ORIGINAL ARTICLE

Variable cosmological term $\Lambda(t)$

J. Socorro$^1$ · M. D’oleire$^2$ · Luis O. Pimentel$^3$

Received: 26 August 2015 / Accepted: 5 October 2015 / Published online: 20 October 2015
© Springer Science+Business Media Dordrecht 2015

Abstract We present the case of time-varying cosmological term $\Lambda(t)$. The main idea arises by proposing that as in the cosmological constant case, the scalar potential is identified as $V(\phi) = 2\Lambda$, with $\Lambda$ a constant, this identification should be kept even when the cosmological term has a temporal dependence, i.e., $V(\phi(t)) = 2\Lambda(t)$. We use the Lagrangian formalism for a scalar field $\phi$ with standard kinetic energy and arbitrary potential $V(\phi)$ and apply this model to the Friedmann-Robertson-Walker (FRW) cosmology. Exact solutions of the field equations are obtained by a special ansatz to solve the Einstein-Klein-Gordon equation and a particular potential for the scalar field and barotropic perfect fluid. We present the evolution on this cosmological term with different scenarios.

Keywords Cosmological term · Scalar field cosmology

1 Introduction

The present phase of an accelerated expansion of the universe stands as one of the most challenging open problems in modern cosmology and astrophysics. This acceleration is characterized by which is popularly known as dark energy. Among many possible alternatives, the simplest candidate for dark energy is the vacuum energy which is mathematically equivalent to the cosmological constant. Models with different decay laws for the variation of cosmological term were investigated during the last two decades in a non covariant way (Chen and Wu 1990; Abdel-Rahman 1990; Pavon 1991; Carvalho et al. 1992; Kalligas et al. 1992; Lima and Maia 1994; Lima and Carvalho 1994; Lima and Trodden 1996; Arbab and Abdel-Rahaman 1994; Birkel and Sarkar 1997; Silveira and Waga 1997; Starobinsky 1998; Overduin and Cooperstock 1998; Vishwakarma 2000, 2001; Arbab 2001, 2003, 2004; Cunha and Santos 2004; Carneiro and Lima 2005; Fomin et al. 2005, 2006; Sola and Stefanic 2005, 2006; Pradhan et al. 2007; Jamil and Deb Nath 2011; Mukhopadhyay et al. 2011); in particular, in Fomin et al. (2005) there are several evolution relations for $\Lambda$ which many author have used, also in Overduin and Cooperstock (1998) appears a table with these relations and the corresponding references where they were considered. Anisotropic cosmological models, also has been treated in this formalism from different points of view (Beesham 1993, 1994; Arbab 1997; Singh et al. 1998; Pradhan and Kumar 2001; Pradhan 2003, 2007, 2009; Pradhan and Pandey 2003, 2006; Pradhan et al. 2007, 2008; Carneiro 2005; Esposito et al. 2007; Bal and Singh 2008; Belinchón 2008; Singh et al. 2008, 2013; Shen 2013; Tripathy 2013; Rahman and Ansary 2013).

In this work we present an analysis in covariant way, using the Lagrangian density of standard scalar field. The main
idea arises by proposing that as in the cosmological constant case, the scalar potential is identified as \( V(\phi) = 2\Lambda \), with \( \Lambda \) a constant. So, we extend this idea and suggest that this correspondence is valid even when this cosmological term has a temporal dependence, i.e., \( V(\phi(t)) = 2\Lambda(t) \).

We include a barotropic equation state between the pressure and energy density of the scalar field, \( p_\phi = \omega_\phi \rho_\phi \), quantities that we shall define in the following lines. In order to build up the analysis presented here, initially we solve the Klein-Gordon equations, whose solution implies that the energy density of a scalar field has a wide range of scaling behavior, \( \rho_\phi \sim A^{-m} \) with \( A \) the scale factor of the FRW model (Ferreira and Joyce 1998; Liddle and Sharrer 1998; Copeland et al. 1998), that emerges as a proportionality law model (Ferreira and Joyce 1998; Liddle and Sharrer 1998; Copeland et al. 1998), that emerges as a proportionality law nonzero matter (2), (3) and one state equation, are the fundamental tools to do the research, considering a variable cosmological term in non covariant way, because there is no a Lagrangian density that reproduce these field equations (2), (3).

In the present treatment we take into account the corresponding Lagrangian density with a scalar field

\[
\mathcal{L}[g, \phi] = \sqrt{-g} \left( R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + V(\phi) \right) + \sqrt{-g} \mathcal{L}_{\text{matter}}
\]

where \( R \) is the Ricci scalar, \( \mathcal{L}_{\text{matter}} \) corresponds to a barotropic perfect fluid, \( p = \omega \rho \), \( \rho \) is the energy density and \( p \) is the pressure of the fluid in co-moving frame and \( \omega \) is the barotropic constant.

The corresponding variation of (4), with respect to the metric and the scalar field gives the Einstein and Klein-Gordon field equations

\[
R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{2} \left( \nabla_\alpha \phi \nabla_\beta \phi - \frac{1}{2} g_{\alpha\beta} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) + \frac{1}{2} g_{\alpha\beta} V(\phi) - 8\pi G T_{\alpha\beta},
\]

(5)

\[
\square \phi - \frac{\partial V}{\partial \phi} = 0.
\]

(6)

From (5) it can be deduced that the energy-momentum tensor associated with the scalar field is

\[
8\pi G T^{(\phi)}_{\alpha\beta} = \frac{1}{2} \left( \nabla_\alpha \phi \nabla_\beta \phi - \frac{1}{2} g_{\alpha\beta} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) - \frac{1}{2} g_{\alpha\beta} V(\phi)
\]

(7)

and the corresponding tensor for a barotropic perfect fluid becomes

\[
T_{\alpha\beta} = (p + \rho) u_\alpha u_\beta + g_{\alpha\beta} p
\]

here \( u_\alpha \) is the four-velocity in the comoving frame. The line element to be considered in this work is the FRW

\[
ds^2 = -N(t)^2 dt^2 + A(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

\[
= -d\tau^2 + A(\tau)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],
\]

(8)

where we identify the time transformation \( N(t) dt = d\tau \), this transformation will be used in the whole work, and in special gauge we recover directly the cosmic time \( \tau \)
2 Field equations

Making use to the metric (8) and a comoving fluid, the equations (5) and (6) becomes (a dot mean a time derivative)

\[
\frac{3\dot{A}^2}{A^2} + \frac{3\kappa N^2}{A^2} - 8\pi G \rho - \frac{1}{4} \phi^2 - \frac{N^2 V(\phi)}{2} = 0, \quad (9)
\]

\[
\frac{2\dot{A}^2}{N^2} + \frac{A^2}{N^2} = \frac{2\dot{A}^2}{N^2} = \frac{2\dot{A}^2}{N^2} + \frac{\kappa}{A^2} + 8\pi G A^2 p
\]

\[
+ \left(\frac{3\dot{A}^2}{A^2} - \frac{1}{2} A^2 V(\phi) = 0, \quad (10)
\]

we will make now the assumption that the scalar is a barotropic fluid: \( p_\phi = \omega_\phi \rho_\phi \), where \( \omega_\phi \) is a constant that play the same role of the \( \omega \) parameter in the barotropic perfect fluid. Under this proposal, the field equations are

\[
\frac{3\dot{A}^2}{A^2} + \frac{3\kappa}{A^2} - 8\pi G \rho + \frac{1}{4} \phi^2 - \frac{N^2 V(\phi)}{2} = 0, \quad (18)
\]

\[
\frac{2\dot{A}^2}{A^2} + \frac{\kappa}{A^2} + 8\pi G p = 0, \quad (19)
\]

\[
\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \phi} \frac{\partial \phi}{\partial \phi} = \frac{V'}{\phi}. \quad (20)
\]

\[
\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \phi} \frac{\partial \phi}{\partial \phi} = \frac{V'}{\phi}. \quad (21)
\]

\[
\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \phi} \frac{\partial \phi}{\partial \phi} = \frac{V'}{\phi}. \quad (22)
\]

\[
\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \phi} \frac{\partial \phi}{\partial \phi} = \frac{V'}{\phi}. \quad (23)
\]

In the literature there are some articles where the authors try to solve these field equation in general way, for instance, in Chimento and Jakubi (1996), the authors present an elaborate technique for solve the Klein-Gordon equation (14), and in Reyes (2008) they use an algebraic method to obtain exact solutions taking as the basic variable the energy density of the scalar field.

In order to solve this set of equations, we introduce the ansatz, of considering that the energy density of the field \( \phi \) is proportional to the energy density of the barotropic perfect fluid, \( \rho_\phi = m_\phi \rho \), where \( m_\phi \) is a positive constant. The scaling behavior occurs when \( m_\phi < 1 \), otherwise, the quintessence field is dominant.

The energy density and pressure of the field \( \phi \) are given as

\[
16\pi G \rho_\phi = \frac{1}{2} \phi^2 + V(\phi), \quad \quad 16\pi G p_\phi = \frac{1}{2} \phi^2 - V(\phi)
\]

now, Eqs. (12), (13) are rewritten as

\[
\frac{3\dot{A}^2}{A^2} + \frac{3\kappa}{A^2} - 8\pi G (\rho + \rho_\phi) = 0, \quad (16)
\]

\[
\frac{2\dot{A}^2}{A^2} + \frac{\kappa}{A^2} + 8\pi G (p + p_\phi) = 0. \quad (17)
\]
density of the perfect fluid, and the proportionality of the \( p_\phi \) with \( \rho_\phi \) then the potential \( V, \phi^2/2 \) and all the densities \((\rho, \rho_\phi, \rho_T)\) are proportional to each other. From Eqs. (18), (19) we know that

\[
\rho_T = c_T A^{-3(\omega_T + 1)}.
\]

Then as a consequence of the proportionality between \( V \) and \( p_\phi \) the exponents in Eqs. (24), (23) should be equal, that corresponds to \( m = n \) case,

\[
\omega_T = \omega_\phi \implies \omega = \omega_\phi = \omega_T.
\]

### 3 General solution for flat space

In this section we present solutions to the field equations for the flat case. Equation (20) is in the flat case written as

\[
\frac{d}{d\tau} \left[ \ln(A^6 V^{\frac{2}{3}}) \right] = 0 \implies V(\tau) = c_\omega A^{-3(1+\omega)}, \quad (26)
\]

using the well-known time evolution of the scalar factor for the barotropic fluid, reported in different places, in particular in Berbena et al. (2007) that for future convenience we write as

\[
A_\omega(\tau) = \begin{cases} 
[a_\omega \tau]^{3(1+\omega)}, & \omega \neq -1 \\
2^{\frac{2}{3}} \pi G \alpha M_{-1} \tau e^{2\sqrt{\frac{2}{3}} G \alpha M_{-1} \tau}, & \omega = -1
\end{cases}
\]

The Hubble function \( H = \frac{\dot{A}}{A} = N \frac{\dot{A}}{A} \) and the deceleration parameter

\[
q = -\frac{\ddot{A}}{A^2} = -\frac{\dot{H} + H^2}{H^2} = -\left[ \frac{A'' A}{A^2} + \frac{A N'}{A' N} \right].
\]

can be calculated for our model, and are depending of the gauge shift function \( N \), which should be important employing the observational data of Supernova type Ia, see Riess et al. (1998) and Perlmutter et al. (1999).

In the gauge \( N = 1 \), the Hubble function becomes

\[
H_\omega(t) = \begin{cases} 
\frac{2}{3(\omega + 1)} \frac{1}{\tau}, & \omega \neq 1 \\
2^{\frac{2}{3}} \pi G \alpha M_{-1} \tau e^{2\sqrt{\frac{2}{3}} G \alpha M_{-1} \tau}, & \omega = -1
\end{cases}
\]

and the corresponding deceleration parameter

\[
q_\omega = \begin{cases} 
\frac{1+3\omega}{2}, & \omega \neq -1 \\
-1 & \omega = -1
\end{cases}
\]

in this gauge, the deceleration parameter have a positive value for all values \( \omega \neq -1 \), and only in the exponential behavior have a negative value.

These results are in agreement with the solution at the deceleration parameter (28) when we take the election that this one is time dependent (Pradhan et al. 2012), or in more general sense, we can choose that \( q(H) = F(H) \), where \( H \) is the Hubble function. By example, when we choose that \( F(H) = -1 \), we recover that the scale factor have a exponential behavior, or \( F(H) = \ell = \text{constant} > 0 \), we can recover the power law in the scale factor.

Employing the gauge \( N \rightarrow \Lambda^3 \), we obtain the following

\[
H_\omega(t) = \begin{cases} 
\frac{2}{3(\omega + 1)} \frac{1}{\tau} e^{\frac{1}{\tau}}, & \omega \neq -1 \\
2^{\frac{2}{3}} \pi G \alpha M_{-1} e^{\frac{6\sqrt{\frac{2}{3}} G \alpha M_{-1} \tau}} & \omega = -1
\end{cases}
\]

and the corresponding deceleration parameter

\[
q_\omega = \begin{cases} 
\frac{1+3\omega}{2} e^{\frac{6\sqrt{\frac{2}{3}} G \alpha M_{-1} \tau}} & \omega \neq -1 \\
-1 & \omega = -1
\end{cases}
\]

Over these last results, is worthy to mention that is necessary to choose an appropriate gauge for obtain a negative deceleration parameter in the transformed time \( \tau \).

#### 3.1 Case \( \omega \neq \pm 1 \)

The cases \( \omega = \pm 1 \) will be considered below, in separate way.

Following with our analysis, the temporal dependence of the potential becomes

\[
V(\tau) = c_\omega \frac{1}{(a_\omega \tau)} \implies \Delta \phi = \ell_\omega \ln(\tau),
\]

where the constants value \( c_\omega \) and \( \ell_\omega \) are determined after substitution into the complete set of Einstein equations, with the scale factor solution (27), being the general solutions for any \( \omega \neq \pm 1 \) the following relations are obtained

\[
V_\omega(\tau) = \frac{2m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi \tau^2},
\]

\[
\iff \quad A(\tau) = \frac{m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi \tau^2},
\]

\[
\Delta \phi(\tau) = \frac{4m_\phi}{3(1 + \omega) a_\phi} \ln(\tau),
\]

when the time \( \tau \) is eliminated between the two last equations we obtain the potential \( \phi \) as a function of the scalar field

\[
V(\phi) = \frac{2m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi} e^{-\Delta \phi} \quad \iff \quad A(\phi) = \frac{m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi} e^{-\frac{3(1 + \omega)}{a_\phi} \Delta \phi},
\]

\[
V(\phi) = \frac{2m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi} e^{-\frac{3(1 + \omega) + m_\phi}{a_\phi} \Delta \phi} \quad \iff \quad A(\phi) = \frac{m_\phi (1 - \omega)}{3(1 + \omega)^2 a_\phi} e^{-\frac{3(1 + \omega) + m_\phi}{a_\phi} \Delta \phi}.
\]
The corresponding scalar potential that emerges from the temporal solution has, as is cited in the literature, an exponential behavior (Lucchin and Matarrese 1985; Halliwell 1985; Burd and Barrow 1988; Weetterich 1998; Wand et al. 1993; Ferreira and Joyce 1997; Copeland et al. 1998). We observe here that for all values of the barotropic parameter we have a decreasing cosmological function in time (remember the relation between the potential energy and the cosmological term, \( V(\phi) = 2\Lambda(\phi) \)), and that as function of the scalar field we have an exponential.

### 3.2 Case \( \omega = -1 \)

In this case, the barotropic equation of state implies that the scalar field is constant as is the potential and we are back to \( \Lambda = \) constant case, also the total energy density is constant and we have the exponential expansion factor of Eq. (27).

### 3.3 Case \( \omega = 1 \)

In this case, we obtain in our proposal that \( V(\phi) = 0 \) and then we have a free scalar field that is the simplest case of the k-essence theory or Saéz-Ballester, see Sabido et al. (2010) and references therein for complete solutions for FRW cosmological model and Socorro et al. (2014) for the corresponding Bianchi type I anisotropic cosmological model.

In what follows we consider particular cases, divided in two branches of the barotropic parameter \( 0 < \omega < 1 \), and \(-1 < \omega < 0\).

### 4 Particular solutions

In this section we consider particular cases of the solutions with specific values of the barotropic coefficient that are of interest in cosmology and considering positive and negative branches.

#### 4.1 Positive branch: \( 0 \leq \omega < 1 \)

The relation between the kinetic term and the potential energy of the scalar field is

\[
\frac{1}{2} \dot{\phi}^2 = \frac{1 + \omega}{1 - \omega} V(\phi)
\]

and we have the following particular case for the barotropic parameter that are of interest in cosmology and astrophysics.

1. Dust scenario, \( \omega = 0 \).

The set of Eqs. (34), (35), (36) have the following form

\[
V_\omega(\tau) = \frac{2m_\phi}{3\alpha_\phi} \frac{1}{\tau^2},
\]

\[
\Delta \phi(\tau) = \sqrt{\frac{4m_\phi}{3\alpha_\phi}} \ln(\tau),
\]

\[
V(\phi) = \frac{2m_\phi}{3\alpha_\phi} e^{-\sqrt{3(1 + m_\phi)} \Delta \phi},
\]

together with the following quantum factor

\[
A(\tau) = [a_0 \tau]^\frac{1}{2}, \quad a_0 = \sqrt{6\pi G\alpha_\phi M_0}.
\]

Also this behavior is found using dynamical system and fitting that one critical point will be an attractor, obtaining that the corresponding factor \( \lambda \) in the exponential function \( V(\phi) \approx e^{\lambda \phi} \), will be less than \(-\sqrt{3} \), (Hernández-Aguayo and Ureña López 2011). This value for the \( \lambda \) parameter was found using others techniques, in quantum solutions and in supersymmetric quantum solutions in quantum cosmology, for the same flat FRW cosmological model (Socorro and D’oleire 2010; Socorro et al. 2013). Chimento and Jakubi (1996) found the value \( \lambda = -\sqrt{2} \), for inflationary era, solving the Einstein field equations as power law.

Is common say that when the scalar potential have a exponential behavior as in this case, the universe must have a fast growing scale factor. Remembering that \( \alpha_\phi = 1 + m_\phi \), and considering that ordinary matter is only 4% of the total, and assuming that the scalar field account for the remaining density, we need that \( \alpha_\phi \) near to 18 when we consider the dark energy scenario. In that case, the scale factor is fast growing and the quintessence field is dominant in the evolution in the universe.

Comparing the different scale factor in Fig. 1, the temporal potential term to the left graphics, have a big negative slope, making that the universe roll faster, having a fast growing when the parameter \( m_\phi = 18 \), in other values this slope is attenuated, making a moderate expansion in the scale factor.

In particular, in the temporal potential field there is a different behavior in the axe \( m_\phi \) in the \([0, 1]\), region where the ordinary matter have a dominant behavior; in the other region, the quintessence field is dominant in the evolution of the universe.

![Fig. 1](image-url) In the dust scenario, the scale factor has a fast growth for large values of the \( m_\phi \) parameter, in the plot we choose the values 0.5, 1, 4 and 18, going from **bottom to top** in the **left graphic**. The corresponding behavior of the cosmological term (potential term) is shown in the **right graphic**.
In the radiation scenario, the scale factor has a fast growth for large values of the $m_\phi$ parameter, in the plot we choose the values 0.5, 1, 4 and 18, going from bottom to top in the left graphic. The corresponding behavior of the cosmological term (potential term) is shown in the right graphic.

Fig. 2

In the inflation like scenario, the scale factor has a fast growth for large values of the $m_\phi$ parameter, in the plot we choose the values 0.5, 1, 4 and 18, going from bottom to top in the left graphic. The corresponding behavior of the cosmological term (potential term) is shown in the right graphic.

Fig. 3

2. Radiation, $\omega = \frac{4}{3}$.

The set of Eqs. (34), (35), (36), for this value of the barotropic parameter, have the following form

$$V_\omega(\tau) = \frac{m_\phi}{4\alpha_\phi} \frac{1}{\tau^2},$$

(42)

$$\Delta\phi(\tau) = \sqrt{\frac{m_\phi}{\alpha_\phi}} \ln(\tau),$$

(43)

$$V(\phi) = \frac{m_\phi}{4\alpha_\phi} e^{-2\sqrt{1+\frac{1}{m_\phi}}\Delta\phi},$$

(44)

and the corresponding scale factor

$$A_{\frac{4}{3}}(\tau) = \left[ a_{\frac{4}{3}} \right]^2 \tau^{2}, \quad a_{\frac{4}{3}} = \frac{4}{3} \sqrt{\frac{6\pi G \alpha_\phi M}{3}}.$$  

(49)

The temporal dependence of the cosmological term goes as $\frac{1}{\tau^2}$, a result that is reported in all references that used a proportional relation between the energy density of the scalar field and the energy density to the barotropic perfect fluid, in non covariant theory.

In Fig. 2, the corresponding behavior to $A(t)$ and $\Lambda$ are shown.

4.2 Negative branch: $-1 < \omega < 0$

In this case we write the relation between the field pressure and density as

$$p_\phi = -\omega \rho_\phi \quad \rightarrow \quad \frac{1}{2} \phi^2 = \beta_\omega V(\phi), \quad \beta_\omega = \frac{1-|\omega_\phi|}{1+|\omega_\phi|}$$

(45)

and we consider two particular values of $\omega$.

1. For instance, when we choose the case $\omega_\phi = -\frac{2}{3}$, i.e., $|\omega_\phi| = \frac{2}{3}$.

The set of Eqs. (34), (35), (36) have the following form

$$V_\omega(\tau) = \frac{10m_\phi}{\alpha_\phi} \frac{1}{\tau^2},$$

(46)

$$\Delta\phi(\tau) = 2 \sqrt{\frac{m_\phi}{\alpha_\phi}} \ln(\tau),$$

(47)

$$V(\phi) = \frac{10m_\phi}{\alpha_\phi} e^{-\sqrt{1+\frac{1}{m_\phi}}\Delta\phi},$$

(48)

with the law for the scale factor

$$A_{-\frac{2}{3}}(\tau) = \left[ a_{-\frac{2}{3}} \right]^2 \tau^{2}, \quad a_{-\frac{2}{3}} = \frac{2}{3} \sqrt{\frac{6\pi G \alpha_\phi M}{3}}.$$  

(53)

We consider that in this phenomenological scenario, the values of $m_\phi$ are in the interval $(0, 1)$, for instance when $m_\phi = 1$, we recover the potential for the scalar field given by Chimento and Jakubi (1996) with the corresponding scale factor. In Fig. 3, the corresponding behavior to $A(t)$ and $\Lambda$ are shown for this scenario.

2. When we choose $\omega = -\frac{1}{3}$, i.e., $|\omega_\phi| = \frac{1}{3}$, we have

$$V(\tau) = \frac{2m_\phi}{\alpha_\phi} \frac{1}{\tau^2},$$

(50)

so, the scalar field is

$$\Delta\phi = \sqrt{\frac{2m_\phi}{\alpha_\phi}} \ln(\tau),$$

(51)

thus, we can write $V(\phi)$

$$V(\phi) = \frac{2m_\phi}{\alpha_\phi} e^{-\sqrt{2(1+\frac{1}{m_\phi})}\Delta\phi},$$

(52)

with a linear evolution for the scale factor

$$A_{-\frac{1}{3}}(\tau) = \left[ a_{-\frac{1}{3}} \right] \tau, \quad a_{-\frac{1}{3}} = \frac{2}{3} \sqrt{\frac{6\pi G \alpha_\phi M}{3}}.$$  

(53)

5 Conclusions

In this work we have characterized the cosmological term $\Lambda(\tau)$ as proportional the potential for the scalar field. The main idea arises by proposing that as in the cosmological constant case, the scalar potential is identified as $V(\phi) = 2\Lambda$, with $\Lambda$ a constant, this identification should be kept
even when the cosmological term has a temporal dependence, i.e., \( V(\phi(t)) = 2\Lambda(t) \). Assuming a proportionality between the energy density of the scalar field and the density of a barotropic fluid of the matter content, an also assuming that the pressure and density of the scalar field satisfy a barotropic law, so that the field equation in the case of the FRW metric reduces to the standard cosmology in term of a total energy density and pressure that also satisfy a barotropic law. We found that for consistency all the different barotropic parameters should be the same. In the case of flat space we were able to find general exact solutions. A common characteristic of all the solutions presented here is that the dynamic cosmological “constant” is decreasing in time as \( \frac{1}{t^2} \) and that it is an exponential function of the scalar field. We also found the exponential behavior in the scalar field in the evolution of the universe, and in particular case, the dust era \( \omega_\phi = 0 \), the scalar potential have a time dependence in agreement with others results that uses directly quintessence field in the dark energy frame, signal that the universe must have a fast growing scale factor. This fast growing scale factor correspond to \( m_\phi > 1 \), that is when the quintessence field dominates in the universe, by instance we claim that if the percent to usual matter becomes as 4 %, in the dark energy and dark matter scenario we have that \( \alpha_\phi \) is near to 18. However the case \( m_\phi < 1 \) corresponds to scaling behavior with the usual matter. In all epochs analyzed in this work using the relation between the energy density of the scalar field and the energy density of the ordinary matter, the behavior of the cosmological term goes as \( \frac{1}{t^2} \), these results were found by other authors in a non covariant way (Chen and Wu 1990; Abdel-Rahman 1990; Pavon 1991; Carvalho et al. 1992; Kalligas et al. 1992; Lima and Maia 1994; Lima and Carvalho 1994; Lima and Trodden 1996; Arbab and Abdel-Rahman 1994; Birkel and Sarkar 1997; Silveira and Waga 1997; Starobinsky 1998; Overduin and Cooperstock 1998; Vishwakarma 2000, 2001; Arbab 2001, 2003, 2004; Cunha and Santos 2004; Carneiro and Lima 2005; Fomin et al. 2005; Sola and Stefancic 2005, 2006; Pradhan et al. 2007; Jamil and Deb Nath 2011; Mukhopadhyay et al. 2011). We consider that this behavior is dependent on the relation between the energy densities considered in this work and others.

Acknowledgements This work was partially supported by CONACYT 167335, 179881, 237351 grants. PROMEP grants UGTO-CA-3 and UAM-1-43. This work is part of the collaboration within the Instituto Avanzado de Cosmologia and Red PROMEP: Gravitation and Mathematical Physics under project Quantum aspects of gravity in cosmological models, phenomenology and geometry of space-time. Many calculations where done by Symbolic Program REDUCE 3.8.

References

Abdel-Rahman, A.M.M.: Gen. Relativ. Gravit. 22, 655 (1990)
Arbab, A.I.: Gen. Relativ. Gravit. 29, 61 (1997)
Arbab, A.I.: Spacetime Subst. 1, 39 (2001)
Arbab, A.I.: Class. Quantum Gravity 20, 93 (2003)
Arbab, A.I.: Astrophys. Space Sci. 291, 141 (2004)
Arbab, A.I., Abdel-Rahman, A.M.M.: Phys. Rev. D 50, 7725 (1994)
Bal, R., Singh, J.P.: Int. J. Theor. Phys. 47, 3288 (2008)
Beesham, A.: Phys. Rev. D 48, 5359 (1993)
Beesham, A.: Gen. Relativ. Gravit. 26, 159 (1994)
Belinchón, J.A.: Int. J. Mod. Phys. A 23, 5021 (2008)
Berbena, S.R., et al.: Rev. Mex. Fis. 53(2), 115 (2007)
Birkel, M., Sarkar, S.: Astropart. Phys. 6, 197 (1997)
Burd, A.B., Barrow, J.D.: Nucl. Phys. B 308, 929 (1988)
Carneiro, S.: Int. J. Mod. Phys. A 20, 2465 (2005)
Carneiro, S., Lima, J.A.S.: Int. J. Mod. Phys. A 20, 2465 (2005)
Carvalho, J.C., et al.: Phys. Rev. D 46, 2404 (1992)
Chen, W., Wu, Y.S.: Phys. Rev. D 41, 695 (1990)
Chimento, L.P., et al.: Commun. Theor. Phys. 1, 757 (1991)
Copeland, E.J., et al.: Phys. Rev. D 57, 4686 (1998)
Cunha, J.V., Santos, R.C.: Int. J. Mod. Phys. D 13, 1321 (2004)
Cunha, J.V., Santos, R.C.: Int. J. Mod. Phys. D 13, 1321 (2004)
Esposito, G., et al.: Class. Quantum Gravity 24, 6255 (2007)
Ferreira, P.G., Joyce, M.: Phys. Rev. Lett. 79, 4740 (1997)
Ferreira, P.G., Joyce, M.: Phys. Rev. D 58, 023503 (1998)
Fomin, P.I., et al.: Preprint arXiv:gr-qc/0509042 (2005)
Halliwell, J.: Phys. Lett. B 185, 331 (1987)
Hernández-Aguayo, C., Ureña-López, L.A.: AIP Conf. Proc. 1473, 68 (2011)
Jamil, M., Debnath, U.: Int. J. Theor. Phys. 50, 1602 (2011)
Kalligas, D., et al.: Gen. Relativ. Gravit. 24, 351 (1992)
Liddle, A.R., Sharrer, R.J.: Phys. Rev. D 59, 023509 (1999)
Lima, J.A.S., Carvalho, J.C.: Gen. Relativ. Gravit. 26, 900 (1994)
Lima, J.A.S., Maia, J.M.F.: Phys. Rev. D 49, 5597 (1994)
Lima, J.A.S., Trodden, M.: Phys. Rev. D 53, 4280 (1996)
Lucchin, F., Matarrese, S.: Phys. Rev. D 32, 1316 (1985)
Mukhopadhyay, U., et al.: Int. J. Theor. Phys. 50, 752 (2011)
Overduin, J.M., Cooperstock, F.I.: Phys. Rev. D 58, 043506 (1998)
Pavon, D.: Phys. Rev. D 43, 375 (1991)
Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
Pradhan, A.: Int. J. Mod. Phys. D 12, 941 (2003)
Pradhan, A.: Fizika B 16, 205 (2007)
Pradhan, A.: Commun. Theor. Phys. 51, 367 (2009)
Pradhan, A., Kumar, A.: Int. J. Mod. Phys. D 10, 291 (2001)
Pradhan, A., Pandey, A.P.: Int. J. Mod. Phys. D 12, 1299 (2003)
Pradhan, A., Pandey, A.P.: Astrophys. Space Sci. 301, 127 (2006)
Pradhan, A., et al.: Int. J. Theor. Phys. 46, 2774 (2007)
Pradhan, A., et al.: Braz. J. Phys. 38, 167 (2008)
Pradhan, A., et al.: Astrophys. Space Sci. 337, 401 (2012)
Pradhan, A., et al.: Astrophys. Space Sci. 337, 401 (2012)
Pradhan, A., et al.: Astrophys. Space Sci. 337, 401 (2012)
Rahman, M.A., Ansary, M.: Prespaceime J. 4, 871 (2013)
Reyes, M.A.: Preprint arXiv:0806.2299 (2008)
Riess, A.G., et al.: Phys. Rev. D 66, 1973 (2002)
Riess, A.G., et al.: Astron. J. 116, 1009 (1998)
Sabolio, M., et al.: Fizika B 19(4), 177–186 (2010)
Shen, M.: Int. J. Theor. Phys. 52, 178 (2013)
Singh, J., et al.: Gen. Relativ. Gravit. 30, 573 (1998)
Singh, J.P., et al.: Astrophys. Space Sci. 314, 83 (2008)
Sola, J., Stefancic, H.: Phys. Lett. B 624, 147 (2005)
Sola, J., Stefancic, H.: Mod. Phys. Lett. A 21, 479 (2006)
Starobinsky, A.A.: JETP Lett. 8, 757 (1998)
Tripathy, S.K.: Int. J. Theor. Phys. 52, 4218 (2013)
Vishwakarma, R.G.: Class. Quantum Gravity 17, 3833 (2000)
Vishwakarma, R.G.: Gen. Relativ. Gravit. 33, 1973 (2001)
Wand, D., et al.: Ann. N.Y. Acad. Sci. 685, 647 (1993)
Weerterrich, C.: Nucl. Phys. B 302, 668 (1998)