Probing texture zeros with scaling ansatz in inverse seesaw

Ambar Ghosal; Rome Samanta
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India

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Abstract

We investigate neutrino mass matrix phenomenology involving scaling ansatz and texture zeros adhering inverse seesaw mechanism. It is seen that four is the maximum number of zeros in $m_D$ and $\mu$ to obtain viable phenomenology. Depending upon the generic nature of the effective neutrino mass matrices we classify all the emerged matrices in four categories. One of them is ruled out phenomenologically due to inappropriate value of reactor mixing angle after breaking of the scaling ansatz. The mass ordering is inverted in all cases. One of the distinguishable feature of all these categories is the vanishingly small value of CP violation measure $J_{CP}$ due to small value of $\delta_{CP}$. Thus those categories will be ruled out if CP violation is observed in the leptonic sector in future experiments.
1 Introduction

Among the variants of the seesaw mechanism, inverse seesaw [1]-[11] stands out as an attractive one, due to its characteristic feature of generation of small neutrino mass without invoking high energy scale in the theory. Although to realize such feature one has to pay the price in terms of incorporation of additional singlet fermions, nevertheless, in different GUT models accommodation of such type of neutral fermions are natural. Furthermore, such mechanism appeals to the foreseeable collider experiments to be testified due to its unique signature. The $9 \times 9$ neutrino mass matrix in this mechanism is written as

$$m_\nu = \begin{pmatrix}
0 & m_D & 0 \\
m_D^T & 0 & M_{RS} \\
0 & M_{RS}^T & \mu
\end{pmatrix}$$

with the choice of basis ($\nu_L, \nu_R^c, S_L$). The three matrices appeared in $m_\nu$ are $m_D$, $M_{RS}$ and $\mu$ among them $m_D$ and $M_{RS}$ are the Dirac type whereas $\mu$ is Majorana type mass matrix. After diagonalization, the low energy effective neutrino mass comes out as

$$m_\nu = m_D M_{RS}^{-1} \mu (m_D M_{RS}^{-1})^T$$

$$= F_{\mu} F^T$$

where, $F = m_D M_{RS}^{-1}$. Such definition resembles the above formula as a conventional type-I seesaw expression of $m_\nu$.

However, the above expression contains large no of parameters and it is possible to fit them with neutrino oscillation experimental data [12]-[14]. Our goal in this work is to find out a phenomenologically viable texture of $m_D$ and $M_{RS}$ with minimum number of parameters or equivalently maximum number of zeros. At the outset of the analysis, we choose a basis where the charged lepton mass matrix and $M_{RS}$ are diagonal. We further bring two ideas to find out a minimal texture are
i) Scaling ansatz [15]-[25],
ii) Texture Zeros [26]-[39].

We start by assuming the scaling property in the elements of $m_D$ and $\mu$. To further reduce the number of parameters we accommodate as much as possible zeros in the above matrices. We are not addressing the origin of such texture zeros in the present work. We restrict ourselves within the frame work of $SU(2)_L \times U(1)_Y$ gauge group however, explicit realization of such scheme obviously more elusive which will be studied elsewhere.
2 Scaling property and texture zeros

We consider scaling property between the second and third row of $m_D$ matrix and the same for $\mu$ matrix also. Explicitly the relationships are written as

$$\frac{(m_D)_{2i}}{(m_D)_{3i}} = k_1 \quad (2.1)$$

$$\frac{(\mu)_{2i}}{(\mu)_{3i}} = k_2 \quad (2.2)$$

where, $i = 1, 2, 3$ is the column index. We would like to mention that although we have considered different scale factors for $m_D$ and $\mu$ matrices, however, the effective $m_\nu$ is still scaling invariant and leads to $\theta_{13} = 0$. Thus, it is obvious to further break the scaling ansatz. In order to generate nonzero $\theta_{13}$ it is necessary to break the ansatz in $m_D$ since, breaking in $\mu$ does not affect the generation of nonzero $\theta_{13}$ although in some cases it provides $m_3 \neq 0$.

Another point is to be noted that, since the $\mu$ matrix is complex symmetric whereas $m_D$ is asymmetric, the scale factors considered in $\mu$ matrix is different from that of $m_D$ to keep the row wise invariance as dictated by Eqn.(2.1) (for $m_D$), and Eqn.(2.2) (for $\mu$). Finally since the texture of $M_{RS}$ matrix is diagonal, it is not possible to accommodate scaling ansatz considered in the present scheme.

Let us now turn to further constrain the matrices assuming zeros in different entries. Since, in our present scheme the matrix $M_{RS}$ is diagonal, we constrain the other two matrices. We start with the maximal zero textures with scaling ansatz of general $3 \times 3$ matrices and different cases are listed systemetically in Table 1.

Table 1: Texture zeros with scaling ansatz of a general $3 \times 3$ matrix

| 7 zero texture | $m_1^7$ | $m_2^7$ | $m_3^7$ |
|----------------|--------|--------|--------|
| $m_1^7$        | $k_1c_1$ \ 0 \ 0 \ 0 | $k_1c_2$ \ 0 \ 0 \ 0 | $k_1c_3$ \ 0 \ 0 \ 0 |
| $m_2^7$        | $0$ \ 0 \ 0 \ 0 | $k_1c_2$ \ 0 \ 0 \ 0 | $0$ \ 0 \ $k_1c_3$ \ 0 |
| $m_3^7$        | $0$ \ 0 \ $k_1c_3$ \ 0 | $0$ \ 0 \ $c_3$ \ 0 | $0$ \ 0 \ 0 \ 0 |
| \( m_1^6 \) & \( m_2^6 \) & \( m_3^6 \) |
|---|---|---|
| \( \begin{pmatrix} d_1 & 0 & 0 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) |
| \( m_4^6 \) & \( m_5^6 \) & \( m_6^6 \) |
| \( \begin{pmatrix} d_1 & 0 & 0 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) |
| \( m_7^6 \) & \( m_8^6 \) & \( m_9^6 \) |
| \( \begin{pmatrix} d_1 & 0 & 0 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) |

| \( m_1^5 \) & \( m_2^5 \) & \( m_3^5 \) |
|---|---|---|
| \( \begin{pmatrix} 0 & 0 & 0 \\ k_1c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & 0 \\ k_1c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_1c_1 & k_1c_3 \\ 0 & c_1 & c_3 \end{pmatrix} \) |
| \( m_4^5 \) & \( m_5^5 \) & \( m_6^5 \) |
| \( \begin{pmatrix} d_1 & d_2 & 0 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & d_3 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) & \( \begin{pmatrix} d_1 & 0 & d_3 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) |
| \( m_7^5 \) & \( m_8^5 \) & \( m_9^5 \) |
| \( \begin{pmatrix} d_1 & d_2 & 0 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & d_3 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} d_1 & 0 & d_3 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) |
| \( m_{10}^5 \) & \( m_{11}^5 \) & \( m_{12}^5 \) |
| \( \begin{pmatrix} d_1 & d_2 & 0 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & d_3 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} d_1 & 0 & d_3 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) |

| \( m_1^4 \) & \( m_2^4 \) & \( m_3^4 \) |
|---|---|---|
| \( \begin{pmatrix} d_1 & 0 & 0 \\ 0 & k_1c_2 & k_1c_3 \\ 0 & c_2 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ 0 & k_1c_2 & k_1c_3 \\ 0 & c_2 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ 0 & k_1c_2 & k_1c_3 \\ 0 & c_2 & c_3 \end{pmatrix} \) |
| \( m_4^4 \) & \( m_5^4 \) & \( m_6^4 \) |
| \( \begin{pmatrix} d_1 & 0 & 0 \\ k_1c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ k_1c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ k_1c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \) |
| \( m_7^4 \) & \( m_8^4 \) & \( m_9^4 \) |
| \( \begin{pmatrix} d_1 & 0 & 0 \\ k_1c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & d_2 & 0 \\ k_1c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} 0 & 0 & d_3 \\ k_1c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \) |
| \( m_{10}^4 \) & \( m_{11}^4 \) & \( m_{12}^4 \) |
| \( \begin{pmatrix} d_1 & d_2 & d_3 \\ k_1c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix} \) & \( \begin{pmatrix} d_1 & d_2 & d_3 \\ 0 & k_1c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix} \) & \( \begin{pmatrix} d_1 & d_2 & d_3 \\ 0 & 0 & k_1c_3 \\ 0 & 0 & c_3 \end{pmatrix} \) |
To start with, we consider all the matrices listed in Table 1 as the Dirac type matrices \( m_D \). As the lepton number violating mass matrix \( \mu \) is complex symmetric, therefore, the maximal number of zeros with scaling invariance is 5. Therefore, only \( m_D^5 \) and \( m_5^5 \) type matrices can be made complex symmetric with the scaling property and are shown in Table 2 where they are renamed as \( \mu_1^5 \) and \( \mu_2^5 \) with a different scale factor \( k_2 \).

Table 2: Maximal zero texture of \( \mu \) matrix

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & k_2^2 s_3 & k_2 s_3 \\
0 & k_2 s_3 & s_3
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & k_2 s_3 & s_3 \\
k_2 s_3 & 0 & 0 \\
s_3 & 0 & 0
\end{pmatrix}
\]

Now using Eqn. (1.2) we can construct \( m_\nu \) and it is found that all the mass matrices constructed out of these matrices are not suitable to satisfy the neutrino oscillation data and the reason goes as follows:

**Case A:** \( m_D \) (7,6 zero) + \( \mu_1^5, \mu_2^5 \) (5 zero):

We cannot generate nonzero \( \theta_{13} \) by breaking the scaling ansatz because in this case all the structures of \( m_D \) are scaling ansatz invariant. This can be understood in the following way: if we incorporate scaling ansatz breaking by \( k'_1 \rightarrow k_1(1+\epsilon) \) all the structures of \( m_D \) are still invariant and \( m_\nu \) matrix will still give \( \theta_{13} = 0 \) as breaking of scaling in \( \mu_1^5 \) and \( \mu_2^5 \) play no role for the generation of nonzero value of \( \theta_{13} \). To generate nonzero \( \theta_{13} \) it is necessary to break scaling in the Dirac sector.

**Case B:** \( m_D \) (5 zero) + \( \mu_1^5, \mu_2^5 \) (5 zero):

For the matrices of the last three row(\( m_1^5 \) to \( m_1^{12} \)) in the ‘5 zero texture’ part of Table 1 are ruled out due to the same reason mentioned in (Case A) while, the matrices of the first row i.e \( m_1^5, m_2^5 \) and \( m_3^5 \) give the structure of \( m_\nu \) as

\[
A_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & * & * \\
0 & * & *
\end{pmatrix}
\]  \quad (2.3)

where ‘\*’ corresponds to some nonzero entries in \( m_\nu \). This structure leads to a complete disappearance of one generation, moreover it has been shown in [26] that if the number of independent

\[^1\text{From now on we use } m^n \text{ as a mass matrix, where } n(= 4, 5, 6, 7) \text{ is the number of zeros in that matrix.}\]
zeros in an effective neutrino mass matrix \((m_\nu)\) is \(\geq 3\), that matrix doesn’t favour the oscillation data and hence ‘\(A_1\)’ type mass matrix is not allowed.

**Case C:** \(m_D\) (4 zero) + \(\mu^5\) (5 zero):

There are 12 \(m_D\) matrices with 4 zero texture and they are designated as \(m^1_D,...,m^4_D\) in Table 1. Due to the same reason as discussed in Case A \(m^1_D,m^2_D\) and \(m^4_D\) are not considered. Furthermore, \(m_\nu\) arises through \(m^1_D,m^4_D\) and \(m^7_D\) also corresponds to the ‘\(A_1\)’ type matrix (shown in Eqn. (2.3)) and hence also discarded. Finally, remaining six \(m_D\) matrices \(m^2_D,m^3_D,m^5_D,m^6_D,m^7_D\) and \(m^9_D\) lead to the structure of \(m_\nu\) with two zero eigenvalues and hence are also neglected.

**Case D:** \(m_D\) (4 zero) + \(\mu^5\) (5 zero):

In this case, for \(m^2_D\) and \(m^3_D\) the low energy mass matrix \(m_\nu\) comes out as a null matrix while for \(m^4_D\) the structure of \(m_\nu\) is given by

\[
A_2 = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}
\]  
(2.4)

which is also neglected since the number of independent zeroes \(\geq 3\).

On the other hand rest of the \(m_D\) matrices (\(m^4_D\) to \(m^9_D\)) correspond to the structure of \(m_\nu\) as

\[
A_3 = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}
\]  
(2.5)

Interestingly, apriori we can not rule out the matrices of type \(A_3\), however it is observed that \(m_\nu\) of this type fails to generate \(\theta_{13}\) within the present experimental bound (details are mentioned in section (6.2.3)).

It is also observed that in this scheme to generate viable neutrino oscillation data, four zero texture of both \(m_D\) and \(\mu\) matrices are necessary. Therefore, now on we discuss extensively the four zero texture in both the sectors (Dirac as well as Majorana sector).

### 3 4 zero texture

There are 126 ways to choose 4 zeros out of 9 elements of a general \(3 \times 3\) matrix. Hence there are 126 textures. Incorporation of scaling ansatz in these textures leads to a drastic reduction to only
12 as given in the Table 1

In our choosen basis since $M_{RS}$ is taken as diagonal therefore the structure of $m_D$ leads to the same structure of $F$. On the other hand the lepton no violating mass matrix $\mu$ is complex symmetric and therefore from the matrices listed in Table 1 only $m^4_1$ and $m^4_{10}$ type matrices are acceptable. We renamed those matrices as $\mu^4_1$ and $\mu^4_2$ and explicit structures of them are presented in Table 3.

Table 3: Four zero texture of $\mu$

|   | $\mu^4_1$ | $\mu^4_2$ |
|---|------------|------------|
|   | $r_1$ 0 0 | $r_1$ $k_2 s_3$ $s_3$ |
|   | 0 $k_2 s_3$ $k_2 s_3$ | $k_2 s_3$ 0 0 |
|   | 0 $k_2 s_3$ $s_3$ | $s_3$ 0 0 |

There are now $2 \times 12 = 24$ types of $m_\nu$ due to both the choices of $\mu$ matrices. We discriminate different types of $m_D$ matrices in the following way:

i) First of all the texture $m^4_{10}$, $m^4_{11}$ and $m^4_{12}$ are always scaling ansatz invariant due to the same reason mentioned earlier in Case A and hence are all discarded.

Next the matrices $m^4_1$, $m^4_2$ and $m^4_3$ are also ruled out due to the following:

a) When $\mu^4_1$ matrix is taken to generate $m_\nu$ along with $m^4_1$, $m^4_2$ and $m^4_3$ as the Dirac matrices, then the structure of the effective $m_\nu$ appears such that, one generation is completely decoupled thus leading to two mixing angles are zero for the matrix $m^4_1$, and two eigenvalues are zero when we consider $m^4_2$ and $m^4_3$ matrices.

b) In case of $\mu^4_2$ matrix, the form of $m_\nu$ for $m^4_1$ comes out as

\[
A_4 = \begin{pmatrix}
* & * & * \\
* & 0 & 0 \\
* & 0 & 0
\end{pmatrix}
\]

(3.1)

which is phenomenologically ruled out and for other two matrices ($m^4_2$ and $m^4_3$) $m_\nu$ is a null matrix.

For a compact view of the above analysis we present the ruled out and the survival structures of $m_\nu$ symbolically in Table 4.
Table 4: Compositions of the discarded and survived structures of $m_\nu$

| $m_D$ | $m^{4}_{1}$ | $m^{4}_{2}$ | $m^{4}_{3}$ | $m^{4}_{4}$ | $m^{4}_{5}$ | $m^{4}_{6}$ | $m^{4}_{7}$ | $m^{4}_{8}$ | $m^{4}_{9}$ | $m^{4}_{10}$ | $m^{4}_{11}$ | $m^{4}_{12}$ |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\mu_1$ | × | × | × | ✔ | ✔ | ✔ | ✔ | ✔ | × | × | × | × |
| $\mu_2$ | × | × | × | ✔ | ✔ | ✔ | ✔ | ✔ | × | × | × | × |

Thus we are left with same six textures of $m_D$ for both the choices of $\mu$ and they are renamed in Table 5 as $m^{4}_{D1}$, $m^{4}_{D2}$, $\ldots$, $m^{4}_{D6}$.

Table 5: Four zero textures of the Dirac mass matrices

| $m^{4}_{D1}$ | $m^{4}_{D2}$ | $m^{4}_{D3}$ | $m^{4}_{D4}$ | $m^{4}_{D5}$ | $m^{4}_{D6}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $\begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$ | $\begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & k_1 c_1 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$ | $\begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$ |

The above analysis finally leads to altogether 12 effective $m_\nu$ matrices arising due to six $m_D$ ($m^{4}_{D1}$ to $m^{4}_{D6}$) and two $\mu$ ($\mu^{4}_{1}$ and $\mu^{4}_{2}$) matrices.

4 Parametrization

Depending upon the composition of $m_D$ and $\mu$ we subdivided those 12 $m_\nu$ matrices in four broad categories and each category is again separated in few cases and the decomposition is presented in Table 6 and Table 7.

Throughout our analysis we consider the matrix $M_{RS}$ as

$$M_{RS} = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}.$$  \hspace{1cm} (4.1)

Following Eqn.(4.2) the $m_\nu$ matrix arises in Category A and Category B can be written in a
Table 6: Different Composition of $m_D$ and $\mu_1$ matrices to generate $m_\nu$.

| Category A | Category B |
|------------|------------|
| Matrices   | $I_A$      | $II_A$ | $I_B$ | $II_B$ | $III_B$ | $IV_B$ |
| $m_D$      | $m_{D2}$   | $m_{D6}$ | $m_{D1}$ | $m_{D3}$ | $m_{D4}$ | $m_{D5}$ |
| $\mu$      | $\mu_1^2$ | $\mu_1^4$ | $\mu_1^4$ | $\mu_1^4$ | $\mu_1^4$ | $\mu_1^4$ |

Table 7: Different Composition of $m_D$ and $\mu_2$ matrices to generate $m_\nu$.

| Category C | Category D |
|------------|------------|
| Matrices   | $I_C$      | $II_C$ | $I_D$ | $II_D$ | $III_D$ | $IV_D$ |
| $m_D$      | $m_{D1}$   | $m_{D4}$ | $m_{D2}$ | $m_{D3}$ | $m_{D5}$ | $m_{D6}$ |
| $\mu$      | $\mu_3^2$ | $\mu_3^2$ | $\mu_3^2$ | $\mu_3^2$ | $\mu_3^2$ | $\mu_3^2$ |

The generic way as

$$m_\nu^{AB} = m_0 \begin{pmatrix} 1 & k_1p & p \\ k_1p & k_1^2(q^2+p^2) & k_1(q^2+p^2) \\ p & k_1(q^2+p^2) & (q^2+p^2) \end{pmatrix}$$

with the definition of parameters as following:

**Set I_A**: $m_0' = \frac{d_3^2 s_3}{p_3^2}, p' = \frac{p_3 c_2}{p_2 d_3}, q' = \frac{c_1 p_3}{d_3 p_3} \sqrt{\frac{r_1}{s_3}}, m_0 = m_0', p = k_2 p', q = q'$

**Set II_A**: $m_0' = \frac{d_1 r_1}{p_1^2}, p' = \frac{p_2 c_2}{p_3 d_2}, q' = \frac{c_1 p_2}{d_2 p_2} \sqrt{\frac{r_1}{s_1}}, m_0 = m_0', p = p', q = q'$

**Set I_B**: $m_0' = \frac{d_3^2 s_3}{p_3^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_3 p_1}{d_1 p_3} \sqrt{\frac{s_3}{r_1}}, m_0 = m_0', p = p', q = q'$

**Set II_B**: $m_0' = \frac{d_3^2 s_3}{p_3^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_3 p_1}{d_1 p_3} \sqrt{\frac{s_3}{r_1}}, m_0 = m_0', p = p', q = q'$

**Set III_B**: $m_0' = \frac{d_1 r_1}{p_1^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_1 p_2}{d_1 p_2} \sqrt{\frac{r_1}{s_1}}, m_0 = m_0', p = p', q = k_2 q'$

**Set IV_B**: $m_0' = \frac{d_1 r_1}{p_1^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_1 p_2}{d_1 p_2} \sqrt{\frac{r_1}{s_1}}, m_0 = m_0', p = p', q = \frac{q'}{k_2}$

Similarly the $m_\nu$ matrix arises in Category C can be written as

$$m_\nu^C = m_0 \begin{pmatrix} 1 & k_1(p+q) & p+q \\ k_1(p+q) & k_1^2(2pq+p^2) & k_1(2pq+p^2) \\ p+q & k_1(2pq+p^2) & (2pq+p^2) \end{pmatrix}$$
with the following choice of parameters

\[
Set\ I_C: m_0' = \frac{d_1^2 r_1}{p_1^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_2 p_1}{d_1 p_2} \sqrt{\frac{s_3}{r_1}}, m_0 = m_0', p = p', q = k_2 q' \\
Set\ II_C: m_0' = \frac{d_1^2 r_1}{p_1^2}, p' = \frac{c_1}{d_1}, q' = \frac{c_3 p_1}{d_1 p_2} \sqrt{\frac{s_3}{r_1}}, m_0 = m_0', p = p', q = q'.
\]

(4.5)

For Category D the effective \( m_\nu \) comes out as

\[
m_\nu^D = m_0 \begin{pmatrix}
0 & k_1 p & p \\
0 & k_1 (q^2 + 2 p r) & k_1 (q^2 + 2 r p) \\
p & k_1 (q^2 + 2 r p) & (q^2 + 2 r p)
\end{pmatrix}
\]

(4.6)

with the definition of parameters as

\[
Set\ I_D: m_0' = \frac{d_1^2 r_1}{p_1^2}, p' = \frac{c_1 p_1 s_3}{d_2 p_2 r_1}, q' = \frac{c_1}{d_1}, r' = \frac{c_2}{d_2}, m_0 = m_0', p = k_2 p', q = q', r = r' \\
Set\ II_D: m_0' = \frac{d_3^2 r_1}{p_1^2}, p' = \frac{c_1 p_1 s_3}{d_3 p_3 r_1}, q' = \frac{c_1}{d_1}, r' = \frac{c_2}{d_2}, m_0 = m_0', p = p', q = q', r = r' \\
Set\ III_D: m_0' = \frac{c_1 p_1 s_3}{d_3 p_3 r_1}, p' = \frac{c_1}{d_1}, q' = \frac{c_1}{d_1}, r' = \frac{c_2}{d_2}, m_0 = m_0', p = k_2 p', q = q', r = r' \\
Set\ IV_D: m_0' = \frac{d_1^2 r_1}{p_1^2}, p' = \frac{c_1 p_1 s_3}{d_2 p_2 r_1}, q' = \frac{c_1}{d_1}, r' = \frac{c_2}{d_2}, m_0 = m_0', p = k_2 p', q = q', r = r'.
\]

(4.7)

In general, we consider all the parameters \( m_0, k_1, p, r \) and \( q \) are complex.

**5 Phase Rotation**

As mentioned earlier, all the parameters of \( m_\nu \) are complex and therefore we can rephase \( m_\nu \) by a phase rotation to remove the redundant phases. Here, we systematically study the phase rotation for each category.

**Category A,B**

The Majorana type mass matrix \( m_\nu \) can be rotated in phase space through

\[
m^\prime_{\nu AB} = P^T m^{AB}_\nu P
\]

(5.1)

where \( P \) is a diagonal phase matrix and is given by \( P = \text{diag}(e^{i\Phi_1}, e^{i\Phi_2}, e^{i\Phi_3}) \).

Redefining the parameters of \( m_\nu \) as

\[
m_0 \to m_0 e^{i\alpha m}, p \to p e^{i\theta_p}, q \to q e^{i\theta_q}, k_1 \to k_1 e^{i\theta_1}
\]

(5.2)

with

\[
\Phi_1 = -\frac{\alpha_m}{2}, \Phi_2 = -(\theta_1 + \theta_p + \frac{\alpha_m}{2}), \Phi_3 = -(\theta_p + \frac{\alpha_m}{2})
\]

(5.3)
the phase rotated $m'^{AB}_\nu$ appears as

$$m'^{AB}_\nu = m_0 \begin{pmatrix}
1 & k_1 p & k_1 (q^2 e^{i\theta} + p^2) & p \\
\left( k_1 p \right) & k_1^2 (q^2 e^{i\theta} + p^2) & k_1 (q^2 e^{i\theta} + p^2) & p \\
\left( p \right) & k_1 (q^2 e^{i\theta} + p^2) & (q^2 e^{i\theta} + p^2) & (q^2 e^{i\theta} + p^2)
\end{pmatrix}$$ (5.4)

where $\theta = 2(\theta_q - \theta_p)$ and all the parameters $m_0, p, q$ and $k_1$ are real. Thus there is only a single phase parameter in $m'^{AB}_\nu$.

**Category C**

In a similar way, the mass matrix of **Category C** can be rephased as

$$m'^{C}_\nu = m_0 \begin{pmatrix}
1 & k_1 (p + q e^{i\theta}) & 0 \\
\left( k_1 (p + q e^{i\theta}) \right) & k_1^2 (2pq e^{i\theta} + p^2) & k_1 (2pq e^{i\theta} + p^2) \\
\left( 0 \right) & k_1 (2pq e^{i\theta} + p^2) & (2pq e^{i\theta} + p^2)
\end{pmatrix}$$ (5.5)

with the same set of redefined parameters as mentioned in Eqn.(5.2) and (5.3), diagonal phase matrix mentioned in the previous case and with $\theta = \theta_q - \theta_p$.

**Category D**

For this **Category** the rephased mass matrix comes out to be

$$m'^{D}_\nu = m_0 \begin{pmatrix}
0 & k_1 p & k_1 (q^2 e^{i\alpha} + 2rpe^{i\beta}) \\
\left( k_1 p \right) & k_1^2 (q^2 e^{i\alpha} + 2rpe^{i\beta}) & k_1 (q^2 e^{i\alpha} + 2rpe^{i\beta}) \\
\left( 0 \right) & k_1 (q^2 e^{i\alpha} + 2rpe^{i\beta}) & (q^2 e^{i\alpha} + 2rpe^{i\beta})
\end{pmatrix}$$ (5.6)

with $r \to re^{i\theta}, \alpha = 2(\theta_q - \theta_p), \beta = (\theta_r - \theta_p)$ and the rest of the parameters are defined as in Eqn.(5.2) and Eqn.(5.3).

### 6 Breaking of the scaling ansatz

Since the neutrino mass matrix obtained in Eqn.(5.4), (5.5) and (5.6) are all invariant under scaling ansatz and thereby give rise to $\theta_{13} = 0$ as well as $m_3 = 0$. Although vanishing value of $m_3$ is yet not ruled out however, the former, $\theta_{13} = 0$ is refuted by the reactor experimental results. Popular paradigm is to consider $\theta_{13} = 0$ at the leading order and by further perturbation nonzero value of $\theta_{13}$ is generated. We follow the same way to produce nonzero $\theta_{13}$ through small breaking of scaling ansatz. It is to be noted in our scheme, generation of nonzero $\theta_{13}$ necessarily needs breaking in $m_D$. To generate nonzero $m_3$ breaking in $\mu$ matrix is also necessary along with $m_D$, however, in **Category B** since $\text{det}(m_D = 0)$ even after breaking in the $\mu$ matrix $m_\nu$ still gives one of the eigenvalue equal to zero. On the other hand for **Category C** and **Category D**, $\mu_2^4$ has always zero determinant because of being scaling ansatz invariant and therefore, leads to one zero eigenvalue.
as that of Category B. It is the Category A for which we get nonzero $\theta_{13}$ as well as nonzero $m_3$ after breaking the scaling ansatz in both the matrices ($m_D$ and $\mu$).

In the following, we invoke breaking of scaling ansatz in all four categories through
i) breaking in the Dirac sector ($\theta_{13} \neq 0, m_3 = 0$)
ii) breaking in the Dirac sector as well as Majorana sector ($\theta_{13} \neq 0, m_3 \neq 0$) and later we discuss separately both the cases.

6.1 Breaking in the Dirac sector

6.1.1 Category A,B

We consider minimal breaking of the scaling ansatz through a dimensionless real parameter $\epsilon$ in a single term of $m_D$ matrices of those categories as

\[
m_D^{2} = \begin{pmatrix}
0 & d_2 & 0 \\
k_1(1 + \epsilon)c_1 & 0 & k_1c_3 \\
c_1 & 0 & c_3
\end{pmatrix}, ~ m_D^{6} = \begin{pmatrix}
0 & 0 & d_3 \\
k_1(1 + \epsilon)c_1 & k_1c_2 & 0 \\
c_1 & c_2 & 0
\end{pmatrix}
\]

for Category A and

\[
m_D^{1} = \begin{pmatrix}
d_1 & 0 & 0 \\
k_1c_1 & 0 & k_1(1 + \epsilon)c_3 \\
c_1 & 0 & c_3
\end{pmatrix}, ~ m_D^{3} = \begin{pmatrix}
0 & 0 & d_3 \\
k_1(1 + \epsilon)c_1 & 0 & k_1c_3 \\
c_1 & 0 & c_3
\end{pmatrix}
\]

\[
m_D^{4} = \begin{pmatrix}
d_1 & 0 & 0 \\
k_1c_1 & k_1(1 + \epsilon)c_2 & 0 \\
c_1 & c_2 & 0
\end{pmatrix}, ~ m_D^{5} = \begin{pmatrix}
0 & d_2 & 0 \\
k_1(1 + \epsilon)c_1 & k_1c_2 & 0 \\
c_1 & c_2 & 0
\end{pmatrix}
\]

for Category B. We further want to mention that breaking considered in any element of the second row are all equivalent. For example, if we consider breaking in the ‘23’ element of $m_D^{2}$ it is equivalent to as considered in Eqn.(6.1). Neglecting the $\epsilon^2$ and higher order terms, the effective $m_\nu$ matrix comes out as

\[
m_\nu^{AB\epsilon} = m_0 \begin{pmatrix}
1 & k_1p & p \\
k_1p & k_1^2(q^2e^{i\theta} + p^2) & k_1(q^2e^{i\theta} + p^2) \\
p & k_1(q^2e^{i\theta} + p^2) & (q^2e^{i\theta} + p^2)
\end{pmatrix} + m_0\epsilon \begin{pmatrix}
0 & 0 & 0 \\
0 & 2k_1^{-2}q^2e^{i\theta} & k_1q^2e^{i\theta} \\
0 & k_1q^2e^{i\theta} & 0
\end{pmatrix}.
\]

As mentioned earlier, that for Category B, det($m_D$) = 0 and it is not possible to generate $m_3 \neq 0$ even if we consider breaking in the $\mu$ matrices. On the other hand, the matrices in Category A posses det($m_D$) $\neq 0$ and thereby give rise to $m_3 \neq 0$.

Now to calculate the eigenvalues, mixing angles, $J_{CP}$ and Dirac and Majorana phases we utilize the results obtained in ref. [40], for a general complex matrix. We should mention that the formula
obtained in ref. [40], for Majorana phases is valid when all three eigenvalues are nonzero. However, when one of the eigenvalue is zero (in this case $m_3 = 0$) one has to utilise the methodology given in ref. [16], which shows, a general Majorana type mass matrix $m_\nu$ can be diagonalized as

$$U^\dagger m_\nu U^* = \text{diag}(m_1, m_2, m_3) \quad (6.4)$$

or, alternatively,

$$m_\nu = U \text{diag}(m_1, m_2, m_3) U^T \quad (6.5)$$

where,

$$U = U_{CKM} P_M. \quad (6.6)$$

The mixing matrix $U_{CKM}$ is given by (following PDG [41] convention)

$$U_{CKM} = \begin{pmatrix}
   c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta_{CP}} \\
   -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta_{CP}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta_{CP}} & c_{13} s_{23} \\
   s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i \delta_{CP}} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i \delta_{CP}} & c_{13} c_{23}
\end{pmatrix} \quad (6.7)$$

with $c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$ and $\delta_{CP}$ is the Dirac CP phase. The diagonal phase matrix $P_M$ is parametrized as

$$P_M = \text{diag}(1, e^{\alpha_M}, e^{i(\beta_M + \delta_{CP})}) \quad (6.8)$$

with $\alpha_M$ and $\beta_M$ are the Majorana phases.

Writing Eqn.(6.5) explicitly with $m_3 = 0$ we can have expressions for six independent elements of $m_\nu$ in terms of the mixing angles, two eigenvalues and the Dirac CP phase, from which the $m_{11}$ element can be written as

$$m_{11} = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{2i\alpha_M} \quad (6.9)$$

and therefore the Majorana phase $\alpha_M$ comes out as

$$\alpha_M = \frac{1}{2} \cos^{-1} \left\{ \frac{|m_{11}|^2}{2 c_{12}^2 s_{12}^2 c_{13}^2 m_1 m_2} - \frac{(c_{12}^4 m_1^2 + s_{12}^4 m_2^2)}{2 c_{12}^2 s_{12}^2 m_1 m_2} \right\}. \quad (6.10)$$

The Jarlskog measure of CP violation $J_{CP}$ is defined in usual way as

$$J_{CP} = \frac{\text{Im}(h_{12} h_{23} h_{31})}{(\Delta m_{21}^2)(\Delta m_{32}^2)(\Delta m_{31}^2)} \quad (6.11)$$

where, $h$ is a hermitian matrix constructed out of $m_\nu$ as $h = m_\nu m_\nu^\dagger$. 

13
6.1.2 Category C

In this case breaking is considered in \( m_D \) as

\[
\begin{align*}
\begin{pmatrix}
  d_1 & 0 & 0 \\
  k_1(1+\epsilon)c_1 & k_1c_2 & 0 \\
  c_1 & c_2 & 0
\end{pmatrix},
\begin{pmatrix}
  d_1 & 0 & 0 \\
  k_1(1+\epsilon)c_1 & 0 & k_1c_3 \\
  c_1 & 0 & c_3
\end{pmatrix}
\end{align*}
\]

and the scaling ansatz broken \( m_\nu \) appears as

\[
\begin{align*}
\begin{pmatrix}
  1 & k_1(p + qe^{i\theta}) & p + qe^{i\theta} \\
  k_1(p + qe^{i\theta}) & k_1^2(2pqe^{i\theta} + p^2) & k_1(2pqe^{i\theta} + p^2) \\
  p + qe^{i\theta} & k_1(2pqe^{i\theta} + p^2) & (2pqe^{i\theta} + p^2)
\end{pmatrix}
\end{align*}
\]

\[
+ m_0 \epsilon \begin{pmatrix}
  0 & k_1q e^{i\theta} & 0 \\
  k_1q e^{i\theta} & 2k_1^2pq e^{i\theta} & k_1pq e^{i\theta} \\
  0 & k_1pq e^{i\theta} & 0
\end{pmatrix}
\]

(6.13)

6.1.3 Category D

Breaking in the \( m_D \) in this case incorporated as

\[
\begin{align*}
\begin{pmatrix}
  0 & d_2 & 0 \\
  k_1c_1 & 0 & k_1(1+\epsilon)c_3 \\
  c_1 & 0 & c_3
\end{pmatrix},
\begin{pmatrix}
  0 & 0 & d_3 \\
  k_1c_1 & 0 & k_1(1+\epsilon)c_3 \\
  c_1 & 0 & c_3
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix}
  0 & d_2 & 0 \\
  k_1c_1 & k_1(1+\epsilon)c_2 & 0 \\
  c_1 & c_2 & 0
\end{pmatrix},
\begin{pmatrix}
  0 & 0 & d_3 \\
  k_1c_1 & k_1(1+\epsilon)c_2 & 0 \\
  c_1 & c_2 & 0
\end{pmatrix}
\end{align*}
\]

and the corresponding \( m_\nu \) comes out as

\[
\begin{align*}
\begin{pmatrix}
  0 & k_1p \\
  k_1p & k_1^2(q^2e^{i\alpha} + 2rpe^{i\beta}) & k_1(q^2e^{i\alpha} + 2rpe^{i\beta}) \\
  p & k_1(q^2e^{i\alpha} + 2rpe^{i\beta}) & (q^2e^{i\alpha} + 2rpe^{i\beta})
\end{pmatrix}
\end{align*}
\]

\[
+ m_0 \epsilon \begin{pmatrix}
  0 & 0 & 0 \\
  0 & 2k_1^2rpe^{i\beta} & k_1rpe^{i\beta} \\
  0 & k_1rpe^{i\beta} & 0
\end{pmatrix}
\]

(6.15)

6.2 Numerical Analysis

In order to execute the numerical analysis to obtain allowed parameter space we utilize the neutrino oscillation data obtained from global fit shown in Table 8.
Table 8: Input experimental values \[14\]

| Quantity       | 3\(\sigma\) ranges other constraints                                      |
|----------------|--------------------------------------------------------------------------|
| \(|\Delta m^2_{31}|\) N | \(2.31 < \Delta m^2_{31}(10^3\text{eV}^{-2}) < 2.74\)                   |
| \(|\Delta m^2_{31}|\) I | \(2.21 < \Delta m^2_{31}(10^3\text{eV}^{-2}) < 2.64\)                   |
| \(\Delta m^2_{21}\)    | \(7.21 < \Delta m^2_{21}(10^3\text{eV}^{-2}) < 8.20\)                  |
| \(\theta_{12}\)       | \(31.3^o < \theta_{12} < 37.46^o\)                                     |
| \(\theta_{23}\)       | \(36.86^o < \theta_{23} < 55.55^o\)                                     |
| \(\theta_{13}\)       | \(7.49^o < \theta_{13} < 10.46^o\)                                     |

6.2.1 Category A,B

We first consider Category A, B, for which the neutrino mass matrix is given in Eqn.(6.3). For our analysis a small value of breaking parameter \((\epsilon = 0.1)\) is sufficient to encompass the extant data. In Fig.1 we plot the variation of \(p\) and \(q\) with \(k_1\) and the allowed ranges are obtained as \(2.11 < p < 3.37, 2.13 < q < 3.29\) and \(0.7 < k_1 < 1.3\).

Fig. 1: plot of \(p\) vs \(k_1\) (left), \(q\) vs \(k_1\) (right) for the Category A, B.

It is interesting to note a typical feature of this category is that the Dirac CP phase \(\delta_{CP}\) comes out too tiny and thereby generating almost vanishing value of \(|J_{CP}| \approx 10^{-6}\) while the range of the only Majorana phase in this category obtained as \(79^o < \alpha_M < 89^o\).
As one of the eigenvalue $m_3 = 0$ therefore, the hierarchy of the masses is clearly inverted in this category, while in Fig. 2 we plot the sum of the three neutrino masses $\Sigma_i m_i (= m_1 + m_2 + m_3)$ with $|m_{11}|$ which predicts the value of the two quantities below the present experimental ranges.

In a nutshell, distinguishable characteristics of this category is i) tiny $|J_{CP}|$ and $\delta_{CP}$ ii) inverted hierarchy of the neutrino masses. At the end of this section we will further discuss the experimental testability of these quantities for all the categories.

### 6.2.2 Category C

In this case it is found that a small breaking ($\epsilon = 0.05$) is sufficient to accommodate all the oscillation data. We explore the parameter space and the ranges obtained as $3.42 < p < 6.07$, $1.68 < q < 3.02$, and $0.7 < k_1 < 1.32$ shown in Fig. 3. The hierarchy in this case is also inverted due to the vanishing value of $m_3$. 

---

**Fig. 2**: Plot of $|m_{11}|$ vs $\Sigma_i m_i$ for Category A, B.
6.2.3 Category D

In case of Category D, although at a glance it is not possible to rule out $m_D^{\epsilon}$ without going into the detailed numerical analysis, however in this case, even if with $\epsilon = 1$ it is not possible to accommodate the neutrino oscillation data. Specifically, the value of $\theta_{13}$ is always beyond the reach of the parameter space. Exactly for the same reason the $m_\nu$ matrix of type $A_3$ in Eqn. (2.5) is phenomenologically ruled out.

6.3 Breaking in Dirac+Majorana sector

In this section we focus on the phenomenology of the neutrino mass matrix where the scaling ansatz is broken in both the sectors. This type of breaking is only relevant for the Category A since in this case $m_D$ is nonsingular after breaking of the ansatz. In all the other categories due to the singular nature of $m_D$, inclusion of symmetry breaking in the Majorana sector will not generate $m_3 \neq 0$. Thus we consider only Category A under this scheme.
We consider the breaking in $m_D$ as mentioned in Eqn.(6.1) and the ansatz broken texture of $\mu_1^4$ matrix is given by

$$
\mu_1^4 = \begin{pmatrix}
 r_1 & 0 & 0 \\
 0 & k_2^2 s_3 & k_2 (1 + \epsilon') s_3 \\
 0 & k_2 (1 + \epsilon') s_3 & s_3
\end{pmatrix}
$$

(6.16)

where, $\epsilon'$ is a dimensionless real parameter. The effective neutrino mass matrix $m_\nu$ comes out as

$$
m_{\nu eff}^A = m_0 \begin{pmatrix}
 1 & k_1 p & p \\
 k_1 p & k_1^2 (q^2 e^{i\theta} + p^2) & k_1 (q^2 e^{i\theta} + p^2) \\
 p & k_1 (q^2 e^{i\theta} + p^2) & (q^2 e^{i\theta} + p^2)
\end{pmatrix} + m_0 \epsilon \begin{pmatrix}
 0 & 0 & 0 \\
 0 & 2k_1^2 q^2 e^{i\theta} & k_1 q^2 e^{i\theta} \\
 0 & k_1 q^2 e^{i\theta} & 0
\end{pmatrix} + m_0 \epsilon' \begin{pmatrix}
 0 & k_1 p & p \\
 k_1 p & 0 & 0 \\
 p & 0 & 0
\end{pmatrix}.
$$

(6.17)

6.3.1 Numerical results

To obtain parameter space for this category we consider small breaking in the Majorana sector along with $m_D$. Parameter space obtained in this case for both values of $\epsilon$ and $\epsilon'$ equals to 0.1 and the ranges of the parameters are $1.89 < p < 3.07$, $1.87 < q < 3.09$ and $0.67 < k_1 < 1.31$. Interestingly although all the eigenvalues are nonzero in this case, the hierarchy is as usual inverted as one can see from Fig. 5 (left).

![Fig. 5: Plot of $(m_1/m_3)$ vs $(m_2/m_1)$ (left) and $\beta_M$ vs $\alpha_M$ (right) after breaking of the scaling ansatz in both the sectors of Category A.](image)

$|J_{CP}|$ is found to be tiny ($\approx 10^{-6}$) again due to small value of $\delta_{CP}$. In the right panel of Fig. 5(right) the Majorana phases are shown. The bounds on $\Sigma_i m_i$ and $|m_{11}|$ obtained as
0.09eV < Σ_i m_i < 0.11eV and 0.011eV < |m_{11}| < 0.015eV which are well below the present experimental upper bounds.

Some comments are in order regarding predictions of the present scheme:

1. After precise determination of θ_{13} taking full account of reactor neutrino experimental data, it is shown that the hierarchy of the light neutrino masses can be probed through combined utilization of NO\nuA and T2K [42] neutrino oscillation experimental results in near future. Thus the speculation of hierarchy in the present scheme will be clearly verified. Moreover, taking the difference of probabilities between \( P(\nu_\mu \rightarrow \nu_e) \) and \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \) information on the value of \( |J_{CP}| \) can be obtained using neutrino and antineutrino beams.

2. More precise estimation of the sum of the three light neutrino masses will be obtained utilizing a combined analysis with Planck data [43] and other cosmological and astrophysical experiments [44], such as, Baryon oscillation spectroscopic survey, The Dark energy survey, Large Synoptic Survey Telescope or the Euclid satellite data [45] etc. Such type of analysis will push \( \Sigma_i m_i \sim 0.1 \) eV (at the 4σ level for inverted ordering) and \( \Sigma_i m_i \sim 0.05 \) eV (at the 2σ level for normal ordering). Thus the prediction of the value of \( \Sigma_i m_i \) in the different categories discussed in this work will also be tested in the near future.

Furthermore, the NEXT-100 [46] will probe the value of \( |m_{11}| \) up to 0.1 eV, a more precise value than the EXO-200 [47] experimental range (0.14-0.38 eV).

7 Summary and conclusion

In this work we explore the phenomenology of neutrino mass matrix obtained due to inverse seesaw mechanism adhering i) Scaling ansatz, ii) Texture zeroes within the framework of \( SU(2)_L \times U(1)_Y \) model with three right handed neutrinos and three left chiral singlet fermions. Throughout our analysis we choose a basis in which the charged lepton mass matrix and the \( M_{RS} \) matrix (appeared in inverse seesaw mechanism due to the coupling of \( \nu_R \) and \( S_L \)) are diagonal. It is found that four is the maximum number of zeros can be allowed in \( M_D \) and \( \mu \) matrices to obtain viable phenomenology. We classified different four zero texture in four different categories depending upon their generic form. Since scaling ansatz invariance always give rise to \( \theta_{13} = 0 \) we have to break such ansatz. We consider breaking in \( M_D \) and also in \( \mu \) matrices. We explore the parameter space and it is seen one category (Category D) is ruled out phenomenologically. The hierarchy obtained in all the cases is inverted and it is interesting to note that all such categories give rise to tiny \( CP \) violation measure \( |J_{CP}| \) due to small value of \( \delta_{CP} \). The vantage point of our analysis is that further
observation of hierarchy of neutrino masses and CP violation in the leptonic sector in the forthcoming experiments will conclusively refute or admit all these categories obtained in the present scheme.

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