Application of Block Sparse Bayesian Learning in Power Quality Steady-State Data Compression

Wenjian Hu 1,2,*, Mingxing Zhu 1,2 and Huaying Zhang 3

1 College of Electrical Engineering and Automation, Anhui University, Hefei 232000, China; zhumingxing@ahu.edu.cn
2 Power Quality Engineering Research Center, Ministry of Education, Hefei 232000, China
3 New Smart City High-Quality Power Supply Joint Laboratory of China Southern Power Grid, Shenzhen Power Supply Co., Ltd., Shenzhen 518000, China; zhyszpower@163.com
* Correspondence: z21514036@stu.ahu.edu.cn; Tel.: +86-137-0559-2785

Abstract: In modern power systems, condition monitoring equipment generates a great deal of steady-state data that are too large for data transmission and, thus, data compression is needed. Therefore, there is a balance to strike between compression quality and data accuracy. Greedy algorithms are effective but suffer from low data reconstruction accuracy. This paper proposes a block sparse Bayesian learning (BSBL)-based data compression method. Based on the prior distribution and posterior probability of the sparse signals, it uses the Bayesian formula to excavate the block structure of these signals. This paper also adds two indicators to the evaluation process to validate the proposed method. The proposed method is effective in terms of signal-to-noise ratio (SNR), relative root mean square error (RRMSE), amplitude error, energy recovery percentage (ERP), and angle error. The first three indicate better performance of the proposed method than the traditional method by giving the same compression ratio. Therefore, the method validates the possibility of a more accurate and economical solution to power quality assurance.

Keywords: block sparse Bayesian learning; compressed sensing; data compression; evaluation indicators; power quality steady-state data

1. Introduction

As more power electronic equipment and impact loads are connected to the power system, more power quality problems also emerge. When investigating and solving these problems, the data from the time of the accident are needed. When data cannot be stored indefinitely, methods to achieve better reconstruction performance at a low compression ratio become a key issue [1,2].

The current methods for power quality data compression are divided into the traditional methods and those based on compressed sensing (CS). The traditional method is based on wavelet theory. Fundamentally, it is to keep the coefficients larger than the set threshold and discard the coefficients smaller than the set threshold after a wavelet transformation of the data [3,4]. It is found that the magnitudes of wavelet-transformed coefficients associated with disturbance events are much larger than those of disturbance-free coefficients; this fact leads to an effective method to compress power quality disturbance data [3]. The compressed data size is one-sixth to one-third of that of the original data. Ref. [5] uses two-dimensional lifting wavelet transformations based on a one-dimensional wavelet to improve the computational efficiency of the compression algorithm. However, this method is based on the Nyquist theorem, and its high sampling rate and high hardware cost will hinder the efficiency of data transmission and analysis.

Based on the above problems, Donoho proposed the compressed sensing theory [6]. It is known from the CS theory that after the signal is sparsely transformed, the signal can be sampled, compressed, and reconstructed far below the standard of the Nyquist sampling
frequency. Presently, many methods for dealing with power quality problems are based on CS theory.

Greedy algorithms represented by the orthogonal matching pursuit (OMP), sparsity adaptive matching pursuit (SAMP), and compressed sampling matching pursuit (CoSAMP) algorithms are widely used, which could compress power quality data. The core idea is to go step-by-step and make the best choice based on an optimization measure according to the current situation [7–11].

Sparse Bayesian learning (SBL) and its improved algorithms are also based on CS theory. They combine existing data to make short-term predictions, compress and reconstruct data, or classify and identify events. Ref. [12] combines the sparse Bayesian learning and extreme learning machine (ELM) algorithms to propose an improved algorithm for power quality event classification, and the test data proves that the algorithm has a faster calculation speed and generalization performance. Ref. [13] proposes a hyperparameter adaptive group Bayesian learning algorithm, which can locate the harmonic source with a small number of measured values and estimate the magnitude and angle of the harmonic source. Ref. [14] proposes a joint clustering and sparse Bayesian learning algorithm when the clustering partition is unknown. The method can adaptively divide clusters, and different clusters can learn different time correlation coefficients, thereby reducing the interference between clusters. Ref. [15] proposes a complex spatiotemporal sparse Bayesian learning (CSTSBL) algorithm to reconstruct angular positions using the spatiotemporal correlation structure of the solution matrix, which can achieve high accuracy and resolution performance at low signal-to-noise ratios.

In addition to statistical features, general sparse vectors also have certain structural features that can provide additional information for algorithm convergence. Based on this idea, the block SBL (BSBL) algorithm is applied to regression and classification in machine learning. Ref. [16] uses the variational Bayesian block sparse method to estimate the radar target scattering center and achieves good results in combination with simulation. Ref. [17] proposes a clustered block sparse Bayesian learning (CBSBL) algorithm for millimeter wave channel estimation by exploiting the correlation between mm wave channels. The algorithm improves the accuracy and robustness of mm wave channel estimation.

BSBL can not only recover signals with block structure but also consider intra-block correlation. Therefore, its recovery of sparse signals can be good. It has been successfully applied to monitor fetal electrocardiograms and electroencephalograms via wireless body area networks [18]. However, there is not much research on the related field of power quality signals. Based on this, this paper proposes a reconstruction method that combines BSBL and sparsity in the transform domain. Different transform domains are used to sparsely transform different power quality steady-state signals. The experimental results using the constructed signal and the measured signal show that the BSBL algorithm outperforms the traditional CS-based algorithms in relative performance indicators.

However, the composition of actual power quality is complex. If it is compressed using a greedy algorithm, it is difficult to pre-estimate the sparsity of the data before finding a suitable dictionary matrix. This causes larger errors in the reconstruction after data compression. Aiming to solve this problem, this paper proposes a BSBL method for compressing steady-state power quality data. It uses the known prior distribution and posterior probability of the signal to mine the correlation within the block and improve the reconstruction accuracy of the signal. For indicators of the reconstructed signal performance comparison, SNR (signal-to-noise ratio), ERP (energy recovery percentage), and RRMSE (relative root mean square error) are more commonly used. However, there is usually a lack of indicators established by combining its signal characteristics to analyze the reconstruction performance. In this paper, two performance indicators are proposed according to the characteristics of the power quality steady-state signal and the relative standard, and their practicability is demonstrated. The Section 2 briefly introduces the BSBL framework and the concept of power quality steady-state data. The Section 3 introduces the flow chart of BSBL and the proposed performance indicators. The Section 4 provides case
studies and performance comparisons with other algorithms. The Section 5 gives the recommended transform domain and CR values for the proposed algorithm in different engineering signals.

2. BSBL Algorithm and Power Quality Steady-State Data

2.1. CS Theory

CS theory points out that when a signal \( f \in \mathbb{R}^{N \times 1} \) of length \( N \) is sparse or its sparse representation on a certain orthogonal transform domain \( \Psi \) is \( x, f = \psi x \). Using a measurement matrix \( \Phi \in \mathbb{R}^{M \times N} \), the measurement signal of \( f \) is

\[
y = \Phi f = \Phi \psi x = Ax
\]

where \( M \) is the signal observation dimension, and \( M \ll N \). Furthermore, \( A = \Phi \psi \) is the sensing matrix, and \( y \) is the observation value of the sparse \( x \) under \( A \). Ref. [18] used mathematics to prove that the measurement matrix \( \Phi \) must meet the requirements of the restricted isometry property (RIP):

\[
(1 - \delta) \|y\|_2^2 \leq \|\Phi y\|_2^2 \leq (1 + \delta) \|y\|_2^2
\]

where \( \delta \) is the restricted equidistant constant and \( \delta \in (0, 1) \) and \( \|y\|_2 \) are the norms of \( y \). The equivalent condition of RIP is that \( y \) is not related to \( \Phi \). The Gaussian random matrix, Bernoulli random matrix, sparse binary matrix, and other matrices are nearly irrelevant to any sparse matrix at present.

2.2. BSBL Framework

For the sparse \( x \) in Equation (1), its non-zero elements often have block structure characteristics [19]. In this structure, suppose it is divided into \( s \) continuous blocks, the number of elements in the \( i \)-th block is \( t_i \), and there are

\[
N = \sum_{i=1}^{s} t_i
\]

When the number of elements in each block is equal, there are

\[
x = [x_{11}, x_{12}, \ldots, x_{1t}, x_{t+11}, x_{t+12}, \ldots, x_{t2}, \ldots, x_{(s-1)t+11}, x_{(s-1)t+12}, \ldots, x_{st}]
\]

That is, \( t_1 = t_2 = \ldots = t_s \) and \( N = st \), assuming that there are \( k \) blocks in these blocks that are non-zero (\( k \ll s \)), then this vector is called block \( k \)-sparse.

Assuming that the blocks are independent of each other and obey Gaussian distribution, \( p(x_i; \gamma_i, B_i) \sim N(0, \gamma_i B_i), i = 1, 2, \ldots, s \)

where \( \gamma_i \) is a hyperparameter. Most \( \gamma_i \) tend to zero, and \( \gamma_i \geq 0 \). \( B_i \) is a positive definite matrix. The prior distribution with \( \gamma_i \) and \( B_i \) as parameters are

\[
p(x; \{\gamma_i, B_i\}_{i=1}^{s}) \sim N(0, \Sigma_0)
\]

\[
\Sigma_0 = \text{diag}\{\gamma_1 B_1, \gamma_2 B_2, \ldots, \gamma_s B_s\} = \begin{pmatrix}
\gamma_1 B_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \gamma_s B_s
\end{pmatrix}
\]

Therefore, the compression sampling process for signal \( x \) is

\[
y = Ax + \epsilon
\]
where \( \varepsilon \) is the noise signal in the sampling process and satisfies the Gaussian distribution with a mean value of 0 and a variance of \( \lambda \) (\( \lambda > 0 \)),

\[
p(\varepsilon_i) \sim N(0, \lambda E)
\]

where \( E \) is the identity matrix. According to the Bayesian formula, the posterior probability distribution of \( x \) is obtained as a Gaussian distribution with mean \( \mu_x \) and variance \( \Sigma_x \):

\[
p(x|y; \lambda, \{\gamma_i, B_i\}_{i=1}^s) \sim N(\mu_x, \Sigma_x)
\]

\[
\mu_x = \frac{1}{\lambda} \Sigma_x A^T y
\]

\[
\Sigma_x = (\Sigma_0^{-1} + \frac{1}{\lambda} A^T A)^{-1} = \Sigma_0 - \Sigma_0 A^T (\lambda E + A \Sigma_0 A^T)^{-1} A \Sigma_0
\]

Thus far, the maximum posterior probability estimate of \( x \) is \( \hat{x} \), and \( \hat{x} = \mu_x \).

2.3. Steady-State Power Quality Signal

The steady-state indicators of power quality include voltage deviation, frequency deviation, harmonics, three-phase voltage unbalance, voltage fluctuation and flicker, etc. [20]. In this paper, common and harmful harmonics, voltage fluctuations, and flicker are used to construct the steady-state signal of power quality.

After performing a discrete Fourier transform (DFT) on a periodic non-sinusoidal AC signal, the components greater than integer multiples of the fundamental frequency are called harmonics, and the non-integer multiples are called interharmonics. The analytical expression is

\[
x(t) = \sum_{h=0.1}^M A_h \cos(2\pi f_0 h t + \varphi_h)
\]

where \( h \) is the (inter)harmonic order, \( f_0 \) is the power frequency, \( A_h \) and \( \varphi_h \) are the amplitude and phase of the corresponding order, respectively.

Voltage fluctuation is the phenomenon of a series of changes or continuous changes in the root mean square value of the voltage. Flicker is the cumulative effect of voltage fluctuations over a period of time, which is reflected by the visual perception caused by unstable light illumination. Flicker becomes a kind of harm caused by voltage fluctuations. Because the voltage fluctuation period is generally greater than the power frequency period, the power frequency signal can be regarded as the carrier wave, and the fluctuation signal can be regarded as the modulating wave. The analytical expression of the voltage fluctuation is as follows:

\[
x(t) = U_0[1 + m \cos(2\pi f_s t + \varphi_s)] \cos(2\pi f_0 t + \varphi_0)
\]

where \( U_0 \) is the power frequency voltage amplitude, \( m \) is the modulation indicator, and \( m < 1 \), \( f_s \) and \( \varphi_s \) are the modulation wave frequency and initial phase, respectively.

3. Algorithm Implementation Process and Evaluation Indicators

3.1. Parameter Learning Rules under the BSBL Algorithm

In order to determine the value of the hyperparameter \( \Theta = (\gamma_i, B_i, \lambda) \) (\( i = 1, 2, 3, \ldots, s \)), expectation-maximization (EM) can be used to obtain the maximum value of \( p(y; \Theta) \) [21,22]. The problem of obtaining the maximum value of \( p(y; \Theta) \) can also be transformed into obtaining the minimum value of \( L(\Theta) \).

\[
L(\Theta) = y^T \Sigma_y^{-1} y + \log|\Sigma_y|
\]

\[
\Sigma_y = \lambda E + A \Sigma_0 A^T
\]
For signals with no noise or noise close to 0, \( \lambda \) can be set to 0 directly. When the signal-to-noise ratio (SNR) is low, the learning rule is [23]

\[
\lambda \leftarrow \frac{\|y - A\mu_x\|_2^2 + \sum_{i=1}^{s} \text{Tr}(\Sigma_i \mu_x^T A^i) + s \sum_{i=1}^{s} \text{Tr}(\Sigma_i x (A_i)^T A_i))}{M}
\]

(17)

where the symbol “\( \leftarrow \)” means assigning the result on the right to the symbol on the left. Furthermore, \( \text{Tr} \) is the trace of the matrix, \( A_i \) is the sub-matrix corresponding to the \( i \)-th block of \( x \), and \( \Sigma_i \) is the \( i \)-th main diagonal block matrix of the \( \Sigma_x \). In other cases, the learning rule of \( \lambda \) is

\[
\lambda \leftarrow \frac{\|y - A\mu_x\|_2^2 + \text{Tr}(\Sigma_x A^T A)}{M}
\]

(18)

The learning rules of \( B_i \) are

\[
B_i \leftarrow \frac{\Sigma_i + \mu_i^T (\mu_i^T)^T}{\gamma_i}
\]

(19)

However, for different block numbers \( i \), if different \( B_i \) are used, over-fitting will occur. Therefore, for any \( i \), the learning rule of setting the average value of the parameters is

\[
B \leftarrow \frac{1}{m} \sum_{i=1}^{s} \frac{\Sigma_i + \mu_i^T (\mu_i^T)^T}{\gamma_i}
\]

(20)

where \( \mu_i \) is the \( i \)-th block of \( \mu_x \).

At the same time, in order to better find the intra-block correlation, a first-order auto-regressive model can be used. At this time, the matrix form in the corresponding block is

\[
B = \text{Toeplitz}([1, r, r^2 \ldots, r^{t-1}]) = \begin{pmatrix}
1 & r & \ldots & r^{t-1} \\
& 1 & \ldots & r^{t-2} \\
& & \ldots & \ldots \\
& & & 1
\end{pmatrix}
\]

(21)

where \( \text{Toeplitz}([1, r, r^2 \ldots, r^{t-1}]) \) is the corresponding Toeplitz matrix, and \( r \) is the autoregressive coefficient.

The learning rules of \( \gamma_i \) are

\[
\gamma_i \leftarrow \frac{1}{t_i} \text{Tr}(B_i^{-1} (\Sigma_i + \mu_i^T (\mu_i^T)^T))
\]

(22)

### 3.2. The BSBL Algorithm Process

The BSBL algorithm process calculates \( \mu_x \) and \( \Sigma_x \), according to Equations (11) and (12), as well as the initialization parameters or return values. Then, the parameters \( \lambda, B, \) and \( \gamma_i \) are continuously iterated according to the method shown in Section 2.3 until the \( \mu_x \) and \( \mu_x' \) calculated twice in a row are less than a certain threshold \( \mu_{th} \), the final \( \mu_x \) is output, and the iteration ends. The flowchart is shown in Figure 1.

It can be seen from the flowchart that BSBL will perform better due to the exploration of the intra-block correlation of the signal and continuous parameter iteration. At the same time, since each iteration will find the inverse of the \( \Sigma_x \) (Equation (12)), the running time will inevitably increase.
3.2. The BSBL Algorithm Process

The BSBL algorithm process calculates $\mu$ and $\Sigma$, according to Equations (11) and (12), as well as the initialization parameters or return values. Then, the parameters $\lambda$, $B$, and $\gamma_i$ are continuously iterated according to the method shown in Section 2.3 until the $\mu$ and $\mu'$ calculated twice in a row are less than a certain threshold $\mu_{th}$, the final $\mu$ is output, and the iteration ends. The flowchart is shown in Figure 1.

FIGURE 1. Flowchart of the BSBL algorithm.

3.3. Evaluation Indicators of Power Quality Data Compression Performance

The compression performance evaluation indicator is an objective evaluation of the performance of the compression algorithm, which can judge the approximation degree of the reconstructed signal to the original signal. The compression performance evaluation indicators generally include the compression ratio (CR), SNR, ERP, and RRMSE.

(1) The calculation equation of the CR is

$$CR = \frac{N_R}{N_O} \times 100\%$$

where $N_R$ and $N_O$ are the reconstructed signal and the original signal, respectively.
(2) The equation for calculating the SNR is:

$$\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{N} |x(i)|^2}{\sum_{i=1}^{N} [x(i) - \bar{x}(i)]^2}$$ (24)

where $x(i)$ is the original signal without white noise, and $\bar{x}(i)$ is the reconstructed signal; the unit of the SNR is the decibel (dB). The larger the value, the smaller the noise contained in the reconstructed signal.

(3) The calculation equation of the ERP is

$$\text{ERP} = \frac{\sum_{i=1}^{N} |\bar{x}(i)|^2}{\sum_{i=1}^{N} |x(i)|^2} \times 100\% \quad (25)$$

The closer the ERP is to 1, the closer the energy recovery is to the original signal.

(4) The calculation equation of relative root mean square error is

$$\text{RRMSE} = \sqrt{\frac{\sum_{i=1}^{N} [x(i) - \bar{x}(i)]^2}{\sum_{i=1}^{N} |x(i)|^2}} \times 100\% \quad (26)$$

The smaller the RRMSE, the smaller the relative error of the reconstructed signal.

The above indicators are only proposed for compression and reconstruction errors of general signals. However, the voltage harmonics of the power quality steady-state signal will cause voltage deviation; the amplitude and phase of the harmonics will affect the analysis of the three-phase voltage imbalance. In addition to the error between the reconstructed and original signals, we must also evaluate the characteristics of the power quality signal. For example, Section 5.2 of IEC41000-4-30 Testing and measurement techniques—Power quality measurement methods stipulates that the measurement uncertainty of the supply voltage magnitude should not exceed 0.1% of $U_{\text{din}}$ (declared input voltage). Meanwhile, Section 5.1 of IEC41000-4-7 Testing and measurement techniques—General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment connected thereto stipulates that the measurement error of the harmonic voltage/current of the Class I harmonic measuring instrument shall not exceed 5%. According to the requirements of these standards and the characteristics of the power quality signal, two new evaluation indicators are proposed: the relative error of the harmonic amplitude (hereafter referred to as the amplitude error) and the relative error of the harmonic phase angle (hereafter referred to as the phase angle error). The calculation equations are

Amplitude error:

$$\text{Amplitude error} = \sqrt{\frac{\sum_{h=0.1}^{N} (A_h - \hat{A}_h)^2}{\sum_{h=0.1}^{N} A_h^2}} \times 100\% \quad (27)$$

Angle error:

$$\text{Angle error} = \sqrt{\frac{\sum_{h=0.1}^{N} (\phi_h - \hat{\phi}_h)^2}{\sum_{h=0.1}^{N} \phi_h^2}} \times 100\% \quad (28)$$
where $A_h$ and $\phi_h$ are divided into the amplitude and phase angle of each harmonic and interharmonic of the original data after the DFT. The values of $\hat{A}_h$ and $\hat{\phi}_h$ are divided into the amplitude and phase angle of each harmonic and interharmonic of the reconstructed signal after the DFT. Thus, amplitude and phase angle errors can reflect the degree of harmonic distortion as the judgment basis after signal compression and reconstruction and meet the relevant national standards, thereby increasing the credibility of the algorithm performance evaluation results and engineering applications.

4. Case Study

4.1. Construction of the Steady-State Power Quality Signal

This article takes the signal length $N = 2560$ ($N$ is the number of sampling points) and the signal sampling frequency of 12.8 kHz, and Gaussian white noise is added to the signal to make the $SNR = 30$ dB. For the choice of the two parameters, $\mu_{th}$ and block length, their being too large or too small will degrade the relative performance of the results. Therefore, a sensitivity analysis was carried out on these parameters by using synthetic data and measured data. The results are shown in Figures 2 and 3. It can be seen from Figure 2 that the value of the SNR gradually decreases with the increase of block length. When $\mu_{th} = 10^{-4}$, the value of the SNR can be greater than 30 dB. It can be seen from the comparison in Figure 3 that the value of the SNR first increases and then decreases with the increase of block length. When block length = 128, the SNR of both data can be greater than 30 dB. Therefore, we set $\mu_{th} = 10^{-4}$ and block length = 128. The measurement matrix was selected as a sparse binary matrix, and the transform domains used by the signal were the DFT domain and the discrete cosine transform (DCT) domain. Suppose that the harmonic signal contains harmonics of 250 Hz, 350 Hz, 1900 Hz, 2100 Hz, 3950 Hz, and 4050 Hz and interharmonics of 85 Hz and 115 Hz. In the voltage fluctuation signal, we set $m = 0.1$, $f_s = 20$ Hz, $\varphi_s = \frac{\pi}{4}$. The signal waveforms and expressions are shown below (normalized basic amplitude to 1 p.u.) and in Figure 4.

\[
\begin{align*}
    x_1(t) &= \sin(2\pi \times 50t) + 0.1 \sin(2\pi \times 250t - 2\pi/3) + 0.1 \sin(2\pi \times 350t + 2\pi/3) \\
    &+ 0.08 \sin(2\pi \times 1900t + 5\pi/6) + 0.08 \sin(2\pi \times 2100t + 5\pi/6) \\
    &+ 0.05 \sin(2\pi \times 3950t + \pi/3) + 0.05 \sin(2\pi \times 4050t + \pi/3) \\
    &+ 0.06 \sin(2\pi \times 85t) + 0.06 \sin(2\pi \times 115t) \\
    x_2(t) &= [1 + 0.1 \cos(10\pi t) + \pi/3] \times \cos(100\pi t)
\end{align*}
\]

(29)

(30)

Figure 2. Sensitivity curve of $\mu_{th}$ in different data. (a) Sensitivity curve of $\mu_{th}$ in measured data. (b) Sensitivity curve of $\mu_{th}$ in synthetic data.
Figure 3. Sensitivity analysis of block length in different data. (a) Sensitivity curve of block length in measured data. (b) Sensitivity curve of block length in synthetic data.

Figure 4. Signal waveform of harmonic and voltage fluctuation.

For harmonic signals and voltage fluctuation signals, the DFT and DCT were used to perform sparse transformations and then compressed and reconstructed. The coefficients in the DFT and DCT domains are shown in Figures 5 and 6, respectively.
For harmonic signals and voltage fluctuation signals, the DFT and DCT were used to perform sparse transformations and then compressed and reconstructed. The coefficients in the DFT and DCT domains are shown in Figures 5 and 6, respectively.

Figure 5. Different transform basis coefficients of the harmonic signal.

Figure 6. Different transform basis coefficient of voltage fluctuation signal.

It can be seen from Figures 5 and 6 that after the transformation of the above two signals, the non-sparse components in the DCT domain were more concentrated in several blocks. Therefore, after compression and reconstruction by the BSBL algorithm, its performance should be slightly better than that under the DFT.

4.2. BSBL Algorithm Reconstruction Performance Analysis

The amplitude error was set up to ensure that the performance of the reconstructed signal meets Section 5.1 of IEC 41000-4-7. SNR is an important indicator for signal compression and reconstruction. However, there is no standard for this indicator at present. In the following cases, when the SNR of the reconstructed signal is more than 30 dB and the amplitude error is less than 5%, the minimum CR value is considered to be the best.

4.2.1. Performance Analysis of the Harmonic Signal

The various indicators of the harmonic signal in the DFT and DCT domains when different CR compressions and reconstructions were used are shown in Tables 1 and 2.
It can be seen from Tables 1 and 2 that in the harmonic signal, when the CR was 30% to 50%, the reconstruction effect of the DCT was better than that of the DFT. In the DFT domain, the value of the ERP differed from 1 by less than 0.5%, the amplitude error was less than 5%, and the phase error was less than 2%. The amplitude error meets the allowable error standard for the measurement of Class I instruments in IEC 41000-4-7, and the situation is similar in the DCT domain. After the DCT, when CR = 40%, the result of the SNR is more than 30 dB. Therefore, for the constructed harmonic signal CR = 40% is an ideal choice under the DCT, and CR = 50% is a good choice under the DFT.

4.2.2. Performance Analysis of Voltage Fluctuation Signal

The various indicators of the voltage fluctuation signal in the DFT and DCT domains when different CR compressions and reconstructions were used are shown in Tables 3 and 4.

For the constructed voltage fluctuation signal, the reconstruction performance after the DCT and DFT is good, the SNR can reach 50 dB or more, the amplitude error is less than...
0.5%, and the difference between the ERP and 1 approaches 0.5%. Therefore, compression reconstruction with CR = 30% is an ideal choice for this case.

4.2.3. Performance Analysis When the Signal Is Not Sparsely Transformed

The BSBL algorithm also has a good effect on non-sparse signals. The signal does not use any sparse transform domain and directly performs compression and reconstruction. However, the performance cannot meet the requirement of SNR > 30dB when \( \mu_{th} = 10^{-4} \), so we set \( \mu_{th} = 10^{-5} \). The performance indicators and reconstruction error of each point are shown in Table 5.

Table 5. Reconstruction results of the harmonic signals without a sparse transform.

| Transform Domain | No Transform Domain |
|------------------|---------------------|
| CR (%)           | 30 40 50 60         |
| SNR (dB)         | 25.40 26.83 28.10 30.05 |
| RRMSE (%)        | 5.37 4.55 3.93 3.17 |
| Amplitude error (%) | 4.26 3.56 3.05 2.43 |
| ERP (%)          | 100.62 100.15 100.03 100.02 |
| Angle error (%)  | 1.09 1.02 0.96 0.91 |

It can be seen from Table 5 that the performance of direct compression and reconstruction of the signal is worse than the result in the DFT or DCT domain. When the CR = 60%, the amplitude error can still be less than 5%, the value of the SNR could be more than 30 dB, the value of the ERP differs from 1 by less than 0.5%, and the phase error is less than 2%. Therefore, in the case of low accuracy and lack of corresponding hardware facilities, the proposed algorithm can directly compress and reconstruct harmonic signals.

4.3. Comparison of Reconstruction Performance with Other Algorithms

In order to judge the performance of the proposed algorithm more scientifically and objectively, this paper compares the performance of this algorithm with other data compression algorithms. For the performance comparison between different algorithms, the priorities of comparison were amplitude error, SNR, RRMSE, ERP, and angle error. This experiment selected the harmonic signal; the CR was selected as 40%; the DCT transform domain was selected; and the CoSaMP, SbOMP [24], and SAMP-SD [25] algorithms were compared. The sparsity (K) was set to 16, and the number of iterations was set to 40. Its comparative performance is shown in Table 6.

Table 6. Performance comparison of reconstruction algorithms (the harmonic signal).

| Algorithms   | SNR (dB) | RRMSE (%) | Amplitude error (%) | ERP (%) | Angle error (%) |
|--------------|---------|-----------|---------------------|---------|----------------|
| Traditional  | SAMP-SD | CoSaMP    | SbOMP               | BSBL    |
| SNR (dB)     | 39.41   | 38.01     | 35.94               | 40.58   |
| RRMSE (%)    | 1.11    | 1.15      | 1.53                | 0.93    |
| Amplitude error (%) | 1.08 | 1.12 | 1.47 | 0.85 |
| ERP (%)      | 100.06  | 100.09    | 100.13              | 100.16  |
| Angle error (%) | 1.39 | 1.43 | 1.40 | 1.39 |

From Table 6, it can be seen that the SNR of the proposed algorithm was better than that of the traditional algorithm. Both the RRMSE and amplitude error were less than 1%, while other algorithms were more than 1%. This shows that the proposed algorithm can improve the first three indicators.

4.4. Compression and Reconstruction of Measured Signals
4.4.1. Signals with Harmonics

In order to further verify the practicability of the BSBL algorithm, the real-world data of the capacitive current of a wind farm retrieved from Ningxia Province, China were used
for compression and reconstruction. It can be seen from Figure 7 that the signal segment contains rich (inter)harmonic content. The various indicators of reconstruction are shown in Tables 7 and 8.

**Figure 7.** Measured original signal waveform and its DFT and DCT coefficients.

**Table 7.** Reconstruction results of the harmonic signal in the DFT domain.

| Transform Domain | DFT |
|------------------|-----|
| CR (%)           | 30  | 40  | 50  | 60  |
| SNR (dB)         | 22.24 | 25.10 | 29.66 | 37.53 |
| RRMSE (%)        | 7.73 | 5.56 | 3.29 | 1.33 |
| Amplitude error (%) | 7.62 | 5.49 | 3.24 | 1.31 |
| ERP (%)          | 103.78 | 101.63 | 100.84 | 100.28 |
| Angle error (%)  | 1.34 | 1.36 | 1.35 | 1.32 |

**Table 8.** Reconstruction results of the harmonic signal in the DCT domain.

| Transform Domain | DCT |
|------------------|-----|
| CR (%)           | 30  | 40  | 50  | 60  |
| SNR (dB)         | 25.91 | 28.25 | 30.51 | 32.28 |
| RRMSE (%)        | 6.10 | 4.72 | 2.95 | 2.85 |
| Amplitude error (%) | 5.83 | 4.33 | 3.16 | 2.51 |
| ERP (%)          | 102.33 | 101.03 | 100.30 | 100.30 |
| Angle error (%)  | 1.15 | 1.13 | 1.07 | 1.03 |

It can be seen from Tables 7 and 8 that for the measured harmonic data, no matter which transformation was adopted, its sparsity was much lower than that of the constructed signal. Therefore, under the same conditions, its performance indicators were lower than those of the constructing signals. When the CR was $\geq 50\%$, the amplitude error met the IEC 40000-6-7 standard. The SNR after the DCT was more than 30 dB, and the SNR after the DFT was 30 dB. In this case, CR = 50% was selected.

The following compares the performance of the proposed algorithm and the traditional algorithm applied to real-world data. The traditional algorithm was set to CR = 40%. Since the sparsity of the measured data cannot be estimated, the sparsity was cycled from 40 to 300. The final result of the traditional algorithm is the result of the largest SNR under the DFT and DCT and the sparsity cycle as the experimental result to compare with the proposed algorithm. The proposed algorithm takes CR = 40%, the result in the DCT domain.
It can be seen from Table 9 that the first three indicators of the proposed algorithm are far better than the traditional ones and because there is no need to predict the sparsity in advance. The process of the proposed algorithm is also simpler. The comparison result shows that the proposed algorithm can be applied to engineering.

Table 9. Performance comparison of reconstruction algorithms (real-world harmonic signal).

| Algoritms     | SAMP-SD | CoSaMP | SbOMP | BSBL |
|---------------|---------|--------|-------|------|
| SNR (dB)      | 23.46   | 21.56  | 20.98 | 28.25|
| RRMSE (%)     | 6.34    | 8.35   | 7.43  | 4.72 |
| Amplitude error (%) | 6.28  | 8.23   | 7.36  | 4.33 |
| ERP (%)       | 100.48  | 100.60 | 100.92| 101.03|
| Angle error (%)| 1.19   | 1.22   | 1.26  | 1.13 |

4.4.2. Fluctuant Signal

Figure 8 shows the fluctuant signal. The application scenario of these data is the sub-synchronous oscillation caused by the super-synchronous interharmonic of the port power electronic converter group. The various indicators of reconstruction are shown in Tables 10 and 11.

Table 10. Reconstruction results of the fluctuant signal in the DFT domain.

| Transform Domain | DFT |
|------------------|-----|
| CR (%)           | 30  | 40  | 50  | 60  |
| SNR (dB)         | 23.59| 36.56| 42.13| 43.76|
| RRMSE (%)        | 6.61 | 1.49 | 0.78 | 0.65 |
| Amplitude error (%)| 6.55 | 1.48 | 0.76 | 0.64 |
| ERP (%)          | 111.51| 102.03| 100.34| 100.04|
| Angle error (%)  | 1.28 | 1.30 | 1.35 | 1.37 |

For the measured fluctuation signal, the performance of the DFT was better than that of the DCT. Taking CR = 40% under the DFT can be satisfied.

Table 12 shows the performance comparison between the proposed and traditional algorithms with parameter settings the same as those in Section 4.4.1.
Table 11. Reconstruction results of the fluctuant signal in the DCT domain.

| Transform Domain | DCT |
|------------------|-----|
| CR (%) | 30  | 40  | 50  | 60  |
| SNR (dB) | 23.52 | 26.58 | 35.75 | 40.76 |
| RRMSE (%) | 6.67 | 4.69 | 1.63 | 0.92 |
| Amplitude error (%) | 5.98 | 4.14 | 1.33 | 0.71 |
| ERP (%) | 103.71 | 100.81 | 100.20 | 100.03 |
| Angle error (%) | 1.18 | 1.16 | 1.02 | 0.90 |

Table 12. Performance comparison of reconstruction algorithms (real-world fluctuant signal).

| Algorithms       | Traditional Methods | Proposed Method |
|------------------|---------------------|-----------------|
|                  | SAMP-SD             | CoSaMP          | SbOMP          | BSBL |
| SNR (dB)         | 35.23               | 34.72           | 35.33          | 36.56 |
| RRMSE (%)        | 1.85                | 1.83            | 1.72           | 1.49  |
| Amplitude error (%) | 1.80          | 1.67            | 1.68           | 1.48  |
| ERP (%)          | 100.10              | 100.04          | 100.1          | 102.03 |
| Angle error (%)  | 1.36                | 1.22            | 1.29           | 1.30  |

It can be seen from the corresponding coefficients in Figures 7 and 8 that the sparsity of the fluctuant signal was much smaller than that of the harmonic signal, so the performance of each algorithm was good, and the performance of the proposed algorithm was slightly better than the traditional algorithm. Furthermore, from the comparison of these two experiments, the reconstruction performance of the proposed algorithm was better than other algorithms for signals with more harmonic content.

4.5. Analysis of Results

The reconstruction performance of the voltage fluctuation signal and harmonic signal (Tables 1–4) shows that the smaller the sparsity of the original signal after sparse transformation, the higher the reconstruction performance. For the harmonic signal, when the compression ratio increases, the performance of each indicator also increases. For the above constructed and measured data, the trend was consistent with CR. However, the measured signal performed worse than the construction signal because of its rich harmonic content. By comparing the constructed signal with the measured signal, with any transformation and harmonic signal with CR = 50%, the amplitude error met the requirements of IEC41000-4-7; the SNR was more than 30 dB; and the performance results of the other indicators, such as RRMSE, ERP, and angle error, were also good. There was little difference between the two transformation domains. For fluctuation signals, when the CR = 40%, the amplitude error met the requirements, but other indicators under the DCT were lower, so using the DFT is recommended.

When compared with the performance of greedy algorithms (SbOMP, SAMP-SD, CoSAMP), the BSBL algorithm does not need to estimate the sparsity of the steady-state power quality signal in advance. Furthermore, the BSBL algorithm has a more prominent effect when the harmonic component in the signal is large. The proposed algorithm can be used normally without sparse transformation, but its performance is slightly worse than with sparse transformation. Therefore, when a large number of steady-state power quality data need to be recorded, sampled, and transmitted in engineering, BSBL is a good solution.

5. Conclusions

This paper studies the application of BSBL in steady-state power quality data compression, converts data into sparse data through different transformation domains (DFT, DCT), and compresses and reconstructs it using BSBL. Based on the relative standard, two indicators of power quality data characteristics, namely relative amplitude error and relative phase angle error, are also presented in this paper. Then, the algorithm’s validity
is verified by constructing and measuring signals. The results show that CR = 50% can meet the standard for harmonic signals in engineering, no matter which transformation is used. For fluctuation signals, using the CR = 40% selection after the DFT is recommended to obtain a better result. The results also show that when $\mu_0 = 10^{-5}$ and CR = 60% are selected, the standard requirement can be met without sparse transformation of signals. Finally, compared with other greedy algorithms (SbOMP, SAMP-SD, CoSAMP), we show that the proposed algorithms are superior to other greedy algorithms in terms of relevant indicators. Therefore, in the case of conforming to the standard, the proposed algorithm can meet the requirements of steady-state signal compression of power quality in engineering. Meanwhile, in the performance comparison with other algorithms, the two indicators of the ERP and angle error are not improved. This will be the direction of our next research study.

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