Cosmology, Thermodynamics and Matter Creation

(In Honor of Prof. Jayme Tiomno)

J. A. S. Lima

Universidade Federal do Rio Grande do Norte (UFRN)
Departamento de Física Teórica e Experimental, C. P. 1641
59072 - 970, Natal, RN, Brazil

M. O. Calvão and I. Waga

Universidade Federal do Rio de Janeiro (UFRJ)
Instituto de Física, Ilha do Fundão, C. P. 68528
21945, Rio de Janeiro, RJ, Brazil

Abstract

Several approaches to the matter creation problem in the context of cosmological models are summarily reviewed. A covariant formulation of the general relativistic imperfect simple fluid endowed with a process of matter creation is presented. By considering the standard big bang model, it is shown how the recent results of Prigogine et alii [1] can be recovered and, at the same time their limits of validity are explicited.

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"e-mail:limajas@astro.iag.usp.br
"e-mail:orca@if.ufrj.br
"e-mail:iwaga@if.ufrj.br
1 Introduction

The study of matter creation processes in the context of the cosmological models for the universe is fairly old. Seemingly, there is a generalized belief that if the mass of the particles or the number of particles itself are time-dependent functions, the scale of time driving such processes could, in principle, be established only in the domain of a given cosmological model. In this sense, the matter creation phenomenon is, from the very beginning, closely related to the cosmological problem.

In the steady-state model [2], for instance, the observed cosmic expansion, together with the perfect cosmological principle, leads necessarily to a continuous creation of matter. In this model the energy conservation law seems to be saved by adding a new term (C-field) to the standard Einstein-Hilbert action. Also, after Dirac [3], in the majority of variable gravity theories, the time variation of the gravitational constant has as a consequence a time-dependence of the mass and/or the particle number.

Recently, following a quite different approach, Wesson [4] proposed a Kaluza-Klein-type variable-mass gravity theory. He argued that just as the speed of light $c$ is used to define $x^0 = ct$, putting time in an equal footing with the space coordinates, one can consider the gravitational constant $G$ to define a new coordinate $x^4 = Gm/c^2$ and use it as the fifth coordinate of a five-dimensional space-time-mass (manifold). In this case, the variation rate of the mass will be related to the “velocity” $\frac{dx^4}{dx^0} = \frac{G}{c^3} \frac{dm}{dt}$ of the particle in the fifth dimension. However, some simple arguments developed by Wesson himself strongly suggest that the mass dimension is again important only in the realm of cosmology.

At first sight, the several approaches mentioned above seem to be quite different, but, to a certain extent, there is a unifying route among them, namely, only the dynamic aspects of the matter creation process have been investigated.

More recently, Prigogine and Collaborators [1] taking advantage of the
thermodynamic theory of irreversible processes, dealt with the matter creation problem in the framework of the standard hot big bang cosmology. The main goal of their paper was to construct a coherent phenomenological approach in which both matter and space-time curvature were simultaneously generated. In this case, as originally envisaged by Tryon [5] the singularity can be avoided, with the universe now observed by us arising from a kind of instability of Minkowski space-time.

In the present work we reexamine the main results of Prigogine et al.’s paper from a more general point of view. Particularly, it will be shown that, if the specific entropy (per particle) is not constant, several of their results are no longer valid.

2 Thermodynamics and Matter Creation for a Simple Fluid

The basic macroscopic variables describing the thermodynamic states of a relativistic simple fluid are the energy-momentum tensor $T_{\alpha\beta}$, the particle flux vector $N^\alpha$, and the entropy flux vector $s^\alpha$.

We will restrict our considerations to an energy-momentum tensor of the form

$$T_{\alpha\beta} = (\rho + P)u^\alpha u^\beta - Pg^{\alpha\beta}, \quad (1)$$

satisfying the conservation law

$$T_{\alpha\beta}^{\alpha\beta} = 0. \quad (2)$$

In (1), $\rho$ is the energy density, $P$ is the (isotropic) dynamic pressure, and $u^\alpha$ is the fluid four-velocity. Here and henceforth we assume spatial isotropy.

The dynamic pressure $P$ may be decomposed as

$$P = p + \Pi \quad (3)$$
where $p$ is the equilibrium (thermostatic) pressure and $\Pi$ is a correction term present in dissipative situations.

The particle flux vector will be assumed to have the form

$$N^\alpha = nu^\alpha, \quad (4)$$

where $n$ is the particle density. It satisfies the balance law

$$N^\alpha = \Psi, \quad (5)$$

where $\Psi$ is a particle source ($\Psi > 0$) or sink ($\Psi < 0$) term. Usually in cosmology it is taken equal to zero.

The entropy flux vector is supposed to be

$$S^\alpha = n\sigma u^\alpha, \quad (6)$$

where $\sigma$ is the specific entropy (per particle). It satisfies the second law of thermodynamics

$$S^\alpha_{\alpha} \geq 0. \quad (7)$$

For this system, the Gibbs relation reads

$$nT d\sigma = d\rho - \frac{\rho + p}{n} dn, \quad (8)$$

where $T$ is the temperature.

From the equations above, it is easy to show that

$$S^\alpha_{\alpha} = -\frac{\Pi}{T} - \frac{\mu}{T} \Psi, \quad (9)$$

where $\Theta = u^\alpha_{\alpha}$ is the expansion of the fluid, and $\mu$ is its chemical potential defined by Euler’s relation

$$\mu = \frac{\rho + p}{n} - T\sigma. \quad (10)$$

Eq. (9), as far as we know, is presented here for the first time in the literature.
When the particle number is conserved (\( \Psi = 0 \)), \( \Pi \) stands for the bulk viscosity of the fluid. This kind of viscosity occurs in almost every situation. For instance, it is a well-know result from kinetic theory that a relativistic simple gas of weakly interacting point particles does not expand adiabatically. We can account for this phenomenon if we conceive the gas as a mixture of two components with different specific heats, each one expanding in an adiabatic manner during a time of the order of the mean free time. Each component will cool down at a different rate, thus producing a kind of “microscopic temperature gradient” over distances of the order of the mean free path. The heat flux tending to reequalize the temperatures is the mechanism of dissipation, which, in this context, is identified with the bulk viscosity [6].

Besides the usual classical meaning described above, the “viscous pressure” \( \Pi \) is also relevant when there occur processes in which a variation of the total number of particles takes place (\( \Psi \neq 0 \)). In this case, it is commonly denoted as a “creation pressure”. For instance, in an expanding universe in which there is matter creation, we must expect that the energy in a comoving volume decreases more slowly than in the ordinary situation. This property is taken account of by means of a reduction in the effective dynamic pressure. Formally, Eq. (2) implies that

\[
\frac{d(\rho \Delta V)}{dt} = -P \frac{\Delta V}{dt},
\]

Assuming \( \frac{\Delta V}{dt} > 0 \) (expansion), we must have \( P < p \), so that \( \rho \Delta V \) falls more slowly than in equilibrium.

Both processes described above are scalar ones and may take place simultaneously. However, from now on, we will restrict ourselves to the process of matter creation only. We shall suppose that the particles spring up into space-time in such a way that they turn out to be in thermal equilibrium with the already existing ones. The entropy production is then due only to the matter creation. It is obvious that, when \( \Psi = 0 \), we shall expect that the creation pressure vanish and so also the entropy production. We shall express this by admitting a kind of phenomenological ansatz
where \( \alpha \) is positive, so as to guarantee that, as argued above, in the case \( \Psi > 0 \) and \( \Theta > 0 \) we shall have \( \Pi < 0 \). With the choice (12), Eq. (9) is rewritten as

\[
S_{\alpha}^{\alpha} = \frac{\Psi}{T}(\alpha - \mu),
\]

or still, using Euler’s relation,

\[
S_{\alpha}^{\alpha} = \Psi \sigma + (\alpha - \frac{\rho + p}{n}) \frac{\Psi}{T}.
\]

It is convenient to compare (14) with the expression obtained by covariant differentiation of (6), that is,

\[
S_{\alpha}^{\alpha} = \Psi \sigma + n \dot{\sigma}.
\]

It is immediate that

\[
\dot{\sigma} = \frac{\Psi}{nT}(\alpha - \frac{\rho + p}{n}).
\]

3 The Results of Prigogine et Alii

The main results presented in Ref. [1] may be recovered if we constrain the formulation of the previous section to the case in which the specific entropy (per particle) \( \sigma \) is constant

\[
\dot{\sigma} = 0.
\]

Indeed, in this case, (16) implies that \( \alpha = (\rho + p)/n \) and the creation pressure assumes the form

\[
\Pi = -\frac{\rho + p}{n\Theta} \Psi,
\]

which is exactly Eq. (13) of Ref. 1.
Notice now that Eq. (8) with condition (17) reduces to

\[ \dot{\rho} = -3H(\rho + p), \]  

(19)
of the FRW models. It must be clear, however, that (19) only holds in the
case \(\sigma = \text{const.} \). Only then do \(\rho\) and \(n\) determine the equilibrium pressure \(p\).
In fact, from (19), we have, for example, that

\[ \rho = mn \Rightarrow p = 0; \]  

(20)

\[ \rho = aT4, n = bT3 \Rightarrow p = \rho/3. \]  

(21)

In the general case, however, we obtain from (8), that

\[ \dot{\rho} = \frac{\dot{n}}{n}(\rho + p) + nT\dot{\sigma}, \]  

(22)

and \(\dot{\sigma}\), or equivalently (in our formulation) \(\alpha\), must also be known for the
pressure to be fixed.

From condition (17), we still obtain that

\[ \frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \frac{\Psi}{n}, \]  

(23)

where \(S\) is the total entropy and \(N\) the total number of particles. Since \(S\) and
\(n\) are positive, and the total entropy cannot decrease, we must have \(\Psi \geq 0\)
and, in this case, as the authors of Ref. 1 have concluded, the space-time can
only create matter, the reverse process being thermodynamically forbidden.
Nevertheless, this result only holds if \(\dot{\sigma} \leq 0\). As a matter of fact, from (15),
we see that the second law of thermodynamics demands only

\[ \Psi \geq -n\frac{\dot{\sigma}}{\sigma}, \]  

(24)

which is compatible with matter destruction (\(\Psi < 0\)) provided \(\dot{\sigma} > 0\).
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