A Bi-Level Scheduling Model of the Distribution System With a Distribution Company and Virtual Power Plants Considering Grid Flexibility

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ABSTRACT With the emergence of virtual power plants (VPPs) in distribution systems, distribution companies (DISCOs) can manage distributed generators scattered across their systems. Interactions between each DISCO and VPP are expected to be crucial for securing system reliability without increasing operational complexity. This article proposes a bi-level scheduling model for these interactions, considering the ramping flexibility of the distribution system. In the upper level, the DISCO minimizes the distribution system’s operational cost and the cost of procuring flexibility by varying the price of the power traded with the VPP. In the lower level, the VPP maximizes its profit by managing the amount of power it trades with the DISCO. We transform the bi-level model into a single-level problem by using Karush–Kuhn–Tucker optimality conditions and linearization techniques. We present case studies of interactions between the DISCO and VPPs, with the assumption that the DISCO can purchase active power from a wholesale market. The results demonstrate that a win–win situation for both the DISCO and VPP can be achieved by managing the power exchanged while procuring flexibility.

INDEX TERMS Virtual power plant (VPP), bi-level programming, flexibility, distribution system, electricity market.

NOMENCLATURE

A. INDICES

i, j Index of bus in [1 : N^B].
t Index of time step in [1 : T].
g Index of distributed generation unit at each bus i owned by the distribution company in [1 : N^G].
k Index of virtual power plant at each bus i in [1 : N^VPP].
k_g Index of distributed generation unit aggregated by the virtual power plant k in [1 : N^DG].
l Index of distribution line in [1 : N^L].
n Index of segment in [1 : N^seg].

B. PARAMETERS

\( \pi_{PCC}^t \) Energy price of the wholesale market at time t. Energy is traded via point of common coupling.

\( c_{g,n} \) Unit production cost of \( n^{th} \) segment of distributed generator \( g \).

\( r_{ij}/x_{ij} \) Resistance/reactance of distribution line \( ij \).

\( \alpha_{PCC}^i \) Indicator for point of common coupling at bus \( i \).

\( p_{L,i,t}^f/q_{L,i,t}^f \) Active/reactive power load in bus \( i \) at time \( t \).

\( S_{ij,max} \) Line capacity of distribution line \( ij \).

\( V_{i,min}/V_{i,max} \) Minimum/maximum voltage of bus \( i \).

\( p_{PCC}^{L,\min}/p_{PCC}^{L,\max} \) Minimum/maximum active power flow limit via point common coupling at time \( t \).

\( P_{D,g,min}^D/P_{D,g,max}^D \) Minimum/maximum output limit of distributed generator \( g \) owned by distribution company.

\( RR_g \) Ramp rate of distributed generator \( g \).

\( S_{g,max} \) Maximum capacity of distributed generator \( g \) owned by distribution company.
C. VARIABLES

| Symbol | Description |
|--------|-------------|
| $T_{RES}^{i,t}$ | Forecasted electrical power of renewable energy in bus $i$ at time $t$. |
| $P_{ess, ch}^{i, min} / P_{ess, ch}^{i, max}$ | Maximum charging/discharging capacity of energy storage system at bus $i$. |
| $E_{ess, i, min} / E_{ess, i, max}$ | Minimum/maximum stored energy of energy storage system at bus $i$. |
| $n_i^{ess}$ | Efficiency of energy storage system at bus $i$. |
| $FRU_t / FRD_t$ | Required up/down flexibility of distribution system at time $t$. |
| $c_{kg,k}$ | Unit production cost of distributed generator $kg$ aggregated by virtual power plant $k$. |
| $PM_{k, min} / PM_{k, max}$ | Minimum/maximum profit margins for the distribution company exchanging power with virtual power plant $k$. |
| $u_{k,n}$ | Utility function of $n^{th}$ segment of demand response of virtual power plant $k$. |
| $S_{ld,fix}^{k,t} / |q_{k,t}|$ | Power factor of virtual power plant $k$. |
| $P_{DG, kg, min} / P_{DG, kg, max}$ | Minimum/maximum active power limit of distributed generator $kg$ in virtual power plant. |
| $Q_{DG, kg, min} / Q_{DG, kg, max}$ | Minimum/maximum reactive power limit of distributed generator $kg$ in virtual power plant. |
| $T_{RES}^{k,t}$ | Forecasted electrical power of renewable energy in virtual power plant $k$ at time $t$. |
| $P_{ld, min}^{k,t} / P_{ld, max}^{k,t}$ | Minimum/maximum load energy limit in virtual power plant $k$ at time $t$. |
| $\delta^{VPP}$ | Maximum capacity of virtual power plant $k$. |
| $\lambda_{(1-18),(k,kg),t} / |\mu_{(1-18),(k,kg),t}$ | Lagrange multipliers of the Karush-Kuhn-Tucker condition. |

D. BINARY VARIABLES

| Symbol | Description |
|--------|-------------|
| $\delta_{ess}^{i,t}$ | Status of charging/discharging operation of energy storage system at bus $i$ and time $t$. |

I. INTRODUCTION

Modern power system operators should cope with challenges in managing power systems related to the interconnection of various distributed energy resource (DER) technologies, such as intermittent and uncontrollable generation, controllable and flexible demands, and energy storage [1]. With the widespread penetration of new technologies, unidirectional or radial supply systems in the transmission and distribution level are undergoing changes into bidirectional or loop supply systems [2]. Although DER technologies can be helpful for allowing system operators to secure supply to customers through these bidirectional systems, the reliable monitoring and control of DERs should be available to operators to ensure effective utilization [3]. To improve efficiency, the concept of a virtual power plant (VPP) was introduced to facilitate aggregated DERs that could be measured and controlled in a manner similar to conventional bulk generators [4]. System operators and aggregators can utilize VPPs to improve their operation strategies.

Microgrids are introduced in the distribution system with increase in the DERs. The microgrids are formed by the disconnection from the distribution system when a fault occurs in the adjacent systems [5], [6]. In the disconnected system, the microgrid operator can maintain the balance between the
supply and the demand with its own resources. Although both the microgrids and the VPPs are developed as an efficient way to manage the distribution system, there are differences between the two forms. The VPPs cannot be disconnected from the distribution system like the microgrids. In other words, the VPPs are always in the grid-connected form and rely on the software systems coordinating the DERs [7], [8].

The VPP can function as an active prosumer unit for maximizing the aggregators’ profit. By grouping producers and consumers, the aggregators can adjust their production and consumption to maximize the net production traded on markets [9], [10]. From the perspective of profit-based VPPs, the aggregators would not be concerned about the stability or reliability of the connected grid. These aggregators are often regarded as commercial VPPs. Alternatively, the VPP can act as a resource that system operators utilize to secure the system’s reliability. The aggregators consider the operational constraints of connected grids and offer services to operators for system stabilization. Contrary to the profit-based VPPs, these aggregators are interested in both profits and system operation [11]. Therefore, these aggregators are regarded as technical VPPs. With the increase in intermittent DERs, technical VPPs are expected to have greater roles in grid operation than are commercial VPPs.

Similar to technical VPPs, the coordination strategies between distribution companies (DISCOs) and VPPs have been researched as an alternative for managing distribution systems. DISCOs are required to manage VPPs, while purchasing energy from an interconnected wholesale market and then delivering this to energy users [12]. The framework of DISCOs interacting with VPPs and the wholesale market has been described by modeling bidding strategies for the DISCOs and VPPs [13], [14]. A DISCO can trade in a wholesale market in a traditional manner, in which the DISCO bids a certain amount of energy to the day-ahead or real-time market; after the market closes, the market clearing result is obtained. These interactions between the DISCO and VPPs include the expectation that, after the VPPs forecast the DER output and commit the aggregated generation to the DISCO, the DISCO determines the amount of generation that each VPP should provide. In this process, the DISCO considers the wholesale market clearing result and determines the value of the energy from the VPP [15].

Bi-level approaches for modeling the behavior of DISCOs and VPPs have been introduced to determine the optimal amount and price of the energy [14], [16]–[19]. Such approaches are appropriate for the optimization of system operations in both types of entities because they have different operational interests [14]. A DISCO in a hierarchical optimization problem would minimize its overall net costs when it supplies the energy to consumers and purchases the energy from the wholesale market and VPPs. During the decision-making process, while considering its interaction with the wholesale market, the DISCO would issue the appropriate price to the VPPs [16]–[18]. The optimization process that the DISCO conducts is also known as the upper-level problem. In contrast to this perspective, the VPPs would minimize their cost or maximize their benefit when the issued price is stated by the DISCO [14], [16]. Along with the VPPs, consumers and prosumers would minimize their energy purchasing cost from the DISCO [17], [19]. During the decision-making process, the VPPs, consumers, and prosumers can control the level of generation or consumption. The optimization process that the VPPs conduct is also known as the lower-level problem. By solving the upper- and lower-level problems, all participants in the distribution system can optimize their costs and revenues.

The dynamic interactions of the VPPs can contribute to the distribution system’s reliability. With the increasing presence of DERs in the distribution system, the proportion of controllable DERs is expected to increase, which could lead to improved system reliability by procuring flexibility for the transmission or distribution systems [20]. Similar to DISCOs, microgrid operators could support interconnected grid operation by managing power exchange and ramping capability [21]. For example, methods for utilizing the ramping capability of VPPs have been proposed to provide flexibility in distribution systems [13], [15], [22]–[24]. Along with the flexibility necessary for operating reliable systems, the new market framework with VPPs promotes liquidity and competition [23], [24]. The active participation of a VPP in a distribution system can contribute to the overall flexibility of the system, including the distribution and transmission systems. The required flexibility in distribution system operation can be secured by the DISCOs who manage such systems; it can also be secured by the use of trading platforms, instead of the local flexibility markets discussed in [20]. Furthermore, as presented in [21], the DISCO in a distribution system connected with a transmission system should mitigate variations in the connection.

In previous studies, it was assumed that the operator in the distribution system could only request the amount of the required flexibility or dispatch its own resources for procuring such flexibility. Contrary to this operator-oriented approach for flexibility, DISCOs are expected to operate flexibly by considering the power exchange with the interconnected system and the generation scheduling of a VPP. Decision processes are required to consider the interactions among wholesale markets, DISCOs, and VPPs; however, no previous study has focused on these under the bi-level approach, as summarized in Table 1. The table compares the operation strategies considering the interactions, flexibility, and network topology in each of the above-mentioned studies [14]–[24]; it also clarifies the major differences between the previous studies and the present study.

In this paper, we introduce a bi-level scheduling model of the distribution system with a DISCO and VPP, in which the DISCO secures flexibility by considering the operation of the VPP and power exchange with the wholesale market. In the proposed model, the DISCO determines the amount of energy traded between itself and the wholesale market, while considering the required flexibility of the distribution system.
and the power flow through the connected line. Additionally, to procure flexibility, the DISCO can issue the optimal energy price to the VPP and indirectly affect the generation of the VPP. The VPP can also decide its amount of energy, based on the price issued by the DISCO. The major contributions of this study are as follows:

1) We propose a bi-level scheduling model of the distribution system with the DISCO and VPPs considering grid flexibility. The bi-level model is formulated as mixed-integer linear programming that can be efficiently solved.

2) The optimal scheduling strategy considers the distribution system’s flexibility via the ramping capability of the DISCO and VPPs.

3) The ramping requirements for distribution networks are analyzed where there are capacity limits on the transferred power between the DISCO and VPPs.

The rest of this paper is organized as follows. The problem structure of the bi-level scheduling model for the DISCO and VPP is described in Section II. The detailed mathematical formulation of the model and decision process are presented in Section III. Section IV describes the solution approach of the decentralized model, and Section V analyzes the results from a case study. Section VI outlines the study’s conclusions.

II. DESCRIPTION OF PROBLEM STRUCTURE

A. ASSUMPTIONS

This study proposes a bi-level scheduling model for the DISCO and VPP, both of which are at the level of the distribution system; one of the distribution buses is connected to the transmission system. The assumptions of the proposed model are:

1) The whole energy system consists of a transmission system and a distribution system. The transmission system is assumed to be managed by the wholesale market operator. The distribution system is assumed to be owned and managed by the DISCO. The DISCO can optimize the operation of its system by interacting with the wholesale market operator and controlling its own resources, including renewable energy sources (RESs), some distributed generation (DG), and an energy storage system (ESS) [17]–[21].

2) The price in the wholesale market is assumed to be obtained from forecasting results. Thus, the DISCO participates in the wholesale market as a price-taker entity because the energy volume of the DISCO’s region is sufficiently small that it does not influence the wholesale market price [25]. Furthermore, only active power is considered in the interactions between the DISCO and the wholesale market.

3) Among resources in the distribution system, photovoltaic generators, wind generators, and demand response (DR) can be aggregated by the VPP. The VPP can manage the aggregated resources based on their operation parameters, in accordance with the price issued by the DISCO [14].

B. MODEL DESCRIPTION

Considering the above assumptions, the proposed business model is depicted in Fig. 1. In this figure, the information and DISCO decision content flows are indicated by red and blue lines, respectively. The electricity flows among the systems, including the VPP and the distribution and transmission systems, are indicated as black lines.
The detailed procedures are:

1) The VPP arranges resources (including photovoltaic and wind generators, as well as DR) into an aggregated profile. Then, it computes the amount of generation and submits the corresponding parameters to the DISCO, based on this computation.

2) The DISCO owns the distribution system; it forecasts renewable energy generation, load, and the system’s operational conditions. Additionally, the DISCO dispatches its own resources and secures the system flexibility. Based on the operational information from the VPP and distribution system conditions, the DISCO establishes the most economic operational scheme for its system. The operational decisions of the DISCO include the pricing result provided to the VPP, scheduling plan provided to the wholesale market operator, and dispatching plan provided to all resources in the distribution system.

3) The wholesale market operator conducts a clearing process for energy and issues decisions on the adequate quantity and price to the DISCO via consideration of the scheduling information submitted by that DISCO and other participants.

4) After deciding the price and scheduling information, the VPP disaggregates the dispatching trajectory to individual resources. Moreover, the DISCO sends the dispatching plan to resources in its own system and issues the pricing results to the VPP.

The proposed model can be realized as a decentralized or centralized decision model. The decentralized decision model reflects the business model of the proposed operational strategy. This model allows interactions among the VPP, DISCO, and wholesale market operator; it will be formulated under the bi-level approach. In contrast, the centralized decision model considers the minimization of the total operating costs across the distribution system. In this model, a single hypothetical entity owns all resources and centrally optimizes the operational decisions in the system. This concept, in which each decision is made by a central entity, is difficult to implement because of the computational complexity involved. In this paper, we also mathematically formulate the centralized model; this will serve as a benchmark to discuss the effectiveness of the proposed approach.

C. DECENTRALIZED DECISION MODEL

The decentralized decision model considers the interests of each participant, considering that the DISCO and VPP have different objectives for their operational strategies. This model focuses on the interaction between the DISCO and VPP. Specifically, the DISCO interacts with the wholesale market operator through the wholesale market price and tie-line information. The bi-level model describes these interactions, as shown in Fig. 2. The upper-level agent (DISCO) issues the energy prices for each time slot to the lower-level agent (VPP). Then, the lower-level agent, responding to this issued price, submits the amount of power to be exchanged to the upper-level agent.

In the upper-level process, the DISCO makes optimal decisions to minimize the operational cost of the distribution system. This consists of the production and start-up or shut-down costs of its own resources, along with the wholesale market and VPP costs. In this level, the upper-level agent could consider operational constraints, including the price limit for the VPP, power flow in the distribution system, generation limit of its own resources, tie-line flow limit, and flexibility of the distribution system. While considering the network topology of the distribution system connected to the transmission system, the DISCO decides the price for the VPP and establishes dispatch commands for the DISCO-owned resources.

In the lower-level process, the VPP agent maximizes its own profit and decides the amount of power to be exchanged by considering the power price result issued from the DISCO. The lower-level agent considers its operational constraints, including the power balance of the aggregated resources and operational model for the RESs and DR.

D. CENTRALIZED DECISION MODEL

In contrast to the decentralized decision model, the centralized model assumes that the VPP no longer aggregates resources. In this model, instead of the VPP, the DISCO can dispatch all resources in its own system. The DISCO minimizes its operational cost by deciding the amount of generation and addressing part of the load via DR. The power price issued by the DISCO no longer applies. Overall, the objective is the minimization of the total cost when only one entity (i.e., the DISCO) procures electrical energy generation and the flexibility of the distribution system while ensuring the system’s reliability.

III. MATHEMATICAL FORMULATION

A. DECENTRALIZED DECISION MODEL FOR DISCO

The upper-level problem minimizes the operating costs of the distribution system, while matching supply and demand;
it also satisfies various system operational constraints. The objective function of the DISCO consists of the revenue from selling power to the VPPs, generation cost of the DG, and power purchasing cost from the wholesale market. This function is formulated as:

\[
p_{\text{PCC},t}^{\text{DISCO}} = \min \left\{ \sum_i P_{i,t}^{\text{DG}} + \sum_{k \in \text{VPP}} P_{k,t}^{\text{VPP}} + \sum_{i,j} F_g(P_{g,i}^D) + \sum_{i} \left( \pi_{i,\text{PCC},t} V_{i} + \pi_{i,\text{VPP},t} V_{i} \right) \right\}
\]

where \( F_g(\cdot) \) is the operation cost of DG \( i \) formulated by a piecewise linear function:

\[
P_{i,t}^{\text{DG}} = \sum_{g} c_{g,n} \cdot P_{g,i}^{D} \quad \forall g, t \tag{2}
\]

\[
P_{g,i}^{D} = \sum_{v} P_{v^i,g}^{D,\text{VPP}} \quad \forall g, i \tag{3}
\]

The following constraints are required to model the upper-level problem. Note that some constraints are nonlinear which will be linearized later in Section IV.

1) POWER FLOW EQUATION

The distribution system topology and power flow constraints are modeled from a linearized distribution network model proposed in [14]. The active and reactive power injection of bus \( i \) at time \( t \) are represented by \( P_{i,t} \) and \( Q_{i,t} \) respectively, formulated as:

\[
P_{i,t} = \sum_{j=1, j \neq i}^{N_B} \left( \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (V_{i,t} - V_{j,t}) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_{i,t} - \theta_{j,t}) \right), \quad \forall i, t \tag{4}
\]

\[
Q_{i,t} = \sum_{j=1, j \neq i}^{N_B} \left( \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (V_{i,t} - V_{j,t}) - \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_{i,t} - \theta_{j,t}) \right), \quad \forall i, t \tag{5}
\]

All buses in the distribution system should satisfy the active and reactive power balance equations represented as:

\[
P_{i,t} = a_{i}^{\text{PCC},t} P_{i,t}^{\text{PCC}} + \sum_{g \in G_i} P_{g,i}^{D} + P_{i,t}^{\text{RES}}
\]

\[
+ P_{i,t}^{\text{ess,dc}} + \sum_{k \in \text{VPP}} P_{k,t}^{\text{VPP}}, \quad \forall i, t \tag{6}
\]

\[
Q_{i,t} = \sum_{g \in G_i} Q_{g,i}^{D} + \sum_{k \in \text{VPP}} Q_{k,i}^{\text{VPP}} - q_{i,t}, \quad \forall i, t \tag{7}
\]

Each of which consists of a power production part and a power dissipation part. The power production part in (6) includes the active power from the point of common coupling (PCC), generated power from DG and RESs, and power that is discharged by the ESS. The power production part also contains the output of the VPPs. The power dissipation part in (6) consists of the inelastic load of bus \( i \), power charging the ESS, and the output of the VPPs. Similarly, the reactive power equation in (7) includes the power produced by the DG, the reactive power of the VPPs and the inelastic load of bus \( i \).

2) BRANCH CONSTRAINTS

The active and reactive power flows in the branch connecting buses \( i \) and \( j \) can be expressed as:

\[
P_{ij,t} = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (V_{i,t} - V_{j,t}) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_{i,t} - \theta_{j,t}), \quad \forall i, j, t \tag{8}
\]

\[
Q_{ij,t} = \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (V_{i,t} - V_{j,t}) - \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_{i,t} - \theta_{j,t}), \quad \forall i, j, t \tag{9}
\]

Additionally, the sum of the square of the active and reactive power flows should satisfy the branch capacity:

\[
P_{ij,t}^2 + Q_{ij,t}^2 \leq S_{ij}^{\text{max}}, \quad \forall i, j, t \tag{10}
\]

The voltage at \( i \) should be kept within the standard range as follows:

\[
V_{i,\text{min}} \leq V_{i,t} \leq V_{i,\text{max}}, \quad \forall i, t \tag{11}
\]

The voltage angle difference between two connected buses \( i \) and \( j \) should be constrained as follows:

\[
\theta_{i,t} + \theta_{j,t} \leq \theta_{i,t} - \theta_{j,t} \leq \theta_{i,t} + \theta_{j,t}, \quad \forall i, j, t \tag{12}
\]

The volume of the active power purchasing from the wholesale market is constrained by the active power flow limit via PCC at time \( t \):

\[
p_{\text{PCC},t}^{\text{DISCO}} \leq p_{\text{PCC}}^{\text{PCC}}, \quad \forall t \tag{13}
\]

3) DG AND RES CONSTRAINTS

The power output of each DG unit should be within the allowable range considering the up and down ramping capability at time \( t \):

\[
p_{g,i}^{D} + u_{g,i}^{D} \leq p_{\text{g},i}^{\text{max}}, \quad \forall g, t \tag{14}
\]

\[
p_{g,i}^{D} - d_{g,i}^{D} \leq p_{\text{g},i}^{\text{min}}, \quad \forall g, t \tag{15}
\]

The up and down ramping capabilities are limited by the ramp rate of each DG unit:

\[
0 \leq u_{g,i}^{D} \leq RR_{g}, \quad \forall g, t \tag{16}
\]

\[
0 \leq d_{g,i}^{D} \leq RR_{g}, \quad \forall g, t \tag{17}
\]

The power change in consecutive time steps is also limited by the ramp rate of each DG unit:

\[
-RR_{g} \cdot \Delta t \leq p_{g,i+1}^{D} - p_{g,i}^{D} \leq RR_{g} \cdot \Delta t, \quad \forall g, t \tag{18}
\]

Importantly, each DG unit’s operation at \( t \) should satisfy its capacity limit:

\[
(p_{g,i}^{D})^2 + (q_{g,i}^{D})^2 \leq (S_{g,i}^{\text{max}}), \quad \forall g, t \tag{19}
\]

The power output of the RES delivered to the grid is limited by the RES forecasting:

\[
0 \leq p_{i,t}^{\text{RES}} \leq P_{i,t}^{\text{RES}}, \quad \forall i, t \tag{20}
\]
4) ESS CONSTRAINTS
The ESS operations can be modeled as (21) – (23). The charging and discharging power of ESS are limited to the maximum charging and discharging rate, expressed in (21) and (22), respectively. The constraint (23) forces the battery cannot charge and discharge at the same time.

\[0 \leq P_{i,t}^{\text{ess, ch}} \leq P_{i,\text{max}}^{\text{ess}}, \forall i, t, \quad (21)\]

\[0 \leq P_{i,t}^{\text{ess, dch}} \leq P_{i,\text{max}}^{\text{ess}} \cdot (1 - \delta_{i,t}^{\text{ess}}), \quad \forall i, t, \quad (22)\]

\[\delta_{i,t}^{\text{ess}} \in [0, 1], \quad \forall i, t. \quad (23)\]

The stored energy is limited by the minimum and maximum capacities of the ESS; the energy volume can be calculated by considering the charging and discharging operations as follows:

\[E_{i,t}^{\text{ess, min}} \leq E_{i,t}^{\text{ess}} \leq E_{i,t}^{\text{ess, max}}, \quad \forall i, t, \quad (24)\]

\[E_{i,t}^{\text{ess}} = E_{i,t-1}^{\text{ess}} + \eta_{i}^{\text{ess}} \cdot P_{i,t}^{\text{ess, ch}} - \frac{e_{i,t}^{\text{ess, dch}}}{\eta_{i}^{\text{ess}}}, \quad \forall i, t > 1. \quad (25)\]

5) FLEXIBILITY REQUIREMENTS
When the DISCO procures the flexibility of its own system, the total amount of up and down ramping capabilities should be larger than the flexibility required at \( t \). The ramping requirement constraints are represented as:

\[\sum_{\forall g} u_{k,g,t}^{\text{D}} + \sum_{\forall i} \left( P_{i,\text{max}}^{\text{ess, ch}} - P_{i,t}^{\text{ess, dch}} \right) \geq FRU_{t} + \sum_{\forall k} \left( P_{k,t}^{\text{VPP}} - P_{k,t-1}^{\text{VPP}} \right) - \left( P_{t}^{\text{PCC}} - P_{t-1}^{\text{PCC}} \right), \quad t > 1 \quad (26)\]

\[\sum_{\forall g} d_{k,g,t}^{\text{D}} + \sum_{\forall i} \left( P_{i,t}^{\text{ess, ch}} - P_{i,\text{max}}^{\text{ess, dch}} \right) \geq FRD_{t} - \sum_{\forall k} \left( P_{k,t}^{\text{VPP}} - P_{k,t-1}^{\text{VPP}} \right) + \left( P_{t}^{\text{PCC}} - P_{t-1}^{\text{PCC}} \right), \quad t > 1. \quad (27)\]

6) PRICE RANGE CONSTRAINTS
The DISCO must issue the appropriate price to the VPPs, which is applied to the power to be exchanged with the lower-level agents (VPPs). The VPP prices are associated with the extent to which the DISCO can increase the price, compared with the wholesale market price \((\pi_{i}^{\text{VPP}} = \pi_{i}^{\text{PCC}} \cdot PM_{i,k,\text{min}})\). The profit margins for the DISCO are limited to a certain range; thus, the DISCO cannot simply raise or lower the VPPs' prices as far it might want to, in accordance with the following constraints:

\[\pi_{i}^{\text{PCC}} \cdot PM_{k,\text{min}} \leq \pi_{i}^{\text{VPP}} \leq \pi_{i}^{\text{PCC}} \cdot PM_{k,\text{max}}, \quad \forall i. \quad (28)\]

B. DECENTRALIZED DECISION MODEL FOR VPPs
The lower-level problem maximizes the net profit of the VPP \( k \) at the bus \( i \). The lower-level agent considers the utility function of the elastic demands and pays for the power received from the DISCO. The agent also estimates the operating cost of its own aggregated resources. The objective function of the VPP consists of the utility function of the elastic demands, power purchasing cost from the DISCO, and generation cost of the aggregated resources. The function can be formulated as:

\[\max_{t} \sum_{i=1}^{T} \left( U_{k}^{\text{p}, \text{var}} \cdot p_{k,i}^{\text{VAR}} - \sum_{k_{g} \in G_{k}} (c_{kg} \cdot p_{k,g}^{\text{DG}}) \right), \quad (29)\]

where \( U_{k} (\cdot) \) is the utility function of the DR, which can be represented by a piecewise linear function as in (30). Lagrange multipliers for applying the Karush–Kuhn–Tucker (KKT) optimality conditions appear next to each constraint, separated by a colon:

\[U_{k} (p_{k,i}^{\text{var}}) = \sum_{n=1}^{N_{\text{DR,seg}}} u_{k,n} \cdot p_{k,m,t}^{\text{var}}, \quad \forall k, t : \lambda_{1,k}, \quad (30)\]

\[p_{k,i}^{\text{var}} = \sum_{n=1}^{N_{\text{DR,seg}}} p_{k,m,n}^{\text{var}}, \quad \forall k, t : \lambda_{2,k}. \quad (31)\]

The elastic demands in the VPP are assumed to be utilized within a constant power factor \( s_{k}^{\text{var}} \):

\[p_{k,i}^{\text{min}} \leq p_{k,i}^{\text{var}} \leq p_{k,i}^{\text{max}}, \quad \forall k, t : \lambda_{3,k}, \quad (32)\]

\[q_{k,i}^{\text{min}} \leq q_{k,i}^{\text{var}} \leq q_{k,i}^{\text{max}}, \quad \forall k, t : \lambda_{4,k}. \quad (33)\]

\[q_{k,i}^{\text{var}} = s_{k}^{\text{var}} \cdot p_{k,i}^{\text{var}}, \quad \forall k, t : \lambda_{5,k}. \quad (34)\]

The following constraints are required to model the lower-level problem.

1) POWER BALANCE EQUATION
The active and reactive power in the VPP \( k \) at \( t \) should be balanced:

\[p_{k,i}^{\text{VPP}} + p_{k,i}^{\text{RES}} + \sum_{k_{g} \in G_{k}} p_{k,g}^{\text{DG}} = p_{k,i}^{\text{fix}} + p_{k,i}^{\text{var}}, \quad \forall k, t : \lambda_{6,k}. \quad (35)\]

\[q_{k,i}^{\text{VPP}} + \sum_{k_{g} \in G_{k}} q_{k,g}^{\text{DG}} = q_{k,i}^{\text{fix}} + q_{k,i}^{\text{var}}, \quad \forall k, t : \lambda_{7,k}. \quad (36)\]

2) DG, RES, DR CONSTRAINTS
Each DG unit in the VPP should satisfy its power output range as follows:

\[p_{k,g,\text{min}} \leq p_{k,g}^{\text{DG}} \leq p_{k,g,\text{max}}, \quad \forall k, g, t : \lambda_{8,k,g}. \quad (37)\]

\[Q_{k,g,\text{min}} \leq q_{k,g}^{\text{DG}} \leq Q_{k,g,\text{max}}, \quad \forall k, g, t : \lambda_{9,k,g}. \quad (38)\]

The power output of the RES in the VPP delivered to the grid is also limited by the RES forecasting:

\[0 \leq p_{k,i}^{\text{RES}} \leq F_{k,i}^{\text{RES}}, \quad \forall k, t : \lambda_{10,k}. \quad (39)\]

The total elastic load in the considered horizon is constrained by the maximum and minimum levels:
3) VPP POWER EXCHANGE CONSTRAINTS
The power exchanged by the VPP and DISCO is limited by the rated capacity of the feeder:

\[ (p_{k,t}^{\text{VPP}})^2 + (q_{k,t}^{\text{VPP}})^2 \leq (s_{k,t}^{\text{VPP}})^2, \forall k, t : \lambda_{18, k, t} \]

\[ (41) \]

C. CENTRALIZED DECISION MODEL
The centralized problem minimizes the overall operating costs of the single entity, which manages the resources that are assumed to be owned by the DISCO and VPPs. The objective function of the single entity consists of the utility function of the consumers, generation costs of the DG, and power purchasing costs of the wholesale market. The optimization problem is formulated as:

\[ \min_{\pi_t \cdot P_{\text{PCC}} \cdot q_{g,t} \cdot d_{g,t}} \sum_{t=1}^{T} U_t(p_{k,t}^{\text{ld, var}}) + \sum_{t=1}^{T} \sum_{k \in G_k} (c_{kg} \cdot P_{k,t}^{\text{DG}}) \]

\[ + \sum_{t=1}^{T} \sum_{k \in G_k} F_{g,t}^{N_{\text{seg}}} + \sum_{t=1}^{T} (\pi_t^{\text{PCC}} \cdot p_{t}^{\text{PCC}}) \]

subject to the constraints (2)-(27), (32)-(41).

\[ (42) \]

IV. SOLUTION APPROACH OF THE DECENTRALIZED MODEL
A. LINEARIZATION OF QUADRATIC CONSTRAINTS
The above decision models are nonlinear and nonconvex problems because of the quadratic constraints, such as (10), (19), and (41). A linearization algorithm is required to transform each nonlinear and nonconvex problem into a mixed integer linear programming model to reduce the computational burden.

The quadratic functions, which can be represented by a circle, can be transformed into a polygon bounded by line segments [14]. Using this transformation, the quadratic constraints in the upper- and lower-level problems can be linearized. The branch capacity constraint (10) can be linearized as:

\[ e_{n, 1}^{ij} = \begin{cases} 
\cos \theta_i^{ij} - \cos \theta_{i+1}^{ij}, & \text{if } n \leq N_{\text{seg}} - 1 \\
\cos \theta_i^{ij} - \cos 2\pi, & \text{otherwise} 
\end{cases}, \forall i, j, n, \]

\[ (43) \]

\[ e_{n, 2}^{ij} = \begin{cases} 
\sin \theta_i^{ij}, & \text{if } n \leq N_{\text{seg}} - 1 \\
\sin (2\pi - \theta_i^{ij}), & \text{otherwise} 
\end{cases}, \forall i, j, n \]

\[ (44) \]
B. TRANSFORMATION OF THE BI-LEVEL MODEL INTO AN EQUIVALENT SINGLE LEVEL MODEL

The transformation of a bi-level problem into an equivalent single-level problem is a popular approach for solving bi-level programming problems. In this approach, the lower-level problem can be replaced with a set of equations based on the KKT optimality conditions. Using the above linearization of the quadratic constraints, the lower-level problem expressed in (29)–(41) can be formulated as a linear programming model, for which the KKT conditions are necessary and sufficient to ensure optimality. The detailed KKT conditions of the lower-level problem used for the equivalent single-level model can be derived as follows.

\[
\delta L \overset{\text{p,\text{var}}}{\delta p_{k,t}} = -\frac{dU_k(\cdot)}{dp_{k,t}} + \lambda_1 \frac{dU_k(\cdot)}{dp_{k,t}} - \lambda_3 \\
+ \lambda_4 - \lambda_7 a \cdot s_k + \lambda_7 b \cdot s_k^{\text{var}} \\
+ \lambda_9 a - \lambda_9 b - \Delta_1 \cdot \lambda_16 + \Delta_1 \cdot \lambda_17 = 0
\]

\[
\delta L \overset{\text{d,\text{var}}}{\delta d_{k,t}} = -\lambda_5 + \lambda_6 + \lambda_7 a - \lambda_9 b - \lambda_9 a - \lambda_9 b = 0
\]

\[
\delta L \overset{\text{p,\text{VPP}}}{\delta p_{k,t}} = \pi_{VPP,k} - \lambda_8 a + \lambda_8 b + \lambda_8 (1) \cdot \epsilon_{t,1} \\
+ \lambda_8 (2) \cdot \epsilon_{2,1} + \ldots + \lambda_8 (n) \cdot \epsilon_{n-x,1} = 0
\]

\[
\delta L \overset{\text{d,\text{VPP}}}{\delta d_{k,t}} = \epsilon_{k,p} - \lambda_8 a + \lambda_8 b + \lambda_8 (1) \cdot \epsilon_{t,2} \\
+ \lambda_8 (2) \cdot \epsilon_{2,2} + \ldots + \lambda_8 (n) \cdot \epsilon_{n-x,2} = 0
\]

\[
\delta L \overset{\text{d,\text{Gg}}}{\delta d_{k,t}} = -\lambda_9 a + \lambda_9 b - \lambda_9 a + \lambda_9 b = 0
\]

Using the constraints obtained from the KKT conditions, the bi-level optimization model described in Section III can be transformed into a single-level problem, as shown in (65). Note that the decision variables of the equivalent single-level problem (65), represented as X, include all variables introduced in the upper- and lower-level problems. The objective function of (65) is equivalent to (1): the objective function of the upper-level problem. Importantly, all constraints are mixed integer linear expressions; however, the objective function is not such an expression because of the second term, \( \pi_{VPP,k,t} \cdot p_{k,t} \), which is a product of two continuous variables.

\[
\min X \quad -\sum_{i=1}^{T} \sum_{k\in N_{VPP}} (\pi_{VPP,k} \cdot p_{k,t}) \\
+ \sum_{i=1}^{T} \sum_{l=1}^{N_B} \sum_{g \in G_i} (F_g(p_{g,t}) \cdot \pi_{L} \cdot p_{L}) \quad \text{(65)}
\]

subject to (2) – (9), (11) – (18), (20) – (28), (32) – (40), (43) – (64) (65)

C. LINEARIZATION OF THE OBJECTIVE FUNCTION

The product of two continuous variables, \( \pi_{VPP,k} \cdot p_{k,t} \), can be reformulated as mixed integer linear equations using a method of special ordered sets of type 2. However, in this study, we approximate the term in a different manner. To address computational concerns at the expense of finding a highly accurate price with which both the DISCO and VPP agree, it is assumed that the price for the VPP (\( \pi_{VPP,k} \)) only have a value among candidate prices. Accordingly, this approach attempts to find a solution reliably and efficiently while retaining the accuracy of the results. The nonlinear term, \( \pi_{VPP,k} \cdot p_{k,t} \), can be reformulated as:

\[
\pi_{VPP,k} \cdot p_{k,t} \approx \sum_{l=1}^{L} \theta_{VPP,k,t,l} \quad \forall k, t
\]

\[
0 \leq \theta_{VPP,k,t,l} \leq M \cdot b_{k,t,l} \quad \forall k, t, l
\]

\[
-M \cdot (1 - b_{k,t,l}) \leq \theta_{VPP,k,t,l} - \pi_{VPP,k} \cdot p_{k,t} \leq M \cdot (1 - b_{k,t,l}) \quad \forall k, t, l
\]

\[
\sum_{l=1}^{L} b_{k,t,l} = 1, \quad \forall k, t
\]

where \( b_{k,t,l} \) are auxiliary binary variables that are introduced to indicate the optimal \( \pi_{VPP,k} \), while \( \pi_{VPP,k} \) is a necessary parameter with values that represent candidate prices. If the candidate prices for the VPP are uniformly determined between the minimum and maximum allowable price, \( \pi_{VPP,k} \) is set to the minimum price for the VPP (\( \pi_{PCC,k} \cdot PM_{k,min} \)) while \( \pi_{VPP,k} \) is set to the maximum price for the VPP (\( \pi_{PCC,k} \cdot PM_{k,max} \)). According to (67) and (68), if the binary variable \( b_{k,t,l} \) becomes 1, \( \theta_{VPP,k,t,l} \) should equal \( \pi_{VPP,k} \cdot p_{k,t} \). Conversely, if \( b_{k,t,l} \) becomes zero the determination of \( \pi_{VPP,k} \) (and the corresponding power flow between the DISCO and VPP) is not affected. The last constraint (69) requires that only one VPP price is chosen as an optimal solution.

Finally, a formulation for the equivalent single-level model of the bi-level model can be derived as (70). This formulation is represented as a mixed integer linear programming problem, which can be efficiently solved by state-of-the-art solvers (e.g., CPLEX, Gurobi, or Mosek).

\[
\min X, \theta_{VPP,k,t,l} \cdot b_{k,t,l} \quad T \sum_{i=1}^{L} \sum_{k \in N_{VPP}} \sum_{l=1}^{L} \theta_{VPP,k,t,l} \\
+ \sum_{i=1}^{T} \sum_{l=1}^{N_B} \sum_{g \in G_i} (F_g(p_{g,t}) \cdot \pi_{L} \cdot p_{L}) \quad \text{(70)}
\]

subject to (2) – (9), (11) – (18), (20) – (28), (32) – (40), (43) – (64), (66) – (69)
V. CASE STUDY
A. TEST SYSTEM AND MAIN ASSUMPTIONS
To verify the effectiveness of the proposed optimization method, we implemented it on a simulation of the IEEE 33-bus radial standard distribution system with DG, VPPs, and a load [27], as shown in Fig. 3. The load in the system is served by the main grid via a PCC; it is also served by two DG units, three wind turbines, an ESS, and two VPPs connected to various buses. Details of the distribution line and the load data used in the test are provided in Appendix.

We focused on the effects of the price to the VPPs under the DISCO and the VPPs’ operations for 1 hour with 15 minutes intervals. For simplicity, the prices applied to all VPPs were assumed to be identical.

The proposed optimization method was solved by GAMS software using the Gurobi solver. In this study, the dual gap of the optimization problem was set to 0.1%, in other words, the error between the obtained solution and the optimal solution was guaranteed to be less than 0.1%.

B. SIMULATION RESULTS
To focus on the effects of \(\pi_{VPP}^{t}\) on the DISCO and VPP operations, we compare the optimal solution of the bi-level problem via the proposed approach (Case I) with the simulation results of the conventional approach where \(\pi_{VPP}^{t}\) is set to be \(\pi_{PCC}^{t} \cdot PM_{k,\text{max}}\), which is the highest allowable price offered by the DISCO (Case II). In Case II, all conditions other than the price setting are treated identically to Case I. The power supplied via the PCC is constrained to zero in Cases I and II for the purpose of investigating strategies between the DISCO and the VPPs. We relaxed this constraint on the PCC in subsequent case studies. Comparisons of the optimization results for Cases I and II are presented in Fig. 4-7 and Tables 2−5.

As shown in Table 2, a win–win situation for the DISCO and VPP can be achieved with the proposed approach. Although the DISCO has reduced the price at which the VPP purchases, such a reduction can also benefit the DISCO (Table 3). This is because the proposed method can optimize the DISCO pricing strategy while ensuring optimal scheduling of resources in the distribution network. From the DISCO perspective, the lowering of prices in time slots T1 and T2 results in increased production costs; this loss can be entirely offset by the growth in the sales revenues. The VPP can also benefit from the lower prices, which lead to an increase in the VPP’s purchasing volume; consequently, the VPP can...
produce less power to save operating costs. The detailed cost structures for the DISCO and VPP are provided in Tables 4-5 and Fig. 5-6. All bus voltage levels in the distribution system are within the allowable ranges, as shown in Fig. 7.

Next, we evaluated the effectiveness of the proposed method under the condition that the DISCO can exchange power with the wholesale market. The simulation was repeated under the condition that the limit for the exchangeable power \( P_{PCC} \) was relaxed to 10 MW; we list the results in Tables 6-7 and Fig. 8. Similar to Cases I and II, Case III denotes that the prices applied to the VPPs are optimally determined by the DISCO, while such prices are fixed at their highest values in Case IV. As shown in Table 6, the optimal strategy for the DISCO is to set the VPP prices lower than the highest allowable value. Consequently, both the DISCO and the VPPs can reduce their operating costs via the proposed method. Importantly, when the DISCO can purchase power from the wholesale market at a low price, this also benefits the VPP; the VPPs have lower operating costs in Case III than in Case I.

For comparison, the centralized decision model was also tested on the same distribution system (Case V). As described in Section III.C, in this hypothetical model, the DISCO
optimally schedules all power outputs of the DG in the system and establishes the purchasing volume from the wholesale market, considering the network constraints and operating conditions of each unit. The dispatched volumes of DG are listed in Tables 8-9, and Fig. 9-10. The results indicate that the proposed method (Case III) yields the same schedules for all DG, including the generators owned by the VPPs, compared to schedules from the centralized model (Case V). Thus, although the resources (e.g., the DR, photovoltaic generators, and DG in the VPP) are not visible to the DISCO, the proposed bi-level decision model can improve operational performance of the entire system.

In the analysis above, the ramping requirements ($FRU_t$, $FRD_t$) were calculated based on the variability of the net loads and uncertainty in forecasts, as represented in (71) [26].

$$FRU_t = NL_t - NL_{t-1} + \sigma_t$$
$$FRD_t = NL_{t-1} - NL_t + \sigma_t$$

(71)

Here, the uncertainty term ($\sigma_t$), which can be determined using historical data or error distribution models, indicates the necessary ramping flexibility to cover the overall forecasting error in the operation of the distribution system. In this study, $\sigma_t$ was set to 6MW considering the renewable energy output and load demand. The value can be adjusted according to the system condition.

| Level | Operating costs($) |
|-------|-------------------|
| Upper (distribution company) | 8915.83 |
| Lower (virtual power plant) | 1732.43 |

VI. CONCLUSION

This paper presents a bi-level scheduling model of a distribution system that includes a DISCO and VPPs, with considerations of operational flexibility in the system. The proposed model can be realized by describing the operational strategy using a decentralized decision model. The model consists of upper- and lower-level problems. The upper-level problem requires minimization of the distribution system’s operational cost for the DISCO by managing the price of the power traded with the VPPs. The lower-level problem requires maximization of the profit for each VPP by managing the amount of power traded with the DISCO. We developed a mathematical formulation of the proposed model to evaluate the upper- and lower-level problems. The bi-level problem was transformed into a single-level mixed integer linear programming problem by using KKT optimality conditions and linearization methods. Due to the reformulation of the original problem, we expect that our method can be applied to the practical unit commitment decision process which is generally formulated in mixed-integer linear programming model. Although the proposed approach is developed based on the linearized distribution network model [28], it can be easily extended to the model that can consider loss factors and load shift factors for distribution system by using the method proposed in [28]. Case study results verified the cost-effectiveness of...
The proposed bi-level scheduling model has the following features:

- A win-win situation for the DISCO and VPPs can be achieved with the proposed operational strategy. In accordance with the price offered by the DISCO, each VPP reacts to the price by managing its purchasing volume.

- The DISCO can procure the required flexibility from each VPP by considering the variability and uncertainty. Using the proposed operational strategy, the interactions between the DISCO and VPPs contribute to reduction of the overall generation cost.

- When using the proposed operational strategy, all bus voltage levels and power flow in the distribution system were maintained within their allowable ranges.

Further work will be required concerning various power market structures, including a wholesale ramping market.
and operating reserves market. The proposed model can be applied to various strategies that the DISCO and VPPs can develop to maximize their revenue depending on novel market structures.

**APPENDIX**

PARAMETERS FOR TEST DATA

See Tables 11–18.

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