Effect of the Equivalence Between Topological and Electric Charge on the Magnetization of the Hall Ferromagnet.

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The dependence on temperature of the spin magnetization of a two-dimensional electron gas at filling factor unity is studied. Using classical Monte Carlo simulations we analyze the effect that the equivalence between topological and electrical charge has on the behavior of the magnetization. We find that at intermediate temperatures the spin polarization increases in a thirty per cent due to the Hartree interaction between charge fluctuations.

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The physics of a two-dimensional electron gas (2DEG) in a magnetic field \( B \) is determined by the number of Landau levels occupied by the electrons. Since the degeneracy of the Landau levels increases with \( B \), the electron density can be accommodated in the lowest Landau level for strong enough fields. At filling factor unity, \( \nu = 1 \), and zero temperature, \( T = 0 \), the ground state of the 2DEG is an itinerant ferromagnet. \[1\] The Zeeman coupling between the electron spin and the magnetic field determines the orientation of the ferromagnet polarization. For zero Zeeman coupling the interaction between the carriers produces a spontaneous spin magnetic moment.

Recently the magnetization of the quantum Hall system was measured \[2\] for filling factors \( 0.66 < \nu < 1.76 \) and temperatures \( 1.55K < T < 20K \). At \( \nu = 1 \) and very low temperatures the system is fully polarized, while for other filling factors the magnetization is reduced. The demagnetization for \( \nu \neq 1 \) is related to the existence of Skyrmions in the system. \[3\] Further experiments have verified the existence of Skyrmions using transport, capacity and optical experiments. \[4\]

For \( \nu = 1 \), the temperature dependence of the magnetization \( M(T) \), has been measured using NMR techniques \[5\] and magneto-optical absorption experiments. \[6\] Different theoretical approaches have been used for the study of \( M(T) \): i) Read and Sachdev \[7\] have studied the \( N \to \infty \) limit of a quantum continuum field theory model for the spin vector field, \( \mathbf{m}(r) \). This model describes the long-wavelength collective behavior of the electronic spins. This work has been extended by Timm et al. \[8\] The field theory is expected to be accurate at low temperatures and weak Zeeman coupling. Using \( SU(N) \) and \( O(N) \) symmetries in the large \( N \) limit, and using the spin stiffness \( \rho \), as a parameter Read and Sachdev obtained results for \( M(T) \) which are in reasonable good agreement with the experimental data at low temperatures. ii) Kasner and MacDonald \[9\] calculated \( M(T) \) using many-particle diagrammatic techniques which include spin-wave excitations and electron spin-wave interaction. This theory is a good improvement on the one-particle Hartree-Fock theory, but it gives a polarization for the system too high compared with the experimental one. Progress in the diagrammatic approach, including temperature dependence screening, has been done by Haussman. \[10\] iii) The dependence of the magnetization on \( T \) has also been obtained by exact diagonalization of the many-particle Hamiltonian for a small (up to 9) number of electrons on a sphere. \[11,12\] These calculations have important finite size effects at low temperature and weak Zeeman coupling. iv) Finally, quantum Monte Carlo (MC) techniques have been used in order to calculate \( M(T) \) for a spin 1/2 quantum Heisenberg model on a square lattice with exchange interactions adjusted to reproduce the spin stiffness of the quantum Hall ferromagnet. \[13\] These calculations are essentially exacts and probably are free of finite size effects.

A unique property of the quantum Hall ferromagnets is the equivalence between the topological charge associated with \( \mathbf{m} \) and the electrical charge. \[14\] This equivalence make the Skyrmions to be the relevant charged excitation of the 2DEG at \( \nu = 1 \). Charge conservation implies that at a given \( \nu \) the integral of the topological charge over all the space should be constant, independently of \( T \). At \( \nu = 1 \) this constant is zero. A spin vector field texture produces a modulation of the topological charge density. Spatial spin fluctuations increase with temperature and produce a modulation of the topological charge density. In this way thermal fluctuations can produce a strong charge fluctuation. Because of the equivalence between topological and real charge, the modulation of the charge density costs Hartree energy. The models described above: diagrammatic techniques, quantum field theory and quantum MC calculations, do not take into account the Hartree contribution to the energy of the ferromagnet.

In this work we study the effect that the Hartree energy has on the temperature dependence of the magnetization. We perform classical MC simulations of \( M(T) \) for the energy functional of the Hall ferromagnet. From the comparison of the results with and without Hartree energy we conclude that at moderate temperatures the inclusion of this term modifies the value of the magnetization up to a thirty per cent. For realistic values of the Zeeman coupling, we find that at intermediate-high
temperature the Hartree energy is near one third of the Zeeman and exchange energies.

The classical model can not describe correctly the low temperature behavior of $M(T)$. The classical dynamics of the electron spins neglects several effects, in particular the quantum description of the spin-density waves which is extremely important for describing $M(T)$ at low temperatures. The inclusion, in the classical model of a temperature dependent low-energy cutoff simulates quantum effects. As we show latter, in the Hall ferromagnet this cutoff appears naturally when discretizing the continuum model. In this work we are interested in the effect that the inclusion of the Hartree term has on $M(T)$. We expect that in the quantum Heisenberg model, the Hartree energy should have the same effect than in the classical model.

The long-wavelength and low-energy properties of the $\nu = 1$ Hall ferromagnet can be described by a functional $E$ of the unit vector field $\mathbf{m}(\mathbf{r})$ which describes the local orientation of the spin magnetic order. The functional $E$ has three terms \[ E = E_x + E_z + E_c \]

where $E_x = \frac{\mu_B}{2} \int d^2r (\nabla \mathbf{m})^2$ is the Zeeman coupling, $E_z = \frac{t}{2\pi\hbar^2} \int d^2r [1 - m_z(\mathbf{r})]$ is the Zeeman energy and $E_c = \frac{1}{2} \sum_{\mathbf{r} \neq \mathbf{r}'} d^2r d^2r' n(\mathbf{r}) v(|\mathbf{r} - \mathbf{r}'|)$ is the Coulomb interaction and $n(\mathbf{r})$ is the charge density and it is given by \[ n(\mathbf{r}) = \frac{1}{8\pi \epsilon_0} \mathbf{m}(\mathbf{r}) \cdot [\partial_\mathbf{r} \mathbf{m}(\mathbf{r}) \times \partial_{\mathbf{r}'} \mathbf{m}(\mathbf{r})] \]

In this continuum model the total topological charge, $Q$, is given by the integral over all the space of $n(\mathbf{r})$, and it represents the number of times $\mathbf{m}(\mathbf{r})$ winds around the sphere $S^2$. Skyrmions are non trivial extrema solutions of the functional $E$ with $Q \neq 0$. The continuum model gives a good description of the Skyrmions with moderate and large spins.\[E = -\rho \sum_{\mathbf{i},\mathbf{j}} \Omega_i \Omega_j - t \sum_i \sum_{\mathbf{j}} (\Omega_{i,z} + 1) + \frac{1}{2} \sum_{\mathbf{i},\mathbf{j}} q_i V_{i,j} q_j.\]

Here, $\Omega_i$ is the unit vector at site $i$, $q_i$ is the topological charge attached to the unit cell $i$ and $V_{i,j}$ is the Coulomb interaction between two charges unity distributed uniformly in the cells $i$ and $j$. The cell $i$ is defined by the points $1:(i_x,i_y)$, $2:(i_x+1,i_y)$, $3:(i_x+1,i_y+1)$ and $4:(i_x,i_y+1)$. The expression of $q_i$ as a function of the unit vectors at the points 1-4 is \[ q_i = \frac{1}{4\pi} \left\{ (\sigma A)(\Omega_1, \Omega_2, \Omega_3) + (\sigma A)(\Omega_1, \Omega_3, \Omega_4) \right\}, \]

where $(\sigma A)(\Omega_1, \Omega_2, \Omega_3)$ denoted the signed area of the spherical triangle with corners $\Omega_1, \Omega_2$ and $\Omega_3$. Apart from a set of ‘exceptional’ configurations of measure zero, this prescription for the topological charge yields well defined integer values for the total topological charge. For smooth spin texture the continuum and discrete expression for the density of topological charge give the same value. In order to analyze the effect of the Hartree term we study also the functional $E_0 = E_x + E_z$ that is the classical version of the quantum Heisenberg Hamiltonian studied by Henelius et al.\[E = -\rho \sum_{\mathbf{i},\mathbf{j}} \Omega_i \Omega_j - t \sum_i \sum_{\mathbf{j}} (\Omega_{i,z} + 1) + \frac{1}{2} \sum_{\mathbf{i},\mathbf{j}} q_i V_{i,j} q_j.\]

In that work the Hartree term is not taken into account.

A comment about the significance of $a_L$ is in order. In two dimensions the exchange term and the topological charge are scale invariants and do not depend on $a_L$. The Zeeman energy increases quadratically with the lattice parameter and $V_{i,j}$ is inversely proportional to $a_L$. An increment in $a_L$ is similar to an increment of the Zeeman strength and the lattice parameter acts as a low energy cutoff that controls the dynamics of the classical Heisenberg model. We obtain the value of $a_L$ by fitting the magnetization obtained from the functional $E_0$ to the magnetization obtained from quantum MC simulations.\[E = -\rho \sum_{\mathbf{i},\mathbf{j}} \Omega_i \Omega_j - t \sum_i \sum_{\mathbf{j}} (\Omega_{i,z} + 1) + \frac{1}{2} \sum_{\mathbf{i},\mathbf{j}} q_i V_{i,j} q_j.\]

Now we describe briefly the MC procedure used for obtaining $M(T)$. The MC simulations were performed by using the techniques due to Metropolis et al.\[E = -\rho \sum_{\mathbf{i},\mathbf{j}} \Omega_i \Omega_j - t \sum_i \sum_{\mathbf{j}} (\Omega_{i,z} + 1) + \frac{1}{2} \sum_{\mathbf{i},\mathbf{j}} q_i V_{i,j} q_j.\] we study a cluster $N \times N$ with periodic boundary conditions (PBC). The use of PBC diminish the finite size effects. For studying $M(T)$ at $\nu = 1$ we consider as a starting configuration a completely ordered ferromagnet, i.e. $\Omega_i = (0, 0, 1)$ for all sites $i$. In this way the initial $Q$ in the system is zero. The sites to be considered for a change in the spin orientation are randomly chosen, avoiding artificial correlations that could distort the results. Once a site is selected for a spin reorientation, we perform the following operations: i) a small change in the direction of $\Omega_i$, ii) calculation of the change induced in the topological charge by the spin reorientation. Only changes that do not modify $Q$ are accepted. iii) evaluation of the energy variation $\Delta E$. iv) acceptance or not acceptance of the new spin direction. If $\Delta E < 0$ the change is accepted. If $\Delta E > 0$ the change is accepted depending if $e^{-\Delta E/kT}$ exceeds a random number. We realize a large number of MC steps until reach the equilibrium situation and perform the average of the different statistical properties: the magnetization $M(T)$, the different contri-
tions to the total energy per electron, $<E_x>$, $<E_z>$ and $<E_c>$ and a measure of the charge fluctuations, 

$$\delta q = \sqrt{\frac{\sum q_i^2}{N \times N}}.$$  \hspace{1cm} (8)

In Fig.1 we plot the magnetization as a function of temperature as obtained from the classical MC simulation for the functional $E_0 = E_x + E_z$ with different value of the lattice parameter. The results correspond to a 2DEG of zero layer thickness, $\rho = 0.0249e^2/\ell$ and a Zeeman coupling $t = 0.008e^2/\ell$. The calculations are performed in a cluster $20 \times 20$. We have checked that for this cluster size and PBC, the results are free of size effects. In the same figure we plot $M(T)$ as obtained from quantum MC simulations [13]. This calculation does not include the Hartree term and it is the quantum version of the functional $E_0$. By comparing the classical and the quantum results we can estimate $a_L$ in the different temperature ranges. We are interested in temperatures in the range $0.075e^2/\ell < T < 0.2e^2/\ell$. At lower temperatures the quantum effects are very important and at higher temperatures the effective functional Eq.(1) it is not longer valid. In order to describe the dynamic in this range of temperatures values of $a_L$ in the range $2a_1 < a_L < 2.5a_1$ are necessary. For simplicity, in the calculation we use a constant value of the lattice parameter. We use $a_L = 2.5a_1$, which is the value of $a_L$ in the range $2a_1 < a_L < 2.5a_1$ where the effects of the Hartree interaction are weaker.

In Fig.2 we plot $M(T)$ as obtained from the classical MC simulation for the full functional $E = E_x + E_z + E_c$ and for the functional without Hartree energy $E_0 = E_x + E_z$, the results corresponds to a lattice parameter $a_L = 2.5a_1$. At low temperatures, $T < 0.05e^2/\ell$, the spins are not very disordered, the charge modulation is very weak and therefore the Hartree term has a small effect on $M(T)$. For higher temperatures the spin fluctuations and consequently the charge fluctuations are stronger and the Hartree term becomes a important contribution to the internal energy. We find that at intermediate temperatures, in the range $0.05e^2/\ell < T < 0.15e^2/\ell$ the magnetization obtained with the full functional $E$ is near thirty per cent higher than the obtained without the Hartree term. At even higher temperatures, $T > 0.15e^2/\ell$, the spin disorder is very large and the magnetization calculated with or without Hartree term is small, although $M(T)$ obtained with $E$ is always higher than the obtained with $E_0$.

As commented above the quantum MC calculation describe correctly the spin density waves and at low temperatures it should give an appropriate $M(T)$ for the $\nu = 1$ Hall ferromagnet. However we expect that for intermediate temperatures the Hartree energy term would modify the quantum MC data in a similar amount that it modifies the classical results. In order to understand the experimental results at intermediate temperatures it is necessary to take into account the charge fluctuations induced by the temperature.

In figure 3 we plot the different contributions to the total energy per electron as a function of the temperature. The parameters are the same than the used in figure 2. At very high temperatures the spin are completely random and the exchange and Zeeman energies tend to their fully disorder values $2\rho/a_L^2$ and $t$ respectively. Note that intermediate temperatures the Hartree energy is near one third the Zeeman energy. For smaller values of the Zeeman coupling and high temperatures we have found that the Hartree energy can be the more important contribution to the internal energy.

Figure 4 shows the variation of the charge fluctuations, $\delta q$ as a function of the temperature. Charge fluctuations cost Hartree energy and they are weaker when the full functional is considered. Observe that at intermediate temperatures $\delta q$ is of the order of 0.1, that is $\sim 10\%$ of the charge in each cell.

In closing, we have studied the effect that the Hartree energy term has on the temperature dependence of a 2DEG at $\nu = 1$. We find that at intermediate temperatures the spin fluctuations are weakened by the Hartree energy and the magnetization is near thirty per cent bigger than the obtained by neglecting the Hartree energy term. At intermediate temperatures the Hartree energy is an important contribution to the internal energy of the Hall ferromagnet.

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FIG. 1. $M(T)$ as obtained by using the functional $E_0$ with different lattice parameters. We also plot the quantum MC (QMC) results. The results correspond to a 2DEG of zero layer thickness and a Zeeman coupling $t = 0.008\epsilon/\ell$.

FIG. 2. $M(T)$ as obtained by using the functional $E$ and as obtained by neglecting the Hartree term. The results correspond to a lattice parameter $a_L = 2.5\sqrt{2}\pi\ell$.

FIG. 3. Different contributions to the internal energies per electron.

FIG. 4. Variation of $\delta q$, equation (8), as a function of the temperature for the full functional $E$ and for the functional without Hartree energy $E_0$. 

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