An Approach to the Cosmological Constant Problem(s). *

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March 26, 2022

*Talk given G.L. Kane at Rencontres de Moriond, 2004. To appear in the proceedings of this meeting
Abstract

We propose an approach to explaining why naïve large quantum fluctuations are not the right estimate for the cosmological constant. We argue the universe is in a superposition of many vacua, in such a way that the resulting fluctuations are suppressed by level repulsion to a very small value. The approach combines several aspects of string theory and the early history of the universe, and is only valid if several assumptions hold true. The approach may also explain why the effective cosmological constant remains small as the universe evolves through several phase transitions. It provides a non-anthropic mechanism leading to a small, non-zero cosmological constant.

1 Introduction

There are a number of “cosmological constant problems”.

(1) The naïve quantum fluctuations in a given vacuum state are divergent and therefore huge. We assume that if the universe is in a supersymmetric state the vacuum energy density vanishes. Conventionally one can write the vacuum energy density, which one would identify with the density of dark energy as

$$\rho_{de} \sim (n_{\text{bosons}} - n_{\text{fermions}}) m_{pl}^4 + (\tilde{m}^2 - m_{SM}^2) m_{pl}^2 + ...$$ (1)

where \(\tilde{m}\) is an appropriate supersymmetry mass scale, \(m_{SM}\) is the associated Standard Model mass, and \(m_{pl}\) is the Planck mass. We will see in the following that for our work the relevant time is at the beginning of inflation, and the inflaton energy density breaks supersymmetry \(\mathbb{R}\) so for us the parameter \(\tilde{m}\) will be the Hubble parameter \(H_{inf}\), which characterizes the inflation energy density at the beginning of inflation. We assume \(H_{inf}\) is in the range \(10^{13} - 10^{16}\) GeV at the beginning of inflation, a few orders of magnitude smaller than the Planck mass.

(2) The action for gravity plus a scalar field is

$$S \sim -\frac{1}{16\pi G_N} \int (R - 2\Lambda) \sqrt{g} d^4x + \int [-\frac{1}{2} (\partial \phi)^2 - V(\phi)] \sqrt{g} d^4x,$$ (2)

so a change in the origin of the potential is equivalent to generating a contribution to \(\Lambda\) since a constant term in \(V\) can just be shifted into the first term. This will happen at both the electroweak and QCD phase transitions as well as any others that may exist. Thus one can also ask, independently of (1), why does \(\rho_{de}\) stay small \((\lesssim 3 \times 10^{-3} eV)\) as the universe evolves through several effective theories where the typical scales are of order 100 GeV or 1 GeV or other relatively large energies?

(3) Physics also needs to explain the observed value of the cosmological constant, \(\rho_{obs}^{de} \simeq +(3 \times 10^{-12} GeV)^4\), the infamous “why now” problem since \(\rho_{obs}^{de}\) is similar in value to the observed amount of baryonic matter and of dark matter.
These problems are logically separate. Our approach \[2\], \[4\] addresses the first two, and possibly will be relevant for the third. It is worth reminding the reader that approaches to (3) must also explain (1) and (2) to have a full solution. There are many approaches to these problems, and we do not wish to criticize them. Perhaps our approach is complementary to some. In particular, it could well be that our approach could explain the first two problems, giving a very small dark energy density, and some alternative approach could account for the observed dark energy.

The validity of our approach will depend on the validity of several assumptions we make. If our approach turned out to be increasingly convincing, it would be evidence for the validity of our assumptions. However, it will be clear that most or all of our assumptions can be generalized and allow the approach to remain valid. We are focusing now on what we hope are sufficient conditions for the approach to apply. The assumptions can be broadened later. We hope at this stage to convince the reader that our approach is not obviously wrong, and is worthy of serious study.

2 The Basic Approach

Here we outline the basic approach. In the following sections we give more details and arguments for most of the points.

◦ We assume an underlying string theory, with a compactification that includes a 6D Calabi-Yau space with N=1 supersymmetry.

◦ We assume the arguments for a huge number of degenerate string vacua \[3\] are correct, with estimates of the number of possible vacua being \(10^{138}\) or perhaps many more.

◦ We assume that at \(t=0\) three space dimensions begin to inflate, giving an inflaton (we do not need to specify the nature of the inflaton for this argument) energy density \(\rho_{inf}\sim H_{inf}^2 m_{pl}^2\) and breaking supersymmetry. This will generate a potential with maxima and minima. We expect that the vacua will remain degenerate to within \(T_{Hawking} \sim \sqrt{\Lambda_{inf}} \sim H_{inf}\). Naively we expect the potential heights and depths to be of order \(H_{inf}\) since the potential becomes flat as \(H_{inf}\to0\). We expect \(H_{inf}/m_{pl}\sim10^{-3}\) to \(10^{-6}\). Thus naively all the minima should be at positive energy, though we do not have a derivation of this; our basic results do not depend strongly on this simple picture of the potential. This potential is similar to but not the same as the often discussed “landscape” because for us supersymmetry is broken by the inflaton energy density and we use the resulting potential at the beginning of inflation.

◦ We expect that the wavefunction of the universe cannot be confined to one vacuum. It must be a superposition over many vacua. This is the default from general principles, and any other assumption such as expecting the universe to be in one particular vacuum would require detailed justification.

◦ We expect the vacua to mix at the beginning of inflation since the universe is in a finite de Sitter space. We have calculated the tunneling rate for the de
Sitter double well and shown it is not significantly suppressed [4]. The tunneling is given in terms of the Hawking-Moss instanton [5]

- We expect, because of the mixing, an energy spectrum to arise analogous to the band spectrum in a solid. If \( N \) vacua mix significantly there will be \( N \) levels, and the lowest will have an energy density \( \sim H^2_{\text{inf}} m^2_{\text{pl}} / N \). One can think of this as a generalized level repulsion – if two levels mix, one would be lowered and the other raised, if three mix the bottom level is lowered more, etc.

- All this “occurs” in the first “instants” after inflation begins, where an instant is presumably of order a few Planck times.

- Then the universe relaxes to the ground state. This occurs rapidly, by several mechanisms. It could occur before the end of inflation, or take somewhat longer – more thinking and analysis is needed here. Mechanisms include graviton pair emission, normal tunneling driven by the Hawking temperature fluctuations, the fact that the levels have widths, and perhaps some kind of stringy mechanisms [6].

- It is important to understand that once the universe has relaxed to the ground state (or near it) inflation occurs in a normal way, ending with the release of the inflaton energy in a Big Bang. The universe is then no longer in inflating de Sitter space. The finiteness of de Sitter space is used in our mechanism only at the beginning of inflation.

- At the electroweak and QCD phase transitions and any others, the levels are rearranged and then the universe again relaxes to the ground state.

- The cosmological constant is small in the ground state, even though it would be large perturbatively in any particular vacuum. We emphasize that we are proposing an approach to solving the problem of why the cosmological constant is not large from quantum fluctuations. For our approach to be correct our assumptions must be basically correct. We have made concrete assumptions as outlined above. Most of them can probably be generalized. It may be that with further work the assumptions can be independently verified – most of them are consistent with respected thinking in the relevant subfields.

3 What is the Residual Dark Energy?  “Why Now?”

If the underlying string theory and the potential resulting from the inflaton energy density and consequent supersymmetry breaking were understood, we could, in principle, calculate the ground state energy density. In practice, of course, this is not possible. Perhaps the resulting energy density is positive but small compared to \( \rho_{\text{obs}}^{\text{de}} \), and another source of dark energy accounts for the current accelerated expansion of the universe, while our approach explains why the cosmological constant is not large. Perhaps, though, the ground state energy from our mechanism is equal to the observed one and our mechanism can also explain “why now?”. We cannot yet argue this, but of course we will try to find an argument for it.
4 Quantum Theory Analogy

To illustrate how the energy levels arise in a calculable example, we consider the way band spectra arise in a solid. The calculation is described in more detail in reference [2]. Consider a particle of mass $m$ moving in a periodic potential $V(x) = V_0(1 - \cos(2\pi x))/2$. The bottom of each well is like a simple harmonic oscillator, with frequency $\omega = \sqrt{2\pi^2 V_0/m}$. If there were only one such well it would have a ground state energy $\approx \hbar \omega/2$. But Bloch’s theorem says the ground state wave function is non-vanishing in all minima, not concentrated in one. Then the mixing generates a level repulsion and an energy spectrum. The instanton calculation of the spectrum is well known. Consider states $|n\rangle$ which are the simple harmonic oscillator states at the minimum $x = n$. One takes a trial Bloch wave function $|\theta\rangle = \sqrt{1/2}\pi\sum_n e^{in\theta} |n\rangle$ and calculates the matrix element $M_{\theta,\theta'} = \langle \theta' | e^{-\mathcal{H}t_E/\hbar} | \theta \rangle$ where $\mathcal{H}$ is the Hamiltonian and $t_E$ the Euclidean time. For large $t_E$, $M_{\theta,\theta'} \to \delta(\theta - \theta') e^{-E(\theta)t_E/\hbar}$, where $E(\theta)$ is the energy of the ground state. One finds $E(\theta) = \hbar \omega/2 - 2\hbar K \cos \theta e^{-S/\hbar}$. $K$ is a calculable determinant operator given by the product of the eigenvalues, and $S = \int \sqrt{2V(x)} dx$ is the instanton action. The lowest energy level occurs for $\theta = 0$, and the band has width $\Delta E = 4\hbar K e^{-S/\hbar}$. If there were $N$ discrete minima the boundaries would be the same for $N \gg 1$, but there would be $N$ discrete levels separated by typically $\Delta E/N$.

5 What is the Correct Generalization to the Early Universe?

A crucial physical point for us is what is the correct generalization to the early universe? First, energy should be replaced by energy density for field theory. If we were in infinite Minkowski space the action would be proportional to an infinite volume factor, so the factor $e^{-V_0}$ would reduce the tunneling to zero, and no band structure would form. But we argue strongly that the correct physical picture to have here is that our universe at the beginning of inflation has a positive energy density and thus is described by de Sitter space where space is of finite extent. There is general agreement [7] that for a scalar field theory with many minima the tunneling rate is not suppressed in de Sitter spacetime, and that the theory always has Euclidean instanton solutions such as the Hawking-Moss one, with non-zero action. As far as we can see the actual de Sitter double well calculation has not previously been reported, so we have carried out the calculation of the tunneling rate and confirmed it is indeed not suppressed [4]. We are in the process of extending the calculation to an asymmetric double well so we can quantitatively discuss a potential with minima of variable depth, and also the case with fermions so we can discuss the supersymmetric limit.
6 Estimate of the Ground State Energy

Here we follow the approach of the quantum theory example, and generalize to the string theory type of vacuum landscape. The calculation is described in detail in [2]. Assume the potential minima can be described by a hypercube lattice of minima at field points $n_1, ..., n_d$, with each connected to $d$ others by de Sitter tunneling, and altogether $N$ connected by multiple transitions. If each were connected to only a single nearest neighbor, then $d = 1$. If all vacua could tunnel directly to all others, then $d = N$. With

$$M_{\theta_1',...\theta_d',\theta_1...\theta_d} = \langle\{\theta_i\}\mid e^{-HtE/\hbar}\mid\{\theta_i\}\rangle \rightarrow \prod_i \delta(\theta_i' - \theta_i)e^{-\rho_{de}VtE/\hbar}$$

where $V$ is the volume of space and

$$\rho_{de} \approx H^2_{inf}m^2_{pl} - 2 \sum_d H^4_{inf} \cos \theta_i e^{-S/\hbar}.$$

Thus just as for the solid analogy, there is an energy spectrum. The spread of levels is of order $4H^4_{inf}d$, and there are $N$ levels so the lowest level has $\rho_{de} \sim 4H^4_{inf}d/N$. It is important to understand that the tunneling here is not spacetime tunneling from one metastable state to another, but a mechanism to calculate the energy spectrum including non-perturbative effects, and is only used at the beginning of inflation.

We cannot, of course, actually calculate this ground state energy at the present time. The best we can do is to see what is required for the approach to be consistent, which implies $\rho_{de} \leq \rho_{obs}^{\text{de}} = (3 \times 10^{-12} GeV)^4$. This works out if $d \leq 10^{12}$ and $N \gtrsim 10^{110}$. In the "landscape" these are quite reasonable numbers, small compared to the expected number of string vacua. Next we look at the implications of such numbers.

7 The Wavefunction of the Universe is a superposition of Many Vacua!

How can it make sense that the wave function of the universe is a superposition of many vacua? Can we imagine features such as the number of families or the SM gauge group emerging as the expectation value of a superposition? We would like to think that the vacua that mix have certain properties the same, perhaps in some sort of “superselection” sense. These should include the number of families, the gauge group, $N=1$ supersymmetry, massless quarks and leptons, and perhaps more. We note however that recently it has been argued that such quantities are not definite indices labeling string vacua. For a recent statement of this and references to other relevant work, see $\text{[3]}$. If our approach is valid such questions are a fascinating set of issues to study. One might imagine that quantities such as the dilaton vev that sets the values of the gauge couplings...
could more meaningfully emerge as a quantum mechanical expectation value. But even for this perhaps the dilaton vev is determined by a self-dual fixed point. Our result that \( N \gtrsim 10^{110} \) vacua are superimposed could alternatively be stated as saying that less than \( 10^{-30} \) (and perhaps far less if the larger estimates of Douglas et al are valid) of the vacua are connected, and that seems like a reasonable result. If all the vacua that are in the superposition share a set of properties that essentially determine the 4D effective field theory at the string scale the goal of calculating the detailed properties of our universe would remain a reasonable one.

8 Relaxation Time and Phase Transitions

We expect the universe to relax to its ground state in a short time. We cannot yet do better than dimensional analysis for estimating the relaxation time, but one can list several mechanisms that seem to allow very rapid cascading. Firstly, each individual level will have some width which is correlated with its time to decay to some lower energy level. Additionally, although the results for a full calculation of the energy spectrum would give a very complicated set of energy levels, since there are so many levels, they may effectively overlap. The presence of the Hawking temperature \( \sim H_{\text{inf}} \) provides a thermal driving of transitions that is very large for the higher levels. The emission of pairs of gravitons, which will occur on a scale of order the Planck time, should be an important mechanism. There could also be stringy mechanisms operating as well [6].

Phenomenologically the relaxation from the initial scale of order \( \sqrt{H_{\text{inf}} m_{pl}} \) to a scale of order 100 GeV would have to occur by about \( 10^{-12} \) seconds (when the electroweak phase transition occurs), which is a long time on the scale of Planck times. At such a phase transition the levels would presumably be rearranged and the universe would end up at some energy density scale, and then continue relaxing to the ground state. There could be earlier transitions such as a GUT one or a supersymmetry-breaking one, and the QCD transition occurs at a scale of order 1 GeV. Our approach seems to have the capability of incorporating these aspects of the history of the universe and explaining the apparent fine tunings needed to understand the history. The simplest outcome would be if the relaxation to the ground state occurred before the end of inflation (presumably of order \( 10^{-35} \) sec), and quite rapidly after each phase transition, but the relevant calculations have to be done to check that.

A useful analogy may be to think of the universe as a protein folding. The number of minima in the landscape of the potential of a protein can be of the same order as the number of string minima, and proteins normally find their ground state in a millisecond or less, which compared to typical atomic transitions suggests that it is reasonable to imagine a very rapid transition.
9 Comment on the Anthropic Principle

The properties of our world are determined by the physics of being in the ground state of the system of vacua, not by any particular vacuum. For simplicity we assume for the moment that the main global properties such as the number of families, etc., are shared by the vacua that mix significantly in the de Sitter space. Then the effective 4D theory at the string scale, which is presumably not far from the Planck scale, is (in principle) sufficient to allow calculation of all dimensionless ratios of parameters. Supersymmetry breaking and mediation are determined by the Lagrangian, and then electroweak symmetry breaking follows, all occurring with the universe in the ground state. We see no reason for any observable to be anthropic – the cosmological constant is the energy density of the ground state and could be calculated if we knew enough about string theory, the fermion mass ratios such as $m_u/m_d$ are determined by the superpotential, the Higgs vev by radiative electroweak symmetry breaking (for which the $\mu$ parameter and $\tan\beta$ are calculated from the high scale theory), and so on.

This leads to no discomfort about why the values of some parameters (and only some) seem to be rather near what is required for life in our universe. Some time ago it was realized that once inflation occurred it would be "eternal" [8] in that bubbles would inflate and separate from any inflating universe, giving an exponential growth of the number of universes. Physical parameters in each of them can be different since they depend on vacuum expectation values which can form differently, so the many many universes will sample a range of dimensionless ratios. Life will occur in all universes where it can – there is no need for detailed probability calculations.

10 Issues

Before we could be confident our approach can explain the long-standing problem of the apparent huge cosmological constant, much needs to be done. Here are some of the issues that have to be understood better:

- Extend the de Sitter tunneling analysis to asymmetric potentials, to be sure the result is not unduly sensitive to fluctuations in the potential depths and heights, and repeat the analysis with fermion fields so the supersymmetric limit can be examined.
- Understand the shape of the potential better, and whether the naïve argument that its heights and depths are of order $H_{inf}$ is correct for the case of interest to us, at the beginning of inflation.
- Understand the timescales for the universe to relax to the ground state.
- Understand the energy budget as a function of time: that is understand the energy of the ground state, the total energy, and relate them to energy scales.
later that involve supersymmetry breaking and electroweak symmetry breaking and the QCD condensation.

- Understand the superposition of many minima that could provide support for the wavefunction of the universe, and whether only vacua with three families, etc., contribute significantly to the wavefunction.
- Generalize the assumptions to less specific ones where possible.
- Find phenomenological tests of specific assumptions and of the whole picture.

If the approach is valid, these issues, while challenging, will be exciting to study.

11 Acknowledgements

GK would like to thank the organizers for their warm hospitality, particularly J. Tran Thanh Van and Jean-Marie Frere. We are grateful to a number of people for discussions and comments, A. Sevrin, L. Randall, N. Arkani-Hamed, G. Ross, J. March-Russell, M. Einhorn, M. Peskin, M. Douglas, M. Duff, J. Wells, J. Liu, R. McNees and Q. Shafi.

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