Didactical design of integers: an elementary school teachers creation viewed from didactical situation perspective

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Abstract. Integers was first introduced in the fourth grade of primary school. According to teachers, the main difficulty is how to introduce 0 and negative numbers for the first time. The use of recreational beach contexts that involve activities such as swimming, diving, and parasailing greatly help children understand the position of something refers to sea level. Prediction of teachers about the response as well as the alleged learning trajectory (HLT), proved to be an invaluable reference for developing both didactical and pedagogical interventions. Appropriate intervention would help students to create adaptation and acculturation processes, so the child's learning process takes place in good ways.

1. Introduction
Teaching integers in the fourth grade of primary school is an interesting event to be analyzed, at least based on three reasons. First, Students already know the positive integers since grade one, where they begin to learn counting (sound of numbers, one, two, three and so on), mapping the sound of numbers with the number of objects, compare numbers based on amount of objects, and perform arithmetic operations with simple addition and subtraction. Second, when the integers will be introduced in their entirety by adding elements 0 and negative integers, teachers and of course the children have difficulties because it is not easy to represent negative integers with concrete approach. In fact, the approach used by most teachers today as the debt to illustrate the negative numbers or forward and backward to represent positive and negative numbers, still leaves a problem in children. Third, didactical design model that exists today in textbooks issued both by government and the books that circulated widely, generally use a number line their explanation. With this direct approach, students do not have the opportunity to experience the thought process that leads to sense making for the meaning of zero and negative integers.

Teachers of an Elementary School in Bandung, Indonesia have been trying to address these issues, using the context of marine sports such as diving, swimming, and parasailing in the hope of encouraging good meaning of integers, including 0 and negative integers. The main objective of teaching was for the students to compare two integers and to sort the integers on a number line. This paper presents information about this information based on observations from the author in the teaching process and discussion with teachers after the lesson. Thus, the main description of this paper covers the theoretical framework as well as the analysis and discussion of the data of the observations. The theoretical framework used as the basis for in-depth look at each event in the classroom,
especially from the perspective of the theory of didactical situations, learning obstacle, learning trajectory, and the didactical contract.

2. Theoretical Framework
Review about the theory of didactical situations that include a variety of didactical situations, learning obstacle, learning trajectory, and the didactical contract is described briefly as a frame of reference to analyze the implementation of the didactic design of the introduction of integers for fourth grade students.

Learning is always initiated action situations [2]. This situation is very important to make space for the student to use the experience and prior knowledge, so the perception of environment and action on the environment can occur well and the encapsulation process in the formation of new mental object can be facilitated [6]. When new mental objects began to form, the situation as a formulation situation, which substantially is the abstraction result of mental acts that occurred [2]. The process of interaction among students and between teacher and students will allow the emergence of negotiation of meaning so as to encourage the development of arguments, statements, or representations that drive the process of both internal and external validation. The learning process does not always go smoothly as expected. The situation and the plot developed to learn there are times when the cause of difficulties for children. Likewise, the intervention of both the didactic and pedagogical teacher, can came difficulties or obstacles to the learning process. There are three types of difficulties or barriers to learning (learning obstacles) ie ontogenic, epistemological and didactical obstacles that may occur in the learning process [2]. Ontogenic obstacle is the kind of difficulties related to the child's readiness to learn. Based on the author's experience observing the learning process for 3 years, there are at least three types of learning difficulties; psychological, instrumental, and conceptual. Ontogenic psychological obstacle is the lack of students' concentration in learning due to the psychological aspects such as low motivation and interest of the material being studied. Ontogenic instrumental obstacle is the technical difficulties that caused the student cannot fully follow the situation in learning as a result he does not understand the technical issues are key to the learning process. Meanwhile ontogenic conceptual obstacle is the kind of conceptual difficulties with regard to the level contained in the design less familiar corresponds to the situation of children seen from previous learning experience. Thinking or conceptual demands are too high can cause a child to lose their orientation to learn that frustration. Conversely, too low a conceptual challenge can be the cause of an underachiever in learning.

The second obstacle is epistemological obstacle. According to Duroux, this type of learning difficulties is more due to the limited context that is used when the concept was first studied [2]. For example, when the triangle concept was introduced to elementary school students, the teacher tends to use the default image as shown below (Figure 1). Segment AB is usually introduced as the base of the triangle and point C as a peak. As a result, students often have difficulty when faced with another triangle, especially when students are asked to determine the area of a triangle with the base and the height are not standard. The difficulty is not caused by not understanding the concept, but rather the lack of understanding related solely to the specific context (in this case a triangle in the standard form).
Figure 1: Picture a triangle that is often used

To achieve the learning goals that have been assigned by a teacher, every student has a trajectory of learning (learning trajectory) which is unique. Differences in the learning experience as well as a variety of mental objects formed from each of these experiences have a major impact on students’ learning trajectory variations. The teacher can facilitate a variety of trajectories of learning that appear at the time of teaching, he/she should be able to predict the various possible students responses, so that the path of learning (Hypothetical Learning Trajectory or HLT) may be the main reference of facilitation process in the form of didactical or pedagogical interventions. A HLT was developed based on the objectives to be achieved, as well as the stages of learning in form of a series of continuous mutual didactical situation towards the achievement of the objectives [3], as shown below (Figure 2).

Figure 2: Hypothetical Learning Trajectory of Teaching LCM

In a process of teaching, teachers and students each have different ideas and expectations relating to the didactical situation that develop in the classroom. It needs the number of rules that determine a teacher and students responsibility related to didactical situation or a didactical contract [2]. The division of roles and responsibilities can span from teacher as the center to the students as center. There are three types of didactical contracts, namely ostention contract, mayeutic Socratic contract, and potential a-didactical contract [1].

The first types of contracts represent teaching process, where the teacher is a main actor in explaining the concept, demonstrating application of the concept, providing examples, as well as preparing the exercises. The second type, represent a teaching process where the teacher does not completely dominate, but to help students through the submission of the key questions in accordance with didactic situation that develop. The third type is a didactic contract is where the teacher provides space for the child as much as possible in accordance with their capacity, to encourage students' thinking independency through the teacher's didactical design. In practice, these three types of
didactical contracts can occur alternately as needed and the design model. In addition, in the implementation process, termination of the didactical contract can occur (contract breaking) when there is a mismatch between the expectations of the teacher and the response shown by students during teaching process. For teachers who want to continue to grow, the situation of termination of the contract may be a trigger for improvements of design and didactical intervention model that are more appropriate for students.

A brief description of the theoretical study of the above can be used to see, interpret, and weave the didactic situations. The information can be used to make a better didactical design. Figure 3 below illustrates the thinking process of teachers using the theoretical perspective, prior learning (Reflection for action), during the learning process (reflection in action), and after learning (reflection of action). At this stage of reflection for action, teachers develop conjectures about the students' response that may occur on the didactical situation in the classroom. Based on these allegations, it could developed general possible students' learning track also called hypothetical learning trajectory (HLT). It can also developed both didactical and pedagogical anticipations based on the allegations about the various possible responses and grooves of students learning.

The process of thinking when teaching occurs is based on a didactical situation that developed as well as allegations regarding about the students' response to the situation. When the teacher noticed the action by a student together with the learning environment, the milieu is formed so that the teacher can consider the process of interaction and negotiation of meaning that develop in the milieu, to provide a didactical or pedagogical intervention.

![Figure 3: Thinking Process of Teacher](image)

Decision of the teacher on the results of these considerations may represent the type of contract didactic, which can be ostention, mayeutic socratic, or a-didactical potential contract. In the event of termination of the contract (contract breaking), the teacher can decide to give a didactical intervention, so that the acculturation of the learning process can occur. However, if only a pedagogical intervention that be done, then there is a process of adaptation. Teachers also need to stay focused so that a series of situations and thinking processes that develop can shape a student's learning groove integrity of both structural (relation between concepts), and functional (linkages between the thought process) [4], [5]. In other words, teachers get to ensure that the aspect of coherence on a series of circumstances and the student's response in this situation. Given that the experience and previous knowledge students are diverse, the teacher should also consider the aspect of flexibility in choosing the type and level of intervention according to the evolving circumstances.

3. Analysis and Discussion
Didactical design on the introduction of integers developed by the teacher, in outline consists of four phases, the respective aim of each is to: (1) determine the position of objects with reference to sea level, (2) complete a vertical number line, and (3) compare the two positions object on a vertical number line. All of the stages form a groove of learning, so that its implementation in the classroom would be illustrated in the variety of didactical situation, learning obstacle, learning trajectory, and didactical contract.

In the first stage, the teacher asks the students to determine the position of the swimmer (R), diver (P), and parasailer (L) in a diagram similar to the one in figure 4. As the teacher predicted, not all students can put position R, P, and L correctly. This is evident from two student responses that be chosen by the teacher, where they put parasailer position is in incorrect position. From the third student response that be chosen by the teacher, where described positioning of the diver as below sea level, the swimmer at sea level, and the parasailer at above sea level. From the teacher’s to the third student, obtained the answer that the parasailer should be placed on the surface of the ocean because of parasailer usually fly in the air. During the interviewing process, other students agreed with the correct student and made changes to the result of their work.

Students’ difficulty relating to the placement of parasailing in the right position shows that there is a learning obstacle that can be indicated as an ontogenic obstacle. For a student who does not know the meaning of parasailing, the term of “parasailing” would prevent him/her be a source of trouble from understanding assignment of teacher. Teacher strategy featuring the first two responses was effective in revealing the source of the difficulty. Selection of a third child who was able to put parasailer in the correct position and give the right his reason was helped other students to reach an understanding. Teacher Strategy to facilitate the students’ thinking through the discussion about three cases that were be selected, can created a situation in which the adaptation depicts the a-didactical contract. In this process, the students were able to reach an understanding as expected due to the pedagogical intervention by exploiting the growing response from the students themselves.

![Figure 4: Placing of positions R, P, L in the picture](image_url)

After the students were able to put the P, R, and L in the correct position, the teacher continued to the next didactic intervention that ask the students to determine at which position P, R, and L were placed based on the scale provided (distance between two points is considered as one meter). A Variety of responses were given by students on this task. For example, a student was using sea level as a reference for determining the position of the diver, so that by his calculations, the diver was at position 7 meters from sea level. However, to determine the position of the parasailer placed above sea level, he continued to count from sea level up by counting 8, 9, 10 and so on until he find a right number for the parasailer. Another response shown that the difficulty was the use of the lowest point as a reference, the diver was considered to be in a position 1 meter from the lowest point. From that point, he continued to count upwards so that the swimmer and parasailer positions were found. Another interesting response was the use of the sea level as a reference for determining the P and L position so that P in the position of 6 meters and L are also in position 6 meters. Being aware of these
responses is very important, the teacher then asked the question: "what is the difference of 6 meters at the position of divers and 6 meters at the position parasailer?" Most students responded that the 6 meters on the diver is at below of sea level, and the 6 meters for the parasailer is above sea level. The change in students' understanding as a result of the key questions asked by the teacher, describe the process of acculturation. In this process, teacher's didactical intervention (asking a question) has become a trigger of a change in students' understanding. The process can be interpreted as a Socratic mayeucitc contract.

Based on the experience learned in the first stage, the teacher begins the second phase by providing a new task that asked students to label a vertical line as seen in Figure 5 by putting the appropriate numbers. In general there were three categories of response of the students, namely: (1) students had difficulty to interpret the point at the sea surface, (2) students who were trying to use the previous learning experience to describe each point on the line, and (3) students who used experience learned elsewhere so that they do not refer to the thinking sequence developed by the teacher.

In the first case, the student seems hesitant to put a number below the parasailer for the position at sea level, and also for the position of the diver. This indicates that the selection of the proper context does not necessarily help all the children to come to the understanding expected by the teacher. In this case there was a potential didactical obstacle, particularly related to the meaning of a point on the position of the swimmer. For students who had a case like this, still need to be helped by didactical intervention that can help him to interpret the point as a reference to determine the position above the sea level or below of the sea level.

In the second case, the students can use the context of the surface to be interpreted as a reference point and use to determine the point positions that were located above and below the sea level. Figure 5 illustrates one of the cases of the student response. The response shows that the student was able to represent the differences of numbers based on their position referring to the sea level. The position of the surface of the sea itself became a point of reference, represented by the statement "fit at the sea surface".

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  5 meters above sea surface
  4 meters above sea surface
  3 meters above sea surface
  2 meters above sea surface
  1 meter above sea surface
  1 meter below sea surface
  2 meters below sea surface
  3 meters below sea surface
  4 meters below sea surface
  5 meters below sea surface
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"Fit at the sea surface"
Figure 5: Completing the vertical number line

In the third case, a student used his experience from other places. He put 0 at the point position sea level, a positive integer at points above it, and negative integers at the points below. This response seems to have nothing to do with the thinking sequence that developed in previous activities. This situation seems to be a dilemma for the teacher, because on the one hand there were many students who had difficulty completing the points on the vertical number line correctly, on the other hand there was a student (a person) who had familiar with integers even using the positive and negative symbols. The three types of responses are in fact very important to create the adaptation process. The second response was used by the teacher to intermediate for students’ understanding to the third response, by taking a deal so that the representation of the second type can be simplified by using a positive sign for above the sea level and a negative sign for below the sea level.

In the third stage, the teacher presents a problem to compare the positive numbers with a positive (A and B), a positive number with a negative (C and D), and negative numbers with the negative (E and F) based on the position of objects on the number line vertically along with reasons (Figure 6). Based on the responses given by the students, generally they were able to determine the greater number based on its position. From these activities, students were able to express a conclusion, that the higher the position, the greater the value and the shorter the position, the smaller the value. This represents good adaptation, because through the pedagogical interventions of by using a particular student’s response, other students were able make a good conclusion. From the standpoint of didactical contract, these events were included in the a-didactical contract.

Figure 6. Comparing the two numbers

To ensure that students understood the conclusion, the teacher asked few questions regarding the comparison of two numbers. The teacher realized that the students had a good understanding using a vertical number line. The teacher gave the next challenge, if the number line is rotated clockwise so that a horizontal number line is obtained, can students complete it by putting integer in corresponding position?

4. Conclusion
Students' learning process was not always smooth in line with expectations. Situation and path of learning developed by a teacher can be the cause of obstacles to student learning. Similarly, the teacher improper intervention may also be a cause of students learning. The learning obstacles can be categorized into three types, namely ontogenic, epistemological and didactical obstacles. Ontogenic obstacle is the kind of difficulty related with student’s readiness to learn; epistemological obstacle is the kind of difficulty related with limited context that was used when the concept was learnt the first
time, and didactical obstacle is the kind of difficulty related to didactical design developed by a teacher or an improper didactical intervention in the classroom.

In the case of students difficult to put the parasailer in the right position indicated that students had an ontogenic obstacle, because of the parasailing was unknown to him. In the case of students difficult to interpret a point at the right position of the sea surface, there was a potential didactical obstacle, particularly relating to the meaning of a point at the swimmer position. One student who had knowledge of integers and even used the positive and negative symbols. The fact that many students had difficulty to complete the points on the vertical number line could indicate a potential pedagogical obstacle.

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