Some new results on “jet” stopping in AdS/CFT

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How stopping length scales with energy  (massless case)

weak coupling: \( \alpha_s \sim \alpha_s \) small \( l_{\text{stop}} \propto E^{1/2} \) (up to logs)

mixed coupling: \( \begin{align*}
\alpha_s & \quad \text{BIG} \\
\alpha_s & \quad \text{small}
\end{align*} \)
\( l_{\text{stop}} \propto E^{1/2} \) (believed)

all strong coupling: \( \alpha_s = \alpha_s \) BIG \( l_{\text{stop}} \propto E^{1/3} \)
( \( \mathcal{N}=4 \) SYM, etc.)

Interesting: Exponent in \( l_{\text{stop}} \propto E^\nu \) can depend on \( \alpha_s \).
Strongly Coupled Case Revisited

BIG $\alpha_s = \alpha_s$: Large-$N_c$ $\mathcal{N}=4$ SYM, etc. with $N_c \alpha_s \to \infty$
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Strongly Coupled Case Revisited

\[ \alpha_s = \alpha_s : \quad \text{Large-} N_c \quad \mathcal{N} = 4 \quad \text{SYM, etc. with } N_c \alpha_s \to \infty \]
Previous AdS Calculations

Example: Classical string calculation of Chesler, Jensen, Karch, Yaffe (2008)
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**Something slightly dissatisfying:** What is the ?

The initial configuration has been expressed in the gravity dual. How precisely do I set up the problem in the 3+1 dim. field theory?

- **AdS₅– Schwarzschild**
Our Method

In the field theory, think impressionistically of

\[ \gamma^* \rightarrow_{\text{large } E \approx p} q - \bar{q} \]  

or  

\[ W^+ \rightarrow_{\text{large } E \approx p} u - \bar{d} \]

Treat \( \sim \) as a localized external field:

\[ \mathcal{L}_{\text{QFT}} \rightarrow \mathcal{L}_{\text{QFT}} + \mathcal{O}(x) \Lambda_L(x) e^{i\vec{k} \cdot x} \]

with \( \vec{k}^\mu = (E, 0, 0, E) \)

some source operator  
e.g. \( j_\mu(x) \)

smooth envelope function localizing source in space and time

Definition for purposes of this talk:

“jet” = localized, high-\( p \) excitation moving through the plasma.
The response is measured by a **3-point correlator**. A crude way to understand this:

\[
|\text{jet}\rangle = \text{fig} \mid \text{plasma}\rangle.
\]

So we want

\[
\langle \text{jet}\mid \text{fig} \mid \text{jet}\rangle = \langle \text{fig} \mid \text{plasma}\rangle.
\]

For **finite-temperature** AdS/CFT calculations:

- lots in literature on computing 2-point correlators
- almost nothing on 3-point correlators
\[ = 5\text{-dim. SUGRA vertex} \]

\[-\cdots- = \text{a Heun function} \rightarrow \text{hard to make any analytic or numeric progress!} \]

*Fortunately, in our problem, ...*
high-energy source $\rightarrow$ high-$k$ approximation (WKB / geometric optics)

want to observe late-time diffusion $\rightarrow$ low-$k$ approximation

Can do calculation!
Our Result

The farthest a jet will ever go is indeed $\propto E^{1/3}$.

But almost all jets will instead stop sooner at $\propto (EL)^{1/4}$

where $L$ is the size of the space-time region in which the jet was initially created.
Q: What does the size $L$ of the source have to do with it?
A: It determines how off-shell the source is.

\[ \propto \Lambda_L(x) e^{i\vec{k} \cdot x} \quad \text{with} \quad \vec{k}^\mu = (E, 0, 0, E) \]

implies that has Fourier components

Typical stopping distance $(EL)^{1/4}$ really means $(E^2 / q^2)^{1/4}$

where \[ q^2 = \text{typical virtuality of the source} \]