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Evaluation of the Exposure to Electromagnetic Fields in Gjirokastra Region*

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ABSTRACT

Nowadays, the increasing use of electric equipments (mobile phones, PC-s, home appliances, radio-TV, etc.), apart of their benefit, has raised the concern of possible health danger when it comes to non ionizing electromagnetic fields they emit. This paper is presenting the research done for the identification of the sources of such fields in the city of Gjirokastra, as well as for mapping the intensity of the field emitted. This study is mainly motivated by the absence of laws limiting field intensity in Albania and by the uncontrolled proliferation of emission sources. The position, the number and the different typologies of emitting sources are determined and afterwards, measurements of the intensity of these fields are done in the adjacency of the sources. Finally, the measured values were compared with the European limits of protection and the results of the measurements indicate that the level of exposure in this variety of sources is low.

Keywords: Electromagnetic Field, Electromagnetic Pollution, Health Effects, European Directive

1. Introduction

The use of computers and mobile phones are significant examples of the radical changes in our lifestyle by such technological breakthroughs of the last decades. This developments result on the increasing of occupational and public exposure to electromagnetic fields.

The increasing number of the sources of such fields, as well as controversies, often lacking scientific arguments, stimulates the misinformation on the actual status of research and achieved results. In the last years the public awareness towards possible health hazards related to the increasing exposure to non ionizing electromagnetic fields is growing continuously. In Gjirokastra town too, the so called electromagnetic pollution has attracted the attention of the local authorities and the inhabitants.

A great number of studies about possible hazardous effects of electromagnetic fields of high frequencies on human health, have given negative results [1-4]. Other epidemiologic studies about stimulation of leukaemia in children exposed to the low frequency fields, have given very contradictory evidence: some studies give negative [5,6] and others give positive results [7,8]. Regardless all these, the exposure to non ionizing radiation is nowadays the main environmental and health concern.

The criterions used for health evaluation, are related to the concepts of biological effect and health danger: the first happens when the interaction of the electromagnetic fields with human body produces a reaction by this latter and; usually the second is defined as a biological effect beyond the normal physiological compensation. This evaluation can be done using the guides based on EC/EU recommendations, which stimulate too the research in this area as well as the raise of public awareness. Two fundamental issues face each other: the public health care and technological progress. Hence, waiting the final results by scientific community on possible hazardous effects of non ionizing radiation, a special attention should be paid to the mechanisms of danger perception and to the ways to inform the public.

For Albania this is not one of the most urgent problem, however the state should adopt the European models of limiting the exposure to such radiation for the public [9] and workers [10]. That’s why this study motivates the accomplishment of researches regarding the layout and intensity of electromagnetic fields.

2. Materials and Methodology

The results presented in this paper are the outcome of direct measurements of electric and magnetic fields of low frequencies (50 Hz) as well as those electromagnetic of high frequencies (RF). Initially, the number and the location of the sources of such fields were defined in
Gjirokastra town (Figure 1) and the data on the technical parameters were collected. The direct measurements of these emitted fields are carried out near the power lines of high voltage, near radio base stations (RBS) of mobile telephony, near the antennas of radio-TV broadcasting, relaxing (bars, restaurants, etc.) and working places (power transforming stations, schools, etc.).

2.1 Low Frequency Measurements

2.1.1 Places of Interest
Power consumption, and the associated electromagnetic field levels, varies over time. The peak periods of power demand on the circuit are likely to be between 8:00 a.m. and 10:30 a.m. and between 4:30 p.m. and 6:30 p.m. Measurements are taken in two power supply stations in Gjirokastra town, near the transformers, generators, transforming cabins, working desk-places, outdoors and in the road passing near the stations. The new power station (built in 2007), is not yet working in its full capacity; while the old power station (built in the '70) provides the majority of power need for the town.

2.1.2 Equipments Used for Measurements
1) Anisotropic sensor for electric and magnetic fields ELF, model EMFields PRO. It has a digital readout and measures electric and magnetic fields with frequencies from 10 to 2000 Hz (± 3 dB). The magnetic scale has a resolution of 0.01 μT and a full scale of 19.99 μT. The electric scale has a resolution of 1 V/m and a full scale of 1999 V/m (with accuracy of ± 2%). This hand-held instrument can be used, at home and in the workplace, to measure the fields from electric power sources-overhead lines, substations, underground cables, house wiring, electrical appliances and equipment, etc.

2) Anisotropic sensor for magnetic fields ELF, model Chauvin-Arnoux CA40. Its frequency response range is 30 Hz–300 Hz. The three scales and the respective accuracy in which this sensor can measure the values of magnetic inductance are: 0.01 μT – 20 μT (± 4%); 0.1 μT – 200 μT (± 5%); and 1 μT – 2000 μT (± 10%).

2.2 High Frequency Measurements

2.2.1 Places of Interest
Measurements of electromagnetic field intensity are taken near sources of various typologies which emit non ionizing radiations in the radio-frequency range, as well as in the vicinity of dwelling and relaxation places in the most exposed areas in Gjirokastra territory.

As the most suitable time to do the measurements are chosen the hours when the measured quantities reach their maximum values. In the case of radio base stations, the mornings of working days are selected (09:00-13:00), which is the time of the greatest use of mobile phones; meanwhile for the sources of radio-TV broadcasting, evening is selected in the interval 19:00-21:00 hrs.

Measuring process generally took an interval of one hour, after which, based on the collected data, the 6 minutes period in which the fields reached the maximum values is selected to be target of analysis.

The places selected for the measurements, besides their geographic distribution in the town, present a significantly dense number of the sources, or are populated for more than four hours per day (schools: S1, S2, S3; working H1, H2; and relaxing places: H3, H4). Generally, as measuring spots are chosen the ones with the greatest coverage by the cone of the field emitted by the source. First measurements of the intensity of electric component (E) of the field of frequency in the range of radio- and micro-waves, as well as of the power density (S), are taken in various spots of Gjirokastra town, in highly attended places by public and/or near small RBS-s.

Other measurements are taken in St.Triadha Mountain (Figure 2(a)), facing the town, a few kilometres from national road, in the vicinity of three radio-base stations (RBS) of three mobile phone operators. It is worth to emphasize the fact that the third RBS in this spot is built up recently (about seven months ago), after the third mobile telephony operator entered the market.

Other measurements are taken in Këculla hill (Figure 2(b)), a high altitude place overlooking Gjirokastra town, which is the site of many RBS-s for mobile telephony and antennas for radio-TV broadcasting. Such spot is almost visible by every point of the town.

2.2.2 Equipments Used for Measurements
Isotropic sensor for radio-frequencies, model Chauvin-Arnoux CA43, operative in the frequency range between
100 kHz and 2.5 GHz. The sensor is capable to measure values of electric field from 0.1 V/m to 199.9 V/m with a resolution of 0.1 V/m and accuracy 1 dB. It can measure also the power density between 0.1 μW/cm² and 1999 μW/cm²; with resolution 0.1 μW/cm² and accuracy 1 dB in the range 0.1 – 199.9 μW/cm² and, with resolution 1 μW/cm² and accuracy 2 dB in the range 200 – 1999 μW/cm². It is capable also to store data in a PC (Figure 3).

3. Results and Discussions

3.1 Low Frequency Fields

The old power station, located in the entrance of the town (L2), is identified as one of the sources emitting low frequency radiation. Meanwhile, in the suburbs of Gjirokastra is functioning, still with reduced capacity, another transforming station (L1) transforming from 110 kV to 20 kV. It is worth emphasizing the great evident difference of the respective technologies and equipments used in these two stations, which obviously influences the exposure level of the working staff.

Measurements are done indoor of each station, where the employees spend most of the time and the European guides [11] are taken as reference. Maximal values, correspond to the measures in the vicinity (~ 30 cm) of the transformers.

Values presented in Table 1 (L1–for the new station; L2–for the old one) show the maximum of the intensity of the magnetic field. From health effect point of view, this is the quantity to be more attentively considered regarding the low frequencies range.

Table 1. Magnetic inductance in the new (L1) and old (L2) power transforming stations in Gjirokastra

| Place of measurement          | L1 | L2 |
|------------------------------|----|----|
| Staff room (control panel)    | 0.06| 0.68|
| Outdoor transformers          | 5.71| 72.6|
| Outdoor control cabin         | 0.16| 7.12|
| Main entrance (next to the road) | 0.31| 0.45|

3.2 High Frequency Fields

Measurements of the intensity of RF electromagnetic fields are done in the St.Triadha mountain (H1) where in front of Gjirokastra town are positioned three RBS-s of mobile telephony (Figure 2(a)); in Këculla (H2) hill near four pillars with a great number of antennas of various typologies: radio-TV broadcasting and RBS-s (Figure 2(b)) and in other places in the town as shown in Table 2.

Measurements are taken in accordance with European guidelines [12]. The use of spectrum analyser is not considered necessary because the measured values result lower than respective European limits.

Maximal measured values of the intensity of electric field and power density are presented in Table 2 where only the $E (V/m)$ is shown, because the values of magnetic inductance are easily calculated by $B = E / c$ [13].

4. Conclusions

By the measurements done for the fields of low frequency (50 Hz) in two electrical power stations, the electric and magnetic fields result with values below the occupational limits (500 μT) of European directives for the places occupied more than 4 hours per day. The greatest values are recorded in the vicinity of the transformers, but these are places where workers stay for only a few minutes for maintenance purposes. Values measured in the new station (L1), much smaller than in the old one (L2), can be explained with the new modern equipments used there and with the reduced capacity that the station is working currently.

Regarding the high frequency emissions, despite the great number of the sources and their typology, the electromagnetic field results with values in consistency with the EU directives for places permanently populated. This can be easily verified by comparing the measured values (Table 2) with limits presented in Table 3 [14].

Even in the case of bar “First” where the clients are practically under a RBS, the measured values are small enough as not to present a worry about health effects for the staff and even less for the clients of the bar.
Table 2. Maximal values of electric field intensity $E$, and power density $S$

| Place of measurement | $E_{\text{max}}$ (V/m) | $S_{\text{max}}$ (W/m$^2$) |
|----------------------|------------------------|--------------------------|
| H1 St. Triadha mountain (RBS) | 6.3 | 0.119 |
| H2 Këculla hill (RBS, TV) | 9.1 | 0.162 |
| H3 Terrace of bar “First” (RBS) | 4.5 | 0.152 |
| H4 Restaurant “Fantazia” | 2.4 | 0.008 |
| S1 High School “H. Tahsin” | 1.2 | 0.003 |
| S2 Elementary School “K. Hoxhi” | 0.9 | 0.002 |
| S3 University of Gjirokastra | 1.6 | 0.006 |

Table 3. European public and occupational limits for exposure to high frequency fields [14]

| $E$ (V/m) | $S$ (W/m$^2$) | $f$ (Hz) | $E$ (V/m) | $S$ (W/m$^2$) |
|----------|-------------|----------|----------|-------------|
| 28       | 2           | 10 – 400 MHz | 61       | 10          |
| 1,375 $f^{1/8}$ | $\beta 200$ | 400 – 2000 MHz | 3 $f^{1/4}$ | $\beta 40$  |
| 61       | 10          | 2 – 300 GHz | 137      | 50          |

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First Evidence of Surface SH-Wave Propagation in Cubic Piezomagnetics

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ABSTRACT

This theoretical work provides with results of characteristics calculation of the ultrasonic surface Zakharenko waves (USZWs) existing in piezomagnetic cubic monocrystals of class m3m that can be readily used for non-destructive testing. The piezomagnetic waves propagate in direction [101] corresponding to relatively easy magnetization for the following piezomagnetics: Galfenol, Terfenol-D, and CoFe$_2$O$_4$ with cubic structures. The phase velocities of the USZW-waves and the coefficient of magnetomechanical coupling (CMMC) $K^2$ were calculated for the crystals. It was found that the coefficient $K^2$ for piezomagnetics with $K_m^2 > 1/3$ and $K_m^2 >> 1/3$ is about 8% to 9%, where $K^2 = 2 \left( V_{USZW,0} - V_{USZW,c} \right) V_{USZW,0}^{1/2}$ and $K_m^2 = h^2/(C_4)$. Knowledge of piezomagnetic properties of cubic crystals makes possible the use of them in new products utilizing the phenomenon called the magnetoelectric effect. Also, this study is useful for possible application of cubic piezomagnetics in composite structures consisting of piezoelectric and (or) piezomagnetic materials and in the microwave technology. This broadens choice of possible piezomagnetic materials for utilization in various technical devices.

Keywords: Piezomagnetic Cubic Monocrystals, Galfenol and Terfenol-D, Ultrasonic Surface Zakharenko Waves

1. Introduction

Ferroic materials can display a spontaneous magnetization (ferromagnetic), polarization (ferroelectric) and strain (ferroelastic, shape memory alloy). Materials possessing two “ferro” properties simultaneously are called “biferroics”. There are three kinds of biferroic materials: electroelastic, magnetoelastic, and magnetoelectric materials. The magnetoelastic materials (composites) can simultaneously possess ferromagnetic and ferroelectric properties. Composite materials or structures consisting of piezoelectric and piezomagnetic phases are able to facilitate the conversion of energy between electric and magnetic fields. Such phenomenon is called magnetoelectric (ME) effect. The ME effect of the composites was first reported by van Suchtelen [1]. The ME effect was then studied by van den Boomgaard [2], van Run et al. [3], and van den Boomgaard et al. [4] for BaTiO$_3$/CoFe$_2$O$_4$ composites. Possible applications of ME materials include magnetic-electric energy converting components; solid state non-volatile memory, multi-state memory, which can find application in quantum computing area; and electrical/optical polarization components, which can find applications in communication, light computing, and solid state memories based on spintronics [5,6]. Also, this research topic can be found in a recent review paper [7] by Fiebig. Note that the ME effect was originally predicted by P. Curie [8] in 1894.

Surface acoustic wave (SAW) devices are widely used in numerous branches of science and technology and their investigation especially in the case of interconnected physical fields is an important and actively developing branch of research and applications. About 120 years ago the first type of SAW was described by Lord Rayleigh [9] in connection with the problem of earthquakes. Bleustein [10] and Gulyaev [11] theoretically predicted that a pure shear-horizontal (SH) surface wave can be guided by the free surface of a piezoelectric half-space. Later, Maerfeld and Tourois [12] investigated SH acoustic waves guided by the interface of two half-spaces, and Danicki [13] described the acoustic waves which can be guided by an embedded conducting plane in the electro-elastic materials of class 6 mm. It is thought that the simplest type of SH-SAW represents Love waves [14] in layered systems, consisting of an isotropic layer and an isotropic substrate. Recent developments in physics and technology made possible to construct new magneto-electro-elastic materials which demonstrate interconnection between magnetic, electric,
and elastic fields [15,16]. When the electric field was connected with the elastic one (piezoelectric materials) it brought up new and unexpected possibilities for science and technology. Connecting the magnetic field with the electric and elastic ones in magneto-electro-elastic materials suggests a range of new possibilities. Alshits et al. [17] conducted a qualitative investigation on the existence of surface waves in half-infinite anisotropic elastic media with piezoelectric, piezomagnetic, and ME effects. Soh and Liu [18] gave the existence conditions of interfacial SH-waves in a piezoelectric/piezomagnetic bi-material. Although much attention has been recently concentrated on magneto-electro-elastic materials and several dynamic problems have been solved by Hu and Li [19], Li [20], Chen, et al., [21], etc., the investigation of SAW propagation in transversely isotropic magneto-electro-elastic materials [22-25] of class 6 mm is currently an open actual subject. In addition to the transversely isotropic materials there are piezoelectric and piezomagnetic crystals with cubic symmetries, in which the surface BG-waves [10,11] cannot exist, according to [26] by Gulyaev and Hickernell. That is also true for the interfacial MT-waves [12]. However, new SH-SAWs called the ultrasonic surface Zakharenko waves (USZWs) were recently discovered in [27], which can propagate in cubic piezoelectrics. That is also true for cubic piezomagnetics. The purpose of this paper is to show the first evidence of the USZW existence in cubic piezomagnetics. It is possible to introduce piezomagnetic cubic crystals which are today highly-called for different technical devices.

The Cobalt Ferrite CoFe₂O₄ (CoO·Fe₂O₃ or CFO) has the spinel ferrite structure with a stoichiometry denoted by AB₂O₄, where A is a divalent transition metal ion or a mixture of a trivalent and a monovalent metal ion. A spinel (see Figure 1 and [28]) is a cubic structure in which O²⁻ ions form an fcc-lattice. It requires eight Formula equivalent AB₂O₄ to form a repeating unit cell which contains 32 O²⁻ ions, 64 tetragonal sites (A sites) and 32 octahedral sites (B sites). However, only 8 tetragonal and 16 octahedral sites are occupied so that the ratio of A-atoms/B-atoms is 1:2. CFO is known to have highly anisotropic magnetic properties and has been widely used in magnetic media and microwave device applications.

In 1971, the U.S. Navy developed an alloy of rare metals Tb and Dy with iron to create Terfenol-D (TbₓDy₁₋ₓFe₈). The name “Terfenol-D” comes from a combination of TER for terbium, FE for iron, NOL for Naval Ordnance Laboratory, and D for dysprosium. Recently an iron/gallium alloy termed Galfenol has also been discovered at the Naval Surface Warfare Center (former NOL), where the magnetostrictive material Terfenol-D was discovered. It was found that magnetostriction in iron/gallium alloys peaks at a volume fraction of ~17% gallium. Unlike Terfenol-D representing a giant magnetostrictive material useable in practical operating conditions, FeGa-alloys are tough and not toxic, and can be machined and used without any special handling in devices where Terfenol-D may fracture. The cubic structure of Galfenol is shown in Figure 2. Terfenol-D has been commercially available since the late 1980’s and has since grown into an international industry. The Terfenol-D rods can be readily manufactured using the freestand-zone-melt process resulting in a cubic crystal structure [29] featuring a large magnetic anisotropy. Also, FeGa-alloys appear promising in filling the role as a mechanically robust material with substantial magnetostrictive capability [30]. Additional benefits of FeGa-alloys include substantially lower material costs than rare-earth based magnetostrictive alternatives. For example, the Terfenol-D (Tb₂ₓDy₁₋ₓFe₈) raw material cost is approximately $ 0.50/g and current production techniques utilize carefully controlled crystal growth processes [31]. For comparison, the raw material cost for FeₓGa₁₋ₓ is approximately $ 0.08/g [31]. Indeed,
Terfenol-D can be used in magnetic transducers. It is well-known that magnetic transducers are increasingly considered as actuators and sensors for numerous aerospace, aeronautic, automotive, industrial, and biomedical applications, for example see [32-34]. Recent progress in studying the cubic structure piezomagnetic crystals Galfenol and Terfenol-D of class m3m for utilization in transducers and sensors can be additionally found in [35-38]. Section 2 theoretically describes SAW propagation in piezomagnetic cubic crystals.

2. Theory

The rectangular coordinate system \((x_1, x_2, x_3)\) is shown in Figure 3. The suitable crystal cut and propagation direction was obtained by \(45^\circ\)-rotation around the \(x_2\)-axis directed along the lowest odd-order symmetry axis of a piezomagnetic cubic crystal. Therefore, the piezomagnetic waves with the anti-plane polarization propagate along the crystal surface in direction \([101]\). It is noted that the \(x_2\)-axis is perpendicular to the sagittal plane. Theoretical description of wave propagation in cubic piezomagnetics is similar to that for cubic piezoelectrics, for example see [27,39,40]. The constitutive equations for a piezomagnetic material can be expressed in terms of the strains \(\epsilon\) and the magnetic field \(H\). Strains are related to the mechanical displacements: \(\epsilon_{ij} = (\partial U_i/\partial x_j + \partial U_j/\partial x_i)/2\) [41]. The governing mechanical equilibrium is \(\partial \sigma_{ij}/\partial x_j = 0\) and the governing magnetostatic equilibrium is \(\partial B_i/\partial x_i = 0\) where \(\sigma_{ij}\) and \(B_i\) are the stress tensor and magnetic flux, respectively. A piezomagnetic medium possesses the elastic \(C_{ijkl}\) and piezomagnetic \(h_{ijk}\) coefficients as well as the magnetic permeability coefficients \(\mu_{ij}\) and the medium density \(\rho\).

It is necessary to write governing equations for the linear case. Constitutive relations are written as follows:

\[
\sigma_{ij} = C_{ijkl} \epsilon_{kl} - h_{ijm} H_m
\]  

(1)

\[
B_m = h_{ijm} \tau_{ij} + \mu_{ijm} H_m
\]  

(2)

in which \(\sigma_{ij}\) and \(\tau_{ij}\) are the stress and strain tensors, respectively; \(B_m\) and \(H_m\) are components of the magnetic induction (i.e. magnetic flux) and magnetic field: \(H_m = -\partial \psi/\partial x_m\) where \(\psi\) is the magnetic potential. The indices \(i, j, k, l, m, \) and \(n\) run from 1 to 3. According to Voigt’s usual notation, \(C_{ijkl}, h_{ijm}\), and \(\mu_{ijm}\) can be written as \(6 \times 6\), \(3 \times 6\), and \(3 \times 3\) matrices standing for the elastic, piezomagnetic, and magnetic tensors, respectively. They are thermodynamically defined as follows:

\[
C_{ijkl}^H = \left( \frac{\partial \sigma_{ij}}{\partial \epsilon_{kl}} \right)_H
\]

(3)

\[
h_{ijm} = h_{ijm} = -\left( \frac{\partial \sigma_{ij}}{\partial H_m} \right)_t = \left( \frac{\partial B_m}{\partial \tau_{ij}} \right)_H
\]

(4)

\[
\mu_{ijm} = \frac{\partial B_m}{\partial H_m}
\]

(5)

where the elastic constants \(C_{ijkl}\) are defined at constant magnetic field and the magnetic constants \(\mu_{ijm}\) are defined at constant strain.

In the quasi-static approximation, the equations of motion of an elastic medium and magnetostatics are

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 U_i}{\partial t^2} \quad \text{and} \quad \frac{\partial B_i}{\partial x_j} = 0
\]

(6)

In Equation (6), \(U_i\) denote the mechanical displacement components; \(t\) is time. Using Equations (1), (2), (6) and \(\psi = U_4\), one can write the coupled equations of motion for a piezomagnetic medium in the following form:

\[
\rho \frac{\partial^2 U_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 U_j}{\partial x_k \partial x_l} + h_{ijm} \frac{\partial^2 \psi}{\partial x_m \partial x_k} + h_{ijm} \frac{\partial^2 \psi}{\partial x_m \partial x_l}
\]

(7)

\[
0 = h_{ijk} \frac{\partial^2 U_j}{\partial x_k \partial x_i} - \mu_{ijm} \frac{\partial^2 \psi}{\partial x_m \partial x_l}
\]

Solutions of homogeneous partial differential Equations (7) of the second order are found in the following plane wave form: \(U_i = U_i^0 \exp[j(\mathbf{k} \cdot \mathbf{r} - \omega t)]\) and \(\psi = \psi^0 \exp[j(\mathbf{k} \cdot \mathbf{r} - \omega t)]\) where the index \(i\) runs from 1 to 3. \(U_i^0\) and \(\psi^0\) are initial amplitudes; \(j = (-1)^{1/2}\). \(\mathbf{k}, \mathbf{r}\) denotes the scalar multiplication of two vectors and \(\omega\) is the angular frequency. \(\{k_1, k_2, k_3\} = k \{n_1, n_2, n_3\}\) are the components of the wavevector \(\mathbf{k}\), \(\{x_1, x_2, x_3\}\) are the components of the real space vector \(\mathbf{r}\), and \(\{n_1, n_2, n_3\}\) are the directional cosines.

The coupled equations of motion can be readily written in the following simplified form, leaving only equations for waves with polarization perpendicular to the sagittal plane as well as non-zero components of the material ten-
sors for the studied direction [101] of wave propagation:

\[
\rho \frac{\partial^2 U_z}{\partial t^2} = C_{44} \left( \frac{\partial^2 U_z}{\partial x_1^2} + \frac{\partial^2 U_z}{\partial x_2^2} - h_{14} \frac{\partial^2 \psi}{\partial x_1^2} + h_{34} \frac{\partial^2 \psi}{\partial x_3^2} \right) - h_{16} \frac{\partial^2 U_z}{\partial x_1 \partial x_3} + h_{36} \frac{\partial^2 U_z}{\partial x_3^2} - \mu \left( \frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2} \right)
\]

Equation (8)

In Equation (8), the mechanical displacement component \(U_z\) is directed along the \(x_3\)-axis, see Figure 3:

\[
U_{2,4} = U_{0,2,4}^0 \exp \left[ jk (n_1 x_1 + n_3 x_3 - V_{ph} t) \right]
\]

Equation (9)

where the phase velocity is defined as \(V_{ph} = \omega / k\) (\(k\) is the wavenumber in direction of wave propagation). For piezomagnetic cubic crystals, it is possible to write \(C_{44} = C_{66} = C\) and \(\mu_{11} = \mu_{33} = \mu\). It is noted that many propagation directions perpendicular to direction [100] can exist. It is also possible to cut a cubic crystal in order to study wave propagation in direction [101] with \(h_{14} = h_{36} = 0\) and the non-zero piezomagnetic constants \{-16\} that is shown in Equation (8). The dependence of the normalized piezomagnetic constants on the cubic crystal cuts and propagation directions is shown in Figure 4.

Substituting the mechanical displacement \(U_z\) and magnetic potential \(\psi = U_3\) of Equation (9) into Equation (8), the equations of motion can readily be written in the well-known tensor form, using corresponding GL-components in the Green-Christoffel equation, \((GL_{\gamma\nu} - \delta_{\gamma\nu} \rho V_{ph}) U_r = 0\) \([39,41]\):

\[
GL_{22} = C(1 + n_1^2), \quad GL_{24} = GL_{42} = -h(1 - n_3^2) \quad \text{and} \quad GL_{44} = -\mu(1 + n_3^2) \quad \text{with} \quad n_3 = k_3 / k.
\]

In the GL-equation, \(r\) and \(w\) run from 1 to 4, \(\delta_{\gamma\nu}\) is the Kronecker delta for \(r < 4\) and \(w < 4, \delta_{44} = 0, U_r = \{U_{1r}, U_{2r}, U_{3r}, \psi_r\}\). Therefore, the following system of two homogeneous equations for pure SH-waves can be written as:

\[
\begin{bmatrix}
GL_{22} - (V_{ph}^2 / V_{14}^2) & GL_{24} \\
GL_{42} & GL_{44}
\end{bmatrix}
\begin{bmatrix}
U^0_n \\
\psi^0_n
\end{bmatrix}
= 0
\]

Equation (10)

In Equation (10), the directional cosines are defined as follows: \(n_1 = 1, n_2 = 0 \) and \(n_3 = n_3\). The velocity \(V_{ph}\) is also defined as follows: \(V_{ph} = (C_{44} \rho)^{1/2}\). Setting the matrix determinant equal to zero in Equation (10), the suitable phase velocity \(V_{ph}\) satisfying boundary conditions discussed in the following section and four polynomial roots \(n_3^{(o)}(V_{ph})\), as well as the functions \(U_{n}^{(o)}(V_{ph})\) and \(\psi^{(o)}(V_{ph})\), can then be found. For example, the functions can be taken in the following form: \(\psi^0 = GL_{42}\) and \(U^0 = -GL_{44}\). It is noted that for the piezomagnetic surface Bleustein-Gulyaev waves, the GL-24 and GL-42 components are as follows: \(GL_{24} = GL_{42} = h(1 + n_3^2)\).

Substituting \(m = 1 + n_3^2\) and expanding the matrix determinant in Equation (10), the following polynomial can be obtained from Equation (10):

\[
(1 + K_m^2)m_1^2 - Dm_1 + 4K_m^2 = 0 \quad \text{with} \quad D = \left( \frac{V_{ph}}{V_{14}} \right)^2 + 4K_m^2
\]

Equation (11)

of which two roots can be found as follows

\[
m_1^{(1,2)} = \frac{D \pm \sqrt{D^2 - 16K_m^2(1 + K_m^2)}}{2(1 + K_m^2)}
\]

Equation (12)

giving four polynomial roots of Equation (10)

\[
n_3^{(1,2,3,4)} = \pm \sqrt{-1 + m_1^{(1,2)}(1,2)}
\]

Equation (13)

The polynomial roots \(n_3^{(1,2,3,4)}\) actually represent the eigenvalues. For each eigenvalue \(n_3^{(o)}\), the corresponding eigenvector can be found in the following form: \((U_n^{(o)}, \psi_n^{(o)})\) where the index \(n\) runs from 1 to 4. The coefficient \(K_m^2\) in Equations (11) and (12) is the static coefficient of the magnetomechanical coupling (CMMC):

\[
K_m^2 = h^2 / (C_{44}H_{11})
\]

Equation (14)

It is also noted that the speed \(V_{14}\) of the bulk SH-wave is given by the following Formula:

\[
V_{14} = \sqrt{V_{ph}^2 (1 + K_m^2)^{1/2}}
\]

Equation (15)

Analyzing the roots for propagation direction [101] in Equations (12) and (13), it can be found that all complex roots will be calculated when the expression under the square root in Equation (12) is negative. That fulfills for velocities \(V_{ph}\) being lower than some velocity \(V_{km}\) obtained solving the following Equation from (12):

\[
D^2 - 16K_m^2(1 + K_m^2) = 0
\]

Equation (16)

and defined by the following formula:

\[
V_{km} = A_{km} V_{14} \quad \text{with} \quad A_{km} = 2K_m\sqrt{1 + K_m^2 - K_m^2}
\]

Equation (17)

It is clearly seen in Equation (17) that the factor \(A_{km}\)
shown in Figure 5 is a function of the CMMC $K_m^2$. The other function $f(K_m^2) = (1 + K_m^2)^{1/2} = V_{tm}/V_{ph}$ from Equation (15) is also shown in Figure 5. The function $A_{km}(K_m^2 = K_0^2 = 1/3)$ approaches the function $f(K_m^2) = (1 + K_m^2)^{1/2}$ giving the following equality: $V_{km} = V_{tm}$ (see Figure 5).

It is noted that only complex polynomial roots can exist when $V_{ph} < V_{km}$. The CMMC $K_0^2 = 1/3$ is readily found by substituting the velocity $V_{tm}$ from Equation (15) instead of the phase velocity $V_{ph}$ in Equation (16). Note that when $K_m^2 < K_0^2$ there are all imaginary roots for $V_{ph} > V_{km}$, but a big $K_m^2 > K_0^2$ gives real roots when $V_{ph} > V_{km}$. It is also noted that only complex or imaginary roots with negative imaginary parts are chosen in order to cope with wave damping towards the depth of a crystal corresponding to negative values of the $x_3$-axis shown in Figure 3.

Table 1 lists experimental data for Galfenol (83% of Fe and 17% of Ga) obtained in [42,43]. It is clearly seen that the static CMMC $K_m^2$ for cubic Galfenol can be as high as~0.42 that is larger than 1/3, see also Table 2. The Galfenol density $\rho$ was taken from [31], which can depend on material synthesis. For example, [31] gives a Galfenol density of~5900 kg/m$^3$ for a typical sintered disk that is only 74% of maximum possible density. It is noted that the material density of Fe is~7848 kg/m$^3$ (see [44]). According to [31,42] as well as many Galfenol Workshops at the University of Maryland, the shear elastic constant $c_{44} = c$ for Galfenol relatively slightly depends on both temperature and concentration of ga in Fe. The elastic constant $c$ can have values from~120 GPa to~135 gPa. According to [31], the static CMMC $K_m^2$ for cubic Terfenol-D can be usually about two times larger than that for Galfenol under the same conditions. However, it is interesting to compare some characteristics of Galfenol and Terfenol-D. Therefore, Terfenol-D with a very big $K_m^2$=1.2 was chosen from [45] for numerical calculations suggesting that Terfenol-d has the cubic symmetry. The piezomagnetics CoFe$_2$O$_4$ [16,46] is also listed in Table 2 for comparison, suggesting the cubic symmetry for the crystal.

Table 2. The elastic $C_{44}$ and piezomagnetic $h_{14}$ constants as well as the material density $\rho$ and magnetic permeability coefficient $\mu$ for the piezomagnetic cubic crystals: Galfenol [42,43], CoFe$_2$O$_4$ [16,46], and Terfenol-D [45]. The coefficient of magnetomechanical coupling (CMMC) $K_m^2$ in the last column was also announced in [42,43].

| Material (symmetry group) | Density, $\rho$ [kg/m$^3$] | $C_{44}$, $10^{10}$ [N/m$^2$] | $h_{14}$ [N/(A×m)] | $\mu_1/\mu_0$, $10^{-4}$ [N/A$^2$] | $K_m^2$ |
|---------------------------|-----------------------------|-----------------------------|---------------------|---------------------------------|--------|
| Galfenol (m3m)            | 7973                        | 12.70                       | 3331.34             | 206.83                          | 0.4225 |
| Galfenol (m3m)            | 7973                        | 12.70                       | 2395.05             | 154.90                          | 0.2916 |
| Galfenol (m3m)            | 7973                        | 12.70                       | 1722.32             | 105.74                          | 0.2209 |
| Galfenol (m3m)            | 7973                        | 12.70                       | 1273.98             | 72.45                           | 0.1764 |
| CoFe$_2$O$_4$ (m3m)       | 5300                        | 4.53                        | 550.0               | 157.00                          | 0.0425 |
| Terfenol-D (m3m)          | 9250                        | 0.60                        | 167.7               | 3.97                            | 1.1807 |

Since the experimental data listed in table 1 is given for different experimental conditions, it is possible to calculate the single piezomagnetic constant $h = h_{14}$ for cubic Galfenol, using known values of $K_m$ and $C$, as well as the measured values of magnetic permeability $\mu$ given in Table 1. The strain $\tau$ and magnetic flux $B$ can be also written based on the following linear constitutive piezomagnetic equations:

\[
\tau = q B + \mu \sigma = \mu \sigma + \mu_1 B = \mu_0 \mu \sigma + \mu_1 B. \]

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\[
\tau = S^\mu \sigma + qH 
\]
\[
B = q^* \sigma + \mu^\varepsilon H 
\]
where \(S^\mu\) is the compliance at constant magnetic field \(H\) and \(\mu^\varepsilon\) is the magnetic permeability at a constant stress \(\sigma\). Note that Equation (19) incorporates the direct magnetostrictive effect in which magnetic flux is generated by stresses in the piezomagnetic material. The piezomagnetic constants \(q\) and \(q^*\) are thermally defined as follows, assuming \(q = q^*\):
\[
q = (\partial \tau / \partial H)_s \quad \text{and} \quad q^* = (\partial B / \partial \sigma)_m \tag{20}
\]
The piezomagnetic coefficient \(q\) symbolizes the axial magnetostriction coefficient and represents the change in strain per change in magnetic field at a constant stress. The parameter \(q^*\) represents the change in magnetic induction per change in stress at a constant magnetic field.

The short circuit elastic modulus \(E^\mu\) represents the stiffest-material condition, which occurs when all available magnetic energy has been transduced into elastic potential energy. When energy is transferred from the elastic to the magnetic regime the effective modulus decreases to the value of \(E^\mu\). It has been shown by invoking linear transduction considerations that these two elastic moduli are related by the magnetomechanical coupling coefficient \(K_{nm}^2\) (0 \(\leq K_{nm}^2 \leq 1\)) as follows [47]:
\[
E^\mu = E^\mu (1 - K_{nm}^2) \tag{21}
\]

In a similar manner, the intrinsic or uncoupled magnetic permeability \(\mu^\varepsilon\) of Equation (19) can be reduced to a value corresponding to the constant strain permeability \(\mu^\varepsilon\) because of the energy conversion from the magnetic to the elastic regime. The permeabilities are related by
\[
\mu^\varepsilon = \mu^\varepsilon (1 - K_{nm}^2) \tag{22}
\]

It is emphasized that expressions (21) and (22) are only valid for linear systems and significant corrections are necessary in order to extend these formulae to the full nonlinear regime. However, they are highly useful to illustrate the exchange mechanisms taking place in magnetostrictive transduction.

The static CMMC \(K_{nm}^2\) can be also written as follows:
\[
K_{nm}^2 = q^* / (S^\mu \mu^\varepsilon) \tag{23}
\]
Therefore, the single non-zero piezomagnetic constant \(h\) can be calculated with the following formula, using Equations (14), (22), and (23):
\[
h = K_{nm} \sqrt{C \mu^\varepsilon (1 - K_{nm}^2)} \tag{24}
\]

Table 2 lists the piezomagnetic constant \(h\) for cubic Galfenol corresponding to the experimentally measured values of the magnetic permeability \(\mu^\varepsilon\) at constant stress \(\sigma\).

For the free space, Laplace’s equation of type \(\Delta \psi = 0\) is written in the following form: \((k_1^2 + k_2^2) \psi_0 = 0\) where \(\psi_0\) is the magnetic constant for the free space. The magnetic potential for the free space can be written as follows:
\[
\psi_0 = F(0) \exp(-k_3 x_3) \exp[j(k_3 x_3 - \omega t)] \tag{20}
\]

The short circuit elastic modulus \(E^\mu\) of Equation (19) can be reduced to a value that is solely valid for linear systems and significant corrections are necessary in order to extend these formulae to the full nonlinear regime. However, they are highly useful to illustrate the exchange mechanisms taking place in magnetostrictive transduction.

The short circuit elastic modulus \(E^\mu\) represents the stiffest-material condition, which occurs when all available magnetic energy has been transduced into elastic potential energy. When energy is transferred from the elastic to the magnetic regime the effective modulus decreases to the value of \(E^\mu\). It has been shown by invoking linear transduction considerations that these two elastic moduli are related by the magnetomechanical coupling coefficient \(K_{nm}^2\) (0 \(\leq K_{nm}^2 \leq 1\)) as follows [47]:
\[
E^\mu = E^\mu (1 - K_{nm}^2) \tag{21}
\]

In a similar manner, the intrinsic or uncoupled magnetic permeability \(\mu^\varepsilon\) of Equation (19) can be reduced to a value corresponding to the constant strain permeability \(\mu^\varepsilon\) because of the energy conversion from the magnetic to the elastic regime. The permeabilities are related by
\[
\mu^\varepsilon = \mu^\varepsilon (1 - K_{nm}^2) \tag{22}
\]

It is emphasized that expressions (21) and (22) are only valid for linear systems and significant corrections are necessary in order to extend these formulae to the full nonlinear regime. However, they are highly useful to illustrate the exchange mechanisms taking place in magnetostrictive transduction.

The static CMMC \(K_{nm}^2\) can be also written as follows:
\[
K_{nm}^2 = q^* / (S^\mu \mu^\varepsilon) \tag{23}
\]
Therefore, the single non-zero piezomagnetic constant \(h\) can be calculated with the following formula, using Equations (14), (22), and (23):
\[
h = K_{nm} \sqrt{C \mu^\varepsilon (1 - K_{nm}^2)} \tag{24}
\]

Also, there are the magnetic boundary conditions: continuity of the normal component \(B_3\) of the magnetic flux at \(x_3 = 0\) being the interface between a vacuum \((B_3 = 0)\) and the crystal surface \((B_3 = B'_3)\) where
\[
B_3 = \sum_{p=1,2} F(p) \left[ h_{k_3}^{(p)} U_4^{(p)} - \mu_{k_3}^{(p)} U_4^{(p)} \right] \tag{25}
\]
and continuity of the magnetic potential \(U_4 = \psi\) at \(x_3 = 0\) (\(\psi = \psi'\)) where
\[
\psi = \sum_{p=1,2} F(p) \psi^{(p)} \quad \text{and} \quad \psi' = F(0) \psi'_0 \tag{27}
\]

Therefore, using Equations (25)-(27), two homogeneous equations for the case of a magnetically closed surface can be readily written as follows:
\[
\begin{bmatrix}
C_{k_1}^{(1)} U_2^{(1)} + h k_{k_3}^{(1)} U_4^{(1)} \\
h k_{k_3}^{(1)} U_1^{(1)} - (\mu_{k_3}^{(1)} - j \mu_{k_3}^{(1)} k_{k_1}^{(1)}) U_4^{(1)}
\end{bmatrix}
\begin{bmatrix}
F^{(1)}
\end{bmatrix}
= 0
\tag{28}
\]
\[
\begin{bmatrix}
C_{k_1}^{(2)} U_2^{(2)} + h k_{k_3}^{(2)} U_4^{(2)} \\
h k_{k_3}^{(2)} U_1^{(2)} - (\mu_{k_3}^{(2)} - j \mu_{k_3}^{(2)} k_{k_1}^{(2)}) U_4^{(2)}
\end{bmatrix}
\begin{bmatrix}
F^{(2)}
\end{bmatrix}
= 0
\tag{29}
\]
Therefore, the corresponding boundary-condition determinants (BCDs) of matrices in Equations (28) and (29) for both cases represent complex numbers.

The complete mechanical displacement $U_{s,4}$ and magnetic potential $\psi^s$ can be written in the plane wave form as follows:

$$U_{s,4} = \sum_{p=1,2} F^{(p)}(q) U_{s,4}^{(p)} \exp \left[ j k (n_x x_1 + n_y y_1 - \rho \phi t) \right]$$ (30)

The corresponding weight functions $F^{(1)}$ and $F^{(2)}$ are found from Equations (28) and (29), which can give the same eigenvectors ($U_{s,4}^{(1)}$, $\psi_s^{(1)}$) and ($U_{s,4}^{(2)}$, $\psi_s^{(2)}$) for two equal eigenvalues $n_1^{(1)} = n_1^{(2)}$, and hence $F^{(1)} = -F^{(2)}$. It is obvious that for this case the weight factors $F^{(1)} = -F^{(2)}$ will zero the complete mechanical displacement $U_{s,4}$ and magnetic potential $\psi$ in Equations (28) and (29) giving “latent” characteristics in Equation (30). On the other hand, unequal eigenvalues $n_1^{(1)}$ and $n_1^{(2)}$ give different eigenvectors ($U_{s,4}^{(1)}$, $\psi_s^{(1)}$) and ($U_{s,4}^{(2)}$, $\psi_s^{(2)}$). It is thought that the $\psi_s$-solutions cannot be found when the surface Bleustein-Gulyaev waves are studied that is an additional difference for finding the BG-waves and USZW-waves.

4. Results and Discussions

Acoustic wave propagation along the surface of piezoelectric material in direction [101] as an instability problem can sustain the surface SH-waves in cubic piezoelectrics that was recently discovered in [27]. Concerning piezomagnetics, it is thought that piezomagnetic SH-SAW can also propagate in direction [101] in cubic piezomagnetics, for instance, crystals of class m3m. Indeed, all cubic crystals of the class possess no piezoelectricity according to [48,49]. Note that concerning transversely-isotropic materials, solutions for the surface BG-waves in piezomagnetics can be written in an identical manner to the wave solutions in piezoelectrics [50]. It is also noted that the velocity $V_{B}^{Gmc}$ of piezomagnetic surface BG-wave propagating along the magnetically open surface of transversely-isotropic piezomagnetics (treating the simplest case) can be found with the following well-known formula:

$$V_{B}^{Gmc} = V_{tm} \left( 1 - \frac{K_m^2}{1 + K_m^2} \right)^{1/2}$$ (31)

The velocity $V_{B}^{Gmc}$ of piezomagnetic BG-wave with the magnetically closed surface can be calculated with the following formula:

$$V_{B}^{Gmc} = V_{tm} \left( 1 - \frac{K_m^2}{1 + K_m^2} \right)^{1/2}$$ (32)

Note that Formula (31) for the velocity $V_{B}^{Gmc}$ is not applicable for the case of cubic piezomagnetics. For example, formula (31) gives $V_{B}^{Gmc}$~4760.093893 m/s for Galfenol with the largest value of $K_m^2 > 1/3$ in the first row in Table 3. This value of $V_{B}^{Gmc}$ is close to that of $V_{tm}$, but not to the value of $V_{km}$ (see Table 3) representing the right result for cubic piezomagnetics with $K_m^2 > 1/3$. The same there is for Terfenol-D with $K_m^2 < 1/3$ in Table 2, for which the incorrect velocity calculated with Formula (31) is $V_{B}^{Gmc}$~1179.199962 m/s.

For cubic piezomagnetics with $K_m^2 < 1/3$, Formula (31) also gives values of $V_{B}^{Gmc}$ being closer to corresponding values of $V_{tm}$ than true values of $V_{USZW,o}$. For example, the velocity $V_{USZW,o}$ for Galfenol in the fourth row in Table 3 with the smallest value of $K_m^2 < 1/3$ (see also Table 2) is $V_{USZW,o}$~4328.763803 m/s. However, Formula (31)

Table 3. The velocities $V_{km}$, $V_{tm}$ and $V_{km}$ (all in m/s) as well as the velocities $V_{USZW,o}$ and $V_{USZW,c}$ of the ultrasonic surface Zakharenko waves for [101] propagation direction in the piezomagnetic cubic crystals. The last column gives values of the $K^2$ calculated with Formula (33). All the Galfenols correspond to those listed in Tables 1 and 2.

| Material (symmetry group) | Structure | $V_{km}$ | $V_{tm}$ | $V_{USZW,o}$ | $V_{USZW,c}$ | $K^2$, % |
|---------------------------|-----------|--------|-------|-------------|-------------|------|
| Galfenol (m3m)            | bcc       | 4740.796256 | 3991.085010 | 4760.101550 | 4740.750567 | 4545.299858 | 8.24 |
| Galfenol (m3m)            | bcc       | 4530.187138 | 3991.085010 | 4535.795584 | 4535.727322 | 4418.693612 | 5.16 |
| Galfenol (m3m)            | bcc       | 4360.495418 | 3991.085010 | 4409.913035 | 4409.878451 | 4337.133082 | 3.30 |
| Galfenol (m3m)            | bcc       | 4217.255080 | 3991.085010 | 4328.798060 | 4328.763803 | 4279.858496 | 2.26 |
| CoFe2O4 (m3m)             | Spinel    | 2396.906402 | 2923.554311 | 2985.080691 | 2985.080691 | 2982.595557 | 0.17 |
| Terfenol-D (m3m)          | -         | 1048.738635 | 805.3872662 | 1189.319685 | 1047.759141 | 999.920958 | 9.17 |

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gives $V_{BGm} = 4328.783915$ m/s that is closer to $V_{m} \approx 4328.798060$ m/s. Therefore, the differences $\Delta V_{BG} = V_{BGm} - V_{m}$ and $\Delta V_{USZW} = V_{USZW,o} - V_{m}$ are $-0.014$ m/s and $-0.034$ m/s, respectively. For comparison, the piezomagnetic CoFe$_2$O$_4$ with the smallest value of $K_m^2 = -4\%$ from Table 2, the true velocity $V_{USZW,o}$ in Table 3 is already very close to the velocity $V_{BGm} = 2985.08072$ m/s, which is also very close to the velocity $V_{m}$. Note that the relative magnetic permeability $\mu/\mu_0$ in Formula (31) for piezomagnetics is about one order larger than the corresponding relative dielectric constant $\varepsilon/\varepsilon_0$ for piezoelectrics for the same $K_m^2 = K_e^2$, where $K_e^2$ is the static coefficient of electromechanical coupling (CEMC), and $\varepsilon$ and $\varepsilon_0$ are the single electric constants for a piezoelectric and free space, respectively. That can result in mm/s-difference for piezomagnetics compared with cm/s-difference for piezoelectrics between the corresponding BG-wave velocity and bulk wave velocity $V_P$. That can result in significantly larger penetration depth of surface BG and USZW-waves for piezomagnetics towards negative values of the $x_3$-axis in Figure 3. It is also noted that the velocity $V_{BGm}$ for a magnetically closed surface can be readily calculated with Formula (32), with which the velocity of interfacial Maerfeld-Tournois waves [12] is also calculated. The interfacial Maerfeld-Tournois waves can propagate along the interface of two transversely-isotropic half-spaces of class 6 mm. Concerning cubic piezomagnetics, it is expected that some interfacial waves can also propagate that can be further researched.

Using the velocities $V_{USZW,o}$ and $V_{USZW,c}$ for the USZW-waves, it is possible to evaluate the coefficient of netomechanical coupling (CMMC) $K^2$ for the piezomagnetic cubic crystals listed in Table 3. Indeed, it is thought that it is possible to use the well-known formula that is used for piezoelectrics. Therefore, the coefficient $K^2$ can be evaluated with the following formula:

$$K^2 = 2 \left( \frac{V_{USZW,o} - V_{USZW,c}}{V_{USZW,o}} \right)$$ (33)

For Galfenol in Tables 2 and 3, the evaluated value of $K^2$ is as high as 2.26% for the smallest value of $K_m^2$. The value of $K^2$ increases with increase in $K_m^2$ to the biggest value of 8.25%. This value of 8.25% for the piezomagnetic Galfenol in the first row of Table 3 can be compared with that for Terfenol-D in the Table: $K^2$(Terfenol-D) $= 9.13\%$. It is clearly seen in the second and third tables that a big value of $K_m^2 = 1.2$ for Terfenol-D being about three times larger than the biggest value of $K_m^2 = 0.65$ for Galfenol does not give a significant rise in the coefficient $K^2$. The rise is smaller than 1%. That can be explained using the following fact: the velocity $V_{USZW,o}$ for cubic piezomagnetics with a giant coefficient $K_m^2$ is situated slightly below the speed $V_{Kom}$, but not slightly below the speed $V_{m}$ (see also Figure 5). That results in lower coefficient $K^2$. Therefore, it can be concluded that the coefficient $K^2$ for cubic piezomagnetics with a giant $K_m^2$ is about 8% to 9% and cannot be larger than 10%. It is thought that the coefficient $K^2$ for the surface BG-waves in transversely-isotropic piezomagnetics with a giant $K_m^2$ can be larger than 10%, according to Formulae (31)–(33) and Figure 5. The sample behaviors of the boundary-condition determinant (BCD3) around the phase velocity solutions for the piezomagnetic monocrystals CoFe$_2$O$_4$ and Galfenol are shown in Figures 6 and 7, respectively. Figure 8 shows the dependence of the $V_{ph}$ on the coefficient $K_m$ for the Fe-17Ga alloy. Note that any change in the material density will change both the velocities $V_{sl}$ and $V_{m}(V_{sl})$, but not the coefficients $K_m^2$ and $K^2$.

5. Conclusions
This paper addresses the theoretical investigations of piezomagnetic acoustic waves propagating along the surface of cubic crystals. The ultrasonic surface Zakharenko waves (USZW) can propagate in cubic piezomagnetics when direction [101] is parallel to the crystal surface. The computer simulation was done for the widely-called piezomagnetic crystals Galfenol, Terfenol-D, and
values of zomagnetics Galfenol and Terfenol-D can possess large VUSZW,o that for a cubic crystal with a large KM netomechanical coupling. The simulations have shown CoFe2O4 suggesting the cubic symmetry. The cubic piezomagnetics Galferol and Terfenol-D can possess large values of Km, where Km is the static coefficient of magnetomechanical coupling. The simulations have shown that for a cubic crystal with a large Km > 1/3 (even Km > 1 for Terfenol-D) the coefficient K 2 is as big as 8%-9% and not larger than 10%. It is noted that a big Km > 1 does not give a significant rise to the coefficient K 2 for the case of Km > 2/3. That is due to the fact that velocity solutions for large values of Km > 1/3 are found just below the velocity Vkm but not just below the velocity Vtm. This theoretical study of cubic piezomagnetics allows use of suitable surfaces of piezomagnetic cubic crystals in non-destructive testing and evaluation that was also recently done for cubic piezoelectrics.

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Magnetic Oil Thermal Behavior under Electromagnetic Induction for Energy Efficient Heating System Design

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ABSTRACT

This study describes thermal behavior of magnetic lubricant oil under electromagnetic induction. Experimental setup include oil pump, oil tank, induction heating unit, and heat exchanger. It is a closed loop system where the oil pump circulates oil through oil tank to the system, at the same time induction heating unit heats up to the heat exchanger where the lubricant oil thermal behavior is examined. The unit has been largely studied and tested both magnetic and regular motor oil in a laboratory environment and promising results have been obtained for an actual indoor floor to space heating system design.

Keywords: Electromagnetic Induction, Electroheat, Lubricant Oil, Heat Transfer, Fluid Dynamics, Induction Heated Iron, Radiant Heater

1. Introduction

Induction heating (IH) applied systems are developed using electromagnetic induction which was first discovered by Michael Faraday in 1831. Electromagnetic induction refers to the phenomenon by which electric current is generated in a closed circuit by the fluctuation of current in another circuit placed next to it. The basic principle of induction heating, which is an applied form of Faraday’s discovery, is the fact that AC current flowing through a circuit affects the magnetic movement of a secondary circuit located near it. The fluctuation of current inside the primary circuit provided the answer as to how the mysterious current is generated in the neighboring secondary circuit. Faraday’s discovery led to the development of electric motors, generators, transformers, and wireless communications devices. Its application, however, has not been flawless. Heat loss, which occurs during the induction heating process, was a major headache undermining the overall functionality of a system. Researchers sought to minimize heat loss by laminating the magnetic frames placed inside the motor or transformer. Faraday’s Law was followed by a series of more advanced discoveries such as Lentz’s Law. This law explains the fact that inductive current flow inverse to the direction of changes in induction magnetic movement [1].

Heat loss, occurring in the process of electromagnetic induction, could be turned into productive heat energy in an electric heating system by applying this law. Many industries have benefited from this new breakthrough by implementing induction heating for furnacing, quenching, and welding [2].

In these applications, induction heating has made it easier to set the heating parameters without the need of an additional external power source. This substantially reduces heat loss while maintaining a more convenient working environment. Absence of any physical contact to heating devices precludes unpleasant electrical accidents. High energy density is achieved by generating sufficient heat energy within a relatively short period of time.

The demand for better quality, safe and less energy consuming products is rising. Products using IH include electronic rice cookers, pans and ovens. Safe, efficient and quick heating appliances attract more customers [3]. This study describes using induction heating system to heat lubricant oil up in a close loop floor to space heating system which will be an alternative environmental, low cost, energy efficient heating system for indoor usage.
2. Types of Electrical Heating Process

Prior to describing indoor induction heating system, some types of electric process heating are explained below to help you understand normally used heat sources [4].

The types of electric heating systems are as follows:
1) Resistance Heating;
2) Conduction Heating;
3) Infrared Radiation Heating;
4) Induction Heating;
5) Dielectric Hysteresis Heating;
6) Electric Arc Heating;
7) Plasma Heating;
8) Electron Beam Heating;
9) Laser Heating.

Resistance heating is the most common type of electric process heating. It uses the relationship between the voltage and current of resistance in Joule’s Law [5]. Conduction heating exploits the heat energy generated when an object is placed between two electric poles, which is another application of Joule’s Law. In this case, however, a different relationship exists between voltage and current, especially when the circuit current is high, because the object itself contains both resistance and inductance features.

The main idea of this study is induction heated iron, which is a combination of electromagnetic induction, the skin effect, and the principle of heat transfer. In short, induction heating refers to the generation of heat energy by the current and eddy current created on the surface of a conductive object (according to Faraday’s Law and the skin effect) when it is placed in the magnetic field, formed around a coil, where the AC current flows through (Ampere’s Law) [6]. Detailed descriptions of induction heating system are presented in the following sections of the study.

3. Indoor Electromagnetic Floor to Space Induction Heating System Design

Indoor induction heating system principle schematic diagram and actual experimental setup picture are shown in Figure 1 and 2 respectively. System has 4 major blocks and the types of blocks are as follows:
1) Oil Tank
2) Oil Pump with DC Servo
3) Induction Heating Unit
4) Heat Exchanger

Close loop system has oil pump which circulates lubricant oil through heat exchanger to the tank. When oil is circulated by the pump, simultaneously induction heating unit is turned on and started to heat up the heat exchanger. While lubricant oil travels through heat exchanger absorbs heat and accumulate heat content on the oil tank. Experimental setup photo can be seen from Figure 2.

While the experiment is running three different temperature data is recorded. First one is the heat exchanger surface temperature (TC2), second one is lubricant oil temperature in the tank (TC1), and third one is the lubricant oil input temperature to the oil tank (TC3).

Electromagnetic induction unit transformer has been design using ferrite U cores which has included 12 U cores in a one row and picture with U Cores and Heat Exchanger can be both seen in Figure 3. Primary coil is surrounded on U Cores body on the top and secondary coil is only the heat exchanger by itself. Number of ferrite U cores has been chosen power of the electromagnetic induction unit accordingly.

4. Indoor Induction Heating Theory, System Performance and Test Results

Alternate power supply and concept of flux and eddy current circuit is shown in Figure 4 at the top and, Ferrite-core,
induction heated iron as heat exchanger and magnetic flux is shown in Figure 4 at the bottom respectively. The cross-section $S_{1,2}$ of the flux box surrounded and controlled with the parameters thickness $d$, permeability $\mu$, and volume resistivity $\rho$ of the iron [7]. The basic formulas of self & mutual inductances of eddy current distributions are calculated from the magnetic flux resistances. Changing above four parameters, the phenomena of induction heated iron are investigated using the equation of complex resistance circuits.

Theory behind the induction heated iron has been implemented to heat lubricant oil and promising results have been obtained actual floor to space heater. First magnetic motor oil (Castrol Magnetec 10W-40) has been circulated in the system to see that magnetic property of the oil will observe more heat content then regular motor oil (Castrol 10W-40) which is used as secondary lubricant oil on the experiment to see the difference between them. Both experiments have been performed under room temperature 25 °C Celsius and oil tank has fill up 3.7854118 liters (a gallon) of lubricant oil in the system on both experiments which have been performed 700 rpm for both liquids to keep the same amount of liquid travel inside the heat exchanger at the same time period. Each experiment has been run 90 minutes and temperature data collected respectively. Results are depicted in Figure 5 (excluded very first data, temperature jumps very quickly because the nature of induction heating) which are the temperature measurement by thermocouples (from Figure 1) TC1-oil tank, TC2-surface of the heat exchanger (in the middle), and TC3-oil tank input.

Experiments are clearly shows that magnetic motor oil much better thermal observant then regular motor oil. Results are clear that magnetic property of the lubricant makes better heat observer, however the amount of the magnetization is questionable in terms of clogging the system. This has to be further investigated to find out optimum level of magnetic property for the lubricant oil. Power consumption is the induction heating unit by itself is only 330Watts which makes attractive heating system when compare around the electrical heaters. The system can easily turn to be a floor to space heater. This can be done by replacing oil tank to the radiant tubes which car- ries the lubricant oil under the floor. Isolated room (under 10 m²) can be heated only 330 Watts power consumption

Figure 3. U-Core and heat exchanger from experimental setup

Figure 4. EMI-Heater system principle circuit schematic and induction transformer with heated iron as a heat exchanger

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from the oil tank temperature data calculations. This makes the heating system energy efficient and comparable to all other electrical heaters available on the market.

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Economic Design of Three-Phase Induction Motor by Particle Swarm Optimization

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ABSTRACT

A Particle Swarm Optimization (PSO) based design of three-phase induction motors are proposed. The induction motor design is treated as a non-linear and multivariable constrained optimization problem. The annual material cost and the total annual cost of the motor are chosen as two different objective functions. The PSO is used to find a set of optimal design variables of the motor which are then used to predict performance indices and the objective functions. The proposed method is demonstrated for two sample motors, and it is compared with the genetic algorithm (GA) and the conventional design methods. The results show that the PSO-based method effectively solved the induction motor design problems and outperforms the other methods in both the solution quality and computation efficiency.

Keywords: Design Optimization, Induction Motor, Particle Swarm Optimization

1. Introduction

The ever increasing imbalance between the demand and supply of energy has focused our attention towards energy conservation. Numerous attempts have been made to achieve this either by harnessing energy from renewable sources or by improving the operating efficiency of devices used in generation, transmission and utilization of electrical energy. Induction motors are used in large number in a variety of applications. Any significant improvement in the operating efficiency of induction motor will, therefore, help our effort at energy conservation. This can be achieved by taking recourse to design optimization techniques.

Induction motors are the main energy consuming devices in industries contributing to more than 80% of electromechanical energy conservation. However, their operating efficiency is often far from satisfactory. The design optimization of energy efficient induction motor is therefore the need of the day. In the past, the design of induction motor has been attempted for achieving better performance characteristics and/or reducing the cost. These were trial and error based which were solely attributed by professional experience. Digital computer has made it possible to use well known optimization techniques in the design of electrical machines.

For the design optimization of induction motors, the most frequently used objective functions are material cost and the operating cost of the motor. If the objective function represents the manufacturer’s viewpoint, then it is equal to the material cost [1-5]. The objective function, in the form of a weighted sum of the material and operating costs, represents the consumer’s viewpoint [4-6].

Several techniques such as GA [7], evolutionary algorithm [8], neural networks [9] and fuzzy logic [10] are used to solve the induction motor design problems. However, these techniques do not always guarantee the globally optimal solution, they will provide suboptimal solution.

The PSO [11-15] algorithm is one of the modern evolutionary algorithms. This algorithm was first proposed by Kennedy and Eberhart. PSO is a population-based search algorithm characterized as conceptually simple, easy to implement and computationally efficient. As it is reported in [13], this optimization technique can be used to solve many problems as GA and does not suffer from some of GA’s difficulties. PSO has also been found to be robust in solving problems featuring non-linearity, non-differentiability and high dimensionality.

In this paper, a PSO based approach to the induction motor design problem is proposed. The method is applied on two sample motors and the results are compared with the GA and the conventional design method [16,17].
2. Optimization Problem

The design optimization of induction motor can be expressed mathematically as follows:

Find \( X (x_1, x_2, \ldots, x_n) \) such that \( F(X) \) is a minimum

\[
\text{Subject to:} \quad \begin{align*}
g_j(X) & \geq 0 \quad j = 1, 2, \ldots, m \\
x_{iL} & \leq x_i \leq x_{Ui} \quad i = 1, 2, \ldots, n
\end{align*}
\]

where \( X (x_1, x_2, \ldots, x_n) \) is the set of independent design variables with their lower and upper limits as \( x_{iL} \) and \( x_{Ui} \), for all 'n' variables. \( F(X) \) is the objective function to be optimized and \( g_j(X) \) are the constraints imposed on the design.

2.1 Design Variables

The following quantities are chosen as the principle design variables for the optimization (X):

1) Stator bore diameter \( (x_1) \);
2) Average air gap flux density \( (x_2) \);
3) Stator current density \( (x_3) \);
4) Air gap length \( (x_4) \);
5) Stator slot depth \( (x_5) \);
6) Stator slot width \( (x_6) \);
7) Stator core depth \( (x_7) \);
8) Rotor slot depth \( (x_8) \);
9) Rotor slot width \( (x_9) \).

The remaining parameters can be expressed in terms of these variables or may be treated as fixed for a particular design.

2.2 Objective Functions

The objective function to be minimized is \( F(X) \). This may be based on the performance indices, weight, or some form of cost of the motor. In the present work, the following two objective functions have been considered:

1) Annual active material cost;
2) Total annual cost of the motor (Annual material cost, annual power loss cost and annual energy loss cost).

The expression of the objective function, in terms of the motor design variables, are as follows:

2.2.1 Annual Active Material Cost

Annual iron material cost,

\[
C_i = \alpha c_i (M_{isc} + M_{isc} + M_{isc} + M_{int}) \quad (2)
\]

where,

\[
M_{isc} = 0.88\pi \times W_i K_{isc} L x (x_{11} + 2x_{12} + x_{13})
\]

\[
M_{isc} = 0.88\pi \times W_i K_{isc} L x (\pi (x_{11} + x_{12}) - N_i x_6)
\]

\[
M_{isc} = 0.88\pi \times W_i K_{isc} L d_{isc} (D_i - 2x_i - d_{isc})
\]

\[
M_{int} = 0.88W_i K_{isc} L d_{int} (\pi (D_i - d_{int}) - N_i x_9)
\]

\[
M_{int} = 0.88W_i K_{isc} L d_{int} (\pi (D_i - d_{int}) - N_i x_9)
\]

Annual copper material cost,

\[
C_c = \alpha c_c (M_{isc} + M_{isc} + M_{isc}) \quad (3)
\]

where,

\[
M_{isc} = W_e K_{isc} x y N_i \left[0.0635 + 0.472 \left(\frac{x_1}{p} + L\right)\right]
\]

\[
M_{isc} = 1.02W_e K_{isc} L x y N_i \left(x_9 - d_{isc}\right)
\]

\[
M_{isc} = 1.9W_e K_{isc} x y N_i \left(x_9 - d_{isc}\right)
\]

Annual active material cost is given by

\[
C_m = C_i + C_c \quad (4)
\]

2.2.2 Annual Active Power Loss Cost

Annual iron loss cost,

\[
C_p = \alpha c_p (P_{isc} + P_{int}) \quad (5)
\]

where,

\[
P_{isc} = P_{isc} M_{isc}
\]

\[
P_{int} = P_{int} M_{int}
\]

where, \( P_{isc} \) and \( P_{int} \) are the specific iron loss corresponding to \( B_{isc} \) and \( B_{int} \) respectively. \( B_{isc} \) and \( B_{int} \) are given as follows

\[
B_{isc} = \frac{\pi x_1 x_2}{1.76K_{isc} x_3 p}
\]

\[
B_{int} = \frac{1.5x_1 x_2}{0.88K_{isc} \left[x_1 + \frac{2x_2}{3} - \frac{N_i x_6}{\pi}\right]}
\]

Annual copper loss cost,

\[
C_p = \alpha c_p (P_{isc} + P_{int} + P_{er}) \quad (6)
\]

where,

\[
P_{isc} = \frac{x_3^2 \rho_i M_{isc}}{W_e}
\]

\[
P_{int} = \frac{\delta x_3^2 \rho_i M_{int}}{W_e}
\]

\[
P_{er} = \frac{(\delta K_e)^2 \rho_i K_{isc} M_{er}}{W_e}
\]

Annual friction and windage loss cost,

\[
C_{fp} = \alpha c_p P_f \quad (7)
\]

where,
The stray loss is assumed to reduce the efficiency by 0.5%, so that:

\[ C_{sp} = \alpha c_p P_s \]  

where,

\[ P_s = \frac{0.005 \times W}{\eta} \]

The total annual active power loss cost is thus:

\[ P_{cc} = P_{i} + P_{c} + P_{sp} + P_{cc} \]  

2.2.3 Annual Energy Loss Cost

\[ C_e = \frac{c_e}{ac_p} T C_p \]  

The first objective function is given by:

\[ F(X) = C_m \]  

The second objective function is given by:

\[ F(X) = C_m + C_p + C_e \]  

2.3 Constraints

The following constraints (g1, ….. g6) are imposed on the design optimization problem:

1) Maximum to full-load torque ratio (g1);
2) Starting to full-load torque ratio (g2);
3) Starting to full-load current ratio (g3);
4) Full-load efficiency (g4);
5) Full-load power factor (g5);
6) Maximum temperature rise (g6);

The expression of the constraint functions is as follows:

2.3.1 Maximum to Full-Load Torque Ratio

\[ g_1 = \frac{S_{fl}}{2R_2} \left[ \left( R_{th} + \frac{R_2}{S_{fl}} \right)^2 + \left( X_{th} + X_2 \right)^2 \right] \]  

2.3.2 Starting to Full-Load Torque Ratio

\[ g_2 = \frac{S_{fl}}{2R_2} \left[ \left( R_{th} + \frac{R_2}{S_{fl}} \right)^2 + \left( X_{th} + X_2 \right)^2 \right] \]  

2.3.3 Starting to Full-Load Current Ratio

\[ g_3 = \frac{S_{fl}}{g_2} \]  

2.3.4 Full Load Power Factor

\[ g_4 = \cos \left[ \tan^{-1} \left( \frac{X_{th} + X_2}{R_{th} + \frac{R_2}{S_{fl}}} \right) \right] \]  

2.3.5 Full Load Efficiency

\[ g_5 = \frac{W}{W + P_i} \]  

where, \( P_i = P_{isc} + P_{st} + P_{sc} + P_{th} + P_{er} + P_i \)

2.3.6 Maximum Temperature Rise

\[ g_6 = \frac{P_{th} + \frac{L_p}{L_o}}{C_{soc} + \frac{\pi D_s L}{C_{sci}} + \frac{\pi X_1 L}{4 C_{scv}}} \]  

where, \( V_{th} = \frac{V_{ph} X_{m}}{X_1 + X_m} \), \( R_{th} = \frac{R_{1} X_{m}}{X_1 + X_m} \), \( X_{th} = \frac{X_{1} X_{m}}{X_1 + X_m} \) 

The equivalent circuit parameters \( R_1, R_2, X_1, X_2 \) and \( X_m \) can be found in terms of the design variables [16,17].

3. Overview of PSO

PSO is one of the modern heuristic algorithms developed by Kennedy and Eberhart. It is a multi-agent search technique that traces its evolution to the emergent motion of a flock of birds searching for food. It uses a number of particles that constitute a swarm. Each particle traverses the search space looking for the global minimum (or maximum). In a PSO system, particles fly around in a multidimensional search space. During flight, each particle adjusts its position according to its own experience, and the experience of neighboring particles, making use of the best position encountered by itself and its neighbors. The swarm direction of a particle is defined by the set of particles neighboring the particle and its history experience. Compared to other evolutionary techniques, the advantages of PSO are as follows:

1) PSO is easy to implement, and only few parameters have to be adjusted;
2) Unlike the GA, PSO has no evolution operators such as crossover and mutation;
3) In GAs, chromosomes share information so that the whole population moves like one group, but in PSO, only global best particle (gbest) gives out information to the others. It is more robust than GAs;
4) PSO can be more efficient than GAs; that is, PSO often finds the solution with fewer objective function evaluations than that required by GAs;
5) Unlike GAs and other heuristic algorithms, PSO has the flexibility to control the balance between global and
local exploration of the search space.

Let X and V denote the particle’s position and its corresponding velocity in search space respectively. At iteration K, each particle i has its position defined by $X_{i,n}^K = [X_{i,1}, X_{i,2}, ..., X_{i,N}]$ and a velocity is defined as $V_{i,n}^K = [V_{i,1}, V_{i,2}, ..., V_{i,N}]$ in search space N. Velocity and position of each particle in the next iteration can be calculated as:

$$
V_{i,n}^{k+1} = W \times V_{i,n}^k + C_1 \times \text{rand}_i \times (\text{pbest}_{i,n} - X_{i,n}^k)
$$

$$
+ C_2 \times \text{rand}_i \times (g\text{best}_{i,n} - X_{i,n}^k)
$$

$$
= X_{\text{min},i,n} \text{ if } X_{i,n}^{k+1} \leq X_{\text{min},i,n}
$$

$$
= X_{\text{max},i,n} \text{ if } X_{i,n}^{k+1} > X_{\text{max},i,n}
$$

where, m number of particles in the swarm;  
N number of dimensions in a particle;  
K pointer of iterations (generations);  
$V_{i,n}$ velocity of particle i at iteration k;  
W weighting factor;  
$C_j$ acceleration factor;  
$\text{rand}_i$ random number between 0 and 1;  
$X_{i,n}^k$ current position of particle i at iteration k;  
$\text{pbest}_i$ personal best of particle I;  
$g\text{best}$ global best of the group.

In the above procedures, the convergence speed of each particle could be influenced by the parameters of acceleration factors $C_1$ and $C_2$. The optimization process will modify the position slowly, if the value of $C_1$ is chosen to be very low. On the other hand, the optimization process will modify the position slowly, if the value of $C_2$ is chosen to be very high. The first term of Formula (2) the initial velocity of particle which reflects the memory behavior of particle; the second term “cognition part” which represents the private thinking of the particle itself; the third part is the “social” part which shows the particles behavior stem from the experience of other particles in the population.

Suitable selection of weighting factor W provides a balance between global and the local explorations. In general, the weighting function is set to be the following equation:

$$
W = W_{\text{max}} - (W_{\text{max}} - W_{\text{min}}) \times \text{Iter}/\text{Iter}_{\text{max}}
$$

where $W_{\text{max}}$ and $W_{\text{min}}$ are initial and final weight respectively. $\text{Iter}_{\text{max}}$ is maximum number of generations and Iter is current number of generations.

4. Constraints Handling Strategy

Induction motor design problem is associated with inequality constraints. In this paper, a penalty-parameter-less penalty approach is used for constraints handling. The fitness function is given by:

$$
J(X) = F(X) \text{ if } X \text{ is feasible}
$$

$$
F_{\text{max}} + CV(X) \text{ otherwise}
$$

where, $F_{\text{max}}$ is the objective function of the worst feasible solution in the population. $CV(X)$ is the overall constraint violation of solution X. It is calculated as follows:

$$
CV(X) = \max(0, g_1(s) - g_1(c)) + \max(0, g_2(s) - g_2(c))
$$

$$
+ \max(0, g_3(s) - g_3(c)) + \max(0, g_4(s) - g_4(c))
$$

$$
+ \max(0, g_5(s) - g_5(c)) + \max(0, g_6(s) - g_6(c))
$$

where c and s denote the computed and specified constraint values respectively.

In this method, the objective function value is not computed for any infeasible solution. Since all feasible solution has zero constraint violation and all infeasible solutions are evaluated according to their constraint violation only, both objective function and constraint violation are not combined in any solution in the population. Thus, there is need to have any penalty parameters for this approach.

5. Design Procedure of Induction Motor Using PSO

In this design, the PSO is used to find a set of design variables which ensure that the function $F(X)$ has a minimum value and all the constraints are satisfied. The penalty-parameter-less approach is used to optimize the design. Hence the optimal design problem reduces to obtaining the design variables which correspond to the minimum value of an unconstrained function $J(X)$.

The procedure for optimal design of induction motor is as follows:

1) Read specifications and performance indices of the motor;
2) Initialize PSO parameters such as $W_{\text{max}}$, $W_{\text{min}}$, $C_1$, $C_2$ and $\text{Iter}_{\text{max}}$;
3) Generate initial population of N particles (design variables) with random positions and velocities;
4) Compute objective value and performance indices of the motor;
5) Calculate fitness: Evaluate the fitness value of current particle using Equation (22);
6) Update personal best: Compare the fitness value of each particle with its pbests. If the current value is better than pbest, then set pbest value to the current value;
7) Update global best: Compare the fitness value of each
particle with gbest. If the current value is better than gbest, set gbest to the current particle’s value;
8) Update velocities: Calculate velocities $V_{k+1}^{g}$ using Equation (19);
9) Update positions: Calculate positions $X_{k+1}^{g}$ using Equation (20);
10) Return to step (4) until the current iteration reaches the maximum iteration number;
11) Output the optimal design variables and their corresponding annual costs of the motor in the last iteration.

6. Results and Discussions
To illustrate the performance of the proposed method, two sample motors (5 HP and 10 HP motors) are tested. The specifications of the sample motors are given in Appendix B. The results of the proposed method are also compared with the GA and the conventional design methods. The value of design constants is given in Appendix C. The parameters of PSO used were the following: Population size = 20; initial inertia weight $w_{\text{max}} = 0.9$; final inertia weight $w_{\text{min}} = 0.1$; acceleration factor $C_1 = C_2 = 1.5$ and maximum iteration $\text{Iter}_{\text{max}} = 50$.

Two different designs (objectives) are considered as follows:
Design 1: The annual material cost is used as objective.
Design 2: the total annual cost of the motor is used as objective.

The results obtained from PSO, GA and conventional based design methods are given in Tables 1 and 3 respectively for motors 1 and 2. From these results, the following important points can be noted.

| Table 1. Optimum design results of motor 1 for different methods |
|---------------------------------------------------------------|
| Variables/ indices/cost | Conventional method | Design 1 | Design 2 | Design 2 |
|-------------------------|---------------------|---------|---------|---------|
| **Independent variables** |                     |         |         |         |
| Stator bore diameter (mm) | 150          | 145      | 145.7    | 138      | 137      |
| Average air gap flux density (Wb/m²) | 0.46        | 0.476    | 0.456    | 0.427    | 0.435    |
| Stator current density (A/mm²) | 4           | 4.2      | 4.02     | 3.54     | 3.65     |
| Air gap length (mm) | 0.43          | 0.41     | 0.39     | 0.35     | 0.33     |
| Stator slot depth (mm) | 24.15        | 22.8     | 22.74    | 28       | 27.8     |
| Stator slot width (mm) | 6.92        | 7.2      | 7.15     | 7.6      | 7.8      |
| Stator core depth (mm) | 24.94        | 26.6     | 26.4     | 29.5     | 29.7     |
| Rotor slot depth (mm) | 10           | 10       | 12       | 13.6     | 11       |
| Rotor slot width (mm) | 5            | 4.6      | 5        | 6        | 6        |
| **Dependent Variables** |                     |         |         |         |
| Gross iron length (mm) | 89           | 92.6     | 95.8     | 11.4     | 112.7     |
| Rotor current density (A/mm²) | 7.74       | 7.6      | 7.4      | 6.1      | 6.4      |
| **Performance index** |                     |         |         |         |
| Maximum to full-load torque ratio | 2.21      | 2.57     | 2.7      | 3.3      | 2.6      |
| Starting to full-load torque ratio | 1.27      | 1.6      | 1.37     | 1.23     | 1.15     |
| Starting to full-load current ratio | 4.15    | 4.92     | 4.68     | 4.1      | 4.2      |
| Full-load efficiency | 81.57       | 82.32    | 83.47    | 86.15    | 85.77    |
| Full-load power factor | 0.86      | 0.82     | 0.84     | 0.88     | 0.89     |
| Maximum temperature rise | 52        | 50.68    | 49.68    | 46.6     | 46.7     |
| **Annual Cost** |                     |         |         |         |
| Material cost (Rs) | 487.1       | 460.42   | 499.43   | 564.19   | 517.02   |
| Power loss cost (Rs) | 981.02     | 912.56   | 890.01   | 847.28   | 844.56   |
| Energy loss cost (Rs) | 5115.24   | 4758.3   | 4640.8   | 4418     | 4403.79  |
| Total cost (Rs) | 6583.36     | 6131.28  | 6030.2   | 5829.5   | 5765.37  |

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The optimum value of the annual material cost is observed to be higher one for the Design 2 as compared to the Design 1. The total annual cost is found to decrease marginally in Design 2. The air gap density and stator current density values are decreased in Design 2. These design variables are inversely proportional to the efficiency of motor. Thus the efficiency of the Design 2 is improved. But the maximum torque and the starting torque are affected. However, their values remain within the limits.

Table 2. Comparison of different methods for motor 1 (20-trials)

| Compared item       | Design 1 | Design 2 |
|---------------------|----------|----------|
| Maximum cost (Rs)   | 6423.63  | 6217.36  |
| Minimum cost (Rs)   | 6131.28  | 6030.2   |
| Mean cost (Rs)      | 6293.2   | 6134.8   |
| Standard deviation of cost (Rs) | 81.2     | 61.61    |

Table 3. Optimum design results of motor 2 for different methods

| Variables/ indices/cost         | Conventional method | Design 1 | Design 2 |
|---------------------------------|---------------------|----------|----------|
|                                 | GA | PSO | GA | PSO |
| Independent variables           |    |     |    |     |
| Stator bore diameter (mm)       | 165| 163 | 164| 139 |
| Average air gap flux density (Wb/m²) | 0.45| 0.465| 0.466| 0.445| 0.45 |
| Stator current density (A/mm²)  | 4  | 4.04| 4.17| 3.9  | 4.02 |
| Air gap length (mm)             | 0.35| 0.388| 0.38| 0.33 | 0.37 |
| Stator slot depth (mm)          | 25 | 26.84| 26.9| 27.88| 27.3 |
| Stator slot width (mm)          | 7  | 7.5 | 7.4 | 6.5  | 6.6  |
| Stator core depth (mm)          | 26 | 27.5| 26.7| 27.89| 22   |
| Rotor slot depth (mm)           | 13 | 13  | 10 | 14   | 12.8 |
| Rotor slot width (mm)           | 4  | 3.8 | 5  | 5    | 6.8  |
| Dependent Variables             |    |     |    |     |
| Gross iron length (mm)          | 133.2 | 122| 130.2| 189.8| 186.6 |
| Rotor current density (A/mm²)   | 5.13| 6.07| 6.36| 4.6  | 4.84 |
| Performance index               |    |     |    |     |
| Maximum to full-load torque ratio| 2.5 | 2.8 | 2.73| 3.04 | 2.06 |
| Starting to full-load torque ratio| 0.975 | 1.25| 1.28| 1.01 | 1.02 |
| Starting to full-load current ratio| 3.6 | 4.8 | 4.92| 4.6  | 4.7  |
| Full-load efficiency            | 85.5| 85.45| 85.08| 86.3 | 85.62 |
| Full-load power factor          | 0.9 | 0.92| 0.92| 0.92 | 0.91 |
| Maximum temperature rise        | 60 | 61.2| 60.08| 55.53| 56   |
| Annual Cost                     |    |     |    |     |
| Material cost (Rs)              | 815.19 | 752.5| 757.2| 940.19| 820.91 |
| Power loss cost (Rs)            | 1533.55| 1524.32| 1523| 1461.4 | 1476.7 |
| Energy loss cost (Rs)           | 7996.47| 7948.33| 7940| 7620.2 | 7700  |
| Total cost (Rs)                 | 10345.21| 10225.1| 10220| 10022| 9997.6 |
Table 4. Comparison of different methods for motor 2 (20-Trials)

| Compared item          | Design 1 | PSO  | Design 2 | PSO  |
|------------------------|----------|------|----------|------|
| Maximum cost (Rs)      | 10499.46 | 10413.23 | 10299.65 | 10161.12 |
| Minimum cost (Rs)      | 10225    | 10220 | 10022    | 9997.6 |
| Mean cost (Rs)         | 10338    | 10319 | 10154    | 10060 |
| Standard deviation of cost (Rs) | 87.67 | 55.76 | 84.23 | 50.6 |

Computational efficiencies of GA and PSO methods are compared based on the average CPU times taken by each algorithm are given in Table 5. From table, it is evident that the average convergence time for PSO is minimum.

7. Conclusions

The PSO based design optimization of induction motors is presented in this paper. Two different objective functions such as annual material cost and total annual cost of the motor are considered. The method has been applied in two sample motors and the results are compared with the GA and the conventional design methods. The results show that the PSO method is simple, robust and reliable for the design optimization of induction motors.

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Appendix A: List of Symbols

- $C_i$ annual iron material cost (Rs)
- $C_p$ annual iron loss cost (Rs)
- $C_{cp}$ annual copper loss cost (Rs)
- $C_{fp}$ annual friction and windage losses cost (Rs)
- $C_s$ annual copper material cost (Rs)
- $C_p$ annual active power loss cost (Rs)
- $C_e$ annual energy loss cost (Rs)
- $C_m$ annual active material cost (Rs)
- $M_{isc}$, $M_{ist}$ core and tooth iron masses in stator (Kg)
- $M_{irc}$, $M_{irtb}$, $M_{irtt}$ core, tooth bodies and tooth tips iron masses in rotor (Kg)
- $b_{M}$, $e_{rM}$, $s_{cM}$ bars, end rings and stator conductor copper masses (Kg)
- $isc_p$, $ist_p$ specific iron loss of stator core and tooth (W/Kg)
- $isc_p$, $isc_p$ core and teeth iron power loss in stator (W)
- $p_{b}$, $p_{er}$, $p_{sc}$ bars, end rings and stator conductors copper power losses (W)
- $f_P$, $s_P$ friction and stray power losses (W)
- $K_{ef}$, $K_{er}$ rotor and stator slot copper insulating factors
- $\delta_i$ rotor current densities (A/mm$^2$)
- $P$ number of poles
- $T$ motor running time per year (hr)
- $A$ annual rate of interest and depreciation
- $\eta_{fl}$ full-load efficiency
- $W$ rated power (W)
- $K_{er}$ end ring non-uniformity current distribution factor
- $W_c$, $W_i$ copper and iron specific masses (Kg/m$^3$)
- $\rho_c$, $\rho_i$ stator and rotor copper resistivities (\Omega.m)
- $B_{sc}$, $B_{at}$ Flux density of stator core and teeth (Tesla)
- $K_i$ iron insulation factor
- $K_{ej}$ end ring to bar current density ratio
- $f$ supply frequency (Hz)
- $V_{ph}$ Voltage per phase (V)
- $N_r$, $N_s$ rotor and stator number of slots
- $c_c$, $c_i$ specific copper and iron material costs (Rs/Kg)
- $c_e$ specific energy loss cost (Rs/Wh)
- $c_p$ specific power loss cost (Rs/W)
- $d_{ic}$ rotor core depth (m)
- $d_{is}$ rotor slot opening depth (m)
- $w_{rs}$ rotor slot opening width (m)
- $D_i$ rotor inner diameter (m)
- $D_o$ stator outer diameter (m)
- $D_r$ rotor diameter (m)
- $L$ gross iron length (m)
- $L_{ai}$ active iron length (m)
- $L_o$ Length of the conductor overhang (m)
- $R_1$, $R_2$ resistances of stator and rotor (\Omega)
- $X_1$, $X_2$, $X_m$ stator, rotor and magnetizing reactances (\Omega)
- $V_{th}$, $R_{th}$, $X_{th}$ Thevenin’s equivalent voltage, resistance and reactance
- $S_0$ full-load slip
- $S_{max}$ Slip at which maximum torque occurs
- $C_{isc}$, $C_{ist}$, $C_{scv}$ cooling coefficients for stator core outer, inner and ventilating ducts
- $M$ number of particles in the swarm
- $N$ number of dimensions in a particle
- $K$ pointer of iterations (generations)
- $V_{kn}$ velocity of particle i at iteration k
- $W$ weighting factor
- $C_j$ acceleration factor
- $r_{np}$ random number between 0 and 1
- $X_{ic}^k$ current position of particle i at iteration k
- $p_{best}$ personal best of particle i
- $g_{best}$ global best of the group
- $Iter$ current iteration number
- $\max_{iter}$ maximum iteration number

Appendix B: Specification of Test Motors

Sample Motor 1

- Capacity 5 HP
- Voltage 400V
- Current 7.8A
- Frequency 50 Hz
- No. of Poles 4
- Full load power factor 0.8
- Full load efficiency 83%

Sample Motor 2

- Capacity 10 HP
- Voltage 415V
- Current 13.68A
- Frequency 50Hz
- No. of Poles 4
- Full load power factor 0.87
Full load efficiency 87%

Appendix C: Assumed Design Constants

\[ \alpha = 0.2, \quad W_i = 7600 \text{ Kg/m}^3, \quad W_c = 8900 \text{ Kg/m}^3, \quad c_i = 35 \text{ Rs/Kg}, \quad c_e = 250 \text{ Rs/Kg}, \quad c_p = 0.002 \text{ Rs/Wh}, \quad c_{p} = 7 \text{ Rs/W}, \quad \rho_r = 2.1 \times 10^{-8}, \quad \rho_s = 2.51 \times 10^{-8}, \quad K_i = 0.9, \quad K_j = 1; \quad T = 3650 \text{ hr}, \quad N_s = 36, \quad N_r = 30. \]
Finite Element Based Analysis of Magnetic Forces between Planar Spiral Coils

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ABSTRACT

This paper elaborates on the magnetic forces between current carrying planar spiral coils. Direct and concentric rings methods are employed in order to calculate the magnetic force between these coils. The results obtained by two calculation methods show the efficiency of the replaced rings method in both simplicity and calculation time. Simulations using the Finite Element Method (FEM) are carried out to analyze the distribution of the magnetic flux density around the coils. Also, coils with precise size have been constructed and tested. The experimental results as well as the results obtained by FEM are used to validate the accuracy of the calculations.

Keywords: Magnetic Force, Finite Element Method, Concentric Rings Method, Planar Spiral Coils

1. Introduction

Regarding the extensive application of planar spiral coils in communication and robotics, determination of magnetic fields around them and forces between these coils are interesting for engineers. In these systems, to have a high inductance and flat configuration, spiral windings are employed [1-3]. Besides, these coils have an extensive application in power electronics and DC/DC converters due to their flatness and special configuration; so, they are better replacement for the ordinary inductances in order to reduce the volume of the converter [4-7].

In recent decades, spiral coils are employed in casting industries to form the thin metal sheets. In [8] the finite difference method is employed to calculate the force between them; furthermore, in this reference to calculate the magnetic force, spiral coils are replaced by concentric rings, but there is no study and discussion on the precision of the method. In [3] these forces are obtained just by test. In [9, 10] the force between circular coaxial coils has been investigated. Recently, the mesh-matrix method has been employed in order to calculate the force between spiral coils [11]. In this paper, using concentric rings instead of spiral coils, an effective and simple procedure is proposed to calculate the magnetic force between these coils. Using the results obtained from the numerical solution of the direct calculation method, the precision of the proposed method is investigated and finally compared with experimental results.

2. Calculation of Magnetic Force between Planar Spiral Coils Using Direct Method

Suppose a system of two spiral coils as shown in Figure 1. To calculate the magnetic force between them, we should first calculate the vector magnetic potential resulting from one of the coils in any given point like P (see Figure 2).

Vector magnetic potential of spiral coil 1 in any given point P is obtained by the following equation [12]:

\[ A = \frac{\mu_0 I_1}{4\pi} \int \frac{dl'}{R_1} \]

where \( I_1 \) is the current of the coil, \( dl' \) is the longitudinal differential component, and \( R_1 \) is the distance between this differential component and point P.

The coordinates marked by prime are related to the source. With suitable substitutions for \( dl' \), the following equation for vector magnetic potential is obtained:

\[ A = \frac{\mu_0 I_1}{4\pi} \int \frac{[-a_1 \sin \phi' + a_1 \cos \phi']r'd\phi' + [a_1 \cos \phi' + a_1 \sin \phi']dr'}{R_1} \]

To calculate the integral in (2), one of the integral variables must be replaced by another one according to the
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Figure 1. The two spiral coils in z distance of each other

Figure 2. Calculation of the vector magnetic potential of spiral coils in any given point like P

relations between them. The variables $\phi'$ and $r'$ have a linear relation; consequently, we can write [13,14]:

$$\phi' = K_1 r'$$

where $K_1$ is a constant coefficient that is called “compression factor” of coil 1 in this paper. This factor depends on the diameter of the wire used and the structure of the coil and determines its compression. Having the vector magnetic potential, magnetic field is calculated using the following equation [12]:

$$B = \nabla \times A$$

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The force acted on the coil 2 is [12]:

$$F_{21} = I_2 \oint_{C_2} dl_2 \times B$$

(5)

In the above equation, $dl_2$ is longitudinal differential component on coil 2. Substituting proper expression for $dl_2$ and employing (4) in (5) and doing some mathematical calculations, we get:

$$F_{21} = a_x f_x + a_y f_y + a_z f_z$$

(6)

where $f_x$, $f_y$, and $f_z$ are the components of the force in directions $x$, $y$, and $z$, respectively, and equal to (7)-(9) at the bottom of the page.

In (7)-(9), the parameters $r'_1$ and $r_1$ are the inner radii of coil 1 and 2, respectively, and $r'_2$ and $r_2$ are the outer radii of coil 1 and 2, respectively. Also, the following equation has been used [14]:

$$\phi = K_2 r$$

(10)

where $K_2$ is compression factor of coil 2 determined with regard to the compression of the coil and the diameter of the wire used in it.

3. Concentric Rings Method

In calculation of the magnetic force between spiral coils, as Equations (7)-(8) show, using the analytical method is slightly complex and time-consuming. Furthermore, the obtained integrands are not smooth functions, and we have some difficulties in calculation of their integrals. To overcome this problem, we can replace the spiral coils with concentric rings and then calculate the forces between them [13,15]. To do this, the force between two concentric current carrying rings should be determined.

Suppose rings 1 and 2 with radiuses $a$ and $b$ while carrying currents $I_1$ and $I_2$, respectively (see Figure 3). The force between the two rings is given by (11):

$$F_{21} = -a \left[ \frac{\mu_0 I_1 I_2}{2 \sqrt{ab} (1-k^2)} \right] \left[ (1-k^2) K_2(k) - \left(1 - \frac{1}{2} k^2 \right) E(k) \right]$$

(11)

$$f_x = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1} r \sin (K_2 r - K_1 r') - K_1 r' \cos (K_2 r - K_1 r') + K_1 r'^2 \frac{\sin (K_2 r) + K_1 r \cos (K_2 r)}{\left[ (\cos (K_2 r) - r' \cos (K_1 r'))^2 + (r \sin (K_2 r) - r' \sin (K_1 r'))^2 + z^2 \right]^{3/2}} dr' dr$$

(7)

$$f_y = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1} r \cos (K_2 r - K_1 r') - K_1 r' \sin (K_2 r - K_1 r') + K_1 r'^2 \frac{\cos (K_2 r) + K_1 r \sin (K_2 r)}{\left[ (\cos (K_2 r) - r' \cos (K_1 r'))^2 + (r \sin (K_2 r) - r' \sin (K_1 r'))^2 + z^2 \right]^{3/2}} dr' dr$$

(8)

$$f_z = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1} \left( 1 + K_1 r r' \right) \frac{\cos (K_2 r - K_1 r') - K_2 r' \sin (K_2 r - K_1 r')}{\left[ (\cos (K_2 r) - r' \cos (K_1 r'))^2 + (r \sin (K_2 r) - r' \sin (K_1 r'))^2 + z^2 \right]^{3/2}} dr' dr$$

(9)
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In the above equation, \( \mu_0 \) is the permeability of vacuum and \( k \) is a constant parameter which is equal to:

\[
k = \sqrt{\frac{4ab}{(a+b)^2 + z^2}}
\]  

and \( K(k) \) and \( E(k) \) are the first and the second order elliptic integrals, respectively, and are equal to:

\[
K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k^2 \sin^2 \theta)^{\frac{3}{2}}}
\]

\[
E(k) = \int_0^{\frac{\pi}{2}} \left(1-k^2 \sin^2 \theta\right)^{\frac{1}{2}} d\theta
\]

Now, having the force between the two rings, we can calculate the force between the two spiral coils with replacing them by concentric rings (Figure 4). The magnetic force between the two coils (the force exerted on coil 2 from coil 1 in Figure 4) will be as follows:

\[
F_{21} = I_1 I_2 \sum_{j=0}^{n_2-1} \sum_{i=0}^{n_1-1} f_{21}(j,i)
\]

where \( n_1 \) and \( n_2 \) are the number of turns of coil 1 and 2, respectively, and \( f_{21}(j,i) \) is equal to:

\[
f_{21}(j,i) = a_i \left(\frac{\mu_0 z k'}{2 \sqrt{a_i a_j (1-k'^2)}}\right) \left[(1-k'^2)K(k') - \left(1 - \frac{1}{2} k'^2\right)E(k')\right]
\]

In the above equation, \( z \) is the distance between the two coils, and the parameters \( a_i, b_j \) and \( k' \) are defined as:

\[
a_i = a_0 + \frac{1}{2} + i \frac{s_1}{2}
\]

\[
b_j = b_0 + \frac{1}{2} + j \frac{s_2}{2}
\]

where \( a_0 \) and \( b_0 \) are the inner radius of coils 1 and 2 and \( s_1 \) and \( s_2 \) are the distance between two neighboring turns in coils 1 and 2, respectively. If the coils are wound compressively, then \( s_1 \) and \( s_2 \) must be replaced by diameter of the wires used in coils 1 and 2, respectively.

4. Calculation Results

In Section 2, the force between two spiral coils was analytically obtained (6). Suppose that the compression factors of the coils are high. In this case, the force values in \( x \) and \( y \) directions are almost zero, and the component of the force in \( z \) direction is non-zero [13,15] which is given by (9). The force in this relation is the force exerted on coil 2 from coil 1 as it is shown in Figure 1.

Although we use precise analytical relations to obtain the force in (9), its integral has no analytical solution, and numerical integration techniques must be used to solve it. The integrand of (9) has some “semi-poles” which depend on the value of the compression factors \( K_1 \) and \( K_2 \). In [11] it was shown that integration of (9) is much more difficult because in order to obtain higher precisions, one needs to increase the number of iterations of numerical integration intensively which, in turn, requires much longer computational time to solve such a problem.

Now, we compare the results of direct calculation of the force using (9) with that of replaced concentric rings method. To calculate the integral in (9), we used recursive adaptive Simpson Quadrature method. In the replaced concentric rings method, the radius of each ring is assumed to be the average of the inner and the outer radii of each turn of spiral coils. In Tables 1 and 2 the results of calculation of the force using two methods for different values of turn number and different center to center
Table 1. Comparison of the force calculation methods between two spiral coils (inner radii and compression factor of the coils are 0 and $2\pi / 0.002$, respectively)

| Number of Turns or Rings Per Coil | 2          | 5          | 10         | 20         | 50         | 100        |
|----------------------------------|------------|------------|------------|------------|------------|------------|
| Direct Method (N)                | $2.9854 \times 10^{-6}$ | $2.3725 \times 10^{-4}$ | $5.6356 \times 10^{-3}$ | $7.1760 \times 10^{-2}$ | $1.0079$   | $5.4610$   |
| Replaced Rings Method (N)        | $1.3416 \times 10^{-6}$ | $2.2745 \times 10^{-4}$ | $5.6091 \times 10^{-3}$ | $7.1713 \times 10^{-2}$ | $1.0078$   | $5.4610$   |
| Error (%)                        | 55.1       | 4.1        | 0.47       | 0.07       | 0.01       | 0          |
| Z = 2 cm                         | $4.9389 \times 10^{-7}$ | $2.3834 \times 10^{-5}$ | $9.2665 \times 10^{-4}$ | $2.2472 \times 10^{-2}$ | $5.8029 \times 10^{-1}$ | $4.0310$   |
| Direct Method (N)                | $9.0210 \times 10^{-8}$ | $2.1251 \times 10^{-5}$ | $9.1686 \times 10^{-4}$ | $2.2445 \times 10^{-2}$ | $5.8024 \times 10^{-1}$ | $4.0310$   |
| Replaced Rings Method (N)        | $5.7461 \times 10^{-9}$ | $1.5053 \times 10^{-6}$ | $8.6061 \times 10^{-5}$ | $3.6745 \times 10^{-3}$ | $2.1791 \times 10^{-1}$ | $2.3208$   |
| Error (%)                        | 94.6       | 29.6       | 2.9        | 0.27       | 0.02       | 0          |

Precision of the calculations in numerical integration for rings of 2 to 100 turns are $0.5 \times 10^{-13}$, $0.5 \times 10^{-10}$, $0.5 \times 10^{-7}$, $0.5 \times 10^{-4}$ and $0.5 \times 10^{-1}$, respectively.

Table 2. Comparison of the force calculation methods between two spiral coils (inner radii and compression factor of the coils are 2.5 cm and $2\pi / 0.002$, respectively)

| Number of Turns or Rings Per Coil | 2          | 5          | 10         | 20         | 50         | 100        |
|----------------------------------|------------|------------|------------|------------|------------|------------|
| Direct Method (N)                | $1.8844 \times 10^{-3}$ | $1.3230 \times 10^{-2}$ | $5.8839 \times 10^{-2}$ | $2.5524 \times 10^{-1}$ | $1.7033$   | $7.0917$   |
| Replaced Rings Method (N)        | $1.8851 \times 10^{-3}$ | $1.3234 \times 10^{-2}$ | $5.8849 \times 10^{-2}$ | $2.5526 \times 10^{-1}$ | $1.7033$   | $7.0917$   |
| Error (%)                        | -0.04      | -0.03      | -0.02      | -0.01      | 0          | 0          |
| Z = 2 cm                         | $5.1474 \times 10^{-4}$ | $4.0012 \times 10^{-3}$ | $2.0961 \times 10^{-2}$ | $1.1654 \times 10^{-1}$ | $1.0846$   | $5.4020$   |
| Direct Method (N)                | $5.1465 \times 10^{-4}$ | $4.0009 \times 10^{-3}$ | $2.0961 \times 10^{-2}$ | $1.1654 \times 10^{-1}$ | $1.0846$   | $5.4020$   |
| Replaced Rings Method (N)        | +0.02      | +0.01      | 0          | 0          | 0          | 0          |
| Error (%)                        | +0.12      | +0.08      | +0.04      | +0.02      | +0.002     | 0          |
| Z = 4 cm                         | $7.6578 \times 10^{-5}$ | $6.6545 \times 10^{-4}$ | $4.1678 \times 10^{-3}$ | $3.1257 \times 10^{-2}$ | $4.8080 \times 10^{-1}$ | $3.2851$   |
| Direct Method (N)                | $7.6487 \times 10^{-5}$ | $6.6492 \times 10^{-4}$ | $4.1660 \times 10^{-3}$ | $3.1252 \times 10^{-2}$ | $4.8079 \times 10^{-1}$ | $3.2851$   |
| Replaced Rings Method (N)        | +0.12      | +0.08      | +0.04      | +0.02      | +0.002     | 0          |

Precision of the calculations in numerical integration for rings of 2 to 100 turns are $0.5 \times 10^{-13}$, $0.5 \times 10^{-10}$, $0.5 \times 10^{-7}$, $0.5 \times 10^{-4}$ and $0.5 \times 10^{-1}$, respectively.

distance of coils are compared. In these tables, the current in both coils is 20 Amperes, the diameter of the wires is 2 mm, and the compression factor for both coils is assumed to be $2\pi / d$, where $d$ is the diameter of the wires in both coils. In Table 1, it is assumed that the coils start to grow from point (0, 0). Comparing the results of the two methods in this table, it is seen that for the fewer number of turns the error is high, but by increasing the number of turns, the error gradually decreases, and when the turn number approaches to 100, the error becomes zero. In Tables 1 and 2 the precision of the calculations is adjusted according to the numerical value of the results. For instance, for the first column of Table 1 the calculated numbers are in the range of $10^{-13}$ (their minimum value). To compare the calculation time in two approaches, it suffices to mention that the required calculation time using the adaptive Simpson method for 100 turns in Table 1 for precision of $10^{-8}$ is 28000 times...
more than that of using replaced concentric rings method. As seen in the table, the results precisely coincide with each other. Another interesting point about Table 1 is that by increasing the distance between the two coils, the calculation error increases showing that in large distances, the replaced concentric rings method does not present a proper approximation of the force.

In Table 2, the comparison between two methods is made for the case in which the inner radius of two coils are equal to 2.5 cm; in other words, the coils start to wind from \( r = 2.5 \text{ cm} \). As seen from the results of the table, the errors in this case are less than the corresponding errors in Table 1. For example, the force error for 2 turn coils in distance of 8 cm reduced from 94.6% in Table 1 to 0.12% in Table 2. These fewer errors for lower turn numbers decrease expeditiously to zero by increasing the turn numbers.

According to the results of Tables 1 and 2, generally for turn numbers higher than 10 turns in each coil, using the replaced concentric rings presents good approximations while having much simpler and faster calculations compared with that of direct method and using (9).

Also the calculation have been done for the case in which there is smaller compression factor for the coils compared with previous cases, i.e. for each turn of coils or for change of \( 2\pi \) Radians in the value of variable \( \phi \) in cylindrical coordinate, the change in the value of variable \( r \) is more than the diameter of the wires used in the coils. At first, it seemed that by decreasing the compression factor the calculation error increases, but this assumption is not true because by decreasing the compression factor, the relative error of calculations with replaced rings method decreases [11].

5. Analysis of the Magnetic Force between Planar Spiral Coils Using FEM

To demonstrate the effectiveness of the concentric rings method, in this section the spiral and replaced concentric coils are simulated using 3-D finite element method. Figure 5 shows the configuration of the two coils in two methods. The distribution of the flux density for spiral coils and replaced concentric rings with the same current and turn number are illustrated in Figures 6 and 7, respectively. It is clear from these figures that the magnetic flux distribution of the spiral coil is similar to that of the replaced concentric rings with the same turn number. The calculations of the force between two coils have been done using concentric rings method and compared with FEM results. Also, the coils have been constructed in laboratory and the force between them has been measured. The characteristics of the constructed coils are given in Table 3. The calculation, FEM, and experimental results for the mentioned coils at different distances are compared in Table 4. As seen in this table, the results of the force measurement and the results obtained using FEM are in good agreement with the results of the calculations. It is noted that in calculating the force between two coils using FEM, given in Table 4, the number of meshes were doubled; however, it did not have impact on the accuracy of the force. This indicates that the number of selected meshes is enough for the calculations.
Figure 6. Flux density distribution for spiral coils

Figure 7. Flux density distribution for replaced concentric rings
Table 4. The experimental and FEM results and their comparison with calculation results of the replaced rings method

| Current of Coils (A) | Force (N) | Measured     | Calculated   | FEM         |
|---------------------|-----------|--------------|--------------|-------------|
|                     | 5.1       | 0.1570       | 0.1593       | 0.1581      |
|                     | 6.5       | 0.2551       | 0.2587       | 0.2568      |
|                     | 8.6       | 0.4513       | 0.4529       | 0.4523      |
|                     | 11.2      | 0.7554       | 0.7681       | 0.7686      |
|                     | 14.7      | 1.3244       | 1.3231       | 1.3239      |
| Z = 1 cm            |           | Z = 1 cm     |              |             |
|                     | 0.0589    | 0.0883       | 0.1570       | 0.2747      |
|                     |           |              |              | 0.4709      |
|                     | 0.0566    | 0.0920       | 0.1610       | 0.2730      |
|                     |           |              |              | 0.4703      |
|                     | 0.0581    | 0.0891       | 0.1621       | 0.2741      |
|                     |           |              |              | 0.4713      |
|                     | 0.0196    | 0.0294       | 0.0589       | 0.0883      |
|                     |           |              |              | 0.1570      |
|                     | 0.0190    | 0.0309       | 0.0541       | 0.0917      |
|                     |           |              |              | 0.1579      |
|                     | 0.0198    | 0.0302       | 0.0557       | 0.0898      |
|                     |           |              |              | 0.1583      |
| Z = 5 cm            |           | Z = 5 cm     |              |             |
|                     | 0.0196    | 0.0294       | 0.0589       | 0.0883      |
|                     |           |              |              | 0.1570      |
|                     | 0.0190    | 0.0309       | 0.0541       | 0.0917      |
|                     |           |              |              | 0.1579      |
|                     | 0.0198    | 0.0302       | 0.0557       | 0.0898      |
|                     |           |              |              | 0.1583      |
| Z = 10 cm           |           | Z = 10 cm    |              |             |
|                     | 0.0196    | 0.0294       | 0.0589       | 0.0883      |
|                     |           |              |              | 0.1570      |
|                     | 0.0190    | 0.0309       | 0.0541       | 0.0917      |
|                     |           |              |              | 0.1579      |
|                     | 0.0198    | 0.0302       | 0.0557       | 0.0898      |
|                     |           |              |              | 0.1583      |

6. Conclusions

The force between planar spiral coils is analyzed using finite element and concentric rings methods. The results of 3-D FEM simulations show that the magnetic flux density distribution for concentric rings is similar to that of the spiral coils. Further investigations on the calculation of the magnetic force between spiral coils confirm the effectiveness of the replaced concentric rings method when compared with the direct calculation method. Therefore, the former method simplifies the calculation procedure and decreases the computation time. According to the obtained results, the accuracy of the replaced rings method for the number of turns more than 10, which is the case of many practical applications, is acceptable; hence, the method is applicable for variety of spiral coils. Also, the calculation results are in good agreement with the experimental and FEM results, validating the precision of the replaced rings method.

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Analysis of Reflectivity and Shielding Effectiveness of Absorbing Material–Conductor Laminate for Electromagnetic Compatibility

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ABSTRACT

An absorbing material–conductor laminate is widely used for electromagnetic compatibility of electronic circuits at microwave frequencies. Such a laminate when properly designed will exhibit good results in terms of electromagnetic interference and compatibility. In this paper, microwave absorbing materials like 1) Ca-NiTi hexa ferrite composites \( \text{Ca} \left( \text{NiTi}_{x} \text{Fe}_{12-2x} \text{O}_{19} \right) \) for \( x = 0.4 \), 2) M-Type Barium ferrites \( \text{BaFe}_{12-2x} \text{A}_{x} \text{Co}_{x} \text{O}_{19} \) for the tetravalent A ions, \( \text{Ru}^{4+} \) is chosen), 3) MnZn ferrite-Rubber composites with volume fraction \( v_{f} = 0.4 \), 4) Carbonyl-Iron particle composites with volume fraction \( v_{f} = 40\% \) and conducting materials like copper, stainless steel are considered to form the interface in the laminate. Mathematical formulations are carried out for the estimation of reflectivity and shielding effectiveness of absorbing material–conductor laminate at microwave frequencies. Analysis is also carried out for various thicknesses of the microwave absorbing material and conducting material in the laminate. The reflectivity and shielding effectiveness depends not only on the type of the selected material in the laminate, but also their thickness in the laminate and frequency of operation.

Keywords: Electromagnetic Compatibility, Reflectivity, Shielding Effectiveness, Propagation Constant, Attenuation Constant, Reflection Coefficient, Transmission Coefficient, Laminate, Interface

1. Introduction

Electromagnetic compatibility of an electronic circuit at microwave frequencies is of prime concern in the design of microwave circuits. External electromagnetic radiations should not interfere with the basic circuit performance and as well the circuit should not radiate electromagnetic energy to interfere with other neighboring circuits. The best method of achieving such an electromagnetic compatibility is to house the circuit in an enclosure made with a laminate which attenuates the radiations from the circuit and stops the external radiations interfering with the circuit. A laminate of microwave absorbing material-metallic conductor is considered to improve the electromagnetic compatibility capability of the circuit. This laminate is designed such that the radiation from the microwave circuit is attenuated to a very large extent before it propagates out of the circuit housing and simultaneously, also shields the microwave circuit from external radiation interferences. A well designed conducting metal with appropriate thickness provides the required shielding ability and a suitably selected microwave absorbing material with high attenuation constant arrests the radiations from the circuit. The thickness of the material layers in the laminate are so designed such that the reflectivity and shielding effectiveness of the laminate are achieved as per the requirement of the circuit compatibility considerations.

Reflectivity is nothing but the total reflection coefficient of the laminate looked from absorbing material layer direction and shielding effectiveness is the total attenuation offered by the laminate. Appropriate equations are derived for the determination of reflectivity and shielding effectiveness of the proposed laminate. Analysis is carried for the estimation of these parameters for different materials, thickness of the material layers and at different microwave frequencies. Various types of popularly available microwave absorbers and metals are considered for the laminate to give best performance in terms of reflectivity and shielding effectiveness. The thickness of the laminate is to be optimized for the required performance of the electromagnetic compatibility.
2. Reflectivity

Reflectivity of the lamination can be estimated using the transmission line analysis for normal incidence. Reflectivity coefficient \([1]\) at an interface of absorbing material-conducting metal can be given as

\[
\Gamma_1 = \frac{n_e - n_d}{n_e + n_d}
\]

where \(n_e\) is the intrinsic impedance of metallic conductor and \(n_d\) is the intrinsic impedance of the microwave absorbing material. The intrinsic impedance \(\[2\]\) of metallic conductor can be estimated as

\[
n_e = (1 + j) \sqrt{\frac{\pi f \mu_e}{\sigma}}
\]

where permeability of the metal, \(\mu_e = \mu_s \mu_r\)

conductivity of the metal, \(\sigma = \sigma_o \sigma_r\)

\(\mu_s\) is relative permeability of the absorbing material,

\(\sigma_r\) is the relative conductivity of the conductor with respect to copper,

\(\mu_r\) is free space permeability, \(\sigma_o\) is conductivity of copper and \(f\) is the frequency of operation.

The intrinsic impedance of absorbing material can be derived to be

\[
n_d = \sqrt{-\frac{(2\pi f \mu_d)^2}{\gamma_d^2}}
\]

where \(\mu_d\) is the permeability of absorbing material

\(\gamma_d\) is the propagation constant of the absorbing material \(\[3\]\) which can be derived as

\[
\gamma_d = j \left( \frac{2\pi f}{c} \right) \sqrt{\mu_A e_{ia}} = j \left( \frac{2\pi f}{c} \right) \sqrt{\mu_A' e_{ia}' - j \mu_A'' e_{ia}''}
\]

Relative permeability, \(\mu_{ia} = \mu_{ia}' - j \mu_{ia}''\)

Relative permeability, \(e_{ia} = e_{ia}' - j e_{ia}''\)

c is the speed of light in free space.

The reflection coefficient at the interface of free space-absorbing material is very small and is neglected in this analysis.

The reflectivity at the interface of absorbing material-conducting metal in the laminate, as shown in the Figure 1, is the path loss of the electromagnetic energy while it propagates from free space to absorbing material-metal interface and back after reflection by the metal in the interface.

Thus, the reflectivity of the laminate can be derived as

\[
r = e^{-\alpha_d t_d} \Gamma_1 e^{-\alpha_d t_d}
\]

\[
e^{-2\alpha_d t_d} \Gamma_1
\]

Where \(t_d\) is the thickness of the absorbing material and \(\alpha_d\) is the attenuation constant of the absorbing material \(\[3\]\) can be given as

Reflectivity expressed in \(dB\) is

\[
R = 20 \log_{10} (r) \ dB
\]

3. Shielding Effectiveness

Shielding effectiveness of the laminate of microwave absorbing material and conducting metal is the total attenuation loss of the electromagnetic energy while it propagates through the interface. In other words, the shielding effectiveness is nothing but transmission coefficient of the interface of microwave absorbing material and conducting metal in the laminate.

The transmission coefficient of the interface at two boundaries (absorber-conductor & conductor-free space) \(\[2\]\) is given as

\[
p = \frac{8n_o n_A n_L}{(n_o + n_A)(n_A + n_c)(n_c + n_o)}
\]

Where \(n_o\) is the free space intrinsic impedance = \(120\pi\) ohms.

Reflection coefficient \(\[2\]\) at absorbing material-metal interface, \(q_1\), and that at conductor-free space interface \(q_2\) can be given as

\[
q_1 = \frac{(n_d - n_o)(n_d - z(c))}{(n_d + n_o)(n_d + z(c))}
\]

\[
q_2 = \frac{(n_c - n_o)(n_c - z(c))}{(n_c + n_o)(n_c + z(c))}
\]

where \(z(c)\), impedance to the right of the absorber-metal interface and can be derived as

\[
\alpha_d = \frac{\sqrt{2\pi f}}{c} \sqrt{\left(\mu_d'' e_{ia}' - \mu_d' e_{ia}\right)^2 + \left(\mu_d'' e_{ia}' - \mu_d' e_{ia}\right)^2 + \left(e_{ia}' + \mu_d'' e_{ia}\right)^2}
\]
4. Results and Conclusions

Analysis is carried out for the estimation of reflectivity (equation 11) of the laminate of microwave absorbing material-metallic conductor for various types of absorbing materials, conducting materials and thickness of material layers in the laminate. Popular and widely used microwave absorbing materials like 1) Ca-NiTi hexa ferrite composites (Ca(NiTi)xFe12-2xO19) for x = 0.4 [4], 2) M-Type Barium ferrites (BaFe12-2xAxCoxO19 for the tetravalent A ions, Ru4+ is chosen) [5], 3) MnZn ferrite-Rubber composites with volume fraction v_f = 0.4 [6] and 4) Carbonyl-Iron particle composites with volume fraction v_f = 40% [3,7] along with copper as conductor is considered for the estimation of optimum reflectivity.

Figures 3 & 4 show the variations of reflectivity (equation 11) with frequency for different absorbing materials at a thickness of 5 and 10 mm respectively. Figures 5 & 6 are the plots for variations of reflectivity with thickness of layer of absorbing materials (BaFe12-2xAxCoxO19 for the tetravalent A ions, Ru4+ is chosen and ferrite-Rubber composites with volume fraction v_f = 0.4) at different frequencies. Reflectivity is estimated at 12GHz for various absorbing materials at different thicknesses and is presented as in Figure 7.

Reflectivity of the laminate mainly depends upon the absorption properties of the microwave absorbing material and its thickness. Thus, the microwave absorbing material, M-Type Barium ferrites exhibits excellent reflectivity (around 20 dB better than Carbonyl-Iron particle composites with volume fraction v_f = 40%) over the entire frequency range compared to other types of absorbing materials. Since attenuation constant is very high for that of M-Type Barium ferrites.
Analysis of Reflectivity and Shielding Effectiveness of Absorbing Material–Conductor Laminate for Electromagnetic Compatibility

Figure 4. The variation of the reflectivity as a function of frequency for different microwave absorbers at absorber thickness of 10 mm

Figure 5. The variation of the reflectivity as a function of thickness for the BaFe$_{12-x}$Al$_{x}$Co$_{0.5}$O$_{19}$–copper laminate at different frequencies

Figure 6. The variation of the reflectivity as a function of thickness for the MnZn ferrite-Rubber composite-copper laminate at different frequencies

Figure 7. The variation of the reflectivity as a function of thickness for different microwave absorbers at 12 GHz frequency

It can also be deduced that the reflectivity of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$ for the tetravalent A ions, Ru$^{4+}$ is chosen and MnZn ferrite-Rubber composites with volume fraction $v_f = 0.4$ is better by approximately 20 dB in the X-band range of frequencies and it decreases at high frequencies.

Investigations are carried out to determine the shielding effectiveness (Equation (18)) of the laminate of microwave absorbing material-metal for different combinations of materials at different thicknesses of layers in the laminate. Figures 8 & 9 are the plots for variation of shielding effectiveness of laminate of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$ (for the tetravalent A ions, Ru$^{4+}$ is chosen) and MnZn ferrite-Rubber composites with volume fraction $v_f = 0.4$ for 1, 5 and 10 mm thicknesses of absorbing material and 1 mil layer thickness of copper respectively. Figures 10 & 11 are the plots for variation of shielding effectiveness of laminate of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$ (for the tetravalent A ions, Ru$^{4+}$ is chosen) and MnZn ferrite-Rubber composites with volume fraction $v_f = 0.4$ for 1, 5 and 10mm thicknesses of absorbing material and 1 mil layer thickness of stainless steel respectively.

Shielding effectiveness is also estimated for different layer thicknesses of the copper in the laminate and is presented in Figures 12 & 13 for microwave absorbing material layer of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$ for $A= Ru^{4+}$, MnZn ferrite-Rubber composites with volume fraction $v_f = 0.4$ for $x = 0.4$ (thickness of the absorbing material = 5 mm). The plot of shielding effectiveness with frequency for different absorbing materials of the interface with copper as conducting layer material is shown in Figure 14.

The shielding effectiveness of the conductor dominates that of microwave absorbing material in the laminate and this in turn depends on the thickness of the conducting layer almost linearly. This is because the attenuation constant of the conductor is extremely high.
Analysis of Reflectivity and Shielding Effectiveness of Absorbing Material–Conductor Laminate for Electromagnetic Compatibility

Figure 8. The variation of the Shielding effectiveness as a function of frequency of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$–copper laminate for different thicknesses of microwave absorber

Figure 9. The variation of the S.E as a function of frequency of MnZn ferrite-Rubber composites–copper laminate for different thicknesses of microwave absorber

Figure 10. The variation of the Shielding effectiveness as a function of frequency of BaFe$_{12-2x}$A$_x$Co$_{0.5}$O$_{19}$–stainless steel laminates for different thicknesses of microwave absorber

Figure 11. The variation of the Shielding effectiveness as a function of frequency of MnZn ferrite-Rubber composites–stainless steel laminate for different thicknesses of microwave absorber

Figure 12. The variation of the Shielding effectiveness as a function of frequency of BaFe$_{12-2x}$A$_x$Co$_{0.5}$–copper for different thicknesses of copper

Figure 13. The variation of the Shielding effectiveness as a function of frequency of MnZn ferrite-Rubber composites–copper for different thicknesses of copper
Analysis of Reflectivity and Shielding Effectiveness of Absorbing Material–Conductor Laminate for Electromagnetic Compatibility

Figure 14. Comparison of shielding effectiveness of different absorbers as a function of frequency

compared that of the absorbing material. The shielding effectiveness of the laminate using stainless steel as conducting layer is much higher than that of a laminate using copper as conducting material layer.

From the above analysis for reflectivity and shielding effectiveness, it may be deduced that laminate comprising of M-Type Barium ferrites as absorbing material layer and stainless steel as conducting material layer exhibits very good reflectivity as well as shielding effectiveness characteristics. It may be concluded that the reflectivity and shielding effectiveness primarily depend upon the material characteristics. The thickness of the material can be selected for a given application according to the requirements of the problem and subject to the availability of the materials and mechanical constraints of the circuit under consideration.

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Generalized Alternating-Direction Implicit Finite-Difference Time-Domain Method in Curvilinear Coordinate System*

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ABSTRACT

In this paper, a novel approach is introduced towards an efficient Finite-Difference Time-Domain (FDTD) algorithm by incorporating the Alternating Direction Implicit (ADI) technique to the Nonorthogonal FDTD (NFDTD) method. This scheme can be regarded as an extension of the conventional ADI-FDTD scheme into a generalized curvilinear coordinate system. The improvement on accuracy and the numerical efficiency of the ADI-NFDTD over the conventional nonorthogonal and the ADI-FDTD algorithms is carried out by numerical experiments. The application in the modelling of the Electromagnetic Bandgap (EBG) structure has further demonstrated the advantage of the proposed method.

Keywords: Alternating Direction Implicit Technique, Numerical Instability, Nonorthogonal FDTD

1. Introduction

The Finite-Difference Time-Domain (FDTD) Method [1] has been proven to be an effective algorithm in computational electromagnetics. The first FDTD algorithm was introduced by Yee [2] in 1966. Since then, extensions of this method [3-8] have been made continuously. The nonorthogonal FDTD (NFDTD) method [9-11] is a standard generalized FDTD algorithm based on the curvilinear coordinate system. As far as a curved geometry is concerned, the NFDTD algorithm has demonstrated improved efficiency over the conventional Yee’s algorithm [12], due to the fact that considerably fewer cells are needed in the former by using conformal meshes instead of employing staircase approximation.

As is the case in the Cartesian FDTD algorithm, the time interval \( dt_{\text{max}} \) used in the NFDTD simulation should not exceed the Courant criterion \( dt_{\text{max},i,j} \) given by (1), which is the minimum value of \( dt_{\text{max},i,j} \) in order to guarantee that the CFL condition is satisfied for every cell in the NFDTD meshes. The orthogonal CFL condition is a special case when \( \theta = 90^\circ \).

For orthogonal meshes, it is straightforward to calculate \( dt_{\text{max}} \). However, with nonorthogonal meshes, it is not trivial to find the minimum value of \( dt_{\text{max},i,j} \) as diverse cells exist. More to the point, when the meshes contain globally large but locally very small or skewed cells, the using of the smallest \( dt_{\text{max},i,j} \) may result in low efficiency in the NFDTD simulations.

\[
dt_{\text{max}} = \min \left[ dt_{\text{max},i,j} \right] = \min \left[ \min \left( \frac{\sin \theta_{i,j}}{c \sqrt{(\Delta \xi)^2 + (\Delta \eta)^2}} \right) \right] (1)
\]

In addition, it has been proven that the NFDTD approach suffers late-time numerical instabilities [13-15]. Various efforts have been made to alleviate this problem [16-18].

The alternating direction implicit (ADI) technique has been successfully applied to the orthogonal FDTD instead of the Yee’s leapfrogging scheme, resulting in the

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removal of the CFL stability condition and an improvement in the efficiency of FDTD method. In [19], the ADI principle as applied in [6] and [7] is extended to the curvilinear coordinates based on Lee’s NFDTD scheme and a novel NFDTD method that is free of the CFL stability condition is briefly introduced. Similar idea was proposed based on Holland’s NFDTD formulation in [20]. In this paper, the formulation of the generalized ADI-NFDTD as in [19] is presented in details and its numerical efficiency and accuracy is further studied. Compared to the conventional NFDTD method, the numerical efficiency can be improved by using an increased \( dt \), and the late time instability of the NFDTD scheme has been greatly reduced. Compared with the conventional orthogonal ADI-FDTD, this generalized scheme is not unconditionally stable. However, it shows improved numerical efficiency in modelling curved structures by requiring less computer memory and computer run time, because coarser conformal grids are employed. In order to demonstrate its efficiency, the proposed ADI-NFDTD is applied in calculating the bandgap diagram of the Electromagnetic Bandgap structure (EBGs).

2. Formulation

The Maxwell curl equations can be written as the following for a source-free space:

\[
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (2)
\]

\[
\nabla \times \mathbf{E} = -\mu \cdot \frac{\partial \mathbf{H}}{\partial t} \quad (3)
\]

where \( \mu \) is the medium permeability. Both of the equations can be casted into partial differential equations in the curvilinear coordinates as shown in [15]. A 2-D TE wave module is used to illustrate the scheme. The formulation for the TM module can be obtained in a similar way.

Two procedures are used to solve the differential equations. The first procedure is based on (4)-(6), and the second procedure is based on (7)-(9).

1) The First Procedure:

\[
D^{\ast} \left( i, j + \frac{1}{2}, \frac{1}{2} \right) - D^{\ast} \left( i, j + \frac{1}{2}, \frac{1}{2} \right) = -\frac{dt}{2 \sqrt{g \left( i, j + \frac{1}{2}, \frac{1}{2} \right)}} \]

\[
H^{\ast} \left( i + \frac{1}{2}, j + \frac{1}{2}, \frac{1}{2} \right) - H^{\ast} \left( i + \frac{1}{2}, j + \frac{1}{2}, \frac{1}{2} \right) \]

\[
E^{\ast} \left( i + \frac{1}{2}, j + \frac{1}{2}, \frac{1}{2} \right) - E^{\ast} \left( i + \frac{1}{2}, j + \frac{1}{2}, \frac{1}{2} \right) \]

where \( E, \ E', \ H \) are the covariant electric and magnetic field components which represent the flow of field along the grid, while \( D', \ D' \) and \( H' \) are the contravariant electric fluxes and magnetic field component which represent the flow going through the facets of the grid. \( \mu_c \) is the medium permeability for calculating \( H' \) and \( g \) is the metric tensor computed from the local curvilinear coordinates. The covariant field components \( (E_c, E_e) \) must be calculated from the contravariant flux \( (D' \ and \ D') \) by interpolating equations as introduced in the conventional NFDTD scheme [9-10]. As the covariant field compo-
nents should be valued in the same position with their contravariant pairs, the neighbouring averaging projection scheme is employed. For a 2-D model, $g_x$ and $g_y$ are zeros, and $g_z$ equals to unity for every cell. Hence we obtain (10)-(14).

$$E_x\left(i, j \frac{1}{2}\right) = g_x(i, j) \cdot D_x^\ast \left(i + \frac{1}{2}, j\right) + g_y(i, j) \cdot 4 \cdot e_x(i, j) \cdot \left[D_x^\ast \left(i, j - \frac{1}{2}\right) + D_x^\ast \left(i + 1, j - \frac{1}{2}\right) + D_x^\ast \left(i, j + \frac{1}{2}\right) + D_x^\ast \left(i + 1, j + \frac{1}{2}\right)\right]$$

$$E_y\left(i, j \frac{1}{2}\right) = g_y(i, j) \cdot D_y^\ast \left(i + \frac{1}{2}, j\right) + g_x(i, j) \cdot 4 \cdot e_y(i, j) \cdot \left[D_y^\ast \left(i + \frac{1}{2}, j - 1\right) + D_y^\ast \left(i + 1, j - \frac{1}{2}\right) + D_y^\ast \left(i + \frac{1}{2}, j + 1\right) + D_y^\ast \left(i + 1, j + \frac{1}{2}\right)\right]$$

$$E_z\left(i, j \frac{1}{2}\right) = g_x(i, j) \cdot D_z^\ast \left(i + \frac{1}{2}, j\right) + g_y(i, j) \cdot 4 \cdot e_z(i, j) \cdot \left[D_z^\ast \left(i, j - \frac{1}{2}\right) + D_z^\ast \left(i + 1, j - \frac{1}{2}\right) + D_z^\ast \left(i, j + \frac{1}{2}\right) + D_z^\ast \left(i + 1, j + \frac{1}{2}\right)\right]$$

where $e_x(i, j)$ is the material permittivity in the $x$- or $y$- direction. By substituting (10), (11) and (13) into (6) and the resulting expression for $H_z^\ast \frac{1}{2}$ into (5), one can obtain (15). From (15) the cell indices $(i, j)$ are used in the equations for a neat presentation, e.g., $D_y^\ast \left(i, j \frac{1}{2}\right)$ is used instead of $D_y^\ast \left(i, j \frac{1}{2}\right)$. With $D_y^\ast \frac{1}{2}$ readily obtained by (4), (15) is a tri-diagonal system of equation that can be easily solved.

Thus $D_y^\ast \frac{1}{2}$ is updated. Then $H_z^\ast \frac{1}{2}$ can be calculated by (6).

The field components in the second procedure can be updated in a similar way, by the use of (11), (12), (14) and (7)-(9).

The proposed ADI-NFDTD algorithm is an extension of the ADI-FDTD in the curvilinear coordinate system. As a result, if the grid is uniform and orthogonal, this algorithm will reduce to the conventional orthogonal ADI-FDTD, as illustrated in (16)-(21).

We apply the following relations to the derived ADI-NFDTD equations:

$$\frac{D_y(i, j) \cdot \sqrt{g_y(i, j)}}{\epsilon_y(i, j)} = E_y(i, j)_{\text{orth}} \quad (16)$$

$$\frac{E_z(i, j)}{\sqrt{g_y(i, j)}} = E_z(i, j)_{\text{orth}} \quad \text{and} \quad H_z(i, j)_{\text{orth}}$$

where $E_x(i, j)_{\text{orth}}$, $E_y(i, j)_{\text{orth}}$ and $H_z(i, j)_{\text{orth}}$ denote
the corresponding electric and magnetic field components in the Cartesian coordinates respectively.

The $g$ tensors have the relationship of (17) under 2-D orthogonal meshes.

$$
g(i, j) = g_{xx}(i, j) \cdot g_{yy}(i, j)
g_{xx}(i, j) = d\mathbf{x}(i, j)^{2}$$

Substituting (16)-(17) into (15) yields (18), which is the ADI-FDTD updating equation under arbitrary orthogonal meshes.

Given that the FDTD grid is uniform, and denoting the spatial increments along the x- and y- axes as $\Delta x$ and $\Delta y$, respectively, (19) are yielded from (17).

$$
g_{xx}(i, j) = \Delta x^{2}
g_{yy}(i, j) = \Delta y^{2}, \text{and}$$

Then substituting (19) into (18) and multiplying $-4\Delta x^{2} \cdot \Delta y \cdot \mu_{z}(i, j)$ on both side of the equation yields (20). Equation (20) is the general equation for orthogonal ADI-FDTD scheme with uniform grid. Equations (18) and (20) can treat homogeneous or inhomogeneous, isotropic or anisotropic media. In a medium where $\mu_{z}(i, j) = \mu_{z}(i-1, j)$, (20) further reduces to the formula given in [6].

### 3. Numerical Results

#### 3.1 Comparison of the Late Time Instability and the Numerical Accuracy of the ADI-NFDTD Algorithm with the Conventional NFDTD Algorithm

The resonant modes of a cylindrical perfect electric conductor (PEC) cavity resonator are calculated by using both the conventional NFDTD and the proposed ADI-NFDTD schemes. The radius of the resonator is 0.15m and the structure is infinitely long. The cavity is filled with vacuum of permittivity $\varepsilon_r = 1$. The outer material is PEC, and the whole computational region is 0.56 m × 0.56 m meshed by 25 × 26 cells. A sine modulated cosine pulse (21) is excited inside the cavity to provide a wide band excitation to excite all the possible TE modes:

$$
Source(n) = A_m \cdot \sin(2\pi f \cdot n \cdot dt) \cdot (1 - \cos(2\pi f \cdot n \cdot dt))
$$

where $A_m$ relates to the amplitude of the signal, $f$ is the frequency parameter, $dt$ is the time increment and $n$ is the iteration index.

The temporal signatures of the magnetic fields inside the resonator are recorded. The first 8,000 time steps results from both the conventional and the ADI-NFDTD algorithms are normalized by their own peak values and are plotted in Figure 2.

More simulations were performed using different $dt$ values to study the stability and the accuracy of the ADI-NFDTD scheme. It is shown that unlike the conventional...
Figure 2. The normalized H field results of the first 8000 time steps from the two schemes

ADI-FDTD, this generalized ADI-NFDTD scheme is not an unconditionally stable scheme. Instability in the temporal results is observed when small dt (i.e., comparable with or smaller than the dt_max required in the NFDTD scheme) is used. However, compared to the NFDTD scheme, the unstable result in the ADI-NFDTD occurs much later and the spurious energy grows much slower than that in the conventional NFDTD scheme. This is demonstrated in Figure 3.

The removal of the CFL stability condition on dt in the ADI-NFDTD method is also observed. When dt reaches 16 ps in the simulations, the conventional NFDTD cannot provide reasonable results, indicating that the CFL stability condition is violated, while ADI-NFDTD scheme can do so, even using a greater dt value.

In order to compare the accuracy of the ADI-NFDTD with the conventional NFDTD algorithm, the resonant frequencies are calculated using the Fast Fourier Transformation (FFT) and are compared with the analytical results. Since both the NFDTD and the ADI-NFDTD methods suffer numerical instability in late time steps, the temporal data are truncated to 40,000 and 100,000 time steps respectively. Such numbers are chosen in order to obtain the best frequency resolutions, and avoid too much spurious energy in the spectra at the same time. Since dt = 1 ps, the frequency resolutions of the conventional and the ADI-NFDTD results are 0.025 GHz and 0.01 GHz respectively. The calculated resonant frequency spectra are plotted in Figure 4.

As can be seen, it is possible to distinguish two closely-spaced frequency spectra from the results obtained by using the ADI-NFDTD, while it is hard to do so in the conventional NFDTD due to the limited frequency resolution. Moreover, the noise level of the ADI-NFDTD is lower than that of the conventional NFDTD, if dt used in the two simulations are comparable.
the calculated resonant frequency with the theoretical one is divided by the latter, and expressed in percentage, forming the relative error rate for each resonant mode. Then, the standard deviation of the relative error rates of all the modes within frequency band of interest (0-3 GHz) are calculated as the averaged relative error rate of the numerical results. The curve of the averaged relative error rate as the function of time interval is plotted in Figure 5.

In theory, the ADI-NFDTD may not necessarily provide better accuracy than the conventional NFDTD. However, to the best of the authors’ knowledge, the former can always provide more stable temporal results than the latter. As the FFT being performed with a sufficient frequency resolution, the ADI-NFDTD shows smaller relative error rate at some dt values in Figure 5. The removal of the CFL condition on NFDTD algorithm can be seen when dt is greater than 16 ps. However, according to the authors’ simulation experience, dt should not exceed the corresponding Courant criterion t_{max} in order to maintain the accuracy in ADI-NFDTD simulations. By using the same dt, the computational efficiency is decreased in the ADI-NFDTD simulation than in the NFDTD one.

### 3.2 Comparison of the Numerical Efficiency and Accuracy of the ADI-NFDTD Algorithm with the ADI-FDTD Algorithm

A two-dimensional cavity resonator is modelled using both the ADI-NFDTD and the conventional orthogonal ADI-FDTD algorithms, in order to demonstrate the numerical efficiency improvement of the former scheme. The radius of the cavity is 0.15 m. The cavity is filled with vacuum of relative permittivity \( \varepsilon_r = 1 \). The outer material is copper, with relative permittivity \( \varepsilon_r = 1 \) and conductivity \( \sigma = 5.8 \times 10^{7} \) S/m. The whole computational region is 0.6 m × 0.6 m, meshed by the ADI-NFDTD method using 46 × 46 cells, and by the orthogonal ADI-FDTD method using 60 × 60, 70 × 70 and 80 × 80 cells respectively. The time step is chosen to be 2 ps and 25,000 iterations are run in each simulation. A modulated Gaussian pulse as shown in (22) is excited inside the cavity. The temporal responses of probed points inside the cavity are recorded and Fourier Transformed, after which the resonant frequencies is identified as peaks in the frequency spectra.

\[
\text{Source}(n) = A_m \cdot \exp\left(j2 \pi f \cdot n \cdot dt\right) \cdot \exp\left[-\left(\frac{(n \cdot \text{delay}) \cdot dt}{r_2}\right)^2\right]
\]

(22)

where \( A_m \) is the maximum amplitude, \( f \) is the frequency parameter, \( n \) is the iteration index, \( dt \) is the time increment, \( \text{delay} = \frac{1}{2}(dt \cdot f) \) is the delay of the pulse in time step, and \( r_2 = \frac{1}{2}(8/f) \) is the pulse half-duration at the 1/e point. In the simulation, \( A_m = 1 \) and \( f = 0.5 \) GHz.

To calculate the relative error rates (RERs), the frequencies of all the genuine modes are compared with the theoretical results [22], and the results are shown in Table 1.

Table 2 presents a comparison of numerical errors, computer resources used in all simulations in the same computer under the same programming environment of Matlab 6. The averaged relative error rate calculated from the genuine and the spurious modes and that from only the genuine modes are denoted as ‘ARER’ and ‘ARER-G’, respectively.

### Table 1. Comparison of the numerical accuracy in terms of the genuine modes calculated by the ADI-NFDTD and orthogonal ADI-FDTD algorithms under different spatial resolutions

| Theoretical results [22] (GHz) | Frequency of the genuine mode (GHz)/RER(%) |
|-------------------------------|------------------------------------------|
|                              | ADI-NFDTD  | Orthogonal ADI-FDTD |
| 46 × 46  | 60 × 60   | 70 × 70   | 80 × 80   |
| 0.5863  | 0.56/4.49 | 0.56/4.49 | 0.56/4.49 |
| 0.9721  | 0.86/11.53| 0.88/9.47 | 0.88/9.47 |
| 1.2198  | 1.16/4.90 | 1.18/9.47 | 1.16/4.90 |
| 1.3372  | 1.32/7.86 | 1.34/2.10 | 1.38/3.20 |
| 1.69    | 1.64/2.96 | 1.62/4.14 | 1.64/2.96 |
| 1.7     | 1.78/4.71 | 1.72/1.18 |
| 2.043   | 2.06/0.83 | 2.18/2.79 | 2.0/2.10  |
| 2.136   | 2.16/1.12 | 2.1/1.69  |
| 2.234   | 2.28/2.06 | 2.2/1.52  |
| 2.389   | 2.34/2.05 | 2.36/1.21 |
| 2.5533  | 2.58/1.06 | 2.52/0.78 |
| 2.718   | 2.72/0.07 | 2.74/0.81 |
| 2.732   | 2.72/0.44 | 2.74/0.29 |
| 2.956   | 2.92/0.81 | 2.98/0.81 |

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Figure 5. The averaged relative error rate of the resonant frequency calculated by the conventional NFDTD and the proposed ADI-NFDTD schemes as the function of relative time interval dt/dt0 (dt0 = 1ps). After [19]
Table 2. Comparison of the accuracy and computer resources of the ADI-NFDTD and the orthogonal ADI-FDTD in calculating the resonant modes of the copper cavity

|                     | ADI-NFDTD | Orthogonal ADI-FDTD |
|---------------------|-----------|---------------------|
| Mesh size           | 46 × 46   | 60 × 60 70 × 70 80 × 80 |
| dt (ps)             | 2         | 2 2 2 2          |
| nmax                | 25000     | 25000 25000 25000 |
| Frequency Resolution(GHz) | 0.02     | 0.02 0.02 0.02 0.02 |
| Number of Spurious Modes | 1        | 7 3 5          |
| ARER-G (%)          | 3.04      | 3.91 3.84 3.49 |
| ARER (%)            | 4.24      | 6.31 4.55 6.02 |
| Computer Memory (MB)| 132       | 160 182 205 |
| Simulation Run Time (minutes) | 53       | 127 288 318 |

It can be seen that the ADI-NFDTD method can provide results with less numerical error while using less computer memory and less simulation run time. This demonstrates the accuracy and computing efficiency improvement of the ADI-NFDTD algorithm over the orthogonal ADI-FDTD. However, compared to the orthogonal ADI-FDTD algorithm, the ADI-NFDTD method is not an unconditionally stable method. The late time instability inherited from the NFDTD scheme is a major drawback of the ADI-NFDTD method.

3.3 Application of ADI-NFDTD Algorithm in Bandgap Structure Simulations

In this section, the ADI-NFDTD scheme is applied to model an electromagnetic bandgap (EBG) structure and the simulation results are compared with those from the NFDTD modelling.

The two-dimensional EBG structure comprises of metallic rods periodically loaded in square lattice in free space. The unit cell approach [23] is used in this model.

This EBG structure is infinite in the x- and the y-direction with a lattice constant (period) a. In the z-direction, the rods are infinitely long. Each rod is made from copper with relative permittivity \( \varepsilon_r = 1 \) and conductivity \( \sigma = 5.8 \times 10^7 \) S/m. The ratio of the radius \( r \) to the lattice constant \( a \) is chosen to be \( r/a = 0.2 \). The unit cell, the Brillouin Zone and the nonorthogonal mesh file used in both the ADI-NFDTD and NFDTD algorithms are shown in Figure 6.

The dispersion diagram calculated by NFDTD is plotted in Figure 7. In the NFDTD simulation, while calculating the frequency spectra for vectors near vector \( k = k_1 \) (Figure 6(b)) for example. The temporal results at the same probe position obtained by using the ADI-NFDTD and the conventional NFDTD are compared in Figure 8. As can be seen from Figure 8(b), the ADI-NFDTD result shows resemblance with the NFDTD one at the beginning of simulations. It is noted that, after the first 50 nanoseconds, the results obtained via the ADI-NFDTD still remain stable while those from the NFDTD experience an exponential growth signaling numerical instability.

Figure 6. (a) The unit cell of the EBGs; (b) the Brillouin Zone in the reciprocal lattice with \( k = k_1 \) shown by a red vector; (c) the conformal mesh of the real lattice.

Figure 7. Dispersion diagram of ADI-NFDTD and NFDTD.

Figure 8. Temporal results of ADI-NFDTD and NFDTD comparison.
Consequently, the frequency spectra obtained using the ADI-NFDTD show a reduced noise level and hence lead to more accurate solutions for identifying eigen-modes in the dispersion diagram, as shown in Figure 9. For instance, the mode at a normalized frequency of 1.36 (highlighted using the datatip in Figure 7) can be hardly detected from the NFDTD results as the NFDTD spectrum is heavily buried by the noise. However, this mode is clearly shown as a distinct peak in the ADI-NFDTD spectra.

4. Conclusions

In this paper, the ADI-FDTD scheme is extended into the nonorthogonal coordinate system to achieve an ADI-NFDTD algorithm. The stability, accuracy and efficiency of the proposed scheme are validated by the cylindrical resonator simulations. It is shown that the CFL condition of NFDTD is removed by the use of the ADI scheme. Secondly, although the ADI-NFDTD is not unconditionally stable, the unstable result occurs at a much later stage compared with the conventional NFDTD algorithm, which is a significant improvement in terms of late time instability.

In this way, the ADI-NFDTD is able to provide more precise frequency spectra with a smaller FFT resolution and a lower noise level than those obtained in the conventional NFDTD. This is of great benefit in distinguishing resonant or eigen-modes closely located in the frequency spectra. As a result, the ADI-NFDTD is quite suitable in the modelling of resonating or periodic structures, in which the energy is hardly dissipated, and in which the resonant or the eigen-modes are key factors to be examined in the study.

Without having to employ staircasing approximation, the curved structures or oblique surfaces are more accurately and efficiently modelled using the ADI-NFDTD scheme than using the orthogonal ADI-FDTD.

Although dt is not limited by the CFL condition and the conformal meshes can be coarser than those in the orthogonal FDTD, the increase of dt value and the decrease of the spatial resolution will result in decreased accuracy in the ADI-NFDTD results. Therefore, in ADI-NFDTD, the dt value and the mesh profile should be carefully chosen for the combined consideration of the efficiency and accuracy.

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Generalized Alternating-Direction Implicit Finite-Difference Time-Domain Method in Curvilinear Coordinate System

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Progressing of Quantum Tomography for Quantum Information Acquisition*

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ABSTRACT

In this paper we review a number of recent developments in the study of quantum tomography which is one of the useful methods for quantum state estimation and quantum information acquisition, having sparked explosion of interest in recent years. The quantum process tomography is also analyzed. At the same time, some success experiments and applications of quantum tomography are introduced. Finally, a number of open problems and future directions in this field are proposed.

Keywords: Quantum Tomography, Quantum Information Acquisition, Quantum State Estimation, Review and Expectation

1. Introduction

In the past few years, quantum tomography has attracted considerable attention as well as experimental research. Much of this work has already been summarized in several reviews [1-6].

The state of a physical system is the mathematical description of our knowledge of it. If we can acquire the state we’ll get complete information on its past and future. The knowledge of the state is equivalent to know the result of any possible measurement on the system. A state estimation technique is a method that provides the complete description of a system, thus allowing one to make at least the best predictions on the results of any measurement that may be performed on the system. In classical physics it is always possible, at least in principle, to devise a procedure that fully recovers the state of a system. But, in Quantum Mechanics this is no longer possible, and this impossibility is inherently related to fundamental features of the theory, such as the Heisenberg uncertainty principle and no-cloning theorem. On one hand, the no-cloning theorem [7] forbids us to create perfect copies of an unknown state in order to make different measurements on the same state. On the other hand, the Heisenberg uncertainty principle [8] indicates that one cannot perform an arbitrary sequence of measurements on a single system without disturbing it in some way. Therefore, it is not possible, even in principle, to determine the quantum state of a system without any prior knowledge on it. [9] This is consistent with the very definition of a quantum mechanical state, which prescribes how to gain information about the state: for a quantum mechanical system it is possible to estimate the unknown state of a system when many identical preparations taken from the same statistical ensemble are available, so that a different measurement can be performed on each of the copies. A procedure of this kind is called Quantum Tomography (QT), which is described as an inverse statistical problem in which the quantum state of a system is the unknown parameter and the data are given by the results of measurements performed on identical quantum states [4]. The quantum state can be represented as an infinite dimensional density matrix. Thus, one can acquire information about the state by reconstructing the density matrix \( \rho \).

The first systematic approach for inferring state of a quantum system was U. Fano’s work in the late fifties of last century. [10] In the last decade a constantly increasing interest has been devoted to the subject. On one side, new developments in experimental techniques, especially in the fields of photodetection and nonlinear optics, resulted in a set of novel and beautiful experiments about quantum mechanics. On the other side, increasing attention has been directed to quantum information technology, such as error correction, purification, fault tolerant quantum computing, long distance teleportation and cryptography. In particular, a characterization of quantum channels relies heavily on quantum tomography techniques. Tomographic methods were initially employed only for measuring radiation states. However, they can

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In quantum process tomography, a device of some sort repeatedly prepares many instances of a quantum system in a fixed quantum state $\rho$. An experimentalist who wishes to characterize the operation of the device or to adjust it for future use may be able to perform measurements on the systems it prepares. He lets an incompletely specified device act on a quantum system prepared in an input state by his choice, and then performs a measurement on the output system. This procedure is repeated many times, with possibly different input states and different measurements, in order to accumulate enough statistics to assign a quantum operation to the device. Then learning about the state will also be learning about the device. The goal of the experimenter is to perform enough measurements, namely enough kinds of measurements on a large enough sample, to estimate $\rho$ and characterize operation of the device. All the operations we mentioned in this paper are trace-preserving completely positive linear map.

Basing on the results of theoretical research, a few experimentalists began to enter the realm of experiment of quantum tomography and quantum process tomography, and the first experiments performed by Michael Raymer’s group at the University of Oregon [14-15], which already showed the results for reconstructions of coherent and squeezed states. In the next decade, there is a quick emergence of many experiments results [2,16-25]. The first exact technique was given for measuring experimentally the matrix elements of the density operator in the photon-number representation [26] by simply averaging functions of homodyne data. After that, the method was further simplified [27], and the feasibility for non-unit quantum efficiency of detectors above some bounds was established. The exact homodyne method has been implemented experimentally to measure the photon statistics of a semiconductor laser [28], and the density matrix of a squeezed vacuum [29]. The success of optical homodyne tomography has then stimulated the development of state-reconstruction procedures for atomic beams [30], the experimental determination of the vibrational state of a molecule [31], of an ensemble of helium atoms [32], and of a single ion in a Paul trap [33]. Thus, the utility of these theories has been verified by relevant experiments. Meanwhile, quantum process tomography has been demonstrated experimentally in liquid state nuclear magnetic resonance [24,34], and recently a number of optical experiments [35-36] have implemented entanglement assisted quantum process tomography. In the third section some important scheme and results about experiments are specified.

This paper aims to review the chief relevant theory and techniques about quantum tomography which has provoked renewed interest in fundamental quantum mechanics. It will consist of two major sections. The first presents the general theory about quantum tomography and quantum process tomography; the second illustrates some experiments and application about them. Finally, a number of challenges for future work are summarized. A number of leading experts have cooperated to describe the main features in this novel field. We will attempt to proceed in a constructive way, making connections with previously presented notions and techniques when possible. If this paper can make more relevant researchers pay attention to these we’ll feel gratified.

2. Quantum Tomographic Methods

What is quantum state? The interpretation, well stated by Leonhardt, is that “knowing the state means knowing the maximally available statistical information about all physical quantities of a physical object”. Typically by “maximally available statistical information” we mean probability distributions. Hence, knowing the state of a system means knowing the probability distributions corresponding to measurements of any possible observable pertaining to that system. Since knowing the state means knowing all the statistical information about a system, is the inverse true? If one knows all of the statistical information about a system, does one then know the quantum state of that system? Clearly, a real experiment cannot measure all possible statistical information. A more practical question is then: can one infer reasonably well the quantum state of a system by measuring statistical information corresponding to a finite number of observables?

The answer to this question is a resounding yes. But there are some caveats. By now it is well established that the state of an individual system cannot, even in principle, be measured [37]. This is easily seen by the fact that a single measurement of some observable yields a single value, corresponding to a projection of the original state onto an eigenstate that has nonzero probability. Clearly this does not reveal much information about the original state. This same measurement simultaneously disturbs the individual system being measured, so that it is no longer in the same state after the measurement. This means that subsequent measurements of this same system are no longer helpful in determining the original state.

Since state measurement requires statistical information, multiple measurements are needed, each of which disturbs the system being measured. Here each member of an ensemble of systems is prepared by the same state-preparation procedure. Each member is measured only once, and then discarded. Thus, multiple measurements can be performed on systems all in the same state, without worrying about the measurement apparatus disturbing the system. A mathematical transformation is then
applied to the data in order to reconstruct, or infer, the state. The relevant interpretation of the measured state in this case is that it is the state of the ensemble. Because in quantum state measurement the collected data is analyzed using a mathematical technique that is very similar to the tomographic reconstruction technique used in medical imaging, and because all techniques are necessarily indirect, a generally accepted term for quantum state measurement has become Quantum State Tomography, for short, quantum tomography. The aim of quantum tomography is to reconstruct the density matrix of quantum states. Some relevant theories are introduced in the following section.

2.1 General Theory of Quantum Tomography

2.1.1 Wigner Function

As a method to express the density operator in terms of c-number functions, the Wigner functions often lead to considerable simplification of the quantum equations of motion. Using the Wigner function one can express quantum-mechanical expectation values in form of averages over the complex plane (the classical phase-space), where the Wigner function plays the role of a c-number function. The maximum value of the generalized Wigner function for \( s < -1 \) as a consequence of the positivity of the \( Q \) function. The maximum value of \( s \) keeping the generalized Wigner functions as positive can be considered as an indication of the classical nature of the physical state [39].

An equivalent expression for \( W_s(\alpha,\alpha^*) \) can be derived as follows. Equation (1) can be rewritten as:

\[
W_s(\alpha,\alpha^*) = \text{Tr}[\rho D(\alpha) \hat{W}_s(\alpha)]
\]

Where,

\[
\hat{W}_s = \int_c d^2 \lambda e^{i\lambda^T \frac{\alpha}{\sqrt{2}} - s} D(\lambda)
\]

Thus, the density matrix can be recovered from the generalized Wigner functions and, in particular, for \( s = 0 \) one has the inverse of the Glauber formula:

\[
\rho = 2 \int_c d^2 \alpha W(\alpha,\alpha^*) D(2\alpha)(-\alpha^*)
\]

whereas, for \( s = 1, \rho \) can be recovered according to (5).
2.1.2 Balance Homodyne Detection [14,40-41]

The balanced homodyne detector provides the measurement of the quadratures of the field $X_\phi$ in (7). It was proposed by Yuen and Chan [42], and subsequently demonstrated by Abbas, Chan and Yee [43].

The scheme of a balanced homodyne [27,44] detector is depicted in Figure 1. The signal mode $a$ interferes with a strong laser beam mode $b$ in a balanced 50/50 beam splitter. The mode $b$ is so-called local oscillator (LO) mode of the detector. It operates at the same frequency of $a$, and is excited by the laser in a strong coherent state $|z\rangle$. Since in all experiments that use homodyne detectors the signal and the LO beams are generated by a common source, we assume that they have a fixed phase relation. In this case the LO phase provides a reference for the quadrature measurement, namely, we identify the phase of the LO with the phase difference between the two modes. As we will see, by tuning $\phi = \arg z$ we can measure the quadratures $X_\phi$ at different phases. After the beam splitter the two modes are detected by two identical photodetectors (usually linear avalanche photodiodes), and finally the difference of photocurrents at zero frequency is electronically processed and rescaled by $2|z|$, the modes at the output of the 50/50 beam splitter are written:

$$c = \frac{a - b}{2}, \quad d = \frac{a + b}{2}$$  \hspace{1cm} (12)

hence the difference of photocurrents is given by the following operator:

$$I = d^*d - c^*c = \frac{a^*b + b^*a}{2|z|}$$  \hspace{1cm} (13)

Thus, the probability distribution of the output photocurrent $I$ for a generic state $\rho$ of the signal mode $a$ can be evaluated.

2.2 Qubit Quantum Tomography [5,45-47]

Much like its classical counterpart, which aims at reconstructing three-dimensional images via a series of two-dimensional projections along various ‘cuts’, quantum tomography characterizes the complete quantum state of a particle through a series of measurements in different bases. While the characterization of a classical object can perform a series of measurements on the same subject, measuring a single quantum particle disturbs its state, often making its further investigation uninformative. For this reason, quantum tomography must be carried out on a number of identical copies of the same state separately.

Before states can be analyzed, it is necessary to understand their representation. In particular, the reconstruction of an unknown state is often simplified by a specific state parametrization. In general, any single-qubit in a pure state can be represented by:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$  \hspace{1cm} (14)

where $\alpha$ and $\beta$ are complex and $|\alpha|^2 + |\beta|^2 = 1$ [48]. If the normalization is written implicitly and the global phase is ignored, this can be rewritten as:

$$|\psi\rangle = \cos \left(\frac{\theta}{2}\right) |0\rangle + \sin \left(\frac{\theta}{2}\right) e^{i\phi} |1\rangle$$  \hspace{1cm} (15)

These representations are sufficient to enable the description of the action of any operator (e.g., projectors or unitary rotations) on a pure state, and therefore to carry out tomography on that state. Meanwhile, mixed states may be described by a probabilistically weighted incoherent sum of pure states. In other words, it is as if any particle in the ensemble has a specific probability of being in a given pure state, and this state is distinguishably labeled in some way. A mixed state can be represented by a density matrix $\rho$:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \begin{bmatrix} A & Ce^{i\phi} \\ Ce^{-i\phi} & B \end{bmatrix}$$  \hspace{1cm} (16)

While any ensemble of pure states can be represented in this way, it is also true that any ensemble of single-qubit states can be represented by an ensemble of only two orthogonal pure states. Any physical density matrix can be diagonalized, such that:

$$\rho = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} = E_1 |\psi\rangle\langle\psi| + E_2 |\psi^+\rangle\langle\psi^+|$$  \hspace{1cm} (17)

where $\{E_1, E_2\}$ are the eigenvalues of $\rho$, and $\{|\psi\rangle, |\psi^+\rangle\}$ are the eigenvectors. Thus the representation of any quantum state, no matter how it is constructed, is identical to that of an ensemble of two orthogonal pure states [49]. For example, order horizontal $|H\rangle = |0\rangle$, vertical $|V\rangle = |1\rangle$ and...
\[ |D\rangle = \left(|H\rangle + |V\rangle\right)/\sqrt{2} \]
\[ |A\rangle = \left(|H\rangle - |V\rangle\right)/\sqrt{2} \]
\[ |R\rangle = \left(|H\rangle + i|V\rangle\right)/\sqrt{2} \]
\[ |L\rangle = \left(|H\rangle - i|V\rangle\right)/\sqrt{2} \]
Then
\[ \rho = \frac{1}{6} \left( \begin{array}{cc}
3 & 1 \\
1 & 3 \\
\end{array} \right) = \frac{2}{3} |D\rangle \langle D| + \frac{1}{3} |A\rangle \langle A| \] (19)

which, as predicted, is a sum of only two orthogonal states. In particular, any single-qubit density matrix \( \rho \) can be uniquely represented by four Stokes parameters \( \{S_0, S_1, S_2, S_3\} \):

\[ \rho = \frac{1}{2} \sum_{i=0}^{3} S_i \sigma_i \] (20)

where

\[ \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

and the \( S_i \) values are given by:

\[ S_i = \text{Tr} \{ \sigma_i, \rho \} \] (22)

For all pure states \( \sum_{i=1}^{3} S_i^2 = 1 \); for mixed states \( \sum_{i=1}^{3} S_i^2 < 1 \); for the completely mixed state \( \sum_{i=1}^{3} S_i = 0 \).

Due to normalization, \( S_0 \) will always equal one.

Although reconstructive tomography of any size system follows the same general procedure, beginning with tomography of a single qubit allows the visualization of each step using the Block Sphere, in addition to providing a simpler mathematical introduction. Exact single-qubit tomography requires a sequence of three linearly independent measurements. Each measurement exactly specifies one degree of freedom for the measured state, reducing the free parameters of the unknown state’s possible Hilbert space by one.

As an example, consider measuring \( |H\rangle \), \( |D\rangle \) and \( |R\rangle \) on the mixed state:

\[ \rho = \begin{pmatrix}
5 & -i \\
8 & \sqrt{2} \\
i & 3 \\
\sqrt{2} & 8
\end{pmatrix} = \frac{1}{2} \left( \sigma_0 + \frac{1}{\sqrt{2}} \sigma_2 + \frac{1}{4} \sigma_3 \right) \] (23)

This form allows us to read off the normalized Stokes parameters corresponding to these measurements:

\[ S_0 = 0, \quad S_2 = \frac{1}{\sqrt{2}}, \quad \text{and} \quad S_3 = \frac{1}{4} \] (24)

As always, \( S_0 = 1 \) due to normalization. Measuring \( R \) first, and looking to the Block Sphere, we see that the unknown state must lie in the \( z = 1/\sqrt{2} \) plane \( (S_2 = 1/\sqrt{2}) \). A measurement in the D basis further constrains the state to the \( y = 0 \) plane \( (S_1 = 0) \), resulting in a total confinement to a line parallel directly above the \( x \) axis. The final measurement of \( H \) pinpoints the state to the \( x = 1/4 \) plane \( (S_3 = 1/4) \). This process is illustrated in Figure 2. Obviously the order of the measurements is irrelevant: it is the intersection point of three orthogonal planes that defines the location of the state.

If instead measurements are made along non-orthogonal axes, a very similar picture develops, as indicated in Figure 3. The first measurement always isolates the unknown state to a plane, the second to a line, and the third to a point.

Of course, in practice, the experimenter has no knowledge of the unknown state before a tomography. The set of the measured probabilities, transformed into the Stokes parameters as above, allow a state to be directly reconstructed.

### 2.3 Quantum Process Tomography

In quantum process tomography [11-13], an experimenter lets a specified device act on a quantum state prepared in an input state by his choice, and then performs a measurement on the output state. This procedure is repeated many times for different input states and different measurements, in order to accumulate enough statistics to assign a quantum operation to the device. In fact quantum
process tomography is equivalent to quantum state tomography in a larger state space [50].

In the usual description of process tomography, it is assumed that the device performs the same unknown quantum operation \( \varepsilon \) every time, and an experimenter’s prior information about the device is expressed via a probability density \( p(\varepsilon) \). Here, we restrict our attention to devices for which the input and output have the same Hilbert space dimension \( D \). \( H_D \) denotes a \( D \)-dimensional Hilbert space, \( H_D^{\otimes N} = H_D \otimes \cdots \otimes H_D \) denotes its \( N \)-fold tensor product and \( L(V) \) denotes the space of linear operators on a linear space \( V \). The set of density operators for a \( D \)-dimensional quantum system is a convex subset of \( L(H_D) \). The action of a device on inputs state is then described by a trace-preserving completely positive map:

\[
\varepsilon^{(N)} : L(H_D^{\otimes N}) \to L(H_D^{\otimes N})
\]

(25)

which maps the input state to the output states. In analogy to the definition of exchangeability for quantum states, a quantum operation \( \varepsilon^{(N)} \) is exchangeable if it is a member of an exchangeable sequence of quantum operations.

Any \( N \)-system density operator \( \rho^{(N)} \) can be expanded in the form:

\[
\rho^{(N)} = \sum_{j,k} \rho_{j,k}^{(N)} \otimes |j\rangle \langle k| \tag{26}
\]

where \( \{|j\rangle, \ldots, |k\rangle\} \) denotes an orthonormal basis for the Hilbert space \( H_D \) of the \( i \)-th state, and \( \rho_{j,k}^{(N)} \) are the matrix elements of \( \rho^{(N)} \) in the tensor product basis. We define the action of the permutation \( \pi \) on the state \( \rho^{(N)} \) by:

\[
\pi \rho^{(N)} = \sum_{j,k} \rho_{j,k}^{(N)} \otimes |j\rangle \langle k| = \sum_{j,k} \rho_{j,k}^{(N)} \otimes |j\rangle \langle \pi(k)| \tag{27}
\]

With this notation, we can make the following definition. A sequence of quantum operations, \( \varepsilon^{(k)} : L(H_D^{\otimes k}) \to L(H_D^{\otimes k}) \), is called exchangeable. For \( k = 1, 2, \ldots \),

1) \( \varepsilon^{(k)} \) is symmetric, i.e.,

\[
\varepsilon^{(k)}(\rho^{(i)}) = \pi(\varepsilon^{(i)}(\pi^{-1}(\rho^{(i)})))
\]

(28)

for any permutation \( \pi \) of the set \( \{1, \ldots, k\} \) and for any density operator \( \rho^{(i)} \in L(H_D^{\otimes i}) \).

2) \( \varepsilon^{(k)} \) is extendible, i.e.,

\[
\varepsilon^{(k)}(T_{L_{k+1}}(\rho^{(k+1)})) = T_{L_{k+1}}(\varepsilon^{(k+1)}(\rho^{(k+1)}))
\]

(29)

for any state \( \rho^{(k+1)} \).

In words, these conditions amount to the following. Condition (1) indicates the requirement that the quantum operation \( \varepsilon^{(k)} \) commutes with any permutation operator \( \pi \) acting on the state \( \rho^{(k)} \): It does not matter what order the states are sent through the device; as long as they are rearranged into the original order at the end, the resulting evolution will be the same. Condition (2) says that it does not matter if we consider a larger map \( \varepsilon^{(N+1)} \) acting on a larger collection of states, or a smaller \( \varepsilon^{(N)} \) on some subset of those states: The upshot of the evolution will be the same for the relevant states.

A quantum operation \( \varepsilon^{(N)} : L(H_D^{\otimes N}) \to L(H_D^{\otimes N}) \) is an element of an exchangeable sequence if and only if it can be written in the form:

\[
\varepsilon^{(N)} = \int p(\varepsilon) e^{\otimes N} d\varepsilon \quad \text{for all } N \tag{30}
\]

where the integral ranges over all single-shot quantum operations \( \varepsilon : L(H_D) \to L(H_D) \), \( d\varepsilon \) is a suitable measure on the space of quantum operations, and the probability density \( p(\varepsilon) \geq 0 \) is unique. The tensor product \( e^{\otimes N} \) is defined by \( \rho^{(N)} = e^{\otimes N}(\rho_1 \otimes \cdots \otimes \rho_N) = e(\rho_1) \otimes \cdots \otimes e(\rho_N) \) for all \( \rho_1, \ldots, \rho_N \) and by linear extension for arbitrary arguments.

This result allows certain latitude in how quantum process tomography can be described. One is free to use the language of an unknown quantum operation if the condition of exchangeability is met by one’s prior \( \varepsilon^{(N)} \) but it is not required: in particular, the known quantum operation \( \varepsilon^{(N)} \) is the only meaningful quantum operation in the problem. The proof of the process tomography theorem is specified in this book [6].

### 2.4 Error Analysis [51]

Error analysis of reconstructed density matrices is in practice a non-trivial process. The traditional method of error analysis involves analytically solving for the error in each measurement due to each source of error, then propagating these errors through a calculation of any derived quantity. In the photon case, for example, errors in counting statistics are analyzed [52], giving errors in both density matrices and commonly derived quantities, such as the tangle and the linear entropy. In practice, however, these errors appear to be too large: In [52],
measurements had been repeated many times, and observed a spread in the value of derived quantities which is approximately an order of magnitude smaller than the spread predicted from an analytic calculation of the uncertainty. Thus it is worthwhile to discuss alternate methods of error analysis.

One promising numerical method is the ‘Monte Carlo’ technique, whereby additional numerically simulated data is used to provide a statistical distribution over any derived quantity. Once an error distribution is understood over a single measurement, a set of ‘simulated’ results can be generated. These results are simulated using the known error distributions in such a way as to produce a full set of numerically generated data which could feasibly have come from the same system. Many of these sets of data are numerically generated (at the measured counts level), and each set is used to calculate a density matrix via the maximum likelihood technique [40, 53-56]. This set of density matrices is used to calculate the standard error on any quantity implicit in or derived from the density matrix.

As an example, consider the application of the Monte Carlo technique to the down-conversion results [57]. Two polarization encoded qubits are generated within ensembles that obey Poisson statistics, and these ensembles are used to generate a density matrix using the maximum likelihood technique [40, 55-56]. In order to find the error on a quantity derived from this density matrix, 36 new measurement results are numerically generated, each drawn randomly from a Poisson distribution with mean equal to the original number of counts. These 36 numerically generated results are then fed into the maximum likelihood technique, in order to generate a new density matrix, from which, the tangle may be calculated. This process is repeated many times, generating both many density matrices and a distribution of tangle values, from which the error in the initial tangle may be determined. In practice, additional sets of simulated data must be generated until the error on the quantity of interest converges to a single value.

Clearly, the problem of error analysis in state tomography is an area of continuing research. The development of adaptive tomography techniques could allow both specific measurements and the data collection times to be reduced in order to optimize for each state to be measured. In addition, because the number of measurements necessary to perform grows exponentially with the number of qubits, it will eventually become necessary to partially characterize states with fewer measurements. Finally, each distinct qubit implementation provides a myriad of unique challenges. Nevertheless, the discussions presented here will be useful for characterizing quantum systems in a broad spectrum of qubit realizations.

3. Research on Experiments

In order to verify the correction of the theory, many experimenters began to focus on the research on experiment and there were more and more satisfying results emerging. The theoretical efforts were complemented by a number of important experimental works suggesting the feasibility and importance of quantum tomography.

The first exact technique was given for measuring experimentally the matrix elements of the density operator in the photon-number representation [26] by simply averaging functions of homodyne data. After that, the method was further simplified [27], and the feasibility for non-unit quantum efficiency of detectors above some bounds was established. The exact homodyne method has been implemented experimentally to measure the photon statistics of a semiconductor laser [28], and the density matrix of a squeezed vacuum [29]. The success of optical homodyne tomography has then stimulated the development of state-reconstruction procedures for atomic beams [30], the experimental determination of the vibrational state of a molecule [31], of an ensemble of helium atoms [32], and of a single ion in a Paul trap [33]. In this section some experiments about quantum tomography and quantum process tomography are introduced.

3.1 An Example: Density Matrix Reconstruction in NMR [58-59]

In NMR quantum computation, the liquid ensemble is described by the density matrix. The state of the NMR computer can be obtained by the state-tomography technique [60]. In order to extract the density matrix, for example, for a 2-qubit system, for a 2-qubit system, 18 read-outs have to be performed. In general, for an n-qubit system, construction of the density matrix requires $3^n \times n$ read-outs. After signal readout, the area of the spectrum is integrated and the density matrix is reconstructed through numerical methods. Obviously, the amount of work in experiment is huge when n becomes moderately large. So, in practice, when n is small the scheme is feasible.

In an NMR measurement, each read-out pulse can only give some off-diagonal matrix elements of the density matrix. To obtain the rest of the matrix elements, one has to rotate the original density matrix through rotational operations. For a 2-qubit system, in order to construct the density matrix, one needs to perform the following operations: $I_2$, $I_3$, $I_4$, $I_5$, $I_6$, $I_7$, $I_8$, $I_9$, $I_{10}$, $I_{11}$, $I_{12}$, $I_{13}$, $I_{14}$, $I_{15}$, $I_{16}$, $I_{17}$, $I_{18}$, $I_{19}$, $I_{20}$, $I_{21}$, $I_{22}$, $I_{23}$, $I_{24}$, $I_{25}$, $I_{26}$, $I_{27}$, $I_{28}$, $I_{29}$, $I_{30}$, $I_{31}$, $I_{32}$, $I_{33}$, $I_{34}$, $I_{35}$, $I_{36}$, $I_{37}$, $I_{38}$, $I_{39}$, $I_{40}$, $I_{41}$, $I_{42}$, $I_{43}$, $I_{44}$, $I_{45}$, $I_{46}$, $I_{47}$, $I_{48}$, $I_{49}$, $I_{50}$, $I_{51}$, $I_{52}$, $I_{53}$, $I_{54}$, $I_{55}$, $I_{56}$, $I_{57}$, $I_{58}$, $I_{59}$, $I_{60}$. Here, $I$, $X$, $Y$, and $Z$ stand for, respectively, the identity operation, a 90° rotation about the x-axis, and a 90° rotation about the y-axis.
Thus, in a quantum state tomography, these operations are performed before NMR measurements. Suppose that the nuclear spins of H and P in a phosphorous acid are used for qubits. For a usual NMR system, only one nuclear spin can be measured at a time, the measurement has to be performed separately for the two nuclear spins, H and P. Next, we restart the computation, but this time the operation $IX$ is performed at the required state before measurement. This process is carried out separately for H and P nuclear spins, and the nine operations are successively performed on each of them. This means that $9 \times 2 = 18$ read-outs are acquired. Suppose the density matrix is following form:
The NMR read-out signal can only give $\rho_{12}$ and $\rho_{34}$ from the nuclear spin of P, and $\rho_{13}$ and $\rho_{24}$ from the nuclear spin of H. To obtain other elements in the density matrix, one of the nine operations is performed on the state so that the desired elements are transformed to the positions which can be given by read-out. Altogether, $4 \times 9 \times 2 = 72$ equations with 16 unknowns are obtained:

$$\sum_{i=1}^{16} A_{\alpha i} x_i = B_{\alpha} \quad \{\alpha = 1, 2, 3, \ldots, 72\} \quad (32)$$

namely:

$$AX = B \quad (33)$$

There are certainly redundant expressions in (32) since the number of equations is more than the number of unknowns. The standard way of dealing with this problem is to use the least-square-fitting procedure which is widely used in various problems in science and engineering. We minimize the quantity $\chi^2$ defined as:

$$\chi^2 = \sum_{\alpha} \left( \sum_{i=1}^{16} A_{\alpha i} x_i - B_{\alpha} \right)^2 \quad (34)$$

To find the minimum, a variation procedure on $\chi^2$ with respect to all parameters is carried out, which gives:

$$CX = B^* \quad (35)$$

here, $C = A^T A$, $B^* = A^T B$. According to $UCU^* = C_d$, the formula (35) becomes:

$$C_d Y = B^* \quad (36)$$

Here, $Y = UX$, $B^* = UB^T$, $U$ is the unitary matrix which diagonalizes $C$, $C_d$ is diagonal matrix.[61] So, the equation becomes:

$$y_i = \frac{(B^*)_i}{(C_d)_i} \quad (37)$$

According to $Y$ we can acquire the $Y$. Thus, the density matrix is constructed.

### 3.2 Quantum Process Tomography Applied in Quantum Channel

In quantum communication, when the state $\rho$ passes through the quantum channel the physical transformation can be described by a super-operator $\varepsilon$. The process can be formulated

$$\rho_{\text{out}} = \varepsilon(\rho_{\text{in}}) \quad (38)$$

Basing on the relationship between the input and output the super-operator can be characterized as in Figure 5.

Because of physical reasonability, the super-operator must accord with the following three characters: trace-preserving, positivity, and linearity. Then, (38) can be rewritten

$$\rho_{\text{out}} = \varepsilon(\rho_{\text{in}}) = \sum_i E_i \rho_{\text{in}} E_i^+ \quad (39)$$

where, $\sum_i E_i^i E_i = I$, $E_i$ is the presentation of Kraus operator.

While the operator cannot be measurement directly in physics, the super-operator, $\varepsilon$, which represents the character of quantum channel must be parameterized. The Kraus operator, $E_i$ is represented by a set of basis $d^2$:

$$E_i = \sum_m e_{im} M_m \quad m = 1, 2, \ldots, d^2 - 1 \quad (10)$$

where, $d$ is the dimension of the input quantum state. So we get

$$\rho_{\text{out}} = \varepsilon(\rho_{\text{in}}) = \sum_i E_i \rho_{\text{in}} E_i^+$$

$$= \sum_{mn} e_{im} M_m \rho_{\text{in}} e_{in}^* M_n^+ = \sum_{mn} \alpha_{mn} M_m \rho_{\text{in}} M_n^+ \quad (41)$$

where, $\alpha_{mn} = \sum_i e_{im} e_{in}^*$.

From above format we know that acquiring the matrix $\alpha$ is equivalent to characterizing the quantum channel. Matrix $\alpha$ has different form according to choice of the
basis. For example, we choose \( \{ I, \sigma_x, -i\sigma_y, \sigma_z \} \) as the basis \( \{ M_n \} \); then

\[
\alpha = \Lambda \left[ \begin{array}{ccc}
\alpha(0)\langle 0 | & \alpha(0)\langle 1 | \\
\alpha(1)\langle 0 | & \alpha(1)\langle 1 |
\end{array} \right] \Lambda
\]

(42)

where, \( \Lambda = \frac{1}{2} \left[ \begin{array}{cc} I & \sigma_z \\ \sigma_z & -I \end{array} \right] \), \( d = 2 \).

Let the input states \( \rho_{in} = |k\rangle\langle l| \) \( (k, l = 1, 2, \ldots, d) \) pass through the quantum channel and measure the corresponding output states \( \rho_{out} = e(|k\rangle\langle l|) \). Then the matrix \( \alpha \) can be obtained according to (42). It is noticeable that the density matrix such as \( \rho = |k\rangle\langle l| \) is illegal. Luckily, they can be calculated indirectly because super operator is linear

\[
e(|k\rangle\langle l|) = e(|+\rangle\langle +|) + ie(|-\rangle\langle -|) = \frac{1 + i}{2} e(|k\rangle\langle k|) - \frac{1 + i}{2} e(|l\rangle\langle l|)
\]

\[
e(|l\rangle\langle k|) = e(|+\rangle\langle +|) - ie(|-\rangle\langle -|) = \frac{1 - i}{2} e(|k\rangle\langle k|) - \frac{1 - i}{2} e(|l\rangle\langle l|)
\]

(43)

where, \( |+\rangle = \frac{|k\rangle + |l\rangle}{\sqrt{2}}, \quad |\rangle = \frac{|k\rangle + |l\rangle}{\sqrt{2}} \quad k < l \).

The polarization state of a photon is a natural experimental realization of a two-level quantum system-a qubit. In following section, an experiment scheme in which the polarization state of photon is selected for experimental object is demonstrated. We choose the input states

\[
\rho_{in}^0 = |0\rangle\langle 0|, \quad \rho_{in}^1 = |1\rangle\langle 1|,
\]

(44)

where, \( |0\rangle = (1, \ 0)^T, \quad |1\rangle = (0, \ 1)^T, \quad |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, \quad |\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \).

The density matrix of output state is acquired by following steps:

1) Let a large number of output photons from quantum channel pass through the vertical-polarization waveplate and record the number at back: \( n_0 = N\langle 0 | \rho_{out} | 0 \rangle \);

2) Let a large number of output photons from quantum channel pass through the horizontal-polarization waveplate and record the number at back: \( n_1 = N\langle 1 | \rho_{out} | 1 \rangle \);

3) Let a large number of output photons from quantum channel pass through the left-rotation waveplate and record the number at back: \( n_+ = N\langle + | \rho_{out} | + \rangle \);

4) Let a large number of output photons from quantum channel pass through the right-rotation waveplate and record the number at back: \( n_- = N\langle - | \rho_{out} | - \rangle \).

The density matrix of output photon is reconstructed according to \( n_0, n_1, n_+, n_- \):

\[
\rho_{out} = \frac{n_0}{n_0 + n_1} |0\rangle\langle 0| + i\frac{n_+}{n_0 + n_1} |+\rangle\langle +| - i\frac{n_-}{n_0 + n_1} |-\rangle\langle -|
\]

(45)

by this mean, the corresponding output density matrices for input, \( \rho_{in}^0 = |0\rangle\langle 0|, \quad \rho_{in}^1 = |1\rangle\langle 1|, \quad \rho_{in}^+ = |+\rangle\langle +|, \quad \rho_{in}^- = |-\rangle\langle -| \) are obtained, \( \rho_{out}^0(n_0, n_1, n_0, n_0), \quad \rho_{out}^1(n_0, n_1, n_0, n_0), \quad \rho_{out}^+(n_0, n_1, n_0, n_0), \quad \rho_{out}^-(n_0, n_1, n_0, n_0) \), after that, the needed transforms are made:

\[
\rho_{out}^{0'} = |0\rangle\langle 0| = \rho_{in}^0, \quad \rho_{out}^{1'} = |1\rangle\langle 1| = \rho_{in}^1, \quad \rho_{out}^{+'} = i|+\rangle\langle +| = i\rho_{in}^+, \quad \rho_{out}^{-'} = -i|-\rangle\langle -| = -i\rho_{in}^-
\]

(46)

corresponding transforms on output density matrices:

\[
e(|0\rangle\langle 0|) = \rho_{out}^{0'} = e(\rho_{in}^0) = \rho_{out}^0
\]

\[
e(|1\rangle\langle 1|) = \rho_{out}^{1'} = e(\rho_{in}^1 + i\rho_{in}^+) = -\frac{1 + i}{2}(\rho_{out}^0 + \rho_{out}^1)
\]

(47)

\[
e(|+\rangle\langle +|) = \rho_{out}^{+'} = e(\rho_{in}^- - i\rho_{in}^+) = -\frac{1 - i}{2}(\rho_{out}^0 + \rho_{out}^1)
\]

Finally, the parameter matrix \( \alpha \) on the basis \( \{ I, \sigma_x, -i\sigma_y, \sigma_z \} \) is achieved:

\[
\alpha = \Lambda \left( \begin{array}{cc}
\rho_{out}^{0'} & \rho_{out}^{1'} \\
\rho_{out}^{+'} & \rho_{out}^{-'}
\end{array} \right) \Lambda
\]

(48)

4. Conclusions and Future Challenges

The state of a physical system is the mathematical object that provides complete information on the system. The knowledge of the state is equivalent to know the result of any possible measurement on the system. Quantum tom-
Figure 6. The design of quantum feedback control system using quantum tomography for quantum information acquisition

Quantum tomography offers a method to estimate a generic quantum system from the measurement of a suitable set of observables, a quorum, on repeated preparations of the system.

Quantum state tomography has come of age. Many theoretical methods exist for converting measured results into information for the quantum state. Numerous experiments demonstrating the utility of these theories have been performed. These experiments have used several different detection technologies, and have measured various quantities ranging from Wigner functions to photon number and phase distributions. According to the characters of quantum tomography, there will be some expectant applications as fellow:

4.1 Preparation of Quantum States

For many experiments in quantum theory and quantum information it is very important to develop reliable sources of arbitrary polarization quantum states. Quantum tomography is important for the development of new quantum sources, since the quantum state reconstruction techniques are natural means of calibration and tuning of experimental apparatuses.

4.2 Quantum Information Acquisition and Characterization of Control System in Quantum Control System

In classical control, the information acquisition and feedback are very important concept. But, in quantum control system, the carrier of information is the quantum states and information is encoded in it. The state, however, is not an observable in quantum mechanics [62] and, thus, a fundamental problem arises: the desired transformation is performed on the input state by the quantum system controller, then, the information has to be read out, in other words, the output state has to be determined in order to judge the performance of control system and make new decision for following action. In quantum control system there is another problem: the parameters of controller are not exact, so we must try to find out and modify them according to our demand.

In some ways, quantum tomography can solve above problem. Figure 6 shows that the output state can be determined by quantum tomography that assists to change control strategy better; the parameters of controller are obtained from quantum process tomography at the necessary time and one is able to modify them on his demand. Thus, the quantum system will evolve at the desired direction. The work in this area is really just beginning, and we expect it to have a bright future.

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