Opening a new window for warm dark matter

Takehiko Asaka\textsuperscript{1}, Mikhail Shaposhnikov\textsuperscript{2,3} and Alexander Kusenko\textsuperscript{4}
\textsuperscript{1}Department of Physics, Tohoku University, Sendai 980-8578, Japan
\textsuperscript{2}Institute of Theoretical Physics, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
\textsuperscript{3}Theory Division, CERN, CH-1211 Geneva 23, Switzerland
\textsuperscript{4}Department of Physics and Astronomy, University of California, Los Angeles, CA 90095-1547, USA

PACS numbers: 14.60.St, 95.35.+d

Introduction. While the Minimal Standard Model (MSM)\textsuperscript{1} of electroweak interactions is in good agreement with accelerator experiments, it is known to be incomplete in the neutrino sector. Neutrino oscillations have been observed in a number of experiments demonstrating that neutrinos have non-zero masses and mixings (for a review of the current constrains on neutrino mass matrix see, e.g.\textsuperscript{2}). In addition, the origin of matter-antimatter asymmetry and the nature of dark matter have no explanation in the framework of the MSM. It has been shown recently that all these three phenomena can be explained in a simple extension of the Standard Model achieved by adding three gauge singlet fermions, the sterile neutrinos, with masses smaller than the electroweak scale\textsuperscript{2,4}. One of these sterile neutrinos can be dark matter, while the other two allow for baryogenesis. This model was called νMSM for the addition of extra neutrinos to the MSM.

The motivation for the νMSM is further strengthened by the preponderance of indirect hints, each of which individually may not be very compelling, but which collectively make a strong case for taking the model seriously. As shown in Ref.\textsuperscript{3}, the minimal number of sterile neutrinos that can explain dark matter in the universe, while being consistent with the experimental data on neutrino oscillations, is \(N = 3\). The lightest of these three states \((N_1)\) can be produced in the early universe in just the right amount to account for cosmological dark matter. The same particle would be emitted from a supernova explosion with the anisotropy that is sufficient to explain the observed velocities of pulsars\textsuperscript{2,6}. X-ray photons from the decays of these particles can speed up the formation of molecular hydrogen and precipitate the early star formation and reionization\textsuperscript{2}, in agreement with the WMAP observations\textsuperscript{8}. The two heavier sterile neutrinos \((N_2\) and \(N_3\)) can lead\textsuperscript{2} to generation of asymmetries between sterile neutrinos and left-handed leptons via sterile neutrino oscillations\textsuperscript{9}. The asymmetry in left-handed leptons is then converted to baryon asymmetry due to the anomalous electroweak fermion number nonconservation at high temperatures\textsuperscript{10}.

Given that several seemingly unrelated phenomena are explained in the framework of a simple model, it is worthwhile to examine the full range of consequences of νMSM. It has been already shown that an absolute scale of active neutrino masses in this model can be fixed\textsuperscript{2,11}. In addition, a constraint on the rate of neutrinoless double β decay has been derived\textsuperscript{12}.

In this paper we will show that the rate of parameters for warm dark matter (WDM) in the νMSM is wider that was previously determined\textsuperscript{13,14,15,16,17,18,19,20,21,22} for dark-matter sterile neutrinos in the context of an extention of the MSM by just one sterile neutrino. In particular, we will demonstrate that the dark-matter neutrino is allowed to have a smaller mass and a larger mixing angle with active neutrinos in νMSM. This has important implications for experimental detection of dark matter. Moreover, we will argue that the dark matter sterile neutrino momentum distribution function can be more complicated than it was previously thought; this result has ramifications for structure formation in WDM cosmology. In addition, we will show that the constraints on other WDM particle candidates (such as light gravitino), which were in thermal equilibrium above the electroweak scale, are relaxed. We also point out that if the primordial production of sterile neutrinos takes place, no upper limit on their mass can be set.

WDM in the model with one sterile neutrino. Let us summarize here the constraints on mass \(M_s\) and mixing angle \(\theta\) of the WDM particle derived for the Standard Model augmented by only one sterile neutrino\textsuperscript{1}.

\begin{footnote}{1 Note that this model cannot accommodate the data on neutrino oscillations since two active neutrinos are predicted to be mass-}

\end{footnote}
This particle will be denoted by $N_1$.

First, the mixing angle $\theta$ between an active and a sterile neutrino must be small enough to avoid overclosing the universe by sterile neutrinos produced in active-sterile neutrino oscillations $^{14,15,16}$. According to $^{22}$, the corresponding limit can be presented in the form:

$$\theta < \theta_{\text{max}}(M_s) = 1.3 \times 10^{-4} \left(\frac{1 \text{ keV}}{M_s}\right)^{0.8}$$

(1)

for the central value of the parameters for the dark matter abundance $\Omega_{DM} = 0.22$ and the QCD cross-over transition temperature $T_{QCD} = 170$ MeV. For such small mixing angles, the sterile neutrinos are out of thermal equilibrium for the entire range of temperatures at which the MSM with one extra fermion can remain a good effective theory. Indeed, the maximum of sterile neutrino production occurs around the temperature $^{14}$

$$T_p \simeq 130 \left(\frac{M_s}{1 \text{ keV}}\right)^{\frac{1}{2}} \text{ MeV},$$

(2)

and these sterile neutrinos come in thermal equilibrium only if $\theta > 5 \times 10^{-4} (1 \text{ keV}/M_s)^{1/2}$.

Another limit comes from the observation $^{15}$ that radiative decays $N_1 \rightarrow \gamma\nu, \gamma\bar{\nu}$ can be observed as a feature in the spectrum of cosmic X-ray radiation. This effect is similar to the ultra-violet radiation resulting from decays of active neutrinos, proposed in $^{22}$ as a possible signal from dark matter composed of ordinary neutrinos. According to Ref. $^{14,22}$, the analysis of X-ray emission from Virgo cluster gives a constraint $\theta < 1.6 \times 10^{-3} (1\text{ keV}/M_s)^2$, for $1 \text{ keV} < M_s < 10 \text{ keV}$, whereas the study of the diffuse X-ray background $^{24}$ gives $\theta < 5.8 \times 10^{-3} (1\text{ keV}/M_s)^{5/2}$, for $1 \text{ keV} < M_s < 100 \text{ keV}$.

Further constraints on the parameters of dark-matter sterile neutrino depend on the assumptions about the initial concentration of $N_1$ at some high energy scale which we will take to be much larger than $M_W \sim 100$ GeV. We are going to distinguish three different scenarios.

I. Sterile neutrinos are not produced at high temperatures, i.e. their abundance is zero at $T > 1$ GeV. This can happen if the couplings of the singlet fermions to the fields beyond the Standard Model, including the inflaton, are sufficiently weak. In this case it is required that $\theta = \theta_{\text{max}}(M_s)$ in order to produce the right amount of dark matter $^{14,15,16,22}$. This requirement, together with an upper bound on the angle $\theta$ from the Virgo cluster observations $^{17}$, gives an upper limit on the mass, which is, according to $^{22}$, $M_s < 8 \text{ keV}$.

The sterile neutrinos produced in neutrino oscillations have a non-negligible free-streaming length. For masses below $M_{\text{min}} = 2 \text{ keV}$, the free-streaming would erase small-scale structure known to exist from cosmic microwave background radiation, Lyman-\(\alpha\) forest, and Sloan Digital Sky Survey data $^{19,20,21}$. Therefore, the allowed range of masses is $2 \text{ keV} < M_s < 8 \text{ keV}$.

II. Sterile neutrinos were in thermal equilibrium at high temperature, and their abundance at the electroweak scale coincides with that of the active neutrinos. This case is excluded since, even if the constraint $^{11}$ is satisfied, the present concentration of sterile neutrinos is only a factor $g^*(M_W)/g^*(T_d) \sim \mathcal{O}(10)$ smaller than the concentration of active neutrinos, where $g^*(T)$ is the number of effectively massless degrees of freedom at temperature $T$, $T_d \simeq 1 \text{ MeV}$ is the decoupling temperature of active neutrinos. The mass of the sterile neutrino, required in this case, $M_s \sim 100 \text{ eV}$, is at odds with the structure formation and with a very conservative Tremaine-Gunn bound $^{25}$ on the mass of any fermionic dark matter, which requires $M_s > M_{QCD} \approx 0.3 \text{ keV}$ when applied to the dwarf spheroidal galaxies $^{22,27}$.

III. The sterile neutrinos are produced at some high energy scale in just the right amount to be dark matter by some processes that have nothing to do with the active-sterile neutrino oscillations. This scenario, though quite trivial, has not been considered previously. In this case, there is no upper limit on their mass, while the lower limit is as small as $M_{QCD}$. The angle $\theta$ is bounded from above by eq. $^{11}$ and by the X-ray observations$^2$.

The mixing angles $\theta < 2 \times 10^{-4}$, required for a dark matter sterile neutrino in this model, leave little hope for a direct detection of sterile neutrinos in a laboratory experiment $^3$. The most promising detection strategy is based on the observation of X-rays from sterile neutrino decays $^{15,17,22,24}$.

Let us now discuss how these constraints are modified in the $\nu$MSM.

**The model.** The Lagrangian of the $\nu$MSM contains the Standard Model fields and gauge singlets $N_I$, as well as the following set of terms:

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{MSM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha l} \Phi T_\alpha N_I - \frac{M_f}{2} \bar{N}_I N_I + \text{h.c.},$$

(3)

where $\Phi$ and $L_\alpha (\alpha = e, \mu, \tau)$ are the Higgs and lepton doublets, respectively, $\langle \Phi \rangle = v = 174 \text{ GeV}$ is the Higgs vacuum expectation value, $M^D = F v$ and $M_f$ are Dirac

---

$^2$ For $\theta < 3 \times 10^{-6}$ the sterile neutrinos cannot explain the pulsar kicks and the early reionization.

$^3$ Of course, none of these bounds apply if the universe has undergone a low scale inflation and was never reheated to a temperature above MeV $^{28}$. Even in this case, one could envision some way to produce dark matter in the form of sterile neutrinos, for example, from a direct coupling of the inflaton to $N_1$. 

...less.
and Majorana masses, respectively. An obvious requirement for the parameters of the $\nu$MSM is that the masses of active neutrinos, given by the see-saw expression

$$m_\nu = -M_D^0 \frac{1}{M_I} [M_D^0]^T, \quad (4)$$

must be in agreement with neutrino oscillation experiments. We will assume that this model is a viable effective field theory up to some very high energy scale, which can be as large as the Planck scale, but is not smaller than $O(1)$ TeV or so, which is required for the validity of the baryogenesis mechanism via sterile neutrino oscillations. The mixing angle $\theta$ is expressed through the parameters of the $\nu$MSM as

$$\theta^2 = \frac{1}{M_s^2} \sum_{\alpha=e,\mu,\tau} |M_{s\alpha}|^2, \quad (5)$$

where $M_s$ is the lightest Majorana mass $M_I$. The masses of the heavier sterile neutrinos are free parameters of the model. To explain the baryon asymmetry of the universe, $N_2$ and $N_3$ must be highly degenerate in mass to amplify the coherent CP-violating effects in sterile neutrino oscillations. Moreover, their (common) mass $M$ cannot exceed greatly the electroweak scale since large Majorana masses enhance the rate of the lepton number non-conservation, which leads to a dilution of the baryon asymmetry.

**Constraints on the sterile neutrinos in the $\nu$MSM.** For reasons explained below we will take $M \sim O(1 - 10)$ GeV. There are limits on sterile neutrinos for masses below 400 MeV [2], but for larger masses there are no laboratory constraints.

As in the case of the Standard Model with one extra singlet fermion, to discuss the allowed range of parameters, one has to fix the concentrations of extra sterile neutrinos at temperatures higher than the electroweak scale. The analysis of all possible cases is beyond the scope of this paper. We will assume here that at least one of the heavy states $N_{2,3}$ was in thermal equilibrium at some temperature greater than $M$. This assumption is certainly true if the Yukawa coupling constants $f_I^2 = \sum_{\alpha} |F_{I\alpha}|^2$ are large enough. Heavier sterile neutrinos can be created in the Higgs decays or in collisions of $t$-quarks at the electroweak scale, and they equilibrate at $T \sim M_W$ if $f_I^2 > 2 \times 10^{-14}$ [2]. For smaller Yukawa couplings

$$f_I^2 > 10^{-19} \left( \frac{M}{1 \text{ GeV}} \right), \quad (6)$$

they equilibrate at $T_D \sim O(20)$ GeV, according to eq. [2] (the estimate [2] is valid for masses 1 GeV $< M < 10$ GeV). The Yukawa couplings even smaller than these are allowed if $N_2$ and $N_3$ have sufficiently strong additional interactions at some high energy scale.

The concentration of $N_2$ and $N_3$ at time of their decay, corresponding to temperatures $T_r \simeq O(1)$ MeV (see below) is given by

$$n_{2,3} = \frac{g^*(T_r)}{g^*(T_D)} \frac{6\zeta(3)g_N^*}{7\pi^2} T^3, \quad (7)$$

where $g_N^* = 2 \times 2 \frac{3}{5}$ is the degeneracy factor for $N_{2,3}$, and we take $g^*(T_r) = 10.75$ and $g^*(T_D) = 86.25$, which accounts for quark degrees of freedom up to the $b$ quark. This equation is valid, provided eq. (6) is satisfied. For smaller Yukawa couplings, according to our assumption, one should replace $g^*(T_D)$ by $g^*(M_W) \simeq 106.75$.

The sterile neutrinos $N_{2,3}$ are unstable and can decay in many channels, depending on their mass. In general, $N_{2,3}$ decays lead to the entropy production, given by a factor $S > 1$. If the decay temperature is smaller than the temperature at which the dark-matter sterile neutrino $N_1$ is produced, the density of $N_1$ is diluted by $S$ and also the momentum distribution is redshifted by a factor $S^3$. This dilution allows for a wider range of allowed masses, with the lower bound $M_s > M_{\text{min}}/S^3$. In addition to this, the constraint on the mixing angle $\theta^2$ becomes weaker by a factor $S$. Moreover, at sufficiently large $S$ even an equilibrium concentration of sterile neutrino $N_1$ can be sufficiently diluted. This means that the mixing angle is limited only by the X-ray constraints [17, 24] discussed above.

Let us show that the entropy production factor as large as $S \sim O(100)$ does not affect the predictions of the Big Bang nucleosynthesis (BBN).

The ratio of entropy produced from the $N_{2,3}$ decays to the rest of the entropy is $S = [g^*(R)^3]_{\text{final}}/[g^*(R)^3]_{\text{initial}}$, where $R$ is the scale factor. The value of $S$ depends on the dimensionless combination $A = \Gamma M_0 (g^*(T_D)/M)^2$, were $\Gamma$ is the width of the decaying particle, and $M_0 = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. It was computed in [30] for different situations. For our case it is given by

$$S \simeq \left[ 1 + (1.37/A)^{2/3} \right]^{3/4}. \quad (8)$$

To find what maximal allowed value of $S$, the Big Bang nucleosynthesis should be reanalyzed in the presence of $N_{2,3}$ decays, taking into account the specific branching ratios for different final states. A similar problem has been addressed in a number of papers [32, 33, 34, 35]. In these papers the initial state of the universe before the BBN was assumed to contain nothing but a decaying scalar field, which produced particles of the Standard Model and created all the entropy. In other words, the factor $S$ was assumed to be very large. In our case $N_{2,3}$ decay in plasma, and, therefore, the constraints must be weaker. We will take a conservative approach and adopt the constraints from Refs. [32, 33, 34, 35], even though they are too strong for our case.
According to \[3\], the reheating temperature, defined as \(T_r = 0.554 \sqrt{\frac{M_{\nu}}{10}}\), can be as small as 0.7 MeV without a conflict with BBN. In this case the effective number of active neutrino degrees of freedom at the time of BBN is less than three. There is some indication that this may, indeed, be required for the BBN predictions to agree with observations.\[3\] A more stringent constraint \(T_r > 4\) MeV was derived in Ref. \[3\], where the information about CMB anisotropies and matter distributions was incorporated into the analysis. We will present our results for both \(T_r = 0.7\) MeV and \(T_r = 4\) MeV.

Since the dilution of dark matter depends on the parameter \(A\), let us determine the allowed range of decay widths \(\Gamma_2\) and \(\Gamma_3\) of the heavy sterile neutrinos \(N_2\) and \(N_3\), respectively. The neutral current diagrams (\(Z\) exchange) give the decay modes \(N_{2,3} \rightarrow \nu \nu, \nu l (l = \mu, e, \tau), \bar{\nu} l, \bar{\nu} \bar{\nu} l l\) plus charge conjugated channels, while the charged currents (\(W\) exchange) channels produce \(l q \bar{q}\) in the final state (here \(q\) corresponds to quarks and \(l\) to charged leptons). In general, these decay widths are given by

\[
\Gamma_I = \frac{G_F^2 M^3}{192\pi^2} \sum_a A_a |M_{aI}|^2,
\]

where \(G_F\) is the Fermi constant, and the coefficients \(A_a \sim 1\) depend on the kinematically allowed channels. For example, for a 10 GeV sterile neutrino one can show that \(\Gamma_I < 22G_F^2 M^3 f^2_{3\text{sol}} v^2/(192\pi^3)\).

The smaller is the Yukawa coupling \(f_I\), the more entropy production one can expect. However, not all choices of the Yukawa couplings are consistent with the data on neutrino oscillations, in particular with the mass scale differences. Applying the general analysis of the see-saw formula made in \[3\], one can prove that it is impossible to have very small \(f^2_{3\text{sol}} \ll f^2_{2\text{sol}}\) and to reproduce the data (here \(f^2_{3\text{sol}} = \sqrt{\Delta m_{23}^2 M/v^2} \approx 3 \times 10^{-9} M/\text{GeV} = \sqrt{\Delta m_{23}^2 \approx 9 \times 10^{-3} \text{eV}}\). In other words, only one of the particles \(N_{2,3}\) can reheat the universe considerably. We take it to be \(N_3\) for definiteness and choose \(f^2_{3} \ll f^2_{2\text{sol}}\). There is no lower bound on \(f^2_{2}\) from experiment. However, in order to reproduce the observed mass differences one must have \(f^2_{2}\sqrt{\Delta m_{23}^2} M < f^2_{2\text{sol}}\sqrt{\Delta m_{23}^2} M\) of the order of \(\sqrt{\Delta m_{23}^2} \approx 10^{-2} \text{eV}\). Moreover, repeating the arguments of Ref. \[3\], one can prove that for such a small value of \(f_2\) the lightest active neutrino must be much lighter than \(\sqrt{\Delta m_{23}^2}\). Hence, the predictions of the active neutrino masses made in Ref. \[3\] remain valid\[4\].

Now we are ready to proceed to numerical estimates of the entropy generation factor \(S\). Taking the Yukawa coupling from eq. \[6\], we get that for \(M = 5\) (11) GeV and \(S = 29\) (10) for the reheating temperature 0.7 (4) MeV. In this case the heavy sterile neutrino decoupling temperature is \(O(20)\) GeV. At temperature \(O(1)\) MeV, the energy density of sterile neutrinos dominates by a factor 10 or 4, respectively. For smaller Yukawa couplings, \(N_3\) is out of thermal equilibrium for all temperatures in the \(\nu\)MSM. Of course, the \(N_3\) abundance can be large due to some physics beyond the \(\nu\)MSM. Then, for \(\Theta < 10^{-10}\) the entropy production factor can exceed \(S = 100\), with the energy dominance factor greater than 20.

This range of \(S\) opens new possibilities for the sterile neutrino production. First, \(N_1\) can be produced from neutrino oscillations, as in Refs. \[13\]-\[16\], \[22\], but for some larger mixing angles \(\theta\). As long as the lightest neutrinos are never in equilibrium in the early universe, the amount of dark matter produced from neutrino oscillations is proportional to \(\theta^2\).\[14\]. Now, the admissible angle can be larger than \(\Theta\) by a factor of \(\sqrt{S}\).

The second possibility is an even more drastic departure from the usual scenario. The lightest sterile neutrinos could, in fact, achieve the equilibrium density in the early universe due to some high-scale physics, new interactions, or because the mixing angle \(\theta\) is large enough. Then at some temperature \(T_{d, s}\) sterile neutrinos freeze out from equilibrium. Their density would then be

\[
\Omega_s = 0.26 \left(\frac{M_\nu}{12 \text{eV}}\right) \left(\frac{1}{S}\right) \left(\frac{g^*(T_d)}{g^*(T_{d, s})}\right).
\]

This gives the right dark matter density for \(M_\nu \sim 2(10)\) GeV. In the absence of entropy production, masses below 2 GeV correspond to dark matter that is too warm. Moreover, the dilution and red shift also change the free-streaming length of the sterile neutrinos by a factor \(S^{1/3}\). Let us, therefore, consider the limits from large-scale structure in more detail.

Some bounds on warm dark matter come from the observation of small-scale structure in the Lyman-\(\alpha\) forest, as well as from SDSS data. The small-scale structure is sensitive to the free-streaming length of sterile neutrinos, \(\lambda_{FS} = f^0_1(v(t')/R(t')) dt'\), where \(v\) is the velocity of sterile neutrinos. For the parameters of interest, the lightest sterile neutrino becomes non-relativistic at the time \(t_{NR}\) (redshift \(z_{NR}\)) long after the decay of the heavier sterile neutrinos and before the matter-radiation equality, which occurs at \(t_{eq}\). Therefore, one can neglect the contribution to \(\lambda_{FS}\) from the matter-dominated phase prior to heavy neutrino decay, and the integral for the free-streaming length has the usual expression:

\[
\lambda_{FS} \approx t_{NR} z_{NR} [2 + \ln(t_{eq}/t_{NR})]
\sim 2 \text{Mpc} \left(\frac{\text{keV}}{M_\nu}\right) \left(\frac{T_s}{T_{eq}}\right) \left(\frac{1}{s}\right)^{1/3},
\]

where \(T_s\) is the temperature of the dark-matter sterile
neutrinos in the absence of dilution and $T_\nu$ is the temperature of active neutrinos.

For a numerical example let us assume that the Lyman-$\alpha$ constraint in the absence of dilution is $M_s > M_{\text{min}} \simeq 2$ keV [20]. For Scenario I we have $T_s \simeq T_\nu$, and for $S \simeq 13$ all masses $M_s > 0.8$ keV are allowed. In this case the sterile neutrinos are out of thermal equilibrium and their mixing angle is smaller than $\theta < 5.4 \times 10^{-4}$.

For larger mixing angles sterile neutrinos come to thermal equilibrium and can be considered as the “generic warm dark matter”.

The maximal possible mixing angle in this case comes from consideration of active neutrino masses and is of order of $\theta^2 \sim \sqrt{\Delta m^2_{\text{atm}}/M_{\text{min}}} \simeq 8 \times 10^{-5}$ where $M_{\text{min}} = 0.55$ keV so that the free streaming length becomes too small to affect the observed structure [20]. Then, in this case, sterile neutrino can be the dark matter for $(g^*(T_{d,s})/g^*(T_\nu))S \simeq 55$.

We, therefore, conclude that, for large enough $S$, sterile neutrinos can be the dark matter for masses $M_s > 0.55$ keV and mixing angles $\theta > \theta_{\text{max}}(M_s)$. These mixing angles are probably within the reach of laboratory experiments [28].

The late entropy production opens a possibility to have several component WDM even with a single dark matter candidate. Namely, a fraction of sterile neutrinos could be created above the electroweak scale, and then a separate population of them could be produced in active–sterile neutrino oscillations. The two populations can have effective temperatures different by a factor of $S^{1/2}$, leaving a fingerprint on a distribution of matter at scales of the order of a few Mpc.

Some comments are now in order. (i) The dilution of the sterile neutrino abundance leads to the decrease of the baryon asymmetry at the same time. This effect, however, can be compensated by an increase in the degeneracy of the states $N_2$ and $N_3$. (ii) The mixing angle $\theta$ is still bounded from above by the X-ray observations [17, 24]. These limits, however, are very weak for $M_s < 1$ keV, as discussed in Ref. [28]. (iii) The large value of the total entropy production factor in the $\nu$MSM opens a possibility to have any type of a warm dark matter candidate with mass being larger than 0.55 keV with decoupling temperature above the electroweak scale, including the light gravitino or other weakly interacting massive particles [21]. (iv) The scenario III, in which the dark matter sterile neutrinos are produced not in the active–sterile transitions but in some other processes, is valid for the $\nu$MSM as well. In this case the mixing angle $\theta$ is constrained by the X-ray observations, while the mass $M_s$ should satisfy the Tremaine-Gunn bound or the Lyman-$\alpha$ bound, depending on the mechanism of their production.

Conclusions. To summarize, in the Standard Model extended by three singlet fermions ($\nu$MSM), the lightest sterile neutrino is a viable dark-matter candidate. For the case when the initial concentration of sterile neutrinos is assumed to be zero, the main reason allowing for a new window for WDM parameters is the decay of a heavier neutrino before the BBN epoch. These decays can produce enough entropy to (i) dilute the density of sterile neutrinos and (ii) reduce their temperature. The second effect, cooling of the sterile neutrino spectrum, reduces the free-streaming length and weakens the bounds from structure formation on non-thermal sterile neutrino. In addition, a sterile neutrino with any mass satisfying the Lyman-$\alpha$ bound, that was produced by a mechanism not related to active–sterile oscillations, is perfectly allowed.

As has been already discussed in [3, 4, 11], the $\nu$MSM with three singlet fermions can be falsified by the data in neutrino physics. In spite of the efforts of two authors of the present paper (T.A. and M.S.) a range of parameters in which the $\nu$MSM can give the dark matter, explain the baryon asymmetry of the Universe, and the data on neutrino oscillations including the LSND anomaly, has not been found. The LSND anomaly can be accommodated to the $\nu$MSM, provided the dark matter or the baryon asymmetry explanations are given up. Thus, if MiniBoone experiment confirms the LSND results, the $\nu$MSM should be extended by at least one extra sterile neutrino, if the requirements to be consistent with cosmology are kept. The same conclusion is true if the active neutrinos are found to be degenerate in mass.

The work of A.K. was supported in part by the DOE grant DE-FG03-91ER40662 and by the NASA ATP grants NAG 5-10842 and NAG 5-13399. The work of T.A. was supported by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture of Japan, No. 16081202. The work of M.S. was supported in part by the Swiss Science Foundation. M.S. thanks Alexey Boyarsky and Oleg Ruchayskiy for many helpful discussions and to the UCLA, where part of this work was done, for hospitality. We thank Oleg Ruchayskiy for kindly providing us a figure with constraints on the properties of sterile neutrino.

Note added. After this work has been submitted for publication, a number of papers on sterile neutrinos appeared [37, 38, 39, 40, 11]. In [37] a new analysis of the Lyman-$\alpha$ forest data has been performed. In [38, 39, 40] new constraints on the mixing angle $\theta$, coming from the analysis of X-rays from galaxy clusters and from our own galaxy have been derived. These results represent a considerable improvement of the constraints existing previously and discussed in the text.

In Fig. 11 we show the limits from [39], based on the statistical analysis of the X-ray data with the method described in [24], and the constraints from [37]. The limits on the mixing angle, derived in [40], are coming from the requirement that the flux in the dark matter emission line is smaller than the total observed flux. These constraints are weaker than those of ref. [39] and allow
more space for sterile neutrinos, as discussed in [41].

A combination of new X-ray and Lyman-\(\alpha\) bounds can rule out a number of scenarios discussed in this paper, unless eq. (1), derived in [22], gets strongly modified by largely unknown dynamics of the strongly interacting plasma at temperatures \(T \sim 150\) MeV or the bound of [37], \(M_s > m_0/S_{\text{tot}}^{1/3}\) with \(m_0 \approx 14\) keV, involving complicated hydrodynamical simulations, gets relaxed because of some reason. Here factor \(S_{\text{tot}}\) is defined as

\[
S_{\text{tot}} = \frac{(g^*(T_{d,s})/g^*(T_d))}{S}
\]

If \(m_0\) is as large as in [37] (\(m_0 \approx 2\) keV according to the earlier works, as discussed in the text), the Dodelson-Widrow scenario (scenario I of this work), is excluded. Indeed, if eq. (1) is combined with the X-ray data, the upper limit of 8 keV, discussed in the text, is replaced by 2.8 keV [31], which is well below the Lyman-\(\alpha\) bound. The scenario II, which was impossible for the MSM with addition of one sterile neutrino and possible for the \(\nu\)MSM, would require to have the entropy production factor greater than \(S \approx 25\) and sterile neutrino mass greater than \(m_s \approx 2.5\) keV. On the contrary, if the concentration of the dark matter sterile neutrino is postulated to be zero at temperatures above few hundreds MeV, then even the \(\nu\)MSM with entropy production cannot reconcile the new constraints. In particular, the scenario with large mixing angle is excluded (it would be allowed if \(m_0 \approx 5\) keV). If true, this points out towards the production mechanism of sterile neutrinos, not related to active-sterile neutrino oscillations (scenario III of the present work). Indeed, in [41] was demonstrated that the inflaton decays can produce effectively the dark matter sterile neutrinos which satisfies all the constraints of [37, 38, 39, 40].

[1] S. L. Glashow, Nucl. Phys. 22 (1961) 579 ; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264 ; Salam, A., in Elementary Particle Physics ed. Svartholm, N.; Almqvist and Wiksell; Stockholm; 1968; S. L. Glashow, Nucl. Phys. 22 (1961) 579 ; S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2 (1970) 1285.
[2] A. Strumia and F. Vissani, Nucl. Phys. B 726 (2005) 294.
[3] T. Asaka, S. Blanchet and M. Shaposhnikov, Phys. Lett. B 631 (2005) 151.
[4] T. Asaka and M. Shaposhnikov, Phys. Lett. B 620 (2005) 17.
[5] A. Kusenko and G. Segrè, Phys. Lett. B 396 (1997) 197; M. Barkovich, J. C. D’Olivo and R. Montemayor, Phys. Rev. D 70 (2004) 043005.
[6] G. M. Fuller, A. Kusenko, I. Mocioiu, and S. Pascoli, Phys. Rev. D 68 (2003) 103002.
[7] P. L. Biermann and A. Kusenko, Phys. Rev. Lett. 96 (2006) 091301.
[8] A. Kogut et al., Astrophys. J. Suppl. 148 (2003) 161.
[9] E. K. Akhmedov, V. A. Rubakov and A. Y. Smirnov, Phys. Rev. Lett. 81 (1998) 1359.
[10] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.
[11] A. Boyarsky, A. Neronov, O. Ruchayskiy and M. Shaposhnikov, Pis’ma v ZhETF 83 (2006) 165 arXiv:hep-ph/0601098.
[12] F. Bezrukov, Phys. Rev. D 72 (2005) 071303.
[13] K. A. Olive and M. S. Turner, Phys. Rev. D 25 (1982) 213.
[14] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. 72 (1994) 17.
[15] A. D. Dolgov and S. H. Hansen, Astropart. Phys. 16 (2002) 339.
[16] K. Abazajian, G. M. Fuller and M. Patel, Phys. Rev. D 64 (2001) 023501.
[17] K. Abazajian, G. M. Fuller and W. H. Tucker, Astrophys. J. 562 (2001) 593.
[18] K. N. Abazajian and G. M. Fuller, Phys. Rev. D 66 (2002) 023526.
[19] S. H. Hansen, J. Lesgourgues, S. Pastor and J. Silk, Mon. Not. Roy. Astron. Soc. 333 (2002) 544.
[20] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. D 71 (2005) 063534.
[21] K. Abazajian, Phys. Rev. D 73 (2006) 063513.
[22] K. Abazajian, Phys. Rev. D 73 (2006) 063506.
[23] A. De Rujula and S. L. Glashow, Phys. Rev. Lett. 45 (1980) 942.
[24] A. Boyarsky, A. Neronov, O. Ruchayskiy and M. Shaposhnikov, arXiv:astro-ph/0612509.
[25] S. Tremaine and J. E. Gunn, Phys. Rev. Lett. 42 (1979) 407.
[26] D. N. C. Lin and S. M. Faber, Astrophys. J. 266 (1983)
[27] J. J. Dalcanton and C. J. Hogan, Astrophys. J. 561 (2001) 35.
[28] G. Gelmini, S. Palomares-Ruiz and S. Pascoli, Phys. Rev. Lett. 93 (2004) 081302.
[29] A. D. Dolgov, S. H. Hansen, G. Raffelt and D. V. Semikoz, Nucl. Phys. B 590 (2000) 562; A. Kusenko, S. Pascoli and D. Semikoz, JHEP 0511 (2005) 028.
[30] R. J. Scherrer and M. S. Turner, Phys. Rev. D 31 (1985) 681.
[31] J. McDonald, Phys. Rev. D 43 (1991) 1063.
[32] M. Kawasaki, K. Kohri and N. Sugiyama, Phys. Rev. Lett. 82 (1999) 4168.
[33] M. Kawasaki, K. Kohri and N. Sugiyama, Phys. Rev. D 62 (2000) 023506.
[34] S. Hannestad, Phys. Rev. D 70 (2004) 043506.
[35] K. Ichikawa, M. Kawasaki and F. Takahashi, Phys. Rev. D 72 (2005) 043522.
[36] G. Steigman, Int. J. Mod. Phys. E 15 (2006) 1.
[37] U. Seljak, A. Makarov, P. McDonald and H. Trac, arXiv:astro-ph/0602430
[38] A. Boyarsky, A. Neronov, O. Ruchayskiy and M. Shaposhnikov, arXiv:astro-ph/0603368
[39] A. Boyarsky, A. Neronov, O. Ruchayskiy, M. Shaposhnikov and I. Tkachev, arXiv:astro-ph/0603660
[40] S. Riemer-Sorensen, S. H. Hansen and K. Pedersen, arXiv:astro-ph/0603661
[41] M. Shaposhnikov and I. Tkachev, arXiv:hep-ph/0604236