Development of strapdown inertial navigation system with MEMS sensors, barometric altimeter and ultrasonic range meter

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Abstract. The results of strapdown inertial navigation system (SINS) tests with 9 degrees of freedom MEMS sensor MPU-9150 (triaxial accelerometer, gyroscope and magnetometer), pressure sensor LPS331 and ultrasonic range meter HC-SR04, implemented on the FPGA Altera Cyclone-II evaluation board DE1 is considered. SINS measures the spatial coordinates and altitude relative to the starting point, the orientation angles and distances to obstacles along the way. It is shown that the relative error of the spatial coordinates estimation does not exceed 1.1% in interval of some minutes.

1. Angular coordinates estimation
The classical parameters that allow to define the angular position of a rigid body uniquely are three Euler - Krylov angles: yaw \( \psi \), pitch \( \theta \) and roll \( \phi \). As the result of rotation in three-dimensional space is not invariant to the sequence of turns, we consider the case when the resulting rotation is specified by a sequence of rotations \( \psi \rightarrow \theta \rightarrow \phi \) [1].

It is suitable to use quaternions – 4-component vectors \( \mathbf{q} = [q_0, q_1, q_2, q_3]^T \) – for solving the problem of integrating of measured gyro angular velocities:

\[
\mathbf{q} = q_0 + iq_1 + jq_2 + kq_3, \quad \|\mathbf{q}\| = 1,
\]

\[
ij = k, \quad ji = -k, \quad k = i, \quad k = j, \quad ik = -j, \quad i^2 = j^2 = k^2 = -1,
\]

\[
q_0 = \cos(\psi/2)\cos(\theta/2)\cos(\phi/2) - \sin(\psi/2)\sin(\theta/2)\sin(\phi/2),
q_1 = \cos(\psi/2)\cos(\theta/2)\sin(\phi/2) + \sin(\psi/2)\sin(\theta/2)\cos(\phi/2),
q_2 = \sin(\psi/2)\cos(\theta/2)\cos(\phi/2) + \cos(\psi/2)\sin(\theta/2)\sin(\phi/2),
q_3 = \cos(\psi/2)\sin(\theta/2)\cos(\phi/2) - \sin(\psi/2)\cos(\theta/2)\sin(\phi/2).
\]

For a right-handed coordinate system vectors used in MPU-9150 (\( Y \)-axis is directed forward, \( X \)-axis is directed right and \( Z \)-axis is directed up) the quaternion rate of change according to \[2\] is
\[
\dot{q} = \frac{1}{2} \begin{bmatrix}
-q_1 & -q_2 & -q_3 \\
q_0 & -q_3 & q_2 \\
-q_3 & q_0 & q_1 \\
-q_2 & q_1 & q_0
\end{bmatrix} \begin{bmatrix}
w_x \\
w_y \\
w_z
\end{bmatrix},
\]

where \([w_x, w_y, w_z]^T\) is the gyroscope data.

Integration of (3) gives

\[
q_{i+1} = q_i \circ q_{rot},
\]

where the symbol " \(\circ\) " denotes the quaternions multiplication according to the rules (1), \(q_{rot} = [\gamma + i\Delta \alpha + j\Delta \beta + k\Delta \gamma] \) is quaternion rotation during the time interval \(T_i\) between points of time \(i\) and \((i + 1)\), \(|q_{rot}| = 1\), \(\Delta k = 0.5w_k T_i\), \(\gamma = [1 - (\Delta \alpha^2 + \Delta \beta^2 + \Delta \gamma^2)]/8\), \(k = \{x, y, z\} \) or [2]

\[
q_{i+1} = \frac{q_i + q_{rot}}{|q_i + q_{rot}|}.
\]

Rodrigues - Hamilton parameters and Euler - Krylov angles are associated by relations

\[
\psi = \text{atan}2(q_0 q_2 - q_1 q_3, q_0^2 + q_1^2 - 0.5),
\]

\[
\theta = \text{atan}2(q_0 q_1 - q_2 q_3, q_0^2 + q_2^2 - 0.5),
\]

\[
\varphi = \text{arcsin}(2q_1 q_2 + 2q_0 q_3).
\]

The accelerometer and the magnetometer are used to determine the initial values of Euler - Krylov angles:

\[
\varphi_a = -\text{atan}2(a_x, a_z), \quad \theta_a = \text{atan}2(a_y, a_z \cos \varphi_a - a_x \sin \varphi_a),
\]

\[
\psi_m = \text{atan}2(H_x, H_y),
\]

where \(H_x = m_x \cos \varphi_m + m_y \sin \varphi_m, H_y = m_x \sin \varphi_m \sin \theta_m + m_z \cos \varphi_m \cos \theta_m \) and \([a_x, a_y, a_z]^T\) and \([m_x, m_y, m_z]^T\) is the accelerometer and magnetometer data respectively.

As the expression (8) allows to determine the deviation from the Earth magnetic pole, not from geographic, so it must be also taken account of magnetic declination by evaluating the course in (8) [3].

2. Estimation of travelled distance, altitude and distance to the obstacles

In the presence of the speed sensor according to the method of dead reckoning the course from the known current coordinates \([X_i, Y_i, Z_i]^T\) at a discrete point of time \(i\) and the ground speed vector \(V_i\) projections \([V_{ix}, V_{iy}, V_{iz}]^T\) the coordinates along the axis \(X, Y\) and \(Z\) at next discrete point of time \((i + 1)\) can be estimated at the following:

\[
[X_{i+1}, Y_{i+1}, Z_{i+1}]^T = [X_i, Y_i, Z_i]^T + T_i [V_{ix}, V_{iy}, V_{iz}]^T,
\]

where \(V_{ix} = V_x \sin \theta_i, V_{iy} = V_x \cos \psi \cos \theta_i, V_{iz} = V_y \sin \psi \cos \theta_i.\)

Except the inertial sensors in the considered SINS were also used the ultrasonic range meter to determine the distance to obstacles and barometric pressure sensor for measuring the altitude.

To reduce the measurement error the sound velocity \(c\) was constantly adjusted depending on the temperature change:

\[
R(t_d, t) = c(t) t_d / 2 = (331 + 0.6t) t_d / 2,
\]
where $t_d$ is the delay time of the ultrasonic signal. The atmospheric pressure sensor was used to determine the altitude $H$ of the object (in meters) relative to the start point [4]:

$$H = 443077 \left[ 1 - \frac{P(H)}{P_a} \right]^{0.190284},$$

where $P(H)$ is the pressure on the current height in mbar and $P_a = 1013$ mbar is the atmospheric pressure.

3. Fusion of navigation information from various sensors

For SINS on MEMS sensors are generally used either the explicit complementary filter [5, 6] or AHRS algorithm [7]. In [8] it is shown that the use of AHRS by the high level of accelerometer noise (for example, by driving on uneven surfaces) gives a more noisy estimate of the roll and pitch than the use of the algorithms [5] or [6], that’s why the explicit complementary filter from [8] was used for the developed SINS with proportional and integral gains $k_p = 0.28$ and $k_i = 0.015$ respectively.

4. Experiment results

During the research of the developed SINS (its block diagram indicating the types of sensors and communication protocols is shown at figure 1), a number of experiments was made: the passage of wheeled platform attached to it SINS through a closed loop with a avoiding obstacle from the left and the right side and with a single stop at start point (figure 2), movement of the platform in a vertical plane, distancing from the obstacles and approach to it. The accelerometer, the gyroscope and the magnetometer were previously calibrated according to the procedures [9-11]. Because the area where the experiment was carried out attended by a large number of massive metal objects the magnetometer data for course correction is not used. Assembled under the scheme of incremental encoder, the speed sensor produced a pulse signal for every 7 mm of travelled distance. The output data rate of the inertial sensors and the speed sensor was 100 Hz, and the output data rate of the ultrasonic range meter and the pressure sensor was 25 Hz.

For MEMS gyro temperature drift compensation used simple linear approximation [12]:

$$b(t) = b(t_0) + \beta_i (t - t_0),$$

where $b(t)$ is gyro bias, $t_0$ is start temperature and $\beta_i$ is a linear term coefficient determined during gyro calibration.

![Figure 1. Block diagram of the investigated SINS.](image1)

![Figure 2. Track built by SINS.](image2)

Estimates of yaw, pitch and roll as well as distance to the obstacles are shown in figures 3-5. Simple threshold filter that the values in the rate gyro signal which lie under a certain threshold are filtered out and set to zero [13] is not used.
High level of pitch and roll fluctuations (figure 4) is due to bad quality (roughness) of asphalt surface on the area where experiment was carried out.

![Figure 3. Yaw estimation.](image)

![Figure 4. Pitch and roll estimation.](image)

![Figure 5. Distance to obstacle estimation.](image)

Approximately similar results were obtained for the twice more velocity of the platform.

By the relative error on the end of moving obtained result the worse than result in [6] (where yaw correction by magnetometer signals is used), but comparable with the results obtained in [14] (where additional correction by optical system is used) and [15] after 50 meters from start point.

5. Conclusion

The results of several experiments have shown:

1) in the range of several minutes and by vibration less than 1 m/s² the root mean square error (RMSE) of angular coordinates is less than 0.9° and the relative spatial coordinates error is less than 1.1%;

2) at a distance up to 2.5 m (9) provides an absolute measurement error less than 10 mm and the RMSE less than 3.2 mm;
3) RMSE measurement of barometric altitude \(10\) by temperature correction according to [4] is less than 0.8 m.

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