Theoretical correction to the neutral $B^0$ meson asymmetry

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Abstract

Certain types of asymmetries in neutral meson physics have not been treated properly, ignoring the difference of normalization factors with an assumption of the equality of total decay width. Since the corrected asymmetries in $B^0$ meson are different from known asymmetries by a shift in the first order of CP- and CPT-violation parameters, experimental data should be analyzed with the consideration of this effect as in $K^0$ meson physics.

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I. INTRODUCTION

To search for CP, CPT violation in neutral meson physics, asymmetries are suggested in $K^0$ meson [1] and heavy meson [2–6]. Though asymmetries of decay modes from the same initial particles are not affected by the normalization of decay modes, asymmetries of $P^0$ (e.g. $K^0$, $B^0$, $D^0$) and $\bar{P}^0$ should be considered with the difference of the normalization factors as in $K^0$ meson physics. The normalization procedure will be derived in detail and the assumption of the equality of total decay width will be criticized. The correct theoretical asymmetries in $B^0$ meson to be analyzed with experimental data will be suggested and discussed.

II. THE NORMALIZATION OF NEUTRAL MESON

The effective Hamiltonian for the $P^0, \bar{P}^0$ system has eigenvectors given by:

$$
|P_S⟩ = \frac{[(1 + \epsilon_P + \delta_P)|P^0⟩ + (1 - \epsilon_P - \delta_P)|\bar{P}^0⟩]}{\sqrt{2}}
$$

$$
|P_L⟩ = \frac{[(1 + \epsilon_P - \delta_P)|P^0⟩ - (1 - \epsilon_P + \delta_P)|\bar{P}^0⟩]}{\sqrt{2}}
$$

The parameter $\epsilon_P$ represents a CP violation with indirect T violation, while the parameter $\delta_P$ represents a CP violation with indirect CPT violation.

We consider a coherent mixture of $P_S$ and $P_L$ whose amplitude at its proper time $\tau$ is described by the wave function

$$
Ψ(\tau) = a_S e^{-i(m_S - i\gamma_S^2)\tau}|P_S⟩ + a_L e^{-i(m_L - i\gamma_L^2)\tau}|P_L⟩
$$

The time evolution of the state is determined by an equation of the Schrödinger form:

$$
i \frac{d}{d\tau}Ψ = (M - \frac{1}{2}iΓ)Ψ
$$

where two Hermitian operators $M$ and $Γ$ called the mass and decay matrices. At any instant decays will occur to specified final state $f$ with a probability proportional to the square of the transition matrix element. The total decay rate is given by summing it over all final states $f$ consistent with energy and momentum conservation. This must be compensated by a decrease in the probability of the initial state [7] [12].

$$
- \frac{1}{N} \frac{d}{d\tau} [Ψ^†Ψ] = \sum |⟨f|T|Ψ(\tau)⟩|^2
$$

where $f$ is the final state and $\bar{f}$ is the CP-conjugate state of $f$. This equation should be normalized in a way that if we integrate over proper time, we should get the number of initial particles. To make Eq. (2.4), we can introduce the normalization factors.

$$
- \frac{1}{N} \frac{d}{d\tau} [Ψ^†Ψ] = \frac{\sum |⟨f|T|Ψ(\tau)⟩|^2}{\int \sum |⟨f|T|Ψ(\tau)⟩|^2}
$$

where $N = - \int \frac{d}{d\tau} [Ψ^†Ψ] d\tau$. The normalization factors is mentioned in Refs. [8] and implemented in the monte carlo simulation of OPAL collaboration [9].
Let’s consider two independent decays where the initial states are $P^0$ and $\overline{P}^0$. Since these two decays are independent, they should be normalized separately.

\[-\frac{1}{N} \frac{d}{d\tau} \langle \overline{\Psi} \Psi \rangle = \frac{\sum P_f(\tau) + P_\overline{f}(\tau)}{\sum R_f + R_\overline{f}} \]
\[-\frac{1}{N} \frac{d}{d\tau} \langle \overline{\Psi} \Psi \rangle = \frac{\sum \overline{P}_f(\tau) + \overline{P}_\overline{f}(\tau)}{\sum R_f + R_\overline{f}} \]

where $P_f = |\langle f|T|\Psi(\tau)\rangle|^2$, $R_f = \int d\tau P_f(\tau)$. Note that we do not have to assume that the normalization factors are the same for the independent decay modes of $P^0$ and $\overline{P}^0$. The decay rate is not $P_f(\tau)$ but $\frac{P_f(\tau)}{\sum R_f + R_\overline{f}}$. We still can compare $P_f(\tau)$ and $P_\overline{f}(\tau)$, since they have the same normalization factors. However we can not compare $P_f(\tau)$ and $\overline{P}_f(\tau)$ without considering the normalization factors. Since these normalization factors have CP- and CPT-violation parameters with the opposite signs, we will have the shift in the first order of CP- and CPT-violation parameters with certain types of asymmetries. It is suggested that the total decay widths of $P^0$ and $\overline{P}^0$ are equal by the CPT theorem \([10]\) so that we do not have the difference in normalization factors.

\[\int_0^\infty d\tau \sum_f P_f(\tau) + P_\overline{f}(\tau) = \int_0^\infty d\tau \sum_f \overline{P}_f(\tau) + \overline{P}_\overline{f}(\tau) \quad (2.7)\]

Note that the total decay widths are the normalization factors. It is claimed that this assumption is the only constraint, that is, we do not assume the equality of decay width in each corresponding decay mode of $P^0$ and $\overline{P}^0$. Even though we accept the equality of total decay width, we have to consider the difference of decay width in a specific decay modes. However, the lifetime equality of CPT symmetry was proved without consideration of the CP violation in the mixing of neutral meson and tested where the mixing is not involved \([11]\). With the mixing of neutral meson, we can not clearly define the lifetime of neutral meson since it decays in three different decay modes of short decay, long decay and interference of these two. The total decay widths with the mixing are also not as simple as in direct CPT symmetry due to indirect CP- and CPT-violation. Since we still have the difference between the total decay widths even without CPT violation as long as CP symmetry is violated, CPT theorem does not guarantee the equality of total decay widths in neutral meson with the mixing. The equality of total decay widths is an assumption not based on CPT theorem. Since we do not have any reason to assume the normalization factors of two independent decay modes are the same, I suggest to investigate asymmetries without assumption of the equality of total decay widths.

We can build asymmetries of $P^0$ and $\overline{P}^0$ with three different types of normalization methods. Experimentally, the decay rate we get from the decay channel is $N \frac{P_f}{\sum R_f + R_\overline{f}}$ where $N$ is the number of initial particles that we could get from the tagging. We can normalize by total number of initial particles $N$ or by the number of events of its own channel $N \frac{R_f + R_\overline{f}}{\sum R_f + R_\overline{f}}$ or by those of other channel $N \frac{R_g + R_\overline{g}}{\sum R_f + R_\overline{f}}$. 

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\[ A_1(\tau) \equiv \frac{\bar{P}_{\tau} \tau - \sum R_f}{\bar{P}_{\tau} \tau + \sum R_f} \]  
(2.8)

\[ A_2(\tau) \equiv \frac{\bar{P}_{\tau} \tau - \sum R_g}{\bar{P}_{\tau} \tau + \sum R_g} \]  
(2.9)

\[ A_3(\tau) \equiv \frac{\bar{P}_g \tau - \sum R_f}{\bar{P}_g \tau + \sum R_f} \]  
(2.10)

Time-integrated asymmetries are similar to these. Though the first type of normalization could be attained in the experiment by production tagging, the effects due to the different normalization factors are not clear unless all decay modes are similar, since the difference of the normalization factors depends on the different types of decay channels. The third type of normalization was suggested in Refs. [12] which is normalized by \((K, \bar{K})^0 \rightarrow \pi \pi\) decay channel. CP and CPT test in CPLEAR was analyzed based on this types of normalization, though the detailed calculations does not agree with Refs. [12].

III. B MESON ASYMMETRY

Considering the decays to flavor-specific final states \(f\) without direct CP, CPT violation such as semileptonic decay mode

\[ \langle f|T|B^0 \rangle = F_f \quad , \quad \langle f|T|\bar{B}^0 \rangle = 0 \]

\[ \langle \bar{f}|T|B^0 \rangle = F_{\bar{f}}^* \quad , \quad \langle \bar{f}|T|\bar{B}^0 \rangle = 0 \]  
(3.1)

Due to mixing, a produced \(B^0\) can decay to the final state \(\bar{f}\) (mixed event) in addition to the final state \(f\) (unmixed event). Here I followed the notation in [13]. The time-dependent decay rates are given by:

\[ P_f(\tau) \equiv |\langle f|T|B(\tau) \rangle|^2 \]

\[ = \frac{1}{4}|F_f|^2 \left\{ (1 + 4 \text{Re} \delta_B) \exp(-\gamma_S \tau) + (1 - 4 \text{Re} \delta_B) \exp(-\gamma_L \tau) + 2 \cos \Delta m \tau - 4 \text{Im} \delta_B \sin \Delta m \tau \exp(-\gamma \tau/2) \right\} \]  

\[ \bar{P}_f(\tau) \equiv |\langle \bar{f}|T|B(\tau) \rangle|^2 \]

\[ = \frac{1}{4}|F_f|^2 \{ \exp(-\gamma_S \tau) + \exp(-\gamma_L \tau) - 2 \cos \Delta m \tau \exp(-\gamma \tau/2) \} \]  

\[ P_{\bar{f}}(\tau) \equiv |\langle \bar{f}|T|\bar{B}(\tau) \rangle|^2 \]

\[ = \frac{1}{4}|F_{\bar{f}}|^2 \{ \exp(-\gamma_S \tau) + \exp(-\gamma_L \tau) - 2 \cos \Delta m \tau \exp(-\gamma \tau/2) \} \]  

\[ \bar{P}_{\bar{f}}(\tau) \equiv |\langle \bar{f}|T|\bar{B}(\tau) \rangle|^2 \]

\[ = \frac{1}{4}|F_{\bar{f}}|^2 \{ \exp(-\gamma_S \tau) + \exp(-\gamma_L \tau) - 2 \cos \Delta m \tau \exp(-\gamma \tau/2) \} \]  

(3.2)
with \( \gamma = \gamma_S + \gamma_L, \Delta m = m_L - m_S, \Delta \gamma = \gamma_S - \gamma_L, b^2 = (\Delta m)^2 + (\gamma^2)/4 \). The time-integrated decay rates of \( B^0 \) and \( \bar{B}^0 \) are:

\[
R_f \equiv \int_0^{\infty} d\tau P_f(\tau) = \frac{1}{4}|F_f|^2 \left[ \frac{1}{\gamma_S \gamma_L} - \frac{1}{b^2} \right] \left( 1 - 4 \text{Re}\, \epsilon_B \right),
\]

\[
\overline{R}_f \equiv \int_0^{\infty} d\tau \overline{P}_f(\tau) = R_f(\delta_B \to -\delta_B),
\]

\[
R_T \equiv \int_0^{\infty} d\tau P_T(\tau) = \frac{1}{4}|F_f|^2 \left[ \frac{1}{\gamma_S \gamma_L} - \frac{1}{b^2} \right] (1 - 4 \text{Re}\, \epsilon_B).
\]

The time-dependent asymmetries normalized by a specific decay channel should be

\[
A'_{CP}(T) = A_{CP}(T) - \frac{\triangle}{\Box},
\]

where the unnormalized time-dependent asymmetries are:

\[
A_{CP}(T) = \frac{\overline{P}_f - P_T}{P_f + P_T} \approx 4 \text{Re}\, \epsilon_B
\]

\[
A_{CP(T)}(T) = \frac{\overline{P}_f - P_T}{P_f + P_T} \approx 4 \text{Im}\, \delta_B \sin \Delta m \tau / (1 + \cos \Delta m \tau)
\]

in the case of \( \Delta \gamma \simeq 0 \). Note that these unnormalized asymmetries are not attainable from experiment due to the difference in normalization factors. Even we accept the equality of total decay width, if it is normalized by a specific decay mode, we get experimental data corresponding to \( A'_{CP(T)} \) not \( A_{CP(T)} \). In \( B^0 \) meson, the shift \( \frac{\triangle}{\Box} \) is \( \text{Im}\, \delta_B + 0.7 \text{Re}\, \epsilon_B \) where \( \tau_B = 1.548 \text{ps} \Delta m_d = 0.472 \text{ps}^{-1} \). Since this shift is not negligible, it should be considered in fitting procedure. Since \( A_{CP(T)} \) is also sensitive to CP parameter \( \epsilon_B \) besides CPT parameter \( \delta_B \), even though \( A'_{CP(T)} \) is zero consistent, it doesn’t mean CPT symmetry is conserved.

\[\text{IV. CONCLUSION}\]

Since the suggested asymmetries without the assumption of the equality of total decay width are different from the known asymmetries by a shift of the first order of CP- and CPT-violation parameters, this effect can not be ignored in the analysis of CP and CPT tests in neutral B meson.
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