A Review of Discrete Element Method Research on Particulate Systems

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Abstract. This paper summarizes research done using the Discrete Element Method (DEM) and explores new trends in its use on Particulate systems. The rationale for using DEM versus the traditional continuum-based approach is explained first. Then, DEM application is explored in terms of geotechnical engineering and mining engineering materials, since particulate media are mostly associated with these two disciplines. It is concluded that no research to date had addressed the issue of using the DEM to model the strength and weathering characteristics of peaty soil-slag-Portland cement-fly ash combinations.

Keywords: Discrete element, numerical modelling.

1. Introduction
Although granular media are a multiphase particulate system, they have generally been modeled as a continuum. Problems occur with this assumption due to the known granular nature of soils and their multi-force multiphase nature. The problem is especially evident with non-linear stress-strain behavior or when significant local cracks and yielding appear in compression tests.

Numerical methods are desired over experimental setups in modeling particulate media since the latter are more expensive to build and operate. Numerical modeling has advantages over physical testing: parametric studies can be performed with ease and non-destructive and obtrusive sample testing can be performed. Also, boundary conditions can be controlled explicitly, sample reproducibility can be guaranteed, and tests can be stopped and restarted to suit a particular need [1]. This paper discusses the implementation and use of the Discrete Element Method (DEM) for the analysis of particulate systems, with emphasis on soils.

2. Continuum Based Numerical Methods
Traditionally, continuum based numerical methods have been used qualitatively and quantitatively to analyze particulate and granular media. The most notable example in soil mechanics is the Finite Element Method and its various hybrids. Although the finite element method was used to study discontinuous problems, it has certain disadvantages. For example this method is continuous in nature, which means that it cannot efficiently mimic the discontinuity property. Also, as the accuracy of the model is as good as the assumptions used, reproducing complex behavior with continuous methods requires complex constitutive models, containing sometimes dozens of parameters and/or internal variables in order to capture discontinuous behavior. This can improve results, but on the other hand they are computationally intense and time consuming [2].

Due to its particulate nature, soil can experience significant local deformations and bifurcations. Continuum based methods cannot compensate for such anomalies without incremental analysis involving non-linear elasto-plasticity based constitutive relations. Even so this approach is lacking in adequately modeling the most rudimentary behavior of soil, including non-linear deformations and local yielding [1].
2.1 The Discrete Element Method

It was not until the 1970s, that Cundall and Strack developed the Discrete Element Method, [3, 4, 5]. The method models particulate media as a discrete collection of particles. This random collection of particles interacts through contact forces. The method calculates the displaced positions and rotations of these particles at discrete time steps. The DEM-simulation is started by first generating a model, which results in the random orientation of particles with assigned initial velocities. The forces and moments acting on each particle are computed from the initial data and the relevant physical laws and contact models. Generally, the simulation consists of three parts: the initialization, the explicit time stepping and post processing.

In DEM the microstructure of the system is modeled rather than using constitutive laws or complicated elements. Using DEM the changes in microstructure, shape and deformations, dynamics and forces within the system can be captured in real time and in detail. Compared to conventional continuum methods, the DEM uses fundamental and much fewer parameters when modeling discontinuous behavior [2].

The method is capable of analyzing multiple interacting continuous, discontinuous or deformable interacting bodies undergoing large displacements and rotations. In the scheme developed by [5] the particles are not allowed to deform, instead they overlap and the method monitors each contact between particles and computes the new positions and orientations accordingly. Also since the algorithm can model dynamic stress propagation from particle to particle, it can be used to analyze dynamic as well as static soil behavior [6]. In addition, data evaluation at a macroscopic level can be achieved [1].

Matar [2] states that DEM is more suited for discontinuous materials for the following reasons:

1) Model is localized by nature, so there is currently no better tool to model discontinuous material than DEM,
2) Modeling discontinuous material is straightforward,
3) Any type of inter-particle forces can be incorporated,
4) Any particle shape can be considered,
5) Results obtained are more related to discontinuous materials than other methods,
6) It can be coupled with other methods to model continuous-discontinuous properties.

Cundall [7] made the case that continuum methods, such as finite element, for rock and soil might be completely replaced by particle models, such as DEM, in 10-20 years.

2.2 General Discrete Element Method Theory

The calculations performed in the discrete element method alternate between the application of Newton’s second law to the discs that represent soil particles, and a force-displacement method at the contacts. Newton’s second law gives the motion of a particle resulting from the forces acting on it. The force-displacement law is used to find contact forces from displacements [5].

The DEM simulation consists of an assembly of particles, which has a certain shape contained in a bounded area. Those particles interact with each other under certain conditions based on a specific relational equation. The bounded area can be two or three dimensional. The DEM method involves the cyclic calculation of the inter-particle force between particles in contact.

The simulation starts by assuming some initial configuration of particle positions and then contact detection module finds which set of particles are interacting. Based on type of interaction, forces are applied to these interacting particles. The simulation then proceeds by stepping in time. Velocities, orientations and positions are calculated based on Newton’s second law of motion [2].

Velocities and incremental displacements are calculated by integrating the equation of motion. The equation of motion for the center of mass of body (i) is given by:

\[ m_i \frac{d v_i}{d t} = \sum_{j=1}^{n} F_{x ij} + m_i g \]  

Where:
- \( m_i \) = Mass of body i
- \( v_i \) = Velocity of body i
- \( n \) = Total number of particles
- \( F_{x ij} \) = Inter-particle force between body i and j
- \( g \) = Acceleration of gravity
The angular velocity is given by:

\[ I_i \frac{d\omega_i}{dt} = \sum_{j=1}^{n} L_{xij} \]  

(2)

Where:
- \( I_i \) = Inertia of body \( i \)
- \( \omega_i \) = Angular velocity of body \( i \)
- \( L_{xij} \) = Angular momentum for body \( i \) and \( j \)

3. DEM Applications in Geotechnical Engineering

After the pioneering work of Cundall and Strack [3, 4, 5] in developing the first discrete element scheme for analyzing particulate and granular media, researchers started evaluating and improving the technique. Dobry and Ng [8] performed a literature survey of publications which have used the DEM of compliant particles or blocks for simulations of granular media during the years from 1982-1992.

A major conference conducted by the American Society for Civil Engineers (ASCE) in 2002 in New Mexico, USA, presented several major trends in the DEM modeling and practice. Participants discussed and analyzed several topics in this field including the following:

1) Theory and algorithms: [9, 10, 11, 12, 13],
2) Model generation: [14, 15, 16],
3) Simulation environments: [17, 18, 19],
4) Solid continuum and discrete element methods [20, 21, 22, 23],
5) Fluid discrete element methods: [24, 25, 26, 27],
6) Experimental validation: [28, 29, 30],
7) Cohesive materials: [31, 32, 33],
8) Granular mechanics: [34, 35],
9) Powders and soils: [36, 37, 38],
10) Rock: [39, 40, 41],

Later on Potyondy and Cundall [42] presented a numerical model for rock using the DEM. They showed that the model reproduced many features of rock behavior including elasticity, fracturing, acoustic emission, damage accumulation producing material anisotropy, hysteresis, dilation, post peak softening and strength increase with confinement.

Matar et al. [43] modeled the evolution of particle subdivision in Montmorillonite clay using 3 dimensional DEM. They wrote a program using ANSI C++ that studied the swelling and swelling pressure response with various amounts of particle breakdown in a unidirectional swelling cell.

Chen et al. [44] performed a 3 dimensional modeling of sinkhole repair using the DEM. They showed that the DEM was a reasonable method to investigate sinkhole repair procedure.

More recently, researchers used the DEM to model geosynthetic reinforced soils [45], geotextile reinforced soils [46] and geogrid reinforced soils [47].

In the area of extraterrestrial geotechnical engineering, Schwartz et al [48] used the DEM in a simulation to model the contact forces between particles in granular materials. While, Lichtenheldt and Schafer [49] developed a simulation model for planetary exploration using the 3 dimensional DEM. They developed an inter-particle contact model and showed that the newly developed model is applicable to wheel soil interaction problems. Then, Kulchitsky et al [50] used the DEM in the numerical modeling of void ratio (packing density) to simulate lunar regolith properties. Later on, Kulchitsky et al. [51] used the DEM in the simulation of boulder extraction from an asteroid and developed an equation linking pulling strength to boulder diameter.

Manne and Satyam [52] reviewed in detail the discrete element modeling of dynamic laboratory tests for liquefaction assessment. They identified gap areas in these laboratory tests related to numerical modeling.

4. DEM Applications in Mining Engineering

Most DEM applications dealt with either rock or soil materials. However a few researchers investigated the use of mine tailings as granular media.
McBride et al. [28] performed an experimental and numerical study using a laboratory ball mill. In the experimental part they recorded the three dimensional trajectory of particles using an automated tracking technique and bi-planar X-ray filming. This was followed by 2 dimensional DEM modeling to determine the parameters needed for the 3 dimensional DEM simulation to avoid the extra computational overhead. Then 3 dimensional DEM simulation was performed. The methods presented provided rigorous benchmarking of DEM’s predictive capabilities based on 3 dimensional trajectory data and the experimental measurement of material properties.

Sarracino et al. [53] performed another experimental and numerical study of mills in their granular form. In the experimental work, they measured incident and final velocities, final angular velocity and the coefficient of restitution of tumbling mills. The experiments were then modeled numerically in order to find contact and damping models that best reproduced the experimental results. These two research studies represent a noticeable incursion into the area of DEM modeling of tailings in their granular form. However, beyond the granular form simulation of mills, no investigation of hardened tailings simulation and analysis had been performed.

Another study by Tannant and Wang [54] modeled using 2 dimensional Particle Flow Code (PFC), [55] spray-on, rapid setting polymeric liner materials for underground rock support in Canadian mines. Two dimensional tensile and block punch tests were modeled. The authors showed that the PFC models of liners were capable of exhibiting many features in the field including: progressive de-bonding, liner bending and elongation, isolated zones of high tensile load, progressive liner rupture in tension and eventual liner failure. The study did not investigate the mechanical properties of the actual mining residues or the mine rock.

5. Conclusions
This shows that application of the DEM method to various rock and soil materials was found to be most promising and adequate. The limited amount of work done on simulating mine tailings also appears to be going on the right track with initial good results.

The DEM method seems to be the most adequate for particulate systems. It should be noted that the DEM has never been tested for strength and weathering characteristics. It also has never been tested on Peat soils. Hence it is suggested to address these issues by doing the following:
1) Determining the unconfined compressive strength and weathering characteristics of several types of Portland cement-fly ash-slag stabilized/solidified Peaty soil combinations,
2) Determining the freezing/thawing and wetting/drying resistance of these combinations,
3) Simulating the strength and weathering characteristics of the Peaty soils using the Discrete Element Method.

References
[1] M. Khwaja (1996). Discrete Element Method: micro-mechanical and large scale modeling. M.Sc. thesis, University of Massachusetts Lowell, Massachusetts.
[2] M.I Matar (2005). Modeling of Montmorillonite clay-water interactions with particle subdivisions using three dimensional Discrete Element Method. Ph.D. thesis, North Dakota State University, Fargo, North Dakota.
[3] P.A Cundall (1971). A computer model for simulating progressive large scale movements in block rock systems. In Proceedings of the Symposium of the International Society of Rock Mechanics, Nancy, France. Article 8.
[4] P.A. Cundall (1974). A computer model for rock-mass behavior using interactive graphics for the input and output of geomechanical data. Report for Contract number DACW45-74-C-0066, for the Missouri River Division, US Army Corps of Engineers, University of Minnesota, Minneapolis, Minnesota.
[5] P. A Cundall, and O.D.L. Strack, A discrete numerical model for granular assemblies. Geotechnique, 29(1), (1979). 47-65.
[6] J.M. Ting, B.T. Corkum, C.R. Kauffman, and C.A. Greco, Discrete numerical model for soil mechanics. Journal of Geotechnical Engineering. ASCE, 115(3) (1989) 379-398.
[7] Peter A. Cundall, A discontinuous future for numerical modeling in soil and rock. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.
[8] R. Dobry and T.-T. Ng, Discrete modeling of stress-strain behavior of granular media at small and large strains. Engineering Computations, 9(2) (1992) 429-143.
[9] J.F. Favier, and M. Kremmer, Modeling a particle metering device using the Finite Wall Method. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[10] Y.T. Feng and D.R.J. Owen, An energy based corner to contact algorithm. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[11] Mark A. Hopkins, Discrete element modeling based on mathematical morphology. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[12] Matthew R. Kuhn, A torus primitive for particle shapes with the Discrete Element Method. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[13] T-T. Ng, Hydrostatic boundaries in Discrete Element Methods. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[14] D. Boutt, and B. McPherson, The role of particle packing in modeling rock mechanical behavior using discrete elements. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[15] Y.T. Feng, K. Han and D.R.J. Owen, An advancing front packing of polygons, ellipses and spheres. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[16] S. Johnson and J. Williams, Formation of packing structures in discrete element modeling with disks. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[17] Youssef M.A. Hashash, and J. Ghaboussi, Discrete element modeling for the development of a real-time soil model in a virtual reality environment. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[18] Petros I. Komodromos and John R. Williams, Utilization of Java and database technology in the development of a combined discrete and finite element multi-body dynamics simulator. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[19] S. Masala, D. Chan, H. Lu and R. Chalaturnyk (2002). A Java-based graphical user interface for a 2-D Discrete Element program. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September 2002. Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[20] T. Bangash, and A. Munjiza, A computationally efficient beam element for FEM/DEM simulations of structural failure and collapse. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[21] Jeen-Shang Lin, A unified framework for discrete and continuum analysis. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[22] M. Reza Salami and F. Amini, Numerical model for the implementation of discontinuous deformation analysis in Finite Element mesh. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.
Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[23] Federico A., Plesha Tavarez, E. Michael and Lawrence C. Bank, Discrete Element Method (DEM) for modeling solid and particulate media. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[24] Benjamin K. Cook, David R. Noble, and John R Williams, A coupled DEM-LB model for the simulation of particle-fluid systems. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[25] Toshihiro Kawaguchi, Satoshi Kajiyama, Toshitsugu Tanaka, and Yutaka Tsuji, DEM simulation of 2-D fluidized bed using similarity model. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[26] Hiroshi Mori, Yoshimi Ogawa and Guoqiang Cao, Liquefaction analysis of river dike with Discrete Element Method. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[27] Colin Thornton and D. Kafui, 3D DEM simulations of gas-solid fluidised beds. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[28] A. McBride, I. Govender, M. Powell, and V. Balden, Three-dimensional validation of DEM using a laboratory ball mill. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen. pp. 207-211.

[29] Takashi Murakami, Akira Murakami, Munae Hori, and Hide Sakaguchi, Inverse analysis of stress developed in a granular assemblage under trap-door conditions and its validation using the Discrete Element Method. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[30] C. O'Sullivan, J.D. Bray and M.F. Riemer, 3-D DEM validation using steel balls with regular packing arrangements. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[31] A. Anandarajah and M. Yao, Three-dimensional Discrete Element Method of analysis of clays. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen. pp. 237-241.

[32] J. Lechman, G.G.W. Mustoe, K.T. Miller, Ning Lu and K. Eccleston, Comparison of DEM simulations and physical experiments for direct measurement of strongly attractive particle-particle interactions. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[33] T.A. Newson, and A. Duliere, The compression behaviour of structured clayey soils. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[34] Hayley H. Shen, Regimes of granular shear flows. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[35] Duan Z. Zhang, Effects of particle interaction history in dense and slow granular flows. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.
[36] Y.P. Cheng, M.D. Bolton and Y. Nakata, The modelling of soil plasticity. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[37] Erfan G. Nezami and Youssef M. A. Hashash, The use of static Discrete Element Method to simulate biaxial compression test. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[38] Morched Zeghal, Tuncer B Edil and Michael E. Plesha, Discrete Element Method for sand-structure interaction. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[39] Mary M. MacLaughlin and Kathryn K. Clapp, Discrete Element Analysis of an underground opening in blocky rock: an investigation of the differences between UDEC and DDA results. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[40] Jon Narayanasamy Olson, Jon R. Holder, Alan Rauch and B. Comacho, DEM study of wave propagation in weak sandstone. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[41] David Potyondy, A bonded-disk model for rock: relating micro properties and macro properties. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[42] D.O. Potyondy and P.A. Cundall, A bonded-particle model for rock. International Journal of Rock Mechanics and Mining Sciences, 41(8) (2004) 1329–1364.

[43] M.I. Matar, D.R. Katti and K.S. Katti, Modeling the evolution of Montmorillonite clay particulate structure: A Discrete Element modeling study. In Proceedings of ASCE, GEO-Denver (2007): New Peaks in Geotechnics, GSP 173 Advances in Measurement and Modeling of Soil Behavior. 18-21 February 2007. Denver, Colorado.

[44] F. Chen, E.C. Drumm, and G. Guiochon, 3D DEM analysis of graded rock fill sinkhole repair: particle size effects on the probability of stability. In Proceedings of the 88th Annual Meeting of the Transportation Research Board. 11-15 January (2009). Transportation Research Board. Washington, DC.

[45] H.I. Park and S.R. Lee, Evaluation of bearing capacity for multi-layered clay deposits with geosynthetic reinforcement using Discrete Element Method. Marine Georesources and Geotechnology, 28(4) (2010) 363–374.

[46] A. Bhandari and J. Han, Investigation of geotextile–soil interaction under a cyclic vertical load using the Discrete Element Method. Geotextiles and Geomembranes, 28(1) (2010) 33–43.

[47] J. Han and A. Bhandari, Evaluation of geogrid-reinforced pile-supported embankments under cyclic loading using Discrete Element Method. Geotechnical Special Publication No. 188. Editors: J. Han, G. Zheng, V.R. Schaefer, M.S. Huang. In Proceedings of US-China Workshop on Ground Improvement Technologies, ASCE, Orlando, Florida. (2009) pp. 73-82.

[48] Stephen R. Schwartz, Derek C. Richardson and P. Michel. An implementation of the soft-sphere discrete element method in a high-performance parallel gravity tree-code. Granular Matter 14 (2012) 363–380.

[49] R. Lichtenheldt and B. Schafer, Locomotion on soft granular soils: A Discrete Element based approach for simulations in planetary exploration. III International Conference on Particle-based Methods-Fundamentals and Applications. PARTICLES (2013). M. Bischoff, E. Onate, D.R.J. Owen, E. Ramm & P. Wriggers (Eds).

[50] Anton V. Kulchitsky, J. B. Johnson and A. Wilkinson. Controlling bulk material packing density in COUP DEM model to simulate lunar regolith: POSTER. The virtual 2013 Lunar Science Forum, July 16-18, (2013).

[51] A. Kulchitsky, J. Johnson, D. Reeves, and A. Wilkinson. Discrete Element Method simulation of a boulder extraction from an asteroid. Proceedings of the 14th ASCE Biennial International Conference
on Engineering, Science, Construction, and Operations in Challenging Environments, St. Louis, Missouri, USA (2014): pp. 485-494.

[52] A. Manne and N. Satyam. A review on the Discrete Element modelling of dynamic laboratory tests for liquefaction assessment. Electronic Journal of Geotechnical Engineering. 20.1 (2015) 21-46.

[53] R.S. Sarracino, H.-J. Dong and M.H. Moys, An experimental and theoretical study of ball-wall impact for DEM modeling of tumbling mills. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[54] Dwayne D. Tannant and Caigen Wang, Thin rock support liners modeled with Particle Flow Code. In Proceedings of the 3rd International Conference on Discrete Element Methods, Discrete Element Methods: Numerical Modeling of Discontinua. 23-25 September (2002). Santa Fe, New Mexico. Edited by Benjamin K. Cook and Richard P. Jensen.

[55] Itasca (1999), Itasca Consulting Group Inc., 111 Third Ave. South, Suite 450 Minneapolis, Minnesota 55401. www.itascacg.com.