Mathematical Modeling of the Linear Stationary Process of Drying with Accuracy of Evaporation and Temperature Relation on the Surface of the Layer in Infrared Heating

Jasur S*, Shaxnoza S and Gani D
Tashkent State Technical University, Uzbekistan

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Corresponding author: Jasur S, Tashkent State Technical University, Tashkent 100095, Republic of Uzbekistan, Email: jasursafarov@yahoo.com

Abstract
The interaction of surface evaporation and the movement of moisture in the inner layers together with changes in the temperature field is a well-known task. In this paper, explicit finite formulas are obtained the behavior of this system under infrared irradiation for stationary problems. The obtained results of the interaction of the temperature field and the humidity field against the background of the irradiation surface by infrared rays.

Keywords: Drying process; Model; Infrared heating; Temperature; Evaporation

Introduction
At certain stages of drying, there comes a period for the stationary of drying processes, this is due to the fact that the thin time layer of moisture and temperature transition is small compared to the total drying time [1,2].

It is known that the boundary conditions on the surface of the dewatered material are complex, since the surface temperature and the moisture concentration on the surface with increasing their value increase the evaporation process [3,4]. On the other hand, surface evaporation, by lowering the temperature and humidity of a thin layer of the surface, affects the internal processes of redistribution of humidity and temperature [1-4]. In known works (Voronezh, Oka Lync). Such a circumstance was not investigated because of the nonlinear interaction of temperature and humidity with the help of analytical studies.

Materials and Methods
We have equations for a plane isotropic material

\[ C_s T_x - \varepsilon W_x = \lambda T_x - C[D W_x + D T_x] + Q \]
\[ (1 - \varepsilon) W_x = (D W_x + D T_x) \]

(1)

Coefficients \( C_s, \varepsilon, \lambda, C, D, D_s \) – constant characterizing properties of materials when they are isotropic.

We will seek the stationary solution (1), provided that, the rate of change of temperature and humidity are constant in time \( t \). This allows us to investigate (1) in the form:

\[ \lambda T_x - C[D W_x + D T_x] + Q = 0 \]
\[ D W_x + D T_x = A \]

(2)

Then, from (2) where \( A \) is a function dependent only on \( t \), which is obtained by integrating the second equation (2)

From (2) we have:

\[ \lambda T_x - C A T_x + Q = 0 \]
\[ D W + D T_x = Ax + B \]

(3)

This problem is interesting in that the boundary conditions are mutually related, i.e.

\[ \left[ KT_x \right] = q_x T + q_x W \]
\[ \left[ KT_x \right] \]  

In this case, as the origin, the plane was chosen at \( x = 0 \), and...
as the surface was chosen the plane located on the upper part of the material layer.

In (4), \( K \) is the coefficient of thermal conductivity, \( q_T, q_w \) - coefficients that characterize the amounts of heat flux from the surface and its relationship to the moisture flux and the surface temperature of the drying material. Let the surface be heated with the power of irradiation of infrared waves, in the form

\[
\lambda = Q_0 e^{-\alpha t(l-x)}.
\]

Where \( \lambda \) - the intensity ratio of the depth attenuation.

Consider the first equation (3), where the time enters the con parameter, so in this expression there are no time-produced, therefore, the integral with respect to \( x \), the function in the interval variation of \( x \), we can consider both a constant coefficient.

So,

\[
T_x = -\frac{CA}{\lambda} T_s = -Q_0e^{-\alpha t(l-x)} \tag{5}
\]

As a result, we have,

\[
T = -\frac{C_0}{\omega} + C_1 e^{\alpha x} - \frac{e^{\alpha x} Q}{\lambda - \omega} e^{\lambda x} \tag{6}
\]

Where \( \omega = \frac{CA}{\lambda} \)

Now, to determine \( C_0 \) and \( C_1 \), which are constants of integration of the function (6), subordinate (agree) with the boundary conditions (4)

\[
K \left[ C_0 \omega + \frac{e^{\alpha x}}{\lambda - \omega} \right] = 0
\]

From here

\[
C_1 = -\frac{e^{\alpha x}}{\lambda - \omega} \tag{7}
\]

Now we use the first equation for \( x = \ell \) in (4), i.e.

\[
K \left[ C_0 e^{\alpha x} + \frac{e^{\alpha x}}{\lambda - \omega} \right] = q_\ell \left[ -\frac{C_0}{\omega} + C_1 e^{\alpha x} + \frac{e^{\alpha x}}{\lambda - \omega} \right] + q W_x(t) \tag{8}
\]

Here, there are connections for the flow of moisture

But from (3) we have

\[
W_x = \frac{A}{D} - \frac{D_T}{D} T_s \tag{9}
\]

Substituting this into (8) with \( x = \ell \)

We have

\[
Ke^{\alpha x} \left[ C_0 + \frac{e^{\alpha x}}{\lambda - \omega} \right] = q_\ell \left[ -\frac{C_0}{\omega} + C_1 e^{\alpha x} + \frac{e^{\alpha x}}{\lambda - \omega} \right] + q \left[ \frac{A}{D} - \frac{D_T}{D} \right] \tag{10}
\]

But from (6), \( T(t) = \omega C_1 e^{\alpha x} + \frac{e^{\alpha x}}{\lambda - \omega} \tag{11} \)

From (10) and (11) we have,

\[
C_0 = \omega C_1 e^{\alpha x} + \frac{e^{\alpha x}}{\lambda - \omega} \tag{12}
\]

We integrate equation (5)

\[
\left\{ \begin{aligned}
T_x = -\frac{CA}{\lambda} T_s &= -Q_0 \int e^{-\alpha t(l-x)} dx = -Q_0 \int e^{-\lambda x} dx \\
T_s = -\frac{CA}{\lambda} T &= +Q_0 e^{-\lambda x} \frac{e^{\alpha x}}{\lambda} + C_0
\end{aligned} \right. \tag{13}
\]

Where \( C_0 \) - is a certain integration constant

We introduce \( T = ye^{\alpha x} \) from (13) have:

\[
T = C_1 e^{\alpha x} \frac{1}{\lambda (\omega - \ell)} + C_0 + e^{\alpha x} \tag{14}
\]

Where \( C_1 \) - also the integration constant \( \omega = \frac{CA}{\lambda} \)

To determine the constants \( C_1 \) and \( C_2 \), we use the boundary conditions of the problem

\[
KQ_\ell e^{\alpha x} - \omega Q_\ell e^{\alpha x} + KQ_\ell e^{\alpha x} = q \left[ Q_\ell e^{\alpha x} + \frac{e^{\alpha x}}{\lambda} \right] \eta (W_x)] \tag{15}
\]

From the second equation (4), have:

\[
Q_\ell e^{\alpha x} \frac{1}{\lambda (\omega - \ell)} + C_0 + oC_1 = 0
\]

Hence we find

\[
C_0 = -Q_\ell e^{\alpha x} \frac{1}{\lambda (\omega - \ell)} - oC_1 \tag{16}
\]

From (15) and (16), we find \( C_1 \), for this, still a manifestation of the attitude (15), where from the second equation with \( x = \ell \) have,

\[
DW_x \bigg|_{x=\ell} + D_T T_x \bigg|_{x=\ell} = A \tag{17}
\]

From here

\[
W_x = \frac{A}{D} - \frac{D_T}{D} T_s, \text{ this equality is substituted in equation (15), and we have } C_1 = \frac{L_1}{L_2}
\]

where

\[
L_1 = e^{\alpha x} \left[ k \frac{q_\ell \eta (W_x)]}{x (\omega - \ell)} e^{\alpha x} + \frac{q_\ell \eta (W_x)]}{x (\omega - \ell)} e^{\alpha x} \right], \frac{q_\ell \eta (W_x)]}{x (\omega - \ell)} e^{\alpha x} + \frac{A}{D} e^{\alpha x} + \frac{D_T}{D} e^{\alpha x} \tag{18}
\]

We have a concrete meaning \( C_1 \) and \( C_2 \).

Now the function is uniquely determined by the solution

From the second equation (2):

\[
W = \frac{A}{D} + \frac{A}{D} \frac{D_T}{D} \tag{19}
\]
This determines the distribution of moisture during the drying process. The amount of moisture inside is determined by integrating over the layer:

\[ W = \int_{0}^{\ell} W dt = \frac{1}{D} \int_{0}^{\ell} \left( T - \frac{D}{\lambda} \right) dx \]

\[ (20) \]

Where, \( T \) - an explicit function of \( x \), from (14).

**Conclusion**

As can be seen from (20), increasing the layer thickness by moisture transfer affects more than the heat transfer process. Obtained explicit formulas also show (14, 20) this moisture distribution is more dependent on the thermal conductivity than the temperature distribution. The obtained results of the interaction of the temperature field and the humidity field against the background of the irradiation surface by infrared rays.

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