Aspects of Inflationary Reconstruction

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Abstract. I review various aspects of techniques for reconstructing the potential of the inflaton field from observations, with special emphasis on difficulties which might arise. While my view is that if inflation is to prove viable then most likely it will be one of the simplest models, it is important to consider the impact should we need to move to a more complicated model-building realm.

1. Introduction

One of the most exciting aspects of the rapidly improving observational situation in cosmology is the hope that we might learn of processes happening in the very early Universe, and thus learn of physics at energies inaccessible to terrestrial experiments. A key idea in early Universe cosmology is inflation, a period of accelerated expansion thought to be driven by the potential energy of one or more scalar fields. Assuming all goes well with upcoming experiments, particularly satellite projects MAP and PLANCK aiming to accurately measure microwave background anisotropies, one can hope to receive a limited, but non-trivial, amount of information concerning the inflationary mechanism. That is to say, one can hope to reconstruct part of the inflationary potential.

Although inflation (or other early Universe ideas such as cosmic strings) is often discussed more or less independently of cosmological parameters such as the Hubble parameter $h$ and the density parameter $\Omega_0$, it is in fact crucial to the entire enterprise of using such observations to constrain cosmology. The reason is that, contrary to the impression given by a fair fraction of the literature, measurements of the microwave background in isolation tell you nothing about cosmological parameters. This is because the influence of the parameters is on the dynamics, whereas the microwave background anisotropies give us a single snapshot. In order to predict the parameters, we need a theoretical prejudice as to the initial conditions, which are processed by dynamical evolution into the anisotropies we see. Consequently, fitting for the cosmological parameters and for the initial conditions for structure formation are not independent tasks which can be decoupled. Rather, they must be done together.

2. The standard paradigm

In this article I will be discussing a few of the ways in which inflation, while being essentially correct, might turn out to be more complicated than envisaged. However, I stress that it is probably much more likely, if inflation proves correct at all, that it is one of the simpler models which is true. If so, then as Neil Turok said at this meeting...
'The person who fits the data with the fewest parameters is the winner' and the game is over. So let’s begin by quickly reviewing the simplest scenario.

It arises when the dynamics of inflation (both classical and quantum) are dominated by a single scalar field evolving in a nearly flat potential. If so it is well established [4, 5] that to a good approximation the two types of perturbations, scalar and tensor, will take on a power-law form, with the tensors giving a subdominant (and quite conceivably negligible) contribution. This is certainly expected to be valid for present data, unless we have a ‘designer’ model with a very strong spectral feature present on observable scales (as discussed in this session by Lesgourgues).

The two power laws require four parameters for their specification. However, there is one consistency relation linking the two spectra which means that the tensor spectral index is not independent of the other parameters; disappointingly this redundancy is unlikely to be useful as almost certainly the tensor spectral index cannot be measured anyway. The remaining three parameters can be taken as the overall perturbation amplitude, the spectral index $n$ of the scalar perturbations and the relative impact $r$ of tensors as opposed to scalars on large-angle microwave background anisotropies. In a given model they are readily calculated, for example via the slow-roll expansion [6].

3. Simplest extension: scale-dependent spectral index

High-accuracy observational data makes stringent demands on theory, so eventually the power-law approximation may prove inadequate. There are some theoretical reasons to believe that slow-roll might not be all that good; in supergravity models the slow-roll parameter $\eta$ which must be small for inflation to proceed, takes the form

$$\eta = 1 + \text{‘other terms’},$$

(1)

Even if the other terms manage to partly cancel the 1, it may be unlikely that they do so to high accuracy. A particular example of this point in action is the running-mass models of inflation introduced by Stewart [7], where slow-roll is due to an accidental, and temporary, cancellation.

If the slow-roll approximation is only weakly satisfied, then higher-order corrections [8] to the formulae for the spectral index etc become significant and have to be accounted for [9]. More pertinent, the power-law approximation is likely to break down [10], and has to be replaced by a more general analysis such as a truncated Taylor expansion of the spectrum about some scale. (Some kind of expansion must be done to describe the spectra with a finite number of parameters which can be fit from the data.)

We investigated corrections to power-law behaviour in Ref. [11]. Adding in extra parameters will always worsen the uncertainty on all parameters, but we found that the likely impact on uncertainties in parameters such as $h$ is small, while as a bonus we have given ourselves one or more extra inflationary parameters to constrain early Universe physics with. In terms of our being able to constrain our models, it appears therefore that a breakdown in slow-roll should be regarded as a good thing, and we should hope that if inflation is correct it is a model of that type.

4. Isocurvature models

A much more disastrous turn of events would be if the best models include isocurvature perturbations. Many of the most popular inflation models have more than one...
dynamically important field, and as soon as that happens we have the possibility of isocurvature modes. These significantly complicate the calculation of the microwave background anisotropies, and a particular disadvantage is that these models appear to defy reconstruction, in the sense that given a set of observations it would be very hard to decide what sort of inflation model gave rise to them. One would have to test candidate models against the data on a one-by-one basis. Three regimes are possible:

- **Pure isocurvature models.**
  The basic idea here is that the field which eventually becomes the cold dark matter already exists during the inflationary era, and acquires perturbations by the usual mechanism. The idea has quite a long history [12], recently revived by Linde & Mukhanov [13] and by Peebles [14].

- **Mixed adiabatic and isocurvature.**
  If both modes are present we need more parameters to describe the initial conditions [15, 16]. Calculationally complex; in particular one usually needs to know the whole evolution of the Universe after inflation to compute predictions, whereas for adiabatic alone one needs evolve only until the modes are well outside the horizon.

- **Low-level isocurvature.**
  A small isocurvature contribution might not be directly detectable, yet be an extra noise source leading to deterioration of cosmological parameter determination.

Isocurvature models pose two difficulties. The first is that the power spectra from the models must be parametrized so they can be fit from the observational data, and in such models it is not clear how many parameters one may need to introduce, e.g. treating them as power-laws may be inadequate. More importantly, unlike the case of single-field inflation models there is no direct connection between these parameters and the inflation model in the form of a set of equations. Even if we have a successful fit of model parameters it may be a difficult task to deduce the form of the inflation model giving rise to them, particularly if details of post-inflation processing of perturbations need to be included.

5. **Open inflation models**

Open inflation is another complicated scenario, either in the original single-bubble Universe models [17, 18, 19] or the more recently-devised instanton models [20]. These models are readily testable insofar as the geometry of the Universe is measurable, and encouragingly already the indications are very much in favour of a flat or nearly flat Universe, both from the microwave background and measurements of the apparent magnitude–redshift relation for type Ia supernovae. If they do remain viable, they pose a similar set of technical problems to those posed in the isocurvature case.

6. **Reconstruction without slow-roll**

I end with a separate topic not closely related to the rest of the article. With the increasingly widespread use of numerical technology in cosmology, and bearing in mind the possibility that slow-roll may not work all that well, a new approach to reconstruction is suggested in the single-field case. The traditional approach relies on computing a parametrized form of the perturbation spectra, which can be input into the CMBFAST program [21] to give the microwave anisotropy spectrum. The drawback
is that the analytic computation of the spectra is only approximate, and ultimately this leads to a biased estimation of the inflaton potential. This can be circumvented by instead using exact computation of the spectra, obtained by numerically solving the mode equations wavenumber by wavenumber as demonstrated in Ref. [22]. These are input directly into a modified form of CMBFAST to give microwave anisotropy predictions which are exact up to the assumption of linear perturbation theory. By obtaining the anisotropies directly from a parametrized form of the potential, one can estimate the uncertainties in the parameters describing the reconstructed potential, and the covariances between the uncertainties on different parameters, directly. This will be described in more detail in a forthcoming publication.

7. Discussion

This article stresses that models of the initial perturbation spectra are an integral part of cosmological parameter estimation, and we rely on our present understanding being a good one. It is reasonable to hope, and even expect, that everything will work out quite simply, but I’ve outlined a few ways in which things might be more complicated. On the plus side, I noted that at least if things are just a little more complicated, especially in the form of departures from perfect power-law spectra, that is likely to be seen as a good thing as it increases the amount of readily accessible information about early Universe physics.

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References

[1] A. H. Guth, Phys. Rev. 23, 347 (1981); A. D. Linde, Particle Physics and Inflationary Cosmology, Harwood Academic, Chur, Switzerland (1990); E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley, Redwood City, California (1990).
[2] E. J. Copeland, E. W. Kolb, A. R. Liddle and J. E. Lidsey, Phys. Rev. D 48, 2529 (1993).
[3] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barriero and M. Abney, Rev. Mod. Phys 69, 373 (1997).
[4] A. R. Liddle and D. H. Lyth, Phys. Lett. B 291, 391 (1992).
[5] A. R. Liddle and D. H. Lyth, Phys. Rep 231, 1 (1993).
[6] A. R. Liddle, P. Parsons and J. D. Barrow, Phys. Rev. D 50, 7222 (1994).
[7] E. D. Stewart, Phys. Lett. B 391, 34 (1997); E. D. Stewart, Phys. Rev. D 56, 2019 (1997).
[8] E. D. Stewart and D. H. Lyth, Phys. Lett. B 302, 171 (1993).
[9] E. J. Copeland, E. W. Kolb, A. R. Liddle and J. E. Lidsey, Phys. Rev. D 49, 1840 (1994).
[10] A. Kosowsky and M. Turner, Phys. Rev. D 52, 1739 (1995).
[11] E. J. Copeland, I. J. Grivell and A. R. Liddle, Mon. Not. R. Astron. Soc. 298, 1233 (1998).
[12] J. E. Lidne, Phys. Lett. B 158, 375 (1985); L. Kofman, Phys. Lett. B 173, 400 (1986).
[13] A. D. Linde and V. Mukhanov, Phys. Rev. D 56, 535 (1997).
[14] P. J. E. Peebles, preprint astro-ph/9805194 (1998).
[15] R. Stompor, K. M. Górski and A. J. Banday, Astrophys. J. 463, 8 (1996).
[16] M. Kawasaki, N. Sugiyama and T. Yanagida, Phys. Rev. D 54, 2442 (1996).
[17] J. R. Gott, Nature 295, 304 (1982).
[18] M. Sasaki, T. Tanaka, K. Yamamoto and J. Yokoyama, Phys. Lett. B 317, 510 (1993).
[19] M. Bucher, A. S. Goldhaber and N. Turok, Phys. Rev. D 52, 3314 (1995).
[20] W. Hawking and N. Turok, Phys. Lett. B 425, 25 (1998).
[21] U. Seljak and M. Zaldarriaga, Astrophys. J. 469, 1 (1996).
[22] I. J. Grivell and A. R. Liddle, Phys. Rev. D 54, 7191 (1996).