Quantum tunneling across spin domains in a Bose-Einstein condensate

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Quantum tunneling was observed in the decay of metastable spin domains in gaseous Bose-Einstein condensates. A mean-field description of the tunneling was developed and compared with measurement. The tunneling rates are a sensitive probe of the boundary between spin domains, and indicate a spin structure in the boundary between spin domains which is prohibited in the bulk fluid. These experiments were performed with optically trapped $F = 1$ spinor Bose-Einstein condensates of sodium.

A metastable system trapped in a local minimum of the free energy can decay to lower energy states in two ways. Classically, the system may decay by acquiring thermal energy greater than the depth of the local energy well (the activation energy). Yet, according to quantum mechanics, the system may decay even in the absence of thermal fluctuations by tunneling through the classically forbidden energy barrier. Quantum tunneling describes a variety of physical and chemical phenomena [1,2] and finds common applications in, for example, scanning tunneling microscopy. In these systems, tunneling dominates over thermal activation because the energy barriers are much larger than the thermal energy.

Bose-Einstein condensates of dilute atomic gases [3] offer a new system to study quantum phenomena. Recently, metastable Bose-Einstein condensates were observed in which a configuration of phase-separated component domains persisted for tens of seconds in spite of an external force which favored their rearrangement [4]. The metastability was due both to the restriction of motion to one dimension by the narrow trapping potential and also to the repulsive interaction between the domains. Thermal relaxation to the ground state was identified and found to be extremely slow, even at temperatures ($\sim$100 nK) much larger than the energy barriers responsible for metastability ($\sim$5 nK), due to the scarcity of non-condensed atoms, to which the thermal energy is available.

In this article, we examine the decay of metastable spin domains in an $F = 1$ spinor condensate via quantum tunneling. The tunneling rates provide a sensitive probe of the boundary between spin domains and of the penetration of the condensate wavefunction into the classically forbidden region. Tunneling barriers are formed not by an external potential, but rather by the intrinsic repulsion between two immiscible components of a quantum fluid. These energy barriers are naturally of nanokelvin-scale height and micron-scale width in the presence of weak magnetic field gradients, and are thus a promising tool for future studies of quantum tunneling and Josephson oscillations [1,2].

We begin by considering the one-dimensional motion of a Bose-Einstein condensate comprised of atoms of mass $m$ in two different internal states, $|A\rangle$ and $|B\rangle$. The condensate is held in a harmonic trapping potential which has the same strength for each component. In a mean-field description, the condensate wavefunction $\psi_i(z)$ is determined by two coupled Gross-Pitaevskii equations

$$
\left(-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V_i(z) + g_{n_i(z)} + g_{A,B_{n_i}(z)} - \mu_i \right) \psi_i(z) = 0
$$

where $V_i(z)$ is the trapping potential, $n_i(z)$ the density and $\mu_i$ the chemical potential of each component ($i,j = \{A,B\}, i \neq j$). The constants $g_A, g_B$, and $g_{A,B}$ (all assumed positive) are given by $g = 4\pi\hbar^2a/m$ where $a$ is the $s$-wave scattering length which describes collisions between atoms in the same ($a_A$ and $a_B$) or different ($a_{A,B}$) internal states. Bulk properties of the condensate are well described by neglecting the kinetic energy (Thomas-Fermi approximation). Under the condition $g_{A,B} > \sqrt{g_{A,B}}$, the two components tend to phase-separate (as observed in [5]). The ground state configuration consists of one domain of each component on opposite sides of the trap (Fig. 1A). The chemical potentials are determined by the densities at the boundary $n_i^b$ as $\mu_i = g_i n_i^b$, and are related to one another by the condition of equal pressure, $\mu_A^2/2g_A = \mu_B^2/2g_B$.

Within the Thomas-Fermi approximation, the domain boundary is sharp and the two components do not overlap. Yet, the kinetic energy allows each component to penetrate within the domain of the other. The energy barrier for component $A$ (similar for $B$) is $\Delta E_A(z) = V_A(z) + g_{A,B}n_B(z) - \mu_A$. Neglecting slow variations in $V_A$ and $n_B$ gives the barrier height

$$
\Delta E_A = \mu_A \left( \frac{g_{A,B}}{\sqrt{g_{A,B}}} - 1 \right).
$$

In this work we consider a condensate of atomic sodium in the two hyperfine states $|A\rangle = |F = 1, m_F = 0 \rangle$ and $|B\rangle = |F = 1, m_F = 1 \rangle$, with scattering lengths of...
Consider that a state-selective force \(-F\hat{z}\) displaces the trapping potential \(V_B(z)\) from \(V_A(z)\) (Fig. 1B). Due to the energy barrier discussed above, the atoms cannot, classically, move to the other end of the trap and thus the condensate is left in a high-energy configuration. This configuration can decay by tunneling. At a boundary where \(V_A = V_B\) the width of the barrier becomes \(z_B = \Delta E_A/F\times 2gB/(g_{A,B} + g_B)\). Tunneling from the metastable spin domains is analogous to the field emission of electrons from cold metals \[\text{[2]}\], where the energy barrier height corresponds to the work function of the metal and the force arises from an applied electric field. The tunneling rate \(dN_A/dt\) of atoms in state \(|A\rangle\) from the metastable spin domain is then given by the Fowler-Nordheim relation \[\text{[2]}\]

\[
\frac{dN_A}{dt} = \gamma \exp\left(-2\sqrt{\frac{2m}{\hbar^2}} \int_0^{z_B} \sqrt{\Delta E_A(z)} dz\right)
\]

\[
= \gamma \exp\left(-\frac{4}{3}\sqrt{\frac{2m}{\hbar^2}} \frac{2gB}{g_{A,B} + g_B} \frac{\Delta E_A^{3/2}}{F}\right)
\]

where \(\gamma\) is the total attempt rate for tunneling, and the exponential is the tunneling probability.

The rate of quantum tunneling was studied in three steps. First, condensates of sodium in the \(|F = 1, m_F = -1\rangle\) hyperfine state were created in a magnetic trap \[\text{[13]}\] and transferred to a single-beam infrared optical trap \[\text{[14]}\] with a \(1/e^2\) beam radius of 12 \(\mu m\), an aspect ratio (axial / radial length) of about 60, and a depth of 1 \(- 2\ \mu K\). Chirped radio-frequency pulses were used to create two-component condensates with nearly equal populations in the \(|m_F = 0\rangle\) and \(|m_F = 1\rangle\) states \[\text{[3,4]}\]. Shortly afterwards, the two components were separated into two domains by the application of a strong (several \(G/cm\)) magnetic field gradient along the axis of the trap in a 15 \(G\) bias field. The spin domains were typically 100 \(- 200\ \mu m\) long.

Second, the condensates were placed in a metastable state by applying a magnetic field gradient \(B'\) in the opposite direction of that used to initially separate the components \[\text{[4]}\]. This metastable state corresponds to that shown in Fig. 1B, where we identify the states \(|A\rangle = |m_F = 0\rangle\) and \(|B\rangle = |m_F = 1\rangle\). The field gradient exerted a state-selective force \(F = g\mu_B m_F B'\) where \(g = 1/2\) is the Landé g-factor and \(\mu_B\) is the Bohr magneton. The condensate was then allowed to evolve freely at the gradient \(B'\) and a bias field \(B_0\) for a variable time \(\tau\) of up to 12 seconds.

Finally, the condensate was probed by time-of-flight absorption imaging combined with a Stern-Gerlach spin separation \[\text{[4,10]}\]. The radial expansion of the condensate in time-of-flight allowed for independent measurement of the chemical potentials \(\mu_0\) and \(\mu_1\) \[\text{[13]}\], while the axial distribution allowed for measurement of the number of atoms in the metastable and ground-state domains of each spin state.

The mean-field description of tunneling from the metastable spin domains was tested by measuring the tunneling rate across energy barriers of constant height and variable width. Condensates in a 15 \(G\) bias field with a chemical potential \(\mu_0 = 300\ \mu K\) were probed after 2 seconds of tunneling at a variable field gradient \(B'\) (Fig. 2). Thus, the energy barrier for tunneling had a constant height of 5 \(\mu K\), and a width between 4 and 20 \(\mu m\). As the barrier width was shortened by increasing \(B'\), the fraction of atoms in the \(m_F = 0\) metastable spin domains decreased. As expressed in Eq. \[\text{[3]}\], the number of atoms which tunnel from the metastable to the ground state domains in a time \(\tau\) should vary as \(\gamma\tau e^{-\alpha/B'}\) where \(\gamma\) and \(\alpha\) were determined by fits to the data as \(\gamma = 1.5(5)\times 10^7\ \text{s}^{-1}\) and \(\alpha = 1.5(2)\ \text{cm}/G\). This value of \(\alpha\) gives a tunneling probability of about \(e^{-4}\) for \(B' = 370\ \text{mG/cm}\), at which the metastable domains were fully depleted.

\[
\text{[1]}\ a_1 = a_{0,1} = 2.75\ \text{nm} \quad \text{[11]} \quad \text{and} \quad (a_1 - a_0) = 0.10\ \text{nm} \quad \text{[12]}
\]

The barrier height for atoms in the \(|m_F = 0\rangle\) state is then 0.018 \(\mu K\), a small fraction of the chemical potential.

\[
\text{[2]}\ A\quad B\quad \text{[A]}
\]

\[
\text{[B]}\quad V_B\quad A\quad B\quad V_A\quad \text{[B]}
\]

\[
\text{[C]}\mu_A^A\quad A\quad B\quad B^A\quad \text{[C]}
\]

\[
\text{[D]}\ A\quad B\quad \text{[D]}
\]

\[
\text{[E]}\ A\quad B\quad \text{[E]}
\]

FIG. 1. Metastable spin domains and the energy barrier for decay. (A) The ground state of a two-component condensate consists of two phase-separated domains. (B) A state-selective force \(F\) displaces the trap potential \(V_B\) from \(V_A\), creating metastable spin domains. Atoms tunnel from the metastable spin domains (direction of arrows) through an energy barrier (C) of maximum height \(\Delta E_A\) and width \(z_B \simeq \Delta E_A/F\) (similar for component \(B\)). (D) Tunneling proceeds from the metastable domains (inner) to the ground-state domains (outer) until (E) the condensate has completely relaxed to the ground state.
The measured value of $\alpha$ can be compared with the prediction of the Fowler-Nordheim equation (Eq. 3). Using the scattering lengths above and $\mu_0 = 300$ nK, we obtain $\alpha = 1.5(2)$ cm/G, in agreement with our measurement (the error reflects a 10% systematic uncertainty in $\mu_0$).

In addition, $g_1 > g_0$ implies $\mu_1 > \mu_0$ and thus the tunneling rate of $m_F = 1$ atoms across the $m_F = 0$ domain should be slower than that of the $m_F = 0$ atoms across the $m_F = 1$ domain. The data in Fig. 3 show evidence for this behavior.

The dependence of the tunneling rate on the energy barrier height was probed by varying the condensate density. For this, the number of trapped atoms was varied between about $10^5$ and $10^6$ by allowing for a variable duration of trap loss, before creating the metastable state. Figure 3 shows data collected at two different settings of the optical trap depth $U$ and tunneling time $\tau$ (see caption). For each data series, at a given field gradient $B'$, there was a threshold value of the chemical potential $\mu_0$ below which the condensates had relaxed completely to the ground state, and above which they had not. Since the total condensate number and the attempt rate $\gamma$ should both scale as $\mu_0^{5/2}$, one expects the threshold chemical potential for complete tunneling to the ground state to vary as $\mu_0 \propto B'^{2/3}$. The data shown in Figure 8 suggest a slightly steeper dependence.

Thus, we have shown the decay of the metastable spin domains at high magnetic fields (15 G) to be due to quantum tunneling in a two-component condensate. At lower magnetic fields, a dramatic change in the tunneling behavior was observed. Metastable spin domains of initial chemical potential $\mu_0 = 600$ nK were prepared at a constant field gradient of $B' = 130$ mG/cm, and a 15 G bias field. The field was then ramped down to between 0.4 and 2 G within 10 ms, and held at a constant value $B_0$. After a variable tunneling time $\tau$ of up to 12 seconds, the condensates were probed and evaluated as to whether they had fully decayed to the ground state. During the tunneling time, the chemical potential dropped due to the loss of atoms from the trap. As the field was lowered below about 1 G, relaxation to the ground state occurred at earlier times (Fig. 4A), and thus at higher chemical potentials (Fig. 4B).

The increase in the tunneling rates at lower magnetic fields is inconsistent with the dynamics of a two-component condensate. Our measurements thus serve as a probe of the spin domain boundary and reveal the presence of the third $F = 1$ spin component ($m_F = -1$). Atoms in the $|m_F = -1\rangle$ state are created by spin-
relaxation wherein two $m_F = 0$ atoms collide to produce an $m_F = 1$ and an $m_F = -1$ atom [14]. Due to the quadratic Zeeman effect, the magnetic energy of two $m_F = 0$ atoms is lower than that of their spin-relaxation product by $2g = 2 \times 20 \text{nK} \times (B_0/G)^2$. Interactions give rise to a spin-dependent energy term $c(F)^2$, where $c = \Delta g n / 2$, $\Delta g = g_1 - g_0$, and $n$ is the condensate density.

Neglecting the kinetic energy, atoms in the $|m_F = -1\rangle$ state are excluded from the domain boundary when $q > c/2$, i.e. at fields $B_0 \gtrsim 250 \text{ mG}$ for typical conditions ($n \sim 3 \times 10^{14} \text{cm}^{-3}$) [14]. However, when the kinetic energy is considered, atoms in the $|m_F = -1\rangle$ state are found to populate the boundary even at fields $B_0 > 250 \text{ mG}$. Within the boundary region, the average magnetization $\langle F_z \rangle$ must vary smoothly. Minimizing the energy functional $q\langle F_z^2 \rangle + c(F)^2$ at constant magnetization $\langle F_z \rangle$ indicates that, for $q/2c \gg 1$, the fraction of atoms in the $|m_F = -1\rangle$ state scales roughly as $B_0^{-2}$ [14]. At a field of 1 G, the fraction of atoms in the domain boundary in the $|m_F = -1\rangle$ state is at most $\sim 2\%$ (about 300 atoms).

The presence of the $m_F = -1$ atoms in the barrier weakens the effective repulsion between the spin domains. Consider a two-component system as before where $|A\rangle = |m_F = 0\rangle$ and $|B\rangle = \cos \theta |m_F = 1\rangle - \sin \theta |m_F = -1\rangle$ where $0 \leq \theta \leq \pi/2$. Evaluating the spin-dependent interaction energy [17,18] one finds $g_B = g_0 + \Delta g \cos^2 2\theta$ and $g_{0,B} = g_0 + \Delta g (1 - \sin 2\theta)$. Thus, as the fraction of atoms in the $|m_F = -1\rangle$ state is increased, the repulsion of the $m_F = 0$ atoms at the domain walls is weakened, increasing the tunneling rate.

In conclusion, we have identified and studied quantum tunneling across phase-separated spin domains in a Bose-Einstein condensate. The energy barriers due to the interatomic repulsion are a small fraction of the chemical potential, and their width is simply varied by the application of a weak force. The tunneling rates at high field ($B_0 > 1\text{G}$) were described by a mean-field model and an application of the Fowler-Nordheim equation, while the tunneling at lower fields reveals a change in the spin-state composition of the domain boundaries. Future studies using metastable spin domains as tunneling barriers may focus on the roles of coherence and damping in quantum tunneling. In the current setup, rapid Josephson oscillations might be expected at frequencies ($\sim 1 \text{ kHz}$) given by the energy difference between the metastable and ground state spin domains. Over long time scales such oscillations are presumably damped as the system evolves toward the ground state. While no evidence for oscillatory behavior was found in the present work, the use of smaller spin domains and better time resolution is warranted.

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