General Relativity and Quantum Jumps:
The Existence of Nondiffeomorphic Solutions to the Cauchy Problem in Nonempty Spacetime and Quantum Jumps as a Provider of a Canonical Spacetime Structure

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Abstract

It is shown that in spite of a generally accepted concept, there exist nondiffeomorphic solutions to the Cauchy problem in nonempty spacetime, which implies the necessity for canonical complementary conditions. It is nonlocal quantum jumps that provide a canonical global structure of spacetime manifold and, by the same token, the canonical complementary conditions.

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Introduction

In the literature, there exists no disagreement on the question of the completeness of the Einstein equation. In all the books on general relativity in which the completeness problem is considered (see [1-13]), the latter is treated as resolved once and for all. The reasoning is this. In the Cauchy problem, there are six independent time evolution equations for six components $g_{ij}$ ($i, j = 1, 2, 3$) of metric, whereas the remaining four components $g_{0\mu}$ ($\mu = 0, 1, 2, 3$) remain arbitrary. But diffeomorphic metrics (ones connected by diffeomorphisms) are physically equivalent, and a diffeomorphism involves exactly four degrees of freedom. Hence it is concluded that all the solutions to the Cauchy problem are diffeomorphic, i.e., equivalent. Therefore the only task is to introduce four complementary conditions fixing a solution. The complementary equations may be to a great extent arbitrary, the only requirement is that they be quasilinear. We will call this resolution of the completeness problem a property of diffeomorphic connectedness of the set of the solutions. For a Ricci flat spacetime, diffeomorphic connectedness does take place [14,15].

The Einstein equation is local, so that as long as it is complete, general relativity is a local theory and does not determine any global structure of spacetime manifold. This gives rise to the problem of compatibility of general relativity with quantum jumps, which are inherently nonlocal (see [16]).

In this paper, we argue that the coincidence of the number of degrees of freedom of a diffeomorphism with the number of the components $g_{0\mu}$ (or missing equations) does not imply diffeomorphic connectedness for nonempty spacetime. Here is a simple counterexample. Let $f$ be a function on a manifold $M^4$ and let $F : M^4 \rightarrow M^4$ be a diffeomorphism, so that $(F^*f)(p) = f(Fp)$, $p \in M^4$. Let $\tilde{f}$ be another function. Is it true that there exists an $F$ such that $\tilde{f} = F^*f$? We have only one equation $\tilde{f}(p) = f(Fp)$ and four degrees of freedom for $F$. But if the ranges of $\tilde{f}$ and $f$ are different, ran $\tilde{f} \neq$ ran $f$, $F$ does not exist. This counterexample may be straightforwardly extended to the problem of diffeomorphic connectedness. Introduce complementary conditions of the form $g_{0i} = 0$ ($i = 1, 2, 3$), $R = f$ where $R$ is the scalar curvature and $f$ is a function. The quasilinearity requirement is respected. If ran $\tilde{f} \neq$ ran $f$, $\tilde{g}$ and $g$ are not diffeomorphic.

The breakdown of the diffeomorphic connectedness implies the necessity for canonical complementary conditions. It is quantum jumps that provide the latter. Nonlocal quantum jumps click out a universal cosmological time $t \in T$, so that spacetime manifold has a canonical global structure: $M^4 = T \times S$ where $S$ is a cosmological space. The canonical complementary conditions are of the form $dt = g(\partial/\partial t, \cdot)$, which corresponds to a synchronous frame.

1 The problem of the completeness of the Einstein equation and diffeomorphic connectedness

In this section, we summarize known facts and conventional concepts on the problem of the completeness of the Einstein equation and the Cauchy problem.
1.1 The underdetermination of the Einstein equation

The Einstein equations

\[ G_\nu^\mu - \Lambda g_\nu^\mu = T_\nu^\mu, \quad \mu, \nu = 0, 1, 2, 3 \]  

(1.1.1)

form a system of ten equations in the ten metric components \( g_{\mu\nu} \) and their first and second derivatives. However the covariant divergence of each side vanishes identically, i.e.,

\[ (G_\nu^\mu - \Lambda g_\nu^\mu)_;^\nu = 0 \]  

(1.1.2)

and

\[ T_\nu^\mu;_\nu = 0 \]  

(1.1.3)

hold independently of the field (metric and matter) equations. Thus (1.1.1) provides only six independent equations for the ten components \( g_{\mu\nu} \).

1.2 The underdetermination of the Cauchy problem

In the Cauchy problem, the equations of time evolution are

\[ G_j^i - \Lambda g_j^i = T_j^i, \quad i, j = 1, 2, 3 \]  

(1.2.1)

The equations

\[ G_\mu^0 - \Lambda g_\mu^0 = T_\mu^0, \quad \mu = 0, 1, 2, 3 \]  

(1.2.2)

are constraints on initial conditions for \( g_{\mu\nu} \) and

\[ \dot{g}_{ij} = g_{ij,0} = \partial g_{ij}/\partial t, \quad t = x^0 \]  

(1.2.3)

Thus there are only six dynamical equations for the ten components \( g_{\mu\nu} \), so that a solution to the Cauchy problem is determined up to four degrees of freedom, i.e., four arbitrary functions.

1.3 Diffeomorphic connectedness

It is the commonly accepted concept that the set of all solutions to the Cauchy problem possesses the property of diffeomorphic connectedness: for any two solutions, \( g \) and \( \tilde{g} \), there exists a diffeomorphism \( F : M^4 \rightarrow M^4 \) such that \( \tilde{g} = F^*g \). Since diffeomorphic metrics describe the same physical situation, the latter is uniquely determined in the Cauchy problem. The ground for the diffeomorphic connectedness is this: There are four degrees of freedom in the solution, and a diffeomorphism involves exactly four degrees of freedom; thus the solution is determined up to a diffeomorphism.

For a Ricci flat spacetime, diffeomorphic connectedness does take place \([14,15]\).

1.4 Complementary conditions

As long as diffeomorphic connectedness takes place, i.e., all the solutions to the Cauchy problem are physically equivalent, the only task is to introduce four complementary conditions fixing a solution.

In the literature, a variety of the complementary conditions is presented: harmonic coordinates \([1,5]\); lapse and shift functions (ADM formalism) \([2]\); synchronous coordinates \([9,6]\); equations for \( \tilde{g}_{0\mu} \) \([8]\). All those are noncovariant and exploit a coordinate system.
2 The invalidity of diffeomorphic connectedness in nonempty spacetime

In this section, we introduce a counterexample to show that in nonempty spacetime diffeomorphic connectedness is not valid.

2.1 A time-space hypoframe and frame

As long as it is possible, it is reasonable to use a coordinate-free approach. For this purpose, we introduce the following objects:

- A timelike vector field \( b_0 \) and a 1-form \( \beta \) such that \( \beta(b_0) > 0 \), or \( \beta = f_0 \beta^0 \), \( \beta^0(b_0) = 1 \) and a function \( f_0 > 0 \);
- A frame \( (b_a : a = 0, 1, 2, 3) \) such that \( \beta^0(b_a) = \delta^0_a \).

We will call \( (b_0, \beta) \) a time-space hypoframe and \( (b_a) \) a time-space frame.

2.2 A synchronous hypoframe and four noncovariant conditions

A time-space hypoframe \( (b_0, \beta^0) \) will be called a synchronous hypoframe. It provides a means of introducing four noncovariant conditions

\[
g(b_0, \cdot) = \beta^0 \tag{2.2.1}
\]

or

\[
g(b_0, b_a) = \delta_{0a}, \quad a = 0, 1, 2, 3 \tag{2.2.2}
\]

2.3 Scalar curvature invariant condition

Consider a nonempty, i.e., not a Ricci flat spacetime. Introduce a complementary condition of the form

\[
R = f_R \tag{2.3.1}
\]

where \( f_R \) is a function on \( M^4 \). This condition is not only coordinate-free but invariant as well. It plays a crucial role in the argumentation that follows.

2.4 (3+1) conditions and a holonomic frame

In addition to (2.3.1), introduce three noncovariant conditions of the form

\[
g(b_0, b_j) = 0, \quad j = 1, 2, 3 \tag{2.4.1}
\]

where \( (b_a) \) is a time-space frame. We will call conditions (2.4.1) and (2.3.1) (3+1) conditions.

Now we introduce a holonomic frame:

\[
(b_a) \rightarrow (b_\mu), \quad b_\mu = \partial_\mu = \partial/\partial x^\mu \tag{2.4.2}
\]

where \( x = (x^\mu) \) are (local) coordinates. Now the (3+1) conditions take the form

\[
g_{0j}(x) = 0, \quad j = 1, 2, 3 \tag{2.4.3}
\]
\[ R(x) = f_R(x) \] (2.4.4)

In the case that all the four conditions are noncovariant (equations (2.2.2) are an important example), all the four degrees of freedom in metric may be described by coordinate functions \( x^\mu = x^\mu(p), \ p \in M^4 \). If one of the conditions is invariant, those functions describe only three degrees of freedom; one degree of freedom is described by a function involved in the invariant condition. The only invariant condition is (2.3.1) or (2.4.4) since \( R \) is the only first order invariant. Thus it is the function \( f_R \) that describes one degree of freedom.

### 2.5 A system of quasilinear differential equations for metric components

The complementary equations may be to a great extent arbitrary. The only requirement is that these equations, like the Einstein equations (1.2.1), be quasilinear. Then there will be a complete system of quasilinear differential equations for metric components. Let us verify that the \((3+1)\) conditions do respect that requirement.

The Ricci tensor

\[ R_\mu^\sigma = g^{\sigma \kappa} R_{\kappa \mu}, \quad R_{\kappa \mu} = g^{\eta \lambda} R_{\eta \kappa \lambda \mu} \] (2.5.1)

the Riemann tensor

\[ R_{\eta \kappa \lambda \mu} = \frac{1}{2} (g_{\eta \mu, \kappa \lambda} + g_{\lambda \kappa, \eta \mu} - g_{\eta \lambda, \kappa \mu} - g_{\kappa \mu, \eta \lambda}) + (g^{\nu \rho} \Gamma_{\nu \kappa \lambda} \Gamma_{\rho \eta \mu} - g^{\nu \rho} \Gamma_{\nu \kappa \mu} \Gamma_{\rho \eta \lambda}) \] (2.5.2)

the Christoffel symbol

\[ \Gamma_{\eta \kappa \lambda} = \frac{1}{2} (g_{\eta \kappa, \lambda} + g_{\eta \lambda, \kappa} - g_{\kappa \lambda, \eta}) \] (2.5.3)

From noncovariant conditions (2.4.3) it follows that

\[ g_{0j} = 0, \quad g^{0j} = 0, \quad g^{00} = 1/g_{00} \] (2.5.4)

We find

\[ R_{m}^p = -\frac{1}{2} g^{00} g_{pk} g_{km,00} + \frac{1}{4} (g^{00})^2 g_{pk} g_{km,00,0} + \tilde{R}_{m}^p \] (2.5.5)

\[ R_{0}^0 = -\frac{1}{2} g^{00} g_{km} g_{km,00} + \frac{1}{4} (g^{00})^2 g_{km} g_{km,00,0} + \tilde{R}_{0}^0 \] (2.5.6)

where

\[ \tilde{R}_{0}^0 = g^{00} g_{km} \tilde{R}_{0km}, \quad \tilde{R}_{m}^p = g^{00} g_{pk} \tilde{R}_{0km} + g^{00} g_{il} \tilde{R}_{i klm} \] (2.5.7)

\[ \tilde{R}_{0km} = -\frac{1}{2} g_{00,km} + \frac{1}{4} g^{00} g_{00,k} g_{00,m} + \frac{1}{4} g^{mr} g_{nk,0} g_{rm,0} + \frac{1}{2} g^{nr} \Gamma_{nk} g_{00,r} \] (2.5.8)

\[ \tilde{R}_{i klm} = \tilde{R}_{ikl m} = \frac{1}{2} (g_{im,kl} + g_{kl,im} - g_{il,km} - g_{km,il}) + (g^{nr} \Gamma_{nk} \Gamma_{rim} - g^{nr} \Gamma_{nk} \Gamma_{ril}) + \frac{1}{4} g^{00} (g_{kl,0} g_{im,0} - g_{km,0} g_{il,0}) \] (2.5.9)

Finally,

\[ R = -g^{00} g^{mk} g_{km,00} + \frac{1}{2} (g^{00})^2 g^{mk} g_{km,00,0} + \tilde{R} \] (2.5.10)
where

$$\tilde{R} = \tilde{R}_\mu^\mu = \tilde{R}_0^0 + \tilde{R}_m^m$$  \hspace{1cm} (2.5.11)

The quantities $\tilde{R}_m^p$ and $\tilde{R}$ involve $g_{ij}$, $g_{ij,k}$, $g_{ij,kl}$, $g_{ij,0}$, $g_{00}$, $g_{00,ij}$, i.e., in fact, $g_{ij}$, $g_{ij,0}$; $g_{00}$. Thus, in view of (2.5.5) and (2.5.10), the seven equations

$$R_j^i - \frac{1}{2} \delta_j^i R - \delta_j^i \Lambda = T_j^i, \quad R = f_R$$  \hspace{1cm} (2.5.12)

form a system of quasilinear equations for the seven metric components $g_{ij}$, $g_{00}$, the higher time derivatives being $\ddot{g}_{ij}$ and $\dot{g}_{00}$, respectively. Initial conditions are those for $g_{ij}$, $\dot{g}_{ij}$ and $g_{00}$ obeying constraints (1.2.2).

### 2.6 The existence of nondiffeomorphic solutions to the Cauchy problem in nonempty spacetime

Let $g$ and $\bar{g}$ be solutions to the system of equations (2.5.12) with functions $f_R$ and $\bar{f}_R$, respectively. If the ranges of the functions are different,

$$\text{ran} \bar{f}_R \neq \text{ran} f_R$$  \hspace{1cm} (2.6.1)

g and $\bar{g}$ are nondiffeomorphic. Indeed, $\bar{g} = F^* g$ implies $\bar{R} = F^* R$ so that

$$\text{ran} \bar{R} = \text{ran} R$$  \hspace{1cm} (2.6.2)

which does not hold. Thus the set of all the solutions to the Cauchy problem in nonempty spacetime is not diffeomorphically connected. It is the degree of freedom described by the function $f_R$ that brings about breaking diffeomorphic connectedness.

It is necessary to point out the following. From (1.1.1) follows

$$R = -T + 4\Lambda$$  \hspace{1cm} (2.6.3)

where $T = T_\mu^\mu$. Therefore if matter dynamics had been independent of metric, it would have been impossible to introduce an arbitrary function $f_R$ since the scalar curvature would have been prescribed. But that independence holds only in an empty spacetime.

In a nonempty spacetime, (2.6.3) and (2.3.1) are two independent equations for metric and matter.

For example, in the case of one scalar matter field $\varphi$, we have eight functions: $(g_{ij})$, $g_{00}$ and $\varphi$, and eight equations: (1.2.1), (2.3.1) and the wave equation

$$\Box \varphi + m^2 \varphi + \xi R \varphi = 0$$  \hspace{1cm} (2.6.4)

### 2.7 The misleadingness of infinitesimal diffeomorphisms

One must not be misled by infinitesimal diffeomorphisms.

Let $g$ and $\bar{g} = g + h$ with $h$ infinitesimal be two solutions to the Cauchy problem. We will show that there exists an infinitesimal local diffeomorphism $F$ such that

$$\bar{g} = F^* g$$  \hspace{1cm} (2.7.1)
In (local) coordinates \((x^\mu)\) we have

\[
g_{\mu\nu} + h_{\mu\nu} = (F^* g)_{\mu\nu}
\]  
(2.7.2)

or

\[
g_{\mu\nu} + h_{\mu\nu} = g(F_* \partial_\mu, F_* \partial_\nu)
\]  
(2.7.3)

For an infinitesimal \(F\), we have

\[
F_* \partial_\mu = \partial_\mu + L_{v^\lambda \partial_\lambda} \partial_\mu = \partial_\mu + [v^\lambda \partial_\lambda; \partial_\mu] = \partial_\mu - v^{\lambda,\mu} \partial_\lambda
\]  
(2.7.4)

where \(v^\lambda \partial_\lambda\) is an infinitesimal vector field. Thus we obtain

\[
g_{\mu\lambda} v_{\lambda,\nu} + g_{\nu\lambda} v_{\lambda,\mu} = -h_{\mu\nu}
\]  
(2.7.5)

or

\[
\begin{align*}
\text{for } & \mu\nu = 00 \quad 2g_{00} v^{\lambda,0} = -h_{00} \\
\text{for } & \mu\nu = 0j \quad g_{00} v^{\lambda,j} + g_{j\lambda} v^{\lambda,0} = -h_{0j} \\
\text{for } & \mu\nu = ij \quad g_{i\lambda} v^{\lambda,j} + g_{j\lambda} v^{\lambda,i} = -h_{ij}
\end{align*}
\]  
(2.7.6-2.7.8)

Let \((x^\mu)\) be synchronous coordinates for \(g\),

\[
g_{0\mu} = \delta_{0\mu}
\]  
(2.7.9)

then

\[
v^{0,0} = -\frac{1}{2} h_{00}
\]  
(2.7.10)

\[
g_{ji} v^{i,0} = -h_{0j} - v^{0,j}, \quad v^{i,0} = g^{ij} (h_{0j} + v^{0,j})
\]  
(2.7.11)

\[
h_{ij} = -(g_{il} v^{l,j} + g_{jl} v^{l,i})
\]  
(2.7.12)

whence

\[
v^0(t, \bar{x}) = v^0(0, \bar{x}) - \frac{1}{2} \int_0^t h_{00}(t', \bar{x}) dt'
\]  
(2.7.13)

\[
v^i(t, \bar{x}) = v^i(0, \bar{x}) - \int_0^t [g^{ij} (h_{0j} + v^{0,j})](t', \bar{x}) dt'
\]  
(2.7.14)

Initial conditions are

\[
h(0, \bar{x}) = 0, \text{ i.e., } h_{0\mu}(0, \bar{x}) = 0, \quad h_{ij}(0, \bar{x}) = 0
\]  
(2.7.15)

\[
h_{ij,0}(0, \bar{x}) = 0
\]  
(2.7.16)

The components \(h_{0\mu}(t, \bar{x})\) are given with \(h_{0\mu}(0, \bar{x}) = 0\).

Put

\[
v^{\mu}(0, \bar{x}) = 0
\]  
(2.7.17)
then
\[ v^{\mu},(0, \vec{x}) = 0 \] (2.7.18)
and
\[ v^l_{,j0} = v^l_{,0j} = -[g^{lk}(h_{0k} + v^0_{,k})]_{,j} = 0 \quad \text{for} \quad x = (0, \vec{x}) \] (2.7.19)

From (2.7.12), (2.7.18), (2.7.19) follows
\[ h_{ij}(0, \vec{x}) = 0, \quad h_{ij,0}(0, \vec{x}) = 0 \] (2.7.20)

Thus the formulas
\[ v^0(t, \vec{x}) = -\frac{1}{2} \int_0^t h_{00}(t', \vec{x}) dt' \] (2.7.21)
\[ v^i(t, \vec{x}) = -\int_0^t [g^{ij}(h_{0j} + v^0_{,j})](t', \vec{x}) dt' \] (2.7.22)
give \( F, \partial_\mu \) (2.7.4).

Now put in the (3+1) conditions
\[ \tilde{f}_R = f_R + w \] (2.7.23)
with \( w \) infinitesimal such that (2.6.1) is fulfilled; e.g.,
\[ \text{ran} f_R = [a, \infty), \quad \text{ran} \tilde{f}_R = [\tilde{a}, \infty), \quad \tilde{a} = a + c, \quad c \neq 0, \quad c \text{ infinitesimal} \] (2.7.24)

Then \( \tilde{g} \) and \( g \) are not diffeomorphic. On the other hand, \( \tilde{g} = g + h \) with \( h \) infinitesimal so that infinitesimally \( \tilde{g} = F^*g \) whence \( R = F^*R \), i.e., \( \tilde{f}_R = F^*f_R \) in spite of (2.6.1).

Again, \( f \) and \( \tilde{f} = f + w \) with \( w \) infinitesimal are, in general, infinitesimally diffeomorphic. The equality
\[ \tilde{f} = F^*f \] (2.7.25)
means
\[ f(x) + w(x) = f(Fx) \] (2.7.26)

Put
\[ Fx = x + \xi(x) \] (2.7.27)
so that
\[ f(x) + w(x) = f(x + \xi(x)) \] (2.7.28)

Infinitesimally
\[ f(x + \xi(x)) = f(x) + [\partial_\mu f] \xi^\mu(x) \] (2.7.29)
whence
\[ (\partial_\mu f) \xi^\mu = w \] (2.7.30)

The solvability criterion is \( \partial_\mu f \neq 0 \).

The consideration carried out demonstrates that an infinitesimal (local) diffeomorphism may be misleading.
3 Quantum jumps and a canonical spacetime manifold structure

In this section, we show that quantum jumps provide a canonical structure of spacetime manifold and, by the same token, canonical complementary conditions.

3.1 The necessity for canonical complementary conditions

Had diffeomorphic connectedness taken place, all complementary conditions would have been equivalent, and choosing some of them would have been a matter of convenience. The breakdown of the connectedness necessitates the introduction of specific, canonical conditions. When introducing the latter, we should be guided by both mathematical and physical reasons.

3.2 Synchronous conditions

We restrict our choice to global coordinate-free complementary conditions involving no arbitrary functions. Such conditions are provided by a synchronous hypoframe

$$ (b_0, \beta^0), \quad \beta^0(b_0) = 1 $$

(3.2.1)

and are of the form of (2.2.1):

$$ g(b_0, \cdot) = \beta^0 $$

(3.2.2)

We call this conditions synchronous since in a holonomic frame

$$ b_0 \to \partial/\partial t, \quad b_j \to \partial/\partial x^j $$

(3.2.3)

they take the form of (2.7.9), which corresponds to synchronous coordinates.

There is a family of synchronous hypoframes and, by the same token, a family of synchronous conditions. A synchronous hypoframe (3.2.1) has seven degrees of freedom, in view of which we will not raise the question on diffeomorphic connectedness. Our goal is to introduce a specific, canonical hypoframe.

3.3 Semiclassical gravity and quantum jumps

It is quantum jumps that, due to their nonlocality, provide a canonical global structure of spacetime manifold and, by this structure, a canonical hypoframe.

In semiclassical gravity, the energy-momentum tensor

$$ T = \left( \Psi, \hat{T}\Psi \right) $$

(3.3.1)

where $\hat{T}$ is the energy-momentum tensor operator and $\Psi$ is a state vector. A quantum jump is that of the state vector (from here on see [17]):

$$ \Psi_{\text{before jump}} =: \Psi^< \rightarrow \Psi^> := \Psi_{\text{after jump}} $$

(3.3.2)
A jump of $\Psi$ results in that of $T$:

$$\Delta T = \left( \Psi^>, \hat{T} \Psi^> \right) - \left( \Psi^<, \hat{T} \Psi^< \right)$$ (3.3.3)

under the assumption that $\hat{T}$ is continuous. Discontinuity of $T$ causes a violation of the Einstein equation (1.1.1). The components $G^i_j$ involve the second time derivatives $\ddot{g}_{ij}$, which makes it possible to retain the six equations (1.2.1). Jumps of $T^i_j$ will result in those of $\ddot{g}_{ij}$.

### 3.4 A canonical spacetime manifold structure

A quantum jump of the state vector gives rise to a set of events—jumps of $\ddot{g}_{ij}$. Those events are, by definition, simultaneous, which allows for synchronizing clocks and thereby furnishing a universal time. The latter, in its turn, implies the product spacetime manifold:

$$M = M^4 = T \times S, \quad M \ni p = (t, s), \quad t \in T, \quad -\infty \leq t_{\text{min}} \leq t \leq t_{\text{max}} \leq \infty, \quad s \in S$$ (3.4.1)

The one-dimensional manifold $T$ is the universal cosmological time, the three-dimensional manifold $S$ is a cosmological space. By (3.4.1), the tangent space $M_p$ at a point $p \in M$ is

$$M_p = T_t \oplus S_s, \quad p = (t, s)$$ (3.4.2)

In special relativity, the concept of simultaneity relating to quantum jumps makes no operationalistic sense. Taking gravity into account endows the concept with an operationalistic content—the simultaneity of the jumps of $\ddot{g}_{ij}$. On the other hand, it is the simultaneity related to quantum jumps that provides the global structure (3.4.1) for spacetime manifold. General relativity per se is a local theory: “Indeed general relativity does not prescribe the topology of the world . . .” (Weyl [18]). Thus general relativity and quantum jumps complement each other.

### 3.5 A canonical synchronous hypoframe and canonical complementary conditions

We introduce a projection function on $M^4$:

$$t : M^4 \to T, \quad p = (s, t) \mapsto t(p) = t$$ (3.5.1)

Now we put

$$\beta^0 = dt, \quad b_0 = \partial/\partial t$$ (3.5.2)

($t$ is a coordinate function). For any frame $(b_j)$ on $S$, we have

$$\beta^0(b_a) = \delta^0_a$$ (3.5.3)

The hypoframe

$$(b_0, \beta^0) = (\partial/\partial t, dt)$$ (3.5.4)

is the canonical one.
The canonical complementary conditions take the form
\[ dt = g(\partial/\partial t, \cdot) \] (3.5.5)

In view of the global character of time \( t \), these conditions are global and, in fact, coordinate-free. We may rewrite (3.5.5) in the explicitly coordinate-free form:
\[ \beta^0 = g(b_0, \cdot) \] (3.5.6)

In connection with this, we quote Weyl [18]: “The introduction of numbers as coordinates . . . is an act of violence whose only practical vindication is the special calculatory manageability of the ordinary number continuum with its four basic operations.”

By (3.5.5), metric is of the form
\[ g = dt \otimes dt - h_t \] (3.5.7)

where \( h_t \) is a Riemannian metric on \( S \) depending on \( t \). Thus we have
\[ T_t \perp S_s, \quad (t, s) \in T \times S \] (3.5.8)

The problem of the underdetermination of the Einstein equation is resolved by the canonical complementary conditions, which are provided by nonlocal quantum jumps. Thus, quantum jump nonlocality not only does not contradict relativity, but it is essential for general relativity to be a complete theory. Quantum jumps occur in nonempty spacetime—just where the underdetermination problem arises.

### 3.6 A canonical decomposition of the energy-momentum tensor

The canonical structure of spacetime manifold implies a canonical decomposition of the energy-momentum tensor:
\[ T = T_E + T_P + T_S \] (3.6.1)

\[ T_E^{\mu\nu} = \delta^{\mu0}\delta^{\nu0}T_{00}, \quad T_P^{\mu\nu} = \delta^{\mu0}\delta^{\nu j}T_{0j} + \delta^{\nu0}\delta^{\mu j}T_{0j}, \quad T_S^{\mu\nu} = \delta^{i\mu}\delta^{j\nu}T_{ij} \] (3.6.2)

\( T_E, T_P, \) and \( T_S \) are the energy, momentum, and stress tensors, respectively. We have correspondences:
\[ T_E \leftrightarrow T_{00}, \quad T_P \leftrightarrow \vec{K}, \quad K_j = T_{0j} \] (3.6.3)

\( T_{00} \) and \( \vec{K} \) are energy and momentum densities, respectively. Energy
\[ E = \int_S d\vec{x} \sqrt{|h|} T_{00} \] (3.6.4)

momentum
\[ \vec{P} = \int_S d\vec{x} \sqrt{|h|} \vec{K} \] (3.6.5)

where \( |h| = \det(h_{ij}) \).
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