B and not L in Supersymmetry: New U(1) Gauge symmetry and Dark Matter

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Abstract

To enforce the conservation of baryon number $B$ and not lepton number $L$ in supersymmetry, a new $U(1)_X$ gauge symmetry is recommended. An example is offered with new particles interacting under $U(1)_X$ which are good candidates for the dark matter of the Universe.
In an unrestricted supersymmetric extension of the Standard Model (SM) of particle interactions, there are the following well-known allowed bilinear and trilinear terms:

\[ L_i \Phi_2, \quad L_i L_j e^c_k, \quad L_i Q_j d^c_k, \quad u^c_i d^c_j d^c_k, \quad (1) \]

where \( L_i = (\nu_i, e_i) \), \( Q_i = (u_i, d_i) \), \( \Phi_1 = (\phi_0^1, \phi_1^-) \), \( \Phi_2 = (\phi_2^+, \phi_0^2) \), etc. This is of course unacceptable because both baryon number \( B \) and lepton number \( L \) are not conserved, and rapid proton decay cannot be avoided. The conventional choice of the Minimal Supersymmetric Standard Model (MSSM) is to impose by hand the notion of \( R \) parity for each particle, which is just the product of its multiplicative baryon number \( (-)^3B \), its multiplicative lepton number \( (-)^L \), and its intrinsic spin parity \( (-)^{2j} \). As a result, all four terms of Eq. (1) are forbidden. This has the desirable consequence of an absolutely stable particle odd under \( R \) which is a candidate for the dark matter of the Universe.

Another choice is to forbid only the last term of Eq. (1) by hand, thereby conserving \( B \), and allow the other three terms, thereby violating \( L \). This is acceptable because \( B \) conservation by itself is sufficient to forbid proton decay. Such models of \( R \) parity violation have been discussed extensively in the literature. Of course, there is no longer any dark-matter candidate, and no better understanding as to why \( B \) is conserved and not \( L \). In any case, whether \( R \) parity is conserved or not, there remains the puzzle of the allowed bilinear term \( \mu \Phi_1 \Phi_2 \). It is not understood why \( \mu \) should be of order TeV or less, and not some much larger fundamental scale, as would be expected.

In this note, a new \( U(1)_X \) gauge symmetry is proposed, whereby \( B \) is conserved but not \( L \), and the scale of \( \mu \) is determined by the spontaneous breaking of \( U(1)_X \). In addition, new particles exist which are good dark-matter candidates. The idea of using a particular new \( U(1) \) gauge symmetry to explain the \( \mu \) puzzle and to prevent proton decay is not new \[1, 2, 3, 4\]. Recently, it has also been applied to enforcing either \( B \) conservation or \( L \) conservation or both \[5\]. Here, the proposal is to conserve \( B \) and not \( L \), in keeping with
most phenomenological studies of $R$ parity violation, and to accommodate dark matter.

Consider first the particles of the MSSM and their transformations under $U(1)_{X}$ as shown in Table 1, where three families of quarks and leptons are understood. The terms $Qu^{c}{\Phi}_{2}$ and $Qd^{c}{\Phi}_{1}$ are already allowed. To have $Le^{c}{\Phi}_{1}$ as well as $LLe^{c}$ and $LQd^{c}$ terms, $n_{4} = -n_{1} - n_{3}$ and $n_{5} = 2n_{1} + 2n_{3}$ are needed. To forbid $u^{c}d^{c}d^{c}$, the condition $n_{2} + 2n_{3} \neq 0$ is required.

Table 1: MSSM particle content of proposed model.

| Superfield | $SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y}$ | $U(1)_{X}$ |
|------------|---------------------------------|------------|
| $Q \equiv (u, d)$ | $(3,2,1/6)$ | $n_{1}$ |
| $u^{c}$ | $(3^{*},1,-2/3)$ | $n_{2}$ |
| $d^{c}$ | $(3^{*},1,1/3)$ | $n_{3}$ |
| $L \equiv (\nu, e)$ | $(1,2,-1/2)$ | $n_{4}$ |
| $e^{c}$ | $(1,1,1)$ | $n_{5}$ |
| ${\Phi}_{1} \equiv (\phi_{1}^{0}, \phi_{1}^{-})$ | $(1,2,-1/2)$ | $-n_{1} - n_{3}$ |
| ${\Phi}_{2} \equiv (\phi_{2}^{+}, \phi_{2}^{0})$ | $(1,2,1/2)$ | $-n_{1} - n_{2}$ |

Consider the addition of a pair of color-triplet superfields $(h, h^{c})$ and one electroweak triplet superfield $\Sigma = (\Sigma^{+}, \Sigma^{0}, \Sigma^{-})$.

\[
\begin{align*}
    h & \sim (3,1,-1/3,n_{6}), \quad (2) \\
    h^{c} & \sim (3^{*},1,1/3,n_{7}), \quad (3) \\
    \Sigma & \sim (1,3,0,n_{8}). \quad (4)
\end{align*}
\]

The absence of the $[SU(3)]^{2}U(1)_{X}$, $[SU(2)]^{2}U(1)_{X}$, and $[U(1)_{Y}]^{2}U(1)_{X}$ anomalies requires

\[
\begin{align*}
    6n_{1} + 3n_{2} + 3n_{3} + (n_{6} + n_{7}) &= 0, \quad (5) \\
    4n_{1} - n_{2} - 4n_{3} + 4n_{8} &= 0, \quad (6) \\
    24n_{1} + 21n_{2} + 30n_{3} + 2(n_{6} + n_{7}) &= 0. \quad (7)
\end{align*}
\]
As a result,

\[ n_1 = -\frac{5}{4}n_2 - 2n_3, \quad n_4 = \frac{5}{4}n_2 + n_3, \quad n_5 = -\frac{5}{2}n_2 - 2n_3, \]
\[ n_6 + n_7 = \frac{9}{2}(n_2 + 2n_3), \quad n_8 = \frac{3}{2}(n_2 + 2n_3). \]  

The absence of the \( U(1)_Y[U(1)_X]^2 \) anomaly implies

\[ 45n_2^2 + 144n_2n_3 + 108n_3^2 - 4n_6^2 + 4n_7^2 = 9(n_2 + 2n_3)(5n_2 + 6n_3 - 2n_6 + 2n_7) = 0. \]  

Hence \( n_2 + 2n_3 \neq 0 \) implies

\[ n_6 = \frac{7}{2}n_2 + 6n_3, \quad n_7 = n_2 + 3n_3. \]  

The crucial condition \( n_2 + 2n_3 \neq 0 \) also forbids the trilinear terms \( u^c d^c \bar{d}^c, u^c d^c h^c, Q Q h, \) and \( L Q h^c \), as well as the bilinear terms \( \Phi_1 \Phi_2, L \Phi_2, \) and \( d^c h \). On the other hand, the terms \( Q u^c \Phi_2, Q d^c \Phi_1, L e^c \Phi_1, L L e^c, \) and \( L Q d^c \) are allowed, thus conserving \( B \) but not \( L \).

At this point, the model is incomplete because of the lack of mass terms for \( \Phi_1 \Phi_2, h h^c, \) and \( \Sigma \Sigma \). Singlet superfields \( \chi_{3,6,9} \) are then required, transforming under \( U(1)_X \) as \( -3, -6, -9 \) respectively in units of \( (n_2 + 2n_3)/2 \). To allow the exotic \( h, h^c \) quarks to decay, another \( \chi_7 \sim -7 \) is needed so that \( d^c h \chi_7 \) is possible. With this particle content, the sum of \( U(1)_X \) charges is \( -5(n_2 + 2n_3) \) and the sum of \( [U(1)_X]^3 \) charges is \( -80(n_2 + 2n_3)^3 \). To cancel these anomalies, the following singlets may be added: one copy each of \( \chi_1 \sim -1, \chi_4 \sim -4, \chi_{10} \sim 10, \) three copies of \( \chi_5 \sim -5, \) and ten copies of \( \chi_2 \sim 2, \) again in units of \( (n_2 + n_3)/2 \). Note that these singlets are chosen so that there are no bilinear terms (i.e. no two with opposite charges), otherwise the analog of a \( \mu \) term would be allowed, thereby defeating the purpose of having a new \( U(1)_X \) gauge symmetry to forbid such terms in the first place. Allowed trilinear terms are \( \chi_2 \chi_2 \chi_4, \chi_1 \chi_9 \chi_{10}, \chi_3 \chi_7 \chi_{10}, \chi_4 \chi_6 \chi_{10}, \) and \( \chi_5 \chi_5 \chi_{10}. \) Since their scalar components may all have nonzero vacuum expectation values, their fermion components all
become massive at that energy scale. Thus an explicit and completely consistent example exists for an anomaly-free $U(1)_X$ which conserves $B$ but not $L$.

As a byproduct of $U(1)_X$, dark-matter candidates also exist. For example, $\chi_2$ or $\chi_5$ may be assigned odd under an exactly conserved $Z_2$ symmetry, in which case they must also have zero vacuum expectation values. They may annihilate in the early Universe through the $U(1)_X$ gauge boson into the usual quarks and leptons with a cross section characterized by the scale of $U(1)_X$ breaking, i.e. of order TeV. Their elastic interaction with nuclei in direct-search experiments may be suppressed at the same time by choosing $n_3 = -3n_2/4$ so that the $U(1)_X$ coupling to an isoscalar combination of quarks is purely axial-vector.

An even better candidate for dark matter [6, 7] is the $\Sigma^0$ fermion, without which $n_2 + 2n_3 \neq 0$ cannot be realized. The mass difference $m_{\Sigma^\pm} - m_{\Sigma^0}$ is given by

$$\Delta m_{\Sigma} = \frac{\alpha m_W^2 m_{\Sigma}}{\pi \sin^2 \theta_W} \left[ \frac{1}{m_{\Sigma}^2 - m_W^2} \ln \frac{m_{\Sigma}^2}{m_W^2} - \frac{1}{m_{\Sigma}^2 - m_Z^2} \ln \frac{m_{\Sigma}^2}{m_Z^2} \right]. \quad (12)$$

This splitting is always positive and has a maximum of about 115 MeV for $m_{\Sigma} = 40$ GeV. Hence the decay of $\Sigma^+$ will likely be exclusively into $\Sigma^0 e^+ \nu$.

In conclusion, it has been shown that $R$ parity violation, in the sense of $B$ conservation but not $L$, may be enforced by a new $U(1)_X$ gauge symmetry which also forbids all bilinear mass terms. The scale of $U(1)_X$ breaking as well as electroweak symmetry breaking are then related to that of supersymmetry breaking. A consistent example is presented, with new particles transforming under $U(1)_X$ as dark-matter candidates.

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