Hybrid-PIC Simulation of LaB₆ Hollow Cathode Self-Heating Characteristics

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Numerical simulation of plasma flow and the self-heating characteristics of a LaB₆ hollow cathode were performed using a hybrid-PIC model. For a discharge current of 30 A and mass flow rate of 3 mg/s, the influences of an emitter temperature profile and model parameter included in an anomalous resistivity model on the plasma flow and energy flux were investigated. In the simulation, the discharge voltage was fixed at a predetermined value and the maximum emitter temperature was periodically adjusted to keep the discharge current constant. The results show that the present model predicts the keeper floating voltage within an accuracy of 20%. It is found that the main reason for the emitter temperature to rise is due to ion bombardment and accompanying recombination energy, and that the maximum emitter temperature can be kept lower as the emitter temperature profile becomes uniform. It is also shown that thermal input into the emitter is decreased when anomalous resistivity increases.

Key Words: Electric Propulsion, Hollow Cathode, Numerical Simulation, Thermal Analysis

Nomenclature

- \( b \): model parameter
- \( D \): modified Richardson-Dushman constant
- \( e \): elementary charge
- \( E \): electric field
- \( j \): current density
- \( k \): Boltzmann constant
- \( L \): length
- \( m \): mass of particle
- \( n \): number density
- \( \hat{n} \): unit vector normal to wall
- \( p \): pressure
- \( q \): heat flux
- \( Q \): energy consumption
- \( r \): radial coordinate
- \( T \): temperature
- \( u' \): mean velocity
- \( v \): particle velocity
- \( V \): reaction energy
- \( z \): axial coordinate
- \( \Gamma \): flux
- \( \alpha \): constant
- \( \varepsilon_0 \): permittivity in vacuum
- \( \mu \): mobility
- \( v \): collision frequency
- \( \rho \): mass density
- \( \phi \): electric potential

Subscripts

- \( ab \): absorption
- \( b \): thermionic electron
- \( e \): electron
- \( em \): emitter
- \( i \): ion
- \( iz \): ionization
- \( k \): keeper
- \( n \): neutral
- \( orf \): orifice
- \( rcm \): recombination
- \( sh \): sheath
- \( w \): work function
- \( 0 \): sheath edge

1. Introduction

Development of a durable high-current hollow cathode is a key technology to realize a high-power Hall thruster capable of raising the orbit of all-electric satellites. This is because the hollow cathode is a very sensitive component and can be a bottleneck to the robustness of the overall propulsion system.¹,² Since the discharge current is basically extracted from an emitter that should be maintained at a high temperature through self-heating in order to emit sufficient thermionic electrons, understanding the self-heating characteristics of the emitter and cathode tube are quite important for designing the hollow cathode.

To clarify the self-heating characteristics, the plasma flow and energy flux, especially into the emitter, should be under-
stood. A number of experimental studies have been conducted for years in an effort to determine the physics inside the cathode,\textsuperscript{3–6} and recently numerical simulation has been developed as a powerful tool to predict the plasma flow of hollow cathodes. Mikellides et al. developed OrCa2D, which uses fluid approximation coupled with a collisionless neutral model in the plume region.\textsuperscript{7–12} This can estimate energy fluxes on the walls, and then those are utilized as boundary conditions for thermal analysis.\textsuperscript{13} Another fluid simulation code has also been developed at University of Toulouse, where a thermal model is self-consistently coupled with plasma simulation.\textsuperscript{14}

In this study, the hollow cathode plasma was simulated by using a hybrid particle-in-cell (PIC) model that treats the heavy species (neutrals and ions) and electrons as particles and fluid, respectively.\textsuperscript{15} Since the hybrid-PIC model can be applied to both dense and rarefied plasma, the model is available to use for the whole domain as it is contrary to the fluid model, which requires special treatment in the plume region regarding rarefied effect.\textsuperscript{11} Although the computational cost of hybrid-PIC is much higher than the fluid approach, recent advances in computation technology and parallel computation enable us to utilize it within a realistic time. The target of the simulation is our laboratory model of the LaB\textsubscript{6} hollow cathode which utilizes a radiative heater for heating the emitter in expectation of robustness against heater failure.\textsuperscript{16} In this study, we performed plasma simulations to investigate the plasma properties and self-heating characteristics of the hollow cathode using the hybrid-PIC model. The influences of the emitter temperature profile and a model parameter included in an anomalous resistivity model in the plasma flow and self-heating characteristics are discussed in this paper.

2. Numerical Modeling

2.1. Hollow cathode and simulation domain

Figure 1 shows a schematic view of the hollow cathode discussed here. As described earlier, LaB\textsubscript{6} is used as the emitter because it has a higher robustness than barium-oxide dispenser cathodes against poisoning by oxidizing impurities. The length and inner diameter of the emitter are 25 mm and 6.3 mm, respectively, which were set to the same as the JPL 1.5-cm class LaB\textsubscript{6} cathode.\textsuperscript{17} A radiative heater made of graphite is wrapped around the 14-mm-dia. graphite cathode tube with a short gap. Around the heater, the heat transfer to the graphite keeper is reduced by a heat shield made of tantalum.

The computational domain is shown as the dashed red line in Fig. 1. The flow is assumed to be axisymmetric, thus only the upper-half region of the cross-section view is considered. The upstream boundary, from which the working gas is fed, corresponds to the beginning of the emitter. The computational domain is extended to the plume region by several centimeters.

2.2. Hybrid-PIC model

The numerical model used here assumes an axisymmetric and quasi-neutral flow in the computational domain. Therefore thin sheath approximation is imposed on the wall boundaries. The heavy species (neutrals and ions) and electrons are treated as particles and fluid, respectively. The trajectories of ions are traced using the PIC method while considering elastic collisions (n-n, n-i, i-i) and charge exchange collision (CEX). The collisions with regard to neutrals are dealt with using the Direct Simulation Monte Carlo (DSMC) method. The Coulomb collision between ions is dealt with using the Monte Carlo Collision (MCC) method to reduce the computational cost. In the CEX model, the effect of the polarization potential, which leads to a large scattering angle, is taken into account.\textsuperscript{18} Since the nominal weight of a macroneutral is much larger than that of a macroion, a mass decay model is utilized as the ionization model.\textsuperscript{19}

Given \( u_e \gg u_n, u_i \), the mean velocity of the electrons is approximated using the drift-diffusion model.

\[
\nabla \cdot \mathbf{E} = -\frac{\mu_e}{e} \nabla n_e, \quad \mathbf{E} = -\nabla \phi
\]

The electron density is equal to the ion density determined from the ion PIC. The electron mobility is calculated using the effective collision frequency, which takes into account two-stream instability\textsuperscript{9}.

\[
\mu_e = \frac{e}{m_e v_e}, \quad v_e = v_en + v ei + v_a
\]

where the following anomalous collision frequency is used.

\[
v_a = \left( \frac{m_e}{m_i} \right)^{1/3} f(M_e)\sqrt{2\omega_{pe}}, \quad M_e = \left| u_e \right| / \sqrt{\kappa T_e/m_e}
\]

\[
f(M_e) = \alpha M_e^b (M_e < 1.3), \quad f(M_e) = 1 \quad (M_e \geq 1.3)
\]

Here, \( u_e \gg u_i \) is assumed. Unless stated otherwise, in this paper, the constants \( \alpha \) and \( b \) are set at 0.171 and 1.25, respectively, which were determined based on the Stringer diagram under the assumption of \( T_i/T_e = 0.01 \).\textsuperscript{9} According to the Stringer diagram, the growth rate of instability is not so sensitive to \( T_i/T_e \) in the typical range for hollow cathodes. Therefore, we used the above precedent values. As to modeling the anomalous resistivity in cathodes, more advanced models have been proposed such as the self-consistent model
derived from kinetic theory, but the conventional model is currently used for simplicity. With Eq. (1) and the current conservation law, the elliptic differential equation with regard to electric potential can be derived.

\[
\nabla \cdot n_e \mu_e \nabla \psi = \nabla \cdot \left( n_e u_e + \frac{\mu_e}{e} \nabla p_e \right)
\]

(4)

The electron temperature is determined from the steady-state energy conservation law:

\[
\nabla \cdot \left[ \left( \frac{1}{2} \rho_e u_e^2 + \frac{5}{2} n_e k T_e \right) u_e - \frac{5 n_e k^2 T_e}{2 e} \mu_e \nabla T_e \right] = J_e \cdot E - \sum_{s=n,i} 3 n_s k \frac{m_e}{m_s} v_{cs} (T_e - T_s) - e V_{ic} k_c n_i n_e
\]

(5)

where, \( k_c \) is the reaction rate coefficients of ionization. Deriving thermal conductivity assumes a weakly ionized plasma. The temperature of heavy species is evaluated as:

\[
T_s = \frac{m_k}{3k} \left( \langle v_s^2 \rangle - \langle v_i^2 \rangle \right), \quad s = n, i
\]

(6)

where, \( \langle x \rangle \) denotes the average of \( x \) in a cell. Equations (4) and (5) are discretized using the finite element method and are separately solved using a direct solver. The computation of particle tracing is parallelized by means of the MPI technique, where the computation domain of each process is dynamically decomposed to keep the computational cost for each process uniform.

### 2.3. Boundary conditions

The boundary conditions are summarized in Fig. 2. The working gas is fed from the inflow boundary \((z/L_{em} = 0)\) as neutrals at local emitter temperature and a constant mass flow rate. Once a neutral macroparticle collides with the wall, the particle is completely thermalized to the wall temperature and is reflected isotropically, which decelerates mean velocity near the wall and forms a shear layer near the walls owing to momentum exchange collisions considered in DSMC. This results in considering viscosity effect which is important to obtain a realistic internal plasma flow. Ions colliding with the wall recombine with electrons and are reflected as thermalized neutrals. Since the wall boundary should be interpreted as a sheath edge, the ion/electron density on the wall boundary is replaced with \( j_i \cdot \hat{n} / e u_{Bohm} \) to assure Bohm, for impinging ions. Generally, \( u_{Bohm} \) is given by \( \sqrt{kT_i/m_i} \); thus, the ion/electron density can be calculated straightforward. Strictly speaking, however, \( u_{Bohm} \) should be modified because of the effect of thermionic electrons on the emitter surface. The Bohm condition on the emitter surface is described in the Appendix. At the outflow boundaries, the particles can freely escape from the computational domain. To consider the background pressure, when the neutral pressure becomes less than a specified value, which is set to a value determined from experiments, macroneutrals are supplemented with zero mean velocity to maintain the minimum pressure.

The discharge current and discharge voltage are specified based on an experiment conducted by the authors. The cathode shown in Fig. 1 was tested in a vacuum chamber 0.6 m in diameter and 1.0 m long, located at a JAXA facility. The cathode and a ring-shaped anode were configured with a gap of 37 mm. The discharge voltage was set as the electric potential on the downstream boundary. To keep the discharge current constant, the maximum emitter temperature \( T_{em,max} \) was periodically adjusted. Since we had not measured the emitter temperature distribution of our own cathode, the normalized temperature profile \( T_{em}(z)/T_{em,max} \) was assumed. To determine the sensitivity of the temperature profile, two profiles were simulated here; uniform and quadratic profiles. The latter profile is based on measurements for a 1.5-cm class LaB6 hollow cathode in JPL. Since our cathode interior was designed based on this cathode and had almost the same dimensions, we simply referred to this cathode. On the wall boundaries, a sheath model was imposed because the present model used thin sheath approximation. On the emitter surface, from which the thermionic electrons are supplied, the electron flux is given by:

\[
\frac{n_e u_e \cdot \hat{n}}{e u_{Bohm}} = \Gamma^{ab} - \Gamma^{em}
\]

(7)

\[
\Gamma^{em} = 1/4 n_e \sqrt{8kT_e / \pi m_e} \exp \left( -\frac{e\phi_{sh}}{kT_e} \right), \quad \Gamma^{em} = \frac{i_{em}}{e}
\]

(8)

where, \( \phi_{sh} \) denotes the sheath potential drop on the wall. The quantity \( i_{em} \) represents the thermionic electron current from the emitter given by the Richardson-Dushman formula:

\[
\Gamma_{em} = DT_{em}^2 \exp \left( -\frac{e\phi_{eff}}{kT_{em}} \right), \quad \phi_{eff} = \phi_e - \frac{eE_{em}}{4\pi\varepsilon_0}
\]

(9)

where, \( E_{em} \) denotes the electric field on the emitter surface. The constant \( D \) and work function \( \phi_e \) were set to 29 A/cm²/K² and 2.67 eV, respectively. The formula to evaluate \( E_{em} \) was obtained by integrating the Poisson equation. On the other wall boundaries, Eq. (7) was applied as it was if \( \Gamma^{em} \) was set as zero. Since the keeper is assumed to be electrically floated in accordance with the experiment, the keeper voltage \( \phi_k \) had to be known to evaluate \( \phi_{sh} \) in Eq. (8); where \( \phi_{sh} \) on the keeper is defined as the difference between the potential at the boundary nodes and \( \phi_k \). In this simulation, the keeper voltage was artificially specified and was periodically adjusted to keep the net current into the keeper zero. When \( \phi_{sh} \) became negative, which corresponds to an ion-repelling sheath, the exponential factor in Eq. (8) was removed. Then, only a few ions that have kinetic energy larger than the...
sheath potential reached the wall and were reflected as thermalized neutrals.

The electron energy flux lost from the bulk plasma toward the emitter is evaluated as follows.

\[ q_e = \Gamma_e^{\text{in}} (2kT_e + e\phi_{sh}) - \Gamma_e^{\text{em}} (2kT_{em} + e\phi_{sh}) \]  

(10)

On the other wall boundaries, Eq. (10) can be used as it is if \( \Gamma_e^{\text{em}} \) is set as zero. At the inflow and outflow boundaries, the electron temperatures are fixed at typical values.\(^7\)

2.4. Calculation conditions

The calculation conditions of the plasma simulation are summarized in Table 1, and the emitter temperature profiles assumed are shown in Fig. 3. As mentioned earlier, two temperature profiles were simulated: uniform and quadratic. The quadratic profile is a fitting curve of the data points measured for the JPL 1.5-cm class LaB\(_6\) cathode working at 30 A and 25 sccm,\(^5\) which seems the closest condition to the present study among the available data set. Since parameter \( b \) in Eq. (3) has a strong impact on the potential distribution,\(^9\) its sensitivity was also investigated. The simulation cases are named as listed in Table 2.

The minimum mesh size within the cathode tube is 0.1 mm. Although this is larger than the mean free path between ions (i.e., approximately 1 \( \mu \)m), which is the shortest among the considered collisions, the mesh size is larger than the characteristic length of the spatial variation of plasma property. In addition, the time interval is set to approximately \( 1 \times 10^{-9} \) sec, which is less than the collision times of all collisions. Therefore, the requirement for collision treatment is fulfilled.

3. Results

3.1. Plasma characteristics

Figure 4 shows the electron number density, electric potential, and electron temperature for Case A. The electron number density peaks at approximately \( 1.1 \times 10^{21} \) m\(^{-3}\) in front of the entrance of the orifice, and then rapidly decreases through the orifice and keeper region. At the end of the plume boundary (\( z/L_{em} = 2 \)), the density reaches approximately \( 8.5 \times 10^{17} \) m\(^{-3}\). The electric potential is gradually increased within the emitter region from 9 to 19 V. Through the orifice region, the increasing rate of the potential becomes relatively high, and it reaches the discharge voltage near the keeper’s orifice. The electron temperature has a peak of
approximately 2.2 eV near the keeper, but it recedes to the boundary condition of 2.0 eV in the plume region.

Figure 5 shows the anomalous and classical collision frequency of electrons along the centerline, and the electron Knudsen number and electron Mach number are also plotted. The Knudsen number is based on classical collisions.

\[ Kn = \frac{\partial n_e}{\partial z} \sqrt{\frac{2kT_e}{m_e}} \frac{1}{v_{en} + v_{ei}} \]  

(11)

Classical collisions are dominant in the emitter region \( z/L_{em} < 1 \), and decrease rapidly from the oriﬁce entrance towards the plume region. The anomalous collision frequency begins to rise in front of the oriﬁce entrance and saturates outside of the oriﬁce. Although sharp drops appear in the Kn plot due to the noisy distribution of electron density, this trend is basically consistent with a preceding study.

### 3.2. Energy flux

The energy flux into the emitter \( q_{em} \) is classiﬁed into contributions from the electrons, neutrals, and ions as follows.

\[ q_{em} = q_{ab}^e + q_{ab}^{bw} + q_{em}^e + q_{em}^{mw} + q_n + q_i + q_i^w + q_{rcm} \]  

(12)

\[ q_{ab}^e = T_e^{ab} 2kT_e, \quad q_{ab}^{bw} = T_e^{ab} e\phi_w \]  

(13)

\[ q_{em}^e = -\Gamma_e^{em} 2kT_{em}, \quad q_{em}^{mw} = -\Gamma_e^{em} e\phi_w \]  

(14)

\[ q_n = \frac{1}{dt} \frac{1}{dS} \left( \sum_{j=m} \frac{1}{2} m_n v_{nj}^2 - \sum_{j=ion} \frac{1}{2} m_n v_{nj}^2 \right) \]  

(15)

\[ q_i = \frac{1}{dt} \frac{1}{dS} \sum_{j=ion} \frac{1}{2} m_i v_{ji}^2 \]  

(16)

\[ q_i^w = -\frac{1}{dt} \frac{1}{dS} \sum_{j=ion} e\phi_w \]  

(17)

\[ q_{rcm} = \frac{1}{dt} \frac{1}{dS} \sum_{j=ion} eV_{iz} \]  

(18)

Here, \( dt \) and \( dS \) are the time interval used in the plasma simulation and area of the surface element with which the particles collide, respectively. On the right-hand side of Eq. (12), the first and second terms are related to the electrons absorbed into the emitter, and the third and fourth terms are derived from the electrons emitted. Since the neutrals and ions are modeled as particles, the energy flux is given by summing up the kinetic energy of each particle colliding with the wall. Note that the neutrals leaving the wall must be considered, which includes not only the reﬂected neutrals, but also the recombed ions. The kinetic energy of the ions involves the incremental energy added through the sheath region.

The energy ﬂuxes on the emitter and oriﬁce of the cathode tube are plotted in Figs. 6 and 7, respectively. The results are an average of 1000 time steps. On the emitter surface, the total energy ﬂux peaks at approximately \( z/L_{em} = 0.9 \) because of the large contribution from ion bombardment and accompanying recombination energy. The energy ﬂuxes regarding electrons are much lower than those of the ions. Since \( |q_{ei}^w| \ll 1 \), its line is overlapped with the line of \( q_{ab}^{bw} \). On the oriﬁce of the cathode tube, the total energy ﬂux monotonically decreases in the axial direction. The degree of contribution of each term in Eq. (12) is basically the same as that on the emitter. The total thermal inputs into the emitter and oriﬁce are 270 W and 39.5 W, respectively. As the thermal input to the oriﬁce, only the radial energy ﬂux from the straight flow channel is considered. The percentages of each total thermal input into the emitter and oriﬁce are tabulated in Table 3. The negative values correspond to the heat released...
from the body.

In Figs. 6 and 7, the energy fluxes of the ions and neutrals were evaluated in terms of particle description using Eqs. (15) and (16). The energy flux of ions can also be described using the following expressions,

\[ q_i = j_i \cdot \hat{n} \left( \frac{kT_e}{2} + e\phi_{th} \right) \]  
\[ q_i = 0.6n_i \sqrt{\frac{kT_e}{m_i}} \left( \frac{kT_e}{2} + e\phi_{th} \right) \]

In Eq. (19), \( j_i \) is the ion current density evaluated using the PIC method at boundary nodes. On the other hand, the ion flux in Eq. (20) is replaced with the flux derived from the ion saturation current. The first term of the right-hand-side of both equations denotes the ion energy at the sheath edge, whereas the modification due to the thermionic electrons is ignored because of its negligible effect, shown later. The energy fluxes of ions evaluated by Eqs. (16), (19), and (20) are shown in Fig. 8. Since \( j_i \cdot \hat{n}/e \) in Eq. (19) inherits the particle description, the results agree well with that of Eq. (16), and it is smoothed due to the averaging effect of PIC. On the other hand, Eq. (20) gives a lower energy flux than the others, especially around the peak of the energy flux.

Figure 9 shows the effective work function defined in Eq. (9), and the relative error of Bohm velocity as defined by

\[ u_{\text{Bohm}} = \sqrt{\frac{kT_e}{m_i}} \]

From Figs. 8 and 9, we can see that the axial position where the effective work function is minimized is near the position of the energy flux peak. The minimum work function is 0.1 V less than the original value. As to the Bohm velocity, the effect of the modification caused by the thermionic electrons seems negligible.

3.3. Influence of emitter temperature profile and model parameter

Figure 10 shows the electron density and electric potential along the centerline for Cases A–D. The electron densities for all of the cases are nearly the same in \( z/L_{em} > 0.5 \), but the electron density for Cases B and D is slightly higher than the other cases near the cathode orifice \( (z/L_{em} \approx 1) \). In \( z/L_{em} < 0.5 \), the uniform emitter temperature (Cases A and B) causes higher electron density than the quadratic cases (Cases C and D). Comparing Case A with Case B, we can see that the higher \( b \) results in a slightly higher electron density, and this is also true for Cases C and D. On the other hand, the electric potential for Cases A and C is higher than that for Cases B and D inside the cathode.

The comparison of the thermal input, maximum emitter temperature \( T_{em,max} \), and keeper floating voltage \( \phi_k \) is shown in Table 4; where, \( \phi_k \) measured in the experiment, is also shown. The value measured was time-averaged in 1 sec at

| Emitter | Orifice | Emitter | Orifice |
|---------|---------|---------|---------|
| \( q_{e}^{a} \) | 0.68 | \( q_{e}^{b} \) | -14 |
| \( q_{e}^{c} \) | 0.63 | \( q_{e}^{d} \) | 76 |
| \( q_{e}^{e} \) | -15 | \( q_{e}^{f} \) | -14 |
| \( q_{e}^{g} \) | -13 | \( q_{e}^{h} \) | 65 |

Fig. 8. Ion energy flux on emitter (Case A).

Fig. 9. Effective work function and relative error of Bohm velocity (Case A).

Fig. 10. Influence of emitter temperature and model parameter on electron density and electric potential along the centerline.
a sampling rate of 10 msec. It can be seen that the uniform emitter temperature profile results in a lower maximum emitter temperature. It is also shown that the lower $b$ leads to lower thermal input to the emitter and higher thermal input to the orifice. Additionally, $\phi_l$ tends to decrease with a lower $b$ and approaches the value measured.

4. Discussion

Table 4 shows that the numerical simulation predicts the keeper floating voltage within an accuracy of 20%. Considering various assumptions in the numerical model, this prediction accuracy is considered reasonable. This suggests that the fundamental plasma properties seen in Figs. 4 and 10 are consistent with the actual plasma flow. However, the keeper floating voltage will be affected by the boundary condition in the plume region. In this simulation, we ignored the actual anode configuration for simplicity and assumed a uniform potential profile in the radial direction at the rightmost boundary. According to previous research on plume diagnostics, however, the potential has a radially increasing profile outside the cathode end. Furthermore, it is known that the axial potential profile near the cathode depends on the anode configuration. In the future, we need to investigate the effect of realistic potential boundary conditions and actual anode configuration on the flow structure.

According to Table 3, the dominant component of energy flux into the emitter and orifice is due to the ion bombardment and accompanying recombination energy. The heat loss related to the electrons is quite low, probably due to the high sheath potential of more than 10 V along the most of the emitter and orifice as seen in Fig. 4(b). Since the electron temperature near the emitter is less than 2 eV, as shown in Fig. 4(c), most of the electrons cannot overcome such a large sheath potential drop. For example, when $T_e = 1.5$ eV, the electrons having a kinetic energy of more than 10 eV is less than 0.4%. The low contribution of electrons to heating was also witnessed in a previous study.

One of the reasons for the underestimated ion energy flux when using Eq. (20) would be the existence of a relatively high electric field near the orifice entrance. It promotes ionization, ion acceleration and heating in the presheath region. These effects are included in the simulation, but are ignored when deriving the ion saturation current. Another possible reason might be the deviation of electrons from the Boltzmann distribution assumed in the derivation of the ion saturation current because of collisions with heavy particles.

In Fig. 10, the higher electron density for Cases A and B in the upstream region is associated with the higher emitter temperature in the upstream region (see Fig. 3). Since a higher emitter temperature results in more thermionic electrons in the plasma, Joule heating leading to ionization can be promoted in the upstream region. The reason for that the electric potential is decreased inside the cathode with lowering $b$ is the growth of the anomalous resistivity. Since the electron Mach number is suppressed to lower than unity when using the present anomalous resistivity model, as shown in Fig. 5, lowering $b$ leads to higher anomalous resistivity, which contributes to the growth of potential gradient through the orifice region. In the present simulation, the potential at the plume boundary is fixed. Therefore, the increase in potential gradient results in lowering the potential inside the cathode. The low space potential of Cases B and D leads to low sheath voltage on the walls, and hence, ion energy flux into the emitter is suppressed as shown in Table 4. Instead, the increase in potential gradient around the orifice invokes Joule heating and ionization, which causes orifice heating to increase. These results indicate that increasing anomalous resistivity is detrimental to self-heating of the emitter under constant discharge voltage conditions. Table 4 also suggests that the maximum emitter temperature can be kept lower when the emitter temperature profile is more uniform, which will be beneficial for cathodes in terms of lifetime.

5. Conclusion

The plasma flow and self-heating characteristics of the LaB$_6$ hollow cathode were numerically investigated using a hybrid-PIC model that treats heavy particles (neutrals and ions) and electrons as particles and fluid, respectively. In the plasma simulation, the discharge voltage and discharge current are specified based on experimental results (24 V, 30 A). Additionally, the maximum emitter temperature was periodically adjusted to keep the discharge current constant, where the temperature profile normalized by the maximum temperature was fixed. This study investigated the influences of emitter temperature profile and model parameter included in the anomalous resistivity model on the results. As the temperature profile, two profiles were simulated: uniform, and quadratic.

From the plasma simulation, it was shown that the dominant components of the energy flux are attributed to ion bombardment and the accompanying recombination energy. It was found that the maximum emitter temperature can be kept lower when a uniform temperature profile is used. It is also shown that thermal input into the emitter decreases when anomalous resistivity increases. Therefore, under constant discharge voltage conditions, it can be said that growth of the anomalous resistivity is detrimental for self-heating of the emitter. The results show that the present model predicts the keeper floating voltage within an accuracy of 20%. Considering the various assumptions used in modeling, this accuracy seems reasonable. However, there is speculation that this is affected by boundary conditions in the plume region.
Since the present boundary conditions are quite simplified, we should investigate the effect of realistic boundary conditions on the flow structure in future work.

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Appendix: Bohm Condition on Emitter Surface

Considering the thermionic electrons as well as the plasma electrons and ions, the energy conservation and continuity relation on an emitter surface can be described as follows:10,32)

Thermionic electrons:

\[ \frac{1}{2} m_e v_i^2 = 2kT_{em} + e(\phi_{sh} - \phi) \]  \hspace{1cm} (A.1)

Plasma electrons:

\[ j_{em} = e n_b v_b \]  \hspace{1cm} (A.2)

Ions:

\[ \frac{1}{2} m_i v_i^2 = e (\phi_{Bohm} + \phi) \]  \hspace{1cm} (A.3)

Here, \( \phi \) represents the potential drop in the sheath region viewed from the sheath edge, and \( \phi_{Bohm} \) is the potential drop within the presheath region. From Eqs. (A.1) and (A.2), we obtain

\[ n_b = \frac{j_{em}}{e} \left( \frac{2kT_{em} + e(\phi_{sh} - \phi)}{m_e} \right)^{1/2} \]  \hspace{1cm} (A.6)

and from Eqs. (A.4) and (A.5).
\[ n_i = n_{i,0} \left( \frac{\phi_{\text{Bohm}}}{\phi_{\text{Bohm}} + \phi} \right)^\frac{1}{2} \]  
\quad (A.7)

Assuming charge neutrality at the sheath edge, we obtain
\[ s_i = \frac{n_{i,0}}{n_{e,0}} = 1 + \frac{s_j}{\sqrt{2}} J_e^{m}(2\theta - \tilde{\phi}_{\text{sh}})^{-\frac{1}{2}} \]  
\quad (A.8)

where,
\[ J_e^{m} = \frac{j_{e m}}{e n_i \sqrt{k T_e / m_e}} \quad \tilde{\phi}_{\text{sh}} = -\frac{e \phi_{\text{sh}}}{k T_e} \quad \theta = \frac{T_e}{T_e} \]  
\quad (A.9)

Equation (A.8) can be reduced to
\[ s_i = \left( 1 - \frac{J_e^{m}}{\sqrt{4\theta - 2\tilde{\phi}_{\text{sh}}}} \right)^{-1} \]  
\quad (A.10)

In addition, assuming that the derivative of the charge density with respect to \( \phi \) is zero at the sheath edge, we can derive the following relation.
\[ \frac{e \phi_{\text{Bohm}}}{k T_e} = \frac{s_j}{2 - \frac{s_j}{\sqrt{2}} J_e^{m}(2\theta - \tilde{\phi}_{\text{sh}})^{-\frac{1}{2}}} \]  
\quad (A.11)

Since the Bohm velocity \( u_{\text{Bohm}} \) is given by
\[ u_{\text{Bohm}} = \sqrt{\frac{2e \phi_{\text{Bohm}}}{m_i}} \]  
\quad (A.12)

\( u_{\text{Bohm}} \) depends on \( n_{i,0} \), which is equal to the total electron density at the sheath edge. Therefore, when enforcing the Bohm condition on the emitter surface, \( n_{i,0} \), obtained by numerically solving the following equation, should be substituted at the boundary node.
\[ j_i \cdot \hat{n} - e n_{i,0} u_{\text{Bohm}} = 0 \]  
\quad (A.13)

Here, \( u_{\text{Bohm}} \) is given through Eqs. (A.9)–(A.12), and \( j_i \) is the ion current density determined using the PIC method.