Complexity and Approximation of Fixing Numerical Attributes in Databases Under Integrity Constraints

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Abstract. Consistent query answering is the problem of computing the answers from a database that are consistent with respect to certain integrity constraints that the database as a whole may fail to satisfy. Those answers are characterized as those that are invariant under minimal forms of restoring the consistency of the database. In this context, we study the problem of repairing databases by fixing integer numerical values at the attribute level with respect to denial and aggregation constraints. We introduce a quantitative definition of database fix, and investigate the complexity of several decision and optimization problems, including DFP, i.e. the existence of fixes within a given distance from the original instance, and CQA, i.e. deciding consistency of answers to aggregate conjunctive queries under different semantics. We provide sharp complexity bounds, identify relevant tractable cases; and introduce approximation algorithms for some of those that are intractable. More specifically, we obtain results like undecidability of existence of fixes for aggregation constraints; MAXSNP-hardness of DFP, but a good approximation algorithm for a relevant special case; and intractability but good approximation for CQA for aggregate queries for one database atom denials (plus built-ins).

1 Introduction

Integrity constraints (ICs) are used to impose semantics on a database with the purpose of making the database an accurate model of an application domain. Database management systems or application programs enforce the satisfaction of the ICs by rejecting undesirable updates or executing additional compensating actions. However, there are many situations where we need to interact with databases that are inconsistent in the sense that they do not satisfy certain desirable ICs. In this context, an important problem in database research consists in characterizing and retrieving consistent data from inconsistent databases [4], in particular consistent answers to queries. From the logical point of view,

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consistently answering a query posed to an inconsistent database amounts to evaluating the truth of a formula against a particular class of first-order structures [2], as opposed to the usual process of truth evaluation in a single structure (the relational database).

Certain database applications, like census, demographic, financial, and experimental data, contain quantitative data, usually associated to nominal or qualitative data, e.g. number of children associated to a household identification code (or address); or measurements associated to a sample identification code. Usually this kind of data contains errors or mistakes with respect to certain semantic constraints. For example, a census form for a particular household may be considered incorrect if the number of children exceeds 20; or if the age of a parent is less than 10. These restrictions can be expressed with denial integrity constraints, that prevent some attributes from taking certain values [10]. Other restrictions may be expressed with aggregation ICs, e.g. the maximum concentration of certain toxin in a sample may not exceed a certain specified amount; or the number of married men and married women must be the same. Inconsistencies in numerical data can be resolved by changing individual attribute values, while keeping values in the keys, e.g without changing the household code, the number of children is decreased considering the admissible values.

We consider the problem of fixing integer numerical data wrt certain constraints while (a) keeping the values for the attributes in the keys of the relations, and (b) minimizing the quantitative global distance between the original and modified instances. Since the problem may admit several global solutions, each of them involving possibly many individual changes, we are interested in characterizing and computing data and properties that remain invariant under any of these fixing processes. We concentrate on denial and aggregation constraints; and conjunctive queries, with or without aggregation.

Database repairs have been studied in the context of consistent query answering (CQA), i.e. the process of obtaining the answers to a query that are consistent wrt a given set of ICs [2] (c.f. [4] for a survey). There, consistent data is characterized as invariant under all minimal forms of restoring consistency, i.e. as data that is present in all minimally repaired versions of the original instance (the repairs). Thus, an answer to a query is consistent if it can be obtained as a standard answer to the query from every possible repair. In most of the research on CQA, a repair is a new instance that satisfies the given ICs, but differs from the original instance by a minimal set, under set inclusion, of (completely) deleted or inserted tuples. Changing the value of a particular attribute can be modelled as a deletion followed by an insertion, but this may not correspond to a minimal repair. However, in certain applications it may make more sense to correct (update) numerical values only in certain attributes. This requires a new definition of repair that considers: (a) the quantitative nature of individual changes, (b) the association of the numerical values to other key values; and (c) a quantitative distance between database instances.
Example 1. Consider a network traffic database $D$ that stores flow measurements of links in a network. This network has two types of links, labelled 0 and 1, with maximum capacities 1000 and 1500, resp.

| Traffic | Time | Link | Type | Flow |
|---------|------|------|------|------|
| 1.1     | a    | 0    | 1100 |
| 1.1     | b    | 1    | 900  |
| 1.3     | b    | 1    | 850  |

Database $D$ is inconsistent wrt this IC. Under the tuple and set oriented semantics of repairs [2], there is a unique repair, namely deleting tuple $Traffic(1.1, a, 0, 1100)$. However, we have two options that may make more sense than deleting the flow measurement, namely updating the violating tuple to $Traffic(1.1, a, 0, 1000)$ or to $Traffic(1.1, a, 1, 1100)$; satisfying an implicit requirement that the numbers should not change too much. □

Update-based repairs for restoring consistency are studied in [24]; where changing values in attributes in a tuple is made a primitive repair action; and semantic and computational problems around CQA are analyzed from this perspective. However, peculiarities of changing numerical attributes are not considered, and more importantly, the distance between databases instances used in [24, 25] is based on set-theoretic homomorphisms, but not quantitative, as in this paper. In [24] the repaired instances are called fixes, a term that we keep here (instead of repairs), because our basic repair actions are also changes of (numerical) attribute values. In this paper we consider fixable attributes that take integer values and the quadratic, Euclidean distance $L_2$ between database instances. Specific fixes and approximations may be different under other distance functions, e.g. the “city distance” $L_1$ (the sum of absolute differences), but the general (in)tractability and approximation results remain. However, moving to the case of real numbers will certainly bring new issues that require different approaches; they are left for ongoing and future research. Actually it would be natural to investigate them in the richer context of constraint databases [17].

The problem of attribute-based correction of census data forms is addressed in [10] using disjunctive logic programs with stable model semantics. Several underlying and implicit assumptions that are necessary for that approach to work are made explicit and used here, extending the semantic framework of [10].

We provide semantic foundations for fixes that are based on changes on numerical attributes in the presence of key dependencies and wrt denial and aggregate ICs, while keeping the numerical distance to the original database to a minimum. This framework introduces new challenging decision and optimization problems, and many algorithmic and complexity theoretic issues. We concentrate in particular on the “Database Fix Problem” ($DFP$), of determining the existence of a fix at a distance not bigger than a given bound, in particular considering the problems of construction and verification of such a fix. These problems are highly relevant for large inconsistent databases. For example, solving $DFP$ can help us find the minimum distance from a fix to the original instance; information that can be used to prune impossible branches in the process of materialization of a fix. The CQA problem of deciding the consistency of query answers is studied wrt decidability, complexity, and approximation under several alternative semantics.
We prove that $DFP$ and $CQA$ become undecidable in the presence of aggregation constraints. However, $DFP$ is $NP$-complete for linear denials, which are enough to capture census like applications. $CQA$ belongs to $Π_2^p$ and becomes $Δ_2^p$-hard, but for a relevant class of denials we get tractability of $CQA$ to non aggregate queries, which is again lost with aggregate queries. With approximation, we prove that $DFP$ is $MAXSNP$-hard in general, and for a relevant subclass of denials we provide an approximation within a constant factor that depends on the number of atoms in them. All the algorithmic and complexity results, unless otherwise stated, refer to data complexity [1], i.e. to the size of the database that here includes a binary representation for numbers. For complexity theoretic definitions and classical results we refer to [20].

This paper is structured as follows. Section 2 introduces basic definitions. Sections 3 presents the notion of database fix, several notions of consistent answer to a query; and some relevant decision problems. Section 4 investigates their complexity. In Section 5 approximations for the problem of finding the minimum distance to a fix are studied, obtaining negative results for the general case, but good approximation for the class of local denial constraints. Section 6 investigates tractability of $CQA$ for conjunctive queries and denial constraints containing one database atom plus built-ins. Section 7 presents some conclusions and refers to related work. Proofs and other auxiliary, technical results can be found in Appendix A.1.

2 Preliminaries

Consider a relational schema $Σ = (U, R, B, A)$, with domain $U$ that includes $\mathbb{Z}$, $R$ a set of database predicates, $B$ a set of built-in predicates, and $A$ a set of attributes. A database instance is a finite collection $D$ of database tuples, i.e. of ground atoms $P(\bar{c})$, with $P ∈ R$ and $\bar{c}$ a tuple of constants in $U$. There is a set $F ⊆ A$ of all the fixable attributes, those that take values in $\mathbb{Z}$ and are allowed to be fixed. Attributes outside $F$ are called rigid. $F$ need not contain all the numerical attributes, that is we may also have rigid numerical attributes.

We also have a set $K$ of key constraints expressing that relations $R ∈ R$ have a primary key $K_R$, $K_R ⊆ (A \setminus F)$. Later on (c.f. Definition 2), we will assume that $K$ is satisfied both by the initial instance $D$, denoted $D \models K$, and its fixes. Since $F \cap K_R = \emptyset$, values in key attributes cannot be changed in a fixing process; so the constraints in $K$ are hard. In addition, there may be a separate set $IC$ of flexible ICs that may be violated, and it is the job of a fix to restore consistency wrt them (while still satisfying $K$).

A linear denial constraint [17] has the form $\forall \bar{x}.¬(A_1 \land \ldots \land A_m)$, where the $A_i$ are database atoms (i.e. with predicate in $R$), or built-in atoms of the form $xθc$, where $x$ is a variable, $c$ is a constant and $θ ∈ \{=, \neq, <, >, ≤, ≥\}$, or $x = y$. If $x ≠ y$ is allowed, we call them extended linear denials.

Example 2. The following are linear denials (we replace $\land$ by a comma): (a) No customer is younger than 21: $\forall Id, Age, Income, Status.¬\langle Customer(Id, Age,$

\footnote{With simple denial constraints, numbers can be restricted to, e.g. $\mathbb{N}$ or $\{0, 1\}$.}
We consider aggregation constraints (ACs) \[22\] and aggregate queries with sum, count, average. Filtering ACs impose conditions on the tuples over which aggregation is applied, e.g. \( \text{sum}(A_1 : A_2 = 3) > 5 \) is a sum over \( A_1 \) of tuples with \( A_2 = 3 \). Multi-attribute ACs allow arithmetical combinations of attributes as arguments for \( \text{sum} \), e.g. \( \text{sum}(A_1 + A_2) > 5 \) and \( \text{sum}(A_1 \times A_2) > 100 \). If an AC has attributes from more than one relation, it is multi-relation, e.g. \( \text{sum}_{R_3}(A_1) = \text{sum}_{R_5}(A_1) \), otherwise it is single-relation.

An aggregate conjunctive query has the form \( q(x_1, \ldots, x_m; \text{agg}(z)) \leftarrow B(x_1, \ldots, x_m, z, y_1, \ldots, y_n) \), where \( \text{agg} \) is an aggregation function and its non-aggregate matrix (NAM) given by \( q'(x_1, \ldots, x_m) \leftarrow B(x_1, \ldots, x_m, z, y_1, \ldots, y_n) \) is a usual first-order (FO) conjunctive query with built-in atoms, such that the aggregation attribute \( z \) does not appear among the \( x_i \). Here we use the set semantics. An aggregate conjunctive query is cyclic (acyclic) if its NAM is cyclic (acyclic) \[1\].

Example 3. \( q(x, y, \text{sum}(z)) \leftarrow R(x, y), Q(y, z, w), w \neq 3 \) is an aggregate conjunctive query, with aggregation attribute \( z \). Each answer \( (x, y) \) to its NAM, i.e. to \( q(x, y) \leftarrow R(x, y), Q(y, z, w), w \neq 3 \), is expanded to \( (x, y, \text{sum}(z)) \) as an answer to the aggregate query. \( \text{sum}(z) \) is the sum of all the values for \( z \) having a \( w \), such that \( (x, y, z, w) \) makes \( R(x, y), Q(y, z, w), w \neq 3 \) true. In the database instance \( D = \{ R(1, 2), R(2, 3), Q(2, 5, 9), Q(2, 6, 7), Q(3, 1, 1), Q(3, 1, 5), Q(3, 8, 3) \} \) the answer set for the aggregate query is \( \{(1, 2, 5 + 6), (2, 3, 1 + 1)\} \).

An aggregate comparison query is a sentence of the form \( q(\text{agg}(z)), \text{agg}(z) \theta k \), where \( q(\text{agg}(z)) \) is the head of a scalar aggregate conjunctive query (with no free variables), \( \theta \) is a comparison operator, and \( k \) is an integer number. For example, the following is an aggregate comparison query asking whether the aggregated value obtained via \( q(\text{sum}(z)) \) is bigger than 5: \( Q : q(\text{sum}(z)), \text{sum}(z) > 5 \), with \( q(\text{sum}(z)) \leftarrow R(x, y), Q(y, z, w), w \neq 3 \).

3 Least Squares Fixes

When we update numerical values to restore consistency, it is desirable to make the smallest overall variation of the original values, while considering the relative relevance or specific scale of each of the fixable attributes. Since the original instance and a fix will share the same key values (c.f. Definition 2), we can use them to compute variations in the numerical values. For a tuple \( k \) of values for the key \( K_R \) of relation \( R \) in an instance \( D, \bar{t}(k, R, D) \) denotes the unique tuple \( \bar{t} \) in relation \( R \) in instance \( D \) whose key value is \( k \). To each attribute \( A \in \mathcal{F} \) a fixed numerical weight \( \alpha_A \) is assigned.

**Definition 1.** For instances \( D \) and \( D' \) over schema \( \Sigma \) with the same set \( \text{val}(K_R) \) of tuples of key values for each relation \( R \in \mathcal{R} \), their square distance is

\[
\Delta_\alpha(D, D') = \sum_{k \in \text{val}(K_R)} \alpha_A \left( \pi_A(\bar{t}(k, R, D)) - \pi_A(\bar{t}(k, R, D')) \right)^2,
\]

where \( \pi_A \) is the projection on attribute \( A \) and \( \bar{a} = (\alpha_A)_{A \in \mathcal{F}} \).
Definition 2. For an instance $D$, a set of fixable attributes $\mathcal{F}$, a set of key dependencies $\mathcal{K}$, such that $D \models \mathcal{K}$, and a set of flexible ICs $\mathcal{IC}$: A fix for $D$ wrt $\mathcal{IC}$ is an instance $D'$ such that: (a) $D'$ has the same schema and domain as $D$; (b) $D'$ has the same values as $D$ in the attributes in $A \setminus \mathcal{F}$; (c) $D' \models \mathcal{K}$; and (d) $D' \models IC$. A least squares fix (LS-fix) for $D$ is a fix $D'$ that minimizes the square distance $\Delta_{\bar{\alpha}}(D, D')$ over all the instances that satisfy (a) - (d).

In general we are interested in LS-fixes, but (non-necessarily minimal) fixes will be useful auxiliary instances.

Example 4. (example 1 cont.) $\mathcal{R} = \{\text{Traffic}, A = \{\text{Time}, \text{Link}, \text{Type}, \text{Flow}\}$, $\mathcal{K}_{\text{Traffic}} = \{\text{Time}, \text{Link}\}$, $\mathcal{F} = \{\text{Type}, \text{Flow}\}$, with weights $\bar{\alpha} = (10^{-5}, 1)$, resp. For original instance $D$, $\text{val}(\mathcal{K}_{\text{Traffic}}) = \{(1.1, a), (1.1, b), (1.3, b)\}$, $\text{val}(\{\text{Time}, \text{Flow}\}) = (1, 1, a, 0, 1100)$, etc. Fixes are $D_1 = \{(1.1, a, 0, 1000), (1.1, b, 1, 900), (1.3, b, 1, 850)\}$ and $D_2 = \{(1.1, a, 1, 1100), (1.1, b, 1, 900), (1.3, b, 1, 850)\}$, with distances $\Delta_{\bar{\alpha}}(D, D_1) = 100^2 \times 10^{-5} = 10^{-1}$ and $\Delta_{\bar{\alpha}}(D, D_2) = 1^2 \times 1$, resp. Therefore, $D_1$ is the only LS-fix.

The coefficients $\alpha_i$ can be chosen in many different ways depending on factors like relative relevance of attributes, actual distribution of data, measurement scales, etc. In the rest of this paper we will assume, for simplification, that $\alpha_i = 1$ for all $A \in \mathcal{F}$ and $\Delta_{\bar{\alpha}}(D, D')$ will be simply denoted by $\Delta(D, D')$.

Example 5. The database $D$ has relations $\text{Client}(ID, A, M)$, with key $ID$, attributes $A$ for age and $M$ for amount of money; and $\text{Buy}(ID, I, P)$, with key $\{ID, I\}$, $I$ for items, and $P$ for prices. We have denials $\text{IC}_1 : \forall ID, P, A, M \models \neg(\text{Buy}(ID, I, P), \text{Client}(ID, A, M), A < 18, P > 25)$ and $\text{IC}_2 : \forall ID, A, M \models \neg(\text{Client}(ID, A, M), A < 18, M > 50)$, requiring that people younger than 18 cannot not spend more than 25 on one item nor spend more than 50 in the store.

We added an extra column in the tables with a label for each tuple. $\text{IC}_1$ is violated by $\{t_1, t_4\}$ and $\{t_1, t_5\}$; and $\text{IC}_2$ by $\{t_1\}$ and $\{t_2\}$. We have two LS-fixes (the modified version of tuple $t_1$ is $t_1'$, etc.), with distances $\Delta(D, D') = 2^2 + 1^2 + 2^2 + 1^2 = 10$, and $\Delta(D, D'') = 3^2 + 1^2 = 10$. We can see that a global fix may not be the result of applying “local” minimal fixes to tuples.
The built-in atoms in linear denials determine a solution space for fixes as an intersection of semi-spaces, and LS-fixes can be found at its “borders” (c.f. previous example and Proposition A.1 in Appendix A.1). It is easy to construct examples with an exponential number of fixes. For the kind of fixes and ICs we are considering, it is possible that no fix exists, in contrast to [2, 3], where, if the set of ICs is consistent as a set of logical sentences, a fix for a database always exist.

Example 6. \( R(X, Y) \) has key \( X \) and fixable \( Y \). \( IC_1 = \{ \forall X_1 X_2 Y \neg (R(X_1, Y), R(X_2, Y), X_1 = 1, X_2 = 2), \forall X_1 X_2 Y \neg (R(X_1, Y), R(X_2, Y), X_1 = 1, X_2 = 3), \forall X_1 X_2 Y \neg (R(X_1, Y), R(X_2, Y), X_1 = 2, X_2 = 3), \forall X Y \neg (R(X, Y), Y > 3), \forall X Y \neg (R(X, Y), Y < 2) \} \) is consistent. The first three ICs force \( Y \) to be different in every tuple. The last two ICs require \( 2 \leq Y \leq 3 \). The inconsistent database \( R = \{(\bar{1}, -1), (2, 1), (3, 5)\} \) has no fix. Now, for \( IC_2 \) with \( \forall X, Y \neg (R(X, Y), Y > 1) \) and \( sum(Y) = 10 \), any database with less than 10 tuples has no fixes. \( \Box \)

Proposition 1. If \( D \) has a fix wrt \( IC \), then it also has an LS-fix wrt \( IC \). \( \Box \)

4 Decidability and Complexity

In applications where fixes are based on changes of numerical values, computing concrete fixes is a relevant problem. In databases containing census forms, correcting the latter before doing statistical processing is a common problem [10]. In databases with experimental samples, we can fix certain erroneous quantities as specified by linear ICs. In these cases, the fixes are relevant objects to compute explicitly, which contrasts with CQA [2], where the main motivation for introducing repairs is to formally characterize the notion of a consistent answer to a query as an answer that remains under all possible repairs. In consequence, we now consider some decision problems related to existence and verification of LS-fixes, and to CQA under different semantics.

Definition 3. For an instance \( D \) and a set \( IC \) of ICs:
(a) \( \text{Fix}(D, IC) := \{D' | D' \text{ is an LS-fix of } D \text{ wrt } IC\} \), the fix checking problem.
(b) \( \text{Fix}(IC) := \{(D, D') | D' \in \text{Fix}(D, IC)\} \).
(c) \( NE(IC) := \{D | \text{Fix}(D, IC) \neq \emptyset\} \), for non-empty set of fixes, i.e. the problem of checking existence of LS-fixes.
(d) \( NE := \{(D, IC) | \text{Fix}(D, IC) \neq \emptyset\} \).
(e) \( DFP(IC) := \{(D, k) | \text{there is } D' \in \text{Fix}(D, IC) \text{ with } \Delta(D, D') \leq k\} \), the database fix problem, i.e. the problem of checking existence of LS-fixes within a given positive distance \( k \).
(f) \( DFOP(IC) \) is the optimization problem of finding the minimum distance from an LS-fix wrt \( IC \) to a given input instance. \( \Box \)

Definition 4. Let \( D \) be a database, \( IC \) a set ICs, and \( Q \) a conjunctive query\(^2\).
(a) A ground tuple \( \bar{t} \) is a consistent answer to \( Q(\bar{x}) \) under the: (a1) skeptical semantics if for every \( D' \in \text{Fix}(D, IC) \), \( D' \models Q(\bar{t}) \). (a2) brave semantics if there

\(^2\) Whenever we say just “conjunctive query” we understand it is a non aggregate query.
exists $D' \in \text{Fix}(D, IC)$ with $D' \models Q(\bar{t})$. (a3) majority semantics if $|\{D' \mid D' \in \text{Fix}(D, IC) \text{ and } D' \models Q(\bar{t})\}| > |\{D' \mid D' \in \text{Fix}(D, IC) \text{ and } D' \not\models Q(\bar{t})\}|$.

(b) That $\bar{t}$ is a consistent answer to $Q$ in $D$ under semantics $S$ is denoted by $D \models_S Q$. If $Q$ is ground and $D \models_S Q$, we say that yes is a consistent answer, meaning that $Q$ is true in the fixes of $D$ according to semantics $S$. $\text{CA}(Q, D, IC, S)$ is the set of consistent answers to $Q$ in $D$ wrt IC under semantics $S$. For ground $Q$, if $\text{CA}(Q, D, IC, S) \neq \{\text{yes}\}$, $\text{CA}(Q, D, IC, S) := \{\text{no}\}$.

(c) $\text{CQA}(Q, IC, S) := \{(D, \bar{t}) \mid \bar{t} \in \text{CA}(Q, D, IC, S)\}$ is the decision problem of consistent query answering, of checking consistent answers.

**Proposition 2.** NE(IC) can be reduced in polynomial time to the complements of $\text{CQA}(\text{False, IC, Skeptical})$ and $\text{CQA}(\text{True, IC, Majority})$, where False, True are ground queries that are always false, resp. true.

In Proposition 2, it suffices for queries False, True to be false, resp. true, in all instances that share the key values with the input database. Then, they can be represented by $\exists Y R(\bar{c}, Y)$, where $\bar{c}$ are not (for False), or are (for True) key values in the original instance.

**Theorem 1.** Under extended linear denials and complex, filtering, multi-attribute, single-relation, aggregation constraints, the problems NE of existence of LS-fixes, and CQA under skeptical or majority semantics are undecidable.

The result about NE can be proved by reduction from the undecidable Hilbert’s problem on solvability of diophantine equations. For CQA, apply Proposition 2. Here we have the original database and the set of ICs as input parameters. In the following we will be interested in data complexity, when only the input database varies and the set of ICs is fixed [1].

**Theorem 2.** For a fixed set $IC$ of linear denials: (a) Deciding if for an instance $D$ there is an instance $D'$ (with the same key values as $D$) that satisfies $IC$ with $\Delta(D, D') \leq k$, with positive integer $k$ that is part of the input, is in NP. (b) $\text{DFP}(IC)$ is NP-complete. (c.f. Definition 3(e))

By Proposition 1, there is a fix for $D$ wrt $IC$ at a distance $\leq k$ iff there is an LS-fix at a distance $\leq k$. Part (b) of Theorem 2 follows from part (a) and a reduction of Vertex Cover to $\text{DFP}(IC_0)$, for a fixed set of denials $IC_0$. By Theorem 2(a), if there is a fix at a distance $\leq k$, the minimum distance to $D$ for a fix can be found by binary search in $\log(k)$ steps. Actually, if an LS-fix exists, its square distance to $D$ is polynomially bounded by the size of $D$ (c.f. proof of Theorem 3). Since $D$ and a fix have the same number of tuples, only the size of their values in a fix matter, and they are constrained by a fixed set of linear denials and the condition of minimality.

**Theorem 3.** For a fixed set $IC$ of extended linear denials: (a) The problem NE(IC) of deciding if an instance has an LS-fix wrt $IC$ is NP-complete, and (b) CQA under the skeptical and the majority semantics is coNP-hard.
For hardness in (a), (b) in Theorem 3, linear denials are good enough. Membership in (a) can be obtained for any fixed finite set of extended denials. Part (b) follows from part (a). The latter uses a reduction from 3-Colorability.

**Theorem 4.** For a fixed set \( IC \) of extended linear denials: (a) The problem \( \text{Fix}(IC) \) of checking if an instance is an LS-fix is \( \text{coNP} \)-complete, and (b) \( \text{CQA} \) under skeptical semantics is in \( \Pi_2^P \), and, for ground atomic queries, \( \Delta_2^P \)-hard. \( \square \)

Part (a) uses 3SAT. Hardness in (b) is obtained by reduction from a \( \Delta_2^P \)-complete decision version of the problem of searching for the lexicographically Maximum 3-Satisfying Assignment (M3SA): Decide if the last variable takes value 1 in it [16, Theo. 3.4]. Linear denials suffice. Now, by reduction from the Vertex Cover Problem, we obtain

**Theorem 5.** For aggregate comparison queries using \( \text{sum} \), \( \text{CQA} \) under linear denials and brave semantics is \( \text{coNP} \)-hard. \( \square \)

## 5 Approximation for the Database Fix Problem

We consider the problem of finding a good approximation for the general optimization problem \( \text{DFOP}(IC) \).

**Proposition 3.** For a fixed set of linear denials \( IC \), \( \text{DFOP}(IC) \) is \( \text{MAXSNP} \)-hard. \( \square \)

This result is obtained by establishing an \( L \)-reduction to \( \text{DFOP}(IC) \) from the \( \text{MAXSNP} \)-complete [21, 20] B-Minimum Vertex Cover Problem, i.e. the vertex cover minimization problem for graphs of bounded degree [15, Chapter 10]. As an immediate consequence, we obtain that \( \text{DFOP}(IC) \) cannot be uniformly approximated within arbitrarily small constant factors [20].

**Corollary 1.** Unless \( P = NP \), there is no Polynomial Time Approximation Schema for \( \text{DFOP} \). \( \square \)

This negative result does not preclude the possibility of finding an efficient algorithm for approximation within a constant factor for \( \text{DFOP} \). Actually, in the following we do this for a restricted but still useful class of denial constraints.

### 5.1 Local denials

**Definition 5.** A set of linear denials \( IC \) is **local** if: (a) Attributes participating in equality atoms between attributes or in joins are all rigid; (b) There is a built-in atom with a fixable attribute in each element of \( IC \); (c) No attribute \( A \) appears in \( IC \) both in comparisons of the form \( A < c_1 \) and \( A > c_2 \).\(^3\)

\(^3\)To check condition (c), \( x \leq c, x \geq c, x \neq c \) have to be expressed using \( <, > \), e.g. \( x \leq c \text{ by } x < c + 1 \).
In Example 5, IC is local. In Example 6, IC₁ is not local. Local constraints have the property that by doing local fixes, no new inconsistencies are generated, and there is always an LS-fix wrt to them (c.f. Proposition A.2 in Appendix A.1). Locality is a sufficient, but not necessary condition for existence of LS-fixes as can be seen from the database \( \{P(a,2)\} \), with the first attribute as a key and non-local denials \( \neg(P(x,y), y < 3), \neg(P(x,y), y > 5) \), that has the LS-fix \( \{P(a,3)\} \).

**Proposition 4.** For the class of local denials, DFP is NP-complete, and DFOP is MAXSNP-hard.

This proposition tells us that the problem of finding good approximations in the case of local denials is still relevant.

**Definition 6.** A set \( I \) of database tuples from \( D \) is a violation set for \( ic \in IC \) if \( I \not\models ic \), and for every \( I' \subsetneq I \), \( I' \models ic \). \( \mathcal{I}(D, ic, t) \) denotes the set of violation sets for \( ic \) that contain tuple \( t \). □

A violation set \( I \) for \( ic \) is a minimal set of tuples that simultaneously participate in the violation of \( ic \).

**Definition 7.** Given an instance \( D \) and ICs \( IC \), a local fix for \( t \in D \), is a tuple \( t' \) with: (a) the same values for the rigid attributes as \( t \); (b) \( S(t,t') := \{I \mid \text{there is } ic \in IC, I \in \mathcal{I}(D, ic, t) \text{ and } (I \setminus \{t\} \cup \{t'\}) \models ic \} \not= \emptyset \); and (c) there is no tuple \( t'' \) that simultaneously satisfies (a), \( S(t,t'') = S(t,t') \), and \( \Delta(\{t\}, \{t''\}) \leq \Delta(\{t\}, \{t'\}) \), where \( \Delta \) denotes quadratic distance. □

\( S(t,t') \) contains the violation sets that include \( t \) and are solved by replacing \( t' \) for \( t \). A local fix \( t' \) solves some of the violations due to \( t \) and minimizes the distance to \( t \).

### 5.2 Database fix problem as a set cover problem

For a fixed set \( IC \) of local denials, we can solve an instance of DFOP by transforming it into an instance of the Minimum Weighted Set Cover Optimization Problem (MWSCP). This problem is MAXSNP-hard [19, 20], and its general approximation algorithms are within a logarithmic factor [19, 8]. By concentrating on local denials, we will be able to generate a version of the MWSCP that can be approximated within a constant factor (c.f. Section 5.3).

**Definition 8.** For a database \( D \) and a set \( IC \) of local denials, \( G(D, IC) = (T, H) \) denotes the conflict hypergraph for \( D \) wrt \( IC \) [7], which has in the set \( T \) of vertices the database tuples, and in the set \( H \) of hyperedges, the violation sets for elements \( ic \in IC \).

Hyperedges in \( H \) can be labelled with the corresponding \( ic \), so that different hyperedges may contain the same tuples. Now we build an instance of \( MWSCP \).
Proposition 7. The transformation of DFOP into MWSCP, and the construction of database instance $D(C)$ from a cover $C$ for $(U, S, w)$ can be done in polynomial time in the size of $D$. 

Example 7. (example 5 continued) We illustrate the reduction from DFOP to MWSCP. The violation sets are $\{t_1, t_4\}$ and $\{t_1, t_5\}$ for IC$_1$ and $\{t_2\}$ for IC$_2$. The figure shows the hypergraph. For the MWSCP instance, we need the local fixes. Tuple $t_1$ has two local fixes $t'_1 = (1, 15, 50)$, that solves the violation set $\{t_1\}$ of IC$_2$ (hyperedge B), and $t''_1 = (1, 18, 52)$, that solves the violation sets $\{t_1, t_4\}$ and $\{t_1, t_5\}$ of IC$_1$, and $\{t_1\}$ of IC$_2$ (hyperedges A, B, C), with weights 4 and 9, resp. $t_2$, $t_4$ and $t_5$ have one local fix each corresponding to: $(2, 16, 50)$, $(1, CD, 25)$ and $(1, DVD, 25)$, resp. The consistent tuple $t_3$ has no local fix.
The MWSCP instance is shown in the table, where the elements are rows and
the sets (e.g. $S_1 = S(t_1, t'_1)$), columns. An entry 1 means that the set contains
the corresponding element; and a 0, otherwise. There are two minimal covers,
both with weight 10: $C_1 = \{S_2, S_3\}$ and $C_2 = \{S_1, S_3, S_4, S_5\}$. $D(C_1)$ and $D(C_2)$
are the two fixes for this problem.

If we apply the transformation to Example 6, that had non-local set of ICs and
no repairs, we will find that instance $D(C)$, for $C$ a set cover, can be constructed
as above, but it does not satisfy the flexible ICs, because changing inconsistent
tuples by their local fixes solves only the initial inconsistencies, but new
inconsistencies are introduced.

5.3 Approximation via set cover optimization

Now that we have transformed the database fix problem into a weighted set cover
problem, we can apply approximation algorithms for the latter. We know, for
example, that using a greedy algorithm, MWSCP can be approximated within a
factor $\log(N)$, where $N$ is the size of the underlying set $U$ [8]. The approximation
algorithm returns not only an approximation $\hat{w}$ to the optimal weight $w^*$, but
also a -non necessarily optimal- cover $\hat{C}$ for problem $(U, S, w)$. As in Definition
10, $\hat{C}$ can be used to generate via $(\hat{C})^\star$, a fix $D(\hat{C})$ for $D$ that may not be LS-
minimal.

Example 8. (examples 5 and 7 continued) We show how to to compute a solution
to this particular instance of DFOP using the greedy approximation algorithm
for MWSCP presented in [8]. We start with $\hat{C} := \emptyset$, $S_1^0 := S_1$; and we add to $C$ the
$S_i$ such that $S_i^0$ has the maximum contribution ratio $|S_i^0|/w(S_i^0)$. The alternatives
are $|S_1|/w(S_1) = 1/4$, $|S_2|/w(S_2) = 3/9$, $|S_3|/w(S_3) = 1$, $|S_4|/w(S_4) = 1/4$ and
$|S_5|/w(S_5) = 1$. The ratio is maximum for $S_3$ and $S_5$, so we can add any of them
to $C$. If we choose the first, we get $\hat{C} = \{S_3\}$. Now we compute the $S_1^1 := S_1^0 \setminus S_3^0$,
and choose again an $S_i$ for $\hat{C}$ such that $S_i^1$ maximizes the contribution ratio. Now $S_5$ is added to $\hat{C}$, because $S_5^1$ gives the maximum. By repeating this process until
we get all the elements of $U$ covered, i.e. all the $S_i^k$ become empty at some
iteration point $k$, we finally obtain $\hat{C} = \{S_3, S_5, S_1, S_4\}$. In this case $\hat{C}$ is an
optimal cover and therefore, $D(\hat{C})$ is exactly an LS-fix, namely $D'$ in Example 5.
Since this is an approximation algorithm, in other examples the cover obtained
might not be optimal. □
Proposition 8. Given database instance $D$ with local ICs $IC$, the database instance $D(\hat{C})$ obtained from the approximate cover $\hat{C}$ is a fix and it holds $\Delta(D, D(\hat{C})) \leq \log(N) \times \Delta(D, D')$, where $D'$ is any LS-fix of $D$ wrt $IC$ and $N$ is the number of of violation sets for $D$ wrt $IC$. \hfill \Box

In consequence, for any set $IC$ of local denials, we have a polynomial time approximation algorithm that solves $DFOP(IC)$ within an $O(\log(N))$ factor, where $N$ is the number of violation sets for $D$ wrt $IC$. As mentioned before, this number $N$, the number of hyperedges in $G$, is polynomially bounded by $|D|$ (c.f. Proposition 7). $N$ may be small if the number of inconsistencies or the number of database atoms in the ICs are small, which is likely the case in real applications.

However, in our case we can get even better approximations via a cover $\hat{C}$ obtained with an approximation algorithms for the special case of the $MWSCP$ where the number of occurrences of an element of $U$ in elements of $S$ is bounded by a constant. For this case of the $MWSCP$ there are approximations within a constant factor based on “linear relaxation” [15, Chapter 3]. This is clearly the case in our application, being $m \times |F| \times |IC|$ a constant bound (independent from $|D|$) on the frequency of the elements, where $m$ is the maximum number of database atoms in an IC.

Theorem 6. There is an approximation algorithm that, for a given database instance $D$ with local ICs $IC$, returns a fix $D(\hat{C})$ such that $\Delta(D, D(\hat{C})) \leq c \times \Delta(D, D')$, where $c$ is a constant and $D'$ is any LS-fix of $D$. \hfill \Box

6 One Atoms Denials and Conjunctive Queries

In this section we concentrate on the common case of one database atom denials (1AD), i.e. of the form $\forall\neg(A, B)$, where atom $A$ has a predicate in $R$, and $B$ is a conjunction of built-in atoms. They capture range constraints; and census data is usually stored in single relation schemas [10].

For 1ADs, we can identify tractable cases for $CQA$ under LS-fixes by reduction to $CQA$ for (tuple and set-theoretic) repairs of the form introduced in [2] for key constraints. This is because each violation set (c.f. Definition 6) contains one tuple, maybe with several local fixes, but all sharing the same key values; and then the problem consists in choosing one from different tuples with the same key values (c.f. proof of Theorem 7). The transformation preserves consistent answers to both ground and open queries.

The “classical” -tuple and set oriented- repair problem as introduced in [2] has been studied in detail for functional dependencies in [7, 11]. In particular, for tractability of $CQA$ in our setting, we can use results and algorithms obtained in [11] for the classical framework.

The join graph $G(Q)$ [11] of a conjunctive query $Q$ is a directed graph, whose vertices are the database atoms in $Q$. There is an arc from $L$ to $L'$ if $L \neq L'$ and there is a variable $w$ that occurs at the position of a non-key attribute in $L$ and also occurs in $L'$. Furthermore, there is a self-loop at $L$ if there is a variable that occurs at the position of a non-key attribute in $L$, and at least twice in $L$. 
When $Q$ does not have repeated relations symbols, we write $Q \in C_{\text{Tree}}$ if $G(Q)$ is a forest and every non-key to key join of $Q$ is full i.e. involves the whole key. Classical CQA is tractable for queries in $C_{\text{Tree}}$ [11].

**Theorem 7.** For a fixed set of 1ADs and queries in $C_{\text{Tree}}$, consistent query answering under LS-fixes is in $\text{PTIME}$. □

We may define that a aggregate conjunctive query belongs to $C_{\text{Tree}}$ if its underlying non-aggregate conjunctive query, i.e. its NAM (c.f. Section 2) belongs to $C_{\text{Tree}}$. Even for 1ADs, with simple comparison aggregate queries with $\text{sum}$, tractability is lost under the brave semantics.

**Proposition 9.** For a fixed set of 1ADs, and for aggregate queries that are in $C_{\text{Tree}}$ or acyclic, CQA is $\text{NP}$-hard under the brave semantics. □

For queries $Q$ returning numerical values, which is common in our framework, it is natural to use the range semantics for CQA, introduced in [3] for scalar aggregate queries and functional dependencies under classical repairs. Under this semantics, a consistent answer is the pair consisting of the $\text{min-max}$ and $\text{max-min}$ answers, i.e. the supremum and the infimum, resp., of the set of answers to $Q$ obtained from LS-fixes. The CQA decision problems under range semantics consist in determining if a numerical query $Q$, e.g. an aggregate query, has its answer $\leq k_1$ in every fix ($\text{min-max}$ case), or $\geq k_2$ in every fix ($\text{max-min}$ case).

**Theorem 8.** For each of the aggregate functions $\text{sum}$, $\text{count distinct}$, and $\text{average}$, there is a fixed set of 1ADs and a fixed aggregate acyclic conjunctive query, such that CQA under the range semantics is $\text{NP}$-hard. □

For the three aggregate functions one 1AD suffices. The results for $\text{count distinct}$ and $\text{average}$ are obtained by reduction from $\text{MAXSAT}$ [20] and $\text{3SAT}$, resp. For $\text{sum}$, we use a reduction from the Independent Set Problem with bounded degree 3 [13]. The general Independent Set Problem has bad approximation properties [15, Chapter 10]. The Bounded Degree Independent Set has efficient approximations within a constant factor that depends on the degree [14].

**Theorem 9.** For any set of 1ADs and conjunctive query with $\text{sum}$ over a non-negative attribute, there is a polynomial time approximation algorithm with a constant factor for CQA under $\text{min-max}$ range semantics. □

The factor in this theorem depends upon the ICs and the query, but not on the size of the database. The acyclicity of the query is not required. The algorithm is based on a reduction of our problem to satisfying a subsystem with maximum weight of a system of weighted algebraic equations over the Galois field with two elements $GF[2]$ (a generalization of problems in [12, 23]), for which a polynomial time approximation similar to the one for MAXSAT can be given [23].

7 Conclusions

We have shown that fixing numerical values in databases poses many new computational challenges that had not been addressed before in the context of
consistent query answering. These problems are particularly relevant in census like applications, where the problem of data editing is a common and difficult task (c.f. http://www.unece.org/stats/documents/2005.05.sde.htm). Also our concentration on aggregate queries is particularly relevant for this kind of statistical applications. In this paper we have just started to investigate some of the many problems that appear in this context, and several extensions are in development. We concentrated on integer numerical values, which provide a useful and challenging domain. Considering real numbers in fixable attributes opens many new issues, requires different approaches; and is a subject of ongoing research.

The framework established in this paper could be applied to qualitative attributes with an implicit linear order given by the application. The result we have presented for fixable attributes that are all equally relevant \(\alpha_A = 1\) in Definitions 1 and 2) should carry over without much difficulty to the general case of arbitrary weighted fixes. We have developed (but not reported here) extensions to our approach that consider minimum distribution variation LS-fixes that keep the overall statistical properties of the database. We have also developed optimizations of the approximation algorithm presented in Section 5; and its implementation and experiments are ongoing efforts. More research on the impact of aggregation constraints on LS-fixes is needed.

Of course, if instead of the \(L_2\) distance, the \(L_1\) distance is used, we may get for the same database a different set of (now \(L_1\)) fixes. The actual approximations obtained in this paper change too. However, the general complexity and approximability results should remain. They basically depend on the fact that distance functions are non-negative, additive wrt attributes and tuples, computable in polynomial time, and monotonically increasing. Another possible semantics could consider an epsilon of error in the distance in such a way that if, for example, the distance of a fix is 5 and the distance to another fix is 5.001, we could take both of them as (minimal) LS-fixes.

Other open problems refer to cases of polynomial complexity for linear denials with more that one database atom; approximation algorithms for the DFOP for non-local cases; and approximations to CQA for other aggregate queries.

For related work, we refer to the literature on consistent query answering (c.f. [4] for a survey and references). Papers [24] and [10] are the closest to our work, because changes in attribute values are basic repair actions, but the peculiarities of numerical values and quantitative distances between databases are not investigated. Under the set-theoretic, tuple-based semantics, [7, 6, 11] report on complexity issues for conjunctive queries, functional dependencies and foreign key constraints. A majority semantics was studied in [18] for database merging. Quite recent papers, but under semantics different than ours, report research on fixing numerical values under aggregation constraints [9]; and heuristic construction of repairs based on attribute values changes [5].

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A Appendix

A.1 Proofs

Those auxiliary technical results that are stated in this appendix, but not in the main body of the paper, are numbered in the form A.n, e.g. Lemma A.1.

Proof of Proposition 1: Let $\rho$ be the square distance between $D$ and $D'$ in Definition 1. The circle of radius $\rho$ around $D$ intersects the non empty “consistent” region that contains the database instances with the same schema and key values as $D$ and satisfy $IC$. Since the circle has a finite number of instances, the distance takes a minimum in the consistent region. □

The following lemma proves that if a tuple is involved in an inconsistency, the set of constraints is consistent and there is at least one fixable attribute in each integrity constraint, then there always exists a local fix (see Definition 7) for it.

Lemma A.1. For a database $D$ and a consistent set of linear denial constraints $IC$, where each constraint contains at least one built-in involving a flexible constraint and there are equalities or joins only between rigid attributes. Then, for every tuple $t$ with at least one fixable attribute and at least one $ic$ in $IC$, $I(D, ic, t) \neq \emptyset$, there exists at least one local fix $t'$ (see Definition 7) □

Proof: Each constraint $ic \in IC$ has the form $\forall \bar{x}.\neg(P_1(\bar{x}), \ldots, P_n(\bar{x}), A_i < c_i, A_j \geq c_j, A_k = c_k, A_l \neq c_l, \ldots)$ and can be rewritten as a clause only with $<$, $>$ and $=$:

$$\forall \bar{x}.(\neg P_1(\bar{x}) \vee \ldots \vee \neg P_n(\bar{x}) \vee A_i \geq c_i \vee A_j < c_j \vee A_k < c_k \vee A_k \geq c_k \vee A_l = c_l \vee \ldots)$$

(1)

This formula shows that since the repairs are done by attributes updates, the only way we have of solving an inconsistency is by fixing at least one of the values of a fixable attribute. Let $ic$ be a constraint in $IC$ such that $I(D, ic, t) \neq \emptyset$ and $I$ be a violation set $I \in I(D, ic, t)$. Now, since $ic \in IC$, $ic$ is a consistent constraint. Then for each fixable attribute $A$ in $ic$ we are able to derive an
interval \([c_l, c_u]\) such that if the value of \(A\) is in it, we would restore the consistency of \(I\). For example if we have a constraint in form of equation (1) with \(A \leq 5\), then, if we want to restore consistency by modifying \(A\) we would need to have \(A \in (-\infty, 5]\). If the constraint had also \(A \geq 1\) the interval would be \([1, 5]\). Since \(t\) has at least one fixable attribute and each fixable attribute has an interval, it is always possible to adjust the value of that fixable attribute to a value in the interval \([c_l, c_u]\) and restore consistency. By finding the adjustment that minimizes the distance from the original tuple we have find a local fix for the tuple \(t\).

The borders of an attribute in an extended linear denial correspond to the surfaces of the semi-spaces determined by the built-in atoms in it.

**Proposition A.1.** Given a database \(D\) and a set of linear denials \(IC\), where equalities and joins can only exist between rigid attributes, the values in every fixable attributes in a local fix \(t'\) (c.f. Definition 7) of a tuple \(t \in D\) will correspond to the original value in \(t\) or to a border of a constraint in \(IC\). Furthermore, the values in every attributes of a tuple \(t' \in D'\) will correspond to the original value of the attribute in the tuple in \(D\) or to a border of a constraint in \(IC\).

**Proof:** First we will replace in all the constraints \(X \leq c\) by \(X < (c+1)\), \(X \geq c\) by \(X > (c-1)\) and \(X = c\) by \((X > (c-1) \land X < (c+1))\). We can do this because we are dealing with integer values. Then, a constraint \(ic\) would have the form \(\forall \bar{x} \neg(P_1(\bar{x}),...,P_n(\bar{x}), A_i < c_i, A_j > c_j, A_k \neq c_k, \ldots)\) and can be rewritten as

\[
\forall \bar{x}(\neg P_1(\bar{x}) \lor \ldots \lor \neg P_n(\bar{x}) \lor A_i \geq c_i \lor A_j \leq c_j \lor A_k = c_k \lor \ldots)
\]

This formula shows that since the repairs are done by attributes updates, the only way we have of solving an inconsistency is by fixing at least one of the values of a fixable attribute. This would imply to change the value of a fixable attribute \(A_i\) to something equal or greater than \(c_i\), to change the value of a fixable attribute \(A_j\) to a value equal or smaller than \(c_j\) or to change the value of attribute \(A_k\) to \(c_k\).

If \(D\) is consistent wrt \(IC\) then there is a unique LS-fix \(D' = D\) and all the values are the same as the original ones and therefore the proposition holds. If \(D\) is inconsistent wrt \(IC\) then there exists a tuple \(t\) with at least one fixable attribute and a set \(IC_t \subseteq IC\) such that for every \(ic \in IC_t\) it holds \(T(D, ic, t) \neq \emptyset\).

If \(IC_t\) is an inconsistent set of constraints then there exists no local fix and the proposition holds. If \(IC_t\) is consistent but there is at least one constraint with no fixable attributes involved then, since it is not possible to modify any attribute in order to satisfy the constraint, there is no local fix and the proposition holds.

So we are only missing to prove the proposition for \(IC_t\) consistent and with at least one fixable attributes for each \(ic\) in \(IC_t\). From Lemma A.1 we know that there exists a local fix for \(t\). Also, since \(IC_t\) is consistent, using the same arguments as in proof of Lemma A.1, it is possible to define for each fixable attribute \(A\) an interval such that if the value of \(A\) is in it we would restore the consistency of the violation sets for constraints in \(IC_t\) involving \(t\). Then, we need to prove that if a value of an attribute, say \(A\), of a local fix \(t'\) of \(t\) is different
than the one in \( t \), then the value corresponds to one of the closed limits of the interval for \( A \). Let us assume that an attribute \( A \) is restricted by the constraints to an interval \([c_l, c_u]\) and that the local fix \( t' \) takes for attribute \( A \) a value strictly smaller than \( c_u \) and strictly greater than \( c_l \). Without lost of generality we will assume that the value of attribute \( A \) in \( t \) is bigger than \( c_u \). Let \( t'' \) be a tuple with the same values as \( t' \) except that the attribute \( A \) is set to \( c_u \). \( t'' \) will have the same values in the rigid attributes as \( t \) and also \( S(t, t') = S(t, t'') \) since the value of \( A \) in \( t'' \) is still in the interval. We also have that \( \Delta(\{t\}, \{t''\}) \leq \Delta(\{t\}, \{t'\}) \). This implies that \( t' \) is not a local fix and we have reached a contradiction.

For the second part of the proposition, the proof of the first part can be easily extended to prove that the values in \( D' \) will correspond to a border of a constraint in \( IC \), because the LS-fixes are combination of local fixes. \)

**Proof of Theorem 1:** Hilbert’s 10th problem on existence of integer solutions to diophantine equations can be reduced to our problem. Given a diophantine equation, it is possible to construct a database \( D \) and a set of ICs \( IC \) such that the existence of an LS-fix for \( D \) wrt \( IC \) implies the existence of a solution to the equation, and viceversa. An example can be found in Appendix A.2.

**Proof of Proposition 2:** First for the skeptical semantics. Given a database instance \( D \), consider the instance \( (D, no) \) for \( CQA(False, IC, Sk) \), corresponding to the question “Is there an LS-fix of \( D \) wrt \( IC \) that does not satisfy False?” has answer Yes iff the class of LS-fixes of \( D \) is empty. For the majority semantics, for the instance \( (D, no) \) for \( CQA(True, IC, Maj) \), corresponding to the question “Is it not the case that the majority of the LS-fixes satisfy True?” we get answer yes iff the set of LS-fixes is empty.

**Proof of Theorem 2:** (a) First of all, we notice that a linear denial with implicit equalities, i.e. occurrences of a same variable in two different database atoms, e.g. \( \forall X, Y, Z \neg(R(X,Y), Q(Y,Z), Z > 3) \), can be replaced by its explicit version with explicit equalities, e.g. \( \forall X, Y, Z, W \neg(R(X,Y), Q(W,Z), Y = W, Z > 3) \).

Let \( n \) be the number of tuples in the database, and \( l \) be the number of attributes which participate in \( IC \). They are those that appear in built-in predicates in the explicit versions of the ICs that do not belong to a key or are equal to a key (because they are not allowed to change). For example, given the denial \( \neg(P(X,Y), Q(X,Z), Y > 2) \), since its explicit version is \( \neg(P(X,Y), Q(W,Z), Y > 2, X = W) \), the number \( l \) is 1 (for \( Y \)) if \( X \) is a key for \( P \) or \( Q \), and 3 if \( X \) is not a key (for \( Y, X, W \)).

If there exist an LS-fix \( D' \) with \( \Delta(D, D') \leq k \), then no value in a fixable attribute in \( D' \) differs from its corresponding value (through the key value) in \( D \) by more than \( \sqrt{E} \). In consequence, the size of an LS-fix may not differ from the original instance by more than \( l \times n \times \text{bin}(k)/2 \), where \( \text{bin}(k) \) is the size of the binary representation of \( k \). Thus, the size of an LS-fix is polynomially bounded by the size of \( D \) and \( k \). Since we can determine in polynomial time if \( D' \) satisfies the ICs and if the distance is smaller than \( k \), we obtain the result.
(b) Membership: According to Proposition 1, there is an LS-fix at a square distance \( \leq k \) iff there is an instance \( D' \) with the same key values that satisfies \( IC \) at a square distance \( \leq k \). We use Proposition 2.

Hardness: We can reduce Vertex Cover (VC) to \( DFP(\text{IC}_0) \) for a fixed set of denials \( \text{IC}_0 \). Given an instance \( (\mathcal{V}, \mathcal{E}) \), \( k \) for VC, consider a database \( D \) with a relation \( E(X, Y) \) and key \( \{X, Y\} \) for the edges of the graph, and a relation for the vertices \( V(X, \text{Chosen}) \), where \( X \) is the key and attribute \( \text{Chosen} \), the only fixable attribute, is initially set to 0. The constraint \( IC: \forall X, Y, C_1, C_2 - (E(X, Y) \land V(X, C_1) \land V(Y, C_2) \land C_1 < 1 \land C_2 < 1) \) expresses that for any edge, at least one of the incident vertices is covered. A vertex cover of size \( k \) exists iff there exists an LS-fix of \( D \) wrt \( IC \) at a distance \( \leq k \). The encoding is polynomial in the size of the original graph. \( \square \)

**Proof of Theorem 3:** (a) For hardness, linear denials are good enough. We reduce the graph 3-colorability problem to \( NE(\text{IC}_0) \), for a fixed set \( \text{IC}_0 \) of ICs. Let \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) be an undirected graph with set of vertexes \( \mathcal{V} \) and set of edges \( \mathcal{E} \). Consider the following database schema, instance \( D \), and set \( \text{IC}_0 \) of ICs:

1. Relation \( \text{Vertex}(\text{Id}, \text{Red}, \text{Green}, \text{Blue}) \) with key \( \text{Id} \) and domain \( \mathbb{N} \) for the last three attributes, actually the only three fixable attributes in the database; they can be subject to changes. For each \( v \in \mathcal{V} \) we have the tuple \((v, 0, 0, 0)\) in \( \text{Vertex} \) (and nothing else).

2. Relation \( \text{Edge}(\text{id}_1, \text{id}_2) \); and for each \( e = (v_1, v_2) \in \mathcal{E} \), there are the tuples \((v_1, v_2), (v_2, v_1)\) in \( \text{Edge} \). This relation is not subject to any fix.

3. Relation \( \text{Tester}(\text{Red}, \text{Green}, \text{Blue}) \), with extension \((1, 0, 0), (0, 1, 0), (0, 0, 1)\).

4. This relation is not subject to any fix.

4. Integrity constraints:

\[
\forall ixyz - (\text{Vertex}(i, x, y, z), x < 1, y < 1, z < 1); \forall ixyz - (\text{Vertex}(i, x, y, z), x > 1) \quad \text{(the same for } y, z); \forall ixyz - (\text{Vertex}(i, x, y, z), x = 1, y = 1, z = 1); \forall ixyz - (\text{Vertex}(i, x, y, z), x = 1, y = 1) \quad \text{etc.}
\]

The graph is 3-colorable iff the database has an LS-fix wrt \( \text{IC}_0 \). The reduction is polynomial in the size of the graph. If there is an LS-fix of the generated instance, then the graph is 3-colorable. If the graph is colorable, then there is a consistent instance with the same key values as the original instance; then, by Proposition 1, there is an LS-fix.

For membership, it suffices to prove that if an LS-fix exists, then its square distance to \( D \) is polynomially bounded by the size of \( D \), considering both the number of tuples and the values taken by the fixable attributes.

We will show that if an LS-fix \( D' \) exists, then all the values in its fixable attributes are bounded above by the maximum of \( n_1 + n + 1 \) and \( n_2 + n + 1 \), where \( n \) is the number of tuples in the database, \( n_1 \) is the maximum absolute value in a fixable attribute in \( D \), and \( n_2 \) is the maximum absolute value of a constant appearing in the ICs.

The set of denial ICs put in disjunctive form gives us a representation for all the ways we have to restore the consistency of the database. So, we have a
constraint of the form \( \varphi_1 \land \varphi_2 \cdots \varphi_m \), where each \( \varphi_i \) is a disjunction of negated database atoms and inequalities, e.g. something like \( \neg P(X,Y,Z) \lor \neg R(X_1,Y_1) \lor X \leq c_1 \lor Y \leq c_2 \lor Z \neq Y_1 \). Since fixes can be obtained by changing values of non key attributes, each tuple in a fix is determined by a set of constraints, each of which is a disjunction of atoms of the form \( X_i \theta c_m \) or \( X_i \neq Y_j \), where \( \theta \) is an inequality of the form \( \leq, \geq, <, > \). E.g. from \( \neg P(X,Y,Z) \lor \neg R(X_1,Y_1) \lor X \leq c_1 \lor Y \leq c_2 \lor Z \neq Y_1 \) we get \( X \leq c_2 \lor Z \neq Y_1 \) if \( X \) is part of the key and its specific value for the tuple at hand does not satisfy \( X \leq c_1 \) (otherwise we drop the constraint for that tuple). In any case, every tuple in a fix can take values in a space \( S \) that is the intersection of the half-spaces defined by inequalities of the form \( X_i \theta c_m \) minus the set of points determined by the non-inequalities \( X_i \neq Y_j \).

If there is a set of values that satisfies the resulting constraints, i.e. if there is an instance with the same key values that satisfies the ICs, then we can find an LS-fix at the right distance: if the difference between any value and \( \max(c_1, \cdots, c_l) \) is more than \( n+1 \) (the most we need to be sure the inequalities \( X_i \neq Y_j \) are satisfied), then we systematically change values by 1, making them closer to the borders of the half-spaces, but still keeping the points within \( S \).

(b) \( \text{coNP} \)-hardness follows from Proposition 2 and part (a). \( \square \)

**Proof of Theorem 4:** (a) We reduce 3-SAT’s complement to LS-fix checking for a fixed schema and set of denials IC. We have a table \( \text{Lit}(l, l) \) storing complementary literals (only), e.g. \( (p, \neg p) \) if \( p \) is one of the variables in the instance for SAT. Also a table \( \text{Cl} \) storing tuples of the form \( (\varphi, l, k) \), where \( \varphi \) is a clause (we assume all the clauses have exactly 3 literals, which can be simulated by adding extra literals with unchangeable value 0 if necessary), \( l \) is a literal in the clause, and \( k \) takes value 0 or 1 (the truth value of \( l \) in \( \varphi \)). The first two arguments are the key of \( C \). Finally, we have a table \( \text{Aux}(K, N) \), with key \( K \) and fixable numerical attribute \( N \), and a table \( \text{Num}(N) \) with a rigid numerical attribute \( N \).

Given an instance \( \Phi = \varphi_1 \land \cdots \land \varphi_m \) for 3-SAT, we produce an initial extension \( D \) for the relations in the obvious manner, assigning arbitrary truth values to the literals, but making sure that the same literal takes the same truth value in every clause, and complementary literals take complementary truth values. \( \text{Aux} \) contains \((0, 0)\) as its only tuple; and \( \text{Num} \) contains \((s+1)\), where \( s \) is the number of different propositional variables in \( \Phi \).

Consider now the following set of denials:
(a) \( \neg(\text{Cl}(\varphi, L, U), U > 1); \neg(\text{Cl}(\varphi, L, U), U < 0) \) (possible truth values).
(b) \( \neg(\text{Cl}(\varphi, L, U), \text{Cl}(\psi, L, V), U \neq V) \) (same value for a literal everywhere).
(c) \( \neg(\text{Cl}(\varphi, L, U), \text{Cl}(\psi, L', V), \text{Lit}(L, L'), U = V) \) (complementary literals).
(d) \( \neg(\text{Cl}(\varphi, L, U), \text{Cl}(\varphi, L', V), \text{Cl}(\varphi, L', W), U = V = W = 0, L \neq L', \ldots, \text{Aux}(K, N), N = 0) \) (each clause becomes true).
(e) \( \neg(\text{Num}(Z), \text{Aux}(K, N), N \neq 0, N \neq Z) \) (possible values).

It holds that the formula is unsatisifiable iff the instance \( D' \) that coincides with \( D \) except for \( \text{Aux} \) that now has the only tuple \((0, s+1)\) is an LS-fix of \( D \) wrt IC. Thus, checking \( D' \) for LS-fix is enough to check unsatisfiability.
For membership to \(\text{coNP}\), for an initial instance \(D\), instances \(D'\) in the complement of \(\text{Fix}(IC)\) have witnesses \(D''\) that can be checked in polynomial time, namely instances \(D''\) that have the same key values as \(D\), satisfy the ICs, but \(\Delta(D, D'') < \Delta(D, D')\).

(b) For the first claim on CQA, let \(IC\) and a query \(Q\) be given. The complement of CQA is in \(\text{NP}^{\text{coNP}}\): Given an instance \(D\), non deterministically choose an instance \(D'\) with \(D' \neq Q\) and \(D'\) a fix of \(D\). The latter test can be done in \(\text{coNP}\) (by part (a)). But \(\text{NP}^{\text{coNP}} = \Sigma^p_2 = \Pi^p_2\). In consequence, CQA belongs to \(\text{coNP}\).

For the second claim, we prove hardness of CQA by a \(\text{LOGSPACE}\)-reduction from the following problem [16, Theo. 3.4]: Given a Boolean formula \(\psi(X_1, \ldots, X_n)\) in 3CNF, decide if the last variable \(X_n\) is equal to 1 in the lexicographically maximum satisfying assignment (the answer is false if \(\psi\) is not satisfiable).

Given the clauses \(C_1, \ldots, C_m\) in \(\psi\), we create a database \(D\) with relations \(\text{Var}(id, tr, fa, \text{weight}), \text{Cl}(id, \text{var}_1, \text{var}_2, \text{var}_3, \text{var}_4, \text{var}_5)\) and constraints:

1. \(\forall id, tr, fa \neg(\text{Var}(id, tr, fa, \_ \_ \_ ) \land tr \leq 0 \land fa \leq 0)\)
2. \(\forall id, tr, fa \neg(\text{Var}(id, tr, fa, \_ \_ \_ ) \land tr \geq 1 \land fa \geq 1)\)
3. \(\forall id, tr, fa, w \neg(\text{Var}(id, tr, fa, w) \land fa = 1 \land w > 0)\)
4. \(\forall id, v_1, x_1, v_2, x_2, v_3, x_3(\text{Cl}(id, v_1, x_1, v_2, x_2, v_3, x_3) \land \text{Var}(\_, v_1, x_1') \land \text{Var}(\_, v_2, x_2') \land \text{Var}(\_, v_3, x_3') \land x_1 \neq x_1' \land x_2 \neq x_2' \land v_3 \neq v_3')\)

The extended denial constraint in 4. could be replaced by eight non-extended denial constraints.

For each variable \(X_i\), insert a tuple \((X_i, 0, 0, 2^{n-i})\) into \(\text{Var}\). In binary encoding, the values \(2^{n-i}\) are polynomial in the size of original formula. For each clause \(C_i = l_{i_1} \lor l_{i_2} \lor l_{i_3}\), insert a tuple \((C_i, X_{i_1}, \tilde{l}_{i_1}, X_{i_2}, \tilde{l}_{i_2}, X_{i_3}, \tilde{l}_{i_3})\) into \(\text{Cl}\), where \(\tilde{l}_{i_j}\) is equal to 1 in case of positive occurrence of variable \(X_{i_j}\) in \(C_i\) and equal to 0 for negative occurrence. For example, for \(C_6 = X_6 \lor \neg X_9 \lor X_{12}\), we insert \((C_6, X_6, 1, X_9, 0, X_{12}, 1)\).

It is easy to see that all the fixes \(D\) represent satisfying assignments for \(\psi\), such that in case a tuple \((X_i, 1, 0, \_\_\_ )\) is in a fix, then value \(true\) must be assigned to variable \(X_i\); and value \(false\) must be assigned to \(X_i\) in case \((X_i, 0, 1, \_\_\_ )\) is in the fix. If \(\psi\) is unsatisfiable, then there are no fixes.

Let us now consider the cost of a repair. Assume that \(S_1 = x_{i_1}^{1}, \ldots, x_{i_m}^{1}\) and \(S_2 = x_{i_1}^{2}, \ldots, x_{i_m}^{2}\) are satisfying assignments with \(S_1 < S_2\) under lexicographical order. Since \(S_1 < S_2\), there exists an integer \(m\) such that \(i_m^1 < i_m^2\), while for all \(j < m\), \(i_j^1 = i_j^2\), by definition of lexicographical order. The cost of repair \(S\) is equal to \(2^{n-i_1} + 2^{n_i^2} + \ldots + 2^{n_{i_m}^2} + n\), because (a) we have to update attribute \(\text{weight}\) for each variable that is assigned value \(true\), and (b) for each tuple in relation \(\text{Var}\), attribute \(tr\) or attribute \(fa\) is changed by 1.

Because of (a), there exists a term \(2^{n-i_m^1}\) in the cost of \(S_1\) that is bigger than the sum of all terms in \(S_2\), with index \(i_m^2\). So, the cost of the fix representing \(S_1\) is greater than the cost of the fix representing \(S_2\). In consequence, the minimal fix would be the maximum according to the lexicographical order of satisfying assignments.
The answer to the ground atomic query \( \text{Var}(X_n, 1, 0, 1) \) is true iff \( X_n \) takes the value 1 in the l.o. greatest assignment. \( \square \)

**Proof of Theorem 5:** The reduction can be established with a fixed set \( IC_0 \) of ICs. Given an undirected graph \( G = (V, E) \), consider a database with relations \( V(X, Z), E(U, W) \), where \( X \) is a key and \( Z \) is the only fixable attribute and takes values in \( \{0, 1\} \) (which can be enforced by means of the linear denials \( \forall X \forall Z \neg (V(X, Z), Z > 1), \forall X \forall Z \neg (V(X, Z), Z < 0) \) in \( IC_0 \)). Intuitively, \( Z \) indicates with 1 if the vertex \( X \) is in the cover, and with 0 otherwise. Attributes \( U, V \) are vertices and then, non numerical.

In the original database \( D \) we have the tuples \( V(e, 0) \), with \( e \in V \); and also the tuples \( E(e_1, e_2) \) for \( (e_1, e_2) \in E \). Given the linear constraint

\[
\forall X_1Z_1X_2Z_2 \neg (V(X_1, Z_1), V(X_2, Z_2), E(X_1, X_2), Z_1 = 0, Z_2 = 0)
\]

in \( IC_0 \), the LS-fixes of the database are in one-to-one correspondence with the vertex covers of minimal cardinality.

For the query \( Q^{(k)}: q(\text{sum}(Z)), \text{sum}(Z) < k \), with \( q(\text{sum}(Z)) \leftarrow V(X, Z) \), the instance \( (D, \text{yes}) \) for consistent query answering under brave semantics has answer \( \text{No} \), (i.e. \( Q^{(k)} \) is false in all LS-fixes) only for every \( k \) smaller than the minimum cardinality \( c \) of a vertex cover.

In the previous proof, the first two ICs in \( IC_0 \) can be eliminated, and the third one is local (c.f. Definition 5). In consequence, the theorem also holds for local denials.

**Proof of Proposition 3:** By reduction from the MAXSNP-hard problem B-Minimum Vertex Cover (BMVC), which asks to find a minimum vertex cover in a graph whose nodes have a bounded degree [15, chap. 10]. We start by encoding the graph as in the proof of Theorem 5. We also use the same initial database \( D \). Every LS-fix \( D' \) of \( D \) corresponds to a minimum vertex cover \( V' \) for \( G \) and vice versa, and it holds \( |V'| = \Delta(D, D') \). This gives us an \( L \)-reduction from BMVC to DFP [20]. \( \square \)

**Lemma A.2.** Given a database \( D \) and a set of consistent local denials \( IC \), there will always exist an LS-fix \( D' \) of \( D \) wrt \( IC \). \( \square \)

**Proof:** As shown in proof of Lemma A.1 for every fixable attribute in \( F \) it is possible to define, using the integrity constraints in \( IC \), an interval \([c_l, c_u]\) such that if the value of attribute \( A \) is in that interval there is no constraint \( ic \in IC \) with a built-in involving \( A \) such that \( I(D, ic, t) \neq \emptyset \). Let \( D'' \) be a database constructed in the following way: for every tuple \( t \in D \) such that the value of a fixable attribute does not belong to its interval, replace its value by any value in the interval. Clearly \( D' \) will be a fix but will not necessarily be an LS-fix. By Proposition 1 we know there exists an LS-fix \( D' \) for \( D \) wrt \( IC \). \( \square \)

**Definition 11.** Given a database \( D \) and a set of ICs \( IC \), a local fix \( t' \) for a tuple \( t \) does not generate new violations if \( \bigcup_{ic \in IC} (\bigcup_{l \in D} I(D', ic, l) \cap \bigcup_{l \in D} I(D, ic, l)) = \emptyset \) for \( D' = (D \cup \{t\}) \cup \{t'\} \). \( \square \)
Lemma A.3. For a set IC of local denials, if t' is a local fix of a tuple t, then
t' does not generate new violations\(^4\) in database D wrt IC. Furthermore, this
holds also for t' a “relaxed” local fix where the distance to t is not necessarily
minimal

Proof: Tuple t' can only differ from t in the value of fixable attributes. Let us
assume that one of the modified values was for an attribute A. Since we have
local constraints, attribute A can only be in the constraints related either to <
and ≤ or to > and ≥, but not both. Without lost of generality, we will assume
that the constraint is written as in equation 1 and that A is related only to
> and ≥. Since t' is a local fix, S(t, t') is not empty and there is a set ICt of
constraints for which t' solves the inconsistency in which t' was involved. There
is an interval \([c_l, +\infty)\) for A that can be obtained by the limits given in ICt that
show the values of A that would force the satisfaction of the constraints in ICt
that have attribute A in an inequality. This shows that the value of attribute A
in t' is bigger than the value of A in t.

For \(D' = (D \setminus \{t\}) \cup \{t'\}\) we need to prove that \(\bigcup_{ic \in IC} (\bigcup_{I \in IC} IC(D', ic, l) \setminus
\bigcup_{I \in IC} IC(D', ic, l)) = \emptyset\). By contradiction let us assume that for a constraint
ic \(\in IC\) there exists a violation set I such that \(I \in \bigcup_{I \in IC} IC(D', ic, l)\) and
\(I \notin \bigcup_{I \in IC} IC(D', ic, l)\). There are two cases to consider (with \((I, ic, l)\), we indicate
that I is a violation set for IC ic):

- \((I, ic) \in S(t, t')\). Then \(I \in IC(D', ic, l)\), but since we wanted an \(I \notin \bigcup_{I \in IC} IC(D',
ic, l)\) this is not possible.
- \((I, ic) \notin S(t, t')\). Then we have two possibilities \(I \notin IC(D', ic, l)\) or \((IC \setminus \{t\}) \cup
\{t'\}) \notin ic.

  - Let us consider first that \(I \notin IC(D', ic, l)\). We have that \(I \in \bigcup_{I \in IC} IC(D',
ic, l)\) and since t' is the only difference between D and D' we have \(I \in IC(D', ic, l)\). Since all the constraints can only have attribute A with >
or ≥ we now that in particular ic does. Since \(I \notin IC(D', ic, l)\) we know
that A satisfied the condition in ic and since we know that t' has a bigger
value than in t, it is not possible to generate an inconsistency in D'.
We have reached a contradiction.

  - Let us consider \((IC \setminus \{t\}) \cup \{t'\}) \notin ic\). Then \(I \in IC(D', ic, l)\). From our
assumption \(I \notin \bigcup_{I \in IC} IC(D', ic, l)\). This is the same situation analyzed in
previous item.

In all the cases we have reached contradiction and therefore the proposition is
proved. Since we never used the property of minimal distance between t' and t,
the second part of the Lemma is also proved. □

Proposition A.2. For local denials it always exists an LS-fix for a database
D; and for every LS-fix D', D' \(\prec D\) is a set of local fixes. Furthermore, for each
violation set \((I, ic)\), there is a tuple \(t \in I\) and a local fix \(t'\) for \(t\), such that
\((I, ic) \in S(t, t')\).

Proof: Since each attribute A can only be associated to < or > built-ins, but
not both, it is clear that set of local denials is always consistent. By Lemma A.2,

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\(^4\) c.f. Definition 11
there always exists an LS-fix \( D' \). Now we need to prove that \( D' \setminus D \) is a set of local fixes. By contradiction assume that \( t' \in (D' \setminus D) \) is not a local fix of the tuple \( t \). This can happen in the following situations:

- \( t \) was consistent. From Lemma A.3 we know that no new inconsistencies can be added by the modifications done to the other tuples and therefore \( t \) is not related to any inconsistency. Then \( D^* = D' \setminus \{t'\} \cup \{t\} \) is also consistent and \( \Delta(D, D^*) < \Delta(D, D') \). But \( D' \) is an LS-fix so this is not possible.
- \( t \) is involved at least in one violation set. If \( S(t, t') = \emptyset \) then \( t' \) is not solving any violation set and therefore \( D^* = D' \setminus \{t'\} \cup \{t\} \) is also consistent and \( \Delta(D, D^*) < \Delta(D, D') \). But \( D' \) is an LS-fix so this is not possible. Now, if \( S(t, t') \neq \emptyset \), from Lemma A.2, considering \( D = \{t\} \) and \( IC = \{ic | (I, ic) \in S(t, t')\} \), there exists an LS-fix \( D' \) of \( D \), i.e. there exists a local fix \( t'' \) such that \( S(t, t'') = S(t, t') \). Since \( t'' \) is a local fix we know that \( \Delta(t, \{t''\}) < \Delta(t, \{t'\}) \). They cannot be equal that would imply that \( t' \) is a local fix and it is not. Then \( D^* = D' \setminus \{t'\} \cup \{t\} \) is also consistent and \( \Delta(D, D^*) < \Delta(D, D') \). Again, this is not possible because \( D' \) is an LS-fix.

The second part of the proposition can be proved using Lemma A.2 and considering a database \( D = I \) and a set of constraints \( IC = \{ic\} \).

**Proof of Proposition 4:** For the first claim, membership follows from Theorem 2(b); and for hardness, we can do the same reduction as in Theorem 2(b), because the ICs used there are local denials. For the second claim, it is not difficult to see that the non-local denials in the proof of Proposition 3 can be eliminated. \( \square \)

**Proposition A.3.** For a database \( D \) and a set of local denial constraints \( IC \):

1. For a set of local fixes \( \{t_1, \ldots, t_n\} \) of a tuple \( t \) there always exists a local fix \( t^* \) such that \( S(t, t^*) = \bigcup_{i=1}^n S(t, t_i) \).
2. For local fixes \( t', t'' \) and \( t''' \) of a tuple \( t \) with \( S(t, t''') = S(t, t') \cup S(t, t'') \), it holds that \( \Delta(t, \{t''\}) \leq \Delta(t, \{t'\}) + \Delta(t, \{t''\}) \). \( \square \)

**Proof:** First we prove item (1). Let \( IC_\iota = \{ic | (I, ic) \in S(t, t') \} \neq \emptyset \) and \( IC_\iota(t, t') = \{ic | (I, ic) \in S(t, t') \} \). From Lemma A.2, considering \( D = \{t\} \) and \( IC \) any subset of \( IC_\iota \), there always exists an LS-fix \( D' \) of \( D \). This LS-fix is a local fix of tuple \( t \) with \( IC_\iota(t, t') = IC \). Since we can find a local fix for any \( IC \subseteq S_t \) then clearly the lemma can be satisfied.

Now we will prove item (2). If the fixable attributes that where modified in \( t' \) and \( t'' \) are disjoint, then \( t''' \) when combining the modifications I’ll get \( \Delta(t, \{t''\}) = \Delta(t, \{t'\}) + \Delta(t, \{t''\}) \). Now, we will consider the case were \( t' \) and \( t'' \) have at least one fixable attribute, say \( A \) that is modified by both local fixes. In this case \( t''' \) will have a value in \( A \) that solves the inconsistencies solved by and \( t' \) and \( t'' \). This value will in fact correspond to the value of \( A \) in \( t' \) or \( t'' \) and therefore we will have that \( \Delta(t, \{t''\}) < \Delta(t, \{t'\}) + \Delta(t, \{t''\}) \). Let \( M \) be the set of attributes that are modified both by \( t' \) and \( t'' \), we can express the relation as follows: \( \Delta(t, \{t''\}) = \sum_{A \in M} (\pi_A(t) - \pi_A(t''))^2 = \sum_{A \in M} (\pi_A(t) - \pi_A(t'))^2 + \sum_{A \notin M} (\pi_A(t) - \pi_A(t''))^2 \).
Proposition A.4. If an optimal cover \( C \) for the instance \((U, S)\) of MWSCP has more than one \( S(t, t') \) for a tuple \( t \), then there exists another optimal cover \( C' \) for \((U, S)\) with the same total weight as \( C \) but with only one \( t' \) such that \( S(t, t') \in C \). Furthermore, \( D(C) \) is an LS-fix of \( D \) wrt IC with \( \Delta(D, D(C)) \) equal to the total weight of the cover \( C \).

Proof: To prove the first part, let us assume that \( S(t, t'), S(t, t'') \in C \). From Proposition A.3 there exists an \( S(t, t'''') \in S \) such that \( S(t, t''') = S(t, t') \cup S(t, t'') \), i.e. such that it covers the same elements as \( S(t, t') \) and \( S(t, t'') \). From Proposition A.3 \( \Delta(S(t, \{t''\})) \leq \Delta(S(t, \{t'\}) + \Delta(S(t, \{t''\})) \) and therefore that weight of \( S(t, t''') \) is smaller or equal than the sum of the weight of the original two sets. If \( \Delta(S(t, \{t''\})) < \Delta(S(t, \{t'\}) + \Delta(S(t, \{t''\})) \) we would have that \( C \) is not an optimal solution so this is not possible. Then \( \Delta(S(t, \{t''\})) = \Delta(S(t, \{t'\}) + \Delta(S(t, \{t''\}) \). Then, if we define \( C' = (C \setminus \{S(t, t'), S(t, t'')\}) \cup \{S(t, t''')\} \) we will cover all the elements and we will have the same optimal weight.

Now we need to prove that given \( D(C) \) is an LS-fix. \( D(C) \) is obtained by first calculating \( C' \) and therefore we have an optimal cover with at most one \( S(t, t') \) for each tuple \( t \). Then \( D(C) \) is obtained by replacing \( t \) by \( t' \) for each \( S(t, t') \in C \). It is direct that \( D(C) \) has the same schema as \( D \) and that it satisfies the key constraints. Now, since \( C' \) covers all the elements, all the inconsistencies in \( D \) are solved in \( D(C) \). From Lemma A.3 the local fixes \( t' \) do not add new violations and therefore \( D(C) \models IC \) and \( D(C) \) is a fix. We are only missing to prove that \( D(C) \) minimizes the distance from \( D \). Clearly \( \Delta(D, D(C)) = \sum_{t \in C} \Delta(S(t, \{t'\})) = \sum_{S(t, t') \in C} w_{S(t, t')} = w \). So, since the optimal solution minimizes \( w \), \( \Delta(D, D(C)) \) is minimum and \( D(C) \) is an LS-fix.

Proof of Proposition 5: From Propositions A.3 and A.4.

Proof of Proposition 6: To prove it is enough to construct this optimal cover. Let \( C = \{S(t, t') \mid t' \in (D' \setminus D)\} \). By definition \( C' = C \) and \( D(C) = D' \). We need to prove that \( C \) is an optimal cover. Since \( D' \) is consistent, all the violation sets were solved and therefore \( C \) is a cover. Also, since \( \Delta(D, D') = \Delta(D, D(C)) = w \) and \( \Delta(D, D') \) is minimum, \( C \) minimizes the weight and therefore is an optimal cover.

Proof of Proposition 7: We have to establish that the transformation of DFOP into MWSCP given above is an L-reduction [20]. So, it remains to verify that the reduction can be done in polynomial time in the size of instance \( D \) for DFP(\( IC \)), i.e. that \( G \) can be computed in polynomial time in \( n \), the number of tuples in \( D \). Notice that if \( m_i \) the number of database atoms in \( iC_i \in IC \), and \( m \) the maximum value of \( m_i \), there are at most \( n^m \) hyperedges associated to \( iC_i \in IC \), each of them having between 1 to \( m \) tuples. We can check that the number of sets \( S(t, t') \) are paired and that their weights are polynomially bounded by the size of \( D \). There is one \( S(t, t') \) for each local fix. Each tuple may have no more than \(|F| \times |IC|\) local fixes, where \( F \) is the set of fixable attributes.

The weight of each \( S(t, t') \) is polynomially bounded by the maximum absolute value in an attribute in the database and the maximum absolute value of a constant appearing in \( IC \) (by an argument similar to the one given in the proof of Proposition 2).
With respect to $D(\mathcal{C})$, the number of sets in $\mathcal{S}$ is polynomially bounded by the size of $D$, and since $C \subseteq S$, $\mathcal{C}$ is also polynomially bounded by the size of $D$. To generate $\mathcal{C}'$ it is necessary to search through $\mathcal{S}$. Finally, in order to replace $t$ in $D$ for each tuple $t'$ such that $S(t, t') \in \mathcal{C}$ we need to search through $D$. □

Proof of Proposition 8: Using the same arguments as in the proof of Proposition A.4 we have that since $\mathcal{C}$ is a cover then $D(\mathcal{C})$ is a fix of $D$ wrt $IC$. We need to prove that $\Delta(D, D(\mathcal{C})) \leq \log(N) \times \Delta(D, D')$. We know that $\Delta(D, D(\mathcal{C})) = \sum_{t \in D} \Delta(\{t\}, \{t'\}) = \sum_{s(t, t') \in \mathcal{C}, w_{s(t, t')}}$. As described in definition 10, $\mathcal{C}'$ is obtained from $\mathcal{C}$ by replacing, for each $t$, all the sets $S(t, t_i) \in \mathcal{C}$ by a unique set $S(t, t^*)$ such that $S(t, t^*) = \bigcup_i S(t, t_i)$. Since we are using euclidian distance to calculate the local fixes, $\Delta(\{t\}, \{t^*\}) \leq \sum_i \Delta(\{t\}, \{t_i\})$. Then,

$$
\Delta(D, D(\mathcal{C})) = \sum_{s(t, t') \in \mathcal{C}, w_{s(t, t')}} \leq \sum_{s(t, t') \in \mathcal{C}, w_{s(t, t')}} = \hat{w}.
$$

Thus, $\Delta(D, D(\mathcal{C})) \leq \hat{w} \leq \log(N) \times \omega = \log(N) \times \Delta(D, D')$, for every LS-fix $D'$ of $D$. □

Proof of Theorem 7: Based on the tractability results in [11], it suffices to show that the LS-fixes for a database $D$ are in one-to-one and polynomial time correspondence with the repairs using tuple deletions [2, 7] for a database $D'$ wrt a set of key dependencies.

Since we have 1ADs, the violation sets will have a single element, then, for an inconsistent tuple $t$ wrt a constraint $ic \in IC$, it holds $I(D, ic, t) = \{t\}$. Since all the violation sets are independent, in order to compute an LS-fix for $D$, we have to generate independently all the local fixes $t'$ for all inconsistent tuples $t$ such that $(\{t\}, ic) \in S(t, t')$, with $ic \in IC$; and then combine them in all possible ways.

Those local fixes can be found by considering all the candidate fixes (not necessarily LS-minimal) that can obtained by combining all the possible limits for each attribute provided by the ICs (c.f. Proposition A.1); and then checking which of them satisfy $IC$, and finally choosing those that minimize $\Delta(\{t\}, \{t'\})$. There are at most $2^{\vert F\vert}$ possible candidate fixes, where $F$ is the set of fixable attributes. Let us now define a database $D'$ consisting of the consistent tuples in $D$ together with all the local fixes of the inconsistent tuples. By construction, $D$ and $D'$ share the same keys. Since each inconsistent tuple in $D$ may have more than one local fix, $D'$ may become inconsistent wrt its key constraints. Each repair for $D'$, obtained by tuple deletions, will choose one local fix for each inconsistent tuple $t$ of $D$, and therefore will determine an LS-fix of $D$ wrt $IC$. □

Proof of Proposition 9: The NP-complete PARTITION problem [12] can be reduced to this case for a fixed set of 1ADs. Let a $A$ be a finite set, whose elements $a$ have integer sizes $s(a)$. We need to determine if there exists a subset $S$ of $A$, such that $\sum_{a \in S} s(a) = n := (\sum_{a \in A} s(a))/2$.

We use two tables: $Set(\text{Element}, \text{Weight})$, with key $\{\text{Element}, \text{Weight}\}$, containing the tuples $(a, s(a))$ for $a \in A$; and $\text{Selection}(\text{Element}, X, Y)$, with key $\text{Element}$, fixable numerical attributes $X, Y$ (the partition of $A$) taking values
0 or 1 (which can be specified with 1ADs), and initially containing the tuples 
\((a, 0, 0)\) for \(a \in A\). Finally, we have the 1AD \(\forall E, X, Y \sim (Select(E, X, Y), X < 1, Y < 1)\).

There is a one-to-one correspondence between LS-repairs of the original 
database and partitions \(X, Y\) of \(A\) (collecting the elements with value 1 in either 
\(X\) or \(Y\)). Then, there is a partition with the desired property if the query 
\(Q : (Set(E, W), Select(E, X, Y), X = 1, sum(W) = n)\) has answer yes under 
the brave semantics. The query used in this proof is acyclic and belongs to the 
class \(C_{tree}\).

For the proof of Theorem 8 we need some preliminaries. Let us define a function 
\(F\), with domain \(G \times S\), where \(G = (V, E)\) is a graph and \(S\) is a subset of vertexes 
of graph \(G\), and range non-negative integers. The function is defined as the 
summation over all the vertices \(v \in S\), of cubes of the number of edges connecting 
\(v\) to vertexes in the complement of \(S\).

**Definition 12.** Given a graph \(G = (V, E)\) and subset of its vertexes \(S \subseteq V\)

\[- F^l(S, v) = |T(S, v)|^3 \text{ where } T(S, v) = \begin{cases} \{v' | v' \in (V \setminus S) \wedge (v, v') \in E\}, & v \in S \\ \emptyset, & v \notin S \end{cases} \]

\[- F(G, S) = \sum_{v \in S} F^l(S, v) \]

**Lemma A.4.** Given a fixed regular undirected graph \(G = (V, E)\) of degree 3, the 
maximal value of \(F(G, S)\) on all possible sets \(S \subseteq V\) is \((3^3 \times |I|)\) for \(I\) a maximal 
independent set.

**Proof:** Let us first assume that \(S\) is an independent set, not necessarily maximal. 
In this case the answer to \(F(G, S)\) will be \(3^3 \times |S|\), because each element \(v \in S\) 
is connected to three vertices in \(V \setminus S\). Then, among independent sets, the 
maximum value for \(F(G, S)\) is \(3^3 \times m\), where \(m\) is the maximum cardinality of 
an independent set.

Let \(G[S] = G(S, E_S)\) where \(E_S\) are all the edges \((v, v') \in E\) such that \(v, v' \in S\).

Now, if \(S\) is not an independent set, there exists a maximum independent set 
\(I_S\) of \(G[S]\). Every \(v \in (V \setminus S)\) is adjacent to at least one vertex in \(I_S\), otherwise 
\(I_S \cup \{v\}\), would be an independent set contained in \(S\) and with more vertices 
than \(I_S\), contradicting our choice of \(I_S\). Now let us define \(F_{ext}(S, v) = (F^l(S, v) + 
\sum_{(v, v') \in E} F^l(S, v'))\). Since every edge \(v' \in (S \setminus I_S)\) is adjacent to \(I_S\), it is easy 
to see that:

\[ F(G, S) \leq \sum_{v \in I} F_{ext}(S, v) \] (3)

We want to prove that \(F(G, S) \leq F(G, I_S)\). This, combined with equation 
(3) shows that it is enough to prove that \(\sum_{v \in I_S} F_{ext}(S, v) \leq F(G, I_S)\). Since 
\(F(G, I_S) = \sum_{v \in I_S} F^l(I_S, v)\), we need to prove \(\sum_{v \in I_S} F_{ext}(S, v) \leq \sum_{v \in I_S} F^l(I_S, v)\) 
and then, it would be sufficient to prove that \(F_{ext}(S, v) \leq F^l(I_S, v)\) is true for 
every \(v \in I_S\). For \(v \in I_S\) and \(S' = (S \setminus I_S)\), we have the following cases:

1. If \(v\) is adjacent to one vertex in \(S'\) then \(F_{ext}(S, v) \leq 2^3 + 2^3 = 3^3\) and \(F^l(I_S, v) = 3^3\) 
therefore \(F_{ext}(S, v) \leq (F^l(I_S, v) - 11)\).
2. If \( v \) is adjacent to two vertexes in \( S' \) in analogous way to item (1) we get 
\[
F_{ext}(S, v) \leq (F'(I_S, v) - 10).
\]
3. If \( v \) is adjacent to three vertexes in \( S' \) in analogous way to item (1) we get 
\[
F_{ext}(S, v) \leq (F'(I_S, v) - 3).
\]

Then, we have proved that \( F_{ext}(S, v) \leq F'(I_S, v) \) and therefore that 
\[
F(G, S) \leq F(G, I_S). \]
We also know that, since \( I_S \) is an independent set, that 
\[
F(G, I_S) \leq 3^k \times m. \]

Proof of Theorem 8: (a) For sum: By reduction from a variation of Independence Set, for graphs whose vertices have all the same degree. It remains NP-hard as a special case of Independence Set for Cubic Planar Graphs [13].

Given an undirected graph \( G = (V, E) \) with degree 3, and a minimum bound \( k \) for the size of the maximal independent set, we create a relation \( Vertex(V, C_1, C_2) \), where the key \( V \) is a vertex and \( C_1, C_2 \) are fixable and may take values 0 or 1, but are all equal to 0 in the initial instance \( D \). This relation is subject to the denial \( IC : \forall C_1, C_2 \neg (Vertex(V, C_1, C_2), C_1 < 1, C_2 < 1) \).

\( D \) is inconsistent wrt this constraint and in any of its LS-fix each vertex \( v \) will have associated a tuples \( Vertex(v, 1, 0) \) or \( Vertex(v, 0, 1) \) but not both. Each LS-fix of the database defines a partition of \( V \) into two subsets: \( S \) with \( (v, 1, 0) \) and \( S' \) with \( (v, 0, 1) \), where clearly \( S \cup S' = V \) and \( S \cap S' = \emptyset \).

Let us define a second relation \( Edge(V_1, V_2; W) \), with rigid attributes only, that contains the tuples \( (v_1, v_2, 1) \) for \( (v_1, v_2) \in E \) or \( (v_2, v_1) \in E \). Every vertex \( v \) appears in each argument in exactly 3 tuples.

Consider the ground aggregate conjunctive query \( Q \):

\[
\begin{align*}
q(sum(W_0)) & \leftarrow Vertex(V_1, C_{11}, C_{12}), C_{11} = 1, \\
& \quad Edge(V_1, V_2, W_0), \quad Vertex(V_2, C_{21}, C_{22}), C_{21} = 0, \\
& \quad Edge(V_1, V_3, W_1), \quad Vertex(V_3, C_{31}, C_{32}), C_{31} = 0, \\
& \quad Edge(V_1, V_4, W_2), \quad Vertex(V_4, C_{41}, C_{42}), C_{41} = 0.
\end{align*}
\]

The query \( Q \), computes the sum of cubes of the number of vertexes of \( S' \) adjacent to vertices in \( S \), i.e. it calculates the function from graph to nonnegative numbers corresponding to \( F(G, S) \) from Definition 12 with \( Q(D') = F(G, S) \) for \( D' \in Fix(D, IC) \) and \( S = \{ v \mid Vertex(v, 1, 0) \in D' \} \).

We are interested in the minimum and maximum value for \( Q \) in \( Fix(D, IC) \), i.e. the min-max answer introduced in [3]. Since the function is nonnegative and since its value is zero for \( S = \emptyset \) and \( S = V \) we have that its minimum value is zero. We are only missing to find its maximum value.

From Lemma 4 we have that the answer to query \( Q \) is at most \( 3^k \times |I| \) with \( I \) a maximum independent set. In consequence, the min-max answer for \( Q \) is \( (0, 3^k \times m) \), with \( m \) the cardinality of the maximum independent set; and then there is an independent set of size at least \( k \) if and only if the min-max answer to \( Q \) is at least \( k \times 3^k \).

(b) For count distinct: By reduction from MAXSAT. Assume that an instance for MAXSAT is given, consisting of a set \( U \) of propositional variables, a collection \( C \) of clauses over \( U \) and a positive integer \( k \). The question is whether at least \( k \) clauses can be satisfied simultaneously, which will get answer yes exactly when a question of the form \( countd \leq (k - 1) \), with \( countd \) defined by an aggregate
query over a database instance (both of them to be constructed below), gets
answer no under the min-max semantics.

Define a relation \( \text{Var}(u, c_1, c_2) \), with (rigid) first key attribute, and the second
and third fixable (the denial below and the minimality condition will make them
take values 0 or 1). The initial database has a tuple \((u, 0, 0)\) for every \(u \in U\).

Another relation \( \text{Clause}(u, c, s) \), has no fixable attributes and contains for every
occurrence of variable \(u \in U\) in a clause \(c \in C\) a tuple \((u, c, s)\) with \(s\) an
assignment for \(u\) satisfying clause \(c\). The IC is \( \forall u, c_1, c_2 \neg (\text{Var}(u, c_1, c_2), c_1 < 1, c_2 < 1) \). The acyclic query is

\[
q(\text{countd}(c)) \leftarrow \text{Var}(u, c_1, c_2), \text{Clause}(u, c, s), c_1 = s,
\]

where \text{countd} denotes the “count distinct” aggregate function. Its answer tells
us how many clauses are satisfied in a given LS-fix. The max value taken on a
LS-fix, i.e. the min-max answer, will be the max number of clauses which may
be satisfied for MAXSAT.

(c) For average: By reduction from 3SAT. We use the same table \( \text{Var}(u, c_1, c_2) \)
and IC as in (a). Now, we encode clauses as tuples in a fixed relation

\[
\text{Clause}(\text{val}, \text{var}_1, \text{val}_1, \text{var}_2, \text{val}_2, \text{var}_3, \text{val}_3),
\]

where \(\text{var}_1, \text{var}_2, \text{var}_3\) are the variables in the clause (in any order), \(\text{val}_1, \text{val}_2, \text{val}_3\) all possible combinations of
truth assignments to variables (at most 8 combinations per clause). And \(\text{val}\) is
the corresponding truth value for the clause (0 or 1). Now, consider the acyclic
query

\[
q(\text{avg}(\text{val})) \leftarrow \text{Clause}(\text{val}, \text{var}_1, \text{val}_1, \text{var}_2, \text{val}_2, \text{var}_3, \text{val}_3),
\text{Var}(\text{var}_1, \text{val}_1, \text{val}_1'), \text{Var}(\text{var}_2, \text{val}_2, \text{val}_2'), \text{Var}(\text{var}_3, \text{val}_3, \text{val}_3').
\]

Then value of \(q\) is maximum in a LS-fix, taking value 1, i.e. the min-max answer
to \(q\) is 1, iff the formula satisfiable. \(\square\)

Proof of Theorem 9: First we reduce CQA under range semantics for
aggregate queries with sum to RWAE2, a restricted weighted version of the
problem of solving algebraic equations over \(GF[2]\), the field with two elements.
Next, we prove that such an algebraic problem can be solved within constant
approximation factor.

(A). Reduction to RWAE2. In order to define polynomial equations, we need
variables. We introduce a set \(\mathcal{V}\) of variables \(X_{k,t}^R\), taking values in \(GF[2]\), for
every tuple \(t_i\) in an LS-fix corresponding to a tuple \(t\) (a ground database atom
in the database) with key \(k\) in a relation \(R\) in the original database, i.e. \(t_i\)
belongs to some LS-fix and \(t_i\) share the key values \(k\). For example if the tuple
\(t\) is consistent or admits only one local fix (one attribute can be changed and in
only one way), only one variable is introduced due to \(t\). Denote with \(\text{bag}(t)\) the
set of variables introduced due to a same initial tuple \(t\).

Consider a conjunctive query

\[
Q(\text{sum}(z)) : -R_1(x), \ldots, R_m(x).
\]
Throughout the proof $\psi$ is the body of the query as a conjunction of atoms, $m$ is the number of database predicates in $\psi$, $n$ is the number of tuples in the database, $k$ is the maximal number of attribute comparisons in the ICs (and the maximal number of fixes of a given tuple).

We may consider all the possible assignments $\beta$ from database atoms in the query to grounds tuples in fixes that satisfy $\psi$. The number of assignments is polynomial in the size of the database, actually $\leq n^m$. Notice that the number of LS-fixes of a database may be exponential, but the number of local fixes of each original tuple is restricted by the number of attributes of the tuple. So, the number of all possible LS-fixes of tuples is polynomial in the size of the original database (even linear). Here we are using the fact that we have 1ADs.

Now we build a system $E$ of weighted algebraic equations. Each such assignment $\beta$ is associated with a combination of tuples $t^{R_1}_{k_1,i_1}, \ldots, t^{R_m}_{k_m,i_m}$ satisfying $\psi$. For each combination put the following equation $E^\beta$ over $GF[2]$ into $E$:

\[
\underbrace{X^{R_1}_{k_1,i_1} \ldots X^{R_m}_{k_m,i_m}}_{\text{selected}} \prod_{i \neq i_1} (1 - X^{R_1}_{k_1,i_1}) \ldots \prod_{i \neq i_m} (1 - X^{R_m}_{k_m,i_m}) = 1. \quad (4)
\]

The first product in (4), before the first $\prod$, contains the variables corresponding to the tuples selected by $\beta$. The rest of the product contains variables for the those tuples that were not selected, i.e. if $t_1$ appears in the first product, with $t_1 \in \text{bag}(t)$, and $t_2 \in \text{bag}(t)$, with $t_1 \neq t_2$, then the variable $X_2$ corresponding to $t_2$ appears as $(1 - X_2)$ in the second part of the product. This captures the restriction that no two different tuples from the same bag can be used (because the share the key values). For each combination $\beta$ of tuples in LS-fixes there is no more then one equation, which in turn has a polynomial number of factors.

Equation (4) gets weight $w(E^\beta)$ that is equal to the value of aggregation attribute $z$ in $\beta$.

In this way we have an instance of the RWAE2. It requires to find the maximum weight for a subsystem of $E$ that can be (simultaneously) satisfied in $GF[2]$, where the weight of the subsystem is the sum of the weights of the individual equations. Of course, this problem also has a version as a decision problem, so as CQA under range semantics.

Claim: The maximal weight of a satisfied subsystem of $E$ is the same as the maximal value of $Q(\text{sum}z)$ over all possible LS-fixes of $D$.

Assume that query $Q$ takes a maximum value over all possible LS-fixes of $D$ on an LS-fix $D'$. Under 1ADs a database LS-fix $D'$ is a set union of local fixes, with one local fix selected for every original tuple. Consider an assignment $A$ defined on $V$ that maps variables corresponding a selected local fix to 1 and all other variables to 0.

Consider all sets of local fixes which simultaneously satisfy $\psi$. If local fixes $t_1, \ldots, t_m$ satisfy $\psi$, then there exist exactly one equation $e$ for that given set of local fixes. The equation $e$ will be satisfied since variables corresponding to selected local fixes have value 1, and “non-selected” variables have value 0. So,
for every set of local fixes satisfying the query body, there would be a satisfied equation with weight equal to the value of aggregated attribute. It means, that a solution to the algebraic equation problem is bigger or equal to the maximal query answer (min-max answer).

Consider an assignment \( A \) which is a solution of algebraic equation problem. It maps elements of \( V \) to \( \{0, 1\} \) in such a way that the weight of satisfied equations of \( E \) is maximum over all possible assignment for \( V \).

First we prove that if there exists a bag \( B \) such that more than one of its variables is mapped to 1, then there exist an assignment \( A' \) with the same weight of satisfied equations of \( E \) as \( A \), but \( B \) contains no more then one variable mapped to 1.

Assume that for a bag \( B \) more then two variables (let us say \( X_i, X_j \)) are mapped to 1. It means that every equation which contains variables from \( B \) will be unsatisfied, since it contains either \((1 - X_i)\) or \((1 - X_j)\) as factors in the equation. If we change a value of one of the variables (say \( X_i \)) to 0, then no satisfied equation become unsatisfied, since satisfied equations do not contain \( X_i \). No unsatisfied equation becomes satisfied, because due to the assumption of maximality of the weight of the satisfied subset of \( E \) for \( A \).

In a second step, we prove that if \( A \) is a maximal assignment and there exist a bag \( B \) such that all of its variables are mapped to 0, then there exist an assignment \( A' \), which satisfies the same subset of \( E \) as \( A \), but at least one variable from that \( B \) is mapped to 1.

If all variables from a bag \( B \) are mapped to 0, then all equations which contain variables from \( B \) are unsatisfied. If we change a value of one variable to 1, then no satisfied equation becomes unsatisfied since all satisfied equations do not contain variables from \( B \). No unsatisfied equation becomes satisfied due to maximality assumption of the weight of satisfied equation for \( A \). Taking step by step all bags from \( V \), for given a maximum assignment \( A \), we produce an assignment \( A' \), which has exactly one variable from each bag mapped to 1.

Now, construct a database \( D' \) which is a set of local fixes corresponding to variables mapped to 1. It is obviously a LS-fix, and \( w(E(A)) \leq Q(D') \).

**B.** A deterministic approximation algorithm for RWAE2. The construction and approximation factor obtained are similar those in the approximation of MAXSAT. C.f. [23, 20]. In two steps, first a randomized algorithm is produced, that is next de-randomized.

**B1.** Randomized approximation algorithm. Assume that from each bag we select one variable with probability \( 1/k \), where \( k \) is the number of variables in the bag. We map selected variable to 1 and all other variables in the bag to 0. For each equation \( e \), random variable \( W_e \) denotes the weight contributed by \( e \) to the total weight \( W \). Thus, \( W = \sum_{e \in E} W_e \) and \( E[W_e] = w_e \cdot Pr[e \text{ is satisfied}] \), where \( E \) is a mathematical expectation and \( Pr \) is a probability.

If the query contains \( m \) predicates, then each equation contains no more than \( m \) variables from different bags (never two different variables from the same bag),
then $E(W_e) \geq k^{-m}w_e$. Now, by linearity of expectation,

$$E[W] = \sum_{e \in E} E[W_e] \geq k^{-m} \sum_{e \in E} w_e \geq k^{-m} \cdot OPT.$$  

**B2. De-randomization via conditional expectation.** We first establish

**Claim:** The RWAEE2 problem is self-reducible [23, Chap. A.5].

In fact, assume $A'$ is a partial assignment from $V$, such that variables $X_1, \ldots, X_i$ are mapped to $\{0, 1\}$. Let $E^s$ be the set of equations satisfied by $A'$ with total weight $W[E^s]$, and $E^u$ is the set of equations which cannot be satisfied under $A'$. Let $E''$ be a set of equations from $E \setminus (E^s \cup E^u)$, such that variables from $X_1, \ldots, X_i$ are replaced by their values. By additivity of the weight function and the independence of the variables, the maximal weight of satisfied equations under an assignment which extends $A'$ is $W[E^s] + \max W[E'']$, where $W[E'']$ is a solution of the RWAEE2 problem restricted to $E''$. It is good enough to consider the self-reducibility trees $T$ such only one variable from each bag gets value 1 along any path in the tree. This establishes our claim.

Assume that a self-reducibility tree $T$ is given, with each node in it corresponding to a step of the self-reduction. Each node $v$ of $T$ is labelled with $X_1 = a_1, \ldots, X_i = a_i$, a partial assignment of values to variables $X_1, \ldots, X_i \in V$ associated to the step for $v$ of the self-reduction. Since this is a partial assignment, some of the equations in $E$ become immediately satisfied, other unsatisfied, and some other undetermined. The latter become a set of equations $E'$ associated to $v$ on variables $V \setminus \{X_1, \ldots, X_i\}$, obtained from $E$ by giving to the variables $X_1, \ldots, X_i$ their values $a_1, \ldots, a_i$. By construction, these equations inherit the weight of the corresponding equations in $E$.

For example, if the set of equations consists of: (1) $yp(1 - x) = 1$, (2) $2xz(1 - y) = 1$, (3) $3zw(1 - y) = 1$, with variables $x, y, z, p, w$, and the partial assignment, at some step of self-reduction for $v$ is $x = 1, y = 0, w = 1$, then equation (1) becomes unsatisfiable, (2) is not satisfied but possibly satisfiable with an appropriate value for $z$; and (3) satisfied. So, $E'$ contains equation (2), but with $x, y$ replaced by their values 1, 0, resp.

The conditional expectation of any node $v$ in $T$ can be computed via its sets of equations $E'$ we just described. Clearly, the expected weight of satisfied equations of $E'$ under a random assignment of values in $GF[2]$ to $V \setminus X_1, \ldots, X_i$ can be computed in the polynomial time. Adding to this the weight of the equations in $E$ already satisfied by the partial assignment $X_1 = a_1, \ldots, X_i = a_i$ gives the conditional expectation.

Then we compute in polynomial time a path from the root to a leaf, such that the conditional expectation of each node on this path is $\geq E[W]$. This can be done as in the construction in [23, Theorem 16.4].

In consequence, we can find a deterministic approximate solution to the RWAEE2 problem in polynomial time. It approximates the optimum solution with a factor greater then $k^{-m}$. It means that we can approximate the maximal value of aggregate conjunctive query within a factor $k^{-m}$, which depends on integrity constraints and a query, but not depend on the size of the database. This ends the proof.
For example, the query with \textit{sum} used in the proof of the \textit{NP}-hardness in Theorem 8 has \( m = 4, k = 2 \), then it can be approximated within the factor \( 2^{-4} \).

\( \square \)

A.2 An Example for Theorem 1

Consider the diophantine equation

\[
2x^3y^2 + 3xy + 105 = x^2y^3 + y^2. \tag{5}
\]

Each term \( t \) in it will be represented by a relation \( R(t) \) with 8 attributes taking values in \( \mathbb{N} \): three, \( X_1, X_2, X_3 \), for the maximum exponent of \( x \), three, \( Y_1, Y_2, Y_3 \), for the maximum exponent of \( y \), one, \( C \), for the constant terms, plus a last one, \( K \), for a key. Value 0 for a non-key attribute indicates that the term appears in \( t \), otherwise it gets value 1. We introduce as many tuples in \( R \) as the coefficient of the term: they differ only in the key value. We will see that only the 0 values will be subject to fixes. These are the relations and their ICs:

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
R(2x^3y^2) & X_1 & X_2 & X_3 & Y_1 & Y_2 & Y_3 & C & K \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 \\
\hline
\end{array}
\]

For this table we have the following set, IC(2\( x^3y^2 \)), of ICs:

\begin{align*}
\forall x_1 \cdots x_8 & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land x_1 \neq x_2), \forall x_1 \cdots x_8 & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land x_2 \neq x_3), \\
\forall x_1 \cdots x_8 & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land x_5 \neq x_6), \forall x_1 \cdots x_8 & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land x_4 \neq 1), \\
\forall x_1 \cdots x_{16} & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land R(2x^3y^2)(x_9, \ldots, x_{16}) \land x_1 \neq x_9) \\
\forall x_1 \cdots x_{16} & \neg (R(2x^3y^2)(x_1, \ldots, x_8) \land R(2x^3y^2)(x_9, \ldots, x_{16}) \land x_5 \neq x_{13}). \\
\end{align*}

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
R(3xy) & X_1 & X_2 & X_3 & Y_1 & Y_2 & Y_3 & C & K \\
\hline
1 & 1 & 0 & 1 & 1 & 0 & 1 & 3 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 4 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 5 \\
\hline
\end{array}
\]

\textit{IC}(3\textit{xy}): \begin{align*}
\forall x_1 \cdots x_{16} & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land R(3\textit{xy})(x_9, \ldots, x_{16}) \land x_3 \neq x_{11}), \\
\forall x_1 \cdots x_{16} & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land R(3\textit{xy})(x_9, \ldots, x_{16}) \land x_6 \neq x_{14}), \\
\forall x_1 \cdots x_8 & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land x_1 \neq 1), \forall x_1 \cdots x_8 & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land x_2 \neq 1), \\
\forall x_1 \cdots x_8 & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land x_4 \neq 1), \\
\forall x_1 \cdots x_8 & \neg (R(3\textit{xy})(x_1, \ldots, x_8) \land x_5 \neq 1). \\
\end{align*}

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
R(105) & X_1 & X_2 & X_3 & Y_1 & Y_2 & Y_3 & C & K \\
\hline
1 & 1 & 1 & 1 & 1 & 1 & 105 & 6 \\
\hline
\end{array}
\]
∀x₁ \cdots x₈ ¬ (R(105)(x₁, \ldots, x₈) \land x₁ \neq 1), ∀x₁ \cdots x₈ ¬ (R(105)(x₁, \ldots, x₈) \land x₃ \neq 1),
∀x₁ \cdots x₈ ¬ (R(105)(x₁, \ldots, x₈) \land x₄ \neq 1),
∀x₁ \cdots x₈ ¬ (R(105)(x₁, \ldots, x₈) \land x₅ \neq 1),
∀x₁ \cdots x₈ ¬ (R(105)(x₁, \ldots, x₈) \land x₆ \neq 1),
∀x₁ \cdots x₈ ¬ (105(x₁, \ldots, x₈) \land x₇ \neq 105).

Similar tables $R(x^2y^3)$ and $R(y^2)$ and corresponding sets of ICs are generated for the terms on the RHS of (5).

Next we need ICs that are responsible for making equal all $x$s and $y$s in all terms of the equation:
∀x₁ \cdots x₁₆ ¬ (R(2x^3y^2)(x₁, \ldots, x₈) \land R(3xy)(x₉, \ldots, x₁₆) \land x₁ \neq x₁₁),
∀x₁ \cdots x₁₆ ¬ (R(2x^3y^2)(x₁, \ldots, x₈) \land R(3xy)(x₉, \ldots, x₁₆) \land x₃ \neq x₁₃),
∀x₁ \cdots x₁₆ ¬ (R(2x^3y^2)(x₁, \ldots, x₈) \land R(x^2y^3)(x₉, \ldots, x₁₆) \land x₁ \neq x₁₀),
∀x₁ \cdots x₁₆ ¬ (R(2x^3y^2)(x₁, \ldots, x₈) \land R(x^2y^3)(x₉, \ldots, x₁₆) \land x₅ \neq x₁₂),
∀x₁ \cdots x₁₆ ¬ (R(2x^3y^2)(x₁, \ldots, x₈) \land R(y^2)(x₉, \ldots, x₁₆) \land x₅ \neq x₁₃).

Now we construct a single table $R(equ)$ that represents equation (5) by appending the previous tables:

\[
\begin{array}{cccccccc}
R(equ) & X_1 & X_2 & X_3 & Y_1 & Y_2 & Y_3 & C & K \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & \cdot \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & \cdot \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 3 & \cdot \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 4 & \cdot \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 5 & \cdot \\
1 & 1 & 1 & 1 & 1 & 1 & 105 & 6 & \cdot \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & \cdot \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 8 & \cdot \\
\end{array}
\]

We need ICs stating the correspondence between the terms in the tables $R(t)$ and table $R(equ)$:
∀x₁ \cdots x₁₆ ¬ (R(equ)(x₁, \ldots, x₈) \land R(2x^3y^2)(x₉, \ldots, x₁₆) \land x₈ = x₁₆ \land x₁ \neq x₉),
∀x₁ \cdots x₁₆ ¬ (R(equ)(x₁, \ldots, x₈) \land R(2x^3y^2)(x₉, \ldots, x₁₆) \land x₃ = x₁₆ \land x₂ \neq x₁ₐ),
\ldots
∀x₁ \cdots x₁₆ ¬ (R(equ)(x₁, \ldots, x₈) \land R(y^2)(x₇ \cdots x₁₆) \land x₈ = x₁₆ \land x₇ \neq x₁₅).

Finally, we have one aggregate constraint that is responsible for making equal the LHS and RHS of equation (5):
\[
\text{sum}_{R(equ)}(x₁ \cdot x₂ \cdot x₃ \cdot x₄ \cdot x₅ \cdot x₆ \cdot x₇ : x₆ < 7) = \text{sum}_{R(equ)}(x₁ \cdot x₂ \cdot x₃ \cdot x₄ \cdot x₅ \cdot x₆ \cdot x₇ : x₆ > 6).
\]

If the database has an LS-fix, then there is an integer solution to the diophantine equation. If the equation has a solution $s$, then there is an instance $R(equ)'$ corresponding to $s$ that satisfies the ICs. By Proposition 1, there is an LS-fix of the database.

The reduction could be done with the table $R(equ)$ alone, making all the ICs above to refer to this table, but the presentation would be harder to follow.