Lattice results on dibaryons and baryon–baryon interactions

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We present the recent study on dibaryons at the almost physical pion mass in lattice QCD by the HAL QCD potential method.

Keywords: Dibaryon; HAL QCD potential method; almost physical pion mass

1. Introduction

A diabaryon is a bound state (or resonance) with a baryon number \( B = 2 \). The deuteron, made of a proton and a neutron, is only a stable dibaryon observed in Nature so far. Thus it is interesting to ask whether other dibaryons exist in Nature or not.

A dibaryon can be classified in the flavor SU(3) representation as

\[
8 \otimes 8 = 27 \oplus 8_a \oplus 1 \oplus 10 \oplus 8_a
\]

for the octet–octet baryons, where a deuteron belongs to the \( 10 \) representation, while a \( H \)-dibaryon was predicted in the \( 1 \) representation\(^1\), and was recently studied in lattice QCD\(^2\text{--}^6\). Classifications including decuplet (10) baryons are

\[
10 \otimes 8 = 25 \oplus 8 \oplus 10 \oplus 27,
\]

where \( N\Omega \) and \( N\Delta \) dibaryons were predicted in \( 8 \) and \( 27 \) representations, respectively\(^7,^8\), and

\[
10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus 10\!
\]

where \( \Omega\Omega \) was predicted in the \( 28 \) representation\(^9\), while \( \Delta\Delta \) dibaryon were predicted in the \( 10 \) representation\(^10,\!11\), whose candidate \( d^*(2380) \) has indeed been observed\(^12\). Note however that only \( \Omega \) is a stable decuplet baryon against strong decays.
2. HAL QCD potential method

A fundamental quantity in the HAL QCD method\textsuperscript{13–15} is an equal-time Nambu–Bethe–Salpeter (NBS) wave function in the center of mass system, which is given for a two nucleon system as

\[ \psi_k(r) = \langle 0 | N(r/2, 0)N(-r/2, 0) | NN, W_k \rangle, \] (4)

where \(| NN, W_k \rangle\) is a two-nucleon eigenstate in QCD having the relative momentum \(k\) and the center of mass energy \(W_k = 2\sqrt{k^2 + m_N^2}\) with the nucleon mass \(m_N\), and \(N(x)\) with \(x = (x, t)\) is the nucleon operator. In our study, we usually take the local operator in terms of quark fields as \(N(x) = \epsilon^{abc}(u_a(x)C\gamma_5d_b(x))q_c(x)\), where \(u_a(x)\) (\(d_a(x)\)) is an up (down) quark field with color \(a\) while \(q = u\) (\(d\)) corresponds to a proton (neutron), and \(C = \gamma_2\gamma_4\) is a charge conjugation matrix acting on spinor indices. A choice of the nucleon operator is a part of the definition (or scheme) for the potential.

In the HAL QCD method, we restrict the total energy below the lowest inelastic threshold as \(W_k < W_{\text{th}} \equiv 2m_N + m_\pi\) with a pion mass \(m_\pi\), so that only the elastic \(NN\) scatterings can occur.

It can be shown\textsuperscript{16,17} that an asymptotic behavior of the NBS wave function at large \(r \equiv |r|\) is given by

\[ \psi_k(r) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{lm}(\Omega_r), \ k \equiv |k|, \] (5)

where \(Y_{lm}\) is a spherical harmonic function for the solid angle of \(r\) (\(\Omega_r\)). For simplicity, we ignore the spin of nucleons here, but you can find a complete formula in Refs.\textsuperscript{14,18}. It is important to note that \(\delta_l(k)\) is the phase of the \(S\)-matrix in QCD for the partial wave with the angular momentum \(l\), which is encoded in the asymptotic behavior of the NBS wave function, similar to the scattering phase shift of the scattering wave in quantum mechanics.

Using this property, we define the energy–independent potential with derivatives from NBS wave functions as

\[ [E_k - H_0] \psi_k(r) = V(r, \nabla) \psi_k(r), \ E_k = \frac{k^2}{m_N}, \ H_0 = -\frac{\nabla^2}{m_N}, \] (6)

for \(W_k < W_{\text{th}}\). The potential at the next-to-leading order (NLO) takes a form as

\[ V(r, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + O(\nabla^2), \] (7)

where \(\sigma_i\) is a spin operator acting on the \(i\)-th nucleon, \(S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - (\sigma_2 \cdot \sigma_1)\) with \(\hat{r} = r/r\) is the tensor operator, \(L = r \times \nabla\) and \(S = (\sigma_1 + \sigma_2)/2\).
The Schrödinger equation with the potential $V(r, \nabla)$ give a correct QCD phase shift $\delta_1(k)$, since NBS wave functions are solutions to the equation by construction. Note that non-relativistic approximation is not employed here since the Klein–Gordon operator reduces to the Helmholtz operator for a given center of mass energy as $-\Box - m^2 = (W_k/2) + \nabla^2 - m^2 = k^2 + \nabla^2$.

We determine local functions such as $V_X(r)$ ($X = 0, \sigma, T, \text{LS}$) order by order. For example, the leading order (LO) potential

$$V^{\text{LO}}_0(r) = V_0(r) + V_G(r) \sigma_1 \cdot \sigma_2 + V_T(r) S_{12}$$

by construction. Note that non-relativistic approximation is not employed.

The Schrödinger equation with the potential $V(r)$ gives the wave function as

$$\hat{H}_0 \psi_k(r) = E_k \psi_k(r)$$

where an argument $\psi_k$ of the potential represents an input for its determination. If $V^{\text{LO}}_0(r; \psi_k) \approx V^{\text{LO}}_0(r; \psi_q)$ for $|k| < |q|$, it turns out that the LO approximation is good at $p$ in $|k| \leq |p| \leq |q|$. If $V^{\text{LO}}_0(r; \psi_k) \neq V^{\text{LO}}_0(r; \psi_q)$, on the other hand, the NLO term can be determined from two equations given by

$$[E_k - H_0] \psi_p(x) = [V^{\text{NLO}}_0(r) + V^{\text{NLO}}_{\text{LS}}(r) \mathbf{L} \cdot \mathbf{S}] \psi_p(x), \quad p = k, q,$$

where a superscript NLO represent the order of the approximation to determine these terms. We can continue this procedure to increase accuracy of the determination. Once the potential is approximately obtained, physical observables such as scattering phase shift can be extracted.

In lattice QCD, a NBS wave function is extracted from a 4-pt correlation function as

$$F(r, t) = \langle 0| N(r/2, t) N(-r/2, t) \bar{J}_{NN}(0)|0 \rangle = \sum_n A_n \psi_{k_n}(r) e^{-W_{k_n}t} + \cdots$$

$$\simeq A_0 \psi_{k_0}(r) e^{-W_{k_0}t}, \quad t \to \infty,$$  \hspace{1cm} (10)

where $\bar{J}_{NN}(t)$ is an operator which creates two-nucleon elastic states at $t$ with an overlap factor $A_n = \langle NN, W_{k_n} | \bar{J}_{NN}(0) |0 \rangle$, an ellipsis represents contributions form inelastic states, and $W_{k_n}$ is an energy of the $NN$ ground state. In practice it is very difficult to take a large $t$ due to a bad signal-to-noise ratio for two baryons, but a use of smaller $t$ may introduce large systematic errors due to contaminations from elastic excited states to the grand state, which is a very serious problem for the conventional method.

In Ref.\textsuperscript{25}, an improved method to extract potentials has been proposed. We define the normalized 4-pt function as

$$R(r, t) = \frac{F(r, t)}{G_N(t)^2} = \sum_n \bar{A}_n \psi_{k_n}(r) e^{-\Delta W_{k_n}t} + \cdots, \quad \Delta W_{k_n} = W_{k_n} - 2m_N.$$
where $G_N(t)$ is a nucleon 2-pt function at rest, which behaves as $Ze^{-m_N t}$ as long as inelastic contributions to the 2-pt function can be neglected, and $\bar{A}_n = A_n / Z^2$. Since all NBS wave functions, $\phi_{kn}$, below inelastic threshold satisfy the same Shrödinger equation (6), we obtain
\begin{equation}
\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N^2} \frac{\partial^2}{\partial t^2}\right\} R(r,t) = V(r,\nabla)R(r,t) \simeq V_{LO}^0(r)R(r,t), \quad (11)
\end{equation}
where we use a relation $\Delta W_k = k^2 / m_N - (\Delta W_k)^2 / (4m_N^2)$, and we need to take a moderately large $t$ satisfying $W_{th} t \gg 1$ to ignore inelastic contributions. Note that $V(r,\nabla)$ extracted from the above equation should be $t$ independent. Therefore, the $t$ dependence for the LO potential $V_{LO}^0(r)$, for example, indicates either an existence of inelastic contributions or contributions from higher order terms in the derivative expansion.

3. Dibaryons at the almost physical pion mass

As an application of the HAL QCD potential method, we presents some results on dibaryons.

In our studies on dibaryons, we employ (2+1)-flavor gauge configurations generated on a $L^3 \times T = 96^3 \times 96$ lattice with the RG-improved Iwasaki gauge action and non-perturbatively $O(a)$-improved Wilson quark action, at $a \simeq 0.085$ fm (thus $La \simeq 8.1$ fm) with $(m_\pi, m_K, m_N) \simeq (146, 525, 955)$ MeV, which correspond to the almost physical point.

We first consider the $\Omega^-\Omega^-$ system in the $^1S_0$ channel, which belongs to the $28$ representation.

In Fig. 1 (Left), we show $\Omega^-\Omega^-$ potentials at $t/a = 16, 17, 18$, which has qualitative features similar to the central potentials for $NN$. We notice,
Fig. 2. (Left) The $N\Omega$ potential $V_C(r)$ in the $^5S_2$ channel at $t/a = 11, 12, 13, 14$, with the same lattice setup for $\Omega\Omega$. (Right) The binding energy and the root-mean-square distance for the $n\Omega^-$ (red open circle) and $p\Omega^-$ (blue open square). Taken from $^{28}$.

however, that its repulsion is weaker and attraction is shorter-ranged than the $NN$ case. With this potential, we obtain one shallow bound state, whose binding energy is shown in Fig. 1 (Right) as a function of the root-mean-square distance, with and without Coulomb repulsion between $\Omega^-\Omega^-$ as $\alpha/r$, denoted by red circle and blue square, respectively. Such a bound state may be searched experimentally by two-particle correlations in future relativistic heavy-ion collisions$^{27}$.

We next consider the $N\Omega^-$ system with $S = -3$ in the $^5S_2$ channel, which belongs to the 8 representation$^{28}$. At the almost physical pion mass, $N\Omega(^5S_2)$ may couple to $D$-wave octet-octet channels below the $N\Omega$ threshold such as $\Lambda\Xi$ and $\Sigma\Xi$. We thus assume that such couplings are small.

In Fig. 2 (Left), we plot the $N\Omega^-$ potential at $t/a = 11–14$, showing attraction at all distances without repulsive core. Thus there is a chance to form a bound state, and indeed one bound state is found to exist in this channel. Fig. 2 (Right) shows the binding energy as a function of the the root-mean-square distance for $n\Omega^-$ with no Coulomb interaction (red) and $p\Omega^-$ with Coulomb attraction (blue). These binding energies are found to be much smaller than $B = 18.9(5.0)(^{+12.1}_{-10.8})$ MeV at heavier pion mass $m_\pi = 875$ MeV$^{29}$. Such a $N\Omega$ state can be searched through two-particle correlations in relativistic nucleus-nucleus collisions$^{27}$, if indeed exists. Actually, some indications in experiments were recently reported$^{30}$, and more will be expected to come.

From potentials we obtain for $\Omega\Omega(^1S_0)$ and $N\Omega(^5S_2)$, we calculate the scattering length $a_0$ and the effective range $r_{eff}$ for these systems. Fig. 3 shows the ratio $r_{eff}/a_0$ as a function of $r_{eff}$ for $\Omega\Omega(^1S_0)$ (blue diamond)
Fig. 3. The ratio of the effective range and the scattering length $r_{\text{eff}}/a_0$ as a function of $r_{\text{eff}}$ for $\Omega\Omega(^1S_0)$ (blue open diamond) and $N\Omega(^3S_2)$ (red open circle) obtained in lattice QCD, as well as for $NN(^3S_1)$ (purple open up-triangle) and $NN(^1S_0)$ (green open down-triangle) in experiments. Taken from 28 and $N\Omega(^5S_2)$ (red circle) obtained in lattice QCD near the physical pion mass, together with the experimental values for $NN(^3S_1)$ (deuteron, purple up-triangle) and $NN(^1S_0)$ (di-neutron, green down-triangle). For all cases, $|r_{\text{eff}}/a_0|$ is small, indicating that these systems are located close to the unitary limit. It will be interesting to understand why dibaryons or dibaryon candidates appear in the unitary region near the physical pion mass.

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References

1. R. L. Jaffe, Perhaps a Stable Dihyperon, Phys. Rev. Lett. 38, 195 (1977), [Erratum: Phys. Rev. Lett. 38, 617 (1977)].
2. T. Inoue, N. Ishii, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, K. Murano, H. Nemura and K. Sasaki, Bound H-dibaryon in Flavor SU(3) Limit of Lattice QCD, Phys. Rev. Lett. 106, p. 162002 (2011).
3. T. Inoue, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, N. Ishii, K. Murano, H. Nemura and K. Sasaki, Two-Baryon Potentials and H-Dibaryon from 3-flavor Lattice QCD Simulations, *Nucl. Phys.* **A881**, 28 (2012).

4. T. Inoue, N. Ishii, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, K. Murano, H. Nemura and K. Sasaki, Baryon-Baryon Interactions in the Flavor SU(3) Limit from Full QCD Simulations on the Lattice, *Prog. Theor. Phys.* **124**, 591 (2010).

5. S. R. Beane et al., Evidence for a Bound H-dibaryon from Lattice QCD, *Phys. Rev. Lett.* **106**, p. 162001 (2011).

6. A. Francis, J. R. Green, P. M. Junnarkar, C. Miao, T. D. Rae and H. Wittig, Lattice QCD study of the H dibaryon using hexaquark and two-baryon interpolators, *Phys. Rev.* **D99**, p. 074505 (2019).

7. J. T. Goldman, K. Maltman, G. J. Stephenson, Jr., K. E. Schmidt and F. Wang, STRANGENESS –3 DIBARYONS, *Phys. Rev. Lett.* **59**, p. 627 (1987).

8. M. Oka, Flavor Octet Dibaryons in the Quark Model, *Phys. Rev. D38*, p. 298 (1988).

9. Z. Y. Zhang, Y. W. Yu, P. N. Shen, L. R. Dai, A. Fuessler and U. Straub, Hyperon nucleon interactions in a chiral SU(3) quark model, *Nucl. Phys.* **A625**, 59 (1997).

10. F. Dyson and N. H. Xuong, Y=2 States in Su(6) Theory, *Phys. Rev. Lett.* **13**, 815 (1964).

11. T. Kamae and T. Fujita, Possible Existence of a Deeply Bound Delta-Delta System, *Phys. Rev. Lett.* **38**, 471 (1977).

12. P. Adlarson et al., ABC Effect in Basic Double-Pionic Fusion — Observation of a new resonance?, *Phys. Rev. Lett.* **106**, p. 242302 (2011).

13. N. Ishii, S. Aoki and T. Hatsuda, The Nuclear Force from Lattice QCD, *Phys. Rev. Lett.* **99**, p. 022001 (2007).

14. S. Aoki, T. Hatsuda and N. Ishii, Theoretical Foundation of the Nuclear Force in QCD and its applications to Central and Tensor Forces in Quenched Lattice QCD Simulations, *Prog. Theor. Phys.* **123**, 89 (2010).

15. S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki, Lattice QCD approach to Nuclear Physics, *PTEP 2012*, p. 01A105 (2012).

16. C. J. D. Lin, G. Martinelli, C. T. Sachrajda and M. Testa, $K \rightarrow \pi\pi$ decays in a finite volume, *Nucl. Phys.* **B619**, 467 (2001).

17. S. Aoki et al., I=2 pion scattering length from two-pion wave functions, *Phys. Rev. D71*, p. 094504 (2005).
18. N. Ishizuka, Derivation of Luscher’s finite size formula for N pi and NN system, PoS LAT2009, p. 119 (2009).
19. T. Iritani et al., Mirage in Temporal Correlation functions for Baryon-Baryon Interactions in Lattice QCD, JHEP 10, p. 101 (2016).
20. S. Aoki, T. Doi and T. Iritani, Lüscher’s finite volume test for two-baryon systems with attractive interactions, PoS LATTICE2016, p. 109 (2017).
21. T. Iritani, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, H. Nemura and K. Sasaki, Are two nucleons bound in lattice QCD for heavy quark masses? Consistency check with Lüscher’s finite volume formula, Phys. Rev. D96, p. 034521 (2017).
22. S. Aoki, T. Doi and T. Iritani, Sanity check for NN bound states in lattice QCD with Lüscher’s finite volume formula – Exposing Symptoms of Fake Plateaux –, in 35th International Symposium on Lattice Field Theory (Lattice 2017) Granada, Spain, June 18-24, 2017, 2017.
23. T. Iritani, S. Aoki, T. Doi, S. Gongyo, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, H. Nemura and K. Sasaki, Systematics of the HAL QCD Potential at Low Energies in Lattice QCD, Phys. Rev. D99, p. 014514 (2019).
24. T. Iritani, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, H. Nemura and K. Sasaki, Consistency between Lüscher’s finite volume method and HAL QCD method for two-baryon systems in lattice QCD, JHEP 03, p. 007 (2019).
25. N. Ishii, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, K. Murano, H. Nemura and K. Sasaki, Hadron-hadron interactions from imaginary-time Nambu-Bethe-Salpeter wave function on the lattice, Phys. Lett. B712, 437 (2012).
26. S. Gongyo et al., Most Strange Dibaryon from Lattice QCD, Phys. Rev. Lett. 120, p. 212001 (2018).
27. K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya and A. Ohnishi, Probing ΩΩ and pΩ dibaryons with femtoscopic correlations in relativistic heavy-ion collisions (2019).
28. T. Iritani et al., NΩ dibaryon from lattice QCD near the physical point, Phys. Lett. B792, 284 (2019).
29. F. Etminan, H. Nemura, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, K. Murano and K. Sasaki, Spin-2 NΩ dibaryon from Lattice QCD, Nucl. Phys. A928, 89 (2014).
30. J. Adam et al., The Proton-Ω correlation function in Au+Au collisions at √s_{NN}=200 GeV, Phys. Lett. B790, 490 (2019).