The Interplay between $\theta$ and $T$

W. Fischler, E. Gorbatov, A. Kashani-Poor, R. McNees, S. Paban, P. Pouliot

Department of Physics
University of Texas, Austin, TX 78712
E-mail: fischler, elie, kashani, mcnees
paban, pouliot@physics.utexas.edu

Abstract: We extend a recent computation of the dependence of the free energy, $F$, on the noncommutative scale $\theta$ to theories with very different UV sensitivity. The temperature dependence of $F$ strongly suggests that a reduced number of degrees of freedom contributes to the free energy in the non-planar sector, $F_{np}$, at high temperature. This phenomenon seems generic, independent of the UV sensitivity, and can be traced to modes whose thermal wavelengths become smaller than the noncommutativity scale. The temperature dependence of $F_{np}$ can then be calculated at high temperature using classical statistical mechanics, without encountering a UV catastrophe even in large number of dimensions. This result is a telltale sign of the low number of degrees of freedom contributing to $F$ in the non-planar sector at high temperature. Such behavior is in marked contrast to what would happen in a field theory with a random set of higher derivative interactions.

Keywords: noncommutative geometry, quantum field theory, supersymmetry, string theory, thermodynamics, winding states.
1. Introduction

Noncommutative field theories have proven to exhibit fascinating properties ([1]-[22]).

As we noted in a recent paper [6], there is a drastic reduction in the number of degrees of freedom that contribute to the free energy in the non-planar sector, $F_{np}$, at high temperatures. By the free energy in the non-planar sector, we mean those contributions to the free energy which arise from non-planar Feynman graphs. This reduction can be read off by looking at the temperature dependence of $F_{np}$. In this paper, we will present further evidence for this reduction, and study two further theories with very different UV sensitivity to test how generic this high temperature behavior is.

We will take $\theta_0 = 0$ throughout.

In [6], we also noted the existence of winding states in noncommutative field theories. Though we have no clean way of treating these modes separately from the conventional degrees of freedom of commutative field theory, it appears plausible that $F_{np}$ is indicative of these additional degrees of freedom, especially at high temperature, at which the winding states become light.
In studying the reduction of the degrees of freedom that contribute to $F_{np}$, we emphasize that this phenomenon does not depend on the sensitivity of the corresponding commutative theory to the ultraviolet [5]. Indeed, this reduction occurs in theories that are extremely insensitive to the ultraviolet, like the $N = 4$, $D = 4$ SYM theory, as well as theories with strong ultraviolet sensitivity, as eg. $\lambda \phi^3$ in $D = 6$.

In a nutshell, what we think happens is that as the temperature rises above the noncommutativity scale, the modes of the various fields with momenta of order the temperature, $T$, do not contribute to $F_{np}$. This occurs because, as the thermal wavelengths drop below the noncommutativity scale $\theta^{1/2}$, as far as their contribution to $F_{np}$ is concerned, there is no way to distinguish these high momenta modes from each other and thus no way to count their contribution individually. Another way of thinking about this is that the contributions to $F_{np}$ are sensitive to the phase space structure of space: the uncertainty principle among the spatial coordinates $x_i$ renders the identification of the degrees of freedom with regard to their contribution to $F_{np}$ impossible; this contribution seems to be summarized by a few quantum degrees of freedom contributing per noncommutative cell whose area is $\theta$. In the following, we will refer to this cell as the Moyal cell.

In the next section, we briefly review the results for the free energy of the Wess-Zumino model. We introduce a tool to test whether a reduction of degrees of freedom contributing to $F_{np}$ in fact takes place, the calculation of the classical statistical mechanics approximation to $F$, and discuss when we expect this approximation to be valid.

In the third section, we discuss the case of $N = 4$, $D = 4$ SYM. This theory also shows a reduction of degrees of freedom at high temperatures, although the theory is rather insensitive to the UV. We see that the nonplanar sector again contains winding states.

In the fourth section, we show how these results generalize to higher dimensions where there can be more noncommutative spatial directions. We show that the classical approximation remains valid for $\lambda \phi^3$ in spite of the high sensitivity of the corresponding commutative theory to the UV.

Finally, we end with conclusions.

2. The thermodynamics of the noncommutative Wess-Zumino model in perturbation theory

The Lagrangian density for the Wess-Zumino model is:
\[ \mathcal{L} = i \partial_{\mu} \bar{\psi} \gamma^\mu \psi + A^* \Box A - \frac{1}{2} M \psi \bar{\psi} - \frac{1}{2} M \bar{\psi} \psi - g \psi \psi A - g \bar{\psi} \bar{\psi} A^* - F^* F, \]

where \( F \) is given by

\[ F = -MA^* - gA^* A *. \]

We showed in a previous paper that in the noncommutative Wess-Zumino model, one can distinguish two regimes of high temperature, \( \beta M \ll 1 \).

These two regimes are: \( \theta T^2 \ll 1 \) and \( \theta T^2 \gg 1 \).

In the case \( \theta T^2 \ll 1 \), the thermal wavelength is larger than the noncommutativity scale. Here, \( F_{\text{np}} \) behaves as

\[ \left. \frac{F}{V} \right|_{\text{np}} \sim -g^2 T^4. \]  

(2.1)

In the other limit, the free energy density behaves as

\[ \left. \frac{F}{V} \right|_{\text{np}} \sim -g^2 \frac{T^2}{\theta} \log T^2 \theta. \]  

(2.2)

This expression for the free energy shows a dramatic reduction of the degrees of freedom contributing to \( F_{\text{np}} \).

Such a reduction has a natural interpretation in terms of the novel phase space structure due to the noncommutative nature of space. At any temperature, the typical momenta of the fields have magnitudes of order the temperature or smaller. As the temperature rises, the thermal wavelengths become smaller. When the temperature reaches values of \( O(\theta^{-1/2}) \), modes of the field with momenta of \( O(T) \) no longer contribute to \( F_{\text{np}} \).

More precisely, what we think happens is that at temperatures \( T^2 \gg 1/\theta \), the modes \( \phi_{k_1, k_2, k_3} \) with \( k_{1,2} \geq \theta^{-1/2} \) cannot be distinguished from each other insofar as their contribution to \( F_{\text{np}} \) is concerned, and therefore will not be counted individually in the non-planar contribution to the partition function. These modes lose their individual identity in the non-planar sector because the noncommutativity between \( x_1 \) and \( x_2 \), \( [x_1, x_2] \neq 0 \), implies an uncertainty relationship which renders impossible their separate identification. This may explain the reduction in the temperature dependence of the free energy from \( T^4 \) to \( T^2 / \theta \).

The non-planar sector yields a contribution to the free energy which, could this sector be isolated, would suggest that it has the number of degrees of freedom of a

\[ \text{We will assume that the Compton wavelength of the fields is bigger than the noncommutative scale, } \theta M^2 \ll 1. \]
1+1 dimensional field theory at temperatures $T \gg 1/\theta^{1/2}$. This is reminiscent of the result obtained by Atick and Witten [23] in the context of string theory. There is also a subleading, logarithmic dependence on the temperature in equation (2.2) for the free energy, which is all that remains from the contributions of the high momenta components of the fields.

As announced in the introduction, one can compare this computation to a classical statistical mechanics calculation of the free energy. The classical calculation can be done by just keeping the zero frequency term in the sum over frequencies. The result is

$$\left. \frac{F}{V} \right|_{\text{np}} \sim -g^2 T^2 \log \Lambda,$$

(2.3)

where $\Lambda$ is an ultraviolet cutoff.

The classical calculation captures the correct power law dependence in the temperature but replaces the logarithmic dependence on temperature obtained in quantum mechanics by $\log \Lambda$.

We should remind the reader that classical statistical mechanics is a good approximation at high temperatures for the free energy of systems with few degrees of freedom. For example, the high temperature behavior of the free energy for a particle of mass $m$ in a potential $V(x)$ is well approximated by classical statistical mechanics when the thermal de-Broglie wavelength $\lambda_{DB} \sim \frac{1}{(mT)^{1/2}}$ is smaller than the length scale over which the potential varies. In the path integral formulation, this would correspond to actually shrinking the temporal circle down to zero size and performing a dimensional reduction. In Feynman diagrams, this amounts to only keeping the contributions of the zero frequency components in the sum over frequencies.

In contrast, if one uses classical statistical mechanics to evaluate the free energy of systems with a field theory number of degrees of freedom at high temperature, one inevitably encounters a UV catastrophe.

On the other hand, classical statistical mechanics is a good approximation in field theory when calculating thermal correlation functions. An example is the calculation of the two point correlation function between two operators, $O_1(x)$ and $O_2(y)$,

$$\langle O_1(x)O_2(y) \rangle - \langle O_1(x) \rangle \langle O_2(y) \rangle .$$

Indeed, the modes that primarily contribute to this correlation function have wavelength commensurate with the distance $|x - y|$ separating the probes. As one heats the system, the population of these modes increases and hence their contribution to the correlation function is well approximated by classical statistical mechanics. So the classical approximation at high temperature becomes applicable in field theory when
there is a bound on the wavelengths of the modes that contribute to the quantity we are calculating.

The fact that $F_{np}$ is well approximated by classical statistical mechanics thus suggests that only a reduced set of degrees of freedom contributes to $F_{np}$ at high temperatures. This should be contrasted to the cases of field theories with an arbitrary infinite series of higher derivative terms in the Lagrangian. In such generic cases, if one attempts to approximate the free energy by a classical statistical calculation, one encounters enhanced UV catastrophes as compared to the theory without the higher derivative terms. The averting of the UV catastrophe due to the special choice of higher derivative terms in noncommutative field theories is therefore quite remarkable.

3. The thermodynamics of $N = 4$, $D = 4$ Super Yang-Mills

Our interest in examining this case is to see how crucial the ultraviolet sensitivity of a noncommutative theory is to the existence of winding states and the high temperature behavior of $F_{np}$. We will show that the existence of winding states as well as the reduction at high temperatures in the number of degrees of freedom contributing to $F_{np}$ are independent of the UV behaviour of the noncommutative field theory.

The calculation of the free energy is straightforward, the result can be found in the literature [16, 9]. We present for completeness some of the steps in the calculation of the free energy in appendix A.

The contribution to the free energy coming from the nonplanar sector is:

$$\left. \frac{F}{V_{np}} \right|_{np} = -4g^2N \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{e^{ip\theta k}}{4\omega_p \omega_k} \left( n_B(\omega_p) + n_F(\omega_p) \right) \left( n_B(\omega_k) + n_F(\omega_k) \right). \quad (3.1)$$

Up to an overall factor, this is exactly the result we found in the Wess-Zumino case [6].

The presence of winding states can again be detected in the non-planar sector. Indeed, by performing the integration over one set of momenta in eq. (3.1), one finds contributions to the free energy that are weighted by the length of the temporal circle. This can be seen for example by considering the following factor in eq. (3.1):

$$\int \frac{d^3k}{(2\pi)^3} \frac{e^{ip\theta k}}{\omega_k} n_B(\omega_k) = \sum_{n=1}^{\infty} \frac{1}{n^2 \beta^2 + (\theta p)^2}. \quad (3.2)$$

We can again compare the limits $T^2 \theta \ll 1$ and $T^2 \theta \gg 1$. In the case where the thermal wavelength is larger than the noncommutativity scale, the free energy behavior is:
In the limit where the thermal wavelength is within the noncommutative scale,
\[ \left. \frac{F}{V} \right|_{np} \sim -g^2 N T^4. \] (3.3)

This reveals again a reduction in the number of degrees of freedom contributing to \( F_{np} \).

There seems to be a universal behavior that whenever the inverse momenta fit into a Moyal cell, the associated components of the field cannot be distinguished in their contributions to \( F_{np} \). What is then left over seems to be the contribution due to a few quantum degrees of freedom per Moyal cell.

This picture will be further tested in the next section, where we will consider a higher dimensional example.

4. The thermodynamics of \( g\phi^3 \) in \( D = 6 \) dimensions

Strictly speaking, this theory does not have good thermodynamic behavior since it lacks a ground state. Therefore, in order to discuss the thermodynamics of this system, we will take the coupling constant to be very small and the temperature to be small enough\(^2\) such that the excursions of the field \( \phi \) away from the local minimum of the potential are within the bounded region of the potential. This implies that the temperature satisfies the inequality \( T \ll \frac{M}{g^{1/2}} \).

The Lagrangian density for this system is:
\[ \mathcal{L} = \int d^6x \left( (\partial \phi)^2 - M^2 \phi^2 - g \phi \ast \phi \ast \phi \right). \]

The free energy in this case has no infrared divergences to leading order in an expansion in the coupling constant \( g \). This is because the high dimensionality of the theory softens the IR sensitivity of the free energy.

The non-planar contribution to the free energy at \( O(g^2) \) is:
\[ \left. \frac{F}{V} \right|_{np} = -g^2 T^2 \sum_{n,l} \int \frac{d^5p}{(2\pi)^5} \frac{d^5k}{(2\pi)^5} \frac{e^{i p \theta k}}{( \frac{4\pi^2 p^2}{\beta^2} + k^2)( \frac{4\pi^2 p^2}{\beta^2} + p^2)( \frac{4\pi^2 (n+l)^2}{\beta^2} + (k+p)^2)} \]. (4.1)

Again, one finds winding states in this theory. This can easily be seen by rewriting eq. (4.1) in the form\(^3\):

\(^2\)while still keeping it much larger than the mass
\(^3\)for details, see appendix B
where we can think of $\alpha_2 + \alpha_1 x (1 - x)$ as the proper time for the propagator of momentum states and $\frac{1}{\alpha_1}$ as the proper time for the propagator of winding states.

Under the restriction that $\theta_{0i} = 0$, the most general case in $D = 6$ is, by a convenient choice of coordinate system, given by nonvanishing $\theta_{12}$ and $\theta_{34}$. If we take these two parameters to be of the same order of magnitude, one can distinguish two cases:

1. $T^2 \ll \frac{1}{\theta_{12}}, \frac{1}{\theta_{34}}$

2. $T^2 \gg \frac{1}{\theta_{12}}, \frac{1}{\theta_{34}}$

In the first case, the contribution to the free energy from the nonplanar sector scales, as a function of temperature, like $T^6$. This is because the exponential involving the Moyal phase does not oscillate much when the momenta are distributed according to the thermal distributions.

In the high temperature limit where $T^2 \gg \frac{1}{\theta_{12}}, \frac{1}{\theta_{34}}$, the possibility of estimating the behavior of the free energy using classical physics arises. Indeed, as discussed above, if the system has many fewer degrees of freedom contributing to the $\theta$ dependence of the free energy, as was the case in the four dimensional examples, then this approximation is valid. Whether or not the approximation is valid is decided a posteriori: if the remaining integrals are finite, then indeed classical statistical mechanics is a good approximation and gives the dominant contribution at high temperature.

Performing the classical statistical mechanics calculation, i.e. evaluating the following expression,

\[
\left. \frac{F}{V} \right|_{np} \sim -g^2 T^2 \int \frac{d^5k}{(2\pi)^5} \frac{d^5p}{(2\pi)^5} e^{i p \theta k} \, e^{i p \theta k} ,
\]

(4.3)

we find no UV divergences.

When $\theta \sim \theta_{12} \sim \theta_{34}$,

\[
\left. \frac{F}{V} \right|_{np} \sim -g^2 \frac{T^2}{\theta_{34}^2 - \theta_{12}^2} \log \frac{\theta_{12}}{\theta_{34}} \sim -\frac{g^2 T^2}{\theta^2} ,
\]

(4.4)

This behavior is again consistent with the picture on how degrees of freedom, as their inverse momenta fall into Moyal cells, do not contribute to $F_{np}$. 

7
5. Conclusions

At high temperatures, one observes a substantial reduction in the number of degrees of freedom that contribute to $F_{np}$, the free energy due to the non-planar sector of the theory. The picture that emerges from the previous sections is that this phenomenon can be traced to the modes of the fields with momenta larger than the noncommutativity scales. What happens is that once the wavelengths are within the Moyal cells, there is no way to distinguish and count the separate contributions of these modes to $F_{np}$. What is left over seems to be the contribution of a single degree of freedom per Moyal cell.

Because of this severe reduction in the number of degrees of freedom contributing to $F_{np}$, this quantity can be calculated at high temperature using classical statistical mechanics.

This behavior does not appear to depend on the details of the noncommutative field theory, the crucial element is the existence of a phase space structure for space.

Acknowledgments

We thank Nathan Seiberg for very useful discussions. The work of WF, EG, RMcN, AK-P, SP, PP is supported in part by the Robert Welch Foundation and the NSF under grant number PHY-9511632. SP is also supported by NSF grant PHY-9973543.

A. The free energy of $N=4, D=4$ SYM

We are working in Euclidean spacetime.

Let’s begin with the commutative case. We choose the decomposition of the 32 dimensional $\Gamma$-matrices given in [24]. This yields the dimensionally reduced Lagrangian

$$\mathcal{L} = 2 \text{Tr} \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{i}{2} \bar{\chi}_K \sigma^\mu D_\mu \chi_K - \frac{1}{2} D_\mu \phi_i D^\mu \phi_i - \frac{1}{2} D_\mu \varphi_i D^\mu \varphi_i \right)$$

$$- \frac{i}{2} g \alpha_{KL}(\chi_K [\phi_i, \chi_L] + \bar{\chi}_K [\phi_i, \bar{\chi}_L]) - \frac{i}{2} g \beta_{KL}(\chi_K [\varphi_i, \chi_L] + \bar{\chi}_K [\varphi_i, \bar{\chi}_L])$$

$$- \frac{1}{4} g^2 ([\phi_i, \phi_j] [\phi_i, \phi_j] + [\varphi_i, \varphi_j] [\varphi_i, \varphi_j] + 2 [\phi_i, \varphi_j] [\phi_i, \varphi_j]),$$

where $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu], D_\mu = \partial_\mu + ig [A_\mu, \cdot]$ and $\mu, \nu$ range from 1 to 4. Remember that this theory contains one N=1 vector multiplet and three N=1 chiral multiplets. Thus $i$ ranges from 1 to 3, $K$ and $L$ from 1 to 4. $\gamma_\mu, \alpha_i$ and $\beta_i$ satisfy $\{\gamma_\mu, \gamma_\nu\} = -2 \delta_{\mu \nu}, \{\alpha_i, \alpha_j\} = -2 \delta_{ij}, \{\beta_i, \beta_j\} = -2 \delta_{ij}$ and commute among each other.
The diagrams that contribute to the free energy at two loop are depicted in the following figure:

At \( T = 0 \), their contribution must vanish. It is easy to see that each diagram is proportional to

\[
g^2 f_{abc} f_{abc} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \frac{1}{k^2 p^2}.
\]

The coefficients of the various contributions are given in the following table:

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
|   | -3 | \( \frac{9}{7} \) | 4 | -1 | \( \frac{9}{7} \) | \( \frac{9}{7} \) | 12 | 0 |

We pass to finite temperature by replacing the integration over energies by a sum over even or odd Matsubara frequencies, for bosonic, fermionic degrees of freedom respectively.

The noncommutative case does not require any additional calculation. We obtain the noncommutative theory by replacing the commutators in the Lagrangian (A.1) by Moyal brackets. This yields, for arbitrary fields \( A, B \), which take values in the fundamental representation,

\[
[A, B]_* (x) = \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \hat{A}^a (p) \hat{B}^b (k) e^{i(p+k)x} \left( [t^a, t^b] \cos \frac{k \theta p}{2} + i\{t^a, t^b\} \sin \frac{k \theta p}{2} \right).
\]

For the gauge group \( U(N) \), both the commutator and the anticommutator of generators of the fundamental close within the algebra:

\[
[t^a, t^b] = i f_{abc} t^c, \quad \{t^a, t^b\} = d_{abc} t^c.
\]
The noncommutative result is thus obtained from the commutative one by replacing
\[ f_{abc} \to f_{abc} \cos \frac{k\theta p}{2} + d_{abc} \sin \frac{k\theta p}{2}. \]

To obtain (3.1), we need to perform the sums \( f_{abc}f_{abc} \) and \( d_{abc}d_{abc} \). The first is of course identical to the \( SU(N) \) result, \( N(N^2 - 1) \). The second can be obtained from the \( SU(N) \) result, \( N(N - \frac{1}{N}) \), where \( \{t^a, t^b\} = d_{abc}^{SU(N)} t^c + \frac{1}{N} \delta_{ab} \), by including the \( U(1) \) generator \( t^{N^2} = \frac{1}{\sqrt{2N}} \). This gives \( N(N^2 + 1) \).

**B. The free energy of the \( D = 6, g\phi^3 \) theory**

We will first briefly show how the winding states appear in this case, then we will perform the classical statistical mechanics calculation of the non-planar contribution to \( O(g^2) \) to the free energy.

The nonplanar contribution to the free energy to \( O(g^2) \) is
\[
-g^2 T^2 \sum_{n,l} \int \frac{d^5p}{(2\pi)^5} \frac{d^5k}{(2\pi)^5} \frac{e^{i(\theta_1(p_1k_2 - p_2k_1) + \theta_3(k_1k_4 - p_3k_4))}}{(p^2 + 4\pi^2 n^2)(k^2 + 4\pi^2 l^2)((p + k)^2 + 4\pi^2 (n + l)^2)}. \tag{B.1}
\]

The appearance of winding contributions in this integral can be shown by introducing a Feynman parameter, \( x \), and Schwinger parameters \( \alpha_1 \) and \( \alpha_2 \):
\[
-g^2 T^2 \sum_{n,l} \int \frac{d^5p}{(2\pi)^5} \frac{d^5k}{(2\pi)^5} \int_0^1 dx \int_0^\infty d\alpha_1 d\alpha_2 \alpha_1 \alpha_2 e^{-\alpha_2(p^2 + 4\pi^2 n^2)} e^{-\alpha_1 k^2} e^{-(\theta p)^2} e^{-\alpha_1 x(1-x)}(p^2 + 4\pi^2 x^2 + 2(1-x)^2) e^{2\pi i n l x}, \tag{B.2}
\]

Performing the integral over \( k \) followed by a Poisson resummation over \( l \) gives
\[
\frac{F}{V}_{np} = -g^2 T^2 \sum_{n,l} \int \frac{d^5p}{(2\pi)^5} \int_0^1 dx \int_0^\infty d\alpha_1 d\alpha_2 \frac{e^{-(\alpha_2 + \alpha_1 x(1-x))x^2 + 4\pi^2 x^2 + 2(1-x)^2}}{\alpha_1} e^{2\pi i n l x}, \tag{B.3}
\]

which is equation (4.2).

We now turn to evaluating the dominant contribution to this expression at high temperature. As explained in the text, this is given by the \( n = l = 0 \) contribution in equation (B.1). We will for simplicity limit our discussion to this mode. We introduce Feynman and Schwinger parameters:
\[
-g^2 T^2 \int_0^\infty d\alpha_2 \int_0^1 dx \int_0^{1-x} dy \int d^5p d^5k e^{-\alpha((1-y)p^2 + (1-x)k^2 + 2(1-x-y)p.k) + i p \theta k}. \tag{B.4}
\]
The Gaussian integrals can now be performed with the result:

\[-g^2T^2 \int_0^\infty \alpha \, d\alpha \int_0^1 \, dx \int_0^{1-x} \, dy \frac{1}{f(x,y)^{1/2}} \frac{1}{\alpha^2 f(x,y) + \theta_{12}^2} \frac{1}{\alpha^2 f(x,y) + \theta_{34}^2}, \tag{B.5}\]

where \(f(x,y) = x(1-x) + y(1-y) - xy\). The Feynman and Schwinger integrals can also be done exactly and they give:

\[-g^2T^2 \frac{1}{\theta_{12}^2 - \theta_{34}^2} \log \left(\frac{\theta_{12}}{\theta_{34}}\right). \tag{B.6}\]

This is the general result when \(M/g^{1/2} \gg T \gg M\).

We note in passing that the zero temperature vacuum energy of this theory is finite:

\[g^2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \frac{e^{-i(\theta_{12}(p_1 k_2 - p_2 k_1) + \theta_{34}(p_3 k_4 - p_4 k_3))}}{p^2 k^2 (p+k)^2} = g^2 \frac{1}{\theta_{12} + \theta_{34}} \frac{1}{\theta_{12} \theta_{34}}. \tag{B.7}\]
References

[1] R. Gopakumar, S. Minwalla and A. Strominger, “Noncommutative Solitons,” hep-th/0003160.

[2] Y. Kiem and S. Lee, “UV/IR Mixing in Noncommutative Field Theory via Open String Loop,” hep-th/0003145.

[3] O. Andreev and H. Dorn, “Diagrams of noncommutative Phi**3 theory from string theory,” hep-th/0003113.

[4] M. Van Raamsdonk and N. Seiberg, “Comments on noncommutative perturbative dynamics,” [hep-th/0002186].

[5] A. Matusis, L. Susskind and N. Toumbas, “The IR/UV connection in the noncommutative gauge theories,” hep-th/0002075.

[6] W. Fischler, J. Gomis, E. Gorbatov, A. Kashani-Poor, S. Paban and P. Pouliot, “Evidence for winding states in noncommutative quantum field theory,” hep-th/0002067.

[7] H. Grosse, T. Krajewski and R. Wulkenhaar, “Renormalization of noncommutative Yang-Mills theories: A simple example,” hep-th/0001182.

[8] S. Terashima, “On the equivalence between noncommutative and ordinary gauge theories,” hep-th/0001111.

[9] G. Arcioni and M. A. Vazquez-Mozo, “Thermal effects in perturbative noncommutative gauge theories,” JHEP 0001, 028 (2000) [hep-th/9912140].

[10] S. Minwalla, M. Van Raamsdonk and N. Seiberg, “Noncommutative perturbative dynamics,” [hep-th/9912072].

[11] I. Chepelev and R. Roiban, “Renormalization of quantum field theories on noncommutative R**d. I: Scalars,” hep-th/9911098.

[12] A. Hashimoto and N. Itzhaki, “On the hierarchy between non-commutative and ordinary supersymmetric Yang-Mills,” JHEP 9912, 007 (1999) [hep-th/9911057].

[13] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909, 032 (1999) [hep-th/9908142].

[14] E. Hawkins, “Noncommutative regularization for the practical man,” hep-th/9908052.

[15] M. Chaichian, A. Demichev and P. Presnajder, “Quantum field theory on noncommutative space-times and the persistence of ultraviolet divergences,” hep-th/9812180.
[16] A. Fotopoulos and T. R. Taylor, “Comment on two-loop free energy in N = 4 supersymmetric Yang-Mills theory at finite temperature,” Phys. Rev., D59, 1999 [hep-th/9811224].

[17] J. C. Varilly and J. M. Gracia-Bondia, “On the ultraviolet behavior of quantum fields over noncommutative manifolds,” Int. J. Mod. Phys. A14, 1305 (1999) [hep-th/9804001].

[18] S. Cho, R. Hinterding, J. Madore and H. Steinacker, “Finite field theory on noncommutative geometries,” hep-th/9903239.

[19] M. M. Sheikh-Jabbari, “Renormalizability of the supersymmetric Yang-Mills theories on the noncommutative torus,” JHEP 9906, 015 (1999) [hep-th/9903107].

[20] C. P. Martin and D. Sanchez-Ruiz, “The one-loop UV divergent structure of U(1) Yang-Mills theory on noncommutative R**4,” Phys. Rev. Lett. 83, 476 (1999) [hep-th/9903077].

[21] A. Connes, M. R. Douglas and A. Schwarz, “Noncommutative geometry and matrix theory: Compactification on tori,” JHEP 9802, 003 (1998) [hep-th/9711162].

[22] T. Filk, “Divergencies in a field theory on quantum space,” Phys. Lett. B376, 53 (1996).

[23] J. J. Atick and E. Witten, “The Hagedorn Transition And The Number Of Degrees Of Freedom Of String Theory,” Nucl. Phys. B310, 291 (1988).

[24] Hugh Osborn “Topological charges for N=4 supersymmetric gauge theories and monopoles of spin 1,” Phys. Lett., B83, 1979.