Ways to Improve Wear Resistance and Damping Properties of Radial Bearings Taking into Account Inertial Forces

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Abstract. This article considers a radial sliding bearing of infinite length whose moving part consists of the support, the porous layer, and the liquid lubricant. The analysis of the existing design calculation methods for such sliding bearings shows that they are very approximate because they do not consider the inertial forces applied, the electric conductivity of the lubricant, the permeability anisotropy, as well as the impacts of the electric field vector, magnetic induction vector, and incomplete filling of the working gap (pre-accident condition). The authors demonstrate how these factors impact the stable operation of the device facilitating the hydrodynamic regime. The authors find the asymptotic solution for the zero, first, and second approximation taking into account the inertial forces for the “thin layer”. By solving the produced equations using the Gauss-Seidel method, the authors determine the key operating parameters of the friction couple in question: the carrying capacity and the friction force. The authors provide an impact assessment for the parameters characterizing the permeability of the porous coating, the electric conductivity, and viscosity of the lubricant, as well as the length of the loaded area and the impacts of inertial forces on the carrying capacity and the friction force.

1. Introduction
The development of improved sliding bearing models with a porous coating of the pivot or the support ring was covered in many works [1-25]. The use of porous-body bearings to facilitate the hydrodynamic regime of lubrication goes ahead of the theoretical developments, which calls for the development of multi-parameter calculation models for sliding bearings with porous coatings. This, it is necessary to develop calculation models for porous bearings taking into account additional factors.

This article is an attempt to fill this gap and account for the anisotropy of porous coatings, the correlations between the viscosity of the electrically conductive lubricant and the permeability of porous coating and the pressure, and the impact of inertial forces and the incomplete filling of the working gap (pre-accident condition).

2. Statement of problem
We review the laminar flow of the electrically conductive lubricant. This allows for the working gap to be only partially filled with the lubricant (pre-accident condition). The porous-body shaft rotates at a speed of Ω. (Fig. 1)

Within the coordinate system \( r, \theta \), circuit equations can be expressed as follows:
Figure 1. The functional diagram for the radial bearing.

In the problem statement formulae, we express the correlation between the viscosity, electric conductivity, and permeability of the porous coating and the pressure as follows:

$$
\mu' = \mu_0 e^{i\eta'}, \quad \sigma' = \sigma_0 e^{i\eta'}, \quad k' = k_0 e^{i\eta'}.
$$

(2)

3. Reference conditions

To solve this problem, we use well-known dimensionless non-linear liquid flow equations for the “thin layer” accurate to $O(\delta N)$, the continuity equation, and Darcy’s equation describing the flow of the lubricant in a porous coating of the shaft neck with the respective boundary conditions:

$$
\frac{\partial p}{\partial r} = 0, \quad \frac{\partial^2 v}{\partial r^2} = \frac{dp}{d\theta} e^{-i\eta'} - A + Nv + e^{-i\eta} \Re \left( u \frac{\partial v}{\partial r} + v \frac{\partial u}{\partial \theta} \right),
$$

$$
\frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \quad \frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2} \frac{\partial P}{\partial r} + \frac{1}{r^3} \frac{\partial^2 P}{\partial \theta^2} = 0,
$$

$$
\left. u \right|_{r=0} = M \frac{\partial P}{\partial r} \left|_{r=0} \right., \quad \left. v \right|_{r=0} = 1, \quad p = P \left|_{r=0} \right., \quad \left. \frac{\partial P}{\partial r} \right|_{r=0} = 0, \quad p(0_1) = p(0_2) = 0,
$$

(3)

$$
A = \frac{\sigma_0 B_0 \delta^2 E'}{\mu_0' \delta^2}, \quad N = \frac{6B_0^2 \delta^2 \sigma_0}{\mu_0', \delta^2}, \quad \Re = \frac{\rho \Omega \delta^2}{\mu_0}, \quad M = \frac{k_0' \delta^3}{\delta}, \quad \eta = \frac{e}{\delta}.
$$

(4)
The transition to dimensionless values in the lubricant and porous layers can be performed using the following analytical expressions:

\[
\begin{align*}
v'_x = \Omega \delta u, & \quad v'_0 = \Omega \delta v, \quad r' = r_0 + \delta r, \quad \delta = r_1 - r_0, \quad p' = p^* p, \quad p^* = \frac{\mu e \Omega r_0^2}{\delta^2}, \\
\beta = \beta' p^*, & \quad \mu' = \mu_0 \mu, \quad \sigma' = \sigma_0 \sigma, \\
P' = p^* P, & \quad r' = \bar{H} r^*.
\end{align*}
\]

Values \(E'(r,0)\) and \(B'(r,0)\) are set in line with the Maxwell equation:

\[
div \bar{B} = 0, \quad rot \bar{E} = 0.
\]

In this case,

\[
E' = \text{const}; \quad B' = \frac{B_0}{r'}, \quad B_0 = \text{const}.
\]

4. Problem solution

If we solve this problem by taking into account the inertial forces, it is necessary to write down the conditions for the bearing surface. Express the boundary conditions as a relative eccentricity range:

\[
\begin{align*}
u(1 + \eta \cos \theta) &= u|_{r=-1} + \left. \frac{\partial u}{\partial r} \right|_{r=1} \eta \cos \theta + \left. \frac{\partial^2 u}{\partial r^2} \right|_{r=1} \eta^2 \cos^2 \theta + \ldots = 0, \\
v(1 + \eta \cos \theta) &= v|_{r=-1} + \left. \frac{\partial v}{\partial r} \right|_{r=1} \eta \cos \theta + \left. \frac{\partial^2 v}{\partial r^2} \right|_{r=1} \eta^2 \cos^2 \theta + \ldots = 0.
\end{align*}
\]

According to (8), we search for the solution to the system of equations (3) using the relative eccentricity:

\[
u = \sum_{k=0}^{\infty} u_k \eta^k, \quad v = \sum_{k=0}^{\infty} v_k \eta^k, \quad p = \sum_{k=0}^{\infty} p_k \eta^k, \quad P = \sum_{k=0}^{\infty} P_k \eta^k.
\]

Considering (9) and (8) in the equation system (3), we get the following algebraic expressions for the zero approximation:

\[
\begin{align*}
\frac{\partial p_0}{\partial r} &= 0, \quad \frac{\partial^2 v}{\partial r^2} = -A + N v, \quad u = 0, \quad p_0(\theta_1) = p_0(\theta_2) = 0; \\
v_0 &= 0 \text{ при } r = 1; \quad v_0 = -1 \text{ при } r = 0.
\end{align*}
\]

We solve equation (10) together with (11) and obtain the following values for the zero approximation velocity field:

\[
v_0 = -\left( N + A \right) \frac{r^2}{2} + \left( N + A \right) \frac{L}{2} + r - 1.
\]

For the first approximation:

\[
\begin{align*}
\frac{\partial p_1}{\partial r} &= 0, \quad \frac{\partial^2 v_1}{\partial r^2} = \frac{dp_1}{d\theta} \left( 1 - \beta p_0 + \frac{\beta^2 p_0^2}{2} \right) + \text{Re} \left( u_0 \frac{\partial v_0}{\partial r} + v_0 \frac{\partial v_1}{\partial \theta} \right).
\end{align*}
\]
The solution for the right part of the system (13)–(14) can be found as follows:

\[ v_1 = \frac{\partial v_0}{\partial r} \cos \theta, \quad u_1 = 0, \quad v_1 = 0 \quad \text{при} \quad r = 0, \quad \tilde{u}_1 = M \frac{\partial P_1}{\partial r} \frac{r^{\text{**} - \frac{\text{**} 0}{H}}}{r^{\text{**} - \frac{\text{**} 0}{H}}}, \]

\[ P_1 = P_1^{(\text{**} - \frac{\text{**} 0}{H})}, \quad \frac{\partial P_1}{\partial r} \frac{r^{\text{**} - \frac{\text{**} 0}{H}}}{r^{\text{**} - \frac{\text{**} 0}{H}}} = 0, \quad p_1(\theta_1) = p_1(\theta_2) = 0. \]

The solution for the right part of the system (13)–(14) can be found as follows:

\[ v_1 = R_1(r)\cos \theta + R_2(r)\sin \theta; \]

\[ u_1 = R_3(r)\cos \theta + R_4(r)\sin \theta; \]

\[ p_1 = D_1\sin \theta + D_2\cos \theta; \]

\[ P_1 = R_5(r^*)\cos \theta + R_6(r^*)\sin \theta. \]

To determine \( R_i (i = 1...6) \), we insert (15) in (13)–(14):

\[ \frac{d^2 R_1}{dr^2} = D_1 \left( 1 - \beta p_0 + \frac{\beta^2 R_0^2}{2} \right) + \text{Re} \left( R_0(r) \frac{\partial v_0}{\partial r} + R_1(r) v_0 \right), \]

\[ \frac{d^2 R_2}{dr^2} = D_2 \left( 1 - \beta p_0 + \frac{\beta^2 R_0^2}{2} \right) + \text{Re} \left( R_0(r) \frac{\partial v_0}{\partial r} + R_1(r) v_0 \right), \]

\[ \frac{dR_3}{dr} + R_3 = 0, \quad \frac{dR_4}{dr} - R_4 = 0, \]

\[ \frac{d^2 R_5}{dr^2} + \frac{1}{r} \frac{dR_5}{dr} + \frac{R_5}{r^2} = 0, \]

\[ \frac{d^2 R_6}{dr^2} + \frac{1}{r} \frac{dR_6}{dr} + \frac{R_6}{r^2} = 0. \]

\[ R_1(1) = -\frac{\partial v_0}{\partial r} \frac{r^{\text{**} - \frac{\text{**} 0}{H}}}{r^{\text{**} - \frac{\text{**} 0}{H}}}, \quad R_2(1) = 0, \]

\[ R_3(1) = 0, \quad R_4(1) = 0, \quad D_1 = R_6^{(\text{**} - \frac{\text{**} 0}{H})}; \]

\[ D_2 = R_4^{(\text{**} - \frac{\text{**} 0}{H})}, \quad R'_5 = 0 \quad \text{при} \quad r^* = \frac{r_0}{H} - 1; \]

\[ R'_6 = 0 \quad \text{при} \quad r^* = \frac{r_0}{H} - 1; \]

\[ R_5(0) = M R_5^{(\text{**} - \frac{\text{**} 0}{H})}, \quad R_4(0) = M R_4^{(\text{**} - \frac{\text{**} 0}{H})}. \]

For the second approximation:
\[ \frac{\partial v_2}{\partial r} = 0, \quad \frac{\partial^2 v_2}{\partial r^2} = \left( 1 - \beta p_0 + \frac{\beta^2 p_0^2}{2} \right) \frac{\partial v_2}{\partial r} + \beta p_1 \left( \beta^2 p_0^2 - 1 \right) \frac{\partial p_1}{\partial \theta} + \left( 1 - \beta p_0 + \frac{\beta^2 p_0^2}{2} \right) \text{Re} \left( u_2 \frac{\partial v_0}{\partial r} + u_1 \frac{\partial v_0}{\partial \theta} + v_0 \frac{\partial v_1}{\partial \theta} + v_1 \frac{\partial v_1}{\partial \theta} \right) + v_0 \frac{\partial v_1}{\partial \theta} + v_1 \frac{\partial v_1}{\partial \theta} + \beta p_1 \left( \beta^2 p_0^2 - 1 \right) + \text{Re} \left( u_0 \frac{\partial v_0}{\partial r} + v_0 \frac{\partial v_1}{\partial \theta} \right); \]

\[ \frac{\partial u_2}{\partial r} + \frac{\partial v_2}{\partial \theta} = 0, \quad \frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial p_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p_2}{\partial \theta^2} = 0; \]

(18)

\( v_2(0) = 0, \quad u_2 \bigg|_{r=0} = M \frac{\partial P_2}{\partial r} \bigg|_{r=0} \), \quad \( P_2 \big( \theta_1 \big) = P_2 \big( \theta_2 \big) = 0. \)

(19)

The solution for (18)–(19) can be found as:

\[ v_2 = R_7(r) + R_6 \cos \theta + R_6(r) \cos^2 \theta + R_{10}(r) \sin \theta + R_{11}(r) \sin^2 \theta; \]

\[ u_2 = R_{12}(r) \sin \theta + R_{13}(r) \sin^2 \theta + R_{14}(r) \cos \theta + R_{15}(r) \cos^2 \theta; \]

\[ P_2 \big( r^*, \theta \big) = R_{17}(r^*) \sin \theta + R_{18}(r^*) \sin^2 \theta + R_{20}(r^*) \cos \theta + R_{20}(r^*) \cos^2 \theta. \]

(20)

To determine \( R_i \ (i = 7...20) \), we insert (20) in (18)–(19):

\[ \frac{d^2 R_i}{dr^2} = 2B \left( 1 - \beta p_0 + \frac{\beta^2 p_0^2}{2} \right) \frac{\partial v_0}{\partial r} + \left( 1 - \beta p_0 + \frac{\beta^2 p_0^2}{2} \right) \frac{\partial v_0}{\partial \theta} + v_0 \frac{\partial v_1}{\partial \theta} + \frac{\partial v_1}{\partial \theta} + \beta \left( \beta^2 p_0^2 - 1 \right) \frac{\partial v_1}{\partial \theta}; \]

(21)
Replace arbitrary summands (16), (21) with a finite-difference formula and determine the system produced by a Gauss-Seidel equation.

5. Finding the carrying capacity and the friction force
Considering (12), (15), and (20), we get the following values for the carrying capacity and the friction force:

\[
R_\ast = p^* n_\ast \int_0^\theta \left( \eta p_1 + \eta^2 p_2 \right) \cos \theta \, d\theta;
\]

\[
R_\gamma = p^* n_\ast \int_0^\theta \left( \eta p_1 + \eta^2 p_2 \right) \sin \theta \, d\theta;
\]

\[
L_{op} = \frac{\mu \Omega r}{\delta} \left[ \frac{\partial \vec{v}_1}{\partial r} \bigg|_{r=0} + \eta \frac{\partial \vec{v}_1}{\partial r} \bigg|_{r=0} + \eta^2 \frac{\partial \vec{v}_2}{\partial r} \bigg|_{r=0} \right] \, d\theta.
\]

(23)

For the numerical analysis, we used the following values of factor variables:

\[
\tilde{H} = 0.0055 \text{ m}; \quad p_g = 0.2 \text{ MPa}; \quad r_i = 0.02 \div 0.35 \text{ m}; \quad \mu = 0.0608 \div 0.0078 \frac{\text{H} \cdot \text{c}}{\text{M}^2}.
\]

Based on the results of the numerical analysis, we constructed the graphs in Figure 2.
The maximum carrying capacity, considering the inertial forces, shifts towards the lower value of $\beta$ and equals $\approx 0.5$. Besides, the friction force increases more intensively in the interval in question due to the inertial forces.

![Figure 2.](image)

These factors show that inertial processes make a huge impact on the operation of the bearings in question.

It is important that accounting for the inertial forces when calculating the second component of carrying capacity $R_x$ also shows the maximum from parameter $\beta$, which confirms the inertia value for this component. Thus, considering the impact of inertia approaches the obtained theoretical results to reality.

During the experiment, we reviewed a radial sliding bearing with a porous coating (see the table).

| No. | Friction coefficient | Theoretical Studies | Experiment |
|-----|----------------------|---------------------|------------|
|     | Bearing without coating | Bearing with coating | Bearing with porous coating |
| 1   | 0.0044                | 0.0026              | 0.0028      |
| 2   | 0.0045                | 0.0025              | 0.0031      |
| 3   | 0.0048                | 0.0024              | 0.0034      |
| 4   | 0.0049                | 0.0023              | 0.0035      |
| 5   | 0.0052                | 0.0024              | 0.0037      |

We determined the value of the friction coefficient that helps assess the hydrodynamic friction regime for the bearing with a lubricant and the porous coating. We analyzed the temperature regime and the transition of the hydrodynamic friction regime to the boundary friction. The analysis of experimental data shows that the impact of porous coatings on the friction coefficient is 2.5-4 times greater than that of the lubricant used.
6. Conclusions
1. We obtained an improved calculation model for the radial sliding bearing with a porous shaft neck coating operating under hydrodynamic lubrication with an electrically conductive lubricant under an external electromagnetic field.

2. We demonstrated a significant impact of inertial forces on the values of the key tribotechnical parameters of the radial bearings in question.

3. We found that the carrying capacity increases significantly due to the growth of parameter $\beta$ characterizing the correlation between the viscosity and pressure, with the explicit maximum at $\beta=0.5$, as well as a significant correlation with parameters $A$ and $N$.

4. We proved that the friction force increases according to an almost linear law along with the growth of parameter $\beta$. When inertial forces increased, this growth became 2 times more intensive.

Symbols
$r_1$ – bearing radius; $r_0$ – shaft radius; $H$ – porous layer thickness; $H$ – lubricant thickness; $A$ – value determined by the presence of an electric field; $N$ – Hartmann number; $Re$ – Reynolds number; $\mu_0$ – typical viscosity; $\mu'$ – dynamic viscosity coefficient; $\sigma'$ – electric conductivity; $P'$ – hydrodynamic pressure in the lubricant layer; $v'_r, v'_o$ – speed vector components; $E$ = \{0,0,E'\} – electric field vector; $P'$ – pressure in the porous layer; $\rho$ – lubricant density; $B$ = \{0,B',0\} – magnetic induction vector.

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