Emittance fluctuation of mesoscopic conductors in the presence of disorders

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Abstract

We report an investigation of the dynamic conductance fluctuation of disordered mesoscopic conductors including one-dimensional, two-dimensional, and quantum dot systems. Our numerical results show that in the quasi-ballistic regime the average emittance is negative, indicating the expected inductive-like behavior. However, in the diffusive and localized regime, the average emittance is still negative. This disagrees qualitatively with the result obtained from random matrix theory. Our analysis suggests that this counterintuitive result is due to the appearance of non-diffusive elements in the system, the necklace states (or the precursor of necklace states in the diffusive regime) whose existence has been confirmed experimentally in an optical system.

The universal behavior of sample-to-sample fluctuation of mesoscopic conductors has attracted intensive theoretical and experimental studies in the last two decades [1]. For dc conductance, it is well known that the quantum interference gives rise to a reproducible fluctuation in coherent mesoscopic structures. When the sample size $L$ is smaller than the coherence length but greater than the elastic mean free path, the system is in the diffusive regime and the conductance fluctuation exhibits a universal behavior with a universal conductance fluctuation (UCF) that is independent of the Fermi energy, disorder strength, and system size. The numerical results of conductance fluctuation for various systems [2] are in good agreement with the UCF values obtained from diagrammatic perturbation theory [1] and random matrix theory [3]. For ac transport, due to the long range Coulomb interaction, the displacement current should be included in the ac current in a self-consistent manner [4]. Hence it is important to examine the role played by the displacement current in the dynamic conductance fluctuation. For chaotic quantum dots (QDs), the fluctuating admittance and capacitance have been studied using random matrix theory [5, 6]. Careful experiments have been carried out to measure this dynamic conductance in clean samples [7, 8]. At low frequencies, the dynamic response of a system to the external bias is characterized by emittance which is the imaginary part of the low-frequency admittance and is proportional to the partial density of state (DOS) of the system [4]. Physically, for a conductor with large transmission coefficient, the system responds like an inductor and the emittance is negative, while for a conductor with low transmission coefficient the emittance is positive and the system behaves like a capacitor. For a ballistic conductor, the emittance $E_{\mu}$ is found to be $E_{\mu} = -(1/4) e^2 dN/dE$, while for a metallic diffusive wire the average emittance is $E_{\mu} = (1/6) e^2 dN/dE$, where $dN/dE$ is the total DOS of the scattering region [9]. For a chaotic QD, the emittance is also positive from the calculation of random matrix theory [5], showing capacitive-like response. For a disordered two-dimensional (2D) waveguide, De Jesus et al have calculated the emittance distribution using a continuous model [10]. The emittance distribution is Gaussian at weak disorder and there is a crossover to a non-Gaussian distribution with a tail in the negative emittance region at strong disorder.

For low-dimensional systems, it has been predicted by Pendry [11] that there exist quasi-extended states known as necklace states in strongly disordered regime. These necklace states are due to the multi-resonance and are very rare events. Recently, the existence of the analogous optical necklace states in disordered 1D systems has been confirmed experimentally in the Anderson localized regime [12]. Since these necklace states come from multi-resonance, the electron dwells a long time with large DOS in the scattering region, giving rise to...
whose Hamiltonian for the disordered SWNT reads

\[ H = \sum_{ij} t_{ij} c_i \hat{c}_j^\dagger + \sum_{ij} (t_{ij} c_i \hat{c}_j + \text{h.c}), \]

where \( c_i \) (\( \hat{c}_j^\dagger \)) is the creation (annihilation) operator for an electron on the carbon site \( i \) and \( t_{ij} \) is the hopping integral. For a clean sample (in the leads region), the on-site energy is zero, i.e., \( \epsilon_i = 0 \). For a disordered sample, static Anderson-type disorder is added to \( \epsilon_i \) with a uniform distribution in the interval \([-W/2, W/2]\), where \( W \) denotes the disorder strength. To reduce the parasitic capacitance, we used two clean SWNTs to function as electrodes.

The conductance of the nanotube is calculated via the Landauer–Buttiker formula, \( G(0) = (2e^2/h)T \), where \( T = \text{Tr}(\Gamma_1 G^\dagger \Gamma_2 G^\alpha) \) is the transmission coefficient, \( G^\alpha = [E - H - \Sigma_1^r - \Sigma_1^\alpha]^{-1} \) is the retarded Green’s function of the central disordered SWNT, and the relationship between the self-energy and the linewidth function is \( \Gamma_1 \alpha = i \Sigma_1^r - \Sigma_1^\alpha \).

At low frequency, the dynamic conductance \( G(\omega) \) can be expanded in terms of frequencies,

\[ G(\omega) = G(0) - i \omega e^2 E_\mu + \cdots, \]

where \( G(0) \) is the static conductance and \( E_\mu \) is the emittance. With the help of the density of states given by Green’s functions, we can express the emittance as [15]

\[ E_\mu = -\text{Tr} \left( \text{Im}[G^\dagger \Gamma_2 G^\alpha G^\alpha_1 xx - D_{1xx} D_{2xx} / D_{1xx} + D_{2xx}] \right). \tag{1} \]

where \( D_{\alpha} = G^\dagger \Gamma_\alpha G^\alpha \), \( D_{\alpha \epsilon} = [D_{\alpha}]_{1xx} \), and \( [\cdots]_{1xx} \) denotes the diagonal element of the relevant square matrix. The quantities in the first and second terms in equation (1) have clear physical meaning. The first term (unscreened term) is the partial DOS \( dN_{12}/dE \) (up to a sign) describing the DOS of the electron coming from the second lead and exiting the first lead, while the second term consists of the local DOS [9] and is due to the displacement current. The dynamic response given by \( E_\mu \) is either positive for small transmission coefficient or negative for large transmission coefficient, giving rise to an inductive-like or capacitive-like behavior. For instance, for a small transmission coefficient, \( dN_{12}/dE \) is very small and the sign of the emittance is dominated by the second term, which is positive. Note that in arriving at equation (1), a quasi-neutrality approximation has been used, i.e., the local charge at any point is zero.

We first examine the emittance fluctuation defined by the root mean square (rms) values as \( \text{rms}(E_\mu) = (\langle E_\mu^2 \rangle - \langle E_\mu \rangle^2)^{1/2} \), where the \( \langle \cdots \rangle \) denotes averaging over an ensemble of samples, with different configurations of the same disorder strength. Figure 1 shows the averaged emittance and its fluctuation versus disorder strength with the energy fixed at the subband center. We also plot the average conductance

\[ E_\mu = -\text{Tr} \left( \text{Im}[G^\dagger \Gamma_2 G^\alpha G^\alpha_1 xx - D_{1xx} D_{2xx} / D_{1xx} + D_{2xx}] \right). \tag{1} \]
Figure 2. The histograms of emittance for an SWNT at the subband center $E = 3.2$ eV for different disorder strength $W = 0.5, 1.0, 2.0, 4.0, 5.0, 6.0$ and $10.0$ eV, respectively. Panel (h) is the same as panel (g) but in a log scale of $P(E_{μ})$ with one million configurations.

The main panel of figure 1 shows that the average emittance is always negative. This is a very counterintuitive result since we expect that a conductor with small average conductance gives a capacitive-like behavior corresponding to a positive emittance. It is true that for any specific configuration, the conductance and emittance are well correlated from our numerical result, i.e., large conductance corresponds to negative emittance and small conductance corresponds to positive emittance. The reason that the average emittance gives an inductive-like response is due to the so-called necklace states (or precursor of necklace states in the diffusive regime) that are rare events in the disordered systems. Through these states, electrons can tunnel through the disordered system of size $L$ via multiple scattering. The larger the system size, the smaller the number of necklace states is. The existence of these extended states in the localized regime was predicted by Pendry [11]. In contrast to single-resonant tunneling, the necklace state is a necklace of $n$ quasi-extended states stretching from one site of the disordered sample to other. They occupy only a fraction of sites with fractal dimension $1/2$ in one dimension, reminiscent of a percolation path [11]. The analogous optical necklace states have been observed experimentally in disordered multilayer systems [12], where the high transmission peaks show up in the transmission spectrum, while the average of the logarithm of the transmission coefficient decays linearly with the thickness. Since the necklace states (multi-resonant states) are rare events, they have very long lifetime, giving rise to a large negative emittance. To make this point clear, we plot in figure 2 the distribution functions $P(E_{μ})$ versus emittance $E_{μ}$ at different disorder strengths $W$. For this figure we have used data from 200 000 different random configurations, except in panel (h). We see that, at low disorder strengths, when $W \leq 2$ the distribution is approximately Gaussian. In this disorder range, as the disorder strength increases, the values of emittance are all negative and the mean emittance gradually increases towards a positive value. For the larger disorder strength $W = 4$ in panel (d), the distribution is still approximately Gaussian but the emittance can be positive, while the mean emittance is negative. When $W = 5, 6$, the distribution deviates gradually from the Gaussian distribution and the mean emittance is around zero, but negative. At even stronger disorder $W = 10$, the distribution is sharply peaked at positive emittance with a long tail in the negative emittance (see figure 2(h)). Similar behavior has been found in [10]. To be more precise, the distribution is asymmetric with large probability at small positive emittance while the probability of negative emittance is small but remains finite even for large negative emittance, resulting in a negative averaged emittance.

Why has the effect of necklace states not been seen in the conductance fluctuation but is important for the dynamic response? This is because, at large disorders, there exists a
Figure 3. The mean and rms values of emittance $-E_\mu$ of the 2D system in panel (a) at $E = 0.62$ and the QD in panel (c) at $E = 3.0$ for different $W$ (one million configurations averaged). The corresponding mean and rms values of conductance in units of $2e^2/h$ are shown in panels (b) and (d), respectively, for the 2D system and the QD.

Figure 4. The average emittance $-E_\mu$ and the localization length $\xi/L$ versus $W$ (100000 configurations averaged) for quasi-1D and 2D systems are shown, respectively, in panels (a) and (b). The other parameters are the same as in figures 1 and 3(a).

A small number of necklace states with small probability. For these necklace states the magnitude of emittance can be very large, while the largest possible value of conductance is around one, since it is unlikely for all multi-channels to tunnel through. Therefore the contribution of necklace states to the average conductance is very small, but is very large for the average emittance. For instance, in figure 2(h) with $W = 10$, the average emittance is $-0.11$ and the total conductance from 11 channels is $G = 0.24$. Out of one million configurations, the total probability of these necklace states with emittance $E_\mu < -100$ is 0.3%. Their contributions to the average emittance and average conductance are, respectively, $E_\mu = -0.39$ and $G = 0.002$. Clearly, these necklace states are important for average emittance and negligible for average conductance. Without these necklace states the average emittance would be positive. For exactly the same reason, the emittance fluctuation
is greatly enhanced due to the existence of these necklace states. We point out that these necklace states exist as long as the system size $L$ is finite. In addition to the quasi-1D system, we have also studied the averaged emittance of a disordered 1D tight-binding chain. Our result indicates that our conclusion remains, i.e., the average emittance is always negative for all disordered strengths.

Note that the necklace states exist in the localized regime and our analysis above on these states is also done in the same regime. Now we wish to argue that the average emittance $E_\mu$ is negative in the diffusive regime as well. Since $E_\mu$ is a monotonic function of disorder strength, $E_\mu$ in the localized regime is larger than $E_\mu$ in the diffusive regime. The fact that $E_\mu$ is negative in the localized regime means that $E_\mu$ is negative in all regimes, including the diffusive regime. Physically, in the diffusive regime, the system is roughly described by a diffusive conductor in parallel with multi-resonant channels connecting the leads (the precursor of necklace states). It is the appearance of these non-diffusive elements in the diffusive system, the precursor of necklace states, that changes the sign of $E_\mu$ in the diffusive regime from positive to negative.

Now we study the emittance using a tight-binding model for a QD and a 2D system with disorder $W$ that is uniformly distributed from $[-W/2, W/2]$. The system size is $L = 40\alpha$, where $\alpha$ is the lattice spacing and two electrodes of width $L_1$ are attached to the scattering region. For the QD, $L_1 = L/4$, while for the 2D system $L_1 = L$. The average conductance and average emittance and their fluctuations for the QD and the 2D system are depicted in figure 3, where we have collected one million configurations for each data point. From figure 3 we see that conductance fluctuation in the diffusive regime exhibits plateaus with universal values that are close to the theoretical predictions $\text{UCF} = 0.70e^2/h$ for a QD [3] and $\text{UCF} = 0.86e^2/h$ for a 2D system [1]. For emittance, our results show that for both the QD and the 2D system, the emittances are negative, independent of disorder strengths, which disagrees with the theoretical predictions. The emittance fluctuations for a QD and a 2D system are consistent with the 1D case.

Our results for the average emittance clearly disagree with the theoretical results obtained from random matrix theory. This is because in order to use an analytic approach, one has to make the following approximation on the second term in equation (1) so that all the quantities involved are global:

$$\text{Tr} \left[ \frac{D_{1xx}D_{2xx}}{D_{1xx} + D_{2xx}} \right] \approx \frac{\text{Tr}(D_1)\text{Tr}(D_2)}{\text{Tr}(D_1 + D_2)},$$  \hspace{1cm} (2)$$

It is easy to see that this approximation greatly overestimates the second term in equation (1) at large disorders. For weak disorders with large transmission coefficient $D_{1xx}$ and $D_{2xx}$ are in the same order of magnitude; hence equation (2) is a good approximation. However, at strong disorders there is a large mismatch between $D_{1xx}$ and $D_{2xx}$, for quite a number of sites $x$. Since $D_{1xx}D_{2xx}/(D_{1xx} + D_{2xx}) \approx D_{2xx}$ when $D_{1xx} \gg D_{2xx}$, the left-hand side of equation (2) is approximately given by $\text{Tr}([D_{1xx}, D_{2xx}])$, which is much smaller than the right-hand side of equation (2) at strong disorders. Using this approximation, our numerical results are shown in figure 4. We see that, indeed, for both quasi-1D and 2D systems the average emittance with the approximation equation (2) (solid square) underestimated the second term in equation (1) so that the results are very different from that obtained using equation (1). This is because the necklace states have been washed out.

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Figure 5. The calculation of localization length $\xi$ at specific disorder strength. $\xi$ is determined by plotting the logarithm of the average conductance versus the system size for the 2D system (panel (a)) and the CNT system (panel (b)).

(This figure is in colour only in the electronic version)
in equation (2). In addition, figure 4 shows that, using the approximation equation (2), the emittance is positive in the diffusive regime, in agreement with the theoretical predictions.

In conclusion, we have studied the fluctuation of emittance and its distribution for disordered 1D, 2D, and QD systems. It is known that a specific sample with a large transmission coefficient gives rise to negative emittance corresponding to an inductive-like response. On the other hand, the emittance is positive for a sample with a small transmission coefficient showing a capacitive-like response. However, the average emittance shows an unexpected behavior when the average conductance is small. Our numerical results indicate that the average emittance is always negative, independent of disorder strengths. Our findings disagree qualitatively with the theoretical results obtained from random matrix theory. The disagreement is due to the existence of non-diffusive elements, necklace states or the precursor of necklace states, that are important for the dynamic response and negligible for the conductance fluctuation. If we assume that the partial density of states is position independent it will wash out the necklace states. For this reason, the necklace states cannot manifest themselves if equation (2) is used to calculate the emittance, as was done in [5]. In the presence of disorder, the conductance fluctuation has been measured experimentally [16]. Similarly, the emittance distribution can be measured using setups of [7, 8].

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