New Decoupled 2-Dof Parallel Mechanism with Fully Spherical Workspace

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Abstract. Decoupled 2-dof spherical parallel mechanism is an important valued spherical parallel mechanism. It is one of the preferred solutions in the field of large-angle spherical workspaces. By studying the configuration of the exiting kinematic decoupled spherical parallel mechanism, it is found that the key issues that it cannot achieve a fully spherical workspace is that the output angle range of the “linear input – rotation output” kinematic chain cannot reach 180°. Based on the input and output characteristics of the double rocker mechanism, two new ideas for constructing the configuration of the “linear input – rotation output” kinematic chain are presented. The output angle range of the new “linear input – rotation output” kinematic chain can reach 180°. Based on the new design idea for “linear input – rotation output” kinematic chain, two new kinematic chains are designed: PSR and PRR. Based on the above kinematic chains, two 2-dof decoupled parallel mechanism with a fully spherical workspace are designed, i.e. RR&P5R parallel mechanism and RR&PRR parallel mechanism.

1. Introduction

The spherical parallel mechanism is the most typical spatial positioning mechanism, which has its own unique superior value in the application of spherical workspace. A spherical parallel mechanism can be one of the preferred solutions in some practical applications of actual spherical workspace requirements [1-3], such as bionic joints [4-6], positional pointing [7-8], and bionic eyes [9-10] etc. However, due to the complexity of the traditional spherical parallel mechanism structure, there was still the problem of a relatively small workspace, maximization of the workspace has always been a challenging subject [11].

Decoupled 2-dof spherical parallel mechanism has been widely used in the field of spherical workspace due to its simple structure and convenient control. Figure 1 shows the general geometry of a 2-dof spherical decoupled parallel mechanism [12], which consists of two kinematic chains: (1) the rotary chain, which consists of the revolute joint R₁ and R₂; (2) the linear input chain, which consists of a prismatic pair P and revolute joints R₃ and R₄. The axes of the revolute joints R₁ and R₂ are orthogonal to each other, and the moving platform rotates around the revolute joints R₁ and R₂. Thus, the mechanism has decoupling characteristics. The existing spherical parallel mechanism has two kinds of workspace ranges: one is that the upper and lower ends both have unreachable working areas, and the other is that only one end has an unreachable working area.
2. Idea for Type Design of Decoupled Parallel Mechanism with Fully Spherical Workspace

In previous studies, decoupled 2-dof spherical parallel mechanisms could not form a fully spherical workspace, since the linear input chain output angle range was less than $[0, 180^\circ]$. Due to the rotary input chain always achieve $360^\circ$ rotation, therefore, if the problem of the linear input chain achieving an output range greater than $[0, 180^\circ]$ is solved, the decoupled parallel mechanism can form a fully spherical workspace.

As shown in Figure 2, when a longer member CD is used as an input and a shorter member AB is used as an output, the double rocker mechanism can achieve a larger output angle $\beta$ at a smaller input angle $\alpha$, i.e. the theoretical output angle $\beta$ can be greater than $180^\circ$.

In order to meet the requirements of the linear input chain of the decoupled 2-dof parallel mechanism, the above double rocker mechanism can be modified to realize a linear displacement input and a rotational displacement output. The specific variants were as follows:

1) The input member CD of the double rocker mechanism was connected to the crank slider mechanism to convert the rotation input into a linear input. A linear input chain based on the double rocker mechanism was obtained, i.e. P5R kinematic chain, as shown in Figure 3.
Assuming that the output angle of the double rocker mechanism is $\theta_1$, the input angle is $\theta_2$, and the radius of the friction circle at point A is $\rho$. In order to ensure that the range of the output angle $\theta_1$ is greater than 180°, the line where the member BC is located cannot intersect the friction circle at the point A during the movement. According to the dot-line distance formula, the following conditions were obtained.

$$\left| \frac{l_1 \sin \theta_2 (l_1 \cos \theta_2 - l_3 \cos \theta_1 - l_4) - (l_1 \sin \theta_2 - l_3 \sin \theta_1)(l_1 \cos \theta_1 + l_4)}{\sqrt{\left(l_1 \cos \theta_1 + l_4 - l_1 \sin \theta_1 \sin \theta_2 \right)^2 + \left(l_1 \sin \theta_2 - l_3 \sin \theta_1 \right)^2}} \right| - \rho > 0 \quad (1)$$

(2) If the input member CD tends to be infinitely long, and the point D is at infinity, the motion trajectory of the point C is converted into a straight line by the arc, and the double rocker mechanism is converted into the offset crank slider mechanism. A linear input chain based on the offset crank slider was obtained, i.e. PRR kinematic chain, as shown in Figure 4.

Assuming that the output angle of the offset crank slider is $\theta$, the input amount of the prismatic pair P is $q$. The distance between the guide path of the prismatic pair P and the point A in the vertical direction is e, and the radius of the friction circle at the point A is $\rho$. Similarly, in order to ensure that the range of the output angle $\theta$ is greater than 180°, the line where the member BC is located cannot intersect the friction circle at the point A during the movement. According to the dot-line distance formula, the following conditions were obtained.

$$\left| \frac{l_1 \cdot (q \sin \theta - e \cos \theta)}{\sqrt{(e - l_1 \sin \theta)^2 + (q - l_1 \cos \theta)^2}} \right| - \rho > 0 \quad (2)$$
3. Analysis of Mechanism Position

3.1. RR&P5R Mechanism

Figure 5 shows the decoupled RR&P5R 2-dof parallel mechanism with a fully spherical workspace based on the P5R kinematic chain. The input amount of the revolute joint $R_1$ is $q_1$, and the input amount of the prismatic pair $P$ is $q_2$. The position of the revolute joint $R_7$ on the member $L_3$ is represented by $k$, and the value range is $(0, 1)$.

Thus, the component geometry can be expressed as:

\[ l_2^2 = (l_4 \cos \theta_3 + l_4 - l_i \cos \theta_2)^2 + (l_i \sin \theta_3 - l_i \sin \theta_2)^2 \]  

(3)

\[ q_2 = kl_i \sin \theta_3 - \sqrt{l_i^2 - (kl_i \cos \theta_3 + l_i - e)^2} \]  

(4)

3.1.1. Forward solution of mechanism position

When the input quantities $q_1$, $q_2$ and the structure size were determined, the output angular displacements $\theta_1$ and $\theta_2$ were the forward position solution. According to Eqs. (3) and (4), the forward solution of the RR&P5R mechanism position can be calculated by:

\[
\begin{align*}
\theta_1 &= q_1 \\
\theta_2 &= \arcsin \frac{C^2 + D^2 + l_2^2 - l_2^2}{2l_1 \sqrt{C^2 + D^2}} - \arcsin \frac{C}{\sqrt{C^2 + D^2}} \\
\theta_3 &= \arcsin \frac{k^2 l_i^2 + q_2^2 + (l_4 - e)^2 - l_i^2}{2kl_i \sqrt{q_2^2 + (l_4 - e)^2}} - \arcsin \frac{l_4 - e}{\sqrt{q_2^2 + (l_4 - e)^2}}
\end{align*}
\]

(5)

Where, $C = l_i \cos \theta_3 + l_4$, $D = l_i \sin \theta_3$.

3.1.2. Inverse solution of mechanism position

When the output angular displacements $\theta_1$ and $\theta_2$ and structure size were given, the input quantities $q_1$ and $q_2$ were the inverse position solution. According to Eqs. (3) and (4), the inverse solution of the RR&P5R mechanism position can be obtained by:
\[
\begin{align*}
q_1 &= \theta_1 \\
q_2 &= kl_3 \sin \theta_3 - \sqrt{l_3^2 - (kl_3 \cos \theta_3 + l_4 - e)^2} \\
\theta_3 &= \arcsin \frac{l_3^2 - l_4^2 - A^2 - B^2}{2l_3 \sqrt{A^2 + B^2}} - \arcsin \frac{A}{\sqrt{A^2 + B^2}}
\end{align*}
\]

(6)

Where, \( A = l_4 - l_1 \cos \theta_2 \), \( B = -l_1 \sin \theta_2 \).

3.2. RR&PRR Mechanism

Figure 6 shows the decoupled RR&PRR 2-dof parallel mechanism with a fully spherical workspace based on the PRR kinematic chain. \( \beta \) is the angle between the plane of the moving platform and the member L1. \( \theta_2 \) is the angle between the plane of the moving platform and the X axis. The input of the revolute joint R1 is \( q_1 \), and the input of the prismatic pair P is \( q_2 \).

![Figure 6. Decoupled RR&PRR parallel mechanism with fully spherical workspace](image)

Therefore, the coordinates of the revolute joint R3 in the fixed coordinate system O-XYZ can be expressed as:

\[
\begin{align*}
x &= l_1 \cos (\theta_2 + \beta) \\
y &= 0 \\
z &= -l_1 \sin (\theta_2 + \beta)
\end{align*}
\]

(7)

The geometric relationship between components can be expressed as:

\[
l_2^2 = \left[ q_2 - l_1 \cos (\theta_2 + \beta) \right]^2 + \left[ l_1 \sin (\theta_2 + \beta) - e \right]^2
\]

(8)

3.2.1. Forward solution of mechanism position

According to Eq. (8), the forward solution of the RR&PRR mechanism position can be calculated by:

\[
\begin{align*}
\theta_1 &= q_1 \\
\theta_2 &= \arcsin \frac{q_2^2 + e^2 + l_1^2 - l_2^2}{2l_1 \sqrt{q_2^2 + e^2}} - \arcsin \frac{q_2}{\sqrt{q_2^2 + e^2}} - \beta
\end{align*}
\]

(9)
3.2.2. Inverse solution of mechanism position

According to the Eq. (8), the inverse solution of RR&PRR mechanism position can be calculated by:

\[
\begin{align*}
q_1 &= \theta_1 \\
q_2 &= \sqrt{l_2^2 - [e - l_3 \sin(\theta_2 + \beta)]^2} + l_4 \cos(\theta_2 + \beta)
\end{align*}
\]

(10)

4. Size Optimizations

The optimization objectives were: (1) Under the premise of a fully spherical workspace, the input quantity \(q_2\) of the linear input chain was minimized. (2) The prismatic pair P had the smallest input speed variation range.

4.1. Optimization for the RR&P5R Mechanism

At first, the size of the double rocker mechanism was optimized. It was found that when the length of the member \(L_1\) reached a minimum value, the angular displacement of the member \(L_3\) was the smallest. The optimization objective was the minimum range of the input angular velocity of the member \(L_3\). Assuming that the output angular velocity of the member \(L_1\) is determined, the member optimization results for different iterations are shown in Table 1.

Table 1. The optimization results for different iterations

| Iterations | Objection | DV_1 | DV_2 | DV_3 |
|------------|-----------|------|------|------|
| 0          | 0.30205   | -200.00 | 250.00 | -350.00 |
| 1          | 0.11991   | -111.64 | 116.88 | -400  |
| 2          | 0.097212  | -149.84 | 165.61 | -400  |
| 3          | 0.091479  | -172.38 | 166.05 | -400  |
| 4          | 0.087909  | -168.13 | 153.20 | -400  |
| 5          | 0.087909  | -168.13 | 153.20 | -400  |

Then, the size of the offset crank slider was optimized. This paper only studied the case where the revolute joint \(R_7\) was at the midpoint of the member \(L_3\), i.e. \(k=0.5\). The optimization objective was to minimize the displacement of the prismatic pair P. Thus, the member optimization results of different iterations are shown in Table 2.

Table 2. The optimization results of different iterations

| Iterations | Objection | DV_4 | DV_5 |
|------------|-----------|------|------|
| 0          | 61.772    | 50.00 | 200.00 |
| 1          | 53.278    | 122.32 | 279.90 |
| 2          | 48.639    | 150.00 | 214.61 |
| 3          | 41.076    | 150.00 | 53.312 |
| 4          | 34.027    | 150.00 | -3.366 |
| 5          | 34.027    | 150.00 | -3.366 |

Through the above optimizations, the optimal member length ratio of the decoupled RR&P5R parallel mechanism with a fully spherical workspace was about \(l_1: l_2: l_3: l_4: l_5: e \approx 1.68: 2.78: 4.03: 2: 1.98: 1.5\).

4.2. Optimization of the RR&PRR Mechanism

As shown in Figure 6, the assumption was made that the angle \(\beta\) was 30°. According to Eqs. (2), \(e_{\text{min}}=51.89\). \(e\) was set to 51.89, 100, 150, 200 for optimization analysis. The optimization objective was to minimize the displacement of the prismatic pair P. The optimization results are shown in Table 3. The results showed that when \(\beta=30^\circ\) \(e=51.89\), the optimal member length ratio of the decoupled
RR&PRR parallel mechanism with fully spherical workspace was about \( l_1 : l_2 : e \approx 2 : 1.83 : 0.5 \).

### Table 3. The optimization results for different values of \( e \)

| \( e \) | Objection | \( DV_6 \) |
|---|---|---|
| 50 | 122.59 | -350.00 |
| 100 | 135.58 | -458.6628 |
| 150 | 143.52 | -526.5067 |
| 200 | 149.30 | -589.5507 |

### 5. Conclusions

(1) In this paper, the type design of a decoupled 2-dof parallel mechanism with a fully spherical workspace was studied. Based on the input and output characteristics of the double rocker mechanism, a new idea for the type design of kinematic chain that can be used to design a decoupled 2-dof spherical parallel mechanism was presented. The kinematic chain has the characteristics of accepting a linear input and producing a rotary output. On this basis, two feasible kinematic chains (P5R and PRR) were derived. Furthermore, two new decoupled 2-dof parallel mechanisms (RR&P5R and RR&PRR) with fully spherical workspaces were designed, and the position analysis of these two mechanisms was elucidated.

(2) Taking the minimum value of input displacement \( q_2 \) as the optimization objective, size optimization analysis of RR&P5R and RR&PRR mechanisms was carried out by Adams software. The optimization results were as follows: the optimal member length ratio of the RR&P5R mechanism was about \( l_1 : l_2 : l_3 : l_4 : l_5 : e \approx 1.68 : 2.78 : 4.03 : 2 : 1.98 : 1.5 \), the optimal member length ratio of the RR&PRR mechanism was about \( l_1 : l_2 : e \approx 2 : 1.83 : 0.5 \).

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