MOLECULES AT HIGH REDSHIFT: THE EVOLUTION OF THE COOL PHASE OF PROTOGALACTIC DISKS

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Received 1996 March 13; accepted 1996 November 25

ABSTRACT

We study the formation of molecular hydrogen, after the epoch of reionization, in the context of canonical galaxy formation theory due to hierarchical clustering. There is an initial epoch of $\mathrm{H}_2$ production in the gas phase through the $\mathrm{H}^-$ route that ends at a redshift of order unity. We assume that the fundamental units in the gas phase of protogalaxies during this epoch are similar to diffuse clouds found in our own Galaxy, and we restrict our attention to protogalactic disks, although some of our analysis applies to multiphase halo gas. Giant molecular clouds are not formed until lower redshifts. Star formation in the protogalactic disks can become self-regulated. The process responsible for the feedback is the heating of the gas by the internal stellar radiation field that can dominate the background radiation field at various epochs. If the gas is heated to above 2000–3000 K, the hydrogen molecules are collisionally dissociated, and we assume that in their absence the star formation process is strongly suppressed because of insufficient cooling. As we demonstrate by the analysis of phase diagrams, the $\mathrm{H}_2$-induced cool phase disappears. A priori, the cool phase with molecular hydrogen cooling can only achieve temperatures $\geq 300$ K. Consequently, it is possible to define a maximum star formation rate during this epoch. Plausible estimates give a rate of $\lesssim 0.2$–2 $M_\odot$ yr$^{-1}$ for condensations corresponding to 1 $\sigma$ and 2 $\sigma$ initial density fluctuations. For more massive structures, this limit is relaxed and in agreement with observations of high-redshift galaxies. Therefore, the production of metals and dust proceeds slowly in this phase. This moderate epoch is terminated by a phase transition to a cold, dense, and warm neutral/ionized medium once the metals and dust have increased to a level $Z \approx 0.03$–0.1 $Z_\odot$. Then (1) atoms and molecules such as C, O, and CO become abundant and cool the gas to below 300 K; (2) the dust abundance has become sufficiently high to allow shielding of the molecular gas; and (3) molecular hydrogen formation can occur rapidly on grain surfaces. This phase transition occurs at a redshift of approximately 1.5, with a fiducial range of $1.2 \leq z \leq 2$, and initiates the rapid formation of molecular species, giant molecular clouds, and stars. Consequently, the delayed initiation of the cold phase in the interstellar medium of protostellar disks at a metallicity of $Z \approx 0.1$ $Z_\odot$ is a plausible physical reason why the formation phase of the stellar disks of the bulk of the galaxies occurs only at a redshift of order unity. The combination of feedback and a phase transition provides a natural resolution of the G-dwarf problem.

Subject headings: early universe — galaxies: formation — molecular processes — stars: formation

1. INTRODUCTION

Despite numerous searches for molecules at high redshift, the positive results are sparse. The only confirmed observations of molecules at high redshift seen in emission are associated with two gravitational lenses: FSC 10214+4724 (Scoville et al. 1995; Frayer 1995) and the Cloverleaf (Barvainis et al. 1997). In absorption, $\mathrm{H}_2$ has been seen in PKS 0528−250 (Foltz, Chaffee, & Black 1988) at a column density of $10^{18}$ cm$^{-2}$, but in no other damped Ly$\alpha$ system. This is surprising since the column densities of atomic hydrogen inferred for damped Ly$\alpha$ systems are $\sim 10^{21}$ cm$^{-2}$. In comparison with diffuse clouds in our own Galaxy, such columns seem sufficient to produce $\mathrm{H}_2$. Galactic diffuse clouds cover the disk with a covering factor between 20% and 50%, where the lower limit is from the Copernicus data (Savage et al. 1977) and the upper limit comes from assuming all the infrared Cirrus is associated with diffuse clouds. Other molecular species, including $\mathrm{HCO}^+$, HCN, CO, $13\mathrm{CO}$, HNC, CN, CS, and $\mathrm{H}_2\mathrm{CO}$, have been seen below $\sim 1$ redshift in the millimeter band (Combes & Wiklind 1996). These absorption techniques are effective for any redshift, provided a bright millimeter continuum background exists. Nevertheless, all the absorbers known to date are in lensed systems with correspondingly small impact parameters.

At lower redshift $z \leq 0.2$, there are well-established observations of many molecular species in emission in the nuclei of starburst galaxies (cf. Scoville & Soifer 1991). For large nearby galaxies such as M31 and NGC 891 (cf. Young 1990), CO maps have been obtained. For our own Galaxy, about a hundred different molecular species have been detected in interstellar space. Many di- and tri-atomic molecules can be observed toward diffuse clouds through the visual and millimeter absorption lines they produce in the continua of nearby stars. Some quasars ($\sim 30\%$) used as millimeter calibration sources are known to exhibit absorption lines as well (de Geus, Hogerheijde, & Spaans 1996).

These data pose interesting questions about the prevailing chemical balance and the physical structure of the ambient medium as a function of redshift (cf. Norman & Braun 1995). Previous work on this subject has focused on very high redshift in the epoch prior to reionization, where light-element hydrides such as DH and LiH have been discussed in the context of observations of deuterium abun-
dances and faint structure in the cosmic background radiation field (Maoli et al. 1996; Dalgarno, Kirby, & Stancil 1996; Stancil, Lepp, & Dalgarno 1996). The importance of H\textsubscript{2} cooling for the formation of the first stars has been discussed by many authors (cf. Haiman, Rees, & Loeb 1996a; Haiman, Thoul, & Loeb 1996b; Tegmark et al. 1997). Although the self-consistent formation of H\textsubscript{2} above a redshift of $z \sim 6$ due to gas-phase processes yields very low abundances of molecular hydrogen $H\textsubscript{2}/H \sim 10^{-6}$ (Lepp & Shull 1984), these may be sufficient to sustain the collapse of initial perturbations. In the post reionization epoch, the problem of H\textsubscript{2} formation has been discussed by Black, Chaffee, & Foltz (1987) for the QSO PHL 957—a damped Ly\alpha system with an atomic H\textsc{i} column density of $2.5 \times 10^{21}$ cm$^{-2}$—where the H\textsubscript{2} abundance was more than 5 orders of magnitude lower than values representative of our own Galaxy (cf. Levshakov & Varshalovich 1985; Lanzetta, Wolfe, & Turnshek 1989). Their analysis indicated that two essential parameters are (1) the background radiation field, which is inferred to be an order of magnitude higher than typical intergalactic values at $z \sim 2.3$, and (2) the dust-to-gas ratio, which is constrained to be no more than $\frac{1}{3}$ of typical Galactic values. Many new results have recently become available on the UV background radiation field (cf. Haardt & Madau 1996), the dust content of damped Ly\alpha systems (Pettini et al. 1994; Fall & Pei 1993), and the merging history of dark matter halos (Kauffmann & White 1993). QSO absorption-line studies have also made remarkable progress in the last few years. The high-redshift ($z > 2$) dependence of the H\textsc{i} column density distribution function contains valuable information on the early stages of structure formation during which large systems are likely to be in the process of aggregating themselves from smaller sub-units. The obvious candidates to focus on are those systems with H\textsc{i} column densities above $N$(H\textsc{i}) $\geq 10^{21}$ cm$^{-2}$ (Wolfe 1995; Storrie-Lombardi et al. 1994). These systems are often associated with protostellar disks and are believed to contain a reasonable fraction of the baryons of the universe.

These new developments motivate us to reinvestigate the formation of molecular hydrogen after reionization and to assess its role in the standard galaxy formation theory due to hierarchical clustering and the subsequent formation of stars. If the latter process is to occur, a diffuse cool ($T \lesssim 500$ K) phase needs to be supported that ultimately leads to the formation of dense molecular clouds and stars. Obviously, the formation of stars produces an internal radiation field that will heat its surroundings and influence the chemical balance. For star formation to continue, the cool phase should persist. In fact, this process will determine the maximally sustained star formation rate. If the associated timescale is long (comparable to a Hubble time), the abundance of metals and dust will increase slowly until a threshold is reached at which the cooling is dominated by atoms and ions like O and C$^+$, as well as molecular species like CO. In addition, H\textsubscript{2} formation on grains proceeds more rapidly and dominates in regions where the bulk of the material is in neutral form. Also, the increased columns of dust absorb the radiation of newly formed stars and sustain a star formation cycle as in our own Galaxy. The onset of the bulk of the star formation at low redshifts and moderate metallicity is consistent with current constraints on disk formation. In this paper, we have restricted our attention to protogalactic disks since they are the likely sites of the cold molecular phase we are focusing on. Multiphase halos have been discussed extensively for galaxy formation (cf. Kang et al 1990; Ikeuchi & Norman 1991), and our results in §§ 2.5, 3, 4, and 5 are relevant for such studies, although we do not pursue them further here.

This paper is organized as follows. In § 2, the formation and subsequent evolution of structure in a cold dark matter–dominated universe is discussed, and the redshift dependence of the UV background radiation field, the stellar radiation field, the gas pressure, the radiation pressure, and the ionization parameter are presented. The gas-phase epoch of molecular hydrogen formation is calculated in § 3, including the production of metals and dust. The feedback effect due to star formation is analyzed in § 4, and upper limits to the star formation rate and metal and dust production rate are derived. In § 5, we discuss the transition to a multiphase interstellar medium (ISM) and the onset of rapid star formation. The epoch when H\textsubscript{2} formation on grains becomes important is discussed in § 6. The formation of metals and subsequent molecular cooling to massive molecular clouds is also discussed there. Section 7 discusses the implications for the study of the ISM in protogalaxies and star formation, as well as the G-dwarf problem. We associate the onset of star formation in disks with both a redshift, $z \sim 1–2$, and a metallicity, $Z \sim 0.03–0.1$, as also indicated in a perceptive paper by Wyse & Gilmore (1988). Where possible, we give reasonable analytical estimates and then use the full (numerical) details of the recently calculated metagalactic and stellar radiation fields in our calculations.

2. BASIC COSMOLOGICAL MODEL

2.1. Galaxy Formation

For the present investigation, the specific details of structure formation are of minor importance, and we adopt the semianalytical relations presented by White & Frenk (1991) that agree well with more detailed N-body simulations (cf. Lacy & Cole 1993, 1994; Kauffmann, Guiderdoni, & White 1993, 1994; Heyl et al. 1995). In the numerical calculations, we also include the merging history of dark matter halos through the conditional probability that a halo of mass $M_0$ at redshift $z_0$ has previously been in a halo of mass $M_1$ at $z_1$, as presented in Kauffmann & White (1993).

Using simple top-hat models, the collapse to virial equilibrium of a perturbation that has become nonlinear at redshift $z$ results in a density enhancement $\delta$. In cold dark matter (CDM) scenarios, the fiducial value is $\delta = 178$ (Narayan & White 1988). We assume that for protogalactic disks to form, cooling takes place and a fraction of the gas originally contained in the mass perturbation collapses to a centrifugally supported disk with a collapse factor of $\lambda^{-1}$, where $\lambda$ is the spin parameter from tidal torques (cf. Peebles 1993) with a canonical value of $\lambda = 0.07$. Note that there are details of the cooling and infall to the protogalactic disk that we have omitted, but they are not essential to the argument given here. For a density perturbation of mass $M$, the resulting column density is given by

$$N = \left(\frac{4\pi}{3}\right)^{-1/3} \delta^{2/3} \lambda^{-2} \Omega_{b,0} \left(\frac{M}{\mu m_p}\right)^{1/3} n_0^{2/3}(1 + z)^2, \quad (1)$$

yielding

$$N = 1 \times 10^{21} \left(\frac{\Omega_{b,0}}{0.01}\right)^{2/3} \left(\frac{M}{5 \times 10^{11} M_\odot}\right)^{1/3} (1 + z)^2 \text{ cm}^{-2}, \quad (2)$$
with $\Omega_{b,q}$ the baryonic mass fraction in the protogalactic disk relative to the total perturbation. Our numerical estimates are sensitive to this parameter. In fact, $\Omega_{b,q}$ depends on the merging history of the dark matter halos and is a function of redshift. Kauffmann (1996, her Fig. 12) presents the redshift distribution of the baryonic fraction, including the conditional probability for halo merging quoted above.

There is a clear maximum between redshift 2 and 3 for a biasing parameter $1 < b < 2$. A value of $b = 1-1.5$ seems most consistent with the latest derivations of Storrie-Lombardi & McMahon (1996). Therefore, we adopt her results in the numerical work presented below.

To characterize the density, we assume that the scale height, $H$, to disk size, $R$, denoted by $H/R$, of the collapsed systems is approximately constant, with a typical value of $\eta \sim 1/100$. It follows that the density in the disk is approximately equal to

$$n = \delta \chi^{-3} \eta^{-1} \Omega_{b,q} n_0 (1 + z)^3,$$

yielding

$$n = S \left( \frac{100}{\eta} \right) \left( \frac{\Omega_{b,q}}{0.01} \right) (1 + z)^3 \text{ cm}^{-3}.$$

The canonical mass value is chosen to be $M = 5 \times 10^{11} M_\odot$. Our estimates of the redshift of molecule formation are only weakly dependent on this choice if we focus on the bulk of the matter that is contained in the 1 $\sigma$ and 2 $\sigma$ density perturbations, $M < 10^{12} M_\odot$. We also assume that at high redshift, the formation of stars has not been effective in locking up the baryons. The ratio $\psi = \text{(gas/gas + stars)}$ should be of order unity for these protogalactic objects. Of course, in more evolved systems such as our Galaxy, the ratio is only $\sim 10\%$. The actual frequency distribution of objects of mass $M$ is determined by the cosmological model in the context of the Press-Schechter hierarchical clustering picture (cf. White & Frenk 1991). For a cosmic density parameter $\Omega = 1$, it is assumed here that the total baryonic fraction in galaxies $\Omega_{b,q}$, at a given redshift, cools to a disk while the dark matter remains in the halo. The fraction of baryons in galaxies is typically $10\%$ of the total number of baryons available. The cosmic density of baryons $n_{b0}$ is found to be $n_{b0} = (3H_0^2/8\pi G)(\Omega_{b,q}/\mu_0) = 10^{-6}(\Omega_b/0.1)h^2$, where $H_0$ is the Hubble constant, $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_b$ is the baryonic fraction of matter in the universe, and $\mu$ denotes the reduced mass of the primordial gas for a helium abundance by number of $10\%$.

The above formulation is for single objects of a given mass, and it is therefore worthwhile to formulate a statistical criterion that is more directly related to the initial conditions of structure formation. Equation (1) can also be written as

$$N = n_0 r_0 \delta^{2/3} \chi^{-2} \Omega_{b,q} (1 + z)^2,$$

where $r_0$ is the initial radius and $r_0 = (3M/4\pi)^{1/3}$. Using the Press-Schechter formalism (see White & Frenk 1991, eqs. [1] and [2]), the abundance of halos $n(M, z)$ with mass between $M$ and $M + dM$ is given by

$$n(M, z) = \left( \frac{2}{\pi} \right)^{1/2} \left( \frac{\rho_0}{M^2} \right)^{y} d \log y \frac{d \log M}{d \log T} \exp \left( - \frac{y^2}{2} \right),$$

where $y = \delta(1 + z)/\sigma(M)$; we take $\delta = 1.8$ and, for standard CDM cosmology, $\sigma(M) = 16.3 h^{-1}(1 - 0.3909(0.09)^{0.1} + 0.4814r_0^{0.2})^{-10}$, where $r_0$ is in units of Mpc and is given by $r_0 = 1(M/10^{12} M_\odot)(\Omega_b h^2)^{-1/3}$ Mpc. The function $\sigma(M)$ can be fitted by an approximate power law $\sigma = \sigma_0(M/M_\star)^q$, leading to power-law distribution functions $n(M, z) \propto M^{-4/3}(1 + z)$ for the halo mass distribution below a characteristic turnover mass defined by $\sigma[M_\star(z)] = \delta(1 + z)$. Gas cooling and resultant disk formation can only occur if the cooling time of the virialized gas in the dark halo at a temperature $T = 2 \times 10^6(M/10^{12} M_\odot)(1 + z) K$ is shorter than the dynamical time at a given redshift. This leads to an effective cooling mass as a function of redshift denoted $M_{cool}(z)$. In Figure 1, we show the halo abundance distribution $n(M, z)$. Also shown in the lower $M$-$z$ plane of Figure 1 are the turnover mass, $M_\star$, and the limiting mass of the dark halo, $M_{cool}$, in which there is gas, with $\Omega_{b,h} = 0.05$, that can cool at the given redshift (see also § 5).

### 2.2. Background UV Radiation Field

The background UV radiation field is well approximated for $0.5 < z < 2.5$ by

$$J = J_{-21}(1 + z)^{1+q},$$

where $J_{-21}$ is the fiducial value for the intergalactic background in units of $10^{-21}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$. The value of $q$ lies between 0.5 $< q < 1$, and it reflects the uncertainty in the detailed spectral shape of the UV continuum of quasars. Current observations seem to favor a value $q = 0.5$ (Tytler et al. 1995). For $z < 3$, equation (5) is accurate to within 30% over the 912–2000 Å range, which is the rele-

![Figure 1](image-url)

**Fig. 1**—The abundance of halos as a function of mass and redshift for standard CDM with biasing parameter $b = 1$. The lower $M$-$z$ plane shows, as functions of redshift, the characteristic turnover mass, $M_\star$ (solid curve), in CDM and the limiting mass of the dark halo, $M_{cool}$ (dashed curve), in which gas can cool.
vant wavelength region for the $H_2$ photodissociation process. Above a redshift of 3, the quasar population turns over, and the metagalactic background decreases with increasing redshift.

In order to more clearly distinguish the effects of $H_2$ dissociation and ionization heating, the background radiation field is separated into two components, one above and one below the Lyman limit at 912 Å, denoted by $J^-\alpha_0$ and $J^+\alpha_0$, respectively. The actual calculation of the radiation field as a function of both wavelength and redshift requires a numerical integration of the cosmological radiative transfer equation in the absorption-line forest (Haardt & Madau 1996). We call this radiation field $J_\alpha$, which is the one used in the numerical computations.

2.3. Stellar Radiation Field

When star formation is initiated in the protogalaxy, the internally generated radiation field can become significant. For example, at 1000 Å the Draine-Habing estimate for the radiation field in our Galaxy, $J_{J\alpha}$, gives an effective radiation field of $J_{-21.\alpha H} = 30 - 50$. We calculate the luminosity, $L_{J\alpha}$, and spectral energy distribution for a continuous star formation rate of $1 M_\odot$ yr$^{-1}$, with a Salpeter initial mass function (IMF), and a lower mass cutoff of $1 M_\odot$ (Leitherer & Heckman 1995). To obtain an intensity, we assume that the stars are being formed uniformly throughout a disk with area $A$ whose nominal value is 100 kpc$^2$. In the numerical results, we include the more detailed estimates of Kauffmann (1996, her Figs. 8 and 13) for the redshift-dependent size of the disk and the radial $H_\alpha$ column density distribution. We denote the stellar radiation field by $J_\alpha$, which is given by

$$J_\alpha = L_\alpha \left( \frac{S}{A} \right),$$

where $S$ is the actual star formation rate. The total radiation field is then

$$J = J_\alpha + J_\alpha^*.$$

Figure 2 presents the shape and magnitude of the overall radiation field $J$ for various epochs and various star formation rates. For $S > 0.02$, the stellar component dominates, which strongly increases the intensity in the 912–1110 Å region where $H_2$ is dissociated. The increase is most pronounced for wavelengths longward of the Lyman limit, leading to variations in the relative contributions to $J^-$ and $J^+\alpha$.

2.4. Dust and Metallicity Content

The visual extinction to distant QSOs for damped Ly$\alpha$ systems is not more than a few tenths of a magnitude (Fall & Pei 1993). Fundamental limits have been obtained on the depletion of elements like Zn and Cr in damped Ly$\alpha$ systems at a redshift of order $z \sim 3$. Inferred dust-to-gas ratios, $\xi_{dg}$, are of the order of $\xi_{dg} \sim 0.01 - 0.1 \xi_{cr}$, where $\xi_{cr}$ is the mean Galactic value (Pettini et al. 1994). The metallicities at these high redshifts are down by approximately the same factor, $Z \sim (1/30) Z_\odot$. An accurate analytical model for the redshift dependence of $\xi_{dg}$ and $Z$ is difficult to construct because of the essential role played by star formation. Nevertheless, this dependence should be included since it is a crucial ingredient in our analysis of molecule formation with cosmic epoch. In our numerical work, the metallicity is calculated from the star formation rate in the closed box limit and also in the limit of the ratio of stars to gas being small, i.e., $1 - \psi = \text{star/(star + gas)} \sim \text{star/gas} \ll 1$. Consequently, we can write

$$Z = \left( \frac{yS t}{M} \right) \left( \frac{2yS}{3H_0 M} \right)(1 + z)^{-3/2},$$

where $y \approx 0.02$ is the yield of metals like C and O (Woosley & Weaver 1995).

The production of dust follows by assuming that a fraction, $\xi_{dg}$, of metals is instantly locked up into grains (typically silicate cores). Therefore, we find that the dust-to-gas ratio $\xi_{dg}$ is given by

$$\xi_{dg} = \xi_{cr} Z = \left( \frac{\xi_{cr} yS t}{M} \right) \left( \frac{2\xi_{cr} yS}{3H_0 M} \right)(1 + z)^{-3/2}. $$

As we shall show later, the critical metallicities in our analysis turn out to be $\sim 0.03 - 0.1 Z_\odot$. Therefore, we have neglected the significantly lower initial metallicity that may have come from the formation of the halo.

2.5. Gas Pressure, Radiation Pressure, and Ionization Parameter

The pressure, $P$, in the midplane of the protogalactic disk can be estimated by assuming that the mass in the disk is in the gas phase and by using the Archimedean formula $P = \rho gh$, where $g$ is the gravitational acceleration normal to the disk. For a thin disk, $g$ is given by $g = 2\pi G\Sigma$, where $G$ is Newton’s constant and $\Sigma$ is the surface density of matter in
the disk. Therefore, one can write

\[ P = 2 \pi \mu^2 m_p^2 G N^2, \]

which can be written using equation (1) as

\[ P = 2^{-1/3} 3^{-2/3} \pi^{1/3} \mu^{1/3} m_p^{1/3} \delta^{1/3} \lambda^{-4} \Omega_b M^{2/3} n_0^{4/3} (1 + z)^4, \]

with numerical value

\[ \bar{P} = 1200 \left( \frac{\Omega_b \cdot 0.01}{5 \times 10^{11} M_\odot} \right)^{2/3} (1 + z)^4 \text{ cm}^{-3} \text{ K}, \]

where \( \bar{P} = P/k \), and \( k \) is Boltzmann’s constant. The \((1 + z)^4\) dependence of the pressure is slightly misleading because of the essential role played by \( \Omega_{b,g} \). In fact, due to the maximum in the redshift dependence of the latter, the sizes of protogalactic disks are smaller at redshifts \( z \geq 3 \) compared with \( z \sim 1 \). Note also that at redshift \( z \sim 1 \), the pressure calculated from the weight of the gas is an order of magnitude greater than the thermal pressure calculated from taking \( nT \). In fact, if one takes the radial dependence of the column density distribution into account and averages over only the inner 3 kpc of the disk, then the gravitational pressure is 2 orders of magnitude larger than the thermal pressure. This is also the case in our Galaxy, and consequently there must be some turbulent pressure that holds up the gas & Cox & Ferrara (Boulares 1990; Norman & Ferrara 1996). The turbulent pressure is due to kinetic motions from supernovae and superbubbles that is ultimately due to massive star formation. For low star formation rates, the turbulent pressure may not be effective, and the midplane pressure acting on the thermal component may increase, leading to a density increase of more than an order of magnitude. This needs to be kept in mind when discussing the phase diagrams that describe the state of the protogalactic gas.

The pressure, \( \bar{P}_h \), in a virialized halo with a gas fraction \( \Omega_{b,g} \) is given by

\[ \bar{P}_h = 80 \left( \frac{M}{5 \times 10^{11} M_\odot} \right)^{2/3} (1 + z)^2 \text{ cm}^{-3} \text{ K}. \]

We therefore neglect the halo pressure relative to the disk pressure. More detailed models require the consideration of the pressures and density profiles in the halo, and the role of cooling flows to precipitate material from halo to disk. These complex issues will not be addressed further here (cf. Norman & Meiksin 1996).

The radiation pressure is given by

\[ P_{rad} = \frac{4 \pi J V}{3 c}, \]

and we verify that it is always less than the gas pressure in the protodisks for each of our calculations. We define the ionization parameter used here by \( U = J/n \), with \( n \) the total number density, and a numerical value defined by \( U_{-21} = J_{-21}/n \). The dependence of the pressures and the ionization parameter are shown in Figure 3. The radiation pressure is typically much less than the gas pressure for the epochs studied here, but it is of the same order of magnitude for \( z \approx 0 \). The ionization parameter varies by a factor of 5 over the 1 < z < 3 range, because of the turnover in the quasar population around \( z = 2.5 \).

3. GAS-PHASE FORMATION OF \( \text{H}_2 \) WITH BACKGROUND RADIATION

We now turn to examine the epoch when molecular hydrogen is produced in the gas phase. We first use a simple model (Donahue & Shull 1991) to find the approximate analytical expressions for the molecular hydrogen abundance through the gas-phase \( \text{H}^+ \) route:

\[ \frac{n(\text{H}_2)}{n} = 1.4 \times 10^{-5} U_{-21}^{0.88} x, \]

where \( x = n_e/n \) is the electron abundance and is given by

\[ x = 0.027 U_{-21}^{0.54}. \]

These equations assume that the \( \text{H}^+ \) abundance is in equilibrium and that its dominant destruction channel is the reaction with neutral hydrogen yielding \( \text{H}_2 \). Conversely, the dominant destruction channels of molecular hydrogen are UV photodissociation and collisional dissociation at temperatures \( T > 2500 \text{ K} \). A simple model for the ionization heating yields

\[ T = 3 \times 10^3 U_{-21}^{0.88} n_{-21}^{0.12}, \]

and the molecular hydrogen abundance is given by

\[ \frac{n(\text{H}_2)}{n(\text{H})} = 10^{-2} U_{-21}^{0.54} n_{-21}^{0.14}. \]

For our two-component model, where the radiation field is split into a component longward of 912 Å and one shortward of the Lyman limit, we find the temperature to be

\[ T = 3 \times 10^3 (U_{-21})^{0.27} (U_{-21})^{0.54} n_{-21}^{0.12} \text{ K}, \]

and the molecular hydrogen abundance to be equal to

\[ \frac{n(\text{H}_2)}{n(\text{H})} = 10^{-2} (U_{-21})^{0.85} (U_{-21})^{-0.3} n_{-21}^{0.15}. \]

In the numerical work, the optically thin \( \text{H}_2 \) dissociation rate assumed in the above equation has been corrected for...
the fact that at abundances of the order of 1 column density in excess of $10^{19}$ cm$^{-2}$, H$_2$ self-shielding plays a role (see below).

4. GAS-PHASE FORMATION OF H$_2$ WITH STAR FORMATION

4.1. Feedback

It is generally expected that feedback mechanisms may be important in galaxy formation (cf. Norman & Ikeuchi 1996). The following model is considered here. If the internal star formation generates a dominant radiation field, then the gas will be heated to above 2000–3000 K, and the molecules will be collisionally dissociated. We assume that the absence of molecular hydrogen formation would strongly limit the efficiency of star formation. From the simple model used above, we find that for the stellar radiation field, the temperature can be estimated to be

$$T = 7.2 \times 10^3 \left( \frac{S}{M_\odot \text{yr}^{-1}} \right)^{0.8} \times \left( \frac{100 \text{ kpc}^2}{A} \right)^{0.8} \left( \frac{n}{1 \text{ cm}^{-3}} \right)^{-0.88},$$

(23)

giving a limiting star formation rate of

$$S_{\text{crit}} = 0.08 \left( \frac{T}{1000 \text{ K}} \right)^{1.25} \left( \frac{n}{1 \text{ cm}^{-3}} \right)^{1.1} \times \left( \frac{A}{100 \text{ kpc}^2} \right) M_\odot \text{yr}^{-1}. \quad (24)$$

Using the estimate of the density from equation (2), we find that

$$S_{\text{crit}} = 0.5 \left( \frac{T}{1000 \text{ K}} \right)^{1.25} \left( \frac{100}{\eta} \right)^{1.1} \left( \frac{\Omega_{b,\phi}}{0.01} \right)^{1.1} \times \left( \frac{A}{100 \text{ kpc}^2} \right) (1 + z)^{3.3} M_\odot \text{yr}^{-1}. \quad (25)$$

For a Schmidt star formation law where $\dot{S} \propto N_p$, with $1 < p < 2$, this corresponds to a critical column density

$$N_{\text{crit}} = 1 \times 10^{21} \left[ 0.5 \left( \frac{T}{1000 \text{ K}} \right)^{1.25} \left( \frac{100}{\eta} \right)^{1.1} \left( \frac{\Omega_{b,\phi}}{0.01} \right)^{1.1} \right]^{1/p} \times (1 + z)^{3.3 - 2p/3} \text{ cm}^{-2}. \quad (26)$$

For a protogalaxy of a given mass, we find that, using the arguments outlined in § 2, the radius of the disk decreases with redshift as

$$R = 100 \left( \frac{M}{5 \times 10^{11} M_\odot} \right)^{1/3} \left( \frac{\Omega_{b,\phi}}{0.01} \right)^{-1/3} (1 + z)^{-1} \text{ kpc}, \quad (27)$$

yielding, with $A = \pi R^2$,

$$S_{\text{crit}} = 1 \left( \frac{T}{1000 \text{ K}} \right)^{1.25} \left( \frac{100}{\eta} \right)^{1.1} \left( \frac{M}{5 \times 10^{11} M_\odot} \right)^{2/3} \times \left( \frac{\Omega_{b,\phi}}{0.01} \right)^{0.43} (1 + z)^{1.3} M_\odot \text{yr}^{-1}. \quad (28)$$

For star formation to proceed at all, $S$ should be a factor of a few smaller.

4.2. Bursts of Star Formation

We have analyzed the feedback effects in terms of a fixed, continuous star formation rate. We now discuss the burst mode where we envisage the following cycle: the star formation rate is sufficiently high, so that all molecules are dissociated; the star formation turns off; the molecules reform; and another burst occurs.

The timescale, $\tau_D$, for the dissociation of molecular hydrogen, assuming no self-shielding, is

$$\tau_D = 2 \times 10^3 \left( \frac{T}{1000 \text{ K}} \right)^{1/2} \left( \frac{1 \text{ cm}^{-3}}{n} \right) \text{yr}.$$

(29)

and the timescale, $\tau_P$, for the formation of molecular hydrogen is

$$\tau_P = 4 \times 10^6 \left( \frac{1000 \text{ K}}{T} \right)^{1/2} \left( \frac{1 \text{ cm}^{-3}}{n} \right) \text{yr}. \quad (30)$$

Consequently, the duty cycle, $D = (\tau_P/\tau_D)$, of on-burst to off-burst is

$$D = 8 \times 10^{-3} \left( \frac{T}{1000 \text{ K}} \right). \quad (31)$$

Since the amplitude of the radiation field is set by the level of star formation rate, it follows that in any burst, the mass of the stars formed, $M_{*,\text{burst}}$, is constant and given by

$$M_{*,\text{burst}} = S \tau_D \approx 10^6 M_\odot. \quad (32)$$

Thus, small bursts can occur, but with a very low mean effective star formation rate. It should be emphasized that once a multiphase medium has been established, bursts can be sustained (Spaans & Norman 1997).

5. MULTIPHASE STRUCTURE OF THE ISM

5.1. Numerical Model

We adopt the heating and cooling curves presented by Donahue & Shull (1991). We include cooling by [O I] 63 $\mu$m, [C II] 158 $\mu$m, and rotational transitions of CO for temperatures below 3000 K. These terms are only of importance for $z < 1.5$. Heating due to photoelectric emission from grains is included and contributes to $z < 1$. At every redshift, equation (9) is used to determine the metallicity, and expression (8) yields the ambient radiation field. The latter is integrated over the hydrogen ionization and H$_2$ photodissociation cross sections. The ionization, chemical, and thermal balance is solved iteratively until convergence is better than 1% in the temperature and H$_2$ abundance. The redshift dependence of the baryonic mass fraction is included in the calculation of the column density. In deriving the H$_2$ abundances, we have included the radial N(H I) profiles of protogalactic disks as presented by Kauffmann (1996).

5.2. Moderate Epoch

The results for a fixed star formation rate $\dot{S} = 0.1 M_\odot$ yr$^{-1}$ are presented in Figure 4. Between $z = 4$ and $z = 2$, the gas is kept at a fairly constant temperature of approximately 500 K. This confirms the suggestion by Haiman et al. (1996a, 1996b) that H$_2$ cooling can facilitate the collapse of high-mass objects and initiate the formation of structure in the early universe. It is also apparent that due to the importance of the stellar radiation field, massive star formation will not occur since a cool phase of $T < 300$ K gas is not present above a redshift of 2. In fact, the existence of this
moderate epoch ensures that the initial cooling time of the disk is not shorter than the dynamical time at redshifts larger than 3. If this would not be so, then the star formation rates at high redshifts would be much too large.

These effects are further reflected in the abundance of CO that remains low, down to a redshift of unity. When the metallicity reaches a threshold of approximately 0.03 $Z_{\odot}$, the magnitude of the cooling curve is strongly enhanced. A multiphase medium is now expected to result. Note that to simplify equations (10) and (11) for the metal enrichment and dust-to-gas ratio development, we have assumed a Schmidt law with index, $p = 1$, implying that the star formation rate per unit mass, $S/M$, is a constant.

5.3. Phase Transition

Figure 5 presents phase diagrams for various epochs and star formation rates. For early epochs, $H_2$ is the only coolant below 3000 K, and only single-temperature solutions exist for certain pressures. As the metallicity increases, the $P-n$ curve attains its characteristic S shape, and lines of constant pressure cut it at three different temperatures. That is, a multiphase medium has formed in which cool and warm components are stable (Field, Goldsmith, & Habing 1969), and can facilitate large-scale star formation. From Figure 5, one finds that the transition to a multiphase ISM in the bulk of the galactic disk occurs at $z \approx 1.5$. At this point, it is timely to mention the recent observations of galaxies in the Hubble Deep Field presented by Madau et al. (1996) and Mobasher et al. (1996) for the star formation history of the universe. Although the published results are preliminary, they seem to suggest evidence for a star formation epoch around $z \sim 1-2$.

Assuming that there is one supernova per 100 $M_{\odot}$ of stars formed, the filling factor of the hot gas component due

**Fig. 4.—** The results of the calculations for the gas temperature and the abundance of molecular hydrogen, CO, dust ($Z_m$), and metals ($Z_m$) as functions of redshift for a star formation rate $S = 0.1 M_{\odot} \text{yr}^{-1}$.

**Fig. 5.—** Pressure, $P$, and gas density, $n_H$, phase diagrams for redshifts 1, 2, and 3, respectively, and star formation rates of 0.01, 0.1, and 1 $M_{\odot} \text{yr}^{-1}$. The curve in the upper left panel denotes constant pressure. The symbol $Z$ indicates the metallicity with respect to Galactic at that epoch for a particular star formation rate. The labels F, G, and H denote the thermal equilibria, of which G is unstable.
to supernovae is (McKee & Ostriker 1977)

\[ Q = 0.5E_{51}^{28} \left( \frac{\dot{S}}{1 \text{ M}_\odot \text{ yr}^{-1}} \right) \left( \frac{5 \times 10^{11} \text{ M}_\odot}{M} \right) \left( \frac{0.01}{\Omega_{0,\phi}} \right) \times \left( \frac{n}{25 \text{ cm}^{-3}} \right)^{1.14} \left( \frac{\bar{\rho}}{2.5 \times 10^4 \text{ cm}^{-3} \text{ K}} \right)^{-1.70}, \]  

(33)

which can be written, using the critical star formation rate, as

\[ Q = 0.5E_{51}^{28} \left( \frac{5 \times 10^{11} \text{ M}_\odot}{M} \right) \left( \frac{0.01}{\Omega_{0,\phi}} \right)^{0.14} \times \left( \frac{\eta}{100} \right)^{1.14} \left( \frac{T}{1000 \text{ K}} \right)^{-0.45}. \]  

(34)

For a Schmidt star formation law, and using equation (1), we find \( Q \propto M^\epsilon(1 + z)^{-0.38} \), where \( 0 < \epsilon < \frac{1}{5}, \) and consequently the estimate is fairly robust.

Therefore, for high star formation rates at the critical value (i.e., more massive objects), the filling factor of hot gas can be significant in these protogalactic disks since the pressure is low, and the supernova bubbles can expand and fill large volumes. The above calculation is indicative only, since the star formation is usually clumped, and the supernova bubbles generally break out of the disk and vent their energy into the halo of the protogalaxy. This process can also result in a feedback that inhibits star formation (Norman & Ikeuchi 1989). However, for high star formation rates, a multiphase medium may develop where the hot phase is driven by supernovae energy input. For star formation rates of \( 0.1-0.3 \text{ M}_\odot \text{ yr}^{-1} \) and below, the hot phase is not significant, and a two-phase mode can occur.

6. FORMATION OF H₂ ON GRAINS

An additional important ingredient in the regulation of the star formation is the formation of H₂ on dust grains, which renders its abundance independent of the electron fraction. These effects have been included in the models above, but an analytic model is presented here to facilitate a connection with observations.

In the presence of dust and at low densities, molecular hydrogen is formed through grain surface reactions. In steady state, the local H₂ density is given by (Tielens & Hollenbach 1985; van Dishoeck & Black 1986)

\[ n(H_2) = \frac{An_H^2}{1 + 2An_H}, \]  

(35)

where \( n_H = n(H) + 2n(H_2) \) is the total hydrogen density. We have discussed the mean density in the disk, but it is necessary to make a good estimate of the mean density in the diffuse clouds in the cold neutral medium component where the H₂ formation is most likely to be initiated. We choose to parameterize the clumpiness of the cold neutral clouds by a diffuse cloud covering factor \( f_c \). If, as we have indicated above, the protostellar disks are not subject to substantial massive star formation before molecules are formed, then we do not expect the development of a supernova-driven hot \( (~10^6 \text{ K}) \) component. Consequently, the temperature and density contrast between cloud and intercloud component will be modest. We infer that the covering factor of the diffuse cloud component is of order \( \sim 1 \).

The controlling parameter, \( An_H \), is the ratio of the formation rate of molecular hydrogen on grains to the photodissociation rate of H₂. \( A \) is given by

\[ A = \frac{k_s T^{1/2} \xi}{I_{UV} R_{\text{thin}} \rho(0) e^{-\tau_{\text{UV}}}}, \]  

(36)

where \( I_{UV} \) denotes the enhancement of the average UV background with respect to the interstellar field at \( z = 0 \). \( \tau_{\text{UV}}, C \) is the optical depth in the UV continuum \( (\tau_{\text{UV}}, C = 2.5 \xi A_q) \), \( R_{\text{thin}} \) is the unattenuated H₂ photodissociation rate, and \( \beta \) is the self-shielding function. The rate constant, \( k_s \), depends on the nature of the grains. The value of \( R_{\text{thin}} \) depends on the precise slope of the UV background between 912 and 1100 Å.

For the present discussion, we want to determine which epoch molecular hydrogen self-shields in the cold diffuse clouds created in the phase transition, and therefore we adopt the constraint

\[ A \geq \frac{2}{n_H}, \]  

(37)

yielding a fractional H₂ abundance of 0.4. For self-shielding, the constraint (37) reduces to

\[ I_{UV} R_{\text{thin}} \tau_{\text{UV},L} \exp \left( -\tau_{\text{UV},C} \right) \leq \left( \frac{1}{2} \right) k_s T^{1/2} \xi n_H. \]  

(38)

Explicitly including the redshift dependence and the dependence on covering factor, we find that

\[ I_{UV,0} R_{\text{thin}} f_c^2 \tau_{\text{UV},L,0} \exp \left[ -\tau_{\text{UV},C,0} f_c^{-1}(1 + z)^{1/2} \right] \times (1 + z)^{1/2 + \eta} \leq \left( \frac{1}{2} \right) k_s T^{1/2} \xi_0 n_{H,0}. \]  

(39)

Subscripts with zero refer to present time values.

For \( \dot{S} \sim 0.03-0.1 \text{ M}_\odot \text{ yr}^{-1} \), the internally generated radiation field dominates at lower redshifts, \( z \sim 1 \). In the limiting case where \( q = 1 \), the total extinction varies slowly with redshift since the lower gas-to-dust ratio of the absorber compensates for its higher column density. In this case, the critical redshift for H₂ formation, \( z_{\text{mol}} \), is given by

\[ (1 + z_{\text{mol}}) \leq \left[ I_{UV,0} R_{\text{thin}} f_c^{-2} \tau_{\text{UV},L,0}^2 \right]^{1/2} \exp \left[ \tau_{\text{UV},C,0} f_c^{-1} k_s T^{1/2} \xi_0 n_{H,0} \right], \]  

(40)

with a numerical value \( z_{\text{mol}} = 1.6 \).

Including variations in the value of \( \dot{S} \sim 0.1-0.5 \) yields a fiducial range for H₂ formation of \( 1.2 \leq z \leq 2.0 \). These estimates have the advantage that they are weakly dependent on the input parameters. The strongest dependence is on covering factor and star formation rate. Obviously, for small covering factors, molecules may form at higher redshift, but they will be less observable. Note that the epoch of significant dust shielding and cooling are similar since the dust production and metallicity production are closely related.

The increasing dust columns can absorb a significant fraction of the internal stellar radiation field, and this will influence the timescale on which subsequent star formation proceeds. Using the formalism of McKee (1989), it is possible to estimate the effect of the dust column on the star formation rate. There is a critical extinction of approximately 4 mag, necessary to absorb most of the radiation of newly formed stars. The timescale to convert all available mass into stars for a density of roughly 1000 cm⁻³ is given
by

\[ 3.2 \times 10^7 \left( \frac{1}{2 \sigma^2} \right)^{0.5 + (1.25/4\nu)} \]

\[ \times \left[ \left( \frac{1}{1 + \frac{8 \sigma^2}{3}} \right)^{0.5} + \left( \frac{1}{\left( \frac{8 \sigma^2}{3} \right)} \right)^{0.5} \right]^{1 - (2.5/4\nu)} e^{16/4\nu} \, \text{yr}, \] (41)

and the dust extinction \( A_V = 4z(1 + z)^2 \). For \( z > 2 \), this timescale is much larger than the local Hubble time, but for epochs later than 1.7, star formation can proceed efficiently in the bulk of the galaxies.

6.1. Statistical Approach

The covering factor discussed above is ultimately determined by the process of galaxy formation. Using the statistical approach discussed in § 2, for any given redshift, the condition (37) for \( H_2 \) formation on grains can be written as

\[ r_0 > r_{0,\text{crit}}(z). \] (42)

The fraction of galaxies that contain molecules, \( f_{\text{mol}}(z) \), is then given by

\[ f_{\text{mol}}(z) = \int_{r_{0,\text{crit}}}^{\infty} \frac{\epsilon^{-x^2}}{\int_{0}^{\infty} \epsilon^{-x^2} \, dx}, \] (43)

where \( \epsilon(r_0, z) = \left[ \delta(1 + z)/2^{1/2} \sigma(r_0) \right] \). The definition of the statistical epoch of molecule formation is taken to be \( f_{\text{mol}}(z) \approx 0.5 \). Inversion of this condition yields \( \langle z_{\text{mol}} \rangle \).

For the parameter values adopted above, and putting the biasing parameter \( b = 1 \), the relations \( r_{0,\text{crit}} = 0.17(1 + z)^{2 + 3.2} \) Mpc and \( \langle z_{\text{mol}} \rangle = 1.5 \) approximately hold. Note that \( r_{0,\text{crit}} \) does not depend on biasing, but the value of \( \langle z_{\text{mol}} \rangle \) does.

Finally, the probability, \( \mathcal{P} \), that a given line of sight to a QSO will show molecular absorption by an intervening damped Ly\( \alpha \) cloud in a (proto)galaxy is given by (Peebles 1993)

\[ \mathcal{P} \approx \int \sigma n_0 c H_0^{-1} \Omega_h^{-1/2} (1 + z)^{1/2} \, dz, \] (44)

where the integral extends over redshifts between 0 and \( \langle z_{\text{mol}} \rangle \), and \( \sigma = \pi r_g^2 \). The characteristic radius (bright part) of the galaxy is chosen equal to \( r_g = 10 \, \text{h}^{-1} \) kpc. With the parameters given above, this yields \( \mathcal{P} \approx 8 \times 10^{-2} \).

Once the conditions are satisfied for molecular hydrogen formation, the timescale is of order \( \sim 1/(n_H \xi) \) Gyr \( \sim 10^8(1 + z)^{-(p + 1)} \) yr. The ion chemistry proceeds much faster on a timescale of approximately \( 3 \times 10^3(1 + z)^{-6} \) yr. Thus, once the hydrogen molecules can be formed on grains, the full range of diffuse cloud species follows immediately.

7. CONCLUSIONS, DISCUSSION, AND IMPLICATIONS

Initially motivated by QSO absorption-line observations indicating an absence of molecular hydrogen at high redshift, we have examined the formation of molecular hydrogen, after the epoch of reionization, in the context of canonical galaxy formation theory due to hierarchical clustering. The issue of molecular hydrogen at high redshift has been discussed in recent interesting papers by Haiman et al. (1996a, 1996b) and Tegmark et al. (1997). Whereas they concentrated more on the redshifts above \( z \gtrsim 10 \), we are focusing on the physics of the phases of the protogalactic gas at redshifts \( z \sim 1–10 \). We have been able to use recently computed accurate models for the cosmic background radiation field (Haardt & Madau 1996) and for the radiation field from a stellar population with a Salpeter IMF, and with a 1 M\( \odot \) lower mass cutoff in a continuous star-forming mode (Leitherer & Heckman 1995). We have also used a simple model for a protogalactic disk and followed conventional CDM cosmology, extended by more recent numerical calculations, when considering the spectrum of disk masses and column densities. We have concentrated mainly on structures associated with 1 σ and 2 σ density fluctuations that are expected to form the bulk of the stars.

As with the recent work by Haiman et al. (1996a, 1996b) and Tegmark et al. (1997), we find that there is an initial epoch of \( H_2 \) production in the gas phase produced through the H\(^{-}\) channel route, where the abundance of molecular hydrogen is approximately 1% and given simply in terms of the ionization parameter by equations (20) and (22). Predicting the details of the state of the interstellar medium of protogalaxies is a complex task. We have normalized to what is known from work on the neutral phase of our Galaxy, and we assume that the fundamental units in the gas phase of protogalaxies during this epoch are akin to the diffuse clouds found in our own Galaxy. We have shown until the metallicity of the gas achieves \( Z \sim 0.03–0.1 \) at a redshift of 1–2, cold giant molecular clouds are not formed because of inefficient cooling. We have found that star formation in the protogalaxies can become self-regulated because of heating of the gas by the internal stellar radiation field. We have given a simple analytic model for the feedback process in § 4.1. It is possible to define a maximum star formation rate during this epoch. Plausible estimates give a rate of \( \lesssim 1 M_\odot \, \text{yr}^{-1} \), and therefore we have considered an appropriate fiducial star formation rate to be \( \sim 0.1–0.3 M_\odot \, \text{yr}^{-1} \). The production of metals and dust proceeds slowly in this self-regulated mode. This slow star formation phase was shown to terminate once the metal abundance increased to a level of approximately \( Z \sim 0.03–0.1 Z_\odot \).

From an analysis of the phase diagrams in Figure 5, we found that species such as C, O, and CO become sufficiently abundant and can cool the gas below 300 K to \( \sim 10–30 \) K. At this point, a phase transition can occur in the protogalactic gas. For the low fiducial star formation rates discussed above, we find this to be a transition to a two-phase medium, as described by Field et al. (1969). Dense molecular clouds can form, and the star formation is no longer self-regulated in the manner described above since the UV radiation does not penetrate the dense cores of the clouds. We expect that rapid, massive star formation ensues, and the abundance of metals and dust increase concomitantly. The dust abundance also becomes sufficiently high to allow molecular hydrogen formation on grain surfaces. With the increased star formation rates, the ISM will change to one dominated by supernovae energy input (McKee & Ostriker 1977), with significant exchange of mass, energy, and metallicity from the disk to the halo (Norman & Ikeuchi 1989). In a subsequent paper, we will investigate in more detail the effects of this phase transition on the evolution of dwarf galaxies and the importance of metal loss driven by supernova explosions.

Our analysis may be of relevance to the G-dwarf problem: the simple model (Pagel 1989 and references therein) for chemical enrichment overproduces the number of metal-poor stars (Cowley 1995; Worthey, Dorman, & Jones 1996 and references therein). Of the many solutions
proposed for the G-dwarf problem, the simplest appears to be that, by the time a few percent of the gas mass of a galaxy is assembled into stars, the remaining gas reservoir is already enriched in metals and has not yet experienced any star formation. The moderate phase due to feedback identified in this work is likely to cause star formation in an inhomogeneous way. The very nature of the feedback mechanism dictates that star formation in one location strongly suppresses additional star formation in its vicinity. The subsequent phase transition then causes rapid star formation throughout the nearly unprocessed ISM that now has a metallicity close to 10% of solar.

In our analysis, we have assumed a fixed value for the collapse factor $\lambda^{-1}$. In reality, the distribution of spin parameters may be quite wide (Warren et al. 1992; Dubinsky & Carlberg 1991). If we view the disk as being formed from a spherical object sustaining a low-$H_2$ abundance, driven by the extragalactic UV background, then an increase in the collapse factor will increase the total and $H_2$ column densities quadratically, and the local density like $\lambda^2$. Consequently, more of the stellar radiation can be absorbed locally, preserving the $H_2$ abundance and suppressing the global heating, and a higher star formation rate can be sustained. We estimate, although tentatively, that for objects with $\lambda \sim 0.02$, a factor of 3.5 below our canonical value, the redshift at which the phase transition occurs, can be as high as 3. More detailed knowledge of the $\lambda$ distribution is necessary to assess how common such high-redshift objects are.

In summary, from our elementary cosmological model, we conclude that this new mode of star formation, where objects now akin to giant molecular clouds in our Galaxy become the sites of star formation, occurs at a redshift of approximately 1.5, with a value higher by a factor of 2 if more massive initial perturbations or larger collapse factors are considered. The observed phase transition in the interstellar medium of protogalactic disks as analyzed in this paper is now a plausible physical reason why the formation of disks of galaxies occurs at a redshift of order unity, with a significant increase in star formation after the metallicity has achieved a value of order $Z \sim 0.03 - 0.1 Z_\odot$. These findings are consistent with the recent studies of the Hubble Deep Field (cf. Madau et al. 1996; Mobasher et al. 1996). The combination of feedback and a phase transition can provide a natural solution to the G-dwarf problems.

We are grateful to Tim Heckman, David Neufeld, and Rosemary Wyse for illuminating and stimulating discussions that contributed significantly to our understanding, and to Andrea Ferrara, Cliff Leitherer, and Piero Madau for such discussions and also for providing us with their excellent data on low-metallicity cooling curves, the spectrum of radiation from stellar populations, and the cosmic background radiation field. We are also grateful to Piero Rosati for his assistance in the presentation of the numerical results. We thank the anonymous referee for his detailed and valuable comments. M. S. also acknowledges with gratitude the support of NASA grant NAGW-3147 from the Long Term Space Astrophysics Research Program.