Metal liner-driven cylindrically convergent isentropic compression of cryogenic deuterium

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Abstract. In order to take advantage of geometrical convergence, we investigated a method, where a beryllium liner drives a cylindrical shockless compression in a cryogenic deuterium fill. The metal liner acts as a current carrier as well as a pressure boundary to the fill. The required driving pressure was obtained through a fictitious flow (FF) simulation [D S Clark and M Tabak 2007 Nucl. Fusion 47 1147]. A current model that can recreate the FF compression inside the liner by shaping the current pulse, is then introduced. This method also allows efficient compression of hydrogen at low entropy, enabling the recreation of conditions present in the interior of gas giants and potentially the observation of a transition into a metallic state. Our two-dimensional simulations show that thick liners remain robust to magneto-Rayleigh-Taylor instability growth, suggesting that cylindrical isentropic ramp compression is a promising scheme for extending deuterium’s experimentally measured equation of state.

1. Introduction
Knowledge of the properties of hydrogen (H) at pressures of the order of several hundred GPa is key to understanding the interior of gas giants [1] and to modelling ICF experiments [2]. In order to reproduce these conditions in the laboratory we investigated the use of metal liner implosions, which have gained considerable interest in the field of material science in recent years [3, 4].

Placing cryogenic H or deuterium (D) inside a metal liner and measuring its properties during implosion, or conducting a point measurement at peak compression is an attractive way of reaching high geometrical convergence, while ohmic heating of the sample is avoided by the high conductivity of the metal liner.

Motivated by ICF schemes we concentrate on D fills in this work. However, the presented liner-driven quasi-isentropic compression technique can be applied to H and other materials in a straightforward manner, making it valuable to Planetary Science and High Energy Density Physics in general. In this work we focus on a design where a peak current of 5.1 MA compresses D to a pressure of around 1 TPa at temperatures between 2500 and 5000 K. Successfully reaching these conditions through liner-driven compression requires the initiation of a cylindrically convergent ramp wave in the D through a controlled implosion of the liner by shaping the current pulse. Degenerate and potentially metallic D is obtained if shock formation is avoided and the D is kept on a low entropy adiabat. In section 2 we present an external pressure history for achieving this.

In section 3 we will derive a set of equations to determine the current required to recreate the previously described pressure history inside a metal liner. Simulations of the robustness of
Figure 1. The evolution of $P_D$, the pressure at the boundary of the real flow during the FF simulation, in comparison with $P_{DC}$ from liner-driven compression.

Figure 2. A sine squared current with 350 ns rise time $I_s$ (red), compared to the optimized current $I_o$ (green).

the implosion to magneto-Rayleigh-Taylor (MRT) growth are presented in section 4 before we finally draw our conclusions.

2. A pressure history for driving a cylindrically convergent isentropic compression in deuterium

By considering the requirements on accuracy of the pulse shape, efficiency of the implosion and mitigation of MRT growth we determined the following dimension of the D sample. A cylindrical D shell with an outer radius of 2.7 mm and an aspect ratio of 3, where the aspect ratio is defined as the ratio of the outer radius and the thickness of the shell.

The goal of the experiment is to compress the sample on a given low-entropy isentrope and obtain nearly uniform and degenerate D at peak compression. We obtained the pressure history $P_D(t)$ that is required at the outer surface of the D through Clark and Tabak’s fictitious flow (FF) method [5, 6]. The desired implosion is obtained by initializing a fictitious flow (FF) behind the D shell. This FF drives an isentropic ramp wave in the D that enables the D shell to approach the self-similar solution of a collapsing bubble. The pressure history $P_D(t)$ at the D/FF interface is recorded during the simulation and demonstrates features of an initial shock, the arrival of a rarefaction wave and an outgoing shock that establishes uniform density at peak compression, as shown in figure 1. We opted for an initial shock, that raises the speed of sound of the D. This allows a steeper isentropic ramp wave and thereby a reduction of the total implosion time [6].

3. Derivation of a current shape for liner-driven quasi-isentropic compression

We aim to reproduce the pressure history from the previous section inside a metal liner by shaping the drive current of a pulsed power generator. For choosing a suitable liner thickness a compromise between stability and efficiency of the implosion has to be made. Due to inevitable surface imperfections, MRT growth starts at the outer liner wall and progresses towards the inner liner wall during the course of the implosion. Thick liners can prevent the MRT instability from penetrating into the D, while thin liners require less drive energy, due to lower kinetic energy of the liner. The choice of low-density liner material can further increase the efficiency of the implosion. For this study we have chosen an aspect ratio 6 Be liner, which results in a liner thickness of 540 $\mu$m, due to the inner radius of 2.7 mm as stated in section 2.
In order to calculate the required current $I(t)$ for recreating $P_D(t)$ at D’s outer surface, we for now assume the liner as a perfect conductor, and obtain the following magnetic pressure $P_m$ applied at the outer liner wall:

$$P_m = \frac{\mu_0 I^2}{8\pi r^2},$$

where $r$ is the radius of the outer liner wall. The magnetic pressure launches sound waves into the liner and subsequently into the D fill. Due to the sound impedance mismatch between Be and D, the sound waves are weakened by the sound absorptivity $\alpha$:

$$\alpha = \frac{2Z_D}{Z_{Be} + Z_D},$$

where $Z_D = \rho_D c_D$ is D’s and $Z_{Be} = \rho_{Be} c_{Be}$ is Be’s sound impedance. In the calculation of $\alpha$ as a function of $P_m$ we assume that D and Be remain on their principal isentropes. Early in time $Z_D$ is much lower than $Z_{Be}$ and much of the sound energy remains in the liner ($\rho_D$ is about 10 times lower than $\rho_{Be}$). Later in time the D becomes more absorptive ($\rho_D$ is around 80% of $\rho_{Be}$). $\alpha(P_m)$ then determines the required magnetic pressure $P_m$ for a given D surface pressure $P_D$:

$$\alpha(P_m) P_m(t_1) = P_D(t_D),$$

where $t_D = t_l + t_s$. $t_s$ is the time it takes for a sound wave to propagate through the liner. $t_s$ decreases from 55 ns at the start to 30 ns towards the end of the implosion. Due to the liner’s finite conductivity, the current diffuses from the outer surface towards the inner liner wall. Therefore, the pressure waves originate from a distributed region within the liner. We assume the sound waves emerge from an effective position $r_c$. Here we approximate $r_c$ by the current’s mean position:

$$r_c = \frac{\int r j(r) dr}{\int j(r) dr}.$$

We then obtain an approximate equation for the time it takes sound waves to cover the distance $r_c - r_D$ by taking into account the sound speed in the Be and the velocity of the magnetic piston $v_c$:

$$t_l = t_D - t_s = t_D - \frac{r_c(t_l) - r_D(t_D)}{c_{Be}(P_m) - v_c(t_l)},$$

where Be’s Eulerian sound speed $c_{Be}$ is evaluated on its principal isentrope at the required pressure $P_m$ and $r_D$ is taken from the FF simulation. Equation 5 assumes that $c_{Be}$ is constant between $r_c$ and $r_D$, but that was nevertheless sufficient to obtain a good estimate for the time $t_l$.

These equations are the essential ingredients for approximating the current $I_o(t)$, in order to obtain the required pressure $P_D(t_D)$ on D’s outer surface. By evaluating this set of equations during a resistive MHD Gorgon [7, 8] simulation that used Lee-More-Desjarlais transport coefficients [9, 10] we calculated the required current shape. Additional to this current model, a suitable prepulse that provides a smooth transition into the current model for $t < 0$ was found through trial and error. For $t > 550$ ns, when the outgoing shock wave has reached D’s outer surface, the liner’s inertia applies a sufficient boundary pressure $P_D(t)$ and no further increase in current is required. Therefore, we reach the peak current $I_p = 5.1$ MA at $t = 550$ ns. We then smoothly drop the current to $I_o = 0$ within 150 ns. The resulting current shape is shown in figure 2 compared to a 350 ns sine squared pulse, which creates similar peak pressures for the same target.

A comparison between D’s boundary pressure in the FF simulation $P_D$ and the liner-driven compression $P_{DC}$ in figure 1 shows that a good match is achieved for $t > 100$ ns. The deviation...
Apart from a small hot region on-axis the D is fairly uniform with densities ranging from 2,950 kg/m$^3$ to 3,150 kg/m$^3$.

The assumption of a mean position $r_c$ for the origin of these sound waves is less obvious, but a comparison between the boundary pressure of FF $P_D$ and liner-driven case $P_{DC}$ for $100 \text{ ns} < t < 550 \text{ ns}$ shows that this definition does indeed give good results. For $t > 550 \text{ ns}$, when the outgoing shock wave reaches the FF, $P_{DC}$ becomes larger than $P_D$ due to the higher inertia of the liner compared to the FF and the remaining current continues to apply a magnetic pressure, which results in higher peak compression.

These deviations from the FF simulation late in time, together with our choice of a low aspect ratio D shell, as discussed in section 2, contribute to the formation of a small lower density hot-spot of 1,000 kg/m$^3$ at around 95,000 K at peak compression, shown in figure 3. The D surrounding this hot region is almost isochoric at densities around 3,000 kg/m$^3$ with temperatures ranging between 2,500 and 5,000 K.

4. MRT

We investigated the impact of MRT growth on the liner-driven quasi-isentropic compression and the properties of the D at stagnation by conducting 2D Gorgon simulations in cylindrical (RZ) coordinates [7, 8]. By comparing our simulations of an empty Be liner with radiography measurements [11] we found a suitable perturbation that was applied to the outer surface of the liner [6].

The design discussed in the previous sections stagnates at a radius of around 0.5 mm. The initial outer radius of the D sample is therefore reduced by a factor of around 5.4. MRT growth at the outer liner wall will have a negligible effect on the D/Be interface at this convergence ratio. In figure 4a) we see little variation in the density in axial direction behind the flow reversing shock for $r < 1.1 \text{ mm}$.

In order to demonstrate the effect of MRT growth at a higher convergence ratio we present the results of a design for a similar target with a peak current of 10.8 MA, where a convergence ratio of around 10 is reached. In figure 4b) we clearly see axial variations in the density. However, MRT bubbles do not break through into the D and the D cylinder maintains a stable expansion after peak compression.

During the flow reversal the D decelerates the Be liner. Since D is the lighter fluid the
D/Be interface is subject to Rayleigh-Taylor growth during this phase. The RT growth rate at the D-Be interface during the deceleration of the liner is determined by the Atwood number $A_t = \frac{(\rho_{Be} - \rho_D)}{(\rho_{Be} + \rho_D)}$. The mean $A_t$ for the 10.8 MA design is around 0.1 at peak compression. The total density map 20 ns after peak compression shown in figure confirms that this is sufficient for maintaining a fairly smooth D/Be interface.

We compared this implosion to a liner filled with D gas of density 10 kg/m$^3$. In a simulation this liner was imploded by applying the same current pulse and slow preheating of the gas to 15 eV resulted in the same peak convergence ratio as for the quasi-isentropically compressed liquid D. However, during stagnation the lower density of the D gas compared to the ramp compressed liquid D and subsequently larger mean $A_t$ of around 0.8 resulted in faster RT growth during flow reversal, which prevented a stable deceleration of the liner.

5. Conclusion

A scheme has been presented for assembling cold and degenerate D inside a metal liner by shaping the current of a pulsed power machine. We obtained a cylindrically convergent, self-similar, isentropic implosion of a D shell through a FF simulation and presented the required external pressure history for driving this implosion in section 2.

A set of equations that determine the current shape for recreating this FF implosion inside a metal liner was derived in section 3. Knowledge of the speed of sound in the liner and treatment of resistive diffusion by evaluating the mean position of the current during a Gorgon simulation gave control over the compression wave in the D. Apart from a small central hot region at 95,000 K the D is kept below the Saturn isentrope at temperatures between 2,500 and 5,000 K [12]. Densities in this cold region are around 3,000 kg/m$^3$ at a stagnation pressure of around 1 TPa.

In section 4 results of 2D Gorgon simulations show that the D is nearly unaffected by MRT growth at peak compression and subsequently during its expansion. By comparing the liner-driven ramp compression to a gas-filled target we showed that the high density of the ramp compressed D mitigates RT growth during flow reversal.

From the simulations in section 3 we determined a required peak voltage to drive the implosion presented of around 30 kV, a peak rate of increase in current of around 0.03 MA/ns and a total electromagnetic energy delivered to the load of around 12 kJ. These parameters are in line with
several of today’s pulsed power generators.

We conclude that the method presented is a very promising way of creating highly degenerate and potentially metallic D on pulsed power machines. Experimental realization of this method can lead to an extension of existing EOS data for D and other H isotopes, which would significantly improve modelling of ICF implosions as well as our understanding of planetary cores.

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