COMPETING LOCAL AND GLOBAL INTERACTIONS IN SOCIAL DYNAMICS: HOW IMPORTANT IS THE FRIENDSHIP NETWORK?

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MOTIVATION

- **sources of social influence**
  - local: friends, family
  - global: mass media, online reviews, aggregate measures

- **study on movie ratings**
  "Do I Follow My Friends or the Crowd? Information Cascades in Online Movie Ratings"
  Y.-J. Lee, K. Hosanagar, Y. Tan, Manag. Sci. 61(9), 2241 (2015)

- **empirical findings**
  - friends' ratings
    - herding behavior \(\rightarrow\) conformity
  - strangers' ratings
    - herding or differentiation behavior \(\rightarrow\) anticonformity
Q-VOTER MODEL

- **agent-based models**
  - network of interacting $N$ agents
  - source of social influence: $q$ agents
    conformity, anticonformity

- **binary-state model**
  - positive attitude
  - negative attitude

\[ s_i = 1 \quad s_i = -1 \]

\[ i = 1, 2, \ldots, N \]

R. H. Willis, Sociometry 26(4), 499 (1963)

R. W. Robins, R. C. Fraley, and R. F. Krueger,
„Handbook of research methods in personality psychology“

C. Castellano et al., Phys. Rev. E 80, 041129 (2009)
P. Nyczka, K. Sznajd-Weron, and J. Cisło, Phys. Rev. E 86, 011105 (2012)
**Q-VOTER MODEL**

The Q-Voter model is defined by two parameters:

- \( p \) – probability of anticonformity
- \( q \) – size of the influence group

**Conformity**

- \( 1 - p \)
  - A unanimous \( q \)-panel
  - OTHERWISE
  - \( s_i = 1 \)
  - \( s_i = -1 \)
  - \( s_i = \pm 1 \)

**Anticonformity**

- \( p \)
  - A unanimous \( q \)-panel
  - OTHERWISE
  - \( s_i = 1 \)
  - \( s_i = -1 \)
  - \( s_i = \pm 1 \)

*P. Nyczka, K. Sznajd-Weron, and J. Cisło, Phys. Rev. E 86, 011105 (2012)*
LOCAL AND GLOBAL INTERACTIONS

- 4 different q-voter models:
  - **GAGC** – global anticonformity and global conformity
  - **GALC** – global anticonformity and local conformity
  - **LALC** – local anticonformity and local conformity
  - **LAGC** – local anticonformity and global conformity

mean-field model study on movie ratings
WHAT DO WE STUDY?

- quantities of interest
  
  \[ m = \frac{1}{N} \sum_{i=1}^{N} s_i \]

- phase transitions
  
  \[ m = 0 \text{ – disordered phase } (c = 0.5) \]
  
  \[ m \neq 0 \text{ – ordered phase } (c \neq 0.5) \]

\[ c = \frac{1}{2} (1 + m) \]

concentration of positive agents

\[ p \text{ – probability of anticonformity} \]
ANALYTICAL APPROACH

• pair approximation

\[
\frac{dc}{dt} = \sum_{j \in \{1,-1\}} c_j \sum_k P(k) \sum_{i=0}^{k} \binom{k}{i} \theta_j^i (1 - \theta_j)^{k-i} f(j,k,i) \Delta_c
\]

\[
\frac{db}{dt} = \sum_{j \in \{1,-1\}} c_j \sum_k P(k) \sum_{i=0}^{k} \binom{k}{i} \theta_j^i (1 - \theta_j)^{k-i} f(j,k,i) \Delta_b
\]

\[\Delta_c = -j \quad \Delta_b = \frac{2}{\langle k \rangle} (k - 2i)\]

\[j - \text{opinion} \quad k - \text{node degree} \quad i - \text{number of active bonds}\]

\[c_1 \equiv c \quad c_{-1} \equiv 1 - c\]

\[\theta_1 = \frac{b}{2c} \quad \theta_{-1} = \frac{b}{2(1 - c)}\]

• assumption:
  • active bonds binomially distributed

• steady states: \[\frac{dc}{dt} = 0 \land \frac{db}{dt} = 0\]
PHASE DIAGRAMS

GALC model

LALC model

LAGC model

\langle k \rangle \text{ increase direction, } \langle k \rangle \in \{8, 10, 16, 30\}, q = 3

$c$ – concentration of positive agents
$p$ – probability of anticonformity
\langle k \rangle – average node degree
$q$ – size of the influence group

only average degree matters
MODEL COMPARISON

• how to choose $q$?

$q = 3$

$q = 5$

$q = 8$

$c$ – concentration of positive agents

$p$ – probability of anticonformity

$\langle k \rangle$ – average node degree

$q$ – size of the influence group

„Statistical Physics Of Opinion Formation: Is it a SPOOF?”
A. Jędrzejewski, K. Sznajd-Weron, C. R. Physique 20(4), 244 (2019)
MONTE CARLO SIMULATIONS

- Watts-Strogatz network model: $\beta$ – rewiring probability

$c$ – concentration of positive agents
$p$ – probability of anticonformity
$q$ – size of the influence group
$\langle k \rangle$ – average node degree

GALC model
LALC model
LAGC model

$q = 4, \langle k \rangle = 50, N = 28160$
CONCLUSIONS

• differentiation between interaction lengths

• global anticonformity and local conformity
  • most sensitive to network structure
  • most difficult to achieve agreement

• local anticonformity and global conformity
  • low impact of network structure
  • average node degree $\langle k \rangle$ matters

study on movie ratings

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Chaos 30, 073105 (2020)