A New Figure of Merit for Dark Energy Studies

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We introduce a new figure of merit for comparison of proposed dark energy experiments. The new figure of merit is objective and has several distinct advantages over the Dark Energy Task Force Figure of Merit, which we discuss in the text.

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I. INTRODUCTION

The measurements of the Cosmic Microwave Background (CMB) anisotropies, most notably by the Wilkinson Microwave Anisotropy Probe (WMAP) mission \[1\], but also by ground based and balloon borne experiments, such as VSA \[2\], CbI \[3\], AcbaR \[4\], SPt \[5\], QuaD \[6\] are in spectacular agreement \[7\] by predictions of the standard cosmological model. Measurements of the low-redshift universe including data from large spectroscopic surveys like SDSS \[8, 9\] analysed using a variety of methods \[10, 11\], measurements of the luminosity distance to type Ia supernovae \[12, 13\] and the galaxy-galaxy lensing \[14, 15\] strengthen the standard picture. These datasets provide constraints on the cosmological model using a variety of techniques with very different systematic issues, which nevertheless converge, within the error-bars, to give a variety of techniques with very different systematic issues, which nevertheless converge, within the error-bars, to give a standard model of cosmology.

However, motivated by the need to establish new and much needed gaps in literature, various authors have considered alternatives to cosmological constant, which most often include new degrees of freedom. Such models generally predict effective equation of state \(w = p/\rho\) for these novel components of the Universe that can deviate from the value for the cosmological constant, namely \(w = -1\) by arbitrarily small amounts \[18\]. Concurrently with these efforts, an industry of phenomenological models has been established. One of the most popular descriptions for the dynamical dark energy is the \(w_{0}-w_{a}\) parametrisation, in which the equation of state is postulated to evolve with cosmological scale factor \(a = (1 + z)^{-1}\) (where \(z\) is redshift) as \[19\]

\[
w(a) = w_0 + (1 - a)w_a. \tag{1}
\]

At the same time, motivated by the need to attract funding, experimentalists have begun proposing various experiments that will measure the value of \(w\) and its derivatives with an ever increasing precision. In fact, designing cosmological experiments around measuring neutrino masses from cosmology has a distinct disadvantage in that sum of neutrino mass eigenstates has a lower limit given by the ground-based experiments \[20, 22\]. The same holds true for constraining theories of inflation by measuring the running of the spectral index, which is expected to be of \(O(10^{-4})\) in the simplest inflationary models, given the current limits on the tilt of the spectral index \(n_s \sim 0.96\) \[23\]. Measuring \(w\) poses no such difficulties: since there is a strong theoretical prejudice that \(w = -1\), one can hope to improve limits on deviation from the cosmological constant value for decades to come.

An interesting question, worth every penny of scientific funding, is the question of comparison of various dark energy experiments. Several methods have been established for this purpose, the most common is the Dark Energy Task Force (DETF) Figure of Merit (FoM) \[24\]. In this paper we propose a new metric, that has several advantages. We discuss the DETF FoM and our new metric in Section \[II\]. We conclude in Section \[III\]. Finally, we also note that clusters of galaxies are the most massive gravitationally bound objects in the Universe.

II. A NEW FIGURE OF MERIT: \(\phi\)

We start by considering the well-established DETF FoM. This figure of merit is defined as

\[
\text{DETF FoM} = (\det C)^{-1/2}, \tag{2}
\]

where \(C\) is the \(2 \times 2\) covariance matrix of the errors on the \(w_0-w_a\) plane

\[
C = \begin{pmatrix}
\sigma_{w_0}^2 & \sigma_{w_0}^2 w_a \\
\sigma_{w_0}^2 w_a & \sigma_{w_a}^2
\end{pmatrix}. \tag{3}
\]

This figure of merit can be interpreted as the inverse of the area of the error ellipse on the \(w_0-w_a\) plane. Now come our ingenious and novel idea. We introduce our new figure of merit, whose value is proportional to the inverse of the circumference of the error ellipse on the \(w_0-w_a\) plane. To calculate this quantity, we first note that the semi major and semi minor axes of the error...
TABLE I: Comparison of standard and improved Figures of Merit for a selection of proposed experiments. Systematics issues were ignored when calculating these Figures of Merit, due to difficulties in modelling them. This table illustrates superiority of $\tilde{\varphi}$ over DETF FoM.

| Experiment            | DETF FoM | $\tilde{\varphi}$ |
|-----------------------|----------|--------------------|
| Somewhat good experiment | 95       | 39                 |
| Very good experiment | 403      | 80                 |
| CNDEM$^a$            | 845      | 132                |
| FMIE$^b$             | 1693     | 180                |

$^a$Lorentz violating Chuck Norris in space, breathing aether and watching galaxies with his naked eyes. For Chuck Norris using specs, add 40\% to the DETF FoM and 18\% to $\tilde{\varphi}$

$^b$Fisher Matrix Itself Experiment

The circumference is then given by

$$s = 4r_+ E(e),$$

where eccentricity of the ellipse is given by

$$e = \sqrt{1 - \frac{r^-}{r^+}},$$

and $E(e)$ is the complete elliptic integral of the second kind. Our new figure of merit is then given by

$$\tilde{\varphi} = s^{-1}.$$  

The symbol for our new figure of merit is $\tilde{\varphi}$ which is to be pronounced as phiu and not “phi double-dot”. We illustrate the two figures of merit in the Figure 1.

Our new and improved figure of merit has several advantages over DETF FOM:

- When two experiments have the same DETF FoM, the $\tilde{\varphi}$ quantity will favour one with less correlated errors on the $w_0$-$w_a$ plane. Since this plane is well motivated by the fundamental physics, it is clear that uncorrelated errors should be favoured;
- Area grows proportionally to the square of the linear dimensions, while circumference grows only linearly. This makes $\tilde{\varphi}$ more linear;
- Calculation of the new figure of merit entails calculating elliptic integrals of the second kind, which makes the method more scientific;
- Correct pronunciation of $\tilde{\varphi}$ allows one to shower the opponents face in one’s saliva, thus quickly and effectively dispersing any doubts about the superiority of the experiment proposed by the speaker.

We also note that both figures of merit can be generalised to models with more than two parameters. For the DETF FoM, this has been performed in [25], but our new figure of merit is considerably more complicated and so we defer this work for future publication.

We compare the two figures of merit in Table I for a couple of proposed experiments. We note that the experiments which are further into the future have better figures of merit. We also note that the more expensive experiments have better figures of merit. The table demonstrates the superiority of the new figure of merit.

III. CONCLUSIONS AND DISCUSSION

In this paper we have introduced a new figure of merit, $\tilde{\varphi}$, which is proportional to the inverse of the circumference.
ence of the error ellipse.

As discussed in the text and we discuss it here again, the new figure of merit has several advantages over the old one. You and your dog should use it. If you do not use it and think it is a pointless number, you should nevertheless cite this paper or I will write you hassling emails. If worse come to worse, I’ll resort to the crowbar and smash your 30 inch liberal screen.

This work opens clear avenues for further research. The quantity $\tilde{\phi}$ can and should be calculated for many future experiments and further compared to the DETF FoM. Rigorous extension of this work into models of dark energy with more than two parameters remains to be performed. Theoretical basis for similarities and differences between the two figures of merit should be established and elaborated. Different parametrization of the dark-energy should be integrated into the new figure of merit resulting in a multitude of useful figures of merit. The best in the science of figures of merit has yet to come!

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