Bounds on hep neutrinos

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Abstract

The excess of highest energy solar-neutrino events recently observed by Superkamiokande can be in principle explained by anomalously high hep-neutrino flux $\Phi_{hep}$. Without using SSM calculations, from the solar luminosity constraint we derive that $\Phi_{hep}/S_{13}$ cannot exceed the SSM estimate by more than a factor three. If one makes the additional hypothesis that hep-neutrino production occurs where the $^3$He concentration is at equilibrium, helioseismology gives an upper bound which is (less then) two times the SSM prediction. We argue that the anomalous hep-neutrino flux of order of that observed by Superkamiokande cannot be explained by astrophysics, but rather by a large production cross-section.

I. INTRODUCTION

In the recent observations of Superkamiokande some excess of high energy solar-neutrino events was detected. This excess is difficult to interpret as distortion of Boron neutrino spectrum due to neutrino oscillations. It might indicate that the hep neutrino flux, $\Phi_{hep}$, is significantly larger (by a factor $\sim$ 30) than the SSM prediction $\Phi_{hep}^{SSM}$.

Apart from $S_{13}$, the zero-energy astrophysical S-factor of the $p + ^3He \rightarrow ^4He + e^+ + \nu$ cross-section, the prediction of the hep neutrino flux in the SSM is rather robust. Bahcall and Krastev estimate this flux as:

$$\Phi_{hep} = 2.1(1 + 0.03) \left( \frac{S_{13}}{S_{13,SSM}} \right) \cdot 10^3 cm^{-2} s^{-1}$$

We remark that $S_{13}$ is not reliably calculated. In the SSM the value $S_{13}^{SSM} = 2.3 \cdot 10^{-20}$ keV b is used following the most recent calculations by Schiavilla et al, though due to...
complexity of the calculations (see Carlson et al. [3], Schiavilla et al. [4]) the uncertainties are rather large: $0.5 < S_{13}/S_{13}^{SSM} < 1.5$, according to Ref. [5]. In a short review of the calculations [3], the authors conclude that from the first-principle physics it is difficult to exclude that the cross-section is an order of magnitude larger.

The 3% error in Eq. (1) accounts for the estimated uncertainties in the solar age, chemical composition, luminosity, radiative opacity, diffusion rate and in all nuclear quantities, except $S_{13}$. This small error follows from the fact that $\Phi_\nu(hep)$ depends rather weakly on all astrophysical variables such as temperature $T$, density $\rho$ and the chemical composition. Besides, all these quantities, except $^3$He abundance, are smooth functions of the radial distance $r$ in the solar region where most of hep-neutrinos are produced, $0.1 < r/R_\odot < 0.2$. It is hard to conceive that SSM’s uncertainties in $T$, $\rho$ and $X$ can result in a considerable change of $\Phi_\nu(hep)$.

The only exception is $^3$He abundance, which radial behaviour is not that smooth. In fact it increases by an order of magnitude when moving from $r = 0.1R_\odot$ to $r = 0.2R_\odot$, so that $\Phi_\nu(hep)$ is sensitive to the $^3$He distribution, see Figs.4.2 and 6.1 of [4]. This abundance is not limited by helioseismic data and in non-standard models it can be, in principle, high in the hep-neutrinos production zone.

We have thus analyzed the astrophysical uncertainties in the flux of hep neutrinos in an approach beyond the SSM. In Section II we derive an upper limit for $\Phi_\nu(hep)/S_{13}$ directly from the solar-luminosity constraint. In Section III we impose the more restrictive assumption of local $^3$He equilibrium in the hep-neutrino production zone and use the helioseismic constraints.

**II. THE SOLAR-LUMINOSITY CONSTRAINT**

The production rate $Q_\nu(hep)$ for the hep neutrinos and the solar-luminosity constraint can be written down as follows

$$Q_\nu(hep) = \int dr 4\pi r^2 n_1(r)n_2(r)\lambda_{13}(T(r)),$$

$$\frac{1}{2}\Delta_1 \int dr 4\pi r^2 n_1(r)^2\lambda_{11}(T(r)) + \frac{1}{2}\Delta_2 \int dr 4\pi r^2 n_3(r)^2\lambda_{33}(T(r)) \leq L_\odot$$

where $\lambda_{ij}$ are energy-averaged reaction rates between nucleus $i$ and $j$,

$$\lambda_{ij}(T) = \int dE dE' f(E,T)f(E',T)(\sigma v)_{ij},$$

$f(E,T)$ is the normalized Maxwell distribution function, $n_i$ is the number density of nuclei with atomic mass number $i$, $(\sigma v)_{ij}$ is the reaction rate between nuclei $i$ and $j$, and the two $\Delta$ correspond respectively to the two values of the heat release: when a $^3$He nucleus is produced ($3p + e^- \rightarrow ^3 He + \nu$), and when two $^3$He are merged ($^3He + ^3He \rightarrow ^4 He + 2p$),

$$\Delta_1 = 3m_p + m_e - m_{^3He} - < E_\nu >_{pp} = 6.7 \text{ MeV}$$

$$\Delta_3 = 2m_{^3He} - m_{^4He} - 2m_p = 12.9 \text{ MeV}$$

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The temperature dependence of the reaction rates (4) can be parametrized as:

$$\lambda_{ij}(T) = \lambda_{ij} \left( \frac{T}{T_0} \right)^{\alpha_{ij}}$$

(6)

where generally speaking $T_0$ is an arbitrary temperature scale. We shall fix $T_0$ at the position of maximum of nuclear-energy production in the SSM ($T_0 = 1.336 \cdot 10^7 K$), which guarantees that that expansion (6) is related to a narrow temperature range. The values of $\lambda_{ij}$ and $\alpha_{ij}$ (this latter rounded to the nearest integer) are given in Table I. They are calculated using the values of astrophysical $S$-factors given in Ref. [7] and for $T_0 = 1.336 \cdot 10^7 K$. Note that uncertainties in $\lambda_{ij}$ depend only on cross-sections; in particular $\lambda_{ij} \propto S_{ij}$, where $S_{ij}$ are astrophysical factors.

From Eq.(3), by using the inequality $\frac{1}{2}(f_1^2 + f_2^2) \geq f_1 f_2$ and the parametrization (6) one obtains

$$\int dr 4\pi r^2 n_1(r)n_3(r)\lambda_{13}(T(r)) \left( \frac{T(r)}{T_0} \right)^{\alpha} \leq \frac{\lambda_{13}}{\sqrt{\lambda_{11}\lambda_{33}}\sqrt{\Delta_1\Delta_2}} L_\odot,$$

(7)

where $\alpha = (\alpha_{11} + \alpha_{33})/2 - \alpha_{13} \approx 2$

One can see that the integrand in the lhs of Eq.(7) is different from that of Eq.(2) only by a factor $(T(r)/T_0)^2$. Inequality (7) further strengthens if this factor is taken as $(T_{min}/T_0)^2$, where $T_{min}$ is the minimum temperature in the hep-production zone. In the SSM at the temperature $T_{min} = 7 \cdot 10^6 K$ (corresponding to $r/R_\odot = 0.3$) the probability for a proton to undergo a nuclear reaction during the solar age is as small as 0.1%. Then from Eq.(7) one obtains

$$\Phi_\nu(hep) < K_\odot \frac{\lambda_{13}}{\sqrt{\Delta_1\Delta_3}\sqrt{\lambda_{11}\lambda_{33}}} \left( \frac{T_0}{T_{min}} \right)^2$$

(8)

where $K_\odot = L_\odot/(4\pi D^2)$ and $D$ is the distance between Sun and Earth.

Note that the weak inequality (7) has turned into a stronger inequality (8) due to substitution $T(r) \to T_{min}$ in lhs of Eq.(7), while actually one should use $T(r) \to \langle T \rangle$.

Using the values from Table 1, one obtains numerically

$$\Phi_\nu(hep) < 6.5 \left( \frac{S_{13}}{S_{SSM}^{13}} \right) \cdot 10^3 \text{ cm}^{-2} \text{ s}^{-1},$$

(9)

a factor three larger than in the SSM calculations, Eq.(1).

### III. LOCAL $^3$HE EQUILIBRIUM AND HELIOSEISMOLOGY

A more restrictive upper bound can be obtained from helioseismic constraints, with an additional assumption that hep neutrinos are produced in a region where the $^3$He concentration is at local equilibrium. This assumption, which is valid for a wide class of stellar models, implies:

$$\frac{1}{2}n_1^2\lambda_{11}T(r) = n_3^2\lambda_{33}T(r).$$

(10)
Putting $n_3(r)$ from this equation into (2) and using $n_1(r) = X(r)\rho(r)/m_p$ one obtains

$$\Phi_\nu(hep) = \frac{\lambda_{13}}{4\pi D^2} \sqrt[4]{\frac{\lambda_{11}}{2\lambda_{33}}} \frac{1}{m_p^2} \sqrt{dr 4\pi r^2 \rho(r)^2 [X(r)T(r)/T_0]^2}.$$ \hfill (11)

In the energy production zone, the equation of state (EOS) for the solar interior can be approximated, with an accuracy better than 1% , by the EOS of a fully ionized classical perfect gas:

$$P(r) = \rho(r)T(r)(k_B/m_p)[2X(r) + \frac{3}{4} Y(r) + \frac{1}{2} Z(r)],$$ \hfill (12)

where $P$ denotes the pressure and $k_B$ is the Boltzmann constant. Using Eqs. (11) and (12) one obtains

$$\Phi_\nu(hep) \leq \frac{1}{4\pi D^2} \frac{1}{4(kT_0)^2} \frac{\lambda_{13}}{2\lambda_{33}} \sqrt{dr 4\pi r^2 P^2(r)}. \hfill (13)$$

It is known \cite{8} that inverting helioseismic data one can derive the (isothermal) sound speed squared, $u = P/\rho$ and $\rho$ with an accuracy of 1% or better for all $r/R_\odot$ of interest. This implies that also pressure $P$ is known with a comparable accuracy. Since SSMs are in agreement with helioseismology, one can use the SSM-calculated pressure $P(r)$ to evaluate the integral in Eq.(13). It gives

$$\Phi_\nu(hep) < 3.5 \left( \frac{S_{13}}{S_{13}^{SSM}} \right) \cdot 10^3 \text{ cm}^{-2} \text{ s}^{-1}. \hfill (14)$$

This upper bound is (less then) two times the SSM prediction. In fact the agreement is even better. Neglecting $Z$ in Eq.(12) and using $Y \approx 1 - X$ one obtains from Eq.(12)

$$X(r)T(r) = \frac{4m_pP(r)}{5k_B\rho(r)} - \frac{3}{5} T(r) \hfill (15)$$

The first term on rhs of Eq.(15) is determined by helioseismic measurements and thus can be taken as in the SSM. Temperature profile $T(r)$ cannot differ from that of the SSM more than by 2 – 3%. Then $[X(r)T(r)]^2$, the only unknown function in the integral (11), can differ from the SSM value by a few percent only and so does $\Phi_\nu(hep)$.

**IV. CONCLUSIONS**

We have derived an upper limit on $\Phi_\nu(hep)/S_{13}$ directly from the solar-luminosity constraint. It is only three times the SSM prediction. If one additionally assumes that hep neutrino production occurs in the region where the $^3$He concentration is at local equilibrium, helioseismology provides a formal upper bound (less than) two times the SSM prediction. More realistically, in this case $\Phi_\nu(hep)/S_{13}$ can be only a few percent higher than in the
SSM. Our limits can be violated only in very exotic models of non-stationary sun with non-stationary transport of $^3$He in the inner core from outside. This transport should not be accompanied by any noticeable transport of other elements, such as $^1$H or $^4$He, otherwise the seismically observed sound speed in the inner core will be affected. We doubt that such models can be constructed.

In principle, the $hep$-neutrinos can be resolved in the high precision experiments. We argue that the anomalous $hep$-neutrino flux of order of that observed by Superkamiokande cannot be explained by astrophysics, but rather by a large production cross-section.

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REFERENCES

[1] SuperKamiokande Coll., Y. Suzuki, in : Neutrino 98, Proceedings of the XVIII International Conference on Neutrino Physics and Astrophysics, Takayama, Japan, 4-9 June 1998, Y. Suzuki and Y. Totsuka eds. To be published in Nucl. Phys. B (Proc. Suppl.).

[2] J.N.Bahcall, P.I.Krastev and A.Yu.Smirnov, hep-ph 9807216

[3] J.N. Bahcall and P.I. Krastev, hep-ph/9807525 (1998).

[4] J.N. Bahcall, Neutrino Astrophysics, Cambridge University Press, Cambridge, 1989.

[5] R. Schiavilla, R.B. Wirings, V.R. Pandharipande, J. Carlson, Phys. ReV. C 45 (1992) 2628.

[6] J. Carlson, D.O. Riska, R. Schiavilla and R.B. Wiringa, Phys. ReV. C 44 (1991) 619.

[7] E.G. Adelberger et al., astro-ph/9805121, to appear on Rev. Mod. Phys. (1998).

[8] W. A. Dziembowski, P. R Goode, A. A. Pamyatnykh and R. Sienkiewicz Ap. J. 432 (1994) 417.
TABLES

TABLE I. Parameters of the reaction rates (6)

| i,j | $\lambda_{ij}$ [cm$^3$s$^{-1}$] | $\alpha_{ij}$ |
|-----|-------------------------------|---------------|
| 1,1 | $8.34 \times 10^{-44}$         | 4             |
| 1,3 | $4.31 \times 10^{-47}$         | 8             |
| 3,3 | $5.88 \times 10^{-35}$         | 16            |