Interaction induced mergence of Dirac points in Non-Abelian optical lattices

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We study the properties of an ultracold Fermi gas loaded in a square optical lattice and subjected to an external and classical non-Abelian gauge field. We calculate the energy spectrum of the system and show that the Dirac points in the energy spectrum will remain quite stable under on-site interaction of certain strength. Once the on-site interaction grows stronger than a critical value, the Dirac points will no longer be stable and merge into a single hybrid point. This merger implies a quantum phase transition from a semimetallic phase to a band insulator. The on-site interaction between ultracold fermions could be conveniently controlled by Feshbach resonances in current experiments. We proposed that this remarkable interaction induced mergence of Dirac points may be observed in the ultracold fermi gas experiments.

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One of the most interesting properties of graphene [1], a single layer of carbon atoms packed in a honeycomb lattice, lie in the fact that the low energy excitations obey a linear dispersion relation [2] around the so-called Dirac points, and thus can be used as a testbed for the relativistic quantum electrodynamics. Consequently, it is now possible to observe many remarkable phenomena in tabletop experiments, such as Klein tunneling [3-4] and the relativistic extension of Landau levels [5-7], which usually only occur in high-energy physics [8]. This advantage of graphene has stimulated a great interest in the investigation of Dirac points [8] in many other systems. In particular, ultracold atoms in optical lattices provide a versatile playground where the properties of condensed matter systems can be simulated [9,10] in a highly controllable manner, such as the superfluid-Mott insulator transitions of Hubbard models [11-13]. A quantum-optical analogue of graphene can be achieved by loading ultracold fermionic atoms such as 40K or 6Li [12,13] into a hexagonal optical lattice [14]. The effects of Dirac points were discussed in the context of ultracold atoms in honeycomb lattice [14] and T3 (rhombic) lattices [12]. Moreover, much more intriguing phenomena arise when these systems are subjected to artificial non-Abelian gauge fields [14, 25-31]. And one thing worthy to be mentioned here is that once the two Dirac points merge, the final dispersion relation becomes quite exotic—it is linear in one direction but parabolic in the other orthogonal direction. Very recently, a well-designed experiment has been carried out by Leticia et al. [32], in which the creating, moving and merging of Dirac points has been realized with a Fermi gas loaded into a tunable honeycomb lattice. While the creating, moving and merging of Dirac points in the experiment [32] is generated by designing complex lattice geometries, the mergence of the Dirac points in our model is induced by strong repulsive on-site interaction. Since pairwise interactions could be conveniently controlled by means of Feshbach resonances [33], we propose that the interesting mergence of Dirac points in our model may be experimentally observed and characterized in non-Abelian optical lattices. The non-Abelian optical lattice could be prepared by generalizing the recent experiment [16], as proposed in [17,18].

Here we would like to emphasize a fact that the non-Abelian artificial gauge field also provide an interesting setup where Dirac points emerge in a square optical lattice [22], which is originally limited to staggered fields [23,24]. In this article, we consider a similar system in which two-component (two-color) ultracold fermionic atoms are trapped in a square optical lattice. And dramatic difference between the two systems comes from the repulsive on-site interaction introduced into the model by us. Works [21,22] mentioned above mainly dwelled on free Fermi gas on 2D optical lattice. We study the effects of repulsive on-site interaction on the energy spectrum of the 2D fermi gas loaded into a square optical lattice and subjected to a non-Abelian artificial gauge field. Implementing a self-consistent mean-field theory, we show that the Dirac points in the energy spectrum remain quite stable under repulsive on-site interaction of certain strength. When the on-site interaction grows stronger than a critical value, the Dirac points will no longer be stable and two Dirac points merge into a single one. This merging indicates a quantum phase transition between a semimetallic phase and a band insulator [14,25,31]. And one thing worthy to be mentioned here is that once the two Dirac points merge, the final dispersion relation becomes quite exotic—it is linear in one direction but parabolic in the other orthogonal direction. Very recently, a well-designed experiment has been carried out by Leticia et al. [32], in which the creating, moving and merging of Dirac points has been realized with a Fermi gas loaded into a tunable honeycomb lattice. While the creating, moving and merging of Dirac points in the experiment [32] is generated by designing complex lattice geometries, the mergence of the Dirac points in our model is induced by strong repulsive on-site interaction. Since pairwise interactions could be conveniently controlled by means of Feshbach resonances [33], we propose that the interesting mergence of Dirac points in our model may be experimentally observed and characterized in non-Abelian optical lattices. The non-Abelian optical lattice could be prepared by generalizing the recent experiment [16], as proposed in [17,18].

We consider a two-component (two-color) Fermi gas trapped on a square optical lattice and subjected to an artificial non-Abelian gauge potential. Fermionic atoms in the system are interacting with repulsive on-site interaction. It is well known that this pairwise interactions can be freely tuned by means of Feshbach resonances [33] in nowadays ultracold atoms experiments. The Hamiltonian of the system reads,
we constrain ourselves to the non-Abelian regime, where the Wilson loop $|W| < 2$ and we set the Abelian flux $\Phi = 0$.

In the noninteracting limit of Hamiltonian (11), i.e. the case in which fermions hop freely between neighbor sites without any interaction, Hamiltonian of the system is a beautiful quadratic form and can be analytically solved by Bogoliubov transformations. Energy spectrum of this case has been beautifully analyzed in literature [22], where the fermion gas becomes a collection of noninteracting quasiparticles and the spectrum develops four independent Dirac points at $K_D \in \{(0,0), (\pi,0), (0,\pi), (\pi,\pi)\}$ in the vicinity of marginally Abelian regime ($\Phi_\alpha, \Phi_\beta \approx \pi/2$). However, once the on-site interaction is taking into account in Hamiltonian (11), it’s not a quadratic form any more and therefore can’t be solved by Bogoliubov transformation directly. This is right the case we consider in this article. We study the repulsively interacting fermions on a square optical lattice subjected to a non-Abelian gauge field by the means of a self-consistent mean field theory. Our startpoint is Eq. (2).

We mainly consider the repulsive interaction regime in this paper. As the on-site interaction grows stronger and stronger, fermionic atoms with different colors on the square lattice tend to repel each other and avoid staying on the same site. Once the interaction strength is over a critical point, the square optical lattice at half-filling will enter a phase in which each site of the lattice is occupied by single atom (see Fig. 1b). Therefore, we define $\Delta_r$ as our order parameter. Under this mean-field approximation, the Hamiltonian (2) can be written as a quadratic form,

$$H_{MF} = -t \sum_{<rr'>} \sum_{\tau,\tau'} (c_{r\tau}^\dagger c_{r'\tau'} + H.c.) + V \sum_r n_{r\uparrow} n_{r\downarrow},$$

(2)

where $t$ is the hopping amplitude, $c_{r\sigma}(c_{r\sigma}^\dagger)$ is the fermionic annihilation (creation) operator at site $r \in \text{optical lattice}$, $\tau = \uparrow, \downarrow$ can be regarded as pseudospin index, $\langle rr' \rangle$ denotes that the sum is over nearest neighbors and $V$ is the strength of the on-site interaction between fermionic atoms. The coordinate of a fermion is given by $r = (m_a n_a, m_b n_b)$, where $m, n$ are integers and $a$ is the lattice constant of the square optical lattice. Here we set $\hbar = c = 1$. The external gauge potential has the following form, $A = \frac{B_0}{\pi} (y, x) + a (B_\alpha \sigma_y, B_\beta \sigma_x)$, in which $B_0, B_\alpha, B_\beta$ are experimentally controllable parameters and $\sigma_{x,y}$ are the Pauli matrices. This intriguing artificial gauge field can be realized following the proposals [17, 18, 34], along the lines of the recent experiment [16]. After some algebra, the original Hamiltonian (11) becomes

$$H = -t \sum_{<rr'>} \sum_{\tau,\tau'} (c_{r\tau} c_{r'\tau'}^\dagger + H.c.) + V \sum_r n_{r\uparrow} n_{r\downarrow},$$

(2)

where $U_{rr'}$ is the matrix form of a nontrivial unitary operator accompanying the hopping between nearest neighbor $r$ and $r'$. If the hopping is along $x$ axis, $U_{rr'} = U_x(m) = e^{-i\Phi_\alpha m \sigma_y}. \Phi_\alpha = B_\alpha a^2$ is the Abelian magnetic flux, and $\Phi_\alpha, \Phi_\beta = B_\alpha, B_\beta a^2$ is the non-Abelian flux. Fermions hopping around an elementary square indicate a unitary transformation [22] $U = U_x(m)U_y(n+1)U_y^\dagger (m+1)U_x^\dagger (n)$. The boundary between Abelian regime and non-Abelian regime is well defined by the gauge-invariant Wilson loop [22, 34] $W = trU$. Here

$$\Phi = 0.$$
We solve the set of BdG equations self-consistently via exact diagonalization method in real space. The system size of $24 \times 24$ is used in the calculation and the convergence criterion of $\Delta_n$ is set to be $10^{-4}$ in unit of nearest-neighbor hopping $t$. We find that the order parameter is uniform ($\Delta_c = \Delta$, where $\Delta$ is real) in the vicinity of the $\pi$-flux regime. Our calculations (see Fig. 1b) show that as the on-site interaction $V$ increases from zero, the order parameter turn out to be non-zero at $V_c = 5.88t$ and the system undergoes a quantum phase transition from a semimetallic phase to a band insulator.

By the above mentioned self-consistent mean-field theory, we transform the original Hamiltonian into a quadratic form Eq. (3). Implementing appropriate Fourier transformations, Eq. (3) can be easily diagonalized in momentum space. The corresponding energy spectra are shown in Fig. 2. As the mean-field order parameter $\Delta$ grows stronger, the two originally separate Dirac points (Fig. 2a) will first move closer (Fig. 2b), then merge into a single hybrid point at $k_h = (\pi/2a, 0)$. Around this hybrid point $p = k - k_h$, the low-energy properties of the system are accurately described by a Dirac Hamiltonian

$$H_{eff} = \sum_p \Psi^\dagger_p H_D \Psi_p, \quad H_D = 2\sigma_y p_x - \sigma_x p_y^2$$

(6)

where $\Psi_p = (c_{p\uparrow}, c_{p\downarrow})^T$ is the relativistic spinor. From this Hamiltonian we can see that the energy spectrum is linear in $k_x$ direction but quadratic in $k_y$ direction (Fig. 2b). The mergence of the two Dirac points signals a quantum phase transition from semimetallic phase to a band insulator. If the order parameter $\Delta$ grows even stronger, a gap will be opened (Fig. 2d), which indicates two normal Dirac points. (b) $\Delta = 1.0$. The two Dirac points moves closer. (c) $\Delta = 2.0$. Two Dirac points merge into a single hybrid point, which signals the quantum phase transition. (d) $\Delta = 2.5$. A gap is opened.

Top: Portrait of the energy spectrum in $k_x$ direction. Bottom: Variations process of the two Dirac points. (a) $\Delta = 0$. There are two normal Dirac points. (b) $\Delta = 1.0$. The two Dirac points moves closer. (c) $\Delta = 2.0$. Two Dirac points merge into a single hybrid point, which signals the quantum phase transition. (d) $\Delta = 2.5$. A gap is opened.

FIG. 2: (Color online) Merging of Dirac points in the band structure of the system as $\Delta$ grows stronger ($\Phi_\alpha = \Phi_\beta = \pi/2$).

For the density of states in Fig. 2, we find that dramatic difference appeared at the occasion when two Dirac points merge into a hybrid one. While the DOS displays two peaks for the case of $\Delta = 0$ (Fig. 3a), there are four peaks in the case of $\Delta = 2$ (Fig. 3d). Moreover, for $\Delta = 2$ (Fig. 3d), DOS turns out to be non-zero at $E = 0$. These obvious differences are supposed to be good indexes for the mergence of the Dirac points. In Fig. 4, we give out the cyclotron mass $m_c$ for different values of $\Delta$, correspondingly, different strengths of on-site interaction.

FIG. 3: Density of state (DOS) of the system vs. order parameter $\Delta$. (a) $\Delta = 0$; (b) $\Delta = 1.0$; (c) $\Delta = 2.0$; (d) $\Delta = 2.5$. 
From Fig. 4, we find that besides the difference in the numbers of peaks, the curve of \( m_c \) changes from convex around the zero-energy point as shown in the inset of Fig. 4. This change signals a quantum phase transition between the semimetallic phase and band insulator phase.

From Fig. 1b, we see that by increasing the strength of the on-site interaction, the mean-field order parameter can be tuned. And it’s well-known that the on-site interaction could be modified by Feshbach resonances indirectly. Therefore, this exotic mergence of the Dirac points induced by on-site interaction is supposed to be observed in ultracold fermi gas experiments nowadays.

FIG. 4: (Color online) Cyclotron mass \( m_c \) vs. \( \Delta \). (a) \( \Delta = 0 \); (b) \( \Delta = 1.0 \); (c) \( \Delta = 2.0 \); (d) \( \Delta = 2.5 \). Inset: Comparison between (a) and (c) around the zero-energy point.

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