Parametrization for chemical freeze-out conditions from net-charge fluctuations measured at RHIC

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Abstract. We discuss details of our thermal model applied to extract chemical freeze-out conditions from fluctuations in the net-electric charge and net-proton number measured at RHIC. A parametrization for these conditions as a function of the beam energy is given.

1. Introduction
Over the past decade, statistical hadronization model (SHM) approaches, cf. e.g. [1], were quite successful in describing the particle multiplicities measured in heavy-ion collisions at various beam energies $\sqrt{s}$, allowing one to deduce the chemical freeze-out parameters as unique functions of the beam energy. As a complementary method for determining the conditions at freeze-out, fluctuations in the conserved charges of QCD, i.e. baryon number $B$, electric charge $Q$ and strangeness $S$, were proposed [2]. The latter can be calculated from first-principles in lattice QCD [3, 4] and compared with corresponding measurements of the moments of multiplicity distributions. Experimental data on fluctuations in the net-electric charge and the net-proton number (not a conserved charge but often assumed to be a good proxy for net-baryon number) became recently available from the beam energy scan program at RHIC [5, 6]. Thermal models provide another possibility to extract the freeze-out parameters from fluctuation observables. In the approach discussed here, we consider a grandcanonical ensemble of hadrons and resonances as listed in [7]. This implies that charge conservation is respected in the means but not in the higher-order moments. Given that this as well as other non-included fluctuation sources, e.g. critical fluctuations, have a negligible influence on the observables we study, we show that an extraction of the chemical freeze-out parameters is feasible within our approach.

2. Freeze-out conditions from moments of net-charge distributions
In this work, we concentrate on the lowest moments of the measured multiplicity distributions and determine the conditions at chemical freeze-out such that the experimental data [5, 6] for mean $M$ and variance $\sigma^2$ are reproduced within error bars. For the net-electric charge one has $M_{\text{net-}Q} = \sum_i Q_i \langle N_i \rangle$, where $\langle N_i \rangle$ is the average of the number $N_i$ of particles of type $i$ with electric charge $Q_i$, and $\sigma^2_{\text{net-}Q} = \sum_i \sum_j Q_i Q_j \langle \Delta N_i \Delta N_j \rangle$ with $\Delta N = N - \langle N \rangle$. The sums run...
over all charged hadrons contained in the measurement. As electric charge is dominated by $\pi^+, \pi^-, K^+, K^-, p$ and $\bar{p}$, we restrict our model to this ensemble. For the net-proton number one has $M_{p-\bar{p}} = \langle N_p \rangle - \langle N_{\bar{p}} \rangle$ and $\sigma^2_{p-\bar{p}} = \langle (\Delta N_p)^2 \rangle + \langle (\Delta N_{\bar{p}})^2 \rangle - 2\langle \Delta N_p \Delta N_{\bar{p}} \rangle$, where the covariance between protons and anti-protons $\langle \Delta N_p \Delta N_{\bar{p}} \rangle$ vanishes in our approach. To take the actual experimental situation more accurately into account, several physically relevant aspects are included in our study as explained in the following.

2.1. Influence of kinematic cuts and resonance decays
The experimental coverage in rapidity (or pseudo-rapidity) and transverse momentum is limited by the detector design and the demands from particle reconstruction and identification. To account for this acceptance limitation, it was proposed in [8] to restrict the phase-space integrals by the detector design and the demands from particle reconstruction and identification. To account for this acceptance limitation, it was proposed in [8] to restrict the phase-space integrals by the detector design and the demands from particle reconstruction and identification. To actual experimental situation more accurately into account, several physically relevant aspects are included in our study as explained in the following.

In Eqs. (1) and (2), $N^*_i$ and $N^*_R$ denote the primordial (before resonance decay) numbers of stable particles $i$ and resonances $R$, while $\langle n_i \rangle_R = \sum_i b^R_i n^R_i$, and $\langle \Delta n_i \Delta n_j \rangle_R = \sum_r b^R_{i,r} n^R_{i,r} - \sum_r b^R_{j,r} n^R_{j,r}$, are decay-channel averages, with $r$-th channel branching ratio $b^R_r$ and $n^R_{i,r}$ the number of $i$ produced in that decay-channel of $R$. As the decay of a resonance is a probabilistic process, which itself causes event-by-event fluctuations in the final hadron multiplicities, the averages in $M$ and $\sigma^2$ involve an averaging over the thermal ensemble and over resonance decays. The thermal, primordial means and (co)variances are obtained in our model by derivatives of the pressure $P$ with respect to the particle chemical potentials $\mu_l$ as (indices $i$ and $j$ stand for hadrons or resonances here)

$$\langle N^*_i \rangle_T = VT^3 \frac{\partial (P(T, \{\mu_l\})/T^4)}{\partial (\mu_i/T)}$$

$$\langle \Delta N^*_i \Delta N^*_j \rangle_T = VT^3 \frac{\partial^2 (P(T, \{\mu_l\})/T^4)}{\partial (\mu_i/T) \partial (\mu_j/T)}.$$ (3)

Although in the grandcanonical ensemble the primordial covariances $\langle \Delta N^*_i \Delta N^*_j \rangle_T$ between different particle species vanish, resonance decays can induce correlations such that the covariances between different hadrons affect the moments of the net-electric charge distribution. Due to baryon-number conservation this is, however, not the case for net protons. Interestingly, the form of the factors $\langle \Delta n_i \Delta n_j \rangle_R$ implies that the fluctuation contributions associated with the probabilistic nature of resonance decays can be small or zero depending on the charge under consideration: if the number of produced charges is the same in each decay channel of $R$, they will exactly vanish. In the case of the net-electric charge this means that although individual factors such as $\langle \Delta n_{+} \Delta n_{-} \rangle_R$ can be non-zero, the probabilistic contributions cancel each other exactly in the double-sum over all charged hadrons in $\sigma^2_{net-Q}$ because electric charge is conserved in each decay. Since we restrict the ensemble of considered hadrons, however, we expect some but sub-dominant contributions from resonances decaying, in addition, into charged hyperons. In the case of the net-proton number, probabilistic fluctuation contributions are significant and vanish only for the $\Delta^{++}$-resonances. A systematic study for net protons was presented in [9].
2.2. Influence of the isospin randomization of nucleons

Final state effects may significantly influence fluctuations, in particular, in the net-proton number. The regeneration and subsequent decay of $\Delta$-resonances via $p(n) + \pi \rightarrow \Delta \rightarrow n(p) + \pi$ can change the isospin identity of the nucleons $p$ and $n$ (similar for anti-nucleons) and, thus, influence their multiplicity distributions. As the electric charge is conserved in these reactions, they do not affect the fluctuations in the net-electric charge. Also, the mean $M_{p-\bar{p}}$ is not altered, but the higher-order moments of the net-proton distribution are influenced. For high enough pion densities and a long enough hadronic phase (before free streaming), such that a full isospin randomization can be assumed, correction expressions for the moments of the net-proton distribution based on the moments of baryon and anti-baryon distributions can be derived, cf. [10, 11]. It was argued in [10, 11] that the necessary conditions are satisfied for $\sqrt{s} \gtrless 10$ GeV.

When we exclude weak decay contributions from hyperons explicitly, these correction expressions may be formulated in terms of the moments of the nucleon and anti-nucleon distributions. Accordingly, the effect of isospin randomization is included in our model as

$$\sigma_{p-\bar{p}}^2 = \langle (r \cdot \Delta N_N - \bar{r} \cdot \Delta N_{\bar{N}})^2 \rangle + \langle r(1-r)N_N + \bar{r}(1-\bar{r})N_{\bar{N}} \rangle \quad (4)$$

with $r = \langle N_p \rangle / \langle N_N \rangle$, $\bar{r} = \langle N_\bar{p} \rangle / \langle N_{\bar{N}} \rangle$ and $N_N = N_p + N_o$, $N_{\bar{N}} = N_\bar{p} + N_o$. The entering quantities are calculated via Eqs. (1) - (3). As shown in [9], the main effect of these corrections is to make the net-proton distribution look Poissonian independent of whether the probabilistic fluctuation contributions from resonance decays are taken into account or not.

3. Parametrization for the chemical freeze-out conditions

With the abovementioned refinements in our thermal model, one can determine the values for temperature $T$ and baryo-chemical potential $\mu_B$ at chemical freeze-out by comparing the measured moment ratios $\sigma_{\text{net}-Q}^2 / M_{\text{net}-Q}$ and $\sigma_{p-\bar{p}}^2 / M_{p-\bar{p}}$ with the model results. In addition, physical constraints on $M_{\text{net}-Q}/M_{\text{net}-B}$ and $M_{\text{net}-S}/M_{\text{net}-B}$ have to be fulfilled. In the moment ratios, the volume $V$ which appears in Eq. (3) cancels when fluctuations in $V$ are ignored. The corresponding parameter values for $T$ and $\mu_B$ were reported in [12] and contrasted with different results from lattice QCD and with SHM results in [13].

In the left panel of Fig. 1 we show the quality of the description of the double-ratio

$$R_{12}^{\text{net}-Q}/R_{12}^{p-\bar{p}} = (\sigma_{p-\bar{p}}^2 / M_{p-\bar{p}}) / (\sigma_{\text{net}-Q}^2 / M_{\text{net}-Q})$$

of the measured moment ratios with our model obtained for the freeze-out parameters [12]. Since we assume full isospin randomization independent of the beam energy, we restrict our study to $\sqrt{s} \gtrless 11.5$ GeV. The ratio $\mu_B/T$ of our freeze-out parameters (with error bars based on the reported errors in the experimental data [5, 6]) is shown in the right panel of Fig. 1 (symbols). A suitable parametrization of these results as a function of $\sqrt{s}$ within $11.5 \leq \sqrt{s}/\text{GeV} \leq 200$ can be given in the form

$$\frac{\mu_B}{T} = \frac{a_0}{(\sqrt{s}/\text{GeV})^{a_1}} + a_2 + a_3(\sqrt{s}/\text{GeV}) \quad (5)$$

with $a_0 = 57.24$, $a_1 = 1.345$, $a_2 = 0.276$ and $a_3 = -0.00080$, which describes accurately our central values for $\mu_B/T$ (cf. solid curve in the right panel of Fig. 1).

4. Conclusion

We described in detail the physically relevant aspects taken into account in our thermal model in order to successfully deduce chemical freeze-out parameters from a comparison of model results with the moment ratios $\sigma^2 / M$ of the measured net-electric charge and net-proton distributions. We provided a suitable parametrization for the ratio $\mu_B/T$ of extracted freeze-out conditions as a function of $\sqrt{s}$. As discussed in [13], other sources of fluctuations that are not included in
Figure 1. (Color online) (a): double-ratio of measured $\sigma^2/M$-ratios for net protons and net-electric charge [5, 6] (circles) as a function of $\sqrt{s}$, compared with our thermal model results (crosses) using the chemical freeze-out parameters from [12]. (b): ratio $\mu_B/T$ of these freeze-out parameters (crosses) as a function of $\sqrt{s}$ together with a suitable parametrization thereof (solid curve) of the form Eq. (5) with parameter values $a_0 = 57.24^{+73.78}_{-35.91}$, $a_1 = 1.345^{+0.298}_{-0.386}$, $a_2 = 0.276^{+0.114}_{-0.251}$ and $a_3 = -0.00080^{+0.00040}_{-0.00082}$ (errors define the shaded band).

Our approach, such as critical fluctuations, volume fluctuations or the influence of exact charge conservation, have a negligible impact on these results for the highest beam energies considered in our analysis.

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References

[1] Cleymans J, Oeschler H, Redlich K and Wheaton S 2006 Phys. Rev. C 73 034905
[2] Karsch F 2012 Central Eur. J. Phys. 10 1234
[3] Bazavov A et al 2012 Phys. Rev. Lett. 109 192302
[4] Borsanyi S, Fodor Z, Katz S D, Krieg S, Ratti C and Szabo K K 2013 Phys. Rev. Lett. 111 062005; 2014 Phys. Rev. Lett. 113 052301
[5] Adamczyk L et al [STAR Collaboration] 2014 Phys. Rev. Lett. 112 032302
[6] Adamczyk L et al [STAR Collaborartion] 2014 Phys. Rev. Lett. 113 092301
[7] Eidelman S et al [Particle Data Group] 2004 Phys. Lett. B 592 1
[8] Garg P, Mishra D K, Netrakanti P K, Mohanty B, Mohanty A K, Singh B K and Xu N 2013 Phys. Rev. Lett. B 726 691-696
[9] Nahrngang M, Bluhm M, Alba P, Bellwied R and Ratti C 2014 Impact of resonance regeneration and decay on the net-proton fluctuations in a hadron resonance gas Preprint arXiv:1402.1238 [hep-ph]
[10] Kitazawa M and Asakawa M 2012 Phys. Rev. C 85 021901
[11] Kitazawa M and Asakawa M 2012 Phys. Rev. C 86 024904; 2012 Phys. Rev. C 86 069902
[12] Alba P, Alberico W, Bellwied R, Bluhm M, Mantovani Sarti V, Nahrngang M and Ratti C 2014 Phys. Lett. B 738 305-310
[13] Bluhm M, Alba P, Alberico W, Bellwied R, Mantovani Sarti V, Nahrngang M and Ratti C 2014 Determination of freeze-out conditions from fluctuation observables measured at RHIC Preprint arXiv:1408.4734 [hep-ph]