Periodic Metallic Stepped Slits for Entire Transmission of Optical Wave and Efficient Transmission of Terahertz Wave

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ABSTRACT
The presence of metallic structures with array of slits, when slits have dimensions much less than the wavelength of the incident optical wave, typically causes extraordinary power transmission. This excellent power transmission indeed comes from the surface waves on the periodic structure excited by the transverse magnetic wave. Here we show that if slits are stepped, then the incident wave power is transmitted into the substrate at desired optical frequency entirely and simultaneously in terahertz frequency band by 70%, for In₀.₅₃Ga₀.₄₇As substrate. Power transmitted through the proposed structure is derived both in a closed form by an analytical model as well as numerically by finite element method. It is found that with the increase of field intensity in the substrate, the structure with array of stepped slits (as opposed to uniform slits) favorably has no reduction of frequency range at maximum transmitted power.

KEYWORDS
Analytical model; Optical wave; Periodic array; Stepped slits; Terahertz wave; Transmitted power

1. INTRODUCTION
Since the pioneering work of Ebbesen et al. [1] on holes, extraordinary power transmission through periodic metallic structures with dimensions much less than the wavelength of incident wave, has attracted growing interests among researchers [2–5]. This, unusual at the same time interesting phenomena, happens due to excited surface waves in subwavelength structure which mitigate its power reflection. Extraordinary power transmission has brought subwavelength structures into practical applications also, in terahertz sources and detectors [6], chemical sensing [7], spectroscopy and imaging techniques [8–11], ultrafast photodetectors [12–14] and high efficiency solar cells [15, 16], to name a few. Plasmonic structures are extensively used in THz generating, biomedical, and chemical sensing devices. The performance of these devices is directly limited by the value of optical power transmission [17, 18]. Therefore, the design of a plasmonic structure with maximum optical power transmission can push the performance of optical and THz devices beyond the currently existing boundaries. To be more specific on the subject, photoconductive antennas, a class of terahertz sources and detectors are well known to suffer from low radiated power [19–24]. Their performance can be improved by the use of subwavelength structures which was proposed for the first time by Jarrahi and co-workers [25–27], as a result of two facts. First, the time required for photocarriers (electrons and holes) generated in photoconductive area to arrive at radiating electrodes decreases. Second, power reflection at optical and terahertz frequencies can be reduced, as will be shown here. In our previous work [28], three-stepped rods for low-temperature grown GaAs (LT-GaAs) substrate at \( \lambda = 0.8 \, \mu \text{m} \) were designed for their dimensions and the power transmission was studied.

In this work, we more comprehensively propose and study the use of stepped slits for the reduction of power reflection at both optical frequency as well as terahertz frequency band. The main topics covered in this work include: theory of transmission power through stepped slits, effect of each step on power transmission and frequency bandwidth, and most importantly, design method of periodic stepped slits with the best possible performance (maximum power transmission and frequency bandwidth) at a favorable frequency. The contents of this paper are as follows: Section 2 develops a theoretical model and subsequently obtains a closed-form expression for the power reflected from the structure. Section 3 then discusses how to design a structure with maximum power transmission at desired frequencies. Finally, results from theoretical model are compared with finite element method calculations for specific values of
dimensions of a structure with In_{0.53}Ga_{0.47}As substrate at 1.5 μm wavelength (the substrate here is different from that of our previous work [28]).

2. THEORY

The X–Y cross view of the proposed periodic metallic structure is shown in Figure 1; all dimensions stay the same through the Z-direction. The structure consists of two nanoscale-layered metallic rods with similar height \( \xi = w_Y/2 \) and different widths. The widths of the upper and bottom rods are, respectively, \( w_x \) and \( w'_x \). These rods are repeated along the X-axis with periodicity \( d_x \). The rods are considered as perfect metals with infinite permittivity in order to keep the generality of theoretical model presented here. In the case of physical metallic rods with complex permittivity, magnetic boundary condition (in Equation 7a) will be modified by adding continuity condition of tangential magnetic fields at interfaces of the rods with the regions 1 and 4. The X–Y cross section can be seen as consisting of four different regions, shown by 1 through 4 in Figure 1. Regions 1 and 4 are infinite uniform mediums, and regions 2 and 3 are periodic regions. Electromagnetic wave incident to the periodic structure can be considered as either transverse electric (TE) with electric field or transverse magnetic (TM) with magnetic field in the Z-direction. In this paper, the TM-polarized incident wave, which excites surface wave and subsequently contributes in extraordinary power transmission, is considered for analysis. The theoretical model is built up by expressing electromagnetic fields in both uniform regions 1 and 4 as well as parallel plate waveguide (PPWG) regions 2 (top PPWG) and 3 (bottom PPWG), as Fourier series expansions. Scattering matrices are considered for different interfaces existing in the structure; \( S^{(1–2)} \) for the interface between air medium 1 and the top PPWG, \( S^{(2–3)} \) for the interface between the top and bottom PPWGs, and \( S^{(3–4)} \) for the interface between the bottom PPWG and the dielectric substrate 4. In order to analyze the performance of the proposed structure on impinging TM-polarized wave, scattering matrices \( S^{(i–j)} \), \( (i, j) = (1, 2), (2, 3), \) and \( (3, 4) \), are first calculated and then combined based on cascade networks rule as in the followings.

2.1 Calculation of \( S^{(1–2)} \) Matrix

To obtain the \( S^{(1–2)} \) matrix, magnetic fields inside regions 1 and 2, respectively, are expressed as Equations (1) and (2) ([29]):

\[
H_x^{(1)}(x, y) = \sum_{n=-\infty}^{+\infty} \left( a_n^{(1)}(y) e^{-jK_0 x} + b_n^{(1)}(y) e^{jK_0 x} \right) \times e^{-j(2\pi n/d_x) x} / \sqrt{d_x},
\]

\[
H_x^{(2)}(x, y) = \sum_{n=0}^{+\infty} \left( a_n^{(2)}(y) e^{-jK_0 x} + b_n^{(2)}(y) e^{jK_0 x} \right) \times \sqrt{2 / d_x - w_x} \cos \left( n\pi (x - w_x^2) / 2 \right),
\]

where \( j = \sqrt{-1} \), and \( a_n^{(i)}(y) \) and \( b_n^{(i)}(y) \) are \( n \)-th-order diffraction coefficients for incident and reflected waves, respectively, in the region \( i \), \( i = 1, 2 \). Propagation constant, \( K_{y,n}^{(i)} \), along Y-axis for \( n \)-th-order diffraction mode in region \( i \) is

\[
K_{y,n}^{(1)} = \sqrt{K_0^2 - \left( 2\pi n / d_x \right)^2}, \quad (3)
\]

for region \( i = 1 \), and

\[
K_{y,n}^{(2)} = \sqrt{K_0^2 \varepsilon_s - \left( K_{x,0} + n\pi / (d_x - w_x) \right)^2}, \quad (4)
\]

for region \( i = 2 \). \( \varepsilon_s \) is the relative permittivity constant of dielectric in regions 2, 3 and 4, and \( K_0 \) is free-space wavenumber. \( K_{x,0} \) in Equation (4) is the initial propagation constant along X-axis. For the special case of normally incident TM-polarized wave, \( K_{x,0} \) is zero, as will be assumed in this study. It is noted that to propagate \( n \)-th-order diffraction mode in regions \( i = 1 \) and 2, its propagation constant \( K_{y,n}^{(i)} \) along Y-direction must be real. Modes with pure imaginary values of \( K_{y,n}^{(i)} \) decay rapidly by distance from \( y = 0 \). Based on Equations (3) and (4), at high enough wavelength values \( \lambda \) (low enough values of \( K_0 \)), all values of propagation constant \( K_{y,n}^{(i)} \) along Y-axis in regions \( i = 1 \) and 2 are pure imaginary except for fundamental mode, \( n = 0 \), that \( K_{y,0}^{(i)} \) is real. In fact, all

![Figure 1: X–Y cross view of the proposed stepped-slit periodic metallic structure](image-url)
non-zero orders \((n \neq 0)\) of diffraction modes are evanescent in regions 1 and 2. Therefore from here on we will focus on the fundamental mode only. By substituting Equations (1) and (2) into the Maxwell’s equation, \(\nabla \times \vec{H} = j \epsilon_0 \omega \vec{E}\), tangential electric field components \(E_x^{(i)}\) in regions \(i = 1\) and \(2\) are calculated as

\[
E_x^{(1)}(x,y) = \frac{1}{j \omega \epsilon_0} \sum_{n=-\infty}^{+\infty} (jK_{y,n}^{(1)}(-a_n^{(1)}(y) e^{-jK_{y,n}^{(1)} y} + b_n^{(1)}(y) e^{jK_{y,n}^{(1)} y}) e^{-j(2\pi n/d_x) x} + \frac{1}{\sqrt{d_x}} e^{-j(2\pi n/d_x) x},
\]

\(5\)

\[
E_x^{(2)}(x,y) = \frac{1}{j \omega \epsilon_i} \sum_{n=0}^{+\infty} (jK_{y,n}^{(2)}(-a_n^{(2)}(y) e^{-jK_{y,n}^{(2)} y} + b_n^{(2)}(y) e^{jK_{y,n}^{(2)} y}) e^{-j(2\pi n/d_x) x} \times \cos \left( \frac{n \pi}{d_x - w_x} (x - \frac{w_x}{2}) \right).
\]

\(6\)

In order to obtain coefficient \(b_n^{(i)}(y)\) in regions \(i = 1\) and \(2\) \((y = 0)\) are used:

\[
H_z^{(1)}(x,0) = H_z^{(2)}(x,0), \quad \frac{w_x}{2} < x < d_x - \frac{w_x}{2},
\]

\(7a\)

\[
E_x^{(1)}(x,0) = \begin{cases} 
0, & 0 \leq x \leq \frac{w_x}{2} \text{ or, } d_x - \frac{w_x}{2} \leq x \leq d_x, \\
\frac{w_x}{2} < x < d_x - \frac{w_x}{2}.
\end{cases}
\]

\(7b\)

By considering \(H_z^{(1)}\) and \(H_z^{(2)}\) in the forms of Equations (1) and (2), respectively, boundary condition (7a) can be written as

\[
\frac{1}{\sqrt{d_x}} \sum_{n=-\infty}^{+\infty} \left( a_n^{(1)}(0) + b_n^{(1)}(0) \right) e^{-j(2\pi n/d_x) x} = \sqrt{\frac{2}{d_x - w_x}} e^{-j(2\pi n/d_x) x} + \frac{e^{j(2\pi n/d_x) x}}{\sqrt{d_x - w_x}} \sum_{n=0}^{+\infty} \left( a_n^{(2)}(0) + b_n^{(2)}(0) \right) \cos \left( \frac{n \pi}{d_x - w_x} (x - \frac{w_x}{2}) \right).
\]

\(8\)

Equations (5) and (6):

\[
\frac{1}{j \omega \epsilon_0} \sum_{n=-\infty}^{+\infty} (jK_{y,n}^{(1)}(-a_n^{(1)}(0) + b_n^{(1)}(0)) e^{-j(2\pi n/d_x) x} + \frac{1}{\sqrt{d_x}} e^{-j(2\pi n/d_x) x} \times \cos \left( \frac{n \pi}{d_x - w_x} (x - \frac{w_x}{2}) \right).
\]

\(9\)

Multiplying both sides of Equations (8) and (9) by \(\cos(m \pi/d_x - w_x (x - (w_x/2)))\) and integrating over a period from \(x = 0\) to \(x = d_x\) would lead to the following system of linear algebraic equations for coefficients \(a_n^{(1)}(0), a_n^{(2)}(0), b_n^{(1)}(0)\) and \(b_n^{(2)}(0)\):

\[
a_m^{(2)}(0) + b_m^{(2)}(0) = (d_x - w_x)^{3/2} \sum_{n=0}^{+\infty} \frac{-j \sqrt{d_x} e^{-j\pi(2d_x + w_x)/d_x} / \pi}{(d_x(m + 2n) - 2nw_x) + (-1 + (-1)^m e^{2j\pi w_x/d_x}) (a_n^{(1)}(0) + b_n^{(1)}(0)),
\]

\(10\)

\[
\frac{K_{y,m}^{(2)}}{K_{y,m}^{(1)}} \left( a_n^{(2)}(0) - b_m^{(2)}(0) \right) = \sqrt{d_x} (d_x - w_x)^{3/2} \sum_{n=-\infty}^{+\infty} \left[ \begin{array}{c}
2j(n(d_x - w_x)) \cos \left( \frac{m \pi w_x}{2(d_x - w_x)} \right) \\
- \cos \left( \frac{m \pi(-2d_x + w_x)}{2(d_x - w_x)} \right) \\
2\pi (d_x(m + 2n) - 2nw_x) (d_x(m - 2n) + 2nw_x) \\
\end{array} \right] + \frac{d_x m \sin \left( \frac{m \pi(-2d_x + w_x)}{2(d_x - w_x)} \right)}{2(d_x - w_x)} + \frac{m \pi w_x}{2(d_x - w_x)} \right) \right) \left( \begin{array}{c}
\frac{2\pi (d_x(m + 2n) - 2nw_x)}{(d_x(m - 2n) + 2nw_x)} \\
\end{array} \right) \right)
\]

\(11\)

As mentioned in the following of Equation (4), at high enough wavelength values, \(K_{y,n}^{(1)}\) and \(K_{y,n}^{(2)}\) for \(n \neq 0\) are pure imaginary; and so, they decay rapidly in \(Y\)-direction.
distance from \( y = 0 \). Therefore, incident and reflected coefficients, \( a_0^{(1)}(y), a_0^{(2)}(y), b_0^{(1)}(y), b_0^{(2)}(y) \), of guided order diffraction mode \( m = 0 \) and \( n = 0 \), are considered in Equations (10) and (11) only. By substituting \( m = 0 \) and \( n = 0 \), coefficients of zeroth-order diffraction mode are calculated in matrix form as

\[
\begin{pmatrix}
 b_0^{(1)}(0) \\
 b_0^{(2)}(0)
\end{pmatrix} = S^{(1-2)} \begin{pmatrix}
 a_0^{(1)}(0) \\
 a_0^{(2)}(0)
\end{pmatrix},
\]

with

\[
S^{(1-2)} = \begin{pmatrix}
 1 + \frac{2(-d_x + w_x)}{d_x - w_x + dx \sqrt{\epsilon_s}} & \frac{2 \sqrt{d_x \sqrt{d_x - w_x}} - \sqrt{\epsilon_s}}{d_x - w_x + dx \sqrt{\epsilon_s}} \\
 \frac{2 \sqrt{d_x \sqrt{d_x - w_x}} - \sqrt{\epsilon_s}}{d_x - w_x + dx \sqrt{\epsilon_s}} & -1 + \frac{2(d_x - w_x)}{d_x - w_x + dx \sqrt{\epsilon_s}}
\end{pmatrix}.
\]

Reflection coefficient \( b_0^{(1)}(-w_y/4) \) at \( y = -w_y/4 \) and transmission coefficient \( b_0^{(2)}(w_y/4) \) at \( y = w_y/4 \) can be obtained from

\[
\begin{pmatrix}
 b_0^{(1)}(-w_y/4) \\
 b_0^{(2)}(w_y/4)
\end{pmatrix} = T^{(1-2)} S^{(1-2)} T^{(1-2)} \begin{pmatrix}
 a_0^{(1)}(-w_y/4) \\
 a_0^{(2)}(w_y/4)
\end{pmatrix},
\]

where \( T^{(1-2)} \) is the transferring matrix:

\[
T^{(1-2)} = \begin{pmatrix}
 e^{-jK_0 w_y} w_y & 0 \\
 0 & e^{-jK_0 \sqrt{\pi} w_y/4}
\end{pmatrix}.
\]

### 2.2 Calculation of \( S^{(2-3)} \) Matrix

In order to calculate the scattering matrix at the interface of regions 2 and 3, \( S^{(2-3)} \), magnetic fields in the upper and lower sides of PPWG (in the middle of stepped slit) interface are required. Similar to Fourier series expansion (2) which was valid for region 2, magnetic field in region 3 can be expanded as

\[
H_z^{(3)}(x, y) = \sum_{n=0}^{+\infty} \left( -a_n^{(3)}(y) e^{-jK_{1,\gamma}^{(3)} y} - b_n^{(3)}(y) e^{jK_{1,\gamma}^{(3)} y} \right) \sqrt{2} \frac{n\pi}{d_x - w_x} \cos \left( \frac{n\pi}{d_x - w_x} \left( x - \frac{w_x}{2} \right) \right),
\]

where the minus sign comes to be consistent with the convention of Figure 2. \( K_{1,\gamma}^{(3)} \) is the propagation constant along \( Y \)-direction for \( n \)-th order diffraction mode in region 3 written as

\[
K_{1,\gamma}^{(3)} = \sqrt{K_0^2 \epsilon_s - \left( \frac{n\pi}{d_x - w_x} \right)^2}.
\]

To obtain the \( x \)-component of electric field in region 3, \( E_x^{(3)} \), a similar procedure as the one that led to Equation (6) should be carried out, which will give

\[
E_x^{(3)}(x, y) = \frac{1}{j\omega\epsilon_s} \sum_{n=0}^{+\infty} \left( jK_{1,\gamma}^{(3)} \right) \left( a_n^{(3)}(y) e^{-jK_{1,\gamma}^{(3)} y} - b_n^{(3)}(y) e^{jK_{1,\gamma}^{(3)} y} \right) \frac{2}{d_x - w_x} \cos \left( \frac{n\pi}{d_x - w_x} \left( x - \frac{w_x}{2} \right) \right).
\]

Reflection coefficients \( b_n^{(i)}(y), i = 2, 3, \) can be calculated from the boundary conditions at the interface of regions 2 and 3; i.e. \( H_z \) and \( E_x \) should vary continuously across \( y = w_y/2 \):

\[
H_z^{(2)}(x, w_y/2) = H_z^{(3)}(x, w_y/2), w_x/2 < x < d_x - w_x/2,
\]

\[
E_x^{(2)}(x, w_y/2) = \begin{cases} 0, 0 \leq x \leq w'_x/2, \text{or,} \\
 d_x - w'_x/2 \leq x \leq d_x, \\
 E_x^{(3)}(x, w_y/2), w'_x/2 \lesssim x < d_x - w'_x/2. 
\end{cases}
\]

By substituting Equations (2), (6), (16) and (18) into Equations (19a) and (19b), and multiplying both sides by \( \cos(n\pi/d_x - w'_x/(x - (w'_x/2))) \) and subsequently integrating over the slot of bottom PPWG \( (w'_x/2 < x < d_x - (w'_x/2)) \), the following system of algebraic equations will be obtained for reflection and transmission coefficients \( b_0^{(2)}(w_y/2) \) and \( b_0^{(3)}(w_y/2) \) for the fundamental mode:

\[
\begin{pmatrix}
 b_0^{(2)}(w_y/2) \\
 b_0^{(3)}(w_y/2)
\end{pmatrix} = S^{(2-3)} \begin{pmatrix}
 a_0^{(2)}(w_y/2) \\
 a_0^{(3)}(w_y/2)
\end{pmatrix},
\]

where \( S^{(2-3)} \) is

\[
S^{(2-3)} = \begin{pmatrix}
 (w_x - w'_x) & -2\sqrt{d_x - w_x} \sqrt{d_x - w'_x} \\
 -2d_x + w_x + w'_x & -2d_x + w_x + w_x \\
 -2\sqrt{d_x - w_x} \sqrt{d_x - w'_x} & 2\sqrt{d_x - w_x} \sqrt{d_x - w'_x} \\
 -2d_x + w_x + w'_x & -2d_x + w_x + w_x
\end{pmatrix}.
\]
and 4 can be calculated in a similar fashion as what we did from successive cascade of transferred interface scatterings describes the relation between $b_{0}^{(2)}(y)$ and $b_{0}^{(3)}(y)$ at $y = w_y/4$ and $y = 3w_y/4$ can be obtained from

$$
\begin{pmatrix}
  b_{0}^{(2)} \\
  b_{0}^{(3)}
\end{pmatrix}
= T^{(2-3)} S^{(2-3)} T^{(2-3)}
\begin{pmatrix}
  a_{0}^{(2)} \\
  a_{0}^{(3)}
\end{pmatrix},
$$

(22)

where the transferring matrix

$$
T^{(2-3)} = \begin{pmatrix}
  e^{-jk \sqrt{\varepsilon}(w_y/4)} & 0 \\
  0 & e^{-jk \sqrt{\varepsilon}(3w_y/4)}
\end{pmatrix}
$$

(23)

describes the relation between $b_{0}^{(2)}(y)$ at $y = w_y/2$ and $y = w_y/4$ as well as $b_{0}^{(3)}(y)$ at $y = w_y/2$ and $y = 3w_y/4$.

### 2.3 Calculation of $S^{(3-4)}$ Matrix

The scattering matrix $S^{(3-4)}$ at the interface of regions 3 and 4 can be calculated in a similar fashion as we did in Section 2.1 for calculating $S^{(1-2)}$. Note that here region 4 is the substrate with relative permittivity constant, $\varepsilon_s$.

### 2.4 Calculation of Total Scattering Matrix, $S^{(1-4)}$

The total scattering matrix $S^{(1-4)}$ can be calculated by successive cascade of transferred interface scatterings from

$$
T^{(1-4)} S^{(1-4)} T^{(1-4)} = \text{Cas} \left\{ \text{Cas} \left\{ T^{(1-2)} S^{(1-2)} T^{(1-2)} \right\}, T^{(2-3)} S^{(2-3)} T^{(2-3)} \right\}, T^{(3-4)} S^{(3-4)} T^{(3-4)} \right\},
$$

(24)

where here-defined operator $\text{Cas}[A, B]$ operates on two by two matrices $A$ and $B$ as ([30])

$$
\text{Cas}[A, B] = \begin{pmatrix}
  A_{11} + A_{12}B_{11}A_{21} & A_{12}B_{12} \\
  B_{21}A_{21} & A_{22}B_{22} + B_{21}A_{22}A_{21}
\end{pmatrix}.
$$

(25)

Subsequently, the total reflection coefficient $b_{0}^{(1)}(0)$ at $y = 0$ and total transmission coefficient $b_{0}^{(4)}(w_y)$ at $y = w_y$ can be obtained as

$$
\begin{pmatrix}
  b_{0}^{(1)}(0) \\
  b_{0}^{(4)}(w_y)
\end{pmatrix} = S^{(1-4)} \begin{pmatrix}
  a_{0}^{(1)}(0) \\
  a_{0}^{(4)}(w_y)
\end{pmatrix}.
$$

(26)

Here, for the sake of brevity, only the expression for $b_{0}^{(1)}(0)$ is presented

$$
b_{0}^{(1)}(0) = e^{jK_0 w_y} \left( \frac{1}{-1 + c - \sqrt{\varepsilon_s}} + \frac{\Psi_1}{\Psi_2} \right) a_{0}^{(1)}(0),
$$

(27a)

with

$$
\begin{align*}
\Psi_1 &= -(-2 + c')(-2 + c + c') e^{jK_0 w_y \sqrt{\varepsilon_s}} \\
&\quad \left( (-1 + c) c - \sqrt{\varepsilon_s} - \varepsilon_s \right) \\
&\quad + c'(-2 + c + c') ((-1 + c) c + \\
&\quad (3 - 2c) \sqrt{\varepsilon_s} + \varepsilon_s) - 2(c - c') \\
&\quad \times e^{jK_0 w_y \sqrt{\varepsilon_s}} ((-1 + c) c + \\
&\quad (3 + c(-2 + c') - 2c') \sqrt{\varepsilon_s} + \varepsilon_s - c' \varepsilon_s), \\
\Psi_2 &= (-2 + c')(-2 + c + c') \\
&\quad \times e^{jK_0 w_y \sqrt{\varepsilon_s}} (1 - c + \sqrt{\varepsilon_s})^2 \\
&\quad + 2(c + c')(-2 + c + c') e^{jK_0 w_y \sqrt{\varepsilon_s}} \\
&\quad \times (-1 + c - (-1 + c') \\
&\quad \times \sqrt{\varepsilon_s}) - c'(-2 + c + c') ((-1 + c)^2 - \varepsilon_s),
\end{align*}
$$

(27b)

where $c = w_x/d_x$ and $c' = w'_y/d_y$ are, respectively, the normalized widths of top and bottom metallic rods. In order to verify the accuracy of this theoretical model, transmission power of the structure proposed in the reference [33] is calculated by the model. The obtained results along with FEM simulation results are shown in Figure 3. The results of this figure have a good agreement with that reported in the reference [33].
3. RESULTS AND DISCUSSION

3.1 Design Method

Analytical expression for the reflection coefficient \( b_0^{(1)}(0) \) of zeroth-order diffraction mode for the impinging TM-polarized wave on the structure was calculated in the last section as Equation (27a). The power \( 1 - |b_0^{(1)}(0)|^2 \) transmitted into the substrate depends on the wavelength of the incident wave, periodicity \( d_x \) of the structure, width \( w_x \) of the top metallic rod, width \( w'_x \) of the bottom metallic rod, height \( w_y \) of stepped slit, and relative permittivity constant \( \epsilon \) of the dielectric of structure. In this section, the dielectric is assumed to be In\(_{0.53}\)Ga\(_{0.47}\)As, with relative permittivity constant 11.7 [31] and absorption coefficient 8000 cm\(^{-1}\) [32]. The ultimate goal of the design is to achieve a structure with maximum transmitted power of incident TM-polarized wave at a specific optical frequency and all the range of terahertz frequency bandwidth, by determining the associated parameters. Figure 4 shows the transmitted power calculated by our theoretical model with respect to height of the proposed structure \( w_y/\lambda \) (normalized to wavelength) at several widths \( w_x \) and \( w'_x \) of metallic rods; different curves correspond to different values of \( w_x \) and \( w'_x \). Several resonant guided modes can be observed in the figure. However, first guided mode, at \( w_y/\lambda \) close to zero, is a non-resonant mode. At this mode, the wavelength of the impinging wave is much more than the height \( w_y \) of structure, and metallic rod widths have no effect on the power transmitted into the substrate. Maximum transmission power at this mode is

\[
T_{\text{trans}} = \frac{4\sqrt{\epsilon}}{(1 + \sqrt{\epsilon})^2}. \tag{28}
\]

Frequency values of the resonant modes in Figure 4 agree well with \( \lambda_n = 2w_y/n\sqrt{\epsilon} \) \((n = 1, 2, \ldots)\) proposed by the reference [33]. However for certain ratios of \( w_x \) and \( w'_x \) (e.g. \( w'_x/w_x = 5 \)) transmit power with much larger values than those reported in [33] for uniform-slit structure, can be obtained at certain frequencies. This is an interesting property of stepped periodic metallic structures and will be discussed further later. It is also noted that frequency values of resonant modes depend on \( w_y \) only, whereas transmitted power depends on solely \( w_x \) and \( w'_x \). Therefore at the first step of design, the height \( w_y \) of the structure is determined, based on the optical frequency desired to transmit maximum electromagnetic power. Maximum power transmission is read at \( w_y/\lambda = 0.14 \) from Figure 4.

Field intensity, \( I \) in unit depth of the substrate can be calculated from

\[
I = \frac{P_{\text{trans}}}{d_x - w'_x}, \tag{29}
\]

where \( P_{\text{trans}} \) is the power transmitted into the substrate, and \( d_x - w'_x \) is the bottom slit width. According to the above equation \( I \) increases with the increase of \( w'_x \). Figure 4 contains \( P_{\text{trans}} \) for uniform-slit structure, A and B, as well as stepped slit structures C and D. First comparing two uniform slit structures A and B, since \( w'_x^{(A)} > w'_x^{(B)} \) and \( P^{(A)}_{\text{trans}} > P^{(B)}_{\text{trans}} \) at \( w_y/\lambda = 0.28 \), so \( I^{(A)} > I^{(B)} \). Frequency bandwidth of A though has a smaller value than that of B. Therefore, in metallic arrays of uniform slits, field intensity \( I \) and frequency bandwidth have opposite trends of change, such that simultaneous enhancement of both quantities field intensity
and bandwidth is impossible, as has been also pointed out by [33]. Second comparing stepped-slit structure D with uniform-slit structure A, $P_\text{trans}^{(A)} = P_\text{trans}^{(D)} = 0.7$ at $w_y/\lambda = 0.28$, and $w_x^{(A)} = w_x^{(D)}$, therefore $I^{(A)} = I^{(D)}$. At the same time, frequency bandwidth of D is larger than that of A. As a result, as opposed to uniform-slit arrays, interestingly metallic arrays with stepped slits can be utilized for increasing the frequency bandwidth, while preserving field intensity $I$ in the substrate constant.

In the second step of design, suitable widths $w_x$ and $w_x'$ should be determined. To this end, transmitted power of TM-incident wave has been calculated from Equation (27a), and is plotted in Figure 5 as a function of $w_x/d_x$ and $w_x'/d_x$, for $K_0\frac{w_y}{\lambda} = 2\pi(0.14)$, $\epsilon_s = 11.7$ and absorption coefficient 8000 cm$^{-1}$. The equi-level surfaces are triangles, and the one corresponding to maximum power transmission of 99% is shown by dark red color with the base at $w_x/d_x = 0$. Note that the special case of $w_x = w_x'$ reduces the stepped slit to uniform slit with maximum power transmission of 70%. Therefore, interestingly, almost entire power transmission is possible for metallic array of stepped slits, which was not the case for uniform slits. Here we assume $w_x/d_x = 0.6 < 1$. Figure 6 demonstrates transmitted power of TM-incident wave as a function of $w_y/\lambda$ and $w_x'/d_x$. Power transmission as high as even 99% is possible for certain values of $w_x'/d_x$. Moreover, the magnitude of power transmissivity at THz frequency (non-resonant mode) in Figure 6 is independent of metallic rods widths ($w_x$ and $w_x'$) and its value is about 70%; this fact was also observed in Figure 4 for small values of $w_y/\lambda$. According to Figure 6, at $w_y/\lambda = 0.14$, maximum power transmission of 99% corresponds to $w_x'/d_x = 0.8$.

### 3.2 Results of the Designed Structure

In the previous section, normalized structure parameters were designed to be $w_y/\lambda = 0.14$, $w_x/d_x = 0.6$, and $w_x'/d_x = 0.8$. For the sake of comparison, the same values for $\lambda$ and $d_x$ are adopted here in the analytical and numerical models as those of reference [33]; $\lambda = 1.5\ \mu\text{m}$ and $d_x = 400\ \text{nm}$, leading to $w_y = 200\ \text{nm}$, $w_x = 240\ \text{nm}$, and $w_x' = 320\ \text{nm}$. Figure 7 shows the designed...
structure as well as its transmitted power calculated both analytically using Equation (27a) and numerically by the finite element method (FEM) within COMSOL package with details listed in Table 1.

Transmitted power for a uniform-slit array having similar dimensions $d_x = 400 \text{ nm}$, $w_y = 200 \text{ nm}$ as the stepped-slit array but with $w_x = 320 \text{ nm}$ has been also plotted in Figure 7 for comparison purpose. Analytical results of Figure 7 show good agreements with FEM results for both stepped-slit and uniform-slit arrays. A slight difference between the resonance wavelength $\lambda$ of green and brown curves corresponding to uniform-slit array can be observed in Figure 7. This difference can be also seen in the literature that uses mode matching method, e.g. reference [33]. The authors suspect that the guided mode of PPWG slightly penetrating into the air ($y = 0^-$) and the substrate ($y = w_y^+$) is responsible for this difference. This fact can be think of as considering the effective value of $w_y$ in the FEM model to be larger than the value of $w_y$ in the analytical model, i.e. $w_y^{\text{fem}} > w_y^{\text{analytic}} (= w_y)$.

On the other hand, according to Figure 4, at first resonance mode $w_y^{\text{analytic}}/\lambda = w_y^{\text{fem}}/\lambda = 0.14$. Therefore, $\lambda^{\text{fem}} > \lambda^{\text{analytic}}$, which justifies the difference between analytic and FEM resonance wavelengths of Figure 7 for uniform-slit arrays. A similar reasoning applies to the difference between resonance wavelength of blue and red curves associated with stepped-slit arrays. As for comparison between the performance of stepped- and uniform-slit arrays, maximum transmitted power of stepped-slit array reaches to the impressive value of about 100% at $\lambda = 1.5–1.6 \mu \text{m}$. For the case of uniform-slit array, this value is only 70%. This fact is also confirmed by Figure 8 which shows that the value of electric field inside the substrate of stepped-slit arrays is larger than that inside uniform-slit arrays. At terahertz frequency band though, both stepped- and uniform-slit arrays have maximum transmitted power of 70%. Therefore, the designed stepped-slit array can transmit entire power of incident wave at optical frequency, while simultaneously transmitting 70% at terahertz frequency band.

4. CONCLUSION

Periodic metallic structures have attracted a lot of interest due to their practical implications for power transmission at optical frequencies. Surface waves excited by the incident transverse magnetic wave are the origin of power transmission through structure into the substrate. In this paper, metallic structure with periodic array of stepped slits was suggested. A closed-form expression for the power transmitted through the structure was obtained. The expression takes on structure height, top and bottom slit widths, permittivity constant of dielectric substrate, and frequency as unknown parameters. Conversely, for given optical wavelength corresponding to the entire power transmission, dimensions of the structure can be designed. As an example, for typical optical wavelength of 1.5 $\mu \text{m}$, results show that 70% of incident power is transmitted to In$_{0.53}$Ga$_{0.47}$As substrate in terahertz frequency band, simultaneous to the entire power transmission at the optical frequency of design. As opposed to uniform slits, with stepped-slit field intensity in the substrate can be increased while preserving frequency range of maximum power transmission constant. All results are confirmed by finite element simulations. It was shown that the proposed structure embedded in rods of a plasmonic photoconductive antenna increases the efficiency as well as the radiated terahertz power.

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DISCLOSURE STATEMENT

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