Improved cosmological constraints on the curvature and equation of state of dark energy

Nana Pan1, Yungui Gong1, Yun Chen2 and Zong-Hong Zhu2

1 College of Mathematics and Physics, Chongqing University of Posts and Telecommunications, Chongqing 400065, People’s Republic of China
2 Department of Astronomy, Beijing Normal University, Beijing 100875, People’s Republic of China

E-mail: pannn@cqupt.edu.cn, gongyg@cqupt.edu.cn and zhuzh@bnu.edu.cn

Received 5 March 2010, in final form 17 May 2010
Published 18 June 2010
Online at stacks.iop.org/CQG/27/155015

Abstract
We apply the Constitution compilation of 397 supernova Ia, the baryon acoustic oscillation measurements including the $A$ parameter, the distance ratio and the radial data, the five-year Wilkinson microwave anisotropy probe and the Hubble parameter data to study the geometry of the Universe and the property of dark energy by using the popular Chevallier–Polarski–Linder and Jassal–Bagla–Padmanabhan parameterizations. We compare the simple $\chi^2$ method of joined contour estimation and the Monte Carlo Markov chain method, and find that it is necessary to make the marginalized analysis on the error estimation. The probabilities of $\Omega_K$ and $w_0$ in the Chevallier–Polarski–Linder model are skew distributions, and the marginalized $1\sigma$ errors are $\Omega_m = 0.279^{+0.015}_{-0.008}$, $\Omega_k = 0.005^{+0.006}_{-0.011}$, $w_0 = -1.05^{+0.23}_{-0.06}$, and $w_a = 0.5^{+0.3}_{-1.5}$. For the Jassal–Bagla–Padmanabhan model, the marginalized $1\sigma$ errors are $\Omega_m = 0.281^{+0.015}_{-0.01}$, $\Omega_K = 0.000^{+0.007}_{-0.006}$, $w_0 = -0.96^{+0.25}_{-0.18}$ and $w_a = -0.6^{+1.9}_{-1.6}$. The equation of state parameter $w(z)$ of dark energy is negative in the redshift range $0 \leq z \leq 2$ at more than $3\sigma$ level. The flat $\Lambda$CDM model is consistent with the current observational data at the $1\sigma$ level.

PACS numbers: 98.80.−k, 98.80.Es

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The accelerating expansion of the Universe was first discovered by the type Ia supernova (SN Ia) observations [1, 2]. The phenomena of acceleration could be explained straightforwardly by introducing an exotic source of matter with negative pressure, the so-called dark energy, which dominates the total matter content of the Universe at the present epoch and causes the expansion to accelerate. During the past decade, in addition to the simple cosmological
constant model, a lot of dynamical dark energy models, such as the quintessence [3], phantom [4], k-essence [5], tachyon [6], quintom [7], h-essence [8], Chaplygin gas [9], holographic dark energy [10], $f(R)$ [11], Dvali–Gabadadze–Porrati [12] models, etc, have been proposed. Although a lot of efforts have been made to understand the driving force of the accelerating expansion and the property of dark energy, whether dark energy is dynamical or not is still an open question. Therefore, it is necessary to study the nature of dark energy such as the evolutions of its energy density and state.

Apart from phenomenological models, another effective approach to study dark energy is through the observational data. Recently, based on the popular Chevallier–Polarski–Linder (CPL) parametrization of dark energy [13], it was found that the flat $\Lambda$CDM model is inconsistent with the current data at more than 1σ level [14, 15]. In [14], it was suggested that the cosmic acceleration is slowing down from $z \sim 0.3$. In [15], it was claimed that dark energy suddenly emerged at redshift $z \sim 0.3$. Furthermore, possible oscillating behavior of dark energy was found in [16]. However, no evidence for dark energy dynamics was found in [17–19]. It was argued that the systematics in different data sets heavily affected the fitting results from observational data [18, 19]. To further study the dynamics of dark energy, it is necessary to apply more complimentary observational data. In this paper, we combine the Constitution sample of 397 SN Ia data [20], the model independent $A$ parameter from the baryon acoustic oscillation (BAO) measurements [21], the two BAO distance ratios at $z = 0.2$ and $z = 0.35$ [22], the radial BAO measurements at $z = 0.24$ and $z = 0.43$ [23], the five-year Wilkinson microwave anisotropy probe data (WMAP5) [24] and the Hubble parameter $H(z)$ data [25, 26] to probe the geometry of the Universe and the nature of dark energy by using the CPL and Jassal–Bagla–Padmanabhan (JBP) [27] parameterizations. We first use the simple $\chi^2$ method of joined contour estimation to obtain the constraints on the model parameters. However, the simple $\chi^2$ method by fixing other parameters at their best fit values has some drawbacks because we neglect the correlation effects between the parameters and the degeneracy between parameters was not considered. When the parameters are strongly correlated, the errors of some parameters will be under-estimated if we fix the other parameters at their best fit values. So we also apply the Monte Carlo Markov chain (MCMC) method to obtain the marginalized errors of the model parameters. The advantage of the MCMC method is that it considers the correlations between the model parameters and the result is more reliable.

The paper is organized as follows. In section 2, we present the SN Ia data [20], the BAO data [21–23], the WMAP5 data [24] and the $H(z)$ data, and all the formulas related with these data. In section 3, We use the $\Lambda$CDM model as an example to show how to apply the data to constrain cosmological models. In section 4, we use the CPL model to study the geometry of the Universe and the property of dark energy. The JBP model is used to probe the geometry of the Universe and the evolution of dark energy in section 5. We conclude the paper in section 6.

2. Fitting procedure

To use the Constitution compilation of 397 SN Ia data [20], we minimize

$$\chi^2 = \sum_{i=1}^{397} \frac{[\mu(z_i) - \mu_{\text{obs}}(z_i)]^2}{\sigma_i^2},$$

where the extinction-corrected distance modulus $\mu(z) = 5 \log_{10}[d_L(z)/\text{Mpc}] + 25$, $\sigma_i$ is the total uncertainty in the SN Ia observation, the luminosity distance $d_L(z)$ is

$$d_L(z) = \frac{1 + z}{H_0}\frac{\sin n \left(\sqrt{\Omega_k} \int_0^z \frac{dz}{E(z)}\right)}{\sqrt{\Omega_k}},$$

where $H_0$ is the Hubble constant and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$. The parameter $\Omega_m$ is the matter density parameter, $\Omega_\Lambda$ is the dark energy density parameter, and $\Omega_k$ is the curvature density parameter.
the dimensionless Hubble parameter $E(z) = H(z)/H_0$ and

$$\frac{\sin(\sqrt{|3\Omega_k|}x)}{\sqrt{|3\Omega_k|}} = \begin{cases} \sin(\sqrt{|3\Omega_k|}x)/\sqrt{|3\Omega_k|}, & \text{if } \Omega_k < 0, \\ x, & \text{if } \Omega_k = 0, \\ \sinh(\sqrt{|3\Omega_k|}x)/\sqrt{|3\Omega_k|}, & \text{if } \Omega_k > 0. \end{cases}$$ (3)

Due to the arbitrary normalization of the luminosity distance, the nuisance parameter $h$ in the SN Ia data is not the observed Hubble constant. So we marginalize the nuisance parameter $h$ with a flat prior, after the marginalization, we get [28]

$$\chi^2_{SN}(p) = \sum_{i=1}^{3} \frac{\alpha_i^2}{\sigma_i^2} - \left( \frac{\sum_i \alpha_i / \sigma_i^2 - \ln 10/3}{\sum_i 1/\sigma_i^2} \right)^2 - 2 \ln \left( \frac{\ln 10}{5} \sqrt{\sum_i \sigma_i^2} \right).$$ (4)

where $\alpha_i = \mu_{\text{obs}}(z_i) - 25 - 5 \log_{10}[H_0 d_L(z_i)]$, and $p$ denotes the fitting parameters in the model. When using the SN Ia data, the radiation term can be neglected because its contribution is negligible.

In addition to the Constitution SN Ia data, we use the BAO distance measurements from the oscillations in the distribution of galaxies. From the BAO observation of the galaxy power spectra, Percival et al measured the distance ratio

$$d_z^v = \frac{r_s(z_d)}{D_V(z)}$$ (5)

at two redshifts $z = 0.2$ and $z = 0.35$ to be $d_{0.2}^v = 0.1905 \pm 0.0061$ and $d_{0.35}^v = 0.1097 \pm 0.0036$, respectively [22]. Here the effective distance is

$$D_V(z) = \left[ \frac{d^2_z(z)}{(1+z)^2 H(z)} \right]^{1/3}.$$ (6)

the drag redshift $z_d$ is fitted as [29]

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}],$$ (7)

$$b_1 = 0.313(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}], \quad b_2 = 0.238(\Omega_m h^2)^{0.223},$$ (8)

the comoving sound horizon is

$$r_s(z) = \int_z^\infty c_s(z) \frac{dz}{E(z)},$$ (9)

the sound speed $c_s(z) = 1/\sqrt{3[1 + \bar{R}_b/(1+z)]}$ and $\bar{R}_b = 3(\Omega_b h^2)/(4 \times 2.469 \times 10^{-5})$. To use these BAO data, we calculate

$$\chi^2_{\text{BAO2}}(p, \Omega_b h^2, h) = \Delta x_i \text{Cov}^{-1}(x_i, x_j) \Delta x_j,$$ (10)

where $x_i = (d_{z=0.2}, d_{z=0.35})$, $\Delta x_i = x_i - x_i^{\text{obs}}$, and $\text{Cov}(x_i, x_j)$ is the covariance matrix for the two parameters $d_{0.2}$ and $d_{0.35}$ [22]. Besides the model parameters $p$, we need to add two more parameters $\Omega_b h^2$ and $\Omega_m h^2$ when we use the BAO data. In [22], they used the priors of $\Omega_b h^2 = 0.02273 \pm 0.00061$ and $\Omega_m h^2 = 0.1099 \pm 0.0063$.

From the measurement of the radial (line-of-sight) BAO scale in the galaxy power spectra, the cosmological parameters were determined from the measured values of

$$\Delta z_{\text{BAO}}(z) = \frac{H(z) r_s(z_d)}{c}$$ (11)
at two redshifts $z = 0.24$ and $z = 0.43$, which are $\Delta z_{\text{BAO}}(z = 0.24) = 0.0407 \pm 0.0011$ and $\Delta z_{\text{BAO}}(z = 0.43) = 0.0442 \pm 0.0015$, respectively [23]. Therefore, we add $\chi^2$ with

$$
\chi^2_{\text{BAO}}(p, \Omega_b h^2, h) = \left( \frac{\Delta z_{\text{BAO}}(0.24) - 0.0407}{0.0011} \right)^2 + \left( \frac{\Delta z_{\text{BAO}}(0.43) - 0.0442}{0.0015} \right)^2.
$$

(12)

When we add these BAO data to the fitting, we also need to use the parameters $\Omega_b h^2$ and $\Omega_m h^2$. The values $\Omega_b h^2 = 0.02273 \pm 0.0066$ and $\Omega_m h^2 = 0.1329 \pm 0.0064$ were used in [23].

In addition to the above two BAO data sets, the BAO $A$ parameter [30] is usually used. The BAO $A$ parameter is defined as

$$
A = \frac{\sqrt{\Omega_m}}{z_{\text{BAO}}} \frac{H_0 D_V(z = 0.35)}{z = 0.35} = \frac{\sqrt{\Omega_m}}{0.35} \left( \frac{1}{E(0.35)} \right) \frac{1}{|\Omega_k|} \sin^2 \left( \sqrt{|\Omega_k|} \int_0^{0.35} \frac{dz}{E(z)} \right)^{1/3},
$$

(13)

and it was measured to be $A = 0.493 \pm 0.017$ [21], so we add $\chi^2$ with

$$
\chi^2_{\text{BAO},A}(p) = \left( A - 0.439 \right)^2 / 0.017^2.
$$

(14)

Note that the BAO $A$ parameter depends on the model parameters $p$ only; it does not depend on the baryon density $\Omega_b h^2$ and the Hubble constant $h$. Although the radiation density depends on $h$, the contribution to the Hubble parameter $E(z)$ is negligible at the redshift $z = 0.35$, so we can neglect the radiation component when we use the BAO $A$ data.

Both the SN Ia and the BAO data measure the distance up to redshift $z < 2$; we need to consider the distance at high redshift in order to determine the property of dark energy. Therefore, we implement the WMAP5 data. To use the full WMAP5 data, we need to add some more parameters which depend on inflationary models, and this will limit our ability to constrain dark energy models. So we only use the WMAP5 measurements of the derived quantities, such as the shift parameter $R(z^*)$, the acoustic scale $l_A(z^*)$ and the decoupling redshift $z^*$, to obtain

$$
\chi^2_{\text{CMB}} = \Delta x_i \cdot \text{Cov}_{2^{-1}}(x_i, x_j) \Delta x_j,
$$

(15)

where the three parameters $x_i = (R(z^*), l_A(z^*), z^*)$, $\Delta x_i = x_i - x_i^{\text{obs}}$ and $\text{Cov}(x_i, x_j)$ is the covariance matrix for the three parameters [24]. The shift parameter $R$ is expressed as

$$
R(z^*) = \frac{\sqrt{|\Omega_k|}}{\sqrt{|\Omega_k|}} \sin \left( \frac{\sqrt{|\Omega_k|}}{\int_0^{z^*} \frac{dz}{E(z)}} \right) = 1.710 \pm 0.019.
$$

(16)

The acoustic scale $l_A$ is

$$
l_A(z^*) = \frac{\pi d_L(z^*)}{(1 + z^*) r_s(z^*)} = 302.1 \pm 0.86,
$$

(17)

and the decoupling redshift $z^*$ is fitted by [31]

$$
z^* = 1048 \left[ 1 + 0.00124 (\Omega_b h^2)^{-0.738} \right] \left[ 1 + g_1 (\Omega_m h^2)^{g_2} \right] = 1090.04 \pm 0.93,
$$

(18)

$$
g_1 = \frac{0.0783 (\Omega_b h^2)^{-0.238}}{1 + 39.5 (\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1 (\Omega_b h^2)^{1.81}}.
$$

(19)

In [24], it was found that $\Omega_m h^2 = 0.2273 \pm 0.0062$ and $h = 0.719^{+0.026}_{-0.027}$.

The SN Ia data, the BAO data and the WMAP5 data use the distance measurement to determine the cosmological parameters. To get the distance scale, we need to integrate the equation of state parameter $w(z)$ twice, so the process of double integration smooths out the variation of the equation of state parameter $w(z)$ of dark energy. To alleviate the problem, we
add the Hubble parameter $H(z)$ data. The Hubble parameter $H(z)$ at nine different redshifts was obtained from the differential ages of passively evolving galaxies in [25], and three more Hubble parameter data $H(z = 0.24) = 76.69 \pm 2.32$, $H(z = 0.34) = 83.8 \pm 2.96$ and $H(z = 0.43) = 86.45 \pm 3.27$ were determined recently in [26]. Therefore, we add these $H(z)$ data to $\chi^2$:

$$
\chi^2_h(p, h) = \sum_{i=1}^{12} \frac{(H(z_i) - H_{\text{obs}}(z_i))^2}{\sigma_{hi}^2},
$$

(20)

where $\sigma_{hi}$ is the 1σ uncertainty in the $H(z)$ data. The model parameters $p$ are determined by applying the maximum likelihood method of $\chi^2$ fit. We use the publicly available MINUIT code for minimization and contour calculation [32]. Basically, the model parameters are determined by minimizing

$$
\chi^2 = \chi^2_{\text{in}} + \chi^2_{\text{BAO}1} + \chi^2_{\text{BAO}2} + \chi^2_{\text{CMB}} + \chi^2_{h}.
$$

(21)

For the convenience of numerical fitting, we take $\Omega_b h^2 = 0.02273$ determined from the WMAP5 data [24]. For the Hubble constant $h$, two different values were observed. The Hubble key project found that $h = 0.72 \pm 0.08$ [33], and recently Riess et al obtained $h = 0.742 \pm 0.036$ by using a differential distance ladder method [34]. To account for the uncertainty of the Hubble constant, we treat it as a free parameter and then fix it at its best fit value.

3. $\Lambda$CDM model with curvature

For the cosmological constant, the equation of state parameter $w = p/\rho = -1$, and the energy density $\rho_\Lambda$ is a constant. In a curved $\Lambda$CDM model, the curvature term $k \neq 0$, ordinary pressureless dust matter, radiation and the cosmological constant contribute to the total energy. The Friedmann equation is

$$
E(z) = \frac{H(z)}{H_0} = [\Omega_k (1 + z)^2 + \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 + \Omega_\Lambda]^{1/2},
$$

(22)

where the Hubble constant $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = (8\pi G \rho_m)/(3H_0^2)$ is the current matter component, the current radiation component $\Omega_r = (8\pi G \rho_r)/(3H_0^2) = 4.1736 \times 10^{-5}h^{-2}$ [24], the current curvature component $\Omega_k = -k/(a_0^2 H_0^2)$ and $\Omega_\Lambda = 1 - \Omega_m - \Omega_k - \Omega_r$. In this model, we have two parameters $p = (\Omega_m, \Omega_k)$ and one nuisance parameter $h$. For the fitting to the SN Ia data, the contribution to the Hubble expansion from the radiation is negligible and we usually neglect the radiation term.

By fitting the $\Lambda$CDM model to the above observational data, we get $\chi^2 = 482.13$, $\Omega_m = 0.280^{+0.014}_{-0.011}$ and $\Omega_\Lambda = 0.001 \pm 0.005$. By fixing the nuisance parameter $h$ at its best fit value $h = 0.702$, we obtain the contours of $\Omega_m$ and $\Omega_k$. The joint contour plots of $\Omega_m$ and $\Omega_k$ or $\Omega_\Lambda$ are shown in figure 1. Compared with WMAP5 fitting results [24], we find that the current data make a little improvement on the constraints of $\Omega_m$ and $\Omega_k$. The improvement is due to more SN Ia and BAO data in addition to the $H(z)$ data. The result tells us that the flat $\Lambda$CDM model is consistent with current observational data at the 1σ level.

4. CPL parametrization with curvature

In order to investigate the equation of state of dark energy for a curved Universe by observational data, in this section we study the popular CPL parametrization [13]

$$
w(z) = w_0 + \frac{w_z z}{1 + z}.
$$

(23)
Figure 1. The 1σ, 2σ and 3σ joint contour plots of Ω_m and Ω_Λ (Ω_k) for the curved ΛCDM model. The straight line in the left panel denotes the flat ΛCDM model.

Figure 2. The 1σ, 2σ and 3σ contour plots of w_0 and w_a for the curved CPL parametrization. ‘+’ denotes the point corresponding to the ΛCDM model. (a) Joint contours of w_0 and w_a by fixing the other parameters at their best fit values. (b) Marginalized contours of w_0 and w_a obtained from the MCMC method.

The dimensionless Hubble parameter including the contributions from dark energy, ordinary pressureless dust matter and radiation is

\[ E(z) = \frac{H(z)}{H_0} = (\Omega_k (1 + z)^2 + \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 + \Omega_{DE})^{1/2}, \]

where the dimensionless dark energy density is

\[ \Omega_{DE}(z) = (1 - \Omega_m - \Omega_k - \Omega_r) \times (1 + z)^3 (1 + w_0 + w_a) \exp[-3 w_a z/(1 + z)]. \]

In this model, we have four model parameters \( p = (\Omega_m, \Omega_k, w_0, w_a) \). By applying the observational data discussed in the previous section to the CPL model, we are able to get the observational constraint on the model parameters \( p = (\Omega_m, \Omega_k, w_0, w_a) \). The best fit is \( \chi^2 = 481.64, \Omega_m = 0.278, \Omega_k = 0.006, w_0 = -1.04, w_a = 0.42 \) and \( h = 0.70 \). By fixing the parameters \( \Omega_m, \Omega_k \) and \( h \) at their best fit values, we obtain the contours of \( w_0 \) and \( w_a \) and they are shown in figure 2(a). From figure 2(a), we see that the ΛCDM model is
The marginalized distributions of the CPL model parameters are shown in (a). The solid lines are marginalized probabilities and the dotted lines represent mean likelihoods. (b) Reconstructed evolution of $w(z)$ and the shaded areas are the $1\sigma$, $2\sigma$ and $3\sigma$ errors.

Figure 3.

Table 1. The marginalized estimates of the model parameters in CPL and JBP models.

| Model | $\chi^2$ | $\Omega_m$ | $\Omega_k$ | $w_0$ | $w_a$ |
|-------|---------|-----------|-----------|-------|-------|
| CPL   | 481.27  | 0.279$^{+0.015}_{-0.008}$ | 0.005$^{+0.006}_{-0.011}$ | $-1.05^{+0.23}_{-0.06}$ | $-1.5^{+0.7}_{-1.9}$ |
| JBP   | 481.46  | 0.281$^{+0.015}_{-0.012}$ | 0.000$^{+0.007}_{-0.006}$ | $-0.96^{+0.25}_{-0.18}$ | $-0.6^{+1.9}_{-1.6}$ |

excluded by the observational data at more than $3\sigma$ level. As we discussed in the previous section, we see that the $\Lambda$CDM model is consistent with the observational data. The totally different conclusions suggest that the simple $\chi^2$ error estimation by fixing other parameters at their best fit values has some drawbacks because we neglect the correlation effects of the other parameters. The degeneracy between parameters was not considered in the above method. When the parameters are strongly correlated, the error of some parameters will be underestimated if we fix the other parameters at their best fit values. To verify this point, we apply the MCMC method to constrain the parameter space $p$ and the nuisance parameters $h$ and $\Omega_ch^2$. Our MCMC code [28] is based on the publicly available package COSMOMC [35]. By using the MCMC method, we get $\chi^2 = 481.27$; the marginalized $1\sigma$ errors are $\Omega_m = 0.279^{+0.015}_{-0.008}$, $\Omega_k = 0.005^{+0.006}_{-0.011}$, $w_0 = -1.05^{+0.23}_{-0.06}$ and $w_a = 0.5^{+0.7}_{-1.9}$. These results are summarized in table 1. The marginalized $1\sigma$, $2\sigma$ and $3\sigma$ contour plots of $w_0$ and $w_a$ are shown in figure 2(b). From figures 2(a) and 2(b), we find that the marginalized $1\sigma$ contour obtained by using the MCMC method includes the $3\sigma$ contour in figure 2(a). From figure 2(b), we see that the $\Lambda$CDM model is consistent with the CPL model at $1\sigma$ level.

The marginalized distributions of the model parameters are shown in figure 3(a). The solid lines are marginalized probabilities and the dotted lines represent mean likelihoods. From figure 3(a), we find that the likelihood of $\Omega_k$ has a local maximum around $\Omega_k \sim 0.02$. 
Even we take $\Omega_k = 0.02$, the value of $\chi^2$ is not far from the minimum value of $\chi^2$. Due to the degeneracy between model parameters, if we fix the other model parameters at their best fit values, then the joined contours of $w_0$ and $w_a$ are under-estimated, and the conclusion drawn from the under-estimated contours is not reliable. The results in figures 2(b) and 3(a) verify this point. By using the marginalized contours of $w_0$ and $w_a$, we reconstruct the evolution of $w(z)$ in figure 3(b). From figure 3(b), we find that $w(z) < 0$ at more than $3\sigma$ confidence level up to redshift $z = 2$, and the $\Lambda$CDM model is consistent with the CPL model at the 1$\sigma$ level.

To account for the correlations between model parameters, we need to use the marginalized probability. To see whether this happens only for the CPL model, we analyze the JBP model in the next section.

5. JBP parametrization with curvature

In this section, we consider the JBP parametrization [27] for dark energy with the equation of state in the form below:

$$w(z) = w_0 + \frac{w_a z}{(1 + z)^2}. \tag{26}$$

The corresponding dimensionless dark energy density is then

$$\Omega_{DE}(z) = (1 - \Omega_m - \Omega_k - \Omega_{\gamma}) \times (1 + z)^{(1 + w_0) \exp[3w_a z^2/2(1 + z)^2]}. \tag{27}$$

In this model, we also have four parameters $p = (\Omega_m, \Omega_k, w_0, w_a)$. We first use the simple $\chi^2$ method to fit the model. The best fit is $\chi^2 = 481.84$, $\Omega_m = 0.281$, $\Omega_k = 0.0015$, $w_0 = -0.97$, $w_a = -0.03$ and $h = 0.70$. By fixing the parameters $\Omega_m$, $\Omega_k$ and $h$ at their best fit values, we obtain the contours of $w_0$ and $w_a$ and they are shown by the solid lines in figure 4(a). Unlike the CPL model, the $\Lambda$CDM model is consistent with the JBP model at the $1\sigma$ level. To verify this conclusion, we also apply the MCMC method to the JBP model. By using the MCMC method, we get $\chi^2 = 481.46$; the marginalized $1\sigma$ errors are $\Omega_m = 0.281^{+0.015}_{-0.001}$, $\Omega_k = 0.000^{+0.007}_{-0.006}$, $w_0 = -0.96^{+0.24}_{-0.18}$ and $w_a = -0.6^{+1.9}_{-1.6}$. These results are summarized in table 1. The marginalized $1\sigma$, $2\sigma$ and $3\sigma$ contour plots of $w_0$ and $w_a$ are shown in figure 4(b).
From figures 4(a) and (b), we see that the contours of $w_0$ and $w_a$ are consistent although the constraints from the MCMC method are a little larger because we consider the correlations among all the parameters in the MCMC method. The $\Lambda$CDM model is also consistent with the JBP model at the 1$\sigma$ level.

The marginalized distributions of the model parameters $p$ are shown in figure 5(a). The solid lines are marginalized probabilities and the dotted lines represent mean likelihoods. From figure 5(a), we find that the probability distributions of the parameters are more or less Gaussian. By using the marginalized contours of $w_0$ and $w_a$, we reconstruct the evolution of $w(z)$ in figure 5(b). From figure 5(b), we find that $w(z) < -0.2$ at more than 3$\sigma$ confidence level up to redshift $z = 2$, and the $\Lambda$CDM model is consistent with the JBP model at the 1$\sigma$ level.

6. Conclusions

Applying the simple $\chi^2$ method, we fitted the CPL and JBP models to the combined SN Ia, BAO, WMAP5 and $H(z)$ data, and obtained the constraint on the property of dark energy. In both CPL and JBP models, there are four parameters $p = (\Omega_m, \Omega_k, w_0, w_a)$. When we apply the BAO and WMAP5 data, we need to add two more parameters $\Omega_b h^2$ and $h$. If we make the joint error analysis, we have six parameters and it will be hard to get a good joint constraint on all these parameters. Therefore, we take $\Omega_b h^2 = 0.02273$, and find out the best fit values of the parameters $p$ and $h$ which minimize $\chi^2$; then we fix the parameters $\Omega_m, \Omega_k$ and $h$ at their best fit values to obtain the joint constraints on $w_0$ and $w_a$. For the CPL model, the contours of $w_0$ and $w_a$ (see figure 2(a)) show that the $\Lambda$CDM model is excluded at more than 3$\sigma$ level. The JBP model is consistent with the $\Lambda$CDM model at the 1$\sigma$ level. Since we get the contours of $w_0$ and $w_a$ by fixing the other parameters at their best fit values, we neglect the correlation...
effects of the parameters and the conclusion based on this method may not be reliable. To confirm this, we use the MCMC method to analyze the CPL and JBP models and obtain the marginalized probabilities of the parameters. For the CPL model, the probability distributions of $\Omega_m$ and $w_0$ are skew distributions, and the marginalized 1σ errors are $\Omega_m = 0.279_{-0.008}^{+0.015}$, $\Omega_k = 0.005_{-0.011}^{+0.006}$, $w_0 = -1.05_{-0.06}^{+0.23}$, and $w_a = 0.5_{-0.3}^{+1.3}$. In the CPL model, the probability distributions of $\Omega_k$ has a local maximum in addition to a global maximum. The uncertainties in $\Omega_k$ and the degeneracies between $\Omega_k$, $w_0$ and $w_a$ lead to under-estimation of the error contours of $\Omega_k$ and $w_a$ if we fix $\Omega_k$ at its global best fit value, and the wrong conclusion that the $\Lambda$CDM model is excluded at more than 3σ level. However, this does not happen for the JBP model. For the JBP model, the parameters have Gaussian distributions, and the marginalized 1σ errors are $\Omega_m = 0.281_{-0.011}^{+0.015}$, $\Omega_k = 0.000_{-0.006}^{+0.007}$, $w_0 = -0.96_{-0.18}^{+0.25}$ and $w_a = -0.6_{-1.6}^{+1.9}$.

In summary, in addition to use the usual SN Ia, BAO A or BAO distance ratio, and WMAP data, we also use the radial BAO measurements and the $H(z)$ data to fit the CPL and JBP models. We find that the equation of state parameter of dark energy $w(z) < 0$ at more than 3σ level in the redshift range $0 < z < 2$, and the flat $\Lambda$CDM model is consistent with the current observational data at the 1σ level. Furthermore, we find that we need to perform the marginalized analysis to estimate the errors of the model parameters.

Acknowledgments

NP was partially supported by the project A2008-58 of the Scientific Research Foundation of Chongqing University of Posts and Telecommunications, and the NNSF of China under grant no 10935013, the National Basic Research Program of China under grant no 2009BA4050. ZZ was partially supported by the NNSF Distinguished Young Scholar project under grant no 2009CB833004, and the Natural Science Foundation Project of CQ CSTC under grant no 2009BA4050. YG was partially supported by the NNSF key project of China under grant no 10825313, and the National Basic Research Program of China under grant no 2010CB833004, and the Scientific Research Foundation of Chongqing University of Posts and Telecommunications, and the NNSF of China under grant no 10947178. NP was partially supported by the project A2008-58 of the Scientific Research Foundation of Chongqing University of Posts and Telecommunications, and the NNSF of China under grant no 10935013, the National Basic Research Program of China under grant no 2009BA4050. ZZ was partially supported by the NNSF Distinguished Young Scholar project under grant no 10825313, and the National Basic Research Program of China under grant no 2007CB815401.

References

[1] Riess A G et al 1998 Astron. J. 116 1009
[2] Perlmutter S et al 1999 Astrophys. J. 517 565
[3] Wetterich C 1988 Nucl. Phys. B 302 688
[4] Caldwell R R 2002 Phys. Lett. B 545 23
[5] Ardmedariz-Picon C, Damour T and Mukhanov V 1999 Phys. Lett. B 458 209
[6] Padmanabhan T 2002 Phys. Rev. D 66 021301
[7] Feng B, Wang X L and Zhang X M 2005 Phys. Lett. B 607 35
[8] Wei H, Cai R G and Zeng D F 2005 Phys. Rev. D 72 123507
[9] Kamenshchik A Y, Moschella U and Pasquier V 2001 Phys. Lett. B 511 265
[10] Li M 2004 Phys. Lett. B 603 1
[11] Capozziello S 2002 Int. J. Mod. Phys. D 11 483
[12] Nojiri S and Odintsov S D 2003 Phys. Rev. D 68 123512
[13] Hsu S D H 2004 Phys. Lett. B 123512
[14] Hu W and Sawicki I 2007 Phys. Rev. D 76 064004
[15] Caldwell R R, Dave R and Steinhardt P J 1998 Phys. Rev. Lett. 80 1582
[16] Caldwell R R 2002 Phys. Lett. B 545 23
[17] Armendariz-Picon C, Damour T and Mukhanov V 1999 Phys. Lett. B 458 209
[18] Padmanabhan T 2002 Phys. Rev. D 66 021301
[19] Bagla J S, Jassal H K and Padmanabhan T 2003 Phys. Rev. D 67 063504
[20] Bento M C, Bertolami O and Sen A A 2002 Phys. Rev. D 66 043507
[21] Bento M C, Bertolami O and Sen A A 2002 Phys. Rev. D 66 043507
[22] Bento M C, Bertolami O and Sen A A 2002 Phys. Rev. D 66 043507
[23] Bento M C, Bertolami O and Sen A A 2002 Phys. Rev. D 66 043507
[14] Shafieloo A, Sahni V and Starobinsky A A 2009 Phys. Rev. D 80 101301
[15] Huang Q G, Li M, Li X D and Wang S 2009 Phys. Rev. D 80 083515
[16] Cai R G, Su Q P and Zhang H-B 2010 J. Cosmol. Astropart. Phys. JCAP04(2010)012
[17] Serra P et al 2009 Phys. Rev. D 80 121302
[18] Gong Y G, Cai R G, Chen Y and Zhu Z-H 2010 J. Cosmol. Astropart. Phys. JCAP01(2010)019
[19] Gong Y G, Wang B and Cai R G 2010 J. Cosmol. Astropart. Phys. JCAP04(2010)019
[20] Hicken M et al 2009 Astrophys. J. 700 1097
[21] Reid B A et al 2010 Mon. Not. R. Astron. Soc. 404 60
[22] Percival W J et al 2010 Mon. Not. R. Astron. Soc. 401 2148
[23] Gaztañaga E, Miquel R and Sánchez E 2009 Phys. Rev. Lett. 103 091302
[24] Komatsu E et al 2009 Astrophys. J. Suppl. Ser. 180 330
[25] Simon J, Verde L and Jimenez R 2005 Phys. Rev. D 71 123001
[26] Gaztañaga E, Cabré A and Hui L 2009 Mon. Not. R. Astron. Soc. 399 1663
[27] Jassal H K, Bagla J S and Padmanabhan T 2005 Mon. Not. Roy. Astron. Soc. 356 L11
[28] Gong Y G, Wu Q and Wang A 2008 Astrophys. J. 681 27
[29] Eisenstein D J and Hu W 1998 Astrophys. J. 496 605
[30] Eisenstein D J et al 2005 Astrophys. J. 633 560
[31] Hu W and Sugiyama N 1996 Astrophys. J. 471 542
[32] James F and Roos M 1975 Comput. Phys. Commun. 10 343
[33] Freedman W L et al 2001 Astrophys. J. 553 47
[34] Riess A G et al 2009 Astrophys. J. 699 539
[35] Lewis A and Bridle S 2002 Phys. Rev. D 66 103511