The vector-scalar mixing in nuclear medium and the two quark component of scalar meson from QCD sum rules

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Abstract

We derive the QCD sum rules for the vector and scalar meson mixing in nuclear medium, using a two quark interpolating field for both mesons. Modeling the mixing via a nucleon hole contribution with known coupling constant, the sum rule can be used to determine the overlap of the interpolating field and the scalar meson. In the I=0 channel, we find a stable Borel curve and an overlap that is about 10% of the corresponding value in the pseudo scalar or vector channel. The sum rule in the I=1 channel is less reliable but also consistent with a small value for the overlap. These results suggest that both the $\sigma$ and $a_0$ have a small two quark and thus probably a large tetraquark components. We discuss the possibility of observing these scalar mesons from vector mesons emanating from the nuclear medium.

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I. INTRODUCTION

The properties of vector meson immersed in nuclear medium is expected to change due to partial chiral symmetry restoration and/or many body nuclear effects. Expected changes range from the decrease in the mass\cite{1, 2}, increase in the width\cite{3, 4, 5, 6, 7}, and appearance of new structures\cite{8, 9}, every aspect of which if measured will lead to a new understanding of strong interaction. Several experiments are reporting preliminary results\cite{10, 11, 12}, which indeed hints to nontrivial changes and new structures appearing at nuclear matter. With further refined experiments and with exclusive measurement of final state particles, the background could be substantially reduced and vector meson kinematics controlled. Then detailed information on the changes of vector meson properties inside nuclear matter can be extracted, which will then serves as a basis for understanding symmetry restoration in QCD, generation of hadron masses, and QCD phase transition\cite{13}. While the prospects of looking directly into the nuclear matter through vector meson seems so exciting, a careful model independent theoretical analysis on the vector meson properties at nuclear matter have to be carried out simultaneously before any conclusion on medium effects can be made.

An important feature of the vector meson when immersed into the nuclear matter is its coupling to the scalar meson with the same isospin. Therefore, the longitudinal mode of $\omega$ meson will couple to $\sigma$, and that of $\rho$ to $a_0$. The coupling is possible because the nuclear matter expectation value of operators with Lorentz index (spin) can be non vanishing. Therefore, in isospin symmetric nuclear matter, mesons with different spins can couple as long as their parity and isospin are identical. The coupling between the scalar and the vector meson have been extensively studied in the Walecka model in the isospin 0 channel\cite{14, 15}, and in the isospin 1 channel\cite{16, 17}. On the other hand, such mixing have not been extensively studied further in any other model independent way. In this respect, it is important to derive model independent constraints in QCD that can be applied to constrain the phenomenological parameters in any model calculation. Here, we will construct the QCD sum rule for the vector-scalar correlation function and derive a constraint equation for their mixing. Such procedure have been previously used to constrain the model parameters in the momentum dependent part of the light vector meson self energy at nuclear matter\cite{18}.

One caveat in working with the scalar meson is the heated discussion on its dominant quark content, which could be a tetraquark\cite{19, 20}. In the lattice approaches, most of the calculations using a two quark interpolating field seem to predict the ground state mass of scalar particle in the isospin 1 channel to be above 1.3 GeV\cite{21, 22, 23, 24, 25}, while some predict it to be around 1 GeV\cite{26, 27}. In contrast, most lattice calculations based on a four quark interpolating field current consistently predict the mass to be around 1 GeV for the $f_0, a_0$\cite{28, 23} and around 600 MeV for the $\sigma$\cite{24}. Some calculations based on two quark current also give some consistent prediction for $\sigma$\cite{29, 30}. So while the lattice calculations seems to favor a tetraquark picture for the scalar, a two quark component is not ruled out. This is in a sense not surprising as the true scalar should have both a two quark and a four quark component, and the real question is what their relative contribution to the total wave function should be.

In the QCD sum rule approach for the scalar meson, a previous work based on a two quark interpolating field\cite{31}, is known to be controversial\cite{32}. This is so because the single instanton configuration is expected to contribute to the correlation functions between the scalar currents or between the pseudo scalar currents, and spoil the respective convergence of the operator product expansion (OPE). In the present analysis, we will investigate the
correlation function between the scalar and the vector current, where both interpolating fields are composed of a quark and an anti-quark. Thus, the OPE are free from the single instanton contribution and a reliable QCD sum rule analysis will be possible. As we will see, by investigating the scalar through a two quark current, and modeling the phenomenological side through a particle hole contribution with known couplings, we find that for both \(a_0(980)\) and \(\sigma\) meson, its overlap to the two quark current is less than 20% of the values typically expected from the usual meson. Hence, our result suggests that the scalar nonet have a dominant tetraquark component[19]. Furthermore, our analysis confirms that the scalar vector mixing indeed takes place in the physical time-like region. While the \(\sigma\) meson is too wide to be observable in the \(\omega\) meson emanating from nuclear matter, the \(a_0\) will be in the \(\rho\).

II. OPE

We start from the correlation function between the scalar and vector meson.

\[
\Pi_\mu(q) = i \int d^4x e^{iqx} \langle T[J(x), J_\mu(0)]\rangle
\]

where \(J^\sigma a_0 = \frac{1}{2}(\bar uu \pm \bar dd)\) and \(J_\omega^\nu = \frac{1}{2}(\bar u\gamma_\mu u \pm \bar d\gamma_\mu d)\).

In the vacuum, the correlation function is zero as there can not be any coupling between the scalar and vector current. However, in nuclear medium Eq.(1) does not vanish, as the medium provides the necessary four momentum.

The OPE of Eq.(1) at nuclear matter can be obtained in the standard way[2, 33, 34]. To leading order in coupling, the first non-vanishing operator occurs at dimension 6. Taking the quark operators only, it is given as

\[
\Pi^\mu = \frac{(g^{\mu\nu} - q^\mu q^\nu/q^2)}{q^4} \left(- \frac{160\pi}{81} \alpha_s\right) \left(\langle \bar uu\rangle_0 \langle \bar u\gamma^\mu u\rangle_{n.m.} + \langle \bar dd\rangle_0 \langle \bar d\gamma^\mu d\rangle_{n.m.}\right),
\]

for both the isospin 1 and 0 channel. Here, \(\langle \cdot \rangle_0 (\langle \cdot \rangle_{n.m.})\) denotes the vacuum (nuclear matter) expectation value. We have assumed vacuum saturation to extract the vacuum expectation value of the quark condensates. For a symmetric nuclear matter at rest, the expectation value becomes to leading order in nuclear density,

\[
\left(\langle \bar uu\rangle_0 \langle \bar u\gamma^\mu u\rangle + \langle \bar dd\rangle_0 \langle \bar d\gamma^\mu d\rangle\right) \to \langle \bar qq\rangle_0 \rho_N 3\delta^{\alpha_0},
\]

where \(\rho_N, m_N\) are the symmetric nuclear matter density and nucleon mass respectively. Here we have made use of the linear density approximation \(\langle \cdot \rangle_{n.m.} = \langle \cdot \rangle_0 + \frac{\rho_N}{2m_N} \langle N| \cdot |N\rangle\).

III. PHENOMENOLOGICAL SIDE

Consider taking the nuclear matter expectation value of the correlation function in Eq.(1). Since this correlator vanishes in the vacuum, it is just the nucleon expectation value times the density factor in the linear density approximation. If we saturate the intermediate states by hadronic states, the contribution from the ground states becomes as follows

\[
\Pi^\mu = \frac{\rho_N}{2m_N} \frac{if_S}{q^2 - m_S^2} M^\nu_{S+N \to \nu+N} \left(g^{\mu\nu} - q^\nu q^\mu/q^2\right) \frac{if_V}{q^2 - m_V^2}.
\]
Here,
\[
\langle 0 | J^a S \rangle = f_S \\
\langle 0 | J^\mu V \rangle = \epsilon^\mu f_V,
\]
where \( S, V \) is the scalar and vector meson, and \( \epsilon^\mu \) the polarization tensor of the vector meson. \( M^\nu_{S+V} \rightarrow V+V \) is the forward scattering matrix element.

Let us assume the following phenomenological Lagrangian,
\[
\mathcal{L} = \left( g_V \bar{N} \gamma_\mu \tau^a N - \frac{\kappa}{2m_N} \bar{N} \sigma_{\mu\nu} \tau^a N \partial_\nu \right) V^\mu + g_S \bar{N} S \tau^a N,
\]
where we define \( \tau^a = \sigma^a \) (Pauli matrices) for the isospin 1 channel and \( \tau^a = 1 \) for the isospin 0 channel. Using this, the forward scattering matrix element has the following form.
\[
M^\nu_{S+V} \rightarrow V+V = g_V g_S \left( g^\nu\mu - q^\nu q^\mu / q^2 \right) \frac{4m_N}{q^2 - 4(p \cdot q)^2 / q^2} \bar{N} \gamma_\mu \tau^a N \]
\[
+ \frac{\kappa}{m_N} g_S \left( g^\nu\mu - q^\nu q^\mu / q^2 \right) \frac{q^2}{q^2 - 4(p \cdot q)^2 / q^2} \bar{N} \gamma_\mu \tau^a N
\]
(7)
Substituting Eq.(7) into Eq.(4), one finds that Eq.(4) can be written in the following form,
\[
\Pi^\mu = \Pi(q^2) \times \left( g^\nu\mu - q^\nu q^\mu / q^2 \right) \bar{N} \gamma_\mu \tau^a N
\]
(8)
For the nucleon at rest, the tensor part combined with the nucleon expectation value is proportional to \(|q|^2\). This means that the vector scalar coupling vanishes when \( q \rightarrow 0 \).

The sum rule constraint is obtained from identifying the \( \Pi_{\text{phen}} \) in Eq.(8) to the corresponding OPE in Eq.(2). In the limit where \( q = (\omega, 0, 0, 0) \), we find the following sum rule.
\[
( - \frac{160\pi}{27} \alpha_s ) \langle \bar{q} q \rangle_0 \frac{1}{\omega^4} = - \left( f_S f_V g_S g_V \right) \frac{1}{(\omega^2 - m_f^2)} \frac{1}{(\omega^2 - m_V^2)} \frac{4m_N}{\omega^2 - 4m_N^2} \\
- \left( f_S f_V g_S \frac{\kappa}{m_N} \right) \frac{1}{(\omega^2 - m_f^2)} \frac{1}{(\omega^2 - m_V^2)} \frac{\omega^2}{(\omega^2 - 4m_N^2)} + ....
\]
(9)
where the dots represent contributions from excited meson states.

The first term in the right hand side of Eq.(9) represents the contributions from the vector coupling in Eq.(6), while the second that from the tensor coupling. It should be noted that since we have calculated only the leading term of the OPE, the QCD sum rule will constrain only one parameter in the phenomenological side.

IV. SUM RULES

A. \( \omega - \sigma \) mixing

Let us start from considering the isospin 0 channel. Here, the mixing of the \( \omega \) will be to the \( \sigma \). As discussed before, many phenomenological and theoretical considerations
lead us to believe that the dominant quark content of the scalar nonet is a tetraquark\cite{19}. The previous QCD sum rule calculation for the isospin 0 and 1 scalar meson with a two quark interpolating current, seemed to reproduce the masses of $a_0, f_0$ to be of 1 GeV and degenerate\cite{31}. However, direct instanton contribution was found to break the degeneracy, such that in the isospin 1 channel, the sum rule couple dominantly to $a_0(1450)$, and in the isospin 0 channel to a state with a mass smaller than 1 GeV, which could be the $\sigma(600)$\cite{35}. On the other hand, recent calculation with tetraquark interpolating currents do seem to reproduce the masses of the scalar nonet correctly, if direct instanton contributions are taken into account\cite{37, 38, 39, 40}.

The culprit for the complication in the scalar channel is the direct contributions from a single instanton\cite{32}. In the present work, since we are considering the non diagonal correlation function between the scalar and the vector current, no direct instanton will contribute to the zero-modes. Moreover, since the ground state will dominate the sum rules, we can get direct information on $f_S$, which is the overlap of the ground state scalar meson to the two quark interpolating field.

The mixing in the isospin 0 channel is slightly complicated because the $\sigma$ width is very large and the $\omega$ will also mix strongly with $f_0(980)$. On the other hand, we have calculated only the leading term in the OPE, and consequently can constrain only one parameter. Therefore, as a first approximation, we will take the $\sigma$ width to be small, neglect the contribution from $f_0(980)$, and assume that the $\omega$ nucleon coupling is dominated by the vector part such that $\kappa = 0$, as is phenomenologically motivated. We then perform the Borel transformation, which will suppress the contribution from $f_0(980)$, and then compare the OPE to the phenomenological side.

After the Borel transformation, the sum rule for the $\omega - \sigma$ mixing becomes,

$$
\left(-\frac{160\pi}{27}\alpha_s\right)\langle\bar{q}q\rangle_0 \frac{1}{M^2} = (f_\sigma f_\omega g_\sigma g_\omega 4m_N) \left[ \frac{1}{(m_\sigma^2 - m_\omega^2)(m_\sigma^2 - 4m_N^2)}e^{-m_\sigma^2/M^2} \right. \\
+ \frac{1}{(4m_N^2 - m_\sigma^2)(4m_N^2 - m_\omega^2)}e^{-4m_N^2/M^2} \\
+ \left. \frac{1}{(m_\omega^2 - 4m_N^2)(m_\omega^2 - m_\sigma^2)}e^{-m_\omega^2/M^2} \right].
$$

The sum rule in Eq. (10) can be used to constrain the whole coefficient $f_\sigma f_\omega g_\sigma g_\omega$. However, phenomenologically $g_\omega, g_\sigma, f_\omega$ are all rather well known. Therefore, we will use the sum rule to constrain the parameter $f_\sigma$. For the parameters, we take $g_\omega = g_N g_\omega = 11.5, g_\sigma = g_N g_\sigma = 9.4\cite{11}$, $m_\sigma = 0.55$ GeV, and $f_\omega = 0.12$ GeV$^2\cite{14}$. The same value of $f_\omega$ can be obtained from the QCD sum rule calculation for the vector mesons\cite{33} from which we also take $f_\omega = f_\rho$. For the parameters appearing in the OPE, we will take the following values,

$$
\langle\bar{q}q\rangle_{\mu=1\text{GeV}} = -(0.23\text{GeV})^3, \\
\alpha(M^2) = \frac{4\pi}{9\ln(M^2/(0.2\text{GeV})^2)}.
$$

Fig 1 shows the sum rule result for $f_\sigma$ normalized by $\frac{1}{2}\langle 0|\bar{q}i\gamma^5\tau^0q|\pi^0\rangle = 0.13\text{GeV}^2$ calculated in the soft pion limit. One notes the sum rule has a plateau at Borel mass of around $1.0 - 2.0$ GeV$^2$, at which the overlap is about 10% of that of the pion. This value fits very well to the picture that the scalar nonet is mainly a tetraquark and has a very small quark anti-quark component. Because the parameters for $\sigma$ have some uncertainties, we check the
sensitivity of our result to $m_\sigma$ and $g_\sigma$. For $m_\sigma$ uncertainty, we reanalyze the Borel plot for $m_\sigma \simeq 0.4 \text{–} 0.8 \text{GeV}$, and confirm that the change of the result is less than 10%. This seems surprising at first when looking at the individual terms in the right hand side of Eq. (10). But it just reflects the fact that for the overall sum, the dependence on $m_\sigma$ is weak, as can be seen from expanding the right hand side of Eq. (10) in $1/M^2$ and noting that the leading term has no dependence in $m_\sigma$. For the uncertainty of $g_\sigma$, we consider the larger value of $g_\sigma = 14 \text{–} 17$ which is recently obtained [42, 43]. However, using these values leads to reducing our result of $f_\sigma$ by about a factor of $1/2$, which indicates the quark anti-quark component is further suppressed.

Finally, we comment that the overlap $f_\sigma$ to that of the pion overlap constant ($= 0.13 \text{GeV}^2$) is not special as the corresponding overlap value for the vector meson is $f_\rho = f_\omega = 0.12 \text{GeV}^2$ and $f_{a_1} \simeq 0.17 \text{GeV}^2$ [31] for the axial vector meson, and therefore, $f_\sigma$ is much smaller than any of these values.

B. $\rho - a_0$ mixing

The sum rule for the isospin 1 channel can be obtained similarly. In this case, however, the $\rho$ coupling to the nucleon is dominated by the large tensor coupling and we will assume $g_\rho = g_{NN\rho} = 0$. For the other couplings, we will take $\kappa = 14.2$ and $g_{a_0} = g_{NNa_0} = 2.8$ from the Bonn potential [41]. For the overlap constant for the vector meson, we again take $f_\rho = 0.12 \text{GeV}^2$ [44]. The Borel sum rule for this case then becomes,

$$
\left( - \frac{160\pi}{27} \alpha_s \right) \langle \bar{q}q \rangle_0 \frac{1}{M^2} = \left( f_{a_0} f_\rho g_{a_0} \kappa / m_N \right) \left[ \frac{m_{a_0}^2}{(m_{a_0}^2 - m_\rho^2) (m_{a_0}^2 - 4m_N^2)} \frac{1}{e^{-m_{a_0}/M^2}} \right. \\
+ \frac{4m_N^2}{(4m_N^2 - m_\rho^2)} \frac{1}{e^{-4m_N^2/M^2}} \\
\left. + \frac{m_\rho^2}{(m_\rho^2 - 4m_N^2)} \frac{1}{e^{-m_\rho^2/M^2}} \right]. \quad (12)
$$

The sum rule for $f_{a_0}$ can be obtained by dividing the right hand side of Eq. (12) by its left side apart from $f_{a_0}$. Fig. 2 shows the the value for $f_{a_0}$, normalized by the corresponding pion value. Unfortunately, there is no stable Borel region from which we can reliably determine the value. Adding the vector coupling of the $\rho$ will not change much. The problem in this case, could be due to the nontrivial contribution from $a_0(1450)$, which is expected to be a dominant quark anti-quark state. In fact, previous QCD sum rule calculation with two quark current in the isospin 1 channel find the ground state mass to be around 1400 MeV [36], suggesting that the $a_0(1450)$ dominantly contributes to the sum rule. Therefore, in the sum rule in Eq. (12), while the Borel transformation suppresses the contribution from the $a_0(1450)$, its contribution might not be suppressed due to the large overlap $f_{a_0}(1450)$. Unfortunately, the relevant couplings for $a_0(1450)$ are not known, and can not be subtracted out from the sum rule to obtain $f_{a_0}$.

Another reason why the sum rule in Eq. (12) is less reliable than that in Eq. (10), can be seen from the phenomenological side. Apart from the couplings, the right hand side of Eq. (12) can be obtained by taking $M^4 d/dM^2$ of the right hand side of Eq. (10). Such derivatives tend to enhance the contributions from higher energy states, as additional factor of
resonance mass is multiplied in front of the exponential suppression factors. Therefore, while the contribution proportional to $e^{-m_s^2/M^2}$ is larger than that proportional to $e^{-m_\rho^2/M^2}$ and that to negligible $e^{-4m_s^2/M^2}$ in Eq.(10), the contribution proportional to $e^{-m_{a_0}^2/M^2}$ becomes less dominant in Eq.(12), suggesting the importance of contributions from excited states.

However, since we have calculated the OPE to the leading power corrections only, including additional terms in the phenomenological side becomes meaningless. Moreover, we will take the asymptotic value at higher Borel masses, where the approximation of taking the leading power correction becomes more reliable. At larger Borel mass, the curve in Fig. (2) approach an asymptotic value for $f_{a_0}$ at about 20\% of the corresponding pion value. This is then quite consistent with the case for the $\sigma$ case, and with the fact that the dominant quark content of the scalar nonet is a tetraquark.

With all our result, we have shown that a consistent picture emerges from the QCD sum rule analysis where there is a nontrivial coupling between the $a_0$ and the $\rho$ in the nuclear medium, whose strength can be reliably estimated with previously determined coupling constants. Therefore, such corrections should always be included in estimating the $a_0$ contribution to the dilepton spectrum from heavy ion collision[16, 17].

V. SUMMARY

We have derived the QCD sum rules for the vector scalar mixing at nuclear matter in both the isospin triplet and singlet channel. Since the phenomenological parameters are well known except for the overlap of the scalar interpolating field to the corresponding ground state scalar mesons, the sum rule can be used to calculate the value of the overlap. We find that the overlap in both the I=0 and 1 channels are very similar but less than 20\% of the corresponding value in the pseudo scalar channel. This result confirms that both the $\sigma$ and $a_0(980)$ and probably the remaining scalar nonets have a small quark anti-quark component and thus suggests a large tetraquark component. If the coupling of the scalar nonet to the nucleon is very large as in the estimates in [43, 45], the overlap would be even smaller.

The mixing between different spin states originates from the fact that the nuclear medium provides a 4 momentum for the different spin states to couple. If the nuclear medium is not isospin symmetric, then there could be mixing between different isospin states also. So for example, in neutron rich matter, there could be coupling between different isospin and spin states, such as between the $\omega$ and the $a_0(980)$.

Experimentally, these mixing poses new challenges, as the extra peaks from mesons with different spins could be observable from the vector mesons emanating from the nuclear medium. As we have shown within the QCD sum rule analysis, a consistent picture of mixing between scalar and vector meson emerges, whose strength can be consistently estimated with previously determined phenomenological couplings. Therefore, while the $\sigma$ peak is too wide to be observable from the $\omega$ emanating from the nuclear medium, the $a_0$ peak will appear nontrivially in the $\rho$ meson channel.

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Figures
FIG. 1: Sum rule for $f_{\sigma} = \frac{1}{2} \langle 0|\bar{u}u + \bar{d}d|\sigma \rangle$ normalized to $\frac{1}{2} \langle 0|\bar{u}i\gamma^5 u - \bar{d}i\gamma^5 d|\pi^0 \rangle$.

FIG. 2: Sum rule for $f_{a_0} = \frac{1}{2} \langle 0|\bar{u}u - \bar{d}d|a_0 \rangle$ normalized to $\frac{1}{2} \langle 0|\bar{u}i\gamma^5 u - \bar{d}i\gamma^5 d|\pi^0 \rangle$. 
### TABLE I: Parameters for the $\omega - \sigma$ ($\rho - a_0(980)$) mixing in the isospin 0 (1) channel.

| Isospin | $f_V$   | $g_s$ | $g_V$ | $\kappa$ | $\frac{f}{\langle 0|\bar{u}\gamma^5u-\bar{d}\gamma^5d|\pi^0}\rangle$ from present work |
|---------|---------|-------|-------|-----------|-------------------------------------------------|
| 0       | 0.12 GeV$^2$ | 9.4   | 11.5  | 0         | 0.10                                            |
| 1       | 0.12 GeV$^2$ | 2.8   | 0     | 14.2      | 0.2                                             |