Explicit Design For Strategy Formulation Frameworks*

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ABSTRACT
This paper deals with the explicit design of the strategy formulations to make the best strategic choices from a conventional matrix form. The explicit strategy formulation is a new mathematical model which provides the analytical strategy framework to find the best moment for strategy shifting for preparing the rapid market changes. Analytically tractable results are obtained by using the fluctuation theory. These results enables to predict the moment for changing strategy in a matrix form and even predict the prior moment of changes. This explicit model could be adapted into practically every strategic decision making situations which are described as a matrix form with the quantitative measures of the decision parameters. This research helps strategy decision makers who want to find the optimal moments when the present strategy should be shifted.

Keywords: Strategy; strategy formulation; fluctuation theory; first exceed model; BCG growth-share matrix; business level strategy

1. INTRODUCTION
The strategy is the set of actions to execute core competencies and gain a competitive advantage. Choosing a proper strategy means firms make choices among competing alternatives as the pathway for deciding how they will pursue strategic competitiveness [1]. It is also dealing with the determination of the long-run goals, the adaptation of series of actions, and the allocation of resource necessary for carrying out these goals [2]. The strategic management is a series of managerial decisions and actions that effect on the business performance of a company. Environmental scanning, strategy formulation, strategy implementation, and evaluation and control are major parts of strategic management [3]. The strategic management follows the process which contains the full set of commitments, decisions, and actions required for a company to achieve strategic competitiveness and gain the break-even returns [1].

Strategy formulation is a particular process to choose the most appropriate strategic action for the realization of organizational goals and objectives and thereby achieving the organizational vision [4]. The strategy formulation process is the vital for the success of the company because its results provide the blueprint for the strategic actions to achieve the goals of a company. Strategy formulation forces a company to carefully check at the moment of changing environment and to be prepared for the possible strategy shifts that may occur [5]. Generally, it contains defining the corporate mission, specifying achievable objectives, developing strategies, and setting policy guidelines [3]. The strategy formulation is based on the sources of its strategic

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inputs (i.e., decision parameters). With this information, the company develops its vision and mission and formulates one or more strategies [1].

This paper deals with analytical model of the strategic formulation. A matrix form has been widely applied in various strategy formulations which are not even recognized by researchers from the business and the management area. The general framework for formulating the strategy choices by using a matrix form are introduced. The main contribution of this paper provides the general framework for formulating the strategy and this framework is explicitly designed and mathematically proven. The theory to determine the probability generating function of the strategy shifting moments is included in this paper. The mathematical model in this paper explicitly identify the best moment of shifting the strategic choice of the company. It even counts the long-term strategy shift which is the one step prior of passing the thresholds of strategic decision parameters. One of powerful mathematical tools in the stochastic modeling is applied to find these best moments of the strategy shifting. The fluctuation theory is originally to introduce non-equilibrium thermodynamics [13]. Fluctuations in macro-states are described by Gaussian probabilities and a quadratic form for the second or transition entropy [13]. The first exceed model is another type of the fluctuation model that is the behavior of a two-dimensional compound renewal process about some fixed two-dimensional level [14-15]. The process will evolve until one of the components hits (i.e. reaches or exceeds) its assigned level for the first time. The following associated random variables are considered: the first passage time, the first excess level (i.e. the value of the process at the first passage time) and the termination index [14]. Dhshalalow [14-15] has developed a joint transformation of the first exceed level, first passage time, and the index of the point process. The first exceed model has been applied into various application including the antagonistic games [16-18], the stochastic defense system [19] and the Blockchain Governance Game [20]. Solving the probability distribution of the opponent by using the basic fluctuation theory [15-17] is another core contribution of this research.

The matrix form is atypical way to describe the set of strategic choices based on the decision parameters (i.e., the category of the strategy input) in the strategy formulation. Although the matrix has been developed from the mathematics, it has been widely used for describing the strategic management issues. Strategic group mapping is usually described as a matrix form. Strategic groups could be categorized in RD intensity and export focus. Decision maker choose the proper strategic choice based on these two decision parameters [2].
Stakeholders can be also described as a matrix form based on the interests of shareholders and their involvement power. Strategy makers should choose strategic alternatives that minimize external pressures and maximize the probability of gaining stakeholder support [3]. This matrix form is called the stakeholder priority matrix [6]. Alternatively, stakeholders could be mapped differently even with the same categories. The stakeholder mapping is the matrix form of the strategic choice to deal with stakeholders based on the same categories of the shareholder mapping [2]. Both strategic decision issues could be formulated by the matrix forms as you could see in Fig. 2 and 3.
Besides of the above the strategy formulations, a lot of strategic decision making matrixes are massively developed including the Issues Priority Matrix by Lederman [7], the business level strategy by Porter [8], the Strategy Clock by Bowman [9], the Product and Market Expansion Grid by Ansoff [10] and the business portfolio matrix by the Boston Consulting Group [11]. In addition, the matrix form could be also applied in the outsourcing strategy [3], the value-creation diversification strategies [1], and the performance matrix [12].

The paper is organized as follows: Section 2 provides the literature review for the strategic decision matrix. In this section, various strategic formulation tools are introduced to show how the formulations could be intuitively described by a simple matrix form. The mathematical model by using the fluctuation theory is analyzed in Section 3. This mathematical model is analytically proved to find the optimal moments of the strategy shifting based on the thresholds. The solutions are mostly obtained by analytically solving the special process rather than from quantitative data. In addition, the initial conditions for adapting this mathematical model into actual strategy formulations are also mentioned in this section. The actual adaptation into the BGC growth-share matrix with the scales of the decision parameters [11] is demonstrated in Section 4. You can find how the new explicit mathematical could be practically applied into the BGC growth-share matrix. Finally, the conclusion is included in Section 5.

2. Literature Review

Total 11 most popular strategy formulation tools are introduced in the literature review. These strategic formulation tools in this section includes the business level strategy by Porter [8], the product-market matrix by Ansoff [10] and the growth-share matrix by the Boston Consulting Group [26]. The purpose of this section is showing how simple matrix form could adapted into various strategic formulations instantly. Hence, the actual applications for using each strategic formulation will not covered in this section.

**Strategic Group Mapping:** As it is mentioned in the previous section, strategic groups are groups with similar strategic characteristics, following similar strategies within an industry or sector [2]. The strategic group matrix maps based on the RD intensity and export focus that distinguish between strategic groups (see Fig. 1).

**Stakeholder Priority Matrix:** Stakeholder Priority Matrix [6] is also mentioned in the previous section. As shown in Fig. 2, each stakeholder group can be shown graphically based on its level of interest (from low to high) and on its relative power (from low to high). However managers take some stakeholders for granted but it may lead to serious problems later [1].

**Stakeholder Mapping** [2] formulates the different strategies even based on the same decision parameters of the Stakeholder Priority Matrix. As it has been mentioned on the previous section, this matrix is more targeted how to deal with stakeholders. (see Fig. 3).

**Issues Priority Matrix** is the matrix form to identify and to analyze developments in the external environment [3][7]. The issues priority matrix is applied for helping
managers to decide which environmental trends should be merely scanned (low priority) and which should be monitored as strategic factors (high priority). The decision parameters are the probable impacts and the chances of occurrences (see Fig. 4).

![3x3 Matrix](image)

**Fig. 4. Issues Priority Matrix adapted from Lederman [7]**

**Value-Creation Diversification Matrix** is the matrix form of a corporate-level strategy specifies actions [1]. Corporate-level strategies help firms to select new strategic positions and it specifies actions of company to gain a competitive advantage by selecting the proper strategy. Operational relatedness and corporate relatedness are the decision parameters to determine the strategy. The decision parameter of the vertical dimension in Fig. 5 is opportunities to share operational activities between businesses while the horizontal dimension suggests opportunities for transferring corporate-level core competencies.

![2x2 Matrix](image)

**Fig. 5. Value-Creation Diversification Matrix adapted from [1]**

**Business Level Strategy** [1][3][8] focuses on improving the competitive position of products or services within the specific industry. It indicates the choices the firm has made about how it intends to compete in individual product markets. Recent research shows that the business unit effects have double the impact on overall company performance than do either corporate or industry effects [3]. Therefore, business level
strategy is critical matters. Based on the competitive advantages and scopes, a company chooses either one of four business-level strategies to establish and defend their desired strategic position against competitors: cost leadership, differentiation, focused cost and focused differentiation (see Fig. 6).

![Fig. 6. Business Level Strategy Matrix adapted from Porter [8]](image)

**Strategy Clock** is a model that explores the options for strategic positioning which is developed by Bowman [9]. It indicates how a product should be positioned to give it the most competitive position in the market [9]. This clock is targeted to illustrate that a business will have a variety of options of how to position a product based on two dimensions based on the price and the perceived value [2]. The original strategy clock looks like a clock but it could be presented as a matrix form in Fig 7.

![Fig. 7. Strategy Clock Strategy Matrix [9]](image)

**Outsourcing Strategy:** A company should consider outsourcing any activity or function that has low potential for competitive advantage based on the present and potential capacities of a company and it could be also described as a matrix form. It should be purchased on the open market if that activity constitutes only a small part of the total value of the products of the company. The fraction of total value added that
the activity under consideration represents and on the amount of potential competitive advantage of the company determines the proper outsourcing strategy.

**Product-Market Matrix** (aka. *Ansoff Matrix*): This model is essential for strategic marketing planning where it can be applied to look at opportunities to grow revenue for a business through developing new products. It is also known as the Ansoff Matrix which is one of most widely used marketing models. The matrix is mainly targeted to evaluate opportunities for companies to increase their sales through showing alternative combinations for new markets against products and services offering four strategies as shown in Fig. 8 [10].

![Fig. 8. Product-Market Matrix adapted from Ansoff [10]](image)

**GE-McKinsey Nine-box Matrix** is another matrix form of strategy that describes a systematic approach for the decentralized corporation to determine where best to invest its cash [22]. The company can judge a unit by two factors that will determine whether it is going to do well in the future: the attractiveness of the relevant industry and the competitive strength within that industry [23]. This 3-by-3 matrix has been developed by McKinsey to consult manage huge and complex portfolio of unrelated GE products in 1970s [24]. The candidate strategies are mainly *Invest (or Grow)*, *Leave (or Hold)* and *Drop (or Divest)* which are determined based on the industry attractiveness and the business unit strength in Fig. 9.
BCG Product Portfolio Matrix (aka. Growth-share Matrix): This matrix which is given to the various segments within their mix of businesses [1] is formulated by the Boston Consulting Group (BCG) since 1970s [26]. It is targeted to help with long-term strategic planning, to help a business consider growth opportunities by reviewing its portfolio of products to decide where to invest, to discontinue or develop products. The decision parameters of the growth-share matrix are rate of market growth and market share and the matrix plots four strategy in a 2-by-2 matrix form as shown in Fig. 10.

As it has been mentioned, there four strategies to choose based on the decision parameters:

- **Dogs (I):** The usual marketing advice here is to aim to remove any dogs from your product portfolio as they are a drain on resources.
- **Cows (II):** The simple rule here is to "Milk these products as much as possible without killing the cow!" Often mature, well-established products.
- **Stars (III):** The market leader though require ongoing investment to sustain. They generate more ROI than other product categories.
- **Question Marks (IV):** These products often require significant investment to push them into the star quadrant.

In addition, the BCG growth-share matrix contains the quantitative scale of each decision parameter [11]. Therefore, it could be easily adapted into the explicit design which is newly introduced in this paper. As it is mentioned at the beginning, actual implementation of the mathematical model to the BCG growth-share matrix is provided in Section 4.

### 3. Explicit Mathematical Model

#### 3.1 Preliminaries
Let $(\Omega, \mathcal{F}(\Omega), P)$ be probability space $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_\tau \subseteq \mathcal{F}(\Omega)$ be independent $\sigma$-subalgebras and $a_j$, $b_k$ be the level of thresholds of two strategic selection factors. Suppose:

$$A := \sum_{j \geq 0} a_j \varepsilon_{s_j}, \quad B := \sum_{k \geq 0} b_k \varepsilon_{s_k}, \quad s_0(=0) < s_1 < s_2 < \cdots, \text{ a.s.} \quad (3.1)$$

are $\mathcal{F}_A$-measurable and $\mathcal{F}_B$-measurable marked Poisson processes ($\varepsilon_a$ is a point mass at $a$) with respective intensities $\lambda_a$ and $\lambda_b$ and point independent marking. These two values are related with the market transitions. The market is observed at random times in accordance with the point process:

$$T := \sum_{i \geq 0} \varepsilon_{\tau_i}, \quad \tau_0(>0), \tau_1, \ldots, \quad (3.2)$$

which is assumed to be delayed renewal process. If

$$(A(t), B(t)) := A \otimes B([0, \tau_k]), \quad k = 0, 1, \ldots, \quad (3.3)$$

forms an observation process upon $A \otimes B$ embedded over $T$, with respective increments

$$(a_k, b_k) := A \otimes B([\tau_{k-1}, \tau_k]), \quad k = 1, 2, \ldots, \quad (3.4)$$

and

$$a_0 = A_0, \quad b_0 = B_0. \quad (3.5)$$

The observation process could be formalized as

$$A_\tau \otimes B_\tau := \sum_{k \geq 0} (a_k, b_k) \varepsilon_{\tau_k}, \quad (3.6)$$

where

$$A_\tau = \sum_{i \geq 0} a_i \varepsilon_{\tau_i}, \quad B_\tau = \sum_{i \geq 0} b_i \varepsilon_{\tau_i}, \quad (3.7)$$

and it is with position dependent marking and with $a_k$ and $b_k$ being dependent with the notation

$$\Delta_k := \tau_k - \tau_{k-1}, \quad k = 0, 1, \ldots, \tau_0 = 0, \quad (3.8)$$

and

$$\gamma(z, g, \theta, \vartheta) = \gamma_a(z, \theta) \gamma_b(g, \vartheta), \quad (3.9)$$

$$\gamma_0(z, g, \theta, \vartheta) = \gamma^0_a(z, \theta) \gamma^0_b(g, \vartheta), \quad (3.10)$$

where

$$\gamma_a(z, \theta) = \mathbb{E}[z^a e^{-\theta \Delta}], \quad \gamma_b(g, \vartheta) = \mathbb{E}[g^b e^{-\vartheta \Delta}], \quad (3.11)$$

$$\gamma^0_a(z, \theta) = \mathbb{E}[z^{a_0} e^{-\theta \tau_0}], \quad \gamma^0_b(g, \vartheta) = \mathbb{E}[g^{b_0} e^{-\vartheta \tau_0}], \quad (3.12)$$
By using the double expectation,
\[
\gamma(z, g, \theta, \vartheta) = \delta(\theta + \vartheta + \lambda_a(1 - g) + \lambda_b(1 - z)),
\]
and
\[
\gamma_0(z, g, \theta, \vartheta) = \delta_0(\theta + \vartheta + \lambda_a(1 - g) + \lambda_b(1 - z)),
\]
where
\[
\delta(\theta) = \mathbb{E}[e^{-\theta \Delta}], \quad \delta_0(\theta) = \mathbb{E}[e^{-\theta \tau_0}],
\]
are the magical transform of increments \(\Delta_1, \Delta_2, \ldots\). The game in this case is a stochastic process \(A, \otimes, B, \tau\) describing the evolution of a strategy matrix between two strategic decision parameters \(A\) and \(B\) known to an observation process \(T = \{\tau_0, \tau_1, \ldots\}\). The strategic choice will be shifted when either the thresholds of the decision parameter \(A\) passes on the \(j\)-th observation epoch \(\tau_j\) or the thresholds of the decision parameter \(B\) passes on the \(k\)-th observation epoch \(\tau_k\). To further formalize the game, the exit index is introduced:
\[
\mu := \inf\{j : A_j = A_0 + a_1 + \cdots + a_j \geq m\},
\]
\[
\nu := \inf\{k : B_k = B_0 + b_1 + \cdots + b_k \geq n\},
\]
and the joint functional of the blockchain network model is as follows:
\[
\Phi_{(m,n)} = \Phi_{(m,n)}(z, g, \theta_0, \theta_1, \vartheta_0, \vartheta_1)
\]
\[
= \mathbb{E}[z^\mu \cdot g^\nu \cdot e^{-\theta_0 \tau_{\mu-1}} e^{-\theta_1 \tau_{\mu}} \cdot e^{-\vartheta_0 \tau_{\nu-1}} e^{-\vartheta_1 \tau_{\nu}} \cdot 1_{\{A_\mu \leq m\}} \cdot 1_{\{B_\nu \leq n\}}],
\]
where \(m\) and \(n\) are the threshold of each strategic decision options. This functional will represent the status of the strategy analysis upon with the shifting time \(\tau_\mu\) and \(\tau_\nu\), one observation prior to this. The Theorem 1 establishes an explicit formula \(\Phi_{(m,n)}\) from (3.13)-(3.14). The first exceed model by Dshahalow [14, 15] has been adopted and its operators are defined as follows:
\[
\mathcal{D}_{(h,i)}[f(h,i)](x,y) := (1-x)(1-y) \sum_{h \geq 0} \sum_{i \geq 0} f(h,i)x^hy^i,
\]
then
\[
f(h,i) = \mathcal{D}_{(h,i)} \{\mathcal{D}_{(x,y)} \{f(h,i)\}\},
\]
where \(\{f(h,i)\}\) is a sequence, with the inverse
\[
\mathcal{D}_{(h,i)}(\bullet) = \begin{cases} 
\left(\frac{1}{h!i!}\right) \lim_{(x,y) \to 0} \frac{\partial^h \partial^i}{\partial x^h \partial y^i} \frac{1}{(1-x)(1-y)}(\bullet), & h \geq 0, i \geq 0, \\
0, & \text{otherwise}.
\end{cases}
\]

The functional \(\mathcal{D}\) is defined on the space of all analytic functions at 0 and it has the following properties:
(i) $\mathcal{D}_x^j$ is a linear functional with fixed points at constant functions,

$$\mathcal{D}_x^m \sum_{j=0}^{\infty} a_j x^j = \sum_{j=0}^{m} a_j.$$  \hspace{1cm} (3.22)

**Theorem 1:** The functional $\Phi_{(m,n)}$ of the process of (3.18) satisfies the following expression:

$$\Phi_{(m,n)} = \mathcal{D}_{(x,y)}^{(m,n)} \left[ \Psi(x, y) \right].$$ \hspace{1cm} (3.23)

where

$$\Psi(x, y) = \frac{(\delta_0 - \varphi_a)(\delta_0 - \varphi_b)(z-s_{10})g_{10} + g_{10} g_{01} + g_{01}}{(1-s_{10})(1-g_{01})} \hspace{1cm} (3.24)$$

and

\begin{align*}
\Gamma_a &= \gamma_a(x, \theta_0 + \theta_1) = \delta_{\theta_0} \cdot \varphi_a, \\
\varphi_a &= \gamma_a(x, \theta_1), \\
\delta_\theta &= \delta(\theta), \quad \delta_0^0 = \delta_0(\theta), \\
\Gamma_b &= \gamma_b(y, \theta_0 + \theta_1) = \delta_{\theta_0} \cdot \varphi_b, \\
\varphi_b &= \gamma_b(y, \theta_1).
\end{align*}

(3.25) \hspace{1cm} (3.26) \hspace{1cm} (3.27) \hspace{1cm} (3.28) \hspace{1cm} (3.29)

From (3.138) and (3.23)-(3.24), we can find the PGFs (probability generating functions) of the exit index $\mu$ (and $\nu$):

\begin{align*}
\mathbb{E}[z^\mu] &= \Phi_{(m,n)}(z, 1, 0, 0, 0, 0), \\
\mathbb{E}[g^\mu] &= \Phi_{(m,n)}(1, g, 0, 0, 0, 0), \\
\mathbb{E}[e^{-\theta_0 \tau_{\nu-1}}] &= \Phi_{(m,n)}(1, 1, \theta_0, 0, 0, 0), \\
\mathbb{E}[e^{-\theta_1 \tau_{\nu}}] &= \Phi_{(m,n)}(1, 1, 0, \theta_1, 0, 0), \\
\mathbb{E}[e^{-\varphi_0 \tau_{\nu-1}}] &= \Phi_{(m,n)}(1, 1, \varphi_0, 0, 0, 0), \\
\mathbb{E}[e^{-\varphi_1 \tau_{\nu}}] &= \Phi_{(m,n)}(1, 1, 0, \varphi_1, 0, 0).
\end{align*}

(3.30) \hspace{1cm} (3.31) \hspace{1cm} (3.32) \hspace{1cm} (3.33) \hspace{1cm} (3.34) \hspace{1cm} (3.35)

It is noted that the Theorem 1 has been mathematically proved and it has been carefully reviewed by the editor and the reviewers. The proof of the Theory 1 could be found in Appendix A.

### 3.2. General Matrix Frameworks for Strategy Formulation

As it has been mentioned in the previous section, the matrix form is atypical way to describe the set of strategic choices based on the decision parameters (i.e., the category of the strategy input) in the strategy formulation. Most strategy formulation could be described by the matrix. The section deals the 2-by-2 conventional matrix to combine the fluctuation theory for determine the proper strategic decision making. The stakeholder mapping [2] in Fig. 2, the product-market matrix in Fig. 6, the business level strategy in Fig. 8 and BCG product portfolio are intuitive described by the 2-by-2 matrix form. The conventional 2-by-2 strategy matrix form is as follows in Fig. 11.
Generally, the 2-by-2 matrix provides the four strategic choices (I-IV) and the optimal strategic choice depends on a present or a future position of the company. Let us assume that the decision parameters are quantitative and the thresholds are exists which are equivalent with \( m \) and \( n \) from (3.16)-(3.17). The strategy I becomes the best when \( A_j \leq m, B_k \leq n \) and strategy II is the best when \( A_j > m, B_k \leq n \). Formally speaking, the best strategic choice could be chosen as follows:

\[
\text{Best Strategy} := \begin{cases} 
I, & A_j \leq m, B_k \leq n, \\
II, & A_j > m, B_k \leq n, \\
III, & A_j > m, B_k > n, \\
IV, & A_j \leq m, B_k > n,
\end{cases}
\]

from (3.16)-(3.17). The moment of selecting the strategic choice becomes the critical matter and the first exceed model could analytically solve when the strategic decision maker takes the action. Generally, the moment of shifting the strategy is the time when the decision parameters pass their thresholds \( \{m, n\} \) which could be determined analytically from (3.16) and (3.17). But a strategic choice could be shifted before passing the strategic thresholds which are \( \tau_{\mu-1} \) and \( \tau_{\nu-1} \) instead of the first exceed moments \( \tau_{\mu} \) and \( \tau_{\nu} \).

### 3.3. Find The Moment of The Strategy Shifts

It is assumed that the observation process has the memoryless properties which might be a special condition but very practical for actual implementation on a decision making system. It implies that the observation process does not have any dependence with the strategic decision parameters. Recall from (3.19), the operator is defined as follows:

\[
F(x) = D_k \left[ g(k) \right](x) := (1-x) \sum_{k \geq 0} g(k) x^k, \\
D_{(j,k)} \left[ f_1(j) f_2(k) \right](x, y) := (1-x)(1-y) \sum_{j \geq 0} \sum_{k \geq 0} g_1(j) g_2(k) x^j y^k \\
= D_j \left[ g_1(j) \right] D_k \left[ g_2(k) \right],
\]

then
\[ g(j, k) = \mathcal{D}_{(x,y)}^{j,k} \{ \mathcal{D}_{(x,y)} \{ g(j, k) \} \}, \quad (3.38) \]

\[ g_1(j)g_2(k) = \mathcal{D}_x^j \{ g_1(j) \} \mathcal{D}_y^k \{ g_2(k) \}, \quad (3.39) \]

where \( \{ f(x), (f_1(x)f_2(y)) \} \) are a sequence, with the inverse (3.21) and

\[
\mathcal{D}_x^j(\bullet) = \begin{cases} \frac{1}{j!} \lim_{x \to 0} \frac{\partial^j}{\partial x^j} \frac{1}{(1-x)(\bullet)}, & m \geq 0, \\ 0, & \text{otherwise}, \end{cases} \quad (3.40)
\]

and

\[
\mathcal{D}_{(x,y)}^{j,k}[F_1(u)F_2(v)] = \mathcal{D}_x^j[F_1(x)]\mathcal{D}_y^k[F_2(y)]. \quad (3.41)
\]

The marginal mean of \( \tau_\mu \) and \( \tau_\nu \) are the moment of the strategy chances and it could be straightforward once the exit index is found. Each exit index of the decision parameters \( A \) and \( B \) could be found as the Lemma 1 and 2:

**Lemma 1**: The probability generating function (PGF) of the exit index for the decision parameter \( A \) could be found as follows:

\[
\mathbb{E}[z^\mu] = z + \left( \frac{1-\kappa}{1+\delta_0, \lambda_0} \right) \left[ \sum_{j=0}^{m} \left( \frac{\tilde{\delta}_0, \lambda_0}{1+\delta_0, \lambda_0} \right)^j \right] + \left( \frac{\kappa}{1+\delta_0, \lambda_0} \right) \left[ \sum_{j=0}^{m} \left( \frac{\tilde{\delta}, \lambda_0}{1+\delta_0, \lambda_0} \right)^j \right] - \frac{(1-z)}{(1+\delta_0, \lambda_0)} \left[ \sum_{k=0}^{n} \left( \frac{(\tilde{\delta}, \lambda_0)}{(1+\delta_0, \lambda_0)-z} \right)^k \right], \quad (3.42)
\]

and the PGF of the exit index for decision parameter \( B \) could be found as the Lemma 2:

\[
\mathbb{E}[g^\mu] = g + \left( \frac{1-\kappa^1}{1+\delta_0, \lambda_0} \right) \left[ \sum_{j=0}^{m} \left( \frac{\tilde{\delta}_0, \lambda_0}{1+\delta_0, \lambda_0} \right)^j \right] + \left( \frac{\kappa^1}{1+\delta_0, \lambda_0} \right) \left[ \sum_{j=0}^{m} \left( \frac{\tilde{\delta}, \lambda_0}{1+\delta_0, \lambda_0} \right)^j \right] - \frac{(1-g)}{(1+\delta_0, \lambda_0)-g} \left[ \sum_{k=0}^{n} \left( \frac{\tilde{\delta}, \lambda_0}{(1+\delta_0, \lambda_0)-g} \right)^k \right] \quad (3.43)
\]

where

\[
\Gamma_b^0 = \gamma_0^b(y, \vartheta_0 + \vartheta_1) = \frac{1}{(1+\delta_0, \lambda_0+\delta_0, \lambda_0) - \delta_0, \lambda_0} = \frac{\beta_0^b}{1-\alpha^b y}, \quad (3.44)
\]

\[
\Gamma_b = \gamma_b(x, \vartheta_0 + \vartheta_1) = \frac{1}{(1+\delta, \lambda_0+\delta, \lambda_0) - \delta_0, \lambda_0} = \frac{\beta_b}{1-\alpha y}, \quad (3.45)
\]

\[
\alpha_b = \frac{\tilde{\delta}, \lambda_0}{1+\delta_0, \lambda_0}, \quad \beta_b = \frac{1}{1+\delta_0, \lambda_0}, \quad (3.46)
\]

\[
\alpha_0^b = \frac{\delta_0, \lambda_0}{1+\delta_0, \lambda_0}, \quad \beta_0 = \frac{1}{1+\delta_0, \lambda_0}, \quad (3.47)
\]

\[
\alpha_b = \frac{\tilde{\delta}, \lambda_0}{1+\delta_0, \lambda_0}, \quad \beta_b = \frac{1}{1+\delta_0, \lambda_0}, \quad (3.48)
\]
\[ \alpha_b^0 = \frac{\delta_0 \lambda_b}{1 + \delta_0 \lambda_b}, \beta_a = \frac{1}{1 + \delta_0 \lambda_b}, \]  
\[ \kappa_1 = \frac{1}{\lambda_b (\delta_0 - \delta)}, \]  
\[ \widetilde{\delta}_0 = \mathbb{E} [\tau_0], \widetilde{\delta} = \mathbb{E} [\Delta_1]. \]

From (3.42),
\[ \mathbb{E} [\mu] = \frac{1}{\delta \cdot \lambda_a}, \mathbb{E} [\nu] = \frac{1}{\delta \cdot \lambda_b}, \]
and
\[ \mathbb{E} [\tau_\mu] = \widetilde{\delta}_0 + \widetilde{\delta} \cdot \mathbb{E} [\mu - 1] = \widetilde{\delta}_0 + \frac{1}{\lambda_a} - \widetilde{\delta}, \]
\[ \mathbb{E} [\tau_\nu] = \widetilde{\delta}_0 + \widetilde{\delta} \cdot \mathbb{E} [\nu - 1] = \widetilde{\delta}_0 + \frac{1}{\lambda_b} - \widetilde{\delta}. \]

### 3.4. Initial Conditions For Mathematical Modeling

Since, we are dealing with the mathematical approach, it is not possible to apply the theoretical model into the practical implementation without any modification. There are some conditions and the assumptions to apply the explicit design into strategy formulations. First, all decision parameters should be quantitative and have the thresholds for changing the strategies. Although all strategy formulations in the literature review are described as a matrix form, not all of them have quantitative decision parameters. The thresholds should be measurable otherwise the analytical model could not be applied. It is noted that the scale of decision parameters might be modified for sustaining the linearity of the decision parameter values. Second, the process for each decision parameter is following the Poisson compound process. It is one of mandatory condition that makes the mathematical model analytically solvable. The generally distributed process of a decision parameter could be considered instead of the Poisson process if the related data are practically obtained and numerical approach is considered instead of an analytical approach. Third, the observation process could have the memoryless properties which ends up that the inter-arrival process is exponentially distributed. It implies that the observation process does not contain any information from the past.

### 4. BCG Growth-Share Matrix

In the literature review, the BCG growth-share matrix contains the quantitative scale of each decision parameter. Using the BCG growth-share matrix with the quantitative scale has been shown in Fig. 12. It is one of intuitive way to portray a company's portfolio strategy [11]. According to Hedley [11], the separating areas of high and low relative competitive position is set at 1.5 times. If a product has relative strengths of this magnitude to ensure that it will have the dominant position needed to be a Star or a Cow. On the other hand, a product has a relative competitive position less than 1.0 has a Dog status. Each status shall have a different strategy and the strategy should be shifted when the status is changed by a cycle.
The scale of the BCG matrix should be modified to adapt the mathematical model and the modified scale could be found in Fig. 13.

From (3.36), the best strategy for the BCG matrix is determined as follows:

\[
\text{Best Strategy} := \begin{cases} 
\text{Dogs (I),} & A_j \leq 0, B_k \leq 10, \\
\text{Cows (II),} & A_j > 0, B_k \leq 10, \\
\text{Stars (III),} & A_j > 17.6, B_k > 10, \\
\text{Question Marks (IV),} & A_j \leq 17.6, B_k > 10, 
\end{cases}
\]  

and the optimal moments to shift the strategy \( E[\tau_\mu] \) and \( E[\tau_\nu] \) are determined from (3.52) and (3.53).

5. CONCLUSION

The objective of this paper is establishing the theoretical framework of the strategy formulations with the explicit equations for general matrix forms of strategies. The core part of research including the Theorem 1, the explicit functionals for finding the
moment of the strategy shift and the special case of the observation process are fully deployed in this paper. This analytical approach supports the theoretical background of the strategy shifting and the strategic choice to make the proper decision making from conventional strategy formulations. In addition, this newly proposed explicit model has been applied into the BCG growth-share matrix to demonstrate the actual implementation of the theoretical model. This research will be helpful for whom is looking for execute the proper strategy on time based on rapid changed values of the decision parameters. Actual implementation of this model into the strategy decision making based on the related data could one of future research topics

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