Bar formation in simulations of interacting galaxies

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Abstract. In this work we present a study of interacting galaxies using \textit{N}-body simulations. The initial condition of galaxies are such that they are composed of a bulge, a disc (Freeman model, with no gas), and a halo. For bulge and halo we follow the Dehnen density-pair spherical models. Galaxies are set in a parabolic encounter characterised by the impact parameter and the collision angle subtended by the planes containing each individual galactic discs. The evolution of galaxies are given in terms of the morphology (bar formation, geometry of the bar, minor and major axis length), and the kinematical bar rotation. We show how this characteristics depend on the collision geometry. The dynamics of the collision is given in terms of individual rotation curves, dispersion of velocities of the disc and mass function as functions of the distance to the center of mass of each individual galaxy.

1. Introduction

In the present work, we pursue to study bars in spiral galaxies. Observations of spiral galaxies indicate that the presence of a bar is a common feature \cite{1, 2}. In particular, in \cite{2} it is found that 29.4 per cent of the galaxy sample have a bar. Instabilities in isolated stellar and gaseous discs lead to bar formation; see \cite{3} for pioneer studies and \cite{4, 5} for a modern view. The bar formation in isolated models has been widely studied both analytically and numerically \cite{6, 7, 8, 9, 10, 11, 12, 13}. In this paper we consider the dynamical effects of non-isolated systems which are found in clusters of galaxies. In this sense, it has been suggested that the observed bar in many spirals is the result of the gravitational interaction between two or more nearby galaxies. For instance, \cite{14} has found that during the collision of two galaxies and between the first and the second closest approaches, the disc takes a transient bar shape. The gravitational interaction between the two galaxies gives rise to perturbations in the orbits of the stars that results in the formation of the bar.

Bar formation in stellar discs depends upon various simultaneous effects. In the case of collisions, simulations have shown that these factors are \cite{15}: rotation curve shape, disc-halo mass ratio, perturbation force and geometry. Additionally, simulations suffer from numerical effects such as low spatial and temporal resolution, too few particles representing the system, and an approximate force model. These effects were studied in references \cite{5, 16}, where it was...
shown that specific parameter choices may change bar properties. Once numerical effects are controlled, we may investigate all the other model parameters, which in our case are: geometric parameters such as impact parameter and the angle between the disc planes.

In the present paper we study the formation of bars as a product of instabilities that result of the collision of two spirals. In particular we study the morphology and kinematics of bars that are formed during a binary collisions. Morphology is given by finding major and minor axis evolution. Kinematics is studied by the computation of the angular velocity of the bar. Dynamics of the disc is studied through the computations of several quantities, like rotation curve, velocity dispersion and the mass histogram as function of distance to the center of mass of each colliding galaxy.

We organise our work in the following form: In the next section we present a general way of constructing an isolated galaxy following Hernquist method [17]; and explain how to set up the parabolic collision geometry. Next, we discuss our results for various collision cases like off-axis impacts and at different angles of collision of two disc galaxies and show the resultant properties of tidally formed bars. In the final section we write our conclusions of this work.

2. Initial conditions and geometry of the collision
We use the standard procedure to construct a galaxy model with a Newtonian potential described in [5, 16]. An individual galaxy consists of a disc, halo, and bulge and its initial condition is constructed using a bulge and a Freeman disc composed of stars immerse in a Hernquist halo model (a Dehnen’s family member with \( \gamma = 1 \), see [18]) that acts on them gravitationally. We do not consider gas.

The spatial distribution of particles are constructed using density profiles: The bulge density profile is given by (Hernquist, 1990):

\[
\rho_{b}(r) = \frac{M_{b}a_{b}}{2\pi} \frac{1}{r(r + a_{b})^{3}},
\]

and for the halo we use a Dehnen density profile with \( \gamma = 0 \) (Dehnen, 1993):

\[
\rho_{h}(r) = \frac{3M_{h}}{4\pi} \frac{a_{h}}{(r + a_{h})^{4}}.
\]

We assume that disk follows the exponential profile (Freeman, 1970):

\[
\rho_{d}(r, z) = \frac{M_{d}a^{-1}}{4\pi z_{0}} e^{-a r} \text{sech}^{2} \left( \frac{z}{z_{0}} \right).
\]

In these equations \( M_{b}, a_{b} \) and \( M_{h}, a_{h} \) are the mass and length of bulge and halo respectively, and \( M_{d}, \alpha^{-1} \) and \( z_{0} \) are the mass, length scale and the thickness length scale of the disk, respectively.

For the spherical distribution of particles, velocities are obtained using the Schwarzschild distribution,

\[
\mathcal{f}_{B,H}(v_{r}, v_{\phi}, v_{\theta}) \propto \exp \left[ -\frac{v_{r}^{2}}{2\sigma_{r}^{2}} - \frac{v_{\phi}^{2}}{2\sigma_{\phi}^{2}} - \frac{v_{\theta}^{2}}{2\sigma_{\theta}^{2}} \right]
\]

where \( \sigma_{r}, \sigma_{\phi}, \text{ and } \sigma_{\theta} \) are the dispersion of velocities and in general they are functions of \( r \). For an isotropic ellipsoid the above velocity distribution is the Maxwell distribution.

For a spherically symmetic mass distribution and without rotation the dispersion of velocities is obtained using Jeans’ equation

\[
\frac{d}{dr} \left( \rho(r) \sigma^{2}_{r} \right) + \rho(r) \frac{\sigma^{2}_{r}}{r} \left[ 2\sigma^{2}_{\phi} + (\sigma^{2}_{\phi} + \sigma^{2}_{\theta}) \right] = -\rho(r) \frac{d\Phi}{dr}
\]
If the distribution of velocities is isotropic
\[ \sigma_r^2 = \sigma_\theta^2 = \sigma_\phi^2 \]  \hspace{1cm} (6)

The above equation can be integrated to give a general expression for the dispersion of velocities:
\[ \sigma_r^2(r) = \frac{1}{\rho(r)} \int_r^\infty \rho(r') \frac{d\Phi}{dr'} dr' \]  \hspace{1cm} (7)

Particles velocities can be found by inverting the equation
\[ f_{\nu,\nu}(v, r) = \frac{4\pi}{(2\pi\sigma^2)^{2/3}} v^2 \exp \left[ -\frac{v^2}{2\sigma^2} \right] \]  \hspace{1cm} (8)

In practice it is convenient to cut the Gaussian distribution at some finite value. A natural choice is the escape velocity \( V_e \).

For axisymmetric distribution we have that the velocity profiles for the disk are computed using the epicyclic approximation, which consists in assuming that velocity dispersions are small \( \sigma_R, \sigma_z, \sigma_\phi \ll R\omega \):
\[ f_D(v_R, v_z, v_\phi) \propto \exp \left[ -\frac{v_R^2}{2\sigma_R^2} - \frac{v_z^2}{2\sigma_z^2} - \frac{(v_\phi - V_0)^2}{2\sigma_\phi^2} \right] \]  \hspace{1cm} (9)

Observations in the exterior of disk galaxies suggest that the radial dispersion is proportional to the surface radial density:
\[ \sigma_R^2 \propto \exp(-\alpha R) \]  \hspace{1cm} (10)

The vertical dispersion in the isothermal shell approximation is also related to the surface density of the disk:
\[ \sigma_z^2 = \pi G z_\phi \Sigma(r) \]  \hspace{1cm} (11)

The ratio \( \sigma_R^2/\sigma_z^2 \) is constant through the disk and is considered equal to 4, i.e.,
\[ \sigma_R^2 = 4\sigma_z^2 \]  \hspace{1cm} (12)

The azimuthal dispersion is simply related to radial dispersion through the epicyclic approximation for the Schwarzschild velocity distribution
\[ \sigma_\phi^2 = \frac{\kappa^2}{4\omega^2} \sigma_R^2 \]  \hspace{1cm} (13)

where \( \omega \) is the angular frequency, computed from the potential
\[ \omega = \frac{\partial \Phi(R)}{\partial R} \]  \hspace{1cm} (14)

and \( \kappa \) is the epicyclic frequency defined by
\[ \kappa^2(R) = 4\omega^2(R) + R \frac{d}{dR} \left[ \omega^2(R) \right] \]  \hspace{1cm} (15)

For an exponential surface density profile, the azimuthal drift velocity is given approximately by
\[ V_0^2 = V_c^2 + \sigma_R^2 - \sigma_\phi^2 - 2\alpha R \]  \hspace{1cm} (16)
Table 1. Parameters of the galaxy model. The system of units is such that the unit of mass, length, time, and velocity are: \(2.2 \cdot 10^{11} \text{ M}_\odot\), 40 kpc, 0.2558 Gyr, 153 km/s, respectively.

| Component | Mass   | Number of particles | Cutoff radius | Scale-length |
|-----------|--------|---------------------|---------------|--------------|
| Bulge     | 0.0025 | 1024                | 0.308         | 0.008        |
| Disc      | 0.1017 | 29491               | 0.5           | 0.045        |
| Halo      | 1.6    | 245760              | 11.55         | 0.3          |

Table 2. Geometry of the numerical experiments.

| ID model | Mass proportion | Impact parameter p | Collision angle | Prograde/Retrograde |
|----------|-----------------|--------------------|-----------------|---------------------|
| IC602    | 1:1             | 0.1                | 30°             | N/A                 |
| IC60255  | "               | "                  | 0°              | Prograde            |
| IC60266  | "               | "                  | 0°              | Retrograde          |
| IC6027   | "               | "                  | 45°             | N/A                 |
| IC6028   | "               | "                  | 90°             | N/A                 |
| IC6034   | "               | 0.3                | 30°             | N/A                 |
| IC6035   | "               | "                  | 0°              | Prograde            |
| IC6036   | "               | "                  | 0°              | Retrograde          |
| IC6037   | "               | "                  | 45°             | N/A                 |
| IC6038   | "               | "                  | 90°             | N/A                 |
| IC6039   | "               | 0.6                | 30°             | N/A                 |
| IC6040   | "               | "                  | 0°              | Prograde            |
| IC6041   | "               | "                  | 0°              | Retrograde          |
| IC6042   | "               | "                  | 45°             | N/A                 |
| IC6043   | "               | "                  | 90°             | N/A                 |

where \(V_c^2 = R \omega\) is the azimuthal circular velocity of the disk.

Once velocity dispersions are computed, the velocity components of particles in the disk can be found by inverting the above Gaussian distribution which includes the drift velocity \(V_0\).

Finally, the galaxy is built, numerically, using a Monte Carlo procedure by choosing six random numbers that when transformed according to the corresponding spatial and velocity distribution functions give us the vector position and the vector velocity of each particle. This is repeated \(N\) times with \(N\) the number of particles to use in the simulation. The number of particles in each component are assigned in proportion to their masses (see table 1, where values for other parameters are given). This initial galaxy condition is relaxed up to a time of 3.0 in units of the code (0.7674 Gyrs). The geometry of the collision is such that we set both galaxies at parabolic collision orbit defined by the impact parameter \(p\) and the angle between disc planes. All experiment runs were performed with smoothing lengths \(\epsilon_b = \epsilon_h = 0.003, \epsilon_d = 0.004\) (bulge, halo, and disc), time step \(\delta t = 0.01\) (but this is the maximum step size, we have using an adaptive time step scheme). In table 2 we show the experiments that we will discuss in the next section.

3. Results

All collision models have \(N = 552,550\) total particles. We consider three sets of simulations followed up to time \(t = 1.56\) Gyr. The sets are defined by the impact parameter. First set is for \(p = 0.1\), the second is for \(p = 0.3\), and the third is for \(p = 0.6\). The proportion of galaxy masses is 1:1. See table 2 for more details. In order to obtain reasonable simulations, conservation of
Figure 1. Contour density map of a snap shot of one of the collision model IC 60255.

total energy and total angular momentum were computed. Total energy is the sum of all kinetic and potential energies of the individual particles (halo, bulge or disc) of both galaxies. And similar for the total angular momentum. We compute at the beginning of the simulation total energy and total angular momentum values and they are our reference values, and then through all the simulation time we keep track of the values of the energy and angular momentum. At the end of the simulation we compute the evolution of the relative differences:

$$\frac{|E(t) - E_0|}{E_0}$$

and

$$\frac{|L(t) - L_0|}{L_0}.$$  

Where $E_0$ and $L_0$ are total energy and magnitude of total angular momentum at the beginning of the simulation and $E(t)$ and $L(t)$ are the corresponding quantities as functions of time. All models show a good energy and angular momentum conservation. The relative error is at most $\approx 0.4\%$ for the total energy and $\approx 0.8\%$ for the total angular momentum. We have not followed a convergence criteria in the sense of studying the influence of the numerical parameters: smoothing potential parameter, number of particles $N$ or the time step size. This study was done in reference: [16]. Here we use numerical parameters values that are consistent with the criteria found reliable in that reference.

To detect the presence of a bar in a galaxy and characterise quantitatively its amplitude, we use the following method. We compute isodensity curves of the disc in a 128 $\times$ 128 mesh, i.e., a contour map of the density field around the center of mass of the disc. We tune the contour map to have a good resolutions map (see figure 1). With this contour map we scan the contour lines following several radial trajectories starting at the center of mass. The longest trajectory that intersects the first set of contour lines and that is in the same direction of the gradient at that point is the major axis. The size of this trajectory is the size of the major axis. Now, assuming that the bar is rectangular in shape we follow a trajectory perpendicular to the one that defines the major axis, and the intersection with the same set of contour lines will define the minor axis and we will obtain its size. This method allows us to detect any non-axisymmetric deformations such as bars in the disc plane. We have found traces of a bar for times between the first and second encounters. In this time interval we compute the angular velocity of the bar by computing the rate of change of the angle between the major axis and the $x$-axis.

In figure 2 we show the evolution of the center of separation for the three impact parameter values. The behaviour of the this curve is the same for all simulations for a given impact
parameter. This is so because the dominant component is the halo, which is spherically symmetric and the same for each galaxy. The first and second encounters, for \( p = 0.1 \), are, approximately, at time 0.38 Gyr and at 1.05 Gyr, respectively. Increasing impact parameter increase the times of the first and second encounters. For all the numerical experiments we look for a bar formation in each of the galaxies for times between the first and second encounters and we analyse the morphology and kinematics of these bars at five times, chosen uniformly in that time interval. The dynamics of the galaxies are studied through the computation of the rotation curves, velocity dispersion and histogram of mass of each of the colliding galaxies, at small set of times before the first encounter and at times chosen in the same way as the ones chosen for analysing the morphology and the kinematics.

In figure 3 we show the morphology and kinematics of the bar formed in the galaxy 1 with the disc in \( x-y \) plane. Morphology of the bar (size of the major and minor axes) is almost the same for \( p = 0.3 \) and \( p = 0.6 \). For \( p = 0.1 \) the major (minor) axis is longer (smaller) than the
Figure 3. Morphology and kinematics of the bar formed in galaxy 1. Three models: \( p = 0.1 \), \( p = 0.3 \), and \( p = 0.6 \). In the first three panels both galaxies have the disc in the \( x-y \) plane. In the last two panels we have discs at different angle, first the collision angle was \( 30^\circ \) and for the last one \( 45^\circ \).

ones for \( p = 0.3 \) and \( p = 0.6 \). On the average, bars for \( p = 0.3 \) and \( p = 0.6 \) move faster than the case of \( p = 0.1 \). Whereas changing the angle between disc planes does not change morphology or kinematics for \( p = 0.1 \). The angular momentum of the disc of galaxy 1 increases when impact parameter increases. In the cases in which impact parameter is \( p = 0.3 \) and \( p = 0.6 \) we have observed that changing the angle among galactic discs do not have influence on bar morphology similar as the case with \( p = 0.1 \). Bar’s axis size is not exceeding values of 0.08, for the mayor axis, and 0.04 for the minor axis. The angular momentum, for the same cases, has values not exceeding 0.005, which is in the same range as the case, \( p = 0.1 \).

In figure 4 we show the dynamics of the disc in galaxy 1 (with the disc in \( x-y \) plane). The rotation curve increases for \( p = 0.3 \) and \( p = 0.6 \) with respect to the case of \( p = 0.1 \). The opposite behaviour occurs for the dispersion of velocities. Dynamics of the disc, for a given impact parameter, is independent of angle of collision.

We have also observed that it is clear from the results analysis that there is a correlation between morphology of the bar and dispersion of velocities of the disc. When dispersion is higher bar is longer and thinner.

4. Conclusions
Our simulations have shown that tidal forces are an efficient mechanism to generate bars in spirals. Morphology, kinematics and dynamics of the barred spirals that result from the collision do not show strong dependence on the geometry of the collision.

There is an indication of a correlation between morphology of the bar and dispersion of velocities of the disc. When the dispersion of velocity increases (in general) in the spatial range we measured, major axis increases and minor axis decreases.

References
[1] Elmegreen B G and Elmegreen D M 1983 ApJ 267 31
[2] Masters K L, et al. 2011 Mon. Not. R. Astron. Soc. 411 2026
[3] Toomre A 1964 ApJ 139 1217
[4] Barnes J E 1998 In Galaxies: Interaction and Induced Star Formation ed D Friedly, L Martinet and D Pfenniger (Berlin: Springer-Verlag) p 275
Figure 4. Dynamics of the bar formed in galaxy 1. Three models: $p = 0.1$, $p = 0.3$, and $p = 0.6$. In the first three panels both galaxies have the disc in the $x$-$y$ plane. In the last two panels we have discs at different angle, first the collision angle was $30^\circ$ and for the last one $45^\circ$.

[5] Gabbasov R F 2006 *Numerical simulation of bars in interacting galaxies* Doctoral thesis (Toluca, México: Universidad Autónoma del Estado de México)
[6] Hohl F 1971 ApJ 168 343
[7] Sellwood J A 1981 A&A 99 362
[8] Sellwood J A and Carlberg R G 1984 ApJ 282 61
[9] Sellwood J A and Athanassoula E 1986 MNRAS 221 195
[10] Athanassoula E and Sellwood J A 1986 MNRAS 221 213
[11] Weinberg M D 1985 MNRAS 213 451
[12] Debattista V P and Sellwood J A 2000 ApJ 543 704
[13] Weinberg M D and Katz N 2002 ApJ 580 627
[14] Nogushi M 1987 MNRAS 228 635
[15] Salo H 1990 A&A 243 118
[16] Gabbasov R F, Rodríguez-Meza M A, Cervantes-Cota J L and Klapp J 2006 Astron. & Astrophys. 449 1043
[17] Hernquist L 1990 ApJ 356 359
[18] Rodríguez-Meza M A and Cervantes-Cota J L 2004 MNRAS 350 671