Development of a virtual balance for didactic purposes applied to mass metrology

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Abstract. Inmetro provides technical training in mass metrology area, it seeks to increase the number of hours of hands-on activities. Virtual resources to simulate practices may be used. The objective of this work is to develop a virtual balance that works on a PC and simulates real equipment’s behavior. It has been obtained an indication mathematical model of a real balance through study of physical parameters that influence on weighing and it has been programmed this mathematical model in Virtual Balance Graphical Interface. Mathematical model was validated by comparing Virtual Balance weighing results and experimental results obtained using a real balance.

Keywords. mass metrology; mathematical model; graphical interface

1. Introduction
The National Institute of Metrology, Quality and Technology (Inmetro), aware of its role in society and the importance for technician’s professional qualification, it offers technical courses in metrology [1]. One of these courses is analytical balance calibration course, which involves laboratory calibration practices, where the student applies the knowledge learned using analytical balances [2].

Inmetro Mass Laboratory (Lamas), which it provides technical training in mass metrology area, seeks to increase the number of hours of hands-on activities in analytical balance calibration training. One alternative to that is to use virtual resources to simulate practices [3].

The objective of this work is to develop a virtual balance that works on a personal computer and simulates the behavior of real equipment, which a technician can interact with through a graphical interface, and regarding that the Virtual Balance is a simulator, to adapt the trainings to varied situations through parameters configuration.

Thus, development of a Virtual Balance will enable more practical lessons with great versatility of setting operating conditions - allowing analyzing its influence on virtual weighing results - with lower costs, since it will not be necessary to have real balances or real environments. It will enable practical lessons and trainings to be performed by students at home or in a library via a computer.

Virtual balance is a computer program that displays an image of a balance on its screen. The balance has an indicator and a weighing pan, which is sensitive to the application of a virtual object (standard weight) [4], and thus user can interact with the balance by applying virtual objects to weighing pan by picking up, placing and then releasing the weight(s) by means of the mouse.
Weighing virtual objects available in Graphical Interface program allows Virtual Balance results to be compared to the behavior of a real balance.

2. Methodology
For the development of this work the following steps have been established:

- Study an analytical balance behavior through experimental tests, identified as balance reacts to physical effects that influence the weighing [5].
- Mathematical modelling of the physical effects that have been studied from real balance experimental measurements.
- Correlate mathematical model through Indication General Equation of Virtual Balance.
- Program a graphical interface (software) that enables user interaction with the Virtual Balance and the available objects within the weighing environment allowing to simulate the handling operations and showing the weighing results of an analytical balance.
- Validate the mathematical model present in the Graphical Interface by comparing the real and virtual weighing results.

3. Mathematical Modeling
In this work, it has been obtained a mathematical model of the balance indication from weighing data acquired from of a real balance analyzing the influence of various physical parameters that influence the weighing including environmental parameters and it has been programmed this mathematical model in Virtual Balance Graphical Interface. The following influence parameters have chosen for this work:

- Force due to weight [2];
- Effect of air buoyancy [2] [6];
- Eccentricity error when applying an object onto the balance weighing pan [2];
- Damping effect of mechanical oscillation caused by the impact, or smooth application, of object on the balance weighing pan [7];
- Balance indication drift or zero drift [2]; and
- Balance errors of indication in conditions before adjustment and after adjustment [2].

Mathematical model that correlate balance indication to these physical effects have been obtained by performing experimental weighing tests on a high accuracy mass comparator Sartorius model CCE6 which it has maximum capacity of 6 g and a resolution of 0.1 μg [8]. From here called CCE6 comparator. CCE6 comparator, which works by the EMFC principle [9], is extremely sensitive to weighing factors [2].

Experimental tests that supported the mathematical model of balance indication are described below:

Force due to weight [2] has been obtained from conventional mass [10] of standard weights used in experimental tests. The value of standard weight density influences the conventional mass value.

Effect of air buoyancy [2] is obtained from theoretical equation of air density [6] and it depends on temperature, humidity and pressure. User can set these input parameters in Graphical Interface.

Eccentricity error [2] has been obtained from experimental measurements with weights of 1 g, 2 g and 5 g. The eccentricity error is obtained by the difference between the indication when weight is loaded at the center and when it is loaded at close to the edge of the weighing pan. [2] The eccentricity error has been measured at eight positions close to the edge of the CCE6 comparator weighing pan, but for simplicity in this work, the mathematical model present in the Graphical Interface considers only a one-dimensional weighing pan (horizontal line) and the largest eccentricity errors.

Damping effect [7] has been obtained from the indication of balance response when placing or removing a weight on the weighing pan. Balance response has been compared to theoretical equation of damped harmonic oscillator. Due to the variability of the results obtained, the mathematical model
present in the Graphical Interface consists of a damped harmonic oscillator equation whose parameters can be set by the user in the Graphical Interface.

Balance indication drift or zero drift behavior has been obtained experimentally from continuous measurement of unloaded balance indication over several days. Thus, the daily zero drift behavior of CCE6 comparator has been obtained. In the Graphical Interface, the zero drift variation is added to the indication of Virtual Balance every second. This zero drift variation depends on the time of day.

Balance errors of indication in conditions before adjustment and after adjustment [2] have been determined experimentally by the difference between the indication of balance and the mass value of reference standard weights at five test points distributed over the weighing range of CCE6 comparator.

4. Indication General Equation of Virtual Balance
The Indication General Equation of Balance is present in the Graphical Interface program and the calculated output parameters are indicated in the Virtual Balance display. The results of the General Equation calculations indicated in display reflect the effects of eccentricity, error of indication, zero drift, damping, conventional mass and air buoyancy. The Indication General Equation of Virtual Balance is given by (1).

\[ I_{\text{Gen}} = (E_{\text{Ind}} + m_c E_{\text{Buo}} + E_{\text{Ecc}}) \times E_{\text{Dam}} + E_{\text{Dri}} \]  

where:
- \( I_{\text{Gen}} \) : Indication General Equation of Virtual Balance
- \( E_{\text{Dam}} \) : Variation due to damping
- \( E_{\text{Dri}} \) : Variation due to zero drift
- \( E_{\text{Ind}} \) : Error of Indication
- \( m_c \) : Conventional mass of standard weight
- \( E_{\text{Buo}} \) : Variation due to air buoyancy
- \( E_{\text{Ecc}} \) : Eccentricity Error

5. Graphical Interface
Finally, with the definition the Graphical Interface parameters together with the definition of the mathematical modeling parameters and the Indication General Equation of Virtual Balance, an algorithm has been developed that, based on the input data, is able to simulate the behavior of a balance through the Graphical Interface. The flowchart shown in figure 1 represents the schematic process to obtain the indication value in Graphical Interface.

Programming Graphical Interface of Virtual Balance has been performed using the Visual Basic language (VB) through the Microsoft Visual Studio program. Visual Basic with the Visual Studio package can be obtained free of charge from Microsoft's website [11].

A Graphical Interface has been developed that enables user interaction with the Virtual Balance and it simulates analytical balance behavior during measurements. Graphical Interface responds to physical effects variations and it can perform weighing procedure generating measurement results and measurement uncertainty [12] as well as a balance. Figure 2 shows the developed Graphical Interface of Virtual Balance [13].
Figure 1 – Flowchart that represents the process to obtain the indication value in Graphical Interface.

Figure 2 – Graphical Interface of Virtual Balance.

The following are the components and features of the Virtual Balance Graphical Interface. Standard weights of 100 mg, 500 mg, 1 g, 2 g and 5 g and a Virtual Balance with a one-dimensional weighing pan that reflect the largest eccentricity errors. Weights can be moved to the weighing pan by means of the mouse and the Virtual balance indicates the mass in milligrams. The Graphical Interface has “Set” button that opens a dialog box where it is possible modify the input variables of damping.
equation [7], weights density and environmental conditions. The Graphical Interface presents a graph showing the temporal response of the balance indication. The Graphical Interface has the “Adjust” button which, when clicked, simulates the Virtual Balance auto adjustment procedure [14].

6. Validation

After programming the Graphical Interface final version, it is necessary to evaluate if Virtual Balance is able to simulate a real equipment behavior, that is, if the developed Virtual Balance behaves like CCE6 comparator. It is necessary perform validation of the mathematical model present in developed Graphical Interface.

The acceptance criterion established for this work is: the difference between measurement results in CCE6 comparator and measurement results in Virtual Balance must be less than five times the resolution of CCE6 comparator. The acceptance criterion has been chosen so that the Virtual Balance Graphical Interface can adequately represent a real micro balance [15] [16]. Equation (2) presents the acceptance criterion established for this work.

\[
\Delta R = |R_{Exp} - R_{Virtual}| < 0,0005 \text{ mg}
\] (2)

where,
\[
\Delta R \quad \text{Difference between experimental result and Virtual Balance result}
\]
\[
R_{Exp} \quad \text{Experimental result}
\]
\[
R_{Virtual} \quad \text{Virtual Balance result}
\]

6.1. Error of indication

The error of indication is obtained by the difference between the indication of balance and the mass value of reference standard [1] [2]. Figure 3 shows the results obtained for Virtual Balance and experimentally using the CCE6 comparator in error of indication tests under the conditions: before adjustment and after adjustment.

![Figure 3 – Graph of error of indication tests for Virtual Balance and CCE6 comparator: before adjustment and after adjustment.](image-url)

Table 1 shows the values obtained for Virtual Balance and experimentally using the CCE6 comparator in error of indication tests under the conditions: before adjustment and after adjustment.
Table 1 – Values obtained in error of indication tests for Virtual Balance and CCE6 comparator: before adjustment and after adjustment.

| Nominal value | Error of indication [mg] | Criterion of acceptance [mg] |
|---------------|--------------------------|-----------------------------|
| CCE6 | Virtual Balance | $\Delta R$ | Criterion | Status |
| 100 mg | 0,00060 | 0,00048 | 0,00012 | 0,0005 | Ok |
| 500 mg | 0,00332 | 0,00304 | 0,00028 | 0,0005 | Ok |
| 1 g | 0,00542 | 0,00534 | 0,00008 | 0,0005 | Ok |
| 2 g | 0,01120 | 0,01106 | 0,00014 | 0,0005 | Ok |
| 3 g | 0,01634 | 0,01628 | 0,00006 | 0,0005 | Ok |
| 5 g | 0,02442 | 0,02448 | 0,00006 | 0,0005 | Ok |
| 100 mg | 0,00008 | -0,00006 | 0,00014 | 0,0005 | Ok |
| 500 mg | 0,00042 | 0,00024 | 0,00018 | 0,0005 | Ok |
| 1 g | -0,00056 | -0,00074 | 0,00018 | 0,0005 | Ok |
| 2 g | -0,00140 | -0,00148 | 0,00008 | 0,0005 | Ok |
| 3 g | -0,00260 | -0,00224 | 0,00036 | 0,0005 | Ok |
| 5 g | -0,00168 | -0,00178 | 0,00010 | 0,0005 | Ok |

From error of indication tests results obtained experimentally in the CCE6 comparator and obtained by the Virtual Balance Graphical Interface, the Virtual Balance presents a similar behavior to real balance and meets the acceptance criterion stipulated in this work.

6.2. Eccentricity

Eccentricity Tests for Virtual Balance Graphical Interface validation have been performed similarly to CCE6 comparator eccentricity tests for mathematical modeling, through measurements performed at the center and at eight points at close to the edge of weighing pan (angles $0^\circ$, $45^\circ$, $90^\circ$, $135^\circ$, $180^\circ$, $225^\circ$, $270^\circ$, $315^\circ$).

Table 2 presents the results for eccentricity tests obtained experimentally and the results obtained in the Virtual Balance. Experimental results have been obtained at $90^\circ$ and $270^\circ$ angles, because they reflect largest eccentricity errors in the CCE6 comparator and mathematical model used in the Graphical Interface has based on this data from these angles [13].

Table 2 – Results for eccentricity tests validation.

| CCE6 | Virtual Balance | Criterion of acceptance |
|------|----------------|-------------------------|
| 90°  | 270° | 90°  | 270° | $\Delta R$ | Criterion | Status | $\Delta R$ | Criterion | Status |
| [mg] | [mg] | [mg] | [mg] | [mg] | [mg] | Status | [mg] | [mg] | Status |
| 5 g  | 0,00031 | -0,00037 | 0,00029 | -0,00079 | 0,00002 | 0,00050 | Ok | 0,00042 | 0,00050 | Ok |
| 2 g  | 0,00022 | -0,00039 | 0,00010 | -0,00033 | 0,00012 | 0,00050 | Ok | 0,00005 | 0,00050 | Ok |
| 1 g  | 0,00008 | -0,00010 | 0,00006 | -0,00019 | 0,00003 | 0,00050 | Ok | 0,00009 | 0,00050 | Ok |
Figure 4 shows graph with comparative results between eccentricity tests obtained in the Virtual Balance and eccentricity tests obtained experimentally in the CCE6 comparator for angles 90° and 270°.

![Figure 4](image_url)

Figure 4 – Eccentricity tests results for Virtual Balance and CCE6 comparator.

According to eccentricity tests results presented in table 2 and figure 4, Virtual Balance presents significant differences in eccentricity value compared to values obtained in CCE6 comparator for the weight of 2 g at angle 90 ° and weight of 5 g at angle 270 °. Despite meeting the acceptance criterion, the results difference value for weight of 5 g at angle 270 ° is more than four times the CCE6 comparator resolution value and it is close to the criterion limit stipulated in this work.

6.3. Repeatability

The repeatability test consists of repeatedly loading and unloading of a single standard weight [4] in order to determine the degree of agreement between measurements using a statistical parameter, which is sample standard deviation [2]. Repeatability tests have been performed on Virtual Balance Graphical Interface with virtual weight of 5 g and 2 g and these results have been compared with the results obtained experimentally through the CCE6 comparator tests performed by the equipment manufacturer prior to their delivery to Inmetro Mass Laboratory [17].

In Repeatability Test the standard deviation is calculated by six averages from 12 differences with 13 successive mass comparisons of the same weight. Table 3 shows more clearly how repeatability test calculations are performed.

| Weighing | Indication [mg] | Difference [mg] | Mean value [mg] | Average of Mean values [mg] | Standard deviation [mg] |
|----------|----------------|----------------|----------------|-----------------------------|-------------------------|
| $i$      | $A_i$          | $d_{i1} = B_i - A_i$ | $x_i = \frac{d_{i1} + d_{i2}}{2}$ | $\bar{x} = \frac{1}{6} \sum_{i=1}^{6} x_i$ | $s = \sqrt{\frac{1}{6} \sum_{i=1}^{6} (x_i - \bar{x})^2}$ |
| $B_i$    | $d_{i2} = B_i - A_{i+1}$ |                |                |                             |                         |

Table 4 presents repeatability tests results for weight of 5 g and 2 g experimentally obtained in CCE6 comparator and results obtained in Virtual Balance Graphical Interface.
Table 4 – Repeatability tests results for weight of 5 g and 2 g experimentally obtained in CCE6 comparator and results obtained in Virtual Balance Graphical Interface.

| Load | Average/ Standard deviation | Result [mg] | Load | Average/ Standard deviation | Result [mg] |
|------|-------------------------------|-------------|------|-------------------------------|-------------|
| 5 g  | \( \bar{x} \) 0.000 02         | 0.000 07    | 5 g  | \( \bar{x} \) 0.000 03         | 0.000 10    |
| 2 g  | \( \bar{x} \) 0.000 12         | 0.000 10    | 2 g  | \( \bar{x} \) -0.000 02        | 0.000 14    |

From results presented in table 4 of repeatability tests performed in CCE6 comparator and Virtual Balance Graphical Interface, the values of standard deviations calculated is close to CCE6 comparator resolution value which is 0.0001 mg and the difference between the results are smaller than resolution of CCE6 comparator meeting the acceptance criterion [8].

Thus, from these repeatability test results, it is evident that the Virtual Balance presents a behavior similar to Sartorius CCE6 real balance regarding the degree of agreement between the measured values.

7. Conclusion

In this work a Virtual Balance has been developed that simulate the Sartorius CCE6 real balance behavior.

CCE6 comparator effects of eccentricity, zero drift, damping and error of indication have been studied from many measurements that generated a large quantity of experimental data which supported the real instrument behavior simulation through mathematical model. The characteristics obtained experimentally from a real balance have been mathematically correlated through Indication General Equation of Virtual Balance and also by the study of other external influences such as weight and buoyancy.

A Graphical Interface has been developed that enables user interaction with the Virtual Balance and it simulates analytical balance behavior during measurements. Graphical Interface responds to physical effects variations and it can perform weighing procedure generating measurement results and measurement uncertainty as well as a real instrument.

The validation of the mathematical model present in the Graphical Interface has been performed by comparing real and virtual mass measurement process. From the results obtained in the validation tests, it is noticed that measurements results performed in Virtual Balance and results performed experimentally in CCE6 comparator are similar meeting the acceptance criterion. It is concluded that the mathematical model present in Graphical Interface is able to simulate, within the limitations of the project, the behavior of a real instrument.

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