The Color Glass Condensate at RHIC

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Abstract. The Color Glass Condensate formalism and its application to high energy heavy ion collisions at RHIC are discussed. We argue that the RHIC data supports the view that the Color Glass Condensate provides the initial conditions for gold-gold collisions at RHIC while final state (Quark Gluon Plasma) effects are responsible for the high $p_t$ suppression in mid rapidity. At forward rapidities in deuteron-gold collisions, however, Color Glass Condensate is the underlying physics of the observed suppression of the particle spectra and their centrality dependence.

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1. Introduction

The recent data from forward rapidity deuteron-gold collisions at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory (BNL) has generated much interest and has led to intense debate on the underlying physics of the forward rapidity processes in deuteron-gold collisions. While the conventional models of multiple scattering based on Glauber theory predicted an enhancement of the Cronin effect in the forward rapidity region, the data shows a clear suppression of the negatively charged hadron spectra, as compared to the normalized proton-proton collisions, in agreement with the predictions based on the Color Glass Condensate formalism. The most recent data on centrality dependence of the suppression in the forward rapidity region also agrees with the predictions of the Color Glass Condensate formalism and is in clear contradiction with expectations based on the conventional models.

The Color Glass Condensate formalism is an effective theory of QCD at high energy (small $x$). At high energy or equivalently (for fixed $Q^2$) at small $x$, gluons are the most abundant partons. While one can not calculate parton distribution functions in QCD using perturbation theory, one can calculate their evolution with $x$ or $Q^2$ in pQCD. The evolution of the gluon distribution function is described by the DGLAP evolution equation which sums $\alpha_s \log Q^2$ type corrections to the gluon distribution function. In DGLAP formalism, the gluon distribution function $xG(x, Q^2)$ grows fast with both $x$ and $Q^2$ as shown in Eq. for fixed a coupling constant $\alpha_s$. It is worth noting that DGLAP evolution equation is just the renormalization group equation for the parton number operator.
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\[ xG(x, Q^2) \sim \exp \left[ \sqrt{\alpha_s \log \frac{1}{x} \log Q^2} \right] \]  

(1)

If the phase space in \( Q^2 \) is limited and there is a large phase space available in \( x \), then BFKL is the applicable formalism. In BFKL approach, one sums terms of the form \( \alpha_s \log \frac{1}{x} \) which leads to an even faster grows of the gluon distribution function with \( x \) (energy) compared to the DGLAP gluons

\[ xG(x, Q^2) \sim e^{\lambda \log \frac{1}{x}} \]  

(2)

with \( \lambda \equiv \frac{N_c \alpha_s}{\pi} 4 \log 2 \) for a fixed coupling constant. In other words, pQCD radiation leads to a fast growth of the gluon distribution function with energy or \( x \) which makes a hadron a dense system of gluons at high energy. The gluon density in a high energy nucleus is even larger by a factor of \( \sim A^{1/3} \) due to the Lorenz contraction of the longitudinal size of the nucleus at high energy. However, this fast growth can not go on forever since it would lead to violation of unitarity for physical cross sections.

Both DGLAP and BFKL approaches are linear in the gluon density. In other words, they only include radiation of gluons. As the phase space density of gluons increases, it becomes as probable for a gluon at small \( x \) to recombine with another gluon at small \( x \) and become a gluon at a higher \( x \) as it is for a gluon at larger \( x \) to radiate another gluon at a smaller \( x \). This recombination of gluons slows down the growth of the gluon distribution function and leads to its eventual unitarization. This was first investigated by Gribov, Levin and Ryskin (GLR) and later on by Mueller and Qiu (MQ). The evolution equation for the gluon distribution function is written as (in the Double Logarithm Approximation)

\[ \frac{\partial^2 xG(x, Q^2)}{\partial \log \frac{1}{x} \partial \log Q^2} = \frac{N_c \alpha_s}{\pi} xG(x, Q^2) - \frac{4\pi^3}{(N_c^2 - 1)} \left( \frac{N_c \alpha_s}{\pi} \right)^2 \frac{1}{Q^2} x^2 G^2(x, Q^2) \]  

(3)

Here \( G^2(x, Q^2) \) is the four point function of gluons while the standard gluon distribution function \( xG(x, Q^2) \) is the two point function of gluons. It is quite customary to factorize the four point function \( G^2 \) in terms of the gluon distribution function \( G \) as \( G^2(x, Q^2) \sim \frac{1}{\pi R^2} [G(x, Q^2)]^2 \) where \( R \) is a scale which can not be determined from pQCD. This factorization is an approximation where one ignore gluon correlations, valid at large \( N_c \). The fact that this is a higher twist correction to the gluon distribution function is manifested in the presence of the \( 1/Q^2 \) factor in the non linear correction.

As the phase space density of gluons becomes large, even the GLR-MQ approach breaks down since all higher point functions become as large as the two point function and one must keep all higher order terms in the evolution equation. The Color Glass Condensate is an effective field theory approach to QCD at high energy (small \( x \)) which extends the applicability of pQCD to a dense system of gluons.
2. The Color Glass Condensate

The effective action for QCD at small $x$ is given by (for a recent review, see [1])

$$S = i \int d^2 x t F[\rho^a(x_t)] - \int d^4 x \frac{1}{4} G^2 + i \frac{G}{N_c} \int d^2 x_t dx^- \delta(x^-) \rho^a(x_t) \text{tr} T_a W_{-\infty,\infty}[A^-](x^-, x_t)$$

(4)

where $G^{\mu\nu}$ is the gluon field strength tensor

$$G^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f_{abc} A^\mu_b A^\nu_c$$

(5)

$T_a$ are the $SU(N)$ color matrices in the adjoint representation, and $W$ is the path ordered exponential along the $x^+$ direction in the adjoint representation of the $SU(N_c)$ group

$$W_{-\infty,\infty}[A^-](x^-, x_t) = \mathcal{P} \exp \left[ -ig \int dx^+ A^-_a(x^-, x_t) T_a \right]$$

(6)

In order to calculate a physical quantity, one averages over the gluonic field configuration

$$\langle O \rangle = \frac{\int [D\rho^a][DA^\mu_a] O(A) \exp \{iS[\rho, A] \}}{\int [D\rho^a][DA^\mu_a] \exp \{iS[\rho, A] \}}$$

(7)

The weight function

$$Z \equiv \exp \left\{ -\int d^2 x_t F[\rho^a] \right\}$$

(8)

appearing in (7) is the statistical weight of a particular configuration of the two dimensional color charge density $\rho^a(x_t)$ inside the hadron. Initially, this statistical weight is taken to be a Gaussian

$$F[\rho(x_t)] = \frac{1}{2\mu^2} \rho^2_0(x_t)$$

(9)

The Gaussian approximation is valid as long as the color charge density is large and random. Also, if one is interested in the gluon correlation at the same rapidity, one needs to include the longitudinal structure of the sources into account. This is done in [2] where one solves the classical equations of motion in the presence of sources which have a longitudinal size. The result for the two point function is

$$G_{ij}(y, x^\perp; y', x'^\perp) = \langle A_i(y, x^\perp) A_j(y', x'^\perp) \rangle$$

(10)

Performing the color averaging with the extended sources, we get

$$G^{aa}_{ii} = \frac{4(N_c^2 - 1)}{N_c x^2_{\perp}} \left[ 1 - \left( \frac{x^2_{\perp} \Lambda_{QCD}^2}{8\pi^2} \right)^2 \right]$$

(11)

The momentum space distribution of the gluon distribution function obtained from the Fourier transform of Eq. (11) has a much milder divergence at low $k_t$ than the pQCD distributions due to the non linearities of the gluons included in a classical description. One can also show that gluons typically have a transverse momentum $Q_s(x, b_t, A)$, called the saturation scale, which can be much larger than $\Lambda_{QCD}$. This is shown in Fig. (1) which illustrates the fact that most gluons reside in a state with a momentum $\sim Q_s$ described by a strong classical field. As one goes to smaller $x$, one needs to include
quantum evolution effects, namely, sum the $\alpha_s \log 1/x$ terms which become large. This has been done and leads to a non linear functional equation for the weight functional $F(\rho)$ which can then be used to write an evolution equation for any desired physical observable. The JIMWLK equation for the weight functional can be written in a compact form as (first written in this form by H. Weigert)

$$\partial_y Z_y[U] = -\frac{1}{2} i \nabla_a \chi_{x_t y_t} \nabla_b Z_y[U]$$

with $y \equiv \log 1/x$. These equations are highly non linear and coupled so that one has to develop approximate techniques to solve them. They have also been investigated numerically. One particular limit which simplifies the non linear JIMWLK equations, is the large $N_c$ limit which was used in to derive an equation for the dipole scattering cross section. This equation is known as the BK equation and is widely regarded as the simplest possible non linear equation which respects perturbative unitarity. The BK equation is simplest in the momentum space where it can be written as

$$\frac{\partial \tilde{N}(x, k_t, b_t)}{\partial y} = 2\tilde{\alpha} \chi(-\frac{\partial}{\partial \log k}) \tilde{N}(x, k_t, b_t) - \tilde{\alpha} \tilde{N}^2(x, k_t, b_t)$$

where $\tilde{N}$ is related to the dipole cross section via

$$\tilde{N}(x, k_t, b_t) \equiv \int \frac{d^2 r_t}{2\pi r_t^2} e^{-i k_t \cdot r_t} \sigma(x, r_t, b_t)$$

and the dipole cross section $\sigma(x, r_t, b_t)$ is defined in terms of the expectation value of the correlator of two Wilson lines

$$\sigma(x, r_t, b_t) \equiv \frac{1}{N_c} Tr < 1 - U(x_t) U^\dagger(y_t) >$$

with $r_t = x_t - y_t$ and $b_t = (x_t + y_t)/2$. The BK equation has been investigated both numerically and analytically and approximate analytic solutions as well as complete numerical solutions have become available.
3. Applications to RHIC

The Color Glass Condensate was first applied at RHIC to the measured hadron multiplicities in nucleus-nucleus collisions at $\sqrt{s} = 130 \text{ GeV}$ \cite{5}. The starting point is the $k_t$ factorized form of gluon production cross section in heavy ion collisions

$$E \frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2 - 1} p_t^2 \int dk_t^2 \alpha_s \phi(x_1, k_t^2)\phi(x_2, (p_t - k_t)^2)$$

where $x_1, x_2 = \frac{p_t}{\sqrt{s}} \exp(\pm \eta)$. To get particle multiplicities, one integrates (16) over the transverse momentum and divides by the inelastic cross section of nucleus-nucleus collisions. The result is shown in Fig. (2) where a good agreement with the data at $\sqrt{s} = 130 \text{ GeV}$ is obtained. The centrality and energy dependence are also in good agreement with the RHIC data. It should be noted that multiplicities are dominated by low transverse momenta. For example, more than 95% of produced particles are produced with transverse momentum less than 1 GeV which indicates that multiplicities probe $x_1, x_2$ of the nucleus wave functions which are smaller than 0.01 at mid rapidity.

Eq. (16) was also applied to hadron transverse momentum spectrum in mid rapidity nucleus-nucleus collisions at $\sqrt{s} = 130 \text{ GeV}$ and a reasonable agreement was obtained. This would indicate that the observed suppression of the hadron spectra in mid rapidity nucleus-nucleus collisions is due to initial state (and not Quark Gluon Plasma) effects. Deuteron-gold collisions were performed in order to distinguish initial state from final state effects. The observed back to back hadron correlations and a lack of suppression in hadron spectra in mid rapidity deuteron-gold collisions basically rule out high gluon density effects at high $p_t$ in mid rapidity gold-gold collisions and are a strong indication of formation of a dense, strongly interacting medium which is most likely partonic in origin.

![Figure 2. Hadron multiplicity](image)
As such, the Color Glass Condensate provides the initial conditions for the formation of the Quark Gluon Plasma \[6\]. However, the physics of mid rapidity nucleus-nucleus collisions is mostly final state interactions between produced partons. Therefore, it is not the best place to look for manifestation of the Color Glass Condensate because, at high $p_t$, the final state effects in mid rapidity nucleus-nucleus collisions are clearly more important than initial state effects.

On the other hand, the forward rapidity region in deuteron-gold collisions is mostly free of these final state interactions since there is no Quark Gluon Plasma expected to be formed there. Also, in the forward rapidity region in deuteron-gold collisions, one probes the smallest $x$ in the nucleus wave function kinematically possible. This is where the Color Glass Condensate effects are expected to be the strongest. Another reason why the forward rapidity region is the best place to look for the Color Glass Condensate effects in a collider environment is that in forward rapidity, the experimental $p_t$ coverage is limited by the kinematics. Therefore, evolution in $p_t$ is not very important while the effects of $x$ evolution are maximized. This is crucial since the Color Glass Condensate formalism does not include $\alpha_s \log Q^2$ effects (summed by DGLAP) which become important at high $p_t$ (the double log region where one sums $\alpha_s \log Q^2 \log 1/x$ terms is indeed included in CGC).

In the forward rapidity region of deuteron-gold collisions one probes the large $x$ part of the deuteron wave function. To simplify the discussion, we will focus on proton-nucleus collisions. In this kinematic region, proton is a dilute system of partons with their distributions well described by pQCD. On the other hand, the nucleus is probed at very small $x$ so therefore, we treat it as a Color Glass Condensate. The starting point is to consider scattering of a valence quark of the proton on the nucleus \[7\]. One could also consider radiation of a gluon from the quark scattering on the nucleus \[8\]. The properties of these cross sections have been studied in detail in \[9\] \[10\] where it shown that Color Glass Condensate formalism, at the classical level, leads to the Cronin effect while inclusion of $x_{bj}$ evolution leads to a suppression of the nuclear enhancement factor. Since the case when the valence quark scatters on the nucleus is simpler (and possibly the most important in the forward rapidity and low $p_t$), we consider scattering of a quark on the nucleus described as a Color Glass Condensate. The differential cross section is given by
\[
\frac{d\sigma^{qA \rightarrow qX}}{d^2b_t \, dq_t \, dq^-} = \frac{1}{(2\pi)^2} \delta(p^- - q^-) \int d^2r_t e^{iq_t \cdot r_t} \sigma(x, r_t, b_t) \tag{17}
\]
where the dipole cross section $\sigma(x, r_t, b_t)$ is given by \[15\]. It is interesting to note that this cross section is finite as $q_t \to 0$ unlike the pQCD cross sections which are divergent. To obtain cross sections for hadron production, one needs to convolute \[17\] with the quark distribution function in a proton and a quark-hadron fragmentation function
\[
\frac{d\sigma^{pA \rightarrow \pi(k)X}}{dk^- \, d^2k_t} \equiv \int dx_q dz q_p(x_q) \frac{d\sigma^{qA \rightarrow qX}}{dq} \frac{d^2D_{q/\pi}}{dz} D_{q/\pi}(z) \tag{18}
\]
In Fig. \[3\] we show the result of a recent calculation \[11\] based on \[15\] for minimum biased deuteron-gold collisions in forward rapidity ($y = 3.2$) and compare it with the
nuclear modification factor $R_{dA}$ for negatively charged hadrons measured by BRAHMS [12] (our calculation is for the sum of positively and negatively charged hadrons, but in the kinematic region considered here, this does not make a big difference). The agreement at low $p_t$ is quite striking specially since there is absolutely no free parameter used here.

![Graph showing $R_{dA}$ for forward rapidity ($y = 3.2$) deuteron-gold collisions.](image)

Figure 3. Minimum bias $R_{dA}$ for forward rapidity ($y = 3.2$) deuteron-gold collisions.

The physics of particle production in forward rapidity and low $p_t$ can be understood in simple terms. The valence quark of the deuteron scatters from the fully developed (as allowed by the kinematics) wave function of the nucleus, characterized by $Q_A^s(y) = \sqrt{2}e^{\lambda_y/2} \sim 2.2 \text{GeV}$ (at $y = 3.2$) which is roughly how much transverse momentum the scattered quark accumulates. Assuming $< z > \sim 0.8$ in the fragmentation process, this means quark-nucleus scattering can describe particle production up to $\sim 1.8 \text{GeV}$. As one goes to higher $p_t$, one needs to include radiation of gluons in analogy with DGLAP radiation. In [8], gluon radiation from a quark scattering on the nucleus is calculated and must be included at higher $p_t$. Indeed, the suppression of hadron multiplicities in deuteron-gold collisions in the forward rapidity region had been predicted [10] based on the behavior of the gluon production cross section in different $p_t$ regions as was the striking centrality dependence of the hadron spectra in different rapidity regions. The fact that the cross section for scattering of quarks on the nucleus has similar properties is not surprising since both quark scattering and gluon production cross section depend on the (fundamental vs. adjoint) dipole cross section.

4. Summary

The recent results from the Relativistic Heavy Ion Collider have provided much excitement and enthusiasm in the high energy heavy ion community. On the one hand, there is overwhelming evidence for the formation of a deconfined state of matter, most likely of partonic nature in thermal equilibrium, in mid rapidity heavy ion collisions. On the other hand, we have, very likely, seen the first unambiguous evidence for the Color
Glass Condensate in the forward rapidity deuteron-gold collisions. Even though the Color Glass Condensate correctly predicted rapidity, energy and centrality dependence of hadron multiplicities in gold-gold collisions, the more conventional models, based on Glauber multiple scattering, were also able to accommodate the data.

The spectacular failure of the conventional models applied to the forward rapidity region in deuteron-gold collisions and the agreement of the Color Glass Condensate predictions with the observed suppression of the nuclear modification factor and its centrality dependence have provided strong evidence in favor of the Color Glass Condensate formalism. To put this on a more firm footing experimentally, one needs to measure more observables, such as photons, dileptons, etc. [13, 14]. Another deuteron-gold run at RHIC will go a long way towards establishing the properties of the Color Glass Condensate and is urgently needed.

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