Control of neural chaos by synaptic noise

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Abstract

We studied neural automata —or neurobiologically inspired cellular automata— which exhibits chaotic itinerancy among the different stored patterns or memories. This is a consequence of activity-dependent synaptic fluctuations, which continuously destabilize the attractor and induce irregular hopping to other possible attractors. The nature of the resulting irregularity depends on the dynamic details, namely, on the intensity of the synaptic “noise” and on the number of sites of the network that are synchronously updated at each time step. Varying these details, different regimes occur from regular to chaotic. In the absence of external agents, the chaotic behavior may turn regular after tuning the noise intensity. It is argued that a similar mechanism might be at the origin of the self-control of chaos in natural systems.

1 The model and its motivation

We report on the complex dynamics and possible applications of a novel neural automaton or cellular automaton [Wolfram(1984)] inspired in neurobiology. The model exhibits dynamic associative memory, including kind of switching behavior that has been reported for neural networks with dynamic synapses [Pantic et al.(2002), Cortes et al.(2004), Abbott and Regehr(2004)]. Our automaton incorporates fast fluctuations of synaptic intensities which depend on neuron activity. Such “noise” induces instability of the recalling dynamics in a way that mimics how the brain efficiently solves some complex tasks. In fact, a rapid response to highly changing stimuli is believed to play a functional role during both attention and sequential processing of parallel sensory information [Cortes et al.(2005)]. In this report, we adapt a previous proposal [Marro et al.(2005)] to show that fast synaptic noise can control the complexity and chaoticity of dynamics and, in particular, the details of the temporal oscillations of the neural activity. Unlike in earlier work [Molgedey et al.(1992), Schiff et al.(1994), Freeman et al.(1997)] the noise intensity in the present mechanism varies autonomously, which could be more relevant to the self-control of chaos in neural systems as well as in other cases.

The model consists of $N$ cooperative and, for simplicity, fully-connected neurons with stochastic dynamics$^1$. A main feature is that, at each time step $t$, the individual states of $n \leq N$ neurons are simultaneously updated. This is performed according to a modification of the Hopfield prescription [Amari(1972), Hopfield(1982), Amit(1989)]. We assume that each neuron $s_i$ endures a current or a local field [Gardiner(2004)].$^1$

$^1$Some consequences of other network topologies have been studied in [Torres et al.(2004)], for instance.
where \( \xi \) patterns that are assumed hereafter to be stored in the system. It then naturally follows stochastic of parameters. In particular, the case analytically and numerically for certain choices instance. [Abbott and Regehr(2004)] for a technical justification of (1), and [Abbott and Regehr(2004)] for a recent discussion on the role of synaptic noise, for instance.

This model has already been analyzed both analytically and numerically for certain choices of parameters. In particular, the case \( n = 1 \) of “sequential updating” was shown to exhibit complex hopping between the attractors in some cases [Cortes et al.(2005)], and we recently demonstrated [Marro et al.(2005)] that the hopping may become chaotic for Littledynamics, namely, \( n = N \). We here illustrate a typical situation between these two limits by means of computer simulations. The case with \( 1 < n < N \) for which we present some results here happens to be relevant to understand the possibility of controlling chaos of the neural activity by means of synaptic “noise”.

In order to deal with model simulations that remain versatile enough, we need to introduce some simplifications in the following; notice, however, that some of them may turn irrelevant to the resulting emergent behavior. Most convenient is to restrict ourselves to binary neurons, i.e., \( s_i = \pm 1 \), which are known to capture the essentials of cooperative phenomena [Abbott and Kepler(1990)]. Concerning the stochastic variable, we need to determine both its nature and its distribution. A simple choice is to assume that synaptic intensities are of the form \( w_{ij} = w_{ij}^{l} x_j \) where \( w_{ij}^{l} \) are average weights which, also for the sake of simplicity, we shall consider to be of the Hebbian type. That is, \( w_{ij}^{l} = N^{-1} \sum_{\mu} \xi_{ij}^{\mu} \xi_{ij}^{\mu} \), where \( \xi_{ij}^{\mu} \) (with \( \mu = 1, ..., M \)) stands for \( M \) (binary) patterns that are assumed hereafter to be stored in the system. It then naturally follows stochasticity of the presynaptic currents in (1) which are
given by \( h_{ij}(S, X) = \sum_{j \neq i} w_{ij}^{l} x_j s_j \). This is consistent with actual features of natural systems such as, for example, variations of the glutamate concentration in the synaptic cleft, and differences in the potency released from different locations on the active zone of the synapses [Franks et al.(2003)]. These and similar “noises” which cause synaptic fluctuations are typically very fast compared to the time relaxation of the whole neuron system. Therefore, it seems sensible to assume that, in the time scale for the neuron activity, neurons behave as in the presence of a steady distribution for the synaptic fluctuations. This is taken into account by means of the distribution \( P(X|S) \) in (1), a situation which is discussed with further detail in Marro and Dickman(1999).

### 2 Synaptic noise

Recent neurobiological findings [Abbott and Regehr(2004)], concerning activity-dependent processes may help in determining \( P(X|S) \). In particular, it was reported short-time synaptic depression [Tsodyks et al.(1998)], i.e., that synaptic weights tend to decrease under repeated presynaptic activation. A simple way of implementing this in (1) is by taking

\[
P(X|S) = \prod_{i} \{ p(\tilde{m}) \delta(x_j + \Phi) + [1 - p(\tilde{m})] \delta(x_j - 1) \},
\]

where the factorization is for simplicity and \( \tilde{m} = \tilde{m}(S) \) is the \( M \)-dimensional overlap vector of components \( m^\mu(S) = N^{-1} \sum_{i} \xi_{ij}^{\mu} s_i \). In accordance with the mentioned observation, (2) implies that increasing the mean firing rate, which will increase the probability function \( p(\tilde{m}) \), will make more likely that synaptic intensities decrease by a factor of \( \Phi \). The Hopfield model, for which such depressing noise is absent, corresponds here to the limit \( \Phi \to -1 \). Finally, in order to fully determine the model, one may use the choice [Cortes et al.(2005)]

\[
\zeta(\tilde{m}) = (1 + \alpha)^{-1} \sum_{\mu} [m^\mu(S)]^2, \text{ where } \alpha = M/N \text{ is the network load parameter [Hertz et al.(1991)].}
\]

After some straightforward algebra, one obtains the effective currents as

\[
\eta_i(S) = \left( 1 - \frac{1 + \Phi}{1 + \alpha} \sum_{\mu} [m^\mu(S)]^2 \right) \sum_{\mu} \xi_{ij}^{\mu} m^\mu(S).
\]
This shows the time variation of the overlap $m \equiv m^1(S)$ between the current neural activity, $S$, and the only pattern which is stored in the synaptic weights, i.e., for $M = 1$, as obtained in a Monte Carlo simulation with $N = 3600$ neurons and a depressing factor $\Phi = 0.043$. The top graph is for $T = 0$, i.e., in the absence of thermal fluctuations, while the bottom graph is for $T = 0.51$.

In addition to the discussed synaptic stochasticity, that we represent here by means of the variable $x$, there are independent causes for assuming a stochastic dynamics of the neuron system. That is, a neuron may sometimes remain silent even if it endures a large current. This is naturally modelled by introducing a “temperature” parameter $T$. In practice, one usually assigns a probability which depends on $(h_i - \theta_i)/T$, where $\theta_i$ is a threshold, to the change according to $\text{sig}(h_i) = s_i$ at time $t$. This mechanism is equivalent to assume the existence of a hypothetical “thermal bath” which induces stochasticity of the neuron activity by means of a master equation. In general, this equation implies a tendency towards equilibrium. However, in the present case, the canonical tendency competes with the stochastic changes of $h_i$, which impedes equilibrium, and the system goes asymptotically to a non-equilibrium steady state [Marro and Dickman (1999)]. This complex, non-equilibrium situation is at the origin of the intriguing behavior we describe next.

Figure 2: Monte Carlo simulations in the absence of thermal fluctuations, $T = 0$, for a single stored pattern and $N = 3600$ neurons showing the effect of varying the synaptic noise parameter $\Phi$. The resulting hopping shows dramatic variations of temporal scale and degree of complexity as one varies $\Phi$.

3 Computer simulations

The above programme was implemented in the computer by iterating the following Monte Carlo algorithm:

1. Store $M$ different patterns $\xi^\mu$ in the average weights $w_{ij}^L$ according to the chosen, e.g., Hebb’s learning rule.

2. Set any state $S = \{s_i\}$ at random.

3. Compute the $N$ local fields $\overline{h_i}(S)$ as defined in [9].

4. Choose a site (neuron) at random, repeat the choice $N$ times and keep only the $n < N$ sites which differ from each other (this procedure lets you with $n \approx \frac{1}{2}N$ sites — for the values of
Figure 3: The entropy function, as defined in the main text, for different time series obtained during Monte Carlo simulations of neural automata for different values of the synaptic noise parameter $\Phi$. Decreasing values of the entropy indicate a tendency towards regularization of the complexity of the time series. The graph reveals different regimes of chaoticity.

$N$ of interest here). \(^{2}\)

5. Perform the changes $s_i \rightarrow -s_i$ at the chosen $n$ sites using the standard rate $\omega(s'_i \rightarrow s_i) = \frac{1}{2}\{1 - s'_i \tanh[\beta h_i(S')]\}$.

6. Increase time in one unit, and go to step 3.

Figure 1 illustrates the resulting behavior for a single pattern, i.e., it corresponds to the limit $\alpha \rightarrow 0$. This shows a complex hopping process between the pattern, $\xi$, and the anti-pattern, $-\xi$. The figure compares the evolution at a finite temperature with that in the absence of thermal fluctuations to demonstrate that hopping is not a consequence of the latter. Consequently, in order to avoid the short–length oscillations shown in the bottom graph of figure 1 which are induced by the thermal noise, we are concerned in the following with simulations at $T = 0$.

Figure 2 illustrates a main result, namely, that the frequency and other details of the hopping strongly depend on the value of the parameter $\Phi$ which modulates the fast synaptic noise. An appropriate measure of the associated entropy will provide a quantitative description of the complexity of this hopping. Using standard fast Fourier transform algorithms, we computed the power spectra $P(\eta)$. The normalized probability $p_\eta = P(\eta)/\sum_\eta P(\eta)$ then allows one to define a regular entropy as $S = -\sum_\eta p_\eta \log_2 p_\eta$. This quantity has been used before to detect regularity out of chaotic activity in actual neurons [Varona et al.(2001)]. As a matter of fact, $S > 0$ is to be associated with chaotic behavior while $S = 0$ would correspond to periodic dynamics.

Figure 3 depicts the entropy which results in our case as a function of $\Phi$. This shows a minimum which corresponds to the smallest degeneration in the time series of figure 2 (second graph from the top). Decreasing $S$ indicates a tendency to regularization or smaller chaoticity, while higher chaos and irregularity in the time series corresponds to larger values of $S$.

4 Conclusions

We have introduced a class of hybrid neural automata with two main features. On one hand, these models provide a convenient arena to analyze the influence of fast synaptic noise on the retrieval process. On the other hand, they may describe a continuous transition from sequential, single–neuron updating to the case of Little dynamics or parallel updating as one varies the model parameter $n$. The synaptic noise is modelled trying to mimic recent observations, namely, the noise occurs in a short–time scale and conveniently couples to the neuron activity to induce synaptic depression. Depending on the intensity of this depression, the model exhibits a varied emergent behavior, including chaotic hopping between the attractors. This results in a rather complex pattern of neural activity. Monitoring the entropy suggests how a fast noise might provide a mechanism to control chaos in living systems [Garfinkel et al.(1992), Schiff et al.(1994)]. The design of a mechanism in which noise intensity varies autonomously could be useful to the self-control of chaos. Notice in this respect that manipulating $n$ in the model might be convenient for the purpose. That is, two main cases follow altogether from the present analysis and some previous work [Cortes et al.(2005), Marro et al.(2005)]. (1) $n = 1$, for which the system is sensible to an external

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\(^2\)Both Monte Carlo simulations and analytical results [Cortes et al.(2006)] are in full-agreement and, in the thermodynamic limit, one has that $n/N = 1 - 1/e$. 

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stimulus, which may destabilize the attractor, but it does not exhibit autonomous hopping between attractors; and (2) \( n > 1 \), for which hopping occurs autonomously, without the need for any external stimulus. In the latter case, as far as \( n < N \), the parameter \( \Phi \) allows for a control of the hopping, while this always occurs at high frequency for \( n = N \). For \( n \approx \frac{4}{3}N \), the case for which we report some results here, the time the neuron activity stays at or near each attractor may be varied by tuning \( \Phi \), as illustrated in figure 2.

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