A new type of supersymmetric twistors and higher spin chiral multiplets

A.A. Zheltukhin\textsuperscript{a,b}

\textsuperscript{a} Kharkov Institute of Physics and Technology, 61108 Kharkov, Ukraine
\textsuperscript{b} Institute of Theoretical Physics, University of Stockholm
SE-10691, AlbaNova, Stockholm, Sweden

Abstract

A new type of supersymmetric twistors is proposed and they are called \(\theta\)-twistors versus the supertwistors. The \(\theta\)-twistor is a triple of spinors including the spinor superspace coordinate \(\theta\) instead of the Grassmannian scalar in the supertwistor triple. The superspace of the \(\theta\)-twistors is closed under the superconformal group transformations except the (super)conformal boosts. Using the \(\theta\)-twistors in physics preserves the auxiliary field \(F\) in the chiral \((0,\frac{1}{2})\) supermultiplet contrarily to the supertwistor description. Moreover, it yields an infinite chain of higher spin chiral supermultiplets \((\frac{1}{2},1), (1,\frac{3}{2}), (\frac{3}{2},2), \ldots, (S,S+\frac{1}{2})\) generalizing the scalar massless supermultiplet.

1 Introduction

The twistor approach \cite{1} and its supersymmetric generalization \cite{2}, \cite{3} were recently effectively applied for the calculation of multigluon amplitudes in Yang-Mills theory \cite{4}, \cite{5}, \cite{6}, \cite{7}. An efficiency of the twistor methods in supersymmetric fields theories and superstrings appeals to further studying the (super)twistors and their role in the description of superspace structures and superfields \cite{8}, \cite{9}, \cite{10}, \cite{11}, \cite{12}, \cite{13}, \cite{14}, \cite{15}.

Here we discuss the supertwistor origin and derive them starting from the known Cartan differential form \cite{16} in the chiral superspace. Then the supertwistor \cite{2} arises as a triple including two commuting spinors and the anticommuting scalar \(\eta = \nu^\alpha \theta_\alpha\), where \(\nu^\alpha\) is commuting Penrose spinor. Thus, the spinor superspace coordinate \(\theta\) is presented in supertwistor by its projection breaking a democracy among the supertwistor components. It leads to the elimination of auxiliary \(F\)-field from the chiral supermultiplet resulting in closure of supersymmetry transformations on the mass-shell of the spinor field \cite{2}. Then the problem appears how to preserve auxiliary fields during the Penrose twistor supersymmetrization. This problem is studied here on the example of \(D = 4\), \(N = 1\) supersymmetry assuming the supersymmetrization process to preserve all \(\theta\) components in a twistor superspace. To this end we introduce a new supersymmetric triple named the \(\theta\)-twistor and constituted from \textit{three} spinors which form a nonlinear representation of the supersymmetry. We reveal that both the \(\theta\)-twistors and supertwistors appear as the general solutions of two different supersymmetric and Lorentz covariant constraints, admissible in the chiral superspace extended by the Penrose spinors \(\nu\) and \(\bar{\nu}\), which generalize the standard chirality.
constraint. The symmetry properties of the $\theta$-twistor superspace are investigated and its closure under the superconformal group transformations except the (super)conformal boosts is established. Thereupon the Penrose contour integration is used to get the superfields independent of $\nu^a$ and $\bar{\nu}^{-\dot{a}}$. These superfields describe the scalar supermultiplet $(0, \frac{1}{2})$ including the desired $F$-field and yield an infinite chain of massless higher spin chiral supermultiplets $(\frac{1}{2}, 1), (1, \frac{3}{2}), (\frac{3}{2}, 2), \ldots, (S, S + \frac{1}{2})$ generalizing the scalar supermultiplet. The superfields realize the $R$-symmetry transformations and may be used for the construction of renormalizable Lagrangians. The studied $D = 4, N = 1$ example is straightforward generalized to the case of $SU(N)$ internal symmetry and higher dimensions $D = 2, 3, 4(mod 8)$ by analogy with the supertwistors [2], [3], [17].

2 Supersymmetry

In agreement with [18] we accept $D = 4 N = 1$ supersymmetry transformations in the form

$$\delta \theta_\alpha = \varepsilon_\alpha, \quad \delta x_{a\dot{a}} = 2i(\varepsilon_\alpha \bar{\theta}_{\dot{a}} - \theta_\alpha \bar{\varepsilon}_{\dot{a}}).$$

(1)

The supercharges $Q_\alpha$ and $\bar{Q}_{\dot{a}}$ induced by (1) are presented by the differential operators

$$Q_\alpha = \frac{\partial}{\partial \theta_\alpha} + 2i\bar{\theta}_{\dot{a}}\bar{\partial}^{\dot{a}\alpha}, \quad \bar{Q}_{\dot{a}} \equiv -(Q^{\alpha})^* = \frac{\partial}{\partial \bar{\theta}_{\dot{a}}} + 2i\theta_\alpha \bar{\partial}^{\dot{a}\alpha}, \quad [Q_\alpha, \bar{Q}_{\dot{a}}]^+_+ = 4i\bar{\partial}^{\dot{a}\alpha},$$

(2)

where $\bar{\partial}^{\dot{a}\alpha} \equiv \frac{\partial}{\partial \bar{\theta}_{\dot{a}}}$. The correspondent supersymmetric odd derivatives $D, \bar{D}$ are

$$D^\alpha = \frac{\partial}{\partial \theta_\alpha} - 2i\bar{\theta}_{\dot{a}}\bar{\partial}^{\dot{a}\alpha}, \quad \bar{D}\dot{a} \equiv -(D^\alpha)^* = \frac{\partial}{\partial \bar{\theta}_{\dot{a}}} - 2i\theta_\alpha \bar{\partial}^{\dot{a}\alpha}, \quad [D^\alpha, \bar{D}\dot{a}] = -4i\bar{\partial}^{\dot{a}\alpha},$$

(3)

$$[Q_\alpha, D^\beta]^+_+ = [Q^\alpha, D^\beta]^+_+ = [\bar{Q}_{\dot{a}}, D^\beta]^+_+ = [\bar{Q}_{\dot{a}}, \bar{D}\dot{a}]^+_+ = 0.$$  

The supersymmetric even Cartan differential one-form invariant under (1) is

$$\omega_{a\dot{a}} = dx_{a\dot{a}} - 2i(\theta_\alpha d\bar{\theta}_{\dot{a}} - d\theta_\alpha \bar{\theta}_{\dot{a}}).$$

(4)

A chiral superfield $F$ is defined by the chirality constraint $\bar{D}\dot{a} F = 0$ which introduces the complex chiral coordinates $y_{a\dot{a}}$

$$y_{a\dot{a}} = x_{a\dot{a}} - 2i\theta_\alpha \bar{\theta}_{\dot{a}}$$

(5)

satisfying the conditions

$$D^\beta y_{a\dot{a}} = 0, \quad D^\beta y_{a\dot{a}} = -4i\delta^\beta_{\dot{a}} \theta_\alpha.$$  

(6)

In the complexified superspace $(y_{a\dot{a}}, \theta_\alpha, \bar{\theta}_{\dot{a}})$ the supersymmetric derivatives and supercharges are presented in the known form [15]

$$D^\alpha = \frac{\partial}{\partial \theta_\alpha} - 4i\bar{\theta}_{\dot{a}}\bar{\partial}^{\dot{a}\alpha}, \quad \bar{D}\dot{a} = \frac{\partial}{\partial \bar{\theta}_{\dot{a}}} + 4i\theta_\alpha \bar{\partial}^{\dot{a}\alpha},$$

(7)

where $\bar{\partial}^{\dot{a}\alpha} \equiv \frac{\partial}{\partial \bar{\theta}_{\dot{a}}}$. It yields a complex superfield $\Phi(y_{a\dot{a}}, \theta_\alpha)$ as the general solution of the chirality constraint. The chiral superspace $(y_{a\dot{a}}, \theta_\alpha)$ is closed under the transformations (1)

$$\delta y_{a\dot{a}} = -4i\delta^a_{\dot{a}} \bar{\varepsilon}_\alpha, \quad \delta \theta_\alpha = \varepsilon_\alpha$$

(8)

which preserve the invariant chiral Cartan one-form (1)

$$\omega_{a\dot{a}} = dy_{a\dot{a}} + 4idx_{a\dot{a}}.$$  

(9)

The form (9) is a suitable mathematical object to reveal the supertwistors introduced in [2].
3 The supertwistors

By analogy with the Penrose twistors [1] the notion of the $D = 4, N = 1$ supertwistor [2] is based on the extension of the standard superspace $(x_{\alpha\dot{\alpha}}, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ by the even Weyl spinors $\nu_\alpha$ and $\bar{\nu}_{\dot{\alpha}}$ which are inert under the supersymmetry transformations (10)

$$\delta \nu_\alpha = 0, \quad \delta \bar{\nu}_{\dot{\alpha}} = 0.$$ \hspace{1cm} (10)

To introduce the supertwistors it is suitable to start from the complex superspace $(y_{\alpha\dot{\alpha}}, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ and the invariant Cartan form (9) of which contraction with $\nu_\alpha^{\dot{\alpha}}$ yields the supersymmetric spinor one-form $s_{\dot{\alpha}}$.

$$s_{\dot{\alpha}} \equiv \nu_\alpha^{\dot{\alpha}} y_{\alpha\dot{\alpha}} = d\bar{q}_{\dot{\alpha}} - d\nu_{\alpha\dot{\alpha}} - 4i\bar{\eta}\bar{\theta}_{\dot{\alpha}} d\eta,$$ \hspace{1cm} (11)

yields the supersymmetric spinor one-form $s_{\dot{\alpha}}$. The even spinor $\bar{q}_{\dot{\alpha}}$ in (11) is

$$\bar{q}_{\dot{\alpha}} \equiv \nu_\alpha^{\dot{\alpha}} x_{\alpha\dot{\alpha}} = \nu_\alpha^{\dot{\alpha}} x_{\alpha\dot{\alpha}} - 2i\bar{\eta}\theta_{\dot{\alpha}},$$ \hspace{1cm} (12)

and the odd scalar variable $\eta$ together with the c.c. coordinate $\bar{y}_{\alpha\dot{\alpha}}$ are defined by

$$\eta \equiv \nu_\alpha^{\dot{\alpha}} \theta_\alpha, \quad \bar{y}_{\alpha\dot{\alpha}} \equiv (y_{\alpha\dot{\alpha}})^* = x_{\alpha\dot{\alpha}} + 2i\theta_\alpha \bar{\theta}_{\dot{\alpha}}.$$ \hspace{1cm} (13)

These new composite objects are characterized by the relations

$s_{\dot{\alpha}}^* s_{\dot{\alpha}} = q_{\dot{\alpha}}^* q_{\dot{\alpha}} = \eta^2 = 0.$ \hspace{1cm} (14)

The contraction of the one-form $s_{\dot{\alpha}}$ (11) with $\bar{v}^{\dot{\alpha}}$ results in the supersymmetric scalar form

$s \equiv \bar{v}^{\dot{\alpha}} s_{\dot{\alpha}} = \bar{v}^{\dot{\alpha}} d\bar{q}_{\dot{\alpha}} - 4i\bar{\eta} d\eta - d\nu_{\alpha\dot{\alpha}}^{\dot{\alpha}} q_{\dot{\alpha}},$ \hspace{1cm} (15)

where $q_{\dot{\alpha}}$ is the complex conjugate spinor for $\bar{q}_{\dot{\alpha}}$

$$q_{\dot{\alpha}} \equiv (q_{\dot{\alpha}})^* = \bar{v}^{\dot{\alpha}} \bar{y}_{\alpha\dot{\alpha}} = \nu_\alpha^{\dot{\alpha}} x_{\alpha\dot{\alpha}} - 2i\bar{\eta}\theta_\alpha.$$ \hspace{1cm} (16)

We observe that the invariant differential form $s$ (15) may be rewritten as

$s \equiv (\nu_\alpha^{\dot{\alpha}} \bar{v}^{\dot{\alpha}}) = -iZ_A d\bar{Z}^A \equiv s(Z, d\bar{Z}),$ \hspace{1cm} (17)

where the triples $Z_A$ and $\bar{Z}^A$ are formed by the following composite coordinates

$$Z_A \equiv (-i q_{\dot{\alpha}}, \bar{v}^{\dot{\alpha}}, 2\bar{\eta}), \quad \bar{Z}^A \equiv (\nu_\alpha^{\dot{\alpha}}, i\bar{q}_{\dot{\alpha}}, 2\eta).$$ \hspace{1cm} (18)

The triples $Z_A$ and $\bar{Z}^A$ coincide with the supetwistor and its c.c. proposed in [2] as a supersymmetric generalization of the Penrose twistors. The space of $Z_A$ is a complex projective superspace because of the equivalency relation

$$Z_A \sim cZ_A,$$ \hspace{1cm} (19)

where the constant $c$ is a non-zero complex number rescaling the original spinor $\nu_\alpha$.

Next we note that the differential form $s(Z, d\bar{Z})$ (17) may be presented as

$$is = Z_A d\bar{Z}^A = ZC^A d\bar{Z}^A = Z_A C^{AB} (d\bar{Z}_B)^*,$$ \hspace{1cm} (20)

where

$$C^{AB} = \begin{pmatrix}
0 & \varepsilon_{\alpha\beta} & 0 \\
\varepsilon^{\dot{\alpha}\dot{\beta}} & 0 & 0 \\
0 & 0 & 1
\end{pmatrix},$$ \hspace{1cm} (20)
where $C$ is treated as a flat metric of the supertwistor space. We observe that $C$ may be used for the introduction of a global linear form

\begin{equation}
 s(Z, Z') = -\bar{s}(Z', \bar{Z}) \equiv -iZ_A \bar{Z}^A = -q_\alpha \nu^\alpha + \tilde{\nu}^\dot{\alpha} \tilde{q}_{\dot{\alpha}} - 4i\bar{\eta} \eta',
\end{equation}

on the supertwistor space after the substitution of $\bar{Z}^A$ for $d\bar{Z}^A$. The triple $\bar{Z}^A$ is the supertwistor, but with $\nu'$ substituted for $\nu$

\begin{equation}
 \bar{Z}^A \equiv (\nu'^\alpha, i\tilde{q}_{\dot{\alpha}}, 2\eta'), \quad \tilde{q}_{\dot{\alpha}} = \nu'^\alpha q_{\alpha\dot{\alpha}}, \quad \eta' = \nu'^\alpha \theta_\alpha.
\end{equation}

It is easy to check that the scalar form $s(Z, Z')$ is a supersymmetric null form

\begin{equation}
 s(Z, Z') \equiv -iZ_A \bar{Z}^A = 0
\end{equation}

which coincides with the bilinear form previously used in [2]. We see that the chiral superspace extension by an even spinor results in the appearance of the bilinear null form $s(Z, Z')$ associated with the triples $Z$ and $\bar{Z}'$ defining the supertwistor space. The global form $s(Z, Z')$ arises from the scalar Cartan differential form $s(Z, d\bar{Z})$ in the enlarged chiral superspace. It sharpens the role of chiral superspaces for the construction of supersymmetric twistors.

4 The superconformal symmetry

The supertwistor components form a linear complex representation of the supersymmetry as it follows from the transformation relations

\begin{equation}
 \delta \tilde{q}_{\dot{\alpha}} = -4i\bar{\eta} \tilde{\epsilon}_{\dot{\alpha}}, \quad \delta \eta = \nu^\alpha \epsilon_\alpha, \quad \delta \nu^\alpha = 0
\end{equation}

and their complex conjugate. The generators of the supersymmetry transformations are given by the differential operators

\begin{equation}
 Q^\alpha = 4i\bar{\eta} \frac{\partial}{\partial \tilde{q}_{\dot{\alpha}}} + \nu^\alpha \frac{\partial}{\partial \eta}, \quad \bar{Q}^\dot{\alpha} = -(Q^\alpha)^* = 4i\bar{\eta} \frac{\partial}{\partial q_{\alpha\dot{\alpha}}} + \tilde{\nu}^\dot{\alpha} \frac{\partial}{\partial \bar{\eta}},
\end{equation}

\begin{equation}
 \{Q^\alpha, \bar{Q}^\dot{\beta}\} = 4iP^{\beta\dot{\alpha}}, \quad P^{\beta\dot{\alpha}} \equiv \nu^\alpha \frac{\partial}{\partial \tilde{q}_{\dot{\beta}}} + \tilde{\nu}^\dot{\alpha} \frac{\partial}{\partial \tilde{q}_{\dot{\beta}}},
\end{equation}

where $P^{\beta\dot{\alpha}}$ is the hermitian generator of translations. One can observe the invariance of the bilinear form $s(Z, Z')$ under the supersymmetry transformations. Moreover, it was found in [4] that the superconformal symmetry $SU(2, 2|1)$, or $SU(2, 2|N)$ for the case of $N$-extended $D = 4$ supersymmetry, is the complete symmetry of $s(Z, Z')$. To prove this we firstly note the symmetry in the positions $q$ and $\nu$, as well as $\eta$ and $\bar{\eta}$ in the bilinear form $s(Z, Z')$. Then the interchange of these coordinates in the generators transforms them into the superconformal boost generators $S^\alpha, \bar{S}^\dot{\alpha}$

\begin{equation}
 S^\alpha = 4i\bar{\eta} \frac{\partial}{\partial \nu^\alpha} + q^\alpha \frac{\partial}{\partial \eta}, \quad \bar{S}^\dot{\alpha} \equiv -(S^\alpha)^* = 4i\bar{\eta} \frac{\partial}{\partial \bar{\nu}_{\dot{\alpha}}} + \bar{q}^\dot{\alpha} \frac{\partial}{\partial \bar{\eta}}.
\end{equation}

The superconformal boosts act as “square roots” of the special conformal generator $K^{\dot{\alpha}\alpha}$

\begin{equation}
 \{S^\alpha, \bar{S}^\dot{\beta}\} = 4iK^{\dot{\alpha}\alpha}, \quad K^{\dot{\alpha}\alpha} \equiv q^\alpha \frac{\partial}{\partial \nu^\beta} + \bar{q}^\dot{\beta} \frac{\partial}{\partial \nu_{\dot{\alpha}}},
\end{equation}

\begin{equation}
 [K^{\dot{\alpha}\alpha}, K^{\dot{\beta}\beta}] = \{S^\alpha, S^\beta\} = \{\bar{S}^\dot{\alpha}, \bar{S}^\dot{\beta}\} = 0.
\end{equation}
The correspondent transformations of the supertwistor components are given by the relations
\[ \delta \nu^\alpha = -4i \eta \xi^\alpha, \quad \delta \eta = \bar{q}^\dagger \xi, \quad \delta \bar{q}^\dagger = 0, \]
\[ \delta \nu^\alpha = -\kappa^\dagger q^\alpha, \quad \delta \eta = 0, \quad \delta \bar{q}^\dagger = 0 \] (28)
supplemented by their complex conjugate, where \( \xi^\alpha \) and \( \kappa^\dagger q^\alpha \) are parameters of the superconformal and conformal boosts. One can check the invariance of (21) under (28). The commutator of the translations (25) and the conformal boosts (27)
\[ [P^{\beta \gamma}, K^\alpha] = \varepsilon^{\alpha \beta} L^{\hat{\alpha} \hat{\beta}} + \varepsilon^{\hat{\alpha} \hat{\beta}} L^{\alpha \beta}, \]
\[ L^{\alpha \beta} = q^\alpha \frac{\partial}{\partial q^\beta} + \nu^\beta \frac{\partial}{\partial \nu^\alpha}, \quad \bar{L}^{\hat{\alpha} \hat{\beta}} = \bar{q}^{\dagger \hat{\alpha}} \frac{\partial}{\partial \bar{q}^{\dagger \hat{\beta}}} + \bar{\nu}^{\hat{\beta}} \frac{\partial}{\partial \bar{\nu}^{\dagger \hat{\alpha}}} \] (29)
closes by the generators \( L^{\alpha \beta} \) and \( \bar{L}^{\hat{\alpha} \hat{\beta}} \) satisfying by the commutation relations
\[ [L^{\alpha \beta}, L^\gamma] = \varepsilon^{\alpha \beta} L^{\gamma \delta} - \varepsilon^{\beta \gamma} L^{\alpha \delta}, \]
\[ [L^{\alpha \beta}, Q^\gamma] = \varepsilon^{\gamma \alpha} Q^\beta, \quad [\bar{L}^{\hat{\alpha} \hat{\beta}}, Q^\gamma] = 0, \]
\[ [\bar{L}^{\hat{\alpha} \hat{\beta}}, \bar{Q}^{\hat{\gamma}}] = \varepsilon^{\hat{\gamma} \hat{\alpha}} \bar{Q}^{\hat{\beta}}, \quad [L^{\alpha \beta}, \bar{Q}^{\hat{\gamma}}] = 0 \] (30)
and identified with the Lorentz rotations, dilatations and phase transformations presented by the generators \( J^{\alpha \beta}, \bar{J}^{\hat{\alpha} \hat{\beta}} \) and \( D, \bar{D} \)
\[ L^{\alpha \beta} \equiv J^{\alpha \beta} + \frac{1}{2} \varepsilon^{\alpha \beta} D, \quad \bar{L}^{\hat{\alpha} \hat{\beta}} \equiv \bar{J}^{\hat{\alpha} \hat{\beta}} + \frac{1}{2} \varepsilon^{\hat{\alpha} \hat{\beta}} \bar{D}, \]
\[ J^{\alpha \beta} \equiv \frac{1}{2}(L^{\alpha \beta} + L^{\beta \alpha}), \quad D \equiv q_\alpha \frac{\partial}{\partial q^\alpha} - \nu^\alpha \frac{\partial}{\partial \nu^\alpha}, \]
\[ \bar{L}^{\hat{\alpha} \hat{\beta}} \equiv \frac{1}{2}(\bar{L}^{\hat{\alpha} \hat{\beta}} + \bar{L}^{\hat{\beta} \hat{\alpha}}), \quad \bar{D} \equiv \bar{q}_{\hat{\alpha}} \frac{\partial}{\partial \bar{q}^{\dagger \hat{\alpha}}} - \bar{\nu}^{\dagger \hat{\alpha}} \frac{\partial}{\partial \bar{\nu}^{\dagger \hat{\alpha}}} \] (31)
which are the symmetries of the bilinear form too. The dilatation \( D_R \) and the phase rotation \( D_I \) generators are presented by the hermitian combinations of \( D \) and \( \bar{D} \)
\[ D_R = (D + \bar{D}), \quad D_I = i(D - \bar{D}) \] (32)
and produce correspondent finite transformations of the supertwistor components
\[ q_\beta' = e^{\varphi} q_\beta, \quad q_{\hat{\beta}}' = e^{\varphi} q_{\hat{\beta}}, \quad \bar{q}_\beta' = e^{-\varphi} \bar{q}_\beta, \quad \bar{q}_{\hat{\beta}}' = e^{-\varphi} \bar{q}_{\hat{\beta}}, \]
\[ \nu_{\beta}' = e^{-\varphi} \nu_{\beta}, \quad \bar{\nu}_{\hat{\beta}}' = e^{-\varphi} \bar{\nu}_{\hat{\beta}}, \quad \nu_{\hat{\beta}}' = e^{-\varphi} \nu_{\hat{\beta}}, \quad \bar{\nu}_{\beta}' = e^{-\varphi} \bar{\nu}_{\beta} \] (33)
where \( \varphi = \varphi_R + i \varphi_I \) is a complex parameter. Eqs. (33) results in the relation
\[ Z'_A = e^{i \varphi_I}(-i q_\alpha e^{\varphi_R}, \bar{\nu}_\beta e^{-\varphi_R}, 2\eta e^{-i \varphi_I}), \quad \bar{Z}'^A = e^{-i \varphi_I}(\nu_\alpha e^{-\varphi_R}, i \bar{q}_{\hat{\alpha}} e^{\varphi_R}, 2 \bar{\eta} e^{i \varphi_I}). \] (34)
proving the invariance of the bilinear form (21). The phase symmetry turns out to be trivial, because of the projectivity of the supertwistor space. To prove this observation we note that the anticommutator of the supercharge with the superconformal boost
\[ \{Q^\alpha, \bar{S}^\hat{\beta}\} = 0, \]
\[ \{Q^\alpha, S^\beta\} = 4i L^\beta^\alpha + 4\varepsilon^{\alpha \beta} U_5 \] (35)
yields one additional symmetry of the bilinear form described by the hermitian generator \( U_5 \)
\[ U_5 = i(\eta \frac{\partial}{\partial \eta} - \bar{\eta} \frac{\partial}{\partial \bar{\eta}}) \] (36)
which produces the phase rotations of the odd variables $\eta$ and $\bar{\eta}$

$$\eta' = e^{i\lambda} \eta, \quad \bar{\eta}' = e^{-i\lambda} \bar{\eta}. \quad (37)$$

The generator $U_5$ has non-zero commutation relations only with $Q^\beta, S^\beta$ and their c.c.

$$[U_5, Q^\beta] = -iQ^\beta, \quad [U_5, \bar{Q}^\beta] = i\bar{Q}^\beta, \quad [U_5, S^\beta] = iS^\beta, \quad [U_5, \bar{S}^\beta] = -i\bar{S}^\beta \quad (38)$$

and its appearance does not break the closure of superalgebra preserving the twistor bilinear form. Using the $U_5$ symmetry may compensate the phase factor in front of $\eta$ and $\bar{\eta}$ components of $Z'^A$ and $\bar{Z}'^A$ in (34) resulting in

$$Z'_A = e^{i\varphi I} Z_A, \quad Z'^A = e^{-i\varphi I} Z^A, \quad (39)$$

where the dilatation parameter $\varphi_R = 0$. This shows that the effect of the phase transformation associated with the generator $D_I$ is equivalent to the multiplication of the supertwistor $Z$ and $\bar{Z}$ by a phase factor. But, the $Z$-space is a projective space and it means that the phase symmetry (33) is excluded from the list of the bilinear form symmetries.

So, we get 15 real symmetries associated with the bosonic sector of the supertwistor space: $4(P) + 4(K) + 6(J, \bar{J}) + 1(D_R) = 15$. Next we observe that the pure phase part of (33) can be treated as the axial or $\gamma_5$ rotation of $\theta_\alpha$ and $\bar{\theta}_\dot{\alpha}$

$$\theta'_\beta = e^{i\kappa_5} \theta_\beta, \quad \bar{\theta}'_{\dot{\beta}} = e^{-i\kappa_5} \bar{\theta}_{\dot{\beta}}. \quad (40)$$

In the addition to the above mentioned 15 bosonic symmetries there are 8 real symmetries associated with the fermionic sector of the supertwistor space: $4(Q, \bar{Q}) + 4(S, \bar{S}) = 8$, that complete the list of symmetries of superconformal group $SU(2, 2|1)$.

Let us remark that the action of the superconformal and conformal symmetries on the coordinates of the chiral superspace $(y_{\beta\dot{\alpha}}, \theta_{\beta\dot{\alpha}})$ is realized by the nonlinear transformations

$$\delta y_{\alpha\dot{\alpha}} \equiv (\xi_\alpha S^{\alpha} + \bar{\xi}_\dot{\alpha} S^{\dot{\alpha}}) y_{\alpha\dot{\alpha}} = 4i\theta_\alpha (\xi^\beta y_{\beta\dot{\alpha}}), \quad \delta \theta_\alpha = y_{\alpha\beta} \bar{\xi}^\beta - 4i\theta_\alpha (\xi^\beta \theta_{\beta\dot{\alpha}}), \quad (41)$$

$$\delta y_{\alpha\dot{\alpha}} \equiv (\kappa_{\beta\dot{\beta}} K^{\beta\dot{\beta}}) y_{\alpha\dot{\alpha}} = -(y_{\kappa\dot{\kappa}})_{\alpha\dot{\alpha}}, \quad \delta \theta_\alpha = - (y_{\kappa\dot{\kappa}})_{\alpha\dot{\alpha}},$$

where $(y_{\kappa\dot{\kappa}})_{\alpha\dot{\alpha}} = y_{\alpha\beta} \kappa^{\beta\dot{\beta}} y_{\beta\dot{\alpha}}$ and $(y_{\kappa\dot{\kappa}})_{\alpha\dot{\alpha}} = -y_{\alpha\beta} \kappa^{\beta\dot{\beta}} \theta_{\beta\dot{\alpha}}$, and $S, \bar{S}, K$ are the (super)conformal generators. So, we conclude that the extension of the chiral superspace by $\nu$ and $\bar{\nu}$ transforms the nonlinear action of the superconformal group to the linear action realized by the composite coordinates in the enlarged chiral superspace belonging to the supertwistor components.

In the next section we present another composite triple in the enlarged chiral superspace.

5 Supersymmetric $\theta$-twistors versus supertwistors

Having a well defined base for the supertwistor conception we are ready now to introduce an alternative supersymmetric triple in the enlarged chiral superspace who we shall call $\theta$-twistor and that realizes a nonlinear representation of the supersymmetry. To this end we observe that the spinor differential one-form $s_\alpha$ which is complex conjugate of $\bar{s}_{\dot{\alpha}}$ (11)

$$s_\alpha = (\bar{s}_{\dot{\alpha}})^* = \omega_{\alpha\dot{\alpha}} \bar{\nu}^{\dot{\alpha}} = dq_\alpha - y_{\alpha\dot{\alpha}} d\nu^{\dot{\alpha}} - 4i\theta_\alpha d\bar{\eta}, \quad (42)$$
may be presented in the equivalent form

\[ s_\alpha = dl_\alpha - y_{a\dot{\alpha}} d\bar{v}^{\dot{\alpha}} + 4i\theta_\alpha \bar{\eta}, \]  

(43)

where the composite even spinor \( l_\alpha \) is defined by the relation

\[ l_\alpha \equiv y_{a\dot{\alpha}} \bar{v}^{\dot{\alpha}} = x_{\alpha\dot{\alpha}} \bar{v}^{\dot{\alpha}} - 2i\theta_\alpha \bar{\eta}. \]  

(44)

The spinor \( l_\alpha \) is connected with the spinor supertwistor component \( q_\alpha \) by the shift

\[ l_\alpha = q_\alpha - 4i\theta_\alpha \bar{\eta}. \]  

(45)

The shift provides new transformation properties of \( l_\alpha \) under the supersymmetry

\[ \delta l_\alpha = -4i\theta_\alpha (\bar{v}^{\dot{\beta}} \bar{\epsilon}_{\dot{\beta}}), \quad \delta \theta_\alpha = \epsilon_\alpha, \quad \delta \bar{\nu}_{\dot{\alpha}} = 0 \]  

(46)

supplemented by their c.c. relations. In contrast to the linear transformations (24), realized by the supertwistor components, the new triple \( \Xi_A \) and its c.c. \( \bar{\Xi}^A \)

\[ \Xi_A \equiv (-il_\alpha, \bar{\nu}^{\dot{\alpha}}, \theta^\alpha), \quad \bar{\Xi}^A \equiv (\nu^\alpha, i\bar{l}_{\dot{\alpha}}, \bar{\theta}^{\dot{\alpha}}) \]  

(47)

yield a nonlinear representations of the supersymmetry. One can see that the triple \( \Xi_A \) is obtained from the supertwistor \( Z_A \) by the substitution \( q_\alpha \rightarrow l_\alpha \) and \( 2\bar{\eta} \rightarrow \theta^\alpha \). Then the substitution of \( l_\alpha \) for \( q_\alpha \) in the linear form (21) with using Eq. (45) results in the relation

\[ s(Z, Z) \equiv -iZ_A Z^A = -i\Xi_A \circ \bar{\Xi}^A \equiv \tilde{s}(\Xi, \Xi'), \]  

(48)

where the nonlinear form \( \tilde{s}(\Xi, \Xi') \) in the \( \Xi \)-triple superspace (17) is defined as

\[ \tilde{s}(\Xi, \Xi') = -i\Xi_A \circ \bar{\Xi}^A \equiv -l_\alpha \nu^\alpha + \bar{v}^{\dot{\alpha}} \bar{l}_{\dot{\alpha}} - 4i\eta' \bar{\eta}. \]  

(49)

The relation (49) differs from (21) by the changes of the sign in front of the term \( \eta' \bar{\eta} \) and the substitution of \( l \) for \( q \). The form (19) becomes a nonlinear in the \( \Xi \)-triple superspace (17)

\[ \tilde{s}(\Xi, \Xi') = -i\Xi_A \circ \bar{\Xi}^A \equiv -l_\alpha \nu^\alpha + \bar{v}^{\dot{\alpha}} \bar{l}_{\dot{\alpha}} - ig_{a\bar{\alpha}} \theta^\alpha \bar{\theta}^{\dot{\alpha}} = 0, \quad g_{a\bar{\alpha}} \equiv 4\nu'_{\dot{\alpha}} \bar{\nu}_{\dot{\alpha}} \]  

(50)

where we observe the “metric” factor \( g_{a\bar{\alpha}} \) appearance in front of the odd component contribution. By analogy with (20) the nonlinear form \( \Xi_A \circ \bar{\Xi}^A \) may be rewritten as

\[ \Xi_A \circ \bar{\Xi}^A = \Xi G \Xi^A \equiv \Xi_A G^{AB}(\Xi_B^*)^*, \]  

\[ G^{AB} = \begin{pmatrix} 0 & \frac{\epsilon_{\alpha\beta}}{\epsilon_{\dot{\alpha}\dot{\beta}}} & 0 \\ \frac{\epsilon_{\dot{\alpha}\dot{\beta}}}{\epsilon_{\alpha\beta}} & 0 & 0 \\ 0 & 0 & 4\nu'_{\dot{\alpha}} \bar{\nu}_{\dot{\alpha}} \end{pmatrix}. \]  

(51)

where \( G \) plays the role of a non-flat “metric” in the \( \Xi \)-triple space. Trying to consider (51) as a natural nonlinear form in the \( \Xi \)-superspace one could resume that the transition to \( \Xi \)-triples might be interpreted as a curving the supertwistor space described by \( g_{a\bar{\alpha}} \). This effective interpretation is partially supported by the relation between the supersymmetric one-form (17) and the nonlinear differential form \( \Xi G d\Xi^+ \)

\[ \Xi G d\Xi^+ = ZCdZ^+ - 4(d\nu^\alpha \theta_\alpha) \bar{\eta} \]  

(52)
which may be rewritten in an equivalent form as
\[ ZCdZ^+ = \Xi(Gd + A)\Xi^+, \quad A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 4d\nu_\alpha \bar{\nu}_\beta \end{pmatrix}. \] (53)

This representation introduces the “connection” \( A \) needed for the construction of the covariant differential in the “curved” \( \Xi \)-superspace. Another remark is that the odd components of the \( \theta \)-twistors break the projective character of its bosonic components. In this connection it is important to know whether the superconformal symmetry survives the transition to the \( \theta \)-twistors from the supertwistors. We shall consider this problem below.

### 6 Symmetries of the \( \theta \)-twistor superspace

To study the properties of the \( \Xi \)-space under the superconformal symmetry we need to present its generators in terms of the \( \theta \)-twistor components. The generators of the supersymmetry transformations (54) just have the required form
\[ Q^\alpha = \frac{\partial}{\partial l^\alpha} + 4i\nu^\alpha(\bar{\theta}_\beta \frac{\partial}{\partial \bar{q}_\beta}), \quad \bar{Q}^{\dot{\alpha}} \equiv -(Q^\alpha)^* = \frac{\partial}{\partial \bar{l}^{\dot{\alpha}}} - 4i\bar{\nu}^\dot{\alpha}({\theta}_\beta \frac{\partial}{\partial q_\beta}) \] (54)

with the anticommutator of \( Q \) and \( \bar{Q} \) closed by the translation generator \( P^{\dot{\alpha}\alpha} \)
\[ \{Q^\alpha, \bar{Q}^{\dot{\alpha}}\} = 4iP^{\dot{\alpha}\alpha}, \quad \{P^{\dot{\alpha}\alpha}, P^{\dot{\beta}\beta}\} = 0, \quad \{Q^\alpha, P^{\dot{\beta}\beta}\} = 0. \] (55)

The generators (55) preserve the Lorentz invariant form (51) and transform the \( \Xi \)-space into itself. The Lorentz generators \( J^{\alpha\beta}, \bar{J}^{\dot{\alpha}\dot{\beta}} \) constructed from the \( \theta \)-twistor components are presented by the symmetric combinations of the new differential operators \( L^{\alpha\beta}, \bar{L}^{\dot{\alpha}\dot{\beta}} \) generalizing the supertwistor generators (24)
\[ L^{\alpha\beta} = l^\alpha \frac{\partial}{\partial l^\beta} + \nu^\alpha \frac{\partial}{\partial \nu_\beta} + \theta^\alpha \frac{\partial}{\partial \theta_\beta}, \quad \bar{L}^{\dot{\alpha}\dot{\beta}} = \bar{l}^{\dot{\alpha}} \frac{\partial}{\partial \bar{l}^{\dot{\beta}}} + \bar{\nu}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\nu}_\beta} + \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_\beta}, \]
\[ [L^{\alpha\beta}, L^{\gamma\delta}] = \varepsilon^{\alpha\delta} L^{\beta\gamma} - \varepsilon^{\beta\gamma} L^{\alpha\delta}, \]
\[ [L^{\alpha\beta}, \bar{Q}^{\gamma}] = \varepsilon^{\gamma\alpha} Q^{\beta}, \quad [L^{\alpha\beta}, \bar{Q}^{\dot{\gamma}}] = 0, \quad [\bar{L}^{\dot{\alpha}\dot{\beta}}, P^{\gamma\delta}] = \varepsilon^{\gamma\dot{\alpha}} P^{\dot{\beta}\delta}. \] (56)

Another irreducible combinations of \( L^{\alpha\beta}, \bar{L}^{\dot{\alpha}\dot{\beta}} \) yield the scale and the phase transformation generators \( D, \bar{D} \) that together with \( J^{\alpha\beta}, \bar{J}^{\dot{\alpha}\dot{\beta}} \) are given by
\[ L^{\alpha\beta} \equiv J^{\alpha\beta} + \frac{1}{2}\varepsilon^{\alpha\beta} D, \quad \bar{L}^{\dot{\alpha}\dot{\beta}} \equiv \bar{J}^{\dot{\alpha}\dot{\beta}} + \frac{1}{2}\varepsilon^{\dot{\alpha}\dot{\beta}} \bar{D}, \]
\[ J^{\alpha\beta} \equiv \frac{1}{2}(L^{\alpha\beta} + L^{\beta\alpha}), \quad D \equiv l^\alpha \frac{\partial}{\partial l^\alpha} - \nu_\alpha \frac{\partial}{\partial \nu_\alpha} + \theta_\alpha \frac{\partial}{\partial \theta_\alpha}, \]
\[ \bar{J}^{\dot{\alpha}\dot{\beta}} \equiv \frac{1}{2}(\bar{L}^{\dot{\alpha}\dot{\beta}} + \bar{L}^{\dot{\beta}\dot{\alpha}}), \quad \bar{D} \equiv \bar{l}^{\dot{\alpha}} \frac{\partial}{\partial \bar{l}^{\dot{\alpha}}} - \bar{\nu}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\nu}_\alpha} + \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_\alpha}, \] (57)

and contain only the \( \theta \)-twistor components. It is easy to check that \( J^{\alpha\beta}, \bar{J}^{\dot{\alpha}\dot{\beta}} \) have the standard commutators with the supercharges (54)
\[ [J^{\alpha\beta}, \bar{Q}^{\gamma}] = \frac{1}{2}(\varepsilon^{\gamma\alpha} Q^{\beta} - \varepsilon^{\beta\gamma} Q^{\alpha}), \quad [\bar{J}^{\dot{\alpha}\dot{\beta}}, \bar{Q}^{\dot{\gamma}}] = 0, \]
\[ [J^{\alpha\beta}, \bar{Q}^{\dot{\gamma}}] = \frac{1}{2}(\varepsilon^{\dot{\gamma}\dot{\alpha}} \bar{Q}^{\dot{\beta}} + \varepsilon^{\dot{\beta}\dot{\gamma}} \bar{Q}^{\dot{\alpha}}), \quad [J^{\alpha\beta}, \bar{Q}^{\dot{\gamma}}] = 0. \] (58)
and with $P^{\dot{\alpha}\alpha}$ showing a realizability of the super Poincare algebra by the $\theta$-twistors. The dilatation $D_R$ and the phase rotation $D_I$ generators are formed by the hermitian combinations of $D$ and $\bar{D}$

$$D_R = (D + \bar{D}), \quad D_I = i(D - \bar{D}).$$

(59)

The finite transformations of the $\theta$-twistor components generated by $D$ and $\bar{D}$ are

$$l'_\beta = e^{\varphi}l_\beta, \quad \bar{l}'_\beta = e^{\varphi}\bar{l}_\beta, \quad \nu'_\beta = e^{-\varphi}\nu_\beta, \quad \bar{\nu}'_\beta = e^{-\varphi}\bar{\nu}_\beta,$$

$$\theta'_\beta = e^{\varphi}\theta_\beta, \quad \bar{\theta}'_\beta = e^{\varphi}\bar{\theta}_\beta,$$

(60)

or equivalently

$$\Xi'_A = e^{\varphi}(-il_\alpha, \bar{\nu^\dot{\alpha}}e^{-2\varphi_R}, \theta^\alpha), \quad \bar{\Xi}'^A = e^{\varphi}(\nu_\alpha e^{-2\varphi_R}, i\bar{l}\dot{\alpha}, \bar{\theta}^\alpha).$$

(61)

It yields the transformation of the form $i\bar{s}(\Xi, \bar{\Xi})$ spanned on the $\theta$-twistors $\Xi$ and $\bar{\Xi}$

$$\Xi' \circ \bar{\Xi}'^A = e^{2\varphi_R}(-il_\alpha, \bar{\nu^\dot{\alpha}}e^{-2\varphi_R}, \theta^\alpha)(\nu_\alpha e^{-2\varphi_R}, i\bar{l}\dot{\alpha}, \bar{\theta}^\alpha)$$

$$= \Xi \circ \bar{\Xi}^A + 4i(e^{-2\varphi_R} - 1)\bar{\eta}\tilde{\eta}$$

(62)

which shows that the form (51) is not invariant under the dilatations (52) generated by $D_R$. From the other hand the phase transformations generated by $D_I$ preserve the form (51), because they yield multiplication of the $\Xi$-space by the phase factors

$$\Xi'_A = e^{i\varphi_5}\Xi_A, \quad \bar{\Xi}'^A = e^{-i\varphi_5}\bar{\Xi}^A,$$

(63)

as it follows from (62) with $\varphi_R = 0$. As the $\Xi$-triple space don’t form projective superspace the phase transformation (63) is not a trivial symmetry of the $\Xi$-space. In spite of the scale symmetry breaking for the nonlinear form (52) the dilatations transform the $\theta$-twistor space into itself. Moreover, the generators $D_R$ and $D_I$ yield the proper commutators with the supercharges $Q^\alpha, \bar{Q}^{\dot{\alpha}}$ (54) and $P^{\dot{\alpha}\alpha}$ (55)

$$[D, Q^\alpha] = -Q^\alpha, \quad [D, \bar{Q}^{\dot{\alpha}}] = 0, \quad [D, P^{\dot{\alpha}\alpha}] = -P^{\dot{\alpha}\alpha},$$

$$[\bar{D}, Q^\alpha] = -\bar{Q}^{\dot{\alpha}}, \quad [\bar{D}, \bar{Q}^{\dot{\alpha}}] = 0, \quad [\bar{D}, P^{\dot{\alpha}\alpha}] = -P^{\dot{\alpha}\alpha},$$

(64)

proving the closure of the superalgebra realized by the $\theta$-twistors.

Similarly to the supertwistors the form (51) is invariant under the $\gamma_5$ rotations (50) carrying out phase rotations of $\theta$ and $\bar{\theta}$

$$U_5 = i(\theta_\alpha \frac{\partial}{\partial \theta^\alpha} - \bar{\theta}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}).$$

(65)

The phase rotation generator $U_5$ coincides with the $\theta$ dependent part of the generator $D_I$ (58) and it has proper commutation relations with $Q^\beta, \bar{Q}^{\dot{\beta}}$ (54) and $J^{\dot{\alpha}\beta}$ (51)

$$[U_5, Q^\alpha] = -iQ^\alpha, \quad [U_5, \bar{Q}^{\dot{\alpha}}] = i\bar{Q}^{\dot{\alpha}}, \quad [U_5, P^{\dot{\alpha}\alpha}] = 0,$$

$$[U_5, L^{\dot{\alpha}\beta}] = 0, \quad [U_5, L^{\dot{\alpha}\beta}] = 0.$$

(66)

Thus, the $\Xi$-space turns out to be invariant under the super Poincare algebra, scaling and phase transformations (60) accompanied by the axial rotations (65).
It is easy to see the inconsistency of Eq. (70) due to the dependence of its l.h.s. on the identities \( \nu \). Repetition of this trick anew does not provide the proper r.h.s. of the commutator \([P^\beta, K^{\hat{a}a}]\), because of loss of the \( \theta \)-contribution in the Lorentz and dilatation generators. So, we have to seek \( K^{\hat{a}a} \) in a form of general \( \Xi, \bar{\Xi} \) dependent hermitian differential operator
\[
K^{\hat{a}a} = (F_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \nu_\gamma} + \bar{F}_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \bar{\nu}_\gamma}) + (G_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \theta_\gamma} + \bar{G}_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \bar{\theta}_\gamma}) + (\Psi_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \theta_\gamma} + \bar{\Psi}_{\gamma}^{\hat{a}a} \frac{\partial}{\partial \bar{\theta}_\gamma}).
\] (67)

The unknown complex functions \( F = F(\Xi, \bar{\Xi}), G = G(\Xi, \bar{\Xi}), \Psi = \Psi(\Xi, \bar{\Xi}) \) and their c.c. \( \bar{F}, \bar{G}, \bar{\Psi} \) in (67) have to be fixed by the superconformal algebra with the \([K, P]\) commutator fixed by
\[
[P^\beta, K^{\hat{a}a}] = \varepsilon^{\alpha\beta} \bar{L}^{\dot{\alpha}\dot{\beta}} + \varepsilon^{\dot{\alpha}\dot{\beta}} L^{\alpha\beta}.
\] (68)

The substitution of (67) into (68) gives the contribution of the terms linear in \( \frac{\partial}{\partial \nu_\gamma} \) and \( \frac{\partial}{\partial \bar{\nu}_\gamma} \)
\[
[P^\beta, \Psi^{\hat{a}a}] \frac{\partial}{\partial \theta_\gamma} + \bar{\Psi}^{\hat{a}a} \frac{\partial}{\partial \bar{\theta}_\gamma} = [P^\beta, \Psi^{\hat{a}a}] \frac{\partial}{\partial \theta_\gamma} + [P^\beta, \bar{\Psi}^{\hat{a}a}] \frac{\partial}{\partial \bar{\theta}_\gamma}.
\] (69)

and next using \( L^{\alpha\beta}, \bar{L}^{\dot{\alpha}\dot{\beta}} \) and \( P^\beta \) transforms Eq. (68) to the equation
\[
(\nu_\beta \frac{\partial}{\partial l_\beta} + \bar{\nu}_\beta \frac{\partial}{\partial \bar{l}_\beta}) \Psi^{\hat{a}a} \frac{\partial}{\partial \theta_\gamma} + (\nu_\beta \frac{\partial}{\partial l_\beta} + \bar{\nu}_\beta \frac{\partial}{\partial \bar{l}_\beta}) \bar{\Psi}^{\hat{a}a} \frac{\partial}{\partial \bar{\theta}_\gamma} = \varepsilon^{\dot{\alpha}\dot{\beta}} \theta^\alpha \frac{\partial}{\partial \theta_\beta} + \varepsilon^{\alpha\beta} \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_\beta}.
\] (70)

It is easy to see the inconsistency of Eq. (70) due to the dependence of its l.h.s. on the even spinors \( \nu_\beta, \bar{\nu}_\beta \) contrarily to its r.h.s.. This \( \nu \)-dependence can’t be removed because of the identities \( \nu_\beta \nu^\beta = \bar{\nu}_\beta \bar{\nu}^\beta = 0 \) and impossibility to construct antisymmetric tensors in the l.h.s. of (70) which might absorb \( \nu, \bar{\nu} \). So, we run into the problem of the conformal boost generator construction by the \( \theta \)-twistors resulting in the superconformal symmetry breakdown up to its subgroup including the super Poincare, scaling and axial symmetries.

7 Chiral supermultiplets of higher spin fields

As it was above discussed the supertwistor appears as a natural object in the chiral coordinate superspace \((y_{\alpha\dot{a}}, \theta_\alpha, \bar{\theta}_{\dot{a}})\) extended by the even Majorana spinor \((\nu_\alpha, \bar{\nu}^\dot{\alpha})\).

But, one can arrive to the supertwistors starting from the supersymmetric constraints
\[
\bar{D}^\dot{\alpha} F(x, \theta, \bar{\theta}) = 0 \rightarrow F = F(y, \theta),
\]
\[
\nu_\alpha D^\alpha F(y, \theta, \nu) = 0 \rightarrow F = F(Z^A),
\] (71)
whose general solution is a chiral superfield \( F(Z^A) \) depending just on the supertwistor \( Z^A \).

It is a consequence of the observation that \( Z^A \equiv (\nu^\alpha, i\bar{q}_{\dot{a}}, 2\eta) \) is the general solution of the second constraint in Eqs. (71)
\[
\nu_\alpha D^\alpha Z^A = 0
\] (72)
as it follows from Eqs. (6) and (12) and the relations
\[
\bar{D}^\dot{\alpha} \bar{q}_{\dot{\beta}} = \bar{D}^\dot{\alpha} \eta = 0, \quad D^\alpha \bar{q}_{\dot{\beta}} = -4i\nu^\alpha \bar{\theta}_{\dot{\beta}}, \quad D^\alpha \eta = \nu^\alpha, \quad \frac{\partial}{\partial y_{\alpha\dot{a}}} \bar{q}_{\dot{\beta}} = \delta^\dot{\alpha}_{\dot{\beta}} \nu_\alpha.
\] (73)
But, if we choose other supersymmetric constraints including $\bar{\nu}$ instead of $\nu$ as in (71)

$$\bar{D}^{\dot{\alpha}} F(x, \theta, \bar{\theta}) = 0 \longrightarrow F = F(y, \theta),$$

$$\bar{\nu}_a \frac{\partial}{\partial x_a} F(y, \theta, \bar{\nu}) = 0 \longrightarrow F = F(\Xi_A),$$

we get a chiral superfields $F(\Xi_A)$, depending on the $\theta$-twistors, to be their general solution because $\Xi_A \equiv (-i\ell_a, \bar{\nu}^\dot{\alpha}, \bar{\theta}^\alpha)$ is the general solution of the second constraint in Eqs. (74)

$$\bar{\nu}_a \frac{\partial}{\partial x_a} \Xi_A = 0,$$

as it follows from Eqs. (8) and (44), and the relations

$$\bar{D}^{\dot{\alpha}} l_\beta = 0, \quad \frac{\partial}{\partial y_{\dot{\alpha}}} l_\beta = 6^\dot{\alpha} \bar{\nu}_\alpha.$$

The both superfields $F(\bar{Z}^A)$ and $F(\Xi_A)$ describe massless supermultiplets because they obey the Klein-Gordon equations

$$\partial_m \partial^m F(\bar{Z}) = 0, \quad \partial_m \partial^m F(\Xi) = 0,$$

where $\partial_m \equiv (\sigma_m)_{\dot{\alpha} \alpha} \frac{\partial}{\partial x_{\dot{\alpha}}} \equiv (\sigma_m)_{\dot{\alpha} \alpha} \frac{\partial}{\partial x_{\dot{\alpha}}}$, $\partial^m = -\frac{1}{2} \bar{\sigma}^{\dot{\alpha} \alpha} \partial_m$. Eqs. (77) are satisfied due to the latter relations in (73) and (76) that provide the vanishing multipliers $\nu^\alpha \nu_\alpha = \bar{\nu}^{\dot{\alpha}} \bar{\nu}_{\dot{\alpha}} = 0$.

For the description of massless higher spin fields in supertwistor space the contour integral

$$G^{\alpha_1 \ldots \alpha_{2S}}(y, \theta) = \oint (dv^\gamma \nu_\gamma) \nu^{\alpha_1 \ldots \alpha_{2S}} F(\nu^\beta, iv^\beta y_{\beta \dot{\beta}}, 2v^\beta \theta_{\beta \dot{\beta}})$$

generalizing the Penrose integral [1] was used in [2]. The superfunction $F(\bar{Z})$ was supposed to be a generalized complex analytic function in the supertwistor space associated with the integral or half-integral spin $S$. To give meaning for (78) as a projective space integral $F(\bar{Z})$ was proposed to be homogenous of degree $-2(S-1)$ with a $\nu$-contour enclosing singularities of $F$ for each fixed value of $(y, \theta)$. Next one can expand $F(\bar{Z})$ in a power series in $\eta$

$$F(\nu^\beta, iv^\beta y_{\beta \dot{\beta}}, 2v^\beta \theta_{\beta \dot{\beta}}) = g_0(\nu^\beta, iv^\beta y_{\beta \dot{\beta}}) + 2v^\lambda \theta_{\lambda \dot{\lambda}} g_1(\nu^\beta, iv^\beta y_{\beta \dot{\beta}})$$

and then inserting Eq. (78) into (78) we get the component expansion for $G$

$$G^{\alpha_1 \ldots \alpha_{2S}}(y, \theta) = g_0^{\alpha_1 \ldots \alpha_{2S}}(y) + 2\theta_{\lambda \dot{\lambda}} g_1^{\alpha_1 \ldots \alpha_{2S} \lambda}(y),$$

where

$$g_0^{\alpha_1 \ldots \alpha_{2S}}(y) = \oint (dv^\gamma \nu_\gamma) \nu^{\alpha_1 \ldots \alpha_{2S}} g_0(\nu^\beta, iv^\beta y_{\beta \dot{\beta}}),$$

$$g_1^{\alpha_1 \ldots \alpha_{2S} \lambda}(y) = \oint (dv^\gamma \nu_\gamma) \nu^{\alpha_1 \ldots \alpha_{2S}} \nu^\lambda g_1(\nu^\beta, iv^\beta y_{\beta \dot{\beta}}).$$

We see that the component fields (81) are totally symmetric in their spinor indices having the same chiralities. So, they describe massless fields with spin $S$ and $S + \frac{1}{2}$, because of the zero mass Klein-Gordon equations similar to (77). Moreover, $g_1^{\alpha_1 \ldots \alpha_{2S} \beta}(y)$ satisfies to the chiral Dirac equation

$$\varepsilon_{\alpha \alpha_1} \bar{\sigma}^{\dot{\alpha} \alpha_1} g_1^{\alpha_1 \ldots \alpha_{2S} \lambda}(x) = 0.$$

It results in the absence of the auxiliary field together with the $\theta^2 \bar{\theta}^2 g_0^{\alpha_1 \ldots \alpha_{2S}}(x)$ and $\theta^2 (\partial_m g_1^{\alpha_1 \ldots \alpha_{2S}}(x) \sigma^m \bar{\theta})$ terms in the further expansion of (80) at the point $x_m$

$$G^{\alpha_1 \ldots \alpha_{2S}}(y, \theta) = g_0^{\alpha_1 \ldots \alpha_{2S}}(x) + 2\theta_{\lambda \dot{\lambda}} g_1^{\alpha_1 \ldots \alpha_{2S} \lambda}(x) - i(\theta \sigma^m \bar{\theta}) g_0^{\alpha_1 \ldots \alpha_{2S}}(x).$$
So, the supersymmetry transformations of the component fields have the reduced form

\[ \delta g_0^{\alpha_1...\alpha_{2S}}(x) = 2\varepsilon_\lambda g_1^{\alpha_1...\alpha_{2S}\lambda}(x), \quad \delta g_1^{\alpha_1...\alpha_{2S}\lambda}(x) = 2i\varepsilon_\lambda \partial^{\lambda\lambda} g_0^{\alpha_1...\alpha_{2S}}(x) \]  

(84)

and close on the mass shell of \( g_1^{\alpha_1...\alpha_{2S}\lambda} \). For the case \( S = 0 \) Eqs. (83) coincide with the transformation rules for the scalar or chiral supermultiplet \( \delta g_0^{\alpha_1...\alpha_{2S}} \) on the mass shell of the spinor field \( g_1^\lambda \) and the vanishing auxiliary field. Thus, the using supertwistors imposes rather severe constraints on the supermultiplet fields and gives rise a question whether it is possible to weaken these constraints. Below we shall give a positive answer for this question based on using the above introduced \( \theta \)-twistors instead of the supertwistors.

To this end let us firstly note that the discussed contour integral method may be also applied for complex analytic functions in the \( \Xi \)-triple space

\[ F(\Xi) \equiv F(-il_\alpha, \bar{\nu}^\alpha, \theta^\alpha) = f_0(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta) - 2\theta^\lambda f^\lambda(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta) + \theta^2 f_2(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta), \]  

(85)

where \( \theta^2 \equiv \theta^\gamma\theta^\gamma, \theta^\alpha\theta^\beta = \frac{1}{2}\varepsilon_{\alpha\beta} \). Next we observe that the supersymmetry generators \( \delta f_i \) in (85) must have the same degree of homogeneity if they are accepted to be homogenous functions. Then one can consider a contour integral

\[ \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y, \theta) = \oint (d\bar{\nu}^\gamma\bar{\nu}_\gamma)\bar{\nu}^{\dot{\alpha}_1}...\bar{\nu}^{\dot{\alpha}_{2S}} F(\bar{\nu}^\beta, -i\bar{\nu}^\beta y_\beta, \theta_\beta) \]  

(86)

similar to the supertwistor integral (78), where \( F(\Xi) \) is supposed to be a complex analytic function in the \( \theta \)-twistor space with the fixed degree of homogeneity \(-2(S - 1)\) and a \( \bar{\nu} \)-contour enclosing singularities of \( F \) for each fixed point \((y, \theta)\). Inserting (85) into (86) we get

\[ \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y, \theta) = f_0^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) - 2\theta_\lambda f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) + \theta^2 f_2^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) \]

(87)

where

\[ f_0^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) = \oint (d\bar{\nu}^\gamma\bar{\nu}_\gamma)\bar{\nu}^{\dot{\alpha}_1}...\bar{\nu}^{\dot{\alpha}_{2S}} f_0(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta), \]

\[ f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) = \oint (d\bar{\nu}^\gamma\bar{\nu}_\gamma)\bar{\nu}^{\dot{\alpha}_1}...\bar{\nu}^{\dot{\alpha}_{2S}} f^{\lambda}(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta), \]

\[ f_2^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) = \oint (d\bar{\nu}^\gamma\bar{\nu}_\gamma)\bar{\nu}^{\dot{\alpha}_1}...\bar{\nu}^{\dot{\alpha}_{2S}} f_2(-iy_\beta\bar{\nu}^\beta, \bar{\nu}^\beta). \]

(88)

Comparing the expansion (83) with (87) we observe two new elements. The first of them is the survival of the auxiliary field and the second is the appearance of the spinor index \( \lambda \) carrying chirality opposite to the chiralities carried by the indices \( \dot{\alpha}_1...\dot{\alpha}_{2S} \) in the spin \( S + \frac{1}{2} \) field \( f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}} \). The chiral index \( \lambda \) in \( f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}} \) has as its origin the spinor \( \theta_\lambda \) contrarily to the antichiral indices \( \dot{\alpha}_1,...,\dot{\alpha}_{2S} \) originating from the twistor component \( \bar{\nu}_\dot{\alpha} \). In view of this difference the Dirac equation similar to (82) is satisfied only for the antichiral indices \( \dot{\alpha}_1,...,\dot{\alpha}_{2S} \)

\[ \varepsilon_{\dot{\alpha}\dot{\alpha}_1} \partial^{\dot{\alpha}} f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}}(x) = 0, \]

(89)

and doesn’t affect on the supersymmetry transformation rules for the auxiliary fields. It is clear, because the component expansion of the chiral superfund \( \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}} \) (79) is going in \( \theta_{\lambda} \) whose chirality is opposite to the \( \bar{\nu}_{\dot{\alpha}} \) chirality. To find the supersymmetry transformation law for the components of the superfund \( \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y, \theta) \) (77)

\[ \delta \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y, \theta) \equiv (\varepsilon_\gamma Q^\gamma + \varepsilon_\dot{\gamma} \bar{Q}^{\dot{\gamma}}) \Phi^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y, \theta) = \]

\[ \delta f_0^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) - 2\theta_\lambda \delta f^{\lambda\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) + \theta^2 \delta f_2^{\dot{\alpha}_1...\dot{\alpha}_{2S}}(y) \]

(90)
we use the chiral representation (7) for $Q^{\gamma}$ and $\bar{Q}^{\gamma}$ and finally find

\[
\delta f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) = -2\varepsilon_\lambda f^\lambda \hat{\alpha}_1\ldots\hat{\alpha}_2S(x),
\]
\[
\delta f^\lambda \hat{\alpha}_1\ldots\hat{\alpha}_2S(x) = 2i\bar{\varepsilon}_\lambda \sigma^\hat{\gamma} f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) + \varepsilon_\lambda f_2^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x),
\]
\[
\delta f_2^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) = -4i\bar{\varepsilon}_\lambda \sigma^\hat{\gamma} f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x).
\]

The further expansion of $\Phi^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(y, \theta)$ at the point $x_m$ is given by

\[
\Phi^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(y, \theta) = f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) - 2\theta_\lambda f^\lambda \hat{\alpha}_1\ldots\hat{\alpha}_2S(x) - 2i\bar{\theta}_\gamma \bar{\sigma}^\gamma f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) -
2i\theta^2 \bar{\theta}_\gamma \bar{\sigma}^\gamma f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) + \theta^2 f_2^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x),
\]

where the term $\frac{1}{2}\theta^2 \bar{\theta}^2 \bar{\sigma}^\gamma \bar{\sigma}_\gamma f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x)$ was dropped because of the zero mass constraint (77)

\[
\square \Phi^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(y, \theta) = 0 \rightarrow \square f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S}(x) = 0.
\]

For sewing together of our results with the known case of $S = 0$ chiral supermultiplet it is suitable to use notations for the component $f$-fields similar to used in [15]

\[
f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S} = \sqrt{2} A^{\hat{\alpha}_1\ldots\hat{\alpha}_2S} \equiv \sqrt{2} A^{\ldots}, \quad f_2^{\hat{\alpha}_1\ldots\hat{\alpha}_2S} = \sqrt{2} F^{\hat{\alpha}_1\ldots\hat{\alpha}_2S} \equiv \sqrt{2} F^{\ldots},
\]

\[
f_0^{\hat{\alpha}_1\ldots\hat{\alpha}_2S} = \psi^{\lambda} \hat{\alpha}_1\ldots\hat{\alpha}_2S \equiv \psi^{\ldots},
\]

where $(... \equiv (\hat{\alpha}_1\ldots\hat{\alpha}_2S)$). Using these notations we present Eq. (92) in a compact form

\[
\frac{1}{\sqrt{2}} \Phi^{\ldots}(y, \theta) = A^{\ldots}(x) + i(\theta \sigma^m \bar{\theta}) \partial_m A^{\ldots}(x) + \sqrt{2} \varepsilon_\lambda \bar{\psi}^{\ldots} -
\frac{i}{\sqrt{2}} \theta^2 \bar{\theta} \bar{\psi}^{\ldots} + \theta^2 F^{\ldots}.
\]

which coincides with the massless chiral supermultiplet [15] if $S = 0$. For $S \neq 0$ Eq. (95) presents chiral supermultiplets of massless higher spin fields with the particle content

\[
\left(\frac{1}{2}, 1\right), \left(\frac{3}{2}, 2\right), \ldots, (S, S + \frac{1}{2})
\]

accompanied by the corresponding auxiliary fields for any of chosen spin $S = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$ from the infinite integral/half spin chain. The transformation rules (91) for the higher spin multiplets (95) rewritten in the notations (94) are

\[
\delta A^{\ldots} = \sqrt{2} \varepsilon_\lambda \bar{\psi}^{\ldots},
\]
\[
\delta \bar{\psi}^{\ldots} = i\sqrt{2}(\sigma_m \varepsilon)^\lambda \partial^m A^{\ldots} + \sqrt{2} \varepsilon_\lambda F^{\ldots},
\]
\[
\delta F^{\ldots} = i\sqrt{2}(\varepsilon \bar{\sigma}_m \partial^m \bar{\psi}^{\ldots})
\]

and coincide with the transformation rules for the $S = 0$ chiral multiplet of the weight $n = \frac{1}{2}$ [15] if we assume $A^{\ldots} = A$, $F^{\ldots} = F$ and $\bar{\psi}^{\ldots} = \psi^{\ldots}$ in the relations (96).

As it was above shown the $\theta$-twistor superspace is invariant under the axial rotations (63) and thus one can consider these phase transformations as inducing the $R$-symmetry transformations for the superfield $F(\Xi)$ (85)

\[
F'(-i\lambda, \tilde{\nu}^\hat{\alpha}, \epsilon^\psi) = e^{2i\varphi} F(-i\lambda, \tilde{\nu}^\hat{\alpha}, \theta^\alpha),
\]
\[
\delta F(\Xi) = 2i\varphi \delta F(\Xi) = \delta \varphi U_5 F(\Xi),
\]

\[13\]
where \( n \) is the correspondent \( R \) number. Then taking into account (86) we get the \( R \)-symmetry transformation of \( \Phi^\dot{\alpha}_1...\dot{\alpha}_2S(y, \theta) \)
\[
\Phi^\dot{\alpha}_1...\dot{\alpha}_2S(y, \theta) = e^{2in\varphi} \Phi^\dot{\alpha}_1...\dot{\alpha}_2S(y, e^{-i\varphi} \theta),
\]
resulting in the \( R \)-symmetry transformation for the component fields similar to (95)
\[
f_0^\dot{\alpha}_1...\dot{\alpha}_2S(x) = e^{2in\varphi} f_0^\dot{\alpha}_1...\dot{\alpha}_2S(x), \quad f_2^\dot{\alpha}_1...\dot{\alpha}_2S(x) = e^{2i(n-1)\varphi} f_2^\dot{\alpha}_1...\dot{\alpha}_2S(x),
\]
\[
f_\lambda^\dot{\alpha}_1...\dot{\alpha}_2S(x) = e^{2i(n-\frac{1}{2})\varphi} f_\lambda^\dot{\alpha}_1...\dot{\alpha}_2S(x).
\]

Having defined the \( R \)-symmetry transformations for the chiral superfields (95) one can construct supersymmetric Langrangians by analogy with the case of scalar multiplet [15] and to expect these Lagrangians to be renormalizable in view of the \( R \)-symmetry.

8 Conclusion

The supertwistor conception based on using the supersymmetric chiral Cartan form was discussed resulting in the construction of a new type of supersymmetric twistor called \( \theta \)-twistor versus the supertwistor. The \( \theta \)-twistor was defined as a triple including two commuting spinors supplemented by the anticommuting spinor \( \theta \) to form a non-linear supersymmetry representation. We revealed that the \( \theta \)-twistors and supertwistors appear as the general solutions of two different supersymmetric and Lorentz covariant constraints admissible in the chiral superspace extended by the Penrose spinors \( \nu \) and \( \bar{\nu} \). The symmetry properties of the \( \theta \)-twistor superspace were studied and its closure under the superconformal group, except the (super)conformal boosts, was established. Using the Penrose contour integral the known chiral superfields depending only on \( x \) and \( \theta \) were restored and their generalization to higher spins was found. The new superfields describe an infinite chain of massless higher spin chiral supermultiplets \((\tfrac{1}{2}, 1), (1, \tfrac{3}{2}), (\tfrac{5}{2}, 2), ..., (S, S + \tfrac{1}{2})\) generalizing the scalar supermultiplet \((0, \tfrac{1}{2})\). These supermultiplets contain auxiliary fields which are absent in the supertwistor description and their supersymmetry transformations generalize the known transformations of the scalar supermultiplet \((0, \tfrac{1}{2})\). Unlike the supertwistor description, where all superfields carry only chiral (or antichiral) spinor indexes, the introduced here superfields carry both the chiral and antichiral indexes. For simplicity the \( D = 4 N = 1 \) supersymmetry case was studied here. However, the construction has a straightforward generalization to more general case of the \( SU(N) \) internal symmetry and higher \( D = 2, 3, 4 \) (mod8), allowing the Majorana spinors, by using the change: \((\theta_\alpha, \bar{\theta}_\dot{\alpha}) \rightarrow \theta_\alpha^i\), where \( a \) is the Majorana spinor index and \( i \in SU(N) \). Moreover, the \( \theta \)-twistor conception is naturally generalized for the tensorial superspaces used for the on-shell description of higher spin field (see e.g. [19] and refs. there). The generalizations as well as studying Langrangians built from the considered superfields will be considered in other place.

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