Metastability in graded and step like variation of field and anisotropy of the Blume-Capel ferromagnet

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Abstract: The metastable behaviour of two dimensional anisotropic Blume-Capel ferromagnet under the influence of graded and stepped magnetic field has been investigated by extensive Monte Carlo simulation using Metropolis single spin flip algorithm. Starting from the initial perfectly ordered state, reversal of magnetisation has been studied in presence of the field. Metastable lifetime or the reversal time of magnetisation for the system having uniform anisotropy and in the presence of both graded and stepped field has been studied. Also the same has been explored for a graded and stepped anisotropic system in presence of a uniform field. Finite size effect is analyzed for the variation of reversal time with gradient of field (\(G_h\)) and a scaling relation \(\langle \tau \rangle \sim L^{-\beta} f(G_h L^\alpha)\) is found out. Spatial variation of density profile for the projection states (i.e., +1, 0 and -1) has also been studied at the moment of reversal of magnetisation. The spatial variation of the number of spin flips per site is studied. Motion of the interface (or domain wall) with the gradient of field and gradient of anisotropy are investigated and are found to follow the hyperbolic tangent like behaviours in both cases. Since, both the anisotropy and the applied field have significant impact on the metastable lifetime, an interesting competitive scenario is observed for a graded anisotropic system in graded field and a stepped anisotropic system in stepped field. A line of marginal competition has been found out for both the cases separating the regions of field dominated reversal and anisotropy dominated reversal.

Keywords: Blume-Capel model, Monte Carlo simulation, Metropolis algorithm, Magnetic anisotropy, Magnetisation reversal, Graded field, Graded anisotropy.
I. Introduction

In the technology of magnetic recording \[1,2\], the magnetisation switching time (or the reversal time) plays the crucial role, to determine the speed of recording. This prompted the theoretical and experimental researchers to study the reversal phenomena with intense care. The reversal of magnetisation and the dependence of the reversal time on the applied magnetic field were historically analysed by Becker-Döring theory \[3, 4\]. The theory was well tested \[5\] in large scale computer simulation in Ising ferromagnets using multispin coding techniques. To study the reversal mechanism and for better understanding of the microscopic spin flip, the Ising ferromagnet was chosen as the simplest model. The longevity of metastable states in kinetic Ising ferromagnet is studied \[6\] to find its dependence on the applied field and system size. The rates of growth and decay of the clusters of different sizes, have been studied \[7\] as functions of external field and temperature. The rate of nucleation of critical nuclei, its speed of expansion and the corresponding changes of free energy is related and is described by famous Kolmogorov–Johnson–Mehl–Avrami law \[8–10\].

The switching in the environment of nonuniform parameters (field, temperature etc.) was investigated recently. The heat assisted magnetisation reversal was investigated \[11\] in ultra thin films for ultra-high-density information recording media. The statistical behaviours of magnetisation reversal was studied \[12\] in random field Ising model by Monte Carlo simulation to investigate the role of quenched impurity in the switching phenomena. The reversal with nonuniform (over space) magnetic field was also investigated \[13\] and a marginally competitive line was drawn \[14\] in the plane formed by the gradient of field and the gradient of temperature. The switching of magnetisation was investigated in Ising ferromagnets by periodic pulsed \[15,16\] magnetic field and a time dependent field like spreading Gaussian \[17\]. The relaxation of Ising ferromagnet after a sudden reversal of applied magnetic field is also studied \[18\].

The above mentioned studies are basically done on Ising model. In the case of spin- 1/2 Ising model, the reversal is mainly governed by the microscopic spin flip. Due to the application of external magnetic field, the spin would flip its direction. The metastability and its lifetime is function of the temperature and the magnitude of external magnetic field. How does the reversal will mutate if the spin component \(S_z\) has a perpendicular component with respect to the direction of applied magnetic field ?

Precisely, how does the anisotropy take part in reversal mechanism in the spin-1 anisotropic Blume-Capel \[19, 20\] model ? Metastability and nucleation in the Spin-1 Blume-Capel (BC) ferromagnet was studied and found the different mechanism of transition \[21\]. The position of the tricritical point was determined by studying \[22\] the method of geometric mutual information. The dynamical phase transition was studied \[23\] in a randomly diluted single site anisotropic BC model in presence of time varying oscillating magnetic field. Recently, the role of single site anisotropy to the reversal of Blume-Capel ferromagnet was investigated \[24\] by extensive Monte Carlo simulation. The thermally activated magnetisation switching of small ferromagnetic particles driven by an external magnetic field has been investigated and interestingly a crossover from coherent rotation to nucleation for a classical anisotropic Heisenberg model, has been reported \[25\].

Recently, in the field of experimental research in magnetisation switching, the nonuniform field and temperature were introduced. Temperature gradient-induced magnetisation reversal of single ferromagnetic nanowires was studied \[26\] in \(Co_{39}Ni_{61}\). The techniques of confinement of magnetic nanoparticles (sub-100 nm) by using localised field gradients was reported \[27\] recently. Continuously graded anisotropy in single Fe-Pt-Cu composites films was incorporated experimentally to have a conclusion that an anisotropy gradient can be realized, and tailored, in single continuous
films without the need for multilayers [28].

These are the main motivation of the present study. In this article, reversal is studied (by Monte Carlo simulation) in the Blume-Capel ferromagnet having the graded field, stepped field, graded anisotropy and stepped anisotropy. The manuscript is organised as follows: the model is introduced in the next section-II, the results and analysis are reported in section-III and the paper ends with a summary in section-IV.

II. The Model and Simulation method:

The spin-1 Blume-Capel model can be described by the following Hamiltonian,

\[ H = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z + \sum_i D(i) (\sigma_i^z)^2 - \sum_i h(i) \sigma_i^z \]  

(1)

where \( \sigma_i^z \) is the component of spin along z-direction at i-th lattice site and can take values +1, 0 and -1. The first sum is restricted to the interaction between nearest neighbour spins only with uniform ferromagnetic interaction coefficient \( (J > 0) \). Second term considers the effect of single site anisotropy (or magneto-crystalline anisotropy) arising from crystal structure. \( D(i) \) is the strength of the anisotropy experienced at i-th lattice site in the system. Third term tells us about the Zeeman energy involving the interaction of externally applied magnetic field (in z direction) with each individual spin. We have simulated a two dimensional ferromagnetic square lattice of size \( L \times L \) with open boundary condition on the both directions. In this paper we have studied some behaviour of the system in presence of two different kinds of both the field and anisotropy.

- **Graded field and graded anisotropy:** A gradient of field is applied here throughout the lattice sites (from left edge to right edge) along x-direction. So the form of the field is simply taken as,

\[ h(x) = G_h \ast x + b \]  

(2)

where \( h(x) \) varies only along the x direction i.e. for a fixed \( x \), lattice sites along y direction are equifield sites. \( G_h = \frac{dh}{dx} \) is the gradient of the field. So if a field of strength \( h_l \) is applied at the left boundary of the lattice and \( h_r \) at the right boundary, then the lattice experiences a gradient of field \( G_h = \frac{h_r - h_l}{L} \) since \( h(x) = b = h_l \) at \( x = 0 \) (left boundary) and \( h(x) = h_r \) at \( x = L \) (right boundary). In a similar way the gradient of anisotropy is also considered here,

\[ D(x) = G_D \ast x + c \]  

(3)

where \( D(x) \) varies along the x direction i.e. for a fixed \( x \), lattice sites along y direction are equianisotropic sites. So the gradient of the anisotropy of the system is \( G_D = \frac{D_r - D_l}{L} \) where \( D_l \) is the anisotropy set at the left boundary of the lattice and \( D_r \) is at right. \( D(x) \) is always taken positive throughout our work.

- **Stepped field and stepped anisotropy:** In that case, the field acts in a stepped way where half of the lattice sites along x direction are influenced by a field \( h_{sl} \) (at left side) and another half are influenced by \( h_{sr} \) (at right side).

\[ h(x) = h_{sl} \quad \text{for} \quad 1 \leq x \leq \frac{L}{2}, \forall y \]  

(4)
$$= h_{sr} \quad \text{for} \quad \frac{L}{2} < x \leq L, \forall y$$

Similarly as graded field, for a fixed $x$ all the $h(x)$ along $y$ direction are same. **Step difference** $(S_h)$ is simply calculated by, $S_h = |h_{sr} - h_{sl}|$. In the same way, stepped anisotropy is,

$$D(x) = D_{sl} \quad \text{for} \quad 1 \leq x \leq \frac{L}{2}, \forall y$$

$$= D_{sr} \quad \text{for} \quad \frac{L}{2} < x \leq L, \forall y$$

if half of the lattice sites at left side have the strength of anisotropy $D_{sl}$ and half at the right side have $D_{sr}$ then **step difference** is simply $S_D = |D_{sr} - D_{sl}|$.

Initially the system is considered to be in perfectly ordered state $(T < T_c)$ where all the spins are $\sigma_i^z = +1 \ \forall i$. Lets discuss the way lattice is updated (by random updating scheme). A lattice site (i-th say) has been chosen randomly. Say initially the spin at that site is $\sigma_i^z(\text{initial})$. The updated spin state may be any of the three states (+1, 0 and -1) which has been determined with equal probability in random way. Let the final state be $\sigma_i^z(\text{final})$. Now whether the spin $\sigma_i^z(\text{initial})$ will be updated to the state $\sigma_i^z(\text{final})$ ultimately that is decided by the Metropolis transition probability [29],

$$P(\sigma_i^z(\text{initial}) \rightarrow \sigma_i^z(\text{final})) = \text{Min}[1, e^{-\Delta H/k_BT}]$$

where $\Delta H$ is the change in energy due to the change in spin state from $\sigma_i^z(\text{initial})$ to $\sigma_i^z(\text{final})$. $k_B$ is the Boltzmann constant and $T$ is the temperature of the system measured in the unit of $J/k_B$. For simplicity, $J$ and $k_B$ has been taken as $J = 1$ and $k_B = 1$. $P(\sigma_i^z(\text{initial}) \rightarrow \sigma_i^z(\text{final}))$ is determined by calculating $\Delta H$ from the Hamiltonian and then compared it to a called random number $r_n$ (uniformly distributed in the range $[0:1]$). The final state $\sigma_i^z(\text{final})$ is accepted if $P \geq r_n$ otherwise the system is considered to be in initial state $\sigma_i^z(\text{initial})$. In this way total $L^2$ number of randomly chosen lattice sites are updated and thus one Monte Carlo Step per Spin (MCSS) is completed which acts as the unit of time throughout the whole study. At each MCSS, the magnetisation of the system is determined by,

$$m(t) = \frac{1}{L^2} \sum_{i} L^2 \sigma_i^z$$

$t$ is the time in unit of MCSS.

**III. Simulational Results**

At $T < T_c$ starting from the perfectly ordered state $(m(t) = 1, \text{all spins are +1})$, variation of magnetisation with time is studied (fig.IIa) under the influence of three different kinds of external field applied in the opposite direction. An anisotropic system having a strength of anisotropy $D = 1.6$ is considered here. In presence of a uniform field, spins try to flip along the direction of the applied field resulting decrement in $m(t)$. Lets define the time taken by the system by which magnetisation starts to acquire negative value $(m(t) \approx 0)$ as the **reversal time or metastable**
presence of a graded field. A gradient of field $G$ along the right edge of the lattice by applying the field $h_l = -0.8$ at the left boundary and $h_r = -0.1$ at the right boundary of the lattice. Since we are dealing with negative field, in presence of such $G$, the field strength (or magnitude of the field) actually decreases from $h_l = -0.8$ gradually along the right side (positive x-direction). So obviously the reversal time rises (blue line) in that case in comparison to the case for uniform field. In presence of stepped field (green curve), $h_{sl} = -0.8$ (taken same as $h_l$ as considered in the case of graded field) is applied to the half of the lattice sites (left side) and $h_{sr} = -0.1$ (taken same as $h_r$ as considered in the case of graded field) is applied to the right half. For the presence of $h_{sr} = -0.1$ at the right half lattice sites, definitely the reversal time will be higher than that for uniform field. In addition, we have also noticed that $\tau$ for stepped field is a little bit smaller compared to that for graded field.

In fig 1b image-plot of the two types of applied magnetic field used in the fig.1a has been illustrated. Left imageplot of fig 1b reflects the presence of a graded field having gradient $G_h = 0.0028$ ($h_l = -0.8; h_r = -0.1$) along the right side. Right imageplot of fig 1b reflects the presence of a step of the applied field. Left half lattice sites are excited by $h_{sl} = -0.8$ (black colour) and the right half sites are excited by $h_{sr} = -0.1$ (yellow colour).

Let's focus on the spin reversal mechanism responsible for the reversal of magnetisation in presence of a graded field. A gradient of field $G_h = 0.0028$ ($h_l = -0.8; h_r = -0.1$) acting towards right boundary is applied to the system. Now if we consider a micro-lattice of size $10 \times 10$ at the top left (named ‘A’) and top right corner (named ‘B’) of the main lattice then the field at the left edge of the microlattice A is $(h_l)_A = -0.8$ and that at right edge is $(h_r)_A = -0.772$. Same for the microlattice B will be $(h_l)_B = -0.128$ and $(h_r)_B = -0.1$. So the effective field acting on such microlattices can be assumed as almost uniform field. In microlattice-A (fig 2a), due to the strong field, reversal occurs through the formation of many clusters of spin ‘-1’ (multi-droplet regime according to the Becker-Doring analysis of classical nucleation [4]) results in relatively shorter $\tau$. Comparatively for microlattice-B, (fig 2b) reversal takes place by the growth of a single droplet (nucleation or single droplet regime) of spin ‘-1’ causing a relatively higher $\tau$. So when applying such a graded field to the lattice, spins have greater tendency to get flipped on the left side first.

We have studied the variation of average reversal time $\langle \tau \rangle$, obtained from randomly different 1000 samples, with the strength of the gradient of field $G_h$ directed towards right edge. At the left boundary applied field is kept fixed at $h_l = -0.8$ while that at right boundary is varied from $h_r = -0.7$ to $+0.7$ to vary the $G_h$. $\langle \tau \rangle$ is found to increase exponentially (fig 3a) with the increase in $G_h$ and also the presence of a crossover (in the rate of exponential growth) is noticed. Actually whenever the $h_r$ is set positive to produce a stronger $G_h$, some of the lattice sites near right edge do not participate in the reversal resulting a higher reversal time. That seems to be the reason of the appearance of the crossover. $\langle \tau \rangle$ is also studied (fig 3b) with the step difference $S_h$ of the applied stepped field and it follows a straight line $\langle \tau \rangle \sim 15.98 S_h$. Step difference is varied here by keeping the field at the left half sites fixed at $h_{sl} = -0.8$ and varying at right half from $h_{sr} = -0.7$ to $-0.1$. The linear variation of $\langle \tau \rangle$ is probably the reflection of the regular modulation of $h_{sr}$. Same study has been explored over the graded and stepped anisotropic system under the influence of a uniform field. In presence of a gradient of anisotropy (fig 3c) acting towards right edge, reversal time decreases exponentially $\langle \tau \rangle \sim e^{-243 G_D}$ with the increase in $G_D$. Though the constant factor (243) is quite large in the exponential, we have to keep in mind the value of $G_D$, which is very small. In stepped anisotropic system (fig 3d), variation of $\langle \tau \rangle$ with $S_D$ follows a stretched exponential $\langle \tau \rangle \sim e^{-1.7 (S_D)^{0.5}}$. 

\[ \text{lifetime } \tau \text{ of magnetisation of the system. Black arrows denote the reversal time for each case. Red curve presents the evolution of } m(t) \text{ in presence of a uniform field of strength } h = -0.8 \text{ interacting with the spin at each lattice site. Now a small gradient of field } G_h = 0.0028 \text{ is generated along the right edge of the lattice by applying the field } h_l = -0.8 \text{ at the left boundary and } h_r = -0.1 \text{ at the right boundary of the lattice. Since we are dealing with negative field, in presence of such } G_h, \text{ the field strength (or magnitude of the field) actually decreases from } h_l = -0.8 \text{ gradually along the right side (positive x-direction). So obviously the reversal time rises (blue line) in that case in comparison to the case for uniform field. In presence of stepped field (green curve), } h_{sl} = -0.8 \text{ (taken same as } h_l \text{ as considered in the case of graded field) is applied to the half of the lattice sites (left side) and } h_{sr} = -0.1 \text{ (taken same as } h_r \text{ as considered in the case of graded field) is applied to the right half. For the presence of } h_{sr} = -0.1 \text{ at the right half lattice sites, definitely the reversal time will be higher than that for uniform field. In addition, we have also noticed that } \tau \text{ for stepped field is a little bit smaller compared to that for graded field.} \]

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Since we are dealing with a lattice of small finite size \((L = 250)\), the system must suffer from the finite size effect. For that, variation of \(\langle \tau \rangle\) with \(G_h\) has been investigated (fig.5b) for 4 different system sizes (data in the plot are simply joined). The system exhibits same kind of behaviour for each case. Additionally \(\langle \tau \rangle\) shows a scaling relation \(\langle \tau \rangle \sim L^{-\beta} \sigma(G_hL^\alpha)\) where all the data for different lattice sizes are collapsed (fig.5b) to a single plot by the exponents \(\alpha \approx 1.06\) and \(\beta = 0\). The scaled plot is now fitted to the exponential function separately in two regions. Some snapshots (fig.6) of a single sample (of size \(L = 100\)) at \(\tau\) are taken for 4 different values of \(G_hL^\alpha\) from which it can be inferred that the crossover appears when the interface becomes smoother.

Some snapshots of the lattice of an anisotropic \((D = 1.6)\) system are taken at the reversal time \(\tau\) in presence of uniform field and graded field. Under uniform field (fig.7a), all the spins \(1\), \(0\) and \(-1\) are uniformly distributed throughout the lattice. As we apply some graded field (fig.7b,c,d), spins on left side try to flip first due to stronger field. That is why spin \(-1\) get accumulated strongly on left side with the increase in strength of \(G_h\). As a result, an interface (or domain wall) appears between the two regions (left region is dominated by spin \(-1\) whereas the right by spin \(1\)).

To have a clear idea on spin-dynamics in an anisotropic system to achieve reversal, spatial variation (along \(x\) direction only) of the density of spin \(1, 0\) and \(-1\) \((\rho_1, \rho_0\) and \(\rho_{-1}\)) has been checked out. To study this, the lattice has been divided into 25 strips (each strip of size \(10 \times 250\)) along \(x\)-direction. In each strip \(\rho_{-1}\) has been obtained at the time of reversal by calculating total number of spin \(-1\) in a strip and dividing it by the total number of spins contained in the strip. And finally the data are averaged over 1000 random samples. \(\rho_1\) and \(\rho_0\) are also calculated in the same fashion. In presence of uniform field (fig.8a), since the impact of the field on each lattice site is equal, density of all the spins remain spatially uniform throughout the lattice. And \(\rho_0\) is higher than \(\rho_1\) or \(\rho_{-1}\) because of the strength of the anisotropy (positive) present in the system. When applying a graded field (fig.8b), obviously there is a spatial variation of the applied field throughout the lattice. Stronger field on left side compels spins to get flipped from \(1\) or \(0\) state to \(-1\) state which rises \(\rho_{-1}\) and reduces both \(\rho_0\) and \(\rho_{-1}\) on that side. Whereas on right side, due to the weaker field, most of the spins remain in spin \(1\) state which results in a high \(\rho_1\) and low \(\rho_0, \rho_{-1}\). That is why \(\rho_{-1}\) decreases from a higher value at left edge to a lower value at right edge. A contrasting nature of \(\rho_1\) is observed. At the middle position of the lattice (around \(x_i \approx 125\)), \(\rho_1\) and \(\rho_{-1}\) become equal. In addition \(\rho_0\) bends near the two edges i.e. \(\rho_0\) becomes almost same in the region of strong or weak field. So actually the effect of anisotropy becomes insensitive to the strength of the field. As the \(G_h\) is increased (fig.8c,d) by reducing the magnitude of field \(h_r\), the ability of spin-flip on right side goes down. Then most of the spins from left side take part in reversal. As a result, a sharp fall of \(\rho_{-1}\) as well as a sharp rise of \(\rho_1\) near the middle of the lattice is found out. In the same way, such kind of density profile is also investigated for stepped field. When a stepped field of small step difference (fig.9a) is applied, \(\rho_0\) remains almost same as that in case of uniform field. \(\rho_{-1}\) gets higher at left half side for stronger \(h_{sl}\) and lower at right for weaker \(h_{sr}\). In case of large \(S_h\) (fig.9b), \(\rho_0\) is reduced because most of the spins at left half side participate in reversal since \(h_{sr}\) is very weak. So a similar kind of behaviour as in case of graded field is also observed here. \(\rho_0\) remains almost spatially uniform in spite of the presence of a step difference in field. Distortion near edges of the lattice in each plot is the reflection of the open boundary condition.

Spatial variation of number of spin flip up to the reversal time per lattice site \(\langle n_f \rangle\) is also investigated (fig.10) in the same way described above by considering the lattice composed of some vertical narrow strips. Total number of spin flip \(N_f\) (here considering the flipping \(\sigma_i^z = +1\) to \(-1\) only) up to \(\tau\) is calculated in each strip. Then dividing it by the total number of sites in each strip
we get the \( n_f = N_f/site \). \( n_f \) is also finally averaged over 1000 random samples. In presence of uniform field (fig-10a) it remains spatially uniform. But under the graded field it becomes higher in the left side compared to the right side (fig-10b,c). When the \( G_h \) is quite strong, a peak of the \( n_f \) is observed near the interface (fig-10a). Actually for stronger gradient spins near the left boundary flip very earlier and remains stable whereas the flipping possibility near the right boundary is very less due to the weak field. In between these two regions, spins try to become stable by balancing the effect of two regions which causes higher flipping near the interface. In presence of a stepped field (fig-11), \( n_f \) is almost uniform having a higher value in the left half lattice. In comparison it remains uniform in the right half with a lower value since left half of the lattice is under the influence of a stronger field than the right half. Same study (fig-12) has been explored over a graded anisotropic system (for 2 different strength of \( G_D \)) in presence of a uniform field \( h = -0.8 \). The strength of \( D \) at the left edge is set to \( D_l = 0.4 \) and that at the right edge is varied \( (D_r > D_l) \) to vary the \( G_D \). Since the strong anisotropy accelerates the spin-flip (causes reduction in \( \tau \)), the number of spin flip rises towards right side. Similarly for the system having stepped anisotropy in presence of a uniform field (fig-13), \( n_f \) remains uniform in the right half lattice with a higher value than the left side due to the stronger anisotropy at the right half. The finite discontinuity in the number of spin flips is obvious. Distortions near edges are present in each plot since we are dealing with open lattice.

Let’s study about the behaviour of the interface (or domain wall) appearing under graded field or graded anisotropy. For the system possessing uniform anisotropy under a particular gradient of field, average position of the interface is determined. From the snapshots (fig-14), clearly at the time of reversal, the left side of the lattice is dominated by the spin `-1’ and the right side by spin `+1’. Now to determine the position of the interface, position of each point on the interface should be found out first. To do so, we have checked the 10 nearest neighbour spins on both sides of each lattice site. The site for which the number of spin `-1’ on left side is almost equal to the number of spin `+1’ on the right side is considered as a point on the interface. Thus all the 250 points are found out for whole lattice and averaged to get the average position of the interface \( \langle x_{if} \rangle \). Then it is obtained for 1000 samples and averaged to get final \( \langle x_{if} \rangle \) whose variation with the \( G_h \) follows a tangent hyperbolic function \( \langle x_{if} \rangle \simeq 124.3 \tanh (418.2 \ G_h) \) (fig-14a). As the \( G_h \) is increased, the interface shift towards right and finally get fixed near the middle of the lattice. Stronger gradient reduces the roughness of the interface and follows the exponential curve \( \sigma(x_{if}) \sim e^{0.12 \ G_h^{0.5}} \) (fig-14b). Roughness of the interface is determined simply by taking the standard deviation of the points on the interface and also averaged over 1000 samples. In same fashion, above study has been analyzed for a graded anisotropic system (fig-15a) in a uniform field, where also \( \langle x_{if} \rangle \) varies as a tangent hyperbolic function of \( G_D \), \( \langle x_{if} \rangle \simeq 124.9 \tanh (328.6 \ G_D) \) and the roughness (defined as the standard deviation of the interfacial positions) \( 13 \) of the interface decreases exponentially with the \( G_D \) (fig-15b), which follows \( \sigma(x_{if}) \sim \exp(-299.8 \ G_D) \).

Since the reversal time of a system is largely dependent on the applied field and also on the anisotropy of the system, an interesting study (to incorporate the competitive behaviour of graded field and graded anisotropy) can be worked out on the behaviour of the system possessing graded anisotropy under the influence of a graded field. A gradient of field \( G_h = 0.0016 \) towards right edge is set up by applying \( h_l = -0.8 \) and \( h_r = -0.4 \). So higher field near the left edge will try to flip the spins. In contrast if we take a graded anisotropic system having anisotropy \( D_l = 0.4 \) and that at right edge is set to some higher value \( (D_r > D_l) \), then the stronger anisotropy near right edge will also try to flip the spins. As a result, there would be a competitive scenario regarding reversal influenced by both field and anisotropy simultaneously. To explore this competitive behaviour
quantitatively, a competition factor [14] can be defined as,

\[ C_F = \frac{\left( \sum_{i=1}^{L^2} R_i \ast \sigma_i \right) + L^2}{2L^2} \]  \hspace{1cm} (8)

where \( R_i \) is the spin at i-th lattice site of a reference lattice [14] whose left half sites are filled with spin ‘-1’ only and the right half by spin ‘+1’. \( \sigma_i \) is the spin at i-th lattice site of the main lattice we are studying. Now obviously at \( \tau \), if the left side of the main lattice is dominated by spin ‘-1’ and the right by spin ‘+1’ (field dominated reversal) then \( \left( \sum_{i=1}^{L^2} R_i \ast \sigma_i \right) \to L^2 \), so \( C_F \to 1.0 \). On the other hand if the left side of the lattice is dominated by spin ‘+1’ and the right by spin ‘-1’ (anisotropy dominated reversal) then \( \left( \sum_{i=1}^{L^2} R_i \ast \sigma_i \right) \to -L^2 \), so \( C_F \to 0.0 \). If the small clusters of spin ‘1’ and ‘-1’ are equally and randomly distributed throughout the lattice then \( \left( \sum_{i=1}^{L^2} R_i \ast \sigma_i \right) \to 0 \), so \( C_F \to 0.5 \). Keeping the \( G_h \) fixed, some snapshots are taken (fig-16) for a single sample at the time of reversal for 3 different \( G_D \) (acting competitively with applied field). In fig-16a reversal is clearly dominated by graded field where \( C_F > 0.5 \) whereas in fig-16b reversal is dominated by the anisotropy where \( C_F < 0.5 \). But in the fig-16c field and anisotropy equally participate in reversal possessing \( C_F \approx 0.5 \) which is set as the benchmark of marginal competition. Now the value of \( C_F \) is averaged over 100 samples and its variation with time (fig-17) is also checked for the 3 cases. The arrow denotes the reversal time for each case (\( C_F \) need not to be maximum at \( \tau \)). \( C_F \) is almost constant (\( \approx 0.5 \)) with time for equally competing field and anisotropy. By determining \( C_F \) at \( \tau \), the value of equally competing \( G_D \) for several \( G_h \) are found out by simple trial-and-error method and studied the variation (fig-18) of them. It follows a straight line \( G_D \sim 1.88 \, G_h \). We can call this line as the ‘line of marginal competition’ below which field plays dominating role in reversal and above which anisotropy plays dominating role. Same study (fig-19 20) have been explored over a stepped anisotropic system in presence of a stepped field where equally competing step differences \( S_D \) and \( S_h \) are plotted. It also follows a straight line \( S_D \sim 2.04 \, S_h \).

What will be the temporal behaviours of metastable volume fraction? Avrami’s law [10] regarding the decay of metastable volume fraction has been investigated here for the anisotropic system in presence of 3 different kinds of field. According to this law, for a d-dimensional system (closer to the critical temperature \( T = 0.8 \, T_c \) here) metastable volume fraction (relative abundance of \( \sigma_i^z = +1 \)) decays exponentially with time \( t^{d+1} \). So here (fig-21) the logarithm of the metastable volume fraction (\( \ln N/t \), \( N_i \) is the number of spin ‘+1’ and \( N \) is the total number of spin in the system) has been studied as function of the third power of dimensionless time \( (1/T)^3 \) at temperature \( T = 0.8 \) [30].

IV. Summary

In this article, the behaviours of metastable lifetimes and the switching of magnetisation in the anisotropic Blume-Capel ferromagnet has been studied by extensive Monte Carlo simulation using Metropolis single spin flip algorithm. The switching mechanism is governed generally by the application of uniform external magnetic field. How does the switching behave in the spatially modulated external magnetic field and the anisotropy of the system, is the main objective of this study.

In the first part, the two different kinds of spatial modulation (namely graded and stepped like) of external magnetic field are considered with uniform anisotropy of the system. The reversal
time (or switching time) was found to increase in the case of graded field as compared to that for uniform field. In the case of graded field a major portion of the lattice experiences weaker magnetic field which plays the role of such delayed reversal. Here, an interesting phenomenon was observed in support of classical nucleation theory (or Becker-Doring theory). Considering two small parts of the whole lattice where the fields are relatively weaker and relatively stronger. In the microlattice where the field is relatively weaker, the growth of single nucleating cluster was observed. On the other hand, in the microlattice where the applied field is relatively stronger, the coalescence of multiple droplets was observed. In the case of graded field the mean reversal time was found to follow the $<\tau>\sim e^{bGh}$. A crossover in the value of the rate ($b$) of this exponential growth was observed from the lower value of the gradient to higher value of the gradient of the external magnetic field. In the case of stepped magnetic field, a linear growth of mean reversal time was observed. The dependences of the mean reversal time on the system size was found to obey a scaling relation $<\tau>\sim L^{-\beta}f(GhL^\alpha)$ with $\alpha \simeq 1.06$ and $\beta \simeq 0.62$.

The nonuniform spatial variation of the mean densities of spin projections was observed in the case of graded magnetic field. The density of $+1$ was found to decrease from left to right in contrary to that of $-1$. The density of 0 gets peaked nearly in the middle.

The mean numbers of spin flip also shows a nonuniform spatial variation in the case of graded magnetic field. Interestingly, for higher gradient this shows a very sharp peak in the middle. In the case of stepped field, this shows a finite discontinuity in the middle, clearly reflecting the stepped discontinuity of the applied magnetic field.

The mean position and the roughness of the domain wall was studied as function of the gradient of the applied magnetic field. In the case of graded field, the mean position of the domain wall was observed to shift towards right in a hyperbolic tangent fashion with the value of the gradient and the roughness of the interface of the domains was found to decay in a stretched exponential manner.

The role of spatial modulation of the anisotropy of the system was also studied by using graded and stepped anisotropy where the external magnetic field was uniform over the space. In the case of graded anisotropy, the mean reversal time $<\tau>$ was found to decrease exponentially with the gradient ($G_D$) of the anisotropy. However, in the case of step like variation of the anisotropy, it was found to decay in a stretched exponential manner ($<\tau>\sim e^{-x_{0.5}}$).

Mean numbers of spin flips were found to increase from left to right side of the lattice in the case of graded anisotropy and it shows finite discontinuity in the middle for the case of stepped anisotropy.

The mean position and the roughness of the domain wall was studied as function of the gradient of the anisotropy. In the case of graded anisotropy, the mean position of the domain wall was observed to shift towards right in a hyperbolic tangent fashion (like the case of graded field) with the value of the gradient and the roughness of the interface of the domains was found to decay exponentially (unlike the case of graded field).

Having the knowledge of the reversal of magnetisation by graded field and graded anisotropy, the joint effects of both were investigated. Here, a competitive scenario was observed and the line of marginal competition was drawn in the plane of gradients of field and anisotropy. The linear boundary was found to separate the regions of field dominated reversal and anisotropy dominated reversal.

The joint effects of stepped field and anisotropy were investigated. Here also, like the case of graded variations, a competitive scenario was observed and the line of marginal competition was drawn in the plane of step discontinuities of field and anisotropy. The linear boundary was found to separate the regions of field dominated reversal and anisotropy dominated reversal.
Finally, the decay of metastable volume fraction was studied in the case of uniform, graded and stepped field to check the Avrami kind of variation [31]. In all cases the logarithmic volume fractions were found to decay as \(- (t/\tau)^{3}\) supporting Avrami behaviours. However, the decay becomes relatively slower (with smaller decay rate) after the reversal.

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Figure 1: (a) Variation in magnetisation (m(t)) with time (t) in presence of a uniform field \( h = -0.8 \) (red line, \( \tau = 12 \) MCSS), gradient of field (blue line; \( h_l = -0.8, h_r = -0.1, G_h = 0.0028, \tau = 28 \) MCSS) and stepped field (green line; \( h_{sl} = -0.8, h_{sr} = -0.1, S_h = 0.7, \tau = 23 \) MCSS). (b) Image plot of the applied Left: graded field having gradient \( G_h = 0.0028 \) (\( h_l = -0.8, h_r = -0.1 \)). Right: stepped field of step difference \( S_h = 0.7 \) (\( h_{sl} = -0.8, h_{sr} = -0.1 \)). Anisotropy of the system is \( D = 1.6 \). Temperature is kept fixed at \( T = 0.8 \).
Figure 2: (a) In presence of gradient of field $G_h = 0.0028$, snapshots of the \textbf{top left} corner (10 $\times$ 10 microlattice) of the main lattice are taken at several time steps (arranged \textbf{clockwise} starting from top left). \textit{Top left:} $\tau = 9$, \textit{top right:} $\tau = 14$, \textit{bottom right:} $\tau = 24$, \textit{bottom left:} $\tau = 32$ MCSS. Clearly the reversal occurs via the coalescence of many droplets of spin ‘-1’. (b) Under $G_h = 0.0028$, snapshots of the \textbf{top right} corner (10 $\times$ 10 microlattice) of the main lattice are taken at several time steps (arranged \textbf{clockwise} starting from top left). \textit{Top left:} $\tau = 360$, \textit{top right:} $\tau = 380$, \textit{bottom right:} $\tau = 423$, \textit{bottom left:} $\tau = 500$ MCSS. Here the single nucleating cluster grows. Anisotropy of the system is $D = 1.6$. Temperature is $T = 0.8$. 
Figure 3: (a) Variation of mean reversal time $\langle \tau \rangle$ with the strength of gradient of field $G_h$ in semi-logarithmic scale. To vary $G_h$, $h_l$ is kept fixed at $h_l = -0.8$ and $h_r$ is varied from $h_r = -0.7$ to $+0.7$. (b) Variation of reversal time $\langle \tau \rangle$ with the step difference $S_h$ of the stepped field. To vary $S_h$, $h_{sl}$ is kept fixed at $h_{sl} = -0.8$ and $h_{sr}$ is varied from $h_{sr} = -0.7$ to $-0.1$. Anisotropy of the system is $D = 1.6$. Temperature is $T = 0.8$.

Figure 4: (a) Semi-logarithmic plot of the variation of the mean reversal time $\langle \tau \rangle$ with the gradient of anisotropy $G_D$. To vary $G_D$, $D_l$ is kept fixed at $D_l = 0.4$ and $D_r$ is varied from $D_r = 0.6$ to $1.6$. (b) Variation of $\langle \tau \rangle$ with the step difference of stepped anisotropy $S_D$. $D_{sl}$ is kept fixed at $D_{sl} = 0.4$ and $D_{sr}$ is varied from $D_{sr} = 0.6$ to $1.6$ to vary $S_D$. Applied uniform field is kept fixed at $h = -0.8$. Temperature is $T = 0.8$. 
Figure 5: (a) Variation of $\langle \tau \rangle$ with $G_h$ for 4 different lattice sizes ($L = 50, 150, 250$ and $350$). Data are simply joined. Anisotropy of the system is fixed at $D = 1.6$. Temperature is $T = 0.8$. (b) Data collapse of the first plot by the exponent $\alpha = 1.06$ and $\beta = 0$ which has been fitted to the exponential function separately in 2 regions.

Figure 6: Imageplot (arranged clockwise starting from top left) of the lattice of size $L = 100$ at the time of reversal for 4 different values of scaled gradient of field $G_h L^\alpha$ where $\alpha = 1.06$ from previous scaled plot. **Top left:** $G_h L^\alpha = 1.0; \tau = 28$ MCSS, **Top right:** $G_h L^\alpha = 1.5; \tau = 73$ MCSS, **Bottom right:** $G_h L^\alpha = 1.6; \tau = 95$ MCSS, **Bottom left:** $G_h L^\alpha = 2.0; \tau = 313$ MCSS.
Figure 7: Snapshots are taken **at the time of reversal** in presence of uniform field and 3 different strengths of gradients of field (a) uniform field $h = -0.8$, $\tau = 12$ MCSS, (b) $h_l = -0.8$, $h_r = -0.3$, $G_h = 0.002$, $\tau = 21$ MCSS, (c) $h_l = -0.8$, $h_r = -0.1$, $G_h = 0.0028$, $\tau = 28$ MCSS, (d) $h_l = -0.8$, $h_r = +0.3$, $G_h = 0.0044$, $\tau = 71$ MCSS. Anisotropy of the system is $D = 1.6$. Temperature is set to $T = 0.8$. 
Figure 8: Variation of density of spin '+1' ($\rho_1$), spin '0' ($\rho_0$), spin '-1' ($\rho_{-1}$) with position along x direction ($x_i$) at the time of reversal in presence of (a) uniform field $h = -0.8$, (b) graded field, $h_l = -0.8$, $h_r = -0.3$, $G_h = 0.002$, (c) graded field, $h_l = -0.8$, $h_r = -0.1$, $G_h = 0.0028$, (d) graded field, $h_l = -0.8$, $h_r = +0.3$, $G_h = 0.0044$. Anisotropy of the system is $D = 1.6$. Temperature is set to $T = 0.8$. 
Figure 9: Variation of density of spin '+1' ($\rho_1$), spin '0' ($\rho_0$), spin '-1' ($\rho_{-1}$) with position along x direction ($x_i$) at the time of reversal in presence of stepped field (a) $h_{sl} = -0.8$, $h_{sr} = -0.6$, $S_h = 0.2$, (b) $h_{sl} = -0.8$, $h_{sr} = -0.2$, $S_h = 0.6$. Anisotropy of the system is $D = 1.6$. Temperature is set to $T = 0.8$. 
Figure 10: Spatial variation of number of spin flip (considering flipping from '+1' to '-1' only) upto reversal time per lattice site $n_f$ for the system in presence of uniform and graded field. (a) uniform field $h = -0.8$, (b) graded field $h_l = -0.8$, $h_r = -0.2$, $G_h = 0.0024$, (c) graded field $h_l = -0.8$, $h_r = 0.2$, $G_h = 0.0040$, (d) graded field $h_l = -0.8$, $h_r = 0.6$, $G_h = 0.0056$. Anisotropy of the system is $D = 1.6$. Temperature is set to $T = 0.8$. 
Figure 11: Spatial variation of number of spin flip (considering flipping from '+1' to '-1' only) up to reversal time per lattice site $n_f$ for the system in stepped field (a) $h_{sl} = -0.8$, $h_{sr} = -0.6$, $S_h = 0.2$, (b) $h_{sl} = -0.8$, $h_{sr} = -0.2$, $S_h = 0.6$. Anisotropy of the system is $D = 1.6$. Temperature is set to $T = 0.8$. 
Figure 12: Spatial variation of number of spin flip (considering flipping from '+1' to '-1' only) upto reversal time per site $n_f$ for graded anisotropic system (a) $D_l = 0.4$, $D_r = 1.0$, $G_D = 0.0024$, (b) $D_l = 0.4$, $D_r = 1.6$, $G_D = 0.0048$. Applied field is fixed at $h= -0.8$. Temperature is $T= 0.8$.

Figure 13: Spatial variation of number of spin flip (considering flipping from '+1' to '-1' only) upto reversal time per site $n_f$ for the stepped anisotropic system (a) $D_{sl} = 0.4$, $D_{sr} = 1.0$, $S_D = 0.6$, (b) $D_{sl} = 0.4$, $D_{sr} = 1.6$, $S_D = 1.2$. Applied field is fixed at $h= -0.8$. Temperature is set to $T= 0.8$. 
Figure 14: (a) Variation of average position of interface at the time of reversal $\langle x_{if} \rangle$ with gradient of field $G_h$ and (b) Variation of roughness $\sigma(x_{if})$ of interface with the $G_h$. Although the fitting parameter ‘b’ is positive here, due to the factor $x^{-0.5}$ in the exponential, roughness is decreasing as $G_h$ increases. Anisotropy is $D=1.6$. Temperature is set to $T=0.8$. To vary $G_h$, keeping $h_l = -0.8$ fixed, $h_r$ is varied from 0.0 to +0.7.

Figure 15: (a) Variation of average position of interface at the time of reversal $\langle x_{if} \rangle$ with gradient of anisotropy $G_D$ and (b) Variation of roughness of interface $\sigma(x_{if})$ with $G_D$. To vary $G_D$, keeping $D_l = 1.6$ fixed $D_r$ is varied such that $D_r < D_l$. Applied field is kept fixed at $h = -0.8$. Temperature is set to $T=0.8$. 
Figure 16: Snapshots are taken for 3 different graded anisotropic (where anisotropy act competitively with field) system at the time of reversal in presence of a fixed $G_h$. Gradient of field is fixed at $G_h = 0.0016$ ($h_l = -0.8; h_r = -0.4$). Keeping $D_l = 0.4$ fixed, $D_r$ is varied to change $G_D$. (a) $D_r = 1.0$, $G_D = 0.0024$, $\tau = 131$ MCSS, (b) $D_r = 1.35$, $G_D = 0.0038$ (for which $C_F \sim 0.5$ at the $\tau$), $\tau = 85$ MCSS, (c) $D_r = 1.6$, $G_D = 0.0048$, $\tau = 61$ MCSS. Temperature is set to $T = 0.8$.

Figure 17: Evolution of the competition factor $C_F$ (averaged over 100 random samples) with time (MCSS) for the system in fixed $G_h = 0.0016$ and 3 different $G_D$. Black arrow denotes the reversal time for each and the horizontal black line indicates $C_F = 0.5$. Temperature is set to $T = 0.8$. 

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Figure 18: Relation of competitive gradient of anisotropy $G_D$ for which $C_F \sim 0.5$ with the gradient of field $G_h$. 

$$f(x) = ax + b$$

$a = 1.88 \pm 0.05$

$b = 0.0007 \pm 0.0001$
Figure 19: Snapshots are taken for 3 different stepped anisotropic (where anisotropy act competitively with field) system at the time of reversal in presence of a fixed $S_h$. $h_{sl} = -0.8$ and $h_{sr} = -0.4$; so the step difference of the applied field is $S_h = 0.4$. (a) $D_{sl} = 0.4$, $D_{sr} = 1.0$, $S_D = 0.6$, $\tau = 123$ MCSS, (b) $D_{sl} = 0.4$, $D_{sr} = 1.32$, $S_D = 0.92$ (for which $C_F \sim 0.5$ at the $\tau$), $\tau = 86$ MCSS, (c) $D_{sl} = 0.4$, $D_{sr} = 1.6$, $S_D = 1.2$, $\tau = 51$ MCSS. Temperature is set to $T = 0.8$.

Figure 20: Relation of competitive stepped anisotropy having step difference $S_D$ for which $C_F \sim 0.5$ with step difference $S_h$ of the applied stepped field.
Figure 21: Variation of logarithmic metastable volume fraction $\frac{N_1}{N}$ ($N_1$ is the number of spin +1, N is the total number of spin) with third power of time is plotted (a) upto reversal time $t = \tau$ where data are fitted near $\tau$. (b) upto $t = 3^{1/3}\tau$ where the solid lines are the same lines as in '(a)'. Now the data are fitted in the later times which are represented by the dotted lines. Anisotropy of the system is $D=1.6$. Temperature is set to $T=0.8$. 