Calculation and Spectra Analysis of Horizontal Acceleration Correction (HACC) for Airborne Gravimetry

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ABSTRACT On the basis of a sinusoidal model of the disturbed horizontal acceleration, the spectrum characteristics of misaligned angle and horizontal acceleration correction are analyzed. In an airborne gravimetry test, the misaligned angle of platform and horizontal acceleration correction are calculated. They are 5° and 3 mGal, respectively, when the flight is stable.

KEYWORDS airborne gravimetry; airborne gravimeter; horizontal acceleration correction; misaligned angle of the platform; GPS

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Introduction

In the traditional airborne gravimetry system, the gravimeter is mounted on the stable platform so as to keep the sensitive axis of sensor along the direction of gravity. It seems that the horizontal acceleration of aircraft does not affect the observations of gravimeter if the platform keeps perfectly horizontal in the local coordinate system. But it is not true because the platform is not stable enough in moving environment. Thus the gravity signal, vertical acceleration of aircraft and the component of horizontal acceleration along the gravity direction are sensed together by gravimeter. The latter is named horizontal acceleration correction (HACC) and some researches show that it is a major factor for limiting the accuracy improvement of airborne gravimetry. There are two methods for deducing its effect: one is to keep the aircraft with a constant speed and along a straight line, and the other is to make platform perfectly horizontal. To keep the flight stable depends on the performance of aircraft, and to keep the platform horizontal depends on the improvement of performance of platform. But at present it is difficult to make such a perfect system, thus evaluating horizontal acceleration correction is one of techniques considered by users.

On the basis of a sinusoidal model of disturbed horizontal acceleration, the misaligned angle, the average HACC and the instantaneous HACC are estimated, respectively. They are analyzed in the frequency domain, and then two methods for the calculation of horizontal acceleration correction are presented.

1 Mathematical model

If there is a small angle θ (called misalignment angle) between the actual platform and its horizontal position, then two types of errors are caused. One is the difference between the true gravity and the sensed gravity, and the relation of true gravity and sensed one is expressed as

\[ g_z = g \cdot \cos \theta \]  \hspace{0.5cm} (1)

where \( g_z \) is the sensed gravity; \( g \) is the true gravity.

And the other is the component of the horizontal acceleration along the vertical direction:

\[ g_z = a_z \cdot \sin \theta \]  \hspace{0.5cm} (2)

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where $g_z$ is the component of horizontal acceleration; $a_z$ is the horizontal acceleration of the platform.

The two errors above are named the horizontal acceleration correction (HACC) $g_t$:

$$g_t = g \cdot \cos \theta - a_z \cdot \sin \theta - g \approx -a_z \cdot \theta - \frac{E}{2} \theta^2$$

(3)

In Eq. (3), the horizontal acceleration $a_z$ can be measured by GPS, and $g_t$ can be calculated if the misaligned angle $\theta$ is given.

2 Spectra analysis of misaligned angle

The role of the gravimeter platform is to keep the gravimeter sensitive-axis along the direction of gravity. The LaCoste-Romberg (L&R) gravimeter is mounted on two-axis dampened platform, and it keeps horizontal by two adjustment circles. The first circle feeds the signal of the angle acceleration sensed by the gyro into the torque motor so as to drive the platform to horizontal position. But the platform soon deviates the horizontal position due to the gyro shifts. Therefore, the second one processes the accelerometer signals and feeds them into the gyro. The signals consist of two parts, one is proportional to the accelerometer signal and the other is proportional to integrated accelerometer singal. The equation for this type of circle is

$$\dot{\theta} + 2f \omega_0 (\theta + \frac{a_z}{g}) + \omega_0^2 \int (\theta + \frac{a_z}{g}) dt = 0$$

(4)

Eq. (5) is derived from Eq. (4):

$$\dot{\theta} + 2f \omega_0 (\theta + \frac{a_z}{g}) + \omega_0^2 (\theta + \frac{a_z}{g}) = 0$$

(5)

where $f$ is the dampened factor of the platform; $\omega_0$ is the inherent frequency of the platform.

If the frequency of the sinusoidal horizontal acceleration is $\omega$, the stable solution of Eq. (5) is

$$\theta = \frac{\omega_0^2 + 2f \omega_0 \omega a_z}{\omega_0^2 + \omega^2 + 2f \omega_0 \omega} \cdot \frac{a_z}{g} = H_s(i\omega) \cdot \frac{a_z}{g}$$

(6)

where $H_s(i\omega)$ is the transfer function of the system, and it is

$$H_s(s) = \frac{\omega_0^2 + 2f \omega_0 s}{\omega_0^2 + s^2 + 2f \omega_0 s}$$

(7)

and $H_s(i\omega)$ can be expressed with amplitude and phase angle, namely $H_s(i\omega) = A \cdot e^{i\varphi}$, thus we have the following two equations:

$$A = \sqrt{\frac{\omega_0^2 + 4f^2 \omega_0^2 \omega^2}{\omega_0^4 + 2a_z \omega_0^2 (2f^2 - 1) + \omega^4}}$$

$$\varphi = \arctan \left[ \frac{2f \omega_0 \omega a_z^2}{\omega_0^2 + 2a_z \omega_0^2 (2f^2 - 1) + \omega_0^2 \omega^2} \right]$$

(8)

(9)

With the dampened coefficient $f$, being $1/\sqrt{2}$ given by L&R, the amplitude response and phase response of transfer function of L&R are, respectively, depicted in Fig. 1. From Fig. 1, we can find that if the frequency $\omega$ is smaller than $\omega_0$, the magnitude of misaligned angle approximately equals to the value $a_z/g$. The misaligned angle is greater than $5^\circ$ when the amplitude of acceleration is 100 Gal, which will cause a big error. Fortunately there is almost $90^\circ$ difference between the phase of the horizontal acceleration and that of the misaligned angle, thus some errors are deducted and they can be removed through a filter if its frequency is different from the gravity signal.

3 Spectra analysis of HACC

3.1 Average HACC

Assuming that the disturbed horizontal acceleration is a sinusoidal function:

$$a_z = a_0 \cdot \cos \omega t$$

And the equation of misaligned angle can be re-
written as follows, according to Eq. (6):
\[
\theta = A \cdot \left( \frac{a_0}{g} \right) \cdot \cos(\omega t + \varphi)
\]
Substituting the above function into Eq. (3), we have
\[
g_s = \frac{a_0^2}{2g} \left[ -2A \cos(\omega t) \cdot \cos(\omega t + \varphi) - A^2 \cos^2(\omega t + \varphi) \right]
\] (10)
Then the following equation can be obtained by triangular transform
\[
g_s = \frac{a_0^2}{2g} \left[ -2A \cos \varphi - A^2 \right] \cdot \cos(\omega t + \varphi) - \sin \varphi \cdot \sin(2\omega t + 2\varphi)
\] (11)
Thus the average HACC is
\[
\langle g_s \rangle = \left( -2A \cos \varphi - A^2 \right) \cdot \langle \frac{a_0^2}{2g} \rangle = C \cdot \langle \frac{a_0^2}{2g} \rangle
\] (12)
where \( \langle \rangle \) is an averaging operator.
Eq. (13) can be deduced from Eq. (8) and Eq. (9):
\[
C = \frac{a_0^4 + 2a_0^2 \omega^2 (2f^2 - 1)}{a_0^4 + \omega^4 + 2a_0^2 \omega^2 (2f^2 - 1)} = 1 - \frac{\omega^4}{a_0^4 + \omega^4 + 2\omega^2 (2f^2 - 1)}
\] (13)
To minimize the average HACC, the value \( C \) will firstly be minimized. From Eq. (13) we can see that the value \( C \) is minimized if \( f = 1/\sqrt{2} \), and then we have
\[
C \left( f = 1/\sqrt{2} \right) = \frac{a_0^4}{a_0^4 + \omega^4}
\] (14)
\[
\langle g_s \rangle = \frac{a_0^4}{a_0^4 + \omega^4} \cdot \langle \frac{a_0^2}{2g} \rangle
\] (15)
The above function depicts the relationship of average HACC, inherent period of platform and the changing period of horizontal acceleration, and can be used for the calculation of magnitude of the HACC. The shorter the inherent period of the platform is, or the longer the changing period of the HACC is, the larger the horizontal acceleration is, as shown in Fig. 2.

\[
\text{Fig. 2 Relationship between } \langle g_s \rangle \text{ and } \omega
\]

### 3.2 Instantaneous HACC

Eq. (10) can be rewritten as:
\[
g_s = \frac{a_0^2}{4g} \left[ C - AD \sin(2\omega t + 2\varphi + \alpha) \right]
\] (16)
And define
\[
\sin \alpha = \frac{2 \cos \varphi + A}{D}, \quad \cos \alpha = \frac{2 \sin \varphi}{D}, \quad \text{where}
\]
\[
D^2 = (2 \cos \varphi + A)^2 + 4 \sin^2 \varphi = 4 + 4 \cos \varphi + A^2
\] (17)
thus we have
\[
\langle g_s \rangle = \frac{a_0^2}{4g} \left[ C - AD \sin(2\omega t + 2\varphi + \alpha) \right]
\] (18)
\[
\langle g_s \rangle = \frac{a_0^2}{4g} \cdot C = \frac{\omega_0^4}{\omega_0^4 + \omega^4} \cdot \frac{a_0^2}{4g}
\] (19)
Qmitting some deduction, we have the instantaneous horizontal acceleration correction:
\[
g_s = \frac{a_0^2}{4g} \cdot \frac{\omega_0^4}{\omega_0^4 + \omega^4} - \frac{a_0^2}{4g} \cdot \sqrt{\frac{\omega_0^4}{\omega_0^4 + \omega^4} \cdot \frac{\omega_0^4}{\omega_0^4 + \omega^4}} \cdot \sin(2\omega t + 2\varphi + \alpha)
\] (20)
where
\[
k \cdot \left( f = 1/\sqrt{2} \right) = \sqrt{1 + 8 \left( \frac{\omega_0^4}{\omega_0^4} \right)}
\] (21)
Because there is the condition \( k>1 \), the amplitude \( k \cdot \langle g_s \rangle \) of the instantaneous HACC \( g_s \) is greater than its average value. The max value of \( g_s \) is
\[
\langle g_{s, \text{max}} \rangle = (1 + k) \cdot \langle g_s \rangle
\] (22)
In order to compare the value \( \langle g_s \rangle \), the amplitude of \( g_s \) and the change of \( g_{s, \text{max}} \), we define some coefficients as
\[
G_s = \frac{a_0^2}{a_0^2 + \omega^4}
\]
\[
G_0 = k \cdot G_s
\]
\[
G_1 = (1 + k) \cdot G_s
\] (23)
The changes of \( G_s \), \( G_0 \) and \( G_1 \) with the frequency are shown in Fig. 3. From these figures, we can see that the average HACC declines rapidly with increasing frequency. Assuming \( a_0 = 0.1 \text{ m/s}^2 \), when \( \omega_0 > \omega_0^* \), the average value is smaller than 1 mGal, when \( \omega = \omega_0^* \), it is smaller than 0.5 mGal. The instantaneous HACC almost does not decline when \( \omega < 3 \omega_0^* \), and after this it declines slowly. The max value of HACC is the sum of the previous two terms, when the frequency is from 0 to \( \omega_0 \), its amplitude is twice the average value, and when \( \omega = 3 \omega_0 \), the max value is still larger than 2 mGal. Obviously, the in-
stantaneous HACC is larger than the value evaluated by Eq. (15), and its magnitude is much larger when \( \omega \gg \omega_0 \). But the instantaneous HACC has periodic characteristics, so it can be eliminated by filter.

\[
\begin{align*}
\text{Fig. 3} & \quad \text{Coefficients for the average value, amplitude} \\
& \quad \text{and the maximum value of the HACC}
\end{align*}
\]

3.3 Data processing

The following data are taken from an actual airborne gravimetry test, for which new gravimeter L8-R is used. In the test, the sampling rate of gravimeter is 1 Hz, the dammed coefficient \( f \) is \( 1/\sqrt{2} \), and the inherent period of platform is 4 min. There are nine surveying lines which include five east-west (E-W) surveying lines and four south-north (S-N) survey lines. The height of flight is 3.4 km and the flying velocity is about 400 km/h. We only analyze the data of surveying-line 1 (an E-W surveying line). The N-S and E-W horizontal accelerations of surveying-line 1 are shown in Fig. 4, and their corresponding amplitude spectra are depicted in Fig. 5. The N-S horizontal acceleration is selected as example because its change is larger than W-E ones. In Fig. 5(a), there are two peaks at 100 s and 60 s, respectively: one is 0.04 m/s² and the other is 0.05 m/s², and most power is concentrated there.

According to Eq. (20), the amplitude spectrum of instantaneous HACC can be obtained if the square of amplitude spectrum (shown in Fig. 5(a)) is divided by 4g and then multiplied by \( G_0 \), and it is shown in Fig. 6. If \( \omega_0 = 2\pi/240s \), the value \( G_0 \) varies with frequency is depicted in Fig. 7.

\[
\begin{align*}
\text{Fig. 4} & \quad \text{N-S and W-E horizontal acceleration for line 1}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 5} & \quad \text{Amplitude spectrum for the N-S and W-E horizontal acceleration}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 6} & \quad \text{HACC amplitude in the frequency domain}
\end{align*}
\]

From Fig. 6 we can see that the magnitude of horizontal acceleration correction spectrum is about 5 mGal at 100 s and 60 s, and its shape is the same as the spectrum of horizontal acceleration because the component whose spectrum is
shorter than 100 s is declined by \( G_0 \). Thus we can draw the conclusion that, to eliminate the acceleration correction completely by filter, the filter's period needs to be longer than 100 s, which will obviously reduce the resolution of the airborne gravimetry.

4 Calculation of HACC

4.1 Calculation of misaligned angle \( \theta \)

If the misaligned angle \( \theta \) has been given, HACC can be estimated by Eq. (3). In Eq. (6), if the value \( (a_\omega / g) \) is filtered by a digital filter with transfer function \( H_\theta(i\omega) \), the output is the misaligned angle. Thus a second-order recursive filter designed by the bilinear transfer method is as follows.

The \( j \)th output \( q_j \) can be calculated from the \( j \)th input \( p_j \) and the \((j-1)\)th and \((j-2)\)th output and input.

\[
q_j = c_0 p_j + c_1 p_{j-1} + c_2 p_{j-2} + d_1 q_{j-1} + d_2 q_{j-2}
\]

where

\[
\begin{align*}
    c_0 &= (a+b)/(4+a+b) ;
    c_1 &= (2b)/(4+a+b) ;
    c_2 &= (b-a)/(4+a+b) ;
    d_1 &= (8-2b)/(4+a+b) ;
    d_2 &= (a-b-4)/(4+a+b) ;
    a &= 4\omega_0 \Delta t ;
    b &= (\omega_0 \Delta t)^2 ;
\end{align*}
\]

For the airborne gravimetry test, if \( \omega_0 = 2\pi/240s \), the time interval meets \( \Delta t = 1 \) s, and the horizontal acceleration is the same as the one shown in Fig. 4, the misaligned angle is evaluated and shown in Fig. 8. From Fig. 8, we can see that the max value of misaligned angle is about 0.5°. Comparing Fig. 4(a) with Fig. 8, we can find that the misaligned angle changes greatly while the horizontal acceleration varies largely. Near 525 400 s, the aircraft experiences a larger roll due to an air onflow, thus a greater misaligned angle along N-S direction is aroused in following time (W-E flight). Fig. 9 is the rectangular figure of the N-S misaligned angle. The statistical result indicates that the average value of N-S misaligned angle in all surveying lines is 8°, above 70 percent of misaligned angles are smaller than 12°, and the values of the misaligned angles smaller than 20° make up 90 percent. In stable flight (before 525 400s), the average value is about 5°, and the W-E one is about 3°.

4.2 Direct calculation method

This method is used to evaluate the misaligned angles of platform but not used in actual test because there is a greater difference between its recursive filter and the FIR filter.
accelerations including gravity are the same in the both coordinate systems. And we have
\[ \sum_{i} a_{i}^2 = \sum_{i} g_{i}^2 + a_{i}^2 \]  \hspace{1cm} (25)
where \( g_{i} \) is the sensed gravity; \( a_{i} \) is the cross horizontal acceleration; \( a_{i} \) is the along-track horizontal acceleration; \( a_{i} \) is the eastward acceleration deduced from GPS; \( a_{i} \) is the northward acceleration; \( G \) is the sum of gravity and vertical acceleration.

Then we have
\[ G = \left( \sum_{i} g_{i}^2 + \sum_{i} a_{i}^2 \right) + \frac{\sum_{i} a_{i}^2 - \sum_{i} a_{i}^2}{2g_{i}} \]  \hspace{1cm} (26)
Because the horizontal acceleration is greatly smaller than the gravity, we approximately have
\[ G = g_{i} + \frac{\sum_{i} a_{i}^2 - \sum_{i} a_{i}^2}{2g_{i}} \]  \hspace{1cm} (27)
So HACC
\[ g_{i} = \frac{\sum_{i} a_{i}^2 - \sum_{i} a_{i}^2}{2g_{i}} \]  \hspace{1cm} (28)
where \( g_{i} \) can be approximately substituted by \( g \).

Before calculating the horizontal acceleration correction by Eq. (28), all accelerations should be filtered by a filter.

The horizontal acceleration correction calculated by the method is shown in Fig. 10. Similar to the instance of the misaligned angle, the critical position is near 525 400 s. Before this position the magnitude of horizontal acceleration correction is smaller, and after this it is larger. The average of absolute value of HACC of all line is about 11 mGal, but in stable segment the average value is about 3 mGal, and the absolute values smaller than 40 mGal make up more than 90 percent.

5 Conclusions

1) The magnitude of misaligned angle is dependent on both the magnitude of disturbed horizontal acceleration and the spectrum characteristic of frequency of the platform. If the frequency of the disturbed horizontal acceleration is smaller than the inherent frequency of the platform, the magnitude of misaligned angle is about \( a_{i}/g \).

2) The horizontal acceleration correction is evaluated by average HACC, but we find that, in fact, the amplitude of the instantaneous HACC changes considerably, hence the average HACC cannot reflect the reality.

3) In the spectrum figure of the N-S horizontal acceleration, there are two significant peaks. One is 0.04 m/s\(^2\) and the other 0.05 m/s\(^2\) at 100 s and 60 s, respectively. And most power is concentrated at them. It means that, in order to eliminate them, the filter period must be larger than 100 s. Therefore it is very important to find the appropriate spectral windows for gravity recovery.

4) If the flight is straight and stable, the misaligned angle and the average value of the horizontal acceleration correction are about 5\(^{\prime}\) and 3 mGal, respectively.

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