ENHANCEMENT AND SUPPRESSION OF HEAT TRANSFER BY MHD TURBULENCE

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ABSTRACT

We study the effect of turbulence on heat transfer within magnetized plasmas for energy injection velocities both larger and smaller than the Alfvén speed. We find that in the latter regime the heat transfer is partially suppressed, whereas in the former regime the effects of turbulence depend on the intensity of driving. In fact, the scale \( l_\perp \) at which the turbulent velocity is equal to the Alfvén velocity is an important new parameter. When the electron mean free path \( \lambda \) is larger than \( l_\perp \), the stronger the turbulence, the lower the thermal conductivity by a factor of 30. When \( \lambda \) is smaller than \( l_\perp \), the single-scale model does not apply. The turbulent motions, however, induce their own advective heat transport, which, for the parameters of intracluster medium, provides effective heat diffusivity that exceeds the classical Spitzer value.

Subject heading: galaxies: clusters: general — ISM: general — MHD — turbulence

1. ASTROPHYSICAL MOTIVATION

Heat transfer in turbulent magnetized plasma is an important astrophysical problem that is relevant to a wide variety of circumstances, from mixing layers in the Local Bubble (see Smith & Cox 2001) and Milky Way (Begelman & Fabian 1990) to cooling flows in the intracluster medium (ICM; Fabian 1994). The latter problem has been subjected to particular scrutiny, as observations do not support the evidence for the cool gas cooling flows in the intracluster medium (ICM; Fabian 1994). This is not surprising as the largest scale for the GS95 turbulence, i.e., \( l_{\perp,\min} \), with eddies \( 1/3 \) of the scale at which the magnetic field is dynamically im-

2. THERMAL CONDUCTIVITY: STATIC MAGNETIC FIELD

2.1. Basics of Heat Transfer in Magnetized Plasma

Following NM01, we initially disregard the dynamics of fluid motions on heat transfer, i.e., consider thermal conductivity induced by electrons moving along static magnetic fields. Magnetized turbulence in the GS95 model is anisotropic with eddies elongated along \( \parallel \) the direction of local magnetic field. Consider isotropic injection of energy at the outer scale \( L \) and dissipation at the scale \( l_{\perp,\min} \), where \( \perp \) denotes the direction of perpendicular to the local magnetic field. NM01 observed that the separations of magnetic field lines for \( r_e < l_{\perp,\min} \) are mostly influenced by the motions at the scale \( l_{\perp,\min} \), which result in Lyapunov-type growth: \( \sim r_e \exp(\ell \l_\perp) \). This growth is similar to that obtained in earlier models with a single scale of turbulent motions (Rechester & Rosenbluth 1978; Chandran & Cowley 1998). This is not surprising as the largest shear that causes field line divergence is provided by the marginally damped motions at the scale around \( l_{\perp,\min} \). In NM01 \( r_e \) is associated with the size of the cloud of electrons of the electron Larmor radius \( r_{\text{Lar,electr}} \). They find that the electrons should travel over the distance

\[
L_{\text{RR}} \sim l_{\perp,\min} \ln \left(l_{\perp,\min}/r_{\text{Lar,electr}}\right)
\]

to get separated by \( l_{\perp,\min} \).

Within the single-scale model that formally corresponds to \( L = l_{\perp,\min} = l_{\perp,\min} \) the scale \( L_{\text{RR}} \) is called the Rechester-Rosenbluth distance. For the ICM parameters the logarithmic factor in equation (1) is of the order of 30, and this causes a 30 times decrease of thermal conductivity for the single-scale models. In the multiscale models with a limited (e.g., a few decades) inertial range, the logarithmic factor stays of the same order, but it does not affect the thermal conductivity, provided that \( L \gg l_{\perp,\min} \). Indeed, for the electrons to diffuse isotropically, they should spread from \( r_{\text{Lar,electr}} \) to \( L \). The GS95 model of turbulence operates with field lines that are sufficiently stiff; i.e., the deviation of the field lines from their original direction is of the order unity at scale \( L \) and less for smaller scales. Therefore, to get separated from the initial distance of \( l_{\perp,\min} \) to a distance \( L \) (see eq. [5] with \( M_\perp = 1 \)), at which the motions get uncorrelated, the electron should diffuse a distance slightly larger (as field lines are not straight) than \( 2L \) (NM01: also see § 2.3.), which is much larger than the extra travel distance \( \sim 30 l_{\perp,\min} \). Explicit calculations in NM01 support this intuitive picture.

2.2. Heat Transfer for \( M_\perp > 1 \)

Turbulence with \( M_\perp > 1 \) evolves along the hydrodynamic isotropic Kolmogorov cascade, i.e., \( V_t \sim (\ell/\lambda)^{5/3} \) over the range of scales \( [\lambda, l_\perp] \), where

\[
l_\perp \approx L(V_t/\lambda)^{3/5} \equiv LM_\perp^{-3}
\]

is the scale at which the magnetic field gets dynamically important, i.e., \( V_t = V_c \). This scale plays the role of the injection scale for the GS95 turbulence, i.e., \( V_t \sim (\ell/\lambda)^{5/3} \), with eddies

1 For the single-scale model \( L_{\text{RR}} \sim 30 \) and the diffusion over distance \( \Delta \) takes \( L_{\text{RR}}/\Delta \) steps, i.e., \( \Delta \sim L_{\text{RR}}/\lambda \), which decreases the corresponding diffusivity coefficient \( k_{\text{diffusive}} \sim \Delta/\Delta t \) by a factor of 30.
at scales less than \( l_\lambda \) getting elongated in the direction of the local magnetic field. The corresponding anisotropy can be characterized by the relation between the semimajor axes of the eddies

\[
l_\parallel \sim L (l/L)^{2/3} M_\lambda^{-1}, \quad M_\lambda > 1,
\]

where \( \parallel \) and \( \perp \) are related to the direction of the local magnetic field. In other words, for \( M_\lambda > 1 \), the turbulence is still isotropic at scales larger than \( l_\lambda \) but develops \( (l/L)^{1/3} \) anisotropy for \( l < l_\lambda \).

For the electron mean free path \( \lambda \gg l_\lambda \), electrons stream freely over the distance of \( l_\lambda \). For electrons at distance \( l_{\text{min}} \) to get separated by \( L \) the required travel is the random walk with the step \( l_\lambda \), i.e., the mean-squared displacement of a thermal electron until it enters an independent large-scale eddy \( \Delta r \sim L^2 / (l_\parallel) \), where \( L/l_\parallel \) is the number of steps. These steps require time \( \Delta t \sim (L/l_\parallel)^2 / (C_v \nu_{\text{elec}}) \), the electron thermal velocity and the coefficient \( C_v = \frac{3}{2} \) accounts for one-dimensional character of motion along magnetic field lines. Thus, the electron diffusivity coefficient is

\[
\kappa_{\text{elec}} = \Delta r \approx \frac{1}{2} l_\lambda \nu_{\text{elec}}, \quad l_\lambda < \lambda,
\]

which for \( l_\lambda \ll \lambda \) constitutes a substantial reduction of conductivity compared to its Spitzer (unmagnetized) value \( \kappa_{\text{Spitzer}} = \lambda \nu_{\text{elec}} \). We assumed in equation (4) that \( L \gg 30 l_{\text{min}} \) (see § 2.1).

For \( \lambda \ll l_\parallel \ll L \), \( \kappa_{\text{elec}} \approx \frac{1}{2} \kappa_{\text{Spitzer}} \) as both the \( L_{\text{LR}} \) and the additional distance for electron to diffuse because of magnetic field being stiff at scales less than \( l_\parallel \) are negligible compared to \( L \). For \( l_\parallel \rightarrow L \), when magnetic field has rigidity up to the scale \( L \), it gets around \( \frac{1}{2} \) of the Spitzer value according to NM01.

Note that even dynamically unimportant magnetic fields do influence heat conductivity over short time intervals. For instance, over time intervals less than \( l_\parallel^2 / C_v \kappa_{\text{Spitzer}} \) the diffusion happens along stiff magnetic field lines and the difference between parallel and perpendicular diffusivities is large.\(^2\) This allows the transient existence of sharp small-scale temperature gradients.

2.3. Heat Transfer for \( M_\lambda < 1 \)

It is intuitively clear that for \( M_\lambda < 1 \) turbulence should be anisotropic from the injection scale \( L \). In fact, at large scales the turbulence is expected to be \textit{weak} (see Lazarian & Vishniac 1999, hereafter LV99). Weak turbulence is characterized by wave packets that do not change their shape but develop structures perpendicular to the magnetic field, i.e., decrease \( l_\parallel \). This cannot proceed indefinitely, however. At some small scale the GS95 condition of “critical balance,” i.e., \( l_\parallel / V_\parallel \approx l_\parallel / V_\perp \), becomes satisfied. This perpendicular scale \( l_{\text{trans}} \) can be obtained by substituting the scaling of weak turbulence (see LV99) \( V_\parallel \sim V_\perp (l/L)^{1/2} \) into the critical balance condition. This provides \( l_{\text{trans}} \sim L M_\lambda^{-2} \) and the corresponding velocity \( V_{\text{trans}} \sim V_\perp M_\lambda^{-1} \). For scales less than \( l_{\text{trans}} \) the turbulence is \textit{strong}, and it follows the scalings of the GS95 type, i.e., \( V_\parallel \sim V_\perp (l/L)^{1/2} M_\lambda^{-1} \) and

\[
l_\parallel \sim L (l/L)^{2/3} M_\lambda^{-4/3}, \quad M_\lambda < 1.
\]

For \( M_\lambda < 1 \), a magnetic field wandering in the direction perpendicular to the mean magnetic field (along the \( y \)-axis) can be described by \( d \langle y^2 \rangle / dx \sim \langle y^2 \rangle / l_\parallel \) (LV99), where\(^3\) \( l_\parallel \) is expressed by equation (5) and one can associate \( l_\parallel \) with 2\( \langle y^2 \rangle \),

\[
\langle y^2 \rangle^{1/2} \sim \frac{X^{3/2}}{3L^{1/2} M_\lambda^2}, \quad l_\parallel < l_{\text{trans}}.
\]

For weak turbulence \( d \langle y^2 \rangle / dx \sim \lambda M_\lambda^{-1} \) (LV99), and thus

\[
\langle y^2 \rangle^{1/2} \sim \frac{X^{3/2}}{L^{1/2} M_\lambda^2}, \quad l_\parallel > l_{\text{trans}}.
\]

Equation (6) differs by the factor \( M_\lambda^{-1} \) from that in NM01, which reflects the gradual suppression of thermal conductivity perpendicular to the mean magnetic field as the magnetic field gets stronger. Physically, this means that for \( M_\lambda < 1 \) the magnetic field fluctuates around the well-defined mean direction. Therefore, the thermal conduction gets anisotropic with the coefficient of thermal conduction parallel to the mean field \( \kappa_{\parallel, \text{elec}} \approx \frac{1}{2} \kappa_{\text{Spitzer}} \) being larger than \( \kappa_{\perp, \text{elec}} \) for the thermal conductivity in the perpendicular direction.

Consider the coefficient \( \kappa_{\perp, \text{elec}} \) for \( M_\lambda \ll 1 \). As NM01 showed, electrons become uncorrelated if they are displaced over the distance \( L \) in the direction perpendicular to magnetic field. To do this, an electron has first to travel \( L_{\text{LR}} \) (see eq. [11]), where equation (5) relates \( l_{\text{min}} \) and \( l_{\text{min}} \). Similar to the case in § 2.1, for \( L \gg 30 l_{\text{min}} \) the additional travel arising from the logarithmic factor is negligible compared to the overall diffusion distance \( L \). At larger scales electrons have to diffuse \( -L \) in the direction parallel to the magnetic field to cover the distance of \( \lambda M_\lambda^{-1} \) in the direction perpendicular to the magnetic field. Therefore, the separation of electrons over the turbulence driving scale \( L \) in \( \perp \) direction would require \( L/L_{\text{LR}} M_\lambda^2 = M_\lambda^2 \) random steps perpendicular to the mean field. Thus, the three-dimensional displacement for electrons initially at \( \nu_{\text{LR, elec}} \) to get separated by \( L \) in \( \perp \) direction is \( \sim L M_\lambda^2 \). If \( \lambda \ll L \), this diffusion requires time \( \Delta t \sim L^2 / (M_\lambda D) \), where \( D \) is the diffusion coefficient, which is \( \nu_{\text{elec}} L / 3 \). As a result,

\[
\kappa_{\perp, \text{elec}} = L^2 / \Delta t \approx \frac{1}{2} \nu_{\text{elec}} M_\lambda^{-2}, \quad M_\lambda < 1,
\]

where we disregarded the distance to travel in the direction perpendicular mean magnetic field, i.e., \( L \), compared to the distance to travel parallel to magnetic field, i.e., \( L M_\lambda^{-2} \). For \( M_\lambda \) of the order of unity this is not accurate, and one should account for the actual three-dimensional displacement (see § 2.1 and NM01).

3. FLUID VERSUS ELECTRON MOTIONS

Turbulent motions themselves can advectively transport heat. In Cho et al. (2003) we dealt with the turbulence with \( M_\lambda \sim 1 \) and estimated

\[
\kappa_{\text{dyne}} \approx C_{\text{dyne}} L V_\parallel, \quad M_\lambda > 1,
\]

where \( C_{\text{dyne}} \approx 0(1) \) is a constant, which for hydroturbulence is

\(^3\) The fact that one gets \( l_{\text{min}} \) in eq. (1) is related to the presence of this scale in this diffusion equation.
around $\frac{1}{4}$ (Lesieur 1990). For fully ionized nondegenerate plasma we assume $C_{\text{dyn}} \approx \frac{1}{2}$ to account for the advective heat transport by both protons and electrons. Thus, equation (9) covers the cases of both $M_{s} > 1$ up to $M_{s} \sim 1$. For $M_{s} < 1$ one can estimate $\kappa_{\text{dyn}} \approx d^{2}\omega$, where $d$ is the random walk of the field line over the wave period $\sim \omega^{-1}$. As the weak turbulence at scale $L$ evolves over time $\tau \approx M_{s}^{3} \omega^{-1}$, $\langle y^{2} \rangle$ is the result of the random walk with a step $d$, i.e., $\langle y^{2} \rangle \sim \tau d^{2}$. According to equation (7), the field line is displaced over time $\tau$ by $\langle y^{2} \rangle \sim M_{s}^{2} \tau$, which is similar to the diffusion arising from strong turbulence at scales less than $l_{\text{trans}}$, i.e., $\kappa_{\text{dyn}} \approx C_{\text{dyn}} l_{\text{trans}} V_{\tau}$. The total diffusion is the sum of the two, i.e., for plasma

$$\kappa_{\text{dyn}} \approx (\beta/3) L V_{M_{s}^{3}}, \quad M_{s} < 1, \quad (10)$$

where $\beta \approx 4$.

The schematic of the parameter space for $\kappa_{\text{elect}} < \kappa_{\text{spitzer}}$ is shown in Figure 1, where the Mach number and the Alfven Mach number $M_{s}$ are the variables. For $M_{s} < 1$, the ratio of thermal conductivities arising from fluid and electron motions is $\kappa_{\text{dyn}}/\kappa_{\text{elect}} \sim \beta \alpha M_{s} (L/\lambda)$ (see eqs. [8] and [10]), which provides the separation line between the two regions in Figure 1. $\alpha M_{s} \sim (\lambda L)^{1/3}$, where $\alpha = (m_{e} v_{\text{ion}}^{2})^{1/2}$ is the square root of the ratio of the electron to proton mass. For $1 < M_{s} < (L/\lambda)^{1/3}$ the mean free path is less than $l_{\text{a}}$, which results in $\kappa_{\text{elect}}$ being some fraction of $\kappa_{\text{spitzer}}$, while $\kappa_{\text{dyn}}$ is given by equation (9). Thus, $\kappa_{\text{dyn}}/\kappa_{\text{elect}} \sim \beta \alpha M_{s} (L/\lambda)$, i.e., the ratio does not depend on $M_{s}$. (Fig. 1, horizontal line). When $M_{s} > (L/\lambda)^{1/3}$ the mean free path of electrons is constrained by $l_{\text{a}}$. In this case $\kappa_{\text{dyn}}/\kappa_{\text{elect}} \sim \beta \alpha M_{s} M_{s}^{3}$ (see eqs. [9] and [4]). This results in the separation line $\beta \alpha M_{s} \sim M_{s}^{3}$ in Figure 1.

4. TURBULENCE AND HEAT TRANSFER IN ICM

It is generally believed that ICM is turbulent. The considerations below can be used as guidance. In unmagnetized plasma with the ICM temperatures $T \sim 10^{8}$ K and density $10^{-3}$ cm$^{-3}$ the diffusivity $\nu_{\text{visc}}/\nu_{\text{ion}} \sim v_{\text{ion}}/\lambda_{\text{ion}}$, where $v_{\text{ion}}$ and $\lambda_{\text{ion}}$ are the velocity of an ion and its mean free path, respectively, would make the Reynolds number $Re \equiv L V_{s}/\nu$ of the order of 30. This is barely enough for the onset of turbulence. For the sake of simplicity we assume that the ion mean free path coincides with the proton mean free path and both scale as $\lambda \approx 3 l_{\text{a}}^{3/2} n_{i}^{-1/2}$ kpc, where the temperature $T_{s} \equiv k T/3$ keV and $n_{i} \equiv n/10^{-3}$ cm$^{-3}$. This provides $\lambda$ of the order of 0.8–1 kpc for the ICM (see NM01).

It is accepted, however, that magnetic fields decrease the diffusivity. Somewhat naively assuming the maximal scattering rate of an ion, i.e., scattering every orbit (the so-called Bohm diffusion limit) one gets the viscosity perpendicular to magnetic field $\nu \sim v_{\text{ion}} l_{\text{Larmor, ion}}$, which is much smaller than $\nu_{\text{visc}}$. Provided that the ion Larmor radius $l_{\text{Larmor, ion}} \ll \lambda_{\text{ion}}$. For the parameters of the ICM this allows essentially inviscid motions of magnetic lines parallel to each other, for example, Alfven motions.

\[5\] This gets clear if one uses the heat flux equation $q = -\kappa \nabla T$, where $\kappa = m_{e} k_{\text{B}} V_{\text{ion}}^{3}$ is electron number density, and $k_{\text{B}}$ is the Boltzmann constant for both electron and advective heat transport.

\[6\] A regular magnetic field $B_{0} \approx (2 m_{e} \kappa T^{2})^{1/2} c / \lambda_{\text{ion}}$ that makes $l_{\text{Larmor, ion}}$ less than $\lambda_{\text{ion}}$ and therefore $\nu < \nu_{\text{visc}}$ is just $10^{-26}$ G. A turbulent magnetic field with many reversals over $l_{\text{Larmor, ion}}$ does not interact efficiently with a proton, however. As a result, the protons are not constrained until $l_{\text{a}}$ is of the order of $l_{\text{Larmor, ion}}$. This happens when the turbulent magnetic field is of the order of $2 \times 10^{9} (V_{s} / 10^{6} \text{ km s}^{-1})$. At this point, the step for the random walk is $\sim 2 \times 10^{-8}$ pc, and the Reynolds number is $5 \times 10^{10}$.

\[7\] One can imagine dynamo action in which super-Alfvenic turbulence generates magnetic field until $l_{\text{a}}$ gets large enough to shut down the turbulence.

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**Fig. 1.**—Sonc Mach number $M_{s}$ is plotted against the Alfven Mach number $M_{a}$. The heat transport is dominated by the dynamics of turbulent eddies above the curve and by thermal conductivity of electrons below the curve. Here $\lambda$ is the mean free path of the electron, $L$ is the driving scale, and $\alpha = (m_{e} v_{\text{ion}}^{2})^{1/2}$. $\beta \approx 4$. The panel in the top right corner of the figure illustrates heat transport for the parameters of a cool core Hydra cluster using data from EV06 (point “F”), “V” corresponds to the illustrative model in EVP05.

In spite of the substantial progress in understanding the ICM (see Ensslin et al. 2006, hereafter EV05; Ensslin & Vogt 2006, hereafter EV06 and references therein), the basic parameters of ICM turbulence are known within a factor of 3 at best. For instance, the estimates of injection velocity $V_{s}$ varies in the literature from 300 to $10^{4}$ km s$^{-1}$, while the injection scale $L$ varies from 2 to 200 kpc, depending on whether the injection of energy by galaxy mergers or galaxy wakes is considered. EVP05 considers an illustrative model in which the magnetic field with the $10^{-9}$ G fills 10% of the volume, while 90% of the volume is filled with the field of $B \sim 1 \mu$G. Using the latter number and assuming $V_{s} = 10^{4}$ km s$^{-1}$, $L = 100$ kpc, and the density of the hot ICM is $10^{-3}$ cm$^{-3}$, one gets $V_{s} \approx 70$ km s$^{-1}$, i.e., $M_{a} > 1$. Using the numbers above, one gets $l_{\text{a}} \approx 30$ pc for the 90% of the volume of the hot ICM, which is much less than $\lambda_{\text{ion}}$. The diffusivity of ICM plasma gets $\nu_{\text{visc}} / \nu_{\text{ion}}$, which for the parameters above provides $Re \approx 2 \times 10^{4}$, which is enough for driving super-Alfvenic turbulence at the outer scale $L$. However, as $l_{\text{a}}$ increases proportional to $B_{0}^{2}$, $Re$ gets around 50 for the field of $4 \mu$G, which is at the border line of exciting turbulence. However, the regions with higher magnetic fields (e.g., $10^{-9}$ G) can support Alfvenic-type turbulence with the injection scale $l_{\text{a}}$ and the injection velocities resulting from large-scale shear $V_{s}(l_{s}/l_{a}) \sim V_{s} L_{a}^{3}$.

For the regions of $B \sim 1 \mu$G the value of $l_{\text{a}}$ is smaller than the mean free path of electrons $\lambda$. According to equation (4) the value of $\kappa_{\text{elect}}$ is 100 times smaller than $\kappa_{\text{spitzer}}$. On the contrary, $\kappa_{\text{dyn}}$ for the ICM parameters adopted will be $\sim 30 \kappa_{\text{spitzer}}$, which makes the dynamic diffusivity the dominant process. This agrees well with the observations of Voigt & Fabian (2004). Figure 1 shows the dominance of advective heat transfer for the parameters of the cool core of Hydra A ($B = 6 \mu$G, $n_{i} = 0.056$ cm$^{-3}$, $L = 40$ kpc, $T = 2.7$ keV according to EV06), point “F” and for the illustrative model in EVP05, point “V”, for which $B = 1 \mu$G.

Note that our stationary model of MHD turbulence in § 2 is not directly applicable to transient wakes behind galaxies. The ratio of the damping times of the hydroturbulence and the time of straightening of the magnetic field lines is $\sim M_{a}^{-1}$. Thus,
for $M_A > 1$, the magnetic field at scales larger than $l_A$ will be gradually straightening after the hydroturbulence has faded away over time $L/V_c$. The process can be characterized as an injection of turbulence at velocity $V_c$ but at scales that increase linearly with time, i.e., as $l_A + V_c t$. The study of heat transfer in transient turbulence and magnetic field “regularly” stretched by passing galaxies will be provided elsewhere.

5. DISCUSSION AND SUMMARY

In this Letter we attempted to describe the heat transfer by electron and turbulent motions for $M_A < 1$ and $M_A > 1$. Unlike earlier papers, we find that turbulence may both enhance heat conduction and suppress it. For instance, when $\lambda$ gets larger than $l_A$, the conductivity of the medium $\sim M_A^{3/4}$ and therefore the turbulence inhibits heat transfer, provided that $k_{\text{elect}} > k_{\text{dyne}}$. Along with the plasma effects that we mention below, this effect can indeed support sharp temperature gradients in hot plasmas with weak magnetic field.

As discussed above, rarefied plasma, for example, ICM plasma, has large viscosity for motions parallel to magnetic field and marginal viscosity for motions that induce perpendicular mixing. Thus, fast dissipation of sound waves in the ICM does not contradict the medium being turbulent. The latter may be important for the heating of central regions of clusters caused by the active galactic nucleus (AGN) feedback (see Churazov et al. 2001, Nusser et al. 2006, and the references in EV06). Note that models that include both heat transfer from the outer hot regions and an additional heating from the AGN feedback look rather promising (see Ruzkowski & Begelman 2002; Piffaretti & Kaastra 2006). We predict that the viscosity for $1 \mu G$ regions is less than that for $10 \mu G$ regions, and therefore heating by sound waves (see Fabian et al. 2005) could be more efficient for the latter. Note that the plasma instabilities in collisionless magnetized ICM arising from compressive motions (see Schekochihin & Cowley 2006; Lazarian & Beresnyak 2006) can resonantly scatter electrons and protons and decrease $\lambda$ for both species compared to the classical plasma values ($\lambda$ gets different for electrons and protons in this case). This further decreases $k_{\text{elect}}$ compared to $k_{\text{Spitzer}}$ but increases $\text{Re}$. In addition, we disregarded mirror effects that can reflect electrons back (see Malyskin & Kulsrud 2001 and references therein), which can further decrease $k_{\text{elect}}$.

All in all, we have shown that it is impossible to characterize the heat transfer of magnetized plasma by a single fraction of Spitzer’s value. The actual heat transport depends on sonic and Alfvén Mach numbers of turbulence and may be much higher or much lower than the classical one. As the result, turbulence can inhibit or enhance heat conductivity, depending on the plasma magnetization and turbulence driving. Our study indicates that in many cases related to ICM, the advective heat transport by dynamic turbulent eddies dominates thermal conductivity.

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In the above-mentioned Letter, equation (8) has the wrong power of the Alfvén Mach number \( M_A \), which is defined as the ratio of the injection velocity to the Alfvén velocity. The correct expression is

\[
\kappa_{\perp, \text{electr}} \approx \frac{1}{4} v_{\text{electr}} \lambda M_A^4, \quad M_A < 1,
\]

where \( v_{\text{electr}} \) is the electron velocity and \( \lambda \) is electron mean free path. A simple justification for this equation is that, while diffusing along magnetic field lines over a scale \( L \), the electron diffuses the distance \( LM_A^2 \) in the direction perpendicular to magnetic field lines (see eq. [7] of the original Letter).

The correction strengthens the conclusions of the original Letter, namely, that in a turbulent magnetized medium, the transfer of thermal energy by an electron moving along turbulent magnetic fields is often subdominant compared to heat advection by turbulent motions. In particular, Figure 1 has been modified in order to demonstrate that the difference in the effective conductivities induced by the above two processes for cluster cores is even higher than the original Letter claimed. The data used for the points in the top right panel of the figure are the same as in the original Letter.

![Figure 1](image_url)

**Fig. 1.**—Sonic Mach number \( M_s \) is plotted against the Alfvén Mach number \( M_A \). The heat transport dominated by the dynamics of turbulent eddies is above the curve, and the heat transport dominated by the thermal conductivity of electrons is below the curve. Here \( \lambda \) is the mean free path of the electron, \( L \) is the driving scale, and \( \alpha = (m_e/m_i)^{1/2} \), \( \beta \approx 4 \). The panel in the top right corner of the figure illustrates heat transport for the parameters for a cool core Hydra cluster (point “F”); “V” corresponds to the illustrative model of a cluster core in Ensslin et al. (2005).