A superweak solution of the Strong CP Problem *

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Abstract

A non-axion solution to the Strong CP Problem is proposed that works even in the context of gravity-mediated supersymmetry breaking. Both $\epsilon'/\epsilon$ and indirect CP violation in the $B - \bar{B}$ are predicted to be unobservably small. $\mu \rightarrow e\gamma$ is predicted to arise, typically, with branching ration $3 \times 10^{-12}$. A new source of dark matter is also predicted in the model.

No known solution to the Strong CP Problem seems to be completely satisfactory. A combination of laboratory, astrophysical, and cosmological bounds has squeezed invisible axion models into the narrow window $10^{10} \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$ [1]. Moreover, preserving the low mass of the axion against corrections from Planck-scale physics is not a simple matter [2]. On the other hand, non-axion solutions to the Strong CP Problem have their own difficulties. The mechanism proposed by Nelson in [3] does not appear to work in the context of supersymmetry if the supersymmetry breaking occurs at high scales and is mediated by gravitational effects as in supergravity.

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models, as was pointed out by Dine, Leigh, and Kagan [4]. A different non-axion mechanism proposed by the present author and Zee [5] does not appear to work in supersymmetry no matter how supersymmetry is broken. Other solutions to the Strong CP Problem have been proposed [6,7,8], but none is compelling. Given this state of affairs, it is worth looking for other, testable ideas. The idea that is proposed here has the virtue, besides simplicity, that it can be implemented in the context of supersymmetry, whether supersymmetry-breaking is mediated by gauge or gravitational interactions. It also leads to interesting and distinctive low-energy phenomenology.

As background, it is helpful to understand what goes wrong with some non-axion solutions to the Strong CP Problem in the context of supersymmetry. Most of these models are based on the idea that $\theta$ is kept small naturally by a discrete symmetry, either CP or P, which is spontaneously broken. It can be arranged in various ways that $\theta$ vanishes at tree level and is induced at one-loop, or sometimes not until two-loop level. There are two kinds of models of this type. In one type, to which the models of [3], [7], and [8] belong, the quark mass matrices have large CP-violating phases, but have real determinants at tree level. Such models explain the Kaon-system CP violation using the standard Kobayashi-Maskawa mechanism. In the other type, to which the models of [5] and [6] belong, the quark mass matrices themselves are real at tree level, and the $\epsilon_K$ parameter arises from some millicharge or superweak interaction rather than from the KM phase, which is extremely small.

One of the key problems faced by all such non-axion models in the context of supersymmetry is that $\theta$ receives a contribution from the phase of the gluino mass. In the models of the type proposed by Nelson [3], this phase can arise from the one-loop diagram shown in Figure 1(a), due to a mismatch between the complex quark-mass and squark-mass matrices. If supersymmetry breaking occurs at high scales and is mediated to the observable sector by gravity effects, this one loop diagram has no natural suppression, and leads to $\theta \sim \alpha_s/4\pi$, which is many orders of magnitude too large [4]. In the context of gauge-mediated supersymmetry breaking, the quark-squark mismatch is very small, and the diagram of Fig. 1(a) can be harmless [9].

In models where the quark mass matrix is real at tree level [5,6], and where consequently the KM phase is negligible, the Kaon-system CP violation is often mediated by colored scalar fields. A characteristic difficulty of these models is that the fermionic partners of these fields, being colored fermions,
also contribute to $\bar{b}$, and generally far too much.

The common difficulty of all these models is that the CP-violating phases enter into the masses of colored fermions other than the quarks — gluinos and/or colored Higgsinos. The question is whether it is possible to insulate the sector of colored fields more effectively from the sector in which CP is spontaneously broken, and yet have CP violation appear in the Kaon system.

Suppose that in addition to the fields of the Standard Model there exists a vectorlike pair of quark fields, $D'$ and $D'^c$, in the $SU(3) \times SU(2) \times U(1)$ representations $(3, 1, -\frac{1}{3}) + (\bar{3}, 1, +\frac{1}{3})$. For anomaly freedom there need to be additional fields, which are most economically chosen to be lepton doublets, $L'$ and $L'^c$, in the representations $(1, 2, -\frac{1}{2}) + (1, 2, +\frac{1}{2})$. Thus the new fermions can come from the complete $SU(5)$ multiplets $\mathbf{5} + \mathbf{\bar{5}}$. The scalar sector of the Standard Model is augmented by a pair of complex singlets, denoted $h_1$ and $h_2$. The Lagrangian contains the following terms involving the new quark and scalar fields:

$$L' = - \sum_{ia} \lambda_{ia} d_i^c D' h_a - M' D'^c D' - \sum_{ab} m^2_{ab} h_a^\dagger h_b.$$  \hspace{1cm} (1)

The masses $M'$ and $m^2_{ab}$ are of order the weak scale. Note that the terms given in Eq. 1 have a global $U(1)$ symmetry under which $D'$ has charge +1, and $D'^c$ and $h_a$ have charge $-1$. Since the singlet scalars $h_a$ are assumed to have vanishing vacuum expectation values, this global symmetry, which we shall call $U(1)_h$, remains unbroken. This symmetry prevents any mixing in the mass matrix between the new quarks and the known quarks $d$, $s$, and $b$.

It is assumed that CP is a spontaneously broken symmetry, so that the Lagrangian is CP invariant. Thus, all the quark masses, including $M'$, are real, as are the Yukawa couplings $\lambda_{ia}$. However, it is assumed that when CP breaks (at sufficiently high scales that the resulting domain walls are inflated away) a CP-violating phase appears in the mass parameter of the singlet scalars, that is in $m^2_{12} = m^2_{21}$. If we call the fields that break CP spontaneously $S_A$, then the phase in $m^2_{12}$ could arise through higher-dimension operators involving factors of $S_A/M_G$. We will return to this point when we discuss implementing these ideas in supersymmetry. Because $\langle h_a \rangle = 0$, the CP-violating phase does not appear at tree level in quark masses.

Since the only CP-violating phase at low energy is in the mass of the singlet scalars $h_a$, the $\epsilon$ parameter of the neutral Kaon system must arise by means of the box diagram shown in Fig. 2. In estimating CP-violating
effects, we will work in a basis where the mass matrix of the scalars \( h_a \) is real and diagonal, and where there are consequently phases in the couplings \( \lambda_{ia} \). For simplicity, it will be assumed that the lighter mass eigenstate of the singlet scalars (with mass \( m \)) dominates in loop diagrams. In this case, the value of \( \epsilon \) is given approximately by

\[
\epsilon \approx \left( \frac{B_K m_K f_K^2}{3\sqrt{2} \Delta m_K} \right) \frac{1}{192\pi^2} \frac{\text{Im}(\lambda_d^* \lambda_s^2)}{M^2 + m^2} g(m^2/M^2),
\]

where \( \lambda_d \) and \( \lambda_s \) are the couplings of the lighter singlet scalar to \( d^cD' \) and \( s^cD' \) respectively, and where \( g(x) = 3x(1+x) \ln x/(1-x)^3 + \frac{3}{2}(1+x)^2/(1-x)^2 \).

Thus, one has that \( \text{Im}(\lambda_d^* \lambda_s^2)/(M^2 + m^2) \approx 8 \times 10^{-12} \text{ GeV}^{-2} \). If one assumes that \( M \sim m \sim 300 \text{ GeV} \), and that the CP-violating phase is of order unity, then \( |\lambda_d \lambda_s| \sim 10^{-3} \).

In the non-supersymmetric model just described, the leading contribution to the strong CP-violating angle \( \theta \) comes from the two-loop diagram shown in Fig. 3(a). A crucial point is that the \( h_a \) only couple to the right-handed components of the known quarks, so that the one-loop diagram of Fig. 3(b) only multiplies the quark mass matrix by a hermitian factor, and therefore does not contribute to \( \theta \). In Fig. 3(a), the loop factors \((16\pi^2)^{-2}\), and the Yukawas \( \lambda^2 \sim 10^{-3} \), together already give a suppression of \( \theta \) by about \( 10^{-7} \). Given that the quartic coupling of the term \( H^\dagger H h^\dagger h \) can be small, it is apparent that \( \theta \) can be sufficiently small without any fine tuning.

It is easy to show that the only place where CP violation would show up in experiments in the foreseeable future is in the parameter \( \epsilon \). \( \epsilon'/\epsilon \) must arise also from one-loop diagrams involving the new particles, and comes out in the range of \( 10^{-5} \) to \( 10^{-6} \). In other words, the predictions are essentially those of a superweak model. Similarly, one would expect no evidence of indirect CP violation to appear in the \( B - \bar{B} \) systems.

However, the model does lead to interesting new physics of other kinds. One interesting possibility is \( \mu \rightarrow e\gamma \). By far the simplest way for anomalies to cancel, as we have noted, is for the new vectorlike quarks to be accompanied by new vectorlike leptons that fit together with them into the \( SU(5) \) multiplets \( 5 + \bar{5} \). In that case, there should also be couplings of the form \( \sum_{ia} \bar{\lambda}_{ia} L_i L^c h_a + \bar{M} L L^c + \text{H.c.} \). These allow the one-loop diagrams shown in Fig. 4, which lead to the following branching ratio for \( \mu \rightarrow e\gamma \):
where $h(x) = (x + 1)(x - 1)^{-4}(2x^3 + 3x^2 - 6x + 1 - x^2 \ln x)$. $h(x)$ is a very slowly varying function, with $h(1) = 1$, $h(0) = 1$, and $h(\infty) = 2$. If one assumes a grand unified model, and that the ratio $(\tilde{\lambda}_i/\tilde{M})/(\lambda_i/M)$ does not run much in going from the unification scale to the weak scale, then one can estimate the expression in the last equation by replacing $\tilde{M}$ by $M$ and $\tilde{\lambda}_{e,\mu}$ by $\lambda_{d,s}$. If, then, $M \sim m \sim 300$ GeV, and $|\lambda_{d,s}| \sim 10^{-3}$, as required to get $\epsilon$ to come out right, one finds that $BR(\mu \rightarrow e\gamma) \approx 3 \times 10^{-12}$. Of course, this branching ratio depends on various unknown parameters, but one can see from this estimate that values only about an order of magnitude below present bounds are typical.

One would also expect that the new particles in this type of model would contribute to the dark matter density of the universe. Because the symmetry $U(1)_h$ is unbroken, the lightest particle carrying its charge is stable. We shall assume that this particle is one of the singlet scalars, which we shall call simply $h$. The relic abundance of $h$ depends on the annihilation cross section, which is given by

$$\sigma v \cong (m^2/8\pi)(3(\sum_i |\lambda_i|^2)^2(M^2 + m^2)^{-2} + 2(\sum_i |\tilde{\lambda}_i|^2)^2)(\tilde{M}^2 + m^2)^{-2}). \quad (4)$$

If one assumes that the Yukawas are largest for the third generation, and that $\lambda_3/\tilde{M} \approx \lambda_3/M$, then one has that $\sigma v \approx (5/8\pi) |\lambda_3|^4 m^2/(M^2 + m^2)^2$. One can then derive a constraint [10] on the parameters from the requirement that the relic $h$ not contribute more than about 0.3 to $\Omega$. One finds that

$$\lambda_3 \gtrsim M/\sqrt{m(1.7 \times 10^4 \text{GeV})}. \quad (5)$$

If $M \sim 300$ GeV, then $\lambda_3 \gtrsim \sqrt{5 \text{GeV}/m}$, which is certainly a reasonable value.

The supersymmetrization of this model is straightforward. The CP-violating mass of the singlet scalars can arise in the following way. Let the fields whose vacuum expectation values violate CP spontaneously be gauge singlets called $S_A$. Suppose that the $h_a$ superfields get supersymmetry-invariant masses from higher-dimension operators which arise from integrating out fields at some high scale, such as the unification scale $M_G$. These
effective operators could be of the form $S_A S_B h_a h_b / M_G$, or even higher order in powers of $S_A / M_G$. If the expectation values of the singlets $S_A$ are less than $M_G$, these higher-dimensional operators can induce masses of the $h_a$ that are of order the weak scale. The phases of the expectation values of the $S_A$ will then show up in the supersymmetric contributions to the off-diagonal mass, $m_{12}$ of the $h_a$. The singlets denoted $S_A$ could have $U(1)_h$ charge of +2, while those denoted $S_B$ could have vanishing $U(1)_h$ charge. It is simple to construct superpotentials for the $S_B$ that spontaneously violate CP.

The crucial point is that all the CP violating phases are appearing in the sector of the gauge-singlet fields $h_a$ and $S_A$. Therefore, these phases can only show up in the masses of color fields in a rather indirect way. It is clear, for instance, that the one-loop diagram in Fig. 1(a) does not contribute to the phase of the gluino mass, since it does not involve the singlets scalars. (The fact that the $h_a$ do not have vacuum expectation values is crucial, as otherwise the quarks would directly get complex masses from the terms in Eq. (1).) To get a contribution to the phase of the gluino mass one must go to the two-loop diagram shown in Fig. 1(b). This gives a contribution to $\theta$ that is of order $\delta \theta \sim (\alpha_s / 64 \pi^3) \lambda_3^2 (A m^2 / m_\tilde{g} M^2)$. Here $M$ is the mass of the heaviest virtual particle in Fig. 1(b); $A$ is a typical $A$-parameter of the trilinear SUSY-breaking terms; and $m$ is the mass of one of the $h_a$. The loop factors and Yukawa coupling factors give a number of order $6 \times 10^{-7}$. The ratios of masses can, for certain choices of parameters, bring this contribution to $\theta$ down to the $10^{-9}$ level. One does not expect, however, that $\theta$ will be much below the present bound.

In a supersymmetric version of this model, there will be other diagrams contributing to $\epsilon$ and to $\mu \longrightarrow e \gamma$ besides the ones shown in Fig. 2 and Fig. 4, but these will not affect the qualitative conclusions of the earlier discussions.

In conclusion, we have proposed a kind of model in which CP is spontaneously broken in a gauge-singlet sector that is somewhat isolated from the sector of colored fields. Some of these gauge singlets couple ordinary down-type quarks to new vectorlike down-type quarks. The CP violation observed in the Kaon system arises from box diagrams involving the gauge singlet fields and the new vectorlike quarks. $\epsilon' / \epsilon$ is unobservably small, so that these models are effectively superweak in their phenomenology. One also does not expect to see indirect CP violation in the B systems. The $\theta$ parameter only arises at two loops, and can be smaller than, but not much
smaller than, $10^{-9}$. The lightest singlet scalar should make a significant contribution to the dark matter density of the universe. $\mu \rightarrow e\gamma$ is expected at a level not much below $3 \times 10^{-12}$ branching ratio.

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Figure Captions

**Fig. 1.** (a) A one-loop diagram that can contribute to the phase of the gluino mass, and therefore to $\bar{\theta}$ in supersymmetric models. (b) In the model proposed in this paper, the lowest order contribution to the phase of the gluino mass is this two-loop diagram.

**Fig. 2.** In the proposed model, $\epsilon_K$ arises from this box diagram. The CP-violating phases appear in the masses of the gauge-singlet scalars $h_a$.

**Fig. 3.** (a) A two-loop contribution to the phase of the quark masses. (b) This one-loop diagram does not affect $\bar{\theta}$.

**Fig. 4.** A contribution to $\mu \rightarrow e\gamma$. 

Fig. 1(a)

Fig. 1(b)
Fig. 2
Fig. 3(a)

Fig. 3(b)
Fig. 4