Charm quark system in 2 + 1 flavor lattice QCD using the PACS-CS configurations

– Progress Report –

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1 Introduction

PACS-CS collaboration reaches the physical point of dynamical $ud$, $s$ quarks.

This is so because parallel talks by D.Kadoh, N.Ukita on Mon, and plenary talks by K-I.Ishikawa on Wed, Y.Kuramashi on Fri.

Our next step is the heavy quark system.

- The standard model parameters such as quark masses and CKM matrix elements are needed as inputs to search for signals beyond the standard model.

However, heavy quarks are hard to be treated on the lattice due to $O(ma)$ corrections. One famous problem in the heavy quark system is that lattice QCD fails to explain the charmonium hyperfine splitting $m_{J/\psi} - m_{\eta_c}$.

We try to solve this problem using a relativistic heavy quark (RHQ) action on the PACS-CS configurations.
Simulation setup

\[ N_f = 2 + 1 \text{ full QCD configurations} \]

- Action: RG improved gauge + non-perturbatively \( O(a) \) improved Clover fermion
- Algorithm: Domain-Decomposed HMC \(^\text{M.Lüscher, 2003}\) + Hasenbusch trick \(^\text{M.Hasenbusch, 2001}\) + Chronological inverter \(^\text{R.Brower et al, 1997}\) + Deflation \(^\text{M.Parks et al, 2006}\)
- Machine: PACS-CS(10 TFlops), T2K(76 TFlops) @Univ. of Tsukuba, T2K(83 TFlops) @Univ. of Tokyo

Developments of algorithms and machines allow us to simulate QCD on the physical point.
• Large lattice size: $32^3 \times 64$ ($L = 3$ fm, $a^{-1} = 2.2$ GeV ($\beta = 1.90$))
• Realistic sea quark masses: $m_{ud} = 3 - 10$ MeV, $m_s = 75 - 80$ MeV
  ($m_\pi = 155 - 300$ MeV, $m_\pi L = 2.3 - 4.3$)

| $\kappa_{ud}$ | $\kappa_s$ | $m_{ud}^{AWT}$ [MeV] | $m_s^{AWT}$ [MeV] | $N_{conf}$ (MD time) |
|---------------|-------------|-----------------------|-------------------|---------------------|
| 0.13770       | 0.13640     | 10                    | 80                | 700 (1750)          |
| 0.13781       | 0.13640     | 3                     | 80                | 330 (825)           |
| 0.137785      | 0.13660     | 3                     | 75                | 310 (775)           |
We use Tsukuba-type RHQ action for heavy quarks. S.Aoki et al, 2001

1-loop (tadpole improved) values are employed for $r_s, C_{SW}^{s,t}$. S.Aoki et al, 2003

\( C_{SW}^{s,t} \) are non-perturbatively improved at the massless point,

\[
C_{SW}^{s,t} = C_{SW}(NP, m = 0) - C_{SW}^{s,t}(PT, m = 0) + C_{SW}^{s,t}(PT, m \neq 0). 
\]

\nu is non-perturbatively tuned. (\( \nu \) is relevant for hyperfine splittings.)

\rightarrow For details, see the next slide.

\[
S_{RHQ} = \sum_{x,y} \bar{q}(x)D(x,y)q(y),
\]

\[
D(x, y) \equiv \delta_{x,y} - \kappa \left\{ (1 - \gamma_4)U_4(x)\delta_{x+4,y} + (1 + \gamma_4)U_4^\dagger(x)\delta_{x,y+4} \right\}
\]

\[
+ \sum_i \left\{ (r_s - \nu \gamma_i)U_i(x)\delta_{x+i,y} + (r_s + \nu \gamma_i)U_i^\dagger(x)\delta_{x,y+i} \right\}
\]

\[ -\delta_{x,y}\kappa \left\{ C_{SW}^t \sum_i \sigma_{4i} F_{4i} + C_{SW}^s \sum_{i<j} \sigma_{ij} F_{ij} \right\}. \]
Non-perturbative tuning of $\nu$

- $\nu$ is tuned so that an effective speed of light becomes unity, $C_{eff} = 1$.
- $C_{eff}$ is determined by a linear slope of a dispersion relation.
  
  $$E^2(|p|) - E^2(0) = C_{eff}^2 |p|^2, \quad |p| = \frac{2\pi}{N_s}(1, \sqrt{2}).$$

- Dispersion relations are deformed by doublers. But, the contribution is small, 1.3% for $|p| = 1$ and 2.6% for $|p| = \sqrt{2}$. 

\[ 32^3 \times 64, \; \kappa_{ud} = 0.13770, \; \kappa_s = 0.13640 \]
\[ \kappa_{-heavy} = 0.11022 \]
3 Results

[Effective masses]

- A good plateau is observed in $t = [13, 32]$. 

\[
\begin{align*}
32^3 \times 64, \, \kappa_{ud} &= 0.13770, \, \kappa_s = 0.13640 \\
\kappa_{\text{heavy}} &= 0.11022
\end{align*}
\]
[Interpolation to the physical point of the charm quark]

- At each $\kappa_{ud}, \kappa_s$, we linearly interpolate our results to the physical point of the charm quark,
  \[ M = A + \frac{B}{\kappa_{\text{heavy}}} \].

- The physical point of the charm quark is determined by the spin-averaged mass,
  \[ M(1S) \equiv \left( M_{\eta_c} + M_{J/\psi} \right)/4 = 3.0677(3)[\text{GeV}] \]. \text{PDG, 2007}
3.1 Orbital excitation

- We first check an orbital excitation $m_{\chi_1}(1P) - m_{J/\psi}(1S)$.
- No clear sea quark mass dependence is observed within our mass range of $m_{ud} = 3 - 10$ MeV, $m_s = 75 - 80$ MeV. 
  → We perform a very short chiral extrapolation using a linear function of quark masses, 
  $$m_V - m_{PS} = A + Bm_{ud} + Cm_s.$$ 
- Our results reproduce the experimental value. [PDG, 2007]
3.2 **Hyperfine splitting, $m_{J/\psi} - m_{\eta_c}$**

- No clear sea quark mass dependence is observed within our mass range of $m_{ud} = 3 - 10$ MeV, $m_s = 75 - 80$ MeV.

\[ \to \quad \text{We perform a short chiral extrapolation using a linear function of quark masses,} \]

\[ m_V - m_{PS} = A + Bm_{ud} + Cm_s. \]

- Our data are slightly smaller than the experimental value. PDG, 2007

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**Graphical Representation:**

- $m_V - m_{PS}$ versus bare $m_{ud}$
- Experimental data
- NP mass at physical point
- NP mass

*Graphical data points and labels are as follows:*

- $\beta = 1.90$, $32^3 \times 64$
- $m_V - m_{PS}$ in GeV
- bare $m_{ud}$ in units of $m_{AW}^4$
[Comparison of $N_f = 2 + 1$ data with $N_f = 0, 2$ data]

- $N_f = 2 + 1$ results are closer to the experimental value.  
  $\rightarrow$ Dynamical quarks give significant contribution to the hyperfine splitting.

- (While $N_f = 2 + 1$ results are obtained with non-perturbative $\nu$, $N_f = 0, 2$ data are with perturbative $\nu$.)

![Graph showing comparison of $N_f = 2 + 1$ data with $N_f = 0, 2$ data.](image-url)
3.3 Heavy-light system

- Our simulation is performed on the physical point of \( ud, s \) and \( c \) \((\kappa_{ud} = 0.137785, \kappa_s = 0.1366, \kappa_{charm} = 0.11236)\).

- Our statistics is small yet (40 conf).

- We employ 1-loop values for renormalization factors. S.Aoki et al, 2004

- Our results are consistent with experiments. (Note that CLEO group assumes \(|V_{cd}| = |V_{us}| \) for experimental analysis of \( f_D \) CLEO, 2008).
4 Summary

We performed calculations of a charm quark system using RHQ action on $N_f = 2+1$ PACS-CS configurations.

- Orbital excitations are reproduced well.
- Our data of the hyperfine splitting are closer to the experimental value, than those in $N_f = 0, 2$.
  $\rightarrow$ Dynamical quarks give significant contribution to the hyperfine splitting.
- Our data of the hyperfine splitting are slightly smaller than the experimental value.
  $\rightarrow$ More statistics are needed for definite conclusion.
  (Possible origins of the discrepancy are $O(g^2a)$ effects in RHQ action, dynamical charm quark effects, disconnected loop contributions.)
- Heavy-light calculations are ongoing.
