Second order contributions to the absorption of massive particles

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Abstract

Recently, in analogy with multiphoton ionization, it has been suggested that multiparticle ionization can also be induced by massive systems. We explore in this paper the possibility that multiparticle absorption processes can also take place for massive particles. To study it we consider, in a perturbative way, a model of absorption which illustrates the analogies with Glauber’s scheme for photons and previous analysis on matter-waves coherence. A major advantage of this approach is that the dependence of the absorption rates on the wavefunction of the incident system can be analyzed in an explicit way. The calculations confirm the form of the second order (two-particle) contributions.

PACS: 03.65.-w; 42.50.Hz; 03.75.Be

Keywords: Absorption of massive particles; Glauber’s model; Multiparticle processes

1 Introduction

Multiphoton processes have been an active area of research during the last decades [1]. Recently, it has been suggested [2] that similar events can occur in the case of massive particles. An example where this possibility can be easily visualized was presented in Ref. [2]. It is the process of ionization. One incident particle can ionize the atoms or molecules of a piece of matter or a gas. This is the usual way we think about ionization. However, there can be other ways. For instance, if two particles arrive on the piece of matter or gas one of them
can excite the atoms, with the second one producing the definitive ionization. The similarity with multiphoton ionization is clear. This type of process can be important because of the recent arrival of atom lasers and other atomic systems showing high degrees of coherence, in close analogy with light lasers.

Massive and massless particles can be absorbed by matter in various physical processes. Absorption has been extensively studied in the case of photons. By analogy with the considerations about ionization we can presume that multiparticle absorption could also take place in the massive case. We shall analyze in this paper this possibility. In Ref. [2] it was shown that multiparticle processes would imprint a clear mark that would distinguish them from single particle events: the probability of a multiple process is not proportional to the squared modulus of the wavefunction, as it is the case for single events. In this paper we want to confirm that result in the case of absorption, but using a more physical approach to the problem. Instead of relying on the formal approach of Ref. [2] where only the probability of detection was considered, we shall introduce here a model where the explicit form of the interaction is presented. Our analysis will be developed in analogy with Glauber’s scheme. Many years ago, Glauber [3] introduced a mathematical description of photon absorption which has been extensively verified and provides a solid basis for the understanding of the detection of light beams in photomultipliers and other measurement devices.

We extend in this paper Glauber’s scheme to the case of massive particles. This is not the first time that the ideas of Glauber have been applied in the framework of matter waves. In Ref. [4] different types of atomic coherence were considered in connection with different types of atomic measurements. Even in Ref. [5] the mutual coherence of optical and matter waves have been studied. In particular, these authors considered three types of measurements, fluorescence, nonresonant imaging and ionization. However, these authors did not consider the case explicitly studied here, the absorption of massive particles. Another important difference between the approach of Refs. [4, 5] and our analysis is that the first one was mainly concerned with the properties of matter waves, whereas the second one is mainly aimed at the study of the dependence of the absorption.

We emphasize the last point. Our approach to multiparticle absorption is focused on the dependence of the absorption probabilities on the wavefunction of the incident beam. This point of view is novel in the case of incident massive beams. Previous approaches to the problem rely on either (i) a classical treatment of the intensity of the incident beam, or (ii) a quantum description in the framework of scattering theory. Imaginary absorption coefficients or optical-type potentials [6] are introduced to simulate the interaction. However, in the first case no information can be obtained about the relationship between the
absorption and the form of the wavefunction, and in the second one it is very difficult to extract.

The plan of the paper is as follows. In Sect. 2 we introduce our model of detection of massive particles by extension of Glauber’s scheme and in analogy with previous studies of matter-waves coherence. Sections 3 and 4 are devoted, respectively, to the evaluation of the first and second order of the perturbative expansion of the model. At first order we obtain the transition rate for incident one-particle beams, which is proportional to the squared modulus of the wavefunction. At second order, which describes two-particle absorptions, we find a different type of dependence on the wavefunction. Finally, in the last section we discuss the main results of the paper.

2 A model for the absorption of massive particles

In this section we present our model for the absorption of massive particles. As signaled before, the model is an extension of Glauber’s theory for the absorption of photons to the case of particles ruled by Schrödinger's equation. Then our starting point will be a short review of the Glauber approach.

2.1 Glauber’s scheme

The theory of absorption of photons was originally introduced by Glauber [3]. It is a very general description and with minor modifications can be applied to a large number of processes. For instance, it has been extensively used for the description of detection devices such as photomultiplier tubes, photographic plates or bolometers. In this scheme, the absorption is mathematically represented by the action of the annihilation operator of the photon under consideration.

The interaction between the incident beam and the device is also specified. For instance, in the case of the photomultiplier it is given by the electric dipole interaction Hamiltonian:

$$\hat{H}_{\text{int}}^{\text{pho}} = -d \hat{E}(r_{\text{pho}})$$

(1)

where $d$ is the atomic electric dipole and $\hat{E}$ is the electric field operator at the location $r_{\text{pho}}$ of the photomultiplier. As it is well-known the electric field operator is decomposed in the form $\hat{E} = \hat{E}^+ + \hat{E}^-$, where each part can be expressed in terms of the creation and annihilation operators of the photons.

Using the interaction Hamiltonian and the expression for the incident beam in Fock’s space it is possible to calculate the transition rate for the absorption.
2.2 The model

We consider massive particles that interact with a system or medium, which has the capacity of absorbing them. Mathematically, the absorption of photons is represented by annihilation operators. Similarly, we shall describe the absorption of massive particles by the action of the annihilation operators of the particles. The annihilation operators describe absorption processes but without any spatial localization. However, in general, we consider absorptions well-localized in space. For instance, we can consider absorbing mediums of small size. The description of well-localized absorption processes can be achieved using \( \hat{\psi}^+ \) and \( \hat{\psi} \), the raising and lowering field operators (the last one, also named Schrödinger’s field operator): the \( \hat{\psi}^+(\mathbf{r}, t) \) (\( \hat{\psi}(\mathbf{r}, t) \)) operator represents one that takes the state of the system to one with one more (less) particle located at \( \mathbf{r} \) at instant \( t \) (with spin \( \Omega \)) [7]. As it is well-known the \( \hat{\psi}(\mathbf{r}, t) \) operator can be written in the form:

\[
\hat{\psi}(\mathbf{r}, t) = \int d^3q \psi_{q\Omega}(\mathbf{r}) \hat{a}_{q\Omega}(t)
\]

where \( \psi_{q\Omega}(\mathbf{r}) \) is a complete set of orthonormal stationary wave functions, labelled by the index \( q \) (if the index is not a continuous one the integral must be replaced by a sum). Note that the time dependence is carried by the annihilation operator. The most common choice for this set is given by plane waves of momentum \( \mathbf{p} \):

\[
\hat{\psi}(\mathbf{r}, t) = \frac{1}{(2\pi \hbar)^{3/2}} \int d^3p e^{i\mathbf{p}.\mathbf{r}/\hbar} \hat{a}_{p\Omega}(t) e^{-iEt/\hbar}
\]

with \( \hat{a}_{p\Omega} = \hat{a}_{p\Omega}(t = 0) \) and \( E = p^2/2m \) the energy.

The raising and lowering operators act on the Fock space of the incident particles, which is constructed in the usual way, \( |0\rangle, |1\rangle,...|n\rangle \), with \( n \) the number of particles in the beam.

The absorbing medium is made of atoms and, in accordance, we assume that it must obey the laws of quantum mechanics. The state of the medium previous to the interaction will be represented by \( |M\rangle \). If after the interaction one incident particle is absorbed by the medium, its state will be \( |M_1\rangle \). Finally, if in the incident beam there are \( N \) particles and all of them are absorbed after the interaction the state will be \( |M_N\rangle \). We can think of \( |M\rangle, |M_1\rangle \) and \( |M_N\rangle \) as collective states of the medium. Note that they can be but do not need to be described in the Fock space of the medium (for instance, the description of the atoms of the photomultiplier is done in the first quantization formalism, whereas that of the photons is in the second one [8]).
We must now introduce the interaction between the particles and the medium. The interaction results in the absorption of a particle at the point where the medium is placed (we assume by simplicity, a very small size medium). Then the interaction must be expressed in terms of the lowering operator, which, as discussed before, represents the absorption at definite points. The particular form in which the lowering and raising operators will be present in the interaction Hamiltonian will be dictated by analogy with the massless case. In that case it is the product of the field operator by another operator representing properties of the absorbing atoms. Similarly, we propose an interaction Hamiltonian which is the product of the lowering and raising operators by another operator related to the absorbing medium. Therefore, we propose the following interaction Hamiltonian (in the tensorial product of both spaces)

\[
\hat{H}_I(Q, t) = \alpha \hat{M}(t) \otimes \hat{\psi}_\Omega(Q, t) + h.c. = \\
\alpha \hat{M} \otimes \hat{\psi}_\Omega(Q, t) + \alpha^* \hat{\psi}_\Omega^\dagger(Q, t)
\]

where \(\alpha\) is a constant depending on the type of particle and medium considered, which gives a phenomenological measure of the strength of the interaction between particle and absorbing system. On the other hand \(\hat{M}\) is the operator representing the physical reaction of the medium to the incident particles (in the case of photons it would correspond to the atomic electric dipole). With this choice we are adopting a phenomenological point of view: we do not care about the precise physical mechanism that produces the interaction and absorption, but only on the fact that \(\hat{M}\) is responsible (through these complex mechanisms) for the process.

The only definitive justification of this choice for \(\hat{H}_I\) would be done by the experimental verification of its physical consequences, which will be derived in next section. This is a clear advantage of the model, it is a testable one.

In the above expression \(Q\) is a parameter that indicates the position at which the medium is placed. We introduce this notation of \(Q\) as a parameter instead of the position variable \(r\) to remark that \(\hat{H}_I(Q, t)\) is not a function of position but only depends on the value of the field operator at \(Q\) \((\hat{\psi}(Q, t))\). When we place the medium at another position \(Q^*\) we must use a different \(\hat{H}_I(Q^*, t)\). For every value of \(Q\) we have a different problem (see, for instance, Ref. [7] for the use of \(Q\) as a parameter). The introduction of the position variable as a parameter is a necessary consistency condition for the cases where the problem is completely described in the second quantization formalism (when the \(|M, >\) states associated with the medium belong to Fock’s space) because the Hamiltonian must be position independent in the second quantization formalism.

The similarity between our approach and that presented in Refs. [4, 5] is
clear. In both cases, the interaction Hamiltonian is expressed in terms of raising and lowering operators.

Before the interaction between the beam and medium the state of the complete system is given by the product $|n> |M>$. When both systems interact the state becomes an entangled one of the type $|n, M>$. This entanglement describes the correlation between both systems.

The Schrödinger equation ruling the evolution of the complete system is:

$$i\hbar \frac{\partial}{\partial t} |n, M> = (\hat{H}_o + \hat{H}_I(Q, t))|n, M>$$  \hspace{1cm} (5)

with $\hat{H}_o$ the free Hamiltonian of the complete system.

The similarities between our model and the optical scheme are evident. One important difference between them must be remarked now. Instead of the electric field operator we have in the interaction Hamiltonian the raising and lowering operators of the incident beam. The last two operators are expressed in terms of the solutions of the Schrödinger equation of the system, which rules the behaviour of massive particles. Photons cannot be encompassed in this category, because they do not obey Schrödinger’s equation.

3 Detection probabilities

Using Eq. (5) we can calculate the transition rate between different states at a given position, i.e., the transition rate at $Q$ from the initial state $|n> |M>$ to the state $|n - N> |M_N>$ with $N$ an integer number. The transition rate is given by [8]

$$w(Q, t) = \frac{d}{dt} | <n - N | < M_N | \hat{U}(Q, t) | M > | n > |^2$$ \hspace{1cm} (6)

where $\hat{U}(Q, t)$ is the evolution operator of (5).

This equation must be computed in a perturbative way. The use of perturbation theory in this problem can be easily justified. The interaction between the atoms and molecules of the medium and the incident particles are of the electromagnetic type, and it is well-known that electromagnetic interactions can be described by perturbation theory.

We shall evaluate the first and second order of the perturbative development.

3.1 First order

To first order of perturbation theory Eq. (6) becomes [8]:

$$w^{(1)}(Q, t) = \frac{2\pi}{\hbar^2} | < n - N | < M_N | \hat{H}_I(Q) | M > | n > |^2$$ \hspace{1cm} (7)
(in this expression, just as in the rest of transition rates below, we also have a Dirac’s delta between the energy of the initial and final states, which is not explicitly included in order to simplify the notation).

Taking into account the dependence of \( \hat{H}_I(Q, t) \) on the raising and lowering operators the above matrix element is only different from zero for the lowering one and \( N = 1 \). We consider as the initial state of the particle \( |n > = |1_f \xi > \) with

\[
|1_f \xi > = \int d^3b f(b) \hat{a}_{b\xi}^+ |0 >
\]

(8)

where \( f(b) \) is the momentum distribution of the particle, obeying the normalization condition \( \int d^3b |f(b)|^2 = 1 \). In the first quantization formalism this particle is represented by the wavefunction

\[
\psi_{f\xi}(Q) = \int d^3b f(b) \psi_{b\xi}(Q)
\]

(9)

With that state Equation (7) becomes \( w^{(1)}(Q, t) = 2\pi |\Upsilon(Q, t)|^2/\hbar^2 \) with

\[
\Upsilon(Q, t) = \alpha < M_1 |\hat{\mathcal{M}}| M > \times
\]

\[
\int d^3q \int d^3b f(b) \psi_{q\xi}(Q) < 0 |\hat{a}_{q\xi} \hat{a}_{b\xi}^+ |0 >
\]

(10)

with \( \hat{a}_{b\xi}^+ \) the creation operator at \( t = 0 \).

Using the well-known (anti)commutation relations \( [\hat{a}_{b\xi}, \hat{a}_{q\xi}^+] = \delta_{\xi\Omega} \delta^3(b - q) \), where the upper sign is valid for bosons and the lower one for fermions, we have \( < 0 |\hat{a}_{b\xi} \hat{a}_{q\xi}^+ |0 > = \delta_{\xi\Omega} \delta^3(b - q) \) (we assume the vacuum state to be normalized \( < 0 |0 > = 1 \)). Finally we obtain

\[
w^{(1)}(Q, t) = \beta \delta_{\xi\Omega} |\psi_{f\xi}(Q)|^2
\]

(11)

with

\[
\beta = \frac{2\pi}{\hbar^2} |\alpha |M_1 |\hat{\mathcal{M}}| M >|^2
\]

(12)

Then the absorption probability (which is easily obtained from the transition rate by integrating on time \( w^{(1)} \)) derived from the model is proportional to the usual Born’s distribution, which establishes the probability to find a particle described by the wave function \( \psi(Q) \) at a point \( Q \) to be \( |\psi(Q)|^2 \). In addition, we have the coefficient \( \beta \). A similar factor is present in quantum optics where \( w \) is the product of the efficiency factor and \( I \) with \( I \) the quantum intensity of the field. Therefore, \( \beta \) is to be identified with an efficiency factor. It depends on \( \alpha \), a function of the medium and the type of particles considered, and on \( < M_1 |\hat{\mathcal{M}}| M > \) the matrix element of the operator \( \mathcal{M} \). The efficiency factor incorporates into the theory the possibility of interaction processes between medium and particle that do not lead to absorption. This is one of the possible channels of the particle-medium interaction.
3.2 Second order

We consider now the second order of the theory. Physically, it corresponds to incident beams with two particles (or two different incident beams).

To second order the transition rate is 

\[ w^{(2)}(Q, t) = \frac{2}{\hbar^2} |\Xi(Q, t)|^2 n > |n > \]

\[ \Xi(Q, t) = \sum_i (E_i)^{-1} < n - z | M_z | H_i(Q) | M_z - 1(I_i) > | (n - (z - 1))(I_i) > \times \]

and \( z \) an integer. The sum on \( i \) refers to all the intermediate states of the incident beam and medium. The energy \( E_i \) is the difference between the total (particle plus medium) initial energy and that of the intermediate state \( E_i = E_{po} + E_{Mo} - E_{pi} - E_{Mi} \) because we have taken the energy of the detector in the initial state as the origin of energies, i.e., \( E_{Mo} = 0 \). \( I_i \) in the intermediate states refers to the two possible ways of absorbing the two particles.

We take now as initial incident state

\[ |2> = \int d^3bf(b) \int d^3dg(d) \hat{a}^+_{bc} \hat{a}^+_{d\mu}|0> \]

The final state is \( |n - z> = |n - 2> = |0> \). The first term in the r. h. s. of Eq. (13) is zero because of the presence of only one lowering operator in the interaction Hamiltonian. If we assume that the intermediate states \( |M_z - 1(I_i) > \) of the medium are non-degenerate there are only two contributions to the second term in the r. h. s. of Eq. (13). In the first contribution the order of absorption is first \( |1_f\xi > \) and later \( |1_g\mu > \), whereas in the second contribution the order is the opposite one. The intermediate states of the beam and medium are, respectively, \( |1_g\mu > \) and \( |M_1(f\xi) > \) and \( |1_f\xi > \) and \( |M_1(g\mu) > \).

Using the explicit expressions of the states and field operators and applying repeatedly the (anti)commutation relations we obtain the following relations (\( q \) and \( k \) are the integration variables in the field operator)

\[ \delta_{\Theta q} \delta^3(q - u) (\delta_{\mu \eta} \delta_{\xi \Theta} \delta^3(d - u) \delta^3(k - b) + \delta_{\xi \Theta} \delta_{\mu \eta} \delta^3(b - u) \delta^3(k - d)) \]

Replacing \( u \) by \( b \) and \( d \) we can evaluate the transition rate. After a simple but lengthy calculation we obtain:

\[ w^{(2)}(Q, t) = \frac{2}{\hbar^2} |\alpha|^4 |W(d, \mu) + W(b, \xi)|^2 \]
with

\[ W(d, \mu) = \langle M_2 | \hat{M} | M_1 (d, \mu) > < M_1 (d, \mu) | \hat{M} | M > \times \]

\[ (E_i (d, \mu))^{-1} \delta_{\xi \Omega} \psi_{f \Omega} (Q) (\delta_{\xi \mu} \delta_{\mu \xi} < f | g > \psi_{g \Omega} (Q) \pm \delta_{\xi \mu} \psi_{f \Omega} (Q)) \quad (17) \]

and

\[ W(b, \xi) = \langle M_2 | \hat{M} | M_1 (b, \xi) > < M_1 (b, \xi) | \hat{M} | M > \times \]

\[ (E_i (b, \xi))^{-1} \delta_{\mu \Omega} \psi_{f \Omega} (Q) (\pm \delta_{\xi \mu} \delta_{\mu \xi} < f | g > \psi_{g \Omega} (Q) + \delta_{\xi \Omega} \psi_{f \Omega} (Q)) \quad (18) \]

with \( E_i (b, \xi) = E_p (b, \xi) - E_{M_1 (b, \xi)}, E_i (d, \mu) = E_p (d, \mu) - E_{M_1 (d, \mu)} \) and \( < f | g > = \int d^3 b f^*(b) g(b) \).

We shall consider two extreme cases where the implications of the new effects can be seen more clearly. The first situation occurs when the particles have no common modes, \( < f | g > = 0 \) and they are in the same spin state, \( \xi = \mu = \Omega \). Then the transition rate is:

\[ w^2 (Q) \sim |\psi_{f \Omega} (Q)|^2 |\psi_{g \Omega} (Q)|^2 \quad (19) \]

The second case is when both incident particles are in the same state, \( f = g \) and \( \xi = \mu = \Omega \) (note that this situation only refers to bosons, because of the impossibility of preparing two indistinguishable fermions in the same state). We obtain

\[ w^2 (Q) \sim |\psi_{f \Omega} (Q)|^4 \quad (20) \]

We see that the second order theory contributes to the transition rate in the form \( |\psi_{f \Omega}|^2 |\psi_{g \Omega}|^2 \), or for bosons in the same state as \( |\psi_{f \Omega}|^4 \). This dependence on the wavefunction differs from that of a single incident particle, which is given by \( |\psi_{f \Omega}|^2 \).

The dependence of the absorption rate on the wavefunction is reminiscent of that of photons on the quantum intensity \( I \). In this case the transition rate is proportional to \( I \) for single absorptions and to \( I^2 \) for double absorptions.

We also see that the multiabsorption rate for massive particles differs for bosons and fermions. Because of the double sign in all the above expressions the two terms add for bosons, whereas they must be subtracted for fermions. In particular, when the states \( f \) and \( g \) are close, the transition rate tends to the form \( |\psi_{f \Omega}|^4 \) for bosons and to 0 for fermions.

### 4 Discussion

We have suggested in this paper the possibility of multiabsorption processes in the case of incident massive particle beams. We have presented a simple model
to evaluate the probabilities of absorption. We have corroborated the results
obtained in [2] showing the existence of second order corrections, associated
with multiparticle processes, to the usual expressions for one-particle processes.

The model presented here is simply the extension of Glauber’s scheme for the
absorption of photons to the case of massive particles. With this generalization
we obtain a unified view for the absorption of both types of particles. The
model also has notorious similarities with the studies on matter-waves coherence
in Refs. [4, 5]. However, these authors did not consider explicitly the case of
massive particles absorption. Moreover, they were more interested in coherence
properties than in the dependence of the absorption rate on the form of the
incident wavefunction.

The model gives a more clear picture of the physical processes involved than
the formal approach to the problem given in Ref. [2]. For instance, we can
relate the parameters characterizing the strength of single and double absorp-
tions (the equivalent to parameters $\alpha_1$ and $\alpha_2$ in [2] introduced there only in a
phenomenological way) with the physical variables of the problem ($\mathcal{M}$, $\alpha$, states
of the medium and particle...). A more detailed model than the one introduced
here would allow, in principle, for an explicit calculation of the parameters.

Our approach to the absorption problem is fully quantum and provides the
possibility of analyzing in a simple way the dependence of the absorption rates
on the wavefunction.

The model can be tested experimentally. Sending one-particle beams in
different states towards the same absorbing medium (for instance, placing the
medium at different positions after an interferometric arrangement, through
which the incident beam has previously passed) we can check the dependence
given by Eq. (11). This would corroborate the form of the interaction Hamilton-
ion chosen here.

To our knowledge this is the first time that an analysis of multiabsorption
phenomena for massive particles is presented in the literature. It can be impor-
tant in situations with highly coherent beams, such as those generated by atom
lasers [9], Bose-Einstein condensates [10]... In the case of identical bosons in the
same state the transition rate of the double absorption process is proportional
to $|\psi|^4$, clearly different from the usual one for one-particle processes ($\sim |\psi|^2$).
Moreover, the second order transition rate shows very different behaviours for
fermions and bosons. We remark that this dependence on $|\psi|^4$ provides the basis
for a test, in principle experimentally feasible, of the multiparticle effects and
of the model. With atom lasers the realization of an experiment which could
distinguish between the $|\psi|^2$ and $|\psi|^4$ behaviours seems to be a realistic goal.
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