Students’ thinking styles and their proof writing levels

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Abstract. This study was done to investigate the relationship between thinking styles and proof writing levels among university students. Participants in this study were 124 students who were selected via multi-stage cluster sampling. Sternberg and Wagners’ thinking styles inventory was used for assessing students’ thinking styles and their proof writing levels were assessed by the proof-writing scale. The results indicated that there was a significant relationship between thinking styles and proof writing levels and the level of significance was 0.95. That means students’ thinking styles have the ability to predict their proof writing levels. There was also a positive and meaningful correlation between executive thinking style and the total scores of proof writing levels. The result was as follow: students who have external proofs schemes (44%), empirical proofs schemes (52%) and analytical proofs schemes (6%). There was a significant difference, in term of their thinking style, among students who had varying levels of proof-writing.

1. Introduction

Mathematics is a field of science that filled with various statements in the form of theorems or propositions. People do not know whether the statement about the propositions is true or not until they had proved it. Therefore, the proof is fundamental to develop mathematics. [1] Proof that provides assurance of mathematical information, in making and understanding of mathematics is a basic activity. Mathematical proof is an essential activity in doing and understanding mathematics, and it gives a warranty for mathematical knowledge [2]. The proof is not only the heart of mathematics education but also an essential tool for promoting students’ mathematical understanding. According to [3] the teaching and learning of proof are recognized internationally as a key component of mathematics education and mathematics curricula. Therefore for anyone involved in mathematics, one important activity is reading a mathematical proof with the aim of determining whether or not it is valid.

With the increase in the role and significance of proof in mathematics, the developments and mental processes of students in various age groups when they prove have been the subject of research in mathematics education. [4] examined students’ proof schemes through their considerations examples and counterexamples and reasoning in proof construction. Research shows that the student has difficulty in proving mathematics in general, especially in the field of geometry [5], and they show difficulties when trying to read proofs [6]. One of the important thing to develop students ability in proves can be done by looking at proof as an intellectual activity that involves the process of thinking.

When students do the proof, they should be able to understand and explain the facts in mathematics. It is explaining the facts of mathematics each person has different ways, some people explain by drawing sketches or graphics, while others use structures, patterns or formulas [7]. There are three aspects to
considering to develop students proof and proving. The first aspect is a proof as a structural object, the second aspect a proof is an intellectual activity and the third aspect is the roles, function, and meaning of proof and proving [5]. Proof and proving as an intellectual activity is a support by students thinking styles. Thinking styles correlated positively with a student success in a variety of academic tasks. Successes and failure attributed to abilities often stem from styles [8].

1.1. Writing Proof

Harel and Sowder [9] conducted a study about thinking processes that classifies solution levels during proofing by interrogating the reasons behind them. The proof schemes can be grouped into three parts as follows.

Analytical or deductive proof schemes: students who hold analytical proof schemes prove and validate theorems by means of logical deductions. These schemes with involve assumptions with logical inferences and reasoning [10]. The characteristics of this scheme are that students can use various strategies. They can make generalizations and be able to utilize mathematical relationships between sections in mathematics.

Empirical proof schemes: this scheme describes student proof schemes who believed in evidence from examples or perceptions rather than logical reasoning. According to Harel and Sowder [9] students proof by giving number values to expressions or use similar instances in the confirmation process and prefer to account for some situations with their intuitions. There are two parts of proof in this category namely inductive and perceptual proof schemes. Inductive proof schemes held by students who are convinced by seeing a few examples. The perceptual proof is a scheme evidenced by a student who is convinced of a theorem’s truth by a rudimentary, mental image or perception [11].

External proof schemes: students in this type are who believe in the truth of a theorem or statements that made by external factors such as mathematicians, teachers, or they classmates. According to [10], students perform proof based on the accuracy of their knowledge of books, rules that apply or to others who considered more expert. In general types of the proof scheme in this category are a ritual, non-referential symbolic, and authoritative. Students ritual proof schemes are convinced by the form or appearance of a proof. Non-referential symbolic.

On the other hand, Balacheff [12] divided mathematical proof into three levels: (1) Pragmatic Proofs: students who have pragmatic level of proof performance unrelated responses, proof completely based on numerical example, responses based simply on the writing of what is given; (2) Intellectual Proofs: students in this types proofs based on formulation, proofs are given through the use of incomplete mathematical language and demonstration, write proof with general argument; and Demonstration Proofs: in this type student completely accurate proofs that have to be organized with a theory or the use of information commonly accepted by a community. In this study level of proofs that be used is a combination of the two opinions the above.

1.2. Thinking Style

Research on the style of thinking has evolved since its emergence Sternberg's work. Some examples of research related to thinking styles include [13]. There is a significant relationship between adopted from students and their thinking styles [14]. Thinking styles have a relationship with problem-solving, decision-making, academic achievement, etc., and variables such as culture, gender, age, etc. [8]. Thinking styles can affect student achievement in mathematics [15].

Thinking styles is a particular procedure by which something done to a person. Strenberg [16] defines thinking styles is the way of thinking and describes how the individuals use their capacities. The theoretical basis for this research is based on Stenberg's theory. According to Stenberg [17] thinking styles dimensions divided into five categories, namely forms, functions, levels, learning, and scopes. The dimensions of functions are the basic of thinking styles and support students abilities in the study. In this dimensions there are three styles as follows:
• Legislative style: students who have legislative style prefer the problems that are requires them to devise new strategies and to create their own laws and they enjoy giving commands. [18]. Legislative Likes to create, discovery, design; do things using own method; and less structure.
• Executive style: students in this style prefer to use the ways that already exist to solve problems, application, and implementation of the rules. They will be able to start work if they know about the five ‘W’- when, why, where, who and whose-. [19]. Executive Likes to follow instructions; does what is requested; structure must be given.
• Judicial Style: students of this method care about the stages of the work and the result. The question that they often asked such as why? What is the reason? [20] Judicial Like to criticise and evaluate people and things.

The questionnaire about ways of thinking by Strenberg’s consist 65 item at an average 5 item for each thinking style. Table 1 shows the distribution of item in the ways of thinking:

| Types     | Items                | Types     | Items                | Types     | Items                |
|-----------|----------------------|-----------|----------------------|-----------|----------------------|
| Legislative | 1,14,27,40,53        | Local     | 5,18,31,44,57        | Monarchic | 9,22,35,48,61        |
| Executive  | 2,15,28,41,54        | Liberal   | 6,19,32,45,58        | Oligarchic| 10,23,36,49,62       |
| Judicial   | 3,16,29,42,55        | Conservative | 7,20,33,46,59 | Anarchic  | 11,24,37,50,63       |
| Global     | 4,17,30,43,56        | Hierarchic| 8,21,34,47,60        | Internal  | 12,25,38,51,64       |
| External   | 13,26,39,52,65       |           |                      |           |                      |

The aims of this study is to investigate the relationship between thinking styles and proof writing levels among university students at Muhammadiyah University of Bengkulu based on Strenberg’s ways of thinking.

2. Methodology
This study was both of qualitative and quantitative design with descriptive research. The participants in this study were 124 students from mathematics study program of Muhammadiyah University of Bengkulu in academic year 2016/2017. There were three tools for gathering the data: 1) questionnaire; 2) interview to obtain data on the student's thinking style; and 3) test to obtain data in student’s writing proofs. The questionnaire made based on every indicator of thinking style.

Both a qualitative and a quantitative was use to collect the data through questionnaire, interviews and documents analysis. The questionnaire was based on an existing standardized instrument, which aimed to determine the different strategies used by the students to proofs the theorems or proposition, execute tasks and make decisions. The questionnaire consisted the functions dimensions of thinking style which three type. For each type, there were five questions on 1 – 4 point Likert scale. The score was then averaged over each type. The second phase of this study by conducting one-to-one interviews with the students based on their answer in the questionnaire. The aim was to ascertain their thinking style when doing the proof and proving. The data about students proofs levels taken from their work documents.

3. Result and Discussion
3.1. Thinking Style
Table 1 shows the distribution of item in the ways of thinking. Each of thinking style type, legislative, executive and judicial have 5 item. One of the item in the legislative type asked the students when they discussing or writing idea. Are the students have many ideas or not? And most of the students answer that they stick only to one idea. In general, the result of the questionnaire and one-to-one interview show in Table 2. Table 2 show students thinking styles based on the types of thinking style.
Table 2. Students thinking style

| Type of Thinking Style | Frequency |
|------------------------|-----------|
| Legislative            | 65        |
| Executive              | 47        |
| Judicial               | 12        |
| Total                  | 124       |

Table 2 shows that most of the students in mathematics study program of Muhammadiyah University of Bengkulu have legislative thinking style. Students who have judicial type thinking style only 12. From that we knew that most of the students stick to standard rules or ways of doing things. This result supported by [21] who claimed that mathematics requires that students be able to think in the terms, thus students should have a preference to create, discover and design which the indicator legislative thinking style.

3.2. Writing Proofs

Students writing proofs were determined based on the combination in [9] and [12]. The combination shown in Table 3.

Table 3. Levels of Students Proof

| Levels of Proofs    | Indicators/Students Response                                                                 |
|---------------------|---------------------------------------------------------------------------------------------|
| Analytic or Deductive| completely accurate proofs that have to be organized with a theory or the use of information commonly accepted by a community |
| Empirical           | 1. based on formulation, 2. proofs are given through the use of incomplete mathematical language and demonstration, 3. write proof with the general argument |
| External            | 1. unrelated responses, 2. proof completely based on the numerical example, 3. responses based simply on the writing of what is given |

Based on Table 3, we made students respond when they doing proofs. For examples when students asked about the set of the natural number is a group or not. They proving about the closure properties in natural number with the example $4 + 5 = 9$, 4 is a natural number, 5 is natural number and 9 is natural number so natural number is closed. Based on the answer we said the the students in external level because the response is unrelated. In general, the students level of proofs shows in table 4.

Table 4. Students Levels of Proofs

| Level of Proofs | Frequency | %  |
|-----------------|-----------|----|
| Analytical      | 6         | 4.8|
| Empirical       | 54        | 43.6|
| External        | 64        | 51.6|

From the Table 4, students who have an analytical level of proofs only 4.8%, it means most students have low abilities in proofs. Students in Muhammadiyah University have the external level of proofs. They often gave the unrelated respond when doing the proofs. The combination students thinking style and levels of proofs show in Table 5.
Table 5. Students Thinking Style and Their Writing Proofs Levels

|           | External | Empirical | Analytical |
|-----------|----------|-----------|------------|
| Legislative | 37       | 28        | 0          |
| Executive  | 14       | 29        | 4          |
| Judicial   | 3        | 7         | 2          |

Table 5 shows that students who have legislative thinking style most have an external level of proof. It is clear because in legislative thinking style students does things using their own method even that is true or not. This result is also supported by [22] who said that the complexity of mathematics can make extensive demands on the reasoning, interpretive and strategic skills of students.

Therefore, based on the data tested correlational to see the relationship between student's thinking style with writing proofs levels. The results indicated that there was a significant relationship between thinking styles and proof writing levels with the correlation 0.67 and the level of significance was 0.95.

In this study, it was seen that students who have external type of writing proofs are 29.8% from legislative, 11.3% from executive and 2.41% from judicial thinking styles. Students who have empirical type of writing proof are from legislative 22.6%, executive 23.4% and 5.7% from judicial thinking styles. The analytical levels of writing proofs are 3.2 from executive and 1.6 from judicial thinking styles. No ones in analytical levels from legislative thinking styles. According to result, there are differences the levels of writing proofs with related to thinking style. The result of this study agreed with the result of a study by [5].

4. Conclusion

Generally, students have different thinking styles, and it is can affect their ability in writing proofs. However, students thinking styles could be taken into consideration as guidelines when teachers give a task writing proofs. Based on the finding of this research, further research is required to develop students writing proofs in the mathematics class.

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