Supergravity approach to tachyon condensation on the brane-antibrane system

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Abstract

We study the tachyon condensation on the D-brane–antiD-brane system from the supergravity point of view. The non-supersymmetric supergravity solutions with symmetry ISO(\(p,1\)) \(\times SO(9-p)\) are known to be characterized by three parameters. By interpreting this solution as coincident \(N\) Dp-branes and \(\bar{N}\) Dp-branes we give, for the first time, an explicit representation of the three parameters of supergravity solutions in terms of \(N,\bar{N}\) and the tachyon vev. We demonstrate that the solution and the corresponding ADM mass capture all the required properties and give a correct description of the tachyon condensation advocated by Sen on the D-brane–antiD-brane system.

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It is well-known that a coincident D-brane–antiD-brane pair (or a non-BPS D-brane) in Type II string theories is unstable due to the presence of tachyonic mode on the D-brane world-volume [1]. As a result, these systems decay and the decay occurs by a process known as tachyon condensation [2]. Tachyon condensation is well understood in the open string description using either the string field theory approach [3, 4] or the tachyon effective action approach [5] on the brane. However a closed string (or supergravity) understanding of this process is far from complete and the purpose of this paper is precisely to have a closed string understanding of the tachyon condensation. An earlier attempt in this direction has been made in [6] by giving an interesting interpretation to the previously known [7, 8] non-supersymmetric, three parameter supergravity solutions with a symmetry $\text{ISO}(p,1) \times \text{SO}(9-p)$ in ten space-time dimensions as the coincident $D_p-\bar{D}_p$ system. The three parameters in this solution were argued [6] to be related (although the exact relations were not given) to the physically meaningful parameters, namely, the number of $D_p$-branes ($N$), number of $\bar{D}_p$-branes ($\bar{N}$) and the tachyon vev$^3$ of the $D_p-\bar{D}_p$ system.

In this paper we use the static counterpart of the asymptotically flat time dependent supergravity solution obtained in [9]. This is also a non-BPS, three parameter solution with the symmetry $\text{ISO}(p,1) \times \text{SO}(9-p)$ and is related to the solution given in ref.[6]. This solution can also be naturally interpreted as the coincident $N$ $D_p$-branes and $\bar{N}$ $\bar{D}_p$-branes system given its aforementioned symmetry. In contrast to the attempt made in [6], we approach the problem by giving, for the first time, an explicit representation of the parameters of the solution in terms of $N$, $\bar{N}$ and the tachyon vev of the $D_p-\bar{D}_p$ system.

We proceed as follows. Once the supergravity solution under consideration is realized to represent the $N$ D-brane and $\bar{N}$ anti D-brane system, we can gain information about the parameters of the solution by examining how it reduces to a supersymmetric configuration which either corresponds to a BPS $N$ $D_p$-branes (for $\bar{N} = 0$) or BPS $\bar{N}$ $\bar{D}_p$-branes (for $N = 0$) or the final supersymmetric state at the end of tachyon condensation. We also expect in taking the BPS limit that only one parameter corresponding to the number of branes remains and the other parameters of the solution get automatically removed. For general case when both $N$ and $\bar{N}$ are non-zero, the solution is not supersymmetric and there must be a tachyon on the world-volume of $D_p-\bar{D}_p$ system belonging to the complex $(N, \bar{N})$ representation of the gauge group $U(N) \times U(\bar{N})$. The end of the tachyon condensation should give BPS $(N - \bar{N})$ D-branes if $N > \bar{N}$ or BPS $(\bar{N} - N)$ anti D-

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$^3$By tachyon vev we mean the classical value of the tachyon for which the total energy of the system takes a particular value which includes the extremum as well as off-shell values [6].
branes if $\bar{N} > N$. We will give a general description for arbitrary $N$ and $\bar{N}$ where $N = \bar{N}$ appears as a special case. We will show how the interplay of the parameters describes the tachyon condensation in accordance with the conjecture made by Sen [1, 2] for the $Dp$–$\bar{D}p$ system. The recognition of having a supersymmetric background at the end of the tachyon condensation is crucial for us to find explicit representation of the parameters of the solution in terms of $N, \bar{N}$ and the tachyon $^4$ vev $T$.

In order to understand the tachyon condensation, we look at the expression of the total ADM mass of the solution representing the total energy of the system. We then express this total energy in terms of the three physical parameters namely, $N$, $\bar{N}$ and $T$ of the $Dp$–$\bar{D}p$ system using the aforementioned relations. The total energy can be seen to be equal or less than the sum of the masses of $N$ $Dp$-branes and $\bar{N}$ $\bar{D}p$-branes, indicating the presence of tachyon contributing the negative potential energy to the system. We will see that the energy expression gives the right picture of tachyon condensation conjectured by Sen [1, 2]. We will reproduce all the expected results from this general mass formula under various special limits for $N \neq \bar{N}$ as well as $N = \bar{N}$ at the top and at the bottom of the tachyon potential. We will also show how the various known BPS supergravity configurations can be reproduced in these special limits.

The static, non-BPS supergravity $p$-brane solution analogous to the time dependent solution obtained in [9] has the form in $d = 10$,

$$ds^2 = F^{-7/8} (-dt^2 + dx_1^2 + \ldots + dx_p^2) + F^{7/8} \left( H \tilde{H} \right)^{7/8} (dr^2 + r^2 d\Omega_{8-p}^2)$$

$$e^{2\phi} = F^{-a} \left( \frac{H}{\tilde{H}} \right)^{2\delta}$$

$$F_{[8-p]} = b \text{Vol}(\Omega_{8-p})$$  \hspace{1cm} (1)

where we have written the metric in the Einstein frame. Note that the metric has the required symmetry and represents the magnetically charged $p$-brane solution. The corresponding electric solution can be obtained from (1) by dualizing the field strength $F_{[p+2]} = e^{a\phi} \ast F_{[8-p]}$, where $\ast$ denotes the Hodge dual. Also in the above

$$F = \cosh^2 \theta \left( \frac{H}{\tilde{H}} \right)^{\alpha} - \sinh^2 \theta \left( \frac{\tilde{H}}{H} \right)^{\beta}$$

$$H = 1 + \frac{\omega_{7-p}}{r^{7-p}}, \quad \tilde{H} = 1 - \frac{\omega_{7-p}}{r^{7-p}}$$  \hspace{1cm} (2)

$^4$When $N = \bar{N} = 1$, tachyon is a complex field and $|T|^2 = TT^\ast$. But for $N, \bar{N} > 1$, tachyon is a matrix and in that case we follow [6] to define $|T|^2 = \frac{1}{N} \text{Tr}(TT^\ast)$. Here and in the rest of the paper we denote $|T|$ as $T$ for simplicity.
with the parameter relation

\[ b = (\alpha + \beta)(7 - p)g_s\omega^{7-p} \sinh 2\theta \]  

(3)

Here \( \alpha, \beta, \theta, \) and \( \omega \) are integration constants and \( g_s \) is the string coupling. Also \( \alpha \) and \( \beta \) can be solved, for the consistency of the equations of motion, in terms of \( \delta \) as

\[
\begin{align*}
\alpha &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}\delta^2 + \frac{a\delta}{2}} \\
\beta &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}\delta^2 - \frac{a\delta}{2}}.
\end{align*}
\]  

(4)

These two equations indicate that the parameter \( \delta \) is bounded as

\[ |\delta| \leq \frac{4}{7-p} \sqrt{\frac{2(8-p)}{p+1}}. \]  

(5)

The solution (1) is therefore characterized by three parameters \( \delta, \omega \) and \( \theta \). In (1) \( a \) is the dilaton coupling to the \( (8-p) \)-form field strength and is given as \( a = (p-3)/2 \) for the \( D_p \)-branes and \( a = (3-p)/2 \) for the NSNS branes. Also \( b \) is the magnetic charge and the \( \text{Vol}(\Omega_{8-p}) \) is the volume-form of the \( (8-p) \)-dimensional unit sphere. Without any loss of generality, we take \( (\alpha + \beta), b, \theta \geq 0 \) as we did already in the above. We note from (2) that the solution has a curvature singularity at \( r = \omega \) and the physically relevant region is \( r > \omega \). As \( r \to \infty \), \( H, \tilde{H} \to 1 \) and so, \( F \to 1 \). The solution is therefore asymptotically flat. In order to obtain the electrically charged solution we dualize \( F_{[8-p]} \) in (1) and get the gauge field \( A_{[p+1]} \) as,

\[ A_{[p+1]} = \sinh \theta \cosh \theta \left( \frac{C}{F} \right) dx^0 \wedge \ldots \wedge dx^p \]  

(6)

where \( C \) is defined as,

\[ C = \left( \frac{H}{\tilde{H}} \right)^\alpha - \left( \frac{\tilde{H}}{H} \right)^\beta. \]  

(7)

From the form of the metric in (1), it is clear that the solution is non-supersymmetric [10] because of the presence of \( (H\tilde{H})^{2/(7-p)} \) factor in the last term. This is also consistent with our interpretation of the solution as \( N D_p \)-brane and \( \bar{N} \bar{D}_p \)-brane system. With this interpretation, we can express the parameter \( b \) in terms of \( N, \bar{N} \) as

\[ Q_0^p|N - \bar{N}| = \frac{b\Omega_{8-p}}{\sqrt{2\kappa_0}} \Rightarrow b = \frac{\sqrt{2\kappa_0}Q_0^p|N - \bar{N}|}{\Omega_{8-p}} \]  

(8)
where \( Q_0^p = (2\pi)^{(7-2p)/2}\alpha'(3-p)/2 \) is the unit charge on a Dp-brane, \( \sqrt{2\kappa_0} = (2\pi)^{7/2}\alpha'^2 \) is related to 10 dimensional Newton’s constant and \( \Omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2) \). Note that \( b \to 0 \) as \( N \to \bar{N} \).

For the solution (1), the supersymmetry will be restored if and only if \( H \bar{H} \to 1 \) which always requires \( \omega^{7-p} \to 0 \). We have the following cases for which supersymmetry can be restored: (1) either \( N = 0 \) or \( \bar{N} = 0 \) or both (the trivial case); (2) when both \( N \) and \( \bar{N} \) are non-vanishing susy can be restored at the end of tachyon condensation. For the second case, we have two subcases. The first is the familiar one with \( N = \bar{N} \) for which the end of tachyon condensation should give an empty spacetime with maximal supersymmetry and the second with \( N \neq \bar{N} \) for which we expect the final configuration to represent BPS \((N - \bar{N})\) Dp-branes if \( N > \bar{N} \) or BPS \((\bar{N} - N)\) \(\bar{D}\)p-branes if \( \bar{N} > N \). For all these cases, we expect \( \omega^{7-p} \to 0 \). This observation is crucial to express the three parameters of the solution in terms of \( N, \bar{N} \) and the tachyon vev \( T \). Given the case (1) above along with the fact that \( \omega^{7-p} \) should be symmetric with respect to \( N \) and \( \bar{N} \), we expect that the leading behavior of \( \omega^{7-p} \) should be \( \omega^{7-p} \sim (N\bar{N})\gamma \) with a positive parameter \( \gamma \). Keeping other factors in mind (for example, \( \omega^{7-p} \neq 0 \) as \( N = \bar{N} \)), we make our educated guess for \( \omega^{7-p} \) as \( \omega^{7-p} = f(N\bar{N})^{1/2} \) with \( f \) depending only on the tachyon vev \( T \) and some other known constants. Considering case (2) above, we choose \( f \) to depend on \( T \) as \( f \sim \cos T \) with \( T = \pi/2 \) as the end point of the tachyon condensation. Putting everything together, \( \omega^{7-p} \) therefore takes the form,

\[
(7-p)\omega^{7-p} = \sqrt{\frac{7-p}{2(8-p)}} (N\bar{N})^{1/2} \frac{2\kappa_0^2}{\Omega_{8-p}} T \cos T.
\]

(9)

Now we come to determine the parameter \( \delta \) which is expected to be related to \( N, \bar{N} \) and the tachyon vev \( T \) as well. As indicated in eq.(5), the parameter \( \delta \) is bounded, therefore it cannot depend on \( N + \bar{N}, N - \bar{N} \) and/or \( N\bar{N} \) or any of the inverse powers of them in a simple fashion since either \( N \) or \( \bar{N} \) or both can take arbitrary large or zero value which cannot give a bounded contribution. Therefore, if \( \delta \) depends on \( N, \bar{N} \) at all, they must appear in such way that when the terms involving \( N \) and \( \bar{N} \) get large they must cancel each other to give a bounded contribution to \( \delta \). Also when the tachyon vev \( T \) takes specific values, the terms involving \( N \) and \( \bar{N} \) should give bounded contribution. Given the above and considering the bound (5) and the special case of \( p = 3 \), our educated

\[\text{At the end of the condensation all of the eigenvalues of the matrix } TT^* \text{ were argued to be the same[11, 6] as } T_0^2 \text{ and we take } T_0 = \pi/2 \text{ here.}\]
guess for $\delta$ is

$$
\delta = \frac{a}{|a|} \sqrt{\frac{8-p}{2(7-p)}} \left[ \frac{|a|}{\sqrt{\cos^2 T + \frac{(N - \bar{N})^2}{4NN \cos^2 T}}} - \sqrt{a^2 \left( \cos^2 T + \frac{(N - \bar{N})^2}{4NN \cos^2 T} \right) + 4 \sin^2 T} \right].
$$

(10)

With this we can read off $\alpha + \beta$ and $\alpha - \beta$ from (see eq.(4))

$$
\alpha + \beta = 2 \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}} \delta^2,
$$

(11)

and

$$
\alpha - \beta = a \delta,
$$

(12)
in terms of $N, \bar{N}$ and the tachyon vev $T$ explicitly.

With (9) and (11), we can now determine the parameter $\theta$ from (3) as

$$
\sinh 2\theta = \frac{|N - \bar{N}|}{c(\alpha + \beta)(NN)^{1/2} \cos T}
$$

$$
\cosh 2\theta = \sqrt{1 + \frac{(N - \bar{N})^2}{c^2(\alpha + \beta)^2NN \cos^2 T}},
$$

(13)

where we have used (8) for $b$ and the constant $c = \sqrt{(7-p)/2(8-p)}$.

So, we have postulated in the above how the two parameters $\omega$ and $\delta$ are related to $N, \bar{N}$ and the tachyon vev $T$ based on the expected properties of the solution and the characteristic behavior of the tachyon condensation. For given $N$ and $\bar{N}$, each value of $T$ labels a static solution, therefore we have a one-parameter family of solutions with $0 \leq T \leq \pi/2$. At this point, we cannot be certain that our educated guess for them is really correct and we have to do some consistency check. In the following, we will write down the ADM mass for the solution (1) representing the total energy of the system, and check whether with the above parameter relations we can produce all the required properties of the total energy, the solution as well as the tachyon condensation according to Sen conjecture.

The total ADM mass of the system can be calculated using the formula given in [12] and for the metric in (1) we obtain,

$$
M = \frac{\Omega_{8-p}}{2\kappa_5^2} (7-p) \omega^{7-p} \left[ (\alpha + \beta) \cosh 2\theta + (\alpha - \beta) \right].
$$

(14)

with $\alpha$ and $\beta$ given in terms of $\delta$ as in eq. (4). Using (9), (10), and (13) for the parameter $\omega, \delta$ and $\theta$ in terms of $N, \bar{N}$ and $T$, the mass can be expressed in a surprisingly simple
form as,

\[
M = T_p \left( \frac{N\bar{N}}{N} \right)^{1/2} \sqrt{4 \cos^4 T + \frac{(N - \bar{N})^2}{NN}} \\
= T_p \sqrt{(N + \bar{N})^2 - 4N\bar{N}(1 - \cos^4 T)} \leq T_p(N + \bar{N}).
\]  

(15)

Thus we note that the total mass is less or equal to the sum of the masses of \(N\) \(Dp\)-branes and \(\bar{N}\) \(\bar{Dp}\)-branes. The difference is the tachyon potential energy \(V(T)\) which is negative. One can easily see that \(T = 0\) gives the maximum of the energy, therefore the maximum of the tachyon potential (actually \(V(T) = 0\) here) while \(T = \pi/2\) gives the corresponding minima.

Let’s check one by one if the above mass formula produces all required properties of the solution and the tachyon condensation. At \(T = 0\), i.e., at the top of the tachyon potential, \(\cos T = 1\) and we have from the above \(M = (N + \bar{N})T_p\), producing the expected result. For this case, \(\delta = 0\), \(\alpha = \beta = \sqrt{2(8 - p)/(7 - p)}\) and \(\omega\) remains finite as can be seen from (9), therefore the corresponding solution breaks all the susy as it should be. At the end of the condensation, i.e., \(T = \pi/2\), \(M = |N - \bar{N}|T_p\), again producing the expected result. As \(T \to \pi/2\), \(\omega \to 0\) and the solution becomes BPS \((N - \bar{N}) Dp\)-branes if \(N > \bar{N}\) or \((\bar{N} - N) \bar{Dp}\)-branes if \(\bar{N} > N\).

Let us discuss in a bit detail how the solution behaves at the end of the condensation. Here \(\delta = 0\) for all \(p\) except for \(p = 3\) (for which \(\delta \to \pm \sqrt{2(8 - p)/(7 - p)}\) as can be seen from (10)). Therefore we again have from eq.(4) \(\alpha = \beta = \sqrt{2(8 - p)/(7 - p)}\) except for \(p = 3\). On the other hand for \(p = 3\), \(\alpha = \beta \to 0\). Even though the parameters \(\alpha, \beta\) and \(\delta\) are different for \(p \neq 3\) and for \(p = 3\), we have a uniform limit for the function \(F\) in (2) at the end of the tachyon condensation as

\[
F \to \bar{H} = 1 + \frac{\bar{\omega}^{7-p}}{r^{7-p}},
\]

(16)

with \(\bar{\omega}^{7-p} = b/g_s(7 - p)\) finite and \(H, \bar{H} \to 1\). The corresponding configuration, as can be seen from (1) with (16), is either BPS \((N - \bar{N}) Dp\)-branes if \(N > \bar{N}\) or BPS \((\bar{N} - N) \bar{Dp}\)-branes if \(\bar{N} > N\).

The tachyon condensation can also be seen for the special case of \(N = \bar{N}\). In this case for \(T = 0\), we have \(M = 2NT_p\) and the corresponding configuration breaks all the susy while at the end of the condensation, i.e., at \(T = \pi/2\), \(M = 0\), corresponding to an empty spacetime preserving all the susy. This can also be seen from the configuration (1) since now \(\bar{\omega}^{7-p} = 0\), therefore \(\bar{H} = 1\).

Finally let us check whether the mass formula (15) produces the expected result for \(N = 0\) or \(\bar{N} = 0\) or both. We can read off from (15) that \(M = NT_p\) when \(\bar{N} = 0\) and
$M = \bar{N} T_p$ when $N = 0$ and $M = 0$ when $N = \bar{N} = 0$ all as expected. It is not difficult to check from (1) that the corresponding configuration is either $N$ BPS $Dp$-branes if $\bar{N} = 0$ or $\bar{N}$ BPS $\bar{Dp}$-branes if $N = 0$ or an empty spacetime when both $N = 0$ and $\bar{N} = 0$. The tachyon vev decouples automatically as expected.

In summary, the supergravity solution (1) is naturally interpreted as coincident $N$ $Dp$-branes and $\bar{N}$ $\bar{Dp}$-branes system given its symmetry and the number of parameters characterizing this solution. Based on the physical properties of the solution and the characteristic behavior of tachyon condensation, we give, for the first time, an explicit representation of the three parameters of the solution in terms of $N$, $\bar{N}$ and the tachyon vev $T$ which produces all the required properties of the ADM mass and the solution as discussed in the paper. In this respect, we capture the right picture of the tachyon condensation using closed string or supergravity description.

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