Local Particle-Ghost Symmetry

Yoshiharu Kawamura

Department of Physics, Shinshu University, Matsumoto, Japan
Email: haru@azusa.shinshu-u.ac.jp

Received 16 June 2015; accepted 27 September 2015; published 30 September 2015

Copyright © 2015 by author and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY).
http://creativecommons.org/licenses/by/4.0/

Abstract

We study the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. The particle-ghost symmetries are new type of fermionic symmetries between ordinary particles and their ghost partners, different from the space-time supersymmetry and the BRST symmetry. There is a possibility that they are useful to explain phenomena of elementary particles at a more fundamental level, by extension of our systems. We show that our systems are formulated consistently or subsidiary conditions on states guarantee the unitarity of systems, as the first step towards the construction of a realistic fundamental theory.

Keywords

Fermionic Symmetry, BRST Quantization, Quartet Mechanism

1. Introduction

Graded Lie algebras or Lie superalgebras have been frequently used to formulate theories and construct models in particle physics. Typical examples are supersymmetry (SUSY) [1]-[4] and BRST symmetry [5]-[7].

The space-time SUSY [8] [9] is a symmetry between ordinary particles with integer spin and those with half-integer spin, and the generators called supercharges are space-time spinors that obey the anti-commutation relations [10] [11].

The BRST symmetry is a symmetry concerning unphysical modes in gauge fields and abnormal fields called Faddeev-Popov ghost fields [12]. Though both gauge fields and abnormal fields contain negative norm states, theories become unitary on the physical subspace, thanks to the BRST invariance [13] [14]. The BRST and anti-BRST charges are anti-commuting space-time scalars.

Recently, models that contain both ordinary particles with a positive norm and their counterparts obeying different statistics have been constructed and those features have been studied [15]-[18]. Models have fermionic symmetries different from the space-time SUSY and the BRST symmetry. We refer to this type of novel sym-
metrics as “particle-ghost symmetries”.

The particle-ghost symmetries have been introduced as global symmetries, but we do not need to restrict them to the global ones. Rather, it would be meaningful to examine systems with local particle-ghost symmetries for the following reasons. It is known that any global continuous symmetries can be broken down by the effect of quantum gravity such as a wormhole [19]. Then, it is expected that a fundamental theory possesses local symmetries, and global continuous symmetries can appear as accidental ones in lower-energy scale. In the system with global particle-ghost symmetries, the unitarity holds by imposing subsidiary conditions on states by hand. In contrast, there is a possibility that the conditions are realized as remnants of local symmetries in a specific situation. Hence, it is interesting to investigate features of particle-ghost symmetries more closely and widely, and to apply them on a microscopic theory beyond the standard model.

In this paper, we study the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. We show that our systems are formulated consistently or subsidiary conditions on states guarantee the unitarity of systems, as the first step towards the construction of a realistic fundamental theory. The conditions can be originated from constraints in case that gauge fields have no dynamical degrees of freedom.

The contents of this paper are as follows. We construct models with local fermionic symmetries in Section 2, and carry out the quantization of the system containing scalar and gauge fields in Section 3. Section 4 is devoted to conclusions and discussions on applications of particle-ghost symmetries. In the Appendix Section, we study the system that gauge fields are auxiliary ones.

2. Systems with Local Fermionic Symmetries

2.1. Scalar Fields with Local Fermionic Symmetries

Recently, the system described by the following Lagrangian density has been studied [15]-[18],

\[
\mathcal{L}_{\phi, \psi} = \partial_{\mu} \phi^* \partial^{\mu} \phi - m^2 \phi^* \phi + \partial_{\mu} c_{\phi}^+ \partial^{\mu} c_{\phi} - m^2 c_{\phi}^+ c_{\phi},
\]

where \( \phi \) is an ordinary complex scalar field and \( c_{\phi} \) is the fermionic counterpart obeying the anti-commutation relations. The system has a global \( \mathcal{OSp}(2|2) \) symmetry that consists of \( U(1) \) and fermionic symmetries. The unitarity holds by imposing suitable subsidiary conditions relating the conserved charges on states.

Starting from (1), the model with the local \( \mathcal{OSp}(2|2) \) symmetry is constructed by introducing gauge fields. The resultant Lagrangian density is given by

\[
\begin{align*}
\mathcal{L} &= \mathcal{L}_M + \mathcal{L}_G \\
\mathcal{L}_M &= \left\{ \partial_{\mu} \phi^* \partial^{\mu} \phi - g C_{\mu}^+ \phi^* \phi + g^* \phi \right\} \\
&\quad + \left\{ \partial_{\mu} \phi^* \partial^{\mu} \phi - g C_{\mu}^+ \phi^* \phi + g^* \phi \right\} \\
&\quad - m^2 \phi^* \phi - m^2 c_{\phi}^+ c_{\phi},
\end{align*}
\]

\[
\mathcal{L}_G = -\left\{ \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + ig \left( C_{\mu}^+ C_{\nu} - C_{\nu}^+ C_{\mu} \right) \right\} \left\{ \partial^{\mu} B_{\nu} - \partial^{\nu} B_{\mu} \right\} \\
&\quad - \frac{1}{2} \left\{ \partial_{\mu} C_{\nu}^+ - \partial_{\nu} C_{\mu}^+ + 2ig \left( B_{\mu} C_{\nu}^+ - B_{\nu} C_{\mu}^+ \right) \right\} \\
&\quad - \left\{ \partial^{\mu} C_{\nu} - \partial^{\nu} C_{\mu} + 2ig \left( B_{\mu} C_{\nu} - B_{\nu} C_{\mu} \right) \right\},
\]

where \( A_{\mu} \) and \( B_{\mu} \) are the gauge fields relating the (diagonal) \( U(1) \) symmetries, \( C_{\mu}^+ \) and \( C_{\mu}^- \) are gauge fields relating the fermionic symmetries, and \( g \) is the gauge coupling constant. The quantized fields of \( C_{\mu}^\pm \) obey the anti-commutation relations.

The \( \mathcal{L} \) is invariant under the local \( U(1) \) transformations,

\[
\begin{align*}
\delta_{\chi} \phi &= -ic \phi, \quad \delta_{\chi} \phi^* = ic \phi^*, \\
\delta_{\chi} c_{\phi} &= -ic c_{\phi}, \quad \delta_{\chi} c_{\phi}^+ = ic c_{\phi}^+,
\end{align*}
\]

\[
\begin{align*}
\delta_{\chi} A_{\mu} &= \frac{-1}{g} \partial_{\mu} \epsilon, \quad \delta_{\chi} B_{\mu} = 0, \quad \delta_{\chi} C_{\mu}^+ = 0, \quad \delta_{\chi} C_{\mu}^- = 0.
\end{align*}
\]
\[ \delta_\epsilon \varphi = -i \xi \varphi, \quad \delta_\epsilon \varphi^\dagger = i \xi \varphi^\dagger, \quad \delta_\xi c_\nu = i \xi c_\nu, \quad \delta_\xi c_\nu^\dagger = -i \xi c_\nu^\dagger, \]
\[ \delta_\mu A_\mu = 0, \quad \delta_\mu B_\mu = \frac{1}{g} \partial_\mu \xi, \quad \delta_\mu C_\mu^+ = -2i \xi C_\mu^+, \quad \delta_\mu C_\mu^- = 2i \xi C_\mu^-, \] (5)

and the local fermionic transformations,
\[ \delta_\epsilon \varphi = -\zeta \varphi, \quad \delta_\epsilon \varphi^\dagger = 0, \quad \delta_\zeta c_\nu = 0, \quad \delta_\zeta c_\nu^\dagger = \zeta \varphi^\dagger, \]
\[ \delta_\epsilon A_\mu = -i \zeta C_\mu^+, \quad \delta_\epsilon B_\mu = 0, \quad \delta_\epsilon C_\mu^+ = 2i \zeta B_\mu + \frac{1}{g} \partial_\mu \zeta, \quad \delta_\epsilon C_\mu^- = 0, \] (6)
\[ \delta_\epsilon \varphi = 0, \quad \delta_\epsilon \varphi^\dagger = \zeta^\dagger c_\nu, \quad \delta_\epsilon \varphi = 0, \quad \delta_\epsilon \varphi^\dagger = 0, \]
\[ \delta_\epsilon A_\mu = -i \zeta^\dagger C_\mu^+, \quad \delta_\epsilon B_\mu = 0, \quad \delta_\epsilon C_\mu^+ = 0, \quad \delta_\epsilon C_\mu^- = -2i \zeta^\dagger B_\mu + \frac{1}{g} \partial_\mu \zeta^\dagger. \] (7)

where \( \epsilon \) and \( \xi \) are infinitesimal real functions of \( x \), and \( \zeta \) and \( \zeta^\dagger \) are Grassmann-valued functions of \( x \).

The \( \mathcal{L}_m \) and \( \mathcal{L}_{\bar{c}} \) are simply written as
\[ \mathcal{L}_m = \left( D_\mu \Phi \right)^\dagger \left( D^\mu \Phi \right) - m^2 \Phi^\dagger \Phi \quad \text{and} \quad \mathcal{L}_{\bar{c}} = -\frac{1}{4} \text{Str} \left( F_{\mu \nu} F^{\mu \nu} \right), \] (8)

respectively. In \( \mathcal{L}_m \), \( D_\mu \) and \( \Phi \) are the covariant derivative and the doublet of fermionic transformation defined by
\[ D_\mu \equiv \left( \partial_\mu + ig A_\mu + ig B_\mu \right) \quad \text{and} \quad \Phi \equiv \left( \phi \atop c_\nu \right). \] (9)

respectively. In \( \mathcal{L}_{\bar{c}} \), \( \text{Str} \) is the supertrace defined by \( \text{Str} = a - d \) where \( M \) is the \( 2 \times 2 \) matrix given by
\[ M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \] (10)

The \( F_{\mu \nu} \) is defined by
\[ F_{\mu \nu} \equiv \frac{1}{ig} \left[ D_\mu, D_\nu \right] = \begin{pmatrix} A_{\mu \nu} + B_{\nu \mu} & -i C^{\mu \nu}_{\nu} \\ i C^{\mu \nu}_{\mu} & A_{\mu \nu} - B_{\nu \mu} \end{pmatrix}, \] (11)

where \( A_{\mu \nu}, \ B_{\nu \mu}, \ C^{\mu \nu}_{\nu} \) and \( C^{\mu \nu}_{\mu} \) are the field strengths given by
\[ A_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig \left( C^{\mu \nu}_{\mu} C_\nu^+ - C^{\mu \nu}_{\nu} C_\mu^+ \right), \] (12)
\[ B_{\nu \mu} = \partial_\nu B_\mu - \partial_\mu B_\nu, \] (13)
\[ C^{\mu \nu}_{\nu} = \partial_\mu C_\nu^+ - \partial_\nu C_\mu^+ + 2ig \left( B_{\mu \nu} C_\nu^+ - B_{\nu \mu} C_\mu^+ \right), \] (14)
\[ C^{\mu \nu}_{\mu} = \partial_\mu C_\nu^+ - \partial_\nu C_\mu^+ - 2ig \left( B_{\mu \nu} C_\nu^+ - B_{\nu \mu} C_\mu^+ \right). \] (15)

Under the transformations (4)-(7), the field strengths are transformed as
\[ \delta_\xi A_{\mu \nu} = 0, \quad \delta_\xi B_{\mu \nu} = 0, \quad \delta_\xi C^{\mu \nu}_{\nu} = 0, \quad \delta_\xi C^{\mu \nu}_{\mu} = 0, \] (16)
\[ \delta_\zeta A_{\mu \nu} = 0, \quad \delta_\zeta B_{\mu \nu} = 0, \quad \delta_\zeta C^{\mu \nu}_{\nu} = -2i \xi C^{\mu \nu}_{\mu}, \quad \delta_\zeta C^{\mu \nu}_{\mu} = 2i \xi C^{\mu \nu}_{\nu}, \] (17)
\[ \delta_\epsilon A_{\mu \nu} = -i \xi C^{\mu \nu}_{\nu}, \quad \delta_\epsilon B_{\mu \nu} = 0, \quad \delta_\epsilon C^{\mu \nu}_{\nu} = 2i \xi B_{\mu \nu}, \quad \delta_\epsilon C^{\mu \nu}_{\mu} = 0. \] (18)
\[ \delta^a_\mu A^a_{\mu} = -i \xi^a \gamma^\mu \xi^a_{\mu}, \quad \delta^a_\mu B^a_{\mu} = 0, \quad \delta^a_\mu C^a_{\mu} = 0, \quad \delta^a_\mu D^a_{\mu} = -2i \xi^a \gamma^\mu B^a_{\mu}. \]  

Using the global fermionic transformations,

\[
\begin{align*}
\delta^a_\mu &\varepsilon = -c^a_\mu, \\
\delta^a_\mu \phi = 0, \\
\delta^a_\mu C^a_{\mu} = 0, \\
\delta^a_\mu \xi^a_{\mu} = \phi^a.
\end{align*}
\]

\[ \delta^a_\mu A^a_{\mu} = -i C^a_{\mu}, \quad \delta^a_\mu B^a_{\mu} = 0, \quad \delta^a_\mu C^a_{\mu} = 2i B^a_{\mu}, \quad \delta^a_\mu \xi^a_{\mu} = 0, \]

\[ \delta^a_\mu \phi = 0, \quad \delta^a_\mu \phi^a = c^a_{\mu}, \quad \delta^a_\mu C^a_{\mu} = 0, \quad \delta^a_\mu \xi^a_{\mu} = 0, \]

\[ \delta^a_\mu A^a_{\mu} = -i C^a_{\mu}, \quad \delta^a_\mu B^a_{\mu} = 0, \quad \delta^a_\mu C^a_{\mu} = 0, \quad \delta^a_\mu \xi^a_{\mu} = -2i B^a_{\mu}. \]

\[ \mathcal{L} \text{ is rewritten as} \]

\[ \mathcal{L} = \delta^a_\mu \delta^b_\mu \mathcal{L}_{\phi, A} = -\delta^a_\mu \delta^b_\mu \mathcal{L}_{\phi, A}, \]

where \[ \mathcal{L}_{\phi, A} \text{ is given by} \]

\[ \mathcal{L}_{\phi, A} = \left\{ \left( \partial_{\mu} - ig A_{\mu} + ig B_{\mu} \right) \phi^a - g C^a_{\mu} \phi \right\} \left\{ \left( \partial_{\mu} + ig A^a_{\mu} + ig B_{\mu} \right) \phi + g C^{a\mu} \phi \right\} - m^2 \phi^a \phi - \frac{1}{4} A_{\mu} A^{\nu}. \]

### 2.2. Spinor Fields with Local Fermionic Symmetries

For spinor fields, we consider the Lagrangian density,

\[ \mathcal{L}_{\psi, c_\psi} = i \overline{\psi} \gamma^\mu \partial_{\mu} \psi - m \overline{\psi} \gamma^\mu \partial_{\mu} \psi - m \overline{\psi} c_\psi, \]

where \[ \psi \text{ is an ordinary spinor field and } c_\psi \text{ is its bosonic counterpart obeying commutation relations. This system also has global } U(1) \text{ and fermionic symmetries, and the unitarity holds by imposing suitable subsidiary conditions on states.} \]

Starting from (24), the Lagrangian density with local symmetries is constructed as

\[ \mathcal{L}' = \mathcal{L}'_{\psi, c_\psi} + \mathcal{L}_{\psi}, \]

\[ \mathcal{L}'_{\psi, c_\psi} = i \overline{\psi} \gamma^\mu \left\{ \left( \partial_{\mu} + ig A_{\mu} + ig B_{\mu} \right) \psi + g C^a_{\mu} c_\psi \right\} - m \overline{\psi} \gamma^\mu \partial_{\mu} \psi - m \overline{\psi} c_\psi, \]

where \[ \mathcal{L}_{\psi} \text{ is given by} \]

\[ \mathcal{L}'_{\psi} = 0, \quad \overline{\psi} \psi \equiv \overline{\psi} \gamma^0, \quad \overline{\psi} c_\psi \equiv c_\psi \gamma^0 \text{ and } \gamma^\mu \text{ are the } \gamma \text{ matrices satisfying } \left\{ \gamma^\mu, \gamma^\nu \right\} = 2 \eta^{\mu \nu}. \]

The \[ \mathcal{L}'_{\psi, c_\psi} \text{ is rewritten as} \]

\[ \mathcal{L}'_{\psi, c_\psi} = i \overline{\psi} \Gamma^\mu \partial_{\mu} \psi - m \overline{\psi} \Psi, \]

where \[ \Gamma^\mu \text{ and } \Psi \text{ are the extension of } \gamma \text{-matrices and the doublet of fermionic transformation defined by} \]

\[ \Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix} \text{ and } \Psi = \begin{pmatrix} \psi \\ c_\psi \end{pmatrix}, \]

respectively.

The \[ \mathcal{L}' \text{ is invariant under the local } U(1) \text{ transformations,} \]

\[ \begin{align*}
\delta_{\psi} &\psi = -i \epsilon \psi, \\
\delta_{\psi}^\dagger &\psi = i \epsilon \psi^\dagger, \\
\delta_{\psi} &c_\psi = -i \epsilon c_\psi, \\
\delta_{\psi}^\dagger &c_\psi = i \epsilon c_\psi^\dagger.
\end{align*} \]

\[ \delta_{\psi} A_{\mu} = \frac{1}{g} \partial_{\mu} \epsilon, \quad \delta_{\psi} B_{\mu} = 0, \quad \delta_{\psi} C_{\mu} = 0, \quad \delta_{\psi} D_{\mu} = 0. \]
\[
\delta_\psi \psi = -i \xi \psi, \quad \delta_\psi \psi^\dagger = i \xi \psi^\dagger, \quad \delta_\psi c_\mu = i \xi c_\mu, \quad \delta_\psi c_\mu^\dagger = -i \xi c_\mu^\dagger,
\]
\[
\delta_\psi A_\mu = 0, \quad \delta_\psi B_\mu = -\frac{1}{g} \partial_\mu \xi, \quad \delta_\psi C_\mu = -2i \xi C_\mu^\dagger, \quad \delta_\psi C_\mu^\dagger = 2i \xi C_\mu
\]

(29)

and the local fermionic transformations,
\[
\delta_\psi \psi = -\zeta c_\mu, \quad \delta_\psi \psi^\dagger = 0, \quad \delta_\psi c_\mu = 0, \quad \delta_\psi c_\mu^\dagger = -\zeta \psi^\dagger,
\]
\[
\delta_\psi A_\mu = -i \zeta C_\mu^\dagger, \quad \delta_\psi B_\mu = 0, \quad \delta_\psi C_\mu^\dagger = 2i \zeta B_\mu + \frac{1}{g} \partial_\mu \zeta, \quad \delta_\psi C_\mu = 0,
\]
\[
\delta_\psi \psi = 0, \quad \delta_\psi \psi^\dagger = -\zeta^\dagger c_\mu^\dagger, \quad \delta_\psi c_\mu = \zeta^\dagger \psi, \quad \delta_\psi c_\mu^\dagger = 0,
\]
\[
\delta_\psi A_\mu = -i \zeta^\dagger C_\mu^\dagger, \quad \delta_\psi B_\mu = 0, \quad \delta_\psi C_\mu^\dagger = 0, \quad \delta_\psi C_\mu = -2i \zeta^\dagger B_\mu + \frac{1}{g} \partial_\mu \zeta^\dagger,
\]

(30)

(31)

where \( \epsilon \) and \( \xi \) are infinitesimal real functions of \( x \), and \( \zeta \) and \( \zeta^\dagger \) are Grassmann-valued functions of \( x \).

Using the global fermionic transformations,
\[
\delta_\psi \psi = -c_\mu, \quad \delta_\psi \psi^\dagger = 0, \quad \delta_\psi c_\mu = 0, \quad \delta_\psi c_\mu^\dagger = -\psi^\dagger,
\]
\[
\delta_\psi A_\mu = -i C_\mu^\dagger, \quad \delta_\psi B_\mu = 0, \quad \delta_\psi C_\mu^\dagger = 2i B_\mu, \quad \delta_\psi C_\mu = 0,
\]
\[
\delta_\psi \psi = 0, \quad \delta_\psi \psi^\dagger = -c_\mu^\dagger, \quad \delta_\psi c_\mu = \psi, \quad \delta_\psi c_\mu^\dagger = 0,
\]
\[
\delta_\psi A_\mu = -i C_\mu^\dagger, \quad \delta_\psi B_\mu = 0, \quad \delta_\psi C_\mu^\dagger = 0, \quad \delta_\psi C_\mu = -2i B_\mu,
\]

(32)

(33)

\( \mathcal{L}^a \) is rewritten as
\[
\mathcal{L}^a = \delta_\psi \delta_\psi^\dagger \mathcal{L}_{\psi,4} = -\delta_\psi \delta_\psi \mathcal{L}_{\psi,4},
\]

(34)

where \( \mathcal{L}_{\psi,4} \) is given by
\[
\mathcal{L}_{\psi,4} = i \bar{\psi} \gamma^\mu \left( \partial_\mu + igA_\mu + igB_\mu \right) \psi + g \bar{C}_\mu c_\mu^\dagger \left( m \bar{\psi} \psi - \frac{1}{4} A_\mu A^\mu \right).
\]

(35)

3. Quantization

We carry out the quantization of the system with scalar and gauge fields described by \( \mathcal{L} = \mathcal{L}_M + \mathcal{L}_G \).

3.1. Canonical Quantization

Based on the formulation with the property that the hermitian conjugate of canonical momentum for a variable is just the canonical momentum for the hermitian conjugate of the variable [16], the conjugate momenta are given by

\[
\pi = \left( \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right)_R = \left( \partial_0 - igA_0 - igB_0 \right) \phi^\dagger - g \bar{C}_0 c_\phi^\dagger,
\]

(36)

\[
\pi^\dagger = \left( \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right)_L = \left( \partial_0 + igA_0 + igB_0 \right) \phi + g \bar{C}_\phi c_\phi,
\]

(37)

\[
\pi_\psi = \left( \frac{\partial \mathcal{L}}{\partial \dot{C}_\phi} \right)_R = \left( \partial_0 - igA_0 + igB_0 \right) c_\phi^\dagger - g \bar{C}_\phi \phi^\dagger,
\]

(38)

\[
\pi_\psi^\dagger = \left( \frac{\partial \mathcal{L}}{\partial \dot{C}_\phi} \right)_L = \left( \partial_0 + igA_0 - igB_0 \right) c_\phi - g \bar{C}_\phi \phi,
\]

(39)
\[
\Pi^\mu_A = \left( \frac{\partial L}{\partial A^\mu} \right)_L = 2B^\mu, \quad \Pi^\mu_B = \left( \frac{\partial L}{\partial B^\mu} \right)_R = 2A^\mu, \quad (40)
\]

\[
\Pi^\mu_C = \left( \frac{\partial L}{\partial C^\mu} \right)_L = C^\mu, \quad \Pi^\mu_C = \left( \frac{\partial L}{\partial C^\mu} \right)_R = C^\mu, \quad (41)
\]

where \( \hat{O} = \partial O/\partial t \), and R and L stand for the right-differentiation and the left-differentiation, respectively. From (40) and (41), we obtain the primary constraints,

\[
\Pi^0_A = 0, \quad \Pi^0_B = 0, \quad \Pi^0_C = 0, \quad \Pi^0_C = 0. \quad (42)
\]

Using the Legendre transformation, the Hamiltonian density is obtained as

\[
H = \pi \phi + \phi^\dagger \pi^\dagger + \pi^\dagger \mathcal{C}_\phi + \mathcal{C}_\phi^\dagger \pi^\dagger + A_\lambda \Pi^\mu_A + B_\lambda \Pi^\mu_B + C_\lambda^+ \Pi^\mu_C^+ + C_\lambda^- \Pi^\mu_C^- - L
\]

\[
+ \lambda^0_A \Pi^0_A + \lambda^0_B \Pi^0_B + \lambda^0_C^+ \Pi^0_C^+ + \lambda^0_C^- \Pi^0_C^-
\]

\[
= \pi \pi^\dagger + \pi^\dagger \mathcal{C}_\phi + \mathcal{C}_\phi^\dagger \pi + \left(D \Phi) (D \Phi) + m^2 \Phi^\dagger \Phi - igA_\lambda \left( \pi \phi - \phi^\dagger \pi^\dagger + \pi^\dagger \mathcal{C}_\phi - \mathcal{C}_\phi^\dagger \pi \right)
\]

\[
- igB_\lambda \left( \pi \phi - \phi^\dagger \pi^\dagger - \pi^\dagger \mathcal{C}_\phi + \mathcal{C}_\phi^\dagger \pi \right) + 2C_\lambda^+ \Pi^\mu_C^+ - 2C_\lambda^- \Pi^\mu_C^-
\]

\[
- gC^\dagger_0 \left( \pi \phi - \phi^\dagger \pi^\dagger + iC^\dagger_1 \Pi^\mu_C^+ - 2iB_\lambda \Pi^\mu_C^+ \right) - g \left( \pi^\dagger \mathcal{C}_\phi - \pi^\dagger \mathcal{C}_\phi - iC^\dagger_1 \Pi^\mu_C^+ + 2iB_\lambda \Pi^\mu_C^+ \right) C^\dagger_0
\]

\[
+ \Pi^\mu_A \Pi^\mu_B + A_\lambda B_\lambda \Pi^\mu_A + B_\lambda \Pi^\mu_B + C_\lambda^+ \Pi^\mu_C^+ + \frac{1}{2} C_\lambda^+ C^\dagger_0 + \Pi^\mu_A \Pi^\mu_B + \Pi^\mu_C^+ \Pi^\mu_C^- + \Pi^\mu_C^- \Pi^\mu_C^-
\]

\[
+ \lambda^0_A \Pi^0_A + \lambda^0_B \Pi^0_B + \lambda^0_C^+ \Pi^0_C^+ + \lambda^0_C^- \Pi^0_C^-
\]

where Roman indices \( i \) and \( j \) denote the spatial components and run from 1 to 3, \( A_\lambda, B_\lambda, C_\lambda^+ \) and \( C_\lambda^- \) are Lagrange multipliers, and \( \lambda^0_A, \lambda^0_B, \lambda^0_C^+ \) and \( \lambda^0_C^- \) are rewritten as \( \lambda^0_A, \lambda^0_B, \lambda^0_C^+ \) and \( \lambda^0_C^- \) in the final expression.

Secondary constraints are obtained as follows,

\[
\frac{d\Pi^0_A}{dr} = \left\{ \Pi^0_A, H \right\}_PB = ig \left( \pi \phi - \phi^\dagger \pi^\dagger + \pi^\dagger \mathcal{C}_\phi - \mathcal{C}_\phi^\dagger \pi \right) + \partial_i \Pi^0_i = 0. \quad (44)
\]

\[
\frac{d\Pi^0_B}{dr} = \left\{ \Pi^0_B, H \right\}_PB = ig \left( \pi \phi - \phi^\dagger \pi^\dagger + \pi^\dagger \mathcal{C}_\phi - \mathcal{C}_\phi^\dagger \pi \right) + \partial_i \Pi^0_i = 0, \quad (45)
\]

\[
\frac{d\Pi^0_C^+}{dr} = \left\{ \Pi^0_C^+, H \right\}_PB = g \left( \pi \phi + \phi^\dagger \pi - \pi^\dagger \mathcal{C}_\phi^\dagger + \mathcal{C}_\phi \pi^\dagger \right) + \partial_i \Pi^0_i = 0, \quad (46)
\]

\[
\frac{d\Pi^0_C^-}{dr} = \left\{ \Pi^0_C^-, H \right\}_PB = g \left( \pi \phi - \phi^\dagger \pi^\dagger + \pi^\dagger \mathcal{C}_\phi - \mathcal{C}_\phi^\dagger \pi \right) + \partial_i \Pi^0_i = 0. \quad (47)
\]

where \( H \) is the Hamiltonian \( H = \int \mathcal{H} dl^3 x \) and \( \{ A, B \}_PB \) is the Poisson bracket. The Poisson bracket for the system with canonical variables \( (Q_i, P_i) \) and \( (Q'_i, P'_i) \) is defined by [16]

\[
\{ f, g \}_PB = \sum_k \left[ \left\{ \frac{\partial f}{\partial Q_k} \right\}_R \left( \frac{\partial g}{\partial P_k} \right)_L - \left\{ -\partial f/\partial Q_k \right\}_L \left( \frac{\partial g}{\partial P_k} \right)_R \right] + \left\{ -\partial f/\partial Q_k \right\}_L \left( \frac{\partial g}{\partial P_k} \right)_R \right] = \sum_k \left[ \left\{ \frac{\partial f}{\partial Q_k} \right\}_R \left( \frac{\partial g}{\partial P_k} \right)_L - \left\{ -\partial f/\partial Q_k \right\}_L \left( \frac{\partial g}{\partial P_k} \right)_R \right], \quad (48)
\]

where \( |Q| \) is the number representing the Grassmann parity of \( Q_k \), i.e., \( |Q| = 1 \) for the Grassmann odd \( Q_k \) and \( |Q| = 0 \) for the Grassmann even \( Q_k \). There appear no other constraints, and all constraints are first class ones and generate local transformations.

We take the gauge fixing conditions,
\( A^0 = 0, \quad B^0 = 0, \quad C^{i0} = 0, \quad C^{-i0} = 0, \quad \partial_i A' = 0, \quad \partial_i B' = 0, \quad \partial_i C^{+i} = 0, \quad \partial_i C^{-i} = 0. \quad (49) \)

The system is quantized by regarding variables as operators and imposing the following relations on the canonical pairs,

\[
\begin{align*}
&\{\varphi(x,t),\pi(y,t)\} = i\delta^3(x-y), \quad \{\varphi^\dagger(x,t),\pi^\dagger(y,t)\} = i\delta^3(x-y), \\
&\{c_\sigma(x,t),\pi_\sigma(y,t)\} = i\delta^3(x-y), \quad \{c_\sigma^\dagger(x,t),\pi_\sigma^\dagger(y,t)\} = -i\delta^3(x-y), \\
&\left[ A_i(x,t),\Pi_{ij}(y,t) \right] = i\left( \frac{\delta_j - \delta_i}{\Delta} \right)\delta^3(x-y), \\
&\left[ B_i(x,t),\Pi_{ij}(y,t) \right] = i\left( \frac{\delta_j - \delta_i}{\Delta} \right)\delta^3(x-y), \\
&\left[ C_i^+(x,t),\Pi_{ij}^+(y,t) \right] = -i\left( \frac{\delta_j - \delta_i}{\Delta} \right)\delta^3(x-y), \\
&\left[ C_i^-(x,t),\Pi_{ij}^-(y,t) \right] = i\left( \frac{\delta_j - \delta_i}{\Delta} \right)\delta^3(x-y).
\end{align*}
\]

where \([\mathcal{O}_1,\mathcal{O}_2] = \mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1, \quad \{\mathcal{O}_1,\mathcal{O}_2^\dagger\} = \mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_2\mathcal{O}_1\), and only the non-vanishing ones are denoted. Here, we define the Dirac bracket using the first class constraints and the gauge fixing conditions, and replace the bracket with the commutator or the anti-commutator.

On the reduced phase space, the conserved \(U(1)\) charges \(N_a\) and \(N_b\) and the conserved fermionic charges \(Q_\xi\) and \(Q_\xi^\dagger\) are constructed as

\[
\begin{align*}
N_a &= -i \int d^3x \left( \pi_\varphi - \varphi^\dagger \pi^\dagger + \pi_\sigma c_\sigma - c_\sigma^\dagger \pi_\sigma^\dagger \right), \\
N_b &= -i \int d^3x \left( \pi_\varphi - \varphi^\dagger \pi^\dagger - \pi_\sigma c_\sigma + c_\sigma^\dagger \pi_\sigma^\dagger + 2C_i^+ \Pi_{ij}^{-i} - 2\Pi_{ij}^+ C_i^- \right), \\
Q_\xi &= -\int d^3x \left( \pi c_\sigma - \varphi^\dagger \pi^\dagger + ic_i^+ \Pi_{ij}^+ - 2iB_i \Pi_{ij}^i \right), \\
Q_\xi^\dagger &= -\int d^3x \left( c_i^+ \pi^\dagger - \varphi^\dagger \pi^\dagger - iC_i^- \Pi_{ij}^+ + 2iB_i \Pi_{ij}^i \right).
\end{align*}
\]

The following algebraic relations hold:

\[
\begin{align*}
N_a^2 &= 0, \quad Q_\xi^{\dagger 2} = 0, \quad \{Q_\xi,Q_\xi^\dagger\} = N_a, \quad [N_a,Q_\xi] = 0, \quad [N_a,Q_\xi^\dagger] = 0, \\
[N_b,Q_\xi] &= -2Q_\xi, \quad [N_b,Q_\xi^\dagger] = 2Q_\xi^\dagger, \quad [N_a,N_b] = 0.
\end{align*}
\]

The above charges are generators of global \(U(1)\) and fermionic transformations such that

\[
\delta_\varphi^\xi \mathcal{O} = i\xi_0 N_a \mathcal{O}, \quad \bar{\delta}_\varphi^\xi \mathcal{O} = i\xi_0 N_b \mathcal{O}, \quad \delta_\sigma \mathcal{O} = i\xi_0 Q_\sigma \mathcal{O}, \quad \bar{\delta}_\sigma \mathcal{O} = i\xi_0 Q_\sigma^\dagger \mathcal{O}, \quad \delta_\xi^\dagger \mathcal{O} = i\xi_0 Q_\xi \mathcal{O}, \quad \bar{\delta}_\xi^\dagger \mathcal{O} = i\xi_0 Q_\xi^\dagger \mathcal{O},
\]

where \(\xi_0\) and \(\xi_0^\dagger\) are real parameters, and \(\xi_0\) and \(\xi_0^\dagger\) are Grassmann parameters. Note that \(\bar{\delta}_\xi\) and \(\bar{\delta}_\xi^\dagger\) in (20) and (21) are related to \(\delta_\xi\) and \(\delta_\xi^\dagger\) as \(\delta_\xi = \xi_0 \bar{\delta}_\xi^\dagger, \quad \delta_\xi^\dagger = \bar{\xi}_0 \delta_\xi\).

The system contains negative norm states originated from \(c_\sigma, \quad \bar{c}_\sigma^\dagger\) and \(C_i^\pm\). In the presence of negative norm states, the probability interpretation cannot be endured. To formulate our model in a consistent manner, we use a feature that conserved charges can be, in general, set to be zero as subsidiary conditions. We impose the following subsidiary conditions on states by hand,

\[
N_a \mid \text{phys} \rangle = 0, \quad N_b \mid \text{phys} \rangle = 0, \quad Q_\xi \mid \text{phys} \rangle = 0, \quad Q_\xi^\dagger \mid \text{phys} \rangle = 0.
\]
In the Appendix, we point out that subsidiary conditions corresponding to (62) can be realized as remnants of local symmetries in a specific case.

### 3.2. Unitarity

Let us study the unitarity of physical $S$ matrix in our system, using the Lagrangian density of free fields,

$$\mathcal{L}_0 = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi + \partial_\mu c_\mu^\dagger c_\mu - m^2 c_\mu^\dagger c_\mu - 2\partial_\mu A_i^\dagger B^i - \partial_\mu C_i^\dagger \partial^\mu C^{-i},$$  \hspace{1cm} (63)

where the gauge fixing conditions (49) are imposed on. The $\mathcal{L}_0$ describes the behavior of asymptotic fields of Heisenberg operators in $\mathcal{L} = \mathcal{L}_M + \mathcal{L}_G$.

From (63), free field equations for $\phi$, $\phi^\dagger$, $c_\mu$, $c_\mu^\dagger$, $A_i$, $B_i$ and $C_i^\pm$ are derived. By solving the Klein-Gordon equations, we obtain the solutions

$$\varphi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( a(k) e^{-ikx} + b^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (64)

$$\varphi^\dagger(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( a^\dagger (k) e^{ikx} + b(k) e^{-ikx} \right),$$  \hspace{1cm} (65)

$$\pi(x) = i\int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( a^\dagger (k) e^{ikx} - b(k) e^{-ikx} \right),$$  \hspace{1cm} (66)

$$\pi^\dagger(x) = -i\int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( a(k) e^{-ikx} - b^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (67)

$$c_\mu(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( c(k) e^{-ikx} + d^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (68)

$$c_\mu^\dagger(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( c^\dagger (k) e^{ikx} + d(k) e^{-ikx} \right),$$  \hspace{1cm} (69)

$$\pi_\mu(x) = i\int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( c^\dagger (k) e^{ikx} - d(k) e^{-ikx} \right),$$  \hspace{1cm} (70)

$$\pi_\mu^\dagger(x) = -i\int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( c(k) e^{-ikx} - d^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (71)

where $k_0 = \sqrt{k^2 + m^2}$ and $k\mu = k^\mu x_\mu$.

In the same way, by solving the free Maxwell equations, we obtain the solutions,

$$A_i(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( e_i^\mu a_\mu (k) e^{-ikx} + e_i^\mu a_\mu^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (72)

$$B_i(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( e_i^\mu b_\mu (k) e^{-ikx} + e_i^\mu b_\mu^\dagger (k) e^{ikx} \right),$$  \hspace{1cm} (73)

$$C_i^\pm(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left( e_i^\mu c_\mu (k) e^{-ikx} + e_i^\mu d_\mu (k) e^{ikx} \right),$$  \hspace{1cm} (74)
\[ C^-_i(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3}} \left( \epsilon^\alpha_i \epsilon^\beta_{\alpha} k e^{i\mathbf{k} \cdot \mathbf{x}} + \epsilon^\beta_{\alpha} d^\beta_{\alpha} (k) e^{-i\mathbf{k} \cdot \mathbf{x}} \right), \] (75)

\[ \Pi^+_i(x) = 2i \int d^3k \left( \epsilon^\alpha_i \epsilon^\beta_{\alpha} k e^{i\mathbf{k} \cdot \mathbf{x}} - \epsilon^\beta_{\alpha} a^\beta_{\alpha} (k) e^{-i\mathbf{k} \cdot \mathbf{x}} \right), \] (76)

\[ \Pi^+_i(x) = 2i \int d^3k \left( \epsilon^\alpha_i \epsilon^\beta_{\alpha} k e^{i\mathbf{k} \cdot \mathbf{x}} - \epsilon^\beta_{\alpha} a^\beta_{\alpha} (k) e^{-i\mathbf{k} \cdot \mathbf{x}} \right), \] (77)

\[ \Pi^+_i(x) = i \int d^3k \left( \epsilon^\alpha_i \epsilon^\beta_{\alpha} k e^{i\mathbf{k} \cdot \mathbf{x}} - \epsilon^\beta_{\alpha} a^\beta_{\alpha} (k) e^{-i\mathbf{k} \cdot \mathbf{x}} \right), \] (78)

\[ \Pi^+_i(x) = -i \int d^3k \left( \epsilon^\alpha_i \epsilon^\beta_{\alpha} k e^{i\mathbf{k} \cdot \mathbf{x}} - \epsilon^\beta_{\alpha} a^\beta_{\alpha} (k) e^{-i\mathbf{k} \cdot \mathbf{x}} \right), \] (79)

where \( k_0 = |\mathbf{k}| \) and \( \epsilon^\alpha_i \) are polarization vectors satisfying the relations,

\[ k_0 \epsilon^\alpha_i = 0, \quad \epsilon^\alpha_i \epsilon^\alpha_j = \delta^{\alpha\beta} \delta^{ij} \quad \sum_\alpha \epsilon^\alpha_i \epsilon^\alpha_j = \delta^i_j - \frac{k_j k_i}{k^2}. \] (80)

The index \( \alpha \) represents the helicity of gauge fields.

By imposing the same type of relations as (50) - (55), we have the relations,

\[ \left[ a(k), a^\dagger(l) \right] = \delta^\alpha_\alpha (k - l), \quad \left[ b(k), b^\dagger(l) \right] = \delta^\alpha_\alpha (k - l), \] (81)

\[ \left[ c(k), c^\dagger(l) \right] = \delta^\alpha_\alpha (k - l), \quad \left[ d(k), d^\dagger(l) \right] = -\delta^\alpha_\alpha (k - l), \] (82)

\[ \left[ a_\alpha (k), b^\dagger_\alpha (l) \right] = -\frac{1}{2} \delta_\alpha_\alpha \delta^\alpha_\alpha (k - l), \quad \left[ b_\alpha (k), a^\dagger_\alpha (l) \right] = -\frac{1}{2} \delta_\alpha_\alpha \delta^\alpha_\alpha (k - l), \] (83)

\[ \left[ c_\alpha (k), c^\dagger_\alpha (l) \right] = \delta_\alpha_\alpha \delta^\alpha_\alpha (k - l), \quad \left[ d^\dagger_\alpha (k), d^\dagger_\alpha (l) \right] = -\delta_\alpha_\alpha \delta^\alpha_\alpha (k - l), \] (84)

and others are zero.

The states in the Fock space are constructed by acting the creation operators \( a^\dagger (k), b^\dagger (k), c^\dagger (k), d^\dagger (k), \) \( a^\dagger_\alpha (k), b^\dagger_\alpha (k), c^\dagger_\alpha (k), d^\dagger_\alpha (k) \) on the vacuum state \( |0\rangle \) where \( |0\rangle \) is defined by the conditions \( a(k)|0\rangle = 0, \quad b(k)|0\rangle = 0, \quad c(k)|0\rangle = 0, \quad d(k)|0\rangle = 0. \) The projection operator \( P^{\alpha_\alpha} \) on the states with \( n \) particles is given by

\[ P^{\alpha_\alpha} = \frac{1}{n} \left\{ a^\dagger P^{(n-1)} a + b^\dagger P^{(n-1)} b + c^\dagger P^{(n-1)} c - d^\dagger P^{(n-1)} d + \sum_\alpha \left( -2a^\dagger a P^{(n-1)} a + 2b^\dagger b P^{(n-1)} a + c^\dagger c P^{(n-1)} a - d^\dagger d P^{(n-1)} a \right) \right\}, \] (86)

where \( n \geq 1 \) and we omit \( k \), for simplicity. Using the transformation properties,
\[ \tilde{\delta}_a a = -c, \quad \tilde{\delta}_a a^\dagger = 0, \quad \tilde{\delta}_b b = 0, \quad \tilde{\delta}_b b^\dagger = -d', \]
\[ \tilde{\delta}_c c = 0, \quad \tilde{\delta}_c c^\dagger = a', \quad \tilde{\delta}_d d = b, \quad \tilde{\delta}_d d^\dagger = 0, \]
\[ \tilde{\delta}_a a_a = -id_a, \quad \tilde{\delta}_a a^\dagger_a = -i c_a, \quad \tilde{\delta}_b b_a = 0, \quad \tilde{\delta}_b b^\dagger_a = 0, \]
\[ \tilde{\delta}_c c_a = 2i b_a, \quad \tilde{\delta}_c c^\dagger_a = 0, \quad \tilde{\delta}_d d_a = 0, \quad \tilde{\delta}_d d^\dagger_a = 2i b^\dagger_a, \]
\[ P^{(n)} \] is written in a simple form as
\[ P^{(n)} = i \{ Q, R^{(n)} \}, \]
where \[ R^{(n)} \] is given by
\[ R^{(n)} = \frac{1}{n} \left( c^\dagger P^{(n-1)} a + b^\dagger P^{(n-1)} d + i \sum_a \left( a^\dagger_a P^{(n-1)} c_a + d^\dagger_a P^{(n-1)} a_a \right) \right). \]

From (88), we find that any state with \[ n \geq 1 \] is unphysical from \[ \langle \text{phys} \mid P^{(n)} \mid \text{phys} \rangle = 0. \] Then, we understand that every field becomes unphysical, and only \[ \mid 0 \rangle \] remains as the physical state. This is also regarded as a field theoretical version of the Parisi-Sourlas mechanism \[ [20] \].

The system is also formulated using hermitian fermionic charges defined by \[ Q_1 = Q_a + Q_b^\dagger \] and \[ Q_2 = i (Q_a - Q_b^\dagger) \]. They satisfy the relations \[ Q_1 Q_2 + Q_2 Q_1 = 0, \quad Q_1^2 = N_1, \quad Q_2^2 = N_2 \]. Though \[ Q_1, Q_2 \] and \[ N_1, N_2 \] form elements of the \[ N = 2 \] (quantum mechanical) SUSY algebra \[ [21] \], our system does not possess the space-time SUSY because \[ N_1 \] is not our Hamiltonian but the \[ U(1) \] charge \[ N_2 \]. Only the vacuum state is selected as the physical states by imposing the following subsidiary conditions on states, in place of (85),
\[ N_1 \mid \text{phys} \rangle = 0, \quad N_2 \mid \text{phys} \rangle = 0, \quad Q_1 \mid \text{phys} \rangle = 0, \quad Q_2 \mid \text{phys} \rangle = 0. \]

It is also understood that our fermionic symmetries are different from the space-time SUSY, from the fact that \[ Q_1 \] and \[ Q_2 \] are scalar charges. They are also different from the BRST symmetry, as seen from the algebraic relations among charges.

The system with spinor and gauge fields described by \[ \mathcal{L}^S = \mathcal{L}_M^S + \mathcal{L}_G \] is also quantized, in a similar way. We find that the theory becomes harmless but empty leaving the vacuum state alone as the physical state, after imposing subsidiary conditions corresponding to (62).

### 3.3. BRST Symmetry

Our system has local symmetries, and it is quantized by the Faddeev-Popov (FP) method. In order to add the gauge fixing conditions to the Lagrangian, several fields corresponding to FP ghost and anti-ghost fields and auxiliary fields called Nakanishi-Lautrup (NL) fields are introduced. Then, the system is described on the extended phase space and has a global symmetry called the BRST symmetry. We present the gauge-fixed Lagrangian density and study the BRST transformation properties.

According to the usual procedure, the Lagrangian density containing the gauge fixing terms and FP ghost terms is constructed as
\[ \mathcal{L}_t = \mathcal{L}_M + \mathcal{L}_G + \mathcal{L}_\text{gf} + \mathcal{L}_\text{FP}, \]
\[ \mathcal{L}_G = -\partial_\mu b_\mu A^\mu + \partial_\mu b_\mu B^\mu + C^{\mu\nu} \partial_\mu \phi_\nu + \partial_\mu \phi_\nu C^{-\mu} + \frac{1}{2} \alpha \left( b^2 + b^2 + 2 \phi \phi \right), \]
\[ \mathcal{L}_\text{gf} = -\partial_\mu c_\mu A^\mu - \partial_\mu b_\mu B^\mu + C^{\mu\nu} \partial_\mu \phi_\nu + \partial_\mu \phi_\nu C^{-\mu} + \frac{1}{2} \alpha \left( b^2 + b^2 + 2 \phi \phi \right), \]
\[ \mathcal{L}_\text{FP} = -i \partial_\mu \overline{c}_\mu \left( \overline{\phi} c_\mu - i g \phi C^{\mu\nu} + i g \phi \phi \overline{C}^{\mu\nu} \right) - i \partial_\mu \overline{c}_b \phi_\mu - i \partial_\mu \overline{\phi}_b - i \partial_\mu \overline{\phi}_b^\dagger \left( \overline{\phi} \phi^\dagger - 2 ig c_b C^{\mu\nu} - 2 ig \phi^\dagger B^\mu \right), \]
where \[ c_\mu, c_\mu, \phi, \phi^\dagger \] are FP ghosts, \[ \overline{c}_\mu, \overline{c}_b, \overline{\phi} \] and \[ \overline{\phi}^\dagger \] are FP anti-ghosts, \[ b_\mu, b_\mu, \phi_\mu \] and \[ \phi_\mu^\dagger \] are NL fields, and \[ \alpha \] is a gauge parameter. These fields are scalar fields. \[ c_\mu, c_\mu, \overline{c}_\mu, \overline{c}_b \] are fermionic, and \[ b_\mu \] and \[ b_\mu \] are bosonic. In contrast, \[ \phi, \phi^\dagger, \overline{\phi} \] and \[ \overline{\phi}^\dagger \] are bosonic, and \[ \phi_\mu \] and \[ \phi_\mu^\dagger \] are fermionic because the relevant symmetries are fermionic.
The $L_{\tau}$ is invariant under the BRST transformation,
\[
\delta_{\text{BRST}}\varphi = -ig_c\varphi - ig_c\varphi - g\phi \varphi, \quad \delta_{\text{BRST}}\varphi^\dagger = ig_c\varphi^\dagger + ig_c\varphi^\dagger - g\varphi^\dagger \varphi^\dagger,
\]
\[
\delta_{\text{BRST}}c_\mu = -ig_c c_\mu + ig_c c_\mu - g\phi c_\mu^\dagger, \quad \delta_{\text{BRST}}c_\mu^\dagger = ig_c c_\mu^\dagger - ig_c c_\mu^\dagger + g\varphi^\dagger \varphi^\dagger,
\]
\[
\delta_{\text{BRST}}c_A = -ig \varphi^\dagger \varphi, \quad \delta_{\text{BRST}}c_B = 0, \quad \delta_{\text{BRST}}\varphi = -2ig_c \varphi, \quad \delta_{\text{BRST}}\varphi^\dagger = 2ig_c \varphi^\dagger,
\]
\[
\delta_{\text{BRST}}A_\mu = \partial_\mu c_A - ig \phi C_\mu^\dagger + ig\phi \varphi, \quad \delta_{\text{BRST}}B_\mu = \partial_\mu c_B,
\]
\[
\delta_{\text{BRST}}C_\mu^\dagger = -2ig_c \varphi C_\mu^\dagger + 2ig \phi B_\mu + \partial_\mu \phi, \quad \delta_{\text{BRST}}C_\mu = 2ig c_\varphi C_\mu + 2ig \phi^\dagger B_\mu - \partial_\mu \phi^\dagger,
\]
\[
\delta_{\text{BRST}}\varphi_A = ib_A, \quad \delta_{\text{BRST}}\varphi_B = ib_B, \quad \delta_{\text{BRST}}\varphi = i\varphi, \quad \delta_{\text{BRST}}\varphi^\dagger = -i\varphi^\dagger,
\]
\[
\delta_{\text{BRST}}b_A = 0, \quad \delta_{\text{BRST}}b_B = 0, \quad \delta_{\text{BRST}}\varphi = 0, \quad \delta_{\text{BRST}}\varphi^\dagger = 0,
\]
where the transformations for $\varphi$, $\varphi^\dagger$, $c_\mu$, $c_\mu^\dagger$, $A_\mu$, $B_\mu$, $C_\mu^\dagger$ and $C_\mu$ are obtained by regarding the sum of transformations $\varphi_0 + \varphi_1$, $c_\mu^\dagger$, $c_\mu$ as $\text{BRST} \varphi$, and replacing $\varepsilon$, $\xi$, $\zeta$ and $\zeta^\dagger$ with $g_c A$, $g_c B$, $g\phi$ and $-g\varphi^\dagger$, and those for $\varphi$, $c_\mu$, $\varphi$ and $\varphi^\dagger$ are determined by the requirement that $\delta_{\text{BRST}}$ has a nilpotency property, i.e., $\delta_{\text{BRST}}^2 = 0$.

The sum of the gauge fixing terms and FP ghost terms is simply written as
\[
L_{\text{gf}} + L_{\text{FP}} = i\delta_{\text{BRST}} \left\{ \partial_\mu \varphi A^\mu + \partial_\mu \varphi B^\mu + C_\mu \partial_\mu \varphi + \partial_\mu \varphi^\dagger \varphi^\dagger C_\mu - \frac{1}{2} \partial_c \left( \partial_\mu b_A + \partial_\mu b_B - \varphi^\dagger \varphi^\dagger \varphi^\dagger \varphi^\dagger \right) \right\},
\]
According to the Noether procedure, the BRST current $J_{\text{BRST}}^\mu$ and the BRST charge $Q_{\text{BRST}}$ are obtained as
\[
J_{\text{BRST}}^\mu = b_A \left( \partial_\mu c_A - ig \phi C_\mu^\dagger + ig \phi \varphi C_\mu^\dagger \right) - c_A \partial_\mu b_A + b_B \partial_\mu c_B - c_B \partial_\mu b_B
\]
\[
- \varphi^\dagger \left( \partial_\mu \varphi - 2ig_c \varphi C_\mu^\dagger + 2ig \phi B_\mu \right) + \varphi^\dagger \varphi^\dagger \varphi^\dagger
\]
\[
- 2gc_\varphi \varphi \varphi^\dagger \varphi^\dagger \varphi^\dagger - 2g\varphi^\dagger \varphi^\dagger \varphi^\dagger \varphi^\dagger - \partial_\nu \left( c_A A_\nu^\dagger \right) - \partial_\nu \left( \varphi C_\nu^\dagger \right) - \partial_\nu \left( \varphi C_\nu^\dagger \right)
\]
\[
Q_{\text{BRST}} = \int \! d^4x j_{\text{BRST}}^\mu = \int \! d^4x \left\{ b_A \left( \partial^\mu c_A - ig \phi C_\mu^\dagger + ig \phi \varphi C_\mu^\dagger \right) - c_A \partial^\mu b_A + b_B \partial^\mu c_B - c_B \partial^\mu b_B
\]
\[
- \varphi^\dagger \left( \partial^\mu \varphi - 2ig_c \varphi C_\mu^\dagger + 2ig \phi B_\mu \right) + \varphi^\dagger \varphi^\dagger \varphi^\dagger
\]
\[
- 2gc_\varphi \varphi \varphi^\dagger \varphi^\dagger \varphi^\dagger - 2g\varphi^\dagger \varphi^\dagger \varphi^\dagger \varphi^\dagger - \partial_\nu \left( c_A A_\nu^\dagger \right) - \partial_\nu \left( \varphi C_\nu^\dagger \right) - \partial_\nu \left( \varphi C_\nu^\dagger \right)
\]
\[
= 0,
\]
respectively. Here we use the field equations. The BRST charge is a conserved charge ($dQ_{\text{BRST}} / dt = 0$), and it has the nilpotency property such as $Q_{\text{BRST}}^2 = 0$.

By imposing the following subsidiary condition on states,
\[
Q_{\text{BRST}} \left| \text{phys} \right\rangle = 0,
\]
it is shown that any negative norm states originated from time and longitudinal components of gauge fields as well as FP ghost and anti-ghost fields and NL fields do not appear on the physical subspace, through the quartet mechanism. There still exist negative norm states come from $c_\varphi$, $c_\varphi^\dagger$ and $C_\mu^\dagger$, and it is necessary to impose additional conditions corresponding to (62) on states in order to project out such harmful states.
4. Conclusions and Discussions

We have studied the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. There exist negative norm states come from fermionic scalar fields (or bosonic spinor fields) and transverse components of fermionic gauge fields, even after reducing the phase space due to the first class constraints and the gauge fixing conditions or imposing the subsidiary condition concerning the BRST charge on states. By imposing additional subsidiary conditions on states, such negative norm states are projected out on the physical subspace and the unitarity of systems hold. The additional conditions can be originated from constraints in case that gauge fields have no dynamical degrees of freedom.

The systems considered are unrealistic if this goes on, because they are empty leaving the vacuum state alone as the physical state. Then, one might think that it is better not to get deeply involved them. Although they are still up in the air at present, there is a possibility that a formalism or concept itself is basically correct and is useful to explain phenomena of elementary particles at a more fundamental level. It is necessary to fully understand features of our particle-ghost symmetries, in order to appropriately apply them on a more microscopic system.

We make conjectures on some applications. We suppose that particle-ghost symmetries exist and the system contains only a few states including the vacuum as physical states at an ultimate level. Most physical particles might be released from unphysical doublets that consist of particles and their ghost partners. A release mechanism has been proposed based on the dimensional reduction by orbifolding [17].

After the appearance of physical fields, $Q^t_s$-singlets and $Q^t_d$-doublets coexist with exact fermionic symmetries. The Lagrangian density is, in general, written in the form as

$$\mathcal{L}_{\text{total}} = \mathcal{L}_s + \mathcal{L}_d + \mathcal{L}_{\text{mix}} = \mathcal{L}_s + \hat{\Delta} \mathcal{L}_{\text{mix}} + \hat{\Delta} \mathcal{L}_s \quad (\Delta \mathcal{L}) .$$

Here, $\mathcal{L}_s$, $\mathcal{L}_d$ and $\mathcal{L}_{\text{mix}}$ stand for the Lagrangian density for $Q^t_s$-singlets, $Q^t_d$-doublets and interactions between $Q^t_s$-singlets and $Q^t_d$-doublets. Under the subsidiary conditions $N^t_s |\text{phys}\rangle = 0$, $N^t_d |\text{phys}\rangle = 0$, $Q^t_s |\text{phys}\rangle = 0$, and $Q^t_d |\text{phys}\rangle = 0$ on states, all $Q^t_d$-doublets become unphysical and would not give any physical effects on $Q^t_s$ singlets. Because $Q^t_s$ singlets would not receive any radiative corrections from $Q^t_d$ doublets, the theory is free from the gauge hierarchy problem if all heavy fields form $Q^t_d$ doublets [15].

The system seems to be same as that described by $\mathcal{L}_s$ alone, and to be impossible to show the existence of $Q^t_d$-doublets. However, in a very special case, an indirect proof would be possible through fingerprints left by symmetries in a fundamental theory. The fingerprints are specific relations among parameters such as a unification of coupling constants, reflecting on underlying symmetries [15] [22].

In most cases, our ghost fields require non-local interactions [15] and the change of degrees of freedom can occur in systems with infinite numbers of fields [17]. Then, they might suggest that fundamental objects are not point particles but extended objects such as strings and membranes. Hence, it would be interesting to explore systems with particle-ghost symmetries and their applications in the framework of string theories1.

Acknowledgements

The author thanks the Editor and the referee for their comments. This work was supported in part by scientific grants from the Ministry of Education, Culture, Sports, Science and Technology under Grant No. 22540272.

References

[1] Neveu, A. and Schwarz, J.H. (1971) *Nuclear Physics B*, 31, 86-112. [http://dx.doi.org/10.1016/0550-3213(71)90448-2](http://dx.doi.org/10.1016/0550-3213(71)90448-2)
[2] Ramond, P. (1971) *Physical Review D*, 3, 2415-2418. [http://dx.doi.org/10.1103/PhysRevD.3.2415](http://dx.doi.org/10.1103/PhysRevD.3.2415)
[3] Aharonov, Y., Casher, A. and Susskind, L. (1971) *Physics Letters B*, 35, 512-514. [http://dx.doi.org/10.1016/0370-2693(71)90386-8](http://dx.doi.org/10.1016/0370-2693(71)90386-8)
[4] Gervais, J.L. and Sakita, B. (1971) *Nuclear Physics B*, 34, 632-639. [http://dx.doi.org/10.1016/0550-3213(71)90351-8](http://dx.doi.org/10.1016/0550-3213(71)90351-8)
[5] Becchi, C., Rouet, A. and Stora, R. (1975) *Communications in Mathematical Physics*, 42, 127-162. [http://dx.doi.org/10.1007/BF01614158](http://dx.doi.org/10.1007/BF01614158)
[6] Becchi, C., Rouet, A. and Stora, R. (1976) *Annals of Physics*, 98, 287-321. [http://dx.doi.org/10.1016/0003-4916(76)90156-1](http://dx.doi.org/10.1016/0003-4916(76)90156-1)

1Objects called ghost D-branes have been introduced as an extension of D-brane and their properties have been studied [23] [24].
Y. Kawamura

[7] Tyutin, V. (1975) Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism. Lebedev Physical Institute Report No.39, arXiv:0812.0580 [hep-th].

[8] Wess, J. and Zumino, B. (1974) *Nuclear Physics B*, 70, 39-50. http://dx.doi.org/10.1016/0550-3213(74)90355-1

[9] Wess, J. and Bagger, J. (1991) Supersymmetry and Supergravity. 2nd Edition. Princeton University Press, Princeton.

[10] Salam, A. and Strathdee, J. (1975) *Physical Review D*, 11, 1521-1535. http://dx.doi.org/10.1103/PhysRevD.11.1521

[11] Haag, R., Lopuszanski, J.T. and Sohnius, M. (1975) *Nuclear Physics B*, 88, 257-274. http://dx.doi.org/10.1016/0550-3213(75)90279-5

[12] Faddeev, L.D. and Popov, N. (1967) *Physics Letters B*, 25, 29-30. http://dx.doi.org/10.1016/0370-2693(67)90067-6

[13] Kugo, T. and Ojima, I. (1978) *Physics Letters B*, 73, 459-462. http://dx.doi.org/10.1016/0370-2693(78)90765-7

[14] Kugo, T. and Ojima, I. (1979) *Progress of Theoretical Physics Supplement*, 66, 1-130. http://dx.doi.org/10.1143/PTPS.66.1

[15] Kawamura, Y. (2015) *International Journal of Modern Physics A*, 30, Article ID: 1550153. http://dx.doi.org/10.1142/S0217751X15501535

[16] Kawamura, Y. (2014) Fermionic Scalar Field. http://arxiv.org/abs/1406.6155

[17] Kawamura, Y. (2015) *International Journal of Modern Physics A*, 30, Article ID: 1550056. http://dx.doi.org/10.1142/S0217751X15500566

[18] Kawamura, Y. (2015) Quantization of Systems with OSp(2|2) Symmetry. http://arxiv.org/abs/1502.00751

[19] Giddings, S. and Strominger, A. (1988) *Nuclear Physics B*, 306, 890-907. http://dx.doi.org/10.1016/0550-3213(88)90446-4

[20] Parisi, G. and Sourlas, N. (1979) *Physical Review Letters*, 43, 744-745. http://dx.doi.org/10.1103/PhysRevLett.43.744

[21] Witten, E. (1982) *Journal of Differential Geometry*, 17, 661-692.

[22] Kawamura, Y. (2015) *International Journal of Modern Physics A*, 30, Article ID: 1550109. http://dx.doi.org/10.1142/S0217751X15501092

[23] Okuda, T. and Takayanagi, T. (2006) *Journal of High Energy Physics*, 2006, 62.

[24] Terashima, S. (2006) *Journal of High Energy Physics*, 2006, 67.
Appendix
A1. System with Auxiliary Gauge Fields
Let us study the system without $G$ described by

$$\mathcal{L}_M = \left\{ \left( \partial_{\mu} - igA_{\mu} - igB_{\mu} \right) \varphi^\dagger - gC_{\mu} c_{\varphi} \right\} \left\{ \left( \partial_{\mu} + igA^{\mu} + igB^{\mu} \right) \varphi + gC^{-\mu} c_{\varphi} \right\} + \left\{ \left( \partial_{\mu} - igA_{\mu} + igB_{\mu} \right) c_{\varphi} \right\} \left\{ \left( \partial_{\mu} + igA^{\mu} - igB^{\mu} \right) c_{\varphi} - gC^{-\mu} \varphi \right\}$$

$$- m^2 \varphi \varphi - m^2 c_{\varphi} c_{\varphi} .$$

In this case, gauge fields do not have any dynamical degrees of freedom, and are regarded as auxiliary fields. The conjugate momenta of $\varphi$, $\varphi^\dagger$, $c_{\varphi}$ and $c_{\varphi}^\dagger$ are same as those obtained in (36) - (39). The conjugate momenta of $A_{\mu}$, $B_{\mu}$, $C_{\mu}^+$ and $C_{\mu}^-$ become constraints,

$$\Pi_{A} = 0, \quad \Pi_{B} = 0, \quad \Pi_{C}^{+} = 0, \quad \Pi_{C}^{-} = 0 .$$

Using the Legendre transformation, the Hamiltonian density is obtained as

$$\mathcal{H}_M = \pi \varphi^\dagger \pi + \pi_{c_{\varphi}} c_{\varphi} + \pi_{c_{\varphi}^\dagger} c_{\varphi}^\dagger + \pi \Delta_{\mu}^{\mu} + \pi B_{\mu} + \pi C_{\mu}^+ + \pi C_{\mu}^- - \mathcal{L}$$

$$= \pi \pi^\dagger + \pi_{c_{\varphi}} c_{\varphi} + \pi_{c_{\varphi}^\dagger} c_{\varphi}^\dagger + \left( D_{\mu} \Phi \right) \left( D_{\mu} \Phi \right)^\dagger + m^2 \Phi^\dagger \Phi$$

$$- igA_{\mu} \left( \pi \varphi^\dagger \pi + \pi_{c_{\varphi}} c_{\varphi} + \pi_{c_{\varphi}^\dagger} c_{\varphi}^\dagger \right) - igB_{\mu} \left( \pi \varphi^\dagger \pi + \pi_{c_{\varphi}} c_{\varphi} + \pi_{c_{\varphi}^\dagger} c_{\varphi}^\dagger \right)$$

$$- gC_{0} \left( \pi_{c_{\varphi}} c_{\varphi}^\dagger - \pi_{c_{\varphi}} c_{\varphi} \right)$$

$$+ \lambda_{A_{\mu}} \Pi_{A}^{\mu} + \lambda_{B_{\mu}} \Pi_{B}^{\mu} + \lambda_{C_{\mu}}^{+} \Pi_{C}^{+ \mu} + \lambda_{C_{\mu}}^{-} \Pi_{C}^{- \mu} ,$$

where $\lambda_{A_{\mu}}$, $\lambda_{B_{\mu}}$, $\lambda_{C_{\mu}}^{+}$ and $\lambda_{C_{\mu}}^{-}$ are Lagrange multipliers.

Secondary constraints are obtained as

$$\frac{d\Pi_{A}^{\mu}}{dt} = \left\{ \Pi_{A}^{\mu}, H_M \right\}_{PB} = gj_{A}^{\mu} = 0 ,$$

$$\frac{d\Pi_{B}^{\mu}}{dt} = \left\{ \Pi_{B}^{\mu}, H_M \right\}_{PB} = gj_{B}^{\mu} = 0 ,$$

$$\frac{d\Pi_{C}^{+ \mu}}{dt} = \left\{ \Pi_{C}^{+ \mu}, H_M \right\}_{PB} = gj_{C}^{+ \mu} = 0 ,$$

$$\frac{d\Pi_{C}^{- \mu}}{dt} = \left\{ \Pi_{C}^{- \mu}, H_M \right\}_{PB} = gj_{C}^{- \mu} = 0 ,$$

where $H_M$ is the Hamiltonian $H_M = \int H_M dx$, and $j_{A}^{\mu}$, $j_{B}^{\mu}$, $j_{C}^{+ \mu}$ and $j_{C}^{- \mu}$ are the currents of $U(1)$ and fermionic symmetries given by

$$j_{A}^{\mu} = i \left( \pi \varphi^\dagger \pi + \pi_{c_{\varphi}} c_{\varphi} - \pi_{c_{\varphi}^\dagger} c_{\varphi}^\dagger \right) ,$$

$$j_{B}^{\mu} = i \left[ \left( \partial_{\mu} - igA^{\mu} - igB^{\mu} \right) \varphi^\dagger - \pi \varphi \right] \left( \partial_{\mu} + igA^{\mu} + igB^{\mu} \right) \varphi + gC^{-\mu} c_{\varphi}$$

$$+ \left( \partial_{\mu} + igA^{\mu} - igB^{\mu} \right) c_{\varphi} - gC^{-\mu} \varphi \right] ,$$

$$j_{C}^{+ \mu} = i \left[ \left( \partial_{\mu} - igA^{\mu} - igB^{\mu} \right) \varphi^\dagger - \pi \varphi \right] \left( \partial_{\mu} + igA^{\mu} + igB^{\mu} \right) \varphi + gC^{-\mu} c_{\varphi}$$

$$+ \left( \partial_{\mu} + igA^{\mu} - igB^{\mu} \right) c_{\varphi} - gC^{-\mu} \varphi \right] ,$$

$$j_{C}^{- \mu} = i \left[ \left( \partial_{\mu} - igA^{\mu} - igB^{\mu} \right) \varphi^\dagger - \pi \varphi \right] \left( \partial_{\mu} + igA^{\mu} + igB^{\mu} \right) \varphi + gC^{-\mu} c_{\varphi}$$

$$+ \left( \partial_{\mu} + igA^{\mu} - igB^{\mu} \right) c_{\varphi} - gC^{-\mu} \varphi \right] ,$$
\[ j^0_a = i \left( \pi \varphi - \varphi^\dagger \pi^\dagger - \pi_{\varphi} c_{\varphi} + c_{\varphi}^\dagger \pi_{\varphi}^\dagger \right), \]
\[ j^0_b = \left[ \left( \varphi^\dagger - igA^i - igB^i \right) \varphi^\dagger - gC^i c_{\varphi} + c_{\varphi}^\dagger \left( \varphi^\dagger + igA^i + igB^i \right) \varphi + gC^i c_{\varphi} + c_{\varphi} \right], \]
\[ j^0_c = \pi c_{\varphi} - \varphi^\dagger \pi^\dagger, \]
\[ j^i_a = \left( \varphi^\dagger - igA^i - igB^i \right) \varphi^\dagger - gC^i c_{\varphi} - \varphi^\dagger \left( \varphi^\dagger + igA^i + igB^i \right) \varphi + gC^i c_{\varphi}, \]
\[ j^i_b = c_{\varphi}^\dagger \pi^\dagger - \pi_{\varphi} \varphi, \]
\[ j^i_c = c_{\varphi}^\dagger \left( \varphi^\dagger + igA^i + igB^i \right) \varphi + gC^i c_{\varphi} - \left( \varphi^\dagger - igA^i + igB^i \right) c_{\varphi}^\dagger - gC^i \varphi. \]

In the same way, tertiary constraints are obtained as
\[ \frac{d}{dt} j^0_a = \{ j^0_a, H^g_M \}_H = -\partial_j j^0_a = 0, \]
\[ \frac{d}{dt} j^0_b = \{ j^0_b, H^g_M \}_H = -\partial_j j^0_b = 0, \]
\[ \frac{d}{dt} j^0_c = \{ j^0_c, H^g_M \}_H = -\partial_j j^0_c = 0, \]
\[ \frac{d}{dt} j^i_a = \{ j^i_a, H^g_M \}_H = -\partial_j j^i_a = 0, \]
\[ \frac{d}{dt} j^i_b = \{ j^i_b, H^g_M \}_H = -\partial_j j^i_b = 0, \]
\[ \frac{d}{dt} j^i_c = \{ j^i_c, H^g_M \}_H = -\partial_j j^i_c = 0, \]

from the invariance under the time evolution of \( j^0_a = 0, j^0_b = 0, j^0_c = 0 \) and \( j^0_c = 0 \). On the other hand, the conditions \( \frac{d}{dt} j^0_a = \{ j^0_a, H^g_M \}_H = 0, \frac{d}{dt} j^0_b = \{ j^0_b, H^g_M \}_H = 0, \frac{d}{dt} j^0_c = \{ j^0_c, H^g_M \}_H = 0 \) and \( \frac{d}{dt} j^i_a = \{ j^i_a, H^g_M \}_H = 0 \) are not new constraints but the relations to determine \( \lambda^\dagger_a, \lambda^\dagger_b, \lambda^\dagger_c \), and \( \lambda^\dagger_i \). Furthermore, new constraints do not appear from the conditions \( d(\partial_j j^0_a)/dt = 0, d(\partial_j j^0_b)/dt = 0, d(\partial_j j^0_c)/dt = 0 \) and \( d(\partial_j j^i_a)/dt = 0 \).

The constraints are classified into the first class ones
\[ \Pi^0_a = 0, \Pi^0_b = 0, \Pi^0_c = 0, \Pi^0_c = 0, \]
and the second class ones
\[ \Pi^0_a = 0, \Pi^0_b = 0, \Pi^0_c = 0, \Pi^0_c = 0, \]
\[ j^i_a = 0, j^i_b = 0, j^i_c = 0, \]
\[ j^i_a = 0, j^i_b = 0, j^i_c = 0, \]
\[ \partial_j j^0_a = 0, \partial_j j^0_b = 0, \partial_j j^0_c = 0, \]
\[ \partial_j j^i_a = 0, \partial_j j^i_b = 0, \partial_j j^i_c = 0. \]

The determinant of Poisson bracket between second class ones does not vanish on constraints.

Using \( j^0_a, j^0_b, j^0_c, j^i_a, j^i_b, j^i_c \), the conserved \( U(1) \) and fermionic charges are constructed as
\[ N_a = -i \int \! dt \, j^0_a, \quad N_b = -i \int \! dt \, j^0_b, \quad N_c = -i \int \! dt \, j^0_c, \]
\[ Q_a = -i \int \! dt \, j^i_a, \quad Q_b = -i \int \! dt \, j^i_b, \quad Q_c = -i \int \! dt \, j^i_c. \]
\[ Q^I_\ell = -\int d^3x j^\alpha = -\int d^3x \left( c_\alpha^I \pi^\ell - \pi^\ell_\alpha \phi \right). \] (118)

The same algebraic relations hold as those in (60).

The above charges are conserved and generators of global $U(1)$ and fermionic transformations for scalar fields. They satisfy the relations,

\[ \{ N_\ell, \phi^\ell \} = 0, \quad \{ N_\ell, \phi^\ell_\ell \} = 0, \quad \{ Q^I_\ell, \phi^\ell \} = 0, \quad \{ Q^I_\ell, \phi^\ell_\ell \} = 0, \] (119)

where \( \phi^\ell \) are first class constraints (113) and the Hamiltonian \( H_M \). From (101) - (104) and (119), following relations can be considered as first class constraints,

\[ N_A = 0, \quad N_B = 0, \quad Q_I = 0, \quad Q^I_\ell = 0. \] (120)

After taking the following gauge fixing conditions for the first class ones (113),

\[ A^0 = 0, \quad B^0 = 0, \quad C^{\alpha 0} = 0, \quad C^{\beta 0} = 0, \] (121)

the system is quantized by regarding variables as operators and imposing the same type of relations (50) and (51) on the canonical pairs. From (120), it is reasonable to impose the following subsidiary conditions on states,

\[ N_A \vert \text{phys} \rangle = 0, \quad N_B \vert \text{phys} \rangle = 0, \quad Q_I \vert \text{phys} \rangle = 0, \quad Q^I_\ell \vert \text{phys} \rangle = 0. \] (122)

Then, they guarantee the unitarity of our system, though it contains negative norm states originated from \( c_\alpha \) and \( c^{\dagger}_\alpha \).