DCT-like Transform for Image Compression
Requires 14 Additions Only

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Abstract

A low-complexity 8-point orthogonal approximate DCT is introduced. The proposed transform requires no multiplications or bit-shift operations. The derived fast algorithm requires only 14 additions, less than any existing DCT approximation. Moreover, in several image compression scenarios, the proposed transform could outperform the well-known signed DCT, as well as state-of-the-art algorithms.

Keywords

DCT Approximation, Fast algorithms, Image compression

1 Introduction

The discrete cosine transform (DCT) is an essential tool in digital signal processing (DSP). In recent years, signal processing literature has been populated with low-complexity methods for the efficient computation of the 8-point DCT [1]. Prominent approximation-based techniques include the signed DCT (SDCT) [2], the level 1 approximation by Lengwehasatit-Ortega [3], the Bouguezal-Ahmad-Swamy (BAS) series of algorithms [4–6], and the DCT round-off approximation [7].

In general, the transformation matrix entries required by approximate DCT methods are only \{0, \pm1/2, \pm1, \pm2\}. This implies null multiplicative complexity, because the involved operations can be implemented exclusively by means additions and bit-shift operations.

In this letter, we introduce a low-complexity DCT approximation that required only 14 additions. The proposed algorithm attains the lowest computational complexity among available methods found in literature. At the same time, the proposed transform could outperform state-of-the-art approximations.

2 Proposed transform

The proposed approximation is based on the approximate DCT introduced in [7]; hereafter referred to as CB-2011 matrix. After judiciously replacing elements of the CB-2011 matrix with zeros, we obtained the

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Figure 1: Signal flow graph for $T$. Input data $x_n$, $n = 0, 1, \ldots, 7$, relates to output $X_k$, $k = 0, 1, \ldots, 7$, according to $X = T \cdot x$. Dashed arrows represent multiplication by $-1$.

The following matrix:

$$T = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0
\end{bmatrix}.$$  

Above matrix furnishes the approximate DCT expressed by: $\hat{C} = D \cdot T$, where $D = \text{diag}\left(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

The entries of $T$ are $\{0, \pm 1\}$. This is an attestation of its null multiplicative complexity. Moreover, bit-shift operations are fully absent. Not only $\hat{C}$ inherits the low computational complexity of $T$, but it is also orthogonal. In terms of complexity assessment, matrix $D$ may not introduce any computational overhead [3–7]. In image compression, the DCT operation is a pre-processing step for subsequent coefficient quantisation. In this context, matrix $D$, in the form of $D^2$, can be merged into the quantisation matrix. Moreover, all elements of $D^2$ are negative powers of two $\{1/2, 1/4, 1/8\}$. Therefore, any implementation of the quantisation step for the exact DCT can be easily adapted to the proposed method by adequately bit-shifting the elements of the quantisation matrix.

A fast algorithm based on sparse matrix factorization leads to $T = P \cdot A_3 \cdot A_2 \cdot A_1$, where:

$$\begin{align*}
A_1 &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, & A_2 &= \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, & A_3 &= \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, & P &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\end{align*}$$

Signal flow graph for $T$ is shown in Fig. [4].
Table 1: Arithmetic complexity analysis

| Method                  | Add. | Mult. | Shifts | Total |
|-------------------------|------|-------|--------|-------|
| Proposed transform      | 14   | 0     | 0      | 14    |
| SDCT [2]                | 24   | 0     | 0      | 24    |
| Level 1 approximation   | 24   | 0     | 2      | 26    |
| BAS-2008 transform [4]  | 18   | 0     | 2      | 20    |
| BAS-2009 transform [5]  | 18   | 0     | 0      | 18    |
| BAS-2011 transform [6]  | 18   | 0     | 2      | 20    |
| CB-2011 transform [7]   | 22   | 0     | 0      | 22    |

Arithmetic complexity assessment and comparisons with state-of-the-art DCT approximations are shown in Table 1. Demanding only 14 additions, proposed transform Ĉ possesses 22.2%, 30.0%, and 41.7% lower arithmetic costs than the BAS-2009 transform [5], the BAS-2011 transform [6], and the SDCT, respectively. Notice that the BAS-2011 transform is the most recent algorithm in the BAS series.

DSP literature contains the DCT approximation described in [8], which is claimed to require 16 additions. However, we could not reproduce the performance results shown in [8]. Indeed, contrary to [8], such approximation could not be verified to be orthogonal. Thus, we could not consider [8] for any meaningful comparison.

3 IMAGE COMPRESSION

To assess the performance of the proposed transform for image compression, we used the methodology described in [2] and supported by [3-7]. A set of 45 512×512 8-bit greyscale images obtained from a standard public image bank [9] was considered. We implemented the JPEG image compression technique for the 8×8 matrix case. Each image was divided into 8×8 sub-blocks, which were submitted to the two-dimensional transforms. This computation furnished 64 coefficients in the approximate transform domain for each sub-block. According to the standard zigzag sequence only the r initial coefficients in each block were employed to reconstruct the image [7]. All the remaining coefficients were set to zero. We adopted 2 ≤ r ≤ 45, which corresponds to compression ratios between 96.875% and 29.690%, respectively. The inverse procedure was then applied to reconstruct the processed data and image degradation was assessed.

For the sake of image compression performance assessment, the peak signal-to-noise ratio (PSNR) and mean square error (MSE) were utilized as figures of merit. However, in contrast with the numerical experiments described in [2-6], we adopted the average quality measure from all considered images instead of the results obtained from particular images. Thus, our analysis is more robust; being less prone to variance effects and fortuitous input data. Among available algorithms, we separate the SDCT [2] and BAS-2011 [7] for comparison. The SDCT is a classic reference in the field [10] and the BAS-2011 transform is the most recent method in the BAS series of algorithms. The parametric transform BAS-2011 was considered with parameter a = 0.5 [6].

Fig. 2(a) shows the resulting PSNR measures. The proposed approximation Ĉ has comparable performance to the SDCT at high compression rates and could indeed outperform it at low compression rates. At the mid-range compression ratios (20 < r ≤ 35), the proposed matrix outperformed both the BAS-2011 transform and the SDCT. This similar result could be achieved in despite of requiring only 70.0% and 58.3%
of the arithmetic cost of the BAS-2011 transform and the SDCT, respectively. Fig. 2(b) depicts the absolute percentage error (APE) relative to the exact DCT for the average MSE. According to this metric, the proposed approximation led to a better performance at compression ratios ranging from 90.625% (r = 6) to 45.310% (r = 35), in which popular compression ratios are included.

Fig. 3 shows a qualitative comparison including the DCT, the proposed transform, the BAS-2011 transform, and the SDCT. A 60.937% compression (r = 25) was applied to the standard Lena image. Proposed transform offered results that are comparable to those furnished by the exact DCT.

4 Conclusion

This letter introduced an 8-point transform suitable for image compression. The proposed transform requires only 14 additions and has comparable or better image compression performance than the classic SDCT and the state-of-the-art BAS-2011 transform.

References

[1] Lecuire, V., Makkaoui, L. and Moureaux, J.-M.: ‘Fast zonal DCT for energy conservation in wireless image sensor networks’, Electron. Lett., 2012, 48, (2).

[2] Haweel, T. I.: ‘A new square wave transform based on the DCT’, Signal Process., 2001, 82, pp. 2309–2319.

[3] Lengwehasatit, K. and Ortega, A.: ‘Scalable variable complexity approximate forward DCT’, IEEE Trans. Circuits Syst. Video Techno., 2004, 14, (11), pp. 1236–1248.

[4] Bouguezel, S., Ahmad, M. O. and Swamy, M. N. S.: ‘Low-complexity 8×8 transform for image compression’, Electron. Lett., 2008, 24, (21), pp. 1249–1250.

[5] Bouguezel, S., Ahmad, M. O. and Swamy, M. N. S.: ‘A fast 8×8 transform for image compression’, Int. Conf. Microelectronics, 2009, pp. 74–77.

[6] Bouguezel, S., Ahmad, M. O. and Swamy, M. N. S.: ‘A low-complexity parametric transform for image compression’, Proc. 2011 IEEE Int. Symp. Circuits and Systems, 2011.
Figure 3: Compressed Lena image using (a) the DCT, (b) the proposed transform, (c) the BAS-2011 transform, and (d) the SDCT, for $r = 25$. 
[7] Cintra, R. J. and Bayer, F. M.: ‘A DCT Approximation for Image Compression’, *IEEE Signal Process. Lett.*, 2011, **18**, (10), pp. 579–582.

[8] Brahimi, N. and Bouguezel, S.: ‘An efficient fast integer DCT transform for images compression with 16 additions only’, 7th International Workshop on Systems, Signal Process. and their Applications, 2011, pp. 71–74.

[9] USC-SIPI Image Database, University of Southern California, Signal and Image Processing Institute, http://sipi.usc.edu/database/.

[10] Britanak, V., Yip, P. and Rao, K.R.: *Discrete Cosine and Sine Transforms*, Academic Press, 2007.