Event generator to construct cross sections for the multiphonon excitation of a set of collective vibrational modes

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The construction of differential cross sections as a function of excitation energy for systems with a collection of low- and high-lying intrinsic vibrational modes has been attempted in the past. A prescription is proposed that simplifies the implementation of such calculation schemes with a remarkable reduction in computational time.

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I. INTRODUCTION

The possibility of exciting collective vibrational modes in heavy ion reactions has captivated the attention of researchers in the field for several decades [1]. Of especial interest has been the challenge of extending the set of participant states (i.e. the open channels) well beyond the familiar low-lying surface modes and into the range of the nuclear giant resonances of the lowest multipolarities. Because of adiabatic considerations these processes required, at the bombarding energies available during the 1970’s and 80’s, very short effective collision times. Only the sharp exponential drop of the nuclear radial formfactors was able to recreate these conditions at that time, a fact that was noted (and exploited) in the early studies of Deep Inelastic Collisions [3, 4].

Nowadays the accelerators provide much higher beam energies and thus the population of giant resonances can also be mediated by the Coulomb excitation mechanism. Under these circumstances the long range of the Coulomb-coupling matrix elements forces us to incorporate – in the theoretical analyses of the process – a considerably larger number of impact parameters or partial waves. This, naturally, increases the chances for exciting the high-energy part of the nuclear response.

Historically, the multiple-phonon excitation of high-lying modes was actively promoted for being the source of characteristic structures in the experimental distribution of cross sections as a function of excitation energy [2]. The study of these patterns became, in turn, a quite convenient source of information for learning about the actual features of the giant resonances (energies of the modes, widths, strengths, anharmonicity, etc).

This link motivated an important body of recent theoretical work [5, 6] where microscopic calculations for the structural aspects of the giant modes have been combined with standard reaction formalisms to yield concrete predictions for the shape of the distributions $d\sigma/dE$. Unfortunately, as it was stated earlier, a much larger number of impact parameters are now needed to compute accurately the nuclear and Coulomb components of the excitation processes. This, together with the fact that there exists a distinct possibility of exciting a multiple number of collective phonons, has resulted into rather complicated and time-consuming coupled-channel schemes.

Upon close inspection of the results, however, one realizes that for practically the entire range of relevant partial waves the excitation probabilities are very small. It is also possible to conclude that, in leading order, the different modes can be considered as being independent from each other. Or, in other words, that one can ignore terms in the hamiltonian that involve simultaneously the coordinates of two or more collective variables $\alpha_\lambda\mu$. Notice that this is not the same as claiming that one works within the perturbation limit; we have already stressed the relevance of multi-step events. Multiphonon processes may occur as the excitation of the same mode (two, three phonons) or as the simultaneous excitation of two or more different modes [2].

Considering the harmonic modes as being essentially uncoupled to each other has a significant practical advantage; it can be exploited to design an event generator that allows for a much simpler, and yet accurate, method for constructing the differential cross sections $d\sigma/dE$. We shall describe this idea in detail in the following sections.

Such an approximation scheme is of course bound to fail for the very central impact parameters. However, these partial waves are unlikely to participate directly in the population of the inelastic channels explicitly taken into account. Within the semiclassical formalism their contribution is, in fact, strongly suppressed by the global absorption associated with the imaginary part of the optical potential. To take into account properly the grazing
impact parameters may perhaps require a complete procedure as the one exploited by the authors of [5]. Notice that this would be necessary, at worst, for only a narrow window of impact parameters; for most of the relevant range (extending up to hundreds of fermis) the method we are proposing in this paper is appropriate and it is precisely here that the major savings of computational time can be achieved.

We elaborate further on these considerations in Sect. II, where we also recall a previous work that proves quite essential to the development of our proposal. In Sect. III we explain what is truly specific about the technique for generating the folding of probabilities that we need. In Sect. IV we take, as an example, the reaction $^{40}\text{Ar} + ^{208}\text{Pb}$ at 40 MeV per nucleon and include two low-lying and three high-lying resonances. The purpose is here mainly to show that the function $d\sigma/dE(E)$ indeed reflects correctly the assumed input to the problem. We reserve Sect. V for a brief summary of the contribution and some closing remarks.

II. BRIEF BACKGROUND

A few years back we developed a general formalism for the excitation of a single collective vibrational mode [7]. We begin the presentation mentioning this reference because it proved quite essential to motivate the present contribution. Exploiting well-tested approximations we were able in [7] to propose a semiclassical prescription to estimate the dynamical effects associated with a spreading width $\Gamma$ of the mode. Modifications in the predicted cross sections due to the presence of an eventual anharmonicity, manifested by an apparent ratio of state energies $\varepsilon$

$$\nu = \frac{\varepsilon(2 \text{ phonon state})}{\varepsilon(1 \text{ phonon state})} \neq 2,$$

were also investigated. We should mention here that the development of this program – aiming mostly to the description of single- and double-phonon giant resonances – was done within the framework of perturbation theory.

Without entering into details in the implementation of ref. [7] let us briefly recall what was the input of the calculation scheme and what could be obtained as a result.

Given a single collective vibrational mode of multipolarity $\lambda$, energy $\hbar\omega_\lambda$, width $\Gamma_\lambda$ and anharmonicity $\nu_\lambda$ a run of the program densely sampling impact parameters $\rho$ belonging to the interval $[\rho_{\text{min}}, \rho_{\text{max}}]$ generated the total differential cross section $d\sigma/dE_\lambda(E)$. The energies of the transitions $0 \rightarrow 1$ phonon $1 \rightarrow 2$ phonon, affected as they are by the anharmonicity factors and the spreading widths, introduced interesting dynamical consequences which were the main object of investigation in [7].

To construct the differential cross sections $d\sigma/dE(E)$ it was necessary to define a procedure to distribute the probabilities of inelastic transitions for the one- and two-phonon vibrational states over the relevant excitation energy ranges. This prescription is quite analogous (except for its generalization from one mode to several modes) to the one we later use for our event generator. We can thus defer its presentation to the next Section.

The effect of the absorption was taken into account in [7] by means of a multiplicative depletion factor that

![FIG. 1: (Color online) Treatment of the absorption. The figure on the top shows the behaviour of $T(b)$ as function of the impact parameter $b$. In the middle frame, the excitation probability of a single phonon is plotted as function of $b$ for the three states reported in the legend. The functions obtained as a product of the two previous quantities is plotted in the figure at the bottom.](image)
rapidly falls to zero as the overlap between the reacting nuclei increases for the lower impact parameters. This is – following standard practice – constructed from an integral along the trajectory \( r(t) \),

\[
T(b) = \exp\left\{ -\frac{2}{\hbar} \int_{-\infty}^{+\infty} W(r(t')) dt' \right\}
\tag{2}
\]

of the imaginary part of the optical potential, \( W \) (see for instance, \[8\] and refs. quoted therein). For the current application the absorptive component was chosen following the prescription of refs. \[7, 9\].

The projection into the subspace of explicitly considered channels is now able to reverse the tendency of the second-order amplitudes to yield too large probabilities for impact parameters at or inside the grazing distance. Notice that this uncomfortable situation would not become obvious when solving for coupled-channel amplitudes (even if the numerics may be equally absurd) because of a prescribed conservation of the norm by the integration algorithm.

The interplay between these contrasting effects can be appreciated in Fig. 1 for the reaction \(^{38}\text{Ar} + ^{208}\text{Pb}\) at 40 MeV per nucleon. The figure shows the attenuation factor that defines what fraction of the contribution of a given impact parameter is actually retained in the inelastic channels. The top frame shows the transmission coefficient \( T(b) \) as a function of impact parameter and puts in evidence that the transition from \( T(b) \approx 1 \) to \( T(b) \approx 0 \) indeed occurs over an interval \( \Delta b \) that spans only a couple of fermis. The middle frame displays the excitation probabilities as a function of the impact parameter in three situations. These are, namely, the low-lying quadrupole mode (full curve), the giant quadrupole resonance (GQR, dashed curve) and a high energy octupole resonance (HEOR, dotted curve). The lower frame corresponds to the actual distribution \( d\sigma/db \), constructed from the information displayed in the two plots above.

### III. FORMALISM

We consider a pair of reacting heavy ions that accumulate a number \( N \) of intrinsic surface vibrational modes. Each one is characterized by their excitation energy \( \hbar \omega_i \), multipolarity \( \lambda_i \), strength \( \beta_i \), and a width \( \Gamma_i \). That is,

\[
\hbar \omega_i, \quad \lambda_i, \quad \beta_i, \quad \Gamma_i \quad \text{with} \quad 1 \leq i \leq N.
\tag{3}
\]

We proceed immediately to define two separate groups of these modes: high- and low-lying modes. The mean reason for establishing the subdivision has to do with the way people in the field have traditionally dealt with their widths. The differentiation is actually an old story that dates back to the 70’s and the use of surface vibrational models to describe specific dynamical features of Deep Inelastic Reactions \[3, 4\]. It aims to reflect two clear experimental facts:

- **Low-lying modes** (excitation energies \( \hbar \omega_\lambda < 5-6 \text{ MeV} \)) are such that at the zero- and one-phonon level are sharp and display no width. Clearly the energy range quoted above is only qualitative. The modes we have in mind are those, for example, known – in the harmonic oscillator terminology for even multipolarities – as \( \Delta N=0 \) excitations. At the two-phonon level they show a spread which is mainly associated with the anharmonicity of the mode. Let us be a bit more explicit. Suppose we have a low-lying quadrupole mode with \( \hbar \omega_{\lambda=2} \approx 4 \text{ MeV} \). At about double that excitation energy it is found a multiplet of states \( 0^+, 2^+, 4^+ \) spanning an interval of energy that we shall call \( \Delta E_{\lambda=2} \). Typically this quantity has an order of magnitude of about 0.5 MeV.

Finally, the last piece of experimental evidence that one wishes to incorporate in the formalism is that there are practically no known three-phonon states associated with low-lying modes. This can be formally done by assuming that, at the three-phonon level, the mode assumes a width that equals the separation energy \( \hbar \omega_\lambda \).

All of these features are best implemented by assigning to the mode an energy-dependent width (see below).

- **High-lying modes** (i.e. giant resonances). In this situation the zero-phonon state is taken to be sharp, while at the one-phonon level the state displays the well-known spreading width \( \Gamma_\lambda \). The distribution of the excitation amplitudes to higher levels is achieved by a straightforward folding (see below).

The prescription that emerges from the two items listed above may appear, at first, difficult to grasp. However it has led to practical conclusions in the treatment of Deep Inelastic Collisions that are in very good agreement with the experimental evidence. Obviously one could come up with different but somewhat equivalent operating procedures. It is easier and reasonable, however, to adhere to this established practice since the details of its implementation have already been described and tested in the literature (see for example ref.\[4\]). A practical reminder of the spreading prescription as it is applied for low- and high-lying modes is summarized in Table 1.

The event-generator works in practice just like any other similar device in a wide variety of physics sub-

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**TABLE I**: Width prescription as it is used in the construction of the cross section shown in figures 2 and 3.

| \( n_\lambda \) | 0     | 1     | 2     | 3     |
|-----------------|-------|-------|-------|-------|
| high-lying      | 0     | \( \sqrt{2} \Gamma \) | \( \sqrt{3} \Gamma \) | \( \Gamma \) |
| low-lying       | 0     | 0     | \( \Delta E \) | \( E \) |
FIG. 2: (Color online) Width prescription as applied to low- and high-lying modes. The distributions of the total differential cross section \( d\sigma/dE(E) \) is constructed as an example for two separate quadrupole modes with \( \hbar \omega = 3 \) and 12 MeV. In this figure we can appreciate the different prescriptions for low- and high-lying modes summarized in Table I. The giant quadrupole state (\( \Gamma(n = 1) = 5 \) MeV) clearly shows the second phonon with a wide and increasing width. The low-lying mode (\( \Delta E = 1 \) MeV) displays, on the other hand, a much narrower structure at the two-phonon level and has the three phonon structure practically washed out (not visible in the figure). A large number of events and an impact parameter range of 100 fm was used for this illustration.

We take a given impact parameter and consider a very large number of possible “events”, \( N_{ev} \). In every instance one generates the probability of occurrence of each independent mode by “throwing a Poisson dice” \( N \) times, in agreement with an average number of phonons \( \langle n_i \rangle \). This means that the random number generator is designed to return a number of phonons \( n_i \) for each of the independent modes consistent with the law

\[
P(n_i) = \frac{(\langle n_i \rangle)^{n_i}}{(n_i)!} \exp\{-\langle n_i \rangle\}.
\]

The differential cross section we search for is to be constructed for a large range of relevant impact parameters \([\rho_1, \rho_2]\), that are sampled with a uniform interval \( \Delta \rho \). The average number of phonons for the \( i^{th} \) independent collective surface mode is previously calculated, for each impact parameter \( \rho_i \), using the formalism developed in ref. [3]. These figures are collected in an auxiliary data-file that has the structure

\[
\rho_1, \langle n_{i=1} \rangle, ..., \langle n_{i=N} \rangle
\]

\[
... \quad \rho_k, \langle n_{i=1} \rangle, ..., \langle n_{i=N} \rangle
\]

\[
... \quad \rho_2, \langle n_{i=1} \rangle ..., \langle n_{i=N} \rangle.
\]

The smaller impact parameter, \( \rho_1 \) should be such that any possible contribution from it is definitely eliminated by the absorption. Similarly, one should verify that contributions from \( \rho > \rho_2 \) can also be neglected.

Adding to this prepared data-set the characteristic information that specifies the different vibrational modes \( i \) one is ready to run the event generator and construct \( d\sigma(E)/dE \).

For each impact parameter in the file (5) the probability assigned to the current event is, naturally,

\[
P = \prod_{i=1}^{N} P(n_i).
\]

It is in the energy scale that we have to be careful with the character of low-lying vibrational state or giant resonance of the particular mode \( i \). Following the prescription summarized in Table 1 we assign a spread \( \Gamma_i \) to the contribution to the excitation energy, \( \epsilon_i \), of this mode. This quantity is thus defined as

\[
\epsilon_i = n_i \hbar \omega_i + G(0, \Sigma_i),
\]

where \( G(0, \Sigma_i) \) is a random number obtained from a normal distribution with zero centroid and standard deviation \( \Sigma_i \approx \Gamma_i/2.3 \). Slightly different prescriptions could once more be obtained by replacing \( G \) by a similar type of random number generator, but these choices are not of much consequence at the level of approximation we have chosen to maintain.

The total excitation energy for the collection of \( N \) independent modes is then simply given by

\[
E = \sum_{i=1}^{N} \epsilon_i.
\]

abiss to which – in a properly designed histogram – we accumulate a weighted version of the probability previously given in eq. (6). The proper units for the differential cross section as a function of energy are obtained by multiplying that number by \((20\pi \rho_0 \Delta \rho)/(\Delta E N_{ev})\), where \( \rho_0 \) is the impact parameter sampled at this moment, and \( \Delta \rho, \Delta E \) are the extents of the impact parameter mesh and the energy mesh respectively. The final result is then given in mb/MeV and a drawing of this histogram represents the predicted distribution of cross section \( d\sigma(E)/dE \).

To test the validity of our method, a comparison with a more sophisticated method like the one of ref. [2] is in order. In this approach one starts with a Hartree-Fock plus Random Phase Approximation calculation in order to identify the most collective one-phonon states. For each of these chosen states one calculates their transition densities and the corresponding form factors. These are used in a semiclassical coupled channel scheme to determine the excitation probabilities for all the possible one-, two- and three-phonon states that one can construct.

In Table II we compare some results for the two approaches. The calculations are performed for the system
We take a simple example where only two one-phonon states are considered as input: the low-lying $3^-$ state (E=4.9 MeV) and the GQR (E=16.9 MeV). The range of impact parameter used in the calculations (15-100 fm) corresponds to the peripheral region where the Coulomb interaction yields the most important contribution. This is also the region where the excitation probability distribution are of a Poisson type. The results of the two methods are very close.

So, one can envisage a calculation performed in two steps: Make use of the method of ref. [5] in the inner region where the nuclear interaction plays an important role and then use our novel approach in the peripheral region for large impact parameters which is the most time consuming part.

Finally, we would like to stress that in the case one wants to take into account the contribution of anharmonicities, our method does not apply.

IV. APPLICATION

We now proceed to illustrate the possibilities of the event generator with an application to the specific reaction $^{38}$Ar + $^{208}$Pb at 40 MeV per nucleon. We take two low-lying modes (one quadrupole, one octupole) with energies $\hbar \omega$=3 and 5 MeV respectively, and a common value $\Delta E$=1 MeV. The deformation parameter assumes for the low-lying modes a value $\beta=0.1$. Three giant resonances are then added; a dipole, a quadrupole and an octupole mode with energies $\hbar \omega$=18, 17, and 31 MeV and widths $\Gamma$=6, 6, and 8 MeV respectively. The effective spread of all these modes is found in all cases following the prescriptions described in Table 1. Accumulating the cross section for a range of impact parameters from $\rho_{\text{min}}=12$ fm to $\rho_{\text{max}}=100$ fm in steps of 0.5 fm we obtain the distribution shown in Fig. 3.

The characteristics of the curve reflect the assumptions made and, in addition, are reminiscent of what is obtained by the time-demanding method of ref. [5] in similar circumstances. Just to stress the advantage of the proposed method let us note that one can save about two orders of magnitude in computing time by using the event generator.

| states        | events generator | ref.[5] calculations |
|---------------|------------------|-----------------------|
| $3^-$         | 1.10             | 1.05                  |
| GQR           | 58.7             | 58.6                  |
| $3^- \times 3^-$ | $0.11 \times 10^{-3}$ | $0.10 \times 10^{-3}$ |
| GQR $\times$ GQR | $0.14 \times 10^{-1}$ | $0.14 \times 10^{-2}$ |

TABLE II: Total excitation cross section (in mb) calculated with the events generator method (second column) and with the method of ref. [5] (third column) for the states shown in the first column.

V. CONCLUDING REMARKS

The developments described and the results shown in this Report were motivated by sheer necessity. In fact, while the complete calculations performed by Lanza et al. are very important the absolute times involved in the computation of cross sections by the procedure described in ref. [5] are quite large. This becomes even more critical when one takes into account that the practical implementation of such a prescription involves a considerable amount of leeway that can only be sorted out by trying alternative calculations with equally acceptable sets of parameters. That appears to be the only sensible way to learn, by gaining experience, how the various input numbers do indeed affect the calculated distributions of cross section. Even if a final presentation is contemplated with the full procedure of [5] a number of previous calculations exploiting the event generator would be – no doubt – very convenient to prepare the ground.

We consider that the use of an event generator like the one described in these pages is highly advisable for that class of problems that can use suggestive results to judge the soundness of the answers they provide. Actually this should be done even before embarking in sophisticated schemes without the proper benefit of an educated intuition.

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