The use of thin-walled structures for the sustainable development of agro-industrial systems and facilities

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Abstract. The issues of optimal use of strength resources of materials from which elements of thin-walled objects of the agro-industrial complex are made for the purpose of sustainable development of agricultural production are currently coming to the fore. In this regard, the development of modern algorithms for the numerical analysis of the processes of physically nonlinear deformation of thin-walled objects of agro-industrial complex of various sizes and shapes is becoming a rather urgent topic. The article proposes an algorithm for the finite element analysis of physically nonlinear deformation of thin-walled objects of the agro-industrial complex, taking into account the transverse shear, based on the hypotheses of the theory of plastic flow. The element of discretization of the thin-walled object of the agricultural and industrial complex was a four-node fragment of its middle surface. The stiffness matrix of this sampling unit was obtained in the process of minimizing the Lagrange functional with respect to the sought nodal parameters, which were the components of the step vector of displacement and their first-order partial derivatives, as well as the components of the step vector of the angles of rotation of the normal.

1. Introduction

As the experience of recent years shows, in the modern world there is a steady demand for food, and with a pronounced tendency to an increase in its cost. Russia, with its huge potential of land resources and a variety of climatic zones, is becoming one of the leading exporters of food and agricultural raw materials. Sustainable development of the agricultural sector and the growth of agricultural production volumes require adequate development of engineering systems of the agro-industrial complex, such as irrigation and drainage networks, bunkers, granaries and others. The overwhelming majority of such systems and objects of the agro-industrial complex belong to the class of thin-walled structures. In this regard, the task of developing modern computing technologies for the nonlinear analysis of the stress-strain state of such objects and systems with optimal design and reconstruction of the latter becomes quite urgent. Taking into account the current trend in resource saving, the most in demand are computational algorithms that take into account the possibility of the used material of the thin-walled structure of the agro-industrial complex operating beyond the elastic limit, which ultimately ensure the sustainable development of the agricultural sector.
The presented study presents a computational algorithm for determining the stress-strain state of engineering objects of the agro-industrial complex, which can be classified as thin-walled (water conduits, hangars, bunker structures, tanks, etc.) taking into account their deformation beyond the elastic limit. The research tool is the finite element method (FEM) [1-26]. When constructing a finite element model, transverse shear deformations are taken into account [27]. The provisions of the theory of plastic flow [28] are used as the theory of plasticity.

2. Materials and methods

2.1. Geometric relationships
The surface equidistant from the outer and inner boundaries of the investigated thin-walled agricultural and industrial complex (AIC) object is specified by the radius vector

$$\vec{R} = x(a^1, a^2)\hat{x} + y(a^1, a^2)\hat{y} + z(a^1, a^2)\hat{z},$$

(1)

where \(a^1, a^2\) curvilinear coordinates of the above surface.

Using the formulas of differential geometry [29], one can obtain the basis vectors of the point \(M^0\) of the surface

$$\vec{a}_1^0 = \vec{R}^0_{,a^1}; \quad \vec{a}_2^0 = \vec{R}^0_{,a^2};$$
$$\vec{a}_3^0 = \vec{a}_1^0 \times \vec{a}_2^0 / [\vec{a}_1^0 \times \vec{a}_2^0].$$

(2)

(3)

If, at the point \(M^0\), restore the normal to the contacting plane and fix the point \(M^{0\kappa}\) on this normal at a distance \(\zeta\) from the point \(M^0\), then the radius vector of the point \(M^{0\kappa}\) can be represented as the sum

$$\vec{R}^{0\kappa} = \vec{R}^0 + \zeta \vec{a}_3^0,$$

(4)

When analyzing the stress-strain state of thin-walled objects of the agro-industrial complex, both geometrically and physically nonlinear formulation, a stepwise application of an external load is usually used. After sequential loading in \(n\) steps, point \(M^{0\kappa}\) will move to point \(M^\kappa\), and after the next \((n + 1)\)-th step of loading - to point \(M^{\kappa\prime}\). The positions of the points \(M^\kappa\) and \(M^{\kappa\prime}\) can be fixed by the corresponding radius vectors

$$\vec{R}^\kappa = \vec{R}^{0\kappa} + \vec{V} ; \quad \vec{R}^{\kappa\prime} = \vec{R}^\kappa + \vec{W},$$

(5)

The vectors \(\vec{V}\) and \(\vec{W}\) characterizing the displacements of the point \(M^{0\kappa}\) from the initial position to the deformed ones after the \(n\) and \((n + 1)\)-th loading steps can be represented by vector sums

$$\vec{V} = \vec{v} + \zeta \vec{y}; \quad \vec{W} = \vec{w} + \zeta \Delta \vec{y},$$

(6)

where \(\vec{v} = v^m \vec{a}_m^0; \quad \vec{w} = w^m \vec{a}_m^0\), \((m = 1, 2, 3)\) - displacement vectors of point \(M^0\) of the surface (1); \(\vec{y} = y^\rho \vec{a}_\rho^0; \quad \Delta \vec{y} = \Delta y^\rho \vec{a}_\rho^0\) \((\rho = 1, 2)\) - vectors of slope angles of the normal after \(n\) loading steps and increment of this vector at the \((n + 1)\)-th loading step.

The basis vectors at the points \(M^{0\kappa}\), \(M^\kappa\) and \(M^{\kappa\prime}\) can be obtained by applying the operation of differentiation with respect to the curvilinear coordinates of the surface (1) to the formulas of radius vectors (4), (5)

$$\vec{g}_\rho^0 = \vec{R}^{0\kappa}_{,\rho}; \quad \vec{g}_3^0 = \vec{R}^{0\kappa}_{,\zeta}; \quad \vec{g}_\rho^\kappa = \vec{R}^\kappa_{,\rho}; \quad \vec{g}_3^\kappa = \vec{R}^\kappa_{,\zeta}; \quad \vec{g}_\rho^{\kappa\prime} = \vec{R}^{\kappa\prime}_{,\rho}; \quad \vec{g}_3^{\kappa\prime} = \vec{R}^{\kappa\prime}_{,\zeta},$$

(7)

Using the fundamental formula of continuum mechanics [25], one can obtain expressions for the covariant components of the strain increment tensor at the point \(M^{0\kappa}\) of a thin-walled AIC object after the \((n + 1)\)-th loading step

$$\Delta \varepsilon^\kappa_{mn} = (g^*_{mn} - g_{mn})/2, (m, n = 1, 2, 3)$$

(8)

or in matrix form

$$\{\Delta \varepsilon^\kappa_{mn}\} = [D] \{W\},$$

(9)

where \([W]^T = \{w^1, w^2, w^3\}\) a column matrix of the components of the vector of the \((n + 1)\)-th step movement; \([D]\) is a matrix that includes algebraic and differential operators in its structure.

The covariant components of the metric tensors in (8) at the points \(M^\kappa\) and \(M^{\kappa\prime}\) are calculated by performing the scalar products (7)

$$g_{mn} = \vec{g}_m \cdot \vec{g}_n; \quad g^*_{mn} = \vec{g}_m^* \cdot \vec{g}_n^*.$$

(10)
2.2. Physical relationships
According to the hypothesis of the theory of plastic flow, the increments of plastic deformations are proportional to the stress deviator [28]

\[ \Delta \varepsilon_{mn}^p = \frac{3}{2} \epsilon_i^{\Delta p} \left( \sigma_{mn} - \frac{1}{3} g_{mn} I_1 \right), \]  

(10a)

where \( I_1 \) is the first invariant of the stress tensor; \( \sigma_i \) - stress intensity; \( \epsilon_i^{\Delta p} \) is the intensity of the increments of plastic deformations, which, as a rule, is replaced by an increment in the intensity of plastic deformations \( \epsilon_i^{\Delta p} \approx \Delta \varepsilon_i^p \), which in turn is determined by the difference between the increment in the intensity of deformations and the increment in the intensity of elastic deformations

\[ \Delta \varepsilon_i^p = \Delta \varepsilon_i - \varepsilon_i^p. \]  

(11)

Using the deformation diagram of the material of a thin-walled AIC object, relation (11) at the \((n+1)\)-th loading step can be represented in the following form

\[ \Delta \varepsilon_i^p = \frac{\Delta \sigma_i - \sigma_i}{E_k} = \Delta \sigma_i \left( \frac{1}{E_k} - \frac{1}{E_o} \right) = \Delta \sigma_i \psi, \]  

(12)

where \( \Delta \sigma_i \) is the increment in stress intensity; \( E_o, E_k \) - initial and tangent modules of the deformation diagram.

Representing the increments of deformations at a point in the form of the sum of increments of elastic and plastic deformations when using the orthonormal local basis of the point of the surface of a thin-walled AIC object, we can write the following physical relations at the \((n+1)\)-th loading step

\[ \begin{align*}
\Delta \varepsilon_{11}^p &= \frac{1}{E} \left( \Delta \sigma_{11} - \mu \Delta \sigma_{22} \right) + \frac{3}{2} \epsilon_i^{\Delta p} \psi (\sigma_{11} - \sigma_o); \\
\Delta \varepsilon_{22}^p &= \frac{1}{E} \left( \Delta \sigma_{22} - \mu \Delta \sigma_{11} \right) + \frac{3}{2} \epsilon_i^{\Delta p} \psi (\sigma_{22} - \sigma_o); \\
\Delta \varepsilon_{33}^p &= -\mu \left( \Delta \sigma_{11} + \Delta \sigma_{22} \right) - \frac{3}{2} \epsilon_i^{\Delta p} \psi \sigma_{11}; \\
2 \Delta \varepsilon_{12}^p &= \frac{\Delta \sigma_{12}}{G} + \frac{3}{2} \epsilon_i^{\Delta p} \psi \sigma_{12}; \\
2 \Delta \varepsilon_{13}^p &= \frac{\Delta \sigma_{13}}{G} + \frac{3}{2} \epsilon_i^{\Delta p} \psi \sigma_{13}; \\
2 \Delta \varepsilon_{23}^p &= \frac{\Delta \sigma_{23}}{G} + \frac{3}{2} \epsilon_i^{\Delta p} \psi \sigma_{23},
\end{align*} \]  

(13)

where \( E, G \) - elastic moduli of the first and second kind, \( \mu \) - Poisson's ratio; \( \sigma_o = \left( \sigma_{11} + \sigma_{22} \right)/3 \).

When assembling (13), the hypothesis of the equality of normal stresses and their increments in the direction normal to the surface of the AIC object, generally accepted in the calculations of thin-walled structures, was taken into account.

\[ \sigma_{33} = 0; \Delta \sigma_{33} = 0. \]  

(14)

The increment in stress intensity included in (13) can be expressed in terms of the increments in stresses using the formula for the total differential

\[ \Delta \sigma_i = \frac{\partial \sigma_i}{\partial \sigma_{11}} \Delta \sigma_{11} + \frac{\partial \sigma_i}{\partial \sigma_{22}} \Delta \sigma_{22} + \frac{\partial \sigma_i}{\partial \sigma_{12}} \Delta \sigma_{12} + \frac{\partial \sigma_i}{\partial \sigma_{13}} \Delta \sigma_{13} + \frac{\partial \sigma_i}{\partial \sigma_{23}} \Delta \sigma_{23}. \]  

(15)

Partial derivatives of stress intensity with respect to stresses \( \partial \sigma_i/\partial \sigma_{mn} \) can be represented by functions of total stresses accumulated over \( n \) previous loading stages

\[ \begin{align*}
\frac{\partial \sigma_i}{\partial \sigma_{11}} &= \frac{1}{2} \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{12}} &= \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{13}} &= \frac{3}{2} \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{22}} &= \frac{1}{2} \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{23}} &= \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{12}} &= \frac{3}{2} \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{13}} &= \sigma_i; \\
\frac{\partial \sigma_i}{\partial \sigma_{23}} &= \frac{3}{2} \sigma_i.
\end{align*} \]  

(16)

Physical relations (13), taking into account (16), can be formalized in the form of matrix dependencies.
\[
\{\Delta \varepsilon_{mn}\} = [N]\{\sigma_{mn}\};
\]
\[
\Delta \varepsilon_{33}^\xi = \{G\}^T\{\Delta \sigma_{mn}\},
\]
where \(\{\Delta \varepsilon_{mn}\}^T = \{\Delta \varepsilon_{11}^\xi, \Delta \varepsilon_{22}^\xi, 2\Delta \varepsilon_{12}^\xi, 2\Delta \varepsilon_{13}^\xi, 2\Delta \varepsilon_{23}^\xi\}; \{\Delta \sigma_{mn}\}^T = \{\Delta \sigma_{11} \Delta \sigma_{22} \Delta \sigma_{12} \Delta \sigma_{13} \Delta \sigma_{23}\}.
\]

Applying the operation of referring to (17), one can obtain the required plasticity matrix at the \((n + 1)\)-th stage of step loading
\[
\{\Delta \sigma_{mn}\} = [C_n]\{\Delta \varepsilon_{mn}\},
\]
where \([C_n] = [N]^{-1}\).

When implementing the computational algorithm, it should be noted that, using the strength parameters of the previous \((n + 1)\)-th loading step, the stress increments are first determined by means of (19), and then, using (18), \(\Delta \varepsilon_{33}^{(n)}\) is calculated and, by summation, the total deformations in a direction perpendicular to the median surface
\[
\Delta \varepsilon_{33}^{(n)} = \Delta \varepsilon_{33}^{(n-1)} + \Delta \varepsilon_{33}^{(n)}.
\]

The value (20) is used to calculate the intensity of deformations [28]
\[
\varepsilon_i = \frac{\sqrt{2}}{3} \left( (\varepsilon_{11}^\xi - \varepsilon_{22}^\xi)^2 + (\varepsilon_{22}^\xi - \varepsilon_{33}^\xi)^2 + (\varepsilon_{33}^\xi - \varepsilon_{11}^\xi)^2 + \frac{3}{2} (2\varepsilon_{12}^\xi)^2 + (2\varepsilon_{13}^\xi)^2 + (2\varepsilon_{23}^\xi)^2 \right).
\]

Having the values \(\varepsilon_i^{(n)}\) (21), we can determine \(\sigma_i, E_k\) from the deformation diagram and again, using formula (19), compose the plasticity matrix \([C_n]\) of the joint venture corresponding to the \((n + 1)\) step of step loading.

3. Results and discussion

The stiffness matrix of a four-node discretization element at the \((n + 1)\)-th stage of step loading is composed based on the Lagrange functional [27]
\[
L = \int_V \{\Delta \varepsilon_{mn}\}^T \{\{\sigma_{mn}\} + \{\Delta \sigma_{mn}\}\}dV - \int_F \{W\}^T \{\{P\} + \{\Delta P\}\}dF,
\]
where \(\{P\}\) and \(\{\Delta P\}\) is the total surface load accumulated over \(n\) stages of sequential loading and the increment of this load at the \((n + 1)\)-th loading step.

The strain increments at the point \(M^i\) at the \((n + 1)\)-th stage of step loading can be represented as a matrix product
\[
\{\Delta \varepsilon_{mn}\} = [\Gamma]\{\Delta \varepsilon_{mn}\},
\]
where \([\Delta \varepsilon_{mn}\}^T\) are the increments of deformations at the point of the median surface \(M\) of the thin-walled APC object, and the matrix \([\Gamma]\) has the form
\[
[\Gamma]_{5\times10} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \xi & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \xi & 0
\end{bmatrix}.
\]

Using the Cauchy relations at the \((n + 1)\)-th stage of step loading (8), the increments of deformations at the point of the median surface \(M\) can be expressed in terms of the column \([W_y]\) of the sought quantities at the nodes of the four-node sampling element, which are the components of the step vector of displacement \(\vec{w} = w^m \hat{\vec{a}}^m\), their first-order partial derivatives and the components of the step vector of the slope of the normal \(\Delta \vec{y} = \Delta y^p \hat{\vec{a}}^p\)
\[
\{\Delta \varepsilon_{mn}\} = [B]\{W_y\},
\]
Functional (22), taking into account (19), (23), and (24), can be transformed to the form
where $[A]$ is a matrix containing polynomial functions of the form.

Applying to (25) the procedure of minimization with respect to $\{W_p\}^T$, we can obtain the following matrix relation

$$[M]\{W_p\} = \{Q\} - \{NR\},$$

where $[M] = \int_{F} [B]^T [\Gamma]^T \{\sigma_{mn}\} + [C_n][\Gamma][B]\}dV - \{W_p\}^T \{A\}^T \{\Delta P\}dF$ - stiffness matrix of a four-node sampling element at the $(n + 1)$-th step of step loading; $\{Q\} = \int_{F} [A]^T \{\Delta P\}dF$ - column of nodal forces at this stage of loading; $\{NR\} = \int_{F} [B]^T [\Gamma]^T \{\sigma_{mn}\}dV - \int_{F} [A]^T \{P\}dF$ - Newton-Raphson correction at the $(n + 1)$ th stage of step loading.

4. Conclusions

It should be noted that the algorithm described above for determining the strength parameters of thin-walled AIC objects, taking into account the plastic deformation of the material from which the objects under study are made, provides for the calculation of the total stresses accumulated over $n$ stages of step loading by summing the stress increments at each step

$$\sigma_{mn} = \sum_{i=1}^{n} \Delta \sigma_{mn}.$$  

Despite the absence of physical relationships between total stresses and total strains, which are present in the deformation theory of plasticity, the stresses calculated from the relations of the theory of plastic flow implemented in the above algorithm are in better agreement with the experimental results.

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