configuration cannot gain the binding energy. Note that the binding energy of the above coupled-channels calculation in Model T is only $0.88$ MeV lower than the more precise value, $-25.29$ MeV, of the SVM calculation. Moreover, this difference could be reduced further if more extended basis sets are used for $^3\text{He}$, $^3\text{H}$, and $d$.

In Model C, the $u$ parameter of the MN potential is set to $u=1.0$ and the $LS$ term is omitted. The resulting energy is $-29.94$ MeV for the combined configurations of $\{^3\text{He}+n\}+\{^3\text{H}+p\}+(d+d)+(2n+2p)$, and $-29.91$ MeV for $\{^3\text{He}+n\}+\{^3\text{H}+p\}$, respectively, whereas the SVM energy is $-29.94$ MeV. Model C thus produces almost fully convergent energy. The contribution of the $d+d$ and $2n+2p$ channels is only $0.03$ MeV, which is much smaller than in Model T.

Now we come to the $^3\text{H}(1/2^+)+p$ S-wave $0^+$ elastic scattering phase shift. Figure 6 compares the phase shifts between Model T and Model C. The solid, dotted, and dash-dotted lines denote the results using the configuration of $\{^3\text{H}+p\}+\{^3\text{He}+n\}$, $\{^3\text{H}+p\}+\{^3\text{He}+n\}+(d(1^+)+d(1^+))$, and $\{^3\text{H}+p\}+\{^3\text{He}+n\}+(d+d)+(2n+2n)$, respectively. The contribution of the $d+d$ channel is very different, depending on the model. It is very large in Model T but much less significant in Model C.

In order to discuss how much the $d+d$ configuration is different from the $^3\text{H}+p$...
configuration between Models T and C, we calculate the following overlap

\[
\langle A \left\{ \left[ \Phi_{3h}^{\nu} \Phi_{p}^{\nu} \right] I=0 \Gamma_{\nu}(\nu, \rho) \right\} | A \left\{ \left[ \Phi_{d}^{\nu} \Phi_{d}^{\nu} \right] I=0 \Gamma_{\nu}(\nu', \rho') \right\} \rangle ,
\]

where the wave functions in bra and ket are both normalized to unity. We take a single Gaussian basis function with a common parameter \( \nu = \nu' \). The overlaps for \( b = 1/\sqrt{\nu} \) = 1.0, 2.0, 3.0, and 4.0 fm are 0.53, 0.90, 0.96, and 0.93 in Model T, respectively, while they are 0.60, 0.92, 0.97, and 0.93 in Model C. The \( d+d \) configuration has a smaller overlap with the \( ^3H+p \) configuration in Model T than in Model C when the two \( d \) clusters come closer than \( b < 1 \) fm.

We understand the different contribution of the \( d+d \) channel between Model T and Model C as follows. In the bound or resonance state where the four nucleons are localized in the interaction region through the attractive interaction of the clusters, various types of correlations like \( 3N+N \) and \( 2N+2N \) are equally important. If the overlap between the different configurations is large and the structure of the state is relatively simple, a particular channel can accommodate most of the indispensable configurations fairly well as in Model C. However, once the tensor force is explicitly taken into account and the higher partial waves are included in the cluster intrinsic wave functions, the overlap between the different configurations becomes smaller at the short cluster relative distance and in addition the structure of the bound or resonance state becomes more complicated. As a result, the state cannot be well described with a single configuration. Contrary to the bound or resonance state, the non-resonant state in the \( ^3He+p \) S-wave scattering is well approximated with the single configuration because the two clusters feel a repulsive interaction in the scattering and the chance of coupling with the other channel becomes small.

\section{Summary}

We have calculated the \(^3He+p\) S- and P-wave elastic scattering phase shifts in two different microscopic cluster models, Model T and Model C. The s-shell cluster intrinsic function includes the D-waves through the tensor force in Model T, while it is described with only the S-wave Gaussian function in Model C. These models have also been applied to the \(^0^++^4He\) and the \(^3H+p\) S-wave elastic scattering phase shift in order to elucidate the role of different cluster channels.

We have found that, in Model T using a realistic nucleon-nucleon interaction, the inclusion of the \( d+2p \) channel is very important to reproduce the \(^3He+p\) P-wave resonant phase shifts, whereas the single \(^3He(1/2^+)+p\) channel alone can reproduce the S-wave non-resonant phase shifts fairly well.

In contrast to the realistic interaction case, in Model C where an effective interaction is used, the role of the \( d+2p \) channel depends on how realistically the \(^3He\) wave function is described. If it is given by the simple \((0s)^3\) harmonic-oscillator function, the \( d+2p \) channel has contributed significantly to both the \( P\)- and \( S\)-wave phase shifts. This is because the distortion effect of the \(^3He\) cluster cannot be taken into account sufficiently by the simple \((0s)^3\) harmonic-oscillator function and the \( d+2p \) channel indirectly modifies the \(^3He\) cluster intrinsic wave function. However, if it
is improved with the $p+p+n$ three-body wave function, we have confirmed that the contribution of the $d+2p$ channel is greatly suppressed even in the $P$-wave resonant phase shifts. In comparison with the model T, these results suggest that the $d+2p$ channel is renormalized mostly as the the $^3\text{He}(1/2^+)+p$ channel in the resonance region in the model C.

We have obtained similar results for both the binding energy of $^4\text{He}$ and the $^3\text{H}+p$ $S$-wave elastic scattering phase shift. The $d+d$ channel has a significant contribution in Model T, while it plays a minor role in Model C. We have shown that in Model T the $d+d$ channel is important to improve the short-range behavior of the four-nucleon wave function. Without the tensor force, the energy gain due to the $d+d$ channel caused by the central potential is much smaller than the energy loss of the kinetic energy, resulting in the minor contribution of the $d+d$ channel.

\section{5. Acknowledgements}

This work presents research results of Bilateral Joint Research Projects of the JSPS (Japan) and the FNRS (Belgium).

\section*{References}

1) H. Horiuchi and K. Ikeda, Cluster Model of Nucleus, “Cluster Models and Other Topics", Int. Rev. of Nucl. Phys. Vol. 4 (1986) (World Scientific, Singapole), 1.
2) K. Wildermuth and Y. C. Tang, A Unified Theory of the Nucleus, (Vieweg, Braunschweig, 1977).
3) K. Langanke, in Advances in Nuclear Physics, Vol. 21, edited by J. W. Negele and E. Vogt, (Plenum, New York, 1994), 85.
4) D. R. Thompson, M. LeMere, and Y. C. Tang, Nucl. Phys. A 286 (1977), 53.
5) H. Furutani, H. Horiuchi, and R. Tamagaki, Prog. Theor. Phys. 62 (1979), 981.
6) A. Csótó and R. G. Lovas, Phys. Rev. C 46 (1992), 576.
7) A. Csótó and R. G. Lovas, and A. T. Kruppa, Phys. Rev. Lett. 70 (1993), 1389.
8) B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, R. B. Wiringa, Phys. Rev. C 56 (1997), 1720.
9) H. Kamada, et al. Phys. Rev. C 64 (2001), 044001.
10) Y. Suzuki, W. Horiuchi, M. Orabi, and K. Arai, Few Body Systems 48 (2008), 33.
11) G. G. Ryzhikh, R. A. Eramzhyany, V. I. Kukulin, and Yu. M. Tchuvil’sky, Nucl. Phys. A 563 (1993), 247.
12) T. Myo, K. Katô, and K. Ikeda, Prog. Theor. Phys. 113 (2005), 763.
13) D. R. Tilley, H. R. Weller, and G. M. Hale, Nucl. Phys. A 541 (1992), 1.
14) A. Deltuva and A. C. Fonseca, Phys. Rev. C 75 (2007), 014005.
15) B. Plützinger, H. M. Hofmann, and G. M. Hale, Phys. Rev. C 64 (2001), 044003; C. ReiB and H. M. Hofmann, Nucl. Phys. A 716 (2003), 107.
16) H. Kanada, T. Kaneko, S. Saito, and Y. C. Tang, Nucl. Phys. A 444 (1985), 209.
17) R. Beck, F. Dickmann, and A. T. Kruppa, Phys. Rev. C 30 (1984), 1044.
18) K. Arai, P. Descouvemont, and D. Baye, Phys. Rev. C 63 (2001), 044611.
19) D. Baye, P.-H. Heenen, and M. Libert-Heinemann, Nucl. Phys. A 291 (1977), 230.
20) R. Tamagaki, Prog. Theor. Phys. 39 (1968), 91.
21) V. I. Kukulin and V. M. Krasnopol’sky, J. of Phys. G3 (1977), 795.
22) K. Varga, Y. Suzuki, and R. G. Lovas, Nucl. Phys. A 571 (1994), 447; K. Varga and Y. Suzuki, Phys. Rev. C 52 (1995), 2885.
23) K. Arai, P. Descouvemont, D. Baye, and W. N. Catford, Phys. Rev. C 68 (2003), 014310; K. Arai, Phys. Rev. C 74 (2006), 064311.
24) I. Reichstein and Y. C. Tang, Nucl. Phys. A 158 (1970), 529.
25) A. Csótó, Phys. Rev. C 48 (1993), 165.
26) T. A. Tombrello, Phys. Rev. 138 (1965), B40.
27) K. Arai, Y. Suzuki, and R. G. Lovas, Phys. Rev. C 59 (1999), 1432.