Photoinduced electronic and spin properties of quantum Hall systems with Rashba spin-orbit coupling

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We theoretically investigate photoinduced phenomena induced by time-periodic driving fields in two-dimensional electron gases under perpendicular magnetic fields with Rashba spin-orbit coupling. Using perturbation theory, we provide analytical results for the Floquet-Landau energy spectrum appearing due to THz radiation. By employing the resulting photo-modulated states, we compute the dynamical evolution of the spin polarization function for an initially prepared coherent state. We find that the interplay of the magnetic field, Rashba spin-orbit interaction and THz radiation can lead to inversion of the spin polarization. The dynamics also induces fractional revivals and non-trivial beating patterns in the autocorrelation function due to interference of the photo-modulated quantum states. We also calculate the transverse photo-assisted conductivity in the linear response regime using Kubo formalism and analyze the impact of the radiation field and Rashba spin-orbit interaction. In the static limit, we find that our results reduce to well-known expressions of the conductivity in non-relativistic and quasi-relativistic (topological insulator surfaces) two-dimensional electron gas thoroughly described in the literature. We discuss the possible experimental detection of our theoretical prediction and their relevance for spin-orbit physics at high magnetic fields.

I. INTRODUCTION

Over the past years, photoinduced properties of two-dimensional systems have been a subject of tremendous study, especially after the identification of non-trivial topological properties that can be induced by periodic driving fields. Another reason for the considerable amount of attention devoted to these systems, both theoretically and experimentally, comes from the fact that driving fields can be used to dynamically control material and topological properties, i.e. one can by simply shining monochromatic laser light promote the system to different topological phase (“Floquet engineering” [7]). In this regard, non-trivial light-induced phases have been shown to exist in several two-dimensional systems such as monolayer graphene [1], [2], silicene [3], [4], transition metal dichalcogenides [5], [6] or topological insulator surfaces [7]–[9].

When the electrons in a two-dimensional plane are additionally subjected to a static perpendicular magnetic field, their periodic motion translate into discrete (Landau) levels due to quantization of the electronic kinetic energy [21]. This quantization is at the origin of dramatic consequences for macroscopic transport properties at low temperatures, the most famous being the perfect quantization of the Hall conductance in plateaus of integral multiples of the conductance quantum [22]. The robustness of the conductance quantization has been addressed by considering the effect of THz driving or spin-orbit (SO) interaction of the Rashba type [23]–[25] among other types of perturbations. The latter appears due to asymmetric confinement of electron gases in low-dimensional nanostructures and can be tuned using local external electric fields [26]. A physically relevant and interesting scenario then occurs when both of these tunable perturbations are simultaneously present in the system, which can happen when two-dimensional surface gases existing in materials with heavy atoms such as InSb [27] or BiSb monolayers [28] are irradiated by a periodic time-dependent field.

In this work, we consider this scenario and study the combined effect of periodic driving (Floquet) and Rashba SO interaction in clean two-dimensional electron gases (2DEG) under perpendicular magnetic fields. Note that, in this situation, one is also required to incorporate the effect of the Zeeman coupling, which might affect the structure of the energy levels. Thanks to the periodicity of the radiation field, we apply Floquet’s theorem and transform the dynamical equations of motion into an exact time-independent problem. Our approach has then the advantage that the dynamics can be tackled without the need of addressing an infinite-dimensional eigenvalue problem. We investigate the emergence of light-modulated Landau energy levels (dubbed Floquet-Landau levels), similar to the static Landau levels but with radiation renormalizing both the Rashba SO parameter and the Zeeman coupling. Using the driven eigenstates, we compute the dynamical evolution of relevant physical observables such as the spin polarization or the autocorrelation function and investigate the effect of SO coupling in the linear response photoconductivity. We further use our results for the photoconductivity to explore different physical regimes characterized by the strength of the Rashba SO interaction. At small values
of the Rashba parameter, we recover the results from the ordinary photo-excited 2DEG. At large values of the SO coupling strength, we obtain expressions for the conductivity of graphene / single surface of topological insulators previously described in the literature. Finally, we discuss the possible experimental probe of our theoretical predictions in realistic systems.

The paper is organized as follows: In Sec. II we present the model Hamiltonian and study the effect of the radiation field on the spectral properties by using perturbation theory. In Sec. III we consider relevant observables and study the time-evolution of the spin polarization and the autocorrelation function when the system is initially prepared in a coherent state. In Sec. IV we obtain the photoconductivity of the Floquet system using Kubo formula and analyze its behavior for several regimes of the effective SO interaction. Finally, in Sec. V we give concluding remarks. We complement the paper by showing explicit algebraic derivations of our main results in the appendix.

II. MODEL

A. Static Hamiltonian

We consider a single electron of spin 1/2, electronic charge $q = -e$ (here $e > 0$) and effective mass $m^*$ confined to a two-dimensional (2D) plane under a perpendicular and uniform magnetic field, $\mathbf{B} = B\hat{z}$. The single-particle Hamiltonian in the presence of SO coupling of the Rashba type and Zeeman interaction is given by

$$H_0 = \frac{\pi^2}{2m^*} \otimes \mathbb{1}_2 + \lambda [\pi \times \sigma]_z + \frac{\Delta}{2} \otimes \sigma_z. \quad (1)$$

Here, the first term corresponds to the spin-diagonal Hamiltonian for a free single electron, with $\mathbb{1}_2$ being the $2 \times 2$ identity matrix in spin space, $\pi$ the gauge-invariant momenta with components $\pi_j = p_j + eA_j(r)$ [$j \in \{x, y\}$], $r = (x, y)$ is the position of the electron and $A(r)$ the electromagnetic vector potential. The latter is related to the external magnetic field through the constitutive relation $\nabla \times \mathbf{A}(r) = \mathbf{B}$. The second term is the Rashba Hamiltonian describing the coupling between spin and orbital degrees of freedom

$$H_R = \lambda [\pi \times \sigma]_z = \lambda [\pi_x \otimes \sigma_y - \pi_y \otimes \sigma_x], \quad (2)$$

with $\lambda$ being the spatially averaged Rashba parameter and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ the vector of Pauli matrices. Finally, the third term is the Zeeman coupling between the spin of the electron and the external magnetic field characterized by the Zeeman gap $\Delta$. We omit in what follows the tensor product symbol and the identity matrix $\mathbb{1}_2$.

Introducing the magnetic length $l_B = \sqrt{\hbar/(eB)}$ and the cyclotron frequency $\omega_c = \hbar/(m^*l_B^2)$, as well as the annihilation and creation operators

$$a = \frac{l_B}{\sqrt{2\hbar}} (\pi_x - i\pi_y), \quad (3a)$$
$$a^\dagger = \frac{l_B}{\sqrt{2\hbar}} (\pi_x + i\pi_y), \quad (3b)$$

it can be easily shown that the Hamiltonian in Eq. (1) can be rewritten as the Jaynes-Cummings Hamiltonian well-known in quantum optics

$$H_0 = \hbar \omega_c N_a - \frac{\delta}{2} \sigma_z - i\lambda_B (a\sigma_+ - a^\dagger \sigma_-). \quad (4)$$

Here $\lambda_B = \sqrt{2}\hbar\lambda/l_B$ characterizes the strength of the SO interaction in the presence of magnetic field (i.e. the SO interaction strength per magnetic length), $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, the detuning is given by $\delta = \hbar\omega_c - \Delta$ and we have defined the operator $N_a := a^\dagger a + (1 + \sigma_z)/2$, which commutes with the Rashba SO Hamiltonian. We assume in this paper that $\delta > 0$ as the cyclotron energy is typically the dominant energy scale in comparison to the Zeeman gap.

After straightforward diagonalization of Eq. (4), we get the energy levels (Landau levels) given by

$$E_{sn} = \hbar \omega_c n + \frac{s}{2} \Delta_n. \quad (5)$$

where $\Delta_n = \sqrt{4n\lambda_B^2 + \delta^2}$. The energy levels are characterized by a positive integer, $n \geq 0$, the Landau level index and the SO quantum number $s = s(n)$ which takes the value $s = \pm 1/2$ if $n \geq 1$ and $s = +$ when $n = 0$. The quantum number $s$ can be interpreted as the “spin” index projection along the axis defined by the Rashba SO interaction. As in the spinless case, the degeneracy of each level per unit area is equal to $n_B = 1/(2\pi l_B^2)$.

In Fig. 1 we show the energy spectrum (normalized to the cyclotron energy) as a function of $\lambda_B/(\hbar \omega_c)$. We have considered values for the electron effective mass, $m^* = 0.02m_0$, normalized detuning, $\delta/(\hbar \omega_c) \approx 1.2$, and Rashba SO interaction typical for 2DEG that can be found in BiSb monolayers which have SO coupling strength $\hbar\lambda = 2.3$ eVÅ. For small values of the renormalized SO interaction relative to the cyclotron energy, the energy levels are distributed as Zeeman-split pairs of Landau levels. Once the effect of the magnetic-field on the SO coupling becomes relevant, each pair of levels splits off with similar to the case of graphene the level with quantum numbers $(0, +)$ is independent of the SO coupling. The energy spectrum then presents non-equidistant levels. Consequently, it also exhibits level crossings between different pairs of Landau levels $(n, s)$ and $(n', -s)$ [31]. Note that necessarily energy levels with the same SO quantum number never cross and that there is a unique energy level labeled by the quantum numbers $(1, -)$ that never crosses with any other Landau level. The accidental degeneracies produced by the level crossings are expected to lift once Landau level mixing occurs due to, i.e. disorder potential.
where $\mathcal{E}$ and $\Omega$ are, respectively, the amplitude and frequency of the electric field. The expression of the incident field can be easily obtained from the standard relation $\mathcal{E}(t) = -\partial_t \mathcal{A}(t)$. For simplicity, we also assume that the light beam is right-handed circularly polarized, extension to left-handed circular polarization being straightforward by considering the transformation $\Omega \to -\Omega$.

Thus, considering the total vector potential $\mathcal{A}(r, t) = \mathcal{A}(r) + \mathcal{A}(t)$, we apply the minimal coupling prescription $p \to p + e\mathcal{A}(r, t)$ in order to obtain an interaction potential term with the driving field

$$V(t) = \xi (\sigma_y \cos \Omega t - \sigma_x \sin \Omega t).$$

This term enters into the time-dependent Hamiltonian $H(t) = H_0 + V(t)$ with an effective coupling constant $\xi = e\lambda \mathcal{E}/\Omega$. Observe that within our approach the additional time-dependent contribution independent of the SO coupling term is neglected. It can be easily checked that this is valid under the assumption $\omega_c \ll \Omega$, an assumption that holds in part of the THz and infrared spectral range for not very large magnetic field strength.

Inclusion of the $V(t)$ term makes the full Hamiltonian $H(t) = H_0 + V(t)$ periodic in time, $H(t + T) = H(t)$, with $T = 2\pi/\Omega$ being the period of oscillation of the driving field. We now apply Floquet’s theorem and write the evolution operator of the system in the form

$$U(t) = P(t) e^{-iH_F t},$$

with $P(t)$ a periodic unitary matrix and $H_F$ a time-independent dynamical generator referred to as the Floquet Hamiltonian. The eigenvalues of the Floquet Hamiltonian $H_F$ form the so-called quasienergy spectrum of the periodically driven system.

For the system under consideration, it can be shown that $P(t) = \exp(-iN_\sigma \Omega t)$ generates a time-dependent unitary transformation, $|\Psi(t)\rangle = P(t)|\Phi(t)\rangle$, such that the time-dependent Schrödinger equation

$$i\hbar \partial_t |\Psi(t)\rangle = H(t)|\Psi(t)\rangle,$$

becomes

$$i\hbar \partial_t |\Phi(t)\rangle = H_F |\Phi(t)\rangle,$$

where $|\Phi(t)\rangle$ are the Floquet eigenstates. Doing the explicit calculation (see Appendix A), $H_F$ is found to be given by

$$H_F = \hbar \omega_- N_\sigma - \frac{\delta}{2}\sigma_z + i\lambda_B (a^\dagger \sigma_- - a \sigma_+) - \xi \sigma_y,$$

where we have introduced the frequency $\omega_- = \omega_c - \Omega$. We first notice that when the resonant condition, $\Omega = \omega_c$, is fulfilled the resonant Hamiltonian $H_r$ expressed in terms of shifted operator $b = a - \beta$, with $\beta = \xi/\lambda_B$, can be written as

$$H_r = i\lambda_B (b^\dagger \sigma_- - b \sigma_+) - \frac{\delta}{2}\sigma_z.$$
Therefore, when the resonance condition is satisfied, the spectrum is the same as in Eq. (5), but an integer number of excitations $\hbar\omega_-$ have been resonantly absorbed from the system. This would be an $m$ photon resonance.

When the system is not at resonance, we apply perturbation theory to obtain an effective Floquet Hamiltonian that allows us to study the full frequency response of the system. We consider as small parameter the effective radiation strength \( \kappa = \xi/\hbar\omega_c \) and transform the Hamiltonian in Eq. (13) as \( H = \exp[-i(\kappa/2)I_+]|H_F|\exp[i(\kappa/2)I_+] \) where the operator \( I_\pm = a^\dagger\sigma_- + a\sigma_+ \) commutes with \( N_a \). Evaluation up to first order in \( \kappa \) using the Baker-Campbell-Hausdorff formula (see Appendix A) gives the effective Floquet Hamiltonian

\[
H_F^{\text{eff}} \simeq \hbar\omega_cN_c - \left( \frac{\delta - 2\kappa\lambda_B N_c}{2} \right) \sigma_z + i \left( \lambda_B + \frac{\kappa\delta}{2} \right) (c^\dagger\sigma_- - \sigma_+),
\]

where \( c = a - \gamma, \gamma = 2\xi/(\kappa\delta + 2\lambda_B) \) and the shifted number operator \( N_c = c^\dagger c + (1 + \sigma_z)/2 \). Higher order terms in \( \gamma \) and \( \kappa \) can be dealt, in principle, by using higher order perturbation theory. The condition \( \kappa = \xi/\hbar\omega_c \ll 1 \) can be easily met for realistic systems. To check this explicitly, let us take as typical values for the parameters \( \xi \approx 0.15 \text{ MV/m}, \hbar\Omega \approx 10 - 20 \text{ meV} \), and Rashba coupling constant typical for 2DEG existing on BiSb monolayer or InSb surface gases \( (\hbar\lambda = 0.7 \text{ eVÅ}) \). This yields for a magnetic field \( B = 1 \text{ T} \) values for the perturbative parameter \( \kappa \approx 10^{-2} - 10^{-3} \).

C. Floquet-Landau energy spectrum

We proceed by diagonalization of the effective Hamiltonian (15) in order to obtain the discrete Floquet-Landau energy spectrum (i.e., Landau levels dressed by the radiation)

\[
\varepsilon_{sm} = m\hbar\omega_+ + \frac{s}{2} \sqrt{4m\lambda_B^2 + \Delta_m^2},
\]

with \( \lambda_B = \lambda_B + \kappa\delta/2 \) and \( \Delta_m = \delta - 2m\kappa\lambda_B \). As in the static case, Sec. II A the (Floquet) band index \( m \geq 0 \) is an integer (that plays the role of the Landau level index) and the new SO quantum number \( s = s(m) \) is equal to \( s = \pm 1 \) if \( m \neq 0 \) and \( s = + \) when \( m = 0 \). By comparing this result to Eq. (5), we find that the radiation field affects both the strength of the SO and the (Zeeman-related) detuning: the magnetic field renormalized SO interaction increases due to coupling to the Zeeman term via the radiation; correspondingly, the detuning is reduced by the SO coupling and becomes dependent on the Floquet band index.

We show in Fig. 2 the Floquet-Landau spectrum for the same set of parameters used in Fig. 1. Similar to the static Landau energy levels, the Floquet-Landau energies present multiple level crossings between pairs of levels \( (m, s) \) and \( (m', -s) \) occurring as a function of the SO coupling \( \lambda \) or the magnetic field. Due to the radiation field, the position of the crossings is drastically altered. In addition, most of the energy levels are substantially shifted in energy by the radiation except \( (0, +) \) which remains unaffected. Note also that, as a consequence of the shift in the Floquet-Landau levels, the levels with positive SO projection \( (0, +) \) and \( (1, +) \) never cross with any other energy level.

The corresponding Floquet eigenstates for any Floquet-Landau level labeled by \( (m, s) \) can be written in a similar way to Eq. (6) and read

\[
|\psi_{sm}\rangle = \left( -isb_{sm}|m-1\rangle \right) / b_{sm}|m\rangle,
\]

where we have defined the coefficients

\[
b_{sm} = \sqrt{\frac{1}{2} \left( 1 + \frac{\Delta_m}{\varepsilon_{sm}} \right)},
\]

with \( \varepsilon_{m} = |\varepsilon_{sm} - m\hbar\omega_-| \). Observe that the energy level degeneracy given by the continuous quantum number \( k \) is not affected and therefore it will remain implicit in our expressions.
III. SPIN AND AUTOCORRELATION FUNCTION DYNAMICS

A. Spin polarization

We begin by considering the dynamics of the spin-polarization, whose time-average $\langle \sigma_z \rangle$ can be defined by

$$\langle \sigma_z \rangle = \frac{1}{T} \int_0^T dt \langle \Psi(0) | U(t) \sigma_z U(t) | \Psi(0) \rangle,$$  
(19)

where $| \Psi(0) \rangle$ is the state of the system prepared at $t = 0$. Taking into account that $[\sigma_z, P(t)] = 0$, we use Eq. (10) to further write the last expression using the Floquet Hamiltonian

$$\langle \sigma_z \rangle = \frac{1}{T} \int_0^T dt \langle \Psi(0) | e^{iH_F t/\hbar} \sigma_z e^{-iH_F t/\hbar} | \Psi(0) \rangle.$$  
(20)

Note that, whenever the initial state is an eigenstate of the Floquet Hamiltonian the expectation value of the spin polarization in Eq. (19) is constant over one period of the radiation field. Yet, in an experimental setup, it is more feasible to prepare the initial state of the system either in a linear combination of eigenstates of the static Hamiltonian [1]. One particular experimentally relevant case is an initial state configuration $| \Psi(0) \rangle$ given by the coherent state $| \alpha \rangle$, for which the mean excitation number is determined by the coherent state parameter $\alpha = \sqrt{\langle |N_\alpha |^2 \rangle}$. The coherent state parameter $\alpha$ in this context is analogous to the mean photon number in quantum optics.\(^{36}\) The coherent state is explicitly given by the expression

$$| \alpha \rangle = e^{-|\alpha|^2/2} \left( | \varphi_0 \rangle + \frac{1 + \sum_{n=1}^{\infty} a^n}{\sqrt{n!}} | \varphi_{s,n} \rangle \right),$$  
(21)

with $| \varphi_{s,n} \rangle$ being the eigenstates of the static Hamiltonian at zero detuning ($\delta = 0$),

$$| \varphi_{s,n} \rangle = \frac{1}{\sqrt{2}} \left( | s \rangle | n-1 \rangle \right),$$  
(22)

for $n \neq 0$ whereas $| \varphi_{s,0} \rangle = | \varphi_{0,0} \rangle$.

Observe that the coherent state also satisfies $\langle \alpha | \sigma_z | \alpha \rangle = -1$. Thus, any change (oscillation, decay, etc.) in the spin polarization as a function of time is either related to spin flipping due to Rashba SO interaction or to fluctuations in the excitation number due to the periodic driving. This initial state has been considered previously to study photoinduced effects on the Landau levels of monolayer graphene.\(^{33}\) We now explore the effect on the dynamics of “real” spin by the interplay of photoinduced renormalized Rashba SO interaction and Zeeman term.

After straightforward calculations (see Appendix B for detailed derivations), the time-dependent spin polarization $\sigma_z(t) = \langle \alpha | U(t) \sigma_z U(t) | \alpha \rangle$ for the coherent state is given by the expression

$$\sigma_z(t) = -e^{-|\alpha|^2} \left( 1 + \sum_{m=1}^{\infty} \frac{|\alpha|^{2m}}{m!} \left( \frac{\Delta_m}{\varepsilon_m} \right)^2 \cos \left( \frac{2\varepsilon_m t}{\hbar} \right) \right).$$  
(23)

We plot Eq. (23) in Fig. 3 considering an effective dimensionless radiation strength $\kappa = 0.25$, along with $\lambda_B/\hbar \Omega = \delta/\hbar \Omega = 1$ and $B = 1$ T, for different values of the coherent state parameter $\alpha$. At low values of $\alpha$ [panels (a) and (b) in Fig. 3], we find small amplitude Rabi oscillations. This is due to the fact that the dynamics is mostly dictated by the interference of the lowest Floquet-Landau levels. However, for larger values of $\alpha$, the higher Floquet-Landau levels contribute with larger weight to the interference and dynamical localization effects appear. This result is explicitly shown in Fig. 3 (c) and (d) where a strong beating pattern as a function of time is present. This situation is qualitatively similar to the behavior of the pseudo-spin polarization in graphene under periodic illumination.\(^{33}\) However, here the dominant time scale for the dynamical localization is not related to the cyclotron frequency but to the photon frequency, $T = 2\pi / \Omega$. It is also interesting to observe that when collective behavior for the charge carriers sets in the spin polarization can change its sign from $\sigma_z(0) = -1$ (i.e., spin down) to $\sigma_z(t) > 0$. Thus, due to periodic driving we obtain dynamical polarization inversion induced by SO interaction. The latter is the responsible of producing the exchange of angular momentum between orbital...
and spin degrees of freedom necessary to inverse the sign of the spin density.

We can get further insight on the interplay of the Rashba SO interaction and the radiation field by calculating the mean polarization \( \langle \sigma_z \rangle \). This quantity is obtained by averaging the expression \( \langle \sigma_z \rangle \) over one period of oscillation of the radiation field [see Eq. (19)] and reads

\[
\langle \sigma_z \rangle = -e^{-|\alpha|^2} \left( 1 + \sum_{m=1}^{+\infty} \frac{|\alpha|^{2m}}{m!} \left( \frac{\Delta_m}{\epsilon_m} \right)^2 + \frac{2\pi}{\Omega} \left[ 1 - \left( \frac{\Delta_m}{\epsilon_m} \right)^2 \sin\left( \frac{4\pi \epsilon_m}{\hbar \Omega} \right) \right] \right). \tag{24}
\]

In Fig. 4 we show a contour density plot of \( \langle \sigma_z \rangle \) in the \( B - \alpha \) parameter space. As in Fig. 3, we set \( \delta/\hbar \Omega = 1, \kappa = 0.25 \) and consider the photon energy such that \( m^* \lambda^2/\hbar \Omega = 1 \). For BiSb surface gases, this corresponds to THz radiation where \( \hbar \Omega \simeq 10 \text{ meV} \). The general trend observed is that at any value of the static magnetic field \( B \), a large magnitude of the average polarization \( \langle \sigma_z \rangle \) is achieved for both small and large values of the mean occupation (coherent state parameter) \( \alpha \). The value of \( \langle \sigma_z \rangle \) is always negative, \textit{i.e.} there is no polarization inversion in the average signal. For intermediate values of \( \alpha \) the contribution of more Floquet-Landau levels yields clear oscillations of \( \langle \sigma_z \rangle \) as a function of \( B \). For specific values of \( B \), we also find polarization inversion in the average signal. We illustrate this behavior of \( \langle \sigma_z \rangle \) by taking “snapshots” at fixed values of the mean occupation parameter in Fig. 5.

![Graph showing expectation value of spin polarization vs. B parameter space.](image1)

**FIG. 4.** Expectation value of the spin polarization, \( \langle \sigma_z \rangle \) in \( B - \alpha \) parameter space. We consider an effective radiation strength \( \kappa = 0.25 \) and fix the radiation frequency of the incident light beam by setting \( m^* \lambda^2/\hbar \Omega = 1 \).

![Graph showing magnetic field response of spin polarization.](image2)

**FIG. 5.** Magnetic field response of the expectation value of the spin polarization at four representative values of the coherent state parameter \( \alpha \). Values of the effective radiation strength, \( \kappa \), and the radiation frequency, \( \Omega \), are set as in Fig. 4.

### B. Autocorrelation function

We complement the physical picture of the spin dynamics by looking at the autocorrelation function

\[
C(t) = \langle \Psi(0)| \Psi(t) \rangle = \langle \Psi(0)| U(t) \Psi(0) \rangle,
\]

which is simply the overlap between the initial and the time-evolved wave packet. The absolute value of \( C(t) \) provides additional insight on (fractional) quantum revivals induced by the dynamics whenever the time-dependent overlap is close to its maximum value. Analogously to the case of the spin polarization, we consider the coherent state and compute \( C(t) = \langle \alpha | \alpha(t) \rangle \). For that purpose, we first compute the time-evolved coherent state, \( | \alpha(t) \rangle \), by considering the projection of the dynamics into the basis of the static Hamiltonian. The time evolution of the coherent state is then expressed as

\[
| \Psi(t) \rangle = \sum_{s'} f^s_{s'}(t) e^{-|s| \Omega \hbar} | \psi_{s' s} \rangle,
\]

with

\[
f^s_{s'} = c_{-s} b_{-s'} + s s' c_{s} b_{s'}.
\]

After straightforward algebra (see details in Appendix C) we find that the coherent state autocorrelation function

\[
C(t) = \sum_{s'} f^s_{s'}(t) f^{s'}_{s}(0).
\]
Adopting the form
\[ C(t) = e^{-|\alpha|^2} \left( 1 + \sum_{m=1}^{+\infty} \frac{|\alpha|^{2m}}{m!} \{ \cos \left( \frac{\varepsilon_m t}{\hbar} \right) \right. \]
\[ \left. - i \left[ \frac{\delta}{\Delta_m} \sqrt{1 - \left( \frac{\Delta_m}{\varepsilon_m} \right)^2} + \frac{\Delta_m}{\varepsilon_m} \sqrt{1 - \left( \frac{\delta}{\Delta_m} \right)^2} \right] \right) \times \sin \left( \frac{\varepsilon_m t}{\hbar} \right) \right) \} \right) \right) . \] (27)

It can be easily checked that this expression reduces to the result quoted in Ref. 33 for graphene in the limit \( \lambda_B/(\hbar\omega_c) \to +\infty \).

We plot the time evolution of \( |C(\alpha)(t)|^2 \) in Fig. 6. To make the comparison with the time-dependent spin polarization explicit, we consider the same values of \( \alpha \) as in Fig. 3 as well as the same parameters. For small values of \( \alpha \), as shown in Fig. 6 panels (a) and (b), the autocorrelation function presents a strong oscillations reminiscent of the Rabi oscillations between the two lowest Floquet bands. For the considered radiation frequency, our result is different from graphene since here the Rabi oscillations involve the quantum interference of more than two Floquet-Landau levels as it can be seen in the beating pattern of \( |C(\alpha)(t)|^2 \). For larger values of \( \alpha \), see Fig. 6 panels (c) and (d), and especially for \( \alpha \geq 4.0 \), the autocorrelation function shows clear fractional revivals. These revivals are correlated in time with the dynamical localization of the spin polarization shown in Fig. 3.

The fractional revivals occur periodically with a period roughly equal to \( T_R \approx 7T/4 \) when the mean Landau level occupation increases. This means that a full reconstruction of the wavepacket never occurs due to the presence of dephasing. In the present case, compared to a purely relativistic energy spectrum, this dephasing is produced to the radiation-dressed detuning \( \Delta_m \). After several periods, the dephasing between the components of the wave packet increases and the value of \( |C(\alpha)(t) = \mu T)|^2 \) with \( \mu \in N_{>0} \) decreases almost in a linear fashion. The decrease in the amplitude of the local maxima of the autocorrelation function in time is a manifestation of the broadening of the spin polarization signal observed in Fig. 3(d) for times \( t/T \geq 2 \).

**IV. PHOTOCONDUCTIVITY**

We turn now our attention to a transport-related observable, the transverse photoconductivity, \( \sigma_{xy}(\Omega) \). The observable is computed using the Kubo formula (see details for the derivation in Appendix C). We find
\[ \sigma_{xy}(\Omega) = \frac{e^2}{\hbar} (\hbar\omega_c)^2 \sum_{m=0}^{+\infty} \sum_{s' s} \left\{ n_F(\varepsilon_{s' s' m+1}) - n_F(\varepsilon_{s s m}) \right\} \left[ s' \sqrt{m + 1} B_{s' s' m+1}^{s m} + \frac{\lambda_B}{\hbar\omega_c} s' B_{s' s' m+1}^{s m} \right] \] (28)

where \( \Omega \) is the photon frequency and \( \Gamma \) is a parameter describing an effective Floquet-Landau level broadening due to residual scattering of the electron with impurities. To simplify the notation, we have defined in Eq. (28) the combination of wavefunction weights \( B_{s' s' m+1}^{s m} = b_{s m} b_{s' s' m+1} ^* \), and denoted by
\[ n_F(E) = \frac{1}{1 + \exp[(E - \mu)/(k_B T)]}, \] (29)

the Fermi-Dirac distribution at temperature \( T \) and constant chemical potential \( \mu \) (\( k_B \) denotes the Boltzmann constant).

We use this analytical expression to study the combined impact of the radiation and the SO interaction. In Fig. 7(a), we show the transverse photoconductivity obtained from Eq. (28) in the static limit. Different curves correspond to several values of the parameter \( \lambda_B/(\hbar\omega_c) \). For simplicity, we assume a vanishing Zeeman field, \( \Delta = 0 \), taking into consideration a non-vanishing Zeeman field being straightforward. For \( \lambda_B/(\hbar\omega_c) = 0 \) (i.e. at vanishing Rashba SO interaction), we recover from Eq. (25) the expected sequence of quantized Hall plateaus. For non-vanishing Rashba SO coupling, \( \lambda_B/(\hbar\omega_c) \neq 0 \), we observe the appearance of additional plateaus at odd values of the conductance quantum. These plateaus have different widths since the Floquet-Landau levels in the presence of SO interaction are no longer equidistant in
FIG. 7. (a) Static transverse conductivity, \( \text{Re} [\sigma_{xy}(0)] \), as a function of the normalized chemical potential \( \mu / (\hbar \omega_c) \). Each curve corresponds to a different value of the parameters \( \lambda_B / (\hbar \omega_c) \) (measuring the SO coupling strength per magnetic length normalized to the cyclotron energy): 0 (solid line), 0.1 (dotted line), 0.2 (dashed line), 0.3 (dot-dashed line). The effective level broadening is taken to be \( \Gamma / (\hbar \omega_c) = 0.05 \) and the temperature is chosen as \( k_B T / (\hbar \omega_c) = 0.05 \). (b) Transverse photoconductivity, \( \text{Re} [\sigma_{xy}(\Omega)] \) represented as a function of \( \mu / (\hbar \omega_c) \) and \( \omega / \omega_c \). Here, we have taken \( \lambda_B / (\hbar \omega_c) = 0.3 \), and consider the same temperature and level broadening as in panel (a).

FIG. 8. (a) Transverse conductivity, \( \text{Re} [\sigma_{xy}(\Omega)] \), as a function of the quantity \( \mu / (\hbar \Omega_c) \) (where \( \Omega_c = \lambda_B / \hbar \) is a SO-dependent characteristic frequency) obtained in the limit of strong SO interaction, \( \lambda_B / (\hbar \omega_c) \rightarrow +\infty \). The effective energy level broadening and the temperature are chosen as in Fig. 7. We show results for the static (\( \Omega / \Omega_c = 0 \), black line) and the photoinduced cases (\( \Omega / \Omega_c = 0.4 \), red line). (b) Transverse photoconductivity, \( \text{Re} [\sigma_{xy}(\Omega)] \), represented as a function of \( \mu / (\hbar \Omega_c) \) and \( \Omega / \Omega_c \). We consider the same energy level broadening and temperature as in panel (a).

energy. We observe also that for higher Landau levels, the conductance seem to show deviations from the expected quantization in units of \( e^2 / h \). These small deviations, which occur when the Rashba SO interaction start to be relevant compared to the kinetic (quadratic) term in the Hamiltonian, have already been pointed out in previous works combining quadratic and linear dispersion and have been attributed to the perturbative (Kubo) formulation of transport theory used here.\(^{[23]}\)

Fig. 7 (b) shows the photoinduced transverse conductivity on the \( \mu - \Omega \) parameter space (normalized to the cyclotron energy / cyclotron frequency) for \( \lambda_B / (\hbar \omega_c) = 0.3 \) (which corresponds to a magnetic-field renormalized SO interaction typical for BiSb surface gases). An important feature of \( \sigma_{xy}(\Omega) \) is that for fixed chemical potential it exhibits a strong resonant structure close to the cyclotron energy, \( \Omega \simeq \omega_c \). Away from the resonant structure, a plateau-like structure is preserved but at finite frequency the conductivity is no longer quantized in integer units of \( e^2 / h \).

From Eq. [28], we now also consider two limiting cases in which (i) the SO interaction is dominant (ii) the SO interaction vanishes. In both situations, we have recovered from our general expression well-known analytical formulas in the static limit.\(^{[24, 25, 39, 40]}\) In Fig. 8 we show the transverse conductivity in the limit \( \lambda_B / (\hbar \omega_c) \rightarrow +\infty \) (this corresponds to the simultaneous formal limits \( \Delta \rightarrow 0 \) and \( m^* \rightarrow +\infty \)). In this limit, the energy scale related to the Rashba SO interaction dominates and the spectrum becomes gapless in the absence of external magnetic and electric fields. In the presence of magnetic field but no coupling to the radiation field, the Floquet-Landau levels reduce to the well-known expression \( \varepsilon_{sm} = s \hbar \Omega_c \sqrt{m} \). Here, \( \Omega_c = \lambda_B / h = \lambda \sqrt{2} / l_B \) is a SO-dependent characteristic frequency analogous to the graphene cyclotron frequency, with \( \lambda \) playing the role of a constant Fermi velocity. Fig. 8 (a) displays the transverse conductivity in this limit for the static (\( \Omega = 0 \)) and dynamic
mula (see Appendix D). The fact that these resonances matically, they arise from poles in the conductivity for-

quasi-relativistic energy levels in the spectrum. Mathe-

structure with additional resonances. These resonances

σ(b). Compared to Fig. 7 (b),

to the cyclotron energy / cyclotron frequency) in Fig. 8

can be correlated to fractional revivals in the autocorre-

ation field. Using the Floquet-Landau states, we have

carriers and the radiation field mediated by SO interac-

the exchange of angular momentum between the charge

Zeeman interaction results in the appearance of addi-

tional plateaus, all of them having the same width. As

reported previously, we obtain a robust step-like struc-

under illumination departing from the exact quanti-

zation in units of $e^2/\hbar$. This step structure can also be seen in Fig. 9 (b) for the full normalized parameter

space $\mu - \Omega$, together with a clear resonance for $\Omega \simeq \omega_c$. The presence of a single resonance peak occurs as well in the case $\lambda_B \neq 0$, as shown in Fig. 7 (b), suggesting that the impact of the SO interaction in the photo-induced conductivity is, at most, of quantitative nature.

\section{Conclusion and Outlook}

In conclusion, we have studied photoinduced phenomena in 2DEG under perpendicular magnetic fields with Rashba SO interaction. Our work provides perturbative analytical expressions for physical observables valid in the THz / low infrared regime. We have first presented the photoinduced modulation to the Landau energy levels in the presence of a continuous and periodic radiation field. Using the Floquet-Landau states, we have considered the dynamical features on the spin polarization and the autocorrelation function. Assuming that the initially prepared state is a coherent state, which possesses a static finite spin polarization, we have shown that the exchange of angular momentum between the charge carriers and the radiation field mediated by SO interaction (which couples spin and orbital degrees of freedom) can yield spin polarization inversion for certain magnetic field strengths. For large enough values of the coherent state parameter $\alpha$ we have demonstrated that the time evolution of the spin polarization has periodic beating patterns due to dynamical localization and interference of Floquet-Landau levels. Dynamical localization effects can be correlated to fractional revivals in the autocorrelation function. Using linear response (Kubo) formalism, we have also computed the transverse photoconductivity of the Floquet system. We have found from the analytical expression that the SO interaction does not drastically change the resonance structure of the transverse photoconductivity (as compared to the 2DEG under illumina-
Appendix A: Perturbative calculation of the effective Floquet Hamiltonian

In order to obtain the effective Floquet Hamiltonian given in Eq. (15), we evaluate the following expression

\[
H = e^{-i\kappa/2\lambda_1} H_F e^{i\kappa/2\lambda_1},
\]

with the operators \( I_\pm \) (resp. Hermitian and anti-Hermitian) defined as

\[
I_\pm = a^\dagger \sigma_- \pm a \sigma_+.
\]

Using the Baker-Campbell-Hausdorff formula we have

\[
H = H_F - \frac{i\kappa}{2} [I_+, H_F] + \frac{1}{2!} \left( \frac{i\kappa}{2} \right)^2 [I_+, [I_+, H_F]] + \ldots
\]

To leading order in \( \kappa \), only the first commutator needs to be evaluated. We find

\[
[I_+ , H_F ] = -\frac{\delta}{2} [I_+, \sigma_z] + i\lambda_B [I_+, I_-] - \xi [I_+, \sigma_y],
\]

where we have used that \( [I_+, \sigma_y] = 0 \). It is a straightforward task to evaluate the commutators in Eq. (A4), we obtain \( [I_+, \sigma_z] = I_-, [I_+, I_-] = 2\kappa \sigma_z \) and \( [I_+, \sigma_y] = i(a + a^\dagger) \sigma_z \).

Upon substitution of these results in Eq. (A3), we get

\[
H_F^{\text{eff}} = \hbar \omega_c N_a - \left( \frac{\delta - 2\kappa \lambda_B N_c}{2} \right) \sigma_z
\]

\[
+ i \left( \lambda_B + \frac{\alpha \delta}{2} \right) I_- - \frac{\kappa \xi}{2} (a^\dagger + a) \sigma_z - \xi \sigma_y.
\]

We now define the shifted ladder operators \( c = a - \gamma \), with \( \gamma = \frac{2\xi}{\sqrt{2\lambda_B + \kappa \delta}} \). With this definition, the following relations are satisfied

\[
N_a = N_c + \gamma (c^\dagger + c) + \gamma^2,
\]

\[
a^\dagger + a = c^\dagger + c + 2\gamma,
\]

and allow us in turn to write the perturbative Hamiltonian as

\[
H_F^{\text{eff}} = \hbar \omega_c N_c + \gamma \hbar \omega_c (c^\dagger + c) + \gamma^2 \hbar \omega_c
\]

\[
- \left( \frac{\delta - 2\kappa \lambda_B N_c}{2} \right) \sigma_z + \kappa \lambda_B \gamma (c^\dagger + c) + \kappa \lambda_B \gamma^2
\]

\[
+ i \left( \lambda_B + \frac{\kappa \delta}{2} \right) (c^\dagger \sigma_- - c \sigma_+)
\]

\[
- \frac{\kappa \xi}{2} (c + c^\dagger + 2\gamma) \sigma_z.
\]

Our effective Floquet Hamiltonian

\[
H_F^{\text{eff}} = \hbar \omega_c N_c - \left( \frac{\delta - 2\kappa \lambda_B N_c}{2} \right) \sigma_z
\]

\[
+ i \left( \lambda_B + \frac{\kappa \delta}{2} \right) (c^\dagger \sigma_- - c \sigma_+),
\]

where we have neglected higher order terms in \( \gamma \) and \( \xi \)

\[
\Delta H = \frac{2\gamma (\kappa \lambda_B + \hbar \omega_c) - \kappa \xi \sigma_z}{2} (c^\dagger + c)
\]

\[
+ \gamma^2 (\kappa \lambda_B + \hbar \omega_c) - \kappa \gamma \xi \sigma_z.
\]
Appendix B: Calculation details of the spin polarization

We want to evaluate the mean spin polarization

\[ \langle \sigma_z \rangle = \frac{1}{T} \int_0^T dt \sigma_z(t), \]  
(B1)

where \( \sigma_z(t) = \langle \Psi(0)| U^\dagger(t) \sigma_z U(t) |\Psi(0)\rangle \), for the initial state of the system given by the coherent state \( |\alpha\rangle = |\Psi(0)\rangle \),

\[ |\alpha\rangle = e^{-|\alpha|^2/2} \left( |\varphi_0\rangle + \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |\varphi_{sn}\rangle \right). \]  
(B2)

We begin by transforming the coherent state to the eigenbasis of the effective Floquet Hamiltonian

\[ |\alpha\rangle = |\psi_0\rangle \langle \psi_0 | \alpha \rangle + \sum_{n=1}^{\infty} |\psi_{sn}\rangle \langle \psi_{sn} | \alpha \rangle, \]  
(B3)

with expansion coefficients

\[ \langle \psi_0 | \alpha \rangle = e^{-|\alpha|^2/2}, \]  
(B4a)

\[ \langle \psi_{sn} | \alpha \rangle = e^{-|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}} b_{s,n}. \]  
(B4b)

Using the approximation \( H_F \approx \hbar \), we get

\[ e^{-iHT/\hbar} |\alpha\rangle = e^{-|\alpha|^2/2} \left[ e^{-i\delta t/2\hbar} |\psi_0\rangle \right. \]  
\[ \left. + \sum_{m=1}^{\infty} \sum_s \frac{\alpha^m}{\sqrt{m!}} b_{s,m} e^{-i(s\epsilon_m + m\hbar\omega_\tau t)/\hbar} |\psi_{sm}\rangle \right] \]

(B5)

and the results

\[ \langle \psi_0 | \sigma_z | \psi_0 \rangle = -1, \]
\[ \langle \psi_{s'm'} | \sigma_z | \psi_{sm} \rangle = (ss'b_{s-m} b_{s-m} - b_{s'm} b_{s'm}) \delta_{m'm'}, \]

we obtain

\[ \sigma_z(t) = -e^{-|\alpha|^2} \left[ 1 + \sum_{m=1}^{\infty} \sum_{s,s'} \frac{|\alpha|^{2m}}{m!} b_{s,m} b_{s-m} \right. \]  
\[ \times e^{i(s'-s)\epsilon_m} \left( b_{s'm} b_{s'm} - b_{s'm} b_{s'm} \right) \right]. \]  
(B7)

Performing the double sum in \( s \) and \( s' \) we get

\[ \sigma_z(t) = -e^{-|\alpha|^2} \left[ 1 + \sum_{m=1}^{\infty} \frac{|\alpha|^{2m}}{m!} \left( \frac{\Delta_m}{\epsilon_m} \right)^2 \right. \]  
\[ \left. + \left( 1 - \frac{\Delta_m^2}{\epsilon_m} \right) \right]. \]

It can be easily checked that this expression reduces to the result obtained for the pseudospin polarization in irradiated graphene under perpendicular magnetic fields from Ref. [33] in the limit \( \lambda_B/(\hbar \omega_c) \to +\infty \). Finally, upon of integration of \( \sigma_z(t) \) over one period of oscillation of the radiation field, \( T = 2\pi/\Omega \), we obtain the result quoted in Eq. [23].

Appendix C: Calculation details of the autocorrelation function

We want to evaluate the autocorrelation function \( C(t) = \langle \Psi(0)| \Psi(t) \rangle \) for the initial state of the system given by the coherent state \( |\alpha\rangle \). We begin by writing the states in Eq. [6] in terms of the Floquet basis \( |\alpha⟩ \) for which the time evolution is trivial. We find \( |\alpha(t)\rangle \) to be

\[ |\alpha(t)\rangle = e^{-|\alpha|^2} \left[ e^{-i\delta t/2\hbar} |\psi_0\rangle + \frac{1}{\sqrt{2}} \sum_{m=1}^{\infty} \sum_{s,s'} \frac{|\alpha|^{2m}}{m!} \right. \]
\[ \times \left( c_{s'm} b_{s-m} + s's'c_{s'm} b_{s'm} e^{-i(s's'\epsilon_m + m\hbar\omega_\tau t)/\hbar} |\psi_{s'm}\rangle \right]. \]
\( \)  
(C1)

Using Eq. [21] and taking into account the orthogonality properties of the Floquet eigenstates we obtain for \( C(t) \)

\[ C_\alpha(t) = e^{-|\alpha|^2} \left\{ 1 + \frac{1}{2} \sum_{m=1}^{\infty} \sum_{s,s'} \frac{|\alpha|^{2m}}{m!} e^{-i\delta t/\hbar} \right\} \]
\[ \left[ (c_{s'm} b_{s-m})^2 + (c_{s'm} b_{s-m})^2 + (c_{s'm} b_{s-m})^2 \right] \]
\[ + 2(c_{s'm} b_{s-m} + c_{s'm} b_{s-m}) (c_{s'm} b_{s-m} - c_{s'm} b_{s-m}) \}. \]
\( \)  
(C2)

Performing the summation over \( s \) and using Eqs. [7] and [18], we get the expression quoted in Eq. [27].

Appendix D: Calculation details of the photoconductivity

**Generalities.** We compute the photoconductivity for the Floquet system using the Kubo formula

\[ \sigma_{\mu\nu}(\Omega) = \frac{i e^2 \hbar}{L^2} \sum_{\epsilon_a,\epsilon_b} \frac{n_F(\epsilon_b) - n_F(\epsilon_a)}{\epsilon_b - \epsilon_a} \]
\[ \times \left[ \frac{\nu_{ab}^\mu \nu_{ab}^\nu}{\epsilon_b - \epsilon_a - (\hbar \Omega + i\Gamma)} - \frac{\nu_{ab}^\mu \nu_{ab}^\nu}{\epsilon_b - \epsilon_a + (\hbar \Omega + i\Gamma)} \right], \]
\( \)  
(D1)

where the Fermi-Dirac distribution function, \( n_F(E) \), is given by Eq. [29]. In Eq. [D1], \( \nu_{ab}^\mu \) represent the matrix elements of the velocity operator, \( \nu_{ab}^\mu := \langle \psi_a | v_{\mu \nu} | \psi_b \rangle \), where the states \( |\psi_a\rangle \) are given by Eq. [17]. We also consider an effective energy level broadening due to scattering with impurities with the phenomenological parameter \( \Gamma \). This parameter is considered to be substantially smaller than the photon energy \( \hbar \Omega \), which is valid for THz radiation and low impurity concentration. Note that in the standard Kubo formula \( \langle D1 \rangle \), there are no contributions from the Floquet replicas, and therefore, the static limit of the conductivity can be obtained from the limit \( \Omega \to 0 \).

The components of the velocity operator are easily obtained from the equation of motion \( v_{\mu} = [r_{\mu}, H(t)]/(i\hbar) \),
with $r_{\mu}$ ($\mu = x, y$) being the components of the position operator and $H(t)$ given by the full Hamiltonian, $H(t) = H_0 + V(t)$. The commutator can be calculated trivially and the velocity thus reads

$$v_{\mu} = \frac{1}{m^*} \pi_{\mu} + \lambda \epsilon_{\mu\nu\lambda} \sigma_{\nu}, \quad (D2)$$

where $\epsilon_{\mu\nu\lambda}$ is the Levi-Civita symbol. We transform to the Floquet basis $\{c^\dagger, c\}$ to find

$$v_x = \frac{\hbar}{\sqrt{2}\mu m^*}(c^\dagger + c) + \lambda \sigma_y, \quad (D3a)$$

$$v_y = \frac{\hbar}{\sqrt{2}\mu m^*}(c^\dagger - c) - \lambda \sigma_x, \quad (D3b)$$

where we neglected additive terms proportional to $\gamma$ [as they correspond to higher order perturbation terms for $\sigma_{xy}(\Omega)$]. We compute the matrix elements $v_{\mu}^{m,m'}$ using the eigenstates in Eq. (17). As expected, the conductivity comes from the off-diagonal velocities of the $x$ component. The explicit calculation of the velocity matrix elements shows this feature, e.g. the matrix element for the $x$ component is given by

$$v_{x,m,m'} = \left[ \sqrt{\mu'} B_{ss'}^{m,m'} + s s' \sqrt{\mu} \right] \delta_{m,m'+1}$$

$$+ \mu B_{ss'}^{m,m'} \delta_{m,m'-1} + \lambda B_{ss'}^{m,m'} \delta_{m,m'+1}. \quad (D4)$$

Here, we defined the combination of wavefunction weights $B_{ss'}^{m,m'} = b_{sm} b_{s'm'}$. Using that $v_{x}^{a,b} v_{y}^{b,a} = -v_{x}^{a,b} v_{y}^{b,a}$, and taking into account the cancellation of terms due to products of delta functions, we obtain Eq. (28) of the main text.

**Calculation details for the $\lambda_B/(\hbar \omega_c) \to +\infty$ limit.**

When the energy scale related to the Rashba SO interaction dominates over the cyclotron energy, $\lambda_B/\hbar \omega_c \gg 1$, the Floquet-Landau spectrum in Eq. (16) reduces to

$$\varepsilon_{sm} = s \lambda_B \sqrt{m + m^2 \kappa^2}. \quad (D5)$$

We recognize that $\lambda_B$ plays the role of a “graphene-like” cyclotron energy with a SO defined cyclotron frequency $\Omega_c := \lambda_B/\hbar = \lambda \sqrt{2}/l_B$. We consider for simplicity the limit of weak-coupling to the radiation field, $k \ll 1$ and $\varepsilon_{sm} \approx s \hbar \Omega_c \sqrt{m}$. Using that $b_{sm} \simeq 1/\sqrt{2}$ and considering only the dominating term proportional to $\lambda_B$ in Eq. (D1), we find

$$\sigma_{xy}(\Omega) \approx \frac{c^2}{\hbar} \sum_{m=0}^{+\infty} \sum_{s s'} \left( \frac{s' \sqrt{m} + 1}{s' \sqrt{m}} \right)^2 \frac{\rho_F(\varepsilon_{s'+1} - \varepsilon_{sm})}{\rho_F(\varepsilon_{sm})} \left( \frac{\Omega}{\Omega_c} + i \frac{\Gamma}{\Omega_c} \right)^2. \quad (D6)$$

In the static limit ($\Omega \to 0$) with $\Gamma = 0$ this expression reduces to the result presented in, i.e., Refs. [21, 23] and [11] for single surfaces of topological insulators under perpendicular magnetic field.

**Calculation details of the $\lambda_B/(\hbar \omega_c) \to 0$ limit.**

In the limit of vanishing SO interaction, $\lambda_B/\hbar \omega_c \to 0$, the spin, $\sigma = \pm$ becomes a good quantum number (instead of the SO quantum number, $s$). It is easy to check that we can reintroduce the picture of Zeeman-split spin polarized Landau levels provided that we map the energy levels as $(m, s) \to (m, \sigma)$ where $m_\sigma = m - (1 + \sigma)/2$. The eigenenergies (16) then reduce to

$$\varepsilon_{sm} = \hbar \omega_c \left( m + 1 + \frac{1}{2} + \frac{\sigma}{2} \frac{\Delta}{\hbar \omega_c} \right), \quad (D7)$$

where we also assume weak-coupling to the radiation field (so that the photon energy only enters in the poles of the Kubo formula). Using that the coefficients $b_{sm} \simeq \sqrt{(1 - s)}/2$, the transverse conductivity (28) reduces to

$$\sigma_{xy}(\Omega) \approx \frac{c^2}{\hbar} \sum_{m=0}^{+\infty} \sum_{s s'} \omega_c^2 \left( \frac{\omega_c}{\Omega + i \Gamma/\hbar} \right)^2 \rho_F(\varepsilon_{sm}). \quad (D8)$$

a result given in Ref. [23].

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