Ballistic electron quantum transport in the presence of a disordered background

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Abstract

The effect of a complicated many-body environment is analyzed on the electron random scattering by a 2D mesoscopic open ballistic structure. A new mechanism of decoherence is proposed. The temperature of the environment is supposed to be zero, whereas the energy of the incoming particle $E_{\text{in}}$ can be close to or somewhat above the Fermi surface in the environment. The single-particle doorway resonance states excited in the structure via external channels are damped not only because of escape through such channels but also due to the ulterior population of the long-lived environmental states. Transmission of an electron with a given incoming energy $E_{\text{in}}$ through the structure turns out to be an incoherent sum of the flow formed by the interfering damped doorway resonances and the retarded flow of the particles re-emitted into the structure by the environment. Though the number of particles is conserved in each individual event of transmission, there exists a probability that some part of the electron’s energy can be absorbed due to environmental many-body effects. In such a case the electron can disappear from the resonance energy interval and elude observation at the fixed transmission energy $E_{\text{in}}$ thus resulting in the seeming loss of particles, violation of the time-reversal symmetry and, as a consequence, suppression of the weak localization. Both decoherence and absorption phenomena are treated within the framework of a unit microscopic model based on the general theory of the resonance scattering. All the effects discussed are controlled by the only parameter: the spreading width of the doorway resonances that uniquely determines the decoherence rate.

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1. Introduction

With the advent of the ability to fabricate mesoscopic analogs of the classical billiards—eminent systems often used to illustrate the characteristic features of the classical dynamical chaos, the opportunity appeared to directly observe the signatures of chaos in classically chaotic quantum systems. An excellent possibility thus has arisen to experimentally verify the theoretical concepts developed in numerous theoretical investigations of ‘quantum chaos’ phenomena. Extensive studies of the electron transport through ballistic meso-structures [1, 2] (see also [3, 4] and references therein) have fully confirmed the correctness of the basic ideas [5–7] of the theory of the chaotic quantum interference as well as relevance [3, 4, 8] of the random-matrix approach [9–12] to the problem of the universal fluctuations in open mesoscopic set-ups. Nevertheless, experiments with the ballistic quantum dots [13–16] reveal noticeable and persisting up to zero temperature loss of the quantum-mechanical coherence in contravention of predictions of the standard semiclassical and random-matrix scattering theories.

A number of different methods of accounting for the decoherence in ballistic quantum transport processes have been suggested. In the phenomenological voltage-probe model that goes back to Büttiker’s papers [17] a subsidiary dephasing lead with vanishing mean current is attached to the cavity. Generally speaking, the fictitious lead can support an arbitrary number $M_\phi$ of channels with some transmission coefficients $T_\phi$. To get rid of the arising ambiguity a special procedure has been worked out in [18]: the limit $M_\phi \to \infty$, $T_\phi \to 0$ has been considered where the product of these two numbers remains constant and is fixed via the connection $\gamma_\phi = \frac{2\pi}{D} M_\phi T_\phi$ by the decoherence rate $\gamma_\phi$—the only parameter of the actual physical meaning. (Here $D$ stands for the mean level spacing in the mesoscopic cavity.) It should be noted in this connection that the construction proposed implicitly suggests a complicated internal structure of the dephasing probe which should, in particular, possess a dense energy spectrum with the mean level spacing $\delta D \ll D$. Otherwise, the assumed limit could hardly be physically justified. If so, the typical time $\tau_d = \frac{2\pi}{D}$ spent by the electron inside the probe constitutes a new time scale different from the mean time delay $\tau_D = \frac{2\pi}{D}$ in the cavity.

Another approach has been proposed in [19] where the decoherence phenomenon is linked to the electron absorption. The absorption can readily be modeled by including in the Hamiltonian a spatially uniform imaginary potential whose strength $\kappa_0$ is directly connected to the decoherence rate. Obviously, the model in which the number of particles is not conserved violates unitarity of the scattering matrix. Nevertheless, the authors of [18] have managed to accord and combine the two mentioned models by compulsory restoring on average conservation of the number of particles with a given energy so that the only effect of such a dephasing probe consists in erasing the phase memory. However, in a wider aspect, a voltage probe allows for energy dissipation also. An elucidative comparison of the properties of dissipative voltage probes on the one hand and the energy conserving dephasing probes on the other hand has been given in [20]. Still, the probe models formulated in such a way are not, as has been noticed in [21], entirely satisfactory. Indeed, only the mean value of the total electron flow through the probe can be forced to vanish. The unitarity of the scattering matrix is not perfectly restored and the number of electrons is not conserved in each individual act of scattering.

An alternative phenomenological model of dephasing has therefore been proposed in [21] with a closed long dephasing stub instead of an opening lead. Thereby unitarity of the scattering matrix is guaranteed and none of the electrons is lost at any individual measurement. At last, the dephasing in the stub has been supposed to be induced by a spatially random time-
dependent external electric field that breaks the phase coherence in the way similar to that
known from the theory of dephasing in bulk disordered conductors [22]. As a result the
electrons once penetrated the stub return back in the cavity without any phase memory.

In spite of the advantages of the stub model the necessity of introducing ad hoc an
external time-dependent potential seems to be somewhat artificial. In this paper we discuss
within the framework of a unit microscopic model a simple alternative mechanism of the
decohere and dissipation phenomena induced by a time-independent weak interaction with
a disordered environment. This disorder arises due to relatively rare irregular impurities in
the semiconductor heterostructure to whose interface region the electrons are confined. We
suggest that though the electron’s mean free path exceeds the size of the dot there exists
some finite probability for an electron to be scattered by such an impurity during its stay
inside the dot. Still such a scattering cannot by itself destroy the phase coherence. Our
model also takes into account unavoidable energy averaging because of the finite accuracy
with which the electron’s energy can be measured in any individual scattering event. In
section 2 we present our model and describe the fine-scale fragmentation of the electron
resonant states in a mesoscopic structure induced by interaction with a disordered environment.
A resonant representation of the scattering matrix in the presence of a disordered environment
is derived. In section 3 we carry out the fine-structure averaging and show how this averaging
results in the decoherence. The cases of isolated and overlapping resonances are analyzed at
zero temperature of the environment. The role of environmental many-body effects and the
energy absorption is then considered when the energy of the incoming electron $E_{\text{in}}$ noticeably
exceeds the Fermi energy in the environment. Ensemble averaging and suppression of the
weak localization are discussed in section 4. We summarize our findings in section 5.

2. Fragmentation of the doorway resonance states due to interaction with a disordered
background

Let $H^{(s)}$ be the Hamiltonian of an ideal ballistic mesoscopic cavity with perfectly reflecting
walls and no environment. Let us suppose that the cavity is attached to two long leads
which altogether support $M$ transversal (channel) modes. An open system of such a kind
is described by the effective non-Hermitian Hamiltonian $\mathcal{H}^{(s)} = H^{(s)} - \frac{1}{2} A A^\dagger$ [9, 25, 26]
where the rectangular matrix $A$ consists of $M$ column vectors of transition amplitudes
between $N$ internal states excited in the cavity during the scattering at some electron energy
$E$ and $M$ channel states. Let us further suppose that our cavity is imbedded in a many-
body environment described by the $N^{(e)} \times N^{(e)}$ Hermitian Hamiltonian matrix $H^{(e)}$ with a
very small mean level spacing $\delta$, to which our system is coupled via a rectangular matrix
$V$. The dimension $N^{(e)} \gg N^{(s)}$ so that the spacing $\delta$ fixes the smallest energy scale. This
spacing will be kept finite throughout the paper to ensure unitarity of the scattering matrix
$S(E) = I - iT(E)$. The total system: the open cavity interacting with the environment is
described by the extended non-Hermitian effective Hamiltonian

$$
\mathcal{H} = \begin{pmatrix}
H^{(s)} & V^\dagger \\
V & H^{(e)}
\end{pmatrix}.
$$

The corresponding transition matrix equals $T(E) = A^\dagger G_D(E) A$ where $G_D(E)$ stands for the
upper-left block

$$
G_D(E) = \frac{I}{E - H^{(s)} - \Sigma(E)}
$$

of the resolvent $G(E) = \frac{1}{E - \mathcal{H}}$ of the extended non-Hermitian Hamiltonian $\mathcal{H}$. The subscript
$D$ means ‘doorway’ and marks the resonance states in the ideal open cavity that are directly
connected to the scattering channels [23, 26]. In the zero approximation \( V \equiv 0 \), only these states are unstable and have complex eigenenergies, \( \varepsilon_n = \varepsilon_n - \frac{i}{2} \Gamma_n \). The environment states get excess to the leads only due to their mixing with doorway resonances exclusively through which they can be excited or relaxed. The matrix \( \Sigma(E) = V \frac{1}{E - H} V \) accounts for the transitions cavity ↔ environment and remains Hermitian (and, correspondingly, the scattering matrix remains unitary) as long as the energy spectrum of the environment is discrete, i.e. the mean level spacing \( \delta \neq 0 \).

In the mean-field single-particle approximation \( H(e) \approx H(e)_{sp} \), an electron penetrating into the environment moves in a mean field that is random because of impurities. So we suppose that the coupling matrix elements are random Gaussian quantities:

\[
\langle V_{\mu m} \rangle = 0, \quad \langle V_{\mu m}^* V_{\nu n} \rangle = \frac{1}{2} \Gamma_s \delta_{\mu \nu} \delta_{mn}.
\]

The subscripts \( m, n \) and \( \mu, \nu \) mark the doorway and the background single-particle states, respectively. As the condition \( \delta \neq 0 \) holds the quasi-particle’s energy spectrum is discrete with a mean level spacing \( d \) that satisfies the inequalities \( \delta \ll d \ll D \). The spreading width \( \Gamma_s = 2\pi \frac{1}{\Gamma_s} \) characterizes the fine-scale fragmentation of the doorway states because of the coupling to the environment. It is understood that the spreading width \( \Gamma_s \gg d \).

Using equation (3) we can substitute with the accuracy \( \frac{1}{N(e)_{sp}} \) the matrix \( \Sigma(E) \) in (2) by the averaged value:

\[
\Sigma(E) \Rightarrow \frac{1}{2} \Gamma_s g(E), \quad g(E) = \frac{1}{\pi} \text{Tr} \frac{1}{E - H_{sp}^{(e)}}.
\]

Here \( N_{sp}(e) \) is the dimension of the Hilbert space of a quasi-electron in the environment. Since the spectrum of the quasi-particle’s Hamiltonian \( H_{sp}^{(e)} \) is discrete the loop function \( g(E) \) is real so that the V-averaging does not destroy unitarity of the scattering matrix \( S(E) \).

The transition amplitudes reduce after that to a sum of the doorway resonant contributions

\[
T_{ab}(E) = \sum_n \frac{A_{a n}^* A_{b n}}{E - \varepsilon_n - \frac{1}{2} \Gamma_s g(E)} \equiv \sum_n \frac{A_{a n}^* A_{b n}}{D_n(E)}.
\]

For some time, we restrict ourselves to the case of the systems with time-reversal symmetry. The decay amplitudes \( A_{a n} \) are real in this case and the matrix of the non-Hermitian effective Hamiltonian \( \hat{\mathcal{H}}^{(s)} \) is symmetric. The amplitudes \( A_{a n} \) in equation (5) are the matrix elements of the coupling matrix \( A = \Psi^T A \), with \( \Psi \) being the complex orthogonal \( (\Psi^T \Psi = 1) \) matrix of the eigenstates of the effective Hamiltonian \( \hat{\mathcal{H}}^{(s)} \). Therefore, unlike the real elements of the matrix \( A \), those of the matrix \( A \) are complex quantities [25, 26].

The exact resonance spectrum \( \{ \varepsilon_n \} \) is now found by solving \( N_{sp}(e) \) independent equations

\[
D_n(\varepsilon_n) = \varepsilon_n - \varepsilon_n - \frac{1}{2} \Gamma_s g(\varepsilon_n) = 0.
\]

Each doorway state is thus fragmented onto \( \sim \Gamma_s / d \gg 1 \) narrow fine-scale resonances. Finally, transition amplitudes can be represented as the coherent sums of \( N_{sp}(e) = N^{(e)} \cdot \Gamma_s / d \) interfering resonant contributions

\[
T_{ab}(E) = \sum_a \frac{A_{a n}^* A_{b n}}{E - \varepsilon_a}.
\]

As distinct from the case of the ideal cavity the interference pattern now depends on two additional parameters: the spreading width \( \Gamma_s \) and the fine-scale level spacing \( d \). Up to this point our consideration has been general enough and is applicable in quite a wide scope. In what follows we re-examine in this framework the problems of the decoherence and absorption in 2D mesoscopic devices.
3. Energy averaging over the fine-structure scale

Let us now take into account that the energy resolution $\Delta E$ is not perfect and does not allow for resolving the fine structure of the doorway resonances, $d \ll \Delta E$ though of course $\Delta E \ll D$. Then only the averaged cross sections

$$\bar{\sigma}^{ab}(E) = \frac{1}{\Delta E} \int_{E - \frac{1}{2} \Delta E}^{E + \frac{1}{2} \Delta E} dE' \sigma^{ab}(E')$$

are observed. To carry out the energy averaging explicitly, we neglect the level fluctuations on the fine-structure scale and assume the uniform spectrum, $\varepsilon_\mu = \mu d$ (the picket fence approximation). This yields immediately $g(E) = \cot \left( \frac{\pi E}{d} \right)$.

3.1. Isolated doorway resonance

In the case of an isolated doorway resonance with width $\Gamma_1 = \sum_c \Gamma^c \ll D$, that is situated close to the Fermi energy, $E_{\text{res}} = 0$, the transition cross section equals

$$\sigma^{ab}(E) = |T^{ab}(E)|^2 = \frac{\Gamma^a \Gamma^b}{\left[ E - \frac{1}{2} \Gamma_s \cot \left( \frac{\pi E}{d} \right) \right]^2 + \frac{1}{4} \Gamma^2},$$

and the fine-scale energy averaging yields

$$\bar{\sigma}^{ab}(E) = \frac{\Gamma^a \Gamma^b}{E^2 + \frac{1}{4} (\Gamma + \Gamma_s)^2} + \frac{\Gamma^a \Gamma^b}{\Gamma} \frac{\Gamma_s}{E^2 + \frac{1}{4} (\Gamma + \Gamma_s)^2}.$$

The phase coherence is destroyed by the averaging and the result consists of two incoherent contributions. The first one corresponds to the excitation and subsequent decay of the doorway resonance widened because of leaking into the environment. This effect is described by shifting in the upper part of the complex energy plane by the distance $\frac{1}{2} \Gamma_s$. The second term accounts for the particles re-injected from the background. There is no net loss of the particles. The environment looks from outside as a black box which swallows particles and spits them back in the cavity after some time.

The transport through the cavity is characterized by the quantity [3, 4]

$$G(E) = \sum_{a \in I, d \in 2} \bar{\sigma}^{ab}(E) = \frac{\Gamma_1 \Gamma_2}{\Lambda(E)} + \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\Gamma_s}{\Lambda(E)} = T_{12} + \frac{T_{1s} T_{2s}}{T_{1s} + T_{2s}},$$

where $\Lambda(E) = E^2 + \frac{1}{4} (\Gamma + \Gamma_s)^2$. The second term in the final expression is expressed in terms of the subsidiary transition probabilities

$$T_{sk}(E) = \frac{\Gamma_k}{\Lambda(E)}, \quad \Gamma_k = \sum_{c \in k} \Gamma^c, \quad k = 1, 2, \quad \Gamma_1 + \Gamma_2 = \Gamma.$$

One can interpret equations (11) and (12) by introducing an additional fictitious $(M^{(s)} + 1)$st channel with the transition amplitude $A^{(s)} = \sqrt{\Gamma_s}$ that connects the resonance state to the environment. It is easy to check that the fictitious scattering matrix $\tilde{S}(E) = 1 - i \tilde{T}(E)$ built in such a way is unitary. Expression (11) is formally identical to that obtained with Büttiker’s voltage-probe model [17] of the decoherence phenomenon. At that the decoherence rate $\gamma_s = \frac{\Gamma_s}{\tau_D} = \Gamma_s \tau_D$ is unambiguously defined by the only parameter—the spreading width of the doorway resonance.

The single-particle approximation used up to now is well justified only when the scattering energy $E$ is very close to the Fermi surface in the environment. For higher scattering energies, many-body effects should be taken into account. They show up, in particular, in a finite
lifetime of the quasi-particle with the energy \( E > E_F = 0 \). The simplest way to account for this effect is to attribute some imaginary part to the quasi-particle’s energy, \( \varepsilon_0 = \mu d - \frac{i}{2} \Gamma_s \).

The resonant denominator then equals [27]

\[
D_{\text{res}}(E) = E - E_{\text{res}} - \frac{1}{2} \Gamma_s (1 - \xi^2) \frac{\eta}{1 + \xi^2 \eta^2} + \frac{i}{2} \left( \Gamma_s \xi^2 \left( \frac{1}{1 + \xi^2 \eta^2} \right) \right),
\]

where \( E_{\text{res}} \) is the position of the doorway resonance and the following notations are used:

\[
\xi = \tanh \left( \frac{\pi \Gamma_s}{2d} \right), \quad \eta = \cot \left( \frac{\pi E}{d} \right).
\]

The averaged transport cross section \( G(E) \) still retains its form (11) but the subsidiary transition probabilities look as

\[
T_{sk}(E) \Rightarrow T_{sk}(E; \kappa) = \frac{\Gamma_s \Gamma_k}{\Lambda(E; \kappa)}
\]

instead of (12). The factor

\[
\frac{1}{\Lambda(E; \kappa)} = \frac{1}{\Lambda(E)} \frac{1}{1 + \kappa \frac{\Delta E}{\Gamma_s}}
\]

depends on the new parameter \( \kappa \) which accounts for inelastic effects in the background,

\[
\kappa = \frac{4 \xi}{(1 - \xi)^2} = e^{\gamma_e} - 1
\]

\[
\approx \begin{cases} 
\gamma_e \ll 1, & \text{if } \tau_e \gg \tau_d, \\
\gamma_e \gg 1, & \text{if } \tau_e < \tau_d, 
\end{cases} \quad (\gamma_e = \tau_d \Gamma_s),
\]

where \( \tau_e = 1/\Gamma_e \) is the lifetime of the quasi-electron in the environment.

Near the doorway resonance energy \( E_{\text{res}} \) the influence of the absorption is negligible within the range \( 0 \leq \kappa \lesssim \kappa_c \approx \frac{4 \Gamma_s}{\Gamma_1} \). The critical value \( \kappa_c \) reaches its maximum possible, \( \kappa_c = 1 \), when \( \Gamma = \Gamma_s \) and becomes small if one out of the two widths noticeably exceeds another. In these cases the interval of weak absorption is very restricted and the absorption begins to play an important role. If the resonance is so narrow that \( \Gamma \ll \Gamma_s \), then \( \kappa_c \approx \frac{4 \Gamma_s}{\Gamma_1} \ll 1 \) and the subsidiary probabilities (14) at the resonance energy \( E = E_{\text{res}} \) and \( \kappa \lesssim \kappa_c \) are small, \( T_{sk}(E = E_{\text{res}}; \kappa) \approx \frac{16 \Gamma_1 \Gamma_k}{\Gamma_s} \ll 1 \). On the other hand, the very quasi-particle concept is self-consistent only if \( \gamma_e = \tau_d \Gamma_e \lesssim 1 \) so that the physically feasible interval of the strong absorption regime is \( \kappa_c \approx \frac{4 \Gamma_s}{\Gamma_1} \lesssim \kappa \lesssim 1 \). In this interval only the contribution \( T_{12}(E) \) remains in equation (11) and our approach reproduces the result of Efetov’s model [19] with the strength of the imaginary potential \( -\frac{1}{2} \gamma_s \).

Under the opposite condition \( \Gamma_s \ll \Gamma \) which complies with the assumption that the interaction with the background is weak, the regime of strong absorption arises when \( \kappa_c \approx \frac{4 \Gamma_s}{\Gamma_1} \ll \kappa \lesssim 1 \). Similar consideration leads to Efetov’s model again. The decoherence rate equals \( \gamma_0 = \gamma_s \) in both cases and coincides with that obtained in the weak absorption limit.

Strictly speaking, the assumed quasi-particle decay, that implies infinite density of the final states in the background, seems to destroy the unitarity of the scattering matrix in contradiction with what has been stated before. In Efetov’s limit the resulting expressions for the transition probabilities are formally identical to those obtained [28] in the case of the analog 2D microwave resonators with resistive walls [29]. The walls absorb the electromagnetic energy thus widening the resonance lines in exactly the same way as has been described above. However, it has to be stressed that the complete identity between the Maxwell equations in 2D microwave cavities and the Schrödinger equation for an electron in a mesoscopic billiard exists only in the case of perfectly reflecting walls. At the same time, quite different physics stays
behind the absorption processes in these two cases. While absorbed photons fully disappear in the environment, an electron preserves its individuality there to a certain extent. It can only lose, because of the many-body effects, a part of its energy but inevitably returns sooner or later in the cavity and escapes finally via one of the leads.

In fact, a single-particle state in the environment with a very dense but, nevertheless, discrete spectrum is not a stationary state with a given energy \( \epsilon_\mu \). Precisely, such a state being once excited evolves after that quite similar to a quasi-stationary state till the time \( 2\pi/\delta \gg \tau_e = 1/\Gamma_e \). Only after this time recovery of the initial non-stationary state begins. There exists a good probability for an electron to be re-emitted in the cavity with some energy \( E_{\text{out}} < E_{\text{in}} \approx E_{\text{res}} \) within the much shorter time interval \( \tau_d = \frac{2\pi}{\delta} \). In an individual event of scattering with a given energy \( E_{\text{in}} \) such an electron does not make resonant contribution if \( E_{\text{res}} - E_{\text{out}} > \Gamma + \Gamma_s \) and therefore escapes observation. The portion of energy lost by such a retarded electron dissipates inside the environment. As a result, the background temperature jumps slightly up during each act of the scattering. However, supposing that the environment system is bulky enough, we can disregard the corresponding very slow increase of the environment temperature. Alternatively, we can suppose that a special cooling technique is in use.

3.2. Overlapping doorway resonances

In the regime of overlapping doorway resonances, the fine-scale averaged cross section reads (see equation (5))

\[
\frac{1}{D_{n'}(E) D_n(E)} = \frac{1}{D_{n'}(E) D_n(E)} \left[ 1 + \frac{\Gamma_n^2}{\Gamma_s^2} \left( \epsilon_{n'} - \epsilon_n \right) + \kappa \right].
\]

(18)

where \( \tilde{D}_n(E) = E - E_n + \frac{i}{2} (\Gamma_n + \Gamma_s) \).

Let us consider at first the idealized case \( \kappa = 0 \). In that case an incoming electron with an energy \( E_{\text{in}} \) excites via the doorway states a long-lived single-(quasi)electron state in the environment. Such a quasi-particle escapes from the device in two steps: first, it repopulates a number of the doorway states in the cavity after a while, they decay, finally, through the leads. The appearance of mesoscopic fluctuations in processes of such a kind has been demonstrated in [24].

Taking into account the two following identities:

\[
\frac{i}{\epsilon_{n'} - \epsilon_n} = \int_0^\infty dt e^{i(E_{\text{out}} - \epsilon_n)t},
\]

\[
\frac{e^{-iE_{\text{in}}t}}{D_n(E)} = -i e^{-i(E_{\text{in}} + \frac{1}{2} \Gamma_s)t} \int_0^\infty dt e^{iE_{\text{out}} - i(E_{\text{in}} - \frac{1}{2} \Gamma_s)t},
\]

we represent the fine-structure-averaged cross section (17) as a sum, \( \overline{\sigma^{ab}(E)} = \sigma_d^{ab}(E) + \sigma_r^{ab}(E) \), of incoherent flows the first of which,

\[
\sigma_d^{ab}(E) = \left| \sum_n A_n^a A_n^b \frac{1}{D_n(E)} \right|^2,
\]

(19)
describes the contribution of the overlapping doorway states damped because of the electron capture by the environment when the second one,

\[ \sigma_{r}^{ab}(E) = \Gamma_s \int_0^\infty dt \sigma_{r}^{ab}(E; t), \]

\[ \sigma_{r}^{ab}(E; t) = \left| \sum_n \frac{A_{n}^a A_{n}^b}{D_n(E)} e^{-iE_n t} \right|^2, \]  
(20)

accounts for the particles that spend some time \( t_r \) in the background, repopulate the doorway levels and finally escape via the channel \( a \). Particles delayed for different times \( t_r \) contribute incoherently.

With the help of the Bell–Steinberger relation

\[ \frac{1}{\varepsilon_n^* - \varepsilon_n} = -i \sum_n A_n^c A_n^c \]

contribution \( G_r(E) = \sum_{a \in 1, \alpha \in 2} \sigma_{r}^{ab}(E) \) of the re-injected particles can be transformed to

\[ G_r(E) = \sum_{n \in n} U_{n'n} \sqrt{U_{n'n} U_{nn}} \sum_{a \in 1} \Phi_{n'}^a \Phi_{n}^a \Phi_{n}^b \Phi_{n}^b \]

\[ \sum_{a \in 1} \Phi_{n'}^a \Phi_{n}^a + \sum_{b \in 2} \Phi_{n'}^b \Phi_{n}^b, \]
(21)

Here \( U = \Psi \Psi \) is the matrix of non-orthogonality of the overlapping doorway states and the subsidiary amplitudes

\[ \Phi_{n}^a(E) = \sqrt{\Gamma_s A_n^a / \sqrt{U_{nn}(E)}} \]

(22)

implicate the fictitious channel. The quantities \( \Gamma^a = \frac{1}{\Delta_{nn}} |A_n^a|^2 \) satisfy the condition \( \sum_n \Gamma^a = \Gamma \) and are the partial widths. The returning particles cannot escape directly, but rather repopulate before the doorway states that finally decay through the external channels.

In the case of moderately overlapping doorway resonances the matrix \( U_{n'n} \approx \delta_{n'n} \) and only the terms that contain the probabilities \( |\Phi_{n}^b(E)|^2 \) contribute. The result obtained in such a way is a direct extension of equation (11). However, contributions of different doorway resonances in (21) nevertheless interfere when the overlapping is strong.

4. Ensemble averaging

Since the electron motion in the cavity is supposed to be classically chaotic the ensemble averaging \( \langle \cdots \rangle \) in the doorway sector is appropriate. It is easy to see that, as long as the inelastic effects in the background are fully neglected, such an averaging perfectly eliminates the dependence of all mean cross sections on the spreading width. Indeed, the ensemble averaged cross section (19) is expressed in the terms of the \( S \)-matrix two-point correlation function \( C_{V}^{ab}(\varepsilon) = C_{V}^{ab}(\varepsilon - i\Gamma) \) as

\[ \{\sigma_{d}^{ab}(E)\} = C_{V}^{ab}(0) = C_{0}^{ab}(-i\Gamma) = \int_0^\infty dt e^{-\Gamma t} K_{0}^{ab}(t). \]

(23)

The subscript \( V \) indicates the coupling to the background and the function \( K_{0}^{ab}(t) \) is the Fourier transform of the correlation function \( C_{0}^{ab}(\varepsilon) \). On the other hand, using the identity

\[ \frac{e^{-it\varepsilon}}{D_n(E)} = \frac{1}{2\pi} e^{i\xi_n/2} \int_0^\infty dt e^{iEt} \int_{-\infty}^\infty dE' e^{-iE'(t+\xi_n)} \]

one can convince oneself that

\[ \{\sigma_{r}^{ab}(E; t_r)\} = \int_0^\infty dt e^{-\Gamma t} K_{0}^{ab}(t + t_r). \]

(24)
Therefore, finally,
\[
\langle \sigma_{ab}(E) \rangle = \int_0^\infty dt \ e^{-\Gamma t} K_0^{ab}(t) + \Gamma \int_0^\infty dt_1 \int_0^\infty dt_2 \ e^{-\Gamma t_1} K_0^{ab}(t_1 + t_2) = \int_0^\infty dt \ K_0^{ab}(t) = |\sigma_{ab}^0(E)|. \tag{25}
\]

The ensemble averaging, being equivalent to the energy averaging over the doorway scale \(D\), suppresses all interference effects save the elastic enhancement because of the time-reversal symmetry, which manifests itself in the weak localization phenomenon. This symmetry is violated only owing to the energy absorption in the environment.

5. Quasi-particle decay and suppression of the weak localization

Reverting now to equations (17) and (18) we rewrite the ensemble-averaged cross sections as
\[
\langle \sigma_{ab}(E) \rangle = \sigma_{ab}^0(E) + \Delta \sigma_{ab}(E; \kappa)
\]
where the correction caused by the absorption looks as
\[
\Delta \sigma_{ab}(E; \kappa) = \kappa \sum_{n,n'} \left\langle \frac{\delta \delta^* \delta \delta^*}{\epsilon_{n}^* - \epsilon_n} \frac{1}{\epsilon_{n'}^* - \epsilon_{n'} + i \frac{\kappa}{\Gamma_{ns}} \tilde{D}_n(E) \tilde{D}_n(E)} \right\rangle.
\tag{26}
\]

It turns out that this correction (that is not at all necessarily positive definite) can still be expressed in terms of the Fourier transform \(K_0^{ab}(t)\) of the two-point correlation function \(C_0^{ab}(\epsilon)\). To show this we expand first expression (26) into power series with respect to the parameter \(\kappa\) and then make use of the relations
\[
\frac{1}{(\epsilon_{n}^* - \epsilon_n)^{k+1}} = -\frac{i}{k!} \int_0^\infty dt \ e^{\frac{\epsilon}{\Gamma} (-it)^k} e^{-i\tilde{D}_n(E)t} \tilde{D}_n(E); \\
\tilde{D}_n^{k}(E) \tilde{D}_n^{k'}(E') = \left( -i \frac{d}{dt} \right)^k e^{i\tilde{D}_n(E)t}; \\
e^{i\tilde{D}_n(E)t} = -\frac{e^{iE}}{2\pi i} \int_{-\infty}^{\infty} dE' \ e^{-iE'} \tilde{D}_n(E).
\]

Subsequent summation brings us to the result
\[
\Delta \sigma_{ab}(E; \kappa) = -\frac{\kappa}{(2\pi)^2} \int_0^\infty dt \int_0^\infty dE_1 \int_0^\infty dE_2 \ e^{iE_1(t_1 + t_2)} C_0^a(E_1 - E_2 - i\Gamma) \int_{t_1=t_2=0}.
\]

After the change of variables:
\[
\bar{E} = \frac{1}{2} (E_1 + E_2), \quad \bar{t} = \frac{1}{2} (t_1 + t_2), \\
\epsilon = E_1 - E_2 \quad \tau = t_1 - t_2
\]
and integration over the variables \(\bar{E}\) and \(\epsilon\) this yields
\[
\Delta \sigma_{ab}(E; \kappa) = -\frac{\kappa \Gamma_s}{4\pi} \int_0^\infty dt \int_0^\infty \frac{dt'}{\sqrt{t'}} e^{-\frac{1}{4\pi\tau} \left( \frac{\bar{t}}{\Gamma} - \frac{1}{\Gamma} \right)^2} K_0^{ab}(t)
\]
\[
= -\frac{\kappa \Gamma_s}{4} \int_0^\infty \frac{dt'}{\sqrt{\frac{1}{\tau} + \frac{\epsilon}{\Gamma_s} \left( \frac{1}{\Gamma} - \Gamma \right)^2}} K_0^{ab}(t).
\]
(Note that the form-factors \( K_{0}^{ab}(t) \) monotonically decrease with the time \( t \).) Being presented in such a form this expression is equally valid for both the orthogonal (GOE) as well as the unitary (GUE) symmetry classes.

To simplify subsequent calculation we will consider the case of an appreciably large number \( M \gg 1 \) of statistically equivalent scattering channels, all of them with the maximal transmission coefficient \( T = 1 \). Then the channel indices \( a, b \) can be dropped. The characteristic decay time \( t_{W} = 1/\Gamma_{W} = \tau_{D}/M \) (dwell time) of the function \( K(t) \) is much shorter than the mean delay time \( \tau_{D} \) (here \( \Gamma_{W} = \frac{D}{2\pi} M \) is the so called Weisskopf width).

It is convenient to represent the function \( K_{0}(t) \) that is real, positive definite, monotonously decreases with the time \( t \) and satisfies the conditions

\[
K_{0}(t < 0) = 0, \quad K_{0}(0) = 1 \text{ in the form of the mean-weighted decay exponent} \ [30] \]

\[
K_{0}(t) = \int_{0}^{\infty} d\Gamma e^{-\frac{\Gamma}{\Gamma_{1}}} w(\Gamma), \quad \int_{0}^{\infty} d\Gamma w(\Gamma) = K_{0}(0) = 1. \tag{27}
\]

Rigorously, the weight functions \( w(\Gamma) \) have different forms before \( (t < \tau_{D}) \) and after \( (t > \tau_{D}) \) the mean delay (Heisenberg) time \( \tau_{D} \). However, contribution of the latter interval is as small as \( e^{-M} \) [30]. Neglecting such a contribution we obtain in any inelastic channel

\[
\Delta\sigma(E; \kappa) = -\sqrt{\frac{\kappa\Gamma_{s}}{4\pi\tau_{W}}} \int_{0}^{\infty} d\Gamma \frac{w(\Gamma)}{\Gamma (\Gamma + \Gamma_{s})} \cdot \tag{28}
\]

In the strong absorption limit \( \kappa \gg \frac{4\Gamma_{W}}{\Gamma_{s}^{2}}, \frac{4\Gamma_{W}}{\Gamma_{s}^{2}} \) the parameter \( \kappa \) disappears from the found expression and the latter reduces to

\[
\Delta\sigma(E; \kappa) \Rightarrow -\Gamma_{s} \int_{0}^{\infty} d\Gamma \frac{w(\Gamma)}{\Gamma (\Gamma + \Gamma_{s})} = -\Gamma_{s} \int_{0}^{\infty} dt \int_{0}^{\infty} e^{-\Gamma_{s}t} K_{0}^{ab}(t + t_{e}). \tag{29}
\]

According to equations (23) and (25) this brings us to the result

\[
\langle \sigma(E) \rangle = \int_{0}^{\infty} d\Gamma w(\Gamma) \frac{1}{\Gamma + \frac{\kappa}{\Gamma_{1}} + \frac{\kappa}{\Gamma_{1}}}. \tag{30}
\]

identical to that of Efetov’s imaginary-potential model [19]. It follows from (30) that the averaged cross section approaches \( 1/\Gamma_{s} \) independently of the symmetry class if the spreading width \( \Gamma_{s} \) noticeably exceeds the typical widths contributing to the integral over \( \Gamma \).

In the opposite limit of weak absorption \( \kappa \ll \frac{4\Gamma_{W}}{\Gamma_{s}^{2}}, \frac{4\Gamma_{W}}{\Gamma_{s}^{2}} \) the expression (28) reads

\[
\Delta\sigma(E; \kappa) \Rightarrow -\sqrt{\frac{\kappa\Gamma_{s}}{4\pi\Gamma_{W}}} \int_{0}^{\infty} d\Gamma \frac{w(\Gamma)}{\Gamma_{1}^{2}}. \tag{31}
\]

so that

\[
\langle \sigma(E) \rangle = \int_{0}^{\infty} d\Gamma w(\Gamma) \left( 1 - \sqrt{\frac{\kappa\Gamma_{s}}{4\Gamma_{W}}} \right). \tag{32}
\]

In the case of time-reversal symmetry (GOE) the asymptotic expansion [9] of the two-point correlation function gives [30]

\[
u^{(\text{GOE})}(\Gamma) = \delta(\Gamma - \Gamma_{W}) - \frac{2}{\Gamma_{W}} \delta'(\Gamma - \Gamma_{W}) + \frac{M}{2\Gamma_{W}} \delta''(\Gamma - \Gamma_{W}) + \cdots \tag{33}
\]

whereas in the case of the absence of such a symmetry (GUE)

\[

\nu^{(\text{GUE})}(\Gamma) = \delta(\Gamma - \Gamma_{W}) + \cdots. \tag{34}
\]
Figure 1. Weak localization versus absorption parameter $\kappa$. Upper/blue line $\gamma_s = 25,$ lower/red line $\gamma_s = 64.$
(This figure is in colour only in the electronic version)

In both cases contributions of the omitted terms are $O(1/(M^{-7/2}))$. With such an accuracy, formula (28) yields for the weak localization the expression

$$
\Delta G \equiv G^{(\text{GUE})} - G^{(\text{GOE})} = M_1 M_2 \left(2 \frac{d}{d\mu} + \frac{\mu}{2} \frac{d^2}{d\mu^2}\right) \left\{ \frac{1}{\mu} \left[1 - \frac{\sqrt{\frac{\kappa}{4}}}{\sqrt{\mu + \frac{\kappa}{4\gamma_s} (\mu + \gamma_s)^2}}\right]\right\} \bigg|_{\mu=M}, \quad (35)
$$

which is valid for arbitrary values of the parameters $\kappa, \gamma_s$ and $M$. The unfolded explicit expression is a bit too lengthy. Therefore, we visualize this result in figure 1 for two different values of the (dimensionless) spreading width $\gamma_s$ and $M_1 = M_2 = 2; M = 4$.

In fact, the condition $\gamma_s \gg M$ can hardly hold if the number of channels is very large, $M \gg 1$. In this case a natural estimate of the absorption parameter is $\kappa \approx 4\frac{\gamma_s^2}{\Gamma_W} = 4\frac{\gamma_s^2}{\Gamma_W}$. With this accuracy we obtain for the transport probabilities

$$
G^{(\text{GOE})} = \frac{M_1 M_2}{M} \left[\left(1 - \frac{\gamma_s}{M}\right) - \frac{1}{M} \left(1 - \frac{9 \gamma_s}{8 M}\right)\right],
$$

$$
G^{(\text{GUE})} = \frac{M_1 M_2}{M} \left(1 - \frac{\gamma_s}{M}\right).
$$

(36)

The principal terms are identical and the difference

$$
\Delta G \equiv G^{(\text{GUE})} - G^{(\text{GOE})} = \frac{M_1 M_2}{M^2} \left(1 - \frac{9 \gamma_s}{8 M}\right) \quad (37)
$$

describes a slight suppression of the weak localization.

The effect of decoherence becomes more and more pronounced as the number of channels decreases. However, the asymptotic expansions (33) and (34) are not justified in the case of few number of channels. Besides, violation of analyticity of the function $K_0(t)$ at the point $t = \tau_D$ becomes essential [30]. The problem calls for a special consideration in this case.

6. Summary

In this paper we have described a possible mechanism of decoherence and absorption phenomena in the electron quantum transport through an open ballistic 2D mesoscopic cavity. These effects are induced by a weak time-independent interaction with a bulky disordered environment with a very dense but, nevertheless, discrete spectrum. Due to such an interaction,
each doorway resonance state in the cavity gets fragmented onto a large number $\sim \Gamma_s/d$ (the spreading width $\Gamma_s$ characterizes the strength of the coupling to the environment when $d$ is the single-quasi-particle mean level spacing) of very narrow resonances that cannot be resolved experimentally. Only the cross sections averaged over the fine ($\sim d$) structure scale are measured. The observable cross sections at a given energy of incoming electron $E_{in}$ turn out to be incoherent sums of flows, the first of which corresponds to the scattering with excitation and subsequent decay of the doorway resonances broadened because of the internal friction induced by interaction with the environment. Such a broadening by the spreading width $\Gamma_s$ simulates absorption. The second flow accounts for the particles re-injected in the cavity after some retardation time spent in the environment. The degree of decoherence of the electrons contributing to these two flows is described by the decoherence rate $\gamma_\phi = \gamma_s = \Gamma_s \tau_D$ uniquely defined by the spreading width.

Formally, the transition amplitudes (5) obey the unitarity condition at any given scattering energy $E$. This implies, in particular, that the number of electrons is conserved during the stationary scattering. All of them enter into and escape from the cavity only through the attached open leads. However, long-lasting ($\tau \sim \tau_\delta = \frac{2\pi}{\delta}$) evolution of the unsteady state of a (quasi)electron once penetrated into the many-body environment entails loss of the electron’s energy in any individual act of scattering lasting a much shorter time $\sim \tau_d = \frac{2\pi}{d} \ll \tau_\delta$. So, some electrons leave the cavity with energies well below the energy of the incoming particles and elude observation made at the resonant scattering energy $E_{res} \approx E_{in}$ (fixed with accuracy worse than $d$). Rigorously speaking, the energy dissipation causes slow heating of the environment. We disregard the latter effect supposing that the environment is bulky enough or, alternatively, a special cooling procedure is used. Disappearance of the particles from the resonant scattering interval not only imitates a seeming loss of particles but also yields real violation of the time-reversal symmetry and, as a consequence, suppression of the weak localization the both effects being controlled by one and only parameter—the decoherence rate $\gamma_\phi$.

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References

[1] Jalabert R A, Baranger H U and Stone A D 1990 Phys. Rev. Lett. 65 2442
[2] Marcus C M, Rimberg A J, Westervelt R M, Hopkins P F and Gossard A C 1992 Phys. Rev. Lett. 69 506
[3] Beenakker C W J 1997 Rev. Mod. Phys. 69 731
[4] Alhassid Y 2000 Rev. Mod. Phys. 72 895
[5] Izrailev F M 1990 Phys. Rep. 5, 6 299
[6] Gutzwiller M C 1990 Chaos in Classical and Quantum Mechanics (Berlin: Springer)
[7] Haake F 2001 Quantum Signatures of Chaos (Berlin: Springer)
[8] Barabger H U and Mello P A 1994 Phys. Rev. Lett. 73 142
[9] Mello P A and Barabger H U 1999 Waves Random Media 9 105
[10] Verbaarschot J J M, Weidenmüller H A and Zimbauer M R 1985 Phys. Rep. 129 367
[11] Mello P A, Pereyra P and Seligman T H 1985 Ann. Phys., NY 161 254
[12] Mehta M N 2004 Random Matrices (New York: Academic)
[13] Fyodorov Y V and Sommers H-J 1997 J. Math. Phys. 38 1918
[14] Marcus C M, Westervelt R M, Hopkins P F and Gossard F C 1993 Phys. Rev. B 48 2460
[14] Bird J P, Ishibashi K, Ferry D K, Ochiai Y, Aoyagi Y and Sugano T 1995 Phys. Rev. B 51 18037
[15] Clarke R M, Chan I H, Marcus C M, Duruöz C I, Harris J S Jr, Campman K and Gossard A C 1995 Phys. Rev. B 52 2656
[16] Huibers A G, Switkes M, Marcus C M, Campman K and Gossard A C 1998 Phys. Rev. Lett. 81 200
[17] Büttiker M 1986 Phys. Rev. B 33 3020
     Büttiker M 1988 IBM J. Res. Dev. 32 63
[18] Brouwer P W and Beenakker C W J 1997 Phys. Rev. B 55 4695
[19] Efetov K B 1995 Phys. Rev. Lett. 74 2299
[20] Pilgram S, Samuelsson P, Förster H and Büttiker M 2006 Phys. Rev. Lett. 97 066801
     Pilgram S, Samuelsson P, Förster H and Büttiker M 2007 Phys. Rev. B 75 035340
[21] Beenakker C W J and Michaelis B 2005 J. Phys. A: Math. Gen. 38 10639
[22] Altshuler B L and Aronov A G 1985 Electron–Electron Interaction in Disordered Systems (Amsterdam: North-Holland)
     Aleiner I L, Altshuler B L and M E Gershenson M E 1999 Waves Random Media 9 201
[23] Feshbach H 1968 Reaction Dynamics (London: Gordon and Breach)
[24] Amir A 2008 Phys. Rev. B 77 050101
[25] Sokolov V V and Zelevinsky V G 1989 Nucl. Phys. A 504 562
[26] Sokolov V V and Zelevinsky V G 1992 Ann. Phys., NY 216 323
[27] Sokolov V V and Zelevinsky V G 1997 Phys. Rev. C 56 311
[28] Fyodorov Y V, Savin D V and Sommers H-J 2005 J. Phys. A: Math. Gen. 38 10731
[29] Kuhl U, Stöckmann H-J and Weaver R 2005 J. Phys. A: Math. Gen. 38 107433
[30] Sokolov V V 2008 Proc. Int. Workshop on Nuclei and Mesoscopic Physics WNMP 2007, AIP Conf. Proc. 995 85