Learning Controllers for Performance Through LMI Regions

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Abstract—In an experiment, an input sequence is applied to an unknown linear time-invariant system (in continuous or discrete time) affected also by an unknown-but-bounded disturbance sequence; the corresponding state sequence (and state derivative sequence, in continuous time) is measured. The goal is to design directly from the input and state sequences a controller that enforces a certain performance specification on the transient behavior of the unknown system. The performance specification is expressed through a subset of the complex plane where closed-loop eigenvalues need to belong, a so-called linear matrix inequality (LMI) region. For this control design problem, we provide here convex programs to enforce the performance specification from data in the form of LMIs. For generic LMI regions, these are sufficient conditions to assign the eigenvalues within the LMI region for all possible dynamics consistent with data, and become necessary and sufficient conditions for special LMI regions. In this way, we extend classical model-based conditions from a work in the literature to the setting of data-driven control from noisy data. Numerical examples substantiate the analysis.

Index Terms—Control design, data-driven control, linear feedback control systems, linear matrix inequalities, Lyapunov methods, matrix stability, noisy data, performance specification, robust control, transient behavior, uncertain systems.

I. INTRODUCTION

Whenever it is cumbersome to derive a model for a process to be controlled or to identify unambiguously its parameters, a viable alternative is to bypass these two steps altogether and, from data collected on the process, design directly a (feedback) controller [30]. Direct data-driven control was conceived within the discipline of system identification, and is enjoying renewed popularity thanks to a result by Willems et al. [33, Th. 1]. This result has inspired several (indirect and direct) data-driven control methods, such as predictive control [10], state- and output-feedback design [12], and optimal control [12], [24], [32]. The resulting line of research has been relevant to address noisy data for stabilization [11], [12], linear quadratic regulation [13], [34], and dynamic performance [2], [3], [31]. Closely related results are in [1] and [23].

This article contributes to the literature on data-driven control by addressing the problem of data-based control design for performance specifications. Dynamic performance has been generally less investigated, and the main directions have been $H_2$ performance [3], [31], $H_{\infty}$ performance [2], [3], [31], and model-reference control [6], [15], [17], [18], [20]. An alternative and general method to impose performance specifications is to impose that the closed-loop eigenvalues belong to specific subsets of the complex plane that determine salient characteristics of the closed-loop transient response (see Example 1 later). The fundamental work in [7] showed that imposing all closed-loop eigenvalues within certain subsets of the complex plane is equivalent to solving a linear matrix inequality (LMI) [7, Th. 2.2] (recalled later in Fact 1), hence the name of LMI regions for these subsets; moreover, the intersection of LMI regions is also equivalently associated with the conjunction of the respective LMIs [7, Cor. 2.3] (recalled later in Fact 2), with the result that LMI regions are dense in the set of convex regions symmetric with respect to the real axis [7, Sec. I.C]. (Mathematically, LMI regions correspond to one instance of several notions of matrix stability [21].) Besides convexity and this approximation property, another appealing feature of this approach for performance is the intuitive link between desired characteristics of the time response and regions of the complex plane that are well known to a control engineer familiar with frequency methods and loop shaping, as their effective use in experimental applications [9], [26], [27], [29] would suggest.

All these positive features in the model-based case appear promising and have motivated us to study how to impose performance specifications through LMI regions in the context of data-driven control with noisy data. Since noise prevents exact identification and one has a set of systems consistent with data and noise model, as in set-membership identification [14], [25], the challenge becomes to guarantee, from data, robust performance, i.e., to assign eigenvalues in an LMI region for all the systems in such consistency set. From a conceptual viewpoint, this approach is similar in nature to [8], which investigates robustness of pole clustering in LMI regions with respect to complex unstructured and real structured uncertainty (such as parameter uncertainty); here we consider robustness with respect to a different type of uncertainty, namely, that induced by noisy data. Our contribution is to develop a data-driven approach based on LMI regions, which has not been investigated so far; we provide sufficient conditions to design a controller enforcing robust eigenvalue assignment in spite of noisy data for generic LMI regions and their intersections, under an energy bound on the whole noise sequence of the experiment; moreover, we obtain that these sufficient conditions become also necessary for special LMI regions; finally, all these results hold both for continuous and discrete time and are given in terms of convenient LMIs. In a nutshell, the proposed approach features an experiment for data collection, performance specifications are intuitively expressed as LMI regions, and the proposed convex programs design the controller to enforce the specification automatically; we then believe that the approach has the potential to incentivize these new data-based techniques among control engineers.

Structure: In Section II, we report the notions needed from the model-based setting in [7]. In Section III we formulate the data-based problem. In Section IV, we give sufficient conditions for generic LMI regions, whereas, for special LMI regions, necessary and sufficient conditions are given in Section V. How these conditions compare is investigated numerically in Section VI.
Notation: \(\mathbb{N}_{\geq 1}\) denotes the natural numbers 1, 2, ..., \(\mathbb{R}\) denotes the real numbers; and \(\mathbb{C}\) denotes the complex numbers. For \(z \in \mathbb{C}\), \(\bar{z}\) denotes the complex conjugate of \(z\). Given \(n \in \mathbb{N}_{\geq 1}\), \(I_n\) (or \(I\)) denotes an identity matrix of dimension \(n\) (or of suitable dimension). A matrix of all zeros of suitable dimensions is denoted by \(0\). For a square matrix with real entries, the Hermitian/transposition operator is \(\text{He} \ A := A + A^\top\). For symmetric matrices \(A\) and \(C\), we sometimes abbreviate the symmetric matrix \([\begin{array}{cc} A & B \\ B & C \end{array}\]) as \([\begin{array}{cc} A & z \\ z & C \end{array}\]) or \([\begin{array}{cc} A & B \\ B & C \end{array}\}]^\top\).

Positive definiteness (semidefiniteness, respectively) of a symmetric matrix \(A\) is indicated as \(A > 0\) (\(A \succeq 0\), respectively). For \(A = A^\top \succeq 0\), \(A^{1/2}\) denotes the unique positive semidefinite root of \(A\). The Kronecker product is denoted by \(\otimes\) and the standard properties of the Kronecker product that we use can be found in [19, Sec. 4.2].

II. Review on LMI Regions

This section recalls the notions we need from [7].

Definition 1: [7, Def. 2.1] A subset \(S\) of the complex plane is called an LMI region if for some \(s \in \mathbb{N}_{\geq 1}\), there exist a symmetric matrix \(\alpha \in \mathbb{R}^{s \times s}\) and a matrix \(\beta \in \mathbb{R}^{s \times s}\) such that

\[
S = \{ z \in \mathbb{C} : \alpha + z\beta + \bar{z}\beta^\top \succeq 0 \} \quad (1)
\]

where the matrix \(\alpha + z\beta + \bar{z}\beta^\top\) is Hermitian. \((\alpha, \beta)\) are called data of \(S\).

Definition 2: [7, p. 359] For \(S \subseteq \mathbb{C}\), the matrix \(A\) is \(S\)-stable if all eigenvalues of \(A\) lie in \(S\).

We recall a first elegant result from [7].

Fact 1: [7, Th. 2.2] For an LMI region \(S \subseteq \mathbb{C}\) with data \((\alpha, \beta)\), the matrix \(A\) is \(S\)-stable if and only if there exists a symmetric matrix \(P\) such that

\[
P > 0, \alpha \otimes P + \beta \otimes (AP + \beta^\top (PA^\top) < 0. \quad (2)
\]

Fact 1 enables treating continuous and discrete time simultaneously. For an LMI region \(S\) with data \((\alpha, \beta)\), define its characteristic matrix \(M_S\) as

\[
M_S(A, P) := \alpha \otimes P + \beta \otimes (AP + \beta^\top (PA^\top) < 0. \quad (3)
\]

so that \(M_S(A, P) < 0\) precisely is the main condition in (2). This brings us to a second key result from [7], recalled next.

Fact 2: [7, Cor. 2.3] Given two LMI regions \(S_1\) with data \((\alpha_1, \beta_1)\) and \(S_2\) with data \((\alpha_2, \beta_2)\), a matrix \(A\) is both \(S_1\)-stable and \(S_2\)-stable if and only if there exists a symmetric positive definite matrix \(P\) such that \(M_{S_1}(A, P) < 0\) and \(M_{S_2}(A, P) < 0\).

When designing a controller \(u = Kx\) for \(\dot{x} + Ax = Bu\), one considers a closed-loop matrix \(A + BK\) in (2) and looks for \(P = P^\top > 0\) and \(K\) such that

\[
\alpha \otimes \beta + \beta \otimes ((A + BK)P + \beta^\top (P(A + BK)^\top) < 0. \quad (4a)
\]

This inequality is not linear in \(P\) and \(K\), so one uses instead

\[
\text{find } \quad P = P^\top > 0, Y \quad (4a) \\
\text{s.t. } \quad \alpha \otimes \beta + \text{He } \{\beta \otimes (AP + BY)\} < 0 \quad (4b)
\]

with (4b) now linear in \(P\) and \(Y\). From the underlying change of variables, \(K = Y^{-1} P\). In the presence of \(r\) LMI regions \(S_i\) with data \((\alpha_i, \beta_i), i = 1, \ldots, r\), (4) extends by Fact 2 to

\[
\text{find } \quad P = P^\top > 0, Y \quad (5a) \\
\text{s.t. } \quad \alpha_i \otimes \beta + \text{He } \{\beta_i \otimes (AP + BY)\} < 0, i = 1, \ldots, r. \quad (5b)
\]

LMI regions in Definition 1 have a generic \(s \in \mathbb{N}_{\geq 1}\). However, \(s = 2\) is a convenient tradeoff between tractability and expressiveness since it encapsulates a plethora of well-known quadratic curves, the most common of which are shown in Fig. 1. By expressing \(z\) in (1) in terms of its real and imaginary parts, we write (1) for \(s = 2\) as

\[
S = \{ x + jy \in \mathbb{C} : \quad \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} \prec 0 \}
\]

\[
= \{ x + jy \in \mathbb{C} : \quad \alpha_{11} \alpha_{22} - \alpha_{12}^2 < 0, \quad 2x \alpha_{11} + 2y \alpha_{12} < 0, \quad x^2 \alpha_{11} + y^2 \alpha_{22} < 0. \quad (6)
\]

Based on this expression, the \(\alpha \text{ and } \beta\) corresponding to the regions in Fig. 1 are reported in [4, Table III]. Some LMI regions of Fig. 1, such as halfplanes, disks, and cones, are well known to prescribe a certain desired transient behavior, as we show in the next Example 1. Other LMI regions of Fig. 1 are presented to show the expressivity of even the case \(s = 2\) and because, in view of Fact 2, they can be intersected to approximate subsets of the complex plane relevant for control purposes.

Example 1: For suitable parameters \(\ell > 0, \rho > 0, \theta \in (0, \pi/2)\), consider the subset \(S(\ell, \rho, \theta)\) depicted in Fig. 2, left, \(S(\ell, \rho, \theta) = \{ z \in x + jy \in \mathbb{C} : x < -\ell \} \cup \{ z \in x + jy \in \mathbb{C} : x^2 + y^2 < \rho^2 \} \cup \{ z \in x + jy \in \mathbb{C} : (\cos \theta)|y| < < (\sin \theta)|x| < 0\}\); hence, \(S(\ell, \rho, \theta)\) guarantees a minimum convergence rate of \(\ell\) (halfplane), a maximum natural frequency of \(\rho\) (disk), and a minimum damping ratio \(\cos \theta\) (cone). In terms of performance, these correspond to upper bounds on settling time, overshoot, frequency of oscillatory modes, and magnitude of high-frequency poles [7], [16, Sec. 3.3-3.4]. By Fact 2, the eigenvalues of \(A_{3d}\) belong to
to $\mathcal{S}(\ell,\rho,\theta)$ if and only if there exists $P = P^\top \succ 0$ such that
$$
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \otimes P + \text{He} \left\{ [1/2 \ 0] \otimes (A_0 P) \right\} \prec 0, \quad \text{He} \left\{ [0 \ 1/2] \otimes (A_0 P) \right\} \prec 0,
$$
where each condition is read off [4, Table III].

### III. Data-Driven Control by LMI Regions

In this section we formulate the data-based control design problem for $\mathcal{S}$-stability and obtain equivalent forms of a key set in this work.

#### A. Problem Formulation

Consider a linear time-invariant system
$$
x^\ell = Ax + Bu + d
$$
where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $d \in \mathbb{R}^n$ is the disturbance, for which we consider a bound $D$, as in (9) later, and $x^\ell$ represents the time derivative $\dot{x}$ of the state, in continuous time, or the update $x_{t+1}^\ell$ of the state, in discrete time. For convenience, we call $x^\ell$ state preview. The matrices $A$, $B$, and $D$ are unknown to us, and we rely instead on data collected through an experiment on the system. Specifically, we apply an input sequence $u(t_0), u(t_1), \ldots, u(t_{T-1})$ and measure the corresponding state and state preview sequences $x(t_0), x(t_1), \ldots, x(t_{T-1})$ and $x^\ell(t_0), x^\ell(t_1), \ldots, x^\ell(t_{T-1})$, where the preview sequence is the sequence of state derivatives $\dot{x}(t_0), \dot{x}(t_1), \ldots, \dot{x}(t_{T-1})$, in continuous time, and the sequence of state updates $x(t_1), x(t_2), \ldots, x(t_T)$, in discrete time. During the experiment, a disturbance sequence $d(t_0), d(t_1), \ldots, d(t_{T-1})$ acts on system (7) and affects its evolution. This sequence is also unknown to us, and due to its influence on the state evolution, we say that data are noisy. In summary, we collect the measured sequences in matrices
$$
U_0 := \begin{bmatrix} u(t_0) & \ldots & u(t_{T-1}) \end{bmatrix}, \quad X_0 := \begin{bmatrix} x(t_0) & \ldots & x(t_{T-1}) \end{bmatrix}, \quad X_1 := \begin{bmatrix} x^\ell(t_0) & \ldots & x^\ell(t_{T-1}) \end{bmatrix},
$$
and the unknown disturbance sequence in $D_0 := [d(t_0) \ldots d(t_{T-1})]$. The times $t_0, t_1, \ldots, t_{T-1}$ are taken as the $0, 1, \ldots, T-1$ multiples of a period $T$; this is a natural choice in discrete time since these times correspond to periodic sampling, and we adopt the same choice also in continuous time (although this is not necessary). Since the data generation mechanism is (7), the data points in the experiment satisfy
$$
X_1 = A X_0 + B U_0 + D_0.
$$
Now that we have illustrated the considered data generation mechanism, we briefly discuss an alternative one in the next remark.

**Remark 1:** Based on the data generation mechanism in (7), we focus here on the case of a process disturbance. Alternatively, measurement noise can be considered and $x^\ell = Ax + Bu + d$, corresponding to (7) in discrete time, would be replaced by $x^\ell = Ax + B \omega + \omega$, where $\omega$ is the measurement noise and the measured variable is no longer $x$ but $z$. This case of measurement noise can be treated similarly, as shown in [12, Sec. V.A], by observing that the dynamical equation for $z$ is $z^\ell = z + \omega = A x + B \omega + (\omega - A \omega)$ and this equation is analogous to (7) after replacing $d$ with a filtered measurement noise. Hence, we focus on the case of a process disturbance for simplicity.

As for the disturbance sequence $D_0$, we know only that, for some $p \in \mathbb{N}_{\geq 1}$ and some matrix $\Delta \in \mathbb{R}^{p \times p}$, it belongs to $\mathcal{D} := \{ D \in \mathbb{R}^{n \times n} : DD^\top \preceq \Delta \Delta^\top \}$;

in other words, we know only an energy bound on disturbance sequences. The pairs $[A \ B]$ that could generate the data points $U_0, X_0, X_1$ for a disturbance sequence $D \in \mathcal{D}$ correspond to the set
$$
\mathcal{C} := \{ [A \ B] : X_1 = AX_0 + BU_0 + D, D \in \mathcal{D} \}
$$
and $\mathcal{C}$ is called the set of matrices consistent with data. Since $D_0 \in \mathcal{D}$, we have $[A \ B] \in \mathcal{C}$.

We make the next assumption on matrix $[X_0 \ U_0]$.

**Assumption 1:** Matrix $[X_0 \ U_0]$ has full row rank.

A necessary condition for Assumption 1 to hold is $T \geq n + m$. Full row rank of $[X_0 \ U_0]$ can be verified directly from data and when it does not hold, one can typically enforce it by collecting more data points, thereby adding columns to $[X_0 \ U_0]$. This rank condition is intimately related to persistence of excitation of the input and disturbance sequences, see a discussion in [5, Sec. 4.1].

The objective of this work is to design a linear feedback controller $u = K x$ that assigns the eigenvalues of the closed-loop matrix $A + B K$ within a certain subset $\mathcal{R}$ of the complex plane. Since $A$ and $B$ are unknown, this is achieved by imposing that $K$ assigns the eigenvalues of $A + B K$ within $\mathcal{R}$ for all $[A \ B] \in \mathcal{C}$. First, we address the case of $\mathcal{R}$ given by a single LMI region $\mathcal{S}$ with data $(\alpha, \beta)$, where the objective becomes
$$
\text{find } P = P^\top \succ 0, K
$$
s.t.
$$
\begin{align}
\alpha & \otimes \beta \otimes (A + BK P) \\
+ \beta^\top & \otimes (PA + BK P)^\top \prec 0 \ \forall [A \ B] \in \mathcal{C},
\end{align}
$$
In words, solving this problem yields a certificate and a controller for $\mathcal{S}$-stability, i.e., the Lyapunov-like matrix $P$ and the gain $K$. Second, we address the case of $\mathcal{R}$ given by the intersection of $r$ LMI regions $\mathcal{S}_i$ with data $(\alpha_i, \beta_i), i = 1, \ldots, r$, that is, $\mathcal{R} := \bigcap_{i=1}^r \mathcal{S}_i$. The objective becomes
$$
\text{find } P = P^\top \succ 0, K
$$
s.t.
$$
\begin{align}
M_{S_1} & (A + BK, P) \prec 0, \\
\ldots, \quad M_{S_r} & (A + BK, P) \prec 0 \ \forall [A \ B] \in \mathcal{C},
\end{align}
$$
where the definition of characteristic matrices $M_{S_1}, \ldots, M_{S_r}$ is in (3). Equation (12) is the natural extension of (11) by taking into account Fact 2 for an intersection of LMI regions.

#### B. Equivalent Forms of Set $\mathcal{C}$

The set $\mathcal{C}$ introduced in (10) plays a key role in our developments and, for it, we present three different forms equivalent to each other in this section. The first form is
$$
\mathcal{C} = \{ [A \ B] : Z^\top : [I_n \ Z^\top] \left[ \begin{bmatrix} B & K \end{bmatrix} \right] \left[ \begin{bmatrix} I_n \ Z^\top \end{bmatrix} \right]^\top \preceq 0 \}
$$
(13a)
$$
\mathcal{C} := X_1 X_1^\top - \Delta \Delta^\top
$$
(13b)
$$
B := - [X_0 \ U_0]^\top, \quad A := [X_0 \ U_0] [X_0 \ U_0]^\top.
$$
(13c)
This form can be obtained with algebraic computations from the definition of $\mathcal{C}$ in (10) by expressing $D$ there as $D = X_1 - AX_0 - BU_0$, substituting this $D$ in the condition defining $\mathcal{D}$ in (9), and collecting $[I_n \ A \ B] = [I_n \ Z^\top]$ to the left and its transpose to the right. By Assumption 1 on full-row rank of $[X_0 \ U_0]$, it holds $A \succ 0$ and, by $A \succ 0$, algebraic computations yield the second form of the set $\mathcal{C}$ as
$$
\mathcal{C} = \{ [A \ B] : Z^\top : (Z - \zeta) A (Z - \zeta) \preceq Q \}
$$
(14a)
$$
\zeta := -A^{-1} B, \quad Q := B^\top A^{-1} B - C.
$$
(14b)
We just noted that $A \succ 0$; the sign definiteness of $Q$ is also a structural property as claimed next.

**Lemma 1:** Under Assumption 1, $A \succ 0$ and $Q \succeq 0$.

**Proof:** See the proof of [5, Lemma 1].

With Lemma 1, we can give the third form of $C$ as

$$C = \{(\zeta + A^{-1/2} \Upsilon Q^{1/2})^\top : \Upsilon^\top \Upsilon \preceq I_n\}. \tag{15}$$

The fact that the set $C$ in (14) rewrites equivalently as in (15) is straightforward for $A \succ 0$ and $Q \succeq 0$; it is less so for $A \succ 0$ and $Q \succeq 0$ and the proof for this case is in [5, Prop. 1]. The third form of $C$ in (15) is the one we typically need to obtain our main results.

In the next remark, we point out that the case of noise-free data yields a singleton for set $C$.

**Remark 2:** The case of noise-free data corresponds to $\Delta = 0$ in (9), so that $D = \{0 \in \mathbb{R}^{n \times T}\}$ and $C = \{[A B] : X_1 = [A B] \{X_0 \, X_0^\top\}^\top \}$ from (10). Since we have that $D_0 \in D$, the data generation mechanism reduces to $X_1 = A X_0 + B U_0 = [A_s B_s] \{X_0 \, X_0^\top\}$, and thus, $[A_s, B_s] \in C$. By Assumption 1, $\{X_0 \, X_0^\top\}$ has right inverse $\{X_0^\top \, X_0\}^{-1} \{X_0^\top \, X_0\}^{-1} = Z_0^\top$. This shows that (i) $[A_s, B_s] = Z_0^\top$, (ii) if $[A, B] \in C$, then $[A_s, B_s] = Z_0^\top$. (iii) and (ii), we conclude that under Assumption 1 and for noise-free data, $C = \{Z_0^\top\}$. As discussed in [5, Sec. 4.2], $Z_0 = -A^{-1}B$ is nothing but the least square estimate of the system dynamics.

### IV. SUFFICIENT CONDITION FOR GENERIC LMI REGIONS

In this section, we consider generic LMI regions and, for them, look for data-based counterparts of Facts 1 and 2, which will result in the sufficient conditions of Theorem 1 given next and of Corollary 1 given later.

**Theorem 1:** Let Assumption 1 hold and $S$ be an LMI region with data $(\alpha, \beta)$. Equation (11) is feasible if the next program is feasible

$$\begin{align*}
\text{find} \quad P & = P^\top > 0, \quad \text{Y}^\top \text{Y} \preceq I_n, \\
\text{s.t.} \quad & \begin{bmatrix}
\{\beta \beta^\top \} \otimes Q + \alpha \otimes P + \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ I_s \otimes (A^{-1/2}) (I_s \otimes \{\zeta \}^\top)
+ \lambda (\beta \otimes Q^{1/2}) (\beta^\top \otimes Q^{1/2}) < 0.
\end{bmatrix}
\end{align*} \tag{16}$$

If (16) is feasible, a controller gain in (11) is $K = Y P^{-1}$.

**Proof:** The proof shows how to encode the condition of Fact 1 for all matrices $[A B] \in C$. Apply Schur complement to (16b) and obtain equivalently, since $A_s \succ 0$ by Assumption 1,

$$\begin{align*}
\{\beta \beta^\top \} \otimes Q + \alpha \otimes P + \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ I_s \otimes (A^{-1/2}) (I_s \otimes \{\zeta \}^\top)
+ \lambda (\beta \otimes Q^{1/2}) (\beta^\top \otimes Q^{1/2}) < 0.
\end{align*} \tag{17}$$

We use the result known as Petersen’s lemma [28] in the version reported in [5, Fact 1]. The existence of $\lambda > 0$ such that (17) holds is equivalent by [5, Fact 1 to]

$$\begin{align*}
\alpha \otimes P + \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{(\beta \otimes Q^{1/2}) Y^\top (I_s \otimes (A^{-1/2} \{\zeta \}^\top))\}
< 0 \quad \forall Y : Y^\top Y \preceq I_{sn}.
\end{align*} \tag{18}$$

For $Y$ with $Y^\top Y \preceq I_n$ as in (15), consider the block-diagonal matrix

$$Y = I_s \otimes \text{Y}, \quad Y^\top = \begin{bmatrix}
I_n & 0 \\
0 & Y^\top Y
\end{bmatrix} \preceq \begin{bmatrix}
I_n & 0 \\
0 & I_n
\end{bmatrix} = I_{sn}, \quad \text{s.t. (18)}$$

implies for the block-diagonal $Y$ that

$$\begin{align*}
0 > \alpha \otimes P + \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{\beta \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{\beta \otimes \{A^{-1/2} \{\zeta \}^\top\}\}
\end{align*} \tag{19}$$

Equivalently, we have by (15) that

$$0 > \alpha \otimes P + \text{He} \{\beta \otimes \{A \mid B \} \{\zeta \}^\top\} \forall [A B] \in C. \tag{20}$$

In summary, feasibility of (16) implies feasibility of

$$\begin{align*}
\text{find} \quad P = P^\top > 0, Y \quad \text{subject to (20)}.
\end{align*}$$

Feasibility of this problem is equivalent to feasibility of (11) by the change of variables $Y = K P$.

The feasibility program in (16) is convenient since the constraint (16b) is a linear matrix inequality in the decision variables $P, Y$. The other quantities in (16b) are performance specifications given by $(\alpha, \beta)$ and data $U_0, X_0, X_1$, which yield $A, C, Q$ from (13c) and (14b). Theorem 1 enforces performance for all systems consistent with data in a set-membership sense; we regard the approach in Theorem 1 as direct because it returns an end-to-end condition for controller design from data. With respect to the model-based condition in Fact 1, Theorem 1 no longer gives a necessary and sufficient condition because, from (18) to (19) in the proof, we used that for matrices $D = D^1, E, G$

$$0 > D + EFG + G^T F^T E^T \forall F : F^T F \preceq I \tag{21a}$$

implies

$$0 > D + E (I \otimes f) G + G^T (I \otimes f^T) E^T \forall f : f^T f \preceq I \tag{21b}$$

but is not implied by it in general, see [4, Footnote 1]. The larger the number of blocks $f$ on the diagonal of the matrix $I \otimes f$ (of unit norm) is, the sparser that matrix is and the more conservative it is potentially to replace that matrix with the full matrix $F$ (of unit norm). In the case of Theorem 1, the number of blocks on the diagonal is the dimension $s$ of the LMI region; hence, the chances of feasibility of (16) decrease with $s$.

From Theorem 1, which is the data-based counterpart of Fact 1, we obtain the next data-based counterpart of Fact 2.

**Corollary 1:** Let Assumption 1 hold and $S$, for $i = 1, \ldots, r$, be an LMI region with data $(\alpha_i, \beta_i)$. (12) is feasible if the next program is feasible

$$\begin{align*}
\text{find} \quad P = P^\top > 0, Y_i \\
\text{s.t.} \quad & \begin{bmatrix}
\{\beta \beta^\top \} \otimes Q + \alpha_i \otimes P + \text{He} \{\beta_i \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ \text{He} \{\beta_i \otimes \{\zeta \otimes \{\zeta \}^\top\}\}
+ I_s \otimes (A^{-1/2} \{\zeta \}^\top)
+ \lambda (\beta \otimes Q^{1/2}) (\beta^\top \otimes Q^{1/2})
< 0
\end{bmatrix}
\end{align*} \tag{22a}$$

for $i = 1, \ldots, r$. (22b)

If (22) is feasible, a controller gain in (12) is $K = Y P^{-1}$.

**Proof:** Equation (12) is equivalent, by the definition in (3), to find $P = P^\top > 0, K$

$$\begin{align*}
\alpha_1 \otimes P + \text{He} \{\beta_1 \otimes ((A + BK)P)\} < 0 \quad \forall [A B] \in C
\end{align*} \tag{23a}$$

$$\begin{align*}
\vdots
\end{align*}$$

$$\begin{align*}
\alpha_r \otimes P + \text{He} \{\beta_r \otimes ((A + BK)P)\} < 0 \quad \forall [A B] \in C.
\end{align*} \tag{23b}$$
Equation (22) implies feasibility of this program by Theorem 1. □

V. NECESSARY AND SUFFICIENT CONDITION FOR SPECIAL LMI REGIONS

In Section IV, sufficient conditions for data-driven stabilization within generic LMI regions and their intersections have been presented. In this section, we show that necessary and sufficient conditions can be found for special LMI regions and their intersections. We will show that the latter are vertical halfplanes, disks centered on the real axis and intersections of such halfplanes and disks, and that these regions can inner-approximate subsets of the complex plane of practical interest.

In Section IV, the source of conservatism leading to sufficient conditions was substituting the block diagonal uncertainty $I \otimes f$ in (21b) with the full uncertainty $F$ in (21a). No conservatism is introduced in this step when we can actually transform $I \otimes f$ into $1 \otimes f = f$, and this occurs for $f$ of rank 1 as we now show. By considering (19) in the proof of Theorem 1 and setting $D := \alpha \otimes P + \text{He}(\beta \otimes (\zeta^\top \otimes [\frac{\cdot}{\cdot}]))$, we have

$$0 > D + \text{He}(\beta \otimes (Q^{1/2}Y^\top A^{-1/2} [\frac{\cdot}{\cdot}])) \quad \forall Y : Y^\top Y \leq I_n.$$ (23)

hence, if $\beta = \eta \gamma^\top$ for some vectors $\eta$ and $\gamma$ in $\mathbb{R}^n$, (23) is equivalent to having that for all $Y$ such that $Y^\top Y \leq I_n$.

$$0 > D + \text{He}(\{\eta \otimes Q^{1/2} (1 \otimes \gamma^\top) (\gamma^\top \otimes [\frac{\cdot}{\cdot}])) \quad \forall \eta : \eta^\top \eta \leq I_n.$$ (23)

where $1 \otimes \gamma^\top = \gamma^\top$ appears as intended to show. This discussion is summarized in the next assumption.

Assumption 2: Let $S$ be an LMI region with data $(\alpha, \beta)$. For $\beta \in \mathbb{R}^{n \times n}$, there exist $\eta$ and $\gamma \in \mathbb{R}^n$ such that $\beta = \eta \gamma^\top$, i.e.,

$$\begin{bmatrix} \eta_1 \gamma_1 & \cdots & \eta_n \gamma_n \\ \vdots & \ddots & \vdots \\ \eta_n \gamma_1 & \cdots & \eta_n \gamma_n \end{bmatrix}.$$ (24)

For an LMI region $S$ satisfying Assumption 2, the program (11) is reformulated equivalently, as in the next result.

Theorem 2: Let Assumption 1 hold and $S$ be an LMI region with data $(\alpha, \beta)$ satisfying Assumption 2, i.e., $\beta = \eta \gamma^\top$ for some $\eta$ and $\gamma$ in $\mathbb{R}^n$. Equation (11) is feasible if and only if the next program is feasible

$$P = P^\top > 0, Y_{11}$$ (26a)

s.t.

$$\begin{bmatrix} \{\eta \otimes I_n\}Q(\gamma^\top \otimes I_n) + \alpha \otimes P \\ + \text{He}(\{\eta \otimes I_n\} \gamma^\top \otimes [\frac{\cdot}{\cdot}]) \end{bmatrix} \prec 0.$$ (26b)

If (24) is feasible, a controller gain in (11) is $K = Y P^{-1}$.

Proof: By Schur complement and $A > 0$ by Assumption 1, (24b) is equivalent to

$$0 > \alpha \otimes P + \text{He}(\{\eta \otimes I_n\} \gamma^\top \otimes [\frac{\cdot}{\cdot}]) + \lambda Y P + Y Y^\top \text{Y}$$

$$\text{and} \lambda Y P + Y Y^\top \text{Y}$$

$$\text{if } (24) \text{ is feasible, a controller gain in (11) is } K = Y P^{-1}.$$ (27)

Motivated by Theorem 2 and Corollary 2, we examine the subsets of the complex plane to which LMI regions satisfying Assumption 2 give rise. A generic LMI region with $s = 1$ and data $(\alpha, \beta)$ has trivially $\beta$ of rank 1, and is $S = \{z = x + jy : C : \alpha_{11} + \beta_{11}(z + \bar{z}) = 0 \}$, which can express vertical halfplanes. For $s = 2$, the subsets of $C$ that can be expressed with $\beta$ of rank 1 are determined in the next lemma.

Lemma 2: Let $S$ be an LMI region with $s = 2$ and data $(\alpha, \beta)$ satisfying Assumption 2, i.e., $\beta = \eta \gamma^\top$ for some $\eta$ and $\gamma$ in $\mathbb{R}^2$. Then, a nontrivial $S$ (i.e., different from $0$ and $C$) can only be: a vertical strip, a vertical halfplane, a disk centered on the real axis or an intersection of the last two.

Proof: Specialize the expression in (6) of a generic LMI region with $s = 2$ for the $S$ considered in the statement with $\eta := [\eta_1]_2, \gamma := [\gamma_1]_2$, and $\beta := \eta \gamma^\top$. Then, $S$ rewrites after some algebraic computations as the set of points $x + jy \in C$ such that

$$0 > \alpha_{11} + 2 x(\eta_1 \gamma_1)$$ (27a)

$$0 < \alpha_{11} \alpha_{22} - \alpha_{12}^2 + 2 x(\alpha_{11} \eta_2 \gamma_2 + \alpha_{22} \eta_1 \gamma_1)$$

$$- \alpha_{12}(\eta_1 \gamma_2 + \eta_2 \gamma_1) - (x^2 + y^2)(\eta_1 \gamma_2 - \eta_2 \gamma_1)^2.$$ (27b)

If $\eta_1 \gamma_2 - \eta_2 \gamma_1 = 0$, $S$ is constituted from (27) by two linear inequalities in the real part $x$, hence a nontrivial $S$ can be a vertical halfplane or a vertical strip depending on the values of $\alpha, \gamma$. Otherwise, divide by $(\eta_1 \gamma_2 - \eta_2 \gamma_1)^2$ in (27b) and complete the square, which can be done for each $\alpha, \gamma, \eta$ with $\eta_1 \gamma_2 - \eta_2 \gamma_1 \neq 0$; after some computations, (27) rewrites equivalently as

$$0 > \alpha_{11} + 2 x(\eta_1 \gamma_1), \sigma \geq (x - x_0)^2 + y^2$$

$$x_0 := (\alpha_{11} \eta_2 \gamma_2 + \alpha_{22} \eta_1 \gamma_1 - \alpha_{12}(\eta_1 \gamma_2 + \eta_2 \gamma_1)) (\eta_1 \gamma_2 - \eta_2 \gamma_1)^2$$

$$\sigma := (\alpha_{11} \eta_2 \gamma_2 + \alpha_{22} \eta_1 \gamma_1 - \alpha_{12}(\eta_1 \gamma_2 + \eta_2 \gamma_1)) (\eta_1 \gamma_2 - \eta_2 \gamma_1)^2.$$ (27b)
If $\sigma$ is nonpositive, $S$ is an empty set. Otherwise, a nontrivial $S$ can be a disk centered on the real axis or an intersection of it with a halfplane.

Lemma 2 shows that (26) is a necessary and sufficient condition to solve (12) for arbitrary intersections of halfplanes left or right of any real part, and disks with any center on the real axis and any radius. When considering the regions expressed by $s = 1$ and $s = 2$, one can expect that increasing $s$ further yields more complex subsets of the complex plane, but finding their analytic expressions seems tractable (numerical investigations aside), so an extension of Lemma 2 for $s > 2$ is an open question. Although Assumption 2 limits expressivity in the case $s = 2$, we would like to note that subsets of the complex plane relevant for control purposes can be inner-approximated by intersection of halfplanes and disks. In Fig. 3, we show a way to inner-approximate the region $S(\ell, \rho, \theta)$ of Example 1, which is relevant for performance specifications, by an intersection of disks as detailed in [4, Example 5]. We can then impose eigenvalues within such an intersection of disks for all matrices consistent with data using the lossless Corollary 2. Similar considerations hold for discrete time, as in the next example.

Example 2: The discrete-time analogue of the continuous-time $S(\ell, \rho, \theta)$ is in Fig. 2, right, and it cannot be represented as an LMI region in general since LMI regions are convex [7, p. 359]. On the other hand, one can still give an inner-approximation through a disk, as we will do in Section VI-B.

VI. NUMERICAL INVESTIGATION

For continuous and discrete time, we illustrate our findings and compare their feasibility in this section.

A. Continuous Time

The following elements constitute our setting.

1) We consider the dynamical system

$$\dot{x} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.4 & 0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u + d$$

(28)

taken from [16, Sec. 9.5] and representing a digital tape transport. This model is used only to generate data points and simulate the closed-loop response since no data-driven design relies on knowing it.

2) We know that the disturbance satisfies the squared norm bound $|d|^2 \leq \epsilon$. This can be embedded in the disturbance model $D$ in (9) with $\Delta = \sqrt{\epsilon} T L$. We use $\epsilon = 2.5 \cdot 10^{-6}$, so that $|d| \leq \sqrt{\epsilon} = 1.58 \cdot 10^{-3}$.

3) The performance specification is given by a region $S(\ell, \rho, \theta)$ with parameters $\ell = 0.3$, $\rho = 2$, $\theta = \pi/5.7$ and the three $(\alpha, \beta)$ corresponding to this region are as in Example 1. As indicated in Fig. 3 and [4, Example 5], this region can be inner-approximated by the intersection of the disks $\{x + jy \in \mathbb{C} : x^2 + y^2 = \rho^2\}$ and $\{x + jy \in \mathbb{C} : (x - x_i)^2 + y^2 = (\sin \theta)^2 x_i^2\}$ with $x_i = -1.4992$ (computed numerically). The two $(\alpha, \beta)$ corresponding to these two disks are immediately determined based on [4, Table III].

4) A single experiment for data collection is performed on (28) under these conditions. The input is obtained by interpolating linearly a sequence that is a realization of a Gaussian variable with zero mean and unit variance. The disturbance is obtained in the same way from a realization of a random variable uniformly distributed in $|d| \leq 1.58 \cdot 10^{-3}$. These continuous-time signals are then sampled with $T_s = 0.1$ to obtain $T = 200$ data points of the matrices in (8).

For this outlined setting, we compare on the same dataset the designs proposed in the previous sections:

1) sufficient conditions in Corollary 1 for performance specification $S(\ell, \rho, \theta)$ and disturbance model $D$;
2) necessary and sufficient condition in Corollary 2 for the inner-approximation of $S(\ell, \rho, \theta)$ and $D$;
3) model-based condition in (5) for $S(\ell, \rho, \theta)$.

Note that the model-based condition would correspond to the ideal case of knowing $[A, B]$ exactly, which cannot be achieved by identification, because of noisy data. All controller designs are obtained by YALMIP [22] and MOSEK ApS in MATLAB R2019b.

The resulting controller designs in terms of eigenvalues are in Fig. 4. All methods manage to move the eigenvalues into the desired $S(\ell, \rho, \theta)$ or its inner-approximation, and the eigenvalue locations imposed by the different methods appear comparable. To appreciate differences, we show in Fig. 5 the time response of (28) with $d = 0$ in closed loop with a controller designed model-based or data-based with Corollary 1. The responses are consistent with the specification imposed by $S(\ell, \rho, \theta)$; e.g., the exhibited convergence rates of around 15 time units are consistent with $\ell = 0.3$. The model-based solution, which does not need to robustly stabilize a set of consistent matrices, shows on the other hand a smaller overshoot. To visualize robust stabilization, we sample 1000 realizations $\hat{Y}$ (corresponding to a real random matrix uniformly distributed in $\mathbb{R}^n$) and, from such $\hat{Y}$, obtain $[A B] \in \mathbb{C}^n$ as in (15); for each such $[A B]$, we depict the eigenvalues of $A$ (open-loop eigenvalues) and those of $A + BK$ (closed-loop eigenvalues) with $K$ designed by Corollary 1; the resulting eigenvalue clouds are in Fig. 6 and the closed-loop eigenvalues are within $S(\ell, \rho, \theta)$ for all realizations.
Next, we examine which of the conditions can withstand the largest disturbance bound $\epsilon$ and summarize the results in Table I for the specific system and performance specification. Although no conservatism is introduced by Corollary 2 for the special LMI regions satisfying Assumption 2, conservatism is introduced by inner-approximating the performance specification by special LMI regions, and this source of conservatism appears to be actually more significant.

### B. Discrete Time

The following elements constitute our setting, where we mention only those different than Section VI-A.

1. Similarly to [32, Sec. 6], we consider the dynamical system

   $$\dot{x} = (I - L) x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u + d, \quad L := \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

   (29)

   where $L$ is the Laplacian matrix of an underlying digraph.

2. We consider the same disturbance model, as in Section VI-A, now with $\epsilon = 1 \cdot 10^{-5}$ and $|d| \leq \sqrt{\epsilon} = 3.16 \cdot 10^{-3}$.

3. The performance specification is given by a disk with center $(0.04+0.2, 0)$ and radius $\sqrt{0.04+0.2}$, which is a disk contained in the performance region described in Example 2 and in Fig. 2, right.

   The $(\alpha, \beta)$ corresponding to this disk is immediately determined based on [4, Table III].

4. A single experiment for data collection is performed on (29) under these conditions and for $T = 200$. The input is a realization of a Gaussian variable with zero mean and unit variance and the disturbance is a realization of a random variable uniformly distributed in $|d| \leq 3.16 \cdot 10^{-3}$.

   For this setting, we consider the same designs as in Section VI-A and compare them for the disk giving the performance specification in discrete time. The resulting controller designs in terms of eigenvalues are in Fig. 7. All methods manage to move the eigenvalues into the desired disk, and the eigenvalue locations imposed by the different methods appear comparable. In Fig. 8, the time responses of (29) with $d = 0$ in closed loop with a controller designed model-based or data-based with Corollary 1 are consistent with the specification and show a smaller overshoot in the model-based case. We visualize robust stabilization, as described in Section VI-A; the resulting clouds of open-loop and closed-loop eigenvalues are in Fig. 9 and the latter are within the desired disk for all realizations.

Next, we examine different values of $\epsilon$ as in Section VI-A, and summarize the results in Table II. The conclusions are analogous to those in Section VI-A, except for the fact that we now apply Corollary 2 on the set corresponding to the performance specification instead of on an inner-approximation of it. In this case, Corollary 2 gives a necessary and sufficient condition unlike the sufficient conditions of Corollary 1, and is then able to withstand a larger $\epsilon$.
Fig. 9. Open-loop (red) and closed-loop (green) eigenvalues for 1000 realizations of $[A \ B] \in \mathcal{C}$, for the controller designed with Corollary 1.

TABLE II
FOR $\epsilon$, FEASIBILITY (✓) OR INFEASIBILITY (✗) OF METHODS

| $\epsilon$ | $1 \cdot 10^{-4}$ | $5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ |
|------------|------------------|------------------|-------------------|-----------------|
| Cor. 1     | ✓                | ✓                | ✓                 | ✓               |
| Cor. 2     | ✓                | ✓                | ✓                 | ✓               |

VII. CONCLUSION

The classical result [7] showed how to enforce performance specifications through eigenvalue assignment in LMI regions when a model of the plant to be controlled is known. We have extended this approach to the case where noisy data are used as a proxy for the unknown plant model. The proposed conditions guarantee to assign the closed-loop eigenvalues within LMI regions robustly for all plant matrices consistent with data. These conditions are LMIs that depend only on the performance specification given by the LMI region and on the collected data and, when feasible, they return as a solution both controller and Lyapunov-like function certifying the (robust) eigenvalue assignment in the LMI region. Besides providing sufficient conditions in terms of LMIs, we have determined when feasibility of these conditions actually becomes necessary and sufficient to solve the problem. Future work includes investigating whether these necessary and sufficient conditions may be extended to additional types of LMI regions, and proof-of-concept implementations on physical systems.

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