We develop an advanced method of solving homogeneous and inhomogeneous Bethe-Salpeter equations by using the expansion over the complete set of 4-dimensional spherical harmonics. We solve Bethe-Salpeter equations for bound and scattering states of scalar and spinor particles for the case of one meson exchange kernels. Phase shifts calculated for the scalar model are in agreement with the previously published results. We discuss possible manifestations of separability for one meson exchange interaction kernels.

1 Introduction and preliminaries

Obtaining solutions of inhomogeneous Bethe-Salpeter (BS) equations is a long standing problem with clear motivation – BS amplitude in continuum (or half-off-shell $T$-matrix) is a necessary ingredient for description of final state interaction in numerous processes with the correlated nucleon-nucleon pair in final state. As an example one can mention, for instance, charge-exchange reaction on deuteron $pD \rightarrow n(pp)$ [1], deuteron breakup with fast forward $pp$-pair [2], electro- and photo-disintegration of deuteron [3] etc. One of the possible ways to obtain the BS amplitude in continuum is the so-called one iteration approximation scheme [4], when the relativistic $P$-components are obtained as first iteration from the non relativistic wave function with a certain interaction kernel. In [5] the inhomogeneous BS equation for spinor particles is regularly treated with a 2-dimensional Gaussian mesh. Although there are no doubts in correctness of these results, it can be shown that, in general, inappropriate choice of a 2-dimensional mapping in integration procedure can lead to under- or overestimation of the solutions. In this sense a 2-dimensional change of variables that one has to apply in partial BS equation for transforming the limits of integration to finite ones is not well defined procedure. Thus, the search of the rigorous method to solve such an equations should be continued.

In [6] the expansion of the BS amplitude and interaction kernel over hyperspherical basis is employed to formulate a method of solving the BS equation for bound states of scalar particles,

\[
\Gamma(k) = i \int \frac{d^4p}{(2\pi)^4} V(k,p) S(p_1) S(p_2) \Gamma(p),
\]

\( p_{1,2} = P/2 \pm p \), and \( S(p) = (p^2 - m^2 + i\varepsilon)^{-1} \). The corresponding expansions after the Wick rotation looks like

\[
\Gamma(ik_4, k) = \sum_{nltm} \varphi_{nlm}^r(\hat{k}) Z_{nlm}(\omega_k),
\]
\[ V(k, p) = \frac{g^2}{(k - p)^2 + \mu^2} = 2\pi^2 \sum_{nlm} \frac{1}{n+1} V_n(\tilde{k}, \tilde{p}) Z_{nlm}(\omega_k) Z^*_{nlm}(\omega_p), \tag{3} \]

\[ V_n(a, b) = \frac{4g^2}{(\Lambda_+ + \Lambda_-)^2} \left( \frac{\Lambda_+ - \Lambda_-}{\Lambda_+ + \Lambda_-} \right)^n, \]

\[ \Lambda_\pm = \sqrt{(a \pm b)^2 + \mu^2}, \]

where by definition \( \tilde{k} = \sqrt{k^2 + \mu^2} \), and \( \omega_k = (\chi, \theta, \phi) \) - angles of vector \( k = (k_4, \mathbf{k}) \) in 4-dimensional Euclidean space. Hyperspherical harmonics

\[ Z_{nlm}(\chi, \theta, \phi) = X_{nl}(\chi) Y_{lm}(\theta, \phi), \quad X_{nl}(\chi) \sim \sin^l \chi C_{n-l}^{l+1}(\cos \chi), \]

are proportional to the product of usual spherical harmonics for orbital momentum \( l \) and Gegenbauer polynomials \( C_{n-l}^{l+1} \).

Performing a Wick rotation in (1) and substituting (2) and (3) one reduces the BS equation to the infinite set of 1-dimensional integral equations. Thus, there is only one degree of freedom left in the choice of the mapping for integration. It is shown in [6] that this approach is very effective for numerical purpose. In particular, due to rapid convergence of hyperspherical expansion, the consideration of only first few equations from the set is sufficient.

## 2 Inhomogeneous BS equation for scalar particles

In this paper the method introduced in [6] is applied to the scalar inhomogeneous BS equation, and generalized to spinor equations, both homogeneous and inhomogeneous. Let us first consider the BS equation for the scattering of scalar particles:

\[ T(k) = V(k, q) + i \int \frac{d^4 p}{(2\pi)^4} V(k, p) S(p_1) S(p_2) T(p), \tag{4} \]

Here \( T(k) \) is half-off-shell \( T \)-matrix for the scattering of real particles with momenta \( q_{1,2}, \) and in c.m.s., where the real 3-momentum of particles \( \vert q \vert = \hat{p}, \)

\[ s = (q_1 + q_2)^2 = (2E_{\hat{p}})^2, \quad E_{\hat{p}} = \sqrt{m^2 + \hat{p}^2}, \quad q = \frac{q_1 - q_2}{2}, \quad (2m)^2 < s < (2m + \mu)^2 \]

Before Gegenbauer decomposition of type (2) and (3) the Wick rotation should be applied. It is more convenient to perform firstly in (4) the usual partial expansion over the spherical harmonics \( Y_{lm} \) (see also [7]):

\[ T_i(k_0, k) = g^2 Q_l(k_0, k; 0, \hat{p}) - i \int \frac{dp_0 dp}{(2\pi)^3} g^2 Q_l(k_0, k; p_0, p) S(p_1) S(p_2) T_i(p_0, p). \tag{5} \]

Here \( Q_l(k_0, k; p_0, p) \equiv Q_l([k^2 + p^2 + \mu^2 - (k_0 - p_0)^2 - i\varepsilon]/2kp) \) is the adjoint Legendre function of the 2nd kind. The normalization of \( T \)-matrix is fixed by the free term in (5) and leads to the following expression for the phase shifts:

\[ T_i(0, \hat{p}) = 16 \pi \hat{p} \sqrt{s} e^{i\delta_l} \sin \delta_l. \tag{6} \]
To eliminate the removable singularity at \((p_0, p) = (0, \hat{p})\), we present the \(T\)-matrix in the following factorized form [9]:

\[ T_l(k_0, k) \sim \varphi_l(k_0, k) t_l(s), \quad t_l(s) \sim e^{i\delta_l} \sin \delta_l, \]

thus obtaining from [3] the equation for \(\varphi_l\),

\[
\varphi_l(k_0, k) = g^2 Q_l(k_0, k; 0, \hat{p}) - i \int \frac{dp_0 \, dp}{(2\pi)^3} \{ g^2 Q_l(k_0, k; p_0, p) \\
- \frac{g^2}{Q_l(s)} Q_l(k_0, k; 0, \hat{p}) Q_l(0, \hat{p}; p_0, p) \} S(p_1) S(p_2) \varphi_l(p_0, p),
\]

\(Q_l(s) \equiv Q_l(0, \hat{p}; 0, \hat{p})\). In the following only the cases \(l = 0, 1\) will be considered. After the Wick rotation the Gegenbauer decomposition of \(\varphi_l\) for \(l = 0\) takes the form

\[
\varphi_0(ik_4, k) = k\hat{p} \sum_{j=1}^{\infty} g_j(\tilde{k}) X_{2j-2,0}(\chi),
\]

and for the coefficient functions \(g_j\) the system of integral equations is obtained:

\[
g_j(\tilde{k}) = \frac{\pi}{2j - 1} V_{2j-2}(\tilde{k}, \hat{p}) X_{2j-2,0} \left( \frac{\pi}{2} \right) \\
+ \sum_{l=1}^{\infty} \int \frac{d\bar{p} \, \bar{p}^3}{8\pi^2} \left[ \frac{1}{2j - 1} V_{2j-2}(\tilde{k}, \hat{p}) S_{2j-2,2l-2}(\bar{p}) \\
- \frac{\bar{p}^2}{2j - 1} V_{2j-2}(\tilde{k}, \hat{p}) X_{2j-2,0} \left( \frac{\pi}{2} \right) \frac{1}{Q_0(s)} N_{0,2l-2}(\tilde{p}, \hat{p}) \right] g_l(\bar{p}) \\
- \int_{0}^{\hat{p}} \frac{dp}{8\pi^2} E_p \sqrt{s - 2E_p} \left[ \frac{p}{\bar{p}} g^2 W_{2j-2,0}(\tilde{k}, p) \\
- \frac{2}{Q_0(s)} \frac{\pi}{2j - 1} V_{2j-2}(\tilde{k}, \hat{p}) X_{2j-2,0} \left( \frac{\pi}{2} \right) Q_0(0, \hat{p}; p^0_2, p) \right] \tau_0(p),
\]

\[
\tau_0(k) = g^2 Q_0(k^0_2, k; 0, \hat{p}) + g^2 \hat{p} \sum_{l=1}^{\infty} \int \frac{d\bar{p} \, \bar{p}^3}{(2\pi)^3} [k N_{0,2l-2}(\tilde{k}, \bar{p}) \\
- \frac{\hat{p}}{Q_0(s)} Q_0(k^0_2, k; 0, \hat{p}) N_{0,2l-2}(\tilde{p}, \bar{p}) \right] g_l(\bar{p}) \\
- g^2 \int_{0}^{\hat{p}} \frac{dp}{8\pi^2} E_p \sqrt{s - 2E_p} [k p U_0(k, p) \\
- \frac{2}{Q_0(s)} Q_0(k^0_2, k; 0, \hat{p}) Q_0(0, \hat{p}; p^0_2, p) \right] \tau_0(p).
\]

The additional unknown function \(\tau_0\) comes after the Wick rotation from the residue at the pole \(p^0_2 = \sqrt{s}/2 - E_p + i\epsilon\), which is able to cross the imaginary \(p_0\) axis. It is
proportional to the $T$-matrix in Minkowsky space for certain value of relative energy, $\tau_0(p) \sim T_0(\sqrt{s}/2 - E_p, p)$. The partial kernels are

$$S_{k'k}(\bar{p}) = \int_0^\pi d\chi \sin^2 \chi \frac{X_{k'k}(\chi)X_{kl}(\chi)}{(\bar{p}^2 - \bar{p}^2)^2 + s\bar{p}^2 \cos^2 \chi},$$

$$N_{ml}(k, \bar{p}) = \int_0\pi d\chi_p \sin^2 \chi_p \frac{1}{2kp} \left\{ Q_m(k_2^0, k; ip_4, p) + Q_m(-k_2^0, k; ip_4, p) \right\} \frac{X_{im}(\chi_p)}{(\bar{p}^2 - \bar{p}^2)^2 + s\bar{p}^2 \cos^2 \chi_p},$$

$$W_{nl}(\bar{k}, p) = \int_0^\pi d\chi_k \sin^2 \chi_k X_{nl}(\chi_k) \frac{1}{kp} \left\{ Q_l(ik_4, k; p_2^0, p) + Q_l(ik_4, k; -p_2^0, p) \right\},$$

$$U_l(k, p) = \frac{1}{kp} \left\{ Q_l(k_2^0, k; p_2^0, p) + Q_l(k_2^0, k; -p_2^0, p) \right\}.$$

It is easy to see that all the partial kernels are real expressions.

### 3 Numerical results and separability

As all the integrations in system (9), (10) are 1-dimensional, the integrals can be replaced by finite sums by using the gaussian mesh. In this way this system is reduced to the usual system of linear equations, which can be solved by any appropriate method. Besides, the solution of integral equations (9), (10) can be obtained by iterating the free term, thus constructing the Neumann series, which represents the solution of an inhomogeneous integral equation. It was explicitly checked that these two procedures of finding a solution lead to the same results.

Taking the first five terms in the expansion (8), the system (9), (10) was numerically considered for the following set of parameters:

$$\frac{g_2^2}{4\pi} = 4\pi, \quad m = \mu = 1 \text{ GeV}, \quad \bar{p} = 0.77 \text{ GeV}/c$$

In Fig. 1 the numerical results for the first three coefficient functions from (8) are shown. It was established that convergence of this expansion is quite rapid, that is why only 3 components are presented and $g_2$ and $g_3$ are multiplied by 10 and 100 respectively. Centered symbols correspond to the points of the mesh, where the solution is defined. Lines connecting the points reproduce the results of the fit for the obtained Gegenbauer components with the fitting functions

1. For $g_1(p), p \equiv \bar{p}$

$$F(p) = \sum_{j=1}^4 \frac{a_j^1 p^{2j-2}}{(p^2 + b_1^2)^j}, \quad (11)$$

2. For $g_2(p)$

$$F(p) = \frac{p^2}{p^2 + b_2^2} \sum_{j=1}^4 \frac{a_j^2 p^{2j-2}}{(p^2 + b_2^2)^j}, \quad (12)$$
3. For \( g_3(p) \)

\[
F(p) = \left[ \frac{p^2}{p^2 + b^2_3} \right]^2 \sum_{j=1}^{4} a_j^3 p^{2j-2} (p^2 + b^2_3)^j
\]  

(13)

For the sake of brevity we don’t show here the numerical values of the parameters \( a_j^{1,2,3} \) and \( b_{1,2,3} \), which can be adjusted by any appropriate method (for instance, the built-in fitting procedure in Microcal Origin 7.0 can be employed). It should be mentioned that the quality of this analytical fit is excellent in the whole range of the argument \( \tilde{p} \) despite the very simple form of the functions (11)-(13). This fact can be treated as an indication to some kind of separability in the one meson exchange interaction and will be discussed below. Besides, it was found that such a fit is valid for Gegenbauer components of solutions of any BS equation in ladder approximation (scalar and spinor equations, both homogeneous and inhomogeneous). Hence, it is very useful for practical purposes, e.g. to represent the numerical results in a compact form.

To discuss the manifestations of separability in more details, let us now turn to the homogeneous BS equation (1). After Wick rotation and usual partial decomposition it gives

\[
\Gamma_l(ik_4, k) = \int \frac{dp_4 dp p^2}{(2\pi)^3} \, V_l(ik_4, k; ip_4, p) \, S(p_1)S(p_2) \, \Gamma_l(ip_4, p).
\]  

(14)

It is generally known that for the conventional separable kernel of the form (see e.g. [9])

\[
V_l(ik_4, k; ip_4, p) = \sum_{ij} \lambda_{ij}(s) \, g_i(\tilde{k}^2) \, g_j(\tilde{p}^2)
\]

the vertex functions are expressed in terms of functions \( g \):

\[
\Gamma_l(ik_4, k) = \sum_i c_i \, g_i(\tilde{k}^2).
\]

In our case, solving equation (14) for kernel (3) by the method given above and fitting the calculated Gegenbauer components with analytical expressions like (11)-(13), we obtain the following approximate representation for the solution (cf. (8)):

\[
\Gamma_l(ik_4, k) = \sum_{j=l}^{N} c_{jl}^l \, R_{jl}(\tilde{k}^2) \, X_{jl}(\chi_k),
\]  

(15)

where the fitting functions are

\[
R_{jl}(\tilde{k}^2) = \frac{(\tilde{k}^2)^{\mu_{jl}}}{(k^2 + \beta_{jl}^2)^{\nu_{jl}}}.
\]

It is clear that constructing the expression for the separable kernel like

\[
V_l(ik_4, k; ip_4, p) = \sum_{ij} \lambda_{ij}(s) \, g_{il}(p_4, p) \, g_{jl}(k_4, k)
\]  

(16)

with

\[
g_{jl}(k_4, k) = R_{jl}(\tilde{k}^2) \, X_{jl}(\chi_k)
\]  

(17)

one identically reproduces the analytical form of the solution (15). Therefore, the kernel (16), (17) can be referred to as some general form for constructing separable kernels. Note that it has the following properties:
• A dependence on mass (or s) in the functions g

• An explicit dependence on \( k_4 \) in g coming from Gegenbauer polynomials being the functions of \( \cos \chi_k = k_4/k \)

• At low \( k^2 \) (16) tends to a Yamaguchi kernel, because \( g(k^2) \to \frac{C}{k^2 + \beta^2} \)

A concrete example of the solution of eq. (14) is given in Table 1. The parameters \( \lambda_{ij} \) [GeV\(^5\)] for the partial kernel (16) of rank 3 are

\[
\begin{align*}
\lambda_{11} &= 0.19613 & \lambda_{22} &= 0.13734 & \lambda_{33} &= -0.01644 \\
\lambda_{12} &= -0.09675 & \lambda_{13} &= 0.00236 & \lambda_{23} &= -0.00034.
\end{align*}
\]

| \( j \) | \( c_j^0 \) | \( \beta_j^0 \) [GeV] | \( \mu_{j0} \) | \( \nu_{j0} \) |
|-------|-------------|-----------------|---------|--------|
| 0     | 0.17904     | 0.39736         | 0       | 1      |
| 2     | -0.00923    | 0.41533         | 1       | 2      |
| 4     | 0.00127     | 0.43362         | 2       | 3      |
| 6     | -0.00055    | 0.45127         | 3       | 4      |
| 8     | 0.00017     | 0.47019         | 4       | 5      |

Table 1. Numerical values of parameters in (15) for \( l = 0, m = 1 \) GeV, mass of bound state \( M = 1.9 \) GeV, \( \mu = 0.1 \) GeV. Coefficients \( c_j^0 \) for odd values of \( j \) are equal to zero.

Analyzing the obtained solutions for the system (9), (10) it is also informative to calculate corresponding phase shifts. The results for \( l = 0 \) and \( l = 1 \) are presented in Figs. 2 and 3, respectively, for different values of coupling constant \( \lambda \) connected with \( g^2 \) as

\[
\frac{g^2}{4\pi} = 4\pi\lambda.
\]

It is obvious, that for \( \lambda = 0.7 \) there are no bound states in this system due to the Levinson’s theorem. For the values \( \lambda = 1, 3, 5 \) there is one bound state, and two bound states for \( \lambda = 7 \). For \( l = 1 \) no bound states found for these values of \( \lambda \), as it follows from Fig. 4. Obtained results for the phase shifts are in a good agreement with [8].

4 Spinor BS equations

Above the procedure of solving the BS equation and handling with its solutions is described for the case of scalar particles. Spinor equations are much more complicated, and so we do not show here the detailed formulas like (9), (10). Nevertheless, it should be
stressed that all the basic steps remain the same. Below we briefly mention only the main results for spinor case.

The problem of obtaining the BS amplitude in the continuum (or half-off-shell $T$-matrix) for spinor particles as a solution of the inhomogeneous BS equation with realistic interaction kernel seems to be of great importance. We present the results of solving the BS equation in the $^1S_0$ channel

$$\hat{T}(k) = V(k, q) \Gamma(1) \Gamma^{++}(q) \Gamma(2) + i \int \frac{d^4p}{(2\pi)^4} V(k, p) \Gamma(1) \hat{S}(p_1) \hat{T}(p) \hat{S}(p_2) \Gamma(2)$$

(18)

in the ladder approximation for the cases of scalar ($\Gamma(1, 2) = 1$) and pseudoscalar ($\Gamma(1, 2) = \gamma_5$) one meson exchanges without cut-off. Here

$$\hat{S}(p) = \frac{\hat{p} + m}{p^2 - m^2 + i\varepsilon}, \quad \hat{S}(p) = \frac{\hat{p} - m}{p^2 - m^2 + i\varepsilon}.$$ 

In (18) the BS vertex function $\hat{T}(k)$ being the matrix $4 \times 4$ before Gegenbauer decomposition of type (2) and (3) should be expanded over some full set of matrices. In $^1S_0$ channel there are only 4 such a matrices usually called spin-angular harmonics (for their explicit form see [10]),

$$\Gamma_{^1S_0^{++}}(-k), \quad \Gamma_{^1S_0^{--}}(-k), \quad \Gamma_{^3P_1}(-k), \quad \Gamma_{^3P_0}(-k),$$

(19)

and corresponding expansion after the Wick rotation reads like

$$\hat{T}(ik_4, k) = \sum_{\alpha} t_{\alpha}(ik_4, |k|) \Gamma_{\alpha}(-k).$$

(20)

Unfortunately, after the expansion (20) the Gegenbauer decomposition of resulting partial equations for $t_{\alpha}(ik_4, |k|)$ can not be performed in a simple analytical way. To make it possible, another set of spin-angular matrices for $^1S_0$ channel was found:

$$\mathcal{T}_1(k) = \frac{\gamma_5}{2}, \quad \mathcal{T}_2(k) = \frac{\gamma_0 \gamma_5}{2}, \quad \mathcal{T}_3(k) = -\frac{(k, \gamma)}{2 |k|} \gamma_0 \gamma_5, \quad \mathcal{T}_4(k) = -\frac{(k, \gamma)}{2 |k|} \gamma_5,$$

(21)

and, similarly to (20),

$$\hat{T}(ik_4, k) = \sum_{j=1}^{4} g_j(ik_4, |k|) \mathcal{T}_j(k).$$

(22)

It can be easily shown that there is a non-degenerate transformation from set (21) to (19). The point of using these two bases in the $^1S_0$ subspace is that the set (21) allows us a very simple Gegenbauer decomposition of partial equations for $g_j(ik_4, |k|)$ in an analytical form, while the set (19) is preferable in calculating the matrix elements of the real processes [1, 2], because of its clear physical meaning.

From the statement above follows that the half-off-shell $T$-matrix $\hat{T}(k)$ is expanded over the basis (21) and the derived set of integral equations for the scalar coefficients after the Wick rotation is decomposed into Gegenbauer polynomials like in (8)-(10). We solve this set of equations numerically, for $m = 0.938$ GeV, $\mu = 0.14$ GeV, $\sqrt{s} = 2m + 0.1$ GeV, and here the final results for solutions are presented in the form of partial
components $t_i(ik_4,|k|)$ of (20). In Figs. 4-7 these functions are shown for the case of a scalar interaction, and in Figs. 8-11 for a pseudoscalar one. For scalar exchange the ratio between different components looks very familiar: $t_{1S^+_0}$ is the largest one, the other components are much smaller. Note that this situation does not hold for pseudoscalar exchange: here $t_{1S^-_0}$ becomes the main component, and the $P$-wave $t_{3P^0}$ is larger than $t_{1S^+_0}$. Although such a ratio between components seems to be exotic, it is valid also for the bound state problem in the $^1S_0$ channel (eq. (22) with $\Gamma(1,2) = \gamma_5$). Moreover, it qualitatively reproduces the previous results (huge $P$-wave) for the BS amplitude in continuum obtained in [2] within the one iteration approximation scheme.

Finally, the homogeneous spinor BS equation of the form

$$\Psi(k) = i \int \frac{d^4p}{(2\pi)^4} V(k,p) \Gamma(1) \hat{S}(p_1) \Psi(p) \hat{S}(p_2) \Gamma(2),$$  

was examined for one meson exchange interaction kernels with $\Gamma(i), i = 1, 2$ as scalar, pseudoscalar, vector, tensor vertices. Only $^1S_0$ bound states were considered. In this study we followed the motivations of [11], and the obtained results will be a subject of a separate publication.

## 5 Conclusion

The method described before was successfully applied to solve both, homogeneous and non homogeneous, BS equations for scalar and spinor particles. It shows a high efficiency and accuracy, and, in particular, does not require any extraordinary computer facilities. The obtained results are in a good agreement with calculations of other groups [8] and also agree with our previous results [2]. Also the manifestations of separability for one meson exchange kernel are discussed. From the analysis of the obtained exact solutions, a possible general form of the separable kernel is suggested.

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Figure 1: Gegenbauer components from (8). Functions $g_2$ and $g_3$ are multiplied by 10 and 100 respectively in view of their smallness.
Figure 2: Phase shifts (in the units of $\pi$) for $l = 0$ and different values of coupling constant.
Figure 3: The same as in Fig. 2 for $l = 1$. 
Figure 4: $^1S_0^{++}$ partial component (the largest one) of the solution of eq. (18) for scalar meson exchange interaction.
Figure 5: $^1S_0^{--}$ partial component for scalar meson exchange.
Figure 6: $^3P_1^e$ partial component for scalar meson exchange.
Figure 7: $^{3}P_{1}^{o}$ partial component for scalar meson exchange.
Figure 8: $^{15}\Sigma^{++}$ partial component of the solution of eq. (18) for pseudoscalar meson exchange interaction.
Figure 9: $^{1}S_{0}^{-}$ partial component (the largest one) for pseudoscalar meson exchange.
Figure 10: $^3P_1$ partial component for pseudoscalar meson exchange.
Figure 11: $^3P^0_1$ partial component for pseudoscalar meson exchange.