Comment on “Manipulating the frequency-entangled states by an acoustic-optical modulator”

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CANADA

Abstract

A recent theoretical paper [1] proposes a scheme for entanglement swapping utilizing acousto-optic modulators without requiring a Bell-state measurement. In this comment, we show that the proposal is flawed and no entanglement swapping can occur without measurement.
I. INTRODUCTION

Entanglement swapping, a term coined by [2], is the process of creating entanglement between two particles that have never before interacted. When one generates entangled states, a pair of particles are created in tandem. The most common processes used to generate these entangled pairs are atomic cascades [3] and spontaneous parametric downconversion [4, 5]. In an entanglement swapping scheme, one begins with a pair of 2-particle entangled pairs. In successful schemes to date [6], one performs a Bell measurement on two of the particles - one from each entangled pair. A successful Bell-state measurement collapses the remaining particles into a new entangled state - even though the particles have not directly interacted. The process of entanglement swapping was central to the experimental realization of quantum teleportation [7].

This comment is on a recent proposal [1] for performing entanglement swapping with acousto-optic modulators (AOMs) without requiring a Bell measurement. The authors make a faulty assumption about the transformation an AOM performs on its input photon modes, which leads to incorrect conclusions. In this comment, we describe generally how one should treat the interaction of an AOM with its two input light fields quantum mechanically. Then we apply this type of interaction to the proposed scheme, and show that no entanglement swapping can take place.

II. THEORY

A. General theory

An acousto-optic modulator can be used to couple two modes of an electromagnetic field by means of a phonon field. A simple diagram from [1] shows this schematically (figure 1). The interaction between two input fields and an acousto-optic modulator will be described by an effective Hamiltonian, \( \mathcal{H}_{\text{eff}} \), of the form:

\[
\mathcal{H}_{\text{eff}} = gb(\delta)a_d(\omega + \delta) + g^*b^\dagger(\delta)a^\dagger_d(\omega)a_d(\omega + \delta),
\]

where \( b \) and \( b^\dagger \) are the annihilation and creation operators for the phonon field, \( a \) and \( a^\dagger \) are the annihilation and creation operators for the photon field, and \( g \) is the coupling constant. The subscripts \( t \) and \( d \) refer to the mode labels shown in figure 1, and \( \omega \) and \( \omega + \delta \) are the photon frequencies. The first term in this Hamiltonian describes the destruction of a phonon of frequency \( \delta \), and a photon of frequency \( \omega \) in mode \( d \), and the creation of a photon with frequency \( \omega + \delta \) in mode \( t \). The second term in the Hamiltonian describes the creation of a phonon of frequency \( \delta \), and a photon of frequency \( \omega \) in mode \( d \), and the destruction of a photon in mode \( t \). This Hamiltonian is manifestly hermitian, and the propagator that follows from it must be unitary. If one assumes that the phonon field in the AOM is a classical field, which is a reasonable approximation for a coherent state of phonons with a high average phonon number, then we can replace the phonon operators with c-numbers \( \beta \) and \( \beta^* \). The Hamiltonian then becomes:
\[ H_{\text{eff}} = g\beta a_\omega a_\omega^\dagger(\omega + \delta) + g^* \beta^* a_t^\dagger(\omega) a_d(\omega + \delta). \] (2)

Over an infinitesimal interaction time, \( dt \), the AOM will perform the following transformations:

\[ |\omega\rangle_1 \rightarrow |\omega\rangle_t - i\frac{\hbar}{\hbar} g\beta |\omega + \delta\rangle_d dt \]
\[ |\omega + \delta\rangle_1' \rightarrow |\omega + \delta\rangle_d - i\frac{\hbar}{\hbar} g^* \beta^* |\omega\rangle_t dt. \] (3)

Over longer times, the transformation becomes:

\[ |\omega\rangle_1 \rightarrow \cos \left( \frac{|g \beta| t}{\hbar} \right) |\omega\rangle_t - i\frac{g\beta}{|g\beta|} \sin \left( \frac{|g \beta| t}{\hbar} \right) |\omega + \delta\rangle_d \]
\[ |\omega + \delta\rangle_1' \rightarrow \cos \left( \frac{|g \beta| t}{\hbar} \right) |\omega + \delta\rangle_d - i\frac{g^* \beta^*}{|g\beta|} \sin \left( \frac{|g \beta| t}{\hbar} \right) |\omega\rangle_t. \] (4)

We can choose the interaction time to create equal superpositions of the outgoing modes and define the phase angle, \( \phi = \text{arg}(g\beta) \). The transformations then become:

\[ |\omega\rangle_1 \rightarrow \frac{1}{\sqrt{2}} \left[ |\omega\rangle_t - ie^{i\phi} |\omega + \delta\rangle_d \right] \]
\[ |\omega + \delta\rangle_1' \rightarrow \frac{1}{\sqrt{2}} \left[ |\omega + \delta\rangle_d - i e^{-i\phi} |\omega\rangle_t \right]. \] (5)

**B. An AOM cannot result in entanglement swapping without a Bell measurement**

The authors of [1] claim that an AOM (figure 1) can be modelled by taking 2 input modes, 1 and 1’, and transforming them to two output modes as follows:

\[ |\omega\rangle_1 \xrightarrow{\text{AOM}} \frac{1}{\sqrt{2}} \left[ |\omega\rangle_t + |\omega + \delta\rangle_d \right] \]
\[ |\omega + \delta\rangle_1' \xrightarrow{\text{AOM}} \frac{1}{\sqrt{2}} \left[ |\omega\rangle_t + |\omega + \delta\rangle_d \right]. \] (6)

The equations above actually differ from equations (1) and (2) from [1] due to a presumed typographical error in the left side of the second equation, but are consistent with the rest of their paper. However, such a transform is not allowed by quantum mechanics as it is non-unitary. In other words, the two input states are orthogonal, and must remain so by any unitary transformation. As one can see from the proposed transformation, the final states are not orthogonal - in fact they are identical. Such transformations destroy information and lead to paradoxes such as superluminal signalling. In the present case, if the proposed scheme were correct, a decision by Alice of whether or not to perform the transformation could instantaneously affect a measurement by Bob of whether or not his photon pair was entangled.
Instead of the transformation given in [1], one should model the AOM by the unitary transformation described previously. We use the transforms from equation (3) and make the assumption that \( \phi = 0 \), without loss of generality. To put the transform into the same form as equation (3), the second transform is multiplied by a phase of \( \exp(i\pi/2) \):

\[
|\omega\rangle_1 \rightarrow \frac{1}{\sqrt{2}} [|\omega\rangle_t - i |\omega + \delta\rangle_d] \\
|\omega + \delta\rangle_{1'} \rightarrow \frac{1}{\sqrt{2}} [|\omega\rangle_t + i |\omega + \delta\rangle_d].
\]

The negative sign in the first term ensures that the final states remain orthogonal, preserving angle in the 2-dimensional Hilbert space. We now follow through the calculations from [1] and describe the separate 2-particle entangled states (see figure 2) as:

\[
|\phi\rangle = \frac{1}{\sqrt{2}} [|\omega\rangle_1 |\omega + \delta\rangle_2 + |\omega + \delta\rangle_1 |\omega\rangle_2']
\]

\[
|\psi\rangle = \frac{1}{\sqrt{2}} [|\omega\rangle_3 |\omega + \delta\rangle_4 + |\omega + \delta\rangle_3 |\omega\rangle_4'].
\]

The states \( |\phi\rangle \) and \( |\psi\rangle \) refer to the states of the particles created at the entangled-photon sources 1 and 2 respectively. The primed and unprimed subscripts refer to the spatial modes of the photons (figure 2), and the labels \( \omega \) and \( \omega + \delta \) refer to their angular frequencies. The two photons described in these states are not only entangled in their energy (frequency) but also in their spatial paths. We can now apply the following unitary transformations to the modes that interact with the AOMs in the scheme:

\[
|\omega + \delta\rangle_2 \xrightarrow{AOM1} \frac{1}{\sqrt{2}} [|\omega\rangle_{T_1'} + i |\omega + \delta\rangle_{T_1}]
\]

\[
|\omega\rangle_3 \xrightarrow{AOM1} \frac{1}{\sqrt{2}} [|\omega\rangle_{T_1'} - i |\omega + \delta\rangle_{T_1}]
\]

\[
|\omega + \delta\rangle_3' \xrightarrow{AOM2} \frac{1}{\sqrt{2}} [|\omega\rangle_{T_2} + i |\omega + \delta\rangle_{T_2'}]
\]

\[
|\omega\rangle_{2'} \xrightarrow{AOM2} \frac{1}{\sqrt{2}} [|\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'}].
\]

AOM1 and AOM2 simply refer to the transformation applied by the AOMs marked 1 and 2 in figure 2.

Using these transformations, the initial state describing the four photons, \( |\phi\rangle \otimes |\psi\rangle \), will become:

\[
|\phi\rangle \otimes |\psi\rangle = \frac{1}{2} \left\{ 
|\omega\rangle_1 \left( |\omega\rangle_{T_1'} + i |\omega + \delta\rangle_{T_1} \right) + |\omega + \delta\rangle_{1'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'} \right) \right\} \otimes \\
\left\{ 
|\omega\rangle_1 \left( |\omega\rangle_{T_1'} + i |\omega + \delta\rangle_{T_1} \right) + |\omega + \delta\rangle_{1'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'} \right) \right\}
\]

\[
= \frac{1}{2} \left\{ 
|\omega\rangle_1 |\omega + \delta\rangle_{1'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'} \right) + |\omega + \delta\rangle_{1'} |\omega\rangle_{1'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'} \right) + \\
|\omega\rangle_1 |\omega + \delta\rangle_{1'} \left( |\omega\rangle_{T_2} + i |\omega + \delta\rangle_{T_2'} \right) + |\omega + \delta\rangle_{1'} |\omega\rangle_{1'} \left( |\omega\rangle_{T_2} + i |\omega + \delta\rangle_{T_2'} \right) + \\
|\omega + \delta\rangle_{1'} |\omega\rangle_{1'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_2'} \right) + |\omega + \delta\rangle_{1'} |\omega\rangle_{1'} \left( |\omega\rangle_{T_2} + i |\omega + \delta\rangle_{T_2'} \right) \right\}.
\]
The authors propose to discard the cases where both photons go through the same AOM (the first and fourth terms in the above equation) and are left with only the remaining 2 terms. These terms are:

\[
|\omega + \delta\rangle_{1'} |\omega + \delta\rangle_{4'} \left( |\omega\rangle_{T_2} - i |\omega + \delta\rangle_{T_{2'}} \right) \left( |\omega\rangle_{T_1'} - i |\omega + \delta\rangle_{T_1} \right) + \\
|\omega\rangle_{1} |\omega\rangle_{4'} \left( |\omega\rangle_{T_1'} + i |\omega + \delta\rangle_{T_1} \right) \left( |\omega\rangle_{T_2} + i |\omega + \delta\rangle_{T_{2'}} \right).
\]

It is apparent that when the proper transformation is used, the terms describing the light after the AOM do not factor out and no entanglement swapping has occurred between photons 1 and 4. In fact, the states describing the light after passing through the AOM have an overlap of zero, precluding even partial entanglement swapping.

In [1], a different AOM scheme is used to create a Greenberger-Horne-Zeilinger (GHZ) 3-particle entangled state using a pair of 2-photon entangled states. Unfortunately, the same transformation as shown in equation (1) is used to model the AOM. When the appropriate transformation is applied instead to their scheme, there is no 3-particle entanglement.

**III. CONCLUSION**

The authors of [1] used a non-unitary transformation to describe the action of an AOM on a pair of input photon modes, and this appeared to lead to unconditional entanglement swapping. We have shown that when a unitary transformation is used instead, as required by quantum mechanics, no entanglement between the photons from different sources is achieved. Due to the same erroneous transformation, the claim that an AOM could create a GHZ state using a pair of 2-photon entangled states is also incorrect. In general, no unitary transformation on one pair of photons can ever modify the reduced density matrix of a different pair. This is why effects such as entanglement swapping [3] and quantum teleportation [6] always require a nonunitary interaction (measurement).

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Figure 1. The two input modes, 1 and 1', enter an AOM and are converted to two output modes, $t$ and $d$.

Figure 2. The schematic for the proposed entanglement swapping scheme.
