On the Spontaneous CP Breaking at Finite Temperature in a Nonminimal Supersymmetric Model

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Abstract

We study the spontaneous CP breaking at finite temperature in the Higgs sector in the Minimal Supersymmetric Standard Model with a gauge singlet. We consider the contribution of the standard model particles and that of stops, charginos, neutralinos, charged and neutral Higgs boson to the one-loop effective potential. Plasma effects for all bosons are also included. Assuming CP conservation at zero temperature, so that experimental constraints coming from, e.g., the electric dipole moment of the neutron are avoided, and the electroweak phase transition to be of the first order and proceeding via bubble nucleation, we show that spontaneous CP breaking cannot occur inside the bubble mainly due to large effects coming from the Higgs sector. However, spontaneous CP breaking can be present in the region of interest for the generation of the baryon asymmetry, namely inside the bubble wall. The important presence of very tiny explicit CP violating phases is also commented.

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1. Introduction

It is well known that for generating the observed baryon asymmetry of the Universe (BAU) three basic ingredients are necessary \[1\]: baryonic violating interactions, departure from thermal equilibrium and C and CP violation. Even if these conditions can be easily fulfilled in grand unified theories (GUT’s) \[2\], severe complications are present. First of all, any non zero fermion number \((B + L)\) created at some superheavy scale is almost completely arisen by the anomalous electroweak \((B + L)\) violating processes \[3\], which are in equilibrium down to a temperature of about \(10^2\) GeV. Moreover, if \((B - L)\) violating processes are present and in equilibrium at high temperatures, then also an eventual \((B - L)\) component of the BAU vanishes before the onset of the electroweak era, unless severe constraints on the \((B - L)\) violating couplings are imposed \[4\] or ad hoc ways out are found \[5\].

All these considerations make the possibility of generating the BAU during the electroweak phase transition (EWPT) very appealing \[6\]. Here the difficulty is threefold. First of all, it is still an open question if the necessary amount of CP violation to produce enough baryon asymmetry is present in the standard model (SM) \[7\]; secondly, it is not obvious that the phase transition is enough strongly first order to make the baryon number production effective; third, one must avoid the wiping out of the baryon asymmetry by anomalous processes, which leads to an upper bound on the mass of the SM Higgs boson \[8\] already ruled out by LEP results \[9\].

The situation does not appear much more appealing when the minimal supersymmetric extension of the standard model (MSSM) \[10\] is considered. The MSSM contains two extra CP violating phases with respect to the SM. The requirement that these phases provide the necessary amount of CP violation for the generation of the BAU gives rise to additional strong constraints on the parameter space of the model \[11\]. Indeed, the electric dipole
moment of the neutron must be larger than $10^{-27}$ e-cm, while an improvement of the current experimental bound on it by one order of magnitude would constrain the lightest chargino and the lightest neutralino to be lighter than 88 and 44 GeV, respectively. As far as the spontaneous CP breaking (SCPB) in the MSSM is concerned, it can be triggered by radiative corrections \cite{12} at zero temperature. As established on general grounds by Georgi and Pais \cite{13}, it requires the existence of a pseudoscalar Higgs boson with a mass of a few GeV \cite{14}, which has been ruled out by LEP \cite{15}. Nevertheless, it has been recently realized \cite{16} that temperature effects can trigger SCPB in the MSSM at the critical temperature of the EWPT and that the right amount of the baryon asymmetry can be generated with the help of tiny CP violating phases giving rise to a neutron electron dipole moment well below its present experimental limit \cite{17}. Even if this mechanism can work in a wide region of the parameter space without any fine tuning, it requires, as a general tendency, small values of the mass of the pseudoscalar Higgs boson and large values of $\tan \beta = v_2/v_1$, where $v_1$ and $v_2$ being the vacuum expectation values (VEV’s) of the two Higgs doublets present in the model, whereas recent results on the phase transition in the MSSM \cite{18} seem to point out towards the opposite direction in order not to wash out the generated baryon asymmetry.

In the framework of supersymmetric grand unified theories, the MSSM is not the most general low energy manifestation of supersymmetric GUT’s. Indeed, it is possible that at low energy the theory contains an additional gauge singlet field, the so-called next-to-minimal supersymmetric standard model (NMSSM) \cite{19}, as predicted in many superstring models based on $E_6$ \cite{20} and $SU(5) \otimes U(1)$ \cite{21} GUT groups. Incidentally, the presence of an additional gauge singlet allows to get rid of the so called $\mu$ problem in the MSSM, namely the presence of an unknown supersymmetric mass term $\mu$ in its superpotential.

The study of the EWPT in the NMSSM has been performed in ref. \cite{22} where it has been shown that the order of the transition is determined by the trilinear soft super-
symmetric breaking terms and that it is possible to preserve the baryon asymmetry from the wiping out of sphaleron interactions for masses of the lightest scalar well above the experimental limit.

In this paper we want to address the question of the SCPB in the NMSSM at finite temperature, which is one of the key ingredients to generate the BAU at the electroweak phase transition. Indeed, in Supergravity inspired models with canonical Kahler potentials it is expected that CP violation in the parameters of the Higgs potential is not very significant \cite{23} and the presence of only explicit CP violation is not enough to produce the BAU, so that any CP violation in the Higgs sector must be spontaneous. Moreover, it has been shown by Romao \cite{24} that the potential of the NMSSM at the tree level and at zero temperature has no CP violating minimum. The point is that at the CP violating extremum there is always a mode with negative mass squared. When one-loop corrections to the tree level potential at zero temperature are added, the CP violating saddle point is turned into an absolute minimum \cite{25}, where some of the Higgs fields are necessarily very light since they correspond to the modes having negative mass squared when one-loop corrections are not present. Quantitatively, the presence of a CP violating minimum requires two neutral and one charged Higgs bosons to have masses smaller than $\sim 100$ GeV, which rules out much of the parameter space \cite{25}. The aim of this paper is to investigate whether finite temperature effects can trigger SCPB in the NMSSM in regions of the parameter space which at zero temperature correspond to non CP violating minima, \textit{i.e.} to mass eigenvalues for the lightest Higgses not dangerously light.

The paper is organized as follow. In Section 2 we discuss the SCPB in the Higgs sector of the NMSSM on very general grounds, \textit{i.e.} using the most general gauge invariant potential containing two Higgs doublets and the gauge singlet. In the same Section we also show how to renormalize the parameters present in the model giving all the details in the Appendix. This general discussion will make easier to understand why at zero temperature
CP cannot be broken spontaneously at the tree level and why, including zero temperature one-loop corrections, SCPB can occur only in a very restricted region of the parameter space. In Section 3 we introduce the corrections coming from the finite temperature effective potential. In Section 4 we present and discuss our results making use of what learned in Section 2. Finally, Section 5 contains our conclusions.

2. Spontaneous CP Violation in the NMSSM

2.1. General Analysis

The superpotential involving the superfields \( \hat{H}_1, \hat{H}_2 \) and \( \hat{N} \) in the NMSSM is

\[
W = \lambda \hat{H}_1 \hat{H}_2 \hat{N} - \frac{1}{3} k \hat{N}^3 + h_t \hat{Q} \hat{H}_2 \hat{U}^c,
\]

where the \( \hat{N}^3 \) term is present to avoid a global \( U(1) \) symmetry corresponding to \( \hat{N} \to \hat{N} e^{i\theta} \) and \( \hat{H}_1 \hat{H}_2 \to \hat{H}_1 \hat{H}_2 e^{-i\theta} \), and \( \hat{Q} \) and \( \hat{U}^c \) denote respectively the left-handed quark doublet and the (anti) right handed quark singlet of the third generation. Note that a \( Z_3 \) symmetry under which any Higgs superfield \( \hat{\phi} \) transforms as \( \hat{\phi} \to \alpha \hat{\phi} \) with \( \alpha^3 = 1 \) is still present. This is also the typical structure emerging from superstring inspired scenarios. This symmetry, when spontaneously broken in the vacuum, could create a serious cosmological problem due to the appearance of domain walls. However, it has been shown in ref. [26] that nonrenormalizable terms like \( \sim N^4/M \), \( M \) is some superheavy scale as the GUT or the Planck scale, would prevent the density of domain walls from becoming large enough to create any cosmological danger, while being too small to have any impact in the low energy phenomenology as well as in the following discussions.

The tree level potential is given by

\[
V_{\text{tree}} = V_F + V_D + V_{\text{soft}},
\]
\[ V_F = |\lambda|^2 \left[ |N|^2 \left( |H_1|^2 + |H_2|^2 \right) + |H_1 H_2|^2 \right] + |k|^2 |N|^4 \\
- \left( \lambda k H_1 H_2 N^* \right) + h.c., \]
\[ V_D = \frac{1}{8} \left( g^2 + g' \right) \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{1}{2} g^2 |H_1 H_2|^2, \]
\[ V_{soft} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_N^2 |N|^2 \\
- \left( \lambda A \lambda H_1 H_2 N + h.c. \right) - \left( \frac{1}{3} k A k N^3 + h.c. \right), \]

where

\[ H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \]

and \( g \) and \( g' \) are the gauge couplings of \( SU(2)_L \) and \( U(1)_Y \), respectively.

Redefining the global phases of \( H_2 \) and \( N \), it can be shown that all the parameters in eq. (2) can be made real, except the ratio \( r = A \lambda / A_k \). We assume this parameter to be real. Indeed, in Supergravity models with a canonical Kähler potential it turns out that \( r = 1 \) and then real at the unification scale. \( r \) can develop a phase through the renormalization group equations thanks to complex gaugino masses, but this effect is small due to the constraints on the gaugino phase coming from the electric dipole moment.

If we define

\[ \langle H_1^0 \rangle \equiv v_1 e^{i \theta_1}, \quad \langle H_2^0 \rangle \equiv v_2 e^{i \theta_2}, \quad \langle N \rangle \equiv x e^{i \theta_3}, \]

and

\[ 3 \theta_3 \equiv 2 \varphi_3, \quad \theta_1 + \theta_2 + \theta_3 \equiv 2 \varphi_1, \quad \theta_1 + \theta_2 - 2 \theta_3 \equiv 2 \varphi_1 - 2 \varphi_3, \]

we can write the most general gauge invariant (under \( SU(2)_L \otimes U(1)_Y \otimes Z_3 \)) potential in the vacuum

\[ \langle V \rangle = D_1 v_1^4 + D_2 v_2^4 + D_3 v_1^2 v_2^2 + D_4 v_1^2 x^2 + D_5 v_2^2 x^2 + D_6 x^4 \\
+ D_7 v_1 v_2 x^2 \cos (2 \varphi_1 - 2 \varphi_3) + D_8 m_1^2 v_1^2 + D_9 m_2^2 v_2^2 + D_{10} m_N^2 x^2 \\
+ D_{11} v_1 v_2 x \cos (2 \varphi_1) + D_{12} x^3 \cos (2 \varphi_3), \]

\(^4\)Note however that the presence of even very small phases for \( r \) is crucial for the generation of the BAU, see [17] and below.
where it is easy to derive the $D$ coefficients in the case of the tree level potential just comparing eqs. (2) and (6).

The potential which depends on the phases

$$V(\varphi_1, \varphi_3) = D_7 v_1 v_2 x^2 \cos(2\varphi_1 - 2\varphi_3) + D_{11} v_1 v_2 x \cos(2\varphi_1) + D_{12} x^3 \cos(2\varphi_3), \quad (7)$$

is minimized by the following values of $\varphi_1$ and $\varphi_3$ [24, 25]

$$\cos(2\varphi_1) = \frac{1}{2} \left( \frac{A_{13}A_{23}}{A_{12}^2} - \frac{A_{23}}{A_{13}} - \frac{A_{13}}{A_{23}} \right),$$

$$\cos(2\varphi_3) = \frac{1}{2} \left( \frac{A_{13}A_{12}}{A_{23}^2} - \frac{A_{12}}{A_{13}} - \frac{A_{13}}{A_{12}} \right),$$

$$\cos(2\varphi_1 - 2\varphi_3) = \frac{1}{2} \left( \frac{A_{12}A_{23}}{A_{13}^2} - \frac{A_{12}}{A_{23}} - \frac{A_{23}}{A_{12}} \right), \quad (8)$$

where

$$A_{12} \equiv D_{11} v_1 v_2 x, \quad A_{13} \equiv D_7 v_1 v_2 x^2, \quad A_{23} \equiv D_{12} x^3, \quad (9)$$

if $|A_{12}A_{13}|$, $|A_{12}A_{23}|$ and $|A_{13}A_{23}|$ form a triangle and if

$$\frac{A_{12}A_{13}}{A_{23}} > 0. \quad (10)$$

Defining now the new variable [24]

$$\Sigma \equiv v_1^2 + v_2^2, \quad \Delta \equiv v_1^2 - v_2^2, \quad (11)$$

one can show that the extremum in the full parameter space $(\Sigma, \Delta, \varphi_1, \varphi_3)$ is a minimum if the following 3x3 matrix is definite positive

$$B = \begin{pmatrix}
    2B_4 & B_6 & B_7 \\
    B_6 & 2B_5 & B_8 \\
    B_7 & B_8 & 2B_9
\end{pmatrix}, \quad (12)$$

where

$$B_4 = \frac{D_1 + D_2}{4} - \frac{1}{4} D_3 + \frac{1}{8} \frac{D_7 D_{11}}{D_{12}}. \quad (13)$$
\[ B_5 = \frac{D_1 + D_2}{4} + \frac{1}{4} D_3 - \frac{1}{8} D_7 D_{11}, \]
\[ B_6 = \frac{D_1 - D_2}{2}, \]
\[ B_7 = \frac{D_4 - D_5}{2}, \]
\[ B_8 = \frac{D_4 + D_5}{2}, \]
\[ B_9 = D_6 - \frac{1}{2} \frac{D_7 D_{12}}{D_{11}}. \]  

(13)

For the tree level case, we have the particular values for the \( B \) parameters

\[ B_4 = -\frac{1}{4} \lambda^2 + \frac{1}{8} \left( g^2 + g'^2 \right) - \frac{3}{4} \lambda^2 r, \]
\[ B_5 = \left( \frac{1}{4} + \frac{3}{4} r \right) \lambda^2, \]
\[ B_6 = B_7 = 0, \]
\[ B_8 = \lambda^2, \]
\[ B_9 = k^2 \left( 1 + \frac{1}{3r} \right). \]  

(14)

It is now easy to understand why CP cannot be broken spontaneously at the tree level. Indeed, one has to satisfy the following system of conditions

\[ \frac{A_{12} A_{13}}{A_{23}} > 0 \Rightarrow r < 0 \text{ plus } \begin{cases} B_9 > 0 \Rightarrow r < -\frac{1}{3} \text{ or } r > 0, \\ B_5 > 0 \Rightarrow r > -\frac{1}{3}, \\ 4B_5 B_9 - B_8^2 > 0 \Rightarrow (1 + 3r)^2/3r > \lambda^2/k^2 > 0, \end{cases} \]  

(15)

which does not have any solutions in the \( r \) space. The key point here is the following: once the condition given in eq. (10) is satisfied, which assures that the nonvanishing phases in eq. (8) correspond to a minimum, then the \( B \) matrix is never definite positive at the tree level. When one-loop corrections are considered, while the parameters involving the condition (10) receive small corrections because the latter are always proportional to \( \lambda \) or \( k \) which are taken to be small (see below), among the elements of \( B \), \( B_5 \) may receive very large corrections from the top-stop sector and the Higgs sector (see the discussion of Section 4). It is then clear that, to have SCPB, the correction to the tree level potential
should act predominantly on the $B_5$ term to drive it and $(4B_5B_9 - B_5^2)$ to positive values. This fact will be crucial for the following and especially for the discussion about the finite temperature corrections to the tree level potential.

2.2. How to Renormalize the Tree Level Potential

In this Subsection we want to give some details on how to calculate the corrections to the tree level coefficients of the most general potential, eq. (6). A complete full analysis will be given in the Appendix. Note also that the following discussion can be extended to any model and to any potential.

The one-loop effective potential can be written as

$$V_{\text{eff}} = V_{\text{tree}} + V_1,$$  \hspace{1cm} (16)

where $V_1(H_1^0, H_2^0, N)$ can indicate either the one-loop correction to the tree level potential at zero temperature or the one-loop correction at finite temperature. The $SU(2)_L \otimes U(1)_Y \otimes Z_3$ invariant and independent operators in the NMSSM are the dimension two operators $|H_1|^2$, $|H_2|^2$ and $|N|^2$ and the dimension three operators $2 \text{Re}(N^3)$, $2 \text{Re}(H_1 H_2 N^* N^2)$ and $2 \text{Re}(H_1 H_2 N)$. Let’s collectively indicate them with $\varepsilon_i$ and $\epsilon_i$, respectively. The effective potential $V_{\text{eff}}$ can be then written as follows

$$V_{\text{eff}} = \sum_i a_i \varepsilon_i + \sum_{i<j} a_{ij} \varepsilon_i \varepsilon_j + \sum_i b_i \epsilon_i,$$

where $a_i$, $a_{ij}$ and $b_i$ refer to dimension two, four and three operators, respectively.

Since $V_1$ receives its contributions from all the particles whose masses are field dependent and therefore is a function of such masses, one can easily calculate the coefficients of each gauge invariant operator in the following way (formulas for the dimension two operators are analogous to those for the dimension three operators)
• dimension three operators: $2 \text{Re}(N^3), 2 \text{Re}(H_1H_2N^*^2)$ and $2 \text{Re}(H_1H_2N)

$$b_i = \frac{\partial V_{eff}}{\partial \epsilon_i} \bigg|_0 = \frac{\partial V_{tree}}{\partial \epsilon_i} \bigg|_0 + \sum_a \frac{\partial V_a}{\partial m_a^2} \frac{\partial m_a^2}{\partial \epsilon_i} \bigg|_0,$$

• dimension four operators: $|H_1|^4, |H_2|^4, |N|^4, |H_1|^2 |H_2|^2, |H_1|^2 |N|^2$ and $|H_2|^2 |N|^2$

$$a_{ij} = \frac{\partial^2 V_{eff}}{\partial \epsilon_i \partial \epsilon_j} \bigg|_0 = \frac{\partial^2 V_{tree}}{\partial \epsilon_i \partial \epsilon_j} \bigg|_0 + \sum_a \left[ \frac{\partial V_a}{\partial m_a^4} \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} + \frac{\partial V_a}{\partial m_a^2} \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} \right] \bigg|_0.$$

In the above expressions the subscript 0 indicate that all the derivatives must be evaluated for vanishing fields and the sum over the index $a$ includes all the particles contributing to $V_1$. The complete details on how to calculate $\partial m_a^2 / \partial \epsilon_i, \partial m_a^2 / \partial \epsilon_i, (\partial^2 m_a^2 / \partial \epsilon_i \partial \epsilon_j)$ and $(\partial^2 m_a^2 / \partial \epsilon_i \partial \epsilon_j)$ are given in the Appendix.

Let’s now consider the zero temperature contribution to $V_1$, $V_1^{T=0}$. In the $\overline{\text{DR}}$ scheme of renormalization, it reads

$$V_1^{T=0} = \frac{1}{64\pi^2} \text{Str} \left\{ \mathcal{M}^4(\phi) \left[ \ln \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right] \right\},$$

(17)

where $\mathcal{M}^2(\phi)$, with $\phi \equiv (H_1^0, H_2^0, N)$, is the field dependent squared mass matrix, the supertrace $\text{Str}$ properly counts the degree of freedom, $Q$ is the renormalization point and the $Q^2$ dependence in eq. (17) is compensated by that of the renormalized parameters, so that the full effective potential is independent of $Q^2$ up to the next-to-leading order.

If we consider in $V_1^{T=0}$ only the largest contributions [25], namely those coming from the top and the two stops with masses

$$m_t^2 = h_t^2 \left| H_2^0 \right|^2, \quad m_{\tilde{t}_L, \tilde{t}_R}^2 = M_{\tilde{S}}^2 + h_t^2 \left| H_2^0 \right|^2,$$

where we have taken a common soft mass term $M_{\tilde{S}}$ for the left and right stop, one can show that the $B_5$ parameter receives, at the minimum, a large and positive correction proportional to $h_t^4 \ln (M_{\tilde{S}}^2 / m_t^2)$ [25]. This is the reason why, when large zero temperature
one-loop corrections to $V_{\text{tree}}$ coming from the top-stop sector are considered, one can achieve SCPB only for large values of $M_S$ [25]. This result does not contradict the Georgi-Pais theorem [13] which is based on the assumption that one-loop corrections to $V_{\text{tree}}$ are small. Indeed, here one-loop corrections are large enough to turn the CP violating saddle point into a minimum. Note also that in the limit $\lambda \to 0$, the $N$ singlet tends to decouple from the $(H_1, H_2)$ sector and a CP violating minimum is found for $(\varphi_1, \varphi_3) = (\pm \pi/4, \pm \pi/2)$ for $A_k < 0$ and $A_k > 0$, respectively. In such a case, a Peccei-Quinn like symmetry is present in the superpotential involving $H_1$ and $H_2$ and it corresponds to the flat direction and then to the massless (at the tree level) pseudoscalar boson predicted by the Georgi-Pais theorem. This is the reason why in ref. [25] CP violating minima are found only for small values of $\lambda$.

Let’s now come to the renormalization of the $D$ coefficients at finite temperature.

3. One-loop Corrections at Finite Temperature

The one-loop effective potential at finite temperature is given by [27]

$$V_{\text{eff}} = V_{\text{tree}} + V_{1}^{T=0} + V_{1}^{T \neq 0},$$

$$\left( V_{1}^{T=0} + V_{1}^{T \neq 0} \right)_{\text{fer}} = - \sum_{i} n_{i,f} \left[ \frac{m_i(\phi)^2 T^2}{48} + \frac{m_i(\phi)^4}{64 \pi^2} \left( \frac{\ln Q^2 A_f T^2 + 3}{2} \right) \right],$$

$$\left( V_{1}^{T=0} + V_{1}^{T \neq 0} \right)_{\text{bos}} = \sum_{i} n_{i,b} \left[ \frac{m_i(\phi)^2 T^2}{24} - \frac{m_i(\phi)^4}{64 \pi^2} \left( \ln \frac{Q^2}{A_b T^2} + 3 \right) \right] - \frac{T}{12 \pi} \left( m_i(\phi)^2 + \Pi_i \right)^{3/2},$$  (18)

where $A_b = 16 A_f = 16 \pi^2 (3/2 - 2 \gamma_E)$, $\gamma_E$ being the Euler constant, and $n_{i,f(b)}$ counts the effective fermionic (bosonic) degrees of freedom. Note that in the bosonic part $\Pi_i$ denotes the thermal polarization mass for bosons contributing to the Debye mass [28]. It arises when one resums at least the leading infrared-dominated higher-loop contributions to $V_{1}^{T \neq 0}$, associated to the so called daisy diagrams [29] whose inclusions amounts to a resummation to all orders in $\alpha \sim (g^2/2\pi)(T^2/m^2)$, where $g$ and $m$ denote the generic
gauge coupling and mass respectively, and to neglect subleading contributions controlled by the parameter $\beta \sim (g^2/2\pi)(T/m)$.

Note also that in eq. (18) we have made use of the high temperature expansion for $V_{\text{eff}}$ which turns to be a very good approximation for $m_{i(f)(\phi)} \lesssim 1.6 (2.2) T$ [30].

Accordingly to what explained in the previous Subsection, the corrections to the coefficients $a_i$ and $a_{ij}$ result as follow

- dimension three operators: $2 \text{ Re}(N^3), 2 \text{ Re}(H_1H_2N^2)$ and $2 \text{ Re}(H_1H_2N)$

$$b_i = \left. \frac{\partial V_{\text{eff}}}{\partial \epsilon_i} \right|_0 = \left. \frac{\partial V_{\text{tree}}}{\partial \epsilon_i} \right|_0 + \sum_{a,\text{fer}} n_{a,f} \frac{\partial m_a^2}{\partial \epsilon_i} \left[ \frac{T^2}{48} + \frac{m_a^2}{32\pi^2} \left( \ln \frac{Q^2}{A_f T^2} + \frac{3}{2} \right) \right]_0$$

$$a_{ij} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial \epsilon_i \partial \epsilon_j} \right|_0 = \left. \frac{\partial^2 V_{\text{tree}}}{\partial \epsilon_i \partial \epsilon_j} \right|_0 + \sum_{a,\text{fer}} n_{a,f} \left\{ \frac{1}{32\pi^2} \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} \left( \ln \frac{Q^2}{A_f T^2} + \frac{3}{2} \right) \right\}_0$$

$$+ \frac{\partial^2 m_a^2}{\partial \epsilon_i \partial \epsilon_j} \left[ \frac{T^2}{48} + \frac{m_a^2}{32\pi^2} \left( \ln \frac{Q^2}{A_f T^2} + \frac{3}{2} \right) \right]_0$$

$$+ \sum_{a,\text{bos}} n_{a,b} \left\{ \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} \left[ -\frac{1}{32\pi^2} \left( \ln \frac{Q^2}{A_b T^2} + \frac{3}{2} \right) - \frac{T}{8\pi} \left( m_a^2 + \Pi_a \right)^{1/2} \right] \right\}_0$$

$$+ \frac{\partial^2 m_a^2}{\partial \epsilon_i \partial \epsilon_j} \left[ \frac{T^2}{24} \frac{m_a^2}{32\pi^2} \left( \ln \frac{Q^2}{A_b T^2} + \frac{3}{2} \right) - \frac{T}{8\pi} \left( m_a^2 + \Pi_a \right)^{1/2} \right]_0.$$  

(19)

- dimension four operators: $|H_1|^4, |H_2|^4, |N|^4, |H_1|^2 |H_2|^2, |H_1|^2 |N|^2$ and $|H_2|^2 |N|^2$

$$a_{ij} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial \epsilon_i \partial \epsilon_j} \right|_0 = \left. \frac{\partial^2 V_{\text{tree}}}{\partial \epsilon_i \partial \epsilon_j} \right|_0 + \sum_{a,\text{fer}} n_{a,f} \left\{ \frac{1}{32\pi^2} \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} \left( \ln \frac{Q^2}{A_f T^2} + \frac{3}{2} \right) \right\}_0$$

$$+ \frac{\partial^2 m_a^2}{\partial \epsilon_i \partial \epsilon_j} \left[ \frac{T^2}{48} + \frac{m_a^2}{32\pi^2} \left( \ln \frac{Q^2}{A_f T^2} + \frac{3}{2} \right) \right]_0$$

$$+ \sum_{a,\text{bos}} n_{a,b} \left\{ \frac{\partial m_a^2}{\partial \epsilon_i} \frac{\partial m_a^2}{\partial \epsilon_j} \left[ -\frac{1}{32\pi^2} \left( \ln \frac{Q^2}{A_b T^2} + \frac{3}{2} \right) - \frac{T}{8\pi} \left( m_a^2 + \Pi_a \right)^{1/2} \right] \right\}_0$$

$$+ \frac{\partial^2 m_a^2}{\partial \epsilon_i \partial \epsilon_j} \left[ \frac{T^2}{24} \frac{m_a^2}{32\pi^2} \left( \ln \frac{Q^2}{A_b T^2} + \frac{3}{2} \right) - \frac{T}{8\pi} \left( m_a^2 + \Pi_a \right)^{1/2} \right]_0.$$  

(20)

Working in the 't Hooft-Landau gauge and in the $\overline{DR}$ scheme, in the bosonic part we have summed over gauge bosons, stops, neutral and charged Higgs scalars, whereas in the fermionic part we have summed over top, neutralinos and charginos. Again we refer the reader to the Appendix for the complete analysis of all the technical points.

The coefficients $b_i$ and $a_{ij}$ given in the above eqs. (19) and (20) are the ones which
determine the $D$ coefficients in the eq. (6) and then determine whether CP may be spontaneously broken at finite temperature.

In the next Section we shall focus on the results about the SCPB obtained when finite temperature corrections are taken into account showing that the picture can be considerably different from the case in which only $T = 0$ corrections are considered. In particular, we shall address the question whether it is possible to have SCPB at finite temperature in regions of the parameter space which correspond to CP conserving minima at $T = 0$, thus avoiding any limits coming from having very light Higgs bosons or electric dipole moment of the electron and neutron close to their experimental bounds.

4. Results and Discussion

As we have stressed in the Subsection 2.1, to have SCPB the corrections to the tree level potential should act predominantly on the $B_5$ term, see eq. (15). As a matter of fact, since $B_5 < 0$ (when $r < -1/3$) at the tree level, the corrections to $V_{\text{eff}}$ should be large enough to trigger positive values for $B_5$. This is what happens in the case of $T = 0$ top-stop large one-loop corrections, as pointed out in the Subsection 2.2.

In the case of finite temperature corrections to $V_{\text{tree}}$, we have proceeded as follows. First of all, we have fixed the set of the parameters given by $(\lambda, k, M_{\tilde{u}}, m_1^2, m_2^2, \tan \beta, A_t)$, where $M_{\tilde{u}}$ is the soft breaking mass for the stop right and $A_t$ is the trilinear soft breaking mass relative to the $h_t \hat{Q} \hat{H}_2 \hat{U}^c$ term in the superpotential. Imposing the minimization conditions at $T = 0$

$$\frac{\partial V_{\text{tree}}}{\partial \phi_1} + \frac{\partial V_1^{T=0}}{\partial \phi_1} = 0, \ (\phi_1 = v_1, \phi_2 = v_2, \phi_3 = x), \ (21)$$

allows one to express the parameters $(A_k, A_{\lambda}, m_2^2)$ as functions of the already fixed parameters and of two free parameters which we have chosen to be $x$ and $M_{\tilde{q}}$, the latter being the soft breaking mass of the stop left. In the minimization conditions we have included only the relevant contributions to $V_1^{T=0}$, namely those coming from the top-stop sector.
The corresponding bottom contributions are negligible for \( \tan \beta \lesssim 20 \) and the gauge sector does not play any important role. This also holds for the extended Higgs sector since the Yukawa couplings \( \lambda \) and \( k \) are taken to be small as suggested by the requirement that they remain in the perturbative regime up to a large scale (say \( 10^{16} \) GeV) \([19]\) or to forbid the breaking of electromagnetism \([31]\). Note, however, that even if the Higgs sector does not play any role in the \( T = 0 \) corrections, it will be fundamental in the finite temperature one-loop contributions as we shall explain later.

In Figs. 1a) and 1b) are displayed the plots of the experimentally allowed regions in the \((M_\tilde{q}, x)\) plane for fixed values of the other parameters\(^5\).

The first constraint is that the lightest Higgs CP even particle \( h^0 \) has not been produced in the decay \( Z^0 \to Z^0 + h^0 \). This gives the conservative bound \( m_{h^0} \gtrsim 60 \text{ GeV} \) \([9]\), dashed line (the exact bound depends on the coupling of \( h^0 \) to \( Z^0, R_{Z^0 Z^0 h^0} \), and then it is weaker, \( m_{h^0} \gtrsim R_{Z^0 Z^0 h^0}^2 60 \text{ GeV} \)). In addition, the lightest pseudoscalar \( A^0 \) should be heavier than \( \sim 20 \) GeV \([15]\) and the dashed-dot line corresponds to \( m_{A^0} = 40 \text{ GeV} \). Note that the allowed regions correspond to \( T = 0 \) CP conserving minima.

When finite temperature corrections are added to \( V_{\text{tree}} \), one can define the critical temperature \( T_c \) as the value of \( T \) at which the origin of the field space becomes a saddle point for the effective potential,

\[
\text{Det} \left[ M^2_{S,T\neq 0}(T_c) \right]_{\phi=0} = 0,
\]

where the effective mass matrix is given by the second derivatives of the full one-loop finite temperature potential with respect to the scalar fields. In the origin of the field space and

\(^5\)Very recently Ellwanger, R. de Traubenberg and Savoy \([19]\) have scanned the complete parameter space searching for the allowed region compatible with different constraints. In particular, they have found that only very large values of \( x \) are permitted, \( x \gtrsim 800 \text{ GeV} \). On the other hand this result is based on different assumptions, \( e.g. \) universality in the soft supersymmetry breaking at the GUT scale. As a consequence, we believe that it is safe to consider also smaller values for \( x \).
in the basis \((\text{Re}H_0^0/\sqrt{2}, \text{Re}H_2^0/\sqrt{2}, \text{Re}N/\sqrt{2})\), it reads

\[
\mathcal{M}_S^{2,T \neq 0}(T_c) = \text{Diag}(m_1^2, m_2^2, m_N^2),
\]

\[
m_1^2 = m_1^2 + \frac{1}{8} \left( 3g^2 + g'^2 + \frac{4}{3}\lambda^2 \right) T^2,
\]

\[
m_2^2 = m_2^2 + \frac{1}{8} \left( 3g^2 + g'^2 + 6h^2_t + \frac{4}{3}\lambda^2 \right) T^2,
\]

\[
m_N^2 = m_N^2 + \frac{1}{3} \left( \lambda^2 + k^2 \right) T^2.
\]

\(\text{(23)}\)

The critical temperature is then given by the highest temperature for which one of the \(m_i^2\) \((i = 1, 2, N)\) vanishes. Obviously, the effective potential becomes flat at the origin only along directions corresponding to negative soft masses. Due to the heavy top \([32]\) \(m_2^2\) can run to negative values at low energy, whereas, for small \(\lambda\) and \(k\), both \(m_1^2\) and \(m_N^2\) remain positive. As a consequence, the EWPT is expected to occur first along the \(H_0^2\) direction and immediately afterwards nonvanishing VEV’s for \(H_0^1\) and \(N\) are driven by the \(A_\lambda\) term \([22]\).

In Figs. 1a) and 1b) we have fixed the temperature at \(T_c = 150\) GeV and shown the curve corresponding to \(m_2^2(T_c) = 0\), solid line. The points in the \((M_\tilde{q}, x)\) plane lying on the curve are then the points for which \(T_c = 150\) GeV. Indeed, since the EWPT is known to be first order, it occurs when \(m_2^2\) is still positive, \(i.e.\) at a temperature \(T^*\) higher than \(T_c\): since all the points in the \((M_\tilde{q}, x)\) plane below the solid line correspond to \(m_2^2 > 0\), then they correspond to the region of the parameter space where the EWPT occurs at temperatures smaller or equal to 150 GeV.

We want to point out that the contribution from the Higgs sector, where plasma effects are crucial to make the effective potential real at the origin, does have an infrared singularity proportional to \((T/m_2^2)\), responsible for the failure of perturbative expansion \([27]\) for values of \((M_\tilde{q}, x)\) such that \(m_2^2 = 0\). The curve \(m_2^2 = 0\) is then representing an upper bound contour, for fixed \(T\), in the \((M_\tilde{q}, x)\) plane which severely constrains the region of the parameters. We have also checked that in the experimentally allowed regions below the
curve $m_2^2 = 0$ and for the choice of $(M_{\tilde{q}}, x)$ adopted for figures 2a) and 2b) (see below), the theory remains perturbative in the sense that the perturbative expansion $\beta \sim g^2(T/2\pi m_2)$ remains smaller than 1.

Once the temperature has fallen down to the tunneling temperature $T^*$, the EWPT proceeds via bubble nucleation. In the centre of the bubbles, to avoid the wiping out of the baryon asymmetry by anomalous interactions, we have to impose that $\tilde{v}(T^*) = \sqrt{v_1^2(T^*) + v_2^2(T^*)} \gtrsim T^*$, which is easily satisfied in the model under consideration [22]. Even if the SCPB in the bubble is not interesting as far as the generation of the baryon asymmetry is concerned, it is however interesting per se’ asking whether the system can tunnel first to a CP violating minimum and then, as the temperature decreases, reach the CP conserving minimum which represents our initial condition at $T = 0$.

4.1. SCPB inside the Bubble

Scanning several sets of points in the allowed region in the $(M_{\tilde{q}}, x)$ plane, we have numerically minimized the effective potential and looked for portions in the plane where conditions (15) could be satisfied. This research has turned out to be fruitless in the sense that very small and, in practice, negligible, allowed regions in the $(M_{\tilde{q}}, x)$ plane have shown up. Even if this result might be quite surprising at a first sight, it can be explained as the effect of different phenomena happening at finite temperature.

We know that one-loop corrections to $V_{eff}$ should trigger positive values for $B_5$ in order to have SCPB. First of all, we have seen that at zero temperature the top-stop sector gives a large positive contribution to $B_5$ proportional to $h_t^4 \ln(M_S^2/m_t^2)$ for large $M_S$. At finite temperature, the corresponding contribution is reduced to $h_t^4 \ln(A_b/A_f)$.

Secondly, the Higgs sector plays a crucial role in determining the sign of $B_5$. Us-

\footnote{In the usual scenarios for the generation of BAU at the EWPT, the only source of baryon violations lies in sphaleron interactions whose rate is imposed to be small enough in the propagating bubble.}
ing eqs. (6), (13), the formalism developed in the Appendix and taking into account that near the critical temperature $m_2 \ll m_1, m_N$, one can realizes that the corrections to $B_5$ are dominated by negative terms such as $-(T/m_2)(\partial m_2/\partial |H_1^0|^2)^2$, with $i = 1, 2$ and by $-(T/m_2)(\partial m_2/\partial |H_1^0|^2)(\partial m_2/\partial |H_2^0|^2)$, so that the largest Higgs contribution to $B_5$, $\Delta B_{5, \text{max}}$, is

$$\Delta B_{5, \text{max}} = -\frac{1}{8\pi} \frac{T}{m_2} \frac{1}{16} \left( \frac{25}{4} g^4 + \frac{30}{4} g'^4 - \frac{19}{2} g^2 g'^2 \right) < 0. \quad (24)$$

Thus, $B_5$ receives a large and negative contribution from the one-loop corrections at finite temperature from the Higgs scalar sector. Since this contribution turns out to be, in absolute values, the largest among the different particles, this prevents $B_5$ to become positive and, consequently, the matrix $B$ to be definite positive. This is the main reason why SCPB cannot occur in the centre of the propagating bubble.

### 4.2. SCPB inside the Bubble Wall

Fortunately, the situation changes when one moves away from the centre of the bubble towards the bubble wall. As a matter of fact, since the EWPT is of the first order, the temperature keeps constant until the Universe is in broken phase, whereas $v_1, v_2$ and $x$ change their values from zero to $v_1(T^\ast), v_2(T^\ast)$ and $x(T^\ast)$, respectively, when a bubble wall passes through a fixed point in space. Incidentally, we define the bubble wall as the region in which sphalerons are active, that is in which $0 \lesssim \tilde{v}(T^\ast) \lesssim T^\ast$ and we remind the reader that, since here the BAU can be created through, e.g., the reflection baryogenesis mechanism [3], it is just in this region that one needs SCPB.

To describe what is happening inside the bubble wall one should solve a system of differential equations involving $v_1, v_2, x, \varphi_1$ and $\varphi_3$ at $T^\ast$. For instance, the equation for
\( \varphi_3 \), should read
\[
\frac{d}{dz} \left[ x^2(z, T^*) \frac{d}{dz} \varphi_3(z, T^*) \right] + \frac{4}{9} \frac{\partial V_{\text{eff}}}{\partial \varphi_3(z, T^*)} = 0,
\]
(25)

where we are approximating the bubble wall to an infinite plane propagating perpendicularly to the \( z \) axis with a width \( L_w \). Nevertheless, one can make the analysis much simpler envisaging the following situation. In the unbroken phase and up to the edge of the advancing bubble wall, at \( z = z_w \), \( v_1 \) and \( v_2 \) are constantly equal to zero and then there is no SCPB; in the broken phase and up to the other edge of the bubble wall, at \( z = z_w + L_w \), \( v_1 \) and \( v_2 \) are nearly constantly equal to their minimum values at the centre of the bubble and, as shown before, again no SCPB is occurring; finally from inside to outside of the bubble wall, the VEV’s are decreasing from their values inside the bubble to zero.

The key point now is that in the bubble wall, where equations like eq. (25) are valid, the VEV’s \( v_1(z, T^*) \), \( v_2(z, T^*) \) and \( x(z, T^*) \) do not have to satisfy any longer the severe constraint \( B_5 > 0 \) coming from the requirement of a mass matrix with positive determinant, but one can simply impose that at the two edges of the bubble wall, where, e.g., eq. (25) reduces to \( \partial V_{\text{eff}}/\partial \varphi_3(z, T^*) = 0 \), \( \cos 2 \varphi_3(z_w + L_w, T^*) > 1 \) and \( \cos 2 \varphi_3(z_w, T^*) < -1 \). So doing, even if the solution of eq. (25) is not known, one is assured that maximal SCPB is occurring inside the bubble wall, that is an observer in a fixed point in space would experience a change \( \Delta \varphi_3 = \pi/2 \). Imposing \( \cos 2 \varphi_3(z_w + L_w, T^*) > 1 \) and \( \cos 2 \varphi_3(z_w, T^*) < -1 \) at the edges of the bubble wall is equivalent to impose
\[
0 < \tilde{v}_- < \tilde{v}(T^*) < \tilde{v}_+ < \tilde{v}(z_w + L_w, T^*),
\]
(26)

where \( \tilde{v}_\pm \) are found imposing \( \cos 2 \varphi_3(T^*) = \pm 1 \), respectively, and are given by
\[
\tilde{v}_\pm = \frac{\sqrt{2}}{|\sin 2\beta|} \left[ \pm \frac{2}{D_{12}} \frac{D_{12}}{D_7 D_{11}} x^3(z, T^*) + \frac{D_{12}^2}{D_7^2} x^2(z, T^*) + \frac{D_{12}^2}{D_{11}^2} x^4(z, T^*) \right]^{1/4}.
\]
(27)

At \( \tilde{v}(z, T^*) = \tilde{v}_\pm \) eq. (25) is then equivalent to \( \partial V_{\text{eff}}/\partial \varphi_3(z, T^*) = 0 \) and also eq. (10) must be satisfied. Naturally, similar relations are valid for \( \tilde{v}^2(z, T^*) \) and \( x(z, T^*) \) once one imposes similar conditions on the angles \( 2 \varphi_1 \) and \( 2(\varphi_1 - \varphi_3) \).
In Figs. 2a) (2b)) we present the region in the \( (\tilde{v}(z,T^*), x(z,T^*)) \) plane corresponding to the conditions (26-27) for the choice \( x = 600 \) (800) GeV, \( m_N = 10, 300 \) GeV and \( M_{\tilde{q}} = 200 \) (300) GeV. The points inside the regions I and II correspond to values of \( \tilde{v}(z,T^*) \) and \( x(z,T^*) \) for which a maximal SCPB is occuring in the bubble wall, \( i.e \), at least one of the angles \( 2\varphi_3, 2\varphi_1 \) and \( 2(\varphi_1 - \varphi_3) \) change by an amount \( \pi \) inside the bubble wall.

As pointed out in ref. [17], the only presence of SCPB inside the bubble wall is not sufficient to create a net BAU. Indeed, if CP is spontaneously broken, any phase, let’s call it \( \delta \), can take two opposite values, corresponding to two exactly degenerate vacua, since the effective potential depends only on \( \cos \delta \). This degeneracy between the two vacua \( \pm \delta \) would induce an equal number of nucleated bubbles carrying phases with opposite signs, which in turn generate baryon asymmetries of opposite signs and an overall BAU equal to zero when averaging over the entire volume of the Universe. However, the presence of a very small and explicit phase in the effective potential, for instance in \( r \), of order of \( 10^{-6} - 10^{-5} \), is sufficient to lift the degeneracy leading to a difference between nucleation rates of the two kind of bubbles and to a baryon asymmetry of the right order of magnitude [17]. This is due to the fact that, being the nucleation rate proportional to \( \exp(-\Delta F/T) \), where \( \Delta F \) is the difference in free energy between the two vacua with phases \( \pm \delta \), it is very sensible to even small changes in \( \Delta F \). The smallness of the explicit phase does not change quantitatively our conclusions on SCPB and give rise to negligible contributions to the neutron electric dipole moment.

5. Conclusions

In the present paper we have investigated the possibility of SCPB at finite temperature
in the MSSM. After having performed a systematical analysis of the renormalization of the operators present in the potential, we have shown that SCPB at finite temperature cannot occur inside the propagating bubble walls which appear after the EWPT. This is due to the fact that the Higgs contribution turns out to be crucial and pushing towards the wrong direction.

SCPB can occur inside the bubble wall, which is the interesting region as far as the generation of the BAU is concerned, without any fine-tuning and in regions of the parameter space which correspond at $T = 0$ to no SCPB, i.e. to regions where no constraints are present coming from experimental bounds on the electron and/or neutron electric dipole moment and from searching for the lightest Higgs bosons at LEP.

Very small explicit phases are necessary to generate a nonvanishing BAU and in any case they give rise to an amount of CP violation which is well below the current experimental limits.

2.1. Acknowledgements

It is a pleasure to thank R. Hempfling for useful and enlightening discussions.

Appendix.

In this Appendix we want to give a complete description of all the techniques necessary to calculate the corrections to the coefficients $a_i$ and $a_{ij}$. Note that this description is quite general and does not depend at all on the particular model we are working with.

As explained in the text, to calculate the renormalized $a_i$ and $a_{ij}$ coefficients, we need to calculate $\partial m_a^2 / \partial \varepsilon_i$ and $(\partial^2 m_a^2 / \partial \varepsilon_i \partial \varepsilon_j)$, $m_a^2$ being the field dependent mass eigenvalues of the particles contributing to the effective potential. Since $m_a^2$ are by definition the squared
mass eigenvalues of an $N \times N$ hermitian mass matrix $M^2_N$, they obey the following equation

$$
\mathcal{U}[m^2_a(\phi), c_n(\phi)] \equiv \text{Det} [M^2_N(\phi) - m^2_a(\phi)] = \sum_{n=0}^{N} c_n(\phi)m^{2n}_a(\phi) = 0. \quad (A. 1)
$$

Let’s analyze first the case for a non degenerate mass matrix $M^2_N$. Taking the derivative of both sides of eq. (A.1) with respect to the gauge invariant and independent operator $\varepsilon_i$, one can easily finds that (analogous expressions are valid for the dimension three operators $\varepsilon_i$)

$$
\frac{\partial m^2_a}{\partial \varepsilon_i} = -\frac{\sum_{n=0}^{N} m^{2n}_a \frac{\partial c_n}{\partial \varepsilon_i}}{\sum_{n=0}^{N} n c_n m^{2(n-1)}_a}, \quad (A. 2)
$$

while, taking the second derivatives with respect to the operators $\varepsilon_i$ and $\varepsilon_j$, one finds

$$
\frac{\partial^2 m^2_a}{\partial \varepsilon_i \partial \varepsilon_j} = -\frac{1}{\sum_{n=0}^{N} n c_n m^{2(n-1)}_a} \sum_{n=0}^{N} \left[ n(n-1)c_n \frac{\partial m^2_a}{\partial \varepsilon_i} \frac{\partial m^2_a}{\partial \varepsilon_j} m^{2(n-2)}_a \right]
+ \sum_{n=0}^{N} m^2_a \frac{\partial^2 c_n}{\partial \varepsilon_i \partial \varepsilon_j} + n m^2_a (n-1) \left( \frac{\partial c_n}{\partial \varepsilon_i} \frac{\partial m^2_a}{\partial \varepsilon_j} + \frac{\partial c_n}{\partial \varepsilon_j} \frac{\partial m^2_a}{\partial \varepsilon_i} \right). \quad (A. 3)
$$

In the case of degenerate squared mass matrices, e.g. the 6x6 squared mass matrix for the neutral Higgs bosons (see below), one cannot use the equation

$$
\frac{\partial \mathcal{U}}{\partial \varepsilon_i} = \frac{\partial \mathcal{U}}{\partial m^2_a} \frac{\partial m^2_a}{\partial \varepsilon_i} + \sum_{n=0}^{N} m^{2n}_a \frac{\partial c_n}{\partial \varepsilon_i} = 0 \quad (A. 4)
$$

to invert in favour of $(\partial m^2_a/\partial \varepsilon_i)$ since now $(\partial \mathcal{U}/\partial m^2_a)$ is vanishing. Nevertheless, for $m^2_a$ eigenvalues with degree of degeneracy equal to two, one can take the derivative of eq. (A.4) with respect to $m^2_a$ and $\partial^2 m^2_a/\partial \varepsilon_i\varepsilon_j$ then obtains

$$
\frac{\partial m^2_a}{\partial \varepsilon_i} = -\frac{\sum_{n=0}^{N} n m^{2(n-1)}_a \frac{\partial c_n}{\partial \varepsilon_i}}{\sum_{n=0}^{N} n(n-1)c_n m^{2(n-2)}_a}, \quad (A. 5)
$$

and

$$
\frac{\partial^2 m^2_a}{\partial \varepsilon_i \partial \varepsilon_j} = -\frac{1}{\sum_{n=0}^{N} n(n-1)c_n m^{2(n-2)}_a} \sum_{n=0}^{N} \left[ n(n-1)(n-2)c_n \frac{\partial m^2_a}{\partial \varepsilon_i} \frac{\partial m^2_a}{\partial \varepsilon_j} m^{2(n-3)}_a \right]
+ \sum_{n=0}^{N} n m^2_a (n-1) \left( \frac{\partial c_n}{\partial \varepsilon_i} \frac{\partial m^2_a}{\partial \varepsilon_j} + \frac{\partial c_n}{\partial \varepsilon_j} \frac{\partial m^2_a}{\partial \varepsilon_i} \right). \quad (A. 6)
$$
What one really needs to calculate the corrections to the coefficients $a_i$ and $a_{ij}$, which in their turn determine the possibility of having SCPB at finite temperature, are then the coefficients for each squared mass matrix whose eigenstates contribute to the effective potential. In the following we give all the details about the properties of the particles which must be included in $V_{eff}$ and from which the coefficients $c_n(\phi)$ can be extracted.

- Top: $n_t = -12$, $m_t^2 = h_t^2 |H_2^0|^2$.
- Stop: $n_{\tilde{t}_1} = n_{\tilde{t}_2} = 6$; the squared mass matrix is

$$M_t^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix}, \quad (A. 7)$$

where

$$m_{LL}^2 = M_{\tilde{q}}^2 + \Pi_{\tilde{q}} + h_t^2 |H_2^0|^2 + \left( \frac{g^2}{12} - \frac{g'^2}{4} \right) \left( |H_2^0|^2 - |H_1^0|^2 \right),$$

$$m_{RR}^2 = M_{\tilde{\tilde{u}}}^2 + \Pi_{\tilde{\tilde{u}}} + h_t^2 |H_2^0|^2 - \frac{g'^2}{3} \left( |H_2^0|^2 - |H_1^0|^2 \right),$$

$$M_{LR}^2 = h_t \left( A_t H_2^0 + \lambda N^* H_1^* \right), \quad (A. 8)$$

and $[18]$

$$\Pi_{\tilde{q}} = \Pi_{\tilde{\tilde{u}}} \simeq \frac{4}{9} g_s^2 T^2 \quad (A. 9)$$

are the thermal polarization squared masses for the squarks calculated in the limit of heavy gluinos.

- Gauge bosons: $n_{W^\pm} = 6$, $n_{W^\pm_L} = 2$, $n_{Z^0} = 3$, $n_{Z^0_L} = n_{\gamma_L} = 1$,

$$m_{W^\pm}^2 = \frac{g^2}{2} \left( |H_1^0|^2 + |H_2^0|^2 \right), \quad (A. 10)$$

$$m_{Z^0}^2 = \frac{g^2 + g'^2}{2} \left( |H_1^0|^2 + |H_2^0|^2 \right),$$

$$m_{W^\pm_L}^2 = m_{W^\pm} + \Pi_{W^\pm_L},$$

$$m_{Z^0_L,\gamma_L}^2 = \frac{1}{2} \left[ m_{Z^0}^2 + \Pi_{W^\pm_L} + \Pi_{B_L} \right. \pm \sqrt{ \frac{1}{4} \left[ \frac{g^2 - g'^2}{2} \left( |H_1^0|^2 + |H_2^0|^2 \right) + \Pi_{W^\pm_L} - \Pi_{B_L} \right]^2 + \frac{g g'}{2} \left( |H_1^0|^2 + |H_2^0|^2 \right)^2 } \bigg],$$

21
where \[ \Pi_{W_L}^2 = \frac{5}{2} g^2 T^2, \quad \Pi_{B_L} = \frac{47}{10} g' T^2. \] (A. 11)

Note that only the longitudinal components of the gauge bosons do have a Debye mass proportional to \( T^2 \).

- **Charginos:** \( n_{\chi^\pm} = 2 \); in the basis \((-i\lambda^+,\psi_{H^0_2},-i\lambda^-,\psi_{H^0_1}^\dagger)\) the mass matrix is

\[
\mathcal{M}_{\chi^\pm} = \begin{pmatrix}
0 & 0 & M_2 & g H^0_1 \\
0 & 0 & g H^0_2 & \lambda N \\
M_2 & g H^0_2 & 0 & 0 \\
g H^0_1 & \lambda N & 0 & 0
\end{pmatrix}, \tag{A. 12}
\]

where \( M_2 \) is the gaugino soft mass for the gauge group \( SU(2)_L \).

- **Neutralinos:** \( n_{\tilde{\chi}^0} = 2 \); in the basis \((\tilde{W}^3, \tilde{B}^0, \tilde{H}^0_1, \tilde{H}^0_2, \tilde{N})\) the mass matrix is

\[
\begin{array}{cccc}
M_2 & 0 - \frac{g}{2} H^0_1 & \frac{g}{2} H^0_2 & 0 \\
0 & M_1 & \frac{g}{2} H^0_2 & -\frac{g^2}{2} H^0_2 \\
-\frac{g}{2} H^0_1 & \frac{g}{2} H^0_2 & 0 & \lambda N \\
\frac{g}{2} H^0_2 & -\frac{g}{2} H^0_2 & \lambda N & 0 \\
0 & 0 & \lambda N & \lambda H^0_1 \end{array}
\]

where \( M_1 \) is the gaugino soft mass for the gauge group \( U(1)_Y \).

- **Charged Higgs scalars:** \( n_{H^\pm} = 2 \); in the basis \((H^-_1, H^+_2)\) the squared mass matrix reads

\[
\begin{align*}
\mathcal{M}_{H^\pm,11}^2 &= m_1^2 + \lambda^2 |N|^2 + \frac{1}{4} (g^2 + g'^2) \left( |H^0_1|^2 + |H^0_2|^2 \right), \\
\mathcal{M}_{H^\pm,22}^2 &= m_2^2 + \lambda^2 |N|^2 - \frac{1}{4} (g^2 + g'^2) \left( |H^0_1|^2 + |H^0_2|^2 \right) + \frac{g^2}{2} |H^0_1|^2, \\
\mathcal{M}_{H^\pm,12}^2 &= \mathcal{M}_{H^\pm,21}^2 = \lambda A_{\alpha N} + \lambda k |N|^2 + \left( \frac{1}{2} g^2 - \lambda^2 \right) H^0_1 H^0_2. \tag{A. 14}
\end{align*}
\]

- **Neutral Higgs scalars:** \( n_{H^0} = 1 \); in the basis \((H^0_1, H^0_1, H^0_2, H^0_2, N, N^*)\) the squared mass matrix is given by

\[
\begin{align*}
\mathcal{M}_{H^0,11}^2 &= m_1^2 + \lambda^2 |N|^2 + \lambda^2 |H^0_2|^2 + \frac{1}{2} (g^2 + g'^2) |H^0_1|^2 - \frac{1}{4} (g^2 + g'^2) |H^0_2|^2, \\
\mathcal{M}_{H^0,12}^2 &= \mathcal{M}_{H^0,21}^2 = \mathcal{M}_{H^0,34}^2 = \frac{1}{4} (g^2 + g'^2) H^0_1 H^0_2, \\
\end{align*}
\]
\[
\mathcal{M}_{H^0,13} = \mathcal{M}_{H^0,31} = \mathcal{M}_{H^0,24} = \left[ \lambda^2 - \frac{1}{4} \left( g^2 + g'^2 \right) \right] H_1^0 H_2^0, \\
\mathcal{M}_{H^0,14} = \mathcal{M}_{H^0,41} = \mathcal{M}_{H^0,23} = \left[ \lambda^2 - \frac{1}{4} \left( g^2 + g'^2 \right) \right] H_1^0 H_2^0 - \lambda k N^2 - \lambda A \lambda N^*, \\
\mathcal{M}_{H^0,15} = \mathcal{M}_{H^0,51} = \mathcal{M}_{H^0,26} = \lambda^2 H_1^0 N^* - \lambda k H_2^0 N, \\
\mathcal{M}_{H^0,16} = \mathcal{M}_{H^0,61} = \mathcal{M}_{H^0,25} = \lambda^2 N H_1^0 - \lambda A \lambda H_2^0, \\
\mathcal{M}_{H^0,22} = \frac{m_1^2}{\mathcal{M}_{H^0,40}} + \lambda^2 |N|^2 + \lambda^2 |H_2^0|^2 + \frac{1}{2} \left( g^2 + g'^2 \right) |H_1^0|^2, - \lambda k \left( g^2 + g'^2 \right) |H_2^0|^2, \\
\mathcal{M}_{H^0,33} = \frac{m_2^2}{\mathcal{M}_{H^0,40}} + \lambda^2 |N|^2 + \lambda^2 |H_1^0|^2 + \frac{1}{2} \left( g^2 + g'^2 \right) |H_2^0|^2 - \lambda k \left( g^2 + g'^2 \right) |H_1^0|^2, \\
\mathcal{M}_{H^0,35} = \frac{m_2^2}{\mathcal{M}_{H^0,53}} = \frac{m_2^2}{\mathcal{M}_{H^0,46}} = \lambda^2 H_2^0 N^* - \lambda k H_1^0 N, \\
\mathcal{M}_{H^0,36} = \frac{m_2^2}{\mathcal{M}_{H^0,63}} = \lambda^2 H_2^0 - \lambda A \lambda H_1^0, \\
\mathcal{M}_{H^0,44} = \frac{m_2^2}{\mathcal{M}_{H^0,44}} + \lambda^2 |N|^2 + \lambda^2 |H_1^0|^2 + \frac{1}{2} \left( g^2 + g'^2 \right) |H_2^0|^2 - \lambda k \left( g^2 + g'^2 \right) |H_1^0|^2, \\
\mathcal{M}_{H^0,55} = \frac{m_2^2}{\mathcal{M}_{H^0,55}} + \lambda^2 |H_1^0|^2 + \lambda^2 |H_2^0|^2 + 2k^2 |N|^2, \\
\mathcal{M}_{H^0,56} = \frac{m_2^2}{\mathcal{M}_{H^0,65}} = 2k^2 N^2 + 2\lambda k H_1^0 H_2^0 - 3k A_0 N^*, \\
\mathcal{M}_{H^0,66} = \frac{m_2^2}{\mathcal{M}_{H^0,66}} + \lambda^2 |H_1^0|^2 + \lambda^2 |H_2^0|^2 + 4k^2 |N|^2. \tag{A. 15}
\]
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Figure Caption

**Fig. 1a)** For the following values of the parameters: $\lambda = 0.2$, $k = 0.4$, $h_t = 1$, $M_q = M_q + 10$ GeV, $m_N = 10,300$ GeV, $m_1 = 250$ GeV, $\tan \beta = 2.5$, $A_t = 10$ GeV, $M_1 = M_2 = 150$ GeV and $T = 150$ GeV, and in the plane $(x, M_\tilde{q})$, the solid line corresponds to the curve $\overline{m}_2(T) = 0$, the dot-dashed line corresponds to $m_{A^0} = 40$ GeV and the dashed line to the conservative bound $m_{h^0} \gtrsim 60$ GeV. Regions I and II correspond to $m_N = 10,300$ GeV, respectively.

**Fig 1b)** The same as in Fig. 1a) but for $\lambda = 0.3$, $k = 0.5$, $\tan \beta = 1.2$, $m_1 = 150$ GeV, $M_1 = M_2 = 250$ GeV.

**Fig. 2a)** Values in the plane $(\tilde{v}(z,T^*), x(z,T^*))$ for which a maximal SCPB occurs. Region I and II correspond to the choice of the parameters given in Fig. 1a) with $x = 600$ GeV and $M_{\tilde{q}} = 200$ GeV.

**Fig. 2b)** Values in the plane $(\tilde{v}(z,T^*), x(z,T^*))$ for which a maximal SCPB occurs. Region I and II correspond to the choice of the parameters given in Fig. 1b) with $x = 800$ GeV and $M_{\tilde{q}} = 300$ GeV.
This figure "fig1-1.png" is available in "png" format from:

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This figure "fig1-2.png" is available in "png" format from:

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