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THE DECLINE OF THE SOURCE POPULATION OF GAMMA-RAY BURSTS AND THEIR LUMINOSITY FUNCTION

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ABSTRACT

The source population of gamma-ray bursts (GRBs) declines toward the present epoch, being consistent with the measured decline of the star formation rate. We show this using the brightness distribution of 3255 long BATSE GRBs found in an off-line scan of the BATSE continuous 1.024 s count rate records. The significance of this conclusion is enhanced by the detection of three GRBs with known redshifts brighter than $10^{52}$ ergs s$^{-1}$ during the last two years. This is an argument in favor of the generally believed idea that GRBs are strongly correlated with star production, at least on cosmological timescales, and favors the association of long GRBs with collapses of supermassive stars. However, we still cannot rule out neutron star mergers if the typical delay time for binary system evolution is relatively short. If we assume a steep decline of the GRB population at $z > 1.5$, then their luminosity function can be clearly outlined. The luminosity function is close to a power law, $dN/dL \propto L^{-1.4}$, for low luminosities over at least 1.7 orders of magnitude. Then, the luminosity function breaks to a steeper slope or to an exponential decline around $L \sim 3 \times 10^{51}$ ergs s$^{-1}$ in the 50–300 keV range, assuming isotropic emission.

Subject headings: galaxies: high-redshift — gamma rays: bursts — methods: data analysis

1. INTRODUCTION

In spite of the remarkable progress that has proved the cosmological origin of gamma-ray bursts (GRBs), there remain a number of extremely important issues that are still not resolved. Some of the main issues dealt with in this work are (1) What is the cosmological evolution of the source population of GRBs? Does it evolve as the star formation rate (hereafter the SF rate) or does it have its own specific evolution? (2) What is the luminosity distribution (or function) of GRBs? (3) What is the total rate of GRBs in the universe?

There are two competing approaches for such studies: (1) the "statistical" one, using large samples of poorly localized GRBs, and (2) the "individual" one, using the small sample of optically followed-up GRBs, in which we have additional information per event, including the redshift and the intrinsic luminosity.

The main data array for statistical studies was supplied by the Burst and Transient Source Experiment (BATSE; Fishman et al. 1989) on board the Compton Gamma Ray Observatory (CGRO). The BATSE sample is a few times larger than the yield of all other experiments that have detected GRBs. It includes 2702 events in its final form.

Fitting the BATSE data to various cosmological/evolutionary models has been the subject of many studies since the start of the BATSE operation in 1991. For references to early works on fitting the BATSE brightness distribution of GRBs to cosmological models, see Bulik (1999). For a number of results setting an upper limit to the width of the luminosity function of GRBs, see, e.g., Hakkila et al. (1996). Subsequently, Loredo & Wasserman (1998) demonstrated that the third BATSE catalog does not constrain the luminosity function. Later work using the larger sample of the fourth BATSE catalog gave few constraints. Krumholtz, Thorsett, & Harrison (1998), fitting the BATSE sample at peak fluxes $P > 0.42$ photons s$^{-1}$ cm$^{-2}$, found that both a nonevolving GRB source population and SF evolutionary models fitted the data, even using standard-candle GRBs. Wijers et al. (1998) demonstrated the same for the SF model. Totani (1999) showed that the SF model does not fit the data using the standard-candle assumption. Using the SF model and taking into account the time delay due to binary system evolution, Panchenko (1999) estimated the minimal width of the luminosity function to be 2 orders of magnitude in luminosity. Finally, the recent work of Porciani & Madau (2001) dealt with a larger sample, including the BATSE nontriggered bursts of Krommers et al. (2000), extending the peak flux down to 0.18 photons s$^{-1}$ cm$^{-2}$. They found that the data fit could not distinguish between different variants of the GRB source evolution at large redshifts. However, they did not check how sensitive the fit was to the evolution at low redshifts.

The general impression arising from fitting the BATSE data using cosmological models was that this approach had little future. Indeed, the "statistical" approach demonstrated an agreement between the data and a wide set of models. The very few constraints obtained were trivial.

The main reason for the poor progress so far is the insufficient depth of the BATSE sample, i.e., the too narrow brightness range. The brightest burst has a peak photon flux of $160$ photons s$^{-1}$ cm$^{-2}$. Bursts useful for a usual least-$\chi^2$ fit are, however, at $P < 30$ photons s$^{-1}$ cm$^{-2}$. The BATSE trigger threshold is at $0.2$ photons s$^{-1}$ cm$^{-2}$, but the bursts near the threshold are difficult to use because of poorly known threshold effects. All works cited above used the peak flux range above 0.4 or even above 1 photon s$^{-1}$ cm$^{-2}$, which is then narrower than 2 orders of magnitude. Loredo & Wasserman (1998) showed that this is too narrow a range for obtaining constraints from the fits.

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The catalog is available at http://gammaray.msfc.nasa.gov/batse/grb/catalog/current.
The “individual” approach has given a wealth of important data. By itself, however, this approach is still unable to resolve the issues stated above. Besides having too poor statistics (currently, we have only 17 GRBs with known redshifts), the approach is subjected to very strong selection biases. Nevertheless, this small sample tells us that the luminosity function is at least 2.5 orders of magnitude wide and that it extends up to $\sim 3 \times 10^{52}$ erg s$^{-1}$.

After most previous work in this area was completed, the following progress concerning the data accumulation has taken place:

1. BATSE obtained additional data until the deorbiting of CGRO on 2000 June 4.

2. Searches for nontriggered bursts were performed by Schmidt (1999), Koomers et al. (2000), and Stern et al. (2000, 2001). In the latter work, the statistics of useful GRB events was increased by a factor of 1.7, and the threshold effect was measured. Thus, the useful fitting range was extended down to 0.1 photons s$^{-1}$ cm$^{-2}$.

3. A sample of GRBs with known intrinsic luminosities (currently 17 events) appeared as a result of optical afterglow observations.

In this use, we work these advances, including the “individual” GRB data. In addition, we include the brightest peak flux interval, which is statistically poor, into the maximum likelihood fit and find that this peak flux interval is very informative. Thus, we extend the fitting brightness range to 3 orders of magnitude. This allows us to obtain a number of conclusive results.

In §§2–4 we describe the set of fitted data, the fitting models including the cosmology, the source evolution, the luminosity function of GRBs, and finally, the fitting procedure. In §5 we present results of the fits and show that the scenario of a nonevolving population of GRBs does not fit the data. Instead, we demonstrate that the GRB population should decline approximately as fast as the star formation rate. We also determine the approximate shape of the luminosity function and give an estimate for the lower limit of the total rate of GRBs as being 3000 GRBs yr$^{-1}$ in the visible universe (that is, up to a reasonably large redshift).

2. THE DATA

Probing various cosmological and evolutionary models, we fitted the sample of 3255 BATSE GRBs longer than 1 s found by Stern et al. (2000, 2001) in the off-line scan of the BATSE continuous daily records in 1.024 s time resolution. This sample, which is selected from the catalog of Stern and Tikhomirova, is essentially uniform and has a corresponding efficiency matrix (measured by a test burst method), which is needed when fitting the weak end of the log $N$–log $P$ distribution. (Hereafter, the term log $N$–log $P$ distribution means the differential distribution of GRBs vs. the logarithm of the peak photon flux.) We excluded short bursts (consisting of one 1.024 s bin) from the analysis for two reasons: (1) short and long bursts could be separate phenomena and (2) the sample is incomplete regarding short bursts, as they have a lower detection efficiency and an incorrect brightness estimate in 1.024 s time resolution.

By excluding one-bin events, we make our sample more homogeneous.

The brightness distribution of the GRBs in this sample was fitted using a hypothetical brightness distribution folded with the detection efficiency matrix described in Stern et al. (2001). This matrix was obtained using a sample of 11,000 artificial test bursts, which were superimposed on the BATSE continuous records and then passed through the same procedure of search and processing as real GRBs. The efficiency matrix is approximately given by

$$F(c_e, c_m) = E(c_e) \frac{1}{\sigma \sqrt{\pi}} \exp \left[ -\frac{\log^2 (c_m/c_{n0})}{2\sigma^2} \right],$$

where $c_e$ is the expected and $c_m$ the measured count rate in units of counts s$^{-1}$ cm$^{-2}$, $E(c_e) = 0.70 \{1 - \exp[-(c_e/c_{e0})^2]\}$ is the efficiency function with fitted parameters $c_{e0} = 0.97$ counts s$^{-1}$ cm$^{-2}$ and $\nu = 2.34$, the lognormal factor describes the relative error of the measured count rate $\sigma = 0.09 (0.08/c_e)^{1/2}$, and the selection bias is crudely expressed as $c_{n0} = c_e + 0.05 \exp(-c_e/0.05)$.

In order to constrain the intrinsic luminosity function (hereafter the luminosity function or the LF), we used the sample of gamma-ray bursts with measured redshifts. We cannot infer the LF from this sample, as it is subjected to strong selection biases. This is demonstrated in §3.3. The redshift data, however, give us a useful piece of information, i.e., the existence of very intrinsically bright GRBs. Three of the intrinsically brightest bursts are GRB 990123, GRB 991216, and GRB 000131 (named by dates), with redshifts 1.6 (Djorgovski et al. 1999), 1.02 (Vreeswijk et al. 1999), and 4.5 (Andersen et al. 2000), respectively, and with BATSE peak fluxes 16.4, 67.5, and 6.3 photons s$^{-1}$ cm$^{-2}$ in the 50–300 keV range, respectively (estimated using the BATSE catalog). If they were emitted at $z = 1$, their peak fluxes with the “K-correction” taken into account (i.e., the correction due to the spectral redshift effects on a fixed spectral band of the detector) would be 45, 69, and 84 photons s$^{-1}$ cm$^{-2}$, respectively, assuming the cosmological parameters ($\Omega_M, \Omega_\Lambda$) = (1, 0). Hereafter, we use the photon peak flux at redshift $z = 1$, $I$, as a measure of the intrinsic brightness. We furthermore use the intrinsic brightness interval of these three events, $I > 40$ photons s$^{-1}$ cm$^{-2}$, to constrain the LF when fitting the BATSE log $N$–log $P$ distribution. The choice of the three brightest events for this purpose is somewhat arbitrary. We cannot use a much wider brightness interval because of a brightness-dependent selection bias. On the other hand, we can neglect such problems for the narrow brightness range of these three events. The data we fitted are presented in Figure 1.

In order to impose a proper constraint on the LF, we must estimate the sampling function for strong GRBs. By the sampling function, we mean the probability that a burst will be detected and localized, its afterglow observed, and its redshift measured. This function evolves with time. It was zero before 1997. Then, this function was limited by the field of view of the two BeppoSAX Wide-Field Cameras, $\sim 0.08$ of the sky, as this was the main instrument supplying precise coordinates of GRBs during 1997–1998.

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5 See http://www.astro.su.se/groups/head/grb_archive.html.

6 See, e.g., http://www.aip.de/~jcg/grb.html.

7 See http://www.sdc.asi.it/bepposax.
In 1999 and 2000, many precise localizations were made by other systems, with most of them being made by the interplanetary network (IPN) Ulysses/Konus/Near Earth Asteroid Rendezvous (see the IPN home page). This means that in this period, the sampling function became larger, and for very bright events, it could in principle approach unity, as all instruments of the IPN had a 4π field of view. Actually, it should be considerably less, as in the same period, three very strong BATSE events (triggers 7301, 7491, 7595) were not localized. For one strong BATSE event (trigger 7954, GRB 000115), an X-ray transient was found, but no optical transient. With this background, let us take a conservatively high estimate of the sampling function, $S_{40} = 0.5$ for GRBs with $I > 40$ photons s$^{-1}$ cm$^{-2}$, and a conservatively low estimate of the rate of these GRBs, $N_{40} = 3$ yr$^{-1}$, in the visible universe.

3. FITTING MODELS

3.1. Cosmology

We tried two sets of cosmological parameters: the flat matter-dominated universe, commonly used in most previous works ($[\Omega_M, \Omega_\Lambda] = (1, 0)$), and the vacuum-dominated cosmology, which is supported by recent data ($[\Omega_M, \Omega_\Lambda] = (0.3, 0.7)$) (see, e.g., Lukash 2002). Hereafter, these two models are denoted as M-models and Λ-models, respectively. The distribution of GRBs over redshift for a nonevolving (NE) population for $\Omega_M + \Omega_\Lambda = 1$ is

$$\frac{dN}{dz} \propto \frac{1}{1+z} \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3} \times \left[ \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}} \right]^2.$$  (2)

The photon number “luminosity” distance is defined by

$$d_L = \frac{c}{H_0} \sqrt{1+z} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}},$$  (3)

where $H_0$ is the Hubble constant, which is assumed to be 75 km s$^{-1}$ Mpc$^{-1}$ when estimating the luminosities of GRBs. For the standard luminosity distance, the factor $(1+z)^{1/2}$ is replaced with $1+z$.

3.2. Evolution of the Source Population

Another important component of the model is the evolution of the population of GRB sources. We probed four cases: a nonevolving population and three evolution functions correlated with the history of star formation following Porciani & Madau (2001). The declining phase of the SF rate at $z < 1.5$ is a relatively well-measured function of $z$. Its history at $z > 2$ is, however, controversial. This issue is discussed in Porciani & Madau (2001), giving the relevant references. Below we reproduce three versions of the SF evolution suggested in that work:

$$R_{\text{SF1}}(z) = \frac{0.3e^{3.4z}}{(e^{3.8z} + 45)} \; M_\odot \; \text{yr}^{-1} \; \text{Mpc}^{-3},$$  (4)

i.e., decreasing SF at $z > 1.5$,

$$R_{\text{SF2}}(z) = \frac{0.15e^{3.4z}}{(e^{4.2z} + 22)} \; M_\odot \; \text{yr}^{-1} \; \text{Mpc}^{-3},$$  (5)

i.e., roughly constant SF at $z > 2$, and

$$R_{\text{SF3}}(z) = \frac{0.134e^{3.05z}}{(e^{2.93z} + 15)} \; M_\odot \; \text{yr}^{-1} \; \text{Mpc}^{-3},$$  (6)

i.e., increasing SF at large $z$.

Hereafter, models are denoted as NE,$M$; SF1,$\Lambda$; and so on, where $M$ and $\Lambda$ denote the two types of cosmologies.

The next step is the generation of the standard-candle log $N$–log $P$ distributions. At this step, we introduce an additional broadening of the observed brightness distribution due to the K-correction that depends on the type of GRB spectrum. For this purpose, we obtained the log $N$–log $P$ distributions using Monte Carlo simulations. To each simulated GRB, we prescribed one of 54 spectra of bright BATSE bursts parametrized by the Band expression (Band et al. 1993). All these template spectra were assumed to be emitted at $z = 1$. Then, we sampled the $z$ of the burst, and the corresponding K-correction for the 50–300 keV band was applied to the apparent brightness of the simulated GRB.

The resulting log $N$–log $P$ distributions are shown in Figure 2. If we neglect the K-correction that depends on the GRB spectrum, then each distribution in Figure 2 is a direct reflection of the corresponding redshift distributions of

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8 See http://ssl.berkeley.edu/ipn3/index.html.
GRBs. The standard-candle luminosity (before applying the K-correction) corresponds to a peak photon flux of \(1 \text{ photon s}^{-1} \text{ cm}^{-2}\) in the 50–300 keV band at \(z = 1\).

If we vary this value when fitting the data, none of the eight models (two cosmological cases, four evolutionary cases) would still fit the observed \(\log N – \log P\) distribution of 3255 long BATSE GRBs (shown by the crosses in Fig. 2). The fact that the evolution of the SF1 type with the standard-candle luminosity function cannot fit data was shown by Totani (1999) and Lloyd & Petrosian (1999). The NE model gives the smallest deviation in this case, but the value of \(\chi^2\) is still unacceptable (61 for 27 degrees of freedom).

However, as is shown below, the discrepancy in the case of the NE model cannot be compensated for by any hypothesis due to the superposition of cosmological and evolutionary effects.

The slopes of the three \(\log N – \log P\) distributions for an evolving SF rate are close to the Euclidean slope of \(-3/2\), starting from a peak flux \(P \sim 1 \text{ photon s}^{-1} \text{ cm}^{-2}\), which at this standard-candle brightness corresponds to \(z = 1\). The agreement with the Euclidean slope is an accidental coincidence due to the superposition of cosmological and evolutionary effects.

3.3. Parameterization of the Luminosity Function

As was stated above, the luminosity distribution of events with known \(z\) cannot be used as a base for the LF model in the whole brightness range. This fact is clear from Figure 3, in which we present \(\log N – \log P\) distributions that would give the sample of 17 GRBs with known absolute luminosities for the NE, SF2, and SF3 evolutionary models. All models give a striking disagreement with the data. The only way to reduce this disagreement is to assume an unreasonably sharp increase of GRBs at large redshifts that, in turn, will contradict the redshift data. Therefore, the shape of a hypothetical LF remains arbitrary (except for the constraints imposed on the brightest end of the LF).

In order to get a handle on the LF of GRBs, we tried different types of functions that describe common shapes of wide distributions in nature: the lognormal distribution (LGN), a truncated power law (TPL), a power law with an exponential cutoff (PLexp), and a broken power law (BPL).

\[ \text{LGN: } dN/dI = C \exp[-\ln^2(I/I_0)/2\sigma^2], \]
\[ \text{with three free parameters } I_0, \sigma, \text{ and C.} \]
\[ \text{TPL: } dN/dI = C I^{\alpha-1} \text{ for } I_1 < I < I_0 \text{ and 0 outside this interval. Free parameters are } \alpha, I_1, I_0, \text{ and C. In some fits, } I_1 \text{ was fixed to } 0, \text{ leaving three free parameters.} \]
\[ \text{PLexp: } dN/dI = C I^{\alpha-1} \exp(-I/I_0), \]
\[ \text{with three free parameters } \alpha, I_0, \text{ and C.} \]
\[ \text{BPL: } dN/dI = C I^{\alpha-1} \text{ for } I_1 < I < I_0, dN/dI = C, I^{\alpha-1} \text{ for } I_0 < I < I_2, \text{ and } dN/dI = 0 \text{ outside the } [I_1, I_2] \text{ interval. Free parameters are } \alpha, \beta, I_1, I_0, \text{ and C, while } I_2 \text{ is fixed to a value above the maximum observed GRB brightness.} \]

We also considered a smoothed version of the broken power law:

\[ \text{SBPL: } dN/dI = C I^{\alpha-1}/[1 + (I/I_0)^B]. \]

For technical convenience, we measure the intrinsic brightness as peak count rate or peak photon flux \(I\) in the 50–300 keV range produced by a GRB at a distance corresponding to \(z = 1\). The absolute peak luminosity of the GRBs is related to \(I\) as \(L = I (3 \times 10^{50}) \text{ ergs s}^{-1}\), assuming isotropic emission. Below we present the main results for the LF both in \(I\) and in absolute luminosity units.

4. THE FITTING PROCEDURE

We used the forward-folding method when fitting the observed distribution of GRBs, i.e., the hypothetical
brightness distribution was convolved with the efficiency matrix from equation (1) and fitted to the observed distribution of GRBs over peak count rate (crosses in Fig. 2). This distribution was represented by 29 data points spaced by 0.1 in log $P$ in the interval 0.067–50 photons s$^{-1}$ cm$^{-2}$ in the 50–300 keV range. During 9.1 yr, there are three GRBs detected by BATSE that are brighter (the rightmost cross in Fig. 2 and one GRB at 160 photons s$^{-1}$ cm$^{-2}$, which is not shown). We treat this brightness range separately, estimating the likelihood function of the fit for each peak flux interval. For the main interval, this is the standard $\chi^2$ probability function. For the tail of the brightness distribution, the likelihood is the Poisson probability of sampling not more than three events brighter than 50 photons s$^{-1}$ cm$^{-2}$ at the given number of such events for the full observation period, $M_{50}$, predicted by the model. The final likelihood function is the product of these two factors.

Instead of the usual $\chi^2$ minimization procedure, we explore the parameter space, sampling ~10$^3$ random points. This is sufficient to find the minimum of $\chi^2$ with a good accuracy while at the same time investigating the $\chi^2$ “topography” of the parameter-space region. This method does not work if the “valley” of the minimum has a very small volume in parameter space. For our cases, the minima are wide and smooth enough.

The maximum likelihood point for the whole sample of points in parameter space represents the unconstrained fit, in which the requirement of the redshift data, $N_{40} > 3$ yr$^{-1}$, was ignored. The subsample selected including this requirement represents the constrained fit. The effective number of degrees of freedom in the second case is smaller by unity as compared to the unconstrained fit. We present results of both the constrained and the unconstrained fits in order to demonstrate the role of the redshift data and the possible effects of the uncertainty in the estimate of the rate of intrinsically strong GRBs.

The best fit parameters are presented in Tables 1 and 2. Table 1 includes fits using various LF models and selected cosmological models. Table 2 summarizes fits using a broken power-law LF for all cosmological models and provides data to compare their relevance using a Bayesian approach.

5. RESULTS

5.1. Rejection of Models without Source Evolution

The best unconstrained fit using the NE models has $\chi^2 = 39.8$ for 25 degrees of freedom (Table 1, truncated power law [TPL]), which is marginally acceptable if we ignore the tail of the brightness distribution. For the tail, the model predicts $M_{50} = 11.5$, while the real number is 3 yr$^{-1}$ and the corresponding maximum likelihood drops down to $1.3 \times 10^{-4}$ (see Table 1). When we impose the redshift data constraint, $N_{40} > 3$ yr$^{-1}$, the maximum likelihood factor that we can obtain using the NE, $\Lambda$ model is $3.2 \times 10^{-6}$, which is for the case of a broken power-law LF (see Table 2). For the NE, $M$ model, the results are even worse.

The likelihood factor is not yet a rejection factor for the NE model, because we cannot exclude some bias or contamination that would increase the $\chi^2$. To reject the NE hypothesis, we should demonstrate a good fit for other equally simple and reasonable models. Indeed, data fits with the SF models are much better (see Tables 1 and 2). Their maximum likelihood factor is about 0.02. This is the case when we can apply the Bayesian approach. The estimate of the rejection level for models with a non-evolving GRB source population is the ratio of the maximum NE likelihood factor (NE, $\Lambda$ model in Table 2) to that for SF models (e.g., SF1, $\Lambda$ in Table 2), which is $1.4 \times 10^{-4}$.

Figures 4, 5, and 6 demonstrate the differences between fits using NE and SF models. As seen in Figure 5, the high brightness slope of the log $N$–log $P$ distribution for the broken power-law NE model is too flat, and it cannot be made considerably steeper by modifying the LF. Note that the NE standard-candle log $N$–log $P$ distribution with $I = 1$ photon s$^{-1}$ cm$^{-2}$ is already flatter than the observed log $N$–log $P$ distribution. Then, if the LF is extended to ~100 photons s$^{-1}$ cm$^{-2}$, the model tail will be considerably flatter than the observed tail, independently of how the LF is extended.

Looking at the integral distribution of real GRBs in Figure 6, one notes that it declines faster than a Euclidean distribution. However, this is still not a statistically significant fact. The probability of such a deviation from the $-3/2$ slope by chance is 0.1, which is relatively large (integral distributions are known to produce an illusion of statistically

| Model          | Luminosity Function | $\chi^2$ | $M_{50}$ | Likelihood | $\alpha$ | $\delta$ | $\Delta\delta$ |
|----------------|---------------------|----------|----------|------------|----------|----------|----------------|
| NE, $\Lambda$ | TPL                 | 39.8, 36.8 | 11.5     | $1.3 \times 10^{-4}$ | $-0.37 \pm 0.27$ | $0.11$ | $2.8, 10.7$ |
| NE, $\Lambda$ | LGN                 | 44.3, 42.5 | 16.6     | $1.2 \times 10^{-6}$ | $1.9$ | $0.032, 0.36$ |
| NE, $\Lambda$ | PLexp               | 54.0, 49.7 | 18.7     | $2 \times 10^{-8}$ | $-0.21 \pm 0.12$ | $0.09$ | $15, 21$ |
| SF1, $\Lambda$| LGN                 | 44.5, 39.8 | 6.0      | $3 \times 10^{-4}$ | $...$ | $2.0$ | $0.032, 0.12$ |
| SF1, $\Lambda$| PLexp               | 31.2, 30.7 | 7.0      | $0.022$ | $-0.43 \pm 0.09$ | $14, 28$ |
| SF2, $\Lambda$| PLexp               | 34.2, 31.7 | 6.7      | $0.016$ | $-0.36 \pm 0.10$ | $11, 33$ |
| SF2, $M$      | PLexp               | 32.7, 31.4 | 6.9      | $0.018$ | $-0.36 \pm 0.08$ | $12, 26$ |

Note.—For models and LF parameterizations, see §§ 3.1–3.3. The fit with the LF unconstrained by the redshift data is given only for the TPL case. For all other given fits, the LF is constrained by $N_{40} > 3$ yr$^{-1}$. The two $\chi^2$ values are given at the maximum likelihood and the minimum $\chi^2$ points, respectively. The “likelihood” is the product of the $\chi^2$ probability and the Poisson probability of sampling not more than three events at a given expected number $M_{50}$. The value of $M_{50}$ is given at the maximum likelihood point. $\alpha$ is the slope of the power-law part of the LF, $\delta$ is the width of the lognormal distribution (see § 3.3), and $I_0$ is the upper cutoff brightness of the truncated power-law LF, the center of the lognormal distribution, or the exponential cutoff energy for the PLexp LF (see § 3.3). The errors in $\alpha$ and the confidence interval, $\Delta\alpha$, for $I_0$ correspond to $\Delta\chi^2 = 4$ (corresponding to 2 or in the conventional interpretation. However, one should be careful with such an interpretation when the results depend on missing data points below the threshold).
significant features from fluctuations). Most probably, we are just dealing with a moderate fluctuation. Nevertheless, the observed slope could really be steeper than the Eudclidean one if the decline of GRBs is steep enough. More data are required to clarify this issue.

The rejection of the NE models is significant, even without the redshift data. The unconstrained rejection factor, taken as the ratio of the unconstrained likelihoods of the NE, A and SF1, A models, is \( \sim 2 \times 10^{-3} \) (see Table 2). This means that the result is not very sensitive to the value of the constraining \( N_{40} \). If we, e.g., overestimate the rate of intrinsically strong GRBs by a factor of 3 (suppose that it is a fluctuation) and assume that \( N_{40} = 1 \), then the rejection factor is \( 4 \times 10^{-4} \).

While the NE models are rejected using the low-redshift behavior, one does not obtain any preference for a certain kind of SF evolution at large redshifts. Different SF models give similar likelihood results (see Tables 1 and 2), except for the SF3 scenario, which gives a slightly worse fit. Furthermore, the data do not allow us to distinguish between matter-dominated and vacuum-dominated cosmologies.

### 5.2. The Shape of the Luminosity Function

Once the NE models have been rejected at a significant level, we now concentrate on the SF models, i.e., models with evolution of the GRB source population. There are

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**TABLE 2**

| Model          | \( \chi^2 \) | \( M_{50} \) | Likelihood        | Unconstrained Likelihood | \( \alpha \) | \( \Delta I_0 \) | \( \Delta \chi^2 \) | \( N_{\text{tot}} \) yr\(^{-1} \) |
|----------------|-------------|-------------|------------------|--------------------------|-------------|----------------|-----------------|------------------|
| NE, M          | 39.1        | 16.8        | \( 6 \times 10^{-7} \) | \( 4 \times 10^{-5} \) | -0.32 ± 0.50 | 0.23, 3.3     | 7               | 2800             |
| NE, A          | 40.4        | 14.5        | \( 3.2 \times 10^{-6} \) | \( 0.87 \times 10^{-4} \) | -0.33±0.04  | 0.43, 5.0     | 5               | 3000             |
| SF1, M         | 29.0        | 7.2         | 0.015            | 0.043                    | -0.49±0.08  | 2.8, 9.2      | 33              | 1800             |
| SF1, A         | 29.4        | 6.7         | 0.023            | 0.050                    | -0.48±0.09  | 2.9, 16       | 28              | 2000             |
| SF2, M         | 30.2        | 6.8         | 0.018            | 0.031                    | -0.44±0.11  | 2.9, 15       | 25              | 2700             |
| SF2, A         | 30.3        | 6.0         | 0.021            | 0.028                    | -0.43±0.12  | 3.4, 21       | 19              | 3000             |
| SF3, M         | 30.9        | 7.2         | 0.014            | 0.20                     | -0.38±0.12  | 2.9, 11       | 21              | 4400             |
| SF3, A         | 30.7        | 6.55        | 0.014            | 0.17                     | -0.34±0.17  | 4.2, 21       | 16              | 4500             |

**Note.**—The LF is parametrized as a truncated broken power law with four free parameters (the BPL model in § 3.3). The number of degrees of freedom is 25. Many details are given in the note of Table 1. Here \( \chi^2 \) is the minimum \( \chi^2 \). The index \( \alpha \) is the slope of the power-law part of the LF below the break. The errors correspond to a 1 confidence interval, and \( \Delta I_0 \) is the confidence interval (\( \Delta \chi^2 = 4 \)) of the break in the LF, \( I_0 \), in photons s\(^{-1} \) cm\(^{-2} \) at \( z = 1 \). Introducing a break in a single power law gives the \( \chi^2 \) reduction, \( \Delta \chi^2 \), in the next column. The term \( N_{\text{tot}} \) is the lower limit of the total GRB rate per year.

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**Fig. 4.**—Fits of the bright tail of the \( \log N - \log P \) distribution using a broken power-law LF, showing the value of \( \chi^2 \) vs. the predicted number of bright events \( M_{50} \) at \( N_{40} > 3 \) yr\(^{-1} \). The actual value of \( M_{50} \) is 3. Parameters \( I_1, I_0, \alpha, \) and \( \beta \) are random. The models are NE, A (top panel) and SF1, A (bottom panel).

**Fig. 5.**—Best fits of data to models with no evolution (top panel: NE) and with SF1, A (bottom panel) GRB source evolution. Crosses are observed data points corrected using the efficiency matrix from eq. (1). **Solid curves:** Models; **dotted curves:** model LF (a broken power law); **dashed line:** Eudclidean -3/2 slope. The LF corresponds to a GRB luminosity distance at \( z = 1 \) with an arbitrary normalization of the rate. The cosmological model has \( (\Omega_M, \Omega_{\Lambda}) = (0.3, 0.7) \). For fitting parameters, see Table 2.
two clear features of the LF that we see for all SF models: a near-power-law interval at the lower brightness range and a break or turnover toward the bright end of the distribution. Attempting to replace this construction of the LF by a log-normal LF gives a decrease of the maximum likelihood by 2 orders of magnitude (see, e.g., the two SF1 models in Table 1).

If we study the \( C^2 \) topography for the broken power-law LF, we find a power-law fragment below the break at least 1.7 orders of magnitude wide, and there is no upper limit on its width. It can be arbitrarily extended to lower brightnesses (see Fig. 7). Indeed, one can see from Figure 7 that the \( C^2 \) distribution reaches its asymptotic shape: extending the LF further to lower brightnesses does not affect the model in the range of the data points.

On the other hand, the slope of this power law is surprisingly well constrained, especially for the SF1 model (Fig. 7), and it is slightly sensitive to the chosen SF behavior at large redshifts. However, the shape of the \( C^2 \) minimum changes. Larger SF rates at high redshifts allow flatter slopes \( \alpha \) (see Fig. 8). The \( C^2-\alpha \) plots for a matter-dominated cosmology, which are not shown, are very close to those shown in Figure 8. They are just slightly narrower and more symmetric.

A break or a turnover is necessary at a high significance level. Its removal increases \( C^2 \) by \( \Delta C^2 \sim 30 \) for SF1 and \( \Delta C^2 \sim 19-25 \) for SF2 models (see Table 2). The properties of the break are, however, less certain than the parameters of the power-law fragment. The \( C^2 \) minimum is nonparabolic, asymmetric, and relatively wide, as seen in Figure 9. All we can say is that some turnover in the power-law LF is required at an intrinsic brightness of about 10 photons s\(^{-1}\) cm\(^{-2}\) at \( z = 1 \), or \( \sim 3 \times 10^{31} \) ergs s\(^{-1}\) for isotropic emission. The fitted position of the break is slightly sensitive to the cosmological model, but the difference is within the statistical errors. We, however, cannot distinguish between a power-law break and an exponential cutoff of the LF. Both

![Fig. 6.](image_url)  
Fig. 6.—Same best-fit functions as in Fig. 5, but in integral form. Histogram: raw peak count rate distribution in integral form \( N(>P) \) is the number of GRBs with peak flux larger than \( P \); solid curve: NEA model; dotted curve: SF1A model; dashed line: Euclidean distribution. The model curves are convolved with the efficiency matrix to correspond to the raw data in this figure.

![Fig. 7.](image_url)  
Fig. 7.—Top: Fits of the log \( N - \log P \) distribution for a broken power-law LF for the SF1A model, showing \( C^2 \) vs. the "width parameter" \( \log \left( I_0/I_1 \right) \) of the low brightness power-law fragment. Minimum \( C^2 \) is 29.4. Bottom: Confidence area for parameters of the power-law fragment of the broken power-law LF. The confidence area corresponds to \( \Delta C^2 < 6.17 \), which formally corresponds to a 2 \( \sigma \) confidence interval. However, see the notes of Table 1.

![Fig. 8.](image_url)  
Fig. 8.—Profile of the \( C^2 \) minima for the power-law slope \( \alpha \). Left panels: for a broken power-law LF; right panels: for the PLexp LF. From bottom to top: SF1A, SF2A, and SF3A models.
give a good $\chi^2$ and the same maximum likelihood (compare Tables 1 and 2). Summarizing, we can claim that the behavior of the LF below $I \sim 3$ photons s$^{-1}$ cm$^{-2}$ or 10$^{51}$ ergs s$^{-1}$ (in the 50–300 keV range) is close to the power law $dN/dI \propto I^{-\alpha}$ (or, more exactly, the power-law index $\alpha \sim 1$ can vary from $\sim 1.35$ to $\sim 1.5$, depending on the model). Then, it breaks to a steeper slope. A smooth break (i.e., the parameterization SBPL in § 3.3) also gives a good result. The likelihood is 0.016 for the SF2,Λ model when the difference in the slopes is large enough, i.e., $\beta < -3$.

Figure 10 shows a set of the best-fit LFs for different models. The reason why the best-fit LFs are more or less well defined is clear when comparing the LF curves with the standard-candle curves in Figure 2. For the SF1 and SF2 models, the standard-candle curves are “narrower” than the BATSE log $N$–log $P$ distribution. Therefore, the LF should roughly correspond to the main features of the observed log $N$–log $P$ distribution: a power law with a turnover. In the case of the SF1 model with narrower redshift distribution, the LF is closer to the observed brightness distribution (see Fig. 10). It is natural that the required turnover of the LFbelow $I \sim 3$ photons s$^{-1}$ cm$^{-2}$ at $z=1$. Crosses show real data points vs. the apparent brightness, where the distance for each event is unknown. Thus, the ordinate for real data has a different meaning.

5.3. The Lower Limit on the Total GRB Rate

The last column of Table 2 shows the lower limits on the total rate of GRBs in the visible universe. These limits correspond to $\Delta\chi^2 = 7$, i.e., to $1$-$\sigma$ of a $\chi^2$ distribution with 25 degrees of freedom. Note that their values are obtained using an abrupt cutoff of the LF at the dim end. For the SF2,Λ model, the highest allowed cutoff is at $\sim 0.4 \times 10^{50}$ ergs s$^{-1}$. A more realistic smoother cutoff would give a higher lower limit. We suggest that the value 3000 GRBs yr$^{-1}$ gives a realistic estimate of the minimum GRB rate. This estimate is for long GRBs only. The result depends on the SF model in a natural way, predicting a larger result for a higher SF rate at large redshifts.

For the SF2,Λ model of the GRB source evolution, the highest minimum comoving GRB rate at large redshifts is $\sim 3$ yr$^{-1}$ Gpc$^{-3}$. At the present epoch, it becomes 0.13 yr$^{-1}$ Gpc$^{-3}$. This lower limit coincides with the value claimed by Porciani & Madau (2001) as the estimate derived from a fit of the data of Kommers et al. (2000). It is, however, very difficult to compare results, as the slopes and the ranges of the LF are different. In fact, the shape of the LF derived by Porciani and Madau has the same parameterization as one of our models, a power law with an exponential cutoff, but nevertheless contradicts our results, predicting a different slope and a finite estimate for the low brightness cutoff of the LF. The latter could be a consequence of the different behavior of the log $N$–log $P$ distribution near the threshold in Kommers et al. (2000).

Our estimate is close to the result of Schmidt (2000), which is $\sim 0.2$ yr$^{-1}$ Gpc$^{-3}$ at $z = 0$, derived assuming that the present GRB rate is 10 times less than at $z \sim 1.5$ (in our case, the decline is a factor of 23). On the basis of this estimate, we can discuss the possible association of GRB 980425 and the supernova SN 1998bw (Bloom et al. 1999; see also Lamb 1999 for a discussion). Schmidt (2000) estimates the probability of events such as GRB 980425 if the association is real as being as low as $10^{-3}$ yr$^{-1}$. Indeed, the distance to SN 1998bw is $\sim 40$ Mpc. The corresponding
sampling volume is $2.7 \times 10^{-5}$ Gpc$^3$. Our estimate of the GRB rate is 0.13 yr$^{-1}$ Gpc$^{-3}$, with the cutoff $\sim 3 \times 10^{46}$ ergs s$^{-1}$. The 50–300 keV peak luminosity of GRB 980425 is $3 \times 10^{46}$ ergs s$^{-1}$ (our rough estimate). If the power-law LF continues down to this luminosity with the determined slope $\alpha = 1 - 1.4$, then the rate of GRBs above $10^{46}$ ergs s$^{-1}$ will be $\sim 20$ times larger, i.e., 2.6 yr$^{-1}$ Gpc$^{-3}$ at $z = 0$, or $0.7 \times 10^{-4}$ yr$^{-1}$ in the sampling volume. Furthermore, one should take into account the probability that such events will be localized with the accuracy of a few arcseconds, which reduces the estimate by at least an extra order of magnitude. Thus, the probability of occurrence and good localization of an event such as GRB 980425 within $\sim 40$ Mpc is below $10^{-2}$ yr$^{-1}$ if the derived power-law behaviour of the LF extends down to $10^{46}$ ergs s$^{-1}$. Such a small probability can hardly be compensated for by a break in the LF below the observational cutoff. The break must be so steep that it will affect the observed log \( L \sim \log P \) distribution near the BATSE threshold. This is a strong argument that GRB 980425 might represent a different phenomenon that does not overlap with classic GRBs in its luminosity function (but at the same time is indistinguishable from typical GRBs in its general appearance). A more probable possibility is that we are dealing with an accidental coincidence.

6. CONCLUSIONS

The source population of GRBs sharply declines from $z \sim 1.5$ toward the present epoch. This fact is established at the confidence level of $10^{-4}$. The measured decline of the star formation rate being used as the evolution hypothesis for the source population of GRBs fits the data satisfactorily.

This is what is expected according to the prevailing view of GRBs being the product of stellar evolution. Such a scenario was already successfully applied for the description of the observed log \( N \sim \log P \) evolution, e.g., by Wijers et al. (1998), Panchenko (1999), Kommers et al. (2000), Schmidt (2000), and Porciani & Madau (2001). However, nobody has quantitatively demonstrated that an evolution similar to the SF rate at some range of \( z \) is necessary to describe the data. We can only mention the work of Schmidt (2000), in which the SF rate hypothesis fits the bright tail of the log \( N \sim \log P \) distribution better (by a visual impression) than the NE hypothesis. However, no quantitative comparison of different scenarios has been done. We have demonstrated here that the decline of the GRB population consistent with the SF decline is a necessary requirement.

The main issue is whether the GRBs are associated with the collapse of massive stars (collapsars) or the merging of neutron stars (mergers). On the small scale, these scenarios differ regarding the expected correlations of GRBs with star-formation regions in galaxies: collapsar GRBs should be well correlated and merger GRBs should show no correlation with the star formation. This fact stimulated searches for such correlations using optical GRB afterglows (see, e.g., the review of van Paradijs, Kouveliotou, & Wijers, 2000). On the cosmological scale, such correlations should exist in both cases. However, in the merger scenario, the occurrence of GRBs would be described by the SF rate convolved with a delay function. If the latter extends to a few billions of years, then the decline of the GRB population will be considerably flatter. For estimates of the delay function for mergers, see Portegies Zwart & Yungelson (1998) and Panchenko et al. (1999).

Qualitatively, our results favor the collapsar scenario. A very interesting situation occurs if the very steep (steeper than $-3/2$) slope of the log \( N \sim \log P \) tail, as shown in Figure 5, persists to larger \( P \). (Ulysses data reduced by Atteia, Boer, & Hurley 1999 indicate that this can be the case). Then one will have to accept that the GRB progenitors are very massive collapsars whose population can decline faster than the general SF rate.

However, at present we do not have sufficient statistical arguments to rule out mergers. We believe that the data can give tight constraints on the delay function for the merger scenario and the latter can be challenged by such constraints. However, we leave such estimates for future studies for the reason that in order to obtain solid conclusions, it is worthwhile to incorporate the Ulysses data, which would provide at least a doubling of the statistics of the brightest GRBs.

Our results concerning the luminosity function of GRBs confirm the conclusion of Loredo & Wasserman (1998) that the width of the luminosity function is not constrained by the BATSE data being wider than 2 orders of magnitude. The shape of the LF tried in most previous works was a truncated power law or a lognormal distribution that satisfied earlier data in a narrower brightness range. Schmidt (2000) used a broken power-law hypothesis similar to one of our models. The position of the break is consistent with our results.

The interpretation of the shape of the luminosity function is beyond the scope of this work. In principle, such types of distributions—broken or exponentially cut power laws—are common in nature. Then, the break implies some physical limit such as a finite energy source. We believe that the outlined shape of the LF might be a useful clue in the development of physical models for the GRB emission.

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