Optical Levitation of a Droplet under Linear Increase of Gravitational Acceleration

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Abstract

Optical levitation of a liquid droplet in gas phase was investigated under time-dependent change of the gravitational acceleration with specific flight pattern of an airplane. Through multiple trials under linear increase of effective gravitational acceleration, we performed the experiment of optical trapping of a droplet from $0.3 g_0$ to $0.9 g_0$, where $g_0 = 9.8 \text{ m/s}^2$. During such change of the effective gravitational acceleration, the trapping position on a droplet with the radius of $14 \mu \text{m}$ was found to be lowered by ca. $100 \mu \text{m}$. The essential feature of the change of the trapping position is reproduced by a theoretical calculation under the framework of ray optics. As far as we know, the present study is the first report on optical levitation under time-dependent gravitational change.

1 Introduction

Since the pioneering study of Ashkin on optical manipulation, trapping by focused laser has been actively studied. It has been shown that laser trapping serves as a useful experimental tool on the manipulation of micro systems in...
liquid solutions. In contrast to active studies on the optical trapping in liquid phase, only a few studies on optical levitation in gas phase have been reported (see ref.[1] and the references in ref.[2]). Optical injection can generate at most only 10 nN as theoretical limit when the power of light is 1 W[3]. Thus, in the case of ground-based experiments, the size of levitation object is limited up to the order of tens of µm.

As for the levitation in gas phase beside optical trapping, several methodologies have been examined such as electrostatics[4], electromagnetics[5], aerodynamics[6], and ultrasonics[7]. With these methods it is still difficult to trap and manipulate a desired single object. Thus, it may be of scientific value to perform the experiment of optical levitation in gas phase toward the optimization of the efficiency as well as the improvement of the performance.

We have reported optical trapping of a water droplet with the radius of µm by using highly converged laser in gas phase on the ground[2]. In the present study, we adapt the low converged laser in order to obtain large working distance. We will report the result of optical levitation under time dependent change of effective gravitational acceleration with a specific designed flight of an airplane. This study carried out in toward the future application for the experiments under microgravity in the International Space Station (ISS).

2 Experimental

We performed the trapping experiments in a jet airplane (Mitsubishi MU-300, operated by Diamond Air Service Co., Aichi, Japan). The flight pattern was selected so as to change the acceleration of the airplane along vertical axis $d^2 H/dt^2$ linearly as shown in Fig.1. It is to be noted that such a change of the gravitational acceleration is largely different from the conventional parabolic flight (see the broken line in the Fig.1) usually adapted for the experiment of “microgravity”.

Experimental setup is schematically illustrated in Fig.2. In order to improve the dispersibility of droplets, aqueous solution containing 30 vol% of ethanol was used. The droplets were injected into glass cell (10 mm × 10 mm × 50 mm) with an atomizer during effective gravitational acceleration in the airplane $g (= d^2 H/dt^2 + g_0$, where the gravitational acceleration on the ground $g_0 = 9.8 \text{ m/s}^2$) was 0.01$g_0$ (e.g. between 0 and 20 s in Fig.1). For optical levitation, slightly converged continuous wave laser beam (wavelength $\lambda = 532 \text{ nm}$, TEM$_{00}$ mode, Millennia, Spectra Physics) at 150 mW was focused from the beneath into the cell with achromatic lens. The convergence angle was 5 degrees. The time course on the manner of optical trapping of droplets was monitored with a CCD camera. The experiments are carried out at ca. 298 K.
Fig. 1. Schematic diagram of the flight pattern. The lower diagram shows the time-trace of height $H$ from the ground, and the upper one shows acceleration $d^2H/dt^2$ along vertical axis. The broken line in the upper diagram shows acceleration in the case of the conventional parabolic flight pattern. In the present study, because of the technical reason we analyze the region of the linear increase of the effective gravitational acceleration, where the point, $t = 0$, is the time-zero adapted in Figs. 3 and 4.

Fig. 2. Schematic diagram of the experimental setup.

3 Results and Discussion

Fig.3 shows the result of optical levitation on a droplet, where the time $t = 0$ and position $z = 0$ are taken at the moment at which the droplet is begun to be trapped. Without laser irradiation, these droplets tended to fall down and the rate of the downward motion was increased with the increase of the effective gravitational acceleration $g$. With laser irradiation from the beneath, under microgravity condition, droplets on the optical corn were scattered to the laser direction. When $g = 0.3g_0$, a droplet was spontaneously trapped. As in the figure, the trapping position was gradually lowered accompanied with
Fig. 3. (Top): Change of the optical microscopic images on the trapped droplet during the linear increase of the effective gravitational acceleration. (Middle): Spatio-temporal representation on the trapped droplet. (Bottom): Time dependent change of the height $z$ of the trapped droplet.

the time development which is correlated with the increase of $g$. The spatio-temporal image indicates the gradual change of the trapping position, where many other floating droplets outside the trapping position were continuously falling down.

Fig. 4(a) shows the change of the trapping position, $z$, with the change of the effective gravitational acceleration, where $z$ changes gradually until $t = \text{ca. 4.5 s}$. Then, the droplet tends to fall down, which is attributed to the escape from the trapping potential due to the increase of $g$. Open circles in Fig. 4(b) shows the correlation between $z$ and the relative value $g/g_0$.

In the present experiment, it was difficult to estimate the actual size of the trapped droplet from the image itself, because of the low magnification of the objective lens on the CCD camera. Thus, we will have the discussion to evaluate the size of the trapped droplet from the experimental observation on the change of the trapping position depending on the change of the effective gravitational acceleration. The motion equation of the droplet is:

$$m\ddot{z} = F_g + F_l + F_v$$ \hspace{1cm} (1)

the inertial force $m\ddot{z}$ ($m$: mass of a droplet), the gravitational force $F_g$, the force induced by converged laser $F_l$ along the optical axis and the viscous force $F_v$. Since the radius of the trapped droplet is on the order of 10 $\mu$m, we can assume the viscous limit: $m\ddot{z} \simeq F_v$, and obtain the relation:

$$F_g + F_l = 0$$ \hspace{1cm} (2)

$$F_g = -\frac{4}{3}\pi r^3 \rho g$$ \hspace{1cm} (3)
Fig. 4. (a) Time trace of the effective gravitational acceleration in the flight given in Fig. 1, where \( g = \frac{d^2H}{dt^2} + g_0 \); \( g_0 \) and \( g \) are the gravity acceleration on the ground and effective gravitational acceleration in the airplane. The longitudinal axis is given as the relative value \( g/g_0 \). The change of the height \( z \) of the trapped droplet is also shown. (b) Change of the height \( z \) of the trapped droplet with respect to \( g/g_0 \). Experimental and theoretical results are given as open circles and solid line, respectively. The broken line is the expected curve from a theoretical consideration. In the theoretical calculation, following parameters are adapted: laser power \( P = 150 \) mW, convergence angle \( \phi = 7.0 \) degrees, injected ratio of laser to the lens \( \sigma/L = 1.5 \), droplet radius \( r = 14.0 \) µm, refractive index of a medium \( n_1 = 1.00 \), refractive index of a droplet \( n_2 = 1.35 \), density of a droplet \( \rho = 9.65 \times 10^2 \) kg/m\(^3\), gravitational acceleration on the ground \( g_0 = 9.80 \) m/s\(^2\). (c) Schematic illustration of the relationship between the droplet’s motion and the shift of potential minimum. The broken arrow shows the change of the net potential profile including both the optical and gravitational contribution: The solid arrow shows the shift of potential minimum position.
where $g$: effective gravitational acceleration and $\rho$: density of a droplet. Additionally, the radius $r$ of the droplet is enough larger than the wavelength $\lambda$. So we can use the ray optics regime in calculating $F_l$,

$$F_l = F_l(r, z; P, \phi, \sigma/L, n_1, n_2)$$  \hfill (4)

where $r$: radius of a droplet, $z$: distance between focal point of laser and the center of a droplet, $P$: laser power, $\phi$: convergence angle of laser, $\sigma/L$: injected ratio of TEM$_{00}$ mode laser to the lens ($\sigma$ is deviation of TEM$_{00}$ mode laser and $L$ is lens radius), $n_1$: refractive index of a medium, $n_2$: refractive index of a droplet, $Q_z$: trapping efficiency along optical axis, and $c$: velocity of light, respectively. In this case, $r$ and $z$ are variable, and the other parameters are fixed.

$F_l$ is found as follows. In the framework of the ray optics, a TEM$_{00}$ mode laser can be divided to rays which are suffixed with $i$ and each power $n_1 P_i/c$ is related to $\sigma/L$. Each rays hit the surface of the droplet at different incident angles $\phi_i$ ($0 \leq \phi_i \leq \phi$), repeat reflections and transmissions in the droplet until the intensity of rays reduce to zero of limit, and give the momentum with a certain efficiency along z-axis, $Q_i = Q_i(r, z; \phi_i, n_1, n_2)$:

$$Q_i = \sin \phi_i \left\{ R_i \sin 2\theta_i - \frac{T_i^2 \left[\sin(2\theta_i - 2r_i) + R_i \sin 2\theta_i\right]}{1 + R_i^2 + 2R_i \cos 2r_i} \right\}$$

$$+ \cos \phi_i \left\{ 1 + R_i \cos 2\theta_i - \frac{T_i^2 \left[\cos(2\theta_i - 2r_i) + R_i \cos 2\theta_i\right]}{1 + R_i^2 + 2R_i \cos 2r_i} \right\}$$  \hfill (5)

where $\theta_i$: incident angle, $r_i$: refractive angle, $R_i$: reflection coefficient and $T_i$: transmission coefficient. These parameters are obtained by considering geometric relation between the droplet and the direction of a beam (see ref.[7]).

Now, assuming the spherical droplet with radius $r$, whose center is located at distance $z$ from the focal point of the laser, we calculate the details of reflections and transmissions concerning all the paths of laser beam.

The total force $F_l$ is

$$F_l = \sum_i \frac{n_1 P_i}{c} Q_i = \frac{n_1 P}{c} Q_z$$  \hfill (6)

where $Q_z = Q_z(r, z; \phi, \sigma/L, n_1, n_2)$. Thus, we obtain the equation:

$$\frac{4}{3} \pi r^3 \rho g = \frac{n_1 P}{c} Q_z$$  \hfill (7)
By changing the radius $r$ in the above equations, we have tried to find the optimal curve to fit the experimental observation. Fig.4(b) shows the change of the trapping position, i.e., minimum on the potential, together with the experimental result on the height $z$ of the trapped droplet with respect to the relative value $g/g_0$. From this result, we can qualitatively understand the motion of the trapped droplet as the shift of the potential minimum point (see Fig.4(c)).

From the above analysis, it has become clear that the essential feature of the optical levitation has been theoretically reproduced in a satisfactory manner. Using eq.(7) at the critical value at $g = 1.0g_0$ to levitate the droplet which is found by experimental result, we obtain maximum trapping efficiency along optical axis $Q_z$ as 0.22. It is noted that this efficiency is one order of magnitude greater than those in previous reports (see the table 1 of ref.[2]), except to our recent result on the ground[2]. Since the water droplet was atomized in the microgravity condition in this experiment, the inertia of the droplet was small and reduced by viscous drag. Additionally, the viscous limit was accomplished by slowly changing the gravitational acceleration. We could, thus, evaluate the critical value of the gravitational acceleration on the optical levitation.

In the present study we have adapted the small converged laser, indicating the experimental condition with rather large working distance on the order of cm. The combination of the small converged laser and microgravity condition may afford interesting experimental system on the optical levitation. We have a plan to improve the experimental system by using multiple laser sources with small converged laser.

4 Conclusion

We investigated the change of the trapping position of a droplet under time-dependent change of the gravitational acceleration. It was shown that theoretical calculation based on ray optics reproduced the experimental trend. The present result may contribute in designing the manipulating system on the International Space Station (ISS), including the experiments of protein crystal growth[?] and water droplet growth[2].

5 Acknowledgements

The authors thank Mr. S. Watanabe for helpful suggestions and Ms. Hayata, Messrs. Fujii, Kawakatsu, and Takahashi for technical assistance. This
research was supported by the Grant-in-Aid for the 21st Century COE “Center for Diversity and Universality in Physics” from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan, and “Ground-based Research Announcement for Space Utilization” promoted by the Japan Space Forum.

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