Production of $f_0(1710)$, a theoretical endeavor of pure scalar glueball state, is studied in detail from exclusive rare $B$ decay within the framework of perturbative QCD. The branching fraction for $B^\pm \to K^{*\pm} f_0(1710) \to K^{*\pm} (K\bar{K})$ is estimated to be about $8 \times 10^{-6}$, while for $B^\pm \to K^{\pm} f_0(1710) \to K^{\pm} (K\bar{K})$ it is smaller by roughly an order of magnitude. With the accumulation of almost 1 billion $B\bar{B}$ pairs from the BABAR and Belle experiments to date, hunting for a scalar glueball via these rare decay modes should be attainable.
From the modern point of view, properties of pseudoscalar mesons can be understood as Nambu-Goldstone bosons due to the spontaneous symmetry breaking of chiral symmetry. Their low energy dynamics can be described by the chiral lagrangian. On the other hand, scalar mesons are not governed by any low energy symmetry like chiral symmetry and thus they cannot take advantage of the power of a symmetry. Indeed, their $SU(3)$ classification, the quark content of their composition, as well as their spectroscopy are not well understood for scalar mesons [1]. Moreover, possible mixings of the $q\bar{q}$ states with a pure glueball state [2,3,4,5,6,7,8] must be taken into consideration.

Recent quenched lattice simulation [9] predicted the lowest pure glueball state has a mass equals $1710 \pm 50 \pm 80$ MeV and $J^{PC} = 0^{++}$. The first error is statistical while the second is due to approximate anisotropy of the lattice. This suggests that before mixing, a glueball mass should be closed to $1710$ MeV, instead of the earlier lattice result of $1500$ MeV [2]. This makes $f_0(1710)$ a strong candidate for a lowest pure glueball state as advocated in [10] based on argument of chiral suppression in $f_0(1710)$ decays into pair of pseudoscalar mesons. The next two pure glueball states predicted by the quenched approximation [9] have masses $2390 \pm 30 \pm 120$ MeV and $2560 \pm 35 \pm 120$ MeV with $J^{PC} = 2^{++}$ and $0^{-+}$ respectively. Mixings between the nearby three isosinglet scalars $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ and the isovector scalar $a_0(1450)$ have been studied in detail in [2] with the following main result: In the $SU(3)$ symmetry limit, $f_0(1500)$ is a pure $SU(3)$ octet and degenerate with the isovector scalar meson $a_0(1450)$, whereas $f_0(1370)$ is mainly a $SU(3)$ singlet with a small mixing with $f_0(1710)$ which is composed predominantly by a scalar glueball.

Important production mechanism of glueballs is the decay of heavy quarkonium [11,12,13]. In fact, the observed enhancement of the mode $J/\psi \to f_0(1710)\omega$ relative to $f_0(1710)\phi$ and the copious production of $f_0(1710)$ in the radiative $J/\psi$ decays are strong indication that $f_0(1710)$ is mainly composed of glueball [2]. Another interesting mechanism is the direct production from $e^+e^- \to \gamma^* \to G_JH$ [14], where $G_J$ stands for a glueball state of spin $J = 0$ or $2$ and $H$ denotes a $J/\psi$ or $\Upsilon$. Recently, glueball production from inclusive rare $B$ decay [15] has also been studied. Ironically, scalar glueball state has never been observed in the gluon-rich channels of $J/\psi(1S)$ decays or $\gamma\gamma$ collisions\(^1\).

In this article, we will study glueball production via exclusive $B$ decay using perturbative

\(^1\) For a summary of the non-$q\bar{q}$ candidates from the Particle Data Group, see p949 of Ref. [16].
QCD (PQCD). Firstly, we will ignore mixing effects and treat $f_0(1710)$ as a pure scalar glueball suggested by the quenched lattice data. At the end of the paper, we will demonstrate the mixing effects are minuscule. At quark level, the effective Hamiltonian for the decay $b \to sq\bar{q}$ can be written as \[17\]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right],$$

(1)

where $V_q = V_{q\bar{q}}V_{qb}$ denotes the product of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the operators $O_1$-$O_{10}$ are defined as

$$O_1^{(q)} = (\bar{s}_a q_2)\Gamma_{\mu A}(\bar{q}_b b_\alpha)\Gamma_{\nu A}, \quad O_2^{(q)} = (\bar{s}_a q_2)\Gamma_{\mu A}(\bar{q}_b b_\alpha)\Gamma_{\nu A},$$

$$O_3 = (\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A}, \quad O_4 = (\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A},$$

$$O_5 = (\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A}, \quad O_6 = (\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A},$$

$$O_7 = \frac{3}{2}(\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A}, \quad O_8 = \frac{3}{2}(\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A},$$

$$O_9 = \frac{3}{2}(\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A}, \quad O_{10} = \frac{3}{2}(\bar{s}_a b_\alpha)\Gamma_{\mu A} \sum_q (\bar{q}_b q_2)\Gamma_{\nu A},$$

(2)

with $\alpha$ and $\beta$ being the color indices and $C_1$-$C_{10}$ the corresponding Wilson coefficients. In addition, the gluonic penguin vertex for $b(p) \to s(p')g^*(q)$ with next-to-leading QCD corrections is given by \[18\]

$$\Gamma^{\mu a} = -\frac{G_F}{\sqrt{2}} \frac{g_s}{4\pi^2} V_{tb} V_{sb} \bar{s}(p') [\Delta F_1(q^2\gamma^\mu - q^\mu \gamma) L - i m_b F_2 \sigma^{\mu\nu} q_\nu] T^a b(p),$$

(3)

where $g_s$ is the strong coupling constant, $m_b$ is the $b$-quark mass, $T^a$ is the generator for the color group, and $L(R) = (1 + \gamma_5)/2$, $\Delta F_1 = 4\pi(C_4^{\text{eff}}(q, \mu) + C_6^{\text{eff}}(q, \mu))/\alpha_s(\mu)$ and $F_2 = -2C_{8g}^{\text{eff}}(\mu)$. Explicit formulas for $C_4^{\text{eff}}, C_6^{\text{eff}}$, and $C_{8g}^{\text{eff}}$ can be found in Ref. \[19\]. Since the ground state scalar glueball is composed of two gluons, the effective interaction between a scalar glueball and gluons can be written as \[10\]

$$\mathcal{L}_{\text{eff}} = f_0 G_0 G_0^a G_0^{a\mu\nu},$$

(4)

where $f_0$ stands for an unknown effective coupling constant, $G_0$ denotes the scalar glueball field, and $G_0^{a\mu\nu}$ is the gluon field strength tensor. With these 4-quarks operators $O_1 - O_{10}$ \[2\] and the two effective couplings \[3\] and \[4\], we can embark upon the computation of the
decay rates for $B \rightarrow K^{(*)}G_0$ using PQCD. The flavor diagrams for $B \rightarrow K^{(*)}G_0$ decays are displayed in Fig. 1. Fig.(1a) denotes contribution from the 4-quarks operators $O_{1-10}$ given in Eq.(2), whereas Fig.(1b) involves the gluonic penguin vertex contribution of Eq.(3). Both diagrams are of the same order in $\alpha_s$. In the heavy quark limit, the production of light meson is supposed to respect color transparency [20], i.e., final state interactions are subleading effects and negligible. We will work under this assumption in what follows. Moreover, diagrams like Fig.2 that are of higher order in $\alpha_s$ will be ignored.

![Flavor diagrams for the $B \rightarrow K^{(*)}G_0$.](image1)

**FIG. 1:** Flavor diagrams for the $B \rightarrow K^{(*)}G_0$.

![Other flavor diagrams for the $B \rightarrow K^{(*)}G_0$ at higher order in $\alpha_s$.](image2)

**FIG. 2:** Other flavor diagrams for the $B \rightarrow K^{(*)}G_0$ at higher order in $\alpha_s$.

To deal with the transition matrix elements for exclusive $B$ decays, we employ PQCD [21, 22] factorization formalism to estimate the hadronic effects. By the factorization theorem, the transition amplitude can be written as the convolution of hadronic distribution amplitudes and the hard amplitude of the valence quarks, in which the distribution amplitudes absorb the infrared divergences and represent the effects of nonperturbative QCD. As usual, the hard amplitudes can be calculated perturbatively by following the Feynman rules. The nonperturbative objects can be described by the nonlocal matrix elements and
are expressed as [23, 24, 25]

\[
\begin{align*}
\int \frac{d^4z}{(2\pi)^4} e^{-ik \cdot z} \langle 0 | \bar{\psi}_B(0) q_\alpha(z) | B(p_B) \rangle &= -\frac{i}{\sqrt{2N_c}}[(\not{p}_B + m_B)\gamma_5]_{\alpha\beta} \phi_B(k), \\
\int \frac{d^4z}{(2\pi)^4} e^{-ixp_K \cdot z} \langle K(p_K) | \bar{q}_\beta(z) s_\alpha(0) | 0 \rangle &= -\frac{i}{\sqrt{2N_c}} \left\{ [\gamma_5 \not{p}_K]_{\alpha\beta} \phi_K(x) + [\gamma_5]_{\alpha\beta} m_0^0 K^\beta(x) + m_0^0 \gamma_5 (\not{n}_+ - \not{n}_-) \right\} \phi_K(x), \\
\int \frac{d^4z}{(2\pi)^4} e^{-ixp_{K^*} \cdot z} \langle K^*(p_{K^*}, \epsilon_L) | \bar{q}_\beta(z) s_\alpha(0) | 0 \rangle &= \frac{1}{\sqrt{2N_c}} \left\{ m_{K^*} [\not{\epsilon}_L]_{\alpha\beta} \phi_{K^*}(x) + [\not{\epsilon}_L, \not{K}^\beta]_{\alpha\beta} \phi_{K^*}(x) + m_{K^*} [1]_{\alpha\beta} \phi_{K^*}(x) \right\}
\end{align*}
\]

for $B$, $K$, and $K^*$ mesons, respectively, where $N_c$ is the number of color, $n_\pm$ are two light-like vectors satisfying $n_+ \cdot n_- = 2$, and $\epsilon_L$ is the longitudinal polarization vector of $K^*$. $\phi_B(x, b)$, the distribution amplitude of $B$ meson, is constructed as follows [25]

\[
\phi_B(x, b) = \int dk^+d^2k_\perp e^{ik_\perp \cdot \mathbf{b}} \phi_B(k)
\]

with $x = k^-/p_B^+$. $\phi_{K^*}(x)$ and $\phi_{K^*}^{\sigma_\beta}(x)$ are the twist-2 and 3 distribution amplitudes of $K^*$ mesons with the argument $x$ stands for the momentum fraction. Finally, $m_B$ and $m_{K^*}$ are the masses for the $B$ and $K^*$ with $m^0_{K^*} = m^2_{K^*}/(m_q + m_s)$ where $m_q$ and $m_s$ denote the light quark masses. The meson distribution amplitudes are subjected to the following normalization conditions

\[
\int_0^1 dx \phi_B(K^*) = \frac{f_B(K^*)}{2\sqrt{2N_c}}, \quad \int_0^1 dx \phi^{\sigma_\beta}_{K^*}(x) = \frac{f^{(T)}_{K^*}}{2\sqrt{2N_c}}, \quad \int_0^1 dx \phi^{\sigma_\beta}_{K^*}(x) = 0
\]

where $\phi_B(x) = \phi_B(x, 0)$ and $f_B(K^*)$ and $f^{(T)}_{K^*}$ are the decay constants. We do not introduce transverse momenta for the light mesons $K$ and $K^*$ here which we will justify later when we discuss the end-point singularities of the decay amplitudes.

In the light-cone coordinate system, we can pick the two light-like vectors to be $n_+ = (1, 0, 0_\perp)$ and $n_- = (0, 1, 0_\perp)$, and the momenta of the $B$ and $K$ mesons can be written as

\[
p_B = \frac{m_B}{\sqrt{2}} (1, 1, 0_\perp), \quad p_K = \frac{m_B}{\sqrt{2}} (1 - r^2_{G_0})(1, 0, 0_\perp),
\]

with $r_{G_0} = m_{G_0}/m_B$. For the vector meson $K^*$, we take

\[
p_{K^*} = \frac{m_B}{\sqrt{2}} (1 - r^2_{G_0}, r^2_{K^*}, 0_\perp), \quad \epsilon_L = \frac{1}{\sqrt{2r_{K^*}}}(1 - r^2_{G_0}, -r^2_{K^*}, 0_\perp),
\]

with $r_{K^*} = m_{K^*}/m_B$ in which the physical condition $\epsilon_L \cdot p_{K^*} = 0$ is satisfied for massive vector particle. The momenta of the spectator quarks with their transverse momenta included are
given by

\[ k_1 = \left( 0, \frac{m_B}{\sqrt{2}} x_1, \vec{k}_{1\perp} \right), \quad k_2 = \left( \frac{m_B}{\sqrt{2}} (1 - r_{C_0}^2) x_2, 0, \vec{k}_{2\perp} \right). \]  \hspace{1cm} (10)

With these light-cone coordinates and distribution amplitudes defined, we can study the transition matrix elements for \( B \to MG_0 \) (\( M = K, K^* \)). We first analyze Fig. 1(a). Within the PQCD approach, we find that Fig. 1(a) is directly proportional to \( x_1 \). Since \( x_1 \) is the momentum fraction of the valence quark inside the \( B \) meson and its value is expected to be \( x_1 \approx \bar{\Lambda}/m_B \ll 1 \) with \( \bar{\Lambda} = m_B - m_b \). Comparing to \( x_2 \sim O(1) \) (Fig. 1(b)), its contribution belongs to higher power in heavy quark expansion. As an illustration, we can use the operator \( O_4 \) in Eq. (2) to demonstrate this effect. Thus, one finds

\[
M_{O_4} \propto \frac{4 f_0 g_2^2 C_F}{\sqrt{2}} \sqrt{N_c f_K m_B^6} \int dx_1 dx_2 \frac{d\vec{k}_{1\perp}}{(2\pi)^2} \frac{d\vec{k}_{2\perp}}{(2\pi)^2} \left( 1 - \frac{m_{G_0}^2}{m_B^2} \right) (2 - x_2) x_2 x_1 \phi_B(x_1, \vec{k}_{1\perp}) \times \frac{1}{(m_{G_0}^2 - m_B^2 x_1) - |\vec{k}_{1\perp}|^2} \cdot \frac{1}{m_B^2 x_1 x_2 - |\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2} \cdot \frac{1}{m_{G_0}^2 (1 - x_2) - |\vec{k}_{2\perp}|^2 |\vec{k}_{2\perp}|^2},
\]  \hspace{1cm} (11)

where \( C_F = (N_c^2 - 1)/(2N_c) \). It has been shown in [25] that under Sudakov suppression arising from \( k_{\perp} \) and threshold resummations, the average transverse momenta of valence quarks are \( \langle k_{\perp} \rangle \sim 1.5 \) GeV and the end point singularities at \( x_{1,2} \to 0 \) in Eq.(11) can be effectively removed. With an explicit factor of \( x_1 \) appearing in the numerator of Eq.(11), this contribution is regarded as a higher power effect in \( 1/m_B \) and therefore can be neglected. We note that this situation is quite similar to the flavor singlet mechanism to the \( B \to \eta' \) form factor [26]. According to the PQCD analysis in Ref. [27], contribution from the possible gluonic component inside \( \eta' \) to the \( B \to \eta' \) form factor also has similar behavior. Its numerical value is two orders of magnitude smaller than the \( B \to \pi \) form factor. Similarly, other operators \( O_{1-3} \) and \( O_{5-10} \) give the same behavior. Therefore, to the leading power in \( \Lambda_{QCD}/m_B \), the contributions from Fig. 1(a) can be neglected. We will concentrate on the contribution of Fig. 1(b) in what follows.

By using the introduced nonlocal matrix elements for mesons and the light-cone coordinates given above, the transition matrix element for \( B \to MG_0 \) (\( M = K, K^* \)) can be obtained from Fig. 1(b) as

\[
A_M = \frac{G_F m_B^3}{\sqrt{2}} V_{tb} V_{tb}^* \mathcal{M}_M
\]  \hspace{1cm} (12)
with the decay amplitude function \( \mathcal{M}_M \) given by

\[
\mathcal{M}_M = \frac{m_B}{\pi} f_0 C_F \int_0^{\infty} b db \int_0^1 dx_1 \int_0^1 dx_2 \phi_B(x_1, b) \times x_2 \left\{ e_M^{(1)} \phi_M(x_2) + e_M^{(2)} \phi_M(x_2) + e_M^{(3)} \phi_M(x_2) \right\} E(t) h(x_1, x_2, b) 
\]

(13)

\[
e_K^{(1)} = \Delta F_1(t)(1 - r_G^2)[1 + 2r_G^2 + 2(1 - r_G^2)x_2] - 3r_b(1 - r_G^2)F_2(t),
\]

\[
e_K^{(2)} = 3r_K[-2\Delta F_1(t)(r_G^2 + (1 - r_G^2)x_2) + r_bF_2(t)(1 + r_G^2 + (1 - r_G^2)x_2)],
\]

\[
e_K^{(3)} = r_b r_K(1 - r_G^2)(1 - x_2)F_2(t),
\]

(14)

for the pseudoscalar \( K \), and

\[
e_K^{(1)} = e_K^{(1)}, \quad e_K^{(2)} = \frac{r_K^*}{r_K}(1 - r_G^2)e_K^{(3)}, \quad e_K^{(3)} = \frac{r_K^*}{r_K} e_K^{(2)},
\]

(15)

for the vector meson \( K^* \). Here we have introduced the dimensionless variables \( r_b = m_b/m_B \), \( r_K = m_K/m_B \), and \( r_{K^*} = m_{K^*}/m_B \). The hard function \( h(x_1, x_2, b) \) in Eq. (13) is given by

\[
h(x_1, x_2, b) = \frac{1}{X_{12} + Y_{12}} \left[ K_0 \left( \sqrt{m_B^2 Y_{12}} b \right) - i \frac{\pi}{2} H_b^{(1)} \left( \sqrt{m_B^2 X_{12}} b \right) \right]
\]

(16)

with \( X_{12} = (1 - x_1)[r_G^2 + (1 - r_G^2)x_2] \) and \( Y_{12} = (1 - r_G^2)x_1 x_2 \). The evolution factor \( E(t) \) in Eq. (13) is defined by

\[
E(t) = \alpha_s(t)e^{-S_B(t) - S_K(t)},
\]

(17)

where \( \exp(-S_{B(K)}) \) is the Sudakov exponents that resummed large logarithmic corrections to the \( B(K) \) meson wave functions \[28, 29\]. Their explicit forms are given by

\[
S_B(t) = s(x_1 p_B^+, b) + \frac{5}{3} \int_1^t \frac{d\mu}{\mu} \gamma(\alpha_s(\mu)),
\]

\[
S_K(t) = s(x_2 p^+, b) + s((1 - x_2)p^+, b) + 2 \int_1^t \frac{d\mu}{\mu} \gamma(\alpha_s(\mu)),
\]

(18)

where \( \gamma(\alpha_s(\mu)) \) is the anomalous dimension. To leading order in \( \alpha_s \), \( \gamma(\alpha_s(\mu)) \) equals \(-\alpha_s/\pi\).

The function \( s(Q, b) \) in Eq. (18) is given by \[30, 31\]

\[
s(Q, b) = \int_{1/b}^Q \frac{d\mu}{\mu} \left[ \ln \left( \frac{Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right],
\]

(19)

where

\[
A = C_F \frac{\alpha_s}{\pi} + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} f + \frac{2}{3} \beta_0 \ln \left( \frac{e^{\gamma_E}}{2} \right) \right] \left( \frac{\alpha_s}{\pi} \right)^2,
\]

\[
B = \frac{2}{3} \left( \frac{\alpha_s}{\pi} \right) \ln \left( \frac{e^{2\gamma_E - 1}}{2} \right)
\]

(20)
with $f = 4$ being the active flavor number and $\gamma_E$ is the Euler constant. As mentioned before, $x_1 \approx \bar{\Lambda}/m_B \ll 1$, we have dropped all terms related to $x_1$ in the above expressions for $\{e_M^{(i)}\}$. Since $r_{K^*} \ll 1$, we have retained only those terms in the above formulas for $\{e_M^{(i)}\}$ that are at most linear in $r_{K^*}$. The scale $t$ where the strong coupling $\alpha_s(t)$ in (17), the Sudakov exponents in (18), and the $\Delta F_1(t)$ and $F_2(t)$ in (14) are evaluated will be discussed later. For comparison, we also present the formula of the decay amplitude function $\mathcal{M}_M$ with $k_\perp = 0$ in Appendix A.

For estimating our numerical results, we take the values of theoretical parameters to be: $f_B = 190$ MeV, $m_b = 4.4$ GeV, $(m_B, m_K, m_{K^*}, m_{G_0}) = (5.28, 0.493, 0.892, 1.71)$ GeV, $V_{ts}V_{tb}^* = -0.041$. For the $B$ meson distribution amplitude, we use \[ \phi_B(x_1, b) = N_B x_1^2 (1 - x_1)^2 \exp \left[ -\frac{1}{2} \left( \frac{m_B x_1}{\omega_B} \right)^2 - \frac{1}{2} \omega_B^2 b^2 \right] \tag{21} \]
with $N_B = 111.2$ GeV and $\omega_B = 0.38$ GeV. For the distribution amplitudes of the light pseudoscalar $K$ and vector mesons $K^*$, we refer to their results derived by the light-cone QCD sum rules in [32, 33, 34]. Their explicit expressions and relevant values of parameters are collected in the Appendix B for convenience.

According to the results of light-cone QCD sum rules, at small $x_2$, the behavior of twist-2 distribution amplitude obeys the asymptotic form $\phi_{K^*}(x_2) \propto x_2(1 - x_2)$, whilst those of twist-3 distribution amplitudes approach a constant $\phi_{K^*}^{p, \sigma}(x_2) \propto \text{const.}$ Consequently, at small $x_2$, the decay amplitude function contributed by the twist-2 distribution amplitudes of $K^*$ behaves like
\[
\mathcal{M}_{K^*}^{tw2} \propto \frac{x_2 \phi_B(x_1) \phi_{K^*}(x_2)}{k^2 q^2} \propto \frac{x_2 x_1^2 (1 - x_1)^2 x_2 (1 - x_2)}{x_1 x_2 (r_{G_0}^2 + (1 - r_{G_0}^2) x_2)} = \frac{x_1 (1 - x_1)x_2 (1 - x_2)}{(r_{G_0}^2 + (1 - r_{G_0}^2) x_2)}. \tag{22} \]

Obviously, even if one sets $r_{G_0}$ to be zero, the effects from twist-2 distribution amplitudes of $K^*$ are well-defined at the end point $x_2 \to 0$. Similarly, the contribution from twist-3 distribution amplitudes to the decay amplitude function at small $x_2$ behaves like
\[
\mathcal{M}_{K^*}^{tw3} \propto \frac{x_2 \phi_B(x_1) \phi_{K^*}^{p, \sigma}(x_2)}{k^2 q^2} \propto \frac{x_2 x_1^2 (1 - x_1)^2}{x_1 x_2 (r_{G_0}^2 + (1 - r_{G_0}^2) x_2)} = \frac{x_1 (1 - x_1)^2}{(r_{G_0}^2 + (1 - r_{G_0}^2) x_2)}. \tag{23} \]

Whence $r_{G_0} \to 0$, one will suffer logarithmic divergences from the twist-3 distribution amplitudes. In practice, $r_{G_0} \sim 0.32$, the divergence will not occur. This implies that the influence of $k_\perp$ can only be mild. As a common practice, we do not introduce transverse momenta for the valence quarks to suppress large effects from end point singularities.
Since the Wilson coefficients are $\mu$ scale dependence, for smearing its dependence, we include the values of Wilson coefficients with the next-to-leading QCD corrections \[19\]. However, even so, the $C_{4,6,8g}^{\text{eff}}$ are still slightly $\mu$-dependence. Due to this reason, determination of the scale of exchanged hard gluons in Fig. is also one of the origins of theoretical uncertainties. For the gluon that attached to the penguin vertex $b \to sg^*$, it carries a typical momentum of $\sqrt{q^2} = m_B ((1 - x_1)(r_{G_0}^2 + (1 - r_{G_0}^2)x_2)^{1/2}$. When $x_1 \sim \bar{\Lambda}/m_b$ and $x_2$ is $O(1)$, say $x_2 = 0.5$, we get $\sqrt{q^2} \sim 3.9$ GeV. However, the gluon attached to the spectator quark carries roughly a typical momentum of $\sqrt{-k^2} = m_B ((1 - r_{G_0}^2)x_1x_2)^{1/2} \sim 1.4$ GeV. We note that a suitable range of $x_2$ in PQCD is often taken as $\sim 0.3 - 0.7$. For definiteness, we take the democratic average value $t = (\sqrt{q^2} + \sqrt{-k^2})/2$ as the hard scale, in which the allowed value is within the range $t \approx 2.45 \pm 0.45$ GeV. This justifies somewhat the validity of the PQCD approach and we will take this range of $t$ as our theoretical uncertainties. For illustration, we present the involving Wilson coefficients at different values of $\mu$ scale in Table I.

| Wilson coefficient | $\mu = 2.1$ GeV | 2.5 GeV | 3.0 GeV |
|--------------------|------------------|---------|---------|
| $C_{4}^{\text{eff}}$ | $-(6.17 + 0.78i) \times 10^{-2}$ | $-(5.80 + 0.89i) \times 10^{-2}$ | $-(5.48 + 0.89i) \times 10^{-2}$ |
| $C_{6}^{\text{eff}}$ | $-(7.69 + 0.78i) \times 10^{-2}$ | $-(7.19 + 0.89i) \times 10^{-2}$ | $-(6.77 + 0.89i) \times 10^{-2}$ |
| $C_{8g}^{\text{eff}}$ | $-0.170$ | $-0.165$ | $-0.161$ |

Effective interactions between a scalar glueball $G_0$ and the pseudoscalars have been studied using chiral perturbation theory \[15, 35\]. By using the current experimental data \[16\] $\Gamma_{\text{total}}(f_0(1710)) = 137 \pm 8$ MeV and $\text{BR}(f_0(1710) \to K\bar{K}) = 0.38^{+0.09}_{-0.10}$, this allows us to get an estimate of the unknown coupling $f_0 = 0.07^{+0.009}_{-0.018}$ GeV$^{-1}$ \[15\]. This result of $f_0$ should be taken as a crude estimation. For one thing, the data of the branching ratio $\text{BR}(f_0(1710) \to K\bar{K})$ was not used for averages, fits, limits, etc. by the PDG \[16\]. Instead the following two ratios were used in the PDG analysis:

$$R_{\eta/K} \equiv \frac{\Gamma(f_0(1710) \to \eta\eta)}{\Gamma(f_0(1710) \to K\bar{K})} = 0.48 \pm 0.15,$$

(24)

$$R_{\pi/K} \equiv \frac{\Gamma(f_0(1710) \to \pi\pi)}{\Gamma(f_0(1710) \to K\bar{K})} < 0.11.$$

(25)

Within the approach of chiral perturbation theory \[15\], it would be difficult to accommodate
these two ratios of Eqs. (24) and (25), since the leading term in the chiral Lagrangian is flavor blind. Here we will present another approach to estimate $f_0$. At quark level, the amplitude for $G_0 \to q\bar{q}$ is proportional to the quark mass $m_q$ and therefore chirally suppressed. Its explicit form is given by \[ A(G_0 \to q\bar{q}) = -f_0 \alpha_s \frac{16\pi\sqrt{2}m_q}{3} \beta \ln \left( \frac{1 + \beta}{1 - \beta} \right) \bar{u}_q v_q, \] (26)

where $\beta$ denotes the velocity of the quark and $u_q (v_q)$ is the quark (anti-quark) spinor. It has been argued in \[10\] that the chiral suppression of the amplitude $A(G_0 \to q\bar{q}) \propto m_q$ persist in all order of $\alpha_s$. One may treat the coefficient of this decay amplitude as the short-distance coefficient of the strong decay $G_0 \to PP$ where $P$ stands for a pseudoscalar meson like $\pi$, $K$, or $\eta$ etc, as illustrated in Fig. 3 Thus,

$$
\langle PP|H_{\text{eff}}|G_0\rangle = -f_0 m_q Y_{PP}(m_{G_0}^2),
$$
(27)

with, to leading order in $\alpha_s$,

$$
Y = \alpha_s(m_{G_0}^2) \frac{16\pi\sqrt{2}}{3} \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right),
$$
(28)

and $F_{PP}(m_{G_0}^2)$ is the time-like form factor $\langle PP|\bar{q}q|0\rangle$ evaluated at $Q^2 = m_{G_0}^2$. For the case $P = \eta$, we include the quark-flavor mixing effect according to $\eta = \cos \phi \eta_q - \sin \phi \eta_s$ and $\eta' = \sin \phi \eta_q + \cos \phi \eta_s$ with $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\eta_s = s\bar{s}$ \[36, 37\], and $\phi = 41.4^o$ \[38\]. Using Eq. (27), the following ratios of the partial decay rates can be obtained

$$
R_{\pi/K} = \frac{\Gamma(G_0 \to \pi\pi)}{\Gamma(G_0 \to KK)} = \frac{3 (m_u + m_d)^2 |F^\pi\pi(m_{G_0}^2)|^2}{8 m_s^2 m_{G_0}^2} \frac{(1 - 4m_s^2/m_{G_0}^2)^{1/2}}{1 - 4m_K^2/m_{G_0}^2}^{1/2},
$$

$$
R_{\eta/K} = \frac{\Gamma(G_0 \to \eta\eta)}{\Gamma(G_0 \to KK)} = \frac{(1 - 4m_\eta^2/m_{G_0}^2)^{1/2}}{(1 - 4m_K^2/m_{G_0}^2)^{1/2}} \times \frac{|(m_u + m_d) \cos^2 \phi F^{\eta\eta}(m_{G_0}^2)/2 + m_s \sin^2 \phi F^{\eta\eta}(m_{G_0}^2) |^2}{2m_s^2 |F^{KK}(m_{G_0}^2)|^2}. \tag{29}
$$
By taking the flavor SU(3) approximation, one finds that \( F^{\pi\pi}/F^{KK} \approx f_{\pi}^2/f_K^2 \), \( F^{\eta\eta}/F^{KK} \approx f_{\eta}^2/f_K^2 \), and \( F^{\eta\eta}/F^{KK} \approx f_{\eta}^2/f_K^2 \). With \( m_u = m_d = 10 \), \( m_s = 120 \), \( f_\pi = 130 \), \( f_K = 160 \), \( f_q = 140 \), \( f_s = 180 \) (all in unit of MeV), one deduces

\[
R_{\pi/K} = 0.006, \quad R_{\eta/K} = 0.37.
\]

Identifying \( G_0 \) to be \( f_0(1710) \), these ratios are consistent with the current experimental data quoted in Eqs. (24) and (25). Using Eq. (26), the following expression of \( f_0 \) can be obtained

\[
f_0^2 = \frac{8\pi m_G \Gamma_{G_0}}{|F^{KK}(m_G^2) m_s Y|^2} \left( 1 - \frac{4m_F^2}{m_G^2} \right)^{1/2} \text{BR}(G_0 \rightarrow KK),
\]

where \( \Gamma_{G_0} = 137 \pm 8 \) MeV is identified as the width of \( f_0(1710) \). The time-like form factor \( F^{KK}(m_G^2) \) has been extracted in Ref. [39] by performing the data fitting to non-resonant \( B \rightarrow KKK \) decays with the following form

\[
F^{KK}(Q) = \frac{v}{3} \left( 3F^{(1)}_{NR} + 2F^{(2)}_{NR} \right) + \sigma_{NR} \exp(-\alpha_{NR}Q^2),
\]

\[
F^{(n)}_{NR} = \left( \frac{x_1^{(n)}}{Q^2} + \frac{x_2^{(n)}}{Q^4} \right) \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-1},
\]

where \( v = (m_K^2 - m_{\pi}^2)/(m_s - m_d) \), \( \Lambda = 0.3 \) GeV, \( x_1^{(1)} = -3.26 \) GeV\(^2\), \( x_2^{(1)} = 5.02 \) GeV\(^4\), \( x_1^{(2)} = 0.47 \) GeV\(^2\), \( x_2^{(2)} = 0 \), \( \sigma_{NR} = 4.4e^{i\pi/4} \) GeV, and \( \alpha_{NR} = 0.13 \) GeV\(^{-2}\). By using \( \text{BR}(G_0 \rightarrow KK) = 0.38^{+0.09}_{-0.19} \) [16], the value for \( f_0 \) is estimated to be \( f_0 = 0.086^{+0.010}_{-0.020} \), which is only slightly larger than the value obtained from the chiral Lagrangian approach.

In passing, we note that, using light-cone distribution amplitudes, it has been argued in Ref. [35] that \( G_0 \rightarrow \pi\pi, K\bar{K} \) might be dominated by the 4-quark process of \( G_0 \rightarrow q\bar{q}q\bar{q} \) which is not chirally suppressed. Using this 4-quark mechanism and PQCD factorization scheme, one would predict a large ratio of \( R_{\pi/K} \approx (f_\pi/f_K)^4 \approx 0.48 \). For further discussion of this issue, we refer our reader to Refs. [35, 40, 41].

Using the matrix element defined by Eq. (13) with the above chosen values of parameters, the values of \( \mathcal{M}_{K^{(*)}} \) are given in Table III for \( f_0 = 0.086 \) GeV\(^{-1}\) and three different values of \( \mu \) scale. For comparisons, we also present the results with \( k_\perp = 0 \) in Table III.

The branching fractions for \( B^+ \rightarrow (K^+, K^{*+})G_0 \) decays are tabulated in Table III. From Table III we find that the branching fraction for \( B^+ \rightarrow K^{*+}G_0 \) is about one order of magnitude larger than that for \( B^+ \rightarrow K^+G_0 \). The difference arises not only from the values of the decay constants \( f_K \) and \( f_{K^*} \) entered in the distribution amplitudes, but also from the
TABLE II: Decay amplitude $M_M$ (in units of $10^{-4}$) for $B^+ \rightarrow (K^+, K^{*+})G_0$ with and without $k_\perp$ at $f_0 = 0.086$ GeV$^{-1}$ and three different choices of $\mu = 2.1$, 2.5, and 3.0 GeV. Numbers given in brackets are without $k_\perp$.

| Mode    | $\mu = 2.1$ GeV | 2.5 GeV | 3.0 GeV |
|---------|------------------|---------|---------|
| $K^+G_0$ | $-3.54 - 0.42i$  | $-3.34 - 0.44i$ | $-3.22 - 0.48i$ |
|         | ($-3.51 - 0.38i$) | ($-3.28 - 0.41i$) | ($-3.08 - 0.43i$) |
| $K^{*+}G_0$ | $-11.13 - 1.51i$ | $-12.56 - 1.51i$ | $-12.40 - 1.70i$ |
|         | ($-10.90 - 1.17i$) | ($-10.18 - 1.25i$) | ($-9.60 - 1.33i$) |

The effects of $e_K^{(2)}\phi_K^p(x_2)$ and $e_K^{(3)}\phi_K^p(x_2)$ in the $K^+G_0$ mode, which are switched to $e_K^{(2)}\phi_K^p(x_2)$ and $e_K^{(3)}\phi_K^p(x_2)$ respectively in the $K^{*+}G_0$ mode. We also find that the $k_\perp$ influence on $B^+ \rightarrow K^{*+}G_0$ decay is stronger than $B^+ \rightarrow K^+G_0$. In addition, when $\mu$ is smaller, $k_\perp$ has lesser effects on the decay $B^+ \rightarrow K^{*+}G_0$.

TABLE III: Branching fractions (in units of $10^{-6}$) for $B^+ \rightarrow (K^+, K^{*+})G_0$ with and without $k_\perp$ at $f_0 = 0.086$ GeV$^{-1}$ and three different choices of $\mu = 2.1$, 2.5, and 3.0 GeV. Numbers given in brackets are without $k_\perp$.

| Mode    | $\mu = 2.1$ GeV | 2.5 GeV | 3.0 GeV |
|---------|------------------|---------|---------|
| $K^+G_0$ | 3.05             | 2.72    | 2.53    |
|         | (2.99)           | (2.62)  | (2.34)  |
| $K^{*+}G_0$ | 29.07           | 35.94   | 36.06   |
|         | (26.50)          | (23.21) | (20.69) |

The branching fractions for the decay chains $B^+ \rightarrow K^+G_0 \rightarrow K^+(K\bar{K})G_0$ and $B^+ \rightarrow K^{*+}G_0 \rightarrow K^+(K\bar{K})G_0$ are tabulated in Table IV where the errors are coming from the experimental data of BR($f_0(1710) \rightarrow K\bar{K}$). From Table IV we learn that one has a better chance to look for the ground state of glueball through the three-body decays $B \rightarrow K^*KK$, since its branching fraction could be more than a factor of 10 larger than $B \rightarrow KKK$. Recently, Babar had reported the following branching ratio for $B^\pm \rightarrow (K^+K^-)K^\pm$ where the $(K^+K^-)$ pair coming from the $f_0(1710)$ \cite{42}

$$
BR(B^\pm \rightarrow (K^+K^-)f_0(1710)K^\pm) = (1.7 \pm 1.0 \pm 0.3) \times 10^{-6}.
$$

(33)
TABLE IV: Branching fractions (in units of $10^{-6}$) for $B^+ \to (K^+, K^{*+})(K\bar{K})_{G_0}$ at $\mu = 2.1, 2.5,$ and 3.0 GeV. Numbers given in brackets are without $k_\perp$.

| Mode          | $\mu = 2.1$ GeV | $2.5$ GeV | $3.0$ GeV |
|---------------|-----------------|-----------|-----------|
| $K^+(K\bar{K})_{G_0}$ | $1.16^{+0.63}_{-0.88}$ | $1.03^{+0.56}_{-0.78}$ | $0.96^{+0.52}_{-0.73}$ |
|              | $(1.13^{+0.62}_{-0.85})$ | $(1.00^{+0.53}_{-0.76})$ | $(0.89^{+0.48}_{-0.67})$ |
| $K^{*+}(K\bar{K})_{G_0}$ | $11.05^{+5.98}_{-8.36}$ | $13.66^{+7.39}_{-10.34}$ | $13.70^{+7.42}_{-10.37}$ |
|              | $(10.07^{+5.45}_{-7.62})$ | $(8.81^{+4.77}_{-6.66})$ | $(7.86^{+4.20}_{-5.96})$ |

From the first and second rows in Table IV, identifying $G_0$ to be $f_0(1710)$, one can see that our predictions are consistent with the experimental data.

Before we close, we want to address the issue of mixing effects. Although we have treated $f_0(1710)$ as a pure gluonic state, it should be interesting to consider its mixing effects with other $q\bar{q}$ states. To deal with the mixture of a pure glueball with the $q\bar{q}$ quarkonia state, we follow Ref. [2] to express the $f_0(1710)$ state as the following combination

$$|f_0(1710)\rangle = C_N|N\rangle + C_S|S\rangle + C_G|G\rangle \quad (34)$$

where $|G\rangle$ is the pure glueball state, $|N\rangle = (u\bar{u} + d\bar{d})/\sqrt{2}$, and $|S\rangle = s\bar{s}$. Accordingly to one of the mixing schemes [2], the coefficients took the following values: $C_N = 0.32$, $C_S = 0.18$, and $C_G = 0.93$. The quoted results of these coefficients are similar to those obtained by others in Refs. [6, 43]. The corresponding flavor diagrams for the decays $B \to K^{(*)}(N, S)$ are shown in Fig. 4. Since the distribution amplitude and decay constant for $f_0(1710)$ are uncertain, for simplicity, we use factorization assumption to estimate the hadronic effects for these two-body $B$ decays. In terms of the operators in Eq. (2), one can easily show that the contributions from diagram Fig. 4(a) and (d) are associated with the matrix element $\langle N(S)|\bar{q}\gamma_\mu q|0\rangle$. Since the scalar $N$ or $S$ is $C$-even while $\bar{q}\gamma_\mu q$ is $C$-odd, the contributions from Fig. 4(a) and (d) must vanish because charge conjugation is a good quantum number in strong interaction. On the other hand, the contributions from Fig. 4(b) and (c) are
FIG. 4: Flavor diagrams for the $B \to K^{(*)}(N, S)$. (a) and (b) are from QCD and electroweak penguin diagrams, while (c) and (d) denote the tree contributions.

non-vanishing and they can be derived as

$$A_{KN} = \frac{G_F f_K c_N}{\sqrt{2}} (m_B^2 - m_N^2) [V_{ts} V_{tb}^* (a_4^u - \rho_K a_6^u) - V_{us} V_{ub}^* a_1] F_0^{BN}(m_K^2),$$  

$$A_{KS} = -\frac{G_F f_K c_N}{\sqrt{2}} V_{ts} V_{tb}^* \left[ 2m_s f_S C_S \frac{m_B^2 - m_K^2}{m_b + m_s} a_6^s \right] F_0^{BK}(m_S^2),$$  

$$A_{K+N} = \frac{G_F f_K c_N}{\sqrt{2}} (m_B^2 - m_N^2) [V_{ts} V_{tb}^* a_4^u - V_{us} V_{ub}^* a_1] F_0^{BN}(m_{K^*}^2),$$  

$$A_{K+S} = \frac{G_F f_K c_N}{\sqrt{2}} V_{ts} V_{tb}^* \left[ 2m_s f_S C_S \frac{m_B^2 - m_K^2}{m_b - m_s} a_6^s \right] A_0^{BK^*}(m_S^2)$$

for $B^+ \to K^+(N, S)$ and $B^+ \to K^{*+}(N, S)$ decays, respectively, where $\rho_K$, $a_1$, and $a_4^u, a_6^s$ are defined by

$$\rho_K = \frac{2 m_K^2}{(m_s + m_u)(m_b + m_u)},$$

$$a_1 = C_2 + \frac{C_1}{\sqrt{N_c}},$$

$$a_4^q = C_4 + \frac{C_3}{N_c} + \frac{3}{2} e_q \left( C_{10} + \frac{C_9}{N_c} \right),$$

$$a_6^q = C_6 + \frac{C_5}{N_c} + \frac{3}{2} e_q \left( C_8 + \frac{C_7}{N_c} \right).$$

$e_q$ is the electric charge of quark $q$ and $F_0^{BM}$ with $M = N, S$ and $A_0^{BK^*}$ correspond to the
\( B \to (M, K^*) \) form factors parametrized by \([44, 45]\)

\[
\langle N(p)|b_{\gamma\mu}\gamma_5q|B(p_B)\rangle = -i \left[ \left( P_\mu - \frac{m_B^2 - m_N^2}{q^2} q_\mu \right) F_1^{BN}(q^2) \right. \\
\left. + \frac{m_B^2 - m_N^2}{q^2} q_\mu F_0^{BN}(q^2) \right],
\]

\[
\langle K(p)|b_{\gamma\mu}q|B(p_B)\rangle = \left[ \left( P_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu \right) F_1^{BK}(q^2) \right. \\
\left. + \frac{m_B^2 - m_K^2}{q^2} q_\mu F_0^{BK}(q^2) \right],
\]

\[
\langle K^*(p, \varepsilon_{K^*})|b_{\gamma\mu}\gamma_5s|B(p_B)\rangle = i \left\{ 2m_{K^*}A_{0K^*}^{BK}(q^2) \frac{\varepsilon_{K^* \cdot q}}{q^2} q_\mu \\
+ (m_B + m_{K^*})A_4^{BK*}(q^2) \left( \frac{\varepsilon_{K^* \mu} - \varepsilon_{K^* \cdot q}}{q^2} q_\mu \right) \\
- A_2^{BK*}(q^2) \frac{\varepsilon_{K^* \cdot q}}{m_B + m_{K^*}} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \right\}.
\]

\( A_1^{BK*}(q^2) \) and \( A_2^{BK*}(q^2) \) are two other form factors that are not relevant in our analysis.

With \( \mu = 2.0 \) GeV, \( V_{us} = 0.22 \), \( V_{ub} = 3.6 \times 10^{-3} e^{-i\phi_3} \), \( \phi_3 = 72^\circ \), \( m_N = 1.47 \) GeV, \( m_S = 1.50 \) GeV \([2]\), \( F^{BN}(m_K^2) = 0.26 \), \( F^{BN}(m_{K^*}^2) = 0.28 \), \( F^{BK}(m_S^2) = 0.38 \), \( A_0^{BK*}(m_S^2) = 0.42 \) \([45]\), and \( f_S = -280 \) MeV \([46]\), one has the following estimation

\[
A_{KN} + A_{KS} \approx \frac{G_F m_B^3}{\sqrt{2}} V_{ts} V_{tb}^* \left( -8.50 + 1.37 i \right) \times 10^{-5},
\]

\[
A_{K^*N} + A_{K^*S} \approx \frac{G_F m_B^3}{\sqrt{2}} V_{ts} V_{tb}^* \left( 1.17 + 0.19 i \right) \times 10^{-4}.
\]

Comparing these values to those coming from the contribution of purely gluonic state given in Table II one can conclude that the \( q\bar{q} \) quarkonia contributions can be safely ignored.

In summary, we have studied the scalar glueball production in exclusive \( B \) decays by using PQCD factorization approach. The typical momenta carried by the exchanged gluons in the process is about half of the meson mass. One thus expects our perturbative results are trustworthy. According to our analysis, we find that the branching fraction for \( B^+ \to K^+ G_0 \) is a few \( \times 10^{-6} \); however, for \( B^+ \to K^{*+} G_0 \) it can be as large as \( 3 - 4 \times 10^{-5} \).

As a result, the branching fraction for the decaying chain \( B^+ \to K^{(*)+} G_0 \to K^{(*)+} (K\bar{K}) G_0 \) is \( \sim 0.66(7.79) \times 10^{-6} \). With the experimental inputs of Eqs.\((21)\) and \((25)\), we also expect the branching ratios for \( B^+ \to K^{(*)+} (\eta\eta) G_0 \) and \( B^+ \to K^{(*)+} (\pi\pi) G_0 \) are about 50% and less than 10% of \( B^+ \to K^{(*)+} (K\bar{K}) G_0 \) respectively. In this work, we have focused on the charged \( B \) mesons. Similar conclusions can be drawn for the neutral \( B \) mesons where the only difference is their lifetimes. Their mass difference \( (m_{B^0} - m_{B^+}) \) is merely \( 0.33 \pm 0.28 \) MeV \([16]\) and will not affect our numerical results significantly. Thus dividing the branching fractions given in Table III and Table IV by the ratio \( \tau_{B^+}/\tau_{B^0} = 1.071 \pm 0.009 \) from
direct measurements, one would obtain the corresponding branching fractions for the neutral \( B \) meson modes. Experimentally, the mode \( B^\pm \rightarrow (K\bar{K})_{f_0(1710)}K^\pm \) has been detected at BABAR with a branching ratio consistent with our PQCD prediction. This work suggests that detection of the resonant three-body mode \( B \rightarrow (K\bar{K})_{f_0(1710)}K^* \) with a predicted larger branching ratio can give further support of \( f_0(1710) \) is a pure scalar glueball.

**APPENDIX A: DECAY AMPLITUDE FUNCTION \( M_M \) WITH \( k_\perp = 0 \)**

Since the mass of glueball is much larger than those of ordinary pseudoscalars, we find that the influence of transverse momentum on the two-body decay \( B \rightarrow K^{(*)}G_0 \) is not as large as in the case of \( B \) decays into two lighter mesons. Setting \( \vec{k}_{1\perp} \) and \( \vec{k}_{2\perp} \) in the momenta of the spectator quarks in Eq. (8) to be zero, the decay amplitude function \( M_M \) given in Eq. (13) can be simplified to be

\[
M_M = \frac{f_0 C_F}{\pi m_B} \int_0^1 dx_1 \int_0^1 dx_2 \phi_B(x_1) \\
\times \left\{ e_M^{(1)} \phi_M(x_2) + e_M^{(2)} \phi_M^p(x_2) + e_M^{(3)} \phi_M^\sigma(x_2) \right\} \alpha_s(t) h(x_1, x_2),
\]

with the hard function \( h(x_1, x_2) \) given by

\[
h(x_1, x_2) = \frac{1}{x_1(1-x_1)(r_{G_0}^2 + (1-r_{G_0}^2)x_2)}.
\]

**APPENDIX B: DISTRIBUTION AMPLITUDES FOR \( K^{(*)} \)**

In this appendix, we compile the light-cone distribution amplitudes that entered in our calculations. The distribution amplitudes for \( K \), defined in Eq. (5), are expressed as follows
\[
\phi_K(x) = \frac{f_K}{2\sqrt{2}N_c} 6x(1-x) \left[1 + a_1^K C_1^{3/2}(\xi) + a_2^K C_2^{3/2}(\xi)\right],
\]

\[
\phi''_K(x) = \frac{f_K}{2\sqrt{2}N_c} \left[1 + 3\rho_+^K (1 + 6a_2^K) - 9\rho_-^K a_1^K + C_1^{1/2}(\xi) \left(\frac{27}{2}\rho_+^K a_1^K - \rho_-^K \left(\frac{3}{2} + 27a_2^K\right)\right) + C_2^{1/2}(\xi) (30\eta_{3K} + 15\rho_+^K a_2^K - 3\rho_-^K a_1^K) + C_3^{1/2}(\xi) \left(10\eta_{3K} \lambda_{3K} - \frac{9}{2}\rho_-^K a_2^K\right) - 3\eta_{3K} \omega_{3K} C_4^{1/2}(\xi) + \frac{3}{2}(\rho_+^K + \rho_-^K) (1 - 3a_1^K + 6a_2^K) \ln(1-x) + \frac{3}{2}(\rho_+^K - \rho_-^K) (1 + 3a_1^K + 6a_2^K) \ln x\right],
\]

\[
\phi'^{q}_{K}(x) = \frac{f_K}{2\sqrt{2}N_c} \left\{\xi \left[b_1 + b_2 C_1^{3/2}(\xi) + b_3 C_2^{3/2}(\xi) + b_4 C_3^{3/2}(\xi)ight.ight.
- 30b_3 x(1-x) + b_5 \ln(1-x) + b_6 \ln x\left.\right] + x(1-x) \left[-6b_2 + 5b_4 (-21(1 - 2x)^2 + 3)\right] + \frac{1}{6} (-xb_5 + (1-x)b_6)\right\}, \quad (B1)
\]

where \(\xi = 1 - 2x\) and the Gegenbauer Polynomials \(C_n^\nu\) are given by,

\[
C_1^{1/2}(t) = t, \quad C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_3^{1/2}(t) = \frac{3}{2}\left(\frac{5}{3}t^3 - t\right),
\]

\[
C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), \quad C_5^{1/2}(t) = 3t, \quad C_6^{1/2}(t) = \frac{3}{2}(5t^2 - 1), \quad C_7^{1/2}(t) = \frac{5}{2}(7t^3 - 3t). \quad (B2)
\]

The coefficients \(\{b_i\}\) are defined as

\[
b_1 = 1 + \frac{3}{2}\rho_+^K + 15\rho_+^K a_2^K - \frac{15}{2}\rho_-^K a_1^K, \quad b_2 = 3\rho_+^K a_1^K - \frac{15}{2}\rho_-^K a_2^K, \quad b_3 = 5\eta_{3K} - \frac{1}{2}\overline{\eta}_{3K} \omega_{3K} + \frac{3}{2}\rho_+^K a_2^K, \quad b_4 = \eta_{3K} \lambda_{3K}, \quad b_5(6) = 9(\rho_+^K \pm \rho_-^K) (1 \mp 3a_1^K + 6a_2^K), \quad \rho^K_+ = \frac{(m_u + m_d)^2}{m_K^2}, \quad \rho^K_- = \frac{m_u^2 - m_d^2}{m_K^2}. \quad (B3)
\]

with \(m_q\) being the mass of \(m_u\) or \(m_d\) since \(m_u \approx m_d\) is assumed. Since \(m_q \ll m_s\), in our numerical estimations, we take \(\rho_+^K = \rho_-^K = \rho^K\). We display the values of decay constant, mass of strange quark, and relevant coefficients of distribution amplitudes for \(K\) meson at \(\mu = 1\) GeV in Table [V].
TABLE V: The decay constant, mass of strange quark (in units of MeV) and coefficients of distribution amplitudes for $K$ meson at $\mu = 1$ GeV.

| $f_K$ | $m_s$ | $a_1^K$ | $a_2^K$ | $\rho^K$ | $\eta_{3K}$ | $\omega_{3K}$ | $\lambda_{3K}$ |
|-------|-------|---------|---------|---------|------------|------------|-------------|
| 160   | 120   | 0.06    | 0.25    | 0.076   | 0.016      | -1.2       | 1.6         |

Similarly, the distribution amplitude for $K^*$ can be expressed as \[33, 34\]

\[
\begin{align*}
\phi_{K^*}(x) &= \frac{f_{K^*}}{2\sqrt{2N_c}} 6x(1-x) \left[ 1 + 3a_1 a_2 \xi + 3a_2 \xi^2 \right], \\
\phi_{K^*}^p(x) &= \frac{f_{K^*}^p}{2\sqrt{2N_c}} \left[ 3\xi^2 + 3a_1 a_2 \xi^2 \right] + 3a_2 \xi^2 + 70\xi^2 C_4(\xi) \\
&\quad + \frac{3}{2}\delta_+ \left( 1 + \xi \ln \left( \frac{x}{1-x} \right) \right) + \frac{3}{2}\delta_- \left( 2 + \ln (1 - x) + \ln x \right), \\
\phi_{K^*}^\sigma(x) &= \frac{f_{K^*}^\sigma}{4\sqrt{2N_c}} \left\{ 6\xi \left[ 1 + a_1 a_2 \xi + \left( \frac{1}{4}a_2^2 + \frac{35}{6}\zeta_3 T \right) (-20x(1 - x) + 5\xi^2 - 1) \right] \\
&\quad - 12a_1 x (1 - x) + 3\delta_+ \left( 3\xi - 2 \ln (1 - x) - 2 \right) \right\}.
\end{align*}
\]

The values of the decay constants and relevant coefficients of the distribution amplitudes for the $K^*$ meson are shown in Table VI.

TABLE VI: The decay constants (in units of MeV) and coefficients of distribution amplitudes for $K^*$ meson at $\mu = 1$ GeV.

| $f_{K^*}$ | $f_{K^*}^p$ | $a_1^{(L)}$ | $a_2^\parallel$ | $a_2^\perp$ | $\zeta_3 T$ | $\delta_+$ | $\delta_-$ |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 210     | 170         | 0.10        | 0.09        | 0.13        | 0.024       | 0.24        | -0.24       |

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