Have Superkamiokande Really Measured the Direction of the Atmospheric Neutrinos which Produce Fully Contained Events and Partially Contained Events?

E. Konishi 1, Y. Minorikawa 2, V.I. Galkin 3, M. Ishiwata 4 and A. Misaki 5, 6

1 Department of Electronics and Information System Engineering, Hirosaki University, 036-8561, Hirosaki, Japan
2 Department of Science, School of Science and Engineering, Kinki University, Higashi-Osaka, 577-8502 Japan
3 Department of Physics, Moscow State University, 119992, Moscow, Russia
4 Department of Physics, Saitama University, 338-8570, Saitama, Japan
5 Advanced Research Institute for Science and Engineering, Waseda University, 169-0092, Tokyo, Japan
6 Innovative Research Organization for the New Century, Saitama University, 338-8570, Saitama, Japan

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Quasi Elastic Scattering (QEL) is the dominant source for producing both Fully Contained Events and Partially Contained Events in the Superkamiokande (SK) detector for the atmospheric neutrinos, in the range \(\sim 0.1\) GeV to \(\sim 10\) GeV. In the analysis of SK events, it is assumed that the direction of the incident neutrino is the same as that of the detected charged lepton. In the present letter, we derive the distribution function for the scattering angle of the charged leptons, their averaged scattering angle and their standard deviation due to QEL. Then, it is shown that the SK assumption for the scattering angle of the charged leptons in the QEL is not valid. Further, we examine the influence of the azimuthal angle of the charged leptons over their zenith angle. As the result, we conclude that the zenith angle distribution of the neutrino under the SK assumption does not reflect the real zenith angle distribution of the atmospheric neutrino which produces Fully Contained Events and Partially Contained Events. This result has clear implication for attempts to detect neutrino oscillations from the analyses of Fully Contained Events and Partially Contained Events in Superkamiokande.

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I. INTRODUCTION

In the experiment in which they try to detect the neutrino oscillation, by using the size of the Earth and measuring the zenith angle distribution of the atmospheric neutrino events, such as, Superkamiokande experiment[1] – hereafter, simply SK –, it is demanded that the measurements of the direction of the incident neutrino are being carried out as reliably as possible. Among the experiments concerned on the neutrino oscillation, the analysis of Fully Contained Events in SK is regarded as mostly ambiguity-free one, because the essential information to extract clear conclusion is stored inside the detector. In SK, they assume that the direction of the neutrino concerned is the same as that of the produced charged lepton (hereafter, simply SK assumption)[2,3]. However, the SK assumption does not hold in the just energies concerned for neutrino events produced inside the detector, which is shown later.

In the energy region where Fully Contained Events and Partially Contained Events (single ring events) are analysed, quasi elastic scattering of neutrino interaction (QEL) is the dominant source for the atmospheric neutrino concerned[4]

II. THE DIFFERENTIAL CROSS SECTION FOR QEL AND THE SCATTERING ANGLE OF THE CHARGED LEPTON

The differential cross section for QEL is given as follows [5].

\[
\frac{d\sigma}{dQ^2} = \frac{G_F^2 \cos^2 \theta_C}{8\pi E_\nu^2} \left\{ A(Q^2) \pm B(Q^2) \left[ \frac{s - u}{M^2} \right] + C(Q^2) \left[ \frac{s - u}{M^2} \right]^2 \right\}.
\]  

(1)

The signs + and - refer to \(\nu_{\mu(e)}\) and \(\bar{\nu}_{\mu(e)}\) for charged
randomly sample $Q$ angle from Eq. (1). Subsequently we obtain the scattering interested in, say standard deviations and consequently we cannot neglect the incident neutrino, being accompanied by rather large scattering angles largely deviate from the direction of clearly from the figure and the table that the average deviations, too. We give them in Table 1. It is shown obtain the average scattering angles and their standard deviations, too. We give them in Table 1. It is shown angles of the charged particles in QEL play a decisive role in the determination of their zenith angles as well as the translation from Fully Contained Events to Partially Contained Events (vice versa) which are mentioned later.

In order to examine the influence of the azimuthal angle of the charged leptons over their zenith angle, let us denote the direction cosines of the incident neutrino ($\ell, m, n$) and denote the scattering angle of the charged lepton, $\theta_s$, and the azimuthal angle, $\phi$, with regard to the axis of the incident neutrino. Then, $(\ell_r, m_r, n_r)$, the direction cosines of the charged lepton which correspond to $(\ell, m, n)$ are given as

$$
\begin{pmatrix}
\ell_r \\
m_r \\
n_r
\end{pmatrix} = \begin{pmatrix}
\ell m - m n \\
\sqrt{\ell^2 + m^2} \\
\sqrt{\ell^2 + m^2}
\end{pmatrix} \begin{pmatrix}
\ell
\\m
\\n
\end{pmatrix} - \begin{pmatrix}
\ell
\\m
\\n
\end{pmatrix} \begin{pmatrix}
\ell m
\\\sqrt{\ell^2 + m^2}
\\\sqrt{\ell^2 + m^2}
\end{pmatrix} \begin{pmatrix}
\ell
\\m
\\n
\end{pmatrix}

\begin{pmatrix}
\sin \theta_s \cos \phi \\
\sin \theta_s \sin \phi \\
\cos \theta_s
\end{pmatrix},
$$

while SK assume

$$(\ell_r, m_r, n_r) = (\ell, m, n)$$

IV. MONTE CARLO PROCEDURE, TAKING ACCOUNT OF THE EFFECT OF THE AZIMUTHAL ANGLE AND DISCUSSIONS

By using Eq. (4), we carry out a Monte Carlo calculation to examine the influence of the azimuthal angle of the charged leptons over their zenith angle. The scatter plots between $\cos \theta$, cosines of the zenith angles of the charged leptons and fractional energies $E_\mu/E_\nu$ of the charged leptons for different directions of the incident neutrinos are given in Figs. 2 to 4. For a given $Q^2$ in

TABLE I: The average values $< \theta_s >$ for scattering angle of the emitted charged leptons and their standard deviations $\sigma_s$ for various primary neutrino energies $E_\nu$

| $E_\nu$ (Gev) | angle (degree) | $\nu_\mu$ | $\nu_\mu$ | $\nu_e$ | $\nu_e$ |
|---------------|----------------|---------|---------|--------|--------|
| 0.2           | $< \theta_s >$ | 89.86   | 67.29   | 89.74  | 67.47  |
|               | $\sigma_s$     | 38.63   | 36.39   | 38.65  | 36.45  |
| 0.5           | $< \theta_s >$ | 72.17   | 50.71   | 72.12  | 50.78  |
|               | $\sigma_s$     | 37.08   | 32.79   | 37.08  | 32.82  |
| 1             | $< \theta_s >$ | 48.44   | 36.00   | 48.42  | 36.01  |
|               | $\sigma_s$     | 32.07   | 27.05   | 32.06  | 27.05  |
| 2             | $< \theta_s >$ | 25.84   | 20.20   | 25.84  | 20.20  |
|               | $\sigma_s$     | 21.40   | 17.04   | 21.40  | 17.04  |
| 5             | $< \theta_s >$ | 8.84    | 7.87    | 8.84   | 7.87   |
|               | $\sigma_s$     | 8.01    | 7.33    | 8.01   | 7.33   |
| 10            | $< \theta_s >$ | 4.14    | 3.82    | 4.14   | 3.82   |
|               | $\sigma_s$     | 3.71    | 3.22    | 3.71   | 3.22   |
| 100           | $< \theta_s >$ | 0.38    | 0.39    | 0.38   | 0.39   |
|               | $\sigma_s$     | 0.23    | 0.24    | 0.23   | 0.24   |

FIG. 1: Distribution function for the scattering angle $\theta_s$ of the muon for muon-neutrino.

III. THE INFLUENCE OF THE AZIMUTHAL ANGLE OF THE CHARGED LEPTON OVER THEIR ZENITH ANGLE

In addition to the scattering angle of the charged leptons, it should be emphasized that the azimuthal current(c.c.) interaction, respectively. The $Q^2$ denotes four momentum transfer between the incident neutrino and the charged lepton. As for details of other symbols, see the text [5]. The relation among $Q^2$ and $E_\nu$, the incident energy of neutrino, $E_\ell$, the energy of the emitted charged lepton ((anti)muon or (anti)electron) and $\theta_s$, the scattering angle of the charged lepton, is given as

$$Q^2 = 2E_\nu E_\ell (1 - \cos \theta_s). \quad (2)$$

Also, the energy of the charged lepton is given by

$$E_\ell = E_\nu - \frac{Q^2}{2M}. \quad (3)$$

For a given energy $E_\nu$ of the incident neutrino, we randomly sample $Q^2$ through the Monte Carlo procedure from Eq. (1). Subsequently we obtain the scattering angle $\theta_s$ of the charged lepton concerned by Eqs. (2) and (3). Thus, we obtain the distribution functions for scattering angle of the charged lepton. In Fig. 1, we give such distribution functions for different incident neutrino energies. Through such a Monte Carlo procedure, we obtain the average scattering angles and their standard deviations, too. We give them in Table 1. It is shown clearly from the figure and the table that the average scattering angles largely deviate from the direction of the incident neutrino, being accompanied by rather large standard deviations and consequently we cannot neglect the scattering angle in the energy region where SK was interested in, say $\sim 0.1$ GeV to $\sim 10$ GeV.

The average values $< \theta_s >$ for scattering angle of the emitted charged leptons and their standard deviations $\sigma_s$ for various primary neutrino energies $E_\nu$.
FIG. 2: The scatter plot between the fractional energies $E_\mu/E_\nu$ of the emitted muons and the cosine of zenith angles, $\cos \theta$, for muon neutrinos with 1 GeV. The incident direction of the neutrinos is vertical. The sampling number is 1000.

FIG. 3: The scatter plot between $E_\mu/E_\nu$ and $\cos \theta$. The incident direction of the neutrinos is horizontal. The other quantities are the same as in Fig. 2.

FIG. 4: The scatter plot between $E_\mu/E_\nu$ and $\cos \theta$. The incident direction of the neutrinos is diagonal. The other quantities are the same as in Fig. 2.

Eq. (1), the energy $E_\ell$ of the charged lepton and its scattering angle $\theta_\ell$ is uniquely determined due to the two body kinematics. In Fig. 2, we give the case of vertically incident neutrinos ($\theta_\nu = 0^\circ$). Here, as the zenith angles of the charged leptons are measured from the vertical direction in the SK case, the azimuthal angles of the charged leptons never influence over their zenith angle, and consequently the relation between their fractional energies and their zenith angles is uniquely determined as mentioned above. In Fig. 3, we give the case of horizontally incident neutrinos ($\theta_\nu = 90^\circ$). Here, the azimuthal angle of the charged leptons has a potent influence on their zenith angle through the operation of Eq. (4). As is seen clearly from the figure, the $\cos \theta$ is widely distributed even to the backward for the same energy of the charged lepton. In Fig. 4, we give the intermediate case of the diagonal incidence ($\theta_\nu = 43^\circ$).

To connect our results with the analysis of the real experimental data, we finally need to take account of the energy spectrum of the incident neutrino in our calculation. For this purpose, we adopt the neutrino energy spectrum at Kamioka site obtained by Fiorentini et al.[6] and have carried out the following Monte Carlo procedure for a given $\cos \theta_\nu$ of the incident neutrino.

Procedure A: we randomly sample the energy of the incident neutrino from the probability function which is composed of the combination of the neutrino energy spectrum by Fiorentini et al., which covers from 0.1 GeV to 100 GeV at Kamioka site, with the corresponding total cross section for QEL.

Procedure B: we decide whether the neutrino concerned should be attributed to the particle or the anti-particle by the random sampling from the corresponding probability functions given in Procedure A.

Procedure C: we randomly sample $Q^2$ of the neutrino concerned for a given energy of the neutrino from Eq. (1).

Procedure D: we decide the energy of the charged lepton, $E_\ell$, from Eq. (3) and its scattering angle, $\theta_\ell$, from Eq. (2).

Procedure E: we randomly sample the azimuthal angle, $\phi$, by using a random number between (0,1).

Procedure F: we decide the direction cosines of the charged leptons from Eq. (4), being accompanied by the scattering angle and the azimuthal angle obtained from Procedures D and E.

We repeat a chain of the procedures A to F until we attain at the required trial number (ten thousands samplings per each case).

In Figs. 5 to 7 for the three cases of $\theta_\nu$ ($\theta_\nu = 0^\circ$, 90$^\circ$ and 43$^\circ$), we give the zenith angle distribution of the sum of $\mu^+$ and $\mu^-$ for a given $\theta_\nu$, obtained by Procedures A to F. If the SK assumption is valid, even if it may be of approximation, the zenith angle distribution of the charged lepton should be of the delta function type with a peak around the direction of the incident neutrino. However, it is clear from the figures that the real zenith angle distributions of the charged leptons deviate far from the delta function type distribution, and further the back-
ward scattering is too serious to neglect, which means rather large mixture of down going muon events into upward going events. It is also noticed that the azimuthal angles critically influence the translation from Fully Contained Events to Partially Contained Events (vice versa) which is strongly dependent on their generation point inside the detector.

In Figs. 8 to 10, we give similar results for $e^-$ and $e^+$ for $\nu_e$ and $\bar{\nu}_e$, taking account of the overall neutrino spectrum at Kamioka site. The direction of the incident neutrino is horizontal. The other quantities are the same as in Fig. 8.

The charcteristics of the Figs. 8 to 10 for (anti-)electron is essentially the same as in Figs. 5 to 7.
(anti-)muon.

V. CONCLUSION

From these figures 5 to 10, it is surely concluded that the zenith angle distributions of the charged leptons under the SK assumption does not reflect the real zenith angle distribution of the incident neutrinos concerned. Of course, it is common sense that the direction of individual charged lepton cannot determine that of the neutrino concerned uniquely, but only statistically. However, our present results clearly shows that the direction of the charged leptons under the SK assumption cannot determine those of the incident neutrino, even if they accumulate experimental data statistically enough. Thus, it is surely concluded that we could not extract any definite conclusion around the neutrino oscillation from the measurement on the charged leptons under SK assumption which occur inside the detector and is regarded as the most reliable one from the point of the experimental accuracy.

The relation between the zenith angle distributions of the charged leptons which are really measured experimentally and those of the parent neutrinos, taking account of the scattering angle and the azimuthal angle of the charged leptons correctly, will be shown in a subsequent paper.

[1] Y.Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998).
Y.Fukuda et al, Phys. Rev. Lett. 82, 2642 (1999).
[2] T.Kajita and Y.Totsuka, Rev. Mod. Phys. 73, 85 (2001).
See p. 101.
[3] M.Ishitsuka, Ph.D thesis, University of Tokyo (2004). See p. 138.
[4] P.Lipari,M.Lusignoli and F.Sartogo, Phys. Rev. Lett. 74, 4384 (1995).
[5] P.Renton, Electro Weak Interaction (Cambdrige University Press,1990). See p. 405.
[6] G.Fiorentini,V.A.Naumov and F.L.Vilante, Phys . Lett, B510, 173 (2001).