Interface Fluctuations on a Hierarchical Lattice

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Abstract: We consider interface fluctuations on a two-dimensional layered lattice where the couplings follow a hierarchical sequence. This problem is equivalent to the diffusion process of a quantum particle in the presence of a one-dimensional hierarchical potential. According to a modified Harris criterion this type of perturbation is relevant and one expects anomalous fluctuating behavior. By transfer-matrix techniques and by an exact renormalization group transformation we have obtained analytical results for the interface fluctuation exponents, which are discontinuous at the homogeneous lattice limit.

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I. Introduction

Recently there is a growing interest in such natural and artificial systems which are organized in a hierarchical way. Examples can be found in economical organizations[1] and stock-market exchanges[2], in geological processes before major earthquakes[3], in studies of relaxation phenomena of proteins[4], spin glasses[5] and computer architectures[6]. Theoretically much effort have been devoted to understand the linear dynamics (i.e. the diffusion process) in a system with hierarchically organized energy barriers. According to numerical[7,8] and exact[9,10] results the diffusion in such systems can be anomalous (which is often called as ”ultradiffusion”[7]), furthermore in several models there is a dynamical phase transition[8] separating regions with normal and anomalous diffusion. For a comprehensive review on the subject see Ref[11].

Another subject of theoretical interest is the properties of (static) phase transitions on hierarchical lattices. For these and other non-periodic (quasi-periodic or more generally aperiodic) systems a relevance-irrelevance criterion has recently been proposed[12] on the analogy of the Harris criterion[13] for random magnets. The cross-over exponent corresponding to a non-periodic perturbation is given by:

$$\Phi = 1 + \nu D(\Omega - 1)$$

in terms of the $\nu$ correlation length exponent of the unperturbed system and the wandering exponent of the sequence $\Omega$[14]. Here $D$ denotes the number of coordinates on which the couplings depend, c.f. for a layered system $D = 1$. The perturbation is then expected to be relevant (irrelevant) if $\Phi > 0$ ($\Phi < 0$), which was indeed found in a series of exact studies on two-dimensional layered Ising models[15,16]. For marginal sequences, where $\Phi = 0$, continuously varying critical exponents and anisotropic scaling behavior was observed[17].

As far as the critical behavior on hierarchical lattices is concerned mainly the two-dimensional layered Ising model with a one-dimensional Huberman-Kerszberg (HK) sequence[7] and the corresponding Ising quantum chain were studied. In numerical[18] and exact[19,20] calculations non-universal critical behavior were found in accordance with the vanishing cross-over exponent in eq(1), which follows from the fact that the fluctuation exponent of the HK sequence is $\Omega = 0$[20].

In this paper we consider the interface fluctuation problem on a layered lattice, where the couplings between the layers follow the HK hierarchical sequence. As far as interface
wandering on non-periodic lattices are concerned the work by Henley and Lipowsky[21] has to be mentioned, who considered the interface roughening in two-dimensional quasicrystals. On a layered lattice with Fibonacci-type quasi-periodicity non-universal interface fluctuations were observed, with a continuously varying interface wandering exponent. This behavior is again in accord with the relevance-irrelevance criterion, since with $\Omega = -1$ and $\nu = \nu_\perp = 1/2$ the cross-over exponent in eq(1) is $\Phi = 0$. In our problem, on the HK lattice $\Omega = 0$, thus $\Phi = 1/2 > 0$ and the perturbation is relevant. Therefore one expects anomalous interface fluctuations on this lattice.

The structure of the paper is the following. We define the model in Sec.II. The results of the transfer matrix calculations and that of an exact renormalization group (RG) transformation are presented in Sections III. and IV., respectively. The results are discussed in the final section.

II. Formalism

We consider a diagonally layered ferromagnetic spin model (c.f. the Ising model) on the square lattice with hierarchically organized interactions. The couplings in the $h$-th diagonal $K_h = J_h/k_BT$ are selected from a set $(\kappa_0, \kappa_1, \kappa_2, \ldots)$ and $\kappa_n = n\kappa_0$, such that

$$K_h = \kappa_n \quad , \quad h = 2^n(2m + 1).$$

(2)

This type of structure of the couplings (Fig.1), which shows the typical features of ultrametric topology[5] was introduced by Huberman and Kerszberg[7] following the work in Ref[1].

The boundary spins on the (1,1) surfaces are fixed in different orientations (Fig.1) and we are interested in the fluctuations of the interface separating the (+) and (-) regions. The interface is considered as a continuous structurless string and complicated interface configurations, such as overhangs and bubbles are omitted. It is generally accepted that to study interfacial fluctuations it is sufficient to keep only Solid-on-Solid (SOS) type interface configurations. In this so called SOS model the interface is geometrically represented by a directed walk or polymer[22].

In the SOS model the interface is characterised by its $h(x)$ height at site $x$ and the interfacial energy is specified by the Hamiltonian:

$$-H/k_BT = \sum_x 2K_{h(x)} \quad ,$$

(3)
where surface effects are omitted. The thermodynamic properties of the interface are conveniently studied in the transfer matrix formalism\[23,24\]. For our model the transfer matrix in the \(x\)-direction, parallel with the boundaries, is given by:

\[
T_{h,l} = \delta_{h,l-1}e^{-2K_h} + \delta_{h,l+1}e^{-2K_l}.
\] (4)

Here according to eq(2) the matrix-elements are from a set \((\epsilon_0, \epsilon_1, \epsilon_2, \ldots)\) and the ratio of successive terms is constant: \(\epsilon_{n+1}/\epsilon_n = R < 1\). For the homogeneous system \(R = 1\), whereas for hierarchical lattices \(R\) measures the strength of inhomogeneity. The interface is not likely to visit sites with a matrix-element \(\epsilon_n, n \gg 1\), since the corresponding probability is weighted by a factor of \(R^n\).

The interfacial free energy \(\sigma\) and the longitudinal correlation length \(\xi_\parallel\), which is measured parallel with the boundaries, are given in terms of the leading and the next-to leading eigenvalues of the transfer matrix \(\lambda_0\) and \(\lambda_1\) as:

\[
\sigma = -\log \lambda_0
\] (5)

and

\[
\xi_\parallel^{-1} = \log(\lambda_0/\lambda_1)
\] (6)

The fluctuations of the interface grow on a power law scale:

\[
< [h(0) - h(x)]^2 > \sim x^{2w},
\] (7)

where \(w\) is the wandering or fluctuation exponent, which is \(w = 1/2\) for homogeneous two-dimensional systems\[22\].

Another quantity of interest is the probability \(P_0(x)\) that the interface after \(x\)-steps has the same position, i.e. \(h(0) = h(x)\). For a walk or diffusion problem, where \(x\) plays the role of the time, \(P_0(x)\) is the autocorrelation function, which has the asymptotic behavior \(P_0(x) \sim x^{-\gamma}\). For homogeneous two-dimensional lattices \(\gamma = 1/2\) and generally \(w = \gamma[8]\). It could be shown by slightly modifying the derivation in Ref.[8] that the autocorrelation function averaged over the starting positions of the interface can be expressed through the spectrum of the transfer matrix as:

\[
P_0(x) = \frac{1}{L} \sum_i \left( \frac{\lambda_i}{\lambda_0} \right)^x = \int_{-\infty}^{1} g(\lambda) \left( \frac{\lambda}{\lambda_0} \right)^x d\lambda
\] (8)
where \( g(\lambda) = 1/L \sum_i \delta(\lambda - \lambda_i) \) is the density of states and \( L \) denotes the width of the system in the \( h \) direction, thus it is the dimension of the transfer matrix.

The eigenvalues of the transfer matrix are dense at the top of the spectrum and one can develop a scaling theory in terms of these critical eigenvalues. We consider a critical level \( \lambda_i \) of a system with a finite width \( L \) and denote by \( \Delta \lambda_i = \lambda_0 - \lambda_i \) its difference from the top of the spectrum. Changing lengths by a factor of \( b=2 \), i.e. with \( L' = L/2 \) the \( i \)-th eigenvalue will be \( \lambda'_i \), and the difference \( (\Delta \lambda_i)' \) will scale with a factor of \( b^{y_\lambda} \), thus

\[
(\Delta \lambda_i)' = 2^{y_\lambda} \Delta \lambda_i
\]

(9)

where \( y_\lambda \) is the gap exponent. We stress that the statement in eq(9), that all critical levels scale with the same factor is a scaling hypothesis, which will be verified by actual calculations in the following sections.

Using eq(9) the transformation law for the density of states is given by

\[
g(\Delta \lambda) = 2^{y_\lambda - 1} g'[(\Delta \lambda)']
\]

(10)

which is compatible with a power law dependence of the density of states at the top of the spectrum:

\[
g(\Delta \lambda) \sim (\Delta \lambda)^{1/y_\lambda - 1}
\]

(11)

Now putting this expression into eq(8) and evaluating the autocorrelation function one gets \( \gamma = 1/y_\lambda \).

From the scaling behavior of the spectrum in eq(9) one obtains for the finite size corrections to the largest eigenvalues:

\[
\lambda_0 - \lambda_i(L) \sim L^{-y_\lambda}
\]

(12)

thus from eqs(6) and (12) the longitudinal correlation length is \( \xi_\parallel \sim L^{y_\lambda} \). In a finite system the correlation length perpendicular to the \( (1,1) \) surface is limited by the width of the strip \( \xi_\perp \sim L \), therefore the interface wandering exponent in eq(7), which can be alternatively defined as \( \xi_\perp \sim \xi_\parallel^w \), is given by:

\[
w = 1/y_\lambda
\]

(13)

Thus indeed \( w = \gamma \), as expected from scaling considerations.

In the following we calculate the interface fluctuations on the HK lattice by two methods. First we study numerically the spectrum of the transfer matrix, verify the
validity of the scaling hypothesis and determine the interfacial tension and the wandering exponent. Then we apply an exact renormalization group transformation and calculate analytical expressions for the critical exponents.

III. Numerical Study of the Transfer Matrix

The transfer matrix of the interface problem in eq(4) is tridiagonal and could be diagonalised by powerful methods[25]. In the specific problem, however, due to the hierarchical structure of the transfer matrix one can implement a very fast algorithm to calculate the roots of the corresponding determinant.

We consider a finite system of size \( L = 2^l \) and express the corresponding determinant \( D(2^l) \) by two subdeterminants of sizes \( 2^{l-1} \) and \( 2^{l-1} - 1 \), respectively, in the form:

\[
D(2^l) = D(2^{l-1})D(2^{l-1} - 1) \quad (14a)
\]

The symmetric determinant \( \tilde{D}(2^l - 2) \) of size \( 2^l - 2 \), which is obtained from \( D(2^l) \) by leaving out the first and last rows and columns, can be similarly expressed as:

\[
\tilde{D}(2^l - 2) = D(2^{l-1} - 1)D(2^{l-1} - 1) - \tilde{D}(2^{l-1} - 2)\tilde{D}(2^{l-1} - 2)\epsilon_{l-1}^2 \quad (14b)
\]

Finally:

\[
D(2^l - 1) = D(2^{l-1})D(2^{l-1} - 1) - D(2^{l-1} - 1)\tilde{D}(2^{l-1} - 2)\epsilon_{l-1}^2 \quad (14c)
\]

These relations supplemented with \( D(1) = -\lambda \), \( D(2) = \lambda^2 - \epsilon_0^2 \) and \( \tilde{D}(2) = \lambda^2 - \epsilon_1^2 \) define a fast procedure to calculate the value of the determinant for very large sizes. For example we could treat with this method slightly perturbed systems with \( R \approx 1 \) up to sizes \( L = 2^{30} - 2^{40} \).

The largest eigenvalues calculated by this method all have the same type of finite size dependence, thus the scaling hypothesis in Sec.II is indeed satisfied. The leading eigenvalues calculated on the largest finite lattices are accurate at least up to 10-12 digits. The gap exponents, describing the finite size dependence of \( \lambda_i(L) \) in eq(12), however could be obtained from the raw data with a comparatively smaller accuracy, up to 5 digits. In this case to increase accuracy we used sequence extrapolation methods, such as the van den Broeck and Schwartz and the Bulirsch and Stoer methods[26].
The leading eigenvalue of the transfer matrix, which is connected to the interfacial tension in eq(5) and the extrapolated values of the the interface wandering exponent are listed on Table 1. One can see that both the leading eigenvalue and the wandering exponent are monotonically decreasing as $R$ goes from one to zero. In the limit $R \to 0$ the interfacial tension in eq(5) together with the wandering exponent go to zero, which is due to the fact that the system tends to be separated into disconnected parts. More interesting is the behavior of the wandering exponent around the homogeneous lattice point. As the value of $R$ is lowered below one the wandering exponent jumps by a finite amount of $\Delta w = 0.0432799$ from $w = 1/2$. In the renormalization group language such type of behavior corresponds to a relevant perturbation, which brings the system into another stable fixed point. In the next section we shall explicitly construct the RG transformation and determine exactly the wandering exponent.

**IV. Renormalization Group Calculation**

We are going to study the scaling behavior of the largest eigenvalues of the transfer matrix in eq(4), which satisfy the second order difference equation:

$$0 = T_{i,i+1}\psi_{i+1} - \lambda \psi_i + T_{i-1,i}\psi_{i-1} ,$$

where in the thermodynamic limit the boundary terms are omitted. The structure of the couplings which are connected to $T_{i,i+1}$ in eq(4) are shown on Fig.1. To construct an exact recursion we decimate out those sites, which are connected to a $\kappa_1$ coupling or equivalently to an $\epsilon_1$ matrix-element (denoted by crosses on Fig.1). We note, that the same type of decimation was used by Maritan and Stella in their study of the diffusion problem on the HK lattice[10]. One can see that after a decimation step the $(\epsilon_0,\epsilon_1,\epsilon_0)$ triplet will play the role of the renormalized $\epsilon_0'$, whereas the other couplings will renormalize as $\epsilon'_n = \epsilon_{n+1}$, keeping the value of $R$ and together the structure of the transfer matrix unchanged.

Performing the RG transformation first we denote the two neighbouring sites to be decimated out by $i$ and $i+1$ and express $\psi_i$ and $\psi_{i+1}$ as:

$$\psi_i = A\psi_{i-1} + B\psi_{i+2}$$

$$\psi_{i+1} = B\psi_{i-1} + A\psi_{i+2} ,$$

(16)
where \( A = \epsilon_0 \lambda / (\lambda^2 - \epsilon_1^2) \) and \( B = \epsilon_0 \epsilon_1 / (\lambda^2 - \epsilon_1^2) \). Then the difference equations in terms of the remaining, non-decimated spins have the same form as in eq(15), provided the eigenvalue and the couplings transforms as:

\[
\chi' = \frac{\lambda - A \epsilon_0}{B}, \quad \epsilon'_n = \frac{\epsilon_{n+1}}{B}, \quad n = 1, 2, ...
\]

and \( \epsilon'_0 = \epsilon_0 \). Thus the ratio of the sequence remains invariant \( R' = R \), as expected. As a consequence in the RG transformation besides \( \lambda \) only one coupling, say \( \epsilon_1 \) is enough to be considered and the RG transformation can be written as a two-parameter recursion:

\[
\chi' = \lambda \frac{\lambda^2 - \epsilon_1^2 - \epsilon_0^2}{\epsilon_0 \epsilon_1}, \quad \epsilon'_1 = R \frac{\lambda^2 - \epsilon_1^2}{\epsilon_0},
\]

where \( \epsilon_0 \) is the input value of the largest matrix-element.

The physically relevant fixed point of the transformation with \( \lambda > 0 \) is given by:

\[
\left( \begin{array}{c} \epsilon_1 \\ \epsilon_0 \end{array} \right)^* = \frac{R}{1 - R}, \quad \left( \begin{array}{c} \chi \\ \epsilon_0 \end{array} \right)^* = \frac{\sqrt{1 - R + R^2}}{1 - R},
\]

which is stable for \( 0 < R < 1 \). The eigenvalues of the linearised fixed point transformation are roots of a quadratic equation and they are given as:

\[
\Lambda_{1,2} = \frac{1}{R} + R + \frac{1}{2} \pm \left[ \left( \frac{1}{R} + R + \frac{1}{2} \right)^2 - 2 \right]^{1/2}.
\]

The leading eigenvalue \( \Lambda_1 > 1 \) determines the scaling behavior of the spectrum of the transfer matrix and the \( y_\lambda \) scaling dimension is given by:

\[
y_\lambda = \frac{\log \Lambda_1}{\log 2}.
\]

The second eigenvalue of the RG transformation is \( \Lambda_2 < 1 \) and the corresponding scaling field is irrelevant, thus the fixed point in eq(21) is attractive and governs the critical properties of the physical model with \( \epsilon_1 = R \epsilon_0 \). It is seen from eq(19) that the fixed point of anomalous interface fluctuations does not exist at the homogeneous point \( R = 1 \), where the fluctuations are characterised by the normal wandering exponent \( w = 1/2 \). Comparing
the analytical results for \( w = 1/y_\lambda \) with those obtained by finite size calculations in Table 1, we can say that the numerical results are indeed very accurate they correspond to that in eq(21) at least up to six digits.

V. Discussion

In this paper we studied interface fluctuations on a layered hierarchical lattice. The perturbation caused by inhomogeneous couplings is relevant according to a linear stability analysis and the observed interface fluctuations are indeed anomalous. The wandering exponent \( w \) is a monotonically decreasing function of \( R \) and discontinuous at \( R = 1 \). The fact that \( w(R) < 1/2 \) can be understood, since the interface preferentially stays on \( \kappa_0 \) lines and the probability to visit a \( \kappa_n \) line is rapidly decreasing with \( n \). Consequently the interface fluctuations are damped by the inhomogeneously distributed couplings.

One can estimate \( w(R) \) in the limit \( R \to 0 \), when the probability of a large interface fluctuation of height \( h = 2^n \) is primary given by \( p_n \sim \epsilon_n \), i.e. by the probability to have one step on the \( \kappa_n \) line. For such a fluctuation the interface approximately takes \( x \sim p_n^{-1} \sim R^{-n} \) steps, thus the wandering exponent in leading order is: \( w(R) = -\log 2/\log R \), which corresponds to the asymptotic behavior of the analytical result in eq(22). We note that in the \( R \to 0 \) limit the interface fluctuations can be described by a Markovian process and then our problem is equivalent to the diffusion of a particle in a hierarchical lattice, as studied in Ref[7-11].

The HK-sequence used in this paper can be generalized, by having a general \( \nu \)-ary character[27], instead of \( \nu = 2 \) used in eq(2). Then one has in eq(2) \( h = R^\mu (\nu m + \mu) \), with \( \mu = 1, 2, ..., \nu - 1 \). According to our numerical and analytical investigations for \( \nu = 3 \) and \( \nu = 4 \) the main characteristics of interface fluctuations remain the same as for \( \nu = 2 \): the wandering exponent has a jump at \( R = 1 \) and varies with \( R \). For \( \nu = 3 \) we obtained the analytical result:

\[
w_{\nu=3} = \frac{\log 3}{\log \Lambda_{\nu=3}}
\]

\[
\Lambda_{\nu=3} = 2 \left( \frac{1}{R} + R + 1 \right) + \left[ 4 \left( \frac{1}{R} + R + 1 \right)^2 - 3 \right]^{1/2}.
\]

As mentioned before the problem studied in this paper is related to the diffusion process on hierarchical lattices[11]. Our problem, however, can be formulated as the quantum-mechanical diffusion process of a particle which is represented by a wave packet and placed
on a one-dimensional HK-potential. Then $x$ and $h(x)$ correspond to the $t$ time and the position of the particle at the given time step, respectively, while the transfer matrix describes time evaluation. According to our results in a one-dimensional hierarchical potential the width of the wave packet will grow in time anomalously as $t^{w(R)}$.

Our final remark concerns some similarities of our results to that of interface fluctuations in a repulsive, inhomogeneous surface potential, decaying as $\sim l^{-\omega}$, where $l$ measures the distance from the surface[28]. In two-dimensions for $\omega < 2$ the perturbation is relevant and the interface wandering exponent takes the anomalous value: $w = 1/\omega > 1/2[29]$. In this problem, however, the perturbation is confined to the surface, furthermore the wandering exponent is continuous at $\omega = 2$.

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| $R$   | $\lambda_0/\epsilon_0$ | $w = 1/y_{\lambda}$ |
|-----|-----------------|-----------------|
| 1.  | 2.              | 0.5             |
| 0.999 | 1.99800894    | 0.4567199       |
| 0.9  | 1.82853274    | 0.4551092       |
| 0.75 | 1.62218648    | 0.4451438       |
| 0.5  | 1.35286081    | 0.4004540       |
| 0.25 | 1.14948652    | 0.3110577       |
| 0.1  | 1.05381456    | 0.2272971       |
| 0.001 | 1.00050038   | 0.0911867       |

Table 1

The leading eigenvalue and the corresponding interface fluctuation exponent from numerical diagonalization of the transfer matrix for different values of the hierarchical parameter.
Figure captions

Fig.1: Structurless interface on a diagonally layered square lattice. The values of the couplings, which follow the hierarchical HK sequence in eq(2) are indicated below. Sites to be decimated out in the RG transformation are marked by $X$. 