Polytopic Model Weighted $H_\infty$ Output Feedback Control of Switched LPV System Based on MDADT

Tao Wang1 and Xuehai Wang1, *
1School of Electrical Engineering, Southwest Jiaotong University, Chengdu 610031, China

*Email: xnjtdx216510@163.com

Abstract. This paper provides and validates a class of weighted $H_\infty$ output feedback controllers of switched LPV (Linear Parameter Varying) systems based on the MDADT (Modal Dependent Average Dwell Time) method. First of all, the article uses a multi-Lyapunov function to convert the linear parameter varying system into a polytopic model, and designs a switching law that can guarantee the system to be globally uniform and exponentially stable on the MDADT method. It can be shown that when the state is stable and can be measured in real time, the switching law and weighted $H_\infty$ controllers for each subsystem under the MDADT limit can be calculated. Finally, a numerical example is given to verify the feasibility of this method and the stability of the switched LPV system.

1. Introduction
Switched system is a sort of hybrid system [1] and generally composed of several subsystems. Each subsystem has different parameters and can be switched one by one. The system can be operated stably by choosing a right switching law and useful controllers. However, variable parameters of switched system are nonlinear in some real working places [2]. Thus methods like Jacobian-linearization are usually used to convert the system into LPV (Linear Parameter Varying) model [3]. Bei Lu proposed a new conception that combined the method of LPV control and the idea of switched system. This new proposal has given a new approach of switching LPV control based on Lyapunov function [4].

The state feedback controllers of switched LPV system based on ADT (Average Dwell Time) [5] is not easy to implement in real world. Moreover, ADT demands each subsystem sharing the same time lower limitation. Thus MDADT (Modal Dependent Average Dwell Time) as an advanced conception evolved from ADT is provided by Xudong Zhao in [6]. MDADT is better than ADT because MDADT is used for each subsystem but not for the whole switched system. Each subsystem has each time limitation by each mode of MDADT but not the same limitation at all.

This paper is interested in switched LPV system. There are three main contributions. Firstly, the polytopic model is used to replace the modeling of a linear variable-parameter system. The subsystem corresponding to each vertex is used to represent the entire system, and the problem of system parameter changes is solved. Secondly, the sufficient conditions for the existence of MDADT-based output feedback controller are given, and the correctness of the condition is proved. Finally, the design method of output feedback controller for disturbance suppression is given and a numerical simulation example is given to prove our results.
2. Multicellular Decomposition of Switched LPV System

Given a complete switched LPV system

\[
\begin{align*}
\dot{x}(t) &= A_{\sigma(t)}(\rho)x(t) + B_{1\sigma(t)}(\rho)w(t) + B_{2\sigma(t)}(\rho)u(t) \\
y(t) &= C_{1\sigma(t)}(\rho)x(t) + D_{1\sigma(t)}(\rho)w(t) + D_{2\sigma(t)}(\rho)u(t), \\
z(t) &= C_{2\sigma(t)}(\rho)x(t) + D_{2\sigma(t)}(\rho)w(t) + D_{2\sigma(t)}(\rho)u(t),
\end{align*}
\]

(1)

where \(x(t) \in \mathbb{R}^n\) and \(u(t) \in \mathbb{R}^r\) denote the state vector and the input vector respectively, and \(y(t) \in \mathbb{R}^p\) denotes the controlled output vector. \(w(t) \in \mathbb{R}^q\) is the disturbance input vector, \(\rho(t) = [\rho_1(t), \rho_2(t), \ldots, \rho_i(t)]^T\) is the time varying parameter vector that can be measured in real time and has the solution in the compact set. \(A_{\sigma(t)}(\rho), B_{1\sigma(t)}(\rho), B_{2\sigma(t)}(\rho), C_{1\sigma(t)}(\rho), C_{2\sigma(t)}(\rho), D_{1\sigma(t)}(\rho), D_{2\sigma(t)}(\rho), D_{2\sigma(t)}(\rho), D_{2\sigma(t)}(\rho), D_{2\sigma(t)}(\rho), D_{2\sigma(t)}(\rho), D_{2\sigma(t)}(\rho)\) are the functions of time-varying parameter matrix \(\rho(t)\). \(\sigma(t)\) denotes the control signal of the switched system and \(\sigma(t):[0, \infty) \rightarrow \mathbb{N} = \{1, 2, \ldots, N\}\). \(N\) is a set of numbers, each number stands for a subpart of subsystem. For the switched LPV system under the control of switching signal \(\sigma(t)\), when \(\sigma(t) = i\), the subsystem is activated, the polytopic model is as follows

\[
\begin{bmatrix}
A_{\sigma(t)}(\rho) & B_{1\sigma(t)}(\rho) & B_{2\sigma(t)}(\rho) \\
C_{1\sigma(t)}(\rho) & D_{1\sigma(t)}(\rho) & D_{2\sigma(t)}(\rho) \\
C_{2\sigma(t)}(\rho) & D_{2\sigma(t)}(\rho) & D_{2\sigma(t)}(\rho)
\end{bmatrix}
= \sum_{m=1}^{n} \alpha_{\sigma(t)m}(t)
\begin{bmatrix}
A_{\sigma(t)m} & B_{1\sigma(t)m} & B_{2\sigma(t)m} \\
C_{1\sigma(t)m} & D_{1\sigma(t)m} & D_{2\sigma(t)m} \\
C_{2\sigma(t)m} & D_{2\sigma(t)m} & D_{2\sigma(t)m}
\end{bmatrix}
\]

(2)

Then, process of designing controllers of each switched LPV subsystem represented by their vertices with the help of MDADT is given. For each vertex \(m\) of (2), according to the properties of the variable parameters, \(\rho(t) \in \mathbb{P} \subset \mathbb{R}^r\), \(\rho(t) = [\rho_1(t), \rho_2(t), \ldots, \rho_i(t)]^T\), there is a polytopic description as \(\text{Col}(\alpha_{m}, m=1, 2, \ldots, r) = \{\sum_{m=1}^{r} \alpha_{m}(x)\alpha_{m} : \alpha_{m} \geq 0, \sum_{m=1}^{r} \alpha_{m}(x) = 1\}\) about this vector. Then for each LPV subsystem \(i\) of the switched LPV system, when \(\sigma(t) = i\), the subsystem is activated, the polytopic model is as follows

\[
\begin{bmatrix}
\dot{x}_{i}(t) \\
u(t)
\end{bmatrix}
= \begin{bmatrix}
A_{K_{im}}(\rho) & B_{K_{im}}(\rho) \\
C_{K_{im}}(\rho) & D_{K_{im}}(\rho)
\end{bmatrix}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
\]

(3)

\(\dot{x}(t)\) denotes the state vector of controllers, \(A_{K_{im}}(\rho), B_{K_{im}}(\rho), C_{K_{im}}(\rho), D_{K_{im}}(\rho)\) are parameter-dependent matrices of the controllers of each subsystem, (3) denote matrices of controllers in each vertex \(m\) of each subsystem \(i\). After the decomposition of polytopic model, \(A_{K_{im}}, B_{K_{im}}, C_{K_{im}}, D_{K_{im}}\) all have no variable parameters and can be calculated offline. After adding these controllers, \(x_{im}(t)\) denotes state of vertex \(m\) of of subsystem \(i\), \(\dot{x}_{im}(t)\) denotes controller state of vertex \(m\) of subsystem \(i\), closed-loop switched LPV system model converted from the open-loop original is as follow

\[
\begin{bmatrix}
\dot{z}_{im}(t) \\
\dot{z}_{im}(t)
\end{bmatrix}
= \begin{bmatrix}
A_{G_{im}} + B_{G_{im}}(\rho) \\
C_{G_{im}} + D_{G_{im}}(\rho)
\end{bmatrix}
\begin{bmatrix}
z_{im}(t) \\
v_{im}(t)
\end{bmatrix}
\]

(4)

where \(A_{G_{im}}, B_{G_{im}}, C_{G_{im}}, D_{G_{im}}\) denote closed-loop switched LPV system parameter matrix and

\[
\begin{bmatrix}
\dot{z}_{im}(t) \\
\dot{z}_{im}(t)
\end{bmatrix}
= \begin{bmatrix}
A_{G_{im}} + B_{G_{im}} + D_{G_{im}}(\rho) \\
C_{G_{im}} + D_{G_{im}}(\rho)
\end{bmatrix}
\begin{bmatrix}
z_{im}(t) \\
v_{im}(t)
\end{bmatrix}
\]

(5)

\[
A_{G_{im}} = \begin{bmatrix}
A_{im} + B_{2i}D_{K_{im}}C_{2i} + B_{2i}D_{K_{im}}C_{2im} + B_{2i}D_{K_{im}}D_{2i} \\
B_{2i} + D_{K_{im}} + B_{2i}D_{K_{im}}D_{2i}
\end{bmatrix}
\]

(6)

\[
C_{G_{im}} = \begin{bmatrix}
C_{2im} + D_{1i}D_{K_{im}}D_{2i} + D_{1i}D_{K_{im}}D_{2i} + D_{1i}D_{K_{im}}D_{2i} \\
D_{1i} + D_{1i}D_{K_{im}}D_{2i} + D_{1i}D_{K_{im}}D_{2i} + D_{1i}D_{K_{im}}D_{2i}
\end{bmatrix}
\]

(7)
3. Polytopic model weighted $H_\infty$ Output Feedback Controllers Based on MDADT

**Lemma 1:** [7] For the switched system (1), $0 < \alpha_0 < 1$, $\mu > 1$, $\gamma > 0$ are given constants. Assuming there exist a positive definite continuous differentiable function $V_{\alpha(i)}: \mathbb{R}^n \to \mathbb{R}$ that satisfies the zero initial condition $V_{\alpha(i)}(0) = 0, \forall (i, j) \in N \times N, i \neq j$. $V_j(x(t)) \leq \mu V_j(x(t))$, $\forall i \in N$, we easily get in (9) by defining disturbances to be $\Gamma(t) = y^T y - \gamma^2 w^T w$

$$\dot{V}_j(x(t)) + \Gamma(t) \leq -\alpha V_j(x(t))$$

Then from the definition of MDADT the system is globally uniformly asymptotically stable, and for any switching signal satisfying $N_{i,m}(t,T) \leq N_{o,i} + (T - t)/\tau_{ai}$ in [8], the signal can be obtained as

$$\tau_{ai}^* \geq \tau_{ai} = \frac{\ln \mu}{\alpha_i}$$

**Theorem 1.** For each vertex $m$ of subsystem $i$, and given constants $0 < \alpha_i < 1$, $\mu_i > 1$, $\gamma > 0$, the switching system (1) is globally uniformly exponentially stable and has output feedback controllers with performance $\gamma$, if there exist $X_{C_{i,j}}$ such that

$$\begin{bmatrix} A_{C_{i,m}}^T X_{C_{i,m}} + X_{C_{i,m}} A_{C_{i,m}} + \gamma \mu_i X_{C_{i,j}} X_{C_{i,j}} A_{C_{i,m}} & C_{C_{i,m}}^T \gamma I & 0 \\ B_{C_{i,m}}^T X_{C_{i,j}} & -\gamma I & D_{C_{i,m}}^T \gamma I \\ C_{C_{i,m}} & -\gamma I & 0 \end{bmatrix} \leq 0$$

$$X_{C_{i,j}} \leq \mu X_{C_{i,j}}$$

**Proof:** For each vertex $m$ of subsystem $i$, assuming its quadratic form Lyapunov function $V_i = x^T X_i$, $\forall (i, j) = i \in N$

$$\dot{V}_i(x) + \alpha_i V_i(x) = x^T \Lambda x$$

When there is no disturbance $\sigma(t) = 0$, we can easily get $\dot{V}_i(x) + \alpha_i V_i(x) = x^T \Lambda x$, and define $\Lambda = A_{C_{i,j}(\rho)}^T X_{C_{i,j}} + X_{C_{i,j}} A_{C_{i,j}(\rho)} + \alpha X_{C_{i,j}}$, where $A_{C_{i,j}(\rho)}$ is a closed-loop system matrix, therefore, we can get the system is possible to be stable if $\Lambda < 0$. When there is disturbances in the system, we have

$$\dot{V}_i(x) + \alpha_i V_i(x) + y^T y - \gamma^2 w^T w$$

$$= \sum_{m=1}^{N_i} \alpha_{\sigma(i)}(t) \left[ x^T w^T \right] \begin{bmatrix} A_{K_{i,m}}^T X_{C_{i,m}} + X_{C_{i,m}} A_{K_{i,m}} + \gamma \mu_i X_{C_{i,j}} X_{C_{i,j}} A_{K_{i,m}} & C_{K_{i,m}}^T \gamma I & 0 \\ B_{K_{i,m}}^T X_{C_{i,j}} & -\gamma I & D_{K_{i,m}}^T \gamma I \\ C_{K_{i,m}} & -\gamma I & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}$$

$$= \sum_{m=1}^{N_i} \alpha_{\sigma(i)}(t) \left[ x^T w^T \right] \Theta \begin{bmatrix} x \\ w \end{bmatrix}$$

From theorem (1), if $0 < \alpha$, the system will be stable, and $\tau_{ai}^* \geq \tau_{ai} = \ln \mu / \alpha_i$. So (10) and (11) must be verified with a feasible solution. Next solve a set of LMIs by using elimination. Introduce following matrices to simplify (10)

$$K_{i,m} = \begin{bmatrix} A_{K_{i,m}} & B_{K_{i,m}} \\ C_{K_{i,m}} & D_{K_{i,m}} \end{bmatrix}, A_{i,m} = \begin{bmatrix} A_{i,m} & 0 \\ 0 & 0 \end{bmatrix}, B_{0,i,m} = \begin{bmatrix} B_{i,m} \\ 0 \end{bmatrix}, C_{0,i,m} = \begin{bmatrix} C_{i,m} & 0 \\ 0 & I \end{bmatrix}, \overline{B}_{I,m} = \begin{bmatrix} 0 & B_{2,i,m} \\ I & 0 \end{bmatrix}, \overline{D}_{I,m} = \begin{bmatrix} 0 & D_{12,i,m} \\ 0 & D_{21,i,m} \end{bmatrix}$$

$$C_{2,i,m} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \overline{D}_{2,i,m} = \begin{bmatrix} 0 & D_{12,i,m} \\ 0 & D_{21,i,m} \end{bmatrix}$$
Then we can get

\[
\Xi X_{Cl,i}(\bar{B}_{i+2m} + \bar{D}_{i+2m} \bar{C}_{i+2m} \bar{C}_{i+2m}) (C_{i+2m} + \bar{D}_{i+2m} \bar{C}_{i+2m} \bar{C}_{i+2m})^T \leq 0, \tag{16}
\]

where \(\Xi = (\bar{B}_{i+2m} + \bar{D}_{i+2m} \bar{C}_{i+2m}) X_{Cl,i} + X_{Cl,i}(\bar{B}_{i+2m} + \bar{D}_{i+2m} \bar{C}_{i+2m}) + \alpha_i X_{Cl,i} \). In (9), \(X_{Cl,i}\) and \(K_{i,m}\) can be solved.

Define assistant matrices \(H_{Cl,i,m}, M_{X_{Cl,i}}, N_{i,m}, T_{Cl,i,m}\)

\[
H_{Cl,i,m} = \begin{bmatrix}
A_{i,m}^T X_{Cl,i} + X_{Cl,i} A_{i,m} + \alpha_i X_{Cl,i} X_{Cl,i} B_{i,m}^T C_{i,m}^T \\
0 & -\gamma I & -\gamma I \\
0 & -\gamma I & -\gamma I
\end{bmatrix},
\]

\[
M_{X_{Cl,i}} = \begin{bmatrix}
\bar{B}_{i,m} X_{Cl,i} & 0 & \bar{D}_{i,m} X_{Cl,i} \\
0 & 1 & 0
\end{bmatrix},
\]

\[
T_{Cl,i,m} = \begin{bmatrix}
X_{Cl,i}^T & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix} = H_{Cl,i,m}
\]

Then (16) can be rewritten as

\[
H_{Cl,i,m} + M_{X_{Cl,i}}^T K_{i,m} N_{i,m} + N_{i,m}^T K_{i,m} M_{X_{Cl,i}} < 0 \tag{19}
\]

Using the properties of nuclear space, \(M_{X_{Cl,i}}^T\) and \(N_{i,m}^T\) are matrices constructed by the kernel space basis vectors of the corresponding matrices. To solve the problem more easily, using \(T_{Cl,i,m}\), we can get

\[
\begin{bmatrix}
M_{X_{Cl,i}}^T \\
N_{i,m}^T
\end{bmatrix}
\begin{bmatrix}
T_{Cl,i,m} \\
H_{Cl,i,m}
\end{bmatrix} < 0
\]

Since the matrix \(X_{Cl,i}\) is a real symmetric matrix with dimensions \((n + n_K) \times (n + n_K)\), \(n\) and \(n_K\) denote orders of system model and controller, the decompose \(X_{Cl,i}\) and \(X_{Cl,i}^{1/2}\) :

\[
X_{Cl,i} = \begin{bmatrix}
X_{1,i} & X_{2,i} \\
X_{2,i}^T & X_{3,i}
\end{bmatrix},
\]

\[
X_{Cl,i}^{1/2} = \begin{bmatrix}
Y_{1,i} & Y_{2,i} \\
Y_{2,i}^T & Y_{3,i}
\end{bmatrix}
\]

Taking back (21) into \(H_{Cl,i,m}\) and \(T_{Cl,i,m}\) and defining \(L_{b,i,m}\) and \(L_{c,i,m}\) are matrices constructed by column vectors which are arbitrary sets of basis vectors in space \(\ker([C_{b,i,m} D_{b,i,m}])\) and \(\ker([B_{c,i,m} D_{c,i,m}])\) respectively. We can get

\[
\begin{bmatrix}
L_{c,i,m}^T & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
A_{i,m} Y_{1,i} + \alpha_i Y_{1,i} Y_{1,i} C_{1,i,m}^T + B_{i,m} \\
0 & -\gamma I & -\gamma I \\
0 & -\gamma I & -\gamma I
\end{bmatrix}
\begin{bmatrix}
L_{c,i,m} & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix} \leq 0 \tag{22}
\]

\[
\begin{bmatrix}
L_{b,i,m}^T & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
A_{i,m} X_{i} + \alpha_i X_{i} X_{b,i,m} C_{i,m}^T \\
0 & -\gamma I & -\gamma I \\
0 & -\gamma I & -\gamma I
\end{bmatrix}
\begin{bmatrix}
L_{b,i,m} & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix} \leq 0 \tag{23}
\]

After verifying the feasibility of \(X_{Cl,i}\), solutions of \(A_{K,i,m}, B_{K,i,m}, C_{K,i,m}, D_{K,i,m}\) can be solved. The controllers of each subsystem are obtained as (26).

\[
K_{i,m} = \sum_{m=1}^{\infty} \alpha_{i,m} \begin{bmatrix}
A_{K,i,m} B_{K,i,m} \\
C_{K,i,m} D_{K,i,m}
\end{bmatrix} \tag{26}
\]
There exist various MDADTs for each vertex \( m \) of each switched LPV subsystem, and the system can be exponentially stable as long as the average ADT of each subsystem is larger than MDADT. If the MDADTs are chosen the same by its property, their different vertices of the subsystem can be switched synchronously and the shape of the system can be maintained. For polytopic model of switched LPV system with multiple vertices that has the same parameter range, by using the same MDADT the whole switched LPV system can realize exponential stability.

4. Simulation
In this section MDADT will be calculated and weighted \( H_\infty \) output feedback controllers will be obtained. A numerical example with two is adopted and the initial state \( x_0 = [4 -7]^T \).

Subsystem 1

\[
\begin{align*}
A_1 &= \begin{bmatrix} -3 -0.1\rho(t) -1 \end{bmatrix}, \\
B_{11} &= \begin{bmatrix} 1 \\ 1.5 \\ 1.5 \\ 0 \end{bmatrix}, \\
B_{21} &= \begin{bmatrix} 1 \\ 1.5 \\ 1.5 \\ 0 \end{bmatrix}, \\
C_{11} &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{bmatrix}, \\
C_{21} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
D_{111} &= D_{121} = D_{211} = D_{221} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
D_{112} &= D_{122} = D_{212} = D_{222} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{align*}
\]  

(27)

Subsystem 2

\[
\begin{align*}
A_2 &= \begin{bmatrix} -2 -0.1\rho(t) -1.5 \end{bmatrix}, \\
B_{12} &= \begin{bmatrix} 1.2 \\ 1.3 \\ 0.62 \\ 0.88 \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} 1.2 \\ 1.3 \\ 0.62 \\ 0.88 \end{bmatrix}, \\
C_{12} &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 \end{bmatrix}, \\
C_{22} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
D_{112} &= D_{122} = D_{212} = D_{222} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
D_{111} &= D_{121} = D_{211} = D_{221} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{align*}
\]  

(28)

Variable parameter vector \( \rho(t) = \sin(t), \rho(t) \in (0,1) \). We decompose the system according to the section 2 to get the Polytopic model, and then get its vertex controller as follows

Vertex 1 controller for subsystem 1

\[
\begin{align*}
A_{K_{11}} &= \begin{bmatrix} -2.0011 & 0 \\ 0 & -2.0011 \end{bmatrix}, \\
B_{K_{11}} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
C_{K_{11}} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
D_{K_{11}} &= \begin{bmatrix} 3.6218 & -0.0544 \end{bmatrix}
\end{align*}
\]  

(29)

Vertex 2 controller for subsystem 1

\[
\begin{align*}
A_{K_{12}} &= \begin{bmatrix} -2.0042 & 0 \\ 0 & -2.0042 \end{bmatrix}, \\
B_{K_{12}} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
C_{K_{12}} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
D_{K_{12}} &= \begin{bmatrix} 4.1262 & 0.0066 \end{bmatrix}
\end{align*}
\]  

(30)

Vertex 1 controller for subsystem 2

\[
\begin{align*}
A_{K_{21}} &= \begin{bmatrix} -5.2573 & 0 \\ 0 & -5.2573 \end{bmatrix}, \\
B_{K_{21}} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
C_{K_{21}} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
D_{K_{21}} &= \begin{bmatrix} -5.1347 & 3.7732 \end{bmatrix}
\end{align*}
\]  

(31)

Vertex 2 controller for subsystem 2

\[
\begin{align*}
A_{K_{22}} &= \begin{bmatrix} -5.0205 & 0 \\ 0 & -5.0205 \end{bmatrix}, \\
B_{K_{22}} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
C_{K_{22}} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
D_{K_{22}} &= \begin{bmatrix} -5.231 & 4.2468 \end{bmatrix}
\end{align*}
\]  

(32)

Figure 1. Disturbance Input

Figure 2. Switching Single
The external disturbance signal of the system is shown in Figure 1. Let $a_1 = a_2 = a = 0.5$, $\mu_1 = 1.22$, $\mu_2 = 1.98$, $\gamma = 1$ and MDADT of each vertex subsystem $\tau_{u1} = \tau_{u12} = 0.3977$, $\tau_{u2} = \tau_{u21} = 1.3662$. In this time, the switching single of the system is shown in Figure 2, the state response and the output of the system is shown in Figure 3 and Figure 4. Therefore, the polytopic model weighted $H_\infty$ output feedback control based on MDADT proposed in this paper can make the switched LPV system stable.

5. Conclusions
The purpose of this paper is to design weighted $H_\infty$ output feedback controllers of switched LPV system based on MDADT. For the switched LPV system, LPV part of the system can be divided into different vertices of each switched subsystem by using polytopic model. Then design output feedback controllers for each vertex based on MDADT of each subsystem so that the system can switch synchronously and maintain its shape and the region of variable parameters. Finally all vertices are integrated into a complete system and the exponential stability of the system can be achieved because of the switching signal restricted by MDADT. The feasibility of this method is verified by simulation. Comparing with the state feedback controller, this approach has a better practical significance in the working places and is easier to implement.

Acknowledgement
This work is supported by the National Natural Science Foundation of China (51477146, 61603312)

References
[1] Sun Z, Ge S S 2005 Analysis and synthesis of switched linear control systems. Automatica, 41(2):181-195.
[2] Liberzon D 2003 Switching in Systems and Control. Berlin: Birkhauser.
[3] Baranyi P 2004 TP model transformation as a way to LMI-based controller design. IEEE Transactions on Industrial Electronics, 51(2):387-400.
[4] Lu B, Wu F 2004 Switching LPV control designs using multiple parameter-dependent Lyapunov functions. Automatica, 40(11): 1973-1980.
[5] Hasapanha J P, Morse A S 1995 Stability of switched systems with average dwell time. Paper presented at the Proceeding of 38th IEEE Conference on Decision and Control, Phoenix.
[6] Zhao X D, Liu H 2013 Weighted $H_\infty$ performance analysis of switched linear systems with mode-dependent average dwell time. International Journal of Systems Sciences, 44(11):2130-2139.
[7] Lu Q, Zhang L, Karimi H R, Shi Y 2013 $H_\infty$ control for asynchronously switched LPV systems with mode-dependent average dwell time. IET Control Theory and Applications. 7(5):673-683.
[8] Zhao X D, Liu H 2013 Weighted $H_\infty$ performance analysis of switched linear systems with mode-dependent average dwell time. International Journal of Systems Sciences, 44(11):2130-21