PAPER

Quasideterminant solutions of 2-component non–commutative complex coupled integrable dispersionless system

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Abstract

We present a non–commutative extension of two-component complex coupled integrable dispersionless (NC-CCID) system. We studied matrix Darboux transformation and generate multiple soliton solutions by using the notion of quasideterminants. Further, we obtain single and double soliton solutions in terms of quasideterminants for NC-CCID system. In the commutative limit, we obtain soliton solutions for CCID system as ratio of ordinary determinants.

1. Introduction

The nonlinear differential equations (integrable or non-integrable) describe various phenomenon in physics and mathematics, such as, plasma physics, fiber optics, condensed matter physics, fluid mechanics, string theory etc [1–3]. During last three decades, there has been an increasing interest in the study of dispersionless integrable systems, due to their abundant applications in the fields of mathematics and physics such as string theory, theories of fields, conformal maps over complex plane, theory of solitons, etc [4–8]. Ordinary integrable systems with a dispersion term reduce into dispersionless integrable systems under semi-classical limits. The coupled integrable dispersionless (CID) system is an important equation of two-dimensional field theory. The coupled integrable equations have attracted a great deal of attraction during recent past [8–22]. In this work, we present a non–commutative extension\(^2\) of two-component complex coupled integrable dispersionless (NC-CCID) system:

\[
q_t + \frac{1}{2}(q_x^+ r_1^+ + r_1^+ q_+ - r_1^- q^+ r_1^- r_2^+ + r_2^+ q_+ r_2^+ - r_2^+ q^- r_2^-) = 0,
\]

along with constraints \(r_2^+ r_1^- - r_1^+ r_2^- = r_2^- r_1^- = r_2^- r_1^+ = r_2^+ r_1^+ - r_1^+ r_2^+\) vanish in commutative limit and subscripts \(x, t\) denote the partial derivatives. Here \(q(x, t)\) is a real-valued and \(\eta(x, t), r(x, t)\) are complex-valued anti-commuting scalar fields. If we take \(r = 0\), and \(\eta = r\), the above system (1.1) reduces into following one-component NC-CCID system:

\[
r_t + \frac{1}{2}(r_x^+ r^+ + r^+ r_x^+ + r^+_x + r^+_x r^+) = 0,
\]

If we take \(r^+ = r\) (a real scalar field), we obtain a non–commutative real coupled integrable dispersionless (NC-CID) system recently reported in [21]:

\[
r_t - qr - rq = 0.
\]

\(^2\)A non–commutative system is obtained by considering the scalar field variables to be non–commuting.
\[ q_t + r_s r + r_s = 0, \]
\[ r_x - q r - r_q = 0. \]  
(1.3)

In [21], authors have constructed multiple soliton solutions for one-component NC-CID (1.3) by using Darboux transformation.

In the commutative limit (1.1) reduces into
\[ q_t + (r_2 q_1^2 + r_2 q_1^2), \]
\[ r_x - 2q q_x = 0, \]
\[ r_x - 2q q_x = 0, \]  
(1.4)

which represents usual commutative two-component CCID system [22].

The NC-CCID system (1.1) can be represented as consistency condition of following Wadati Konno Ichikawa (WKI) type Lax pair
\[ \varphi^x = U \varphi, \]  
(1.5)
\[ \varphi^t = V \varphi, \]  
(1.6)

where \( \varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)^T \) and the 4 \( \times \) 4 coefficients \( U, V \) are given by
\[ U = -i \lambda Z, \quad V = V_0 + \frac{i}{2\lambda} V_1, \]  
(1.7)

with
\[ Z = \begin{pmatrix} \frac{Q_1^*}{Q} \frac{R_1^*}{R} \frac{R_2^*}{R} \frac{Q_2^*}{Q} \end{pmatrix}, \quad V_0 = \begin{pmatrix} \frac{O_1^*}{O} \frac{R_1^*}{R} \frac{R_2^*}{R} \frac{Q_2^*}{Q} \end{pmatrix}, \quad V_1 = \begin{pmatrix} \frac{I_1^*}{I} \frac{O_1^*}{O} \frac{R_1^*}{R} \frac{Q_2^*}{Q} \end{pmatrix}, \]

along with
\[ Q = \begin{pmatrix} q & 0 \\ 0 & q \end{pmatrix}, \quad R = \begin{pmatrix} \eta_1 & \eta_2 \\ \eta_2^* & \eta_1^* \end{pmatrix} = T^T. \]

Note: \( O_2 \) and \( I_2 \) are 2 \( \times \) 2 null and identity matrices respectively. The integrability condition of (1.5)–(1.6) becomes a zero-curvature condition i.e.
\[ U_t - V_x + UV - VU = 0. \]  
(1.8)

If we substitute \( U \) and \( V \) into (1.8) we obtain a coupled system (1.1).

Darboux transformation (DT) is a powerful solution generating technique [23–28]. In this paper, we apply DT and construct multiple soliton solutions of 2-component NC-CCID system (1.1) in terms of quasideterminants. In section 2, we apply DT on \( \varphi \) (a matrix-valued solution) to the linear system (1.5)–(1.6) and generate multi-soliton solutions in terms of quasideterminants [29–31]. In commutative limit, we obtain multi-soliton solutions as ratios of ordinary determinants for CCID system (1.4). In section 3, we compute explicit expressions of single and double soliton solutions for NC-CCID system (1.1). In order to illustrate our results we also draw different profiles of soliton solutions for CCID system (1.4). Section 4, deals with concluding remarks and future work.

2. Darboux transformation and multiple soliton solutions

DT enables us to compute a family of new solutions to linear eigenvalue problem from a known one. If \( \varphi \) is a known solution of linear system (1.5)–(1.6), then under the action of DT the new solution \( \varphi[1] \) is given by
\[ \varphi \rightarrow \varphi[1] = (\lambda^{-1} I_4 - \Gamma) \varphi, \]  
(2.1)
where \( I_4 = \text{diag}(1, 1, 1, 1) \) and \( \Gamma \) be a 4 \( \times \) 4 matrix-valued function to be determined. The new solution also satisfies system (1.5)–(1.6), that is,
\[ \varphi^x[1] \equiv U[1] \varphi[1] = -i \lambda Z[1] \varphi[1], \]  
(2.2)
\[ \varphi^t[1] \equiv V[1] \varphi[1] = \begin{pmatrix} V_0[1] + \frac{i}{2 \lambda} V_1 \end{pmatrix} \varphi[1], \]  
(2.3)
the transformed matrix-valued functions \( Z[1] \) and \( V_0[1] \) are obtained by replacing \( Q \) and \( R \) by \( Q[1] \) and \( R[1] \) in matrices \( Z \) and \( V_0 \), respectively. The compatibility condition of (2.2)–(2.3) yields into NC-CCID equation (1.1) for the new scalar variables \( q[1], \eta[1] \) and \( r[1] \).

If we substitute equation (2.1) into (2.2)–(2.3), we obtain transformation on matrix potential \( Z \) as
\[ Z[1] = Z - i \partial_t \Gamma, \]  
(2.4)
along with following conditions on $\Gamma$:

\[
\Gamma_x = i\Gamma Z \Gamma^{-1} - iZ,
\]
\[
\Gamma_t = [V_0, \Gamma] + \frac{i}{2} [V_t, \Gamma] \Gamma.
\] (2.5) (2.6)

We would like to construct $\Gamma$ for this, let $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ are four nonzero distinct eigenvalues (real/complex) and $|\alpha_1\rangle, |\alpha_2\rangle, |\alpha_3\rangle$ and $|\alpha_4\rangle$ are constant column vectors, such that,

\[
\Omega \equiv (\varphi(\lambda_1)|\alpha_1\rangle, \varphi(\lambda_2)|\alpha_2\rangle, \varphi(\lambda_3)|\alpha_3\rangle, \varphi(\lambda_4)|\alpha_4\rangle) = (|\omega_1\rangle, |\omega_2\rangle, |\omega_3\rangle, |\omega_4\rangle).
\] (2.7)

Each column $|\omega_i\rangle = \varphi(\lambda_i)|\alpha_i\rangle$ satisfies (1.5)–(1.6) for $\lambda = \lambda_i$, i.e.,

\[
|\omega_i\rangle_x = -i\lambda_i Z|\omega_i\rangle,
\]
\[
|\omega_i\rangle_t = \left(V_0 + \frac{i}{2\lambda_i} V_t\right)|\omega_i\rangle.
\] (2.8) (2.9)

Let's choose $\Gamma$-matrix as

\[
\Gamma = \Omega \Lambda^{-1} \Omega^{-1}, \quad \text{where} \quad \Lambda = \text{diag} (\lambda_1, \lambda_2, \lambda_3, \lambda_4).
\] (2.10)

The matrix of particular solutions $\Omega$ satisfies following linear system

\[
\Omega_x = -iz\Omega \Lambda,
\]
\[
\Omega_t = V_0\Omega + \frac{i}{2} V_t\Omega \Lambda^{-1}.
\] (2.11) (2.12)

It is straightforward to verify that $\Gamma$ given by (2.10) satisfies both conditions (2.5) and (2.6). Therefore, $\Gamma = \Omega \Lambda^{-1} \Omega^{-1}$ is a required solution of equations (2.5)–(2.6).

Summarizing our results as: if $\{\varphi, Z\}$ be a trivial (or known) solution set of (1.5)–(1.6), then $\{\varphi[1], Z[1]\}$ would be a nontrivial solution of the same linear problem, i.e.,

\[
\varphi[1] = (\Lambda^{-1} I_4 - \Omega\Lambda^{-1} \Omega^{-1}) \varphi,
\]
\[
Z[1] = Z - i(\Omega\Lambda^{-1} \Omega^{-1})_z,
\] (2.13) (2.14)

is the required one-fold DT for NC-CCID system (1.1).

Using properties of quasideterminants (for more details see Appendix), one can re-express equation (2.13) as:

\[
\varphi[1] = \begin{vmatrix} \Omega & I_4 \\ \Omega^{(i)} & I_4 \end{vmatrix} \begin{vmatrix} I_4 \\ I_4 \end{vmatrix} \varphi,
\] (2.15)

where $\Omega^{(i)} = \Omega\Lambda^{-1}$ and $I_4^{(i)} = \Lambda^{-1} I_4$. Similarly equation (2.14) yields into

\[
Z[1] = Z + i \begin{vmatrix} \Omega & I_4 \\ \Omega^{(i)} & I_4 \end{vmatrix} \begin{vmatrix} I_4 \\ I_4 \end{vmatrix} \varphi.
\] (2.16)

Note: $I_4$ and $O_4$ are $4 \times 4$ identity and null matrices respectively. The $M$-times successive application of DT on $\varphi$ leads into the following quasideterminant formula

\[
\varphi[M] = \begin{vmatrix} \Omega_1 & \Omega_2 & \cdots & \Omega_M & I_4 \\ \Omega_1^{(i)} & \Omega_2^{(i)} & \cdots & \Omega_M^{(i)} & I_4^{(i)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Omega_1^{(M)} & \Omega_2^{(M)} & \cdots & \Omega_M^{(M)} & I_4^{(M)} \end{vmatrix} \varphi.
\] (2.17)

Similarly, $M$-times iteration of DT on matrix-valued potential $Z$ becomes

\[
Z[M] = Z + i \begin{vmatrix} \Omega_1 & \Omega_2 & \cdots & \Omega_M & O_4 \\ \Omega_1^{(i)} & \Omega_2^{(i)} & \cdots & \Omega_M^{(i)} & O_4 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Omega_1^{(M)} & \Omega_2^{(M)} & \cdots & \Omega_M^{(M)} & I_4 \end{vmatrix} \varphi.
\] (2.18)
Here we have used the notation $\Omega^{(k)} = \Omega \Lambda^{-1}$ and $f^{(k)} = \lambda^{-1} I_4 (k, i = 1, 2, \ldots, M)$ and $\Omega_i$ be a particular solution of (2.11)–(2.12) at $A = \Lambda_i (i = 1, 2, \ldots, M)$. One can easily prove above expressions (2.17)–(2.18) by using mathematical induction (For more details see e.g. [32, 33]). Equation (2.18) can also be re-written as

$$Z[M] = Z + i \partial \Gamma^{(M)}, \quad (2.19)$$

where

$$\Gamma^{(M)} = \begin{pmatrix} \Omega \\ \Omega^{(M)} \end{pmatrix} \mathcal{E}^{(M)} = \begin{pmatrix} \Gamma_1^{(M)} \\ \Gamma_2^{(M)} \end{pmatrix},$$

with

$$\Omega = \begin{pmatrix} \Omega_1 & \Omega_2 & \ldots & \Omega_M \\ \Omega_1 & \Omega_2 & \ldots & \Omega_M \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{M-1} & \Omega_{M-1} & \ldots & \Omega_{M-1} \end{pmatrix} \in \mathbb{C}^{M \times M},$$

$$\Omega^{(M)} = \begin{pmatrix} \Omega_1^{(M)} \\ \Omega_2^{(M)} \\ \vdots \\ \Omega_{M-1}^{(M)} \end{pmatrix} \in \mathbb{C}^{M \times 1},$$

$$\mathcal{E}^{(M)} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ \vdots \\ \mathcal{E}_{M-1} \end{pmatrix} \in \mathbb{C}^{(M-1) \times 1}.$$

Furthermore, we have

$$\Omega_m = \begin{pmatrix} \omega_{1,1}^{(2m-1)} & \omega_{1,2}^{(2m-1)} & \omega_{1,3}^{(2m-1)} & \omega_{1,4}^{(2m-1)} \\ \omega_{2,1}^{(2m-1)} & \omega_{2,2}^{(2m-1)} & \omega_{2,3}^{(2m-1)} & \omega_{2,4}^{(2m-1)} \\ \omega_{3,1}^{(2m-1)} & \omega_{3,2}^{(2m-1)} & \omega_{3,3}^{(2m-1)} & \omega_{3,4}^{(2m-1)} \\ \omega_{4,1}^{(2m-1)} & \omega_{4,2}^{(2m-1)} & \omega_{4,3}^{(2m-1)} & \omega_{4,4}^{(2m-1)} \end{pmatrix},$$

$$\Lambda_m = \begin{pmatrix} \lambda_{2m-1} & 0 & 0 & 0 \\ 0 & \lambda_{2m-2} & 0 & 0 \\ 0 & 0 & \lambda_{2m-3} & 0 \\ 0 & 0 & 0 & \lambda_{2m-4} \end{pmatrix},$$

$$\Omega_m^{(j)} = \Omega_m \Lambda_m^{-j} = \begin{pmatrix} \lambda_{2m-1}^{j} & 0 & 0 & 0 \\ 0 & \lambda_{2m-2}^{-j} & 0 & 0 \\ 0 & 0 & \lambda_{2m-3}^{-j} & 0 \\ 0 & 0 & 0 & \lambda_{2m-4}^{-j} \end{pmatrix}.$$

equation (2.19) gives us following transformations on $Q$, $R$ and $T$:

$$Q[M] = Q + i \partial \Gamma_3^{(M)}, \quad Q[M] = Q - i \partial \Gamma_4^{(M)}, \quad R[M] = R + i \Gamma_5^{(M)}, \quad T[M] = T + i \Gamma_6^{(M)}, \quad (2.20)$$

along with

$$\Gamma_1^{(M)} = \begin{pmatrix} \Gamma_1^{(M)} \\ \Gamma_2^{(M)} \end{pmatrix}, \quad \Gamma_2^{(M)} = \begin{pmatrix} \Gamma_3^{(M)} \\ \Gamma_4^{(M)} \end{pmatrix}, \quad \Gamma_3^{(M)} = \begin{pmatrix} \Gamma_1^{(M)} \\ \Gamma_2^{(M)} \end{pmatrix}, \quad \Gamma_4^{(M)} = \begin{pmatrix} \Gamma_3^{(M)} \\ \Gamma_4^{(M)} \end{pmatrix}. \quad (2.21)$$
Further expressions (2.20)–(2.21) give us following transformations on scalar potentials \( q \) and \( r_i \):

\[
q[M] = q + i \partial_q \Gamma_{11}^M, \quad q[M] = q + i \partial_q \Gamma_{22}^M,
\]

\[
q[M] = q - i \partial_q \Gamma_{33}^M, \quad q[M] = q - i \partial_q \Gamma_{44}^M,
\]

\[
\eta_1[M] = \eta_1 + i \Gamma_{13}^M, \quad r_2[M] = r_2 + i \Gamma_{14}^M,
\]

\[
r_1^*[M] = r_1^* + i \Gamma_{24}^M, \quad r_2^*[M] = r_2^* - i \Gamma_{23}^M,
\]

\[
\eta_1[M] = \eta_1 + i \Gamma_{42}^M, \quad r_2[M] = r_2 - i \Gamma_{32}^M,
\]

\[
r_1^*[M] = r_1^* + i \Gamma_{33}^M, \quad r_2^*[M] = r_2^* + i \Gamma_{41}^M.
\]

From above transformations, we obtain following reduction conditions

\[
\Gamma_{12}^M = \Gamma_{21}^M = \Gamma_{34}^M = \Gamma_{43}^M = 0,
\]

\[
\Gamma_{11}^M = \Gamma_{22}^M = \Gamma_{33}^M = \Gamma_{44}^M = 0,
\]

\[
\Gamma_{11}^M = \Gamma_{22}^M = - \Gamma_{33}^M = - \Gamma_{44}^M = 0,
\]

\[
\Gamma_{13}^M = - \Gamma_{14}^M = \Gamma_{23}^M = \Gamma_{24}^M = 0,
\]

\[
\Gamma_{34}^M = - \Gamma_{32}^M = \Gamma_{43}^M = \Gamma_{41}^M = 0.
\]

The quasideterminant formulae (2.22) permit us to compute multiple soliton solutions of NC-CCID as well as for commutative CCID. In next section, we’ll compute explicit expressions of single and double solitons by using quasideterminant formulae (2.22).

3. Explicit soliton solutions

Let us compute expressions for single and double-soliton solutions of NC-CCID (1.1), we begin with \( q = 1 \) and \( r_i = 0 \) as seed solutions (or \( Q = I_2 \) and \( R = O_2 \)). The associated Lax pair (1.5)–(1.6) reduces into

\[
\varphi_x = - i \lambda \left( \frac{I_2}{O_2} \right) \varphi,
\]

\[
\varphi = \frac{i}{2 \lambda} \left( \frac{I_2}{O_2} \right) \varphi.
\]

The integration of above system yields

\[
\varphi = \left( e^{\eta I_2} \frac{O_2}{O_2 e^{\eta I_2}} \right),
\]

where \( \eta = - i \lambda x + \frac{i}{2 \lambda} t + \alpha \), here \( \alpha \) be a constant of integration.

3.1. Single-soliton solution

To construct an explicit expression of single-soliton solution take \( M = 1 \) into (2.22), we obtain

\[
q[1] = 1 + i \partial_q \Gamma_{11}^{(1)},
\]

\[
\eta_1[1] = i \Gamma_{13}^{(1)},
\]

\[
r_2[1] = i \Gamma_{14}^{(1)}.
\]
For one-soliton solution, we have

\[
\Omega = \Omega_1 = \begin{pmatrix}
\omega^{(1)}_{1,1} & \omega^{(1)}_{1,2} & \omega^{(2)}_{1,3} & \omega^{(2)}_{1,4} \\
\omega^{(2)}_{2,1} & \omega^{(2)}_{2,2} & \omega^{(2)}_{2,3} & \omega^{(2)}_{2,4} \\
\omega^{(3)}_{3,1} & \omega^{(3)}_{3,2} & \omega^{(3)}_{3,3} & \omega^{(3)}_{3,4} \\
\omega^{(4)}_{4,1} & \omega^{(4)}_{4,2} & \omega^{(4)}_{4,3} & \omega^{(4)}_{4,4}
\end{pmatrix}
\]

\[
\Lambda^{-1}_1 = \begin{pmatrix}
\lambda^{-1}_1 & 0 & 0 & 0 \\
0 & \lambda^{-1}_1 & 0 & 0 \\
0 & 0 & \lambda^{-1}_2 & 0 \\
0 & 0 & 0 & \lambda^{-1}_2
\end{pmatrix}, \quad E^{(1)} = I_4 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\Omega^{(1)}_4 = \Omega_1 \Lambda^{-1}_1 = \begin{pmatrix}
\lambda^{-1}_1 \omega^{(1)}_{1,1} & \lambda^{-1}_1 \omega^{(1)}_{1,2} & \lambda^{-1}_1 \omega^{(2)}_{1,3} & \lambda^{-1}_1 \omega^{(2)}_{1,4} \\
\lambda^{-1}_1 \omega^{(2)}_{2,1} & \lambda^{-1}_1 \omega^{(2)}_{2,2} & \lambda^{-1}_1 \omega^{(2)}_{2,3} & \lambda^{-1}_1 \omega^{(2)}_{2,4} \\
\lambda^{-1}_1 \omega^{(3)}_{3,1} & \lambda^{-1}_1 \omega^{(3)}_{3,2} & \lambda^{-1}_1 \omega^{(3)}_{3,3} & \lambda^{-1}_1 \omega^{(3)}_{3,4} \\
\lambda^{-1}_1 \omega^{(4)}_{4,1} & \lambda^{-1}_1 \omega^{(4)}_{4,2} & \lambda^{-1}_1 \omega^{(4)}_{4,3} & \lambda^{-1}_1 \omega^{(4)}_{4,4}
\end{pmatrix}
\]

The matrix element \( \Gamma^{(1)}_{11} \) is given by

\[
\Gamma^{(1)}_{11} = \begin{pmatrix}
\omega_{1,1}^{(1)} & \omega_{1,2}^{(1)} & \omega_{1,3}^{(2)} & \omega_{1,4}^{(2)} & 1 \\
\omega_{1,2}^{(2)} & \omega_{2,2}^{(1)} & \omega_{2,3}^{(2)} & \omega_{2,4}^{(2)} & 0 \\
\omega_{1,3}^{(3)} & \omega_{1,2}^{(3)} & \omega_{3,3}^{(3)} & \omega_{3,4}^{(3)} & 0 \\
\omega_{1,4}^{(4)} & \omega_{1,2}^{(4)} & \omega_{4,3}^{(4)} & \omega_{4,4}^{(4)} & 0 \\
\lambda^{-1}_1 \omega_{1,1}^{(1)} & \lambda^{-1}_1 \omega_{1,2}^{(1)} & \lambda^{-1}_2 \omega_{1,3}^{(2)} & \lambda^{-1}_2 \omega_{1,4}^{(2)} & 0
\end{pmatrix}
\]

Now substitute \( \Gamma^{(1)}_{11} \) into (3.3), we obtain

\[
q[1] = 1 + i
\]

Similarly, one can easily compute following expressions

\[
r_1[1] = i
\]

\[
r_2[1] = i
\]

Equations (3.6)–(3.8) represent single-soliton solution for NC-CCID (1.1).
In commutative limit, equations (3.6)–(3.8) reduce into ratios of ordinary determinants, i.e,

\[ q[1] = 1 + i\theta \left( \frac{\Delta_1}{\Delta} \right), \quad n[1] = i \left( \frac{\Delta_2}{\Delta} \right), \quad r_1[1] = -i \left( \frac{\Delta_3}{\Delta} \right), \]  

(3.9)

where

\[ \Delta_1 = \begin{vmatrix} \lambda_1^{-1} \omega_{1,1}^{(1)} & \lambda_2^{-1} \omega_{1,2}^{(1)} & \ldots & \lambda_4^{-1} \omega_{4,4}^{(1)} \\ \omega_{2,1}^{(1)} & \omega_{2,2}^{(2)} & \omega_{2,3}^{(2)} & \omega_{2,4}^{(2)} \\ \omega_{3,1}^{(1)} & \omega_{3,2}^{(2)} & \omega_{3,3}^{(2)} & \omega_{3,4}^{(2)} \\ \omega_{4,1}^{(1)} & \omega_{4,2}^{(2)} & \omega_{4,3}^{(2)} & \omega_{4,4}^{(2)} \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} \omega_{1,1}^{(1)} & \omega_{1,2}^{(1)} & \omega_{1,3}^{(2)} & \omega_{1,4}^{(2)} \\ \omega_{2,1}^{(1)} & \omega_{2,2}^{(2)} & \omega_{2,3}^{(2)} & \omega_{2,4}^{(2)} \\ \omega_{3,1}^{(1)} & \omega_{3,2}^{(2)} & \omega_{3,3}^{(2)} & \omega_{3,4}^{(2)} \\ \omega_{4,1}^{(1)} & \omega_{4,2}^{(2)} & \omega_{4,3}^{(2)} & \omega_{4,4}^{(2)} \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} \omega_{1,1}^{(1)} & \omega_{1,2}^{(1)} & \omega_{1,3}^{(2)} & \omega_{1,4}^{(2)} \\ \omega_{2,1}^{(1)} & \omega_{2,2}^{(2)} & \omega_{2,3}^{(2)} & \omega_{2,4}^{(2)} \\ \omega_{3,1}^{(1)} & \omega_{3,2}^{(2)} & \omega_{3,3}^{(2)} & \omega_{3,4}^{(2)} \\ \omega_{4,1}^{(1)} & \omega_{4,2}^{(2)} & \omega_{4,3}^{(2)} & \omega_{4,4}^{(2)} \end{vmatrix}, \quad \Delta = \begin{vmatrix} \omega_{1,1}^{(1)} & \omega_{1,2}^{(1)} & \omega_{1,3}^{(2)} & \omega_{1,4}^{(2)} \\ \omega_{2,1}^{(1)} & \omega_{2,2}^{(2)} & \omega_{2,3}^{(2)} & \omega_{2,4}^{(2)} \\ \omega_{3,1}^{(1)} & \omega_{3,2}^{(2)} & \omega_{3,3}^{(2)} & \omega_{3,4}^{(2)} \\ \omega_{4,1}^{(1)} & \omega_{4,2}^{(2)} & \omega_{4,3}^{(2)} & \omega_{4,4}^{(2)} \end{vmatrix}. \]

In what follows next, we would like to compute an explicit expression of single-soliton solutions in terms of elementary function. Let us take

\[ |\alpha_3\rangle = (1 \ 0 \ a \ -b)^T, \quad |\alpha_4\rangle = (0 \ 1 \ b \ a)^T, \quad |\alpha_3\rangle = (a \ b \ -1 \ 0)^T, \quad |\alpha_4\rangle = (b \ -a \ 0 \ 1)^T \]

be constant column vectors and \( a, b \in \mathbb{R} \). \( \Omega \) becomes

\[ \Omega = \begin{pmatrix} e^{\eta_1} & 0 & ae^{\eta_1} & be^{\eta_2} \\ 0 & e^{\eta_1} & be^{\eta_2} & -ae^{\eta_1} \\ ae^{-\eta_1} & be^{-\eta_1} & -e^{-\eta_2} & 0 \\ -be^{-\eta_1} & ac^{-\eta_1} & 0 & e^{-\eta_2} \end{pmatrix}. \]
One can easily calculate following elements

\[
\Gamma_{11}^{(1)} \equiv \frac{\Delta_i}{\Delta} = \left\{ \begin{array}{l}
- a^3 e^{\eta_i} \lambda_i - 2 a^2 b^2 e^{\eta_i} \lambda_i + b^2 e^{\eta_i} \lambda_i - a^2 e^{2\eta_i + 2\eta_0} \lambda_i \\
- b^2 e^{2\eta_i - 2\eta_0} \lambda_i - e^{\eta_i} \lambda_i - a^2 e^{2\eta_i + 2\eta_0} \lambda_i - b^2 e^{2\eta_i + 2\eta_0} \lambda_i \\
\end{array} \right.
\]

\[
\Gamma_{13}^{(1)} \equiv \frac{\Delta_i}{\Delta} = \left\{ \begin{array}{l}
- a^3 e^{\eta_i} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i + ab^2 e^{\eta_i + 3\eta_0} \lambda_i \\
- ab^2 e^{\eta_i + 3\eta_0} \lambda_i - ab^2 e^{\eta_i + 3\eta_0} \lambda_i - b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
\end{array} \right.
\]

\[
\Gamma_{14}^{(1)} \equiv \frac{\Delta_i}{\Delta} = \left\{ \begin{array}{l}
- b^2 e^{\eta_i + 3\eta_0} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i - b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
- b^2 e^{\eta_i + 3\eta_0} \lambda_i - b^2 e^{\eta_i + 3\eta_0} \lambda_i + b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
\end{array} \right.
\]

\[
\Gamma_{24}^{(1)} \equiv \frac{\Delta_i}{\Delta} = \left\{ \begin{array}{l}
- a^3 e^{\eta_i} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i - ab^2 e^{\eta_i + 3\eta_0} \lambda_i \\
- a^2 e^{\eta_i + 3\eta_0} \lambda_i - ab^2 e^{\eta_i + 3\eta_0} \lambda_i + b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
\end{array} \right.
\]

The reduction conditions (2.23) allow us to take \( \lambda_i^* = -\lambda_i \), implies \( \eta_i^* = \eta_i \) (for \( i = 1, 2 \)). Let us verify the first condition in fourth row of (2.23), i.e. \( \Gamma_{13}^{(1)} = - \Gamma_{24}^{(1)} \), take complex conjugate

\[
\Gamma_{24}^{(1)} = \frac{\Delta_i}{\Delta} \lambda_i^* \lambda_i^* (e^{\eta_i} - a^3 e^{\eta_i} - a^2 b^2 e^{\eta_i} - b^2 e^{\eta_i} - 2 a^2 e^{2\eta_i + 2\eta_0} - 2 b^2 e^{2\eta_i + 2\eta_0}),
\]

Similarly, one can easily verify other conditions given by expression (2.23). Using \( \Gamma_{11}^{(1)} \), \( \Gamma_{13}^{(1)} \) and \( \Gamma_{14}^{(1)} \) from (3.10) equation (3.9) become

\[
q[1] = 1 + i \delta_i \left\{ \begin{array}{l}
- a^3 e^{\eta_i} \lambda_i - 2 a^2 b^2 e^{\eta_i} \lambda_i + b^2 e^{\eta_i} \lambda_i - a^2 e^{2\eta_i + 2\eta_0} \lambda_i \\
- b^2 e^{2\eta_i - 2\eta_0} \lambda_i - e^{\eta_i} \lambda_i - a^2 e^{2\eta_i + 2\eta_0} \lambda_i - b^2 e^{2\eta_i + 2\eta_0} \lambda_i \\
\end{array} \right.
\]

\[
\eta[1] = i \frac{\lambda_i}{\lambda_i^*} \left\{ \begin{array}{l}
- a^3 e^{\eta_i} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i + ab^2 e^{\eta_i + 3\eta_0} \lambda_i \\
- ab^2 e^{\eta_i + 3\eta_0} \lambda_i - ab^2 e^{\eta_i + 3\eta_0} \lambda_i - b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
\end{array} \right.
\]

\[
r_2[1] = - i \frac{\lambda_i}{\lambda_i^*} \left\{ \begin{array}{l}
- b^2 e^{\eta_i + 3\eta_0} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i - b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
+ b^2 e^{\eta_i + 3\eta_0} \lambda_i - a^2 e^{\eta_i + 3\eta_0} \lambda_i + b^2 e^{\eta_i + 3\eta_0} \lambda_i \\
\end{array} \right.
\]

represents single-soliton solution for commutative CCID system (1.4). The different profiles of dark and bright soliton solutions (3.11)–(3.13) have been plotted in figure 1.

Our results are consistent with the results obtained in [18].

3.2. Double-soliton solution

For \( M = 2 \) equation (2.22) gives two-fold transformation on scalar potential \( q, r_1 \) and \( r_2 \)

\[
q[2] = 1 + i \delta_i \Gamma_{11}^{(2)}
\]

\[
\eta[2] = i \Gamma_{13}^{(2)}
\]

\[
r_2[2] = i \Gamma_{14}^{(2)}
\]
In this case, we have

\[
\Omega = \begin{pmatrix}
\omega^{(1)}_{1,1} & \omega^{(1)}_{1,2} & \omega^{(1)}_{1,3} & \omega^{(1)}_{1,4} & \omega^{(2)}_{1,1} & \omega^{(2)}_{1,2} & \omega^{(2)}_{1,3} & \omega^{(2)}_{1,4} & \omega^{(3)}_{1,1} & \omega^{(3)}_{1,2} & \omega^{(3)}_{1,3} & \omega^{(3)}_{1,4} & \omega^{(4)}_{1,1} & \omega^{(4)}_{1,2} & \omega^{(4)}_{1,3} & \omega^{(4)}_{1,4}
\end{pmatrix}
\]

\[
\Lambda_{m}^{-1} = \begin{pmatrix}
\lambda_{2m-1}^{-1} & 0 & 0 & 0 \\
0 & \lambda_{2m-1}^{-1} & 0 & 0 \\
0 & 0 & \lambda_{2m-1}^{-1} & 0 \\
0 & 0 & 0 & \lambda_{2m-1}^{-1}
\end{pmatrix}
\]

\[
\Omega^{(2)} = \begin{pmatrix}
\lambda_{1}^{-2}\omega^{(1)}_{1,1} & \lambda_{1}^{-2}\omega^{(1)}_{1,2} & \lambda_{1}^{-2}\omega^{(1)}_{1,3} & \lambda_{1}^{-2}\omega^{(1)}_{1,4} & \lambda_{2}^{-1}\omega^{(2)}_{1,1} & \lambda_{2}^{-1}\omega^{(2)}_{1,2} & \lambda_{2}^{-1}\omega^{(2)}_{1,3} & \lambda_{2}^{-1}\omega^{(2)}_{1,4} & \lambda_{3}^{-1}\omega^{(3)}_{1,1} & \lambda_{3}^{-1}\omega^{(3)}_{1,2} & \lambda_{3}^{-1}\omega^{(3)}_{1,3} & \lambda_{3}^{-1}\omega^{(3)}_{1,4} & \lambda_{4}^{-1}\omega^{(4)}_{1,1} & \lambda_{4}^{-1}\omega^{(4)}_{1,2} & \lambda_{4}^{-1}\omega^{(4)}_{1,3} & \lambda_{4}^{-1}\omega^{(4)}_{1,4}
\end{pmatrix}
\]

Equation (3.14) becomes

\[
q[2] = 1 + i
\]

\[
\eta[2] = i
\]
The above expressions represent interaction of two-soliton for NC-CCID system (1.1). In commutative limit, we obtain

\[
q[2] = 1 + i\lambda_2 \left( \frac{\Delta_x}{\Delta_y} \right), \quad \eta_1[2] = i \left( \frac{\Delta_y}{\Delta_x} \right), \quad \eta_2[2] = -i \left( \frac{\Delta_x}{\Delta_y} \right)
\]

(3.15)
where

\[
\begin{align*}
\Delta_6 &= \begin{bmatrix}
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\end{bmatrix},
\end{align*}
\]

\[
\Delta_7 &= \begin{bmatrix}
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\end{bmatrix},
\end{align*}
\]

\[
\Delta_8 &= \begin{bmatrix}
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\end{bmatrix},
\end{align*}
\]

\[
\Delta_9 &= \begin{bmatrix}
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\omega_{1,1} & \omega_{1,2} & \omega_{2,1} & \omega_{2,2} & \omega_{3,1} & \omega_{3,2} & \omega_{4,1} & \omega_{4,2} \\
\end{bmatrix},
\end{align*}
\]

Figures 2–3 shows different profiles of interactions of double-soliton solutions (3.15) for $\lambda_{2k} = \lambda_{2k-1}^*$. The results obtained are in agreement with the results obtained in [18, 19].

### 4. Concluding Remarks

In this article, we have presented a non–commutative extension of complex coupled integrable dispersionless (NC-CCID) system which reduces into usual two-component complex coupled integrable dispersionless (CCID) system in commutative limit. The Darboux transformation has been applied to find a solution for the linear eigenvalue problem and we obtain multi-soliton solutions in terms of quasideterminants for NC-CCID system. We computed single and double-soliton solutions for NC-CCID system (1.1). The soliton solution, further reduces into ratio of ordinary determinants for the usual commutative CCID system in commutative limit. The exact solution obtained in this paper may be helpful to understand various interesting physical
phenomena of dispersionless slow light propagations in periodic medium. It would be fascinating to compute soliton solutions of other non–commutative and supersymmetric integrable systems by using matrix Darboux transformation.

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Appendix

Let us consider a square matrix $D$ with non–commuting entries, $D = [d_{kl}]$ where $k \in K$ and $l \in L$. The quasideterminant $|D|_{pq}$ is defined as [29–31]

$$|D|_{pq} = d_{pq} - r^p(D)_{pq}^{-1}d_{pq}^q,$$  \hspace{1cm} (A.1)

where $D_{pq}$ denotes a submatrix that could be obtained by deleting $p$–th row and $q$–th row and column of matrix $D$ respectively. Here $r^p$ and $d_{pq}^q$ represents $p$–th row and $q$–th row and column vectors respectively of $D$ obtained by deleting $d_{pq}$ entry. In case if the entries of $D$-matrix are commuting then

$$|D|_{pq} = (-1)^{p+q} \frac{\det D}{\det D_{pq}}.$$  \hspace{1cm} (A.2)

Non–commutative Jacobi identity for quasideterminants is

$$\begin{vmatrix} R & V & A \\ S & P & X \\ Q & U & \Box \end{vmatrix} = \begin{vmatrix} R & A \\ Q & \Box \end{vmatrix} - \begin{vmatrix} R & V \\ Q & S \end{vmatrix} \begin{vmatrix} R & V \\ S & P \end{vmatrix}^{-1} \begin{vmatrix} R & A \\ S & X \end{vmatrix}.$$  \hspace{1cm} (A.3)

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