Low-lying $s = +1$ Pentaquark states in the Inherent Nodal Structure Analysis

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Abstract

The strangeness $s = +1$ pentaquark states as $qqqg\bar{q}$ clusters are investigated in this letter. Starting from the inherent geometric symmetry, we analyzed the inherent nodal structure of the system. As the nodeless states, the low-lying states are picked out. Then the S-wave state $(J^P, T) = (\frac{1}{2}^-, 0)$ and P-wave state $(J^P, T) = (\frac{1}{2}^+, 0)$ may be the candidates of low-lying pentaquark states. By comparing the accessibility of the two states and referring the presently obtained K-N interaction potential, we propose that the quantum numbers of the observed pentaquark state $\Theta^+$ may be $(J^P, T) = (\frac{1}{2}^+, 0)$ and $L = 1$.

Keywords: pentaquark, quark model, inherent nodal structure, QCD

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It has been proved that the quantum chromodynamics (QCD) is the underlying theory for strong interaction, and the quark model is remarkably successful in classifying the hadrons as composite systems of quark and antiquark. Meanwhile the existence of color-singlet multiquark systems \( q^n\bar{q}^m, m + n > 3 \) has also been predicted in QCD. This subject is very important, since it is an appropriate place to investigate the quark behavior in the short distance, which may shed a light on new physics.

Recently, LEPS\[1\], DIANA\[2\], CLAS\[3\], SAPHIR\[4\] and many others\[5, 6, 7, 8, 9\] reported that they all observed a new resonance \( \Theta \), with strangeness \( s = +1 \). This is probably a 4-quark and 1-antiquark system. If it is really a pentaquark state, it will be the first multiquark state observed in experiment. There have been then many theoretical works to explain the properties of the \( \Theta^+ \). For example, the mass of \( \Theta^+ \) has been calculated in the chiral soliton model\[10, 11, 12\], Skyrme model\[13\], diquark-triquark cluster model\[14, 15, 16\], constituent quark model\[17, 18, 19\], chiral SU(3) quark model\[20\], QCD sum rules\[21, 22, 23\], Lattice QCD\[24, 25, 26\], large \( N_c \) QCD\[27\], chiral perturbation approach\[28\], and so on. Meanwhile, the general framework of QCD\[29, 30, 31, 32, 33\] and the group theoretical classification\[34, 35\] have also been implemented to explore the quantum numbers of the pentaquark state \( \Theta^+ \). Nevertheless, the quantum numbers of the \( \Theta^+ \) have not yet been determined uniquely due to the dynamical model dependence. Especially, the parity of the lowest-lying pentaquark state is more controversial. On the other hand, just based on the symmetry of the intrinsic color-flavor space, many configurations are possible for the pentaquark state (see, for example, Ref.\[35\]). If one can fix which configuration is the practical building block of the state, it will be much helpful to assign the quantum numbers and to perform the numerical calculation. If one can determine the quantum numbers (including the parity) model independently, the result may be more reliable. Fortunately, it has been known that the features of quantum states depend on the distribution of wave-functions in the coordinate space, more specifically, on the inherent nodal structure of the wave-functions. The applications of the inherent nodal structure analysis approach to six-nucleon and six-quark systems\[36, 37\] show that analyzing the inherent nodal structure can uncover why a state with a specific set of quantum numbers is lower or higher, and how the wave-function
of a specific state is distributed in the specific way in coordinate space. Then one can pick out the real accessible configuration from the quite large configuration space and fix the quantum numbers. We will thus take the inherent nodal structure analysis approach to fix the quantum numbers of the low-lying pentaquark states model independently in this letter.

It has been well known that, in quantum physics, the point for the wave-function to be zero is a node. The set of the variables for the node to appear spans a nodal surface. Meanwhile, for usual quantum system, the less nodes the configuration contain, the lower energy the state has. For example, for the particle in an infinite well with width $a$, the relation between the number $n$ of the nodes of the wave-function and the energy of the state is $E_n = \frac{(n+1)^2\pi^2\hbar^2}{2ma^2}$. For the particle in a harmonic oscillating potential, the relation is $E_n = (n + \frac{1}{2})\hbar\omega$. The nodal surface is usually classified into the dynamical one and the inherent one. The dynamical nodal surface depends on the dynamics of the system. The inherent nodal surface (INS) relies on the inherent geometric configuration of the system.

Let $\mathcal{A}$ be a geometric configuration in the multi-dimensional coordinate space of a quantum system, $\hat{O}_i$ be an element of the operation on wave function $\Psi(\mathcal{A})$ of the system, we have

$$\hat{O}_i \Psi(\mathcal{A}) = \Psi(\hat{O}_i \mathcal{A}).$$  \hspace{1cm} (1)

If this set operation \{\hat{O}_i\} (with $m$ elements) leaves the configuration $\mathcal{A}$ invariant, i.e., $\hat{O}_i \mathcal{A} = \mathcal{A}$, we have

$$\hat{O}_i \Psi(\mathcal{A}) = \Psi(\mathcal{A}).$$  \hspace{1cm} (2)

Since the $\Psi(\mathcal{A})$ spans a representation of $\hat{O}_i$, Eq. (2) can be written in a matrix form, which is in fact a set of homogeneous linear algebraic equations. Therefore, associated with the $m$ operators, there are $m$ sets of homogeneous equations, that the $\Psi(\mathcal{A})$ must obey. However, a set of homogeneous equations does not always have non-zero solutions. In the case that common non-zero solutions fulfilling all the $m$ sets of equations do not exist, all $\Psi(\mathcal{A})$ must be zero at $\mathcal{A}$. Then a nodal surface appears. Since such a nodal surface is determined by the inherent geometric and intrinsic configuration but not by the dynamics at all, it is referred
to inherent nodal surface (INS). It indicates that, when the particles form a shape with a specific geometric symmetry, specific constraints are imposed on the wave-function. Only the inherent nodeless components are the accessible ones to the state. Then, with the inherent nodal structure analysis we can pick out the main components from the whole configuration space in a way independent of dynamics.

The wave function of the five-particle systems can usually be written as a coupling of the orbital part and the internal part. Since quark and antiquark are not identical to each other, the wave-function is not antisymmetric via the interchange between them. However, if we consider only the permutations among the 4 quarks, it should be antisymmetric, i.e. it has the symmetry $[1^4]$. It has been well known that the $s = +1$ system with light quarks possesses the internal symmetry $\text{SU}_{CTS}(12) \supset \text{SU}_C(3) \otimes \text{SU}_T(2) \otimes \text{SU}_S(2)$. Let $[f]_O$, $[f]_C$ and $[f]_{TS}$ be the irreducible representation(irrep) of the group associated with the orbital, color and isospin-spin space, respectively, we shall have

$$[1^4] \in [f]_O \otimes [f]_C \otimes [f]_{TS}.$$  

The lack of direct experimental observation of free “color charge” suggests that all the observable states should be SU(3) color-singlets. We can thus restrict our study to the systems whose 4 quarks have a color symmetry $[f]_C = [211]$. Then the configuration of the $[f]_O$ and $[f]_{TS}$ can be fixed with the group theoretical method. The obtained possible $[f]_{TS}$ and $[f]_O$ are listed in Table I. It is obvious that such a orbital and isospin-spin configuration space is very large. We should pick out the important ones for the low-lying pentaquark state. As mentioned above, we can do so by taking the inherent nodal structure analysis approach.

It has been known that the wave-function of the state with total angular momentum $J$, orbital parity $\pi$ and total isospin $T$ can be written as

$$\Psi = \sum_{L,S,\lambda} \Psi^S_{L\pi\lambda}, \quad (3)$$

with

$$\Psi^S_{L\pi\lambda} = \sum_{i,M,M_S} C^S_{LM,SM_S} F^{\pi \lambda \chi}_{LM \chi TS MS}, \quad (4)$$
Table 1: The irreducible representations of the isospin-spin symmetry corresponding to each possible orbital symmetry with color singlet restriction.

| \([f]_O\) | SU\(_{TS}(4)\) |
|----------|---------------|
| [4]      | [3 1]         |
| [3 1]    | [4], [3 1], [2 2], [2 1 1] |
| [2 2]    | [3 1], [2 1 1] |
| [2 1 1]  | [3 1], [2 2], [2 1 1], [1^4] |
| [1^4]    | [2 1 1]       |

where \(F_{LM}^{\pi \lambda_i}\) is a function of the spatial coordinates, \(\lambda\) denotes a representation of the \(S_4\) group (permutations among 4 quarks), and \(i\) specifies a basis state of this representation. The \(L, S\) is the total orbital angular momentum, the total spin of the 4 quarks, respectively. They are coupled to \(J'\) via the Clebsch-Gordan coefficients, and total spin \(J\) is formed by coupling the \(J'\) and the antiquark’s spin. The \(M, M_S\) and \(M_{J'}\) are the \(Z\)-components of \(L, S\) and \(J'\), respectively. The orbital parity of pentaquark is given by \(\pi = (-)^L\), and the total parity \(P = -\pi = (-)^{L+1}\), since the antiquark holds an instinct negative parity. \(\chi_{TSM_{\lambda}}\) is a state in the isospin-spin space with good quantum numbers \(T\) and \(S\), and belonging to the \(\tilde{\lambda}\)-representation, the conjugate of \(\lambda\). The \(\lambda\) contained in \(\Psi\) is determined by \(S\) and \(T\). The result is listed in Table 2. Such a \(\Psi_{S_{T \pi \lambda}}^{L}\) is usually denoted as a \(\lambda\)-component of \(\Psi\).

Let \(i'-j'-k'\) be a body frame, the spatial wave-functions can be expanded as

\[
F_{LM}^{\pi \lambda_i}(1234) = \sum_Q D_{QM}^L(-\gamma, -\beta, -\alpha)F_{LQ}^{\pi \lambda_i}(1'2'3'4'),
\]  
(5)

where \(D_{QM}^L\) is the Wigner function, \(\alpha, \beta\) and \(\gamma\) are the Euler angles to specify the collective rotation, \(Q\) is the component of \(L\) along \(k'\), (1234) and (1'2'3'4') denote that the coordinates in \(F_{LM}^{\pi \lambda_i}\) and \(F_{LQ}^{\pi \lambda_i}\) are related to the laboratory frame or to the body frame, respectively. It turns out that the \(\{F_{LQ}^{\pi \lambda_i}\}\) span a representation of the rotation group, space inversion group, and permutation group \((S_4)\). Thus the transformation property of the \(F_{LQ}^{\pi \lambda_i}\) with
Table 2: Some of the allowed $\lambda$ and $[f]_{TS}$ for isospin $T = 0, 1, 2$ states of the four $u$- and $d$-quark system

| $(S, T)$ | $[f]_{TS}$ | $\lambda$ |
|---------|------------|-----------|
| (0, 0)  | [4]        | [3 1]     |
|         | [2 2]      | [3 1], [2 1 1] |
| (1, 0)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
|         | [2 1 1]    | [3 1], [2 2], [2 1 1], [1] |
| (2, 0)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
|         | [2 2]      | [3 1], [2 1 1] |
| (0, 1)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
|         | [2 1 1]    | [3 1], [2 2], [2 1 1], [1] |
| (1, 1)  | [4], [3 1] | [4], [3 1], [2 2], [2 1 1] |
|         | [2 2]      | [3 1], [2 1 1] |
| (2, 1)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
| (0, 2)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
|         | [2 2]      | [3 1], [2 1 1] |
| (1, 2)  | [3 1]      | [4], [3 1], [2 2], [2 1 1] |
| (2, 2)  | [4]        | [3 1]     |

respect to the operations of the above groups is prescribed. This fact will impose a very strong constraint on the $F_{LQ}^{\pi\lambda}$, from which we can fix the practically accessible configuration in orbital space.

Since the quarks are not identical to the antiquark, we can consider only a special kind of configurations, in which the four quarks form a geometric shape, and the antiquark locates at its center due to the mechanical balance (analogous to that in the diquark-antiquark model\[16\]). Considering the geometric configuration of the four quarks, one can imagine that the linkages among the quarks may form a tetrahedron, a tetragon, or others. Recalling the
lattice QCD result of the color flux-tube structure of three-quark system \[38\], we know that there exists genuine three-body interaction among the three quarks, and the linkages between every two quarks may form a equilateral triangle (the interaction is in Y shape). Extending such a geometric feature to the four-quark system, we take the equilateral tetrahedron (ETH) and the square into account in this paper. In fact, as we shall discuss below, if the geometric configuration is not so regular, less constraint is imposed to the system.

For the geometric configuration in equilateral tetrahedron (ETH, denoted also as \(A\) in the following), which is illustrated in figure 1, we denote \(O\) as the center of mass of the four quarks (where the antiquark is located at), \(O'\) as the center between particles 1 and 2, \(O''\) as the center between particles 3 and 4, \(r_{12} \perp k'\) and \(r_{34} \perp k'\). Referring to \(R_{k}^{\vec{u}}\) as a rotation about the axis along the vector \(\vec{u}\) by an angle \(\delta\), \(p_{ij}\) as an interchange of the particles \(i\) and \(j\), \(p_{ijk}\) as a permutation among the particles \(i, j\), and \(l\), and \(\hat{P}\) as a space inversion, we know that the ETH is invariant to the operations

\[
\hat{O}_1 = p_{12}p_{34}R_{k'}^{\vec{u}} , \quad (6)
\]

\[
\hat{O}_2 = p_{12}R_{x}^{\vec{u}} \hat{P} , \quad (7)
\]

\[
\hat{O}_3 = p(1423)R_{k'}^{\vec{u}} \hat{P} , \quad (8)
\]

\[
\hat{O}_4 = p(243)R_{2x}^{\vec{u}} . \quad (9)
\]

Inserting these to Eq. (2) respectively, we have

\[
\sum_{i'} \left\{ g_{ii'}^{\lambda} \left[ p_{12}p_{34}(-1)^Q - \delta_{ii'} \right] \right\} F_{LQ}(A) = 0 , \quad (10)
\]

\[
\sum_{i'} \left\{ g_{ii'}^{\lambda} \left[ p_{12} - \delta_{ii'} \right] \right\} F_{LQ}(A) = 0 , \quad (11)
\]

\[
\sum_{i'} \left\{ g_{ii'}^{\lambda} \left[ p_{1423}(-i)^Q - \delta_{ii'} \right] \right\} F_{LQ}(A) = 0 , \quad (12)
\]

\[
\sum_{i'Q'} \left\{ g_{ii'}^{\lambda} \left[ p_{243} \sum_{Q''} D_{Q''Q}^{L}(0, \theta, 0) e^{i \frac{2\pi}{3} Q''} D_{Q''Q'}^{L}(0, \theta, 0) - \delta_{ii'} \delta_{QQ'} \right] \right\} F_{LQ'}(A) = 0 , \quad (13)
\]

where \(\{ g_{ii'}^{\lambda} \}\) are the matrix elements of the representation \(\lambda\) and the \(\theta\) with restriction \(\cos \theta = \sqrt{1/3}\).

Eqs. (10)-(13) are the equations that the \(F_{LQ}(A)\) have to fulfill. They are homogeneous linear algebraic equations depending on \(L, \pi\) and \(\lambda\). Because the rotational energy of the
state with angular momentum $L$ is $E_r \propto \frac{L(L+1)}{r^2}$, and the size of quark system is very small, we take only the cases with $L < 2$ into account. Since the search for the non-zero solutions of the homogeneous equations is trivial, we neglect describing the evaluating process but list in Table 3 directly whether non-zero solutions $F_{LQ}^{\pi \lambda i}$ satisfying the above constraints exist (marked with a letter “A”) at the configuration $\lambda$ or not (marked with a letter “−”). The table shows obviously that, for only a few cases, there is a set of non-zero solutions $F_{LQ}^{\pi \lambda i}$ satisfying all these equations. It implies that the associated $\lambda$-component $\Psi^\lambda$ is non-zero at the ETH configurations. We may then say that this $\lambda$-component is ETH-accessible. In other cases, there are no non-zero solutions, all the $F_{LQ}^{\pi \lambda i}$ must be zero at the ETH configuration regardless of its size and orientation. In such cases, the $\lambda$-component is ETH-inaccessible.

Table 3: The accessibility of the ETH and the square configurations to the $(L\pi\lambda)$ wave-functions.

| $L^\pi$ | [4] | [3 1] | [2 2] | [2 1 1] | [1 4] |
|---------|-----|-------|-------|---------|-------|
| ETH     | 0$^+$ | A | − | − | − |
| square  | 0$^+$ | A | − | A | − |
| ETH     | 1$^-$ | − | A | − | − |
| square  | 1$^-$ | − | A | − | A | − |

Although the wave-functions are strongly constrained at the ETH, they are less constrained in the neighborhood of the ETH. For example, when the shape in Fig. 1 is prolonged along $k'$, which can be called a prolonged tetrahedron, it is invariant to $\hat{O}_1$, $\hat{O}_2$ and $\hat{O}_3$, but not $\hat{O}_4$. Hence, the $F_{LQ}^{\pi \lambda i}$ should fulfill the Eqs. (10) to (12). Evidently, a common non-zero solution of Eqs. (10) to (13) is necessarily a common solution of Eqs. (10) to (12). Thus, if a $\Psi^\lambda$ is non-zero at an ETH, it remains non-zero in its neighborhood. In other words, an ETH-accessible component is inherently nodeless in the domain surrounding the ETH.

For the configuration that the linkages among the four quarks form a square (the anti-quark locates at its center), as shown in Fig. 2. It is evident that the square is invariant
These invariants lead also to constraints embodied in four sets of homogeneous equations, and therefore the accessibility of the square can be identified as listed in Table 3. As discussed above, a square-accessible component is inherently nodeless in the domain surrounding the square.

Table 4: The predicted quantum numbers of the inherent nodeless (low-lying) pentaquark states (with $L < 2$)

| state | $J^P$ | $L^\pi$ | $T$ | $S$ | $[f]_{TS}$ | $\lambda$ |
|-------|-------|---------|-----|-----|-------------|-----------|
| A     | $\frac{1}{2}^+$ | 1$^-$ | 0   | 0   | [4], [2 2] | [3 1]     |
| B     | $\frac{1}{2}^-$ | 0$^+$ | 0   | 1   | [3 1] | [4] |
| C     | $\frac{1}{2}^+$ | 1$^-$ | 0   | 1   | [3 1], [2 1 1] | [3 1] |
| D     | $\frac{1}{2}^-$ | 0$^+$ | 1   | 0   | [3 1] | [4] |

Referring to Table 3, we find that, when a wave function $\psi_{L^\pi\lambda}$ possesses quantum numbers $(L^\pi, \lambda) = (0^+, [4]), (1^-, [3 1])$, it can access both the ETH and the square configurations. These and only these $\psi_{L^\pi\lambda}$ are inherently nodeless components in the two important configurations and should be the dominant components of the low-lying pentaquark states. We have then deduced four possible low-lying states without taking any dynamical model. The results can be listed in Table 4. All the other pentaquark states should be remarkably higher in energy, because either they are dominated by $L \geq 2$ components, or they do not contain inherent-nodeless $\lambda$-components. In the case of $L = 0$, two states $(J^P, T) = (\frac{1}{2}^-, 0)$ and $(\frac{1}{2}^-, 1)$ (since the antiquark $\bar{s}$ is located at the center of the ETH or the square, its total
spin is just its intrinsic spin $1/2$) contain both ETH-accessible and square-accessible components. They are denoted as B and D in Table 4. These states, dominated by component with spatial symmetry $[4]$ and isospin-spin symmetry $[3\ 1]$, are the low-lying S-wave states, while other S-wave states must have much higher energies. In the case of $L = 1$, two states with the same quantum numbers $(J^P, T) = (\frac{1}{2}^+, 0)$ contain both ETH-accessible and square-accessible components. As the ones denoted as A and C in Table 4, they are associated with the same spatial configuration $[3\ 1]$ but different isospin-spin symmetry $[f]_{TS}$. According to our analysis, these two states are the low-lying ones with positive parity.

It is evident that if a state does not contain a collective excitation of rotation, i.e. the angular momentum $L$ is zero, it would be usually lower in energy than the state with $L > 0$. This is particularly true for the systems with a very small size since $E_r \propto L(L + 1)/r^2$. It is then reasonable to assume that the low-lying S-wave state B has an energy much lower than all the low-lying P-wave states. It is thus the lowest pentaquark state. However, referring to Table 4, one can easily recognize that, the accessibility in the isospin-spin space for the S-wave state B, D is only 1. Meanwhile the accessibility for both the P-wave state A and C is 2. Then if the coupling in the isospin-spin space can not be neglected, it is possible for the P-wave state to appear as the lowest-lying state (i.e., with an energy lower that of the S-wave state).

On the other hand, in view of the nucleon-meson collision, P-wave resonance may also be important. If the S-wave state $(J^P, T) = (\frac{1}{2}^-, 0)$ has an energy above the threshold of a possible decay channel, this state can not be stable, the pentaquark state will decay into a meson and a nucleon via strong interaction. Furthermore, according to Table 2, a “physical” state $(S, T) = (1, 0)$ has four components associated with orbital symmetry $[4]$, $[3\ 1]$, $[2\ 2]$, and $[2\ 1\ 1]$, respectively, i.e. low-lying state B has three partners with higher energy. This will probably lead to form a wide resonance, which contradicts the experimental results. In the theoretical point of view, the S-wave K-N potential in fall-apart mode makes it very difficult to have a narrow width\textsuperscript{[20, 32]}. Then, the low-lying P-wave states, which possess a centrifugal barrier to confine the nucleon and kaon in a narrow resonant state, may become the stable ones instead. Comparing the P-wave states A and C listed in Table 4,
since the state A holds \((S, T) = (0, 0)\) and unique orbital component \([3 1]\), the state C has \((S, T) = (1, 0)\) and four orbital components \([4, [3 1], [2 2]\) and \([2 1 1]\), we propose that the P-wave state A, whose orbital symmetry is uniquely \([3 1]\), is the lowest-lying stable state because the experimentally observed width is very narrow. It is evident that such a result is consistent with many of the previous predictions, for instance, the most original chiral soliton model prediction\[10\], recent Lattice QCD result\[26\], Karliner and Lipkin’s result\[14\], Stancu and Risks’s result\[18, 19\], Jaffe and Wilczek’s result\[29, 16, 39\], production cross section analysis result\[40\], and so on. By the way, considering the spin-orbital coupling, we propose that the \(J^P = \frac{3}{2}^+\) state may also be the low-lying state.

In summary, with the inherent nodal structure being analyzed for the system including four light quarks and one antiquark, we propose dynamical model independently that the quantum numbers of the lowest-lying pentaquark state \(\Theta^+\) may be \(J^P = \frac{1}{2}^+, L = 1\) and \(T = 0\). Such a result is consistent with many previous predictions obtained in concrete dynamical models. Combining our model independent analysis and the previous dynamical calculations, we would prefer to conclude that the parity of the pentaquark state \(\Theta^+\) is positive. Of course, such a result is only a qualitative result. However, taking the presently assigned configuration into dynamical model calculations can obviously help to release the load of numerical calculation. By the way, it is worth to mention that, if there exists attractive interaction in the nucleon-kaon S-wave channel, the parity of the \(\Theta^+\) may be negative.

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Figure 1: A body frame for the ETH

Figure 2: A body frame for the square