On vortex sheet intensity reconstruction in meshless vortex particle method for two-dimensional flows simulation

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Abstract. The problem of numerical solution of the boundary integral equation arising in 2D Lagrangian meshless vortex particle methods is considered. The Viscous vortex domains method used for viscous flow simulation, that leads to a large number of vortex particles in the flow domain, especially in the near-wall region. From a mathematical point of view, it leads, in turn, to the fast oscillating right-hand side of the integral equation. An original correction technique is developed that permits one to obtain a numerical solution with high resolution without grid refining. It is based on the Galerkin approach where additional terms are included in numerical solution expansion, that correspond to the subgrid-scale of the solution variation.

1. Introduction

The Vortex Particle Method (VPM) initially was developed for loads estimation acting on structures in the flow. The brief history of the vortex method before the 1980s can be found in [1]. Normally, inviscid and incompressible flow was considered in early modifications of VPM. Despite such simplification, for some problems arising in engineering practice, the mentioned approach permits to obtain results, which are in good agreement with experimental data. Low numerical cost of the simulation was the main reason for the popularity of the vortex particle methods in 1970–1980s and early 1990s. Moreover, vortex methods not only allow for steady-state simulation, when it is necessary to estimate the so-called stationary aerodynamic coefficients, but they are suitable for unsteady regimes simulation, again with a low numerical cost. Such efficiency of vortex methods can be easily explained: the position of the separation point (or points) and separation line (or lines) for 2D and 3D cases, respectively, is considered to be preliminary known. It corresponds to corner points and edges for 2D airfoils and edge lines for 3D bodies. As the result, these methods can hardly be applied for flow simulation around bluff bodies, where the position of the separation point/line is unknown and it varies in time. In such cases viscosity plays a decisive role, so the inviscid flow model seems to be too simplified.

The next ‘generation’ of the vortex methods is based on the well-known paper of Chorin [2]. The random-walk method for viscosity effect accounting remains rather popular nowadays due to its simplicity and robustness. The other approach, which is more complicated and requires rather ‘delicate’ implementation, is deterministic. It is also called ‘diffusive velocity method’
and this diffusive velocity describes additional vorticity redistribution in the flow domain due to the viscosity effect. This approach was introduced in [3] and developed significantly in [4], where it was called ‘Viscous Vortex Domains method’. One more approach also should be noted [5] that also seems to be efficient for 2D viscous flows simulation.

According to the Viscous Vortex Domains method, vorticity flux is simulated on the airfoil surface line, and there are a lot of vortex particles in the near-wall region. Their influences are taken into account in the right-hand side of the governing boundary integral equation with respect to vortex sheet intensity. Previously developed numerical schemes for its approximate solution, based on the Galerkin approach, provide 1st–3rd order of accuracy for rather ‘smooth’ (non-oscillating) right-hand side. Such a situation takes place in cases of a potential flow or in presence of vortex particles which are far enough from the airfoil.

The aim of this paper is to develop an accurate numerical scheme, suitable for vortex sheet intensity reconstruction taking into account the influence of closely placed vortex particles. This problem is solved by the analytical estimation of the contribution of closely placed vortex particles when space discretization of the surface grid is not enough for direct solution reconstruction.

2. Vorticity generation simulation

In numerical simulation the airfoil in the flow is modelled with thin vortex sheet on its surface line, which intensity satisfies the boundary integral equation (BIE) of the 1st or 2nd kind (there are 2 mathematically equivalent models [6]):

\[ \oint_K Q_n(r, \xi) \gamma(\xi) d\xi = f_n(r) \quad \text{or} \quad \oint_K Q_\tau(r, \xi) \gamma(\xi) d\xi - \frac{1}{2} \gamma(r) = f_\tau(r), \tag{1} \]

where the kernels have the following form:

\[ Q_n(r, \xi) = Q(r, \xi) \cdot \tau(r), \quad Q_\tau(r, \xi) = -Q(r, \xi) \cdot n(r). \]

Vector-kernel \( Q(r, \xi) \) is similar to the gradient of the Green’s function for 2D case,

\[ Q(r, \xi) = \frac{k \times (r - \xi)}{2\pi |r - \xi|^2}; \]

where \( k \) is unit vector orthogonal to the flow plane; the right-hand sides are \( f_n(r) = f(r) \cdot n(r) \), \( f_\tau(r) = f(r) \cdot \tau(r) \),

\[ f(r) = -\left( V_\infty + \sum_{k=1}^{N_w} Q(r, r_k) \Gamma_k \right). \]

Here \( n(r) \) and \( \tau(r) \) are outer normal vector and tangent vector, chosen such as \( n(r) \times \tau(r) = k; \) \( V_\infty \) is constant incident flow velocity; \( N_w \) is number of vortex particles in the flow domain; \( r_k \) and \( \Gamma_k, k = 1, \ldots, N_w \), are positions and circulations of the vortex particles, respectively.

Both these equations have an infinite set of solutions. To select the unique solution, one should consider an additional condition for the value of the integral from the solution:

\[ \oint_K \gamma(\xi) d\xi = \Gamma. \]

This equation is solved together with the governing BIE by using the approach suggested in [7].

We consider the simplest case when the rigid airfoil is immovable and there are no other walls in the unbounded flow domain. For movable/deformable airfoil and for multiple airfoils, additional terms are added to the BIEs (1), however, all the following results remain applicable.
Because of the properties of the BIEs (1) kernels \( Q_\tau \) is bounded or, at least, integrable; \( Q_n \) is singular and the integral is understood as the Cauchy principal value \([7]\), one should distinguish numerical schemes for their numerical solving. The \( T \)-schemes (based on the solution of the equation with kernel \( Q_\tau \)) provide as a rule much more accurate solution \([8]\).

Note, that in much of existing models and codes, however, the \( N \)-equation is being solved \([7, 9, 10]\), may be due to ‘historical’ reasons: in the 1st generation methods — Discrete Vortex Method and similar approaches — the vortex wake had been considered as thin surface (vortex sheet) in the flow domain where tangent velocity has a discontinuity. This vortex sheet is normally discretized into separate vortex particles, which move in the flow according to the Euler equations (inviscid flow). So there is no vorticity in the near-wall region and the velocity field is quite smooth. This, in turn, makes it possible to solve the BIE with acceptable accuracy even by using rather coarse numerical schemes. The same also concerns velocity field reconstruction.

In opposite, in modern models vorticity is considered as ‘real’ 2D distribution (also modelled with large number of vortex particles). Satisfaction of the no-slip boundary condition, instead of the no-through condition in the above mentioned approaches, leads to formation of a ‘boundary layer’ with essentially non-zero vorticity in the flow domain near the airfoil surface line. The schematic picture is shown in fig. 1

**Figure 1.** The scheme of flow simulation around the airfoil in the framework of Discrete Vortex Method (a) and Viscous Vortex Domains method (b). Vortex wake after the circular cylinder simulated by using VVD method (c)

Numerical simulations show, that the accuracy of the flow modelling using the Viscous Vortex Domains method is usually restricted just by the accuracy of the BIE numerical solution.

In \([11, 12]\) the family of numerical schemes is developed for the \( T \)-type boundary equation, that is based on the Galerkin approach and provides a piecewise-constant and a piecewise-linear (discontinuous or continuous) solution for vortex sheet intensity \( \gamma(\xi) \), as well as a consistent algorithm for velocity reconstruction in the flow domain (including the near-wall region). For the airfoil boundary replaced with a polygon with \( n \) sides (which are usually called ‘panels’), such schemes provide the 1st or the 2nd order of accuracy with respect to \( n \) for vortex sheet intensity and only the 1st order of accuracy for velocity reconstruction.

In practice, it is important to resolve accurately the contributions of separate vortex particles, that form the right-hand side of the BIEs (1), to the numerical solution. To do it ‘straightforwardly’, one should provide the length of panels to be smaller than the distance from the airfoil boundary to the vortex particle. However, brief estimation shows, that for flows simulations at the Reynolds numbers higher than several hundred, we should deal with the number of panels of order \( 10^6 \). Of course, it is impossible from the computational point of view.

At the same time, if we consider a rather simple model problem with circular airfoil and unique separate vortex particle (its exact solution follows from complex mapping theory and mirroring technique \([13]\)), one can see, that the above mentioned numerical schemes provide significant error for the velocity field reconstruction: the most accurate scheme (and simultaneously the most time consuming one, with piecewise-linear discontinuous solution) provides 1 % error for
the vortex element placed at the distance, equal to the panel length \( h \); 3 % for distance \( h/2 \) and more than 10 % for distance \( h/4 \).

Such error follows from the impossibility to reconstruct solution of the BIE using the traditional Galerkin approach. The solution varies the more significantly along the panel the closer vortex particle becomes to the airfoil boundary. In fig. 2 numerical solution is shown in comparison with the exact one for the panels which are close to the vortex element.

Note, that the ‘integral’ characteristics (average values on the panels) of the numerical solution are very close to reference values for all the schemes, however significant ‘local’ errors lead to incorrect velocity field reconstruction in the near-wall region. As the result, we can conclude, that by using known numerical schemes with a reasonable number of panels, it is hardly possible to simulate correctly the flows at a rather high Reynolds number and especially turbulent regimes, even in a simplified statement.

3. The improved technique of the boundary integral equation solution

As it was shown above, to solve the problem of correct velocity field reconstruction, it is necessary to improve somehow the numerical solution of the BIE (1). We will not consider the easiest way — to refine the surface grid and deal with an extremely high number of panels — because it leads to the extremely high-dimension problem (∼ 1 million unknown values for one airfoil).

From the other hand, the developed numerical schemes [11, 12] and the Galerkin method seem to be attractive due to a number of reasons. As the result, the original approach is developed which makes it possible to solve the above-mentioned problem. Firstly, it was shown, that it is impossible in principle to provide the 2nd order of accuracy for velocity reconstruction in case of airfoil surface line discretization with straight (rectilinear) panels. Developed ‘curvilinear’ scheme [14, 15, 16], based on the same Galerkin-type ideas, is more accurate, particularly, because of the possibility of piecewise-quadratic solution reconstruction and correct BIE approximation in case of non-uniform surface line discretization. However, it doesn’t allow for solving the above-mentioned problem.

A fruitful idea of solution correction consists of adding to the numerical solution some term, which takes into account the local behavior of the solution on one panel, caused by the influence of closely-placed vortex particle:

\[
\gamma(\xi) = \sum_k \gamma_k^*(\xi) + \sum_q \gamma^q \phi^q(\xi) \quad \text{(for the particular panel)},
\]

where \( \gamma^* \) is the correction term; \( \phi^q \) is the basis functions (i.e., constant for \( q = 0 \), linear for \( q = 1 \) and quadratic for \( q = 2 \)); \( \gamma^q \) is the unknown coefficients, which can be found from the Galerkin technique. We consider for simplicity orthogonal basis functions, \( \phi^0(\xi) \equiv 1 \) for \( \xi \) lying on the corresponding panel, and variations of \( \phi^1 \) and \( \phi^2 \) are equal to 1. Outside of the panel all the basis functions, associated with it, are equal to zero.

Let us consider the model problem for the unique vortex particle in the flow domain. In the general case, analogous operations are performed for all the vortex particles in the near-wall region.

The idea of the suggested correction procedure is the following: panels should be chosen, which lie rather close to the vortex particle, we consider panels \( k \in [k_0, k_e] \). These panels are replaced with the arcs of osculating circles, having curvatures \( \kappa_k \). So, the conformal mappings theory [13] permits to write down the exact solution for each such panel:

\[
\gamma_k(\xi) = \frac{\Gamma_w}{2\pi} \left( \frac{r_k(\xi) - r_w}{|r_k(\xi) - r_w|^2} - \frac{r_k(\xi) - r_k^m}{|r_k(\xi) - r_k^m|^2} + \frac{r_k(\xi) - r_k^c}{|r_k(\xi) - r_k^c|^2} \right) \cdot n_k(\xi),
\]
Figure 2. Piecewise-constant (a), piecewise-linear discontinuous (b) and piecewise-linear continuous (c) numerical solution (blue line) of the BIE compared to the exact solution (red curve) for the vortex element placed on the distance $h/2$ from the airfoil.

here $\Gamma_w$ is circulation of the vortex particles located at a point $r_w$; $r_k(\xi)$ is a point on the arc of the osculation circle, which corresponds to the point $\xi$ on initial airfoil; $r_\xi$ is the center of the osculating circle; $r_k^{(v)}$ is the position of the mirrored vortex.
Hereinafter we consider only the second of the BIEs (1), that corresponds to the $T$-scheme. Let us also introduce natural parameterization for the airfoil surface line, so all the variables now depend on arc length instead of the corresponding points. Taking into account the above assumptions, the BIE takes the following form:

$$
\sum_{j=1}^{N} \sum_{q=0}^{2} \gamma_{j}^{q} \int_{K_{j}} \phi_{i}^{p}(s)ds \int_{K_{j}} Q_{\tau}(s, \sigma)\phi_{j}^{q}(\sigma)d\sigma + \sum_{k=k_{b}}^{k_{e}} \int_{K_{i}} \phi_{i}^{p}(s)ds \int_{K_{k}} Q_{\tau}(s, \sigma)\gamma_{k}^{*}(\sigma)d\sigma -
$$

$$
- \frac{1}{2} \gamma_{j}^{0} \int_{K_{j}} \phi_{i}^{p}(s)\phi_{j}^{q}(s)ds - \frac{1}{2} I_{i} \int_{K_{i}} \phi_{i}^{p}(s)\gamma_{i}^{*}(s)ds = \int_{K_{i}} \phi_{i}^{p}(s)f(s)ds, \ i = 1, \ldots, N, \ p = 0, 1, 2,
$$

where $N$ is number of panels; $I_{i}$ is an indicator, which value is 1 when $i \in [k_{b}, k_{e}]$ and 0 otherwise. The lower index is always connected with the considering panel; arc length parameters $s$ and $\sigma$ correspond to the points $\tau$ and $\xi$, respectively.

Since $\gamma_{k}^{*}(s)$ are exact solutions on panels $k \in [k_{b}, k_{e}]$, which correspond to the vortex sheet intensity on circular airfoil in presence of a point vortex, the following equality takes place:

$$
\sum_{k=k_{b}}^{k_{e}} \int_{K_{i}} \phi_{i}^{p}(s)ds \int_{K_{k}} Q(s, \sigma)\gamma_{k}^{*}(\sigma)d\sigma - \frac{1}{2} \int_{K_{i}} \phi_{i}^{p}(s)\gamma_{i}^{*}(s)ds = \int_{K_{i}} \phi_{i}^{p}(s)f^{*}(s)ds,
$$

where $f^{*}(s)$ is the contribution of the considered vortex particle to the right-hand side of the BIE.

Now we can write down the resulting linear algebraic system, which right-hand side consists of three terms:

$$
\sum_{j=1}^{N} \sum_{q=0}^{2} A_{ij}^{pq} \gamma_{j}^{q} + D_{ii}^{pq} \gamma_{i}^{p} = (b_{v})_{i}^{p} + (1 - I_{i}) (b_{w})_{i}^{p} + (b_{g})_{i}^{p}, \ i = 1, \ldots, N, \ p = 0, 1, 2.
$$

Here

$$
A_{ij}^{pq} = \int_{K_{i}} \phi_{i}^{p}(s)ds \int_{K_{j}} Q(s, \sigma)\phi_{j}^{q}(\sigma)d\sigma; \ D_{ii}^{pq} = - \frac{1}{2} \int_{K_{i}} \phi_{i}^{p}(s)\phi_{i}^{q}(s)ds;
$$

$$
(b_{v})_{i}^{p} = - \int_{K_{i}} \phi_{i}^{p}(s)(\nabla \cdot \tau(s))ds; \ (b_{g})_{i}^{p} = - \sum_{k=k_{b}}^{k_{e}} \int_{K_{i}} \phi_{i}^{p}(s)ds \int_{K_{k}} Q(s, \sigma)\gamma_{k}^{*}(\sigma)d\sigma, \ (k \neq i);
$$

$$
(b_{w})_{i}^{p} = - \int_{K_{i}} \phi_{i}^{p}(s) \frac{\Gamma_{w}(s) - r_{w} \cdot n(s)}{2\pi |r(s) - r_{2}|^{2}}ds = \Gamma_{w}(q_{w})_{i}^{p}, \ i = 1, \ldots, N, \ p = 0, 1, 2.
$$

The additional condition for the unique solution selection remains the same, except for changing $\Gamma$ to $\Gamma_{w}$ in the right-hand side:

$$
\Gamma_{w} = \Gamma - \sum_{k=k_{b}}^{k_{e}} \tilde{\Gamma}_{w,k};
$$

where

$$
\tilde{\Gamma}_{w,k} = \int_{K_{k}} \gamma_{k}^{*}(s)ds.
$$
Approximate formulae for the coefficients $A^0_{ij}$, $D^0_{ij}$, $(b_i)_i^p$ of the system are remain the same as for numerical scheme without correction [14, 15, 16]. The coefficients $(b_g)_i^p$, $(b_w)_i^p$ on the right side of the system are of more interest.

Coefficients $(b_w)_i^p$ are expressed through the coefficients $(q_w)_i^p$, for which, in turn, the following approximate formulae were obtained

\[
(q_w)_i^0 \approx -\frac{\sin \delta_{iw}}{2\pi} \left( \frac{L_i}{h_{iw}} \right) - \frac{1}{48\pi} \left( 2\sin 3\delta_{iw} - \kappa_i h_{iw} (3\cos 2\delta_{iw} + \kappa_i h_{iw} \sin \delta_{iw}) + \kappa_i' h_{iw}^2 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^3;
\]

\[
(q_w)_i^1 \approx \frac{1}{24\pi} \left( \sin 2\delta_{iw} - \kappa_i h_{iw} \cos \delta_{ih} \right) \left( \frac{L_i}{h_{iw}} \right)^2 + \frac{1}{960\pi} \left( 6\sin 4\delta_{iw} - \kappa_i h_{iw} (12\cos 3\delta_{iw} + 7\kappa_i h_{iw} \sin 2\delta_{iw} - \kappa_i^2 h_{iw}^2 \cos \delta_{iw}) + \kappa_i' h_{iw}^2 (4\cos 2\delta_{iw} + 3\kappa_i h_{iw} \sin \delta_{iw}) - \kappa_i'' h_{iw}^3 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^4;
\]

\[
(q_w)_i^2 \approx \frac{1}{180\pi} \left( 2\sin 3\delta_{iw} - \kappa_i h_{iw} (3\cos 2\delta_{iw} + \kappa_i h_{iw} \sin \delta_{iw}) + \kappa_i' h_{iw}^2 \cos \delta_{iw} \right) \left( \frac{L_i}{h_{iw}} \right)^3.
\]

Here $\kappa_i$, $\kappa_i'$ and $\kappa_i''$ denote the curvature of the airfoil surface line at the centre of the $i$-th panel and its first and second derivatives with respect to the arc length; $h_{iw}$ is the length of the vector $h_{iw} = c_i - r_w$, which connects the vortex particle position with the centre of the $i$-th panel; $\delta_{iw}$ is the angle between the tangent unit vector $\tau_i$ at the centre of the $i$-th panel and the vector $h_{iw}$ (which value is signed and measured in such a way that the positive angle corresponds to a turn from $h_{iw}$ to $\tau_i$ counterclockwise); all the notations are similar to ones introduced in [12].

The coefficients $(b_g)_i^p$ correspond to the influence of the vortex sheet (more precisely, its part that had been estimated analytically through the exact solution for the arc of a circle). Numerical experiments show, that only coefficient $(b_g)_i^0$ has a significant effect while $(b_g)_i^1$ and $(b_g)_i^2$ can be neglected.

Let us introduce the necessary designations: $\Gamma_w$ is the circulation of the vortex element, placed close to the airfoil surface, which contribution is taken into account; $k$ is the index of the panel, for which the correction procedure is performed ($k \in [k_0, k_r]$); $s_{k-\frac{1}{2}} = \frac{1}{2} (s_{k-1} + s_k)$ is the arc length parameter value that corresponds to its centre; $d_{w,k}$ is the distance from the vortex particle to the $k$-th panel (more precisely, to the osculating circle); $s_w$ is the value of the arc length parameter that corresponds to the projection of the vortex element position onto the $k$-th panel, then its ‘shift’ from the panel centre is $u_{w,k} = s_w - s_{i-\frac{1}{2}}$

We consider first the neighbouring panels when $k = i + 1$, i.e., arc length parameters $s_i = s_{k-1}$; the lengths of the $k$-th and the $i$-th panels are $L_i$ and $L_k$, respectively; the curvature of the airfoil surface line at their common point is $\kappa_i$, the derivative of the curvature at the common point with respect to arc length is $\kappa_i'$.

The following approximate formula can be derived for the term $(b_g)_i^0$:

\[
(b_g)_i^0 \approx \frac{\kappa_i L_i}{4\pi} \left( \Gamma_{w,k} + \frac{\kappa_i'}{\kappa_i} \frac{\Gamma_{w,k}^{(3)}}{2\pi} \right) \left( 1 + \frac{\kappa_i'}{\kappa_i} \frac{2L_k^2 - L_i^2}{2(\kappa_i'' + \kappa_i L_i)} \right).
\]

For the non-neighboring panels the corresponding integral can be approximately calculated much easier:

\[
(b_g)_i^0 \approx -\int_{K_i} \phi_0^0(s) ds \int_{K_k} Q(s, \sigma) \gamma_k^2(\sigma) d\sigma \approx \frac{\Gamma_{w,k} h_{ik} \cdot n_i}{2\pi |h_{ik}|^2} L_i,
\]

where $h_{ik} = c_i - c_k$, $c_i$ and $c_k$ are the centres of the $i$-th and the $k$-th panels, respectively; $n_i$ is outer normal unit vector at the centre of the $i$-th panel.
Figure 3. Two neighboring curvilinear panels and vortex element near them

The approximate formula can be also derived for total vorticity, distributed over the \( k \)-th panel due to analytical taking into account of the contribution of the vortex particle:

\[
\hat{\Gamma}_{w,k} = \int_{K_k} \gamma^*_k(s) ds \approx \frac{\Gamma_w}{2\pi} \left( \kappa_k L_k - 2\theta^{(3)}_{w,k} - \frac{d_{w,k}\kappa_k}{2} \left( \sin 2\theta^{(1)}_{w,k} + \sin 2\theta^{(2)}_{w,k} \right) \right).
\]

Here \( \theta^{(1)}_{w,k}, \theta^{(2)}_{w,k} \) and \( \theta^{(3)}_{w,k} \) are the following angles:

\[
\theta^{(1)}_{w,k} = \arctan \left( \frac{d_{w,k}}{\frac{L_k}{2} - u_{w,k}} \right), \quad \theta^{(2)}_{w,k} = \arctan \left( \frac{d_{w,k}}{\frac{L_k}{2} + u_{w,k}} \right), \quad \theta^{(3)}_{w,k} = \pi - \theta^{(1)}_{w,k} - \theta^{(2)}_{w,k},
\]

where \( \theta = \arctan(\eta, \xi) \) means angle for which

\[
\sin \theta = \frac{\eta}{\sqrt{\xi^2 + \eta^2}}, \quad \cos \theta = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}.
\]

4. Numerical experiment

The test problem was considered for elliptic airfoil (circular airfoil seems to be uninteresting because in this case, the described correction procedure leads to the exact solution). Position of the vortex particle is shown in fig. 4 (distance to the airfoil surface line is equal to \( h/2 \)), and three panels are marked, for which correction was performed. Numerical solution of the BIE according to the Galerkin technique without solution correction leads to a significant error on the panels no. 2, 3, 4 (fig. 5, a). After correction (fig. 5, b and c) the numerical solution becomes much closer to the exact one, the maximal difference has order of 1%. The error of the velocity reconstruction has the same order.

Moreover, there is practically no qualitative difference comparing piecewise-linear and piecewise-quadratic numerical schemes. Thus, we can conclude, that there is no practical reason in the usage of the piecewise-quadratic scheme, which, of course, is more complicated in comparison to the piecewise-linear one.

It also should be noted, that the developed correction technique is the more efficient the closer vortex particle is placed to the airfoil boundary. Numerical experiments show that the correction procedure should be performed when this distance is less than the length of one panel. For larger distance, the numerical solution itself is quite smooth, so it is reconstructed accurately by using ‘pure’ Galerkin method.
For the non-neighboring panels the corresponding integral can be approximately calculated much easier: ... scheme (c)

Figure 4. Elliptic airfoil with 2 : 1 semiaxes ratio and vortex particle near it

Figure 5. Numerical solution on the panels no. 1...5 without correction (a) and with correction for piecewise-linear scheme (b) and piecewise-quadratic scheme (c)

5. Conclusion
An original semi-analytic correction technique developed for improving the resolution of the vortex sheet intensity reconstruction. The suggested approach is suitable for 2D flows simulation
using meshless Lagrangian vortex particle method and makes it possible to deal with coarse surface grids. Additional terms that arise in the right-hand side of the boundary integral equation are expressed through the integrals, that cannot be calculated analytically. Approximate formulae are derived for it, that makes it possible to avoid time-consuming numerical integration. Numerical results show, that after implementation of the correction procedure, the accuracy becomes high enough for piecewise-linear solution representation in the Galerkin algorithm, so there is no need to introduce higher-order basis functions.

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