Numerical Simulation of a Thermoelectric Generator for a Road Vehicle Application

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Abstract. This paper presents a numerical model for a thermoelectric generator (TEG) on the tailpipe of commercial (3.5-ton) and heavy duty (40-ton) vehicles. The widths and lengths of the TEG for the two vehicles are 0.10 m × 0.15 m and 0.2 m × 0.15 m respectively. Exhaust gas is used as the heat source and cooling water is the heat sink. Both fluids flow in the same direction. The study examines the improvement in heat transfer from the exhaust gas side. A numerical model is developed that provides a choice between two different heat exchanger configurations with either plain fins or offset strip fins. The influence of fin height, length and spacing is analysed. According to the criteria used in this study, plain fins are a better choice, yielding a maximum electrical power of 107 W for the commercial vehicle and 480 W for the heavy duty vehicle.

1. Introduction

In recent years, the thermoelectric generator (TEG) has emerged as a promising technology for waste heat recovery in the automotive industry due to improvements in thermoelectric materials [1]. The underlying physical phenomenon on which the TEG is based consists of what is known as the Seebeck effect, discovered by Thomson J. Seebeck in 1821. This effect states that a circuit made from two dissimilar materials with their junctions at different temperatures produces a difference in electrical potential. Fitting a TEG in the tailpipe of a road vehicle allows waste heat to be reused and reducing fossil fuel consumption as a result. If we compare the TEG to other technologies, such as the organic Rankine cycle, we notice that the TEG does not have any moving parts. This makes it simpler and less costly to operate and maintain the whole system. Thanks to these qualities, using TEGs offer great potential for future automotive applications. Further investigations are needed in this area.

2. State of the art

Energy transfer from the exhaust gas to the thermocouples represents a major barrier in the overall performance of the TEG. In order to find a solution for this problem, several studies were conducted on this topic with the purpose of finding an internal structure for heat exchangers that best meets the overall requirements for use in a TEG.

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Lu et al. [2] investigated two types of heat transfer enhancement structures, namely rectangular offset strip fins and metal foams for the exhaust gas heat exchanger in a light-duty gasoline vehicle. They used the net power as main comparative parameter. It was concluded that the electrical power from metal foams was significantly higher (c.a. 130-294 W) than that from offset strip fins. However, metal foams caused such a high pressure drop that not only the engine efficiency did not improve, but also energy consumption was greater due to the increase in the required pumping power. The solution that the authors suggested was to develop a device that can be used as both a TEG and a three-way catalytic converter at the same time. Bai et al. [3] used a computational fluid dynamics (CFD) code to investigate the heat transfer and pressure drop for six different internal structures for heat exchangers in a light-duty 1.2 L gasoline engine vehicle. They showed that the serial plate structure presents the most effective heat transfer. However, it also exhibits the highest pressure drop. In contrast, the heat exchanger featuring an empty cavity presented both the lowest heat transfer and the lowest pressure drop. They concluded that there needs to be a compromise between a high heat transfer and a low pressure. The maximum electrical and net powers were not determined. Su et al. [4] compared three internal heat exchanger structures for an automotive vehicle with a four-cylinder engine: fishbone-shaped, accordion-shaped and scatter-shaped. The study demonstrated that the accordion-shape design presents better uniform temperature distribution. Again, the maximum electrical and net powers were not presented. Likewise, Liu et al. [5] focused on the temperature distribution in a heat exchanger mounted on the exhaust of a 2.0 L naturally aspirated gasoline engine. They compared two internal heat exchanger geometries – fishbone-shaped and chaos-shaped – and concluded that the chaos-shaped structure produces better results (maximum electrical power c.a. 183 W).

From the literature review, it is concluded that numerical models have been mostly developed on a component level without conducting deeper, comprehensive analysis of the system and its interactions. Consequently, this paper seeks to fill this gap and present a numerical model for a TEG which compares the direct influence of two different heat exchanger structures, with either plain fins or offset strip fins, on the resulting electrical and net power output. It analyses and discusses the influence of the height, spacing and length of fins. The selected case study examines two diesel vehicles, a 3.5-ton commercial vehicle and a 40-tons heavy duty vehicle, chosen as representative vehicles carriage of goods. Their higher mass flow rates and available energy in the exhaust gas, larger installation space and considerably longer constant driving times compared to passenger cars provide a promising way of introducing TEGs in the automotive sector.

3. Model description

3.1. Thermoelectric generator configuration

A thermocouple is the core component in a TEG. It is composed of a pair of p-type and n-type semiconductor legs, which are connected at least at one point, as shown in Figure 1. Many p-doped and n-doped semiconductor legs are electrically connected in series and thermally connected in parallel to form a thermoelectric module (see Figure 2). Electrical conductor strips are used to connect the multiple legs. Ceramic strips ensure that thermoelectric modules are electrically insulated and structurally supported. Several thermoelectric modules are placed in parallel, forming an array of \( N \times M \) thermocouples. This structure is sandwiched between a heat source and a heat sink. Additional layers of thermoelectric modules may be placed along the y-axis, separated from adjacent ones by a heat source or a heat sink, forming a device known as a TEG. The TEG used in this study is symmetric with respect to its height along the y-axis, which contains 6 layers of thermoelectric modules in that direction. Exhaust gas is used as heat source and the cooling water as heat sink. It is assumed that both fluids have uniform inlet temperature and velocity distributions, and the temperature is assumed to remain uniform along the z-axis.
3.2. Thermal and electrical behaviour of a thermocouple

The thermal analysis presupposes that the TEG is working in steady state, the heat losses due to radiation are negligible, the gaps between the device’s \( n \) and \( p \) legs are optimally insulated and the heat conduction and current flow in the thermocouple are considered one-dimensional. The respective total heat transfer rates through the hot and cold junctions of a thermocouple are calculated using these formulas [6]:

\[
q_h = \alpha_{pn} T_{3,h} I - 0.5 R I^2 + K (T_{3,h} - T_{3,c})
\]

\[
q_c = \alpha_{pn} T_{3,c} I + 0.5 R I^2 + K (T_{3,h} - T_{3,c})
\]

whereby \( \alpha_{pn} \) is the difference between the Seebeck coefficients of the two thermocouple legs, \( T_{3,h} \) and \( T_{3,c} \) are the temperatures at the hot and cold junctions and \( I \) is the electric current in the circuit. The internal (electrical) resistance \( R \) and the thermal conductance in the thermocouples \( K \) are obtained as follows:

\[
R = \frac{\rho_p L_p}{A_p} + \frac{\rho_n L_n}{A_n}
\]

\[
K = \frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}
\]

whereby \( \rho \) represents the electrical resistivity, \( k \) the thermal conductivity, \( A \) the area in the cross-section of a thermocouple leg, and \( L \) its height. Subscripts \( p \) and \( n \) identify the p-type and n-type materials, respectively. The properties of the materials, which depend on the temperature, were evaluated based on the mean temperature at the thermocouple junctions.

The temperature difference across the legs of the thermocouple, \( \Delta T = T_{3,h} - T_{3,c} \), induces a voltage drop due to the Seebeck effect. Moreover, it is known that doped semiconductors have an associated internal resistance. Consequently, each thermocouple can be treated as a temperature-controlled voltage source with internal resistance. The current flow in each thermoelectric module, through which maximum power is obtained, is hereby taken as constant and written as [6]:

\[
I = \alpha_{pn} \Delta T / (R + R_L)
\]
In this equation, \( R_L = R \) represents the external load resistance supported by each thermocouple which is required to obtain the maximum electrical output power \( P_{el} \), which is equal to
\[
P_{el} = R_L I^2.
\]

3.3. Thermal and electrical behaviour in the TEG

The exhaust gas and cooling water ducts are discretized along the flow direction, i.e., the \( x \)-axis, considering a number of control volumes (CVs) equal to the number of thermoelectric modules in that direction, denoted as \( N \) in Figure 2. The length of the CVs along the \( x \)-axis, \( \Delta x \), is equal to the ratio of the length of the TEG to \( N \). This level of discretization is sufficiently accurate because the difference between the temperatures upstream and downstream of a CV is small. Let \( j \) and \( j+1 \) denote the temperatures at the cell faces of the \( j \)th CV, i.e., upstream and downstream of the \( j \)th thermoelectric module, respectively. Assuming that the temperature of the surfaces on the hot and cold fluid ducts, denoted by \( T_{b,h} \) and \( T_{b,c} \), respectively, are uniform within a control volume, the energy balances for the \( j \)th control volume may be written as follows:
\[
\dot{m}_h \left( c_p h \left( T_{h,j} - T_{h,j+1} \right) \right) = h_f A_f \left( T_{h,j} - T_{h,h,j} \right)
\]
\[
\dot{m}_c \left( c_p c \left( T_{c,j+1} - T_{c,j} \right) \right) = h_c A_c \left( T_{h,c,j} - T_{c,j} \right)
\]
whereby \( \dot{m} \) is the mass flow rate, \( c_p \) the specific heat capacity, \( h \) the convective heat transfer coefficient, and \( A \) the heat transfer area. Subscripts \( h \) and \( c \) denote the hot and cold fluids, respectively.

A compact plate fin heat exchanger is used to improve energy transfer from the exhaust gas to the hot surface on the thermoelectric modules. Two different configurations are investigated in this study: one with plain fins and the other one with offset strip fins, as shown in Figure 3. Symbol \( \eta_f \) in equation (7) stands for the efficiency of the fins. In the case of plain fins, the Nusselt number was evaluated using either the correlation suggested by Martin [7], in the case of laminar flow, or the Gnielinski correlation, in the case of turbulent flow. The correlation developed by Manglik and Bergles [8] was used for offset strip fins. The Nusselt number for the cooling water was again calculated using the Gnielinski correlation since the flow is turbulent.

The heat transfer along the \( y \)-axis was determined using a thermal resistance network, as shown in Figure 1 while assuming one-dimensional steady-state conduction, omitting all thermal contact resistances, omitting heat losses due to radiation and taking into account that the gaps between the legs are perfectly insulated. The heat transfer rates through the hot and cold junctions may be set equal to the heat transfer rates through the hot and cold electrical conductor strips:
\[
\left( A_n + A_p \right) k_{Cu} \left( T_{2,h} - T_{3,h} \right) / L_{Cu} = \alpha_{pn} T_{3,h} I - 0.5 R I^2 + k \left( T_{3,h} - T_{3,c} \right)
\]
whereby $k_{Cu}$ and $L_{Cu}$ denote the thermal conductivity and the thickness of the electrical conductor strip made of copper (Cu). Energy balance equations may be written for every inner node of the thermal resistance network (nodes $T_{b,h}$, $T_{1,h}$, $T_{2,h}$, $T_{2,c}$, $T_{1,c}$ and $T_{b,c}$ in Figure 1), yielding 6 equations, which along with equations (1), (2), (9) and (10) allow the calculation of the temperatures of the hot and cold fluids downstream of the $j$th module, and the eight temperatures shown in the resistance network in Figure 1. This system of 10 equations is non-linear since the thermo-physical properties of solid materials and fluids depend on the temperature. The solution of this system was carried out using Newton’s method. The CVs are treated sequentially along x-axis, setting the temperature of the fluids leaving the $j$th CV equal to the inlet temperatures of the fluids entering the ($j+1$)th CV. The electrical power for a thermocouple is calculated using equation (6) using the current intensity and the load resistance computed using equations (3) and (5), respectively. Since a thermoelectric module consists of $M$ thermocouples electrically connected in series, their electrical resistance can be added to obtain the electrical power output of a thermoelectric module. The TEG’s overall electrical output power is determined by adding the electrical power from all thermoelectric modules together.

3.4. Pressure drop

The pumping power $P_{pump}$, required to overcome the pressure drop $\Delta p$, in the heat exchanger can be taken into account to assess the performance of the TEG, and is evaluated as follows for the $j$th thermoelectric module:

$$P_{pump,j} = \dot{V}_j \Delta p_j = \dot{m}_h \varphi \frac{u^2 \Delta k}{2 D_h}$$

whereby $\dot{m}$ and $\dot{V}$ represent the mass and volumetric flow rates across the thermoelectric module, respectively, $u$ is the mean velocity, $D_h$ the hydraulic diameter, $f$ the Darcy friction factor, and $\varphi$ a correction factor for tubes with a rectangular cross-section, which depends on the ratio of the side lengths. The pumping power for the TEG is equal to the sum of the pumping powers for all the thermoelectric modules. Finally, the net output power is the difference between the electrical and the pumping power.

4. Case study

In this paper, 3.5-ton and 40-ton vehicles have been selected as the target study. A constant speed event on a flat road was chosen for these vehicles: at 120 km/h and 90 km/h, respectively. The vehicles were simulated using ADVISOR [9] to obtain the required input parameters for the TEG model described above. The input data for ADVISOR are given in Table 1. The exhaust gas system used in the simulation includes a diesel oxidation catalyst (DOC) and a diesel particle filter (DPF). The output from this simulation is summarized in Table 2. The temperatures presented in this table were obtained immediately after the DPF, where the TEG is positioned. Consequently, the TEG’s performance will be evaluated below based on the exhaust gas temperature shown in Table 2.

| Vehicle (tons) | Frontal area (m²) | Engine displacement (L) | Power (kW) | Torque (Nm) | Transmission |
|---------------|-------------------|-------------------------|------------|-------------|--------------|
| 3.5           | 3.288             | 2.14                    | 95 at 3800 rpm | 305 at 1200-2400 rpm | Manual 5-speed |
| 40            | 8.348             | 15.6                    | 380 at 1600 rpm | 2600 at 1100 rpm    | Manual 5-speed |
Table 2. Output results from ADVISOR for the selected target study.

| Vehicle (tons) | Exhaust gas mass flow rate (g/s) | Exhaust gas temperature at DPF exhaust (K) | Coolant fluid temperature (K) | Engine brake power (kW) | Fuel consumption (L/100km) |
|---------------|---------------------------------|------------------------------------------|-----------------------------|------------------------|---------------------------|
| 3.5           | 80.12                           | 568.93                                   | 368.15                      | 26                     | 7.8                       |
| 40            | 201.48                          | 710.86                                   | 368.15                      | 127                    | 39                        |

Table 3. Thermoelectric materials.

| Vehicle (tons) | n-type                  | p-type                  |
|---------------|-------------------------|-------------------------|
| 3.5           | Bi$_{2}$Se$_{0.3}$Te$_{2.7}$ [10] | Bi$_{0.5}$Sb$_{1.5}$Te$_{3}$ [11] |
| 40            | (Bi$_{0.001}$Se$_{0.999}$Te)$_{0.88}$(PbS)$_{0.12}$ [12] | AgSbTe$_{2}$ [13] |

The selection of thermoelectric materials is a crucial step in ensuring efficiency in a thermoelectric device used to generate electricity. These materials must be selected based on the working temperature range. The thermoelectric materials used in this study are listed in Table 3. They were selected based on the data given in Table 2. It should be noted that these materials are still under development and are only available in laboratory scale. However, it is expected that materials with such properties or even better ones will become available for commercial applications in the near future.

5. Results
The numerical model described in section 3 was applied to both vehicles presented in section 4 to study the influence of internal geometry on the heat exchanger, containing plain fins and offset strip fins. The external geometry is fixed: 3.5-ton vehicle – width = 0.10 m and length = 0.15 m; 40-ton vehicle – width = 0.20 m and length = 0.15 m. The electrical and net power outputs are calculated for all studied cases.

The influence of the plain fins on the TEG performance is shown for different combinations of fin height and fin spacing in Figures 4 and 5. It can be seen that both vehicles behave in a similar way. Small fin spacing increases the electrical power nearly up to its maximum level while a lower height h produces greater electrical power. Moreover, there is a spacing value that maximizes the net output power for each fin height. When fin spacing is reduced below this value, the pumping power starts to increase more rapidly than the electrical power, resulting in a reduction in the net power. When fin spacing is increased above its optimum value, the net power also starts to diminish due to the decrease in electrical power. Figures 4 and 5 further show that the fin spacing corresponding to the maximum net power decreases when a higher fin height is used. Given these findings, the following criteria are proposed to select heat exchanger geometry: the one selected is a combination (fin height, fin spacing) between the lowest height (to guarantee compactness) and a pumping power lower than 30% of the electrical power. The following geometry is selected based on these criteria:

3.5-ton vehicle – fin spacing = 5 mm; fin height = 15 mm,
40-ton vehicle – fin spacing = 5 mm; fin height = 20 mm.

The influence of the offset strip fins on the electrical and net power is shown for both vehicles in Figures 6 and 7. The height of the TEG is influenced by the height of the fins. Consequently, the height of the plain fins previously selected is fixed and is also used for the offset strip fins to allow a fair comparison between plain fins and offset strip fins. The influences of two parameters fin length and fin spacing is analysed in Figures 6 and 7. Note that the dimensions of the fins are maintained within the validity range for the correlations used to estimate the Nusselt number [8]. If we look at the new geometrical variable, i.e. fin length, it can be seen that a lower offset strip fin length not only yields higher electrical output power, but also reduces the net power due to the higher pumping power associated with this geometry. In some cases, this internal heat exchanger geometry leads to a net power that is almost 2/3 of the electrical power. In conclusion, the shorter the offset strip fins are, the
better the heat transfer enhancement is, although a higher pumping power is required. The following criteria are suggested to select the most suitable offset strip fin geometry: the combination (fin spacing, fin length) with the highest electrical output power and a pumping power lower than 30% of the electrical output power is selected. The following geometry is selected based on these criteria:

- **3.5-ton vehicle** – fin spacing = 7.5 mm; fin length = 83 mm,
- **40-ton vehicle** – fin spacing = 10 mm; fin height = 50 mm.
Table 4. Vehicle’s electrical and net output powers.

| Vehicle (tons) | Plain fins          | Offset strip fins |
|---------------|---------------------|-------------------|
|               | Electrical Power (W)| Net Power (W)     | Electrical Power (W)| Net Power (W) |
| 3.5           | 107                 | 78                | 99                 | 73            |
| 40            | 480                 | 357               | 430                | 320           |

The results presented above reveal that the design of both internal heat exchanger structures must offer a compromise to ensure that both high electrical and high net powers are achieved. The electrical and net output powers for the geometries selected above are presented in Table 4. It can be seen that plain fins present a higher electrical and net power.

6. Conclusion
This study developed a numerical model for a TEG to be used in the exhaust gas system in road vehicles. The numerical model presents a high degree of freedom, making it adaptable for both different structural parameters and different boundary conditions. Thanks to its versatility, this model can serve as an excellent tool to evaluate the output performance of the TEG in early design processes. The first part of this research focused on the internal structure of the heat exchanger. Plain fins were compared to offset strip fins. It was concluded that plain fins are a better choice for this application. Maximum electrical output powers of 107 W and 480 W were obtained for 3.5-ton and 40-ton vehicles, respectively. In future studies, the influence of the external geometry of the heat exchanger, and consequently, the external dimensions of the TEG, will be analysed. A more detailed study will also be performed on thermocouple heights. Finally, an exergy analysis needs to be performed to determine the TEG’s recovery efficiency.

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