Superconductivity-enhanced spin pumping: The role of Andreev bound-state resonances

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We describe a simple hybrid superconductor|ferromagnetic-insulator structure manifesting spin-resolved Andreev bound states in which dynamic magnetization is employed to probe spin related physics. We show that, at low bias and below $T_c$, the transfer of spin angular momentum pumped by an externally driven ferromagnetic insulator is greatly affected by the formation of spin-resolved Andreev bound states. Our results indicate that these bound states capture the essential physics of condensate-facilitated spin flow. For finite thicknesses of the superconducting layer, comparable to the coherence length, resonant Andreev bound states render highly transmitting subgap spin transport channels. We point out that the resonant enhancement of the subgap transport channels establishes a prototype Fabry-Pérot resonator for spin pumping.

Introduction.—Spatial variations in the superconducting order in a finite region lead to the formation of spin-degenerate Andreev bound states (ABSs) with discrete excitation energies below the superconducting gap $[1]$. An externally applied magnetic field or proximity to a ferromagnetic order, on the other hand, can induce spin splitting in the ABSs that results into spin-resolved ABSs $[2, 3]$. In this Letter, we consider a normal metal (N) sandwiched between a superconductor (S) and a ferromagnetic insulator (FI) that serves as a simple platform with spin-resolved ABSs, which are localized in the N layer. The non-equilibrium pure spin current engendered from the externally driven FI–spin pumping—is utilized to probe spin transport in an S|N|FI hybrid structure. The spin pumping generated from a time-dependent magnetization, on the other hand, is a flow of spin angular momentum into adjacent materials that dissipates energy of the ferromagnet $[4]$. We suspect that the magnetic damping increase in a superconducting hybrid multilayer reported in Ref. $[5]$ can be understood by the resonant enhancement of the spin pumping discussed here.

In the context of superconducting spintronics, combining an s-wave superconducting order, favoring electrons to form a singlet state, with a ferromagnetic order, favoring spin alignment, leads to a powerful enhancement or reduction of angular momentum transfer $[7, 8]$. The angular momentum transfer, as a central effect in spintronics, is greatly modified on account of two major underlying causes: the itinerant spin-polarized quasiparticles (QPs) with long spin-coherence lengths $[9, 10]$ or the creation of spin-triplet Cooper pairs $[12–14]$ induced at highly spin-active regions or complex magnetic multilayers $[15]$. Here, the spin pumping in an S|N|FI hybrid structure, however, is an interplay between spin-polarized QPs and spin-triplet Cooper pairs, which are dynamically generated by the excited FI $[16, 17]$. The subgap ABSs accommodate the spin-polarized QPs and spin-triplet Cooper pairs that for a sufficiently thin S layer can tunnel across and contribute to the spin current. To collect the spin current we have placed a spin reservoir $N_r$, comprising an $N_r|S|N|FI$ structure, see Fig. 1.

Spin-resolved ABS.—We will determine the subgap spin-resolved ABSs by studying the resonant conditions for the spin transport in an $N_r|S|N|FI$ hybrid structure. In order to establish the pumped spin current in the presence of superconducting and dynamic ferromagnetic orders, we solve a time-dependent scattering problem $[17, 18]$, which accounts for the relevant processes such as Andreev reflection (AR), ABS, and a dynamic triplet-paring generation. For subgap energies, the full scattering matrix develops peak structures marking resonant bound states $[19]$ (ABSs), which result in highly transmitting subgap transport channels for the spin flow.

We proceed with establishing notations and key features of the scattering that is used throughout this Letter. The incident QPs from the reservoir onto the $N_r|S$
interface can either transmit across the S or retroreflect as the quasiholes back into the reservoir, a process known as AR. For a QP with energy \( \varepsilon \) the AR amplitude is given by

\[
r^\infty_A = \begin{cases} e^{-i \arccos(\varepsilon/\Delta)}, & |\varepsilon| < \Delta, \\ e^{i \arccos(\Delta/\varepsilon)}, & |\varepsilon| > \Delta, \end{cases}
\]

where \( \Delta \) is the superconducting pair-potential [2]. The AR amplitude given in Eq. (1), by focusing on the interface, assumes a bulk S (a thick S layer) [17]. A high probability of QP transmission for the thin S reduces the AR amplitude (we shall address this in Eq. (5)). The transmitted QPs with spin \( \sigma \in \{ \uparrow, \downarrow \} \) propagate through the N layer and acquire a spin-dependent phase \( e^{i \vartheta_A} \) upon reflection from the N|FI interface. We point out that the FI region is considered an exchange-splitting insulator for the QPs [21] that causes the full scattering matrix to be merely a reflection matrix. In addition, the full reflection matrix is block diagonal in the spin space due to the conservation of the QP spin during each individual scattering event at the interfaces. Consequently, multiple ARs at the N|S along with spin-dependent reflections at the N|FI interfaces constitute the full scattering matrix.

The spin-active interface N|FI, upon reflection, rotates the QP spin around the FI magnetization axis, which in turn for a driven magnetization leads to generation of a non-equilibrium spin current detected in \( N_e \) [22]. The conductance determining the transport of this spin current, known as the mixing conductance \( g^{\uparrow \downarrow} \), is given by

\[
g^{\uparrow \downarrow} = \text{Tr} \left[ 1 - r_{ee}^+ r_{ee}^{+ \downarrow} + r_{he}^+ r_{he}^{+ \downarrow} \right],
\]

where \( r_{ee}^\sigma \) and \( r_{he}^\sigma \) represent the total electron-to-electron and electron-to-hole reflection amplitudes in the S|N|FI hybrid structure, respectively [16, 18]. The reflection amplitudes are written in the basis where the spin quantization axis is parallel up to the magnetization in the FI and the trace is over all the transverse modes. (The exact expressions for the reflection amplitudes can be found in the Supplemental Material [17]).

Generically, the mixing conductance given in Eq. (2) is a complex number whose real part governs the spin pumping current [18]. After a straightforward calculation three distinct regimes are recognized for subgap energies:

\[
\text{Re}[g^{\uparrow \downarrow}] \approx \begin{cases} 1 - \cos \vartheta, & d_S \ll \xi_0, \\ \frac{\varepsilon^2}{\varepsilon^2 + \beta^2}, & d_S \approx \xi_0, \\ 0, & d_S \gg \xi_0, \end{cases}
\]

where \( \gamma = \exp[-2\sqrt{1 - (\varepsilon^2/\Delta_s^2)}/d_S/\xi_0] \), \( \beta = \cos(4d_N\varepsilon/\xi_0\Delta_0 + 2\vartheta_A) - \cos \vartheta_0/\sqrt{\sin^2 \vartheta_A} \), and \( \vartheta_A \approx -\arccos(\varepsilon/\Delta_0) \). The mixing angle defined as \( \vartheta_0 \equiv \vartheta_0 - \vartheta_1 \) is controlled by the FI exchange interaction for a given QP energy \( \varepsilon \). Here, \( \Delta_0 \) and \( \xi_0 \) are the superconducting gap and coherence length at zero temperature, respectively. Evidently, \( \text{Re}[g^{\uparrow \downarrow}] \) displays a resonant behavior associated with the intermediate S thickness whose energy levels correspond to the ABS energies \( \varepsilon_A \) determined by \( \beta = 0 \):

\[
4d_N\varepsilon_A/\xi_0\Delta_0 + 2\vartheta_A \equiv \pm \vartheta \pmod{2\pi}.
\]

When the exchange interaction is absent, that is, \( \vartheta = 0 \), Eq. (4) yields a pair of solutions \(-\varepsilon_A, \varepsilon_A\), which is a direct consequence of the particle-hole symmetry imposed on the scattering formalism [17]. A non-zero mixing angle \( \vartheta \), on the other hand, by lifting the spin degeneracy of each level results in the spin-resolved ABSs with the following energies \(-\varepsilon_A^\pm, \varepsilon_A^\mp\). As an outcome, the spectral overlap of the spin-split bound states can be controlled by the exchange interaction of the FI \( \Delta_\text{ex} \), e.g., see Fig. 2(a). One can discern that when \( d_N \rightarrow 0 \) Eq. (4) can be reduced to the well known result [23] for ABSs with magnetically active interfaces, \( \varepsilon_A/\Delta_0 = \pm \cos \vartheta/2 \). Hereinafter, in counting the number of the independent solutions for Eq. (4) we will only consider the positive energy ABSs. We emphasize that, following Eq. (3), only at \( d_S \approx \xi_0 \) the resonant ABSs establish highly transmitting transport channels for the spin flow, which dwindle for either a bulk or no S layer, see Fig. 2(b). This is one of the main results of this Letter.

We highlight the fact that Eq. (4) is a condition for a constructive quantum interference in a RowellMcMillan process [14] for the QPs, that is, four times crossing \( N \) with two Andreev conversions as well as two reflections from FI, once as electron and once as hole. It is clear that a constructive interference for QPs inside the normal layer remains intact as long as the probability of the AR...
and \( d_N \) are nonzero. This captures the essential physics of a two-mirror Fabry-Pérot resonator with a resonator length \( 2d_N \), which we shall describe now. The N|FI and S|N interfaces operate as “mirrors” for the QPs that give rise, respectively, to a spin-dependent specular reflection (a spin-dependent mirror) and a phase-conjugating mirror, which retroreflects electrons with energy \( E_F + \varepsilon_A \) as holes with energy \( E_F - \varepsilon_A \) [24]. The resonant enhancement of a Fabry-Pérot device occurs when its mirrors have a near unity reflection probability [25]. Here, the N|FI interface reflects all the incident QPs with probability one, while retroreflection probability of the S|N interface, on the other hand, is determined by the AR probability. Therefore, the Fabry-Pérot enhancement is in accordance with the AR amplitude, which for an S with a thickness \( d_S \) [17] is given by

\[
r_A = \frac{(1 - \gamma) r_A^\infty}{1 - \gamma (r_A^\infty)^2}.
\]

For subgap energies, in the limiting case of \( d_S > \xi_0 \), we get \( \gamma \ll 1 \) or equivalently \( |r_A| \approx 1 \), for which the resonant enhancement of a Fabry-Pérot device is expected. For energies above the gap, on the other hand, the AR reflection amplitude decreases. This results in highly transmissive channels near \( \varepsilon \gtrsim \Delta_0 \), which rapidly decline for higher energies, see Fig. 3.

The mixing conductance oscillation described here is a result of the QP interference in the normal layer (a RowellMcMillan resonance). In addition, above the gap A prior quantum interference inside the S layer takes place.

An incident electron-like QP interferes with a hole-like QP reflected from the other S|N interface, which leads to \( |r_A| = 0 \). The process known as Tomasch resonance [26] occurs for QP energies \( \varepsilon_n/\Delta_0 = \sqrt{1 + (n \pi \xi_0/2d_S)^2} \) with integer \( n \), which modulates the amplitude for the Fabry-Pérot oscillations. The RowellMcMillan and Tomasch based geometric resonances of \( \text{Re}[g^{\uparrow \downarrow}] \) are shown in Fig. 3.

**Spin pumping enhancement.**—The spin pumping is generated by the variations in the magnetization direction \( \mathbf{m}(t) \) [4, 12, 27]. For sufficiently slow variations, to the first order in the pumping parameter frequency \( |\mathbf{m}(t)| \), the spin pumping can be written in terms of the instantaneous mixing conductance in the magnetization coordinate system, that is, Eq. (2). Consequently, the spin pumping current, assuming no voltage bias [18], is given by

\[
I_s(t) = \frac{1}{4 \pi} \text{sign}(\xi) \mathbf{m} \times \partial_t \mathbf{m},
\]

where the effective mixing conductance is defined as follows:

\[
\delta^{\uparrow \downarrow}_{\text{eff}} = \int_{-\infty}^{\infty} d\varepsilon \partial_\varepsilon f(\varepsilon) \text{Re}[g^{\uparrow \downarrow}].
\]

Here, \( f(\varepsilon) = (1 + e^{\varepsilon/k_B T})^{-1} \) is the Fermi-Dirac distribution for a QP with energy \( \varepsilon \) and in order to properly take into account the temperature dependence of the S order, we have considered a temperature dependent S gap \( \Delta(T) \) [21].

In accordance with Eq. (6), the spin pumping current in the direction of \( \mathbf{m} \times \partial_t \mathbf{m} \) is simply determined by
$g'_{\text{eff}}$, which can be regarded as the mixing conductance transformed into the temperature domain. The resultant effective mixing conductance is plotted in Fig. 4. We find that for subgap temperatures $g'_{\text{eff}}$ shows a significant enhancement at the finite thickness of the S layer, i.e., $d_S \approx \xi_0$. As it can be seen in Fig. 4, the enhancement is optimal at the midgap temperatures and diminishes upon approaching the normal state regime at $T \approx T_c$. Assuming a fixed subgap temperature, e.g., $T = 0.4T_c$, we find that the optimal enhancement happens for $\Delta_{\text{ex}} < E_F$ and $d_N \geq \xi_0$, see Fig. 5. The enhancement behavior gradually ceases to exist for a narrower normal layer, i.e., $d_N < \xi_0$. This indicates that the insertion of a normal metal layer not only provides a level of control but is a necessary element for the enhancement of spin flow.

Before concluding this Letter, we explore the effect of interface barrier potentials at the $N_S|S|N$ interfaces on the effective mixing conductance. The dimensionless parameters $Z_1$ and $Z_2$ determine strength of the barrier potentials given by $Z_1 \hbar v_F \delta(x)$ and $Z_2 \hbar v_F \delta(x - d_S)$, where $v_F$ is the Fermi velocity and $\delta(x)$ is the Dirac delta function. Assuming a weak barrier potential, the overall enhancement feature persists. However, for strong barrier potentials the interfaces are highly reflective and the effective mixing conductance decreases, see Fig. 6. A quantum interference instigated by the partially reflective interface at $x = 0$ ($Z_1 \neq 0$) superimposes an oscillation with the Fermi wavelength on the effective mixing conductance. In accord with this description, no oscillation is observed for a barrier potential located at $x = d_S$ ($Z_2 \neq 0$).

**Conclusion and discussion.**—We have shown that superconductivity can enhance spin pumping due to the formation of the resonant spin-resolved ABSs, which result in highly transmitting spin transport channels. For finite thicknesses of the S layer, our setup operates as a Fabry-Pérot resonator for the spin pumping. On the other hand, unfolding $N_S|S|N$ FI effectively maps our setup to an $N_S|S|F|S|N$ structure, where $F$ stands for a normal metal with a ferromagnetic order. For finite thicknesses of the S part in the latter structure, an enhanced damping in the magnetization dynamics has been reported in Ref. [5]. Furthermore, hybrid Josephson junctions realizing ABSs with near unity transmission probability for charge transport have been proposed to coherently manipulate quantum-information devices such as Andreev-level qubit [28, 29]. From this standpoint, unfolding our setup realizes a magnetically active Josephson junction [9] with resonant transport channels, which in turn, can provide a spintronic paradigm for a coherent manipulation of quantum-information devices involving spin-resolved ABSs.

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Throughout this Letter, we consider an energy gap of $1.1E_F$ for up-spin and $1.1E_F + 2\Delta_{ex}$ for down-spin QPs.
Supplementary Material for “Superconductivity-enhanced spin pumping: The role of Andreev bound-state resonances”

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In this supplementary material, we discuss a QP scattering formalism that takes into account a dynamic FI order and superconducting pair-potential in an $N_r|S|N|FI$ hybrid structure, see Fig. S1. We will derive analytical expressions for the scattering amplitudes and elaborate on the dynamic generation of the triplet Cooper pairs in our setup.

**Spin-dependent scattering.**—In the context of QP scattering in hybrid structures generic boundary conditions pertinent to interface scattering and disorder can be taken into account by an insertion of a perfect spacer N between adjacent materials [1, 2]. As such, the full scattering process can be broken down into studying $N_r|S|N$, and $|FI$ structures separately. We bear in mind that propagation through the N layer mixes neither particle-hole nor spin degrees of freedom. For the $N_r|S|N$ structure, following BTK [3], we assume a uniform superconducting pair-potential $\Delta$ inside the S layer that vanishes in the N layers. This way the scattering process takes place only at the interfaces, which essentially contains QPs propagating into the S and Andreev retroreflections. In a more realistic (dirty) normal metal-S interface, however, there is a possibility of a specular reflection for electrons and holes. In order to model partially reflective interfaces, we have assumed a barrier potential in the form of $V(x)$.

Initially, we study point contact scattering in a 1D wire. Physically, this can be justified by imposing a constriction, that is, transverse thickness of the N layers to be on the order of a Fermi wavelength [1]. Later on, we will relax this condition by considering a 3D geometry for which the incident QPs can have a transverse momentum component.

Let us construct a basis for the scattering. The scattering is carried out by spinful QPs, i.e., electrons and holes, whose amplitudes are given by the complex numbers $c_{e,h}^\pm$. The superscript $\pm$ sign indicates the direction of the group velocity (normal component) given by $\pm \zeta^{e,h} \hat{\kappa}$, for which we have defined $\zeta^e = +1$, and $\zeta^h = -1$. We begin with the BdG formalism that accounts for symmetries of the superconducting part for the electron and hole wave functions,

$$H_{\text{BdG}} \Psi = \varepsilon \Psi,$$

with

$$H_{\text{BdG}} = \begin{pmatrix} \hat{H}_0 & \hat{\Delta} \\ \hat{\Delta}^\dagger & -\hat{H}_0^\dagger \end{pmatrix},$$

where $\hat{H}_0 = (p^2/2m + V - E_F)$ is the single-electron Hamiltonian matrix and $\hat{\Delta} = \Delta i\sigma_y$, in which $\Delta$ is a real-valued pairing potential. The Pauli matrix $\sigma_y$ is acting on the spin space $\{|\uparrow,\downarrow\}$ of conduction electrons and holes. Above eigenvalue equation admits the following basic electron and hole QP modes with spin $\sigma$:

$$\Psi_{n,e,\sigma}^\pm = \begin{pmatrix} \chi_{\sigma}^+ \\ A(\varepsilon)(-i\sigma_y)\chi_{\sigma}^+ \end{pmatrix} e^{\pm iq_e \cdot r_{||}^{\sigma}} e^{ik_{||} \cdot r_{\parallel}^{\sigma}},$$

$$\Psi_{n,h,\sigma}^\pm = \begin{pmatrix} A^\dagger(-\varepsilon)(-i\sigma_y)\chi_{\sigma}^+ \\ \chi_{\sigma}^+ \end{pmatrix} e^{\pm iq_h \cdot r_{||}^{\sigma}} e^{ik_{||} \cdot r_{\parallel}^{\sigma}},$$

where $\chi_\uparrow = (1_0)$, $\chi_\downarrow = (-i\sigma_y)\chi_\uparrow$, are two dimensional spinors, and we have defined

$$A(\varepsilon) = \begin{cases} e^{-i\text{arccos}(\varepsilon/\Delta)}, & |\varepsilon| < \Delta, \\ \frac{\varepsilon - \varepsilon \sqrt{1 - \Delta^2/\varepsilon^2}}{\Delta}, & |\varepsilon| > \Delta. \end{cases}$$

Due to a translation symmetry on transverse direction ($r_{\parallel}$) $k_{\parallel}$ is a good quantum number and conserved during the scattering. Owing to the S spectrum one has

$$q_{e,h} = \sqrt{\frac{2m}{\hbar^2} \sqrt{E_F + \zeta^{e,h} \sqrt{\varepsilon^2 - \Delta^2}}}.$$

The full energy of an incoming electron or hole in the normal layer is given by $E = \frac{\hbar^2}{2m} k_{e,h}^2$, where $k_{e,h} = q_{e,h}(\Delta \to 0)$.

The conservation of the QP spin in the scattering leads to a full scattering matrix that is block diagonal in the spin space. Whence, we can work with the following 2 dimensional eigenvectors,

$$\Psi_{S_{ne}}^\pm = \begin{pmatrix} 1 \\ A(\varepsilon) \end{pmatrix} e^{\pm iq_e \cdot r_{||}^{\sigma}} e^{ik_{||} \cdot r_{\parallel}^{\sigma}},$$

$$\Psi_{S_{nh}}^\pm = \begin{pmatrix} A(\varepsilon) \\ 1 \end{pmatrix} e^{\pm iq_h \cdot r_{||}^{\sigma}} e^{ik_{||} \cdot r_{\parallel}^{\sigma}}.$$
It is clear that outside of the S, where $\Delta \to 0$, we get $A(\varepsilon) \to 0$, leading to normal region's basis elements,

$$
\Psi_{Nce}^\pm = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{\pm ik_c x} \Phi(y, z),
$$

$$
\Psi_{Nnh}^\pm = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{\pm ik_h x} \Phi(y, z),
$$

Therefore, the incident and reflected modes in N and S layers can be expanded as

$$
\Psi_N = c_e^+(N)\Psi_{Nce}^+ + c_e^-(N)\Psi_{Nce}^- + c_h^+(N)\Psi_{Nnh}^+ + c_h^-(N)\Psi_{Nnh}^-,
$$

and

$$
\Psi_S = c_e^+(S)\Psi_{Sce}^+ + c_e^-(S)\Psi_{Sce}^- + c_h^+(S)\Psi_{Snh}^+ + c_h^-(S)\Psi_{Snh}^-.
$$

The scattering matrix across the S is obtained by imposing the following conditions: 1) Continuity of the wave functions $\Psi_{Nc}$, $\Psi_S$ and their derivative at the interface $x = 0$. 2) Continuity of the current inside the S, that is, $0 < x < d_S$. 3) Continuity of $\Psi_S$, $\Psi_N$ their derivative at $x = d_S$. A straightforward algebra, under Andreev approximation $\Delta/E_F \ll 1$, yields

$$
\begin{pmatrix}
  c_e^+(N) \\
  c_e^-(N) \\
  c_h^+(N) \\
  c_h^-(N)
\end{pmatrix} = \frac{1}{1 - (r_A^\infty)^2} \begin{pmatrix}
  0 & \kappa \gamma u^2 & (r_A^\infty)(1 - \gamma) & 0 \\
  \kappa \gamma u^2 & 0 & 0 & (r_A^\infty)(1 - \gamma) \\
  (r_A^\infty)(1 - \gamma) & 0 & 0 & \gamma \kappa^{-1} u^2 \\
  0 & (r_A^\infty)(1 - \gamma) & \gamma \kappa^{-1} u^2 & 0
\end{pmatrix} \begin{pmatrix}
  c_e^+(N) \\
  c_e^-(N) \\
  c_h^+(N) \\
  c_h^-(N)
\end{pmatrix},
$$

where

$$
\kappa = e^{ik_F d_S}, \quad \gamma = e^{2\Omega d_S}/\hbar v_F,
$$

$$
u^2 = \frac{\varepsilon}{\Omega} \left( \frac{1}{2} + \frac{\Omega}{2\varepsilon} \right), \quad (r_A^\infty)(\varepsilon) = A(\varepsilon),
$$

in which

$$
\Omega = \begin{cases}
    i\sqrt{\Delta^2 - \varepsilon^2}, & \text{if } |\varepsilon| < \Delta \\
    \varepsilon \sqrt{1 - \Delta^2/\varepsilon^2}, & \text{if } |\varepsilon| > \Delta
\end{cases}; \quad H_0^2 = \Omega^2.
$$

This scattering matrix can be generalized to include nonzero $Z_{1,2}$ on the interfaces [4]. The quantity $(r_A^\infty)$ is the AR amplitude at the interface of a normal metal and a bulk S [1]. For a thin S layer, as it can be checked from the QP scattering matrix for the $N_r|S|N$ structure, this is modified to

$$
r_A = \frac{(1 - \gamma)(r_A^\infty)}{1 - \gamma(r_A^\infty)^2}.
$$
For the normal metal N a width of $d_N$, the electron and hole propagate through and reflect back off the FI interface without mixing particle-hole and spin spaces. In general, this reflection corresponds to a phase shift for the reflected electrons and holes from the magnetically active FI region,

$$c^-_e(N) = r^e_e c^+_e(N),$$
$$c^+_h(N) = r^-_h c^-_h(N),$$  \hspace{1cm} (S13)

in which the electron reflection coefficient is given by $r^e_e(\varepsilon) = e^{i2k_d d_N e^{i\theta\sigma}}$, and particle-hole symmetry yields $r^h_h(\varepsilon) = \left[r^e_e(-\varepsilon)\right]^*$. The spin-dependent phase shift $\theta_\sigma$ is given by

$$e^{i\theta_\sigma} = \frac{\sqrt{E} - i\sqrt{V_\sigma - E}}{\sqrt{E} + i\sqrt{V_\sigma - E}}, \quad E < V_\sigma.$$  \hspace{1cm} (S14)

The energy of an incoming electron or hole is given by $E = E_F + \zeta e^h |\varepsilon|$, where the excitation energy $\varepsilon$ is measured with respect to the Fermi energy $E_F$. Combining the scattering matrix of $N|S|N$ with Eq. (S13) the full scattering matrix obtains, which relates incoming modes to the outgoing ones as follows:

$$\begin{pmatrix} c^-_e(N_f, \varepsilon_n) \\ c^+_h(N_f, \varepsilon_n) \end{pmatrix} = \begin{pmatrix} r^\sigma_{ee} & r^\sigma_{eh} \\ r^\sigma_{he} & r^\sigma_{hh} \end{pmatrix} \begin{pmatrix} c^+_e(N_i, \varepsilon_n) \\ c^-_h(N_i, \varepsilon_n) \end{pmatrix},$$  \hspace{1cm} (S15)

where

$$r^\sigma_{ee} = \frac{\kappa^2_\gamma}{2} r^\sigma_e, \quad r^\sigma_{hh} = \frac{\kappa^2_\gamma}{2} r^\sigma_h,$$
$$r^\sigma_{eh} = \frac{\kappa_\gamma}{A} (1 - \gamma) \left[1 - (r^A_\infty)^2 - (r^A_\infty)^2 r^2_e r^{-2}_h \right].$$

In the above equations we have defined

$$\eta \equiv (1 - (r^A_\infty)^2)^2 - (r^A_\infty)^2 (1 - \gamma)^2 r^2_e r^{-2}_h.$$

It is instructive to note that a constructive RowellMcMillan interference takes place when

$$r^\sigma_e r^{-\sigma}_h (r^A_\infty)^2 = e^{i2n\pi}, \quad n = 0, \pm 1, \pm 2, \cdots$$  \hspace{1cm} (S17)

The extra terms in $\eta$ stem from multiple tunneling and reflection possibilities due to finite thickness of the S, that is, $\gamma \neq 0$. At this point it is refreshing to test the scattering matrix elements against the intuitive circumstances. In the limit of thick S, $d_S \to \infty$ the scattering process is dominated by AR for $\varepsilon < \Delta$. Taking the limit, we get $r^\sigma_{ee} = r^{-\sigma}_{hh} = 0$, and $r^\sigma_{eh} = r^{-\sigma}_{he} = r^A_\infty$. Additionally, in the limit of no S, $d_S \to 0$, particle-hole mixing elements vanish i.e., $r^{-\sigma}_{ee} = 0$.

The spin-flipped scattering matrix elements ($\sigma \to -\sigma$) are obtained via $r^A_\infty \to -r^A_\infty$,

$$r^{-\sigma}_{eh}(\varepsilon) = -r^{-\sigma}_{eh}(\varepsilon), \quad r^{-\sigma}_{he}(\varepsilon) = -r^{-\sigma}_{he}(\varepsilon),$$  \hspace{1cm} (S18)

and from the particle-hole symmetry it follows that

$$\left[r^{\sigma}_{ee}(-\varepsilon)\right]^* = r^{\sigma}_{hh}(\varepsilon), \quad \left[r^{\sigma}_{eh}(-\varepsilon)\right]^* = r^{\sigma}_{he}(\varepsilon).$$  \hspace{1cm} (S19)

In short, the scattering matrix with all the non-zero elements can be written as follows:

$$S(\varepsilon) = \begin{pmatrix} r^\sigma_{ee}(\varepsilon) & 0 & 0 & r^\sigma_{eh}(\varepsilon) \\ 0 & r^\sigma_{ee}(\varepsilon) & r^\sigma_{eh}(\varepsilon) & 0 \\ 0 & r^\sigma_{he}(\varepsilon) & r^{-\sigma}_{hh}(\varepsilon) & 0 \\ r^\sigma_{he}(\varepsilon) & 0 & 0 & r^{-\sigma}_{hh}(\varepsilon) \end{pmatrix}. $$  \hspace{1cm} (S20)

Here, we conclude by noting that the condition given in Eq. (S17) corresponds to the resonant behavior in the mixing conductance, see Fig. S2.

**Time-dependent scattering.**—A comparison between a typical FI precession period $\sim 10^{-10}$ s (FMR period) with the time scales of electron and hole dynamics, $(d_S + d_N)/v_F \ll 10^{-10}$ s, reveals that the magnetization dynamics
FIG. S2. Left panel: The number of resonant ABSs can be controlled by \( d_N \). Right panel: In spite of an overall reduction in the mixing conductance with the interface barrier potentials \( Z_1 = Z_2 = 0.1 \), the underlying resonant behavior persists. Here, we have chosen \( d_S = 1.5\xi_0 \) and \( \Delta_{\text{ex}} = 0.4E_F \).

FIG. S3. The effective mixing conductance per unit area \( L^2 \) shows a similar enhancement feature as the 1D case. For an incident QP with the transverse momentum \( k_\parallel \) on a 2D interface, all the transverse modes are integrated out to obtain an effective 1D setup.

can be taken as an adiabatic evolution during the scattering process. Initially, this reduces the problem to a time-independent process carried out with a snapshot configuration for the magnetization. Nonetheless, one can generalize to incorporate magnetization precession dynamics with a simple time-dependent \( SU(2) \) rotation to the lab frame in which the magnetization direction is varying [5]. Including the precessional motion of the magnetization, which is rotating around \( z \)-axis \( \mathbf{m}(t) = (\sin \theta \cos \omega t, \sin \theta \sin \omega t, \cos \theta) \), can be accomplished by a spinor rotation [5] given by

\[
U(t) = \begin{pmatrix}
    e^{i\omega t \sigma_y} & 0 \\
    0 & e^{i\omega t \sigma_y}
\end{pmatrix} \begin{pmatrix}
    e^{i\omega t \sigma_z} & 0 \\
    0 & e^{-i\omega t \sigma_z}
\end{pmatrix},
\]

where \( \omega \) is the precessional angular speed. Consequently, the time dependent scattering matrix in the lab frame obtains, \( S(\varepsilon, t) = U(t)^\dagger S(\varepsilon) U(t) \).

3D geometry.—Due to the translation symmetry on the interfaces, the full 3D scattering problem can be inferred from our scattering matrix, given in Eq. (S20), by the following transformation \( E_F \to E_F - E_\parallel \), where \( E_\parallel = \hbar^2 k_\parallel^2 / 2m \) refers to the energy associated with the transverse modes. Now for a given incident QP energy \( E_F - E_\parallel + \zeta e^h \varepsilon \), we sum over all \( E_\parallel \) to obtain net 1D spin current per unit area. For sub-gap energies, the mixing conductance has
negligible values except when Eq. (S17) is satisfied. Therefore, integrating over all the energy values in spin current, only selects incident particles with ABS energies irrespective of their incident angle. This way, the mechanisms led to the enhancement of the spin current are preserved, e.g., see Fig. S3.

Dynamic generation of triplet Cooper pairs—We utilize a heuristic argument to describe the triplet pairing generation in our setup. For a thin S layer, $d_S \sim \xi_0$, QPs with the resonant energies can escape the N layer either by tunnelling through the S layer or forming Cooper pairs inside the S layer which is due to the s-wave pairing order of the S. For the latter case, one can obtain [6] the Cooper pair containing the spin-dependent phases as follows:

$$e^{i\alpha} |\uparrow\downarrow\rangle - e^{-i\alpha} |\downarrow\uparrow\rangle,$$

where $\alpha \sim \vartheta$. The transformed Cooper pair contains a spin-triplet component, $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, with an amplitude determined by $\sin \vartheta$. The precessing magnetization $\mathbf{m}$ can dynamically induce spin-polarized triplet Cooper pairs $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$, which is caused by the variations in the direction of the quantization axis for spins [6]. From a mathematical point of view, this can be understood by noting that the spin-polarized and $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ triplet states belong to the same irreducible Hilbert space, as such any unitary transformation on an element will be expanded within the Hilbert space.

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