Enhanced domain wall velocity near a ferromagnetic instability.

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Assuming a Fermi liquid behavior for conduction electrons, we rewrite the extended Landau-Lifshitz-Gilbert equation (LLG) renormalized by interactions through the Landau parameters $F^a_l(s)$ ($l = 0, 1, 2, \cdots$) in an explicit form to describe the dynamic of domain wall (DW) due to spin transfer torque phenomenon. As a Stoner instability is approached, the DW velocity increases and the critical spin current density decreases. By taking the inter-electronic interactions into account, we obtain larger DW velocities, in agreement with experimental results, when compared to those calculated previously without the account of inter-electronic interactions. We are also able to understand different materials having higher DW velocities and lower critical currents which maybe helpful in searching for applied materials.

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I. INTRODUCTION.

In a ferromagnet, a DW is an inhomogeneity that separates regions with magnetizations pointing in different directions, usually in opposite directions. When a certain critical current flows through these inhomogeneities, these become unstable and start moving. Understanding this process is important for applications such as magnetic memories.

One kind of problem involving the construction of a magnetic device is the large drive current necessary to control a DW once the applied one over a DW should be less than 0.5 mA. Another problem is the high DW velocity. The higher the DW moving velocity the faster one can invert the magnetization of a region by passing a current through it.

An usual approach to explain the dynamic of a DW considers a nonadiabatic spin transfer torque component on s-d exchange model proposed by pioneer works of L. Berger and Slonczewski. The system is described by an extended LLG equation where the account of spins in a ferromagnetic state ($\mathbf{M}(r,t)$) while conduction electrons on s-band are non-interacting and paramagnetic. This “nonadiabatic approach” gives a reasonable qualitative explanation for experimental observations of DW velocity, but has quantitative disagreements with it by a factor 2. The authors claim that difference occurs due to impurity or defects.

In 2008 was released an alternative theoretical approach that preserves both s-d exchange model and nonadiabatic spin torque component and goes beyond. It considers the Fermi liquid theory to describe the interaction between spins of the $s$-electrons where a measurement of the interaction is given by $F^a_l(s)$. In such work, the interaction between $s$-electrons leads to collective modes like spin waves on s-band and quantum phase transitions phenomena which can modify the behavior of a DW.

Close to Stoner instability (a quantum critical point) where $F^a_0 \rightarrow -1$, a solution of the dynamic equation shows that a wide DW becomes narrow, a condition required to reduce the drive current. In this work, we follow the alternative approach and we show an enhanced DW velocity for reduced driven current close to Stoner instability point. First, we rewrite the extended LLG equation renormalized by the interaction between spins of $s$-electrons in an explicit form. Second, we show that the hypothesis of Fermi liquid behavior for $s$-electrons solves the discrepancy between theoretical predictions and experimental observations for DW velocities in Permalloy. Third, we analyze the DW velocity near the Stoner instability point for $s$-electrons. Forth, we derive the critical spin current density $j_c^s$ relation dependent of $F^a_l$ and we study its dependence with the $F^a_0$ parameter. In absence of interaction between spins of $s$-electrons, i.e., in the limit where $F^a_0$ and others parameters introduced by Fermi liquid description of the $s$-electrons go to zero, the critical spin current density behaves as in nonadiabatic approach. In the end, we present a tendency where materials with smallest saturation magnetization manifest highest domain wall velocities for smallest applied driven current density near the ferromagnetic instability point.

II. RENORMALIZED EXTENDED LLG EQUATION.

We can recognize an extended LLG equation renormalized by $F^a_l(s)$ from the dynamic equation derived by the alternative approach which considers interaction between $s$-electrons. If we keep only the terms of $O(\delta \mathbf{m})$ on the $\Delta$ term of dynamic equation, we have

$$\frac{\Delta}{D^a_c} = g_{ni}^a \mathbf{M} \times \nabla^2 \delta \mathbf{m}$$

where $\delta \mathbf{m}$ is a small deviation of the equilibrium magnetization $\mathbf{M}(r,t)$ which generates the nonadiabatic torque
component and \( D' \) is the renormalized spin diffusion coefficient.

If we plug Eq. (1) on dynamic equation again, we obtain \( \dot{\delta m} \) term renormalized by Landau parameters,

\[
\frac{1}{\tau_{sf}} \dot{\delta m} + \frac{1}{\tau_{sf} M_s} (\delta m \times M) = -\frac{n_0}{M_s} \frac{\partial M}{\partial t} + \frac{\mu_B P'}{e M_s} (\hat{\mathbf{j}} \cdot \nabla) M. \tag{2}
\]

We assume a spin wave model for the deviation of the equilibrium component, i.e., \( \delta m e^{i(\omega t-k \cdot r)} \) for both s and d bands with the same spin wave vector \( k \). On Eq. (2) \( \mu_B \) is the Bohr magneton, \( P' \) is the renormalized polarization of the spin current density \( j_e \), \( 1/\tau_{sf} \equiv D'_s k^2 + 1/\tau_s \) and \( 1/\tau_{ex} \equiv 1/\tau_{ex} - D'_s k^2 g n'_0 \) are renormalized relaxation times where \( g \) gives the strength of interaction and \( n'_0 \) is the renormalized equilibrium spin density \( \left\langle n \right\rangle \) and \( k \cdot k = k_x^2 + k_y^2 + k_z^2 \) is the spin wave vector which comes from \( \nabla^2 \delta m = -k^2 \delta m \).

If we use the transversal properties of the system, \( \delta m \cdot M = 0 \) and \( (\delta m \times M)/(\tau_{ex} M_s) = \mathbf{T} \). Eq. (2) can be written as a new spin transfer torque due to s-d interaction renormalized by interaction between spins of the s-electrons,

\[
\mathbf{T} = \frac{\tau_{sf} / \tau_{ex}}{1 / \tau_{ex} + (\xi/\tau_{ex})^2} \left[ -\frac{n_0}{M_s} \frac{\partial M}{\partial t} + \frac{\xi n'_0}{(M_s)^2} M \times \frac{\partial M}{\partial t} \right. \\
\left. + b_j (\hat{\mathbf{j}}_e \cdot \nabla) M - \frac{c}{M_s} b_j M \times (\hat{\mathbf{j}}_e \cdot \nabla) M \right], \tag{3}
\]

where \( \xi^* \equiv \tau_{sf}^{*} / \tau_{ex}^{*} \) and \( b_j \equiv \mu_B P' j_e / e M_s \). Here we consider DW typical width and spin diffusion length of s-electrons close in magnitude and then both \( M \) and \( \delta m \) contributes to modify the dynamic of a DW due to the product \( M \times \nabla^2 \delta m \). This condition is required to both s and d bands share the same wave vector \( k \).

Clearly Eq. (3) becomes the nonadiabatic spin torque component in the limit of all \( F_{l}^{(a)} (l = 0, 1, 2 \cdots) \) and \( \tau_{FL} \) equals to zero and for a small spin wave vector \( k \) where we can consider \( k^2 \approx 0 \). Notice these quantities are independent and all three limits are required for equations to become the same as in nonadiabatic approach and we call it nonadiabatic limit \( (F_{l}^{(a)} \to 0, \tau_{FL} \to 0 \) and \( k^2 \approx 0 \)).

If we plug Eqs. (3) on extended LLG equation, we obtain a renormalized extended LLG equation in an explicit form,

\[
\frac{\partial M}{\partial t} = -\gamma^* M \times \mathbf{H}_{eff} + \frac{\alpha^*}{M_s} M \times \frac{\partial M}{\partial t} + \mathbf{T}^*, \tag{4}
\]

where the renormalized nonadiabatic spin transfer torque component is given by

\[
\mathbf{T}^* = b^*_j (\hat{\mathbf{j}}_e \cdot \nabla) M - \frac{c^*_j}{M_s} M \times (\hat{\mathbf{j}}_e \cdot \nabla) M, \tag{5}
\]

with the coefficients,

\[
\gamma^* \equiv -\frac{\gamma}{1+\Gamma n_0}, \\
\alpha^* \equiv \frac{1}{1+\Gamma n_0} \left( \alpha + \Gamma \frac{\gamma^* n_0}{M_s M_r} \right), \\
b^*_j \equiv \frac{\gamma}{1+\Gamma n_0} b_j, \\
c^*_j \equiv \xi b^*_j \text{ and } \\
\Gamma \equiv \frac{\tau_{sf}^*}{\tau_{ex}^* (\xi^*)^2}. \tag{6}
\]

### III. HIGHER DW VELOCITY AND AGREEMENT WITH EXPERIMENTAL RESULTS.

An experimental average DW velocity \( (v_{DW}) \) estimated in Permalloy was 3 ms\(^{-1}\).\(^{10}\)

When \( j_e = 1.2 \times 10^{12} \text{ Am}^{-2} \), \( P = 0.7 \), \( \alpha = 0.1 \), \( M_s = 8 \times 10^5 \text{ Am}^{-1} \), \( n_0/M_s \sim 10^{-2} \), \( \tau_{sf} \sim 10^{-12} \) s, \( \xi = \tau_{ex}/\tau_{ex} \sim 10^{-2} \), the DW velocity predicted by nonadiabatic approach is 6 ms\(^{-1}\) for Permalloy\(^{12}\). The authors of nonadiabatic theory attributed the discrepancy between experimental and theoretical prediction due to defects or impurity not considered in theory development\(^{12}\).

Such disagreement between experimental result and theory prediction can be explained by the hypothesis of interaction between spins of the s-electrons. From Eq. (4), we find a DW velocity renormalized by \( F_{l}^{(a)} \) in the absence of magnetic field

\[
v_{DW}^* = -\frac{c^*_j}{\alpha^*}, \tag{7}
\]

as done by nonadiabatic approach\(^{12}\). The Eq. (7) has a spin wave vector \( k \) implicit dependence (which comes from the \( \Delta \) term) and a Landau parameters \( F_{l}^{(a)}, F_{l}^{(n)}, F_{l}^{(i)} \) implicit dependence \( (F_{0}^{(a)} \text{ appears in } \tau_{sf}^{*}, \text{ and } F_{l}^{(i)} \text{ in } m^* \), the effective mass of the s-electrons).

In order to move a DW, it is required a minimal energy. If we minimizes \( v_{DW}^*(k) \), we find a critical DW velocity at \( k_e = 0 \). At this point, only the dependence of \( v_{DW}^*(k_e = 0) \) on \( F_{0}^{(a)} \) survives. In this regime, our estimation for \( v_{DW}^*(k_e = 0) \) can be seen on Fig. 1.

For \( F_{0}^{(a)} = 1.02 \), Eq. (7) gives \( v_{DW}^* = 3.00 \text{ ms}^{-1} \) in the absence of magnetic fields in agreement with experimental observation in Permalloy as we expect. If we take the limit \( F_{0}^{(a)} \to 0 \) (turn off the interaction between s-electrons), nonadiabatic DW velocity is recovered, i.e., \( v_{DW}^* = 6.07 \text{ ms}^{-1} \).

We can extrapolate our analysis to study the behavior of \( v_{DW}^* \) near the ferromagnetic Stoner instability (see Fig. 2).

Notice we can not control the interaction between s-electrons in the system, so we can not choose it to be near or far from the Stoner instability point. The interaction is fixed a priori and we estimated \( F_{0}^{(n)} = 1.02 \) for Permalloy. Although we can not control the interaction of s-electrons, \( F_{0}^{(b)} \) changes for different materials.
field and it diverges for $F$ necessary to destabilize a DW. As we can see on Fig. 2, $v_{DW} \approx 6.07 \text{ms}^{-1}$ in agreement with nonadiabatic theory prediction.

In this sense, when the material is changed, $F_0^s$ "can be controlled" (see sec. V).

![FIG. 2. $v_{DW}^s$ vs. $F_0^s$ near Stoner instability for s-electrons. At critical point where $k_c = 0$ and $F_0^s \rightarrow -1_+$, we have the DW velocity grows up indefinitely. For $F_0^s \approx -0.99$, $v_{DW}^s \approx 600 \text{ms}^{-1}$. As we can see on Fig. 2, $v_{DW}^s$ grows up indefinitely close to s-electrons ferromagnetic instability. For $F_0^s \approx -0.99$, the $v_{DW}^s \approx 600 \text{ms}^{-1}$ in the absence of external magnetic field and it diverges for $F_0^s \rightarrow -1$.]

**IV. CRITICAL CURRENT**

We derived the follow critical spin current density necessary to destabilize a DW

$$j^* = \frac{eM_s(1+F_0^s)}{\mu_B P} \left(\frac{1}{1-\frac{1}{\alpha}}\right) \frac{1}{2} \times$$

$$\times \sqrt{\frac{2A k^2 + H}{M_s} \left(\frac{2A k^2 + H + 4\pi M_s}{\mu_B}\right)} , \quad (\text{8})$$

similarly to nonadiabatic approach where $k^2 = k_x^2$ and, $k_y = k_z = 0$.

In absence of the external and anisotropic magnetic fields ($H_e = H_k = 0$), $j^*(k)$ has a minimal at point at $k_c = 0$ and eq. (8) becomes

$$j^*_c(k_c = 0) = \frac{eM_s(1+F_0^s)}{\mu_B P} \left(\frac{1}{1-\frac{1}{\alpha}}\right) \times$$

$$\times \sqrt{\frac{2A}{M_s}} \left(\frac{1}{\sqrt{4\pi M_s}} \right). \quad (\text{9})$$

Our result is shown in Fig. 3 where we consider $M_s = 7.96 \times 10^9 \text{Am}^{-1}$, $A = 1.3 \times 10^{-11} \text{Jm}^{-1}$, $P = 0.7$, $\gamma = 1.76 \times 10^{11} \text{s}^{-1}\text{T}^{-1}$, $\alpha = 0.005$, $\tau_{FL} = 10^{-12} \text{s}$, $\tau_{ex}/\tau_{sf} = 10^{-2}$ and $n/M_s = 10^{-2}$ for Permalloy.

![FIG. 3. $j^*_c$ vs. $F_0^s$. At $k_c = 0$, $F_0^s = 1.02$ and $j^*_c \approx 1.20 \times 10^{16} \text{Am}^{-2}$ in disagreement with an applied current to move a DW on Permalloy nanowire experiment. For $F_0^a = 1.02$ we have $j^*_c \approx 1.20 \times 10^{16} \text{Am}^{-2}$. This value is four orders greater than the critical spin current density necessary to move a DW on Permalloy nanowire experiment. Nonadiabatic approach calculations present two kinds of spin current density for Co system, one for the bulk electrons and other for the superficial electrons. The last one has three orders of power less than the first one. In our case, we calculated the spin current density for bulk electrons, which justify the four orders of the difference. The minimal drive current $j^*_c$ at $F_0^a \approx -0.99$ estimated by us is $\approx 1.00 \times 10^{12} \text{Am}^{-2}$. If we take the limit for $F_0^a \rightarrow -1_+$ on Eq. (9), the polarized induced critical spin current density $j^*_c \rightarrow 0$ (see Fig. 3). For nonadiabatic limit Eq. $j^*_c = j_c$.]

**V. MODEL VS REALITY.**

Despite the paramagnetic phase of s-electrons, the system composed by $s$ and $d$ bands is a ferromagnetic system. If we remember that this is a model like “two mixed fluid” where, for the real system, the $s$-band and the $d$-band share the same electrons, this guarantees that if $F_0^a$ becomes closer to $-1_+$ for s-electrons, then the $s$-d band system becomes closer to ferromagnetic instability point. As we pointed earlier, we cannot control the interaction in a specific system. However, we can see a direct manifestation of the limit $F_0^a \rightarrow -1$ as a reduction of the saturation magnetization comparing different ferromagnetic materials. For example, the weak itinerant ferromagnet MnSi has a small magnetic moment of 0.4 $\mu_B$/Mn and...
$F_0^a$ has been estimated as being $-1.10^{[18]}$ while Permalloy has a magnetic moment of $1.61 \mu_B/\text{cell}^{[19]}$ and we estimate $F_0^a = 1.02$ for it. Since the saturation magnetization is directly related to the magnetic moment, we expect a manifestation of larger DW velocities in materials with lower saturation magnetization.

A plot of the data extracted from nonadiabatic approach^{[21]} reveals this tendency of higher DW velocity in materials with smallest saturation magnetization. Such tendency is a way to looking for materials where an application of smallest spin current density reproduces higher DW velocity.

![Figure 4](image.png)

**FIG. 4.** DW velocity ($b_s$) vs saturation magnetization ($M_s$). Each point represent a specific material. It shows a tendency of high DW velocity in materials with smallest saturation magnetization.

VI. CONCLUSIONS

We rewrite the extended LLG equation renormalized by interactions between s-electrons in an explicit form to describe the dynamic of a DW. The Fermi liquid hypothesis gives a successful calculation for DW velocity where we found $v_{DW}^s = 3 \text{ ms}^{-1}$ for $F_0^a = 1.02$ in agreement with real-experimental observations in Permalloy nanowires. We also derived the critical spin current density where we found $j_c^* = 1.20 \cdot 10^{16} \text{ Am}^{-2}$ in the absence of external and anisotropic magnetic fields for $F_0^a = 1.02$. It is four orders bigger than that observed in the real experiment for Permalloy nanowire. Such four orders discrepancy appears because we consider the bulk electrons as nonadiabatic approach Co calculation^{[21]}. An extrapolation of our results for a DW velocity when $F_0^a \rightarrow -1_+$ (close to ferromagnetic instability point) shows that the DW velocity grows up indefinitely. For $F_0^a \approx -0.99$ our estimation gives $v_{DW}^s \approx 600 \text{ ms}^{-1}$. At same time, near ferromagnetic critical point, we found a reduced critical spin current density. For $F_0^a = -0.99$ our estimation is $j_c^* = 1.00 \cdot 10^{12} \text{ Am}^{-2}$, four orders less than that estimated when $F_0^a \approx 1.02$.

We conclude that higher DW velocities are manifested for small driven current density applied in materials with smallest saturation magnetization close to Stoner instability point.

Finally, we reinforce the necessity of the verification if impurities and defects not considered by nonadiabatic approach can explain the direct real-observation of DW velocity so well as the hypothesis of interaction between spins of the s-electrons.

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