Lepton flavor violation decays $\tau^- \rightarrow \mu^- P_1 P_2$ in the topcolor-assisted technicolor model and the littlest Higgs model with $T$ parity

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Abstract

The new particles predicted by the topcolor-assisted technicolor ($TC2$) model and the littlest Higgs model with $T$-parity (called $LHT$ model) can induce the lepton flavor violation ($LFV$) couplings at tree level or one loop level, which might generate large contributions to some $LFV$ processes. Taking into account the constraints of the experimental data on the relevant free parameters, we calculate the branching ratios of the $LFV$ decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ with $P_1 P_2 = \pi^+ \pi^-$, $K^+ K^-$ and $K^0 \bar{K}^0$ in the context of these two kinds of new physics models. We find that the $TC2$ model and the $LHT$ model can indeed produce significant contributions to some of these $LFV$ decay processes.
1. Introduction

In the standard model (SM), because of the unitary of the leptonic analog of Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and the masslessness of three neutrinos, the lepton flavor violation (LFV) processes are forbidden at tree level. Experimentally, the neutrinos acquire small mass for the observation of neutrino oscillations and the LFV processes are possible [1]. Thus, the LFV processes may provide good tests of new physics (NP) beyond the SM. This fact has lead to great amount of the theoretical efforts studying on the underlying NP in the leptonic flavor sector.

In the SM, the $\tau$ lepton is the most heavy particle in the leptonic sector, which is much more sensitive than the leptons $e$ or $\mu$ to NP related to the flavor and mass generation problems [2]. The semileptonic $\tau$ decays related LFV are very interesting and needed to be studied, which could provide a better laboratory to search NP.

Experimentally, with a total data set now exceeding 1.1 ab$^{-1}$ of integrated luminosity and an $e^+e^- \rightarrow \tau^+\tau^-$ cross-section at 10.58 GeV of 0.919 nb [3], B factories have recorded more than $10^9$ tau pairs and contributed significant progress to tau lepton physics. The current experimental limits for the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ at 90% C.L. have been fixed at [4, 5]:

$$Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-) < 2.9 \times 10^{-7},$$

$$Br(\tau^- \rightarrow \mu^- K^+ K^-) < 2.5 \times 10^{-7},$$

$$Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0) < 3.4 \times 10^{-6}. $$

There are a lot of theoretical researches on the LFV $\tau$ decays in many possible extension of the SM. For example, the LFV $\tau$ decays have been studied in supersymmetry (SUSY) model [6, 7, 8, 9], the littlest Higgs model with $T$ parity (called LHT model) [10, 11, 12], and others [13, 14, 15, 16]. In particular, the LFV $\tau$ decays $\tau \rightarrow l P_1 P_2$ ($l = \mu, e$) have been studied in Refs. [6, 9, 17, 18, 19]. However, so far, we have not found discussions on the LFV $\tau$ decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ with $P_1 P_2 = \pi^+ \pi^-$, $K^+ K^-$ and $K^0 \bar{K}^0$ in the framework of the topcolor-assisted technicolor (TC2) model [20] as well as the LHT model [21]. These two models are popular and interesting NP models at
present and the experimental upper limits of the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ have been improved to $O(10^{-7})$ at 90% C.L. \cite{1, 3}. So in this paper, we would like to consider the contributions of the $TC2$ model and the $LHT$ model to the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$.

Among various kinds of dynamical electroweak symmetry breaking ($EWSB$) theories, the topcolor scenario is attractive because it can explain the large top quark mass and provide a possible $EWSB$ mechanism \cite{22}. The $TC2$ model \cite{20} is one of the phenomenologically viable models, which has all essential features of the topcolor scenario. This model predicts the existence of the nonuniversal gauge boson $Z'$ and the top-Higgs $h_0^t$. These new particles treat the third generation fermions differently from those in the first and second generations and thus can lead to the tree level flavor-changing ($FC$) couplings. Thus these new particles might give significant contributions to the LFV semileptonic decays $\tau^- \rightarrow \mu^- P_1 P_2$. Our numerical results show that the contributions of the scalar $h_0^t$ are much small, while the nonuniversal gauge boson $Z'$ can enhance the branching ratio $Br(\tau^- \rightarrow \mu^- P_1 P_2)$ by several orders of magnitude.

The $LHT$ model \cite{21} is one of the attractive little Higgs models, it predicts the existence of the T-odd $SU(2)$ doublet fermions and new gauge bosons. These new fermions and gauge bosons can provide rich phenomenology at present or in future high energy collider experiments \cite{23, 24, 25, 26, 27, 28, 29, 30}. Our numerical results show that the contributions of the $LHT$ model can significantly enhance the branching ratio $Br(\tau^- \rightarrow \mu^- P_1 P_2)$, which might approach its experimental upper limit with reasonable values of the free parameters.

The structure of this paper is as follows. After briefly summarize the relevant couplings of new particles to ordinary particles arising from the $TC2$ model and the $LHT$ model, we calculate the branching ratios of the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ with $P_1 P_2 = \pi^+\pi^-$, $K^+K^-$ and $K^0\bar{K}^0$ generated by these two kinds of NP models in sections 2 and 3, respectively. In our numerical estimation, we have taken into account the constraints of the current experimental data on the model-dependent free parameters and compared our numerical results with the current experimental up limits for $\tau^- \rightarrow \mu^- P_1 P_2$ in these
two sections. Our conclusions and discussions are given in section 4. In appendix A we give the explicit forms of the relevant form factors for the pseudoscalar mesons $P_1$ and $P_2$. The explicit forms of the relevant functions for the TC2 and the LHT models are collected in appendixes B and C, respectively.

2. The TC2 model and the LFV $\tau$ decay processes $\tau^- \rightarrow \mu^- P_1 P_2$

In the TC2 model [22], topcolor interaction is not flavor-universal and mainly couples to the third generation fermions. It generally generates small contributions to EWSB and gives rise to the main part of the top quark mass. Thus, the nonuniversal gauge boson $Z'$ has large Yukawa couplings to the third generation fermions. Such features lead to large tree level $FC$ couplings of the nonuniversal gauge boson $Z'$ to ordinary fermions when one writes the interaction in the fermion mass eigen-basis.

The explicit form for the $LFV$ couplings of the nonuniversal gauge boson $Z'$ to ordinary leptons, which are related our calculation, can be written as [31, 32]:

$$L_{Z'}^{FC} = \frac{1}{2} g_1 K' Z'_\mu [\bar{\tau}_L \gamma^\mu \mu_L + 2 \bar{\tau}_R \gamma^\mu \mu_R],$$  \hspace{1cm} (4)

where $g_1$ is the ordinary hypercharge gauge coupling constant. $K'$ is the mixing factor between the leptons $\tau$ and $\mu$. The relevant flavor-diagonal ($FD$) couplings of $Z'$ to ordinary fermions can be written as [20, 22, 31]:

$$L_{Z'}^{FD} = -\sqrt{4\pi K_1} \left\{ Z'_\mu \left[ \frac{1}{2} \bar{\tau}_L \gamma^\mu \tau_L - \bar{\tau}_R \gamma^\mu \tau_R \right] - \tan^2 \theta' Z'_\mu \left[ \frac{1}{6} \bar{u}_L \gamma^\mu u_L + \frac{2}{3} \bar{u}_R \gamma^\mu u_R 

+ \frac{1}{6} \bar{d}_L \gamma^\mu d_L - \frac{1}{3} \bar{d}_R \gamma^\mu d_R + \frac{1}{6} \bar{s}_L \gamma^\mu s_L - \frac{1}{3} \bar{s}_R \gamma^\mu s_R \right] \right\},$$  \hspace{1cm} (5)

where $K_1$ is the coupling constant and $\theta'$ is the mixing angle with $\tan \theta' = g_l/\sqrt{4\pi K_1}$.

For the TC2 model, the extended gauge groups are broken at the TeV scale, which proposes that $K'$ is an $O(1)$ free parameter. Its value can be generally constrained by the present experimental upper limits on the $LFV$ processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_l$. For example, for the $LFV$ process $\mu \rightarrow 3e$, the decay width arisen from $Z'$ exchange can be written as [33]:

$$\Gamma(\mu \rightarrow 3e) = \frac{25\alpha^5}{384\pi K_1^3 \cos^{10} \theta_W} \frac{m_\mu^5}{M_{Z'}^2} K'^2,$$  \hspace{1cm} (6)
where $\theta_W$ is the Weinberg angle. The current experimental upper limit is $B_{\tau}^{\text{exp}}(\mu \rightarrow 3e) \leq 1 \times 10^{-12}$ \cite{34}, which can give constraints to the free parameters of the TC'2 model. In our following numerical calculation, we will take into account these limits.

Figure 1: The Feynman diagrams contributing to the LFV decay processes

\[ \tau^- \rightarrow \mu^- P_1 P_2. \] 

$V$ represents a photon, a gauge boson or a Higgs boson.

From the above discussions, we can see that the nonuniversal gauge boson $Z'$ can contribute to the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ at tree level and one loop level as shown in Fig. 1. This diagram can be mediated by a photon, a gauge boson or a Higgs boson. Here the effective LFV vertex is represented by a black dot and the hadronic vertex by a gray box. There are also other types of diagrams induced by the gauge bosons $W^\pm$ which have been discussed in Ref. \cite{17}. However, those diagrams do not exist in our calculation, because they are just adapt to models including right-handed neutrinos. The pseudoscalar mesons $P_1$ and $P_2$ in the final state stem from the hadronisation of quark bilinear currents, namely parameterizing by the vector form factors $F^{P_1 P_2}(s)$ \cite{8, 35}. These form factors can be defined through the vacuum-to-$P_1 P_2$ matrix elements of the local quark currents. The relevant formula can be written as \cite{8, 35}:

\[ \langle P_1 P_2|\bar{q} \gamma_\mu q|0\rangle = (p_1 - p_2)_\mu F^{P_1 P_2}(s), \quad (7) \]

and $\sum_{u,d,s} Q_q F^{P_1 P_2}(s) = F^{P_1 P_2}(s)$, where $Q_q$ is the electric charge of the $q$ quark in units of the positron charge $e$ and $s = (p_1 + p_2)^2$, in which $p_1$ and $p_2$ are the momentum of mesons $P_1$ and $P_2$, respectively. The explicit forms of $F^{\pi^+ \pi^-}(s)$, $F^{K^+ K^-}(s)$ and $F^{\bar{K}^0 K^0}(s)$ have been displayed in the appendix A.

In this section, we give the explicit calculation of the $Z'$ contributions to the LFV decay processes $\tau^- \rightarrow \mu^- P_1 P_2$ at both tree level and one loop level.
A. The tree level contributions of the nonuniversal gauge boson $Z'$

From Eq. (4), we can see that the nonuniversal gauge boson $Z'$ can contribute to the $LFV$ decay processes $\tau^{-} \rightarrow \mu^{-}P_{1}P_{2}$ at tree level. The relevant Feynman diagram is similar to Fig. 1. The amplitude mediated by $Z'$ exchange in terms of the final state quarks can be written as:

$$A_{Z'} = \frac{1}{M_{Z'}^{2}} C_{Z'} \bar{\mu}\gamma_{\mu}(v_{l} + a_{l}\gamma_{5})\tau \bar{q}\gamma_{\nu}(v_{q} + a_{q}\gamma_{5})q,$$

in which $v_{l(q)}$ and $a_{l(q)}$ are the constants for the vector and axial-vector couplings of the gauge boson $Z'$ to ordinary leptons (quarks). The coefficient $C_{Z'}$ can be written as:

$$C_{Z'} = \frac{1}{2}g_{1}K'\sqrt{4\pi K_{1}}\tan^{2}\theta'.$$

(8)

Utilizing the hadronisation formula given by Eq. (7), the quark bilinear currents can be written in term of the form factors $F_{q}^{P_{1}P_{2}}(s)$ which correspond to the two mesons $P_{1}$ and $P_{2}$ in the final state. Then, in terms of the final state hadrons, we can obtain the amplitude of the $LFV$ decay processes $\tau^{-} \rightarrow \mu^{-}P_{1}P_{2}$ generated by the nonuniversal gauge boson $Z'$

$$A_{Z'} = \frac{v_{q}}{M_{Z'}^{2}}C_{Z'}F_{q}^{P_{1}P_{2}}(s) \bar{\mu}(p_{1} - p_{2})(v_{l} + a_{l}\gamma_{5})\tau.$$

(9)

The explicit form of the branching ratio $Br(\tau^{-} \rightarrow \mu^{-}P_{1}P_{2})$ can be expressed as [3]:

$$Br(\tau^{-} \rightarrow \mu^{-}P_{1}P_{2}) = \frac{\tau_{\tau}}{64\pi^{4}m_{\tau}^{2}} \int_{s_{\min}}^{s_{\max}} ds \int_{t_{\min}}^{t_{\max}} dt |A_{Z'}|^{2},$$

(11)

where $\tau_{\tau}$ is the lifetime of lepton $\tau$, $t = (p_{\tau} - p_{1})^{2}$, and

$$t_{\max} = \frac{1}{4s} \left[ \left( m_{\tau}^{2} - m_{\mu}^{2} \right)^{2} - \lambda^{1/2} \left( s, m_{P_{1}}^{2}, m_{P_{2}}^{2} \right) \right],$$

$$s_{\min} = 4m_{P}^{2}, \quad s_{\max} = (m_{\tau} - m_{\mu})^{2}, \quad \lambda(x, y, z) = (x + y - z)^{2} - 4xy.$$  

(12)

In above equations we have assumed $m_{P_{1}} = m_{P_{2}} = m_{P}$.

Before giving numerical results, we need to specify the relevant $SM$ parameters. Most of these input parameters are shown in Table 1. The vacuum tilting, the constraints from Z-pole physics, and U(1) triviality require $K_{1} \leq 1$ [36]. The mass of nonuniversal gauge boson $M_{Z'}$ can be generally seen as free parameter. The lower bounds on $M_{Z'}$
\[ G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \]
\[ \alpha = 7.297 \times 10^{-3} \]
\[ \tau_\tau = 2.91 \times 10^{-13} \text{ s} \]
\[ m_\tau = 1.78 \text{ GeV} \]
\[ m_\mu = 0.106 \text{ GeV} \]
\[ m_K = 0.494 \text{ GeV} \]
\[ m_{K^0} = 0.498 \text{ GeV} \]
\[ M_W = 80.43 \text{ GeV} \]
\[ \sin^2 \theta_W = 0.2315 \]

Table 1: Numerical inputs used in our analysis. Unless explicitly specified, they are taken from the Particle Data Group [5].

can be obtained from dijet and dilepton production in the Tevatron experiments [37] or \( B\bar{B} \) mixing [38]. However, these bounds are significantly weaker than those from the precision electroweak data. Ref. [39] has shown that, to fit the precision electroweak data, the \( Z' \) mass \( M_{Z'} \) must be larger than 1 TeV. In the following numerical estimation, we will assume that the values of the free parameters \( M_{Z'} \) and \( K_1 \) are in the ranges of 1000 GeV \( \sim \) 2000 GeV and 0 \( \sim \) 1, respectively.

The branching ratios \( Br(\tau^- \rightarrow \mu^- P_1 P_2) \) with \( P_1 P_2 = \pi^+ \pi^- \), \( K^+ K^- \) and \( K^0 \bar{K}^0 \) contributed by the nonuniversal gauge boson \( Z' \) at tree level are plotted as functions of the mass parameter \( M_{Z'} \) in Fig. 2 in which we have taken \( K_1 = 0.4 \) (Fig. 2a) and 0.8(Fig. 2b), and considered the constraints on the free parameter \( K' \) giving by the current experimental upper limit of \( Br^{exp}(\mu \rightarrow 3e) \), as shown in Eq. (6). From these diagrams we can see that the values of the branching ratios \( Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-) \), \( Br(\tau^- \rightarrow \mu^- K^+ K^-) \) and \( Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0) \) decrease as the mass parameter \( M_{Z'} \) increasing. It is obviously that the branching ratios of the different decay channels satisfy the relation \( Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-) > Br(\tau^- \rightarrow \mu^- K^+ K^-) \geq Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0) \). This is mainly because the mass of the \( K \) meson is larger than that of the \( \pi \) meson and the \( FD \) couplings of the nonuniversal gauge boson \( Z' \) to up-type quarks are different from those for the down-type quarks as
shown in Eq. (5). The max values of the branching ratios for the LFV decay processes $\tau^{-} \rightarrow \mu^{-} P_{1} P_{2}$ and $\tau^{-} \rightarrow \mu^{-} K^{0} \bar{K}^{0}$ can reach $2.41 \times 10^{-10}$ and $1.05 \times 10^{-10}$, respectively. However, these values are much smaller than the corresponding experimental upper limits given in Eqs. (2, 3). While the max value for the LFV decay process $\tau^{-} \rightarrow \mu^{-} \pi^{+} \pi^{-}$ can reach $1.68 \times 10^{-8}$, which might approach its upper limit in the future high energy collider experiments.

**B. The loop level contributions of the nonuniversal gauge boson $Z'$**

The nonuniversal gauge boson $Z'$ predicted by the TC2 model can also generate contributions to the LFV decay processes $\tau^{-} \rightarrow \mu^{-} P_{1} P_{2}$ at one loop level. The relevant Feynman diagrams for the effective LFV vertexes $Z \tau \bar{\mu}$ and $\gamma \tau \bar{\mu}$ have been displayed in Fig. 3.

The effective Hamilton for the LFV decay process $\tau^{-} \rightarrow \mu^{-} P_{1} P_{2}$ including the contributions of $Z'$ at one loop level has the form:

$$H = H_{1} + H_{2},$$

Figure 2: The branching ratios $Br(\tau^{-} \rightarrow \mu^{-} P_{1} P_{2})$ contributed by the nonuniversal gauge boson $Z'$ at tree level as functions of mass parameter $M_{Z'}$ for the parameter $K_{1} = 0.4$ (a) and $K_{1} = 0.8$ (b).
Figure 3: The Feynman diagrams for the effective LFV vertexes $Z\tau\bar{\mu}$ and $\gamma\tau\bar{\mu}$ contributed by the nonuniversal gauge boson $Z'$. 

\begin{align*}
H_1 &= \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} C_1 \bar{\mu} \gamma\mu (v_l + a_l \gamma_5) \tau \bar{q} \gamma\nu (v_q + a_q \gamma_5) q, \quad (14) \\
H_2 &= \frac{G_F}{\sqrt{2}} m_e e^2 Q q C_2 \bar{\mu} \sigma_{\mu\nu} (v_l + a_l \gamma_5) \tau \bar{q} \gamma_\nu q, \quad (15)
\end{align*}

where $H_1$ and $H_2$ represent the $Z'$ contributions mediated by $Z$ gauge boson exchange and the photon exchange, respectively. $k$ represents the photon momentum. The explicit forms of the coefficients $C_1$ and $C_2$ are:

\begin{align*}
C_1 &= \frac{2g_1 K' \sqrt{4\pi K_1}}{g_2^2} \left[ 4F_1(x_\tau) - 2F_2(x_\tau) + \left( 1 + \frac{m_\tau}{m_\mu} F_3(x_\tau) \right) \right], \quad (16) \\
C_2 &= \frac{32g_1 K' M_W^2 \sqrt{4\pi K_1}}{g_2^2 m_\tau} \left[ F_4(x_\tau) + \left( 1 + \frac{m_\tau}{m_\mu} F_3(x_\tau) \right) \right], \quad (17)
\end{align*}

where $g_2$ is the SM $SU(2)_L$ gauge coupling constant. The Inami-Lim functions $F_i(x)$ ($i = 1, 2, 3, 4$) are collected in Appendix B with $x_\tau = m_\tau^2 / M_{Z'}^2$. 

Applying similar hadronisation process to the bilinear quark currents as that for the tree level, the amplitude contributed by the nonuniversal gauge boson $Z'$ at one loop can be written as:

\begin{align*}
A_1 &= \frac{G_F}{\sqrt{2}} \frac{v_\gamma \alpha}{2\pi \sin^2 \theta_W} C_1 F_{P_1 P_2} (s) \bar{\mu} (p_1^\ell - p_2^\ell) (v_l + a_l \gamma_5) \tau, \quad (18) \\
A_2 &= \frac{G_F}{\sqrt{2}} \frac{e^2 m_\tau}{2\pi^2 k^2} C_2 F_{P_1 P_2} (s) \bar{\mu} p_1^h \sigma_{\mu\nu} p_2^\nu (v_l + a_l \gamma_5) \tau. \quad (19)
\end{align*}

In the context of the TC2 model, the expression of the corresponding branching ratio induced by the nonuniversal gauge boson $Z'$ at one loop level can be written as:

\begin{align*}
Br(\tau^- \to \mu^- P_1 P_2) &= \frac{\tau_\tau}{64\pi^3 m_\tau^2} \int_{s_{\text{min}}}^{s_{\text{max}}} ds \int_{t_{\text{min}}}^{t_{\text{max}}} dt \left( |A_1|^2 + |A_2|^2 \right). \quad (20)
\end{align*}
Using the values of the relevant SM input parameters given at Table 1 we present the branching ratios $Br(\tau^- \rightarrow \mu^- P_1 P_2)$, $Br(\tau^- \rightarrow \mu^- K^+ K^-)$ and $Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)$ contributed by the nonuniversal gauge boson $Z'$ at one loop level as functions of the mass parameter $M_{Z'}$ in Fig. 4 in which we have taken $K_1 = 0.4$ (Fig. 4a) and $K_1 = 0.8$ (Fig. 4b). From these diagrams, one can see that the values of the branching ratios $Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-)$, $Br(\tau^- \rightarrow \mu^- K^+ K^-)$ and $Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)$ decrease as the mass parameter $M_{Z'}$ increasing. The value of $Br(\tau^- \rightarrow \mu^- K^+ K^-)$ is close to that of $Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)$ in most of the parameter space of the TC2 model. Comparing this figure to the $Z'$ tree level contributions displayed in Fig. 2 one can see that the contributions of $Z'$ to the LFV decay processes $\tau^- \rightarrow \mu^- \pi^+ \pi^-$, $\tau^- \rightarrow \mu^- K^+ K^-$ and $\tau^- \rightarrow \mu^- K^0 \bar{K}^0$ at one loop level are smaller than those of the tree level diagram by several orders of magnitude in most of the parameter space.

The TC2 model also predicts the existence of the top-Higgs $h_t^0$, which treats the third generation fermions differently from those in the first and second generations and thus can lead to the tree level FC couplings to ordinary fermions. So this kind of new particle can also generate contributions to the LFV semileptonic decays $\tau^- \rightarrow \mu^- P_1 P_2$ at tree level.
and one loop level. However, the LFV coupling $h^0_t \tau \mu$ is suppressed by a factor $m_\tau / \nu$ with the electroweak scale $\nu = 246$ GeV. Thus, the contributions of the top-Higgs $h^0_t$ to the LFV semileptonic decays $\tau^- \to \mu^- P_1 P_2$ are much smaller than those of the nonuniversal gauge boson $Z'$. Our numerical results show that it indeed is this case. The value of the branching ratio $Br(\tau^- \to \mu^- P_1 P_2)$ contributed by the scalar $h^0_t$ is smaller than that of $Z'$ at least by two orders of magnitude.

3. The LHT model and the LFV $\tau$ decay process $\tau^- \to \mu^- P_1 P_2$

In this section, we first review the essential features of the LHT model studied in Ref. [21], which are related our calculation. Then we will consider the contributions of the LHT model to the LFV $\tau$ decay process $\tau^- \to \mu^- P_1 P_2$.

Similar with the LH model, the LHT model is based on an SU(5)/SO(5) global symmetry breaking pattern. A subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ of the SU(5) global symmetry is gauged, and at the scale $f$ it is broken into the SM electroweak symmetry $SU(2)_L \times U(1)_Y$. T-parity is an automorphism which exchanges the $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$ gauge symmetries. The T-even combinations of the gauge fields are the SM electroweak gauge bosons $W^a_\mu$ and $A_\mu$. The T-odd combinations are T-parity partners of the SM electroweak gauge bosons.

After taking into account EW SB, at the order of $\nu^2/f^2$, the masses of the T-odd set of the $SU(2) \times U(1)$ gauge bosons are given as:

$$M_{A_H} = \frac{g_1 f}{\sqrt{2}} \left[ 1 - \frac{5 \nu^2}{8 f^2} \right], \quad M_{Z_H} \approx M_{W_H} = g_2 f \left[ 1 - \frac{\nu^2}{8 f^2} \right],$$

(21)

where $f$ is the scale parameter of the gauge symmetry breaking of the LHT model. Because of the smallness of $g_1$, the T-odd gauge boson $A_H$ is the lightest T-odd particle, which can be seen as an attractive dark matter candidate [23, 41].

To avoid severe constraints and simultaneously implement T-parity, it is need to double the SM fermion doublet spectrum [21, 24]. The T-even combination is associated with the $SU(2)_L$ doublet, while the T-odd combination is its T-parity partner. The masses of the T-odd fermions can be written in a unified manner as:

$$M_{F_i} = \sqrt{2} k_i f,$$

(22)
where \( k_i \) are the eigenvalues of the mass matrix \( k \) and their values are generally dependent on the fermion species \( i \).

The mirror fermions (T-odd quarks and T-odd leptons) have new flavor violating interactions with the SM fermions mediated by the new gauge bosons \( (A_H, W_H^\pm, Z_H) \), which are parameterized by four \( CKM-like \) unitary mixing matrices, two for mirror quarks and two for mirror leptons [27, 28, 42]:

\[
V_{Hu}, \ V_{Hd}, \ V_{Hl}, \ V_{H\nu}, \quad \text{(23)}
\]

they satisfy:

\[
V_{Hu}^+ V_{Hd} = V_{CKM}, \ V_{Hl}^+ V_{H\nu} = V_{PMNS}, \quad \text{(24)}
\]

where the \( CKM \) matrix \( V_{CKM} \) is defined through flavor mixing in the down-type quark sector, while the \( PMNS \) matrix \( V_{PMNS} \) is defined through neutrino mixing. Similar with Ref. [28], we will set the Majorana phases of \( V_{PMNS} \) to zero in our following calculation.

The matrix \( V_{Hl} \) can give rise to the \( LFV \) processes.

From the above discussions, we can see that the \( LHT \) model provides a new mechanism for the \( LFV \) processes, which comes from the flavor mixing in the mirror lepton sector. Thus, the \( LHT \) model might give significant contributions to the \( LFV \) processes \( \tau^- \rightarrow \mu^- P_1 P_2 \). The relevant Feynman diagrams have been shown in Fig. 5 and Fig. 6 in which we just display the effective \( LFV \) vertex without hadronic part. In these diagrams, \( l_i^t, \ \nu^j_i, \ q_i^t \) represent the T-odd partners of three family leptons \( l_i, \ \nu_j \) and quarks \( q_i \), respectively. The Goldstone bosons \( \omega^\pm, \ \omega^0 \) and \( \eta \) are eaten by heavy gauge bosons \( W_H^\pm, \ Z_H \) and \( A_H \), respectively. In this paper we use the ’t Hooft-Feynman gauge, so the Goldstone Boson mass is the same as its corresponding gauge boson, that’s to say: \( M_\omega = M_{W_H}, \ M_\omega^0 = M_{Z_H}, \) and \( M_\eta = M_{A_H} \). The relevant couplings of these new particles to ordinary leptons and their T-odd partners can be found in Ref. [28].

The effective Hamilton for the \( LFV \) decay process \( \tau^- \rightarrow \mu^- P_1 P_2 \) can be written as [28]:

\[
H_3 = \frac{G_F \alpha}{\sqrt{2} 2 \pi sin^2 \theta_W} \bar{X}_{odd} \bar{\mu} \gamma_\mu (1 - \gamma_5) \tau \ q_{\mu} + a_q \gamma_5 q, \quad \text{(25)}
\]

\[
H_4 = \frac{G_F e^2 M_\tau Q q}{\sqrt{2} 4 \pi^2 k^2} \bar{D}_{odd} \bar{\mu} \sigma_{\mu \nu} k^\nu (1 + \gamma_5) \tau \ q_{\mu} q, \quad \text{(26)}
\]
Figure 5: The penguin diagrams for the effective LFV vertexes $Z\tau\bar{\mu}$ and $\gamma\tau\bar{\mu}$ in the LHT model.

with

$$X^u_{\text{odd}} = \left[ \chi_2^{(\tau\mu)} \left( J^{u\bar{u}}(y_2, z) - J^{u\bar{u}}(y_1, z) \right) + \chi_3^{(\tau\mu)} \left( J^{u\bar{u}}(y_3, z) - J^{u\bar{u}}(y_1, z) \right) \right], \quad (27)$$

$$X^d_{\text{odd}} = \left[ \chi_2^{(\tau\mu)} \left( J^{d\bar{d}}(y_2, z) - J^{d\bar{d}}(y_1, z) \right) + \chi_3^{(\tau\mu)} \left( J^{d\bar{d}}(y_3, z) - J^{d\bar{d}}(y_1, z) \right) \right], \quad (28)$$

$$D_{\text{odd}} = -\frac{\nu^2}{8f^2} \sum_i \chi_i^{\tau\mu} \left[ D_0(y_i) - \frac{7}{6} E_0'(y_i) - \frac{1}{10} E_0''(y_i) \right], \quad (29)$$

here

$$J^{u\bar{u}}(y_1, z) = \frac{1}{64} \frac{\nu^2}{f^2} \left[ y_i S_{\text{odd}}(y_i) + F^{u\bar{u}}(y_i, z; W_H) \right]$$
Figure 6: The box diagrams for the LFV decay process $\tau^{-} \rightarrow \mu^{-} P_1 P_2$ in the LHT model.

\begin{equation}
+4 \left( G(y_i, z; Z_H) + G_1(y_i', z'; A_H) - G_2(y_i, z; \eta) \right),
\end{equation}

\begin{equation}
J^{\tilde{d} \tilde{d}}(y, z) = \frac{1}{64 f^2} \left[ y_i S_{\text{odd}}(y_i) + F^{dd}(y_i, z; W_H) \right. \\
\left. -4 \left( G(y_i, z; Z_H) + G_1(y_i', z'; A_H) + G_2(y_i, z; \eta) \right) \right],
\end{equation}

where $y_i = M^2_{l_i}/M^2_{W_H} = M^2_{l_i}/M^2_{Z_H}$, $z = m^2_{q}/M^2_{W_H}$, $y_i(z)' = 5y_i(z)/\tan^2 \theta_W$, $\eta = \tan^2 \theta_W/5$ and $\chi_{\tau\mu} = V^{\text{odd}}_{H^*} V^{\text{odd}}_{H^*}$. The explicit forms of $S_{\text{odd}}(x)$, $F^{u,d}(x)$, $G_1(x)$, $D_0'(x)$ and $E_0'(x)$ are collected in Appendix C.

In the context of the LHT model, the amplitude of the LFV decay process $\tau^{-} \rightarrow$
\(\mu^P_1 P_2\) can be written as:

\[
A_3 = \frac{G_F}{\sqrt{2}} \frac{v_q \alpha}{2 \pi \sin^2 \theta_W} F^e_{P_1 P_2}(s) \bar{x}_{\text{odd}} \bar{\mu}(P_1 - P_2)(1 - \gamma_5) \tau, \tag{32}
\]

\[
A_4 = \frac{G_F}{\sqrt{2}} \frac{e^2 m_\tau}{2 \pi^2 k^2} F^e_{P_1 P_2}(s) \bar{D}_{\text{odd}} \bar{i} \mu P_1^{\nu} \sigma_{\mu \nu} P_2^{\nu} (1 + \gamma_5) \tau. \tag{33}
\]

The contributions of the LHT model to LFV decay process have been extensively studied and compared with the current experimental limits in the literatures \[11, 12, 28, 29\]. It has been shown that the LHT model can enhance the SM prediction values by several orders of magnitude and the experimental measurement data for some LFV decay processes can give constraints on the free parameters of the LHT model. For example, in order to suppress the branching ratio \(Br(\mu \rightarrow e\gamma)\) and \(Br(\mu \rightarrow 3e)\) predicted by the LHT model below the present experimental upper bounds, the relevant mixing matrix \(V_{Hi}\) must be rather hierarchical or mass splitting for the first and second T-odd lepton masses is very small. Ref.\[28\] has shown that there must be \(\sin^2 \theta \leq 0.05\) or \(\delta \leq 5\%\). A complete analysis can be found in Ref.\[28\]. Thus, in our following numerical estimation, we will assume \(M_{\ell H} = M_{\ell H} = M_{\ell H} = M_1 = 800\text{ GeV}, V_{Hi} = V^+_{PMNS}\), and take \(M_{\ell H} = M_{\ell H} = M_2\) and the scale parameter \(f\) as free parameters. Considering the mirror quarks only contribute to the branching ratios of decay \(\tau^- \rightarrow \mu^- P_1 P_2\) in box diagrams, we assume their masses degeneration and take \(M_{q H} = 1\text{ TeV}\).

The branching ratios \(Br(\tau^- \rightarrow \mu^- P_1 P_2)\) with \(P_1 P_2 = \pi^+ \pi^-\), \(K^+ K^-\) and \(K^0 \bar{K}^0\) contributed by the LHT model are plotted as functions of the scale parameter \(f\) for \(M_2 = 500\text{ GeV}\) (Fig. 7 (a)) and \(M_2 = 1500\text{ GeV}\) (Fig. 7 (b)). From these figures, one can see that the values of the branching ratios \(Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-)\), \(Br(\tau^- \rightarrow \mu^- K^+ K^-)\) and \(Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)\) decrease as the scale parameter \(f\) increasing while as the mass of T-odd lepton \(M_{\ell H}\) decreasing. For \(M_{\ell H} = 1500\text{ GeV}\) and \(f = 500\text{ GeV}\), the value of the branching ratio \(Br(\tau^- \rightarrow \mu^- \pi^+ \pi^-)\) can reach \(3.14 \times 10^{-8}\), which is larger than that induced by the TC\(2\) model. For the LFV decay processes \(\tau^- \rightarrow \mu^- K^+ K^-\) and \(\tau^- \rightarrow \mu^- K^0 \bar{K}^0\), the values of their branching ratios are much smaller than the experimental upper limits in all of the parameter space of the LHT model.
Figure 7: In the LHT model, the branching ratio $Br(\tau^- \to \mu^- P_1 P_2)$ as function of $f$ for the parameter $M_2 = 500$ GeV (a), $M_2 = 1500$ GeV (b).

4. Conclusions and Discussions

The experimental upper limits of the LFV decay processes $\tau^- \to \mu^- P_1 P_2$ with $P_1 P_2 = \pi^+ \pi^-$, $K^+ K^-$ and $K^0 K^0$ have been improved to $\mathcal{O}(10^{-7})$ at 90% C.L. [4, 5]. Whether these LFV decay processes exist or not is very important to the neutrino mass problem in the SM. It is well known that the SM does not allow the LFV processes at tree level, while many popular NP models can induce the LFV processes at tree level or loop level which might make their branching ratios significantly larger than those predicted by the SM. So the LFV decay processes $\tau^- \to \mu^- P_1 P_2$ are very suitable for the determination of the free parameters of the NP models. Studying of these decay processes are very interesting and needed.

The TC2 model and the LHT model are two kinds of the popular NP models at present. In this paper, we have calculated their contributions to the branching ratios of the LFV decay processes $\tau^- \to \mu^- P_1 P_2$. We find that the new particles predicted by these two NP models can indeed produce significant contributions to these LFV decay processes. Taking into account the limits of the relevant experimental data on the free parameters, we calculate the $BR(\tau^- \to \mu^- P_1 P_2)$, and have the following conclusions.
i) The TC2 model can induce the LFV decay processes $\tau^- \to \mu^- P_1 P_2$ both at tree level and one loop level, while the LHT model can only give contributions to these processes at one loop level. Furthermore, in the case that the T-odd leptons are degenerate, the LHT model has no contributions to these LFV decay processes.

ii) For these two NP models, the branching ratios satisfy the following hierarchy:

$$\text{Br}(\tau^- \to \mu^- \pi^+ \pi^-) > \text{Br}(\tau^- \to \mu^- K^+ K^-) \gtrsim \text{Br}(\tau^- \to \mu^- K^0 \bar{K}^0).$$

iii) The contributions of the nonuniversal gauge boson $Z'$ at tree level to the LFV decay processes $\tau^- \to \mu^- \pi^+ \pi^-$, $\tau^- \to \mu^- K^+ K^-$ and $\tau^- \to \mu^- K^0 \bar{K}^0$ are larger than those at one loop level by one order of magnitude in most of the parameter space. However, these values are still not large enough to be detected by present high energy experiments, which still need the future experimental verification.

iv) The branching ratios of the LFV decay processes $\tau^- \to \mu^- P_1 P_2$ generated by the LHT model are much larger than those generated by the TC2 model. For $M_{\tau H} = 1500$ GeV and $f = 500$ GeV, the value of the branching ratio $\text{Br}(\tau^- \to \mu^- \pi^+ \pi^-)$ can reach $3.14 \times 10^{-8}$, which might approach the upper limit given in Eqs. (1). However, the values of the branching ratios of the LFV decay processes $\tau^- \to \mu^- K^+ K^-$ and $\tau^- \to \mu^- K^0 \bar{K}^0$ are smaller than $1 \times 10^{-9}$ in most of parameter space of the LHT model.

Our calculation can be extended to the LFV decay process $\tau^- \to e^- P_1 P_2$ by replacing the mass parameter $m_\mu$ to $m_e$. Since the nonuniversal gauge boson $Z'$ treats the first generation fermions same as those in the second generation, the coefficient of the coupling $Z' \tau \mu$ approximately equals to that of the coupling $Z' \tau e$. This feature leads to the fact that the contribution of the TC2 model to the decay $\tau^- \to e^- P_1 P_2$ is nearly the same as that of the decay $\tau^- \to \mu^- P_1 P_2$ channel. For the LHT model, its contributions to the LFV decay processes $\tau^- \to e^-(\mu^-) P_1 P_2$ can only exist at one loop. The relevant flavour mixing matrix elements and the masses of new particles for the LFV decay process $\tau^- \to e^- P_1 P_2$ are different from those of the LFV decay process $\tau^- \to \mu^- P_1 P_2$, which can make the branching ratios of these two decay processes different from each other. However, if we neglect these differences, the value of the branching ratio for the decay process $\tau^- \to \mu^- P_1 P_2$ should approximately equals to that of the decay process $\tau^- \to e^- P_1 P_2$. 

17
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Appendix

A. The relevant functions of the hadronic form factors

In this appendix we list the hadronic form factors that are related to the LFV \( \tau \) decays \( \tau^- \to \mu^- P_1 P_2 \). Their explicit expressions have been given in Ref. [6], we just put the related functions as follows:

\[
F^{\pi^+\pi^-}(s) = F(s) \exp \left[ 2 \Re \left( \hat{H}_{\pi\pi}(s) \right) + \Re \left( \hat{H}_{KK}(s) \right) \right],
\]

\[
F^{K^+K^-}(s) = F_{\rho}(s) + F_{\omega}(s) + F_{\phi}(s),
\]

\[
F^{K^0\bar{K}^0}(s) = -F_{\rho}(s) + F_{\omega}(s) + F_{\phi}(s),
\]

with

\[
F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \left[ 1 + \left( \delta \frac{M_\omega^2}{M_\rho^2} - \frac{s}{M_\rho^2} \right) \frac{s}{M_\omega^2 - s - iM_\omega \Gamma_\omega} \right],
\]

\[
F_{\rho}(s) = \frac{1}{2} \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[ 2 \Re \left( \hat{H}_{\pi\pi}(s) \right) + \Re \left( \hat{H}_{KK}(s) \right) \right],
\]

\[
F_{\omega}(s) = \frac{1}{2} \left[ \sin^2 \theta_V \frac{M_\omega^2}{M_\omega^2 - s - iM_\omega \Gamma_\omega} \right] \exp \left[ 3 \Re \left( \hat{H}_{KK}(s) \right) \right],
\]

\[
F_{\phi}(s) = \frac{1}{2} \left[ \cos^2 \theta_V \frac{M_\phi^2}{M_\phi^2 - s - iM_\phi \Gamma_\phi} \right] \exp \left[ 3 \Re \left( \hat{H}_{KK}(s) \right) \right],
\]

\[
\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F^2} \left[ \sigma_\pi^3(s) \theta(s - 4m_\pi^2) + \frac{1}{2} \sigma_K^3(s) \theta(s - 4m_K^2) \right],
\]

\[
\Gamma_\rho(s) = \Gamma_\rho(M_\rho^2) \frac{s}{M_\rho^2} \left( \frac{\sigma_\pi^3(s) + \frac{1}{2} \sigma_K^3(s) \theta(s - 4m_K^2)}{\sigma_\pi^3(M_\rho^2) + \frac{1}{2} \sigma_K^3(M_\rho^2) \theta(s - 4m_K^2)} \right) \theta(s - 4m_\pi^2). \]
where \( \sigma_P(s) = \sqrt{1 - 4\frac{m_P^2}{s}} \), and the other definitions are:

\[
\begin{align*}
\beta &= \frac{\Theta_{\rho\omega}}{3M_\rho^2}, \\
\gamma &= \frac{F_VG_V}{F^2} (1 + \beta) - 1, \\
\delta &= \frac{F_VG_V}{F^2} - 1, \\
\tilde{H}_{PP}(s) &= \frac{s}{F^2} M_P(s), \\
M_P(s) &= \frac{1}{12} \left( 1 - 4\frac{m_P^2}{s} \right) J_P(s) - \frac{k_P(M_P)}{6} + \frac{1}{288\pi^2}, \\
J_P(s) &= \frac{1}{16\pi^2} \left[ \sigma_P(s) \ln \frac{\sigma_P(s) - 1}{\sigma_P(s) + 1} + 2 \right], \\
k_P(\mu) &= \frac{1}{32\pi^2} \left( \ln \frac{m_P^2}{\mu^2} + 1 \right). \\
\end{align*}
\]

(38)

The contribution of the isospin breaking \( \rho - \omega \) mixing \( \Theta_{\rho\omega} = -3.3 \times 10^{-3} \text{GeV}^2 \), and the asymptotic constraint on the \( N_C \rightarrow \infty \) vector form factor indicates \( F_VG_V \simeq F^2 = F_\pi^2 \).

The mixing between the octet and singlet vector components employed in the construction of the \( I = 0 \) component of the kaon vector form factors is defined by :

\[
\begin{pmatrix}
\phi \\
\omega
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_V & -\sin \theta_V \\
\sin \theta_V & \cos \theta_V
\end{pmatrix}
\begin{pmatrix}
v_8 \\
v_0
\end{pmatrix},
\]

(39)

and the ideal mixing \( \theta_V = 35^\circ \) was used.

**B. The relevant functions in the TC2 model**

In the framework of TC2 model, the Inami-Lim functions that are used in our calculation are given as following.

\[
\begin{align*}
F_1(x) &= \frac{1}{8} \left[ \frac{x^2\ln x}{(x - 1)^2} - \frac{2x\ln x}{(x - 1)^2} + \frac{x}{x - 1} \right]; \\
F_2(x) &= \frac{1}{4} \left[ \frac{x}{x - 1} - \frac{x\ln x}{(x - 1)^2} \right]; \\
F_3(x) &= \frac{1}{32} \left[ \frac{x^2\ln x}{(x - 1)^2} - \frac{x}{x - 1} - \frac{x\gamma_E}{2} - x\ln4\pi - \frac{3x^2}{8} \\
&\quad + \frac{x^4\ln x}{4(x - 1)^2} - \frac{x^2}{4(x - 1)} \right]; \\
\end{align*}
\]

(40) (41) (42)
\[ F_d(x) = \frac{x}{16} \left[ \frac{-1}{4(x-1)} + \frac{3}{4(x-1)^2} + \frac{3}{2(x-1)^3} - \frac{3lnx}{(x-1)^4} \right]. \] (43)

C. The relevant functions in the LHT model

In this appendix we enumerate the functions related our calculation of the LFV \( \tau \) decays \( \tau^- \to \mu^- P_1 P_2 \) in the LHT model, which have been discussed in Ref. [28].

\[ S_{odd}(x) = \frac{x^2 - 2x + 4}{(1-x)^2}lnx + \frac{7-x}{2(1-x)}, \] (44)

\[ F_{uu}^{\alpha}(y_i, z; W_H) = \frac{3}{2} y_i - F_5(y_i, z) - 7F_6(y_i, z) - 9U(y_i, z), \] (45)

\[ F_{dd}^{\alpha}(y_i, z; W_H) = \frac{3}{2} y_i - F_5(y_i, z) - 7F_6(y_i, z) + 3U(y_i, z), \] (46)

\[ F_5(y, z) = \frac{y^3 \log y_i}{(1-y) (z-y_i)} + \frac{z^3 \log z}{(1-z) (y_i-z)}, \] (47)

\[ F_6(y, z) = -\left[ \frac{y_i^2 \log y_i}{(1-y)(z-y_i)} + \frac{z^2 \log z}{(1-z)(y_i-z)} \right], \] (48)

\[ U(y, z) = \frac{y^2 \log y_i}{(y_i-z)(1-y_i)^2} + \frac{z^2 \log z}{(z-y)(1-z)^2} + \frac{1}{(1-y_i)(1-z)}, \] (49)

\[ G(y, z; Z_H) = -\frac{3}{4} U(y, z), \] (50)

\[ G_1(y', z'; A_H) = \frac{1}{25a} G(y', z'; Z_H), \] (51)

\[ G_2(y, z; \eta) = -\frac{3}{10a} \left[ \frac{y^2 \log y_i}{(1-y)(\eta-y_i)(y_i-z)} \right. \]
\[ \left. + \frac{z^2 \log z}{(1-z)(\eta-z)(\eta-y_i)} + \frac{\eta^2 \log \eta}{(1-\eta)(y_i-\eta)(\eta-z)} \right], \] (52)

\[ D'_0(x) = -\frac{3x^3 - 2x^2}{2(x-1)^4}lnx + \frac{8x^3 + 5x^2 - 7x}{12(x-1)^3}, \] (53)

\[ E'_0(x) = \frac{3x^2}{2(x-1)^4}lnx + \frac{x^3 - 5x^2 - 2x}{4(x-1)^3}. \] (54)

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