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To cite this article: Sergey Kovalenko et al 2010 J. Phys.: Conf. Ser. 259 012070

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Lepton number, black hole entropy and $10^{32}$ copies of the Standard Model

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Abstract. Lepton number violating processes are a typical problem in theories with a low quantum gravity scale. In this paper we examine lepton number violation (LNV) in theories with a saturated black hole bound on a large number of species. Such theories have been advocated recently as a possible solution to the hierarchy problem and an explanation of the smallness of neutrino masses. Naively one would expect black holes to introduce TeV scale LNV operators, thus generating unacceptably large rates of LNV processes. We show, however, that this does not happen in this scenario due to a complicated compensation mechanism between contributions of different Majorana neutrino states to these processes. As a result rates of LNV processes are extremely small and far beyond experimental reach, at least for the left-handed neutrino states.

Recently the existence of a large number of copies of Standard Model particles has been proposed as a possibility to lower the Planck scale and solve the electroweak hierarchy problem [1, 2]. In the following we briefly review the main argument of this approach, closely following [2]: Let us assume there exist $N$ copies of the Standard Model, each carrying a separately conserved charge, where $N$ is a large number $\sim 10^{32}$. It is in principle possible to prepare a black hole containing one particle of each species. Now, due to charge conservation, the contained charges have to be revealed in the evaporation process via Hawking radiation. On the other hand, a particle of mass $\Lambda$ can only be emitted if the Hawking temperature is large enough,

$$T_H \simeq \frac{M_B^2}{M_{BH}} \gtrsim \Lambda.$$  \hfill (1)

Moreover, energy conservation bounds the maximal number of particles with mass $\Lambda$ emitted due to the evaporation process of a black hole of mass $M_{BH}$ to be

$$n_{\text{max}} = \frac{M_{BH}}{\Lambda}.$$  \hfill (2)

$^1$ Talk presented by Heinrich Päes.
Assuming that the number of states $N$ saturates the black hole bound we obtain with (1)

$$N = n_{\text{max}} = \frac{M_P^2}{\Lambda^2}.$$  \hfill (3)

Finally, since semiclassically the lifetime of a black hole is given by

$$\tau_{BH} = \frac{1}{N} \int \frac{1}{T_{BH}^2} dM_{BH} \sim \Lambda^{-1}$$ \hfill (4)

we conclude that a black hole of size $\Lambda$ has a life time of order $\Lambda^{-1}$ implying that $\Lambda$ is the scale where the semi-classical treatment breaks down and quantum gravity sets in, namely the true Planck scale,

$$\Lambda \simeq \frac{M_P}{\sqrt{N}},$$ \hfill (5)

where $M_P$ has to be interpreted as the effective Planck scale, which implies $\Lambda \sim O(\text{TeV})$ for $N \simeq 10^{32}$ and thus solves the hierarchy problem.

Finally, in [4] this scenario has been advocated also as a mechanism for generating small neutrino masses, providing an attractive alternative for seesaw, extra dimensional and other known mechanisms. It is assumed that there exists one SM singlet right-handed neutrino $\nu_{Rj}$ per SM copy, so that $j = 1, ..., N$. The mechanism relies on the fact that the right-handed neutrinos, being SM singlets, couple to all the SM copies “democratically”. This SM singlet democracy combined with the requirement of unitarity of the theory implies a $1/\sqrt{N}$ suppression of the corresponding Yukawa couplings to the left-handed neutrinos $\nu_{Lj}$ and thus a suppression of the Dirac type neutrino masses. The minimalistic approach to the problem of smallness of neutrino mass suggests that in this scenario possible $B - L$ violating Majorana masses of $\nu_{Rj}$ are unnecessary and the lepton number is conserved.

However, the assumption of lepton number conservation appears rather ad hoc, as there is no fundamental reason to forbid Majorana masses for the right-handed neutrinos and lepton number conservation appears as accidental symmetry. Quantum gravity breaks global symmetries, and then conserved lepton number requires a gauged $B - L$ symmetry $U_{1(B-L)}$. The latter should be spontaneously broken otherwise there must exist the corresponding massless gauge boson stringently constrained by phenomenology. On the other hand lepton number violation might be helpful for successful baryogenesis.

In the following we analyze some consequences of the $N$-copies SM without lepton number conservation. Interestingly, even in this case, Majorana masses turn out to be suppressed by an exactly analogous reason as the Dirac masses. We assume a
gauge symmetry of the $N$-copies SM. It includes a common anomaly free gauge factor $U_{1(B-L)}$. We introduce this gauge symmetry in order to prevent the appearance of phenomenologically dangerous LNV operators induced by the TeV black holes. An additional permutation symmetry $Z_N$ acting in the space of the SM species is also imposed [4] in the $N$-copy SM scenario. To be unaffected by black holes, this discrete symmetry should be considered as a gauged symmetry in the sense of being a discrete subgroup of some continuous gauge group. The Lagrangian terms relevant for our discussion are the following:

$$\mathcal{L}_{\nu HS} = \lambda_{ij} \overline{\nu_{Rj}} (LH)_i + \beta_{ij} \overline{\nu_{Rj}} \nu_{Rj} S + \kappa_i (H^\dagger H)_i S^i S.$$ \hfill (7)

The model involves $N$ right-handed SM singlet neutrinos $\nu_{Rj}$ and one SM singlet complex scalar field $S$ having the $B - L$-charge equal to +2. Then the trilinear $HHS$ couplings are forbidden
in (7). The $U_1(B-L)$ is spontaneously broken by a vacuum expectation value $\langle S \rangle$ resulting in a Majorana mass term from the second term in Eq. (7). We assume that the scale of $B-L$ breaking lies below the gravity cutoff $\Lambda$. The Dirac mass term considered in ref. [4] originates from the first term after the electroweak symmetry breaking.

Let us consider the $N \times N$ Yukawa coupling matrix $\lambda_{ij}$ of Dirac type following ref. [4]. As the $\nu_{Ri}$ fields are not charged under the SM symmetry they cannot be assigned to a single SM-copy, besides respecting the same transformation properties under a permutation symmetry acting on the space of species. This permutation symmetry constrains the Yukawa coupling matrix to the form

$$\lambda_{ij} = \begin{pmatrix} a & b & b & \cdots \\ b & a & b & \cdots \\ b & b & a & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}.$$  \hfill (8)

This matrix combined in the first term in (7) with the SM Higgs expectation value $\langle H \rangle$ results in the Dirac neutrino mass matrix

$$m^D_{ij} = \lambda_{ij} \langle H \rangle.$$  \hfill (9)

Following ref. [4] we assume that the electroweak symmetry breaking leaves the permutation symmetry unbroken. This implies that the VEVs of all the Higgs species are equal to the same value $\langle H \rangle$. A key point guaranteeing the smallness of neutrino mass matrix entries is the smallness of the Yukawa coupling matrix (8) which follows from the requirement of unitarity of the theory. In fact, let us consider the right-handed neutrino inclusive production in the scattering of the SM particles as shown in Fig. 1(a). At high energies the rate of this process grows like

$$\Gamma \simeq N b^2 E,$$  \hfill (10)

as follows from dimensional analysis. Here we assumed $a \sim b$ which is suggested by an observation that these two quantities are of the same nature and have no fundamental reason to be very different in magnitude [4]. Unitarity below the gravity cutoff is preserved only for

$$b \lesssim \frac{1}{\sqrt{N}}.$$  \hfill (11)

Thus the neutrino mass matrix (9) results in $N-1$ Dirac neutrinos with tiny masses $m^D \simeq \langle H \rangle/\sqrt{N} \lesssim O(\text{eV})$ [4] which fulfill the experimental bounds constraining them to the sub-eV scale. One neutrino state in this framework is very heavy with the mass of the order $M^D \simeq \sqrt{N} \langle H \rangle$ which is comparable with the Plank scale.

Now let us turn to the second term of Eq. (7) which after the $B-L$-symmetry breaking leads to the Majorana mass matrix of the right-handed neutrinos

$$m^M_{ij} = \beta_{ij} \langle S \rangle.$$  \hfill (12)

The permutation symmetry constrains the Majorana type Yukawa coupling $N \times N$ matrix, just as the Dirac type Yukawa couplings had been constrained before, to be of the form

$$\beta_{ij} = \begin{pmatrix} c & d & d & \cdots \\ d & c & d & \cdots \\ d & d & c & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}.$$  \hfill (13)
Figure 1. Diagrams relevant for Unitarity constraints on Neutrino Yukawa couplings: (a) to the Higgs doublet $H_i$ (Lepton number conserving); (b) to a Higgs Singlet (Lepton number violating). $X_i$ denotes some of the SM fields coupled to $H_i$ with a strength $f$.

Figure 2. The diagram leading to the strongest Unitarity constraint on the Majorana neutrino mass term.

Figure 3. Diagram for neutrinoless double beta decay in the presence of $N$ copies of the SM particle content.

The similarity arguments used above to justify $a \sim b$ in Eqs. (8), (10) can equally be applied to
motivate \( d \sim c \). Now, when considering the scattering process of right-handed neutrinos, both final states can be any of the \( N \) copies as in Fig. 1(b) and, thus, the inclusive rate grows like

\[
\Gamma \approx N^2 c^2 d^2 E,
\]

which preserves unitarity below the gravity cutoff only for

\[
c \sim d \lesssim \frac{1}{\sqrt{N}}.
\]

An even more stringent bound originates from the diagram of Higgs doublet scattering in Fig. 2,

\[
\Gamma \approx N^4 \kappa^2 d^4 E,
\]

The scalar quartic couplings \( \kappa_i \) should not be very small since there is no symmetry protecting its smallness. Thus, an assumption \( \kappa_i \sim 1 \) could be used for rough estimations. Then we have

\[
c \sim d \lesssim \frac{1}{N}.
\]

This limit remains unaffected by insertion of additional \( S \)-branches in the diagram in Fig. 2. As a consequence the neutrino Majorana mass matrix entries (12) are even more strongly suppressed than the Dirac masses discussed above.

In order to discuss the phenomenology of LNV processes we now analyze the mass spectrum of the present scenario. The mass matrix written in the basis of the \( 2N \) fields \( \mathcal{N} = \{ \nu_{Li}, \nu_{Ri} \} \) has the following form

\[
\mathcal{M}' = \begin{pmatrix}
0 & m^D \\
m^D & m^M
\end{pmatrix},
\]

where \( m^D \) and \( m^M \) are \( N \times N \) submatrices given by Eqs. (9)-(12). The set of \( 2N \) mass eigenstates \( \nu_i = U_{ij} \nu_j \) of this symmetric matrix split in two groups of \( (N - 1) \) degenerate states \( \nu^+ \) and \( \nu^- \) and another two heavy states \( \nu^\pm \). The details of the diagonalization procedure will be given elsewhere [5].

For consistency with neutrino phenomenology one needs one neutrino at the sub-eV scale, say, \( m_\nu \sim 10^{-2} \)eV. Then, the states \( \nu^\pm \) are pushed in mass towards the Planck scale and, therefore, their phenomenological impact is negligible. The light Majorana states \( \nu^\pm \) having very small mass splitting \( \delta_m \sim 1/\sqrt{N} \) form a quasi Dirac state with the mass \( m_\nu \). Thus they are expected to induce lepton number violating processes at rates \( \sim 1/N \). Moreover due to the structure of the mass matrix (18) with zero submatrix in the upper-left corner LNV processes are even more strongly suppressed. Thus the dominating contributions to neutrinoless double beta decay are

\[
\langle m^3 \rangle = (a - b)^2 (c - d) \langle H \rangle^2 \langle S \rangle \sim N^{-2},
\]

\[
\langle \frac{1}{M_N} \rangle \approx \frac{\langle H \rangle d}{\langle S \rangle^2 b^2 N^2} \sim N^{-2}.
\]

Therefore, the amplitude for neutrinoless double beta decay and other, related processes in the studied framework is extremely small.

In this paper we have addressed a problem arising in any scenario with a low quantum gravity scale: do LNV operators induced by TeV scale black holes invalidate the model? For the case of the \( N = 10^{32} \)-copies SM we have shown that this consequence is avoided due to a non-trivial
cancelation mechanism. This property should be considered as an important benefit of the model.

Nevertheless, the presence of a large number of right-handed Majorana states can have interesting phenomenological consequences. For example, a very naive estimate of the right-handed neutrino decay diagrams on tree and one-loop level, which give rise to the baryon asymmetry in leptogenesis, scale as $(\sqrt{N})^2$ from the Yukawa coupling with $N$ $\nu_{Ri}$ copies contributing potentially to the decay, and the $\nu_{Ri}$ propagator in the loop diagram. So the process may be relevant despite LNV signals being suppressed for the left-handed neutrino states. A similar line of reasoning could apply to single $\nu_{Ri}$ production at the LHC.

We thus conclude that the $N = 10^{32}$-copies SM is safe from LNV in the SM sector, and leave the potentially interesting phenomenology of $\nu_{Ri}$ production and decay for further study.

Acknowledgments

This work has been partially supported by the DFG grant PA-803/5-1 and by the PBCT project ACT-028 “Center of Subatomic Physics” and CONICYT, Programa de Financiamiento Basal para Centros Científicos y Tecnológicos de Excelencia. HP thanks the Universidad Técnica Federico Santa María for hospitality offered while part of this work was carried out.

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