Practical Design for Multiple-Antenna Cognitive Radio Networks with Coexistence Constraint

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Abstract

In this paper we investigate the practical design for the multiple-antenna cognitive radio (CR) networks sharing the geographically used or unused spectrum. We consider a single cell network formed by the primary users (PU), which are half-duplex two-hop relay channels and the secondary users (SU) are single user additive white Gaussian noise channels. All transmitters and receivers are with multiple antennas. In addition, the coexistence constraint which requires PUs’ coding schemes and rates unchanged with the emergence of SU, should be satisfied. The contribution of this paper are twofold. First, we explicitly design the scheme to pair the SUs to the existing PUs in a single cell network. Second, we jointly design the nonlinear precoder, relay beamformer, and the transmitter and receiver beamformers to minimize the sum mean square error of the SU system. In the first part, we derive an approximate relation between the relay ratio, chordal distance and strengths of the vector channels, and the transmit powers. Based on this relation, we are able to solve the optimal pairing between SUs and PUs efficiently where the metric is the sum rate of SUs’ network. In the second part, considering the feasibility of implementation, we exploit the Tomlinson-Harashima precoding instead of the dirty paper coding to mitigate the interference at the SU receiver, which is the known side information at the SU transmitter. To complete the design, we first approximate the optimization problem as a convex one. Then we propose an iterative algorithm to solve it with CVX. This joint design exploits all the degrees of design freedom including the spatial diversity, power, and the side information of the interference at transmitter, which may result in better SU’s performance than those without the joint design. To the best of our knowledge, both the two parts have never been considered in the literature. Numerical results show that the proposed pairing scheme outperforms the greedy and random pairing with low complexity. Numerical
results also show that even if all the channel matrices are full rank, under which the simple zero forcing scheme is infeasible, the proposed scheme can still work well.

I. INTRODUCTION

Recently, many research efforts have been devoted to the studies on the so-called cognitive radio (CR) technology [1], which enables unlicensed (or secondary) users to dynamically sense and locate unused spectrum segments and to communicate via these unused spectrum segments. Most CR systems initially proposed in the literature, e.g., [2] and references therein, adopt the interference avoidance (underlay) based approach, which requires secondary users (SUs) to vacate the spectrum once the primary users (PUs) signals are sensed. The concept of interference avoidance based CR has been standardized, e.g., the IEEE 802.22 Wireless Regional Area Network (WRAN) standard [3] (802.22 or 802.22 WRAN herein). It is aimed at using the CR techniques to allow the sharing of the geographically unused spectrum which is originally allocated to the television multi-cast service, and do no harmful interference to the incumbent operation, i.e., digital TV and analog TV multi-casting. However, the interference avoidance based CR demands fast and accurate sensing of spectrum holes, or it can not operate in the region covered by PU [3]. Furthermore, it may not yield the most efficient spectrum utilization since only one user can access the specific frequency bands at any given time and/or geographical location.

In this paper we consider the practical design of the interference mitigating (overlay) [4] multiple-antenna CR systems [4], where the primary system includes several two-hop relay channels in a single cell. Contrary to the CR systems in [3] which can only use the geographically unused spectrum, here the considered CR systems can additionally access the frequency bands which are geographically used with the coexistence constraint. The detail will be explained in the next section. And the practical scenarios of the considered model may include the homogeneous and heterogeneous networks. In both kinds of networks, SUs help to relay PU’s signals to use PU’s spectrum as a tradeoff. Besides, an apparent benefit of SU’s relay is that when the channel between PU transmitter and receiver is poor or unstable, PU’s transmission can still be successful. On the other hand, several capacity results of the interference mitigation CR utilizing the dirty paper coding (DPC) are discussed in [5] [6] [7]. Although DPC is capacity achieving,
the complexity makes it infeasible for practical systems. Instead of using DPC, in this paper we design the practical interference mitigation based CR by exploiting the Tomlinson-Harashima precoding (THP) \[8\] \[9\]. Note that DPC capacity can be achieved by the nested lattice coding \[10\]. And that means the binning operation in the random coding scheme of DPC can be accomplished by the vector quantization. On the other hand, THP uses a scalar quantization and thus can be treated as a special case of DPC. Due to its simplicity, THP is widely used in channels where the side information is known at the transmitter \[10\]–\[13\], albeit the modulo loss, the shaping loss, and the power loss \[14\] relative to DPC. On the other hand, spatial diversity is also exploited to avoid the interference between CR and PR’s transmission by designing linear precoders and/or receiver beamformers \[15\] \[16\] \[17\]. Also, the relay assisted transmission is considered in CR design \[18\] \[19\].

Different from the above, we consider a much more complete and complicated model: several SU transmitter and receiver pairs coexist with the PU network where all PUs are assumed to be two-hop relay channels. All nodes are assumed to have multiple antennas. Before the SUs can simultaneously transmit with PUs in the same time-frequency slot, each SU should be paired to only one PU, vice versa, to form an interference mitigation cognitive channel. After that, the transmitters and receivers in each single cognitive channel should be designed. Upon solving these two problems, the main contributions of this paper are listed in the following.

1. We derive an approximate representation of the relay ratio in terms of the transmit powers of PU and SU networks and the chordal distance and the strengths of the vector channels. Based on this representation, we are able to design an explicit scheme to pair the SUs to the existing PUs in a single cell network with low complexity, while the sum rate of SUs’ network is maximized. To the best of our knowledge, such pairing design for the overlay CR with coexistence constraint has never been considered in the literature. Numerical results show that the proposed pairing outperforms the greedy algorithm and random pairing with low complexity.

2. We design the SU transmitter and receiver in each cognitive channel by exploiting all the degrees of design freedom such as the side information at transmitter, the spatial diversity, and the transmit power
to jointly design the non-linear precoder, transmit and receive beamformer, and the relay matrix, for
the interference mitigation based CR with coexistence constraint. To the best of our knowledge, such
joint design has never been considered in the literature. To attain this goal, we first form an optimization
problem, where the objective is to minimize the sum mean square error (SMSE) of the CR system, subject
to the coexistence and power constraints. However, since the problem is non-convex, we approximate it and
propose an iterative algorithm to solve it by the CVX [20]. Numerical results show that the performance
of the proposed joint design outperforms that with the precoder designed by the generalized zero forcing,
not to mention the traditional zero forcing. Especially when the channels are full rank, there is no null
space can be used whence the traditional zero forcing scheme can not be applied.

The rest of the paper is organized as follows. In Section II we introduce the considered system model.
In Section III we solve the pairing problem between the SU and the PU’s networks. The practical design
problem of the SU transceiver is discussed in Section IV. In Section V we illustrate the numerical results.
Finally, Section VI concludes this paper.

II. SYSTEM MODEL

The considered cognitive channel is shown in Fig. 1 In this model, all the transmitters and receivers are
with multiple antennas. Assume that the primary system is a half-duplex two-hop relay channel, i.e., in
the odd time slots the base station transmits to the PU-TX, and in the even time slots the PU-TX relays to
the PU-RX. This kind of network plays an important role in reducing the transmit power and/or extending
the coverage of communication networks, which is considered in several wireless standards, e.g., [21],
etc. Contrary to the CR systems in [3] which can only use the geographically unused frequency bands,
here the considered CR systems can access the frequency bands both geographically used or unused with
the coexistence constraint, which is defined as

*Definition 1: Coexistence constraint* [5]: Under CR’s own transmission, there exists a sequence of
$(2^{nR_1}, n)$-codes that can be decoded with a single-user decoder at the primary receiver with vanishing
error probability as $n \to \infty$. 

Thus such coexisted secondary systems can increase the spectral efficiency in a predefined time/frequency/geo-location slot of many of the state-of-the-art wireless communication standards/technologies and the design of such secondary systems is worth investigating. The way to attain the coexistence is explained in the following. SU-TX can overhear the signal transmitted by the base station due to the wireless characteristic. As long as the channel between the base station to the SU-TX is no worse than that between the base station and PU-TX, this signal can be decoded successfully at the SU-TX prior to the transmission of PR-TX. Thus this signal can be treated as a non-causally known side information at SU-TX. The importance of the non-causality are twofold. First, SU-TX can use this non-causally known signal to do the decode and forward relay to maintain the SINR at PU-RX, while SU-TX is simultaneous transmitting his own signal by the same spectrum geographically used by PU. Second, with the knowledge of the non-causal side information, SU-TX can use the advanced coding schemes such as DPC [5][22][23] when there is full channel state information at the transmitter (CSIT) or Gelfand-Pinsker coding [24][25] for the partial CSIT [23] case. This kind of CR is coined by the interference mitigation CR [4], with the most commonly used interference avoidance CR, are rigorously defined in the following

**Definition 2: Interference mitigation based CR [4]:** With the non-causal knowledge of PU, the CR can simultaneously operate in the spectrum occupied by PU without violating the coexistence constraint.

**Definition 3: Interference avoidance based CR [4]:** CR systems are active only when the PU is silent.

Although DPC is capacity achieving for the interference mitigation based CR channel [5], the complexity [26][27] prohibits its use in practical systems widespreadly. Considering the implemental feasibility, in this paper we design the interference mitigation CR by exploiting the Tomlinson-Harashima precoding (THP) [8][9] instead of DPC. In this paper we jointly design the nonlinear precoder, the transmitter and receiver beamformers, and the relay matrix to implement the interference mitigation CR. Note that such joint design exploits all the resources of side information at transmitter, spatial diversity, and power.
The received signals at SU and PU’s receivers can be respectively represented as:

\[
y_S = H_{23} x_S + H_{13} x_P + n_S, \\
y_P = H_{14} x_P + H_{24} x_S + n_P,
\]

where \( x_S \in \mathbb{C}^{N_T} \) is SU’s transmit signal, \( x_P \in \mathbb{C}^{N_T}(0, \Sigma_{x_P}) \) is PU’s transmit signal where \( \Sigma_{x_P} \in \mathbb{C}^{N_T \times N_T} \); \( H_{23} \) and \( H_{14} \) are the channels between the transmitters of the SU and PU to their designated receiver, respectively; \( H_{24} \) and \( H_{13} \) are the channels between SU’s transmitter to PU’s receiver and PU’s transmitter to SU’s receiver, respectively; \( n_S \sim \mathcal{CN}(0, I) \) and \( n_P \sim \mathcal{CN}(0, I) \) are the circularly symmetric complex additive white Gaussian noises at receivers of SU and PU’s networks, respectively. All channel gains in this paper are assumed to be static. We also assume that all receivers perfectly know their channel gains respectively, which can be easily attained by the training process. There are average power constraints for

\[
(x)^+ = \max\{0, x\}.
\]
both SU and PU’s networks respectively

\[
\text{tr}(E[x_Sx_S^H]) \leq P_T, \quad \text{and} \quad \text{tr}(E[x_Px_P^H]) \leq P_P. \tag{3}
\]

The detailed functions of the secondary transmitter and receiver in Fig. 1 are shown in the upper part of Fig. 2. The channel input and output of the secondary systems can be respectively represented by

\[
x_S = Fu + Ax_P,
\]

\[
y_S = H_{23}x_S + H_{13}x_P + n_S = H_{23}Fu + s + n_S, \tag{4}
\]

where \(F\) is the transmitter beamformer at SU-TX to transmit his own signal \(u\) and \(A\) is the relay matrix for relaying PU’s signal \(x_P\).

III. THE PAIRING PROBLEM IN A MULTIUSER NETWORK

To accommodate the commercial systems, we assume that the PU-TXs are served by the base station with the orthogonal multiple access schemes such as the TDMA, FDMA, CDMA, SDMA, etc. We also assume that the transmissions among different PU-TX and PU-RX pairs use the same orthogonal multiple access scheme. And we assume the transmissions of the BS to PU-TXs and also the PU-TXs to PU-RXs are in different time slots. To consider the performance of SUs in a single cell such as the total
throughput of the cell or the quality of services (QoS)\footnote{The reason to consider only the QoS of the secondary users is that the primary users’s rates are not changed due to the coexistence constraint}, where several SUs coexist with PUs, to design a scheme properly pairing SUs to PUs is very critical to the system performance of SUs. For example, an improper pairing may cause the SU-TX to waste large portion of his power to relay PU’s signal and results in a low SU’s rate. In this section we first derive the approximate relay ratio in terms of the channel information and the transmit power, which is critical to the objective to be optimized. Then we formulate an integer programming problem which describes the considered pairing problem. After relaxing we propose an algorithm to numerically solve it. Note that in calculating the objective of the pairing problem, the proposed pairing scheme does not need to know all system parameters which should be designed as shown in Sec. IV. Thus the design process is highly simplified.

A. The metric of pairing

In this section we will first give some observations from the SISO case. After that we will extend this relation to the MIMO case.

1) The observation from the SISO case: Recall that the coexistence constraint of the SISO case\cite{23} can be described by

\[
\frac{|h_{14}xp + h_{24}\sqrt{\frac{\alpha P_x}{P_T}}xp|^2}{1 + |h_{24}|^2(1 - \alpha)P_T} = |h_{14}|^2P_P, \tag{5}
\]

where the left and right hand side are the signal to interference and noise ratio under SU is active and inactive, respectively. Note that the noise variances at both PU and SU RXs are set as unity without loss of generality. Also note that Gaussian signaling is used for PU and SU. Then with the change of variables

\[
P_1 \triangleq |h_{14}|^2P_P, \quad P_2 \triangleq |h_{24}|^2P_T, \quad P_3 \triangleq |h_{23}|^2P_T,
\]

and some arrangements, \(\alpha\) can be solved as

\[
\alpha = \frac{P_1}{P_2} \left( \frac{-1 + \sqrt{1 + P_2 + P_1P_2}}{1 + P_1} \right)^2. \tag{6}
\]

In the following we show how \(h_{14}, h_{24}, P_P\), and \(P_T\) affect SU’s rate through \(\alpha\). In Fig. 3 we first show SU’s rate versus \(P_1\) with different \(P_2\)’s and fixed \(P_T\), i.e., the increasing of \(P_2\) is due to the increasing of \(|h_{24}|\). It can be seen that SU’s rate decreases with increasing \(|h_{24}|\) and also with increasing \(P_1\). In the same
Fig. 3. Cases of SU’s rate versus $P_1$ when $P_T$ is fixed and when $h_{24}$ is fixed.

In the MIMO case, there is no simple trichotomy law between two channels like that in the SISO case. So before solving the pairing problem, first we need to extract the useful properties in vector case from the matrices $H_{14}$ and $H_{24}$. An intuitive way is to consider the spatial correlation of the two subspaces represented by $H_{14}$ and $H_{24}$, which are the only two channels affecting the relay ratio as (6). For example, the principal angle [28] and the chordal distance [29].

**Definition 4:** (Principal Angle): For any two nonzero subspaces $V, W \subseteq \mathbb{C}^n$, the principal angles between $V$ and $W$ are recursively defined to be the numbers $0 \leq \theta_i \leq \pi/2$ such that

$$
\cos \theta_i = \max_{\mathbf{v} \in V_i, \mathbf{w} \in W_i, \|\mathbf{v}\|_2 = \|\mathbf{w}\|_2 = 1} \mathbf{v}^H \mathbf{w}, \ i = 1, \cdots, \min\{\dim(V), \dim(W)\},
$$

where $\theta_i$ are the vectors that construct the $i$th principal angle $\theta_i$, $\|\mathbf{v}_i\|_2 = \|\mathbf{w}_i\|_2 = 1$, $V_i = V_{i-1} \cap \mathbf{v}_{i-1}^\perp$ and $W_i = W_{i-1} \cap \mathbf{w}_{i-1}^\perp$. Furthermore, $\theta_{\min} = \theta_1 \leq \cdots \leq \theta_{\max}$. 
Definition 5: (Chordal distance): The chordal distance between $V$ and $W$ with dimensions $p$ and $q$, respectively, is defined in terms of the principal angles $\theta_i$ as

$$d_c(U,V) = \sqrt{\sum_{i=1}^{\min(p,q)} \sin^2 \theta_i}. \quad (8)$$

In the following we investigate how the spatial correlation between the two channels $H_{14}$ and $H_{24}$ and also the transmit powers $P_P$ and $P_T$ affect the relay ratio. The proof is given in Sec. VII. Later we use this relation to formulate the pairing problem.

Theorem 1: To satisfy the coexistence constraint, the relay ratio and other system parameters approximately follow

$$\alpha_d = \left( \frac{d_{24,min}^2 \text{tr}(D_{14}^2) - \frac{1}{P_T}}{d_{24,max}^2 \left( \frac{1}{\lambda_{max}(\Sigma_{14})} (d_{24}^2 \text{tr}(D_{14}^2) + N_T) + d_{24,min}^2 \text{tr}(D_{14}^2) \right)} \right)^+ \quad (9)$$

where $d_c$ is the chordal distance between the eigen-spaces of $H_{14}$ and $H_{24}$.

Remark 1: Note that for the tractability, we assume the signal distribution at SU-TX is Gaussian, which will be explained in more detailed in Remark 4.

Remark 2: From Theorem 1 we can find that the chordal distance, the eigenvalue spread of $H_{24}$, the strength of the subspaces $\text{tr}(D_{14}^2)$, and also the transmit power are used to characterize the relation with $\alpha$ for satisfying the coexistence constraint. The interplay between these parameters are discussed in the following. From (9) it is clear that to make PU’s rate fixed after SU is active, the relay ratio is approximately inversely proportional to the square of chordal distance. This result is intuitively reasonable, i.e., when the distance between the two subspaces is larger, SU can use less relaying power to keep PU’s rate unchanged. As an extreme example, when $H_{24}$ is orthogonal to $H_{14}$, SU-TX does not need to relay since no SU’s signal will be received by PU-RX under the assumption that PU-RX uses the left singular vectors of $H_{14}$ as the beamformer. Note that when $d_{24,min} \to 0$, $\alpha \to 0$. This is reasonable since there may exist directions causing much less interferences to PU’s RX due to the small eigenvalues, and SU’s TX can transmit his own signal in such directions to reduce the relay ratio, i.e., the power for transmitting his own signal can be increased. In contrast, when $d_{24,max}^2$ is small, there may be no directions with relatively
small enough eigenvalues for SU’s own transmission to reduce the interference to PU’s RX. And thus SU wastes more power on relaying, i.e., $\alpha_d$ increases. Note also that PU’s transmit power has the same effect to $\alpha_d$ as that in the SISO case. It can easily be seen that $\alpha_d$ is proportional to $\lambda_{\text{max}}(\Sigma X_p)$, which is consistent to the SISO case by the fact that $\lambda_{\text{max}}(\Sigma X_p)$ may increase with increasing $P_p$.

B. The pairing problem formulation and solving

Based on the relation in Theorem 1, we can form the following optimization problem with only the knowledge of the channel matrices and the transmit power, but no need to know other system parameters such as the precoder, transmit and receive beamformer, etc., which can not be attained before the pairing. Note that we use $(i, j)$ to denote the $PU-SU$ pair, i.e., the $i$th SU transceiver is paired to the $j$th PU transceiver, and $\alpha_{a_{i,j}}$ is the approximate relay ratio of the pair $(i, j)$. To simplify the notation, we use $H_{23,i}$, $F_i$, and $W_i$ to respectively denote the channel between the SU-TX and SU-RX, the transmit and receive beamformer, where the PU-SU pair is formed by the $i$th SU transceiver.

Corollary 1: The maximization of the sum rate of the SU’s network after pairing can be approximated by

$$\max_{\{i,j\}} \sum_{i=1}^{M} \sum_{k=1}^{N_T} \log (1 + (1 - \alpha_{a_{i,j}}) P_T \lambda_{23,ik}^2),$$

where there are $M$ SU and PU transceivers, respectively, and each node has $N_T$ antennas.

Proof: We may represent the sum rate of the SU’s network as

$$\sum_{i=1}^{M} \log |I + W_i H_{23,i} F_i F_i^H H_{23,i}^H W_i^H| \overset{(a)}{=} \sum_{i=1}^{M} \log |I + (1 - \alpha_i) P_T W_i H_{23,i} F_i F_i^H H_{23,i}^H W_i^H| \overset{(b)}{=} \sum_{i=1}^{M} \log(|I + (1 - \alpha_i) P_T \Lambda_{23,i}^2|) \overset{(c)}{=} \sum_{i=1}^{M} \sum_{k=1}^{N_T} \log (1 + (1 - \alpha_{a_{i,j}}) P_T \lambda_{23,ik}^2),$$

where in (a) we normalize $F_k$ such that $\text{tr}(F_k F_k^H) = 1$; in (b) we assume $W_i = U_{23,i}, F_i = V_{23,i}/\sqrt{N_T}$, where $U_{23,i}$ and $V_{23,i}$ are the left and right singular vectors of $H_{23,i}$, i.e., $H_{23,i} = U_{23,i} \Lambda_{23,i} V_{23,i}^H$. This selection of $W_i$ and $F_i$ is due to the fact that before the pairing operation, we do not know the exact $\{H_{13,i}\}$, $\{H_{23,i}\}$, $\{H_{14,i}\}$, and $\{H_{24,i}\}$. Thus we can not attain $F$ and $W$ by solving the optimization problem $P1$.
in Sec. [IV] And here we use this simple but reasonable choice to make the following derivation tractable; in (c) Theorem 1 is used.

Now we can reform (11) as an explicit optimization problem

$$\max_{\{t_{ij}\}} \sum_{i=1}^{M} \sum_{j=1}^{M} t_{ij} \sum_{k=1}^{N_T} \log (1 + (1 - \alpha_{a,ij}) P_i \lambda_{23,ijk}^2) \quad \text{s.t.} \quad \sum_{j=1}^{M} t_{ij} = 1, \forall j; \sum_{i=1}^{M} t_{ij} = 1, \forall i; \ t_{ij} \in \{0, 1\}, \forall i, j,$$

where the indicator \(t_{ij}\) is 1 if the \(i\)th SU is paired with the \(j\)th PU. And the first and second constraints state that each PU is only paired by one SU, and each SU is only paired by one PU, respectively. Note that this problem is an integer programming one, which is computationally prohibited especially when \(M\) is large. Thus we relax the third constraint as \(t_{ij} \geq 0, \forall i, j\). In the following we resort to the dual method to solve the relaxed problem. By dualizing the first constraint, we have the Lagrangian as

$$L(T, \mu) = \sum_{i=1}^{M} \sum_{j=1}^{M} t_{ij} \sum_{k=1}^{N_T} \log (1 + (1 - \alpha_{a,ij}) P_i \lambda_{23,ijk}^2) + \sum_{i=1}^{M} \mu_i \left(1 - \sum_{j=1}^{M} t_{ij}\right) \triangleq \sum_{i=1}^{M} \sum_{j=1}^{M} t_{ij} (R_{ij} - \mu_j) + \sum_{j=1}^{M} \mu_j,$$

where \(R_{ij} \triangleq \sum_{k=1}^{N_T} \log (1 + (1 - \alpha_{a,ij}) P_i \lambda_{23,ijk}^2)\) and \(\mu \triangleq [\mu_1, \mu_2, \cdots, \mu_M] \in \mathbb{R}^M\) are the dual variables and the \(i\)th row \(j\)th column of the indicating matrix \(T\) is \(t_{ij}\). And the dual objective function is

$$g(\mu) = \max_{T} L(T, \mu), \ s.t. \ \sum_{j=1}^{M} t_{ij} = 1, \forall i; \ t_{ij} \geq 0, \forall i, j,$$

and the dual problem is \(\min_{\mu} g(\mu)\). Since \(R_{ij}\) is independent of \(T\), we can solve the optimal \(T\) for (13) as

$$t_{ij}^{opt} = \begin{cases} 1, & j^* = \arg \max_{j=1, \ldots, M} (R_{ij} - \mu_j) \\ 0, & \text{otherwise}, \end{cases}$$

(14)

for \(i = 1, \cdots, M\). In the final step we use the subgradient method [30] to solve \(\mu\)

$$\mu_{j}^{(l+1)} = \mu_{j}^{(l)} - \gamma_{j}^{(l)} \left(1 - \sum_{i=1}^{M} t_{ij}\right), \ j = 1, \cdots, M,$$

(15)

where the superscript \((l)\) denotes the number of iteration and \(\{\gamma_{j}^{(l)}\}\) is the sequence of step sizes which should be designed properly. With the new \(\mu\) in each iteration, the subcarrier pairing can be updated from (14). Note that (15) will converge to the dual optimum variables [30]. The algorithm in Table II summarized the above steps of solving \(T\) and \(\mu\), where \(S_j\) is the set of indices of non-zero entries in the \(j\)th column of \(T\) and \(Z\) is the set of indices of columns with all zero entries of \(T\). Note that from line
TABLE I
The Proposed Algorithm for the Optimal Pairing of the PU and SU Networks

1: Initialize $\mu^{(0)}$
2. Repeat
3: Compute $\bigg\{ \frac{R_{ij}^{(l)}}{\gamma} = \sum_{k=1}^{N_T} \log \left( 1 + \frac{1}{\alpha_{a_{i,j}}^{(l)}} \right) P_{ij} \lambda_{23,ik}^2 \bigg\}$
4: Compute $\bigg\{ t_{ij}^{opt,(l)}(l) = 1, j^* = \arg \max_{j=1,...,M} (R_{ij} - \mu_{j}^{(l)}); t_{ij}^{opt,(l)} = 0, \text{otherwise} \bigg\}$, for $i = 1, \cdots, M$
5: Compute $\mu_{j}^{(l+1)} = \mu_{j}^{(l)} - \gamma^{(l)} \left( 1 - \sum_{i=1}^{M} t_{ij} \right)$, for $j = 1, \cdots, M$
6: For $j = 1 \sim M$
7: $i^* = \arg \max_{i \in S_j} (R_{ij} - \mu_{j})$, $S_j = S_j \setminus i^*$
8: While $|S_j| \geq 1$
9: $i^* = \arg \max_{i \in S_j} (R_{ij} - \mu_{j})$, $k^* = \arg \min_{k \in Z} |\mu_k - \mu_{j}|$,
10: swap the entries $(i^*, j)$ and $(i^*, k^*)$
11: $S_j = S_j \setminus i^*$, $Z = Z \setminus k^*$
12: End
13: End
14: Until $||\mu^{(l+1)} - \mu^{(l)}|| / ||\mu^{(l+1)}|| < \varepsilon$
15: Return $\{ t_{ij}^{opt} \}$

6 to 13 we try solve the problem that some columns may be allocated with more than one 1 by moving the additional 1s to the all zero columns with least reduction of the objective.

Remark 3: The complexity of brute force pairing is $O(M!)$. In contrast, the proposed scheme highly reduces the complexity to $O(M^2)$ for calculating $R_{ij}$ in each iteration. Assume $T$ iteration is required for the algorithm to converge, the total complexity is $O(TM^2)$, which is much more tractable and feasible, especially when $M$ is large.

IV. Practical Design of the Secondary Systems

After the pairing process discussed in the previous section, each SU transceiver knows the PU he should coexist with and also all the channels in Fig. 1. In this section we discuss our proposed scheme for solving the relay matrix, the non-linear precoder, the transmitter and receiver beamformer. Considering the feasibility of implementation, we adopt the THP as the precoder to mitigate the interference at the SU receiver instead of the dirty paper coding. The THP output for the $k$-th antenna can be represented by

$$u_k = \left( v_k + d_k - \sum_{j=1}^{N_T} w_{kj} s_j - \sum_{j=1}^{k-1} b_{kj} u_j \right) \mod A, \quad k = 1 \sim N_T,$$ (16)
where we assume \( v_k, k = 1 \sim N_T \) are mutually independent and selected from an \( M \)-QAM constellation; the \( k \)th element of \( s \) is denoted by \( s_k \) and \( s = (H_{13} + H_{23}A)x_p \) is the non-causally known side information at SU-TX and \( A \) is the relay matrix; the random dither \( d_k \in \mathbb{C} \) is at the \( k \)-th antenna which is known by both the transmitter and the receiver before the transmission, where \( \text{Re}\{d_k\} \sim \text{unif}(\frac{-A}{2}, \frac{A}{2}) \) and \( \text{Im}\{d_k\} \sim \text{unif}(\frac{-A}{2}, \frac{A}{2}) \). Note that due to the insertion of the dither, the elements of \( u \) are uncorrelated, i.e., \( E[uu^H] = I \), and are uniformly distributed over the Voronoi region [31] which is proved as the Crypto Lemma [10].

The modulo operation is as following. For a vector \( g \), the mod \( A \) operation \( g \mod A = g' \) is applied element-wise to each element \( g_i \) of \( g \) such that \( g'_i = g_i - Q_A(g_i) \), \( \forall i \), where \( Q_A(g_i) \) is the nearest multiple of \( A \) to \( g_i \). And the Voronoi region of \( A \) is set as \([−A/2, A/2)\]. Note that \( B \) is a strict lower triangular matrix, i.e., \( b_{kj} = 0, \forall k \leq j \), to perform the successive interference cancelation, inherited from its dual operation, i.e., the decision feedback equalizer at the receiver. Without loss of generality, we can stack \( u_1, \cdots, u_{N_T} \) in (16) in the vector form

\[
u = (v + d - Ws - Bu) \mod A = (I + B)^{-1}(v + j + d - Ws) \triangleq C^{-1}(v_2 + d - Ws),
\]

where the second equality is from [31] and \( v_2 \triangleq v + j, j \) is the modulo indices. With \( j \), we may represent the considered model as an equivalent one without the modulo at transmitter, as shown in the lower part of Fig. 2. In addition, assume a moderate or high \( M \) is used, then the power loss due to the modulo operation can be neglected. Then the estimated signal after the dither removal at receiver can be represented by

\[
\hat{v}_2 = Wy_S - d = W(H_{23}Fu + s + n) - d.
\]

From (17) and (18), the estimated error can be described by

\[
e \triangleq \hat{v}_2 - v_2 = W(H_{23}Fu + s + n) - d - Cu - Ws + d = (WH_{23}F - C)u + Wn.
\]

Then the sum mean square error (SMSE) among all antennas is

\[
\text{tr}(E) = \text{tr}(E[ee^H]) = \text{tr}((WH_{23}F - C)(WH_{23}F - C)^H + WW^H) = ||m||_2^2,
\]

where in the last equality we define \( m \triangleq (\text{vec}(WH_{23}F - C)^T, \text{vec}(W)^T)^T \).

Here we use the SMSE in (20) as the performance metric, instead of the sum rate of SUs as in Sec. III. In fact, maximizing SMSE is closely related to maximizing the sum rate as following shows, while the
later may be intractable. Note that the error covariance matrices of the minimum MSE-decision feedback equalization (MMSE-DFE) and MMSE-THP are the same [32], under the assumption that the former has correct previous decisions while the latter has negligible precoding loss and valid linear model [12]. In addition, we know that for an MMSE-DFE system,

$$\max \sum_{k=1}^{N_T} \log(1 + \text{SINR}_k) = \max - \log \prod_{k=1}^{N_T} \text{MSE}_k \geq -N_T \log (\min \text{tr}(E)/N_T) ,$$

where the equality is from $\text{SINR}_k = 1/\text{MSE}_k - 1$, $k = 1, \ldots, N_T$ [32] and the inequality comes from arithmetic and geometric means and also (20). From the above, minimizing the sum MSE maximizes the lower bound of the sum rate of the SU’s networks. Therefore we may claim that these two metrics are consistent even in $P_0$ we have additional variable $A$ and constraints compared to the traditional MMSE-THP problem.

Then we can form the optimization problem as

$$\textbf{P0} : \min_{A,B,F,W} \|m\|_2^2$$

s.t. $\log \frac{|I + H_{24}FF^HH_{24}^H + (H_{14} + H_{24}A)[\Sigma_{xp}(H_{14} + H_{24}A)]^H|}{|I + H_{24}FF^HH_{24}^H|} \geq \log(|I + H_{14}[\Sigma_{xp}H_{14}^H]|),$  

$$\text{tr}(FF^H) + \text{tr}(A\Sigma_{xp}A^H) \leq P_T,$$

$$B$$ is a strictly lower triangular matrix,  

where (22) is the coexistence constraint to guarantee that PU’s rate is not decreased when the SU is active and the left hand side (LHS) is the rate of PU when the SU is active; (23) is the total power constraint at the SU transmitter, which is the sum power of SU’s own signal $\text{tr}(FF^H)$ and that of the relay signal $\text{tr}(A\Sigma_{xp}A^H)$. By eigenvalue decomposition we have $\Sigma_{xp} = V\Lambda_{xp}V^H$, where $V$ is selected as the right singular vector of $H_{14}$, i.e., $H_{14} = USV^H$ by singular value decomposition; $\Lambda_{xp} = \text{diag}\{P_1, P_2, \ldots, P_{N_T}\}$ where $P_i = \left(\mu - \frac{1}{\lambda_i}\right)^+$ is the water-filling result according to $H_{14}$, $\lambda_i$ is the $i$th singular value of $H_{14}$ and $\mu$ is chosen such that $\Sigma_{i=1}^{N_T}P_i = P_p$.

Remark 4: Note that the output of THP is uniformly distributed per two-dimension but not Gaussian. So the distribution of the received signal at PU-RX is the convolution of uniform and Gaussian distributions. And thus the PU’s achievable rate is not easy to derive compared to the one with Gaussian input. For
the tractability, we lower bound PU’s rate when SU is active by the concept that Gaussian noise is the worst noise when the signal is Gaussian [33], i.e., treating SU’s signal as Gaussian distributed results in the lowest PU’s rate. Therefore solving $A, B, F,$ and $W$ with this lower bounded constraint is more conservative and is thus feasible for the original problem.

By epigraph and Schur’s Complement Theorem [34], we can transform $P_0$ as

$$
P_1: \min_{A, B, F, W, t_0} t_0, \text{ s.t. (22), (23), (24), } \begin{pmatrix} t_0 & m^H \\ m & I \end{pmatrix} \succeq 0.
$$

(25)

To proceed, in the first step we simplify (24) and solve $W$ in the way without loss of optimality, which can be attained due to the fact that $W$ is at SU-RX and is independent of all the constraints. After some manipulations, we can have the following lemma. The proof is provided in Section VIII.

**Lemma 1:** Given $F,$ we can solve $W$ and $B$ as

$$W = CF^H H_{23}^H (H_{23} F F^H H_{23}^H + I)^{-1},$$

(26)

$$B = (\Delta_1 + \sum_{i=1}^{N_T} S_i^T \mu_i^* e_i^T) (I + \Delta_2),$$

(27)

$$\Delta_1 \triangleq F^H H_{23}^H (H_{23} F F^H H_{23}^H + I)^{-1} H_{23} F - I,$$

(28)

$$\Delta_2 \triangleq F^H H_{23}^H H_{23} F,$$

(29)

where $\mu_i, i = 1, \cdots, N_T$ are the Lagrange multipliers of (24) which can be solved from

$$(1 + \Delta_{2, kk}) S_k S_k^T \mu_k^* + S_k \sum_{i \neq k} \Delta_{2, ik} S_i^T \mu_i^* + c_k = 0, k = 1 \sim N_T,$$

(30)

$\Delta_{2, ij}$ is the entry at the $i$-th row and $j$-th column of $\Delta_2,$ $S_k = [1_k, 0_{k \times (N_T - k)}],$ $c_k \triangleq S_k \Delta_1 (I + \Delta_2) e_k,$ and $e_k$ is the $k$-th column of the identity matrix.

$\begin{pmatrix} A & B \\ B^H & C \end{pmatrix} \succeq 0 \iff C - B^H A^{-1} B \succeq 0,$ where $A > 0$ and $C$ is Hermitian.
Meanwhile, the coexistence constraint (22) can be approximated by the following lemma.

**Lemma 2:** Given $\tilde{F}_2$, we can approximate (22) by

$$
\log(1 + t_1 + t_2) - \text{tr}\left((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}F_2H_{24}^H\right) \geq c_0,
$$

(31)

$$
\begin{pmatrix}
    t_1 & \text{vec}(H_{24}F)^H \\
    \text{vec}(H_{24}F) & I
\end{pmatrix} \succeq 0,
$$

(32)

$$
\begin{bmatrix}
    t_2 & \text{vec}(H_{14} + H_{24}A)^H \\
    \text{vec}(H_{14} + H_{24}A) & I_2 \otimes \Sigma_x
\end{bmatrix} \succeq 0,
$$

(33)

$$
\begin{bmatrix}
    I & F \\
    F^H & F_2
\end{bmatrix} \succeq 0,
$$

(34)

where $c_0 = \log|I + H_{14}\Sigma_xH_{14}^H| + \log|I + H_{24}\tilde{F}_2H_{24}^H| - \text{tr}\left((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}\tilde{F}_2H_{24}^H\right)$.

The proof is provided in Section IX. To summarize the above, we can formulate a relaxed problem as

$$
P_2: \min_{A,F,F_2,t_0,t_1,t_2} t_0, \text{ s.t. } (25), (26), (27), (31), (32), (33), (34).
$$

To proceed, we need to check the convexity of the constraints first. Note that $c_0$ is a constant. Thus (31) is convex of the slack variables $t_1$, $t_2$, and $F_2$ and thus can be solved by CVX [20]. In addition, since (32), (33), and (34) are linear matrix inequalities, they also can be solved by CVX. Note also that from (27) $B$ is not convex of $F$. However, we may fix $B$ and $W$ first, and solve $A$ and $F$, which is then a convex problem. Thus $P_2$ can be solved by CVX with fixed $B$ and $W$. After that we substitute the solved $F$ into (27) and (26) to find $B$ and $W$, respectively. And we repeat this procedure iteratively. As a result we propose the following iterative algorithm in Table II to solve $A$, $B$, $F$, and $W$. Note that from the numerical results we observe that the original coexistence constraint (22) may be violated during the iteration, which is due to the relaxation (however, the relaxed constraints (31), (32), (33), and (34) are all satisfied during the iterations.) We also observe that the LHS of (22) may decrease during the iteration. Thus we insert an additional constraint as Step 6 into the algorithm to help to terminate it.
TABLE II

THE PROPOSED ALGORITHM FOR SOLVING THE PRECODING MATRIX B, THE RELAYING MATRIX A, THE TRANSMIT BEAMFORMER F, AND THE RECEIVE BEAMFORMER W.

1: Initialize $A^{(0)} = 0$, $F^{(0)} = 0$ (thus $B^{(0)} = 0$), $F_2^{(0)} = 0$, $\tilde{F}_2^{(0)} = 0$, and $n = 0$.

2: Repeat

3: $n = n + 1$

4: Given $B^{(n-1)}$, $W^{(n-1)}$, $\tilde{F}_2^{(n-1)}$, solve $A^{(n)}$ and $F^{(n)}$ from $P2$.

5: Update $W^{(n)} = C(F^{(n)}) H H_{23}^{H} (H_{23} F^{(n)}(F^{(n)}) H H_{23}^{H} + I)^{-1}$ and

$$B^{(n)} = (\Delta_1^{(n)} + \sum_{i=1}^{N_T} S^T_i \mu^s_i e_i^T)(I + \Delta_2^{(n)})$$

where $\Delta_1^{(n)}$, $\Delta_2^{(n)}$, and $\mu^s_i$ are from (28), (29), (30), respectively.

Also update $\tilde{F}_2^{(n)} \leftarrow \tilde{F}_2^{(n)}$.

6: Until The equality in (22) is valid.

V. NUMERICAL RESULTS

In this section, we first show the sum rate performance of the SU networks of the proposed pairing scheme. Then we use numerical results to illustrate the sum MSE performance of the proposed joint design of the relay, THP, and transmitter/receiver beamformers. We assume that the PU-TX, PU-RX, SU-TX, and SU-RX are all equipped with 2 antennas. We also assume that with the knowledge of $H_{14}$, PU-TX can find $\Sigma_{x_p}$ as explained in Sec. [V]. The adopted channel model in the pairing simulation is the general Kronecker product form channel model [35]:

$$H = \sqrt{\rho} R_t^{1/2} H_w R_r^{1/2}$$

where $\rho = 1/(2d^\beta)$, $d$ is the distance between the uniformly generated PU and SU nodes and $\beta$ is the path loss exponent which is set as 3; $H_w \in C^{2 \times 2}$ is a zero mean unit variance i.i.d. complex Gaussian matrix between the transmitter and receiver; $R_t$ and $R_r$ are the transmit and receive correlation matrices which are modeled by $[R_t]_{ij} = \gamma_t |i-j|^2$ and $[R_r]_{ij} = \gamma_r |i-j|^2$, respectively, where $\gamma_t$ and $\gamma_r$ are the correlation coefficients, respectively, which are modeled as uniformly distributed variables within $[0, 1]$. The multiplier $\mu$ is randomly initialized within $[0, 2]^M$ and $\gamma^{(l)}$ is selected as $\gamma^{(l)} = 0.05/\sqrt{l}$. In Fig. 4 we compare the sum rate performances of the
proposed method with greedy algorithm and random pairing. The greedy algorithm randomly searches an SU and pairs it to PU in the greedy sense. After removing the paired PU and SU nodes, this procedure is done iteratively. The random pairing selects SU and pairs it to PU both randomly. We consider three numbers of pairs in this example: \( M = 5, 10, \) and \( 20. \) We can find that the proposed scheme outperforms the others in sum rates. Note that owing to the representation of \( \alpha \) in Theorem \( \[1\] \) we can attain the pairing with low complexity.

In the following we demonstrate the performance of the proposed transmission scheme. The SU-TX is assumed to know PU-TX’s selection of \( \Sigma_{x_p} \) to solve \( \textbf{P2}. \) Without loss of generality, we choose the vector channels as

\[
H_{13} = \begin{pmatrix}
-0.7 + 0.28i & 1.82 + 0.2i \\
-0.35 - 0.67i & -1.64 + 1.31i
\end{pmatrix}, \quad H_{14} = \begin{pmatrix}
0.97 - 0.66i & -0.03 + 0.77i \\
-0.07 - 0.05i & 0.3 - 0.32i
\end{pmatrix},
\]

\[
H_{23} = \begin{pmatrix}
-1.12 + 0.57i & 0.41 + 1.7i \\
1.46 - 0.92i & 1.13 - 0.02i
\end{pmatrix}, \quad H_{24} = \begin{pmatrix}
-0.77 + 0.38i & -0.09 - 0.41i \\
-0.84 - 0.61i & 0.63 + 1.12i
\end{pmatrix}. \tag{35}
\]

Note that the above channels are all full rank. Thus it is impossible to use the traditional zero forcing methods as \([15] [16] [36] [37]\) to avoid the interferences from CR’s signals to PU’s receiver. Therefore the coexistence constraint will never be valid. On the other hand, with the aid of the considered relay as discussed previously, it is possible for the CR systems to simultaneously transmit with PU. To overcome the caveat of the traditional ZF, we generalize it as

\[
\textbf{F} = \sqrt{\alpha_F} P_T \left( \sqrt{\gamma_F} V_1 \Sigma_1^{1/2} + \sqrt{1 - \gamma_F} V_2 \Sigma_2^{1/2} \right),
\]

where \( V_1 \) and \( V_2 \) are formed by the singular vectors in \( V_1 \) and \( V_2 \) corresponding to non-zero singular values of \( \Pi_{24} H_{23} = U_1 D_1 V_1^H \) and \( \Pi_{24}^1 H_{23} = U_2 D_2 V_2^H, \) respectively, which are mutually orthogonal; \( \Sigma_1 \) and \( \Sigma_2 \) are derived from water-filling according to the singular values \( D_1 \) and \( D_2 \) respectively with \( \text{tr}(\Sigma_1) = \text{tr}(\Sigma_2) = 1; \) \( \alpha_F \in [0, 1] \) denotes the ratio of the total power allocated to the SU’s signal transmission, and \( \gamma_F \) controls the power allocated in the directions of \( H_{24} \) and its null space; \( \Pi_{24} = H_{24}^H (H_{24} H_{24}^H)^{-1} H_{24} \)

\section*{Footnotes}

\( \dagger \) For transmitting SU’s own signal, it is better to avoid the direction to the PU RX. However, directly choosing the column vectors of \( H_{24}^H \) as the directions for transmitting SU’s own signal may not be good when \( H_{23} \) has similar direction to \( H_{24}. \) Similarly, to design \( \textbf{A}, \) it is better to avoid the direction \( H_{23} \) such that not to interfere SU RX. Again, if \( H_{24} \) has similar direction to \( H_{23}, \) this selection does not work well. So we resort to the linear combination of the desired subspace with its null space and then we optimize the coefficients to find the best directions. However, directions of \( \textbf{F} \) and \( \textbf{A} \) are designed almost separately, which may be inefficient compared to our method.
is a projection matrix projecting the operand to its column space and $\Pi_{24}^\perp = I - \Pi_{24}$. Similarly, we can let $A = \sqrt{\alpha A} P^T \left( \sqrt{\gamma A} V'_3 \Sigma_3^{1/2} + \sqrt{1 - \gamma A} V'_4 \Sigma_4^{1/2} \right)$, where $V'_3$ and $V'_4$ are formed by the eigenvectors in $V_3$ and $V_4$ corresponding to non-zero singular values of $\Pi_{23} H_{24} = U_3 D_3 V'_3$ and $\Pi_{23}^\perp H_{24} = U_4 D_4 V'_4$, respectively. $\gamma A \in [0,1]$ controls the power allocated in the directions of $H_{23}$ and its null space; $\Pi_{23} = H_{23}^H (H_{23} H_{23})^{-1} H_{23}$ is a projection matrix projecting the operand to its column space and $\Pi_{23}^\perp = I - \Pi_{23}$. Note that $\Sigma_3$ and $\Sigma_4$ are derived from water-filling according to the singular values $D_3$ and $D_4$. And $\text{tr}(\Sigma_3) = \text{tr}(\Sigma_4) = 1$. Then based on the line search, we can solve the three variables $\alpha_F$, $\gamma_F$, and $\gamma_A$.

In Fig. 5 we show the sum MSE performance versus $P_T$ under different $P_P$s. From this figure we can find that the minimum sum MSE may increase with increasing $P_P$. This is due to the fact that when increasing $P_P$, i.e., SU-TX needs to allocate more power for relaying PR’s signal to make the coexistence constraint valid. Thus less power can be used by SU-TX to transmit its own signals and the sum MSE performance deteriorates with increasing $P_P$. From the same figure we can easily see that the proposed joint optimization outperforms the generalized ZF. In Fig. 6 we show that how the sum MSE and the coexistence constraint change with the number of iterations with $P_P = 17$dB and $P_T = 20$dB. Note that the coexistence constraint in the figure means the difference between the LHS and RHS of (22). It can be seen that both the coexistence constraint and the sum MSE decreases with increasing number of iterations. These two subplots are consistent since as long as the coexistence constraint decreases, i.e., the LHS of (22) decreases, and more power is allocated to transmit SU’s own signal. And thus SU can have better performance. As mentioned previously, the coexistence constraint decreases with the iterations, thus we need to check its crossover point to stop the iteration. In Fig. 7 we show the cumulative density function (CDF) $F(\text{SMSE})$ of the SMSE under fast fading channels, where the number of random channel realizations is $10^4$ and the four channels are all set as i.i.d. Gaussian matrices with zero mean and unit variance. From this figure we can find the most probable region of the SMSE of the proposed scheme under random channels, which may not be able to reflect in Fig. 5. Due to the complexity of the ZF scheme, we do not show the CDF performance of it in the same figure. In Fig. 8 we compare the symbol

$\dagger$ When $\gamma_F = 0$, SU’s transmission is zero forcing to PU’s reception. Similarly, when $\gamma_A = 0$, the relay of PU’s signal is zero forcing to SU’s reception.
error rate (SER) performances between the proposed scheme and the generalized ZF. The SERs in both cases are averaged over two antennas. It can be easily seen that the performance loss owning to the non-optimized \( A, B, F, \) and \( W \) is large, which is about 5dB and 4.5 dB in the considered cases \( P_T = 50 \) and \( P_T = 100 \), respectively. On the other hand, the SER increases with \( P_T \), which is consistent to that in Fig. 5. In Fig. 9 we compare the relay ratios and the power SU can transmit his own signal versus \( P_T \) between the proposed scheme and the generalized ZF, where the former is denoted by solid lines and the later is denoted by dashed lines. It can be seen that the power SU can use for transmitting his own signal increases with decreasing \( P_T \) and increasing \( P_T \), which is consistent to that of the SISO case in Fig. 3.

![Comparison of different pairing schemes.](image-url)

Fig. 4. Comparison of different pairing schemes.
The proposed scheme

The generalized ZF

Fig. 5. The sum MSE performance versus $P_T$ under fixed channels with different $P_p$'s.

The generalized ZF

The proposed scheme

Fig. 6. The changes of the sum MSE and the coexistence constraint with the number of iterations.
Fig. 7. The CDF of sum MSE performance under fast fading channels with different $P_T$'s.

Fig. 8. The SER performance.
VI. CONCLUSION

In this work we practically designed the multiple-antenna cognitive radio (CR) networks with the coexistence constraint. The considered primary user (PU) is a half-duplex two-hop relay channel and the secondary user (SU) is a single user additive white Gaussian noise channel. The contribution of this paper are twofold. First, we explicitly designed the scheme to pair the SUs to the existing PUs in a cellular network. Second, we jointly designed the nonlinear precoder, relay beamformer, and the transmitter and receiver beamformers to minimize the sum mean square error of the SU system. In the first part, we solve the optimal pairing between SUs and PUs such that the sum rate of SUs is maximized. To achieve it, we derive the relation between the relay ratio, chordal distance and strengths of the vector channels, and the transmit powers. In the second part, we adopt the Tomlinson-Harashima precoding instead of the dirty paper coding to mitigate the interference at the SU receiver. To complete this design, we first approximated the optimization problem as a convex one. Then we proposed an iterative algorithm to solve it with CVX. This joint design exploits all the degrees of design freedom including the spatial diversity, power, and the side information of the interference at transmitter, which may result in better SU’s performance than those without the joint design. To the best of our knowledge, both the two parts have never been considered in...
the literature. Numerical results verify the performance of the proposed schemes.

VII. PROOF OF THEOREM 1

Proof: In this proof we use the upper bound of the LHS of (22) to derive the approximate relay ratio \( \alpha \). We rearrange the LHS of (22) as

\[
\log |I + (I + H_{24}FF^H H_{24}^H)\Sigma x_p (H_{14} + H_{24}A)^H| \\
\leq N_T \log \left( \frac{1}{N_T} \text{tr} (I + (I + H_{24}FF^H H_{24}^H)\Sigma x_p (H_{14} + H_{24}A)^H) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \text{tr} (H_{14} + H_{24}A)\Sigma x_p (H_{14} + H_{24}A)^H) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) \\
\leq N_T \log \left( 1 + \frac{1}{N_T} \lambda_{\text{max}}(Q) \lambda_{\text{max}}(\Sigma x_p) (\text{tr}(A^H H_{24}^H H_{14} + \text{tr}(H_{14}^H H_{24} A) + \text{tr}(H_{14}^H H_{14}^H + \text{tr}(H_{24} A^H H_{24}^H))) \right) 
\]
where (a) uses \( \det(M) \leq (\text{tr}(M)/m)^m \), and \( m \) is the dimension of the square matrix \( M \); (b) is by the properties \( \text{tr}(PQPH) \leq \lambda_{\text{max}}(Q)\text{tr}(PH) \) and \( \text{tr}(PQ) = \text{tr}(QP) \), where \( Q \triangleq (I + H_{24}FF^H H_{24}^H)^{-1} \) and \( P \triangleq (H_{14} + H_{24}A)\Sigma_{x_P}^{-1/2} \); (c) expands the product inside \( \text{tr}(\cdot) \); (d) uses \( |\text{tr}(A^H H_{24}H_{14}) - 1|^2 \geq 0 \); (e) uses \( |\text{tr}(PQ)|^2 \leq \text{tr}(PP^H)\text{tr}(QQ^H) \), and we define \( c = \text{tr}(H_{14}H_{14}^H) + 1 \); in (f) we decompose \( H_{24} = U_{24}D_{24}V_{24}^H \) and \( H_{14} = U_{14}D_{14}V_{14}^H \) by singular value decomposition, and use \( \text{tr}(AB) = \text{tr}(BA) \) and the property: if both \( P \) and \( Q \) are positive semi-definite and Hermitian, then \( 0 \leq \text{tr}(PQ) \leq \text{tr}(P)\text{tr}(Q) \) \([38]\); in (g) we use the property in (f) again; in (h) we define \( T \triangleq U_{24}^HH_{14} \) with eigenvalues \( \{\sin \theta_k\} \), where \( \theta_k \) is the \( k \)th principal angle; in (i) we use \( \text{tr}(AB) \leq \Sigma_i \lambda_i(A)\lambda_i(B) \leq \Sigma_i \lambda_{\text{max}}(A)\lambda_i(B) \); (j) is by definition of the chordal distance in Definition\([5]\) in (k) we use \( \text{tr}(PQPH) \leq \lambda_{\text{max}}(Q)\text{tr}(PH) \) reversely; in (l) \( P_T \) is extracted out to form the term \( \frac{\text{tr}(A\Sigma_{x_P}A^H)}{P_T} \), which is the relay ratio by definition and by the fact \( N_T d_{24,\text{max}}^2 \geq \text{tr}(D_{24}^2) \); (m) is by the fact that \( Q = V(I + D)^{-1}V^H \), where \( H_{24}FF^H H_{24}^H = VDV^H \) by eigenvalue decomposition; in (n) we set \( F = \sqrt{(1-\alpha)P_T\tilde{F}} \) and use the two properties \([39]\): 1. \( AB \) and \( BA \) have the same set of eigenvalues; 2. \( \lambda_{\text{min}}(AB) \geq \lambda_{\text{min}}(A)\lambda_{\text{min}}(B) \); in (o) we define \( c_1 \triangleq \frac{\lambda_{\text{max}}(\Sigma_{x_P})}{\alpha P_T d_{24,\text{max}}^2} c \); the case of \( d_{24,\text{min}} = 0 \) is discussed in \([2]\).

By equating the RHS of \([36]\) to the upper bound of PU’s rate when SU is not active, i.e.,

\[
\log |I + H_{14}\Sigma_{x_P}H_{14}^H| \leq N_T \log \left(1 + \frac{1}{N_T} \lambda_{\text{max}}(\Sigma_{x_P})\text{tr}(D_{14}^2)\right),
\]

we then can find \([9]\). Note that this upper bound is by the properties in step (a) and (b). Note also that to derive \([9]\) we assume that \( \tilde{F} = U_{23} \), where \( H_{23} = U_{23}D_{23}U_{23}^H \) by eigenvalue decomposition. The assumption comes from the fact that before the pairing, optimal \( F \) is unknown and this selection makes the derivation tractable.

VIII. PROOF OF LEMMA \([1]\)

We first rearrange the sum MSE in \([20]\) as

\[
\text{tr}(E) = \text{tr}\left(W(H_{23}FF^HH_{23}^H + I)W^H - CF^H H_{23}^H W^H - WH_{23}FC^H + CC^H\right).
\]

(38)
Note that $W$ is not included in any constraints. Thus the Karush-Kuhn-Tucker (K.K.T.) condition which is the derivative of the Lagrangian with respect to $W$, is identical to

$$\frac{\partial \text{tr}(E)}{\partial W} = W(H_{23}FF^H H_{23}^H + I) - CF^H H_{23}^H = 0$$

and thus $W = CF^H H_{23}^H(H_{23}FF^H H_{23}^H + I)^{-1}$. After substituting $W$ into (20), we get

$$\text{tr}(E) = \text{tr}(CC^H - CF^H H_{23}^H(H_{23}FF^H H_{23}^H + I)^{-1}H_{23}FC^H).$$

To solve $C$ (or equivalently, $B$), the lower triangular constraint should be taken into account, and which can be modeled by

$$S_kBe_k = 0_k, \ k = 1 \sim N_T.$$

Then we can form the Lagrangian of $PO$ as

$$L = \text{tr}((I + B)(I + B)^H - (I + B)F^H H_{23}^H(H_{23}FF^H H_{23}^H + I)^{-1}H_{23}F(I + B)^H) - \sum_{k=1}^{N_T} \mu_k^T S_k Be_k + \lambda \{ \log |I + H_{14}\Sigma x_p H_{14}^H| + \log |I + H_{24}FF^H H_{23}^H| - \log |I + H_{24}FF^H H_{23}^H + (H_{14} + H_{24}A)\Sigma x_p (H_{14} + H_{24}A)^H| \} + \gamma \{ \text{tr}(FF^H) + \text{tr}(A\Sigma x_p A^H) - P_T \},$$

where $\mu_k \in \mathbb{C}$, $\lambda \geq 0$, and $\gamma \geq 0$ are Lagrange multipliers. Note that the lower triangular constraint is an equality constraint here, thus there is no restriction on the sign of the multiplier $\mu_i$. After equating the first derivative of $L$ with respect to $B$ to zero, we have

$$\frac{\partial L}{\partial B} = (I + B)^H F^H H_{23}^H(H_{23}FF^H H_{23}^H + I)^{-1}H_{23}F(I + B)^H - \sum_{k=1}^{N_T} S_k \mu_k^* e_k^T = 0,$$

which can be further arranged as (27) with (28) and (29), where the matrix inversion lemma is used. To get $\mu_i$ we first apply the constraint (41) to $B$ in (27). Then we have

$$S_k(\Delta_1 + \sum_i S_k^T \mu_i^* e_i^T)(I + \Delta_2) e_k = 0, \ k = 1 \sim N_T,$$

where $\Delta_1$ and $\Delta_2$ are defined in (28) and (29), respectively. After expanding we can get

$$S_k \Delta_1 (I + \Delta_2) e_k + S_k \left( \sum_i S_k^T \mu_i^* e_i^T \right) (I + \Delta_2) e_k = 0, \ k = 1 \sim N_T.$$
Then combine the terms of $e_k$ we can get

$$S_k S_k^T \mu_k^* + S_k \left( \sum_i S_i^T \mu_i^* e_i^T \right) \Delta_2 e_k + c_k = 0, \quad k = 1 \sim N_T,$$

where the first term is due to the fact that $e_i^T e_k = \delta(i - k)$, $\delta$ is the Dirac delta function. After combining the terms of $\mu_i^*$, we have (30). Note that for each $k$, there are $k$ scalar equations and thus there are $(N_T + 1)N_T/2$ equations in total. Meanwhile, there are also $(N_T + 1)N_T/2$ scalar unknown variables in the set of variables $\{\mu_1^* \cdots \mu_{N_T}^*\}$. Thus we can solve the multipliers from the linear equations at hand.

**IX. Proof of Lemma 2**

It is known that the function $\log(\det(.))$ is concave in its domain and the term inside the determinant of the numerator on the LHS of (22) is a quadratic form of $F$ and $A$, which is convex. Thus the convexity of the composition of $\log(\det(.))$ and the quadratic form is unknown. To proceed, we first use the inequality \[40\] $\det(I + M) \geq 1 + \text{tr}(M)$, i.e., the numerator on the LHS of (22) can be lower bounded by

$$\log \left( 1 + \text{tr}(H_{24}FF^H H_{24}^H + (H_{14} + H_{24}A) \Sigma_x (H_{14} + H_{24}A)) \right)$$

$$\geq \log \left( 1 + \text{vec}(H_{24}F)^H \text{vec}(H_{24}F) + \text{vec}(H_{14} + H_{24}A)^H (I \otimes \Sigma_x) \text{vec}(H_{14} + H_{24}A) \right)$$

$$\geq \log(1 + t_1 + t_2), \quad (44)$$

where in (a) we use the properties $\text{tr}(MM^H) = \text{vec}(M)^H \text{vec}(M)$ and $\text{tr}(MM^H) = \text{vec}(M)^H (I \otimes \Sigma_x) \text{vec}(M)$; in (b) we introduce the following slack variables $t_1 \triangleq \text{vec}(H_{24}F)^H \text{vec}(H_{24}F)$ and $t_2 \triangleq \text{vec}(H_{14} + H_{24}A)^H (I \otimes \Sigma_x) \text{vec}(H_{14} + H_{24}A)$. Then we use the matrix-lifting semi-definite relaxation \[41\] to relax the above such that $t_1 \geq \text{vec}(H_{24}F)^H \text{vec}(H_{24}F)$ and $t_2 \geq \text{vec}(H_{14} + H_{24}A)^H (I \otimes \Sigma_x) \text{vec}(H_{14} + H_{24}A)$, to accommodate the use of CVX. After this relaxation, we can further rearrange them as the linear matrix inequalities as shown in (32), (33), and (34), respectively by the Schur’s Complement Theorem.

After replacing the numerator on the LHS of (22) by (32), (33), and (34) and substituting $F_2 \triangleq FF^H$ (which is further relaxed by $F_2 \succeq FF^H$) into the denominator on the LHS of (22), we can observe that the LHS of (22) is the difference of two concave functions, and the convexity is still unknown. However, we may resort to the skill in [23] to do the first order Taylor expansion to the denominator of the LHS of (22).
and which will become a linear function of $F_2$. By definition, we know that the first order Taylor expansion of the function $f(X)$ with respect to $X$ at $\tilde{X}$ is $f(X) \leq f(X_0) + \text{tr}(D^H(X - \tilde{X}))$, where $D \equiv \frac{\partial f}{\partial X} \bigg|_{X=\tilde{X}}$. Then with the fact $\frac{\partial \log \det(X)}{\partial X} = (X^{-1})^T$, after applying the above to the denominator on the LHS of (22), we can get

$$\log |I + H_{24}F_2H_{24}^H| \leq \log (I + H_{24}\tilde{F}_2H_{24}^H) + \text{tr}((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}F_2H_{24}^H) - \text{tr}((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}\tilde{F}_2H_{24}^H) - \text{tr}((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}\tilde{F}_2H_{24}^H) - \text{tr}((I + H_{24}\tilde{F}_2H_{24}^H)^{-1}H_{24}\tilde{F}_2H_{24}^H).$$

After substituting the above into (22) and some arrangements, we can get Lemma 2.

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