Collaborative target-tracking control using multiple autonomous fixed-wing UAVs with constant speeds:
Theory and experiments

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Abstract—This paper considers a collaborative tracking control problem using a group of fixed-wing UAVs with constant and non-identical speeds. The dynamics of fixed-wing UAVs are modelled by unicycle-type equations, with nonholonomic constraints by assuming that UAVs fly at constant attitudes in the nominal operation mode. The control focus is on the design of a collective tracking controller such that all fixed-wing UAVs as a group can collaboratively track a desired target’s position and velocity. We construct a reference velocity that includes both the target’s velocity and position as feedback, which is to be tracked by the group centroid. In this way, all vehicles’ headings are controlled such that the group centroid follows a reference trajectory that successfully tracks the target’s trajectory. We consider three cases of reference velocity tracking: the constant velocity case, the turning velocity case with constant speed, and the time-varying velocity case. An additive spacing controller is further devised to ensure that all vehicles stay close to the group centroid trajectory. Trade-offs and performance limitations of the target tracking control due to the constant-speed constraint are also discussed in detail. Experimental results with three fixed-wing UAVs tracking a target rotorcraft are shown to validate the effectiveness and performance of the proposed tracking controllers.

Note to Practitioners: Fixed-wing UAVs have found increasing applications in both civilian and defense fields in recent years. Compared with rotary-wing UAVs, fixed-wing UAVs in general feature significant larger flight ranges and longer flight duration which are more suitable for tasks such as surveillance, circumnavigation and tracking, which has motivated this research. A key challenge in fixed-wing UAV coordination and control is its airspeed constraint that maintains its flying operation on the air. This paper investigates the possibility and applicability of using multiple fixed-wing UAVs with constant speeds to conduct a collaborative target tracking task. Each UAV flies with a constant speed (operated at a nominal and optimal mode), and the airspeeds for different UAVs in the group are possibly non-identical. To perform a successful tracking task, the tracking UAVs’ speeds should be larger than the target speed, while all UAVs must perform certain turning trajectories to track a moving target with a slower speed. We develop a feasible control framework for multiple UAVs to track a moving target. The control involves two parts: a velocity generation control for the group centroid that serves as a representative of all UAVs in tracking a reference velocity (which, in turn, tracks the target’s velocity and position), and a spacing control that regulates all vehicles to stay close to the target. We have performed numerical simulations and real-UAV experiments (with three fixed-wing UAVs tracking a target rotorcraft) to validate the effectiveness and performance of the proposed tracking controllers.

Index Terms—Fixed-wing UAV, target tracking, collaborative tracking, constant speed.

I. INTRODUCTION

A. Background and motivation

Large-scale operations involving search and rescue, disaster response, environmental monitoring and sport coverage are envisioned to be more cost-effective by making full use of networked multi-vehicle systems. One of the most active and important challenges in multi-vehicle systems is the control and coordination of a group of Unmanned Aerial Vehicles (UAVs) [1], [2], in particular, fixed-wing aircraft. A particular constraint complicating cooperative control design arises from their airspeeds. In practice, airspeeds for fixed-wing UAVs should lie in a bounded value interval: a lower bound for the UAV speed that guarantees they will not stall, and an upper bound arising from actuator constraints. In fact, small-sized fixed-wing aircraft typically fly optimally at constant airspeeds, which are usually designated nominal values designed for an optimal operation mode. For example, a constant speed might be given due to the optimization of the lift/drag ratio and as a consequence of having the vehicle’s motor working in a nominal state with a fixed-pitch propeller. Furthermore, there often exists an optimal airspeed which is the most aerodynamically efficient speed for a given airframe of a fixed-wing UAV [3]. Such a speed constraint imposes additional challenges for coordination control of multiple fixed-wing UAVs.
Fig. 1: Footprints of the aircraft and helicopters tracking and covering the 6th stage of Tour de France 2018. Courtesy of www.flightradar24.com.

Tracking control of stationary or moving targets by multiple fixed-wing UAVs has been a benchmark control problem in the field of multi-vehicle coordination control, which has found numerous applications in practice including target localization, surveillance and target orbiting [4]–[8]. The footprints in Fig. 1 show four aircraft and helicopters tracking cyclists in the Tour de France 2018. In many stages the cyclists were split in many different groups, making it almost impossible to track all of them at the same time with only four vehicles. The usage of coordinated UAVs might solve this problem where efficient aircraft must fly at their nominal air-speeds to cover a cycling tour stage that might last several hours. Fig. 2 shows two other typical scenarios involving multi-UAVs and target tracking, performed by the US military in a Perdix UAV swarm demonstration1. The demonstration employed almost 50 UAVs with adjustable cruising speeds and heading velocity, in a seemingly centralized control framework.

These examples motivate us to study and explore new approaches for the coordination control of a team of fixed-wing aircraft to perform a collaborative target-tracking task. By assuming that each vehicle flies at a constant attitude in a nominal operation mode, the dynamics of fixed-wing UAVs can be modelled by 2-D differential equations with nonholonomic motion constraints and constant speeds. This paper focuses on the design of feasible target-tracking controllers for multiple autonomous fixed-wing UAVs with motion and speed constraints to cooperatively track a moving target.

B. Related papers

The above-mentioned coordination problems become much more challenging if all UAVs in the group have speed constraints. In fact, a more realistic model than single- or double-integrators that can describe the nonholonomic constraints of such fixed-wing UAV dynamics is the unicycle model. Early contributions on coordination control of unicycle-type vehicles include consensus-based formation control [9], pursuit formation design [10] and rendezvous control [11]. Other recent papers include different control constraints [5], [6], [12]–[16] to name just a few, but all assume that both the cruising speed and heading angular speed of individual vehicles are adjustable or controllable. For example, collaborative target-tracking guidance with fixed-wing UAVs was discussed in e.g., [4]–[6], [13] via several strategies such as model predictive control or dynamic programming. The tracking control of multiple unicycles was considered in [16], [17], in which a group of unicycles were tasked to track the trajectory of a target with a time-varying velocity and the framework of circular motion control proposed in [18] was employed. A more recent paper [19] relaxed measurement requirements on target information, but still assumed a unicycle-type model with control inputs relating to both cruising speed and angular speed. In this paper we will consider the more challenging tracking control problem when only the orientation can be controlled and the speeds of all the vehicles remain fixed, so as to reflect the consideration of speed constraints in certain types of real fixed-wing UAVs [4], [8].

When a group of constant-speed vehicles are involved in the control task of trajectory tracking, the problem becomes even more challenging. Two fundamental tracking problems (on tracking a straight line trajectory or a circular trajectory without the consideration of velocity matching) were discussed in [18], [20]–[23], which assumed 2-D unicycle-type UAV models with unit speeds. In [18], [21] the authors showed how to control a group of unit-speed unicycles to achieve two behavior primitives (viz. circular motion and translational motion), with the switching between circling and aligned translation control. The proposed technique in [22] studied how to design tracking controllers such that the formation centroid of a group of unit-speed vehicles can track a target vehicle with possibly non-constant velocity. Such strategies were further explored in [24] for vision-based flocking control of multiple autonomous vehicles.

In this paper we further consider a more realistic scenario

1Images in Fig. 2 are captured from the video released in https://www.dvidshub.net/video/504622/perdix-swarm-demo-oct-2016. Also see the news report http://www.bbc.com/news/technology-38569027, dated on 10 January 2017.
where constant speeds in a multi-UAV group are not identical (but may be similar in terms of their nominal values). This is motivated by practical tracking scenarios that a multi-vehicle group may consist of multiple heterogeneous UAVs with different functions or payloads, which will help to perform a comprehensive target-tracking task. Recent efforts towards the coordination of fixed-wing aircraft with non-identical constant speeds were presented in [25]–[27], which demonstrated coordination algorithms based on circular motions, rigid formations and distributed consensus-based flying coordination in practice. A general theory on target-tracking control with multiple fixed-wing UAVs with non-identical constant speeds is however lacking in the literature.

C. Contributions and paper organization

In this paper we aim to provide a systematic method to solve the target-tracking control problem under the constraints that (a) a target with a general trajectory curve is to be tracked, and (b) different vehicles have non-identical constant speeds. The framework for designing tracking controllers in this paper is motivated in part by [18], [22], but several significant extensions are required to deal with heterogeneous vehicles with non-identical speeds with the control task of tracking a general target trajectory. The controller design consists of two parts: reference velocity tracking that aims to regulate the group centroid to track a target’s velocity and position, and a spacing control that ensures all vehicles stay close to the group centroid. Due to the coordination constraints arising from constant speed, trade-offs in the controller design are inevitable and we also present detailed analysis on the performance limitations on using multiple fixed-wing UAVs in a collaborative tracking task.

A preliminary conference version of this paper was presented in [28]. The extensions of this paper compared to [28] include detailed proofs for all key results which were omitted in [28], the convergence analysis of velocity tracking control with mobile targets, and discussions on the trade-offs and tracking limitations of using constant-speed UAVs in the target tracking task. Furthermore, a new section devoted to experimental verifications is also included in this extended version to demonstrate the real-life performance of the tracking controller.

We organize this paper as follows. We introduce the UAV model and problem formulation in Section II. Reference velocity tracking that ensures the UAV group centroid successfully tracks a reference trajectory is discussed in Section III. Section IV proposes the design of a reference velocity and spacing controller to ensure all vehicles move close to the group centroid. Experimental results with fixed-wing UAVs tracking a moving rotorcraft with the proposed collaborative tracking controller are shown in Section VI. Finally, Section VII concludes this paper.

II. BACKGROUND, PRELIMINARY AND PROBLEM DESCRIPTION

A. Notations

The notations used in this paper are fairly standard. The set \( S^1 \) denotes the unit circle and an angle \( \theta_i \) is a point \( \theta_i \in S^1 \).

The \( n \)-torus is the Cartesian product \( T^n = S^1 \times \cdots \times S^1 \). For a complex number \( z \in \mathbb{C} \), \( \text{Re}(z) \) and \( \text{Im}(z) \) denote, respectively, its real part and imaginary part, and \( \bar{z} \) is the complex conjugate of \( z \). For \( z_1, z_2 \in \mathbb{C}^n \), the real scalar product is defined by \( \langle z_1, z_2 \rangle = \text{Re}(z_1^T z_2) \), i.e., the real part of the standard scalar product over \( \mathbb{C}^n \). The norm of \( z \in \mathbb{C}^n \) is defined as \( \| z \| = \sqrt{\langle z, z \rangle} \).

B. Vehicle models

In this paper we consider a group of \( n \) fixed-wing vehicles modelled by unicycle-type kinematics subject to a nonholonomic constraint and constant-speed constraint. By following [4], [8], [13], the kinematic equations of fixed-wing vehicle \( k \) flying in a fixed horizontal plane are described by

\[
\begin{align*}
\dot{x}_k &= v_k \cos(\theta_k) \\
\dot{y}_k &= v_k \sin(\theta_k) \\
\dot{\theta}_k &= u_k
\end{align*}
\]  

where \( x_k, y_k \in \mathbb{R} \) are the coordinates of vehicle \( k \) in the real horizontal plane and \( \theta_k \) is the heading angle. The fixed-wing UAVs have fixed cruising speeds \( v_k > 0 \) which in general are distinct for different vehicles; \( u_k \) is the control input to be designed for steering the orientation of vehicle \( k \). The equation (1) serves as a high-level kinematic model, which captures well motion constraints and vehicle dynamics for fixed-wing UAVs flying at trim conditions (e.g., constant attitudes) [3]. In fact, the model (1) fits fairly well into the dynamics of a small fixed-wing aircraft as we have shown in [26], [27]. When an aircraft flies at its nominal airspeed both its lift and weight are balanced so that there is not change in the vehicle’s altitude. Therefore, the 3D dynamics of the aircraft can be easily decoupled to separate the planar motion parallel to the ground, i.e., the dynamics (1), and the vertical motion. Consequently, we will assume that the aircraft fly at a constant altitude in this paper. Note that when the airspeed is much higher than the windspeed we can consider \( v_k \) as the ground speed [3]. As we will see during the experiments, this is a mild assumption that does not have a substantial impact on the performance of the controller for the coordination of multiple fixed-wing UAVs.

For the convenience of analysis we also rewrite in complex notation the model (1) for vehicle \( k \) as

\[
\begin{align*}
\dot{r}_k &= v_k e^{i \theta_k} \\
\dot{\theta}_k &= u_k
\end{align*}
\]  

where the vector \( r_k(t) = x_k(t) + iy_k(t) := |r_k|e^{i \theta_k(t)} \in \mathbb{C} \) denotes vehicle \( k \)’s position in the complex plane (where \( |r_k| := \sqrt{x_k^2 + y_k^2} \) and \( \phi_k := \arg(r_k) \)). We also define the vectors \( r = [r_1, r_2, \ldots, r_n]^T \in \mathbb{C}^n, \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n \) and \( e^{i \theta} = [e^{i \theta_1}, e^{i \theta_2}, \ldots, e^{i \theta_n}]^T \in \mathbb{C}^n \) to collect the positions and headings of all the vehicles.

C. Problem formulation

We consider the problem of tracking a target trajectory \( r_{\text{target}}(t) \) by a team of fixed-wing UAVs with (possibly non-identical) constant speeds. A reasonable strategy in the multi-vehicle tracking control is to use the centroid of the vehicle
team, denoted by $\tilde{r}(t) = \frac{1}{n} \sum_{k=1}^{n} r_k(t)$, as the surrogate position of the whole vehicle group, which is to be controlled to match the target position $\nu_{\text{target}}(t)$. In fact, we will generate a velocity reference signal $\dot{\nu}_{\text{ref}}(t)$ to be tracked by the centroid in a manner previously introduced in [22]. As will be seen later in Section IV, this velocity reference signal is not necessarily identical to the target’s velocity (unless initially the reference point is collocated with the target’s position). The construction of the reference velocity takes into account both the target’s velocity and the relative position between the target and the group centroid. The control strategy is split into two loops similarly to [29]. In a first phase discussed in Section III, an inner loop for each member of the UAV team controls their orientations so that the velocity of the centroid tracks $\dot{\nu}_{\text{ref}}(t)$. In a second phase discussed in Section IV, an outer loop generates $\dot{\nu}_{\text{ref}}(t)$ using information on the target’s position.

We will see that our proposed controller for coordinating constant-speed UAVs resembles the phase control problem of coupled oscillators. This is due to the fact that we will control angular velocities of the vehicles to regulate their heading orientations. An important measure for such an oscillator network is the so-called order parameter $p_0 = \frac{1}{n} \sum_{k=1}^{n} e^{i \theta_k}$, which is actually the centroid velocity of a group of unit-speed vehicles and is often used to measure the phase coherence or phase synchronization level [18], [30]. In this paper we will use a similar metric called order parameter $\bar{\nu}$, which is actually the velocity of the group centroid point according to the definition of $\tilde{r}$. The authors in [25] employ this metric for the control of circular motions of unicycles with different constant speeds, and one of our proposed controllers can be seen as an extension of that work.

Apart from the velocity reference tracking control by the group centroid, we also need to provide an extra control to steer each vehicle to stay close to the group centroid. For example, if we consider the position of the target is stationary at the origin, one may encounter situations where some or all of the tracking vehicles travel away from the origin while their centroid remains constant at the target, a situation which is not acceptable (except in the short term). We will therefore introduce an additional term to the tracking controller to regulate the spacing among the vehicles as a coherent tracking team. Consequently, all the vehicles will remain at a bounded distance from their centroid. In this work, we will not consider additional terms for collision avoidance. In particular, since one of our goals is to demonstrate the algorithm on fixed-wing aircraft, we can always suppose they are flying at different altitudes. This should not be a strong restriction if we are not aiming at really massive swarms, and it is a common assumption in the literature [22], [25]. In summary, the overall tracking controller will be in the form

$$u_k = \nu_k^{\text{velocity}} + \nu_k^{\text{spacing}}$$

where $\nu_k^{\text{velocity}}$ is the responsible term to track the reference velocity (which is constructed by feedback from a target’s position and velocity) via the group centroid and the term $\nu_k^{\text{spacing}}$ controls the spacing for each individual vehicle so that they can remain with a bounded distance to the centroid. It is obvious that there exist certain trade-offs in the design of these two controllers since, generally speaking, the two sub-tasks (i.e., reference velocity tracking and spacing control) are not likely to be achieved perfectly at the same time. Performance limitations arising from the trade-offs in the controller design will be discussed in more detail in Section III and Section IV.

### III. CONTROLLER DESIGN PHASE I: REFERENCE VELOCITY TRACKING

In this section we discuss Phase I in the controller design, i.e., how to regulate each vehicle’s heading and motion with the dynamics $\dot{\theta}_k = \nu_k^{\text{velocity}}$ so that the velocity of the vehicle group’s centroid achieves a desired reference velocity. The construction of a reference velocity, the combined controller design that stabilizes the spatial error to the target’s position and the spacings between individual vehicles will be discussed in the next section.

#### A. Conditions on constant speeds for a feasible reference velocity tracking

Denote the reference velocity by $\dot{\nu}_{\text{ref}}(t) = \nu_{\text{ref}}(t)e^{i \theta_{\text{ref}}(t)}$, where $\nu_{\text{ref}}(t)$ and $\theta_{\text{ref}}(t)$ are the (possibly time-varying) airspeed and heading direction of the reference, respectively. If initially the reference trajectory coincides with the target trajectory, the target velocity is used as the reference velocity. Otherwise, the construction of the reference velocity should take into account both the target’s position and velocity (which will be elaborated in detail in Section IV). For a group of constant-speed vehicles, one cannot expect that an arbitrary reference velocity can be tracked by the group centroid. In this section we will give several conditions that guarantee a feasible reference velocity tracking.

We recall the definitions of the group centroid position $\bar{r}$ and velocity $\dot{\bar{r}}$ as defined in Section II-C, whose values depend on the simultaneous headings and velocities of each individual vehicle. The maximum value of the group centroid speed, denoted as $\bar{v}_{\text{max}}$, can be achieved when all the vehicles reach a heading synchronization, in which case there holds $\bar{v}_{\text{max}} := \|\dot{\bar{r}}_{\text{max}}\| = \frac{1}{n} \sum_{k=1}^{n} v_k$. However, due to the non-identical constant speeds in the group, even if the maximum group centroid speed $\bar{v}_{\text{max}} = \frac{1}{n} \sum_{k=1}^{n} v_k$ can be achieved, the inter-vehicle distances between individual vehicle will grow larger and larger and eventually unboundedly because of the non-zero differences between individual speeds. Therefore, a strict condition on the individual speeds should be imposed.

For any vehicle in the group, the minimum speed, denoted by $v_{\text{min}} := \min_{k \in \{1,2,\ldots,n\}} v_k$ should be greater than the reference speed $\|\dot{\nu}_{\text{ref}}(t)\|$. Otherwise, the distance between the vehicle with the smallest speed and the target will grow unboundedly and a collective target tracking can not be achieved. Therefore, one should ensure that the reference speed is smaller than the minimum speed in the vehicle group (which trivially ensures that the reference speed is smaller than any constant speed of the vehicle group). Of course, the reference speed cannot exceed the maximum speed of the group centroid $\frac{1}{n} \sum_{k=1}^{n} v_k$. 
Now consider the case that the group centroid is able to achieve a zero centroid velocity, which is not possible if there exists one vehicle in the group whose constant speed is larger than the sum of all other vehicles’ speeds. In order to ensure that the group centroid speed lies in the range \([0, v_{\text{min}}]\), one should have \(v_{\text{max}} \leq \sum_{k=1}^n v_k - v_{\text{max}}\). In summary, the necessary conditions for a feasible reference velocity tracking are shown in the following proposition.

**Proposition 1.** For a feasible reference velocity tracking, the constant speeds for the vehicle group should satisfy the following conditions:

- All vehicles’ speeds should be greater than the reference speed, i.e., \(v_{\text{min}} \geq v_{\text{ref}}\).
- There does not exist one vehicle in the group whose speed is larger than the sum of the speeds of all other vehicles, i.e. there should hold \(v_{\text{max}} \leq \sum_{k=1}^n v_k - v_{\text{max}}\).

In the following subsections, we consider velocity tracking control for the constant-speed vehicle group. We will start from the simple case of constant reference velocity and then extend the controller design result to the time-varying velocity case.

### B. Tracking a constant reference velocity

This subsection solves the control problem of regulating the formation centroid to track and match a constant reference velocity \(\hat{v}_{\text{ref}} = v_{\text{ref}} e^{i\theta_{\text{ref}}}, \) in which both \(v_{\text{ref}}\) and \(\theta_{\text{ref}}\) are constant. The controller involves collectively regulating the heading of each individual vehicle. This control problem can be seen as an extension of the result in [18], [20] which discussed the control problem of regulating a group of unit-speed vehicles to achieve a flocking behavior (i.e. a translational motion along a fixed direction).

The first main result on constant velocity tracking is stated in the following theorem.

**Theorem 1.** Suppose that the constant reference velocity \(v_{\text{ref}}\) and all vehicles’ constant speeds \(v_k\) satisfy the conditions in Proposition 1. Consider the following steering control law for (2)

\[
v_k^{\text{velocity}} = -\gamma \langle \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}, iv_k e^{i\theta_k} \rangle = -\gamma \left\langle \frac{1}{n} \sum_{k=1}^n v_k e^{i\theta_k} - v_{\text{ref}} e^{i\theta_{\text{ref}}}, iv_k e^{i\theta_k} \right\rangle
\]

(4)

where \(\gamma\) is a positive control gain. Suppose that the initial headings of all the vehicles are not aligned with the phase of the reference velocity. Then the equilibrium point for the phase dynamics (2b) at which \(\hat{\mathbf{r}} := \frac{1}{n} \sum_{k=1}^n v_k e^{i\theta_k} = v_{\text{ref}} e^{i\theta_{\text{ref}}}, \) is asymptotically stable and all other equilibria are unstable. Furthermore, the control law (4) almost globally stabilizes the group centroid velocity to the desired constant reference velocity \(\hat{v}_{\text{ref}} = v_{\text{ref}} e^{i\theta_{\text{ref}}}.\)

**Proof.** We first show that the system \(\hat{\theta} = u^{\text{velocity}}\) with the above designed controller (4) describes a gradient flow for the following quadratic potential

\[
V(\theta) = \frac{1}{2} \|\hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}\|^2 = \frac{1}{2} \left( \frac{1}{n} \sum_{k=1}^n v_k e^{i\theta_k} - v_{\text{ref}} e^{i\theta_{\text{ref}}}, \frac{1}{n} \sum_{k=1}^n v_k e^{i\theta_k} - v_{\text{ref}} e^{i\theta_{\text{ref}}} \right)
\]

(5)

The gradient of \(V(\theta)\) can be calculated as

\[
\frac{\partial V(\theta)}{\partial \theta_k} = \left\langle \frac{\partial \hat{\mathbf{r}}}{\partial \theta_k}, \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}} \right\rangle = \langle iv_k e^{i\theta_k}, \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}} \rangle = \langle \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}, iv_k e^{i\theta_k} \rangle
\]

(6)

Hence the phase system (2b) with the designed control law (4) can be written as \(\dot{\theta} = -\gamma \nabla V(\theta)\) which is a gradient descent flow for the potential function \(V(\theta)\). Furthermore, \(V(\theta) \geq 0\) and \(V(\theta)\) is zero if and only if \(\hat{\mathbf{r}} = \hat{\mathbf{r}}_{\text{ref}}\). Thus, \(V(\theta)\) can be used as a Lyapunov function for the convergence analysis. Due to the gradient property of the phase system (2b), there exists no limit cycle under the control \(\hat{\theta} = u^{\text{velocity}}\) at the steady state [31]. Furthermore, the stability analysis of different equilibria (2) can be cast as a critical point analysis of the real analytic potential \(V(\theta)\). Note that the system variable is \(\theta \in \mathbb{T}^n\) where \(\mathbb{T}^n\) is compact and thus the sub-level sets of \(V(\theta)\) are also compact according to its definition in (5). We remark that for the phase dynamics (2b) with the velocity tracking controller (4), the state variable is \(\theta\) while \(r\) is not involved.

The derivative of \(V(\theta)\) along the trajectory of the phase system (2) can be computed as

\[
\dot{V} = \nabla V(\theta)^T \dot{\theta} = -\gamma \frac{n}{n} \left\{ \langle \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}, iv_k e^{i\theta_k} \rangle \right\}^2 \leq 0
\]

(7)

By LaSalle’s Invariance Principle, all solutions of (2) with the controller (4) converge to the largest invariant set contained in

\[
O(r, \theta) = \{(r, \theta) | V = 0\}
\]

(8)

In the following we will show the properties of different sets of critical points. Note that the Jacobian of the right-hand side of (2) with the controller (4) is \(-\gamma H_V\) where \(H_V\) is the Hessian of \(V\). The nature of an equilibrium (of being a minimum, a saddle point or a maximum) can be determined by the signs of the eigenvalues of the Hessian \(H_V\) at that equilibrium assuming that the Hessian is non-singular.

From the above analysis, it is clear that the desired critical points on which \(\hat{\mathbf{r}} := \frac{1}{n} \sum_{k=1}^n v_k e^{i\theta_k} = v_{\text{ref}} e^{i\theta_{\text{ref}}} =:\) \(\hat{\mathbf{r}}_{\text{ref}}\) are global minima of \(V(\theta)\) which are asymptotically stable. We will show other equilibrium sets, which correspond to \(V(\theta) > 0\), or equivalently equilibrium points in the set \(O(r, \theta)\) in (8) with \(\langle \hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}, iv_k e^{i\theta_k} \rangle = 0\) and \(\hat{\mathbf{r}} \neq \hat{\mathbf{r}}_{\text{ref}}\), are unstable. Denote

\[
\dot{\hat{\mathbf{r}}} := \frac{\hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}}{\|\hat{\mathbf{r}} - \hat{\mathbf{r}}_{\text{ref}}\|}
\]

(9)

\[\text{This point will be made clear in Remark 2, in which the control input is equivalently written in real variables that only involve } \theta.\]
where \( \phi := \arg(\hat{r}) \) by definition. We call such critical points for which \( \| \hat{r} \| > 0 \) undesired equilibria since they do not achieve a desired reference velocity tracking. Note that at the undesired equilibria there holds \( \sin(\theta_k - \phi) = 0 \) for all \( k = 1, 2, \ldots, n \), which implies that either \( \theta_k = \phi \mod 2\pi \) or \( \theta_k = \phi + \pi \mod 2\pi \). Let \( m \) be the number of vehicles with phase \((\phi + \pi \mod 2\pi)\) at one of such undesired equilibria. Now consider two extreme cases:

- The case that \( m = 0 \) indicates that all the vehicles have the same phase \((\phi \mod 2\pi)\). An equilibrium with \( m = 0 \) is a global maximum of \( V(\theta) \) and a small variation of any \( \theta_k \) will decrease the value of \( V(\theta) \). Therefore, any equilibrium with \( m = 0 \) is unstable.
- The case that \( m = n \) cannot happen because of the assumption that \( v_{\text{min}} \leq v_{\text{min}} \).

In the following, we will show that all other equilibria with \( 1 \leq m \leq n-1 \) are all saddle points. Without loss of generality, we renumber the vehicles such that the first \( m \) vehicles are with phase \((\phi + \pi \mod 2\pi)\). The diagonal entries of the Hessian of \( V \) can be calculated as

\[
\frac{\partial^2 V}{\partial \theta_k^2} = \begin{cases} 
(1/n)v_k^2 + \|\hat{r}\|v_k, & k \in \{1, \ldots, m\} \\
(1/n)v_k^2 - \|\hat{r}\|v_k, & k \in \{m+1, \ldots, n\}
\end{cases}
\]

and the off-diagonal entries are

\[
\frac{\partial^2 V}{\partial \theta_j \partial \theta_k} = \begin{cases} 
(1/n)v_j v_k \cos(\theta_j - \theta_k), & j, k \in \{1, \ldots, m\} \\
(1/n)v_j v_k, & j, k \in \{m+1, \ldots, n\}, j \neq k \\
-1/n v_j v_k, & \text{else}
\end{cases}
\]

Therefore, the Hessian can be written in a compact form

\[
H_V = \frac{1}{n}v_n^T + \|\hat{r}\| \text{diag}(v)
\]

where the vector \( v \) is defined as

\[
v = [v_1, \ldots, v_m, -v_{m+1}, \ldots, -v_n]^T.
\]

Since there exists at least one diagonal entry in the form of \((1/n)v_k^2 + \|\hat{r}\|v_k\) which is positive, the Hessian \( H_V \) has at least one positive eigenvalue. We will show the Hessian \( H_V \) has at least one negative eigenvalue at any critical point with \( 1 \leq m \leq n-1 \) by proving that there exist vectors \( q^- \in \mathbb{R}^n \) such that \( q^T H_V q^- < 0 \). We first consider the case of \( m = n-1 \), which can happen if and only if the vehicle with maximum speed has phase \((\phi \mod 2\pi)\) which is also the phase for the reference velocity. Define \( q^- = [a_1, \ldots, a_{n-1}]^T \), where the constant \( a_i \) satisfies \( 0 \leq a_i < 1 \) for \( i = 1, \ldots, n-1 \) and \( v^T q^- = 0 \). Note that according to Proposition 1, such negative \( a_i \) always exists and cannot all be zero, which guarantees the existence of the vector \( q^- \). Then a simple calculation yields

\[
q^T H_V q^- = \sum_{i=1}^{n-1} a_i^2 v_i v_n - \sum_{i=1}^{n-1} (a_i^2 v_i - a_i v_n) < 0.
\]

Therefore, an equilibrium point with \( m = n-1 \) is a saddle point and is therefore unstable.

We then consider the case of \( 1 \leq m < n-1 \). Actually there are many options for constructing such a vector \( q^- \). Without loss of generality, let us choose

\[
q^- = [0, \ldots, 0, -v_n, v_{n-1}]^T
\]

The existence of such \( q^- \) in (11) is guaranteed because \( m < n-1 \). Note that there holds \( v^T q^- = 0 \). It then follows that

\[
q^T H_V q^- = \|\hat{r}\| q^- \text{diag}(v) q^- \\
= \|\hat{r}\|(-v_{n-1}^2 v_n - v_{n-1} v_n^2) < 0
\]

Hence, it is proved that such critical points with \( 1 \leq m < n-1 \) are saddle points and therefore are unstable. Consequently, all the undesired equilibria with \( \langle \hat{r} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \rangle = 0 \) and \( \hat{r} \neq \hat{r}_{\text{ref}} \) are unstable.

By summarizing the above arguments, it is thus proved that the desired equilibria at which \( \hat{r} = \hat{r}_{\text{ref}} \) are asymptotically stable, and all other equilibria are unstable. The initial points at which the initial headings of all the vehicles are aligned are excluded in the set of initial positions because these points correspond to unstable equilibria.

\[\square\]

**Remark 1.** The result on the reference velocity tracking in Theorem 1 is relevant and applicable to the target-tracking scenario in which initially the target position is the same as the centroid position, and the target velocity is constant (which, as will be discussed in Section IV, is used as the reference velocity). Extensions for time-varying velocity tracking will be discussed in the sequel. We also remark that since the group centroid is tasked to track a reference trajectory, an average of the positions and headings of all vehicles in the group should be calculated and therefore a complete graph is assumed in the reference velocity control to facilitate the calculation. This is justified by the all-to-all communication, a commonly-used assumption in the literature on fixed-wing UAV coordination control. Such an assumption may be relaxed by some consensus-based estimation technique (which only requires connectivity for an underlying communication graph) but will be traded-off by additional computation overhead for each vehicle, and therefore is not considered in this paper.

**Remark 2.** (Control input in real variables) Theorem 1 can be seen as an extension of [25, Theorem 2], which considered the stabilization problem of the average linear momentum when \( \hat{r}_{\text{ref}} = 0 \). The above controller involves complex numbers and scalar products of complex vectors. For the implementation, one can calculate the control input (4) in real variables:

\[
u_k^{\text{velocity}} = -\gamma \langle \hat{r} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \rangle \\
= -\gamma \sum_{j=1}^{n} v_j e^{i\theta_j} \langle iv_k e^{i\theta_k} \rangle + \gamma \langle \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \rangle \\
= -\gamma \sum_{j=1}^{n} v_j v_k \sin(\theta_j - \theta_k) + v_k v_{\text{ref}} \sin(\theta_{\text{ref}} - \theta_k)
\]

The phase dynamics (2b) with the controller (13) written in real variables has a similar form to the Kuramoto oscillator model which has been studied extensively [30], [32], but the difference is that the speed term is involved in (13) for controlling non-identical constant-speed vehicles. If we assume all vehicles have the same unit speed, the controller (13) then reduces to the one studied in [18], [20], [22] where the oscillator synchronization theory [30], [32] can apply.
C. Tracking a turning reference velocity with constant speed

In this subsection we consider the tracking control with a reference trajectory with constant speed and turning angular velocity, whose dynamics can be described by

\[ \dot{\hat{r}}_{\text{ref}} = v_{\text{ref}} e^{i\theta_{\text{ref}}} \]
\[ \dot{\hat{\theta}}_{\text{ref}} = \kappa_{\text{ref}} \]  

(14)

where \( v_{\text{ref}} \) is the constant speed of the reference velocity and \( \kappa_{\text{ref}} \) is the angular velocity, which can be constant or non-constant and corresponds to the scaled curvature of the trajectory generated by the velocity. In the case of tracking a time-varying reference velocity with a constant speed, the essence of the velocity tracking control is to design a reference velocity matching controller such that the constant reference speed and the trajectory curvature can be tracked. The main result is summarized in the following theorem.

**Theorem 2.** Suppose the turning reference velocity and all vehicles’ constant airspeeds \( v_k \) satisfy the conditions in Proposition 1 and the initial headings of all the vehicles are not aligned with the initial phase of the reference velocity. Consider the following steering control law

\[ u_k^{\text{velocity}} = h_k - \gamma \left( \hat{r} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \right) \]

(15)

where \( \hat{r} \) is the group centroid position, \( \hat{r} \) is the group centroid velocity as defined previously, \( \gamma \) is a positive control gain and the \( h_k \) are any real control terms that satisfy

\[ \frac{1}{n} \sum_{k=1}^{n} iv_k e^{i\theta_k} h_k = iv_{\text{ref}} e^{i\theta_{\text{ref}} \kappa_{\text{ref}}} \]

(16)

Then the equilibrium point for the phase dynamics (2b) at which \( \hat{r} = \hat{r}_{\text{ref}} \) is asymptotically stable and all other equilibria are unstable. The above control law will almost globally asymptotically stabilize the group centroid velocity of the multi-vehicle group (2) to the desired reference velocity \( \hat{r}_{\text{ref}} \) (14).

Theorem 2 is a generalization of Theorem 1. If the time-varying component \( \theta_{\text{ref}} = \kappa_{\text{ref}} \) of the turning reference velocity is zero, then the control (15) can be reduced to the one in Theorem 1 as one can take all \( h_k \) equal to zero.

**Proof.** The proof can be seen as an extension of the proof for Theorem 1. We denote a new variable \( \hat{r} := \hat{r} - \hat{r}_{\text{ref}} \) as the velocity tracking error. Note that

\[ \hat{r} = \frac{1}{n} \sum_{k=1}^{n} v_k e^{i\theta_k} \hat{\theta}_k = \frac{1}{n} \sum_{k=1}^{n} iv_k e^{i\theta_k} u_k^{\text{velocity}} \]

(17)

and

\[ \hat{r}_{\text{ref}} = iv_{\text{ref}} e^{i\theta_{\text{ref}} \kappa_{\text{ref}}} \]

(18)

Then the dynamics for the velocity tracking error can be written as

\[ \ddot{\hat{r}} = \frac{1}{n} \sum_{k=1}^{n} v_k e^{i\theta_k} \dot{\hat{\theta}}_k = \frac{1}{n} \sum_{k=1}^{n} iv_k e^{i\theta_k} u_k^{\text{velocity}} \]

\[ = \frac{1}{n} \sum_{k=1}^{n} iv_k e^{i\theta_k} \gamma \left( \hat{r} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \right) \]

(19)

Consider the same Lyapunov-like function \( V = \frac{1}{2} \parallel \dot{r} - r_{\text{ref}} \parallel^2 \) as used earlier in studying the convergence analysis. Note that \( V(\dot{r}) \geq 0 \) and \( V(\dot{r}) \) is zero if and only if \( \dot{r} = r_{\text{ref}} \), or equivalently \( \parallel \dot{r} \parallel = 0 \).

The derivative of \( V \) along the trajectories of the velocity tracking error system (19) can be calculated as

\[ \dot{V} = \left( \dot{\hat{r}} - \hat{r}_{\text{ref}}, \frac{\dot{\hat{r}} - \hat{r}_{\text{ref}}}{\parallel \dot{r} - r_{\text{ref}} \parallel} \right) \]

\[ = -\gamma \sum_{k=1}^{n} \parallel \dot{\hat{r}} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \parallel^2 \leq 0 \]

(20)

By LaSalle’s Invariance Principle and similar arguments as in the proof of Theorem 1, it can be proven that the solution of (2) converges to a stable equilibrium in the largest invariant set in which \( \dot{r} = 0 \), which is equivalent to \( \hat{r} = \hat{r}_{\text{ref}} \) indicating that the time-varying reference velocity can be successfully tracked in the limit.

**Remark 3.** The conditions in Proposition 1 guarantee the existence of the desired equilibrium \( \hat{r} = \hat{r}_{\text{ref}} \).

We discuss how to calculate the \( h_k \) below in Section III-E.

D. Tracking a time-varying reference velocity

With the preparation of tracking controller analysis in the above subsections, in this subsection we will consider the most general case. We will show the design of a general form of velocity tracking controller to regulate the group centroid velocity that aims to track a desired time-varying reference velocity \( \hat{r}_{\text{ref}}(t) = v_{\text{ref}}(t) e^{i\theta_{\text{ref}}(t)} \). The equation of the reference velocity can be written as

\[ \dot{r}_{\text{ref}} = v_{\text{ref}}(t) e^{i\theta_{\text{ref}}(t)} \]
\[ \dot{\theta}_{\text{ref}} = \kappa_{\text{ref}}(t) \]
\[ \dot{\kappa}_{\text{ref}} = a_{\text{ref}}(t) \]

(21)

where \( \kappa_{\text{ref}}(t) \) and \( a_{\text{ref}}(t) \) can be time-varying functions. To avoid cumbersome notations we will omit the argument \( t \) in the following analysis.

**Theorem 3.** Suppose the time-varying reference velocity (21) satisfies the conditions in Proposition 1 and the initial headings of all the vehicles are not aligned with the initial phase of the reference velocity. Consider the following steering control law

\[ u_k^{\text{velocity}} = -\gamma \left( \hat{r} - \hat{r}_{\text{ref}}, iv_k e^{i\theta_k} \right) + h_k \]

(22)
where \( \gamma \) is a positive control gain and the additional control terms \( h_k \) that are designed for tracking a time-varying reference velocity satisfy
\[
\frac{1}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} h_k = i \nu_{\text{ref}} e^{i \theta_{\text{ref}}} \kappa_{\text{ref}} + a_{\text{ref}} e^{i \theta_{\text{ref}}}
\] (23)
for \( k = 1, \ldots, n \). Then the equilibrium set at which \( \dot{\theta} = \dot{\theta}_{\text{ref}} \) is asymptotically stable and all other equilibria are unstable. The above control law (22) with the additional input (23) will almost globally asymptotically stabilize the group centroid velocity to the desired reference velocity \( \dot{\theta}_{\text{ref}} \).

Theorem 3 is a generalization of both Theorem 2 and Theorem 1 and it treats the most general case for tracking a time-varying reference velocity. If the headings of all the vehicles are not all synchronized or anti-synchronized with each other along time, then the control terms \( h_k \) are guaranteed to exist. We note here that there exist multiple choices for the additional controller term \( h_k \) in the right hand side of (22), which will be discussed in the next subsection.

**Proof.** The proof takes similar steps as that in Theorem 1. The differences lie in the feedforward control term \( h_k \) to address the time-varying terms in the observed velocity \( \dot{\theta} \). We define the velocity tracking error as \( \ddot{\theta} := \ddot{\theta}_{\text{ref}} \). Note that
\[
\ddot{\theta} = \frac{1}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} \dot{\theta}_k = \frac{1}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} \dot{\theta}_k
\] (24)
and
\[
\dot{\theta}_{\text{ref}} = i \nu_{\text{ref}} e^{i \theta_{\text{ref}}} \kappa_{\text{ref}} + a_{\text{ref}} e^{i \theta_{\text{ref}}}
\] (25)
According to (22), the dynamics for the velocity tracking error can be calculated as
\[
\ddot{\theta} = \ddot{\theta} - \dot{\theta}_{\text{ref}}
= -\frac{\gamma}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} \langle \dot{\theta} - \dot{\theta}_{\text{ref}}, i \nu_k e^{i \theta_k} \rangle
\] (26)
We consider the same Lyapunov function \( V = \frac{1}{2} \| \ddot{\theta} - \dot{\theta}_{\text{ref}} \|^2 \) which measures the difference between the current centroid velocity and the desired reference velocity. Note that \( V(\ddot{\theta}) \geq 0 \) and \( V(\dot{\theta}) \) is zero if and only if \( \ddot{\theta} = \dot{\theta}_{\text{ref}} \), or equivalently \( |\ddot{\theta}| = 0 \). The derivative of \( V \) along the trajectories of the velocity tracking error system (26) can be calculated as
\[
\dot{V} = \langle \ddot{\theta} - \dot{\theta}_{\text{ref}}, \ddot{\theta} - \dot{\theta}_{\text{ref}} \rangle
= \left\langle \ddot{\theta} - \dot{\theta}_{\text{ref}}, -\frac{\gamma}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} \langle \dot{\theta} - \dot{\theta}_{\text{ref}}, i \nu_k e^{i \theta_k} \rangle \right\rangle
= -\frac{\gamma}{n} \sum_{k=1}^{n} \langle \dot{\theta} - \dot{\theta}_{\text{ref}}, i \nu_k e^{i \theta_k} \rangle^2 \leq 0
\] (27)
By LaSalle’s Invariance Principle and similar arguments as in the proof of Theorem 1, it can be proven that the desired equilibrium in which \( \ddot{\theta} = 0 \) in the largest invariant set is asymptotically stable, which indicates the time-varying reference velocity can be successfully tracked. Again, Proposition 1 guarantees the existence of the desired equilibrium at which \( |\ddot{\theta}| = 0 \). A similar analysis involving the Hessian matrix shows all other equilibrium points with \( |\ddot{\theta}| \neq 0 \) are unstable.

**E. Discussions**

In this subsection we discuss how to design the control terms \( h_k \) in Theorems 2 and 3. The control terms \( h_k \) in (16) and (22) serve as feedforward controls which are necessary to track the time-varying component of the reference velocity.

By denoting \( h = [h_1, h_2, \ldots, h_n]^T \) and separating a complex variable into real part and complex part in the form \( e^{i \theta} := [\cos(\theta), \sin(\theta)]^T \), one obtains the left-hand side of (16) and (23) in the following equivalent form:
\[
\frac{1}{n} \sum_{k=1}^{n} i \nu_k e^{i \theta_k} h_k
= \frac{1}{n} \left[ -v_1 \sin(\theta_1) - v_2 \sin(\theta_2) \cdots - v_n \sin(\theta_n) \right] h
:= Ah
\] (28)
The right-hand sides of (16) and (23) are rewritten respectively as
\[
b_1 := i \nu_{\text{ref}} e^{i \theta_{\text{ref}}} \kappa_{\text{ref}} + a_{\text{ref}} e^{i \theta_{\text{ref}}}
= \left[ -v_{\text{ref}} \sin(\theta_{\text{ref}}) \kappa_{\text{ref}} + v_{\text{ref}} \cos(\theta_{\text{ref}}) \kappa_{\text{ref}} \right]
\] (29)
\[
b_2 := i \nu_{\text{ref}} e^{i \theta_{\text{ref}}} \kappa_{\text{ref}} + a_{\text{ref}} e^{i \theta_{\text{ref}}}
= \left[ a_{\text{ref}} \cos(\theta_{\text{ref}}) - v_{\text{ref}} \sin(\theta_{\text{ref}}) \kappa_{\text{ref}} \right]
\] (30)
Then the calculation of \( h \) is equivalent to solving a linear equation with a standard form \( Ah = b_1 \) or \( Ah = b_2 \). Since \( A \in \mathbb{R}^{2 \times n} \), a sufficient condition to guarantee the existence of the solution is \( \text{rank}(A) = 2 \). For the \( n \)-vehicle group, the rank condition is satisfied if and only if at least one pair of vehicles whose headings are not aligned or anti-aligned (i.e., \( \theta_i \neq \theta_j \) or \( \theta_i \neq \pi - \theta_j \) for at least one pair of vehicles \( i, j \)). In the case of a two-vehicle group, a unique solution exists as \( h = A^{-1} b \) when the two vehicles are not aligned or anti-aligned in their headings. For an \( n \)-vehicle group, given that the rank condition is satisfied, the solution is not unique and this provides flexibility in the controller design, while a standard least 2-norm solution could be preferred.

**Remark 4.** The reference velocity tracking in the tracking control framework is inspired by the previous papers [18], [22]. Here we are considering a heterogeneous vehicle group with constant non-identical speeds for individual vehicles, which is more general than [18], [22] that discussed tracking control with unit-speed unicycle-type agents. The present results are also extensions of the tracking control strategies for unit-speed unicycles using two special motion primitives (i.e. circular motion and parallel motion) discussed in [18]. A rigorous proof for the convergence of velocity tracking is presented, which is lacking in [22]. Also, the non-identical speed constraints in the group present limitations for a successful tracking; note we have stated explicit necessary conditions on the maximum reference speed and the minimum vehicle speed in Proposition 1 for the tracking controller design.
IV. CONTROLLER DESIGN PHASE II: REFERENCE TRAJECTORY GENERATION AND SPACING CONTROL

A. Reference velocity generation for target trajectory tracking

In the above section we have designed reference velocity tracking controllers so that the group centroid of the fixed-wing UAVs can successfully track a reference trajectory by matching a reference velocity. In order to guarantee a successful tracking of a target, the reference trajectory should include both the target’s trajectory and velocity in the construction of the reference velocity. We propose the following reference velocity

\[ \dot{r}_{\text{ref}} = \dot{r}_{\text{target}} + w(r_{\text{target}} - r(t)) \]  

(31)

where \( w > 0 \) is a weighting parameter on the relative position between the target \( r_{\text{target}} \) and the group centroid \( r(t) \). The weighting parameter \( w \) can be used to adjust the convergence speed of position tracking. A larger value of \( w \) puts more weights on asymptotically tracking the position of the target, which enables a fast track to the target’s trajectory. If initially the group centroid \( r(t) \) is collocated with the target position, then the target velocity can be used as the reference velocity. Otherwise, the relative position term \( (r_{\text{target}} - r(t)) \) is involved in the reference velocity as a feedback term to guarantee a successful tracking to the target. The following lemma shows that by using the constructed reference velocity in (31) and the velocity tracking controller in Section III, the target’s trajectory can be asymptotically tracked.

**Lemma 1.** With the reference velocity tracking controller in (22) designed in Section III, and the constructed reference velocity in (31), the centroid of the fixed-wing UAV group will be asymptotically stabilized to match the target trajectory.

**Proof.** As proved in Theorem 3, the designed reference velocity tracking controller (22) guarantees an asymptotic convergence of the group centroid velocity to the reference velocity. Denote \( \alpha(t) = \dot{r}_{\text{ref}} - \dot{r}(t) \) as the tracking difference between the reference velocity and the centroid velocity. Then one has \( \alpha(t) \to 0 \) by virtue of Theorem 3. From (31) we have the following equivalent equation

\[ \dot{r}(t) - \dot{r}_{\text{target}} = -w(r(t) - r_{\text{target}}) - \alpha(t) \]  

(32)

Denote \( \beta(t) = \dot{r}(t) - r_{\text{target}} \) as the trajectory tracking error between \( r(t) \) and \( r_{\text{target}} \). Then one has \( \dot{\beta}(t) = -w\beta(t) - \alpha(t) \).

Since \( w > 0 \) and \( \alpha(t) \to 0 \), one obtains \( \beta(t) \to 0 \) which indicates that the trajectory tracking error converges to zero asymptotically.

In practice one can design the weight \( w \) as a distance-dependent parameter (i.e., \( w(t) := w(\rho) \) where \( \rho = \|r(t) - r_{\text{target}}\| \)) to adjust the convergence speed in different phases of the tracking process. Furthermore, since in the limit there holds \( \dot{r}_{\text{ref}} \to \dot{r}_{\text{target}} \), in order to ensure the condition in Proposition 1 is satisfied, we should also impose the condition on the target velocity \( v_{\min} \geq \|v_{\text{target}}(t)\| \) to ensure a feasible tracking.

B. Vehicle-target spacing control

In this subsection we design the spacing controller \( u_{\text{spacing}} \) to ensure all vehicles stay with a bounded distance to the group centroid. As noted above, there exists a trade-off between the design of the velocity tracking controller and a spacing control. We will follow a similar idea as in [33], [34] for the spacing controller design. We first present the following condition for an admissible spacing control that does not affect the performance of the centroid velocity tracking control.

**Lemma 2.** Denote the spacing control vector \( u_{\text{spacing}} = [u_1^\text{spacing}, u_2^\text{spacing}, \ldots, u_n^\text{spacing}]^T \) and define the matrix \( A \) as in (28). The additional spacing control \( u_{\text{spacing}} \) satisfying

\[ u_{\text{spacing}} \in \ker(A) \]  

(33)

preserves the asymptotic tracking performance of the reference velocity by the group centroid (i.e., \( \dot{\hat{r}} \to \dot{r}_{\text{ref}} \) as \( t \to \infty \)).

**Proof.** We consider the same Lyapunov function as used in Section III. The controller \( u_{\text{velocity}} \) designed in (22) is used here as an example for the velocity tracking analysis. The time derivative of the Lyapunov function along the solution of the system (2) with the combined controller (3) can be calculated as

\[ \dot{V} = \langle \dot{\hat{r}} - \dot{r}_{\text{ref}}, \dot{\hat{r}} - \dot{r}_{\text{ref}} \rangle - \langle \dot{\hat{r}} - \dot{r}_{\text{ref}}, 1/n \sum_{k=1}^{n} \dot{r}_k e^{i\theta_k u^{\text{velocity}}} \rangle - \langle \dot{\hat{r}} - \dot{r}_{\text{ref}}, 1/n \sum_{k=1}^{n} \dot{r}_k e^{i\theta_k u^{\text{spacing}}} \rangle \]

\[ = \left( \frac{\langle \dot{\hat{r}} - \dot{r}_{\text{ref}}, 1/n \sum_{k=1}^{n} \dot{r}_k e^{i\theta_k u^{\text{velocity}}} \rangle}{\|\dot{\hat{r}} - \dot{r}_{\text{ref}}\|^2} \right) \leq 0 \]  

(34)

Note that the equality in the third line of the above (34) is due to the condition in (33). Therefore, \( \dot{V} \) is invariant for any control \( u_{\text{velocity}} \) of (33), and is negative semidefinite according to the controller property. The remaining analysis is similar to the proof of previous theorems in Section III and is omitted here.

\[ \square \]

For a vehicle group with \( n > 2 \) vehicles with non-aligned headings, the matrix \( A \) is of full row rank and so has a null space of dimension \( n - 2 \), which leaves motion freedoms for designing the spacing controller. However, it is challenging to design an admissible control input that lies in \( \ker(A) \) while also keeping all vehicles within a reasonable spacing around the centroid. In particular, an analytical solution for an admissible spacing control \( u_{\text{spacing}} \) is not available, while a numerical solution is usually expensive. Actually, even for the control of unit-speed vehicles, it is still an open problem to design explicit controllers to satisfy the above constraint and design requirement (see more in-depth discussions in [34, Chapter 2]).

Inspired by [35] and [36], we consider an alternative approach based on the beacon control law proposed by Paley et al. [36], [37] to design an intuitive spacing control. The idea is to allow each vehicle to perform limited circular trajectories around a chosen beacon point in the reference trajectory path.
In this way, the spacing control takes a position feedback from a reference trajectory, and is designed as

$$u_k^{\text{Spacing}} = -\left( \omega_0 + \gamma \omega_0 \langle r_k - r_{\text{ref}}, iv_k e^{i\theta_k} \rangle \right)$$

where $\omega_0$ is a positive parameter for adjusting the period in the circular motion. It has been proved in [26], [36], [37] that the above control (in the absence of velocity tracking control) guarantees constant-speed vehicles performing circular motions around a reference point $r_{\text{ref}}$. It has also been shown in [35] by using simulation examples that the above spacing control will ensure that all the vehicles move and remain close to the centroid. We note that this control is generally not an admissible one in the null space of $A$ satisfying the condition in Lemma 2 and therefore a perfect velocity tracking performance is not guaranteed by the addition of the spacing controller. However, as demonstrated by numerous numerical simulations and experiments in the next sections, such a spacing control law can ensure all agents stay close to the group centroid while the group centroid position tracks the target trajectory.

In fact, a pragmatic solution would consider to projecting (35) onto the non-zero kernel of $A$. The philosophy of designing $u^{\text{Spacing}}$ without affecting the centroid velocity tracking task is inspired by [22], [38], and is actually in the broad framework of null-space-based robotic behavior control proposed in [38]–[40]. In this framework, each sub-task is assigned a certain priority and the control term for a task with a lower priority should live in the null space of the control task space of those with higher priorities. There exist alternative approaches in the joint design of velocity tracking controller and spacing controller. Since an analytical and perfect solution for both subtasks in the target tracking control is hard to find as shown in [34], one may consider ad-hoc solutions (see e.g. [41]) by taking into account different way points in the target trajectory to be used as feedback information and designing a switching tracking controller to ensure tracking convergence and boundedness. However, in this way analytical convergence results are hard to obtain.

V. Simulation examples

We refer the readers to [28] for several typical simulation examples that demonstrate the effectiveness and performance of the proposed tracking controllers for a group of fixed-wing UAVs with constant but non-identical airspeeds. Two sets of numerical simulation examples were given in [28], one for collective tracking of a target with constant velocity, and the other for collective tracking of a target with time-varying (but bounded) velocity. In both cases, the proposed tracking controllers guarantee a desirable tracking performance through vehicle centroid and bounded tracking distance errors for a group of constant-speed vehicles to collaboratively track moving targets.

VI. Experimental verifications

The purpose of this section is to validate the proposed algorithms with fixed-wing aircraft in a series of experiments. This validation is not a mere extension of the theoretical work. In particular, we have reconsidered some assumptions in the theoretical analysis that are no longer satisfied in a real distributed control system. For example, several issues exist in practice such as the presence of delays in the transmission of information, the non-synchronization of clocks, or sensors on different vehicles that are biased with respect each other. Some of these issues may potentially have a significant impact on the performance of the overall system, especially in a decentralized control setting [42], [43]. Therefore, one of the goals of this section is to validate the performance of the proposed algorithms in practice even when some important factors have been omitted in the development of theoretical analysis.

The experimental setup consists of one Parrot Bebop2 rotorcraft (serving as a target to be tracked) and three fixed-wing aircraft labeled as Wing 2, 3 and 4, respectively. All the vehicles are equipped with the open-source autopilot Paparazzi, which allows a rapid prototyping for distributed aerial systems as shown in our recent works [26], [27]. This platform also enables third parties to quickly implement and use for other purposes our proposed algorithms.

We choose the scenario of target trajectory tracking described in Section IV.A since it covers most of the presented results in this paper. In particular, a rotorcraft flies as an independent target at the ground speed of 2 m/s. The three aircraft fly at constant speeds between 10 m/s and 16 m/s while they execute onboard their control actions (22) with $\dot{r}_{\text{ref}}$ given by (31) and a positive distance-dependent weight $w(\rho) = 1 - e^{-0.1\rho}$, and the spacing controller (35) with $\omega_0 = 0.6$ rads/sec. The relative positions between the vehicles are calculated onboard by having the UAVs broadcasting their absolute positions obtained by a GPS. The broadcasting frequency for the target is 5Hz, whereas for the aircraft is 10Hz. [Note that this broadcasting step can be substituted by onboard radar-like sensors although this modification would}
not change the outcome of our experiments.] Indeed, the fact that there exist communication losses (i.e., communication dropouts over short intervals) between aircraft, and that the aircraft process the information at different times, leads to possible discrepancies among the three vehicles about the relative positions, just as in the case of having onboard sensors at different aircraft.

The experiment was performed in a radio control club in Muret, a city close to Toulouse in France with calm weather. We divide the mission in two stages. In the first stage, the rotorcraft target stays stationary on the ground. In the second stage, the target flies by following a closed path with a constant ground speed of 2 m/s.

We first place the target rotorcraft on the ground (black cross in Fig. 4), with the other three tracking aircraft orbiting around a standby point (black circle in Fig. 4). After the three aircraft reach their different designated altitudes, we start the collaborative tracking mission. In particular, during the first five minutes of the mission the aircraft enclose the rotorcraft while it is on the ground. This first stage helps to keep the aircraft within the allowed airspace during the transitory phase of the algorithm, and to save battery energy in the rotorcraft before the convergence of the team, i.e., when its centroid is orbiting around the target as shown in Fig. 5. The convergence of the control signal $r_{ref}$ with dynamics (31) to the position of the rotorcraft target is shown in Fig. 6. We would like to highlight that Theorems 2 and 3 make a conservative assumption by considering that the signal $r_{ref}$ is common to
Fig. 8: The trajectory of the centroid of the team orbiting around the moving target (black cross). The red color represents the last minute’s trajectory of the centroid.

Fig. 9: Convergence of the signals $r_{ref}$ (one per each aircraft in different colors) to the position of the moving rotorcraft target. The second stage when the target starts moving begins at $t = 575$ seconds. Note that the signal $r_{ref}$ has converged in the first stage already by tracking the constant black line denoting the position of the rotorcraft while it is on the ground. We observe how the control signals $r_{ref}$ track the position of the target even while the target rotorcraft exhibits changes in its speed and direction.

all the vehicles. In this experiment, since all the vehicles are able to execute control algorithms on board (22), each vehicle has a different signal $r_{ref}$ to be used onboard. In particular, in order to experimentally show that this is not an issue at all, we have assigned quite different initial conditions to $r_{ref}$ for each aircraft which are noticeable at the beginning of the plots in Fig. 6.

We continue with the second stage of the mission, where the target flies following a closed path. The three aircraft keep close to the target, and in particular the centroid orbits around it as shown in Figs. 7 and 8. The effectiveness of the algorithm is illustrated in Fig. 9, where the control signal $r_{ref}$ (the reference trajectory) from the three aircraft enables a successful tracking of the trajectory of the target with a high accuracy. Finally, we show in Fig. 10 some screenshots from the ground control station of Paparazzi during the mission.

Fig. 10: The screenshot (a) denotes the initial conditions in Fig. 4. The screenshot (b) shows the end of the first stage. The screenshots (c) and (d) show the team of fixed-wing UAVs tracking a moving target as in Figs. 7 and 8.
VII. CONCLUSIONS

In this paper we have discussed the problem of target tracking controller design for a group of fixed-wing UAVs with constant and non-identical speeds to track a moving target. Inspired by previous papers (e.g. [21], [22]), a systematic framework is proposed for the collaborative target-tracking control. We have used the group centroid as a representative of the overall UAV group in the tracking process. The design of the tracking controller consists of two parts: the reference velocity tracking control that regulates the group centroid to track a reference velocity, and a spacing controller that ensures all vehicles keep close to the group centroid. The reference velocity involves the target velocity as well as the relative position to the target as feedback to ensure the group centroid tracks the target trajectory. We have also discussed the trade-offs and limitations of using fixed-wing UAVs to track a moving target, and conditions that ensure a feasible tracking. Numerical simulations and real-life experiments involving a group of three fixed-wing UAVs are performed to demonstrate the performance and effectiveness of the proposed collaborative tracking control laws using multiple fixed-wing UAVs.

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REFERENCES

[1] J. G. Manathara, P. Sujit, and R. W. Beard, “Multiple UAV coalitions for a search and prosecute mission,” Journal of Intelligent & Robotic Systems, vol. 62, no. 1, pp. 125–158, 2011.

[2] B. D. O. Anderson, B. Fidan, C. Yu, and D. Walle, “UAV formation control: theory and application,” in Recent advances in learning and control, pp. 15–33, Springer, 2008.

[3] R. W. Beard and T. W. McClain, Small unmanned aircraft: Theory and practice. Princeton University Press, 2012.

[4] Y. Kang and I. B. Hedrick, “Linear tracking for a fixed-wing UAV using nonlinear model predictive control,” IEEE Transactions on Control Systems Technology, vol. 17, no. 5, pp. 1202–1210, 2009.

[5] S. A. Quintetto, F. Papi, D. J. Klein, L. Chisci, and J. P. Hespanha, “Optimal UAV coordination for target tracking using dynamic programming,” in Proc. of the 2010 49th IEEE Conference on Decision and Control (CDC), pp. 4541–4546, IEEE, 2010.

[6] N. Regina and M. Zanzi, “UAV guidance law for ground-based target trajectory tracking and loitering,” in Proc. of the 2011 IEEE Aerospace Conference, pp. 1–9, IEEE, 2011.

[7] W. Ren and R. W. Beard, “Trajectory tracking for unmanned air vehicles with velocity and heading rate constraints,” IEEE Transactions on Control Systems Technology, vol. 12, no. 5, pp. 706–716, 2004.

[8] N. Regina and M. Zanzi, “Surface target-tracking guidance by self-organizing formation flight of fixed-wing UAV,” in Proc. of the 2013 IEEE Aerospace Conference, pp. 1–10, IEEE, 2013.

[9] Z. Lin, B. Francis, and M. Maggiore, “Necessary and sufficient graphical conditions for formation control of unicycles,” Automatic Control, IEEE Transactions on, vol. 50, no. 1, pp. 121–127, 2005.

[10] J. A. Marshall, M. F. Broucke, and B. A. Francis, “Pursuit formations of unicycles,” Automatica, vol. 42, no. 1, pp. 3–12, 2006.

[11] D. V. Dimarogonas and K. J. Kyriakopoulos, “On the rendezvous problem for multiple nonholonomic agents,” Automatic Control, IEEE Transactions on, vol. 52, no. 5, pp. 916–922, 2007.

[12] S. Mastellone, D. M. Stipanović, C. R. Graakne, K. A. Intlekofer, and M. W. Spong, “Formation control and collision avoidance for multi-agent non-holonomic systems: Theory and experiments,” The International Journal of Robotics Research, vol. 27, no. 1, pp. 107–126, 2008.

[13] N. Regina and M. Zanzi, “A novel solution for overtaking and surveying of a collaborative target with fixed wing UAV,” in AIAA Guidance, Navigation, and Control Conference, p. 4605, 2012.

[14] T. Liu and Z.-P. Jiang, “Distributed formation control of nonholonomic mobile robots without global position measurements,” Automatica, vol. 49, no. 2, pp. 592–600, 2013.

[15] B. Fidan, V. Gazi, S. Zhai, N. Cen, and E. Karatas, “Single-view distance-estimation-based formation control of robotic swarms,” Industrial Electronics, IEEE Transactions on, vol. 60, no. 12, pp. 5781–5791, 2013.

[16] L. Brión-Arranz, A. Seuret, and C. Canudas-de Wit, “Cooperative control design for time-varying formations of multi-agent systems,” Automatic Control, IEEE Transactions on, vol. 59, no. 8, pp. 2283–2288, 2014.

[17] L. Brión-Arranz, Cooperative control design for a fleet of AUVs under communication constraints. PhD thesis, Université de Grenoble, 2011.

[18] R. Sepulchre, D. A. Paley, and N. E. Leonard, “Stabilization of planar collective motion: All-to-all communication,” Automatic Control, IEEE Transactions on, vol. 52, no. 5, pp. 811–824, 2007.

[19] L. Brión-Arranz, A. Seuret, and A. Pascoal, “Target tracking via a circular formation of unicycles,” Proc. of the 20th IFAC World Congress, vol. 50, no. 1, pp. 5782–5787, 2017.

[20] N. Moshtagh and A. Jadbabaie, “Distributed geodesic control laws for flocking of nonholonomic agents,” Automatic Control, IEEE Transactions on, vol. 52, no. 4, pp. 681–690, 2007.

[21] R. Sepulchre, D. A. Paley, and N. E. Leonard, “Stabilization of planar collective motion with limited communication,” Automatic Control, IEEE Transactions on, vol. 53, no. 3, pp. 706–719, 2008.

[22] D. J. Klein and K. A. Morgansen, “Controlled collective motion for trajectory tracking,” in Proc. of the American Control Conference, pp. 5269–5275, IEEE, 2006.

[23] E. W. Justh and P. Krishnaprasad, “Equilibria and steering laws for planar formations,” Systems & control letters, vol. 52, no. 1, pp. 25–38, 2004.

[24] N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, “Vision-based, distributed control laws for motion coordination of nonholonomic robots,” Robotics, IEEE Transactions on, vol. 25, no. 4, pp. 851–860, 2009.

[25] C. Xiong, A. S. Morse, and R. Sepulchre, “Collective motion of unicycle type vehicles with nonidentical constant velocities,” Control of Network Systems, IEEE Transactions on, vol. 1, no. 2, pp. 167–176, 2014.

[26] C. Xiong, A. S. Morse, and R. Sepulchre, “Collective motion of unicycle type vehicles with nonidentical constant velocities,” Control of Network Systems, IEEE Transactions on, vol. 1, no. 2, pp. 167–176, 2014.

[27] Z. Sun, G. S. Seyboth, and B. D. O. Anderson, “Collective control of multiple unicycle-type agents with non-identical constant speeds,” IEEE Transactions on Control Systems Technology, in press, DOI: 10.1109/TCST.2017.2763938, pp. 1–14, 2017.

[28] H. G. de Marina, Z. Sun, M. Bronz, and G. Hattenherger, “Circular formation control of first-flight UAVs with constant speeds,” in Proc. of the 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 5298–5303, IEEE, 2017.

[29] Z. Sun, G. S. Seyboth, and B. D. O. Anderson, “Collective control of multiple unicycle agents with non-identical constant speeds: Tracking control and performance limitation,” in Proc. of the 2015 IEEE Conference on Control Applications (CCA), pp. 1361–1366, IEEE, 2015.

[30] E. Xargay, V. Dobrokhotov, I. Kaminer, A. M. Pascoal, N. Kovakinov, and C. Cao, “Time-critical cooperative control of multiple autonomous vehicles,” Control Systems Magazine, IEEE, vol. 32, no. 5, p. 49, 2012.

[31] F. Dörfler and F. Bullo, “Synchronization in complex networks of phase oscillators: A survey,” Automatica, vol. 50, no. 6, pp. 1539–1564, 2014.

[32] P.-A. Absil and K. Kurdyka, “On the stable equilibrium points of gradient systems,” Systems & control letters, vol. 55, no. 7, pp. 573–577, 2006.

[33] Y. Kuramoto, Chemical oscillations, waves, and turbulence. Dover Publications, 2003.

[34] D. Kingston and R. Beard, “UAV slay state configuration for moving targets in wind,” in Advances in Cooperative Control and Optimization, pp. 109–128, Springer, 2007.

[35] D. J. Klein, Coordinated control and estimation for multi-agent systems: Theory and practice. PhD thesis, University of Washington, 2008.

[36] A. Pongpunwattana, B. Triplett, and K. Morgansen, “Target tracking control with limited communication and steering dynamics,” in Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 3094–3100, IEEE, 2007.

[37] D. A. Paley, N. E. Leonard, and R. Sepulchre, “Collective motion: Bistability and trajectory tracking,” in Proc. of the 43rd IEEE Conference on Decision and Control, vol. 2, pp. 1932–1937, IEEE, 2004.
[37] D. A. Paley, N. E. Leonard, R. Sepulchre, D. Grunbaum, and J. K. Parrish, “Oscillator models and collective motion,” *Control Systems, IEEE*, vol. 27, no. 4, pp. 89–105, 2007.

[38] G. Antonelli, F. Arrichiello, and S. Chiaverini, “Experiments of formation control with multirobot systems using the null-space-based behavioral control,” *Control Systems Technology, IEEE Transactions on*, vol. 17, no. 5, pp. 1173–1182, 2009.

[39] F. Arrichiello, S. Chiaverini, G. Indiveri, and P. Pedone, “The null-space-based behavioral control for mobile robots with velocity actuator saturations,” *The International Journal of Robotics Research*, vol. 29, no. 10, pp. 1317–1337, 2010.

[40] H. Sadeghian, L. Villani, M. Keshmiri, and B. Siciliano, “Task-space control of robot manipulators with null-space compliance,” *IEEE Transactions on Robotics*, vol. 30, no. 2, pp. 493–506, 2014.

[41] D. van der Walle, B. Fidan, A. Sutton, C. Yu, and B. D. O. Anderson, “Non-hierarchical UAV formation control for surveillance tasks,” in *Proc. of American Control Conference*, pp. 777–782, IEEE, 2008.

[42] S. Mou, M.-A. Belabbas, A. S. Morse, Z. Sun, and B. D. O. Anderson, “Undirected rigid formations are problematic,” *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 2821–2836, 2016.

[43] H. G. de Marina, M. Cao, and B. Jayawardhana, “Controlling rigid formations of mobile agents under inconsistent measurements,” *IEEE Transactions on Robotics*, vol. 31, no. 1, pp. 31–39, 2015.