Toward the measurement of the hyperfine splitting in the ground state of muonic hydrogen

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Abstract The recent Lamb shift experiment at PSI and the controversy about proton size revived the interest in measuring the hyperfine splitting in muonic hydrogen and extracting the proton Zemach radius. The efficiency of the experimental method depends on the energy dependence of the muon transfer rate to higher-Z gases in the near epithermal energy range. As long as the available experimental data only give the average transfer rate in the whole epithermal range, and the detailed theoretical calculations have not yet been verified, an experiment has been started for the measurement of the transfer rate in thermalized gas target at different temperatures and extracting from the data an estimate of the transfer rate for arbitrary energies. We outline the underlying mathematical method and estimate its accuracy.

Keywords Muonic hydrogen · Proton Zemach radius · Muon transfer

1 Introduction

The laser spectroscopy measurement of the hyperfine splitting (HFS) of the ground state of the muonic hydrogen atom, first proposed more than two decades ago [1], is considered as complementary to the measurements of the hyperfine splitting in ordinary hydrogen. Since
then, the realization of this fundamental project has been awaiting for the development of a sufficiently powerful tunable near-infrared (NIR) laser and the availability of a muon source of sufficiently high intensity. Both these preconditions are now satisfied, and the work on the project has recently been started [2]. The results of the recent muonic hydrogen Lamb shift experiment at PSI [3] and the discovered incompatibility of the values of the charge radius of the proton extracted from muonic hydrogen spectroscopy on the one hand, and ordinary hydrogen spectroscopy and electron-proton scattering data, on the other, supplied strong additional motivation for the project. The proton Zemach radius extracted from the experimental data on muonic hydrogen will be compared with the value obtained from hydrogen spectroscopy, and this will either give more weight to explanations of the proton size puzzle supposing methodology uncertainties in the proton charge radius extracted from electron-proton data, or boost the search for new physics.

The experimental method for the measurement of the hyperfine splitting in muonic hydrogen by laser spectroscopy exploits specific features of the muon transfer reaction at collision energies above the thermal ones [4] and is compatible with the use of a multi-pass cavity. Its physical background is the following chain of reactions. The muonic hydrogen atoms that absorb a photon of the resonance frequency, undergo a hyperfine para-to-ortho transition and when de-excited back to the para spin state in a collision with a \( \text{H}_2 \) molecule, are accelerated by \( \sim 0.12 \text{ eV} \) [4]. The number of atoms that have undergone the above sequence (and therefore — the tuning of the laser to the resonance hyperfine transition frequency) can be determined by observing the characteristic X-rays emitted after the transfer of the muon from muonic hydrogen to the nucleus of a heavier gas added to the hydrogen target, provided that the transfer rate is energy-dependent in the epithermal range. There are experimental evidences that for some gases (e.g., oxygen, argon, etc.) this is indeed the case. The objective of the first stage of the experimental program, launched in 2014, is to obtain reliable quantitative data on the energy dependence of the rate of muon transfer to these gases — and possibly other as well — in order to select the optimal chemical composition and physical parameters (pressure, temperature) of the hydrogen gas target that will provide the highest accuracy in the future measurements of the hyperfine splitting of \((\mu^- p)_{1s}\) and the Zemach radius of the proton.

It is planned to obtain data about the muon transfer rate \( \lambda(\varepsilon) \) as function of the center-of-mass (CM) collision energy \( \varepsilon \) of \( \mu^- p \) and the higher-Z atoms in the “near epithermal range” \( 0.004 < \varepsilon < 0.13 \text{ eV} \). This interval has been selected to cover the energy range \( \varepsilon_T < \varepsilon < \varepsilon_T + (2/3) \Delta E_{\text{hfs}} \) for gas target temperatures \( T \) between 30 K and 320 K (where \( \varepsilon_T = 3kT/2 \) and \( 2/3 \Delta E_{\text{hfs}} \sim 0.12 \text{ eV} \) is the energy acquired by the muonic atom after the spin de-excitation). The results will set the required firm ground for all following activities and will be used to determine the optimal temperature, pressure, laser shot timing and the chemical composition (admixture gas and its concentration) of the gas target for the measurement of the HFS in muonic hydrogen. The experiment will be performed using a novel method consisting in repeated measurement of the transfer rate from completely thermalized muonic hydrogen atoms at different gas target temperatures. While the experimentally observed muon transfer events will come from hydrogen atoms with Maxwell-Boltzmann distributed energies in a broad range, the mathematical model outlined in [5] allows for extracting from the experimental data the functional dependence of the muon transfer rate on the collision energy, and also provides a rigorous estimate of the uncertainty of the extracted values. In comparison with alternative approaches based on the analysis of the time distribution of the muon transfer events, the proposed method has the advantage to eliminate the uncertainties in the energy distribution of the non-thermalized muonic atoms. This is an important step forward, since the accuracy of the available data about the energy
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Fig. 1 Uncertainty $\Delta \lambda(\varepsilon)$ of the muon transfer rate in the epithermal CM energy range, normalized with the experimental data uncertainty $\Delta$, for quadratic (solid line) and cubic (dotted line) polynomial approximation of the functional dependence. The observable transfer rate $\Lambda(T)$ is measured at the temperatures labelling the curves.

dependence of the transfer rate in the epithermal range is insufficient for modeling and optimizing the muonic hydrogen hyperfine splitting experiment. On the ground of the preceding test measurements in 2014 we have concluded that gas target pressures around 40 bar and incident negative muon beam of momentum around 61 MeV/c are optimal for the proposed experiment. The precise admixture gas concentration, the optimal target temperatures (in the range 70 – 320 K) and a few more parameters will be determined by means of Monte Carlo simulations that are already in progress.

2 Outline of the mathematical model of the experiment

The experimentally observable rate of muon transfer at temperature $T$, $\Lambda(T)$, is the average of $\lambda(\varepsilon)$ with respect to the collision energy $\varepsilon$, evaluated with the probability density $\rho(\varepsilon; T)$:

$$\Lambda(T) = \int \lambda(\varepsilon) \rho(\varepsilon; T) \, d\varepsilon.$$  (1)

We are interested in the inverse transform expressing $\lambda(\varepsilon)$ in terms of $\Lambda(T)$.

In thermal equilibrium the distribution $\rho(\varepsilon; T)$ is nothing but the Maxwell-Boltzmann distribution:

$$\rho(\varepsilon; T) = \rho_{MB}(\varepsilon; T) = \rho_0(\varepsilon/\varepsilon_T)/\varepsilon_T,$$  (2)

where $T$ is the temperature and $k_B$ is Boltzmann’s constant. Leaving aside the general problem of inverting (1), here we focus on determining $\lambda(\varepsilon)$ in the thermal and near epithermal energy range since the efficiency of the adopted experimental method for the measurement of the $\mu^- p$ hyperfine splitting depends on the variations of $\lambda(\varepsilon)$ in this range [4]. To this goal we apply a numerical method which consists in using a polynomial approximation for $\lambda(\varepsilon)$ and expressing its parameters in terms of a finite set of experimental values.
\( \Lambda_k = \Lambda(T_k) \), measured in thermalized gas target at temperatures \( T_k \). We take the polynomial approximation in the form:

\[
\lambda(\varepsilon) = \sum_{k=1}^{N} \Lambda_k P^{(k)}(\varepsilon)
\]

(4)

where \( P^{(k)} \) are polynomials of degree \( N - 1 \)

\[
P^{(k)}(\varepsilon) = \sum_{n=0}^{N-1} \varepsilon^n \frac{4^n}{(2n + 1)!} \left. \frac{\partial^n}{\partial z^n} \prod_{j \neq k} (z - \varepsilon_j) \right|_{z=0} (\varepsilon_k - \varepsilon_j).
\]

(5)

and \( \varepsilon_k = k_B T_k \) stand for the thermal energies at \( T_k \), \( k = 1, \ldots, N \). Equations (4–5) allow for expressing the (approximated) value of the muon transfer rate for any collision energy in the interval of interest in terms of the experimental data \( \{\Lambda_k\} \). Moreover, they give an estimate of the statistical uncertainty \( \Delta \lambda(\varepsilon) \) of the calculated value \( \lambda(\varepsilon) \). Indeed, by assuming that the experimental data are normally distributed: \( \Lambda_k \sim N(\bar{\Lambda}_k, \Delta_k) \), we get

\[
\Delta \lambda(\varepsilon)^2 = \sum_{k=1}^{N} (P^{(k)}(\varepsilon))^2 \Delta_k^2.
\]

(6)

From (6) we see that \( \Delta \lambda(\varepsilon) \) is independent of \( \Lambda_k \) and only depends on the measurement temperatures \( T_k \) and on the degree of the polynomial approximation. For simplicity we may take all experimental uncertainties equal: \( \Delta_k = \Delta \). The specific values of the target temperatures \( (T_k, k = 1, \ldots, N) \) may be predetermined by the availability of cooling devices or may be subject to optimization by imposing specific criteria for the minimization of \( \Delta \lambda(\varepsilon) \), e.g.,

\[
\int_{\varepsilon_l}^{\varepsilon_u} d\varepsilon \left( \Delta \lambda(\varepsilon) \right)^2 = \min.
\]

(7)

On Figs. 1 and 2 we plot \( \Delta \lambda(\varepsilon)/\Delta \) – the ratio of the statistical uncertainty of the calculated and measured rates – for energies in the interval of interest. We see that
The uncertainty grows very quickly with $\varepsilon$. This is not surprising: the contribution to the observable transfer rate $\Lambda(T)$ from the “code” of the Maxwell-Boltzmann distribution $\rho_{MB}(\varepsilon; T)$ decreases exponentially with energy;

- the uncertainty of $\lambda(\varepsilon)$ can be brought down much below the uncertainty of the available data with a reasonable statistics. Under the experimental conditions of Stage 1 of the FAMU project, $10^6$ muon transfer events will be registered within less that an hour that so that a few hours of data acquisition will provide $\Delta k < 0.1\%$ and $\Delta \lambda(\varepsilon)$ of the order of a few percent.

- The uncertainty grows very quickly with the degree of the polynomials $P^{(k)}(\varepsilon)$ and makes inappropriate the use of polynomials of degree higher than $N - 1 = 4$. This is a frequently encountered problem in approximating data with polynomials; to avoid it the degree of the approximating polynomials should be lower than the number of measurements. The estimates of the uncertainty in this case will be discussed elsewhere.

3 Conclusions

The present work is part of the investigations of the muon transfer from hydrogen to heavier gases, aimed at providing the needed firm ground for the measurement of hyperfine splitting in the ground state of muonic hydrogen and the determination of the Zemach radius of the proton with the experimental method of Refs. [4, 6]. We demonstrate that the energy dependence of the muon transfer rate in the near epithermal rate can be reliably determined by means of measurements of the muon transfer rate in completely thermalized gas target, and derive an estimate of the statistical uncertainty of the results that confirms the efficiency of the proposed approach.

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Conflict of interests  The authors declare that they have no conflict of interest.

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