Performance assessment of urban goods vehicles

C. J. Isted, C. Eddy and D. Cebon

University of Cambridge, Cambridge, UK

ABSTRACT
Controlling greenhouse gas emissions is becoming increasingly more important. With road freight contributing to a significant amount of energy usage, finding ways to improve this sector will, in turn, lead to large reductions in carbon dioxide emissions, with one method to achieve this being to use larger vehicles. Currently, prescriptive legislation dictates the dimensions a vehicle can take. An alternative to this is to use ‘Performance-Based Standards (PBS)’. This involves determining a set of manoeuvres and performance metrics that a vehicle must perform and pass in order to be road-worthy, instead of saying a vehicle can be a certain size or a certain weight. Through innovation and optimisation, using this method will then allow larger vehicles that are safe for driving on the road to be built. The research conducted here involved creating a PBS framework based on low-speed manoeuvrability for rigid delivery vehicles as well as assessing the high-speed stability of articulated vehicles to determine whether they would be safe for use on urban roads. Additionally, design changes such as incorporating rear axle steering were considered to determine whether vehicles that had failed the proposed PBS framework could be made to pass.

1. Introduction

In 2017, transport accounted for 33% of carbon dioxide emissions in the UK [1]. Among all the measures that can be used to reduce emissions from heavy goods vehicles, it can been shown that using larger vehicles is one of the most effective (as long as they are near fully loaded), leading to significantly higher energy efficiency [2]. Through correct design, heavy goods vehicles can be made longer while still maintaining or improving their manoeuvrability [3]. This suggests some potential for changing the way the rules are set regarding road legal vehicles.

Currently, legislation set by the EU [4] and UK [5] defines the maximum weights and dimensions that vehicles can have. Such ‘prescriptive’ standards limit opportunities for innovation in vehicle design, which can often yield sub-optimal solutions. An alternative approach is to specify a set of performance tests for new vehicles, with required performance metrics which must be met. For example, this could include being able to perform a U-turn within a specified amount of space. This set of tests would determine whether the
vehicle is allowed to travel on a given class of roads. This approach, so-called ‘Performance-Based Standards (PBS)’, is becoming increasingly popular around the world. It enables laws involving prescribed vehicle dimensions to be relaxed so that larger and/or heavier vehicles could be used. Such vehicles can be more productive and generate lower carbon emissions than conventional vehicles, while being safer and less damaging to the road infrastructure.

1.1. Existing PBS initiatives

Several countries have developed or are currently developing PBS frameworks for High Capacity Vehicles which are detailed here.

1.1.1. Australia

The PBS initiative in Australia was set up in 2003 with the intention of achieving national harmonisation in transport regulations through the use of performance-based regulations, to help overcome the limitations of prescriptive regulations. The development of the Australian PBS system involved six stages; identifying relevant performance measures by analysing the existing heavy vehicle fleet [6], establishing a system where PBS can act as a seamless alternative to prescriptive regulations [7], preparing detailed guidelines on the procedures involved in testing the vehicles using PBS [8], developing appropriate legislation [9,10], demonstrating practical application of PBS through case studies [11], and lastly implementing the system [12]. It is currently the most complete PBS system in the world, including performance tests for low-speed manoeuvrability and static stability, high-speed dynamic performance, braking performance as well as tests to ensure acceptable levels of road wear and loading on infrastructure such as bridges. Depending on the performance of each vehicle, it is permitted to travel on ‘Level 1’, ‘Level 2’, ‘Level 3’ or ‘Level 4’ roads, where ‘Level 1’ refers to unrestricted road access with the most stringent performance criteria, and ‘Level 4’ refers to mainly arterial roads suitable for longer vehicles with the least stringent performance criteria [8].

1.1.2. New Zealand

In 2010, an amendment was made to the Vehicles Dimensions and Mass Rule to allow for the introduction of High Productivity Motor Vehicles (HPMVs) in New Zealand [13]. This enabled vehicles outside the standard size and weight range to be allowed on roads that are able to accommodate them. The PBS approach was used to help facilitate the uptake of HPMVs by developing designs for these larger vehicles whilst ensuring satisfactory performance by specifying standards for drive-train requirements, low-speed manoeuvrability and high-speed dynamics [14]. New Zealand also requires HPMVs, goods service vehicles with a gross vehicle mass of over 10 tonnes and trailers over 12 tonnes have a Static Rollover Threshold (SRT) greater than 0.35 g. SRT is a standard that measures the lateral acceleration required to cause a vehicle to roll [15]. Although PBS has been used extensively in New Zealand, and work is currently being done to create a framework, no formal system currently exists [16].

1.1.3. South Africa

A pilot programme for PBS was set up in South Africa in 2004 with the intent of trialling the introduction of high productivity road freight transport. As of August 2019, there are
322 PBS vehicles participating in the programme that in total have completed 183 million monitored kilometres [17]. To date, the pilot programme has shown that the total number of heavy vehicle trips was reduced, as was the accident rate and the fuel consumption per tonne-km. This also led to reduced greenhouse gas emissions and a reduction in the road wear consumption per tonne-km [18].

1.1.4. Europe
The 'Freight and Logistics in a Multi-modal Context' project (FALCON) [19] was a recent collaborative effort to address the carbon emission targets for road freight transport set up by the European Commission. One of the major objectives of the project was to define a PBS framework for cross-border road freight transport in Europe. The framework would accommodate high capacity vehicles, and the research involved producing a representative fleet of vehicles and simulating these against specific low- and high-speed manoeuvres and calculating their effects on the highway infrastructure (roads and bridges). The research considered a range of vehicles approximately in line with the Australian road access levels 1, 2, and 3. In the FALCON report, the idea of a future performance-based standard for urban roads was introduced and designated 'level 0' to account for the less manoeuvrable city access roads that cannot be sensibly categorised into the Australian road access levels, but no analysis was performed and no recommendations for the relevant performance-based standards were provided.

1.2. Motivation and research objectives
Over half of the global population now live in towns or cities, compared to 30% in the 1950s [20] and is expected to rise to 85% by 2050 [21]. With the world becoming increasingly urban, it is of growing importance to consider urban goods vehicles in PBS initiatives as they become more prominent. Current PBS initiatives have primarily focused on defining standards for Heavy Commercial Vehicles [8,19] with little focus on producing a framework for urban goods vehicles. A PBS framework will ensure that any future vehicle designs can adhere to at least the same safety standards that are present in current roadworthy vehicles.

The aims of this paper are:

1) To propose a framework that defines a set of performance tests and pass levels that goods vehicles must perform in order to drive on urban 'level 0' roads which were not accounted for in the FALCON report [19]. This will be achieved by selecting representative vehicles and appropriate manoeuvres, and simulating them.

2) To assess the dynamic performance of articulated vehicles that the FALCON report showed to be highly manoeuvrable [19], to determine whether they could be allowed to drive on urban roads despite poor high-speed dynamics that may limit their ability to drive safely on higher-speed roads such as motorways.

3) To determine whether existing vehicles that fail the suggested 'level 0' framework could be modified so as to pass. For example, having modified dimensions or rear-axle steering.
2. Methodology

The assessment of vehicles in this study was done through the use of simulation software, requiring the development of dynamic vehicle models with drivers and assessing their performance on a set of carefully chosen manoeuvres representative of proposed ‘Level 0’ European urban roads. In order to use manoeuvres that are relevant to urban streets in the UK, a survey of corners in three UK cities was performed. This gave an insight into the manoeuvrability requirements for city streets of various types. The intention was to mix these tests with some of the ‘conventional’ tests used in other PBS systems – in Australia, South Africa, etc.

2.1. Survey of urban routes in the UK

Delivery routes were found using satellite images on Google Earth [22] and observation of city-centre locations. Four routes were found in each of Cambridge and Oxford, both historic cities with older, narrower streets. An additional four routes were found in Stevenage, a new town, where the urban roads are typically wider and easier to negotiate for heavy vehicles. Figure 1 shows an example route from each of the cities analysed.

The dimensions of the turns that delivery vehicles have to make on each route were determined by fitting a circle to the curb along the turn and then finding the distance between the centre of the circle and the mid-line of the road. The value calculated is the outer radius of the turn. An example of this process is shown in Figure 2.

The outer radii for each corner on each delivery route are plotted in Figure 3. The smallest outer radius of each route is shown with a cross. There is a tight cluster of turns around the 6.5–7 m outer radius, all of which come from the Cambridge and Oxford routes. This suggests that there should be a test at 6.5 m to determine whether the vehicles are suitable for the narrower and older roads.

A wider band appears for larger radii, between 8 and 10.5 m. Most of the routes through Stevenage are in this band. It would therefore make sense to have a second test in this range, to assess the suitability of vehicles on more modern urban streets. The proposed radius is 9 m, so as to allow most vehicles that are commonly used in these conditions in the UK.

For reference, the ‘UK standard roundabout’ has an outer radius of 12.5 m and inner radius of 5.3 m[5]. Being able to traverse this roundabout is a requirement for all lorries using UK roads. It is proposed as one of the performance tests in the FALCON report [4].
2.2. Review of existing low-speed manoeuvres

Several tests for manoeuvrability currently exist. The Australian PBS proposes one test with several performance metrics to define a vehicle's low-speed manoeuvrability [8] and countries in the UK and EU have to adhere to a turning circle test [4].
2.2.1. **12.5 m outer radius right turn**
The test for low-speed manoeuvrability in the Australian PBS [8] is a 12.5 m outer radius right turn. A reference path is defined with a straight entry tangent followed by a 90° circular arc with a radius of 12.5 m, followed by a straight exit tangent perpendicular to the entry tangent. The outer wall of the front outer tyre will drive along the reference path. For rigid trucks, three manoeuvrability performance metrics are measured from the manoeuvre; Tail Swing (TS), Front Swing (FS) and Swept Path Width (SPW) which are defined in Section 2.6. The benefit of such a manoeuvre in defining a vehicle's manoeuvrability lies in the fact that the transients of the turn are considered, as opposed to considering the steady-state turning circle of the vehicle. This approach to assessing manoeuvrability can often fail to consider a vehicle's intended operating conditions, for example, in tight urban areas, turns with lower radii are encountered and manoeuvrability in these conditions are not necessarily accounted for. A second drawback is the failure to consider 'hard' (walls) and 'soft' (curbs) boundaries, for example, the extremities of a bus may encroach on pathways during tight cornering even though the tyres remain on the road.

2.2.2. **EU turning circle**
Vehicles in the UK and EU must be able to complete a 360° turn with an outer radius of 12.5 m, measured from the front outer extremity of the vehicle, and an inner radius of 5.3 m [4]. The EU turning circle does not distinguish between road types and is a basic requirement for all vehicles regardless of road level. This manoeuvre does also not distinguish between hard and soft boundaries, or account for the possibility of how manoeuvrability changes as the turn radius decrease.

2.3. **Manoeuvre selection and simulation**
A number of performance tests were used to establish the levels at which existing UK vehicles perform. Two low-speed manoeuvres, a 90° left turn and a U-turn were considered along with two high-speed tests, a lane change and a steering impulse.

2.3.1. **90° left turn**
A 90° left turn test as shown in Figure 4 was used to imitate the Australian test [8] with suitability for UK roads, where vehicles are driven on the left. The reference path is defined in the same way as the Australian test, but mirrored to provide a left turn instead of a right turn and the radius of the test adjusted to be suitable to urban roads. The outer radius of the circle was varied between 6.5 and 12.5 m, with 6.5 m being representative of the smallest outer radius found in narrow urban streets (see above), and 12.5 m being the ‘level 1’ test proposed in the FALCON report [19].

The vehicle traversed the manoeuvre at a speed of 0.5 m/s with the front outer tyre wall following the reference path. The outputs of the simulation were the position and yaw angle of the vehicle at each time step from which the trajectories of several reference points could be calculated.

2.3.2. **U turn**
The U-Turn was set up in a similar way to the left turn, the difference being that the turn was through 180°. It is shown in Figure 5. The manoeuvre was included for the purpose of
including a performance metric that considers the hard and soft boundaries that a vehicle must manoeuvre, which existing manoeuvres did not consider. This performance metric is defined in Section 2.6.1.

2.3.3. Lane change
The lane change, shown in Figure 6, involved a lateral displacement of 3.5 m over a time of 2.5 s along a sinusoidal path, as per the lane change in the FALCON report [19] and the Australian PBS test regime [8]. The lane change manoeuvre was based on the Single Lane Change, ISO 14791:2000(E). The original test required a steering input frequency of 0.4 Hz and a test speed of 88 km/h[23].
2.3.4. Steering impulse
The steering impulse manoeuvre is taken from ISO 14791:2000(E), and required ramping the steering angle up and back down to zero over a short time interval whilst the vehicle was in motion. The function was the first half of a sine wave, with amplitude of 0.5 rads and a duration of 0.2 s [24]. The function applied to the steering angle is shown in Figure 7.

2.4. Determining high-speed test specification
Since the study was concerned with urban roads, a speed limit of 18 m/s (65 km/h) is imposed. It was therefore decided to use 18 m/s as the vehicle speed for the high speed tests.

In order to be able to compare the performance of the vehicles against the ‘level 1’ criteria which are calculated based on higher speeds, the manoeuvres must be scaled in a way that accurately reflects the drivers behaviour performing the manoeuvres at lower speeds. The lane change is designed to represent an evasive manoeuvre. In order to calculate valid results from the simulation, the frequency of the manoeuvre is kept constant, to ensure the lateral acceleration of the front unit is constant regardless of speed. The same approach was
taken in [25]. The speed at which the steering impulse occurs is dependent on the speed at which the driver can turn the steering wheel, and not on the speed that the vehicle is travelling, therefore the manoeuvre is valid in the current form regardless of vehicle speed and does not require any adjustments.

2.5. Vehicle model

2.5.1. Vehicle

Two dynamic single-track vehicle models were developed in MATLAB [26] for the purpose of vehicle simulation. These were a rigid vehicle, and a tractor with any number of semi-, or full trailers. The appendix provides a derivation of the equations of motion and diagrams visualising the models.

2.5.2. Tyres

A nonlinear tyre model was used to determine the tyre forces. It is based on a brush model with a parabolic pressure distribution [27], modified to include changes in cornering stiffness with lateral load [28]. The resultant equation for vertical tyre force is:

\[
F_y = \begin{cases} 
F_z \bar{C} \alpha - \frac{F_z C_1^2}{3 \mu} |\alpha| \alpha + \frac{F_z C_2^3}{27 \mu^2} \alpha^3 & |\alpha| < 3 \frac{\mu}{\bar{C}} \\
\frac{\alpha \mu F_z}{|\alpha|} & |\alpha| > 3 \frac{\mu}{\bar{C}}
\end{cases}
\] (1)

where \(F_y\) is the lateral tyre force, \(F_z\) is the normal force acting on the tyre, \(\alpha\) is the slip angle, \(\mu\) is the friction coefficient of the tyre and \(\bar{C}\) is defined by

\[
\bar{C} = C_1 + C_2 F_z
\] (2)

where \(C_1\) is the tyre cornering coefficient and \(C_2\) is the tyre curvature coefficient.

2.5.3. Driver

A PID controller was used to emulate the steering of the vehicle, using a single preview point method designed minimise the perpendicular distance from a specified point on the vehicle to the desired path [29]. Further information is in the appendix.

2.6. Performance metrics

The performance metrics measured from the simulation results are detailed below. Maximum Swept Path Width (SPW), Front Swing (FS) and Tail Swing (TS) were all determined from the 90° left turn, Axle Swept Path Width (ASPW) was determined from the U-Turn, Rearward Amplification (RA) and High Speed Transient Off-tracking (HSTO) were measured from the lane change, and Yaw Damping Coefficient (YDC) was measured from the steering impulse. Illustrations to aid the explanation of each metric are shown in Figure 8.

2.6.1. Low-speed performance metrics

Swept Path Width (SPW). SPW is defined as the road width swept out by the extremities of a vehicle as it moves along a path. In this case, the performance metric was the maximum
Figure 8. Illustrations of the performance metrics from each manoeuvre. (a) Left Turn performance metrics. (b) U-Turn performance metrics. (c) RA calculation. (d) YDC calculation. (e) HSTO.

SPW, a measure of the widest road width necessary for the vehicle to complete the turn, which was calculated by finding the largest perpendicular distance between the outermost and innermost paths drawn by the vehicle’s extremities [8]. When a vehicle makes a turn, the rear of the vehicle will follow a path that is inside the path that the front of the vehicle follows, known as low-speed off-tracking. Large off-tracking, and therefore high swept path width are undesirable, as it becomes more likely that the rear of the vehicle will depart from the road and move into another lane or collide with parked cars, pedestrians or road furniture.
Tail swing (TS). Tail swing is defined as the maximum lateral displacement of the rear of the vehicle during the entry tangent to the 90° turn compared to the path that the front outer wheel of the vehicle travels along [8]. If the rear of the vehicle swings out by a significant amount, it could block the adjacent lane or collide with parked cars, street furniture or hit a motorist or cyclist that has turned into the road. Critically, tail swing causes the rear of the vehicle to enter a region not travelled by the front, and for many vehicles, located in the driver’s blind-spot. Therefore hazards are often unrecognised by the driver.

Front swing (FS). When a vehicle turns, the front of the vehicle will deviate away from the path of the steering axle. In terms of the left-turn manoeuvre, front swing is defined as the maximum displacement between the path travelled by the front right outer corner of the vehicle, and the path travelled by the outer tyre wall of the front wheel of the vehicle [8]. A large amount of front swing could cause issues in similar ways to the other criteria – the vehicle could encroach on another lane and potentially cause accidents, and again this would take place in a blind spot.

Axle swept path width (ASPW). The axle swept path width is defined as the narrowest width that the wheelbase is able to perform a U-turn. The importance of this measurement stems from the fact that urban roads can be tight, and if the vehicle is not able to turn sufficiently to navigate them, situations may occur where the wheels hit the curb, or the vehicle becomes stuck. To measure ASPW, the largest perpendicular distance between the paths drawn by the outer wall of the outer tyres and those drawn by the outer wall of the inner tyres is found.

2.6.2. High-speed performance metrics

Rearward amplification (RA). When a vehicle changes lanes, the lateral acceleration of the rear compared to the front is amplified through the motion of each trailer attached to the vehicle. This can be an issue if the value is too high, as it can cause rollover of the rearmost trailer, which is often not felt by the driver until rollover has occurred.

Rearward amplification is defined as the ratio of the maximum lateral acceleration of the centre of mass (COM) of the rearmost unit compared to that of the front steering axle during a lane change, normalised by the vehicle’s SRT [8]. SRT is a measure of the lateral acceleration a vehicle must be subjected to in order to cause it to roll. The requirement for SRT in the FALCON report was greater than 0.35 g [19]. The pass level for SRT was independent of the road level, and so this value has also been taken as the pass level for ‘level 0’ roads [19].

To drive on ‘level 1’ roads, the FALCON report sets a value for RA less than 5.7, tested at highway speeds. RA also decreases with speed [25], suggesting that the value of RA a vehicle exhibits at the test speed is representative of the largest value of RA the vehicle will exhibit (provided it does not travel faster than the test speed). As RA is concerned with lateral stability, a vehicle could be considered safe up to the speed at which RA exceeds 5.7. This further suggests that the RA of a vehicle failing the requirement on ‘level 1’ roads may decrease enough to be safe to travel on ‘level 0’ roads that have a lower speed limit.

High-speed transient off-tracking (HSTO). HSTO is a measure of the maximum deviation of the rear axle in comparison to the path followed by the front axle during a lane change [8]. Limiting HSTO is important because it measures transient overshoot when a vehicle changes lanes at motorway speeds. Under these circumstances, the rear of the vehicle may swing out and hit a vehicle in an adjacent lane causing an accident. HSTO will decrease as
speed decreases [25]. Using the same logic that was applied to RA, this suggests a vehicle can be considered safe at speeds up to the point at which the requirement is exceeded, and consequently a vehicle failing the ‘level 1’ test specification may have a large enough reduction in HSTO as speed decreases to be within safe levels on ‘level 0’ roads. For the purposes of this study, it has been assumed that lane widths for ‘level 1’ roads are similar to ‘level 0’ roads in order to make the pass level for ‘level 1’ roads comparable to ‘level 0’.

**Yaw damping coefficient (YDC).** Yaw damping coefficient is a measure of how quickly the oscillatory motion of the trailing units dies down in response to a sharp change in steering. Vehicles that take a long time for yaw oscillations to decay have low yaw stability, increase the drivers workload and also pose a risk to other drivers. There is also the potential that if yaw damping is too low, a sharp manoeuvre could cause rollover. Yaw damping is calculated by measuring the amplitude of the first six local maximum values from a function of the absolute value of articulation angle, articulation rate and yaw rate for each vehicle unit. Yaw Damping coefficient is defined as [8]:

\[
YDC = \frac{\ln \tilde{A}}{\sqrt{(2\pi)^2 + [\ln \tilde{A}]^2}}
\]  

where

\[
\tilde{A} = \frac{1}{4} \left( \frac{A_1}{A_3} + \frac{A_2}{A_4} + \frac{A_3}{A_5} + \frac{A_4}{A_6} \right)
\]  

and \( A_i \) is the \( i \)th local maximum of articulation angle, articulation rate, or yaw rate. The reported YDC is the lowest value found from the full set. The FALCON report recommended vehicles on ‘Level 1’ roads must have a YDC greater than 0.15, tested at highway speeds [19]. YDC is a dimensionless number concerning the yaw stability of a vehicle that will increase as speed is lowered [8]. Similarly to RA and HSTO, this implies that vehicles that fail the ‘level 1’ test may reach safe levels when the speed is reduced in accordance with what is found ‘level 0’ roads.

### 3. Analysis of existing vehicles

#### 3.1. Vehicle selection

A number of delivery vehicles that are typically driven on urban roads were selected to sample a realistic fleet and cover the range of goods vehicles driven on UK roads [30]. Some non-delivery vehicles were also included such as a refuse truck, fire truck and a coach.

Additionally, the FALCON report [19] identified several articulated vehicles that are highly manoeuvrable but have poor high-speed dynamics. These could possibly be used for urban deliveries, but are not safe to operate on highways at high speeds. These vehicles were investigated using the high-speed manoeuvres. The first two are rigid trucks with single trailers TK6x2-CT2 and TK6x2-CT3, and the third is a rigid truck with two trailers TK8x4-CT3-CT3. The SRT for each of the articulated vehicles as stated in the FALCON report are 0.4, 0.46 and 0.5 g, respectively [19]. Since the three vehicles all passed this criteria, SRT was not considered further in the study.

The schematics of the vehicles considered in the study are shown in Figure 9 and further details listed in Table 1.
The rigid vehicles can be split into two categories, labelled ‘Rigid Vehicle A’ and ‘Rigid Vehicle B’ depending on how many axles the vehicle has. The refuse truck is the only one in the study with a tandem axle and falls under the category B schematic, all other rigid vehicles are category A.

### 3.2. Low-speed tests

Figures 10 and 11 show the simulation results for SPW and TS measurements from the 90° left turn manoeuvre. Some curves are truncated for turns with outer radii where the vehicle did not have the steering capabilities to make the turn.

The largest vehicle that can perform the 6.5 m outer radius turn has a SPW of 5.09 m and TS of 0.2 m, so the pass levels take these worse case scenarios into account, and give a small allowance to make up for the fact that the simulated fleet is not completely representative, giving a SPW pass mark of 5.5 m and a TS pass mark of 0.25 m. This method was applied to the other performance metrics to give pass levels for two tests, at 6.5 and 9 m outer radius, with Table 2 showing the full results and pass levels for the two different turns. If a vehicle did not have the steering capacity to complete the test, it is marked ‘N/A’. In some cases, judgement was needed to exclude vehicles if they were considered outliers, for

---

**Table 1.** Vehicles simulated in study.

| Vehicle name               | Total weight (tonnes) | Leading unit length (m) | First trailer length (m) | Second trailer length (m) |
|----------------------------|-----------------------|-------------------------|--------------------------|---------------------------|
| Iveco ML75E16 [31]         | 11                    | 5.6                     | N/A                      | N/A                       |
| Iveco ML75E16 [31]         | 11                    | 7.6                     | N/A                      | N/A                       |
| Iveco ML75E16 [31]         | 11                    | 8.6                     | N/A                      | N/A                       |
| Fire truck [32]            | 14                    | 8.1                     | N/A                      | N/A                       |
| Refuse truck [33]          | 26                    | 10.4                    | N/A                      | N/A                       |
| Mercedes-Benz Sprinter [34]| 3.5                   | 6.2                     | N/A                      | N/A                       |
| Citan Panel Van [35]       | 3                     | 4                       | N/A                      | N/A                       |
| MAN TG 18.240 [36]         | 18                    | 11                      | N/A                      | N/A                       |
| Coach [37]                 | 19                    | 12.3                    | N/A                      | N/A                       |
| TK6x2-CT2 [19]             | 44.3                  | 10.2                    | 7.8                      | N/A                       |
| TK6x2-CT3 [19]             | 35.9                  | 10.2                    | 6.1                      | N/A                       |
| TK8x4-CT3-CT3 [19]         | 52.4                  | 9.2                     | 6.1                      | 6.1                       |

---

**Figure 9.** Schematic of the vehicles considered. Vehicle diagrams ©2019 CEDR (reproduced from [19]).
Figure 10. Maximum swept path results for the common rigid vehicles.

Figure 11. Tail swing results for the common rigid vehicles.
Table 2. Results of rigid vehicles performing the low-speed manoeuvres.

| Test    | Vehicle                  | SPW max (m) | TS (m) | FS (m) | ASPW (m) |
|---------|--------------------------|-------------|--------|--------|----------|
| 6.5 m   | Citan Panel Van          | 2.63        | 0.01   | 0.37   | 2.25     |
| 6.5 m   | IvecoL75E16 (5.6)        | 3.67        | 0.05   | 0.66   | 2.99     |
| 6.5 m   | Mercedes-Benz Sprinter   | 3.56        | 0.07   | 0.56   | 2.98     |
| 6.5 m   | IvecoL75E16 (7.6)        | N/A         | N/A    | N/A    | N/A      |
| 6.5 m   | Fire truck               | 5.09        | 0.20   | 0.95   | 4.12     |
| 6.5 m   | IvecoL75E16 (8.6)        | N/A         | N/A    | N/A    | N/A      |
| 6.5 m   | Refuse truck             | N/A         | N/A    | N/A    | N/A      |
| 6.5 m   | MAN TG 18.240            | N/A         | N/A    | N/A    | N/A      |
| 6.5 m   | Coach                    | N/A         | N/A    | N/A    | N/A      |
| 6.5 m   | Pass level               | 5.5         | 0.25   | 1.2    | 4.5      |
| 9 m     | Citan Panel Van          | 2.41        | 0.01   | 0.28   | 2.13     |
| 9 m     | IvecoL75E16 (5.6)        | 3.26        | 0.04   | 0.51   | 2.75     |
| 9 m     | Mercedes-Benz Sprinter   | 3.08        | 0.05   | 0.44   | 2.63     |
| 9 m     | IvecoL75E16 (7.6)        | 3.88        | 0.12   | 0.63   | 3.23     |
| 9 m     | Fire truck               | 4.38        | 0.14   | 0.77   | 3.59     |
| 9 m     | IvecoL75E16 (8.6)        | 4.59        | 0.16   | 0.96   | 3.60     |
| 9 m     | Refuse Truck             | N/A         | N/A    | N/A    | N/A      |
| 9 m     | MAN TG 18.240            | 5.56        | 0.29   | 0.88   | 4.71     |
| 9 m     | Coach                    | 6.87        | 0.31   | 1.91   | 4.90     |
| 9 m     | Pass level               | 5.75        | 0.3    | 1.2    | 4        |

Table 3. Results for articulated vehicles performing the 18 m/s high speed tests

| 18 m/s tests | YDC | RA | HSTO (m) |
|----------------|-----|----|----------|
| TK6x2-CT2      | 0.17| 3.45| 0.35     |
| TK6x2-CT3      | 0.17| 2.83| 0.38     |
| TK8x4-CT3-CT3  | 0.05| 3.74| 1.06     |

For example, the coach has a significantly larger front swing than the other vehicles, and so was deemed to have failed the test, and the pass level decided without including this result. For the purposes of this study, it has been assumed that vehicles already legally permitted to operate on these roads can be considered safe, therefore allowing pass levels to be derived from them.

3.3. High-speed tests

Table 3 shows the results of the high-speed tests performed at 18 m/s. The ‘level 1’ pass levels for YDC, RA and HSTO are 0.15, 5.7 and 0.6 m, respectively. The analysis shows that at the imposed speed limit for ‘level 0’ roads, both the TK6x2-CT2 and TK6x2CT3 had adequate yaw damping to be considered stable. The TK8x4-CT3-CT3 failed this test.

Both the TK6x2-CT2 and TK6x2-CT2 had adequately low HSTO of 0.35 and 0.38 m, respectively. Again, the TK8x4-CT3-CT3 failed this test. All three articulated vehicles comfortably passed the ‘level 1’ criteria for RA at the lower speed of 18 m/s.

Overall, under the assumptions of the analysis, the two articulated vehicles with a single articulation point had adequate high-speed dynamics to drive safely on ‘level 0’ roads, whereas the vehicle with two articulation points failed.
4. Design case studies

This section investigates some of the vehicles that failed the proposed level 0 PBS specifications to determine whether they could be modified and made to pass. The modified coach and the TK6x2-CT3 were both able to pass the level 09.0 tests. The Mercedes-Benz Sprinter, which passed the level 06.5 tests was analysed to see how much larger it could be made without failing.

4.1. Coach

The coach failed the 9 m left turn for all three performance criteria, with front swing being particularly poor. The reason for a large amount of front swing is due to the large front overhang. To reduce the front swing, the front overhang must be reduced. In order to reduce the overhang in a way that will not overload the rear axle, the rear axle is also moved back, which consequently also reduces TS. Originally the vehicle had a front overhang of 2.89 m, a wheelbase of 6.09 m, a rear axle load of 11.5 tonnes and a front axle load of 7.5 tonnes [37]. The modified vehicle has a front overhang of 1.28 m, a wheelbase of 8.75 m, with the axles loaded as before. In this configuration, FS is 1 m, TS is 0.1 m and SPW is 8.3 m. This design modification has been effective at reducing the front swing and tail swing to acceptable values, but leads to a higher SPW. With tail swing and front swing under the proposed limits, rear-axle steering was introduced. The steering strategy is known as 'path-following' steering [38] and is illustrated in Figure 12.

The rear axle steers to ensure a point near the rear of the vehicle tracks the path followed by the front axle. A rear axle steering limit of 25° has been assumed based on what has previously been achieved when implementing this strategy [39]. As the follow point is brought in from the rear of the vehicle, SPW and FS are reduced and TS increases due to the effective wheelbase being reduced. In order to bring SPW down as much as possible, the follow point will be located as far in as possible without exceeding the TS limit. Applying this strategy to the bus reduces SPW to 5 m, FS to 0.8 m and TS to 0.3 m, all within the proposed limits. The rear axle requires 31° of steering in order to complete the turn, above the assumed limit. To reduce the amount of rear steering required to 25°, the position of the rear axle must be brought towards the centre of mass by 0.75 m. This will place an extra 1 tonne of load on the rear axle due to the mass distribution of the vehicle.
Table 4. Results of articulated vehicles performing the low-speed manoeuvre.

| Test | Vehicle              | SPWmax (m) | TS (m) | FS (m) | ASPW (m) |
|------|----------------------|------------|--------|--------|----------|
| 6.5 m| TK6x2-CT2            | 7.53       | 0.69   | 1.10   | 6.42     |
| 6.5 m| TK6x2-CT2 steered    | 7.09       | 0.66   | 0.97   | 6.13     |
| 6.5 m| TK6x2-CT3            | 6.93       | 0.32   | 1.04   | 5.89     |
| 6.5 m| TK6x2-CT3 steered    | 6.40       | 0.32   | 0.97   | 5.43     |
| 6.5 m| TK8x4-CT3-CT3        | 7.4        | 0.4    | 1.02   | 6.47     |
| 6.5 m| TK8x4-CT3-CT3 steered| 6.89       | 0.59   | 0.97   | 5.91     |
| 9 m  | TK6x2-CT2            | 6.66       | 0.35   | 0.89   | 5.77     |
| 9 m  | TK6x2-CT2 steered    | 6.48       | 0.41   | 0.81   | 5.67     |
| 9 m  | TK6x2-CT3            | 6.25       | 0.24   | 0.89   | 5.36     |
| 9 m  | TK6x2-CT3 steered    | 5.75       | 0.21   | 0.79   | 4.96     |
| 9 m  | TK8x4-CT3-CT3        | 6.76       | 0.25   | 0.83   | 5.93     |
| 9 m  | TK8x4-CT3-CT3 steered| 6.19       | 0.34   | 0.79   | 5.39     |

4.2. Light goods vehicle

The Mercedes-Benz Sprinter is able to comfortably pass the level 06.5 tests. This implies that the vehicle can be made larger without exceeding the pass level for each performance criteria.

In this case, the weight of the original vehicle is 3.5 tonnes, with a maximum payload of 1 tonne, and a payload density of 90 kg/m³ based on the maximum payload. Increasing the vehicle length by 3 m and strategically positioning the rear axle to give a wheelbase of 5.66 m and rear overhang of 2.57 m whilst also introducing rear axle path-following steering allows the vehicle to perform the 6.5 m outer radius tests with a maximum SPW of 3.79 m, TS of 0.25 m, FS of 0.64 m and ASPW of 3.15 m: all within the proposed limits.

This procedure gives rise to an additional volume of 11.1 m³, or 0.999 tonnes, effectively doubling the original payload.

4.3. Articulated vehicles

It has been shown that the high-speed dynamics of two of the articulated vehicles are within safe levels for travel on ‘level 0’ roads, however, their manoeuvrability is not good enough to pass the proposed PBS framework, as demonstrated in Table 4.

Again, there is scope for manoeuvrability improvement of these vehicles using ‘optimised’ rear axle steering proposed by Jujnovich [38] and illustrated in Figure 13.

In this case, the trailer’s axle will steer to keep a point towards the rear of the trailer fixed on the path drawn by the hitch location of the unit in front of it, the leading unit remains unsteered. The performance of the articulated vehicles for the low-speed manoeuvres can be improved upon, with one vehicle (the TK6x2-CT3) now being capable of passing the 9 m test. The results of low-speed manoeuvrability tests for the articulated vehicles with path-following steering are shown alongside the original articulated vehicles performance in Table 4.

5. Discussion

The full set of proposed performance tests and pass criteria for each performance metric is detailed in Table 5. The ‘level 0’ roads were divided into two categories; level 06.5, representing urban routes with tight turns in which the manoeuvrability tests are performed...
using an outer radius of 6.5 m, and level 0\textsubscript{9.0}, representing urban routes consisting of less severe turns, in which the manoeuvrability tests are performed using an outer radius of 9 m. The high-speed performance criteria are the same for both categories of 'level 0' roads. A vehicle is limited to driving only on 'level 0' roads unless able to successfully pass the high-speed requirements for higher-level roads. Additionally, vehicles passing the level 0\textsubscript{9.0} tests may not drive on level 0\textsubscript{6.5} roads unless they have passed the relevant level 0\textsubscript{6.5} tests.

Table 6 shows the results of all the vehicles tested and whether or not they passed. The cause of failure for rigid vehicles was manoeuvrability constraints. The articulated vehicles without path-following all failed the 'level 0' manoeuvrability tests, with the TK6x2-CT2 and TK6x2-CT3 passing the 'level 0' high-speed tests. The TK6x2-CT3 with path-following control was capable of passing the level 0\textsubscript{9.0} manoeuvrability tests and the high-speed tests giving an overall pass for this road level. The TK8x4-CT3-CT3 both with and without path-following, failed the 'level 0' high speed and manoeuvrability tests.

By analysing current road worthy vehicles and relevant routes, a basis for low-speed manoeuvrability standards for vehicles on urban roads have been produced. It must be noted that the sample size of vehicles and delivery routes considered in the study is relatively small in comparison to the entire road network and fleet of road legal vehicles. In order for such a system to be implementable, it is necessary to apply the methodology to
Table 6. Results of each vehicles performance on corresponding test level.

| Vehicle                  | Level 06.5 | Level 09.0 |
|-------------------------|------------|------------|
| Citan Panel Van         | Pass       | Pass       |
| Iveco 5.6 m             | Pass       | Pass       |
| Mercedes 313CDI         | Pass       | Pass       |
| Iveco 7.6 m             | Fail       | Pass       |
| Fire Truck              | Pass       | Pass       |
| Iveco 8.6 m             | Fail       | Pass       |
| Refuse Truck            | Fail       | Fail       |
| MAN TG 18.240           | Fail       | Fail       |
| Coach                   | Fail       | Fail       |
| Modified Coach          | Fail       | Pass       |
| TK6x2-CT3               | Fail       | Fail       |
| TK6x2-CT2               | Fail       | Fail       |
| TK8x4-CT3-CT3           | Fail       | Fail       |
| Modified TK6x2-CT3      | Fail       | Fail       |
| Modified TK6x2-CT2      | Fail       | Fail       |
| Modified TK8x4-CT3-CT3  | Fail       | Fail       |

a larger, more significant set of appropriate routes and vehicles in order to draw accurate conclusions on suitable test specifications and pass levels.

Two of the three articulated vehicles considered for possible use on urban roads have been shown to have acceptable levels of dynamic stability at the speeds at which they would be operating. With the use of path-following rear axle steering [38], one vehicle demonstrated sufficient manoeuvrability to pass the level 09.0 tests. Although this vehicle may be suitable for use on city roads, the FALCON report [19] showed this vehicle has poor dynamic performance at higher speeds and so may not be usable on higher-level roads. Additionally, to validate the proposed 'level 0' requirement for HSTO, an additional study must be undertaken to determine the relationship in lane widths between 'level 0' and 'level 1' roads to either confirm the suggested criteria is acceptable, or to make adjustments as necessary. A problem arises over the feasibility of having large articulated vehicles that are only roadworthy on small urban roads. Logistically, it may not be possible to develop sensible delivery routes without deviating on to higher-level roads.

Section 4 highlighted the importance of the positioning of a vehicles axle in respect to manoeuvrability and the power of using path-following rear axe steering [38]. It was shown that by manipulating the front and rear axles position, reductions in TS and FS can be achieved, but may not always be implementable. Changing the position of the axles can change the axle loading and potentially lead to overload and require further design modifications in order to support the additional load. It is worth noting that significantly changing the position of the front axle on a coach or bus may not be feasible, as the location of the front door becomes an issue, and it might be too much to expect bus companies to completely revamp their vehicle design to change the location of the door to accommodate this. It, therefore, may be that a separate study of buses and coaches needs to be done, and an additional allowance for front swing made, as was the recommendations for the New Zealand PBS system [16]. As of January 2020, the criterion for FS in the Australian PBS system has changed to include an allowance for busses and coaches [8].

The tests outlined in this study only form a beginning framework of a full system. The standards analysed form a framework partially describing low-speed manoeuvrability and
high-speed stability. The Australian PBS system outlines tests and performance metrics in five categories; driveability, manoeuvrability, high-speed stability, winter conditions and infrastructure, all of which must be analysed with suitable pass levels determine in order to be legally implemented.

6. Conclusions

(1) Simulations were performed using a set of UK heavy vehicles that are suitable for operation on urban roads.
(2) A PBS framework was proposed consisting of a set of low- and high-speed manoeuvres and related performance criteria and ‘pass’ levels.
(3) Three case studies were presented in which design changes were made to vehicles that failed to meet one or more of the proposed performance criteria, or was undersized and passed comfortably. Modifying the wheelbase, axle positioning and size, and introducing path-following rear axle steering were shown to have positive effects on vehicle manoeuvrability.
(4) It was shown that using an appropriate steering control system, articulated vehicles capable of carrying significantly larger payloads may be used safely on urban roads.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This research was supported by the Engineering and Physical Sciences Research Council Grant EP/R035199/1: ‘Centre for Sustainable Road Freight 2018-2023’.

ORCID

C. J. Isted http://orcid.org/0000-0001-6941-1324
C. Eddy http://orcid.org/0000-0002-0512-442X
D. Cebon http://orcid.org/0000-0003-2828-6445

References

[1] Department for Business. Energy and industrial strategy. 2018 UK Greenhouse Gas Emissions; 2018 [cited 2019 May 8]. Available from: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/790626/2018-provisional-emissions-statistics-report.pdf
[2] Odhams AMC, Roebuck RL, Lee YJ, et al. Factors influencing the energy consumption of road freight transport. Proc Inst Mech Eng Part C. 2010;224:1995–2010.
[3] Eddy C. Design of urban freight vehicles to maximise capacity. 15th International Symposium on Heavy Vehicle Transportation Technology; The Netherlands; 2018.
[4] European Council. Directive 97/27/EC of 22 July 1997 on the masses and dimensions of certain categories of motor vehicles and their trailers. European Parliament, Council of the European Union; 1997. p. 1–31.
[5] UK Department for Transport. The road vehicles (construction and use) (amendment) regulations; 2018.
[6] NRTC Australia. Specification of performance standards and performance of the heavy vehicle fleet; 2000.
[7] NRTC Australia. Performance-based standards for heavy vehicle regulation: proposed regulatory framework and processes; 2005.
[8] NRTC Australia. Performance-based standards scheme—the standards and vehicle assessment rules [Online]; 2020 [cited 2020 Jun 20]. Available from: https://www.nhvr.gov.au/files/0020-pbstdsvehassrules.pdf
[9] NRTC Australia. Road transport reform (compliance and enforcement) bill. Regulatory impact statement; 2003.
[10] NRTC Australia. Road transport reform (compliance and enforcement) bill. Model provisions; 2003.
[11] NRTC Australia. Performance based standards for heavy vehicles: assembly of case studies; 1999.
[12] NZ Transport Agency. Vehicle dimensions and mass amendment 2010 (the 2010 VDAM amendments); 2010.
[13] NZ Transport Agency. The PBS requirements for heavy vehicles in New Zealand. 2019.
[14] NZ Transport Agency. Factsheet 13e – static rollover thresholds; 2017.
[15] De Pont J, Hutchinson DN, Taylor GB. Formalising the PBS system in New Zealand. 14th International Symposium on Heavy Vehicle Transportation Technology; New Zealand; 2016.
[16] CSIR. Smart truck portal. [cited 2020 Jun 15]. Available from: http://smarttruckportal.herokuapp.com/dashboard
[17] Nordengen P, Berman R, De Saxe C, et al. An overview of the performance based standards pilot project in South Africa. 15th International Symposium on Heavy Vehicle Transportation Technology; The Netherlands; 2018.
[18] de Saxe CC, Kural K, Schmidt F et al. Definition and validation of a smart infrastructure access policy utilising performance-based standards (CEDR call 2015: freight and logistics in a multimodal context). CEDR; 2019. (CEDR Contractor Report 2019-03, WP3.0001.2018).
[19] United Nations Department of Economic and Social Affairs. Population division. World urbanization prospects. 2014 revision (highlights); 2014.
[20] OECD. Organisation for economic co-operation and development, delivering the goods: 21st century challenges to urban goods transport. Tech. Rep.; 2003.
[21] Satellite imaging and map data obtained from Google Maps; 2019.
[22] single Lane Change. ISO 14791:2000(E); 2000.
[23] Steer Impulse. ISO 14791:2000(E); 2000.
[24] Prem H, de Pont JJ, Pearson B, et al. Performance characteristics of the Australian heavy vehicle fleet. Melbourne; National Road Transport Commission: 2002.
[25] Pacejka HB. Tyre and vehicle dynamics. Amsterdam: Elsevier Butterworth-Heinemann; 2012.
[26] Fancher PS. A Factbook of the mechanical properties of the components for single-unit and articulated heavy trucks. Washington (USA); US Department of Transportation; 1986.
[27] Guo K, Guan H. Modelling of driver/vehicle directional control system. Veh Syst Dyn. 1993;22(3-4):141–184.
[28] Department for Transport. Statistical dataset VEH0521 [Online]. 2018 [cited 2019 Oct 28]. Available from: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/794497/veh0521.ods
[29] Iveco. 75E16 truck technical sheet [Online]. 2009 [2018 Nov 12]. Available from: https://www.iveco.com/uk/collections/technical_sheets/Documents/eurocargo/Eurocargo-75/75E16_Truck.pdf
[30] Scania. Scania water and rescue vehicle [Online]. 2015 [cited 2018 Nov 12]. Available from: https://www.scania.com/group/en/wp-content/uploads/sites/2/2016/01/pdf-C200835_Water-and-Rescue-Vehicle.pdf
[31] Coventry City Council. Waste storage planning technical advice note version 2.2 [Online]. 2017 [cited 2019 May 27]. Available from: http://www.coventry.gov.uk/download/downloads/id/22895/waste_storage_technical_advice_note_2017.pdf
Appendix. Derivation of equations of motion

A.1 Single-unit ‘rigid’ model

Figure A1 shows a generic tractor unit which may be coupled to a trailer via the connection point at the rear, with the relevant forces and velocities for a single unit model. For simplicity, a single rear axle is shown. The model could accommodate any number of axles, as shown in the derivation of equations. When simulating a single-unit vehicle, the rear coupling forces will be equal to zero, since no trailer is attached. The following assumptions were made to derive the equations of motion:

- The equations of motion were determined in a frame of reference rotating with the vehicle body.
- The vehicle had a fixed longitudinal velocity.
- A single-track ‘bicycle model’ was used to reduce complexity in the absence of roll motion [40].
- The effects of vehicle roll and suspension were ignored.
- Tyre forces were represented using a brush model with a parabolic pressure distribution, modified to account for changes in cornering stiffness with lateral load (See Section 2.4.2).

A.1.1 Derivation

The slip angles for the front and rear tyres were defined:

\[ \alpha_f = \arctan \left( \frac{v + a\Omega}{u} \right) - \delta \]
\[ \alpha_{r,k} = \arctan \left( \frac{v - b_k\Omega}{u} \right) \]

where \( \alpha_f \) is the front slip angle, \( \alpha_r \) is the rear steer angle, \( v \) is the lateral velocity of the vehicle, \( u \) is the longitudinal velocity of the vehicle, \( \Omega \) is the angular velocity of the vehicle, and \( a \) and \( b \) are the displacement from the centre of mass of the front and rear tyre, respectively. The subscript \( k \) indicates the \( k \)th rear axle and the following equations must be summed over \( k \). Resolving forces laterally, and taking moments about the centre of mass gives:

\[ \dot{\Omega} = \frac{1}{I} \left[ b_k F_y(\alpha_{r,k}) - a F_y(\alpha_f) \cos(\delta) \right] \]
\[ \dot{v} = -u \Omega - \frac{1}{m} \left[ F_y(\alpha_{r,k}) + F_y(\alpha_f) \cos(\delta) \right] \]
where $m$ is vehicle mass, $I$ is yaw moment of inertia and $\delta$ is the steer angle of the front tyre. Two extra equations were implemented in order to determine the position of the vehicle in a stationary reference frame:

\begin{align*}
\dot{x} &= u \cos(\theta) - v \sin(\theta) \\
\dot{y} &= u \sin(\theta) + v \cos(\theta)
\end{align*}  \tag{A5, A6}

where $\theta$ is the yaw angle of the vehicle and $x$ and $y$ are the position of the vehicle centre of mass in a stationary reference frame. These equations were numerically integrated in MATLAB using an ODE solver (ode45) to give the position and yaw angle of the vehicle at any time.

A.2 Articulated vehicle model

The process of creating a model for an articulated vehicle with any number rigid units was more complex, requiring a force balance and consideration of kinematic constraints for each vehicle unit. The forces and velocities are shown in Figure A1. The assumptions for the multiply-articulated model are the same as the single unit model. The only additional assumption in this case was that each trailer was attached to a fixed point on the vehicle unit directly in front.

A.2.1 Derivation

For simplicity, the derivation below describes a tractor with a single rear axle pulling trailers with a single axle. Modelling of multiple axle groups was achieved by adding extra lateral forces at each axle location by evaluating the slip angle in that position in the same way as was done for the rigid vehicle. The slip angles on the trailer axles are defined in the same way as Equation (A2). Consider
a state vector describing the motion of the tractor and its subsequent trailers:

\[
\mathbf{z} = [u_1 \quad v_1 \quad \Omega_1 \quad \theta_1 \quad \cdots \quad u_n \quad v_n \quad \Omega_n \quad \theta_n]^T \quad (A7)
\]

\[
\dot{\mathbf{z}} = [\dot{u}_1 \quad \dot{v}_1 \quad \dot{\Omega}_1 \quad \dot{\theta}_1 \quad \cdots \quad \dot{u}_n \quad \dot{v}_n \quad \dot{\Omega}_n \quad \dot{\theta}_n]^T \quad (A8)
\]

In order to fully solve the problem, four equations were required from each vehicle unit. The first of these four equations was determined from the angular velocities which appear in both \( \mathbf{z} \) and \( \dot{\mathbf{z}} \):

\[
\dot{\theta}_i = \Omega_i \quad (A9)
\]

The kinematic constraints allowed for equations to be determined for \( \dot{v} \) and \( \dot{u} \) for each of the trailers. Firstly, the articulation angle for each trailer was defined as:

\[
\Gamma_i = \theta_{i+1} - \theta_i \quad (A10)
\]

The velocities for each trailer in their own reference frame was then calculated:

\[
u_{i+1} = u_i \cos(\Gamma_i) + (v_i - (b_i + c_i)\Omega_i) \sin(\Gamma_i)
\]

\[
v_{i+1} = (v_i - (b_i + c_i)\Omega_i) \cos(\Gamma_i) - u_i \cos(\Gamma_i) - \Omega_{i+1} a_{i+1}
\]

These equations were then further differentiated to find \( \dot{u} \) and \( \dot{v} \) for each trailer:

\[
\dot{u}_{i+1} = \dot{u}_i \cos(\Gamma_i) + (\dot{v}_i - (b_i + c_i)\dot{\Omega}_i) \sin(\Gamma_i) + \dot{\Gamma}_i((v_i - (b_i + c_i)\Omega_i) \cos(\Gamma_i) - u_i \sin(\Gamma_i))
\]

\[
\dot{v}_{i+1} = (\dot{v}_i - (b_i + c_i)\dot{\Omega}_i) \cos(\Gamma_i) + \dot{\Omega}_i(\dot{v}_i - (b_i + c_i)\dot{\Omega}_i) \sin(\Gamma_i) + \dot{\Omega}_i \dot{\theta}_i - \dot{\Omega}_{i+1} a_{i+1} + u_i \cos(\Gamma_i))
\]

The full set of equations describing \( \dot{u} \) and \( \dot{v} \) for every vehicle unit was then completed by resolving lateral forces on the tractor and remembering that the longitudinal velocity of the tractor was fixed:

\[
\dot{u}_1 = 0 \quad (A15)
\]

\[
\dot{v}_1 = -u_1 \Omega_1 - \frac{1}{m_1}(F_y(\alpha_{r,1}) + F_y(\alpha_f) \cos(\delta) - Y_1) \quad (A16)
\]

A final group of equations describing \( \dot{\Omega} \) for each unit was needed to fully complete the set. The first step in doing this was to first resolve \( X_i \) and \( Y_i \) for each unit. This was accomplished by starting at the rear unit and observing that there was no force acting on it:

\[
X_n = 0 \quad (A17)
\]

\[
Y_n = 0 \quad (A18)
\]

Next, the lateral and longitudinal forces acting on each units front hitch point was determined and referred to the rear hitch point of the trailer in front of it:

\[
Y_{i-1} = (Y_i - m_i(\dot{v}_i + u_i \Omega_i - F_y(\alpha_{r,i}))) \cos(\Gamma_{i-1}) - (X_i + m_i(\dot{u}_i - v_i \Omega_i)) \sin(\Gamma_{i-1}) \quad (A19)
\]

\[
X_{i-1} = (Y_i - m_i(\dot{v}_i + u_i \Omega_i - F_y(\alpha_{r,i}))) \sin(\Gamma_{i-1}) + (X_i + m_i(\dot{u}_i - v_i \Omega_i)) \cos(\Gamma_{i-1}) \quad (A20)
\]

The final set of equations were then found by resolving moments about the rear hitch point for the front unit, and the front hitch point for each subsequent trailer:

\[
\dot{\Omega}_1 = -\frac{1}{I_1} [c_1 F_y(\alpha_{r,1}) + (c_1 + b_1)m_1(\dot{v}_1 + u_1 \Omega_1) + (c_1 + b_1 + a_1)F_y(\alpha_f) \cos(\delta)] \quad (A21)
\]

\[
\dot{\Omega}_i = \frac{1}{I_i} [a_i m_i(\dot{v}_i + u_i \Omega_i) + (a_i + b_i)F_y(\alpha_{r,i})(a_i + b_i + c_i)Y_i] \quad (A22)
\]

The four groups of independent differential equations relating to the state of the vehicle were programmed in MATLAB (in this case, using ode15i as it was the only solver capable of solving a set of
implicit differential equations) to give variation of the vehicles states over time. The state-space can be expressed as:

\[ \dot{z} = f(z, \delta) \]  \hspace{1cm} (A23)

The velocities of each unit can be integrated according to Equations 8 and 9 to determine the global coordinates of the vehicle. The outputs can then be expressed as:

\[ w(t) = f(z, x, y) \]  \hspace{1cm} (A24)

### A.3 Driver model

A driver model was used to control the front axle steer angle during the manoeuvres. The purpose of the controller was not to replicate human driver input, but instead to keep the vehicle following the manoeuvres’ path as closely as possible. The ‘preview point’ was selected as the position of the outer wall of the front outer tyre for the low-speed manoeuvrability tests, and the front centre of the vehicle for the lane change. A feedback loop was used with the error signal being the lateral displacement between the reference path and the preview point. The PID was manually tuned in order to obtain accurate reference tracking.