Quasinormal modes of a black hole with quintessence-like matter and a deficit solid angle: scalar and gravitational perturbations

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Abstract In the previous paper (Li, X. Z., Xi, P., Zhai, X. H.: Phys. Lett. B666, 125-130 (2008)), we show the solutions of Einstein equations with static spherically-symmetric quintessence-like matter surrounding a global monopole. Furthermore, this monopole become a black hole with quintessence-like matter and a deficit solid angle when it is swallowed by an ordinary black hole. We study its quasinormal modes by WKB method in this paper. The numerical results show that both the real part of the quasinormal frequencies and the imaginary part decrease as the state parameter $w$, for scalar and gravitational perturbations. And we also show variations of quasinormal frequencies of scalar and gravitational fields via different $\epsilon$ (deficit solid angel parameter) and different $\rho_0$ (density of static spherically-symmetric quintessence-like matter at $r = 1$), respectively.

Keywords: quasinormal modes; black hole with a deficit solid angle; quintessence-like matter; WKB method

1 Introduction

Current observations (Hinshaw et al. 2008; Ho 2008; Percival et al. 2007; Kowalski et al. 2008) (cosmic microwave background, Type Ia Supernovae, baryon acoustic oscillation, integrated Sachs-Wolfe effect correlations, etc.) show that our universe is accelerating, which suggest the existence of a spatially homogeneous and gravitationally repulsive energy component referred as dark energy. The simplest dark energy candidate is the cosmological constant $\Lambda$ stemming from energy density of the vacuum, which corresponds to a fluid with a constant equation of state $w = -1$. But the theoretical value of vacuum energy density is much larger than the observed (Weinberg 1989). So, various scalar-field dark energy models (Peebles et al. 1988; Caldwell 2002; Li et al. 2002; Li 2004; Chiba et al. 2000; Piazza et al. 2004) are presented, such as quintessence ($-1 < w < -\frac{1}{3}$), phantom ($w < -1$), etc.. However, these dynamical dark energy candidates lack a concrete motivation from fundamental physics. Therefore, the subject of dark energy is still an attractive topic.

On the other hand, the phase transition in the early universe could have produced different kinds of topological defects, whose cosmological implications are very important (Vilenkin et al. 1994). The global monopole, which has divergent mass in flat space-time, is one of the most interesting defects. When one considers gravity, the linearly divergent mass of the global monopole has an effect analogous to that of a deficit solid angle plus a tiny mass at the origin. It has been shown that this effective mass is actually negative (Shi and Li 1991; Harari and Lousto 1990). However, this monopole become a black hole with a deficit solid angle (global monopole black hole) when it is swallowed by an ordinary black hole (Barriola et al. 1989).

Recently, black holes enveloped by the quintessence field have been widely investigated. Kiselev (Kiselev 2003) provided a new black hole solution using Einstein equations with static spherically-symmetric quintessence-like matter. Then, quasinormal modes of this kind of black hole have been studied (Chen et al. 2005; Zhang et al. 2006), which are believed by some physicists to be a unique fingerprint in directly identifying the existence of a black hole. In this paper, we show the solution of black hole with quintessence-like matter and a deficit solid angle, and study its QNMs by WKB method (Iyer et al. 1985). The numerical results show both the real part and the imaginary part...
The Lagrangian density is
\[ \mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a - \frac{\lambda^2}{4} (\phi^a \phi^a - \phi_0^2)^2. \]  
(1)
where $\phi^a$ is triplet of scalar fields, isovector index $a = 1, 2, 3$. The hedgehog configuration describing a global monopole is
\[ \phi^a = \sigma_0 f (\hat{r}) \frac{x^a}{r}, \quad \text{with} \quad x^a x^a = \hat{r}^2. \]  
(2)
so that we shall actually have a monopole solution if $f \to 1$ at spatial infinity and $f \to 0$ near the origin.

The general static metric with spherical symmetry can be written as
\[ ds^2 = B(\hat{r}) dt^2 - A(\hat{r}) dr^2 - \hat{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]  
(3)
The solutions for a global monopole surrounded by the static spherically-symmetric quintessence-like matter are as follows \cite{Li et al. 2008}:
\[ ds^2 = (1 - \frac{2GM}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}) dt^2 - \frac{1}{1 - \frac{2GM}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]  
(4)
where $\epsilon \equiv \sqrt{8\pi G \phi_0}$ is a dimensionless parameter of a deficit solid angle, a dimensionless $r \equiv \sigma_0 \hat{r}$, and $m \approx -\frac{16\pi \rho_0}{3w} M$.

When such a global monopole is swallowed by an ordinary black hole with mass $\tilde{M}$, a black hole with quintessence-like matter and a deficit solid angle can be formed \cite{Barriola et al. 1989}:
\[ ds^2 = (1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}) dt^2 - \frac{1}{1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}} dr^2 - r^2 (d\theta^2), \]  
(5)
where $M = G\sigma_0 (M - m)$ is the dimensionless parameter of global monopole black hole mass surrounded by quintessence-like matter.

Now, we consider concretely the behaviors of scalar and gravitational perturbations in a black hole with quintessence-like matter and a deficit solid angle, respectively. The propagation of a massless scalar field is described by the Klein-Gordon equation
\[ \Box \Phi = 0, \]  
(6)
Then we separate variables by setting
\[ \Phi(t, r, \theta, \phi) = \frac{1}{r} \psi(r) Y_{lm}(\theta, \phi) e^{-i\omega t}, \]  
(7)
where $Y_{lm}(\theta, \phi)$ are the usual spherical harmonics. Submitting Eq. (7) to (6), we obtain
\[ \frac{d^2 \psi(r)}{dr^2} + (\omega^2 - V_s) \psi(r) = 0, \]  
(8)
where $r_s$ is the tortoise coordinate
\[ r_s = \int \frac{1}{1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}} dr, \]  
(9)
and $V_s$ is the effective potential
\[ V_s = (1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}) \times \frac{[l(l+1) + \frac{2M}{r^3} + \frac{\rho_0 (3w+1)}{3w} r^{-3w-3}]}{r^2}, \]  
(10)
For gravitational perturbations, the metric function is expressed as
\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \]  
(11)
where $\bar{g}_{\mu\nu}$ is the background metric, and $h_{\mu\nu}$ is a small perturbation. Here, We adopt the canonical form for $h_{\mu\nu}$ in classical Regge-Wheeler gauge \cite{Regge and Wheeler 1957}:
\[ h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & h_0(r) \end{pmatrix} e^{-i\omega t} (\sin \theta \frac{\partial}{\partial \theta}) P_l(\cos \theta), \]  
Introducing $Q(r) = \frac{1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}}{r^2} h_1(r)$, we obtain
\[ \frac{d^2 Q(r)}{dr^2} + (\omega^2 - V_g) Q(r) = 0, \]  
(12)
where $V_g$ is the effective potential
\[ V_g = (1 - \frac{2M}{r} - \epsilon^2 + \frac{\rho_0}{3w} r^{-3w-1}) \times \frac{[l(l+1) + 6M + \frac{\rho_0 (3w+1)}{3w} r^{-3w-3}]}{r^2}, \]  
(13)
of the quasinormal frequencies for scalar and gravitational perturbations decrease as the state parameter $\omega$; oscillating frequencies and damping frequencies of scalar and gravitational fields increase with the deficit solid angle parameter $\epsilon$ decreasing and density of static spherically-symmetric quintessence-like matter $\rho_0$ decreasing, respectively.
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3 WKB method and numerical results

Here, we make use of the third-order WKB method to numerical calculation. The QNMs is as follows

$$\omega^2 = [V_0 + (-2V_0')^{1/2}\Delta] - i(n + \frac{1}{2})(-2V_0')^{1/2}(1 + \Omega), \quad (14)$$

where

$$\Delta = \frac{1}{(-2V_0')^{1/2}} \left\{ \frac{1}{8} \left( \frac{V_0^{(4)}}{V_0} \right) \left( \frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left( \frac{V_0''}{V_0} \right)^2 (7 + 60\alpha^2) \right\},$$

$$\Omega = \frac{1}{(-2V_0')^{1/2}} \left\{ \frac{5}{6912} \left( \frac{V_0''}{V_0} \right)^4 (77 + 188\alpha^2) - \frac{1}{384} \left( \frac{V_0''^2}{V_0^3} \right) (51 + 100\alpha^2) + \frac{1}{2304} \left( \frac{V_0^{(4)}}{V_0} \right)^2 (67 + 68\alpha^2) + \frac{1}{288} \left( \frac{V_0^{(5)}}{V_0''^2} \right) (19 + 28\alpha^2) - \frac{1}{288} \left( \frac{V_0''}{V_0} \right) (5 + 4\alpha^2) \right\},$$

and

$$\alpha = n + \frac{1}{2}, \quad V_0^{(n)} = \frac{d^n V}{d r^n} \bigg|_{r_s=r_s(r_p)}$$

Our numerical results of QNMs for scalar and gravitational perturbations are listed in Tables 1-5 and Tables 6-8, respectively. As a reminder, the oscillating quasi-period and the damping time scale are shown in these tables. In Table 1, we fixed $M = 1$, $\epsilon^2 = 0.001$, $\rho_0 = 0.01$ and $l = 1$, and the complex quasinormal frequencies vary with the state parameter $w$. Obviously, the real parts of the quasinormal frequencies decrease with $w$, which means the larger the value of $w$ is, more quickly the global monopole black hole surrounded by quintessence-like matter oscillates. And the magnitude of imaginary part decreases as the absolute value of $w$ increases, corresponding to the case more slowly oscillation of quintessence-like matter surrounding global monopole black hole decays, smaller the state parameter $w$ is. This behavior is different from that in Ref. (Chen et al. 2005), in which it was found that the perturbation damps quicker with the increase of the absolute value of the equation of state of quintessence. The difference may be caused by variation of the second-order derivative of the effective potential at its maximum value with $w$ as shown in Fig. 1. In Table 2, we chose $M = 1$, $\epsilon^2 = 0.001$ and $l = 1$, and listed quasinormal frequencies of global monopole black hole. Comparing Table 2 with Table 1, we find that quasi-period of the oscillation and the imaginary part of quasinormal frequencies in Table 2 are smaller than that in Table 1. In other words, oscillation of global monopole black hole decays more slowly in quintessence-like matter case. In Table 3, we consider the quasinormal frequencies vary with different $\rho_0$ (density of static spherically-symmetric quintessence-like matter at $r = 1$) fixing $M = 1$, $\epsilon^2 = 0.001$, $w = -\frac{2}{3}$ and $l = 1$. The oscillating frequency and the damping frequency both increase when $\rho_0$ decreases. In Table 4, we show how the QNMs behave for various deficit solid angels in $w = -\frac{2}{3}$, $M = 1$, $\rho_0 = 0.01$ and $l = 1$ case. It is clear that the real part of the quasinormal frequencies increases when $\epsilon$ decreases. However, the imaginary part decreases as $\epsilon$. It is interesting to note that when $\epsilon$ small enough, the real part of quasinormal frequencies in Table 4 is very close to that of ordinary black hole with quintessence-like matter in Table 5 (Parameters of this black hole $w$, $M$, $\rho_0$, and $l$ are same as those in Table 4). However, the damping time scales of black hole with a deficit solid angle (in Table 4) are larger than that of black hole without deficit solid angle (in Table 5).

For gravitational perturbations, we study variations of quasinormal frequencies via different $w$, $\rho_0$ and $\epsilon$, respectively, as showed in Tables 6-8. In Table 6, we choose $M = 1$, $\rho_0 = 0.1$, $\epsilon^2 = 0.001$ and $l = 2$, and the complex quasinormal frequencies vary with $w$. The real parts of quasinormal frequencies decrease with the decrease of $w$, but the imaginary parts increase. This conclusion is similar to that in Ref. (Zhang et al. 2006), which means the gravitational perturbations damps more slowly in quintessence-like matter case. In Table 7, we considered the relation between quasinormal modes and the parameter $\rho_0$ with $M = 1$, $w = -\frac{2}{3}$, $\epsilon^2 = 0.001$ and $l = 2$. Evidently, the oscillating quasi-period and damping time scale both decrease as $\rho_0$. In Table 8, we show the real parts and the magnitude of the imaginary parts of quasinormal frequencies both increase as $\epsilon^2$ decrease. To sum up, the variations of quasinormal frequency with $w$, $\rho_0$ and $\epsilon$ for gravitational field are similar to those for scalar field.

Acknowledgements This work is supported by National Science Foundation of China grant No. 10847153.
Table 1 QNMs of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different state parameter \( w \) for scalar perturbations.

| \( w \) | \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|-----|----------------|----------------|----------------|----------------|
| -5/12 | 0.28580-0.09569i | 0.25764-0.30009i | 0.21992-0.51424i | 0.17134-0.73078i |
| -7/12 | 0.30746-0.06531i | 0.28498-0.20031i | 0.26028-0.34243i | 0.22295-0.48887i |
| -9/12 | 0.34370-0.06155i | 0.30762-0.18411i | 0.25074-0.32151i | 0.21620-0.45855i |

Table 2 QNMs of black hole with a deficit solid angle for scalar perturbations.

| \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|----------------|----------------|----------------|----------------|
| 0.29066-0.09780i | 0.26182-0.30680i | 0.22324-0.52574i | 0.17350-0.74710i |

Table 3 QNMs for scalar perturbations of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different \( \rho_0 \) for scalar perturbations.

| \( \rho_0 \) | \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|----------|----------------|----------------|----------------|----------------|
| 0.1      | 0.21180-0.06850i | 0.19486-0.21269i | 0.17140-0.36264i | 0.14100-0.51411i |
| 0.01     | 0.28332-0.09503i | 0.25563-0.29779i | 0.21857-0.51005i | 0.17077-0.72464i |
| 0.001    | 0.28992-0.09753i | 0.26121-0.30591i | 0.22277-0.52417i | 0.17323-0.74486i |

Table 4 QNMs for scalar perturbations of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different \( \epsilon \) for scalar perturbations.

| \( \epsilon \) | \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|-----------|----------------|----------------|----------------|----------------|
| 0.1       | 0.23936-0.07645i | 0.21773-0.23880i | 0.18827-0.40891i | 0.15055-0.58106i |
| 0.01      | 0.27913-0.09326i | 0.25210-0.29216i | 0.21577-0.50040i | 0.16894-0.71094i |
| 0.001     | 0.28322-0.09503i | 0.25563-0.29779i | 0.21857-0.51005i | 0.17077-0.72464i |

Table 5 QNMs of ordinary black hole with quintessence-like matter for scalar perturbations.

| \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|----------------|----------------|----------------|----------------|
| 0.28368-0.09523i | 0.25602-0.29842i | 0.21888-0.51112i | 0.17098-0.72616i |

Table 6 QNMs of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different state parameter \( w \) for gravitational perturbations.

| \( w \) | \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|-----|----------------|----------------|----------------|----------------|
| -5/12 | 0.28580-0.09569i | 0.25764-0.30009i | 0.21992-0.51424i | 0.17134-0.73078i |
| -7/12 | 0.30746-0.06531i | 0.28498-0.20031i | 0.26028-0.34243i | 0.22295-0.48887i |
| -9/12 | 0.34370-0.06155i | 0.30762-0.18411i | 0.25074-0.32151i | 0.21620-0.45855i |

Table 7 QNMs of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different \( \rho_0 \) for gravitational perturbations.

| \( \rho_0 \) | \( \omega \) \((n = 0)\) | \( \omega \) \((n = 1)\) | \( \omega \) \((n = 2)\) | \( \omega \) \((n = 3)\) |
|----------|----------------|----------------|----------------|----------------|
| 0.1      | 0.29370-0.06155i | 0.27752-0.18841i | 0.25074-0.32151i | 0.21620-0.45855i |
| 0.01     | 0.30746-0.06531i | 0.28498-0.20031i | 0.26028-0.34243i | 0.22295-0.48887i |
| 0.001    | 0.34370-0.06155i | 0.30762-0.18411i | 0.25074-0.32151i | 0.21620-0.45855i |

| 0.0001  | 0.37264-0.08902i | 0.34558-0.27429i | 0.30263-0.46998i | 0.24733-0.67135i |
Table 8  QNMs of global monopole black hole surrounded by static spherically-symmetric quintessence-like matter with different $\epsilon^2$ for gravitational perturbations.

| $\epsilon^2$ | $\omega (n = 0)$ | $\omega (n = 1)$ | $\omega (n = 2)$ | $\omega (n = 3)$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| 0.1         | 0.23642-0.04486i| 0.22619-0.13670i| 0.20891-0.23234i| 0.18650-0.33066i|
| 0.01        | 0.28870-0.05998i| 0.27303-0.18352i| 0.24719-0.31306i| 0.21387-0.44643i|
| 0.001       | 0.29379-0.06155i| 0.27752-0.18841i| 0.25074-0.32151i| 0.21620-0.45855i|
| 0.0001      | 0.29379-0.06155i| 0.27752-0.18841i| 0.25074-0.32151i| 0.21620-0.45855i|

Fig. 1  The second-order derivative of effective potential at its maximum value $V_0''$ are plotted for different values of $w$. The variation of $V_0''$ with $w$ in terms of our numerical results is shown in (a); the variation of $V_0''$ with $w$ referring to that in Ref. (Chen et al. 2005) is shown in (b).
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