Combining directional light output and ultralow loss in deformed microdisks

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A drawback of high-quality modes in optical microdisks is their isotropic light emission characteristics. Here we report a novel, robust, and general mechanism that results in highly directional light emission from those modes. This surprising finding is explained by a combination of wave phenomena (wave localization along unstable periodic ray trajectories) and chaotic ray dynamics in open systems (escape along unstable manifolds). The emission properties originating in the chaotic ray dynamics permit directional light output even from microcavities at low thresholds. Moreover, all deformed microdisks can be optimized either w.r.t. directional emission and therefore for more efficient extraction and collection of light. Several shapes have been proposed and realized since then, but only a few lead to directional light output even from high-Q modes in microdisks which does not suffer from the above-mentioned drawback. The key to our finding is to apply what is at the heart of quantum chaos and nonlinear dynamics of open systems, respectively: wave localization along unstable periodic ray trajectories in systems with chaotic ray dynamics and the only recently acknowledged importance of the so-called unstable manifold for the FFPs of microcavities. The wave localization ensures the desired high Q-factors, whereas the unstable manifold provides the directional emission. Concerning the realisation of this scheme in practise, adequate microcavity devices can be expected to be easy to fabricate as no sophisticated adjustment of parameters is required. They are, moreover, well suited for multi-mode laser operation as all high-Q modes of given polarization possess similar FFPs.

A microdisk is a quasi-two-dimensional geometry described by an effective index of refraction $n$. We assume $n = 3.3$ (GaAs) both for transverse magnetic (TM) and transverse electric (TE) polarization. A slight polarization dependence of $n$ is neglected; it could be adjusted in the fabrication process, e.g., by changing the slab thickness. For the boundary value of the deformed microdisk we choose the limaçon of Pascal which reads in polar coordinates 

$$
\rho(\phi) = R(1 + \varepsilon \cos \phi)
$$

The corresponding family of closed cavities is known as
“limaçon billiards” [32]. Ray and wave dynamics in billiards has been extensively discussed in the fields of non-linear dynamics [24] and quantum chaos [25]. The limiting case of vanishing deformation parameter \( \varepsilon \) is the circle with radius \( R \). Figure 1(a) illustrates a whispering-gallery ray trajectory in a circular microdisk trapped by total internal reflection. A two-dimensional phase space representation, the so-called Poincaré surface of section (SOS), is shown in Fig. 1(b). Whenever the trajectory hits the cavity’s boundary, its position \( s \) (arclength coordinate along the circumference) and tangential momentum \( \sin \chi \) (the angle of incidence \( \chi \) is measured from the surface normal) is recorded. For \( \varepsilon = 0 \), rotational invariance of the system implies conservation of the angular momentum \( \propto \sin \chi \). Ignoring wave effects, such a ray never leaves the cavity since it cannot enter the leaky region between the two critical lines for total internal reflection given by \( \sin \chi_c = \pm 1/n \).

Figures 1(c) and (d) show a trajectory in the limaçon cavity for \( \varepsilon = 0.43 \). In contrast to the case of small deformation parameter \( \varepsilon \) [23] the dynamics is predominantly chaotic. Starting with an initial \( \chi \) well above the critical line, a test ray (square, thick dots, and triangle) rapidly approaches the leaky region (\( \sin \chi \) is not conserved) where it escapes according to Snell’s and Fresnel’s laws. Without refractive escape (\( n = \infty \), hard wall or closed billiard limit), the trajectory would fill the phase space in a random fashion (small dots). Periodic ray trajectories do exist but are always unstable, except for the two islands in the leaky region. Whispering-gallery trajectories are confined to the tiny region \( |\sin \chi| \geq 0.99 \).

While in the long-time limit the phase space of closed chaotic systems is essentially structureless, cf. Fig. 1(d), the phase space of an open chaotic system is structured by the so-called “chaotic repeller” [26]. It is the set of points in phase space that never visits the leaky region both in forward and backward time evolution. The stable (unstable) manifold of a chaotic repeller is the set of points that converges to the repeller in forward (backward) time evolution. The unstable manifold therefore describes the route of escape from the chaotic system. In the case of light, Fresnel’s laws impose an additional, polarization dependent weighting factor to the unstable manifold in the leaky region [31], since at each reflection the intensity inside is multiplied by the Fresnel reflection coefficient.

Following Refs. [29, 31] the unstable manifold can be uniquely computed as a survival probability distribution calculated from an ensemble of rays starting uniformly in phase space having identical intensity. Figure 2 depicts the resulting Fresnel weighted unstable manifolds for the limaçon cavity using 50,000 rays. Note that (i) in the leaky region, the manifold is concentrated on very few high-intensity spots. We therefore expect a highly directional FFP. (ii) While in the case of TE polarization one finds one spot with \( \chi > 0 \) (and another symmetry-related one at \( s \to s_{\text{max}} - s, \chi \to -\chi \)), the TM polarization case possesses two of those.

The unstable manifold in the leaky region directly determines the FFP. Mapping the unstable manifold in Fig. 2 to the far field by using Snell’s and Fresnel’s laws (for generalization to curved interfaces, see [33]) we obtain Fig. 3. Note that the FFP is shown only for the upper half space (\( 0^\circ - 180^\circ \)), the lower half space is given by symmetry. For TE polarization, we find directionality around \( \phi = 0 \), whereas in the TM case additional, smaller peaks occur. The ray in the left upper inset represents one typical trajectory emitting to \( \phi \approx 0 \) (marked by arrows). The emitting bounce (marked 1, is the symmetry-related counterpart), the three bounces before and the one after (marked 2) are shown. Whereas the trajectories are equal for both polarizations, their intensities are different:
As visible in the right inset, the rays escaping at 1 hit the line of the Brewster angle $\chi_B = \arctan 1/n < \arcsin 1/n$. In the TE case, transmission is nearly complete and no intensity can reach the next bounce 2. This causes the sharp decrease in the intensity that is more clearly visible in Fig. 2(b).

The Fresnel law for TM polarization does not show the Brewster angle feature. Therefore, a significant percentage of the light is reflected towards bounce 2. Since bounce 2 emits into a different direction, an appreciable amount of intensity collects in a second far-field peak.

To summarize up to this point, we have seen that chaotic ray dynamics can lead to highly directional emission. However, the $Q$-factor is low since the light rays typically leave the cavity very quickly. Does this result of geometric optics carry over to the wave dynamics of the electromagnetic field?

It has been demonstrated that the FFP of optical modes can be strongly influenced by the unstable manifold of the underlying ray dynamics [28, 29, 31]. A consequence is that for fixed polarization and cavity parameters the FFP is independent on the internal mode structure [30]. This raises the hope that the high directionality observed in ray simulations of the limaçon cavity will survive in wave optics. To this end we solve Maxwell equations numerically using the boundary element method [34]. According to the discrete symmetry, even and odd modes are distinguished.

The top panel of Fig. 4 shows near- and far-field pattern of a high-$Q$ TE mode. The normalized frequency $\Omega = \omega R/c = 26.0933$, $c$ being the speed of light in vacuum, corresponds to, e.g., a free-space wavelength of about 900 nm for $R = 3.75 \mu m$. Indeed, as predicted by our ray dynamical analysis the mode exhibits directional light emission around $\phi = 0$. The angular divergence of $24^\circ$ is significantly smaller than the values reported for low-$Q$ disks [21, 22], and also less than in Ref. [24]. Moreover, for fixed polarization and cavity parameters, the FFPs of all high-$Q$ TE modes in this cavity have similar envelope even though the internal mode structure is in general different. This is exemplarily demonstrated in the upper panel of Fig. 4 for an odd-parity mode (dashed line) which is quasi-degenerate with the even-parity solution (solid line). Note that in other types of cavities the even- and the corresponding odd-parity solution have in general a different FFP [24]. For the high-$Q$ modes with TM polarization we also observe an FFP which is independent of the internal mode structure, but a slightly different one, see lower panel of Fig. 4.

Whereas the ray and wave based FFPs in Figs. 3 and 4 respectively, agree remarkably well, other wave properties seem to contradict the ray simulations: (i) the mode does not look chaotic but spatially rather well confined. (ii) the cavity $Q$s are too large if compared to the escape rate from the chaotic repeller, in fact their values reach, or even exceed, the present limit achievable for microdisks with low residual absorption and surface roughness [9].

To further investigate the character of these optical modes we consider the Husimi projection [35], representing the wave analogue of the Poincaré SOS. From ray-wave correspondence one would expect that the Husimi projection is distributed uniformly over the unstable manifold. However, Fig. 5(a) demonstrates that the TE mode is localized around $|\sin \chi| \approx 0.86$ and has only exponentially small intensity in the leaky region which explains the high $Q$-factor. A closer inspection [36] reveals that the mode intensity is enhanced around an unstable periodic ray trajectory (dots and inset in Fig. 5(a)) which is part of the chaotic repeller. This phenomenon is called scarring [27] and has been observed...
in several kinds of physical systems including microcavities [37, 38, 39, 40]. Note that the other high-$Q$ modes found in this system also exhibit localization along – in general other – unstable periodic rays. The Husimi projection of the TM mode in Fig. 4 looks similar (not shown). The localization is even stronger, leading to a higher $Q$-factor.

Even though the Husimi projection has an exponentially small contribution in the leaky region, it is precisely this outgoing light that determines the FFP. Figures 5(b) and (c) show the Husimi projection in the leaky region. The convincing agreement with the unstable manifold in Figs. 2(b) and TM (c) mode; cf. Fig. 2(b) and (c) for the ray simulation results.

Note that, due to the observed agreement between ray and wave simulation, our results are also applicable to larger cavities. The particular deformation parameter $\varepsilon = 0.43$ is the optimum value for the localization of the discussed FFPs with $n = 3.3$ but highly localized FFP and high $Q$-factors can also be found for $0.41 \leq \varepsilon \leq 0.49$ (not shown), i.e. fabrication tolerances are not crucial. Moreover, we tested that our results are robust against variations of the refractive index and remain valid for $2.7 \leq n \leq 3.9$.

In summary, we have proposed a deformed microdisk as a novel cavity design for robust directional light emission from high-$Q$ modes. No complicated adjustment of geometry parameters is necessary, and the emission directionality is largely independent from wavelength, cavity size, refractive index, and the details of the interior mode structure. The latter finding is especially relevant for multi-mode lasing devices. We trace our, at first sight, counterintuitive results back to (i) wave localization along unstable periodic ray trajectories ensuring high $Q$-factors and (ii) escape of rays along the unstable manifold of the chaotic repeller leading to directional emission. The simplicity of the cavity design allows for easy fabrication with a wide range of applications in photonics and optoelectronics. The discussed mechanisms are not restricted to disk-like geometries but can in principle also be exploited for other geometries such as deformed microspheres and microcuberes.

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