On excitation of Görtler vortices due to scattering of free-stream vortices on surface non-uniformities

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Abstract.
The study is devoted to experimental investigation of processes of excitation of non-stationary Görtler vortices in a boundary layer on a concave wall by vortices convected in free stream. In the present set of experiments, performed at controlled disturbance conditions, several cases of possible receptivity mechanisms have been considered: (i) scattering of 2D freestream vortices on localized 3D surface non-uniformities, (ii) scattering of 3D freestream vortices on localized 2D surface non-uniformities, (iii) scattering of 2D freestream vortices on uncontrolled 3D base-flow non-uniformities and (iv) scattering of 3D freestream vortices on natural 2D base-flow non-uniformities. Cases (i) and (ii) represent localized types of the vortex-receptivity mechanisms, while cases (iii) and (iv) correspond to distributed type of vortex-receptivity mechanisms. Accurate measurements are carried out in a broad range of experimental parameters. It is found that in cases (i), (ii), and (iii) no generation of measurable Görtler vortices is observed. However, the interaction of 3D convected vorticity with the growing boundary layer (case (iv)) does lead to excitation of rather intensive Görtler vortices. In this case the distributed receptivity mechanism is able to change significantly the growth rates of the exciting unsteady Görtler vortices. The corresponding receptivity coefficients are evaluated.

1. Introduction

The problem of investigation of mechanisms of excitation of steady and unsteady Görtler vortices in boundary-layer flows over curved surfaces is of great practical importance. This problem has many aspects and was not studied quantitatively in experiments till recently due to several circumstances. In particular, problems with agreement between linear-stability characteristics observed in experiments and in linear stability theories has been existed for a long time. Only recently such agreement has been achieved for both stationary and non-stationary Görtler vortices (Boiko et al., 2010).

New experimental approaches developed past years for investigations of linear receptivity mechanism (by Kachanov (2000), Gaponenko et al. (2002) and in other studies) and Görtler-instability mechanisms (Boiko et al., 2010) gave the possibility to start quantitative experimental studies of mechanisms of excitation of unsteady (in general) Görtler vortices by various external perturbations.

Previous experiments in this series performed by Ivanov et al. (2010a) were devoted to the mechanism of excitation of unsteady (in general) Görtler vortices by surface non-uniformities (vibrations, in general). As the result, values of the corresponding receptivity coefficients have been obtained for the first time for this particular receptivity problem.
The present study is devoted to investigation of another receptivity problem, associated with excitation of unsteady Görtler vortices by freestream vortices in presence of surface and/or base-flow non-uniformities.

2. Description of performed experiments

2.1. Experimental model and disturbance sources

The experiments were performed in the low-turbulence wind-tunnel T-324 of the ITAM SB RAS at flow velocity of $U_e = 9.18$ m/s.

The Görtler flow under investigation was realized on an experimental model with concave wall (2) having radius of curvature of $R = 8.37$ m (figure 1). The leading edge of the plate is cylindrical with radius of 1.5 mm. An adjustable wall bump (5) was mounted above the plate in order to eliminate streamwise pressure gradient. Thus, the boundary layer under investigation was Blasius-like but in presence of the wall curvature. The desirable curvature of the plate and its rigidity were provided by a duralumin frame (6). All main measurements in the flow were performed with a hot-wire probe (8) mounted on a traverse (9).

Figure 1. Sketch of experimental model. 1 - wind-tunnel wall, 2 - concave surface of experimental model, 3 - freestream vortex generator, 4 - generator of surface non-uniformities, 5 - wall bump, 6 - rigid frame of ribs of a fixed radius of curvature, 7 - backup abutments, 8 - hot-wire probe, 9 - traversing mechanism

In the present experiments the main measurements were performed in the range of $x = 312$ to 1050 mm (where $x$ is the streamwise coordinate measured along the curved surface), which corresponds to a range of local Görtler number $G^* = \left(\frac{U_e \delta^*}{\nu}\right) \sqrt{\delta^*/R} = 8.23$ to 20.81 (here $\delta^*$ is the boundary layer displacement thickness and $\nu$ is the kinematic air viscosity).

Two-dimensional vortical disturbances of the freestream were generated with the help of a vibrating wire (3) of either 50 or 200 microns in diameter, mounted parallel to the model leading edge at a distance of 138 mm upstream. The wire was forced to oscillate by two miniature stepping motors mounted on the wind-tunnel walls. The harmonic wire oscillations occurred in the plane normal to the flow direction leaded to excitation of a freestream vortex street. The wall-normal positioning of the wire was adjusted in a way that the vortex street convected along the external boundary-layer edge just above it.

The streamwise elongated (3D) freestream vortices were produced by a spanwise localized non-uniformity (a swelling) made of a lacquer planted on the 50-micron wire. The non-uniformity had about 100 microns in diameter and 5 mm width in span (figure 2). Three-component hot-wire measurements have proved that in the flow motions produced in the wake behind such
non-uniformity had rather intensive streamwise vorticity component (aside the non-uniformity
the vortex street, of course, remained two-dimensional).

![Figure 2. Sketch of excitation of streamwise vortices by vibrating-wire non-uniformity](image)

The disturbance source simulated 3D surface non-uniformities (4 in figure 1) represented
a spanwise row of circular vibrating membranes flush mounted with the model surface (3 in
figure 3). The membranes were made of latex film about 80 microns thick. They were forced to
oscillate under effect of pressure fluctuations produced by closed loudspeakers. The speakers were
positioned outside the wind-tunnel test section, while pressure fluctuations were transmitted to
volumes located under membranes with the help of plastic pipes (Ivanov et al., 2010c). Every
two neighboring membranes vibrated in antiphase, so the modeled surface non-uniformity was
essentially three-dimensional and periodic in span. In the present experiments with 3D non-
uniformities the spanwise period \( \lambda_z \) was equal to 8 mm (see figure 3). The source simulating
two-dimensional surface non-uniformity was also driven pneumatically in a similar way but had
single elongated membrane with dimensions of 9 mm in the streamwise direction and 160 mm
in span.

The two described disturbance sources applied in the present experiments were controlled
with the help of electric signals produced by a special generator. The same generator triggered
also the beginning of the data acquisition process, so all collected data were phase locked, that
allowed performing their ensemble averaging in order to increase accuracy of the measurements.

3. Regimes of measurements

3.1. Regimes associated with vortex-surface interactions
In the present study the receptivity mechanisms of interaction of unsteady freestream vortices
(of frequency \( f_v \)) with unsteady surface non-uniformities (oscillating at frequency \( f_s \)) localized
in the streamwise direction were examined in two particular cases: (i) scattering of two-
dimensional vortices (having basically spanwise vorticity fluctuations) on three-dimensional
surface non-uniformities, and (ii) scattering of three-dimensional vortices (having basically
streamwise vorticity) on two-dimensional surface non-uniformities. It was expected that due
to such interactions the generation of unsteady (in general) Görtler vortices at combination
frequencies \( f = f_v \pm f_s \) could occur in the boundary layer. The frequencies of the freestream
vortices \( f_v \) and surface oscillations \( f_s \) were selected in experiments in a way that the boundary
layer could be unstable to the Görtler vortices having one of the combination frequencies (either
There were several cases of excitation of Görtler vortices. In the case of a rather strong excitation of Görtler vortices, we investigated frequencies \( f_0 = 15 \) and \( 26 \) Hz and \( f_s \) between 2 and 50 Hz in amplitudes of vortical perturbations (measured at the boundary layer edge) about 1\% and amplitudes of the surface oscillations up to 0.7 mm (i.e. up to \( A_s / \delta^* = 0.64 \)).

3.2. Regimes associated with scattering of freestream vortices on base-flow non-uniformities

The experiments on the distributed receptivity mechanisms, associated with scattering of freestream vortices on the natural base-flow non-uniformities, were carried out for both 2D (with predominantly spanwise vorticity) and 3D (with predominantly streamwise vorticity) freestream vortices in the groups of regimes called above (i), (ii), (iii), and (iv), respectively. The case (iii) (2D vortices) turned out to be trivial, as expected, in a sense that no excitation of any Görtler vortices was detected in the boundary layer despite, of course, some weak uncontrolled spanwise non-uniformities of the base flow were present; only excitation of 2D Tollmien-Schlichting waves was found. Meanwhile, in the case (iv) (3D freestream vortices) a rather strong excitation of Görtler vortices was observed. This case was investigated in detail at three freestream vortex frequencies \( f_0 = 15, 20, \) and \( 26 \) Hz (which correspond to the non-dimensional frequency parameters \( F = 17.04, 22.72, \) and \( 29.54 (F = 2\pi f_0 / U_e \cdot 10^9) \)). In the studied ranges of local Görtler numbers \((G^* = 8.23 \) to \( 20.81) \) and dimensionless spanwise wavelengths \( \lambda = (U_e \lambda_s / \nu) \sqrt{\lambda_s / R} = 149 \) to \( 774 \) (with dimensional spanwise wavelengths \( \lambda_s = 8 \) to \( 24 \) mm), the excited unsteady Görtler vortices were either amplified (by the base-flow linear-instability mechanism) or neutral, or attenuated depending on the particular combination of the governing parameters.

4. Excitation and evolution of boundary-layer disturbances

4.1. Weakness of vortex/surface receptivity mechanisms

During the data acquisition in groups of regimes (i) and (ii) the length of realization was typically 30 seconds or longer. The measurements with such long realizations allowed performing ensemble averaging of the studied periodical signals for several hundreds of fundamental periods. At the present experimental conditions of measurements in the T-324 wind-tunnel, this allowed us to measure unsteady boundary layer disturbances with as low amplitudes as \( 10^{-3}\% \) and even lower (depending on frequency). However, no any Görtler vortices were detected in the boundary layer at combination frequencies \( f = f_0 \pm f_s \) in experiments (i) and (ii) indicated above, when the scattering of freestream vortices on surface non-uniformities was studied. It was absolutely unexpected that the excitation of Görtler vortices was not observed even at extremely huge amplitudes of 2D surface vibrations with \( A_s / \delta^* = 64\% \). The only detected boundary-layer perturbations represented, as was shown, the Tollmien-Schlichting instability waves. The most efficient excitation of them was detected at high frequencies of the combination modes (several dozens of Hz, usually), although at low frequencies some attenuating Tollmien-Schlichting waves were also found.

Thus, the excitation of Görtler vortices by means of the receptivity mechanisms associated with scattering of freestream vortices on localized surface non-uniformities is found to be extremely weak. The same turned out to be true for scattering of 2D freestream vortices on uncontrolled 3D non-uniformities of the base-flow itself (the case (iii) indicated above).

Nevertheless, the measurements have shown that the three-dimensional freestream vortices are able to interact rather efficiently with the natural 2D spatial non-uniformity of the boundary layer (case (iv), indicated above). This case is discussed in detail below.
4.2. Properties and evolution of distributedly excited perturbations

The boundary-layer disturbances, excited by the mechanism (iv) were localized in the spanwise direction in the region of existence of the streamwise, three-dimensional controlled vortices (where the non-uniformity on the vibrating wire shown in figure 2 was located).

Figure 4. Amplitude and phase distributions of boundary-layer disturbances. (a) - typical spanwise distribution measured at \( U/U_e = 0.6 \). (b) - wall-normal profile measured at spanwise location I indicated in plot (a). (c) - wall-normal profile measured at location II indicated in plot (a). (d) - magnified near-wall region of plot (c). Thick solid and dashed lines - LST-calculations, performed for the most rapidly growing Görtler vortices (\( \lambda_z = 10 \text{ mm} \)). \( x = 900 \text{ mm} \) (\( G^* = 18.67 \)), \( f = 20 \text{ Hz} \) (\( F = 22.72 \)).

Typical spanwise distributions of disturbance amplitudes and phases measured inside the boundary layer are shown in figure 4a. The measurements are carried out inside the boundary layer at wall-normal distance corresponding to amplitude maximum of unsteady Görtler vortices observed at \( U/U_e = 0.6 \) (see e.g. study of Boiko et al. (2010)). One can see that the measured disturbances represent a spanwise localized wave-train. Its amplitude maximum in the spanwise distribution corresponds to location of the controlled three-dimensional freestream vortex produced by the vibrating wire non-uniformity.

Figure 4b displays the wall-normal amplitude and phase profiles of boundary-layer disturbances measured far from location of the vibrating-wire non-uniformity, i.e. aside the streamwise-elongated freestream vortex. This spanwise position is indicated by line I in figure 4a. Vertical dashed-dotted line displays an approximate position of the boundary layer edge. In absence of the non-uniformity the vibrating wire excites in the free stream a purely 2D antisymmetric vortex street. This freestream disturbance does not lead to excitation of disturbances inside the boundary layer (figure 4b) - the amplitudes of perturbations around the spanwise position I are very low near the wall and decay monotonously towards the wall.
A rather different situation can be seen at the spanwise location of existence of the 3D (streamwise, basically) freestream vortices (figure 4c), indicated in figure 4a as section II. An excitation of some large-amplitude boundary-layer disturbances is observed here leading to appearance of a new maximum in the amplitude profile associated with the Görtler vortices distributedly excited by streamwise oriented freestream vortices.

Shown in figure 4d is a magnified near-wall part of the amplitude and phase profiles presented in figure 4c. In addition to the experimental points, the eigenfunctions of unsteady Görtler vortices, calculated by the local parallel linear stability theory (LST) by A.B. Boiko (Boiko et al., 2010), are plotted here with bold lines. The calculations were carried out for the most amplified vortices with $\lambda_z = 10$ mm ($\Lambda = 209$). The locations of the near-wall amplitude maxima in the experimental and theoretical profiles agree very well with each other. The experimentally measured disturbance phases are also in a good agreement with the LST-calculations in the near-wall region. Subsequent observations have supported the idea that the disturbances measured in the boundary layer do correspond to the unsteady Görtler vortices excited due to the mechanisms of the distributed vortex receptivity. This statement is supported, in particular, by the results presented in figure 5, where a typical Fourier spectrum of measured boundary-layer disturbances, measured at rather large streamwise coordinate, is shown versus the spanwise wavelength. It is seen that the greatest spectral amplitudes $B^d$ correspond to the perturbations with spanwise
scale $\lambda_z = 9$ to 12 mm ($\Lambda = 178$ to 345). According to the linear stability theory, the Görtler vortices of this wavelength range are the most amplified in the boundary layer.

The amplification curves of the excited boundary-layer disturbances, obtained for spanwise-wavenumber modes after spatial Fourier decomposition, are illustrated in figure 6 (symbols) for couple of studied modes (with $f = 15$ Hz) in comparison with the amplification curves calculated for unsteady Görtler vortices by the LST (lines). The results are presented for two spanwise wavelengths $\lambda_z = 10$ and 14 mm. It is seen that the Görtler vortices excited in a distributed way, grow significantly faster than those developing according to the linear stability theory, although in absence of the distributed generation a very good agreement between the measurements and the linear stability theory has been shown recently by Boiko et al. (2010) and Ivanov et al. (2010b).

![Figure 7](image)

**Figure 7.** Sets of streamwise distributions of amplitudes ($a, c, e$) and phases ($b, d, f$) of excited unsteady Görtler vortices.

Similar results are obtained for all frequency regimes in the whole studied range of the spanwise wavelengths. Figure 7 shows the corresponding streamwise distributions of amplitudes and phases of the boundary-layer disturbances excited distributedly by the 3D freestream vortices. One can see that due to the distributed character of the receptivity mechanism, the Görtler vortices are able to grow downstream at all investigated frequencies, despite the linear-instability amplification can not be observed at frequencies of 20 Hz and higher (Boiko et al., 2010). The disturbance phase velocities are close to the value of 0.65 of $U_e$ in agreement with the linear Görtler instability characteristics.
5. Estimation of coefficients of distributed vortex receptivity

5.1. Definition of the distributed receptivity coefficients

In present work coefficients of the distributed vortex receptivity for excitation of Görtler vortices are defined in a similar way as those in experiments (e.g. by Borodulin et al. (2006)) for excitation of TS-waves. The differential equation

$$\frac{d\mathcal{B}_d(x, y_m)}{dx} = i\alpha(x)\mathcal{B}_d(x, y_m) + \mathcal{B}_v(x, y) |_{y=\delta} \mathcal{G}_d(x), \quad (1)$$

describes the evolution of complex amplitudes $\mathcal{B}_d$ of unsteady Görtler vortices with streamwise coordinate $x$. Two mechanisms are responsible for this evolution: (i) the mechanism of linear instability to previously excited disturbances (here $\alpha(x)$ is the complex streamwise wavenumber of the pure Görtler vortices), and (ii) the mechanism of distributed excitation of new portions of Görtler vortices under the influence of freestream vortices. In expression (1) $\mathcal{B}_v(x, y) |_{y=\delta}$ is the complex amplitude of the freestream vortices measured at the boundary layer edge, while $\mathcal{G}_d(x)$ is the sought complex receptivity coefficient.

In the case, when the complex streamwise wavenumber $\alpha(x)$ is assumed to be constant, the solution of equation (1) is well known (see e.g. book by Pontriagin (1994)):

$$\mathcal{B}_d(x) = e^{i\pi x} \left\{ \int_0^x \mathcal{B}_v(s) \mathcal{G}_d(s)e^{-i\pi s} ds + \mathcal{B}_d^0 \right\}, \quad (2)$$

For convenience, the origin of the $x$-coordinate is assigned here in the beginning of the region of measurements, where the complex amplitude of the boundary-layer disturbance is $\mathcal{B}_d^0$.

Similar to experiments by Borodulin et al. (2006), for every given value of the disturbance frequency and spanwise wavenumber the unknown functions $\mathcal{G}_d(x)$ and $\mathcal{B}_d^0$ can be found during approximation of the measured distribution $\mathcal{B}_d(x)$ by the analytical solution (2). It was assumed in the present study that the receptivity coefficients depend linearly on the streamwise coordinate $\mathcal{G}_d(x) = \mathcal{G}_0 + g_1 x$. This assumption was shown to provide a very good approximation of the experimental data. Six parameters were varied during performing the approximation, namely: real and imaginary parts of complex values $\mathcal{G}_0$, $g_1$ and $\mathcal{B}_d^0$.

Two examples of the results of the approximations are shown in figure 8 for disturbance amplitudes (left) and phases (right) with $f = 15$ Hz ($F = 17.04$) and spanwise wavelengths $\lambda_z = 10$ mm ($\Lambda = 208$) and 14 mm ($\Lambda = 345$). Figure 8 shows that the procedure described above tuned to be work rather successfully. Both the amplitude and phase parts of the analytical solution (2) with the found parameters are in a good agreement with the experimental distributions. Similar results are obtained for all other studied combinations of the spanwise wavenumber and frequency.

5.2. Dependence of receptivity coefficients on frequency and spanwise scale

The values of amplitude and phase parts of the distributed receptivity coefficients represents the most important results of the present study. They are presented in figure 9 for two frequencies: 15 and 20 Hz versus the spanwise wavelength. The coefficients are determined at the beginning (figure 9a, b) and at the end (figure 9c, d) of the region of measurements (i.e. for $x = 312$ and 900 mm). It is seen, that the mechanism of the distributed vortical receptivity is the most efficient for Görtler vortices with spanwise wavelength $\lambda_z = 10$ to 14 mm. Note that the same is true for the efficiency of the Görtler instability mechanism (in a sense that the Görtler vortices most amplified due to the linear instability mechanism have spanwise wavelengths in the same
**Figure 8.** Examples of results of approximation of experimental growth curves of Görtler vortices (symbols) by analytical solution (2) (lines). (a, b) - $\lambda_z = 10$ mm ($\Lambda = 208$), (c, d) - $\lambda_z = 14$ mm ($\Lambda = 345$).

**Figure 9.** Amplitude (left) and phase (right) parts of coefficients of distributed receptivity of boundary layer on concave wall to streamwise freestream vortices versus spanwise scale of excited Görtler vortices for two studied frequencies. (a, b) - $x = 312$ mm ($G^* = 8.23$), (c, d) - $x = 900$ mm ($G^* = 18.70$).

range). Thus, the receptivity and the instability mechanisms enhance each other. It is seen also that the receptivity amplitudes are reduced with the streamwise coordinate and in the end of the region of measurements become lower by a factor of two approximately. This observation
correlates with the physical sense of the distributed receptivity mechanism, which efficiency must be reduced when the boundary-layer non-uniformity gets weaker.

6. Summary

(i) The experimental study of a mechanism of generation of Görtler vortices due to scattering of freestream vortices on localized (in streamwise direction) surface non-uniformities is carried out for two different configurations of the freestream and surface perturbations. It is found that despite a great studied range of variation of the problem parameters and a very high accuracy of the present boundary-layer measurements, no excitation of measurable Görtler vortices has been detected. This result assumes that the localized receptivity mechanisms indicated above are extremely weak.

(ii) The mechanism of distributed excitation of unsteady Görtler vortices in a boundary layer on concave wall under the influence of streamwise (3D) freestream vortices is studied experimentally. It is found that this receptivity mechanism is rather efficient and is able to change significantly the growth rates of the Görtler vortices (in comparison with the linear-instability growth rates). In particular, the presence of streamwise freestream vortices can convert attenuating Görtler vortices into the amplified ones.

(iii) For the first time the coefficients of the distributed vortex receptivity of boundary layer with excitation of Görtler vortices are estimated experimentally. It is found, that the distributed receptivity mechanism excites most efficiently those Görtler vortices which have spanwise wavelengths corresponding the most linearly unstable modes. The receptivity amplitudes are found to decay with the streamwise coordinate.

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