Models of recognition operators defined in the space of large dimension attributes

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Abstract. The problems of constructing modified recognition operators are considered taking into account the high dimensionality of the feature space. As a basic algorithm, models of recognition algorithms based on radial functions are considered. The main idea of the model of modified recognition operators is the formation of independent subsets of interrelated objects and the allocation of basic objects for each subset of tightly coupled objects. A distinctive feature of the proposed model of algorithms is to determine the appropriate set of distance functions when building a model of discriminating operators. To test the performance of the proposed object model, experimental studies were carried out in solving a number of problems. The main advantage of the proposed recognition operators is the improvement of accuracy and a significant reduction in the volume of computational operations in recognition of unknown objects, which allows them to be used in the construction of recognition systems operating in real time.

1. Introduction

One of the main and most rapidly developing areas of applied mathematics and computer science is the theory and methods of pattern recognition. This is due to the fact that in recent years the use of these methods has been expanding in solving various applied problems. They help to solve, mainly, the tasks of geological forecasting, the tasks of medical and technical diagnostics, the tasks of identifying a person’s personality through a face image, speech recognition tasks and other applied tasks. Therefore, an ever wider circle of specialists is paying attention to the pattern recognition problem.

In practice, often, the objects to be recognized are described by a large number of signs. This circumstance inevitably entails some correlation of the signs. Classically, when solving such problems, they resorted to some preliminary processing of the attribute space, as a result of which there was often a loss of important information about the structure of objects, which allowed them to be more effectively classified. In this regard, the relevance of the problem of improving, developing
and researching models of recognition algorithms (RAs), oriented to solving problems of diagnosing, predicting and classifying objects defined in a large feature space (FS), is increasing. The goal of this work is to construct recognition operators (ROs) for solving the problem of classifying objects defined in a high-dimensional attribute space. The model of RAs based on radial functions is considered as an initial model of algorithms [1–3].

To achieve this goal it is necessary to solve the following tasks:

- analyze existing models of RAs and determine the range of tasks to be solved;
- develop a model for recognizing operators based on radial functions;
- conduct experimental studies to assess the effectiveness of the developed ROs.

The model based on the use of radial functions is considered as an initial model of RAs. The main idea of the proposed model is to build ROs based on an assessment of the interconnectedness of objects. It should be noted that this work is a logical continuation of the research of the scientific school of Academician of RAS Yu.I. Zhuravlev and undefined concepts used in this work are given in the works [4-7].

The object of the study are recognizing operators based on radial functions. The subject of the study is the construction of modified ROs based on radial functions, taking into account the large dimension of the FS.

Scientifically, the results of this work represent a new solution to a scientific problem related to the construction of RAs under the condition of high dimensionality of the FS. The practical significance of the obtained results lies in the fact that the developed algorithms can be applied when solving applied problems under conditions of high dimensionality of the FS (for example, when a person is identified by a signature image).

2. Analysis of existing solutions

It is known that the development of the theory of pattern recognition is divided into two stages. The first stage of development was in the nature of projects of various technical devices or algorithms for solving specific applied problems. The value of the developed methods of pattern recognition is determined primarily by the achieved experimental results. [8, 9]. The second stage of the development of the pattern recognition theory is characterized by the transition from individual algorithms to the construction of models - a family of algorithms for a unified description of methods for solving classification problems [5, 9]. At this stage of development, Yu.I. Zhuravlyov showed that an arbitrary RA can be represented as a sequential execution of the operators $B$ (recognizing operator) and $C$ (decision rule) [5, 7]:

$$ A = B \circ C. $$

From (1) it follows that each RA $A$ can be divided into two successive stages. At the first stage, the discriminating operator $B$ converts the admissible object $S_u$ into a numerical estimate represented by the vector $\tilde{b}_u$:

$$ B(S_u) = \tilde{b}_u, $$

where $\tilde{b}_u = (\tilde{b}_{u1}, \ldots, \tilde{b}_{uj}, \ldots, \tilde{b}_{ul}).$

In the second stage, the numerical estimate $\tilde{b}_{uj}$ decision rule $C$ determines the value of the predicate $P_j(S_u)$ [5, 7]:

$$ C(\tilde{b}_{uj}) = \begin{cases} 0, & \text{if } \tilde{b}_{uj} < c_1; \\ \Delta, & \text{if } c_1 \leq \tilde{b}_{uj} \leq c_2; \\ 1, & \text{if } \tilde{b}_{uj} > c_2, \end{cases} $$

where $c_1, c_2$ – are the parameters of the decision rule (2). In this case, the estimate $b_{uv}$ is calculated using the operator (3).
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Consider the set of admissible objects $\mathcal{O}$, which consists of $l$ subsets (classes) $\mathcal{K}_1, \ldots, \mathcal{K}_l$:

$$\mathcal{O} = \bigcup_{j=1}^{l} \mathcal{K}_j, \mathcal{K}_i \cap \mathcal{K}_j = \emptyset, i \neq j, i, j \in \{1, 2, \ldots, l\}. \tag{4}$$

In this case, it is assumed that the partition (4) is not completely defined. However, there is only some initial information $I_0$ about the classes $\mathcal{K}_1, \ldots, \mathcal{K}_l, \mathcal{K}_i$:

$$I_0 = \{ S_1, \tilde{\alpha}(S_1), \ldots, S_u, \tilde{\alpha}(S_u), \ldots, S_m, \tilde{\alpha}(S_m) \}, \tag{5}$$

where $\tilde{\alpha}(S_u)$ is the information vector of the object $S_u (S_u \in \mathcal{O})$: $\tilde{\alpha}(S_u) = (\alpha_{u1}, \ldots, \alpha_{uj}, \ldots, \alpha_{ul})$.

Here $\alpha_{uij}$ is the value of the predicate $P_j(S_u) = \"S_u \in \mathcal{K}_j\"$:

$$P_j(S_u) = \begin{cases} 1, & \text{if } S_u \in \mathcal{K}_j; \\ 0, & \text{if } S_u \notin \mathcal{K}_j. \end{cases} \tag{6}$$

Let a set of control objects $S^q = \{ S'_1, \ldots, S'_l, \ldots, S'_k \} (S^q \subset \mathcal{O})$ be given. It is known [5–7, 12] that in the FS $X = (x_1, \ldots, x_i, \ldots, x_n)$ for an arbitrary object $S_u (S_u \in \mathcal{O})$ you can match the $l$-dimensional vector $\mathbf{a} = (a_{u1}, \ldots, a_{ui}, \ldots, a_{ul})$. The dimension of the FS is quite large. The task is to build such a RO $B$, which, using the decision rule $C$, translates the set $(I_0, S^q)$ into the information matrix $[\beta_{uv}]_{q \times l}$, where $\beta_{ij}$ is interpreted as in [5, 7, 9].

4. Proposed approach

The paper considers an original approach to solving the problem of constructing ROs of images defined in a high-dimensional attribute space. Based on this approach, a model of modified ROs based on radial functions was developed [21]. The main idea of the proposed model is to form a set of independent (or weakly dependent) objects based on an assessment of the interconnectedness of objects.

Defining a model of recognizing operators based on an assessment of the interconnectedness of objects includes the following main steps:

1. Definition of “independent” subsets of tightly bound objects. At this stage, a system of “independent” subsets of objects is determined. The studied set of objects belongs to one subset if they are close enough (in a sense) to each other. Otherwise, they belong to different subsets of objects. Let $E_u$ ($u = 1, \ldots, k$) be a subset of tightly bound objects. The measure of proximity $L(E_p, E_q)$ between subsets of $E_p$ and $E_q$ can be specified in various ways, for example:

$$L(E_p, E_q) = \max_{S_i \in E_p, S_j \in E_q} \Phi(S_i, S_j), \tag{7}$$

where $\Phi(S_i, S_j)$ is a function that characterizes the strength of paired bonds between objects $S_i$ and $S_j$, and satisfies the following conditions:

1. $\Phi(S_i, S_j) = \Phi(S_j, S_i)$. \tag{8}
2. $r_1 \leq \Phi(S_i, S_j) \leq r_2$ ($r_1, r_2$ - are real numbers), \tag{9}
3. $\Phi(S_i, S_i) > \Phi(S_i, S_j), i \neq j$. \tag{10}
Undoubtedly, depending on the method of specifying the proximity measure \( L(E_p, E_q) \) between \( E_p \) and \( E_q \) one can obtain various algorithms for the selection of independent subsets of tightly bound objects.

2. **Determination of the base object in each subset of tightly bound objects and the formation of the FS.**

In each subset \( E_u \) the base object \( b_u = (b_{u1}, b_{u2}, ..., b_{un}) \) determined by the formula

\[
b_u = \sum_{q=1}^{m} E_{uq} \left| a_{q1}, ... a_{q1}, ... a_{qT} \right|, S_q \in E_u.
\]

(11)

Let \( b_1, b_2, ..., b_k \) be basic objects. Then the dependency model can be specified as

\[
S = F(\Lambda, \Xi), \quad \Lambda = (\lambda_1, \lambda_2, ..., \lambda_k), \quad \Xi = (b_1, b_2, ..., b_k),
\]

(12)

where \( F \) is a function from some given class \( \{F\} \), for example, linear; \( \Lambda \) is the feature vector of the object \( S \), which defines a specific dependence with a given accuracy.

The calculation of the value of the vector of unknown features \( \Lambda \) determines the description of the object \( S \) in the FS \( \Xi(S = (x_1, ... x_i, ... x_k)) \). In this case, it is assumed that \( \Lambda \in \Xi, \Lambda = (\lambda_1, \lambda_2, ..., \lambda_k) \). Depending on the definition of the parametric type \( F(\Lambda, \Xi) \) and the method of determining \( \Lambda \), we obtain various ROs.

3. **Determination of the proximity function \( d_u(S_p, S_q) \) between the objects \( S_p \) and \( S_q \) in a two-dimensional subspace of features.**

At this stage, a function is defined that characterizes the similarity of objects \( S_p \) and \( S_q \) in a two-dimensional subspace of features \( D = (D_1, ..., D_u, ..., D_n), D_u = (x_{u1}, x_{u2}), x_{u1}, x_{u2} \in E \). The distances between these objects in the subspace of attributes \( D_u \) are determined as follows [13]:

\[
\rho_u(S_p, S_q) = \sqrt{\sum_{i=1}^{2}(\lambda_{p,ui} - \lambda_{q,vi})^2}.
\]

(13)

In pattern recognition problems, the square of the distance (3) can be taken as the measure of similarity in \( D_u \):

\[
d_u(S_p, S_q) = \rho_u^2(S_p, S_q) = \sum_{i=1}^{2}(\lambda_{p,ui} - \lambda_{q,vi})^2.
\]

(14)

Then, using (4), we introduce the concept of a first-level proximity function. The proximity function of the first level is the radial function defined in the two-dimensional subspace of features \( D_u \):

\[
\mu_{u1}(S_p, S_q) = 1/(1 + \gamma_{u1}^2 d_u(S_p, S_q))
\]

(15)

where \( \gamma_{u1} \) is the parameter of the RO used in constructing the first-level proximity function.

The set of first-level proximity functions is denoted by \( \mathcal{B}_1 \).

The second level proximity function is the radial function defined in the subspace of features \( D_u \) and \( D_v \):

\[
\mu_{uv}(S_p, S_q) = 1/(1 + \gamma_{u2}^2 d_u(S_p, S_q) + \gamma_{v2}^2 d_v(S_p, S_q))
\]

(16)

where \( \gamma_{u2} \) and \( \gamma_{v2} \) are the parameters of the RO used when constructing the second-level proximity function.

The set of second-level proximity functions is denoted by \( \mathcal{B}_2 \).

The \( k \)-th level proximity function is the radial function defined in the subspace of features \( D_{u1}, ..., D_{uk} \):

\[
\mu_{ku}(S_p, S_q) = 1/(1 + \gamma_{ku1}^2 d_{u1}(S_p, S_q) + ... + \gamma_{ku2}^2 d_{uk}(S_p, S_q))
\]

(17)

where \( \gamma_{ku} \) is the parameter of the discriminating operator used in constructing the \( k \)-th level proximity function.

The set of \( k \)-level proximity functions is denoted by \( \mathcal{B}_k \).

5. **Calculate the proximity score at each level.** The numerical characteristic \( B_k(S_p, S_q) \), which is called the \( k \)-th proximity, is calculated. The rating is determined by the value of the proximity function by objects in the selected level of the proximity function:

\[
B_k(S_p, S_q) = \sum_{i=1}^{n} v_i \mu_{ki}(S_p, S_q).
\]

(18)
where \( \nu_u \) the given parameter of the RO.

6. Calculate grade for class at each level. Let \( \Gamma_i(S_p, S_q)(S_p \in \mathcal{K}_j) \) be calculated. The grade for the class is determined as follows:

\[
B_k(\mathcal{K}_j, S_q) = \sum_{S_p \in \mathcal{K}_j} \tau_p B_k(S_p, S_q),
\]

where \( \tau_p \) is the given parameter of the RO, indicating the importance of the object \( S_p \).

7. Estimation for class \( \mathcal{K}_j \) for all levels. Let each grade for a class, at each level, correspond to a numeric parameter \( \gamma_i \). The estimate for all levels is

\[
B(\mathcal{K}_j, S_q) = \sum_{k=1}^{t} \xi_k B_k(\mathcal{K}_j, S_q),
\]

where \( \gamma_i \) is the given parameter of the RO.

Thus, we have defined a class of modified ROs based on radial functions. An arbitrary algorithm \( A \) from this model is completely determined by specifying a set of parameters \( \hat{\pi} \). The set of all RAs from the proposed model is denoted by \( A(\hat{\pi}, \mathcal{K}) \). The search for the best recognizing operator is carried out in the parameter space \( \hat{\pi} \) [9, 21].

5. Experiment and Results

For practical use of the considered algorithm, programs for the determination of representative features in the C ++ language have been developed. The efficiency of the developed programs will illustrate the solution of a number of problems, in particular the model and practical problems. The following models of RAs were chosen: the classical model of RAs based on potential functions (\( A_1 \)) [13, 14], and the model (\( A_2 \)), proposed in this work. A comparative analysis of the listed models of RAs for solving the considered problem was carried out according to the following criteria: 1) recognition accuracy of test sample objects; 2) time spent on training; 3) the time spent on the recognition of objects from the control sample. It should be noted that the temporal characteristics of the algorithms are measured in seconds.

To calculate the specified criteria when solving the considered problem, in order to exclude successful (or unsuccessful) splitting of the initial sample into two parts (sample for training and sample for control), the sliding control method is used [22]. After the completion of the sliding exam procedure, the recognition accuracy and temporal indicators were determined as average. The experiments were carried out on a Pentium IV Dual Core 2.2 GHz computer with 1 Gb RAM.

5.1. Model problem

The source data of recognizable objects for a model task is generated in the FS. The number of classes in this experiment is two. The size of the initial sample is 1000 implementations (500 implementations for objects of each class). The number of attributes in the model example is 300. The distribution type is normal.

The following models of RAs were chosen: the classical model of RAs of the type of potential functions \( A_1 \), and the model (\( A_2 \)), proposed in the present work. The recognition accuracy in the control process for \( A_1 \) is 80.5\%, for \( A_2 \) – 95.7\%. The time spent on learning (on recognition of objects from the control sample) for \( A_1 \) is 4.141 (0.017), for \( A_2 \) – 8.272 (0.009).

Comparison of these results shows that the proposed model of RAs \( A_2 \) has improved the accuracy of recognition of objects described in the space of interrelated features (more than 15\% higher than \( A_1 \) and \( A_2 \)). This is due to the fact that the models \( A_1 \) and \( A_2 \) do not take into account the interrelatedness
of features. However, for model $A_2$ there is a certain increase in the learning time due to the implementation of an additional procedure for the formation of independent subsets of interrelated features.

5.2. Practical tasks
One of the main tasks related to the creation and implementation of information systems for the automation of processes is to ensure their stable and safe operation. This problem is solved using appropriate information security measures, for example, user authentication methods. It should be noted that the “friendliness” of the authorization process and the cost of its implementation depend on the choice of user authentication method.

Password and attribute user authentication methods are simple. The advantage of these methods is simplicity and ease of implementation. The disadvantages of these methods include: the ambiguity of user identification, the possibility of cheating the system by stealing or hacking a password or attribute. In addition, they do not provide for the possibility of detecting the substitution of an authorized user.

Methods of user authentication based on biometric parameters are considered more effective and complex. The advantage of these methods is the relative high accuracy of compliance checking, user-friendliness and low risk of replacing an authorized user. The disadvantages include the relative high cost and algorithmic complexity in the implementation.

The method of user authentication using keyboard handwriting makes it possible to ensure high efficiency of the user authentication process at minimal cost. It is known that keyboard typing is a biometric attribute characterizing the way a person works on a keyboard. To estimate it, the following parameters are used [23, 24]:

1. Determination of the time interval between keystrokes. The interval between keystrokes is the time interval $t_{\text{int}}$ between releasing the previous key and pressing the current key:

   $$t_{\text{int}} = t_2 - t_1.$$

2. Determine the key hold time. The key hold time is the time period $t_{\text{hold}}$, during which the key remains pressed: $t_{\text{hold}} = t_1 - t_0$.

3. Determining the number of errors in typing. The number of errors is the number of errors made by $n_{\text{mis}}$ when typing a key phrase. This parameter is determined by pressing the “backspace” or “delete” key when typing a key phrase.

4. Determining the number of overlaps between the keys. Overlap between keys is characterized by events in which the current key is not released and the next key is pressed.

5. Determination of the degree of arrhythmia in recruitment. Arrhythmia when typing is a percentage that characterizes how many percent the average average value of the interval between keys of a training sample differs from the average value of the interval between keys for a recognizable implementation of a key phrase.

   $$S_{\text{ar}} = \left( \frac{t_{\text{int}}^{\text{average}} \times 100}{t_{\text{int}}^{\text{rec}}} \right),$$

   where $t_{\text{int}}^{\text{average}}$ – the average value of the interval between the keys of the training sample for the key phrase; $t_{\text{int}}^{\text{rec}}$ – is the average value of the key spacing for a recognizable implementation of the key phrase.

6. Determining the performance of a set. The performance of a set is characterized by the number of characters typed per unit of time and is determined by the formula

   $$V = \frac{C_W}{t},$$

   where $C_W$ – is the number of characters in the key phrase; $t$ is the time taken to set a key phrase.

To assess the performance of the proposed RAs, experimental studies have been carried out in solving the authentication task. As a key phrase, a phrase consisting of 29 characters is considered. The
number of classes is \( l = 5 \), the sample size is \( m = 250 \) objects. Moreover, the number of objects in each class is the same (that is, \( |R_j| = 50, \quad j = 1, 5 \)).

Table 1 shows the results of experiments on the considered problem. It should be noted that the values of the parameters that characterize the work of the implemented algorithms are presented in averaged form.

Table 1. The results of user authentication by keyboard handwriting.

| Recognizing operators | Time (insec.) | Recognition accuracy (in percents) |
|-----------------------|---------------|-----------------------------------|
| \( A_1 \)            | 1.04776219    | 0.08250595                        | 74.74     |
| \( A_2 \)            | 6.85794636    | 0.03781879                        | 94.03     |

A comparison of the obtained results shows (see Table 1) that the proposed model of RAs \( A_2 \) has improved the recognition accuracy (about 20% higher than with \( A_1 \)). This is explained by the fact that the \( A_1 \) model does not take into account the hierarchical feature of the feature set when comparing them. However, for model \( A_2 \) there is a certain increase in the learning time due to the implementation of an additional procedure for the formation of independent subsets of interrelated features.

**Conclusion**

An approach based on an assessment of the interconnectedness of objects is proposed, and on the basis of this approach a model of modified discriminating operators based on radial functions is constructed. The main idea of the proposed model is based on the definition of independent and basic objects. This model is focused on solving the problem of recognizing objects defined in the space of features of a large dimension.

The results of solving a model and practical problem showed that the proposed model of recognizing operators improves the accuracy of recognizing objects specified in the space of features of a large dimension, and reduces the time spent on recognition of objects from the control sample. It can be used in the preparation of various programs aimed at solving problems of diagnostics, classification and management of objects in difficultly formalized areas of science and technology.

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