The drag-induced resonant capture for Kuiper Belt objects

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\textbf{ABSTRACT}

It has been an interesting question as to why one-third of Kuiper Belt objects (KBOs) are trapped into the 3 : 2 resonance but, in contrast, only a few KBOs are claimed to be associated with the 2 : 1 resonance. In a model proposed by Zhou et al., the stochastic outward migration of the Neptune could reduce the number of particles in the 2 : 1 resonance, and thus the objects in the 3 : 2 resonance become more distinct. As a complementary study, we investigate the effect of protostellar discs on the resonance capture. Our results show that the gaseous drag of a protostellar disc can trap KBOs into the 3 : 2 resonance rather easily. In addition, no objects are captured into the 2 : 1 resonance in our simulation.

\textbf{Key words:} stellar dynamics -- celestial mechanics -- Kuiper Belt -- Solar system: formation -- Solar system: general -- planetary systems.

\section{1 Introduction}

The general picture of the outer Solar system has been changed due to the discovery of Kuiper Belt objects (KBOs; Jewitt & Luu 1993; Williams et al. 1995). The origin of the dynamical properties of KBOs has become a very interesting but controversial issue. Particularly, it is surprising that one-third of the population are engaged into the 3 : 2 resonance with Neptune.

Malhotra (1995) proposed a mechanism of resonance sweeping to explain the large fraction of 3 : 2 resonance objects, plutinos, in the Kuiper Belt. In that model, as Neptune radially migrates outward, its mean motion resonance is swept ahead of it through the Kuiper Belt and captures Pluto, along with plutinos, into the 3 : 2 resonance. This theory predicted that the populations of the 3 : 2 and 2 : 1 resonances should be of the same order. Moreover, the radial migration of Neptune is assumed to be smooth and continuous. Zhou et al. (2002) improved this by introducing a stochastic term to make this outward migration more realistic. They showed that for particular chosen parameters, the number of particles in the 2 : 1 resonance can be reduced and thus the objects in the 3 : 2 resonance become more distinct.

Yeh & Jiang (2001) showed that the scattered planet shall move on an eccentric orbit and thus the pure radial migration is too naive, although it could be an approximation. For example, Neptune’s orbit is not circular during its outward migration, as shown in the simulations by Thommes, Duncan & Levison (1999). However, Neptune’s current orbit is nearly circular and thus one needs to introduce a massive Kuiper Belt to circularize it. Please also see the simulations in Gomes (2003).

In fact, the total mass of Kuiper Belt is itself a controversial issue. In order to form 100-km size KBOs around the current region, one would need a much more massive Kuiper Belt initially because the planetesimal accretion rate has to be high enough to form KBOs in time. However, the material in the Kuiper Belt has to be depleted significantly during the evolution because the observational upper limit of the mass at the Kuiper Belt is only 0.1 Earth mass. To avoid this formation problem, Levison & Morbidelli (2003) proposed a pushing-out model, in which both the KBOs and Neptune were forced to migrate outward from where they were formed initially. They showed that some KBOs were captured into 2 : 1 resonance and could move out together with Neptune in their simulations.

On the other hand, the traditional planet formation theory, i.e. core-accretion model, is facing the competition from the disc-instability model (Boss 1998). Because the disc-instability model can form a Jupiter-mass planet in a few thousand years, it could be that the formation of Neptune and KBOs is not that difficult as the traditional model suggests. Although the solid core formation through dust grain growth probably still plays an important role, the condition to initiate the formation of Neptune and KBOs and thus the formation timescales could be very different. Furthermore, Bryden, Lin & Ida (2000) showed that even by the core-accretion model, it is possible to form Neptune around the current location in $10^6$ or $10^7$ yr, which is comparable to or shorter than the disc depletion time-scale. In any case, the higher density gaseous blobs might shorten the dust-accretion process. Therefore, it is not clear how severe the initial mass problem is.

No matter what the exact formation scenario is, our model here could represent a process that starts either (i) at the time when the proto-Neptune and proto-KBOs were already pushed out after a violent migration and settle down around current places (if it is difficult to form at current locations) or (ii) after the Neptune and KBOs have grown and formed around current regions with only very gentle migrations.
We are particularly interested in the influence from the protostellar disc and thus the possible migration is ignored here. The model will be described briefly in Section 2 and the results are in Section 3. There are some discussions in Section 4. Section 5 concludes the paper.

2 THE MODEL

In our model, the test particle moves in a disc–star–planet system: both the star and planet are performing circular motions with fixed star–planet distance and a disc is added to provide additional forces on the test particle. As we only consider the two-dimensional coplanar orbits, the equations of motion are similar to the ones used in Jiang & Yeh (2004a), but we use a power-law density profile for the disc as in Jiang & Yeh (2004b).

Moreover, we also introduce the drag force as in Murray & Dermott (1999). We choose a formula that the drag force is proportional to the local disc density and also the difference between the velocity of the test particle and the local rotating velocity of the disc (assuming a Keplerian disc). That is, for a test particle located at \((\xi, \eta)\) with velocity \((d\xi/dt, d\eta/dt)\) in the inertial frame, the drag force per unit mass is

\[
F = -\alpha \Sigma(r) \left( \frac{d\xi}{dr} - v_\xi \right) \hat{\xi} - \alpha \Sigma(r) \left( \frac{d\eta}{dr} - v_\eta \right) \hat{\eta},
\]

where \((v_\xi, v_\eta)\) is the local rotating velocity of the disc and \(\Sigma(r)\) is the disc surface density, which is a function of radius only. The proportional constant \(\alpha\) is set to be 1/5.

The disc mass is assumed to be \(M_b = 0.01\). 900 test particles are randomly placed in a belt region \(1.1 \leq r \leq 2.0\) with uniform number distribution, where \(r\) is the radial coordinate. They are all performing circular motions initially. The simulation starts from \(t = 0\) and stop at \(t = t_{\text{end}} = 123\,200\pi \approx 3.8 \times 10^5\), which is about \(10^7\) yr in the real time-scale when the unit of mass is \(M_\odot\) and the unit of length is 30 au.

3 THE RESULTS

Figs 1 and 2 are the results of our simulation. In Fig. 1, crosses mark the initial locations of all test particles in the \(x-y\) plane, i.e. the orbital plane. We calculate the 3 : 2 resonant argument \(\phi\) (and also the 2 : 1 resonant argument \(\phi_1\)) for any \(t\) during the simulation for all particles. We determine both the maximum and the minimum of the 3 : 2 resonant argument \(\phi\), i.e. \(\phi_{\text{max}}\) and \(\phi_{\text{min}}\), respectively, for the final stage when \(t \in [t_f, t_{\text{end}}]\) (where \(t_f = 3.5 \times 10^5\)). If \(\phi_{\text{max}} - \phi_{\text{min}} < 180^\circ\), this particle is in 3 : 2 resonance for our simulation. We then use circles to mark the initial locations of test particles which are in the 3 : 2 resonance. These circles overlap some of the crosses. The same procedure is carried out for the 2 : 1 resonance, but we find that no particle is captured into the 2 : 1 resonance.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The initial locations of all test particles (crosses) and also the initial locations of those particles which are captured into 3 : 2 resonance (circles).
To investigate more details, Fig. 2 provides a combination of different aspects of the results. In Fig. 2(a), we show the positions of all particles on the $a-e$ plane when $t = t_{\text{end}}$. It appears that many orbital eccentricities of the particles are amplified to a larger value for those with $a < 1.35$. Particularly, the particles with $a = 1.33$ have the largest eccentricities among these particles. There is also a small number of particles with slightly larger eccentricities around $a = 1.6$. We find that, in fact, the region of the $3:2$ resonance is close to $a = 1.33$ and the region of the $2:1$ resonance is near $a = 1.6$. In addition to that, there are also concentrations of particles in the regions where $a < 1.33$. These regions are corresponding to the $7:5$, $4:3$, $5:4$ and $6:5$ resonances approximately. These particles could have been influenced by these resonances.

When we plot the positions of all test particles on the $x-y$ plane at $t = t_{\text{end}}$ as in Fig. 2(b), we find that, in deed, there is a concentration of particles at a particular radius about 1.33. Thus, it is likely that many particles are captured into the $3:2$ resonance. During our simulation, all the particles are forced to migrate inward due to the gas drag. For the particles with initial $a > 1.33$, they stop this inward migration at $a = 1.33$ due to the $3:2$ resonance capture. That explains why there are many particles at $a = 1.33$ with higher eccentricities at $t = t_{\text{end}}$. To understand more about this drag-induced inward migration and resonance capture, we make Fig. 2(c), in which we plot the number of particles captured into the resonance as a function of time. The number of particles in a particular resonance at $t_i$ is defined to be the total number of particles with the difference between the maximum and minimum resonance arguments less than $180^\circ$ during $t_{i-1} < t < t_i$. We set $t_0 = 0$, $t_i - t_{i-1} = 11 \times 200\pi$, where $i = 1, 2, \ldots, 11$. We did calculations for both $3:2$ and $2:1$ resonances. We find that the number of particles captured into the $3:2$ resonance keeps increasing and finally becomes a very large fraction of all particles as shown in Fig. 2(c). However, we still find that no particle gets captured into the $2:1$ resonance during the simulation. Fig. 2(d) shows the $3:2$ resonance argument as a function of time for a particle initially located at $(x, y) = (0.940, 1.047)$ marked in Fig. 2(b). It shows that this particle gets captured into the $3:2$ resonance after $t = t_2$.

4 DISCUSSIONS

Our results show that the effect of the drag force from the disc is important. As a comparison, let us discuss the situation when there is no disc. In this case, the test particles are influenced by the central star and Neptune only. Most particles will continue to perform circular motions, although the secular perturbation from the Neptune might affect them but this process is very slow. There are two kinds of particles which might change their orbits quickly: (i) those are very close to Neptune (they are influenced by Neptune quickly and might get scattered); and (ii) those in the resonant regions (their eccentricity might get increased; part of them might leave and some
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of them might stay in the resonant regions). There would be no obvious inward migrations for test particles. Thus, the probability for them to get captured into 3:2 resonance would be much smaller.

The speed of inward migration shall be proportional to the strength of the drag force. Thus, the stronger force could make more particles migrate and get captured by the 3:2 resonance given that there are enough particles in the outer part of the Kuiper Belt. Therefore, the results could be related to the initial distribution of the particles and also the strength of the drag force.

Because the protostellar disc definitely exists and provides the drag force when the proto-KBOs are forming, the mechanism of drag-induced resonant capture seems to be attractive and cannot be ignored. It explains the resonant KBOs in a natural way. The traditional mechanism of sweeping capture by migrating Neptune also works well as demonstrated by the models of stochastic migrations in Zhou et al. (2002). However, in that case, one would need a physical mechanism to explain Neptune’s outward migration. One also has to understand how the migration stops, and thus explain Neptune’s current orbital radius and how the orbit could become nearly circular finally.

The details of the new mechanism of drag induced resonant capture and its combination with the traditional sweeping capture by migrating Neptune would be an interesting future work.

5 CONCLUDING REMARKS

We have used a model of the disc–star–planet system to investigate the effect of a gaseous protostellar disc on the resonance capture. In addition to the force from the central star and planet, the test particle is also influenced by the gravitational and frictional forces from the disc. Particularly, we apply our model to the problem of resonant KBOs and focus on the 3:2 and 2:1 resonances.

Our results show that the drag force plays an important role for the resonant capture; many KBOs can get captured into the 3:2 resonance but not into the 2:1 resonance. This is consistent with the observation that about one-third of the whole population of KBOs are in the 3:2 resonance and only a few are claimed to be in the 2:1 resonance. Therefore, the mechanism we study here might be helpful to understand the history of the resonance capture of KBOs.

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