Three-Fluid Description of the Sympathetic Cooling of a Boson-Fermion Mixture

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Abstract

We present a model for sympathetic cooling of a mixture of fermionic and bosonic atomic gases in harmonic traps, based on a three-fluid description. The model confirms the experimentally observed cooling limit of about $0.2 T_F$ when only bosons are pumped. We propose sequential cooling – first pumping of bosons and afterwards fermions – as a way to obtain lower temperatures. For this scheme, our model predicts that temperatures less than $0.1 T_F$ can be reached.
I. INTRODUCTION

Recent attempts to cool down a trapped gas of fermionic atoms, such as $^{40}$K [1] or $^{6}$Li [2,3], have reached temperatures below the Fermi temperature. Degenerate atomic Fermi gases offer the intriguing possibility to create a paired fermion state such as a Bardeen-Cooper-Schrieffer phase in a novel and highly controllable system. However, until now the temperatures obtained in the experiments with ultracold fermionic atoms are not low enough to observe such phenomena. In this paper, we propose a straightforward description of the cooling mechanism, discuss its inherent limitations, and we investigate strategies to alter the cooling procedure in order to achieve lower temperatures.

The cooling process used to create Bose-Einstein condensates in trapped bosonic gases is evaporative cooling, whereby the most energetic atoms are removed from the gas, leaving the remnant colder after rethermalization. However, due to the antisymmetrization requirement of the fermionic wave function, identical fermionic atoms cannot undergo the $s$-wave collisions necessary for rethermalization, and thus they cannot be directly cooled using evaporative cooling. This problem was circumvented by simultaneously trapping two atom species or two different hyperfine spin states of a given atom species. In [2], $^{6}$Li is trapped together with its bosonic isotope $^{7}$Li. Whereas the $s$-wave scattering length of $^{6}$Li is zero by symmetry, the $s$-wave scattering length for a collision between a $^{6}$Li and a $^{7}$Li atom is 2.2 nm, which is enough to thermalize both species together. This version of evaporative cooling is denominated ‘sympathetic cooling’.

We describe the sympathetic cooling process, using the three-fluid model presented in Sec. II, as a sequence of evaporation and rethermalization steps so that in the evaporation steps the energetic atoms are removed from the mixture and in the rethermalization steps the new (lower) equilibrium temperature is reached, as explained in Sec. III. The results obtained with this model are discussed in Sec. IV, and an improvement on the current sympathetic cooling process is proposed to reach lower temperatures ($< 0.1 \, T/T_F$).

II. THREE-FLUID MODEL

The trapped system which will be subjected to sympathetic cooling consists of $N_b$ bosons with mass $m_b$ and scattering length $a_b$; and $N_f$ fermions with mass $m_f$, all at initial temperature $T$ in a parabolic trap with trapping frequency $\omega$ and characteristic length $a_{HO_{b,f}} = \sqrt{\hbar/ (m_{b,f} \omega)}$. The scattering length for a collision between a boson and a fermion will be denoted by $a_f$. The three-fluid model [4] distinguishes the fermion gas, the Bose-Einstein condensed (BEC) bosons and the non-condensed or ‘thermal’ bosons. The number of BEC bosons is $N_c = N_b (1 - (T/T_C)^3)$ for $T < T_C$ and zero otherwise, where [5]

$$T_C = \frac{\hbar \omega}{k_B} \sqrt{\frac{N_b}{\zeta(3)}} \left( 1 - 0.73 N_b^{-1/3} - 1.33 \frac{a_b}{a_{HO_b}} N_b^{1/6} \right). \quad (1)$$

The number of thermal bosons is $N_t = N_b - N_c$. We use the notation

$$g = \frac{4\pi \hbar^2 a_b}{m_b} \quad \text{and} \quad f = \frac{4\pi \hbar^2 a_f}{m_f} \quad (2)$$
where \( m_r = 2m_b m_f / (m_b + m_f) \) is twice the reduced mass of a fermion and a boson. \( n_c(r) \) is the density of BEC bosons, \( n_t(r) \) is the density of thermal bosons, and \( n_f(r) \) is the density of fermions. In the three-fluid model, the BEC bosons, the thermal bosons and the fermions are described as ideal gases in effective one-body potentials. The effective one-body potential for the BEC bosons \((V_c)\), for the thermal bosons \((V_t)\) and for the fermions \((V_f)\) are given by:

\[
V_c(r) = \frac{m_f \omega^2}{2} r^2 + gn_c(r) + 2gn_t(r) + fn_f(r) \\
V_t(r) = \frac{m_b \omega^2}{2} r^2 + 2gn_c(r) + 2gn_t(r) + fn_f(r) \\
V_f(r) = \frac{m_f \omega^2}{2} r^2 + fn_c(r) + fn_t(r)
\]

These potentials determine the densities

\[
n_c(r) = \frac{1}{g} \left[ \mu_b - \frac{m_f \omega^2}{2} r^2 - 2gn_t(r) - fn_f(r) \right] \\
n_t(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp \left[ \beta \left( \frac{h^2k^2}{2m_b} + V_t(r) - \mu_b \right) \right] - 1} \\
n_f(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{1 + \exp \left[ \beta \left( \frac{h^2k^2}{2m_f} + V_f(r) - \mu_f \right) \right]}
\]

In these expressions, \( \beta = 1/(k_B T) \) and \( \mu_f, \mu_b \) is the chemical potential of the fermion gas and boson gas respectively. The set of equations (3)-(5) and (6)-(8) form a self-consistent description of the three fluids, in which the chemical potentials are set by fixing the total number of fermions and the total number of bosons; \( N_b = \int d^3r n_c(r) + n_t(r) \) and \( N_f = \int d^3r n_f(r) \). Note that the interaction between the bosonic atoms, and the interaction between the bosonic and fermionic atoms, is taken into account on the level of the mean-field approximation.

The total energy of the BEC bosons \((E_c)\), of the thermal bosons \((E_t)\) and the fermions \((E_f)\) is given by

\[
E_c = \int d^3r \left[ \frac{m_f \omega^2}{2} r^2 n_c(r) + \frac{1}{2} g [n_c(r)]^2 \right] \\
E_t = \int d^3r \left[ \int \frac{d^3k}{(2\pi)^3} \frac{\frac{m_f \omega^2}{2} r^2 + \frac{m_b \omega^2}{2} k^2}{\exp \left[ \beta \left( \frac{h^2k^2}{2m_t} + V_t(r) - \mu_b \right) \right] - 1} + g [n_t(r)]^2 + 2gn_t(r) n_c(r) \right] \\
E_f = \int d^3r \left[ \int \frac{d^3k}{(2\pi)^3} \frac{\frac{m_f \omega^2}{2} r^2 + \frac{m_f \omega^2}{2} k^2}{\exp \left[ \beta \left( \frac{h^2k^2}{2m_f} + V_f(r) - \mu_f \right) \right] + 1} + fn_f(r) (n_c(r) + n_t(r)) \right].
\]

From the above set of equations (3)-(11), the temperature, and the chemical potential of the bosons and the fermions can be calculated if the total numbers of bosons and fermions,
and their total energy are given. Note that the Bose-Einstein condensate is treated in the Thomas-Fermi approximation as if it were at zero temperature. For the thermal bosons we have experimented using the Maxwell-Boltzman distribution instead of a Bose-Einstein distribution in the expressions for the density (7) and the energy (10). In the experiment, this approximation is routinely performed in order to determine the temperature of a trapped atomic gas by fitting the Maxwell-Boltzman distribution to the tail of the measured velocity distribution or density profile. Also in our calculations, we found that the Maxwell-Boltzman distribution provides an accurate description of the thermal bosons.

III. SYMPATHETIC COOLING

We describe the cooling process as a two-step process. The first step is the evaporative stage, whereby either just the bosons can be ‘evaporated’, or just the fermions, or both bosons and fermions. The effect of the evaporative stage is truncating the radial distribution by removing the outer atoms. The energy of these atoms is higher than the average energy of the mixture and so this removal allows us to obtain a lower temperature. The second step is the rethermalization stage, where the collisions between the bosons and between bosons and fermions bring the truncated (non-equilibrium) distribution obtained in the evaporative stage into a new equilibrium distribution. This new equilibrium distribution will be at a lower temperature if the atoms removed have more energy than the average atom energy. We will describe the sympathetic cooling as a discrete succession of evaporation and rethermalization stages. In reality, the evaporation and rethermalization goes on simultaneously. However, previous studies have shown that the discrete treatment of the cooling process provides an accurate description [6].

The evaporation is realized by a radiofrequent (or microwave) field which induces a transition between the trapped hyperfine state of a selected atom species and an untrapped hyperfine state. The frequency suited for this transition depends on the distance from the atom to the center of the trap, due to the Zeeman effect of the spatially varying magnetic trapping field. By tuning the frequency, atoms that are beyond a cut-off distance $R$ from the center of the trap are removed. If the bosons are evaporated in this way, then the number of bosons which are removed from the mixture is

$$\Delta N_t = \int_{r>R} d^3 r \ n_t(r),$$

(12)

and the total energy removed in the evaporation is

$$\Delta E_t = \int_{r>R} d^3 r \ \left[ \int \frac{d^3 k}{(2\pi)^3} \ \exp \left[ \beta \left( \frac{\hbar^2 k^2}{2m} + V_t(r) - \mu_b \right) \right] - 1 \right] + g [n_t(r)]^2 + f n_f(r) n_t(r).$$

(13)

If fermionic atoms are removed from the mixture, then the corresponding number of removed fermions $\Delta N_f$ and the total evaporated energy of the fermion gas $\Delta E_f$ are found from
expressions analogous to those above. Note that evaporating the Bose-Einstein condensed bosons will not lead to cooling. The Bose-Einstein condensed bosons are always the lowest-energetic bosons in the mixtures - removing them would only lead to heating of the system. Thus it is important that the cut-off distance $R_C$ is larger than the Thomas-Fermi radius of the condensate.

The evaporative stage results in a new total number of bosons $N'_b = N_b - \Delta N_b$, and a new total energy $E'_{\text{total}} = E_c + E_t + E_f - \Delta E_t$, if only bosons are removed from the trap. If also fermions are evaporated, $\Delta E_f$ has to be subtracted too, and $N'_f = N_f - \Delta N_f$ is the new number of fermions. Using the equations (3)-(11), the new system parameters $N'_f$, $N'_b$, $E'_{\text{total}}$ are expressed as a function of the new temperature $T'$ and the new chemical potentials $\mu'_b$ and $\mu'_f$. The new temperature and chemical potentials are then solved from these equations. After this rethermalization stage, a new evaporative stage can be initiated.

Losses due to three-body collisions lead to heating, since the three-body collisions will be prominent in the condensate which harbors the coldest atoms. Such losses introduce another depletion of the number of atoms. For the bosons,

$$\Delta N_b^{\text{heat}} = K_3 t \int dr \ [n_c(r)]^3,$$

$$\Delta E_b^{\text{heat}} = K_3 t \int dr \ [n_c(r)]^3 \left( \frac{m_b \omega_r^2}{2} r^2 + g n_c(r) + 2 g n_t(r) + f n_f(r) n_c(r) \right).$$

where $K_3$ is the three-body collisional cross section and $t$ is the time duration of one step. For $^7\text{Li}$, $K_3$ is about $2.6 \times 10^{-28} \text{cm}^6/\text{s}$ [7]. This additional loss process can be taken into account during the evaporative stage.

IV. RESULTS AND DISCUSSION

A. Evaporating the bosonic atoms

In the experiments on sympathetic cooling by Schreck et al. [3,8] a mixture of bosons and fermions is cooled from 2 mK to around 9 $\mu$K, starting with $2 \times 10^8$ $^7\text{Li}$ bosons and about $10^6$ $^6\text{Li}$ fermions, in an anisotropic trap ($\omega_r = 2\pi \times 4050 \text{ Hz}$, $\omega_a = 2\pi \times 71.17 \text{ Hz}$). We have simulated this process and compared with the experimental data [8]. The suited number of steps can be determined by noting that the time needed for the system to rethermalize is 200 ms and that the reported cooling process takes 35 s [8]. The number of 200 ms steps must be of the order of 100-200. For the evolution of the cut-off radius in time, we have used the same as in [8]. The agreement between theory and experiment, shown in figure 1, is reasonably good.

In the next simulation we have started from 9 $\mu$K, trying to cool further by evaporating only bosons, as is done in the experiment reported in Ref. [3]. We simulate the evaporative cooling of a system, starting with $N_f = 4 \times 10^3$ fermions and $N_b = 10^6$ bosonic atoms at an initial temperature of $T = 9$ $\mu$K which is 6.12 times the Fermi temperature for this amount of fermions. Figure 2 shows the cooling process as a function of the time (number of evaporation-rethermalization steps), when the evaporation is performed at decreasing cutoff radius starting from 20 $a_{HO}$ and decreasing with 0.32 $a_{HO}$ each time step. The lowest
temperature which is obtained is 0.22 $T_F$, in agreement with the temperature reported by Schreck \textit{et al.} [3]. We have performed the simulation for different values of the three body boson decay constant ($K_3$) and one can see that in our model there is only an effect on the number of bosons, not on the final temperature.

Figure 3 illustrates the influence of the initial cut-off radius on the cooling process, showing the temperature at step 10,20,30,40 and 50 of the cooling process, as a function of the initial cut-off radius. During each step the cut-off radius was decreased by 0.32 $a_{HO}$. An optimal radius exists around 18 $a_{HO}$, which is small enough to remove enough bosons, but sufficiently large in order not to remove too many bosons in the initial steps and not being energy selective. The simulations have a maximum of 50 steps or stop when the cut-off radius becomes smaller than the condensate. In figure 4 we investigate how the temperature obtained by cooling is affected by reducing the cut-off radius with different amounts during the cooling process. At each time step, the cut-off radius is decreased by a size $a$. The different curves in the figure show the temperature as a function of time (step in the cooling process) at different values of $a$. We find that a faster rate of decrease of the radius leads to a faster cooling, but that no lower temperatures can be reached. If the radius is decreased too fast, it reaches the radius of the condensate too soon, and the cooling process is stopped. For a fixed time of the cooling process, which is determined by the loss rate of the trap, an optimal value of $a$ can be determined. For the 50 steps that we took in figure 4, this is $a$ between 0.32 and 0.6 trap oscillator lengths. Figure 5 shows the dependence of the temperature on the initial number of bosons, keeping the number of fermions fixed. We find the counterintuitive result that the initial ratio between bosons and fermions has only a minor effect on the final temperature that can be reached: it is always around 0.2 $T_F$. This can be understood by noticing that the number of thermal bosons (which are responsible for the cooling) is almost independent of the total number of bosons for temperatures below the condensation temperature. ($N_t = N_b \left( \frac{T}{T_c} \right)^3 \approx T^3 \zeta(3)$).

Truscott \textit{et al.} [2] note that the cooling process seems to stall as soon as the specific heat of the bosons is smaller than the specific heat of the fermions, because the energy which can be carried away by evaporating a bosonic atom then becomes smaller than the energy of a fermionic atom. This result is confirmed by the present simulation, which explains why the final temperature is insensitive to the initial ratio of bosons to fermions. Basically, when the number of thermal bosons is small compared to $N_F$, the energy which can be carried away by evaporating the bosons is small compared to the energy of the Fermi gas. The temperature does not decrease significantly and the bosons are quickly lost, because the spatial extent of the cloud remains big.

### B. An improved cooling process: sequential evaporation

This leads us to suggest another cooling process. When evaporative cooling by removing bosons has led to a final temperature, and continued evaporation of the bosons no longer leads to substantial further cooling, we suggest to switch to evaporating the fermions, or fermions and bosons. Removing the bosons from the trap does not cool the system any more because the bosons can no longer carry away the necessary energy - but the fermions still can.
Assume that by the evaporation of bosons, the temperature has already decreased to 0.3 $T_F$, and there are $N_b = 5 \times 10^4$ and $N_f = 5 \times 10^4$ atoms left in the trap. This system is experimentally achievable by evaporating bosons alone. Figure 6 shows the resulting temperature as a function of time, after switching from evaporating the bosons to evaporating only the fermions. The cut-off radius was chosen at $R_C = 15 a_{HO}$ and decreases each time step. In this way the temperature can be lowered to $0.06 T_F$. As also illustrated in the figure, the number of bosons has almost no influence on the cooling process.

Losses due to three particle collisions involving two bosons and one fermion is expected to limit the achievable temperature, as pointed out by Timmermans [9]. The effect of this type of loss can be investigated in the present formalism in the same way as the effect of the three-boson collisions. The results are shown in the inset of figure 6, for a cooling process starting with $5 \times 10^4$ bosons and $5 \times 10^4$ fermions. The different curves correspond to different values of the three particle recombination constant $K_{3BF}$. If the product of the time to achieve equilibrium ($t$) and the recombination constant is $K_{3BF}t \lesssim 10^{-7}$ we find that the lowest achievable temperature is raised by less than 0.05 $T_F$. For $K_{3BF}t \lesssim 10^{-8}$ there is no appreciable effect on the final temperature of the cooling process.

V. CONCLUSIONS

We have proposed a versatile, and straightforwardly implementable three-fluid model to describe the sympathetic cooling of bosons and fermions. Our model reproduces well the cooling observed in an experiment with $^6$Li and $^7$Li, and we find that the final temperature $T/T_F \approx 0.2$ which is reached is not sensitive to the initial ratio of the number of bosons to the number of fermions. Lower temperatures can be reached if the stage where the only the bosonic atoms are evaporated is followed by a stage where also the fermionic atoms are evaporated. When this procedure is applied, a temperature of $T/T_F = 0.06$ could be reached, which in combination with the tunability of the interaction strength through the Feshbach resonance, brings the Fermi gas closer to temperatures where pairing may be observed.

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FIGURES

FIG. 1. The temperature (upper panels) and the number of bosons (lower panels) as a function of the $^7$Li cut energy, starting with $2 \times 10^8$ bosons and $10^6$ fermions. The cut off radius decreases in time. The experimental values are taken from [8].

FIG. 2. The temperature in units of the Fermi temperature is shown for cooling processes with decreasing cut-off radius, with and without three-body decay for the condensate. The initial cut-off radius is $20 \ a_{HO}$ and decreases with $0.32 \ a_{HO}$ each time step. The initial numbers of bosons and fermions are $N_b = 10^6$, $N_f = 4 \times 10^3$.

FIG. 3. The temperature in units of the Fermi temperature is shown at time steps 10, 20, 30, 40 and 50 of the cooling process, as a function of the initial radius $R_0$. At each time step, the cut-off radius is reduced by $0.32 \ a_{HO}$. The initial numbers of bosons and fermions are $N_b = 10^6$, $N_f = 4 \times 10^3$.

FIG. 4. The ratio of the temperature to the Fermi temperature is shown as a function of time (step) in the cooling process. At each step of the cooling process the cut-off radius is reduced by a value $a$. The different curves represent different rates of reduction $a$. The initial radius is always $20 \ a_{HO}$. The initial numbers of bosons and fermions are $N_b = 10^6$, $N_f = 4 \times 10^3$.

FIG. 5. The ratio of the temperature to the Fermi temperature for cooling processes with decreasing radius, for different numbers of initial bosons ($N_{B,0}$). The radius starts at $20 \ a_{HO}$ and decreases with $0.32 \ a_{HO}$ each time step.

FIG. 6. Last steps of a sequential sympathetic cooling process. The temperature relative to the Fermi temperature, $T/T_F$ is shown. For (a)-(c) the number of bosons is 50, 000, for (d), it is 100, 000. The radius is $R = (15 - 0.1 \times \text{step})a_{HO}$ for (a), $R = (15 - 0.2 \times \text{step})a_{HO}$ for (b) and $R = (18 - 0.2 \times \text{step})a_{HO}$ for (c). For (d) the radius is the same as in (a). The inset shows the influence of the loss due to three body collisions involving two bosons and one fermion on $T/T_F$, for the parameters of the best process (a). The considered values of $K_{3BFt}$ are $0$, $10^{-8}$, $10^{-7}$, $5 \times 10^{-7}$ and $10^{-6}$. 

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FIGURE 1

with $10^6$ fermions

no fermions

$\text{Li cut energy (mK)}$

$N \times 10^6$

$T (\mu \text{K})$

100 steps

200 steps

experiments Schreck et. al.
FIGURE 2

- without losses
- $K_3 t = 2.4 \times 10^{-8}$
- $K_3 t = 10^{-5}$

(harmonic oscillator units)
FIGURE 3

- step 10
- step 40
- step 20
- step 50
- step 30

$a = 0.32 a_{HO}$
FIGURE 4
$N_F = 4 \times 10^3$

$N_{B,0} = 10^6$

$N_{B,0} = 5 \times 10^6$

$N_{B,0} = 5 \times 10^5$

FIGURE 5
(a) $R_0 = 15; a = 0.1$

$N_{B,0} = 5 \times 10^4$

(b) $R_0 = 15; a = 0.2$

$N_{B,0} = 5 \times 10^4$

(c) $R_0 = 18; a = 0.2$

$N_{B,0} = 5 \times 10^4$

(d) $R_0 = 15; a = 0.1$

$N_{B,0} = 10^5$