THE ADBVECTIVE-ACOUSTIC INSTABILITY IN TYPE II SUPERNOVAE

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Abstract. The puzzle of birth velocities of pulsars (pulsar kicks) could be solved by an asymmetric explosion of type II Supernovae. We propose a simple hydrodynamical mechanism in order to explain this asymmetry, through the advective-acoustic cycle (Foglizzo 2002) : during the phase of stalled shock, an instability based on the cycle between advected perturbations (entropy / vorticity) and acoustic perturbations can develop between the shock and the surface of the nascent neutron star. Eigenfrequencies are computed numerically, improving the calculation of Houck & Chevalier (1992). The linear instability is dominated by a mode $l = 1$, as observed in the numerical simulations of Blondin et al. (2003) and Scheck et al. (2004). The frequency dependence of the growth rate reveals the presence of the advective-acoustic cycle.

1 Stationary accretion above a solid surface

We consider a shocked accretion flow onto a solid surface (where the velocity of the flow is null) at a constant accretion rate. A cooling region is located above the surface and described by the generic function $L \propto \rho^{\beta-\alpha}P^{\alpha}$ as in Houck & Chevalier (1992), hereafter HC92. The basic equations of the flow are the continuity equation, the Euler equation and the entropy equation. This latter is:

$$\frac{\partial S}{\partial \tau} + (\vec{v} \cdot \nabla)S + \frac{\mathcal{L}}{P} = 0,$$

where a measure of the entropy is defined by $S \equiv 1/(\gamma - 1) \log(P/\rho^\gamma)$. These equations are perturbed and projected onto spherical harmonics $Y^l_m$.

The jump conditions at the shock $r_{sh}$ are given by the Rankine-Hugoniot relations for the stationary quantities. The perturbations at the shock are evaluated taking into account the displacement $\Delta \xi$ of the shock position and its velocity $\Delta v = -i\omega \Delta \xi$. In particular, the perturbations of the transverse velocity are as
follows (Landau & Lifchitz 1987):

\[ \delta v_\theta = \frac{v_1 - v_2}{r_{sh}} \frac{\partial \Delta \xi}{\partial \theta} \]  

(1.1)

\[ \delta v_\phi = \frac{v_1 - v_2}{r_{sh} \sin \theta} \frac{\partial \Delta \xi}{\partial \phi} \]  

(1.2)

where \( v_1 \) and \( v_2 \) are the pre-shock and post-shock velocities of the flow. These transverse velocity perturbations at the shock are not null for non-radial perturbations. This contrasts with Eq. (51) of HC92, who did not allow for transverse velocity perturbations at the shock.

The eigenfrequency \( \omega \) is a complex number \((\omega_r, \omega_i)\) such that the velocity perturbation satisfies a wall type condition \( (\delta v/v = 0) \) at the surface of the accretor. The imaginary part \( \omega_i \) of the eigenfrequency is the growth rate of the perturbations.

2 Calculations of the eigenmodes

We performed several calculations of the fundamental modes (an example with \( \gamma = 5/3, \alpha = 1/2 \) and \( \beta = 2 \) is shown in Fig. 1). The mode \( l = 1 \) is always the most unstable. This result differs from the analysis made by HC92 because of the error in their boundary conditions at the shock.

The advective-acoustic instability is based on the cycle between advected perturbations (entropy / vorticity) and acoustic perturbations between the shock and the surface. A reference timescale \( \tau \) for this mechanism is equal to the accretion time from the shock to the coupling region near the surface plus the time for an acoustic wave to reach the shock:

\[ \tau \equiv \int_{r_s}^{r_{sh}} \frac{1}{1 - M |v|} \frac{dr}{1 - M} \]  

(2.1)

The acoustic time \( t_{ac} \) is defined by the time needed for an acoustic wave to propagate from the shock to the accretor and then back up to the shock, \( \omega_{ac} \equiv 2\pi/t_{ac} \) being the pulsation associated to this acoustic time:

\[ t_{ac} \equiv \int_{r_s}^{r_{sh}} \frac{2}{1 - M^2} \frac{dr}{c} \]  

(2.2)

On Figs. 1-2 the growth rate \( \omega_i \) is at best comparable to \( \omega_{sh} \sim 1/\tau \) and the pulsation \( \omega_r \) of the fundamental unstable modes is close to \( 2\pi/\tau \), as expected in the advective-acoustic mechanism. The frequency dependence of the growth rate of the eigenmodes shows an oscillatory behaviour with a period comparable to \( \omega_{ac} \) (Fig. 2). Such oscillations are expected in the advective-acoustic instability, as a consequence of the modulation of the advective-acoustic cycle by the purely acoustic cycle (Foglizzo 2002). This interpretation is confirmed by measuring the
ratio $\tau/t_{ac} \sim 4.5 - 6.5$ which is comparable to the number of modes found per oscillations (Foglizzo 2002).

We note that for big cavities, the most unstable eigenmodes correspond to low frequencies, in the "pseudo-sound" regime ($\omega_r < \omega_{ac}$). Unstable eigenmodes are also found in the acoustic regime ($\omega_r > \omega_{ac}$), with a smaller growth rate however (Fig. 2).

![Figure 1](image1.png)

**Fig. 1.** For $\gamma = 5/3$, $\alpha = 1/2$ and $\beta = 2$, numerical calculations of the frequency $\omega_r$ and the growth rate $\omega_i$ in units of $\omega_{sh} \equiv -v_{sh}/(r_{sh} - r_*)$ of the fundamental modes $l = 0, 1, 2, 3$ depending on the size of the cavity $x_{sh}/r_* \equiv (r_{sh} - r_*)/r_*$, as in Houck & Chevalier (1992). The degree $l$ of the modes is indicated on each curve.

![Figure 2](image2.png)

**Fig. 2.** For $\gamma = 4/3$, $\beta = 2$, $\alpha = 1/2$, and a cavity $r_{sh}/r_* \sim 12$, numerical calculations of the frequency $\omega_r$ in units of $\omega_{ac}$ and the growth rate $\omega_i$ in units of $\omega_{sh}$ of radial $l = 0$ and non-radial $l = 1, 2$ eigenmodes.
On Fig. 3, for $\gamma = 4/3$ and a cooling function described by $\alpha = 6$, $\beta = 1$, relevant to the phase of a stalled shock in type II Supernovae (Bethe & Wilson 1985), an unstable mode $l = 1$ can also be found for big enough cavities ($r_{sh}/r_* \gtrsim 3.5$). A frequency dependence study, still in progress, shows unstable modes $l = 1$ both in the ”pseudo-sound” regime and in the acoustic regime. The radial modes ($l = 0$) are always stable.

Fig. 3. For $\gamma = 4/3$, $\alpha = 6$ and $\beta = 1$, numerical calculations of the frequency $\omega_r$ in units of $\omega_{ac}$ and the growth rate $\omega_i$ in units of $\omega_{sh} \equiv -v_{sh}/(r_{sh} - r_*)$, of the first unstable mode $l = 1$, depending on the size of the cavity $x_{s0}/r_* = (r_{sh} - r_*)/r_*$. The vertical dashed line correspond to $r_{sh}/r_* = 3.5$ ($x_{s0}/r_* = 2.5$).

3 Conclusion

The stalled accretion shock above a neutron star is unstable, with a domination of a mode $l = 1$ if the shock radius is large enough. The instability is interpreted as an advective-acoustic cycle modulated by a purely acoustic cycle. The instability is also found when the cooling function mimics neutrino cooling in type II Supernovae. The advective-acoustic cycle is thus a good candidate to seed an asymmetric explosion which could lead to an important birth velocity of the neutron star.

References

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