An inventory model for defective items with trade credit and inflation

R. Uthayakumar and M. Palanivel*

Department of Mathematics, The Gandhigram Rural Institute – Deemed University, Gandhigram, Dindigul 624 302, Tamil Nadu, India

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This paper develops an inventory model with imperfect items under the effect of inflation and time value of money in which demand is a deterministic function of selling price and advertisement cost. In this process, a certain fraction of purchased items are defective. These non-conforming items are reworked or refunded if they reach the customer. In this article, the model is considered with finite replenishment rate under progressive payment scheme within the cycle time. Here, the retailer is allowed a trade credit offer by the supplier to buy more items. This model aids in minimizing the total inventory cost by finding the optimal cycle length, the optimal time length of replenishment, and the optimal order quantity. As a particular case, the results of the perfect system (i.e. the system without defective items) are obtained. The optimal solution of the model is illustrated with the help of numerical examples. Also, the effect of changes in the different parameters on the optimal total cost is graphically presented.

Keywords: imperfect production; defective item; trade credit; inflation

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1. Introduction

Inflation plays an essential role for the optimal order policy and influences the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Usually, these spending are on peripherals and luxury items that give rise to demand of those items. As a result, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy. As mentioned above, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. The concept of inflation should be considered especially for long-term investment and forecasting. Buzacott (1975) first developed the Economic Order Quantity (EOQ) model taking inflation into account. Data and Pal (1991) considered the effects of inflation and time value of money on an inventory model with a linear time-dependent demand rate and shortages. Hariga (1995) extended Data and Pal’s (1991) model to consider flexible replenishment time and decreasing demand rate function. Chung (1996) developed an algorithm with finite replenishment and infinite planning horizon. Ray and Chaudhuri (1997) provided an EOQ model with inflation time discounting. Hou and Lin (2006) proposed an EOQ model for deteriorating items
with price- and stock-dependent selling rates under inflation and time value of money. Thangam and Uthayakumar (2010) studied an inventory model for deteriorating items with inflation-induced demand and exponential partial backorders – a discounted cash flow approach. Valliathal and Uthayakumar (2010) discussed the production – inventory problem for ameliorating/deteriorating items with non-linear shortage cost under inflation and time discounting. Sarkar and Moon (2011) developed an imperfect production process for time-varying demand with inflation and time value of money – an EMQ model. Tolgari, Mirzazadeh, and Jolai (2012) studied an inventory model for imperfect items under inflationary conditions by considering inspection errors. Guria, Das, Mondal, and Maiti (2013) formulated an inventory policy for an item with inflation-induced purchasing price, selling price, and demand with immediate part payment. Yang and Chang (2013) developed a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation.

The suppliers offer delay in payment to the retailers to buy more items and the retailers can sell the item before the closing of the delay time. As a result, the retailers sell the items and earn interests. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. This provides opportunities to the retailers to accumulate revenue and earn interest by selling their items during the delay period. This permissible delay in payment provides benefit to the supplier in as much as attracting new customers who consider it to be a type of price reduction and reduction in sells outstanding as some customers make payments on time in order to take advantage of permissible delay more frequently. In this direction, Goyal (1985) extended the EOQ model under the conditions of permissible delay in payments. Liao, Tsai, and Su (2000) developed an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. Alfares (2007) found out inventory model with stock-dependent demand under variable holding cost. Shah, Soni, and Patel (2010) used an integrated approach for optimal unit price and credit period for deteriorating inventory system when the buyer’s demand is price-sensitive. Recently, Musa and Sani (2012) studied an inventory ordering policies of delayed deteriorating items under permissible delay in payments. Sarkar (2012c) framed an EOQ model with delay in payments and time-varying deterioration rate. Maihami and Abadi (2012) derived the joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. Lin, Ouyang, and Dang (2012) developed a joint optimal ordering and delivery policy for an integrated supplier–retailer inventory model with trade credit and defective items.

In the present competitive market, the selling price of an item is one of the important factors in selecting an item for use. It is commonly seen that higher selling price causes decrease in demand, whereas lesser selling price has the reverse effect. Apart from price, the other marketing parameter which affects the demand is advertisement. It is commonly seen that a product is promoted through the advertisement in the well-known print or electronic media or by other means to attract the customers. The purpose of this type of advertisement is to raise the demand of the product. Hence, it can be concluded that the demand of an item is a function of marketing cost and selling price of an item. Kotler (1971) incorporated marketing policies into inventory decisions and studied the relationship between EOQ and decision. Subramanyan and Kumaraswamy (1981), Urban (1992), Goyal and Gunasekaran (1997) and Luo (1998) developed inventory models incorporating the effect of price variations and advertisement on demand. Mondal, Bhunia, and Maiti (2009) studied an inventory model for defective items incorporating marketing decisions with variable production cost. Geetha and
Uthayakumar (2009) developed an optimal inventory control policy for items with time-dependent demand. Bhunia, Pal, Chattopadhyay, and Medya (2011) investigated an inventory model of two-warehouse system with variable demand dependent on instantaneous displayed stock and marketing decision via hybrid RCGA. Soni and Patel (2012) discussed an optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy. Shah, Soni, and Patel (2012) developed an optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Deane and Agarwal (2012) considered a model for scheduling online advertisements to maximize revenue under variable display frequency.

In most of the models, researchers considered that the produced products are perfect in nature. But, in real-life situation, it is often that we see some of the items may be defective in nature. Here, defective items may be a certain fraction of the total production. This production of defective items is due to machine breakdown, labor problem, etc. These non-confirmative items can be dealt in two different ways by the producer, i.e. may be repaired or refunded (if these items are handed over to the customers). Depending on this policy, some researchers like Rosenblatt and Lee (1986), Lee and Park (1991) and Lin (1999) formulated an EPL model considering this type of production process. Salameh and Jaber (2000) and Goyal and Cardenas-Barron (2002) discussed an EPQ model for imperfect quality items. Goyal et al. (2003) discussed an EPQ model for imperfect quality items for a deterministic model. Wee, Yu, and Chen (2007) studied an integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. Min and Zhou (2009) gave a perishable inventory model under stock-dependent selling rate and shortage-dependent partial backlogging with capacity constraint. Sana (2010) developed an EOQ model for perishable item with stock-dependent demand and price discount rate. Begum, Sahoo, and Sahu (2012) proposed a replenishment policy for items with price-dependent demand, time-proportional deterioration, and no shortages. Sarkar (2012a) developed an EOQ model with delay in payments and stock-dependent demand in the presence of imperfect production. Sarkar (2012b) investigated an inventory model with reliability in an imperfect production process. Widyadana and Wee (2012) developed an economic production quantity model for deteriorating items with multiple production setups and rework.

In the present work, a deterministic inventory model for imperfect items with finite replenishment and permissible delay in payments is proposed in which the demand is a deterministic function of selling price and advertisement cost. These imperfect items are reworked or refunded if they reach the customer. Inflation and time value of money are considered in this model. As a particular case, the results of non-defective items are obtained. We have shown the suitable numerical examples to illustrate the model. Sensitivity analysis of the optimal solution with respect to major parameters of the system is carried out. The proposed model extends the model of existing literature with finite replenishment rate, price- and advertisement-dependent demand, permissible delay in payments, and two different scenarios based on imperfect items. To the author’s best knowledge, such type of model has not yet been discussed in the EOQ literature.

The rest of the paper is organized as follows: In Section 2, the notations and assumptions, which are used throughout this article, are described. In Section 3, the mathematical model to minimize the total annual inventory cost is established. Section 4
presents solution procedure to find the optimal length of time and order quantity. Numerical examples are provided in Section 6 to illustrate the theory and the solution procedure. This is followed by sensitivity analysis and conclusion.

2. Notations and assumptions

2.1. Notations

The following notations are used throughout this paper:

- \( t_1 \) Duration of the replenishment rate
- \( T \) The length of the inventory cycle
- \( q_1(t) \) The inventory level at time \( t, 0 \leq t \leq t_1 \)
- \( q_2(t) \) The inventory level at time \( t, t_1 \leq t \leq T \)
- \( K \) The constant supply rate of finished goods by the supplier to the retailer
- \( K_1 \) Total number of defective items
- \( \mu \) The scaling parameter for defective items, where defective items = \( \mu K^\delta \), \( \mu > 0 \) and \( 0 < \delta \leq 1 \)
- \( Q \) The order size per cycle for both scenario I and II
- \( A \) The ordering cost per cycle
- \( C_1 \) The holding cost (excluding interest charges) per unit per unit time
- \( p \) The purchasing cost per unit
- \( C_r \) The rework cost for the defective item (per unit)
- \( C_{rf} \) The refunded cost for the defective item (per unit)
- \( M_1 \) The first offered trade credit period without any charge
- \( M_2 \) The second offered trade credit period with charge
- \( I_e \) The rate of interest earned due to financing inventory
- \( I_{C_1} \) The rate of interest charged in stock by the supplier during \([M_1, M_2]\)
- \( I_{C_2} \) The rate of interest charged in stock by the supplier during \([M_2, T]\)
- \( r \) The discount rate which represents the time value of money
- \( f \) The inflation rate
- \( R \) The net discount rate of inflation, i.e. \( R = r - f \)
- \( TC \) The total cost of the system

2.2. Assumptions

To formulate the mathematical model, the following assumptions are made:

1. A single item is considered over an infinite planning horizon.
2. The demand rate \( D \) is a deterministic function of selling price \( s \) and advertisement cost \( A_C \) per unit of item, i.e. \( D(A_C, s) = A_C^\eta a s^{-b}, a > 0, b > 1, 0 \leq \eta < 1 \), \( a \) is the scaling factor, \( b \) is the index of price elasticity, and \( \eta \) is the shape parameter.
3. The replenishment takes place at finite rate.
4. The permissible delay in payment is offered by the supplier to the retailer.
5. Shortages are not permitted.
6. The effects of inflation and time value of money are considered.
7. The lead time is zero.
8. The imperfect (defective) items are considered.
3. Formulation and solution of the model

In the selling environment, to maintain the goodwill of the firm, the defective items, $K_1$, must be reworked or refunded, if those are sold to the customer. Considering these situations, we investigate the two different scenarios as follows.

3.1. Scenario I

$Q$ units are purchased. All the purchased items including the defective ones are sold to the customers at the rate of $D$ units as good units and later $K_1$ defective units are refunded from the customer with penalty at a cost of $C_{rf}(>s)$ per unit.

3.2. Scenario II

$Q$ units are purchased and $K_1$ purchased defective units are spotted just after purchasing, repaired against the cost of $C_r$ per unit and sold as good items to the customer.

3.3. Model description

The inventory system is developed as follows: the inventory cycle starts at $t=0$ with zero inventory and increases at a rate $K$ and also simultaneously decreases at a rate $D$ up to time $t_1$, and the inventory level is decreasing only due to demand rate in the interval $[t_1, T]$, which finally reaches the zero level at time $T$. The related Figure 1 of the model is as follows.

Based on the above description for scenario I and II, the differential equations representing the inventory status are given by

\[
\frac{dq_1(t)}{dt} = K - D, \quad 0 \leq t \leq t_1
\]

\[
\frac{dq_2(t)}{dt} = -D, \quad t_1 \leq t \leq T
\]

with the boundary conditions $q_1(0) = 0$ and $q_2(T) = 0$.

The solutions of the above differential Equations (1) and (2) are given by

![Inventory level](image)

Figure 1. Graphical representation of the inventory system.
\[ q_1(t) = (K - D)t, \quad 0 \leq t \leq t_1 \] (3)

\[ q_2(t) = D(T - t), \quad t_1 \leq t \leq T \] (4)

Put \( t = t_1 \) in Equations (3) and (4), we find the value of \( t_1 \) as

\[ t_1 = \frac{DT}{K} \] (5)

Since the supply rate occurs in the continuous time span \([0, t_1]\), then the order size in the problem is

\[ Q = Kt_1 \] (6)

As the defective items are being added in the inventory at the rate of \( \mu K^\delta \) per unit time, so the total number of defective items for all scenarios is given by

\[ K_1 = \mu K^\delta t_1. \] (7)

Now we want to find the different inventory costs with the effect of inflation as:

(i) Ordering cost = \( A \). (8)

(ii) The purchase cost

\[ PC = \int_0^{t_1} p Ke^{-Rt} dt = \frac{pK}{R} [1 - e^{-Rt_1}] \] (9)

(iii) Inventory holding cost (HC) for scenario I and II is given by

\[ HC = C_1 \left[ \int_0^{t_1} q_1(t) e^{-Rt} dt + \int_{t_1}^T q_2(t) e^{-Rt} dt \right] \\
= \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} [K + D(e^{-RT} - 1)] \right\} \] (10)

(iv) Refunded cost (RFC) for scenario I is given by

\[ RFC = C_{rf} \int_0^{t_1} \mu K^\delta e^{-Rt} dt = \frac{C_{rf} \mu K^\delta}{R} [1 - e^{-Rt_1}] \] (11)

(v) Reworked cost (RWC) for scenario II is given by

\[ RWC = C_r \int_0^{t_1} \mu K^\delta e^{-Rt} dt = \frac{C_r \mu K^\delta}{R} [1 - e^{-Rt_1}] \] (12)

The following cases arise due to different types of delay periods.

Case (1): If the retailer pays the purchasing cost after \( M_2 \) and before \( T \) (i.e. \( M_2 \leq T \)) for the purchased units from the supplier, then the supplier can charge a rate of interest \( I_{C_2} \) on the unpaid balance.

Case (2): If the retailer pays the purchasing cost after \( M_1 \) and before \( M_2 \) (i.e. \( M_1 \leq T \leq M_2 \)), then the supplier can charge a rate of interest \( I_{C_1} \) on the retailer.

Case (3): If the retailer pays the purchasing cost within the time \( M_1 \) (i.e. \( T \leq M_1 \)), then there is no interest charged by the supplier.
Therefore, the total cost of the system per unit time is given by

\[
TC_{i,j}(T), \quad M_1 \leq T \leq M_2 \quad \text{for } i = \{1, 2\}, \text{ (scenario 1 and 2)}
\]

where \(TC_{i,j}(T), i = \{1, 2\} \) and \(j = \{1, 2, 3\} \) are discussed as follows.

Case 1: \(M_2 \leq T\)

In this case, the retailer pays to the supplier at the end of the second offered credit period, \(M_2\), which is before the inventory is depleted completely. Hence, the retailer still has some stock on hand during the time interval \([M_2; T]\). Therefore, the interest charged by the supplier to the retailer per unit time is

\[
IC_1 = pIC_1 \int_{M_2}^{T} q_2(t) e^{-RT} dt = pIC_1 D \frac{e^{-RT}}{R} + e^{-RM_2} (T - M_2 - (1/R)) \quad (13)
\]

Furthermore, for the items that are already sold but have not yet been paid for during the time interval \([0, M_2]\), the interest per unit time earned by the retailer is

\[
IE_1 = sIE_c \left\{ t_1 - (1/R) + e^{-RM_2} (-T + M_2 + (1/R)) + e^{-RT_1} (T - t_1) \right\} \quad (14)
\]

Case 2: \(M_1 \leq T \leq M_2\)

In this case, the interest charged by the supplier to the retailer is

\[
IC_2 = pIC_1 \int_{M_1}^{T} q_2(t) e^{-RT} dt = pIC_1 D \frac{e^{-RT}}{R} + e^{-RM_1} (T - M_1 - (1/R)) \quad (15)
\]

Interest earned by the retailer is

\[
IE_2 = sIE_c \left\{ t_1 - (e^{-RT} - 1) + e^{-RT_1} (T - t_1) \right\} \quad (16)
\]

Case 3: \(T \leq M_1\)

In this case, the first period of delay in payment (\(M_1\)) is more than the cycle length. The retailer has received all the payment for sold goods from the customers at time \(T\) and makes payment for purchased goods to the supplier at the end of the credit period, \(M_1\). Therefore, the retailer uses the sales revenue to earn interest at a rate of \(I_e\), and no interest is payable (Figures 2–4).

Hence, the retailer’s interest earned per unit time is

\[
IE_3 = sIE_c \left\{ t_1 - \frac{1}{R} (e^{-RT} - 1) + e^{-RT_1} (T - t_1) \right\} + K t_1 (e^{-RT} - e^{-RM_1}) \quad (17)
\]

Hence, the total average cost per cycle = (ordering cost + purchase cost + inventory holding cost + refunded cost (for scenario I) + rework cost (for scenario II) + Interest charged – interest earned)/\(T\).
Figure 2. Inventory level as a function of time for case 1 ($M_2 \leq T$).

Figure 3. Inventory level as a function of time for case 2 ($M_1 \leq T \leq M_2$).

Figure 4. Inventory level as a function of time for case 3 ($T \leq M_1$).
Scenario I:

Case 1: \( M_2 \leq T \)

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + refunded cost + Interest charged – interest earned) / \( T \).

\[
TC_{1,1} = (A + PC + HC + RFC + IC_1 - IE_1) / T
\]
\[
= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} [K + D(e^{-RT} - 1)] \right\} \\
+ \frac{C_{rf} \mu K^5}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{pIC_2 D}{R} \left[ e^{-RT} + e^{-RM_1} (T - M_2 - (1/R)) \right] \\
- \frac{sL_D}{R} \left\{ -(1/R) + t_1 + e^{-RM_2} (-T + M_2 + (1/R)) - e^{-Rt_1} (t_1 - T) \right\} \right\}
\]
(18)

Case 2: \( M_1 \leq T \leq M_2 \)

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + refunded cost + Interest charged – interest earned) / \( T \).

\[
TC_{1,2} = (A + PC + HC + RFC + IC_2 - IE_2) / T
\]
\[
= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} [K + D(e^{-RT} - 1)] \right\} \\
+ \frac{C_{rf} \mu K^5}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{pIC_2 D}{R} \left[ e^{-RT} + e^{-RM_1} (T - M_1 - (1/R)) \right] \\
- \frac{sL_D}{R} \left\{ t_1 - \frac{(e^{-RT} - 1)}{R} + e^{-Rt_1} (T - t_1) \right\} \right\}.
\]
(19)

Case 3: \( T \leq M_1 \)

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + refunded cost – interest earned) / \( T \).

\[
TC_{1,3} = (A + PC + HC + RFC - IE_3) / T
\]
\[
= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} [K + D(e^{-RT} - 1)] \right\} \\
+ \frac{C_{rf} \mu K^5}{R} \left[ 1 - e^{-Rt_1} \right] \\
- \frac{sL_D}{R} \left\{ D \left[ t_1 - \frac{1}{R} (e^{-RT} - 1) + e^{-Rt_1} (T - t_1) \right] + Kt_1 (e^{-RT} - e^{-RM_1}) \right\} \right\}
\]
(20)

Scenario II:

Case 1: \( M_2 \leq T \)

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + reworked cost + Interest charged – interest earned) / \( T \).

\[
TC_{2,1} = (A + PC + HC + RC + IC_1 - IE_1) / T
\]
\[
= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} [K + D(e^{-RT} - 1)] \right\} \\
+ \frac{C_{rf} \mu K^5}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{pIC_2 D}{R} \left[ e^{-RT} + e^{-RM_1} (T - M_2 - (1/R)) \right] \\
- \frac{sL_D}{R} \left\{ - \frac{1}{R} + t_1 + e^{-RM_2} (-T + M_2 + (1/R)) - e^{-Rt_1} (t_1 - T) \right\} \right\}.
\]
(21)
Case 2: $M_1 \leq T \leq M_2$

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + reworked cost + Interest charged – interest earned)/$T$.

$$TC_{2,2} = \frac{(A + PC + HC + RC + IC_2 - IE_2)}{T}$$

$$= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_i} \right] + \frac{C_1}{R} \left\{ e^{-Rt_i} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R^2} \left[ K + D(e^{-RT} - 1) \right] \right\} ight\}$$

$$+ \frac{C_r \mu K^2}{R^2} \left[ 1 - e^{-Rt_i} \right] + \frac{plC_1 D}{R} \left[ e^{-RT} + e^{-RM_t} (T - M_1 - (1/R)) \right]$$

$$- \frac{sIC_2 D}{R} \left\{ t_1 - \frac{(e^{-RT} - 1)}{R} + e^{-Rt_i} (T - t_1) \right\} \right\} \right\} \right\} \right\}$$

(22)

Case 3: $T \leq M_1$

Here, the total cost per cycle per unit time = (ordering cost + purchase cost + inventory holding cost + reworked cost – interest earned)/$T$.

$$TC_{2,3} = \frac{(A + PC + HC + RC - IE_3)}{T}$$

$$= \frac{1}{T} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_i} \right] + \frac{C_1}{R} \left\{ e^{-Rt_i} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} \left[ K + D(e^{-RT} - 1) \right] \right\} ight\}$$

$$+ \frac{C_r \mu K^2}{R^2} \left[ 1 - e^{-Rt_i} \right] - \frac{sIC_2 D}{R} \left[ t_1 + \frac{1}{R^2} (e^{-RT} - 1) + e^{-Rt_i} (T - t_1) \right]$$

$$+ Kt_1 (e^{-RT} - e^{-RM_t}) \right\} \right\} \right\} \right\}$$

(23)

4. Solution procedure

Our aim is to minimize the total inventory cost by finding the optimal cycle length. In order to find the optimal solution $T^*$ and to minimize the annual total relevant cost, we take the first and second order derivatives of $TC_{i,j}(T)$ with respect to $T$, where $i = \{1, 2\}$ and $j = \{1, 2, 3\}$. In other words, the necessary and sufficient conditions for minimization of $TC_{i,j}(T)$ are, respectively, $\frac{dTC_{i,j}(T)}{dT} = 0$ and $\frac{d^2TC_{i,j}(T)}{dT^2} > 0$, where $i = \{1, 2\}$ and $j = \{1, 2, 3\}$.

Now,

$$\frac{dTC_{1,1}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} \left\{ e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} \left[ K + D(e^{-RT} - 1) \right] \right\} ight\}$$

$$+ \frac{C_r \mu K^2}{R^2} \left[ 1 - e^{-Rt_1} \right] + \frac{plC_1 D}{R} \left[ e^{-RT} + e^{-RM_t} (T - M_2 - (1/R)) \right]$$

$$- \frac{sIC_2 D}{R} \left\{ (-1/R) + t_1 + e^{-RM_t} (-T + M_2 + (1/R)) - e^{-Rt_1} (t_1 - T) \right\} \right\} \right\} \right\} \right\} \right\} \right\} \right\}$$

$$+ \frac{D}{RT} \left\{ pRe^{-Rt_1} - C_1 \left\{ \frac{R}{K} e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + e^{-RT} \right\} ight\}$$

$$+ \frac{C_r K^2}{R^2} e^{-Rt_1} + \frac{plC_2}{R} \left[ e^{-RM_t} - e^{-RT} \right]$$

$$+ sIC_2 \left\{ e^{-RM_t} - 1 + \left( \frac{D}{K} - 1 \right) (e^{-Rt_1} (1 - Rt_1) - 1) \right\} \right\} \right\}$$

(24)
\[
\frac{dT_{C_{1,2}}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} [1 - e^{-Rt}] + \frac{C_{1}}{R} \left[ e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + \frac{1}{R} [K + D(e^{-RT} - 1)] \right] \\
+ \frac{C_{r_{f} \mu}}{R} \left[ 1 - e^{-Rt} \right] + \frac{pL_{c_{1}} D}{R} \left[ e^{-RT} + e^{-RM_{t}} (T - M - (1/R)) \right] \\
- s_{L_{c}} D \left\{ t_1 - \frac{(e^{-RT} - 1)}{R} + e^{-Rt} (T - t_1) \right\} \\
+ \frac{D}{RT} \left\{ pRe^{-Rt} - C_{1} \left[ \frac{R}{K} e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + e^{-RT} \right] \right\} \\
+ C_{r_{f} \mu} K^{3} \left( e^{-Rt} - 1 - (1 + e^{-RT}) \right) \\
+ s_{L_{e}} \left\{ \left( \frac{D}{K} - 1 \right) (e^{-Rt} (1 - R_{t}) - 1) + (e^{-RM_{t}} - 1) + RT e^{-RT} \right\}\right\}
\]

(25)

\[
\frac{dT_{C_{1,3}}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} [1 - e^{-Rt}] + \frac{C_{1}}{R} \left[ e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + \frac{1}{R} [K + D(e^{-RT} - 1)] \right] \\
+ \frac{C_{r_{f} \mu}}{R} \left[ 1 - e^{-Rt} \right] - s_{L_{c}} D \left\{ \left( 1/R \right) + t_1 + e^{-RM_{t}} (T - M - (1/R)) \right\} \\
+ \frac{D}{RT} \left\{ pRe^{-Rt} - C_{1} \left[ \frac{R}{K} e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + e^{-RT} \right] \right\} \\
+ C_{r_{f} \mu} K^{3} \left( e^{-Rt} - 1 - (1 + e^{-RT}) \right) \\
+ s_{L_{e}} \left\{ \left( e^{-RM_{t}} - 1 \right) + \left( \frac{D}{K} - 1 \right) (e^{-Rt} (1 - R_{t}) - 1) \right\}\right\}
\]

(26)

\[
\frac{dT_{C_{2,1}}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} [1 - e^{-Rt}] + \frac{C_{1}}{R} \left[ e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + \frac{1}{R} [K + D(e^{-RT} - 1)] \right] \\
+ \frac{C_{r_{f} \mu}}{R} \left[ 1 - e^{-Rt} \right] + \frac{pL_{c_{1}} D}{R} \left[ e^{-RT} + e^{-RM_{t}} (T - M - (1/R)) \right] \\
- s_{L_{c}} D \left\{ \left( 1/R \right) + t_1 + e^{-RM_{t}} (T - M - (1/R)) \right\} \\
+ \frac{D}{RT} \left\{ pRe^{-Rt} - C_{1} \left[ \frac{R}{K} e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + e^{-RT} \right] \right\} \\
+ C_{r_{f} \mu} K^{3} \left( e^{-Rt} - 1 - (1 + e^{-RT}) \right) \\
+ s_{L_{e}} \left\{ \left( e^{-RM_{t}} - 1 \right) + \left( \frac{D}{K} - 1 \right) (e^{-Rt} (1 - R_{t}) - 1) \right\}\right\}
\]

(27)

\[
\frac{dT_{C_{2,2}}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} [1 - e^{-Rt}] + \frac{C_{1}}{R} \left[ e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + \frac{1}{R} [K + D(e^{-RT} - 1)] \right] \\
+ \frac{C_{r_{f} \mu}}{R} \left[ 1 - e^{-Rt} \right] + \frac{pL_{c_{1}} D}{R} \left[ e^{-RT} + e^{-RM_{t}} (T - M - (1/R)) \right] \\
- s_{L_{c}} D \left\{ \left( 1/R \right) + t_1 + e^{-RM_{t}} (T - M - (1/R)) \right\} \\
+ \frac{D}{RT} \left\{ pRe^{-Rt} - C_{1} \left[ \frac{R}{K} e^{-Rt} \left( DT - K \left( t_1 + \frac{1}{R} \right) \right) + e^{-RT} \right] \right\} \\
+ C_{r_{f} \mu} K^{3} \left( e^{-Rt} - 1 - (1 + e^{-RT}) \right) \\
+ s_{L_{e}} \left\{ \left( \frac{D}{K} - 1 \right) (e^{-Rt} (1 - R_{t}) - 1) - (1 + e^{-RT}) \right\}\right\}
\]

(28)
and
\[
\frac{dT_{C_3}(T)}{dT} = -\frac{1}{T^2} \left\{ A + \frac{pK}{R} \left[ 1 - e^{-Rt_1} \right] + \frac{C_1}{R} e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + \frac{1}{R} \left[ K + D(e^{-RT} - 1) \right] \right. \\
+ \frac{C_e \mu K e}{R} \left[ 1 - e^{-Rt_1} \right] \left[ R + K e \right] \left[ \frac{1}{R} t_1 + \frac{1}{R} \right] \left[ D(e^{-RT} - 1) + e^{R(t_1 - T)} \right] \right. \\
+ \frac{D}{RT} \left[ pRe^{-Rt_1} - C_e \frac{R}{K} e^{-Rt_1} \left[ DT - K \left( t_1 + \frac{1}{R} \right) \right] + e^{-RT} \right] \\
+ \frac{C_r \mu K e}{R} e^{-Rt_1} + \frac{pI_{C_1}}{R} [e^{-RM_t} - e^{-RT}] \\
+ sI_e \left\{ \frac{D}{K} - 1 \right\} \left[ e^{-Rt_1} - 1 \right] + \frac{K e}{R} \left[ e^{-Rt_1} - 1 \right] + RTe^{-RT} \right\}
\]

By solving the equations \( \frac{dT_{C_3}(T)}{dT} = 0 \), we get the optimal values of \( T = T_{i,j} \). Moreover, \( T \) satisfies the equations \( \frac{d^2T_{C_3}(T)}{dT^2} > 0 \), where \( i = \{1, 2\} \) and \( j = \{1, 2, 3\} \). From Equations (5–7), the optimal value of \( t_1 = t_1^* \), \( Q = Q^* \) and \( K_1 = K_1^* \) can be found out.

5. For non-defective items

As the defective items are entered at the rate of \( \mu K e \) per unit time, the total number of defective items for all scenarios is given by \( K_1 = \mu K e t_1 \). Note that if \( \mu = 0 \), then \( K_1 = 0 \). Therefore, the items are completely perfect (i.e. free from defective items). Then, the optimal results in this case are obtained from the above expressions using \( \mu = 0 \).

6. Numerical examples

Example 1 (for defective items)

Let \( A_C = $50 \); \( a = 40,000 \); \( b = 2.5908 \); \( s = 0.4 \); \( p = 10 \); \( C_r = $5 \); \( C_{rf} = $20 \); \( A = $100 \); \( C_1 = $3 \); \( I_e = 0.15 \); \( I_{C_1} = 0.18 \); \( I_{C_2} = 0.20 \); \( R = 0.1 \); \( M_1 = 1.142 \) year; \( M_2 = 1.753 \) year; \( \mu = 0.08 \); \( \delta = 0.8 \); and \( K = 300 \).

Then, we obtain the optimal solutions as follows:

For scenario I,

Case 1:
\[ \{ t_1^* = 1.1411 \text{ year}, T^* = 1.9945 \text{ year}, Q^* = 342.3527, K_1^* = 8.7526, TC^* = $1779.6013 \} \]

Case 2:
\[ \{ t_1^* = 0.9972 \text{ year}, T^* = 1.7430 \text{ year}, Q^* = 299.1779, K_1^* = 7.6488, TC^* = $1816.8814 \} \]

Case 3:
\[ \{ t_1^* = 0.3177 \text{ year}, T^* = 0.5553 \text{ year}, Q^* = 95.3299, K_1^* = 2.4372, TC^* = $1753.5804 \} \]

Among the above solutions, the better optimal solution is \( \{ t_1^* = 0.3177 \text{ year}, T^* = 0.5553 \text{ year}, Q^* = 95.3299, K_1^* = 2.4372, TC^* = $1753.5804 \} \). Figure 5 shows that the total cost decreases with \( T \) and \( TC \) attains the minimum value 1753.5804 at \( T = 0.5553 \). If \( T \) crosses 0.5553, the total cost then increases. From Figure 5, it is observed that the total cost is a convex function with respect to \( T \). Similarly, Figures 6 and 7 show the convexity of the total cost function.
For scenario II,

Case 1: \(\{t_1^* = 1.1372\text{ year}, T^* = 1.9877\text{ year}, Q^* = 341.1805, K_1^* = 8.7226, TC^* = $1717.3878\}\)

Case 2: \(\{t_1^* = 0.9870\text{ year}, T^* = 1.7252\text{ year}, Q^* = 296.1299, K_1^* = 7.5709, TC^* = $1754.2169\}\)

Case 3: \(\{t_1^* = 0.3168\text{ year}, T^* = 0.5537\text{ year}, Q^* = 95.0509, K_1^* = 2.4300, TC^* = $1688.7893\}\)

Among the above solutions, the better optimal solution is \(\{t_1^* = 0.3168\text{ year}, T^* = 0.5537\text{ year}, Q^* = 95.0509, K_1^* = 2.4300, TC^* = $1688.7893\}\).

From Figure 8, it is observed that the total cost decreases with \(T\) and it attains the minimum value 1717.3878 at \(T=1.9877\). After that, the total cost will increase. From Figure 8, it is observed that the total cost is a convex function with respect to \(T\). Similarly, Figures 9 and 10 show the convexity of the total cost function.

**Example 2** (for non-defective items)

Using the same parameter values as in Example 1 except that of \(\mu\) and let \(\mu = 0\), then we get the optimal solutions as:

Case 1: \(\{t_1^* = 1.1359\text{ year}, T^* = 1.9854\text{ year}, Q^* = 340.7925, TC^* = $1696.6473\}\).

Case 2: \(\{t_1^* = 0.9837\text{ year}, T^* = 1.7194\text{ year}, Q^* = 295.1355, TC^* = $1733.3218\}\).

Case 3: \(\{t_1^* = 0.3165\text{ year}, T^* = 0.5532\text{ year}, Q^* = 94.9584, TC^* = $1667.1915\}\).

Among the above optimal solutions, the better optimal solution is \(\{t_1^* = 0.3165\text{ year}, T^* = 0.5532\text{ year}, Q^* = 94.9584, TC^* = $1667.1915\}\). From Figure 11, it is observed that when \(\mu = 0\), the total cost decreases with \(T\) and it attains the minimum value 1696.6473 at \(T=1.9854\). Similarly, Figures 12 and 13 show the convexity of the total cost function.

![Figure 5](image-url)  
*Figure 5. Case 1: The total cost with respect to \(T\) for scenario 1.*
Figure 6. Case 2: The total cost with respect to $T$ for scenario 1.

Figure 7. Case 3: The total cost with respect to $T$ for scenario 1.
7. Sensitivity analysis

We now study the effects of changes in the values of the system parameters $A_c, a, b, \eta, C_{rf}, A, C_1, \mu,$ and $R$ on the optimal replenishment policy of case 1 of the Example 1.

Figure 8. Case 1: The total cost with respect to $T$ for scenario 2.

Figure 9. Case 2: The total cost with respect to $T$ for scenario 2.
We change one parameter at a time keeping the other parameters unchanged. The results are summarized in Table 1.

Figure 10. Case 3: The total cost with respect to $T$ for scenario 2.

Figure 11. Case 1: The total cost with respect to $T$ when $\mu = 0$. 
Based on Table 1, we obtain the following managerial phenomena:

1. When the advertisement cost $A_c$ is increasing, the cycle length, the order quantity, and the total cost are highly increasing. That is the minimum advertisement cost will minimize the total cost of the retailer.

Figure 12. Case 2: The total cost with respect to $T$ when $\mu = 0$.

Figure 13. Case 3: The total cost with respect to $T$ when $\mu = 0$. 
When the parameter $a$ is increasing, the cycle length, the order quantity, and the total cost are also increasing.

(3) When the parameter $b$ is increasing, the cycle length, the order quantity, and the total cost are highly decreasing.

(4) When the parameter $\eta$ is increasing, the cycle length and the order quantity are increasing and the total cost changes variably.

Figure 14. Total cost vs. cycle length for different values of $A_c$.

Figure 15. Total cost vs. cycle length for different values of $a$. 
(5) When the refunded cost $C_{r}$ is increasing, the cycle length, the order quantity, and the total cost are also increasing. That is, if the retailer minimizes the refunded cost, then his/her total cost will be minimized.

(6) When the setup cost $A$ is increasing, the cycle length, the order quantity, and the total cost are also increasing. That is, minimum setup cost will minimize the total cost of the retailer.

(7) When the holding cost $C_{1}$ is increasing, the cycle length and the order quantity are decreasing and the total cost is increasing. That is, the minimum cost for holding the items will minimize the total cost of the retailer.
When the length of fresh product time $\mu$ is increasing, the cycle length, the order quantity, and the total cost are also increasing. That is, when the length of fresh product time $\mu$ is decreasing, the total cost and the order quantity are decreasing. Since the order quantity is reduced, automatically holding cost of the items will also be reduced. So, the total cost of the retailer will be minimized.

Figure 18. Total cost vs. cycle length for different values of $C_{cf}$.

Figure 19. Total cost vs. cycle length for different values of $A$. 

(8) When the length of fresh product time $\mu$ is increasing, the cycle length, the order quantity, and the total cost are also increasing. That is, when the length of fresh product time $\mu$ is decreasing, the total cost and the order quantity are decreasing. Since the order quantity is reduced, automatically holding cost of the items will also be reduced. So, the total cost of the retailer will be minimized.
(9) When the net discount rate of inflation $R$ is increasing, the cycle length, the order quantity, and the total cost are highly increasing. That is, for lower values of the net discount rate of inflation ($R$), the total cost of the retailer will be minimized.

Figures 14–22 gives the convexity of the total cost function and the implications with respect to the changes in the parameters $A_c$, $a$, $b$, $\eta$, $C_{rf}$, $A$, $C_1$, $\mu$, $\delta$, and $R$.

Figure 20. Total cost vs. cycle length for different values of $C_1$.

Figure 21. Total cost vs. cycle length for different values of $\mu$. 
Table 1. Sensitivity analysis for various inventory parameters.

| Parameter | Parameter value | \( t_1 \) | \( T \) | \( Q \) | \( TC \) |
|-----------|-----------------|-------|-------|-----|-------|
| \( A_c \) | 25               | 0.8099 | 1.8680 | 242.97 | 1429.4177 |
|           | 50               | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 75               | 1.4502 | 2.1552 | 435.06 | 1982.2283 |
| \( a \)   | 20,000           | 0.5225 | 1.8266 | 156.36 | 993.5821  |
|           | 40,000           | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 60,000           | 2.3700 | 2.7615 | 711    | 2179.2154 |
| \( b \)   | 2.4              | 3.4034 | 3.5482 | 1021.02 | 2133.2487 |
|           | 2.5908           | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
| \( \eta \)| 0.2              | 0.4789 | 1.8303 | 143.67 | 916.9378  |
|           | 0.4              | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 0.6              | 16.1307| 12.8929| 4839.21| 422.2435  |
| \( C_{rf} \)| 10               | 1.1385 | 1.9899 | 341.55 | 1738.1270 |
|           | 20               | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 30               | 1.1438 | 1.9991 | 343.14 | 1821.0704 |
| \( A \)   | 50               | 1.1126 | 1.9446 | 333.78 | 1754.2150 |
|           | 100              | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 150              | 1.1692 | 2.0435 | 350.76 | 1804.3660 |
| \( C_1 \) | 2                | 1.2140 | 2.1219 | 364.2  | 1711.7486 |
|           | 3                | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 4                | 1.0794 | 1.8866 | 323.82 | 1843.9746 |
| \( \mu \) | 0.04             | 1.1385 | 1.9899 | 341.55 | 1738.1270 |
|           | 0.08             | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 0.12             | 1.1438 | 1.9991 | 343.14 | 1821.0703 |
| \( R \)   | 0.05             | 1.0786 | 1.8853 | 323.58 | 1830.9046 |
|           | 0.1              | 1.1412 | 1.9945 | 342.36 | 1779.6013 |
|           | 0.15             | 15.0948| 26.3826| 4528.44| 2845.2377 |
8. Conclusion

In this paper, a model for determining the optimal cycle length, optimal production length, and optimal order quantity for imperfect production process is developed where delay in payment is allowed. In the present situation, inflation and time value of money are also the main factors. In keeping with this reality, these factors are incorporated in the present model. Finite replenishment rate, price, and advertisement-dependent deterministic demand pattern are considered in this model and two different scenarios are discussed. Based on these scenarios, the imperfect items are reworked or if they reach the customer, refunded. As a particular case, the results for a system of without defective items are obtained. Our result shows that when the delay in payment is allowed, the total cost for the retailer decreases. Also, when the items are reworked, the total cost decreases. Therefore, the reworking defective items are better than the refunding the cost for defective items. If the items are non-defective, then the total cost is minimized. Furthermore, numerical examples are provided to illustrate the model. Finally, sensitivity analysis is carried out with respect to the key parameters and useful managerial insights are obtained. The graphical illustrations are also given to analyze the efficiency of the model clearly.

This paper can be extended in several ways, for instance, we could extend the model by considering the non-zero lead time. Also, we may consider time-dependent holding cost in the model. Finally, we could extend this model by allowing shortages.

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