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**Anabelian geometry with étale homotopy types.** (English) Zbl 1405.14053
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In algebraic topology, there are topological spaces that can be rebuilt by their fundamental groups and by their homotopy types with higher homotopy groups in a more general case.

Likewise, in Grothendieck’s anabelian geometry, for an algebraic variety over a field, the absolute Galois group of the field has an action, as outer automorphisms, on the geometric fundamental group of the algebraic variety. In particular, for the case that the algebraic varieties are the spectra of number fields, the Neukirch-Uchida theorem says such varieties are determined by outer isomorphisms between their absolute Galois groups. Grothendieck conjectures that algebraic varieties can be determined by outer isomorphisms between their étale fundamental groups, i.e., the classical anabelian geometry with étale fundamental groups. Such algebraic varieties are now called anabelian varieties.

On the other hand, by Quillen’s simplicial homotopy theories in Grothendieck’s étale site, particularly with étale homotopy type functor from category of algebraic varieties to the homotopy category, one has étale homotopy type and higher étale homotopy groups of algebraic varieties.

In this paper, the authors discuss anabelian geometry with étale homotopy type for algebraic varieties over a finitely generated extension field of the rational field, which is taken as a generalisation of the classical anabelian geometry with étale fundamental groups since in addition there are higher étale homotopy groups of the étale homotopy type for the algebraic varieties that are involved in. By overcoming several unusual technical difficulties, they give a homotopy-theoretic reformulation of Mochizuki’s theorem on anabelian geometry over hyperbolic curves. Several other remarkable results including strongly hyperbolic Artin neighborhoods are also obtained in the paper.

Reviewer: Feng-Wen An (Hubei)

**MSC:**
- 14F42 Motivic cohomology; motivic homotopy theory
- 14H30 Coverings of curves, fundamental group
- 14G32 Universal profinite groups (relationship to moduli spaces, projective and moduli towers, Galois theory)
- 14F35 Homotopy theory and fundamental groups in algebraic geometry

**Keywords:**
anabelian geometry; pro-spaces; étale homotopy theory

**Full Text:** DOI arXiv

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