Rotating AdS black holes in Maxwell-\(f(T)\) gravity

G G L Nashed\(^1,2\) and Emmanuel N Saridakis\(^3,4\)

\(^1\) Centre for Theoretical Physics, The British University, PO Box 43, El Sherouk City, Cairo 11837, Egypt
\(^2\) Mathematics Department, Faculty of Science, Ain Shams University, Cairo 11566, Egypt
\(^3\) Department of Physics, National Technical University of Athens, Zografou Campus GR 157 73, Athens, Greece
\(^4\) CASPER, Physics Department, Baylor University, Waco, TX 76798-7310, United States of America

E-mail: nashed@bue.edu.eg and Emmanuel_Saridakis@baylor.edu

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Abstract
The investigation on higher-dimensional AdS black holes is of great importance under the light of AdS/CFT correspondence. In this work we study static and rotating, uncharged and charged, AdS black holes in higher-dimensional \(f(T)\) gravity, focusing on the power-law ansatz which is the most viable according to observations. We extract AdS solutions characterized by an effective cosmological constant that depends on the parameters of the \(f(T)\) modification, as well as on the electric charge, even if the explicit cosmological constant is absent. These solutions do not have a general relativity or an uncharged limit, hence they correspond to a novel solution class, whose features arise solely from the torsional modification alongside the Maxwell sector incorporation. We examine the singularities of the solutions, calculating the values of various curvature and torsion invariants, finding that they do possess the central singularity, which however is softer comparing to standard general relativity case due to the \(f(T)\) effect. Additionally, we investigate the horizons structure, showing that the solutions possess an inner Cauchy horizon as well as an outer event one, nevertheless for suitably large electric charge and small mass we obtain the appearance of a naked singularity. Finally, we calculate the energy of the obtained solutions, showing that the \(f(T)\) modification affects the mass term.

Keywords: \(f(T)\) gravity, black holes, AdS

(Some figures may appear in colour only in the online journal)
1. Introduction

After the formulation of the AdS/CFT correspondence [1], namely the correspondence between gravity in a higher dimensional space and the gauge theory on its boundary, a large amount of research has been devoted in the investigation of AdS black holes [2–9]. On the other hand, it has been widely discussed that if the higher-dimensional gravitational theory corresponds to modified gravity, then one obtains corrections in the gauge theory side in the strong coupling limit [10]. Finally, the role of the Maxwell sector has also been found to be important in the aforementioned duality structure [11–14].

Spherically symmetric solutions, and in particular AdS black holes, have been studied in many modified gravity theories [15–28], nevertheless almost all of them remain in the framework of curvature-modified gravity, namely in modified-gravity formulations which are based on the standard Einstein–Hilbert action. On the other hand, it was recently realized that one can construct new classes of gravitational modifications starting from the torsional formulation of gravity, that is from the teleparallel equivalent of general relativity (TEGR) [29–69]. Although black hole solutions in torsional modified gravity, such as \( f(T) \) gravity, have been studied in the literature [70–85], the detailed investigation of charged AdS black holes and in particular in higher dimensions is something that still misses.

In the present work we are interested in investigating rotating charged AdS black holes in higher-dimensional \( f(T) \) gravity. Such an analysis will be very useful for the application of AdS/CFT correspondence in torsional gravity, which may have an advantage relating to the boundary structure comparing to the curvature formulation (since the torsion and curvature scalars differ by a boundary term) [29]. Furthermore, note that although curvature-based modified gravity has usually higher-order field equations, which apart from raising ghost and instabilities issues do not allow for the extraction of analytic solutions, in the case of \( f(T) \) gravity the equations are second-order and hence analytic solutions can be easily extracted [29]. This feature becomes important in relation to the arguments that higher derivatives terms are strongly constrained in AdS/CFT framework due to causality constraints [86, 87]. Since \( f(T) \) gravity does not include higher derivatives, it bypasses these constraints, and thus its application to the AdS/CFT correspondence has an additional advantage comparing to curvature modified gravity.

The plan of the work is as follows. In section 2 we briefly review \( f(T) \) gravity and we extract the equations in the presence of the electrodynamics sector. In section 3 we extract uncharged and charged static AdS solutions, analyzing their singularity and horizon structure and calculating their energy. In section 4 we extract the rotating charged AdS solutions in Maxwell-\( f(T) \) gravity. Finally, section 5 is devoted to the conclusions.

2. \( f(T) \) gravity

In this section we present briefly the \( f(T) \) gravitational theory. In torsional formulation of gravity it proves convenient to use the vielbeins fields \( b^i{}_{\mu} \) (tetrads in four dimensions) as dynamical variables, which form an orthonormal basis for the tangent space at each point of the spacetime. These are related to the metric through

\[
g_{\mu\nu} := \eta_{ij} b^i{}_{\mu} b^j{}_{\nu},
\]

with \( \eta_{ij} = (+, −, −, − \cdots) \) being the \( N \)-dimensional Minkowskian metric of the tangent space (Greek indices are used for the coordinate space while Latin indices for the tangent one).
One introduces the curvature-less Weitzenböck connection \( \Gamma^\lambda_{\mu
u} := b^\lambda_i \partial^i b^j_{\mu} \) [88], and thus the torsion tensor is defined as
\[
T^\alpha_{\mu\nu} := \Gamma^\alpha_{\nu\mu} - \Gamma^\alpha_{\mu\nu} = b^\alpha_i \left( \partial^i b^j_{\nu} - \partial^j b^i_{\nu} \right),
\]
which carries all the information of the gravitational field. Finally, contracting the torsion tensor we obtain the torsion scalar as
\[
T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\rho\mu} - T_{\rho \rho\mu} T^{\mu \nu}. \tag{3}
\]
When \( T \) is used as the Lagrangian in the action of teleparallel gravity the obtained theory is the teleparallel equivalent of general relativity (TEGR), since variation with respect to the vielbeins leads to exactly the same equations with general relativity.

Inspired by the \( f(R) \) extensions of general relativity, one can extend \( T \) to \( f(T) \), obtaining \( f(T) \) gravity, determined by the action [29]:
\[
L = \frac{1}{2\kappa} \int |b| f(T) \, d^N x, \tag{4}
\]
where \( |b| = \sqrt{-g} = \det (b^\mu_{i}) \) is the determinant of the metric and \( \kappa \) is a dimensional constant defined as
\[
\kappa = \frac{2}{N-3} \Omega_{N-1} \frac{\kappa}{G_N},
\]
with \( \Gamma \) the \( \Gamma \)-function (in the case \( N = 4 \) we have \( 2/(N-3)\Omega_{N-1} = 8\pi \)).

In this work we desire to study the charged AdS black hole solutions in the framework of \( f(T) \) gravity. Hence, in action equation (4) we add the Maxwell Lagrangian too. Therefore, the considered action in this work is
\[
L = \frac{1}{2\kappa} \int |b| f(T) \, d^N x + \int |b| L_{em} \, d^N x, \tag{6}
\]
where \( L_{em} = -\frac{1}{2} F \wedge \star F \), with \( F = dA \) and \( A = A_\mu dx^\mu \) the electromagnetic potential 1-form [71].

Variation of action equation (6) with respect to the vielbeins leads to [29]:
\[
\xi^\mu_{\nu} = S^\mu_{\rho\nu} \partial^\rho T_{\mu T} + \left[ b^{-1} b^\nu_{\mu} \partial^\mu (b b_{\alpha}^\rho S_{\alpha}^{\mu\nu}) - T^\alpha_{\lambda\mu} S_{\lambda}^{\mu\nu} \right] f_T - \frac{f}{4} \delta^\mu_{\nu} + \frac{1}{2} \kappa e^m_{\mu\nu} \equiv 0, \tag{7}
\]
with \( f := f(T) \), \( f_T := \frac{\partial f(T)}{\partial T} \), \( T_{\mu T} := \frac{\partial^2 f(T)}{\partial T^2} \), \( e^m_{\mu\nu} \) the energy-momentum tensor of the electromagnetic field defined as
\[
e^m_{\mu\nu} = F_{\mu\nu} \cdot \delta^\alpha_{\beta} F_{\alpha\beta},
\]
and \( S^\mu_{\nu\alpha} \) the superpotential tensor, which is anti-symmetric in the last two indices, defined as
\[
S_{\alpha}^{\mu\nu} := \frac{1}{2} \left( K_{\alpha}^{\mu\nu} + \delta^\alpha_{\beta} T^{\nu\beta} - \delta^\nu_{\alpha} T^{\beta\mu} \right), \tag{8}
\]
where \( K_{\alpha}^{\mu\nu} \) is the contortion tensor defined as
\[ K^{\mu\nu}_\alpha := -\frac{1}{2} \left( T^{\mu\nu}_\alpha - T^{\nu\mu}_\alpha - T^\alpha_{\mu\nu} \right). \]  

(9)

Additionally, variation of (6) with respect to \( A_\mu \) gives:

\[ \partial_\nu \left( \sqrt{-g} F^{\mu\nu} \right) = 0. \]  

(10)

The above equations determine Maxwell-\( f(T) \) gravity in arbitrary dimensions.

3. Anti-de-Sitter black hole solutions

In this section we extract AdS charged black hole solutions in general dimensions in the case of \( f(T) \) gravity. Using cylindrical coordinates in \( N \) dimensions \((t, r, \phi_1, \phi_2, \ldots, \phi_n, z_1, z_2 \ldots z_k)\), with \( k = 1, 2 \cdots N - n - 2 \), in which \( 0 \leq r < \infty \), \( -\infty < t < \infty \), \( 0 \leq \phi_n < 2\pi \) and \( -\infty < z_k < \infty \), we consider the vielbein [71]:

\[ (b'_\mu) = \left( \sqrt{B(r)}, \frac{1}{\sqrt{B_1(r)}}, r, r, r \cdots \right), \]  

(11)

which corresponds to the metric

\[ ds^2 = B(r)dt^2 - \frac{1}{B_1(r)}dr^2 - r^2 \left( \sum_{i=1}^{n} d\phi_i^2 + \sum_{k=1}^{N-n-2} dz_k^2 \right), \]  

(12)

where the functions \( B(r) \) and \( B_1(r) \) depend only on the radial coordinate \( r \). We mention here that metric (12) is not the most general one since it includes flat sections and not spherical or hyperbolic ones. This arises from the fact that considering the general metric in four dimensions one obtains additionally the \( r - \theta \) field equation which implies that either \( f_{T T} = 0 \) (which is TEGR i.e. general relativity case), or \( T = \text{const.} \), or that the \( \phi \)-section is flat [72, 89]. Therefore, in order to obtain general new solutions we focus on the metric (12).

Substituting the vielbein form (11) into the torsion scalar definition (3) we find

\[ T = (N - 2) \frac{B'B_1}{rB} + (N - 2)(N - 3) \frac{B_1}{r^2}, \]  

(13)

where \( B'(r) \equiv \frac{dB(r)}{dr} \) and \( B'_1(r) \equiv \frac{dB_1(r)}{dr} \) and from now on we omit the arguments in \( B, B_1, B', B'_1 \). Finally, since the power-law \( f(T) \) form is the one with best agreement with cosmological data [90–92], in the following we focus our analysis to the choice

\[ f(T) = T + \beta T^2 + \gamma T^3 - 2\Lambda, \]  

(14)

with \( \beta \) and \( \gamma \) the model parameters and where we have included an explicit cosmological constant for completeness.

3.1. Asymptotically static AdS black holes

We start our investigation by extracting asymptotically static \( AdS \) black holes in the case of absent electromagnetic sector, namely considering \( F^{\mu\nu} = 0 \). In this case, inserting the vielbein (11) into the general field equations (7) and (10) we obtain the following non-vanishing components:
\[ \xi'_r = 2Tf_T + 2\Lambda - f = 0, \]
\[ \xi^0_{\phi_1} = \xi^0_{\phi_2} = \ldots = \xi^0_{\phi_4} = \xi^1_{\phi_1} = \xi^2_{\phi_1} = \ldots = \xi^{n-1}_{\phi_{n-1}} = \]
\[ \frac{f_T r^2 T + (N - 2)(N - 3)B_1}{(N - 2)\gamma} + \frac{fr}{2^{2\gamma}B_2} \left\{ 2^{2\gamma}B_2 B'' - r^2 B_1 B'' + 2(2N - 5)rB_1B' + r^2 B_1B'_1 \right\} - f + 2\Lambda = 0, \]
\[ \xi^0_r = \frac{2(N - 2)B_1frT^2}{r} + \frac{(N - 2)f_T}{r^2 B} \left\{ 2(N - 3)BB_1 + rB_1B' + rBB'_1 \right\} - f + 2\Lambda = 0. \] (15)

In the case of the \( f(T) \) form (14) these equations reduce to
\[ \xi'_r = T + 3\beta T^2 + 5\gamma T^3 + 2\Lambda = 0, \] (16)
\[ \xi^0_{\phi_1} = \xi^0_{\phi_2} = \ldots = \xi^0_{\phi_4} = \xi^1_{\phi_1} = \xi^2_{\phi_1} = \ldots = \xi^{n-1}_{\phi_{n-1}} = \]
\[ \frac{2(\beta + 3\gamma T)r^2 T + (N - 2)(N - 3)B_1}{(N - 2)\gamma} + \frac{(1 + 2\beta T + 3\gamma T^2)}{2^{2\gamma}B^2} \left\{ 2^{2\gamma}B_2 B'' - r^2 B_1 B'' + 2(2N - 5)rB_1B' + r^2 B_1B'_1 \right\} - T - \beta T^2 - \gamma T^3 + 2\Lambda = 0, \] (17)
\[ \xi'_r = \frac{4(N - 2)(\beta + 3\gamma T)B_1 T}{r} + \frac{(1 + 2\beta T + 3\gamma T^2)(N - 2)}{2^{2\gamma}B^2} \left\{ 2(N - 3)BB_1 + rB_1B' + rBB'_1 \right\} - T - \beta T^2 - \gamma T^3 + 2\Lambda = 0, \] (18)

where \( T' \equiv dT(r)/dr \) is calculated through (13).

A first observation is that equation (16) is a third-order algebraic equation and hence it implies that \( T = T_0 = \text{const.} \). Hence, the differential equation (13) for \( T = \text{const.} \) leads easily to the general solution
\[ B(r) = \Lambda_{\text{eff}} r^2 - \frac{m}{r^{n-3}}, \]
\[ B_1(r) = B(r)B, \] (19)
where \( m \) is an integration constant related to the mass parameter, and the function \( B \) is calculated by inserting (19) into (17) and (18), giving
\[ B = \frac{T_0^\gamma}{(N - 1)(N - 2)} = \text{const.}, \] (20)
in the case where we set the explicit cosmological constant \( \Lambda \) to zero (in which case \( T_0 = -\frac{3\beta \gamma \sqrt{B^2 - 20N}}{10\gamma} \)). In the above expressions the constant \( \Lambda_{\text{eff}} \) is given by
\[ \Lambda_{\text{eff}} = \frac{1}{\gamma}, \] (21)
and we can clearly see that it plays the role of an effective cosmological constant. The important observation here is that we obtain an effective cosmological constant that arises solely from the \( f(T) \) modification, even if the initial explicit cosmological constant is absent. Hence, interestingly enough, the structure of the \( f(T) \) gravity leads to an effective cosmological constant and in the case where it is negative the solution is an AdS one. This feature, namely the
induction of an effective cosmological constant due to the $f(T)$ structure, was indicated to happen in $f(T)$ gravity [72, 93], however in the present work we show robustly that it does appear and moreover in general dimensions.

We mention that the above solution exists only for $\gamma \neq 0$, namely it is a result of the higher-power correction to standard TEGR, i.e to general relativity, and it reveals the effect of such corrections. In the case where $\beta = \gamma = 0$ and $\Lambda \neq 0$ then $\Lambda_{\text{eff}} \propto \Lambda$, that is we recover standard TEGR with a cosmological constant, and its Schwarzschild-(A)dS solution. Lastly, note that although $B(r)$ and $B_1(r)$ differ by a constant, the $g_{tt}$ and $g_{rr}$ components of the metric have the same Killing and event horizons. The solution has a singularity at $r = 0$, while it possesses a horizon at $m = \Lambda_{\text{eff}}r^{N-1}$.

We continue our analysis for the special choice where
\[
\Lambda = \frac{1}{18\beta} \quad \text{and} \quad \gamma = \frac{3\beta^2}{5},
\]
with $\beta \neq 0$, since in this case even for $\Lambda \neq 0$ the solution has $B(r) = B_1(r)$. The investigation of solutions with $g_{tt} = g_{rr}^{-1}$ has an increased interest in the literature since they have increased capability in exhibiting the right change of signature in the $t$ and $r$ components for having an event horizon [20, 94, 95], and additionally they are the ones that are preferred from solar system tests [72, 96]. In particular, following the above steps we extract the solution
\[
\begin{align*}
B(r) &= \Lambda_{\text{eff}} r^2 - \frac{m}{r^{N-5}}, \\
B_1(r) &= B(r),
\end{align*}
\]
where $m$ is an integration constant related to the mass parameter and where
\[
\Lambda_{\text{eff}} = -\frac{1}{3(N-1)(N-2)\beta}.
\]
Similarly to the previous case we obtain an effective cosmological constant which now depends on the parameter $\beta$ and the space dimensionality $N$. In the case where $\beta > 0$ we obtain an AdS solution. The horizon of the solution (24) is again at $m = \Lambda_{\text{eff}}r^{N-1}$.

### 3.2. New charged AdS black hole solutions

Let us now proceed with the analysis of the charged solutions, that is we consider also the electromagnetic Lagrangian $L_{\text{em}}$ in (6), choosing additionally without loss of generality the vector potential to have the general form $A = q(r)dt$. Imposing again the $N$-dimensional vielbeins of (11), the field equations (7) and (10) have the following non-vanishing components:
\[
\begin{align*}
\xi'^r &= 2Tfr + 2\Lambda - f + \frac{2q^2(r)B_1}{B} = 0, \\

\xi^{0\nu} \phi_{\nu} &= \xi^{0\nu} \phi_{\nu} = \cdots = \xi^{0\nu} \phi_{\nu} = \xi^{i2} \phi_{i2} = \cdots = \xi^{i2} \phi_{i2}, \quad i = 1, \ldots, 2N-1, \\
&= \frac{fr[(N-2)(N-3)B_1]T'}{(N-2)r} + \frac{fr}{2\alpha^2B^2} \left\{ 2r^2BB'B'' - r^2B'B'^2 + 4(N-3)^2B^2B_1 \\
&+ 2(2N-5)rBB'B' + r^2BB'B'^2 + 2(N-3)rB^2B_1 \right\} - f + 2\Lambda - \frac{2q^2(r)B_1}{B} = 0, \\
\xi^r &= \frac{2(N-2)B_1frT'}{r} + \frac{(N-2)fr[2(N-3)BB_1 + rB_1B' + rBB_1]}{r^2B} - f + 2\Lambda + \frac{2q^2(r)B_1}{B} = 0, \\
\end{align*}
\]
where \( q' = \frac{dq}{dr} \). In the case of the \( f(T) \) form (14) these equations reduce to

\[
\xi_\gamma = T + 3\beta T^2 + 5\gamma T^3 + 2\Lambda + \frac{2q'^2(r)B_1}{B} = 0,
\]

\[
\xi_\alpha = \xi_\beta = \cdots = \xi_\gamma = \xi_\zeta = \cdots = \xi_{\gamma-1} = \cdots
\]

\[
= 2(\beta + 3\gamma T)[r^2T + (N - 2)(N - 3)B_1T'] + (1 + 2\beta T + 3\gamma T^2) \left\{ \frac{2^2BB_1B'' - r^2B_1B'^2 + 2(2N - 5)rrB_1B'}{2r^2B^2} \right\}
\]

\[
+ r^2BB'r' + 2(N - 3)B^2(2N - 3)B_1(rB_1 + rB_1') - T - \beta T^2 - \gamma T^3 + 2\Lambda - \frac{2q'^2(r)B_1}{B} = 0,
\]

\[
\xi_{\gamma} = \frac{4(N - 2)(\beta + 3\gamma T)B_1T'}{r} + \frac{(1 + 2\beta T + 3\gamma T^2)(N - 2)}{r} \left\{ \frac{2(2N - 3)B_1B + rB_1B' + rB'_1}{B} \right\}
\]

\[
- T - \beta T^2 - \gamma T^3 + 2\Lambda + \frac{2q'^2(r)B_1}{B} = 0. \tag{26}
\]

Although the above equations in the case of a general \( \Lambda \) can be solved only numerically, analytical solutions can still be extracted when \( \Lambda \) is given by (22). In this case the general N-dimensional solution takes the form

\[
B(r) = r^3\Lambda_{\text{eff}} - \frac{m}{r^{N-3}} + \frac{15q(N - 3)}{4(N - 2)r^{2(N - 3)}} + \frac{45(N - 3)q^3}{16(N - 2)(5N - 13)\sqrt{r^2(N - 3)}},
\]

\[
B_1(r) = h(r)B(r), \tag{27}
\]

with

\[
h(r) = \left[ 1 + \frac{2q}{3(N - 3)\sqrt{r^{2(N - 3)}}} + \frac{4q^2}{9(N - 3)^2\sqrt{r^{2(N - 3)}}} + \frac{q^3}{9(N - 3)^3\sqrt{r^{2(N - 3)}}} + \frac{q^4}{36(N - 3)^4\sqrt{r^{2(N - 3)}}} \right]^{-1},
\]

\[
q(r) = \frac{q}{r^{N-3}} + \frac{q^2}{(5N - 13)\sqrt{r^{N(N - 3)}}} + \frac{q^3}{2(N - 3)(7N - 17)\sqrt{r^{N(N - 3)}}}, \tag{28}
\]

where

\[
\Lambda_{\text{eff}} = \frac{81(N - 3)^5}{4(N - 1)(N - 2)q}, \tag{29}
\]

and with \( m, q \neq 0 \) being the constants of integration related to mass and electric charge respectively. We stress that the above solution exists only for \( q \neq 0 \), since for \( q = 0 \) we have the solutions of the previous subsection, thus it arises from the structure of the electromagnetic sector.

Let us now discuss on the properties of the above solution. First of all, inserting (27) into (11) and then into (1) we obtain the metric as
\[\begin{align*}
\mathrm{d}s^2 &= \left[ r^2 \Lambda_{\mu\nu} - \frac{m}{r^2 - 1} + \frac{15q^2(N-3)}{4(N-2)^2 r^{2(N-1)}} + \frac{45(N-3)q^3}{16(N-2)(5N-13)\sqrt{r^{2(N-1)}}} + \frac{9q^4}{8(N-2)(7N-17)\sqrt{r^{2(N-1)}}} \right] \mathrm{d}r^2 \\
&\quad - \left[ \frac{2q}{3(N-3)\sqrt{r^{2(N-1)}}} + \frac{9(N-3)q^2}{9(N-3)\sqrt{r^{2(N-1)}}} + \frac{q^3}{36(N-3)\sqrt{r^{2(N-1)}}} + \frac{q^4}{36(N-3)\sqrt{r^{2(N-1)}}} \right] \mathrm{d}q^2 \\
&\quad \times \left[ r^2 \Lambda_{\mu\nu} + \frac{m}{r^2 + 1} + \frac{15q^2(N-3)}{4(N-2)^2 r^{2(N-1)-1}} + \frac{45(N-3)q^3}{16(N-2)(5N-13)\sqrt{r^{2(N-1)-1}}} + \frac{9q^4}{8(N-2)(7N-17)\sqrt{r^{2(N-1)-1}}} \right]^{-1} \mathrm{d}r^2 \\
&\quad - r^2 \left( \sum_{i=1}^{N-2} \sum_{k=1}^{N-3} \mathrm{d}x_i^2 + \sum_{k=1}^{N-3} \mathrm{d}y_k^2 \right). 
\end{align*}\]

As we observe, in this case the solution is more complicated, however it is still asymptotically AdS or dS according to the sign of \(q\). We mention the interesting feature that the effective cosmological constant is a result of the electric charge. Nevertheless, note that both \(\beta\) and \(\gamma\) are important for the solution structure, and hence this solution does not have a TEGR, i.e. general relativity, limit, nor an uncharged one. Moreover, this solution subclass is also outside the ones obtained in [71, 84], due to the use of more general \(f(T)\) forms in the present work. Therefore, solution (27) corresponds to a new charged AdS black hole in power-law \(f(T)\) gravity.

Additionally, we mention here that apart from the difference in the metric part, the above solution has also a difference in the charge structure too, comparing to those obtained in [70, 71, 84], since the potential \(q(r)\) depends on a monopole and higher-order electromagnetic potential. This electromagnetic potential will have a vanishing value only when the constant \(q = 0\), which is not allowed in the above solution, and thus this implies that within the framework of \(f(T)\) gravity we cannot find a charged solution with monopole only.

We now proceed to the investigation of the singularity structure of the solution, by calculating curvature and torsion invariants. The curvature scalars are calculated from the metric (30) while the torsion scalar is calculated through the vielbeins (11), or straightway from (13). Additionally, observing the solution (13) we deduce that it is adequate to focus our analysis close to the roots of the function \(h(r)\). Calculating the Ricci scalar, the Ricci tensor square, and the Kretschmann scalar, we respectively find:

\[ R = F_1(r) \left( \frac{1}{\sqrt{r^{2(N-2)}}} \right), \quad R^{\mu\nu}R_{\mu\nu} = F_2(r) \left( \frac{1}{\sqrt{r^{4(N-2)}}} \right), \]

\[ K \equiv R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = F_3(r) \left( \frac{1}{\sqrt{r^{4(N-2)}}} \right), \]

while calculating the torsion scalar we respectively obtain:

\[ T(r) = \frac{12\sqrt{r^{2(N-2)}} + \beta[3(N-3)]^2(N-2)}{36 |\beta| \sqrt{r^{2(N-2)}}}, \]

where \(F_i(r)\) are polynomial functions of \(r\). The above invariants first of all show the singularity at \(r = 0\). Close to \(r = 0\) the behavior of these invariants are given by \((K, R_{\mu\nu}R^{\mu\nu}) \sim r^{-4(N-2)}\) and \((R, T) \sim r^{-2(N-2)}\), in contrast to the solutions of the Einstein–Maxwell theory in both general relativity and TEGR formulations which have \((K, R_{\mu\nu}R^{\mu\nu}) \sim r^{-2N}\) and \((R, T) \sim r^{-N}\).

This shows clearly that the singularity of our charged solution is softer than the one obtained in GR and TEGR for the charged case. Finally, notice that although in the solution the \(g_{\mu\nu}\) components of the metric are different, they have the same Killing and event horizons.

In a similar way we can investigate the horizons of equation (27), which can alternatively be calculated examining the roots of \(B(r) = 0\). In the case of four dimensions in figure 1 we
depict \( B(r) \) of solution (27), for various values of the model parameters. From this graph we can see the two roots of \( B(r) \) that define the black hole inner Cauchy horizon \( r_- \) and the black hole outer event horizon \( r_+ \) [97]. As we observe, as \( q \) increases and \( m \) decreases, and in particular for \( q > m \), we enter in a parameter region where there is no horizon, and thus the central singularity is a naked singularity. This is an interesting result of Maxwell-\( f(T) \) gravity (we mention that we see the issue from the mathematical point of view and we do not examine whether such a solution can indeed be formed physically through gravitational collapse), which does not appear in the absence of the electromagnetic sector (indications of this feature had also been discussed in [70, 71]). Moreover, note that for suitable \( m \) and \( q \) the two horizons coincide and become degenerate, namely we obtain \( r_- = r_+ = r_{5d} \).

Finally, in order to present the above features in a different way, in figure 2 we depict the value \( m_+ \) of the parameter \( m \) that corresponds to the horizon \( r_+ \), which is obtained setting \( B(r_+) = 0 \), namely

\[
m_+ = \frac{1}{8}(N-3) \left( \frac{r_+^2}{\Lambda_{\text{eff}}} + \frac{15q^2(N-3)}{4(N-2)r_+^{2(N-1)}} + \frac{45(N-3)q^3}{16(N-2)(5N-13)(\sqrt{r_+^{2(N-1)}} + \sqrt{r_+^{2(N-1)}})8(N-2)(7N-17)(\sqrt{r_+^{2(N-1)}})} \right).
\]

### 3.3. Energy of the AdS black holes

In this subsection we discuss the energy issues of the obtained solutions. In general \( f(T) \) gravity and for general geometry, equation (7) can be rewritten as

\[
d \beta \left[ b S^\alpha \beta f_T \right] = \kappa b b^{\alpha \beta} \left[ r^{\alpha \beta} + T^{\alpha \beta} \right],
\]

where \( t^{\alpha \beta} \) is defined as [98, 99]

\[
t^{\alpha \beta} := \frac{1}{\kappa} \left[ 4 f_T S^{\mu \beta \lambda} T_{\mu \lambda} - g^{\alpha \beta} f \right].
\]

Due to the fact that the superpotential \( S^{\mu \beta \lambda} \) is antisymmetric in the last two indices, we have \( \partial_\alpha \partial_\beta \left[ b S^{\alpha \beta} f_T \right] = 0 \), which using (33) gives \( \partial_\beta \left[ b (r^{\alpha \beta} + T^{\alpha \beta}) \right] = 0 \), which leads to

\[
\frac{d}{dt} \int_V d^{(N-1)} x \ b b^{\alpha \alpha} \left( r^{\alpha \alpha} + T^{\alpha \alpha} \right) + \int_{\Sigma} \left[ b b^{\alpha \alpha} \left( t^{\alpha \alpha} + T^{\alpha \alpha} \right) \right] = 0.
\]

Hence, we can now interpret equation (35) as the conservation law of the energy-momentum tensor \( T^{\alpha \beta} \) and \( t^{\alpha \beta} \). Therefore, \( t^{\lambda \mu} \) can be interpreted as the energy-momentum tensor of the gravitational field in \( f(T) \) theory [98, 99]. Hence, the energy-momentum of \( f(T) \) gravity in \((N - 1)\)-dimensional volume \( V \) writes as

\[
P^\alpha = \int_V d^{(N-1)} x \ b b^{\alpha \alpha} \left( r^{\alpha \alpha} + T^{\alpha \alpha} \right) = \frac{1}{\kappa} \int_V d^{(N-1)} x \partial_\beta \left[ b S^{\alpha \beta} f_T \right].
\]

Note that the above expressions recover the known results of TEGR in the case \( f(T) = T \).

Let us now apply the above general analysis in the specific class of power-law \( f(T) \) gravity and for the AdS black hole solutions obtained above, namely for expressions (23) and (27). Inserting solution (23) into the above general expressions gives
where we have replaced $\kappa$ by $\kappa = 2(N - 3)\Omega_N G_N$. Similarly, inserting (27) into the general energy expressions gives

$$E = \frac{(N - 2)[1 + 2(N - 1)(N - 2)\Lambda_{\text{eff}}^2 + 6(N - 1)(5N^2 - 33N + 58)\Lambda_{\text{eff}}^2]}{4(N - 3)G_N} m + \mathcal{O}\left(\frac{1}{r^{4/3}}\right).$$

As we observe the $f(T)$ modification has an effect on the mass term of the energy of standard TEGR \cite{98, 99}, while the charge term does not appear up to $\mathcal{O}\left(\frac{1}{r^4}\right)$. The charge term will contribute to the calculation of energy starting from $\mathcal{O}\left(\frac{1}{r^4}\right)$, in contrast to Reissner–Nordström spacetime. This difference comes from the contribution of the function $h(r)$ given in relation (27).

4. Rotating black holes in Maxwell-$f(T)$ gravity

We close this work by deriving rotating solutions that satisfy the field equations of power-law $f(T)$ gravity. In order to do so, we will be based on the static solutions extracted above. In particular, we apply the following transformations with $n$ rotation parameters:
\[
\ddot{\phi}_i = -\mathcal{K} \dot{\phi}_i + \frac{a_i}{l^2} t, \quad \ddot{t} = \mathcal{K} t - \sum_{j=1}^{n} a_j \phi_j, \tag{39}
\]

with \(a_i\) the rotation parameters (their number is \(n = \lfloor (N - 1)/2 \rfloor\) where \([\ldots]\) marks the integer part), and where the parameter \(l\) is related to the parameter \(\Lambda_{\text{eff}}\) of the static solution through

\[
l = -\frac{(N - 2)(N - 1)}{2 \Lambda_{\text{eff}}}. \tag{40}
\]

Additionally, \(\mathcal{K}\) is defined as

\[
\mathcal{K} := \sqrt{1 - \sum_{j=1}^{n} a_j^2}. \tag{41}
\]

Applying the transformation (39) to the vielbein (11) we obtain

\[
\begin{pmatrix}
\mathcal{K} \sqrt{B(r)} & 0 & -a_1 \sqrt{B(r)} & -a_2 \sqrt{B(r)} & \cdots & -a_n \sqrt{B(r)} & 0 & 0 & \cdots & 0 \\
0 & 1/\sqrt{B_1(r)} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
-\frac{a_1 r}{\sqrt{B(r)}} & 0 & -\mathcal{K} r & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
-\frac{a_2 r}{\sqrt{B(r)}} & 0 & 0 & -\mathcal{K} r & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & -\mathcal{K} r & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & r & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & r & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & r
\end{pmatrix}.
\]

where \(B(r)\) and \(B_1(r)\) are given by the previously extracted static solution (27). The vielbein (41) coincides with (11) when the rotation parameters \(a_i = 0\). Hence, for the electromagnetic potential (28) we obtain the form

\[
\tilde{q}(r) = -q(r) \left[ \sum_{j=1}^{n} a_j d\phi'_j - \mathcal{K} d\phi' \right]. \tag{42}
\]

Note here that although the transformation (39) leaves the local properties of spacetime unaltered, it does change them globally as has been shown in [100], since it mixes compact and noncompact coordinates. Thus, the vielbein (11) and (41) can be locally mapped into each other but not globally [100, 101].

The metric that corresponds to the vielbein (41) is written as

\[
ds^2 = -B(r) \left[ \mathcal{K} dr' - \sum_{j=1}^{n} a_j d\phi'_j \right]^2 + \frac{dr'^2}{B_1(r)} + \frac{r^2}{\mathcal{K}} \sum_{j=1}^{n} \left[ a_j dr' - \mathcal{K}^2 d\phi'_j \right]^2 + r^2 dz^2 - \frac{r^2}{\mathcal{K}} \sum_{i<j} \left( a_i d\phi'_j - a_j d\phi'_i \right)^2. \tag{43}
\]
where $0 \leq r < \infty$, $-\infty < t < \infty$, $0 \leq \phi_i < 2\pi$, $i = 1, 2 \cdots n$ and $-\infty < z_k < \infty$, and where $dz_k^2$ is the Euclidean metric on $(N - n - 2)$ dimensions with $k = 1, 2 \cdots N - 3$. Note that the static configuration (12) can be recovered as a special case of the above general metric when the rotation parameters $a_j$ are chosen to be vanished, while by inverting the coordinate transformations (39) we get back the static spacetime (30).

We mention that the transformation (39) can be carried out locally but not globally, since the first Betti number of the manifold is one due to the fact that closed curves encircling the horizon cannot be shrunk to zero [102, 103]. In both static (30) and stationary (43) spacetimes there is a timelike Killing field $\xi = \partial/\partial t$. In the static spacetime this corresponds to an exact one-form $V$ inverse to $\xi$ (i.e. $V_\mu \equiv \xi^\mu/|\xi|^2$) given then by $V = dt$ (see [103] for details), while in the stationary spacetime the corresponding one-form is $V = dt + a_\phi \phi_i$ which is a closed one-form but not exact. De Rham’s cohomology theorems then state that since the first Betti number of the manifolds is one, there are global diffeomorphisms which map the $\xi$ of the two manifolds, however there is no such global diffeomorphisms mapping $V$ and $\bar{V}$. Hence, since the metric maps vectors into one-forms, it is implied that metrics (30) and (43) can be locally mapped into each other but not globally, and thus they are distinct.

Additionally, note that the line-element (43) is created when the Minkowskian metric in (1) takes in cylindrical coordinates the form

![Figure 2](image-url). The value $m_+$ of the parameter $m$ that corresponds to the horizon $r_+$, of solution (27) of Maxwell-$f(T)$ gravity in four dimensions, for various values of the electric charge $q$ in units where $\kappa = 1$. The horizontal line at $m_+ = 0$ is drawn for convenience.
\[
\eta_{ij} = \begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 + \frac{a_0^2}{R^2} & -\frac{a_0 a_1}{R^2} & -\frac{a_0 a_2}{R^2} & \ldots & -\frac{a_0 a_{n-1}}{R^2} & 0 & 0 & \ldots & 0 \\
0 & 0 & \frac{a_0 a_1}{R^2} & 1 + \frac{a_1^2}{R^2} & -\frac{a_1 a_2}{R^2} & \ldots & -\frac{a_1 a_{n-1}}{R^2} & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & -\frac{a_0 a_{n-1}}{R^2} & -\frac{a_1 a_{n-1}}{R^2} & -\frac{a_2 a_{n-1}}{R^2} & \ldots & 1 + \frac{a_{n-1}^2}{R^2} & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 \\
\end{pmatrix}
\]

(44)

It is of interest to note that the torsion components of the above Minkowski metric are vanishing.

In summary, we have managed to extract the rotating charged AdS black hole solution in power-law \( f(T) \) gravity. This is a novel solution and one of the main results of the present paper. Concerning the singularity properties, as we observe from the structure of (43), this will be the same with the static solution of (30). Hence, all the results and the discussion in section 3.2 are valid for the rotating solutions above too. Thus, at \( r = 0 \) we obtain a singularity, and close to \( r = 0 \) the behavior of the invariants is \((K, R_{\mu\nu}R^{\mu\nu}) \sim \sqrt{r}^{-4(N-2)}\) and \((R, T) \sim \sqrt{r}^{-2(N-2)}\), in contrast to the charged solutions of general relativity and TEGR theories. Additionally, the horizon structure is qualitatively similar to the one discussed in the end of section 3.2, and we do obtain the appearance of a naked singularity for suitable large \( q \). Finally, concerning the energy of the rotating charged AdS black hole (43), following the procedure of section 3.3 it is calculated as

\[
E = \frac{(N - 2)[1 + 2(N - 1)(N - 2)\Lambda_{\alpha\beta} + 6(N - 1)(5N^2 - 33N + 58)\gamma\Lambda_{\alpha\beta}^2] [\Lambda_{\alpha\beta}^2 \sum_{j=1}^{m} w_j + 3\Lambda^2]m}{12(N - 3)G_N} + O\left(\frac{1}{r^{4N}}\right).
\]

(45)

5. Conclusions

After the formulation of the AdS/CFT correspondence there is an increasing interest in the extraction and study of (higher-dimensional) Anti-de-Sitter black hole solutions. Apart from standard gravity, the investigation extends to various gravitational modifications as well as in the case where the Maxwell sector is also present. Nevertheless, almost all of the works remain in the framework of curvature-modified gravity. Thus, in the present manuscript we investigated static and rotating, uncharged and charged, AdS black holes in higher-dimensional \( f(T) \) gravity, focusing on the power-law ansatz which is the most viable according to observations.

In the case where the electromagnetic sector is absent we extracted AdS static solutions, which are characterized by an effective cosmological constant that depends on the \( f(T) \) modification. In the case where we switch on the Maxwell sector, we analytically obtained charged static solutions, which are asymptotically AdS, characterized by an effective cosmological constant that depends on the parameters of the \( f(T) \) modification, as well as on the electric
charge. Hence, these solution subclasses do not have a TEGR, i.e. general relativity limit, nor an uncharged one, and they correspond to new charged AdS black holes in power-law $f(T)$ gravity, where their features arise solely from the torsional modification alongside the Maxwell sector incorporation. Finally, we showed that in the extracted solutions the potential $q(r)$ depends on a monopole and higher-order electromagnetic potential, and thus within the framework of $f(T)$ gravity we cannot find a charged solution with monopole only.

As a next step we examined the singularity structure of the solutions, calculating the values of various curvature and torsion invariants, showing that they do possess the singularity at $r = 0$, which however is softer comparing to the standard general relativity case due to the $f(T)$ structure. Additionally, we investigated the horizons of the solutions, showing that the solutions possess an event horizon as well as a cosmological horizon. Nevertheless, for suitably large electric charge and small mass we obtain the the appearance of a naked singularity. Finally, we calculated the energy of the obtained solutions, showing that the $f(T)$ modification affects the mass term.

Based on the analysis of static solutions, through suitable transformations we were able to extract rotating AdS solutions of Maxwell-$f(T)$ gravity. Similarly to the static case, the effective cosmological constant arises from the $f(T)$ modification and the the Maxwell sector. Additionally, the singularity and horizon properties are the same with the uncharged case. These solutions also do not have a TEGR or an uncharged limit, and they correspond to a novel class.

The extraction of AdS black holes in $f(T)$ gravity may be very helpful towards the investigation of AdS/CFT correspondence in torsional framework. This direction is expected to have advantages comparing to the standard curvature-based formulation, due to the known relations between curvature and torsion invariants through boundary terms.

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ORCID iDs

Emmanuel N Saridakis https://orcid.org/0000-0003-1500-0874

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