Discretuum versus Continuum Dark Energy

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Abstract

The dark energy equation of state for theories with either a discretuum or continuum distribution of vacua is investigated. In the discretuum case the equation of state is constant $w = p/\rho = -1$. The continuum case may be realized by an action with large wave function factor $Z$ for the dark energy modulus and generic potential. This form of the action is quantum mechanically stable and does not lead to measurable long range forces or violations of the equivalence principle. In addition, it has a special property which may be referred to as super-technical naturalness that results in a one-parameter family of predictions for the cosmological evolution of the dark energy equation of state as a function of redshift $w = w(z)$. The discretuum and continuum predictions will be tested by future high precision measurements of the expansion history of the universe. Application of large $Z$-moduli to a predictive theory of $Z$-inflation is also considered.
1 Multi-Vacua

The fundamental theory of nature may possess many vacua, each with distinct physics. For example, there are believed to be large classes of (meta)stable (non)-supersymmetric string and M-theory vacua [1, 2]. In a multi-vacua theory physical parameters depend on moduli fields which varying between vacua. There are a priori two classes of multi-vacua theories which may be distinguished experimentally. In the first, which has come to be known as the discretuum, the moduli have a significant mass gap or are discrete. In this case the physical parameters in any given vacuum are constant and time independent. In the second class, which will be referred to as the continuum, the moduli interpolate continuously between vacua. If the full potential of the theory is moduli dependent, each modulus value is not strictly an independent vacuum. But if the moduli evolution is slow enough on cosmologically relevant time scales, each value can effectively be considered to be an independent vacuum. In this case the physical parameters may vary and in principle be time dependent.

In this paper we focus on the vacuum energy and distinguish the observable properties of cosmological dark energy in the discretuum and continuum cases. The effects on other physical parameters turn out to be insensitive to the multi-vacua, as discussed below. The continuum case may be realized by a modulus with large wave function factor $Z$ which in principle might arise from infrared physics. This realization of the continuum is quantum mechanically stable and does not lead to observable long range moduli forces or violations of the equivalence principle. In addition, it has a special property we refer to as super-technical naturalness, which in the simplest case leads to a one-parameter family of predictions for the evolution of the dark energy equation of state. This form of moduli action is also applicable to theories of inflation.

Cosmologies which allow a distribution of vacuum energies through a very slowly evolving field have been considered previously in the context of very flat potentials [3, 4, 5]. Large $Z$ Lagrangians which could stabilize moduli [6] or likewise allow a distribution of vacuum energies [7, 8] have also been
considered. Here we address properties of the discretuum and continuum including quantum stability, (super)-technical naturalness, absence of long range forces, implications for evolution of the dark energy equation of state, and effects on other physical parameters.

2 Discretuum versus Continuum

For a discretuum multi-vacua, the vacuum energy in a given vacuum is by definition constant, \( V = V_0 \). The magnitude is not predicted, but with a large enough number of vacua there may be many which happen to have an energy density consistent with cosmological observations. However, the density and pressure are related in the discretuum case to the vacuum energy by \( \rho = V_0 \) and \( p = -V_0 \). So the discretuum leads to the prediction that the dark energy equation of state parameter is constant

\[
w = \frac{p}{\rho} = -1
\]

independent of redshift.

For a continuum multi-vacua, the vacuum energy depends continuously on some moduli fields. Allowing for possible space-time dependence of the moduli fields, the relevant Lagrangian describing the dark energy is then

\[
\frac{1}{2} Z \partial_\mu \phi \partial^\mu \phi - V(\phi)
\]

where \( \phi \) parameterizes the trajectory through moduli space, \( Z \) is the kinetic term wave function, and \( V(\phi) \) is the modulus dependent full quantum potential of the entire theory. Spatial gradients in the moduli fields are redshifted to insignificant levels by an early phase of inflation. Keeping only possible time dependence, the equation of motion for the dark energy modulus from the Lagrangian (2) in an expanding background is then

\[
Z \ddot{\phi} + 3ZH\dot{\phi} - V'(\phi) = 0
\]

where \( H = \dot{a}/a \) is the Hubble constant, \( ' \equiv d/d\phi \), and possible moduli dependence of \( Z \) is neglected. The first term in (3) is the moduli inertial
kinetic term, the second is damping due to expansion of the universe, and
the final term is forcing arising from the potential. The continuum dark
energy equation of state parameter is
\[ w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{1}{2} Z \dot{\phi}^2 - V(\phi) \]
\[ \frac{1}{2} Z \dot{\phi}^2 + V(\phi) \]  
(4)

With possible time dependence from the equation of motion (3) the continuum then allows for the possibility of a time and therefore redshift dependent dark energy equation of state, \( w = w(z) \).

The specific evolution of the continuum equation of state depends on the form the Lagrangian (2). Here we consider a very general form for the potential in which the moduli can range over values of order the Planck scale, and allow for the possibility that the overall magnitude of the potential is smaller than the Planck scale
\[ V(\phi) = m^2 M_p^2 f(\phi/M_p) \]  
(5)

where \( f \) is a generic dimensionless function with order unity range and domain. A small overall magnitude for the potential could arise, for example, from supersymmetry which protects the potential from quantum corrections above the supersymmetry breaking scale. Another possibility is that the fundamental scale is smaller than the four-dimensional Planck scale as occurs in theories with extra dimensions. In either case since the supersymmetry breaking and/or fundamental scale can not be smaller than roughly TeV, the overall magnitude is bounded by \( m^2 M_p^2 \gtrsim \text{TeV}^4 \) so that \( m^2 / M_p^2 \gtrsim 10^{-60} \).

Cosmological observations indicate that the universe has recently entered a phase of accelerated expansion [9]. This implies the existence of a dark energy which is vacuum dominated and a sizeable fraction of critical density, \( V_0 \lesssim 3 H_0^2 M_p^2 \), where \( V_0 = V(\phi_0) \) and \( \phi_0 \) is the present value of the continuum dark energy modulus. Since this is much smaller than the minimum magnitude for the potential given above, \( H_0^2 / M_p^2 \sim 10^{-120} \), the most natural assumption which does not require tuning of any Lagrangian (2) parameters is that the continuum potential (5) at \( \phi_0 \) happens to be near a simple zero \( f(\phi_0) \simeq 0 \) [10].
Vacuum domination of the dark energy implies first that the kinetic energy is somewhat smaller than the potential, $\frac{1}{2} Z \dot{\phi}^2 \lesssim V_0$, and second that the inertial term in the modulus equation of motion (3) is small compared with the friction and potential terms, $\ddot{\phi} \lesssim 3H \dot{\phi}$, so that the equation of motion reduces to $3ZH \dot{\phi} \simeq V'(\phi)$. These conditions, along with the magnitude of the dark energy density given above, for any potential $V(\phi)$ may be written

$$\frac{1}{Z^{n/2}} \left| \frac{d^n V(\phi_0)}{d\phi^n} \right| \lesssim H_0^2 M_p^{2-n}$$

for $n = 0, 1, 2$. These are analogous to the slow roll conditions of inflation. For a generic potential if the conditions on the first and/or second derivative are close to being saturated then vacuum domination over of order a Hubble time requires the conditions (6) to hold for all derivatives, $n \geq 0$. In terms of the dimensionless function in the potential (5) the conditions (6) are

$$\frac{d^n f(x_0)}{dx^n} \lesssim Z^{n/2} \frac{H_0^2}{m^2}$$

where $x = \phi/M_p$. So although the smallness of the overall vacuum energy may arise naturally from a simple zero of the modulus potential, continuum dark energy requires at least one small dimensionless parameter, characterized by $H_0^2/m^2 \ll 1$, in the modulus Lagrangian (2).

### 3 Kinetic Seizing

For a canonically normalized modulus, $Z = 1$, the conditions (7) require possibly a large number of small parameters in order to obtain vacuum dominated continuum dark energy. In this language these conditions may be stated as the requirements that the potential be very flat, very low curvature, and very smooth.

However, in a general basis with wave function $Z$, all the conditions (7) may be satisfied for sufficiently large $Z$ even with a generic function $f$ without any small parameters. The 0-th derivative condition is satisfied automatically near a zero $f(x_0) \lesssim H_0^2/m^2$. The first derivative condition
is satisfied for order one $f'(x_0)$ if $Z^{1/2} \lesssim m^2/H_0^2$. The second and higher order derivative conditions (7) are then automatically satisfied by additional powers of $Z^{(n-1)/2}$ for order one dimensionless derivatives. The required wave function factor ranges from $Z^{1/2} \lesssim 10^{60}$ for $m^2M_p^2 \sim \text{TeV}^4$ to $Z^{1/2} \gtrsim 10^{120}$ for $m^2M_p^2 \sim M_p^4$.

This description is of course formally equivalent to the canonically normalized one by a field redefinition, $\tilde{\phi} = Z^{1/2}\phi$. But in this language it is clear that only a single small parameter, $Z^{-1/2}$, is required to obtain continuum dark energy. And as discussed below, the large $Z$ form of the Lagrangian is super-technically natural which is more restrictive than the most general technically natural Lagrangian which could give rise to vacuum dominated continuum dark energy. Also, vacuum domination in this language arises not from special flatness properties of the potential but from the large kinetic inertia and Hubble damping of the continuum modulus. This seizes its evolution, as is apparent in the vacuum dominated equation of motion, $\dot{\phi} \simeq V'(\phi)/(3ZH)$. In the large $Z$ limit, space-time gradients in the modulus field are suppressed by a large cost in Lagrangian density (2). In the $Z \to \infty$ limit the modulus completely seizes and effectively becomes a parameter.

The most striking feature of continuum dark energy is the specific form of the modulus evolution for large $Z$. The potential may be expanded about the present value of the dark energy modulus

$$V(\phi) = \sum_{n=0}^{\infty} m^2M_p^{2-n} \frac{1}{n!} \frac{d^n f(x_0)}{dx^n}(\phi - \phi_0)^n$$  \hspace{1cm} (8)

Keeping for the moment only the first derivative term, the modulus total change over the history of the universe is

$$\Delta \phi = \int dt \, \dot{\phi} \sim \frac{V'}{3ZH_0^2} = \frac{m^2M_p f'}{3ZH_0^2}$$  \hspace{1cm} (9)

Using this, the contribution of the $n$-th derivative term in the expansion (8) to the total fractional change in the potential is

$$\frac{\Delta V^{(n)}}{V_0} \sim \frac{m^2}{H_0^2} \left( \frac{m^2 f'}{ZH_0^2} \right)^n \frac{d^n f}{dx^n} \lesssim \frac{1}{Z^{n/2}H_0^2} \frac{d^n f}{dx^n}$$  \hspace{1cm} (10)
where for the inequality the first derivative vacuum domination condition (7) has been used. Now since only the first derivative condition (7) is restrictive for large $Z$, implying $Z^{1/2} \gtrsim m^2/H_0^2$, the contributions to (10) of derivative terms beyond linear order are suppressed by additional powers of $Z^{(n-1)/2}$ and therefore have a negligible effect on the evolution of the dark energy modulus over any observable cosmological epoch. This occurs because with seized evolution the modulus does not make large enough excursions for terms beyond the linear one in (8) to be important. And with a linear potential only the ratio $V'/Z$ appears in the equation of motion (3). So for a given dark energy density today, the evolution of the continuum dark energy modulus is then characterized by a single dimensionless seizing parameter which may be taken to be $Z^{-1}|M_pV'/V_0|$.

In order to be at least technically natural, the form of the Lagrangian (2) with large $Z$ and potential (5) must be stable against quantum corrections. Integrating out matter fields to which the moduli may couple does not spoil the form of the potential (5) with a generic order one function $f$ and Planck scale cutoff. In addition, since the continuum modulus propagator involves the inverse wave function

$$\langle 0|\phi(0)\phi(p)|0\rangle = \frac{Z^{-1}}{p^2 - V''(\phi_0)}$$

(11)
corrections from moduli self interactions also do not modify the form of the potential. In fact, with a Planck scale cutoff the potential would be quantum mechanically stable against self coupling corrections and technically natural even if the derivative terms in the expansion (8) were larger by factors of $Z^{(n-1)/2}$ for $n \geq 2$. So the form of the Lagrangian (2) with potential (5) is actually more restrictive than required by technical naturalness. We will refer to the property of the Lagrangian (2) with self interactions (8) which are smaller by factors of $Z^{(n-1)/2}$ than that required solely by technical naturalness as super-technical naturalness [11]. It is super-technical naturalness which is responsible for the negligible effects of higher order derivative terms (8) on the cosmological evolution of the dark energy modulus. It is important to note that super-technical naturalness follows from a single small parameter, $Z^{-1}$, and is therefore not an unnatural tuning beyond technical
naturalness.

Super-technical naturalness and stability against details of ultraviolet physics is enforced by an approximate $\phi \rightarrow \phi + C$ shift symmetry which is recovered in the $Z \rightarrow \infty$ limit. This arises because at fixed space-time gradients in this limit the potential term becomes insignificant compared with the gradient terms. Note that the wave function may vary over the full moduli space of multi-vacua so that the shift symmetry need only be an emergent approximate symmetry in regions of large $Z$.

Finally, note that although the continuum modulus is very light and may couple to matter fields, it does not lead to measurable long range forces or violations of the equivalence principle because of the highly seized propagation, as evidenced by the inverse wave function in the propagator (11).

The features of continuum dark energy presented above are to be contrasted with quintessence models [5]. Most models of this type require multiple small parameters which are not protected by an approximate symmetry. As such they generally are not stable quantum mechanically or technically natural, are sensitive to details of ultraviolet physics, and require unnatural tuning of parameters. In addition, without an approximate symmetry to protect couplings, the light fields could lead to violations of the equivalence principle [12]. Even with an approximate symmetry to protect couplings and stabilize the model, the most general technically natural potential described above which leads to vacuum dominated dark energy saturates all the derivative conditions (6). All the derivatives in the potential are then important and the evolution of the equation of state is in general not a priori predictable. This is in contrast to continuum dark energy which requires only one super-technically natural small parameter and in the simplest version enjoys a one-parameter family of predictions outlined below.

4 Large $Z$

The large wavefunctions required for continuum dark energy are super-technically natural and could be natural if they arise from some dynamics. One possibility is that $Z$ depends on other moduli which dynamically drive it towards
large values. Another possibility is that large $Z$ results from infrared dynamics below the fundamental scale [7]. It remains an open challenge to develop mechanisms or models which are both natural and super-technically natural and realize such large $Z$.

The approximate shift symmetry $\phi \to \phi + C$ which emerges for large $Z$ might seem susceptible to violations by quantum gravity corrections at the fundamental scale. Large $Z$ may also seem equivalent to a trans-Plankian range for the canonically normalized modulus $\tilde{\phi} = Z^{1/2} \phi$ since the potential (5) is then a function of $f(\tilde{\phi}/Z^{1/2}M_p)$. However, if the dynamics which leads to a large $Z$ were due to infrared physics below the fundamental scale, the shift symmetry would be an emergent infrared symmetry and therefore immune from any quantum gravity corrections or questions of trans-Planckian ranges.

5 Continuum Dark Energy

The cosmological evolution which results with continuum dark energy depends on the modulus wave function $Z$ and slope of the potential $V'$. In a general model $Z$ might depend on other moduli and itself be time dependent.

Here for definiteness we make the minimal assumption that $Z$ is constant. In this case the evolution is specified in terms of the single dimensionless seizing parameter $Z^{-1}|M_p V'/V_0|$ discussed above. In order to determine the expansion history, the coupled Friedman and modulus equations of motion (3) are integrated forward from a high redshift in the matter dominated era with initial conditions chosen to yield a given $\Omega_\phi = (\frac{1}{2}Z \ddot{\phi}_0^2 + V(\phi_0))/(3H_0^2 M_p^2)$. The evolution of the scale factor is shown in Fig. 1 for different values of $Z^{-1}|M_p V'/V_0|$. For small values the modulus motion is highly seized and once the universe is dark energy dominated it enters a de-Sitter phase of accelerated exponential expansion for an extended period. For any value of seizing, the dark energy modulus eventually evolves to negative total energy density at which time the Hubble constant changes sign and the expansion reverses. The $3ZH\dot{\phi}$ term in the moduli equation of motion (3) becomes anti-damping in this epoch and pushes the modulus to more negative values.
of the potential, resulting in a rapid crunch [13]. This feature is shared by any model which evolves to negative energy density [14]. For $Z^{-1}|M_p V'/V_0|$ of order unity the de-Sitter phase is brief and the crunch time is a few Hubble times from the beginning of dark energy domination.

It may appear that the continuum modulus initial conditions have to be rather specially chosen in order to yield acceptable continuum dark energy. However, there is a natural dynamical selection effect since only regions of the universe (after inflation say) which are near a zero of the continuum potential can evolve to become large with extended eras of radiation and matter domination followed much later by dark energy domination.

Details of the transition from matter to dark energy domination in general depend on the dark energy equation of state. The continuum dark energy equations of state (4) as a function of redshift for various values of constant seizing parameter are shown in Fig. 2. At high redshift $w_\phi \to -1^+$ since

Figure 1: Scale factor as a function of time for different continuum dark energy seizing parameters for $\Omega_m = 0.3$ and $\Omega_\phi = 0.7$. 
Figure 2: Continuum dark energy equation of state $w_{\phi} = p_{\phi}/\rho_{\phi}$ as a function of redshift for different continuum seizing parameters for $\Omega_m = 0.3$ and $\Omega_{\phi} = 0.7$.

the larger Hubble constants there yield more effective seizing and smaller modulus kinetic energy. For $Z^{-1}|M_pV'/V_0|$ of order unity the vacuum domination condition (7) on the first derivative is just marginally satisfied, and kinetic seizing is only marginally effective at the transition from matter to dark energy domination at a redshift $z \sim 1$. In this case the continuum modulus begins to roll at this transition epoch and the kinetic energy is not too much smaller than the potential energy. The dark energy equation of state can then differ significantly from $w_{\phi} = -1$ at low redshift and show considerable evolution.

The dark energy equation of state $w = w(z)$ is difficult to extract directly from experimental measurements of the expansion history. It is more useful to consider experimental observables directly. Fig. 3 shows the magnitude–redshift relation for various values of constant seizing parameter relative to a constant vacuum energy cosmology. This relation represents a one-parameter
family of predictions for the given cosmological parameters. Future precision measurements of the magnitude–redshift relation such as from the SuperNova Acceleration Probe (SNAP) [15] should be able to measure the seizing parameter at the few % level. This, along with other measurements of the expansion history from structure formation, weak lensing, and the cosmic microwave background should eventually be able to distinguish the continuum from other theories of dark energy. A detailed study of how well such measurements can test continuum dark energy [16] in particular by bounding higher derivatives in the dark energy potential beyond linear order will be presented elsewhere [17].

Finally, it is worth considering what the expectation might be for the value of $Z$. In a multi-vacua theory the distribution of dark energy modulus wave functions $Z$ over all the multi-vacua might be a rapidly falling function at large $Z$. This may in fact be likely since extremely large $Z$ seems rather
exceptional. In this case the most likely value of \( Z \) is one not too much larger than the minimum value consistent with the observed vacuum dominated dark energy. This corresponds to a seizing parameter \( Z^{-1}|M_pV'/V_0| \) not too much smaller than order unity. So in this case the continuum dark energy modulus should have begun to roll by the current epoch, leading to a measurable evolution of the dark energy equation of state [18].

In any multi-vacua theory the magnitude of the dark energy need not be too much smaller than the maximum allowed value [19]. In a continuum multi-vacua theory we see that the time dependent evolution of the dark energy also need not be too much smaller than the maximum allowed value. The expectation that properties of the dark energy are not much smaller than maximally acceptable could be referred to as the principle of living dangerously, since regions in which these properties are significantly exceeded are uninhabitable.

6 Physical Parameters in the Multi-Vacua

For a discretuum, all physical parameters, such as gauge and Yukawa couplings, are constants and independent of time. For a continuum, if these parameters depend on the continuum moduli they are in principle time dependent. However, from (9) the total range of the continuum modulus over the history of the universe is

\[
\Delta \phi / M_p \lesssim Z^{-1/2}
\]

where the inequality results from the first derivative vacuum domination condition (7). So if physical parameters are generic functions of the dark energy modulus with order unit range, \( g = g(\phi / M_p) \), then the changes in these parameters,

\[
\Delta g \simeq g'\Delta \phi / M_p \lesssim g'Z^{-1/2}
\]

where \( g' \equiv d g(x_0)/d x \), are unobservably small even for order one seizing parameter. So measurements of the evolution of the dark energy equation of state appear to be unique experimental probes of the nature of multi-vacua theory.
7  Z-inflation

Large $Z$ moduli are also applicable to inflation. With only a moderate wave function factor, $Z^{1/2} \gtrsim 10$, and a completely general potential, seized evolution of a modulus can lead to an early epoch of inflation with sufficient $e$-foldings to solve the horizon and flatness problems. Since only the constant and linear parts of the potential are important during seizing, the super-technically natural feature of $Z$-inflation yields a predictive relation between the spectral tilt, $n$, derivative of the tilt, $dn/d\ln k$, and tensor to scalar ratio, $T/S$ [20]. A large $Z$ inflaton has been considered previously in the context of Brans-Dicke theory with exit from inflation by bubble nucleation [21] but without regard to quantum stability, or the predictive feature of super-technical naturalness.

8  Conclusions

Multi-vacua theories lead to distinct and testable predictions for properties of cosmological dark energy. In the discretuum case the dark energy equation of state is constant $w = -1$. In the continuum case super-technical naturalness of the large $Z$ dark energy modulus leads to a one-parameter family of predictions for the evolution of the dark energy equation of state, $w = w(z)$, and associated distance–redshift observables. High precision measurements of the expansion history of the universe will therefore provide important guidance as to what type of fundamental theory may describe our universe.

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