Dragging $D$ mesons by hot hadrons

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We evaluate the drag and diffusion coefficients of a hot hadronic medium consisting of pions, nucleons, kaons and eta using open charm mesons as a probe. The interaction of the probe with the hadronic matter has been treated in the framework of effective field theory. It is observed that the magnitude of both the transport coefficients are significant, indicating substantial amount of interaction of the heavy mesons with the thermal hadronic system. The results may have noticeable impact on the experimental observables like the suppression of single electron spectra originating from the semi-leptonic decays of the heavy mesons which are produced from heavy quark fragmentation. Several theoretical attempts [4–15] have been made to explain $R_{AA}$ and $v_2$, where the role of hadronic matter has been ignored. However, to make the characterization of QGP reliable the role of the hadronic phase should be taken into consideration and its contribution must be subtracted out from the observables. Although a large amount of work has been done on the diffusion of heavy quarks in QGP, the diffusion of heavy mesons in hadronic matter has received much less attention so far. Recently the diffusion coefficient of $D$ meson has been evaluated using heavy meson chiral perturbation theory [16] and also by using the empirical elastic scattering amplitudes [17] of $D$ mesons with thermal hadrons.

The present work addresses the relevance of the hadronic sector to some of these issues. The drag and diffusion coefficients for the hadronic phase have been evaluated and their importance to the experimental observables in heavy ion collisions have been discussed. In particular, we consider the interaction of a $D$ meson with a thermal hadronic system composed of pions, nucleons, kaons and eta in a temperature domain relevant for heavy ion phenomenology. It is expected that the relaxation time for heavy mesons are larger than the corresponding quantities for light hadrons. The abundance of the heavy mesons will be low for the temperature ($T$) range under consideration ($T=100$-$180$ MeV), as a result they do not decide the bulk properties of the matter. The thermal production of charm mesons can be ignored for the range of temperature mentioned above. Therefore, the drag ($\gamma$) and diffusion ($B_0$) coefficients of the heavy mesons can be evaluated by using its elastic interaction with the thermal hadrons. For the process, $D(p) + h(q) \rightarrow D(p') + h(q')$ ($h$ stands for pion, nucleon, kaon and eta), the drag $\gamma$ can be calculated by using the following expression [18]:

$$\gamma = p_i A_i / p^2$$  \hspace{1cm} (1)

Nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies are aimed at creating a phase where the bulk properties of the matter are governed by a deconfined state of (light) quarks and gluons known as Quark Gluon Plasma (QGP). The study of the transport properties of QGP is a field of great contemporary interest and the heavy flavors, namely, charm and bottom quarks, play a crucial role in such studies. The weakly interacting picture of the QGP stems from the perception of asymptotic freedom of QCD at high temperatures and densities. However, the experimental data from RHIC, especially the measured elliptic flow indicate that the matter produced in Au+Au collisions exhibit properties which are more like a strongly interacting liquid than a weakly interacting gas. The magnitude of the transport coefficients can be used to understand the strength of the interaction within the QGP. For example, the shear viscosity or the internal friction of the fluid symbolizes the ability to transfer momentum over a distance of about a mean free path. Therefore, in a system where the constituents interact strongly the transfer of momentum is performed over a distance of about a mean free path. Consequently such a system may be characterized by a small value of $\eta/s$ where $s$ is the entropy density. On the other hand, for a weakly interacting system the momentum transfer between the constituents become strenuous which gives rise to a large $\eta$. The importance of viscosity also lies in the fact that it damps out the variation in the velocity and makes the fluid flow laminar. A very small viscosity (large Reynold number) may make the flow turbulent. A lower bound on the value of $\eta/s$ has recently been found using AdS/CFT [1].

The interaction of heavy quarks with the QGP can be used to estimate the value of the transport coefficients. This has recently been performed by experimentally measuring the nuclear suppression factor ($R_{AA}$) [2] and the elliptic flow ($v_2$) [3] for the single electron spectra originating from the semi-
where $A_i$ is given by

$$A_i = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3q'}{(2\pi)^3} \frac{1}{g_D} \sum |M|^2 (2\pi)^4 \delta^4(p + q - p' - q')$$

$$f(q)(p-p')_i = \langle \langle p - p' \rangle \rangle \quad (2)$$

g_D being the statistical degeneracy of the $D$ meson propagating in the medium. The above expression indicates that the drag coefficient is the measure of the thermal average of the square of the invariant amplitude $|M|^2$ weighted by the momentum transfer, $p-p'$. The factor $f(q)$ denotes the thermal phase space for the particle in the medium.

Similarly the diffusion coefficient $B_0$ can be defined as:

$$B_0 = \frac{1}{4} \left[ \langle p^2 \rangle - \langle (p, p')^2 \rangle \right] \quad (3)$$

With an appropriate choice of $T(p\tau)$ both the drag and diffusion co-efficients can be evaluated from the following expression:

$$< T(p) > = \frac{1}{512\pi^4 E_p} \int_0^\infty q^2 dq d\phi(\cos \chi) \int f(q)$$

$$\lambda^2(s, m_{m_{q}}, m_q) \frac{1}{\sqrt{s}} \int_1^{\langle \langle p, p' \rangle \rangle} d\cos \theta_c.m. \left[ \frac{1}{g} \sum |M|^2 \int_0^{2\pi} d\phi_c.m. T(p\tau) \right] \quad (4)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the triangular function and $|M|^2$ in the present case corresponds to the scattering of $D$-mesons from the light mesons $\pi$, $K$, $\eta$ and nucleons. In a hot pion gas this was obtained by Fuchs et al. [19] using experimental information on $D$-meson resonances. Here we evaluate these amplitudes using a covariant formulation of chiral perturbation theory in which the leading term allows for $D$ scattering via $D^*$ meson exchanges in addition to a contact interaction. In nuclear matter the $DN$ scattering amplitudes have been obtained in a coupled channel Bethe-Salpeter approach where $\Lambda_c$ and $\Sigma_c$ appear as dynamically generated [20]. In this work we have obtained the $DN$ scattering amplitudes proceeding via $\Lambda_c$ and $\Sigma_c$ exchanges using the Lagrangian of Ref. [21]. The generic Feynman diagrams for the elastic processes are depicted in Fig. 1. We have included form factors in each of the interaction vertices to take into account the finite size of the hadrons. For the $t$ and $s$-channel diagrams the form factors are taken as $F_t = \Lambda^2/(\Lambda^2 + q^2)$ and $F_s = \Lambda^2/(\Lambda^2 + p_i^2)$ respectively, where $p_i$ is the initial three momentum of the light mesons (pion, kaon and eta) or nucleon. In the four point (contact) vertices a form factor, $F_4 = (\Lambda^2/(\Lambda^2 + q^2))^2$ with $q = 5p_i^2/3$ has been introduced [22]. We have taken $\Lambda = 1$ GeV. The interaction Lagrangian as well as the $|M|^2$ for various processes are detailed in the appendix.

In Fig. 2 variation of the drag coefficient with temperature has been depicted for $D$-mesons. We have observed that the $D - \pi$ meson interaction plays the most dominant role in the drag coefficient primarily because of the larger phase space density of the pions. However, at higher $T$ the contribution from the nucleons become significant. As mentioned before, $\gamma$ is the thermal average of the square of the invariant amplitude weighted by the momentum transfer. Therefore, as the temperature of the thermal bath increases the hadrons move faster and gain the ability to transfer larger momentum during their interaction with the $D$ mesons - resulting in the increase of the drag coefficient. This trend is clearly observed in Fig. 2. It may be mentioned here that the drag increases with $T$ when the system behaves like a gas. In case of a liquid the drag may decrease with temperature (except for very few cases) since a substantial part of the thermal energy goes into making the attraction between the interacting particles weaker. This allows them to move more freely resulting in a smaller drag force. Therefore, the variation of the drag with $T$ may be used to characterize the nature of interaction of the fluid. The large value of the drag coefficient indicates that the interaction of the $D$ meson with the thermal medium is quite significant so that the the $D$ meson may get thermalized in the system and flows with the bulk matter. This may be examined by analyzing the transverse momentum spectra of the $D$ mesons produced in heavy ion collisions [23]. We find that the typical value of the relaxation time ($\sim \gamma^{-1}$) is about 4-5 fm/c. Therefore, if the life time of the hadronic phase is more than this time scale then the $D$ meson may get thermalized indeed.

In the same way it may be argued that the diffusion coefficient involves the square of the momentum transfer - which should also increase with $T$ as seen...
FIG. 2: The variation of drag coefficients with temperature due to the interaction of the $D$ with thermal pions, nucleons, kaons and eta.

FIG. 3: Variation of diffusion coefficient as a function of temperature. The solid line indicates the variation of the diffusion coefficient with temperature obtained from Eqs. 3 and 4. The dashed line stands for the diffusion coefficient obtained from the Einstein relation (Eq. 5).

The dominant contribution comes from the interaction of $D$ mesons with pions. The drag and the diffusion coefficients are related through the Einstein relation as:

$$B_0 = M_D \gamma T,$$

where $M_D$ is the mass of the $D$-meson. The temperature variation of the diffusion coefficient obtained from Eq. 5 is depicted in Fig. 3 (dashed line). The difference between the results obtained from Eq. 4 and Einstein’s relation is about 10% at $T = 180$ MeV. This small difference illustrates the validity of the Einstein relation in the current situation. The value of the spatial diffusion coefficient, $D_x$, may be expressed in terms of drag coefficient as $D_x = T/(M_D \gamma)$. The value of $D_x$ at $T = 180$ MeV is $\sim 2/(2\pi T)$ i.e. 2 times larger than the thermal wave length, $\lambda = 1/(2\pi T)$ which is well within the quantum bound.

The magnitude of the energy dissipation of the $D$ meson in the system may be estimated by using the relation

$$\frac{dE}{dx} = \gamma p.$$

The magnitude of $\gamma$ obtained in the present calculation indicates a substantial loss of energy of the $D$ meson in the medium, which might have observable effects on quantities such as the nuclear suppression factor of single electrons originating from the decays of heavy mesons.

Here it is necessary to point out that though chiral perturbation theory provides a consistent framework for performing perturbative calculations of strong interaction processes such as $D - h$ scattering in this case, it is limited by the abundance of coupling constants appearing in the Lagrangian which have to be determined from experimental data [24]. In the case at hand, the experimental error in the $D^*\rightarrow D\pi$ decay width leads to an uncertainty (11%) in the $D^*D\pi$ coupling, $g$ [25], which results in a significant variation in the value of the drag diffusion coefficients. Interestingly, the lower bound in this value leads to a drag coefficient which agrees reasonably well at high temperatures with that obtained by He et al [17] using empirical elastic scattering amplitudes. In addition to this, chiral symmetry breaking effects in the pseudoscalar decay constants could also contribute to the uncertainty. However, since in our case the pion contribution dominates maximally, this effect will be insignificant.

To summarize, in this work we have evaluated the drag and diffusion coefficients of open charm mesons propagating in a hadronic background composed of pions, kaons, nucleons and eta. We observe that the values of both the transport coefficients increases with temperature and the dominant contributions come from the pions in the medium. However, at higher $T$ the contributions from heavier hadrons become significant. The magnitude of the drag coefficient of the $D$ meson in the hadronic medium reveals that while evaluating the nuclear suppression for the single leptons originating from the decays of $D$ mesons the hadronic contributions should be included. Lattice QCD calculations [26] indicate that at low baryonic chemical potential and high temperature domain there is no phase transition between hadronic matter and QGP - it is a cross over, which means that the hadronic matter can make a continuous transition to QGP in this region of phase diagram. Therefore, the transport coefficients evaluated for the hadronic matter with zero baryonic chemical potential may have vital effects from the quark gluon plasma.
I. APPENDIX

In this appendix we provide the interaction Lagrangian and matrix elements for scattering of $D$ mesons from the light mesons ($\pi, K, \eta$) and nucleons discussed in this work.

The leading order chiral Lagrangian describing the interaction of Goldstone bosons with the heavy-light pseudoscalar ($P$) and vector ($P^*_\mu$) mesons is given by [23]

$$\mathcal{L}_{PP}\Phi = \langle D_\mu P D^\mu P \rangle - m_D^2 \langle PP \rangle$$

$$-\langle D_\mu P P^\mu \rangle + m_D^2 \langle PP^\uparrow \rangle$$

$$+ ig\langle P P^\mu P - P P^\mu P \rangle$$

(7)

where $P = (D^0, D^+, D^{+}_{s})$ and $P^*_\mu = (D^0_\mu, D^+_{s\mu}, D^{+}_{s\mu})$ and $\langle ... \rangle$ denotes trace in flavour space. The covariant derivatives are defined as $D_\mu P_a = \partial_\mu P_a - P a \Gamma^a_{\mu b} D^b$ and $D^a P^*_\mu = \partial^\mu P^a + \Gamma^a_{\mu b} P^b$ with $a, b$ the SU(3) flavour indices. The value of the heavy-light pseudoscalar-vector coupling constant $g = 1177 \pm 137$ MeV is obtained by reproducing the experimental $D^* \to D\pi$ decay width of $\sim 65 \pm 15$ keV with the above interaction. The vector and axial-vector currents are respectively given by $\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger)$ and $u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$ where $u = \exp(i\frac{\Phi}{2F_\pi})$. The unitary matrix $\Phi$ collects the Goldstone boson fields and is given by

$$\Phi = \sqrt{2} \begin{pmatrix} \pi^0 + \frac{\sqrt{6}}{3} \pi^- & \pi^+ - \frac{\sqrt{6}}{3} K^- \\ \pi^+ - \frac{\sqrt{6}}{3} K^- & K^0 - \frac{2\sqrt{6}}{3} \end{pmatrix}.$$  

To lowest order in $\Phi$ the vector and axial-vector currents are

$$\Gamma_\mu = \frac{1}{8F_\pi} \{ \Phi, \partial_\mu \Phi \}, \quad u_\mu = -\frac{1}{F_\pi} \partial_\mu \Phi.$$  

(8)

The above interaction allows elastic scattering of the $D$ meson with the $\pi, K$ and $\eta$ fields through a heavy-light vector meson exchange in addition to a contact interaction as shown in Fig. 1. The form of the contact interaction obtained in the covariant formulation of chiral perturbation theory used here coincides with that in [27]. The invariant amplitudes for $D^+$ elastic scattering from hadrons, $h(D^+(p_1) + h(p_2) \to D^+(p_3) + h(p_4))$ have been obtained as follows:

$$|M_{D^+\pi^0}|^2 = \left( \frac{g^2}{F_\pi} \right)^2 \frac{\{p_1 \cdot p_4 - (p_1 \cdot p_3 + m_\pi^2)^2/m_D^2\}}{s - m_D^2}$$

$$+ \left( \frac{g^2}{3F_\pi} \right)^2 \frac{\{p_1 \cdot p_4 - (p_1 \cdot p_3 + m_\pi^2)^2/m_D^2\}}{t - m_D^2}$$

$$+ \left( \frac{g^2}{4F_\pi} \right)^2 (s - u)$$

(11)

$$|M_{D^+\pi^0}|^2 = \left( \frac{g^2}{F_\pi} \right)^2 \frac{\{p_1 \cdot p_4 - (p_1 \cdot p_3 + m_\pi^2)^2/m_D^2\}}{s - m_D^2}$$

$$+ \left( \frac{g^2}{3F_\pi} \right)^2 \frac{\{p_1 \cdot p_4 - (p_1 \cdot p_3 + m_\pi^2)^2/m_D^2\}}{t - m_D^2}$$

$$+ \left( \frac{g^2}{4F_\pi} \right)^2 (s - u)$$

(12)

In the numerical calculations we have used the physical masses for the particles involved. For the heavy-light mesons we have taken $m_D = 1867$ MeV, $m_{D^*} = 2008$ MeV and $m_{D_s} = 1969$ MeV.

We next discuss scattering of the $D$ meson with nucleons which proceeds via exchange of the charmed baryons $\Sigma_c$ and $\Lambda_c$ according to the Lagrangian [21].

$$\mathcal{L}_{DN} = \frac{f_{DN\Sigma_c}}{m_D} \left[ \bar{N} \gamma_5 \gamma^\mu \Lambda_c \partial_\mu \bar{D} + \partial_\mu D \gamma_5 \gamma^\mu N \right]$$

$$+ \frac{f_{DNS_c}}{m_D} \left[ \bar{N} \gamma_5 \gamma^\mu (\bar{\tau} \cdot \bar{\Sigma_c}) \partial_\mu \bar{D} + \partial_\mu D (\bar{\tau} \cdot \bar{\Sigma_c}) \gamma_5 \gamma^\mu N \right]$$

(15)

where $D = (D^0, D^+) = (\bar{D^0}, D^-)^T$. The amplitudes for $D^+$ mesons elastically scattering from nucleons (and anti-nucleons) via $\Gamma_c$ exchange are obtained as,

$$|M_{D^+\pi^0}|^2 = \frac{1}{2} \left( \frac{f_{DNS_c}}{m_D} \right)^4 X^s_{DNS_c}(m_D, m_N, m_{\Lambda_c})$$

(16)
where

\begin{equation}
X_{BND}^{\ast}(m_D, m_N, m_B) = \frac{4}{(s - m_D^2)^{2}} [m_D^2 \{m_N^2 m_D^2
- 2(p_1 \cdot p_2)(p_1 \cdot p_3)(m_N^2 + m_D^2) + (p_1 \cdot p_3)^2
+ m_D^2 m_N^2)\} + m_B \{2(-m_N^2 m_D^2 + 2(p_1 \cdot p_3)^2 m_N
- (m_N^2 + m_D^2) - 4m_N m_D^4 (p_1 \cdot p_3)\})
+ \{8(p_1 \cdot p_2)(p_1 \cdot p_3) + m_D^2 (p_1 \cdot p_3)^2
+ 2m_D^2 (p_1 \cdot p_3) + m_N^2 m_D^4\} + \{t (m_N^2 m_D^2 + 4m_D^2
(p_1 \cdot p_3) - (m_N^2 + 4(p_1 \cdot p_3)^2)(p_1 \cdot p_3)\})\}
\end{equation}

(21)

The various invariant amplitudes can be expressed in terms of the Mandelstam variables using the relations: $p_1 \cdot p_2 = \frac{s - m_D^2 - m^2}{2}$, $p_1 \cdot p_3 = \frac{m_D^2 + m_N^2 - t}{2}$ and $p_1 \cdot p_4 = \frac{m_D^2 + m_N^2 - u}{2}$, where $m_D$ is the mass of $D$ meson and $m_h$ that of the light hadrons.

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