3D Rough Surface Topography Model of Fractal Interpolation Based on Wavelet Transform

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Abstract. With the development of computer technology, there are many kinds of instruments for obtaining the 3D surface topography such as confocal topography using optical principle, the scanning electron microscope, the transmission electron microscope and the probe contact topography. However, the topography or profile of the rough surface has scale correlation. The certain measurement deviation will be caused due to the limited resolution of the topographer when acquiring a 3D topography. In addition, the dimensions and attributes of the tested sample are strictly required and it is impossible to record the surface topography that changes in real time during friction and wear. In order to overcome the disadvantages of the above surface topography modeling methods, a method combining fractal interpolation and wavelet transform is used in this paper. The accuracy and the resolution of the 3D surface topography model can be controlled by the number of iterations and the selection of the reference data resolution.

1. Introduction

In recent decades, tribology has made continuous progress and the characterization of surface quality has also evolved from one-dimensional parameters to three-dimensional parameters, which can reflect the surface topography more accurately and more reasonably [1]. However, in practical applications, the surface roughness is still selected as the most common parameter to characterize the surface quality. But some surface contours with the same roughness value will also be different and the actual contact area and the retention of lubricating oil will change, resulting in a great difference in wear resistance [2]. Therefore, it can no longer meet the requirements to characterize the workpiece only by the roughness. The surface topography is like the fingerprint of the workpiece surface, which is universal and unique. And the analysis is more robust and reliable by characterizing the surface quality with the surface topography. The surface micro-topography is the general term for the surface geometry at the microscopic angle and it is a representation of the surface height distribution characteristics, including surface roughness, waviness, surface texture, and shape error [3-4]. The microscopic contact, friction, wear and lubrication of the workpiece surface can be explored from a more fundamental perspective with the surface topography. Therefore, the acquisition of surface topography becomes the first step in analyzing the friction and wear characteristics of the workpiece surface.

There are many ways to obtain the micro-topography of the workpiece surface such as the white light interferometer, optical microscope, the scanning electron microscope, the transmission electron microscopy and the atomic force microscopy. However, these instruments all have limitations on the shape, size, or properties of the test sample. And the surface topography that changes in real time during friction and wear cannot be recorded [5]. On the other hand, the changing of the profile height is a non-stochastic random process. The surface micro-morphology acquired by a certain resolution instrument.

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can only reflect the roughness of the resolution level, and the true information of the surface roughness cannot be fully reflected. The friction surface has fractal properties and belongs to a self-similar fractal surface [6]. Therefore, the fractal simulations can be performed on rough surface like fractal Brownian function simulation, inverse Fourier transform simulation WM function simulation and fractal interpolation simulation method. For example, Kopytov VV, et.al. [7] used the Brown method as an adaptive prediction method to calculate the time series of loads in a cluster and used it to predict the fractal dimension. Zhang X, et.al. [8] reconstructed rough surfaces by the inverse Fourier transform based on the specified power spectral density (PSD). Xu Y, et.al. [9] calculated the multi-scale structure and fractal dimension of the rough surfaces by Fourier transform analysis. Jin L, et.al. [10] modeled the microprotrusions on the contact surface based on the WM fractal function. Huimin MI, et.al. [11] simulated the non-Gaussian fluctuating wind pressure with the fractal Weierstrass-Mandelbrot function by considering the fractal feature. Fu Y, et.al. [12] simulated one-dimensional and two-dimensional fractal surfaces by displacement method and W-M fractal function simulation. Chand AKB, et.al. [13] developed the multiresolution analysis caused by the aggregation latent variable fractal interpolation function (CHFIF). Feng Z, et.al. [14] proposed a method for constructing fractal interpolation surfaces with arbitrary interpolation nodes in a rectangular domain.

In order to overcome the disadvantages of the above surface topography modeling methods, a method combining fractal interpolation and wavelet transform is used in this paper. Based on the experimental data, a 3D surface topography model is established to conform to the real features of the surface.

2. Methodology

The fractal dimension reflects the complexity and irregularity of the surface profile. Generally, surface roughness increases with the fractal dimension. Therefore, determining the fractal dimension is the key issue to characterize the fractal features of surface topography. The covariance of the time series and the scale rate of the time interval are used to solve the fractal dimension and scale factor by the RMS method. However, the relationship of the scale rate is established only when the sample length is smaller than the correlation length of the profile [17]. Compare with the RMS method, the wavelet transform method is an important development of Fourier analysis. It is one of the effective means to analyze the fractal structure. It has the functions of enlargement and shift and it is similar to the “magnifier” of mathematics. The more and more abundant details on smaller scales can be observed through the transforming through different scales.

2.1. Wavelet Transform Method

Wavelet analysis is a set of bandpass filters that separate signals of different scales and frequencies through different frequency band channels. The most important thing is to construct the wavelet function, do wavelet transform on the fractal signal and find the wavelet coefficients at each scale. The fractal signal is the height distribution of the surface profile. The signal can be decomposed into a set of uncorrelated random variables satisfying certain conditions. The decomposition formula is:

\[
x(t) = \sum_m \sum_n d_{m,n} \phi_{m,n}^n(t)
\]
In the formula, \( x(t) \) is the fractal signal, \( d^m_n \) is a wavelet coefficient and \( \varphi^m_n(t) \) is a wavelet function. The core of the wavelet transform method is to construct the wavelet function and determine the number of wavelet decomposition layers according to the requirements. The wavelet function satisfies the following form.

\[
\varphi^m_n(t) = 2^{2n} \varphi(2^n t - n)
\]  

(2)

The lengths of the approximate and detail coefficients corresponding to the low-frequency and the high-frequency part of the signal can be obtained after the signal is decomposed by the wavelet, thereby the fractal dimension of the rough outline can be solved. For a fractal signal \( x(t) \), if its variance is \( \sigma^2_x \), then

\[
\text{Var}(d^m_n) = 2^{-m} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sigma^2_x |\varphi(\omega)|^2}{|\alpha|} d\omega
\]  

(3)

From Eq. (3), it can be seen that the double logarithmic curve obtained from the variance of the wavelet coefficients at each scale should be a straight line in an ideal case. Therefore, the linear regression analysis of \( \text{Var}(d^m_n) \) and \( m \) is performed on the double logarithmic coordinates, the slope of the straight line is \( \alpha \), the fractal dimension is \( D = 2 - \alpha \). The straight line intercept is the scale factor.

2.2. Fractal Interpolation of Contours

Interpolation methods are commonly used in numerical simulations and numerical calculations. The classical interpolation method is to analyze the relationship between data points geometrically. There are various interpolation function selection methods like algebraic polynomials, trigonometric polynomials or rational functions. The spline interpolation method can overcome the non-smooth fitting problem of the segmented interpolation function [18]. The image of the surface profile can sometimes not be described well by the Euclidean function, while the fractal interpolation function can be close to these images. Therefore, the fractal interpolation theory can be used to establish the surface contour or 3D topography model. The core of the theory is to construct an iterative function system to make its attractor \( A \) is the graph of the interpolation function [19].

For a two-dimensional coordinate point of a rough surface contour \( \{(x_i, y_i), i = 0, 1, ..., N\} \in R^2 \) the fractal interpolation function is a continuous function \( f[x_i, y_i] \rightarrow R \) interpolated by \( \{(x_i, y_i), i = 0, 1, ..., N\} \) and its graph is an attractor of a hyperbolic iterative function system.

Now construct an iterative function system \( \{R^2: W_i, i = 1, 2, ..., N\} \) with the attractor equal to the interpolation function \( f(x) \).

(1) Select interpolation data points \( \{(x_i, y_i), i = 0, 1, ..., N\} \):

(2) Each function \( W_i \) in the iterated function system be an affine transform whose construction is expressed as \( W_i: R^2 \rightarrow R^2, i = 1, 2, ..., N \), that is,

\[
\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow W_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, i = 1, 2, ..., N
\]  

(4)

(3) The constraint conditions is that the interpolation function value is the same as the original data point at each interpolation point as shown in equation set (5).

\[
\begin{pmatrix} W_{i}(x_i) \\ W_{i}(y_i) \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}, i = 1, 2, ..., N
\]  

(5)

\[
\begin{align*}
a_i x_i + e_i &= x_i \\ a_i x_i + e_i &= x_i \\ c_i x_i + d_i y_i + f_i &= y_i \\ c_i x_i + d_i y_i + f_i &= y_i
\end{align*}
\]  

(6)

(4) From Eq. (4), the five coefficients of affine transformation function should satisfy the equation
set (6).

The equation set (6) contain five variables, so there is one free variable. It is generally chosen $d_i$ as a free variable and it is called the vertical scale factor, which satisfies $0 \leq d_i < 1 (i = 1, 2, ..., N)$. Therefore, the other four coefficients for each function in the iterated function system are:

$$
a_i = \frac{x_j - x_{i+1}}{x_n - x_0}, \quad c_i = \frac{y_j - y_{i+1}}{y_n - y_0}, \quad f_i = \frac{x_jy_{i+1} - x_0y_j}{x_n - x_0}, \quad g_i = \frac{x_jy_0 - x_0y_j}{x_n - x_0}, \quad \epsilon_i = \frac{x_jx_{i+1} - x_0x_j}{x_n - x_0}.
$$

Among them, the approximate expression of the relationship between vertical scale factor and fractal dimension is $d_i = 0.9D - 1$.

### 2.3. Rough outline simulation

The height distribution characteristics of the GZB45BA slider outer raceway surface is obtained by the surface profiler, as shown in Fig.1. The fractal interpolation simulation of the contour curve is performed based on a certain resolution of the contour and the specific process are shown in Fig.2.

![Fig.1 Surface profiler](Image)

![Fig.2 The fractal interpolation contour simulation flow chart based on wavelet transform](Image)

In Fig. 3 and Fig. 4, the original contour is the rough contour measured by the surface profiler and the resolution is 0.125um. According to Fig.2, the fractal dimension of the measured contour is firstly solved by the wavelet transform method and it is 1.435. Then the reference data with 6.25um and 12.5um resolution respectively are selected for fractal interpolation. The number of iterations is 3, 5, 7, 9 and 11.
respectively. The fractal dimension $D$, scale factor $C$ and the characteristic classification parameters $E$ are solved of the fractal interpolation contour under different iterations as shown in Table 1.

![Interpolation with resolution 12.5um](image1.png)

**Fig. 3 Interpolation with resolution 12.5um**

![Interpolation with resolution 6.25um](image2.png)

**Fig. 4 Interpolation with resolution 6.25um**

| No. | Iteration | $D$  | $C$  | $E$  |
|-----|-----------|------|------|------|
| 1   | Original  | 1.4353 | 0.6630 | 2.0705 |
| 2   | 3         | 1.4283 | 0.6678 | 2.0264 |
| 3   | 5         | 1.4346 | 0.6722 | 2.0188 |
| 4   | 7         | 1.4354 | 0.6655 | 2.0570 |
| 5   | 9         | 1.4356 | 0.6576 | 2.1015 |
| 6   | 11        | 1.4356 | 0.6495 | 2.1482 |

| No. | Iterations | $D$  | $C$  | $E$  |
|-----|------------|------|------|------|
| 1   | Original   | 1.4353 | 0.6630 | 2.0705 |
| 2   | 3          | 1.4312 | 0.6634 | 2.0575 |
| 3   | 5          | 1.4351 | 0.6632 | 2.0689 |
| 4   | 7          | 1.4355 | 0.6559 | 2.1109 |
| 5   | 9          | 1.4356 | 0.6479 | 2.1576 |
| 6   | 11         | 1.4356 | 0.6399 | 2.2056 |

By comparison, the higher the number of iterations, the closer the interpolation contour is to the original contour and the corresponding resolution is higher. The number of iterations is an important factor in determining the resolution of the fractal interpolation profile. From Table 1, the fractal dimension increases slightly with the number of iterations. The fractal dimension is closest to the fractal dimension of the original measurement profile when a certain number of iterations is reached. After the certain iteration number, the fractal dimension increases again. With the improvement of the iteration number, the surface topography of the fractal interpolation simulation gradually becomes more and more complicated and the fractal features are more obvious, thus, the fractal dimension gradually increases from small to large. Comparing the fractal interpolation contours of the same iteration times in Fig. 3 and Fig. 4, it is found that the fractal interpolation contour in Fig. 4 has a higher degree of overlap between the interpolation contour and the original contour. It indicates that the higher the resolution of the reference data selected, the more realistic the fractal interpolation simulation contour is. The resolution of the reference data is an important factor influencing the fractal dimension and scale factor of the fractal interpolation profile. The same conclusion can be obtained from Fig. 5 and Fig. 6, which are fractal interpolation simulation with the number of iterations and the reference data resolution as the influencing factors.

![Fig. 5 changing the number of iterations](image3.png)

![Fig. 6 changing the benchmark data resolutions](image4.png)
3. The establishment of a 3D surface topography model
In this paper, the raceway surface of the GZB5BA slider is used to establish the surface topography model. The surface of the raceway is manufactured by the grinding wheel on the grinder and the processing texture of the surface is heterosexual in both vertical directions. Therefore, the 3D surface topography model is established through the superposition of fractal interpolation in x and y directions. From the Fig. 7 to Fig. 9, they are fractal interpolation model of the 3D surface topological for the X-direction, Y-direction and complete surface, respectively. The resolution of the selected reference data is 1.939*100 um and the number of iterations is 7. For the rough surface 3D topography model established, the y-direction resolution of is 1.613 um and the x-direction resolution is 0.01517 um.

![Fig.7 x-direction interpolation model](image1)
![Fig.8 y-direction interpolation model](image2)

![Fig.9 fractal interpolation model](image3)
![Fig.11 3D surface topography by surface profiler](image4)

Using a confocal 3D surface topographer to measure the raceway surface is shown in Fig. 10 and the 3D surface topography is obtained as shown in Fig. 11.

![Fig.10 Confocal 3D surface profiler](image5)

Comparing Fig. 9 with Fig. 11, the 3D surface topography model based on the surface contour measurement data can also completely and accurately represent the topography features, such as obvious features of grooves, protrusions, scratches, etc., and the X, The Y direction has a higher resolution than the profiler.

4. Conclusion
The friction surface has a fractal nature and it belongs to a self-similar fractal surface. The 3D surface
topography is established through the fractal interpolation theory based on wavelet transform in this paper. The fractal features of the interpolation simulation surface can be ensured by the wavelet transform method. Moreover, the accuracy and resolution of the fractal interpolation surface topography can be controlled by the iteration factor and the reference data resolution. Compare the 3D surface topography model with the topography measured by the topography instrument, it is found that the obtained 3D surface topography can not only characterize the topography of the workpiece surface accurately and effectively, including dents, bulges, and scratches, etc., and it does not require a complicated sample preparation process. What is more, it can overcome the measurement error brought about by a limited resolution profiler and it is expected to realize the recording of the surface topography changing in real-time during the friction and wear process.

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