Using Galaxy Cluster Peculiar Velocities to Constrain Cosmological Parameters

A. Peel\textsuperscript{*} and L. Knox\textsuperscript{†}

\textsuperscript{*}UC Davis Cosmology Group, 1 Shields Ave, Davis, CA 95616, USA

Galaxy cluster peculiar velocities can be inferred from high-sensitivity, high-resolution multiple-frequency observations in the 30 to 400 GHz range. While galaxy cluster counts and power spectra are sensitive to the growth factor, peculiar velocities are sensitive to the time-derivative of the growth factor and are hence complementary. Using linear perturbation theory, we forecast constraints on $\Omega_m$, $H_0$ and the dark-energy equation of state parameter, $w$, given 820 densely sampled cluster locations (at $z \simeq 1$) from a $\Lambda$CDM N-body simulation and 820 sparsely sampled cluster locations in a broader redshift range.

1. Introduction

After the cosmic microwave background, galaxy clusters may be our best hope for a precision cosmological probe. Although clusters are highly non-linear objects, the dominant importance of gravitational effects over non-gravitational ones makes them attractive candidates as systems whose properties should be calculable from first principles. Cluster number density evolution has been proposed as a way to determine cosmological parameters including the dark-energy equation of state parameter $w$. Here we investigate constraints on cosmology possible from measurements of cluster peculiar velocities.

Peculiar velocities of galaxies measured via, e.g., the Tully–Fisher and fundamental plane relations have already been used to constrain cosmological parameters. These efforts have been hindered by large statistical and systematic errors in the velocity measurements. In contrast, cluster peculiar velocities determined from the kinetic and thermal Sunyaev-Zel’dovich (SZ) effects are potentially more reliable. In addition, SZ is the only way to determine peculiar velocities over a wide range of redshifts allowing one to observe the evolution of statistical properties.

2. Sunyaev-Zel’dovich Effects

About 1% of the cosmic microwave background (CMB) photons that pass through the gravitationally confined hot gas in the deep potential wells of galaxy clusters Compton scatter off a free electron. On average these photons gain energy causing a spectral distortion of the blackbody nature of the CMB characterized by a deficit of low frequency photons and an excess at high frequencies, with a null near $\nu = 217$ GHz. Specifically the distortion has the frequency dependence:

$$\frac{\Delta I_\nu}{I_\nu} = y \left( \frac{e^x + 1}{e^x - 1} - 4 \right); \quad x = \frac{h \nu}{k T_{\text{CMB}}}; \quad (1)$$

$$y = \frac{k \sigma_T}{m_e c^2} \int dl \, T_e(l) n_e(l) \approx \tau \frac{k T_e}{m_e c^2}. \quad (2)$$

where $T_e(l)$ is the electron temperature at distance $l$ from the observer.

This spectral distortion has been observed in the direction of numerous known clusters, whose locations had been determined by other means (optical and/or X-ray surveys).

The bulk motion of the cluster with respect to the CMB causes the kinetic SZ effect which, to first order, leads to a hotter (colder) blackbody spectrum for a cluster that is moving toward (away) from the observer:

$$\frac{\Delta I_\nu}{I_\nu} = \frac{v_r}{c} \frac{\tau x e^x}{e^x - 1} \left( \frac{\delta T_{\text{CMB}}}{T_{\text{CMB}}} \right)_{\text{SZ}} \approx -\frac{v_r}{c} \tau \quad (3)$$

where $v_r$ is the radial component of the velocity.
Measurements at multiple frequency bands in the 30 to 400 GHz range can be used to simultaneously determine \( y \approx \tau \frac{kT_e}{m_e c^2} \) and \( \tau v_r/c \). These two determinations together with a temperature determination can be used to solve for \( v_r/c \):  
\[
\frac{v_r}{c} = -\frac{kT_e}{m_e c^2} \frac{(\delta T_{\text{CMB}}/T_{\text{CMB}})_{\text{SZ}}}{y}.
\]  
(3)

For hotter clusters there are temperature-dependent corrections to Eq. (3) that allow one to solve for the temperature as well. For cooler clusters, \( X \)-determinations of the temperature may be necessary.

The best determinations of peculiar velocities from \( \text{SZ} \) measurements have errors of \( \sigma_v \approx 1000 \) km/s but can be greatly reduced with more sensitive measurements. Published predictions of achievable \( \sigma_v \) assume that the motions of the electrons in the cluster are simply thermal motions plus one coherent bulk motion. In this case there is indeed a well-defined “velocity of the cluster”, and the error in its measurement is:

\[
\sigma_v \approx 25 \frac{\text{km}}{\text{s}} \left( \frac{0.01}{\tau} \right) \left( \frac{\Delta T_{\text{CMB}}^2 + \Delta T_{\text{noise}}^2}{4 \mu K^2} \right)^{\frac{1}{2}}.
\]  
(4)

In the no-noise (\( \Delta T_{\text{noise}} = 0 \)) limit and with a confusion noise from the Ostriker–Vishniac effect of \( \Delta T_{\text{CMB}} = 2 \mu K \) (achievable for clusters of small angular extent which includes any cluster at \( z \geq 0.2 \), and with high (~1’) angular resolution) then \( \sigma_v \approx 25 \) km/sec is possible.

However, clusters have \textit{many} bulk flows in them, not just one. Integrating over all these internal bulk flows, with the mass-weightings naturally provided by the \( \text{SZ} \) effects, should still lead to what might be called “the velocity of the cluster”. But a sizeable fraction of the mass is along lines of sight with quite low optical depth and hence high velocity errors. The net result is that \( \sigma_v \approx 100 \) km/sec is a more likely error.

### 3. Survey Strategy Issues

We imagine peculiar velocities determined as part of a high-resolution, multi-frequency (30 to 400 GHz) follow-up campaign on a cluster survey. These clusters may have been found from X-ray, \( \text{SZ} \) or even galaxy redshift surveys (e.g., SDSS and DEEPII). Important issues are how and how accurately to determine redshifts, how many velocity measurements to make at each redshift and whether to sample densely to study correlations or sparsely to cover more volume.

Cluster redshifts may be obtained via (i) optical photometric and/or spectroscopic redshifts of galaxy cluster members, (ii) measurement of CO lines in galaxy cluster members, or (iii) X-ray measurements with sufficient spectral resolution. X-ray and optical data also provide valuable information on the dynamic state of the cluster; e.g., has the cluster fully virialized, or is it still merging? Cluster redshifts are already vitally important for determining \( dN/dz \) from thermal \( \text{SZ} \) surveys. Note that interpretation of peculiar velocity correlations is much less sensitive to selection effects than the \( dN/dz/d\Omega \) statistic.

Dense sampling puts much higher demand on the accuracy of the redshift determinations. For a specified error in comoving separation \( \Delta r \) we need a redshift error smaller than \( \Delta z = 0.003h(H(z)/H_0)(\Delta r/(10 \text{ Mpc})) \); in order to calculate the expected correlations accurately we need \( \Delta r \lesssim 10 \text{ Mpc} \). With sparse sampling, all we need is a coarse redshift binning and assurance that the clusters are separated by \( \gtrsim 200 \text{ Mpc} \) (comoving); photometric redshifts are almost certainly sufficient for that case.

There are good arguments for targeting the \( z \approx 0.2 \) to 0.5 range rather than higher redshifts. The tolerance on redshift error decreases only slowly with increasing redshift, whereas the amount of telescope time required typically increases dramatically. In addition, optical and X-ray data on the clusters may be valuable for investigating the dynamical state of the cluster and third X-ray temperatures are necessary for the colder clusters.

### 4. Linear Theory

The evolution of the Fourier-transformed density contrast \( \delta_k \) in the linear regime is separable, allowing us to write:

\[
\delta_k(\eta) = D(\eta)\delta_k(\eta_0)
\]  
(5)
where $D(\eta)$ is the growth factor and $\eta_0$ is the conformal time today. From the continuity equation for matter, $ikv_k = \delta_k$, we see that velocities are probes of the time derivative of $D$.

The correlation between radial velocity components of two clusters at locations $(r_i, \hat{\gamma}_i)$ and $(r_j, \hat{\gamma}_j)$ relative to the observer is given by [11]:

$$
\Psi_{ij} = \langle \hat{\gamma}_i \cdot \mathbf{v}(x_i) \mathbf{v}(x_j) \cdot \hat{\gamma}_j \rangle = \Psi_{i} \cos \theta + \langle \Psi_{\parallel, i} - \Psi_{\perp, i} \rangle R(\theta, r_i, r_j)
$$

$$
R(\theta, r_i, r_j) = \frac{(r_i^2 + r_j^2) \cos \theta - r_i r_j (1 + \cos^2 \theta)}{r_i^2 + r_j^2 - 2r_i r_j \cos \theta}
$$

$$
\Psi_{\perp, i} = \frac{\dot{D}(r_i) \dot{D}(r_j)}{2\pi^2} \int dk |k_\perp|^2 K_{\perp, i}(kr)
$$

where $\cos \theta = \hat{\gamma}_i \cdot \hat{\gamma}_j$, $K_{\parallel}(kr) = j_1(kr)/(kr)$, $K_{\perp}(kr) = j_0(2kr)/(kr)$ is the comoving distance between the clusters and an overdot symbolizes $d/d\eta$. Figure 1 shows $\Psi_{\perp}(\theta)$ for four different flat cosmologies at redshift $z = 1$.

Cosmology dependence in Eq. 6 arises from: (i) the redshift–distance relation $r(z)$ and (ii) the time–derivative of the growth factor, $\dot{D}$. The latter has weak dependence on $w$ as we can see from the highly accurate analytic approximation [10]:

$$
\dot{D}(z) = D(z)a(z)H(z)[\Omega_m(z)]^{\alpha(w)}
$$

because $\alpha$ only ranges from about 6/11 to 3/5 for $w = -1$ to $w = 0$, and because $a(z)D(z)H(z)$ is also fairly insensitive to $w$ for $z \lesssim 0.5$. In contrast, $\dot{D}$ is highly sensitive to $\Omega_m$.

5. Forecasted Parameter Errors

For nearly uncorrelated cluster velocities, from, e.g., a very sparse survey, we can easily estimate the expected error variance on $\Omega_m$ from measurement of $N$ clusters with expected peculiar velocity variance $\Psi_0(z_i)$:

$$
(\Delta \Omega_m)^2 = \left( \sum_i \left( \frac{\partial \Psi_0(z_i)}{\partial \Omega_m} \right)^2 \frac{1}{2(\Psi_0(z_i) + \sigma_v^2)} \right)^{-1}
$$

$$
\simeq \frac{800}{N} (.01)^2
$$

where the last equality assumes $N$ clusters with $\sigma_v^2 \ll \Psi_0$ all at $z = 1$, and $\partial \ln \Psi_0/\partial \Omega_m \simeq 5$.

We now turn to a Fisher matrix analysis of two survey types. For a dense survey, we sample 820 neighboring clusters from the Hubble Volume Lightcone Simulation Cluster Catalog [11] over an area of 80 sq. deg. at redshift $z \simeq 1$. For a sparse survey, we construct a grid of 820 clusters from $z = 0.1$ to 1.0 over an area of 2600 sq. deg such that all comoving cluster separations are greater than 200 Mpc.

We calculate the expected covariances in linear theory for a density field smoothed with a 5 Mpc radius tophat filter. We calculate their partial derivatives by finite difference. The models we difference have fixed COBE normalization [12]. All our results in Figure 2 assume a noise level of 100 km/s.

As expected, $\Omega_m$ is measured very well. Neither $h$ nor $w$ are well constrained; nor do they lead to significant confusion in the $\Omega_m$ measurement. In addition the results are very similar for our two different survey types, with the sparse survey (dashed lines) marginally more constrai-
Figure 2. Forecasted 1 and 2–σ contours for the dense survey (solid) and the sparse survey with $(dN/dz)\Delta z$ as shown (dashed). For each two-parameter contour, the third parameter is held fixed. [The drop in $dN/dz$ in the final bin is due to a quirk in our sampling algorithm.]

6. Non–linear and biasing effects

We expect that linear theory provides only a rough guide to the constraints possible from peculiar velocity measurements. Numerically–determined variances agree with linear theory predictions to \(~40\% in the present epoch [13]. We plan to include non–linear and biasing effects by use of the PINOCCHIO gravitational instability code [14]. With PINOCCHIO we will obtain highly accurate calculations of the velocity correlation functions in several models and once again use finite difference to calculate Fisher matrices.

7. Conclusions

Cluster peculiar velocity measurements are highly sensitive to $\Omega_m$ and largely insensitive to $h$ and $w$. Sparse sampling is probably preferable to dense sampling. Velocity determinations of \(\sim 800\) clusters can potentially be used to determine $\Omega_m$ to \(\sim 0.03\%\). This $w$–independent measurement of $\Omega_m$ may be useful for improving $w$ constraints from observations of type Ia supernovae [15].

We would like to thank B. Holden, G. Holder, M. Kaplinghat, C.-P. Ma, J. Mohr, S. Meyer, M. White, G. Wilson and I. Zehavi for very useful conversations.

REFERENCES

1. Z. Haiman, J. Mohr, G. Holder (2001) ApJ 553: 545
2. e.g., Y.P. Jing, G. Börner, Y. Suto (2002) ApJ 564: 15; S. Zaroubi, et al., (2001) MNRAS 326: 375
3. R.A. Sunyaev, Ya.B. Zel’dovich (1980) Sov. Astron. Lett. 6: 737; M. Birkinshaw (1998) Phys. Reprts. 310: 97
4. L. Grego et al. (2001) ApJ 552: 2
5. W. Holzapfel, et al., (1997) ApJ 479: 17; W. Holzapfel, et al. (1997) ApJ 481: 35
6. N. Aghanim, K. Górski, J.-L. Puget (2001) A&A 374: 1-12; M.G. Haehnelt, M. Tegmark (1996) MNRAS 279: 545
7. J.P. Ostriker, E.T. Vishniac (2000) ApJL 306: 51; W. Hu, M. White (1997) ApJ 479: 568; A.H. Jaffe, M. Kamionkowski (1998) Phys. Rev. D 58: 043001; V. Springel, M. White, L. Hernquist (2001) ApJ 549: 681; C.-P. Ma, J. Fry (2002) PRL 88: 211301
8. G. Holder (2002) in preparation.
9. K. Górski (1988) ApJ 332: L7
10. L. Wang, P. Steinhardt (1998) ApJ 508: 483
11. J.M. Colberg, et al., (The Virgo Consortium) (2000), MNRAS 319: 209
12. E. Bunn, M. White (1997) ApJ 480: 6
13. J.M. Colberg, et al., (The Virgo Consortium) (2000) MNRAS 313: 229
14. P. Monaco, T. Theuns, G. Taffoni (2002) MNRAS 331: 587
15. S. Perlmutter et al., (1999) ApJ 517: 565