An Analysis Model of Helicopter and UAV in Overhead Powerline Inspection

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Abstract. With the development of unmanned aerial vehicle technology, unmanned aerial vehicles have been widely used in inspection of overhead power lines. Compared to traditional helicopters, unmanned aerial vehicles represent advantages of low cost and flexibility that unmanned aerial vehicles can cross complex terrain and security, more important pilots generally take no life risk during tasks. However, unmanned aerial vehicles are significantly limited by inefficiency, flying range and departure spot. Thus energy companies are seeking for a scheme that makes unmanned aerial vehicles and helicopter coordinate in inspection of overhead power lines. This paper presents a mathematic model to assess the performances of unmanned aerial vehicles and helicopter in inspection of overhead power lines in order to provide reference in seeking aerial vehicles coordination scheme.

1. Introduction

Helicopters have always been an essential method in overhead power lines inspection due to the advantages of high technology, efficiency, reliability and insusceptibility to terrain. According to our national geographic features and distribution of high voltage power transmission lines, power grid mainly lines in mountains and hills, far from cities and artery traffic. As the scale of power grid grows up, the present methods of power lines maintenance and management will soon be unable to suit the construction and development of modern power grid. Especially when the power grid encounters failure and extreme climates, manually seeking and processing failure would be more difficult, which does a great harm to the security and stability of power grid operation. Thus aerial vehicles are necessary in inspection of overhead power lines. Applying aerial vehicles reduce the cost of manpower and materials resources so that increase the efficiency and stability of the power grid operation [1].

With unmanned aerial vehicles technology developing significantly during recent years, they already can fulfill the task of power lines inspection. Meanwhile the versatility of unmanned aerial vehicles allows them to inspect under different terrain and complex environment with satisfactory efficiency and cost. Multirotor can be applied to specific inspection and failure examination, while fixed-wing UAV (unmanned aerial vehicle) suits long distance inspection and emergent task [2][3]. Helicopter holds all features of them. Therefore, in most cases UAV can take place of helicopter in inspection task, however, restricted to flying range and duration, in some cases UAV cannot [4-6].

An ideal scheme is to make helicopter and UAV cooperate in inspection task combining the advantages of helicopter and UAV. This paper discusses a task allocation scheme under certain inspection line, environment factors and extra restricts using mathematic modeling.
2. Mathematic model set-up

2.1 Cost function

The aerial performance assessment model is showed as bellow.

\[ Cost = \sum_{i=1}^{n} l_i \cdot C(l_i, x) \]  
\[ Time = \sum_{i=1}^{n} l_i \cdot T(l_i, x) \]

In the formulas above, (1) represents total cost function, \( l_i \) represents ith division of actual lines, \( C(l_i, x) \) represents the sub cost function of this division, parameter \( l_i \) represents the array of environment factors, \( x \) represents the array of the aerial vehicle’s parameters ; (2) represents total time function, \( l_i \) represents ith division of actual lines, \( T(l_i, x) \) represents the sub time function of this division, parameter \( l_i \) represents the array of environment factors, \( x \) represents the array of the aerial vehicle’s parameters [7-9].

In order to simplify the analysis and make the result meet realistic expectation, the model was set up mainly concerning cost and time. The main concern in real life inspection includes cost, time, security and so on [10][11]. The model combines security factor into cost and time, which is because the influence of security and other factors is complex and complicated and cannot be easily quantified. The divisions of lines consider that the environment factors may vary a lot in that area, for instance, on one side of a hill it may rain while the other side sun shines, the performances of aerial vehicles differ significantly under different environment factors. The expected cost of one division is a comprehensive result of environment factors and aerial vehicle’s parameters, thus the sub cost function contains two main parameters. The same concern is with sub time function.

In order to analyze the performances of UAV and helicopter in quantity, the model using mathematic programming, set up a linear programming problem and then compare the performances of UAV and helicopter by finding the minimizer. The total cost function and time function are constructed as bellow [12][13].

\[ C(l, x) = C(e_1, e_2, x) = Cost \cdot C_{\text{wind}}(e_1, x) \cdot C_{\text{wea}}(e_2, x) \]
\[ Time = Time_{\text{wind}}(e_1, x) \cdot Time_{\text{wea}}(e_2, x) \]

In the formulas, \( Cost \) represents basic cost, \( C_{\text{wind}} \) represents the part of the sub cost function which is determined by wind, \( C_{\text{wea}} \) represents the part of the sub cost function which is determined by weather, \( Time \) represents basic cost, \( Time_{\text{wind}} \) represents the part of the sub time function which is determined by wind, \( Time_{\text{wea}} \) represents the part of the sub time function which is determined by weather.

Considering both realistic situation and the construction of analysis model, the model takes wind and weather as two main factors. Helicopter faces more risk considering the crew safety flying in bad weather like heavy rain, fog and snow on the other hand UAV has merely influence about efficiency
and cost. Helicopter maintain stable even in strong wing however UAV are relatively more susceptible
of wind. Although there exists other factors like terrain and altitude influence the performances of
aerial vehicle, the influence is complex and intricate, so the model does not directly take them as main
factors but still takes them into account over the whole construction of analysis model.

2.2 Model analysis

2.2.1 Cost function

Cost function under the influence of wind

The ability to fly against strong wind of UAV and helicopter differs significantly. According to
research, in no wind environment, the cost of UAV is far less than helicopter's. However, with the
increasing of wind scale, we need more expensive UAV to fly in strong wind, which is not practice.
However, the helicopter is designed to be able fly under certain scale of wind, before the wind scale
increases to that scale, the influence of wind can be ignored. So the sub cost function under the
influence of wind is constructed as bellow to meet the model's expectation.

\[ C_{\text{wind}}^{\text{helicopter}} = \ln(1 - \frac{v}{V_1}) \]  
(4)

\[ C_{\text{wind}}^{\text{UAV}} = \left( \frac{v}{v_0} \right)^{\frac{\alpha}{\delta}} \]  
(5)

Cost function under the influence of weather

The ability to fly in bad weather of UAV and helicopter are both susceptible of weather. Considering the crew safety, helicopter abandons the lines with bad weather, while UAV does not have this concern. However, both aerial vehicle cannot fly in extreme bad weather. So the sub cost function under the influence of weather is constructed as bellow to meet the model’s expectation.

\[ C_{\text{weather}}^{\text{helicopter}} = (v_0 \cdot x_0)^{\frac{\alpha}{\delta}} \]  
(6)

\[ C_{\text{weather}}^{\text{UAV}} = -\ln(1 - \frac{x}{x_1})^{\frac{\alpha}{\delta}} \]  
(7)

Fig. 1. Sub cost function under the influence of wind

The function graph with certain coefficients is shown in Fig.1. From the graph we can tell that the
basic cost of helicopter is higher than UAV, with the increase of wind scale the cost of UAV increases
fast and exceeds helicopter’s cost, while the cost of helicopter stays almost still when the scale of wind
is low and increases significantly when the scale of wind reaches a certain level. The function
generally meets the model’s expectation.

Cost function under the influence of weather

The ability to fly in bad weather of UAV and helicopter are both susceptible of weather. Considering the crew safety, helicopter abandons the lines with bad weather, while UAV does not have this concern. However, both aerial vehicle cannot fly in extreme bad weather. So the sub cost function under the influence of weather is constructed as bellow to meet the model’s expectation.

\[ C_{\text{weather}}^{\text{helicopter}} = (v_0 \cdot x_0)^{\frac{\alpha}{\delta}} \]  
(6)

\[ C_{\text{weather}}^{\text{UAV}} = -\ln(1 - \frac{x}{x_1})^{\frac{\alpha}{\delta}} \]  
(7)
The function graph with certain coefficients is shown in Fig. 2. From the graph we can tell that the basic cost of helicopter is higher than UAV, with the increase of bad weather the cost of helicopter increases fast, while the cost of UAV stays almost still when the scale of bad weather is low and increases significantly when the scale of bad weather reaches a certain level. The function generally meets the model’s expectation.

**The final cost function**

The final cost function is as bellow:

\[
C(l, x) = C(e_1, e_2, x) = \text{Cost} \cdot C_{\text{wind}}(e_1, x) \cdot C_{\text{weather}}(e_2, x) \tag{8}
\]

The graph is showed in Fig. 3 with certain coefficients. From the graph we can tell that when the environment is idea the cost of UAV is less than helicopter’s, with the increase of wind scale, the cost of UAV gradually exceeds the cost of helicopter, while in bad weather, the cost of helicopter is still more than UAV’s. The cost function generally meets the model’s expectation.

### 2.2.2 Time function

**Time function under the influence of wind**

On the contrary, on efficiency helicopter has advantages than UAV. According to research, in no wind environment, the time cost of helicopter is far less than UAV’s. With the increase of wind scale, the time cost of UAV increases fast, while the time cost of helicopter increases slowly when the scale
of wind is low and increases significantly when the scale of wind reaches a certain level. So the sub time function under the influence of wind is constructed as bellow to meet the model’s expectation.

\[
Helicopter: T_{wind} = -\ln\left(1 - \left(\frac{x}{e_1}\right)^{x^*}\right) \tag{9}
\]

\[
UAV: T_{wind} = (u_{a,b} X_0)^{x^*} \tag{10}
\]

![Fig. 4. Sub time function under the influence of wind](image1)

The function graph with certain coefficients is shown in Fig.4. From the graph we can tell that the basic time cost of helicopter is less than UAV, with the increase of wind scale the time cost of UAV increases fast, while the time cost of helicopter increases slowly when the scale of wind is low and increases significantly when the scale of wind reaches a certain level. The function generally meets the model’s expectation.

**Time function under the influence of weather**

Just like cost function, helicopter is more susceptible of bad weather. However, both aerial vehicle cannot fly in extreme bad weather. So the sub cost function under the influence of weather is constructed as bellow to meet the model’s expectation.

\[
Helicopter: T_{wea} = (v_{a,b} X_0)^{x^*} \tag{11}
\]

\[
UAV: T_{wea} = -\ln\left(1 - \left(\frac{x}{e_2}\right)^{x^*}\right) \tag{12}
\]

![Fig. 5. Sub time function under the influence of weather](image2)
The function graph with certain coefficients is shown in Fig. 5. From the graph we can tell that the basic time cost of helicopter is less than UAV, with the increase of bad weather the time cost of helicopter increases fast, while the time cost of UAV stays almost still when the scale of bad weather is low and increases significantly when the scale of bad weather reaches a certain level. The function generally meets the model’s expectation.

**The final time function**

The final time cost function is as bellow:

\[ T(l, x) = T(e_1, e_2, x) = Time \cdot T_{\text{min}}(e_1, x) \cdot T_{\text{max}}(e_2, x) \]  

(13)

Fig. 6. Time function of helicopter and UAV

The graph is showed in Fig. 6 with certain coefficients. From the graph we can tell that when the environment is idea the time cost of helicopter is less than UAV’s, with the increase of bad weather, the time cost of helicopter gradually exceeds the cost of UAV’s, while in strong wind, the cost of helicopter is still more than UAV’s. The time cost function generally meets the model’s expectation.

**3. Mathematic programming**

The aim of setting up the model is to build up an aerial vehicle performance assessment system, to achieve this aim we propose a pair of linear programming problem [14] [15].

\[
\begin{align*}
\text{minimize} \quad & \text{Cost} = \sum_{i=1}^{n} l_i \cdot C(l_i, x) \\
\text{s.t.} \quad & C(l_i, x) \leq c_i, \quad i = 1, 2, 3, \ldots, n \\
& T(l_i, x) \leq t_i, \quad i = 1, 2, 3, \ldots, n
\end{align*}
\]  

(14)
\[ \text{minimize}\quad \text{Time} = \sum_{i=1}^n l_i \cdot T(l_i, x) \]
\[ \text{subject to}\quad C(l_i, x) \leq c_i, \quad i = 1, 2, 3, \ldots, n \]
\[ T(l_i, x) \leq t_i, \quad i = 1, 2, 3, \ldots, n \]  

In the formulas above, \( l_i \) represents the divisions of power lines, \( x \) represents the scale of helicopter’s flight during this division, \( c_i \) represents the constraints of cost, \( t_i \) represents the constraints of time.

3.1 Solution without artificial constraints

The problem without artificial constraints is just as above, \( c_i = C_{\text{con}} \) represents the maximal allowed cost \( t_i = T_{\text{con}} \) represents the maximal allowed time cost. According to realistic experience, under environment with no wind and good weather UAV has advantage on cost while helicopter has advantage on time.

Below is the problem and solution with actual data: First we divide a given inspection line into 12 divisions with different environment factors, shown in Table 1-3.

| Table 1. Environment factors settings of lines |
|---|---|---|---|---|
| Line 1 | Line 2 | Line 3 | Line 4 |
| Wind | 0 | 3 | 6 | 9 |
| Weather | 0 | 0 | 0 | 0 |

| Line 5 | Line 6 | Line 7 | Line 8 |
|---|---|---|---|
| Wind | 0 | 3 | 6 | 9 |
| Weather | 5 | 5 | 5 | 5 |

| Line 9 | Line 10 | Line 11 | Line 12 |
|---|---|---|---|
| Wind | 0 | 3 | 6 | 9 |
| Weather | 10 | 10 | 10 | 10 |

Solving the mathematic programming problem under the aim of minimize the cost:

| Table 2. Solution to this problem with minimal cost |
|---|---|---|---|
| Line 1 | Line 2 | Line 3 | Line 4 |
| Choice | UVA | UVA | Helicopter | None |

| Line 5 | Line 6 | Line 7 | Line 8 |
|---|---|---|---|
| Choice | UVA | UVA | UVA | None |

| Line 9 | Line 10 | Line 11 | Line 12 |
|---|---|---|---|
| Choice | None | None | None | None |

Solving the mathematic programming problem under the aim of minimize the time:

| Table 3. Solution to this problem with minimal time |
|---|---|---|---|
| Line 1 | Line 2 | Line 3 | Line 4 |
| Choice | Helicopter | Helicopter | Helicopter | None |

| Line 5 | Line 6 | Line 7 | Line 8 |
|---|---|---|---|
| Choice | UVA | UVA | UVA | None |

| Line 9 | Line 10 | Line 11 | Line 12 |
|---|---|---|---|
| Choice | None | None | None | None |

From the solution we can tell the result generally meets our expectation, though errors come along with the quantification and other procedures, but they are in the permitted range.
3.2 Solution with artificial constraints

After adding the artificial constraints, we use $c_r^i$ and $t_r^i$ to represent that.

\[
\begin{align*}
\text{minimize} \quad \text{Cost} &= \sum_{i=1}^{n} l_i \cdot C(l_i, x) \\
\text{s.t.} \quad C(l_i, x) &\leq c_i, \quad i=1,2,3,\ldots,n \\
C(l_i, x) &\leq c_r^i, \quad i=1,2,3,\ldots,n \\
T(l_i, x) &\leq t_i, \quad i=1,2,3,\ldots,n \\
T(l_i, x) &\leq t_r^i, \quad i=1,2,3,\ldots,n
\end{align*}
\]

(16)

\[
\begin{align*}
\text{minimize} \quad \text{Time} &= \sum_{i=1}^{n} l_i \cdot T(l_i, x) \\
\text{s.t.} \quad C(l_i, x) &\leq c_i, \quad i=1,2,3,\ldots,n \\
C(l_i, x) &\leq c_r^i, \quad i=1,2,3,\ldots,n \\
T(l_i, x) &\leq t_i, \quad i=1,2,3,\ldots,n \\
T(l_i, x) &\leq t_r^i, \quad i=1,2,3,\ldots,n
\end{align*}
\]

(17)

With artificial constraints, the model can be applied to solve some realistic problems, for example, because of certain reasons, the inspection must be done in few weeks. In this case, we add artificial time constraints, and we can find the new cost minimizer that makes time cost is in the permitted range, providing more useful references for scheme.

Below is the problem and solution with actual data: Supposing such a case, an emergent inspection task must been down in a short time, so we set an artificial constraints $t_r^i$ and make $t_r^i = t_i/2$, and the solution of such problem is shown in Table 4.

Table 4. Solution to this problem with artificial constraints

| Line 1 | Line 2 | Line 3 | Line 4 |
|--------|--------|--------|--------|
| Choice | Helicopter | Helicopter | None | None |
| Line 5 | Line 6 | Line 7 | Line 8 |
| Choice | UVA | UVA | None | None |
| Line 9 | Line 10 | Line 11 | Line 12 |
| Choice | None | None | None | None |

From the solution we can tell the solution with artificial constraints varies a lot from the solution without. When the environment factors are bad, no feasible solution exists. When the environment factors are ideal, helicopter performs better than UAV. The result generally meets our expectation.

4. Conclusion

From the model we can tell that helicopter and UAV has its own advantages and disadvantages under different circumstances.

In order to minimize the cost, UAV has advantage under ideal circumstance. With the increase of wind scale the safety of UAV decreases fast, when the wind scale increases to a certain level, helicopter would become a better choice.

In order to minimize the time, helicopter has advantage under ideal circumstance. With the increase of bad weather, the safety of helicopter decreases fast, when the bad weather increases to a certain level, UAV would become an appropriate supplement.
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