Real-time tracing FPGA circuit system of the virtual signals from thermal motion simulations in optical tweezers

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Abstract—Optically trapped nanospheres are demonstrated capable of atto-Newton force sensing, where its experiment need to control the modulation voltages of the laser power according to the positions of the trapped nanosphere in high speed. In this paper, position fluctuations of the trapping nanospheres due to thermal motions are simulated using Monte-Carlo method and finite difference method. Equal-scale amplified transformations of those position sequences are generated as the discrete voltage signal of the virtual position detector. A high-speed digital incremental PID(Proportion-Integration-Differentiation) control system is developed by a low-cost FPGA circuit system, which can generate feedback voltage signals correspondingly. The responsive signal frequency is up to 1MHz with a time delay of 0.3μs and quite high amplitude stability. It is validated to integrate the virtual position detector and the PID feedback system into a low-cost semi-physical system, which can test various feedback cooling mechanisms for the complex systems of optical tweezers in vacuum. It will be a further step relative to the pure simulations in digital computers and provide references for the development of optical tweezers in vacuum.

1. INTRODUCTION
Since the invention of optical trapping in 1970s and optical tweezers in 1986 by A. Ashkin et al., the technology of optical tweezers has been developed with great potentials in biology[1-3], nanotechnology[4-6], and basic physics[7-10]. In the past decade, optical tweezers in vacuum has gradually emerged with remarkable achievements. For optical tweezers either in the liquid or gas environment, feedback control is commonly used to improve the position stability of the trapped particles. The damping in vacuum has been greatly reduced, particles are propelled into harmonic motions by the thermal collisions of the residual gas molecules. In these feedback control methods, the positions of particles is measured and corresponding voltage signals will be used to modulate the optical trapping powers to control or cool the trapped particles.

However, the experimental system of optical tweezers in vacuum is generally complex, costly, and time-consuming[7-8]. Semi-physical simulations of virtual position detector and matched feedback circuit system would be more convenient and a better choice for the new beginners, as well as those in demands. A most important step of the semi-physical simulations is a real-time response and react to the virtual position signals.

Generally the resonant frequencies of optically trapped nanoparticles can reach nearly 1 MHz. FPGA(Field Programmable Gate Array) circuits and digital PID(Proportion-Integration-Differentiation)
control algorithm are common choices to realize the requirements above, as FPGA has the advantages of parallel computing and controllable delays. As an early edition of the semi-physical simulations, the functions are not completed yet, and only real-time responses and signal tracing will be presented here.

In this paper, we firstly introduce the principles of thermal Brownian motion in optical tweezers, incremental digital PID algorithm, and the FPGA implementation. Subsequently we simulate the particle fluctuations due to the thermal motions and generate the virtual detector signal. Then we demonstrate the capability of real-time responses with the designed PID control process in a FPGA circuit, and later show real-time signal tracing according to the virtual position signals. At last, we simply discuss the time delay during the process and introduce the common compensation method.

2. THEORIES AND PRINCIPLES

2.1. Semi-physical simulation system structure

The edition of the semi-physical simulation system here contains mainly three parts: signal generator, FPGA circuit system, and signal responses, as shown in Fig.1. The signal is generated from the virtual position detector, and imported into the FPGA circuit system for signal response and signal tracing.

2.2. Principles of thermal Brownian motion dynamics in Optical tweezers

An optically trapped nano-sphere will be affected by the collisions of the surrounding fluid molecules, and suffer from thermal Brownian motions[7]. The resulted position fluctuations of the nano-sphere can be represented by the Langevin equation[8] as follows,

\[
\frac{d^2 q}{dt^2} + \Gamma_0 \frac{dq}{dt} + \Omega_0^2 q = \frac{F_{\text{fluct}}}{m}
\]

(1)

where \(q\) is the displacement relative to the equilibrium position, \(t\) represents the time, \(m\) is the mass, \(\Gamma_0\) is the fluid damping, \(\Omega_0=\kappa\sqrt{m}\) is the angular resonance frequency, \(\kappa\) is the optical tweezers stiffness, and \(F_{\text{fluct}}\) is the random thermal force. According to Monte-Carlo method and finite difference method, the time can be equally divided into \(n\) steps with a step as \(\Delta t\). Assume \(p\) to be the momentum of the nano-sphere, the discretized relations of position \(q\) and momentum \(p\) between the adjacent time steps can be obtained as follows,

\[
q(i) = q(i-1) + \frac{p(i-1)}{m} \Delta t
\]

(2)

\[
p(i) = 1/(1 + \Gamma_0 \Delta t) \cdot \left( p(i-1) - m\Omega_0^2 \cdot q(i) \Delta t + \sqrt{2k_B T \Gamma_0} \cdot dW(i) \right)
\]

(3)

where \(k_B\) is Boltzmann constant, \(T\) is the ambient Kelvin temperature, and \(W\) is white noise.

2.3. Incremental PID algorithm and its FPGA implementation

PID algorithm is a typical classical control method which controls the target by linear combination of proportions, integrals and differential deviations[11]. The incremental PID algorithm has PID advantages of less computation, less risk of runaway and less accumulated error[12], and the expression can be presented as follows.
In a digital control system, the continuous time quantity is usually separated and dispersed into a time series \( \{t(k)\} \), where \( t(k) \) represents the \( k \) moment and the adjacent time spacing is \( \Delta t_2 \) of \( t(k) \) time, so the incremental PID algorithm is adopted. The expression is:

\[
u(k) = u(k-1) + k_0 e(k) + k_1 e(k-1) + k_2 e(k-2)
\]

where \( m \) represents the time discretization \( t(m) \), \( e(k) \) is the deviation at \( t(m) \), and \( \{k_0,k_1,k_2\} \) are the incremental PID parameters.

A low-cost development circuit system with Xilinx XC7A35T chip is chosen, and full-parallel computing is employed to achieve the fastest control speed\(^\text{[11]}\). A full PID control process in FPGA is shown in Fig.1. Firstly, the variables are initialized to zero, and the targets are sampled from the outside signals. Subsequently the deviations \( e(k) \) between output \( y(k) \) and target \( r(k) \) is calculated and blocked into the registers for \( e(k-1) \) and \( e(k-2) \). The control variable \( \Delta u \) is calculated using the deviations and PID parameters. During this process, a shared state machine is waited to control the timing. The loop will be performed until \( e(k) \) is acceptable.

In order to achieve a well PID control process, the PID parameters are firstly obtained by the attenuation curve method and then optimized to be \( \{k_0=9, k_1=-6, k_2=4\} \).

![Figure 2. The logic diagrams of the PID control process in FPGA](image)

### 3. Experimental Process

#### 3.1. The validation experiments of fast responses and signal trace

As the resonant frequencies of the trapped nano-spheres can reach about 1MHz, a module circuit Alinx AN108 is used to work with FPGA development board, where the sampling frequencies of the 8-bit ADC and 8-bit DAC are 125MHz and 32 MHz respectively.

Sinusoidal waves are generated from signal generator (Rigol DG1022U) with Vpp as 5 volts and frequency of 1MHz(or 500kHz). The signal is conducted into two branches, the signal in one branch is connected into the oscilloscope (Rigol DS1054Z) as a reference signal, and the signal in the other branch is imported into the FPGA circuit system for fast responses and signal tracing. The tracing signal is also connected into the same oscilloscope for comparisons with the reference signal, as shown in Fig. 3.
Figure 3. Real-time responses and signal tracing of Sinusoidal waves with frequency (a) 1 MHz and (b) 500 kHz.

The response and tracing signal has a constant time delay of about 0.3 μs relative to the reference signal. It demonstrate that the designed circuit system can achieve fast responses and signal tracing to the signals with frequencies up to 1 MHz.

3.2. Generation of the virtual position signals and real-time signal responses

According to iterations in equations 2 and 3, numerical simulation is carried out with the following parameters: the radius of the SiO$_2$ is 70 nm with a density of 2.65 g/cm$^3$, the stiffness is 2 pN/μm, and the step number is $10^5$. The initial position is 0 and the initial velocity is assigned according to the energy equipartition theorem. The ambient gas is firstly set as the atmosphere, where the nanosphere is experiencing a over-damped state. The position fluctuations is shown in figure 4.

Then the position fluctuations are transformed into virtual position signal $U(i)$ using the following equation,

$$U(i) = A \cdot q(i)$$  \hspace{1cm} (5)

where $A$ is the alterable transformation constant, and here we have used $A=10V/\mu$m. The signal transfer is similar to those in Fig.4(a), except that the signal generators is changed into the real-time virtual position signal $U(i)$. The tracing signal after the fast response and the reference signal are shown in Fig.5(a). A short sequence of the signals is shown in Fig.5(b) and the tracing signal is added with 2 volts to clarify the signals.
Figure 5. (a) The tracing signal and the reference signal for virtual thermal position fluctuations in atmosphere, (b) a short sequence of the tracing signal and the reference signal.

Assuming other conditions remain the same, the pressure is reduced to $1 \times 10^{-3}$ Pa, which corresponds to an under-damped state. The position fluctuations of the nanosphere is shown in Fig.6.

Figure 6. (a) Simulated position fluctuations in vacuum, (b) a short sequence of the position fluctuations, (c) the histogram of the position fluctuations.

The position fluctuations in vacuum ($1 \times 10^{-3}$ Pa) is different from those in atmosphere due to the different damping state. The position fluctuations in vacuum is close to harmonic motions with an amplitude of 0.393μm and a frequency of 114.9 kHz.

The signal of the virtual position detector is also generated from the position fluctuations in vacuum, and the real-time response and signal tracing is done in our designed system. The tracing signal and the reference signal is shown in Fig.7.
As shown above, the signal generations from virtual position detectors and the fast responses in FPGA circuit system have been achieved. However, the frequencies of the generated signals are limited to a maximum of 50Hz due to the participation of the commercial personal computers (dual core CPU, 2.8 GHz), which will introduce time delays of about 10 ms at least. Thus the next step would be to run the system without the computers and the frequencies are expected to be reached to 0.1-1MHz. At that moment, the designed semi-physical simulations of optical tweezers in vacuum will be greatly developed further and provide more help for the beginners and those in demands.

A constant time delay $\delta t$ as about 0.3$\mu$s is measured in our designed circuit system, which can be converted into the delay phase $\Omega_1=2\pi/\delta t$ relative to the position signals. A common treatment would be to compensate the feedback signal with the phase $(2\pi-\Omega_1)$ to realize the phase matching between the position signals and the feedback signals[8].

4. SUMMARY
The paper has shown an early edition of the designed semi-physical simulation system of optical tweezers in vacuum, which consists of the virtual position detectors and the FPGA feedback circuit system. Virtual position signals are generated from the numerical simulations using Monte-Carlo method and finite difference method. The FPGA feedback circuit system are designed using incremental PID algorithm, and are demonstrated capable of fast responses to the signals with frequencies of 1MHz. Subsequently, the virtual position signals both in atmosphere and vacuum are shown and real-time signal tracing using the designed FPGA circuit system are demonstrated. It will progress towards the semi-physical simulations of optical tweezers in vacuum in a near future, and benefit the beginners and those in demands.

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