ON SURFACE THEORY IN 3-DIMENSIONAL ALMOST CONTACT METRIC MANIFOLD

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Abstract. In this paper, we study surface theory in 3-dimensional almost contact metric manifolds by using cross product defined by Camcı [6]. Camcı also studied the theory of curves using the new cross product on this manifolds. In this study, we have defined unit normal vector field of any surface in $\mathbb{R}^3(−3)$ and then, we investigate shape operator matrix of the surface. Moreover, we calculate the formulas of Gaussian and mean curvatures of a surface in $\mathbb{R}^3(−3)$.

1. Introduction

In contact geometry, a lot of studies have been published about curves such as Legendre curves and finite type curves ([1, 2, 3, 4, 5]). Particularly, the Legendre curves are very important in the studies of contact manifolds where a diffeomorphism is a contact transformation if and only if any Legendre curves in a domain of it go to Legendre curves. Moreover, in a 3-dimensional Sasakian manifold, the Legendre curves are studied by Baikoussis and Blair who gave the Frenet 3-frame in this space ([3]). Then, Camcı has studied the curves theory in contact geometry for any curves ([4]).

But, few studies have been published the surface theory in contact geometry since Camcı defined a new cross product in 3-dimensional almost contact metric manifold and studied the theory of curves using this new cross product in this manifold ([6]). And then, Gök has studied the surface theory in 3-dimensional almost contact metric manifold by using cross product defined by Camcı ([8]).

In this paper, we study surface theory in 3-dimensional almost contact metric manifold by using cross product defined by Camcı ([6]) and we define unit normal vector of any surface in $\mathbb{R}^3(−3)$ and then, we investigate shape operator matrix of the surface. Moreover, we calculate the formulas of Gaussian and mean curvature using the new cross product in this manifold.
2. Preliminaries

Let $M$ be a $(2n + 1)$ dimensional differentiable manifold which has a 1-form $\eta$, such that

$$\eta \wedge (d\eta)^n \neq 0$$

on $M$. In this case, $M$ is called contact manifold and $\eta$ is called a contact 1-form. There exists a unique $\xi$, called characteristic vector field of $\eta$, satisfying $\eta(\xi) = 1$ and $d\eta(\xi, X) = 0$ for all $X \in \chi(M)$. $D$ is said to be contact distribution defined by

$$D = \{ x \in \chi(M) : \eta(X) = 0 \}.$$  

$(\varphi, \xi, \eta)$ is called an almost contact structure on $M^{2n+1}$ where $\varphi, \xi, \eta$ are type $(1, 1), (0, 1)$ and $(1, 0)$ tensor field, respectively, satisfying the equations

$$\varphi^2(X) = -X + \eta(X)\xi, \quad \varphi(\xi) = 0, \quad \eta(\xi) = 1$$

where the endomorphism $\varphi$ has rank $2n$.

$(\varphi, \xi, \eta, g)$ is called an almost contact metric structure on $M^{2n+1}$ where $g$ is a Riemannian metric, satisfying

$$g(\varphi(X), \varphi(Y)) = g(X, Y) - \eta(X)\eta(Y),$$

$$g(X, \varphi(Y)) = d\eta(X, Y),$$

$$\eta(X) = g(X, \xi)$$

for all $X, Y \in \chi(M)$.

Let $M$ be a $(2n + 1)$-dimensional manifold which is called Sasaki manifold if it is endowed with a normal contact metric structure $(\varphi, \xi, \eta, g)$. We know that an almost contact metric structure on $M$ is sasakian structure if and only if

$$(\nabla_X \varphi)(Y) = g(X, Y)\xi - \eta(Y)X$$

for all $X, Y \in \chi(M)$, where $\nabla$ is the Riemannian connection of $g$.

Let $(x, y, z)$ be the standard coordinates on $\mathbb{R}^3$. Let consider the 1-form

$$\eta = \frac{1}{2} (dz - ydx)$$

on $\mathbb{R}^3$ and $\xi = 2\frac{\partial}{\partial z}$ on $\mathbb{R}^3$, then we can easily see that $\xi$ is a characteristic vector field.

If the Riemannian metric is defined by

$$g = \frac{1}{4} (dx^2 + dy^2) + \eta \otimes \eta$$

and the endomorphism of $\varphi$ is defined by the matrix

$$\varphi = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & y & 0 \end{bmatrix},$$
then we know that \((\varphi, \xi, \eta, g)\) is a Sasakian structure and the sectional curvature \(\varphi\) of this space is equal to \(-3\). So, it is defined by \(\mathbb{R}^3\). It is well known that

\[
\psi = \left\{ e = e_1 = 2 \frac{\partial}{\partial y}, \varphi(e) = e_2 = 2 \left( \frac{\partial}{\partial x} + y \frac{\partial}{\partial z} \right), \xi = e_3 = 2 \frac{\partial}{\partial z} \right\} \tag{2.1}
\]

is an orthonormal basis with respect to \(g\) in \(\mathbb{R}^3\). Let \(X = x_1 e_1 + x_2 \varphi(e) + x_3 \xi\) and \(Y = y_1 e_1 + y_2 \varphi(e) + y_3 \xi\) be vector fields in \(\mathbb{R}^3\), then we can easily see that \((\mathbb{R}^3, \varphi, \xi, \eta, g)\) is a 3-dimensional almost contact metric manifold and \(\varphi\) and \(\eta\) satisfying the equations

\[
\begin{align*}
\varphi(X) &= -x_2 e + x_1 \varphi(e), \\
\varphi(Y) &= -y_2 e + y_1 \varphi(e), \\
\eta(X) &= x_3.
\end{align*}
\]

In a 3-dimensional almost contact metric manifold, Camci stated the following definition and theorem.

**Definition 2.1.** Let \(M^3 = (M, \varphi, \xi, \eta, g)\) be a 3-dimensional almost contact metric manifold. The cross product \(\wedge: \chi(M) \times \chi(M) \to \chi(M)\) is defined by

\[
X \wedge Y = -g(X, \varphi(Y))\xi - \eta(Y)\varphi(X) + \eta(X)\varphi(Y) \tag{2.2}
\]

where \(X, Y \in \chi(M)\) ([6]).

**Theorem 2.2.** Let \(M^3 = (M, \varphi, \xi, \eta, g)\) be a 3-dimensional almost contact metric manifold. Then, for all \(X, Y, Z \in \chi(M)\) the cross product satisfying the following properties:

i) The cross product is bilinear and antisymmetric.

ii) \(X \wedge Y\) is perpendicular both of \(X\) and \(Y\).

iii)

\[
Y \wedge \varphi(X) = g(X, Y)\xi - \eta(Y)X,
\hat{\varphi}(X) = \xi \wedge X. \tag{2.3}
\]

iv) Define a mixed product by

\[
(X, Y, Z) = g(X \wedge Y, Z)
\]

\[
= -\left[ g(X, \varphi(Y))\eta(Z) + g(Y, \varphi(Z))\eta(X) + g(Z, \varphi(X))\eta(Y) \right] \tag{2.4}
\]

and

\[
(X, Y, Z) = (Y, Z, X) = (Z, X, Y).
\]

v) Define \(g(X, \varphi(Y))Z + g(Y, \varphi(Z))X + g(Z, \varphi(X))Y = -\det(X, Y, Z)\xi\) and

\[
(X \wedge Y) \wedge Z = g(X, Z)Y - g(Y, Z)X,
\]

\[
(X \wedge Y) \wedge Z + (Y \wedge Z) \wedge X + (Z \wedge X) \wedge Y = 0. \tag{2.5}
\]
\begin{align}
\begin{aligned}
g(X \wedge Y, Z \wedge W) &= g(X, Z)g(Y, W) - g(Y, Z)g(X, W), \\
g(X \wedge Y, X \wedge Y) &= \|X \wedge Y\|^2 = g(X, X)g(Y, Y) - g^2(X, Y),
\end{aligned}
\end{align}

(2.7)

for the proofs of the above equalities (see [6, 8]).

3. Shape operator matrix of a surface in 3-dimensional almost contact metric manifold

In this section, we first recall the definition of a shape operator in general mean and then we investigate shape operator matrix of a surface, the formulas of Gaussian and mean curvatures in the 3-dimensional almost contact metric manifold using its unit normal vector field.

**Definition 3.1.** Let \( M \) be a surface in \( \mathbb{E}^n \). The linear map \( S : \chi(M) \to \chi(M) \) defined by

\[ S(X) := DXN, \quad X \in \chi(M), \]

is called the shape operator on \( M \), where \( D \) is the Riemannian connection in \( \mathbb{E}^n \) and \( N \) is the unit normal vector field of the surface \( M \).

**Proposition 1.** Let \( U \) denote an open set in the plane \( \mathbb{R}^2 \). The open set \( U \) will typically be an open disk or open rectangle. Let

\[ X : U \to \mathbb{R}^3(-3) \]

\[ (u, v) \to X(u, v) = (f_1(u, v), f_2(u, v), f_3(u, v)) \]

be a parameterization at a point \( P \in M \) of a surface \( M \) in \( (\mathbb{R}^3(-3), \varphi, \xi, \eta, g) \). Tangent vectors for the \( u \) and \( v \)-parameter curves are given by differentiating of the \( f_i(u, v) \). According to the basis \( \{ e, \varphi(e), \xi \} \) of \( (\mathbb{R}^3(-3), \varphi, \xi, \eta, g) \), we can write

\[ X_u = \frac{1}{2} f_{2,u} e + \frac{1}{2} f_{1,u} \varphi(e) + \frac{1}{2} (f_{3,u} - f_{2,f_{1,u}}) \xi, \quad (3.1) \]

\[ X_v = \frac{1}{2} f_{2,v} e + \frac{1}{2} f_{1,v} \varphi(e) + \frac{1}{2} (f_{3,v} - f_{2,f_{1,v}}) \xi. \quad (3.2) \]

where \( f_{i,u} \) and \( f_{i,v} \) (\( 1 \leq i \leq 3 \)) mean that the first derivatives of \( f_i(u, v) \) according to the \( u \) and \( v \)-parameters.

**Proof.** Tangent vector of the \( u \)-parameter curve on a surface \( M : X(u, v) \) in \( (\mathbb{R}^3(-3), \varphi, \xi, \eta, g) \) is

\[ X_u = f_{1,u} \frac{\partial}{\partial x} + f_{2,u} \frac{\partial}{\partial y} + f_{3,u} \frac{\partial}{\partial z}, \]

from the equation (2.1) we have

\[ X_u = \frac{1}{2} f_{2,u} e + \frac{1}{2} f_{1,u} \varphi(e) + \frac{1}{2} (f_{3,u} - f_{2,f_{1,u}}) \xi.\]
and similarly

\[ X_v = \frac{1}{2} f_{2,v} e + \frac{1}{2} f_{1,v} \varphi(e) + \frac{1}{2} (f_{3,v} - f_{2,f_1,v}) \xi, \]

which complete the proof. \qed

**Theorem 3.2.** Let \( U \) denote an open set in the plane \( \mathbb{R}^2 \) and let \( X : U \to \mathbb{R}^3(-3) \):

\[ (u,v) \mapsto X(u,v) = (f_1(u,v), f_2(u,v), f_3(u,v)) \]

be a parameterization at a point. \( P \in M \) of a surface \( M \) in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \). The unit normal vector field of \( M \) in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \) is given by

\[
N = \frac{1}{4\sqrt{EG - F^2}} \left[ f_{1,u}(f_{3,v} - f_{2,f_1,v}) - f_{1,v}(f_{3,u} - f_{2,f_1,u}) \right] e \\
+ \frac{1}{4\sqrt{EG - F^2}} \left[ f_{2,u}(f_{3,u} - f_{2,f_1,u}) - f_{2,v}(f_{3,v} - f_{2,f_1,v}) \right] \varphi(e) \\
+ \frac{1}{4\sqrt{EG - F^2}} \left( f_{1,v} f_{2,u} - f_{1,u} f_{2,v} \right) \xi
\]

(3.3)

where

\[
E = \frac{1}{4} f_{2,u}^2 + \frac{1}{4} f_{1,u}^2 + \frac{1}{2} (f_{3,u} - f_{2,f_1,u})^2, \\
G = \frac{1}{4} f_{2,v}^2 + \frac{1}{4} f_{1,v}^2 + \frac{1}{2} (f_{3,v} - f_{2,f_1,v})^2, \\
F = \frac{1}{4} f_{2,u} f_{2,v} + \frac{1}{4} f_{1,u} f_{1,v} + \frac{1}{4} (f_{3,u} - f_{2,f_1,u})(f_{3,v} - f_{2,f_1,v}).
\]

(3.4)

(3.5)

(3.6)

**Proof.** From the Definition (2.1), we know

\[ X_u \wedge X_v = -g(X_u, \varphi(X_u)) \xi - \eta(X_u) \varphi(X_u) + \eta(X_u) \varphi(X_u). \]

By using the Proposition (1) and following equations

\[
\varphi(X_u) = -\frac{1}{2} f_{1,u} e + \frac{1}{2} f_{2,u} \varphi(e), \quad \varphi(X_v) = -\frac{1}{2} f_{1,v} e + \frac{1}{2} f_{2,v} \varphi(e), \\
\eta(X_u) = \frac{1}{2} (f_{3,u} - f_{2,f_1,u}), \quad \eta(X_v) = \frac{1}{2} (f_{3,v} - f_{2,f_1,v})
\]

(3.7)

(3.8)

we have

\[
X_u \wedge X_v = \frac{1}{4} \left[ f_{1,u}(f_{3,v} - f_{2,f_1,v}) - f_{1,v}(f_{3,u} - f_{2,f_1,u}) \right] e \\
+ \frac{1}{4} \left[ f_{2,u}(f_{3,u} - f_{2,f_1,u}) - f_{2,v}(f_{3,v} - f_{2,f_1,v}) \right] \varphi(e) \\
+ \frac{1}{4} (f_{1,u} f_{2,v} - f_{1,v} f_{2,u}) \xi.
\]

(3.9)

Then, via the Theorem (2.2) the norm of \( X_u \wedge X_v \) is given by

\[
\|X_u \wedge X_v\| = \left( g(X_u, X_u) g(X_v, X_v) - g^2(X_u, X_v) \right)^{\frac{1}{2}}
\]
where \( g(X_u, X_u) = E, g(X_v, X_v) = F \) and \( g(X_v, X_v) = G \).

Since \( N = \frac{X_u \wedge X_v}{\|X_u \wedge X_v\|} \), we have

\[
N = \frac{1}{4\sqrt{EG-F^2}} \begin{pmatrix} [f_{1,u}(f_{3,v} - f_{2,f_1,u}) - f_{1,v}(f_{3,u} - f_{2,f_1,u})]e \\
+ [f_{2,u}(f_{3,u} - f_{2,f_1,u}) - f_{2,u}(f_{3,v} - f_{2,f_1,v})] \varphi(e) \\
+ (f_{1,u}f_{2,u} - f_{1,u}f_{2,v}) \xi \end{pmatrix},
\]

which completes the proof.

\( \square \)

**Remark 3.3.** Let \( M \subset X(u,v) \) be a surface in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \). We know that all \( u \) and \( v \)-parameter curves are lines of curvature if and only if \( F = 0 \) and \( m = 0 \). So, we consider that they are not lines of curvature. Because, it can easily convert to preceding case.

**Definition 3.4.** Let \( X = x_1e + x_2\varphi(e) + x_3\xi \) and \( Y = y_1e + y_2\varphi(e) + y_3\xi \) be a differentiable vector fields in an open set \( U \subset M \) of a regular surface \( M \) in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \). By using the Christoffel symbols on \( M \) in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \), we have

\[
\nabla_\epsilon \varphi(e) = \xi = -\nabla_\varphi(e) \epsilon, \\
\nabla_\xi e = -\varphi(e) = \nabla_\varphi(e) \xi, \\
\nabla_\varphi(e) = e = \nabla_\varphi(e) \xi, \\
\nabla_\epsilon e = \nabla_\varphi(e) \varphi(e) = \nabla_\xi \xi = 0,
\]

then the covariant derivative for \( M \) in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \) is defined by

\[
\nabla_XY = X[y_1] e + X[y_2] \varphi(e) + X[y_3] \xi - \eta(Y) \varphi(X) - \eta(X) \varphi(Y) - d\eta(X,Y) \xi, \tag{3.10}
\]

(see [4, 8]).

**Proposition 2.** Let \( M \subset X(u,v) \) be a surface in \( \mathbb{R}^3(-3), \varphi, \xi, \eta, g \). The second order-derivatives \( X_{uu}, X_{uv} \) and \( X_{vv} \) are, respectively,

\[
X_{uu} = \frac{1}{2} \left[ f_{2,vv} + f_{1,u}(f_{3,u} - f_{2,f_1,u}) \right] e + \frac{1}{2} \left[ f_{1,vv} - f_{2,u}(f_{3,v} - f_{2,f_1,v}) \right] \varphi(e) + \frac{1}{2} \left[ f_{3,vv} - f_{2,u}f_{1,u} - f_{1,u}f_{2,v} \right] \xi, \tag{3.11}
\]

\[
X_{uv} = \frac{1}{2} \left[ f_{2,vv} + \frac{1}{2} f_{1,v}(f_{3,u} - f_{2,f_1,u}) + \frac{1}{2} f_{1,u}(f_{3,v} - f_{2,f_1,v}) \right] e + \frac{1}{2} \left[ f_{1,vv} - \frac{1}{2} f_{2,u}(f_{3,u} - f_{2,f_1,u}) - \frac{1}{2} f_{2,u}(f_{3,v} - f_{2,f_1,v}) \right] \varphi(e) + \frac{1}{2} \left[ f_{3,vv} - \frac{1}{2} f_{2,u}f_{1,u} - f_{1,u}f_{2,v} - \frac{1}{2} f_{1,v}f_{2,v} \right] \xi, \tag{3.12}
\]
\[ X_{uu} = \nabla_{X_u} X_u \]
\[ = \frac{1}{2} f_{2,uu} e + \frac{1}{2} f_{1,uu} \varphi(e) + \frac{1}{2} (f_{3,uu} - f_{2,u} f_{1,u} - f_{1,uu} f_2) \xi \]
\[ + \eta(X_u) \varphi(X_u) - \eta(X_u) \varphi(X_u) - g(X_u, \varphi(X_u)) \xi \]
\[ = \frac{1}{2} f_{2,uu} e + \frac{1}{2} f_{1,uu} \varphi(e) + \frac{1}{2} (f_{3,uu} - f_{2,u} f_{1,u} - f_{1,uu} f_2) \xi \]
\[ - \frac{2}{2} (f_{2,uu} + f_{1,u}(f_{3,u} - f_{2} f_{1,u})) e + \frac{1}{2} (f_{3,uu} - f_{2,u} f_{1,u} - f_{1,uu} f_2) \varphi(e) \]
\[ + \frac{1}{2} (f_{3,uu} - f_{2,u} f_{1,u} - f_{1,uu} f_2) \xi. \]

where \( f_{i,uu}, f_{i,uv} \) and \( f_{i,uv} \) \( 1 \leq i \leq 3 \) mean that the second derivatives of \( f_i(u,v) \) according to the \( u \) and \( v \) parameters.

**Proof.** From the definition of covariant derivative

\[ X_{uv} = \frac{1}{2} \left[ f_{2,uv} e + \frac{1}{2} f_{1,u} (f_{3,v} - f_{2} f_{1,v}) + \frac{1}{2} f_{1,v} (f_{3,u} - f_{2} f_{1,u}) \right] \]
\[ + \frac{1}{2} f_{1,uv} e + \frac{1}{2} f_{2,u} (f_{3,v} - f_{2} f_{1,v}) - \frac{1}{2} f_{2,u} (f_{3,v} - f_{2} f_{1,v}) \varphi(e) \]
\[ + \frac{1}{2} \left[ f_{3,uv} - f_{2,u} f_{1,v} - f_{1,uv} f_2 - \frac{1}{2} f_{1,v} f_{2,u} \right] \xi, \]
\[ X_{vv} = \frac{1}{2} \left[ f_{2, vv} e + \frac{1}{2} f_{1,v} (f_{3,v} - f_{2} f_{1,v}) \right] \]
\[ + \frac{1}{2} f_{1, vv} e + \frac{1}{2} f_{2,u} (f_{3,v} - f_{2} f_{1,v}) \varphi(e) \]
\[ + \frac{1}{2} \left[ f_{3, vv} - f_{2,u} f_{1,v} - f_{1, vv} f_2 \right] \xi. \]

These complete the proof. \( \square \)

**Theorem 3.5.** Let \( M : X(u,v) \) be a surface in \( (\mathbb{R}^3,-3), \varphi, \xi, \eta, g \). The shape operator matrix of \( M \) in \( (\mathbb{R}^3,-3), \varphi, \xi, \eta, g \) is

\[
S = \begin{bmatrix}
G_{1,F_m} & E_{m-F_1} \\
E_{m-F_2} & G_{m-F_2}
\end{bmatrix}
\]
where \( l = g(N, X_{uu}) \), \( m = g(N, X_{uv}) \) and \( n = g(N, X_{vv}) \).

**Proof.** We need expressions of \( S(X_u) \) and \( S(X_v) \) in terms of the basis for \( \{X_u, X_v\} \). We can write \( S(X_u) = aX_u + bX_v \) and \( S(X_v) = cX_u + dX_v \). Our aim is to find \( a, b, c \) and \( d \). If we can compute \( g(S(X_u), X_u) \) and \( g(S(X_u), X_v) \), we find

\[
a = \frac{Gl - Fm}{EG - F^2}, \quad b = \frac{Em - Fl}{EG - F^2}
\]

and similarly if we can compute \( g(S(X_v), X_u) \) and \( g(S(X_v), X_v) \), we know

\[
c = \frac{Gm - Fn}{EG - F^2}, \quad d = \frac{En - Fm}{EG - F^2}.
\]

Consequently, since we know that

\[
S(X_u) = \frac{Gl - Fm}{EG - F^2}X_u + \frac{Em - Fl}{EG - F^2}X_v
\]

\[
S(X_v) = \frac{Gm - Fn}{EG - F^2}X_u + \frac{En - Fm}{EG - F^2}X_v
\]

we have

\[
S = \begin{bmatrix}
\frac{Gl - Fm}{EG - F^2} & \frac{Em - Fl}{EG - F^2} \\
\frac{Gm - Fn}{EG - F^2} & \frac{En - Fm}{EG - F^2}
\end{bmatrix}
\]

where the matrix is in terms of the basis for \( \{X_u, X_v\} \). These complete the proof. \( \square \)

**Theorem 3.6.** Let \( M : X(u, v) \) be a surface in \( \mathbb{R}^3(-3, \varphi, \xi, \eta, g) \). According to the shape operator matrix of the surface, the Gaussian curvature of \( M \) is

\[
K = \frac{ln - m^2}{EG - F^2} \tag{3.15}
\]

where \( l, n, m, E, G \) and \( F \) are defined in the above equalities.
Proof. From the definition of Gaussian curvature $K$ for the matrix $S = \begin{bmatrix} Gl - Fm & Em - Fl \\ EG - F^2 & EG - F^2 \end{bmatrix}$, we may write that

\[
K = \det S = \begin{vmatrix} Gl - Fm & Em - Fl \\ EG - F^2 & EG - F^2 \end{vmatrix} = \frac{EG (\ln - n^2) - F^2 (\ln - m^2)}{(EG - F^2)^2} = \frac{(EG - F^2) (\ln - m^2)}{(EG - F^2)^2} = \frac{ln - m^2}{EG - F^2}
\]

which completes the proof. □

Theorem 3.7. Let $M : X(u, v)$ be a surface in $(\mathbb{R}^3(-3), \varphi, \xi, \eta, g)$. According to the shape operator matrix of the surface, the mean curvature of $M$ is

\[
H = \frac{1}{2} \left( \frac{Gl + En - 2Fm}{EG - F^2} \right) \tag{3.16}
\]

where $l, n, m, E, G$ and $F$ are defined previously.

Proof. From the definition of mean curvature $H$ for the matrix $S = \begin{bmatrix} Gl - Fm & Em - Fl \\ EG - F^2 & EG - F^2 \end{bmatrix}$, we may write that

\[
H = \frac{1}{2} \tr S = \frac{1}{2} \left( \frac{Gl - Fm + En - Fm}{EG - F^2} \right)
\]

\[
H = \frac{1}{2} \left( \frac{Gl + En - 2Fm}{EG - F^2} \right)
\]

which completes the proof. □

ÖZET: Bu makalede, 3-boyutlu hemen hemen kontak manifoldlarda Camcı [6] tarafından tanımlanan dış çarpmeyardımyyla yüzeyler gözönünde bulunduruldu. Camcı, çalışmasında tanımladığı bu dış çarpmın kullanarak bu tip manifoldlarda eğriler teorisini çalıştı. Bu çalışmada $\mathbb{R}^3(-3)$ uzayında herhangi bir yüzeyin birim normal vektör alanı tanımlandı ve bu yüzeye ait şekil operatörü matrisi araştırıldı. Dahasi, bu yüzeyin Gauss ve ortalama eğriliklerinin formları hesaplandı.
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