Hořava Gravity and Gravitons at a Conformal Point

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Abstract

Recently Hořava proposed a renormalizable gravity theory with higher derivatives by abandoning the Lorentz invariance in UV. Here, I construct the Hořava model at $\lambda = 1/3$, where a local anisotropic Weyl symmetry exists in the UV limit, in addition to the foliation-preserving diffeomorphism. By considering linear perturbations around Minkowski vacuum for the non-projectable version of the Hořava model, I show that the scalar graviton mode is completely disappeared and only the usual tensor graviton modes remain in the physical spectrum. The existence of the UV conformal symmetry is unique to the theory with the detailed balance and this may explain the importance of the detailed balance condition in quantum gravity.

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I. INTRODUCTION

Recently Hořava proposed a renormalizable gravity theory with higher spatial derivatives (up to sixth order) in four dimensions which reduces to Einstein gravity with a non-vanishing cosmological constant in IR but with improved UV behaviors, by abandoning the Lorentz invariance from non-equal-footing treatment of space and time \[1,2\]. Due to lack of the full diffeomorphism (\textit{Diff}), some extra graviton modes are expected generally but there have been confusions regarding the extra modes and the consistency of the Hořava model \[3–15\]. But, recently I have reconsidered these problems for the non-projectable version of the Hořava model and showed that, in the Minkowski vacuum background, the extra scalar graviton mode can be consistently decoupled from the usual tensor graviton modes, by imposing the (local) Hamiltonian constraint as well as the momentum constraints \[16\], for arbitrary values of the Lorentz breaking parameter \(\lambda\), except the point \(\lambda = 1/3\), where the theory becomes singular and needs a separate consideration. This reduces to the results of Einstein gravity in IR and achieves the consistency of the model.

In this paper, I consider the Hořava model at the particular point \(\lambda = 1/3\), where local anisotropic Weyl rescaling symmetry exists in the UV limit, in addition to the foliation-preserving \textit{Diff}. Due to the existence of the extra (local) symmetry in UV, it was demonstrated that there would be no physical excitation of the extra scalar mode in UV \[2\]. But this analysis was not enough to understand the complete aspects of the system. For example, the Cotton-squared term was crucial for the scalar graviton’s UV decoupling but this peculiar term can only be naturally explained by the detailed balance condition, though its physical meaning was unclear in the non-conformal cases. And also, the analysis was only for the UV limit and so more extension of the analysis was needed to see the complete spectrum of all the graviton modes. Finally, due to the “local” symmetry in UV, it has been noted that the lapse function \(N\) can no longer be a projectable function on the foliation, which means \(N = N(t)\) for the foliation time \(t\), in the conformal case, in contrast to non-conformal cases \[2\]. So, it would be important to see the consequences of the non-projectability in the complete spectrum of graviton modes at the conformal point \(\lambda = 1/3\). So, the object of this paper is twofold. First is, for the first time, the construction of the full Horava action at the conformal point \(\lambda = 1/3\), which needs a separate consideration due to degeneracy of the DeWitt metric, based on the standard framework with the detailed balance condition \[1,2,17,18\]. In Sec. II, I first construct the Hořava model at the conformal point \(\lambda = 1/3\), which needs a separate consideration due to degeneracy of the DeWitt metric, based on the standard framework with the detailed balance condition \[1,2,17,18\]. In Sec. III, I consider the non-projectable version of the model and show that the scalar graviton mode is completely disappeared in the physical spectrum. Only the usual tensor graviton modes remain, as expected, by considering linear perturbations of metric around Minkowski vacuum and imposing the (local) Hamiltonian constraint, as well as the momentum constraints, from the non-projectable lapse function \[16\]. In Sec. IV, I conclude with several discussions.
II. HOŘAVA GRAVITY AT A CONFORMAL POINT

Hořava gravity is defined as a power-counting renormalizable gravity, by introducing the anisotropic UV scaling

$$x \rightarrow sx, \ t \rightarrow st$$ (1)

with $z = 3$ in (3+1) spacetime dimensions [2]. By considering the ADM decomposition of the metric

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right),$$ (2)

the Hořava action reads, formally,

$$S = \int dtd^3x \sqrt{gN} \left\{ \frac{2}{\kappa^2} K_{ij} G^{ijkl} K_{kl} - \frac{\kappa^2}{2} \left[ \frac{1}{\nu^2} C^{ij} - \frac{\mu}{2} \left( R^{(3)ij} - \frac{1}{2} R^{(3)} g^{ij} + \Lambda_W g^{ij} \right) \right] \right\} \times G_{ijkl} \left[ \frac{1}{\nu^2} C^{kl} - \frac{\mu}{2} \left( R^{(3)kl} - \frac{1}{2} R^{(3)} g^{kl} + \Lambda_W g^{kl} \right) \right],$$ (3)

where

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$ (4)

is the extrinsic curvature (the dot (') denotes the derivative with respect to $t$),

$$C^{ij} = \epsilon^{ikl} \nabla_k \left( R^{(3)ij} - \frac{1}{4} R^{(3)} \delta^j_i \right)$$ (5)

is the Cotton tensor, $\kappa, \lambda, \nu, \mu$, and $\Lambda_W$ are constant parameters. The supermetric

$$G^{ijkl} = \delta^{ijkl} - \lambda g^{ij} g^{kl},$$ (6)

with $\delta^{ijkl} \equiv \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk} + g^{ij} g^{kl})$, is the generalized DeWitt metric for a Lorentz symmetry breaking parameter $\lambda$, which is 1 for the usual Lorentz invariant general relativity (GR) and its deviation from 1 measures the violation of Lorentz symmetry in the kinetic term,

$$S_K = \int dtd^3x \sqrt{gN} \frac{2}{\kappa^2} K_{ij} G^{ijkl} K_{kl}.$$ (7)

Another supermetric $G_{ijkl}$ is the inverse of the DeWitt metric, satisfying $G_{ijkl} G_{mnkl} = \delta^{ij}_{kl}$. And in deriving all the remaining terms, other than the kinetic terms, in (3), which is called as the potential terms, I have adopted the “detailed balance” prescription as

$$S_V = -\frac{\kappa^2}{8} \int dtd^3x \sqrt{gN} \frac{\delta W}{\delta g^{ij}} G_{ijkl} \frac{\delta W}{\delta g^{kl}}$$ (8)

with

$$\frac{\delta W}{2\delta g^{ij}} = \frac{1}{\nu^2} C^{ij} - \frac{\mu}{2} \left( R^{(3)ij} - \frac{1}{2} R^{(3)} g^{ij} + \Lambda_W g^{ij} \right)$$ (9)
from the three-dimensional (Euclidean) gravity action \[20, 21\]

\[
W = \frac{1}{\nu^2} \int Tr \left( \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right) + \mu \int d^3x \sqrt{g} (R^{(3)} - 2\Lambda_W). \tag{10}
\]

For \( \lambda \neq 1/3 \) (1/D in D spatial dimensions), the DeWitt metric is not degenerated and the explicit form of the action can be obtained by considering the inverse DeWitt metric \( G_{ijkl} = \delta_{ijkl} - \frac{1}{3\lambda} g_{ij} g_{kl} \) with \( \delta_{ijkl} \equiv \frac{1}{2} (g_{ik} g_{j\ell} + g_{i\ell} g_{k\ell}) \) \[1, 2\]. On the other hand, for \( \lambda = 1/3 \), the DeWitt metric is degenerated and we need to project out the non-degenerate parts only when considering the inverse DeWitt metric. Actually, by using the fact that \( G_{ijkl} \) has a null eigenvector \( g_{ij} \),

\[
G^{ijkl} g_{ij} = 0,
\]

it is easy to see that its inverse \( G_{ijkl} \) is given by

\[
G_{ijkl} = \delta_{ijkl} - \frac{1}{3} g_{ij} g_{kl}, \quad G_{ijkl} g^{ij} = 0, \quad G_{ijmn} G^{mnkl} = \tilde{\delta}^{kl}_{ij}
\]

with the (projected) Kronecker-delta \( \tilde{\delta}^{kl}_{ij} = \delta^{kl}_{ij} - \frac{1}{3} g^{ij} g^{kl} \), satisfying \( \tilde{\delta}^{kl}_{ij} g^{ij} = 0 \).

Then, after some manipulations, one can find the following action, from \[3\],

\[
S = \int dt d^3x \sqrt{g} N \frac{2}{\kappa^2} \left( K^{ij} K_{ij} - \frac{1}{3} K^2 \right) - \frac{\kappa^2}{2\nu^2} C_{ij} C^{ij} + \frac{\kappa^2 \mu^2}{2\nu^2} \epsilon^{ijk} R^{(3)i}_{ik} \nabla_j R^{(3)k}_{j}
\]

\[-\frac{\kappa^2 \mu^2}{8} \left( R^{(3)i}_{ij} R^{(3)ij} - \frac{1}{3} (R^{(3)})^2 \right). \tag{13}
\]

Note that all terms which are proportional to \( \Lambda_W \) are canceled, in sharply contrast to \( \lambda \neq 1/3 \) cases, and consequently there is no term, proportional to \( R^{(3)} \) and cosmological constant term \( \Lambda \) of the usual GR. This is basically due to the “detailed balance” condition and we need “soft” breakings of the detailed balance in order that the usual GR limit may be obtained in IR\(^1\). Then the desired form of the general action would be,

\[
S_g = S + \int dt d^3x \sqrt{g} N \frac{\kappa^2 \mu^2 \dot{\omega}}{8} \left( R^{(3)} - \frac{2\Lambda}{\nu^2} \right) \tag{14}
\]

by introducing the soft breaking terms of the detailed balance, \( R^{(3)}, \Lambda \) and these modify the IR behaviors \[2, 4, 17, 18\].

This action breaks the Lorentz symmetry manifestly even in IR limit where the kinetic term and the last two IR-modification terms in \[14\] dominate, due to the Lorentz non-invariant deformation of the kinetic term with \( \lambda = 1/3 \). However, this action has another symmetry, called “anisotropic” Weyl rescaling symmetry,

\[
g_{ij} \to \Omega^2(t, x) g_{ij}, \quad N \to \Omega^2(t, x) N, \quad N_i \to \Omega^2(t, x) N_i
\]

\[1\] The recovery of GR would require \( \lambda \) to flow from 1/3 in UV to 1 in IR. But this is problematic in the, so-called, “projectable” version of the Hořava gravity, where the scalar graviton exists and it becomes a ghost for \( 1/3 < \lambda < 1 \) \[14\] (see also \[19\]), i.e., in the process of RG-flows.
at each spacetime point: Under this transformations, the measure part is transformed as
\( \sqrt{g^N} \rightarrow \Omega^{3+z} \sqrt{g^N} \) and, for \( z = 3 \), the \( \Omega^{3+z} = \Omega^6 \) factor is canceled by the \( \Omega^{-6} \) factors from the kinetic parts and the highest (6th order) spatial-derivative term \( C_{ij}C^{ij} \). But this symmetry exists only in the UV limit since all other lower-spatial derivative terms violate this symmetry, explicitly; it is interesting to note that the last term in \( (13) \) is, up to boundary terms, the spatial part of the square of the (four-dimensional) Weyl tensor \( C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = \frac{1}{2}(R_{\mu\nu}R^{\mu\nu} - \frac{2}{3}R^2) \) in the conformal gravity, which is invariant under \( (15) \) for \( z = 1 \) but not invariant for \( z = 3 \). So this is an “emergent” symmetry in the UV limit only and this is in contrast to the Lorentz symmetry, which emerges in the IR limit for \( \lambda = 1 \) \([2, 16]\). The existence of the UV conformal symmetry is unique to the theory with the detailed balance: If one does not adopt the detailed balance, one does not have the 6th-order spatial-derivatives term \( C_{ij}C^{ij} \) and the UV symmetry is violated by the potential terms \( S_V \), generally \([8]\). Actually, the conformal symmetry is inherent in the defining properties of spacetimes with the scaling \( (1) \) since the transformation \( (15) \) corresponds to the local version of the rigid anisotropic scaling \( (1) \) with a constant \( \Omega = s \) \([1, 2, 22]\); the detailed balance is a natural way to introduce the desired potential \( C_{ij}C^{ij} \) into the action.

By comparing the IR limit of the general action \( (14) \) with \( \lambda \)-deformed Einstein-Hilbert action \( S_{\lambda EH} \) \([18]\),

\[
S_{\lambda EH} = \frac{c^4}{16\pi G} \int dt d^3x \sqrt{g^N} \left[ \frac{1}{c^2} \left( K_{ij}K^{ij} - \lambda K^2 \right) + R(3) - \frac{2\Lambda}{c^2} \right]
\]

one may obtain the fundamental constants of the speed of light \( c \), the Newton’s constant \( G \) as

\[
c = \sqrt{\frac{\kappa^4 \mu^2 \omega}{16}}, \quad G = \frac{\kappa^2 c^2}{32 \pi}.
\]

In the canonical formulation, the existence of the Weyl symmetry is reflected in the primary constraint

\[
\pi^i_i \equiv g_{ij} \pi^{ij} \approx 0
\]

for the momenta

\[
\pi^{ij} \equiv \frac{\delta S}{\delta \dot{g}_{ij}} = \frac{2\sqrt{\theta}}{\kappa^2} C^{ijkl}K_{kl},
\]

in addition to the usual Hamiltonian and momentum constraints. And the symmetry in UV limit implies that the constraint leads to the first-class constraints system in UV.

**III. GRAVITON MODES**

The action is invariant under the foliation-preserving \textit{Diff},

\[
\delta x^i = -\zeta^i(t, \mathbf{x}), \quad \delta t = -f(t),
\]

\[
\delta g_{ij} = \partial_t \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij},
\]

\[
\delta N_i = \partial_t \zeta^j N_j + \zeta^j \partial_j N_i + \zeta^j g_{ij} + f \dot{N}_i + \dot{f} N_i,
\]

\[
\delta N = \zeta^j \partial_j N + f \dot{N} + \dot{f} N.
\]
Note that this Diff exists for arbitrary spacetime-dependent $N, N_i, g_{ij}$. This implies that the equations of motion by varying $N, N_i, g_{ij}$ are all the “local” equations as in the usual Lorentz invariant Einstein gravity (see [16] for the detailed discussions). Generally, it seems that there are two gauge inequivalent classes of Hořava gravity, i.e., projectable and non-projectable versions, depending on whether $N$ is a function of $t$ only or not. However, from the recovery of GR in IR, which could be problematic in the projectable version, as mentioned in the footnote No.1, I only consider the non-projectable version in this paper. This is in the same spirit as in the previous work on $\lambda \neq 1/3$.

In order to study graviton modes, I need to consider perturbations of metric around some appropriate backgrounds, which are solutions of the full theory (14). Here, I consider only the perturbations around Minkowski vacuum, which is a solution of the full theory (14) in the limit of $\Lambda \to 0$, for simplicity,

$$g_{ij} = \delta_{ij} + \epsilon h_{ij}, \quad N = 1 + \epsilon n, \quad N_i = \epsilon n_i$$  \hspace{1cm} (21)

with a small expansion parameter $\epsilon$.

From the extrinsic curvatures under the perturbations (21),

$$K_{ij} = \frac{\epsilon}{2} \left( \dot{h}_{ij} - \partial_i n_j - \partial_j n_i \right) + \mathcal{O}(\epsilon^2),$$

$$K = \frac{\epsilon}{2} \left( \dot{h} - 2 \partial_i n^i \right) + \mathcal{O}(\epsilon^2)$$  \hspace{1cm} (22)

with $h \equiv \delta^{ij} h_{ij}$, the kinetic part $S_K = \int dt d^3x \sqrt{g} N^2 \frac{\kappa^2}{2} \left( K_{ij} K^{ij} - \frac{1}{3} K^2 \right)$ becomes, at the quadratic order,

$$S_K = \int dt d^3x \frac{\epsilon^2}{2 \kappa^2} \left( \dot{h}_{ij} \dot{h}^{ij} - \lambda \dot{h}^2 - n_i \mathcal{H}^{(i)}_i \right),$$

where

$$\frac{\epsilon}{\kappa^2} \mathcal{H}^{(i)}_i \equiv -2 \epsilon \frac{\kappa^2}{\kappa^2} \left[ \partial_t \left( \partial_j h^{ij} - \frac{1}{3} \delta^{ij} \partial_j h \right) - \frac{1}{3} \partial^i \partial_j n^j - \partial^2 n_i \right] \approx 0$$  \hspace{1cm} (24)

are the momentum constraints at the linear order of $\epsilon$.

On the other hand, the Diff (20) reduces to (see [4, 8] for comparisons)

$$\delta x^i = -\epsilon \xi^i (t, x), \quad \delta t = -\epsilon g(t),$$

$$\delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i,$$

$$\delta n_i = \dot{\xi}_i, \quad \delta n = \dot{g}.$$  \hspace{1cm} (25)

Here, one can choose, by taking time-independent spatial Diff, $\xi^i = \xi^i (x)$,

$$n_i = 0$$  \hspace{1cm} (26)

but this does not mean the absence of the momentum constraints $\epsilon \mathcal{H}^{(i)}_i \approx 0$ [16]. In this case, one can choose the Hořava’s gauge [1, 2] for the perturbed metric $h^{ij}$,

$$\partial_j h^{ij} - \frac{1}{3} \delta^{ij} \partial_j h = 0,$$  \hspace{1cm} (27)
which is time independent, according to the momentum constraints (24). Then, the transverse field

\[ \tilde{H}_{ij} \equiv h_{ij} - \frac{1}{3} \delta_{ij} h, \quad \partial_i \tilde{H}_{ij} = 0 \]  

may be introduced. Note that \( \tilde{H}_{ij} \) is traceless already, i.e., \( \tilde{H}^i_i = 0 \).

From these, one obtains

\[ h_{ij} = \tilde{H}_{ij} + \frac{1}{3} \delta_{ij} h \]  

with the transverse traceless part \( \tilde{H}_{ij} \). Then the kinetic part (23), the quadratic order of \( \epsilon \), becomes [1, 2]

\[ S_K = \frac{\epsilon^2}{2\kappa^2} \int dt d^3x \; \dot{\tilde{H}}_{ij} \dot{\tilde{H}}^{ij}. \]  

Here, it is important to note that there is no kinetic term for the scalar mode \( h \), as in the \( \lambda = 1 \) case [16]. So, there is no physical excitation for the scalar mode at the conformal point \( \lambda = 1/3 \) also.

From the intrinsic curvatures\(^2\) under the perturbations [21],

\[ R^{(3)}_{ij} = \frac{\epsilon}{2} \left( \partial^k \partial_i h_{jk} - \partial^k \partial_j h_{ik} - \partial^k \partial_j h_{ik} - \partial^k \partial_j h_{ik} \right) + R^{(NL)}_{ij}, \]

\[ R^{(3)} = \epsilon \left( \partial_k \partial_i h^{ik} - \partial^2 h \right) + O(\epsilon^3), \]  

the potential part which is second order in the (spatial) derivatives in the flat limit \( \Lambda \to 0 \),

\[ S_{V(2)} = \int dt d^3x \; \sqrt{g} N \frac{\kappa^2 \mu^2 \hat{\omega}}{8} R^{(3)} \]  

becomes

\[ S_{V(2)} = -\frac{\epsilon^2 \kappa^2 \mu^2 \hat{\omega}}{8} \int dt d^3x \; \left[ \frac{1}{4} h_{ij} \left( -\partial^2 h^{ij} + 2 \partial^k \partial^i h^{jk} - 2 \partial^i \partial^j h + \delta^{ij} \partial^2 h \right) - n \mathcal{H}'(\epsilon) \right], \]  

where

\[ \epsilon \mathcal{H}'(\epsilon) \equiv -\epsilon \partial_k (\partial_i h^{ik} - \partial^k h) \approx 0 \]  

is the Hamiltonian constraint at the linear order of \( \epsilon \). Here, I have used [14, 16]

\[ \sqrt{g} R^{(3)} = \delta^{ij} R^{(NL)}_{ij} + \epsilon h_{ij} \left( -R^{ij(L)}_{ij} + \frac{1}{2} \delta^{ij} R^{(L)} \right) + O(\epsilon^3) \]

\[ = \frac{\epsilon}{2} h_{ij} \left( -R^{ij(L)}_{ij} + \frac{1}{2} \delta^{ij} R^{(L)} \right) + O(\epsilon^3), \]  

where \( R^{(L)}_{ij}, R^{(NL)}_{ij} \) denote the linear, non-linear perturbations of \( R^{(3)}_{ij} \), respectively. The action (32), when combined with the Hořava’s gauge (27), reduces to

\[ S_{V(2)} = \frac{\epsilon^2 \kappa^2 \mu^2 \hat{\omega}}{8} \int dt d^3x \; \left[ \frac{1}{4} h_{ij} \partial^2 h^{ij} - \frac{5}{36} h \partial^2 h + n \mathcal{H}'(\epsilon) \right]. \]  

\(^2\) I follow the conventions of Wald [23].
On the other hand, the Hamiltonian constraint (33), when combined with the gauge fixing condition (27), reduces to

$$\mathcal{H}^i_{(c)} = \frac{2}{3} \partial^2 h \approx 0.$$  (36)

From the mode decomposition, the second-order spatial derivative action (35) becomes

$$S_{V(2)} = \omega^2 \kappa^2 \mu^2 \hat{\omega} \int dtd^3x \left[ \frac{1}{4} \tilde{H}_{ij} \partial^2 \tilde{H}^{ij} - \frac{2}{36} h \partial^2 h + n\mathcal{H}^i_{(c)} \right].$$  (37)

Then the second-order derivative action becomes altogether

$$S_{(2)} = \omega^2 \int dtd^3x \left[ \frac{1}{2\kappa^2} \tilde{H}_{ij} \frac{\dot{\tilde{H}}}{2} + \frac{\kappa^2 \mu^2 \hat{\omega}}{32} \tilde{H}_{ij} \partial^2 \tilde{H}^{ij} - \frac{2}{9} \kappa^2 \mu^2 \hat{\omega} h \partial^2 h + \frac{\kappa^2 \mu^2 \hat{\omega}}{16} n\mathcal{H}^i_{(c)} \right].$$  (38)

The first two terms represent the usual transverse traceless graviton modes $\tilde{H}_{ij}$ with the speed of gravitational interaction

$$c_g = \sqrt{\frac{\kappa^4 \mu^2 \hat{\omega}}{16}},$$  (39)

which agrees with the speed of light $c$ in (17) and here it is important to note that the propagation can exist due to the IR modification term with an arbitrary coefficient $\hat{\omega}$, as in the non-conformal cases [4, 16]. The next two terms seem to imply the spatial modulations of the scalar mode $h$ but this mode is completely disappeared in the physical subspace of the Hamiltonian constraint (36).

The UV behaviors are governed by the higher derivative terms in (14) and the quadratic part of the perturbed action is

$$S_{(UV)} = \frac{\epsilon^2}{4} \int dtd^3x \left[ -\bar{a} \tilde{H}_{ij} \partial^6 \tilde{H}^{ij} + \bar{b} \epsilon^{ijk} \tilde{H}_{ij} \partial^4 \tilde{H}^{ik} + \bar{c} \tilde{H}_{ij} \partial^4 \tilde{H}^{ij} + \frac{2\bar{c}}{27} h \partial^4 h \right],$$  (40)

where

$$\bar{a} = -\frac{\kappa^2}{2\nu^4}, \quad \bar{b} = \frac{\kappa^2 \mu}{2\nu^2}, \quad \bar{c} = -\frac{\kappa^2 \mu^2}{8}$$  (41)

are the coefficients of $C_{ij} C^{ij}$, $\epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}$, and $(R^{(3)}_{ij} R^{(3)ij} - \frac{1}{3} R^{(3)} R^{(3)})$, respectively. The first three terms provide the modified dispersion relation $\omega^2 \sim k^6 + \cdots$ for the transverse traceless modes.3 Here, the (UV) detailed balance with the particular values of the coefficients (41) do not have any role. The last term contains higher spatial derivatives of

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3 From the parity violating term, $\bar{b} \epsilon^{ijk} \tilde{H}_{ij} \partial^4 \tilde{H}^{ik}$, as in the non-conformal cases $\lambda \neq 1/3$, the asymmetry in the right and left-handed circular polarization modes $\tilde{H}_{R/L} = \frac{1}{\sqrt{2}} (\tilde{H}_{11} \pm i \tilde{H}_{12})$ propagating along $x^3$ direction ($\tilde{H}_{3i} = 0$), with the (full) dispersion relations $\omega^2_{R/L} = \frac{\kappa^2 \mu^2 \hat{\omega}}{6} h_{R/L}^2 \pm \frac{\kappa^2 \mu^2}{9} h_{R/L}^2 \pm \frac{\kappa^2 \mu^2}{9} h_{R/L}^2$, is also generated such as the gravitational waves are chiral [14, 24]. This is in contrast to the naive expectation from the non-conformal case, which gives the suppression of the chiral modes in the $\lambda \to 1/3$ limit.
the scalar mode \( h \) but this does not appear in the physical subspace again. Here, the non-existence of sixth derivative terms for the scalar mode is the result of the detailed balance in sixth order,

\[
C_{ij}C^{ij} = \alpha \nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk} + \beta \nabla_i R^{(3)}_{jk} \nabla^j R^{(3)ik} + \gamma \nabla_i R^{(3)} \nabla^i R^{(3)}
\]  

(42)

with \( \alpha = 1, \beta = -1, \gamma = -1/8 \). On the other hand, for arbitrary values of \( \alpha, \beta, \gamma \) one obtains

\[
C_{ij}C^{ij} = -\frac{\alpha \epsilon^2}{4} \tilde{H}_{ij} \partial^6 \tilde{H}^{ij} + \frac{\epsilon^2}{4} \left( \frac{2}{3} \right)^2 \left( \frac{3\alpha}{2} + \beta + 4\gamma \right) h \partial^6 h
\]  

(43)

and there are sixth derivative terms for the scalar mode \( h \). But, even in this case, these terms do not produce the propagation in the physical subspace.

IV. CONCLUDING REMARKS

In conclusion, I have constructed the Hořava model for the \( \lambda = 1/3 \) case, where the local anisotropic Weyl symmetry exists in the UV limit, in addition to the foliation-preserving \textit{Diff}, such as the lapse function \( N \) is a non-projectable function on the foliation generally. By considering the linear perturbations around the Minkowski vacuum background, I showed that the scalar graviton mode is completely disappeared in the physical spectrum, as expected, and only the usual tensor graviton modes remain. This situation is analogous to \( \lambda = 1 \), which is Lorentz invariant in the IR limit.

The existence of the UV conformal symmetry is unique to the theory with the detailed balance. This may explain the importance of the detailed balance in quantum gravity. Generally, the parameter \( \lambda \) would run in RG flows due to quantum corrections except for the fixed point where high symmetries occur. In this context, it would be quite probable that \( \lambda = 1/3 \) be the UV fixed point since there is no UV symmetry enhancement outside of \( \lambda = 1/3 \). However, it is not known yet whether this symmetry is high enough for the fixed point or not. It would be a challenging problem to check this explicitly in RG flows as in \( \lambda = 1 \), which seems to be the IR fixed point, similarly.

\textit{Note added:} After the appearance of this paper, a related papers which considered the black hole solutions appeared \cite{25, 26}. Later at the fully non-linear orders for the IR limit of Hořava gravity \cite{16}, it has been also found that the number of physical degrees of freedom is the same as GR, which implies that there is no non-perturbative generation of scalar graviton as well, in the IR limit. \cite{27}

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