ABSTRACT
With recent advancements in edge computing capabilities, there has been a significant increase in utilizing the edge cloud for event-driven and time-sensitive computations. However, large-scale edge computing networks can suffer substantially from unpredictable and unreliable computing resources which can result in high variability of service quality. Thus, it is crucial to design efficient task scheduling policies that guarantee quality of service and the timeliness of computation queries. In this paper, we study the problem of computation offloading over unknown edge cloud networks with a sequence of timely computation jobs. Motivated by the MapReduce computation paradigm, we assume each computation job can be partitioned to smaller Map functions that are processed at the edge, and the Reduce function is computed at the user after the Map results are collected from the edge nodes. We model the service quality (success probability of returning result back to the user within deadline) of each edge device as function of context (collection of factors that affect edge devices). The user decides the computations to offload to each device with the goal of receiving a recoverable set of computation results in the given deadline. Our goal is to design an efficient edge computing policy in the dark without the knowledge of the context or computation capabilities of each device. By leveraging the coded computing framework in order to tackle failures or stragglers in computation, we formulate this problem using contextual-combinatorial multi-armed bandits (CC-MAB), and aim to maximize the cumulative expected reward. We propose an online learning policy called online coded edge computing policy, which provably achieves asymptotically-optimal performance in terms of regret loss compared with the optimal offline policy for the proposed CC-MAB problem. In terms of the cumulative reward, it is shown that the online coded edge computing policy significantly outperforms than other benchmarks via numerical studies.

1 INTRODUCTION
Recent advancements in edge cloud has enabled users to offload their computations of interest to the edge for processing. Specifically, there has been a significant increase in utilizing the edge cloud for event-driven and time-sensitive computations (e.g., IoT applications and cognitive services), in which the users increasingly demand timely services with deadline constraints, i.e., computations of requests have to be finished within specified deadlines. However, large-scale distributed computing networks can substantially suffer from unpredictable and unreliable computing infrastructure which can result in high variability of computing resources, i.e., service quality of the computing resources may vary over time. The speed variation has several causes including hardware failure, co-location of computation tasks, communication bottlenecks, etc [1, 35]. While edge computing has offered a novel framework for computing service provisioning, a careful design of task scheduling policy is still needed to guarantee the timeliness of task processing due to the increasing demand on real-time response of various applications and the unknown environment of the network.

To take advantage of the parallel computing resources for reducing the total latency, the applications are often modeled as a MapReduce computation model, i.e., the computation job can be partitioned to some smaller Map functions which can be distributedly processed by the edge devices. Since the data transmissions between the edge devices can result in large latency delay, it is often the case that the user computes the Reduce function on the results of the Map functions upon receiving the computation results of edge devices to complete the computation job.

In this paper, we study the problem of computation offloading over edge cloud networks with particular focus on unknown environment of computing resources and timely computation jobs. We consider a dynamic computation model, where a sequence of computation jobs needs to be computed over the (encoded) data that is distributedly stored at the edge nodes. More precisely, in an online manner, computation jobs with given deadlines are submitted to the edge network, i.e., each computation has to be finished within the given deadline. We assume the service quality (success probability of returning results back to the user in deadline) of each edge device is parameterized by a context (collection of factors that affect each edge device). The user aims at selecting edge devices from the available edge devices and decide what to be computed by the selected edge devices, such that the user can receive a recoverable set of computation results in the given deadline. Our goal is then to design an efficient edge computing policy that maximizes the cumulative expected reward, where the expected reward collected at each round is a linear combination of the success probability of the computation and the amount of computational resources used (with negative sign).

One significant challenge in this problem is the joint design of (1) data storage scheme to provide robustness against unknown behaviors of edge devices; (2) computation offloading to edge device; and (3) an online learning policy for making the offloading decisions based on the past observed events. In our model, the computation capacities of the devices (e.g., how likely the computation can be returned to the user within the deadline) are unknown to the user.

As the main contributions of the paper, we introduce a coded computing framework in which the data is encoded and stored at the edge devices in order to provide robustness against unknown computation capabilities of the devices. The key idea of coded computing is to encode the data and design each worker’s computation
task such that the fastest responses of any $k$ workers out of total of $n$ workers suffice to complete the distributed computation, similar to classical coding theory where receiving any $k$ symbols out of $n$ transmitted symbols enables the receiver to decode the sent message. Under coded computing framework, we formulate a contextual-combinatorial multi-armed bandit (CC-MAB) problem for the edge computing problem, in which the Lagrange coding scheme is utilized for data encoding [33]. Then, we propose a policy called **online coded edge computing policy**, and show that it achieves asymptotically optimal performance in terms of regret loss compared with the optimal offline policy for the proposed CC-MAB problem by the careful design of the policy parameters. To prove the asymptotic optimality of online coded edge computing policy, we divide the expected regret to three regret terms which are due to (1) exploration phases, (2) bad selections of edge devices in exploitation phases, and (3) good selections of edge devices in exploitation phases; then we bound these three regrets separately.

In addition to proving the asymptotic optimality of online coded edge computing policy, we carry out numerical studies. In terms of the cumulative reward, the results show that the online coded edge computing policy significantly outperforms other benchmarks.

### 1.1 Related Prior Work

Next, we provide a brief literature review that covers three main lines of work: task scheduling over cloud networks, coded computing, and the multi-armed bandit problem.

In the dynamic task scheduling problem, jobs arrive to the network according to a stochastic process, and get scheduled dynamically over time. The first goal in task scheduling is to find a throughput-optimal scheduling policy (see e.g. [7]), i.e. a policy that stabilizes the network, whenever it can be stabilized. For example, Max-Weight scheduling, first proposed in [5, 30], is known to be throughput-optimal for wireless networks, flexible queueing networks [24], data centers networks [22] and dispersed computing networks [31]. Moreover, there have been many works which focus on task scheduling problem with deadline constraints over cloud networks (see e.g. [11]).

Coded computing broadly refers to a family of techniques that utilize coding to inject computation redundancy in order to alleviate the various issues that arise in large-scale distributed computing. In the past few years, coded computing has had a tremendous success in various problems, such as straggler mitigation and bandwidth reduction (e.g., [6, 12, 15, 16, 20, 21, 29, 34]). Coded computing has also been expanded in various directions, such as heterogeneous networks (e.g., [26]), partial stragglers (e.g., [8]), secure and private computing (e.g., [3, 33]) and distributed optimization (e.g., [13]). In a dynamic setting, [32] considers a coded computing framework with deadline constraints and develops a learning strategy that can adaptively assign computation loads to cloud devices. In this paper, we go beyond the two states Markov model considered in [32], and make a substantial progress by combining the ideas of coded computing with contextual-combinatorial MAB, which is a more general framework that does not make any strong assumption (e.g., Markov model) on underlying model for the speed of edge devices.

The multi-armed bandit (MAB) problem has been widely studied to address the critical tradeoff between exploration and exploitation in sequential decision making under uncertainty of environment [14]. The goal of MAB is to learn the single optimal arm among a set of candidate arms of a priori unknown rewards by sequentially selecting one arm each time and observing its realized reward [2]. Contextual bandit problem extends the basic MAB by considering the context-dependent reward functions [18, 27, 28]. The combinatorial bandit problem is another extension of the MAB by allowing multiple-play (select a set of arms) each time [9, 17]. The contextual-combinatorial MAB problem considered in this paper has also received much attention recently [4, 19, 23, 25]. However, [19, 25] assume that the reward of an action is a linear function of the contexts different from the reward function considered in our paper. [23] assumes the arm set is fixed throughout the time but the arms (edge devices) may appear and disappear across the time in edge networks. [4] considers a CC-MAB problem for the vehicle cloud computing, in which the tasks are deadline-constrained. However, the task replication technique considered in [4] is to replicate the “whole job” to multiple edge devices instead of offloading smaller decomposed tasks of the job. This replication technique doesn’t take advantage of parallelism of computational resources that our computation model does. Moreover, the success probability term (for receiving any $k$ results out of $n$ results) of reward function considered in our paper is more general than the success probability term (for receiving any 1 result out of $n$ results) of reward function considered in [4].

### 2 SYSTEM MODEL

#### 2.1 Computation Model

We consider an edge computing problem, in which a user offloads its computation to an edge network in an online manner, and the computation is executed by the edge devices. In particular, there is a given deadline for each round of computation, i.e., computation has to be finished within the given deadline.

As shown in Fig. 1, The considered edge network is composed of a user node and a set of edge devices. There is a dataset $X$ which is divided to $X_1, X_2, \ldots, X_k$. Specifically, each $X_j$ is an element in a vector space $\mathbb{F}$ over a field $\mathbb{F}$. Dataset $X_1, X_2, \ldots, X_k$ is prestoned in each edge device where the prestored data for each edge device can be possibly a function of $X_1, X_2, \ldots, X_k$.

Let $\{1, 2, \ldots, T\}$ be the index of the user’s computation jobs received by the edge network over $T$ time slots. In each round $t$ (or time slot in a discrete-time system), the user has a computation job denoted by function $g_t$. Especially, we assume that function $g_t$ can be computed by

$$g_t(X_1, X_2, \ldots, X_k) = h_t(f_t(X_1), f_t(X_2), \ldots, f_t(X_k))$$

where function $g_t$ and $f_t$ (with degree $\deg(f_t)$) are multivariate polynomial functions with vector coefficients. In such edge network and motivated by a MapReduce setting, the user is interested in computing Map functions $f_t(X_1), f_t(X_2), \ldots, f_t(X_k)$ in each round $t$ and the user computes Reduce function $h_t$ on those results of Map functions to obtain $g_t(X_1, X_2, \ldots, X_k)$. 


2.2 Network Model

We note that the considered computation model naturally appears in many machine learning applications which use gradient-type algorithms. For example, in linear regression problems, the user wants to compute \( f_t(X_j) = X_j^T (X_j \omega_t - \tilde{y}_t) \), which is the gradient of the quadratic loss function \( \frac{1}{2} || X_j \omega_t - \tilde{y}_t ||^2 \) with respect to the weight vector \( \omega_t \) in round \( t \). To complete the update \( \omega_{t+1} = g_t(X_1, \ldots, X_k) = \omega_t - \beta_t \sum_{j=1}^{N} f_t(X_j) \), the user has to collect the computation results \( f_t(X_1), f_t(X_2), \ldots, f_t(X_k) \).

2.2 Network Model

In an edge computing network, whether a computation result can be returned to the user depends on many factors. For example, the computation load of an edge device influences its runtime; the output size of the computation task affects the transmission delay, etc. Such factors are referred to as context throughout the paper. The impact of each context on the edge devices is unknown to the user. More specifically, the computation service of each edge device is modeled as follows.

Let \( \Phi_T \) be the context space of computation tasks which includes the information of computation task, e.g., size of input/output, size of computation, and deadline, etc. Let \( \Phi_S \) be the context space of edge devices which includes the information related to edge devices such as computation speed, bandwidth, etc. Let \( \Phi = \Phi_T \times \Phi_S \) be the joint context space, where \( \Phi = \{0, 1\}^D \) and \( D \) is the dimension of context space \( \Phi \) without loss of generality.

In each round \( t \), let \( \mathcal{V}' \) denote the set of edge devices available to the user for computation, i.e., the available set of devices might change over time. Moreover, we denote \( b^t \) the budget (maximum number of devices to be used) in round \( t \). The service delay (computation time plus transmission time) of each edge device \( v \) is parameterized by a given context \( \phi_v^t \in \Phi \). We denote by \( c_v^t \) the service delay of edge device \( v \), and \( d^t \) the computation deadline in round \( t \). Let \( q_v^t \leq d^t \) be the indicator that the service delay of edge device \( v \) is smaller than or equal to the given deadline \( d^t \) in round \( t \). Also, let \( \mu(q_v^t) = \mathbb{P}[q_v^t \leq d^t] \) be the success probability that edge device \( v \) returns the computation result back to the user within deadline \( d^t \), and \( \mathcal{P} = \{(\phi_v^t), v \in \mathcal{V}'\} \) be the collection of success probabilities of edge devices in round \( t \).

2.3 Problem Statement

Let \( \mathcal{V} = \{1, 2, \ldots, |\mathcal{V}|\} \) be the set of all edge devices in the network. Given context \( \mathcal{P} = \{(\phi_v^t), v \in \mathcal{V}'\} \) of the edge devices available to the user in round \( t \), the goal of the user is to select a subset of edge devices from the available set of edge devices \( \mathcal{V}' \subseteq \mathcal{V} \), and decide what to be computed by each selected edge device, such that a recoverable (or decodable as will be clarified later) set of computation results \( f_t(X_1), \ldots, f_t(X_k) \) can be returned to the user within deadline \( d^t \).

3 ONLINE CODED EDGE COMPUTING

In this section, we introduce a coded computing framework for the edge computing problem, and formulate the problem as a contextual-combinatorial multi-armed bandit (CC-MAB) problem. Then, we propose a policy called online coded edge computing policy, which is a context-aware learning algorithm.

3.1 Lagrange Coded Computing

For the data storage of edge devices, we leverage a linear coding scheme called the Lagrange coding scheme [33] which is demonstrated to simultaneously provide resiliency, security, and privacy in distributed computing. We start with an illustrative example.

In each round \( t \), we consider a computation job which consists of computing quadratic functions \( f_t(X_j) = X_j^T (X_j \omega_t - \tilde{y}_j) \) over available edge devices \( \mathcal{V}' = \{1, 2, \ldots, 6\} \), where input dataset \( X \) is partitioned to \( X_1, X_2, \ldots \). Then, we define function \( m \) as follows:

\[
m(z) = X_1 \frac{z - 1}{0 - 1} + X_2 \frac{z - 0}{1 - 0} = z (X_2 - X_1) + X_1,
\]

in which \( m(0) = X_1 \) and \( m(1) = X_2 \). Then, we encode \( X_1 \) and \( X_2 \) to \( X_v = m(v - 1) \), i.e., \( X_1 = X_2 = X_v \). This means that \( X_1 = -X_2 + 2X_2 \), \( X_3 = -X_4 + 2X_4 \), \( X_5 = -3X_1 + 4X_4 \) and \( X_6 = -4X_1 + 5X_2 \). Each edge device \( v \in \{1, 2, \ldots, 6\} \) prestores an encoded data chunk \( X_v \) locally. If edge device \( v \) is selected in round \( t \), it computes \( f_t(X_v) = X_v^T (X_v \omega_t - \tilde{y}_j) \) and returns the result back to the user upon its completion. We note that \( f_t(X_v) = f_t(m(v - 1)) \) is an evaluation of the composition polynomial \( f_t(m(z)) \), whose degree at most 2, which implies that \( f_t(m(z)) \) can be recovered by any 3 results via polynomial interpolation. Then we have \( f_t(X_1) = f_t(m(0)) \) and \( f_t(X_2) = f_t(m(1)) \).

Formally, we describe Lagrange coding scheme as follows: We first select \( k \) distinct elements \( \beta_1, \beta_2, \ldots, \beta_k \) from \( \mathbb{F} \), and let \( m \) be the respective Lagrange interpolation polynomial

\[
m(z) = \sum_{j=1}^{k} X_j \prod_{l \in [k] \setminus \{j\}} \frac{z - \beta_l}{\beta_j - \beta_l},
\]

where \( u : \mathbb{F} \rightarrow \mathcal{V} \) is a polynomial of degree \( k - 1 \) such that \( m(\beta_j) = X_j \). Recall that \( \mathcal{V} = \bigcup_{t=1}^{T} \mathcal{V}' \) which is the set of all edge devices. To encode input \( X_1, X_2, \ldots, X_k \), we select \( |\mathcal{V}| \) distinct elements \( \alpha_1, \alpha_2, \ldots, \alpha_{|\mathcal{V}|} \) from \( \mathbb{F} \), and encode \( X_1, X_2, \ldots, X_k \) to \( X_v = m(\alpha_v) \).
for all $v \in |V|$, i.e.,

$$
\hat{X}_v = m(a_v) = \sum_{j=1}^{k} X_j \prod_{i \in [j], \{j\}} \alpha_{v} - \beta_{j}.
$$

(3)

Each edge device $v \in V$ stores $\hat{X}_v$, locally. If edge device $v$ is selected in round $t$, it computes $\hat{f}_t(\hat{X}_v)$ and returns the result back to the user upon its completion. Then, the optimal recovery threshold $Y^t$ using Lagrange coding scheme is

$$
Y^t = (k - 1)\deg(f_t) + 1
$$

(4)

which guarantees that the computation tasks $\hat{f}_t(X_1), \ldots, \hat{f}_t(X_k)$ can be recovered when the user receives any $Y^t$ results from the edge devices. The encoding of Lagrange coding scheme is oblivious to the computation task $f_t$. Also, decoding and encoding process in Lagrange coding scheme rely on polynomial interpolation and evaluation which can be done efficiently.

### 3.2 CC-MAB for Coded Edge Computing

Now we consider a coded computing framework in which the Lagrange coding scheme is used for data encoding, i.e., each edge device $v$ prestores encoded data $X_v$. The encoding process is only performed once for dataset $X_1, \ldots, X_k$. After Lagrange data encoding, the size of input data and computation of each user don’t change, i.e., context $\phi'_i$ of each edge device $v$ remains the same.

More specifically, we denote by $\mathcal{A}^t$ the set of devices which are selected in round $t$ for computation. In each round $t$, the user picks a subset of devices $\mathcal{A}^t$ from all available devices $V^t$, and we call $\mathcal{A}^t \subseteq V^t$ the “offloading decision”. The reward $r(\mathcal{A}^t)$ achieved by using offloading decision $\mathcal{A}^t$ is defined as follows:

$$
r(\mathcal{A}^t) = \begin{cases} 1 - \eta|\mathcal{A}^t|, & \text{if } \sum_{v \in \mathcal{A}^t} \phi'_v \geq Y^t \\ -\eta|\mathcal{A}^t|, & \text{if } \sum_{v \in \mathcal{A}^t} \phi'_v < Y^t \end{cases}
$$

(5)

where the term $|\mathcal{A}^t|$ captures the cost of using offloading decision $\mathcal{A}^t$ with the unit cost $\eta$ for using one edge device, and $Y^t$ is the optimal recovery threshold defined in (4). Then, the expected reward denoted by $u(\mu^t, \mathcal{A}^t)$ in round $t$ can be rewritten as follows:

$$
u(\mu^t, \mathcal{A}^t) = \sum_{s=Y^t}^{\mathcal{A}^t} \sum_{\mathcal{A} \subseteq \mathcal{A}^t, |\mathcal{A}|=s} \sum_{v \in \mathcal{A}} \mu(\phi'_v) \prod_{v' \in \mathcal{A} \setminus \mathcal{A}} (1 - \mu(\phi'_v)) - \eta|\mathcal{A}^t|
$$

(6)

where the first term of the expected reward of an offloading decision is the success probability that there are at least $Y^t$ computation results received by the user for LCC decoding.

Consider an arbitrary sequence of computation jobs indexed by $[1, 2, \ldots, T]$ for which the user makes offloading decisions $\{\mathcal{A}^t\}_{t=1}^T$. To maximize the expected cumulative reward, we introduce a contextual-combinatorial multi-armed bandit (CC-MAB) problem for coded edge computing defined as follows:

**CC-MAB for Coded Edge Computing:**

$$
\max_{\{\mathcal{A}^t\}_{t=1}^T} \sum_{t=1}^{T} u(\mu^t, \mathcal{A}^t)
$$

s.t. $\mathcal{A}^t \subseteq V^t$, $|\mathcal{A}^t| \leq b^t$, $\forall t \in [T]
$$

(7) (8)

where the constraint (8) indicates that the number of edge devices in $\mathcal{A}^t$ cannot exceed the budget $b^t$ in round $t$. The proposed CC-MAB problem is equivalent to solving an independent subproblem in each round $t$ as follows:

$$
\max_{\mathcal{A}^t} \sum_{s=Y^t}^{\mathcal{A}^t} \sum_{\mathcal{A} \subseteq \mathcal{A}^t, |\mathcal{A}|=s} \sum_{v \in \mathcal{A}} \mu(\phi'_v) \prod_{v' \in \mathcal{A} \setminus \mathcal{A}} (1 - \mu(\phi'_v)) - \eta|\mathcal{A}^t|
$$

s.t. $\mathcal{A}^t \subseteq V^t$, $|\mathcal{A}^t| \leq b^t$.

We note that the expected reward function considered in [4] is a submodular function, which can be maximized by a greedy algorithm. However, the expected reward function defined in (6) is more general which cannot be maximized by greedy algorithm.

### 3.3 Optimal Offline Policy

We now assume that the success probability of each edge device $v \in V^t$ is known to the user. In round $t$, to find the optimal $\mathcal{A}^t$, we present the following intuitive lemma proved in Appendix E.

**Lemma 3.1.** Without loss of generality, we assume $\mu(\phi'_1) \geq \mu(\phi'_2) \geq \cdots \geq \mu(\phi'_{|V^t|})$ in round $t$. Considering all possible sets $\mathcal{A}_g^t \subseteq V^t$ with fixed cardinality $n_g$, the optimal $\mathcal{A}_g^t$ with cardinality $n_g$ that achieves the largest expected reward $u(\mu^t, \mathcal{A}_g^t)$ is

$$
\mathcal{A}_g^t = \{1, 2, \ldots, n_g\}
$$

(9)

which represents the set of $n_g$ edge devices having largest success probability $\mu(\phi'_i)$ among all the edge devices.

By Lemma 3.1, to find the optimal set $\mathcal{A}^t$, we can only focus on finding the optimal size of $\mathcal{A}^t$. Since there are only $b^t$ choices for size of $|\mathcal{A}^t|$ (i.e., $1, 2, \ldots, b^t$), this procedure can be done by a linear search with the complexity linear in the number of edge devices $|V^t|$. We present the optimal offline policy in Algorithm 1.

**Algorithm 1: Optimal Offline Policy**

**Input:** $V^t$, $b^t$, $Y^t$, $\mu(\phi'_i), v \in V^t$;

**Initialization:** $\mathcal{A} = \emptyset$, $\mathcal{A}_\text{opt} = \emptyset$, $u_\text{opt} = 0$;

Sort $\mu^t : \mu(\phi'_1) \geq \mu(\phi'_2) \geq \cdots \geq \mu(\phi'_{|V^t|})$;

$\mathcal{A} \leftarrow \{1, 2, \ldots, Y^t\}$;

$\mathcal{A}_\text{opt} \leftarrow \{1, 2, \ldots, Y^t\}$;

$u_\text{opt} \leftarrow u(\mu^t, \mathcal{A})$;

for $z \leftarrow Y^t + 1$ to $b^t$ do

$\mathcal{A} \leftarrow \mathcal{A} \cup \{z\}$;

if $u(\mu^t, \mathcal{A}) > u_\text{opt}$ then

$\mathcal{A}_\text{opt} \leftarrow \mathcal{A}$;

$u_\text{opt} \leftarrow u(\mu^t, \mathcal{A})$

end

end

return $\mathcal{A}_\text{opt}$
the regret of the policy which is formally defined as follows:

\[
R(T) = \mathbb{E}\left[\sum_{t=1}^{T} r(\mathcal{A}^t) - r(\mathcal{A}^t)\right]
\]

(10)

\[
= \sum_{t=1}^{T} u(\mu', \mathcal{A}^t) - u(\mu', \mathcal{A}^t).
\]

(11)

In general, the user doesn’t know in advance the success probabilities of edge devices due to the uncertainty of the environment of edge network. In the following subsection, we will propose an online learning policy for the proposed CC-MAB problem which enables the user to learn the success probabilities of edge devices over time by observing the service quality of each selected edge device, and then make offloading decisions adaptively.

3.4 Online Coded Edge Computing Policy

Now, we describe the proposed online edge computing policy. The proposed policy has two parameters $h_T$ and $K(t)$ to be designed, where $h_T$ decides how we partition the context space, and $K(t)$ is a deterministic and monotonically increasing function, used to identify the under-explored context. The proposed online coded edge computing policy (see Algorithm 2) is performed as follows:

**Initialization Phase:** Given parameter $h_T$, the proposed policy first creates a partition denoted by $\mathcal{P}_T$ for the context space $\Phi$, which splits $\Phi$ into $(h_T)^D$ sets. Each set is a $D$-dimensional hypercube of size $\frac{1}{h_T} \times \cdots \times \frac{1}{h_T}$. For each hypercube $p \in \mathcal{P}_T$, the user keeps a counter $C_t(p)$ which is the number of selected edge devices that have context $\phi^t_v$ in hypercube $p$ before round $t$. Moreover, the policy also keeps an estimated success probability denoted by $\hat{\mu}^t(p)$ for each hypercube $p$. Let $Q^t_t = \{q^t_v : \phi^v_v \in p, v \in \mathcal{A}, \tau \in 1, \ldots, t - 1\}$ be the set of observed indicators (successful or not) of edge devices with context in $p$ before round $t$. Then, the estimated success probability for edge devices with context $\phi^t_v \in p$ is computed by $\hat{\mu}^t(p) = \frac{1}{Q^t_t} \sum_{q \in Q^t_t} q$.

In each round $t$, the proposed policy has the following phases:

**Hypercube Identification Phase:** Given the contexts of all available edge devices $\phi^t_v \in \phi^t_v \in \mathcal{V}^t$, the policy determines the hypercubes $p_v^t \in \mathcal{P}_T$ for each context $\phi^t_v$, such that $\phi^t_v \in p_v^t$. We denote by $p_v^t = \{p_v^t\}_{v \in \mathcal{V}^t}$ the collection of these identified hypercubes in round $t$. To check whether there exist hypercubes $p \in p_v^t$ that have not been explored sufficiently, we define the under-explored hypercubes in round $t$ as follows:

\[
\mathcal{P}_T = \{p \in \mathcal{P} : \exists v \in \mathcal{V}, \phi^t_v \in p, C_t(p) \leq K(t)\}.
\]

(12)

Also, we denote by $\mathcal{V}_U$ the set of edge devices which fall in the under-explored hypercubes, i.e., $\mathcal{V}_U = \{v \in \mathcal{V} : p_v^t \in \mathcal{P}_T\}$.

Depending on $\mathcal{V}_U$ in round $t$, the proposed policy then either enters an exploration phase or an exploitation phase.

**Exploration Phase:** If $\mathcal{V}_U$ is non-empty, the policy enters an exploration phase. If set $\mathcal{V}_U$ contains at least $b^t$ edge devices (i.e., $|\mathcal{V}_U| \geq b^t$), then the policy randomly selects $b^t$ edge devices from $\mathcal{V}_U$. If $\mathcal{V}_U$ contains fewer than $b^t$ edge devices ($|\mathcal{V}_U| < b^t$), then the policy selects all edge devices from $\mathcal{V}_U$. To fully utilize the budget $b^t$, the remaining $(b^t - |\mathcal{V}_U|)$ ones are picked from the edge devices with the highest estimated success probability among the remaining edge devices in $\mathcal{V} \setminus \mathcal{V}_U$.

Algorithm 2: Online Coded Edge Computing Policy

```
Input: $T, h_T, K(t)$
Initialization: $\mathcal{P}_T, C(p) = 0, \hat{\mu}(p) = 0, \forall p \in \mathcal{P}_T$

for $t \leftarrow 1$ to $T$
do

\text{Observe edge device $V^t$ and contexts $\phi^t_v$}
\text{Find $p^t_v = \{p^t_v\}_{v \in V^t}, p^t_v \in \mathcal{P}_T$ such that $\phi^t_v \in p^t_v$}
\text{Identify $\hat{\mu}(p^t_v)$ and $\hat{\mu}(p^t_v)$}
\text{if $|\mathcal{V}_U| \neq 0$ then}

\text{if $|\mathcal{V}_U| \geq b^t$ then}
\text{\textbf{Exploitation Phase:} If $|\mathcal{V}_U| \geq b^t$, the policy enters an}
\text{\textbf{Algorithm 2:} Online Coded Edge Computing Policy}

4 ASYMPTOTICAL OPTIMALITY OF ONLINE CODED EDGE COMPUTING POLICY

In this section, by providing the design of policy parameters $h_T$ and $K(t)$, we show that the online coded edge computing policy achieves a sublinear regret in the time horizon $T$ which guarantees an asymptotically optimal performance, i.e., $\lim_{T \to \infty} \frac{R(T)}{T} = 0$.

To conduct the regret analysis for the proposed CC-MAB problem, we make the following assumption on the success probabilities of edge devices in which the devices’ success probabilities are similar if they have similar contexts. This natural property is formalized by the Hölder condition defined as follows:

**Assumption 1 (Hölder Condition).** A real function $f$ on $D$-dimensional Euclidean space satisfies a Hölder condition, when there exist $L > 0$ and $\alpha > 0$ for any two contexts $\phi, \phi' \in \Phi$, such that $|f(\phi) - f(\phi')| \leq L \|\phi - \phi'\|^\alpha$, where $\| \cdot \|$ is the Euclidean norm.

Under Assumption 1, we choose parameters $h_T = \left\lceil \log_{1 - \alpha} T \right\rceil$ for the partition of context space $\Phi$ and $K(t) = t^{\frac{1}{\alpha - 1}}$ in round $t$ for identifying the under-explored hypercubes of the context. We present the following theorem which shows that the proposed
online coded edge computing policy has a sublinear regret upper bound.

**Theorem 4.1 (Regret Upper Bound).** Let \( K(t) = t \frac{\log(t)}{\log(T)} \) and \( h_T = [T \frac{\log(T)}{\log(T)}] \). If the Holder condition holds, the regret \( R(T) \) is upper-bounded as follows:

\[
R(T) \leq (1 + \eta B) \frac{\pi^2}{3} \sum_{k=1}^{B} \binom{|V|}{k} \left( (3D) + \frac{6a + 2D}{2\gamma + D} \right) \text{BMT} + \frac{\text{max} \frac{a}{\gamma + D}}{2\gamma + D},
\]

where \( B = \text{max} \frac{a}{\gamma + D} \) and \( M = \text{max} \frac{a}{\gamma + D} \). The dominant order of the regret \( R(T) \) is \( O(T \text{max} \frac{a}{\gamma + D} \log(T)) \) which is sublinear to \( T \).

Proof. We first define the following terms. For each hypercube \( p \in P_T \), we define \( \overline{p} = \sup_{\phi \in \mathcal{F}} \mu(\phi) \) and \( \mu = \inf_{\phi \in \mathcal{F}} \mu(\phi) \) as the best and worst success probabilities over all contexts \( \phi \in \mathcal{F} \). Also, we define the context at a hypercenter of a hypercube \( p \) as \( \hat{\phi}_p \) and its success probability \( \mu(p) = \mu(\hat{\phi}_p) \). Given a set of available edge devices \( V^t \), the corresponding context set \( \Phi^t = (\Phi_1^t, \text{v} \in V^t) \) and the corresponding hypercube set \( \mathcal{P}^t = \{ (\Phi_1^t, \text{v} \in V^t) \} \). For each round \( t \), we define \( \overline{\mathcal{P}}^t = \{ (\overline{\Phi}_1^t, \text{v} \in V^t) \} \) and \( \mu^\prime = \{ \mu(\Phi_1^t), \text{v} \in V^t \} \). For each round \( t \), we define set \( \mathcal{A}^t \) which satisfies

\[
\mathcal{A}^t = \arg\max_{\mathcal{A} \subseteq V^t, \mu(\mathcal{A}) \geq \mu^\prime} \left( u(\mathcal{A}) \right)
\]

We then use set \( \mathcal{A}^t \) to identify the set of edge devices which are bad to select. We define

\[
\mathcal{L}^t = \{ G : \mathcal{A} \subseteq V^t, |G| \leq \mu(\mathcal{A}) \}
\]

(14) to be the set of suboptimal subsets of edge hypercube set \( \mathcal{P}^t \), where \( A > 0 \) and \( \theta < 0 \) are the parameters which will be used later in the regret analysis. We call a subset \( G \in L^t \) suboptimal and \( \mathcal{A}_{k}^t \setminus L^t \) near-optimal for \( \mathcal{P}^t \), where \( \mathcal{A}_{k}^t \) denotes the subset of \( V^t \) with size less than \( k^t \). Then the expected regret \( R(T) \) can be divided into three summands:

\[
R(T) = \mathbb{E}[R_{C}(T)] + \mathbb{E}[R_{U}(T)] + \mathbb{E}[R_{N}(T)],
\]

where \( \mathbb{E}[R_{C}(T)] \) is the regret due to exploration phases and \( \mathbb{E}[R_{U}(T)] \) and \( \mathbb{E}[R_{N}(T)] \) both correspond to regret in exploitation phases: \( \mathbb{E}[R_{C}(T)] \) is the regret due to suboptimal choices, i.e., the subsets of edge devices from \( L^t \) are selected; \( \mathbb{E}[R_{U}(T)] \) is the regret due to near-optimal choices, i.e., the subsets of edge devices from \( \mathcal{A}_{k}^t \setminus L^t \). In the following, we prove that each of the three summands is bounded.

First, the following lemma (see the proof in Appendix A) gives a bound for \( \mathbb{E}[R_{C}(T)] \), which depends on the choice of two parameters \( z \) and \( \gamma \).

**Lemma 4.2. (Bound for \( \mathbb{E}[R_{C}(T)] \)).** Let \( K(t) = t^2 \log(t) \) and \( h_T = [T^{1/2}] \), where \( 0 < z < 1 \) and \( 0 < \gamma < \frac{1}{z^2} \). If the algorithm is run with these parameters, the regret \( \mathbb{E}[R_{C}(T)] \) is bounded by

\[
\mathbb{E}[R_{C}(T)] \leq (1 + \eta B) \frac{\pi^2}{3} \sum_{k=1}^{B} \binom{|V|}{k} \left( (3D) + \frac{6a + 2D}{2\gamma + D} \right) \text{BMT} + \frac{\text{max} \frac{a}{\gamma + D}}{2\gamma + D},
\]

where \( B = \text{max} \frac{a}{\gamma + D} \).

Next, the following lemma (see the proof in Appendix B) gives a bound for \( \mathbb{E}[R_{U}(T)] \), which depends on the choice of two parameters \( z \) and \( \gamma \) with an additional condition of these parameters which has to be satisfied.

**Lemma 4.3. (Bound for \( \mathbb{E}[R_{U}(T)] \)).** Let \( K(t) = t^2 \log(t) \) and \( h_T = [T^{1/2}] \), where \( 0 < z < 1 \) and \( 0 < \gamma < \frac{1}{z^2} \). If the algorithm is run with these parameters, Assumption 1 holds, and the additional condition \( 2BMt^{-1/2} \leq A \) is satisfied for all \( 1 \leq t \leq T \), the regret \( \mathbb{E}[R_{U}(T)] \) is bounded by

\[
\mathbb{E}[R_{U}(T)] \leq (1 + \eta B) \frac{\pi^2}{3} \sum_{k=1}^{B} \binom{|V|}{k} \left( (3D) + \frac{6a + 2D}{2\gamma + D} \right) \text{BMT} + \frac{\text{max} \frac{a}{\gamma + D}}{2\gamma + D},
\]

(17) where \( B = \text{max} \frac{a}{\gamma + D} \) and \( M = \text{max} \frac{a}{\gamma + D} \).

Lastly, the following lemma (see the proof in Appendix C) gives a bound for \( \mathbb{E}[R_{N}(T)] \), which depends on the choice of two parameters \( z \) and \( \gamma \).

**Lemma 4.4. (Bound for \( \mathbb{E}[R_{N}(T)] \)).** Let \( K(t) = t^2 \log(t) \) and \( h_T = [T^{1/2}] \), where \( 0 < z < 1 \) and \( 0 < \gamma < \frac{1}{z^2} \). If the algorithm is run with these parameters and Assumption 1 holds, the regret \( \mathbb{E}[R_{N}(T)] \) is bounded by

\[
\mathbb{E}[R_{N}(T)] \leq (1 + \eta B) \frac{\pi^2}{3} \sum_{k=1}^{B} \binom{|V|}{k} \left( (3D) + \frac{6a + 2D}{2\gamma + D} \right) \text{BMT} + \frac{\text{max} \frac{a}{\gamma + D}}{2\gamma + D},
\]

where \( B = \text{max} \frac{a}{\gamma + D} \) and \( M = \text{max} \frac{a}{\gamma + D} \).

Now, let \( K(t) = t^2 \log(t) \) and \( h_T = [T^{1/2}] \), where \( 0 < z < 1 \) and \( 0 < \gamma < \frac{1}{z^2} \), and let \( H(t) = BMt^{-1/2} \). Also, we assume that Assumption 1 holds and the additional condition \( 2BMt^{-1/2} \leq A \) is satisfied for all \( 1 \leq t \leq T \). By Lemmas 4.2, 4.3, and 4.4, the regret \( R(T) \) is bounded as follows:

\[
R(T) \leq (1 + \eta B) \frac{\pi^2}{3} \sum_{k=1}^{B} \binom{|V|}{k} \left( (3D) + \frac{6a + 2D}{2\gamma + D} \right) \text{BMT} + \frac{\text{max} \frac{a}{\gamma + D}}{2\gamma + D},
\]

which has the dominant order \( O(T \text{max} \frac{a}{\gamma + D} \log(T)) \).

□

**5 EXPERIMENTS**

In this section, we demonstrate the impact of the online coded edge computing policy by simulation studies.

Given a dataset partitioned to \( X_1, X_2, \ldots, X_n \), we consider the linear regression problem using the gradient algorithm. It computes the gradient of quadratic loss function \( \frac{1}{2} \| X_\text{opt} - \hat{Y}_\text{opt} \|^2 \) with respect to the weight vector \( \hat{Y}_\text{opt} \) in round \( t \), i.e., \( f(X) = X_\text{opt}^\top (X_\text{opt} - \hat{Y}_\text{opt}) \) for all \( 1 \leq j \leq 5 \). The computation is executed over \( |V| = 20 \) edge devices. In such setting, we have the optimal recovery threshold \( T^{1/4} = 9 \) in each round \( t \).

Motivated by the distribution model proposed in [15, 26] for computation time in cloud networks, we model the success probability of each edge device \( v \in V \) as a shifted exponential function
defined as follows
\[ \mu(\phi_t) = \mathbb{P}(c_t^i \leq d^t) = 1 - e^{-\lambda_t^i(d^t - a_t^i)}, \quad \forall d^t \geq a_t^i, \quad (20) \]
where the context of each edge device consists of the deadline \( d^t \), the shift parameter \( a_t^i > 0 \), and the straggling parameter \( \mu_t^i > 0 \) associated with edge device \( v \). Under this model, the dimension of context space \( D \) is 3. Moreover, for function \( \mu \) defined in (20), it can be shown that the Hölder condition with \( \alpha = 1 \) holds. Then, we run the online coded edge computing policy with parameters
\[ h = \lceil T \rceil \text{ and } K(t) = t^2. \]
In each round \( t \), the deadline \( d^t \in [3, 5] \) (sec), the shift parameter \( a_t^i \in [1, 2] \) (sec), and the straggling parameter \( \mu_t^i \in [0.2, 0.8] \) (1/sec) are chosen uniformly at random. Each edge device \( v \) stores an encoded data chunk \( X_v \), using Lagrange coding scheme. The budget \( b^t \) is fixed to 12 throughout the time for simplicity. The penalty parameter \( \eta \) is 0.01.

For comparison with the online coded edge computing policy, we consider the following benchmarks:

1. **Optimal Offline policy**: Assuming knowledge of the success probability of each edge device in each round, the optimal set of edge devices is selected by using Algorithm 1.
2. **LinUCB [18]**: LinUCB is a contextual-aware bandit algorithm which picks one arm in each round. We obtain a set of edge devices where the context of each edge device consists of the deadline \( d^t \), the shift parameter \( a_t^i > 0 \), and the straggling parameter \( \mu_t^i > 0 \) associated with edge device \( v \). Under this model, the dimension of context space \( D \) is 3. Moreover, for function \( \mu \) defined in (20), it can be shown that the Hölder condition with \( \alpha = 1 \) holds. Then, we run the online coded edge computing policy with parameters
\[ h = \lceil T \rceil \text{ and } K(t) = t^2. \]
In each round \( t \), the deadline \( d^t \in [3, 5] \) (sec), the shift parameter \( a_t^i \in [1, 2] \) (sec), and the straggling parameter \( \mu_t^i \in [0.2, 0.8] \) (1/sec) are chosen uniformly at random. Each edge device \( v \) stores an encoded data chunk \( X_v \), using Lagrange coding scheme. The budget \( b^t \) is fixed to 12 throughout the time for simplicity. The penalty parameter \( \eta \) is 0.01.

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\[ h = \lceil T \rceil \text{ and } K(t) = t^2. \]
In each round \( t \), the deadline \( d^t \in [3, 5] \) (sec), the shift parameter \( a_t^i \in [1, 2] \) (sec), and the straggling parameter \( \mu_t^i \in [0.2, 0.8] \) (1/sec) are chosen uniformly at random. Each edge device \( v \) stores an encoded data chunk \( X_v \), using Lagrange coding scheme. The budget \( b^t \) is fixed to 12 throughout the time for simplicity. The penalty parameter \( \eta \) is 0.01.

3. **UCB [2]**: UCB algorithm is a non-contextual and non-combinatorial algorithm. Similar to LinUCB, we repeat UCB \( b^t \) times to select edge devices.
4. **Random**: A set of edge devices with size of \( b^t \) is selected randomly from the available edge devices in each round \( t \).

Fig. 2 provides the cumulative rewards comparison of the online coded edge computing policy with the other 4 benchmarks. Fig. 3 presents the expected regret of the proposed policy. We make the following conclusions from the results:

- The optimal offline policy achieves the highest reward which gives an upper bound to the other policies. After a period of exploration, the proposed online policy is able to exploit the learned knowledge, and the cumulative reward approaches the upper bound.
- The proposed online coded edge computing policy significantly outperforms other benchmarks by taking into account the context of edge computing network. The expected regret achieved by the proposed policy demonstrates the asymptotic optimality.
- Random and UCB algorithms are not effective since they don’t take the context into account for the decisions. Although LinUCB is a contextual-aware algorithm, it achieves similar cumulative regret as random and UCB algorithms. That is because the success probability model is more general here than the linear functions that LinUCB is tailored for.

### 6 CONCLUSION

Motivated by the volatility edge devices’ computing capabilities and quality of service, and increasing demand for timely event-driven computations, we consider the problem of online computation offloading over unknown edge cloud networks without the knowledge of edge devices’ capabilities. Under the coded computing framework, we formulate a combinatorial-contextual multiarmed bandit (CC-MAB) problem, which aims to maximize the cumulative expected reward. We propose the online coded edge computing policy which provably achieves asymptotically-optimal performance in terms of timely throughput, since the regret loss for the proposed CC-MAB problem compared with the optimal offline policy is sublinear. Finally, we show that the proposed online coded edge computing policy significantly improves the cumulative reward compared to the other benchmarks via numerical studies.

There are many interesting directions that can be pursued on the problem of online edge computing in the dark. First, the fundamental limit of achievable regret is not known, and developing a tight lower bound for regret remains to be resolved. Moreover, our proposed problem setting can be potentially extended to other computation models such as replications of jobs or redundant requests, offloading computations of jobs with precedence constraints that are modeled by directed acyclic graphs (DAGs), and other coded computing frameworks such as coded gradient aggregation [29].

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A PROOF OF LEMMA 4.2

Suppose the policy enters the exploration phase in round $t$ and let $\mathcal{P}_T$ be the corresponding hypercubes of edge devices. Then, based on the design of the proposed policy, the set of under-explored hypercubes $\mathcal{P}_T^{\text{ineq}}$ is non-empty, i.e., there exists at least one edge device with context $\phi_t^e$ such that a hypercube $p$ satisfying $\phi_t^e \in p$ has $C(p) = K(t) = \log(t)$. Clearly, there can be at most $|T|^2 \log(T)$ exploration phases in which edge devices with contexts in $p$ are selected due to under-explored $p$. Since there are $(T)^2$ $\mathcal{P}_T^{\text{ineq}}$ hypercubes in the partition, there can be at most $(T)^2|T|^2 \log(T)$ exploration phases. Also, the maximum achievable reward of an offloading decision is bounded by $1 - \eta$ and the minimum achievable reward is $-B\eta$. The maximum regret in one exploration phase is bounded by $1 + \eta (B - \eta) < 1 + B\eta$. Therefore, we have

$$\mathbb{E}[R_s(t)] \leq (1 + B\eta)T^2 \log(T)$$

using the fact that $|T|^2 \log(T) \leq (2T)^{2D} = 2DT^D$. The maximum regret is bounded by $1 + \eta B - 1 < 1 + B\eta$. Therefore, we have

$$\mathbb{E}[R_s(t)] \leq (1 + B\eta)T^2 \log(T)$$

B PROOF OF LEMMA 4.3

For each $t \in [T]$ we define $W_t = \{p^{\text{ineq}, t} = \emptyset\}$ as the event that the algorithm enters the exploitation phase. By the definition of $p^{\text{ineq}, t}$, we have that $C(p) > K(t) = \log(t)$ for all $p \in \mathcal{P}_t^{\text{ineq}}$. Let $V_{G_s}^{t}$ be the event that the subset $G_s \in \mathcal{L}^t$ is selected in round $t$. Then, we have

$$R_s(t) = \sum_{t=1}^{T} \sum_{G \in \mathcal{L}^t} \mathbb{1} \{V_{G}^{t}, W_{t}^{t}\} \times (r(G) - r(G))$$

Since the maximum regret is bounded by $1 + \eta B$, we have

$$R_s(t) \leq \mathbb{E}[R_s(t)] \leq (1 + \eta B) \sum_{t=1}^{T} \sum_{G \in \mathcal{L}^t} \mathbb{1} \{V_{G}^{t}, W_{t}^{t}\}$$

By taking the expectation, the regret can be bounded as follows

$$\mathbb{E}[R_s(t)] \leq (1 + \eta B) \sum_{t=1}^{T} \sum_{G \in \mathcal{L}^t} \Pr(V_{G}^{t}, W_{t}^{t})$$

Now, we explain how to bound $\Pr(V_{G}^{t}, W_{t}^{t})$. Because of the design of policy, the choice of $G$ is optimal based on the estimated $\hat{\mu}_i$. Thus, we have $u(\hat{\mu}_i, G) \geq u(\hat{\mu}_i, \hat{A}_i)$, which implies

$$\Pr(V_{G}^{t}, W_{t}^{t}) \leq \Pr(u(\hat{\mu}_i, G) \geq u(\hat{\mu}_i, \hat{A}_i, W_{t}^{t}))$$
The event \(\{u(\mu', G) \geq u(\hat{\mu}', \hat{A}^t), W^t\}\) actually implies that at least one of the following events holds for any \(H(t) > 0\):

\[
E_1 = \{u(\mu', G) \geq u(\bar{\mu}', G) + H(t), W^t\}
\]

(29)

\[
E_2 = \{u(\mu', \hat{A}^t) \leq u(\hat{\mu}', \hat{A}^t) - H(t), W^t\}
\]

(30)

\[
E_3 = \{u(\mu', G) \geq u(\hat{\mu}', \hat{A}^t), u(\mu', G) < u(\tilde{\mu}', G) + H(t), u(\mu, \hat{A}^t) > u(\hat{\mu}', \hat{A}^t) - H(t), W^t\}. \tag{31}
\]

Therefore, we have \(\{u(\mu', G) \geq u(\hat{\mu}', \hat{A}^t), W^t\} \subseteq E_1 \cup E_2 \cup E_3\).

Then, we proceed to bound the probabilities of events \(E_1, E_2\) and \(E_3\) separately. Before bounding \(\text{Pr}(E_1)\), we first present the following lemma which is proved in Appendix D.

**Lemma B.1.** Given a positive integer \(H(t), \mu_1, \mu_2\) and \(G\), if \(u(\mu_1, G) \geq u(\mu_2, G) + H(t)\), then there exists \(v \in G\) such that

\[
\mu_1(p_v^t) \geq \mu_2(p_v^t) + \frac{H(t)}{BM}. \tag{32}
\]

where \(B = \max_{1 \leq t \leq T} b^t\) and \(M = \max_{1 \leq t \leq T} \left(\frac{B}{t - 1}\right)\).

Thus, by Lemma B.1, we have \(E_1 = \{u(\mu', G) \geq u(\bar{\mu}', G) + H(t), W^t\} \subseteq \{\hat{\mu}'(p_v^t) \geq \hat{\mu}(p_v^t) + \frac{H(t)}{BM}, \exists v \in G, W^t\}\). By the definition of \(\hat{\mu}(p)\), the expectation of estimated success probability for the edge device \(v \in V'\) can be bounded by \(\mathbb{E}[\hat{\mu}(p_v^t)] \leq \hat{\mu}(p_v^t)\). Then, we bound \(\text{Pr}(E_1)\) as follows

\[
\text{Pr}(E_1) \leq \text{Pr}(u(\mu', G) \geq u(\bar{\mu}', G) + H(t), W^t) \tag{33}
\]

(33)

\[
\leq \text{Pr}(\hat{\mu}'(p_v^t) \geq \hat{\mu}(p_v^t) + \frac{H(t)}{BM}, \exists v \in G, W^t) \tag{34}
\]

(34)

\[
\leq \text{Pr}(\hat{\mu}'(p_v^t) \geq \mathbb{E}[\hat{\mu}'(p_v^t)] + \frac{H(t)}{BM}, \exists v \in G, W^t) \tag{35}
\]

(35)

\[
\leq \sum_{v \in G} \text{Pr}(\hat{\mu}'(p_v^t) \geq \mathbb{E}[\hat{\mu}'(p_v^t)] + \frac{H(t)}{BM}, W^t). \tag{36}
\]

(36)

By applying Chernoff-Hoeffding inequality [10] and the fact that there are at least \(t^2 \log(\text{samples drawn})\) samples, we have

\[
\text{Pr}(E_1) \leq \sum_{v \in G} \text{Pr}(\hat{\mu}'(p_v^t) \geq \mathbb{E}[\hat{\mu}'(p_v^t)] + \frac{H(t)}{BM}, W^t) \tag{37}
\]

(37)

\[
\leq \sum_{v \in G} \exp\left(-\frac{2t^2 \log(t)H(t)^2}{BM^2}\right) \tag{38}
\]

(38)

\[
\leq \sum_{v \in G} \exp\left(-\frac{2t^2 \log(t)H(t)^2}{BM^2}\right). \tag{39}
\]

(39)

If we choose \(H(t) = \frac{BMt^{-2/3}}{2} > 0\), we have

\[
\text{Pr}(E_1) \leq B \exp\left(-2t^2 \log(t)H(t)^2\right) \tag{40}
\]

(40)

\[
= B \exp(-2t^2 \log(t)) = Br^{-2}. \tag{41}
\]

(41)

Similarly, we have a bound for \(\text{Pr}(E_2)\):

\[
\text{Pr}(E_2) \leq Br^{-2}. \tag{42}
\]

(42)

Lastly, we bound \(\text{Pr}(E_3)\). Now we suppose that the following condition is satisfied:

\[
2H(t) \leq At^\theta. \tag{43}
\]

Since \(G \in \mathcal{L}^t\), we have \(u(\mu', \hat{A}^t) - u(\bar{\mu}', G) \geq A^t\). With (43), we have \(u(\mu', \hat{A}^t) - H(t) \geq u(\bar{\mu}', G) + H(t)\) which contradicts event \(E_3\). That is, under condition (43), we have \(\text{Pr}(E_3) = 0\).

Under condition (43), using (41) and (42), we have

\[
\text{Pr}(V_G^t, W^t) \leq \text{Pr}(E_1) + \text{Pr}(E_2) + \text{Pr}(E_3) \leq 2Br^{-2}. \tag{44}
\]

(44)

Finally, we complete the regret bound for \(\mathbb{E}[R_k(T)]\) as follows:

\[
\mathbb{E}[R_k(T)] \leq (1 + \eta B) \sum_{t=1}^{T} \text{Pr}(V_G^t, W^t) \tag{45}
\]

\[
\leq (1 + \eta B) |\mathcal{L}| \sum_{t=1}^{T} 2Br^{-2} \leq (1 + \eta B) |\mathcal{L}| |\mathcal{L}| (2B) \sum_{t=1}^{\infty} t^{-2} \tag{47}
\]

\[
= (1 + \eta B)|\mathcal{L}|^2 \frac{\pi^2}{6} \leq (1 + \eta B)B^2\left(\frac{3}{4} \sum_{k=1}^{B} \binom{V}{k} \right). \tag{48}
\]

C PROOF OF LEMMA 4.4

For each \(t \in [T]\), we define \(W^t = \{\mu^{\text{opt}}, t = 0\}\) as the event that the policy enters the exploitation phase. Then, the regret due to near-optimal subsets can be written as

\[
R_n(T) = \sum_{t=1}^{T} \mathbb{I}_{(W^t, G' \in A_{b^{t-2}} \setminus \mathcal{L}^t)} (r(\mathcal{A}^t) - r(G^t)). \tag{49}
\]

Let \(G^t = \{W^t, G' \in A_{b^{t-2}} \setminus \mathcal{L}^t\}\) be the event that a near-optimal subset is selected in round \(t\). Then, we have

\[
\mathbb{E}[R_n(T)] = \sum_{t=1}^{T} \text{Pr}(G^t)\mathbb{E}[r(\mathcal{A}^t) - r(G^t)|G^t] \tag{50}
\]

\[
\leq \sum_{t=1}^{T} (u(\mu^t, \mathcal{A}^t) - u(\mu^t, G^t)). \tag{51}
\]

where \(G^t\) is near-optimal in each round \(t\). By the definition of \(\mathcal{L}^t\), we then have

\[
u(\mu^t, \mathcal{A}^t) - u(\bar{\mu}', G^t) < At^\theta. \tag{52}
\]

By the function \(c\) defined in Appendix D and Assumption 1, we have

\[
u(\mu^t, \mathcal{A}^t) - u(\bar{\mu}', \mathcal{A}^t) = c(\mu^t, \tilde{\mu}^t, \mathcal{A}^t, Y^t) \tag{53}
\]

\[
\leq \sum_{(G_t, G_v, v) \in S(\mathcal{A}^t, Y^t)} |\mu(\phi_v^t) - \mu(\tilde{\phi}_v^{p_v^t})| \tag{54}
\]

\[
\leq \sum_{(G_t, G_v, v) \in S(\mathcal{A}^t, Y^t)} \|\phi_v^t - \tilde{\phi}_v^{p_v^t}\|_2 \tag{55}
\]

\[
\leq \sum_{(G_t, G_v, v) \in S(\mathcal{A}^t, Y^t)} LD^2 \|h_t^\alpha\|_2 \tag{56}
\]

\[
\leq BMLD^2 \|h_t^\alpha\|_2 \tag{57}
\]

Similarly, we have the following inequalities:

\[
u(\mu^t, \mathcal{A}^t) - u(\mu^t, \mathcal{A}^t) \leq BMLD^2 \|h_t^\alpha\|_2 \tag{59}
\]

\[
u(\bar{\mu}', G^t) - u(\mu^t, G^t) \leq BMLD^2 \|h_t^\alpha\|_2 \tag{60}
\]
Now, we bound \( u(\mu^t, A^{t*}) - u(\mu^t, G^t) \) as follows:

\[
\begin{align*}
&u(\mu^t, A^{t*}) - u(\mu^t, G^t) \\
\leq &\leq u(\mu^t, A^{t*}) + BMLD \frac{\alpha}{T} h_2^\alpha - u(\mu^t, G^t) \\
\leq &\leq u(\mu^t, A^t) + BMLD \frac{\alpha}{T} h_2^\alpha - u(\mu^t, G^t) \\
\leq &\leq u(\mu^t, A^t) + 2BMLD \frac{\alpha}{T} h_2^\alpha - u(\mu^t, G^t) \\
\leq &\leq u(\mu^t, A^t) + 3BMLD \frac{\alpha}{T} h_2^\alpha - u(\mu^t, G^t) \\
\leq &\leq 3BMLD \frac{\alpha}{T} h_2^\alpha + At^\beta
\end{align*}
\]

by the definition of \( \hat{A}^t \) and (52). With \( h_T = [T^\gamma] \), we have

\[
\begin{align*}
&u(\mu^t, A^{t*}) - u(\mu^t, G^t) = 3BMLD \frac{\alpha}{T} [T^\gamma]^{-\alpha} + At^\beta \\
&\leq 3BMLD \frac{\alpha}{T} T^{-\alpha T} + At^\beta.
\end{align*}
\]

Thus, we complete the regret bound for \( E[R_n] \) as follows:

\[
E[R_n] \leq \sum_{t=1}^{T} \left( 3BMLD \frac{\alpha}{T} T^{-\alpha T} + At^\beta \right)
\]

\[
\leq 3BMLD \frac{\alpha}{T^{1-\alpha T}} + \frac{A}{1 + \frac{\theta}{T^\gamma}}.
\]

\section*{D PROOF OF LEMMA B.1}

First, we suppose that

\[
\mu_1(p^t_1) - \mu_2(p^t_1) < \frac{H(t)}{BM}, \forall v \in G.
\]

We note that the following equation holds and will be used for analysis later.

\[
\prod_{i=1}^{N} a_i - \prod_{i=1}^{N} b_i = \sum_{i=1}^{N} a_1 \ldots a_{i-1}(a_i - b_i)b_{i+1} \ldots b_N.
\]

\section*{E PROOF OF LEMMA 3.1}

For a fixed integer \( n_g \), we suppose \( \mathcal{A}_1 \) is the optimal set with cardinality \( n_g \) where \( i \notin \mathcal{A}_1 \) and \( 1 \leq i \leq n_g \). Thus, there exists a \( j \in \mathcal{G}_1 \) such that \( j > n_g \). We construct a set \( \mathcal{A}_2 = (\mathcal{A}_1 \setminus \{j\}) \cup \{i\} \), where \( \mathcal{A}_1 \setminus \{j\} = \mathcal{A}_2 \setminus \{i\} \). Then, we have

\[
\begin{align*}
&u(\mu^t, \mathcal{A}_2) = Pr(\sum_{v \in \mathcal{A}_2} q_v' \geq Y^T - \eta_g) - \eta_g = \mu(p^t_1) Pr(\sum_{v \in \mathcal{A}_1 \setminus \{j\}} q_v' \geq Y^T - 1) + (1 - \mu(p^t_1)) Pr(\sum_{v \in \mathcal{A}_2 \setminus \{i\}} q_v' \geq Y^T - 1) + (1 - \mu(p^t_1)) Pr(\sum_{v \in \mathcal{A}_1 \setminus \{j\}} q_v' \geq Y^T - 1).
\end{align*}
\]

which contradicts \( u(\mu_1, G) \geq u(\mu_2, G) + H(t) \), i.e., there exists \( v \in G \) such that

\[
\mu_1(p^t_1) \geq \mu_2(p^t_1) + \frac{H(t)}{BM}.
\]