CONSTRANTS ON THE WARM DARK MATTER MODEL FROM GRAVITATIONAL LENSING

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ABSTRACT

Formation of subgalactic halos is suppressed in a warm dark matter (WDM) model due to thermal motion of WDM particles. This may provide a natural resolution to some puzzles in standard cold dark matter (CDM) theory such as the cusped density profiles of virialized dark halos and the overabundance of low-mass satellites.

One of the observational tests of the WDM model is to measure the gravitationally lensed images of distant quasars below subarcsecond scales. In this Letter, we report a comparison of the lensing probabilities of multiple images between CDM and WDM models using a singular isothermal sphere model for the mass density profiles of dark halos and the Press-Schechter mass function for their distribution and cosmic evolution. It is shown that the differential probability of multiple images with small angular separations down to ~10 mas should allow one to set useful constraints on the WDM particle mass. We discuss briefly the feasibility and uncertainties of this method in future radio surveys (e.g., VLBI) for gravitational lensing.

Subject headings: cosmology: theory — dark matter — galaxies: formation — gravitational lensing

1. INTRODUCTION

While cosmological models with a mixture of roughly 35% cold dark matter (CDM) and 65% vacuum energy have proved remarkably successful at explaining the origin and evolution of cosmic structures on large scales (>1 Mpc), there are a number of independent observations on small scales that seem to be in conflict with the predictions of standard CDM models, such as the cusped density profiles of virialized dark halos and the overabundance of satellite structures. Among many other mechanisms suggested for resolving these discrepancies, the simplest approach is to smear out small-scale power by free-streaming due to thermal motion of particles. Warm dark matter (WDM) is thus invoked and has recently received a lot of attention in the literature (e.g., White & Croft 2000; Colin, Avila-Reese, & Valenzuela 2000; Sommer-Larsen & Dolgov 2001; Bode, Ostriker, & Turok 2000b and references therein).

In the conventional scenario of structure formation by CDM, perturbations on small mass scales \( M \approx 4 \times 10^{11} M_{\odot} h^2 \times \Omega_\chi(R/0.1\,\text{Mpc})^3 \) are damped, relative to CDM models, where \( \Omega_\chi \) is the WDM density parameter and \( R \) denotes the characteristic free-streaming length. As a result, it suppresses the formation of small halos with \( M < M_f \) at high redshift and meanwhile reduces the overall number of low-mass halos in the universe, depending sensitively on the WDM particle mass \( m_\chi \). Observational tests of this scenario and hence robust constraints on \( m_\chi \) are restricted to the studies of less massive systems with \( M < M_f \). The existing limits in the literature are based on the Gunn-Peterson effect and the Ly\( \alpha \) forest, which set \( m_\chi > 750 \) eV (Narayanan et al. 2000). In this Letter, we examine whether the gravitational lensing of high-redshift quasars by foreground dark halos can be used to distinguish CDM and WDM models or place any meaningful constraints on \( m_\chi \). This arises because the probability of gravitational lensing would be lower in the WDM model, as a result of the suppression of formation of small-mass halos on galactic and subgalactic scales, than in the CDM model. For a massive galaxy like the Milky Way as a lens at a cosmological distance, the Einstein radius of a background quasar is typically 1\( ' \). So, it is expected that the difference in lensing properties between WDM and CDM models would occur below subarcsecond scales. We would like to demonstrate quantitatively to what angular separations the two models become to be distinguishable and how sensitively their lensing probabilities depend on the WDM particle mass.

2. LENNSING PROBABILITY

For simplicity, we use a singular isothermal sphere to model the mass distribution inside a virialized halo so that the total mass \( M \) within radius \( r \) is simply \( M = 2\pi\rho rvir r^2/3 \), where \( \rho_v = 3H^2(z)/8\pi G \) is the critical mass density of the universe, \( H(z) = 100 h E(z) \) km s\(^{-1}\) Mpc\(^{-1}\) is the Hubble constant, \( E(z) = 1 + z/(1 + \Omega_M + \Omega_{\Lambda}) \), \( \Omega_M \) denotes the curvature term, and \( \Delta \) represents the overdensity of dark matter with respect to the critical density \( \rho_c \) at the collapse redshift \( z_c \). For a dark halo at \( z \) as a lens, the multiple images of a distant quasar at \( z_s \) with an angular separation of \( \delta\theta = 2\theta_{\text{e}} \) will occur if it happens to lie in the Einstein radius of the dark halo: \( \theta_{\text{e}} = 4\pi (\sigma\chi/c)(D_{L_s}/D_s) \), in which \( D_{L_s} \) and \( D_s \) are the angular diameter distances from the lens and observer to the source, respectively. The lensing cross section for the multiple images is simply \( \pi\theta^2_{\text{e}} \). The probability that a quasar at \( z_s \) will have multiple images with an angular separation greater than \( \delta\theta \) due to all the foreground dark halos is thus

\[
P(\delta\theta) = \int_0^\infty\frac{dV}{dz}\int_{\Delta_{\text{min}}}^{\infty} \frac{\theta^2_{\text{e}}}{2} \frac{dN}{dM} dM. \tag{1}
\]

where \( dV/dz \) is the comoving volume element, and \( M_{\text{min}} \) is related to \( \delta\theta \) through

\[
M_{\text{min}} = 1.50 \times 10^{12} M_{\odot} h^{-1} \times \frac{1}{E(z)} \frac{\Delta^{3/2}}{100} \left( \frac{D_s}{D_L} \right)^{3/2} \left( \frac{\Delta}{200} \right)^{-1/2}. \tag{2}
\]

We use the Press-Schechter (PS) mass function for the abun-
dence and evolution of virialized dark halos:

\[
\frac{dN}{dMdz} = -\sqrt{2} \rho \frac{\delta(z) da}{\pi M \sigma^2} dM \exp \left( -\frac{\delta(z)^2}{2\sigma^2} \right),
\]

where \( \bar{\rho} \) is the present-day average mass density of the universe, \( \delta(z) \) is the linear overdensity of perturbations that collapsed and virialized at redshift \( z \), and \( \sigma \) is the variance of the mass density fluctuation in a sphere of mass \( M = (4\pi/3)\bar{\rho}R^3 \). It is convenient to express \( \delta(z) \) as \( \delta(z) = \delta_{\text{sd}}(1 + z)[g(0)/g(z)] \), where \( g(z) \) is the linear growth factor, for which we take the approximate formula by Carroll, Press, & Turner (1992), and the quantity \( \delta_{\text{sd}} \) has a weak dependence on \( \Omega_c \): \( \delta_{\text{sd}} = 1.6866[1 + 0.01256 \log \Omega_c(z)] \) for a flat universe with a non-zero cosmological constant (Mathiesen & Evrard 1998). Once a power spectrum \( P(k) \) is specified, the mass variance becomes

\[
\sigma^2(M) = \frac{1}{2\pi} \int_0^\infty k^2P(k)W^2(kR)dk,
\]

in which \( W(x) = 3(\sin x - x \cos x)/x^3 \) is the Fourier representation of the window function. We adopt the following parameterization of the WDM power spectrum:

\[
P(k) = Ak^nT^2_c(k)T^2_{\text{CDM}}(k),
\]

where \( n \) is the primordial power spectrum and is assumed to be the Harrison-Zeldovich case \( n = 1 \). \( T_{\text{CDM}}(k) \) is the transfer function of the adiabatic CDM model for which we use the fit provided by Bardeen et al. (1986), and \( T_c(k) \) acts as the CDM to WDM “transfer function” for which we adopt the form provided by Bode et al. (2000b): \( T_c(k) = [1 + (\alpha k)^2]^{-1.5} \), where \( \alpha = 1.2 \) and \( \alpha \) is related to the WDM particle mass \( m_x \) through \( \alpha = 0.048(\Omega_x/0.4)^{1/2}(h/0.65)^{1/2}(\text{keV}/m_x)^{1/2} \). The amplitude \( A \) is determined by equation (4) using the rms mass fluctuation on an 8 h^{-1} Mpc scale, \( \sigma_c \).

We work with a flat cosmological model where \( \Omega_m = \Omega_x + \Omega_b = 0.3 \), \( \Omega_b = 0.03 \), \( \Omega_x = 0.7 \), and \( h = 0.68 \), which fixes the shape parameter \( \Gamma = 0.1 h \exp [-\Omega_x(1 + \sqrt{2}\Omega_b)] = 0.176 \). The normalization parameter is taken from the calibration of cluster abundance, \( \sigma_8 = 0.85 \). We compute the lensing probabilities for a quasar at redshift \( z_s = 1, 2, \) and 3, respectively, and for two choices of the WDM particle mass \( m_x = 350 \text{ eV} \) and 1 keV. As a comparison, we also give the corresponding lensing probabilities for a CDM model with \( \Omega_{CDM} = 0.27 \). In order to highlight their differences, we display in Figure 1 the normalized differential probabilities \( [1/P(\Delta \theta)]dP(\Delta \theta)/d\Delta \theta \) instead of the total probabilities \( P(\Delta \theta) \).

For the latter, the typical value for a quasar at \( z_s = 2, \) with \( \Delta \theta = 1^\prime \) is nearly the same for all three models (two WDM models and the standard CDM model): \( P(>1^\prime) \approx 1 \times 10^{-5} \). This result is consistent with the pioneering statistical study of gravitational lensing by Turner, Ostriker, & Gott (1984). Noticeable departure of the lensing probability in WDM models from that in the CDM model occurs only at small angular separations \( \Delta \theta < 1^\prime \) and \( \Delta \theta < 0.1^\prime \) for \( m_x > 350 \text{ eV} \) and \( m_x > 1 \text{ keV} \), respectively. It is likely that one needs to reach an even smaller angular separation of \( \sim 10 \) mas in order to make an effective distinction between the WDM and CDM models for a realistic WDM particle mass of \( m_x \sim 1 \text{ keV} \). We have applied the lensing probability \( P(\Delta \theta) \) to the quasar luminosity function determined from the 2dF quasar survey over absolute magnitudes \(-26 < M_b < -23 \) and redshifts \( 0.35 < z < 2.3 \) (Boyle et al. 2000). Here we would rather take a less vigorous approach to the problem by neglecting the magnification effect of the double images. It is shown that at a limiting magnitude of \( m_b = 20 \), the fractions of small angular separation events with \( 0.001 < \Delta \theta < 0.01 \) in the total number of doubly imaged quasars are approximately 1/100, 4/1000, and 1/1000 for the CDM model and the WDM models with \( m_x = 1 \text{ keV} \) and \( m_x = 350 \text{ eV} \), respectively. Indeed, observational tests of these fractions on milliarcsecond scales turn out to be difficult and challenging.

3. DISCUSSION AND CONCLUSIONS

In the CDM scenario of structure formation, many small galaxies would form, relative to what are predicted by the WDM model. Consequently, the standard CDM model would predict more multiple, small angular separation images of the gravitationally lensed distant quasars than the WDM model does. This unique property can be used as an effective tool to distinguish these two prevailing models. It has been shown that the significant difference in the distributions of image angular separations between WDM and CDM models occurs at small angular separations \( \Delta \theta \sim 0.01 \) for the WDM particle mass \( m_x \sim 1 \text{ keV} \). Although there are no data available at present to test our prediction, these angle ranges should be accessible by current radio observations such as VLBI. Recall that systematic radio surveys for strong gravitational lensing have proved to be very successful in finding small separation images in the range \( 0.3\text{–}0.6^\prime \) (e.g., Augusto, Wilkinson, & Browne 1998; Augusto & Wilkinson 2001 and references therein). It is hoped that future multiple image surveys down to \( \sim 10 \) mas would allow one to place useful constraints on the WDM particle mass.

We now briefly discuss several major uncertainties in the present predictions. First, we have used a simple singular isothermal sphere model for the mass distributions of dark halos. An alternative is the universal density profile suggested by high-resolution N-body simulations (Navarro, Frenk, & White 1995, hereafter NFW), which should be valid for the virialized dark halos as small as \( 10^5 M_\odot \). Li & Ostriker (2000) have recently compared the lensing probabilities \( P(\Delta \theta) \) produced by
a singular isothermal sphere model and the NFW profile using roughly the same approach as ours. It turns out that the NFW profile results in a significantly small value of \( P(\Delta \theta) \sim 10^{-6} \), insensitive to the image separation \( \Delta \theta \). We have recalculated the lensing probabilities under CDM and WDM models following the approximate treatment of Li & Ostriker (2000) for the image separation by NFW profile, \( \Delta \theta \approx 2\theta_\text{c} \), where \( \theta_\text{c} \) is the solution to the lensing equation with the zero alignment parameter. Unlike Li & Ostriker (2000), who took a constant halo concentration \( c \) for massive halos from numerical simulations, we fix the characteristic density \( \delta_c \) and halo concentration \( c \) for each halo \( M \) using the \( \delta_c-M \) relation via the collapse redshift \( z_\text{coll} \) proposed by Navarro, Frenk, & White (1997). Nevertheless, we have reached a conclusion essentially similar to Li & Ostriker (2000). For example, the lensing probabilities for a distant quasar at \( z_q = 3 \) with \( \Delta \theta \geq 0.001-1'' \) are \( P(\Delta \theta) \approx 3.7 \times 10^{-6}, 3.6 \times 10^{-6}, \) and \( 2.9 \times 10^{-6} \) for the CDM model and the WDM models with \( m_x = 1 \) keV and \( m_x = 350 \) eV, respectively. The lack of small separation events below arcsecond scales will invalidate our original intention that the study of doubly imaged quasars may help to distinguish between CDM and WDM models. However, we note that the lensing probability produced by the NFW profile is unreasonably small (~\( 10^{-6} \)). This implies that the NFW profile may be inappropriate to the description of the mass distributions in the inner regions of dark halos where the multiple images would occur. Indeed, the central mass density of an NFW profile is too shallow to act as an effective lens. The absence of detectable odd images in the known lens systems also lends support to the existence of a steeper inner density profile in galaxies, \( \rho \propto r^{-2} \) (Rusin & Ma 2001). In a word, the singular isothermal sphere model may be a reasonable approximation for the inner mass profiles of galactic and subgalactic halos.

Second, the PS mass function may become inaccurate for small halos. High-resolution \( N \)-body simulations have shown that the PS approximation predicts too many small halos at low redshifts (e.g., Bode et al. 2000a; Jenkins et al. 2001). This will lead to an overestimate of the total lensing probability for multiple images, \( P(\Delta \theta) \), although the differential lensing probability normalized by \( P(\Delta \theta) \), \( [1/P(\Delta \theta)]dP(\Delta \theta)/d\Delta \theta \), is probably less affected. In order to demonstrate the uncertainty, we have used the modified PS mass function by Sheth & Tormen (1999) and the best-fit parameters by Jenkins et al. (2001). It is found that the total lensing probabilities for small separation events in both CDM and WDM models are slightly reduced as compared with those predicted by the standard PS formalism. For example, at \( z_q = 2 \) and \( \Delta \theta = 0.1'' \), the values of \( P(\Delta \theta) \) in terms of the Sheth & Tormen (1999) mass function are 1.2 times smaller than those derived from the PS approach, and this ratio is insensitive to cosmological models. The distribution of the differential lensing probability normalized by \( P(\Delta \theta) \) based on the Sheth & Tormen (1999) mass function remains roughly the same as in Figure 1.

Third, the theoretically predicted distribution of image separations depends critically on the choice of the cosmological parameters (e.g., Li & Ostriker 2000). Therefore, a robust constraint on the WDM particle mass from the study of lensing probability for multiple images with small angular separations needs a priori precise determinations of many other parameters such as \( \Omega_m, \sigma_8, \) and \( H_0 \).

Finally, one may worry about the contamination of the supermassive black holes as lenses in the actual application of the gravitational lensing probability to the observational tests of the WDM model. Indeed, it is believed that all active galactic nuclei contain supermassive black holes with masses \( M \sim 10^{7}-10^{9} M_\odot \) in their centers. The lensing effect by these supermassive black holes leads to an image separation of typically \( \sim 0.1'' \) at cosmological distance, which is indeed of the same order of magnitude as the effect produced by small galaxies discussed in the present Letter. Yet, the lensing probability of a background quasar due to the supermassive black holes residing in active galactic nuclei is considerably low (~\( 10^{-5}; \) Augusto & Wilkinson 2001). Overall, we feel that gravitational lensing should serve as a critical test for the WDM model, although further work will be needed to address in more detail various uncertainties in the issue.

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