Emergent mesoscopic quantum vortex and Planckian dissipation in the strange metal phase

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Keywords: strange metal, mesoscopic vortex, local field, umklapp scattering, Planckian dissipation

Abstract

A major puzzle of condensed-matter physics is the physics behind the linear-in-temperature law of resistivity in many exotic metallic systems, including cuprates, pnictides, and heavy fermions. In this work, we propose, based on a symmetry-breaking analysis, that the strange metal phase is a novel emergent mesoscopic quantum state, beyond Landau’s quasiparticle excitation, which is composed of fluctuating vortices. The model predicts, in a straightforward way, the local magnetic field with a correlation time determined by the Coulomb potential, validated by observations of dynamic muon spin relaxation rates in both 3d cuprates and 5d iridate without fitting parameter. Furthermore, the model resolves the underlying quantum mechanism of the Planckian dissipation in terms of carrier scattering by fluctuating vortex, which predicts a scattering rate proportional to the vortex density, thus deriving both linear-in temperature and linear-in field laws, with a universal scattering coefficient validated by data of several dozens of samples for cuprates and iron pnictides. These findings offer a new phenomenology for non-Fermi liquid in strongly correlated materials.

1. Introduction

Many strongly correlated metallic systems including cuprates [1–3], pnictides [4] and heavy fermions have a linear-temperature ($T$) scaling of resistivity, in contrast to $T^2$ law due to electron–electron scattering for conventional metals, and the physics behind is still a major puzzle of condensed-matter physics. According to [5], the underlying scattering rate of this linear law is solely determined by the thermal energy under quantum uncertainty relation, i.e., $\hbar/\tau \sim k_B T$ (where $\hbar$ is the reduced Planck constant, and $k_B$ is the Boltzmann constant), referred to as Planckian dissipation limit. This limit indicates the absence of an intrinsic transport energy, which then suggests an unconventional quantum dissipation mechanism associated with quantum criticality [6].

Generally speaking, the quantum critical regime is ruled by symmetry-broken orders, which generate critical fluctuations and then induce non-Fermi liquid behaviors [6, 7]. The anomalous scattering of normal state in cuprates has been attributed to such exotic critical fluctuations as singular bosonic modes [8], including spin fluctuations [9], dynamic charge density wave [10, 11], or $d$-wave pairing fluctuations [12], as well as fluctuations of intra-unit-cell loop current [13, 14]. Although the $T$-linear resistivity can be derived, they lack universality for explaining the linear law’s persistence up to 1000 K [3] and a wide range of observations for materials of different dimensionalities and microscopic interactions. Besides, these theoretical arguments have not yielded a simple explanation without macroscopic disorder for recent linear-in-field ($B$-linear) magnetoresistance [4, 15, 16].

In contrast to models of Fermions coupled with critical fluctuations, the many-body theories devote to uncover the collectively entangled quantum states underlying the Planckian dissipation and the corresponding diffusion mechanisms of energy and charge carried by them [17–22]. Many novel collective
behaviors have been discussed, e.g., an island of electrons with random and all-to-all interactions [18], the many-body quantum chaos [20] and turbulent flows on the nanometric scale [22], as well as the similarity between the Planckian dissipation and black holes’ oscillations [21]. These theories exhibit nonquasiparticle excitations and T-linear Planckian dissipation, but their connection to the microscopic state of real-world strange metals is unclear. It is noteworthy that the quantum chaos and the nanoscale turbulence arguments exhibit a similarity to the critical fluctuations of loop current order, thus providing a possibility to synthesize theories of collective quantum states and phenomenological marginal Fermi liquid [13].

We believe that a comprehensive phenomenology requires a unified symmetry-breaking analysis for the collective behaviors of strongly correlated electrons, which may induce the emergence of novel mesoscopic self-organized structures and quantum dissipations beyond Landau’s approximately independent quasiparticle excitations. Inspired by a recent successful theory of wall turbulence, we realize that wall-shear generated turbulent fluid motions are typically strongly correlated due to wall-induced dilation symmetry-breaking, leading to emergent fluctuating eddies whose ensemble-averaged characteristic length determines transport coefficient [23]. Symmetry-breaking analysis alone yields successful predictions of momentum [23] and energy distributions [24] across the entire wall-flow domain. We here search for a similar symmetry-breaking feature in high-temperature superconductors (HTSC) to predict thermodynamics and transport properties of the strongly correlated materials.

Recent high-field measurements for both cuprates and pnictides reported a universal B-linear magnetoerosistance [4, 15]. From a symmetry perspective, normal carriers in fields displaying circulating magnetic vortex breaks time-reversal (T) symmetry, which may be the common cause behind T- and B-linearities. Indeed, it has been confirmed by many recent experimental observations in terms of spontaneous and fluctuating magnetic order [25–28] in pseudogap state of cuprates, qualitatively consistent with loop-current order in a domain [29, 30]. Therefore, a critical question is whether this T-symmetry can spontaneously break in strange metal phase? In fact, Zhang et al indeed observed a signal of magnetic fluctuations in strange metal phase from dynamic muon spin relaxation (μSR) experiments in all samples of YBa$_2$Cu$_3$O$_y$ [28]. This anomalous signal is beyond any systematic error and deserves a sound explanation that does not exist yet.

Here, we propose that the underlying physics of this anomaly is an emergent fluctuating magnetic order with spontaneous T symmetry-breaking. In other words, the loop-current order in the pseudogap state may be further excited, when temperature and magnetic field increase, to be a mesoscopic fluctuating vortex for carriers, which is an ensemble-averaged structure of the quantum chaos and nanoscale turbulence. Comparing to plane wave excitations in conventional metals, these topological excitations may be favored for carriers, which is an ensemble-averaged structure of the quantum chaos and nanoscale turbulence. As discussed above, both magnetoeresistance and μSR measurements support T-symmetry breaking and vortex excitations (either magnetic or thermal) of normal carriers in strange metal. In the following, based on the spontaneous emergent thermal vortex, we derive critical physical properties of slow magnetic fluctuations and their relation to the microscopic Coulomb potential. Furthermore, we uncover the quantum dissipation mechanism due to the carrier scattering by fluctuating thermal and magnetic vortices to clarify the origin of both linear-in temperature and linear-in field laws for resistivity of macroscopic transport.

2. Theory of fluctuating vortex and Umklapp scattering

2.1. Fluctuating vortex patterns

We consider that in the strange metal state for HTSC, normal carriers are driven by a magnetic field or thermal fluctuations to form mesoscopic circulating currents. From a microscopic view, our mesoscopic vortices originate from thermal and magnetic excitations of the intra-unit-cell loop current. It is similar to the loop-current domain discussed in reference [30], but with more specific driven force and fluctuating characteristics. For finite temperature (or field), thermal fluctuations (or magnetic field) introduce phase variations for mean-field order parameters, which may induce the self-organization of four different orientations of the loop-current order to yield mesoscopic currents at the supercell boundaries. As carriers are surrounded by strongly correlated electrons, we propose that the strong Coulomb repulsions generate strong correlations between individual vortices to form vortex arrays, which locally break both T-symmetry and spatial translation symmetry, as shown in figure 1.

As observed in μ SR measurements, the mesoscopic vortex lifetime is of the order of nanosecond, much longer than the timescale of Planckian dissipation, i.e., $\hbar/k_B T$. Therefore, for the microscopic scattering process, the mesoscopic vortex phase is a ‘long-range’ and quasistatic array rather than a gas or liquid. However, due to strong thermal fluctuations, the thermal (or magnetic) vortex array is in a fluctuating state that induces both fluctuating magnetic field and quantum dissipation. Near the quantum critical point
(QCP), where the mean-field of loop current ends, fluctuations of vortex array become quantum critical fluctuations. For the absence of an intrinsic energy scale near QCP, these fluctuations have a singular vortex–vortex correlation length (i.e. periodicity) determined by thermal de Broglie wavelength (or magnetic length) under quantization condition, while dispersion is assumed to be negligible in the leading order. Therefore, the coupling to carriers introduces a scale-invariant scattering rate determining transport properties in the strange metal state.

In this work, TV and MV are used to identify the thermal vortex and the magnetic vortex, respectively. Specifically, for a single quantum magnetic vortex, a contour integral around the vortex yields \( \Delta \theta = \frac{e}{\hbar} \oint d\vec{r} \vec{A} = e\phi_v / b \), which must be \( 2\pi \) considering the single value condition of the wave function, where \( \theta \) is the phase for the wave function of a carrier, \( e \) the electron charge, \( \vec{A} \) the magnetic vector potential, and \( \phi_v = h / e \) a quantum flux. Note that the area occupied by a quantum flux should be \( \pi r_{MV}^2 = \phi_v / B \), thus the characteristic radius of the vortex is \( r_{MV} = \sqrt{2\hbar / eB} \). On the other hand, for a thermal vortex excitation, we assume that the thermal DBW rotates to form a loop current whose stability requires that \( \Delta \theta = \lambda_T / r_{TV} = 2\pi \), i.e., a standing wave condition, where \( \lambda_T = h / m^* v_T \) is the DBW wavelength, \( v_T \) is thermal velocity satisfying \( m^* v_T^2 / 2 = k_B T \) for a 2 dimensional system, \( m^* \) the effective mass. This yields a characteristic radius of the thermal vortex as \( r_{TV} = h / \sqrt{2m^* k_B T} \).

In contrast to the superconducting vortex of Cooper pairs with long-range circulating supercurrents, a fluctuating vortex of normal carriers is composed of a vortex core and a tinylamina of circulating current. Therefore, vortex cores in the strange medal phase pack tightly together and result in the vortex period merely determined by the vortex radius. Since both magnetic and thermal vortices are distorted and fluctuating, as shown in figure 1, the ensemble-averaged periodicity of the magnetic-vortex array equals the diameter of the magnetic quantum vortex

\[
l_{MV} = 2r_{MV} = 2\sqrt{\frac{2\hbar}{eB}}, \tag{1}
\]

while the periodicity of the thermal-vortex array equals \( 2\sqrt{2} \) times of the radius of the thermal vortex

\[
l_{TV} = 2\sqrt{2} r_{TV} = \frac{2\hbar}{\sqrt{m^* k_B T}}, \tag{2}
\]

where \( \sqrt{2} \) is due to the presence of vortex and anti-vortex alternation.

### 2.2. Local magnetic fluctuations of thermal vortex

Now, let us focus on discussing the magnetic properties of the thermal vortex. In a thermal vortex excitation, a carrier circulating the characteristic perimeter of the DBW length \( \lambda_T \) induces a local magnetic field. According to Biot-Savart’s law, the magnetic field strength at the vortex center must be

\[
B_{TV} = \frac{\mu_0 I}{2\pi r_{TV}} = \frac{\mu_0 e}{2\pi \hbar} \left( m^* \right)^{1/2} \left( k_B T \right)^{1/2}, \tag{3}
\]

where \( \mu_0 \) is the permeability of the vacuum, \( I = e v_T / \lambda_T \) is the effective current strength of a carrier. Therefore, we can obtain the characteristic magnetic field of a thermal vortex associated with DBW as

\[
B_{TV} = \frac{1}{\sqrt{2\pi}} \frac{\mu_0 e}{\hbar} \left( m^* \right)^{1/2} \left( k_B T \right)^{1/2}.
\]

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**Figure 1.** Schematic diagrams of carriers’ circulating vortex pattern. (a) A distorted square array of magnetic vortices at the high magnetic field but zero temperature. The period length \( l_{MV} \) is twice the characteristic radius \( r_0 \) of the magnetic vortex (MV). (b) A distorted square array of thermal vortex (TV)-antivortex pairs at high temperature but zero magnetic field. The period length \( l_{TV} \) is \( \sqrt{2} \) times of the characteristic radius \( r_0 \) of the TV.
Equation (3) is a rigorous result in the semi-classical sense. It predicts that the local field strength is merely determined by the effective mass and temperature.

On the other hand, thermal motions would induce frequent annihilations of vortex–antivortex pair, which competes with electron–electron repulsions. For weak correlation limits, thermal fluctuations induce nearly plane wave excitation; thus, the vortex lifetime approaches zero, consistent with simple metals. On the other hand, vortex motions are favored in the strong electron–electron repulsions limit to avoid overlap of electron wave functions. In this case, we postulate that the vortex lifetime is positively correlated with microscopic Coulomb repulsions among electrons and negatively correlated with thermal energy. Therefore, the most natural scaling function for the characteristic lifetime (or correlation time $\tau_c$) of thermal vortex is

$$\tau_{TV} = C_{TV} \frac{U^2}{(k_B T)^2},$$  \hspace{1cm} (4)

where $C_{TV}$ is a constant, $U$ is the on-site Hubbard interaction. In $\mu$ SR experiments, the motionally narrowed dynamic $\mu$ SR rate in zero field is given by

$$\lambda_{ZF} = 2\gamma_\mu B_0^2 \tau_c,$$  \hspace{1cm} (5)

where $\gamma_\mu = 8.52 \times 10^8 \text{s}^{-1} \text{T}^{-1}$. Substituting equations (3) and (4) into (5), we obtain that $\lambda_{ZF} (T) \propto T^{2-n}$. Experimentally, Zhang et al have reported observation that all samples of YBa$_2$Cu$_3$O$_y$ have a constant rate of $\lambda_{ZF} = 1-3 \text{ m s}^{-1}$ at high temperatures [28], thus independent of $T$, yielding $n = 2$. Substituting $n = 2$ into equation (4), together with equations (3) and (5), we obtain that

$$\tau_{TV}(T) = C_{TV} \frac{U^2}{(k_B T)^2},$$  \hspace{1cm} (6)

$$\lambda_{ZF} = \frac{C_{TV}}{\pi^2} \gamma_\mu \mu c^2 \hbar^{-3} U^2 m^*.$$  \hspace{1cm} (7)

\subsection*{2.3. Umklapp scattering by phason mode}

Now, we consider Umklapp scattering by phason modes of vortex arrays, either thermal or magnetic. When the periodicity of vortex arrays is larger than the lattice constant, the wave vector associated with vortex arrays is small enough to connect the two quantum states on the Fermi surface, thus generate Umklapp scattering for carriers. For an Umklapp scattering process between initial state $k$ and final state $k'$, the momentum conservation requires $k' = k + q + nQ$, where $Q_k = 2\pi/l$, is the wave vector of the vortex array, $l$, the periodicity of vortex arrays, $n$ the positive integer. Ignoring anisotropy, we only consider the mean scattering rate. Following Lee and Rice’s widely known theory for Umklapp scattering by phason modes of charge density wave [31], we propose that

$$\frac{1}{\tau} = \frac{2m^*}{\hbar} \sum_{k \neq k'} (v_{k'} - v_k)^2 W_{kk'} \left[ \delta \left( E_k - E_{k'} - \omega_q \right) \frac{\partial f}{\partial E} \left( 1 + n_q - f_k \right) + \delta \left( E_k - E_{k'} + \omega_q \right) \frac{\partial f}{\partial E} \left( 1 + n_q - f_k \right) \right],$$  \hspace{1cm} (8)

where $v$ and $E$ denote the velocity and band energy of carrier, respectively; $W_{kk'}$ the module square of transition matrix describing the scattering, $\omega_q$ the frequency of phason modes; $f$ and $n_q$ the distribution function of carrier and phason modes, respectively.

It is reasonable to assume a linear relation for small momentum difference, i.e.,

$$v_{k'} - v_k \approx s \hbar (k - k')/m^*,$$

where $s$ is a constant proportional to $n$. Meanwhile, comparing to $Q_k$, $q$ of low-lying excitations is always small due to large sound velocity induced by strong electron–electron correlations, yielding $(v_{k'} - v_k)^2 \approx s^2 \hbar^2 Q_k^2 / (m^*)^2$. This yields a scattering rate determined by the order’s wave vector (or period length),

$$\frac{\hbar}{\tau} = \gamma_c \frac{\hbar^2 Q_k^2}{m^*} = \gamma_0 \frac{\hbar^2}{m^* l_0^2},$$  \hspace{1cm} (9)

where $\gamma_c$ is a dimensionless coefficient describing the mean strength of carrier–phason scattering.

$$\gamma_c = \frac{4}{\pi} \sum_{k \neq k'} W_{kk'} \left[ \delta \left( E_k - E_{k'} - \omega_q \right) \frac{\partial f}{\partial E} \left( 1 + n_q - f_k \right) + \delta \left( E_k - E_{k'} + \omega_q \right) \frac{\partial f}{\partial E} \left( 1 + n_q - f_k \right) \right].$$  \hspace{1cm} (10)

Generally speaking, $\gamma_c$ is proportional to the amplitude of phason mode. Since phason modes are fluctuations of the quantum vortex, it is reasonable to assume that $\gamma_c$ is proportional to the strength of the vortex current, which is an important parameter characterizing the change of angular momentum ($L_\nu$) during the excitation process. Thermal vortex is excited by heat with $L_\nu = \hbar$ per carrier, because its perimeter is DBW length $\lambda_T = \hbar/m^* v_T$ with thermal velocity $v_T$, which is universal for different materials.
at QCP. On the other hand, the strong magnetic field introduces considerable Zeeman energies ($\approx 27$ K at 40 T for one Bohr magneton) comparing to the temperature associated with the linear-in field law for electron spins, which induces correlated or collective spin flips for all carriers in one ion due to strong electron–electron correlation. Therefore, it is reasonable to assume that $L_v$ equals twice the total spin of all carriers in one ion, where the double factor represents the maximal spin flip from highest to lowest Zeeman level. Therefore, $L_v = h$ and $4h$ for the magnetic vortex in cuprates and iron-based HTSC, respectively, owing to the $h/2$ and $2h$ total spins of Cu$^{2+}$ and Fe$^{2+}$, respectively. Thus, we propose that

$$\gamma = \frac{L_v}{h}, \quad (11)$$

where $L_v = h$ for the thermal vortex in any strange metal and the magnetic vortex in cuprates, and $L_v = 4h$ for the magnetic vortex in iron-based HTSC. $\gamma$ is a scattering coefficient that may be related to carrier dispersion and carrier–vortex coupling strength. However, in the present work, since the contribution of band dispersion to resistivity is assumed to be renormalized near QCP, $\gamma$ may be a universal constant, consistent with Bruin et al.’s experimental observation of the universality of $T$-slope [32].

Thus, near QCP, substitute equation (9) into the Drude model, i.e., $\sigma = n_c e^2 \tau / m^*$, we predict the characteristic resistivity

$$\rho - \rho_0 = \frac{\gamma}{\varepsilon^* n_c}, \quad (12)$$

where $\rho_0$ is assumed as the ‘residual resistivity’ at zero temperature or field, and the carrier density $n_c = p/k_B$, $\alpha_0$ and $\beta_0$ are in-plane lattice constants, $c_0$ the $c$-axis lattice constant, $K$ is the number of Cu or Fe ion on conducting plane in one unit cell, and $p$ is carrier concentration per such ion.

For thermal vortex array, $l_v = l_{TV}$ and $\gamma = \gamma_{TV}$, thus

$$\rho - \rho_0 = \frac{\pi}{\varepsilon^* n_c} \frac{k_B T}{e n_c}, \quad (13)$$

where $\gamma_{TV}$ is defined as the effective strength of carrier-thermal vortex scattering. For magnetic vortex array, $l_v = l_{MV}$ and $\gamma = \gamma_{MV}$, thus

$$\rho - \rho_0 = \frac{\pi}{4} \frac{\gamma_{MV} B}{e n_c}, \quad (14)$$

where $\gamma_{MV}$ is defined as the effective strength of carrier–magnetic vortex scattering. Defining $\alpha = (\rho - \rho_0) / T$ for thermal effect and $\beta = (\rho - \rho_0) / B$ for magnetic effect, we predict

$$\alpha = \frac{\pi}{2} \frac{k_B}{e} \frac{m^*}{n_c}, \quad (15)$$

$$\beta = \frac{\pi}{4} \frac{\gamma_{MV}}{e n_c}. \quad (16)$$

In equations (11) and (12), the periodicity term $l_v^{-2}$ characterizes the linear temperature and field dependence, the effective mass and carrier density $n_c$ characterize the doping dependence of slope, while $L_v$ characterizes the vortex type dependence in different materials. Therefore, with a universal scattering coefficient $\gamma$, we provide a unified explanation for the $T$-linear and $B$-linear laws of resistivity in strange metal for varying doping, material, and vortex type. In recent literature, the Planckian dissipation scenario [2, 5] or possible quantum diffusion [33] proposes empirically that the relaxation time is solely determined by the extrinsic energy scale. For thermal and magnetic effect, the most natural energy scale is $k_B T$ [5] and $\mu_B B$ [15], which predicts that $h / \tau = C_T k_B T$ or $C_M \mu_B B$. The present model based on $T$-symmetry-breaking analysis provide explicit derivation for the energy scale via equation (9), and predicts $C_T = (\pi/2) \gamma_{TV}$ and $C_M = (\pi/2) \gamma_{MV} m_e / m^*$, where $m_e$ is the electron mass.

3. Results

3.1. Validation for local magnetic fluctuations

Equations (3), (6) and (7) predict three essential physical quantities in experimental observations in $\mu$SR spectroscopy. If we choose $C_{TV} = 1$, these equations reveal that the $\mu$SR spectroscopy is only determined by $m^*$ and the Hubbard $U$, independent of any other details. For YBa$_2$Cu$_3$O$_{y}$, the effective mass is observed from dc transport and infrared spectroscopy in reference [34] to be nearly constant, i.e., $m^* = 2.6 \pm 1.7 m_e$ for various doping, and the previous microscopic simulations found that the Hubbard $U$ is 3–5 eV [35–37]. Therefore, equations (3) and (6) predict $B_{TV} = 0.60 \pm 0.23$ mT and $\tau_{TV} = 4.3 \pm 2.4$ ns at $T = 157 K$ (the boundary between pseudogap and strange metal phases), which are very close to Zhu et al.’s observations...
Figure 2. The temperature dependence of the local field strength in YBa$_2$Cu$_3$O$_{y}$ (a) and Sr$_2$Ir$_{1-x}$Rh$_x$O$_4$ (b). All symbols are data from $\mu$SR spectroscopy experiments in references [38,39]. Solid lines are predictions from equation (3) with $m^* = 2.6 \pm 1.7 m_e$ and $5 \pm 2 m_e$ chosen from the dc transport and infrared spectroscopy in reference [34] and the optical spectra in reference [40], respectively. Dashed lines represent the errorbar due to uncertainties of effective mass. The temperature dependence of the correlation time in YBa$_2$Cu$_3$O$_y$ (c) and Sr$_2$Ir$_{1-x}$Rh$_x$O$_4$ (d). Solid lines are predictions from equation (6) with $C_{TV} = 1$ and the Hubbard $U = 4 \pm 1$ eV and $2 \pm 1$ eV chosen from previous microscopic simulations [35–37, 41, 42], respectively. Dashed lines represent the errorbar due to uncertainties of the Hubbard $U$. 

for YBa$_2$Cu$_3$O$_{6.77}$, i.e., $B_{loc} = 0.71 \pm 0.06$ mT and $\tau_c = 6.7 \pm 1.1$ ns [38]. Furthermore, the constant signal predicted by equation (7) is $\lambda_{ZF} (T) = 4.4 \pm 3$ m s$^{-1}$, which is also very consistent with experimental observation, i.e., $1 \text{–} 3$ m s$^{-1}$.

These agreements are remarkable since no fitting parameter other than $C_{TV} = 1$ is involved. It would be helpful to compare the present analysis with other arguments. Note that Zhang et al. have examined the spin fluctuations idea and found that the Korringa relaxation due to spin fluctuations in band states predicts $\lambda_K \approx h^2 \gamma^2 B_{loc} k_B T / E_F$; to match experimental observations requires Fermi energy $E_F \approx 1$ eV and $B_{loc} \approx 12$ T, which is much too large under any known mechanism [28]. Similarly, from the simplest dimensional analysis, a characteristic vortex involving a flux quantum $(\hbar / 2e)$ on the same area ($\pi r_{TV}^2$) as our thermal vortex induces a characteristic magnetic field as $2m^* k_B T / e \hbar \approx 610$ T for YBa$_2$Cu$_3$O$_y$ at 157 K, which is almost six orders of magnitude larger than data (0.71 mT). It means that the circulating velocity of this vortex with a flux quantum is 10$^6$ times higher than the real thermal velocity in our thermal vortex with an angular momentum quantum. Therefore, both Korringa relaxation due to spin fluctuations and simple dimensional argument can be definitely ruled out.

Furthermore, it is interesting to examine the specific temperature dependence of the local magnetic field and its correlation time predicted by the present theory. As shown in figure 2, the predicted dependence is compared with recent data from YBa$_2$Cu$_3$O$_y$ and Sr$_2$Ir$_{1-x}$Rh$_x$O$_4$ [38, 39], and consistency with most data is found within errorbars, especially above the pseudogap or Néel temperature, i.e., $T \gtrsim 130$ K. Similarly, in these comparisons for Sr$_2$Ir$_{1-x}$Rh$_x$O$_4$, $C_{TV} = 1$, and the effective mass $m^* = 5 \pm 2 m_e$, which are consistently chosen from optical spectra data [40]; the Hubbard $U = 2 \pm 1$ eV is from previous microscopic simulations [41, 42]. Therefore, there is no fitting parameter involved. However, both the magnitudes and temperature scalings are found to be closely consistent with data, especially in figures 2(b) and (c). Since the 5d electrons with more extended orbitals in Sr$_2$Ir$_{1-x}$Rh$_x$O have weaker Coulomb repulsion than 3d electrons in YBa$_2$Cu$_3$O$_y$, equation (6) predicts that $\tau_{TV}$ for Sr$_2$Ir$_{1-x}$Rh$_x$O is smaller than for YBa$_2$Cu$_3$O$_{6.83}$, which is also consistent with data.

In conclusion, the assumption that the underlying quantum state of the strange metal is composed of mesoscopic thermal vortices yields analytical formulas for the local magnetic field strength, characteristic correlation time, and dynamic $\mu$SR rate. Without fitting parameters, predictions agree well with data of
impressive for a single parameter fitting. Since figure 3(a), taking the fitting parameter working field (55 T) for the measurement of result in a nonlinear from the Nernst signal [47, 48], the upper critical field for an optimally doped sample is close to the verification of equation (16).

YBa$_2$Cu$_3$O$_y$ and Sr$_2$Ir$_{1-x}$Rh$_x$O$_4$. However, it is worth mentioning that the present theory makes divergent predictions at zero temperature, which underestimate local magnetic field strength and $\mu$ SR rate but overestimate of correlation time below the pseudogap or Néel temperature. We will discuss the origin of this departure in the end.

3.2. Validation for doping dependence of slopes

In the following, we verify predictions of linear laws in equations (13)–(16). Here, the effective mass for La$_{2-x}$Sr$_x$CuO$_4$ is calculated by $m^*/m_e = 2.2 - 1.6x \pm 0.24$ determined from a linear fit of data in reference [44]. The fitting values of $\rho_0$ are 100, 70, 58, 40, and 20 $\mu$Scm for $p = 0.16$, 0.19, 0.22, 0.26 and 0.30. (b) Doping dependence of $\alpha$. Symbols are experimental values determined from the resistivity data [15,43,45,46]. The error bars are comparable with or smaller than the symbols. The solid blue line is the prediction from equation (15) with $m^*/m_e = 2.2 - 1.6x$ and $\gamma_F = 1.8$, while the dashed blue lines represent the error bar of the effective mass. The dashed orange line represents the prediction at the Planckian limit [2]. Material parameters used for the calculation of carrier density are listed in table S3 of the supplemental material (https://stacks.iop.org/NJP/23/043050/mmedia).

3.3. Validation for universal scattering coefficient

It would be important to determine the characteristic scattering coefficient $\gamma_0$ and verify its universality, especially near the QCP. To show the possible doping and material dependence of scattering coefficients, we need to determine thermal and magnetic scattering coefficients from resistivity slopes with equations (15)

**Figure 3.** Temperature slope of $\rho$ for La$_{2-x}$Sr$_x$CuO$_4$. (a) The temperature dependence of $\rho$. Symbols are data determined from reference [43]. Solid lines are predictions from $\rho_0 = \rho_0 + \alpha T$ and equation (13) with $\gamma_F = 1.8, \rho_0$ and the effective mass $m^*/m_e = 2.2 - 1.6x \pm 0.24$ determined from a linear fit of data in reference [44]. The fitting values of $\rho_0$ are 100, 70, 58, 40, and 20 $\mu$Scm for $p = 0.16, 0.19, 0.22, 0.26$ and 0.30. (b) Doping dependence of $\alpha$. Symbols are experimental values determined from the resistivity data [15,43,45,46]. The error bars are comparable with or smaller than the symbols. The solid blue line is the prediction from equation (15) with $m^*/m_e = 2.2 - 1.6x$ and $\gamma_F = 1.8$, while the dashed blue lines represent the error bar of the effective mass. The dashed orange line represents the prediction at the Planckian limit [2]. Material parameters used for the calculation of carrier density are listed in table S3 of the supplemental material (https://stacks.iop.org/NJP/23/043050/mmedia).
Figure 4. The magnetic field slope of $\rho$ for La$_{2-x}$Sr$_x$CuO$_4$. (a) The field dependence of $\rho$. Symbols are data determined from reference [45], solid lines are predictions from $\rho = \rho_0 + \beta B$ and equation (14) with $\gamma_{MV} = 1.4$ and $\rho_0 = 2.8$ and 11.5 $\mu\Omega$ cm for $p = 0.18, 0.21$ and 0.23, respectively. (b) Doping dependence of field slope $\beta$. Red and black symbols are experimental values determined from resistivity data [15,45]. The error bars are explained in supplemental section 1. Blue solid line is the prediction from equation (16) with $\gamma_{MV} = 1.4$. Material parameters used for calculation of carrier density are listed in table S3 of the supplemental material (https://stacks.iop.org/NJP/23/043050/mmedia).

Figure 5. The universal scattering coefficient in the vicinity of the QCP. Symbols are calculated values of $\gamma_{TV}$ or $\gamma_{MV}$ for cuprates (i.e. La$_{2-x}$Sr$_x$CuO$_4$, Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, YBa$_2$Cu$_3$O$_y$ and YBa$_2$Cu$_4$O$_8$) at the pseudogap QCP $x_c = 0.19$ and pnictides (i.e. BaFe$_2$(As$_{1-x}$P$_x$)$_2$) at the antiferromagnetism QCP $x_c = 0.31$ with equations (17) and (18) using experimental data of resistivity slopes ($\alpha$ and $\beta$) determined from references [15,33,45,46,50], the effective mass determined from references [34,44,51–55], lattice constants and carrier density determined from references [56–58]. The error bars are estimated from equations (17) and (18) by a simple propagation of data error. The solid and dashed black lines represent a condensate of these data, i.e., $\gamma_0 = 1.7 \pm 0.4$. Values of parameters used for predictions are listed in tables S1–S4, see supplemental section 5.

and (16) for each sample

$$\gamma_{TV} = \frac{2 e^2 h}{\pi} \frac{n_1}{k_B m^*} \alpha,$$

$$\gamma_{MV} = \frac{4}{\pi e n_c} \beta.$$  \hspace{1cm} (17) \hspace{1cm} (18)

According to equation (11), to determine $\gamma_0$, one needs to estimate angular momentum for the vortex. As discussed above, $L_v = \hbar$ for the thermal vortex in any strange metal and the magnetic vortex in cuprates. Therefore, $\gamma_0 = \gamma_{TV}$ and $\gamma_0 = \gamma_{MV}$ for both kinds of vortices in cuprates. In figure 5, both $\gamma_{TV}$ and $\gamma_{MV}$ for La$_{2-x}$Sr$_x$CuO$_4$ fluctuate around 1.7 near the pseudogap QCP at $x = 0.19$, revealing that $\gamma_0$ is universal not only for various doping but also for both vortex types. These observations provide vital support to the remarkable similarity between magnetic and thermal vortices near the QCP, and the existence of the criticality in particular.

Furthermore, it is intriguing to question if $\gamma_0$ is universal for iron-based HTSC near QCP. As discussed above, $L_v = 4\hbar$ for the magnetic vortex in iron-based HTSC. Therefore, $\gamma_0 = \gamma_{TV}$ and $\gamma_0 = \gamma_{MV}/4$ for the thermal vortex and the magnetic vortex, respectively. Indeed, the universality of $\gamma_0$ is verified to be true for BaFe$_2$(As$_{1-x}$P$_x$)$_2$ near its antiferromagnetism QCP ($x_c = 0.31$), as shown in figure 5, which further
As compound, predict the temperature and field slopes of resistivity for Bi$_2$Sr$_2$CaCu$_2$O$_8$ near the QCP at close to the pseudogap QCP at exchange couplings [49].

It would be interesting to verify the theory and the universality of the scattering coefficient for other compounds. In supplemental section 1–3, we use the universal scattering coefficient $\gamma_0 = 1.7 \pm 0.4$ to predict the temperature and field slopes of resistivity for Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, YBa$_2$Cu$_4$O$_8$, and YBa$_2$Cu$_3$O$_7$ close to the pseudogap QCP at $p_c = 0.19$, and find quantitative agreements in most samples (figures S1–S3). As shown in figure 5, it is intriguing to find that the mean values of the scattering coefficient for 80% of 48 samples locates in the range of $\gamma_0 = 1.7 \pm 0.4$, supporting the universality of the scattering coefficient. As $\gamma_0$ is close to $\pi/2$, we speculate that its nature may originate geometrically from circular motions on the conducting plane. Future studies should clarify the boundary for this universality, which would contribute to understanding universal versus specific features of strongly correlated materials.

### 3.4. Planckian limit for quantum dissipation

The universality of $\gamma_0$ enables us to highlight the similarity between magnetic and thermal vortices, and further redefine a unified Planckian limit for quantum dissipations in the strange metal. Based on equation (11), this similarity is quantified by the normalized ratio between the scattering coefficient of the magnetic and thermal vortices,

$$\frac{\gamma_{MV}}{\gamma_{TV}} \frac{L_{TV}}{L_{MV}} = 1.$$  \hspace{1cm} (19)

As shown in figure 6, this prediction is quantitatively verified by data in La$_{2-x}$Sr$_x$CuO$_4$ samples (as well as two samples of YBa$_2$Cu$_4$O$_8$, see figure S2 in supplemental section 3) near $x_c = 0.19$ and BaFe$_2$(As$_{1-x}$P$_x$)$_2$ samples near the QCP at $x_c = 0.31$. For cuprates, we find also $\gamma_{MV}/\gamma_{TV} = L_{MV}/L_{TV} = 1$, revealing a physical equivalency between magnetic and thermal vortices. Meanwhile, for iron-based HTSC, $\gamma_{MV}/\gamma_{TV} = L_{MV}/L_{TV} = 4$ revealing a strength of vortex current correction to the exact equivalence between magnetic and thermal vortices.

This similarity cannot be explained by the current Planckian dissipation theory, as we argue below. A simple extension of the Planckian dissipation scenario [2, 5] to magnetic effect yields the similarity as

$$\frac{h}{\tau} = C_T k_B T$$

with $C_T = C_M$ predicting $\gamma_{MV}/\gamma_{TV} = m'/m_c$. Indeed, for BaFe$_2$(As$_{1-x}$P$_x$)$_2$ at $x_c = 0.31$, $C_T = 2.5 \pm 0.8$ and $C_M = 2.7 \pm 0.8$, satisfying the similarity condition. Furthermore, for this compound, $m'/m_c = 4.5 \pm 1.1 \approx L_{MV}/L_{TV} = 4$, yields similar prediction (see magenta dashed line in figure 6) as ours. However, this extension makes an over prediction of $\gamma_{MV}/\gamma_{TV}(L_{TV}/L_{MV})$ for La$_{2-x}$Sr$_x$CuO$_4$ (see dashed black line in figure 6) since $C_T \approx 2.3 \pm 0.3 > C_M \approx 1.1 \pm 0.2$ near the QCP of La$_{2-x}$Sr$_x$CuO$_4$, breaking the similarity condition. This also happen for YBa$_2$Cu$_4$O$_8$ since $C_T \approx 2.5 \pm 0.1 > C_M \approx 1.8 \pm 0.1$. 

![Figure 6. The normalized ratio between scattering coefficients of magnetic and thermal vortices.](image-url)
A further problem for the Planckian dissipation scenario is that scattering rates near QCP in both BaFe$_2$(As$_{1-x}$P$_x$)$_2$ and La$_{2-x}$Sr$_x$CuO$_4$ break the simple Planckian limit, i.e., $C_T = 1$ [2]. As shown in figure 3(b), this Planckian limit makes a significant underestimation (the orange dashed line) for the temperature slope. One way to remedy the discrepancy is to assume a higher effective mass, i.e., $m^*/m_e = 5.1 - 3.7x$, which may be, however, too high to reconcile with experimental measurements from ARPES and theoretical simulation [44]. On the other hand, for the same material, the characteristic magnetic slope requires $C_M \approx 0.48$, which is significantly lower than the magnetic version of the Planckian limit, i.e., $C_M = 1$, revealing there is no way for both $C_T$ and $C_M$ to reach the simple Planckian limit due to $C_T \neq C_M$.

In summary, the former Planckian dissipation scenario from an energy balance perspective hardly provides a unified and precise explanation for magnetic and thermal linear laws in both cuprate and iron-based HTSC. In contrast, the present symmetry-breaking perspective predicts universal scattering energy determined by varying order length of the vortex for Planckian dissipation

$$\frac{\hbar}{\tau} = \gamma_0 \frac{hL_v}{m^*l_v^*}, \quad (20)$$

where $L_v$ is the characteristic angular momentum. It is worthy of emphasizing the importance of the vortex radius determined by magnetic length or thermal de Broglie wavelength to scattering rate. The energy scale equation (20) can yield a unified description for both linear laws in different doping, material, as well as quantifications of magnetic and thermal effects. Furthermore, the universality of the characteristic scattering coefficient $\gamma_0$ enables us to describe a new Planckian limit for the magnetic and thermal effects

$$\frac{\hbar}{\tau} = \frac{\pi}{2} \gamma_0 k_B T, \quad (21)$$

$$\frac{\hbar}{\tau} = \frac{\pi}{2} \gamma_0 \frac{L_{MV}}{h} \frac{m_e}{m^*} \mu_B B. \quad (22)$$

In these two equations, the difference between the magnetic and thermal effects is solely characterized by the angular momentum $L_{MV}$ of the magnetic vortex (or ion spin) and effective mass $m^*$, which are crucial elements to the unification of the Planckian limit. As shown in figure 5, $\gamma_0$ is universal for varying doping and vortex types in both BaFe$_2$(As$_{1-x}$P$_x$)$_2$ and La$_{2-x}$Sr$_x$CuO$_4$ near the QCP, revealing that the temperature and field effects similarly reach the Planckian limit in these samples. Therefore, our theory provides a derivation for the phenomenological proposed Planckian dissipation scenario [2, 5, 33] with a redefinition of the Planckian limit.

4. Discussion and conclusion

In this work, we assert that the strange metal of HTSC is composed of emergent mesoscopic quantum vortices locally breaking $T$-symmetry. As discussed in the introduction, the many-body entangled chaos and the nanoscale turbulence arguments exhibit a vortex similarity to the critical fluctuations of the loop current order. Therefore, our mesoscopic-vortex theory may be viewed as a phenomenological synthesis of the many-body entangled quantum states [20, 22] and the marginal Fermi liquid based on the loop-current fluctuations [13, 14], but with more quantitative prediction power. In contrast to previous works [13, 14, 17–22], the model predicts, in a unified way, the local magnetic field, the vortex lifetime, the linear-in-field resistivity, validated by data of dozens of samples. These findings offer critical fundamental insights regarding the underlying quantum state of strange metal and its unconventional quantum dissipation mechanism. In general, this novel phenomenological paradigm for strongly correlated materials involves a cross-scale coupling between the ‘mesoscopic order’ emergent from the microscopic strong electron–electron correlations, and the ‘quantum dissipation by order fluctuations’ of the non-Fermi liquid.

Contrary to the infrequent observations of the loop-current order, our model explains both the fluctuating magnetism and the Planckian dissipation for a wide class of strongly correlated materials. This is achieved for the following reasons: the fluctuating vortex pattern proposed in this work is a local current excitation at high temperature (e.g., $>100$ K for YBa$_2$Cu$_3$O$_{6,77}$), which is favored by strong electron–electron repulsions for it avoids the excessive double occupations of the extended plane-wave excitations. The mean-field loop-current order is the condensate of these fluctuating vortices but must compete with other ordered phases, such as antiferromagnetism and superconductivity, at lower temperatures (e.g., $<80$ K for YBa$_2$Cu$_3$O$_{6,77}$). Therefore, the low-temperature loop-current order may be infrequent due to its energy disadvantage, but fluctuating vortices may be widespread at the high-temperature metal phase of strongly correlated materials. However, the validity of our mesoscopic
vortex scenario is in principle constrained by the coexistence of the time-reversal and spatial translation symmetry breakings. Therefore, the upper temperature and field limits of this regime are reached when vortices’ periods equal the in-plane lattice constants, after which the spatial translation symmetry is preserved. The metal state above this limit should be determined by the microscopic intra-unit-cell physics rather than the mesoscopic vortex. On the other hand, its lower limits are the onset temperature and field of the mean-field of the loop current order or superconductivity.

In the present work, we mainly focus on linear laws by only considering the high-temperature state without the magnetic field or the strong-field state near zero temperature. But our argument can be readily carried out for intermediate states involving the coupling effect. At the intermediate temperature, an increasing magnetic field would introduce extra flux, driving the system to a mixed-state of magnetic and thermal vortices. Using the vortex Hamiltonian within the Debye–Huckel mean-field approximation [59], we obtain a vortex density as \( \sqrt{\frac{T^2}{h^2}} + \frac{\beta B}{l} \). Based on equation (12), this nonlinear form yields a resistivity as \( \frac{\alpha^2 T^2}{\beta B^2} \), which is also fully confirmed near QCP in BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\) and reported in reference [4]. This interesting result supports the validity of equation (12) and would be quantitatively discussed elsewhere.

Section 2.2 proposes that the vortex lifetime positively correlates with Hubbard \( U \) rather than normalized interactions of the loop-current order. The key reason is that the collective modes predicted by the renormalized quantum model have characteristic energies of 35–110 meV, which results in a much faster fluctuation time (4 to 5 orders of magnitude lower) than experimental observations. More importantly, the vortex-antivortex annihilation is a typical cross-scale process involving both mesoscopic motions and microscopic electron–electron interactions, as figures 2(c) and (d) indicate. Similarly, it is worthy noting that in helium superfluid, the decay of vortex lines involves multiscale dissipative processes, i.e., vortex reconnections, Kelvin waves and phonons (or Caroli–Matricon excitations) excitations [60, 61]. Therefore, in the future, it would be intriguing to specify the cross-scale dissipation mechanism of fluctuating vortices in strange metal states and compare it with the dynamics in liquid helium and the classical turbulent dissipations.

The emphasis of the common mesoscopic physics of the fluctuating vortex breaking \( T \)-symmetry is the cornerstone for understanding the quantum state of the strange metal phase. It would shed light on the development of further comprehensive microscopic theories. For instance, the present work suggests modeling the ensemble-averaged vortex structure in many-body theory of quantum chaos [20], the magnetic-field-induced loop-current order, and its critical fluctuations for the marginal Fermi liquid [13, 14], so as to explain the microscopic nature of the local magnetic field and the linear magnetoresistance. Besides, the microscopic emergence of our mesoscopic vortices from the loop-current order should be quantitatively calculated from the effective Hamiltonian [13, 30, 62] to explain the single-particle spectrum in the strange metal phase. Furthermore, we have only proposed the phenomenology of the fluctuating thermal vortices for the high-temperature state. While for the low-temperature pseudogap phase, Varma has examined the possibility that periodic domains of the loop-current order generate the observed pseudogap and Fermi arcs [30]. In the future, these two states should be combined to describe the transition or crossover from the pseudogap phase to the strange metal phase.

For strongly correlated materials, fluctuating order with local symmetry-breaking is ubiquitous [7]. For the pseudogap phase of cuprate HTSC, there are spin- and charge-density-wave (SDW and CDW), locally breaking spatial translation symmetry. Therefore, it would be reasonable to assume that the carrier Umklapp scattering by fluctuating SDW/CDW is an effective mechanism dominating the anomalous transport in the pseudogap phase. If this scenario, discussed in detail elsewhere, is confirmed, a universal mechanism of the quantum dissipation for non-Fermi liquid in HTSC would rise, breaking through ‘the bottom line’ for cuprates, since ‘the existing theoretical machinery seems inadequate to describe both the rich physics of the pseudogap phase and the nature of the strange metal phase’ [7].

Acknowledgments

We thank Yi-feng Yang and Tao Li for many constructive suggestions. We thank Hui Qian Luo and Lei Shu for helpful discussions. We thank Chandra M Varma for sharing his recent work. This work is partially supported by the National Natural Science Foundation of China (Grants No. 11452002).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).
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