The deceleration of small charged bodies in a rarefied plasma

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Abstract. The purpose of this work is to create a method for determining the aerodynamic characteristics of fine particles in a highly rarefied plasma. The created method is based on the theorem on the change in momentum imparted to the body. The "body+surrounding plasma" system was considered to be closed; therefore, the change in the momentum of the body is equal in the impulse of the incident flow and is opposite to it in direction. The change in the pulse flux is calculated using the statistical method. To compare the results and approbation of the calculation method, the deceleration forces of a spherical body in a flow of neutral gas were determined. Next, we calculated the drag coefficient from the side of the charged plasma component. The calculation results are presented in the graphs.

1. Introduction
Study of rarefied plasma flow around natural and artificial bodies is necessary for solving a number of problems of applied and theoretical nature. So, for example, to calculate the satellites lifetime, assessing the operation of attitude control systems and stabilizing their position on orbit, predicting the movement of spacecraft in near-earth and the interplanetary medium needs to know the forces and moments acting on them. The methods for determining the aerodynamic characteristics of bodies, moving in a rarefied environment are outlined in works [1–5]. Basically, they relate to the around bodies flow with a rarefied neutral gas.

Another area in which knowledge of aerodynamic performance bodies moving in a rarefied plasma is necessary - the study the influence of anthropogenic activity on changes in the composition and structures of the upper layers of the atmosphere.

2. Statement of the problem.
In a rarefied environment, the contribution to the aerodynamic characteristics of neutral and charged particles is additive. In turn, the problem of a charged component flowing around bodies can be conditionally divided into groups, highlighting them for one reason or another. For example, you can divide the modes flow around bodies in relation to the body size to the Debye radius of the incident plasma for the modes of flow around small ($R \ll D$), medium ($R \sim D$) and large ($R \gg D$) bodies.

Naturally, consideration of all possible situations is impossible, therefore the main role in the choice of the range of tasks to be solved is played by practical requests.
In this work, the deceleration force of small charged bodies in a rarefied plasma is determined. Such bodies can be particles of space 'debris' with sizes less than 1 mm. Since these particles are exposed to charging, then the aggregate of these particles is called 'dusty' plasma [6]. The numerical determination of the braking force is the purpose of this work.

The main assumptions for finding the deceleration forces of a dust particle are as follows:
1. Plasma is composed of neutral particles, ions and electrons.
2. Plasma is considered collisionless.
3. The Debye radius is much larger than the dimensions of the streamlined body.
4. In an unperturbed region, the plasma is in an equilibrium state.
5. Average speed of the incident flow related to the thermal speed of the incident flow particles, was considered as a parameter that determines the braking force.
6. External electric and magnetic fields are absent, except for the field created by the dust particle.

3. Calculation of aerodynamic characteristics. Basic formulas.

To calculate the forces and moments acting on bodies moving in rarefied plasma, knowledge of distribution functions on the surface of these bodies is not sufficient since the radius of action of electric and magnetic fields can be comparable, and in some cases exceed the characteristic dimensions of the body. Therefore, the problem of determination of aerodynamic characteristics should be solved based on the general theorems of dynamics. At the same time, it is convenient to break the forces acting on bodies into two groups. These are, firstly, the forces acting on bodies, due to direct collisions of incident particles with the body surface, and, secondly, field forces.

In this way,
\[ \vec{F} = \vec{F}_{ob} + \vec{F}_p, \] (1)

where \( \vec{F}_{ob} \) is the force acting on the surface of the body, \( \vec{F}_p \) - The 'field' force acting from the side of transitory particles. When calculating the aerodynamic characteristics of small bodies, the authors used the general formulas obtained in [5].

\[ \vec{F} = (\vec{P}_{in})_S - (\vec{P}_{out})_S, \] (2)

where

\[ (\vec{P}_{in})_S = \sum_i \int_S \int m_i \vec{\nu} |v_n| f_i(\vec{r}_S, \vec{\nu}) d\vec{\nu} dS. \] (3)

\[ (\vec{P}_{out})_S = \sum_i \int_S \int m_i \vec{\nu} |v_n| f_i(\vec{r}_S, \vec{\nu}) d\vec{\nu} dS. \] (4)

In formulas (3) - (4): \( S \) is a surface, all points of which are at a sufficiently large distance from the surface of the body, that is, all points outside \( S \) lie in an unperturbed region; \( f_i(\vec{r}_S, \vec{\nu}) \) is the velocity distribution function on \( S \); index \( i \) defines the type of particles. These formulas took into account the contribution of the neutral, ionic and electronic components. Since the contribution of the electronic component is small, it was not further considered.

\( (\vec{P}_{in})_S \) is average momentum brought by particles crossing surface \( S \) per unit time from the outside; \( (\vec{P}_{out})_S \) - average momentum carried away by particles crossing the surface \( S \) per unit time from the inside. \( \vec{F} \) - medium strength, acting on the body.

The contribution of the neutral component was calculated to test our proposed method. The results were compared with the data shown in Fig.53 and Fig.54 in the monograph [3]. Good agreement confirmed the correctness of the proposed calculation method. When calculating the
aerodynamic coefficients for a neutral gas, the surface $S^*$, which coincides with the surface of the body, was taken as $S$.

For neutral particles
\[ \vec{F}_{ob} = (\vec{P}_{in})_{S^*} - (\vec{P}_{out})_{S^*}. \]  

(5)

In formula (5)
\[ (\vec{P}_{in})_{S^*} = \int \int_{S} m_i \bar{v} |v_n| f_i(\vec{r}_{S^*}, \vec{v}) d\bar{v} dS^*. \]  

(6)

\[ (\vec{P}_{out})_{S^*} = \int \int_{S^*} m_i \bar{v}' |v_n| f_i(\vec{r}_{S^*}, \vec{v}) T(\vec{v}, \vec{v}') d\bar{v}' d\bar{v} dS^*. \]  

(7)

where $T(\vec{v}, \vec{v}')$ is the boundary scattering function. In this work, this function was taken in the form corresponding to the mirror law of reflection. These formulas written out for one of the components.

Unfortunately the calculation of integrals (3),(4),(6) and (7) by analytical methods encounters serious difficulties, therefore, to calculate the braking force the Monte Carlo method is used.

4. Application of the Monte Carlo method to the problem of deceleration of a small body

Consider the main features of the application of the Monte Carlo method to solving the problem on the calculation of the braking force acting on a small body.

As a function of the particle velocity distribution in the unperturbed region the Maxwellian distribution function was taken with a shift, given by the formula:
\[ f_i(\vec{r}_{S}, \vec{v}) = n_i \left( \frac{m_i}{2\pi kT_i} \right)^{3/2} \exp \left( -\frac{m_i}{2kT_i} (\vec{v} - \vec{V})^2 \right). \]  

(8)

As the surface $S$, we took the surface of a cube whose two faces were are perpendicular to the mean velocity of the incident particles $\vec{V}$.

The contribution of the neutral component to the deceleration force was calculated under the assumption that the streamlined body is a sphere.

When calculating the field forces acting from the ionic component, the bodies dimensions were considered infinitely small. The calculations took into account only the field created by charges on the body. The characteristic size in the calculations was taken in this case, the Debye radius.

The interaction of the body with the oncoming plasma is largely determined by the flow of the number of particles crossing the boundary of the influence region $S$. The number of particles $dN$ flown through the elementary area $dS$:
\[ dN = dS \int_{v_n<0} |v_n| f_i(\vec{r}_{S}, \vec{v}) d\bar{v}. \]  

(9)

Ratio $dN/N$, where $N$ is the total flux of the number of particles entering the zone influence, can be considered as the probability of hitting one separately taken particle into the area bounded by the surface $S$ through the area $dS$. Calculation of the ratio $dN/N$ requires preliminary calculation of $N$.

\[ N = \int \int_{S} |v_n| f_i(\vec{r}_{S}, \vec{v}) d\bar{v} dS. \]  

(10)
Since the choice of the surface $S$ is largely arbitrary, the surface of a cube was chosen as $S$, the two faces of which are perpendicular the mean velocity of the incident flow, and the geometric center coincides in one the case with the center of the sphere, and in the other it coincides with the center of the force field.

The choice of a cube surface as a surface $S$ is made for the purpose of simplification calculating integral (10). For all points lying on one face of the cube, the inner integral has the same value and, as a result, integral (10) is reduced to to the sum of six terms. The flow of the number of particles crossing the first face directed towards the incoming flow for the Maxwell shifted distribution has the form:

$$N_1 = L^2 \int_{v_n < 0} |v_n| n_{\infty} \left( \frac{m}{2\pi kT_{\infty}} \right)^{3/2} \exp \left( -\frac{m}{2kT_{\infty}} \left( \vec{v} - \vec{V} \right)^2 \right) \, d\vec{v} =$$

$$= L^2 \frac{n_{\infty} v_{cp}}{4} \left( \exp(-u^2) + \sqrt{\pi} \, u \, (1 + erf(u)) \right).$$

(11)

where $L$ is the length of the edge of the cube; $n_{\infty}$ is the concentration of particles in the unperturbed areas; $v_{cp} = \sqrt{8kT_{\infty}/(\pi m)}$ - average speed of particles; $u = V/v_t$ is the ratio of the incident flow velocity to the heat the velocity $v_t = \sqrt{2kT_{\infty}/m}$ of either ions or neutral particles.

Similarly, for the second face of the cube opposite the first:

$$N_2 = L^2 \frac{n_{\infty} v_{cp}}{4} \left( \exp(-u^2) - \sqrt{\pi} \, u \, (1 - erf(u)) \right).$$

(12)

For side faces:

$$N_3 = N_4 = N_5 = N_6 = L^2 \frac{n_{\infty} v_{cp}}{4}.$$ (13)

The total number of particles entering the cube per unit of time $N = \sum_{i=1}^{6} N_i$. The probability of a particle entering the cube through the $i$-th face $p_i = N_i/N$.

Having determined the edge through which a randomly selected particle flies into the region influence, we simulate the coordinate of the point through which it flies into the volume. This point is chosen from the condition that the choice of a point on the face is equally probable for each of them. After that, the projections of the particle velocity on the selected coordinate axes are simulated (see Fig.1), in accordance with the Gaussian distribution law. Next, the trajectory of the particle movement inside the cube was built.

5. The contribution to the braking force of the neutral plasma component

For neutral particles, two types of trajectories are possible. First, the trajectories particles that fly through the cube without colliding with the body, located in a cube. These particles make zero contribution to the forces and moments acting on the body. Secondly, the trajectories of
particles colliding with the surface of the body. In our work the mirror law of reflection from the surface of the body after the collision was adopted. Particles of the second type give a nonzero contribution to forces and moments. Trajectories of these particles are broken lines, consisting of two line segments. One of the ends each segment is located on the surface of the body.

Determining the change in the momentum of a particle in the course of a numerical experiment, we determine its contribution to the braking force. In the course of calculations, the contributions of all particles are summed up. After a series of tests, the amounts received were divided by the total number of particles participating in the series. The result obtained determined the average force acting on the body from the side of one particle flying into the zone influence.

Then the resulting number was multiplied by the total number of particles \( N \) entering the zone influence per unit of time. According to earlier calculations

\[
N = L^2 \frac{n_\infty \text{exp}}{4} \left( 2 \exp(-u^2) + 2 \sqrt{\pi} u \text{erf}(u) + 4 \right). \tag{14}
\]

Let us write the expression for the aerodynamic braking force in the generally accepted form:

\[
F_{ob} = C_{st} \frac{m n_\infty V^2}{2} \cdot S_b, \tag{15}
\]

where \( C_{st} \) - aerodynamic drag coefficient; \( S_b \) - the area of the largest cross-section of the body (midelsection).

On the other hand, you can write

\[
F_{ob} = \Delta \vec{p}_1 \cdot N, \tag{16}
\]

where \( \Delta \vec{p}_1 \) is the average momentum transferred to the body by one particle.

The above formulas allow us to write out the following expression for finding \( C_{st} \):

\[
C_{st} = L^2 \frac{\Delta \vec{p}_1 \, (\exp(-u^2) + \sqrt{\pi} u \text{erf}(u) + 2)}{\pi R^2 \sqrt{\pi} u^2}. \tag{17}
\]

The results obtained using formula (17) are shown in Figure 2. They coincide with the results obtained for the sphere by other methods [3].

![Figure 2. \( C_{st} \)-coefficient of the frontal resistance of the sphere for the flow of neutral particles.](image)

6. The contribution to the deceleration force of the charged plasma component

The method for calculating forces and moments for a charged component is in many ways similar to the method for the neutral component. The only difference is that for particles of the charged component in the area of influence, the trajectories of motion and the change in particle velocities was used to determine the momentum transferred to the charged body. Since
the body was considered small, the contribution of particles that collided with body surface was not taken into account. The field in the vicinity of the body was considered to be Coulomb.

The braking force arising from the interaction of the charged component plasma with a charged body can be called the electrodynamic braking force. The results of calculating the braking force are shown in Figure 3. Coefficient drag $C_e$ in the figure is normalized to $m n_\infty V v_t \pi D^2/2$, where $D$ - Debye radius in plasma.

![Figure 3. $C_e$ is the electrodynamic drag coefficient of a charged body normalized to $m n_\infty V v_t \pi D^2/2$.](image)

The problem of calculating the braking force of a small body moving in rarefied plasma is similar to the problem of ‘runaway’ electrons, but different from it [7]. First, in the problem of runaway electrons, the motion was considered electrons in an accelerating electric field and, secondly, the braking force, obtained in the first and second problems, being qualitatively similar, differ from each other quantitatively.

7. Conclusions
1. From the performed calculations and data on the ionosphere, it follows that in the lower layers of the ionosphere the aerodynamic component of the resistance prevails over the electrodynamic one.
2. In the upper layers of the ionosphere and in the interplanetary plasma, the opposite situation takes place: the electrodynamic component prevails over the aerodynamic one.
3. As in the phenomenon called runaway electrons, the coefficient of electrodynamic resistance tends to zero with an increase in the speed of the incoming flow.

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