Accuracy and Grade Demonstration and Research of Elevation Control Network for Long distance Water Diversion Projects in Plateau

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Abstract: According to the characteristics of the Water Diversion Project in Central Yunnan, the application of error theory analysis and simulation calculation method, the influence value of the elevation error of the channel line on the longitudinal slope is obtained. It is deduced that the first-level elevation control network of the project should use the second-level control network. Combined with the analysis and calculation of the error in the weakest point elevation, it is analyzed that the project should use the third-level control network. Finally, through the valuation calculation, it is determined that the Water Diversion Project in Central Yunnan does not need to correct the lunisolar attraction in the elevation control network.

1. Introduction
The Water Diversion Project in Central Yunnan is the first of the key water conservancy landmark projects under construction in China. The project involves 35 cities and 6 districts. The average annual water intake is 3.403 billion cubic meters, and the water receiving area is 36,900 square kilometers[1]. The scale of the project is huge, and the magnificent project must have first-class surveying and mapping support, so that the design, construction and supervision and management units along the line can work under the control of the unified precision and standard framework[2,3]. In meanwhile, the construction control network is the basis of the surveying and mapping work, which can provide unified plane and elevation control benchmarks and precision guarantee for all parties involved in the project.

The length of the Water Diversion Project in Central Yunnan is more than 660km, and the water diversion line is from 2035m to 1400m above sea level. With the characteristics of long span of survey area, high average altitude and large variation of height difference, it is the largest plateau long-distance water transfer project in China at this stage. Among them, the study of the layout of its elevation control network is of great salience. Therefore, this paper systematically carries out the accuracy and grade demonstration of the elevation control network for the characteristics of the Water Diversion Project in Central Yunnan.

2. Layout Analysis of the First-level Elevation Control Network
The elevation control network is divided into a first-level elevation control network and an encrypted elevation control network. By applying the error theory to analyze the influence of the error of the channel elevation measurement on the longitudinal slope, the elevation measurement accuracy and
measurement level can be selected scientifically, reasonably and feasibly[4~6].

Let i be the slope of the channel line of the canal, h is the height difference between the two ends of the channel, and L is the length of the channel. Then:

\[ i = \frac{h}{L} \]

Expressed in logarithmic form as:

\[ \ln i = \ln h - \ln L \]

Differentiate equation (2):

\[ \frac{di}{i} = \frac{dh}{h} - \frac{dL}{L} \]

According to the law of error propagation, convert equation (3) to medium error:

\[ \left( \frac{m_i}{i} \right)^2 = \left( \frac{m_h}{h} \right)^2 + \left( \frac{m_L}{L} \right)^2 \]

Equation (4) shows the relationship between the slope of the channel line, the height difference, and the relative error of the length. It is assumed that the error caused by the measurement affects the slope of the channel line by no more than 1/10 of the slope, and that is:

\[ \frac{m_i}{i} \leq \frac{1}{10} \]

The length measurement error is generally up to 1/100,000, so the error affecting the slope of the channel line is mainly the height difference measurement error \( m_h \). Therefore, equation (5) can be approximated as:

\[ \frac{m_h}{h} \leq \frac{1}{10} \]

Substituting equation (1) into equation (6), the error allowed by the height difference measurement of the channel line is:

\[ m_h \leq \frac{1}{10} \times i \times L \]

The height difference error of the channel line is composed of basic elevation control error \( m_1 \), station elevation control error \( m_2 \), and elevation stakeout error \( m_3 \), which is:

\[ m_h = \left( m_1^2 + m_2^2 + m_3^2 \right)^{1/2} \]

If the elevation stake-out error is strictly required, it can reach ±5mm, and the precision gradient \( m_1/m_2=1/3 \), which can be obtained by formula (7) and formula (8):

\[ m_h \leq \left( \frac{(IL)^2}{1000} - 2.5 \right)^{1/2} \]

Equation (9) reflects the mathematical relationship between the measurement error of the first stage high control network and the length of the channel section and the length of the channel section. The longer the segment L considers, the greater the measurement error allows.

In addition, the medium error of the height difference of the level measurement is obtained as follows:

\[ m = M_w L^{1/2} \]

In the equation, \( M_w \) is the total error measured per kilometer level. The first value is 1.0mm, the second is 2.0mm, the third is 6.0mm, and the fourth is 10.0mm. It can be seen from equation (10) that the value of the error m in the height difference of the segment is also related to the value of L.
Therefore, the determination of the leveling level is closely related to the value of L. According to the requirements of the national leveling specification, a leveling stone should be buried every 4–8 km, that is, 4 km ≤ L ≤ 8 km. The L value is selected within this range and the corresponding level of measurement is selected. If it meets the needs of engineering construction and is easy to implement, it means that the length of the section considered is appropriate. Otherwise, the length of the canal can be recalculated and analyzed until a reasonable length of the canal is selected.

The slope i of the Water Diversion Project in Central Yunnan has a maximum of 1/1000 and a minimum of 1/15000. Therefore, i = 1/15000, 1/6000, 1/1000; L = 4 km; M_w = 2.0, 6.0, 10.0 are respectively substituted into equations (9) and (10), and the calculation results are shown in Table 1. In the table, m_1 is the error in the basic elevation control point, m_2 is the error in the station elevation control, and m_{II}, m_{III}, and m_{IV} are the medium errors in the height difference of the second, third, and fourth level measurement.

| Table 1. Measurement error table of L=4km channel height difference Unit: mm |
|------------------|---|---|---|---|---|
| i                | m_1 | m_2 | m_{II} | m_{III} | m_{IV} |
| 1/15000          | 8.3 | 24.9 |         |         |        |
| 1/6000           | 21.0| 63.0| 4       | 12      | 20     |
| 1/1000           | 126.5| 379.5|         |         |        |

It can be seen from Table 1 that when the distance between the elevation points is about 4 km, the construction control network of the channel line should select the second-level accuracy to meet the requirements of the construction of the water diversion project in the middle. However, the difference between m_1 and m_{II} is large, indicating that the value of L can be appropriately adjusted.

Further, the value of L is changed to 8 km, and the equations (9) and (10) are substituted. The results are shown in Table 2.

| Table 2. Measurement error table of L=8km channel height difference Unit: mm |
|------------------|---|---|---|---|---|
| i                | m_1 | m_2 | m_{II} | m_{III} | m_{IV} |
| 1/15000          | 16.8| 50.4|         |         |        |
| 1/6000           | 42.1| 126.3| 5.7    | 17.0    | 28.3   |
| 1/1000           | 253.0| 759.0|         |         |        |

It can be seen from Table 2 that when the distance between the elevation points is about 8 km, the third-level leveling control accuracy of the channel line elevation construction control network can basically meet the requirements of the construction of the water diversion project in the middle.

The above is an analysis based on theoretical values. In the actual elevation measurement of the water diversion project in the middle of the river, due to the location of the plateau, the elevation point spacing should not be too large. Therefore, it is recommended to use 4 to 8 km spacing and set the second-class standard as the first-level elevation control network for the main canal.

3. Layout Analysis of Encrypted Elevation Control Network

The purpose of the encryption elevation control network setting is to measure the elevation of the measurement station. Generally, an attachment level route is arranged between two adjacent basic level points. The weakest point in the route is in the middle of the level. The error in the weakest elevation is:

\[ m_{\text{weakest}} = \frac{1}{2} M_w S^{1/2} \]  \hspace{1cm} (11)

In the equation, S is the level line length and the unit is km.

Another \( m_{\text{weakest}} \geq m_2 \), take \( m_2 = m_{\text{weakest}} \), substituting it into equation (8) and taking into account equation (7):
\[ M_w \leq 2 \left( \frac{i^2 \times L^2 \times 10^{10} - m_1^2 - m_3^2}{S} \right)^{1/2} \]  \hspace{1cm} (12)

In the equation, \( L \) is the length of the channel, and \( S \) is the length of the level line, and the unit is km.

When the first-level elevation control adopts the second-class level measurement, \( m_1 = 4 \text{mm} \), and \( L = 4 \text{km} \), \( S = 8 \text{km} \). Take the elevation stakeout error \( m_3 = \pm 5 \text{mm} \), \( \pm 10 \text{mm} \) and \( \pm 20 \text{mm} \) respectively, substituting into equation (12), calculate the \( M_w \) value, and list the results in Table 3.

Table 3. Calculation of the error in the measurement of the level of each kilometer Unit: mm

| \( i \) | \( m_3 = \pm 5 \text{mm} \) | \( m_3 = \pm 10 \text{mm} \) | \( m_3 = \pm 20 \text{mm} \) |
|-------|----------------|----------------|----------------|
| 1/15000 | 18.3          | 17.2          | 12.1          |
| 1/6000  | 46.9          | 46.5          | 44.9          |
| 1/1000  | 282.8         | 282.7         | 282.5         |

In general, when the value of \( M_w \) is 6~10mm, the third-level encryption is used. When the value of \( M_w \) is 10~15mm, it is encrypted with four levels. When the value of \( M_w \) is above 15 mm, it is encrypted with five levels. According to the numerical values in Table 3, when the error value in the elevation stakeout is controlled within 10 mm, only the fifth-level encryption is used. However, when the error value in the elevation stakeout is higher than 10 mm, especially to a value of 20 mm, a fourth level of encryption is required. Therefore, the level of four or more can meet the accuracy requirements of the encrypted elevation control network.

On the other hand, due to the numerous buildings in the Water Diversion Project in Central Yunnan, the layout of its encrypted elevation network is mainly to meet the needs of building construction control. The tunnel project accounts for about 92% of the total length of the water diversion project. Therefore, meeting the tunnel elevation control network layout requirements, basically meet the requirements of all building elevation control network deployment. The average length of the tunnel in the Water Diversion Project in Central Yunnan is about 9.6km. According to the Specification for construction survey of water and hydropower projects[7], the elevation construction control network related to the long tunnel of 5km~20km should meet the second-class or third-class precision requirement. Besides, considering the demand for hierarchical network, it is recommended to design the encryption elevation control network, that is, the design level of the building's first-level elevation construction control network, to meet the elevation accuracy requirements of the ground control network for tunnel construction and other building construction measurements.

4. Error Analysis of the Lunisolar attraction in the Elevation Control Network

The direction of gravity at any point on the earth, in addition to the distribution of matter within the Earth and the centrifugal force generated by the Earth's rotation, is also affected by the sun, the moon, and other celestial bodies, as well as the Earth's outer atmosphere. Due to the effect of the lunisolar attraction, the vertical direction of the earth changes instantaneously, that is, the vertical line deviates from the effect, so the same tilt change occurs in the horizontal plane passing through the point. In addition, due to the influence of the lunisolar attraction, there will be ground deformation. In precision elevation leveling, the corrections that estimate the effects of lunisolar attraction are called tidal corrections. The data show that in extreme cases, the influence of lunisolar attraction does not exceed 0.1 mm per kilometer. In most cases, the error per kilometer is about 0.01 mm to 0.02 mm[8,9]. This error is not reflected in the segment closure difference, the route closure difference, and the loop closure difference, but this kind of error will accumulate in the north and south. For China, the cumulative value in the north-south direction can reach 12cm.
The water diversion project in the middle reaches about 370km. Under normal circumstances, according to the error of 0.01mm–0.02mm per kilometer of gravity, the cumulative error of the project can reach 3.7mm–7.4mm. This is much smaller than the accuracy requirements of the first-level elevation control network and the encrypted elevation control network analyzed in the previous section, so it can be ignored. Therefore, the tidal correction is not a must in the Water Diversion Project in Central Yunnan.

5. Conclusion
In this paper, the accuracy and grade demonstration of the elevation control network are systematically carried out on the basis of the characteristics of the Water Diversion Project in Central Yunnan. By analyzing the influence of the channel height measurement error on the longitudinal slope and the simulation results, it is recommended that the first-level elevation control network of the main channel use the second-level level control network. Combined with the analysis and calculation of the error in the weakest point elevation, it is recommended to use the third-class level to encrypt the elevation control network. Finally, through the valuation calculation, it is determined that the Water Diversion Project in Central Yunnan does not need tidal correction of the elevation control network.

Generally speaking, it provides theoretical support for the design and establishment of the elevation control network of the Water Diversion Project in Central Yunnan, and serves the construction of major national water conservancy projects with high precision.

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