A Performance Limit Estimation Framework for Multi-hop Repeated/Regenerated Optical Links

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ABSTRACT We propose and demonstrate a generalized framework for performance limit evaluation and comparison of multihop optical repeated and regenerated links. The model developed is implementation agnostic and applies to multihop optical links of varied forms, including fiber, free space, and underwater links. The framework estimates the best-case performance gains of deploying an all-regenerative link over an all-repeater link for any given implementation. The implementation-independent technique is then illustrated using guided and free-space optical links. An abstract model is developed first with the evolution of signal, noise power, and bit error rate down the link compared and contrasted for both cases. The model is then evaluated using physical parameters for a typical fiber optic intensity-modulated direct detection link, and the obtained all-regenerator link performance advantage is translated to extra reach and lower transmission power requirements. Further, certain approximations are provided to reduce computational complexity and improve the analytical tractability of the procedure, which could be particularly helpful when employed in specialized hardware or for dynamic reconfiguration networks. Finally, the framework’s versatility is established by employing it in analyzing an ideal free-space link and comparing amplify and forward links against decode and forward counterparts. Similar results are also reproduced on a commercial optical link simulation suite. Detailed literature on link analysis is provided for fiber, free space, and underwater links, bringing out their similarities. We conclude by elaborating on various current and emerging application domains and certain limitations of the proposed technique.

INDEX TERMS BER, Cascaded Amplification, EDFA, FSO, Multi-hop, Optical Amplifiers, Optical link, Optical Regenerators, Performance analysis, SNR

I. INTRODUCTION

Fibre optical links have long taken over long-haul high-speed communication infrastructure owing to their very high bandwidth, small size, and flexibility along with excellent Electromagnetic Interference (EMI) immunity [1], [2]. Fiber links are also becoming ubiquitous in intracity and last-mile connectivity and finds application in front-hauling of modern wireless data transport including 5G [3], [4], [5]. The introduction of Erbium-Doped Fiber Amplifiers (EDFA) in the early 1990s was a significant breakthrough as amplification could now be achieved optically rather than having to undergo the costly and cumbersome Optical-Electrical-Optical (OEO) conversion [6], [7]. Further, the wide bandwidth of EDFA enabled Wavelength Division Multiplexing (WDM), which drastically increased fiber bandwidth capacity.

However, optical amplifiers working in the principle of stimulated emission inevitably add noise to the transmission in the form of spontaneous emission, thereby degrading the Signal to Noise Ratio (SNR) and Bit Error Rate (BER) down the link, which ultimately limits the system. Additionally, other impairments like nonlinearities and dispersion are also aggravated, which further limits the link range [1]. Hence OEO optoelectronic regenerators are typically employed after several repeaters - which detect the signals, take bit decisions, and retransmit the newly generated signal down the link. However, as the regenerators decide on noisy signals, they could introduce bit errors that accumulate down the link and ultimately limit the reach. Regenerator spacing was
TABLE 1: Literature on multihop link analysis for different channels

| Channel Fiber Links | System Investigated | Reference(s) |
|---------------------|---------------------|--------------|
| FSO Link            | OEO Repeaters       | [16], [17]   |
|                     | Optical Amplifiers  | [11], [6], [7], [16] |
|                     | Amplify and Forward (AF) Scheme | [26]–[28] |
|                     | AF using EDFA       | [29]–[34]    |
|                     | Decode and Forward (DF) Scheme | [13], [35] |
|                     | All Optical Regenerator and Forward | [36], [37] |
| Underwater Links    | Early proposals     | [38], [39]   |
|                     | Single hop links    | [40]–[44]    |
|                     | AF and DF links     | [45]–[49]    |

around 600-800 km at the dawn of the millennia and has increased since. Nevertheless, OEO regenerators are typically bandwidth limiting, expensive and difficult to implement with WDM [8]. Hence, all-optical 3R regenerators, which are much more power-efficient, have lower latency and integrate much better with optical links, are hotly researched [9], [10], [11], [12] but are yet to mature.

This paper proposes and demonstrates a framework to evaluate the absolute performance gains achievable by employing an all-regenerator link instead of an all-repeater link. The methodology is implementation agnostic and can be used to evaluate any Intensity Modulated Direct Detection (IMDD) multihop optical link, including Free space optical (FSO) and underwater links. We initially focus on the well-established fiber optic links for the framework’s derivation and demonstration but later show how the same can be translated to FSO links. We begin by developing abstract link models for both EDFA and regenerator links - establishing the dramatic performance advantage of the latter. Then this BER advantage is reinterpreted in terms of extra reach and power savings. The abstract model is then extended and evaluated using physical parameters to produce real-world relevant results. The motivation here is to establish the best-case performance limits for both systems, which can serve as a theoretical reference against which future link (e.g., hybrid repeater-regenerator links or all-regenerator links) advancements could be measured.

As pointed out before, the framework is general and can be applied to various IMDD optical links, including FSO and underwater links. We demonstrate this with FSO links in section VII. Multihop FSO links are becoming commercially viable - though with a significantly lower number of hops as compared to fiber-optic links and have already cut out some niche applications domains [13] [14] [15]. We also show in sections VII and VIII that the developed framework is compatible with multihop results, hence proving the generality and adaptability of the developed model. Further, we enlist a few approximations that can reduce computational complexity when implementing on low-powered or purpose-built hardware or making quick decisions as in dynamically routed networks.

Several publications in the literature analyze the multihop optical links from different perspectives. However, none of them proves to be as cross-platform or generalized as the framework we propose here. But some important ones provide accurate, in-depth modeling of system performance but for specific systems. One of the earliest such works is by Schiess et al. [16] for cascaded analog optoelectronic repeaters where the pulse shape, noise, and BER evolutions are analytically modeled. It was proved that even though the cascaded analog OEO repeaters can be made to perform well in dispersion limited systems, beyond a point, cascaded EDFA links outperform because of lower noise accumulation. Later, Ohlen et al. [17] investigated the effect of nonlinearity in the power transfer function of OEO repeaters and proved a small nonlinearity could help in the reshaping of pulse and drastically improve the cascadability of OEO repeaters. These early works predate optical regenerators, but later Mork et al. [18] derived the approximate analytical expressions for BER accumulation over cascaded all-optical 2R regenerators. BER modeling of cascaded 2R regenerators considering noise, finite extinction ratio, and non-linear reshaping is investigated in [19]. Zhu et al. [20] extended this to include timing jitter and pattern dependence. Cascadability of SOA based optical 3R regenerator is discussed in [21] [22]. Further, cascaded operation of advanced modulation (RZ-DPSK) all-optical 3R regenerator based on MZI-SOA is presented in [23]. Extending this to optical packet/burst-switched networks is [24]. Nevertheless, as mentioned above, these models are implementation-specific, complex, and don’t always consider bit-inversion along the link. The framework proposed here is much more general, implementation-independent, simple, and can be used with non-guided channels, making it particularly attractive for emerging fields.

One such field is FSO links which can be seen to follow a similar technology development trajectory as with fibre optical links discussed above. FSO links are attractive due to the high wireless transmission bandwidth, EMI immunity, license-free spectrum availability, security, and ease of establishing connectivity. However, just as with fiber communications, noise accumulation soon becomes the limiting factor for link reach. The earliest suggestion for a multihop FSO link came in the form of a mesh network for broadband wireless network [25], but the focus was on capacity computations with quality of service guarantees. Outage probability of multihop FSO link employing non-regenerative relays (Amplify and Forward relay) was first investigated in [26] for Nakagami Fading channels. The requirement for a regenerative relays was also investigated with different placements and was most effective at lower Signal to Noise Ratios (SNR). Further analysis of Amplify and Forward (AF) links, including closed-form lower bounds for non-identically distributed Nakagami-fading channel, outage probability, and average BER is presented in [27] [28]. Error analysis of multihop decoded relays (comparable to OEO regenerators) followed in [50]. AF links performance is compared to that of decode and forward (DF) relays in strong turbulence channels in [35]. Recently more advanced modulation over mixed RF and
FSO link for 5G was investigated using DF relays in [13].

Early implementations of all-optical multihop FSO links can be found in [29] [30] [31] where EDFA is used in an optical AF FSO link and found to perform better than multiple-transmitter arrangement. The outage probability of such a system in log-normal fading is discussed in [32], where it was proved to be outperforming electrical relays. An optical hard-limit is used along with EDFAs in [33] to reduce the accumulation of amplified spontaneous emission (ASE) noise. Such a multihop link is shown transmitting 10 Gb/s over a turbulent channel in [34]. Further, All-Optical Regenerate and Forward (AORF) links are shown to significantly outperform All-Optical Amplify and Forward links (AOAF) in [36] [37]. As can be observed, multihop FSO developments follow similar trends to multihop fiber links, and all the advantages of our model detailed above hold true for FSO links.

A very similar development arc holds true for Underwater Optical Communication (UWOC) links, too - with the early idea, implementations, and Monte-Carlo simulations explored in [40]. Traditionally, acoustic signals have been ubiquitous with underwater communications, but they are severely limited in bandwidth and latencies, which are drastically improved by using optical wireless links [41] [42], albeit at the cost of a much lower link length. There have been very early investigations [38], and implementations [39] of the same but the field really come alive in the last decade or so and is hotly researched currently [48], [49]. Some popular schemes are discussed in [41] [42], channel modeling in [43] [42] and turbulence modelling in [44] [42]. As attenuation is extremely high (span is typically only tens of meters) in UWOC links, multihop links (also termed relay assisted links in literature) are extremely important and necessary for many applications. [45] shows a DF link can extend the range to hundreds of meters. [46] analyses both AF and DF relays and proposes optimal relay placements. Both are further analysed, including pointing-tracking-acquisition mechanisms and routing techniques in [47]. Very recently, [48] analysed DF links for MIMO systems in a turbulence-induced fading UWOC channel, and [49] derived BER and outage probability of both AF and DF links over turbulence channels.

Evidently that the evolution of these different links (fiber, FSO, UWOC) mostly follow a common thread and the literature discussed above is neatly put together in Table 1. Notice how a generalized framework to estimate the performance ceiling of such repeater/regenerator (AF/DF) links could be hugely insightful not only for these links but also for any such potential multihop links in the future. This is where the proposed framework comes in and tries to bring out the similarities and some general features of the link performance evolution.

This paper is arranged as follows. Section II introduces the abstract model and BER performance of both all repeater/regenerator links and its impact on some other link parameters. Section III introduces some Q function approximations and Sec IV follows it up with relevant results from the preceding sections. A practical fiber optic repeater/regenerator link is modelled and compared in Sec V followed by corresponding results and discussions in Sec. VI. The framework is then used to analyse an ideal FSO link in the next section, Sec VII, followed by corresponding results in Sec VIII. Finally, several additional application domains and the future scope of the model are discussed along with its limitation in Sec. IX.

II. ABSTRACT MODEL
An abstract model for the BER evolution along the link is developed, which is extended to include physical parameters in the next section. As multihop fiber optic links are well established, and components are readily available commercially (albeit 3R all-optical regenerators yet to mature), we have opted to focus primarily on fiber-optic links in this section. However, the same applies to FSO links with minor modifications and is further discussed in Sec. VII.

A. ALL REPEATER LINK (1R)
Consider an M repeated link as in Fig.1 with each repeater having just enough gain to offset the losses of the preceding section. The SNR at the mth hop for any simple modulation scheme (ON-OFF Keying (OOK), Binary Phase Shift Keying (BPSK) or Binary PAM ) would be [51],

\[ SNR_{m} = \frac{SNR_{1}}{M} \]  

(1)

where \( SNR_{1} \) is the single hop SNR. Hence the probability of error (BER) for an IMDD system would be,

\[ P_{e}^{\text{exp}} = Q \left[ \sqrt{\frac{1}{M} \left( \frac{S}{N} \right)} \right] \]  

(2)

where \( Q() \) is the Q-function. This relates the progressive noise build-up of an all-repeater link to the system BER. So to maintain BER while increasing hops would require the transmitter to scale its power linearly. But the transmit power is typically limited owing to other factors like Kerr non-linearity in fiber links and eye-safety regulations in FSO links. So extending transmission range, maintaining BER and transmit power would require shortening span length, hence
increasing hops, which increases the noise accumulation, degrading SNR and BER - ultimately limiting the range. Hence regenerators are introduced to restore signal SNR.

B. ALL REGENERATOR LINK (3R)

Though regenerators recreate the signal and restore the SNR, they are typically expensive and complex, and their all-optical implementation is yet to mature. Hence they are optimally placed after a few repeaters in a hybrid configuration. However, our objective is to derive the maximal performance uplift when using an all-regenerator link. Interestingly, as typical multi-hop FSO links are only a few hops long, there have been interest in using all-regenerator (called decode and forward) link in FSO systems [52] [53] [35] [54]. Recently all-optical regenerators have also been introduced into multi-hop links [37] [36] too, which makes the analysis all the more relevant.

As pointed out earlier, regenerative links are theoretically limited by bit error accumulation. Though they reconstruct the signal and restore the SNR, any bit flip between hops cannot be corrected and accumulates down the link. This could be particularly severe for FSO links where such bit-flips are much more probable because of atmospheric turbulence. Let single-hop BER be [1],

$$BER_1 = \alpha = Q\left[ \sqrt{\left( \frac{S}{N} \right)} \right]$$

Building on the analysis [sec 11.2 [51]], the BER of M regenerative IMDD optical link can be expressed as,

$$P_e^{reg} = \sum_{i\text{ odd } k \leq M} P_{i}(i) = \left( \frac{M}{1} \right) \alpha (1 - \alpha)^{M-1} + \left( \frac{M}{3} \right) \alpha^3 (1 - \alpha)^{M-3} + \ldots + \left( \frac{M}{l} \right) \alpha^l (1 - \alpha)^{M-l}$$

where \(l\) is the largest odd number \(<= M\). When \(\alpha << 1\) and \(M\) not too large - which are very reasonable approximations here, this can be approximated to,

$$P_e^{reg} \approx M\alpha$$

which represents an almost linear BER degradation along the link. But it is also evident that this evolution is much milder compared to that of repeater link, where the per-hop noise addition significantly reduces SNR and hence BER, further along the link. Now, we try to represent this BER advantage in terms of improved reach or lowered launch power.

C. LINK REACH IMPROVEMENT AND REDUCTION IN TRANSMIT POWER

As regenerator links can sustain better BER at a given length - the advantage can be used to enable longer links. As there is no noise addition in intermediate nodes, link length is ideally limited only by noise accumulation. However, practically other factors like jitter accumulation limit this reach, esp for higher bit rate systems. Again, FSO links, owing to their shorter span and lower bit rates, could benefit from such a performance benchmark. Comparing both links for the same BER we have,

$$Q\left[ \sqrt{\frac{1}{M_{rep}} \left( \frac{S}{N} \right)} \right] = \left( \frac{M_{reg}}{1} \right) \alpha (1 - \alpha)^{M_{reg}-1} + \left( \frac{M_{reg}}{3} \right) \alpha^3 (1 - \alpha)^{M_{reg}-3} + \ldots + \left( \frac{M_{reg}}{l} \right) \alpha^l (1 - \alpha)^{M_{reg}-l}$$

The number of hops, \(M_{rep}\) and \(M_{reg}\) for all-repeater link and all-regenerator link, can be estimated numerically.

Alternatively, the BER advantage can be translated to lower launch powers targeting the same BER. The regenerator link power advantage is illustrated by numerically computing and comparing the SNR requirements for target BER. So we have,

$$Q\left[ \sqrt{\frac{1}{M} \left( \frac{SNR_{rep}}{1} \right)} \right] = \sum_{i\text{ odd } i} \left( \frac{M}{i} \right) \alpha^i (1 - \alpha)^{M-i}$$

where \(\alpha = Q\left[ \sqrt{SNR_{reg}} \right]. This essential means that (refer results in section IV-A) the regenerator link can manage to operate at a lower SNR - reducing power requirements of remote nodes which is highly desirable in many installations.

III. APPROXIMATIONS TO BER EXPRESSION

Computing the Q-function (tail distribution of standard normal curve), which is a non-elementary integral for various hops, configurations, and noise conditions, could get computationally involved, especially for lower powered or purpose-made hardware (ASICs, FPGAs). Hence we re-derive the above expressions using a few approximations with varying complexity and accuracy. We begin with (5) and conjecture the following inequality (later verified in results section - IV-A) to be true for large enough SNR,

$$MQ\left( \sqrt{SNR} \right) < Q\left( \sqrt{\frac{SNR}{M}} \right)$$

Now, we approximate the Q-function using different approximations and show the conjecture to be true (refer section IV-B).

1) Using Q() from Chiiani et al.

From [55] we have,

$$Q(x) \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{2x^2}{3}} \quad \forall x > 0$$

Now redefining \(x\) as \(\sqrt{SNR}\) for regenerators (LHS) and \(\sqrt{SNR/M}\) (RHS) for repeaters we have from (8),

$$M\left[ \frac{1}{12} e^{\frac{SNR}{2}} + \frac{1}{4} e^{-\frac{2SNR}{3}} \right] < \frac{1}{12} e^{\frac{SNR}{2M}} + \frac{1}{4} e^{-\frac{2SNR}{3M}}$$

We plot LHS and RHS independently (see section IV-B) and prove the inequality holds true for reasonable values of SNR.
2) Using Q() from Karagiannidis et al. Karagiannidis et al. derived tighter bounds for \( erf() \) in [56]. Converting it to Q function and using reasonable values of the given constants and simplifying we would get,

\[
Q(x) \approx \frac{1 - e^{-1.4x}}{2.8450x} \quad \forall x \geq 0
\]  
(11)

which when used in (8), we have

\[
M \left[ \frac{1 - e^{-1.4\sqrt{SNR}}}{2.8450\sqrt{SNR}} \right] \left[ e^{-\frac{2SNR}{M}} \right] < \frac{1 - e^{-1.4\sqrt{\frac{SNR}{M}}}}{2.8450\frac{\sqrt{SNR}}{M}}
\]

for \( M, SNR > 0 \)  
(12)

which simplifies to,

\[
\sqrt{M} \left( 1 - e^{-1.4\sqrt{SNR}} \right) e^{-\frac{SNR}{2}} < \left( 1 - e^{-1.4\sqrt{\frac{SNR}{M}}} \right) e^{-\frac{SNR}{2M}}
\]

(13)

3) Using Q() from by Abreu et al. Tight bounds derived by Abreu et al. in [57] have upper and lower bounds with the ‘tightness’ adjustable using different parameters. From [57] we have,

\[
e^{-x^2} - \frac{e^{-x^2}}{b_L (x+1)} \leq Q(x) \leq \frac{e^{-x^2}}{a_U} + \frac{e^{-x^2}}{b_U (x+1)}
\]

(14)

Setting typical values of \( b_L \geq \frac{\sqrt{2\pi}}{8b_L \sqrt{2\pi} exp(-0.5)} \approx 2.5 \) and \( a_L = \frac{8b_L \sqrt{2\pi} exp(-0.5)}{4b_L - 3\sqrt{2\pi}} \approx 12.16, a_U = 48.9 \) and \( b_U = 2 \) we have,

\[
\frac{e^{-x^2}}{12.16} + \frac{e^{-x^2}}{2.5 (x+1)} \leq Q(x) \leq \frac{e^{-x^2}}{48.9} + \frac{e^{-x^2}}{2 (x+1)}
\]

(15)

Choosing the worst case (with regard to regenerative links) for comparison, i.e., using the lower Q bound for repeaters and the upper bound on regenerators, we can rewrite (8) as,

\[
M \left( e^{-\frac{SNR}{48.9}} + \frac{e^{-\frac{SNR}{2}}}{2\sqrt{SNR} + 1} \right) < \frac{e^{-\frac{SNR}{12.16}}}{2.5 \left( \sqrt{\frac{SNR}{M}} + 1 \right)}
\]

(16)

It has to be noted that any computational advantage of such approximations on modern hardware is minimal. However, in applications using low specced or battery-powered remote nodes with specialized hardware (FPGA, ASICs), needing to recalculate such computations frequently (e.g., dynamically reconfigurable optical networks), these approximations could conceivably make a difference. Further, the approximation removes the numerical nature of these computations and lends it more analytically tractable for further analysis or extension while also making it compatible with symbolic computational tools (computer algebra).

### IV. RESULTS - ABSTRACT MODEL AND APPROXIMATIONS

This section illustrated the numerical results for analysis from previous sections, Sec. II and III. These are the abstract model for an ideal long-repeated optical link that is implementation agnostic. The approximations provided in Sec. III are also verified against the numerical results of the exact expressions.

#### A. PERFORMANCE ANALYSIS USING ABSTRACT MODEL

We begin by plotting the BER vs SNR for \( M = 10 \) hop all-repeat and all-regenerator links in Fig. 2 based on (2) and (4) respectively. It is evident that BER improves significantly with SNR and that the regenerator link easily outperforms the repeater link. Thus, adding a few bit errors at each link is more desirable than letting the SNR degrade progressively along the link, only to be converted to a much larger BER at the detector. This is further illustrated in Fig. 3, where the build-up of bit errors along the link is plotted for both links (at an SNR of around 14 dBm). Notice the approximately linear BER evolution of regenerator links, as predicted by (5). Not only does the regenerator link incur way lower bit errors per hop, but the error addition is also much more gradual - esp at the initial hops of the link.

With the BER advantage of regenerator links well established, we now see how this translates into other useful link parameters like reach (Fig. 4) and power savings (Fig. 5). Notice in Fig. 4, a minimum SNR is required for any length of the link to be supported, which is determined by the intrinsic noises of a single hop. Beyond this, though, regenerator can support much longer links which increases dramatically for higher SNRs. The step-like response of the repeater link is attributed to it needing much higher SNR margins to support an additional hop. So, \( M \) remains constant for a large range of SNR. The SNR for this comparison had to be limited somewhere below 14 dBm, as large values of regenerator
$M$ (for larger SNRs) were causing memory issues in software. The approximation helps to get over such limitations easily. We have chosen to use a conservative Forward Error Correction (FEC) limit of $10^{-5}$ [58] as the target BER for both links. The number of hops supported by each link while maintaining a BER of $10^{-5}$, $M_{rep}$ and $M_{reg}$, are numerically computed for different values of SNR to generate Fig. 4.

The reduced SNR requirements of regenerator link mean lower transmit power for a given BER and reach, as illustrated in Fig. 5. The power savings are represented as the SNR ratio required for both links to sustain the same number of hops at a given BER. Note that this is not the wall-plug efficiency and only represents the maximum theoretical gains. The actual gains would be very much implementation-specific.

**B. APPROXIMATIONS RESULTS**

Different approximation for $Q()$ from Sec. III is compared here. Note that there are two different approximations used in the paper - the approximation expression for regenerator link (5) and different approximations to $Q$-function from Sec. III. Its the latter that’s explored here.

The conjecture in (8) is verified and compared with different approximations for $M = 10$ in Fig. 6. Plotting both LHS and RHS of (8) proves the conjecture to be true for all SNRs but for its lowest values which are not typical for most IMDD optical links. Comparing the 3 different approximations (as defined in Sec. III) Abreu’s (16) is the closest, with additional flexibility of having tunable ‘tightness’. Karagiannidis’s approximation (13) is a bit off, while Chiani’s (10) was almost as good as Abreu’s for regenerator and is still very good for repeaters. Any of the two could be adopted for acceptable degrees of accuracy and should help in various scenarios as discussed in Sec. III.
V. A PRACTICAL MULTIHOP FIBER OPTIC LINK

This section illustrates the implementation of the proposed framework on a practical multihop fiber optic IMDD link. Different noise sources at the detector are considered, along with Amplified Spontaneous Emission (ASE) noise. These parameters are plugged into the abstract model to analyze the noise build-up, SNR degradation, and BER evolution to compare the performance of all-repeated and all-regenerated fiber IMDD links. We have chosen to use optoelectronic (OEO) regenerators here (comparable to DF regenerators in FSO links), representing the worst-case scenario of all-regenerator links (Fig. 7). An ideal 3R all-optical regenerator would have noise addition only at the detector, and hence its BER throughout any number of equidistant hops would be theoretically constant and equal to $\alpha$ (Eq. 3). This, of course, is not realistic as such a link would well be limited by jitter accumulation or other implementation-specific limitations of the all-optical regeneration technique used (which are yet to mature, but several competing implementations exist, particularly the ones based on Mamychev filters [59] [60]).

SNR at the detector of an optical links is [1],

$$SNR = \frac{\text{average signal power}}{\sigma^2} = \frac{I_p^2}{\sigma^2}$$ (17)

where $P_{in}$ is the incident optical power, $I_p$ is the photocurrent with $R_d$ the responsivity, $\sigma^2$ is the net noise power comprised of thermal ($\sigma_T^2 = 4(k_BT/R_d)F_n\Delta f$) and shot noise ($\sigma_s^2 = 2q(R_dP_{in} + I_d)$) (neglecting the dark current). The constants have their typical notations, $k_B$ - Boltzmann constant, $q$ the charge of electron, $T$ - temperature, $R_L$ - load resistance, $F_n$ - amplifier noise figure at receiver, and $\Delta f$ - effective noise bandwidth.

For an OOK IMDD link with equiprobable '1's and '0's and with Gaussian distributed thermal and shot noises, we have [1],

$$BER = P_e = \frac{1}{2} erfc \left( \frac{Q}{\sqrt{2}} \right)$$ (18)

where, $Q = \frac{I_1-I_0}{\sigma_0 + \sigma_1}$. $I_0$, $\sigma_0$ and $I_1$, $\sigma_1$ represent intensity and net noise variance corresponding to '0' and '1', respectively. For thermal noise limited system (which is typical) $\sigma_0 = \sigma_1 = \sigma$, and assuming $I_0 = 0$ we have, $Q = \frac{I_1}{2\sigma} = \sqrt{\frac{SNR}{2}}$. Using this in (18) we have,

$$BER = \frac{1}{2} erfc \left( \sqrt{\frac{SNR}{2\sqrt{2}}} \right)$$ (19)

A. BER ACCUMULATION - REGENERATOR LINK

Defining single-hop BER using (19), we have

$$BER_1 = \beta \pm \frac{1}{2} erfc \left( \frac{\sqrt{SNR_{reg}}}{2\sqrt{2}} \right)$$ (20)

Modeling the regenerator as a receive-transmit pair, as mentioned above, SNR at the receiver is [1],

$$SNR_{reg} = \frac{(R_P_{in})^2}{\sigma_T^2 + \sigma_s^2}$$ (21)

Plugging Eq. 20 into the analysis from section, II-B we have,

$$BER_{M_{reg}} = \sum_{i \text{ odd } \& < M} P_{i}(i) = \left( M \frac{1}{2} \right) \beta (1 - \beta)^{M-1} + \left( M \frac{3}{2} \right) \beta^3 (1 - \beta)^{M-3} + \ldots + \left( M \frac{M}{2} \right) \beta^M (1 - \beta)^{M-M-1}$$ (22)

which becomes $\beta M$ when $\beta << 1$ and $M$ not too large. Hence we have,

$$BER_{M_{reg}} \approx \frac{M}{2} \left( 1 - \frac{1}{\sqrt{2}} \right)$$ (23)

B. BER ACCUMULATION - REPEATER LINK

EDFA, the most popular optical amplifier in multihop links, is chosen for the repeater link. EDFAs are prominent not just in fiber optic links, but are widely proposed for multihop implementations in FSO links as well [29] [30] [31], as noted earlier. The spontaneously emitted photons in the same direction and mode as the optical signal (but uncorrelated with the signal and emissions it stimulates) get amplified, accumulate over multihop links, and manifest as noise after beating with the signal at the detector. The ASE noise variance at the detector is given by [1],

$$\sigma_{ASE}^2 \approx 4(RGP_{in})(RS_{sp})\Delta f$$ (24)

where $R$ is the responsivity, $G$ amplifier gain, $P_{in}$ the input power, $\Delta f$ the bandwidth, and $S_{sp}(\nu)$ the spontaneous emission noise spectrum, which is typically white and is given by,

$$S_{sp}(\nu) = (G - 1)n_{sp}\nu h$$ (25)

with $n_{sp}$ being the population inversion factor given by $n_{sp} = \frac{N_1}{N_2 - N_1}$. $N_1$ and $N_2$ are the atomic densities at ground and excited states, respectively.

Finally, considering noise power from thermal and shot noise as well, we have the SNR at the detector of M-hop all-repeat links,

$$SNR_{rep}^M = \frac{I^2}{\sigma^2} = \frac{(R_P_{in})^2}{\sigma_T^2 + \sigma_s^2 + M\sigma_{ASE}^2}$$ (26)

Which when used with (19) yields the BER of repeater link as,

$$BER_{M_{rep}} = \frac{1}{2} erfc \left( \frac{\sqrt{SNR_{M_{rep}}^M}}{2\sqrt{2}} \right)$$ (27)
VI. NUMERICAL RESULTS AND DISCUSSION - PRACTICAL FIBER OPTIC IMDD LINK

Results from the above section are numerically investigated using typical physical link parameters, and various inferences are verified. The following parameters are used for the numerical results - $\lambda = 1550$ nm, $R = 1$ A/W, $k_B = 1.38 \times 10^{-23}$ J/K; $T = 300$ K, $F_n = 2$, $\Delta f = 5$ GHz; $R_L = 1$ KΩ, $q = 1.602 \times 10^{-19}$ C; $P_{in} = 100$ nW; $G = 10$; $n_{sp} = 1$; $h = 6.626 \times 10^{-34}$ J.s and $c = 3 \times 10^8$ m/s.

Fig. 8 shows the important BER vs input power for single and multihop ($M = 10$) all-repeated and all-regenerated links. Of course, the performance degrades as compared to single-hop systems for both multihop links, but it is much more drastic with the repeater links. Regenerators link scales very well at $M = 10$ compared to repeater links which require drastically more power to keep operating at the same BER. This is because more signal power is required to combat the accumulating ASE noise, whereas regenerator link clean-up and restores SNR at each hop and degrades only because of the small number of bit-errors added per hop. This is further evident from the BER accumulation analysis in Fig. 9, where BER vs. $M$ for the regenerator link has a very small slope compared to the repeater link, which increases quickly to unusable levels (at $P_{in} = -23$ dBm). The flat region in the repeater links is unusable, and the link would need higher input powers to bring it back to useable levels. Referring back to Fig. 8 it can be quickly accessed that for $M = 10$ the repeater link would require input power $> -10$ dBm to bring it back to acceptable levels of BER. But at these higher input levels, the performance contrast to regenerator link is further exacerbated, as is again discernable from Fig. 8. Further, the maximum input optical power to these links is limited by the onset of different non-linearities. Finally, these results are all in agreement with the abstract model results in Fig. 2 and 3.

Its evident from Fig. 8 that an input power of more than $-17$ dBm would be required to sustain even a single hop repeater link below a BER of $10^{-5}$. From Fig. 10 this would represent an $SNR > 17$. An imaginary horizontal line at this level would hence represent the minimal SNR requirement for a multi-hop repeater link to sustain a BER of $10^{-5}$. Using such a construction, it can be concluded that even an input power level of $-10$ dBm cannot support an $M = 10$ link (thought anything less than say $M = 5$ will be supported). Fig. 10 represents how ASE noise accumulates to degrade SNR in the EDFA link at different input power levels. Such a plot for regenerators would simply be a horizontal line at different levels (corresponding to different input powers) as the signal is regenerated and SNR restored at each hop.

The superior BER performance of regenerator link evident from Fig. 8 can be translated to longer reach or lower power.
FIGURE 11: Link reach vs input power (BER target - $10^{-5}$) for all Repeater/Regenerator links

FIGURE 12: Power required for Repeater/Regenerator link for achieving a particular BER

by plugging parameters derived in Sec. V in Sec. II-C. Figure 11 plots the maximum number of hops achievable by all repeater and all-regenerator links for given input power and a target BER of $10^{-5}$. As was noticed before, beyond a minimum input power (depending on different noises and BER target), the regenerator link can support a very large number of hops, whereas the repeater link shows much more subdued growth. This is because the regenerator link can sustain a larger SNR throughout the link. Plot axes are chosen to show the gradual rise of repeater links clearly. The staircase-shaped features are because the repeater link needs to overcome an added SNR degradation due to ASE noise at each hop, to serve the next hop.

Similarly, analyzing the input power requirements for sustaining particular BER levels across both links, we have Fig. 12. As discussed above, regenerator links barely need additional power to scale to much longer links once the required single-hop SNR is (corresponding to the BER targets here, we have considered three different targets) achieved. Also, decreasing the BER targets requires only a minimal increase in power requirements. Repeater links need more input power to start with, and the requirement keeps rising with the number of hops or with the tightening of BER targets.

The links, with the parameters considered here, are mostly limited by the thermal noise, and we verify this by comparing with a thermally limited system with other parameters remaining the same. Fig. 13 compares the BER at the output of thermally limited $M = 10$ repeater and regenerator links against those in Fig. 8. The plots (regular vs. thermally limited) for both repeater and regenerator links (red and blue, respectively) mostly overlap, signifying the diminishing contribution of shot noise in the SNR of both links. This is particularly true for the repeater links, where the accumulated ASE noise drowns out other noises.

Finally we have the multi-parameter plots (Fig. 14a and Fig. 14b), tracking BER against input power and $M$ simultaneously. This gives a holistic and complete picture of everything discussed above without parametrized plots. Fig. 14a for the repeater link reaffirms its requirement of higher input powers for sustaining even a few hops. The drop in link performance with both $M$ and input power is significantly more severe than with regenerator link in Fig. 14b. The repeater link curves more ‘spherically’ compared to the regenerator link, which is more ‘cylindrical’. This is easily explained by recalling that the regenerator links support much longer links with minor input power increment (refer Fig. 11 and 12), making the hops seem independent of input power; whereas for repeater link both hops and power requirement are strongly tied and rises together. This is further evident by considering equi-BER planes (cross-sections...
VII. ANALYSING MULTIHOP FSO LINKS

As discussed earlier, the same analysis framework could be extended to other domains as well, and we demonstrate this using FSO links in this section. Continuing from the discussion on different types of multihop FSO links in the Introduction, we analyze amplify-and-forward (AF, also called analog relaying) and decode-and-forward (DF, sometimes called digital relaying) links, which are analogous to repeater and OEO regenerator links in fiber communications. As multihop FSO links were introduced much later, optical amplifiers (typically EDFAs) are typically used at each hop, there is no OEO conversion, making them similar to fiber links (other than the channel).

A. ALL-REPEATER (AMPLIFY-AND-FORWARD) FSO LINKS

The SNR at the receiver of an FSO link, where each hop have knowledge about the Channel State Information (CSI) of the preceding hop, is given by [28] [35],

$$\mu = \frac{\prod_{i=1}^{N} s_{i}^2 g_{i-1}^2}{\sum_{i=1}^{N} n_i \left( \prod_{j=i+1}^{N} g_j^2 - 1 \right)}$$  \hspace{1cm} (28)

where $s_i = \eta I_i$ is the instantaneous intensity gain of $i$th hop, $g_i$ represent the $i$th relay gain, $n_i$ the AWGN noise at the input of $i$th relay having power $N_0$, $\eta$ is the photocurrent conversion ratio and $I_i$ is the turbulence induced light intensity. A popular choice of $g_i$ as proposed in [61] is $g_i^2 = \frac{1}{s_i^2 + N_0}$. Under such condition, the end-to-end SNR reduces to [26],

$$\mu_{eq} = \left[ \prod_{n=1}^{N} \left( 1 + \frac{1}{\mu_n} \right) - 1 \right]^{-1}$$  \hspace{1cm} (29)

where $\mu_n = \frac{\eta^2 I^2_n}{N_0}$ is the SNR of the $n$th hop. As our idea is to derive and compare the maximum performance contrast between repeated and regenerated FSO link - the upper bound on the SNR of (29) is [35],

$$\mu = \left( \sum_{i=1}^{N} \frac{1}{\mu_i} \right)^{-1}$$  \hspace{1cm} (30)

where $\mu_i = \frac{\eta^2 I^2_i}{N_0}$ is the SNR of the $i$th hop. This happens when the relay gain is set to $G_n = \frac{1}{\alpha^2}$, which is common in literature [26] [28] [35]. In fact, such a link serves as the benchmark for the performance of all practical non-regenerative systems [26]. Further, the equivalent SNR of the link is related to the harmonic mean of individual SNRs as,

$$\mu_{eq} = \frac{\gamma_H \mu}{N}$$  \hspace{1cm} (31)

where $\gamma_H$ is the harmonic mean of individual SNRs. As we have considered equidistant and equi-performing links for fiber link analysis, using the same here, we get

$$\mu_n = \frac{\mu_i}{N}$$  \hspace{1cm} (32)

which is exactly the relation we got for fiber optical systems (1). Finally, an FSO link is modeled as a Binary Symmetric Channel in [50] with error probability for ON-OFF keying given by,

$$P_{eFSO} = Q \left( a(t) \sqrt{SNR} \right)$$  \hspace{1cm} (33)

where Q() is the error function and $a(t)$ is the total attenuation. Ignoring atmospheric turbulence, the total attenuation is expressed as,

$$a(t) = a_G \cdot a_A(t)$$  \hspace{1cm} (34)
where $a_G$ is attenuation due to geometric spread given by [62] [63],
\[ a_G = d_R^2/(d_T + \theta_s L)^2 \]  
(35)
where $d_R$ and $d_T$ are receiver and transmitter diameters, separated by $L$ distance and $\theta_s$ is the angular divergence of light source. $a_A(t)$ is the atmospheric attenuation - empirically give by [64] [65],
\[ a_A(t) = e^{-\rho R}, \text{ with} \]  
(36)
\[ \rho = 3.91 \frac{\lambda}{V(t)} \left( \frac{550 \text{mm}}{550 \text{nm}} \right)^{-q} \]  
(37)
where $V(t)$ is the atmospheric visibility and $q = 1.3$ for 50 km to 6 km of visibility, and $q = 0.585 V^{1/3}$ for visibilities lower than 6 km. Atmospheric attenuation due to various environmental factors like rain, fog, etc. are accounted for using $V(t)$ the value of which is can be obtained from the International Visibility Codes [66].

Finally, using (32) in (33) and comparing with the BER for fiber repeater links in (2) it can be seen that for a still atmosphere, (33) takes the form of a scaled (2). The assumption is justifiable as FSO links are typically slow-varying [31] and the attenuation of a single bit can be assumed to be constant [50]. Hence, it can be concluded that the same framework used for analyzing fiber optic repeater links is capable and compatible with the analysis of AF FSO links. We now investigate DF links, which are similar to regenerator links.

**B. ALL-REGENERATOR (DECODE-AND-FORWARD) FSO LINKS**

Decode-and-forward links employ either OEO conversion or implement one of the developing optical regeneration techniques. For an $M$ hop all-DF link, the end-to-end BER is given by [50],
\[ P_e^{FSO-DF} = 1 - \left( (1 - p_1)(1 - p_2) \ldots (1 - p_M) \right) \]  
(38)
\[ = 1 - \prod_{i=1}^{M} \left[ 1 - \alpha_i^{FSO} \right] \]  
(39)
where $p_i = \alpha_i$ is the probability of error of individual links. Assuming clear and consistent weather across the link (ie $\alpha_i^{FSO} = \alpha^{FSO}$), we can approximate (38) using binomial expansion as,
\[ P_e^{FSO-DF} = 1 - \left( 1 - \alpha^{FSO} \right)^M \]  
(40)
\[ \approx 1 - \left( 1 - Ma^{FSO} \right) \]  
(41)
\[ \approx Ma^{FSO} \]  
(42)
This is precisely the approximation we arrived at in (5). Hence we can conclude that the best case analytical BER performance of all-repeater and all-regenerator links have the same form across different (channel) links and the framework introduced here to analyze these links is general and implementation agnostic. This could also help estimate the best-case performance from repeater/regenerated multihop optical link in a given scenario. We now use these results to compare and contrast static, ideal AF (repeater), and DF (regenerative) multihop FSO links.

**VIII. NUMERICAL RESULTS - FSO LINKS**

As mentioned above, the general form of the solution for both fiber and FSO links are comparable, and this is reinforced here by illustrating FSO links’ performance in Fig. 15a and 15b. As the performance upper-bounds are the primary focus here, static and ideal conditions are considered. Nevertheless, link performance at much lower visibility, foggy conditions, are also provided for reference. Visibilities, $V$ of 50 km [62] and 150 m [66] are considered (so $q = 1.3$ and 3.108 respectively), which represent very clear skies and thick-fog conditions respectively - as per the International visibility code [65] [66]. Repeater/regenerators are separated by a distance of 2.5 km. FSO links are typically much shorter than fiber links, so $M = 3$ is used. Further, a transmit aperture, $d_T$ of 3 cm, and receiver aperture, $d_R$, of 10 cm [62] with a beam spread of 0.1 mrad [31] is considered.

Fig. 15a shows how such multihop-FSO AF and DF links compare for $M = 3$ links at different visibilities of 50 km and 150 m (representing clear skies and thick-fog respectively with corresponding $q$ values of 1.3 and 3.108). As is expected, the regenerator link outperforms the repeater links, and that margin only increases with longer links (until where the AF link BER gets really poor). Also, lower visibilities result in significantly degraded link BER unless offset using higher SNR (which again is limited by eye-safe operation limits). Besides, note the drastically higher BER requirements for FSO links (compared to fiber links) for any given BER. Similarly, BER at different link lengths is compared when starting off with a 35 dB SNR at visibilities of 50 km ($q = 1.3$, clear sky) and 500 m ($q = 4.643$, foggy sky) in Fig. 15b. BER degrades drastically with link reach for AF link until its degraded beyond usable limits, but DF link degrades more gracefully - which is exactly what was observed with fiber optic links in Fig. 11 and Fig. 12. The result again shows the excellent scalability of the DF links.

These results are further corroborated using a commercial optical simulation suite, OptiSystem by OptiWave, and the results are given in Fig. 16. All the familiar traits discussed before are reproduced here too, though the exact values may differ as the software uses a more physical model and have provisions for more parameters than demanded by our abstract model, where generality is more important than specificity. Also, in Fig. 16b the DF link generates a horizontal line; this is because the software probably uses a BER estimation technique rather than computing the actual bits in error. Hence, the bit error accumulation cannot be faithfully reproduced, resulting in the horizontal line, and is purely a software limitation.

So it can be concluded that FSO systems share much of the same broad characteristics with fiber links under ideal conditions, as is further illustrated by comparing Figs. 2 and 3 with Fig. 15a and 15b. Hence, the technique’s effectiveness...
FIGURE 15: FSO AF/DF link performance comparison

IX. DISCUSSION - LIMITATIONS AND APPLICATIONS

Coherent optical links have long replaced IMDD links in long-haul communications and are typically limited by fiber-non-linearities and not noise accumulation [67]. Commercial systems are evolving from 100G QPSK DWDM to 400G, exploiting advanced modulations, higher baud rates, and using optical super-channels [68] [69] [70]. Linear and, to some extent, nonlinear impairments can be compensated using Digital Signal Processing (DSP) at the receiver [71]. Further, Space Division Multiplexing (SDM) and Multiple In Multiple Out (MIMO) architectures are challenging the nonlinear Shannon limit for single-mode transmission [71] [72]. Even short-haul links are now moving to 400G and beyond employing advanced modulation like PAM4 [73] [74] [75]. But these advanced modulation formats cannot be analyzed using the framework; though it may seem like a severe limitation, there are plenty of current and emerging fields where such an analysis could be beneficial. Several such potential applications are discussed in this section.

For all the promises shown by coherent links, they are still not commercially viable for point-to-point short-haul links. Complexity, cost, noise and non-linearities are still fundamentally limits [76] and hence there is still interest in IMDD systems [77] [78]. For eg: 400G coherent transceiver is ≈ 170% costlier than 4 IMDD links each at 100 Gbps [79]. Also, some medium-haul links (typically 40 to 80 km) like inter-data centre still prefer IMDD [80]. Further, the 400G IEEE 802.3bs specifications propose WDM channels (8x50 Gb/s) at 1300nm window to keep chromatic dispersion in check [80]. This increases attenuation, requiring cascaded optical amplifiers for longer links where the proposed analysis could be relevant.

Another domain where IMDD links are particularly popular is in data centre interconnects, where its simplicity (coherent systems require an additional local oscillator laser) and lower costs are still attractive [81] [82]. As data needs
exploded exponentially, there has been a number of proposed architectural changes in the last decade – many of which require optical amplification at different levels [83]. As high data rates explode, the cost per bit of a lower-powered optical amplifier becomes viable to be used in data centre links and has the additional benefit of relaxing the modulator drive swing and hence reducing overall module power. They can be employed in preamplifiers [84], optical switches and interconnects [85]–[88], and in splitting loss compensation [85]. Optical switches and interconnects are hotly researched, and most popular proposals employ an optical amplifier primarily used to achieve wavelength conversion [85]–[88].

Inter-data-center interconnects (DCI), particularly those over Metro network, are yet another interesting application for the proposed model. DCI traffic is doubling year on year [89], hence requiring the setup of multiple datacenters around the same region (< 80 km compared to < 2 km intra-datacentre). Such systems typically employ an optical booster at the transmitter and an optical preamplifier at the receiver, and many links have inline amplifiers as well [89]. Also, certain low latency applications require some data centers to be closer to the user [89], but infrastructural limitations force other datacentres to be further off. There has been a demonstration of IMDD DCI working at 204 Gb and 140 Gb over 10 km and 80 km DCIs, respectively [90]. Several implementations are using wideband WDM (which seems to be the future for DCI) and many proposals for using optical amplifiers (using different optical amplifiers to amplify different bands) in such links [84], [91]–[93].

Access networks are yet another domain dominated by IMDD links. Passive Optical Network (PON) are IMDD links supporting applications like IPTV, HDTV, GPON, and the next-generation solutions like ITU-T’s NG-PON2 [94] [95]. Conventionally these have been passive links, but there are recent proposals for link reach extensions by introducing optical amplifiers in the link [96], where such an analysis could be employed. There are a number of literature that discusses various aspects, like link budget and splitting ratio compensation, specialized amplifiers etc. [97], [98]. Amplifiers and regenerators are researched to be used as Range Extenders (RE) [99], [100] and are compliant with ITU-T G.984.6 recommendations. Mid-span range extenders are an objective in the XG-PON systems as part of Next-Generation PON (NG-PON) [99], [100], and are compliant with ITU-T G.984.6 recommendations. Mid-span range extenders are an objective in the XG-PON systems as part of Next-Generation PON (NG-PON), and makes use of optical amplifiers and optoelectronic regenerators [100]. Special bidirectional optical amplifier for such extended PON links is hotly debated too [97], [98]. Taking this further, [101] proposes different amplification methodologies, including optical 2R and OEO 3R - which align perfectly with the framework discussed here. Another potential use case in PON would be the usage of optical amplification to support higher splitting ratios in highly dense areas (as a loss of 10 log(N) dB is incurred at each 1:N split) [99], [100]. There is also the ITU-T G.698.2 (11/2018) (Amplified multichannel dense wavelength division multiplexing applications with single-channel optical interfaces) [102] recommendation, which specifies optically amplified DWDM systems. The standard is noted as particularly suited for metro links and has detailed specifications for links with one or multiple optical amplifiers.

IMDD subsystems are still under active development, signaling continued interest in the field. For e.g., a 100 Gb/s NRZ-OOK transmission using silicon photonic Electro-Absorption (EA) modulator is described in [103]. External modulators for high bit rate systems are rapidly evolving, too - [104] used silicon photonic dual-drive traveling wave MZM and a very high-speed DAC to reach 190 Gb/s OOK. Further, 204 Gb/s OOK transmission was achieved using a Distributed Feedback Laser (DFB) modulated using an EA modulator with a 2:1 double heterojunction bipolar transistor selector multiplexer [79]. The detection was done using a 110 GHz PIN diode and 100 GHz ADC. These are just a few among many noteworthy advancements and stand to prove that IMDD links stay relevant, and techniques like the one proposed here could be taken advantage of to analyze these future links.

As mentioned before, the proposed framework is not limited to fiber-optic links, and we have already illustrated its applicability to FSO links. Another manifestation of the same would be in laser satellite communications [105], [106], especially when there is almost another space race in sending thousands of micro-satellites in Low Earth Orbit (LEO) for satellite internet. There are several proposed architectures for the same, and almost all of them include optical amplifiers of some sort – most of them as optical preamplifiers [105], [107] and many as booster or inline amplifiers [108], [109]. Further, [110] analyses the performance of AF multihop optical inter-satellite links, reinforcing the applicability of the proposed system.

Finally, there are other emerging domains, notably under-water multi-hop networks [111] [112] [14], that could take advantage of the proposed framework with minimal modifications. A brief review of the same can be found in Sec. I. E.g., Underwater Visible Light Communication (UVLC) using detect-and-forward and amplify-and-forward relays are investigated in [46] could lend itself well to analysis as described here.

These potential application domains are listed compactly under Table 2. So we expect the proposed framework and the provided approximations to help with current and emerging multi-hop optical links with quick performance evaluations and help with different design directions.

X. CONCLUSION

A generalized framework for analyzing multihop IMDD repeater and regenerator optical links is presented with applicability elaborated for fiber optic, FSO, and potentially UWOC links. Theoretical bounds on BER performance for all-repeaters and all-regenerated links are derived and compared. These bounds represent the best-case performance targets and could also serve as a benchmark to compare future developments. The BER performance superiority of regenerator links for noisy hops is established with detailed expla-
nations on noise build-up and BER accumulation. The BER advantage is then shown to translate into extra reach or lower transmission power. Additionally, certain approximations are introduced to reduce computational cost and facilitating analytical tractability. The abstract model is then used to analyze a typical optically amplified fiber-optic link against an OEO-based regenerator link. Different noise sources and amplifier parameters are considered, and the multihop performance is compared. Notably, the regenerator link could scale to very long links with minimal degradation along the link in contrast to repeater links where the per-hop SNR degradation ultimately limits its reach. Further, we prove the same framework can be directly extended to the FSO link (by analyzing and comparing AF and DF FSO links) and potentially to all multihop links, including underwater links. A detailed literature survey on the analytical model evolution of fiber, FSO, and UWOC links are also provided. We conclude by providing a detailed exploration into the applicability of the framework in current and future multihop optical links with relevant literature.

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TABLE 2: Literature signifying framework applicability and relevance

| Application | Reference(s) |
|-------------|--------------|
| IMDD over coherent detection for short reach | [76]–[80] |
| Data Centre Interconnects (DCI) | [81]–[83] |
| Preamplifiers | [84] |
| Optical switches and interconnects | [85]–[88], [113], [114] |
| Splitting loss compensation | [85] |
| Metro DCI using optical amplifiers | [84], [89]–[93] |
| Access network | [94]–[102] |
| Laser Satellite communications | [105]–[109] |
| Preamplifiers | [105], [107] |
| Booster/radial amplifiers | [108], [109] |
| Amplify and Forward multihop | [110] |
| Thriving IMDD Subsystems | [79], [103], [104] |
| ITU-T Recommendation | [102] |
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