I am not good in circle task: Exploration on student’s semi-relationalist mathematical concepts

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Abstract. This study aims to explore the student’s understanding of mathematical concepts semi-relationalist behaviours category. In the related literature, there are three categories of understanding mathematical concepts behaviour, one of the categories is semi-relationalist category. In this study explored the orientation of semi-relationalist category of understanding mathematical concepts behaviour. Data was obtained from one of the second-grade students of the junior high school as a participant in this study. From the results of the study, it was found that the student as categorized to behave as semi-relationalist in understanding mathematical concept behaviour. Seven indicators have been used as references in determining the understanding mathematical concept behaviour. Further, five written test questions were given to participants. The written test results are analyzed and supplemented with interviews. From the seven indicators, it was found that the student was categorized as having a semi-relationalist in the understanding mathematical concept behaviour. Based on the findings, the student was included in the participant who had the opportunity to improve their understanding of mathematical concepts behaviour.

1. Introduction
Mathematics is a widely known science and students are obligated to learn it from elementary, middle up to high levels. One of the goals of learning mathematics is understanding the concepts and their interrelations. Experts have researched, discovered, and continue to develop the goal achievement of learning mathematics. From various points of view, mathematics is developed to be easily learned and useful for the world community. One study in mathematics delves in how the wider community in the world can understand mathematics. Understanding mathematics is inseparable from understanding the concepts that exist in mathematics.

A concept is an abstract idea used to understand a fact by expressing similarities and differences derived from the fact [1]. There are three types of mathematical concepts, namely (1) basic mathematical concepts, such as writing ‘eight’ in different manners such as 8, VIII, eight; (2) the notation concept, such as writing ‘273’, the number two is hundreds, number 7 is tens, and number 3 is a unit, it is an example of the concept of place value; and (3) the application concept, which is an application of basic mathematical concepts and notation concepts, for example in solving mathematical problems such as area and volume [2].

Mathematics concept understanding is the integrity of the concepts that exist in mathematics with its operations and relationships. For example, in understanding the meaning of a number reduction, students...
build a concept by relating other concepts. The ability to understand important concepts, as Nickerson argued, that students who understand the concept can (1) look at the characteristics of the concept more deeply, (2) view information specifically in fast situations, and (3) represent a situation and see the situation using mental models [3]. Understanding of a concept can also be measured, as Skemp stated, measuring the conceptual understanding and categorized them in an instrumental and relational manner [4]. Hierbert argues that understanding is seen from actions, processes, and results of actions [5], and Duffin believes that people who grasp mathematic concept possess belief, feel comfortable, able to forget what was understood, capable of conveying to others and possess confidence [6]. Developing student understanding, as stated by Piere & Kieren, that concept understanding development occurs through dynamic and recursive processes and levels which include (1) primitive knowing, (2) image making, (3) image having, (4) property noticing, (5) formalizing, (6) observing, (7) structuring, and (8) inventing [7]. Based on the opinions of these experts, a student who understands a concept in mathematics exhibits the following indicators. First, students can represent a concept in a different perspective, students can translate concepts in verbal language into written language and vice versa, students can predict the tendency of a pattern in a concept, students can use the procedure skillfully, and students can associate a concept with another concept.

There have been many studies examining relevant concepts. Researchers have researched involving the categories, development, and character of conceptual understanding, but still, few who see understanding as learning outcome behaviour. Gunawan is one of the researchers who has discussed the behaviour of mathematical concept understanding of second-grade junior high school students. The results of his research state that there are three categories of the behaviour of mathematical concepts understanding namely relationalist behaviour, semi-relationalist behavior, and instrumentalist behaviour [8]. The context is only limited to categorizing the behaviour of understanding concepts. It is necessary specifically developed about those categories. Further research needs to be studied whether semi-relationalist categories can be developed or directed into relationalist categories or even can be instrumentalists, so this study wants to examine semi-relationalist behaviour in one junior high school student. This research will be useful for teachers. They can measure student from their behaviour and then improve student who is categorized as low behaviour.

2. Method
This research used a qualitative method. The researchers were the main instrument to reveal the condition of natural objects. Research approaches were case studies [9]. This study involve one respondent, a second-grade junior high school student in Bandung. The research used a mathematical concept which explored the circle concept. She was given five questions related to the circle as a stimulus to reveal the understanding of the circle concept. Number one is directing to see how to perceive problems and how to solve them. Number two is finding out how students translate problems faced both verbally and in writing. Number three is measuring the way predict patterns students. Number four is measuring how students in carrying out procedures, and number five is measuring the way students associate one concept with another concept in solving the problems they face.

After the student takes the test, we interviewed the subject. The interview is related to the utilization of previous knowledge, control (metacognitive thinking), beliefs, and attitudes — indicators to uncover these categories. Then formulated in a rubric of understanding mathematical concepts behaviour [8]. The behavioural rubric of understanding mathematical concepts that is a reference in this study is presented in Table 1. Meanwhile, to see the subject’s behaviour, we only use the rubric in the semi-relationalist behaviour category.
Table 1. The Rubric of Understanding Mathematical Concepts Behavioral

| Factor              | Indicator                      | Categories of Behaviour                      |
|---------------------|--------------------------------|----------------------------------------------|
| Knowledge ownership | Previous knowledge             | Instrumentalist: Not utilising prior knowledge | Semi-relationalist: There is an attempt to utilise prior knowledge | Relationalist: Make use of prior knowledge |
|                     | Represents concepts in different view | Instrumentalist: Cannot represent concepts from a different view | Semi-relationalist: There is an error in representing a concept from a different point of view | Relationalist: Can represent a concept correctly from a different point of view |
|                     | Translating concepts in the verbal language into writing and vice versa | Instrumentalist: Cannot translate a concept into verbal or written language | Semi-relationalist: There is an error in translating a concept into verbal or written language | Relationalist: Can translate a concept into verbal and written language |
|                     | Predicts trend patterns        | Instrumentalist: Cannot predict the trend of a particular pattern | Semi-relationalist: There is an error in predicting certain pattern trends | Relationalist: Can predict certain pattern trends |
|                     | Use the procedure skillfully   | Instrumentalist: Cannot use the procedure perfectly | Semi-relationalist: There is a mistake in using the procedure | Relationalist: Can use the procedure skillfully |
|                     | Linking one concept with another | Instrumentalist: Unable to associate a concept with another concept | Semi-relationalist: There is an error in associating a concept with another concept | Relationalist: Can link a concept with another concept |
| Control             | Metacognitive thinking         | Instrumentalist: Metacognitive thinking does not appear, either in written or verbal communication | Semi-relationalist: The metacognitive thinking appears verbally | Relationalist: Metacognitive thinking is evident in written and verbal responses |
| Belief              | Belief in the concept he understands | Instrumentalist: Unsure of his conceptual understanding | Semi-relationalist: Can solve confidently some of the problems related to the concept he understands | Relationalist: Sure can solve the problem related to the concept he understands |
| Affective           | Fast and precise in showing his understanding | Instrumentalist: Slow in showing their understanding through answers to questions | Semi-relationalist: Showing doubt in his or her understanding through answers to questions | Relationalist: Fast and precise in showing his understanding through answers to questions |

Furthermore, it will be seen in detail about the subject’s behaviour by referring to the behaviour rubric for the semi-relationalist category.

3. Results and Discussion

3.1. Subject behaviour regarding knowledge ownership aspects
The knowledge ownership aspect in this study is seen from several indicators, in which student is capable of (1) utilize prior knowledge, (2) represent concepts from different points of view, (3) translate concepts in verbal language into written language or vice versa, (4) predict pattern trends, (5) use procedures skillfully, and (6) link one concept to another. Furthermore, the findings from the subject will be elaborated. It is related to the knowledge ownership aspect and its indicators.

Regarding utilizing prior knowledge indicator, Subject attempted to use formulas in solving the problem. Nevertheless, there was an error in the process as exhibited in problem number three.
In Figure 1 (i), it appears that Subject was trying to write the pattern formula to look for the radius pattern of the tenth circle by using geometric sequence formula and get 1024 results, but she is still wrong in completing the final calculation of the problem-solving. The answer is influenced by previous knowledge, she cannot associate previous knowledge to solve the problem, so she cannot acquire her knowledge to solve her problem. Silver and Smith argue that the acquisition of knowledge and its utilization is influenced by the ability of students to link the problems they work with problems that have been encountered before by the mathematical structure [10].

Furthermore, for indicators representing concepts from different points of view, it can be seen in Subject's number one work. Subject has been able to represent the point of view of the problem from a different perspective; the results of Subject's work can be seen in Figure 1 (ii). Subject has the view that to find the area of the shaded area, we searched by subtracting the total area of the square by the area of 1 circle. He is different from the other views which look to find the area shaded by calculating the area of a small square reduced by the area of a quarter circle then multiplied by four.

Subject’s performance in the indicator of translating concepts from the verbal into written language or vice versa can be seen in the results of work for problem number two. From the results of the work, it appears that Subject has been able to translate concepts from written language into symbolic language, as illustrated in Figure 2 (i). Subject has been able to translate word problems in his language through the writing of mathematical symbols. He began writing the circumference of the rectangle divided by the number of distances between trees in diameter. The ability to translate verbal language is important in understanding concepts; it is in line with the results of Daniyanti's research which states that verbal ability contributes to achievement as outlined in writing in solving problems [11].
While the results of Subject's work on problem number three as an answer for indicators can predict pattern trends. Problem number three and Subject's answers are shown in Figure 1 (i). Subject has not been able to finish until the completion correctly even though the subject seems able to make patterns or predict patterns trends. Subject was able to predict to find the area she had to find the radius of the tenth pattern, and she had searched for it by making the first four patterns then she continued by using the geometry formula to look for the tenth pattern of the radius. But she is not perfect.

Indicator skillfully uses the procedure; it is shown from the results of Subject's work on problem number four. The subject did the problem number four imperfectly; the subject couldn't use the procedure perfectly. The results of Subject's work are shown in Figure 2 (ii). She did not write down the steps to look for the sector well. She did not write down how to search the detailed area in detail even though it was directed. She did not write down how to find the area of a triangle and mention what triangle is meant by the problem even further she did not write down how to find the height of a triangle.

While question number five is to see the ability in linking one concept to another. The subject made a mistake in working on problem number five. Subject wrote a broad concept to work on the concept of circles so that there was an error in linking one concept to another. She had written the formula around the rectangle. She also had written around the circle, but still could not answer the question of the problem. The results of Subject's work can be seen in Figure 3.

3.2. Subject's behaviour viewed from the control aspect

The second aspect is to look at the behaviour of understanding concepts, namely the control aspect. This aspect is measured by the indicator of whether or not metacognitive thinking is visible in students. In this study, it was determined that to see the appearance of metacognitive thinking by looking at the results of the work whether or not there are scribbles if there are no scribbles or there are, but the answer is correct, then it is said that the metacognitive is visible. And if there are no scribbles or there are, but the answer is still wrong, so metacognitive thinking is not visible. Subject's work on the problems number one and two can be done correctly without scribbles then in the interview results she could explain the answer so that verbally she can explain, while for problems number three, four, and five she worked without scribbles but the answers are still wrong and based on the results of the interview she also showed doubtful behaviour and could not explain correctly the answer to the problem given, so that verbally she could not explain the answer correctly, so she was more likely not to show metacognitive thinking.
Control aspects of the conceptual understanding behaviour are interpreted as appearing or not thinking metacognitive. Regarding metacognitive thinking [12, 13] argues that students think metacognitive degradation from high, medium and low, these levels can be directed to increase levels.

3.3. Subject's behavior viewed from the belief aspect
The last aspect is to look at the behavior of concept understanding, namely belief aspect. In this aspect, the indicator is that students can show confidence in the concepts they understand. Subject showed hesitant behavior in solving the problems she was facing. She can complete with confidence some of the concepts. The results of the work are shown in Figure 2 (ii). And the following is the interview with Subject for problem number four.

P : What was asked at number 4?
RL : Area of the shaded area
P : What area is the shaded area?
RL : Sector
P : Ok, what is the width of the sector?
RL : (looking at the problem for a long time), I don’t know… eh… angle AOB per 360 equals… eh..I forget..
P : The width of AOB what is it? The width of seg…
RL : Segment
P : Width of segment minus width of…
RL : Sector…eh…a quarter of circle… eh width of triangle
P : What triangle is this?
RL : Equilateral…isosceles (high intonation seems certain)
P : Isosceles or equilateral?
RL : Equilateral then..
P : Why equilateral?
RL : The sides are similar
P : How do you know?
RL : Right triangle, because there’s an elbow
P : Where is the elbow?
From the interview, Subject said the word "uh" more often, and was repeated over and over, which meant that she was not sure what she was answering, so she doubted whether the answer was correct or not. Also, she suspected the triangle was a right triangle, but when asked by the next question the answer changed. Thus Subject behaved uncertain of what she understood. The aspect of beliefs in line with research [14] which results that there is a significant relationship between the impact of beliefs and behaviour.

3.4. Subject's behaviour viewed from the affective aspect

Affective aspects in determining the understanding behaviour of mathematical concepts are characterized by indicators fast and precise in showing their understanding. For problems number one and two, Subject could answer questions correctly but slowly. Meanwhile, for problems, number three, four and five Subject answered the problems slowly and the answers are more likely wrong. The example can be seen in the interview for problem number four, she said more “uh ... just ...” and corrected the answers that had been given. Here are the results of Subject's interview for problem number 2.

P : Question number two, what is asked in problem number two?
RL : Number of trees planted in the border land
P : How can we find out the number of trees planted at the border?
RL : (students seem to think to answer)
P : What is this? (show the written answer sheet $A = 9.5 \times 5.3 = 50.35$
RL : Width of land
P : What about this (while showing the written answer sheet $P = 2 \times (9.5 + 5.3) = 29.6$
RL : This is around the land
P : This? Many trees?
RL : Ya eh..., diameter plus distance between trees,
P : Can you describe the situation of the problem? (Try giving blank paper so that the respondent describes the problem situation)
RL : No, I cannot
P : Describe the situation, (try to motivate to draw)
RL : I can’t
P : Keep trying (a rectangular plot of land)
RL : Try drawing a rectangle using a length of 9.5 and width of 5.3
P : Which one will be planted?
RL : At these borders (showing the image he has made)
P : Guess if the trees don’t cross the line, where is the position of the trees?
RL : They can be here… or maybe here…,
P : Is it a border?
RL : Oh..yes, inside it
P : Try
P : Sure your answer is correct?
RL : Sure.. hehe

From the results of the interview, it appears that Subject showed a slow answer, but in the end, she answered and showed that the answer was correct. Confidence in answering questions is part of the characteristic understanding of students’ concepts; it is in line with the results of Mafakheri’s research which states that insecurity is a factor that influences student achievement [12].

Studies related to behaviour in learning states that learning is a change in observable behaviour. Mathematical abilities that have been studied are problem-solving behaviour of elementary students. From the results of Muir’s research concluded that problem-solving abilities could be observed from the behaviour of students and their level of behaviour can be categorized [15, 16].

Indicators of the behaviour of mathematical concepts understanding are shown by students’ experience degradation resulting in the category level of the behaviour. But from the one level to another can be developed with each other, for example from semi-relationalis can be developed or directed into relationalist behaviour. Subject's behaviour in understanding mathematical concepts shows conditions can be developed towards behaviours that tend to be relationalist. This development can be done by paying more attention to reading problems. This problem is in line with the research conducted by Hite regarding how to direct and develop students in understanding problems by asking them to read the problem carefully and directed [17].

4. Conclusion
Based on the results of the findings in the field then analyzed by referring to the rubric used in this study, it can be concluded that Subject tends to behave semi-rationalist in understanding mathematical concepts. Besides, it can be seen from the results of the interview that Subject could still be directed so that she can potentially develop behaviour into better behaviour than semi-relationalist, perhaps relationalist or perhaps closer to the relationalist. From the conclusion, it can be seen that it is necessary to develop further analysis for relationalist or instrumentalist categories or perhaps new categories may emerge apart from the three categories found in the rubric. For this reason, the researcher is advised to review the categorization of different subjects and places.

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