Neutrino Spin Transitions and the Violation of the Equivalence Principle.

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Abstract

The violation of the equivalence principle (VEP) causing neutrino oscillations is of current interest. We study here the possibility of not only flavor oscillation but spin flavor oscillation of ultra high energy (\(\sim 1 \text{ PeV}\)) neutrinos emanating from AGN due to VEP and due to the presence of a large magnetic field (\(\sim 1 \text{ Tesla}\)) in AGN. In particular we look at the resonance spin flavor conversion driven by the AGN potential. Interesting bounds on the transition magnetic moment of neutrinos may therefore be obtained.
The equivalence principle can be stated, “In any and every local Lorentz frame, anywhere and anytime in the Universe, all the (non-gravitational) laws of physics must take on their familiar special-relativistic forms.” [1]. It follows from this principle that if the gravitational couplings of differently flavored neutrinos are different, the equivalence principle is violated. Neutrinos with non-zero magnetic moment (transition magnetic moment when looking at different flavors) may undergo spin oscillations in the presence of a magnetic field. Both the effects, “violation of the equivalence principle” and “large neutrino magnetic moments” are speculative phenomena. Nonetheless both have considerable importance when studying physics beyond the standard model.

Active Galactic Nuclei (AGN) are by far the strongest sources of ultrahigh energy neutrinos in the Universe [2], producing fluxes detectable with present technology [3]. Neutrinos are expected to be generated mainly through the $\pi \rightarrow \mu \rightarrow e$ decay chain implying that we can expect twice as many muon type neutrinos as electron type neutrinos. A negligible number of tau neutrinos are produced in the AGN environment. The search for such high energy neutrinos by the neutrino telescopes under construction (e.g. AMANDA, NESTOR, BAIKAL etc.) necessitates a clear picture of the expected neutrino fluxes for these objects. AGN have luminosities ranging from $10^{42}$ to $10^{48}$ ergs/sec, corresponding to black hole masses of the order of $10^4$ to $10^{10}M_\odot$, on the natural assumption that they are powered by Eddington-limited accretion onto the black hole. The spherical accretion model (based on works by Kazanas, Protheroe and Ellison [4, 5]) is used in most of the calculations of the neutrino production in central regions of AGN [6, 7].
According to this scenario, close to the black hole the accretion flow becomes spherical and a shock is formed where the ram pressure of the accretion flow is balanced by the radiation pressure. The distance from the AGN center to the shock, denoted as the shock radius $R (\approx$ a few Schwarzschild radii) contains the central engine of AGN. The shock radius is parametrized \([4, 5]\) by $R = x r_g$ where $r_g$ is the Schwarzschild radius of the black hole, and $x$ is estimated to be in the range of 10 to 100 \([4]\).

The matter density at the shock $\rho(R)$ can be estimated from the accretion rate needed to support black hole luminosity, and from the radius and accretion velocity at the shock \([3]\)

$$\rho(R) \simeq 1.4 \times 10^{33} x^{-2} L_{\text{AGN}}^{-1} Q^{-1} \text{gm/cm}^3$$

(1)

where $Q(x) = 1 - 0.1 x^{0.31}$ is the efficiency for converting accretion power into accelerated particles at the shock \([3]\), and $L_{\text{AGN}}$ is the AGN continuum luminosity in units of ergs/sec.

Magnetic effects around AGN have important astrophysical consequences. An estimate of magnetic fields involving the “equipartition” condition for specific models \([8]\) is $B \sim 10^4$ G. We will be interested in the vicinity of the horizon where the pressure scale height is $\sim r_g$. In this region we will assume that $B$ remains at the above value. We will analyse the effects of tiny non-universality in gravitational couplings of neutrinos on such high energy neutrinos. Minakata and Smirnov \([9]\) have looked into this matter but here we also incorporate the effects of the magnetic field in the AGN environment which causes a spin flip in addition to the flavor transition due to the violation of the equivalence principle (VEP). It has been found \([2]\) that the accuracy of testing the equivalence principle, $\Delta f$ is improved by 25
orders of magnitude for massless neutrinos and by 11 orders of magnitude for massive neutrinos due to ultra high neutrino energies and cosmological distances.

We look for bounds on $\Delta f$ when looking into resonance conditions due to VEP leading to spin flavor flip. This mechanism of neutrino oscillation if valid in AGN environment will add to the testimony that tau neutrinos can be detected at energies 1 PeV and beyond [10] by the neutrino telescopes under construction and will also cause a depletion of neutrino flux by oscillation to unobservable right handed neutrinos (provided of course the neutrinos have non-zero transition magnetic moment). A bound on the transition magnetic moment of the neutrino can be calculated which allows resonant spin flavor transitions in the AGN environment.

It has been discussed at length that non-universality in the gravitational couplings of different flavors of neutrinos can give rise to neutrino flavor oscillations [11]. The solution to the solar neutrino problem by VEP has been studied in some detail [12]. The atmospheric neutrino data has also been looked into using this scenario [13].

Here we have given a qualitative study of the resonance spin flavor conversion of ultra high energy neutrinos due to VEP and due to the presence of a large magnetic field in AGN.

We will consider the gravitational effects in the presence of neutrino matter and mixing incorporating the effects of magnetic field assuming neutrinos to have non-zero transition magnetic moment. The main focus of our work is the effect of VEP on spin flavor precession. We find a region of resonance where this conversion will occur unimpeded. The physics of this resonance...
is very similar to the MSW resonance \[14\]. Such transitions due to matter induced resonance has been studied in sufficient detail in reference \[13\]. Matter, magnetic and strong gravitational field effects are found to directly cause resonant spin flip using standard neutrino couplings to gravity in reference \[16\]. Minakata \textit{et.al} \[9\] give an elaborate study of the effects on the cosmic high energy neutrinos due to the breakdown of the equivalence principle but do not study spin precession.

To illustrate resonant spin flavor precession phenomenon, we examine for simplicity only two neutrino flavors, i.e. the $\nu_e - \nu_\mu$ system. Similar results are obtainable for $\nu_\mu - \nu_\tau$ system. The non universality of gravitational couplings to the two flavors is given by $\Delta f$. Using the chiral bases $\nu_{eL}, \nu_{\mu L}, \nu_{eR}, \nu_{\mu R}$, the evolution equation for propagation through the AGN environment is given by

$$
\frac{d}{dr} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix} = H_{\text{eff}} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix}
$$

(2)

where $H_{\text{eff}}$ is a $4 \times 4$ matrix.

Since we are specifically interested in spin flavor precession let us look as an example at the $2 \times 2$ matrix associated with the transition ($\nu_{eL} \rightarrow \nu_{\mu R}$) which is given by

$$
H_{\text{eff}} = \begin{pmatrix}
\frac{1}{2E} \Delta m_{12}^2 \sin^2 \theta_M + V_G \cos^2 \theta_G & \mu_{e\mu} B^* \\
\mu_{e\mu} B & \frac{1}{2E} \Delta m_{12}^2 + V_G \sin^2 \theta_G
\end{pmatrix}
$$

(3)
where $\theta_M$ is the neutrino mixing angle, $\theta_G$ is the gravitational mixing angle and $\Delta m_{12}^2 = m_1^2 - m_2^2$. $B$ is the magnetic field in AGN assumed to be constant over a scale length of the order $r_g$ and $\mu_{e\mu}$ is the corresponding transition magnetic moment. The second term in the diagonal elements is due to the VEP, where $V_G = \frac{1}{2} \Delta f E \phi(r)$ \cite{1,2}, $\phi(r)$ being the gravitational potential and $E$ the neutrino energy. An order of magnitude estimate below shows that the matter effects are negligible compared to this term and so we have not taken them into consideration.

To compare the orders of magnitude of the matter effects and gravitational effects, we consider the spherical accretion model of AGN, density $\rho$ for typical cases is found to be $10^{-10} \text{eV}^4$ \cite{1}. The matter effect is given by a term of the form

$$G_F \rho \sim 10^{-33} \rho \text{eV} \sim 10^{-29} - 10^{-32} \text{eV} \quad (4)$$

where $G_F$ is the Fermi coupling constant, $m_p$ the proton mass and we have used typical values of AGN matter density as given above. To look at the gravitational part we need to know values of gravitational potential $\phi(r)$ and also $\Delta m_{12}^2$. For an order of magnitude estimation, let us look at the AGN potential at the neutrino production point. $\phi(r) = MG/r$ is the gravitational potential at a radius $r$ from an object of mass $M$ in the Keplerian approximation. At the site of neutrino production i.e. at $r = (10 - 100)r_g$, $\phi(r) \sim 5 \times 10^{-3}$ \cite{2}, for a $10^8 M_\odot$ black hole. So for neutrino energy $E = 10^{15}\text{eV}$ (1 PeV), the gravitational part gives

$$E \Delta f \phi(r) \sim 10^{12} \Delta f \text{eV} \quad (5)$$

The optimal sensitivity on VEP one would achieve by solar neutrino ob-
servations by using next generation water cherenkov detectors is \( |\Delta f| \sim 10^{-15} - 10^{-16} \) \cite{17}. This gives the gravitational part to be \( \sim 10^{-3} - 10^{-4} \) eV. Also we will show that at resonance conditions when we are dealing with very high energy neutrinos (1 PeV), \( \Delta f \sim 10^{-28} \Delta m^2_{12}/eV^2 \) which could give \( \Delta f \sim 10^{-38} \) for extremely small value of \( \Delta m^2_{12} = 10^{-10} \) eV \(^2\) (vacuum neutrino oscillation scenario). In that case an estimate of the gravitational part would give \( \sim 10^{-26} \) eV. Clearly in the above two cases the VEP effect dominates over the matter effects and so normal matter induced resonance effects are suppressed. To study the dominance of the VEP part at this high energy we have looked at the highest sensitivities predicted till date for \( \Delta f \), the conclusion clearly holds for lower sensitivities that is larger values of \( \Delta f \). We can therefore neglect the matter effects on neutrinos in this scenario.

We are looking at non-universality of gravity coupling to relevant flavors and are studying spin transitions. Hence it would be sufficient to look at the transitions in (3) for a general idea. Looking at the resonance condition by equating the diagonal terms in (3) we get,

\[ \phi_{res} \approx \frac{\Delta m^2_{12}}{2E^2 \Delta f} \]  

We have assumed that for a fixed \( \Delta m^2_{12} \) and \( V_G \), \( \cos \theta_m \approx \cos \theta_G \approx 1 \). \footnote{For a more realistic study we should consider different values of the mixing angles which has been done in sufficient detail in \cite{9}.}

Assuming that the resonance gravitational potential is the potential at the neutrino production site, \footnote{At the neutrino production point the AGN potential dominates over the supercluster and galactic effects \cite{9}.} estimated to be \( \sim 10^{-3} \), at neutrino energy being 1 PeV,

\[ \Delta f \sim 10^{-28} \left( \frac{\Delta m^2_{12}}{1 \text{eV}^2} \right) \]  

\footnote{1}
This accuracy increases by using still higher energy neutrinos.

The transition matrix can be cast into the form

\[
\begin{pmatrix}
  d & b \\
  b & -d
\end{pmatrix}
\]  

the explicit forms of \( b \) and \( d \) being,

\[ b = \mu_{\text{eq}} B \quad d = (\Delta m^2 \text{ term}) + (\text{VEM term}) \]  

Resonances occur when \( d \) vanishes, in which case the adiabatic transition probability is well described (for sufficiently slowly varying \( d \) and \( b \)) by the Landau-Zener approximation \[ P_{LZ} = \exp \left\{ -2\pi^2 \frac{\beta^2}{\alpha} \right\} \]

where

\[ \beta = b \big|_{\text{res}} \quad \alpha = \dot{d} \big|_{\text{res}} \]  

The condition for these resonances to induce an appreciable transition probability (adiabaticity condition) is

\[ \beta^2 \geq \frac{\alpha}{2\pi^2}. \]  

Also as previously mentioned, in the absence of a detailed model we will assume that the magnetic field remains approximately constant inside a region of size equal to the pressure scale height (\( \sim r_g \) in this case), the average fields of two of these regions will be uncorrelated. Let us define a scale height \( \Lambda \)

\[ \Lambda = \left| \frac{dr}{d(\ln \Delta f E\phi(r))} \right| \]
over which the magnetic field is assumed to have a constant value (estimated to be $\sim 10^4$ G). At resonance $\alpha \sim \Delta m^2/E\Lambda$ and the condition (12) reduces to

$$\mu_{e\mu} \geq \frac{1}{B} \frac{1}{2 \pi^2 E \Lambda} \left| \frac{\Delta m^2_{12}}{12} \right|^{1/2} = \mu_{\text{res}}_{\text{min}} \quad (14)$$

Resonant spin flavor transitions will occur therefore provided the transition magnetic moment satisfies the above bound. This constraint is however independent of the gravitational potential and $\Delta f$. Note that we have assumed the Keplerian approximation where $dV_G/dr = -V_G/r$. Also we take $V_G/r = V_G(r_g)/r_g$ and $r \sim r_g$ since we restrict ourselves to the site of neutrino production ($r \sim r_g$) within which the magnetic field is constant. Taking energy to be 1 PeV and $B = 10^4$ G, considering a $10^8 M_\odot$ black hole, $\mu_{\text{res}}_{\text{min}} \approx 10^{-12} |\Delta m^2_{12}|^{1/2} \mu_B$ (where $\mu_B$ denotes the Bohr Magnetron) which as an example for $\Delta m^2_{12} = 10^{-6}$ (for solar large angle solution) gives, $\mu_{\text{res}}_{\text{min}} = 10^{-15} \mu_B$. This lies comfortably inside the direct experimental bounds ($\mu_\nu \leq 10^{-10} \mu_B$) as well as the astrophysical limits ($\mu_\nu \leq 10^{-11} \mu_B$). In view of this such resonances will induce significant transition probabilities whenever the resonance conditions are satisfied.

Thus the violation of the equivalence principle when applied to ultrahigh energy AGN neutrinos can effectively induce a spin flavor conversion provided the transition magnetic moment is sufficiently large. Matter effects are found to be negligible in comparison to the VEP terms and so we do not look at usual matter induced flavor spin transitions. However the lack of precise modeling of the AGN magnetic field and lacking a better understanding of the parameters of the neutrino system, it is impossible to determine unambiguously whether such transitions do take place.
Ultrahigh energies and strong AGN gravitational potential allow to improve the accuracy of testing the equivalence principle by several orders of magnitude as noted in [9]. For an energy of 1 PeV for instance, resonance conditions lead to \( \Delta f \sim 10^{-28} \Delta m_{12}^2 / \text{1eV}^2 \). We have limited our calculations to the AGN environment only. A more complete study would involve the inclusion of the intergalactic and galactic potentials as done in [9] to study the total VEP effects on the neutrinos for their entire distance of travel (\( \sim 100 \text{ Mpc} \) is a sensible distance to AGN). But our effort was to show that spin flavor transitions can also be looked at in such a scenario which gives a magnetic moment bound independent of the gravitational potential. Our calculated bound for the transition magnetic moment for reasonable values of different parameters is \( \mu_{\text{min}}^{\text{res}} \sim 10^{-12} |\Delta m_{12}^2| \frac{1}{2} \mu_B \). It is of relevance to study possible helicity flips of neutrinos because observable left handed neutrinos could be converted into unobservable right handed ones thereby reducing the expected neutrino flux.
References

[1] C.W. Misner, K.S. Thorne and J.A. Wheeler; *Gravitation* (Freeman, San Francisco, 1973).

[2] V.S. Berezinsky, in *Neutrino 77*, Proceedings of the International Conference, Bakson Valley, USSR (Nauka, Moscow, 1977) 1, 177; D. Eichler, Astrophys. J. 232, 106 (1979); R. Silberberg and M.M. Shapiro, in Proc. 16th Int. Cosmic Ray Conf. Vol 10, 357 (1979); V.S. Berezinsky and V.L. Ginzburg, Mon. Not. Royal Astr. Soc. 194, 3 (1981).

[3] R.J. Wilkes, in Proc. Slac. Summer Institute 1994; H.W. Sobel, Nucl. Phys. B (Proc Suppl), 19, 444 (1991); S. Barwick, F. Halzen, D. Lowder, T. Miller, J. Phys. G: Nucl. Part. Phys. 18, 225 (1992); A. Roberts, Rev. Mod. Phys. 64, 259 (1992); F. Halzen, Nuclear Physics B, 38, 472 (1995).

[4] R.J. Protheroe and D. Kazanas, Astrophys. J. 265, 620 (1983).

[5] D. Kazanas and D.C. Ellison, Astrophys. J., 304, 178 (1986).

[6] A.P. Szabo and R.J. Protheroe, Astropart. Phys. 2, 375 (1994).

[7] M. Sikora and M.C. Begelman, in Proc. of High Energy Neutrino Astrophysics, eds V.J. Stenger et al. (World Scientific, Singapore, 1992).

[8] A.P. Szabo and R.J. Protheroe, Astropart. Phys. 2, 375 (1994); M.C. Begelman, R.G. Blandford, M. Rees, Rev. Mod. Phys. 56, 255 (1984).

[9] H. Minakata and A. Smirnov, Phys. Rev. D54, 3698 (1996).
[10] J.G. Learned and S. Pakvasa, Astropart. Physics, V3, N3, 267 (1995)

[11] M. Gasperini, Phys. Rev. D38, 2635 (1988); Phys. Rev. D39, 3606 (1989).

[12] J.R. Mureika and R.B. Mann, Phys. Lett. B368, 112 (1996); J.R. Mureika and R.B. Mann, Phys. Rev. D54, 2761 (1996); J.N. Bahcall, P.I. Krastev and C.N. Leung, Phys. Rev. D52, 1770 (1995); A. Halprin and C.N. Leung, Phys. Rev. Lett. 67, 1833 (1991); H. Minakata and H. Nunokawa, Phys. Rev. D51, 6625 (1995).

[13] A. Halprin, C.N. Leung and J. Pantaleone, Phys. Rev. D53, 5365 (1996).

[14] L. Wolfenstein, Phys. Rev. D17, 2369 (1978); S.P. Mikheev and A.Yu Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)].

[15] C.S. Lim, W.J. Marciano, Phys. Rev. D37, 1368 (1988).

[16] Dardo Piriz, Mou Roy and Jose Wudka, Phys. Rev. D54, 1587 (1996).

[17] H. Minakata and H. Nunokawa, Phys. Rev. D51, 6625 (1995).

[18] L.D. Landau, Phys. Z. Sowjetunion 1, 426 (1932); C. Zenner, Proc. Roy. Soc. A137, 696 (1932).