Non-parametric reconstruction of cosmological matter perturbations

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Abstract. Perturbative quantities, such as the growth rate \( f \) and index \( \gamma \), are powerful tools to distinguish different dark energy models or modified gravity theories even if they produce the same cosmic expansion history. In this work, without any assumption about the dynamics of the Universe, we apply a non-parametric method to current measurements of the expansion rate \( H(z) \) from cosmic chronometers and high-z quasar data and reconstruct the growth factor and rate of linearised density perturbations in the non-relativistic matter component. Assuming realistic values for the matter density parameter \( \Omega_{m0} \), as provided by current CMB experiments, we also reconstruct the evolution of the growth index \( \gamma \) with redshift. We show that the reconstruction of current \( H(z) \) data constrains the growth index to \( \gamma = 0.56 \pm 0.12 \) (2\(\sigma\)) at \( z = 0.09 \), which is in full agreement with the prediction of the \( \Lambda \)CDM model and some of its extensions.

Keywords: dark energy theory, cosmological parameters from LSS, galaxy clustering

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1 Introduction

The cosmic acceleration, first inferred from type Ia supernovae (SNe Ia) observations in the late 1990s [1, 2], cannot be explained in the framework of the General Relativity (GR) with the material content of the Universe satisfying the strong energy condition. This in turn poses a major challenge for theoretical physics and has led physicists to hypothesize the existence of dark energy (DE), a negative pressure energy component which dominates the energy content of the universe at present (for a review, see [3–7]).

In order to achieve cosmic acceleration, GR equations for a homogeneous and isotropic universe require $w < -((\Omega_{m0}/3\Omega_{DE,0} + 1/3)$, where $w$ is the ratio between the dark energy pressure to its energy density, and $\Omega_{m0}$ and $\Omega_{DE,0}$ stand for the present-day density parameters of the clustered matter and of the dark energy, respectively. Since very little is known about the nature of this DE field (e.g., it is unclear if its energy density is in fact time-independent), alternative explanations for cosmic acceleration have been suggested. These are mainly based on modification of gravity at large scales and examples of them include scalar-tensor gravity [8], $f(R)$ theories [9–11], higher dimensional braneworld models [12–15], among others (an extensive list of DE models and modified gravity theories is discussed in [16] and references therein).

From the observational point of view, it is well known that the accuracy of present background observations — e.g., measurements of luminosity and angular diameter distances and of the cosmic expansion rate — is not enough to distinguish between DE models and scenarios of modified gravity. In reality, even for highly accurate data, it is not possible to decide which one gives the best description of the Universe because different models can produce the same cosmic expansion and, therefore, the same background observables. However, the existing degeneracy at the background level can be lifted by the study of the growth of matter density perturbation, $\delta$ [17]. As well known, in theories of modified gravity the growth rate for $\delta$ is usually different from that predicted by general relativistic models, with the effective gravitational constant $G_{\text{eff}}$, which appears in the source term driving the evolution of $\delta$, changing significantly relative to the Newton’s gravitational constant $G$, usual in the GR regime. It is worth mentioning that the linearized matter perturbation can also be scale-dependent in this kind of scenarios.
On the other hand, given the large number of competing cosmological models and the inherent difficulties of distinguishing between them, parametric and non-parametric methods have been developed with the aim of obtaining independent information about the physics behind cosmic acceleration from observations (see [19–33] and references therein). In this paper, we apply a non-parametric method, namely, Gaussian Processes (GP) [25, 28], to a set of observational data to reconstruct the growth factor, \( g \), and rate, \( f \), of linear perturbations and the growth index, \( \gamma \), following closely the treatment developed in ref. [18]. In our analysis we use cosmological model-independent measurements of the cosmic expansion rate \( H(z) \) lying in the redshift range \( 0.070 \leq z \leq 2.34 \). Currently, most of the \( H(z) \) data available come from measurements of age differences of the so-called cosmic chronometers, i.e., passively evolving galaxies at different \( z \) [34], whose uncertainties are around 10\%–15\% [35–39]. Estimates of the expansion rate have also been obtained from the three-dimensional correlation function of the transmitted flux fraction in the Ly\( \alpha \)-forest of high-\( z \) quasars, as reported in refs. [40–42]. In particular, the application of this latter technique to a large sample of quasars provided measurements of \( H(z) \) within \( \sim 3\% \) accuracy at \( z \approx 2.3 \), which imposes tight bounds on cosmological parameters when combined with current \( H_0 \) measurements and other cosmological data sets (see [43] for a recent analysis). From a subsample of the currently available \( H(z) \) data, we reconstruct perturbative quantities from background observations and investigate possible tensions between current data and the DE models predictions.

This paper is organised as follows: in section 2 we summarise the treatment developed in ref. [18] introducing the basic expressions that govern the matter perturbation growth and the related quantities. We discuss the observational data and the non-parametric method used to reconstruct the cosmic history in section 3. In section 4 we present the reconstructed functions of the perturbative quantities and discuss their compatibility with the standard cosmological description. We end this paper by summarizing the main conclusions in section 5.

2 Matter perturbation equations

The scalar perturbations of a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric are characterised, in the longitudinal gauge, by the line element

\[
ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Psi)a^2(t)d\vec{x}^2, \tag{2.1}
\]

where \( \Phi \) and \( \Psi \) are the gauge invariant potential and curvature perturbation, respectively. In GR these quantities are equal if we neglect any anisotropic stress which could, for instance, be produced by primordial neutrinos.\(^1\) On sub-Hubble scales, the potential satisfies the Poisson equation

\[
\nabla^2 \Phi = 4\pi G a^2 \rho_m \delta, \tag{2.2}
\]

where

\[
\delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \rho(t)}{\rho(t)} \tag{2.3}
\]

is the non-relativistic matter density contrast.

Assuming GR and a background filled with matter and an unclustered component of DE covariantly conserved, the linearised matter density contrast satisfies the second order

\(^1\)If we consider anisotropy stress, then \( \nabla^2(\Phi + \Psi) \neq 0 \), mimicking some models of modified gravity. In our analysis we assume that the dark energy component does not have anisotropic stress and does not couple to matter.
differential equation
\[ \ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m \delta = 0. \tag{2.4} \]

The covariant conservation of the matter energy-momentum tensor implies \( \rho_m \propto (1 + z)^3 \). Using this result and the definition of the dimensionless physical distance given by
\[ D = H_0 \int_t^{t_0} \frac{dt}{a(t)} = H_0 \int_0^z \frac{dz_1}{H(z_1)}, \tag{2.5} \]
eq (2.4) can be rewritten as [18]:
\[ \left( \frac{\delta'}{1 + z(D)} \right)' = \frac{3}{2} \Omega_{m0} \delta, \tag{2.6} \]
where a prime denotes derivative with respect to \( D \) and \( \Omega_{m0} \) is the matter density parameter at the present time. The solution of the eq. (2.6) can be written in terms of a set of integral equations as follows [18, 44]:
\[ \delta(D) = 1 + \delta_0 \int_0^D [1 + z(D_1)] dD_1 \tag{2.7a} \]
\[ \delta'(D) = \delta'_0 [1 + z(D)] \]
\[ + \frac{3}{2} \Omega_{m0} [1 + z(D)] \int_0^D \delta(D_1) dD_1. \tag{2.7b} \]

In order to solve the previous set of integral equations one needs to assume a value for \( \Omega_{m0} \). In our analysis, we adopt two different estimates of this quantity, as provided by Planck and WMAP collaborations (see section 4). Also, we fix the two integration constants to obtain a unique solution of the eq. (2.6). The first one \( \delta_0 = \delta(z = 0) \) is implicitly fixed by eq. (2.7a) since the solution is normalised to its value today, i.e., \( \delta_0 = 1 \) whereas the second one, \( \delta'_0 = \delta'(z = 0) \), is fixed with the requirement that the behaviour of the density contrast at high redshift must be proportional to \( a \). In practice, however, it is easier to fix \( \delta'_0 \) analysing the growth factor, defined as
\[ g(z) \equiv (1 + z)\delta(z). \tag{2.8} \]

A unique solution for eq. (2.6) implies that the cosmic expansion history in GR determines univocally the matter density contrast as pointed out in refs. [17, 45]. Therefore, we can perform an indirect determination of \( \delta \) reconstructing the Hubble parameter from the \( H(z) \) data. As shown in ref. [46] the inverse problem, i.e., the determination of cosmic expansion as a function of the density contrast, \( H(z) = H(\delta(z)) \), has an analytical solution given by
\[ H^2(z) = 3\Omega_{m0}H_0^2 \frac{(1 + z)^2}{(d\delta/dz)^2} \int_z^\infty \frac{\delta|d\delta/dz|}{1 + z} dz. \tag{2.9} \]
The growth rate of linear perturbations is defined as
\[ f(z) \equiv \frac{d\ln \delta}{d\ln a} = -\frac{(1 + z)H_0}{H(z)} \frac{\delta'}{\delta}. \tag{2.10} \]
Note that the values of the growth rate obtained by solving the above equation via reconstruction of \( H(z) \) constitute an independent estimate of this quantity as inferred from the
matter power spectrum or weak gravitational lensing data [47]. A tension between them would be an evidence of non-standard cosmology where the eq. (2.4) is not valid. If this is the case, it would imply that:

- The Universe is not correctly described by a flat FLRW metric. For instance, in the case of a non-flat and inhomogenous universe the Poisson equation is modified, as shown in ref. [48].

- The evolution of the matter density is not proportional to \((1 + z)^3\). It implies that the matter energy-momentum tensor is not covariantly conserved. Typical examples are models with decaying of dark energy into dark matter or vice versa [49–52].

- The GR is not valid and it needs to be modified or the DE is clumping and its effect has to be taken into account. In both cases, it would be possible to define an effective gravitational function, \(G \to G_{\text{eff}}\) which can depend on the scale and time [53–55].

The growth index is written as [56–58]

\[
\gamma = \frac{d \ln f(z)}{d \ln \Omega_m(z)},
\]

where

\[
\Omega_m(z) \equiv \frac{\Omega_{m0}(1 + z)^3 H_0^2}{H^2(z)},
\]

is the matter density parameter as a function of redshift. As mentioned earlier, the growth rate and growth index constitute key quantities to distinguish between modified gravity and DE models [17, 53]. For instance, \(\gamma = 6/11\) for \(\Lambda\)CDM [58, 59], \(\gamma = 11/16\) for the so-called DGP scenarios [59] and lies in the interval \(0.40 < \gamma < 0.43\) [53] for the \(f(R)\) model proposed in ref. [11]. For non clustering DE models, \(\gamma\) is related to a slowly evolving DE equation-of-state \(w\) through \(\gamma \simeq 3(\frac{w - 1}{6w - 5})\) [17].

3 Data and Hubble parameter reconstruction

3.1 Data

Currently, there are different approaches to measure directly the expansion rate of the Universe. One of them is based on the determination of the age difference between passively evolving galaxies at approximately the same redshift and known as cosmic chronometers [34]. This information (redshift and age) provides \(H(z) \simeq -\Delta z / \Delta t(1 + z)\), where \(\Delta t\) is the difference between the age estimates of two galaxies whose redshifts differ by \(\Delta z\). Presently, there are 23 measurements of \(H(z)\) using the differential age approach [35–39]. This method is cosmological model-independent but there can be dependence on stellar population synthesis models at high redshift. In our analysis, we follow the arguments of ref. [60] and consider 15 \(H(z)\) measurements up to \(z \simeq 1.04\). We also increase slightly (20%) the error bar of the highest-\(z\) point to account for the uncertainties of the stellar population synthesis models. We complement our sample with two high-\(z\) quasar data at \(z = 2.34\) [41] and \(z = 2.36\) [42] which were obtained by determining the BAO scale from the correlation function of the Ly\(\alpha\) forest systems (see [40] for more details). The data set used in our analysis is shown in table 1.
| $z$  | $H_{\text{obs}}(z)$ [km s$^{-1}$ Mpc$^{-1}$] | ref. |
|------|---------------------------------|-----|
| 0.100 | 69 ± 12                         | [36] |
| 0.170 | 83 ± 8                          | [36] |
| 0.179 | 75 ± 4                          | [38] |
| 0.199 | 75 ± 5                          | [38] |
| 0.270 | 77 ± 14                         | [36] |
| 0.352 | 83 ± 14                         | [38] |
| 0.400 | 95 ± 17                         | [36] |
| 0.480 | 97 ± 62                         | [37] |
| 0.593 | 104 ± 13                        | [38] |
| 0.680 | 92 ± 8                          | [38] |
| 0.781 | 105 ± 12                        | [38] |
| 0.875 | 125 ± 17                        | [38] |
| 0.880 | 90 ± 40                         | [37] |
| 0.900 | 117 ± 23                        | [36] |
| 1.037 | 154 ± 20                        | [38] |
| 2.34  | 222 ± 7                         | [41] |
| 2.36  | 226 ± 8                         | [42] |

Table 1. Measurements of the expansion rate from 15 cosmic chronometer systems and two high-$z$ quasar data used in the analysis.

3.2 Gaussian Processes

A Gaussian Process is the generalisation of a Gaussian distribution of a random variable to a function space. It constitutes a powerful method to reconstruct the expected function that describes the behaviour of a given data. GP use a few assumptions about the characteristics of the expected function $W(z)$, e.g., a correlation between the $W(z)$ and $W(z')$ values, $z$ and $z'$ being different points (see ref. [28] and references therein for a complete review of the method). In any case, the reconstruction can be made without assuming a model or a parametric function to describe the data.

This method has shown great success, being applied to reconstruct several cosmological quantities like the DE equation of state [28], the deceleration parameter, the duality-distance parameter [61, 62] and to infer the Hubble constant [33, 60, 63]. In GP the variation of the expected function in two different points is not independent and it is characterised by a covariance function $k(z, z')$. The covariance depends on a set of hyperparameters (non model parameters) which determine the correlation between the $W(z)$ and $W(z')$ values. The fact that the $H(z)$ parameter must be infinitely differentiable allows us to choose a Gaussian covariance given by:

$$k(z, z') = \sigma \exp \left( -\frac{(z - z')^2}{2l^2} \right),$$  

(3.1)

where $\sigma$ and $l$ are the so-called hyperparameters related to typical changes in the function values and to the length scale between two points $z$ and $z'$, respectively. In order to perform the non-parametric reconstruction of the cosmic expansion history we use the code Gaussian
Figure 1. Reconstruction of the cosmic expansion (in km/s/Mpc) via Gaussian Processes from cosmic chronometer and high-z quasar data. The black solid line corresponds to the GP reconstruction whereas the shaded regions to the 1σ and 2σ confidence intervals. The data points represent the observational data displayed in table 1. b) The quantity $H(z)/(1+z)$ as a function of $z$.

Processes in Python\(^2\) applied to the $H(z)$ data presented in table 1 (we refer the reader to [25] for more details on GP).

4 Results

For the best-fit values of the GP hyperparameters, the reconstructed $H(z)$ function is shown in figure 1(a) along with the data points used in our analysis (table 1). We also plot the quantity $H(z)/(1+z)$ as a function of $z$ in figure 1(b), which shows a minimum at $z \simeq 0.62$, corresponding to the recent deceleration/acceleration transition. The evolution of $\Omega_m(z)$ [eq. (2.12)] is shown in figure 2(a) adopting $\Omega_{m0} = 0.308 \pm 0.012$, as given by the Planck collaboration [64] and in figure 2(b) adopting $\Omega_{m0} = 0.279 \pm 0.025$, as given by the WMAP collaboration [65]. After reconstructing the expansion rate $H(z)$ we calculate the density contrast solution (2.7a) in an iterative way. We perform the calculation of $\delta(z)$ considering the two different values of the present-day matter density parameter as mentioned above. The only free parameter in eq. (2.7a) is $\delta_0'$ and we need to find an appropriate value for it. From perturbation theory we expect $\delta \propto a = 1/(1+z)$ at high-$z$ or, equivalently, $g(z) \to$ constant. In practice, we calculate the growth factor for many different values of $\delta_0'$ until we obtain this required behaviour at $z \simeq 2.34$, the highest redshift of our data set. For the Planck and WMAP values of $\Omega_m$, respectively, we estimate $\delta_0' = 0.515 \pm 0.003$ and $\delta_0' = 0.485 \pm 0.003$. The calculated $g(z)$ functions are shown in figure 3. We note that they are very similar to the ones obtained in ref. [18] using a non-parametric smooth reconstruction from SNe Ia data. As expected (see section 2), the $g(z)$ reconstructed function depends significantly on the present-day value of the matter density parameter assumed in the analysis.

In the reconstruction of the growth rate $f(z)$, $\delta_0'$ plays an important role because these quantities are related through $f(0) = -\delta_0'$. The resulting reconstruction, the growth rate as a function of $z$, is shown in the figure 4. For comparison, we also display current measurements of this quantity, as discussed in ref. [66]. The clear compatibility between the reconstruction of $f(z)$ from cosmic chronometers data and the measurements of the growth rate from galaxy surveys can be seen as a measure of consistency of the theoretical treatment introduced in ref. [18] as well as of the non-parametric method of reconstruction used in the present

\(^2\)http://www.acgc.uct.ac.za/~seikel/GAPP/index.html
Figure 2. The evolution of the matter density parameter calculated using the reconstruction of $H(z)$ shown in figure 1(a) and current estimates of $\Omega_m$ from the Planck (a) and the WMAP collaborations (b). The shaded regions correspond to 1σ and 2σ confidence intervals.

Figure 3. a) The growth factor on sub-Hubble scale obtained solving eq. (2.7a) using the Planck 2015 $\Omega_m$ value. The solid line corresponds to the reconstruction whereas the shaded regions represent 1σ and 2σ confidence levels. b) The same as in the previous panel assuming the value of $\Omega_m$ given by the WMAP collaboration.

analysis. More importantly, for the values of $\Omega_m$ given by the current CMB experiments, the results of figure 4 show a good agreement with the standard cosmological description, i.e., a general relativistic universe described by the FLRW line element and whose matter content is covariantly conserved (see section 2). Note, however, that this conclusion may change if one considers values of the matter density parameters far from the current CMB interval. This is clearly seen in figure 4c which assumes $\Omega_m = 0.399 \pm 0.025$, as obtained from a recent analysis of type Ia supernova data (assuming the $\Lambda$CDM model) [67]. Quantitatively speaking, a fit of the $f(z)$ data to the $f(z)$ reconstructed curves provides $\chi^2 = 7.51$ and $\chi^2 = 5.20$ for the values of $\Omega_m$ displayed in Panels 4a and 4b, respectively, and $\chi^2 = 25.80$ for the SNe Ia value considered in Panel 4c.

Finally, we also calculate the growth index $\gamma$ using the reconstructed function of $f(z)$ and the CMB values of $\Omega_m$ discussed above. At $z = 0$, we found $\gamma_0 = 0.56 \pm 0.12$ (2σ) and $\gamma_0 = 0.57 \pm 0.13$ (2σ) for the Planck and WMAP values of $\Omega_m$, respectively. From our reconstruction, the growth index is more effectively constrained at $z = 0.09$, i.e., $\gamma = 0.57 \pm 0.11$ (2σ), assuming the interval of $\Omega_m$ given by the Planck collaboration. For the WMAP-9
Figure 4. The growth rate of the matter perturbation. The solid line corresponds to the reconstruction by solving eq. (2.10) whereas the shaded regions represent 1σ and 2σ confidence levels. a) The growth rate obtained assuming the value of $\Omega_{m0}$ given by the Planck collaboration. b) The same as in the previous panel assuming the WMAP $\Omega_{m0}$ value. c) The same as in the previous panels assuming $\Omega_{m0} = 0.399 \pm 0.027$, as obtained from SNe Ia observations [67]. The data points were taken from table II of ref. [66].

Figure 5. The growth index $\gamma(z)$ of matter perturbation. The solid line corresponds to the reconstruction from GP whereas the shaded regions represent 1σ and 2σ confidence levels. a) The growth index obtained assuming the Planck $\Omega_{m0}$ value. b) The same as in the previous panel assuming the value of $\Omega_{m0}$ given by the WMAP collaboration.

estimate of the matter density parameter, we found a very similar value at $z = 0.05$. Note also that the current precision of the $H(z)$ measurements is not enough to place significant constraints on the $\gamma'_0$ value, which could provide a test of the $\Lambda$CDM model [53]. The final reconstruction of the growth index is presented in figure 5.

5 Conclusions

In this work we have performed a non-parametric reconstruction of the cosmic expansion with cosmic chronometer and high-$z$ quasar data using the method of Gaussian Process. As discussed in ref. [60] the cosmic chronometer data until $z \sim 1.2$ are independent of cosmological and stellar population models. We have followed ref. [18] and calculated the most representative perturbative quantities in the GR frame with non-clustering DE, assuming spatial homogeneity and isotropy. For the values of $\Omega_{m0}$ given by the current CMB experiments, we have found a good agreement between current growth rate measurements and the growth rate reconstructed using the $H(z)$ data displayed in table 1 (see figure 4). In other
words, this amounts to saying that no evidence for a deviation from the standard cosmological description has been found in our analysis. On the other hand, a direct comparison of the reconstructed functions \(g(z), f(z)\) and \(\gamma(z)\) assuming different values of the matter density parameter clearly show the significant influence of this quantity in the calculations of the matter perturbations.

We have also derived the value of the growth index at the present epoch, i.e., \(\gamma_0 = 0.56 \pm 0.12\) (2\(\sigma\)), whose evolution is almost constant until \(z = 1\). Such a result is compatible with the \(\Lambda\)CDM expected value \(\gamma = 0.545\) and with its first derivative \(\gamma'_0 \simeq -0.015\) \([53]\). Finally, we have shown that the reconstruction from the subsample of \(H(z)\) data used in our analysis constrains the growth index to the interval \(0.51 < \gamma(z) < 0.62\) (1\(\sigma\)) at \(z = 0.09\). Using a different approach and assuming a constant growth index, ref. \([68]\) found \(0.505 < \gamma < 0.869\) (1\(\sigma\)).

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References

[1] Supernova Search Team collaboration, A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116 (1998) 1009 [astro-ph/9805201] [nSPIRE].

[2] Supernova Cosmology Project collaboration, S. Perlmutter et al., Measurements of Omega and Lambda from 42 high redshift supernovae, Astrophys. J. 517 (1999) 565 [astro-ph/9812133] [nSPIRE].

[3] V. Sahni and A.A. Starobinsky, The Case for a positive cosmological Lambda term, Int. J. Mod. Phys. D 9 (2000) 373 [astro-ph/9904398] [nSPIRE].

[4] P.J.E. Peebles and B. Ratra, The Cosmological constant and dark energy, Rev. Mod. Phys. 75 (2003) 559 [astro-ph/0207347] [nSPIRE].

[5] T. Padmanabhan, Cosmological constant: The Weight of the vacuum, Phys. Rept. 380 (2003) 235 [hep-th/0212290] [nSPIRE].

[6] J.S. Alcaniz, Dark Energy and Some Alternatives: A Brief Overview, Braz. J. Phys. 36 (2006) 1109 [astro-ph/0608631] [nSPIRE].

[7] D.H. Weinberg, M.J. Mortonson, D.J. Eisenstein, C. Hirata, A.G. Riess and E. Rozo, Observational Probes of Cosmic Acceleration, Phys. Rept. 530 (2013) 87 [arXiv:1201.2434] [nSPIRE].

[8] N. Bartolo and M. Pietroni, Scalar tensor gravity and quintessence, Phys. Rev. D 61 (2000) 023518 [hep-th/9908521] [nSPIRE].

[9] S. Capozziello, Curvature quintessence, Int. J. Mod. Phys. D 11 (2002) 483 [gr-qc/0201033] [nSPIRE].

[10] J. Santos, J.S. Alcaniz, M.J. Reboucas and F.C. Carvalho, Energy conditions in \(f(R)\)-gravity, Phys. Rev. D 76 (2007) 083513 [arXiv:0708.0411] [nSPIRE].

[11] A.A. Starobinsky, Disappearing cosmological constant in \(f(R)\) gravity, JETP Lett. 86 (2007) 157 [arXiv:0706.2041] [nSPIRE].
[12] G.R. Dvali, G. Gabadadze and M. Porrati, 4 – D gravity on a brane in 5 – D Minkowski space, Phys. Lett. B 485 (2000) 208 [hep-th/0005016] [inSPIRE].

[13] V. Sahni and Y. Shtanov, Brane world models of dark energy, JCAP 11 (2003) 014 [astro-ph/0202346] [inSPIRE].

[14] J.S. Alcaniz and N. Pires, Cosmic acceleration in brane cosmology, Phys. Rev. D 70 (2004) 047303 [astro-ph/0404146] [inSPIRE].

[15] A. Lue, The phenomenology of dvali-gabadadze-porrati cosmologies, Phys. Rept. 423 (2006) 1 [astro-ph/0510068] [inSPIRE].

[16] E.J. Copeland, M. Sami and S. Tsujikawa, Dynamics of dark energy, Int. J. Mod. Phys. D 15 (2006) 1753 [hep-th/0603057] [inSPIRE].

[17] Y. Wang, Differentiating dark energy and modified gravity with galaxy redshift surveys, JCAP 05 (2008) 021 [arXiv:0710.3885] [inSPIRE].

[18] M. Hicken, W.M. Wood-Vasey, S. Blondin, P. Challis, S. Jha, P.L. Kelly et al., Improved Dark Energy Constraints from 100 New CfA Supernova Type Ia Light Curves, Astrophys. J. 700 (2009) 1097 [arXiv:0901.4804] [inSPIRE].

[19] M. Chevallier and D. Polarski, Accelerating universes with scaling dark matter, Int. J. Mod. Phys. D 10 (2001) 213 [gr-qc/0009008] [inSPIRE].

[20] E.V. Linder, Exploring the expansion history of the universe, Phys. Rev. Lett. 90 (2003) 091301 [astro-ph/0208512] [inSPIRE].

[21] E.M. Barboza, Jr. and J.S. Alcaniz, A parametric model for dark energy, Phys. Lett. B 666 (2008) 415 [arXiv:0805.1713] [inSPIRE].

[22] I. Sendra and R. Lazkoz, SN and BAO constraints on (new) polynomial dark energy parametrizations: current results and forecasts, Mon. Not. Roy. Astron. Soc. 422 (2012) 776 [arXiv:1105.4943] [inSPIRE].

[23] S.D.P. Vitenti and M. Penna-Lima, A general reconstruction of the recent expansion history of the universe, JCAP 09 (2015) 045 [arXiv:1505.01883] [inSPIRE].

[24] A. Montiel, R. Lazkoz, I. Sendra, C. Escamilla-Rivera and V. Salzano, Nonparametric reconstruction of the cosmic expansion with local regression smoothing and simulation extrapolation, Phys. Rev. D 89 (2014) 043007 [arXiv:1401.4188] [inSPIRE].

[25] C.E. Rasmussen and C.K.I. Williams, Gaussian Processes for Machine Learning, MIT Press (2006).

[26] T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib et al., Nonparametric Dark Energy Reconstruction from Supernova Data, Phys. Rev. Lett. 105 (2010) 241302 [arXiv:1011.3079] [inSPIRE].

[27] T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib et al., Nonparametric Reconstruction of the Dark Energy Equation of State from Diverse Data Sets, Phys. Rev. D 84 (2011) 083501 [arXiv:1104.3041] [inSPIRE].

[28] M. Seikel, C. Clarkson and M. Smith, Reconstruction of dark energy and expansion dynamics using Gaussian processes, JCAP 06 (2012) 036 [arXiv:1204.2832] [inSPIRE].

[29] M. Seikel, S. Yahya, R. Maartens and C. Clarkson, Using H(z) data as a probe of the concordance model, Phys. Rev. D 86 (2012) 083001 [arXiv:1205.3431] [inSPIRE].

[30] A. Shafieloo, Crossing Statistic: Reconstructing the Expansion History of the Universe, JCAP 08 (2012) 002 [arXiv:1204.1109] [inSPIRE].

[31] A. Shafieloo, A.G. Kim and E.V. Linder, Gaussian Process Cosmography, Phys. Rev. D 85 (2012) 123530 [arXiv:1204.2272] [inSPIRE].
[32] A. Shafieloo, U. Alam, V. Sahni and A.A. Starobinsky, *Smoothing Supernova Data to Reconstruct the Expansion History of the Universe and its Age*, Mon. Not. Roy. Astron. Soc. 366 (2006) 1081 [astro-ph/0505329] [SPIRE].

[33] Z. Li, J.E. Gonzalez, H. Yu, Z.-H. Zhu and J.S. Alcaniz, *Constructing a cosmological model-independent Hubble diagram of type-IA supernovae with cosmic chronometers*, Phys. Rev. D 93 (2016) 043014 [arXiv:1504.03269] [SPIRE].

[34] R. Jimenez and A. Loeb, *Constraining cosmological parameters based on relative galaxy ages*, Astrophys. J. 573 (2002) 37 [astro-ph/0106145] [SPIRE].

[35] R. Jimenez, L. Verde, T. Treu and D. Stern, *Constraints on the equation of state of dark energy and the Hubble constant from stellar ages and the CMB*, Astrophys. J. 593 (2003) 622 [astro-ph/0302560] [SPIRE].

[36] J. Simon, L. Verde and R. Jimenez, *Constraints on the redshift dependence of the dark energy potential*, Phys. Rev. D 71 (2005) 123001 [astro-ph/0412269] [SPIRE].

[37] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski and S.A. Stanford, *Cosmic Chronometers: Constraining the Equation of State of Dark Energy. I: H(z) Measurements*, JCAP 02 (2010) 008 [arXiv:0907.3149] [SPIRE].

[38] M. Moresco et al., *Improved constraints on the expansion rate of the Universe up to z 1.1 from the spectroscopic evolution of cosmic chronometers*, JCAP 08 (2012) 006 [arXiv:1201.3609] [SPIRE].

[39] M. Moresco, *Raising the bar: new constraints on the Hubble parameter with cosmic chronometers at z 2*, Mon. Not. Roy. Astron. Soc. 450 (2015) L16 [arXiv:1503.01116] [SPIRE].

[40] N.G. Busca et al., *Baryon Acoustic Oscillations in the Ly-α forest of BOSS quasars*, Astron. Astrophys. 552 (2013) A96 [arXiv:1211.2616] [SPIRE].

[41] BOSS collaboration, T. Delubac et al., *Baryon acoustic oscillations in the Ly-α forest of BOSS DR11 quasars*, Astron. Astrophys. 574 (2015) A59 [arXiv:1404.1801] [SPIRE].

[42] BOSS collaboration, A. Font-Ribera et al., *Quasar-Lyman α Forest Cross-Correlation from BOSS DR11: Baryon Acoustic Oscillations*, JCAP 05 (2014) 027 [arXiv:1311.1767] [SPIRE].

[43] O. Farooq and B. Ratra, *Constraints on dark energy from the Ly-α forest baryon acoustic oscillations measurement of the redshift 2.3 Hubble parameter*, Phys. Lett. B 723 (2013) 1 [arXiv:1212.4264] [SPIRE].

[44] V. Sahni and A. Starobinsky, *Reconstructing Dark Energy*, Int. J. Mod. Phys. D 15 (2006) 2105 [astro-ph/0610026] [SPIRE].

[45] S. Nesseris and D. Sapone, *Accuracy of the growth index in the presence of dark energy perturbations*, Phys. Rev. D 92 (2015) 023013 [arXiv:1505.06601] [SPIRE].

[46] A.A. Starobinsky, *How to determine an effective potential for a variable cosmological term*, JETP Lett. 68 (1998) 757 [astro-ph/9810431] [SPIRE].

[47] R. Reyes, R. Mandelbaum, U. Seljak, T. Baldauf, J.E. Gunn, L. Lombriser et al., *Confirmation of general relativity on large scales from weak lensing and galaxy velocities*, Nature 464 (2010) 256 [arXiv:1003.2185] [SPIRE].

[48] S. February, C. Clarkson and R. Maartens, *Galaxy correlations and the BAO in a void universe: structure formation as a test of the Copernican Principle*, JCAP 03 (2013) 023 [arXiv:1206.1602] [SPIRE].

[49] S. Carneiro, C. Pigozzo, H.A. Borges and J.S. Alcaniz, *Supernova constraints on decaying vacuum cosmology*, Phys. Rev. D 74 (2006) 023532 [astro-ph/0605607] [SPIRE].
F.E.M. Costa and J.S. Alcaniz, *Cosmological consequences of a possible $\Lambda$-dark matter interaction*, Phys. Rev. D 81 (2010) 043506 [arXiv:0908.4251] [inSPIRE].

A. Pourtsidou, C. Skordis and E.J. Copeland, *Models of dark matter coupled to dark energy*, Phys. Rev. D 88 (2013) 083505 [arXiv:1307.0458] [inSPIRE].

C. Pigozzo, S. Carneiro, J.S. Alcaniz, H.A. Borges and J.C. Fabris, *Evidence for cosmological particle creation?*, arXiv:1510.01794 [inSPIRE].

R. Gannouji, B. Moraes and D. Polarski, *The growth of matter perturbations in $f(R)$ models*, JCAP 02 (2009) 034 [arXiv:0809.3374] [inSPIRE].

S.M. Carroll, I. Sawicki, A. Silvestri and M. Trodden, *Modified-Source Gravity and Cosmological Structure Formation*, New J. Phys. 8 (2006) 323 [astro-ph/0607458] [inSPIRE].

C. Pigozzo, S. Carneiro, J.S. Alcaniz, H.A. Borges and J.C. Fabris, *Evidence for cosmological particle creation?*, arXiv:1510.01794 [inSPIRE].

R. Gannouji, B. Moraes and D. Polarski, *The dispersion of growth of matter perturbations in $f(R)$ gravity*, Phys. Rev. D 80 (2009) 084044 [arXiv:0908.2669] [inSPIRE].

S. Tsujikawa, R. Gannouji, B. Moraes and D. Polarski, *The dispersion of growth of matter perturbations in $f(R)$ gravity*, Phys. Rev. D 80 (2009) 084044 [arXiv:0908.2669] [inSPIRE].

O. Lahav, P.B. Lilje, J.R. Primack and M.J. Rees, *Dynamical effects of the cosmological constant*, Mon. Not. Roy. Astron. Soc. 251 (1991) 128 [inSPIRE].

V.C. Busti, C. Clarkson and M. Seikel, *Evidence for a Lower Value for $H_0$ from Cosmic Chronometers Data?*, Mon. Not. Roy. Astron. Soc. 441 (2014) 11 [arXiv:1402.5429] [inSPIRE].

S. Santos-da Costa, V.C. Busti and R.F.L. Holanda, *Two new tests to the distance duality relation with galaxy clusters*, JCAP 10 (2015) 061 [arXiv:1506.00145] [inSPIRE].

Y. Zhang, *Reconstruct the Distance Duality Relation by Gaussian Process*, arXiv:1408.3897 [inSPIRE].

S. Nesseris and L. Perivolaropoulos, *Testing Lambda CDM with the Growth Function $\Delta(a)$: Current Constraints*, Phys. Rev. D 77 (2008) 023504 [arXiv:0710.1092] [inSPIRE].