Gravitational waves from in-spirals
of compact objects in binary common-envelope evolution

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Detection of gravitational-wave (GW) sources enables the characterization of binary compact objects and their in-spiral due to GW-emission. However, other dissipative processes can affect the in-spiral. Here we show that the in-spiral of compact objects through a gaseous common-envelope (CE) arising from an evolved stellar companion produces a novel type of GW-sources, whose evolution is dominated by the dissipative gas dynamical friction effects from the CE, rather than the GW-emission itself. The evolution and properties of the GW-signals significantly differ from those of isolated gas-poor mergers. We find characteristic strains of \( \sim 10^{-22} - 10^{-21} \) (10kpc/D) for such sources - observable by next-generation space-based GW-detectors. The evolution of the GW-signal can serve to probe the interior parts of the evolved star, and the final stages of CE-evolution, otherwise inaccessible through any other observational means. Moreover, such CE-mergers are frequently followed by observable explosive electromagnetic counterparts and/or the formation of exotic stars.

As two gravitating bodies orbit each other they emit gravitational waves (GWs), whose amplitude and frequency depend on the relative acceleration of the bodies, which, in turn, depends on their masses and separation. Generally, larger masses and smaller separations give rise to stronger accelerations and thereby to stronger GW emission. Current and upcoming GW-detectors cannot detect the merger of stellar binaries whose radii – and hence separation at merger – are far below their amplitude sensitivity at the relevant frequency scale. Hence, only sufficiently compact (and sufficiently massive) stars could serve as detectable GW sources; current studies typically focus on the mergers of white dwarfs (WDs) [1], neutron stars (NSs), and black holes (BHs) [2, 3]. However, the stellar cores of evolved stars could serve as a novel type of sufficiently compact objects to produce GW sources, once they merge with another compact object (CO). Such mergers can occur following the common-envelope (CE) evolution of evolved stars with compact binary companions, leading to the in-spiral of the compact object and its merger with the core of the evolved stars [4], as we discuss in the following.

During the evolution of stars beyond their main-sequence evolutionary stage, they fuse the hydrogen into their cores into helium and heavier elements, that accumulate in compact cores. The stellar envelopes of the stars then expand to large radii of tens up to hundreds of Solar radii (during the red-giant branch, and asymptotic giant branch, phases of their evolution, as well as the Wolf-Rayet stage for very massive stars). When such an evolved star has a close stellar companion the expanded envelope engulfs the companion and, under appropriate conditions gives rise to a CE of gas embedding both the core of the evolved star and the companion [4]. The stellar companion is then thought to in-spiral inside the envelope due to its gravitational interaction with the gas (see figure [1]). The CE phase could result in either the ejection of the gaseous envelope before the companion arrives to the core, or in a final in-spiral and merger of the companion with the compact stellar core. The former occurs when the evolved stellar envelope is not sufficiently massive and dense compared with the stellar companion, and vice versa. Here we show that when the stellar companion is a compact-object (a WD, a NS or a BH), and the stellar-core is sufficiently compact, CE mergers give rise to diverse range of possible CO-core mergers with detectable GW signals (see [3, 6] for an initial study in the context of NSs). Note that not all the envelope needs to be ejected, and significant fractions of the envelope may be ejected on much longer non-dynamical timescales [3, 8], this, however, is always preceded by the dynamical phase.

The in-spiral of a compact object may proceed on a short dynamical timescale as long as the dynamical friction due to the CE is sufficiently strong and/or the tidal damping effects on the stellar core and CE are efficient. The closest approach accessible for a CO-core binary occurs when the compact object spirals down to the tidal radius of the stellar-core \( r_t \sim (M_{\text{CO}}/M_{\text{core}})^{1/3} R_{\text{CORE}} \), at which point the latter will be tidally disrupted, potentially leading to an explosive event and/or the formation of an exotic star with a compact-object core and a gaseous envelope (e.g. a Thorne-Zytkow object [5]. In order to characterize the GW-detectability of such binary mergers, one needs to derive the strain and frequency of the GW emission of such binaries close to the tidal radius. However, one now also needs to account for the evolution due to the interaction with the gaseous envelope. This process is special, in-so-far as the driving force behind the in-spiral is not energy-loss to gravitational waves, but rather interaction with an environment. As such the gravitational-wave signal should reflect the properties of the environment, and allow for potentially otherwise-inaccessible direct observations of the such environments/processes [10, 11].

In order to study and characterize such GW sources
we first model the evolution as two point masses orbiting inside an envelope with a given constant density profile, where we neglect the back-reaction of the motion of the system on the envelope itself. This simplified assumption will be relaxed in a later study where we will use detailed hydrodynamical simulations to follow the exact evolution of the binary and the gaseous envelope. In our current simplified analysis we model the in-spiral evolution through the effects of gas dynamical friction (GDF), rather than the approach taken previously by Nazin and Postnov, and we include 2.5 post-Newtonian corrections in the calculation of the gravitational interaction of the compact companion and the core. In principle the GW signal would then depend on the density profile of the gaseous CE, thereby enabling the use of GWs to probe the properties of the interior of part of the red-giant/CE.

The in-spiral is driven by GDF and not aerodynamic drag, given the high ratio between the mass of the inspiraling CO and its geometric cross-section. In particular, Grishin and Perets find that the critical radius for the transition between GDF-dominated evolution and aerodynamic-drag dominated evolution is \( R_e \propto v_{\text{rel}}/\sqrt{G\rho_m} \), where \( v_{\text{rel}} \) is the relative velocity between the body and the gas, and \( \rho_m \) is the density of the body. The proportionality constant depends on the dimensionless Reynolds and Mach numbers of the flow, and is of order unity for most plausible situations.

In our case, we assume an initial relative velocity of 100 km s\(^{-1}\) (similar to the Keplerian velocity around a massive star at 1 AU). For a typical WD density of \( \rho_{\text{WD}} \approx 10^6 \) g cm\(^{-3}\), we get that the critical radius is a few per cent of the WD radius, \( R_e \approx 0.05R_{\text{WD}} \), with \( R_{\text{WD}} = 0.01R_\odot \). For typical NS density of \( \rho_{\text{NS}} = 10^{14} \) g cm\(^{-3}\), the critical radius is around \( 4 \times 10^3 \) cm, much less than a typical NS radius. Therefore, for compact objects embedded in envelopes of giant stars, GDF is considerably larger than aerodynamic gas drag.

The paper is structured as follows. We begin by describing the set-up of our models and the equations of motion, which we numerically integrate to calculate the GW signatures of the common-envelope gravitational-wave (CEGW) sources, exploring various configurations of CE-binaries and compact-objects. We then discuss the properties of such CEGW sources and possibility of detecting them, and then we summarize.

**Set-Up:** We work in the centre-of-mass frame of the binary, and neglect any back-reaction of the companion on the envelope, that is, we take the density profile of the envelope to be constant in time, and ‘glued’ to the core of the star, which we model as a point mass. This system may be described as an effective one body, whose position \( \mathbf{r} \) describes the relative separation between the companion and the core. The equations of motion governing the system are

\[
\ddot{\mathbf{r}} = -\frac{G(M + m)}{r^3} \mathbf{r} - \frac{GM_{\text{env}}(r)}{r^3} \mathbf{r} - F(\mathbf{r}, \mathbf{v}) + \text{P.N.}, \quad (1)
\]

where \( M_{\text{env}}(r) \) is the mass of the envelope inside a sphere of radius \( r \) (excluding the core), \( M = M_{\text{core}} \) is the core mass, and \( m = M_{\text{CO}} \) is the companion mass, and ‘P.N.’ denotes any post-Newtonian terms. We further define \( M_{\text{tot}} = M + M_{\text{env}}(R) \), where \( R \) is the radius of the giant. \( F \) describes the effects due to GDF, for which we adopt the model of Ostriker \( \text{[12]} \)

\[
F(\mathbf{r}, \mathbf{v}) = \frac{2\pi G^2 m \rho(\mathbf{r})}{v^3} \begin{cases} \ln \left( \frac{1 + \mathcal{M}}{\mathcal{M} e^{-2\mathcal{M}}} \right), & \mathcal{M} < 1 \ \ \ \ \\ \ln \left( \Lambda^2 - \Lambda^2 \right), & \mathcal{M} > 1. \end{cases} \quad (2)
\]

where \( \mathcal{M} = \nu/c_s \) is the Mach number, where \( c_s \) is the local speed of sound of the envelope. The Coulomb logarithm \( \ln \Lambda \) is given by \( \Lambda = b_{\text{max}}/b_{\text{min}} \text{[14], p. 835] \), where \( b_{\text{min}} = \max \{ Gm/c^2, r_{\text{coll}} \} \), \( r_{\text{coll}} \) is the radius at which it collides with the core. Furthermore, we adopt \( b_{\text{max}} = 2r \), as in Kim and Kim \( \text{[13]} \) rather than \( b_{\text{max}} = R_{\text{env}} \), because the density outside \( r \) is a lot smaller than the density inside \( r \).

**Wave-Forms:** During the in-spiral the binary emits GWs due to changes in its quadrupole moment. As \( r \) decreases, the orbital acceleration increases, and so does the GW-amplitude, right up to the final merger. The GW-strain \( h_{ab}(t) \) is given by the quadrupole formula, to leading order, \( \nu/tz \)

\[
h_{ab}^{TT}(t) = \frac{2G}{Dc^3} \hat{Q}_{ab}^{TT}(t_{ret}), \quad (3)
\]

where \( TT \) denotes the transverse-traceless gauge, and an upper \( TT \) on \( Q \) denotes a projection on the direction of observation; \( \hat{Q}_{ab} = M^{ab} - \delta_{ab}M^{kk}/3 \) is the quadrupole moment, and \( t_{ret} = t - D/c \) is the retarded time, with \( D \) being the distance between the detector and the source. Note that even though one of the bodies is not point-like, it is spherical, so the mass moment is

\[
M^{ij} = m x_1^i(t) x_1^j(t) + M_{\text{tot}} x_2^i(t) x_2^j(t) + \frac{1}{3} S^{ij} C, \quad (4)
\]

where \( C \) is some higher moment of the gas density \( \rho \), and \( x_{1,2} \) is the position of the centre of the CO/giant. The second term vanishes when calculating \( Q_{ab} \), so we are free to ignore it. The contribution of the star is thus equal to that of a point mass, and we may therefore use the formula for \( M^{ij} \) of a binary system in the centre of mass practice, the CE evolution changes the structure of the envelope even when the envelope is significantly more massive.
frame (e.g. given by equation (3.72) of Maggiore \[16\]). The solution to equation (1) defines the wave-form, and the problem of calculating the wave-form is reduced to solving the equations of motion.

**Numerical Calculations:** In order to calculate the CEGW signatures, we integrate equation (1) numerically using a Runge-Kutta integrator (MATLAB’s ode113). In our code, we added relativistic corrections to the motion of the core and the companion up to 2.5PN (to account for GW dissipation\[14\]), as given by Lincoln and Will \[17\]. The orbit can still be described by a point particle with mass $\mu = m M_{\text{tot}} / (m + M_{\text{tot}})$, moving under the influence of the gravitational field of a particle with mass $M + m$ and inside a potential created by the envelope. Here we considered a few combinations of $m, M$, and $M_{\text{env}}$ (see table\[1\] and figure\[1\]).

The density profiles of the stars were obtained from detailed stellar evolution models using the MESA code, evolved from initial masses of 15, 8 and 5 $M_{\odot}$ \[18–20\], until they reached $110 R_{\odot}$; the stars retained almost all of their original mass. The orbital evolution is shown in figure\[1\].

For each of these models we calculate the expected wave-forms emitted by the binary, computed for sources assumed to be at a distance of 10 kpc, with $m = 0.6 M_{\odot}$ to represent a WD companion, and $m = 1.4 M_{\odot}$ – for a NS. The resulting wave-forms are shown in figure\[2\]. All in-spirals show a characteristic evolution beginning with regular low-amplitude oscillations during the slow-in-spiral at the early phases, which then gradually increase in frequency and amplitude down to the final plunge accompanied by a high amplitude burst.

**Detectability:** Let us now consider the detectability of such CE-in-spirals. The size of the stellar core is comparable or larger than that of WDs, and the orbital frequency of the binary before its final merger is therefore lower than the detection range of aLIGO. However, as we show in the following, such GW sources are detectable by next-generation space-based GW-detectors, such as LISA or DECIGO.

Whether the signal we predict could be detected or not is determined by the characteristic strain, which is related to the signal-to-noise ratio (SNR). If $S_n(f)$ is the noise power spectrum density of a detector, and $\hat{h}$ is the physical signal (without noise) then the signal-to-noise ratio is given by Moore et al. \[21\]

$$
\left( \frac{S}{N} \right)^2 = 4 \int_0^\infty \frac{\left| \hat{h}(f) \right|^2}{S_n(f)} df = \int_0^\infty \frac{h_n^2(f)}{h_n^2(f)} df,
$$

(5)

\[2\] These have significant contributions only when they are extremely close to each other, so the post-Newtonian terms do not include the gravity of the gas in the envelope.
TABLE I. Signal-to-noise ratios for the binary in-spirals discussed in the text for LISA, DECIGO and BBO. $M_{\text{tot}}$ is defined as $M + M_{\text{env}}$. Cf. figure [3].

| Envelope | $m (M_{\odot})$ | LISA | DECIGO | BBO | LISA | DECIGO | BBO | LISA | DECIGO | BBO |
|----------|----------------|------|--------|-----|------|--------|-----|------|--------|-----|
| Compact  | 0.6            | 0.57 | 68     | 214 | 0.42 | 32     | 100 | 6    | 690    | 2200|
| Compact  | 1.4            | 1.3  | 180    | 560 | 1.4  | 260    | 840 | 19   | 3500   | 11000|
| Extended | 0.6            | 0.54 | 58     | 180 | 2.3  | 29     | 93  | 1.2  | 450    | 1400|
| Extended | 1.4            | 1.2  | 140    | 450 | 11   | 34     | 100 | 20   | 3700   | 12000|

FIG. 2. The expected wave-forms (for both polarizations) of the GW produced through a CE-in-spiral, shown for the different combinations of compact-objects and evolved star binaries, computed at a distance of 10kpc.

before expected to occur in more extended density profiles than the initial more compact one assumed here, and therefore the density close to the core is expected to be lower relative to the initial density. In this scenario the amplitude of the GDF force on the companion is therefore expected to be smaller, and the amount of time the CO spends very close to the core, emitting the strongest GWs, is increased. In other words, our SNR calculations should not decrease the SNRs.

line in the figure [3]. In a follow-up paper we intend to study the effect of the extended envelope in detailed hydrodynamical simulations, that take the back-reaction on the envelope into account. As a simplified approach we gauge the magnitude of the extended-envelope effect using a simple inflated envelope model, where we again integrate the equations of motion, but we now consider - a neutron star companion of 1 $M_{\odot}$, - a neutron star companion of 1 $M_{\odot}$, and the right - a neutron star companion of 1.4 $M_{\odot}$. The top row displays envelopes that remain compact, while the bottom row shows envelopes dilated by a factor of 10. All quantities are calculated at a distance $D = 10$ kpc. The Fourier Transforms are calculated by sampling the signal at a rate of 1 Hz. Sampling it at higher rates (which is necessary for DECIGO and BBO)
velopes are shown in the second row of figure 3 and the final SNRs are given in table 1.

As evident from figure 3 and table 1 CE-in-spiral occurring in the Galaxy could potentially be observed by future GW detectors LISA, DECIGO and BBO, where the latter two might even be sensitive to extragalactic in-spirals up to distances of a few Mpc.

Summary: In this paper we explored the GW emission from the in-spiral of a compact object in an evolved common-envelope binary, leading to the final merger of the compact-object with the stellar core of the evolved star. As we show the in-spirals significantly differ from GW-in-spirals of binary compact objects evolved in isolation; CE-in-spiral are dominated by the gas-dynamical friction interaction with the envelope and not by the dissipation from the GW-emission itself. Such evolution changes the acceleration of the binary and gives rise to GW-signatures from the gas-driven evolution with unique frequency evolution. In particular such GW-sources could be potentially observable by next-generation GW-detectors such as LISA/DECIGO/BBO, and their unique frequency evolution could provide not only a smoking-gun signature for their origin, but also make use of GW-detections as a probe for mapping the interior density profiles of evolved stars, otherwise inaccessible through electromagnetic observations.

The common-envelope phase is also typically accompanied by explosive electromagnetic transients. The in-spiral might give rise to luminous flaring possibly resembling stellar mergers such as V1309 Scorpii [22]. The merger of a WD (NS) and the degenerate-core might give rise to a thermonuclear explosion possibly observable as peculiar luminous (sub-luminous) type II/Ib supernovae (given the large amounts of hydrogen and helium in the envelopes). Finally, the in-spiral of a NS into the center of the evolved star may produce a Thorne-˙Zytkow object [9]. Systems with heavier compact companions, such as BHs, may generate even stronger signals; these will be addressed in a future paper.

Estimates for the rates of mergers of compact objects with degenerate cores of evolved stars leading to type Ia SNe suggest that they should occur at rates of 10 – 100% of the observationally-inferred rates of type Ia SNe [23], where the rates of CE-GW sources could be higher. Nazin and Postnov [6] estimate a rate of 0.002 yr⁻¹ for the formation of Thorne-˙Zytkow objects, based on [24], implying an overall rate of one CEGW event per a few centuries in our Galaxy. Though less likely to be observed by LISA given the expected Galactic rates, the DECIGO/BBO observatories would enable observing extragalactic CEGW and potentially detect a few such CEGW sources.

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