Noncommutative Supersymmetric Tubes

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We investigate supersymmetric tubular configurations in the matrix theory. We construct a host of BPS configurations of eight supersymmetries. They can be regarded as cylindrical D2 branes carrying nonvanishing angular momentum. For the simplest tube, the world volume can be described as noncommutative tube and the world volume dynamics can be identified as a noncommutative gauge theory. Among the BPS configurations, some describe excitations on the tube and others describe many parallel tubes of different size and center.
1 Introduction

It is now well known how to describe the half BPS D-branes from the viewpoint of matrix theory[1, 2, 3]. The D-brane configurations arise as a flat hypersurface formed by constituent D-particles. There was also some study of a spherical D2 configurations and its variations [4]. When external R-R fields are turned on, the constituent D-particles respond to the external field like a dielectric material in an external electromagnetic field [5, 6]. For example, a spherical D2-brane configuration of finite radius occurs and the geometry involved here is so called a fuzzy sphere. The bending of the surface, in this case, is caused by the external R-R fields. It is found recently a tube configuration from the Born-Infeld theory description, which is self sustained by its worldvolume gauge field [7]. The configuration is a quarter BPS state unlike the other D-brane configurations and also carrying a nonvanishing angular momentum produced by the worldvolume gauge field.

In this paper we like to realize the tube configuration from the viewpoint of the matrix theory. To this end, we first derive a set of BPS equation and find solutions describing tubes. The worldvolume geometry is defined by an algebra describing a noncommutative cylinder. The configuration involves the worldvolume magnetic field as well as electric field, which leads to a nonvanishing angular momentum. The direction of worldvolume electric field is found to agree with the supertube of the Born-Infeld theory [7]. However the precise match of the charges, the strength of the electric field, and so on are not clear because highly nonlinear terms play a role in the Born-Infeld description.

Since both electric and magnetic fields are present in the worldvolume, one may naively expect that its dynamics is described by a spacetime noncommutative gauge theory. But it turns out that the worldvolume dynamics is described by a gauge theory involving spacelike noncommutativity only. However the presence of electric field makes the gauge theory differ from the conventional noncommutative Yang-Mills theory on a cylinder. We develop here also the $*$-product realization of the algebra and operators on the noncommutative supersymmetric tube. This makes the geometrical interpretation clear.

We construct noncommutative soliton configurations describing multiple D0’s out of the noncommutative tube. Here it is not difficult to identify the moduli parameters involved with the solitons. Interestingly, the moduli dimensions are the number of D0’s multiplied by nine—the number of spatial dimensions. Hence the D0 solitons seem to easily fly off the tube at least classically. Unlike ordinary unstable D0-D2 system [8], the configurations are BPS saturated preserving again eight supersymmetries. The tube solution may be generalized to coincident tubes. The worldvolume dynamics here become $U(p)$ noncommutative gauge theory [8]. We also find many tubes of different sizes and centers.

In Section 2, we present the tube solution of the matrix model. The low energy description of the worldvolume gauge theory is investigated in Section 3. In Section 4, we construct the solitonic solutions describing D-particles out of the tube. In addition, we discuss more general configuration of many tubes. Last section comprises conclusions and comments.
2 Supersymmetric Tube Solutions

To construct the tube configurations, we shall begin with the matrix model Lagrangian

\[ L = \frac{1}{2R} \text{tr} \left( (D_0 X_I)^2 + \frac{R^2}{l_{11}^2} [X_I, X_J]^2 + \text{fermionic part} \right) \]  

(1)

where \( I, J = 1, 2, \ldots 9 \) and \( R \) is the radius of tenth spatial direction. Here \( l_{11} \) is the eleven dimensional Planck length, which we will set to unity in the following discussions. The scale \( R \) will be omitted below for simplicity and we shall recover it whenever necessary. As is well known, this Lagrangian can be thought of describing \( N \) D-particles if one takes all the dynamical variable as \( N \times N \) matrices.

Let us first describe relevant BPS equations we like to solve. For this, we shall turn on only first three components of the matrices \( X_I \). Then the Gauss law reads

\[ [X, D_0 X] + [Y, D_0 Y] + [Z, D_0 Z] = 0. \]  

(2)

Using the Gauss constraint, the bosonic part of the Hamiltonian can be written as

\[ H = \frac{1}{2} \text{tr} \left( (D_0 X \pm i[Z, X])^2 + (D_0 Y \pm i[Z, Y])^2 + (D_0 Z)^2 + [X, Y]^2 + 2C_J \right) \geq \text{tr} C_J \]  

(3)

where \( \text{tr} C_J \) is the central charge defined by

\[ \text{tr} C_J = \pm \frac{i}{2} \text{tr} \sum_{i=1}^{3} [X_i, Z(D_0 X_i) + (D_0 X_i)Z]. \]  

(4)

The saturation of the BPS bound occurs if the BPS equations

\[ [X, Y] = 0, \quad D_0 Z = 0, \quad D_0 X \pm i[Z, X] = 0, \quad D_0 Y \pm i[Z, Y] = 0 \]  

(5)

hold together with the Gauss law constraint in (3). On the choice of gauge \( A_0 = \frac{R}{l_{11}} Z \), the BPS equations of the upper sign imply that all the fields are static. Hence in this gauge, the system of equations reduce to

\[ [X, Y] = 0, \quad [X, [X, Z]] + [Y, [Y, Z]] = 0, \]  

(6)

where the latter comes from the Gauss law constraint. Before providing the representation of the algebra, let us count the remaining supersymmetries of the state specified by the nontrivial representation of the the algebra. There are two sixteen supercharges of the matrix model. The 16 components of the kinematical supersymmetry is broken spontaneously by the presence of the longitudinal momentum. The remaining supersymmetric variation of the fermionic coordinates \( \psi \) is given by

\[ \delta \psi = -D_0 X (\Gamma_{01} + \Gamma_{13}) \epsilon - D_0 Y (\Gamma_{02} + \Gamma_{23}) \epsilon. \]  

(7)

Setting this to zero, one finds that the solutions preserve eight of remaining sixteen supersymmetries. Hence in total the configuration preserves a quarter of the 32 supersymmetries of the matrix model.
Among the solutions of the BPS equations, we are particularly interested in the solutions defined by the following algebra,

\[ [z, x] = ily, \quad [y, z] = ilx, \quad [x, y] = 0, \quad (8) \]

with \( X_i = x_i \). This is the algebra defining two dimensional Euclidean group. The length scale \( l \) is the noncommutativity parameter.

The algebra in (8) is realized as follows. Let us introduce variables \( x_\pm \) by

\[ x_\pm = x \pm iy. \quad (9) \]

The algebra is then rewritten as

\[ [z, x_\pm] = \pm lx_\pm, \quad [x_-, x_+] = 0, \quad (10) \]

so it is clear that \( x_- x_+ = x^2 + y^2 \equiv \rho^2 \) is a Casimir operator. We are interested in the following irreducible representation of the algebra,

\[ x_+|n\rangle = \rho |n+1\rangle, \quad z|n\rangle = ln|n\rangle. \quad (11) \]

We use \(|n\rangle (n \in \mathbb{Z})\) to be the basis to represent infinite dimensional matrices.

As \( \rho^2 \) is Casimir operator and can be regarded as a number, we can represent the \( x_\pm \) with the angular variable as follows

\[ x_\pm = \rho e^{\pm i\theta} \quad (12) \]

with periodic hermitian operator \( \theta \). Then \( e^{\pm i\theta}|n\rangle = |n \pm 1\rangle \) and \([z, e^{\pm i\theta}] = \pm le^{\pm i\theta}\). It is obvious that our BPS configuration describes a noncommutative tube of radius \( \rho \) in three dimensions. The coordinates \((\theta, z)\) of this tube would be noncommutative.

Any well-defined operator can be presented as

\[ f(z, \theta) = \sum_{n=-\infty}^{\infty} \int_{-\pi}^{\pi} \frac{dk}{2\pi} \tilde{f}_n(k) e^{in\theta + ikz}. \quad (13) \]

The range of \( k \) is determined by the fact that the \( z \) operator has discrete eigenvalues. Also any operator can be represented as \( f = \sum_{n,m=-\infty}^{\infty} f_{nm}|n\rangle \langle m| \) in the matrix theory. Two representations are related by

\[ f_{nm} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \tilde{f}_{n-m}(k) e^{\frac{\pi}{\lambda}(n+m)k}. \quad (14) \]

For the operation relation, we also get \(|0\rangle\langle 0| = l \int \frac{dk}{2\pi} e^{ikz} \).

The multiplication of operators on the noncommutative tube is well defined. Instead, we can define the \(*\)-product of ordinary functions on the corresponding commutative tube of the same radius. In the fourier representation of ordinary functions, their \(*\)-product should leads to the

\footnote{The same algebra is considered in Ref.[1] in the context of spacetime noncommutativity.}
fourier representation which we would get as the product of operators. Thus, the product of two ordinary functions \( g \) and \( h \) would be

\[
g \ast h = \left[ e^{\frac{i}{l}z(\partial_\theta - \partial_z,\theta')} g(\theta, z)h(\theta', z') \right]_{\theta = \theta', z = z'}.
\]  

(15)

In addition, the spatial integration \( \int d\theta dz \) on the tube corresponds to \( 2\pi l \times \rho \). Since above \( \ast \)-product implies that \( \theta \ast z - z \ast \theta = il \), the minimal area is in a rough sense given by \( \Delta = (2\pi l \times \rho) \). Since the circumference of the tube is \( 2\pi \rho \), one may regard the noncommutativity scale \( l \) as a kind of minimal distance in the \( z \) direction. Indeed the discreteness of the spectrum of \( z \) is consistent with this observation. Moreover, \( 1/\Delta \) corresponds to the area density of the the constituent D0-branes. Namely the total number of D0-branes is \( N = \text{tr} I = \frac{1}{\Delta} \int dz d\theta \rho \). Hence the D0 brane density per unit length in the \( z \)-direction is \( 1/l \).

Now let us reconsider the preliminary discussion of the physical implication of the solution. First of all, the central charge can be reexpressed as

\[
\text{tr} C_J = l^2 \text{tr} x_+ x_- = l^2 \rho^2 \text{tr} I,
\]

(16)

where we have used the algebra in (10). The central charge density per unit length in \( z \)-direction is \( l\rho^2 R/l_{11}^6 \). We compare this with the angular momentum along the \( z \) axis,

\[
\text{tr} J = \text{tr} (X D_0 Y - Y D_0 X) = -l\rho^2 \text{tr} I.
\]

(17)

Thus \( \text{tr} C_J = -l\rho \text{tr} J \) and the system carries a nonvanishing angular momentum density. As we seen above, the configuration describes a tubular configuration whose coordinates may be identified as \( \theta \) and \( z \) in the commutative limit. It is a cylindrical object embedded in a flat 9-dimensional space. Hence we shall call the configuration as noncommutative supersymmetric tube.

From the view of the commutative Born-Infeld action, there exists a nonzero magnetic field along the \( \rho \) direction, which is responsible for the noncommutativity. In addition, there exists an electric field on the world volume along the \( z \) direction. Since \( D_0 X = ly \), \( D_0 Y = -lx \), it might appear that the electric field is applied to the \( \theta \) direction. But this identification is not quite right because the open string metric describing the worldvolume dynamics is twisted due to the presence of background \( B_{\theta z} \) field[11]. In short, the configuration discussed above corresponds to the supertube found recently in the Born-Infeld description[7].

3 Worldvolume Gauge Theory

In this section, we shall describe the low energy dynamics of the worldvolume gauge theory by taking the solution (8) as a background configuration. For this end, let us first consider the field equation governing fluctuation of the transverse scalar. Turning on just one component, \( \phi = X_4 \), the equation reads

\[
[\partial_t - iz, [\partial_t - iz, \phi]] + [x_i, [x_i, \phi]] = 0,
\]

(18)

where, recovering the \( l_{11} \) and \( R \), \( x_i \) is replaced by \( Rx_i/l_{11}^3 = x_i/l_s^2 \) with the string scale \( l_s \). First, let us write the equation in the commutative limit. To this end, we note

\[
[z, \cdot] = -il\partial_\theta, \quad [x, \cdot] = -il y\partial_z + O(l^2),
\]

\[
[y, \cdot] = il x\partial_z + O(l^2).
\]

(19)
Using this relation, the above equation can be written as
\[
(\partial_t - l \partial_\theta)^2 \phi - l^2 \rho^2 \left( \frac{1}{\rho^2} \partial^2_\theta + \partial^2_z \right) \phi + O(t^3) = 0.
\] (20)

This is the equation of motion produced by the Lagrangian density
\[
\mathcal{L}_\phi = \frac{1}{2} (\partial_t \phi)^2 - l \partial_t \phi \partial_\theta \phi - \frac{l^2 \rho^2}{2} (\partial_z \phi)^2 + O(l^4).
\] (21)

The system is a field theoretic analog of electrically charged particles moving in a constant magnetic field. One can find the canonical momentum density
\[
p_\phi = \dot{\phi} - l \partial_\theta \phi
\]
and its Hamiltonian density
\[
H_\phi = (p_\phi + l \partial_\theta \phi)^2 / 2 + \rho^2 (\partial_z \phi)^2 / 2, \text{ which is positive definite.}
\]

As is well known in the case of constant magnetic field, the effect of electric field can be canceled by going to the rotating frame defined by
\[
z' = z, \quad \theta' = \theta + lt, \quad t' = t
\] (22)

The equation then becomes a free field equation on a cylinder:
\[
(\partial^2_{t'} - l^2 \rho^2 \left( \frac{1}{\rho^2} \partial^2_{\theta'} + \partial^2_{z'} \right)) \phi(\theta', z') + O(l^3) = 0.
\] (23)

In fact the scalar equation in (18) can be solved generically without difficulties. To this end, let us use the operator representation
\[
\phi = \sum_n \int \frac{dk}{2\pi} \tilde{\phi}_n(k, t) e^{i(n\theta + lt + ikz)},
\]
where we introduce the time dependent phase to cancel the effect of the background electric field. The scalar equation for each Fourier component becomes trivial and its most general solution is then
\[
\tilde{\phi}_n(k, t) = f_n(k) e^{\pm i [l^2 n^2 + 4 \rho^2 \sin^2(\frac{\theta}{2})]^{1/2} t}
\] (24)

with arbitrary \( f_n(k) \).

We now turn to the description of the world volume gauge theory. The \( U(\infty) \) gauge symmetry of the matrix theory becomes a local gauge symmetry of the world volume theory with local gauge transformation \( U(t, \theta, z) \). Two of \( \Delta X_I \) fluctuations around the supersymmetric tube solution would act as the gauge field and the rest of them as the scalar field. However, the world volume action is more complicated than the naive noncommutative Yang-Mills theory because the tube is imbedded in the three dimensional space and also there is a nonzero background momentum. The resulting Lagrangian is not that illuminating. Thus, we focus on the small fluctuation around on the tube configuration. To derive the Lagrangian governing the dynamics of these fluctuations in the leading order, we begin with the Lagrangian in (1) and turn on again only \( X_I \) for simplicity. We now introduce gauge fields \( a_0, a_\theta, a_z, \) and \( a_\rho \) by
\[
X = x - \frac{1}{2} (ya_z + a_z y) + \frac{1}{2} (xa_\rho + a_\rho x) + O(t^3)
\]
\[
Y = y + \frac{1}{2} (xa_z + a_z x) + \frac{1}{2} (ya_\rho + a_\rho y) + O(t^3)
\]
\[
Z = z - la_\theta, \quad A_0 = z + la_0
\] (25)
In evaluating the Lagrangian, we shall count orders of fields by the noncommutativity scale \( l \) by

\[
\begin{align*}
  a_0, a_\theta, a_z, a_\rho & \sim O(l^0) \\
  [a_\theta, a_z], [a_\theta, a_\rho], [a_\rho, a_\rho] & \sim O(l^0) \\
  \partial_l & \sim O(l).
\end{align*}
\]

Especially the second line might appear unconventional but is consistent. Using \( \Theta \), the commutators become

\[
\begin{align*}
  [X, Y] & = -il^2 \rho^2 \nabla_x a_\rho + O(l^3) \\
  [X, Z] & = ily = l^2 y (\nabla_x, \nabla_\theta - ia_\rho) + il^2 x (\nabla_\theta a_\rho - a_z) + O(l^3) \\
  [Y, Z] & = ilx = -il^2 x (\nabla_x, \nabla_\theta - ia_\rho) + il^2 y (\nabla_\theta a_\rho - a_z) + O(l^3),
\end{align*}
\]

where \( \nabla_\theta \equiv \partial_\theta - i[a_\theta, \cdot], \nabla_\rho \equiv \partial_\rho - i[a_\rho, \cdot] \) and \( \nabla_z \equiv \partial_z - i[a_z, \cdot] \). One recognizes that the world-volume coordinates are twisted from the original matrix coordinate. For example, the commutator \( [X, Y] \) is not related to the z-directional magnetic field. Inserting these to the Lagrangian directly, one would get a Lagrangian where terms of \( O(l^3) \) are present and higher order terms in \( \Theta \) appear in general. Instead, we add appropriate total derivative terms first so that the leading order contribution starts with terms of \( O(l^3) \) and the higher order terms in \( \Theta \) appear only in the next leading order contributions. Following this procedure, one gets

\[
\begin{align*}
  L & = \frac{l^2}{2} \text{tr} 
  \left( (\dot{a}_\theta - l\nabla_\theta \dot{a}_0)^2 + \rho^2 (\dot{a}_z - il \nabla_x \dot{a}_0)^2 + \rho^2 (\dot{a}_\rho - il [\dot{a}_0, a_\rho])^2 - l^2 \rho^4 (\nabla_x a_\rho)^2 \\
  & \quad -2il \rho^2 (\dot{a}_\theta - l\nabla_\theta \dot{a}_0) (\nabla_x, \nabla_\theta - 2l \rho^2 (\dot{a}_\rho - il [\dot{a}_0, a_\rho]) \nabla_\theta a_\rho + 2l^2 \rho^2 \mathcal{L}_{CS} + O(l^5)) \right),
\end{align*}
\]

where \( \dot{a}_0 = a_0 + a_\theta \) and

\[
\text{tr} \mathcal{L}_{CS} = -\partial_l a_z a_\rho + a_z \partial_l a_\rho - 2l \dot{a}_0 (\partial_z a_\rho - i[a_z, a_\rho]).
\]

This is the standard form of the Chern-Simons Lagrangian with \( \partial_\rho = 0 \). As done in the scalar case, we now introduce the rotating coordinate system in \( \Theta \). Then the above Lagrangian becomes

\[
\begin{align*}
  L & = \frac{l^2}{2} \text{tr} 
  \left( -[\nabla_{\theta'}, \nabla_\theta] - \rho^2 [\nabla_{\theta'}, \nabla_z] + l^2 \rho^2 [\nabla_z, \nabla_{\theta'}] + \rho^2 (\nabla_{\theta'}a_\rho')^2 \\
  & \quad -l^2 \rho^2 (\nabla_{\theta'}a_\rho')^2 - l^2 \rho^4 (\nabla_z a_\rho')^2 + 2l^2 \rho^2 \mathcal{L}_{CS} + O(l^5) \right),
\end{align*}
\]

where \( a_{\theta'} = a_\theta, a_{z'} = a_z, a_{\rho'} = a_\rho \) and \( \nabla_{\theta'} \equiv \partial_{\theta'} - i[a_{\theta'}, \cdot] \) with \( a_{\theta'} = a_0 \). The theory contains a Chern-Simons term in addition to the conventional U(1) noncommutative gauge theory on a cylinder with the rotating coordinate \( \theta' = \theta + lt \). The Chern Simons part plays a role in finding D0 solutions in the next section. Namely the solutions of D0 excitations preserve the same supersymmetries of the tube background. If the worldvolume theory were just conventional noncommutative Yang-Mills theories, such higher supersymmetric solutions could not exist.

In the next section, we shall describe the nature of solitonic configurations arising from the tube. The configurations correspond to adding or subtracting multiple D0’s from the tube. To the given order of approximation, these solutions can be found from the above action, but we shall rather seek the solutions from the original equations of motion.
4 Tube-D0 Systems and Multiple Tubes

In this section, we shall first consider other static solutions that approach asymptotically the tube configurations. For this, one may try to solve the equations of motion in the background of the D-tube configuration concentrating on the solutions describing localized profiles. This is the approach taken in finding the noncommutative solitons in various two dimensional models \[^{[12, 13, 8, 14]}\], which may produce BPS configurations in some special cases \[^{[13, 15]}\]. Instead, we will solve directly the BPS equations while preserving the tubular boundary conditions, that is, we solve the equations in \[^{(6)}\]. In fact it is simple to construct such localized profiles on the D-tube background. First let us introduce a shift operator defined by

\[
S_m = \sum_{n=0}^{\infty} |n + m \rangle \langle n| + \sum_{n=-\infty}^{-1} |n \rangle \langle n| .
\]

It satisfies the relations

\[
S_m S_m^\dagger = I - P_m , \quad S_m^\dagger S_m = I ,
\]

where the projection operator \(P_m\) is defined by \(P_m = \sum_{a=0}^{m-1} |a \rangle \langle a|\). Then the solutions describing \(m\) D0-branes are given by

\[
X_i = S_m x_i S_m^\dagger .
\]

Unlike the case of the noncommutative Yang-Mills theory describing a planar D2-brane, the solutions we constructed here are BPS saturated states of eight supersymmetries.

If one computes the central charges corresponding to the D0 configurations, one naively gets \(\text{tr} \, C_J = l^2 \rho^2 \text{tr} \, (I - P_m)\). The difference with the value for the background indicates that the energy has been lowered. This kind of problem is not new. The noncommutative Yang-Mills theory on D2-branes for example may be related to the matrix theory \[^{[12]}\]. Then the matrix theory computation of energy for the D2-D0 system produce the result like \(\text{tr} \, (I - P_m)\). There, from the view point of the noncommutative Yang-Mills theory, the noncommutative D0 solutions are well localized configurations certainly carrying finite energy excited above the vacuum of the noncommutative Yang-Mills theory that is well defined at least classically. Moreover, the fluctuation spectra around these solutions perfectly agree with the worldsheet conformal theory analysis of the superstrings.

We expect that a similar resolution may exist for our present problem. However, as our supersymmetric vacuum \(^{[8]}\) and the excitations discussed above have the same eight supersymmetries, there should be some crucial modification in the world volume field theory. In ordinary case, we expect the excited BPS configuration to have lower number of supersymmetry than the vacuum. This difference needs a further analysis.

The more general solutions including the moduli parameters are given by

\[
X_i = S_m x_i S_m^\dagger + \sum_{a=0}^{m-1} \lambda_i^a |a \rangle \langle a| , \quad X_s = \sum_{a=0}^{m-1} \varphi_s^a |a \rangle \langle a| ,
\]

with the index \(s\) referring to the transverse scalar \(X_4\) to \(X_9\). The moduli are describing the positions of D0 branes in the 9-dimensional space. The appearance of moduli further support the view point
that the configurations are describing not holes in tubes but extra D0-branes that may even fly off the tube.

Next we like to mention briefly another type of solutions which describe many coincident tubes. The solution is given by

\[
X + iY = \rho \sum_{n=-\infty}^{\infty} \sum_{a=0}^{p-1} |(n+1)p+a\rangle\langle np+a|, \quad Z = l \sum_{n=-\infty}^{\infty} \sum_{a=0}^{p-1} n|np+a\rangle\langle np+a|,
\]

(35)

where \( p \) is a positive integer characterizing the solutions. This background makes the worldvolume theory being a \( U(p) \) noncommutative gauge theory. The \( U(p) \) basis can be constructed by writing \( |np+a\rangle\langle mp+b| = |n\rangle'|\langle m|T_{ab} \). Here \( |n\rangle' \) is interpreted as a new basis for the space while \( T_{ab} \) generates \( U(p) \) algebra. For the further details, see Ref.\[9\] where the \( U(p) \) vacuum solutions of the noncommutative gauge theory is constructed in the \( U(1) \) noncommutative gauge theory on a plane.

This type of solutions in (35) can be generalized further to describe many parallel tubes whose centers are located in arbitrary positions on the \((X,Y)\) plane. These solutions are

\[
X + iY = \sum_{n=-\infty}^{\infty} \rho_a \sum_{n=-\infty}^{\infty} |(n+1)p+a\rangle\langle np+a| + \sum_{a=0}^{p-1} \xi_a \sum_{n=-\infty}^{\infty} n|np+a\rangle\langle np+a|, \\
Z = \sum_{n=-\infty}^{\infty} l_a \sum_{n=-\infty}^{\infty} n|np+a\rangle\langle np+a|,
\]

(36)

where \( \rho_a \) is for the radius of each tube, \( l_a \) is for the noncommutative parameter of each tube, and \( \xi_a \) is for the position of the center of each tube. Again they are BPS saturated configurations preserving eight supersymmetries. The identification of the worldvolume gauge theory for these general configurations is not clear at this point, as the noncommutative parameters for tubes can be different from each other. Of course, we can add the position along the other dimensions and also excitations on each tube to get a further generalization of the above solutions.

5 Conclusion

In this note, we found BPS configurations describing tubes from the matrix model. The low energy description of the worldvolume gauge dynamics is also identified. We find soliton solutions describing many D0’s on the tube. In addition, we found BPS solutions describing many coincident tubes, whose worldvolume dynamics is \( U(p) \) noncommutative gauge theory. There are further solutions of many parallel tubes of different size and center.

As we found additional supersymmetric configurations besides the supersymmetric tubes found in Ref.\[7\]. It would be interesting to see whether its analogue exists in the Born-Infeld theory. The set of BPS equations in (3) in a static gauge appears quite simple. Also it would be interesting to see if there exist other category of BPS solutions besides those found here.

The relation between supersymmetry and the world volume dynamics of the noncommutative supersymmetric tubes needs a further consideration as discussed in the previous section. Additional
understanding of the tube dynamics may be obtained from the approach of worldsheet conformal field theory of superstrings\cite{16}. Namely as done in Ref.\cite{8} for the D2-D0, the conformal field theory description may provide detailed dynamical information on the tube-D0 systems. The results then may be compared to the fluctuation analysis around the tube-D0 configurations.

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