Investigation on numerical optimization method for high capacity two-stage 4 K pulse tube cryocooler

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Abstract. Two-stage 4 K pulse tube cryocooler is one of the most promising coolers for low temperature applications. However, an efficient and reliable method is still absent for pulse tube optimization in liquid helium temperatures due to complex heat and mass flow oscillation. In this study, a two-stage pulse tube cryocooler model was built with a commercially-available software, SAGE®. The DC flow effect on the performance was investigated and the filling ratio of second stage regenerator were numerically optimized. With the model, a no-load temperature of 2.7 K and a cooling capacity of 0.5 W at 4.2 K were achieved, which is well-matched with the experimental results.

1. Introduction
Owing to its low vibration and high reliability, pulse tube cryocooler has been one of the most promising cryocoolers for low temperature applications since it achieved a temperature below 4 K [1].

Although lots of great improvements in regenerative materials and configurations have been achieved for a 4 K pulse tube cryocooler [2, 3, 4], an efficient and reliable simulation method is still absent for its design and optimization. In previous works, the coupling effect between the first and second stages was neglected and each stage was simulated independently [5]. Also, the mass flow rate and pressure were simplified as sinusoidal functions and the DC flow effect was neglected [6].

In this study, a two-stage pulse tube cryocooler model was built with SAGE. The DC flow effect on the performance was investigated and the effect of the regenerator filling ratio on the cryocooler performance was numerically optimized.

2. Numerical model
SAGE is a software developed by David Gedeon. It is a one-dimensional frequency domain modeller aimed for oscillating thermodynamic systems. In spite of its limitations for radial flow, SAGE is a fast solver and provides sufficiently accurate results for optimization.

The two-stage 4 K pulse tube cryocooler model was established with SAGE as shown in figure 1. It is based on a 4 K pulse tube cryocooler, RP-062B, which was developed by Sumitomo Heavy Industries, Ltd.
Figure 1. RP-062B model in SAGE.

The valve unit of RP-062B consists of a fixed stem and a rotating disk. The supply or return channel opens as the holes of the disk are overlapped with the holes of the stem. The valves open twice a cycle. The curve of opening section area is similar to a triangle wave instead of a sinusoidal wave. Based on the triangle wave Fourier series equations 1 and 2, valve timing was calculated. As shown in figure 2, the calculation is consistent with the experimental results.

\[
\text{Supply valve: } f(t) = \frac{10.78}{\pi^2} \times \sum_{k=0}^{\infty} (-1)^k \left( \frac{\cos((2k+1)t-90^\circ)}{(2k+1)^2} \right) + 0.35
\]

\[
\text{Return valve: } f(t) = \frac{10.09}{\pi^2} \times \sum_{k=0}^{\infty} (-1)^k \left( \frac{\cos((2k+1)t+90^\circ)}{(2k+1)^2} \right) + 0.26
\]

Figure 2. Experimental and simulation valve open fraction.

3. Results and discussion
In the model, the 4 K pulse tube cryocooler is operated with a charge pressure of 1.70 MPa at 1.2 Hz. The working fluid is helium assumed as ideal gas. The simulation results are shown in table 1.

| Table 1. Experimental performance VS calculation results of RP-062B. |
|---------------------------------------------------------------|
| **Experimental Data** | **Simulation Results** |
| Temperature with Heat Load | Second Stage/0.5 W | 4.0 K | 3.5 K |
|                          | First Stage/30 W   | 61.0 K | 56.3 K |
| Temperature without Heat Load | Second Stage | 2.8 K | 2.7 K |
|                          | First Stage        | 34.0 K | 25.7 K |
| Operation Pressure at Steady State with 0.5W/30W Heat Load | High | 2.30〜2.50 MPa | 2.30 MPa |
|                          | Low               | 0.74〜0.80 MPa | 0.76 MPa |
3.1 Mass flow rate, pressure and energy distribution

The mass flow rates are always leading a little before the pressures at the hot ends of both the first and second regenerators. According to the amplitudes of the mass flow rate, the ratio of the first and the second stage is calculated to be about 10 : 7. Also, the pressure ratio decreased from 2.0 in the first stage to 1.5 in the second stage due to pressure drop in the regenerators, as shown in figure 3.

The enthalpy flow and PV power are described in figure 4. The enthalpy keeps constantly in the regenerators and increases dramatically in cold heat exchangers (Cold_HX) due to the heat load added to the cold ends. Because of the heat conduction of the pulse tube wall, the enthalpy of gas in the first stage slowly decreases as heat is released to the wall. However, gas in the second pulse tube continuously pumps heat from the hot end heat exchanger (Hot_HX) to the cold end, which means that even though the second Hot_HX isolated the heat flow from the second orifice, it is really a heat source for the pulse tube.

![Figure 3. Mass flow rates and pressures in (a) the first, (b) the second stage regenerators.](image)

![Figure 4. Enthalpy and PV power distribution in (a) the first, (b) the second stage of the cryocooler.](image)

3.2 Effect of double inlet orifice diameters on DC flow

Figure 5 shows the effect of the diameter of double inlet (Db) orifices on the localized DC flow ratio (DC/amplitude of AC). DC_1st stage and DC_2nd stage refer to the DC flow in the first and second Db orifices, respectively. DC_1st regenerator is a sum of DC_1st stage and DC_2nd stage. DC_1st stage and DC_2nd stage always changes oppositely to each other.

It is found that the DC flow rate is less than 0.2% of the main flow rate in RP-062B, and the first Db affects weakly on DC_1st stage and DC_2nd stage, but the second Db can change them effectively. It
means that the second Db orifice is the dominant component for the optimization of DC flow in a 4 K pulse tube cryocooler because the cooler performance is more sensitive to DC_2nd stage, as shown in figure 6.

![Figure 5](image1.png)

**Figure 5.** Effect of (a) the first orifice and (b) the second Db orifice diameters on DC flow ratios in the pulse tube cryocooler.

![Figure 6](image2.png)

**Figure 6.** Effect of DC flow ratios on the cryocooler performance with heat load: (a) the DC flow ratios in the first Db; (b) the DC flow ratios in the second Db.

### 3.3 Filling ratio of the 2nd regenerator

Table 2 shows the performance with respect to filling ratio of the second stage regenerator with 30 W heat load at the first stage and 0.5 W at the second stage. \( T_{\text{GOS}} \) denotes the cold end temperature of GOS (Gd_2O_2S) layer, and \( H_{Cp} \) length denotes to the length of GOS layer where temperature is below 5.5 K.

| Case | Pb | HoCu_2 | GOS | \(T_{Pb}\) | \(T_{\text{HoCu}_2}\) | \(T_{\text{GOS}}\) | \(T_1\) | \(T_2\) | \(H_{Cp}\) Length |
|------|----|--------|-----|--------|-----------|--------|------|------|-----------------|
| A    | 54.40% | 28.20% | 17.40% | 15.5 K  | 6.7 K     | 3.7 K   | 56.8 K | 3.68 K | 20.00 mm        |
| B    | 51.70% | 25.50% | 22.80% | 16.2 K  | 7.2 K     | 3.4 K   | 56.3 K | 3.46 K | 23.22 mm        |
| C    | 51.70% | 20.10% | 28.20% | 15.2 K  | 7.6 K     | 3.3 K   | 56.0 K | 3.39 K | 26.83 mm        |
| D    | 51.70% | 18.50% | 29.80% | 14.8 K  | 7.7 K     | 3.3 K   | 56.0 K | 3.40 K | 27.20 mm        |
| E    | 51.70% | 17.40% | 30.90% | 14.5 K  | 7.8 K     | 3.3 K   | 55.9 K | 3.41 K | 28.03 mm        |

Based on figure 7, the ideal boundary temperatures of Pb and HoCu_2 layers are \(T_{Pb}\) of 12 K, and \(T_{\text{HoCu}_2}\) of 5.5 K, respectively. However, the calculation gives a different conclusion. As shown in table
2, although the lowest $T_{\text{HoCu2}}$ of 6.7 K was obtained in Case A, the performance of the second stage was the worst ($T_2=3.68$ K). As the filling ratio of GOS increased, $T_2$ decreased to its lowest value of 3.39 K and then rebounded to 3.41 K as shown in table 2 and figure 8.

Figure 7. Heat capacities of regenerative material.

Figure 8. $T_2$ and $T_{\text{HoCu2}}$ VS GOS ratio.

It implies that for a specified regenerator, the best boundary temperatures of layers are not the cross point of their heat capacity curves. Also, it is not an efficient method for optimizing the ratio of regenerative materials only based on the heat capacity curves or temperature distribution. For RP-062B, the quantity of GOS in $H_{C_p}$ is the most valuable factor when $T_{\text{HoCu2}}$ is below 7.7 K.

4. Conclusions
A SAGE model of RP-062B was established and a theoretical cooling capacity of 30 W at 56.3 K at the first stage and 0.5 W at 3.5 K at the second stage was achieved using SAGE. The results show that the double inlet orifice of the second stage is the main factor for DC flow optimization, and the quantity of GOS in $H_{C_p}$ strongly affects the performance.

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