Modeling and Leveraging Prerequisite Context in Recommendation

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Prerequisites can play a crucial role in users’ decision-making yet recommendation systems have not fully utilized such contextual background knowledge. Traditional recommendation systems (RS) mostly enrich user-item interactions where the context consists of static user profiles and item descriptions, ignoring the contextual logic and constraints that underlie them. For example, a RS may recommend an item on the condition that the user has interacted with another item as its prerequisite. Modeling prerequisite context from conceptual side information can overcome this weakness. We propose Prerequisite Driven Recommendation (PDR), a generic context-aware framework where prerequisite context is explicitly modeled to facilitate recommendation. We first design a Prerequisite Knowledge Linking (PKL) algorithm, to curate datasets facilitating PDR research. Employing it, we build a 75k+ high-quality prerequisite concept dataset which spans three domain. We then contribute PDRS, a neural instantiation of PDR. By jointly optimizing both the prerequisite learning and recommendation tasks through multi-layer perceptrons, we find PDRS consistently outperforms baseline models in all three domains, by an average margin of 7.41%. Importantly, PDRS performs especially well in cold-start scenarios with improvements of up to 17.65%.

CCS Concepts: • Information systems → Recommender systems; Content analysis and feature selection; Collaborative filtering.

Additional Key Words and Phrases: Recommendation Systems, Prerequisites, Context-aware Recommendation

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1 INTRODUCTION

Prerequisites are defined as the necessary contexts that enable downstream activity or state in human cognitive processes [16]. In certain domains — especially education [1, 24, 34] — such requisites are an important consideration that constrains item selection. Context-aware recommendation systems have integrated the use of collaborative filtering with auxiliary metadata about users’ current background or state, such as time and location [22, 32]. However, the role of prerequisite context (represented in the form of concepts [16] describing items) has been neglected in recommendation, where such dependent information is crucial for modeling users’ interests. Take the educational domain example in Figure 1: The item’s key concept Probability Classifier and user’s prior knowledge are both prerequisite contexts for recommendation. By leveraging the implicit prerequisite relationships between them (represented in the form of a prerequisite graph), we can achieve a comprehensive and explainable recommender that connects the user prerequisite context with item prerequisite context. In our example, Probability Classifier is beyond the knowledge of what the user already knows...
Fig. 1. An illustration of prerequisite driven recommendation: in which a recommender (right) incorporates prerequisite knowledge (left), distilled from both user and item prerequisite context. Dashed edges link users to the concepts they have mastered as prior concepts and the desired concepts to acquire as target knowledge.

(Bayes’ Theorem) but on the path towards to the user’s desired target knowledge (Naive Bayes Classifier), thus deserving a higher recommendation probability. Comparatively, items containing the concept of Conditional Random Field — although related to Bayes’ Theorem — would be poor choices since the user lacks the prerequisite prior knowledge of Hidden Markov Model. Given the importance of user and item prerequisite context, we explore the possibility of modeling and leveraging them in recommendation.

Though previous work [12] focused on the sequential modeling between items, there is virtually no work investigating the next-step decision in light of users’ conceptually-mastered knowledge. We fill this gap by capturing the user and item prerequisite contexts through the compilation of a prerequisite graph, and treating the user’s knowledge as static context. Specifically, both the set of prior concepts that a user has already mastered, and the set of target concepts the user aims to acquire, directly influence the sequence of items to recommend. Based on this, we make a key observation that prerequisite driven recommendation requires two subtasks: (1) prerequisite modeling at the concept level, and (2) user modeling that identifies the prior and target concepts at the user–item level. More concretely, concept prerequisite modeling is the identification of prerequisite links among concepts; cf Fig. 1, prerequisite edges link the introductory concept Probability Classifier to the more advanced Naïve Bayes Classifier. Prior and target concept identification is thus the process of inferring the state of knowledge for each user, with respect to the inventory of concepts in the prerequisite graph.

To demonstrate the effectiveness of leveraging prerequisite context for recommendation, we propose a Prerequisite-Driven Recommendation System (PDRS) embodying this formalism. The key challenge here is how to accurately acquire user- and item- forms of prerequisite context for use in recommendation. However, there is an important shortcoming. Explicit prerequisite knowledge is often sparse, requiring laborious effort to compile. It is often also brittle, as items and their relationships with underlying contextual concepts can evolve over time. Assuming manually-labeled prerequisites [33, 34] is often unrealistic due to the heavy cost of human annotation. An automatic means of inferring prerequisites is called for. We address this in two parts by contributing a) an automatic key knowledge concept extractor from item descriptive text, and b) a prerequisite relation constructor for concept pairs by inferring prerequisite weights from both internal and external domain features. Our encoding components finds a suitable representation of a user in terms of prior and target knowledge, leveraging pretrained language models. We then integrate such prerequisite context into the recommendation process by joint training of both the recommendation and prerequisite knowledge learning tasks.

We evaluate our PDRS system on three datasets representing different domains. We find that PDRS achieves performance gains in recommendation not only in domains where prerequisites exist as hard constraints — such as (educational) course recommendation — but also in domains where prerequisites are soft, personal preferences, as in movie and book recommendation. Importantly, we also show that such model makes an especially strong impact in sparse data cold-start scenarios, a pervasive problem in RS.
2 PROBLEM FORMULATION

**Definition 1. Prerequisite Context.** The context of each user $u$ can be seen as a personal concept repository consisting of a set of mastered prior concepts and a set of target concepts to be acquired, represented as sets $\{C^p_u, C^t_u\} \subseteq C$, respectively. Items then contain these concepts: each item $v$’s context are denoted as $C^i_v$. Importantly, items manifest concepts in a latent, implicit manner, such that the item concept inventory must be inferred.

**Definition 2. Prerequisite Graph.** We represent prerequisite context as a graph $G$, having concepts $C$ as nodes, and prerequisite relations $R$ among them as edges, where $r \in R$ represents the confidence towards prerequisite relation linking knowledge $c_i$ to knowledge $c_j$. $G$ can thus be represented as a series of edge tuples; for example, (logic, 0.99, python) means that logic is prior knowledge required for python with a confidence score of 0.99.

The task of Prerequisite-Driven Recommendation (PDR) aims to learn the latent factors not only from user–item interactions $Y$ (where $y_{uv} = 1$ means $u$ has interacted with $v$), but also from prerequisite context. We can view PDR as combining the knowledge linkage prediction task $g$ and context-aware recommendation prediction task $f$. Specifically, it can be formalized as: 1) inferring latent prerequisites $\hat{r}_{c_i, c_j} = g(c_i, c_j|\Phi, G)$, where $\hat{r}_{c_i, c_j}$ represents the predicted prerequisite confidence from concept $c_i$ to $c_j$; and 2) prerequisite-driven recommendation $\hat{y}_{uv} = f(u, v, |c| \in (C^p_u \cup C^t_u \cup C^i_v))|\Theta, \Phi, Y, G)$. Here, $\Phi$ and $\Theta$ are the parameters for encoding knowledge concepts and users/items.

3 PDRS: PREREQUISITE-DRIVEN RS

Our instantiated framework (PDRS) is depicted in Figure 2. It consists of three components (§3.1, 3.2 and 3.3):

3.1 Prerequisite Knowledge Inference

To leverage prerequisite context, we first need to build a prerequisite graph with concept-level prerequisite links. We decompose this process into two subtasks: extracting concepts from item descriptions, and inferring prerequisite relation between concepts from the ordered item documents and general knowledge (topological relations in Wikipedia).

**Concept Extraction.** To extract key concepts from item documents (item description title, and item description content) as prerequisite context, we extend prior work [26] on graph propagation as a three-step subprocess. First, seed concepts are extracted from item $v$’s document titles using TextRank [23] (with an empirically tuned threshold of 50%). Next, candidate concepts are gathered from $v$’s document content, identifying all phrases that match the part-of-speech tag pattern for a noun phrase: $(A|N) + (A|N)^* (NP)?(A|N)^* N$ [13] (here, $A$, $N$, and $P$ indicate adjectives, nouns, and prepositions). Lastly, we construct a fully-connected graph to include both seed concepts and candidate concepts, and expand the seed concept set to cover its relevant concepts. We implement this by iterative propagation, where concepts’
confidence scores (where seed concepts are initially weighted with unit confidence) are propagated to their neighbors\footnote{For computational efficiency, we only propagate when edge scores are above a tunable parameter $\lambda$.} regarding their semantic relatedness, as measured by cosine similarity $\omega(c_i, c_j) = \cosine(h_{BERT}(c_i), h_{BERT}(c_j))$ of the textual meaning $h$ from BERT [7].

\textbf{Inferring Prerequisites.} Knowledge concepts are then associated by prerequisite relations. As shown in Fig 2(a), we obtain the final linking scores by considering three features: i) asymmetric sequential interaction distance, ii) reference distance from general domain, and iii) semantic relatedness (as defined above). We explain the former two features:

(i) \textit{Asymmetric Sequential Interaction Distance.} We observe that concepts covered in subsequently-consumed items are often dependent on those in previous ones; i.e., they are prerequisites. For example, courses that relate to \textit{Bayes’ Theorem} are more likely to appear in a user’s interaction history before those related to \textit{Hidden Markov Model}. We introduce Asymmetric Sequential Interaction Distance (AsyD) to model this, which captures the distributional pattern of knowledge concept pairs. Let’s say $P(c_i, c_j) = \sum_{(s_p, s_q)} P_s h_{c_i}(s_q)h_{c_j}(s_p)$ indicates the probability of $c_i$ preceding $c_j$ in interaction histories, where $P_s = \{(s_p, s_q)|s_u, s_v \in \mathcal{H}_s; ts_u(s_u) < ts_u(s_v)\}$ indicates item pairs ordered by timestamp $ts$ in user historical interaction sequence $\mathcal{H}_s$ and $h(c, v)$ denotes the term frequency of $c$ in item $v$’s documents (e.g., title and description). Specifically, if $c_i$ is a prerequisite of $c_j$, the probability $P(c_i, c_j)$ should be greater than the converse association $P(c_j, c_i)$. Thus, our defined Asymmetric Sequential Interaction Distance is a normalized probability: $\text{AsyD}(c_i, c_j) = \sigma(P(c_i, c_j))/P(c_j, c_i) - 1$. When $c_i$ and $c_j$ have arbitrary consumption order – i.e., when they are independent of each other’s distribution – then $\text{AsyD} = 0.5$.

(ii) \textit{Wikipedia Reference Distance.} Textual evidence of prerequisites in domain documents can be sparse, resulting in noisy learned prerequisites. Contextual knowledge from general information sources, such as Wikipedia, can aid the identification of prerequisite relations by providing supplemental evidence. We observe that related general information that refer to domain concepts can also provide statistical evidence of prerequisites. To capture this, we propose a domain-adaptive Reference Distance (RefD) which builds on Liang’s work [20] on Wikipedia. For example, the in-domain concept $c$ – ”probability classifier” in Wiki corpus. Specifically, when a related concept $t$ is present in Wikipedia that refers to concept $c$, we capture its reference ratio, similar to AsyD, by calculating its normalized distance score. $\text{RefD}(c_i, c_j)$ measures the imbalance between each pair of in-domain concepts $c_i$ and $c_j$ via its associated reference imbalance $\tau$ to general concepts $t$. This process transforms the concept linking task to one of locating related concepts in Wikipedia. The reference imbalance $\tau = 1$ if and only if $t_i$ refers to $t_j$ in any Wikipedia article but where $t_j$ never refers to $t_i$. As such, $\tau$ analogously ranges $[-1, +1]$. Thus, $\text{RefD}$ can smoothly incorporate information from general sources ($\tau$), as well as in-domain sources (represented by AsyD); i.e., $\text{RefD}(c_i, c_j) = \sum_{i,j} \tau(t_i, t_j) \cdot S_{i,j}/\sum_{i,j} S_{i,j}$, where $S_{i,j} = \omega(t_i, c_i)\omega(t_j, c_j)$ is used to represent how closely $c_i$ and $c_j$ are linked to their related Wiki concepts, and the denominator allows $\text{RefD}$ to also be interpreted as a normalized probability.

\textit{Prerequisite Learning.} To obtain the final prerequisite knowledge linkage scores $PKL(c_i, c_j)$, we train logistic regression over seed annotated samples ($n = 300$ per dataset), as shown on the right part of Fig. 2a. The regression is learned by adjusting the contribution weights for the three features $\omega$, AsyD and RefD on manually-labeled concept pairs. We then run the regression to yield output for all knowledge concept pairs. This process reduces noise from the parameters, providing more accurate PKL scores for downstream recommendation (Fig. 2b). Our constructed prerequisite data are available at https://github.com/HoldenHu/PDRS/.
3.2 User/item and Prerequisite Context Encoding

The Encoding modules take the sparse input representations of users, items and concepts identified by PKL and encode them into dense representations (Fig. 2b [left]), employing a multi-layer perceptron (MLP) for both Knowledge Encoding Module (KEM) and Behavior Encoding Module (BEM).

KEM learns the knowledge concept embedding by training pairs of \((c_i, c_j)\) to approximate their previously-assigned prerequisite PKL scores. Our target in this step is to obtain every concept \(c\)'s embedding \(c \in \mathbb{R}^{1 \times d}\) by tuning the learnable parameters \(\Phi\) to achieve the minimized \(L_{KEM}(r_{i,j}, \hat{r}_{i,j})\), where \(r_{i,j}\) is expected prerequisite weight \(PKL(k_i, k_j)\), and \(\hat{r}_{i,j}\) is the estimated output from an MLP. BEM works similarly, learning user/item embeddings by minimizing the difference \(L_{BEM}\) between predicted \(\hat{y}_{i,j} = MLP(u_i, v_j, \Theta)\) and the truth \(y_{i,j}\). We select a mean-squared loss for \(L_{KEM}\) to best model the real-valued prerequisite scores, and a binary cross entropy loss for \(L_{BEM}\).

3.3 Recommendation Module

As shown in Fig. 2b [right], PDRS combines the embedded representation of user context (user prior knowledge concepts \(C_u^p\) and target knowledge concepts \(C_u^t\)) and item context (item concepts \(C_i^t\)) to output the final recommendation prediction \(\hat{y}_{i,j}\). We apply the average pooling for the concept embedding, while a user/item corresponds to multiple knowledge concepts. Formally, \(\hat{y}_{uv} = MLP(u, v, avg(C_u^p), avg(C_i^p), avg(C_u^t))\), where BEM and KEM provide the pre-trained embeddings \(u, v, \) and \(c\) for the user, item and prerequisite contexts, respectively. We further tune the embedding parameters through joint training that alternately applies the objective of \(L_{KEM}\) and the final recommendation \(L_{Rec}\). \(L_{Rec}\) is also a binary cross entropy loss between model prediction \(\hat{y}\) and the ground truth interaction \(y\).

We empirically observe that the joint training of losses \(L_{KEM}\) and \(L_{Rec}\) enables the model to focus on prerequisite information. It also reduces the time complexity to a linear \(O(|Y| + |R|)\) from the total quadratic complexity \(O(\frac{|Y|^2|R|\times (dY+dR)}{dY\times dR})\) of alternatively executing the two objectives (where \(dY\) and \(dR\) represent the batch sizes of interaction and prerequisite pairs, respectively).

4 EXPERIMENTS

To the best of our knowledge, no existing datasets are specifically designed for prerequisite context modeling. Hence, we modify the existing datasets SSG-Data, MovieLens, and Amazon Books\(^2\) which all contain textual descriptions of items. SSG-Data (Course) is an exhaustive anonymized listing of life-long course-taking history of citizen participants in 9 month period in SkillsFuture Singapore (SSG), which is not publicly available yet. Course content and objectives are used as an item’s documents. We also use the public MovieLens 100K and Amazon Books as Movie and Book datasets, where the crawled textual description from IMDB and TMDB\(^3\) serve as movies’ documents, and the crawled textual overview from Goodreads and Book Reviews\(^4\) serve as books’ documents. To align the task with (binary) recommendation, where only implicit feedback is available, we deem interactions with ratings \(\geq 3\) as positive feedback in the movie and book scenarios. We provide detailed dataset statistics in Appendix A.

To obtain the user’s state of knowledge \(C_u^p, C_u^t\), we assume that users have mastered the knowledge contained in items that they have previously interacted with, and take concepts from the documents of first 30% and the last 20% of items they interacted with as their prior and target knowledge, respectively. To maintain strict training and testing separation, we only use the remaining 50% of knowledge concepts for training and testing our PDRS. We also follow the

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\(^2\)https://www.skillsfuture.gov.sg/, \(^3\)https://www.imbd.com, and \(^4\)https://www.goodreads.com/
CARS@RecSys’22, Seattle, WA, USA,  
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| Model | Course | Movie | Book |
|-------|--------|-------|------|
|       | H@2   | N@2  | N@10| H@2  | N@2  | N@10|
| ItemPop | 0.4541 | 0.7813 | 0.3973 | 0.5300 | 0.2564 | 0.5995 | 0.2132 | 0.3496 | 0.2305 | 0.4740 | 0.2083 | 0.2665 |
| ItemKNN | 0.7122 | 0.8076 | 0.6554 | 0.6785 | 0.2631 | 0.4613 | 0.2299 | 0.3902 | 0.0890 | 0.1178 | 0.0818 | 0.0944 |
| SVD | 0.5934 | 0.8256 | 0.5863 | 0.6672 | 0.2760 | 0.6552 | 0.2288 | 0.3789 | 0.2025 | 0.4002 | 0.1791 | 0.2527 |
| BPR | 0.6650 | 0.8396 | 0.6317 | 0.6794 | 0.2882 | 0.6673 | 0.2469 | 0.3958 | 0.2048 | 0.4043 | 0.1820 | 0.2712 |
| NeuMF | 0.6859 | 0.8455 | 0.6257 | 0.6811 | 0.2899 | 0.6641 | 0.2467 | 0.3936 | 0.2178 | 0.4404 | 0.1876 | 0.2748 |
| GCMC | 0.4081 | 0.7661 | 0.3471 | 0.4928 | 0.2518 | 0.5904 | 0.2165 | 0.3511 | 0.2234 | 0.4734 | 0.2234 | 0.2929 |
| DeepFM | 0.5329 | 0.8154 | 0.6430 | 0.5799 | 0.2695 | 0.6457 | 0.2341 | 0.3821 | 0.2236 | 0.4431 | 0.1915 | 0.2751 |
| PDRS | 0.6894 | 0.8789 | 0.6390 | 0.7142 | 0.3290 | 0.6993 | 0.2825 | 0.4281 | 0.2427 | 0.5150 | 0.2110 | 0.3172 |

Table 1. Hit Ratio (H) and NDCG (N) @K on the Course, Movie, and Book datasets. Bold figures highlight best performers.

| C\(p\) | C\(t\) | C\(i\) | Course | Movie | Book |
|------|------|------|--------|-------|------|
|      | H@10 | N@10 | H@10  | N@10  | H@10|
| ✓    | 0.8192 | 0.6117 | 0.6444 | 0.3796 | 0.4460 | 0.2749 |
| ✓    | 0.8626 | 0.6944 | 0.6598 | 0.4003 | 0.4579 | 0.2810 |
| ✓    | 0.8556 | 0.6929 | 0.6714 | 0.4026 | 0.4669 | 0.2859 |
| ✓    | ✓    | 0.8422 | 0.6539 | 0.6731 | 0.3991 | 0.4684 | 0.2826 |
| ✓    | ✓    | ✓    | 0.8682 | 0.7099 | 0.6705 | 0.4043 | 0.4644 | 0.2809 |
| ✓    | ✓    | ✓    | ✓    | 0.8633 | 0.6968 | 0.6940 | 0.4208 | 0.4989 | 0.3053 |
| ✓    | ✓    | ✓    | ✓    | ✓    | 0.8639 | 0.6978 | 0.6851 | 0.4134 | 0.4966 | 0.3062 |
| ✓    | ✓    | ✓    | ✓    | ✓    | 0.8789 | 0.7142 | 0.6993 | 0.4281 | 0.5150 | 0.3172 |

Table 2. Ablation study on PDRS. C\(p\), C\(t\), and C\(i\) represent user prior concepts, user target concepts, and concepts contained by item. Bold figures indicated leading performers; italicized figures, second-best.

We assess our PDRS method against traditional recommendation models (ItemPop, ItemKNN [28], and NeuMF [11]) and lightweight yet effective feature-incorporated models (GCMC [2] with optimized dropout rate 0.5, and DeepFM [10] with optimized dropout rate 0.2). We also compare against two-wide accepted models of GCMC and DeepFM to validate whether our prerequisite context representation is effective by replacing their user and item IDs feature inputs and modifying them to accept our KEM-derived 64-dimension feature embedding (denoted as GCMC+KEM and DeepFM+KEM). Specifically, the feature embedding dimensions, and node dropout rates are tuned for optimal performance, set as \(K\) = 8, 16, and 0.5, respectively. The optimal knowledge concept embedding dimension is set empirically at 64. We also employ two weak baselines: BPR [27] and SVD [14] for a comprehensive comparison.

We split our interaction data into (80%, 10%, 10%) to serve as training, validation and testing, respectively. For training, we use all available interactions. For the user (item) cold-start (also called new item) scenarios, users (items) in validation/testing set do not appear in the training set. For each test case, we follow the common practice and only retain user records with more than three interactions, to ensure each user has at least one item for evaluating recommendation performance and one item each for modeling prior and target knowledge. We also employ two weak baselines: BPR [27] and SVD [14] for a comprehensive comparison.

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We split our interaction data into (80%, 10%, 10%) to serve as training, validation and testing, respectively. For studying warm-start, we leave one item out per user. For the user (item) cold-start (also called new item) scenarios, users (items) in validation/testing set do not appear in the training set. For each test case, we follow the common practice [11], ranking 100 items — 99 negative samples and 1 positive sample. We use Hit Ratio\(@k\) (HR\(@k\)) and Normalized Discounted Cumulative Gain\(@k\) (NDCG\(@k\)) [30] as top-\(k\) ranking-based accuracy measures.

### 4.1 Main Results

Table 1 shows recommendation performance. In general, PDRS outperforms the baselines across all three datasets, in terms of both HR and NDCG, demonstrating the effectiveness of prerequisite context. One exception is that PDRS performs worse than ItemKNN on course recommendation when \(K = 2\). We believe this is due to the differing levels of sparsity on the item side: there are \(\frac{89K}{43} = 20.7\) records per item for courses, but only \(\frac{40K}{70K} = 5.8\) records per item for books. This makes it easier to find similar items in the former, but more difficult in the latter. This gain evaporates when \(K\) increases to 10, as ItemKNN only recommends accurately for users who choose courses similar to previous ones;
however we see that users do manifest individualized learning pathways in courses, which ItemKNN does not generalize to. GCMC and DeepFM perform well, outperforming other baselines, but they improve dramatically with addition of encoded prerequisite information from KEM, validating that prerequisites play an important role in recommendation tasks. The variants utilizing KEM pretraining (GCMC+KEM, DeepFM+KEM, PDRS) all improve over their corresponding base models. Among these three models, PDRS performs best.

To verify which form of side information in PDRS is most responsible for performance gains (i.e., user prior knowledge, user target knowledge, and item concept in Table 2), we conduct an ablation study (Table 2). When no side information is used, the model functions just as matrix factorisation. For all three recommendation scenarios, the results are best when all forms of side information are used, and worst when none are leveraged. The result of introducing a single form of side information shows that user context plays the most important role in course recommendation (+5.3% with \( C_p \), +4.4% with \( C_t \), and +2.8% with \( C_i \)), whereas item context is more helpful for movie and book recommendation (+2.4, 4.1, 4.4% for movies and +2.7, 4.7, 5.0% for books on \{\( C_p, C_t, C_i \}\), respectively. Interestingly, the three datasets exhibit different optimal combinations of two forms of side information. For courses, combining user prior and target knowledge is best, likely because learners do choose necessary bridging courses based on their target course. In contrast, in movie recommendation, the optimal combination is \( C_p \) and \( C_i \). Watchers may choose based more on their experience with relatable plots and characters. For book recommendation, \( C_t \) and \( C_i \) is the best combination. We surmise that readers choose from their preferred book category (correlated in our PDRS by target knowledge).

4.2 Discussion: Is prerequisite context beneficial for Cold-start Problem?

User cold-start (also termed as new-user) and item cold-start problems [15, 29] are major recommendation system concerns. Table 3 compares PDRS in the course recommendation against baselines in user and item cold-start scenarios.

From the table, item cold start is more serious than user cold start, as evidenced by the drastic drop in performance in the former case. Missing item interactions — in addition to high user sparsity — makes item and user representations inaccurate. Note that as ItemKNN locates items similar to users’ previous interactions and ItemPop uses item popularity, both baseline models do not apply to this item cold start scenario. The performance of latent factor based approaches (SVD, BPR) severely drops in such cold start scenarios. NeuMF, being able to learn nonlinear relationships, is aware of more complex information from interaction data, yielding comparable HR@10. PDRS significantly outperforms baselines in both cold start scenarios, validating the value added by prerequisite knowledge modeling in recommendation accuracy. By comparing the use of user side information in user cold start problem, we observe the advantage of user information. The same applies to items with item information linking items to users through knowledge linkage.

4.3 Discussion: Is our induced prerequisite knowledge accurate?

PDRS relies on accurate and comprehensive prerequisite inference from documents. To verify the reliability of our automatically compiled prerequisites, we also conduct a quality evaluation of our induced prerequisite graphs.

Recall that during relation inferring in prerequisite graphs (§ 3.1), we train a logistic regression model to predict the strength of prerequisite edges between concepts. We validate our PKL score fitted from the use of features, and evaluate using precision, recall and F1. We take 80% as training samples and the remaining 20% as test. We compare against two baselines in this knowledge learning prediction task: 1) Hyponym Pattern Method (HPM) [36]: detects whether prerequisites among noun phrases pairs in sentences fulfill 10 lexico-syntactic patterns (e.g., "Python (NP1), one of the Programming Language (NP2)"). 2) Reference Distance (RD) [20]: where we modify RD’s feature extraction
Table 4. Prerequisite extraction performance (%) on our Course dataset. AsyD is an ablated form of our prerequisite inferring component by testing only the Asymmetric Distance (AsyD) module. Bold figures highlight the best performer.

| Approach | Precision | Recall | F1    |
|----------|-----------|--------|-------|
| HPM      | 66.68     | 16.09  | 25.92 |
| RD       | 59.37     | 47.44  | 52.79 |
| AsyD (Ours) | 77.08   | 68.66  | 67.89 |
| PKL (Ours) | 76.00   | 62.29  | 68.47 |

Table 5. Example Prerequisite Knowledge Linkage (PKL) scores for items $PKL('Machine Learning', knowledge)$ from Course, $PKL('Harry Potter', knowledge)$ from Movie, and $PKL('Alien', knowledge)$ from Book.

| Domain | Knowledge | PKL  | Knowledge | PKL  | Knowledge | PKL  |
|--------|-----------|------|-----------|------|-----------|------|
| Course | Python    | 0.38 | Code      | 0.50 | Feature Learning | 0.75 |
|        | Machine Principle | 0.34 | Program    | 0.50 | Deep Learning | 0.78 |
|        | Computer Basic   | 0.34 | Database   | 0.50 | Algorithm Analysis | 0.73 |
| Movie  | Voldemort   | 0.38 | Policeman  | 0.56 | Handsome Boy | 0.75 |
|        | Trinwizard Tournament | 0.34 | Battle     | 0.50 | Europe     | 0.78 |
|        | Evil Dragon     | 0.31 | Merchant   | 0.50 | Fate of Human | 0.93 |
| Book   | Scientist      | 0.39 | Atom Bomb  | 0.50 | Space Travel | 0.75 |
|        | 21$^{th}$ Centuries | 0.35 | Flash      | 0.50 | Ethologist  | 0.80 |
|        | World War      | 0.35 | Earthquake | 0.50 | Star Trek   | 0.87 |

Methodology to be able to apply it to our task (cf.3.1) by empirically tuning a symmetric threshold $\theta = 0.15$, which is used for an $RD(k_i, k_j)$ of $[-1, -\theta], [-\theta, \theta], (\theta, 1]$ denoting a posterior, neutral, or prior prerequisite relationship.

Table 4 shows the macroscopic comparative results against the Course ground truth. Both AsyD and the final PKL outperform the other two baselines by large margins (+15 F1). The performance improvement is mostly attributed to domain-specific features (AsyD is much better than RD), whereas general domain features brought in by the final PKL bring a minor boost. We zoom in on a typical microscopic case study of the machine learning knowledge concept from the Course dataset. Table 5 lists knowledge concepts that frequently co-occur with machine learning. Large, positive PKL scores indicate that machine learning is a prerequisite of the knowledge concept. We see that concepts on the right, high-PKL column have higher probability of being recommended to users who master machine learning, compared to concepts on the left, low-PKL side. As an example, people taking up a course on machine learning usually have learned python. Prerequisite linking in our other two datasets is less intuitive, but meaningful nonetheless.

5 RELATED WORK

Context-aware recommender systems handle static metadata as auxiliary information, such as user profiles [17, 38] and item attributes [4]. Textual content is a typical form with rich contextual information to facilitate RS accuracy. Some works treat item content as raw features by hidden vectors [31, 40], while others select important text pieces, such as item tags [8, 19], and semantic clues [35]. However, few works focus on establishing context causality between user and item, where our approach uses prerequisite context.

Proper Prerequisite Relation Identification is thus crucial for both intrinsic prerequisite representation task and our ultimate extrinsic task of recommendation. Many works rely on statistical methods to determine prerequisites. An early study by Vuong et al. [34] examined the effect of learning curriculum units in various orders. Chen et al. [6] treated prerequisite relations as a Bayesian network, which requires a mapping of courses to fine-grained skill and relevant student performance data. Chen et al. [5] apply probabilistic association rule mining to infer student knowledge from performance data. To make the prerequisite relation identification more feasible, others — including ourselves — tap into generic information sources. Pan et al. [25] utilize a Wikipedia corpus to learn semantic representation of concepts for detecting prerequisites in MOOC. Talukdar and Cohen [33] study how prerequisites can be inferred between Wikipedia entities. Wang et al. [36] use Wikipedia articles and categories for Concept Graph Learning that uses observed prerequisite relation to learn unobserved ones. Although this task has mostly been applied to education [39], our findings emphasize that prerequisites indeed generalize and do not need to be restricted to a particular context.
6 CONCLUSION AND FUTURE WORK

To the best of our knowledge, we are the first to explore the use of prerequisites — an overlooked but crucial form of context — for recommendation. We introduce Prerequisite Knowledge Linking (PKL) method to induce a prerequisite graph automatically, through semi-supervised learning over both general and domain-specific features. We instantiate our formalism in the form of a Prerequisite Driven Recommendation System (PDRS; §3) embodied as a modern neural architecture, which adopts joint training to optimise the model for the twin objectives of knowledge linking prediction and recommendation. We demonstrate that prerequisite context is a functional booster to solve cold-start problem, and can benefit recommenders universally through our experiments on our Course, Movie, and Book datasets (§4).

While our PDRS is a simple instantiation of a prerequisite driven recommendation, its elegance leads to synergistic performance gains. Designing more sophisticated models to leverage captured prerequisite knowledge is open future work. As our prerequisite graph is a structured format of knowledge, future work may seek more complex encoding methods, such as typical translation-based methods (e.g., TransE [3], TransH [37]). Moreover, our work opens the door for studying user’s state of knowledge in dynamic scenarios, such as conversational recommendation and long-term sequential recommendation.

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Appendix A  DATASET STATISTICS

| Dataset  | # Users | # Items | # U–I | # Concepts | # C–C Pairs |
|----------|---------|---------|-------|------------|-------------|
| Course   | 34,235  | 4,322   | 89,565 | 5,131      | 227,515     |
| Movie    | 608     | 7,354   | 49,738 | 12,503     | 8,402,493   |
| Book     | 92,971  | 70,695  | 409,616| 58,492     | 15,889,953  |

Table 6. Statistics of the datasets used in our experiments. U–I: user–item interactions; C–C pairs: concept-concept pairs.

Detailed dataset statistics are shown in Table 6. Our Course dataset is sparser per user, validating observations that educational activities are hard to recommend without any side information. We follow the commonly-used annotation method [9, 21, 25] in inferring prerequisite relations: manually labeling a small number of concept pairs to train a logistic regression to label the rest. We annotated 300 pairs \((c_i, c_j)\) for each task with \(+1\) if \(c_i\) is \(c_j\)'s prerequisite, \(-1\) if \(c_j\) is \(c_i\)'s prerequisite, or \(0\) if no obvious dependency relation exists. Of the 300 labeled samples \(80\% = 240\) are used for model training, and the remaining \(20\% = 60\) are held out for testing. The main purpose of this process is to de-noise the PKL scores by considering three indicators, discussed in § 4.3.

Appendix B  DETAILED MICROSCOPIC CASE STUDY OF GENERATED PREREQUISITE GRAPHS

As shown in Table 5, prerequisite linking in our other two datasets feels less intuitive, but is meaningful nonetheless. Taking selected concepts from our Movie dataset for the film 'Harry Potter' as an example, we see that the film is learned as a prerequisite for the knowledge concept 'Fate of Human'. This means that the source IMDB and TMDoc documents mentioning 'Harry Potter' mention 'Fate of Human' but not vice versa, leading to a large PKL score, hence a prerequisite. Casual inspection of the general Web confirms that 'Harry Potter' is a foil for subsequent discussions about fate and free will, which makes sense. While the prerequisite does not constitute a strictly sequential viewing order among 'Harry Potter' and other movies featuring fate and free will as a central theme, these soft constraints do model inspiration; in Books, people who have read 'Alien'-themed books may go on to choose similarly themed 'Space Travel' books, including 'Star Trek' novels.

We also find that prerequisite graphs have different structural properties. Course prerequisite graphs contain the richest dependency relations (i.e., longer average path and larger average node degree), while graphs for the Movie and Book datasets are sparser and shallower. Our casual observation of the graphs also reveal that longer prerequisite paths correlate with more specific terms (e.g., 'Voldemort'), while the shorter paths refer to more general knowledge ('Love'). Both studies show consistency, in that movies and books have fewer dependencies on required prerequisites, as compared to the formal knowledge acquired in courses, but that such soft constraints still aid recommendation accuracy.

Appendix C  THE ROLE OF PREREQUISITES IN RECOMMENDATION

C.1 What embedding size is most suited for prerequisite representation?

Here, we use Root Mean Squared Error (RMSE) and R Squared (R2) to measure the fidelity of the KEM prediction; that is, after encoding. In the figure, KEM (Knowledge Encoding Module) refers to the results from knowledge encoding module alone (ablating recommendation), RM (Recommendation Module) refers to the training of recommendation target without constraints from KEM, and PDRS applies both sets of constraints.

Both plots in Figure 3 show a consistent trend: adding more latent dimensions improves knowledge representation and prerequisite capture. Furthermore, the joint training in PDRS of both \(L_{PKL}\) and \(L_{Rec}\) shows a slight but consistent
benefit. Both exhibit large improvements compared against optimizing RM alone. Seen this way, good prerequisite capture through PKL is its own reward, but has the free side effect of benefitting downstream recommendation as well.

Fig. 3. Effect of varying the dimensionality of the knowledge concept encoding (x-axes) on downstream recommendation performance (y-axes, as measured in (l) RMSE (lower better) and (r) R2 (higher better).

C.2 Do prerequisites complement user–item interaction?

Unlike collaborative filtering, prerequisites play a decisive and logical factor in driving users’ sequential interactions with items. While collaborative filtering may find quality recommendations, such recommendations may not account for the immediate needs of the user.

Take the first row of Table 7 as an example of the role of such knowledge. Here, the left portion of the table gives the input user’s state of knowledge and the right portion shows the output recommendation from both the interaction-only BEM and our full PDRS. We see this user has handled basic office skills, and thus she may be interested in other related skills such as team work, or soft social skills, which may be recommended by collaborative filtering as is done in BEM. But perhaps their priority is in building on existing credentials: here, 'Microsoft Word' can be seen as a more efficient means of 'text' entry.

| User Context | Model | First Recommended Item’s Knowledge Concepts |
|--------------|-------|---------------------------------------------|
| PriorK: ‘sort’, ‘text’, ‘ribbon’, ‘page orientation’, ‘manipulation’ | PDRS | ‘read’, ‘document’, ‘page’, ‘undo’, ‘microsoft word’ |
| TargetK: ‘file’, ‘combo box’, ‘footnote’, ‘layout’, ‘form’, ‘controller’ | BEM | ‘team’, ‘development’, ‘plan’, ‘social response’, ‘application’ |
| PriorK: ‘muscle’, ‘tissue’, ‘bone’, ‘mobile’, ‘neck’ | PDRS | ‘autonomic nervous system’, ‘profile’, ‘nervous system’, ‘area’ |
| TargetK: ‘privacies’, ‘skill’, ‘certifier’, ‘participant’ | BEM | ‘compliance’, ‘requirement’, ‘response’, ‘investigation’ |
| PriorK: ‘protein’, ‘sugar’, ‘food product’, ‘selection’ | PDRS | ‘bread’, ‘baking’ |
| TargetK: ‘tour’, ‘participation’, ‘competition’, ‘visitor’ | BEM | ‘preventive act’, ‘report’, ‘unit’ |
| PriorK: ‘conversation’, ‘spot’, ‘paper’, ‘detection’ | PDRS | ‘data analysis’, ‘skill’, ‘sql’, ‘in demand’, ‘database’ |
| TargetK: ‘database’, ‘product’, ‘real world’, ‘code’, ‘data feature’, ‘universe’ | BEM | ‘facial’, ‘neck’, ‘eye’, ‘client’, ‘facial car’ |

Table 7. Course examples where PDRS behaves differently. Underlined concepts are strongly induced by PKL.

Can prerequisite knowledge be derived from user–item interaction directly? In short, yes, but much less effectively as we have done in PDRS. To illustrate the fine-grained level of useful prerequisite knowledge captured (item or concept), we also benchmark PDRS with coarser, obvious prerequisites. We build another item dependency graph directly from the interaction sequence order in our Course scenario. We set an aggressive threshold, keeping only the top ~250 item pairs satisfying a minimal number of occurrences and prerequisite strength. We run these identified, salient prerequisites through BEM but find that performance suffers significantly, leading to a 3.6% drop in performance. This finding directly attributes the power of the fine-grained prerequisite capture, and the effectiveness of PDRS in incorporating general features enhance the signal coming from sequential interaction histories.
Appendix D  DISCUSSION: IS PDRS SENSITIVE TO HYPERPARAMETER SETTING?

We examine how PDRS handles both prerequisite context and user/item encodings as the model complexity (in terms of hidden layers) is varied. As shown in Table 8, models with or without pretraining both perform best with $L = 4$. Fewer layers are insufficient to learn the complex relationship between embeddings (especially for PDRS to learn the relation between knowledge embedding from both user and item), whereas larger numbers suggest overfitting.

PDRS with pretraining achieves better performance with more hidden layers, but is worse than the case without pretraining, when number of layers is small (i.e., $L = 1, 2, 3$). It can be seen that using pretraining may yield more accurate user/item and knowledge embeddings as initial values for PDRS. Again, too few layers may be insufficiently rich to model embeddings for recommendation.

| # Layers | With Pre-training | Without Pre-training |
|----------|-------------------|----------------------|
|          | HR@10  | NDCG@10  | HR@10  | NDCG@10  |
| 1        | 0.7965 | 0.5884  | 0.8042 | 0.5961   |
| 2        | 0.8543 | 0.6717  | 0.8542 | 0.6723   |
| 3        | 0.8592 | 0.6937  | 0.8613 | 0.6986   |
| 4        | 0.8703 | 0.7051  | 0.8682 | 0.7028   |
| 5        | 0.8641 | 0.6993  | 0.8633 | 0.6984   |
| 6        | 0.8640 | 0.6989  | 0.8638 | 0.6932   |

Table 8. Performance of PDRS with varying numbers of layers, with and without pretraining. Bold figures indicate the better strategy w.r.t. each number of layers.

We also examine the impact of the combination of the dimension $d$ used for BEM and the dimension $d'$ used for KEM on Course recommendation. As can be seen from the heat map in Figure 4, NDCG@10 improves as $d$ and $d'$ increases. This supports using more factors to store the latent signals and thus improving the model capacity. The best HR@10 is achieved when $d = 128, d' = 64$, and larger models tend to overfit.

![Fig. 4. Top-10 recommendation Hit Ratio and NDCG scores w.r.t. various # latent factor $d$ in BEM and $d'$ in KEM (# layer = 4).](image-url)