Research Article

Optimizing the Subsidy Calculation Model of Urban Public Transport

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In recent years, to alleviate and resolve the traffic problems in China, the government introduced a series of policies to subsidize public transportation. Accordingly, to effectively implement these financial subsidies, the formulation of a scientific model of urban public transport subsidy calculation, strict control of the total amount of financial subsidies, and clear identification of subsidy targets are necessary. In this study, a linear dual model is developed based on the principle of resource allocation and the economic meaning of shadow prices. The public transportation system of a city in China is considered an example, and both public transport operators and public interest stakeholders are considered. The subsidy calculation for service quality improvement is established for public transportation operations and vehicle transformation. Based on the validation using the above example, the calculation result is found to approximate the actual value, indicating the feasibility of the proposed model. Finally, this article presents suggestions pertaining to subsidy mechanism, operating environment, and service quality as a reference for urban public transport subsidies.

1. Introduction

With the rapid development of China’s social economy, urban traffic has increasingly become problematic. To alleviate urban traffic congestion and mitigate environmental pollution, an important measure is to prioritize public transportation development. To promote the coordinated development of man and nature, as well as achieve sustainable economic development, in 2006, four departments—the Ministry of Construction, Ministry of Finance, National Development and Reform Commission, and Ministry of Labor—expressed their opinions on several economic policies on the priority development of urban public transportation [1], emphasizing that the “local people’s governments should increase capital investment in urban public transportation.” In 2019, the State Council issued guidance opinions on the urban priority development of public transport [2], indicating that “the public welfare property of urban public transport is highlighted, and public transport is placed in the primary position of urban transport development.” In 2017, the Ministry of Transport issued the “Opinions on comprehensively and deeply promoting the development of green transportation” [3], requiring the intensification of the public transportation priority policy and the comprehensive promotion of the development of green transportation.

All provinces and cities were obliged to actively advocate the national policy of prioritizing the development of public transportation, vigorously support and develop urban public transportation, increase investment in public transportation, and implement the subsidy mechanism of low ticket prices. In Beijing, for example, since the implementation of the low bus fare system, the cumulative subsidy amount from 2007 to 2015 reached 122.5 billion yuan. In recent years, even after the implementation of the new ticket price, the level of annual financial subsidy remains high at 10 billion yuan [4]. In Shaoxing City, the government’s financial subsidy for public transportation increased from 5,099,800 yuan in 2012 to 93,282,100 yuan in 2019. For seven years, the government’s subsidy for public transportation increased by...
approximately 20 times. In the county, although the public transportation subsidy increment was moderate, it increased by several times. Moreover, despite the relatively flat trend of passenger flow, the growth of financial subsidies for public transportation was rapid. Over the years, with an average growth rate of 5%, the passenger flow in Shaoxing County public transportation has approached a horizontal line. This indicates that the growth ratio of subsidies is considerably greater than that of passenger flow, and the impact of subsidies on passenger flow is insignificant. Accordingly, a bus subsidy status investigation was conducted on several cities across the country (e.g., Beijing, Zhejiang, Shaoxing, Hangzhou, and Huzhou; Suzhou and Wuxi in Jiangsu Province; Shenyang, Liaoyang, and Huludao in Liaoning Province). It revealed a series of problems: the subsidy was not in place; the object of subsidy was vague, resulting in reduced enthusiasm for bus operation; the bus service quality was inadequate; the local financial pressure increased; and some problems remained unresolved by subsidies. Consequently, the means for reducing the local financial pressure and improving the bus service quality have become the research focus of many scholars.

In various countries, traffic congestion has also become a key factor hindering the healthy development of the economy; hence, the research on public transportation has become a mainstream topic. Moreover, because subsidies for public transport are an important means of alleviating traffic problems, many countries have long started research on subsidy calculation models. The following studies were conducted based on population, population density, operating range, ticket prices, and transport costs (certain studies abroad have comprehensively examined the public transport ticket). Stephen Schmidt [5] indicated that, from the perspective of the population and bus operating mileage subsidy model, increased spending on subsidy may lead to low efficiency; accordingly, certain suggestions for improvement were presented. Obeng et al. [6] analyzed the comprehensive effects of subsidies, regulations, and perceived budgets and subsequently established a scientific calculation model. Boitani and Cambini [7] proposed an improved non-Bayesian model based on economics, focusing on the means of reducing operating cost and improving the bus service quality. Tisato [8] analyzed the relationship between passenger cost and optimal bus subsidy, devised a subsidy model, and verified the model’s feasibility. Sakano et al. [9] examined the subsidy problem of urban rail transits in the United States. To solve the low subsidy efficiency problem, they developed a subsidy model based on static cost minimization to identify the internal factors leading to low efficiency in the company. Karlafitis and McCarthy [10] described the performance of a bus system through three attributes: efficiency, effectiveness, and completeness. They found that systems with high scores in one attribute usually perform well in other attributes. They also observed the inverse relationship between system performance and subsidy. Such a relationship may be useful in supporting performance-based subsidy allocations. Jara-Díaz and Gschwender [11] explained that the average cost could decrease with increased sponsorship based on the relationship between cost, pricing, and optimal variables; based on the foregoing, they concluded that an optimal subsidy could be achieved. By comparing the optimal frequency and least operating cost, it was concluded that autonomous financial constraints could interfere with the optimal pricing scheme. To study the impact of passenger number and frequency on subsidies, as well as the optimal pricing of these services [12–14], a microeconomic model was established. Further, a numerical simulation was implemented to aid in explaining and modifying the decisions of operators. In addition to the foregoing, note that the subsidies and policies among cities [15, 16], between developed and developing countries [17] and between small and big cities [18], vary. van Goeverden [19] and Batarce and Galilea [20] studied the advantages and problems of subsidy intensity and pricing policy with or without subsidy and discussed the advantages and disadvantages of the two pricing policies. The commuter subsidy of public transportation is evaluated through the political economy of the commuter subsidy problem in central cities [15] and the equilibrium model of urban spatial mismatch [21]. This subsidy is also assessed in terms of accessibility, affordability, and equity [22], which all improve the welfare of low-income families and increase the net income of those who commute over extended periods or have high transportation costs. The public transport subsidy policy is closely related to simultaneous economic and environmental effects. Shuai et al. [23] and Chaparro et al. [24] proposed an incentive mechanism for bus subsidies by studying the effect of monetary incentives on bus drivers, number of passengers transported, and bus speed. Basso and Jara-Díaz [25] and Vigren and Pyddoke [26] studied the effect of bus contracts with passenger incentive measures and proposed a subsidy scheme to maximize the profit of bus operators without neglecting the welfare of passengers. Sun and Zhang [27] and Stojić [28] optimized the government-funded bus incentive scheme from the perspective of the government and operators and proposed a new analysis model to determine the allocation of funds for quantitative public service compensation to maximize social welfare and profit. Hao et al. [29] indicated the advantages of public transport passenger subsidy classification and established a game model considering the income of residents, enterprise operating costs, and subsidies. They achieved the maximum utility of government subsidy, indicating that the new model was an effective method for measuring urban public transport subsidy. Foreign research results have great reference value for my country to optimize the calculation method of urban public transport financial subsidies. However, it is necessary to construct a scientific and reasonable financial subsidy calculation method according to my country’s special national conditions.

Considerable research on subsidy models has also been conducted in China; different subsidy models have been selected according to the actual situation in various parts of the country. Wang and Zhang [30] presented suggestions to formulate a model of urban bus operation income, station construction, vehicle transformation, and other relevant fiscal subsidies. The suggestions focused on the sources of
urban bus special subsidy funds, construction funds, fuel subsidies, self-raised funds for bus enterprises, and funds derived from other sectors of society. Chen [31] and Gao et al. [32] developed a comprehensive fare subsidy model based on cost regulation from the perspective of economy, efficiency, and benefit of bus subsidy policies. Wang et al. [33] and Hao et al. [34] devised a subsidy calculation model based on passenger utility, enterprise cost, and government financial subsidies, providing a relatively novel idea for the development of public transportation. After analyzing the advantages and disadvantages of the Parry and Small model, Xu [35] optimized it under the cost regulation condition. Zeng [36] created a subsidy model based on the maximization of social welfare, internal rate of return, and reasonable rate of return from the perspective of bus loss and bus characteristics. The Shizuishan bus was considered a case for demonstrating the subsidy calculation. Chen [37] and Ye [38] comprehensively analyzed the advantages and disadvantages of various subsidy measurement models and established a subsidy efficiency assessment and evaluation system using enterprise quality, operating cost, and service quality as indicators. The domestic results mainly use the utility theory and cost-benefit theory to analyze and calculate the subsidies for the policy losses of public transportation companies. However, there is a problem of information asymmetry in the communication process between the government and public transportation companies, which is not conducive to the implementation of subsidies, which in turn causes the government’s financial burden to continue to increase. When there are constraints on government financial subsidy resources, this article applies the resource allocation problem of economics to the subsidy of public transportation based on the principle of the duality of public transportation profitability and public welfare. Satisfaction is fully incorporated into the subsidy linear programming model, which makes the calculation of financial subsidies more accurate.

In summary, domestic and international research scholars have examined public transport subsidies considering three aspects: bus companies, government departments, and passenger analysis problems of public transport subsidies. However, most of these studies only involved qualitative analysis; they lacked quantitative analysis. Moreover, they did not consider the government and public transport enterprise information asymmetry caused by the financial pressure. The existing public transport subsidy calculation model only considers the cost of buses, trip distance, transit passenger flow, and less certain costs. The foregoing requires public transport service quality research; however, the aforementioned model is unable to satisfy the required service quality. Public transportation is a type of quasipublic product involving government departments, operating enterprises, and the public. It has the dual nature of profitability and public welfare. As an operational company, bus companies must be economically viable to maintain normal operations. However, they must ensure that, in addition to sustaining the normal operation of the company, the welfare of passengers is protected. Therefore, as its contribution, this study aims to widen the scope of research on bus service quality based on existing bus operation and vehicle transformation, quantify the bus service quality, and establish an urban bus subsidy mechanism based on the principle of resource allocation in economics. The foregoing objectives involve coordinating the operations of stations, determining the optimum subsidies, calculating the shadow price subsidies, identifying the explicit subsidy object, and reducing the fiscal pressure on the government. This further includes resolving the information asymmetry between the government and public transport enterprise caused by financial accounting problems. Such a resolution increases the enthusiasm of public transport enterprises to operate, improves the quality of fiscal subsidy effect on bus service, and promotes the healthy development of public transport.

2. Model Establishment

2.1. Principles of Public Transport Subsidy Model Formulation. Substantial effort has been devoted to the research on subsidy models both domestically and internationally. However, most studies are not sufficiently systematic to fully reflect the status of subsidies. Many studies take bus operation subsidies and vehicle modification subsidies as the main research objects of subsidies, which are relatively novel, but ignore the importance of service quality.

Therefore, the current study draws on existing research ideas, proceeds on improving optimization based on these concepts, widens the extent of research on the service quality regulation subsidy, and comprehensively analyzes the subsidy factors. Based on the principle of public transportation profitability and public welfare duality, the problem and resolution of resource allocation in economics are applied to the public transportation subsidy. Further, the subsidy model of resource allocation is established through linear programming and based on the economic significance of the shadow price [39]. The subsidies on bus operation, vehicle reconstruction, and passenger satisfaction are allocated to the bus subsidy as three resources (Figure 1), and the shadow unit prices of the subsidies of these resources are derived. By comparing the subsidy amount and operating cost over the years, reasonable and scientific suggestions are presented.

The subsidy mechanism of urban public transportation includes three aspects: the return-on-investment subsidy of bus line operation, one-way subsidy, and service quality adjustment subsidy. The bus line investment-return subsidy includes the bus operation subsidy. Individual subsidies refer to special subsidies for vehicle transformation (such as vehicle renewal, repair, and maintenance) and ticket price concessions [40]. The service quality adjustment subsidy refers to the subsidy amount combined with the subsidy for passenger satisfaction evaluation criteria.

2.2. Linear Programming Model Based on Passenger Flow. Assume an objective function, $F(X_i)$, where $(i = 1, 2, 3)$, $F$ represents the total bus revenue, and $X_i$ represents the unit revenue price of the first type of bus company. This model can resolve two types of problems: maximizing revenue and
minimizing fiscal subsidies. In this model, resource \( R_1 \) is the bus operation subsidy, resource \( R_2 \) is the vehicle modification subsidy, and resource \( R_3 \) is the passenger satisfaction subsidy; all of these are allocated to city bus \( A_1 \), county bus \( A_2 \), and bus \( A_3 \), respectively. The definitions of relevant variables in formula are summarized in Table 1.

The function model of the maximum revenue of the three companies is as follows:

\[
P: \quad \text{max } Z = \text{max } \sum c_i x_i, \quad \text{s.t. } \sum a_{ij} x_i \leq d_j, \quad x_i \geq 0.
\]  

(1)

where \( i = 1, 2, 3 \) represents bus companies and \( j = 1, 2, 3 \) represents resources \( (R_1, R_2, \text{ and } R_3) \), respectively. The linear programming model of the bus company revenue presented above refers mainly to the maximum revenue constrained by certain subsidy resources. In addition to the maximum revenue, this study identifies the minimum subsidy input, that is, the maximum revenue equal to the minimum subsidy. Accordingly, a linear programming model is formulated to determine the minimum subsidy input, that is, the dual linear model of the above model.

The mathematical model of linear programming can also be expressed as

\[
\text{max } Z = CX, \quad \text{s.t. } AX \leq d, \quad X \geq 0.
\]

(6)

Among them, \( A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad C = (c_1, c_2, c_3), \quad X = (x_1, x_2, x_3)^T, \quad \text{and } d = (d_1, d_2, d_3)^T.\]

The dual model of a linear \( P \) problem can be expressed as follows:

\[
D: \quad \text{min } \omega = \min \sum d_j y_j, \quad \text{s.t. } \begin{cases} \sum a_{ij} y_j \leq c_i, \\ y_j \geq 0, \end{cases}
\]

(7)

where \( \omega \) is the total amount of financial subsidies provided by the government and \( y_j \) refers to the unit price of the financial subsidy of resource \( j \).

The dual problem of the linear programming problem can be expressed as

\[
\text{min } \omega = Y d, \quad \text{s.t. } YA \geq c, \quad Y \geq 0.
\]

(8)

Among them, \( Y = (y_1, y_2, y_3) \).

According to the dual theory of linear programming problems, the necessary and sufficient condition for \( P \) to have an optimal solution \( X^* = (x_1^*, x_2^*, x_3^*)^T \) is that \( D \) also has an optimal solution \( Y^* = (y_1^*, y_2^*, y_3^*)^T \), and there is

\[
\text{max } Z = c_1 x_1^* + c_2 x_2^* + c_3 x_3^* = \min \omega = d_1 y_1^* + d_2 y_2^* + d_3 y_3^*.
\]

(9)
Table 1: Definitions of relevant variables in formula.

| Continuous variable | Implication |
|---------------------|-------------|
| $x_i$               | Optimal passenger flow in type $i$ bus company ($i = 1, 2, 3$) |
| $c_i$               | Ridership price per unit revenue of type $i$ bus company ($i = 1, 2, 3$) |
| $p_i$               | Ridership flow subsidized by bus company ($i = 1, 2, 3$) |
| $q_i$               | Ridership flow supported by type $i$ bus company's vehicle modification subsidy ($i = 1, 2, 3$) |
| $w_i$               | Number of passengers supported by satisfaction subsidy of type $i$ bus company ($i = 1, 2, 3$) |
| $a_{ij}$            | Annual growth rate of passenger flow supported by $j$th resource in $i$th bus company ($i = 1, 2, 3; j = 1, 2, 3$) |
| $Z$                 | Total revenue of three bus companies |
| $k_i$               | Average daily passenger flow |
| $m_i$               | Number of vehicles |
| $s_i$, $h_i$, $t_i$, $n_i$ | Average number of passengers transported, daily working hours, annual passenger flow, and passenger satisfaction, respectively |
| $d_j$               | Total passenger flow supported by $j$th resource |

Problem $P$ seeks to maximize the total profit of urban public transportation companies. In dual linear programming problem $D, Y = (y_1, y_2, y_3)$ is a reasonable evaluation of the government’s financial subsidies for urban public transportation companies so that the government can provide the city with the largest total profit of urban public transportation companies. The total amount of financial subsidies for public transportation companies is the least. It can be understood as the lowest subsidy price that urban public transportation companies can accept when the government provides financial subsidies to urban public transportation companies, under the condition that the urban public transportation companies do not operate at a loss, that is, the shadow price of the government’s financial subsidies for urban public transportation.

Assuming that $B$ is the optimal basis of model $P$, then the optimal solution of the objective function is

$$
\max Z = CX^* = \min \omega = Y^* d = C_B B^{-1} d_j
$$

$$
= y_1^* d_1 + y_2^* d_2 + y_3^* d_3. \tag{10}
$$

In the linear programming problem $P$, $Z$ is the linear function of the decision variables $x_1, x_2, x_3$. Since the problem $P$ is to find the maximum value of $Z = c_1 x_1 + c_2 x_2 + c_3 x_3$ under the constraints of formula (1), $\max Z$ is restricted by the resource limits $d_1, d_2, d_3$, which can be expressed as

$$
\max Z = d_1 y_1^* + d_2 y_2^* + d_3 y_3^*. \tag{11}
$$

According to $(\partial Z/\partial d_j) = y_j^* (j = 1, 2, 3)$, it can be seen that $y_j^*$ represents the marginal benefit of government financial subsidies under the optimal allocation plan, reflecting the degree of influence of changes in government financial subsidies on the value of the objective function of the total profit of urban public transportation enterprises; if $B$ is the optimal basis of linear programming problem $P$ and $y_j^* (j = 1, 2, 3)$ is the optimal solution of linear programming problem $D$, then

$$
y_j^* = C_B B^{-1} = \frac{\partial Z}{\partial d_j} = (y_1^*, y_2^*, y_3^*). \tag{12}
$$

The economic significance of this model is as follows. (1) Model $P$ is the means of maximizing the benefits of public transportation under the condition of constant subsidy input and unit revenue. (2) Amount $Z$ reflects the final value and not the sum of the market prices of various resources. (3) Model $D$ indicates that model $P$ reaches the optimal solution through the action of optimal basis $B$; $D$ also reaches the optimal solution; that is, the values are equal. This means that the maximum revenue of public transportation is equal to the minimum value of the financial subsidy. The optimal basis of model $D$ is $y_j^*$, that is, the unit price of shadow subsidy resulting from the optimal decision.

3. Empirical Analysis

3.1. Data Sources. This study employs a city bus, county bus, and automobile bus operating in a city in 2019 to demonstrate the analysis. There are five bus companies in the city. For the research, the city bus, county bus, and automobile bus were selected from these companies because the passenger flows in these buses are high, and the data and bus system are more complete compared to other buses; hence, they are significant to the study. Over a long time, the operational cost has exceeded the operating income. Traffic change has not been considered, but the increase in fiscal subsidies and subsidies for problems that cannot be resolved is outstanding. The reason is that although the government has introduced financial subsidies (without necessarily increasing them) to where they are deemed necessary to improve service quality and mitigate traffic problems, the government and public transport enterprise information is inaccurate. This has led to an increase in public transportation cost or false positive ticket income, thus increasing the government’s financial burden. Hence, the formulation of a scientific and reasonable fiscal subsidy model to obtain a subsidy shadow price as a reference for government subsidies is urgent. This can ensure that government subsidies are not only effective but also maximized. In this study, the bus-related data of three bus enterprises in a city from 2012 to 2019 are collated, as summarized in Tables 2 and 3.
3.2. Result Analysis and Discussion

3.2.1. Correlation Analysis of Subsidies and Various Factors.
According to the correlation analysis of the city’s bus data from 2012 to 2019, as summarized in Table 4, the subsidy is considerably correlated with the bus passenger flow, operating cost, and passenger satisfaction; however, it has a slight negative correlation with bus revenue. The summary also indicates that the bus passenger volume serves as a measure of bus service efficiency and sharing rate index. Through a series of preferential policies, the government aims to improve the sharing rate of public transportation, relieve traffic congestion, promote the sustainable development of public transportation, and exert a positive influence on environmental protection, energy conservation, and emission reduction. The passenger volume is the basis of bus subsidy; when the passenger volume is large, the bus subsidy should correspondingly increase. As a type of quasipublic product, public transportation must not only be profitable but also protect public welfare. The bus fare is lower than the operating cost, and the government pays the price difference by providing financial subsidies to public transportation enterprises. When the bus operation cost is extremely high, cost management should be implemented to ensure that the bus enterprises derive reasonable returns and maintain their normal operation. Passenger satisfaction, as the concrete embodiment of bus service quality, should be regarded by the government as a subsidy assessment index to improve the bus service level. Accordingly, in this study, bus revenue, operating cost, bus passenger flow, and passenger satisfaction are all included in the subsidy model, all of which are conducive to the development of healthy public transport.

3.2.2. Model Calculation. The summarized results in Table 5 are obtained by substituting the data collected over the years into equations (2)–(5):

![Table 2: Three-year data of three bus enterprises in city.](source)

| Company name   | Relevant indicators                          | Maximum value | Minimum value | Average value |
|---------------|---------------------------------------------|---------------|---------------|--------------|
| City bus company | Number of vehicles                          | 729.00        | 303.00        | 631.00       |
|               | Average number of passengers                | 40.00         | 32.00         | 39.00        |
|               | Working hours per day                       | 14.00         | 10.00         | 12.60        |
|               | Daily average passenger flow (×10^4 passengers) | 25.87        | 7.47          | 22.03        |
|               | Unit revenue price (yuan)                   | 1.38          | 1.02          | 1.24         |
|               | Passenger satisfaction                       | 83.70         | 82.00         | 82.47        |
| County bus company | Number of vehicles                          | 264.00        | 226.00        | 249.00       |
|               | Average number of passengers                | 45.00         | 45.00         | 45.00        |
|               | Working hours per day                       | 14.00         | 11.00         | 12.30        |
|               | Daily average passenger flow (×10^4 passengers) | 8.64        | 6.95          | 7.51         |
|               | Unit revenue price (yuan)                   | 2.12          | 1.01          | 1.69         |
|               | Passenger satisfaction                       | 88.50         | 81.50         | 83.80        |
| Bus company   | Number of vehicles                          | 125.00        | 84.00         | 111.00       |
|               | Average number of passengers                | 44.00         | 44.00         | 44.00        |
|               | Working hours per day                       | 14.00         | 11.00         | 12.30        |
|               | Daily average passenger flow (×10^4 passengers) | 4.91        | 3.50          | 4.03         |
|               | Unit revenue price (yuan)                   | 1.84          | 1.75          | 1.79         |
|               | Passenger satisfaction                       | 85.00         | 82.00         | 83.33        |

Source: integrated data of city bus companies from 2012 to 2019.

![Table 3: Total passenger transport and operating costs of public transport.](source)

| Company name | Relevant indicators                          | Maximum value | Minimum value | Average value |
|--------------|---------------------------------------------|---------------|---------------|--------------|
| City bus company | Total passenger volume (×10^4 people)         | 10 150.52     | 2725.45       | 8264.52      |
|               | Operating cost (×10^4 yuan)                  | 26,997.66     | 8,191         | 18,538.51    |
|               | Total subsidy (×10^4 yuan)                   | 10,669.87     | 0             | 5,200.14     |
| County bus company | Total passenger volume (×10^4 people)         | 3520.21       | 2623          | 2760.76      |
|               | Operating cost (×10^4 yuan)                  | 8,623.36      | 5,058         | 6,973.16     |
|               | Total subsidy (×10^4 yuan)                   | 4,195.89      | 7.50          | 2,001.57     |
| Bus company   | Total passenger volume (×10^4 people)         | 1793.52       | 1045.97       | 1365.69      |
|               | Operating cost (×10^4 yuan)                  | 3,423.85      | 2,995.93      | 3,192.11     |
|               | Total subsidy (×10^4 yuan)                   | 1,849.18      | 865.07        | 1,399.31     |

Source: integrated data of city bus companies from 2012 to 2019.

![Table 4: Correlation analysis of bus subsidy and various factors.](source)

| Influencing factors | Correlation coefficient |
|---------------------|-------------------------|
| Bus revenue         | −0.179                  |
| Bus passenger flow  | 0.954                   |
| Operating cost      | 0.933                   |
| Passenger satisfaction | 0.863               |

Note: data calculated by EVIEWS software.
The increment, \( a_{ij} \), in the average annual passenger flow from 2012 to 2019 may be derived from the data listed in Table 5 as summarized in Table 6.

\[
\begin{align*}
a_{11} &= \frac{\sum_{t=2012}^{2019}((P_{1(t+1)} - P_{1t})/P_{1t})}{7}, \\
a_{21} &= \frac{\sum_{t=2012}^{2019}((P_{2(t+1)} - P_{2t})/P_{2t})}{7}, \\
a_{31} &= \frac{\sum_{t=2012}^{2019}((P_{3(t+1)} - P_{3t})/P_{3t})}{7}, \\
a_{12} &= \frac{\sum_{t=2012}^{2019}((Q_{1(t+1)} - Q_{1t})/Q_{1t})}{7}, \\
a_{22} &= \frac{\sum_{t=2012}^{2019}((Q_{2(t+1)} - Q_{2t})/Q_{2t})}{7}, \\
a_{32} &= \frac{\sum_{t=2012}^{2019}((Q_{3(t+1)} - Q_{3t})/Q_{3t})}{7}, \\
a_{13} &= \frac{\sum_{t=2012}^{2019}((W_{1(t+1)} - W_{1t})/W_{1t})}{7}, \\
a_{23} &= \frac{\sum_{t=2012}^{2019}((W_{2(t+1)} - W_{2t})/W_{2t})}{7}, \\
a_{33} &= \frac{\sum_{t=2012}^{2019}((W_{3(t+1)} - W_{3t})/W_{3t})}{7}.
\end{align*}
\]

The optimal financial subsidy model based on passenger flow can be obtained by substituting the 2019 data into equation (1). Model \( P \) is as follows:

\[
P: \quad \max Z = 1.11x_1 + 1.01x_2 + 1.79x_3,
\]

subject to

\[
\begin{align*}
0.3x_1 + 0.05x_2 + 0.19x_3 &\leq 1.54, \\
0.35x_1 + 0.06x_2 + 0.35x_3 &\leq 2.11, \\
0.02x_1 + 0.16x_2 + 0.18x_3 &\leq 1.29, \\
x_1, x_2, x_3 &\geq 0.
\end{align*}
\]

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a_{31} &= \frac{\sum_{t=2012}^{2019}((P_{3(t+1)} - P_{3t})/P_{3t})}{7}, \\
a_{12} &= \frac{\sum_{t=2012}^{2019}((Q_{1(t+1)} - Q_{1t})/Q_{1t})}{7}, \\
a_{22} &= \frac{\sum_{t=2012}^{2019}((Q_{2(t+1)} - Q_{2t})/Q_{2t})}{7}, \\
a_{32} &= \frac{\sum_{t=2012}^{2019}((Q_{3(t+1)} - Q_{3t})/Q_{3t})}{7}, \\
a_{13} &= \frac{\sum_{t=2012}^{2019}((W_{1(t+1)} - W_{1t})/W_{1t})}{7}, \\
a_{23} &= \frac{\sum_{t=2012}^{2019}((W_{2(t+1)} - W_{2t})/W_{2t})}{7}, \\
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0.02x_1 + 0.16x_2 + 0.18x_3 &\leq 1.29, \\
x_1, x_2, x_3 &\geq 0.
\end{align*}
\]
transport services. According to the data summarized in Table 3, the unit cost of city bus operation in 2019 is approximately 2.86 yuan/person. The county bus operation cost is approximately 2.35 yuan/person, and the bus operation cost is approximately 1.76 yuan/person, which is close to the shadow price of the bus operation subsidy unit price and delivers the expected results. In general, the calculation results of the proposed model conform with the unit price of conventional transportation subsidies; hence, the model formulated in this study has considerable potential.

In view of the current status of research and existing problems in the city, three improvement measures are proposed:

1. Establish a scientific subsidy mechanism. The government’s financial subsidies to the city’s three bus companies have been increasing year by year. However, the state of bus operation has not improved, and the government’s financial burden continues to increase. Hence, a dual linear subsidy model based on passenger flow can be considered to achieve the optimal allocation of fiscal subsidies, improve the efficiency of subsidies, and reduce the financial burden on the government.

2. Improve the operating environment of subsidies by allotting financial subsidies for professional supervision and identify where the subsidies have been devoted. Moreover, to promote the sound development of the public transport system, the relationship between buses and taxis and other means of transportation should be strengthened rather than substituting one for the other.

3. Improve the quality of bus service. According to a survey and a study of the city’s satisfaction over the years, the average satisfaction score for 18 evaluation indicators is approximately 82.7 points. Although the overall score is high, the score of some indicators is low. For example, although the station kiosks are aesthetically pleasing and provide protection from the sun and rain, the length of waiting time at bus stations and the degree of crowding have low scores. Based on the results of a passenger satisfaction survey, passenger satisfaction may be linked to the subsidy amount, which must be used mainly for transforming the bus station kiosk, increasing the bus frequency, and reducing the waiting time and passenger crowding.

4. Summary

In this study, three bus companies in a city were selected as research objects. The subsidy calculation was implemented using a linear dual model, and the optimal unit price of shadow subsidy was derived. Compared with the fiscal subsidy amount in 2019 of the city considered in the study, the shadow unit price calculated via the model approximates...
the actual cost. The model affords the following advantages: it can make modifications according to the traffic size, adjust unit price subsidies, calculate subsidies that can achieve the optimal effect, and promote the healthy development of public transport. Using the model, policymakers can make adjustments according to the state of development of the city, economy, and population. The suitable subsidies and subsidies for unit price calculation can be identified, problem of asymmetric information can be resolved, and government and bus enterprises can cooperate in terms of subsidy implementation; thus, subsidy resources can be optimally allocated.

Data Availability

All data included in this study are available upon request by contacting the corresponding author.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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