HEAVY BARYON SPIN 3/2 THEORY
AND RADIATIVE DECAYS OF THE
DECUPELET

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Abstract.

We study the radiative decays of the decuplet \((\frac{3}{2} \rightarrow \frac{1}{2} \gamma)\) using Heavy Baryon Chiral Perturbation Theory (HBChPT). We emphasize the problems faced by the interacting spin \(\frac{3}{2}\) field theory. We argue that, to lowest order in the \(\frac{1}{m}\) expansion, HBChPT provides a framework where R-invariance and the appropriated constraints for the interacting spin \(\frac{3}{2}\) fields are consistently incorporated. We perform a gauge invariant calculation of the decay amplitudes, to lowest order in \(\left(\frac{1}{m}\right)\) and to order \(\frac{\Lambda^2}{\Lambda^2}\) in the chiral expansion and report analytical results for the one-loop contributions to the two form factors involved in the \(\frac{3}{2} \rightarrow \frac{1}{2} \gamma\) transitions. Parameters independent predictions for the SU(3) forbidden decays are presented.

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Chiral Perturbation Theory is a useful tool with which to describe low momentum processes involving Goldstone Bosons \cite{1}. When the formalism is extended to interactions with Baryons (BCHPT) it suffers from two unpleasant characteristics: i) there is no correspondence between the loop and momentum expansion and ii) the expansion parameter turn out to be of order one. A new formalism (HBCHPT) was proposed by Jenkins and Manohar (J-M) which circumvents these problems \cite{2}. The new theory is written in terms of definite velocity baryon fields for which the Dirac equation corresponds to a massless baryon. The Heavy Baryon Lagrangian is expressed in terms of a $\frac{1}{m}$ expansion. The $\frac{1}{m}$ effects in HBCHPT are considered in HBCHPT by the inclusion of higher dimension operators suppressed by inverse powers of $m$.

In addition to the baryon octet, the baryon decuplet is also included \cite{3}. In this case the theory involves spin 3/2 fields whose effective mass is $\Delta m = M - m$ (m-M masses of the baryon octet and decuplet respectively). It is possible to construct an effective theory by integrating out the decuplet fields. Virtual effects of the decuplet in the theory are then considered by higher dimension operators involving only octet baryons and mesons which are suppressed by inverse powers of $\Delta m$. In the real world the decuplet-octet baryon mass difference $\Delta m \approx 300 \text{MeV}$ is small when compared with the hadronic scale of 1GeV. The smallness of $\Delta m$ produces an enhancement of the decuplet effects, therefore it is advantageous to retain explicitly decuplet fields in the effective theory, rather than integrate them out.

The formalism previously described has been recently used by Butler, Savage and Springer (BSS) \cite{4} to carry out a detailed study of the radiative decays of the decuplet ($T \to B\gamma$). The scheme is promising as it provides a systematic field theoretical approach which is able to produce testable predictions. From our point of view, the analysis of BSS can be improved if the following points are addressed:

- It is well known that Quantum Field Theory for spin 3/2 interacting fields suffers of serious inconsistencies. In particular, within the Rarita-Schwinger formalism \cite{6} the R-S spinor $\psi_\mu$ has more degrees of freedom than required, which results in a non-unique classical Lagrangian. In fact there exist a whole family of one-parameter Lagrangians $\mathcal{L}(A)$ from which the Dirac-Fierz-Pauli equation and the necessary free field constraints for $\psi_\mu$ may be obtained \cite{7}.
Furthermore, these Lagrangians remain invariant under point transformations (R-invariance) which mixes the spurious spin 1/2 fields contained in $\psi_\mu$. This invariance ensures the unphysical spin 1/2 degrees of freedom have no observable effects. Generalizations of this scheme to describe interacting spin 3/2 fields requires that the corresponding Lagrangian preserves R-invariance. This introduces new ambiguities into the theory (the so-called “off-shell” parameters) [8,12]. It should also be emphasized that the constraint equations, which achieve the elimination of the unphysical spin 1/2 degrees of freedom in the free case, are no longer valid for interacting fields. As shown below, only the leading order of the $\frac{1}{m}$ expansion is free of the “off-shell” parameters ambiguities.

- The $\frac{1}{m}$ and the chiral $\left( \frac{w}{\Lambda} \right)$ expansions must be separately considered. An interacting spin 3/2 theory which is free of the “off-shell” ambiguities limits to the zeroth order the $\left( \frac{1}{m} \right)$ expansion. On the chiral the expansion must be consistently carried, order by order, including all possible counterterms.

- Existing calculations for $T \rightarrow B\gamma$ amplitudes have been performed in the $\epsilon \cdot v = 0$ gauge. We understand that a non-gauge invariant calculation is valid whenever it is carried in a consistent way. However, a gauge invariant analysis is desirable as it allows an unambiguous determination of the counterterms required, the form factors and thereby of the multipole amplitudes $E1$ and $M2$.

The paper is organized as follows. In section 1 we summarize the kinematics and the invariant amplitudes including the counterterms and the heavy baryon limit. In section 2 we incorporate the electromagnetic interactions to the approach used in [12] to describe the R-invariant interacting spin 3/2 fields. In section 3 we consider Heavy Baryon Chiral Perturbation Theory (HBChPT). The R-invariant Lagrangians as well as the subsidiary conditions on the spin 3/2 field are considered in the light of the $\frac{1}{m}$ expansion. Section 4 is devoted to the radiative decay of the decuplet, details of the calculations are presented as our analytical results differ from previous calculations [4]. Further details on Chiral Perturbation Theory and SU(3) Clebsch-Gordon coefficients are deferred to an appendix.
1.- GENERAL CONSIDERATIONS

Lorentz covariance, gauge invariance and parity dictates the more general form of the invariant amplitude describing the $3/2^+ \rightarrow 1/2^+\gamma$ transitions. For on-shell external baryons the amplitude is parametrized as:

$$iM_{fi} = e\bar{\psi}(q)\Gamma_{\mu\nu}\psi(p)\varepsilon^{\nu},$$

where the vertex tensor $\Gamma_{\mu\nu}$ can be written as:

$$\Gamma_{\mu\nu} = \frac{g_1^D}{2\Lambda}(k_\mu\gamma_\nu - kg_{\mu\nu})\gamma_5 + \frac{g_2^D}{4\Lambda^2} \frac{1}{M + m} (q \cdot kg_{\mu\nu} - k_\mu q_\nu) k\gamma_5.$$

$\Lambda$ is a still unspecified mass scale and $M(m)$ is the $3/2^+(1/2^+)$ baryon mass. This is essentially the characterization used in [9], the only differences are that we use the mass scale $\Lambda$ instead of the spin $1/2$ baryon mass $m$ to normalize the form factors, and $p$ is written in terms of $q$ by using the equations of motion.

The processes we are considering gets contributions from magnetic dipole ($M1$) and electric quadrupole ($E2$) amplitudes. In terms of the form factors $g_1^D, g_2^D$ the multipole amplitudes are given by:

$$M1 = \frac{e}{12\Lambda} \left( \frac{w}{Mm} \right)^{1/2} \left( (3M + m)g_1^D - \frac{M(M - m)}{2\Lambda} g_2^D \right)$$

$$E2 = -\frac{e}{6\Lambda} \frac{w}{M + m} \left( \frac{wM}{m} \right)^{1/2} \left( g_1^D - \frac{M}{2\Lambda} g_2^D \right)$$

where $w$ is the photon energy in the rest frame of the $3/2^+$ particle. The decay width is expressed in terms of the multipoles:

$$\Gamma(T \rightarrow B\gamma) = \frac{w^2}{2\pi} \frac{m}{M} \left[ |M1|^2 + 3|E2|^2 \right].$$

Our purpose in this paper is to work out the predictions of HBChPT, which requires considering the heavy baryon limit of (1) and (2). This is achieved by considering the baryons as fields of definite velocity. Following [3] we write $M = m + \Delta m; \ q = mv + \ell$ and take the $m \rightarrow \infty$ limit to obtain
\[ \Gamma_{\mu\nu}^{HB} = P_+ \Gamma_{\mu\nu} P_+ = \frac{g_1}{2\Lambda} (k_\mu S_\nu - S \cdot k g_{\mu\nu}) + \frac{g_2}{8\Lambda^2} (v \cdot k g_{\mu\nu} - k_\mu v_\nu) S \cdot k \]  

(5)

where \( S_\mu \) is the heavy baryon limit of the spin operator

\[ S_\mu = \frac{i}{2} \gamma_5 \sigma_\mu v^\rho, \]  

(6)

and \( P_+ \) stand for the projector over the positive energy subspace (see Eq. (20) below).

Radiative decays of the decuplet are induced by 1-loop diagrams which are generated with the leading order (dimension 4) HBChPT Lagrangian. Actual calculation (see (24,26) below) shows that the 1-loop induced amplitude is \( \mathcal{O} \left( \frac{\Lambda^2}{\Lambda} \right) \), hence besides the leading order chiral Lagrangian, three counterterms are required since to \( \mathcal{O} \left( \frac{\Lambda^2}{\Lambda} \right) \) one dimension five and two dimension six counterterms contribute to the decuplet radiative decays at tree level.

\[ \mathcal{L}_{CT}^1 = e \frac{\Theta_1}{\Lambda} B^\theta S^\alpha QT^\mu F_{\theta\mu} \]

\[ \mathcal{L}_{CT}^2 = -ie \frac{\Theta_2}{\Lambda^2} B^\alpha S^\theta QT^\mu \partial_\alpha F_{\theta\mu} \]  

\[ \mathcal{L}_{CT}^3 = ie \frac{\Theta_3}{\Lambda^2} B^\theta S^\alpha QT^\mu \partial_\alpha F_{\theta\mu} \]  

(7)

These counterterms reproduce the Lorentz structure indicated in (5). \( \Theta_1 \) is finite whereas \( \Theta_2 \) and \( \Theta_3 \) renormalize the one-loop contributions.

2.- CHIRAL SPIN 3/2 THEORY

We will not review the formalism for Chiral Perturbation Theory involving only spin 1/2 and pseudoscalar fields (for a review see [3]). Instead we will focus our attention on the spin 3/2 sector as we believe the
longly known problems faced by spin 3/2 field theory have not been properly discussed in the light of chiral symmetry and the Heavy Field approximation.

a) Free fields.

Within the Rarita-Schwinger approach, the S=3/2 field is described by a spinor-vector $\psi_\mu$ which is obtained from the tensor product of a spinor and a four-vector. Clearly $\psi_\mu$ involves more degrees of freedom than required. This redundancy is reflected in the formalism in the need for the subsidiary conditions:

$$\gamma^\mu \psi_\mu(x) = 0$$
$$\partial^\mu \psi_\mu(x) = 0.$$  \hspace{1cm} (8)

It has been shown that for the spin 3/2 field, there exist a whole family of one parameter Lagrangians from which the equation of motion and the subsidiary conditions (1) can be derived \[7\]

$$L(A) = \psi^\mu(x) \left\{ i \partial_\alpha \Gamma^\alpha_{\mu\nu}(A) - MB_{\mu\nu}(A) \right\} \psi^\nu(x)$$  \hspace{1cm} (9)

where

$$\Gamma^\alpha_{\mu\nu}(A) = g_{\mu\nu}\gamma^\alpha + Bg_{\mu\nu}\gamma^\alpha\gamma^\nu + A(\gamma_\mu g^\alpha\nu + g_\mu^\alpha\gamma^\nu).$$

$$B_{\mu\nu}(A) = g_{\mu\nu} - C\gamma_\mu\gamma_\nu$$

$$A \neq -1/2, \hspace{0.5cm} B = \frac{3}{2}A^2 + A + \frac{1}{2}, \hspace{0.5cm} C = 3A^2 + 3A + 1.$$  

For $A=-1/3$, $L(A)$ reduces to the Lagrangian originally proposed by R-S \[6\].

By construction the Lagrangian (9) is invariant under the point transformations (R-invariance):
\[
\psi_\mu \rightarrow \psi'_\mu = R_{\mu\alpha}(a)\psi^\alpha \quad \quad A \rightarrow A' = \frac{A - 2a}{1 + 4a}
\]

(10)

where

\[
R_{\mu\nu}(a) = g_{\mu\nu} + a\gamma_\mu\gamma_\nu \quad \quad a \neq -\frac{1}{4}
\]

The R operator acts only on the spin 1/2 components of \(\psi_\mu\). The arbitrariness of the spin 1/2 components of \(\psi_\mu\) is at the origin of the family of one parameter Lagrangians (9).

Notice the factorization property (which in fact can be generalized to the lagrangian including interaction).

\[
\Gamma^\alpha_{\mu\nu}(A) = R^\beta_{\mu}(h(A))\Gamma^\alpha_{\beta\rho}(A = -\frac{1}{3})R^\rho_{\nu}(h(A))
\]

\[
B^\alpha_{\mu\nu}(A) = R^\beta_{\mu}(h(A))B^\alpha_{\beta\rho}(A = -\frac{1}{3})R^\rho_{\nu}(h(A))
\]

with \(h(A) = \frac{1}{2}(1 + 3A)\). Useful properties of the R operator are

\[
R_{\mu\nu}(a)R^\nu_{\lambda}(b) = R_{\mu\lambda}(a + b + 4ab);
\]

\[
R^{-1}_{\mu\nu}(a) = R_{\mu\nu}\left(-\frac{a}{4a + 1}\right)
\]

\[
R_{\mu\nu}(0) = g_{\mu\nu}
\]

(11)

The generalization of (9) to include interactions should lead to physical quantities which are A-independent. On the other hand, the Kamefuchi-O’Raifeartaigh-Salam (K.O.S.) theorem [10] assures the A-independence of the S matrix elements if the whole Lagrangian \(\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}\) is R-invariant. Therefore once we work with an R-invariant Lagrangian we are free to use any value of A, physical quantities being A-independent.
b) Interacting Fields.

Chiral symmetry has been traditionally taken as the guiding principle to construct phenomenological Lagrangians aiming to describe the existing data [11]. Further requirements for any sensible Lagrangians involving spin 3/2 fields are:

- R-Invariance. The full Lagrangian, $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$ must be invariant under contact R-transformations. This ensures, through the K-O-S theorem [10] the $A$ independence of physical quantities.

- Constraints. Once the interactions are included, the Lagrangian must lead to the appropriate subsidiary conditions so that only the correct number of degrees of freedom to describe a spin 3/2 field are left.

A convenient procedure to construct R-invariant Lagrangian describing interacting spin 3/2 fields was discussed in [12]. To lowest order in the chiral expansion, the Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}^{3\frac{1}{2}0} + \mathcal{L}^{3\frac{3}{2}0}. \quad (12)$$

The subindices $\frac{3}{2}0$ and $\frac{3}{2}0$ stand for the spin of the fields included in the corresponding Lagrangian. The free Lagrangian is given in (9) and the R-invariant interactions read

$$\begin{align*}
\mathcal{L}^{3\frac{1}{2}0} &= i\mathcal{C}\bar{\psi}^{\mu}O_{\mu\nu}(A, Z)\psi^{\nu}\Delta^{\nu} + h.c. \\
\mathcal{L}^{3\frac{3}{2}0} &= i\mathcal{H}\bar{\psi}^{\mu}O_{\mu\nu\alpha}(A, X, Y)\psi^{\nu}\Delta^{\alpha}
\end{align*} \quad (13)$$

$\mathcal{C}$ and $\mathcal{H}$ are coupling constants and $\Delta_{\mu}$ is the axial-vector chiral covariant field given in (A4) of the appendix. The vertex tensors can be written as [8,12]

$$\begin{align*}
O_{\mu\nu}(A, Z) &= (g_{\mu\nu} + f(A, Z)\gamma_{\mu}\gamma_{\nu}) \\
O_{\mu\nu\alpha}(A, X, Y) &= R_{\mu\beta}(f(A, X)g^{\beta}_{\sigma}\gamma_{\alpha}\gamma_{5}R^{\sigma}_{\nu}(f(A, Y))
\end{align*} \quad (14, 15)$$
with

\[ f(A, V) = \frac{1}{2} (1 + 4V)A + V \quad V = X, Y, Z \]  

(16)

where \( X, Y, Z \) are arbitrary ("off-shell") parameters. Here and thereafter we omit the SU(3) structure. Details on these vertices as well as the Clebsch-Gordon coefficients are given in the appendix.

We still need to derive the \( T - B - \gamma \) R-invariant interaction, which has the following Lorentz structure

\[ \mathcal{L}_{\frac{4}{2} \gamma} = e \bar{\psi} \gamma^\mu \mathcal{O}_{\mu\alpha\beta}^{(\gamma)} (A) \gamma_5 \psi F^{\alpha\beta}, \]  

(17)

\( \mathcal{O}_{\mu\alpha\beta}^{(\gamma)} (A) \) is splitted as

\[ \mathcal{O}_{\mu\alpha\beta}^{(\gamma)} = \frac{1}{2} g_1 \mathcal{O}_{\mu\alpha\beta}^{(1)} + \frac{1}{2} g_2 \mathcal{O}_{\mu\alpha\beta}^{(2)} \]

with

\[ \mathcal{O}_{\mu\alpha\beta}^{(1)} = R_{\mu\lambda}(\ell(A))(g^\lambda_{\alpha\gamma} \gamma_\beta - g^\lambda_{\beta\gamma} \gamma_\alpha) \]

\[ \mathcal{O}_{\mu\alpha\beta}^{(2)} = R_{\mu\lambda}(\ell(A))(g^\lambda_{\alpha\partial_\beta} - g^\lambda_{\beta\partial_\alpha}) \]

\( \ell(A) \) is, in principle, an arbitrary function of \( A \), however the requirement of R-invariance for the Lagrangian restricts \( \ell(A) \) to the class of functions for which the following identify holds

\[ \ell(A) = (1 + 4a)\ell(A') + a \]

For linear functions \( \ell(A) \), the solution is given by (16) where now \( V \) stand for \( U, W; \) two new arbitrary parameters. Therefore, the R invariant \( T - B - \gamma \) interaction reads:

\[ \mathcal{L}_{\frac{4}{2} \gamma} = e \bar{\psi} \gamma^\mu \{ g_1 (g_{\mu\gamma_\alpha} + f(A, W) \gamma_\mu \gamma_\alpha \gamma_\beta) + g_2 (g_{\mu\alpha} \partial_\beta + f(A, U) \gamma_\mu \gamma_\alpha \partial_\beta) \} \gamma_5 \psi F^{\alpha\beta}. \]  

(18)
Summarizing, R-invariance which ensures the A-independence of physical quantities, can be implemented in the Lagrangian at the price of introducing arbitrary “off-shell” parameters. Sometimes these “off-shell” parameters are arbitrarily fixed, thus for example in [13] R-invariance is implemented by imposing the condition $\gamma^\mu O_{\mu\nu} = 0$ which amounts to pick up the value $Z = -1/4$. Other authors use the “off-shell” parameters to fit the data [9,11].

In addition to R-invariance we need adequate subsidiary conditions in order to eliminate the redundant degrees of freedom contained in $\psi_\mu$. In the following section we will discuss the subsidiary conditions in the light of the Heavy Baryon approach, which is required in order to get a sensible chiral expansion.

3.- HEAVY BARYON SPIN 3/2 THEORY

When the decuplet is introduced in the theory where the octet baryons are treated as heavy fields, the spin $\frac{3}{2}$ fields of definite velocity are defined by:

$$\psi_\mu^v(x) = e^{imv.x} \psi_\mu(x)$$

where $m$ stand for the nucleon mass in the chiral limit, i.e. the decuplet Lagrangian contains an explicit decuplet mass term proportional to $\Delta m = M - m$.

We assume that the HBChPT is defined by the Feynman rules which are obtained from the $\frac{1}{m}$ expansion of the lagrangian derived in the last section.

In the heavy baryon limit ($m \to \infty$), writing $P_\mu = mv_\mu + \ell_\mu$ and keeping leading terms in the $\frac{1}{m}$ expansion, the free lagrangian (9) leads to the propagator [12]:

$$i\Delta_{\mu\nu}(P, A) = \left\{ \begin{array}{c} - g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3} (\gamma_\mu v_\nu - v_\mu \gamma_\nu) \\ + \frac{2}{3} v_\mu v_\nu \end{array} \right\} \frac{P^+}{(v \cdot k - \Delta m + i\epsilon)} + O\left(\frac{1}{m}; A\right)$$

$$\equiv i\Delta^0_{\mu\nu} + O\left(\frac{1}{m}; A\right)$$

(19)
where

\[ P_+ = \frac{1 + \dot{\phi}}{2} \]  

(20)

It is straightforward to check that

\[ \gamma^\mu \Delta^0_{\mu\nu} = \Delta^0_{\mu\nu} \gamma^\nu = 0 \]  

(21)

The important point is that (21) implies that, to zeroth order in the \( \frac{1}{m} \) expansion, terms in the Lagrangian containing the “off-shell” parameters do not contribute to S-matrix elements. Indeed, any calculation regarding the Lagrangian (12), involves the vertices \( O_{\mu\nu} \) and \( O_{\mu\nu\alpha} \) which connect either to an external on-shell heavy baryon or to a heavy baryon propagator \( \Delta^0_{\mu\nu} \).

The constraint for the free field (1), or relation (20) for the heavy propagator ensure in both cases the vanishing of the contributions arising from “off-shell” parameters in the Lagrangian.

These results suggests that to leading order in the \( \frac{1}{m} \) expansion, the constraints \( \gamma^\mu \psi_{\mu} = 0 \) do not get modified by the interacting terms. In fact, in [12] it was shown that modifications to the free constraints due to interactions are \( \mathcal{O}\left(\frac{1}{m}\right) \), therefore, the interacting spin 3/2 field theory is free of “off-shell” ambiguities only to order zero in the \( \frac{1}{m} \) expansion. Furthermore, to that order in the \( \frac{1}{m} \) expansion the elimination of the redundant degrees of freedom is guaranteed since the free field constraints still hold.

To order zero in the \( \frac{1}{m} \) expansion we can just forget the “off-shell” terms, in whose case the interaction reduces to the one commonly used in the literature [3]. It is important to remark however that we get a different free Lagrangian and even more important, our analysis shows that beyond the zeroth order in the \( \frac{1}{m} \) expansion the “off-shell” ambiguities appear and modifications to the subsidiary conditions due to the interactions have to be reconsidered. Consequently we need to keep track separately of the heavy baryon and chiral expansions, even if the corresponding parameters are of the same order of magnitude, \( m \approx 1 \text{ GeV} \), and \( \Lambda_\chi \approx 1 \text{ GeV} \).
4.- \( \frac{3}{2} \to \frac{1}{2} + \gamma \) DECAYS

Previous calculations of the \( T \to B\gamma \) decays in the framework of HBChPT were carried by BSS [4]. The present work is an improvement of such work as we address the problems of the interacting spin \( \frac{3}{2} \) field theory and we perform a gauge invariant calculation which permits to unambiguously identify the form factors. In particular we have argued in the previous section that, if we work to order zero in the \( \frac{1}{m} \) expansion, HBChPT provides a framework to incorporate interacting spin \( \frac{3}{2} \) fields which is both R-invariant and free of the so called “off-shell” ambiguities.

According to these results, below we calculate the amplitude for the \( T \to B\gamma \) decay to lowest order in \( \frac{1}{m} \) and to order \( \frac{w^2}{\Lambda^2} \) in the chiral expansion. This is in contrast with BSS calculation [4] where both expansion are mixed under the argument that \( m \approx \Lambda \chi \). Futhermore, once we decide to work to \( \mathcal{O}\left(\frac{w^2}{\Lambda^2}\right) \) in the chiral expansion, the Lorentz structure of the amplitude (5) indicates that we require 3 counterterms. One finite \( \mathcal{O}\left(\frac{w}{\Lambda}\right) \) and two \( \mathcal{O}\left(\frac{w^2}{\Lambda^2}\right) \), which arise naturally once a gauge invariant calculation is performed.

4a) Loop Calculations

For the processes under consideration there are twelve generic diagrams. Using the Feynman rules derived from the Heavy Baryon lagrangian (summarized in the appendix) it is possible to show that the only non vanishing contribution arise from those diagrams shown in fig. (1). As our analytical results differs from those given in [4] and the differences turn out to be numerically relevant, below we give some details on the calculations. For diagram (a) we obtain

\[
\mathcal{M}_a = -\frac{2e}{f^2} C_{TB^B} \bar{B}S^\theta T^\mu \epsilon^\nu \tau_{\theta\mu\nu} \tag{22}
\]

where

\[
\tau_{\theta\mu\nu} = \int \frac{d^4 \ell}{(2\pi)^4} \frac{((\ell + k) \cdot v + i\xi)((\ell + k)^2 - m_p^2 + i\xi)(\ell^2 - m_p^2 + i\xi)}{(\ell + p) \cdot v + i\xi)((\ell + k)^2 - m_p^2 + i\xi)(\ell^2 - m_p^2 + i\xi)}
\]
$m_p$ denotes the mass of the pseudoescalar entering in the loop and $C_{TBB}$ stand for the products of Clebsch-Gordon coefficients involved in a particular process. Using Feynman’s parametrization and dimensional regularization this expression reduces to

$$\tau_{\theta\mu\nu} = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^\infty d\lambda \left\{ \frac{\Gamma(\varepsilon)(1-x)g_{\mu\nu}k_{\theta} - xg_{\nu\theta}k_{\mu}}{(\lambda^2 + 2\lambda\beta + \gamma)^\varepsilon} \right\}$$

(23)

where $\varepsilon \equiv 2 - \frac{d}{2}$, $\mu$ is the mass scale introduced by dimensional regularization, $\beta \equiv (k \cdot v) x - (p \cdot v)$ and $\gamma \equiv m_p^2 - i\xi$.

Using the identity

$$\frac{d}{dx} \frac{x(1-x)}{\lambda^2 + 2\lambda\beta + \gamma} = \frac{1 - 2x}{\lambda^2 + 2\lambda\beta + \gamma} - \varepsilon \frac{2\lambda x(1-x)(k \cdot v)}{(\lambda^2 + 2\lambda\beta + \gamma)^{1+\varepsilon}}$$

Eq. (22) can be cast in an explicitly gauge invariant form

$$\tau_{\theta\mu\nu} = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^\infty d\lambda \left\{ \frac{(k \cdot v)}{(\lambda^2 + 2\lambda\beta + \gamma)^{1+\varepsilon}} \right\}$$

(24)

where

$$i\mathcal{M}_a = \frac{e}{(4\pi)^2} \bar{B} \{ g_1^{\alpha I_{\mu\nu}} + g_2^{\alpha I_{\mu\nu}} \} S^\theta T^\mu \varepsilon^\nu$$
\[ I_{\mu\nu\theta}^{(1)} = k \cdot v (g_{\nu\theta} k_\mu - g_{\mu\nu} k_\theta) \\ I_{\mu\nu\theta}^{(2)} = (k \cdot v g_{\mu\nu} k_\theta - k_\theta k_\mu v_\nu) \]  

\[
g_1^a \equiv -2(CG)_{TBB} \int_0^1 x dx (4\pi \mu_1^2)^\varepsilon \Gamma(\varepsilon) I(-\varepsilon, b, c) \\ g_2^a \equiv 2(CG)_{TBB} \int_0^1 (1 - 2x) dx (4\pi \mu_1^2)^\varepsilon \Gamma(\varepsilon) I(-\varepsilon, b, c) 
\]

with

\[
I(-\varepsilon, b, c) \equiv \int_0^\infty dx (\lambda^2 + 2\lambda b + c)^{-\varepsilon}.
\]

\[
\mu_1 = \frac{\mu}{k \cdot v}, \quad b = \frac{\beta}{k \cdot v} = x - \frac{p \cdot v}{k \cdot v}, \quad c = \frac{\gamma}{(k \cdot v)^2}.
\]

Before presenting the results of diagram (1b) the following comments are in order:

- In the \( v \cdot \varepsilon = 0 \) gauge the last term in the curly brackets of (23) is absent and the existence of two independent gauge invariant structures is masked [4].

- The Feynman integration parameter in (23) is dimensionfull, which obscures the power counting analysis. It proofs to be convenient to work with the dimensionless integration variable \( \lambda/k \cdot v \).

The second diagram is handled similarly, the only point to keep in mind is that the spin \( \frac{3}{2} \) has an effective mass \( \Delta m = M - m \),

\[
i\mathcal{M}_b = \frac{e}{(4\pi f)^2} \bar{B} (g_1^{\mu\nu\theta} I_{\mu\nu\theta}^{(1)} + g_2^{\mu\nu\theta} I_{\mu\nu\theta}^{(2)}) S^\theta T^\mu \varepsilon^\nu \]  

(26)
where

\[ g^b_1 = -2(CG)_{TTB} \int_0^1 \left( 1 - \frac{2}{3} x \right) dx (4\pi \mu^2_1)^\varepsilon \Gamma(\varepsilon) I(-\varepsilon, b', c) \]

\[ g^b_2 = -2(CG)_{TTB} \int_0^1 \frac{1}{3} (1 - 2x)dx (4\pi \mu^2_1)^\varepsilon \Gamma(\varepsilon) I(-\varepsilon, b', c) \]

where \( b' = x - \frac{p \cdot v}{k \cdot v} + \frac{\Delta m}{k \cdot v} \) and \( (CG)_{TTB} \) stand for the product of Clebsh-Gordon coefficients entering in diagram (b). Notice in this sense that the \( b' - b \) difference appearing in \( g^\alpha_i; \ i = 1, 2; \ \alpha = a, b \) arises from the baryon propagators in the loop. Whereas fig. (1a) involves a massless baryon, in fig. (1b) a decuplet baryon of mass \( \Delta m \) is propagating.

Using the recursion relations worked out by J-M [3] for the integrals \( I(a, b, c) \) we obtain

\[ (4\pi \mu^2_1)^\varepsilon I(-\varepsilon, b, c) = -b \left( \frac{1}{\varepsilon} - \gamma + \ln 4\pi \right) + b \ln \left( \frac{c}{\mu^2_1} - 2 \right) - 2(c - b^2) I(-1, b, c) + O(\varepsilon). \]

Within the modified minimal subtraction scheme, the last equation is divided into a divergent and a finite part.

\[ (4\pi \mu^2_1)^\varepsilon I(-\varepsilon, b, c) = -b \left( \frac{1}{\varepsilon} - \gamma + \ln 4\pi \right) + I_{fin}(b, c) \]

with

\[ I_{fin}(b, c) = b \ln \left( \frac{c}{\mu^2_1} - 2 \right) - 2(c - b^2) I(-1, b, c) \]

In the rest frame of the spin 3/2 particle \( p \cdot v = \Delta m, k \cdot v = w \). On the other hand \( w = \Delta m + O\left( \frac{1}{m} \right) \) so that to order zero in \( \frac{1}{m} \), \( w = \Delta m \). As the \( \frac{1}{m} \) corrections for \( w \) generates \( \frac{1}{m} \) corrections for \( g^a_i \) we have \( b' = x; \ b = x - 1 \).

Combining the contributions of fig. (1) we get \( (g_i^L = g_i^a + g_i^b) \)
\[ g_1^L = \{-2(CG)_{TBB} \int_0^1 (1-x)I_{\text{fin}}(-x,c) \}
- 2(CG)_{TBB} \int_0^1 \left(1 - \frac{x}{3}\right) I_{\text{fin}}(x,c) \} \] (27)

\[ g_2^L = \{-2(CG)_{TBB} \int_0^1 (1-2x)I_{\text{fin}}(-x,c) \}
- 2(CG)_{TBB} \int_0^1 \frac{1}{3}(1-2x) I_{\text{fin}}(x,c) \} . \]

Using the t’Hooft & Veltman [14] conventions to deal with the logarithm branches points of \( I(-1,b,c) \) we calculate the finite part as:

\[ I_{\text{fin}}(x,c) = x \left( \ln \frac{m_p^2}{\mu^2} - 2 \right) + \begin{cases} \sqrt{x^2 - \rho^2} \ln \left| \frac{\eta_+}{\rho} \right| & x > \rho \\ -2\sqrt{\rho^2 - x^2} \arctg \left[ \frac{\rho^2 - x^2}{x^2} \right]^{1/2} & x < \rho \end{cases} \] (28)

\[ I_{\text{fin}}(-x,c) = -x \left( \ln \frac{m_p^2}{\mu^2} - 2 \right) + \begin{cases} -\sqrt{x^2 - \rho^2} \left( \ln \left| \frac{\eta_-}{\eta_+} \right| - 2\pi i \right) & x > \rho \\ 2\sqrt{\rho^2 - x^2} \arctg \sqrt{\frac{\rho^2}{x^2} - 1 - \pi} & x < \rho \end{cases} \]

where

\[ \eta_{\pm} = x \pm \sqrt{x^2 - \rho^2} \quad \rho \equiv \frac{m_p}{w} . \] (29)

Explicit calculations lead to the analytical results reported in table 1.
Table 1

\[
\Delta \rightarrow N\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ (F + D)\ell_i^a(\pi) - (F - D)\ell_i^a(K) - \frac{1}{3} \mathcal{H}(5\ell_i^a(\pi) + \ell_i^b(K)) \right\}
\]

\[
\Sigma^{*0} \rightarrow \Lambda\gamma
\]

\[
g_i^L = 2 \frac{C}{3} \left\{ -D(\ell_i^a(K) + 2\ell_i^a(\pi)) + \mathcal{H}(\ell_i^b(K) + 2\ell_i^b(\pi)) \right\}
\]

\[
\Sigma^{*0} \rightarrow \Sigma^0\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ D\ell_i^a(K) - \mathcal{H}(\ell_i^b(K)) \right\}
\]

\[
\Sigma^{*+} \rightarrow \Sigma^{+}\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ - (D - F)\ell_i^a(\pi) - (D + F)\ell_i^a(K) + \frac{1}{3} \mathcal{H}(\ell_i^b(\pi) + 5\ell_i^b(K)) \right\}
\]

\[
\Sigma^{*-} \rightarrow \Sigma^{-}\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ - (D - F)(\ell_i^a(\pi) - \ell_i^a(K)) + \frac{1}{3} \mathcal{H}(\ell_i^b(\pi) - \ell_i^b(K)) \right\}
\]

\[
\Xi^{*0} \rightarrow \Xi^0\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ - (D - F)\ell_i^a(\pi) - (D + F)\ell_i^a(K) + \frac{1}{3} \mathcal{H}(\ell_i^b(\pi) + 5\ell_i^b(K)) \right\}
\]

\[
\Xi^{*-} \rightarrow \Xi^{-}\gamma
\]

\[
g_i^L = 2 \frac{C}{\sqrt{3}} \left\{ - (D - F)(\ell_i^a(\pi) - \ell_i^a(K)) + \frac{1}{3} \mathcal{H}(\ell_i^b(\pi) - \ell_i^b(K)) \right\}
\]
where

\[ \ell_a^1(P) = r_p^1 \int_0^1 (1 - x) I_{\text{fin}}(-x, c) \]
\[ \ell_b^1(P) = r_p^1 \int_0^1 (1 - \frac{1}{3}x) I_{\text{fin}}(x, c) \]
\[ \ell_a^2(P) = r_p^1 \int_0^1 (1 - 2x) I_{\text{fin}}(-x, c) \, dx \]
\[ \ell_b^2(P) = r_p^1 \int_0^1 \frac{1}{3}(1 - 2x) I_{\text{fin}}(x, c) \, dx. \]

We include the \( r_p \) factor to deal with the \( f_k^2 - f_{\pi}^2 \) difference, \( r_{\pi} = 1 \), \( r_k = \frac{1}{(1.22)^2} \). This completes the one loop calculations, now we turn to the counterterms.

4b) Counterterms

It is clear that 1-loop calculations reproduces the most general amplitude Eq. (5). The counterterms for this calculation can be read by comparing (5) with (24,26). To \( \mathcal{O}\left(\frac{v^2}{\Lambda^2} \right) \) three counterterms contributes to \( \frac{3}{2} \to \frac{1}{2} \gamma \)

\[ \mathcal{L}_{ct}^1 = e \frac{\Theta_1}{\Lambda_\chi} \bar{B} S^\theta Q T^\mu F_{\theta\mu} \]
\[ \mathcal{L}_{ct}^2 = -ie \frac{\Theta_2}{\Lambda_\chi} \bar{B} v^\alpha S^\theta Q T^\mu \partial_\alpha F_{\theta\mu} \]
\[ \mathcal{L}_{ct}^3 = ie \frac{\Theta_3}{\Lambda_\chi} \bar{B} v^\theta S^\alpha Q T^\mu \partial_\alpha F_{\theta\mu} \]

where \( Q \) denotes the SU(3) charge matrix \( Q = \frac{1}{3} \text{diag} (2, -1, -1) \). The \( SU(3) \) structures of the counterterms are discussed in the appendix. Notice that \( \mathcal{L}_{ct}^1 \) is \( \mathcal{O}\left(\frac{v}{\Lambda_\chi} \right) \) while \( \mathcal{L}_{ct}^2 \) and \( \mathcal{L}_{ct}^3 \) are \( \mathcal{O}\left(\frac{v^2}{\Lambda_\chi} \right) \).
From (24,26) we see that the 1-loop contributions are $O\left(\frac{w^2}{\Lambda^2}\right)$, which means that $\Theta_1$ is finite while $\Theta_2$ and $\Theta_3$ are infinite constants which renormalize the 1-loop contributions.

Including both, one-loop and counterterm contributions, we finally obtain:

$$i\mathcal{M} = e\bar{B}\left\{ \frac{g_1}{\Lambda^2} I_{\mu\nu} + \frac{g_2}{\Lambda^2} J_{\mu\nu} \right\} S^{\mu} T^{\nu} \varepsilon^{\nu}$$  \hspace{1cm} (32)

where

$$g_1 = \left( \frac{\Lambda^2}{w} \Theta_1 + \Theta^R_2 \right) Q_{TB} + g^L_1$$
$$g_2 = \Theta^R_3 Q_{TB} + g^L_2$$  \hspace{1cm} (33)

$Q_{TB}$ stand for the Clebsch-Gordon coefficients obtained from Eq. (A8), and $\Theta^R_i i = 2, 3$ are the renormalized constants. By choosing the normalization scale $\Lambda = \Lambda_\chi$ in Eq. (2), and comparing (5), (25) and (32) we obtain:

$$g^D_1 = \frac{2w}{\Lambda_\chi} g_1 \hspace{1cm} g^D_2 = 8g_2.$$  \hspace{1cm} (34)

To leading order in the $\frac{1}{m}$ expansion, from Eq. (2) we obtain the multipole amplitudes and decay widths:

$$M1 = \frac{e}{3} \left( \frac{w^{3/2}}{\Lambda^2_\chi} \right) [2g_1 - g_2]$$
$$E2 = \frac{e}{3} \left( \frac{w^{3/2}}{\Lambda^2_\chi} \right) g_2$$
$$\frac{E2}{M1} = \left( \frac{g_2}{2g_1 - g_2} \right)$$
$$\Gamma = \frac{2\alpha}{9} \left( \frac{w}{\Lambda_\chi} \right)^4 w \{|2g_1 - g_2|^2 + 3|g_2|^2\}$$  \hspace{1cm} (35)

where $\alpha$ denotes the fine structure constant and $w = \Delta m$. 

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It is worth-remarking that the leading contributions to $g_1$ are $\mathcal{O}\left(\frac{w}{\Lambda}\right)$ while, the contributions to $g_2$ are $\mathcal{O}\left(\frac{w^2}{\Lambda^2}\right)$. Thus the $E2$ amplitude is subleading in the chiral expansion.

5. Numerical Results

Using HBChPT we have calculated the contributions to the form factors to leading order in $\frac{1}{m}$ and to $\mathcal{O}\left(\frac{w^2}{\Lambda^2}\right)$ in the chiral expansion. The $g_1$, $g_2$ form factors depend on the low energy constants $D,F,H$ and $C$ and on the renormalization constants $\Theta_1, \Theta_2^R, \Theta_3^R$. In order to make some predictions for the radiative decays of the decuplet it is necessary to fix these parameters. The low energy constants $D,F,H$ and $C$ have been estimated from hyperonic semileptonic and strong decays [2,3] of the decuplet. The values obtained are $|C| = 1.6$, $F = 0.40 \pm 0.05$, $D = 0.61 \pm 0.04$ and $H = -1.9 \pm 0.7$ which are in good agreement with the relations obtained from an SU(6) symmetry for baryons, namely, $F = \frac{3}{2}D$, $C = -2D$, $H = -3D$. For numerical calculations we consider isospin as a good symmetry. SU(3) breaking is introduced by taking the physical values for $f_\pi$ and $f_K$ and also through the mass difference between the isospin multiplets. According to our assumptions we take $w = \Delta m = M - m$ where $M$ and $m$ stand for the average mass of the isospin multiplets of the decuplet and octet respectively.

The $SU(3)$ forbidden decay $\Sigma^{*-} \rightarrow \Sigma \gamma$ and $\Xi^{*-} \rightarrow \Xi^0 \gamma$, for which $Q_{TB} = 0$, do not receive contributions from the counterterms, and therefore are entirely fixed by loop calculations. Phenomenologically these decays are very important since they are induced by SU(3) breaking due to mass terms. For the SU(3) forbidden decays we obtain the ratio of multipole amplitudes and branching ratios shown in table 2. The numerical calculations are carried using the central values for $F,D,C$ and $H$ obtained in [2,3].
The values we obtain for the branching ratios of the $SU(3)$ forbidden decays differs by one order of magnitude from those gives in [4] and are closer to the experimental bounds [16]. We expect these branching ratios to be correct within a factor of 2 due to $\Delta m$ corrections.

For the $SU(3)$ allowed radiative decays of the decuplet, besides the low energy constants $D, F, C, H$ it is necessary to fix $\Theta \equiv \frac{\Lambda}{k\gamma} \Theta_1 + \Theta_2^R$ and $\Theta_3^R$, since $\Theta_1$ and $\Theta_2^R$ enter always in the $\Theta$ combination. However existing experimental information is not enough to fix these constants. In ref. [1] the value of $\Theta_1$ has been fixed from $\Delta \rightarrow N\gamma$ by disregarding the contributions from $\Theta_2^R, \Theta_3^R$ under the argument that loop contributions are enhanced by the chiral logarithm. We do not agree with this procedure as according to the philosophy of Chiral Perturbation Theory all terms contributing to a given order of a process must be included. Furthermore, there are counter examples to the argument of chiral logarithm enhancement [15]. In table 3 we quote the predictions we get for the loop contributions to the form factors. The numerical calculations are similar to the ones performed for the forbidden decays (table 2).

### Table 2

| Process          | $E2/M1$      | $BR (Exp)$ |
|------------------|--------------|------------|
| $\Sigma^+ \rightarrow \Sigma^+\gamma$ | -0.05 - 0.04 i | 0.024 % (?) |
| $\Xi^+ \rightarrow \Xi^-\gamma$ | -0.05 - 0.06 i | 0.13 % (< 4%) |

### Table 3

| Process          | $g_1^L$       | $g_2^L$      |
|------------------|---------------|--------------|
| $\Delta \rightarrow N\gamma$ | 17.55 - 0.67 i | 1.29 + 1.98 i |
| $\Sigma^{*0} \rightarrow \Lambda\gamma$ | -18.05 + 0.39 i | -1.07 - 1.31 i |
| $\Sigma^{*0} \rightarrow \Sigma^{0}\gamma$ | 20.13 | 0.15 |
| $\Sigma^{*+} \rightarrow \Sigma^{+}\gamma$ | -37.28 + 0.02 i | -0.63 - 0.22 i |
\[
\Sigma^* \rightarrow \Sigma^0\gamma \quad 2.97 + 0.02 \ i \quad -0.33 - 0.22 \ i
\]
\[
\Xi^* \rightarrow \Xi^0\gamma \quad 2.58 + 0.05 \ i \quad -0.29 - 0.30 \ i
\]
\[
\Xi^* \rightarrow \Xi^0\gamma \quad -33.13 + 0.05 \ i \quad -0.64 - 0.30 \ i
\]

Conclusions

- We analyzed the interacting spin $3/2$ field theory in the light of R-invariance, which is necessary in order to ensure the non-physical nature of the spin $1/2$ content of the Rarita-Schwinger spinor. We have seen that R-invariance can be implemented at the price of introducing the so called “off-shell” parameters.

- We have argued that to order zero in the heavy baryon expansion, terms involving the arbitrary “off-shell” parameters do not contribute to the S-matrix elements. Even more, to the same order in the heavy baryon expansion, the subsidiary conditions reduces to those of the free theory, ensuring that the interacting theory involves only the $S = \frac{3}{2}$ degrees of freedom.

- The interacting spin $3/2$ theory is free of the “off-shell” ambiguities only to order zero in the heavy baryon expansion. This implies that the heavy baryon and the chiral expansion must be separately considered even if the expansion parameters, the baryon mass $m \approx 1\ GeV$ and $\Lambda_\chi \approx 1\ GeV$ respectively, are of the same order.

- We applied the formalism previously described to the calculation of radiative decays of the spin $3/2$ decuplet. We performed an explicitly gauge invariant calculation which permits to unambiguously identify the form factors as well as the counterterms required by the chiral expansion. The calculation is valid to leading order in $\frac{1}{m}$ and to $O\left(\frac{w^2}{\Lambda^2_\chi}\right)$ in the chiral expansion. We present predictions for the $SU(3)$ forbidden decays $\Sigma^* \rightarrow \Sigma^0\gamma$ and $\Xi^* \rightarrow \Xi^0\gamma$. Our results are one order of magnitude larger than previous estimates [4]. We also report analytical and numerical results for the 1-loop contributions to the $SU(3)$ allowed radiative decays of the decuplet. Unfortunately we are not able to present predictions for the latter decays since it is necessary to fix two low energy constants which is not possible with the available experimental information.
Appendix

The lowest order Heavy Baryon Lagrangian for Spin 1/2 Fields is [3]

\[
\mathcal{L}^{(1)}_8 = <\bar{B}(v \cdot i\partial)B + 2iD\bar{B}S^\mu\{\Delta_\mu, B\} + 2iF\bar{B}S^\mu[\Delta_\mu, B] + \frac{f^2}{4}D_\mu\Sigma D^\mu\Sigma + a\mathcal{M}(\Sigma + \Sigma^+) >
\]  

(A.1)

where < > denotes trace over SU(3) structure, \(\mathcal{M}\) denotes the quark mass matrix and \(B\) is a matrix containing the spin 1/2 baryon fields. Although not explicitly written throughout this section, baryon fields denote definite velocity fields. Pseudoscalar fields enter in the formalism through the matrices \(\Phi\), \(\xi \equiv \exp(i\Phi)\) and \(\Sigma \equiv \xi^2\) where \(\Phi\) contains the pseudogoldstone fields. Explicitly

\[
\Phi = \frac{1}{2}\lambda_\alpha\phi^\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{-\pi^+}{\sqrt{2}} + \frac{n}{\sqrt{6}} & K^+
\frac{\pi^-}{\sqrt{2}} + \frac{n}{\sqrt{6}} & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0
\frac{K^-}{\sqrt{2}} & \frac{-2\eta}{\sqrt{6}} & -\frac{2n}{\sqrt{6}}
\end{pmatrix},
\]

(A.2)

\[
B = \frac{1}{\sqrt{2}}\lambda_\alpha\bar{B}^\alpha = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p
\frac{\Sigma^-}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Xi^- & n
\frac{\Xi^0}{\sqrt{2}} & \frac{-2\Lambda}{\sqrt{6}} & -\frac{2\eta}{\sqrt{6}}
\end{pmatrix},
\]

\[
\bar{B} = \frac{1}{\sqrt{2}}\lambda_\alpha\bar{B}^\alpha = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}}, & \Sigma^- & \Xi^-
\frac{\Sigma^+, \bar{\Sigma}}{\sqrt{2}}, & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Xi^0
\bar{p} & \bar{n} & -\frac{2\Lambda}{\sqrt{6}}
\end{pmatrix}.
\]

With these conventions we have \(f_\pi \approx 93\) MeV. The chiral covariant derivatives contains the chiral conexion

\[
\mathcal{D}_\mu B = \partial_\mu B + [\Gamma_\mu, B].
\]  

(A.3)

This conexion and the chiral covariant axial vector field \(\Delta_\mu\), are given as

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\[\Gamma_\mu = \frac{i}{2}\{\xi \partial_\mu \xi^+ + \xi^+ \partial_\mu \xi\} = \frac{1}{2f^2}[\Phi, \partial^\mu \Phi] + \cdots \quad (A.4)\]

\[\Delta_\mu = \frac{1}{2}\{\xi \partial_\mu \xi^+ - \xi^+ \partial_\mu \xi\} = \frac{1}{f}\partial^\mu \Phi - \frac{1}{6f^3}[\Phi, [\Phi, \partial_\mu \Phi]] + \cdots\]

Interactions with external sources are introduced through covariant derivatives which for electromagnetic interactions reduces to the substitutions

\[
\Gamma_\mu \rightarrow \Gamma_\mu + \frac{ie}{2}A_\mu (\xi Q \xi^+ + \xi^+ Q \xi) \\
\Delta_\mu \rightarrow \Delta_\mu - \frac{e}{2}A_\mu (\xi Q \xi^+ - \xi^+ Q \xi) \\
D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu [Q, \Sigma].
\]

If we consider one-meson exchange only this reduces to

\[
\Gamma_\mu \rightarrow ieQA_\mu \\
\Delta_\mu \rightarrow \frac{1}{f}\partial_\mu \Phi + \frac{ie}{f}A_\mu [Q, \Phi] \\
D_\mu B = \partial_\mu B + ieA_\mu [Q, B].
\]

According to the main text, if we work to leading order in the \(1/m\) expansion we can forget those terms in Eqs. (12,18) containing the “off-shell” parameters. The resulting lagrangian for the spin 3/2 sector is

\[
\mathcal{L}_{10} = \bar{T}^\mu (i\partial_\alpha \Gamma^\alpha_{\mu\nu} - MB_{\mu\nu})HT^\nu + iC(\bar{T}^\mu \Delta_\mu B + \bar{B} \Delta_\mu T^\mu) + 2i\mathcal{H}T^a S_\nu \Delta^\nu T_\mu
\]

where

\[
(D^\nu T^\mu)_{abc} = \partial^\nu T^\mu_{abc} + (\Gamma^\nu)_a \begin{array}{c}{}^a \end{array} T^\mu_{abc} + (\Gamma^\nu)_b \begin{array}{c}{}^b \end{array} T^\mu_{abc} + (\Gamma^\nu)_c \begin{array}{c}{}^c \end{array} T^\mu_{abc}.
\]
The $SU(3)$ structures are

\[ \bar{T} \Delta B + h.c \equiv \varepsilon^{kmn} \bar{T}_{ijk} \Delta^i m B^j n + h.c \]  
\[ T \Delta T \equiv T_{jkl}(\Delta)^j m T^{mkl}. \]  

(A.7)

We use the subindex $HB$ in Eq. (A6) to denote the Heavy-Baryon limit of the first term. According to the main text as far as we consider one meson exchange and leading order terms in the $\frac{1}{m}$ expansion this term reduces to a free term which produces the propagator given in Eq. (19) plus a minimally coupled electromagnetic interaction.

\[ L_{\text{TAT}} = iT^\mu v^\alpha (ieQA_\alpha)T_\mu = -ev \cdot AT^{abc}_{\mu}(QT^\mu)_{abc} \]

where

\[ (QT)_{abc} \equiv Q_a dT_{dbc} + Q_b dT_{adc} + Q_c dT_{abd}. \]

Physical spin 3/2 fields and $SU(3)$ fields are related by

\[ T_{111} = \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}} \Delta^{+}, \quad T_{122} = \frac{1}{\sqrt{3}} \Delta^{0}, \quad T_{222} = \Delta^{-} \]

\[ T_{113} = \frac{1}{\sqrt{3}} \Sigma^{++}, \quad T_{123} = \frac{1}{\sqrt{6}} \Sigma^{*0}, \quad T_{223} = \frac{1}{\sqrt{3}} \Sigma^{*-} \]

\[ T_{123} = \frac{1}{\sqrt{3}} \Xi^{*0}, \quad T_{233} = \frac{1}{\sqrt{3}} \Xi^{*-} \]

\[ T_{333} = \Omega^{-}. \]

In addition to the leading order terms we have the counterterms given in Eq. (31) which have the $SU(3)$ structure

\[ \bar{B}QT \equiv \varepsilon^{kmn} \bar{B}_m i Q^n T_{ijk}. \]  

(A.8)
Feynman rules we require to perform the calculation of the non-nule di-
agrams shown in fig. 1 are

\[ \frac{\sqrt{2}i}{f} (C.G.)_{BB\pi} S \cdot \ell \]

\[ \frac{\sqrt{2}i}{f} (C.G.)_{TT\pi} (S \cdot \ell) g_{\alpha\beta} \]

\[ \frac{i}{\sqrt{2}f} (C.G.)_{TB\pi} \ell_{\mu} \]

\[ - iq(\ell_1 + \ell_2)_{\mu} \]

\[ \frac{i}{\Lambda_1} \frac{\Theta_1}{Q_{TB}} \frac{1}{k \cdot v} f^I_{\mu \nu \theta} S^\theta \]

\[ \frac{i}{\Lambda_2} \frac{\Theta_2}{Q_{TB}} f^I_{\mu \nu \theta} S^\theta \]

\[ \frac{i}{\Lambda_3} \frac{\Theta_3}{Q_{TB}} f^I_{\mu \nu \theta} S^\theta \]

where double (single) lines denotes the spin 3/2 (spin 1/2) fields, dashed
(wavy) lines denotes the pseudoscalar (photon) field and \( q \), \( (C.G) \) denotes the charge of the pseudoscalar entering in the loop and the product of Clebsch-
Gordon coefficients respectively.

With these conventions we obtain the same values for the products of \( (C.G) \)
coefficients and for \( Q_{TB} \) as those listed in ref [4].
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