Acoustic orbital angular momentum transfer to matter by chiral scattering

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Abstract

We report on orbital angular momentum exchange between sound and matter mediated by a non-dissipative chiral scattering process. An experimental demonstration is made possible by irradiating a three-dimensional printed, spiral-shaped chiral object with an incident ultrasonic beam carrying zero orbital angular momentum. Chiral refraction is shown to impart a nonzero orbital angular momentum to the scattered field and to rotate the object. This result constitutes a proof of concept of a novel kind of acoustic angular manipulation of matter.

The mechanical effects of waves on material media can achieve contactless manipulation in a controlled manner, and the technique has already found numerous applications in optics and acoustics. Wave-matter exchange of linear and/or angular momentum that results from radiation force and/or torque can be classified in two categories, dissipative and non-dissipative. To date, the demonstration of all kinds of exchanges mediated by dissipative processes have been carried out experimentally. In optics, this includes the transfer of both spin and/or orbital angular momentum (OAM) [1–3]. Notably in acoustics, the angular manipulation of objects resulting from the absorption of (pseudo [4]) OAM was demonstrated only recently. This included both angular deviation [5, 6] and spinning [7–9] of sound-absorbing systems.

Non-dissipative exchanges of linear and angular momentum between waves and matter are also possible. In optics, this was experimentally realized in 1936 for spin angular momentum [10] and the OAM case was explicitly identified much later [11]. Today, the field of optical angular manipulation is mature; see [12] for a recent review. Intriguingly however, the demonstration of exchange of angular momentum between sound and matter that is not mediated by absorption has not yet been reported. Here, we aim to fill this gap by observing the direct signature of the phenomenon experimentally, both for the material medium and the acoustic field.

Our demonstration relies on the use of a chiral object having the form of a spiral phase plate (SPP). This is the acoustic analog of an optical SPP [13] but for sound waves. It formally consists of a plate having an azimuthally varying thickness $\varepsilon = \ell \phi / \phi$ with $\ell$ an integer and $\phi$ the azimuthal angle in the transverse plane with respect to the beam propagation axis,

$$\varepsilon = \frac{1}{f(\ell \varepsilon - c_1^{-1} - c_2^{-1})},$$

with $f$ the frequency of the sound wave, $c_1$ the sound speed in the medium surrounding the SPP and $c_2$ the sound speed in the material that constitutes the SPP. A normally incident axisymmetric acoustic wave thus emerges from the SPP with a pressure field having an azimuthal dependence of the form $\exp(\imath \ell \phi)$, i.e. an acoustic vortex with topological charge $\ell$. Such a field carries nonzero acoustic OAM [4, 14] leading to the appearance of a nonzero acoustic radiation torque exerted on the SPP due to angular momentum conservation [15]. This is the acoustic analog of optical OAM carried by light beams endowed with phase singularities [16], which is a general feature of wave physics. The mechanical action of the acoustic torque is here demonstrated from the direct...
observation of SPP rotation around the beam propagation axis, whereas OAM generation is identified from pressure field measurements.

We designed a SPP operating at frequency \( f = \omega / 2\pi = 2.55 \) MHz made of polyactic acid (PLA) thermoplastic (sound speed \( c_2 = 2.2 \) km s\(^{-1}\)) and produced using an Ultimaker three-dimensional (3D) printer with 200\(\mu\)m voxel size. The SPP consists of two subparts: a cylindrical pedestal with diameter \( d_p = 20 \) mm and height \( e_p = 0.8 \) mm; and a spiral-shaped part composed of \( \ell \) identical portions of spirals with diameter \( d_s = 12 \) mm, angular width \( 2\pi / \ell \) and height \( e_s = \varepsilon_s \); see figure 1(a) for \( \ell = 4 \). This ensures a \( 2\pi \) phase delay ramp along each angular sector, hence a \( 2\pi \varepsilon \) circulation of the phase delay around the SPP axis. The SPP is placed at the interface between a lower layer of saturated saltwater and an upper layer of mineral oil (sound speed \( c_1 = 1.46 \) km s\(^{-1}\)) as depicted in figure 1(b). The SPP is therefore sustained by buoyancy and capillarity, and is either free to rotate (for rotation experiments) or fixed using a holder (for pressure scans). With present parameters, \( \varepsilon = 1.7 \) mm, a picture of the \( \ell = 4 \) SPP is shown in figure 1(b). In practice, the pedestal ensures a reproducible pinning of the triple contact line along its perimeter and enhances the mechanical stability of the SPP. In addition, an axial hole of diameter \( d_h = 0.8 \) mm is drilled through the SPP in order to maintain its axis along the beam axis owing to a needle.

The SPP is irradiated from below by an acoustic beam produced by a spherical, piezoelectric, mono-element, high-power transducer with radius of curvature \( F = 38 \) mm, diameter \( D = F \), central frequency 2.55 MHz and bandwidth 600 kHz. The ultrasonic transducer is immersed in the lower aqueous layer, as sketched in figure 1(c), and is fed with sinusoidal wave trains with voltage amplitude \( U \) at carrier frequency \( f \), with \( N \) cycles duration at 1 kHz repetition rate \( (N = 10 \) for pressure scans\). Optimization of matter-field interaction generation requires both the incident acoustic power propagating through the hole and that outside the SPP cross-section to be minimized. In practice, the SPP is placed at a distance \( F - H \) from the transducer, with \( H = 10 \) mm as a trade-off value for which 90% of the incident acoustic power is intercepted by the SPP.

The first experiment is dedicated to the demonstration of SPP rotation using the setup sketched in figure 1(c). We choose a freely rotating SPP with \( \ell = 4 \) and \( N = 510 \) for the number of cycles of the wave trains emitted by the transducer at a 1 kHz repetition rate. This corresponds to a 20% duty cycle and we may thus refer to a quasi-continuous acoustic irradiation. At steady state, the SPP rotates at constant angular velocity \( \Omega \) in the clockwise direction (when looking at the SPP from the top), namely \( \Omega = \Omega_z \) with \( \Omega < 0 \). In addition, \( \Omega \) is found to be proportional to \( U^2 \), hence proportional the incident acoustic power \( P \), as shown in figure 2. Unfortunately, because the manufacturer calibrated the transducer in pure water whereas the transducer is immersed in saturated brine in the present experiment—and the emission efficiency of our high-power transducer notably depends on the acoustic impedance of the liquid in contact with its emitting surface—we were unable to determine the acoustic power of the incident beam quantitatively. In addition, the viscous torque exerted on the rotating SPP could not be quantitatively determined theoretically due to the complex geometry of the SPP.

We have also established that the effect is due to the shape of the SPP, by checking that a disk (i.e. an SPP with \( \ell = 0 \)) does not rotate. Moreover, the observed direction of rotation and the linearity of the SPP angular velocity with incident power are both consistent with the balance of angular momentum between the SPP and the

\[5\text{ Sound propagation properties in PLA at frequency have been determined from transmission measurement through a 3D-printed slab using the method described in [22].} \]
emerging pressure field proportional to \( \exp[i(-\omega t + kz + \ell \phi)] \). Indeed, the output field carries an OAM whose projection along the \( z \)-axis has the sign of \( \ell \) and its magnitude is proportional to \( \ell |P| \), where \( P \) is the beam power \([4, 17]\). Consequently, from angular momentum conservation, the SPP experiences a recoil torque with a sign equal to that of \(-\ell\) and a magnitude proportional to \( \ell |P| \).

On the other hand, the basic effect of the SPP on the incident acoustic field can be grasped assuming it behaves as a mask with complex transmittance \( f(t) \). Considering a Gaussian incident beam \([6]\) under the focusing conditions used in our experiments (see figure 1), the incident pressure field in the plane of the SPP \( z = z_{in} = -h \), with \( h \) being the distance between the SPP and the beam focus, is thus written in the complex form as \( p_{in}(r, z_{in}) = p_0 \exp\{-r^2/[w_0^2(1 + z_{in}^2/z_0^2)] + ikr^2/[2z_{in}(1 + z_{in}^2/z_0^2)]\} - \i \arctan(z_{in}/z_0) \}, \) with \( w_0 \) the beam waist radius, and where the unimportant phase factor \( \exp[-i(\omega t - k z_{in})] \) is omitted. Neglecting the reflected field at the brine-SPP and SPP-oil interfaces for the time being (the effect of reflection due to acoustic impedance mismatch will be detailed in the last part of the paper), within the paraxial approximation the transmitted pressure at an altitude \( z > z_{in} \) from the SPP is

\[
p_{out}(\tilde{r}, \phi, \tilde{z}) = \frac{z_0}{i\pi(\tilde{z} + h)} \int_0^{2\pi} \int_0^\infty t(\tilde{\rho}, \theta) p_{in}(\tilde{\rho}, \tilde{z}) \, \tilde{\rho} \, d\tilde{\rho} \, d\theta,
\]

where \( z_0 = kw_0^2/2 \) with \( k \) the wave vector, and where the reduced distances \( \tilde{X} = X/w_0, X = (r, \rho, z) \), are introduced.

In the ideal case of a pure phase mask, i.e. \( t = t_{ideal}(\phi) = \exp(i\ell \phi) \), the spatial distribution of the output pressure magnitude exhibits an axisymmetric, doughnut shape as a consequence of the on-axis phase singularity with topological charge \( \ell \). This is illustrated, for \( \ell = 4 \), in figures 3(a) and (b) that show the calculated magnitude and phase of \( p_{out} \) at altitude \( z = -h/2 \).

The second experiment consists of the direct observation of acoustic vortex generation at \( z = -h/2 \), which is performed using the same setup but holding the SPP. The transverse distribution of the maximum of the pressure magnitude of the transmitted wavetrain is acquired using a needle hydrophone with a 85 \( \mu \)m-diameter needle.
sensitive element with 250 μm spatial resolution. Experimental data are displayed in figure 3(c) and the observed non-axisymmetry contrasts with the expected axisymmetric pattern shown in figure 3(a). In fact, this observation can be explained by accounting for the actual features of the experiment: PLA attenuates sound, the SPP has a hole at its center and the SPP is placed in a holder that completely absorbs sound for \( r > e_s \).

This leads to an azimuthally dependent pressure transmittance expressed as

\[
t(r < r_h, \phi) = 1,
\]

\[
t(r_h < r < r_s, \phi) = \tau_p \tau_s \exp(-\alpha e_s/2) \exp(i\ell_\phi),
\]

\[
t(r > r_s, \phi) = 0,
\]

where \( r_s = d_s/2 \) with \( x = (h, s, p) \), \( \alpha = 400 \text{ m}^{-1} \) is the pressure attenuation coefficient in PLA\(^7\), \( \tau_s = \exp(-\alpha e_s) \) is the pressure transmission factor associated to an SPP thickness \( e_s \) and \( \tau_p = \exp(-\alpha e_p) \) is the pressure transmission factor of the pedestal. The computed spatial distribution of the pressure magnitude, which is displayed in figure 3(d), qualitatively reproduces the experimentally observed square-shaped doughnut-like pressure magnitude pattern. Such non-axisymmetric spatial distribution of the pressure magnitude is associated with splitting of the on-axis phase singularity of topological charge \( \ell \) into \( \ell \) off-axis single charge phase singularities, see figure 3(e), which is a generic feature of the structural instability of high-order vortices \([18, 19]\).

Note that in our experiments the noise of the transmitted wavetrain phase due to unavoidable bulk and surface imperfections of the 3D printing process prevents direct measurement of the transverse phase distribution. This also prevents a quantitative study of the sound driven rotation of the SPP as a function of the topological charge. Still, the above results allow experimental ascertainment of the generation of acoustic vortex generation by the SPP as a result of chiral scattering of sound.

We next review possible mechanical consequences of steady flows induced by sound absorption on SPP rotation. First, let us consider the flow due to the absorption of the incident beam below the SPP \( (z < -h) \), which may contribute to the total torque exerted on the SPP if its bottom facet is not perfectly flat or not coaxial with the incident beam. Since an irradiated disk (i.e. \( \ell = 0 \)) does not rotate, acoustic linear streaming is not at play in our experiment. Second, let us consider acoustic rotational streaming \([9]\), which consists of steady fluid rotation due to absorption of OAM. In our experiment, this occurs above the SPP \( (z > -h) \) as a result of absorption of the generated acoustic vortex. Angular momentum conservation implies that the angular momentum acquired by the SPP and the angular momentum carried by the generated acoustic vortex are opposite. Hence, viscous drag exerted by rotational acoustic streaming on the SPP results in a torque opposite to the acoustic radiation torque induced by OAM transfer. Acoustic rotational streaming is therefore also not at play in our experiment.

\(^7\) See footnote 5.
Until now, the generation of torque by chiral scattering has been described using angular momentum conservation. Interestingly, we note that the total torque exerted on the SPP can also be described from the basics of continuum mechanics, which allows us to quantitatively account for: the generation at the SPP surface of a torque, the acoustic radiation stress exerted by the incident field on the SPP, and the acoustic radiation stress exerted by the transmitted field on the SPP surface. According to the acoustic radiation stress exerted by the incident field on the SPP is calculated by integrating the torque density $\tau$ over its surface, where $\mathbf{f}$ refers to the orthonormal basis of the cylindrical coordinate system $(r, \phi, z)$ (figure 4(a)), namely $\Gamma = \int_0^{2\pi} \int_0^R \gamma r dr d\phi$. Taking into account the experimental parameters $\rho_i = 850$ kg m$^{-3}$ and $\rho_2 = 1190$ kg m$^{-3}$ one finds $f(r, \phi) \cdot \mathbf{u}_z < 0$, and hence $\Gamma_z < 0$, which agrees with experimental observations.

Interestingly, this also agrees with the simple picture for matter-wave interaction drawn in the introduction, where angular momentum conservation is applied accounting only for the transmitted field. However, we stress that such an assertion does not always hold. Indeed, since in our case OAM of the reflected field is opposite to that of the transmitted field, we expect $\Gamma_z > 0$ when acoustic torque is dominated by the contribution of the reflected field rather than that of the transmitted field, which occurs for large enough impedance mismatch. This is grasped by evaluating the dependence versus $Z_i/Z_2$ of the ratio between the torque $\Gamma_z$ and its value in the case of matched impedances $\Gamma_z/\Gamma_z\text{match}$ for the value of $c_i/c_2$ that corresponds to our experiment. As shown in figure 4(b), $\Gamma_z/\Gamma_z\text{match} < 1$ and becomes negative when $Z_i/Z_2$ departs enough from unity.

To summarize, we have experimentally demonstrated the transfer of acoustic OAM to matter mediated by a chiral scattering process. This has been achieved using a 3D-printed SPP irradiated by a focused ultrasonic beam carrying zero OAM. Both the generation of an acoustic vortex and the ensuing radiation torque exerted on the object have been observed. This chirality-based exchange of OAM that is not mediated by absorption completes the acoustomechanical toolbox, with potential applications to tailormade pumping and mixing fluids at small scales. Since the proposed process does not involve heating of the manipulated object, it should promote the use of sound waves for contactless manipulation of fragile entities.

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