Phase-coherent loops in selectively-grown topological insulator nanoribbons

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Abstract
We succeeded in the fabrication of topological insulator (Bi\textsubscript{0.57}Sb\textsubscript{0.43})\textsubscript{2}Te\textsubscript{3} Hall bars as well as nanoribbons by means of selective-area growth using molecular beam epitaxy. By performing magnetotransport measurements at low temperatures information on the phase-coherence of the electrons is gained by analyzing the weak-antilocalization effect. Furthermore, from measurements on nanoribbons at different magnetic field tilt angles an angular dependence of the phase-coherence length is extracted, which is attributed to transport anisotropy and geometrical factors. For the nanoribbon structures universal conductance fluctuations were observed. By performing a Fourier transform of the fluctuation pattern a series of distinct phase-coherent closed-loop trajectories are identified. The corresponding enclosed areas can be explained in terms of nanoribbon dimensions and phase-coherence length. In addition, from measurements at different magnetic field tilt angles we can deduce that the area enclosed by the loops are predominately oriented parallel to the quintuple layers.

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(Some figures may appear in colour only in the online journal)

1. Introduction
Nanoribbons of topological insulators (TI) have attracted considerable interest recently, in particular, since in combination with superconducting contacts spatially separated Majorana excitations are expected to be realized for the purpose of preparing robust topological quantum bits \cite{1–3}. In this context the understanding of coherent quantum transport in such TI nanoribbons is of paramount importance, particularly in connection with the topologically protected surface states \cite{4–6}. So far most studies in this direction have been performed on ribbons which have been grown epitaxially in a bottom-up approach from the gaseous phase \cite{7–13}. In such ribbons well-developed Aharonov–Bohm (AB) oscillations due to magneto-transport within the topologically protected surface states occurred \cite{7, 9–17}. The two-dimensional nature of the surface states in nanoribbons was confirmed.
by observing Shubnikov–de Haas oscillations [14, 18, 19], while phase-coherent transport was revealed by the presence of conductance fluctuations and weak antilocalization [12, 13, 20, 21].

With respect to more circuit flexibility in networks of nanoribbons for topological quantum computation a planar arrangement of these structures prepared by lithography in a top-down process would be advantageous, as compared to the standard bottom-up approach of preparing single free standing nanoribbons. In order to achieve this goal, we fabricated planar (Bi$_{0.57}$Sb$_{0.43}$)$_2$Te$_3$ nanoribbons which are grown selectively by molecular beam epitaxy (MBE) within lithographically patterned nano-grooves. This particular stoichiometry was chosen in order to minimize the bulk conductivity contribution [22]. In the selectively grown nanoribbons the van der Waals bonded quintuple layers are oriented parallel to the nanoribbon axis. Thus, current transport between contacts at both ends of the ribbon occurs parallel to the quintuple layers.

For designing circuits for topological quantum computation it is important to know on which length scale phase coherence is maintained, i.e. particularly if interference effects in loop structures are involved [23]. Information on phase-coherence can be obtained by analyzing transport phenomena such as weak (anti)localization and universal conductance fluctuations in structures of the according length scale [24]. The results are immediately applicable to the requirements that have to be matched when moving towards topological insulator based quantum computing architectures.

The coherent quantum magnetotransport is governed by self-interference of electron partial waves in an ensemble of closed-loop trajectories of different size. The loops are formed by elastic scattering at randomly distributed impurities. In the case of weak localization the constructive interference of electron partial waves in an ensemble of closed-loop trajectories of different size. The loops are formed by elastic scattering at randomly distributed impurities. In the case of weak localization the constructive interference of electron partial waves in an ensemble of closed-loop trajectories of different size. The loops are formed by elastic scattering at randomly distributed impurities. In the case of weak localization the constructive interference of electron partial waves in an ensemble of closed-loop trajectories of different size. The loops are formed by elastic scattering at randomly distributed impurities.
Figure 1. (a) Schematic illustration of a selectively-grown topological insulator (TI) nanoribbon. The nanoribbon is capped by a thin aluminum oxide layer. (b) Scanning electron micrograph of a focused ion beam cut cross section of a 50 nm wide nanoribbon. The silicon oxide layer is not resolved due to its low thickness and the elementary similarity to the substrate. By comparing the lower edge of the TI and the nitride its location can be assumed. On top of the structure platinum layers were deposited by two different techniques in order to stabilize the structure during the cutting procedure. The lower layer of platinum blends into the oxide capping and the nitride layers visually. (c) Scanning electron beam micrograph of a contacted 100 nm wide nanoribbon (indicated in red). The neighboring nanoribbons do not contribute to the transport.

were performed by using lock-in technique and injecting an AC current of 10 nA into the nanoribbon.

3. Results and discussion

3.1. Hall bar structures

In order to characterize the transport properties of the (Bi\(_{0.57}\)Sb\(_{0.43}\))\(_2\)Te\(_3\) film, first, magnetoresistance and Hall effect measurements were performed on a Hall bar structure with a length of 100 \(\mu\)m and a width of 10 \(\mu\)m. In figure 2(a) the magnetoresistance at temperatures ranging from 4 to 35 K is shown.

The magnetic field is oriented perpendicular to the TI layer. The magnetoconductance curves reveal a positive magnetoresistance with a distinct dip at zero magnetic field, which can be attributed to weak antilocalization [25, 26, 34], as it was reported for Bi\(_2\)Se\(_3\) [35–37], Bi\(_2\)Te\(_3\) [22, 38, 39], Sb\(_2\)Te\(_3\) [40, 41], as well as for (Bi\(_x\)Sb\(_{1−x}\))\(_2\)Te\(_3\) layers [22]. As can be seen in figure 2(a), inset, the Hall voltage has a linear negative slope, which indicates that the transport is \(n\)-type [22].

From magnetoresistance and Hall measurements a resistivity of \(\rho = 2.5 \times 10^{-3} \ \Omega \ cm\), a projected 2D carrier concentration of 1.02 \(\times\) \(10^{14}\) cm\(^{-2}\), and a mobility of 139 cm\(^2\)/Vs at a temperature \(T = 4\) K were extracted. The relatively large charge carrier concentration indicates a considerable contribution of bulk charge carriers participating in the transport [22]. From the above values an elastic mean free path of \(l_e = 6\) nm was derived. By increasing the temperature from 4 to 35 K, the electron concentration only increases slightly, by less than 3\%, showing that the transport is in the metallic regime. In order to gain information on phase-coherent transport from the weak antilocalization feature a fit to the Hikami–Larkin–Nagaoka (HLN) model was performed (see figure S2) [25]. From the fit a phase-coherence length \(l_\phi\) of about 150 nm was extracted at 4 K. The temperature dependent decay was found to be proportional to \(T^{-0.5}\) which is in agreement with the Nyquist electron-electron interaction for disordered systems [42]. Furthermore, we find that the transport takes place in a single channel, since the fitted pre-factor \(\alpha\) of the HLN formula of about \(-0.4\) is close to the predicted value of \(-0.5\) for a single channel. This suggests an interpretation of two topological two-dimensional transport channels that are coupled via bulk scattering, or alternatively that device fabrication causes the two channels to have substantially different phase-coherence lengths [22, 43]. In addition,
3.2. Nanoribbons

We now turn to the transport experiments on a selectively-grown nanoribbon with a length of 10 μm, a width of 50 nm, and a thickness of 29 nm. The measurements were performed in a two-terminal configuration at a temperature of 1.4 K. In figure 3 the magnetoresistance is shown as a function of tilt angle θ of the magnetic field. The sample shows a resistance of about 440kΩ at 0 T, corresponding to a resistivity of $\rho = 6.4 \times 10^{-3} \, \Omega \cdot \text{cm}$. We attribute the higher resistivity compared to values obtained from the Hall bar measurements to the additional boundary scattering contribution.

As found in the measurements on Hall bar structures, the curves possess a dip feature at zero magnetic field due to the weak antilocalization effect. In addition to what has been observed in the Hall measurements, one finds reproducible resistance modulations, which will be addressed in detail below. This feature was observed in all nanoribbons under investigation, i.e. a set of measurements on a nanoribbon with a width of 100 nm is shown in figure S3. The resistance modulations only occur in nanostructures and get lost in the transition to microstructures. Since the phase-coherence length $l_\phi$ extracted from the Hall bar measurements exceeds the ribbon width, we applied a model for quasi 1-dimensional structures to fit the experimental data and to extract $l_\phi$ [42, 47, 48]. For dirty metals in the strong spin-orbit scattering limit the correction of...
Figure 4. Phase-coherence length in the nanoribbon as a function of tilt angle of the magnetic field for WAL by fitting the experimental data shown in figure 3 according to equation (1) (red symbols) and from UCF theory (yellow symbols) applied to the same data. Inset: Schematic arrangement of the nanoribbon with two exemplary closed loops and their projected enclosed areas $S_1$ and $S_2$ with respect to a perpendicular magnetic field $B$.

The resistance due to weak antilocalization can be expressed by a quasiclassical approach [49]

$$\Delta R = -\frac{1}{2} \frac{R^2}{L} \frac{e^2}{\pi h} \left( \frac{1}{l_{\phi}^2} + \frac{1}{l_{B,\perp}^2} + \frac{1}{l_{B,\parallel}^2} \right)^{-1/2}$$

(1)

Here, $l_{\phi}$ is the phase-coherence length at a tilt angle $\theta$; $w$ and $d$ are the width and thickness of the ribbon, respectively, and $L$ is the contact separation. Furthermore, the magnetic dephasing lengths for the perpendicular and parallel components of the magnetic field are defined as $l_{B,\perp} = \sqrt{3\hbar/eB\sin(\theta)w}$ and $l_{B,\parallel} = \sqrt{2\pi\hbar/eB\cos(\theta)\sqrt{wd}}$, respectively [42, 47, 49]. These parameters take care of the geometrical limitations in different orientations. A possible contribution of the Zeeman effect was neglected, since it could be shown by weak localization measurements on topological insulator layers that this contribution is not relevant in the low field range [36, 50]. The model was fitted to the data for the different sample orientations. A typical fit is shown in figure S4. Figure 4 shows the results of the fit (red symbols). For a magnetic field aligned perpendicularly to the substrate plane ($\theta = 90^\circ$) we find a phase-coherence length of around 60 nm, which is reduced by 30% to about 40 nm for an in-plane magnetic field. Thus, an angular dependence of $l_{\phi}$ is extracted from the WAL. This indicates an anisotropy of the material favouring coherent transport parallel to the substrate surface [32, 51]. As a matter of fact for similar measurements on InAs nanowires it was found that $l_{\phi}$ is rather constant which shows that here no anisotropy is present [49]. Nevertheless, we cannot completely rule out that the angular dependence of $l_{\phi}$ is also affected by geometrical factors [52]. The phase-coherence length of around 60 nm for a perpendicular field is somewhat smaller than the value gained from the Hall bar structure. A possible reason might be the effect of boundary scattering on the phase-coherence length in case of the nanoribbon.

The modulations of the magnetoresistance $R(B)$ found in figure 3 can be attributed to universal conductance fluctuations [27, 28]. As mentioned above, these fluctuations originate from the Aharonov–Bohm type interference of closed-loop electron trajectories of different size and orientation (see schematic illustration in figure 4) [29]. By analyzing the correlation field $B_C$ of the conductance fluctuations the characteristic phase-coherence length for the interference in the loops can be estimated [27, 53]. The correlation field is determined via the autocorrelation function $F(\Delta B) = \langle \delta G(B + \Delta B)\delta G(B) \rangle$ by $F(B_C) = \frac{1}{2}F(0)$. Here, $\delta G(B)$ are the conductance fluctuations after subtracting the slowly varying background and $\langle \ldots \rangle$ denotes the average over the magnetic field. For a quasi-one-dimensional transport channel in the dirty limit $l_F \ll w$ the relation between $B_C$ and $l_{\phi}$ is expressed by $B_C = \gamma \Phi_0/(l_{\phi}w)$ [53, 54]. For the prefactor $\gamma$ we choose 0.42 [54] for $l_{\phi}$ larger than the thermal length $l_T = \sqrt{\hbar D/k_B T} \approx 7 \text{ nm}$, with $D$ the diffusion constant calculated from the mobility and the charge carrier concentration. In figure 4 the phase-coherence length $l_{\phi}$ extracted from $B_C$ is plotted for different tilt angles $\theta$. We restricted the analysis to tilt angles $\theta \geq 20^\circ$ since for smaller values of $\theta$ too few
fluctuations occurred in the measured magnetic field range. Once again we find a tendency that $l_\phi$ is anisotropic, i.e. $l_\phi$ is smaller for small tilt angles, similar to our analysis of the WAL measurements. However, the values of $l_\phi$ obtained from the UCF measurements are larger by a factor of about two compared to the values gained from the WAL measurements. The significant difference between the two $l_\phi$ values might have several reasons. First, the $l_\phi$ values evaluated from UCFs originate from measurements in large magnetic fields, in contrast to WAL where $l_\phi$ is determined close to zero magnetic field. Consequently, spin-related scattering mechanisms might yield differently strong contributions in both phenomena. Second, there might be uncertainties in the theoretical analysis of both interference phenomena, e.g. the characteristic $\gamma$ factor in the UCF analysis might deviate from the theoretically predicted value [52]. Third, the phase-coherence length $l_\phi$ is in the order of the circumference of the nanoribbon. In that case the interference effects in the magnetoconductance are also limited by the circumference in addition to $l_\phi$ [55].

For the universal conduction fluctuations each individual loop contributes to the magnetoconductance with a characteristic period and thus with a particular frequency. Therefore, by analyzing the frequency spectrum of the magnetoconductance detailed information on the loop sizes, i.e. cross sections $S/\Phi_0$ with respect to an applied magnetic field $\vec{B}$, can be obtained. In order to do so, we calculated the Fourier transform of the magnetoconductance $G(B) = 1/R(B)$:

$$\tilde{G}(S/\Phi_0) = \int dB G(B) \exp(-2\pi iBS/\Phi_0) ,$$

(2)

with $S$ the loop cross sectional area perpendicular to a given orientation of an applied magnetic field. Figure 5(a) gives an overview of the Fourier amplitude vs. $S/\Phi_0$ (‘frequency’) in units of $1/T$ at a magnetic field tilt of $0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$, respectively. The Fourier amplitude generally decreases with increasing $S/\Phi_0$, which reflects the fact that the loop return probability decreases with increasing loop size. Furthermore, towards larger tilt angles the frequency range is extended. This can be explained by the larger allowed projection areas provided for loops at larger tilt angles.

In order to resolve more details in the Fourier spectrum, the logarithm of the Fourier amplitude $\log_{10}(G)$ as a function of $S/\Phi_0$ is plotted in figure 5(b) for tilt angles between $80^\circ$ and $100^\circ$. No background was subtracted from the original signal. It is clear that the Fourier spectrum fluctuates indicating that it contains a large number of different frequencies. The spectrum reflects the contribution of different loops with specific cross sectional areas $S$ to the magnetoconductance. The spectrum is reproducible, and changes gradually when the magnetic field orientation is varied. These changes occur because as the tilt angle changes, each loop’s cross-sectional area perpendicular to the magnetic field vector also changes.

Collecting the traces corresponding to different magnetic field orientations in a color plot confirms that there are systematic patterns in the fluctuations of the Fourier spectrum, as shown in figure 6(a).

One can clearly see that for a perpendicular magnetic field the oscillation frequency range is large whereas for a parallel

Figure 5. (a) Fourier amplitude vs. loop cross sectional area per magnetic flux quantum $S/\Phi_0$ for magnetic field tilt angles of $0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. (b) Logarithm of the Fourier amplitude $\log(G)$ vs. $S/\Phi_0$ taken from the magnetoconductance at magnetic field tilt angles between $80^\circ$ and $100^\circ$. The data is plotted with an offset for better visibility. The bottom curve is without offset.
magnetic field the range is considerably smaller, i.e. the characteristic brighter stripes are getting closer towards smaller magnetic field tilt angles $\theta$. Since the frequency is directly proportional to the loop cross sectional area $S$ with respect to the magnetic field orientation, one can directly deduce that for the perpendicular case larger-area loops contribute. This is indeed plausible, since for that orientation, the maximum phase-coherent loop area is approximately given by $S = l_\phi w$, with $l_\phi$ the phase-coherence length and $w$ the nanoribbon width, assuming $w < l_\phi$. One finds that at $\theta = 90^\circ$ the frequency contributions diminish at about $2.5 \ T^{-1}$ corresponding to an area of approximately $10^4 \ \text{nm}^2$ resulting in a lower estimate for the phase-coherence length $l_\phi$ of 200 nm. This value is close but somewhat larger than the corresponding value determined from the correlation field. For the parallel magnetic field orientation the maximum loop area is given by the cross section $S = wd = 1450 \ \text{nm}^2$ of the ribbon, providing that the thickness $d$ of the (Bi$_{0.5}$Sb$_{0.43}$)$_2$Te$_3$ layer is smaller than $l_\phi$. The corresponding value of $S/\Phi_0 = 0.35 \ T^{-1}$ fits very well to the range found in figure 6(a) at $\theta = 0$. However, we do not find a distinct single Aharonov–Bohm peak in the spectrum at $0.35 \ T^{-1}$, as it would be expected if the transport solely takes place in the topologically protected surface states or in the accumulation layer of massive electrons at the interface. One reason might be that the thickness of the nanoribbons varies by one or two quintuple layers, resulting in a variation of the cross-section along the ribbon. We furthermore observe some frequency contributions below $0.35 \ T^{-1}$ indicating that smaller size loops in the bulk contribute as well.

As mentioned above, in figure 6(a) we find a stripe-like pattern of peaks in the Fourier spectrum which is getting closer towards smaller tilt angles. Each stripe can be assigned to a distinct phase-coherent loop [29]. In order to gain more precise information on the loop orientation with respect to the magnetic field we followed each trace and determined a set of corresponding discrete points in the $S/\Phi_0 - \theta$ plane, as indicated exemplarily by the orange dots in figure 6(b). To each sequence of data points we performed a fit according to

$$S_{\Lambda}(\theta) = \frac{S_{\Lambda, \text{max}}}{\Phi_0} \sin(\theta + \gamma), \quad (3)$$

with $S_{\Lambda}(\theta)$ and $S_{\Lambda, \text{max}}$ the projected and maximum loop area, respectively, while $\gamma$ is the tilt angle of the maximum loop area with respect to the substrate plane. Some exemplary fits are shown in figure 6(b) (red lines).

We find that most of the lines correspond to loops with small tilt angles $\gamma$ in the range of $\pm 7^\circ$. In fact, we could not identify any sine-like lines with larger offset angles $\gamma$, corresponding to loops with a larger tilt angle. This is indeed remarkable, since if we would assume bulk conductance, phase-coherent loops with orientations in a much broader range of tilt angles are expected. This is particularly true for loops enclosing a smaller area, since here the tilt angle should not be limited by geometrical constraints, i.e. the elongated shape of the ribbon. Thus, we can state that the peaks found in the Fourier spectrum correspond to interference loops that are mainly oriented within the plane parallel to the substrate. Most likely, these loops are located within the topologically protected surface states. However, from the fact that we find loops with finite tilt angles, we can also infer that the phase-coherent loops extend over different quintuple layers to a certain extent, i.e. the weak van der Waals bonding does not prevent an extension of the electron waves across the quintuple layers, completely [32]. In the supplementary information, we support our conclusions drawn here, by modelling the Fourier spectrum for a typical set of loops and deducing from that the magnetoconductance fluctuations.

Finally, let us briefly come back to the general properties of the Fourier spectrum. As shown in figure 5(a), the amplitude in the Fourier spectrum is generally decreasing with increasing
projected loop area $S/\Phi_0$. Thus, the number of coherent loops with a certain projection area is decreasing for increasing loop sizes. This general trend reflects the fact that the return probability decreases the more the electron trajectories move away from their origin. Including all contributing loops leads to the observed weak antilocalization feature [29]. However, owing to the small dimensions of the nanoribbon and the according finite number of loops there is a randomness in the actual distribution of the loop sizes. This finally results in the superimposed fluctuations in the amplitude of the Fourier spectrum and the stripe pattern when plotted as a function of tilt angle (cf figure 6). The fact, that the stripe pattern systematically shifts with the tilt angle supports our interpretation that the orientation of all loops contributing to a certain Fourier amplitude is basically the same, i.e. parallel to the quintuple layers. If this would not be the case, the fluctuation pattern would not be shifted systematically but rather randomly.

4. Conclusions

In conclusion, by using selective-area molecular beam epitaxy we succeeded to grow (Bi$_{0.57}$Sb$_{0.43}$)$_2$Te$_3$ topological insulator Hall bar structures as well as nanoribbons. Low temperature magnetotransport experiments on these structures revealed signatures of weak antilocalization. From these measurements a phase-coherence length in the order of 100 nm was extracted. By performing measurements in a tilted magnetic field we extracted a phase-coherence length which is larger for loops oriented parallel to the quintuple layers. This behaviour is attributed to transport anisotropy and geometrical factors. For the nanoribbons we also observed universal conductance fluctuations. By performing a Fourier transform of the fluctuation pattern we were able to identify a series of distinct phase-coherent closed-loop trajectories with areas which can be explained in terms of nanoribbon dimensions and phase-coherence length. From measurements at different magnetic field tilt angles we conclude that these loops are predominately located parallel to the quintuple layers i.e. within the topologically protected surface states.

We expect that the present approach towards analyzing universal conductance fluctuations at different magnetic field orientations by Fourier transforms turns out to be a powerful tool for better understanding conducting pathways in disordered topological materials and nanostructures thereof. In the present samples, we could not avoid a significant bulk conductance contribution due to background doping. In future structures, one should try to minimize the impact of background doping either by using optimized ternary or quaternary topological insulators and by means of heterostructures or by gate control. Latter could also help to avoid a possible two-dimensional contribution of massive electrons from the conduction band in the surface accumulation layer. If that is once achieved it should be possible to observe a single peak in the Fourier spectrum for an in-plane magnetic field resulting from Aharonov–Bohm type oscillations due to phase-coherent transport in the topologically protected surface states.

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