FOUR DIMENSIONAL SPHERICAL SPACE-TIMES AND STRING THEORY.

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Abstract

Some shortcomings in regard to our lack of conceptual understanding of string theory are displayed and prescription to untangle them is proposed. String theory should be a fundamental dynamics of four dimensional symmetric space-times. Properties of the two dimensional equivalent action are studied, in the hydrodynamic approximation. In the pressureless regime it is conformal invariant. Correlations of our proposal to ’t Hooft work on quantization of black holes[7] and work on 2D black hole solutions established by Witten[14] are pointed out as perspectives of the present work.


1 Introduction.

During the last century, concepts, theoretical developments and experimental achievements dealing with structure and dynamics of nature at different energy scales have reached a sufficient level of maturity and clarity to have a good sense of orientation on the long time survey toward the unified theory of nature. It is clear that Whatever its formulation, it should satisfy the following conditions:

i) unify all the known basic interactions in a quantum relativistic frame.

ii) permit only, or almost, one structure of matter and its interactions at any scale or epoch of our universe.

iii) to be free of non-naturalness problems, like the fine tuning one.

iv) to be compatible with the actual standard models. In the particle physics it must be close to the standard model of electroweak and strong interactions at scales $\lesssim 100$ GeV$^1$, It also should include the hot standard Big-Bang model and the inflationary paradigm$^2$.

On this setting, it is commonly accepted that actually string theory shows the most likely aspects which get from a fundamental description of nature. Mainly string theory admits Calabi-Yau solutions with gauge structures very close to the standard model one with parameters entirely deduced from topological considerations$^3$, its effective action is close to the usual N=1 supergravity near the Planck scale and the problem of non-renormalisability of quantum gravity is naturally solved.

Theses viable aspects are certainly encouraging results in this direction and undoubtedly the accompanied sophisticated mathematical developments development of M-theory, duality and p-branes solutions in string theory$^4$, the Holographic principle$^5$ the Ads/CFT correspondence$^6$ and the developments of Hawking-’t Hooft proposal on the construction of Back holes quantum mechanics$^7$, are certainly important advances toward the understanding of string theory in a technical and conceptual context and will be very useful to subsequent developments in physics, but analysis of its building blocks and its historical occurrence show clearly that string theory is highly intuitively founded$^7$ and a deep understanding of its foundation is in order.

The aim of this work is to propose an alternative description of string theory which deals with messes in our understanding of the string theory. In the following section we consider building blocks of string theory in order to list such messes and suggest alternative description; string theory should be interpreted as an effective description of four dimensional spherical space-time. In section three we analysis the global aspect of such effective description and obtains the following scheme: String theory with seemingly two dimensional static target space describes in fact dynamic, including quantum effects, in four dimensional symmetric and time-dependant geometry. Section four deals with the explicit construction of the relevant reduced action principle and study its classical properties in particular we show that the action is conformal invariant in the pressureless regime. In the conclusion we discuss perspectives of the present work.
2 Alternative description of string theory.

To have an explicit look over what may be lost in our understanding of the string theory, let’s first consider its building blocks. It consist of the following:

1) A target space, including space-time, evolving upon two dimensional space, the world sheet $\sum g$ with genus $g$ and metric $h_{\alpha\beta}$; according to a generalized sigma model action

$$S = \sum_{g=0}^{\infty} \int d^2\sigma A_i(Z, \partial Z)\Phi_i(Z)$$

$A_i(Z, \partial Z)$ is a complete set of composite of target coordinates and theirs derivations with respect to the world-sheet coordinates $\sigma^\alpha (\alpha = 1, 2)$. $A_i(Z, \partial Z)$ are in one to one correspondence with states of string theory. $\Phi_i(Z)$ are string backgrounds, target sections or 2D fields coupling constants. The essential (renormalisable) bosonic part of (1) is

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left\{ \sqrt{h^{\alpha\beta}} \partial\alpha X^\mu \partial\beta X^\nu G_{\mu\nu} + \varepsilon^{\alpha\beta} \partial\alpha X^\mu \partial\beta X^\nu B_{\mu\nu} \right\} + \alpha' \int d^2\sigma \sqrt{h}\phi(X)R + S_{\text{int}}$$

$\alpha'$ is the inverse string tension, $X^\mu (\mu = 1, \ldots, D)$ are space time coordinates. Backgrounds $G_{\mu\nu}, B_{\mu\nu}$ and $\phi$ are respectively space-time metric, axion field and the dilaton field. $S_{\text{int}}$ is the internal part of (2). It is usually represented by Landau-Ginsburg model or sigma model with internal space coordinates.

2) The partition function $Z(\phi_i)$, which is close the space time effective action $\Gamma(\phi_i)$,

$$Z(\phi_i) = \sum_{g=0}^{\infty} e^{\rho\chi} \int \left[ dh_{\alpha\beta} \right] \left[ dX^i \right] \left[ dZ \right] e^{iS}$$

$e^\rho$ is the string coupling constant, $\rho$ is the constant part of the dilaton field, $\chi$ is the $\sum_g$ Euler characteristic; $\chi(\sum_g) = 2(g + 1)$.

3) $N$-points correlation functions

$$\frac{\delta^N \Gamma(\Phi_1, \ldots, \Phi_N)}{\delta\Phi_1 \ldots \delta\Phi_N} = \left\{ \prod_{i=1}^{N} A_i \right\}$$

$$= \sum_{g=0}^{\infty} e^{(N+1)\rho\chi} \int \left[ dh_{\alpha\beta} \right] \left[ dX^i \right] \left[ dZ \right] \left\{ e^{iS} \prod_{i=1}^{N} A_i \right\}$$

$$= \sum_{g=0}^{\infty} e^{(N+1)\rho\chi} \int \left[ dh_{\alpha\beta} \right] \left[ dX^i \right] \left[ dZ \right] \left\{ e^{iS} \prod_{i=1}^{N} A_i \right\}$$
with background satisfying the quantum equation of motion

$$\frac{\delta \Gamma(\Phi_i)}{\delta \Phi_i} = 0$$  \hspace{1cm} (5)

the effective action $\Gamma(G_{\mu\nu}, \Phi_i = 0)$, $G_{\mu\nu} \neq \Phi_i$, at level $g=0$, or the string classical equation of motion of the metric, can be expanded in terms of sigma model perturbation theory

$$R_{\mu\nu} + \alpha' (R_{\mu\nu\rho\sigma})^2 + \ldots = 0$$  \hspace{1cm} (6)

where dots represent higher perturbative terms constructed from derivatives and higher powers of the curvature tensor.

There are strong hints to suspect that we need an understanding of the basic components (equations 1→6) which we usually use to describe string theory. Let is display what is actually messy with string theory:

1) The degeneracy of string vacua\cite{8}. There are many potentially viable solutions of string theory.

2) The lack of a consistent mechanism for supersymmetric breaking\cite{8}.

3) The cosmological constant problem.\cite{8}.

4) The recent exciting advances in string theory were limited to strings in time independent backgrounds. They shed any light on time dependant dynamic of the universe, essentially on its expansion and its inflationary phase, nor on the existence of singularities in the strong curvature regime. Moreover attempts to study sigma models with time dependant backgrounds shows that they are too singular \cite{9}; our understanding is limited to backgrounds which admit in asymptotic spatial infinity a global timelike Killing vector\cite{10}. This condition rules out interesting backgrounds like those which are important in cosmology\cite{11}. Understanding cosmological solutions in the context of string theory is interesting both from a conceptual and from a pragmatic point of view. We can hope that through cosmology the much desired connection between string theory and experiment can materialize.

5) The perturbative interpretation of Polyakov series (this interpretation is inherent to the string concept) seems to be incompatible with the usual connection between loops corrections and the planck constant $\hbar$. In principle at each topological level of the Polyakov series a two dimensional perturbation development contains trees terms independent of $\hbar$ and loops corrections which vanish at semiclassical limit $\hbar \rightarrow 0$. So in the space-time sense the effective action admits two dimensional trees contributions from higher topological terms, this suggests a new interpretation of the Polyakov series.

6) In the light-cone gauge\cite{3} the world sheet time variable coincides with the space time variable $x^0$. Evolution of spatial coordinates upon the world sheet is given by $x^i = x^i(\sigma, x^0)$, $\sigma$ is the spatial world sheet coordinate. When the evolution of string in space is concerned this dependance seems natural. But if we are interested about locality in the spatial part of the universe the $\sigma$-dependance of $x^i$ incites to consider $\sigma$ as a scale of the universe in equal footing than the cosmic time $x^0$, this indicates that the natural interpretation of
the world sheet is rather a two dimensional parameter space (base space) upon which the universe - or a part of it - and matter evolve.

7) The last point which goes against the string concept is the fact that actually there is no principles which support this concept or exclude higher dimensional extension of the point particle concept.

Recent development of M-theory, duality and p-branes solutions in string theory[4], the Holographic principle[5] the Ads/CFT correspondence[6] and the developments of Hawking-'t Hooft proposal on the construction of Back holes quantum mechanics[7], are certainly important advances toward the understanding of string theory in a technical and conceptual context. But they are not sufficient by themselves to solve the above listed problems.

In our point of view the appropriate frame to look for an outcome to these points is the low dimensional effective description of initially higher dimensional symmetric dynamical systems. For example when we deal with matter evolving on homogenous isotropic four dimensional space time M4, gravitational equations coupled to matter reduce to a dynamical system with one dimensional base space M1. This reflects the existence of differentiable fibration

\[ f_1 : E \rightarrow M_4 \]
\[ f_2 : M_4 \rightarrow M_1 \]  (7)

The initial dynamical system corresponding to \( f_1 \) is described by a \( G_1 \) gauge invariant and general covariant action. We should get an analog action on \( M_1 \) corresponding to \( f_2 \) of \( f_1 \) with \( G = G_1 \times G_{ext} = SO(3) \), \( G_1 \) and \( G_{ext} \) are respectively the \( M_4 \) internal symmetry and the geometric one; this action describes evolution upon \( M_1 \). So on this setting the basic data are \((E,G)\), gauge invariance and general covariance principle. The fundamental law governs G-orbits evolution upon the orbit space \( E_G = M_1 \), \( d = dim E_G = dim E - dim G \), according to a \( G \)-gauge invariant and generally covariant action

\[ S_d = \int_{M_d} \left\{ \sqrt{g} \phi R + L_{mat} + L_{gauge} + L_{top} \right\} \]  (8)

\( \phi \) is a scalar field (the dilaton), \( \int_{M_d} \left\{ L_{mat} + L_{gauge} \right\} \) is the usual matter and gauge field action and \( \int_{M_d} \left\{ L_{top} \right\} \) is the topological term which picks the vacuum.

The partition function corresponding to (8) is given by the polyakov like series

\[ Z(\phi) = \sum_{M-topology} \left\{ \int_{M_d} \left[ dg_{\alpha \beta} \right] \left[ d\Psi \right] \left[ dZ \right] e^{\frac{i}{\hbar} S_d} \right\} \]  (9)

In this way:
i) With d=p+1 we obtain the p-brane systems\[^{[12]}\]. That means ”string” p=1 or p-branes appear as fundamental description of nature needs physical considerations which pick the initial data the total space and the structure group (E,G).

ii) In the d=2 case we get a string like system:

★ An action given by S\(^2\); in the trivial connection case, \(\int_{M_4} \{ L_{\text{gauge}} \} = 0\), coincides with the string action, the target is the G-orbit space. L\(_{\text{top}}\) correspond to the Kalb-Ramond topological invariant term which is usually represented by the Wess-Zumino-Witten term when the G-orbit space is a Lie group\[^{[13]}\].

★ M\(_2\) is part of space time and what may be space-time static solutions in string SEns are 4D space time and time dependant ones. 

★ Z is the quantum gravity partition function; if the quantum effects do not lead to topology change, contribution to Z comes entirely from one term in (9) corresponding to integration over topologically equivalent metrics.

Interests of this scheme are:

i) To describe dynamic in four dimensional time-dependant geometry we need a string theory with two dimensional static target space, this get over the above listed problems.

ii) Developments of string theory show that this dynamic including quantum effects may entirely be deduced from evolution upon the space-time invariant part M\(_2\) with respect to G\(_{\text{ext}}\), in particular when M\(_2\)=S\(^2\).

To develop this scheme, we first need construction of the two dimensional equivalent action of Einstein equation in four dimensional G\(_{\text{ext}}\) isometric space-time and actions describing evolution of G\(_{\text{ext}}\)-orbits and matter upon M\(_2\). Our purpose is to deal with the two first constructions. In the following section we consider reduction in the case G\(_{\text{ext}}\)=\(\frac{SO(3)}{U(1)}\) and the simplest one of RFW space-times. In the fourth section we consider gravitational action in the case G\(_{\text{ext}}\)=\(\frac{SO(3)}{U(1)}\) and its symmetries in the thermodynamic approximation, they include 2D conformal symmetries in the pressureless regime. We conclude by discussing perspectives of the present work.

3 Fundamental evolution on four dimensional space-times.

Four dimensional isotropic and homogeneous space-times admit a six dimensional isometric group G with isotropic subgroup SO(3). This implies that the four dimensional space-time M\(_4\) splits into three dimensional spatial part which evolves upon a one dimensional time-like M\(_1\) part following a trivial fibration

\[ f_1 : M_4 \rightarrow M_1 \]

with group structure \(\frac{G}{SO(3)}\) and M\(_1\) is the corresponding invariant part. In this case metrics are of RFW type

\[ ds^2 = dt^2 + a(t) dl^2 \quad (10) \]
a(t) is the scale of the universe and \( dl \) is the spatial linear element.

It is well known that Einstein equations corresponding to the Hilbert-Einstein action coupled to a scalar field

\[
S = \frac{1}{2k} \int d^4 x \left\{ \sqrt{g} \left( R + k \left[ g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - 2U(\phi) \right] \right) \right\}
\]  

reduce in the RFW space-time to a simple dynamical system

\[
\frac{dH}{dt} = kU(\phi) - 3H^2 = V(\phi, H)
\]

\[
\frac{d\phi}{dt} = \pm \sqrt{-\frac{2}{k} V(\phi, H)}
\]

with solutions corresponding to expanding, collapsing or stationary universe, this depend on the content of matter and the initially conditions. \( H(t) = \frac{da}{dt} \) is the Hubble expansion rate. This system obtains from the action

\[
S = S_{grav} + S_{mat}
\]

\[
S_{grav} = \int_{M_1} dt \left\{ -\frac{3}{k} a \left( \frac{da}{dt} \right)^2 \right\}
\]

\[
S_{mat} = \int_{M_1} dta^3 \left\{ \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - U(\phi) \right\}
\]

To have the complete action we need to add one which governs evolution of \( G_{SO(3)} \) upon \( M_1 \) or the geodesic flow given by

\[
S_{grav}(x,g) = \int_{M_1} dt \sqrt{h} h^{00} \partial_i x^j \partial_i x^j G^{ij}
\]

So evolution in RFW space-times reduces to a quantum mechanical system

\[
S_{RFW} = S_{grav} + S_{mat} + S_{geo}
\]

where the coordinates \( x^i \), the \( M_1 \)-metric \( h \) and the scalar field \( \phi \) are the basic fields, the metric \( g_{ij} \), the potential couplings and the metric of the field space look like section upon \( G_{SO(3)} \) or field depending couplings. The quantum fluctuation about the classical solutions obtains from the partition function

\[
Z = \sum_{M_1-Iop_{M_1}} \int [dh] [dx] [d\phi] e^{-\left( \frac{i\hbar}{\hbar} S_{RFW} \right)}
\]
This situation, namely reduction of Einstein equation coupled to matter in RFW universe to dynamical law governing evolution upon the cosmic time $\tau$, extend to the spherical case. Four dimensional spherical space-time $M$ admit SO(3) isometry group with its isotropic subgroup SO(2) $\equiv U(1)$. This implies existence of a fibration

$$f_2 : M_4 \longrightarrow M_2. \quad (20)$$

with structure group SO(3)/U(1) and the base space $M_2$ is the invariant part of $M_4$ with respect to SO(3). This fibration is trivial; this means in particular there exist coordinates system upon which the $M_4$ metric writes

$$ds^2 = h_{\alpha\beta} d\sigma^\alpha d\sigma^\beta + g_{ij} dx^i dx^j \quad (21)$$

$(h_{\alpha\beta})$ is the metric with the time-like coordinate $\sigma^0 = \tau$ and the spatial one $\sigma^i = \sigma$. $(g_{ij})$ is the SO(3)/U(1) metric with coordinates $(x^2, x^3)$. In the commoving frame and spherical coordinates $(r, \theta, \varphi)$ the metric writes

$$ds^2 = d\tau^2 + -e^\lambda dr^2 - e^{2\mu} (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (22)$$

$\lambda$ and $\mu$ generally depend on $(r, \tau) : \lambda(r, \tau) \text{ et } \mu(r, \tau)$. The corresponding Einstein equations with hydrodynamic energy-momentum tensor reduces to a simple dynamical system on the phase space with the action

$$S_{gr}(\mu, \lambda) = \int_{M_2} d^2\sigma L(\mu, \dot{\mu}, \lambda, \dot{\lambda}) \quad (23)$$

In the following paragraph we derive the explicit form of this action and point out its symmetries.

The evolution of SO(3)/U(1) orbits upon the base space $M_2$ is two dimensional extension of the above geodesic flow. The natural corresponding action is SO(3)/U(1) gauge invariant and $M_2$ generally covariant. Since the fibration is trivial the Yang-Mills term vanishes and we get SO(3)/U(1) WZW model

$$S_{WZW} = \int_{M_2} d^2\sigma \sqrt{h} \alpha^\beta \partial_\alpha x^i \partial_\beta x^j g_{ij} \quad (24)$$

$x^i, g_{ij}$ are respectively the SO(3)/U(1) coordinates and metric. $\sigma^\alpha$ and $(h_{\alpha\beta})$ are their $M_2$ analogue. Contribution of fields $\phi_i$ gets from the action

$$S_{mat} = \sum_i \int_{M_2} d^2\sigma A_i(x) \phi_i(x) \quad (25)$$

$A_i$ are composites of SO(3)/U(1) coordinates and their derivatives with respect to $M_2$ ones. Then the evolution of Einstein-matter system in four dimensional spherical space-time obtains from the action principle

$$S_2 = S_{gr} + S_{WZW} + S_{mat} \quad (26)$$
The quantum fluctuation about its classical solutions get from the partition function

$$Z_2 = \sum_{M_2-\text{top}} \int [dh] [dx] [d\phi] e^{-\frac{i}{\hbar} S_2}$$

(27)

which reflects that there is uncertainties only in geometry and topology of $M_2$ and position in the orbit coordinates.

From the above analysis emerges the following scheme: at fundamental scales the universe is four dimensional and spherical. Law governing evolution of matter and geometry upon $M_2$ is a formal string theory with the following correspondences:

| String Framework | The present Framework |
|------------------|-----------------------|
| World-sheet $\sum g$ | The SO(3) invariant part of $M_2$ with coordinates including the time variable |
| The perturbative Polyakov Series | The partition function of 2D quantum gravity |
| $The$ action represent a static two dimensional space-time sigma model | $The$ action represent fundamental evolution of matter in four dimensional space-time. |

at relatively large scales spherical space-time loses its center, in particular we reach homogeneity and so fall on RFW universe. This scheme gives an outcome to the above inadequacies, concerning time dependent processes in string theory and the interpretation of its building blocks. It also constitutes a cosmological scenario where we can evaluate quantum effects in particular when $M_2 = S^2$, the tree level in the string sense. To carry out this scenario we need first to determine the action $S_{grav}$.

### 4 Action principle of four dimensional spherical space-time.

In this paragraph we construct the two dimensional action $S_{grav}(\lambda, \mu)$ equivalent to Einstein equations coupled to matter

$$R_{\mu\nu} - \frac{1}{2} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(28)

in four dimensional spherical space-times in the hydrodynamic case

$$T_{\mu\nu} = (p + \varepsilon) v_\mu v_\nu + pg_{\mu\nu}$$

with $p = k\varepsilon$

(29)

and we study its symmetries.
In four dimensional spherical space-times the non vanishing components of the energy-momentum tensor $T_{\mu\nu}$ are $T_{00}, T_{11}, T_{22}, T_{33}$ and $T_{10}$ and the space-time metric writes

$$ds^2 = d\tau^2 + e^{-\lambda}dr^2 - e^{2\mu}(d\theta^2 + \sin^2\theta d\varphi^2)$$ (30)

$T_{\mu\nu}, \nu, \lambda$ and $\mu$ are $(r, \tau)$ functions.

The Einstein equations are given by

$$\frac{1}{4} e^{-\lambda} \left( \frac{\mu' + \mu'\nu'}{2} - e^{-\nu} \left( \frac{\tilde{p}}{2} + \frac{3\hat{u}}{4} \tilde{\mu}^2 \right) - e^{-\mu} = \frac{8\pi G}{C^4} T_1 \right)$$ (31)

$$\frac{1}{4} e^{-\nu} \left( 2\nu' + \nu'^2 + 2u'' + u'^2 - u'\lambda' - \nu'\nu' + u'u' \right)$$

$$+ \frac{1}{4} e^{-\nu} \left( \dot{\nu}\dot{\lambda} + \dot{\nu}\tilde{\mu} - \tilde{\mu}\dot{\lambda} - 2\dot{\lambda} - \lambda^2 - 2\dot{u} - \dot{u}^2 \right) = \frac{8\pi G}{C^4} T_2$$ (32)

$$- e^{-\lambda} \left( u'' + \frac{3}{4} u'^2 - \frac{u'(\lambda')}{4} \right) + \frac{1}{2} e^{-\nu} + \frac{1}{2} \hat{u}^2 + e^{-\mu} = - \frac{8\pi G}{C^4} T_0$$ (33)

Moreover there is covariant conservation equations

$$\nabla^\mu T_{\mu\nu} = 0$$ (34)

which implies additional dependance of $\mu, \nu, \lambda$ and the fields of which depend $T_{\mu\nu}$.

In the hydrodynamic case conservation equations writes

$$\nu' = \frac{2p'}{p + \epsilon}$$ (35)

$$(2\mu' + \lambda') = - \frac{2\dot{\epsilon}}{p + \epsilon}$$ (36)

des these equations and the constraint $p = k\varepsilon$ implies additional dependences of variables $(\mu, \nu, \lambda, \epsilon, p)$. We show that equations $(32-34)$ are equivalent to equations of motion corresponding to the action

$$S_{\text{grav}} = \int_{M_2} L(\lambda, \mu, \mu', \lambda', \nu') d\tau dr$$ (37)

$$L = L_1 + L_2 + L_3$$ (38)

$$L_1 = \left\{ \frac{u'^2}{2} + ku'(\lambda' + 2u') \right\} \exp \left[ -\frac{k+1}{2} \lambda + (k+1)u \right]$$ (39)

$$L_2 = \left\{ \dot{u}^2 + \tilde{\mu}\dot{\lambda} \right\} \exp \left[ -\frac{k-1}{2} \lambda + (1-k)u \right]$$ (40)

$$L_3 = 2 \exp \left[ \frac{k+1}{2} \lambda + ku \right]$$ (41)
with the constraints

\[
\begin{align*}
\tilde{T}_0^1 &= \dot{\lambda} \frac{\partial L}{\partial \lambda'} + \dot{\mu} \frac{\partial L}{\partial \mu'} = \frac{1}{2} e^{-\lambda} \left\{ 2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - (k\lambda' + 2k\mu') \dot{\mu} \right\} = 0 \quad (42) \\
\tilde{T}_1^0 &= \lambda' \frac{\partial L}{\partial \lambda} + \mu' \frac{\partial L}{\partial \mu} = 0 \quad (43)
\end{align*}
\]

\( T_{\alpha\beta} \) is the energy-momentum tensor of \( S_{grav} \) (38).

In term of the horizon "a", equation of motion take a simple form

\[
\begin{align*}
\dot{a} &= -\frac{8\pi G}{c^4} pr^2 \dot{r} \quad (44) \\
\dot{a} &= -\frac{8\pi G}{c^4} pr^2 r' \quad (45) \\
1 - a &= \dot{r} p \frac{k}{r^4} - r^4 \epsilon \frac{k}{r^4} \quad (46)
\end{align*}
\]

(47) extend the expression of the horizon to non-static case, \( r = e^{2\mu} \) is the radius of the spherical system.

In this form, it easy to verify that equations (45 – 47) are invariant with respect to the transformations

\[
\begin{align*}
\tau &\rightarrow \alpha \tau \quad (47) \\
r &\rightarrow \beta r \quad (48)
\end{align*}
\]

under which the energy density \( \epsilon \), the pressure \( p \), the horizon "a" and the radius \( r \) transform like

\[
\begin{align*}
p &\rightarrow \gamma p \quad \epsilon \rightarrow \gamma \epsilon \quad (49) \\
r &\rightarrow \lambda r \quad a \rightarrow \lambda a \quad (50)
\end{align*}
\]

Parameters \( (\alpha, \beta, \gamma, \lambda) \) verify

\[
\begin{align*}
\alpha^2 &= \lambda^{-\frac{\beta}{k} - 2} \beta^2 \quad (51) \\
\gamma &= \lambda^{-2} \quad (52)
\end{align*}
\]

This implies in the particular case when the pressure vanishes, \( K = 0 \), Einstein equations in four dimensional spherical space-times admits a 2d conformal symmetry.
5 Discussion and conclusion.

An alternative description of string theory which deals with insufficiency in our understanding of the string theory is proposed. String theory with seemingly two dimensional static target space describes in fact dynamic, including quantum effects, in four dimensional symmetric and time-dependent geometry. This permits to deal with the problem of time dependent solutions of string theory and the inadequate actual interpretation of its building blocks. The corresponding principle action $S_{grav} + S_{WZW}$ are studied, the action is conformal invariant in the pressureless regime.

The present work should be confronted to works relatives to 't Hooft proposal on quantization of Back Holes, which 'reconstruct' string paradigm from quantization of black holes in symmetrical four dimensional space-times and to the Witten construction of a two dimensional black hole solution \cite{14}, the corresponding action consist of the $\frac{SL(2R)}{U(1)} \equiv SU(1,1)$ WZW model and additional dilaton coupling which we get from the perturbative quantum correction. In the first order this coupling writes

$$\int_{M_2} \phi(r) R^{(2)} \sqrt{h}$$

$\phi(r)$ is the dilaton field, some function of $SU(1,1)$ coordinates, and $R^{(2)}$ is the curvature of the world-sheet metric of $M_2$. Moreover this model admits $W_\infty$ symmetries\cite{15}, there are similarities that shares this model with $S_{grav} + S_{WZW}$. The $\frac{SO(3)}{U(1)}$ WZW model up a $Z_2$ factor, looks like an Euclidean continuation of $\frac{SL(2R)}{U(1)}$ WZW model. $S_{grav}$ is comparable to the dilaton coupling with $r = e^{2u}$, the world-sheet $M_2$ coincides with the invariant part of the four dimensional spherical space-time with respect to $SO(3)$. Moreover $S_{grav} + S_{WZW}$ should admit the above $W_\infty$ (this is the case for the $\frac{SO(3)}{U(1)}$ WZW model). Equations of motion corresponding to $S_{grav}$ constitute a quasi-homogeneous system (similitude) with Weight depending on the ratio $k$ of pressure to the energy density. It is tempting to show that similitude admits $W_\infty$ symmetries.

Following the above work of Witten, identification of $S_{grav}$ to dilaton coupling is likely to carry out the induced gravity idea in four dimensional space-times\cite{16}. It is known that this coupling is induced by perturbative quantum corrections. This implies that fundamental evolution in four dimensional spherical space-times is given at the classical level by the $\frac{SO(3)}{U(1)}$ WZW model. The Hilbert-Einstein action coupled to matter, which is equivalent to $S_{grav}$, get in fact from quantum corrections.
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