AN ELEMENTARY GREEN IMPRIMITIVITY THEOREM FOR INVERSE SEMIGROUPS

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Abstract. A Morita equivalence similar to that found by Green for crossed products by groups will be established for crossed products by inverse semigroups. More precisely, let $G$ be an inverse semigroup, $H$ a finite sub-inverse semigroup of $G$ and $A$ a $G$-algebra or a $H$-algebra. Then the crossed product $A \rtimes H$ is Morita equivalent to a certain crossed product $B \rtimes G$.

1. Introduction

In a classical paper [5], Green showed that for a closed subgroup $H$ of a locally compact group $G$, and a $G$-algebra $A$ there exits a Morita equivalence between $A \rtimes H$ and $C_0(G/H, A) \rtimes G$ via an imprimitivity bimodule over these algebras ([5, Prop. 3]). This useful result was discussed and generalized in many directions, for example, in [13, 17, 11, 4], but these are just a few samples.

In this note we shall establish an analogous imprimitivity theorem for an inverse semigroup $G$ and a finite sub-inverse semigroup $H \subseteq G$ for crossed products in Sieben’s sense [16]. As a corollary of this, we show this holds true also for a given $H$-algebra $A$, and thus this may be usefully combined with induction like in Kasparov [8, 6]. Actually, this note was motivated by the fact that the Baum–Connes map [2] for groups $G$ is a kind of extrapolation of Green–Julg isomorphisms for crossed products by $G$ of induced algebras by compact subgroups $H \subseteq G$, as noted by Meyer and Nest in [10]. In establishing that, Kasparov’s induction plays a fundamental role. To potentially carry this result over from groups to inverse semigroups, we need induction for compact (and thus finite) sub-inverse semigroups $H \subseteq G$, and this is now provided in this note. Our proof is dedicated Section 2, and we give a short summary.

At first we rewrite the inverse semigroup crossed product $A \rtimes H$ as a groupoid crossed product $A \rtimes \mathcal{G}$ to have a group-like construction. Then we adapt and follow Green’s proof

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[5] p. 199-204] in a natural way. The action on a certain quotient space $G_G/G$ ($G/H$ in Green [5]) is similar to the regular representation action by Khoshkam and Skandalis [9]. After establishing Green’s imprimitivity Theorem 2.2, we apply it to the induced algebra (in the sense of Kasparov [3, 6]) $A$ of a $H$-algebra $D$, and restrict to ideals to get the second Green imprimitivity theorem, Corollary 2.3.

2. The imprimitivity theorem

We begin by recalling crossed products in the sense of Khoshkam and Skandalis [9] and Sieben [16], but use several notions from [3]. Let $G$ denote an inverse semigroup. A $G$-algebra $A$ is a $C^*$-algebra $A$ endowed with a $G$-action in the following sense: there exists a semigroup homomorphism $\alpha : G \to \text{End}(A)$, written as $g(a) := \alpha(g)(a)$, such that $g^*(a)b = agg^*(b)$ for all $a, b \in A$ and $g \in G$. Such a $G$-algebra (whose definition is equivalent to [3, Def. 3.1]) is a special case of $G$-algebras in the sense of [16] and [9].

Let $\mathbb{F}(G, A)$, or $\mathbb{F}$ for brevity, be the universal $*$-algebra over $\mathbb{C}$ generated by disjoint copies of $A$ and $G$ such that the $*$-relations of $A$ are respected, the multiplication and involution of $G$ are respected, and the relations $g(a)gg^* = gag^*$, $[gg^*, a] = 0$ (commutator) hold for all $a \in A$ and $g \in G$. The algebraic crossed product $A \rtimes_{alg} G$ denotes the linear span of elements of the form $ag (a \in A, g \in G)$, which are usually denoted by $a \rtimes g$, and is a $*$-subalgebra of $\mathbb{F}$. We denote by $G_0$ the idempotent elements of $G$, and by $E(G)$ the set of the projections $e_0(1 - e_1) \ldots (1 - e_n)$ in $\mathbb{F}$ with $e_0, \ldots, e_n \in G_0$ and $n \geq 0$. The set $G_E := \{gp \in \mathbb{F} | g \in G, p \in E(G)\}$ is an inverse semigroup in $\mathbb{F}$. We also write $a \rtimes g := ag$ when $g \in G_E$. The identity $agp = gg^*(a)gp$ holds in $\mathbb{F}$ for all $a \in A$ and $gp \in G_E$ ($g \in G, p \in E(G)$). An element (or more precisely, an expression) $a \rtimes gp$ with $a \in A_{gg^*} := gg^*(A)$ and $gp \in G_E$ is called standard.

The full crossed product $A \rtimes G$ is the closure of the image of $A \rtimes_{alg} G$ under the universal $*$-representation $\pi$ of $\mathbb{F}$ on Hilbert space ([9, Def. 5.4] or [3, 5.16, 6.2, 8.4]). It is easy to see with the reduced representations [9, p. 271] that $\pi$ is injective on $A \rtimes_{alg} G$, and so the latter is a pre-$C^*$-algebra with a $C^*$-norm. Sieben’s crossed product $A \hat{\rtimes} G$ is defined to be the image of $A \rtimes_{alg} G$ under the universal $*$-representation $\tau$ of $\mathbb{F}$ on Hilbert space satisfying $\tau(g(a) - gag^*) = 0$ (see [16]). We write $a \hat{\rtimes} g$ for $\tau(ag)$. Note, in particular, that $\hat{\rtimes}$ is compatible: $e(a) \hat{\rtimes} g = a \hat{\rtimes} eg$ for all $g \in G_E$ and $e \in E(G)$.

Let us be given a finite sub-inverse semigroup $H'$ of $G$. Denote by $H$ the groupoid associated to $H'$ (cf. [11]). More precisely, let $H^{(0)}$ be the set of nonzero minimal projections of $E(H')$ and $H = \{te \in \mathbb{F} | t \in H', e \in H^{(0)}\} \setminus \{0\}$.
Define \( G_H = \{ ge \in F \mid g \in G, e \in H^{(0)}, g^*g \geq e \} \setminus \{ 0 \} \). We endow \( G_H \) with an equivalence relation: \( g \equiv h \) if and only if there exists \( t \in H \) such that \( gt = h \) (\( g, h \in G_H \)). We denote by \( G_H/H \) the set-theoretical quotient of \( G_H \) by \( \equiv \). We shall exclusively work with representatives in this quotient; writing \( g \in G_H/H \) means implicitly that \( g \in G_H \) and we use no class brackets; if then \( g \in G_H \) is meant or the class \( g \in G_H/H \) becomes apparent from the context.

Let \( C_0(G_H/H) \) denote the commutative \( C^* \)-algebra of (continuous) complex-valued functions vanishing at infinity of the (discrete) set \( G_H/H \) with the pointwise operations. The delta function \( \delta_g \) in \( C_0(G_H/H) \) is denoted by \( g \) (\( g \in G_H/H \)). For an assertion \( \mathcal{A} \) we let \([\mathcal{A}]\) be the real number 0 if \( \mathcal{A} \) is false, and 1 if \( \mathcal{A} \) is true. \( C_0(G_H/H) \) is endowed with the \( G \)-action \( g(h) := [gh \in G_H] \, gh \), where \( g \in G \) and \( h \in G_H/H \) (of course, \( gh \in G_H \) is equivalent to \( g^*g \geq hh^* \)). We let \( A \otimes C_0(G_H/H) \) be the \( C^* \)-algebraic tensor product endowed with the diagonal action by \( G \).

**Lemma 2.1.** (i) If \( g_1, \ldots, g_n \in G_H \) are mutually different then \( \sum_{i=1}^n a_i \times g_i = 0 \) (sum of standard elements) implies \( a_1 = \ldots = a_n = 0 \).

(ii) The \( G \)-action on a \( G \)-algebra \( A \) extends canonically to a \( G_E \)-action on \( A \).

(iii) The formulas \((a \times g)(b \times h) = ag(b) \times gh \) and \((b \times h)^* = h^*(b^*) \times h^* \) hold in \( F \) for all \( g, h \in G_E \), \( a \in A_{gg^*} := gg^*(A) \) and \( b \in A \).

**Proof.** (i) Note first that the claim was true for \( g_i \in G \) by the reduced representation in [9, p. 271]. For the stated case, we may assume that all \( g_i \) have the same source projection in \( H^{(0)} \). Write \( g_i = h_i (1 - e_1) \ldots (1 - e_n) \) with \( h_i \in G \) and \( e_i \in G_0 \). Note that the \( h_i \) are mutually different. Expanding, we get \( 0 = \sum_{i=1}^n a_i g_i = \sum_{i=1}^n a_i h_i - \sum_{i=1}^n a_i h_i e_1 + \ldots \) in \( F \). If, for example, we had \( a_i h_i = a_j h_j e_1 \) then we would have \( a_i g_i = 0 \) and so \( g_i = 0 \) (however \( 0 \notin G_H \)). Consequently, \( \sum_{i=1}^n a_i h_i = 0 \), which yields the claim. (ii) It easy to see that we have a well-defined semigroup homomorphism \( \alpha : E(G) \to \text{End}(A) \) with \( \alpha_{1-e} = id_A - \alpha_e \) and extending the \( G \)-action \( \alpha \) on \( A \). If \( gp = hq \neq 0 \) for \( h, g \in G \) and \( p, q \in E(G) \) then \( gpq = hpq \) and so \( g = h \) by a similar argument as in (i). Then \( g^* gp = g^* gq \) in \( E(G) \). Hence, \( \alpha_{gp} := \alpha_g \alpha_p = \alpha_h \alpha_q \) is well-defined. (iii) We have \( agp bhq = gpq^*(a) g(b) gphq = agp(b) gphq \) in \( F \) for \( g, h \in G \), \( p, q \in E(G) \), \( a \in A_{gg^*} \) and \( b \in A \). \( \square \)
We introduce the spaces
\[ B_0 = A \rtimes_{alg} H := \text{span}\{ a \rtimes t \in A \rtimes_{alg} G | a \in A_{tt^*}, \ t \in H \}, \]
\[ X_0 = \text{span}\{ a \rtimes g \in A \rtimes_{alg} G | a \in A, g \in G_H \}, \]
\[ E_0 = (A \otimes C_0(G_H/H)) \rtimes_{alg} G. \]

The spaces \( B_0 \subseteq A \rtimes G \) and \( E_0 \) are regarded as pre-\( C^* \)-algebras. We make \( X_0 \) to a right pre-Hilbert module over \( B_0 \) (cf. \[14\], Def. 2.8]) by the following operations
\[ X_0 \times B_0 \longrightarrow X_0 : (a \rtimes g)(c \rtimes t) := ag(c) \rtimes gt, \]
\[ X_0 \times X_0 \longrightarrow B_0 : (a \rtimes g, b \rtimes h)_{B_0} := [g^* h \in H] g^*(a^* b) \rtimes g^* h \]
for \( a, b \in A, c \in A_{tt^*}, g, h \in G_H \) and \( t, h \), and to a left pre-Hilbert module over \( E_0 \) by
\[ E_0 \times X_0 \longrightarrow X_0 : (a \otimes r \rtimes s)(b \rtimes h) := [s^r \in G_H] [r \equiv s^r] as(b) \rtimes s^r, \]
\[ X_0 \times X_0 \longrightarrow E_0 : (a \rtimes g, b \rtimes h)_{E_0} := a gh^*(b^*) \otimes g \rtimes gh^* \]
for \( a, b \in A, r \in G_H/H, s \in G \) and \( j, g, h \in G_H \). Note that standard elements go to standard elements and we extend the above formulas by linearity on sums of standard elements.

Straightforward computations show that we have
\[ (x, yb)_{B_0} = (x, y)_{B_0} b, \quad (x, y)_{B_0}^* = (y, x)_{B_0}, \]
\[ (fx, y)_{E_0} = f(x, y)_{E_0}, \quad (x, y)_{E_0} = (y, x)_{E_0}, \]
\[ (fx, y)_{B_0} = (x, f^* y)_{B_0}, \quad (x, yb)_{E_0} = (xb^*, y)_{E_0}, \quad x(y, z)_{B_0} = (x, y)_{E_0} z \]
for all \( x, y, z \in X_0, b \in B_0 \) and \( f \in E_0 \) (cf. \[14\], Def. 6.10]).

For convenience of the reader we sketch the first identity of line \[11\]. We have
\[ \langle (a \otimes r \rtimes s)(b \rtimes g), c \rtimes h \rangle_{B_0} \]
\[ = [sg \in G_H] [r \equiv sg] [g^* s^* h \in H] g^* s^* (s(b^*)a^*) \rtimes g^* s^* h \]
for \( a, b, c \in A, r \in G_H/H, s \in G \) and \( g, h \in G_H \), and
\[ \langle b \rtimes g, (a \otimes r \rtimes s^*)(c \rtimes h) \rangle_{B_0} \]
\[ = \langle b \rtimes g, ([s^* r \in G_H] s^*(a^*) \otimes s^* r \rtimes s^*)(c \rtimes h) \rangle_{B_0} \]
\[ = [g^* s^* h \in H] [s^* r \equiv s^* h] [s^* h \in G_H] [s^* r \in G_H] g^* (b^* s^*(a^*) s^*(c)) \rtimes g^* s^* h. \]
The reader checks easily that line (2) is nonzero if and only if line (3) is nonzero, and so both expressions are identical. One just uses implications like, if $s^*h \in G_H$ then $ss^* \geq hh^*$ (as $h \in G_H$), or if $g^*s^*h \in H$ and $g, s^*h \in G_H$ then $gg^* = (s^*h)(s^*h)^*$ and so $s^*h \equiv g$.

Let $(a_\alpha)_\alpha$ be an approximate identity of $A$. Let $x = \sum_{s=1}^{m} b_s \cdot h_s$ in $X_0$ and choose for every different equivalence class $h_sH$ in $G_H/H$ exactly one representative $g_i := h_s$. Set $x_{i,\alpha} = a_\alpha \cdot g_i \in X_0$. Set $x_\alpha = \sum_{i=1}^{n} (x_{i,\alpha}, x_{i,\alpha})_{E_0} x$. Then a straightforward computation shows that $x = \lim_{\alpha} x_\alpha$ in the norm of $E_0$. Consequently,

$$\langle x, x_\alpha \rangle_{B_0} = \sum_{i=1}^{n} \langle x, x_{i,\alpha} \rangle_{B_0} \langle x, x_{i,\alpha} \rangle_{B_0}^* \geq 0$$

as in Green [5], page 202. The positivity $\langle x, x \rangle_{B_0} \geq 0$ follows by computing that also $\langle x, x_\alpha \rangle_{B_0}$ tends to $\langle x, x \rangle_{B_0}$ in the norm of $B_0$. The argument for the positivity of $\langle x, x \rangle_{E_0}$ is similar (choose, for example, $x_\alpha = x \sum_{e \in H(0)} (a_\alpha \cdot e, a_\alpha \cdot e)_{B_0}$).

We need to verify the identities

$$\langle f, f \rangle_{B_0} \leq \|f\|_{E_0}^2 \langle x, x \rangle_{B_0}, \quad \langle x, x \rangle_{E_0} \leq \|b\|_{B_0}^2 \langle x, x \rangle_{E_0}$$

for all $x \in X_0$, $f \in E_0$ and $b \in B_0$ (cf. [14] Def. 6.10]). For a standard element $f = a \otimes r \otimes s \in E_0$ and $x \in X_0$ we have

$$\|f\|_{E_0} \langle x, x \rangle_{E_0} - \langle f, f \rangle_{E_0} = \langle \|f\|_{E_0} - f^*f, x \rangle_{E_0}$$

$$= \langle zx, zx \rangle_{E_0} + \langle (1-p)x, (1-p)x \rangle_{E_0} \geq 0,$$

where $z := (\|f\|_{E_0} - s^*(a^*a))^{1/2} \otimes s^*r \otimes s^*s$ and $p := \|f\|_{E_0} \otimes s^*r \otimes s^*s$ are elements in $\mathcal{M}(A) \otimes C_0(G_H/H) \ltimes G$; we have (easily) extended the action of $E_0$ on $X_0$ and the first identity of (1) to this space.

We have shown that $\|fx\| \leq \|f\|_{E_0} \|x\|$ (where $\|x\| := \|(x, x)_{B_0}\|^{1/2}$) for elementary elements $f \in E_0$, and by taking sums of such elements we readily obtain $\|fx\| \leq \|f\|_{\ell^1(G, A \otimes C_0(G_H/H))} \|x\|$ for all $f \in E_0$. Since $E_0 \to \mathcal{L}(X_0)$ is an $\ell^1$-contractive representation into a pre-$C^*$-algebra, and the norm closure of $E_0$ is the enveloping $C^*$-algebra of $\ell^1$ (cf. [2]), we get $\|f\|_{\mathcal{L}(X_0)} \leq \|f\|_{E_0}$ and so the first inequality of (1). The second inequality of (1) is proved similar (but is easier as $B_0$ is norm-closed).

Denote by $E_X \subseteq E_0$ the closure of $\langle X_0, X_0 \rangle_{E_0}$, and by $B_X \subseteq B_0$ the closure of $\langle X_0, X_0 \rangle_{B_0}$. We now apply the argument following [15] Prop. 3.1 to see that $X_0$ may be completed in semi-norm $\|x\| = \|(x, x)_{B_0}\|^{1/2}$ (after factoring out the elements of norm 0) to obtain an $E_X - B_X$ imprimitivity bimodule $X$. The $C^*$-algebra $B_X = B_0$ is canonically isomorphic.
to the groupoid crossed product $A \rtimes H$, which is canonically isomorphic to $A \hat{\rtimes} H'$ by [12 Thm. 7.2]. To meet exactly the assumptions in [12], switch to the carrier algebra $\tilde{A} = p(A)$ for $p = \sum_{e \in H(0)} e$ of $A$, which does not change the crossed product, that is, $\tilde{A} \hat{\rtimes} H'$. Denote by $C_0(G_H/H, A)$ the $G$-invariant $C^*$-subalgebra generated by \{a \otimes r \in A \otimes C_0(G_H/H) | a \in A_{rr^*}, r \in G_H/H\}. Note that $C_0(G_H/H, A)$ is an ideal in $A \otimes C_0(G_H/H)$ and so $C_0(G_H/H, A) \hat{\rtimes} G$ embeds in $(A \otimes C_0(G_H/H)) \hat{\rtimes} G$, as can be seen by extending a $G$-action on a $C^*$-algebra to the multiplier algebra (completely analog as in [7 §1.4]).

Let $\sigma : E_X \to C_0(G_H/H, A) \hat{\rtimes} G$ be the canonical map. It is surjective, because given an elementary element $aa^* \otimes r \hat{\rtimes} g$ with $rr^* = gg^*$ (in $P$) in $C_0(G_H/H, A) \hat{\rtimes} G$ ($a \in A_{rr^*}, r \in G_H/H, g \in G_E$), we have

$$aa^* \otimes r \hat{\rtimes} g = \sigma(\langle a \rtimes r, g^*(a) \rtimes g^*r \rangle_{E_0}).$$

If $\sigma$ were not injective, then its kernel $J$ were nonzero, and so would correspond to a nonzero ideal $I$ in $\hat{B}_X$ via the imprimitivity module (see [13 Cor. 3.1]), which then would contain a nonzero element of the form $a \rtimes e$ with $e \in H^{(0)}$. The element $f = \langle a \rtimes e, a \rtimes e \rangle_{E_0}$ would be in $J$, however $\sigma$ is nonzero on $f$. We have obtained our result:

**Theorem 2.2.** Let $H'$ be a finite sub-inverse semigroup of an inverse semigroup $G$, and $A$ a $G$-algebra. Then $X$ is an $E_X - B_X$ imprimitivity bimodule, where $E_X \cong C_0(G_H/H, A) \hat{\rtimes} G$ and $B_X \cong A \hat{\rtimes} H'$.

Now assume that $D$ is a $H'$-algebra. Define, similar as in [8 §5 Def. 2],

$$\text{Ind}_{H'}^G(D) := \{f : G_H \to D | \forall g \in G_H, t \in H \text{ with } gt \in G_H : f(gt) = t^*(f(g)),$$

$$\|f(g)\| \to 0 \text{ for } gH \to \infty \text{ in } G_H/H \}.\text{ }$$

Let $A$ denote $\text{Ind}_{H'}^G(D)$. It is a $C^*$-algebra under the pointwise operations and the supremum’s norm and becomes a $G$-algebra under the $G$-action $(gf)(h) := [g^*h \in G_H] f(g^*h)$ for $g \in G, h \in G_H$ and $f \in A$. Consider the $H'$-invariant ideal $A_0$ of $A$ consisting of all functions which vanish on $G_H/H$. Then $A_0 \rtimes H$ embeds canonically as an ideal $J$ in $A \rtimes H$, and by [15 Cor. 3.1], associated to $J$ is the submodule in $X$ generated by $X_0 = \langle y \in X_0 \mid (y, y)_{X_H} \in J \rangle = \text{span} \{g(a) \rtimes g \in X_0 | a \in A_0, g \in G_H\}$, and the ideal $I$ in $E_X$ generated by $\langle Y_0, X_0 \rangle_{E_X} = \text{span} \{g(a) \otimes g \hat{\rtimes} gh^* \in E_X | a \in A_0, g, h \in G_H\}$.

The ideal $I$ is canonically isomorphic to $K \hat{\rtimes} G$, where $K$ denotes the $G$-invariant ideal in $C_0(G_H/H, A)$ generated by \{$(g(a) \otimes g \in C_0(G_H/H, A)) | a \in A_0, g \in G_H/H\}$. To see that $I \to K \hat{\rtimes} G$ is surjective, write a nonzero element $g(a) \otimes g \hat{\rtimes} s \in K \hat{\rtimes} G$ as $ss^*(g(a) \otimes g) \hat{\rtimes} gg^*s$. 
We have a $G$-equivariant isomorphism $\psi : A \to K$ defined by $\psi(f) = \sum_{g \in G_H/H} f \cdot 1_{gH} \otimes g = \sum_{g \in G_H/H} g^*(f) \cdot 1_H \otimes g$. There is a $H'$-equivariant epimorphism $\Phi : D \to A_0$ given by $\Phi(d)(t) = t^*(d)$ for $t \in H$ and $d \in D$. It is an isomorphism on the carrier algebras of $D$, so that $D \hat{\otimes} H' \cong A_0 \hat{\otimes} H'$. Consequently we have obtained, by restricting to the ideals $I$ and $J$ in Theorem 2.2 and applying [15, Cor. 3.1]:

**Corollary 2.3.** Let $H' \subseteq G$ be a finite sub-inverse semigroup and $D$ a $H'$-algebra. Then $\text{Ind}_{H'}^G(D) \hat{\otimes} G$ and $D \hat{\otimes} H'$ are Morita equivalent.

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