Rate-Splitting Multiple Access for Multi-Antenna Broadcast Channel with Imperfect CSIT and CSIR

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Abstract—Rate-splitting multiple access (RSMA) has appeared as a powerful transmission and multiple access strategy for multi-user multi-antenna communications. Uniquely, this paper studies the optimization of the sum-rate of RSMA with imperfect channel state information (CSI) at the transmitter and the receivers (CSIR). Robustness of the RSMA approach against imperfect CSIT has been investigated in the previous studies while there has been no consideration for the effects of imperfect CSIR. This motivates us to develop a robust design relying on RSMA in the presence of both imperfect CSIT and CSIR. Since the optimization problem for the design of RSMA precoder and power allocations to maximize the sum-rate is non-convex, it is hard to solve directly. To tackle the non-convexity, we propose a novel alternating optimization algorithm based on semidefinite relaxation (SDR) and concave-convex procedure (CCCP) techniques. By comparing simulation results with conventional methods, it turns out that RSMA is quite robust to imperfect CSIR and CSIT, thereby improving the sum-rate performance.

Index Terms—Rate-splitting multiple access (RSMA), sum-rate, multi-user multiple-input single-output (MU-MISO).

I. INTRODUCTION

Due to the increase in data traffic and the number of communicating devices, there is an increasing need to design efficient communication strategies to boost the data rate, spectral efficiency and manage the interference. To that end, multi-antenna/multi-user multi-input multi-output (MIMO) processing is a key technology. To deal with the interference problem in multi-user multi-antenna systems, the perfect channel state information (CSI) at receiver (CSIR) and transmitter (CSIT) are essential. However, it is difficult to obtain accurate CSI due to quantization error, channel mobility, and estimation error. Even with the ideal assumption of perfect CSIR, it is questionable whether a base station (BS) can obtain accurate CSIT.

Rate-splitting multiple access (RSMA) has recently emerged and has been found to have multiple advantages over conventional multiple access methods in terms of robustness against imperfect CSIT [1], and spectral and energy efficiencies [2]. [3]. The key feature of RSMA is the split of the messages into common and private parts. The common parts are encoded in a common stream that can be decoded by multiple users. On the other hand, each of the private messages is encoded in a private stream which is decoded by its respective receiver. Each receiver then decodes the common stream, retrieves its intended common part, then removed the common stream from the received signal using successive interference cancellation (SIC). After removing the common stream, each receiver can decode its intended private stream by treating the remaining private streams as interferences. From the common stream and the private stream, each receiver can reconstruct the original message. The flexibility of RSMA lies in adjusting the content and power allocated to the common and private streams, so as to partially decode interference and partially treat interference as noise [4]. Such flexibility leads to more robustness and performance enhancements in various network and propagation conditions [4].

In [5, 6, it is shown that RSMA can outperform conventional approaches in terms of rate maximization under perfect CSI assumption. Especially in [5], it is shown that RSMA unifies other four strategies (i.e. non-orthogonal multiple access (NOMA), space-division multiple access (SDMA), orthogonal multiple access (OMA) and multicasting) and outperforms them in a two user multiple-input single-output (MISO) broadcast channel (BC) channel.

In case of imperfect CSIT and perfect CSIR scenario, the BS is unable to calculate the achievable rates at the receivers accurately. Thus, the BS should adjust precoding vector and power allocation by using estimated channel and error information. In [7], sample average approximation combined with a weighted minimum mean square error (WMMSE) algorithm is used for sum-rate maximization in RSMA by generating channel error ensembles. In [7], max-min fairness optimization using the worst case rate and WMMSE is proposed with bounded channel error. Both studies show robust transmission of RSMA in multi-user (MU) MISO compared to conventional approaches.

In this paper, we consider both imperfect CSIT and CSIR under the assumption that the BS obtains CSI from the receiver through lossless channel feedbacks. This is the first paper studying the design and optimization of RSMA with both imperfect CSIT and CSIR. We formulate the sum-rate maximization problem in RSMA based MU-MISO system. For converting non-convex problem to convex problem, the algorithm using the two methods semidefinite relaxation (SDR) and concave-convex procedure (CCCP) is proposed. By the proposed algorithm, we jointly optimize precoding vectors and power allocation. In simulation, we show the performance
the proposed RSMA over existing techniques.

The reminder of this paper is organized as follows. In section II, system model and achievable rate in imperfect CSI are described. In section III, the optimization problem for maximizing the sum-rate is formulated and joint precoding vector and power allocation optimization is conducted for sum-rate optimization by the proposed algorithm based on SDR and CCCP. Simulation result are provided in section IV. The paper is concluded in V.

A. Notation

Standard letter indicates scalar, lower case boldface letter denotes vector, and upper case boldface letter denotes matrix. Notation $A \succeq B$ indicates that matrix $A - B$ is positive semidefinite matrix. Superscript $(\cdot)^H$ denotes hermitian (conjugate transpose). Trace of matrix $A$ is denoted by $\text{tr}(A)$ and rank of matrix $A$ is denoted by $\text{rank}(A)$. Notations of $|\cdot|$, $\|\cdot\|$ and $E[\cdot]$ refer to the absolute value, Euclidean norm, and expectation operation, respectively. A matrix $I_n$ denotes a $n$ by $n$ identity matrix.

II. System Model

A. Rate-Splitting Multiple Access Based System

We consider a single cell MU-MISO system operating in downlink where the BS equipped with $N_t$ antennas serves $K$ single antenna users. As shown in Fig. 1] the main idea of RSMA is to split a message for user-$k$ $W_k$, $k = 1, ..., K$ into common and private parts, i.e., $W_k = \{W_p,k, W_c,k\}$. The common part can be decoded by all users and the private part can be decoded by only the corresponding user. All common parts of each user message are combined into one common message $W_c$, i.e. $W_c = \{W_{c,1}, ..., W_{c,K}\}$. The common message $W_c$ is encoded into the common stream $s_c$ by using a codebook known to all users and each private message $W_{p,k}$ is encoded into the private stream $s_k$ by using a codebook known to only the intended receiver. Each stream is assumed to be independent zero mean unit variance Gaussian random variable, i.e. $s_i \sim \mathcal{CN}(0, 1)$, $i \in I \equiv \{c, 1, ..., K\}$. These $K + 1$ streams are linearly precoded by using precoding vector $p_i \in \mathbb{C}^{N_t \times 1}$, $i \in I$. The transmitted signal at the BS is expressed as

$$x = p_c s_c + \sum_{k=1}^{K} p_k s_k,$$

(1)

where a transmitted signal power constraint with a total power $P_t$ is

$$\sum_{i \in I} \|p_i\|^2 \leq P_t.$$

(2)

We refer to $h_k \in \mathbb{C}^{N_t \times 1}$ as a downlink channel vector from the BS to user-$k$ and a received signal at user-$k$ is denoted by

$$y_k = h_k^H x + n_k, \quad k = 1, ..., K,$$

(3)

where $n \sim \mathcal{CN}(0, \sigma_n^2)$ is additive white Gaussian noise (AWGN). Since the common stream can be decoded by all users, users can remove the common stream by SIC. Thus, users decode the private stream after SIC. When decoding the common stream, all private streams are treated as interference. When decoding a private stream, only other private streams are treated as interference, provided that the common stream is completely removed. Each user reconstructs the original message after retrieving the part of its message encoded in the common stream and the part encoded in the private stream.

B. Assumption on Channel State Information

We assume that users cannot accurately estimate the channel vector, i.e. imperfect CSIR. The channel model is given by

$$h_k = \hat{h}_k + e_k,$$

(4)

where $\hat{h}_k$ is an estimated channel and $e_k \sim \mathcal{CN}(0, \Phi_k)$ is a channel error. Also, we assume that the BS has the same CSI with the users because of lossless channel feedback. Thus, all users and the BS know the expectation of the channel, $E[h_k] = \hat{h}_k$, and the covariance of the channel, $E[(h_k - E[h_k])(h_k - E[h_k])^H] = \Phi_k$. In this paper, it is assumed that the covariance matrix of channel error $e_k$ is $\Phi_k = \sigma_e^2 I$. In other words, the channel error is assumed as a vector of independent and identically distributed (i.i.d) random variables.

C. Achievable Rate

It is difficult to determine an explicit achievable rate under the imperfect CSIR assumption, since the users do not know the actual channel. Thus, the concept of generalized mutual information (GMI) is used in order to characterize the achievable rate at a user with imperfect CSI [8, 9]. We first introduce a general form of GMI by considering a point to point case for simplicity. When the input has Gaussian distribution $x \sim \mathcal{CN}(0, \varepsilon_x)$, the output signal is expressed by

$$y = h x + n$$

(5)
where \( h \) is the fading channel and \( n \sim CN(0, N) \) is the noise. When knowing expectation and variance of channel, \( h \) can be broken into \( \hat{h} \) and \( e \), i.e. \( h = \hat{h} + e \) where \( \mathbb{E}[|h|] = \hat{h} \) and \( \mathbb{E}[e] = 0 \). We can intuitively consider \( \hat{h} \) as an estimate of the channel and \( e \) as a channel error having zero mean with variance \( \sigma^2_h \). GMI is defined by

\[
I_{GMI} = \log_2 \left( 1 + \frac{\|\hat{h}\|^2}{\mathbb{E}[|e|^2]|e_x + N} \right),
\]

where \( \mathbb{E}[|e|^2] = \sigma^2_e \). In the case of imperfect CSIR, GMI corresponds to an achievable rate when a user uses a nearest neighbor decoder and the input is Gaussian distribution \([10]\). By using this property, we apply GMI to RSMA based system and derive the achievable rate under imperfect CSI.

In RSMA approach, a user first decodes the common stream and then decodes the corresponding private stream after SIC. By this feature, the rate of the private stream is derived using the received signal after SIC. The received signal at user-\( k \) in (8) is rewritten as

\[
y_k = \hat{h}_k^H x + e_k^H x + n, \quad k = 1, \ldots, K
\]

\[
= \hat{h}_k^H p_c s_c + e_k^H p_c s_c
\]

\[
+ \sum_{j=1}^K (\hat{h}_k^H p_j s_j + e_k^H p_j s_j) + n.
\]

Considering the common stream, the signal received from user-\( k \) in (5) can be re-expressed in the form of (9) as

\[
y_k = \hat{h}_{k,c} s_c + e_{k,c} s_c + z_c
\]

\[
= \hat{h}_{k,c} s_c + z_c.
\]

where \( h_{k,c} = \hat{h}_{k,c} + e_{k,c} \), \( \hat{h}_{k,c} = \hat{h}_k^H p_c \), \( e_{k,c} = e_k^H p_c \), and \( z_c = \sum_{j=1}^K (\hat{h}_k^H p_j s_j + e_k^H p_j s_j) + n \). Due to the independence between each stream and the noise, the expectation and variance of each component are derived as follows:

\[
\mathbb{E}[h_{k,c}] = \hat{h}_{k,c} = \hat{h}_k^H p_c,
\]

\[
\mathbb{E}[e_{k,c}] = 0, \quad \mathbb{E}[z_c] = 0,
\]

\[
\mathbb{E}[|e_{k,c}|^2] = \mathbb{E}[|e_k^H p_c|^2],
\]

\[
\mathbb{E}[|z_c|^2] = \sum_{j=1}^K (\|\hat{h}_k^H p_j\|^2 + \|e_k^H p_j|^2) + \sigma^2_n.
\]

Substituting the values in (11), (13) and (14) into (6), the GMI for the common stream under imperfect CSIT can be obtained as

\[
R_{c,k} = \log_2 \left( 1 + \frac{\|\hat{h}_k p_c\|^2}{\sum_{j=1}^K \|\hat{h}_k^H p_j\|^2 + \sum_{j \in I} \|e_k^H p_j\|^2 + \sigma^2_n} \right).
\]

where \( \mathbb{E}[|e_k^H p_j|^2] = p_j^H \Phi_k p_j \), due to \( \mathbb{E}[e_k^H e_k^H] = \Phi_k \). Note that \( |e_k^H p_c|^2 \) is associated with not only the desired stream but also the channel error. Thus this term is considered as interference when decoding the desired stream, since users do not have any information on the channel error. The same phenomenon occurs when decoding the private streams.

When operating with perfect CSIR, the common stream can be removed perfectly by SIC. However, the common stream cannot be removed perfectly under imperfect CSIR, since the users do not have accurate information the actual channel. Thus, the part of the common stream associated with channel error still remains after SIC. The received signal after SIC with imperfect CSIR is expressed by

\[
y_{k,SIC} = y_k - \hat{h}_k^H p_c s_c
\]

\[
= e_k^H p_c s_c + \sum_{j=1}^K (\hat{h}_k^H p_j s_j + e_k^H p_j s_j) + n.
\]

The rate of the private stream can be obtained in a similar manner as the common part. The received signal after SIC is rewritten as

\[
y_{k,SIC} = \hat{h}_k s_k + e_k s_k + z_k
\]

\[
= h_{k,c} + z_k.
\]

where \( h_k = \hat{h}_k + e_k \), \( \hat{h}_k = \hat{h}_k^H p_k \), \( e_k = e_k^H p_k \), and \( z_k = e_k^H p_c s_c + \sum_{j=1}^K (\hat{h}_k^H p_j s_j + e_k^H p_j s_j) + n \). Thus, the achievable rate of private stream for user-\( k \) with imperfect CSIR is determined as

\[
R_k = \log_2 \left( 1 + \frac{\|\hat{h}_k^H p_k\|^2}{\sum_{j=1}^K \|\hat{h}_k^H p_j\|^2 + \sum_{j \in I} \|e_k^H p_j\|^2 + \sigma^2_n} \right).
\]

### III. SUM-RATE MAXIMIZATION WITH IMPERFECT CSI

In this section, a sum-rate maximization problem is formulated. We transform the optimization problem and propose a novel algorithm for solving the optimization problem that is non-convex.

#### A. Problem Formulation

Our objective is to optimize precoding vectors consisting of power and direction for maximizing the sum-rate. The sum-rate is expressed as

\[
R_s = R_c + \sum_{k=1}^K R_k,
\]

where the common rate \( R_c = \min_{p_k} R_{c,k}, \quad k = 1, \ldots, K \) because the common stream is decoded by all users. The optimization problem for sum-rate maximization is formulated as:

\[
P(1) : \max_{p_k, \forall i} R_s = \sum_{k=1}^K R_k
\]

s.t.

\[
R_{c,k} \geq R_c,
\]

\[
\sum_{\forall i \in I} \|p_i\|^2 \leq P_t.
\]
We first simplify expression of the objective function of \((P1)\) using a stacking method introduced in \([11]\). First, we equivalently transform the expressions of the rate \((15)\) and \((20)\) as

\[
R_{c,k} = \log_2 \left( \frac{\sum_{j \in I} p_j^H (h_k h_k^H + \Phi_k) p_j + \sigma_n^2}{\sum_{j=1}^K p_j^H h_k h_k^H p_j + \sum_{j \notin I} p_j^H \Phi_k p_j + \sigma_n^2} \right)
\]

(23)

and

\[
R_k = \log_2 \left( \frac{\sum_{j=1}^K p_j^H h_k h_k^H p_j + \sum_{j \notin I} p_j^H \Phi_k p_j + \sigma_n^2}{\sum_{j=1}^K p_j^H h_k h_k^H p_j + \sum_{j \notin I} p_j^H \Phi_k p_j + \sigma_n^2} \right).
\]

(24)

By using a combined precoding vector \(p = [p_1^H, \ldots, p_K^H, p_c^H]^H \in \mathbb{C}^{N_t(K+1)\times 1}\), the numerator term in \((23)\) can be expressed by

\[
\sum_{j \in I} p_j^H (h_k h_k^H + \Phi_k) p_j + \sigma_n^2 = p^H A_k p,
\]

(25)

where \(A_k \in \mathbb{C}^{N_t(K+1)\times N_t(K+1)}\) is a block diagonal and positive definite matrix defined by

\[
A_k = \begin{bmatrix}
    h_k h_k^H & 0 & \cdots & 0 \\
    0 & h_k h_k^H & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & h_k h_k^H \\
  \end{bmatrix}
+ \left( \frac{\sigma_n^2}{\sigma_n^2} + \sigma_n^2 \right) I_{N_t(K+1)}
\]

(26)

under the assumption that all transmission power is used, i.e., \(\|p\|^2 = P_t\), and \(\Phi_k = \sigma_n^2 I\). We apply the same approach to the denominator and the numerator terms of \((23)\), \((24)\). Each term can be rewritten as:

\[
\sum_{j=1}^M p_j^H h_k h_k^H p_j + \sum_{j \in I} p_j^H \Phi_k p_j + \sigma_n^2 = p^H B_k p,
\]

(27)

\[
\sum_{j \notin I} p_j^H h_k h_k^H p_j + \sum_{j \notin I} p_j^H \Phi_k p_j + \sigma_n^2 = p^H D_k p,
\]

(28)

where the positive definite matrix \(B_k, D_k\) are defined as

\[
B_k = A_k - \begin{bmatrix}
    0 & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & 0 \\
\end{bmatrix},
\]

(29)

\[
D_k = B_k - \begin{bmatrix}
    0 & \cdots & 0 & \cdots & 0 \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & \cdots & 0 \\
\end{bmatrix}
\]

(30)

The matrix in the second term of \((30)\) is a block diagonal matrix formulated by \(\text{diag}(0, \ldots, h_k h_k^H, \ldots, 0)\) in which \(h_k h_k^H\) is a \(k\)th diagonal sub-block. As the result, the achievable rates are simplified to

\[
R_{c,k} = \log_2 \left( \frac{p^H A_k p}{p^H B_k p} \right), \quad R_k = \log_2 \left( \frac{p^H B_k p}{p^H D_k p} \right),
\]

and the objective function of the optimization problem \((P1)\) can also be simplified to

\[
f(p) = \sum_{k=1}^K \log_2 \left( \frac{p^H B_k p}{p^H D_k p} \right) + \min_{j} \log_2 \left( \frac{p^H A_j p}{p^H A_j p} \right).
\]

(31)

In this case, because \(f(p) = f(\alpha p)\) with non-zero parameter \(\alpha\), the power constraint \((22)\) can be ignored. Each rate is written as the difference of concave functions, e.g., \(R_k = \log_2(p^H B_k p) - \log_2(p^H D_k p)\), which is a non-convex function. Thus, finding the optimal solution is difficult due to the non-convexity of the objective function. In order to solve the non-convex problem, we propose an algorithm based on alternating optimization.

B. Proposed Optimization Algorithm

Before finding a solution, to transform the problem, we derive upper and lower bounds of the denominator and numerator in \((31)\) by using auxiliary variables \(a_k, b_k, c_k, d_k\):

\[
p^H A_k p \geq e^{a_k}, \quad p^H B_k p \geq e^{b_k},
\]

(33)

\[
p^H B_k p \leq e^{b_k}, \quad p^H D_k p \leq e^{d_k}.
\]

(34)

Using these bounds, we can induce the lower bound of objective function \(f(p)\) by

\[
f(p) \geq 1 - \frac{1}{\ln 2} \left[ \sum_{k=1}^K \ln \left( \frac{e^{a_k}}{e^{d_k}} \right) + \min_j \left( \frac{e^{a_j}}{e^{b_j}} \right) \right]
\]

\[
= 1 - \frac{1}{\ln 2} \left[ \sum_{k=1}^K (c_k - d_k) + \min_j (a_j - b_j) \right].
\]

(35)

Finally, by adding one more slack variable \(e\), for term of \(\min_j (a_j - b_j)\), the optimization problem \((P1)\) can be transformed to \((P2)\) denoted as

\[
(P2) : \max_{a, b, c, d, p, l_e} \sum_{k=1}^K (c_k - d_k) + e \]

s.t. \(a_k - b_k \geq l_e, k = 1, \ldots, K\), \((33), (34)\),

where \(a \triangleq [a_1, \ldots, a_K], b \triangleq [b_1, \ldots, b_K], c \triangleq [c_1, \ldots, c_K]\), and \(d \triangleq [d_1, \ldots, d_K]\). The problem is still non-convex, since the constraints \((33), (34)\) are non-convex. For constraint \((33)\), we apply SDR technique which obtains a solution that is close to the optimal solution in non-convex quadratically constrained quadratic program (QCQP) \([12]\). SDR converts a non-convex problem into a convex problem by removing the rank one
The value of \( b \) using CVX toolbox \([14]\).

The constraint functions in (34) are difference of convex (DC) functions that are generally non-convex. For this problem, we approximate the DC function to a convex function by the CCCP method, which guarantees a local optimal solution of the DC problem \([13]\).

For such approximation, we linearize the exponential term in (34), which is concave, by using the first-order Taylor series approximation. Finally, constraints at the \( s \)th iteration can be denoted by

\[
\begin{align*}
\text{tr}(A_kX) &\geq e^{ak}, \quad \text{tr}(B_kX) \geq e^{ck}, \\
\text{tr}(B_kX) &\leq e^{b_k(s-1)}(b_k-b_k(s-1)+1), \\
\text{tr}(D_kX) &\leq e^{d_k(s-1)}(d_k-d_k(s-1)+1).
\end{align*}
\]

The sub-problem at \( s \)th iteration (P3) is a convex problem expressed by

\[
\begin{align*}
\text{(P3)}: \max_{a,b,c,d} \sum_{k=1}^{M} (e_k - d_k) + l_c \\
\text{s.t.} \quad &a_k - b_k \geq l_c, k = 1, \ldots, K, \\
&X \succeq 0, \\
&(38), (39), (40).
\end{align*}
\]

The value of \( b_k(s-1) \) and \( d_k(s-1) \) are updated by solving the sub-problem (P3) and a local optimal solution is obtained with a sufficient number of iterations. The detailed process is expressed in Algorithm 1. The convex problem can be solved using CVX toolbox \([13]\).

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**Algorithm 1** Alternating Optimization based on SDR and CCCP

1. Initialize \( b_k^{(0)}, d_k^{(0)} \) and set \( s = 0 \) and \( \epsilon \) to a small value
2. repeat
3. \( s \leftarrow s + 1 \)
4. Given \( b_k^{(s-1)}, d_k^{(s-1)} \), solve the problem (P3) and obtain optimal \( X^*, a_k^*, b_k^*, c_k^*, d_k^*, l_c^* \)
5. Update \( b_k^{(s)} \leftarrow b_k^*, \quad d_k^{(s)} \leftarrow d_k^* \)
6. until convergence of \( b_k^{(s)}, d_k^{(s)} \)
7. Decompose \( X^* = U\Sigma U^H \)
8. Generate enough random vectors \( r \sim \mathcal{CN}(0, I_{N_t(K+1)}) \)
9. Choose the best \( \bar{f} = U\Sigma^{1/2}r \) as a solution \( p^* \)

---

It is noted that a obtained solution \( X^* \) does not satisfy the constraint \( \text{rank}(X) = 1 \). Therefore, we refine \( X^* \) to satisfy the rank constraint. Since \( X^* \) is a Hermitian positive semidefinite matrix, it can be decomposed in the form \( X^* = U\Sigma U^H \) by the singular value decomposition. Then, we generate sufficient number of random vectors \( r \in \mathbb{C}^{N_t(K+1)} \times 1 \) and obtain \( \bar{f} = U\Sigma^{1/2}r \). Finally, we choose the best \( \bar{f} \) maximizing the objective function \( f(p) \) as a final solution \( p^* \). It has been shown that SDR with sufficiently large number of random vectors guarantees a solution close to the optimal solution \([12]\). Overall step of the proposed algorithm is described in Algorithm 1.

**Remark (Convergence)**: In Algorithm 1, we randomly initialize the values of \( b_k^{(0)} \) and \( d_k^{(0)} \). In Fig. 2 it is numerically confirmed that the proposed algorithm converges to finite point as the number of iterations increases when \( \text{SNR} = 20\text{dB}, \sigma^2_{e,k} = 0.1 \). The value of objective function gradually increased without fluctuation. We can observe that as the number of transmitter antennas and users increases, the number of iterations required for convergence increases.

IV. SIMULATION RESULTS

We assume \( h_k \) has i.i.d complex Gaussian distribution with zero mean and unit variance, i.e. \( h_k \sim \mathcal{CN}(0,1) \). The variance of AWGN is fixed as \( \sigma^2_n = 1 \). The channel error is also distributed by complex Gaussian distribution, i.e., \( e_k \sim \mathcal{CN}(0, \sigma^2_{e,k} I) \). The estimated channel \( \hat{h}_k \) is independent from the channel error and has complex Gaussian distribution with zero mean and variance \( \sigma^2_{h_k} = 1 - \sigma^2_{e,k} I \), i.e. \( \hat{h}_k \sim \mathcal{CN}(0,1 - \sigma^2_{e,k} I) \). We also consider that all channel error has same covariance matrix \( \sigma^2_{e,k} I = \sigma^2_{e} I \).

A. Comparison with Conventional Multiple Access Techniques

In this section, we consider a 2-user scenario and provide simulation results to compare with existing multiple access...
In this section, performances of the precoding vectors are illustrated. As shown in Fig. 3, RSMA has a better performance than other multiple accesses. The gap in sum-rate between RSMA and the other schemes is notable in high SNR. It is worth pointing out that these benefits come from the flexibility of RSMA that generalizes and bridges the conventional schemes for 2-user scenario. Compared to SDMA, the use of the common stream for RSMA, offers more design flexibility in jointly optimizing its precoding vector and power allocation. Under perfect CSIR, there is no increment of interference when a desired signal transmit power is increased. However, under imperfect CSIR, the interference from the channel error is also increased as the desired signal transmit power is increased. Thus, GMI and the sum-rate are saturated at high SNR. It is worth noting that these benefits come from the flexibility of RSMA, which considers channel error information in the BS.

We illustrate the achievable sum-rate of the proposed method according to SNR when the BS has two antennas and $\sigma^2_e = 0.05$. As shown in Fig. 4, RSMA has a better performance than other multiple accesses. The gap in sum-rate between RSMA and the other schemes is notable in high SNR. It is worth pointing out that these benefits come from the flexibility of RSMA that generalizes and bridges the conventional schemes for 2-user scenario. Compared to SDMA, the use of the common stream for RSMA, offers more design flexibility in jointly optimizing its precoding vector and power allocation. Under perfect CSIR, there is no increment of interference when a desired signal transmit power is increased. However, under imperfect CSIR, the interference from the channel error is also increased as the desired signal transmit power is increased. Thus, GMI and the sum-rate are saturated at high SNR. Furthermore, it is described in Fig. 3 that the performance is degraded in the absence of information about channel error and RSMA is less sensitive to the knowledge of channel errors than the other schemes.

### B. Impact of Proposed Precoding in RSMA

In this section, performances of the precoding vectors are analyzed by comparing simulation results of RSMA with the proposed precoding vectors and existing fixed precoding vectors, zero-forcing (ZF) and MRT. ZF beamforming aims to...
remove the interference by nulling, i.e. \( \| \mathbf{h}_j^H \mathbf{p}_j \| = 0, \ j \neq k \) \[15\] and MRT refers to precoding the stream in the same direction with the channel vector. ZF and MRT are applied to the precoding vectors of the private streams. As shown in TABLE [11] ZF and MRT determine the direction of the precoding vector for the private stream based on the estimated channel. As a result, the direction of the precoding vectors for the private stream, \( \mathbf{v}_k = \mathbf{p}_k / \| \mathbf{p}_k \|, \ k = 1, \ldots, K, \) are fixed. Thus, power allocation, \( P_k = \| \mathbf{p}_k \|, \) and precoding vector for the common stream \( \mathbf{p}_c \) should be optimized. We optimize the sum-rate of RSMA-ZF/MRT with the similar approach of the proposed RSMA by formulating an optimization problem for \( P_k \) and \( \mathbf{p}_c \).

Fig. [4] demonstrates the changes in the performance of RSMA with respect to channel error covariance \( \sigma_e^2 \) and SNR, where \( K = 2 \) and \( N_t = 2 \). The results confirm that the rate under imperfect CSI has different characteristics from that obtained under perfect CSI. Under perfect CSI, it is well known that MRT is near optimal in low SNR and ZF is asymptotically optimal in high SNR [16]. However, the imperfect knowledge of channel brings out the residual interference caused by the inaccurate operations of precoding (at the transmitter) as well as coherent detection including SIC (at the receiver), which corresponds to low SNR scenarios. When the variance of channel error, \( \sigma_e^2, \) is dominant, RSMA tends to operate as in low SNR under perfect CSI so that the optimized beamformers approximate to MRT. On the other hand, when channel error variance is very small, e.g. \( \sigma_e^2 \approx 0, \) ZF can provide a better sum-rate performance than that of MRT due to negligible residual interference at high SNR regime. As shown in Fig. [4] RSMA with the proposed precoding vector performs strictly better than RSMA with ZF and MRT precoders regardless of the variance of channel error over the entire range of SNR.

V. CONCLUSION

In this paper, we have studied a robust design of RSMA when perfect CSI is not available at both transmitter and receiver. To tackle the sum-rate maximization problem turned out to be non-convex, the proposed algorithm has utilized the SDR and CCCP in jointly optimizing the precoding vectors and power allocation. The simulations results have numerically shown that the proposed RSMA achieves the enhanced sum-rate performance compared to the conventional multiple access schemes. Also, RSMA with joint optimization of power allocation and the precoding vectors provides the sum-rate improvement over RSMA in which the fixed precoding schemes, ZF and MRT, which are applied to the private streams. From these results, it can be seen that the proposed RSMA is a powerful technique in terms of the sum-rate performance under imperfect CSIR and CSIT.

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