Einstein’s Boxes

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At the 1927 Solvay conference, Albert Einstein presented a thought experiment intended to demonstrate the incompleteness of the quantum mechanical description of reality. In the following years, the experiment was modified by Einstein, de Broglie, and several other commentators into a simple scenario involving the splitting in half of the wave function of a single particle in a box. This paper collects together several formulations of this thought experiment from the literature, analyzes and assesses it from the point of view of the Einstein-Bohr debates, the EPR dilemma, and Bell’s theorem, and argues for “Einstein’s Boxes” taking its rightful place alongside similar but historically better known quantum mechanical thought experiments such as EPR and Schrödinger’s Cat.

I. INTRODUCTION

It is well known that several of quantum theory’s founders were dissatisfied with the theory as interpreted by Niels Bohr and other members of the Copenhagen school. Before about 1928, for example, Louis de Broglie advocated what is now called a hidden variable theory: a pilot-wave version of quantum mechanics in which particles follow continuous trajectories, guided by a quantum wave. David Bohm’s rediscovery and completion of the pilot-wave theory in 1952 led de Broglie back to these ideas: Bohm’s theory has continued to inspire interest in alternatives to the Copenhagen interpretation. Erwin Schrödinger was likewise doubtful that the quantum wave function could alone constitute a complete description of physical reality. His famous “cat” thought experiment was intended to demonstrate quantum theory’s incompleteness by magnifying the allegedly real quantum indefiniteness up to the macroscopic level where it would directly conflict with experience.

By far the most important critic of quantum theory, however, was Albert Einstein. Several of his early thought experiments attacking the completeness doctrine are memorably recounted in Bohr’s reminiscence. Einstein’s most important argument against quantum theory’s completeness is the paper (EPR) he co-authored with Boris Podolsky and Nathan Rosen in 1935.

The purpose of this paper is to resurrect another thought experiment which, like EPR and Schrödinger’s cat, is intended to argue against the orthodox doctrine of quantum completeness. This thought experiment — “Einstein Boxes” — is due originally to Einstein, although it has also been discussed and reformulated by de Broglie, Schrödinger, Heisenberg, and others.

Given its unique simplicity, clarity, and elegance, the relative obscurity of this thought experiment is unjustified. Although generally aiming to establish the same dilemma as that posed in the EPR paper, Einstein’s Boxes establishes this conclusion with a more straightforward logical argument. Our hope is that this paper will aid in re-injecting this old thought experiment into the ongoing discussions of the significance and implications of EPR, the completeness doctrine, and alternatives to Copenhagen such as Bohm’s non-local hidden variable theory. Because of its remarkable simplicity, the Einstein Boxes thought experiment also is well-suited as an introduction to these topics for students and other interested non-experts.

The paper is organized as follows. In Sec. III we introduce Einstein’s Boxes by quoting a detailed description due to de Broglie. We compare it to the EPR argument and discuss how it fares in eluding Bohr’s rebuttal of EPR. In Sec. IV we present a new, Bell-inspired formulation of Einstein’s Boxes and relate the ideas to Bell’s celebrated theorem about hidden variable theories. Section V begins with some comments of Heisenberg on the thought experiment, considers an experimentally realized version of it, and assesses some of Heisenberg’s statements on non-locality. In Sec. VI we conclude with a general discussion.

II. EPR AND DE BROGLIE’S VERSION OF THE BOXES

In a 1964 book de Broglie gave a detailed statement of the Einstein’s Boxes thought experiment.

“Suppose a particle is enclosed in a box $B$ with impermeable walls. The associated wave function $\Psi$ is confined to the box and cannot leave it. The usual interpretation asserts that the particle is “potentially” present in the whole of the box $B$, with a probability $|\Psi|^2$ at each point. Let us suppose that by some process or other, for example, by inserting a partition into the box, the box $B$ is divided into two separate parts $B_1$ and $B_2$ and that $B_1$ and $B_2$ are then transported to two very distant places, for example to Paris and Tokyo. [See Fig. 1.] The particle, which has not yet appeared, thus remains potentially present in the assembly of the two boxes and its wave function $\Psi$ consists of two parts, one of which,
Then separated, at which point the boxes may be opened and the contents examined.

The two half boxes (B1 and B2) are then separated, at which point the boxes may be opened and their contents examined.

$\Psi_1$, is located in $B_1$ and the other, $\Psi_2$, in $B_2$. The wave function is thus of the form $\Psi = c_1 \Psi_1 + c_2 \Psi_2$, where $|c_1|^2 + |c_2|^2 = 1$.

“The probability laws of wave mechanics now tell us that if an experiment is carried out in box $B_1$ in Paris, which will enable the presence of the particle to be revealed in this box, the probability of this experiment giving a positive result is $|c_1|^2$, whilst the probability of it giving a negative result is $|c_2|^2$. According to the usual interpretation, this would have the following significance: because the particle is present in the assembly of the two boxes prior to the observable localization, it would be immediately localized in box $B_1$ in the case of a positive result in Paris. This does not seem to me to be acceptable. The only reasonable interpretation appears to me to be that prior to the observable localization in $B_1$, we know that the particle was in one of the two boxes $B_1$ and $B_2$, but we do not know in which one, and the probabilities considered in the usual wave mechanics are the consequence of this partial ignorance. If we show that the particle is in box $B_1$, it implies simply that it was already there prior to localization,” and therefore, by implication, that it was already not in box $B_2$ before the observation. Generalizing, the measurement at $B_1$ cannot have affected the contents of the box $B_2$, and therefore the contents of $B_2$ (which are known after inspection of $B_1$) must have been already definite before that inspection.

According to at least one commonly held version of the orthodox interpretation, however, it is the very act of observation that disturbs the physical system in question and brings about a definite result. Why should de Broglie think that the act of opening $B_1$ and examining its contents can’t affect the contents of $B_2$? Because $B_2$ is spatially distant. Its contents could not be affected by any reasonable physical mechanism by something done to the well-separated $B_1$. That is evidently the point of the drainage experiment made in Tokyo in box $B_2$. Every other interpretation is absurd. How can we imagine that the simple fact of having observed nothing in Tokyo has been able to promote the localization of the particle at a distance of many thousands of miles away?”

Although de Broglie did not make his reasoning explicit, he gives some hints that allow us to plausibly reconstruct the intended logical structure of this argument for incompleteness. The rhetorical question at the end implies that de Broglie thinks it is impossible (“absurd”) for the negative result of a measurement in Tokyo to have a decisive causal effect on the contents of the box in Paris. Likewise for a positive result: “If we show that the particle is in box $B_1$, it implies simply that it was already there prior to localization,” and therefore, by implication, that it was already not in box $B_2$ before the observation. Generalizing, the measurement at $B_1$ cannot have affected the contents of the box $B_2$, and therefore the contents of $B_2$ (which are known after inspection of $B_1$) must have been already definite before that inspection.

There is thus a locality or separability assumption at work here. Given that $B_2$ is physically separated from $B_1$ by a great distance, nothing we do to $B_1$ can affect the contents of $B_2$. (Indeed, we could consider the contents of $B_2$ at space-time coordinates that lie outside the light cone of the observation event at $B_1$. Then any “disturbance” by the measurement at $B_1$ on $B_2$’s contents would evidently violate relativity’s prohibition on superluminal causation.) So when we open $B_1$ and determine its contents, we learn immediately whether or
not $B_2$ contains the particle. And because $B_2$’s contents cannot have been in any way affected by the opening of $B_1$, $B_2$ must have either contained or not contained the particle all along. And, finally, because “the wave function $\Psi$ gives no information about” the particle’s actual pre-measurement location (in particular, $\Psi$ attributes no definite particle content to $B_2$), that description must have been incomplete.

De Broglie seems to have had this reasoning in mind. Notably, it is structurally identical to the argument of the more famous EPR paper. EPR begins with a 2-particle wave function

$$\psi(x_1, x_2) = \delta(x_1 - x_2 - a) = \int dk e^{ik(x_1-x_2-a)}, \quad (1)$$

which is a simultaneous eigenstate of the relative position operator $(\hat{x}_1 - \hat{x}_2)$ (with eigenvalue $a$) and of the total momentum operator $(\hat{p}_1 + \hat{p}_2)$ (with eigenvalue 0). The authors also assume the two particles to be sufficiently well-separated that any measurement performed on particle 1 can have no physical effect on particle 2. They then argue as follows: if the experimenter chooses to measure the position of particle 1, he will immediately be able to infer the position of particle 2; should he choose instead to measure the momentum of particle 1, he may immediately infer the momentum of particle 2. Therefore, because nothing the experimenter does in the region near particle 1 can affect particle 2 in any way, both the position and momentum of particle 2 must already have had definite values prior to any measurement. Both quantities are “elements of reality” according to the famous EPR criterion of reality: “If, without in any way disturbing a system, we can predict with certainty (that is, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”

Having also defined a necessary condition for the completeness of a theory (“every element of the physical reality must have a counterpart in the physical theory”), EPR concluded that the quantum mechanical description is incomplete because it doesn’t permit simultaneous attribution of definite position and momentum values to particle 2.

Niels Bohr is widely believed to have refuted the EPR argument by pointing out an “essential ambiguity” in EPR’s criterion of reality. The alleged ambiguity was contained in EPR’s phrase “without in any way disturbing a system.” In his response, Bohr appears to concede EPR’s locality/separability principle when he writes that “there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure.” However: “even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system. Because these conditions constitute an inherent element of the description of any phenomenon to which the term ‘physical reality’ can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete.”

Bohr seems to have meant that although we do not “mechanically” disturb the second particle by measuring the position or momentum of particle 1, we do enact a decisive choice when we decide which quantity to measure. For whichever quantity we decide to actually measure, we forgo the possibility of subsequently learning the value of the complementary variable in the distant particle. Bohr discusses this decisive choice in terms of the need for a “discrimination between different experimental procedures which allow of the unambiguous use of complementary classical concepts.” Bohr is evidently referring to the mutually exclusive experimental arrangements needed to measure the position and momentum of particle 1.

In short, although we can choose to learn about the position of particle 2 or the momentum of particle 2, we cannot with any single experimental arrangement learn about both the position and the momentum. And, says Bohr, this fact demonstrates the invalidity of EPR’s simultaneous use of the two concepts in regard to particle 2: the experimental arrangement which warrants use of one concept makes the other simply inapplicable. Bohr believed that this “complementarity” perspective allows one to consistently maintain that the “quantum-mechanical description of physical phenomena ... fulfill[s] ... all rational demands of completeness.” That is, it eludes the EPR argument from locality to incompleteness.

Although widely accepted in the physics community, the adequacy of Bohr’s reply to EPR has been questioned many times by those who have carefully scrutinized the issue. We will not fully engage this long-standing dispute, but we can shed some fresh light on the debate over the validity of EPR’s conclusion by returning to Einstein’s Boxes and seeing how this simpler thought experiment fares in escaping what appears to have been the main thrust of Bohr’s response to EPR.

It seems reasonable to infer from de Broglie’s formulation of the Boxes argument (quoted above) that de Broglie meant to assert the existence of a pre-measurement “element of reality” corresponding to the contents of (at least) the distant box, $B_2$, and that he would argue for its existence on the basis of the EPR criterion of reality: “without in any way disturbing” box $B_2$, we can determine whether it contains a complete particle or nothing. The presence or non-presence of the particle in $B_2$ is therefore an element of reality even before the box $B_2$ is opened and its contents examined. Finally, because the initial quantum mechanical wave function $\Psi$ attributed neither (definite) state to box $B_2$, there is at least one element of reality which fails to “have a counterpart in the physical theory.” And therefore quantum theory is incomplete by precisely the EPR completeness criterion.
The logic of this elaboration of de Broglie’s argument exactly matches the logic of the EPR paper. But because the Boxes thought experiment in no way relies on a choice between two complementary quantities to measure, it seems immune from Bohr’s criticism that there is a kind of “semantic disturbance”16 effected by this choice. There is no question with the Boxes experiment about which quantity will be measured. We are going to open $B_1$ and look for the particle there, period. So there appears to be no room for “an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.”15 Given the assumption of locality/separability (with which Bohr appears to agree given his denial of a “mechanical disturbance”), the Einstein’s Boxes thought experiment establishes the conclusion of incompleteness (that is, it establishes the dilemma between locality and completeness) in a straightforward way that seems immune to Bohr’s criticism of EPR.

III. EINSTEIN AND THE BOXES

Although it is not specifically formulated in terms of a particle being split between a pair of boxes, Einstein presented a thought experiment at the 1927 Solvay Conference17,18 on which de Broglie’s later formulation was based. The experiment involves a particle incident on a diaphragm with a single aperture, behind which lies a large, hemispherical detection screen. (See Fig. 2.) With a sufficiently narrow aperture, the incident electrons will diffract, resulting in essentially spherical Schrödinger waves propagating toward the screen. If a single electron is sent through the aperture, the electron will eventually be detected at some distinct point on the screen. Einstein’s comments focus on the apparent conflict between the spreading spherical wave and the distinct point of eventual detection.

Einstein described two possible interpretations of this scenario. According to Interpretation 1, “The de Broglie-Schrödinger waves do not correspond to a single electron, but to a cloud of electrons extended in space. The theory does not give any information about the individual processes, but only about the ensemble of an infinity of elementary processes.” In Interpretation 2, “The theory has the pretension to be a complete theory of individual processes.”19,20

Implicit in Interpretation 1 is the idea that each particle follows some definite trajectory from the slit to the screen; the spreading of the wave merely indicates ignorance about the exact identity of this trajectory for any particular particle. The wave represents a kind of ensemble average over a large number of such individual trajectories. The standard Born rule can then be understood as follows: $|\Psi|^2$ gives the probability that any particular electron in this (real or imagined) ensemble in fact has (just prior to detection) a given location. Thus, according to Interpretation 1, the description in terms of wave and Born rule probabilities is incomplete because it fails to describe the actual path taken by each individual particle between the slit and the screen.

Interpretation 2, on the other hand, denies that each individual electron has a definite trajectory. Instead, each electron is assumed to be spread out in a way that is accurately described by the spherical wave. This interpretation was probably meant to capture the view held by Schrödinger at the time, according to which $|\Psi|^2$ represented an actual charge density for an individual electron. It also is a fair description of the views of Jordan and (the later) Heisenberg who held that, prior to the moment of detection, there existed a spatially dispersed potentiality for particle localization, these potentialities being triggered into realities by the eventual interaction with a measurement device.21

As Einstein went on to point out, however, the particle is in fact always found at a definite point on the screen. Thus, at the moment it is detected there, the Schrödinger wave (representing charge density or some kind of irreducible potentiality for particle localization) must suddenly collapse to zero at all other points on the screen. And, as long as we hold (with Interpretation 2) that the initially spherical wave accurately and completely captures the physical reality of the propagating particle, this collapse evidently involves a kind of action-at-a-distance.

Max Jammer summarizes Einstein’s objection to Interpretation 2 as follows: “If $|\psi|^2$ is interpreted [this way],

FIG. 2: Einstein’s original version of the boxes thought experiment: particles incident from the left penetrate a narrow aperture, where they diffract, resulting in essentially spherical Schrödinger waves propagating toward the hemispherical detection screen.
then, as long as no localization has been effected, the particle must be considered as potentially present with almost constant probability over the whole area of the screen; however, as soon as it is localized, a peculiar action-at-a-distance must be assumed to take place which prevents the continuously distributed wave in space from producing an effect at two places on the screen. This possibility will be further explored in Sec. IV.

Einstein continued: “It seems to me that this difficulty cannot be overcome unless the description of the process in terms of the Schrödinger wave is supplemented by some detailed specification of the localization of the particle during its propagation. I think M. de Broglie [who, recall, was toying with the pilot-wave theory at this time] is right in searching in this direction. If one works only with Schrödinger waves, the Interpretation 2 of $|\psi|^2$, I think, contradicts the postulate of relativity.”

The relevant aspect of the “postulate of relativity” is of course precisely the locality/ separability assumption mentioned previously: there can be no causal dependence between two space-like separated events. Relativity thus prohibits the action-at-a-distance entailed by Interpretation 2.

Einstein began the letter by clarifying what he meant by locality and completeness. He asked Schrödinger to consider a classical particle (a ball) confined to a box. A partition is then inserted, and the two half-boxes are carried to distant locations where they are separately opened and their contents examined. As in his discussion at the Solvay Conference, Einstein surveyed two possible ways of analyzing this experiment:

“Now I describe a state of affairs as follows: The probability is 1/2 that the ball is in the first box. Is this a complete description?

NO: A complete description is: the ball is (or is not) in the first box. That is how the characterization of the state of affairs must appear in a complete description.

YES: Before I open them, the ball is by no means in one of the two boxes. Being in a definite box only comes about when I lift the covers. This is what brings about the statistical character of the world of experience, or its empirical lawfulness. Before lifting the covers the state [of the distant box] is completely characterized by the number 1/2, whose significance as statistical findings, to be sure, is only attested to when carrying out observations. Statistics only arise because observation involves insufficiently known factors foreign to the system being described.”

Note that the NO and YES alternatives map exactly onto Interpretation 1 and Interpretation 2 from the 1927 discussion. According to the NO view (and Interpretation 1), the description of the state of the system in terms of probabilities is incomplete, there being, in reality, an actual fact of the matter about the location (trajectory) of the particle. According to the YES view (and Interpretation 2), the description in terms of probabilities is complete because the actual fact of the matter regarding the location of the ball (particle) only comes into existence with the act of measurement.

For the case of a classical particle like the ball, the YES view is not very plausible, and Einstein asserts that “the man on the street would only take the first . . . interpretation seriously.” But for a single electron, the YES view is essentially the completeness claim of the Copenhagen interpretation of quantum mechanics. Einstein didn’t accept this view and wanted to argue that the first view (NO) was the correct one not only for the classical particle but for the electron as well. But he recognized that merely arguing from the classical analogy wasn’t adequate, and went on in the letter to Schrödinger (and in several other places over the subsequent decades) to construct a complete argument.

The necessary additional premise — one that was taken for granted for the case of the ball but needed
to be made explicit to make the argument rigorous for the electron — was the locality/separability assumption. Howard translates Einstein, again from the same 1935 letter: “My way of thinking is now this: properly considered, one cannot [refute the completeness doctrine, Interpretation 2] if one does not make use of a supplementary principle: the ‘separation principle.’ That is to say: ‘the second box, along with everything having to do with its contents is independent of what happens with regard to the first box (separated partial systems).’" If one adheres to the separation principle, then one thereby excludes the second . . . point of view, and only the [first] point of view remains, according to which the above state description is an incomplete description of reality, or of the real states.”

Einstein continues: “The preceding analogy [that is, the ball-in-boxes scenario] corresponds only very imperfectly to the quantum mechanical example in the [EPR] paper. It is, however, designed to make clear the point of view that is essential to me. In quantum mechanics one describes a real state of affairs of a system by means of a normed function \( \psi \) of the coordinates (of configuration space). The temporal evolution is uniquely determined by the Schrödinger equation. One would now very much like to say the following: \( \psi \) stands in a one-to-one correspondence with the real state of the real system. The statistical character of measurement outcomes is exclusively due to the measuring apparatus, or the process of measurement. If this works, I talk about a complete description of reality by the theory. However, if such an interpretation doesn’t work out, then I call the theoretical description ‘incomplete’.\textsuperscript{30}

Note that Einstein defines “completeness” here as a one-to-one correspondence between quantum wave functions and real states of real systems. This definition is somewhat different from the one (cited in Sec. \textsuperscript{14} used in the EPR paper; Arthur Fine refers to it as “bijective completeness” in contrast to “EPR completeness.”\textsuperscript{31}

Einstein then dropped the ball-in-boxes example and returned to the original EPR scenario involving two entangled particles. He reconstructed the EPR logic as a simple argument from the separation principle to a failure of bijective completeness, modeled after the discussion of the boxes. The argument is the following: consider the entangled state of the two particles discussed in EPR. Before the measurement on particle 1, the entangled wave function attributes to particle 2 no definite position and no definite momentum. After a measurement on 1 (whatever is measured), the wave function collapses and particle 2 is in a state of some definite position or some definite momentum or perhaps some other quantity altogether — which state of course depends on what kind of measurement is made on particle 1 and what its outcome is. But that aspect (which to Bohr seemed so central to the EPR argument) is now completely irrelevant. All that matters is that the wave function associated with the second particle has changed in a situation where (by the separation principle) the actual physical state of particle 2 has not changed. That it happens to change to a state function that can (according to the standard eigenstate-eigenvalue link) be interpreted as attributing a definite value for some property (an “element of reality”) is simply irrelevant.\textsuperscript{32}

According to Einstein “Now what is essential is exclusively that [the wave functions relating to the distant particle] are in general different from one another. I assert that this difference is incompatible with the hypothesis that the \( \psi \) description is correlated one-to-one with the physical reality (the real state). After the collision, the real state of [the two particle system] consists precisely of the real state of [particle 1] and the real state of [particle 2], which two states have nothing to do with one another. The real state of [particle 2] thus cannot depend upon the kind of measurement I carry out on [particle 1]. (‘Separation hypothesis’ from above.) But then for the same state of [particle 2] there are two (in general arbitrarily many) equally justified \( \psi_{[2]} \), which contradicts the hypothesis of a one-to-one or complete description of the real states.”\textsuperscript{33}

In summary, by the separation principle, the real, physical state of particle 2 is unaffected by the measurement at 1. Yet according to the collapse postulate, the wave function associated with particle 2 changes. At most one of the two wave functions can constitute a complete description. Thus the quantum mechanical description is shown to be, in the general case, incomplete.

Although Einstein dropped the ball-in-boxes thought experiment (and returned to the two-particle entangled state of EPR) when presenting this reformulated argument from locality to incompleteness, he need not have done so. Despite his statement that the ball-in-boxes analogy “corresponds only very imperfectly to the quantum mechanical example in the [EPR] paper,” the EPR dilemma can be established equally well with a quantum mechanical version of the boxes.

Consider the quantum state of the single particle whose wave function has been split between two spatial locations (boxes):

\[
\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2),
\]

(2)

where \( \psi_{1,2} \) are completely localized in two well-separated regions of space \( (B_{1,2}) \). According to the standard recipe, if and when a position measurement is made on the particle, its wave function immediately collapses to zero at every point except the place where the particle was found. Applying this recipe here, imagine that we open box \( B_1 \) and find the particle there. At this moment, the wave function in \( B_2 \) immediately goes to zero (indicating that there is no longer any probability of finding the particle there). But according to the locality premise, the actual physical state of \( B_2 \) (whatever that might have been) cannot have changed. Hence, we have two different quantum mechanical descriptions (in particular, wave functions with different values in \( B_2 \)) of the same one unchanged physical contents of \( B_2 \) — a clear failure.
of Einstein’s (bijective) completeness criterion.

This reformulation of Einstein’s argument for incompleteness using the ball-in-boxes scenario is astonishingly simple compared to the EPR argument for the same conclusion. Our reformulation highlights that, unlike our reconstruction of de Broglie’s boxes argument and unlike the argument presented in EPR, the argument from locality to incompleteness need not rely on actually determining the identity of any elements of reality (by means of the EPR criterion of reality). In the EPR scenario involving the two entangled particles, there is an almost irresistible tendency to attempt to establish definite properties for the distant particle; after all, there is a particle there that one is tempted to think has those properties. With the (one particle) Einstein’s Boxes argument, however, the contents of the distant box are less clear, and the temptation to attribute definite properties is consequently reduced – which is a good thing, from the point of view of emphasizing that it does not matter to the argument what the contents of the box are. All that matters is what according to the locality principle, whatever is there doesn’t change as a result of the measurement performed in $B_1$. That alone is sufficient to establish the conclusion that Einstein’s bijective completeness criterion fails for (an assumed local) quantum theory.

It also bears repeating that Einstein’s actual argument in terms of the pair of previously interacting particles and the reformulation here in terms of the ball-in-boxes scenario both completely avoid any mention of the non-commuting observables that seemed (to Bohr at least) to play such a large role in the actual EPR paper. It is intriguing to speculate about how Bohr would have responded to this simplified version of the argument for incompleteness. We won’t, however, undertake such speculation here. Suffice it to credit Einstein’s Boxes for helping to un-smother Einstein’s true arguments against the quantum completeness doctrine.

Although Einstein strongly believed in the locality/separability premise of this argument and therefore regarded it as an argument for incompleteness, he realized that, strictly speaking, what the argument establishes is only that, for quantum mechanics, locality entails incompleteness. That is, quantum mechanics must give up either the completeness claim or the locality principle:

“By this way of looking at the matter it becomes evident that the paradox forces us to relinquish one of the following two assertions:

1. the description by means of the $\psi$-function is complete.

2. the real states of spatially separated objects are independent of each other.

On the other hand, it is possible to adhere to (2) if one regards the $\psi$-function as the description of a (statistical) ensemble of systems (and therefore relinquishes (1)). However, this view blasts the framework of the ‘orthodox quantum theory’."

Or as Arthur Fine eloquently summarizes this point: “It is important to notice that the conclusion Einstein draws from EPR is not a categorical claim for the incompleteness of quantum theory. It is rather that the theory poses a dilemma between completeness and separation; both cannot be true."

Perhaps this elaboration of the logic of Einstein’s argument against the completeness doctrine — especially the simplified version involving the boxes — helps explain why Einstein never believed that Bohr (or anyone else) had adequately addressed the dilemma between locality and completeness posed in EPR.

### IV. BELL AND THE BOXES

In the last paper of his tragically short life, J. S. Bell prefaced a discussion and derivation of his celebrated inequality with a classical analogy reminiscent of Einstein’s boxes. Bell wrote:

“Most physicists were (and are) unimpressed by [the correlations exhibited in the EPR scenario]. That is because most physicists do not really accept, deep down, that the wavefunction is the whole story. They tend to think that the analogy of the glove left at home is a good one. If I find that I have brought only one glove, and that it is right-handed, then I predict confidently that the one still at home will be seen to be left handed. But suppose we had been told, on good authority, that gloves are neither right- nor left-handed when not looked at. Then that, by looking at one, we could predetermine the result of looking at the other, at some remote place, would be remarkable. Finding that this is so in practice, we would very soon invent the idea that gloves are already one thing or the other even when not looked at. And we would begin to doubt the authorities that had assured us otherwise. That common-sense position was that taken by Einstein, Podolsky and Rosen, in respect of correlations in quantum mechanics. They decided that the wavefunction, making no distinction whatever between one possibility and another, could not be the whole story. And they conjectured that a more complete story would be locally causal.

“However it has turned out that quantum mechanics can not be ‘completed’ into a locally causal theory, at least as long as one allows, as Einstein, Podolsky and Rosen did, freely operating experimenters. The analogy of the gloves is not a good one. Common sense does not work here."
Bell goes on in the paper to derive his famous theorem, in the context of Bohm’s version of the EPR argument. In this version, it is different spin components of the two particles, rather than their positions and momenta, which play the role of quantum mechanically non-commuting variables which an EPR-like argument suggests may nevertheless possess simultaneously definite values.

Assuming a non- (or super-) quantum-mechanical theory in which each particle in the EPR pair carries with it a set of hidden variables (an “instruction set”) telling it how to respond when encountering variously oriented polarizers (that is, a theory with hidden elements of reality specifically the kind suggested by the EPR-Bohm thought experiment), Bell imposes a locality constraint according to which the outcome of the measurement on each side must depend only on the variables in the particles’ instruction set and the orientation of the local apparatus. The dependence of each outcome on the setting of the distant apparatus or the distant outcome is excluded. The mathematical consequence is an inequality restricting the average correlations between the outcomes of measurements on the two particles.

This kind of inequality is violated by the quantum mechanical predictions and, it seems, by real particles in real experiments. This violation means that one (or more) of the assumptions made in deriving the inequality must be false. The two assumptions typically called into question are the assumption of hidden variables and the assumed lack of (relativity-violating) causal dependence of the outcomes on distant settings and distant outcomes. And choosing one of these assumptions to blame has generally been considered no dilemma at all; relativity’s prohibition on super-luminal causation is a central pillar of modern physics, so “obviously” the correct choice is to reject the assumption of hidden variables. Thus, the empirical violations of Bell’s inequalities seem to be regarded by most physicists as the best argument against the hidden variables program and for the completeness doctrine.

For example, N. David Mermin wrote that violations of Bell’s inequalities “fatally undermine the position of EPR” meaning, presumably, EPR’s claim that a (hidden variable) completion of quantum theory might be possible. Daniel Styer, answering an alleged “misconception regarding quantum mechanics,” stated that “The appealing view that a state vector describes an ensemble of classical systems [and therefore an incomplete description of individual systems] was rendered untenable by tests of Bell’s theorem which show that no deterministic model, no matter how complicated, can give rise to all the results of quantum mechanics.” (Bohm’s theory of course provides a straightforward counterexample to this claim, unless Styer intends the tacit premise that the deterministic model must respect locality.) Asher Peres and Daniel R. Terno assert that “Bell’s theorem does not imply the existence of any non-locality in quantum mechanics itself,” only “classical imitations of quantum mechanics” suffer inevitable non-locality. And Eugene Wigner wrote: “In my opinion, the most convincing argument against the theory of hidden variables was presented by J. S. Bell.”

In summary, it is almost universally believed that experimental violations of Bell’s inequalities imply fatal problems with attempts to “complete” quantum theory by adding hidden variables, but pose no particular problem or puzzle for quantum mechanics itself. This attitude evidently accounts for the fact that very few physicists are presently interested in the hidden variables program. It also explains why equally few worry that there is any deep contradiction between standard quantum theory and relativity.

Bell himself, however, did not see it this way. Like Einstein, Bell believed that quantum mechanics itself faced a problematic dilemma between completeness and locality. We will discuss the logical implications of this shortly, in particular, the implications of combining the EPR (or Einstein’s Boxes) locality-completeness dilemma with Bell’s theorem.

First, let us give one final version of the argument for this dilemma by applying Bell’s reasoning to the boxes scenario. Instead of applying this reasoning to an EPR-motivated hidden variable theory (like Bell did), however, we will assume orthodox quantum mechanics, completeness doctrine and all, as our working theory. What we will see is that by using reasoning similar to Bell’s, we can arrive, not at a conclusion equivalent to Bell’s famous theorem, but rather at a new argument for the EPR dilemma: quantum mechanics is either incomplete or non-local.

Let us begin by assuming that the initial wave function,

$$\lambda = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2),$$

is a complete description of the physical reality of the pre-measurement situation. (As before, $\psi_1$ and $\psi_2$ represent states in which the particle is localized respectively in $B_1$ and $B_2$.

Note too that we have used the variable $\lambda$ rather than $\psi$ to denote the pre-measurement wave function. This choice is meant to highlight the similarity to Bell’s derivation, in which $\lambda$ is traditionally used to denote a complete specification of the pre-measurement state of the two particles including any necessary hidden variables. Here, however, we are assuming with Bohr that the wave function alone provides a complete specification of the pre-measurement contents of the boxes.)

Let $N_1 = \{0, 1\}$ represent the number of particles found in box $B_1$ when it is opened; similarly for $N_2$. The Born rule implies the following expressions for the probabilities of finding $N_1 = 1$ (when/if $B_1$ is opened and examined) and of finding $N_2 = 1$ (when/if $B_2$ is opened and examined):

$$P(N_1 = 1 | \lambda) = \frac{1}{2}.$$
and

\[ P(N_2 = 1 \mid \lambda) = \frac{1}{2} \]  

Finally, let us calculate the probability for a double detection, that is, the probability that the particle is found in both of the two half-boxes when they are opened. Call this probability \( P(N_1 = 1 \& N_2 = 1) \). If we follow exactly Bell’s reasoning in deriving an inequality for hidden variable theories, we can (by Bayes’ formula) decompose the joint probability into a product as:

\[ P(N_1 = 1 \& N_2 = 1 \mid \lambda) = P(N_1 = 1 \mid \lambda) \cdot P(N_2 = 1 \mid N_1 = 1, \lambda). \]  

Now, quoting Bell: “Invoking local causality, and the assumed completeness of \( \ldots \lambda \ldots \), we declare redundant certain of the conditional variables in the last expression, because they are at space-like separation from the result in question.” In our situation, this same local causality requirement implies that

\[ P(N_2 = 1 \mid N_1 = 1, \lambda) = P(N_2 = 1 \mid \lambda), \]  

because, as Bell says, the outcome event at \( B_1 \) \((N_1 = 1)\) is (let us assume) at a space-like separation from the event in question \((N_2 = 1)\) at \( B_2 \). Thus, the probability of this event at \( B_2 \) should not depend on the outcome \( N_1 = 1 \); it should instead depend only on those variables \((\lambda)\) that described (by assumption, completely) the pre-measurement contents of the boxes.

Thus, the probability of double-detection simplifies to

\[ P(N_1 = 1 \& N_2 = 1 \mid \lambda) = P(N_1 = 1 \mid \lambda) \cdot P(N_2 = 1 \mid \lambda). \]  

If we substitute the two 50% probabilities on the right-hand side, we evidently find that a double-detection should occur with probability 25%. That is, one quarter of the time, we should find the particle in both boxes.

One expects, of course, that this will never happen. (Such an occurrence would violate several fundamental conservation principles.) Therefore one (or more) of the assumptions leading to this prediction must be wrong. In particular, it seems that we must reject either the assumption of locality or the assumption of completeness – the same dilemma established earlier using the boxes in a different way.

The reader might object that we neglected to collapse the wave function and thus erred in treating the second measurement as based on the same quantum state \((\lambda)\) as the first. As Einstein pointed out in 1927, it is precisely the collapse postulate which prevents a single particle, whose wave function before measurement is spread out over some finite region of space, from being detected at two different places at once.

Our claim, however, is that this collapse violates the locality assumption if the wave function description is regarded as complete. If the physical contents of the second box do not change as a result of the measurement on the first box, and if these physical contents are completely described by the wave function \(\lambda\), then the associated (Born rule) probability cannot change as a result of the outcome of the distant experiment. That is precisely Bell’s locality assumption. To demand the use of a new, collapsed wave function for the second box (one that takes into account the information gathered when the first box was opened) is to concede that the description provided by the initial wave function was incomplete.

If the reader finds this response unconvincing, imagine that the same objection was leveled against the parallel assumption in Bell’s theorem by an advocate of a supposedly local hidden variable theory. Such an advocate might object to Bell on the grounds that, after the first measurement is made (on particle \(A\) say), the state specification of particle \(B\) should be allowed to re-adjust to be in accord with the outcome of (that is, the information obtained during) the experiment on \(A\). “The experimental violation of Bell’s inequalities,” such a person might say, “in no way proves that my local hidden variable theory is untenable. For Bell unjustifiably assumed that the probabilities for various outcomes at \(B\) could depend only on the physical state attributed to \(B\) before the measurement at \(A\). But in my local hidden variable theory, the state attributed to \(B\), and therefore the probability for a particular outcome at \(B\), depends on the result of the measurement at \(A\). Thus Bell’s constraint does not apply and my theory is saved.”

The correct response to such an argument would be to simply deny that this sort of theory is local. A state attribution for \(B\) that changes as a result of the measurement outcome at \(A\) is precisely the kind of non-locality Bell’s locality assumption is meant to exclude. But this response applies equally well to quantum theory itself, if we are assuming that quantum theory provides, with the wave function alone, a complete description of physical reality. Recalling Bell’s reasoning, it is precisely the assumption of “the assumed completeness of \( \ldots \lambda \)” that warrants the removal of the \(A\)-side outcome from the list of variables on which the \(B\)-side result is conditioned.

Despite the conventional wisdom to the contrary, Bell’s reasoning is not uniquely applicable to hidden variable theories or any other particular kind of theory. Rather it is a way of teasing out the implications of any theory that is assumed to be both complete and local. If we use the Einstein’s Boxes scenario, this reasoning (applied to quantum theory without any hidden variables) provides yet another way to establish Einstein’s dilemma: quantum theory either violates relativity’s demand for locality/separability or is incomplete.

Note that this Bell-inspired argument for the quantum completeness-locality dilemma relies on definitions of completeness and locality that are different from those encountered in Secs. \(\text{[I]}\) and \(\text{[III]}\). Locality is defined here formally in terms of the stochastic irrelevance (or redundancy) of facts that are outside the past light cone of a given event: 

\[ P(N_2 = 1 \mid N_1 = 1, \lambda) = P(N_2 = 1 \mid \lambda). \]
And completeness here amounts to the standard assertion that the probability of detecting a particle is given by the absolute square of the wave function, and that this probability cannot be accounted for by ignorance regarding any additional hidden variables – i.e., that there is nothing other than the value of χ in B₂ that determines the probability for detecting a particle there.¹⁷

We return then to the earlier point about the use of Bell’s theorem as an argument against the hidden variables program (that is, its use as an argument for the orthodox completeness doctrine). It seems that we need to reconsider the implications of Bell’s theorem when it is combined with the completeness-locality dilemma. If Bohr can be considered to have adequately refuted the argument for this dilemma, that is, if quantum mechanics can be validly interpreted as both complete and local, then Bell’s theorem would indeed argue strongly against any attempt to supplement the theory with additional variables. But what if (perhaps motivated by Einstein’s Boxes) we regard Bohr’s response as inadequate? That is, what if we agree with Einstein that a dilemma between locality and completeness exists for quantum theory?

There seem to be two options. We might choose to regard the quantum theory as complete but non-local. But then we are no longer in a position to use the required non-locality of hidden variable theories (required, that is, by Bell’s theorem) as a reason to dismiss them. If non-locality is unacceptable to physical theories, not just Bohm’s theory (for example), but quantum theory itself would have to be dismissed as in conflict with relativity theory.

Perhaps we are therefore tempted to back up and take the other horn of the dilemma – to regard quantum mechanics as local but incomplete. But then it seems clear that we ought to attempt to complete it, presumably by adding hidden variables (which, it turns out, will have to be non-local).

Either way, Bell’s theorem ceases to be a valid argument against the hidden variables program – a conclusion opposite the standard interpretation of Bell’s result.

If the argument for the locality-completeness dilemma is sound – and I think the various formulations in terms of Einstein’s Boxes make this argument rather compelling compared to the original EPR argument – then Bell’s result turns into a powerful argument in favor of taking seriously non-local hidden variable theories like Bohm’s.¹⁸

V. HEISENBERG, LOCALITY, AND EXPERIMENTAL METAPHYSICS

Heisenberg also discussed something like Einstein’s Boxes: “... one other idealized experiment (due to Einstein) may be considered. We imagine a photon which is represented by a wave packet built up out of Maxwell waves. It will thus have a certain spatial extension and also a certain range of frequency. By reflection at a semi-transparent mirror, it is possible to decompose it into two parts, a reflected and a transmitted packet. There is then a definite probability for finding the photon either in one part or in the other part of the divided wave packet. After a sufficient time the two parts will be separated by any distance desired; now if an experiment yields the result that the photon is, say, in the reflected part of the packet, then the probability of finding the photon in the other part of the packet immediately becomes zero. The experiment at the position of the reflected packet thus exerts a kind of action (reduction of the wave packet) at the distant point occupied by the transmitted packet, and one sees that this action is propagated with a velocity greater than that of light.”¹⁹

There are several interesting points to note. First, Heisenberg converted the Boxes thought experiment into a more realistic (and experimentally realizable; see the following) situation where the carrying apart of two half-boxes is replaced by the natural motion of a particle after being either transmitted or reflected by a half-silvered mirror. Second, it is interesting that Heisenberg seemed to accept the dilemma between locality and completeness – and I think the various formulations in terms of Einstein’s argument is intended to establish and to favor the “complete but non-local” response to that dilemma.

Yet as he claimed in the sentence just following the previous quote, the admitted “action at a distance” that is exerted by the nearby experimental apparatus doesn’t lead to a contradiction with relativity: “However, it is also obvious that this kind of action can never be utilized for the transmission of signals so that it is not in conflict with the postulates of the theory of relativity.”²⁰

So evidently Heisenberg’s position was to concede to Einstein that quantum mechanics was non-local, but to deny that its particular type of non-locality was at odds with relativity’s prohibitions on superluminal causation. (This view is widely held and was eloquently advocated recently by Abner Shimony²¹ who described the situation as a “peaceful coexistence” between quantum non-locality and relativity.)²² The tacit premise here is that the only kind of superluminal causation that relativity theory forbids is the kind that can be used “for the transmission of signals.”

This particular attempt to wriggle free of the dilemma was assessed by Bell:

“Do we then have to fall back on ‘no signaling faster than light’ as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have lost the idea that correlations can be explained, or at least this idea awaits reformulation. More importantly, the ‘no signaling’ notion rests on concepts which are desperately vague, or vaguely applicable. The assertion that ‘we cannot signal faster than light’ immediately provokes the question:

Who do we think we are?
We who can make ‘measurements’, we who can manipulate ‘external fields’, we who can ‘signal’ at all, even if not faster than light? Do we include chemists, or only physicists, plants, or only animals, pocket calculators, or only mainframe computers?

In addition, Heisenberg’s move here undermines one of the principal arguments against non-local hidden variable theories like Bohm’s. For although that theory does include non-local, action-at-a-distance effects, these effects (just like the similar effects Heisenberg concedes are present in orthodox quantum mechanics) cannot be used to transmit signals. In Bohm’s theory this inability arises from the inevitable uncertainty about the initial locations of particles within their guiding waves. So if, as Heisenberg argues, it is only the possibility of sending superluminal signals that conflicts with relativity, we must evidently conclude that Bohm’s theory and orthodox quantum mechanics enjoy an equally peaceful coexistence with it.

Let us finally return to the previous comment about the apparent experimental realizability of the Heisenberg version of Einstein’s Boxes. According to John Clauser Schrödinger was intrigued by the clarity with which Einstein’s Boxes brought out the locality-completeness dilemma for quantum theory, and persuaded Ádám, Jánossy, and Varga (AJV) to perform Heisenberg’s version of the experiment.

Let us quote from Clauser’s description of the AJV experiment:

“In their experiment, two independent photodetectors are placed respectively in the transmitted and reflected beams of a half-silvered mirror. If photons have a particle-like character, that is, if their detectable components are always spatially bounded and well localized, then photons impinging on the half-silvered mirror will not be split in two at this mirror. On the other hand, if they are purely wavelike in nature, (as with classical waves) then they can and will be split into two independent classical wave packets at the mirror. This fact then implies that if they are purely wavelike (in this classical sense), then the two detectors will show coincidences when a single temporarily localized photon is directed at said mirror. One of these independent wave packets will be transmitted to illuminate the first detector, and the other will be reflected to illuminate the second detector, and both detectors will then have a finite probability of detecting the same photon (classical wave packet) . . . [AJV] found no such anomalous coincidences, and thereby concluded that . . . photons do not split at the mirror and thus do exhibit a particle-like character."

Note that this experiment is precisely an empirical test of Eq. (8). Although the outcome of the experiment is not surprising to anyone today, it is hoped that the preceding discussion underlines the apparent implications of this outcome: if we accept the locality principle, we must evidently conclude that photons do not split at the mirror, but instead choose one or the other path exclusively. That is, we must conclude that the quantum mechanical description of the photon before detection,

\[ \psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2), \]

is incomplete because it fails to attribute a definite position to the particle.

Alternatively, we could uphold the completeness of the wave function description by denying the locality premise and regarding the collapse of the wave function on measurement as describing a real physical change in the state of the distant packet’s referent.

There is thus an interesting parallel between the AJV experiment and the recent experimental tests of Bell’s inequalities. Because Bell’s inequality represents a constraint on local hidden variable theories, empirical violations of the inequalities imply that local hidden variable theories aren’t empirically viable. Likewise, Eq. (8) represents a constraint on a local version of quantum theory (assumed complete). So the AJV finding that this constraint is violated has implications paralleling those of the Bell tests: the local version of quantum theory (assumed complete) is not empirically viable. Either locality or the orthodox completeness assumption must be abandoned. This conclusion is not new, but the same dilemma we have seen now several times.

The extent to which orthodox quantum theory and the hidden variable alternatives are on equal footing is striking: local versions of each are evidently refuted by experiment. Thus it seems the program of “experimental metaphysics” (to use one of Shimony’s apt phrases) began more than 30 years before the recent tests of Bell’s inequality.

VI. DISCUSSION

We have presented several different formulations and revisions of the argument that Einstein first presented at the 1927 Solvay conference. The goal of this argument in all of its versions is to establish an inconsistency between the following two claims:

- The quantum mechanical description of reality is complete.
- Quantum mechanics describes a world with only local interactions.

Although the various formulations of the argument differ slightly in terms of logical structure and definitions of key terms, they point to the same conclusion and ultimately pivot around the same fulcrum: the collapse of the wave function.
The collapse rule itself is absolutely essential to the quantum formalism, as evidenced, for example, by the derivation of Eq. (8) and the subsequent discussion in Sec. IV. However, the exact meaning of the rule is somewhat opaque in the standard interpretation. The essential lesson of Einstein’s Boxes (and this is a point Einstein seems to have understood quite clearly as early as 1927) is that we face a very simple choice when it comes to interpreting the collapse. Either it represents a real, physical change of the physical state of the affected system, or it doesn’t. If it does represent such a physical disturbance, this entails a non-local action-at-a-distance that seems to conflict with relativity’s prohibition on superluminal causation. (Or, if the physical disturbance produced by the collapse were to propagate from the measurement event at a sub-luminal speed, the quantum predictions would be empirically wrong in cases of spatially separated correlations like those illustrated in the AJV experiment.) On the other hand, if the collapse of the wave function does not represent a physical change in the state of the real system, then the quantum mechanical description must be incomplete (in at least Einstein’s bijective sense). For quantum theory would then attribute two distinct state descriptions to one and the same physical state.

The orthodox claim, initiated by Bohr in his reply to EPR, is that quantum mechanics is both complete and local. On further scrutiny, however, it seems this claim can be maintained only with a kind of double-speak about the collapse postulate. To protect the completeness claim, the collapse must be interpreted as a real physical state change, usually backed up by some version of the disturbance theory of measurement. On the other hand, the locality claim can be defended only by interpreting the wave function collapse as merely a change in knowledge.

This double-speak was perhaps first identified by Einstein. In the 1935 letter to Schrödinger that was discussed earlier, Einstein wrote: “The talmudic philosopher doesn’t give a hoot for ‘reality’, which he regards as a hobgoblin of the naive, and he declares that the two points of view differ only as to their mode of expression.” The “Talmudist” here is Bohr, and the “two points of view” are the two interpretations of the boxes discussed previously. But these interpretations map precisely onto the two just-mentioned interpretations of wave function collapse. If, on the one hand, the wave function description is incomplete (Interpretation 1 from Sec. III), then the collapse postulate can be safely interpreted as merely updating our knowledge. But if, on the other hand, the wave function description is regarded as complete (Interpretation 2), then the collapse must represent a real physical change in the state of the system.

Evidently Einstein believed that the claim that wave function collapse represents a physically real change of state, and the claim that it represents a mere change in knowledge, do not “differ only as to their mode of expression.” Of course, the existence of a clean distinction between these two views of wave function collapse presupposes a realist philosophy that Bohr and many of his followers deny (or even claim to have refuted). Bohr, for example, is said to have declared that “there is no quantum world.” Although this anti-realist attitude continues to receive lip service, it seems that most physicists don’t take the literal denial of a quantum world very seriously. Most of us believe in the real existence of atoms, electrons, and other sub-atomic particles. Whatever strange quantum properties they may have, surely there is a “they” that have them.

We will by no means resolve the debate about realism here. But it is important to recognize that anti-realism is not a sensible response to the alleged incompatibility of the completeness and locality claims. It is, rather, a wholesale denial of the very issues of completeness and locality: there is no sense, for example, to a claim that there is no quantum world, but quantum mechanics provides a complete description of it.

One cannot respond to Einstein’s dilemma by claiming that an anti-realist attitude warrants a simultaneous belief in the completeness and locality of quantum mechanics. For according to the anti-realist attitude, there is simply no such issue as completeness (there being no quantum world for quantum mechanics to completely or incompletely describe) and likewise no such issue as locality. This wholesale denial of the issues is not what Bohr seems to have expressed explicitly, and it does not seem consistent with the views of most contemporary defenders of the Copenhagen interpretation who think, for example, that locality is a meaningful and important requirement for physical theories.

It is only from the point of view of the quantum realist that these issues are meaningful; and from this point of view, they are crucial and must be addressed before we can claim to truly understand the meaning and status of quantum theory. That is a program Einstein helped initiate in the 1920s and, sadly, it is one that has been largely abandoned by the physics community – no doubt because many came to believe that the standard interpretation was adequate on both counts. But, from the point of view of Einstein, this adequacy was largely overestimated: the standard interpretation of quantum theory does suffer from a troubling dilemma, one with major implications not only for the theory itself but also for possible alternatives to or extensions of it. Indeed, when combined with the empirical violations of Bell’s inequalities, Einstein’s dilemma has major implications for our understanding of nature.

It is fascinating that, after more than 75 years of heated discussion, Einstein’s criticisms of the quantum theory still seem effective and fresh. There is still much work to be done before we achieve total clarity about the foundations of quantum theory, the kind of clarity that Einstein continuously sought. As we approach the centennial celebration of Einstein’s “miraculous year”, let us acknowledge that Einstein still has much to teach us.
about how this clarity can be achieved. It is hoped that the Einstein’s Boxes argument in particular will continue to stimulate further discussions on these fascinating and important topics.

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4 Erwin Schrödinger “The present situation in quantum mechanics,” translated by John D. Trimmer, in Proc. Amer. Philosophical Soc. 124, 323–38 (1980), reprinted in Ref. 6.
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7 Niels Bohr, “Discussions with Einstein on epistemological problems in atomic physics,” in Albert Einstein: *Philosopher-Scientist*, edited by P. A. Schilpp (*The Library of Living Philosophers*, Evanston, 1949), pp. 200–241.
8 A. Einstein, B. Podolsky, and N. Rosen, “Can quantum mechanical description of physical reality be considered complete?,” Phys. Rev. 47, 777–780 (1935).
9 Credit for this name should probably go to Arthur Fine, who used the phrase in passing in *The Shaky Game* (The University of Chicago Press, 1996), p. 37.
10 Lucien Hardy presented an equally clear account of the thought experiment in a recent paper: “Spooky action at a distance in quantum mechanics,” *Contemporary Phys.* 39 (6), 419–429 (1998). For an interesting theoretical discussion of some potential practical difficulties with this experiment, see Julio Gea-Banacloche, “Splitting the wave function of a particle in a box,” Am. J. Phys. 70 (3), 307–312 (2002) and T. Norsen, “Splitting the wave function,” Am. J. Phys. 70 (7), 664 (2002).
11 L. de Broglie, *The Current Interpretation of Wave Mechanics: A Critical Study* (Elsevier Publishing Company, 1964). Note also that this passage comes just after a discussion of the thought experiment given by Einstein at the 1927 Solvay conference and that de Broglie introduced the discussion with this comment: “In a recent article . . . I have revived the above objections [Einstein’s] in a new form.” The article is L. de Broglie, “L’interprétation de la mécanique ondulatoire,” J. Phys. Rad. 20, 963 (1959).
12 N. Bohr, “Can quantum-mechanical description of physical reality be considered complete?,” Phys. Rev. 48, 696–702 (1935).
13 This is one very plausible and influential interpretation of Bohr’s reply; see, for example, Ref. 11, p. 196. But other commentators have understood Bohr’s rather obscure prose differently or not at all.
14 See, for example, the comments of J. S. Bell: Ref. 1, p. 155–6.
15 Actually, as Tim Maudlin, *Quantum Non-Locality and Relativity* (Blackwell Publishing, 1994), pp. 140–141 has eloquently pointed out, neither does the original EPR thought experiment. The fact that one can, in the EPR situation, predict with certainty just the position of particle 2 (without disturbing it in any way) is sufficient, according to the reasoning set up by EPR, to conclude that, if local, quantum theory is incomplete. For that alone means that the position of particle 2 is an element of reality, and one that has no counterpart in the original, pre-measurement entangled wave function. Maudlin refers to the attempt to (in addition) beat the uncertainty principle by also establishing the real existence of particle 2’s momentum as “an unnecessary bit of grandstanding” which plunged “the previously simple EPR argument into the muddy waters of [modal logic and counterfactual definiteness].” As we have seen, it is precisely this question of counterfactuals that Bohr seems to have jumped on in his reply. In any case, Einstein’s Boxes (which completely avoids the issue of counterfactual definiteness) should help clarify this point.
16 Ref. 6, p. 35.
17 See, for example, Ref. 7, pp. 115–117; Ref. 11, pp. 24–26.
18 L. E. Ballentine, “Einstein’s interpretation of quantum mechanics,” Am. J. Phys. 40, 1763–1771 (1972).
19 L. E. Ballentine translates the French from “Électrons et Photons,” Institut International de Physique Solvay, Rapports et Discussions du Cinqième Conseil de Physique (Gauthier-Villars, Paris, 1928), pp. 253–256.
20 Ref. 6, pp. 24–33.
21 Ref. 6, pp. 44, 160–162.
22 Ref. 6, p. 116.
23 Ref. 6, p. 116. See also Ref. 15, p. 1765.
24 Ref. 6, Chap. 3.
25 Ref. 6, p. 35.
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28 For a complete list of references, see Don Howard, “Einstein on locality and separability,” Stud. Hist. Phil. Sci. 16, 171-201 (1985).
29 Ref. 11, pp. 178–179.
30 Ref. 11, p. 71.
Ref. 9, p. 180.

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J. S. Bell, “La nouvelle cuisine” in Between Science and Technology, edited by A. Sarlemijn and P. Kroes (Elsevier Science (North-Holland), 1990). Bell gives an equivalent classical analog in “Bertleman’s socks and the nature of reality,” reprinted in Ref. 3.

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Ascher Peres and Daniel R. Terno, “Quantum Information and Relativity Theory”, Rev. Mod. Phys. 76 (2004) 93.

E. Wigner. “The interpretation of quantum mechanics,” reprinted in Ref. 3.

Ref. 43, p. 109.

Arthur Fine (private communication) has criticized this derivation on the grounds that it equates factorizability (of the joint, double-detection probability) with locality. (See also “The Shaky Game,” pp. 59–63 and “Do Correlations Need to be Explained?” in Philosophical Consequences of Quantum Theory, edited by J. Cushing and E. McMullin (University of Notre Dame Press, Notre Dame, 1989) pp. 175–194.) He argues that there is no reason (save a kind of primitive metaphysical bias) to insist that distant events can only be statistically correlated if they are causally implicated by one another, that is, either one causes the other or both share a prior common cause. I am not particularly swayed by this objection, because I believe it is a fundamental goal of science to unearth causal explanations of (persistent) observed correlations, and not something that we should apologize for or dismiss as based on “metaphysical bias.” But even if one were to grant this objection, one must grant it equally as applied to both the derivation here of double-detection probabilities for the boxes and to Bell’s original derivation of the famous inequality. That is, standard quantum theory and the hidden variable theories are on equal footing. If Bell’s factorizability assumption (applied to orthodox quantum theory) is invalid, then it is invalid applied to hidden variables theories as well – in which case Bell’s theorem can not be used the way it typically is used – namely, as an argument against the hidden variables program. For then the empirical violations of Bell’s inequalities simply would not count as an empirical refutation of local hidden variable theories.

To avoid possible confusion, let me just state explicitly that this Bell-inspired derivation is not meant to show that Einstein’s Boxes is somehow equivalent to Bell’s theorem. It isn’t. The former establishes that quantum mechanics itself cannot be both complete and local. The latter establishes that no local hidden variable theory can agree with quantum theory’s predictions. Our point here is to show that one can generate a new version of the Einstein’s Boxes argument by borrowing some of the logical steps also used by Bell in his derivation of his inequalities. It will be crucial to the subsequent discussion that these two items – the locality-completeness dilemma and Bell’s theorem – are distinct.

Given that there are, at this point, several distinct definitions of “completeness” and “locality” on the table, the reader may begin to suspect that the dilemma we keep running into is merely due to an improper definition of one of these terms, and that the trouble could be avoided with some clever new definitions. One such attempt will be considered in Sec. 5. For now let me simply caution the reader against this attitude. The various definitions we have seen are not truly distinct; they all focus on different aspects of a single, shared concept of “completeness” (the idea that a complete theory shouldn’t leave anything out of its description of reality) and a single, shared concept of “locality” (that what exists/occurs “over here” shouldn’t instantaneously affect what exists/occurs “over there”). In my opinion, the lesson of the various formulations we are considering is not that new definitions might allow us to elude the dilemma, but rather that any reasonable definitions give rise to the dilemma. (As we will see in the next section, a sufficiently radical redefinition in which the basic meaning is changed can “succeed” in removing the dilemma. But this isn’t true success; it’s cheating.)

It is worth clarifying the logic here. The arguments we have been discussing for the locality-completeness dilemma show that, to maintain locality, one must regard the quantum mechanical description of reality as incomplete. Bell’s theorem then demonstrates that this project of saving locality cannot succeed: no local hidden variable completion of quantum mechanics can reproduce quantum theory’s predictions. Thus, Bell’s result, combined with the quantum locality-completeness dilemma, shows that any theory which reproduces the predictions of quantum mechanics – quantum mechanics itself most certainly included – must necessarily involve non-local actions-at-a-distance. Moreover, because Bell’s inequalities are empirically violated – because the predictions of quantum mechanics are correct – these non-local actions-at-a-distance are known to exist in nature. Of course, this leaves the completeness question unanswered. It doesn’t show that a non-local hidden variable theory like Bohm’s is true – but merely that whatever theory one favors (straight QM or a hidden variable theory), the theory will have to be non-local in order to agree with experiment. The motivation for considering non-local hidden variable theories like Bohm’s, then, (as opposed to simply conceding that quantum mechanics is non-local but still complete) comes from the other jobs (such as solving the measurement problem) the hidden variables can do for us. See Bell’s article “Bertleman’s Socks and the Nature of Reality,” and Sheldon Goldstein, “Bohmian mechanics,” Stanford Encyclopedia of Philosophy, (http://plato.stanford.edu/entries/qm-bohm/) (Sec. 2 in particular) for an especially clear discussion of this point.

Werner Heisenberg, The Physical Principles of the Quantum Theory (Dover Publications, 1949), p. 39.

See, for example, Abner Shimony, “Search for a worldview
which can accommodate our knowledge of microphysics,” in *Philosophical Consequences of Quantum Theory: Reflections on Bell’s Theorem*, edited by James T. Cushing and Ernan McMullin (University of Notre Dame Press, 1989).

Shimony has since acceded to the position taken by Bell in the quote in the next paragraph. See: Abner Shimony, “Bell’s Theorem,” Stanford Encyclopedia of Philosophy, <http://plato.stanford.edu/entries/bell-theorem/>.

Ref. 54, p. 111.

See Ref. 13 for a clear and systematic discussion of the possible positions one might take on this issue of the consistency of quantum non-locality and relativity.

A. Adám, L. Jánossy, and P. Varga, Acta Phys. Hung. 4, 301 [xx I don’t have either the title or exact page numbers for this paper; it’s in an obscure journal, and I only know of it via the reference in Clauser’s paper, xx] (1955).

A similar point is made in Chap. 5 of Ref. 15.

See also: Eric Brannen and H. I. S. Ferguson, Nature, Vol. 9 (1956) 481–482; John F. Clauser, “Experimental Distinction Between the Quantum and classical field-theoretic predictions for the photoelectric effect”, Phys. Rev. D 9, 853–860 (1974); P. Grangier, G. Roger and A. Aspect, “Experimental evidence for a photon anti-correlation effect on a beamsplitter,” Europhys. Lett., Vol. 1 (1986) 173–179; G. Greenstein and A. G. Jozenc, “The Quantum Challenge”, Jones and Bartlett Publishers, 1997, Chapter 2; and J.I. Thorn, M.S. Neel, V.W. Donato, G.S. Bergreen, R.E. Davies and M. Beck, “Observing the quantum behavior of light in an undergraduate laboratory,” Am.J.Phys. 72 1210-1219 (2004).

Here, by “photons,” we mean, strictly speaking, the particles that are actually detected. There is nothing here to rule out the possibility that, in addition, there exists (say) a wave that splits in half at the mirror. This is the story told by Bohm’s theory: the wave packet splits in half at the mirror and the photon particle ends up in one or the other of the two packets.

This is not surprising from the point of view outlined in Ref. 13. Because nature is non-local, it stands to reason that all local theories would be inconsistent with experiment.

After reading an early draft of this paper, Roderich Tumulka (private communication) suggested another version of the argument in which there seems to be no explicit assumption of locality: consider two different Lorentz frames (F and F’), relative to which the observation of the contents of box 1 is simultaneous with two different spacetime points (P and P’, say, with P earlier than P’) on the worldline of box 2. According to the collapse postulate of orthodox quantum theory, an observer in F should attribute a definite particle content to box 2 between P and P’, the measurement on box 1 having collapsed the wave function in box 2 at P. However, according to an observer in F’, box 2 remains in a superposition of containing and not containing a particle until the event P’. It seems this disagreement can be put together with the principle of relativity (according to which all Lorentz frames are equally valid platforms from which to describe reality) to yield

a violation of Einstein’s bijective completeness condition: because the accounts of both observers are equally valid, there are evidently two simultaneously valid but distinct quantum mechanical descriptions of the same physical contents of box 2. (However, this argument involves a switch to an occupation number representation in which we talk of the contents of box 2 rather than the state of the particle per se; it probably therefore deserves further thought and scrutiny.) Whereas in the earlier formulations we had arguments from locality to incompleteness, here we have an argument from the principle of relativity to incompleteness. This argument, perhaps even more clearly than those presented in the main text, brings out the fact that, if quantum mechanics is regarded as complete, the collapse postulate is in deep conflict with relativity as such and not merely with some narrow aspect thereof, for example, “locality,” about whose definition Heisenberg and others are wont to quibble. This point supports the claim made earlier that the dilemma cannot be escaped simply by redefining one or more of the terms involved.

Ref. 28, p. 178.

See N. David Mermin, “What’s Wrong With This Quantum World?”, Phys. Today, Vol. 57 (2), 10–11 (Feb. 2004). Some, however, definitely do take it seriously; see, for example, Ref. 61, and Ole Ulfbeck and Aage Bohr, “Genuine fortuitousness. Where did that click come from?,” Found. Phys. 31 (5), 757–774 (2001).

H. P. Stapp makes this point in “The Copenhagen interpretation,” Am. J. Phys. 40, 1098–1116 (1972), p. 1108. Thanks to Sheldon Goldstein (private communication) for pointing out that Tim Maudlin also stresses this point in “Space-time in the quantum world,” in *Bohmian Mechanics and Quantum Theory: An Appraisal*, edited by James T. Cushing, Arthur Fine, and Sheldon Goldstein (Kluwer Academic Publishers, 1996), p. 305. Maudlin writes that “Physicists have been tremendously resistant to any claims of non-locality, mostly on the assumption (which is not a theorem) that non-locality is inconsistent with Relativity. The calculus seems to be that one ought to be willing to pay any price – even the renunciation of pretensions to accurately describe the world – to preserve the theory of Relativity. But the only possible view that would make sense of this obsessive attachment to Relativity is a thoroughly realistic one! These physicists seem to be so certain that Relativity is the last word in space-time structure that they are willing even to forego any coherent account of the entities that inhabit space-time.”