Are deviation from bi-maximal mixing and none-zero $U_{e3}$ related to non-degeneracy of heavy Majorana neutrinos?

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Abstract

We propose a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two generations of degenerate heavy neutrinos in the seesaw framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. On top of the scenario, we show that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos leads to the non-zero small mixing angle $U_{e3}$ in the PMNS matrix and the little deviation of the atmospheric neutrino mixing angle from the maximal mixing.

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Thanks to enormous progress in solar, atmospheric and terrestrial neutrino experiments, we have now the robust evidence for the existence of neutrino oscillation which provides a window to physics beyond the standard model (SM). Until now, while the atmospheric neutrino deficit still points toward a maximal mixing between the tau and muon neutrinos, however, the solar neutrino problem favors a not-so-maximal mixing between the electron and muon neutrinos. There have been many attempts to explain the origin of the deviation of the solar mixing angle from the maximal mixing. Surprisingly, it has recently been noted that the solar neutrino mixing angle $\theta_{\text{sol}}$ required for a solution of the solar neutrino problem and the Cabibbo angle $\theta_C$ reveal a striking relation $\theta_{\text{sol}} + \theta_C \simeq \frac{\pi}{4}$, which is satisfied by the experimental results within a few percent accuracy, $\theta_{\text{sol}} + \theta_C = 45.4^\circ \pm 1.7^\circ$ [2, 3, 4]. This quark-lepton complementarity (QLC) relation has been simply interpreted as an evidence for certain quark-lepton symmetry or quark-lepton unification as shown in Refs. [1, 5, 6]. But, it can be an accidental phenomenon as pointed out in Ref. [6, 7]. Thus, it is worthwhile to find the possible alternatives to the grand unification origin of the deviation of the solar mixing from the maximal mixing.

In this letter, we propose a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two generations of degenerate heavy neutrinos in the seesaw framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. The maximal atmospheric neutrino mixing and the smallness of $U_{e3}$ may be the trace of the original “bi-maximal” mixing which is presumably supposed to be achieved by some underlying flavor symmetries, and thus the best possible approach to the problem is to start in the limit of the maximal mixing with $U_{e3} = 0$, and understand how the deviation of the solar mixing from the maximal is realized. In our scenario, the primitive “bi-maximal” neutrino mixing is generated only from the neutrino Dirac Yukawa matrix by taking a diagonal form of three degenerate heavy Majorana neutrinos in a basis where the charged lepton mass matrix is real and diagonal. As will be shown, the deviation of the solar mixing can then be generated from breakdown of the degeneracy of the heavy Majorana neutrino masses between the first and the other two generations. The main point in this scenario is that the deviation can be expressed in terms of the ratio between two heavy Majorana neutrino masses. On top of the scenario, we will also show that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos will lead to the small mixing angle $\theta_{13}$ in
the PMNS matrix and the very small deviation of the atmospheric neutrino mixing angle from the maximal mixing.

Before proceeding to our scenario, we wish to motivate one scheme that leads to exact “bimaximal” mixing in the framework of the seesaw mechanism. We study in a basis where the charged lepton mass matrix is real and diagonal. The light neutrino mass matrix $M_\nu$ diagonalized by $U_{\text{bimax}}$ is given through the seesaw mechanism by

$$M_\nu = M_D^T M_R^{-1} M_D,$$

$$= U_{\text{bimax}} M_{\nu}^{\text{diag}} U_{\text{bimax}}^T,$$  

(1)

where $M_D = Y_D v/\sqrt{2}$ with electroweak vacuum expectation value $v$ and the neutrino Dirac Yukawa matrix $Y_D$, and $M_R$ is a mass matrix of heavy Majorana neutrinos. The mixing matrix $U_{\text{bimax}}$ denotes the “bi-maximal” mixing matrix [8]:

$$U_{\text{bimax}} = U_{23} \left( \frac{\pi}{4} \right) U_{12} \left( \frac{\pi}{4} \right)$$

$$= \begin{pmatrix} 
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} 
\end{pmatrix}.$$  

(2)

Then, the “bi-maximal” mixing can be achieved by one of the three possible ways as follows:

- Taking $Y_D$ diagonalized by $U_{\text{bimax}}$ and $M_R = M \cdot I$ with the identity matrix $I$ and a common mass scale $M$.

- Taking $Y_D = y \cdot I$ and $M_R$ diagonalized by $U_{\text{bimax}}$.

- Taking “bi-maximal” mixing pattern from the combination of the nontrivial $Y_D$ and $M_R$.

For the third case, there may exist various origins of the deviation of the solar mixing depending on possible combinations, and some of which have been discussed before [9]. For the other two cases, the modification of the trivial sectors proportional to the unit matrix can be in charge for the origin of the deviation from the maximal mixing. However, since the second case may lead to the undesirable deviation of the atmospheric mixing aside from the deviation of the solar mixing as one can easily see, we only focus on the first case in this letter. In the first case, the “bi-maximal” mixing can be achieved by taking the symmetric matrix
$Y_D$ with specific form. As an example, we present a detailed model of $Y_D$ leading to the “bi-maximal” mixing, while keeping $M_R = M \cdot I$ based on the discrete symmetry $A_4 \otimes Z_2$ \cite{10}. Let the three families of leptons and singlet heavy neutrinos be denoted by $(\nu_i, l_i)_L, l_iR, N_{iR}$ for $i = 1, 2, 3$. In this convention, $\tilde{l}_iLl_jR$ and $\bar{\nu}_iLN_{jR}$ are Dirac mass terms for charged leptons and neutrinos. Under the discrete symmetry $A_4 \otimes Z_2$, the 3 families of leptons transform as $(\nu_i, l_i)_L \sim (3, +), N_{iR} \sim (3, +), l_iR \sim (1', -), (1'', -)$. We introduce Higgs scalar sectors consisted of seven Higgs doublets $\Phi_i$ \cite{11}. From the assignment, the $A_4 \otimes Z_2$ invariant Dirac Yukawa interactions for charged lepton sector, $\tilde{l}_iLl_iR\Phi_j$, leads to a diagonal mass matrix with 3 independent entries each as shown in Ref. \cite{11}. For the mass matrix of the heavy Majorana neutrinos, we can take $MN_{iR}N_{iR}$ with common mass scale $M$ because of $A_4$ symmetry, i.e. $3 \times 3 \sim 1$. The Dirac Yukawa matrix for the neutrino sector, which is invariant under $A_4 \otimes Z_2$ and diagonalized by the “bi-maximal” mixing matrix, can be obtained from the interaction Lagrangian as follows:

$$Y_D = h_1(\bar{\nu}_1N_1 + \bar{\nu}_2N_2 + \bar{\nu}_3N_3)\phi$$

$$+ h_2(\bar{\nu}_1N_2\sigma_3 + \bar{\nu}_2N_3\sigma_1 + \bar{\nu}_3N_1\sigma_2)$$

$$+ h_3(N_1\nu_2\sigma_3 + N_2\nu_3\sigma_1 + N_3\nu_1\sigma_2) + h.c. \quad (3)$$

In order to achieve the symmetric form of the Dirac Yukawa matrix, we require $h_2 = h_3$. The vacuum expectation values for the neutral components of Higgs sector $\sigma^0_i$ can be determined by the Higgs potential invariant under $A_4$,

$$V = m^2\sigma^+_i\sigma_i + \frac{1}{2}\lambda_1(\sigma^+_i\sigma_i)^2$$

$$+ \lambda_2(\sigma^+_i\sigma_1 + \omega^2\sigma^+_3\sigma_2 + \omega\sigma^+_2\sigma_3)(\sigma^+_1\sigma_1 + \omega\sigma^+_2\sigma_2 + \omega^2\sigma^+_3\sigma_3)$$

$$+ \lambda_3[(\sigma^+_2\sigma_3)(\sigma^+_3\sigma_2) + (\sigma^+_3\sigma_1)(\sigma^+_1\sigma_3) + (\sigma^+_1\sigma_2)(\sigma^+_2\sigma_1)]$$

$$+ \left\{\frac{1}{2}\lambda_4[(\sigma^+_2\sigma_3)^2 + (\sigma^+_3\sigma_1)^2 + (\sigma^+_1\sigma_2)^2] + h.c.\right\}, \quad (4)$$

where $\omega = e^{2\pi/3}$. Taking $\sigma^0_1 = 0$ and $\sigma^0_2 = \sigma^0_3 = v$ with $v = \sqrt{\frac{-m^2}{2\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}}$ as well as non-vanishing $\phi^0$ for the Higgs sector $\phi$, we can achieve the final form of the
Dirac Yukawa matrix given as follows,

\[ Y_D = \begin{pmatrix} a & b & b \\ b & a & 0 \\ b & 0 & a \end{pmatrix}. \quad (5) \]

Defining \( Y_D^{\text{diag}} = \text{diag}(x, y, z) \), the neutrino Dirac Yukawa matrix \( Y_D \) diagonalized by \( U_{\text{bimax}} \) is generally given in the symmetric matrix form by

\[ Y_D = U_{\text{bimax}} Y_D^{\text{diag}} U_{\text{bimax}}^T. \quad (6) \]

Here, we consider the case of nonzero values for \( x \) and \( y \), which is crucial to our purpose.

In order to achieve the observed deviation of the solar neutrino mixing from the maximal mixing, we take into account the mass splitting between the first generation of the heavy Majorana neutrino and the other two degenerate ones, for which the mass matrix is given by \( M_R = M_R^{\text{diag}} = (M_1, M_2, M_2) \), which results from the breaking of \( A_4 \) in the heavy neutrino sector and reflects separation of \( N_{iR} \sim N_{1R}(1) \oplus N_{(2,3)R}(2) \) under \( S_3 \) symmetry. Then, the light neutrino mass matrix \( M_\nu \) is presented as follows:

\[ M_\nu = U_{\text{bimax}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{\text{bimax}}^T \begin{pmatrix} M_1^{-1} & M_2^{-1} & 0 \\ M_2^{-1} & M_2^{-1} & 0 \\ 0 & 0 & M_2^{-1} \end{pmatrix} U_{\text{bimax}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{\text{bimax}}^T \\ = U_{\text{bimax}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{12}(\frac{\pi}{4}) \begin{pmatrix} M_1^{-1} & M_2^{-1} & 0 \\ M_2^{-1} & M_2^{-1} & 0 \\ 0 & 0 & M_2^{-1} \end{pmatrix} U_{12}(\frac{\pi}{4}) \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{\text{bimax}}^T \\ = U_{\text{bimax}} M'_\nu U_{\text{bimax}}^T, \quad (7) \]

where the mass matrix \( M'_\nu \) is given by

\[ M'_\nu = \begin{pmatrix} \frac{x^2}{2M_1M_2}(M_1 + M_2) & \frac{xy}{2M_1M_2}(M_1 - M_2) & 0 \\ \frac{xy}{2M_1M_2}(M_1 - M_2) & \frac{y^2}{2M_1M_2}(M_1 + M_2) & 0 \\ 0 & 0 & \frac{z^2}{M_2} \end{pmatrix}. \quad (8) \]

Then, the matrix \( M'_\nu \) can be diagonalized by \( U_{12}(\theta) \), and after diagonalizing \( M'_\nu \), we can
obtain the mixing angle $\theta$ and three neutrino mass eigenvalues as follows:

$$\tan 2\theta = \frac{2xy(M_2 - M_1)}{(x^2 - y^2)(M_1 + M_2)},$$  \hspace{1cm} (9)$$

$$m_{\nu_1} = \frac{1}{2M_1 M_2} [(c^2 x^2 + s^2 y^2)(M_1 + M_2) + 2csxy(M_1 - M_2)],$$

$$m_{\nu_2} = \frac{1}{2M_1 M_2} [(s^2 x^2 + c^2 y^2)(M_1 + M_2) - 2csxy(M_1 - M_2)],$$  \hspace{1cm} (10)$$

$$m_{\nu_3} = \frac{z^2}{M_2},$$

where $c = \cos \theta$, $s = \sin \theta$. Comparing the mixing matrix $U_{12}(\theta)$ with $U_{12}(\pi/4)$ in $U_{\text{bimax}}$, we can get the solar mixing angle $\theta_{\text{sol}}$ which deviates as much as the value of $\theta$ from the maximal mixing. Note that the value of $\theta$ should be negative in order to achieve the desirable deviation of the solar neutrino mixing. We can argue that the generation of the mixing angle $\theta$ due to the splitting between $M_1$ and $M_2$ in seesaw mechanism may be the origin of the deviation of the solar mixing angle from the maximal mixing in the case of nonzero $x$ and $y$. However, since we do not have yet any information on the values of $M_1$ and $M_2$, we cannot immediately test whether the difference between $M_1$ and $M_2$ is really compatible with the deviation of the solar mixing angle from the maximal mixing, but we can make numerical estimate for the size of the ratio of $M_1$ to $M_2$, which accommodates the deviation of the solar mixing based on the experimental results for the neutrino oscillation. From the numerical results, we can also predict the magnitude of the effective Majorana neutrino mass $m_{ee}$, which is the neutrino-exchange amplitude for the neutrinoless double beta decay.

For our purpose, let us define two parameters $\kappa$ and $\omega$ as follows:

$$\kappa \equiv \frac{y}{x}, \quad \omega \equiv \frac{M_1}{M_2}. \hspace{1cm} (11)$$

Then, the expressions for $\theta$ and $m_{\nu_i}$ are given as follows,

$$\tan 2\theta = \frac{2\kappa(1 - \omega)}{(1 - \kappa^2)(1 + \omega)},$$  \hspace{1cm} (12)$$

$$m_{\nu_1} = \frac{x^2}{2M_1} [(c^2 + s^2 \kappa^2)(1 + \omega) + 2cs\kappa(\omega - 1)],$$

$$m_{\nu_2} = \frac{x^2}{2M_1} [(s^2 + c^2 \kappa^2)(1 + \omega) - 2cs\kappa(\omega - 1)],$$  \hspace{1cm} (13)$$

$$m_{\nu_3} = \frac{z^2}{M_2}.$$

In addition, the effective Majorana neutrino mass $m_{ee}$ is presented by

$$m_{ee} = \frac{x^2}{4M_1} [(1 + \kappa)^2 + \omega(1 - \kappa)^2]. \hspace{1cm} (14)$$
As shown in Eq. (12), the non-vanishing value of the mixing angle $\theta$ can arise when $\omega$ is deviated from one, which indicates the splitting between $M_1$ and $M_2$. In fact, the present experimental results are not enough to determine all the parameters introduced. But, if we fix one neutrino mass eigenvalue by hand, we can determine several independent parameters as well as the magnitude of $m_{ee}$ from Eqs. (12,13,14). For our numerical calculation, we set the parameter $\theta$, $\Delta m^2_{21}$ and $\Delta m^2_{32}$ to be $13^\circ$, $8 \times 10^{-5}$ eV$^2$, $2.5 \times 10^{-3}$ eV$^2$, respectively. Those numbers correspond to the best fit values for the measurements of the deviation of the solar mixing angle from the maximal mixing, the mass-squared differences of the solar and atmospheric neutrino oscillations, respectively. By fixing $m_{\nu_1}$ as an input parameter, we can determine the parameter set ($\kappa, \omega, \frac{z^2}{M_1}, \frac{z^2}{M_2}$) for normal hierarchy $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ through the relations (11,12,13).

TABLE I: All numbers corresponding to the mass parameters are given in the unit eV for normal hierarchy.

| $m_{\nu_1}$ (input) | $\kappa$ | $\omega$ | $\frac{z^2}{M_1}$ | $\frac{z^2}{M_2}$ | $m_{ee}$ |
|---------------------|----------|----------|-----------------|-----------------|--------|
| 0.005               | 1.298    | 0.772    | 0.003           | 0.051           | 0.009  |
| 0.01                | 1.118    | 0.897    | 0.006           | 0.052           | 0.013  |
| 0.05                | 1.006    | 0.994    | 0.025           | 0.071           | 0.051  |
| 0.1                 | 1.002    | 0.998    | 0.050           | 0.112           | 0.101  |

In TABLE I, we present our numerical results for normal hierarchy. From the TABLE I, we can see that the values of $\kappa$ and $\omega$ approach to one as $m_{\nu_1}$ increases up to of order 0.1 eV, and one needs fine-tuning to obtain the parameter set satisfying the relations above for the case of such a large $m_{\nu_1} \sim 0.1$ eV. As $m_{\nu_1}$ goes down, the value of $\kappa$ rapidly increases whereas that of $\omega$ decreases. We can also predict the size of the amplitude of the neutrinoless double beta decay $m_{ee}$ as a function of $m_{\nu_1}$, which is presented in the last column of TABLE I. If the neutrinoless double beta decay will be measured in near future, we will be able to determine three neutrino mass eigenvalues and the parameters introduced in Eqs. (12,13,14). For inverted hierarchy $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$, the numerical results are presented in TABLE II. In this case, contrary to the normal hierarchical case, we take $m_{\nu_3}$ as an input.

Next, to generate non-vanishing $U_{e3}$, on top of the above scenario, we consider an interesting possibility that the breaking of the degeneracy between the second and the third
TABLE II: All numbers corresponding to the mass parameters are given in the unit eV for inverted hierarchy.

| $m_{\nu_3}$(input) | $\kappa$ | $\omega$ | $\frac{\kappa^2}{M_1^2}$ | $\frac{\kappa^2}{M_2^2}$ | $m_{ee}$ |
|---------------------|----------|----------|---------------------------|---------------------------|----------|
| 0.005               | 1.672    | 0.585    | 0.011                     | 0.010                     | 0.041    |
| 0.01                | 1.569    | 0.630    | 0.013                     | 0.013                     | 0.042    |
| 0.05                | 1.137    | 0.881    | 0.029                     | 0.051                     | 0.066    |
| 0.1                 | 1.044    | 0.959    | 0.052                     | 0.100                     | 0.109    |

generation masses in the heavy Majorana neutrino sector, i.e., $M_R = \text{diag}(M_1, M_2, M_3)$, can be an origin of the generation of non-vanishing $U_{e3}$. We remark that the value of $U_{e3}$ goes to zero in the limit of $M_2 = M_3$ in this scenario. The effective light Majorana neutrino mass matrix is given by

$$M_\nu = U_{\text{bimax}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{\text{bimax}}^T \begin{pmatrix} M_1^{-1} \\ M_2^{-1} \\ M_3^{-1} \end{pmatrix} U_{\text{bimax}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} U_{\text{bimax}}^T$$ (15)

$$= U_{\text{bimax}} M'_\nu U_{\text{bimax}}^T$$ (16)

Assuming that the mass splitting between $M_2$ and $M_3$ is small enough to accommodate the tiny $U_{e3}$, the mixing matrix, which diagonalizes the neutrino mass matrix $M_\nu$, can be approximately given by

$$U \simeq U_{23} \left( \frac{\pi}{4} \right) U_{12} \left( \frac{\pi}{4} \right) \begin{pmatrix} \cos \sigma & \sin \sigma & \delta \\ -\sin \sigma & \cos \sigma & \eta \\ -\delta & -\eta & 1 \end{pmatrix},$$ (17)

where the mixing angle $\sigma$ corresponds to the (1,2) rotation of $2 \times 2$ submatrix of $M'_\nu$. The mixing angle $\sigma$ is presented by

$$\tan \sigma \simeq \frac{2\kappa(1-\omega-\varepsilon)}{(1-\kappa^2)(1+\omega+\varepsilon)},$$ (18)

where $\varepsilon = M_1/M_3$, and $\omega, \kappa$ are given earlier. This mixing angle $\sigma$ is responsible for the deviation of the solar mixing angle from the maximal mixing. We note that non-vanishing value of $\sigma$ is possible even when $\omega = 1$, i.e., $(M_1 = M_2)$, but this case is undesirable because it leads to negative $\sigma$ which positively contributes to $\theta_{12}$. The mixing angle $\sigma$ is zero when
\[ \omega + \varepsilon = 1, \] but it corresponds to the large hierarchy among three heavy Majorana masses, which is far beyond our purpose. The mixing elements \( \delta \) and \( \eta \) are given by

\[
\begin{align*}
\delta &= c_1 \left( -\frac{1}{M_2} + \frac{1}{M_3} \right), \\
\eta &= c_2 \left( -\frac{1}{M_2} + \frac{1}{M_3} \right),
\end{align*}
\]

(19)

where \( c_1 \) and \( c_2 \) are presented in terms of three light neutrino mass eigenvalues and the parameters \( \kappa, \omega, \varepsilon \). Then, the mixing element \( U_{e3} \) and the deviation of the atmospheric mixing from the maximal mixing are simply presented in terms of \( \sigma \) and \( \eta \) as follows,

\[
\begin{align*}
|U_{e3}| &\simeq \frac{1}{2} |\delta - \eta|, \\
\delta \sin \theta_{23} &\simeq \frac{1}{2} (\delta + \eta).
\end{align*}
\]

(20)

(21)

Imposing the bound on \( |U_{e3}| \) of CHOOZ experiment, \( |U_{e3}| < 0.2 \), and the result of \( \sin^2 \theta_{23} \) from atmospheric neutrino data, \( \sin^2 \theta_{23} = 0.44(1^{+0.41}_{-0.22}) \), we can determine the allowed regions of the ratio \( M_2/M_3 \). In TABLE III, we present the numerical results for the ratio \( M_2/M_3 \) and the prediction for the bound on \( |U_{e3}| \). The second and third columns correspond to the normal hierarchical case, whereas the fourth and fifth columns to the inverted hierarchy. We find that the result for \( \delta \sin^2 \theta_{23} \) constrains \( M_2/M_3 \) more severely than the bound on \( |U_{e3}| \) for \( m_{\nu_1(3)} < 0.05 \) eV. But for \( m_{\nu_1(3)} \sim 0.1 \) eV, both \( \delta \sin^2 \theta_{23} \) and \( |U_{e3}| \) from neutrino data severely constrain the allowed region of \( M_2/M_3 \). The values in the columns for \( |U_{e3}| \) indicate the predictions for the upper bound. As shown in Table III, the allowed region for \( M_2/M_3 \) gets narrowed as \( m_{\nu_1(3)} \) increases, and it becomes nearly one for \( m_{\nu_1(3)} \geq 0.1 \) eV. This implies that such large values of \( m_{\nu_1(3)} \) lead to moderately degenerate light neutrino spectrum realized by almost degenerate heavy Majorana neutrinos.

Finally we note that there could be radiative corrections to neutrino mass matrix which can lead to some modification of our results. However, non-negligible renormalization effects can be expected only in the case of degenerate light neutrino spectrum. The numerical results for \( m_{\nu_1(3)} = 0.1 \) eV in the tables may be significantly modified due to possible renormalization effects, but the detailed investigation on the renormalization effects is not our main interest in this work and we will leave it for the future work.

In summary, we have proposed a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two degenerate ones in the seesaw
framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. Our scheme is based on the assumption that nature presumably started with “bi-maximal” neutrino mixing and then it has been deviated somehow. We have considered the case that the “bi-maximal” mixing is achieved only from the neutrino Dirac Yukawa matrix by taking a diagonal form of three degenerate heavy Majorana neutrinos in a basis where the charged lepton mass matrix is real and diagonal. Allowing the mass splitting between the first and the other two generations of the heavy Majorana neutrinos, we could obtain the deviation of the solar mixing angle from the maximal. In addition, we have also shown that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos leads to the small mixing angle $\theta_{e3}$ in the PMNS matrix and the very small deviation of the atmospheric neutrino mixing angle from the maximal mixing.

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