Observation of coherent oscillation in single-passage Landau-Zener transitions

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Landau-Zener transition (LZT) has broad applications in atomic and molecular physics, quantum optics, condensed matter physics, chemical physics, and quantum information science. For example, LZT has been applied to investigating the jump time and quantum Zeno and anti-Zeno effects of cold atoms in accelerated optical lattices, the behavior of molecular magnets at low temperature, nonequilibrium phase transitions, and it is also exploited as a tunable beam splitter of wave functions to generate entangled multipartite states. LZT also plays a key role in determining whether random optimization problems can be solved using the quantum adiabatic algorithm. Recently, LZT’s potential for robust manipulation of coherent quantum states has attracted much attention in the context of quantum information processing because LZT may provide a simple and effective solution to the realization of high fidelity quantum state control without the need for precise timing.

The time-dependent Hamiltonian describing LZT in quantum two-level systems can be written in the generic form as

\[ H_{LZ}(t) = -\frac{1}{2} \dot{\varepsilon}(t) \sigma_z - \frac{1}{2} \Delta \sigma_x, \]  

where \( \sigma_{x,z} \) are Pauli matrices, \( \dot{\varepsilon}(t) = \nu t \) is the energy difference between the two diabatic (crossing) basis states (i.e., the eigenstates | + \rangle and | - \rangle of the \( \sigma_z \) operator) controlled by an external parameter which depends linearly on time \( t \), and \( \Delta \) is the constant gap between the two instantaneous eigenenergy states | + \rangle and | - \rangle at the center of the avoided crossing \( \varepsilon = 0 \), as depicted in Fig. 1(a). In such systems, when \( \dot{\varepsilon}(t) \) is swept through the avoided crossing, transitions between | ± \rangle with energies \( E_{\pm}(t) = \pm \frac{1}{2} \sqrt{(\dot{\varepsilon}(t))^2 + \Delta^2} \) can occur and the transition probability is given by the well-known Landau-Zener (LZ) formula

\[ P_{LZ} = e^{-\frac{2\pi\Delta}{\nu}}, \]  

where \( \nu = |d\varepsilon/dt| \) is the Landau-Zener speed and we have set the reduced Planck constant \( h = 1 \). Equation (2) gives the probability of finding the system in the excited (ground) state at \( \varepsilon_x = \dot{\varepsilon}(t \to +\infty) \) when it is started in the ground (excited) state at \( \varepsilon_i = \dot{\varepsilon}(t \to -\infty) \). By defining \( \kappa = \Delta^2/4\nu \) as the adiabaticity parameter the LZ formula can be simplified to \( P_{LZ} = \exp(-2\kappa \nu) \). Although analytical solution to the problem cannot be obtained when \( \varepsilon_i \) and/or
avoided crossing is assumed to be determined entirely by starting from the ground (excited) state the probability of finding

(b) Optical micrograph of the sample with Al/AlO states (the solid lines). The constant gap

dundreds of MHz while superconducting qubits the tuning range of energy level spacing is

The formula may become quantitatively inaccurate or even qualitatively incorrect. In spite of some theoretical studies on the effects of finite $|\epsilon_i|/\Delta$ on $P_1$, there is an acute lack of adequate experimental evidence.

On the other hand, understanding LZT with moderate values of $|\epsilon_i|/\Delta$ is in urgent need because this region of parameter space is important to quantum information processing. For instance, in superconducting qubits the tuning range of energy level spacing is usually limited to a couple of GHz or even as narrow as a few hundreds of MHz while $\Delta/2\pi$ could be as large as $10^5$ MHz$^{3,19-26}$. For quantum state control based on sweeping $\epsilon$ through avoided crossings, understanding LZT probability’s dependence on $|\epsilon_i|/\Delta$ and the sweeping time is essential to high fidelity operation. The fidelity of various techniques based on LZT relies critically on the accuracy of the LZ formula which predicts a simple exponential dependence of the LZT probability on $|\epsilon_i|/\Delta$ and the sweeping time is essential to high fidelity operation. The fidelity of various techniques based on LZT relies critically on the accuracy of the LZ formula which predicts a simple exponential dependence of the LZT probability on $|\epsilon_i|/\Delta$ and the sweeping time is essential to high fidelity operation.

As shown in Fig. 2(a), however, the microwave spectrum of the qubit $\Phi_0(\Phi_1)$ has a rather large avoided energy-level crossing at $\Phi_1 \approx -2.8$ m$\Phi_0$ (with respect to the flux bias point at which $\Phi_0(2\pi) \approx 16.348$ GHz) indicating significant interaction between the qubit and a microscopic TLS$^{30}$. The transition frequency between the TLS ground state $|g\rangle$ and excited state $|e\rangle$ and the qubit-TLS coupling strength are $\omega_{TLS}/2\pi = 16.450 \pm 0.002$ GHz and $\Delta/2\pi = 70.0 \pm 0.5$ MHz from the spectrum and vacuum Rabi oscillation, respectively. Note that in this coupled qubit-TLS system the time-dependent energy difference between the two diabatic states involved in LZT is $\epsilon(t) = \epsilon_1(t) - \epsilon_{TLS}$ which depends linearly on the flux bias to a good approximation. The relationship between the flux bias and $\epsilon$ can be found from Fig. 2(a).

Fig. 1(c) illustrates the experimental procedure used to observe coherent LZO. We begin by setting the initial diabatic energy of the effective quantum two-level system $\epsilon$, at about 100 MHz below $\omega_{TLS}$ with a static flux bias. The qubit is prepared in its ground state by waiting for much longer than the energy relaxation time $T_1 \approx 70$ ns of the qubit. A microwave pulse is then applied to the qubit when it is biased at a fixed value $\epsilon_1/2\pi = -100$ MHz. The microwave pulse coherently transfers the population of the qubit-TLS from $|0\rangle$ to one of the system’s eigenstates $|\pm\rangle$ through a process that is discussed in detail in Methods. The lack of oscillation in $T_1$ measurement taken at $\epsilon_1$, as shown in Fig. 2(b) confirms that the initial state of the qubit-TLS system at $t = 0$ is indeed the eigenstate $|\pm\rangle$. As illustrated in Fig. 1(c), a time-dependent flux $\Phi(t) = \Phi_0(t)/\Phi_p$ is then superimposed between $t = 0$ and $t = t_p$ onto the static flux bias to sweep $\epsilon$ linearly from $-100$ MHz to its maximum value $\epsilon_1$. The corresponding LZ speed $v$ is thus $\epsilon_f/\Phi_p$. This is followed immediately by a 5-ns readout pulse which performs a projective measurement of the probability $P_f$ of finding the qubit in state $|1\rangle$ (i.e., the coupled system is in state $|1g\rangle$ corresponding to $|\downarrow\rangle$ in Fig. 1(a)).

### Results

In our experiment we use a superconducting phase qubit. However, since a single phase qubit does not have an intrinsic avoided energy-level crossing, we utilize an avoided level crossing arising from interaction between the qubit and a microscopic TLS$^{30}$. As discussed below in more detail, when the transition frequency of the qubit $\omega_{10}$ is close to that of the TLS $\omega_{TLS}$ which is fixed, the first and second excited states of the coupled qubit-TLS system form an effective quantum two-level system described by the LZ Hamiltonian (1). Note that to make quantitative comparisons between the theory/numerical simulation and the experiment without free parameters, all relevant system parameters, including the energy relaxation and dephasing time of the qubit and the energy gap $\Delta$, are obtained from direct measurements.

A microscopic picture of the superconducting phase qubit is shown in Fig. 1(b). The qubit consists of an $L \approx 770$ pH superconducting loop intersected by a Al/AlO/Al Josephson junction with a critical current $\tilde{I}_c \approx 1.4$ $\mu A$ and a junction capacitance $C \approx 240$ fF. By varying the magnetic flux applied to the superconducting loop the potential energy of the qubit becomes asymmetrical. The ground state and the first excited state in the upper potential well, represented by $|0\rangle$ and $|1\rangle$ respectively, can be used as the computational basis states of the qubit. For an isolated qubit, the transition frequency between $|0\rangle$ and $|1\rangle$, $\omega_{10}$, is a single-valued function of the external flux bias $\Phi$, which is coupled inductively to the superconducting loop through an on-chip flux bias line.

As shown in Fig. 2(a), however, the microwave spectrum of the qubit $\Phi_0(\Phi_1)$ has a rather large avoided energy-level crossing at $\Phi_1 \approx -2.8$ m$\Phi_0$ (with respect to the flux bias point at which $\Phi_0(2\pi) \approx 16.348$ GHz) indicating significant interaction between the qubit and a microscopic TLS$^{30}$. The transition frequency between the TLS ground state $|g\rangle$ and excited state $|e\rangle$ and the qubit-TLS coupling strength are $\omega_{TLS}/2\pi = 16.450 \pm 0.002$ GHz and $\Delta/2\pi = 70.0 \pm 0.5$ MHz from the spectrum and vacuum Rabi oscillation, respectively. Note that in this coupled qubit-TLS system the time-dependent energy difference between the two diabatic states involved in LZT is $\epsilon(t) = \epsilon_1(t) - \epsilon_{TLS}$ which depends linearly on the flux bias to a good approximation. The relationship between the flux bias and $\epsilon$ can be found from Fig. 2(a).
We first measure $P_1$ vs. $t_{sp}$ at a constant value of $\varepsilon_f$ by keeping $\Phi_{LZ}$ fixed while increasing $t_{sp}$ from almost 0 ns to 45 ns. The maximum $t_{sp}$ is selected to avoid too much influence of the qubit's energy relaxation. By stepping $\Phi_{LZ}$ from 0 to $-11$ m$\Phi_0$, the value of corresponding $\varepsilon_f$ is then varied from about $-1.4\Delta$ to $-4\Delta$, in which the condition $|\varepsilon_f|/\Delta$ is no longer satisfied. This procedure is repeated at each $\varepsilon_f$ to obtain $P_1(\varepsilon_f, t_{sp})$. Fig. 3(a) shows the dependence of $P_1$ on $\varepsilon_f$ and $t_{sp}$. It can be seen that $P_1$ vs. $t_{sp}$ decays exponentially for $\varepsilon_f < 0$ ($\Phi_{LZ} \in [-2.8 \text{ m}$$\Phi_0, 0]$) with a characteristic time $T_1$ due to energy relaxation. As $\varepsilon_f$ becomes positive, $P_1$ vs. $t_{sp}$ becomes oscillatory. Since the avoided crossing is traversed only once, the observed oscillation in $P_1$ vs. $t_{sp}$ with constant $\varepsilon_f$ must be a consequence of the moderate value of $\varepsilon_f/\Delta$ and is not caused by the Landau-Zener-Stückelberg interference which requires multiple passages through the avoided crossing.

It is worth noting that the observed oscillation is not a consequence of ill-prepared initial states with non-negligible probability amplitude in the excited state $|\rangle$ of the effective Hamiltonian (1) because the microwave pulse used to initialize the system resonantly couples $|0\rangle$ to $|-\rangle$ and has negligible coupling to $|\rangle$ due to large frequency detuning. Furthermore, this process of transferring the system to the desired initial state $|-\rangle$ via a resonant microwave pulse is robust in the sense that it does not depend sensitively on the accuracy of the pulse duration $t_{MW}$. Deviation in pulse duration simply leaves some probability amplitude in $|0\rangle$ which has no effect on LZO other than reducing the visibility of the oscillation (see Methods for detail on the initial state preparation). Therefore, we are confident that the oscillation observed in the non-adiabatic region of the parameter space arises neither from Landau-Zener-Stückelberg interference nor unwanted probability amplitude of $|\rangle$ in the initial state. This is also supported by the good agreement between the results of experiment and numerical simulation shown in Fig. 3(b), which uses $|-\rangle$ as the initial state at the start of the single passage sweep.

**Discussion**

By replacing $\varepsilon_f$ with $\varepsilon_i + vt$, solving the problem of sweeping $\varepsilon$ in a finite range is transformed to finding $P_1$ at finite time. Previous studies have discovered that LZ transition probability reaches the asymptotic value given by equation (2) at $t \gg t_{LZ}$ (assume $\varepsilon = 0$ at $t = 0$), where $t_{LZ} = 2\sqrt{2}/\Delta \max(1, \sqrt{2})$ is called the Landau-Zener time, and oscillates in the vicinity of avoided crossing (corresponds to $t \approx t_{LZ}$) due to the transient dynamics. Since corrections to the standard LZ formula are significant only if the adiabaticity parameter $\varepsilon \ll 1$, the region of which transient dynamics plays an important role is given by $\varepsilon_f \ll t_{sp}^{-1}$, where “∼” means less than or comparable to. By examining the experimental data we find that the region of most noticeable coherent LZO coincides with $\varepsilon_f \ll t_{sp}^{-1}$ which agrees well with the result of numerical simulation. These results unambiguously show that coherent LZO is originated from the transient dynamics of the LZO.

**Figure 2** Spectroscopy and Rabi oscillation. (a) Microwave spectroscopy measurement of the coupled qubit-TLS system. The splitting is $\Delta/2\pi = 70.0 \pm 0.5$ MHz centered at $\omega_0/2\pi = 16.450 \pm 0.002$ GHz. The beginning point of the flux bias for the single passage LZ sweeping is denoted as $0 \text{ m}$$\Phi_0$, corresponding to $\varepsilon_f/2\pi = (\alpha_{10} - \omega_{TLS})/2\pi = -100$ MHz in the upper abscissa. (b) Rabi oscillation and $T_1$ at $\varepsilon_i$ respectively. The experimental data (the red circles and blue triangles) agree well with the theoretical fits (the black solid lines).

**Figure 3** Coherent LZO. (a) Experimentally measured $P_1$ vs. $\varepsilon_f$ and $t_{sp}$ (b) Numerically calculated $P_1$ vs. $\varepsilon_f$ and $t_{sp}$ with all input parameters obtained from the experiment. The white dashed lines correspond to the value of modified adiabaticity parameter $\varepsilon' = 10$. Notice that in the region below (above) the lower (upper) white dashed line, one has $\varepsilon' \gg 1$ thus the system evolves adiabatically and no oscillation in $P_1$ is expected as confirmed experimentally. The LZ speed $v$ equals the slope of any straight lines originated from the lower-left corner of the $t_{sp}$-$\varepsilon_f$ plane. For example, the yellow dashed line in (a) has $v = 400/19.5 = 20.5$ MHz/ns. For the sake of clarity, the temporal evolution of the system along the yellow dashed line (constant $v$) is presented separately in Fig. 4(b).
Such coherent LZO has little effect on the adiabatic evolution. Because in the true adiabatic regime, by definition the system always stays in the instantaneous ground state and no LNZ could occur. In order to find the region of approximate adiabatic evolution in our experiment, it is necessary to modify the definition of the adiabaticity parameter to 
\[ x' = \frac{(\omega_{LS} - \omega_{00})^2 + \Delta^2}{4\nu}. \]
For the adiabatic theorem to hold \( x' > 1 \) is required. As shown in Fig. 3(a), the white dashed lines represent \( x' = 10 \). It is clear that there is no coherent LNZ observed in the region \( x' > 1 \).

In a popular analogy to optics an avoided crossing acts as an effective beam splitter, with a transmission coefficient corresponding to \( P_{LZ} \) in the LZ formula, for quantum wave functions. This beam splitter analogy has been applied successfully to the visualization and explanation of the behavior of superconducting and semiconductor qubits. In this analogy, a single sweep through the avoided crossing is equivalent to passing a beam of light through the beam splitter only once. When \( |\epsilon_f| \gg \Delta \) and thus a greater LZ speed corresponds to a higher transmission coefficient of the beam splitter according to the LZ formula. But when \( |\epsilon_f| \gg \Delta \) is not satisfied, \( P_L \) differs greatly from \( P_{LZ} \). As an example, \( P_L \) vs. \( \mathcal{t}_{ps} \) and thus the LZ speed \( v \), with \( \epsilon_f/2\pi = 200 \) MHz is shown in Fig. 4(a). The maximum in the difference \( \delta P_1 \) between the experimental \( P_L \) and those obtained from the LZ formula (2), shown in the inset of Fig. 4(a), can reach 0.21. The observation of coherent LNZ strongly suggests that when \( |\epsilon_f| \gg \Delta \) is not met corrections to the LZ formula should be considered to avoid conceptual difficulties.

Coherent LNZ also has significant consequences on the coherent manipulation of quantum states of single qubits and coupled two-qubit systems based on LZT. For this approach of quantum state control, the LZ transition probability \( P_{LZ} \) plays a central role since each single passage through the avoided crossing results in a unitary operation \( U_{LZ} \) given by Ref. 12
\[
U_{LZ} = \begin{pmatrix}
1 - \sqrt{P_{LZ}} e^{-i\varphi}, & -\sqrt{P_{LZ}} \\
\sqrt{P_{LZ}}, & 1 - \sqrt{P_{LZ}} e^{i\varphi}
\end{pmatrix},
\]
where \( \varphi \) is the Stokes phase, which has no effect on the single-passage LZT process discussed here and thus can be set to zero for the sake of convenience. As mentioned above, the transition frequency \( \omega_{LS} \) of most artificial atoms, in particular the superconducting qubits, is limited to a couple of GHz. Because the speed of two-qubit operations is proportional to the inter-qubit coupling strength \( \Delta \), increasing \( |\epsilon_f|/\Delta \) by reducing \( \Delta \) is undesirable. Hence, evaluating \( U_{LZ} \) according to the LZ formula (2) could result in significant errors when \( \epsilon_f/\Delta \) is not satisfied. In order to conduct a quantitative analysis, the experimental data along the yellow dashed line corresponding to constant \( v \), in the Fig. 3(a) and shown in Fig. 4(b). Oscillation in \( P_L \) is clearly observed and it is qualitatively different from the exponential decay predicted by equation (2), when decoherence is taken into account. Suppose the initial quantum state is \( |1\rangle \). Then after a single passage through the avoided crossing, if one replaces \( P_{LZ} \) with \( P_L \) as in the asymptotic situation, the deviation \( \delta P_1 = P_L - P_{LZ} \) would be quite large. For example, when \( \mathcal{t}_{ps} = 12.8 \) ns the deviation \( \delta P_1 = 0.229 \), which is unacceptably large for coherent quantum state transformation.

In conclusion, we have investigated the effect of finite energy (\( \epsilon \)) sweep (or equivalently finite time) on LZT probability \( P_L \) experimentally. Single-passage technique is used to isolate the effect of finite \( \epsilon_f \) on \( P_L \) from that of interference caused by passing the avoided crossing multiple times. We find that \( P_L (\epsilon_f/\Delta, \alpha = \text{const}) \) oscillates when \( \epsilon_f \) is comparable to \( \Delta \) and \( \alpha < 1 \). The good agreement between the experimental and numerical calculation strongly supports the notion that coherent LNZ is caused by the underlying transient dynamics of the finite time LZT which cannot be described by the LZ asymptotic formula. In this region of the LZT parameter space, corrections to the LZ formula must be taken into account, otherwise it will lead to substantial errors in quantum state operations based on LZT. The result also shows that when applying the simple beam splitter analogy one should not automatically assume that greater \( \alpha \) (i.e., faster sweep) corresponds to larger transmission coefficient (i.e., greater \( P_{LZ} \)) as implied by the asymptotic LZ formula.

**Methods**

**Initial state preparation.** We first derive an analytical result explaining the lack of oscillation at the very beginning of LZT. The Hamiltonian of the qubit-TLS system coupled to a microwave field is given as: (in the basis \{0\}, |1\rangle, |0e\}, |1e\})
\[
H = \begin{pmatrix}
\Omega_\alpha \cos\theta & 0 & 0 & 0 \\
0 & \Omega_\alpha \cos\theta & \Delta/2 & 0 \\
0 & \Delta/2 & \Omega_\alpha \cos\theta & 0 \\
0 & 0 & 0 & \Omega_\alpha \cos\theta + \omega_{TLS}
\end{pmatrix}.
\]

![Figure 4](image-url) | \( P_L \) oscillation at constant LZ speed. (a) Measured \( P_L \) as a function of \( \mathcal{t}_{ps} \) (the red squares) with \( \epsilon_f/2\pi = 200 \) MHz which clearly shows oscillation in the region of \( x < 1 \) which compares well with the result of numerical calculation (the solid line). This is in stark contrast to the smooth exponential decay expected from the LZ formula (the dashed line). The inset is the difference \( \delta P_1 = P_L - P_{LZ} \). (b) The measured (the red squares) and numerically calculated (the solid line) \( P_L \) vs. \( \mathcal{t}_{ps} \) with constant LZ speed \( \nu = 20.5 \) MHz/ns corresponding to evolving along the yellow dashed line in Fig. 3(a). Again, \( P_L \) oscillates in the region where the adiabatic condition \( x > 1 \) is not satisfied which is not expected from the LZ formula (the dashed line).
where $\Omega_m$ is the Rabi frequency, $\omega$ is the microwave frequency, $\omega_{\text{TLS}}$ is the energy difference between the ground state $\vert g \rangle$ and the excited state $\vert e \rangle$ of TLS, $\Delta$ is the coupling strength between the qubit and TLS, $\delta = \omega_{\text{TLS}} - \omega$ and $\delta_i = \omega_{\text{TLS}} - \omega_{ji}$ are detunings. By rotating the frame, Hamiltonian (3) can be transformed to the following time-independent form:

$$H_1 = \frac{1}{2} \begin{pmatrix} -\delta - 2\Delta & \Omega_m & 0 \\ \Omega_m & -\delta & \Delta \\ 0 & \Delta & 0 \end{pmatrix}.$$ \hspace{1cm} (4)

Next, we rewrite $H_1$ in which the subspace spanned by $\{\vert g \rangle, \vert 0 \rangle e \}$ is diagonalized:

$$H_2 = \begin{pmatrix} -\frac{\delta + 2\Delta}{2} & -\frac{\Delta}{4N_+} \Omega_m & 0 \\ -\frac{\Delta}{4N_+} \Omega_m & \frac{1}{2} \sqrt{\Delta^2 + \delta_i^2} & 0 \\ 0 & -\frac{\Delta}{4N_-} \Omega_m & -\frac{\Delta}{4N_-} \Omega_m - \frac{\delta + 2\Delta}{2} \end{pmatrix}.$$ \hspace{1cm} (5)

where $N_+ = \sqrt{\frac{\Delta^2}{4} + \frac{\delta + 2\Delta}{2} \pm \sqrt{\Delta^2 + \delta_i^2}} / 2$, from which we obtain the resonant condition:

$$\omega = \frac{\omega_{ji} + \omega_{\text{TLS}}}{2} \pm \frac{1}{2} \sqrt{(\omega_{\text{TLS}} - \omega_{ji})^2 + \Delta^2 \equiv \lambda \chi}.$$ \hspace{1cm} (6)

In the limit of $\omega_{ji} - \omega_{\text{TLS}} \gg \Delta$, we have $\omega = \omega_{ji}$, which corresponds to the usual two-state Rabi oscillation. Note that in this limit there is also a solution $\omega = \omega_{\text{TLS}}$.

However, in this case the coupling strength is $-\frac{\Delta}{2N_+} \Omega_m \rightarrow 0$. The reason is that although the microwave frequency could match that of TLS, coupling between the microwave and TLS is negligible which is confirmed by the absence of Rabi oscillation between the two states of the TLS in a separate experiment. In the other limit of $\omega_{ji} - \omega_{\text{TLS}} = 0$, we have $\omega = \frac{\omega_{ji} + \omega_{\text{TLS}}}{2} \pm \Delta$, and the dynamics have been thoroughly studied in Ref. 30.

In our experiment, we have $(\omega_{\text{TLS}} - \omega_{ji}) \Delta \approx 100$ MHz and $\Delta 2\pi \approx 70$ MHz, which means LITZ occurs in the region where $(\omega_{\text{TLS}} - \omega_{ji}) \ll \Delta$. Because the frequency of the applied microwave is $\omega = \lambda \chi$, which can be determined from the measured energy spectrum shown in Fig. 2(a), $\vert 0 \rangle g$ is resonantly coupled to $\vert - \rangle$, which is the eigenstate of $H_2 = \omega_{ji} \vert 0 \rangle g (\vert 0 \rangle g + \Delta \vert 0 \rangle g) + \frac{\Delta}{2} (\vert 0 \rangle g + \vert 0 \rangle g).$

Although there is in principle also a coupling between $\vert 0 \rangle g$ and $\vert + \rangle$, the effective coupling is much smaller because of the large detuning, as discussed below.

For $\Delta 2\pi \approx 70$ MHz, $(\omega_{\text{TLS}} - \omega_{ji}) 2\pi \approx 100$ MHz, the resonance between $\vert 0 \rangle g$ and $\vert - \rangle$ is at $\Delta \approx 116$ MHz. In our experiments, the coupling strength between $\vert 0 \rangle g$ and $\vert - \rangle$ is about 20 MHz and that between $\vert 0 \rangle g$ and $\vert + \rangle$ would be $\frac{\lambda}{N_+} \approx 20$ MHz $\approx 6.3$ MHz. Because 6.3 MHz is comparable with 20 MHz, one may think that coupling between $\vert 0 \rangle g$ and $\vert + \rangle$ cannot be neglected. However, there is also a large detuning of about 122 MHz between $\vert 0 \rangle g$ and $\vert + \rangle$. Therefore, the effective coupling between $\vert 0 \rangle g$ and $\vert + \rangle$ is reduced to $(6.3 / 122) \approx 0.33$ MHz and thus can be safely neglected. To be more precise, we calculate the population $P_g$ (where $P_g$ is the population of state $\vert g \rangle$) after the application of a $\pi$ pulse numerically, and it is found that $P_g / P_e \approx 5 \times 10^{-5}$. Based on this analysis, when $\omega = \lambda \chi$, the dynamics can be described by the Hamiltonian in the subspace $\{\vert 0 \rangle g, -\rangle\}$:

$$H_3 = \begin{pmatrix} -\frac{\delta + 2\Delta}{2} & -\frac{\Delta}{4N_+} \Omega_m \\ -\frac{\Delta}{4N_+} \Omega_m & -\frac{1}{2} \sqrt{\Delta^2 + \delta_i^2} \end{pmatrix}.$$ \hspace{1cm} (7)

At the resonance $-\frac{\delta + 2\Delta}{2} = -\frac{1}{2} \sqrt{\Delta^2 + \delta_i^2}$, $H_3$ becomes

$$H_3 = -\frac{1}{2} \sqrt{\Delta^2 + \delta_i^2} \times 1 + \begin{pmatrix} -\frac{\Delta}{4N_+} \Omega_m & 0 \\ 0 & -\frac{\Delta}{4N_-} \Omega_m \end{pmatrix}.$$ \hspace{1cm} (8)

where $I$ is a $2 \times 2$ identity matrix.

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**Figure 5** | The effect of $t_{\text{MW}}$ on finite time LITZ (a) Experimentally measured and (b) numerically calculated $P_i$ vs. $t_{\text{MW}}$ and $t_p$ for $\epsilon / 2\pi = 200$ MHz showing that the effect of imprecise $\pi$ pulse is to reduce the visibility of the oscillation of $P_i$ vs. $t_p$ by reducing the probability amplitude of the desired $\vert - \rangle$ state. Because the microwave pulse is resonant with the $\vert 0 \rangle g \rightarrow \vert - \rangle$ transition while largely detuned from the $\vert 0 \rangle g \rightarrow \vert + \rangle$ transition even a significant deviation from a $\pi$ pulse would only result in negligible transfer of population to $\vert + \rangle$. Furthermore, since $\vert 0 \rangle g$ does not participate in the single passage LZ process, the observed oscillation could neither be due to LZS interference nor non-negligible population in $\vert + \rangle$ at the beginning of each $\epsilon$ sweep. For comparison, we also present the numerically calculated $P_i(t_{\text{MW}}, t_p)$ for $\epsilon / 2\pi = 1450 \approx 20.7\Delta$ in (c). The result shows the exponential decay behavior described by the asymptotic LZM formula as expected for $\epsilon / \Delta > 1$.

For initial state $\Psi(t = 0) = \vert 0 \rangle g$, the amplitude of $\vert - \rangle$ is

$$C_+ = i \sin \frac{\Delta}{4N_-} \Omega_{\text{mf}}.$$ \hspace{1cm} (9)

Considering $\vert - \rangle = -\frac{\Delta}{2N_-} \vert 0 \rangle g + \frac{\delta_i + \sqrt{\Delta^2 + \delta_i^2}}{2N_-} \vert 0 \rangle e$, the amplitude of $\vert 1 \rangle$ is

$$C_1 = C_{-1} + C_{+1} = C_1 = -\frac{\Delta}{2N_-} \sin \frac{\Delta}{4N_-} \Omega_{\text{mf}}.$$ \hspace{1cm} (10)

Therefore, with a microwave pulse of duration $t_{\text{MW}}$ which is used to prepare the initial state, we have

$$P_i = |C_1|^2 = \Delta^2 \left( \sin \frac{\Delta}{4N_-} \Omega_{\text{mf}} t_{\text{MW}} \right)^2.$$ \hspace{1cm} (10)

This is the reason why in the experiment we observe a usual Rabi oscillation instead of
Effect of $\nu MW$ and $\nu$ on an LZT. In this section, we discuss two factors that may affect the LZT probability, i.e., the width of the microwave pulse $\nu MW$ used to prepare the initial state at $t=\nu$ and the end of the normalized diabatic energy sweeping $\nu$,$\Delta$, respectively.

After a microwave pulse, by projecting into the subspace $|\uparrow\rangle \pm |\downarrow\rangle$, the system is in the eigenstate $|\phi_t\rangle$. Then the dynamics of $\Lambda$ can be described by $H_0$ with a time-dependent $\omega_{\nu}(t)$, i.e.,

$$H_0 = \left( \frac{\omega_{\nu}(t)}{\Delta/2} \right) ,$$

To investigate the Landau-Zener diffusion factor, we sweep $\omega_{\nu}(t)$ across $\omega_{\nu}t$, i.e.,

$$\omega_{\nu}(t) = \omega_{\nu}(t=0) + vt, \quad (0 \leq t \leq \nu) .$$

When $\omega_{\nu}(t) = \nu MW$, we expect that the Landau-Zener asymptotic formula holds and $P_1 = P_{1\nu}$. Thus no oscillations should occur in $P_1$. This is confirmed by the result of numerical simulation shown in Fig. 5(c), where $\omega_{\nu}t = 1450 \text{ MHz} \approx 20.7\alpha$, reproducing the exponential decay behavior described by the asymptotic LZ formula independent of the initial state of the qubit-TLS system. In this case, the population in the qubit state $|\uparrow\rangle$ can be expressed as

$$P_1 \propto \frac{\Delta^2}{4N^2} \left( \sin \frac{\Delta}{4N} \frac{\omega_{\nu}t}{\Delta} \right)^2 \propto \frac{1}{\nu^2} \propto e^{-4(t_{\nu} + \Delta)} .$$

where the first term reflects the effect of microwave duration $\nu MW$ in preparing the initial state, the second term corresponds to the LZT probability, and the third term represents the relaxation effect. However, as $\omega_{\nu}(t) = \nu MW$ moves towards $\omega_{\nu}t$, the situation $\omega_{\nu}(t=\nu MW) - \omega_{\nu}t \Delta \approx \Delta$ does not hold any more, and we observe oscillation features in the $t_{\nu}$ direction, as shown in Fig. 5(a) (experiment) and 5(b) (numerical simulation). Notice that Fig. 5(a) and 5(b) also confirm that the effect of imprecise pulse is an incomplete transfer of system from $|\uparrow\rangle$ to $|\downarrow\rangle$, which reduces the probability amplitude of $|\downarrow\rangle$ from the maximum value, instead of resulting in non-negligible probability amplitude in the unwanted $|\uparrow\rangle$ state.

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