Approach of the measurement of thermal diffusivity of mural paintings by front face photothermal radiometry

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Abstract. In this paper we present, in an experimental way, the possibilities of front face photothermal radiometry to measure, in situ, the longitudinal thermal diffusivity of mural paintings. First, we present the principle of the method of measurement. Then, we present the experimental device implemented for the study. Finally, we show, using the experimental study of a plaster sample, the photothermal method allows in a particular case, a good approximation of the parameter longitudinal thermal diffusivity.

1. Introduction

In the field of assistance to restoration of mural paintings, our laboratory has worked for about ten year with the detection of separations and air pockets located in mural paintings. Front face photothermal radiometry, already allowed us to detect, in situ, delaminations located in the Saint Christopher of the Campana collection of Louvre, in the painted walls of the church “Saint Florentin de Bonnet”, in the ceilings painted of the abbey of “Saint Savin sur Gartempe” (classified with the world heritage of UNESCO) and finally in the Cocteau frescos of the vault Saint Pierre of “Villefranche sur mer” [1]. These qualitative studies being positive, we want to study the possibilities of the photothermal method for the characterization of the defects. In the long term the objective is to determine at the same time the surface extent of the defect but also the depth at which it is located. For that we plan to proceed by an adjustment theory/experiment using methods of inverse techniques of the type “levenberg marquardt” [2]. To feed the model evoked previously, it is necessary to know the thermophysical properties of studied materials. Two solutions were offered to us: Either to call upon bibliographical values, or to develop a method of measurement of diffusivity usable in situ. For reasons of self life of the materials (ageing, presence of moisture, manufacturing processes…), it is the second solution we chose to implement. To lead to these measurements of diffusivity, we had still two solutions: Either to take a sample of work of art and to implement a traditional method of measurement of this thermophysical parameter (flash method) [3-4]), or to develop a method usable in situ and non destructive for the studied mural painting. For reasons of good conservation of the works of art, it is this last option which we chose to implement. The thickness of the couple coating - wall composing the mural painting is generally of a thickness of tens centimetres, a measurement of transverse thermal
diffusivity is often impossible. Thus, we chose to develop a method of measurement of longitudinal thermal diffusivity of the work of art. This method is presented here. We present initially the principle of the method of measurement, based on the coupled use of integral transformations and quadruple method [4]. Then, we present the experimental device developed for the study. Finally, we show the method allows a good estimation of the longitudinal diffusivity of a plaster sample.

2. Principle of the measurement «in situ” of longitudinal thermal diffusivity.

The general principle of the measurement "in situ" of longitudinal thermal diffusivity by front face photothermal radiometry is the following: An anisotropic sample is subjected on its front face to an excitation temporally close to a function delta of Dirac (T) and unspecified spatial shape f(x,y). We measure the field of temperature of the front face using an infra-red camera of thermography. From the temporal evolution of this field of temperature, we go up, using a mathematical post treatment the values of thermal diffusivity of material according to its directions of anisotropy. Let us examine in details this mathematical post treatment on which this measurement technique is based. \( \lambda_x, \lambda_y \) et \( \lambda_z \) are the thermal conductivity of the studied sample. These thermal conductivities will be supposed to be constant in time and according to the temperature (assumptions of short analyzes and weak temperature variations). \( p \) and \( c_p \) are the density and heat-storage capacity of this sample. \( a_x, a_y \) et \( a_z \) are the thermal diffusivities of the studied sample. \( h_0 \) and \( h_e \) are the convective coefficients of the front face and rear face of the sample. \( e \) is the thickness of the material. This thickness is supposed very weak in front of side dimensions of the sample, which makes it possible to neglect the side convecto-radiative losses of the sample. The sample is initially in thermal balance with its environment. The mathematical translation of these assumptions leads to the following differential system:

\[
\lambda_c (\partial^2 T / \partial x^2) + \lambda_c (\partial^2 T / \partial y^2) + \lambda_c (\partial^2 T / \partial z^2) = \rho c (\partial T / \partial t)
\]

For \( z = 0 \)
\[
\lambda_c (\partial T / \partial z)_{z=0} = h_0 (T(z=0) - T_{ext}) - f(x,y) \delta(t)
\]

For \( z = e \)
\[
\lambda_c (\partial T / \partial z)_{z=e} = -h_e (T(z=e) - T_{ext})
\]

For \( x = 0 \) et \( x = L_x \)
\[
\partial T / \partial x = 0
\]

For \( y = 0 \) et \( y = L_y \)
\[
\partial T / \partial y = 0
\]

At \( t = 0 \)
\[
T = T_{ext}
\]

To resolve this differential system, we chose to implement an integral transformation, i.e. a transformation of Laplace into time associated with a transformation of Fourier into space coordinates \( x \) and \( y \) :

\[
\theta(\alpha_n, \beta_m, z, p) = \int_{x=0}^{L_x} \int_{y=0}^{L_y} \int_{t=0}^{T_{ext}} T(x, y, z, t) \cos(\alpha_n x) \cos(\beta_m y) \exp(-pt) dx dy dt
\]

(2)

with \( \alpha_n = n \pi / L_x \) et \( \beta_m = m \pi / L_y \). By applying this integral transformation to the preceding differential system, the differential equation to solve in space transformed only depends of \( z \) and can thus be solved easily by the method of the thermal quadropoles [6]. We obtain then:

\[
\theta(\alpha_n, \beta_m, z = 0, p) = \frac{F(\alpha_n, \beta_m)(ch(\gamma_{n,m} e) + h_x sh(\gamma_{n,m} e) / (\lambda_c \gamma_{n,m}))}{\lambda_c \gamma_{n,m} sh(\gamma_{n,m} e) + (h_0 + h_e) ch(\gamma_{n,m} e) + h_0 h_e sh(\gamma_{n,m} e) / (\lambda_c \gamma_{n,m})}
\]

(3)

with : \( \gamma_{n,m} = \sqrt{p / a_c + (\lambda_x / \lambda_c) \alpha_n^2 + (\lambda_y / \lambda_c) \beta_m^2} \)
F(α_m, β_m), the transform of Laplace Fourier of exciting flux f(x,y) δ(t).

By now taking the inverse transform of Laplace of the temperature we obtain:

\[ \ln \left( \frac{\theta(\alpha_n, \beta_m, z = 0, t)}{\theta(0,0, z = 0, t)} \right) = \ln \left( \frac{F(\alpha_n, \beta_m)}{F(0,0)} \right) - (a_x \alpha_n^2 t + a_y \beta_m^2 t) \]  \hspace{1cm} (4)

We can notice that longitudinal diffusivities \( a_x \) and \( a_y \) can be simply deduced from the slope of the curve representing the ratio of the logarithm of the coefficients of Fourier traced compared to time.

3. The experimental system implemented for this study

The experimental device implemented for our study is derived from the system of analysis of thin materials by infra-reds of the laboratory. The excitation source is a laser diode emitting at 810 nm associated to an optic of collimation and focalization. Optic of infrared is consisted of a « long waves » bolometer camera, working in macro mode (in order to have a sufficient spatial resolution). This camera is placed perpendicular to the sample, at approximatively 5cm. The beam of light resulting from the laser diode is send in a tilted way on the sample to analyze because of obstruction of the camera. Its shape is slightly elliptic. The laser diode is controlled in electric current in order to make it emit a power of 2W throughout a period of 20ms. Frequency of acquisition of the camera of thermography is 50Hz.

![Figure 1. The experimental device](image)

4. The analyzed sample

The sample is a plaster block of 12 cm * 15 cm side and 2,2 cm thick. It is covered with a fine coat of black paint on the analyzed side, in order to simulate the presence of a pictorial layer. To determine with precision its thermal diffusivity, we initially studied it with the flash diffusivimeter of the LEMTA of Nancy (assumption of isotropy of thermal diffusivity). For that, we machined it in order to reduce its thickness to 6.19 mm. Three traditional modes of examination were implemented, the method of partial times, the method of the temporal moments and an adjustment theory/experiment. The values of thermal diffusivity obtained are: 3,46 \( 10^{-7} \) m\(^2\)/s with the method of partial times, 3,49 \( 10^{-7} \) m\(^2\)/s with the method of the temporal moments and 3,50 \( 10^{-7} \) m\(^2\)/s with an adjustment theory/experiment.

5. Experimental results obtained

On figure 2, we present four infra-red images representative of the temporal evolution of the thermal signature of the laser spot. We notice the thermal signature becomes wider and less intense as time passes.
On figure 3, we present the temporal evolution of the ratio of the logarithms of the coefficients of Fourier, calculated with order 2. It reveals, as it was envisaged by the theory, a line of negative slope. It is equal, in this case to -2.558 s$^{-1}$.

Lastly, to succeed a measurement of longitudinal diffusivity, we calibrated spatially our experimental device. For that, we placed a hold standard on the surface of the object studied. We deduced of the infra-red image obtained, a space dimension of the pixel equals to 161 µm. Formula 4, allowed us to determine the longitudinal value of the thermal diffusivity of the analyzed plaster sample. We found an average value equals to 3.49 $10^{-7}$ m$^2$/s. This value is close to the values obtained with the classic flash method, it seems to show the possibilities of the method as regards measurement "in situ", of this thermophysical parameter.

6. Conclusion
In this work, we tried to approach in an experimental way, the possibilities of photothermal radiometry as regards measurement "in situ" of longitudinal thermal diffusivity of mural paintings. Initially, we presented the principle of the method of measurement. Then we presented the experimental device implemented for the study. Then we showed, using the experimental study of a plaster sample, the photothermal method allows in a particular case, a good approximation of the thermal parameter diffusivity. This experimental result obtained on a particular sample is encouraging, it seems to open the way with the photothermal characterization in situ of works of art. Studies going in this direction are in progress.

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