STRING UNIFICATION OF GAUGE COUPLINGS WITH INTERSECTING D-BRANES

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After reviewing the general structure of supersymmetric intersecting brane world models, we discuss the issue of stringy gauge coupling unification for a natural class of MSSM-like models.

1. Introduction

During the last years string compactifications with intersecting D-branes have been of particular interest to the string phenomenology community for their appealing Standard Model-like features such as chiral fermions, family replication and Standard Model gauge groups.

From the model building point of view, having a low string scale scenario in mind, most effort went into the construction of non-supersymmetric string models, which, as many non-supersymmetric string models do, suffer from perturbative instabilities like tachyons and/or uncanceled disc tadpoles for some of the scalar fields including the dilaton.

Clearly, from the stringy point of view it is much more convincing to start with a supersymmetric intersecting brane configuration, where the string scale is unconstrained and can be everywhere between the weak and the Planck scale. This set-up is closer to the heterotic string, which was the string phenomenologists favorite branch of the M-theory moduli space before the concept of D-branes was introduced. Unfortunately, as concrete model building attempts have revealed, realistic supersymmetric intersecting D-brane models are much more difficult to uncover, as supersymmetry appears to be a fairly rigid constraint.

Despite this lack of simple concrete realizations, one might think about general aspects of this class of models. For instance, the discussion of the general structure of three and four point-functions and the discussion of the proton lifetime in intersecting brane world models (IBWs)
are nice examples of this general approach.

In this letter we review another recent attempt, which deals with the issue of gauge coupling unification in a very natural class of MSSM-like intersecting brane world models. After reviewing briefly the general structure of IBWs and introducing a natural class of MSSM like models, we discuss how perturbative gauge coupling unification might arise. Note that in this article we are using the term “unification” in the loose sense, that the values of the gauge couplings at the string scale are consistent with the stringy tree level predictions. In IBWs this does not necessarily mean, that the couplings really meet at some scale, as in the usual field theoretic GUT scenario.

2. The general set-up

We consider Type IIA string theory compactified on an orientifold background of the form

\[ X = \mathbb{R}^{3,1} \times \frac{M}{\sigma} \]

where \( M \) is a Calabi-Yau manifold and \( \sigma \) denotes an anti-holomorphic involution.

The fixed point locus of \( \sigma \) gives rise to orientifold O6 planes, whose R-R charge must be canceled by introducing D6-branes in the background. The easiest possibility, which has been mostly studied in the literature, is to place the D6-branes directly on top of or at least parallel to the orientifold planes. The whole philosophy about intersecting brane worlds is to give up this restriction and allow the D6 branes to occupy more general positions. This has the effect that they will intersect each other non-trivially leading to both chirality and in general supersymmetry breaking.

The massless modes are localized on defects in the ten-dimensional space-time of different dimensionality. Since the internal space is compact, we have to cancel the R-R tadpoles. In local coordinates the anti-holomorphic involution can be written as \( \sigma : z_i \rightarrow \overline{z_i} \) and the fixed locus is a special Lagrangian (sLag) 3-cycle. Now we introduce general D6-branes wrapped on homology cycles \( \pi_a \) and their \( \Omega \sigma \) images \( \pi'_a \). For such A-type D-branes wrapping sLag cycles with vanishing gauge field strength \( F = 0 \) the R-R 7-form charge on the compact manifold \( M \) vanishes if the total homology class vanishes

\[ \sum_a N_a (\pi_a + \pi'_a) - 4 \pi_{O6} = 0. \]
In a supersymmetric configuration all branes are calibrated with respect to
the same 3-form as the orientifold plane implying that the scalar potential
vanishes as the consequence of the R-R tadpole condition. Note, that the
scalar potential only depends on the complex structure of $M$.

As we show in Table 1 the chiral massless spectrum is entirely deter-
mined by the topological intersection numbers between pairs of D6-branes.

| Sector | Rep. | Number |
|--------|------|--------|
| $aa'$  | $A_a$ | $\frac{1}{2}(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$ |
| $aa'$  | $S_a$ | $\frac{1}{2}(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$ |
| $ab$   | $(\pi_a, N_b)$ | $\pi_a \circ \pi_b$ |
| $ab'$  | $(N_a, N_b)$ | $\pi'_a \circ \pi_b$ |

The non-abelian gauge anomalies cancel automatically and mixed
$U(1)_a - SU(N)^2_a$ anomalies are canceled by a generalized Green-Schwarz
mechanism involving dimensionally reduced R-R forms.

3. Gauge couplings

In contrast to the heterotic string, here each gauge factor comes with its
own gauge coupling, which at string tree-level can be deduced from the
Dirac-Born-Infeld action to be

$$\frac{4 \pi}{g_a^2} = \frac{M_3^3 V_a}{(2\pi)^3 g_{st} \kappa_a}$$

(3)

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can
similarly express the Planck mass in terms of stringy parameters ($M_{pl} =
(G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8 M_s^8 V_6}{(2\pi)^6 g_{st}^2}.$$ 

(4)

A very natural way to embed the Standard Model into intersecting
brane worlds $^{26}$ is to start with four stacks of D6-branes giving rise to the
initial gauge symmetry $U(3) \times SP(2) \times U(1) \times U(1)$ (with $SP(2) \simeq SU(2)$).

Requiring that the chiral spectrum of this model agrees with the three
generation Standard Model augmented by a right handed neutrino uniquely
fixes the intersection numbers between pairs of D-branes. The hypercharge is given by the following linear combination of \( U(1) \) factors

\[
Q_Y = \frac{1}{3}Q_a - Q_c - Q_d. \tag{5}
\]

leading to the gauge coupling

\[
\frac{1}{\alpha_Y} = \frac{1}{6} \frac{1}{\alpha_a} + \frac{1}{2} \frac{1}{\alpha_c} + \frac{1}{2} \frac{1}{\alpha_d}. \tag{6}
\]

From the stringy point of view a natural subclass of such models show some more symmetry among the four appearing gauge couplings. Since the intersection numbers of the first and fourth stack of branes are identical and both branes are calibrated, in the simplest case one gets that the internal volumes agree too, \( V_a = V_d \). Employing the condition that the Green-Schwarz terms still allow for a massless hypercharge, allows one to deduce that the third stack satisfies \( \pi_c' = \pi_c \). Therefore, at the bottom of this simple realization there lies an extended Pati-Salam like model

\[
U(4) \times SU(2) \times SU(2). \tag{7}
\]

where in the following we will also assume that the two gauge couplings of the two \( SU(2) \) factors are identical (which in the simple toroidal models discussed in \(^{26,27}\) is a consequence of supersymmetry). This allows us to derive the following string tree level Pati-Salam like relation among the gauge couplings

\[
\frac{1}{\alpha_Y} = \frac{2}{3} \frac{1}{\alpha_s} + \frac{1}{\alpha_w}. \tag{8}
\]

4. One loop corrections to the gauge couplings

Assuming that all gauge couplings are in the perturbative regime, we now study the one-loop running of the three gauge couplings and determine whether for appropriately chosen string scale the string tree level relation can be consistent with the known values of the Standard Model gauge couplings at the weak scale \( M_w \). The tree level relation at the string scale yields

\[
\frac{2}{3} \frac{1}{\alpha_s(M_w)} + \frac{2 \sin^2 \theta_w(M_w) - 1}{\alpha(M_w)} = \frac{B}{2\pi} \ln \left( \frac{M_w}{M_s} \right) \tag{9}
\]

with

\[
B = \frac{2}{3} b_s + b_w - b_y. \tag{10}
\]

where \( b_s, b_w \) and \( b_y \) are the beta function coefficient for the strong, weak and hypercharge gauge coupling respectively. Note, that the resulting value
of the unification scale only depends on the combination $B$ of the beta-
function coefficients.

In general besides the chiral matter string theory contains also additional vector-like matter. This is also localized on higher dimensional intersection loci of the $D6$ branes and also comes with multiplicities $n_{ij}$ with $i, j \in \{a, b, c, d\}$. Assuming the most general vector-like matter in bifundamental, (anti-)symmetric and adjoint representations of the gauge group, one finds the following contribution to $B$

$$B = 12 - 2n_{aa} - 4n_{ab} + 2n_{a'c} + 2n_{a'd} - 2n_{bb} + 2n_{c'c} + 2n_{c'd} + 2n_{d'd}. \tag{11}$$

$B$ does not depend on the number of weak Higgs multiplets $n_{bc}$. Depending on $B$, one finds the discretuum of unification scales displayed in Table 2.

| $B$  | $M_s[\text{GeV}]$ |
|------|-------------------|
| 18   | $3.36 \cdot 10^{13}$ |
| 16   | $5.28 \cdot 10^{12}$ |
| 14   | $1.82 \cdot 10^{14}$ |
| 12   | $2.04 \cdot 10^{16}$ |
| 10   | $1.51 \cdot 10^{19}$ |
| 8    | $3.06 \cdot 10^{23}$ |

5. Examples

For the MSSM one has the well known values $(b_s, b_w, b_y) = (3, -1, -11)$, i.e $B = 12$ and the unification scale is the usual GUT scale

$$M_X = 2.04 \cdot 10^{16} \text{GeV}. \tag{12}$$

Assuming $g_{st} = g_X$, for the internal sizes of the entire Calabi-Yau manifold and the two 3-cycles where the $SU(3)$ and $SU(2)$ branes are wrapped on, one obtains

$$M_sR = 5.32, \quad M_sR_s = 2.6, \quad M_sR_w = 3.3. \tag{13}$$

The running is shown in figure 1. Note, that in contrast to the heterotic string, where one had to invoke one-loop heavy threshold corrections at the
string scale, here unification of gauge couplings can be achieved with just
the tree level gauge couplings and appropriately chosen internal radii.

As a second example, we consider a model with a second weak Higgs
field, i.e. \( n_{bc} = 1 \), so that we still get \( B = 12 \) but with \((b_s, b_w, b_y) = (3, -2, -12)\). As shown in the left picture of figure 2, the gauge couplings
still "unify" at the usual GUT scale, but do not really meet there.

Finally, let us discuss the case with \( B = 10 \), where one gets the intriguing
value

\[
\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}. \tag{14}
\]

Choosing for instance vector-like adjoint matter \( n_{aa} = 1 \), the beta-function
coefficients read \((b_s, b_w, b_y) = (0, -1, -11)\) leading to the running shown in
the right picture in figure 2. The couplings at the string scale turn out to be

\[
\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_y(M_s) = 0.035. \tag{15}
\]

and the Weinberg angle at the string scale reads \(\sin^2 \theta_w(M_s) = 0.445\). For
the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

\[
M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3. \tag{16}
\]
6. Outlook

We have shown that a general class of realistic IBWs feature perturba-
tive gauge coupling unification. The burning problem remains to really
construct concrete intersecting brane models, which fall into the class of
models discussed in this letter. For a concrete model, it would be inter-
etesting to analyze what the effect of the one-loop threshold corrections\textsuperscript{28}
would be.

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