Asymptotically Deterministic Time of Extinction for a Stochastic System of Spiking Neurons

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Abstract

We consider a countably infinite system of spiking neurons introduced by Ferrari et al. in [8]. In this model each neuron has a membrane potential which takes value in the non-negative integers. Each neuron is also associated with two point processes. The first one is a Poisson process of some parameter $\gamma$, representing the leak times, that is the times at which the membrane potential of the neuron is spontaneously reset to 0. The second point process, which represents the spiking times, has a non-constant rate which depends on the membrane potential of the neuron at time $t$. This model was previously proven to present a phase transition with respect to the parameter $\gamma$ (see [8]). It was also proven in [3] that the renormalized time of extinction of a finite version of the system converges in law toward an exponential random variable when the number of neurons goes to infinity, which indicates a metastable behavior. Here we prove a result which is in some sense the symmetrical of this last result: we prove that when $\gamma > 1$ (super-critical) the renormalized time of extinction converges in probability to 1.

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1 Introduction

In the present paper we consider an infinite system of spiking neurons which is as follows. We have a countably infinite set of neurons indexed by $\mathbb{Z}$. Each neuron can be in two different states, 1 or 0, respectively called active and quiescent. To each neuron $i$ is associated a Poisson process $(N_i^\dagger(t))_{t \geq 0}$ of some parameter $\gamma$, representing the leak times. At any of these leak times the state of neuron $i$ is immediately reset to 0. Another point process $(N_i(t))_{t \geq 0}$ representing the spiking times is also associated to each neuron, which rate at time $t$ is equal to 1 if the neuron is active and to 0 otherwise. Whenever neuron $i$ spikes its state is also reset to 0 and the state of each of its neighbours in the one-dimensional lattice (i.e. neurons $i-1$ and $i+1$) immediately becomes 1. We denote by $\xi(t)$, the state of neuron $i$ at time $t$. The resulting process $(\xi(t))_{t \geq 0}$ is an interacting particle system, that is: a markovian process taking value in $\{0,1\}^\mathbb{Z}$ (see [11]).

This model is a specific instantiation of a model introduced by Ferrari et al. in [8], and we refer to section 2 of this same article for a more formal description. It can be seen as a continuous time variant of the model introduced in [9]. Other continuous time variants of this model have been studied since this first paper, see for example [4], [5] and [6]. We refer to [10] for a general review.

It has been proven in [8] that this model is subject to a phase transition. More precisely the following theorem was proven.
Theorem 1.1  Suppose that for any \( i \in \mathbb{Z} \) we have \( X_i(0) \geq 1 \). There exists a critical value \( \gamma_c \) for the parameter \( \gamma \), with \( 0 < \gamma_c < \infty \), such that for any \( i \in \mathbb{Z} \)

\[
P(N_i([0, \infty[) < \infty) = 1 \text{ if } \gamma > \gamma_c
\]

and

\[
P(N_i([0, \infty[) = \infty) > 0 \text{ if } \gamma < \gamma_c.
\]

In words there exists a critical value \( \gamma_c \) for the leaking rate which is such that the process dies almost surely above it, and survive with positive probability below it.

Moreover this model was also proven to exhibit a metastable behavior in [3]. What we mean by this is that if you consider a finite version of the process \((\xi(t))_{t \geq 0}\), where the neurons are indexed on \( \mathbb{Z} \cap [-N, N] \) instead of \( \mathbb{Z} \), and if you denote \( \tau_N \) the time of extinction of this finite process, then the following holds in the sub-critical region:

\[
\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow{\mathcal{D}} \mathcal{E}(1), \quad N \to \infty,
\]

where \( \mathbb{E} \) denotes the expectation, \( \mathcal{D} \) denotes a convergence in distribution and \( \mathcal{E}(1) \) an exponential random variable of mean 1. To be exact, for technical reasons related to the way the proofs were constructed in [3], this wasn’t proven for any \( 0 < \gamma < \gamma_c \) but only for \( 0 < \gamma < \gamma'_c \), where \( \gamma'_c \) is some value satisfying \( \gamma'_c \leq \gamma_c \).

In this article we consider the super-critical case. We’re aimed to prove that in the super-critical regime the following holds:

\[
\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow{p} 1, \quad N \to \infty,
\]

where the \( p \) denotes a convergence in probability. This is the object of Theorem 3.3 which is our main result.

This result is symmetrical to the result proven in [3]. Indeed the later tells us that the time of extinction in the sub-critical regime is asymptotically memory-less, which means that it is highly unpredictable: knowing that the process survived up to time \( t \) doesn’t give you any information about what should happen after time \( t \). What we prove here is that in the super-critical regime the time of extinction is asymptotically constant, so that it is highly predictable.

We don’t prove this result for the whole super-critical region but only for \( \gamma > 1 \) (it was shown in [3] that \( \gamma_c < 1 \)). This allows us to use a coupling argument with a continuous time branching process, which greatly simplifies the proof. The proof of the general case is not out of reach but we believe that it would require to prove a large set of other intermediary results which would go far beyond the scope of this paper.

The paper is organized as follows. In Section 2 we briefly introduce the notation used in this article. In Section 3 we prove our main result. Finally we give some of the classical results used throughout the proof in Section 4 (which is an annex).

2 Notations and other formalities

For any \( \eta \in \{0, 1\}^\mathbb{Z} \) we denote \((\xi^n(t))_{t \geq 0}\) the process with initial state \( \xi^n(0) = \eta \). By convention, when the initial state is the "all one state", we omit the superscript, writing simply \((\xi(t))_{t \geq 0}\).

In the rest of this article we will repeatedly identify the state space \( \{0, 1\}^\mathbb{Z} \) with \( \mathcal{P}(\mathbb{Z}) \), the set of all subsets of \( \mathbb{Z} \). Indeed any state \( \eta \) of the process, laying in \( \{0, 1\}^\mathbb{Z} \), can be seen as well as an element \( A \) of \( \mathcal{P}(\mathbb{Z}) \), writing \( A = \{i \in \mathbb{Z} \text{ such that } \eta_i = 1\} \). For example we will write \((\xi^0(t))_{t \geq 0}\) to indicate the process which start with the neuron 0 active and all other neurons quiescent. Notice
that putting 0 to indicate the initial state with only neuron 0 active is an abuse of notation as we should write \(\{0\}\) instead. We will also write such thing as \(\xi^0(t) \neq \emptyset\) to indicate the event in which the process didn’t die yet at time \(t\).

The finite process, defined on the window \([-N, N] \cap \mathbb{Z}\), will be written \((\xi_N^0(t))_{t \geq 0}\). As for the infinite process we adopt the convention of omitting the superscript when the initial state is the whole window \([-N, N] \cap \mathbb{Z}\). Note that by elementary results on Markov processes the time of extinction \(\tau_N\) of this process is almost surely finite, since the state space is finite and the state where all neurons are quiescent is an absorbing state.

3 Result

Our main result follows almost entirely from the following proposition, which basically says that the time of extinction in the super-critical regime is asymptotically logarithmic in \(N\). A similar result was proven for the contact process in [2], which served as an inspiration for our own proof.

**Proposition 3.1** Suppose that \(\gamma > 1\). Then there exists a constant \(0 < C < \infty\) depending on \(\gamma\) such that the following convergence holds

\[
\frac{\tau_N}{\log(2N + 1)} \overset{p}{\rightarrow} C, \quad N \rightarrow \infty.
\]

**Proof:** We define the following function

\[
t \mapsto f(t) = \log \left( \mathbb{P} (\xi^0(t) \neq \emptyset) \right).
\]

We also define the following constant

\[
C' = -\sup_{s>0} \frac{f(s)}{s}.
\]

The first step is to show that the function \(f\) is superadditive. For any \(s, t \geq 0\) we have the following inequality

\[
\mathbb{P} (\xi^0(t + s) \neq \emptyset | \xi^0(t) \neq \emptyset) \geq \mathbb{P} (\xi^0(s) \neq \emptyset).
\]

Indeed saying that the process is still alive at time \(t\) is the same as saying that it possesses at least one active neuron at time \(t\), which happens to be the number of active neurons at time \(0\). Then the inequality follows from the fact that having a higher number of active neurons in the initial configuration implies having a higher probability to be alive at any given time \(s\). Moreover this last inequality can be rewritten as follows

\[
\mathbb{P} (\xi^0(t + s) \neq \emptyset) \geq \mathbb{P} (\xi^0(t) \neq \emptyset) \mathbb{P} (\xi^0(s) \neq \emptyset),
\]

and taking the log gives the superadditivity we are looking for. Now from a well-known result about superadditive functions (Lemma 4.3 in the annex) we get the following convergence

\[
\frac{f(t)}{t} \overset{t \rightarrow \infty}{\rightarrow} -C'.
\]

This implies that we have the following inequality

\[
\mathbb{P} (\xi^0(t) \neq \emptyset) \leq e^{-C't} \quad (3.2)
\]

Now notice that while it is clear that \(C' < \infty\), it is not obvious that \(C' > 0\). We show that it is the case using a coupling with a branching process. Let \((Z(t))_{t \geq 0}\) be the branching process
which dynamic is defined in the annex. The coupling is done as follows, at time 0 the only active neuron in $\xi^0(0)$ is coupled with the only individual in $Z(0)$. By this we mean that if this neuron becomes quiescent then the individual dies, and if the neuron spikes, then the individual gives birth to another individual. When a spike occurs, there are three possibilities: two neurons are activated, one neuron is activated, no neuron is activated. In the first case the individual which was coupled to the neuron that just spiked is coupled with one of the newly activated neuron, and the newly born individual is coupled with the other one. In the second and third cases, the supernumerary individuals are given their own independent exponential clocks and then evolve freely (as well as their possible offspring). At any time $t \geq 0$ we obviously have $|\xi^0_t| \leq Z_t$. Using Markov inequality and Proposition 4.1 from the annex it follows that

$$P(\xi^0_t \neq \emptyset) \leq P(Z_t \geq 1) \leq E(Z_t) = e^{-(\gamma - 1)t}.$$  

Then we take the log and divide by $t$ in the previous inequality and we obtain at the limit that $C' \geq \gamma - 1$, and from the assumption that $\gamma > 1$ we get $C' > 0$.

Let us break the suspense and already reveal that the constant $C$ we are looking for is actually simply the inverse of $C'$. Therefore in order to prove our result we are going to prove that for any $\epsilon > 0$ we have the two following convergences

$$P \left( \frac{\tau_N}{\log(2N + 1)} - \frac{1}{C'} > \epsilon \right) \xrightarrow{N \to \infty} 0, \quad (3.3)$$

and

$$P \left( \frac{\tau_N}{\log(2N + 1)} - \frac{1}{C'} < -\epsilon \right) \xrightarrow{N \to \infty} 0. \quad (3.4)$$

Let us start with (3.3), which is the easiest part. Using inequality (3.2) we get

$$P(\xi_N(t) \neq \emptyset) \leq (2N + 1)P(\xi^0(t) \neq \emptyset) \leq (2N + 1)e^{-C't}.$$  

(3.5)

Now, for any $\epsilon > 0$, if you let $t = \left( \frac{1}{C'} + \epsilon \right) \log(2N + 1)$ then the following holds

$$P \left( \frac{\tau_N}{\log(2N + 1)} - \frac{1}{C'} > \epsilon \right) = P(\xi_N(t) \neq \emptyset) \leq e^{-C' \epsilon \log(2N + 1)}.$$  

Then the fact that $C' > 0$ insure us that the term on the right hand of the inequality goes to 0 as $N$ diverges, which proves (3.3).

It remains to prove (3.4). If for some $N \in \mathbb{N}^*$ we take $t = \left( \frac{1}{C'} - \epsilon \right) \log(2N + 1)$, then we can write

$$\mathbb{P} \left( \frac{\tau_N}{\log(2N + 1)} - \frac{1}{C'} < -\epsilon \right) = \mathbb{P}(\xi_N(t) = \emptyset),$$

so that it suffices to show that the right-hand part converges to 0 for this choice of $t$ as $N$ goes to $\infty$.

From (3.1) (and from the fact that $C' > 0$) we get that for any $\epsilon > 0$ and for big enough $t$

$$\frac{f(t)}{t} \geq -(1 + \epsilon)C',$$

which can be written

$$\mathbb{P}(\xi^0_t = \emptyset) \leq 1 - e^{-(1+\epsilon)C't}.$$  

Taking $t = \left( \frac{1}{C'} - \epsilon \right) \log(2N + 1)$ we have
\[ P(\xi_0^t = \emptyset) \leq 1 - \frac{1}{2N + 1}. \]  
(3.6)

Now for any \( k \in \mathbb{Z} \) we define

\[ F_k \overset{\text{def}}{=} \mathbb{Z} \cap [(2k - 1)K \log(2N + 1), (2k + 1)K \log(2N + 1)], \]

where \( K \) is some constant depending on \( N \) which value will be chosen later in order for \( K \log(2N + 1) \) to be an integer. We then consider a modification of the process \((\xi_N(t))_{t \geq 0}\) where all neurons at the border of one of the sub-windows \( F_k \) defined above (i.e. all neurons indexed by \((2k + 1)K \log(2N + 1)\) for some \( k \in \mathbb{Z} \)) are fixed in quiescent state and therefore are never allowed to spike. This modified process is denoted \((\zeta_N(t))_{t \geq 0}\). We also define the following configuration

\[ A_N \overset{\text{def}}{=} \{2kK \log(2N + 1) \text{ for } k \in \mathbb{Z} \cap [-\frac{2N + 1}{2K \log(2N + 1)}, \frac{2N + 1}{2K \log(2N + 1)}]\}. \]

Notice that the fact that the neurons at the borders of the windows \( F_k \) are never allowed to spike makes the evolution of \((\zeta_N(t))_{t \geq 0}\) independent from one window to another. Moreover notice that the integers belonging to \( A_N \) are all at the center of one of these windows.

Now for any \( t \geq 0 \) we define \( r_t \overset{\text{def}}{=} \max \xi_0^t \). Considering the spiking process \((\xi_t)_{t \geq 0}\) with no leaking it is easy to see that the right edge \( r_t \) can be coupled with an homogeneous Poisson process of parameter \( 1 \), which we denote \((M(t))_{t \geq 0}\), in such a way that for any \( m \geq 0 \)

\[ P\left(\sup_{s \leq t} r_s \geq m\right) \leq P\left(M(t) \geq m\right). \]

We have

\[ E\left(e^{M(t)}\right) = e^{t(e-1)}, \]

so taking the exponential, using Markov inequality and taking \( m = K't \) (where \( K' \) is some constant that we are going to fix in a moment) we get

\[ P\left(\sup_{s \leq t} r_s \geq K't\right) \leq e^{t(e-1-K')} \leq e^{t(2-K')}, \]

where in the last inequality we simply used the fact that \( e-1 < 2 \).

Now taking again \( t = \left(\frac{1}{\epsilon \beta} - \epsilon\right) \log(2N + 1) \) and \( K' = 2(1 + C') \) and we get

\[ P\left(\sup_{s \leq t} r_s \geq m\right) \leq e^{-2(1-\epsilon) \log(2N+1)}, \]

and assuming without loss of generality that \( \epsilon < \frac{1}{2} \) we get

\[ P\left(\sup_{s \leq t} r_s \geq m\right) \leq \frac{1}{2N + 1}. \]

(3.7)

It is now possible to fix the value of the constant \( K \) we introduced earlier. We take

\[ K = \inf \left\{ x \in \mathbb{R} \text{ such that } x \geq \frac{K'}{C} \text{ and } x \log(2N + 1) \in \mathbb{N} \right\}. \]
In words we take $K$ equal to $K/\epsilon$ and then enlarge it slightly in order for $K \log(2N + 1)$ to be an integer. We also define the following event

$$E_t \overset{\text{def}}{=} \{ \zeta_0 \text{ doesn’t escape from } \mathbb{Z} \cap [-K \log(2N + 1), \ldots, K \log(2N + 1)] \text{ for any } s \leq t \}.$$  

Now taking $N$ large enough and $t = (\frac{1}{2} + \epsilon) \log(2N + 1)$ we have

$$\mathbb{P}(\xi_N(t) = \emptyset) \leq \mathbb{P}(\zeta_N(t) = \emptyset) \leq \mathbb{P}(\zeta_N(t) = \emptyset \cap E_t) + \mathbb{P}(E_t^c) \leq \left(1 - \left(\frac{1}{2N + 1} \right)^{1 - \epsilon} - \frac{2}{2N + 1}\right)^{2N + 1/(2K \log(2N + 1))}.$$

To obtain the inequality above we used (3.6) and the fact that inside $E_t$ the process $(\zeta_0^t)_{s \in [0,t]}$ evolves just like $(\xi_0^t)_{s \in [0,t]}$, which allows us to bound $\mathbb{P}(\zeta_N(t) = \emptyset)$, and we used (3.7) to bound $\mathbb{P}(E_t^c)$.

Finally we write

$$a_N = \frac{1}{(2N + 1)^{1 - \epsilon^2}} - \frac{2}{2N + 1},$$

and

$$b_N = \frac{2N + 1}{2K \log(2N + 1)},$$

and we can easily verify that $(a_N)_{N \geq 0}$ and $(b_N)_{N \geq 0}$ verify the assumptions of Lemma 4.2 in the annex, so that

$$(1 - a_N)^{b_N} \quad \xrightarrow{N \to \infty} \quad 0.$$

The next step consists in showing that the same convergence holds for the expectation, which is the object of the following proposition.

**Proposition 3.2** Suppose that $\gamma > 1$. Then the following convergence holds

$$\frac{\mathbb{E}(\tau_N)}{\log(2N + 1)} \quad \xrightarrow{N \to \infty} \quad C,$$

where $C$ is the same constant as in Proposition 3.1.

**Proof:** It is well-known that the fact that a sequence of random variables $(X_n)_{n \in \mathbb{N}}$ converges in probability to some random variable $X$ doesn’t necessarily implies that $\mathbb{E}(X_n) \quad \xrightarrow{N \to \infty} \quad \mathbb{E}(X)$. Nonetheless this implication holds true with the additional assumption that the sequence is uniformly integrable (see for example Theorem 5.5.2 in [7] page 259), i.e. if the following holds

$$\lim_{M \to \infty} \left( \sup_{n \in \mathbb{N}} \mathbb{E}\left(|X_n| 1_{(|X_n| > M)}\right) \right) = 0.$$
It is therefore sufficient to show that \( \tau_N/\log(2N + 1) \) is uniformly integrable, and the result will follow from Proposition 3.1. For some \( M > 0 \) and some \( N \in \mathbb{N}^* \) it is easy to see that we have the following

\[
\mathbb{E}\left( \frac{\tau_N}{\log(2N + 1)} \mathbb{1}_{\{\tau_N/\log(2N + 1) > M\}} \right) = \int_0^\infty \mathbb{P}\left( \frac{\tau_N}{\log(2N + 1)} > \max(t, M) \right) dt.
\]

Now using inequality (3.5) and the previously proven fact that \( C' > 0 \) when \( \gamma > 1 \) we have the following

\[
\int_0^\infty \mathbb{P}\left( \frac{\tau_N}{\log(2N + 1)} > \max(t, M) \right) dt = \int_0^M \mathbb{P}\left( \frac{\tau_N}{\log(2N + 1)} > M \right) dt + \int_M^\infty \mathbb{P}\left( \frac{\tau_N}{\log(2N + 1)} > t \right) dt
\leq (2N + 1) \left[ \int_0^M e^{-C' \log(2N + 1) t} dt + \int_M^\infty e^{-C' \log(2N + 1) t} dt \right]
= (2N + 1) \left[ M (2N + 1)^{-C'M} + \frac{(2N + 1)^{-C'M}}{C' \log(2N + 1)} \right]
= (2N + 1)^{1-C'M} \left[ M + \frac{1}{C' \log(2N + 1)} \right],
\]

where \( C' \) is the same constant as in the previous proof. Without loss of generality we assume that \( M > \frac{1}{C'} \), so that the bound above is decreasing in \( N \), from what we get

\[
\sup_{n \in \mathbb{N}^*} \mathbb{E}\left( \frac{\tau_N}{\log(2N + 1)} \mathbb{1}_{\{\tau_N/\log(2N + 1) > M\}} \right) \leq 3^{1-C'M} \left[ M + \frac{1}{C' \log(3)} \right]. \tag{3.8}
\]

Finally the right-hand side of inequality (3.8) goes to 0 when \( M \) goes to \( \infty \), which ends the proof. \( \square \)

From these two propositions we get

**Theorem 3.3** Suppose that \( \gamma > 1 \). Then the following convergence holds

\[
\frac{\tau_N}{\mathbb{E}(\tau_N)} \stackrel{p}{\to} 1, \quad N \to \infty.
\]

**Proof:** This theorem is a trivial consequence of the two previous propositions. \( \square \)
4 Annex

4.1 Continuous time branching process

We define a continuous time branching process as follows. At time 0 we have a single individual. Two independent exponential random clocks of parameter 1 and \( \gamma \) respectively are attached to this individual. If the rate \( \gamma \) clock rings before the other one, then the individual dies. In the contrary case the individual gives birth to another one, to which another couple of exponential clocks is attached and so on. All individuals behave independently of each other.

We denote by \( (Z_t)_{t \geq 0} \) the process corresponding to the number of individuals of the population along the time. Note that by hypothesis we have \( Z_0 = 1 \). We have the following result, giving an explicit value for the expectation at time \( t \).

**Proposition 4.1** For any value of the parameter \( \gamma \), and for any \( t \geq 0 \), we have

\[
E(Z_t) = e^{-(\gamma - 1)t}.
\]

**Proof:** A proof can be found in [1] (chapter 8).

\[\square\]

4.2 Lemmas

**Lemma 4.2** Let \((a_n)_{n \in \mathbb{N}}\) and \((b_n)_{n \in \mathbb{N}}\) be two sequences of real numbers satisfying the following conditions:

\[
\lim_{n \to \infty} a_n = 0, \quad \lim_{n \to \infty} b_n = +\infty \quad \text{and} \quad \lim_{n \to \infty} a_nb_n = +\infty.
\]

Then we have the following convergence

\[
\lim_{n \to \infty} (1 - a_n)^{b_n} = 0.
\]

**Proof:** For any \( n \geq 0 \) we can write \((1 - a_n)^{b_n} = e^{b_n \log(1 - a_n)}\). Now the well-known inequality \( e^{-x} \geq 1 - x \) can be written \( \log(1 - x) \leq -x \), which gives us

\[
(1 - a_n)^{b_n} \leq e^{-b_n a_n},
\]

and this last bound goes to 0 when \( n \) goes to \( \infty \) using the hypothesis. \(\square\)

The following lemma is a classical result in real analysis about superadditive functions, sometime called Fekete lemma.

**Lemma 4.3** Let \( f : \mathbb{R}^+ \to \mathbb{R} \) be a locally bounded function such that for any \( s, t \geq 0 \) the following holds

\[
f(s + t) \geq f(s) + f(t).
\]

Then we have the following

\[
\lim_{t \to \infty} \frac{f(t)}{t} = \sup_{t > 0} \frac{f(t)}{t}.
\]
Proof: Let fix some $s > 0$. Then any $t \geq 0$ can be written $t = q(t)s + r(t)$, where $q(t)$ (the "quotient") is a non-negative integer and $r(t)$ (the "remaining") belongs to $[0, s[$. Iterating the super-additivity property we have $f(q(t)s) \geq q(t)f(s)$ so that

$$f(t) = f(q(t)s + r(t)) \geq q(t)f(s) + f(r(t)).$$

Now, using the assumption that $f$ is locally bounded, we have

$$\frac{q(t)f(s) + f(r(t))}{t} \rightarrow \frac{f(s)}{s},$$

from what it follows that

$$\liminf_{t \rightarrow \infty} \frac{f(t)}{t} \geq \frac{f(s)}{s}.$$  

The inequality above being true for any $s > 0$ we get

$$\liminf_{t \rightarrow \infty} \frac{f(t)}{t} \geq \sup_{t > 0} \frac{f(t)}{t}.$$  

The result then follows from the inequality above together with the trivial inequality

$$\sup_{t > 0} \frac{f(t)}{t} \geq \limsup_{t \rightarrow \infty} \frac{f(t)}{t} \geq \liminf_{t \rightarrow \infty} \frac{f(t)}{t}.$$  

□
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