Examples of Exact Exponential Cosmological Solutions with Three Isotropic Subspaces in Einstein–Gauss–Bonnet Gravity

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Received June 10, 2022; revised July 23, 2022; accepted August 1, 2022

Abstract—We consider (1 + 8)- and (1 + 10)-dimensional Einstein–Gauss–Bonnet models with a cosmological constant. Some new examples of exact solutions are obtained, governed by three non-coinciding constant Hubble-like parameters \( H \neq 0 \), \( h_1 \) and \( h_2 \), obeying the condition \( mH + k_1h_1 + k_2h_2 \neq 0 \), corresponding to factor spaces of dimensions \( m \geq 3 \), \( k_1 > 1 \), and \( k_2 \geq 1 \). In this case, the multidimensional cosmological model deals with four factor spaces: the external 3D (“our”) world and internal subspaces with dimensions \( m - 3 \), \( k_1 \), and \( k_2 \).

DOI: 10.1134/S0202289322040090

1. INTRODUCTION

Even before the Lovelock theory [1], modified theories of gravity were developed. As in the Einstein equations, in Lovelock’s theory the field equations second-order [2]. Lovelock’s modified theory differs from Einstein’s theory in that a higher order of curvature is included in the equation. Under this condition, the action of the field is defined as

\[
S = \sum_{s=0}^{[(d-1)/2]} \alpha_s L_s,
\]

where \( \alpha_s \) and \( L_s \) are the \( s \)-order Lovelock parameters and Lagrangians, respectively.

Lovelock’s gravity [1] is based on the metric and curvature tensors and is one of generalized theories of gravity. The Lagrangian includes the scalar curvature, the cosmological term, and new corrections for each odd space-time dimension above four. The first correction is called the Gauss–Bonnet term,

\[
G = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2.
\]

This invariant defines multidimensional gravity, i.e., gravity with \( D > 4 \). It can be noted here that in the 4D case these corrections are either topological or identically zero, and Einstein’s theory in 4D follows from such a modified theory of gravity.

After the advent of general relativity (GR), it is believed that the Einstein equations are the most effective theory describing physical phenomena from large to small scales [2]. However, recent research data argue that there are some limitations in explaining some of the new observations: based on GR, we cannot explain the nature of dark matter [3] and dark energy [4–6]. Thus we can conclude that to create a new theory of gravity, which describes the dynamics of the universe and the nature of its dark side, is a need of modern theoretical physics.

 Corrections to Einstein’s 4D gravity have different motivations. One of them is that by increasing the dimension from four, one can construct a new theory, similar to Einstein’s (see, e.g., [7, 8]) but containing scalar–tensor couplings that could give cosmologically observable effects [9, 10] or violate the equivalence principle [11]. Another trend for a new alternative theory of gravity is using corrections by adding quadratic combinations of the Riemann tensor (see, e.g., [12]), such as \( R^2 \) and \( R_{\mu\nu}R^{\mu\nu} \), leading to higher-order modification of the field equations. Such theories have been used to model inflation [13, 14] and dark energy [15].

A modified theory which eliminates the need for dark energy and which seems to be stable is considered in [16]. Terms with positive powers of the curvature support the inflationary epoch, while terms with negative powers of the curvature serve as effective dark energy, supporting the current cosmic acceleration. The equivalent scalar–tensor gravity may be compatible with the simplest solar system experiments.

It is possible to construct a unique quadratic combination of the Riemann curvature tensor invariants that, when added to the usual Einstein–Hilbert action, does not increase the differential order of the
equations of motion [1, 17]. The correction is obtained from the Gauss–Bonnet theorem concerning the Euler characteristic of 2D surfaces [18] and is therefore called the Gauss–Bonnet term. In a multidimensional theory of gravity, there is no significant reason to omit it (other than to make it more complicated). In multidimensional gravity, mostly in the context of brane worlds, such theories have been shown to have surprising properties. Without trying to be complete, we can refer to black hole physics (e.g., [19, 20]), gravitational instabilities (e.g., [21]), and the dynamics of brane worlds of codimension one and higher (e.g., [22–24]).

A general model of multidimensional $R^2$ gravity including the Riemann tensor square term (the case $c \neq 0$) is considered in [25]. A number of brane worlds in such a model are constructed (mainly in 5D), and their properties are discussed. Thermodynamics of S-AdS black holes (BHs) is presented with small $c \neq 0$. The entropy, free energy and energy are calculated. For $c \neq 0$, the BH entropy (energy) is not proportional to the area (mass). The brane equation of motion in a BH background is presented as a FRW equation. Using a dual CFT description, it is shown that the dual field theory is not a conformal one when $c \neq 0$.

As already noted, the Gauss–Bonnet term leads to a nontrivial contribution in multidimensional gravity while in 4D it reduces to a full divergence and thus has no dynamic significance. However, in scalar-tensor theory (see, e.g., [26] for a detailed and general discussion), the Gauss–Bonnet term is of general value coupled to the scalar sector, and 4D gravity is modified (see also [27] for the case of the Lorentz and Chern–Simons terms). Therefore, the Gauss–Bonnet term is included in the most general second-order scalar-tensor theory. Such a correction appears if we consider, for example, the case where the scalar field is a moduli field, which comes from toroidal (i.e., flat) Kaluza–Klein (KK) compactification of a $(4+N)$-dimensional theory. But such reasoning about the uniqueness of the Gauss–Bonnet term is valid only at the classical level.

One more motivation is related to low-energy efficient string actions, such as in heterotic string theory. In string coupling, the Gauss–Bonnet term is included as the leading order and the unique (before this order) ghost-free $\alpha'$ correction to the Einstein–Hilbert action [28] (see, e.g., [29] for cosmology and models of brane worlds arising as a result of such actions). Since the Gauss–Bonnet term is included in the scalar-tensor theory, an important question arises: how can current observations constrain such models? It has already been shown that at the background level a coupled Gauss–Bonnet term allows for a viable cosmology [30].

2. THE COSMOLOGICAL MODEL

The action of the model reads

$$S = \int_\mathcal{M} d^Dz \sqrt{|g|} \left\{ \alpha_1(R[g] - 2\Lambda) + \alpha_2\mathcal{L}_2[g] \right\},$$ (3)

where $g = g_{MN} dz^M \otimes dz^N$ is the metric defined on the manifold $\mathcal{M}$, $\dim \mathcal{M} = D$, $|g| = |\det(g_{MN})|$, $\Lambda$ is the cosmological constant, $R[g]$ is the scalar curvature,

$$\mathcal{L}_2[g] = R_{MNPQ} R^{MNPQ} - 4 R_{MN} R^{MN} + R^2$$

is the standard Gauss–Bonnet term, and $\alpha_1, \alpha_2$ are nonzero constants.

We consider the manifold

$$\mathcal{M} = \mathbb{R} \times M_1 \times \cdots \times M_n$$ (4)

with the metric

$$g = -dt \otimes dt + \sum_{i=1}^n B_i e^{2v_i} dy^i \otimes dy^i,$$ (5)

where $B_i > 0$ are arbitrary constants, $i = 1, \ldots, n$, and $M_1, \ldots, M_n$ are one-dimensional manifolds (either $\mathbb{R}$ or $S^1$), and $n > 3$.

The equations of motion for the action (3) give us the set of polynomial equations [31]

$$E = G_{ij} v^i v^j + 2\Lambda - \alpha G_{ijkl} v^l v^k + \psi v^l = 0,$$ (6)

$$Y_i = \left[ 2G_{ij} v^j - \frac{4}{3} \alpha G_{ijkl} v^k v^l \right] \sum_{i=1}^n v^i - \frac{2}{3} G_{ij} v^i v^l + \frac{8}{3} \Lambda = 0,$$ (7)

where $i = 1, \ldots, n$, and $\alpha = \alpha_2/\alpha_1$. Here,

$$G_{ij} = \delta_{ij} - 1, \quad G_{ijkl} = G_{ij} G_{ik} G_{il} G_{jk} G_{jl} G_{kl}$$ (8)

are components of two metrics on $\mathbb{R}^n$ [32, 33]. The first one is a 2-metric, and the second one is a Finslerian 4-metric. For $n > 3$ we get a set of fourth-order polynomial equations.

We note that for $\Lambda = 0$ and $n > 3$, the set of equations (6) and (7) has an isotropic solution $v^1 = \cdots = v^n = H$ only if $\alpha < 0$ [32, 33]. This solution was generalized in [34] to the case $\Lambda \neq 0$.

It was shown in [32, 33] that there are no more than three different numbers among $v^1, \ldots, v^n$ when $\Lambda = 0$. This is also valid for $\Lambda \neq 0$ if $\sum_{i=1}^n v^i \neq 0$ [35].
3. COSMOLOGICAL SOLUTIONS

Here we consider a class of solutions to the set of equations (6), (7) of the following form:

\[ v = \left( \frac{m}{k_1}, \frac{1}{k_1^2}, \frac{k_2}{k_1} \right), \]  

(9)

where \( H \) is the Hubble-like parameter corresponding to an \( m \)-dimensional factor space, \( h_1 \) is the same corresponding to the \( k_1 \)-dimensional factor space with \( k_1 > 1 \), and \( h_2 \) is the same for the \( k_2 \)-dimensional factor space with \( k_2 \geq 1 \). We split the \( m \)-dimensional factor space for \( m > 3 \) into a product of two subspaces of dimensions 3 and \( m - 3 \). The first one is identified with “our” 3D space, while the second one is considered as a subspace of \( (m - 3 + k_1 + k_2) \)-dimensional internal space.

We consider the ansatz (9) with three Hubble-like parameters \( H, h_1, \) and \( h_2 \) which obey the following restrictions:

\[ H \neq h_1, \quad H \neq h_2, \quad h_1 \neq h_2, \]  

(10)

\[ S_1 = mH + k_1h_1 + k_2h_2 \neq 0. \]  

(11)

Among the exact solutions to the set of equations (6), (7), those with \( k_2 = 1 \) are of interest, i.e., with

\[ v = \left( \frac{m}{k_1}, \frac{1}{k_1^2}, \frac{1}{h_1}, \frac{h_2}{h_1} \right). \]  

(12)

There solutions exist for a certain value of \( \lambda = \Delta \alpha \) \((k_2 = 1)\):

\[ \lambda = \frac{(m + 1)(mk_1 + 1) - 4mk_1 + (m - k_1)^2}{8(m - 1)(k_1 - 1)(m + k_1 - 2)}. \]  

(13)

For this value of \( \lambda \), the Hubble-like parameters are determined as follows:

\[ H = \sqrt{\frac{(k_1 - 1)}{2\alpha(m - 1)(m + k_1 - 2)}}, \]  

(14)

\[ h_1 = -\frac{m - 1}{k_1 - 1}H, \]  

(15)

\[ h_2 \in \mathbb{R}. \]  

(16)

Among stable solutions obeying \( H > 0, \ k_1 > 1, \ k_2 > 1 \), and the key stability condition [35]

\[ S_1 = mH + k_1h_1 + k_2h_2 > 0, \]  

(17)

solutions with zero variation of the effective gravitational constant \( G \) were of great interest. (We note that for \( k_1 = 1 \) or \( k_2 = 1 \) the stability analysis from [35] does not work.) Stable solutions that describe exponential expansion in 3D subspace with the Hubble parameter \( H > 0 \) and zero variation of \( G \) were found in [37]. In the present case, the condition of zero variation of \( G \) should be also added to the set of polynomial equations:

\[ (m - 3)H + k_1h_1 + k_2h_2 = 0. \]  

(18)

4. ANALYSIS OF SOLUTIONS FOR \((1 + 8)\)- AND \((1 + 10)\)-DIMENSIONAL MODELS

4.1. \((1 + 8)\)-Dimensional Model

Among the solutions in \((1 + 8)\)-dimensional model, the following four stable solutions with \( H > 0 \) can be identified:

In \([3, 3, 2]\)-splitting, solutions to Eqs. (6), (7) of the following form:

\[ v = (H, H, h_1, h_1, h_1, h_2), \]  

(19)

where \( H \) corresponds to a 3D factor space, \( h_1 \) also to a 3D factor space, and \( h_2 \) to a 2D factor space.

1. In this case, the following two stable solutions among eight can be found for \( H > 0 \) and \( \alpha > 0 \):

**IA.**

\[ h_2 = \frac{1}{\sqrt{15\alpha}} \sqrt{1 - 2\theta_1} > 0, \]  

(20)

\[ h_1 = -\frac{1}{4\sqrt{15\alpha}} \]  

\[ \times \left( \sqrt{1 - 2\theta_1} + 5\sqrt{1 + \frac{2}{5} \theta_1} \right) < 0, \]  

(21)

\[ H = \frac{1}{4\sqrt{15\alpha}} \left( 5\sqrt{1 + \frac{2}{5} \theta_1} - \sqrt{1 - 2\theta_1} \right) > 0, \]  

(22)

\[ S_1 = \frac{1}{2\sqrt{15\alpha}} \sqrt{1 - 2\theta_1} > 0, \]  

(23)

where we have denoted \( \theta_1 \equiv \sqrt{60\lambda - 11} \). The range of \( \lambda \) which occurs for stable solutions with \( H > 0 \), \( h_1 < 0 \), and \( h_2 > 0 \) is

\[ \frac{11}{60} < \lambda < \frac{3}{16}. \]  

(24)

**IB.**

\[ h_2 = \frac{1}{\sqrt{15\alpha}} \sqrt{1 + 2\theta_1} > 0, \]  

(25)

\[ h_1 = -\frac{1}{4\sqrt{15\alpha}} \]  

\[ \times \left( \sqrt{1 + 2\theta_1} + 5\sqrt{1 - \frac{2}{5} \theta_1} \right) < 0, \]  

(26)

\[ H = \frac{1}{4\sqrt{15\alpha}} \left( 5\sqrt{1 - \frac{2}{5} \theta_1} - \sqrt{1 + 2\theta_1} \right) > 0, \]  

(27)

\[ S_1 = \frac{1}{2\sqrt{15\alpha}} \sqrt{1 + 2\theta_1} > 0. \]  

(28)

The range of \( \lambda \) which occurs for stable solutions with \( H > 0 \), \( h_1 < 0 \), and \( h_2 > 0 \), is

\[ \frac{11}{60} < \lambda < \frac{1}{4}. \]  

(29)
The solution 1B was presented (in fact) earlier in [36]. Here we eliminate an error in the upper bound on \( \lambda \) in Eq. (3.21) from [36].

2. In \([m, k_1, 1]\) splitting, solutions to the set of equations (6), (7) are the following:

2.1. \( m = 3, \ k_1 = 4, \ k_2 = 1, \ \lambda = \frac{11}{60} \)
\[
H = \frac{1}{2} \sqrt{\frac{3}{5\alpha}}, \quad h_1 = -\frac{1}{\sqrt{15\alpha}}, \quad h_2 \in \mathbb{R}.
\]

2.2. \( m = 4, \ k_1 = 3, \ k_2 = 1, \ \lambda = \frac{11}{60} \)
\[
H = \frac{1}{\sqrt{15\alpha}}, \quad h_1 = -\frac{1}{2} \sqrt{\frac{3}{5\alpha}}, \quad h_2 \in \mathbb{R}.
\]

2.3. \( m = 5, \ k_1 = 2, \ k_2 = 1, \ \lambda = \frac{23}{80} \)
\[
H = \frac{1}{2\sqrt{10\alpha}}, \quad h_1 = -\frac{1}{\sqrt{5\alpha}}, \quad h_2 \in \mathbb{R}.
\]

3. Using Eqs. (3.17), (3.20), and (3.25) from [37], we can find stable solutions at \( \lambda = \Delta \alpha = 213/980 \) with \([3, 3, 2]\) splitting in dimension \((1+8)\) with zero variation
\[
H = \frac{1}{2\sqrt{35\alpha}}, \quad h_1 = -\frac{1}{\sqrt{35\alpha}}, \quad h_2 = \frac{3}{\sqrt{35\alpha}}.
\]

4.2. \((1+10)\)-Dimensional Model

Among the solutions in \((1+10)\)-dimensional model can be identified as following four stable solutions with \( H > 0 \):

In \([4, 4, 2]\)-splitting, solutions to Eqs. (6), (7) of the form
\[
v = (H, H, H, H, h_1, h_1, h_1, h_1, h_2, h_2),
\]
where \( H \) is the Hubble-like parameter corresponding to a 4D factor space, \( h_1 \) to another 4D factor space, and \( h_2 \) to a 2D factor space.

1. The following two stable solutions among eight can be found for \( H > 0 \), and \( \alpha > 0 \):

1A.
\[
h_2 = \frac{1}{28} \sqrt{\frac{42}{\alpha}} \sqrt{1 - \theta_2} > 0, \quad \lambda = \frac{27}{140},
\]
\[
h_1 = -\frac{1}{4\sqrt{42\alpha}} \times \left( \sqrt{1 - \theta_2} + 7 \sqrt{1 + \frac{1}{7} \theta_2} \right) < 0,
\]
\[
H = \frac{1}{4\sqrt{42\alpha}} \left( 7 \sqrt{1 + \frac{1}{7} \theta_2} - \sqrt{1 - \theta_2} \right) > 0,
\]
\[
S_1 = \frac{1}{\sqrt{42\alpha}} \sqrt{1 - \theta_2} > 0,
\]
where we have denoted \( \theta_2 = \sqrt{336\lambda - 55} \).

The range of \( \lambda \) which occurs for stable solutions with \( H > 0, \ h_1 < 0, \) and \( h_2 > 0 \) is
\[
\frac{55}{336} < \lambda < \frac{13}{48}.
\]

1B.
\[
h_2 = \frac{1}{28} \sqrt{\frac{42}{\alpha}} \sqrt{1 + \theta_2} > 0, \quad \lambda = \frac{27}{140},
\]
\[
h_1 = -\frac{1}{4\sqrt{42\alpha}} \times \left( \sqrt{1 + \theta_2} + 7 \sqrt{1 - \frac{1}{7} \theta_2} \right) < 0,
\]
\[
H = \frac{1}{4\sqrt{42\alpha}} \left( 7 \sqrt{1 - \frac{1}{7} \theta_2} - \sqrt{1 + \theta_2} \right) > 0,
\]
\[
S_1 = \frac{1}{\sqrt{42\alpha}} \sqrt{1 + \theta_2} > 0.
\]

2. In \([m, k_1, 1]\) splitting, solutions to Eqs. (6), (7) are the following:

2.1. \( m = 3, \ k_1 = 6, \ k_2 = 1, \ \lambda = \frac{27}{140} \)
\[
H = \frac{1}{2} \sqrt{\frac{5}{7\alpha}}, \quad h_1 = -\frac{1}{\sqrt{35\alpha}}, \quad h_2 \in \mathbb{R}
\]

2.2. \( m = 4, \ k_1 = 5, \ k_2 = 1, \ \lambda = \frac{55}{336}, \)
\[
H = \frac{1}{\sqrt{21\alpha}}, \quad h_1 = -\frac{1}{4\sqrt{7\alpha}}, \quad h_2 \in \mathbb{R}.
\]

2.3. \( m = 5, \ k_1 = 4, \ k_2 = 1, \ \lambda = \frac{55}{336}, \)
\[
H = \frac{1}{4\sqrt{7\alpha}}, \quad h_1 = -\frac{1}{\sqrt{21\alpha}}, \quad h_2 \in \mathbb{R}.
\]

2.4. \( m = 6, \ k_1 = 3, \ k_2 = 1, \ \lambda = \frac{27}{140} \)
\[
H = \frac{1}{\sqrt{35\alpha}}, \quad h_1 = -\frac{1}{2\sqrt{5\alpha}}, \quad h_2 \in \mathbb{R}.
\]

2.5. \( m = 7, \ k_1 = 2, \ k_2 = 1, \ \lambda = \frac{13}{42} \)
\[
H = \frac{1}{2\sqrt{21\alpha}}, \quad h_1 = -\frac{1}{\sqrt{3\alpha}}, \quad h_2 \in \mathbb{R}.
\]
3. Stable solutions at well-defined values of $\lambda = \Lambda \alpha$ with $[3, 5, 2]$, $[4, 4, 2]$ and $[5, 3, 2]$ splitting in dimensions $(1 + 10)$ with zero variation of $G$ are expressed as follows:

\[
\begin{align*}
3.1. \quad & m = 3, \quad k_1 = 5, \quad k_2 = 2, \quad \lambda = \frac{991}{4732}, \\
& H = \frac{3}{2} \sqrt{\frac{1}{91\alpha}}, \quad h_1 = -\frac{2}{\sqrt{91\alpha}}, \quad h_2 = 5 \sqrt{\frac{1}{91\alpha}}. \\
3.2. \quad & m = 4, \quad k_1 = 4, \quad k_2 = 2, \quad \lambda = \frac{269}{1344}, \\
& H = \frac{1}{2\sqrt{21\alpha}}, \quad h_1 = -\frac{5}{4\sqrt{21\alpha}}, \quad h_2 = 4 \sqrt{21\alpha}. \\
3.3. \quad & m = 5, \quad k_1 = 3, \quad k_2 = 2, \quad \lambda = \frac{589}{2800}, \\
& H = \frac{1}{2\sqrt{70\alpha}}, \quad h_1 = -3 \sqrt{\frac{1}{70\alpha}}, \quad h_2 = 2 \sqrt{\frac{2}{35\alpha}}.
\end{align*}
\]

5. CONCLUSIONS

We have considered the $(1 + 8)$- and $(1 + 10)$-dimensional Einstein–Gauss–Bonnet (EGB) models with a $\Lambda$-term. By using the ansatz with diagonal cosmological metrics, we have found, for different splitting and certain $\lambda = \alpha \Lambda$, a class of explicit solutions with three Hubble-like parameters $H > 0$, $h_1$, and $h_2$, corresponding to submanifolds of dimensions $m \geq 3$, $k_1 > 1$, and $k_2 \geq 1$, respectively. The obtained solutions are exact. They are stable for $k_2 > 1$. As we know, stability plays a predominant role in exact solutions of a set of equations. All exact solutions obtained in this paper have been verified through the results of previously published papers [35, 38].

ACKNOWLEDGMENTS

The author is grateful to Prof. V. D. Ivashchuk for fruitful discussions.

FUNDING

This publication was supported by the RUDN University Strategic Academic Leadership Program.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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