The Singlet Contribution to the Structure Function
\( g_1(x, Q^2) \) at Small \( x \)

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Abstract

The resummation of \( O(\alpha_s^{l+1} \ln^2 x) \) terms in the evolution equation of the singlet part of \( g_1(x, Q^2) \) is carried out. The corresponding singlet evolution kernels are calculated explicitely. The leading small-\( x \) contribution to the three-loop splitting function matrix is determined in the \( \overline{\text{MS}} \) scheme. Relations are derived for the case of \( \mathcal{N} = 1 \) supersymmetric Yang–Mills field theory. Numerical results are presented for the polarized singlet and gluon densities, and the structure functions \( g_1^p(x, Q^2) \) and \( g_1^n(x, Q^2) \). They are compared for different assumptions on the non–perturbative input distributions, and the stability of the results against presently unknown subleading contributions is investigated.

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1 Introduction

The small-$x$ behaviour of polarized structure functions is a largely unexplored subject. The current measurements cover at most the range $x > 5 \cdot 10^{-3}$ \[1\]. At even lower $x$-values both size and sign of the structure functions $g^p_1(x, Q^2) \text{ and } g^n_1(x, Q^2)$ are yet unknown. The initial distributions at a starting scale $Q^2_0$ of the QCD evolution can not be determined within perturbative QCD. Irrespectively of the specific behaviour of these non-perturbative quantities one may consider, however, the QCD evolution of the structure functions and parton densities. Besides the well-known leading order anomalous dimensions \[2\], recently also the next-to-leading order ones have been calculated \[3\]. Furthermore the all-order resummation of leading singular terms in higher-order anomalous dimensions may be important in the small-$x$ range. These terms behave as $\alpha_{l+1}s \ln l x$, corresponding to $N(\alpha_s/N^2)^{l+1}$ for $N \to 0$ in the Mellin moment plane. Their numerical effect on the evolution of non-singlet \[4\] structure functions has been analyzed in refs. \[5, 6\]. Contrary to an earlier expectation in ref. \[7\] it turns out to be very small and is found to depend on the way in which fermion number conservation is imposed, i.e. also on less singular $\ln^k x$ contributions. Recently also an equation for the leading small-$x$ resummed singlet evolution has been obtained \[8\].

In the present paper we study this singlet resummation in the framework of the renormalization group equation. After recalling the general evolution equation and setting up our notations in Section 2, we calculate the anomalous dimension matrix as a series in $\alpha_s$ explicitely in Section 3. A series of properties of this matrix is discussed. The numerical effect of the resummation beyond the known \[3\] next-to-leading order effects on the small-$x$ behaviour of the polarized singlet quark and gluon densities, $\Delta \Sigma(x, Q^2)$ and $\Delta g(x, Q^2)$, and on the structure functions $g^p_1(x, Q^2) \text{ and } g^n_1(x, Q^2)$ is then studied in Section 4, using input distributions as determined in recent analyses \[9\]. We also investigate the stability of the results against yet unknown effects of terms less singular as $x \to 0$. Section 5 contains our conclusions.

2 The Evolution Equation

The evolution equation for the polarized singlet quark and gluon densities ($\Delta \Sigma, \Delta g$) is given by

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Delta \Sigma(x, Q^2) \\ \Delta g(x, Q^2) \end{pmatrix} = P(x, \alpha_s) \otimes \begin{pmatrix} \Delta \Sigma(x, Q^2) \\ \Delta g(x, Q^2) \end{pmatrix}.$$ \tag{1}

Here $\otimes$ stands for the Mellin convolution, and the matrix of the polarized singlet splitting functions $P(x, \alpha_s)$ is specified below. In the following, we will simplify the notation by using the abbreviation $a_s \equiv \alpha_s(Q^2)/4\pi$ for the running QCD coupling. The scale dependence of $a_s$ is obtained from

$$\frac{da_s}{d\ln Q^2} = -\sum_{k=0}^{\infty} a_s^{k+2} \beta_k,$$ \tag{2}

where only $\beta_0 = (11/3) C_A - (4/3) T_F N_f$ and $\beta_1 = (34/3) C_A^2 - (20/3) C_A T_F N_f - 4 C_F T_F N_f$ enter up to next-to-leading order (NLO). Here $C_A = N_c \equiv 3$, $C_F = (N_c^2 - 1)/(2N_c) \equiv 4/3$, $T_F = 1/2$, and $N_f$ denotes the number of flavours. The matrix $P(x, \alpha_s)$ can be represented by the series

$$P(x, \alpha_s) = \sum_{l=0}^{\infty} a_s^{l+1} P^{(l)}(x).$$ \tag{3}
Unlike the LO and NLO (MS) transition matrices $P^{(0)}(x)$ and $P^{(1)}(x)$ [4, 5], the splitting functions $P^{(l=2)}(x)$ are not completely known so far. For the solution of eq. (1) beyond NLO in Section 4, we will use the asymptotic small-$x$ form of the latter matrices as derived in Section 3.

From that solution the structure function $g_1(x, Q^2)$ is finally obtained by the convolution

$$g_1^{p,n}(x, Q^2) = \frac{1}{3} \left\{ c_{l,q}(x, Q^2) \otimes \Delta \Sigma(x, Q^2) + c_{l,g}(x, Q^2) \otimes \Delta g(x, Q^2) \right\} + g_1^{p,n}(x, Q^2) \quad (4)$$

with

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s + \Delta s,$$  

and $\Delta u, \Delta d, \Delta s$ denoting the polarized up-, down-, and strange–quark distributions. We consider only the contribution of the three light flavours. The non–singlet part of $g_1(x, Q^2)$ has been dealt with in refs. [5, 6] already, to which we refer for further details. The coefficient functions $c_1(x, Q^2)$ can be expanded in the strong coupling as

$$c_{l,i}(x, Q^2) = \delta(1 - x) \delta_{lq} + \sum_{l=1}^{\infty} a_s^l c_{l,i}(s). \quad (6)$$

Here it is important to note that in the MS scheme the known coefficient functions $c_{l}^{(i)}(x)$ for both $l = 1$ and $l = 2$ (cf. refs. [7, 8]) behave only like

$$c_{1}^{(1)} \propto \alpha_s \ln \left( \frac{1}{x} \right), \quad c_{1}^{(2)} \propto \alpha_s^2 \ln^3 \left( \frac{1}{x} \right) \quad (7)$$

at small $x$. Therefore a prediction can be made on the small-$x$ behaviour of the three–loop transition matrix in the MS–scheme, $P_{x \to 0}^{(3)}$, see eq. (13) below. Note that in eq. (1) the resummation under consideration is of leading order, i.e. the respective terms emerge only together with $\beta_0$ after rewriting the evolution equation in terms of $a_s$. Details of the solution will be presented elsewhere [9].

### 3 Resummation of dominant terms for $x \to 0$

The resummed transition matrix for the leading singular terms as $x \to 0$, $P(x, a_s)_{x \to 0}$, can be obtained from the solution of eq. (4) via inverse Mellin transformation:

$$P(x, a_s)_{x \to 0} \equiv \sum_{l=0}^{\infty} P^{(l)}_{x \to 0} a_s^{l+1} \ln^l x = \frac{1}{8\pi^2} M^{-1} \left[ F_0(N, a_s) \right] (x). \quad (8)$$

The matrix–valued function $F_0(N, a_s)$ is subject to the relation

$$F_0(N, a_s) = 16\pi^2 a_s \frac{N}{N} M_0 - \frac{8a_s}{N^2} F_s(N, a_s) G_0 + \frac{1}{8\pi^2} \frac{1}{N} F^2_0(N, a_s) \quad (9)$$

derived in ref. [9], where $F_s(N, a_s)$ is the solution of

$$F_s(N, a_s) = 16\pi^2 a_s \frac{N}{N} M_8 + \frac{2a_s}{N} C_G \frac{d}{dN} F_s(N, a_s) + \frac{1}{8\pi^2} \frac{1}{N} F^2_s(N, a_s). \quad (10)$$

The basic colour factor matrices are given by

$$M_0 = \begin{pmatrix} C_F & -2T_F N_f \\ 2C_F & 4C_A \end{pmatrix}, \quad G_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}, \quad M_8 = \begin{pmatrix} C_F - C_A/2 & -T_F N_f \\ C_A & 2C_A \end{pmatrix}. \quad (11)$$
We determine the matrix $P(x, a_s)_{x \rightarrow 0}$ in terms of a series in $a_s$ since this representation is needed for the solution of the singlet evolution equation (1), see Section 4.

The lowest order entries read

$$P^{(0)}_{x \rightarrow 0} = 2 \left( \begin{array}{cc} C_F & -2 T_F N_f \\ 2 C_F & 4 C_A \end{array} \right),$$

(12)

$$P^{(1)}_{x \rightarrow 0} = 2 \left( \begin{array}{cc} C_F (2C_A - 3C_F - 4T_F N_f) & -2T_F N_f (2C_A + C_F) \\ 2C_F (2C_A + C_F) & 8 C_A^2 - 4 T_F N_f C_F \end{array} \right).$$

(13)

They agree with the respective leading $\ln^2 x$ terms of the complete LO and NLO splitting function matrices [4, 5]. We also list the entries of $P^{(2)}_x$ and $P^{(3)}_x$ explicitly in the colour factors:

$$P^{(2)}_{qq} = \frac{2}{3} C_F \left[ -5 C_F^2 - \frac{3}{2} C_A^2 + 6 C_A C_F - 8 T_F N_f C_F - 6 T_F N_f C_A \right]$$

$$P^{(2)}_{qg} = \frac{2}{3} T_F N_f \left[ -15 C_F^2 + 2 C_A^2 - 6 C_F C_A + 8 T_F N_f C_F \right]$$

$$P^{(2)}_{gq} = \frac{2}{3} C_F \left[ 15 C_A^2 - 2 C_F^2 + 6 C_F C_A - 8 T_F N_f C_F \right]$$

$$P^{(2)}_{gg} = \frac{2}{3} \left[ 28 C_A^3 + 2 T_F N_f C_A^2 - 4 T_F N_f C_F^2 - 24 C_F T_F N_f C_A \right],$$

(14)

and

$$P^{(3)}_{qq} = \frac{2}{45} C_F \left[ 6 C_A^3 - 20 C_F C_A^2 + 22 C_A C_F^2 - \frac{19}{2} C_F^3 - 74 T_F N_f C_A^2 - 44 C_F T_F N_f C_A + 2 T_F N_f C_F^2 + 40 (T_F N_f)^2 C_F \right]$$

$$P^{(3)}_{qg} = \frac{2}{45} T_F N_f \left[ -54 C_F C_A^2 - 2 C_F^2 C_A + 40 T_F N_f C_F^2 - 128 C_A^3 - 8 T_F N_f C_A^2 + 15 C_F^3 + 108 T_F N_f C_F C_A \right]$$

$$P^{(3)}_{gq} = \frac{2}{45} C_F \left[ 54 C_F C_A^2 + 2 C_F^2 C_A - 40 T_F N_f C_F^2 + 128 C_A^3 + 8 T_F N_f C_A^2 - 15 C_F^3 - 108 T_F N_f C_F C_A \right]$$

$$P^{(3)}_{gg} = \frac{2}{45} \left[ -288 T_F N_f C_F^2 C_A^2 - 64 C_F^2 T_F N_f C_A + 6 T_F N_f C_A^2 + 64 (T_F N_f)^2 C_F^2 + 20 T_F N_f C_A^2 + 252 C_A^4 \right].$$

(15)

Comparing eqs. (12, 13) it is interesting to note that the leading small-$x$ off-diagonal elements are related by

$$P^{(l)}_{qg} / (T_F N_f) = -P^{(l)}_{gq} / C_F.$$

(16)

We have verified this property analytically up to order $l = 10$. The numerical values of the matrix elements of $P^{(l)}_{x \rightarrow 0}$ up to $l = 10$ are listed in Table 1 for $SU(N_c = 3)$, leaving the number of (massless) partonic flavours as the only free parameter since the corresponding analytical results become rather lengthy. The matrix elements for even higher indices are easily obtainable.

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2We used the program system Maple V [12] for checks and the derivation of higher order coefficients given subsequently.
Table 1: The elements of the coefficient matrices $P^{(l)}_{fg}$ in eq. (9) for $N_f = 3$ and $N_f = 4$.

We have also calculated the matrices $P^{(l)}_{fg}$ for an $\mathcal{N} = 1$ supersymmetric Yang–Mills field theory, i.e. $C_A = C_F = 1, N_f = 1, T_F = 1/2$. One finds that in this case the so-called supersymmetric relation

$$P^{(l)}_{qq}(x) + P^{(l)}_{gq}(x) - P^{(l)}_{qg}(x) - P^{(l)}_{gg}(x) = 0$$

(17)

is fulfilled for the small-$x$ leading terms. We have verified this behaviour explicitly up to order $l = 100$. Beginning with $O(\alpha_s^2)$ even

$$P^{(l)}_{qq} - P^{(l)}_{qg} = 0, \quad P^{(l)}_{gq} - P^{(l)}_{gg} = 0$$

(18)

holds, and the matrix of the small-$x$ transition functions depends only on one single scalar coefficient $p_l$ at each order in $\alpha_s$. Hence one can write

$$P_{x \to 0}^{SUSY} = 2\alpha_s M_1 + \sum_{l=1}^{\infty} a_s^{l+1} \ln^{2l} x p_l M_2$$

(19)

with

$$M_1 \equiv M_0^{SUSY} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}, \quad M_2 = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}.$$
Note that there is a series of relations between the matrices $M_1$ and $M_2$:

\[ M_1 M_2 = 3 M_2, \quad M_2^2 = M_2, \quad [M_1, M_2] = 0, \]  

(21)

which determines the structure of $F_{8}^{\text{SUSY}}(N)$.

We have furthermore investigated whether the dominant terms found in the present small-$x$ expansion can be obtained in the large-$N_f$ expansion partly, where predictions have been obtained on the behaviour of the eigenvalue

\[ e_2 = \frac{1}{2} [P_{qq}(N) + P_{gg}(N)] - \frac{1}{2} \sqrt{[P_{qq}(N) - P_{gg}(N)]^2 + 4P_{qq}(N)P_{gg}(N)} \]  

(22)

of the Mellin transformed all-order singlet transition matrix. However, as it is found for the lowest order in $\alpha_s$ already, the contributions to the large-$N_f$ expansion of $e_2$ can be obtained only by accounting for non-leading terms for $x \to 0$. Therefore the limits $x \to 0$ and $N_f \to \infty$ do not interchange and a further test on the elements of $P^{(i)}(N)$ can not be obtained in this way.

4 Numerical results

After transformation to Mellin moments, the singlet evolution equation (1) is reduced to a system of coupled ordinary differential equations. Unlike in the non-singlet case considered in refs. [5, 6] the solution cannot be given in a closed analytical form beyond LO here, due to the non-commutativity of the matrices $P^{(i)}(N)$ for different orders in $\alpha_s$. Instead the evolution taking into account the leading small-$x$ resummed kernels (8) has to be written down in terms of a power series in $\alpha_s$, yielding

\[ \left( \begin{array}{c} \Delta \Sigma(N, a_s) \\ \Delta g(N, a_s) \end{array} \right) = \left[ 1 + \sum_{i=1} a_s U^{(i)}(N) \right] \left( \frac{a_s}{a_0} \right)^{-P^{(0)}(N)/\beta_0} \left[ 1 + \sum_{i=1} a_0 U^{(i)}(N) \right]^{-1} \left( \begin{array}{c} \Delta \Sigma(N, a_0) \\ \Delta g(N, a_0) \end{array} \right) \]  

(23)

with $a_0 = a_s(Q_0^2)$. The singlet evolution matrices $U^{(i)}(N)$ can be expressed in terms of the splitting function moments $P^{(j<k)}(N)$. Technical details can be found elsewhere [14]. Due to the structure of eq. (1), the solution (23) is necessarily related to an asymptotic expansion. For all practical cases, say $x > 10^{-6}$, however, retaining 8–10 terms in eq. (23) is adequate for obtaining accurate and stable results. The transformation of the outcome back to $x$-space finally affords one standard numerical integration in the complex $N$-plane.

We study the numerical consequences of the resummation (1) using the recent NLO parametrization of ref. [9] (GRSV) as the input at the reference scale $Q_0^2 = 4 \text{ GeV}^2$. The presently large ambiguities due to the virtually unknown polarized gluon distribution will be briefly illustrated by the maximal and minimal $\Delta g$ scenario of the same group. For a short review on current parametrizations of polarized parton densities see, e.g., ref. [15]. The number of flavours $N_f$ in the $\beta$-function is fixed at $N_f = 4$ for all results shown below, and $\Lambda \equiv \Lambda_{\text{MS}}(N_f = 4) = 200 \text{ MeV}$ is employed in the standard approximation to the NLO running of $\alpha_s$,

\[ a_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{\beta_1}{\beta_0^2} \ln \ln(Q^2/\Lambda^2) \right]. \]  

(24)

\[ ^{3}\text{Recall that the non-singlet transition functions in the small-$x$ limit do not depend on $N_f$ at all.} \]
The number of partonic flavours in the splitting functions $P(i)(N)$ is restricted to $N_f' = 3$ \[\text{[9, 10]}\].

We are now ready to present the resummation effects on the polarized singlet quark and gluon densities, $x\Delta\Sigma$ and $x\Delta g$, as well as on the proton and neutron structure functions, $g_1^p$ and $g_1^n$, respectively. The structure functions have been obtained from the (scheme-dependent) parton distributions by the convolution (4), i.e. without subtracting the resulting subleading $a_s^2$ terms. The non–singlet case has been studied for our present input choice in ref. \[\text{[6]}\]. The corresponding resummation effect is negligible (about 1% or less) over the full kinematical region considered here. In Figure 1 the NLO and leading small-$x$ resummed ($Lx$) results are displayed for the singlet parton densities, and Figure 2 depicts the corresponding proton and neutron structure functions $g_1^{p,n}$.

The resummation effects are much larger than for the non–singlet quantities, as to be expected from the comparison of the coefficients in Table 1 to the corresponding non–singlet results. E.g., the ratio $(NLO + Lx)/(NLO)$ amounts to about 1.72 (1.64) for $\Delta\Sigma$ ($\Delta g$), respectively, at $Q^2 = 10$ GeV$^2$ and $x = 10^{-4}$. It should be noted in this context that the small-$x$ evolution strongly depends on the practically unknown gluon input distribution. This is illustrated in Table 2 where the resummed results of Figure 1 are compared at two representative values of $x$ and $Q^2$ to those obtained by evolving in the same way the ‘minimal $\Delta g$’ and ‘maximal $\Delta g$’ distributions of ref. \[\text{[9]}\]. One finds variations up to a factor of almost 5 (10) for $\Delta\Sigma$ ($\Delta g$), respectively, indicating that at present the input ambiguities are the dominant source of uncertainties also at small $x$.

| $Q^2$ | 10 GeV$^2$ | 100 GeV$^2$ |
|------|-----------|-------------|
| $x$  | $10^{-4}$ | $10^{-3}$   |
|      | $10^{-4}$ | $10^{-3}$   |
| $x\Delta\Sigma$ | -0.0100 | -0.0169 |
|      | -0.0285 | -0.0396 |
|      | -0.0473 | -0.0560 |
| $x\Delta g$ | 0.019  | 0.034   |
|      | 0.101  | 0.152   |
|      | 0.201  | 0.294   |
|      | 0.053  | 0.071   |
|      | 0.226  | 0.281   |
|      | 0.432  | 0.528   |

Table 2: A comparison of the resummed evolution of the polarized parton distributions for different assumptions on the gluon distribution $\Delta g$. Upper lines: minimal gluon, middle lines: standard set, lower lines: maximal gluon (and corresponding quark distributions) of ref. \[\text{[9]}\] at $Q_0^2 = 4$ GeV$^2$.

An obvious question concerning the large effects found is whether the resummed $a_s^{l+1}\ln^{2l}x$ terms really dominate with respect to the presently yet uncalculated terms less singular in $\ln x$. Recall that we found for the non–singlet structure functions that this is not the case \[\text{[5, 6]}\]. Lacking any further information on the higher–order splitting functions, e.g. from sum rules as in the non–singlet and unpolarized singlet cases, it appears reasonable to assume that the coefficients of the first less singular term is of roughly the same size but of opposite sign as the leading one. To obtain a first estimate we use

$$\Delta P^{(i>1)} \rightarrow \Delta P^{(i>1)} \cdot (1 - N).$$

This assumption is motivated by corresponding relations in the LO and NLO splitting functions.
where, e.g., for the largest quantity $P_{gg}$:

$$
P_{gg}^{(0)}(N) = \frac{24}{N} - 15 + O(N), \quad P_{gg}^{(1)}(N) = \frac{256}{N^3} - \frac{244}{N^2} + O(1/N)$$

(26)

for three flavours. The corresponding results are also given in Figures 1 and 2, and are denoted by $L_x \ast (1 - N)$. One finds that the corrections due to these new contributions are very sizeable. In the $x$-range considered here the effect of the $L_x$–resummation is practically cancelled. Hence the calculation of also the less singular terms in the higher–order splitting functions is indispensable for arriving at sound conclusions on the small $x$ evolution on the small-$x$ polarized evolution.

5 Conclusions

We have investigated the effect of the resummation of terms of order $\alpha_s^{l+1} \ln^{2l} x$, derived from the infrared evolution equations in [8], on the small-$x$ behaviour of polarized singlet parton distributions and the structure function $g_1$ in deep–inelastic scattering. The comparison with the corresponding contributions obtained in the same order by complete NLO calculations of the splitting function matrix [3] shows the equivalence of both approaches in this limit up order $\alpha_s^2$. Since the coefficient functions up to two–loop order contain only terms less singular in $\ln x$ in the $\overline{\text{MS}}$ scheme, the contributions $\propto \alpha_s^3 \ln^4 x$ in the three–loop $\overline{\text{MS}}$ splitting functions $P^{(2)}(x)$ have been predicted on the basis of this resummation.

As a general result for $SU(N_c)$, the off–diagonal elements of the matrix $P(x, \alpha_s)_{x\to0}$ are found to be proportional by the factor $-T_F N_f / C_F$. The supersymmetric relation holds for the leading small–$x$ terms of the anomalous dimensions in $\mathcal{N} = 1$ supersymmetric Yang–Mills theory. Starting with order $l = 1$ even more constraining relations are found in this case and all anomalous dimensions are related at a given order in $\alpha_s$.

The numerical analysis shows that the all–order resummation of the terms $O(\alpha_s^{l+1} \ln^{2l} x)$ leads to very large corrections at small $x$. Terms less singular in $\ln x$, being not calculated yet, can be expected to contribute in a very significant way even at the smallest $x$–values considered, $x \simeq 10^{-5}$, as in the case of leading and next–to–leading order. Even a full compensation of the effect obtained resumming the most singular terms can not be excluded. Hence solid conclusions on the small–$x$ evolution of polarized singlet parton densities and structure functions can only be drawn if these terms are calculated.

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Figures

Figure 1: The \( Q^2 \) evolution of the polarized singlet quark and gluon momentum distributions \( x\Delta\Sigma \) and \( x\Delta g \) as obtained from the GRSV standard distribution [9] at \( Q_0^2 = 4 \text{ GeV}^2 \). The results are shown for the NLO kernels (full), the leading small-\( x \) resummed kernels (dashed), and a modification of the latter by possible less singular terms discussed in the text (dotted curves).

Figure 2: The \( x \) and \( Q^2 \) behaviour of the polarized proton and neutron structure functions \( g_1^{p,n}(x, Q^2) \) as obtained from the parton densities in the previous figure. The notations are the same as in Figure 1.
Fig. 1

\[ x\Delta \Sigma(x, Q^2) \]

- NLO
- NLO + Lx
- NLO + Lx * (1−N)

\[ x\Delta g(x, Q^2) \]

\[ Q^2(\text{GeV}^2): \]
- 100
- 10
- 4
Fig. 2