Baryon Asymmetry of the Universe and Lepton Mixing

W. Buchmüller and M. Plümacher

Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany

Abstract

Baryogenesis appears to require lepton number violation. This is naturally realized in extensions of the standard model containing right-handed neutrinos. We discuss the generation of a baryon asymmetry by the out-of-equilibrium decay of heavy Majorana neutrinos in these models, without and with supersymmetry. All relevant lepton number violating scattering processes which can inhibit the generation of an asymmetry are taken into account. We assume a similar pattern of mixings and masses for neutrinos and up-type quarks, as suggested by SO(10) unification. This implies that $B - L$ is broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, if $m_{\nu_\mu} \sim 3 \cdot 10^{-3}$ eV, as preferred by the MSW solution to the solar neutrino deficit. The observed baryon asymmetry is then obtained without any fine tuning of parameters.

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1. Sphaleron transitions and thermal equilibrium in the early universe

Due to the chiral nature of the weak interactions baryon number ($B$) and lepton number ($L$) are not conserved in the standard model (SM)\(^1\). At zero temperature this has no observable effect due to the smallness of the weak coupling. However, as the temperature approaches the critical temperature $T_c$ of the electroweak phase transition, $B$ and $L$ violating processes come into thermal equilibrium\(^2\). Their rate is determined by the free energy of sphaleron-type field configurations which carry topological charge. In the standard model they induce an effective interaction of all left-handed fermions (cf. fig. 1) which violates baryon and lepton number by three units,

$$\Delta B = \Delta L = 3.$$  \hspace{1cm} (1)

Sphaleron processes have a profound effect on the generation of the cosmological baryon asymmetry. Eq. (1) suggests that any $B + L$ asymmetry generated before the electroweak phase transition, i.e., at temperatures $T > T_c$, will be washed out. However, since only left-handed fields couple to sphalerons, a non-zero value of $B + L$ can persist in the high-temperature, symmetric phase if there exists a non-vanishing $B - L$ asymmetry.

This is most easily seen in an analysis of the chemical potentials involved in this problem\(^3\). The gauge bosons of unbroken gauge symmetries have vanishing chemical potentials. In the SM with $N_F$ fermion generations and $N_H$ Higgs doublets $\phi_i$ we have $2N_F$ left-handed quark and lepton doublets $q_{iL} = (u_{iL}, d_{iL})$ and $l_{iL} = (\nu_{iL}, e_{iL})$, $3N_F$ right-handed quark and charged lepton singlets $u_{iR}, d_{iR}$ and $e_{iR}$ and $2N_H$ neutral and charged Higgs fields $\varphi^0_i$ and $\varphi^-_i$. However, not all of the corresponding chemical potentials are independent. Cabibbo mixing between the quarks will balance out the chemical potentials of the various up- and down-quark states, respectively. If in addition the mixing between the Higgs doublets is strong enough all the Higgs fields $\varphi^0_i$ and $\varphi^-_i$ have the same chemical potentials $\mu_0$ and $\mu_-$. In thermal equilibrium, perturbative electroweak interactions yield the relations,

\begin{align*}
W^- &\leftrightarrow \varphi^- + \varphi^0 : \quad \mu_0 = -\mu_- , \\
W^- &\leftrightarrow \bar{u}_L + d_L : \quad \mu_{dL} = \mu_{uL} , \\
W^- &\leftrightarrow \bar{\nu}_{iL} + e_{iL} : \quad \mu_{ieL} = \mu_{iv} , \\
\varphi^0 &\leftrightarrow \bar{u}_L + u_R : \quad \mu_{uR} = \mu_{uL} + \mu_0 , \\
\varphi^0 &\leftrightarrow \bar{d}_R + d_L : \quad \mu_{dR} = \mu_{uL} - \mu_0 , \\
\varphi^0 &\leftrightarrow \bar{e}_{iR} + e_{iL} : \quad \mu_{ieR} = \mu_{iv} - \mu_0 .
\end{align*}  \hspace{1cm} (2)

The sphaleron processes (cf. fig. 1) yield the additional condition

$$N_F(\mu_{uL} + 2\mu_{dL}) + \sum_{i=1}^{N_F} \mu_{iv} = 0 .$$  \hspace{1cm} (3)
In thermal equilibrium, if all the chemical potentials are small compared to the temperature, one obtains for the baryon number $B$ and the lepton number $L$, 

$$\langle B \rangle_T = \frac{n_B}{s} - \frac{n_{\bar{B}}}{s} = \frac{15}{\pi^2 g_s T} N_F \left( \mu_{uL} + \mu_{uR} + \mu_{dL} + \mu_{dR} \right),$$  

(4)

$$\langle L \rangle_T = \frac{n_L}{s} - \frac{n_{\bar{L}}}{s} = \frac{15}{\pi^2 g_s T} \sum_{i=1}^{N_F} \left( \mu_{ieL} + \mu_{ieR} + \mu_{i\nu} \right),$$  

(5)

where $g_s$ is the number of relativistic degrees of freedom and $s$ is the entropy density of the universe. From eqs. (2)-(3) it follows that $B$ is proportional to $B - L$, 

$$\langle B \rangle_T = C \langle B - L \rangle_T = \frac{C}{C - 1} \langle L \rangle_T, \quad C = \frac{8N_F + 4N_H}{22N_F + 13N_H}. \quad (6)$$

In the SM, with $N_F = 3$ and $N_H = 1$ one has $C = \frac{28}{79}$. We conclude that $B - L$ violation is needed to obtain a non-vanishing baryon asymmetry. In the standard model, as well as its supersymmetric version and its unified extensions based on the gauge group SU(5), $B - L$ is a conserved quantity. Hence, no baryon asymmetry can be generated dynamically in these models.

In principle, this conclusion could be avoided if the baryon asymmetry were produced directly in a first-order electroweak phase transition. However, a detailed study of the thermodynamics indicates that the phase transition in the SM is too weak for baryogenesis. In the minimal supersymmetric extension of the standard model (MSSM) such a scenario is still conceivable for a limited range of parameters.
2. Standard model with right-handed neutrinos

The cosmological baryon asymmetry appears to require $B - L$ violation, and therefore $L$ violation. Lepton number violation is naturally realized by adding right-handed Majorana neutrinos to the standard model. Heavy right-handed Majorana neutrinos, whose existence is predicted by theories based on gauge groups containing SO(10), can also explain the smallness of the light neutrino masses via the see-saw mechanism.

The most general lagrangian for couplings and masses of charged leptons and neutrinos is given by

$$L_Y = -\bar{l}_L \phi g_l e_R - \bar{l}_L \phi g_\nu \nu_R - \frac{1}{2} \bar{\nu}_R M \nu_R + \text{h.c.} \ .$$

(7)

The vacuum expectation value of the Higgs field $\langle \phi^0 \rangle = v \neq 0$ generates Dirac masses $m_l$ and $m_D$ for charged leptons and neutrinos,

$$m_l = g_l v \ , \quad m_D = g_\nu v \ ,$$

(8)

which are assumed to be much smaller than the Majorana masses $M$. This yields light and heavy neutrino mass eigenstates

$$\nu \simeq K^\dagger \nu_L + \nu_R^C K \ , \quad N \simeq \nu_R + \nu_R^C \ ,$$

(9)

with masses

$$m_\nu \simeq -K^\dagger m_D \frac{1}{M} m_D^T K^* \ , \quad m_N \simeq M \ .$$

(10)

Here $K$ is a unitary matrix which relates weak and mass eigenstates.

The exchange of heavy Majorana neutrinos generates an effective $\Delta L = 2$ interaction at low energies (cf. fig. 2),

$$L_{\Delta L = 2} = \frac{1}{2} \bar{l}_L \phi g_\nu \frac{1}{M} g_\nu^T \phi l_R^c + \text{h.c.} \ .$$

(11)

At finite temperature the corresponding $\Delta L = 2$ processes take place with the rate

$$\Gamma_{\Delta L = 2}(T) = \frac{1}{\pi^3} \frac{T^3}{v^4} \sum_i m_{\nu_i}^2 .$$

(12)

Figure 2: Lepton number violating lepton Higgs scattering
In thermal equilibrium the interaction (11) implies
\[ \mu_0 = \mu_{ie_L} = \mu_{iv} , \] (13)
and therefore
\[ \langle B \rangle_T = \langle B - L \rangle_T = 0 . \] (14)
To avoid this conclusion, the \( \Delta L = 2 \) interaction (11) must not reach thermal equilibrium, which imposes an upper bound on the light neutrino masses \( m_\nu \).

The right-handed neutrinos, whose exchange may erase any lepton asymmetry, can also generate a lepton asymmetry by means of out-of-equilibrium decays\(^{10}\). This lepton asymmetry is then partially transformed into a baryon asymmetry by the sphaleron processes. The decay width of \( N_i \) in its rest frame reads at tree level (cf. fig. 3),
\[
\Gamma_{Di} = \Gamma_{rs} \left( N^i \rightarrow \phi^c + l \right) + \Gamma_{rs} \left( N^i \rightarrow \phi^+ + l^c \right) = \frac{1}{8\pi} \frac{(m^D D m_D)_{ii}}{v^2} M_i .
\] (15)
The decay width is closely related to the light neutrino masses which are therefore constrained by the out-of-equilibrium condition. Requiring \( \Gamma_{Di} < H|_{T=M_i} \), where \( H \) is the Hubble parameter, a rough estimate yields\(^{11}\)
\[ m_{\nu i} < 10^{-3} \text{eV} . \] (16)
The detailed calculations described later will be consistent with this estimate.

Interference between the tree-level amplitude and the one-loop vertex correction (cf. fig. 3) yields the \( CP \) asymmetry
\[
\varepsilon_i = \frac{\Gamma(N_i \rightarrow l^c \phi) - \Gamma(N_i \rightarrow l^c \phi)}{\Gamma(N_i \rightarrow l^c \phi) + \Gamma(N_i \rightarrow l^c \phi)}
= \frac{1}{8\pi v^2} \frac{1}{(m^D D m_D)_{ii}} \sum_j \text{Im} \left[ (m^D D m_D)_{ij}^2 \right] f \left( \frac{M_j^2}{M_i^2} \right) ,
\] (17)
where
\[ f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right]. \tag{18} \]

Note, that self-energy corrections do not contribute to CP asymmetries \[. \]

From the CP asymmetry (17) one obtains a rough estimate of the baryon asymmetry (cf. \[. \])
\[ Y_{B-L} = \frac{n_{B-L}}{s} \sim \frac{\varepsilon}{g_\ast}, \tag{19} \]
where the effective number of degrees of freedom \( g_\ast \approx 100 \) in the SM. This estimate is useful as an upper bound on the generated baryon asymmetry. However, it is easily too large by a factor \( O(100) \). In order to obtain a more accurate result, one has to solve the Boltzmann equations.

3. Boltzmann equations and scattering processes

In a quantitative analysis of baryogenesis one has to integrate the relevant set of Boltzmann equations which are treated in some approximation \[. \] One usually neglects the difference between Bose and Fermi statistics so that the equilibrium phase space density of a particle \( \psi \) with mass \( m_\psi \) is given by Maxwell-Boltzmann statistics,
\[ f^{eq}_\psi (E_\psi, T) = e^{-E_\psi/T}. \tag{20} \]
The corresponding particle density is
\[ n_\psi(T) = \frac{g_\psi}{(2\pi)^3} \int d^3 p_\psi f_\psi, \tag{21} \]
where \( g_\psi \) is the number of internal degrees of freedom. The number of particles \( Y_\psi \) in a comoving volume element is given by the ratio of \( n_\psi \) and the entropy density \( s \). If the universe expands isentropically, \( Y_\psi \) is not affected by the expansion, i.e. \( Y_\psi \) can only be changed by interactions.

One distinguishes elastic and inelastic scatterings. Elastic scatterings only affect the phase space densities of the particles but not the number densities, whereas inelastic scatterings do change the number densities. If elastic scatterings do occur at a higher rate than inelastic scatterings one can assume kinetic equilibrium, i.e., the phase space density is
\[ f_\psi (E_\psi, T) = \frac{n_\psi}{n^{eq}_\psi} e^{-E_\psi/T}. \tag{22} \]
The Boltzmann equation, which describes the evolution of \( Y_\psi \) with temperature, then reads
\[ \frac{dY_\psi}{dz} = - \frac{z}{sH (m_\psi)} \sum_{a, i, j, \ldots} \left[ \frac{Y_\psi Y_a \ldots}{Y^{eq}_\psi Y^{eq}_a \ldots} \gamma^{eq}_\psi (\psi + a + \ldots \rightarrow i + j + \ldots) - \frac{Y_i Y_j \ldots}{Y^{eq}_i Y^{eq}_j \ldots} \gamma^{eq}_\psi (i + j + \ldots \rightarrow \psi + a + \ldots) \right]. \tag{23} \]
Here \( z = m_\psi / T \) and \( H (m_\psi) \) is the Hubble parameter at \( T = m_\psi \).

The right-hand side of eq. (23) describes the interactions in which a \( \psi \) particle takes part, where \( \gamma^{eq} \) is the space time density of scatterings in thermal equilibrium. In a dilute gas we only have to take into account decays, two-particle scatterings and the corresponding back reactions. For a decay one has:

\[
\gamma_D := \gamma^{eq} (\psi \to i + j + \ldots) = n_\psi^{eq} \frac{K_1(z)}{K_2(z)} \tilde{\Gamma}_{rs},
\]

where \( K_1 \) and \( K_2 \) are modified Bessel functions and \( \tilde{\Gamma}_{rs} \) is the usual decay width in the rest system of the decaying particle. The ratio of the Bessel functions is a time dilatation factor.

If one neglects \( CP \) violating effects the same reaction density describes the inverse decays,

\[
\gamma_{ID} := \gamma^{eq} (i + j + \ldots \to \psi) = \gamma_D.
\]

For two body scattering one has

\[
\gamma^{eq} (\psi + a \leftrightarrow i + j + \ldots) = \frac{T}{64 \pi^4} \int ds \hat{\sigma} (s) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right),
\]

where \( s \) is the squared centre of mass energy, and the reduced cross section \( \hat{\sigma} (s) \) for the process \( \psi + a \to i + j + \ldots \) is related to the usual total cross section \( \sigma (s) \) by

\[
\hat{\sigma} (s) = \frac{8}{s} \left[ (p_\psi \cdot p_a)^2 - m_\psi^2 m_a^2 \right] \sigma (s).
\]

In kinetic equilibrium, contributions from elastic scatterings drop out of eq. (23).

Consider now the various processes which are relevant in the leptogenesis scenario. In order to obtain a lepton asymmetry of the correct order of magnitude, the right-handed neutrinos have to be numerous before decaying, i.e., they have to be in

![Diagram](image)

Figure 4: Lepton number conserving processes mediated by the new neutral gauge boson \( Z' \).
Figure 5: Lepton number violating neutrino top-quark scattering

thermal equilibrium at high temperatures. The Yukawa interactions (7) are too weak to achieve this and additional interactions are therefore needed. Since right-handed neutrinos are a necessary ingredient of SO(10) unified theories, it is natural to consider leptogenesis within an extended gauge model, contained in a SO(10) GUT. The minimal extension of the standard model is based on the gauge group

\[ G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'} \subset SO(10) \].

Here \( U(1)_{Y'} \), and therefore \( B - L \), is spontaneously broken, and the breaking scale is related to the heavy neutrino masses. The additional neutral gauge boson \( Z' \) accounts for pair creation and annihilation processes and for flavour transitions between heavy neutrinos of different generations (cf. fig. 4). For appropriately chosen parameters these processes generate an equilibrium distribution of heavy neutrinos at high temperatures.

Of crucial importance are the \( \Delta L = 2 \) lepton number violating scatterings shown in fig. 2 which, if too strong, erase any lepton asymmetry. Similarly, the \( \Delta L = 1 \) lepton number violating neutrino top-quark scatterings shown in fig. 5 have to be taken into account because of the large top Yukawa coupling. Finally, the heavy neutrino decays (cf. fig. 3) as well as the inverse decays have to be incorporated in the Boltzmann equations.

Based on these equations the resulting lepton and baryon asymmetries can be evaluated, and it is known that the observed cosmological baryon asymmetry can be obtained for a wide range of Yukawa couplings in eq. (7)\(^{14,15,16}\). Further, one may ask whether the right order of magnitude of the asymmetry results naturally in the leptogenesis scenario. To address this question one has to discuss patterns of neutrino mass matrices which determine the generated asymmetry.

4. Neutrino masses and mixings

In sect. 1 we argued that a dynamical generation of the cosmological baryon asymmetry requires lepton number violation. This is most easily realized by adding right-handed neutrinos to the standard model. In the context of unified theories one is then led to go beyond the SU(5) GUT and to consider SO(10) as smallest unified
gauge group allowing right-handed neutrinos. In the following we shall therefore assume a similar pattern of mixings and mass ratios for leptons and quarks, which is natural in SO(10) unification.

Such an ansatz is most transparent in a basis where all mass matrices are maximally diagonal. In addition to real mass eigenvalues two mixing matrices then appear. One can always choose a basis for the lepton fields such that the mass matrices $m_l$ for the charged leptons and $M$ for the heavy Majorana neutrinos $N_i$ are diagonal with real and positive eigenvalues,

$$m_l = \begin{pmatrix} m_\ell & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}.$$ (29)

In this basis $m_D$ is a general complex matrix, which can be diagonalized by a biunitary transformation. Therefore, we can write $m_D$ in the form

$$m_D = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger,$$ (30)

where $V$ and $U$ are unitary matrices and the $m_i$ are real and positive. In the absence of a Majorana mass term $V$ and $U$ would correspond to Kobayashi-Maskawa type mixing matrices of left- and right-handed charged currents, respectively.

According to eq. (17) the $CP$ asymmetry is determined by the mixings and phases present in the product $m_D^\dagger m_D$, where the matrix $V$ drops out. Hence, to leading order, the mixings and phases which are responsible for baryogenesis are entirely determined by the matrix $U$. Correspondingly, the mixing matrix $K$ in the leptonic charged current, which determines $CP$ violation and mixings of the light leptons, depends on mass ratios and mixing angles and phases of $U$ and $V$. This implies that there exists no direct connection between the $CP$ violation and generation mixing relevant at high and low energies.

Consider now the mixing matrix $U$. One can factor out five phases,

$$U = e^{i\gamma} e^{i\lambda_3 \alpha} e^{i\lambda_8 \beta} U_1 e^{i\lambda_3 \sigma} e^{i\lambda_8 \tau},$$ (31)

where the $\lambda_i$ are the Gell-Mann matrices. The remaining matrix $U_1$ depends on three mixing angles and one phase, like the Kobayashi-Maskawa matrix for quarks. In analogy to the quark mixing matrix we choose the Wolfenstein parametrization as ansatz for $U_1$,

$$U_1 = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$ (32)
where $A$ and $|\rho + i\eta|$ are $O(1)$, while the mixing parameter $\lambda$ is assumed to be small. For the masses $m_i$ and $M_i$ we assume the same hierarchy which is observed for up-type quarks,

$$m_1 = b\lambda^4 m_3, \quad m_2 = c\lambda^2 m_3, \quad b, c = O(1) \quad (33)$$

$$M_1 = B\lambda^4 M_3, \quad M_2 = C\lambda^2 M_3, \quad B, C = O(1). \quad (34)$$

For the eigenvalues $m_i$ of the Dirac mass matrix this choice is suggested by SO(10) unification. For the masses $M_i$ this is an assumption motivated by simplicity. The masses $M_i$ cannot be degenerate, because in this case there exists a basis for $\nu_R$ such that $U = 1$, which implies that no baryon asymmetry is generated. We shall see in the next section that the precise form of the assumed hierarchy has no influence on the viability of the leptogenesis mechanism.

The light neutrino masses are given by the seesaw formula (10). The matrix $K$, which diagonalizes the neutrino mass matrix, can be evaluated in powers of $\lambda$.

A straightforward calculation gives the following masses for the light neutrino mass eigenstates

$$m_{\nu_e} = \frac{b^2}{|C + e^{i4\alpha} B|} \lambda^4 m_{\nu_e} + O(\lambda^6) \quad (35)$$

$$m_{\nu_\mu} = \frac{c^2 |C + e^{i4\alpha} B|}{BC} \lambda^2 m_{\nu_\mu} + O(\lambda^4) \quad (36)$$

$$m_{\nu_\tau} = \frac{m_3^2}{M_3} + O(\lambda^4). \quad (37)$$

The $CP$-asymmetry in the decay of the lightest right-handed neutrino $N_1$ is easily obtained from eqs. (17) and (32)-(34),

$$\varepsilon_1 = -\frac{1}{16\pi} \frac{B A^2}{c^2 + A^2 |\rho + i\eta|^2} \lambda^4 \frac{m_3^2}{v^2} \text{Im} \left[ (\rho - i\eta)^2 e^{i2(\alpha + \sqrt{3}\beta)} \right] + O(\lambda^6). \quad (38)$$

This yields for the magnitude of the $CP$ asymmetry,

$$|\varepsilon_1| \leq \frac{1}{16\pi} \frac{B A^2 |\rho + i\eta|^2}{c^2 + A^2 |\rho + i\eta|^2} \lambda^4 \frac{m_3^2}{v^2} + O(\lambda^6). \quad (39)$$

How close the value of $|\varepsilon_1|$ is to this upper bound depends on the phases $\alpha$, $\beta$ and $\text{arg} (\rho + i\eta)$. For $\lambda \sim 0.1$ one has $\varepsilon_1 \sim 10^{-6} \cdot m_3^2/v^2$. Hence, a large value of the Yukawa coupling $m_3/v$ will be required by this mechanism of baryogenesis. This holds irrespective of the neutrino mixings and the heavy neutrino masses.
5. Numerical Results

To obtain a numerical value for the produced baryon asymmetry, one has to specify the free parameters in the ansatz (32)-(34). In the following we will use as a constraint the value for the $\nu_\mu$-mass which is preferred by the MSW explanation of the solar neutrino deficit,

$$m_{\nu_\mu} \simeq 3 \cdot 10^{-3} \text{ eV}.$$  

(40)

A generic choice for the free parameters is to take all $O(1)$ parameters equal to one and to fix $\lambda$ to a value which is of the same order as the $\lambda$ parameter of the quark mixing matrix,

$$A = B = C = b = c = |\rho + i\eta| \simeq 1, \quad \lambda \simeq 0.1.$$  

(41)

From eqs. (35)-(37), (40) and (41) one now obtains,

$$m_{\nu_e} \simeq 8 \cdot 10^{-6} \text{ eV}, \quad m_{\nu_\tau} \simeq 0.15 \text{ eV}.$$  

(42)

Finally, a second mass scale has to be specified. In unified theories based on SO(10) the Dirac neutrino mass $m_3$ is naturally equal to the top-quark mass,

$$m_3 = m_t \simeq 174 \text{ GeV}.$$  

(43)

This determines the masses of the heavy Majorana neutrinos $N_i$,

$$M_3 \simeq 2 \cdot 10^{14} \text{ GeV},$$  

(44)

and, consequently, $M_1 \simeq 2 \cdot 10^{10} \text{ GeV}$ and $M_2 \simeq 2 \cdot 10^{12} \text{ GeV}$. From eq. (39) one obtains the $CP$ asymmetry $|\varepsilon_1| \simeq 10^{-6}$, where we have assumed maximal phases. The solution of the Boltzmann equations now yields the baryon asymmetry (see fig. 6a),

$$Y_B \simeq 9 \cdot 10^{-11},$$  

(45)

which is indeed the correct order of magnitude! The precise value depends on unknown phases.

The large mass $M_3$ of the heavy Majorana neutrino $N_3$ (cf. (14)), suggests that $B - L$ is already broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16} \text{ GeV}$, without any intermediate scale of symmetry breaking. This large value of $M_3$ is a consequence of the choice $m_3 \simeq m_t$. To test the sensitivity of the result for $Y_{B-L}$ on this assumption, consider as an alternative the choice $m_3 = m_b \simeq 4.5 \text{ GeV}$, with all other parameters remaining unchanged. In this case one has $M_3 = 10^{11} \text{ GeV}$ and $|\varepsilon_1| = 5 \cdot 10^{-10}$ for the mass of $N_3$ and the $CP$ asymmetry, respectively. Since the maximal $B-L$ asymmetry is $-\varepsilon_1/g*$, it is clear that the generated asymmetry will be too small. The solutions of the Boltzmann equations are shown in fig. 6b. The generated asymmetry,

$$Y_B \simeq 8 \cdot 10^{-14},$$  

(46)
Figure 6: Time evolution of the neutrino number density and the $B-L$ asymmetry for $\lambda = 0.1$ and for $m_3 = m_t$ (a) or $m_3 = m_b$ (b). The equilibrium distribution for $N_1$ is represented by a dashed line, while the hatched area shows the measured value for the asymmetry.

is too small by more than two orders of magnitude. We conclude that high values for both masses $m_3$ and $M_3$ are preferred, which is natural in SO(10) unification.

In eq. (34) we had assumed a mass hierarchy for the heavy Majorana neutrinos like for the up-type quarks. One may also consider a weaker hierarchy, like for the down-type quarks. This corresponds to the choice $B = 10$, $C = 3$. Keeping all other parameters in eq. (41) one obtains for the $\nu_e$ and $\nu_\tau$ masses,

$$m_{\nu_e} \simeq 5 \cdot 10^{-6} \text{ eV}, \quad m_{\nu_\tau} \simeq 0.7 \text{ eV}.$$ \hspace{1cm} (47)

The large Dirac mass \[43\] again leads to a large Majorana mass

$$M_3 \simeq 4 \cdot 10^{13} \text{ GeV},$$ \hspace{1cm} (48)

and, consequently, $M_1 \simeq 4 \cdot 10^{10} \text{ GeV}$, $M_2 \simeq 10^{12} \text{ GeV}$. From eq. (38) one obtains the $CP$ asymmetry $\varepsilon_1 \simeq -10^{-6}$. The corresponding solutions of the Boltzmann equations are shown in fig. 7. The final baryon asymmetry,

$$Y_B \simeq 2 \cdot 10^{-9},$$ \hspace{1cm} (49)

is larger than required, but this value can always be lowered by adjusting the unknown phases. Hence, the possibility to generate a lepton asymmetry does not depend on
Figure 7: Generated lepton asymmetry if one assumes a similar mass hierarchy for the right-handed neutrinos and the down-type quarks.

the special form of the mass hierarchy assumed for the right-handed neutrinos, as long as some kind of mass hierarchy exists.

Models for dark matter involving massive neutrinos favour a $\tau$-neutrino mass $m_{\nu_{\tau}} \simeq 5 \text{ eV}$, which is significantly larger than the value given in (42). Such a large value for the $\tau$-neutrino mass can be accommodated within the ansatz described in this section. However, it does not correspond to the simplest choice of parameters and requires some fine-tuning. For the mass of the heaviest Majorana neutrino one obtains in this case $M_3 \simeq 6 \cdot 10^{12} \text{ GeV}$.

The recently reported atmospheric neutrino anomaly\[1\] may be due to neutrino oscillations. The required mass difference and mixing angle are $\Delta m^2 \sim 0.005 \text{ eV}^2$ and $\sin^2 2\Theta \sim 1$. The preferred solution for baryogenesis discussed above yields (cf. eq. (12)) $m_{\nu_{\tau}}^2 - m_{\nu_{\mu}}^2 \simeq 0.02 \text{ eV}^2$ which, within the theoretical and experimental uncertainties, is certainly consistent with the mass difference required by the neutrino oscillation hypothesis. The $\nu_{\tau}-\nu_{\mu}$ mixing angle is not constrained by leptogenesis and therefore a free parameter in principle. The large value needed, however, is against the spirit of small generation mixings manifest in the Wolfenstein ansatz and would require some special justification.

6. Supersymmetric extension

Without an intermediate scale of symmetry breaking, the unification of gauge couplings appears to require low-energy supersymmetry. Supersymmetric leptogenesis
has already been considered in an approximation where lepton number violating scatterings are neglected which inhibit the generation of lepton number. However, a full analysis of the mechanism including all the relevant scattering processes is necessary in order to get a reliable relation between the input parameters and the final asymmetry. It turns out that the lepton number violating scatterings are qualitatively more important than in the non-supersymmetric scenario and that they can even account for the generation of an equilibrium distribution of heavy neutrinos at high temperatures.

The supersymmetric generalization of the lagrangian (7) is the superpotential

$$ W = \frac{1}{2} N^c M N^c + \mu H_1 \epsilon H_2 + H_1 \epsilon L \lambda_l E^c + H_2 \epsilon L \lambda_\nu N^c, \quad (50) $$

where, in the usual notation, $H_1$, $H_2$, $L$, $E^c$ and $N^c$ are chiral superfields describing spin-0 and spin-$\frac{1}{2}$ fields. The basis for the lepton fields can be chosen as in the non-supersymmetric case. The vacuum expectation values $v_1 = \langle H_1 \rangle$ and $v_2 = \langle H_2 \rangle$ of the two neutral Higgs fields generate Dirac masses for the leptons and their scalar partners,

$$ m_l = \lambda_l v_1, \quad m_D = \lambda_\nu v_2. \quad (51) $$
The heavy neutrinos and their scalar partners can decay into various final states (cf. fig. 8). At tree level, the decay widths read,

\[
\Gamma_{rs}(N_i \to \tilde{l} + H^c) = \Gamma_{rs}(N_i \to l + H_2) = \frac{1}{16\pi} \frac{(m_D^\dagger m_D)_{ii}}{v^2} M_i , \tag{52}
\]

\[
\Gamma_{rs}(\tilde{N}_c \to \tilde{l} + H_2) = \Gamma_{rs}(\tilde{N}_i \to l + \tilde{h}^c) = \frac{1}{8\pi} \frac{(m_D^\dagger m_D)_{ii}}{v^2} M_i . \tag{53}
\]

The $CP$ asymmetry in each of the decay channels is given by\cite{24}

\[
\varepsilon_i = -\frac{1}{8\pi v^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_j \text{Im} [(m_D^\dagger m_D)_{ji}] g \left( \frac{M_j^2}{M_i^2} \right) \tag{54}
\]

\[
g(x) = \sqrt{x} \ln \left( \frac{1 + x}{x} \right) . \tag{55}
\]

It arises through interference of tree level and one-loop diagrams shown in fig. 8. In the case of a mass hierarchy, $M_j \gg M_i$, the $CP$ asymmetry is twice as large as in the non-supersymmetric case.

Like in the non-supersymmetric scenario lepton number violating scatterings mediated by a heavy (s)neutrino have to be included in a consistent analysis, since they can easily reduce the generated asymmetry by two orders of magnitude\cite{23}. A very interesting new feature of the supersymmetric model is that the (s)neutrino (s)top scatterings are strong enough to bring the neutrinos into thermal equilibrium at high temperatures. Hence, an equilibrium distribution can be reached for temperatures far below the masses of heavy gauge bosons.

From the discussion of the out-of-equilibrium condition in sect. 1 we know that the generated baryon asymmetry is very sensitive to the decay width $\Gamma_1$ of $N_1$, and therefore to $(m_D^\dagger m_D)_{11}$. In fact, it turns out that the asymmetry essentially depends on the ratio

\[
\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} . \tag{56}
\]

For the mass matrices discussed in sect. 4 $\tilde{m}_1$ is of the same order as the muon neutrino mass. One easily verifies,

\[
\tilde{m}_1 = \frac{C(c^2 + A^2|\rho + i\eta|^2)}{c^2 |C + e^{i\alpha} B|} m_{\nu_\mu} + \mathcal{O}(\lambda^2) . \tag{57}
\]

In fig. 9 we have plotted the generated lepton asymmetry as function of $\tilde{m}_1$ for three different values of $M_1$, where we have assumed the hierarchy $M_2^2 = 10^3 M_1^2$, $M_3^2 = 10^6 M_1^2$ and the $CP$ asymmetry $\varepsilon_1 = -10^{-6}$.

Fig. 9 demonstrates, first of all, that in the whole parameter range the generated asymmetry is much smaller than the value $4 \cdot 10^{-9}$ which one obtains from the naive
Figure 9: The generated \((B - L)\) asymmetry for \(M_1 = 10^8\) GeV (dotted line), \(M_1 = 10^{10}\) GeV (solid line) and \(M_1 = 10^{12}\) GeV (dashed line). The hatched area shows the measured value for the asymmetry.

estimate (19), neglecting lepton number violating scattering processes. For small \(\tilde{m}_1\) the reason is that the Yukawa interactions are too weak to bring the neutrinos into equilibrium at high temperatures. For large \(\tilde{m}_1\), on the other hand, the lepton number violating scatterings wash out a large part of the generated asymmetry at temperatures \(T < M_1\).

Baryogenesis is possible in the range

\[
10^{-5}\ eV \lesssim \tilde{m}_1 \lesssim 5 \cdot 10^{-3}\ eV .
\]  

This result is independent of any assumptions on the mass matrices, in particular it is not a consequence of the ansatz discussed in sect. 5. This ansatz just implies (cf. (57))

\[
\tilde{m}_1 \simeq m_{\nu_\mu} .
\]  

It is very interesting that the \(\nu_\mu\)-mass preferred by the MSW explanation of the solar neutrino deficit lies indeed in the interval allowed by baryogenesis according to fig. 9.

Consider now again the simplest choice of parameters given by eqs. (40)-(43). The corresponding generated lepton asymmetries are shown in fig. 10a. \(Y_{L_f}\) and
\( Y_L \) denote the absolute values of the asymmetries stored in leptons and their scalar partners, respectively. They are related to the baryon asymmetry by
\[
Y_B = \frac{8}{23} Y_L, \quad Y_L = Y_{L_f} + Y_{L_s},
\] (60)

\( Y_{N_1} \) is the number of heavy neutrinos per comoving volume element, and
\[
Y_{1\pm} = Y_{N_1}^{\pm} \pm Y_{\bar{N}_1},
\] (61)

where \( Y_{N_1}^{\pm} \) is the number of scalar neutrinos per comoving volume element. As fig. (10a) shows, the generated baryon asymmetry has the correct order of magnitude, like in the non-supersymmetric case,
\[
Y_B \approx 10^{-10}.
\] (62)

Lowering the Dirac mass scale of the neutrinos to the bottom-quark scale has again dramatic consequences (cf. fig. (10b)). The baryon asymmetry is reduced by three orders of magnitude
\[
Y_B \approx 10^{-13}.
\] (63)

Hence, like in the non-supersymmetric scenario, large values for both masses \( m_3 \) and \( M_3 \) are necessary.
Figure 11: Generated lepton asymmetry if one assumes a similar mass hierarchy for the right-handed neutrinos and the down-type quarks.

Again like in the non-supersymmetric case the result is insensitive to the precise form of the assumed hierarchy for the right-handed neutrino masses. Repeating the calculation with the parameter choice corresponding to eq. (47) yields the results shown in fig. 11. The final asymmetry reads

\[ Y_B \approx 3 \cdot 10^{-9}. \]  \( (64) \)

Comparing the results (62), (63) and (64) with their non-supersymmetric counterparts (45), (46) and (49), one sees that the larger \( CP \) asymmetry and the additional contributions from the sneutrino decays in the supersymmetric scenario are compensated by the wash-out processes which are stronger than in the non-supersymmetric case. The final asymmetries are the same in the non-supersymmetric and in the supersymmetric case.

7. Summary

Anomalous electroweak \( B + L \) violating processes are in thermal equilibrium in the high-temperature phase of the standard model. As a consequence, asymmetries in baryon and lepton number are related at high temperatures, and the cosmological baryon asymmetry can be generated from a primordial lepton asymmetry. Necessary ingredients are right-handed neutrinos and Majorana masses, which occur naturally in SO(10) unification.
The baryon asymmetry can be computed by standard methods based on Boltzmann equations. In a consistent analysis lepton number violating scatterings have to be taken into account, since they can erase a large part of the asymmetry. In supersymmetric scenarios these scatterings are sufficient to generate an initial equilibrium distribution of heavy Majorana neutrinos.

Baryogenesis implies stringent constraints on the light neutrino masses. Assuming a similar pattern of mixings and masses for neutrinos and up-type quarks, as suggested by SO(10) unification, the observed asymmetry is obtained without any fine tuning. The $\nu_\mu$ mass is predicted in a range consistent with the MSW solution of the solar neutrino problem. $B - L$ is broken at the unification scale. The baryogenesis scale is given by the mass of the lightest of the heavy Majorana neutrinos, which is much lower and consistent with constraints from inflation and the gravitino abundance.

As our discussion illustrates, the cosmological baryon asymmetry is closely related to neutrino properties. Already the existence of a baryon asymmetry is a strong argument for lepton number violation and Majorana neutrino masses. Together with further information about neutrino properties from high-energy physics and astrophysics, the theory of the baryon asymmetry will give us new insights into physics beyond the standard model.

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