A synthesis of Polyakov loop models of the deconfinement transition and quasiparticle models of gluon plasma thermodynamics leads to a class of models in which gluon quasiparticles move in a non-trivial Polyakov loop background. These models are successful candidates for explaining both critical behavior and the equation of state for the $SU(3)$ gauge theory at temperatures above the deconfinement temperature $T_c$. Polyakov loop effects are most important at intermediate temperatures from $T_c$ up to roughly $2.5T_c$, while quasiparticle mass effects provide the dominant correction to blackbody behavior at higher temperatures.

The equation of state is one point where theory, experiment, and lattice calculation all provide valuable contributions to our understanding of finite temperature QCD. Perturbation theory predicts the equation of state at high temperatures, with interactions providing corrections to the blackbody expression. Experiment typically uses phenomenological equations of state as input for modelling and analysis of data. Lattice gauge theory simulations provide a body of results on the critical behavior and thermodynamics of QCD and related theories at finite temperature. These results are among the most important that lattice gauge theory simulations have provided, giving important guidance to both theory and experiment. Analytical work on the thermodynamics of the quark-gluon plasma at zero chemical potential is now benchmarked against results from lattice simulations, but theoretical work derived solely from QCD has had difficulty explaining lattice results at intermediate temperatures, just above the temperature where the quark-gluon plasma forms. Perturbation theory for the pressure has been calculated to $O(g^5)$, but the series is not well-behaved, and the prospects for extracting the $O(g^6)$ contribution from the infinite set of diagrams required are not good. Closely related approaches are dimensional reduction and hard thermal loop (HTL) resummation. The former can extract the $O(g^n \ln g)$ contribution to the equation of state, but not the full $O(g^n)$ behavior; the fit to the lattice data is extremely sensitive to the choice of scale. The latter appears to be capable of fitting lattice results well at high temperatures, but again falls short at intermediate temperatures. More significantly, perturbation theory and its variants cannot account for confinement and chiral symmetry breaking at low temperatures, as well as the rather wide range of phase transitions and other critical behavior observed in lattice simulations of QCD and related models.

Phenomenological models of deconfinement and chiral symmetry restoration at finite temperature have been successful at explaining much of the phase structure of QCD and related models. Linear sigma models and Nambu-Jona Lasinio models have been used extensively to study chiral symmetry as quark properties, e.g., the number of light quarks, are varied. Because these models do not include the gluonic degrees of freedom, they are incapable of completely describing the quark-gluon plasma. Models based on the use of the Polyakov loop as the order parameter for confinement play a somewhat parallel role in modeling the deconfinement transition in pure gauge theories. However, in their simplest form such models have as their principal success the prediction of the order of the phase transition for $SU(2)$ and $SU(3)$, and do not make detailed predictions of thermodynamic behavior.

A third theoretical approach to QCD thermodynamics combines perturbative results with a simple physical picture. Quasiparticle models have been quite successful in fitting lattice results over a large range of temperatures. In the case of pure $SU(3)$ thermodynamics, some models fit lattice results at all temperatures above the deconfinement transition. All such models treat the quark-gluon plasma as a gas of independent quasiparticles with temperature-dependent effective masses accounting for much of the deviation from blackbody thermodynamics. However, many quasiparticle models must introduce ad hoc mechanisms to turn off gluonic degrees of freedom at low temperatures.

Here we focus on pure $SU(3)$ gauge theory, particularly the deconfinement phase transition and the thermodynamics of the gluon plasma. We show that the thermodynamic behavior above the deconfinement temperature $T_c$ can be largely explained in terms of quasiparticle gluons moving in the presence of a non-trivial background Polyakov loop. As will be discussed below, the Polyakov loop naturally plays an important role around the deconfinement transition, but gluon quasiparticles dominate the thermodynamics at higher temperatures.

Given the role of the Polyakov loop as the order parameter for the deconfinement phase transition in pure gauge theories, it is natural to expect that the equilibrium free energy density $f$ can be extended to a function $f(T,P)$
such that for any given temperature, $f(T, P)$ is minimized at the equilibrium value of $P$. Here $P$ is defined as a path-ordered exponential in Euclidean time at finite temperature

$$P = \mathcal{P} \exp i \int_0^\beta dt \, A_0$$

(1)

where $\beta = 1/T$. The deconfined phase of pure gauge theories is associated with spontaneous breaking of a global symmetry under the center of the gauge group. For $SU(3)$, the gauge theory is invariant under a global symmetry transformation in which $P \rightarrow zP$, where $z \in \{1, e^{2 \pi i/3}, e^{4 \pi i/3}\}$. It is convenient to define $L = Tr_F(P)/N_c$, the normalized trace of $P$ in the fundamental representation. If center symmetry is unbroken

$$\langle L \rangle = \langle zL \rangle = z \langle L \rangle$$

(2)

and $\langle L \rangle = 0$. This is the confined, low temperature phase of the gauge theory. In the high temperature gluon plasma phase the symmetry is spontaneously broken. In the spirit of mean field theory, we will take $P$ to be a constant element of $SU(3)$, ignoring spatial fluctuations. Thus gluon quasiparticles will be taken to move in a constant gauge-invariant background.

In this letter, we study a broad class of models with free energies of the form

$$f(T, P) = V(T, P) - p_g(T, P, M(T)) + B(T, P, M(T)).$$

(3)

The first term $V(T, P)$ is phenomenological, and favors $L = 0$ at low temperature. This term is needed to induce a confined phase at low temperatures, because the other two terms favor $L = 1$ at all temperatures. The second term is the negative of the quasiparticle pressure $p_g$, and represents the contribution of gluon quasiparticles moving in the presence of a background Polyakov loop. This term is given by

$$p_g(T, P, M(T)) = -2T \int \frac{d^3k}{(2\pi)^3} Tr_A \ln [1 - e^{-\beta \omega_k} P]$$

(4)

where $Tr_A$ denotes the trace in the adjoint representation. The pressure depends on the quasiparticle mass through the quasiparticle energy $\omega_k$, assumed to obey the dispersion relation $\omega_k^2 = k^2 + M^2(T)$, with a temperature-dependent mass $M(T)$. The third term $B(T, P, M(T))$ is the so-called bag term [22]. It arises because the zero-quasiparticle state has a non-zero, temperature-dependent energy $B$. If we write $B$ in such a way that it depends on $T$ only through the quasiparticle mass $M$, then thermodynamic consistency leads to an expression for $B$ which may be written as

$$B(T_1, P, M(T_1)) - B(T_0, P, M(T_0)) = \int_{T_0}^{T_1} dT \, T \frac{dM^2}{dT} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} Tr_A \ln [1 - e^{-\beta \omega_k} P]$$

(5)

This term has no well-defined connection to bag models of hadrons, although it plays a superficially similar role in the free energy. After minimization of $f(T, P)$ with respect to $P$, the equilibrium pressure is given by the negative of the minimum of the free energy density.

Previous work on quasiparticle models did not include Polyakov loop effects, instead using various ad hoc mechanisms to freeze out the quasiparticle degrees of freedom as the temperature approaches $T_c$. These models have $L = 1$ at all temperatures, and are thus incapable of reproducing the critical behavior of the deconfinement transition seen in lattice simulations.

Modeling both the equation of state and the critical behavior is a much stronger requirement than fitting the equation of state alone. Fitting the lattice thermodynamic results exactly with a phenomenological model is not difficult. Any model with an adjustable parameter which interpolates smoothly between $p = 0$ and the blackbody limit of $\pi^2 T^4/90$ per degree of freedom can be used to fit exactly the lattice results for the pressure. Note that all other thermodynamic quantities are obtainable from the pressure, so a perfect fit to the pressure in principle gives a perfect fit to all. One procedure which exactly fits the lattice pressure data is to determine $M(T)$ by requiring that $p_g(T, P = I, M(T))$ equal the lattice result. The mass so determined is shown in Figure 1. The rise in the quasiparticle mass near $T_c$ is typical of models which rely heavily on the quasiparticle mass to suppress the pressure near $T_c$ [19]. Another approach is to use the function $p_g(T, P, M = 0)$, and determine the Polyakov loop from the lattice results. This leads to a Polyakov loop behaving as shown in Figure 2. This behavior is similar to what is expected for a first-order deconfining transition, and resembles recent results for the renormalized Polyakov loop [23, 24]. While either of these prescriptions fit the lattice results perfectly, the two cannot be combined together without additional assumptions. Nevertheless, these figures give an important clue as to which effects are important.
at different temperatures. In the absence of Polyakov loop effects, a temperature-dependent quasiparticle mass must show a steep increase as the temperature is lowered towards $T_c$ in order to mimic the effects of the deconfinement transition. Above about $2.5T_c$, $M(T)/T$ varies slowly with temperature. On the other hand, the model with a temperature-dependent Polyakov loop shows a significant variation in $L$ in the range $T_c - 2.5T_c$, and $L$ appears to be nearly saturated at higher temperatures. This suggests examining models where Polyakov loop effects are most important in the range $T_c - 2.5T_c$, and the quasiparticle mass is important at higher temperatures.

In previous work\cite{18}, we have developed models for the SU($N$) deconfinement transition in pure gauge theories which fit within the general class specified by equation (3) and give a good representation of the critical behavior for all $N$. However, quasiparticle mass effects were not considered: $M$ was identically zero, and hence $B$ was identically zero as well. Here we study the effect of adding a temperature-dependent mass to one of these models, with a potential $V$, of the form

$$V(T, P) = -\frac{T}{R^3} \ln [\mu(P)] + v_0. \tag{6}$$

The function $\mu(P)$ is the Haar measure on the gauge group, a function of the Polyakov loop which is maximized when $L = 0$. With $V$ dominating the free energy at low temperature, this term gives rise to the confined phase. The parameter $R$ can be interpreted as the distance scale above which the Polyakov loop enforces color neutrality. The limit $R \to \infty$ corresponds to global color neutrality only. In perturbation theory only global color neutrality is enforced: an infinite product of Haar measures is cancelled by ghost contributions in the functional integral \cite{25}. In practice, we use the parameter $R$ to set $T_c$ to the result of lattice simulations; we find in this model $R = 1.38/T_c$. The constant $v_0$ is used to match the pressure at one point on the curve, which we have chosen to be $T_c$. In the absence of a quasiparticle mass, this procedure fixes the only parameters of the model. We show in Figure 3 the dimensionless pressure $p/T^4$ as a function of $T/T_c$ for this model ($M = 0$), along with the results from reference \cite{26}, which gives simulation data extrapolated to the continuum limit.

We initially extend our model by taking $M(T)$ to be a linear function of temperature: $M(T) = cT$. Because the effective mass depends on the temperature, the so-called bag term must be included. A least squares fit to the dimensionless pressure gives $c = 1.23$ and $R = 1.61/T_c$. Figure 4 shows $p/T^4$ for this model. The inclusion of the
quasiparticle mass results in a significant improvement in the goodness of fit over the model with \( M \) identically zero. The least-squares fit shows multiple nearby minima as a function of the fitting parameter \( c \); the fit yielding a global minimum is shown.

Previous quasiparticle models have taken \( M(T) \) to be proportional to \( g(T)T \), where \( g(T) \) is the QCD coupling constant, running with temperature \cite{19, 20, 21}. We now consider a mass of the form \( M(T) = g(T)T \sqrt{2} \), where \( g(T) \) is given by

\[
g^2(T) = \frac{8\pi^2}{11 \ln (T/\Lambda)}
\]

as suggested by one-loop finite temperature perturbation theory \cite{2, 19}. The sole adjustable parameter in fitting to simulation data is the scale parameter \( \Lambda \); as \( \Lambda \) is varied, \( R \) and \( v_0 \) are adjusted to keep \( T_c \) and \( p(T_c) \) fixed. A least-squares fit results in \( \Lambda = 0.121 T_c \), and \( R = 1.63/T_c \). The value of \( \Lambda \) is comparable to that found in \cite{19} and
A direct comparison is difficult, because both these works postulate different forms for the running coupling in order to dramatically reduce quasiparticle effects at low temperature. As may be seen from figure 5, the inclusion of running coupling constant effects does not significantly improve the quality of the fit to lattice simulations over the linear form. The quasiparticle mass is shown as a function of temperature in figure 6; in both cases shown, the mass is well-behaved near $T_c$. In figure 7, we show the dimensionless interaction measure $\Delta/T^4$, where $\Delta = \varepsilon - 3p$, as a function of $T/T_c$ for the $M = 0$ and $M = cT$ fit compared with lattice results. For $\Delta$, the fit to a running mass is so close to the linear fit as to be indistinguishable. Note that all these models correctly reproduce the first-order character of the $SU(3)$ deconfinement transition, but the combination of quasiparticle mass and Polyakov loop effects fit both the latent heat discontinuity and the asymptotic tail. Our previous work suggests that the peak in $\Delta$ is sensitive to the precise form of $V$ \[13\]. On the other hand, the behavior of the order parameter $L$ is similar in all the models considered here, resembling figure 2.

Within the class of models we have studied, neither a mass linear in temperature nor a running quasiparticle mass by themselves give a good fit to simulation results. We have studied the effect of a linear or running quasiparticle mass alone by considering a a model with $V = 0$, so that $L = 1$ at all temperatures. The best fit to the lattice data with $V = 0$ is worse than the model with $M = 0$, which we originally proposed in \[18\].

The models we have studied here with $V \neq 0$ do a very credible job of describing both the deconfining phase transition and thermodynamic behavior in the deconfined phase. It should be apparent that many models will be able to fit lattice thermodynamic results well, if a sufficient number of free parameters are used. The ability to describe both the thermodynamic results and the deconfinement transition with a small number of parameters, while not unique to the models described here, seems very desirable. The introduction of $V$ in the free energy is phenomenological, but based on our understanding of the deconfinement transition. The onus of describing the temperature region just above $T_c$ is placed where it likely belongs, on the confinement mechanism. The Polyakov loop $P$ plays a central role in suppressing thermal quasiparticle excitations at intermediate temperatures. The pressure is naturally obtained by minimizing the free energy as a function of $P$.

We obtain a picture of the deconfined phase in which Polyakov loop effects provide the dominant correction to blackbody thermodynamics in the temperature regime from $T_c$ to approximately $2.5T_c$. At higher temperatures, $L \simeq 1$, and quasiparticle mass effects can account for most of the deviation from blackbody behavior. There is
clear evidence from higher temperatures that $M \propto T$, but the evidence for a one-loop running coupling form is less compelling. Although we have not made a systematic study, we believe that the behavior of the quasiparticle mass at higher temperatures is not sensitive to the precise form of the confining potential term $V$. On the other hand, a better choice for the form of $V$ could improve the fit of the interaction measure $\Delta$ to lattice results just above $T_c$.

There are several possible advances in our understanding of finite temperature gauge theories that would in turn lead to a better understanding of the equation of state. The first, certainly the most fundamental and probably the most difficult, is a better understanding of the mechanism of confinement. An improved understanding would be reflected here as an expression for $V$ based on fundamental physics rather than on phenomenology. This is particularly important at intermediate temperatures just above $T_c$. The second possibility is an analysis of lattice results for the renormalized Polyakov loop in a form suitable for use in the quasiparticle pressure term $p_g$.

A final possibility is a reliable lattice determination of electric and magnetic screening masses. However, screening masses are not synonymous with a gluon quasiparticle masses, and lattice results have been marred by ambiguities associated with lattice gauge fixing.

The extension of these models by the inclusion of quarks is directly relevant to experiment. However, our inability to describe chiral symmetry breaking in a fundamental way forces us to use couplings of Nambu-Jona Lasinio type to mimic QCD’s chiral behavior. Given that any model with a parameter that varies the pressure between zero and the black-body result can give an exact match to simulation results, the real possibility of reading too much out of the lattice data increases as the number of fields and phenomenological parameters grows. Nevertheless, a useful synthesis of our current understanding of deconfinement and chiral symmetry restoration is likely possible. The class of models which we have applied here to pure $SU(3)$ gauge theory also can be extended to other pure gauge theories as well, with $SU(2)$ being most interesting because of its second-order deconfining transition.

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