FDI: Quantifying Feature-based Data Inferability

Shouling Ji†‡, Haiqin Weng‡, Yiming Wu†, Pan Zhou‡, Qinming He†, Raheem Beyah‡, Ting Wang‡
† Zhejiang University
‡ Georgia Institute of Technology
‡ Huazhong University of Science and Technology
‡ Lehigh University
sji@zju.edu.cn

Abstract—Motivated by many existing security and privacy applications, e.g., network traffic attribution, linkage attacks, private web search, and feature-based data de-anonymization, in this paper, we study the Feature-based Data Inferability (FDI) quantification problem. First, we conduct the FDI quantification under both naive and general data models from both a feature distance perspective and a feature distribution perspective. Our quantification explicitly shows the conditions to have a desired fraction of the target users to be Top-$K$ inferable ($K$ is an integer parameter). Then, based on our quantification, we evaluate the user inferability in two cases: network traffic attribution in network forensics and feature-based data de-anonymization. Finally, based on the quantification and evaluation, we discuss the implications of this research for existing feature-based inference systems.

I. INTRODUCTION

Many existing security and privacy applications/techniques can be characterized as a feature-based inference system, e.g., network traffic attribution in network forensic applications, private web search, feature-based data de-anonymization [1]-[8]. To conduct network traffic attribution, usually, a network traffic attribution system is first learned based on the features extracted from historical network traces. Later, when new network traffic comes, features will be extracted from the new traffic first, and then the data will be automatically attributed to the users who generated them by the system based on the features (as shown in Fig.[1]). In fact, the network traffic attribution system can be directly considered as a feature-based inference system, where the system is first learned based on the historical/training data (in detail, features of the historical/training) and then used to infer the new data (in this scenario, users who generate the new traffic) based on their features (as shown in Fig.[2]). Another example is the code stylometry-based de-anonymization attack to programmers proposed in [6]. In this kind of attack, the code stylometry features of training programs are first extracted and used to train a de-anonymization model. Then, this model can be used to de-anonymize the programmers of the target programs based on their code stylometry features. For this example, the code stylometry-based de-anonymization model can also be considered as a feature-based inference system to infer (de-anonymize) target data (programmers of targeting programs).

Now, some interesting questions are brought: how to quantify the performance of those feature-based inference systems for security and privacy applications? and what is the performance of existing feature-based inference techniques relative to the inherent theoretical performance bound? Answering these questions are important to accurately evaluate and understand the performance of existing feature-based inference systems/techniques and further develop improved ones. Unfortunately, although we already have many feature-based inference systems/techniques for various security and privacy applications, the answers to the brought questions remain unclear. Therefore, to address these open problems, in this paper, we study the Feature-based Data Inferability (FDI) quantification for existing feature-based inference systems/techniques in various security and privacy applications. Particularly, we make the following contributions in this paper.

• We first quantify the FDI under a naive data model, where each user-feature relationship is characterized by a binary function (a user either has a feature or does have a feature). Under the naive model, we quantified the conditions to have a target dataset to be ($\delta, K$)-inferable, i.e., to have $\delta m$ target users to be Top-$K$ inferable, where $\delta$ is a parameter in $[0, 1]$, $m$ is the number of overlapped users between the training data of the inference model and the targeting data, (thus, $\delta m$ is the number of users that can be correctly Top-$K$ inferred), and $K$ is an integer specifying the desired inference accuracy.

• Subsequently, we extend our FDI quantification to a general data model. Under the general data model, we quantify the FDI from both the feature distance perspective and the feature distribution perspective to have a target dataset to be ($\delta, K$)-inferable. Our quantification in the general scenarios provides the answers to the raised open problems, and meanwhile, our quantification provides the theoretical foundation for the first time for existing feature-based inference systems/techniques in various security and privacy applications, to the best of our knowledge.

• Based on our FDI quantification, we conduct a large-scale evaluation leveraging on real world data. Specifically, we evaluate the user inferability in two cases: network traffic attribution in network forensics and feature-based data de-anonymization. We explicitly demonstrate the ($\delta, K$)-inferability of users in these two cases and analyze the reasons.

• In terms of our quantification and evaluation, we discuss the implications of this paper to practical feature-based
inference systems/techniques. We also point out the future research directions.

The rest of this paper is organized as follows. In Section II we describe the motivation applications and formalize the problem. In Section III we quantify the FDI under both naive and general data models. In Section IV we evaluate the FDI in two scenarios. We make further discussion in Section V. In Section VI we summarize the related work and we conclude in Section VII.

II. PROBLEM FORMALIZATION

In this section, we formalize the studied problem. To make the problem easily understandable and to further motivate our research, we start from introducing motivation examples that our study is applicable for analysis.

A. Motivation Examples

In this paper, we study data’s feature-based inferability. Our study is motivated by several existing security and privacy applications, e.g., network traffic attribution in network security forensics [1][2][3], linkage attacks and private web search [4][5], and data de-anonymization [6][7][8].

Network traffic attribution is one of the fundamental issues in network security forensics, under which users, who are responsible for the observed activities and behaviors on network interfaces, are inferred [1][3]. Taking the network traffic attribution system Kaleido proposed in [1] and shown in Fig. 1 as an example, a typical network traffic attribution system works as follows: 1, based on the historical network traces, a set of features (corresponding to each user) are extracted; 2, a learning model is designed to learn a discriminant model based on the features of historical network traces, which is used for network traffic attribution and/or new user (could be an intruder) identification; 3, when new network traffic comes, the features of the new network traffic are extracted; and 4, taking the features of the new network traffic as input, the discriminant model either attributes the traffic to a set of candidate users or concludes that the traffic is generated by a new user (a set of new users).

Web searching is one of the most fundamental computer applications, by which users obtain desired knowledge and/or find interested websites. Intuitively, users’ web search traces carry users’ interests and intents. Therefore, potential adversaries (e.g., eavesdroppers) may design some linkage attacks and exploit users’ web search traces to infer users’ profiles and other sensitive information [4][5]. The key idea of a linkage attack is that (i) an adversary first learns a linkage function based on the features of target users’ historical web search data and then (ii) determines whether the new generated web search data/events belong to the target users. To defend against the linkage attack in web search applications, several obfuscation mechanisms have been proposed for private web search [4][5]. The basic idea is to obfuscates users’ web search data by adding some noise, i.e., obfuscating the features of users’ web search data such that the linkage attack cannot effectively infer the generator of the data.

Our study in this paper is also motivated by existing feature-based de-anonymization attacks and techniques, e.g., programmers de-anonymization [6], authorship distribution to underground forums and multi-author detection [7], and movie rating data de-anonymization [8]. In these de-anonymization attacks/techniques, a feature-based de-anonymization model is first learned based on a training dataset. Subsequently, the new coming data (generated by an existing user or a new user) are de-anonymized by the de-anonymization model based on the data’s features.

Mathematically, all the aforementioned security and privacy applications can be reduced to a simple yet general system as shown in Fig. 2. 1, a model is learned based on the features of historical data; 2, the target data are input to the model; and 3, inferences, e.g., candidate users who generate the data and/or identified new users, are concluded based on the results of the learned model. Now, after observing the success of the aforementioned security and privacy applications [1][8], e.g., Kaleido is able to identify the responsible users with over 80% accuracy, two interesting questions are that why these techniques/attacks are success and given the target data, how to determine the performance of these techniques/attacks relative to the intrinsic inferability of the target data, e.g., how good the 80% accuracy of Kaleido is and is that possible to achieve some better accuracy than 80%? To answer the two questions, we study the intrinsic inferability of the target data given the historical data (training data). Therefore, our research in this paper can serve as the theoretical foundation of the aforementioned security and privacy applications. Furthermore, our quantification enables the development of a tool to evaluate the relative performance of the aforementioned techniques/attacks and guides the development of future research (as discussed in Section VII).
B. Problem Formalization and Models

Now, we formalize the studied problem. During the formalization, the basic principle is to make the problem sufficiently general and meanwhile mathematically tractable.

We denote the training data (e.g., the historical data in the network traffic attribution scenario) as $U$. Since we do not distinguish a user and the data generated by that user, we assume $U$ consists of $n$ users (or the data generated by $n$ users), and further assume $U = \{u_i| i = 1, 2, \ldots, n\}$, where $u_i$ is a user (or the data generated by a user). For $\forall u_i \in U$, it represents a user or the data generated by a user depending on the context. To model the feature extraction process (as shown in Fig.1 and Fig.2), we assume there is a feature extraction mechanism $F = \{f^1, f^2, \ldots, f^N\}$, where $f^k$ denotes some particular feature function and $N$ is the dimension of the feature space. Applying $F$ to $U$, we can get the features of $U$, denoted by set $\mathcal{F}(U)$. In this paper, we focus on the scenario that $\mathcal{F}(U)$ is a finite set, i.e., $N$ is some finite value. Specifically, for $\forall u_i \in U$, its features with respect to $F$ are denoted by vector $\mathcal{F}(u_i) = < f^1_{u_i}, f^2_{u_i}, \ldots, f^N_{u_i} >$, where $f^k_{u_i} (1 \leq k \leq N)$ denotes the feature of $u_i$ with respect to the feature function $f^k \in F$.

Similar to formalizing the training data and taking account of the security and privacy applications (1-8), we denote the target data by $V = \{v_j| j = 1, 2, \ldots, m\}$, where $v_j$ is a user (or the data generated by a target user) in the target data and $m$ is the number of users in the target data. As shown in Fig.1 and Fig.2 (1-8), before inferring the users in $V$, we apply the same $\mathcal{F}$ to extract the features of $V$ denoted by $\mathcal{F}(V)$, which is again assumed to be a finite set. For $\forall v_j \in V$, its features with respect to $F$ are denoted by vector $\mathcal{F}(v_j) = < f^1_{v_j}, f^2_{v_j}, \ldots, f^N_{v_j} >$, where $f^k_{v_j}$ denotes the feature of $v_j$ with respect to the feature function $f^k \in F$. After having $\mathcal{F}(V)$, the task now is to infer the users in $V$ using an inference model (e.g., the network traffic discriminant model as shown in Fig.1).

Based on the aforementioned definitions, the studied problem in this paper can be formalized as follows:

**Definition II.1. Feature-based Data Inferability (FDI).** Given $U$, $V$, and $\mathcal{F}$, we quantify the inferability of $V$ with respect to $U$ and $\mathcal{F}$.

In this paper, we study the intrinsic FDI of the security and privacy applications as shown Section 1-A. Mathematically, the FDI study can serve as the theoretical foundation of the applications in Section 1-A, e.g., the network traffic distribution system Kaleido proposed in [1]. Practically, the FDI study can be employed to evaluate the relative performance of the existing techniques in the applications of Section 1-A and guide the development of new/improved techniques.

III. FDI Quantification

In this section, we conduct the FDI quantification. We start the quantification from a naive scenario. Then, we generalize the FDI quantification to the more practical cases.

To make our following discussion easily understandable, we use the network traffic attribution application in network security forensics as the studying context without of dedicated specification in the rest of this paper. Straightforwardly, our discussion is applicable to the scenarios of the linkage attack and private web search (4, 5) and data de-anonymization (6, 7, 8).

A. Preliminary

Following the security and privacy applications in [1]-8, an inferring model can be learned from $\mathcal{F}(U)$ as shown in Fig.1 and Fig.2, e.g., the discriminant model in the network security forensics application [1-3], the linkage attack model in private web searching [4, 5], and the de-anonymization model in [6, 7, 8]. We denote the inference (attack, de-anonymization) model by $M$. Then, $M$ is employed to infer the new coming data, i.e., the target data.

When employing $M$ to infer users (data generated by users) in the target data, $M$ employs some inference function learned from $\mathcal{F}(U)$. We here model the inference function of $M$ by $\phi(\cdot, \cdot)$. Then, $\forall v_j \in V$, when inferring $v_j$ using $M$, we denote the process by $M(v_j: U)$ and denote the inference result by $M(v_j: U) = \{u_i|u_i \in U, \phi(u_i, v_j) \text{ returns true}\} \cup \{\triangle\}$, where $\triangle$ denotes a new user (the data generated by a new user) such that $\triangle \notin U$. We further explain the inference result definition as follows: when employing $M$ to infer the target user (data generated by the target user) $v_j$, it may be inferred to some candidate users in the training data $U$ if the inference function $\phi(\cdot, \cdot)$ is satisfied. Otherwise, $M$ is more confident to infer $v_j$ as a new user that never appeared in $U$. For instance, in the network traffic distribution application, when using Kaleido ($M$ in our definition) to monitor the online network traffic, the inference result could be that the traffic is generated by some existing user (used for training Kaleido) or the traffic is generated by some new user that not appeared before (could be some intruder). Now, we are ready to start our quantification.

B. Warmup: Naive Quantification

In this subsection, we conduct the FDI quantification for a naive scenario, where we assume that $\forall f^k \in F$, $f^k$ is a binary feature function, i.e., $\forall u_i \in U$ or $\forall v_j \in V$, $u_i$ or $v_j$ either has feature $f^k$ or not. Then, we have $\forall w \in U \cup V$, $\mathcal{F}(w) = f^k_w| f^k_w \in \{0, 1\}$, $k = 1, 2, \ldots, N$, i.e., the feature vector of $w$ is a $N$-dimensional 0-1 vector with respect to $F$. Furthermore, for $\mathcal{F}(w)$, we define $\Gamma_w = \sum_{k=1}^{N} f^k_w$. Given two 0-1 vectors $\mathcal{F}(x)$ and $\mathcal{F}(y)$ where $x, y \in U \cup V$, we define...
For $v \in V$ and $u \in U$, we denote $v \approx u$ the scenario that $v$ and $u$ correspond to the same user (or the data generated by the same user) and $v \neq u$ otherwise, e.g., the network traffic generated by the same user in different time windows or not. To conduct the FDI quantification, the first step is to understand and quantify the correlation of the features of $v$ and $u$. Toward this objective, for $v \in V$ and $u \in U$, we assume that $Pr(f^k_x = f^k_u | v \approx u) = p_{x}$ for $1 \leq k \leq N$, i.e., the probability that $v$ preserves the same property of $u$ with respect to a feature is $p$. Now, for $u, w \in U$ and $v \in V$, suppose $v \approx u$ while $v \neq w$. Then, we have the following lemma, which quantifies the inferability of $v$ with respect to $u$ and $w$.

**Lemma 1.** If $p \neq 1/2$ and $\Gamma_{u,w} \geq \frac{16 \ln N + 8}{(1-2p)^2}$, then $\exists M$ such that $M(v: \{u, w\}) = \{u\}$, i.e., $v$ is inferable with respect to $u$ and $w$.

**Proof:** To prove this lemma, we first analyze the difference between $\Gamma_{u,w}$ and $\Gamma_{v,w}$. To facilitate our analysis, we partition the feature space $F$ into four disjoint subsets with respect to $F(u)$ and $F(w)$, denoted by $F_1$, $F_2$, $F_3$, and $F_4$ respectively as shown in Fig. 3 where $F_1 = \{ \{ 1 \} \}$ (the set of features that $u$ has while $w$ does not have), $F_2 = \{ \{ 2 \} \}$ (the set of features that both $u$ and $w$ have), $F_3 = \{ \{ 3 \} \}$ (the set of features that $u$ has while $w$ has), and $F_4 = \{ \{ 4 \} \}$ (the set of features that neither $u$ nor $w$ has). Let $k_i = |F_i|$ for $i = 1, 2, 3, 4$, where $|\cdot|$ is the cardinality of a set. Furthermore, for $x \in \{ u, w, v \}$ and $1 \leq i \leq 4$, let $F_i(x)$ be the feature vector of $x$ with respect to features in $F_i$. Evidently, $F_i(x)$ is a subvector of $F(x)$. Furthermore, let $\Gamma^i_{u,w} = \sum_{x} f^i_{x,F_i}$, then, it is easy to show that $\forall x, y \in \{ u, w, v \}$, $\Gamma_{x \oplus y} = \sum_{i=1}^{4} \Gamma^i_{x \oplus y}$.

Let $\Lambda^u_{v,u} = \Gamma_{v,u} - \Gamma_{u,u}$. Since $\Gamma_{v,u} = \sum_{i=1}^{4} \Gamma^i_{v,u}$ and $\Gamma_{u,u} = \sum_{i=1}^{4} \Gamma^i_{u,u}$, we have $\Lambda^u_{v,u} = \sum_{i=1}^{4} (\Gamma^i_{v,u} - \Gamma^i_{u,u})$. Now, we consider each $F_i$ separately: (1) since both $u$ and $w$ have the features in $F_2$, we have $\Gamma^2_{v,u} - \Gamma^2_{v,w} = 0$; (2) similar to $F_2$, since neither $u$ nor $w$ has any feature in $F_4$, we have $\Gamma^4_{v,u} - \Gamma^4_{v,w} = 0$; (3) for $F_3$, the set of features hold by $u$ while not $w$, statistically, we have $\Gamma^3_{v,u} \sim B(k_1, 1-p)$ and $\Gamma^3_{v,w} \sim B(k_1, p)$, where $B(x, y)$ is a binomial variable with parameters $x$ and $y$; and (4) for $F_3$, the set of features hold by $w$ while not $u$, statistically, we have $\Gamma^3_{v,w} \sim B(k_3, 1-p)$ and $\Gamma^3_{v,u} \sim B(k_3, 1-p)$. Then, we have

$$
\Lambda^u_{v,u} = \sum_{i=1,3} (\Gamma^i_{v,u} - \Gamma^i_{v,w})
$$

Statistically,

$$
\Pr(\Lambda^u_{v,u} \leq 0) = \Pr(B(k_1, p) + B(k_3, p) - B(k_1, 1-p) - B(k_3, 1-p)) = 2 \exp(-2 \ln N - 1) \leq \frac{1}{N^2}.
$$

Thus, according to the Borel-Cantelli Lemma and statistically, we have $\Pr(\Lambda^u_{v,u} \leq 0) \rightarrow 0$, which implies that statistically, $\Pr(\Gamma_{u,w} > \Gamma_{v,u}) \rightarrow 1$.

Second, we consider the case that $p \neq 1/2$. In this case, we have $\Pr(\Gamma_{u,w} < (1-p)\Gamma_{v,u})$. Then, applying the Pedarsani-Grossglauser lemma, we have

$$
\Pr(\Lambda^u_{v,u} \geq 0) \leq 2 \exp(-2 \ln N - 1) \leq \frac{1}{N^2}.
$$

Considering that $\Gamma_{u,w} \geq \frac{16 \ln N^2 + 8}{(1-2p)^2}$, we have

$$
\Pr(\Lambda^u_{v,u} \geq 0) \xrightarrow{\text{statistically}} 0, \text{ i.e., } \Pr(\Gamma_{v,u} < \Gamma_{v,w}) \xrightarrow{\text{statistically}} 1.
$$

According to the Borel-Cantelli Lemma and statistically, we have $\Pr(\Lambda^u_{v,u} \geq 0) \xrightarrow{\text{statistically}} 0$, i.e., $\Pr(\Gamma_{v,u} < \Gamma_{v,w}) \xrightarrow{\text{statistically}} 1$. 

Fig. 3. Feature space partition.
Algorithm 1: A naive implementation of $M$.

1. if $p > \frac{1}{2}$ then
2. $\{v \in \{u, w \} = \arg \min \{x|\Gamma_{v|\Theta_2}, x \in \{u, w \} \}$;
3. else if $p < \frac{1}{2}$ then
4. $\{v \in \{u, w \} = \arg \max \{x|\Gamma_{v|\Theta_2}, x \in \{u, w \} \}$;

Now, we need to show that $\exists M$ such that $M(v : \{u, w \} = \{u \}$. Based on our proof, it is trivial to show that (1) when $p > \frac{1}{2}$, $\Pr(M(v : \{u, w \} = \{u \})$ statistically $\geq 1$ if $M$ is an increasing function with respect to $\Gamma_{v|\Theta_2}$, where $x \in \{u, w \}$; and similarly, when $p < \frac{1}{2}$, $\Pr(M(v : \{u, w \} = \{u \})$ statistically $= 1$ if $M$ is a decreasing function with respect to $\Gamma_{v|\Theta_2}$.

Therefore, for our purpose it is easy to design $M$ using existing techniques [11-13]. To name a naive one, we can set $M$ as shown in Algorithm 1.

In Lemma 1, we quantified the condition to successfully infer user $v$ from $V$ with respect to $\{u, w \} \subseteq U$. We further discuss Lemma 1 as follows. First, one condition is that $p \neq \frac{1}{2}$. This is consistent with our institution. If $p = \frac{1}{2}$, the features of each worker are uniformly and equiprobably distributed in $F$. Then, theoretically, all the users are equivalent with respect to $F$ and thus it is difficult (if not impossible) to successfully infer $v \in V$ based on the features in $F$ by any model. Second, when $p \neq \frac{1}{2}$, we explicitly specify the condition that $v \in V$ is statistically guaranteed to be successfully inferable with respect to $\{u, w \}$. In our proof, we also give how to design $M$. Note that, the specified condition is sufficient while not necessary to have $v$ inferable with respect to $\{u, w \}$. Even if the condition is not satisfied, it is also possible to design $M$ using the same technique as in Lemma 1.

Corollary 1. For $v \in V$ and $u, w \in U$, suppose $v \approx u$ and $v \neq w$. If $p \neq 1/2$, then $\exists M$ such that $\Pr(M(v : \{u, w \}) = \{u \})$ statistically $\geq \max\{0, 1 - 2\exp(-\frac{(1-2p)^2}{8} \Gamma_{v|\Theta_2}) \}$.

Proof: This corollary can be proven using the similar technique as in Lemma 1.

In Lemma 1, we quantify the FDI of $v$ with respect to $\{u, w \}$. Now, we quantify the FDI of $v$ with respect to $U$. In practice, we usually infer $v$ to a set of candidate users in $U$. For instance, in the network traffic distribution system Kaleido [1], the user responsible for the new coming traffic might be inferred to a set of $K$ ($K \in [1, n]$) users. Therefore, given $v \in V$, we define the Top-$K$ candidate set of $v$ as follows.

Definition III.1. Top-$K$ candidate set and Top-$K$ inferable. For $v \in V$, suppose that $\exists u \in U$ such that $u \approx v$. Then, the Top-$K$ candidate set of $v$ ($K \in [1, n]$), denoted by $K_v$, is defined as $K_v \subseteq U$ such that $|K_v| = K$ and $u \in K_v$. $v$ is Top-$K$ inferable with respect to $U$ if $\exists M$ such that $M(v : U) = K_v$, i.e., $M$ returns a subset of $U$ with size $K$ and $u$ is in that subset.

Now, we quantify the Top-$K$ FDI of a user $v \in V$. Let $K_v$ be a subset of $U$ such that $|K_v| = n - K$ and $v \notin K_v$. We show the result in the following lemma.

Lemma 2. For $v \in V$, suppose that $v \approx u \in U$. Then, $v$ is Top-$K$ inferable if $p \neq 1/2$ and $\exists K_v \subseteq U$ such that $\min\{\Gamma_{u|\Theta_2} | w \in K_v \} \geq \frac{16 \ln N + 8 \ln (2N\theta)}{(1-2p)^2}$, where $\theta = \frac{n-K}{n}$.

Proof: We prove this lemma by considering two cases. First, we consider the case that $p > \frac{1}{2}$. We define an event $E_1$ as $\exists w \in K_v$ such that $\Gamma_{v|\Theta_2} \geq \Gamma_{v|\Theta_2}$. Then, we have $\Pr(E_1) = \Pr(\bigcup_{w \in K_v} \Gamma_{v|\Theta_2} \geq \Gamma_{v|\Theta_2}) \leq \sum_{w \in K_v} \Pr(\Gamma_{v|\Theta_2} \geq \Gamma_{v|\Theta_2})$ according to Boole’s inequality. From Lemma 1 when $p > \frac{1}{2}$, we have $\Pr(E_1) \leq \exp(-\frac{(2p-1)^2}{8} \Gamma_{v|\Theta_2})$. Then, we have $\Pr(E_1) \leq \sum_{w \in K_v} \exp(-\frac{(2p-1)^2}{8} \Gamma_{v|\Theta_2}) \leq \sum_{w \in K_v} 2 \exp(-2 \ln N - \ln 2\theta n) = 2\theta n \exp(-2 \ln N - \ln 2\theta n) = 1/N^2.$

According to the Borel-Cantelli Lemma, we have $\Pr(E_1) \rightarrow N \rightarrow 0$, i.e., $\Pr(\forall w \in K_v$, $\Gamma_{v|\Theta_2} < \Gamma_{v|\Theta_2}) \rightarrow N \rightarrow 1$.

Second, we consider the case that $p < \frac{1}{2}$. In this case, we define $E_2$ as an event that $\exists w \in K_v$ such that $\Gamma_{v|\Theta_2} \leq \Gamma_{v|\Theta_2}$. Then, similar to the case that $p > \frac{1}{2}$, we have $\Pr(E_2) = \Pr(\bigcup_{w \in K_v} \Gamma_{v|\Theta_2} \leq \Gamma_{v|\Theta_2}) \leq \sum_{w \in K_v} \Pr(\Gamma_{v|\Theta_2} \leq \Gamma_{v|\Theta_2}) \leq \sum_{w \in K_v} 2 \exp(-\frac{1}{2} \Gamma_{v|\Theta_2}) \leq 1/N^2$.

Again, according to the Borel-Cantelli Lemma, we have $\Pr(E_2) \rightarrow N \rightarrow 0$, i.e., $\Pr(\forall w \in K_v$, $\Gamma_{v|\Theta_2} > \Gamma_{v|\Theta_2}) \rightarrow N \rightarrow 1$.

Now, we discuss how to design $M$ and how to find $K_v$. Based on our proof, if $p \neq 1/2$ and $\exists K_v \subseteq U$ such that $\min\{\Gamma_{u|\Theta_2} | w \in K_v \} \geq \frac{16 \ln N + 8 \ln (2N\theta)}{(1-2p)^2}$, then (1) when $p > 1/2$, $\Pr(\forall w \in K_v$, $\Gamma_{v|\Theta_2} < \Gamma_{v|\Theta_2}) \rightarrow N \rightarrow 1$, which implies that among $U$, there are at least $n - K$ users having their $\Gamma_{v|\Theta_2}$ values greater than $\Gamma_{v|\Theta_2}$; and (2) when $p < 1/2$, $\Pr(\forall w \in K_v$, $\Gamma_{v|\Theta_2} > \Gamma_{v|\Theta_2}) \rightarrow N \rightarrow 1$, there are at least $n - K$ users having their $\Gamma_{v|\Theta_2}$ values smaller than $\Gamma_{v|\Theta_2}$. According to this observation, we give a preliminary implementation of $M$ as shown in Algorithm 2. Basically, if $p > 1/2$, Algorithm 2 returns a set $K_v$ consisting of $K$ users from $U$ that have the top-$K$ minimum $\Gamma_{v|\Theta_2}$ values; and if $p < 1/2$, Algorithm 2 returns a set $K_v$ consisting of $K$ users from $U$ that have the top-$K$ maximum $\Gamma_{v|\Theta_2}$ values. By a contradiction-based technique, we can show that the $M$ shown in Algorithm 2
Algorithm 2: An implementation of $\mathcal{M}$ to have $v$ Top-$K$ inferable.

1. $\mathcal{K}_v \leftarrow \{u_1, u_2, \ldots, u_K\}$
2. $U' \leftarrow U \cup \mathcal{K}_v$
3. if $p > \frac{1}{2}$ then
   for $x \in U'$ do
      $u' = \arg\max\{\Gamma_{v \oplus y} | y \in \mathcal{K}_v\}$
      if $\Gamma_{v \oplus u'} > \Gamma_{v \oplus x}$ then
         $\mathcal{K}_v \leftarrow \mathcal{K}_v \setminus \{u'\}$
         $\mathcal{K}_v \leftarrow \mathcal{K}_v \cup \{x\}$
   else if $p < \frac{1}{2}$ then
      for $x \in U'$ do
         $u' = \arg\min\{\Gamma_{v \oplus y} | y \in \mathcal{K}_v\}$
         if $\Gamma_{v \oplus u'} < \Gamma_{v \oplus x}$ then
            $\mathcal{K}_v \leftarrow \mathcal{K}_v \setminus \{u'\}$
            $\mathcal{K}_v \leftarrow \mathcal{K}_v \cup \{x\}$
   return $\mathcal{K}_v$

returns a Top-$K$ candidate set of $v$, i.e., $v$ is Top-$K$ inferable.

In Lemma 2, the conditions for a user to be Top-$K$ inferable are quantified. If the specified conditions are satisfied, we also provide an implementation of $\mathcal{M}$ in the proof (Algorithm 2). In fact, there are also many other techniques to implement $\mathcal{M}$, e.g., the techniques proposed in [1]-[8]. Further, similar to Lemma 1, the conditions in Lemma 2 are sufficient while not necessary for $v$ to be Top-$K$ inferable. When the conditions are satisfied, it is statistically guaranteed that $v$ is Top-$K$ inferable. Otherwise, $v$ is still Top-$K$ inferable with some probability. Particularly, we show that probability in the following corollary.

Corollary 2. For $v \in \mathcal{V}$, suppose that $v \simeq u \in U$. Then, if $p \neq \frac{1}{2}$, $\Pr(\mathcal{M}(v : U) = \mathcal{K}_v) \geq \max\{0, 1 - 2\theta_n \exp(-\frac{(1 - 2p)^2}{2} \Gamma_{v \oplus w}^{(1, 1)}(\mathcal{T}_v)) \}$, where $\theta_n = \frac{n - K}{n}$ and $\Gamma_{v \oplus w}^{(1, 1)}(\mathcal{T}_v) = \min(\Gamma_{v \oplus w} | w \in \mathcal{K}_v)$.

Now, we consider an even more general scenario where we try to infer multiple users in $V$. A practical application corresponding to this scenario is to distribute the monitored network traffic generated by multiple users in network forensics [1]-[3]. Let $\tilde{V} = \{x | x \in V, \text{ and } \exists y \in U, \text{ s.t. } x \simeq y\}$, i.e., $\tilde{V}$ is a set of users that appeared in both $V$ and $U$. Furthermore, let $\delta$ be a constant and $\delta \in [0, 1]$. Then, we define the $(\delta, K)$-inferability of $V$ (i.e., $V$ is $(\delta, K)$-inferable) as follows.

Definition III.2. $(\delta, K)$-Inferable. $V$ is $(\delta, K)$-inferable if there are at least $\delta \cdot m$ users in $\tilde{V}$ are Top-$K$ inferable.

Then, we quantify the $(\delta, K)$-inferability of $V$ in the following theorem.

Theorem 1. Let $V$ be any subset of $\tilde{V}$ and $|V_\delta| = \delta m$. $V$ is $(\delta, K)$-inferable if $p \neq \frac{1}{2}$ and $\forall v \in V_\delta, \exists \mathcal{K}_v \subseteq U$ such that $|\mathcal{K}_v| = n - K$, and $\min\{\Gamma_{v \oplus u} | u \in U, u \simeq v, \text{ and } w \in \mathcal{K}_v\} \geq \frac{16\ln N + 8\ln(2\delta m n)}{(1 - 2p)^2}$.

Proof: We first prove this theorem for the case that $p > \frac{1}{2}$. For $v \in V_\delta$, suppose $v \simeq u \in U$. Evidently, $|\mathcal{K}_v| = K$. Now, to prove this theorem, it is sufficient to show that $\forall v \in V_\delta$, $v$ is Top-$K$ inferable. Let $E$ be the event that $\exists w \in V_\delta$ such that $v$ is not Top-$K$ inferable. Then, we have

\[
\Pr(E) = \Pr(\bigcup_{v \in V_\delta} v \text{ is not Top-$K$ inferable}) \\
\leq \sum_{v \in V_\delta} \Pr(v \text{ is not Top-$K$ inferable}) \\
\leq \sum_{v \in V_\delta} (1 - \Pr(\forall u \in \mathcal{K}_v, \Gamma_{v \oplus u} < \Gamma_{v \oplus w})) \\
= \sum_{v \in V_\delta} \Pr(\exists w \in \mathcal{K}_v, \Gamma_{v \oplus u} \geq \Gamma_{v \oplus w})
\]

Then, according to Lemma 1 and Lemma 2 we have

\[
\Pr(E) \lessapprox \sum_{v \in V_\delta} \sum_{w \in \mathcal{K}_v} 2\exp(-\frac{(1 - 2p)^2}{2} \Gamma_{v \oplus w}) \\
\leq \sum_{v \in V_\delta} \sum_{w \in \mathcal{K}_v} 2\exp(-2\ln N - \ln(2\delta m n)) \\
= 1/N^2.
\]

Following the Borel-Cantelli Lemma, we have $\Pr(E) \overset{N \to \infty}{=} 0$, i.e., $\Pr(\forall v \in V_\delta, v \text{ is Top-$K$ inferable}) \overset{N \to \infty}{=} 1$ which implies that $V$ is $(\delta, K)$-inferable.

For the case that $p < \frac{1}{2}$, we have

\[
\Pr(E) = \Pr(\bigcup_{v \in V_\delta} v \text{ is not Top-$K$ inferable}) \\
\leq \sum_{v \in V_\delta} \Pr(v \text{ is not Top-$K$ inferable}) \\
\leq \sum_{v \in V_\delta} (1 - \Pr(\forall u \in \mathcal{K}_v, \Gamma_{v \oplus u} > \Gamma_{v \oplus w})) \\
= \sum_{v \in V_\delta} \Pr(\exists w \in \mathcal{K}_v, \Gamma_{v \oplus u} \leq \Gamma_{v \oplus w}).
\]

Then, according to Lemma 1 and Lemma 2 we have

\[
\Pr(E) \lessapprox \sum_{v \in V_\delta} \sum_{w \in \mathcal{K}_v} 2\exp(-\frac{(1 - 2p)^2}{2} \Gamma_{v \oplus w}) \\
\leq 1/N^2.
\]

Again, following the Borel-Cantelli Lemma, we have $\Pr(E) \overset{N \to \infty}{=} 0$, which implies that $V$ is $(\delta, K)$-inferable.

In Theorem 1 we quantify the $(\delta, K)$-inferability of $V$. When comparing Theorem 1 and Lemma 2 we can see that
the conditions specified in Theorem 1 are stronger than that in Lemma 2 with respect to two aspects. First, in Theorem 1 it is required that for all $v \in V_\delta$, there exists one desired $K_v$. This is for the purpose of making $v$ Top-$K$ inferable. Second, the required $\min_{u \in U} \Gamma_v$ is stronger than in Theorem 1 than that in Lemma 2. This can be explained from the statistical perspective. In Lemma 2, the objective is to make one user statistically Top-$K$ inferable while in Theorem 1, the objective is to make all the users in $V_\delta$ statistically Top-$K$ inferable (simultaneously).

If the specified conditions in Theorem 1 are satisfied, an interesting question is how to design a $\mathcal{M}$ to make $V (\delta, K)$-inferable. An preliminary implementation of $\mathcal{M}$ can be built using the procedure in Algorithm 2 for each user $v \in V$, we use Algorithm 2 to find a $K_v$ for it. Then, by the similar argument as in Lemma 2, we can conclude that $V$ is $(\delta, K)$-inferable under $\mathcal{M}$.

In this subsection, we conduct the FDI quantification under the assumption that each feature function is binary. Apparently, this assumption may not hold in many real applications. Nevertheless, the quantification in this subsection can shed light on sophisticated FDI analysis. In the following subsections, we consider general FDI quantification by removing this assumption.

C. General Quantification: From the Distance Perspective

In the previous FDI quantification, we assume that $f^i \in \mathcal{F}$, $f^i$ is a binary function, i.e., $f^i \in \{0, 1\}$. Although this assumption holds in many real applications (e.g., linkage attacks and data de-anonymization attacks), $f^i$ may not be a binary function in many other applications. Therefore, in the following FDI quantification, we assume that $f^i$ can be any function with a real-value output. Furthermore, given $\mathcal{F} = \{f^1, f^2, \cdots, f^N\}$, an inference model $\mathcal{M}$ may assign different weights to each feature (usually, the weights are learned from the features of the training data, i.e., $\mathcal{F}(U)$).

To characterize this situation, we model that each feature $f^i$ in $\mathcal{F}$ corresponds to a weight vector in $\mathcal{M}$, which can be obtained by a weight function $w^i$. In addition, to make our FDI quantification sufficiently general and meanwhile mathematically tractable, we model the correlation between the feature function $f^i$ and the weight function $w^i$ by another function $g(f^i, w^i)$, i.e., $g(\cdot, \cdot)$ is a function defined on $f^i$ and $w^i$.\footnote{Here, to make our model sufficiently general, we do not specify the dedicated definition of $g(\cdot, \cdot)$. In a specific application, $g(\cdot, \cdot)$ can be specified accordingly. For instance, we may have $g(f^i, w^i) = w^i \cdot f^i$ as in a linear regression model.}

Given $M$ learned from $U$, we quantify the FDI of $V$ using $\mathcal{M}$. For instance, $V$ could be the new monitored network traffic or the new collected web search data. For $v \in V$, to infer $v$ to some user in $U$ (or the data in $U$ generated by the same user) or to determine whether $v$ is a new user (or the data generated by a new user), two fundamental approaches are usually employed in $\mathcal{M}$: distance-based approach and distribution-based approach. In the distance-based approach, $\mathcal{M}$ computes the feature distance between $v$ and each $u \in U$, i.e., the distance between $\overrightarrow{F(v)}$ and $\overrightarrow{F(u)}$ for $u \in U$. Then, $\mathcal{M}$ infers $v$ to a subset of candidates in $U$ (either has the minimum or the maximum distance value).

In the distribution-based approach, $\mathcal{M}$ computes the feature distribution similarity between $v$ and each $u \in U$, i.e., the distribution similarity between $\overrightarrow{F(v)}$ and $\overrightarrow{F(u)}$ for $u \in U$. Then, $\mathcal{M}$ infers $v$ to a subset of candidates in $U$ (usually, the users in $U$ who have the most similar feature distributions with that of $v$). In this paper, we quantify the FDI for both approaches. Specifically, in this subsection, we focus on distance-based FDI quantification.

To facilitate our quantification, we first make the following definitions and assumptions. For $x, y \in U \cup \{v\}$, we define their feature distance as $D_{x,y}$. In practice, $D_{x,y}$ can be defined in an application-oriented manner. For instance, $D_{x,y}$ can be defined using the $p$-norm distance as follows:

$$D_{x,y} = \left( \sum_{i=1}^{N} |g(f^i_x, w^i_x) - g(f^i_y, w^i_y)|^p \right)^{1/p}.$$  

Let $E(\cdot)$ be the expectation/mean value of a random variable. Then, we define the expectation value of $D_{x,y}$ as $\mu_{x,y} = E(D_{x,y})$. Furthermore, we assume that $D_{x,y} \in [0, \zeta_{x,y}]$, i.e., the feature distance between $x$ and $y$ is lower bounded by 0 (which is an intuitive assumption) and upper bounded by some value $\zeta_{x,y} \geq 0$. Now, for $v \in V$ and $u, w \in U$, suppose that $v \approx u$ and $v \neq w$. We quantify the inferability of $v$ with respect to $u$ and $w$ in the following lemma.

**Lemma 3.** (1) When $\mu_{v,u} < \mu_{v,w}$, $v$ is inferable if $\min \left\{ \frac{1}{\zeta_{v,u}}, \frac{1}{\zeta_{v,w}} \right\} \geq \frac{1}{2(2\ln N + 1) \mu_{v,u} - \mu_{v,w}}$; (2) When $\mu_{v,u} > \mu_{v,w}$, $v$ is inferable if $\min \left\{ \frac{1}{\zeta_{v,u}}, \frac{1}{\zeta_{v,w}} \right\} \geq \frac{1}{2(2\ln N + 1) \mu_{v,u} - \mu_{v,w}}$.

**Proof:** We start from proving the first conclusion. Let $X = \frac{\mu_{v,u} + \mu_{v,w}}{2}$, $\xi_1 = \frac{\mu_{v,u} - \mu_{v,w}}{2\mu_{v,u}}$, and $\xi_2 = \frac{\mu_{v,w} - \mu_{v,u}}{2\mu_{v,u}}$. When $\mu_{v,u} < \mu_{v,w}$, we have

$$\Pr(\text{D}_{v,u} \geq \text{D}_{v,w}) \leq \Pr(\text{D}_{v,u} \geq X) + \Pr(\text{D}_{v,u} \leq X) = \Pr(\text{D}_{v,u} \geq (1 + \xi_1)\mu_{v,u}) + \Pr(\text{D}_{v,w} \leq (1 - \xi_2)\mu_{v,u}).$$

Applying Chernoff bound (as shown in Lemma 2 in the Appendix), we have

$$\Pr(\text{D}_{v,u} \geq \text{D}_{v,w}) \leq \exp(-\frac{2\xi_1^2 \mu_{v,u}^2}{\zeta_{v,u}^2}) + \exp(-\frac{\xi_2^2 \mu_{v,u}}{\zeta_{v,u}^2}) = \exp(-\frac{(\mu_{v,u} - \mu_{v,w})^2}{2\zeta_{v,u}^2}) + \exp(-\frac{(\mu_{v,w} - \mu_{v,u})^2}{4\zeta_{v,u}^2}) \leq \max\{2\exp(-\frac{(\mu_{v,u} - \mu_{v,w})^2}{2\zeta_{v,u}^2}), 2\exp(-\frac{(\mu_{v,w} - \mu_{v,u})^2}{4\zeta_{v,u}^2})\} \leq 2\exp(-2\ln N - 1) < 1/N^2.$$
Lemma 3 corresponds to an implementation of Algorithm 2 while changing $\Gamma_{v: \mathbf{z}}$ to $D_{v, w}$. Based on our proof, we conclude that the obtained $K_v$ of Algorithm 2 satisfies that $u \in K_v$ and $K_v \subseteq K$ (actually, $K_v = K$). \hfill \Box

In Lemma 4 we quantified the conditions for a user $v \in V$ to be Top-$K$ inferable. Based on Lemma 3 and Lemma 4, we can quantify the $(\delta, K)$-inferability of $v$. We show the result in the following theorem.

**Theorem 2.** Let $V_3$ be any subset of $\tilde{V}$ with $|V_3| = \delta m$. $V$ is $(\delta, K)$-inferable if for all $v \in V_3$, $\exists \tilde{K}_v \subseteq U$ such that $|\tilde{K}_v| = n - K$, $\mu_{v, u} \neq \mu_{v, w}$ for all $x \in \tilde{K}_v$, and $\min\{1 - \epsilon_n x \mid x \in \{u \cup \tilde{K}_v\} \geq 8\ln N + 4\ln 2\delta m n\}$, where $\epsilon_n = \frac{2\ln n}{\ln 2}$. Let $\delta = \frac{2\ln n}{\ln 2}$ and $\min = \min\{ \mu_{v, u} - \mu_{v, w} \mid x \in \tilde{K}_v \}$.

**Proof:** To prove this theorem, we take a similar approach as in proving Theorem 1. Let $E$ be the event that $\exists v \in V_3$, $v$ is not Top-$K$ inferable. Then, we have

$$\Pr(E) = \Pr(\bigcup_{v \in V_3} v \text{ is not Top } - K \text{ inferable})$$

$$\leq \sum_{v \in V_3} \Pr(v \text{ is not Top } - K \text{ inferable})$$

$$\leq \sum_{v \in V_3} 2\exp(-2\ln N - 2\delta m n) = 2\delta m n \cdot \exp(-2\ln N - 2\delta m n) = 1/N^2.$$

Therefore, $\Pr(E) N \rightarrow \infty 0$, which implies that $\Pr(\forall v \in \tilde{K}_v, v$ is inferable with respect to $\{u, w\}) N \rightarrow \infty 1$.
D. General Quantification: From the Distribution Perspective

In the previous subsection, we conduct the FDI quantification for the applications that \( \mathcal{M} \) employs a feature distance-based inference model. In many other applications, \( \mathcal{M} \) may employ a feature distribution-based inference model [1-8], i.e., determine whether to prove that \( \mathcal{M} \) is equivalent to prove that \( \mathcal{M} \) is sufficient while not necessary, it is possible to design some sophisticated \( \mathcal{M} \) to achieve better inference performance.

For each user \( v \in V \). Then, according to the similar argument as in Lemma 2, we can show that \( V \) is \( (\delta, K) \)-inferable under \( \mathcal{M} \). Again, since the conditions quantified in Theorem 2 (as well as in Lemma 3 and Lemma 4) are sufficient while not necessary, it is possible to design some sophisticated \( \mathcal{M} \) to achieve better inference performance.

Furthermore, let \( x = x(v,w) \) be any subset of \( U \) such that \( \mathcal{M}(v,w) \) is Top-\( K \)-inferable. The result is shown in Lemma 6.

**Lemma 6.** \( v \) is Top-\( K \)-inferable if \( \mu \geq \frac{(h-1)\sqrt{N \ln N \ln N}}{k^2} \), where \( \mu = \mathbb{E}(X) \), \( \theta = \frac{n-K}{n} \), and \( \xi \in (0,1) \) is a constant value.

**Proof:** This lemma can be proven based on Lemma 5. Let \( K_v \) be any subset of \( U \) such that \( |K_v| = n - K \). Then, we first prove that \( \mathcal{M}(v,w) \) is Top-K inferable with respect to \( \{u, w\} \). Let \( E \) be the event that \( \exists w \in K_v \) such that \( \cos(v, w) \leq \cos(v, u) \). Then, applying Lemma 5, we have

\[
\Pr(E) = \Pr(\mu \geq \frac{(h-1)\sqrt{N \ln N \ln N}}{k^2})
\]

\[
\leq \frac{\exp(-\frac{\xi^2 \mu^2}{N(h-1)^2})}{1/N^2}
\]

Therefore, \( \Pr(X \leq \mu) \geq \frac{1}{N^2} \), which implies \( v \) is inferable with respect to \( \{u, w\} \).
According to Lemma 6, we have \( \Pr(\forall u \in K_v, \cos(v, u) - \cos(v, w) > 0) \overset{N \to \infty}{=} 1 \). Now, we design a \( \mathcal{M} \) for \( v \) to be Top-\( K \)-inference. Similar to the one in Algorithm 2, we can design a \( \mathcal{M} \) under which the users in \( U \) who have the Top-\( K \) feature distribution similarity scores (Cosine similarity scores) with \( v \) are returned as \( K_v \). Then, based on our proof, we have \( u \in K_v \).

In Lemma 6, the feature distribution-based Top-\( K \) inferability of \( v \) is quantified. When the specified conditions are satisfied, we also discussed how to implement \( \mathcal{M} \) in the proof. Based on Lemma 5 and Lemma 6, we can quantify the \((\delta, K)\)-inference of \( V \). The result is shown in the following theorem.

**Theorem 3.** \( V \) is \((\delta, K)\)-inference if \( \mu \geq \frac{(h-l)\sqrt{N\ln(\delta \theta m n N^3)}}{\xi} \), where \( \theta = \frac{n-K}{n} \) and and \( \xi \in (0, 1) \) is a constant value.

**Proof:** Let \( V_\delta \) be any subset of \( V \) with size \( \delta m \). To prove this theorem, it is sufficient to prove that all the users in \( V_\delta \) are Top-\( K \) inferable. Let \( E \) be the event that \( \exists v \in V_\delta \) such that \( v \) cannot be Top-\( K \) inferable. Then, we have

\[
\Pr(E) = \Pr(\bigcup_{v \in V_\delta} v \text{ is not Top-} K \text{-inference}) \\
\leq \sum_{v \in V_\delta} \Pr(v \text{ is not Top-} K \text{-inference}).
\]

According to Lemma 6, we have

\[
\Pr(E) \leq \sum_{v \in V_\delta} \sum_{w \in K_v} \exp\left(-\frac{\xi^2 \mu^2}{N(h-l)^2}\right) \\
\leq \sum_{v \in V_\delta} \sum_{w \in K_v} \exp\left(-\ln(\delta \theta m n N^3)\right) \\
= \delta m \cdot \theta n \cdot \exp\left(-\ln(\delta \theta m n N^2)\right) \\
= 1/N^2.
\]

Therefore, we have \( \Pr(E) \overset{N \to \infty}{=} 0 \), i.e., statistically, all the users in \( V_\delta \) are \((\delta, K)\)-inference.

In Theorem 3, we quantify the feature distribution similarity-based \((\delta, K)\)-FDI of \( V \). When the specified conditions are satisfied, we can also design a \( \mathcal{M} \) using the one shown in Lemma 6 finding the \( K_v \) for each user \( v \in V \) using the \( \mathcal{M} \) shown in Lemma 6. According to our proof, we can see that \( V \) is \((\delta, K)\)-inference under such a \( \mathcal{M} \). Furthermore, similar to that in Theorem 1 and Theorem 2, the conditions in Theorem 3 are sufficient while not necessary. Therefore, a sophisticated \( \mathcal{M} \) could be implemented to improve the inference performance. Here, our FDI quantification can serve as a theoretical baseline to facilitate and guide the design of better inference models.

**E. Discussion: Inferring New User/Data**

In the previous subsections, we focus on quantifying the feature distance and distribution based FDI of the users that appear in both the training data \( U \) and the target data \( V \). In reality, it is possible that there are some new users/data that appear in \( V \) while not in \( U \). Formally, it is possible that \( \exists v' \in V \) such that \( v' \approx u \). In this case, an ideal inference model \( \mathcal{M} \) will infer \( v' \) as a new user (or data generated by a new user), e.g., an intruder in network forensics applications \([1][4]\). In practical inference models \([1][8]\), a user \( v' \) in \( V \) is inferred as a new user (or data generated by a new user) if the feature distance \( D_{v', u} \) is larger than a threshold for \( \forall v \in U \), or the feature distribution similarity \( \cos(v', u) \) is smaller than a threshold for \( \forall u \in U \).

Theoretically, it is challenging (or, impossible) to quantify the precise inferability of a new user \( v' \in V \) in general with statistical guarantee (that is why an inference system has false positive and false negative). The reason is that theoretically, the feature characteristics of a new user (data generated by a new user) might be arbitrarily similar to an existing user (e.g., the network intruders keep improving their camouflaging techniques). Nevertheless, our FDI quantification still has meaningful implications for inferring new users. For \( x \in V \), \( y \in U \), and \( x \approx y \), let \( \mu_d = \mathbb{E}(D_{x,y}) \) when \( \mathcal{M} \) is a feature distance based model and \( \mu_s^* = \mathbb{E}(\cos(x,y)) \) when \( \mathcal{M} \) is a feature distribution similarity based model. Then, when \( D_{v', u} \) is significantly apart from \( \mu_d^* \) or \( \mu_s^* \) depending on \( \mathcal{M} \) (distance or distribution based), \( v' \) can be inferred as a new user (the data generated by a new user) with a higher confidence, i.e., \( \mu_d^* \) or \( \mu_s^* \) can be set as the threshold values in practical applications. The behind-the-scene reason for this fact can be explained by the following corollary, which is a direct result of the Chernoff bound.

**Corollary 3.** (1) Let \( D_{\min} = \min\{D_{v', x} | v' \in V, x \in U\} \) and \( D_{\max} = \max\{D_{v', x} | v' \in V, x \in U\} \). When \( \mu_d^* \in [0, \xi] \) and \( \mu_d^* \geq \frac{\sqrt{2} \ln N}{\xi} \), \( v' \) is a new user if \( D_{\min} \geq (1 + \xi)\mu_d^* \) or \( D_{\max} \leq (1 - \xi)\mu_d^* \) for all \( \xi > 0 \). (2) Let \( C_{\max} = \max\{\cos(v', x) | v' \in V, x \in U\} \). When \( \mu_s^* \in [l, h] \) and \( \mu_s^* \geq \frac{(h-l)\sqrt{2} \ln N}{\xi} \), \( v' \) is a new user if \( C_{\max} \leq (1 - \xi)\mu_s^* \) for all \( \xi > 0 \).

In practice, the accurate value of \( \mu_d^* \) or \( \mu_s^* \) is usually difficult to be obtained, if not impossible. Frequently, \( \mu_d^* \) or \( \mu_s^* \) can only be estimated based on the observed data and thus it may change with more data coming, i.e., the threshold estimation problem itself is an interesting problem. For our purpose, we propose to quantify the correlation between the threshold setting and the false positive/negative rate of \( \mathcal{M} \) as one of our future research directions.

**IV. Evaluation**

In this section, we evaluate the user inferability of real world security and privacy applications based on our FDI quantification. Specifically, we evaluate two scenarios: network traffic attribution in network forensics and feature-based data de-identification (as shown in Section II-A).

**A. Network Traffic Attribution**

1) Data Collection and Analysis: In this scenario, we evaluate the user inferability of four large-scale network traces generated by the employees of a large enterprise. These four traces are collected in four periods of 2014: April 1 – April 30 which consists of the network traffic generated by 5888 users,
July 1 – July 31 which consists of the network traffic generated by 5610 users, October 1 – October 31 which consists of the network traffic generated by 5268 users, and December 1 – December 31 which consists of the network traffic generated by 5699 users. For each network trace, it is composed of three parts: HTTP request headers, netflow measures, and DNS queries.

Here, we do not consider the network traffic payloads, e.g., the HTTP payloads, for the following reasons. First, those data are highly sensitive and using them may cause some legal issues. Second, although network traffic payloads may provide more information, using our network traces is sufficient to infer many users as shown in our experiments. Finally, as indicated in [1], in most of the common available traces, they do not have those payloads. Therefore, studying the common feature-based data inferability would be more useful and general for security and privacy applications.

2) Feature Extraction: After collecting these four traces, we extract the features of them. Here, we use the feature extraction model proposed in [1]. Although we may extract more features, for our purpose, it is sufficient to extract two kinds of lexical-based features for our FDI analysis: domain feature and path feature (tokenized). Basically, these two features characterize the behaviors of users in terms of what types of websites they have visited and how they interacted with the websites. For instance, given a HTTP request “www.google.com/search?q=ndss+2016&ie=utf-8&oe=utf-8”, we will extract a domain feature as “www.google.com”. For the path features, we tokenize each path (URL) using ‘?’ , ‘=’ , ‘&’ etc. as delimiters and employ a bag-of-word representation of the tokens. We refer to the interested readers for more details of the feature extraction model to [1]. Finally, we show the feature extraction results of the four traces in Table I, where Apr, July, Oct, and Dec represent the four traces collected in April, July, October, and December of 2014, “-Domain” means the domain features, “-Path” means the tokenized path features, \( n \) is the number of users in the dataset, and \( N \) is the number extracted features.

### Table I

|       | \( n \) | \( N \) | \# of user-feature relationships |
|-------|--------|--------|----------------------------------|
| Apr-Domain | 5,888  | 290,537 | 3,968,361                        |
| Apr-Path  | 5,888  | 1,685,439 | 17,389,051                       |
| July-Domain | 5,610  | 391,290  | 3,739,246                        |
| July-Path | 5,610  | 1,855,415 | 16,010,442                       |
| Oct-Domain | 5,268  | 270,604  | 3,868,538                        |
| Oct-Path  | 5,268  | 1,741,781 | 16,895,932                       |
| Dec-Domain | 5,699  | 298,490  | 3,736,956                        |
| Dec-Path  | 5,699  | 2,159,448 | 16,926,145                       |

Now, we define the degree of each user as the number of features this user has and the degree of each feature as the number users that have this feature. Then, we show the user degree distribution and feature degree distribution of the traces in Table I with respect to the domain feature and the path feature in Fig[1] and Fig[2] respectively. From Fig[3] and Fig[4] we have the following observations: both the user degree and feature degree generally follow a power-law-like distribution [11], especially the feature degree distribution, i.e., most of the users have a small number of features while only a few users have many features, and meantime, most of the features only appear in the trace of a few users while a small number of features appear in the trace of a large number of users. These distributions together suggest that these features could be employed to effectively infer the users.

3) Evaluation Methodology: To conduct the FDI evaluation, we basically follow the same process as shown in Fig[1] and Fig[2]. Meanwhile, since we focus on evaluating the statistically inherent FDI, we also make the evaluation process mathematically tractable. Following the models shown in Fig[3] and Fig[4] we first determine the training data and the testing data. Here, instead of partitioning the raw data into two parts for training and testing respectively (as in many existing literature, e.g., [1]), we take another while theoretically equivalent approach: following the FDI quantification in Section III, we first construct the training dataset by keep all the users and features in each trace while sample the user-feature relationships independently and identically using a probability \( p \). Similarly, we construct the testing/targeting dataset using the same process as in obtaining the training dataset. We use this approach to construct the training and testing data for two reasons. First, mathematically, this approach is equivalent to the traditional method in [1]. In the traditional method, the raw data is partitioned into the training data and the testing data and then features are extracted from both datasets. Apparently, the reason that existing inferring techniques can work is that the training data and the testing data share some common features (or similar distributions over a feature space). Therefore, statistically, we can consider the training and testing data as some sampling versions of the original raw data respectively, i.e., each training and testing data partition method mathematically corresponding to one \( p \) here. Second, using this approach to obtain the training and testing/targeting data makes it easier to apply our FDI quantification method. We will make more discussions on closing the gap between theory and practice in Section V.

After obtaining the training and testing data, we quantify the FDI of the four traces using the general scenario FDI quantification technique in Section III. Specifically, for the network traffic attribution application, most the of existing inference

\(^4\)It is not necessary to have the training data and the testing data to have the same group of users or features. If they are not the same, we can either apply our theory to the overlapped users/features, or make them the same by adding isolated users/features that only appeared in the other dataset. Theoretically, different user/feature group will not change the validity of our quantification.
models are based on feature distance [1][2][3]. Therefore, we evaluate the FDI using the distance-based quantification technique here. Following Theorem 2, we can easily construct an inference model $\mathcal{M}$ on top of the procedure of Algorithm 2 as shown in Section III-C. Then, we apply $\mathcal{M}$ to quantify the FDI of each dataset. We also make more discussion on the implications of our quantification as well as the implications of the results in Section IV.

4) Results and Analysis: Now, we evaluate the FDI of the four traces following the above evaluation methodology. To reduce any bias, all the experiments are run 10 times (e.g., for the same $p$). The final results are the average of that of the 10 runs. We show the $(\delta, K)$-inferability of the four datasets with respect to the domain and path features in Fig.6 respectively, where we set $K = 10$, i.e., we are targeting a user to be Top-10 inferable. From Fig.6, we have the following observations.

- With the increase of $p$, $\delta$ also increases, which implies that more and more users become Top-10 inferable. The reason is that a large $p$ implies more common features are shared by the training data and the targeting data, i.e., there is more knowledge available to an inference model. Therefore, statistically, more users can be successfully Top-10 inferable.
- When comparing the domain feature-based data inferability (Fig.6(a)) with the path feature-based data inferability (Fig.6(b)), we find that the path features are more powerful in inferring the users than the domain features. This can be explained based on the results in Table 1, Fig.4 and Fig.5. First, for each dataset, it has much more path features than domain features (Table 1), i.e., much more knowledge can be used to conduct path feature-based inference. Second, the users of each dataset have higher path feature-based degrees than domain feature-based degrees (Fig.4), and meanwhile, both the domain
and the path feature degree distributions generally follow similar power-law-like distributions. Thus, users are more distinguishable with respect to the path features than that of the domain features.

In our evaluation, we also examined the data inferability with respect to other settings: changing the value of $K$ and combining the domain and path features. The results are as expected and we put them in the technical report [12]. Here, we briefly summarize the results. When increasing $K$ (from $0.05n$ to $0.2n$), more users are Top-$K$ inferable given the same $p$. The reason is evident since increasing $K$ implies decreasing the desired inference accuracy. Statistically, more users become Top-$K$ inferable. Furthermore, after combining the domain and path features together, we also have more users inferable compared to the scenario of applying the domain and path features separately. The reason is also straightforward since more features imply more knowledge are available for inferring users, and thus the inference accuracy is improved.

B. Data De-anonymization

Now, we evaluate the users’ feature-based inferability in the data de-anonymization application [6][7][8].

1) Data Collection and Features: In our evaluation, we use three social network datasets, Google+, Facebook, and Twitter, which are publicly available at the Stanford Large Network Dataset Collection [13]. The reason for us to use these datasets is that they are published along with well-defined user features, e.g., birthdays, education, hometown, languages, career, etc. For de-anonymization attacks, an adversary may directly employ these features to de-anonymize users. For our purpose, we can also employ these features to quantify users’ FDI. We show the statistics of these three datasets in Table II. By comparing the datasets in Tables IV and II, we can find that the three social datasets have much less features. Furthermore, for the three datasets in II, there is no weight information associated with the user-feature relationships.

We show the user degree distribution of Google+, Facebook, and Twitter in Fig[7] Basically, the user degree of these three datasets also shows a power-law-like distribution (similar to the datasets in Table II, the feature degree of these three datasets show a power-law-like distribution either [12]). This suggests that the users in the three datasets could be inferred (i.e., de-anonymized here) based on the associated features.

2) FDI Evaluation and Analysis: To evaluate the FDI of Google+, Facebook, and Twitter in Fig[7] we take the same methodology as in the previous subsection. We show the FDI of the three datasets in Fig[8] where $K = 10$, i.e., we also target the Top-10 inferability of users. From the result, we have the following observations.

- Again, with the increase of $p$, more users become Top-10 inferable in the three datasets. The reason is the same as that in analyzing Fig[6].
- Google+ is much less inferable than that of Facebook and Twitter. For instance, when $p = 0.8$, 13.47% Facebook users and 44.65% Twitter users are Top-10 inferable, while only 1.62% Google+ users are Top-10 inferable. Even if $p = 0.9$, only 8.34% Google+ users are Top-10 inferable. This can be explained based on the results in Table II and Fig[7]. First, the user-feature relationship of Google+ is much sparser than the other two datasets. Second, the degree of most of the Google+ users is very low. Therefore, there is not too much information can be
leveraged to infer the Google+ users.

In reality, it is possible to improve the data de-anonymization performance using more auxiliary information (more features). Here, our FDI quantification results can provide a benchmark for evaluating the performance of a data de-anonymization attack.

V. DISCUSSION

In this section, we make more discussion on the proposed FDI quantification technique, followed by pointing out the future research directions.

A. Theory versus Practice

Motivated by many existing security and privacy applications, in this paper, we study the FDI quantification problem. To the best of our knowledge, we provide the first FDI quantification technique for general feature-based inference models from both the distance perspective and the feature distribution perspective. Using our quantification technique, we also evaluate the FDI of feature-based network forensics and data de-anonymization applications.

Our quantification is important in several perspectives. First, our quantification provides the theoretical foundation of many existing feature-based security and privacy applications, e.g., network traffic attribution in network forensics [1][3], linkage attacks and private web search [4][5], and feature-based data de-anonymization [6][7][8]. Therefore, for such kind of applications, our quantification closes the gap between the practice and theory.

Second, our quantification can be employed to evaluate the performance of the existing techniques in the aforementioned security and privacy applications. Note that, we are aiming to quantify the users who can be inferred with statistical guarantee based on their features as well as other users’ features. Meanwhile, we also provide insights on how to design the inference model (as shown in Section III). Therefore, the quantification results (e.g., the evaluation results in Section IV) can serve as a benchmark to evaluate the performance of existing techniques. For instance, to evaluate the performance of the network traffic attribution system Kaleido [1], we can employ the evaluation results in Section IV directly.

Similarly, we can also employ the FDI quantification to evaluate existing feature-based query linkage attacks, private searching techniques, data de-anonymization attacks, etc.

Finally, since our quantification can provide a benchmark of existing feature-based security and privacy applications, it is evident that our quantification is helpful for researchers to study and develop new techniques for these applications.

B. Future Work

In this paper, we take the first step in understanding the theoretical foundation of many existing security and privacy applications to the best of our knowledge. Specifically, we propose the FDI quantification techniques for distance-based and distribution-based inference models. There are still several interesting directions to continue the research. First, it is interesting to further generalize our quantification to the inference models that take account of both feature distance and feature distribution. Second, in addition to the distance/distribution-
based models, it is also meaningful to quantify data’s inferability under other models for more security and privacy applications. Third, it is an interesting and meaningful direction to develop some FDI-based evaluation tool which can friendly and conveniently serve the data inferability analysis for existing feature-based inference-oriented security and privacy applications.

VI. RELATED WORK

In this section, we survey the related work. Since we did not have other literature studying the theoretical foundation or inferability quantification problem for existing feature-based security and privacy applications to the best of our knowledge, we focus on briefly summarizing the applications that our FDI security and privacy applications to the best of our knowledge, not have other literature studying the theoretical foundation or perform user authentication; in [16], Monrose et al. designed another keystroke dynamics (traces) to perform user authentication; in [16], Monrose et al. designed a technique to reliably generate a cryptographic key from a user’s voice while speaking a password; and in [17], Zheng et al. implemented an efficient user verification system based on mouse movement traces.

Linkage Attacks and Privacy-preserving Web Search. In [4], Gervais proposed a quantitative framework to understand the web-search privacy guarantee’s adversary’s background knowledge and attacks. In [13], Peddinti and Saxena analyzed whether query obfuscation can preserve users’ privacy when against an adversarial search engine. In [19], Jones presented attacks to users’ query logs and broke users’ privacy. Recently, Balsa et al. presented a SoK paper on linkage attacks and privacy-preserving web search [5].

Feature-based Data De-anonymization. In [6], Caliskan-Islam et al. presented a novel data de-anonymization attack to programmers leveraging the code stylometry. Afroz et al. presented another stylometry-based de-anonymization attack in [7], by which they can identify anonymous authors of anonymous texts. In [8], Narayanan and Shmatikov presented a new class of statistical de-anonymization attacks to high-dimensional micro-data, e.g., recommendation data, transaction data, and so on. An off-line de-anonymization attack of bubble forms is presented in [20] by Calandrino et al.

Remark. In addition to the aforementioned security and privacy applications, there are also other applications, e.g., feature-based malware detection systems and intrusion detection systems, that our quantification can be applicable for analysis. Although we have many feature-based inference techniques for various security and privacy applications, their theory foundation is remain unclear. Furthermore, there is also no theoretical benchmark to evaluate the performance of existing techniques relative to the inherent performance bound. To remedy the gap, we conduct the first FDI quantification in general scenarios from both distance and distribution perspectives.

VII. CONCLUSION

Considering that many security and privacy applications can be characterized by the feature-based inference problem, we study the FDI issue in this paper. First, we conduct the FDI quantification under a naive data model, under which we demonstrate the conditions to have a desired fraction of target users to be Top-K inferable. Subsequently, we extend our quantification to a general data model by conducting the FDI quantification from both a distance perspective and a distribution perspective. Our quantification addressed several important yet open problems and lies the foundation of existing feature-based inference systems/techniques. Third, based on our quantification, we evaluate the user inferability in both the network traffic attribution case and the feature-based data de-anonymization case. Finally, we point out the implications of this research to existing feature-based inference systems/techniques for various security and privacy applications.

REFERENCES

[1] T. Wang, F. Wang, D. Schales, and R. Sailer, Kaleido: Network Traffic Attribution using Multifaceted Footprinting, S&P 2014.
[2] J. V. Davis, B. Kulis, P. Jain, S. Stra, and I. S. Dhillon, Information-theoretic Metric Learning, ICML 2007.
[3] C. Neasbitt, R. Perdisci, K. Li, and T. Nels, ClickMiner: Towards Forensic Reconstruction of User-Browser Interactions from Network Traces, CCS 2014.
[4] A. Gervais, R. Shokri, A. Singla, S. Capkun, and V. Lenders, Quantifying Web-Search Privacy, CCS 2014.
[5] E. Balsa, C. Troncoso, and C. Diaz, OB-PWS: Obfuscation-based Private Web Search, S&P 2012.
[6] A. Caliskan-Islam, R. Harang, A. Liu, A. Narayanan, C. Voss, F. Yamaguchi, R. Greenstadt, De-anonymizing Programmers via Code Sty-lometry, USENIX Security 2015.
[7] S. Afroz, A. Caliskan-Islam, A. Stolerman, R. Greenstadt, and D. McCoy, Doppleg¨anger Finder: Taking Stylometry to the Underground, S&P 2014.
[8] A. Narayanan and V. Shmatikov, Robust De-anonymization of Large Sparse Datasets, S&P 2009.
[9] P. Pedarsani and M. Grossglauser, On the Privacy of Anonymized Networks, KDD 2011.
[10] M. Goemans, Chernoff Bound, and some Applications, http://math.mit.edu/~goemans/18310S15/chernoff-notes.pdf.
[11] Power Law Distribution, https://en.wikipedia.org/wiki/Power_law.
[12] ***, The FDI project, ***.
[13] Stanford Large Network Dataset Collection, http://snap.stanford.edu/data/index.html.
[14] E. S. Pilli, R. C. Joshi, and K. Niyogi, Network Forensic Frameworks: Survey and Research Challenges, Digital Investigation, 7(1):1427, 2010.
[15] F. Bergadano, D. Gunetti, and C. Picardi, User authentication through keystroke dynamics, ACM TISSEC, 2002.
[16] F. Monrose, M. K. Reiter, Q. Li, and S. Wetzel, Cryptographic key generation from voice, S&P 2011.
APPENDIX

A GENERAL VERSION OF CHERNOFF BOUND

The following version of Chernoff bound applies to bounded variables with any distribution [10].

Lemma 7. Let $X_1, X_2, \ldots, X_n$ be random variables such that $a \leq X_i \leq b$ for all $i$. Let $X = \sum_{i=1}^{n} X_i$ and set $\mu = \mathbb{E}(X)$ (i.e., the expectation value of $X$). Then, for all $\xi > 0$: 
\[
\Pr(X \geq (1 + \xi)\mu) \leq \exp\left(-\frac{\xi^2 \mu^2}{n(b-a)^2}\right), \quad \text{and} \quad 
\Pr(X \leq (1 - \xi)\mu) \leq \exp\left(-\frac{\xi^2 \mu^2}{n(b-a)^2}\right).
\]