FORM FACTOR OF THE RELATIVISTIC SCALAR BOUND STATE CALCULATED IN MINKOWSKI SPACE

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We have calculated the electromagnetic elastic form factor of a two body scalar bound state. The Bethe-Salpeter equation is solved directly in the Minkowski space using the Perturbation Theory Integral Representation. At soft coupling regime the obtained results are compared with those following from a quasipotential spectator (Gross) approximation.

1 Introduction

Relativistic bound states manifest themselves as poles of the scattering matrix at $P^2 = M^2$, where $P$ is the total momentum of the system and $M$ is the rest mass of the bound state. The residua at those poles define so-called vertex functions $\Gamma(p, P)$ satisfying the Bethe-Salpeter equation (BSE),

$$\Gamma(p, P) = i \int \frac{d^4k}{(2\pi)^4} V(p, k, P) G_0(k, P) \Gamma(k, P),$$

(1)

where $p$ and $k$ are relative four-momenta of the constituent pair in final and intermediate states, and the free two-particle propagator $G_0(k, P) = g(k + P/2, m_1)g(k - P/2, m_2)$ is a product of renormalized one body propagators $g$. The kernel $V$ is a sum of all Bethe-Salpeter irreducible diagrams.

For solution, the BSE is usually analytically continued (with the help of the Wick rotation) into the Euclidean space. This avoids the singularities in the kernel and propagators and standard numerical techniques can be employed. An interesting alternative has been developed by Kusaka and Williams. It is based on the Perturbation Theory Integral Representation (PTIR), i.e., on the fact that any n-point Green function can be expressed as a unique integral over spectral variables. Then, the known structure of the singularities can be factorized and the BSE is cast into the real finite integral equation for the real vertex weight function. This approach is, in principle, applicable also for complicated kernels and/or for the case when the propagators are arbitrarily dressed, which may make the Wick rotation difficult or impossible. However, the derivation of the spectral decomposition is nontrivial and has been so far performed only for kernels induced by cubic type - derivative free interaction of scalars.
In this contribution we present our calculation of the electromagnetic (e.m.) form factor of a bound state of two scalars, calculated with the help of PTIR. Our calculations are done with the simplest ladder kernel, only the s-wave ground state being considered. The results are compared with those obtained in the 3-dimensional quasipotential (QP) spectator (Gross) reduction of the BSE.

2 Formalism

The interaction lagrangian of our simple scalar model is

$$\mathcal{L}_{\text{int}} = -g \psi_1(x)\phi(x) - g \psi_2^*(x)\psi_2(x)\phi(x) - e^2 \psi_2^*(x)\psi_2(x)A_\mu(x)A^\mu(x) - e [\psi_2^*(x)\partial_\mu\psi_2(x) + \partial_\mu\psi_2^*(x)\psi_2(x)]A^\mu(x),$$

where $g$ is a (strong) coupling constant, $\psi_1$ and $\psi_2$ are fields of massive scalar particles (with masses $m_1$ and $m_2$), neutral and charged, respectively, $\phi$ is the lighter scalar field mediating the strong interaction, and $A_\mu(x)$ is a photon field. In ladder approximation the t channel kernel is given by

$$V(p, k, P) = \frac{g^2}{m_\phi^2 - (p - k)^2 - i\epsilon}.$$ 

The spectral decomposition of the bound state vertex function reads

$$\Gamma(p, P) = \int_0^\infty d\alpha \int_{-1}^1 dz \rho^{[2]}(\alpha, z) \frac{\rho^{[2]}(\alpha, z)}{m^2 + \alpha - (p^2 + zP \cdot P + \frac{P^2}{4}) - i\epsilon}.$$ 

The general scattering kernel can be expressed as a rather complicated multidimensional spectral integral (for details see [1,2]). From the BSE it then follows for the weight functions:

$$\frac{1}{\lambda} \rho^{[2]}(\alpha, z) = \int_0^\infty d\alpha \int_{-1}^1 dz K^{\text{tot}}(\alpha, z, \alpha, z) \rho^{[2]}(\alpha, z) \rho^{[2]}(\alpha, z),$$

where $\lambda = g^2/(4\pi)^2$ and where all the scattering kernel and propagators weight functions together with appropriate number of spectral integrals are non-trivially hidden in $K^{\text{tot}}(\alpha, \bar{z}, \alpha, z)$. This integral equation for $\rho(\alpha, z)$ has been solved numerically by iterations.

It is interesting to include an annihilation interaction into (3). Consider a generalized annihilation kernel:

$$V_{ab}(P) = -\int_0^\infty d\gamma \int_0^1 d\xi \frac{\rho_{ab}(\gamma, \xi)}{\gamma - P^2 \xi - i\epsilon}.$$
which includes a tree level $V_{\text{th,tree}} = -g^2/(m_\phi^2 - M^2 - i\epsilon)$. From the assumption that any vertex function can be expressed in the PTIR form it follows a surprising fact that the annihilation term cannot contribute to $\langle \bar{\psi}_1 \psi_1 \rangle$, since BSE is derived assuming a zero width of a bound state. This would mean that the presence of the annihilation term does not affect the bound state spectrum and wave functions. On the other hand, for the bound states of $(\psi_1, \psi_1)$ or $(\psi_2, \psi_2)$ the annihilation channel is open and the bound states decay into two or more $\phi$ particles. Clearly, in this case the spectrum and the vertex functions are affected by the annihilation kernel. It is difficult to take this into account in a self consistent way, since the usual derivation of the BSE is based on the factorization of the S-matrix with the real bound state pole, i.e., without the finite width.

The square of the electromagnetic form factor is an observable quantity. It appears that the well known expression for the e.m. form factor of the bound state described by the BSE can be rewritten in terms of the PTIR weight functions as an integral free of singularities. The resulting expression is lengthy, but easy to use numerically and will be given explicitly elsewhere.

3 Results

Figure 1a illustrates the differences between Bethe-Salpeter and QP results at soft coupling regime. We fix the masses of $\psi_1, \psi_2, \phi$ to be equal to the masses of the neutron, proton and pion, respectively (actually, we use $m_n = m_p = 939$ MeV, $m_\phi = 138$ MeV). The coupling constant $\lambda$ is for these masses and for $\epsilon_d = 2.3$ MeV given by $\lambda = 0.118$ (in units of $m_p^2$). For QP we also introduce usual phenomenological form factors $(\Lambda^2 - m_\phi^2)/(\Lambda^2 - p^2)$. Obviously, Bethe-Salpeter and Gross form factors have very similar shape, the difference between them is much smaller than the effect due to the modification of the strong interaction by the phenomenological form factors.

Figure 1b shows the e.m. form factors for several values of the fraction of binding $\eta = M/(m_1 + m_2)$ for the choice of masses $m_1 = m_2 = m = 2m_\phi$ also used by Kusaka et al. Recall, that for the deuteron $\eta_d \simeq 0.9988$. Although the fraction of binding varies dramatically, the form factors look rather similar. It seems that their slopes are much more sensitive to the range of interaction rather than its strength.

4 Conclusions and Outlook

In this contribution the elastic e.m. form factors of scalar bound states have been calculated in Bethe-Salpeter and Gross formalism. We also claim that
annihilation term does not influence the spectrum and wave functions of the bound states.

For more realistic systems the complication of spin degrees of freedom has to be considered. We have formally derived similar equation for fermion-antifermion state. Its solution is being developed.

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References
Figure 2: The BS e.m. form factors for the following fraction of binding: \( \eta = 0.1, 0.2, 0.6, 0.8 \). \( Q^2 \) is given in the units of \( m^2 \).

1. See e.g., N. Nakanishi, *Prog. Theor. Phys., Suppl.* 43, 1 (1969) and references therein.
2. K. Kusaka, K. Simpson, A.G. Williams, *Phys. Rev. D* 56, 5071 (1997) and references therein. The authors of this work used \( K(\bar{\alpha}, \bar{z}, \alpha, z) \) which was derived as at least one dimensional certain integral representation. It was shown by one of the author (V.Š.) that this integration can be performed analytically. Avoiding this additional integration significantly decrease the necessary computer time. The appropriate statement will be proved in the article which one is under preparation.*
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* added in November 2001