Effect of elastic foundations on vibrations of thick-walled hollow poroelastic cylinders

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Abstract. This paper deals with the effect of elastic foundations on the vibrations of thick-walled hollow poroelastic cylinders. The frequency equations of axially symmetric vibrations and non-axially symmetric vibrations, each for pervious and impervious surface are obtained using the analytical model based on Biot’s theory. The plots of frequency versus ratio of thickness to inner radius of composite cylinder for different materials are presented graphically.

1. Introduction
The study of elastic foundations in poroelastic cylinder are fundamental and core structural elements in various fields of Engineering and Technology. Recently they have acquired a paramount and valuable place in practical applications of constructing gas cylinders, pressure vessels, boilers etc. Paliwal et. al [1] studied free vibrations of circular cylindrical shell on Wrinkler and Pasternak foundations. In the said paper, it is concluded that elastic foundation affects radial vibrations mode frequency while pertaining to torsional and longitudinal remains are unaffected. Goncalves and Batista [2] gave a theoretical analysis for the free vibration of simply supported vertical shells partially filled with or submerged in a fluid and vibrations of the shells are examined using the sanders shell theory. Paliwal and Pandey [3] discussed the free vibrations of cylindrical shell on elastic foundation. In the said paper, the frequency equation of thin circular cylindrical shell resting on an elastic foundation is developed by using the first order shell theory of Sanders. Natural frequency of a cylindrical panel on kerr foundation is studied by Cai et. al [4]. Exact analysis of plane-strain vibrations of thick-walled hollow elastic circular cylinder is discussed by Gazis [5]. In the said paper, approximate expressions for the frequencies of free modes are derived for moderately thick shells. Employing Biot’s theory of poroelasticity [6], plane-strain vibrations of thick-walled hollow poroelastic cylinder is investigated by Malla Reddy and Tajuddin [7]. In the said paper, the free vibrations of an infinite thick-walled hollow poroelastic cylinder is studied both for axially symmetric and non-axially symmetric vibrations. Shankar et. al [8] investigated plane strain vibrations in a poroelastic composite hollow cylinder. In the said paper, frequency equation of plane strain vibrations of poroelastic composite hollow cylinder is obtained along with some particular cases such as poroelastic composite hollow cylinder with rigid casing, poroelastic composite solid cylinder with rigid casing, poroelastic composite bore and inner radius of core is discussed. Axially symmetric vibration of finite composite poroelastic cylinders is studied by Shah and Tajuddin [9]. Tajuddin [10] discussed torsional vibrations of finite composite cylinders. In the said paper, frequency equation is obtained for torsional vibrations of composite poroelastic cylinders either
concentric or bonded end to end. Longitudinal shear vibrations of hollow poroelastic cylinder is investigated by Tajuddin and Ahmed shah [11]. In the said paper, frequency equation of longitudinal shear vibrations of a hollow poroelastic circular cylinders of infinite extent is derived and also the frequency is same for a thin poroelastic cylindrical shell in case of axially symmetric shear vibrations, flexural vibrations and non-axially symmetric shear vibrations. Malla Reddy and Tajuddin [12] investigated axially symmetric vibrations of a poroelastic composite cylinder in the context of fretting fatigue and the results are compared with that of Rule of Mixture (RoM). However, to the best of author’s knowledge, effect of elastic foundations on vibrations of thick-walled hollow poroelastic cylinders is not yet investigated. Therefore, in this paper the same is investigated in the framework of Biot’s theory. Non-dimensional frequency against the ratio of thickness to inner radius for different elastic foundations is studied for both axially symmetric and non-axially symmetric vibrations, each for a pervious and an impervious surface respectively, whose axis is in the direction of z-axis. Then the solid displacements $\vec{u}(u,v,w)$ and $\vec{U}(U,V,W)$ are solid and fluid displacements, $\epsilon$ and $\epsilon$ are the dilatations of solid and fluid respectively; the symbols $A, N, Q, R$ are all poroelastic constants; $\rho_{ij}$ are mass coefficients. The constitutive relations are

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Qe)\delta_{ij}, (i, j = 1, 2, 3),$$

$$s = Qe + Re.$$  

In eq. (2), $e_{ij}$’s are strain displacements $\sigma_{ij}$’s are solid stresses and the fluid pressure $s$, $\delta_{ij}$ is the well known Kronecker delta function.

3. Solution of the problem

Let $(r, \theta, z)$ be the cylindrical polar coordinates. Consider a thick-walled homogenous isotropic infinite poroelastic cylinder with inner and outer radii $r_1$ and $r_2$ respectively, whose axis is in the direction of z-axis. Then the solid displacements $\vec{u}(u,v,o)$ which can be evaluated from eq. (1) are

$$u(r, \theta, t) = -[C_1\xi_1(J_{n+1}(\xi_1r) - \frac{n}{\xi_1^2}J_n(\xi_1r)) + C_2\xi_2(Y_{n+1}(\xi_1r) - \frac{n}{\xi_2^2}Y_n(\xi_1r)) + C_3\xi_2(J_{n+1}(\xi_2r) - \frac{n}{\xi_2^2}J_n(\xi_2r)) + C_4\xi_2(Y_{n+1}(\xi_2r) - \frac{n}{\xi_2^2}Y_n(\xi_2r)) - A_1\xi_3J_n(\xi_3r) - B_1\xi_3Y_n(\xi_3r)]\cos(n\theta)e^{i\omega t},$$

$$v(r, \theta, t) = [C_1\frac{n}{\xi_1^2}J_n(\xi_1r) + C_2\frac{n}{\xi_2^2}Y_n(\xi_2r) + C_3\frac{n}{\xi_2^2}J_n(\xi_2r) + C_4\frac{n}{\xi_2^2}Y_n(\xi_2r) + A_1\xi_3J_n(\xi_3r) - B_1\xi_3Y_n(\xi_3r)]\sin(n\theta)e^{i\omega t}.$$  

(3)
In eq. (3) $C_1, C_2, C_3, C_4, A_1, B_1$ are all arbitrary constants, $\omega$ is the frequency of wave, $n$ is the number of waves and $\xi_i = \frac{i \pi}{R}, i = 1, 2, 3$. Here $V_i (i = 1, 2, 3)$ are the dilatational wave velocities of first and second kind and shear wave velocity, respectively. By substituting the displacements functions, $u, v$ in eq. (2), the relevant stresses are

$$\sigma_{rr} + s - Ku - G\Delta u = (M_{11}(r)C_1 + M_{12}(r)C_2 + M_{13}(r)C_3 + M_{14}(r)C_4 + M_{15}(r)A_1 + M_{16}(r)B_1)\sin(n\theta)e^{i\omega t},$$

$$\sigma_{r\theta} = (M_{21}(r)C_1 + M_{22}(r)C_2 + M_{23}(r)C_3 + M_{24}(r)C_4 + M_{25}(r)A_1 + M_{26}(r)B_1)\sin(n\theta)e^{i\omega t},$$

$$s = (M_{31}(r)C_1 + M_{32}(r)C_2 + M_{33}(r)C_3 + M_{34}(r)C_4)\sin(n\theta)e^{i\omega t},$$

$$\frac{\partial s}{\partial r} = (N_{31}(r)C_1 + N_{32}(r)C_2 + N_{33}(r)C_3 + N_{34}(r)C_4)\sin(n\theta)e^{i\omega t}.$$

(4)

Where the coefficients $M_{ij}$ and $N_{ij}$ are

$M_{11}(r) = 2N\left(\frac{n(n-1)}{r^2} - \xi_i^2\right) - (A + Q)\xi_i^2 + (Q + R)\xi_i^2\xi_1^2 - \frac{Kn}{r}J_n(\xi_1r) + (\frac{2N\xi_1}{r} + K\xi_1)J_{n+1}(\xi_1r),$

$M_{15}(r) = \frac{2nN\xi_i}{r}J_{n+1}(\xi_1r) + (\frac{2n(1-n)}{r^2} - \frac{Kn}{r})J_n(\xi_3r),$

$M_{21}(r) = \frac{2Nn(n-1)}{r}J_n(\xi_1r) - \frac{2nN\xi_i}{r}J_{n+1}(\xi_1r),$

$M_{25}(r) = N\left(\frac{2n(1-n)}{r} + \xi_1^2\right)J_n(\xi_3r) - \frac{2n\xi_i}{r}J_{n+1}(\xi_3r),$

$M_{31}(r) = (R\delta_1^2 - Q)\xi_1^2J_n(\xi_1r),$

$N_{31}(r) = \frac{R\delta_1^2 - Q}{r}\xi_3^2J_n(\xi_1r) + (Q - R\delta_1^2)\xi_1^2J_{n+1}(\xi_1r),$

$M_{12}(r) \text{ is similar expression as } M_{11}(r) \text{ with } J_n \text{ and } J_{n+1} \text{ replaced by } Y_n \text{ and } Y_{n+1} \text{ respectively},$

$M_{13}(r) \text{ is similar expression as } M_{11}(r) \text{ with } \delta_1^2 \text{ and } \xi_1 \text{ replaced by } \delta_2^2 \text{ and } \xi_2 \text{ respectively},$

$M_{14}(r) \text{ is similar expression as } M_{11}(r) \text{ with } \delta_2^2 \text{, } \xi_1, J_n \text{ and } J_{n+1} \text{ replaced by } \delta_2^2, \xi_2, Y_n \text{ and } Y_{n+1} \text{ respectively},$

$M_{16}(r) \text{ is similar expression as } M_{15}(r) \text{ with } J_n \text{ and } J_{n+1} \text{ replaced by } Y_n \text{ and } Y_{n+1} \text{ respectively},$

$M_{22}(r) \text{ is similar expression as } M_{21}(r) \text{ with } J_n \text{ and } J_{n+1} \text{ replaced by } Y_n \text{ and } Y_{n+1} \text{ respectively},$

$M_{23}(r) \text{ is similar expression as } M_{21}(r) \text{ with } \xi_1 \text{ replaced by } \xi_2,$

$M_{24}(r) \text{ is similar expression as } M_{21}(r) \text{ with } J_n, J_{n+1} \text{ and } \xi_1 \text{ replaced by } Y_n, Y_{n+1} \text{ and } \xi_2 \text{ respectively},$

$M_{26}(r) \text{ is similar expression as } M_{25}(r) \text{ with } J_n, J_{n+1} \text{ replaced by } Y_n \text{ and } Y_{n+1} \text{ respectively},$

$M_{32}(r) \text{ is similar expression as } M_{31}(r) \text{ with } J_n \text{ replaced by } Y_n,$

$M_{33}(r) \text{ is similar expression as } M_{31}(r) \text{ with } \delta_1^2 \text{ and } \xi_1 \text{ replaced by } \delta_2^2 \text{ and } \xi_2 \text{ respectively},$

$M_{34}(r) \text{ is similar expression as } M_{31}(r) \text{ with } \delta_1^2, \xi_1, \text{ and } J_n \text{ replaced by } \delta_2^2, \xi_2, Y_n \text{ respectively,}$
$N_{32}(r)=$ similar expression as $N_{31}(r)$ with $J_n$ and $J_{n+1}$ replaced by $Y_n$ and $Y_{n+1}$ respectively,

$N_{33}(r)=$ similar expression as $N_{31}(r)$ with $\delta_1^2$, $\xi_1$ replaced by $\delta_2^2$, and $\xi_2$, respectively,

$N_{34}(r)=$ similar expression as $N_{31}(r)$ with $\delta_1^2$, $\xi_1$, $J_n$ and $J_{n+1}$ replaced $\delta_2^2$, $\xi_2$, $Y_n$ and $Y_{n+1}$ respectively.

In the above, $\delta_1^2$ and $\delta_2^2$ are

$$\delta_1^2 = \frac{(Rm_{11} - Qm_{12} - V_1^{-2}(PR - Q^2))}{Rm_{12} - Qm_{22}},$$

$$\delta_2^2 = \text{similar expression as } \delta_1^2 \text{ with } V_1^{-2} \text{ replaced by } V_2^{-2},$$

and $m_{11} = \rho_{11} - i\omega^{-1}, m_{12} = \rho_{12} + i\omega^{-1}$, $m_{22} = \rho_{22} - i\omega^{-1}$.

4. Boundary conditions and frequency equation

The boundary conditions for free vibrations in case of a pervious surface $r = r_1$ and $r = r_2$ is

$$\sigma_{rr} + s - Ku - G\Delta u = \sigma_{r\theta} = s = 0,$$  \hspace{1cm} (5)

The boundary conditions for free vibrations in case of a impervious surface $r = r_1$ and $r = r_2$ is

$$\sigma_{rr} + s - Ku - G\Delta u = \sigma_{r\theta} = \frac{\partial^2 u}{\partial r^2} = 0.$$  \hspace{1cm} (6)

In eq. (5) and (6), $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2}$, $K$ is the foundation modulus. If shear modulus $G$ goes to zero, the Pasternak foundation will reduce to Wrinkler foundation [13]. Eq. (5) and (6) gives a system of six homogeneous equations in $C_1, C_2, C_3, C_4, A_1, B_1$ each for a pervious surface and an impervious surface. A nontrivial solution can be obtained if the determinant of coefficient vanishes. Accordingly, the frequency equation in the case of pervious surface is

$$\begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) \\
M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) \\
M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & 0 & 0 \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) \\
M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) \\
M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & 0 & 0
\end{vmatrix} = 0. \hspace{1cm} (7)
$$

In the case of an impervious surface, the frequency equation is

$$\begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) \\
M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) \\
N_{31}(r_1) & N_{32}(r_1) & N_{33}(r_1) & N_{34}(r_1) & 0 & 0 \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) \\
M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) \\
N_{31}(r_2) & N_{32}(r_2) & N_{33}(r_2) & N_{34}(r_2) & 0 & 0
\end{vmatrix} = 0. \hspace{1cm} (8)
$$

5. Non-axially symmetric vibrations

If $n$ is non-zero, the dilatational and shear vibrations are coupled as indicated by the frequency eqs (7) and (8). Using the eq. (7), we obtain the frequency equation of pervious surface in the following

$$|A_{ij}| = 0, \hspace{0.5cm} (i, j = 1, 2, 3, 4, 5, 6).$$  \hspace{1cm} (9)
Arguing in the similar lines, using eq. (8), we obtain the frequency equation of an impervious surface in the following form

\[ |B_{ij}| = 0, \quad (i, j = 1, 2, 3, 4, 5, 6). \quad (10)\]

### 6. Axially symmetric vibrations

If we set \( n = 0 \), then the frequency eq. (7) for pervious surface degenerates into the product of two determinants each of second and fourth order respectively. The second determinant corresponds to shear vibrations and fourth determinant corresponds to extensional vibrations.

#### 6.1. Shear vibrations

The frequency equation of shear vibrations for both pervious and impervious can be obtained from eq. (7) and (8) in the following form

\[ \begin{vmatrix}
M_{25}(r_1) & M_{26}(r_1) \\
M_{25}(r_2) & M_{26}(r_2)
\end{vmatrix} = 0. \quad (11)\]

Eq. (11), corresponds to zero dilatational potential and is independent of nature of surface. By substituting \( M_{ij}(r) \) into eq. (11), and using recurrence relations [14] the frequency equation simplifies to

\[ J_2(\xi_3 r_1)Y_2(\xi_3 r_2) - J_2(\xi_3 r_2)Y_2(\xi_3 r_1) = 0. \quad (12)\]

#### 6.2. Extensional vibrations

The frequency equation of extensional vibrations for pervious surface can be obtained from eq. (7) is

\[ \begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) \\
M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) \\
M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2)
\end{vmatrix} = 0. \quad (13)\]

Similarly, it can be seen the frequency eq. (8) for an impervious surface yields the product of two determinants each of second and fourth order respectively. The second order determinant is same as in eq. (11), while the fourth order determinant yields the extensional vibrations of an impervious surface is

\[ \begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) \\
N_{31}(r_1) & N_{32}(r_1) & N_{33}(r_1) & N_{34}(r_1) \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) \\
N_{31}(r_2) & N_{32}(r_2) & N_{33}(r_2) & N_{34}(r_2)
\end{vmatrix} = 0. \quad (14)\]

### 7. Numerical results

For the sake of numerical work, the dissipative coefficient \( b \) is taken zero and hence we obtained only real frequency \( \Omega \). To analyze the frequency equations, it is convenient to introduce the following non-dimensional parameters.
\[ a_1 = \frac{p}{H_1}, \quad a_2 = \frac{Q}{H_1}, \quad a_3 = \frac{R}{H_1}, \quad a_4 = \frac{N}{H_1}, \]
\[ d_1 = \frac{\rho_1}{\rho}, \quad d_2 = \frac{\rho_2}{\rho}, \quad d_3 = \frac{\rho_2}{\rho}, \quad d_4 = \frac{\rho_1}{\rho}, \]
\[ x = \left(\frac{V_1}{V_1}\right)^2, \quad y = \left(\frac{V_2}{V_2}\right)^2, \quad z = \left(\frac{V_3}{V_3}\right)^2, \]
\[ b_1 = \frac{P_1}{P_1}, \quad b_2 = \frac{Q_1}{Q_1}, \quad b_3 = \frac{R_1}{R_1}, \quad b_4 = \frac{N_1}{N_1}, \]
\[ g_1 = \frac{(\rho_1)_{1}}{\rho}, \quad g_2 = \frac{(\rho_2)_{1}}{\rho}, \quad g_3 = \frac{(\rho_2)_{1}}{\rho}, \quad m = \frac{C}{h}, \]
\[ \bar{x}_1 = \left(\frac{V_0}{V_1}\right)_1^2, \quad \bar{y}_1 = \left(\frac{V_0}{V_2}\right)_1^2, \quad \bar{z}_1 = \left(\frac{V_0}{V_3}\right)_1^2. \]

In eq. (15), \(C\) is the phase velocity, \(\Omega\) is the non-dimensional frequency, \(C_0\) and \(V_0\) are references velocities \(C_0^2 = \frac{N_1}{\rho_1}, V_0^2 = \frac{H_1}{\rho_1}\) and \(m\) is the non-dimensional phase velocity, \(\rho_1 = (\rho_1)_{1} + 2(\rho_2)_{1} + (\rho_2)_{1}, H_1 = P_1 + 2Q_1 + R_1\). Let \(g = \frac{\omega}{\rho}\) so that and \(\frac{h}{r_1} = g - 1\) and \(\Omega = \frac{\omega h}{\rho}\). In all the above subscript '1' stand for the quantities related to inner cylinder. Employing the non-dimensional quantities in the frequency equation we obtain an implicit relation between frequency and ratio of wall thickness to inner radius. Non-dimensional frequency is calculated for two types of composite cylinders, namely composite cylinder-I and composite cylinder-II, for each pervious and impervious surface using numerical process bisection method performed in MATLAB. The composite cylinder consists of two cylinders one cylinder is made up of sandstone saturated with kerosene[15], and the other is made up of sandstone saturated with water[16], while in composite cylinder-II the materials are reversed. The physical parameters of composite cylinders are given in Table 1. Figures 1-8 plots the non-dimensional frequency against the ratio of thickness to inner cylinder pertaining to elastic foundations 10, 50. From the figures 1-8, it is observed that frequency of cylinder-I is more than the frequency of cylinder-II. From figures 1, 3 it is clear that frequency is greater for impervious surface than that of pervious surface in the case of cylinder-I. Also the frequency is steady and constant beyond the ratio of thickness to inner radius 7 for pervious surface. From figures 2, 4 it is observed that frequency is greater for pervious surface than that of an impervious surface in the case of cylinder-II. Also, the frequency is steady and constant beyond the ratio of thickness to inner radius 7 for pervious surface and frequency is constant beyond 11 for an impervious surface. From figures 5, 7 it is seen that frequency is more for pervious surface than that of an impervious surface in the case of cylinder-I. Also, from figure 7 it is clear that frequency is steady and constant beyond the ratio of thickness to inner radius 11 for pervious surface and frequency is constant beyond 7 for an impervious surface. The trend is reversed in the cases of cylinders this is due to the influence of elastic foundation present in materials.
Figure 1. Non-axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at $K = 10$ (composite cylinder-I).

Figure 2. Non-axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 10$ (composite cylinder-II).

Figure 3. Non-axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 50$ (composite cylinder-I).

Figure 4. Non-axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 50$ (composite cylinder-II).

Figure 5. Axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation $K = 10$ (composite cylinder-I).

Figure 6. Axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 10$ (composite cylinder-II).
Figure 7. Axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 50$ (composite cylinder-I).

Figure 8. Axially symmetric vibrations: variation of frequency with $\frac{h}{r_1}$ at elastic foundation at $K = 50$ (composite cylinder-II).

Table 1. Material parameters

| Material parameters | cylinder-I | cylinder-II |
|---------------------|------------|-------------|
| $a_1$               | 0.445      | 1.819       |
| $a_2$               | 0.034      | 0.011       |
| $a_3$               | 0.015      | 0.054       |
| $a_4$               | 0.123      | 0.780       |
| $d_1$               | 0.887      | 0.891       |
| $d_2$               | 0.001      | 0           |
| $d_3$               | 0.1        | 0.124       |
| $\bar{x}$           | 1.863      | 0.489       |
| $\bar{y}$           | 8.884      | 2.330       |
| $\bar{z}$           | 7.183      | 1.142       |
| $b_1$               | 0.96       | 0.843       |
| $b_2$               | 0.006      | 0.065       |
| $b_3$               | 0.028      | 0.028       |
| $b_4$               | 0.142      | 0.234       |
| $g_1$               | 0.877      | 0.901       |
| $g_2$               | 0          | -0.001      |
| $g_3$               | 0.123      | 0.101       |
| $\bar{x}_1$         | 0.913      | 0.999       |
| $\bar{y}_1$         | 4.347      | 4.763       |
| $\bar{z}_1$         | 2.129      | 3.851       |

8. Conclusion

Effect of elastic foundations on the vibrations of thick-walled hollow poroelastic cylinder are investigated in the framework of Biot’s theory. Non dimensional frequency against ratio of thickness to inner radius is computed for two types of materials. In the case of non-axially symmetric vibrations, extensional and shear vibrations are coupled. The frequency of impervious surface values are greater than that of pervious surface in the case of cylinder-I. The frequency of pervious surface values are greater than that of an impervious surface in the case of cylinder-II. In the case of axially symmetric vibrations, the extensional and shear vibrations exist uncoupled. The frequency of pervious surface values are greater than that of an impervious surface in case of cylinder-I. The frequency of impervious surface values are greater than that of pervious surface in case of cylinder-II. This kind of analysis can be made if pertinent materials are available.
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