2D-DOA Estimation in Arc-Array with a DNN Based Covariance Matrix Completion Strategy

YE TIAN¹,², RURU MEI¹,², YONGHUI HUANG¹,², XIAOGANG TANG³, TIANTIAN CUI⁴

¹National Space Science Center, Chinese Academy of Sciences, Beijing, 100190, China
²University of Chinese Academy of Sciences, Beijing, 100190, China
³School of Aerospace Information, Space Engineering University, Huairou District, Beijing, 101416, China
⁴Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing, 100084, China

Corresponding author: Xiaogang Tang (e-mail: titantxg@163.com).

This work was supported by the National Natural Science Foundation of China under Grant No.62027801

ABSTRACT Two-dimensional direction of arrival (2D-DOA) estimation, or estimating the azimuth and elevation angles of sources simultaneously, is an increasingly important area in array signal processing. This paper presents a 2D-DOA algorithm together with a novel structure of array named by Arc-Array (ArcA). The ArcA is enlarged to a virtual Uniform Circular Array (UCA) through a Deep-Neural-Network (DNN) based covariance matrix completion strategy; afterwards, the MUSIC algorithm is performed with the completed covariance matrix. The proposed method is named as ArcA-DNN, and the performance of ArcA-DNN is evaluated by computer simulations. The simulation results indicate that the performance of 2D-DOA estimation in ArcA is able to approach that of a complete UCA; meanwhile, the number of physical elements is substantially reduced compared to the UCA. Moreover, the proposed ArcA-DNN algorithm gives access to implementing underdetermined 2D-DOA estimation with reasonable results.

INDEX TERMS 2D-DOA Estimation, UCA, ArcA, Covariance Matrix Completion, Deep Learning.

I. INTRODUCTION

THE 2D-DOA estimation is a fascinating problem in the scope of Radar [1], Sonar [2], and Wireless Communication Systems [3], etc. The UCA is extensively utilized for the 2D-DOA estimation due to its attractive advantages of covering 360° azimuthal space, offering elevational information and being easy to set up. Many superior Direction Finding (DF) algorithms have been extended to the UCA scenario such as UCA-RB-MUSIC, UCA-ESPRIT [4], and UCA-RARE [5], etc. Notably, all these algorithms require a sufficiently large number of antenna elements to avoid aliasing in the beamspace [6]. Meanwhile, a large aperture array, formed by a large number of antennas, can provide better resolution and estimation accuracy. However, it is challenging to implement a receiver with multiple coherent channels, which is required by the DF systems; moreover, its hardware cost increases dramatically with the number of radio frequency (RF) chains. Strikingly, a unique array configuration named Sparse Circular Array (SCA) has been proposed [7]–[9] in order to preserve advantages of a large aperture UCA with fewer antenna elements.

Sparse arrays such as the coprime array [10] and nested array [11] have received considerable attention in the community of array signal processing. The sparse array is able to offer additional degrees of freedom (DOFs) over the number of antennas, which means the effective aperture of array is enlarged and underdetermined DOA estimation can be performed. However, the defective structures of sparse arrays lead to new problems. Compressive sensing methods combined with sparse arrays offer a novel perspective on solving these problems. The compressive sensing method is widely used in 1D-DOA estimation [12] and 2D-DOA estimation [13] systems, due to its excellent ability to handle the low SNR signals and sparse arrays. Nevertheless, the majority of articles focus on Sparse Linear Array (SLA) by exploiting the Toeplitz property of its covariance matrix [14], [15], in order to form a coarray [16], [17] with more antennas. Meanwhile, some works focus on the design strategies of SLA to increase DOFs [18], [19].

Unfortunately, the above algorithms originating from SLA are difficult to be extended to the SCA scenarios because of the non-Vandermonde structured steering vector of the circular array. Some efforts have been tried by [8], [20]. In [8], a covariance matrix completion algorithm is proposed...
for a well-designed SCA, yet the DOAs of sources are assumed to be confined in the plane where the SCA is located. Additionally, it is difficult to perform a coarse estimation of DOAs required by the initialization of this algorithm. In [20], a Khatri–Rao (KR) subspace approach has been extended to UCA, which is able to perform underdetermined 1D-DOA estimation of quasi-stationary signals. Apparently, the premises imposed on the sources limit the practical application. So far, a solution for 2D-DOA estimation problem in SCA is still unclear.

To close the gap, we propose a covariance matrix completion method based on DNN. Inspired by [21], the covariance matrix completion problem is formulated as a regression problem. Interestingly, compared with SCA, we find that the Arc-Array (ArcA) is more suitable for DNN to figure out the covariance matrix completion problem. Afterward, a 2D spatial spectrum search is performed, based on the classical MUSIC [22] algorithm. The proposed method is named as ArcA-DNN.

The main contributions of our works are summarized as follows:

1) A novel structure of array named ArcA is proposed to implement 2D-DOA estimation.
2) A DNN based covariance completion strategy is adopted for the 2D-DOA estimation in ArcA, and the 2D spatial spectrum search is performed.
3) Computer simulations of 2D-DOA estimation in ArcA are carried out to demonstrate the proposed ArcA-DNN algorithm is able to preserve the advantages of UCA with fewer antenna elements.
4) The proposed algorithm is able to perform underdetermined 2D-DOA estimation in ArcA with a reasonable performance.

The remainder of this paper is organized as follows. Section II describes the system model of 2D-DOA estimation in ArcA. In Section III, the proposed DNN is introduced. The simulation results and related discussions are included in Section IV. Finally, Section V concludes the paper.

**Notations:** In this paper, boldface lowercase letters such as \( \mathbf{a}, \mathbf{b} \) denote vectors, and boldface uppercase letters such as \( \mathbf{A}, \mathbf{B} \) denote matrices. \( \mathbf{I}_N \in \mathbb{R}^{N \times N} \) is the identity matrix. \( [\mathbf{A}]_{i,j} \) denotes the element of \( \mathbf{A} \) coordinated by \( (i, j) \); \( [\mathbf{a}]_i \) denotes the \( i \)-th component of vector \( \mathbf{a} \). Superscripts \( (\cdot)^{-1}, (\cdot)^*, (\cdot)^T \), and \( (\cdot)^H \) denote the inverse operation, complex conjugate, transpose, and conjugate transpose, respectively. Moreover, \( \text{diag}\{\mathbf{a}, n\} \) denotes a column vector formed by the elements of the \( n \)-th diagonal of matrix \( \mathbf{A} \); meanwhile, \( \text{diag}\{\mathbf{a}, n\} \) denotes a square matrix with the components of vector \( \mathbf{a} \) embedded in its \( n \)-th diagonal; notably, \( n = 0 \) is the main diagonal, \( n > 0 \) is above the main diagonal and \( n < 0 \) is below the main diagonal. \( \|\mathbf{a}\| \) denotes the Euclidean norm of vector \( \mathbf{a} \). \( j = \sqrt{-1}; \text{Re}\{z\} \) and \( \text{Im}\{z\} \) are the real part and imaginary part of the complex number \( z \), respectively. \( (\mathbf{a}, \mathbf{b}) \) is the inner product of vectors \( \mathbf{a} \) and \( \mathbf{b} \).

**II. SYSTEM MODEL**

Consider \( D \) uncorrelated far-field narrowband sources from directions \( \{\theta_d, \varphi_d\}_{d=1}^{D} \) impinging on an ArcA. As shown in Fig. 1, the ArcA consists of \( N_p \) physical isotropic antenna elements distributed over a circle with radius \( R \). The \( N_p \)-element ArcA is part of the \( N \)-element UCA; and the other part of UCA, named as ImCA (Imaginary Circular Array), is filled by \( N_I = N - N_p \) imaginary antennas. The ArcA part is labeled as \( \{1, 2, \ldots, N_p\} \), and the ImCA part is labeled as \( \{N_p + 1, N_p + 2, \ldots, N\} \), in counterclockwise order. We try to recover the covariance matrix of the UCA from snapshots sampled by the ArcA. The array manifold matrix \( \mathbf{A} \) of the UCA is given by

\[
\mathbf{A} = [\mathbf{a}(\theta_1, \varphi_1), \ldots, \mathbf{a}(\theta_D, \varphi_D)] \in \mathbb{C}^{N \times D},
\]

where

\[
\mathbf{a}(\theta_d, \varphi_d) = [a_1(\theta_d, \varphi_d), \ldots, a_N(\theta_d, \varphi_d)]^T \in \mathbb{C}^N,
\]

is the \( d \)-th steering vector, and

\[
a_n(\theta_d, \varphi_d) = e^{j2\pi R \sin \theta_d \cos (\varphi_d - \gamma_n)},
\]

is the phase difference between the array center and the \( n \)-th antenna. The angular coordinate of this antenna is denoted by \( \gamma_n = \frac{2(n - \frac{D}{2})}{N} \pi \). Clearly, the array manifold matrix of ArcA \( \mathbf{A}_P \) is the first \( N_P \) rows of \( \mathbf{A} \). Meanwhile, \( \mathbf{A}_P \) can be augmented into a matrix \( \mathbf{A}_A = [\mathbf{A}_P^T; \mathbf{0}_{N_I \times D}]^T \in \mathbb{C}^{N \times D} \) of the same size as \( \mathbf{A} \).

**FIGURE 1: System Model**

The \( k \)-th snapshot sampled by ArcA is expressed as

\[
\mathbf{x}_A(k) = \mathbf{A}_A \mathbf{s}(k) + \mathbf{n}_A(k), \quad k = 1, 2, \ldots, K,
\]

where \( \mathbf{s}(k) = [s_1(k), \ldots, s_D(k)]^T \) is the signal vector of sources; and \( \mathbf{n}_A(k) = [n_P(k)^T, \mathbf{0}_{N_I \times 1}]^T \) is the augmented noise vector, where

\[
\mathbf{n}_P(k) \sim \mathcal{CN}(0, \sigma^2_n \mathbf{I}_{N_P}),
\]
is the additive white Gaussian noise with zero mean and $\sigma_n^2$ variance. The augmented covariance matrix of the received data can be reasonably estimated as

$$\hat{\mathbf{R}}_A = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_A(k)\mathbf{x}_A(k)^H, \quad (6)$$

where $K$ is the number of snapshots collected by ArcA. The structure of the matrix $\hat{\mathbf{R}}_A$ is plotted in Fig. 2.

Similarly, the $k$-th snapshot received by the complete UCA is written as

$$\mathbf{x}(k) = \mathbf{A} \mathbf{s}(k) + \mathbf{n}(k), \quad k = 1, 2, \ldots, K, \quad (7)$$

with $\mathbf{n}(k) \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I}_N)$. Besides, in the same way, the covariance matrix of UCA is estimated as

$$\hat{\mathbf{R}}_{\text{UCA}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}(k)^H. \quad (8)$$

Furthermore, the 2D-DOA estimation can be conducted with the classical MUSIC algorithm as follows.

$$\hat{\mathbf{R}}_{\text{UCA}} = \hat{\mathbf{U}}_s\hat{\mathbf{A}}_s\hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n\hat{\mathbf{A}}_n\hat{\mathbf{U}}_n^H, \quad (9)$$

where $\hat{\mathbf{U}}_s$ and $\hat{\mathbf{U}}_n$ are the estimated signal and noise subspace, respectively. Accordingly, the 2D spatial spectrum is formed as

$$f(\theta, \varphi) = \frac{1}{\|\mathbf{a}(\theta, \varphi)^\mathbf{H}\hat{\mathbf{U}}_n\|^2}. \quad (10)$$

We try to recover $\hat{\mathbf{R}}_{\text{UCA}}$ by filling in the blanks of $\hat{\mathbf{R}}_A$. This problem has been solved well in the case of SLA due to the Toeplitz property of the covariance matrix. However, this excellent property is not equipped in the covariance matrix of UCA, which leads to a tricky problem.

### III. DEEP NEURAL NETWORK FOR COVARIANCE COMPLETION

In this section, a well-designed DNN is adopted to solve the above covariance matrix completion problem. Obviously, there are complex relationships between the zero and nonzero elements of $\hat{\mathbf{R}}_A$, but this is precisely what the DNN is good at. The DNN is an excellent tool for solving the matrix completion problem. After extensive training, this nonlinear mapping is recorded in the connections between layers. Afterward, the DNN is able to approach $\hat{\mathbf{R}}_{\text{UCA}}$ with $\hat{\mathbf{R}}_A$ as the raw material; the covariance matrix reconstructed from the
outputs of DNN is denoted by $\mathbf{R}_{\text{DNN}}$. This process is briefly abstracted into

$$\mathbf{R}_{\text{DNN}} = \mathcal{M}\mathcal{R}\{\mathcal{N}(\mathbf{R}_A)\},$$  \hspace{1cm} (11)

where $\mathcal{N}(\cdot)$ is the output of the DNN, and $\mathcal{M}\mathcal{R}\{\cdot\}$ denotes a Matrix Reconstruction operator detailed later.

### A. DNN ARCHITECTURE

The architecture of DNN is shown in Fig.3, which begins with an input layer with $N_2^2$ neurons, followed by multiple hidden layers and an output layer with $N_2^2$ neurons. After many trials, the number of hidden layers is set to $L_h = 7$. Both the first and last hidden layers are composed of $N_h = 2048$ neurons, and each of the middle hidden layers is composed of $N_m = 4096$ neurons. The ReLU (rectified linear unit) is employed as the nonlinear activation function of the hidden layers, which is defined as

$$\text{ReLU}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$  \hspace{1cm} (12)

Furthermore, the mapping between the output and the input of the DNN can be written as

$$\mathcal{N}(\mathbf{R}_A) = f^{(L_h+1)}(f^{(L_h)}(\cdots f^{(1)}(\mathcal{M}\mathcal{V}\{\mathbf{R}_A\})),$$  \hspace{1cm} (13)

where $f^{(n)}$ abstractly denotes the mapping from the $(n-1)$-th layer to the $n$-th layer. Similarly, $\mathcal{M}\mathcal{V}\{\cdot\}$ denotes a Matrix Vectorization operator detailed in the next subsection. The parameters of DNN denoted by $w$ are learned and updated through offline training to minimize the loss function.

### B. MATRIX VECTORIZATION

The proposed DNN accepts the vectorized covariance matrix $\mathcal{M}\mathcal{V}\{\mathbf{R}_A\}$ as input; the output is also a vector of the size same as the input. We elaborate on the process of Matrix Vectorization ($\mathcal{M}\mathcal{V}\{\cdot\}$) as follows. The process is visualized in Fig.4. It is well known that the covariance matrix is a Hermitian matrix; therefore, only the upper triangular part has been utilized. The $r$-th diagonal of $\mathbf{R}_A$ is given by

$$d^A_r = \text{diag}(\mathbf{R}_A, r), \, r \in \{0, 1, \cdots, N - 1\}.$$  \hspace{1cm} (14)

Notably, $d^A_r \in \mathbb{R}^N$ is a real vector due to its elements stem from the autocorrelation; yet $d^A_r \in \mathbb{C}^{N - r}$ is a complex vector under $r \in \{1, 2, \cdots, N - 1\}$. In order to be the input of DNN, the complex vector $d^A_r$ needs to be realized as follows,

$$\mathcal{R}\{d^A_r\} = \begin{bmatrix} \text{Re}\{d^A_{r1}\} \\ \text{Im}\{d^A_{r1}\} \\ \text{Re}\{d^A_{r2}\} \\ \text{Im}\{d^A_{r2}\} \\ \vdots \\ \text{Re}\{d^A_{r,N-r}\} \\ \text{Im}\{d^A_{r,N-r}\} \end{bmatrix} \in \mathbb{R}^{2(N-r)},$$  \hspace{1cm} (15)

where the real and imaginary parts of each element of $d^A_r$ are arranged into $\mathcal{R}\{d^A_r\}$, alternately. So far, the vectorization of the upper triangular part of $\mathbf{R}_A$ is given by
which is written as

\begin{equation}
\mathcal{MV}\{\hat{A}\} = \begin{bmatrix}
  d_0^A \\
  \mathcal{R}\{d_1^A\} \\
  \mathcal{R}\{d_2^A\} \\
  \vdots \\
  \mathcal{R}\{d_{N-1}^A\}
\end{bmatrix} \in \mathbb{R}^{N^2}.
\end{equation}

C. MATRIX RECONSTRUCTION

For brevity, the output of DNN $\mathcal{N}(\hat{A})$ is denoted by $h$, which is a real vector of the same size as $\mathcal{MV}\{\hat{A}\}$. Next, the covariance matrix $\hat{R}_DNN$ is reconstructed from $h$. Firstly, the real vector $h \in \mathbb{R}^{N^2}$ is split into $N$ sub-vectors as follows:

\begin{equation}
\begin{aligned}
  h_0 &= [h]_{1:N} \\
  h_r &= [h]_{u_r:v_r}, \quad r \in \{1, 2, \ldots, N-1\},
\end{aligned}
\end{equation}

where $u_r = N + 1 + \sum_{s=1}^{r-1}(N - s)$ and $v_r = N + \sum_{s=1}^{r-1}(N - s)$ are respectively the starting and ending index corresponding to the subvector $\mathcal{R}\{d_s^A\}$ embedded in $\mathcal{MV}\{\hat{A}\}$.

Secondly, each real subvector $h_r \in \mathbb{R}^{2(N-r)}$ is mapped to a complex vector $c_r \in \mathbb{C}^{N-1}$ in terms of

\begin{equation}
c_r = \mathcal{C}\{h_r\},
\end{equation}

where the $k$-th element of $c_r$ is calculated as $[c_r]_k = [h_r]_{2k-1} + j[h_r]_{2k}$. Then the reconstruction of $\hat{R}_{DNN}$ is given as:

\begin{equation}
\begin{aligned}
  \hat{R}_{DNN} &= \mathcal{M}\mathcal{R}\{h\} \\
  &= \text{diag}\{h_0, 0\} \\
  &\quad + \sum_{r=1}^{N-1}\left(\text{diag}\{h_r, r\} + \text{diag}\{h_r^*, -r\}\right).
\end{aligned}
\end{equation}

Next, the MUSIC method can be adopted to perform 2D-DOA estimation (9) based on the completed covariance matrix $\hat{R}_{DNN}$. However, the source enumeration must be implemented before MUSIC, due to the number of sources $D$ is required as prior knowledge in the key step of the MUSIC method, which is the separation of the signal and noise subspaces. Fortunately, there are already many papers that provide methods for source enumeration (Please refer to [23]–[25] for details). Finally, the main steps of the proposed ArcA-DNN algorithm are summarized in Algorithm 1.

D. TRAINING STRATEGY

Similarly, the vectorization of $\hat{R}_{UCA}$ is written as $l = \mathcal{MV}\{\hat{R}_{UCA}\}$ for brevity. In the training stage, $\mathcal{MV}\{\hat{R}_{UCA}\}$ is selected as the label vector. The setup of training is illustrated in Fig.5.

The loss function $\mathcal{L}(h, l)$ is a weighted sum of the mean squared error (MSE) loss and cosine similarity (CS) loss, which is written as

\begin{equation}
\mathcal{L}(h, l) = \eta \times \mathcal{L}_{\text{MSE}}(h, l) + (1-\eta) \times \mathcal{L}_{\text{CS}}(h, l),
\end{equation}

with

\begin{equation}
\mathcal{L}_{\text{MSE}}(h, l) = \|h - l\|^2,
\end{equation}

and

\begin{equation}
\mathcal{L}_{\text{CS}}(h, l) = \frac{\langle h, l \rangle}{\|h\|\|l\|}.
\end{equation}

During the training stage, the parameters of DNN are optimized to minimize the loss function (20). The Adam optimizer together with the error backpropagation (BP) strategy is used for fine-tuning. In order to improve the training efficiency, the learning rate is exponentially decaying as the training proceeds. Meanwhile, the total training data is randomly divided into several batches. In summary, the minimization problem is formulated as follows

\begin{equation}
w_{\text{opt}} = \arg\min_{w} \mathcal{L}(h, l),
\end{equation}

where $w$ is the optimal vector of parameters in the DNN after training with 100 iterations.

The parameters of training for the DNN are listed in Table 1. A stochastic gradient descent (SGD)-based Adam optimizer is adopted. The initial learning rate is set to 0.001 and decayed by 0.04 in each epoch.
IV. SIMULATION RESULTS

A. CONDITION FOR MATRIX COMPLETION

In this subsection, the simulation results are presented to illustrate the selection of \( \{ N, N_p \} \) to ensure successful completion of the covariance matrix.

Firstly, \( N \) is set to an odd number. In this case, there are no such antenna pairs whose coordinates are symmetrical about the center, due to \( \frac{360^\circ}{N} \) is not an integer. Therefore, except for the conjugate elements, each correlation is unique in the covariance matrix of the UCA with an odd \( N \), or the covariance matrix is more informational. The limited number of precious antennas are fully utilized in this case.

Next, we consider the selection of \( N_p \). Based on (16), the proportion of zeros in the input vector of DNN is \( 1 - \left( \frac{N_p}{N} \right)^2 \). Obviously, the number of non-zero elements in the input vector increases with \( N_p \). Since those non-zero elements are the basic materials for the DNN to fill in the blanks of the covariance matrix, the larger \( N_p \) leads to more successful matrix completion. However, our starting point is to use as few antennas in a circular array as possible to perform the 2D-DOA estimation. Therefore, the following simulations are performed to explore the lower bound of \( N_p \) leading to successful matrix completion.

The results are summarized in Table 2. The array configuration selected under each \( \{ N, N_p \} \) is ArcA, that is, the labels of physical antennas are consecutive. The SNR and \( K \) are chosen as 15 dB and 1024 (moderate value), also the number of sources \( D \in [1, N_p] \) is an integer variable. If the combination of \( \{ N, N_p \} \) passes the training phase of the DNN, it is labeled Success, otherwise, it is labeled Failed.

From the above results, we can roughly draw the following empirical conclusions to select \( N_p \) for a successful covariance matrix completion, which is given by:

\[
N_p \geq \frac{N + 1}{2}. \tag{24}
\]

B. RMSE OF DOA ESTIMATION

In this section, we evaluate the performance of the ArcA-DNN algorithm. The estimated values of DOAs \((\hat{\theta}_d, \hat{\varphi}_d)\) result from peak searching in the 2D spatial spectrum obtained by (10). The RMSE of 2D-DOA estimation is defined as

\[
\text{RMSE} = \sqrt{\frac{1}{PD} \sum_{p=1}^{P} \sum_{d=1}^{D} [(\varphi_d^p - \hat{\varphi}_d^p)^2 + (\theta_d^p - \hat{\theta}_d^p)^2]}, \tag{25}
\]

where \((\hat{\theta}_d^p, \hat{\varphi}_d^p)\) is the ground truth value of the \( d \)-th source in the \( p \)-th Monte-Carlo trial. Other related simulation parameters are given in Table 3.

| Parameters | Values |
|------------|--------|
| Number of Antennas in UCA: \( N \) | 9 |
| Number of Antennas in ArcA: \( N_p \) | 5 |
| Number of Sources: \( D \) | 1,2,3,4,5 |
| Number of Snapshots: \( K \) | 128, 256, ..., 8192 |
| Number of Monte-Carlo trials: \( P \) | 500 |
| SNR | 5:30 dB |
| \( \theta \) Search Grid | \( 0^\circ : 0.1^\circ : 90^\circ \) |
| \( \varphi \) Search Grid | \( 0^\circ : 0.1^\circ : 360^\circ \) |

Firstly, we study the RMSE versus SNR and \( K \) in scenarios of various \( D \). The DOAs are randomly selected from the 2D angular space \((\{\theta_d, \varphi_d\} | \theta_d \in (0^\circ, 90^\circ], \varphi_d \in (0^\circ, 360^\circ]\}) \) in each Monte-Carlo trial, in order to avoid the particularity of the simulation. The RMSE of DOA estimation with the completed covariance matrix \( \mathbf{R}_{\text{DNN}} \) is labeled as ArcA-C (Completed Arc-Array); meanwhile, the RMSE of DOA estimation with the covariance matrix obtained by UCA \( \mathbf{R}_{\text{UCA}} \) is compared in each subplot (labeled as UCA), in order to show the deviation of \( \mathbf{R}_{\text{DNN}} \) from \( \mathbf{R}_{\text{UCA}} \); in this way, the UCA curve performs as a lower bound. In addition, the RMSE of DOAs obtained by ArcA, or the non-zero part of the sample covariance matrix (labeled as ArcA), is also plotted in Fig.6.7(a)-(c) for better comparison. While the results of ArcA are not provided in the scenario of \( D = 4 \) (Fig.6.7(d)), due to its extremely low success rate.
As shown in Fig.6, the RMSE of UCA curve together with the ArcA curve declines as the SNR increases under scenarios of various D. In the case of D = 1 (Fig.6(a)), the ArcA curve is lower than the ArcA-C curve; even though, the gap between the two curves is very narrow at SNR varying from 15 dB to 25 dB. The superiority brought by matrix completion becomes apparent under D ≥ 2. As shown in Fig.6(b), the ArcA-C curve is lower than the ArcA curve when the SNR is greater than 5 dB. Furthermore, the gap between the two curves is wider in Fig.6(c), since there are some failed estimations in ArcA. Moreover, the performance of ArcA-C curve gradually degrades as D increases. Notably, the RMSE of ArcA-C in Fig.6(d) fluctuates slightly when SNR is greater than 15 dB.

Secondly, the relationships between RMSE and K are separately shown in Fig.7 under different D when SNR is fixed. Apparently, the RMSE of the three curves declines as K increases. In the case of D = 1 (Fig.7(a)), the performance of ArcA-C is slightly better than the ArcA under K ≥ 2048. When D is greater than or equal to 2, the advantage of ArcA-C can be clearly found. Especially in Fig.7(c), the ArcA curve almost converges to 1.6° while the ArcA-C curve drops to lower than 1°. The above results indicate that K has a significant impact on covariance matrix completion; moreover, it is easier to accumulate more snapshots instead of improving SNR in an actual DF system. In summary, R_{DNN} is able to perform DOA estimation in the same way as R_{UCA} and provide reasonable results; moreover, the matrix completion strategy improves the performance of DOA estimation in ArcA when there are multiple sources impinging on the array.
FIGURE 7: RMSE versus Number of Snapshots $K$, SNR is set to 15 dB.

C. BENEFITS WITH COVARIANCE MATRIX COMPLETION

In this section, we focus on the benefits of DOA estimation that come with the covariance matrix completion. The DOA estimation is performed with $\hat{\mathbf{R}}_{\text{DNN}}, \hat{\mathbf{R}}_{\text{UCA}}$, and $\hat{\mathbf{R}}_{\text{ArcA}}$ (Covariance Matrix Obtained by ArcA). The performance of DOA estimation is evaluated by Success Rate. A successful estimation is defined as follows:

1) the number of peaks in spatial spectrum is equal to $D$;
2) the deviation $(\Delta \theta_d, \Delta \varphi_d)$ of each estimated DOA $(\hat{\theta}_d, \hat{\varphi}_d)$ meets the following condition:

$$\Delta \theta_d = |\theta_d - \hat{\theta}_d| < 3^\circ \text{ and } \Delta \varphi_d = |\varphi_d - \hat{\varphi}_d| < 3^\circ.$$ 

With the above definition, the Success Rate is given by

$$\text{SuccessRate} = \frac{P_{\text{Success}}}{P},$$

where $P_{\text{Success}}$ is the number of successful Monte-Carlo trials in each simulation scenario.

As shown in Fig.8(a), the Success Rate of ArcA-C curve is slightly lower than the UCA curve, yet the ArcA curve is significantly lower than the ArcA-C curve. The gap between the ArcA curve and the ArcA-C curve gradually narrows as the SNR increases in $D = 3$ scenario; while the gap between ArcA and ArcA-C curves is still 5% when SNR is 30 dB. The simulation results under $D = 4$ are shown in Fig.8(b). It is evident that the ArcA curve is far below the ArcA-C curve; moreover, the ArcA curve is always below 20% when the SNR ranges from 5 dB to 30 dB. The above results indicate that, through covariance matrix completion, the effective aperture of ArcA has been enlarged to that of a complete UCA; meanwhile, the performance of DOA estimation has also been enhanced.
D. RESOLUTION

In this section, we study the resolution of two closely spaced sources when DOA estimation is performed with $\mathbf{R}_{\text{DNN}}$, $\mathbf{R}_{\text{UCA}}$ and $\mathbf{R}_{\text{ArcA}}$. The SNR is set to 10dB and $K$ is set to 1024. The DOAs of two sources in each Monte-Carlo trial are selected as follows:

1) The total angular separation of two sources is defined as:
$$\delta_T = \sqrt{\delta_\theta^2 + \delta_\phi^2};$$
2) The angular separation of $\theta$ is randomly selected from $\delta_\theta \in (0^\circ, \delta_T)$, then $\delta_\phi$ can be calculated as
$$\delta_\phi = \sqrt{\delta_T^2 - \delta_\theta^2};$$
3) The DOA of the first source is randomly selected from the two-dimensional angular space:
$$(\theta_1, \phi_1) \in \{ (\theta_d, \phi_d) \mid \theta_d \in (0^\circ, 90^\circ - \delta_T], \phi_d \in (0^\circ, 360^\circ - \delta_T] \};$$
4) The DOA of the second source is obtained by:
$$\theta_2 = \theta_1 + \delta_\theta, \phi_2 = \phi_1 + \delta_\phi.$$

The related results are depicted in Fig.9(a)(b), respectively. Results in Fig.9(a) suggest that the Success Rate of ArcA is no more than 50% when $\delta_T$ is 12$^\circ$; yet the ArcA-C curve is over 90% under $\delta_T = 10^\circ$. As shown in Fig.9(b), the two RMSE curves follow a similar trend versus $\delta_T$; nevertheless, the ArcA-C curve is always below the ArcA curve. The results in this section also prove the superiority of the covariance matrix completion scheme in DOA estimation, as it improves the resolution of the original array.

FIGURE 8: Success Rate versus SNR, $K$ is set to 1024

FIGURE 9: Success Rate, RMSE versus $\delta_T$
E. UNDERDETERMINED DOA ESTIMATION

It is well known that at most \( N - 1 \) sources can be estimated with a \( N \)-element array; estimating \( D \) (\( D \geq N \)) sources is called Underdetermined DOA Estimation. Since the ArcA is enlarged to a UCA with more sensors, it is possible to perform underdetermined DOA estimation. In this section, the number of sources is set to \( D = 5 \) same as the number of physical elements \( N_P = 5 \). The DOAs of these 5 sources are randomly selected from the 2D angular space. The 2D Spatial Spectrum, Success Rate and RMSE are displayed in Fig.10, Fig.11, and Fig.12, respectively.

As shown in Fig.10, the Spatial Spectrum obtained from \( \hat{R}_{UCA} \) is also depicted for comparison. Clearly, the peaks of UCA are finer and higher than those of ArcA-C; furthermore, there are slight deviations between the peaks of ArcA-C and the red markers (ground truth value of DOA). A refinement of the Spatial Spectrum obtained from \( \hat{R}_{DNN} \) will be considered in future work. The Success Rate and RMSE results shown in Fig.11 and Fig.12 also quantitatively manifest the ability of ArcA-C to conduct underdetermined DOA estimation. In summary, the proposed scheme gives a tolerable performance of underdetermined DOA estimation under moderate conditions (SNR=15dB, \( K = 1024 \)).
F. COMPUTATIONAL COMPLEXITY ANALYSIS

The major computational complexity of the proposed algorithm corresponds to the following steps.

1) Matrix Completion with DNN;
2) Eigenvalue Decomposition (EVD) required for MU-SIC;
3) 2D Spatial Spectrum Search required for MUSIC;

We analyze the complexity of the above steps separately. Since the size of the input and output layers is $N^2$, and the number of hidden layers is $L_h$ ($N_h = 2048$ is the size of first and last hidden layers, $N_m = 4096$ is the size of middle hidden layers); then the computational complexity of the DNN is $O(2N_h(N^2 + N_m) + (L_h - 3)N_m^2)$ based on [26]. The EVD step also contributes a part of the computational complexity, which is $O(N^3)$. The computational complexity of the 2D spatial spectrum search is obtained as $O(G_\theta G_\phi (N + 1)(N - D))$ [13], where $G_\theta = 900$ and $G_\phi = 3600$ are the dimensions of $\theta$ and $\phi$ grids, respectively. We evaluate the algorithm under CPU I7-10510U at 2.30 GHz and 12 GB RAM. The computational complexity and the average CPU running time of 500 Monte-Carlo trials are given in Table 4. Obviously, the process of 2D spatial spectrum search is the most time-consuming.
TABLE 4: Computational Complexity and CPU Running Time

| Step | DNN | EVD | 2D-Spatial Spectrum Search |
|------|-----|-----|---------------------------|
| Computational Complexity | $O(2N_0(N^2 + N_h) + (L_h - 3)N^2_h)$ | $O(N^3)$ | $O(G_h G_p (N + 1) | (N - D) )$ |
| CPU Running Time (s) | 0.02625 | 0.0012 | 4.451 |

V. CONCLUSION
In this paper, an algorithm named ArcA-DNN has been proposed, which can perform 2D-DOA estimation in ArcA based on the covariance matrix completion. The non-Toeplitz structured covariance matrix of ArcA can be completed to a covariance matrix of UCA through a DNN; afterward, the 2D-DOA estimation is carried out in the virtual UCA with fewer physical antennas. Simulation results demonstrate that the effective aperture of ArcA has been enlarged; meanwhile, the performance of DOA estimation has been improved in terms of Success Rate, RMSE, and Resolution. Moreover, this algorithm offers access to conduct underdetermined 2D-DOA estimation in ArcA.

REFERENCES
[1] F. Wen, J. Shi, and Z. Zhang, “Joint 2d-dod, 2d-doa, and polarization angles estimation for bistatic emvs-mimo radar via parafac analysis,” IEEE Transactions on Vehicular Technology, vol. 69, no. 2, pp. 1626–1638, 2020.
[2] W. Shi, J. Huang, and Y. Hou, “Fast doa estimation algorithm for mimo sonar based on ant colony optimization,” Journal of Systems Engineering and Electronics, vol. 23, no. 2, pp. 173–178, 2012.
[3] Z. Zheng, W.-Q. Wang, H. Meng, H. C. So, and H. Zhang, “Efficient beamspace-based algorithm for two-dimensional doa estimation of inherently distributed sources in massive mimo systems,” IEEE Transactions on Vehicular Technology, vol. 67, no. 12, pp. 11 776–11 789, 2018.
[4] C. Mathews and M. Zoltowski, “Eigenstructure techniques for 2-d angle estimation with uniform circular arrays,” IEEE Transactions on Signal Processing, vol. 42, no. 9, pp. 2395–2407, 1994.
[5] R. Goossens and H. Rogier, “A hybrid uca-rcar/root-music approach for 2-d direction of arrival estimation in uniform circular arrays in the presence of mutual coupling,” IEEE Transactions on Antennas and Propagation, vol. 55, no. 3, pp. 841–849, 2007.
[6] R. Goossens, H. Rogier, and S. Werbrouck, “Uca root-music with sparse uniform circular arrays,” IEEE Transactions on Signal Processing, vol. 56, no. 8, pp. 4095–4099, 2008.
[7] S. Wandale, T. Basikolo, and K. Ichige, “Super nested sparse circular array for high resolution doa estimation,” in 2019 IEEE International Symposium on Circuits and Systems (ISCAS), 2019, pp. 1–5.
[8] G. Jiang, X.-P. Mao, and Y.-T. Liu, “Underdetermined doa estimation via covariance matrix completion for nested sparse circular array in nonuniform noise,” IEEE Signal Processing Letters, vol. 27, pp. 1824–1828, 2020.
[9] S. K. Yadav and N. V. George, “Underdetermined direction-of-arrival estimation using sparse circular arrays on a rotating platform,” IEEE Signal Processing Letters, vol. 28, pp. 862–866, 2021.
[10] P. P. Vaidyanathan and P. Pal, “Theory of sparse coprime sensing in multiple dimensions,” IEEE Transactions on Signal Processing, vol. 59, no. 8, pp. 3592–3608, 2011.
[11] P. Pal and P. P. Vaidyanathan, “Nested arrays: A novel approach to array processing with enhanced degrees of freedom,” IEEE Transactions on Signal Processing, vol. 58, no. 8, pp. 4167–4181, 2010.
[12] K. Aghababaiyan, V. Shah-Mansouri, and B. Maham, “High-precision omp-based direction of arrival estimation scheme for hybrid non-uniform array,” IEEE Communications Letters, vol. 24, no. 2, pp. 354–357, 2019.
[13] K. Aghababaiyan, R. G. Zefreh, and V. Shah-Mansouri, “3d-omp and 3d-fomp algorithms for doa estimation,” Physical Communication, vol. 31, pp. 87–95, 2018.
[14] X. Wu, W.-P. Zhu, and J. Yan, “A toeplitz covariance matrix reconstruction approach for direction-of-arrival estimation,” IEEE Transactions on Vehicular Technology, vol. 66, no. 9, pp. 8223–8237, 2017.
[15] S. Liu, Z. Mao, Y. D. Zhang, and Y. Huang, “Rank minimization-based toeplitz reconstruction for doa estimation using coprime array,” IEEE Communications Letters, vol. 25, no. 7, pp. 2265–2269, 2021.
[16] C.-L. Liu and P. P. Vaidyanathan, “Robustness of difference coarrays of sparse arrays to sensor failures—part i: A theory motivated by coarray music,” IEEE Transactions on Signal Processing, vol. 67, no. 12, pp. 3213–3226, 2019.
[17] Z. Zheng, Y. Huang, W.-Q. Wang, and H. C. So, “Augmented covariance matrix reconstruction for doa estimation using difference coarray,” IEEE Transactions on Signal Processing, vol. 69, pp. 5345–5358, 2021.
[18] A. Ahmed and Y. D. Zhang, “Generalized non-redundant sparse array designs,” IEEE Transactions on Signal Processing, vol. 69, pp. 4580–4594, 2021.
[19] R. Rajamäki and V. Koivunen, “Sparse symmetric linear arrays with low redundancy and a contiguous sum co-array,” IEEE Transactions on Signal Processing, vol. 69, pp. 1697–1712, 2021.
[20] M.-Y. Cao, L. Huang, C. Qian, J.-Y. Xue, and H.-C. So, “Underdetermined doa estimation of quasi-stationary signals via khatiri–rao structure for uniform circular array,” Signal Processing, vol. 106, pp. 41–48, 2015.
[21] A. Barthelme and W. Utschick, “Doa estimation using neural network-based covariance matrix reconstruction,” IEEE Signal Processing Letters, vol. 28, pp. 783–787, 2021.
[22] R. Schmidt, “Multiple emitter location and signal parameter estimation,” IEEE Trans. Antennas Propag., vol. 34, no. 4, pp. 276–280, 1986.
[23] M. Wax and T. Kailath, “Detection of signals by information theoretic criteria,” IEEE Transactions on acoustics, speech, and signal processing, vol. 33, no. 2, pp. 387–392, 1985.
[24] G. Schwarz, “Estimating the dimension of a model,” The annals of statistics, pp. 461–464, 1978.
[25] H.-T. Wu, J.-F. Yang, and F.-K. Chen, “Source number estimators using transformed gerschgorin radii,” IEEE transactions on signal processing, vol. 43, no. 6, pp. 1325–1333, 1995.
[26] D. Chen, S. Shi, X. Gu, and B. Shim, “Robust doa estimation using denoising autoencoder and deep neural networks,” IEEE Access, 2022.

YE TIAN received the B.S. degree in Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, China, in 2017. He is currently pursuing the Ph.D. degree in electromagnetic field and microwave technology with National Space Science Center (Beijing, China) of Chinese Academy of Sciences. His current research interests include array signal processing, DOA estimation and RF signal geolocation.

RURU MEI was born in Anhui, China. He received the B.Eng. degree in electronic and information engineering from North China University of Technology of China of Beijing, China in 2020. He is currently working towards the Ph.D. degree in electromagnetic field and microwave technology at the University of Chinese Academy of Sciences (UCAS), Beijing, China. His research interests include artificial neural networks and its application for communication systems.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2022.3172478, IEEE Access
YONGHUI HUANG was born in Anshan, China. He received the B.Sc. degree in electronics engineering from Tsinghua University of Beijing, China in 1998. In 2001, he obtained the M.S. degree in aero-spacecraft design from University Chinese Academy of Sciences, China. He achieved the Ph.D. degree in wireless communication from Aalborg University of Aalborg, Denmark in 2008. He is currently a professor with National Space Science Center of Chinese Academy of Science in Beijing, China. From 2002 to 2011, he worked as a Postdoc and research assistant in Aalborg University of Aalborg, Denmark. He is currently a researcher at the National Space Science Center of the Chinese Academy of Sciences. His current research interests include radio frequency machine learning, phased array antenna and transmitter linearization. He is the TPC member of IEEE CCET and IEEE WiSEE.

XIAOGANG TANG received his M.S. and Ph.D. degree in Xi’an Jiaotong University in Shann’xi, Xi’an, China in 2005 and 2014. Now he is an associate professor of Space Engineering University, and his research is mainly about pattern recognition and information intelligent processing. He has published several research papers in scholarly journals in the above research areas and has participated in several conferences.

TIANSHU CUI was born in Shandong, China in 1986. He received the B.S. degree in aircraft manufacturing engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China in 2009, and obtained the M.S. degree in navigation, guidance and control from Beihang University, Beijing, China in 2012. From 2012 to 2021, he worked in National Space Science Center of Chinese Academy of Science in Beijing, China. He achieved the Ph.D. degree in computer application from University of Chinese Academy of Sciences in Beijing, China in 2021. He is doing postdoctoral research in Beijing National Research Center for Information Science and Technology at Tsinghua University, China, and his research interests include radio frequency machine learning, especially radar signal sorting and recognition.

* * *