On Lagrangian Formulation of Higher Spin Theories on AdS

Angelos Fotopoulos\(^a\) *, Kamal L. Panigrahi\(^b\) † and Mirian Tsulaia\(^a\) ‡

\(^a\) Department of Physics, University of Crete, 710 03 Heraklion, Crete, Greece

\(^b\) Department of Physics, Indian Institute of Technology, Guwahati, 781 039 India

Abstract

In this short note we present a Lagrangian formulation for free bosonic Higher Spin fields which belong to massless reducible representations of \(\mathcal{D}\)-dimensional Anti de Sitter group using an ambient space formalism.

*e-mail: afotopou@physics.uoc.gr
†e-mail: panigrahi@iitg.ernet.in
‡e-mail: tsulaia@physics.uoc.gr
1 Introduction

Massless Higher Spin gauge theories are classical field theories which describe (self)-interacting massless fields with an arbitrary spin (see [1] for recent reviews). Being an interesting subject by itself Higher Spin gauge theories recently triggered an increasing interest because of their possible connection with String and M-theory, namely it has been conjectured that massless Higher Spin theory is the most symmetrical phase of String theory the later being spontaneously broken phase of the former.

There are two distinct approaches for the formulation of massless Higher Spin gauge theory. One is a “frame like” formulation [2]–[3] and the other is a “metric like” formulation.[4]–[5] (see also [6] for a nonlocal formulation of massless Higher Spin fields on flat space–time background, [7] for a corresponding local formulation and [8] for a description of Higher Spin fields on a ‘tensorial extension’ of AdS space). The problem of self consistent interactions of massless Higher Spin fields has been effectively solved in the “frame like” formulation (see also [9]), where it has been shown that the consistency of interaction requires the Anti-de Sitter (AdS) background. In order to address this problem in the “metric like” formulation we would like to discuss free Lagrangian and equations of motions first. Until now there are several descriptions available. Apart from the original construction of [5] which involves off shell constraint on basic fields and gauge transformation parameters, and describes single irreducible Higher Spin mode one can obtain a Lagrangian description where all the fields and the gauge parameters are unconstrained [10] for irreducible and [11] for reducible Higher Spin fields on AdS. These descriptions include along with a basic field some auxiliary fields as well and after a partial gauge fixing the Lagrangian of [10] reduces to the one of [5]. In the present paper we give an alternative formulation to [11] which, though describes the same physical polarizations, is new in a sense that it is carried out in a (D + 1)-dimensional ambient space of a D- dimensional AdS.
The field equations for irreducible Higher Spin fields in an ambient space has been extensively studied in [12]. Below we use this formalism for Lagrangian description of reducible Higher Spin fields in the framework of BRST approach. We believe that this formulation might be useful for further studies of Higher Spin fields on AdS background.

The plan of the paper is as follows. In Section 2 we review the free equations, the Lagrangian and the gauge transformations which describe the reducible representations of the Poincare group and of the Anti-de Sitter group, using the triplet method for the description of Higher Spin fields. First we deal with the flat space-time background. Then for the sake of completeness we give some general definitions and a review of various basic facts about fields on \( D \) dimensional AdS space in terms of representations of AdS isometry group. Then in Section 3 we present a Lagrangian formulation of massless reducible bosonic Higher Spin fields using an ambient space formalism when the \( D \) dimensional AdS space is embedded in a \( D + 1 \) dimensional flat ambient space. This formalism is further illustrated through the simple example of a vector field propagating through \( D \) dimensional AdS space. In Subsection 3.2 we make some qualitative remarks on massive fields on AdS. In the Section 4, we present our conclusions and outlook. We collect some useful formulas for calculations in ambient space in the Appendix A.

2 Free Field Theory of Higher Spins

In this section we will review the construction of Higher Spin field theories for massless (and massive) gauge fields. As it will become clear from what follows, there is a particularly simple description of Higher Spin fields based on the triplet construction, see [13]. We deal exclusively with the case of fully symmetric Higher Spin tensors as we stress in several points in what follows.

We now review the system described in [13] for the case of a flat space-time and its AdS deformation given in [11], [14]. This system is named bosonic triplet and describes the propagation of reducible representations of the Poincare and Anti-de-Sitter groups. The name “triplet“ comes about, because a gauge invariant description of massless fields with spins \( s, s - 2, s - 4, \ldots 1/0 \) requires in addition, to a tensor field \( \phi \) of rank \( s \), the presence of two more tensor fields. We denote them as \( C \) (of rank \( s - 1 \)) and \( D \) (of rank \( s - 2 \)). After complete gauge fixing one is left only with physical polarization of Higher Spin fields with spins \( s, s - 2, s - 4, \ldots 0 \) or 1 depending if \( s \) is even or odd.

2.1 Massless Fields on Flat Space-Time.

It is rather instructive, to see how the triplet formulation works for the simplest non-trivial example. Consider a field \( \phi_{\mu\nu}(x) \) of rank 2. Therefore in the triplet there should exist a field \( C_\mu(x) \) of rank 1 and a field \( D(x) \) of rank zero. The triplet
equations take a rather simple form in this case [13]:

\[ \Box \phi_{\mu\nu} = \partial_{\mu} C_{\nu} + \partial_{\nu} C_{\mu} \]  
(2.1)

\[ C_{\mu} = \partial_{\nu} \phi_{\mu\nu} - \partial_{\mu} D \]  
(2.2)

\[ \Box D = \partial_{\mu} C^{\mu}. \]  
(2.3)

The system is invariant under the gauge transformations

\[ \delta \phi_{\mu\nu} = \partial_{\mu} \lambda_{\nu} + \partial_{\nu} \lambda_{\mu}, \quad \delta C_{\mu} = \Box \lambda_{\mu}, \quad \delta D = \partial_{\mu} \lambda^{\mu}. \]  
(2.4)

Let us introduce also a traceless field \( \tilde{\phi}_{\mu\nu} \)

\[ \tilde{\phi}_{\mu\nu} = \phi_{\mu\nu} - \frac{1}{D} \eta_{\mu\nu} \phi', \quad \phi' = \eta^{\nu\rho} \phi_{\rho\nu}. \]  
(2.5)

In order to see the physical polarizations described by these equations one can use the light-cone gauge fixing procedure i.e., eliminate \( \tilde{\phi}_{++}, \tilde{\phi}_{+-} \) and \( \tilde{\phi}_{-+} \) of the field \( \tilde{\phi}_{\mu\nu} \) using gauge transformation parameter \( \lambda_{\mu} \). The other nonphysical polarizations \( \tilde{\phi}_{--}, \tilde{\phi}_{-i} \) as well as the field \( C_{\mu} \) are eliminated by the field equations. Therefore one is left with the physical degrees of freedom \( \tilde{\phi}_{ij} \) \( (i, j, = 1, ..., D - 2) \) which correspond to the spin 2 field and a gauge invariant scalar

\[ \tilde{D} = \phi' - 2D. \]  
(2.6)

The system described above can be generalized to the case of arbitrary spin as well. The field equations\(^8\) turn out to be

\[ \Box \phi = \partial C, \]

\[ C = \partial \cdot \phi - \partial D, \]

\[ \Box D = \partial \cdot C. \]  
(2.7)

along with the gauge transformations

\[ \delta \phi = \partial \lambda, \quad \delta C = \Box \lambda, \quad \delta D = \partial \cdot \lambda, \]  
(2.8)

with a completely unconstrained parameter of gauge transformations describe Higher Spin fields with spins \( s, s - 2, ..., 1/0. \)

The field equations in (2.7) can be obtained as equations of motion from a Lagrangian

\[ \mathcal{L} = \frac{1}{2} \phi \Box \phi + s \partial \cdot \phi C + s(s - 1) \partial \cdot C D - \frac{s(s - 1)}{2} D \Box D - \frac{s}{2} C^2, \]  
(2.9)

\(^8\)Here the total symmetrization with respect to the indexes is assumed. The symbol \( \partial \cdot \) means divergence, while \( \partial \) is symmetrized action of \( \partial_{\mu} \) on a tensor as for example in the r.h.s. of the first of 2.1).
which as it is clear from the discussion above contains except from physical fields, some auxiliary fields as well. Let us note that the case of spin two is somewhat degenerate, since only in this case the combination (2.6) is gauge invariant. For the Higher Spin case we have

\[ \delta (\phi' - 2D) = \partial \lambda'. \]  

(2.10)

If one adds to the triplet equations “by hand“ an extra condition

\[ \phi' = 2D, \]  

(2.11)

then the parameter of the gauge transformations \( \lambda \) is no more unconstrained, but rather it obeys the vanishing trace condition

\[ \lambda' = 0 \]  

(2.12)

Further combining equations (2.7) and equation (2.11) one obtains the condition

\[ \phi'' = 0. \]  

(2.13)

By \( \phi' \) we denote the trace \( \phi'_{\mu...} \) and \( \phi'' \) the double trace \( \phi''_{\mu\nu...} \). The extra condition (2.11) gives the possibility to gauge away the fields \( C, D \) and \( \phi' \) and thus one obtains the propagation of a single irreducible Higher Spin mode. Alternatively using equations (2.7) and (2.11) one can express fields \( C \) and \( D \) via field \( \phi \) and put these expressions back into the Lagrangian (2.9) to obtain the one given by Fronsdal [4] which describes the propagation of a single massless Higher Spin field. Let us note however that one can obtain the equation (2.11) as an equation of motion from a Lagrangian which contains some more auxiliary fields and therefore formulate Lagrangians for Irreducible Massless Higher Spin fields ([15], [11] for bosons, and [16] for fermions) without any off-shell constraints on gauge transformation parameter and basic field \( \phi \).

### 2.2 Fields on AdS

Here we give some basic definitions concerning \( D \) dimensional Anti de Sitter space. More detailed treatment can be found in [17] or in reviews [18].

AdS space is a vacuum solution of Einstein equations with a negative cosmological constant. Its Riemann tensor has a form

\[ R_{\mu\nu\rho\sigma} = \frac{1}{L^2} \left( g_{\mu\rho} g_{\nu\sigma} - g_{\nu\rho} g_{\mu\sigma} \right), \]  

(2.14)

where \( L \) is an AdS radius and \( L \to \infty \) corresponds to a flat space-time limit. It is convenient to represent \( D \) dimensional AdS space with coordinates \( x^\mu (\mu = 0, ..., D - 1) \) and signature \( (1, D - 1) \) as a hyperboloid in \( D + 1 \) dimensional flat space with the signature \( (2, D - 1) \), parameterized by coordinates \( y^A (A = 0, ..., D) \). The coordinates in this ambient space obey the condition
\[
\eta_{AB} y^A y^B = -L^2, \quad \eta_{AB} \eta^{AB} = \mathcal{D} + 1, \quad \eta_{AB} = (-, +, +, \ldots, -).
\] (2.15)

Therefore an isometry group is a pseudo-orthogonal group of rotations \(SO(\mathcal{D} - 1, 2)\)
and the AdS space itself is isomorphic to a coset \(SO(\mathcal{D} - 1, 2)/SO(\mathcal{D} - 1, 1)\). In order to simplify the equations we set a radius of the AdS space to unity.

The AdS isometry group is noncompact and therefore its unitary representations are infinite dimensional. In order to build them it is convenient to rewrite the \(SO(\mathcal{D} - 1, 2)\) algebra

\[
[J_{AB}, J_{CD}] = \eta_{BC} J_{AD} - \eta_{AC} J_{BD} - \eta_{BD} J_{AC} + \eta_{AD} J_{BC},
\]

\[
J_{AB} = -J_{BA}, \quad (J_{AB})^+ = -J_{AB},
\]
in a different form. Namely after taking the following linear combinations

\[
J^\pm_a = (-iJ_{0a} \pm J_{Da}), \quad a = 1, \ldots, \mathcal{D} - 1
\]

\[
H = iJ_{0D},
\]

one obtains the commutation relations

\[
[H, J^\pm_a] = \pm J^\pm_a
\]

\[
[J^-_a, J^+_b] = 2(H\delta_{ab} + J_{ab})
\]

\[
[J_{ab}, J_{\pm c}] = \delta_{bc} J_{\pm a} - \delta_{ac} J_{\pm b}.
\]

(2.19)

as well as

\[
[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}.
\]

(2.20)

From these commutation relations one can conclude that AdS isometry group has a maximal compact subgroup \(SO(2) \otimes SO(\mathcal{D} - 1)\), spanned by generators \(H\) and \(J_{ab}\) respectively. These operators correspond to one dimensional and \(\mathcal{D} - 1\) dimensional rotations. The operator \(H\) is an operator of energy on AdS while time on AdS is defined as a variable conjugate to \(H\). Therefore time variable is compact and energy eigenvalues are quantized having integer values in order a wave function to be single valued. However usually one considers a covering space of AdS, where time is uncompactified. The quadratic Casimir operator in this basis has the form

\[
\mathcal{C}_2 = -\frac{1}{2} J^{AB} J_{AB} = H(H - \mathcal{D} + 1) + \frac{1}{2} J^{ab} J_{ab} - J^+_a J^-_a
\]

(2.21)

Infinite dimensional unitary representations of the AdS group are obtained from the “lowest weight states” \(|\bar{E}_0, s\rangle\), which is a representation of \(SO(2) \otimes SO(\mathcal{D} - 1)\). The later therefore is characterized by an eigenvalue of energy and by a Young tableaux.
\( s = (s_1, s_2, .., s_k), k = \left[ \frac{D-1}{2} \right] \). A lowest weight state is annihilated by all operators \( J_a^- \)

\[
J_a^- |E_0, s\rangle = 0. \tag{2.22}
\]

Then the other states of the representations are obtained by successively applying operators \( J_a^+ \) on a lowest weight state

\[
J_{a_1}^+ J_{a_2}^+ ... J_{a_k}^+ |E_0, s\rangle \tag{2.23}
\]

The crucial point is that representations obtained in this way do not always have positive norm. Therefore, when building new states with the help of operators \( J_a^+ \) one has to check their norm. For some special values of \( E_0 \) and \( s \) the norm is equal to zero. There is a unitarity bound on the energy \( E_0 \) below which the states get negative norms and should be excluded form the physical spectrum. The unitarity bound is saturated (norm of states becomes zero) for states with \( E_0 \) and \( s \) related via

\[
E_0 = s_1 + D - t_1 - 2 \tag{2.24}
\]

where \( t_1 \) is the number of rows of maximal length \( s_1 \) in the corresponding Young tableaux.\(^4\) The states which saturate the unitarity bound are identified with massless fields on AdS space. These states decouple from the original multiplet along with their descendants since their scalar product with the other states is zero. This effect is known as a ‘multiplet shortening” and it is interpreted as an enhancement of gauge symmetry.

Fields whose energy is above the unitarity bound are massive representations of AdS space. Both massive and massless fields on AdS have flat space – time counterparts i.e., one can take an usual flat space limit to obtain massless and massive fields propagating through Minkowski space-time. However there is one more type of fields on AdS, which have no flat space -time analogue. These are called singletons. For example the unitarity bound for spinless singleton is \( E_0 = \frac{1}{2}(D - 3) \). Singletons do not admit a proper field theoretical description in AdS bulk, rather they are described as boundary degrees on freedom.

Let us turn to a field theoretical description of massless fields on AdS. In order to obtain wave equations describing massless fields with an arbitrary integer value of spin on AdS background one has to find a relation between the quadratic Casimir operator and the D’Alembertian. The result for totally symmetric representations of AdS group i.e., when \( s = (s, 0, ..0) \), is [12]

\[
(\nabla^2 - (s - 2)(s + D - 3))F_{A_1, A_2 ... A_s}(y) = 0. \tag{2.25}
\]

\(^4\)There might be some extra states which saturate the unitarity bound. For example in the case of \( D = 4 \) there are two states for scalar massless fields with \( E_0 = 1 \) and \( E_0 = 2 \). These states have the same quadratic Casimir operator but correspond to different asymptotic behavior on the AdS boundary [19]
where
\[ \nabla^A = \theta^{AB} \frac{\partial}{\partial y^B}, \quad \theta^{AB} = \eta^{AB} + y^A y^B, \quad \nabla^2 = \nabla^A \nabla_A. \] (2.26)

A possible way to see where does the condition (2.25) come from is to introduce a auxiliary Fock space spanned by a set of oscillators
\[ [\alpha^A, \alpha^{B+}] = \eta^{AB}, \] (2.27)

and consider a state in this Fock space
\[ |\Phi\rangle = \frac{1}{s!} F_{A_1 A_2 \ldots A_s} \alpha^{A_1+} \alpha^{A_2+} \ldots \alpha^{A_s+} |0\rangle. \] (2.28)

The generators of \( SO(2, D - 1) \) can be represented as
\[ J^{AB} = L^{AB} + M^{AB}, \] (2.29)

where orbital part \( L^{AB} \) and spin part \( M^{AB} \) have the form
\[ L^{AB} = y^A \nabla^B - y^B \nabla^A, \quad M^{AB} = \alpha^A \alpha^B - \alpha^B \alpha^A. \] (2.30)

A field \( |\Phi\rangle \) in this Fock space is required to satisfy the mass–shell condition
\[ (\nabla^2 - m^2) |\Phi\rangle = 0, \] (2.31)

where \( m^2 \) is a “mass – like“ parameter to be determined, divergencelessness condition
\[ \alpha^A \nabla_A |\Phi\rangle = 0, \] (2.32)

and transversality condition
\[ y^A \alpha_A |\Phi\rangle = 0. \] (2.33)

The requirement of invariance of these equations under gauge transformations
\[ \delta |\Phi\rangle = \alpha^{A+} \nabla_A |A_1\rangle + y^A \alpha_{A+}^A |A_2\rangle \] (2.34)

leads to the mass–shell equation (2.25). If one computes the explicit form of quadratic Casimir operator in terms of realization (2.29)–(2.30), one finds its eigenvalues \(< C_2 >\). Comparing equation
\[ (C_2 - < C_2 >) |\Phi\rangle = 0 \] (2.35)

with (2.25) one obtains the expression for the unitarity bound in an alternative way.
3 Triplet in Ambient Space Formulation of AdS

In order to make symmetries of Lagrangian and equations of motion discussed in the previous subsection more transparent, we reformulate them in the ambient space of $D$ dimensional Anti de–Sitter space. The system of free irreducible massless Higher Spin fields propagating on an arbitrary dimensional Anti de–Sitter space in the formalism of the ambient space was considered in [5] at Lagrangian level and in [12] at the level of equations of motion. Below we consider the case of reducible representations of AdS group. Let us note also another approach [20] where a dimensional reduction have been performed from flat $D + 1$ dimensional space to $D$ dimensional Anti de–Sitter space. Alternatively we perform our construction on the Anti de–Sitter space from the very beginning.

The isometry group of AdS space i.e., $SO(2, D − 1)$ can be explicitly realized now in the whole construction. Further for simplicity one can specify the embedding by an extra condition [19]

$$y^A \frac{\partial}{\partial y^A} x^\mu = 0$$

(3.36)
i.e., choose the $x$–space coordinates to be homogeneous functions of degree zero in the ambient space coordinates. The field’s transformations from one coordinate system to another are

$$F_{\mu_1, \mu_2, ..., \mu_s}(x) = \frac{\partial y^{A_1}}{\partial x^{\mu_1}} \frac{\partial y^{A_2}}{\partial x^{\mu_2}} ... \frac{\partial y^{A_s}}{\partial x^{\mu_s}} F_{A_1, A_2, ..., A_s}(y).$$

(3.37)

Using this equation and (A.6) it is simple to find the inverse transformation

$$\frac{\partial x^{\mu_1}}{\partial y^{A_1}} \frac{\partial x^{\mu_2}}{\partial y^{A_2}} ... \frac{\partial x^{\mu_s}}{\partial y^{A_s}} F_{\mu_1, \mu_2, ..., \mu_s}(x) = \theta_{A_1}^{B_1} \theta_{A_2}^{B_2} ... \theta_{A_s}^{B_s} F_{B_1, B_2, ..., B_s}(y).$$

(3.38)

Now we are in a position to reformulate the triplet equations in ambient space. It is convenient to take a state in a Fock space in the form

$$|\bar{\Phi}\rangle = \frac{1}{s!} \bar{\Phi}_{A_1, ..., A_s}(y) \tilde{\alpha}^{A_1}+ ... \tilde{\alpha}^{A_s+} |0\rangle,$$

(3.39)

where the oscillators are defined by the transformation

$$\tilde{\alpha}^A = \theta^A_B \alpha^B$$

(3.40)

and therefore one has a commutation relation

$$[\tilde{\alpha}^A, \tilde{\alpha}^{B+}] = \theta^{AB}.$$

(3.41)

Let us note that a state in ambient space satisfies identically

$$T|\bar{\Phi}\rangle = 0, \quad T = y^A \alpha_A.$$

(3.42)
This means that (3.42) does not impose any further condition on $\tilde{\Phi}_{A_1 \ldots A_s}$.

In order to describe a triplet on $D$ dimensional Anti de–Sitter space it is convenient to introduce the set of oscillators $(\alpha^\mu, \alpha_{\mu}^\nu)$, which can be obtained from the previous ones (2.27) and the AdS vielbein

$$[\alpha^\mu, \alpha_{\mu}^\nu] = g^{\mu\nu}, \quad \alpha_{\mu}^\nu = e_{\alpha}^{\mu} \alpha_{a}^\alpha,$$  \hspace{1cm} (3.43)

where $g_{\mu\nu}$ denotes the AdS metric. An ordinary partial derivative is replaced by an operator

$$p_{\mu} = -i \left( \partial_{\mu} + \omega_{\mu}^{ab} \alpha_{a}^\alpha \alpha_{b}^\nu \right).$$  \hspace{1cm} (3.44)

Acting with $p_{\mu}$ on a state in Fock space

$$|\Phi\rangle = \frac{1}{(s)!} \varphi_{\mu_1 \mu_2 \ldots \mu_s}(x) \alpha_{\mu_1}^{\mu +} \ldots \alpha_{\mu_s}^{\mu +} |0\rangle,$$  \hspace{1cm} (3.45)

produces the proper covariant derivative

$$p_{\mu} |\Phi\rangle = -\frac{i}{(s)!} \alpha_{\mu_1}^{\mu +} \ldots \alpha_{\mu_s}^{\mu +} \nabla_{\mu} \varphi_{\mu_1 \mu_2 \ldots \mu_s}(x) |0\rangle,$$  \hspace{1cm} (3.46)

where in (3.44) $\omega_{\mu}^{ab}$ denotes the spin–connection on AdS and $\nabla_{\mu}$ is the AdS covariant derivative. These operators satisfy commutation relations

$$[p_{\mu}, p_{\nu}] = \alpha_{\mu}^{\nu} \alpha_{\nu}^{\mu} - \alpha_{\nu}^{\mu} \alpha_{\mu}^{\nu},$$  \hspace{1cm} (3.47)

due to the expression (2.14) for the Riemann tensor.

Further let us introduce the following operators

D’Alembertian operator

$$l_0 = g^{\mu\nu} (p_{\mu} p_{\nu} + i \Gamma_{\mu\nu}^{\lambda} p_{\lambda}) = p_{a}^{a} p_{a} - i \omega_{a}^{ab} p_{b}$$  \hspace{1cm} (3.48)

which acts on Fock-space states as the proper D’Alembertian operator

$$l_0 |\Phi\rangle = -\frac{1}{(s)!} \alpha_{\mu_1}^{\mu +} \ldots \alpha_{\mu_s}^{\mu +} \Box \varphi_{\mu_1 \mu_2 \ldots \mu_s}(x) |0\rangle.$$  \hspace{1cm} (3.49)

Divergence operator

$$l = \alpha_{\mu} p_{\mu}$$  \hspace{1cm} (3.50)

which acts on a state in the Fock space as divergence

$$l |\Phi\rangle = -\frac{i}{(s-1)!} \alpha_{\mu_1}^{\mu +} \ldots \alpha_{\mu_s}^{\mu +} \nabla_{\mu} \varphi_{\mu_1 \mu_2 \ldots \mu_s}(x) |0\rangle,$$  \hspace{1cm} (3.51)

Symmetrized exterior derivative operator,

$$l^{+} = \alpha_{\mu}^{\mu +} p_{\mu}$$  \hspace{1cm} (3.52)
\[ l^+|\Phi\rangle = -\frac{i}{(s+1)!}\alpha^{\mu_1+}\alpha^{\mu_2+}\ldots\alpha^{\mu_s+}\nabla_\mu \varphi_{\mu_1\mu_2\mu_3\ldots\mu_s}(x)|0\rangle \quad (3.53) \]

which is hermitian conjugate to the operator \( l \) with respect to the scalar product

\[ \int d^p x \sqrt{-g} \langle \Phi_1|\Phi_2 \rangle. \quad (3.54) \]

It is straightforward to obtain the commutation relations of the algebra generated by these operators. Having this algebra at hand one can construct the corresponding nilpotent BRST charge. Then if one builds the nilpotent BRST charge for this non-linear algebra one arrives to a Lagrangian description of a triplet on AdS according to the lines of [11].

In order to extend the aforementioned construction in the ambient space formulation, we should construct the ambient space operators which are the analogues of \( l, l^+ \), and \( l_0 \). It is straightforward to obtain the following relations between various kinds of derivatives in \( x \) and \( y \) spaces

\[ \nabla^\mu \tilde{\Phi}_{\mu_1\ldots\mu_s} = \frac{\partial y^{A_1}}{\partial x^{\mu_1}} \ldots \frac{\partial y^{A_s}}{\partial x^{\mu_s}} \left( \nabla^A + (D + s)y^A \right) \tilde{\Phi}_{A_1A_2\ldots A_s}, \quad (3.55) \]

\[ \nabla_{(\mu_1} \tilde{\Phi}_{\mu_2\ldots\mu_s)} = \frac{\partial y^{A_1}}{\partial x^{\mu_1}} \ldots \frac{\partial y^{A_s}}{\partial x^{\mu_s}} \left( \partial_{(A_1} \Phi_{A_2\ldots A_s)} + (s-1)y^A \eta_{(A_1A_2} \tilde{\Phi}_{A_3A_4\ldots A_s)} \right), \quad (3.56) \]

\[ \Box \tilde{\Phi}_{\mu_1\ldots\mu_s} = \frac{\partial y^{A_1}}{\partial x^{\mu_1}} \ldots \frac{\partial y^{A_s}}{\partial x^{\mu_s}} \left( \nabla^2 \Phi_{A_1\ldots A_s} - s\Phi_{A_1\ldots A_s} + 2s\partial_{(A_1}y^A \Phi_{A_2\ldots A_s)} \right)
+ s(s-1)y^A y^B \eta_{(A_1A_2} \Phi_{A_3B_4A_5\ldots A_s)} \right), \quad (3.57) \]

\[ \Box \tilde{\Phi}_{A_1\ldots A_s} = \nabla^2 \tilde{\Phi}_{A_1\ldots A_s}. \quad (3.58) \]

Right hand sides of equations (3.55) and (3.56) can be obtained acting on Fock space states by operators

\[ e = i\tilde{\alpha}^A \nabla_A + i(D + \tilde{\alpha}^{A+} \tilde{\alpha}_A)T - iT^+ \tilde{\alpha}^A \tilde{\alpha}_A, \quad (3.59) \]

\[ e^+ = i\tilde{\alpha}^{A+} \nabla_A - iT^+ \tilde{\alpha}^{A+} \tilde{\alpha}_A + i\tilde{\alpha}^{A+} \tilde{\alpha}_A^+ T, \quad (3.60) \]

respectively. Then the operator \( e_0 \) is by definition the one obtained by their commutation

\[ e_0 = [e, e^+] \quad (3.61) \]

and its explicit form is

\[ e_0 = -\nabla^2 + 2(\alpha^{A+} y_A)(\nabla_B \tilde{\alpha}^B) - 2(\tilde{\alpha}^{A+} \nabla_A)(\alpha^B y_B) \]

\begin{align*}
+ D \tilde{\alpha}^{A+} \tilde{\alpha}_A - \tilde{\alpha}^{A+} \tilde{\alpha}_A + (\tilde{\alpha}^{A+} \tilde{\alpha}_A)^2 - (\tilde{\alpha}^{A+} \tilde{\alpha}_A)^2 (\tilde{\alpha}^B \tilde{\alpha}_B) \\
+ 2T^+(\tilde{\alpha}^{A+} \tilde{\alpha}_A)T - (\tilde{\alpha}^{A+} \tilde{\alpha}_A)TT - T^+ T^+(\tilde{\alpha}^{A+} \tilde{\alpha}_A) - DT^+ T
\end{align*}

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The remaining commutation relations look as follows

\[ [e_0, e] = 2(1 - D)e - 4\alpha^A\alpha_Ae + 4e^+(\tilde{\alpha}^A\tilde{\alpha}_A). \] (3.63)

Finally one constructs the BRST charge for this system

\[ Q = c_0(e_0 + 6 - 2D - 4\tilde{\alpha}^A\tilde{\alpha}_A) + ce^+ + c^+e - c^+c_b_0 + (2D + 4\tilde{\alpha}^A\tilde{\alpha}_A - 6)c_b c^+ b - 4c_0c^+b^+(\tilde{\alpha}^A\tilde{\alpha}_A) + 4c_0cb(\tilde{\alpha}^A\tilde{\alpha}_A) + 12c_0c^+b^+cb. \] (3.64)

where we have introduced ghost variables \(c_0, c^+, c\) with ghost number +1 and corresponding antighost variables \(b_0, b, b^+\) with ghost number −1. After defining the ghost vacuum as

\[ b_0|0\rangle_{gh} = c|0\rangle_{gh} = b|0\rangle_{gh} = 0 \] (3.65)

one obtains a Lagrangian

\[ \mathcal{L} = \int dc_0\langle \tilde{\Phi}|Q|\tilde{\Phi}\rangle \] (3.66)

which is invariant under gauge transformations

\[ \delta|\tilde{\Phi}\rangle = Q|\Lambda\rangle. \] (3.67)

The total vacuum is now a direct product of ghost and \(\tilde{\alpha}\) vacua

\[ |0\rangle = |0\rangle_{\tilde{\alpha}} \otimes |0\rangle_{gh}, \quad \tilde{\alpha}^A|0\rangle_{\tilde{\alpha}} = 0 \] (3.68)

and the integration of the ghost zero mode is defined as

\[ \int dc_0\langle 0|c_0|0\rangle = 1. \] (3.69)

One can conclude from (3.66) and (3.67) that in order for the Lagrangian to have ghost number zero the field \(|\tilde{\Phi}\rangle\) must have ghost number zero, while gauge transformation parameter \(|\Lambda\rangle\) must have ghost number −1. Therefore their expansion in terms of ghost variables has the form

\[ |\tilde{\Phi}\rangle = |\phi\rangle + c^+b^+|D\rangle + c_0b^+|C\rangle, \] (3.70)

\[ |\Lambda\rangle = b^+|\lambda\rangle \] (3.71)

where states \(|\phi\rangle, |D\rangle, |C\rangle\) and \(|\lambda\rangle\) are expanded in terms of oscillators \(\tilde{\alpha}^A\).

Using relations above it is straightforward to obtain the corresponding Lagrangian

\[ \mathcal{L} = \langle \phi|e_0 + 6 - 2D - 4\tilde{\alpha}^A\tilde{\alpha}_A|\phi\rangle - \langle D|e_0 + 6 + 2\tilde{\alpha}A\tilde{\alpha}_A|D\rangle + \langle C||C\rangle - \langle \phi|e^+|C\rangle + \langle D|e|C\rangle - \langle C|e|\phi\rangle + \langle C|e^+|D\rangle + 4\langle D|\tilde{\alpha}^A\tilde{\alpha}_A|\phi\rangle + 4\langle \phi|\tilde{\alpha}^A\tilde{\alpha}_A|D\rangle, \] (3.72)
gauge transformations
\[ \delta |\phi\rangle = e^+ |\Lambda\rangle, \quad \delta |C\rangle = e_0 |\Lambda\rangle, \quad \delta |D\rangle = e |\Lambda\rangle, \] (3.73)

and equations of motion
\[ (e_0 + 6 - 2D - 4\tilde{\alpha}^{A+}\tilde{\alpha}_A)|\phi\rangle = e^+ |C\rangle - 4\tilde{\alpha}^{A+}\tilde{\alpha}_A|D\rangle, \] (3.74)
\[ |C\rangle = e|\phi\rangle - e^+ |D\rangle, \] (3.75)
\[ (e_0 + 6 + 2D + 4\tilde{\alpha}^{A+}\tilde{\alpha}_A)|D\rangle = e|C\rangle + 4\tilde{\alpha}^{A}\tilde{\alpha}_A|\phi\rangle. \] (3.76)

Equations (3.73) and (3.74), can be easily written in component notation using (3.55, 3.56). Namely for gauge transformations we have
\[ \delta\phi = (s - 1)\eta(y \cdot \Lambda) + \partial \Lambda, \]
\[ \delta D = (D + s - 2)(y \cdot \Lambda) + \nabla \cdot \Lambda, \] (3.77)
\[ \delta C = \nabla^2 \Lambda + 2(s - 1)\partial (y \cdot \Lambda) + (s - 1)(s - 2)\eta((yy) \cdot \Lambda) \]
\[ + (s - 1)(2 - s - D)\Lambda + 2\eta\Lambda', \]
where as before \( \partial \) means action of the derivative \( \partial A \), \( \nabla \cdot \) means divergence and a total symmetrization with respect to the indices is assumed. In addition we have defined:
\[ (y \cdot \phi)_{A_1...A_{s-1}} = y^A\phi_{AA_1...A_{s-1}}, \quad (\phi')_{A_1...A_{s-2}} = \theta^{AB}\phi_{A,B,A_1...A_{s-2}} \] (3.78)
and
\[ \eta\phi = \eta(\phi'_{A_1...A_s}), \] (3.79)

while equations of motion look as follows:
\[ (\nabla^2 - [(2 - s)(3 - D - s)])\phi + 2s\partial(y \cdot \phi) + s(s - 1)\eta(\eta C \cdot \phi) + 2\eta\phi' = \]
\[ = (s - 1)\eta(y \cdot C) + \partial C + 8\eta D, \] (3.80)
\[ C = (D + s - 1)y \cdot \phi + \nabla \cdot \phi - (s - 2)\eta(y \cdot D) - \partial D, \]
\[ (\nabla^2 - [s(D + s - 1) + 4])D + 2(s - 2)\partial(y \cdot D) + \]
\[ (s - 2)(s - 3)\eta(yy \cdot D) + 2\eta D' = (D + s - 2)y \cdot C + \nabla \cdot C - 4\phi'. \]

Again as it was for the case for a triplet in flat space time one can add by hand an extra condition
\[ \tilde{\alpha}^A\tilde{\alpha}_A|\phi\rangle = 2|D\rangle \] (3.81)

and restrict the parameter of gauge transformations by the condition \( \tilde{\alpha}^A\tilde{\alpha}_A|\Lambda\rangle = 0 \)
to obtain the Lagrangian description [5]
\[ \mathcal{L} = \langle \phi | e_0 - e^+ e + \frac{1}{2} e^+ e^+ \tilde{\alpha}^A\tilde{\alpha}_A + \frac{1}{2} \tilde{\alpha}^{A+}\tilde{\alpha}^+_A e e - \frac{1}{2} \tilde{\alpha}^{A+}\tilde{\alpha}^+_A e_0 \tilde{\alpha}^B\tilde{\alpha}_B \]
\[ - \frac{1}{4} \tilde{\alpha}^{A+}\tilde{\alpha}^+_A e^+ e \tilde{\alpha}^B\tilde{\alpha}_B + 6 - 2D - 4\tilde{\alpha}^{A+}\tilde{\alpha}_A \]
\[ - \frac{1}{2} \tilde{\alpha}^{A+}\tilde{\alpha}^+_A (1 + D + 2\tilde{\alpha}^B\tilde{\alpha}_B) \tilde{\alpha}^C\tilde{\alpha}_C |\phi\rangle \]

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of a single irreducible Higher Spin mode on AdS space. The integration measure for
the action in ambient space is given in [19]
\[ d^{D+1}y\delta^{D+1}(y^2 + 1) = d^D x \sqrt{-g} \] (3.83)

3.1 A Vector Field on AdS Background

It will be useful to show explicitly through a specific example how the formalism
described in the previous subsection leads to a description of massless fields on AdS
background. For this we consider the simplest case of a massless vector field on AdS
background. The Lagrangian (3.72) which in this case contains only one physical
field \( \phi_A \) and an auxiliary field \( C \) is:
\[
\mathcal{L} = \theta^{AB} \phi_A (\nabla^2 + (D - 2)) \phi_B + 2 \phi_A \nabla^A (y^B \phi_B) - C^2 - (\nabla_A C) \phi^A + (\nabla_A \phi^A) C + D C y^A \phi_A. \] (3.84)

The equations of motion in (3.80) become:
\[
\nabla^2 \phi_A + (D - 2) \phi_A + 2 \partial_A (y^B \phi_B) = \partial_A C, \tag{3.85}
\]
\[
\nabla_A \phi^A + D y^A \phi_A = C, \tag{3.86}
\]
with gauge transformation rules (3.77):
\[
\delta \phi_A = \partial_A \lambda, \quad \delta C = \nabla^2 \lambda. \tag{3.87}
\]

One might suppose that when making an embedding into higher dimensional space
some extra degrees of freedom can appear, for example a vector in an ambient space
might correspond to a vector and a scalar in \( x \) space. It is not so however since
in \( D + 1 \) dimensions one has some extra gauge freedom which allows to eliminate
“extra” degrees of freedom. In particular one can impose the gauge condition \( y^A \phi_A = 0 \), which eliminates a scalar from the spectrum. Then one is left with a gauge
parameter \( \tilde{\lambda} \) which is constrained through \( y^A \partial_A \lambda = 0 \). Using this parameter one can
further gauge away the field \( C \) and check that this further gauge fixing procedure is
consistent with equations of motion and therefore is correct [12]. Finally one is left
with equations
\[
(\nabla^2 + D - 2) \phi_A = 0, \quad \partial^A \phi_A = y^A \phi_A = 0, \tag{3.88}
\]
which describe a massless vector field on AdS. It is simple to rewrite this system in
the \( x \)-space. Namely the Lagrangian (3.84), which contains physical field \( \phi_\mu \) and
an auxiliary field \( C \), takes the form
\[
\mathcal{L} = \frac{1}{2} \phi^\mu \nabla_\mu \phi_\mu + (\nabla_\mu \phi^\mu) C - \frac{1}{2} C^2 + \frac{1}{2} (D - 1) \phi^\mu \phi_\mu. \tag{3.89}
\]

This Lagrangian is invariant under gauge transformations
\[
\delta \phi_\mu = \nabla_\mu \lambda, \quad \delta C = \Box \lambda \tag{3.90}
\]

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and gives the following equations of motion

\[ \Box \phi_\mu = \nabla_\mu C - (D - 1) \phi_\mu \]  
\[ \nabla_\mu \phi^\mu = C. \]  

(3.91)

(3.92)

We can use (3.90) to gauge away C and then we get:

\[ (\Box + D - 1) \phi_\mu = 0, \quad \nabla_\mu \phi^\mu = 0. \]  

(3.93)

### 3.2 Massive Fields on AdS

Although it is beyond the main scope of this paper, we discuss briefly how one could proceed in the case of massive fields in AdS. This construction has been outlined in [21] where the detailed study of massive Higher Spin fields on flat space was performed in the framework of BRST approach. We briefly summarize their arguments.

The crucial point in this construction is played by the method of auxiliary representations (see [22] for a review). This method can be more transparently demonstrated in the simpler example for the \( SO(2, 1) \) algebra.

Suppose we want to build a BRST charge for the constraints

\[ M = \frac{1}{2} \tilde{\alpha}^A \tilde{\alpha}_A, \quad M^+ = \frac{1}{2} \tilde{\alpha}^{A+} \tilde{\alpha}_{A^+}, \quad N = \tilde{\alpha}^{A+} \tilde{\alpha}_A + \frac{D}{2}. \]

Though these operators form a closed algebra, the operator \( N \) cannot be included into the total set of constraints since it is strictly positive. The way out, is to introduce an extra oscillator \([b, b^+] = -1\) and build an auxiliary realization of the algebra in terms of these oscillators and some parameter \( h \). The only requirement for the \( M_{\text{aux}}, M^+_{\text{aux}}, N_{\text{aux}} \) is that they obey the same commutation relations as the original operators and that \( N_{\text{aux}} \) depends on \( h \) linearly.

For the particular example under consideration these representations can be chosen as

\[ M_{\text{aux}} = b \sqrt{h + 1} + b^+ b, \quad M^+_{\text{aux}} = b^+ \sqrt{h + b^+ b}, \quad N_{\text{aux}} = -2b^+ b - h. \]  

(3.94)

Next one defines modified operators \( \tilde{M}, \tilde{M}^+, \tilde{N} \) as the sum of old and new operators and build the BRST charge

\[ Q = c_N \tilde{N} + c_M \tilde{M}^+ + c_M^+ \tilde{M} + c_N (2c^+_N b_N + 2b^+_N c_N - 2) - c^+_M c_M b_N \]  

(3.95)

for the system of modified constraints. Finally one considers an auxiliary phase space \((x_h, h)\) such that \([x_h, h] = i\) and performs similarity transformation \( Q \to U^{-1} Q U \) with \( U = e^{i \pi x_h} \) and \( \pi = N - 2b^+ b - 2 + 2c^+_M b_M + 2b^+_M c_M \). In this way one eliminates the dependence of the BRST charge \( Q \) on the offending ghost variables \( c_N \) and \( b_N \) but maintains its nilpotency at the same time.

This procedure is completely general in the sense that it can be used for BRST construction of algebras either linear or nonlinear when some of the Cartan generators (operator \( N \) in our case) are excluded from the set of constraints due to some physical reasons (i.e., because it is strictly positive). The problem thus reduces to the question
of how to build auxiliary representations for the algebra under consideration and as a result a general state in Fock space will depend also on extra oscillators $b_i^\pm$.

An analogous picture appears for the case of massive Higher Spin fields [21] since equation (3.61) becomes now

$$[e, e^+] = \tilde{e}_0 - m^2 ,$$

(3.96)

where $\tilde{e}_0$ is now a “modified D’Alembertian” for massive bosonic fields on AdS i.e., the mass parameter plays the role of “central charge“ in the algebra. The auxiliary representations for the nonlinear algebra under consideration was built in [23] in terms of two sets of oscillators and two parameters which correspond to mass and spin. It is an interesting open problem to carry out explicit calculations for the Lagrangian describing massive Higher Spin fields on AdS, as well as to study its supersymmetric extension and the mechanism of mass generation especially in the framework of AdS/CFT duality [24] – [25].

4 Conclusions

In this paper, we have studied some aspects of Lagrangian formulation for Higher Spin gauge fields in AdS\(_D\) background, by using the triplet method for describing Higher Spin fields. We have embedded AdS\(_D\) into a \(D+1\) dimensional flat spacetime, the ambient space. Consequently we generalized the AdS spacetime triplet formalism to its ambient space analogue. We further demonstrated the equivalence of the two formulations for the simple case of a U(1) gauge field on AdS\(_D\).

The formulation in terms of ambient space might be useful when considering self interaction of the triplet [26] on AdS background and for a further study of the properties of massless and massive Higher Spin fields.

A further application might be in the study of a possible connection between massless and massive Higher Spin theory with superstring theory. Namely it is interesting to understand the properties of string theory in the high energy limit [27]–[28] especially on a highly curved Anti de Sitter background as well as a further study of AdS/CFT duality beyond the Supergravity limit [29]. One more topic of interest is to perform the analogous study for mixed symmetry fields on AdS [30], (see [31] – [32] for a recent discussion for flat space–time in “metric like“ approach).

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A Some Formulas in Ambient Space

One can check some useful relations for an ambient space

\[ \theta^{AB} \theta^{BC} = \theta^{AC}, \quad \nabla^C \theta^{AB} = \theta^{CA} y^B + \theta^{CB} y^A, \quad \nabla^A \theta^{AB} = \mathcal{D} y^B, \quad (A.1) \]

\[ [\nabla_A, \nabla_B] = -y_A \nabla_B + y_B \nabla_A, \quad (A.2) \]

\[ [\nabla^2, y^A] = 2 \nabla_A + \mathcal{D} y^A, \quad [\nabla^2, \nabla^A] = (2 - \mathcal{D}) \nabla^A + 2 y^A \nabla^2. \quad (A.3) \]

\[ y^A \nabla_A = 0, \quad y^A \theta^B_A = 0, \quad \nabla^A y_A = \mathcal{D}. \quad (A.4) \]

The induced metric, its inverse and Christoffel connection look as follows:

\[ g_{\mu\nu} = (\partial_{\mu} y^A) (\partial_{\nu} y^A), \quad g^{\mu\nu} = (\nabla^A x^\mu)(\nabla^A x^\nu), \quad \Gamma^\lambda_{\mu\nu} = \frac{\partial x^\lambda}{\partial y^A} \frac{\partial^2 y^A}{\partial x^\mu \partial x^\nu}. \quad (A.5) \]

We have also

\[ \theta^A_B = \frac{\partial y^A}{\partial x^\mu} \frac{\partial x^\mu}{\partial y^B}, \quad \delta^\mu_\nu = \frac{\partial y^A}{\partial x^\mu} \frac{\partial x^\nu}{\partial y^A}, \quad (A.6) \]

as well as

\[ \theta^{AB} = g^{\mu\nu} (\partial_{\mu} y^A)(\partial_{\nu} y^B), \quad (A.7) \]

which follow from the differentiation rules

\[ \theta^{AB} \frac{\partial}{\partial y^B} = \eta^{AC} \frac{\partial x^\mu}{\partial y^C} \frac{\partial}{\partial x^\mu}, \quad \frac{\partial}{\partial x^\mu} = \frac{\partial y^A}{\partial x^\mu} \frac{\partial}{\partial y^A}. \quad (A.8) \]

Note that

\[ \nabla^A x^\mu = \frac{\partial x^\mu}{\partial y^A} \quad (A.9) \]

due to the embedding condition (3.36). Finally it is straightforward to derive the following relations

\[ \theta^{AB} \frac{\partial x^\mu}{\partial y^B} = g^{\mu\nu} \frac{\partial y^A}{\partial x^\nu}, \quad \nabla^2 x^\mu = -\Gamma^\mu_{\nu\rho} g^{\nu\rho}. \quad (A.10) \]

\[ \nabla^A \nabla_A \Phi^{C_1, C_2, \ldots, C_s} = \nabla^\mu \nabla_\mu \Phi^{C_1, C_2, \ldots, C_s}, \quad \nabla^A \frac{\partial y^A}{\partial x^\nu} = g_{\mu\nu} y^A. \quad (A.11) \]

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