The hypermultiplet low-energy effective action, 
$N=2$ supersymmetry breaking and confinement

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Abstract: Some exact solutions to the hypermultiplet low-energy effective action in $N = 2$ supersymmetric four-dimensional gauge field theories with massive ‘quark’ hypermultiplets are discussed. The need for a spontaneous $N = 2$ supersymmetry breaking is emphasized, because of its possible relevance in the search for an ultimate theoretical solution to the confinement problem.

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1 Introduction

Despite of remarkable recent advances in string duality and brane technology, our knowledge about the non-perturbative string theory (= M-theory) is still very much dependent upon our understanding of non-perturbative quantum field theory like QCD. At high energies the QCD is well described by a perturbation theory because of its asymptotic freedom, in a good agreement with well-known experimental data about deep inelastic scattering and jet production. However, at low energies, the QCD vacuum is essentially non-perturbative, so that some of the most obvious experimental facts about strong interactions, e.g., the confinement of quarks inside hadrons, are still waiting for an ultimate theoretical solution. On the theoretical side, the quantum generating functional (or the effective action) of a non-abelian gauge field theory should be defined in practical terms, which would allow one to get a non-perturbative solution to the theory. Unfortunately, the corresponding path integral is usually defined in many ways beyond the perturbation theory (e.g., lattice field theory, instantons, duality), which makes getting an exact solution to be extremely difficult, if ever possible.

Therefore, it seems to be quite natural to take advantage of the existence of exact solutions to the low-energy effective action in certain $N=2$ supersymmetric gauge field theories since the remarkable discovery of Seiberg and Witten [1], and apply them to the old problem of color confinement in QCD. In fact, it was one of the main motivations in the original work [1]. The most attractive mechanism for color confinement is known to be the dual Meissner effect or the dual (Type II) superconductivity [2]. It takes just three steps to connect an ordinary BCS superconductor to the simplest Seiberg-Witten model in quantum field theory: first, define a relativistic version of the superconductor, known as the (abelian) Higgs model in field theory, second, introduce a non-abelian version of the Higgs model, known as the Georgi-Glashow model, and, third, $N=2$ supersymmetrize the Georgi-Glashow model in order to get the Seiberg-Witten model [1]. Since the t’Hooft-Polyakov monopole of the Georgi-Glashow model belongs to a (HP) hypermultiplet in its $N=2$ supersymmetric (Seiberg-Witten) generalisation, it is quite natural to explain confinement as the result of a monopole condensation (dual Higgs effect), i.e. a non-vanishing vacuum expectation value for the magnetically charged (dual Higgs) scalars belonging to the HP hypermultiplet.

In fact, the exact solutions to the low-energy effective action in quantum gauge field theories are only available in $N=2$ supersymmetry, and neither in $N=1$ supersymmetry nor in the bosonic QCD. Hence, on the one side, it is the $N=2$ supersymmetry that crucially simplifies an evaluation of the low-energy effective action. However, on the other side, it is the same $N=2$ supersymmetry that is obviously incompatible with phenomenology e.g., because of equal masses of bosons and fermions inside $N=2$ supermultiplets (it also applies to any $N\geq 1$ supersymmetry), and the non-chiral nature of $N=2$ supersymmetry (e.g. ‘quarks’ then appear in real representations of the gauge group). Therefore, if we believe in the $N=2$ supersymmetry, we should find a way of judicious $N=2$ supersymmetry breaking.

The $N=2$ supersymmetry can be broken either softly or spontaneously, if one wants to preserve the benefits of its presence (e.g. for the full control over the low-energy effective action) at high energies. As regards the gauge low-energy effective action, the information about it in the Seiberg-Witten approach is encoded in terms of holomorphic functions defined over the quantum moduli space whose modular group is identified with the duality group, while the functions themselves can be calculated exactly. In the $N=2$ supersymmetric QCD, one has to add ‘quark’ hypermultiplets, which have some bare (BPS) masses, flavour and color, i.e. belong to the fundamental representation of the gauge group. In the full theory, one expects an appearance of additional (e.g. magnetically charged) degrees of freedom, to be described by some effective action via strong-weak coupling duality and depending upon the (Coulomb, Higgs or confinement) branch under consideration. The full low-energy effective action in the $N=2$ super-QCD is given by a sum of the gauge and the hypermultiplet parts.

We would like to find a vacuum solution to the full $N=2$ supersymmetric low-energy effective action, which would break supersymmetry due to the non-vanishing vacuum expectation value of a magnetically charged (Higgs) scalar, similarly to that in ref. [1]. The same dual Higgs mechanism may also be responsible for the chiral symmetry breaking and the appearance of the pion effective Lagrangian if the dual Higgs field has flavor charges also [1]. In fact, Seiberg and Witten used a mass term for the
$N = 1$ chiral multiplet, which is a part of the $N = 2$ vector multiplet, in order to softly break $N = 2$ supersymmetry to $N = 1$ supersymmetry. As a result, they found a non-trivial vacuum solution with a monopole condensation and, hence, a confinement. The weak point of their approach is an ad hoc assumption about the existence of the mass gap, i.e. the mass term itself. It would be nice to derive the mass gap from the fundamental theory instead of postulating it. The $N = 2$ supersymmetry may be useful here since it severely constrains all possible ways of its soft (or spontaneous) breaking.

The soft breaking of $N = 2$ supersymmetry is a very practical approach to analyse the consequences of the Seiberg-Witten exact solution towards its possible phenomenological applications like a derivation of the pion lagrangian or the confinement problem in QCD. The general analysis of all possible soft $N = 2$ supersymmetry breaking patterns in the $N = 2$ supersymmetric QCD was recently given by Alvarez-Gaumé, Mariño and Zamora in ref. [3]. Though being quite pragmatic, the soft susy breaking has, however, a limited predictive power and too many free parameters. Hence, it makes sense to search for the patterns of spontaneous $N = 2$ supersymmetry breaking. In practice, this means finding a non-supersymmetric vacuum solution for the $N = 2$ supersymmetric scalar potential at the level of the low-energy effective action in $N = 2$ gauge theories. Since the $N = 2$ supersymmetry remains unbroken for any exact Seiberg-Witten solution in the gauge sector, we should consider the induced (i.e. quantum generated) scalar potentials in the hypermultiplet sector of an $N = 2$ gauge theory. Moreover, once we accepted $N = 2$ supersymmetry in field theory, we can also take into account those brane configurations of the underlying M-theory that are relevant for the four-dimensional $N = 2$ supersymmetric effective physics in the limit $M_{\text{Planck}} \to \infty$. The related brane-technology [4] can provide us with some additional insights into the non-perturbative field theory, as well as supply us with its geometrical interpretation. Since the relevant M-theory brane configurations with eight supercharges arise as the solitonic solutions to the effective equations of motion in the M-theory, their ‘soft’ deformation, which breaks some more of the supersymmetries but still remains to be a solution to the M-theory effective equations of motion, should be interpreted as a spontaneous supersymmetry breaking (see an example in sect. 3).

In sect. 2 we analyse the general problem of constructing the low-energy hypermultiplet effective action in $N = 2$ rigid (global) supersymmetry, by using the $N = 2$ harmonic superspace. In sect. 3 we give two simple (toy) examples of the non-trivial induced scalar potentials for a single matter hypermultiplet.

2 The hypermultiplet low-energy effective action

There are only two basic $N = 2$ supermultiplets (modulo classical duality transformations) in the rigid $N = 2$ supersymmetry (with the $SU(2)_A$ internal symmetry): an $N = 2$ vector multiplet and a hypermultiplet. The $N = 2$ vector multiplet components (in a WZ-like gauge) are $(A, \chi^i_{\alpha}, V_\mu, D^{(ij)})$, where $A$ is a complex Higgs scalar, $\chi^i$ is a chiral spinor (‘gaugino’) $SU(2)_A$ doublet, $V_\mu$ is a real gauge vector field, and $D^{(ij)}$ is an auxiliary $SU(2)_A$ scalar triplet $(i, j = 1, 2)$. Similarly, the on-shell physical components of the Fayet-Sohnius (FS)-type hypermultiplet are $(q^i, \psi_\alpha, \psi^\dagger_{\alpha})$, where $q^i$ is a complex scalar $SU(2)_A$ doublet, and $\psi$ is a Dirac spinor. There exists another (dual) Howe-Stelle-Townsend (HST)-type hypermultiplet, whose on-shell physical components are $(\omega, \omega^{(ij)}, \chi^i_{\alpha})$, where $\omega$ is a real scalar, $\omega^{(ij)}$ is a scalar $SU(2)_A$ triplet, and $\chi^i$ is a chiral spinor (‘quark’) $SU(2)_A$ doublet.

The universal (i.e. most general and off-shell) and manifestly $N = 2$ supersymmetric formulation of all $N = 2$ supersymmetric four-dimensional field theories is only possible in the $N = 2$ harmonic superspace (HSS) [5] (see e.g. ref. [6] for a recent introduction). The $N = 2$ HSS coordinates include extra bosonic variables (called harmonics $u_{ij}$), which parametrize the sphere $S^2 \sim SU(2)/U(1)$, in addition to the standard $N = 2$ superspace coordinates. The harmonics play the role of twistors or spectral parameters known in the theory of integrable systems. In particular, an off-shell FS hypermultiplet in HSS is described by an analytic superfield $q^+_\alpha$ of the $U(1)$-charge (+1), whereas the HST hypermultiplet in HSS is described by a real analytic superfield $\omega$ of vanishing $U(1)$ charge. An $N = 2$ vector gauge multiplet is similarly described by an analytic HSS superfield $V^{++}_{ij}$ of the $U(1)$-charge (+2), which is introduced as a connection to the basic HSS harmonic derivative $D^{++}$ present in the kinetic terms of the hypermultiplet actions (see below).
The power of \( N = 2 \) superspace is clearly seen in the most general form of the \( N = 2 \) gauge low-energy effective action in the Coulomb branch,

\[
\Gamma_{\nu}[W, \bar{W}] = \int_{\text{chiral}} \mathcal{F}(W) + \text{h.c.} + \int_{\text{full}} \mathcal{H}(W, \bar{W}) + \ldots ,
\]

where the abelian field strength \( W(V) \), which is a harmonic-independent, \( N = 2 \) chiral and gauge-invariant superfield, has been introduced. The leading term in eq. (1) is given by the chiral \( N = 2 \) superspace integral over a holomorphic function \( \mathcal{F} \) of \( W \), with the latter being valued in the Cartan subalgebra of the gauge group. The Seiberg-Witten approach provides a solution to the holomorphic function \( \mathcal{F} \) in terms of the auxiliary Riemann surface \( \Sigma_{SW} \). It appears to be a solution to the particular Riemann-Hilbert problem of fixing a holomorphic multi-valued function \( \mathcal{F} \) by its given monodromy and singularities. The number (and nature) of the singularities is the physical input: they are identified with the appearance of massless non-perturbative BPS-like physical states (dyons) like the 't Hooft-Polyakov magnetic monopole. The monodromies are supplied by perturbative renormalization-group \( \beta \)-functions and \( S \)-duality. The next-to-leading-order term in eq. (1) is given by the full \( N = 2 \) superspace integral over a real function \( \mathcal{H} \) of \( W \) and \( \bar{W} \). Some partial results about this function are known \[7\]. The dots in eq. (1) stand for higher-order terms containing the derivatives of \( W \) and \( \bar{W} \).

The most general form of the leading term in the hypermultiplet low-energy effective action can be written down in the \( N = 2 \) HSS as follows:

\[
\Gamma_{H}[q^+, \bar{q}^+; \omega] = \int_{\text{analytic}} \mathcal{K}^{(+4)}(q^+, \bar{q}^+; \omega; u_+^\pm) + \ldots ,
\]

where \( \mathcal{K}^{(+4)} \) is a function of the FS analytic superfield \( q^+ \), its conjugate \( \bar{q}^+ \), the HST analytic superfield \( \omega \) and the harmonics \( u_+^\pm \), with the overall \( U(1) \)-charge \( (+4) \). The action (2) is supposed to be added to the kinetic hypermultiplet action whose analytic Lagrangian is quadratic in \( q^+ \) or \( \omega \), and of \( U(1) \)-charge \( (+4) \). A free FS hypermultiplet action is given by

\[
S[q] = -\int d\zeta^{(-4)} du \bar{q}^+ D^{++} q^+ ,
\]

whereas its minimal coupling to an \( N = 2 \) gauge superfield reads

\[
S[q, V] = -\int d\zeta^{(-4)} du \bar{q}^+ (D^{++} + iV^{++}) q^+ .
\]

Similarly, a free action of the HST hypermultiplet is given by

\[
S[\omega] = -\frac{1}{2} \int d\zeta^{(-4)} du (D^{++} \omega)^2 ,
\]

and it is on-shell equivalent to the standard \( N = 2 \) tensor (or linear) multiplet action in the ordinary \( N = 2 \) superspace \[3\].

The function \( \mathcal{K} \) is called the hyper-Kähler potential. In components, it automatically leads to the \( N = 2 \) supersymmetric non-linear sigma-model for the scalars with a hyper-Kähler metric, just because of the \( N = 2 \) supersymmetry by construction (see the examples in sect. 3). When being expanded in components, the first term in eq. (1) also leads to the certain Kähler non-linear sigma-model in the Higgs sector \((A, \bar{A})\). The corresponding Kähler potential \( K_{\mathcal{F}}(A, \bar{A}) \) is dictated by the holomorphic function \( \mathcal{F} \) as \( K_{\mathcal{F}} = \text{Im}[A \mathcal{F}'(A)] \), so that the function \( \mathcal{F} \) plays the role of a potential for this special Kähler (but not hyper-Kähler) geometry \( K_{\mathcal{F}}(A, \bar{A}) \). As regards the hypermultiplet non-linear sigma-model of eqs. (2)–(5), a relation between the hyper-Kähler potential \( \mathcal{K} \) and the corresponding Kähler potential \( K_{\mathcal{K}} \) is much more involved. It is easy to see that the hyper-Kähler condition on a Kähler potential amounts to a non-linear (Monge-Ampère) partial differential equation. It is remarkable that the HSS approach allows one to get a formal ‘solution’ to any hyper-Kähler geometry in terms of an analytic scalar potential \( \mathcal{K} \).
However, the real problem is now translated into finding the relation between $K$ and the corresponding Kähler potential (or metric) in components, whose determination amounts to solving infinitely many linear differential equations altogether, just in order to eliminate an infinite number of HSS auxiliary fields (sect. 3).

The gauge-invariant functions $F(W)$ and $H(W, \bar{W})$ receive both perturbative and non-perturbative contributions,

$$F = F_{\text{per.}} + F_{\text{inst.}}, \quad H = H_{\text{per.}} + H_{\text{non-per.}},$$

while the non-perturbative corrections to the holomorphic function $F$ are entirely due to instantons. This is an important difference from the (bosonic) non-perturbative QCD whose low-energy effective action is dominated by instanton-antiinstanton contributions.

It is remarkable that the perturbative contributions to the leading and subleading terms in the $N = 2$ gauge effective action (1) come from the one loop only. As regards the leading holomorphic contribution, $N = 2$ supersymmetry puts the trace of the energy-momentum tensor $T_{\mu}^\mu$ and the axial or chiral anomaly $\bar{j}_R^\mu$ of the abelian $R$-symmetry into one $N = 2$ supermultiplet. The $T_{\mu}^\mu$ is essentially determined by the perturbative renormalization group $\beta$-function, $T_{\mu}^\mu \sim \beta(g) FF$, whereas the one-loop contribution to the chiral anomaly, $\partial \cdot j_R \sim C_{1}\text{loop} F^* F$, is known to saturate the exact solution to the Wess-Zumino consistency condition for the same anomaly. Hence, $\beta_{\text{per.}}(g) = \beta_{1\text{-loop}}(g)$ by $N = 2$ supersymmetry also. Since the $\beta_{\text{per.}}(g)$ is effectively determined by the second derivative of $F_{\text{per.}}$, one concludes that $F_{\text{per.}} = F_{1\text{-loop}}$. This simple component argument can be extended to a proof when using the $N = 2$ HSS approach. It then becomes clear that the non-vanishing central charges of the $N = 2$ supersymmetry algebra are of crucial importance for the non-vanishing holomorphic contribution to the gauge effective action (1).

Similarly, the BPS mass of a hypermultiplet can only come from the central charges since, otherwise, the number of massive hypermultiplet components has to be increased. The most natural way to introduce central charges $(Z, \bar{Z})$ is to identify them with spontaneously broken $U(1)$ generators of dimensional reduction from six dimensions via the Scherk-Schwarz mechanism. It naturally leads to the additional ‘connection’ term in the four-dimensional harmonic derivative as

$$D^{++} = D^{++} + v^{++}, \quad \text{where} \quad v^{++} = i(\bar{\theta}^+ \theta^+)\bar{Z} + i(\bar{\theta}^+ \bar{\theta}^+)Z.$$  

Therefore, the $N = 2$ central charges can be equally treated as a non-trivial $N = 2$ gauge background, with the covariantly constant $N = 2$ chiral superfield strength $\langle W \rangle = Z$.

3 Examples

We are still far from presenting a convincing pattern of spontaneous $N = 2$ supersymmetry breaking via the hypermultiplet low-energy effective action. Nevertheless, the examples that we already have, give some reasons for optimism. Our point here is quite simple: given non-trivial kinetic terms in the hypermultiplet low-energy effective action to be represented by the non-linear sigma-model, in a presence of non-vanishing central charges it leads to a non-trivial hypermultiplet scalar potential whose form is entirely determined by the hyper-Kähler metric of the kinetic terms and $N = 2$ supersymmetry.

The first example of this interesting connection was given in ref. [9]. Consider a single charged FS hypermultiplet $q^+$ in the Coulomb branch of the $N = 2$ gauge theory. As was shown in ref. [9], it has a unique non-trivial self-interaction whose form in the $N = 2$ HSS reads

$$\text{LEE}_A[q^+] = \int_{\text{analytic}} \left[ \frac{q^+ D^{++} q^+}{\pi^4} + \frac{\lambda}{2} (q^+)^2 (\bar{q}^+)^2 \right],$$

where the induced coupling constant $\lambda$ is given by

$$\lambda = \frac{g^4}{\pi^2} \left[ \frac{1}{m^2 \Lambda^2} \ln \left( 1 + \frac{m^2}{\Lambda^2} \right) - \frac{1}{\Lambda^2 + m^2} \right].$$
in terms of the gauge coupling constant $g$, the hypermultiplet BPS mass $m^2 = |Z|^2$, and the IR-cutoff $\Lambda$. When using the parametrization
\[
q^+|_{q=0} = f^i(x)u^+_i \exp \left[ \lambda f^{ij}(x)f^{jk}(x)u^+_j u^+_k \right],
\]
the bosonic terms take the form of the one non-linear sigma-model,
\[
\text{LEEA}_{\text{bosonic}}[f] = \int d^4x \left\{ g_{ij}(f)\partial_m f^i \partial^m f^j + \bar{g}^{ij}(f)\partial_m \bar{f}^i \partial^m \bar{f}^j + h_{ij}(f)\partial_m f^i \partial^m \bar{f}^j - V(f) \right\},
\]
whose metric turns out to be that of Taub-NUT or a KK-monopole (modulo field redefinitions), whereas the induced scalar potential is
\[
V(f) = |Z|^2 \frac{f\bar{f}}{1 + \lambda f\bar{f}}.
\]

A non-trivial hypermultiplet self-interaction for a single neutral HST-type $\omega$-hypermultiplet can be non-perturbatively generated in the presence of non-vanishing constant $N = 2$ Fayet-Iliopoulos (FI) term \( \langle D^{(ij)} \rangle \equiv \xi^{(ij)} = \frac{1}{2}(\bar{\tau} \cdot \xi)^{ij} \), where $\bar{\tau}$ are Pauli matrices. The FI-term has a nice geometrical interpretation in the underlying ten-dimensional type-IIB superstring brane picture made out of two solitonic 5-branes located at particular values of $\vec{w} = (x^7, x^8, x^9)$ and some Dirichlet 4- and 6-branes, all having the four-dimensional spacetime $(x^0, x^1, x^2, x^3)$ as the common macroscopic world-volume \[7\]. The values of $\xi$ can then be identified with the angles at which the two 5-branes intersect, $\xi = \vec{w}_1 - \vec{w}_2$, in the type-IIB picture \[7\]. The three hidden dimensions ($\vec{w}$) are identified by the requirements that they do not include the two hidden dimensions $(x^4, x^5)$ already used to generate central charges in the effective four-dimensional field theory, and that they are to be orthogonal (in the effectively $N = 2$ supersymmetric configuration) to the direction $(x^6)$ in which the Dirichlet 4-branes are finite and terminate on 5-branes.

The unique low-energy effective action for the (dimensionless) $\omega$-hypermultiplet in the presence of the FI-term reads \[7\]:
\[
S_{EH}[\omega] = -\frac{1}{2\kappa^2} \int d\xi^{(-4)}du \left\{ (D^{++}\omega)^2 - \frac{(\xi^{++})^2}{\omega^2} \right\},
\]
where $\xi^{++} = u_+^+ u_+^+ \xi^{(ij)}$ is the FI-term, and $\kappa$ is the coupling constant of dimension one (in units of length). After changing the variables to $q^+_a = u_+^+ \omega + u_+^- f^{++},$ and eliminating the Lagrange multiplier $f^{++}$ via its algebraic equation of motion, one can rewrite eq. (13) to the equivalent gauge-invariant form
\[
S_{EH}[q, V] = -\frac{1}{2\kappa^2} \int d\xi^{(-4)}du \left\{ q^{(a)}_A D^{++} q^+_A + V^{++} \left( \frac{1}{2} \varepsilon^{AB} q^{(a)}_A q^{(b)}_B + \xi^{++} \right) \right\},
\]
in terms of two FS hypermultiplets $q^+_a (A = 1, 2)$ and the auxiliary real analytic $N = 2$ vector superfield $V^{++}$ \[10\], where we have introduced the pseudo-real notation $q_a = (\bar{q}^+, q^+)$ and $\varepsilon^{ab} q^{(a)}_b = q^{a+}, a = 1, 2$. It is now straightforward to calculate the bosonic terms in the HSS action (14), in terms of the scalar fields, $q^+_A = f^+_A u^+_i$, and $f^+_A \equiv m^2 A \exp(i \varphi_3^{(A)})$. One finds the constraint
\[
\xi^{(ij)} = f^{(i)} f^{(j)} - f^{(i)}_{\bar{f}^{(j)}} = f^{(i)} f^{(j)} - f^{(i)}_{\bar{f}^{(j)}}\]
leading to the Eguchi-Hanson metric for the kinetic terms, as well as the scalar potential \[10\]
\[
V = \frac{Z^2}{(f_1 f_2 + f_2 f_1)^2} \left[ (f_1 f_2 + f_2 f_1)^2 + (f_1 f_2 + f_2 f_1)^2 - 2 \right].
\]
When choosing the direction $\xi^2 = \xi^3 = 0$ and $\xi^1 = 2i$, it is not difficult to solve the constraint (15) in terms of four independent fields \[10\] $f_2 = m, f_1 = n, \varphi_1 = \theta, \varphi_2 = \phi$, where the local $U(1)$ invariance has been fixed by the gauge condition $\varphi_1 = \varphi_2 = \varphi_1 + \varphi_2$. One finds
\[
V = |Z|^2 \frac{\sin^2(\theta + \phi)}{m^2 + n^2} \left[ \frac{4(m^2 - n^2)^2}{1 + (m^2 + n^2)^2 \sin^2(\theta + \phi)} \right] + \frac{1 + (m^2 + n^2)^2 \sin^2(\theta + \phi)}{\sin^2(\theta + \phi)}.\]
It is clear that the potential $V$ is positively definite, and it is only non-vanishing due to the non-vanishing central charge $|Z|$. It signals the spontaneous breaking of $N = 2$ supersymmetry in our model.
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