ON THE RATES OF STEADY, QUASI-STEADY AND IMPULSIVE MAGNETIC RECONNECTION

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(Received August 8, 2018)
Submitted to ApJ Letter

ABSTRACT

Magnetic reconnection (MR) is considered as an important mechanism for particle energization in astrophysical plasma. Analyses of MR often assume the magnetostatic condition, i.e. \( \partial_t = 0 \), but various studies have concluded that MR cannot be steady. Using Maxwell and Poynting equations, we show: 1) Under the Sweet-Parker-Petschek framework, magnetostatic conditions produce contradictory results suggesting steady state cannot be achieved. In addition, fast MR must be compressible and magnetic flux is not conserved; 2) The quasi-steady MR defined as reconnection electric field being constant, i.e., \( \partial_t E = 0 \), but \( \partial_t B \neq 0 \) or equivalently \( \partial_t j \neq 0 \), better describes the asymptotic behavior of non-turbulent Petschek-like MR. The conservation of mean Poynting flux implies that a fast MR does not require strong dissipation in the diffusion region. The upper limit of MR rate for quasi-steady MR is found to be \( \sim \frac{1}{3\sqrt{3}} \sim 0.2 \). 3) For impulsive MR (\( \partial_t B_r \neq 0 \) or \( \partial_t j_r \neq 0 \) and \( \partial_t E_r \neq 0 \)), the MR rate is not bounded by the limit found for quasi-steady MR. The impulsive MR rate can be higher or lower than \( 1/3\sqrt{3} \) depending on factors such as the evolution stages of the MR and turbulence. Our analysis is independent of mass ratio and dissipation mechanism, thus the above conclusions can be applied to MR in pair plasma.

Keywords: magnetic reconnection — acceleration of particles — plasmas—turbulence
1. INTRODUCTION

Magnetic reconnection (MR) is believed to be an important mechanism for particle energization in magnetospheric substorms (Baker et al. 1996; Zelenyi et al. 2010), solar wind (Zank et al. 2014), and solar flares (Benz 2017), and is drawing increasing interests in its possible roles in astrophysical phenomena such as the origin of the solar wind (Gloeckler et al. 2003; Fisk 2003), γ-ray flares in the Crab Nebula (Bühler & Blandford 2014; Blandford et al. 2017) or pulsar nebulae in general, and γ-ray bursts (Kouveliotou et al. 2012; Blandford et al. 2017).

From the beginning, the study of MR is dominated by two independent approaches (Sonnerup 1979; Biskamp 1993): one considers the driven steady MR, which refers to open, externally forced reconnections (Sweet 1958; Parker 1957; Petschek 1964; Sonnerup 1988; Biskamp 1993), while the second approach concerns unsteady spontaneous MRs that arise from internal current instabilities whose dynamical evolutions only weakly depend on the external coupling (Dungey 1961; Coppi et al. 1966; Galeev 1979). The steady MR approach has attracted wide interests since in many systems, the size of the reconnection region is much smaller than the spatial scale of the system. The coupling between the reconnection region and the external system occurs through the boundary condition imposed on the subsystem. In the collisional Sweet-Parker (Sweet 1958; Parker 1957) and Petschek (Petschek 1964) models, the small region where the ideal MHD frozen-in condition \( \mathbf{E} + \mathbf{U} \times \mathbf{B}/c = 0 \) breaks is called the diffusion region (DR). Later the steady MR model is expanded to include non-collisional terms in the generalized Ohm’s law that break the frozen-in condition (Vasyliunas 1975; Sonnerup 1988; Gurnett & Bhattacharjee 2005; Che et al. 2011). These non-ideal terms include the non-gyrotrropic pressure gradient, the convective momentum transport, the Hall effect, and the anomalous dissipation due to kinetic-scale turbulence.

The fundamental issue in MR is how to achieve the fast magnetic energy conversion seen in observations. The generalized Sweet-Parker and Petschek models offer the theoretical framework to address the problem. The normalized reconnection rate, defined as \( \dot{R} = U_I/c_A \), where \( U_I \) is the speed of inflow plasma from the external system into the diffusion region, and \( c_A \) is the Alfvén speed. Constraints of reconnection rate come largely from numerical simulations, particularly particle-in-cell (PIC) simulations. Some simulations seem to suggest that fast collisionless MR is controlled by Hall effect and has a universal rate 0.1 (Shay et al. 1999). Other simulations show that anomalous effects can accelerate MR processes to be faster than the Hall MR rate (Bhatotcharjee et al. 1999; Che et al. 2011; Che 2017; Muñoz & Büchner 2017) but not always (Daughton et al. 2011; Le et al. 2018). In relativistic pair plasma simulations in which hall effect is zero due to the equal mass of particles, \( U_I/c \) can reach as high as 0.6 (the relativistic \( c_A \) is smaller than \( c \))(Blandford et al. 2017; Papini et al. 2018).

Direct observations of the plasma inflow show that MRs in solar flares are unsteady and the rates vary from 0.01-0.5 (Su et al. 2013), while indirect measurements of the inflow using the motion of magnetic flux tubes at the foot-points of magnetic loops, assuming the MRs being steady and the magnetic fluxes conserved, found the MR rates < 0.1 (Qiu et al. 2002, 2004). A large number of unsteady reconnections called fast flux transfer events (FTEs), have been discovered in the magnetopause since 1970s (Russell & Elphic 1978). In situ Magnetospheric Multiscale Science (MMS) observations of the magnetopause MR events show that the reconnection rate can be > 0.1 facilitated by anomalous effects (Torbert et al. 2017). The rates for impulsive MR in laboratory plasma lie in a large range varying from 0.01 to >0.5 (Fox et al. 2011; Dorfman 2012).

How to reconcile these seemingly controversial results is a profound challenge to our understanding of MR. Studies on MR rate often assumes the magnetostatic condition, i.e. \( \partial_t = 0 \). However, rigorous calculations have shown that steady solutions in the DR, and the boundary conditions can not be self-consistently obtained in the Sweet-Parker model (Biskamp 1993), and the Petschek-like MR is intrinsically not steady (Syrovatskii 1971; Zelenyi et al. 2010). Kulsrud (2001) showed that Petschek MR is equivalent to Sweet-Parker MR if steady condition is imposed, implying MR can not be steady. This has been demonstrated in resistive MHD numerical simulations, which show that steady Sweet-Parker MR is unrealistic (Birn & Hesse 2001), and Petschek-like MR can only be achieved when the resistivity and electric field are centralized near the null-point (Sato & Hayashi 1979; Birn & Hesse 2001). It is obvious that a non-uniform \( E_z \) requires \( \partial_z B \neq 0 \).

In this letter using Maxwell and Poynting equations, we demonstrate that under the Sweet-Parker-Petschek (SPP) framework, steady MR ansatz causes contradictory results, indicating that MR is not intrinsically steady. Quasi-steady MR, defined as reconnection electric field \( \partial_t E_r = 0 \) but reconnection magnetic field \( \partial_t B_r \neq 0 \) (or equivalently the associated current density \( \partial_t j_r \neq 0 \)), better describes the “asymptotic” behavior of non-turbulent MR. The upper limit of reconnection rate for quasi-steady MR is found to be \( 1/3 \sqrt{3} \approx 0.2 \).
turbulent/impulsive reconnection where $\partial_t \mathbf{B}_r \neq 0$ and $\partial_t \mathbf{E}_r \neq 0$, the reconnection rate can be higher than this limit as the magnetic flux piles up in DR. Our conclusions are independent of the mass ratio and dissipation processes in DR, and are applicable to pair plasma.

2. RECONNECTION RATE

The Poynting equation

$$\partial_t W + \mathbf{j} \cdot \mathbf{E} + \nabla \cdot \mathbf{N} = 0,$$

where $W = (B^2 + E^2)/8\pi$ is the field energy, $\mathbf{N} = c\mathbf{E} \times \mathbf{B}/4\pi$ is the Poynting vector and $\mathbf{j} \cdot \mathbf{E}$ the plasma heating, describes two essential processes in MR: the electromagnetic energy conversion in the DR and the transport of Poynting flux into and out of the DR. In this section we show that the Poynting equation provides a short-cut to constrain the reconnection rate without the need to consider the dissipation mechanism in the DR.

Although MR in general is 3D, only the anti-parallel magnetic field components $\mathbf{B}_r$ are involved in the field annihilation, the guide-field $\mathbf{B}_g$ on the other hand may affect the physical processes inside the DR (Swisdak et al. 2005; Sauppe & Daughton 2018). MR with a guide-field is also known as component reconnection (Swisdak et al. 2005). While the original SPP framework describes MR in 2D, it can be considered as a model for the reconnection of the anti-parallel component in a 3D MR. Following the common practice in space and astrophysical plasma (van Ballegooijen 1985; Biskamp 1993; Yamada et al. 2010; Boozer 2018), let $\mathbf{B}_g = B_g \hat{x} + B_g \hat{y}$ be in the $xy$ plane (Fig. 1), and $\mathbf{B}_g = B_g \hat{z}$ in the direction perpendicular to the reconnection plane and is a constant. SPP reconnection is characterized by an X-Type neutral point magnetic field geometry determined by $\nabla \cdot \mathbf{B} = 0$ (Parnell et al. 1996). The DR can be approximated as a box with dimensions of $2\delta \times 2L$. Inside the DR the magnetic field lines break and reconnect as determined by the generalized Ohm’s law (Vasyliunas 1975). Outside the DR, the MHD ideal frozen-in condition $\mathbf{E} + \mathbf{U} \times \mathbf{B}/c = 0$ holds and determines the transport of the magnetic flux into and out of the DR together with the plasma flow. The separatrices demarcate the inflow and outflow regions. We use subscripts/superscripts “I” and “O” to denote quantities in the inflow and outflow regions, respectively. The reconnecting magnetic field in the upper inflow region $\mathbf{B}_I$ is in the $x$-direction and $\mathbf{B}_O = B_O \hat{y}$. From the frozen-in condition, we have in the upper inflow region $E_z^I = -U_I B_I/c\hat{z}$ and in the right outflow region $E_z^O = -U_O B_O/c\hat{z}$.

2.1. Can Magnetic Reconnection Be Steady?

It is commonly assumed that after a fast onset phase MR can eventually reach a steady state, i.e., $\partial_t = 0$, when the reconnection electric field peaks and the magnetic flux brought into the DR by the frozen-in plasma flow balances the merging of the magnetic field inside. This assumption implicitly excludes turbulent MR. In the following we show that the steady state ansatz can produce conflicting results, indicating magnetic reconnection cannot be steady.

$$\partial_t \mathbf{B}_r = 0$$ reduces the Faraday’s law to $\nabla \times \mathbf{B} = 0$, or

$$\partial_z E_z = 0, \quad \partial_y E_z = 0.$$  

The steady Ampere’s law becomes $\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}$. Obviously, the current is also steady, i.e.,

$$\partial_t \mathbf{j} = 0.$$  

Eq. (2) implies that $E_z$ is a constant inside and outside the DR. Using the frozen-in condition, $E_z^I = E_z^O$ gives

$$U_I B_I = U_O B_O,$$  

i.e., the inflow and outflow magnetic fluxes are balanced. This implies that the magnetic flux is conserved (Newcomb 1958), i.e.,

$$\nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}) = 0.$$  

in the DR as the inflow magnetic fluxes move into the null region and out to the outflow region after the field-line reconnection.

Near the X-type neutral point with opening angle $\alpha$, it is easy to show

$$\frac{B_O}{B_I} = \frac{\delta}{L}.$$  

Combining with Eq. (4) we have
\[ \frac{U_I}{U_O} = \frac{\delta}{L}. \] (7)

Steady fluid equations imply that the magnetic pressure can accelerate outflow speed to \( c_A \), and the reconnection rate \( U_I/c_A \leq 1 \). Eq. (7) is the well-known scaling-law of the Sweet-Parker MR under the incompressible condition \( \nabla \cdot U = 0 \) (Sweet 1958; Parker 1957). In other words, steady state implies incompressibility.

Now we investigate the magnetic flux conservation, i.e., Eq. (5) inside the DR. Let’s consider a small region inside the DR adjacent to the inflow boundary. The \( x \)-component of \( U \) and \( y \)-component of \( B \) are negligible, thus \( U = U_y \hat{y}, \quad B = B_x \hat{x}. \)
\[ \nabla \cdot U = 0 \quad \text{and} \quad \nabla \cdot B = 0 \]
reduce to
\[ \frac{\partial U_y}{\partial y} = 0, \quad \frac{\partial B_x}{\partial x} = 0. \] (8)

Expanding Eq. (5), and taking into account \( \nabla \cdot B = 0, \nabla \cdot U = 0 \), and \( \nabla \times E = 0 \), we obtain:
\[ B_x \frac{\partial U_y}{\partial x} \hat{y} - U_y \frac{\partial B_x}{\partial y} \hat{x} = 0. \] (9)

In the DR, for \( y \neq 0, B_x \neq 0 \) and \( U_y \neq 0 \). Therefore,
\[ \frac{\partial U_y}{\partial x} = 0, \quad \frac{\partial B_x}{\partial y} = 0. \] (10)

Thus \( U_y \) and \( B_x \) must be non-zero constants. At the upper and lower boundaries of the DR, \( B_x = B_I, U_y = U_I, \) and \( E_x \) is also a constant as shown earlier. This implies that the only solution to Eq. (9) is the frozen-in condition \( E_x + U_I \times B_I/c = 0 \), suggesting the small region should not be in the DR. Thus we need to redefine a smaller DR that does not include the small region we have carved out. Repeat this process and eventually the DR become infinitesimally small and we rule out the existence of the DR. This contradiction is clearly a consequence of the steady-state assumption. Similar inconsistencies are found in solutions of MHD moment equations of steady MR (Syrovatskii 1971; Biskamp 1993; Zelenyi et al. 2010). That steady conditions prohibit MR is consistent with the resistive MHD MR simulations with a uniform resistivity whose rates are found to be consistently \( \ll 0.1 \) and negligible (Birn & Hesse 2001).

A corollary of our results is that SPP MR is compressible and the magnetic flux is not conserved.

2.2. Reconnection rate of Quasi-steady Magnetic Reconnection

We now relax the requirement of \( \partial_t B = 0 \), and investigate the rate of quasi-steady non-turbulent Petschek-like MR that satisfies \( \partial_t B \neq 0 \) or equivalently \( \partial_t j \neq 0 \), but \( \partial_t E_z = 0 \).

From the Faraday’s Law \( \nabla \times E = -\frac{1}{c} \partial_t B \), the mean \( E_z \) inside the DR is
\[ E_z = -\frac{\delta}{c} \frac{\Delta B}{\Delta t} \hat{z}, \] (11)
where \( E_z \) is estimated at \( y = \pm \delta/2 \). From \( \nabla \times B = \frac{4\pi}{c} j \hat{z} \), the mean current density is
\[ \bar{j}_z = -\frac{c}{4\pi} \frac{B_I}{\delta} \hat{z}, \] (12)
where we neglect the contributions from \( E_x, j_x, E_y \) and \( j_y \) associated with the spatial and temporal variations of \( B_y \). Since \( B_y \) does not participate in the field line merging and \( \partial_t B_y \approx 0 \), we then have
\[ \bar{j}_z \cdot E_z = -\frac{c}{4\pi} \frac{B_I}{\delta} \hat{z} \cdot \frac{\Delta B}{\Delta t} \hat{z} = \frac{B_I \Delta B}{4\pi \Delta t}. \] (13)

The mean decrease of the electromagnetic energy \( \Delta W/\Delta t \) is approximately
\[ \frac{\Delta W}{\Delta t} = -\frac{B_I \Delta B}{4\pi \Delta t}. \] (14)

Using \( j_z E_z \approx \bar{j}_z E_z \), the Poynting equation is approximately \( \nabla \cdot \mathbf{N} = -\partial_t \mathbf{W} - j_z \mathbf{E}_z \), and therefore inside the DR we have
\[ \nabla \cdot \mathbf{N} = 0. \] (15)

Eq.(15) shows that quasi-steady reconnection is a rather delicate state in which the Poynting flux is conserved inside the DR. To achieve such a state the thermal dissipation in the DR must be small so that the annihilated magnetic field is regenerated through the increase of the electric current. Large thermal dissipation in the DR such as collisional resistivity or anomalous resistivity, on the other hand, impedes the increase of the current and tips the balance of the Poynting flux in the DR.

The conservation of Poynting flux \( \int \mathbf{N} \cdot d\mathbf{S} = 0 \) yields:
\[ \frac{|N_O|}{L} = |N_I| \frac{1}{\delta}. \] (16)
By definition
\[ \mathbf{N}_O = -\frac{c}{4\pi} \mathbf{E}_z^O \mathbf{B}_O, \mathbf{N}_I = \frac{c}{4\pi} \mathbf{E}_z^I \mathbf{B}_I. \] (17)
Then we get
\[ \frac{U_I}{U_O} = \left( \frac{\delta}{L} \right)^3, \] (18)
where we used the frozen-in condition \( \mathbf{E}_z^O = -B_O U_O/c \hat{z}, \mathbf{E}_z^I = -B_I U_I/c \hat{z}, \) and \( B_O/B_I = \delta/L \).
We now show that the mean reconnection rate and the aspect ratio of the DR $\delta/L$ can be estimated in quasi-steady MR. $|E_z|$ peaks in the midplane and decreases towards the boundary of the inflow region, thus $|E_z^{\alpha}|$ along the midplane is close to the maximum of $|E_z|$ and $|E_z^{\beta}|$ is close to the minimum of $E_z$. We approximate $E_z \approx (E_z^{\alpha} + E_z^{\beta})/2$, plug it into Eq. (11) and we have

$$\frac{B_tU_1 + B_0U_O}{2c} \sim \frac{\delta \Delta B}{c \Delta t}. \quad (19)$$

During $\Delta t$ the total change of magnetic field in the DR due to the annihilation of the anti-parallel $B_t$ is $\Delta B = 2B_t$, and hence

$$\frac{4\delta}{\Delta t} \sim U_1 + \frac{B_0}{B_t}U_O = U_1 + \frac{\delta}{L}U_O. \quad (20)$$

Using the relation $\delta/\Delta t = U_1$ and $U_O/U_1 = (L/\delta)^3$, we obtain

$$\frac{\delta}{L} \sim \frac{1}{\sqrt{3}}, \quad (21)$$

and the rate of Petschek-like MR is

$$\frac{U_t}{U_O} \sim \frac{1}{3\sqrt{3}} \approx 0.2. \quad (22)$$

It should be noted that since the shrink of current sheet due to $\partial_t j_r \neq 0$ costs part of the released magnetic energy, the ram pressure $mn_0U_t^2/2$ no longer balances the magnetic pressure $B^2/8\pi$ as in steady MR, and hence $U_O \leq c_A$, thus the reconnection rate satisfies

$$R = \frac{U_t}{c_A} \leq \frac{1}{3\sqrt{3}} \approx 0.2. \quad (23)$$

Finally we look at how variable the magnetic field is in quasi-steady MR. Using the frozen-in condition in the inflow region to replace $E_z$ in Eq. (11), we obtain

$$\frac{\Delta B/B_t}{\Omega_t \Delta t} \sim \frac{U_1/c_A}{\delta/d_i} \leq \frac{0.2}{\delta/d_i}, \quad (24)$$

$$\frac{E_z/E_0}{\delta/d_i} \leq 0.2, \quad (25)$$

where $E_0 = B_t c_A/c$, $d_i$ is the ion inertial length and $\Omega_i$ is the ion gyro-frequency. For a current sheet with width $\sim d_i$, the magnetic field varies by $\leq 20\%$ over $t \sim \Omega_i^{-1}$. The corresponding spatial gradient of $E_z$ is also $\leq 20\%$.

### 2.3. Unsteady Magnetic Reconnection

If $\partial_t B_r \neq 0$ and $\partial_t E_r \neq 0$, MR becomes unsteady or impulsive, and the reconnection rate is not bounded by the limit we found for quasi-steady MR. Simulations of unsteady turbulent MR show that the rate can indeed exceed $1/3\sqrt{3} \sim 0.2$ (Che 2017; Blandford et al. 2017), but not all turbulent reconnections have high rates. Under what circumstance could the reconnection rate exceed $0.2$? Let’s consider a turbulent MR in which the mean field reaches a “quasi-steady state”, but some instabilities in the current sheet generate high-frequency waves, so that the MR is unsteady. In this case, we can split $E$, $B$, and $j$ into the slow and fast changing parts, so that $E = \langle E \rangle + \delta E$, and $\langle \delta E \rangle = 0$, etc., where $\langle \ldots \rangle$ represents the ensemble average. The turbulent part of the Poynting equation becomes

$$\partial_t \langle \delta W \rangle + \langle \delta j \cdot \delta E \rangle + \nabla \cdot \langle \delta N \rangle = 0, \quad (26)$$

where $\langle \delta W \rangle = (\langle \delta B^2 \rangle + \langle \delta E^2 \rangle)/8\pi$, and $\langle \delta N \rangle = c\langle \delta E \times \delta B \rangle/4\pi$. In the DR, Anomalous turbulence effects generated by internal current instabilities, whose growth timescale is much shorter than the MR evolution timescale $\sim L/c_A$, enhance the magnetic field and non-thermal plasma heating by wave-particle interactions so that $\nabla \cdot \langle \delta N \rangle < 0$. Examples include anomalous resistivity produced by electrostatic instabilities (Yamada et al. 2010; Che 2017), anomalous viscosity produced by electromagnetic instabilities such as electron velocity shear instability (Che et al. 2011). Since the mean-field is in a “quasi-steady state” so that $\nabla \cdot \langle N \rangle = 0$, we have $\nabla \cdot \langle N \rangle < 0$ in the DR, consequently $U_t/U_O > (\delta/L)^3 > 0.2$.

Clearly, anomalous turbulence effects do not necessarily result in high reconnection rate if the mean field MR does not reach a quasi-steady state or the turbulent enhancement is not strong enough. This is why some turbulent PIC MR simulations show anomalous effects significantly accelerate reconnection while others do not.

### 3. CONCLUSIONS AND DISCUSSIONS

In this letter we revisited the rate of MR under the SPP framework. This model is particularly useful in open astrophysical environment. We show that the couplings between the reconnection electric field and the current, and the electromagnetic energy flux transfer determine the rate of MR. The main conclusions are: 1) The magnetostatic ansatz, i.e., $\partial/\partial t = 0$, leads to contradictory results for SPP MR. This suggests that the steady MR is not possible; 2) Steady state implies magnetic-flux conservation and incompressibility in the MR. A corollary of the first conclusion is that SPP MR is compressible and the magnetic flux is not conserved; 3) Non-turbulent Petschek-like MR can be quasi-steady, i.e., the reconnection electric field satisfies $\partial_t E_r \sim 0$ but the reconnecting magnetic field $\partial_t B_r \neq 0$ or the associated current density $\partial_t j_r \neq 0$. The time variation
of magnetic field is limited by $\Delta B / B_1/\Omega t \leq 0.2 / \delta d_i$. The characteristic of quasi-steady MR is the Poynting flux being nearly conserved in the DR, implying that the dissipation in the DR being small. The MR rate for quasi-steady MR is $U_1/c_A \leq (\delta / L)^3 \leq 1 / 3 \sqrt{3} \sim 0.2 \cdot 4$ For impulsive MR driven by internal current instabilities in which $\partial_t B_r \neq 0$ and $\partial_t E_r \neq 0$, the rate can be higher or lower than 0.2. These results are applicable to both 2D and 3D MR. Guide field may affect the detailed processes in the DR which may affect the reconnection rate (Sauppe & Daughton 2018). However, the conclusions regarding steady, quasi-steady and unsteady MR should not change qualitatively. Note that the equations in this analysis are intrinsically relativistic, and our analysis is independent of the mass ratio and the dissipation processes in the DR, thus the above conclusions are applicable to MR in relativistic pair plasma.

The near conservation of Poynting flux $\nabla \cdot \mathbf{N} \sim 0$ in quasi-steady MR means the dissipation inside the DR must be small, and the annihilated magnetic field is recovered by the reconnection electric field through the inertia $\partial_t I_j \neq 0$. Therefore, the width of the current sheet can not be constant. For example, collisionless MR can be fully supported by inertia without dissipation (Boozer 2018). The current sheet may shrink until it becomes unstable to instabilities driven by the intense magnetic/velocity shears, and subsequently the instabilities may broaden the current sheet. Various non-ideal effects, such as non-gyrotropic pressure and convective momentum transport (Vasyliunas 1975; Kuznetsova et al. 2001) may slow the narrowing of the current sheet on electron inertial scale, but cannot fully stabilize the current sheet since the reconnection electric field centralizes in the electron DR and globally is non-uniform.

In resistive MR, $\nabla \cdot \mathbf{N} \sim 0$ implies a high Lundquist number $S \propto 1 / \eta$, and when $S$ is larger than the corresponding critical value, tearing instability is triggered and the reconnection becomes impulsive (Loureiro & Uzdensky 2016).

Impulsive MR behave like quasi-steady non-turbulent MR when the turbulence fully decays i.e. $\langle \delta N \rangle \sim 0$, or evolves into the fully developed state with the correlation scale comparable or larger than the size of the DR. In the latter case the turbulence effect is close to uniform spatially and thus $\nabla \cdot \langle \delta \mathbf{N} \rangle \sim 0$.

Solar flares are unsteady and commonly impulsive (Fletcher et al. 2011). Assuming conservation of magnetic flux for such systems when measuring reconnection rate can underestimate the merging rate of magnetic field. This may explain the apparent discrepancy between Su et al. (2013) and Qiu et al. (2002, 2004).

HC would like to thank Roald Sagdeev for the constructive discussions. HC also likes to thank the helpful discussions with Russell Kulsrud on the anomalous resistivity, Lev Zeleny on the stability of MR; David Seibek on the unsteady MR in magnetopause, Jiong Qiu and Brian Dennis on the observations of reconnection rate in solar flares, Joachim Birn and Michael Hesse on the compressible MHD simulations of MR. HC also thanks the anonymous referees for the insightful comments that help to improve the clarity of this manuscript. HC is partly supported by NASA grant No. NNX17AI19G and MMS project.

REFERENCES

Baker, D. N., Pulkkinen, T. I., Angelopoulos, V., Baumjohann, W., & McPherron, R. L. 1996, Journal of Geophysical Research: Space Physics, 101, 12975. https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/95JA03753

Benz, A. O. 2017, Living Reviews in Solar Physics, 14, 2

Bhattacharjee, A., Ma, Z. W., & Wang, X. 1999, J. Geophys. Res., 104, 14543

Birn, J., & Hesse, M. 2001, J. Geophys. Res., 106, 3737

Birn, J., Drake, J. F., Shay, M. A., et al. 2001, J. Geophys. Res., 106, 3715

Biskamp, D. 1993, Nonlinear magnetohydrodynamics (Cambridge Monographs on Plasma Physics, Cambridge [England]; New York, NY: Cambridge University Press, −c1993)

Blandford, R., Yuan, Y., Hoshino, M., & Sironi, L. 2017, SSRv, 207, 291

Boozer, A. H. 2018, Journal of Plasma Physics, 84, 715840102

Bühler, R., & Blandford, R. 2014, Reports on Progress in Physics, 77, 066901

Che, H. 2017, Physics of Plasmas, 24, 082115

Che, H., Drake, J. F., & Swisdak, M. 2011, Nature, 474, 184

Coppi, B., Laval, G., & Pellat, R. 1966, PhRvL, 16, 1207

Daughton, W., Roytershtein, Y., Karimabadi, H., et al. 2011, Nature Physics, 7, 539

Dorfman, S. E. 2012, PhD thesis, Princeton University

Dungey, J. W. 1961, Physical Review Letters, 6, 47

Fisk, L. A. 2003, J. Geophys. Res., 108, 11578

Fletcher, L., Dennis, B. R., Hudson, H. S., et al. 2011, SSRv, 159, 19

Fox, W., Bhattacharjee, A., & Germaschewski, K. 2011, Phys. Rev. Lett., 106, 215003. https://link.aps.org/doi/10.1103/PhysRevLett.106.215003
Galeev, A. A. 1979, SSRv, 23, 411
Gloeckler, G., Zurbuchen, T. H., & Geiss, J. 2003, J. Geophys. Res., 108, 1158
Gurnett, D. A., & Bhattacharjee, A. 2005, Introduction to Plasma Physics, 462
Kouveliotou, C., Wijers, R. A. M. J., & Woosley, S. 2012, Gamma-ray Bursts
Kulsrud, R. M. 2001, Earth Planet. Space, 53, 417
Kuznetsova, M. M., Hesse, M., & Winske, D. 2001, J. Geophys. Res., 106, 3799
Le, A., Daughton, W., Ohia, O., et al. 2018, Physics of Plasmas, 25, 062103
Loureiro, N. F., & Uzdensky, D. A. 2016, Plasma Physics and Controlled Fusion, 58, 014021
Muñoz, P. A., & Büchner, J. 2017, ArXiv e-prints, arXiv:1705.01054
Newcomb, W. A. 1958, Annals of Physics, 3, 347
Papini, E., Landi, S., & Del Zanna, L. 2018, ArXiv e-prints, arXiv:1801.10534
Parker, E. N. 1957, J. Geophys. Res., 62, 509
Parnell, C. E., Smith, J. M., Neukirch, T., & Priest, E. R. 1996, Physics of Plasmas, 3, 759
Petschek, H. E. 1964, in The Physics of Solar Flares, ed. W. N. Hess, 425–+
Qiu, J., Lee, J., Gary, D. E., & Wang, H. 2002, ApJ, 565, 1335
Qiu, J., Wang, H., Cheng, C. Z., & Gary, D. E. 2004, ApJ, 604, 900
Russell, C. T., & Elphic, R. C. 1978, SSRv, 22, 681
Sato, T., & Hayashi, T. 1979, Phys. Fluid, 22, 1189
Sauppe, J. P., & Daughton, W. 2018, Physics of Plasmas, 25, 012901
Shay, M. A., Drake, J. F., Rogers, B. N., & Denton, R. E. 1999, Geophys. Res. Lett., 26, 2163
Sonnerrup, B. U. Ò. 1979, in Space Plasma Physics: The Study of Solar-System Plasmas. Volume 2, 879–+
Sonnerrup, B. U. Ò. 1988, Computer Physics Communications, 49, 143
Su, Y., Veronig, A. M., Holman, G. D., et al. 2013, Nature Physics, 9, 489
Sweet, P. A. 1958, in IAU Symp. 6: Electromagnetic Phenomena in Cosmical Physics, ed. B. Lehnert, 123–134
Swisdak, M., Drake, J. F., Shay, M. A., & Mcllhargey, J. G. 2005, J. Geophys. Res., 110, 5210
Syrovatskii, S. I. 1971, Soviet Journal of Experimental and Theoretical Physics, 33, 933
Torbert, R. B., Burch, J. L., Argall, M. R., et al. 2017, Journal of Geophysical Research: Space Physics, n/a. http://dx.doi.org/10.1002/2017JA024579
van Ballegooijen, A. A. 1985, ApJ, 298, 421
Vasyliunas, V. M. 1975, Reviews of Geophysics and Space Physics, 13, 303
Yamada, M., Kulsrud, R., & Ji, H. 2010, Reviews of Modern Physics, 82, 603
Zank, G. P., le Roux, J. A., Webb, G. M., Dosch, A., & Khabarova, O. 2014, ApJ, 797, 28
Zelenyi, L. M., Artemyev, A. V., Malova, K. V., Petrukovich, A. A., & Nakamura, R. 2010, Physics Uspekhi, 53, 933