Statistical separation of weak gravitational lensing and intrinsic ellipticities based on galaxy colour information

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1 INTRODUCTION

The next generation of weak lensing surveys has the goal to measure correlations in the shapes of neighbouring galaxies over a wide range of angular scales and with resolution in redshift in order to investigate the properties of gravity through its influence on cosmic structure formation and in order to carry out a precision determination of cosmological parameters, investigate the properties of gravity on large scales or determine details of dark energy models through their influence on cosmic structure formation. While weak lensing surveys like Euclid or LSST provide exquisite statistical precision due to the vast amount of data, the control of systematics is the primary obstacle in the way of exploiting this information.

Weak cosmic shear refers to the correlated distortion of the cross-section of light bundles that reach us from distant galaxies (Mellier 2000; Bartelmann & Schneider 2001; Refregier 2003; Hoekstra & Jain 2008; Munshi et al. 2008; Kilbinger 2015). Using it as a cosmological probe one primarily determines angular correlations in the ellipticities of galaxies, after subdividing the galaxy sample in redshift bins. Under the assumption of intrinsically uncorrelated shapes the angular ellipticity correlation function measures lensing-efficiency weighted tidal shear correlations in the cosmic large-scale structure. These correlations carry information about the structure formation history as well as the expansion history of the Universe (Matilla et al. 2017), and do not rely on any assumption apart from the validity of a gravitational theory (Huterer 2002; Linder & Jenkins 2003; Amendola et al. 2008; Bernstein 2009; Huterer 2010; Curran et al. 2011; Martinelli et al. 2011; Vanderveld et al. 2012).

Intrinsic alignments of galaxies mimic correlations in the shapes of neighbouring galaxies which would be naively contributed to gravitational lensing (for reviews, Kirk et al. 2015; Kiessling et al. 2015; Joachimi et al. 2015; Schäfer 2009; Troxel & Ishak 2015). If unaccounted for, either by modelling or by mitigation, they would interfere with the parameter inference process and would lead to wrong conclusions about the cosmological model (Kirk et al. 2010, 2011, 2012; Laszlo et al. 2012; Capranico et al. 2011; Schäfer & Mérk 2015; Krause et al. 2016; Tugendhat & Schäfer 2018). While the exact mechanisms of galaxy alignment with the cosmic large-scale structure are not yet clear, tidal alignment models provide a physically motivated way to link the shapes of galaxies to the matter distribution on large-scales. In the case of elliptical galaxies, for which the linear alignment model might be applicable, one assumes a distortion of the galaxy ellipsoid by tidal gravitational forces, which act perturbatively on the galaxy and exert a shearing distortion (Hirata & Seljak 2004, 2010; Blazek et al. 2011, 2017). In this case, the observed ellipticity is proportional to the tidal shear components perpendicular to the line of sight. In contrast, the alignment of spiral galaxies may be due to the quadratic alignment model, where the orientation of the galaxy is linked to the host halo angular momentum, which in turn is generated in the early stages of structure formation by tidal torquing.
2 COSMOLOGY

Under the symmetry assumption of Friedmann-Lemaître-cosmologies all fluids are characterised by their density and their equation of state: In spatially flat cosmologies with the matter density parameter $\Omega_m$ and the corresponding dark energy density $1 - \Omega_m$, one obtains for the Hubble function $H(a) = \dot{a}/a$ the expression,

$$H^2(a) = \frac{\Omega_m}{a^2} + \frac{1 - \Omega_m}{a^{2(1+w)}}. \quad (1)$$

The comoving distance $x$ is related to the scale factor $a$ through

$$x = -c \int_0^a \frac{dz}{a H(z)}, \quad (2)$$

where the Hubble distance $x_H = c/H_0$ sets the distance scale for cosmological distance measures. Small fluctuations $\delta$ in the distribution of dark matter, as long as they are in the linear regime $|\delta| \ll 1$, according to the growth function $D_*(a)$ (Linder & Jenkins 2003),

$$\frac{d^2}{da^2}D_*(a) + \frac{2 - q}{a} \frac{d}{da}D_*(a) - \frac{3}{2a^2}\Omega_m(a)D_*(a) = 0, \quad (3)$$

and their statistics is characterised by the spectrum $\langle \delta(k)\delta'(k') \rangle = (2\pi)^3\delta(k-k')P_g(k)$. Inflation generates a spectrum of the form $P_g(k) \propto k^{n-4}$ with the transfer function $T(k)$ (Bardeen et al. 1986) which is normalised to the variance $\sigma_8$ smoothed to the scale of 8 Mpc/$h$,

$$\sigma_8^2 = \int_0^{k_{max}} \frac{k^3dk}{2\pi^2} W^2(8 \text{ Mpc}/h \times k) P_g(k), \quad (4)$$

with a Fourier-transformed spherical top-hat $W(x) = 3j_1(x)/x$ as the filter function. From the CDM-spectrum of the density perturbation the spectrum of the dimensionless Newtonian gravitational potential $\Phi$ can be obtained,

$$P_\Phi(k) = \left(\frac{3\Omega_m}{2(k_Ha)^3}\right)^2 P_g(k) \quad (5)$$

by applying the comoving Poisson-equation $\Delta \Phi = 3\Omega_m/(2k_H^2)\delta$ for deriving the gravitational potential $\Phi$ (in units of $c^2$) from the density $\delta$. Because our analysis relies on the assumption of Gaussianity, we need to avoid nonlinearly evolving scales and will restrict our analysis to large angular and spatial scales, where the cosmic density field can be approximated to follow a linear evolution, conserving the near-Gaussianity of the initial conditions. We increase the variance of the weak lensing signal and of the intrinsic alignment signal of elliptical galaxies on small scales because of nonlinear structure formation using the description of Smith et al. (2003); Casarin (2011, 2012). Consequently, the cross-correlation between weak lensing and elliptical galaxy shapes will likewise have increased variances on small scales. Shapes of spiral galaxies are set by the initial conditions of structure formation, therefore we did not apply changes to the linear CDM-spectrum $P(k)$.

3 WEAK GRAVITATIONAL LENSING AND INTRINSIC ALIGNMENTS

3.1 Gravitational tidal fields and their statistics

In our investigation, alignments of galaxies with the large-scale structure are both due to gravitational-tidal interactions with shear fields which we will assume to have Gaussian statistics. This assumption will not be applicable at late times and on small scales. Tidal alignment models relate correlations in the shapes of galaxies to correlations in the tidal shear field,

$$\Phi_{\text{talign}}(x) = \frac{\partial^2 \Phi(x)}{\partial x^a \partial x^b}, \quad (6)$$

as a tensor containing the second derivatives of the Newtonian gravitational potential. Correlations of $\Phi_{\text{talign}}(x)$ as a function of distance $r = |x - x'|$ will be described by the correlation function

$$C_{\text{talign}}(r) \equiv \langle \Phi_{\text{talign}}(x)\Phi_{\text{talign}}(x') \rangle \quad (7)$$

which Catelan & Porciani (2011) have shown to take the form

$$C_{\text{talign}}(r) = \delta_{\alpha\beta}\delta_{\alpha\beta} + \delta_{\alpha\beta}\delta_{\alpha\beta} + \delta_{\alpha\beta}\delta_{\alpha\beta} \zeta_2(r) + \hat{\gamma}_2\delta_{\alpha\beta} + 5 \text{ perm.} \zeta_2(r). \quad (8)$$

The fluctuation statistics of the gravitational potential $P_\Phi(k)$ enters through the functions $\zeta_2(r)$,

$$\zeta_2(r) = (-1)^n r^{n-4} \int \frac{dk}{2\pi^2} W_\delta(k) k^{n+2} \tilde{J}_e(kr), \quad (9)$$

as derived by Crittenden et al. (2001). $\hat{\gamma}_2$ is the $\alpha$-component of the unit vector parallel to $r = x - x'$. While the tidal shear fields will directly change the shape of an elliptical galaxy, the effect of tidal fields on a spiral galaxies is to determine its angular momentum direction and consequently the inclination of the galactic disc. In both cases, the relevant components of the tidal shear tensor are those of the traceless part. The
resulting correlation function \( \tilde{C}_{\alpha eta}(r) \) of the traceless tidal shear \( \Phi_{\alpha \beta} = \Phi_{\alpha \beta} - \frac{\Delta \Phi}{3} \delta_{\alpha \beta} \), would be given by

\[
\tilde{C}_{\alpha eta}(r) = C_{\alpha eta}(r) - \frac{1}{3} \left( \delta_{\alpha \beta} \langle 5 \zeta_i(r) + \zeta_i(r) \rangle + \frac{1}{2} \zeta_i(r) \zeta_i(r) \right) \delta_{\alpha \beta},
\]

with the lensing efficiency function

\[
G(\chi) = \int_{\chi} d\chi' \frac{2}{3} \frac{d_\chi}{d\chi'} \frac{1 - \chi}{\chi'},
\]

with \( d_\chi/d\chi' = H(\chi')/c \) and \( n(\chi') \) being the distribution of the sources. Because weak gravitational lensing affects both ellipsoidal and spiral galaxies alike, we scale the resulting lensing spectra \( \Sigma^\ell(t) \) with the total number of galaxy pairs \( n^2 = (n_1 + n_2)^2 \).

### 3.3 Alignments of spiral galaxies

The alignment of spiral galaxies is purely due to their orientation, which in turn is related to the angular momentum correlation of neighbouring galaxies relative to the line of sight (Croft & Metzler 2000; Crittenden et al. 2001). Angular momentum correlations are mainly build up at early times during structure formation and are thus due to initial correlations (Catelan & Theuns 1996; Theuns & Catelan 1997; Catelan & Theuns 1997). The correlated angular momenta result into correlated inclination angles of neighbouring galaxies and thus ultimately into correlated ellipticities (Catelan et al. 2001). Assuming that the symmetry axis of the galactic disc coincides with the direction of the angular momentum \( \ell = L/L \), the ellipticity can be written as

\[
\epsilon = \frac{I^2 - I^2}{1 + I^2} + 2i \frac{I_i I}{1 + I^2}.
\]

Angular momentum is generated by a torque exerted by the ambient large-scale structure onto the protogalactic halo, a mechanism called tidal torquing (White 1984; Barnes & Efthathiou 1987; Schaefer 2009; Stewart et al. 2013). For Gaussian random fields the auto-correlation of angular momenta is given by (Lee & Pen 2001)

\[
\langle I_i I_j \rangle = \frac{1}{3} \left( 1 + \frac{A}{3} \right) \delta_{ij} - \frac{1}{3} \Delta_{\alpha \beta} \Phi_{\alpha \beta} \Phi_{\alpha \beta}.
\]

Since the correlation is determined by the traceless part of the shear tensor \( \Phi_{\alpha \beta} \), the resulting effect is clearly due to orientation effects only. For a Gaussian distribution \( p(L|\Phi_{\alpha \beta})dL \) and the use of eq. (17) one can express the ellipticity in terms of the tidal field

\[
\epsilon(\Phi) = \frac{A}{3} \left( \Phi_{11} \Phi_{22} - \Phi_{12} \Phi_{21} - 2i \Phi_{12} \Phi_{21} \right).
\]

Correlations in the ellipticities can thus be traced back to the 4-point function of the shear field, which is given in eq. (11). For keeping a correct relative normalisation of the shape correlations, we scale the resulting angular ellipticity spectra \( \Sigma^\ell(t) \) with the squared number of spiral galaxies \( n^2 \). It is remarkable that the shapes of spiral galaxies in the quadratic alignment model are in fact sensitive to tidal shear components parallel to the line of sight, in fact, those components determine the magnitude of the alignment effect, in contrast to the alignment of elliptical galaxies in the linear alignment model or to gravitational lensing, which reflect purely the tidal shear components perpendicular to the line of sight.

**Figure 1** gives a visual impression of alignments in the quadratic, angular momentum-based alignment model: From a realisation of a Gaussian random density field \( \delta(x) \) from the CDM-spectrum \( p(k) \) we computed the traceless, unit-normalised tidal shear \( \Phi_{\alpha \beta} \), which is used to determine the variance of the distribution \( p(L|\Phi_{\alpha \beta}) \). Angular momenta are drawn at random locations.
which fix the orientation of the galactic discs. The amount of correlation in the orientation of the galactic discs corresponds determined from the realisation corresponds to the theoretically computed correlation function. Ellipticity correlations between spiral galaxies are rather short-ranged, with a typical correlation length of about 1 Mpc/h (Schafer & Merkel 2012), which makes them a small-scale phenomenon at angular scales of $\ell \approx 10^3$ for Gpc-scale surveys.

3.4 Alignments of elliptical galaxies

For elliptical galaxies we assume a virialised system in which stars move randomly in a gravitational potential $\Phi$ with the velocity dispersion $\sigma^2$. In equilibrium the density profile is a solution of the radially symmetric Jeans equation and scales $\rho \propto \exp(-\Phi/\sigma^2)$. In the presence of a tidal field induced by the ambient large-scale structure, the equilibrium situation is perturbed and the galaxy finds a new equilibrium. Perturbing the Jeans equation at first order in the tidal fields $\delta \Phi/\Phi$ yields the following solution for the density:

$$\rho \propto \exp\left(\frac{-\Phi(x)}{c^2}\right) \left(1 - \frac{1}{2\sigma^2} \frac{\partial^2 \Phi(x)}{\partial x^2} x^2\right). \quad (20)$$

While the reaction of the halo to the tidal fields is determined by the velocity dispersion $\sigma^2$, i.e. how strongly the particles are bound in the gravitational potential, the relationship between tidal shear field and ellipticity needs as well to reflect the luminous profile, which we absorb in the definition of a constant of proportionality $D$. Since this model gives rise to ellipticities being linear in the tidal fields, we absorb in the definition of a constant of proportionality $D$, because those terms would be proportional to a third moment of the tidal field. In contrast, there will be a non-vanishing relationship between gravitational potential, the relationship between tidal shear field and velocity dispersion $\sigma$.

3.5 Cross-alignments between intrinsic shapes and lensing

For spiral galaxies there exist no GI-type terms due to Wick’s theorem, because those terms would be proportional to a third moment of the tidal shear field. In contrast, there will be a non-vanishing cross-correlation between lensing and the intrinsic alignment of elliptical galaxies. We would like to point out that these cross-correlations, $C_{ij}^{\text{GI}}(t)$, are symmetrised with respect to the bin numbers: Naturally, the more distant galaxy is lensed whereas the closer galaxy is intrinsically aligned while the inverse is not possible. We scale the cross-spectra $C_{ij}^{\text{GI}}(t)$ with the number $n_i(n_i + n_j) = n_{ij}$ of pairs involving at least one elliptical galaxy.

It is worth pointing out that there is a straightforward physical difference between the weak lensing and intrinsic alignments. As weak lensing is an integrated effect, there will be nonzero cross-correlations between different tomographic bins, whereas intrinsic alignments will, due to their locality, only show correlations within the same bin. This can already be used as a method of discrimination between $\text{I}$- and $\text{GG}$-spectra (Bernstein & Jain 2004; Huterer & White 2005), but will not get rid of the GI-contribution.

3.6 Shape correlations in a weak lensing survey

We carry out our investigation for a weak lensing survey similar to Euclid’s: The redshift distribution $n(z)dz$ is assumed to have the shape,

$$n(z)dz \propto \left(\frac{z}{z_0}\right)^2 \exp\left(-\frac{z}{z_0}\right) dz, \quad (22)$$

with the choices $\beta = 3/2$ and $z = 1/\sqrt{2}$, which generates a median redshift of unity (Laureijs et al. 2011). Euclid is expected to yield 40 galaxies per arcminute and to observe a fraction of $f_{\text{sky}} = 0.5$ of the sky.

Linking the correlations of the observable ellipticity $\epsilon$ to the tidal shear fields allows to express correlations in $\epsilon$ in terms of correlations in $\delta \Phi/\Phi$. A suitable Limber-projection with the redshift-distribution $n(z)dz$ while introducing a binning allows us to compute angular correlation functions and in the next step, to obtain tomographic angular E-mode spectra $C_{ij}^{\text{EI}}(t)$, $C_{ij}^{\text{EI}}(t)$ and lastly $C_{ij}^{\text{GI}}(t)$, which we can compare to the tomographic weak lensing spectrum $C_{ij}^{\text{GI}}(t)$ (Hu 1999; Hu & White 2001). Due to the locality of intrinsic alignments, the two $\text{II}$-spectra are $\propto \delta_{\text{II}}$, while the GI-spectrum or the weak lensing spectrum does not possess this property (Okumura & Jing 2009). Central to our investigation will be the linear dependence of the ellipticity with tidal shear field for gravitational lensing and for the intrinsic alignment of elliptical galaxies, while the shapes of spiral galaxies depend on squares of the tidal shear. For non-tomographic, 3-dimensional weak lensing surveys, the effects of intrinsic alignments are physically identical (Merkel & Schaefer 2013).

Figure 2 shows the expected $E$-mode spectra for a $\Lambda$CDM-cosmology with a conventional choice of the alignment parameters $A$ and $D$ for tomography with $n_{\text{bin}} = 3$ bins with Euclid, resulting from a Limber-projection (Limber 1954) and subsequent Fourier-transform. For a reasonably deep lensing survey such as Euclid’s, lensing-induced ellipticity correlations dominate over intrinsic alignments. The IA contribution on large angular scales is caused by elliptical galaxies following the tidal shearing model, whereas on small scales the contribution from spiral galaxies, which is described by the tidal torquing model, is most important. Over a wide range of angular scales the negative cross-correlation between gravitational lensing and intrinsic alignments shapes the ellipticity spectrum. It is notable that the shapes of elliptical galaxies and lensing measure tidal field components perpendicular to the line of sight and are proportional to the magnitude of the tidal shear, but that in contrast spiral galaxies reflect with their shapes the tidal field orientation including line of sight-components. For details on the derivation of angular shape correlation functions and $E/B$-mode ellipticity spectra we refer to Caprano et al. (2013), Schaefer & Merkel (2015) and Tugendhat & Schaefer (2018).
Separation of gravitational lensing and IAs

4 SEPARATION BETWEEN WEAK LENSING AND GALAXY ALIGNMENTS

4.1 Idea and formalism

Weak gravitational lensing and intrinsic alignments of galaxies are tidal gravitational effects of the cosmic large scale structure, but the details of how the observed shape of a galaxy is influenced by the tidal field $\partial^2_i \Phi$ generated by the large-scale structure depends on the interaction mechanism: Gravitational lensing is universal and operates on all galaxy shapes in an identical way, elliptical galaxies change their shape in proportion to the tidal gravitational field according to the tidal shearing model, and the shape of spiral galaxies is an orientation effect that depends on the squared, unit-normalised tidal field, as stipulated by the tidal torquing model. The dependences on the linear and squared tidal shear field have the important consequence that there is no cross-correlation between lensing and the intrinsic shape of spiral galaxies, $\langle \gamma \epsilon' \rangle = 0$ and neither between the shapes of spiral and elliptical galaxies, $\langle \epsilon_\alpha \epsilon'_\beta \rangle = 0$, if the tidal shear field follows Gaussian statistics: In this case, the two correlation functions would be proportional to a third moment of a symmetric distribution, which makes them vanish. In contrast, there will be a nonzero cross-correlation between the intrinsic shapes of elliptical galaxies and weak lensing, $\langle \gamma \epsilon'_e \rangle \neq 0$.

Starting with a tomographic observation of the ellipticity field in a range of redshift bins $i$, which contains contributions from weak gravitational lensing and from the two alignment mechanisms and which is sampled including colour-information we define the observed maps $e_{\alpha,i}$ and $e_{e,i}$, where the subscripts denote spiral galaxies and elliptical galaxies. These contain contributions from weak...
lensing $\gamma_i$ in redshift bin $i$ and their respective alignment mechanisms $\epsilon_{i,s}$ and $\epsilon_{i,e}$,

$$e_{i,s} = \gamma_i + \epsilon_{i,s},$$

$$e_{i,e} = \gamma_i + \epsilon_{i,e},$$

respectively. Using the same value of $\gamma_i$ for both shapes assumes that the change in shape due to gravitational lensing does not depend on the type of galaxy. In reality, the situation is more complicated, because an estimate of the shape of the galaxy depends on the brightness distribution in a nonlinear way which is affected by the tidal shear and higher derivatives of the gravitational potential. Similarly, there are dependences of the measured shape on colour. In reality, the situation is more complicated, because a galaxy can not be spiral and elliptical at the same time.

The two maps $e_{i,s}$ and $e_{i,e}$ respectively. Using the same value of $\gamma_i$ for both shapes assumes that the change in shape due to gravitational lensing does not depend on the type of galaxy. In reality, the situation is more complicated, because an estimate of the shape of the galaxy depends on the brightness distribution in a nonlinear way which is affected by the tidal shear and higher derivatives of the gravitational potential. Similarly, there are dependences of the measured shape on colour. In reality, the situation is more complicated, because a galaxy can not be spiral and elliptical at the same time.

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showing that there identical optimised choices for \( \alpha \) from \( C_{ij}^{+}(t) \). Likewise, the contribution \( N_{ij}(t) \) to the covariance matrix due to shape noise changes under rotations by \( \alpha \), implying for the entries

\[
N_{ij}^{+}(t) = \sigma_{n_{bin}}^{2} \left( n_{e} \cos^{2} \alpha + n_{s} \sin^{2} \alpha \right),
\]

as well as

\[
N_{ij}^{-}(t) = \sigma_{n_{bin}}^{2} \left( n_{s} \sin^{2} \alpha + n_{e} \cos^{2} \alpha \right),
\]

and finally

\[
N_{ij}^{0}(t) = \sigma_{n_{bin}}^{2} \left( n_{e} - n_{s} \right) \cos \alpha \sin \alpha,
\]

which effectively corresponds to the expressions for \( C_{ij}^{+}(t) \), \( C_{ij}^{-}(t) \) and \( C_{ij}^{0}(t) \) if one sets the lensing effect to zero and replaces the intrinsic alignment spectra with a Poissonian noise term. With the spectra reflecting the number of galaxy pairs and the shape measurement noise being proportional to the number of galaxies on retains a correct relative normalisation of all spectra and noise terms.

### 4.2 Misclassification

Imperfections in the classification of galaxies can be incorporated by introducing conditional probabilities of the type \( p(b|r) \) indicating that an elliptical galaxy (with the label \( r \), due to the red colour) is wrongly classified as a spiral galaxy (labelled \( b \), due to the blue colour). Consequently, the number of observed spiral galaxies \( n_{s} \) and observed elliptical galaxies \( n_{e} \) is given in terms of the true number of spiral galaxies \( n_{s} \) and elliptical galaxies \( n_{e} \) by

\[
n_{s} = p(b|r)n_{s} + p(r|b)n_{e},
\]

\[
n_{e} = p(r|b)n_{e} + p(b|r)n_{s},
\]

where the normalisation conditions \( p(b|r) + p(r|b) = 1 \) and \( p(r|b) + p(b|r) = 1 \) make sure that the total number of galaxies is conserved, \( n = n_{s} + n_{e} = n_{s} + n_{e} \). A misclassification would take place if \( p(b|r) \) and \( p(r|b) \) were not equal to zero, so in fact only two of the conditional probabilities are independent variables. As a consequence of the misclassification one would obtain (i) wrong amplitudes for the ellipticity correlations of a given galaxy type, (ii) additional contributions from the respective other galaxy type with possible cross-correlations with the lensing signal, and (iii) one would estimate these ellipticity spectra on the basis of the wrongly inferred number of galaxy pairs. The observed ellipticity fields in the tomographic bin \( i \) now contain contributions from wrongly classified galaxies,

\[
e_{ij} = y_{i} + p(b|r)e_{ij} + p(r|b)e_{ij},
\]

\[
e_{ij} = y_{i} + p(r|b)e_{ij} + p(b|r)e_{ij}.
\]

Consequently, the covariance matrix \( C_{ij}(t) = S_{ij}(t) + N_{ij}(t) \) reads

\[
C_{ij}(t) = \begin{pmatrix} C_{ii}^{e}(t) & C_{ij}^{e}(t) \\ C_{ji}^{e}(t) & C_{jj}^{e}(t) \end{pmatrix},
\]

whose signal part \( S_{ij}(t) \) contains contributions from wrongly classified galaxies. The components of matrix in the colour basis assume the following form

\[
S_{ij}^{b}(t) = n_{e}^{2}C_{ij}^{b}(t) + 2p(b|r)C_{ij}^{e}(t) + p(b|r)^{2}C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}n_{s}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{s}n_{e}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}^{2}C_{ij}^{e}(t) + 2p(r|b)C_{ij}^{e}(t) + p(r|b)^{2}C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}n_{s}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{s}n_{e}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}^{2}C_{ij}^{e}(t) + 2p(r|b)C_{ij}^{e}(t) + p(r|b)^{2}C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}n_{s}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{s}n_{e}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}^{2}C_{ij}^{e}(t) + 2p(r|b)C_{ij}^{e}(t) + p(r|b)^{2}C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{e}n_{s}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

\[
S_{ij}^{b}(t) = n_{s}n_{e}C_{ij}^{e}(t) + C_{ij}^{e}(t)
\]

and whose noise part \( N_{ij}(t) \) needs to be updated because the numbers of blue and red galaxies are not equal to the numbers of spirals and ellipticals if \( p(r|b) \neq 0 \) or \( p(b|r) \neq 0 \).

\[
N_{ij}(t) = \sigma_{n_{bin}}^{2} \begin{pmatrix} n_{e} \delta_{ij} & 0 \\ 0 & n_{s} \delta_{ij} \end{pmatrix}.
\]

It should be noted that equations (43) and (44) consistently reduce to equations (26) and (27) if there is no misclassification, \( p(r|b) = p(b|r) = 0 \), such that the covariances are identical, \( C_{ij}(t) = C_{ij}(t) \). Finally one would again rotate \( C_{ij}(t) \) into the new basis with the orthogonal matrix \( U \). We will use for illustration purposes misidentification rates of \( p(b|r) = p(r|b) = 0.1 \). In Figure 4 we show the influence of the misclassification: the dashed line shows a case with all galaxies being identified correctly, while the solid curves have misclassification of 10 %. Clearly the lensing signal itself is unaffected as it does not depend on the colour of the galaxy. The other contributions however change in amplitude and dependence on \( \alpha \). It is worth noting that the position where lensing and the GT term vanish does not depend on the misclassification rate but only on the ratio of elliptical and spiral galaxies in the survey.

### 4.3 Separating out intrinsic alignments

The signal to noise-ratio \( \Sigma \) of the isolated intrinsic alignments \( C_{ij}^{e}(t) + C_{ij}^{s}(t) \) can be determined to be \( \Sigma \approx 60 \) for Euclid: Focusing on the contribution \( e_{ij} \), after setting \( \alpha = 3\pi/4 \) for the case \( n_{s} = n_{e} \) defines the signal covariance \( S_{ij}^{e}(t) \) as the upper left \( n_{bin} \times n_{bin} \)-block of \( S_{ij}(t) \). Consequently, the significance for measuring the intrinsic alignment contribution \( C_{ij}^{s}(t)+C_{ij}^{e}(t) \) is given...
by
\[ \Sigma^2 = f_{\text{sky}} \sum_\ell \frac{2\ell + 1}{2} \text{tr} \left( C^{-1}_{\text{Eff}}(\ell) S^{++}(\ell) C^{-1}_{\text{Eff}}(\ell) S^{++}(\ell) \right). \] (45)

By isolating the $C_{\text{eff}}$ components we throw away the information contained in the other components of the covariance.

Figure 5 shows in detail the attainable significance $\Sigma$ for the intrinsic alignment contribution, both cumulatively and differentially, as a function of multipole moment $\ell$ and for different number of tomographic bins. Clearly, intrinsic alignments generate a significant signal in Euclid’s survey and the magnitude of the alignments, both for spiral and elliptical galaxies, allows a determination of parameters of the alignment model at percent precision. It is also interesting to see that most of the signal is picked up at relatively low multipoles, showing that the assumption of Gaussian random fields can indeed be used. The reason for this is that the intrinsic alignment signal only probes the auto-correlation of different bins and thus the shape-noise dominates the high multipole moments, which becomes dominant earlier compared to the $H$ alignments due to their lower amplitude. Tomography is a large factor in attaining a high signal to noise ratio and therefore, in investigating intrinsic alignment models, as $\Sigma$ increases from 35 with 2 bins to over 60 with 7 bins. These numbers are in agreement with forecasts on alignment amplitudes: As the alignment signal is proportional to $A$ or $D$, the signal to noise-ratio corresponds to the inverse statistical error on these prefactors, such that one should achieve % -level precision from a measurement yielding 100% confidence. It is remarkable that this level can be reached even if one considers a combination of cosmological probe and a complex cosmological model (Kitching et al. 2014; Merkel & Schaefer 2017).

We quantify the difference between the ideal spectra and the one containing wrongly classified galaxies with the average $\langle \Delta \chi^2 \rangle$ between the two models. For that purpose, we identify the $C_{\text{eff}}^{++}(\ell)$ contribution from the rotated covariance matrix $C_\ell(t)$ and the corresponding $X_{\text{eff}}^{++}(\ell)$ from the rotated covariance matrix $C_\ell(t)$ including shape correlations of wrongly identified galaxies, by setting $\alpha = 3\pi/4$.

\[ (\chi^2) = f_{\text{sky}} \sum_\ell (2\ell + 1) \text{tr} \left( \ln C^{++}(\ell) + \text{id} \right) \] (46)

whereas the average $\langle \chi^2 \rangle$ of the wrongly classified model can be computed with

\[ (\chi^2) = f_{\text{sky}} \sum_\ell (2\ell + 1) \text{tr} \left( \ln X^{++}(\ell) + X^{-1}_{\text{eff}}(\ell) C^{++}(\ell) \right), \] (47)

such that the difference $\langle \Delta \chi^2 \rangle = (\chi^2) - (\chi^2)$ between the true and false model in the light of the on average expected data $C_\ell$ yields

\[ \langle \Delta \chi^2 \rangle = f_{\text{sky}} \sum_\ell (2\ell + 1) \left( \ln \left( \frac{\text{det} X^{++}(\ell)}{\text{det} C^{++}(\ell)} \right) + \text{tr} \left( X^{-1}_{\text{eff}}(\ell) C^{++}(\ell) \right) - n_{\text{bin}} \right), \] (48)

where we again assume statistical homogeneity of the $e_{\alpha x}$ and $e_{\alpha y}$ fields, and a number of $2\ell + 1$ statistically uncorrelated modes on a given angular scale $\ell$ as a consequence of statistical isotropy. We scale the resulting $\chi^2$-values or signal to noise-ratios $\Sigma$ by $\sqrt{f_{\text{sky}}}$ if the sky coverage is incomplete. If $X^{++}(\ell) = C^{++}(\ell)$, then $\langle \Delta \chi^2 \rangle = 0$ because $\text{tr} \left( X^{-1}_{\text{eff}}(\ell) C^{++}(\ell) \right) = n_{\text{bin}}$. In the relations above, id refers to the unit matrix in $n_{\text{bin}}$ dimensions.

First of all we expect the obtainable $\Sigma(\ell)$ to be smaller in the case where galaxies have been identified wrongly, since the relative contribution is smaller as it can be seen in Figure 4. This will lead to a difference $\langle \Delta \chi^2 \rangle$ between the true model and the model containing wrongly identified galaxies by expressing the difference in the spectra in units of the cosmic variance, which we show in Figure 6. At $\ell \gtrsim 200$, the difference in $\chi^2$ continues to grow steadily due to the difference in the noise contributions, as the noise is dependent on the galaxy counts, which naturally differ if there is a misclassification of galaxy types. Furthermore, the loss of Gaussianity in the correlations at larger $\ell$ will lead to complicated cross-correlations between the alignment effects that have not been included in the calculation of $\langle \Delta \chi^2 \rangle$ or indeed in the considerations for a misclassified signal $S_\ell(t)$ (cf. Equation 43).
4.4 Boosting the weak lensing spectrum

The weak lensing contribution $C^\ell_{ij}(\ell)$ can be boosted relative to the intrinsic alignments $C^\ell_{ij,ba}(\ell)$ and $C^\ell_{ij,bb}(\ell)$ by choosing a value of $\alpha$ in the vicinity of $\pi/4$. Unlike the previous case, there is no complete cancellation of the intrinsic alignment contribution and one can only hope to optimise the measurement. This optimisation, however, does depend on the particular values of the lensing and alignment spectra and is therefore not model-independent. Because this is inevitably the case, there are two competing effects to be taken care of: Firstly, the amplitude of the contribution of the weak lensing spectrum to $C^\ell_{ij}(\ell, \alpha)$ changes as a function of $\alpha$, giving rise to a change in the statistical precision of the measurement encoded in the Fisher-matrix,

$$F_{\mu\nu}(\alpha) = \frac{1}{2} \sum_\ell \frac{2\ell + 1}{\ell} \text{tr} \left[ C^\ell_{ij,ba}(\ell,\alpha) \partial_\alpha S^\ell_{ii}(\ell,\alpha) C^\ell_{ij,bb}(\ell,\alpha) \partial_\alpha S^\ell_{ii}(\ell,\alpha) \right],$$

(49)

and resulting in changes in the marginalised statistical errors $\sigma^2_{\alpha}(\alpha) = \left[F^{-1}(\alpha)_{\mu\nu}\right]_{\mu\nu}$. We approximate derivatives $\partial_\alpha S^\ell_{ii}(\ell, \alpha)$ with the derivatives at $\alpha = \pi/4$ because weak lensing dominates the spectrum and the derivatives with respect to the cosmological parameters are identical for every choice of $\alpha$ in this limit.

Secondly, the relative contribution of the intrinsic alignment spectra change as well with $\alpha$ such that there is a varying amount of contamination of intrinsic alignments and a corresponding systematic error $\delta_\alpha(\alpha)$. This systematic error is computed using the formalism of Schäfer & Heisenberg (2012) as an extension to Taburet et al. (2009) and Amara & Réfrégier (2008) from

$$\delta_\alpha(\alpha) = \sum_\ell \left(G^{-1}(\alpha)\right)_{\mu\nu} a_\alpha(\ell),$$

(50)

with the matrix $G_{\mu\nu}$

$$G_{\mu\nu}(\alpha) =$$

$$= \frac{1}{2} \sum_\ell \frac{2\ell + 1}{\ell} \text{tr} \left[ C^\ell_{ij,ba}(\ell,\alpha) \partial_\alpha C^\ell_{ij,bb}(\ell,\alpha) - \text{id} \right]$$

$$- \text{tr} \left[ C^\ell_{ij,ba}(\ell,\alpha) \partial_\alpha C^\ell_{ij,bb}(\ell,\alpha) - \text{id} \right],$$

(51)

and the vector $a_\alpha$

$$a_\alpha(\alpha) = f_{\text{sky}} \sum_\ell \frac{2\ell + 1}{2\ell + 2} \text{tr} \left[ C^\ell_{ij,ba}(\ell,\alpha) \partial_\alpha C^\ell_{ij,bb}(\ell,\alpha) \left( C^\ell_{ij,d\alpha}(\ell,\alpha) C^\ell_{ij,d\alpha}(\ell,\alpha) \text{id} \right) \right]$$

(52)

which are both functions of the mixing angle $\alpha$.

A convenient way for expressing the magnitude of the systematic error in units of the statistical error is the figure of bias $Q(\alpha)$,

$$Q^2(\alpha) = \sum_\mu_\nu F_{\mu\nu}(\alpha) \delta_\alpha(\alpha) \delta_\alpha(\alpha),$$

(53)

which is related to the Kullback-Leibler divergence $D_{\text{KL}}$ for Gaussian likelihoods under the assumption of constant covariances,

$$D_{\text{KL}} = \frac{Q^2}{2}. \quad (54)$$

Figure 7 shows $Q$ as a function of $\alpha$: Clearly, there is an optimised choice of $\alpha$ that reduces the intrinsic alignment contribution in order to yield an unbiased measurement of the cosmological parameter set, which we demonstrate by computing $Q$ in scanning through all possible choices of $\alpha$. We calculate the figure of bias $Q$ both with the systematic errors taken in relation to the statistical errors at $\alpha = 0, F_0$, in black and with the Fisher matrix taken at the respective $\alpha$, $F_{\text{res}}$, in blue. It becomes clear that as long as there is a significant lensing contribution, up until $\alpha \approx \pi/2$, the differences are negligible. As soon as the contributions from intrinsic alignments become comparable to the one from weak lensing, the figure of bias forks: while comparing the systematic errors to a co-evolving statistical error, the figure of bias becomes increasingly small. This is not due to a diminishing bias but rather due to extremely large statistical errors as the information from lensing disappears. For a constant statistical error taken at $\alpha = 0$, the curve is therefore more representative of the actual precision of the measurement. As $\alpha$ approaches $\pi$, the curves start to merge again.

Typically, relative contributions of order ten percent of the intrinsic alignment signal to weak lensing cause figures of bias $Q$ of the order up to a few hundred, which can be controlled by our technique. It would be interesting to propagate intrinsic alignments with other systematic effects through the parameter estimation process as outlined by Cardone et al. (2014).

Lastly, we aim to constrain the parameters of the alignment models for the two types of galaxies, specifically the alignment amplitude $D$ that describes the elasticity of elliptical galaxies and the amplitude $A$ which describes the magnitude $A$ of the misalignment between tidal shear and inertia which is responsible for angular momentum generation in spiral galaxies. If one suppresses lensing, including GI-alignment, Figure 8 suggests that both alignment parameters can be constrained at the percent level with the Euclid data set without having to worry about biases due to gravitational lensing. These numbers result from a Fisher-matrix analysis for the parameters $A$ and $D$ with a choice of $\alpha$ that eliminates lensing, and fitting the corresponding intrinsic ellipticity spectra to the remainder, while taking account of cosmic variance and shape noise contributions and keeping all cosmological parameters fixed to their fiducial values.
to obey one of the two alignment mechanisms. Based on the fact that lensing is universal and affects all galaxy shapes in an identical way irrespective of galaxy type it is possible to find linear combinations of ellipticity fields measured for spiral and elliptical galaxies that do not contain any lensing signal and only retain shape correlations due to intrinsic alignments. It should be emphasised that this is possible without any assumption about the details of gravitational lensing or of the intrinsic alignment models. Intrinsic alignments that have been separated in this way from the lensing signal, can be measured on the basis of Euclid’s weak lensing data set with a high statistical significance.

(i) As alignment models we consider tidal shearing for elliptical galaxies and tidal torquing for spiral galaxies, and compute the resulting tomographic angular ellipticity spectra including the non-zero cross-correlation between the shape of elliptical galaxies and weak lensing. Both models have a single free parameter each, which we determined from weak lensing data and from numerical simulations, respectively. In analysing data, we showed that Euclid will allow their measurement at a level of a few percent. In that, we assume constant and scale-independent alignment parameters. The forecasted statistical precision would allow the investigation of alignment models with Euclid’s weak lensing data set, and shed light on (i) the average misalignment of angular momenta with tidal shear fields and (ii) the reaction of a virialised structure to external tidal shear fields.

(ii) We develop a statistical method which allows the separation between weak lensing and both alignment types on the basis of colour or morphological information: We assume that elliptical galaxies, if they are correctly identified, obey exclusively linear tidal shearing as their alignment mechanism, while spiral galaxies are described by the quadratic tidal torquing model. Gravitational lensing is universal as it affects the shapes of spiral and elliptical galaxies identically.

(iii) All mitigation and suppression techniques have in common that statistical precision is traded for systematical accuracy, and our method is no exception: Starting from shape catalogues measured for different types of galaxies it is possible to find linear combinations of the shape measurements that contain no alignment, no elliptical alignment or no gravitational lensing, including in this case no lensing-alignment cross correlation either. With that in mind, it is possible to find a linear combination that eliminates lensing from the data set and leaves only contributions to the shape correlations that differ between elliptical and spiral galaxies, i.e. intrinsic alignments. Taking these cleaned correlation functions, they allow a measurement of the intrinsic alignment signal without any model assumptions to ~ 60% of statistical precision for a tomographic survey such as Euclid’s.

(iv) Suppression of intrinsic alignment contributions for achieving a bias-free weak lensing measurement is possible, but not completely. With an optimised choice of the mixing angle θ one can reduce the systematical bias in units of the statistical error by a large margin, and we show that biases are reduced to amount to typically a few σ for the full ΛCDM parameter set. We quantify the magnitude of the systematic error in units of the statistical error by the figure of bias \( Q^2 = \sum_{\mu \nu} F_{\mu \nu} \delta \rho \delta \), which takes care of the orientation of the systematic error \( \delta \rho \) with respect to the statistical degeneracies encoded in the Fisher-matrix \( F_{\mu \nu} \). Incidentally, \( Q^2 / 2 \) corresponds to the Kullback-Leibler-divergence between the biased and unbiased likelihood \( L \).

(v) Misclassification, i.e. non-zero probabilities \( p(r|b) \) and \( p(b|r) \) do not affect our conclusions strongly even for very high

Figure 7. Total bias \( Q(\alpha) \) in units of the statistical error, \( Q^2 = \sum_{\rho \rho} F_{\rho \rho} \delta \rho \delta \), as a function of the mixing angle \( \alpha \), where the minimum indicates the value of \( \alpha \) that is able to yield the smallest possible systematic error corresponding to the cleanest weak lensing spectrum.

Figure 8. Fisher-constraints of the model parameters \( A \) and \( D \) for the quadratic and linear alignment models respectively for \( \eta_{\text{bias}} = 7 \) in Euclid.

5 SUMMARY

Subject of this investigation was the use of colour information to differentiate between intrinsic ellipticity correlations of galaxies and the weak gravitational lensing signal. We operate under the assumptions that (i) the large-scale structure follows Gaussian statistics on large scales, (ii) intrinsic shapes of elliptical galaxies follows the linear tidal shearing model, (iii) intrinsic shapes of spiral galaxies follows the quadratic tidal torquing model, (iv) gravitational lensing is universal and linear in the tidal shear, and (v) it is possible to classify galaxies on the basis of colour information.
probabilities of misclassification. We quantify this by computing the difference between the resulting covariance in terms of the $\chi^2$-statistic, where instrumental noise and cosmic variance are present as noise sources. At accessible scales below a few hundred in $\ell$, i.e. before the instrumental noise dominates, the integrated $\Delta \chi^2$ is a few ten, which given the sensitivity of a weak lensing survey with respect to parameters such as $\sigma_8$ or $\Omega_m$, would give rise to biases. We estimate that the misidentification probabilities would need to be controlled to the percent-level for the biases not to exceed $1\sigma$ in terms of the statistical error.

Preceding the work presented here, there are quite a few other mitigation techniques: Catelan et al. (2001) proposed tomographic methods to reduce the $II$-alignment signal, by avoiding spatially close galaxies. This exploits the large correlation length of the weak lensing signal, compared to the intrinsic alignment signal. The drawback of this method is the increased cosmic variance and shape noise and thus a loss of statistical power. This technique was used by several authors (e.g. Heymans & Heavens 2003; Heymans et al. 2004; King & Schneider 2002; Heymans et al. 2004; Takada & White 2004; Joachimi et al. 2013a; Heymans et al. 2013).

Another method is to construct a different weighting of the cosmic shear signal, to reduce the contamination by $GI$-alignments. This nulling technique was discussed by Huterer & White (2005); Joachimi & Schneider (2010). It has been proposed as well to null and boost magnification (Heavens & Joachimi 2011; Schneider 2014), which can also bias the parameter inference process. Alternatively, one can also use self-calibration techniques, making use of additional information and the cross-correlation of cosmic shear and galaxy clustering, as well as galaxy clustering auto-correlations, which has been used by Bernstein & Jain (2004); Bernstein (2009); Zhang (2010) or use the large difference in the amplitudes of $E$- and $B$-mode spectra of intrinsic alignments and weak lensing (Crittenden et al. 2002; Schaefer & Merkel 2015).

Our method could as well be extended bispectra of the ellipticity field (Shi et al. 2010; Merkel & Schaefer 2014; Larsen & Challinor 2016), either in order to make the method less dependent on the assumption of Gaussianity of the tidal shear field or by relaxing on the relationship that the observable shape depends on the tidal shear field in a linear or quadratic way. Taking this idea further, it would be very interesting to see if other cross-correlation measurements, for instance correlations between the CMB-lensing field and galaxy shapes at higher redshifts than the ones considered here, would constraint alignment processes as well (Hall & Taylor NRAS).

In summary, there is a wealth of information that helps to differentiate intrinsic alignments and weak lensing, even without sacrificing statistical precision for accuracy. While our investigation assumes that it is possible to assign an alignment model to a given galaxy on the basis of its colour (or other morphological information), inclusion of higher-order statistics, $E/B$-mode decomposition or redshift weighting scheme should enable a thorough understanding of shape correlations. We would consider the possibility of eliminating the lensing signal including the GI-terms from the ellipticity correlation through a suitable choice of $\alpha$ very interesting for the model-independent detection of intrinsic alignments.

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