Multi-period structure of electro-weak phase transition in the 
3-3-1-1 model

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(Dated: May 25, 2018)

The electroweak phase transition (EWPT) is considered in the framework of 3-3-1-1 model for Dark Matter. The phase structure within three or two periods is approximated for the theory with many vacuum expectation values (VEVs) at TeV and Electroweak scales. In the mentioned model, there are two pictures. The first picture containing two periods of EWPT, has a transition $SU(3) \rightarrow SU(2)$ at 6 TeV scale and another is $SU(2) \rightarrow U(1)$ transition which is the like-standard model EWPT. The second picture is an EWPT structure containing three periods, in which two first periods are similar to those of the first picture and another one is the symmetry breaking process of $U(1)_N$ subgroup. Our study leads to the conclusion that EWPTs are the first order phase transitions when new bosons are triggers and their masses are within range of some TeVs. Especially, in two pictures, the maximum strength of the $SU(2) \rightarrow U(1)$ phase transition is equal to 2.12 so this EWPT is not strong. Moreover, neutral fermions, which are candidates for Dark Matter and obey the Fermi-Dirac distribution, can be a negative trigger for EWPT.

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However, they do not make lose the first-order EWPT at TeV scale. Furthermore, in order to the strong first-order EWPT at TeV scale, the symmetry breaking processes must produce more bosons than fermions or the mass of bosons must be much larger than that of fermions.

PACS numbers: 11.15.Ex, 12.60.Fr, 98.80.Cq
Keywords: Spontaneous breaking of gauge symmetries, Extensions of electroweak Higgs sector, Particle-theory models (Early Universe)

I. Introduction

The EWPT is another view of spontaneous symmetry breaking in Theoretical Particle Physics. The latter is a transition of the Higgs field with vanishing VEV to non-zero one. The EWPT plays an important role at early stage of expanding universe; and its issue is also related to hot topics such as Dark Matter (DM) or Dark Energy. From a micro viewpoint and within the current limits, candidate for DM may be a heavy particle. If we accept the symmetry-breaking mechanism as an universal mechanism, then mass of the DM candidate must also be generated through a phase transition process. Moreover, if the mass of the DM candidate is very large so the phase transition process must take place before the EWPT of the Standard Model (SM) and must also follow the gradually decreasing temperature structure of the universe.

As in the SM, the EWPT process has only one phase at the energy level around 200 GeV. This process is accompanied by mass generation of particles. However, at present, the existence of heavy particles is possible only at energy scale larger than 200 GeV. Therefore, the production of these heavy particles interacting with the SM ones, must also be considered.

At present, the mechanism of symmetry-breaking is believed to be accurate, but the Higgs potential is not exactly determined because its form is model dependent.

The EWPT consists of an important question of phase transition which must be a strongly first-order phase one. This is the third Sakharov condition being deviation from thermal equilibrium \[1\]. The mentioned condition together with B, C, CP violations leads to solution of the Baryon Asymmetry of Universe (BAU). The B, C and CP violations can be seen throughout the sphaleron rate and the CKM-matrix in models \[2\] or other CP violation sources as neutrino mixing.

At present, the EWPT is considered at a one-loop level, particularly, in beyond the Standard Model. A new trend nowadays is multi-phase calculations in multi-Higgs scalar potential.

In order to consider the EWPT, we must build the high-temperature effective potential which is usually in the following form
\[ V_{\text{eff}} = D.(T^2 - T_0^2)\nu^2 - E.T\nu^3 + \frac{\lambda T}{4}\nu^4, \]  

where \( \nu \) is the VEV of Higgs boson. The first order EWPT binds that the strength of phase transition should be larger than the unit \( (S = \frac{\nu}{\nu_c} \geq 1, \) where \( \nu_c \) is VEV of Higgs field at a critical temperature \( T_c \).

The effective potential \( V_{\text{eff}} \) in Eq. (1) is a function of temperature and VEVs. It can have one or two minimums when the temperature goes down. At \( T_c \), the two minimums are separated by a potential barrier, the VEV of Higgs field crosses over from vanishing VEV to a non-zero VEV. This transition is called the first order phase transition and it can cause large deviations from thermal equilibrium.

The EWPT has been calculated in the SM [2] and in some extended models [3–17]. It is reminded that DM, heavy particles and neutrino oscillations can be triggers of the EWPT [18]. The most studies of the EWPT are in the Landau gauge. However gauge also made contributions in EWPT as done in Ref. [17]. It is reminded that in some extended models, Higgs sector consists multi-vacuum structure of which the classical example is the Two Higgs Doublet model. This additional Higgs structure can be a new source to answer the BAU puzzles.

Another example of multi-vacuum structure belongs to the models based on \( SU(3)_C \otimes SU(3)_L \otimes U(1)_X \) group [19, 20] called 3-3-1 models for short. There exist two main versions of the 3-3-1 models: the minimal [19] and another with right-handed neutrinos [20]. To provide an explanation for the observed pattern of SM fermion masses and mixings, various 3-3-1 models with flavor symmetries and radiative seesaw mechanisms have been proposed in the literature [1]. However some of them involve non-renormalizable interactions. In addition the 3-3-1 models do not give completely desired answer on the DM issue. In the recently proposed 3-3-1-1 model [23] based on \( SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N \) group has a good advantage in explaining DM. Phenomena of this model such as DM, inflation, leptogenesis, neutrino mass, kinetic mixing effect, and \( B-L \) asymmetry, have been studied in Refs. [24–28]. The 3-3-1-1 model has three Higgs triplets to generate masses of fermions and the mass of new heavy particle with masses around some TeVs. This model fits with candidates for DM. The presence of the above mentioned particles might also lead to interesting consequences such

\[ ^1 \text{With the help of discrete } Z_N \text{ symmetries, the 3-3-1 model with } \beta = \frac{1}{\sqrt{3}} \text{ can provide solutions of neutrino mass and mixing, DM and inflation} \]
as the baryon asymmetry or EWPT which is a subject of this study.

This article is organized as follows. In section II, the matter fields and Higgs bosons in the 3-3-1-1 model are briefly reviewed. In section III, the effective potential having the contribution from heavy bosons a function of temperature and VEVs is derived. In section IV, we analysis in details structure of phase transition, find the first order phase transition and show constraints on mass of charged Higgs boson in the case without neutral fermions. In section V, we discuss the role of neutral fermions in the EWPT problem. Finally, we summarize and make outlooks in section VI.

II. Brief review of the 3-3-1-1 model

Nowadays, heavy particles are widely accepted to be exist. Within their existence, unexplained problems can be caused. They may be a candidate for DM, or just new ones. The 3-3-1-1 model has many new particles inserted in the multiplet of the gauge group $SU(3)C \otimes SU(3)L \otimes U(1)X \otimes U(1)N$, where $U(1)X$ is the gauge group associated with the electromagnetic interaction and $U(1)N$ is the gauge group associated with the conservation of $B - L$ number when combining with $SU(3)L$ charges [23–27].

To keep the model being anomaly free, the fermion content has to have equal number of the $SU(3)L$ triplets and anti-triplets as follows [23]

$$
\psi_{aL} = (\nu_{aL}, e_{aL}, (N_{aR})^c)^T \sim (1, 3, -\frac{1}{3}, -\frac{2}{3}),
$$

$$
e_{aR} \sim (1, 1, -1, -1), \nu_{aR} \sim (1, 1, 0, -1), \ (2)
$$

$$
Q_{aL} = (d_{aL}, -\overline{u}_{aL}, D_{aL})^T \sim (3, 3^*, 0, 0),
$$

$$
Q_{3L} = (u_{3L}, d_{3L}, U_{L})^T \sim (3, 3, 1/3, 2/3),
$$

$$
u_{aR} \sim \left(3, 1, 2, \frac{1}{3} \right),
$$

$$
d_{aR} \sim \left(3, 1, -\frac{1}{3}, \frac{1}{3} \right),
$$

$$
U_{R} \sim \left(3, 1, 2, \frac{4}{3} \right),
$$

$$
D_{aR} \sim \left(3, 1, -\frac{1}{3}, -\frac{2}{3} \right),
$$

where $a = 1, 2, 3$ and $\alpha = 1, 2$ are family indices. $N_{aR}$ is neutral fermions playing a role of candidates for DM. In [2], the numbers in bracket associated with multiplet correspond to number of members in the $SU(3)C$, $SU(3)L$ assignment, its $X$ and $N$ charges, respectively.

The Higgs sector of the model under consideration contains three scalar triplets and one singlet as follows

$$
\eta = (\eta_1^0, \eta_2^0, \eta_3^0)^T \sim (1, 3, -1/3),
$$

$$
\chi = (\chi_1^0, \chi_2^-, \chi_3^0)^T \sim (1, 3, -1/3),
$$

$$
\rho = (\rho_1^+, \rho_2^0, \rho_3^0)^T \sim (1, 3, 2/3),
$$

$$
\phi \sim (1, 1, 0).
$$

Note that in [2], the lepton and anti-lepton lie in the same triplet. Hence, lepton number is not conserved and it should be replaced with new conserved one $L$ [29]. Assuming the
bottom element in lepton triplet \((N_{aR})\) without lepton number, ones have 23
\[
B - L = -\frac{2}{\sqrt{3}} T_8 + N.  
\]  
(5)

Note that in this model, not only leptons but also some scalar fields carry lepton number as seen in Table II.

| Particle | \(\nu\) | \(e\) | \(N\) | \(U\) | \(D_3\) | \(\rho_3\) | \(\chi_1\) | \(\chi_2\) | \(\phi\) |
|----------|--------|--------|--------|-------|-------|-------|-------|-------|-------|
| \(L\)    | 1      | 1      | 0      | -1    | 1     | -1    | 1     | 1     | -2    |

TABLE I: Non-zero lepton number \(L\) of fields in the 3-3-1-1 model.

From Table II we see that elements at the bottom of \(\eta\) and \(\rho\) triplets carry lepton number \(-1\), while the elements standing in two first rows of \(\chi\) triplet have the opposite one \(+1\).

To generate masses for fermions, it is enough that only neutral scalars without lepton number develop VEV as follows
\[
\langle \eta \rangle = \left( \frac{u}{\sqrt{2}}, 0, 0 \right)^T, \quad \chi = \left( 0, 0, \frac{\omega}{\sqrt{2}} \right)^T, \quad \rho = \left( 0, \frac{v}{\sqrt{2}}, 0 \right)^T.  
\]  
(6)

For the future presentation, let us remind that in the model under consideration, the covariant derivative is defined as
\[
D_\mu = \partial_\mu - ig_s t_i G_{i\mu} - ig T_i A_{i\mu} - ig_X X_{i\mu} - ig_N N_{i\mu},  
\]  
(7)

where \(G_{i\mu}, A_{i\mu}, B_{\mu\nu}, C_{\mu\nu}\) and \(g_s, g, g_X, g_N\) correspond gauge fields and couplings of \(SU(3)_C, SU(3)_L, U(1)_X\) and \(U(1)_N\) groups, respectively.

The Yukawa couplings are given as
\[
\mathcal{L}_{Yukawa} = h_{ab}^e \bar{\psi}_a L \rho e_b R + h_{ab}^\nu \bar{\psi}_a L \eta \nu_b R + h_{ab}^\mu \bar{\psi}_a L \nu_b R \mu R + h_{ab}^U \bar{Q}_3 L \chi U_R + h_{ab}^D \bar{Q}_3 L \chi^* D_R \\
+ h_{ab}^U \bar{Q}_3 L \eta u_a R + h_{a}^d \bar{Q}_3 L \rho d_a R + h_{ab}^d \bar{Q}_a L \eta^* d_b R + h_{ab}^u \bar{Q}_a L \rho^* u_b R + H.c..  
\]  
(8)

From Eq. (8), it follows masses of the top and bottom quarks as follows
\[
m_t = \frac{h_t u}{\sqrt{2}}, \quad m_b = \frac{h_b v}{\sqrt{2}},  
\]
while masses of the exotic quarks are determined as
\[
m_U = \frac{\omega}{\sqrt{2}} h_U^U; \quad m_{D_1} = \frac{\omega}{\sqrt{2}} h_{11}^D; \quad m_{D_2} = \frac{\omega}{\sqrt{2}} h_{22}^D.  
\]
The Higgs fields are expanded around the VEVs as follows

\[
\eta = \langle \eta \rangle + \eta', \eta' = \left( \frac{S_\eta + iA_\eta}{\sqrt{2}}, \eta^-, \frac{S'_\eta + iA'_\eta}{\sqrt{2}} \right),
\]

\[
\rho = \langle \rho \rangle + \rho', \rho' = \left( \rho^+, \frac{S_\rho + iA_\rho}{\sqrt{2}}, \rho'^+ \right),
\]

\[
\chi = \langle \chi \rangle + \chi', \chi' = \left( \frac{S_\chi + iA_\chi}{\sqrt{2}}, \chi^-, \frac{S'_\chi + iA'_\chi}{\sqrt{2}} \right),
\]

\[
\phi = \langle \phi \rangle + \phi' = \frac{\Lambda}{\sqrt{2}} + \frac{S_4 + iA_4}{\sqrt{2}}.
\]

It is mentioned that the values \(u\) and \(v\) provide masses for all fermions and gauge bosons in the SM, while \(\omega\) gives masses for the extra heavy quarks and gauge bosons. The value \(\Lambda\) plays the role for the \(U(1)_N\) breaking at high scale; and in some cases, it is larger than \(\omega\).

The scalar potential for Higgs fields is a function of eighteen parameters

\[
V(\rho, \eta, \chi, \phi) = \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\chi^\dagger\chi)^2 + \lambda_3(\eta^\dagger\eta)^2
\]

\[
+ \lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta)
\]

\[
+ \lambda_7(\rho^\dagger\chi)(\rho^\dagger\eta) + \lambda_8(\rho^\dagger\eta)(\rho^\dagger\eta) + \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + f \varepsilon^{mnp}\eta_m\rho_n\chi_p + H.c
\]

\[
+ \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta).
\]

When constructing this Higgs potential, triple scalar self-interactions needs to be limited because it forces us to introduce a \(f\) parameter (\(f\) has a mass dimension the same as \(\omega\)) that can like an interrupt factor for these interactions. In addition, \(f\) can be replaced by one Higgs field or another interaction among three Higgs fields. Thus, the mentioned interaction will become a fourth- or sixth-order coupling. We often do not consider high-order interactions (because these high-order interactions may be difficult to renormalization. However they may be related to other hypothetical offending processes). Therefore we can ignore \(f\) in this article though it may have a different role in other problems. For detailed analysis of the Higgs sector in the model under consideration, the reader is referred to Ref. [23].

In this particular model, the mass of scalar boson depends on not only VEVs, \(\mu_{1,2,3}\) and \(\lambda_i, i = 1, 2, 3, \cdots, 12\) but also \(f\) parameter. Note that \(f\) increases the mass of bosons [23]. Returning to our work, in order to limit the parameter number, as above mentioned, we will ignore \(f\) hereafter.
A. Higgs boson masses

Substituting Eq. (9) into Eq. (10) yields

\[ V(\rho, \eta, \chi, \phi) = V_0 + V_1 + \sum_{i=\rho,\eta,\chi} (V_{S_i} + V_{A_4}) + V_{S_4} + V_{A_4} + \text{Interaction terms}, \]

where \( V_0 \) and \( V_1 \) are the minimum interaction term being independent of scalar fields and

\[ V_0 = \frac{\lambda_2^2 \Lambda^4}{4} + \frac{1}{4} \lambda_{11} \Lambda^2 \omega^2 + \frac{\lambda_2 \omega^4}{4} + \frac{\Lambda^2 \mu^2}{2} + \frac{1}{2} \mu^2 \omega^2 + \frac{\lambda_3 u^4}{4} + \frac{1}{4} \lambda_{12} \Lambda^2 u^2 + \frac{1}{4} \lambda_6 u^2 \omega^2 \]

\[ V_1 = S_\eta \left[ u \mu_3^2 + \lambda_3 u^3 + \frac{1}{2} \lambda_5 u \omega^2 + \frac{1}{2} \lambda_6 u \omega^2 + \frac{1}{2} \lambda_{12} u \Lambda^2 \right] \]

\[ + S_\rho \left[ v \mu_1^2 + \lambda_1 v^3 + \frac{1}{2} \lambda_4 v \omega^2 + \frac{1}{2} \lambda_5 v \omega^2 + \frac{1}{2} \lambda_{10} v \Lambda^2 \right] \]

\[ + S_\chi \left[ \omega \mu_2^2 + \lambda_2 \omega^3 + \frac{1}{2} \lambda_4 \omega v^2 + \frac{1}{2} \lambda_6 \omega u^2 + \frac{1}{2} \lambda_11 \omega \Lambda^2 \right] \]

\[ + S_4 \left[ \Lambda \mu^2 + \lambda \Lambda^3 + \frac{1}{2} \lambda_{10} \Lambda v^2 + \frac{1}{2} \lambda_{11} \Lambda \omega^2 + \frac{1}{2} \lambda_{12} \Lambda u^2 \right] \].

Hence, the potential minimization conditions are obtained by

\[ u(\lambda_{12} \Lambda^2 + \lambda_6 \omega^2 + 2 \mu_3^2 + 2 \lambda_3 u^2 + \lambda_5 v^2) = 0, \]

\[ \omega(\lambda_{11} \Lambda^2 + 2 \lambda_2 \omega^2 + 2 \mu_2^2 + \lambda_6 u^2 + \lambda_4 v^2) = 0, \]

\[ v(\lambda_{10} \Lambda^2 + \lambda_4 \omega^2 + 2 \mu_1^2 + \lambda_5 u^2 + 2 \lambda_1 v^2) = 0, \]

\[ \Lambda(2 \lambda \Lambda^2 + \lambda_{11} \omega^2 + 2 \mu^2 + \lambda_{12} u^2 + \lambda_{10} v^2) = 0. \]
From (11) we get the part for charged Higgs bosons

\[
V_\eta = \lambda_3 (\eta^+ \eta^-)^2 + \left(\frac{\Lambda^2 \lambda_{12}}{2} + \frac{\lambda_6 \omega^2}{2} + \mu_3^2 + \lambda_3 u^2 + \frac{\lambda_8 v^2}{2} + \frac{\lambda_8 v^2}{2}\right) \eta^+ \eta^- + \frac{1}{2} \lambda_8 u \eta^+ \rho^-
\]

\[
= \lambda_3 (\eta^+ \eta^-)^2 + \left(\frac{\lambda_8 v^2}{2}\right) \eta^+ \eta^- + \frac{1}{2} \lambda_8 u \eta^+ \rho^-;
\]

\[
V_\chi = \lambda_2 (\chi^+ \chi^-)^2 + \left(\frac{\Lambda^2 \lambda_{11}}{2} + \lambda_2 \omega^2 + \mu_2^2 + \frac{\lambda_6 u^2}{2} + \frac{\lambda_7 v^2}{2} + \lambda_4 \omega^2 + \frac{\lambda_5 u^2}{2} + \lambda_1 v^2\right) \chi^+ \chi^- + \frac{1}{2} \lambda_7 v \omega \chi^- \rho^+.
\]

\[
= \lambda_2 (\chi^+ \chi^-)^2 + \left(\frac{\lambda_7 v^2}{2}\right) \chi^+ \chi^- + \frac{1}{2} \lambda_7 v \omega \chi^- \rho^+;
\]

\[
V_\rho = \lambda_1 (\rho^+ \rho^-)^2 + \left(\frac{\Lambda^2 \lambda_{10}}{2} + \frac{\lambda_4 \omega^2}{2} + \frac{\lambda_5 \omega^2}{2} + \mu_1^2 + \frac{\lambda_5 u^2}{2} + \frac{\lambda_8 u^2}{2} + \lambda_1 v^2\right) \rho^+ \rho^- + \frac{1}{2} \lambda_8 u v \eta^- \rho^+
\]

\[
= \lambda_1 (\rho^+ \rho^-)^2 + \left(\frac{\lambda_8 u^2}{2}\right) \rho^+ \rho^- + \frac{1}{2} \lambda_8 u v \eta^- \rho^+;
\]

\[
V_{\rho'} = \lambda_1 (\rho'^+ \rho'^-)^2 + \left(\frac{\Lambda^2 \lambda_{10}}{2} + \frac{\lambda_4 \omega^2}{2} + \frac{\lambda_5 \omega^2}{2} + \mu_1^2 + \frac{\lambda_5 u^2}{2} + \frac{\lambda_8 u^2}{2} + \lambda_1 v^2\right) \rho'^+ \rho'^- + \frac{1}{2} \lambda_7 v \omega \chi^+ \rho'^-
\]

\[
= \lambda_1 (\rho'^+ \rho'^-)^2 + \left(\frac{\lambda_7 \omega^2}{2}\right) \rho'^+ \rho'^- + \frac{1}{2} \lambda_7 v \omega \chi^+ \rho'^-.
\]

From the above equations, after some manipulations, the mass terms of charged Higgs bosons are given by

\[
V_{\text{Higgs mass}}^{\text{mass}} = \left(\frac{\lambda_8 v^2}{2}\right) \eta^+ \eta^- + \frac{1}{2} \lambda_8 u v \eta^+ \rho^- + \left(\frac{\lambda_8 u^2}{2}\right) \eta^+ \rho^- + \frac{1}{2} \lambda_8 u v \eta^- \rho^+
\]

\[
+ \left(\frac{\lambda_7 v^2}{2}\right) \chi^+ \chi^- + \frac{1}{2} \lambda_7 v \omega \chi^- \rho^+ + \left(\frac{\lambda_7 \omega^2}{2}\right) \chi^+ \rho^- + \frac{1}{2} \lambda_7 v \omega \chi^+ \rho^-
\]

\[
= u^2 + v^2 \lambda_8 \left(\frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2}} \right) \left(\frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2}} \right)
\]

\[
+ \omega^2 + v^2 \lambda_7 \left(\frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2}} \right) \left(\frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2}} \right)
\]

\[
= \frac{u^2 + v^2}{2} \lambda_8 H_1^+ H_1^- + \frac{\omega^2 + v^2}{2} \lambda_7 H_2^+ H_2^- + \frac{m_{H_1}^2}{2} H_1^+ H_1^- + \frac{m_{H_2}^2}{2} H_2^+ H_2^-;
\]

where

\[
H_1^\pm = \frac{v \eta^\pm + u \rho^\pm}{\sqrt{u^2 + v^2}}; \quad m_{H_1}^2 = \frac{u^2 + v^2}{2} \lambda_8;
\]

\[
H_2^\pm = \frac{v \eta^\pm + \omega \rho^\pm}{\sqrt{\omega^2 + v^2}}; \quad m_{H_2}^2 = \frac{\omega^2 + v^2}{2} \lambda_7.
\]
Similarly, the part of neutral Higgs bosons is given by:

\[ V_{A_4} = \frac{\lambda}{4} A_4^4 + \left( \frac{1}{2} \lambda \Lambda^2 + \frac{1}{4} \omega^2 \lambda_{11} + \frac{\mu_2^2}{2} + \frac{\lambda_{12} u^2}{4} + \frac{\lambda_{10} v^2}{4} \right) A_4^2 \]
\[ = \frac{\lambda}{4} A_4^4, \]
\[ V_{S_4} = \frac{\lambda}{4} S_4^4 + \left( \frac{3}{2} \lambda \Lambda^2 + \frac{1}{4} \omega^2 \lambda_{11} + \frac{\mu_2^2}{2} + \frac{\lambda_{12} u^2}{4} + \frac{\lambda_{10} v^2}{4} \right) S_4^2 \]
\[ = \frac{\lambda}{4} S_4^4 + \lambda \Lambda^2 S_4^2, \]
\[ V_{A_\eta} = \frac{\lambda_3}{4} A_\eta^4 + \left( \frac{\Lambda^2 \lambda_{12}}{4} + \frac{\lambda_6 \omega^2}{4} + \frac{\mu_3^2}{2} + \frac{\lambda_3 u^2}{2} + \frac{\lambda_5 v^2}{2} \right) A_\eta^2 \]
\[ = \frac{\lambda_3}{4} A_\eta^4, \]
\[ V_{A_\eta'} = \frac{\lambda_3}{4} A_\eta'^4 + \left( \frac{\Lambda^2 \lambda_{12}}{4} + \frac{\lambda_6 \omega^2}{4} + \frac{\mu_3^2}{2} + \frac{\lambda_3 u^2}{2} + \frac{\lambda_5 v^2}{2} \right) A_\eta'^2 \]
\[ = \frac{\lambda_3}{4} A_\eta'^4 + \frac{\lambda_9 \omega^2}{4} A_\eta'^2, \]
\[ V_{A_\chi} = \frac{\lambda_2}{4} A_\chi^4 + \left( \frac{\Lambda^2 \lambda_{11}}{4} + \frac{\lambda_2 \omega^2}{2} + \frac{\mu_2^2}{2} + \frac{\lambda_6 u^2}{4} + \frac{\lambda_4 v^2}{4} \right) A_\chi^2 \]
\[ = \frac{\lambda_2}{4} A_\chi^4 + \frac{\lambda_9 u^2}{4} A_\chi^2, \]
\[ V_{A_\chi'} = \frac{\lambda_2}{4} A_\chi'^4 + \left( \frac{\Lambda^2 \lambda_{11}}{4} + \frac{\lambda_2 \omega^2}{2} + \frac{\mu_2^2}{2} + \frac{\lambda_6 u^2}{4} + \frac{\lambda_4 v^2}{4} \right) A_\chi'^2 \]
\[ = \frac{\lambda_2}{4} A_\chi'^4, \]
\[ V_{A_\rho} = \frac{\lambda_1}{4} A_\rho^4 + \left( \frac{\Lambda^2 \lambda_{10}}{4} + \frac{\lambda_4 \omega^2}{4} + \frac{\mu_1^2}{2} + \frac{\lambda_5 u^2}{4} + \frac{\lambda_1 v^2}{2} \right) A_\rho^2 \]
\[ = \frac{\lambda_1}{4} A_\rho^4, \]
\[ = \frac{\lambda_3}{4} S_\eta^4 + u \lambda_3 S_\eta^3 + \lambda_3 u^2 S_\eta^2, \]
\[ V_{S_\chi} = \frac{\lambda_2}{4} S_\chi^4 + \omega \lambda_2 S_\chi^3 + \left( \frac{\Lambda^2 \lambda_{11}}{4} + \frac{3 \lambda_2 \omega^2}{2} + \frac{\mu_2^2}{2} + \frac{\lambda_6 u^2}{4} + \frac{\lambda_4 v^2}{4} \right) S_\chi^2 \]
\[ = \frac{\lambda_2}{4} S_\chi^4 + \omega \lambda_2 S_\chi^3 + \lambda_2 \omega^2 S_\chi^2, \]
\[ V_{S_\rho} = \frac{\lambda_1}{4} S_\rho^4 + v \lambda_1 S_\rho^3 + \left( \frac{\Lambda^2 \lambda_{10}}{4} + \frac{\lambda_4 \omega^2}{4} + \frac{\mu_1^2}{2} + \frac{\lambda_5 u^2}{4} + \frac{3}{2} \lambda_1 v^2 \right) S_\rho^2 \]
\[ = \frac{\lambda_1}{4} S_\rho^4 + v \lambda_1 S_\rho^3 + \lambda_1 v^2 S_\rho^2, \]
\[ V_{S_{\eta}} = \frac{\lambda_3}{4} S_{\eta}^4 + \left( \frac{\lambda_2 \lambda_{12}}{4} + \frac{\lambda_6 \omega^2}{4} + \frac{\lambda_9 u^2}{4} + \frac{\mu_3^2}{2} + \frac{\lambda_3 u^2}{2} + \frac{\lambda_5 v^2}{4} \right) S_{\eta}^2 + \frac{1}{2} \lambda_9 u \omega S_{\eta}^2 S_{\chi} \]
\[ = \frac{\lambda_3}{4} S_{\eta}^4 + \frac{\lambda_9 u \omega}{2} S_{\eta}^2 + \frac{1}{2} \lambda_9 u \omega S_{\eta}^2 S_{\chi}, \]
\[ V_{S_{\chi}} = \frac{\lambda_2}{4} S_{\chi}^4 + \left( \frac{\lambda_2 \lambda_{11}}{4} + \frac{\lambda_6 u^2}{4} + \frac{\lambda_9 u^2}{4} + \frac{\lambda_4 v^2}{4} \right) S_{\chi}^2 \]
\[ = \frac{\lambda_2}{4} S_{\chi}^4 + \frac{\lambda_9 u^2}{2} S_{\chi}^2. \]

Combination among \( S_{\chi} \) and \( S_{\eta} \) yields
\[ V_m(S_{\chi}, S_{\eta}') = \frac{\lambda_9 \omega^2}{4} S_{\eta}^2 + \frac{1}{2} \lambda_9 u \omega S_{\eta} S_{\chi} + \frac{\lambda_9 u^2}{4} S_{\chi}^2 \]
\[ = \frac{\lambda_9}{4} \left( \omega^2 S_{\eta}^2 + 2 u \omega S_{\eta} S_{\chi} + u^2 S_{\chi}^2 \right) \]
\[ = \frac{\lambda_9 (u^2 + \omega^2)}{4} \left( \frac{\omega S_{\eta}^2}{\sqrt{u^2 + \omega^2}} + \frac{u S_{\chi}}{\sqrt{u^2 + \omega^2}} \right)^2 \]
\[ = \frac{\lambda_9 (u^2 + \omega^2)}{4} (H_3)^2 = \frac{1}{2} m_{H_3}^2 (H_3)^2, \] (22)

where physical boson \( H_3 \) is given by
\[ H_3 = \frac{\omega S_{\eta}^2}{\sqrt{u^2 + \omega^2}} + \frac{u S_{\chi}}{\sqrt{u^2 + \omega^2}}; \quad \text{with } m_{H_3}^2 = \frac{\lambda_9 (u^2 + \omega^2)}{2}. \] (23)

The mass of neutral Higgs bosons is presented in Table II

| Neutral Higgs boson | \( S_4 \) | \( A_\eta' \) | \( A_\chi \) | \( S_\eta \) | \( S_{\eta}' \) | \( S_\rho \) | \( H_3 \) |
|---------------------|-----------|----------------|----------------|-----------|-----------|-------|-------|
| Squared mass        | \( 2 \lambda \Lambda^2 \) | \( \frac{\lambda_9 \omega^2}{2} \) | \( \frac{\lambda_9 u^2}{2} \) | \( 2 \lambda_3 u^2 \) | \( 2 \lambda_9 \omega^2 \) | \( 2 \lambda_1 v^2 \) | \( \frac{\lambda_9 (u^2 + \omega^2)}{2} \) |

Remind that the massless Goldstones bosons are: \( X A_4, A_\eta, A_\eta', A_\rho \) in neutral scalar sector and two massless combinations orthogonal to the charged Higgs bosons. It is noted that at the limit \( f \to 0 \), the results given in [24–26] are consistent with those of this study.

**B. Gauge boson sector**

The gauge bosons obtain masses when the scalar fields develop the VEVs. Therefore, their mass Lagrangian is given by
\[ \mathcal{L}_{\text{mass}}^{\text{gauge}} = \sum_{\Phi} (D^\mu \langle \Phi \rangle)^\dagger (D_\mu \langle \Phi \rangle). \]
Substituting the scalar multiplets $\eta, \rho, \chi$ and $\phi$ with their covariant derivative, we obtain

$$L_{\text{mass}} = \frac{g^2 u^2}{8} \left[ \left( A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} - \frac{2}{3} t_X B_{\mu} + \frac{2}{3} t_N C_{\mu} \right)^2 + 2W_\mu^+ W^- + 2X_{\mu}^0 X^{0\mu} \right]$$

$$+ \frac{g^2 v^2}{8} \left[ \left( -A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} + \frac{4}{3} t_X B_{\mu} + \frac{2}{3} t_N C_{\mu} \right)^2 + 2Y_\mu^+ Y^- + 2Y_{\mu}^0 Y^{0\mu} \right]$$

$$+ \frac{g^2 \omega^2}{8} \left[ \left( -A_{3\mu} - \frac{A_{8\mu}}{\sqrt{3}} - \frac{4}{3} t_X B_{\mu} - \frac{2}{3} t_N C_{\mu} \right)^2 + 2Y_{\mu}^+ Y^- + 2Y_{\mu}^0 Y^{0\mu} \right]$$

$$+ 2g^2 N^2 \Lambda^2 C_{\mu}^2,$$

where we have denoted $t_X \equiv \frac{\eta}{g}$, $t_N \equiv \frac{\rho}{g}$, and

$$W_{\pm} = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}}, \quad X_{\mu}^{0,0*} = \frac{A_{4\mu} \mp iA_{5\mu}}{\sqrt{2}}, \quad Y_{\mu} = \frac{A_{6\mu} \mp iA_{7\mu}}{\sqrt{2}}.$$ (24)

The mass Lagrangian can be rewritten as

$$L_{\text{mass}} = \frac{g^2}{4} \left( u^2 + v^2 \right) W^+ W^- + \frac{g^2}{4} \left( v^2 + \omega^2 \right) Y^+ Y^- + \frac{g^2}{4} \left( u^2 + \omega^2 \right) X_{0*}^0 X^0$$

$$+ \frac{1}{2} \left( A_3 A_8 B C \right) M^2 \begin{pmatrix} A_3 \\ A_8 \\ B \\ C \end{pmatrix},$$

where the Lorentz indices have been omitted and should be understood. The squared-mass matrix of the neutral gauge bosons is found to be:

$$M^2 = \frac{g^2}{2} \begin{pmatrix} \frac{1}{2}(u^2 + v^2) & \frac{u^2 - v^2}{2\sqrt{3}} & -\frac{t_X(u^2 + 2v^2)}{3} & \frac{t_N(u^2 - v^2)}{3} \\ \frac{u^2 - v^2}{2\sqrt{3}} & \frac{1}{6}(u^2 + v^2 + 4\omega^2) & -\frac{t_X(u^2 - 2v^2 + 4\omega^2)}{3\sqrt{3}} & \frac{t_N(u^2 + v^2 + 4\omega^2)}{3\sqrt{3}} \\ -\frac{t_X(u^2 + 2v^2)}{3} & -\frac{t_X(u^2 - 2v^2 + 4\omega^2)}{3\sqrt{3}} & \frac{2}{9}t_X^2(u^2 + 4v^2 + \omega^2) & \frac{2}{9}t_X t_N(u^2 - 2(v^2 + \omega^2)) \\ \frac{t_N(u^2 - v^2)}{3} & \frac{t_N(u^2 + v^2 + 4\omega^2)}{3\sqrt{3}} & -\frac{2}{9}t_X t_N(u^2 - 2(v^2 + \omega^2)) & \frac{2}{9}t_N^2(u^2 + v^2 + 4(\omega^2 + 9\Lambda^2)) \end{pmatrix}.$$ 

The non-Hermitian gauge bosons $W^\pm$, $X_{0,0*}^0$ and $Y^\pm$ are physical fields with corresponding masses:

$$m_W^2 = \frac{g^2}{4}(u^2 + v^2), \quad m_X^2 = \frac{g^2}{4}(u^2 + \omega^2), \quad m_Y^2 = \frac{g^2}{4}(v^2 + \omega^2).$$

Because of the constraints $u, v \ll \omega$, we have $m_W \ll m_X \simeq m_Y$. The $W$ boson is identified as the SM $W$ boson. It follows

$$u^2 + v^2 = (246 \text{ GeV})^2.$$
Gauge boson & $W$ & $Y$ & $X$
\hline
Squared mass & \( \frac{g^2}{4}(u^2 + v^2) \) & \( \frac{g^2}{4}(\omega^2 + v^2) \) & \( \frac{g^2}{4}(\omega^2 + u^2) \)
\hline

| TABLE III: The mass of charged gauge bosons. |

The $X$ and $Y$ fields are the new gauge bosons with the large masses given in the $\omega$ scale. The physical charged gauge bosons and their masses are summarized in Table III.

From aforementioned analysis, it follows that the phenomenological aspects of the 3-3-1-1 model can be divided into two pictures corresponding to different domain values of VEVs.

1. Picture (i): $\Lambda \sim \omega \gg v \sim u$

The physical neutral gauge bosons are derived through the following transformation $(A_3, A_8, B, C) \rightarrow (A, Z, Z_2, Z_1)$:

\[
\begin{pmatrix}
A_3 \\
A_8 \\
B \\
C
\end{pmatrix} = U_1 U_2 U_3 \begin{pmatrix}
A \\
Z \\
Z_2 \\
Z_1
\end{pmatrix}.
\]

The above diagonalization is realized through three steps \([24][27]\),

The first step: $M'' = U_1^T M^2 U_1$,

The second step: $M''' = U_2^T M'' U_2$,

The final step: $M'''' = U_3^T M''' U_3 = \text{diag}(0, m_Z^2, m_{Z_2}^2, m_{Z_1}^2)$,

where

\[
U_1 = \begin{pmatrix}
s_W & c_W & 0 & 0 \\
-\frac{s_W}{\sqrt{3}} & \frac{c_W t_W}{\sqrt{3}} & \sqrt{1 - \frac{t_W^2}{3}} & 0 \\
c_W \sqrt{1 - \frac{t_W^2}{3}} & -s_W \sqrt{1 - \frac{t_W^2}{3}} & t_W & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

\[
U_2 \approx \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & \xi \\
0 & -\xi^T & 1
\end{pmatrix},
\]

\[
U_3 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c_\xi & -s_\xi \\
0 & 0 & s_\xi & c_\xi
\end{pmatrix}.
\]
In Eq. (25), the $\mathbf{E}$ is a two-component vector given by \[24–27\]

$$\mathbf{E}_1 = -\frac{\sqrt{4t_X^2 + 3(3\Lambda^2[(2t_X^2 - 3)u^2 + (4t_X^2 + 3)v^2] + t_X^2\omega^2(u^2 + v^2)])}} {4\Lambda^2(t_X^2 + 3)^{3/2}t_N} \ll 1,$$

$$\mathbf{E}_2 = \frac{t_X^2\sqrt{4t_X^2 + 3(u^2 + v^2)}} {8\Lambda^2(t_X^2 + 3)^{3/2}t_N} \ll 1,$$

$$t_2 \delta \simeq \frac{4\sqrt{3 + t_X^2}t_N\omega^2} {(3 + t_X^2)\omega^2 - 4t_N^2(\omega^2 + 9\Lambda^2)},$$

$$s_W = \frac{\sqrt{3t_X}} {\sqrt{3 + 4t_X^2}} \simeq 0.231.$$  

Finally we obtain the masses of neutral gauge bosons as follows

$$m_Z^2 \simeq \frac{g^2(u^2 + v^2)} {4c_W^2},$$ \hspace{1cm} (26)

$$m_{Z_1}^2 \simeq \frac{g^2} {18} \left(3 + t_X^2\right)\omega^2 + 4t_N^2(\omega^2 + 9\Lambda^2) + \sqrt{\left((3 + t_X^2)\omega^2 - 4t_N^2(\omega^2 + 9\Lambda^2)\right)^2 + 16(3 + t_X^2)t_N^2\omega^4},$$ \hspace{1cm} (27)

$$m_{Z_2}^2 \simeq \frac{g^2} {18} \left(3 + t_X^2\right)\omega^2 + 4t_N^2(\omega^2 + 9\Lambda^2) - \sqrt{\left((3 + t_X^2)\omega^2 - 4t_N^2(\omega^2 + 9\Lambda^2)\right)^2 + 16(3 + t_X^2)t_N^2\omega^4}.$$ \hspace{1cm} (28)

From the experimental data $\Delta \rho < 0.0007$, ones get $u/\omega < 0.0544$ or $\omega > 3.198$ TeV \[23\] (provided that $u = 246/\sqrt{2}$ GeV as mentioned). Therefore, the value of $\omega$ results in the TeV scale as expected.

2. **Picture (ii): $\Lambda \gg \omega \gg v \sim u$**

If we assume $\Lambda \gg \omega \gg u \sim v$, three gauge bosons are derived as \[24–27\]

$$m_Z^2 \simeq \frac{g^2(u^2 + v^2)} {4c_W^2},$$ \hspace{1cm} (29)

$$m_{Z_1}^2 \simeq 4g^2t_N^2\Lambda^2,$$ \hspace{1cm} (30)

$$m_{Z_2}^2 \simeq \frac{g^2c_W^2\omega^2} {(3 - 4s_W^2)}.$$ \hspace{1cm} (31)

From the Table \[III\] and Eqs.\[29, 30, 31\], the $W^\pm$ boson and the $Z$ boson are recognized as two famous gauge bosons in the SM. Now we turn to the main object - the effective potential.
III. Effective potential

Within the above assumption, the Higgs potential is given as follows:

\[
V(\rho, \eta, \chi, \phi) = \mu_1^2 \rho^4 + \mu_2^2 \chi^4 + \mu_3^2 \eta^4 + \lambda_1 (\rho^4 \rho) + \lambda_2 (\chi^4 \chi) + \lambda_3 (\eta^4 \eta)^2
+ \lambda_4 (\rho^4 \rho)(\chi^4 \chi) + \lambda_5 (\rho^4 \rho)(\eta^4 \eta) + \lambda_6 (\chi^4 \chi)(\eta^4 \eta)
+ \lambda_7 (\rho^4 \chi)(\chi^4 \rho) + \lambda_8 (\rho^4 \eta)(\eta^4 \rho) + \lambda_9 (\chi^4 \eta)(\eta^4 \chi)
+ \mu^2 \phi^4 \phi + \lambda (\phi^4 \phi)^2 + \lambda_{10} (\phi^4 \phi)(\rho^4 \rho) + \lambda_{11} (\phi^4 \phi)(\chi^4 \chi) + \lambda_{12} (\phi^4 \phi)(\eta^4 \eta)
\]

from which, one obtains \(V_0\) depending on VEVs:

\[
V_0 = \frac{\lambda_0^4}{4} + \frac{\lambda_{11}^2}{4} \phi^2 \phi^2 + \frac{\lambda_{12}}{2} \phi^2 \phi^2 + \frac{\lambda_3}{4} \phi^4 \phi + \frac{\lambda_{12}^2}{4} \phi^2 \phi^2 + \frac{\lambda_6}{4} \phi^2 \phi^2 + \frac{\lambda_{13}}{4} \phi^2 \phi^2 + \frac{\lambda_{14}}{4} \phi^2 \phi^2 + \frac{\lambda_{15}}{4} \phi^2 \phi^2
\]

Here \(V_0\) has quartic form like in the SM, but it depends on four variables \(\phi_1, \phi_2, \phi_u, \phi_v\), and has the mixing terms between them. However, developing the potential \(V_0\), we obtain four minimum equations. Therefore, we can transform the mixing between four variables to the form depending only on \(\phi_1, \phi_2, \phi_u, \phi_v\). Hence, we can write \(V_0(\phi_1, \phi_2, \phi_u, \phi_v) = V_0(\phi_1) + V_0(\phi_2) + V_0(\phi_u) + V_0(\phi_v)\).

In order to derive effective potential, we need the mass spectrum of fields. Starting from the Lagrangian of the scalars (both kinetic and potential terms) and Yukawa interactions, and expanding Higgs fields around VEVs, we obtain the mass terms for all fields in the 3-3-1-1 model.

The gauge sector in the 3-3-1-1 has ten gauge bosons: the photon and nine massive gauge bosons. The latter includes two massive like the SM Z and \(W^\pm\) bosons, and two new heavy neutral \(Z_1, Z_2\) bosons, the charged gauge bosons \(Y^\pm\) and the neutral non-Hermitian bosons: \(X^{0,0^*}\). The Higgs sector contains four charged Higgs \(H_1^+, H_2^+, H_3^+, H_4^+\), seven neutral Higgs \(S_4, A_1', A_2', S_5, S_6, S_7, S_8, S_9, S_10\), \(S_11, S_12, S_13, S_14\), and \(H_3\). The model consists of four heavy quarks \(U, D_1, D_2, S_11\), top quark. Masses of fields in the 3-3-1-1 model are presented in Table IV.

From the mass spectra, we can split masses of particles into four parts as follows

\[
m^2(\phi_1, \phi_2, \phi_u, \phi_v) = m^2(\phi_1) + m^2(\phi_2) + m^2(\phi_u) + m^2(\phi_v).
\]

Taking into account Eqs. (33) and (34), we can also split the effective potential into four parts:

\[
V_{eff}(\phi_1, \phi_2, \phi_u, \phi_v) = V_{eff}(\phi_1) + V_{eff}(\phi_2) + V_{eff}(\phi_u) + V_{eff}(\phi_v).
\]
It is difficult to study the electroweak phase transition with four VEVs, so we assume $\phi_\Lambda \approx \phi_\omega, \phi_u \approx \phi_v$ over space-times. Then, the effective potential becomes

$$V_{\text{eff}}(\phi_\Lambda, \phi_\omega, \phi_u, \phi_v) = V_{\text{eff}}(\phi_\omega) + V_{\text{eff}}(\phi_u).$$

### IV. Electroweak phase transition without neutral fermion

Taking phase transitions in this model into account, it is important to find the activity domain of $\omega, \Lambda, u$ and $v$. Looking at data in Ref. [30, 31], we arrive to assumption: $m_{Z_2} \geq 2.2$ TeV. In addition, from Ref. [23], we also assume $m_{Z_2} < 2.5$ TeV. Hence

$$2.2 \text{ TeV} \leq m_{Z_2} \leq 2.5 \text{ TeV}. \quad (35)$$

From the constraint in (35), we will infer the domain values of $\omega$ and $\Lambda$. It is worth mentioning that in the 3-3-1-1 model, the structure of symmetry breaking which can be divided into two or three periods depending on scale of VEVs as suggesting in the above two pictures.
A. Two periods EWPT in picture (i)

In picture (i), we have assumed $\lambda \sim \omega \gg u \sim v$ meaning that the symmetry breaking or phase transition has two periods. The first transition is $SU(3) \to SU(2)$ through $\omega \sim \Lambda$, which generates masses of the heavy gauge bosons $X^\pm, Y^\pm, Z_1, Z_2$, Higgs bosons $H_2, H_3, A'_\eta, S'_\chi, S_4$, and three exotic quarks. The phase transition $SU(3) \to SU(2)$ only depends on $\phi_\omega \sim \phi_\Lambda$.

When our universe has been expanding and cooling due to $u$ scale, the symmetry breaking $SU(2) \to U(1)$ is turned on, which generates masses of the SM particles and the last part of masses of $H_2, H_3, X^\pm, Y^\pm$. Therefore, phase transition $SU(2) \to U(1)$ only depends on $\phi_u \sim \phi_v$.

1. Phase transition $SU(3) \to SU(2)$

This phase transition involves exotic quarks, heavy bosons, but excludes the SM particles. As a consequence, the effective potential of the EWPT $SU(3) \to U(1)$ is $V_{\text{eff}}(\phi_\omega)$.

Applying the Coleman-Weinberg’s method, the effective potential $V_{\text{eff}}(\phi_\omega)$ is given as

$$ V_{\text{eff}}(\phi_\omega) = D_\omega(T^2 - T_0^2)\phi_\omega^2 - E_\omega T \phi_\omega^3 + \frac{\lambda_\omega(T)}{4} \phi_\omega^4, \quad (36) $$
where

\[ \lambda_\omega(T) = \frac{m_{A_\eta'}^4}{16\pi^2\omega^4} \left( \frac{m_{A_\eta'}^2}{T^2m_{\eta'}} - \frac{m_{H_2}^4}{8\pi^2\omega^4} - \frac{m_{H_3}^4}{16\pi^2\omega^4} - \frac{m_{S_1}^4}{16\pi^2\omega^4} \right) - \frac{m_{A_\eta''}^4}{16\pi^2\omega^4} \left( \frac{m_{A_\eta''}^2}{T^2m_{\eta''}} \right) + \frac{m_{A_\eta'}^2}{2\omega^2} + \frac{m_{A_\eta''}^2}{2\omega^2} + \frac{m_{S_1}^2}{2\omega^2}, \]

\[ E_\omega = \frac{m_{A_\eta'}^3}{12\pi^2\omega^3} + \frac{m_{H_2}^3}{6\pi^2\omega^3} + \frac{m_{H_3}^3}{12\pi^2\omega^3} + \frac{m_{S_1}^3}{12\pi^2\omega^3} + \frac{m_{S_4}^3}{12\pi^2\omega^3} + \frac{m_{\eta'}^3}{2\omega^3} + \frac{m_{\eta''}^3}{2\omega^3} \]

\[ D_\omega = \frac{m_{A_\eta'}^2}{2\omega^2} + \frac{m_{A_\eta''}^2}{2\omega^2} + \frac{m_{H_2}^2}{2\omega^2} + \frac{m_{H_3}^2}{2\omega^2} + \frac{m_{S_1}^2}{2\omega^2} + \frac{m_{S_4}^2}{2\omega^2} + \frac{m_{\eta'}^2}{2\omega^2} + \frac{m_{\eta''}^2}{2\omega^2} \]

\[ F_\omega = \frac{m_{A_\eta'}^4}{32\pi^2\omega^2} - \frac{m_{A_\eta'}^2}{4} - \frac{3m_{D_1}^4}{8\pi^2\omega^2} - \frac{3m_{D_2}^4}{8\pi^2\omega^2} + \frac{m_{H_2}^4}{16\pi^2\omega^2} + \frac{m_{H_3}^4}{32\pi^2\omega^4} + \frac{m_{S_1}^4}{4} + \frac{m_{S_4}^4}{4} - \frac{m_{\eta'}^4}{32\pi^2\omega^2} - \frac{m_{\eta''}^4}{32\pi^2\omega^2} \]

and

\[ T_{\omega}^2 = \frac{F_\omega}{D_\omega}. \]

The minimum conditions are

\[ V_{eff}(0) = \frac{\partial V_{eff}(\phi, \omega)}{\partial \phi} \bigg|_{\omega} = 0; \quad \frac{\partial^2 V_{eff}(\phi, \omega)}{\partial \phi^2} \bigg|_{\omega} = m_{A_\eta'}^2 + m_{A_\eta''}^2 + m_{S_1}^2 + m_{S_4}^2. \]

The values of $V_{eff}(\phi, \omega)$ at the two minima become equal at the critical temperature and the phase transition strength are

\[ T_{\omega} = \frac{T_{\omega}}{\sqrt{1 - E_\omega^2/D_\omega \lambda_{T_{\omega}}}}, \]

\[ S_\omega = \frac{2E_\omega}{\lambda_{T_{\omega}}}. \]

From Eqs. (35), with the limit of $m_{Z_2}$ given in Eq. (35), it follows: $5.856$ TeV $\leq \omega$ $\sim \Lambda \leq 6.654$ TeV.

In this work, we assume $\omega = 6$ TeV, so that $m_{Z_1} = 8.304$ TeV and $m_{Z_2} = 2.254$ TeV. The problem here is that there are nine variables: the masses of $U, D_1, D_2, H_2, H_3$ and $A_\eta', S_\chi', S_4, Z_1$.
However, for simplicity, we assume $m_U = m_{D_1} = m_{D_2} = m_{H_2} \equiv O$, $m_{A\eta} = m_{S'} = m_{H_3} = m_{S_1} \equiv P$. Consequently, the critical temperature and the phase transition strength are the function of $O$ and $P$; therefore we can rewrite the phase transition strength as follows

$$S_\omega = \frac{2E_\omega}{X_{Tc_\omega}} \equiv S_\omega(O, P, S_\omega). \quad (44)$$

In Figure 1, we have plotted the relation between masses of the charged particles $O$ and neutral particles $P$ with some values of the phase transition strength at $\omega = 6$ TeV.

![Figure 1](image_url)

**FIG. 1:** The mass area corresponds to $S_\omega > 1$

The mass region of particles is the largest at $S_\omega = 1$, the mass region of charged particles and neutral particles are

$$\begin{cases}
0 \leq m_{\text{ExoticQuark/ChargedHiggsboson}} \leq 7000 \text{ GeV}, \\
0 \leq m_{H_3} \leq 2600 \text{ GeV}.
\end{cases}$$

From Eq. (44) it follows that the maximum of $S_\omega$ is around 70.

2. Phase transition $SU(2) \to U(1)$

In this period, the symmetry breaking scale equals to $u = 246/\sqrt{2}$ and the masses of the SM particles and apart of masses of $X_1, X_2, X, Y, H_1, H_2, H_3, A_\chi, S_\eta$ are generated.
There are six variables corresponding to the masses of bosons $H_1, H_2, A_χ, A_η, H_3, S_ρ$. For simplicity, we assume: $m_{H_1} = m_{H_2} \equiv K$, $m_{A_χ} = m_{S_η} = m_{H_3} \equiv L$, and $m_{S_ρ} = 125$ GeV.

The effective potential of EWPT $SU(2) \to U(1)$ is given as

$$V_{\text{eff}}(φ_u) = \frac{λ_u(T)}{4} φ_u^4 - E_u T φ_u^3 + D_u T^2 φ_u^2 + F_u φ_u^2.$$  

The minimum conditions are

$$V_{\text{eff}}(0) = \left. \frac{∂V_{\text{eff}}(φ_u)}{∂φ_u} \right|_u = 0; \quad \left. \frac{∂^2V_{\text{eff}}(φ_u)}{∂φ_u^2} \right|_u = m_{A_χ}^2 + m_{H_3}^2 + m_{S_η}^2 + m_{S_ρ}^2,$$

FIG. 2: The mass area corresponds to $S_ω > 1$ with real $T_C$ condition. The gaps on the lines $(S = 1, 2, 3)$ correspond to values that make $T_C$ to be complex.
where

\[
D_u = \frac{m_{A_u}^2}{24u^2} + \frac{m_{H_1}^2}{12u^2} + \frac{m_{H_2}^2}{24u^2} + \frac{m_{S_{\eta}}^2}{24u^2} + \frac{m_W^2}{4u^2} + \frac{m_{\chi}^2}{4u^2} + \frac{m_Y^2}{4u^2} + \frac{m_T^2}{8u^2} + M_4^2,
\]

\[
F_u = \frac{m_{A_u}^4}{32\pi^2 u^2} - \frac{m_{A_u}^2}{4} + \frac{m_{H_1}^4}{16\pi^2 u^2} + \frac{m_{H_2}^4}{16\pi^2 u^2} + \frac{m_{S_{\eta}}^4}{32\pi^2 u^2} - \frac{m_{H_3}^2}{4} - \frac{m_{S_{\eta}}^2}{4} - \frac{m_{S_{\rho}}^2}{4} + \frac{3m_W^4}{32\pi^2 u^2} + \frac{3m_Y^4}{8\pi^2 u^2},
\]

\[
E_u = \frac{m_{A_u}^3}{12\pi u^3} + \frac{m_{H_1}^3}{6\pi u^3} + \frac{m_{H_2}^3}{12\pi u^3} + \frac{m_{S_{\eta}}^3}{12\pi u^3} + \frac{m_{S_{\rho}}^3}{12\pi u^3} + \frac{m_W^3}{2\pi u^3} + \frac{m_Y^3}{2\pi u^3} - \frac{m_T^3}{2\pi u^3} + \frac{m_{\chi}^3}{2\pi u^3},
\]

\[
\lambda_u(T) = -\frac{m_{A_u}^4 \log \left( \frac{m_{A_u}^2}{T^2 a_b} \right)}{16\pi^2 u^4} - \frac{m_{H_1}^4 \log \left( \frac{m_{H_1}^2}{T^2 a_b} \right)}{8\pi^2 u^4} - \frac{m_{H_2}^4 \log \left( \frac{m_{H_2}^2}{T^2 a_b} \right)}{8\pi^2 u^4} - \frac{m_{H_3}^4 \log \left( \frac{m_{H_3}^2}{T^2 a_b} \right)}{16\pi^2 u^4} - \frac{m_{S_{\eta}}^4 \log \left( \frac{m_{S_{\eta}}^2}{T^2 a_b} \right)}{8\pi^2 u^4} - \frac{m_{S_{\rho}}^4 \log \left( \frac{m_{S_{\rho}}^2}{T^2 a_b} \right)}{16\pi^2 u^4} - \frac{m_W^4 \log \left( \frac{m_W^2}{T^2 a_b} \right)}{8\pi^2 u^4} - \frac{m_Y^4 \log \left( \frac{m_Y^2}{T^2 a_b} \right)}{8\pi^2 u^4} - \frac{3m_{\chi}^4 \log \left( \frac{m_{\chi}^2}{T^2 a_b} \right)}{8\pi^2 u^4}.
\]

\[
T_c = \frac{T_0}{\sqrt{1 - \frac{E^2}{2\lambda T_c}}}.
\]

\[
S = \frac{2E}{\lambda T_c}.
\]

The critical temperature and the phase transition strength are given by

Like the phase transition $SU(3) \to SU(2)$, in Fig. 4 we have plotted the relation between masses of the charged particles $K$ and neutral particles $L$ with some values of the phase transition strength.

However, we can fit the mass of heavy particle one again when considering the condition of $T_c$ to be real, so that Fig. 3 is redrew to Fig 4 and the maximum of strength is reduced from 3 to 2.12.

| Strength $S$ | $K[\text{GeV}]$ | $L[\text{GeV}]$ |
|-------------|-----------------|----------------|
| 1.0-2.12    | $195 \leq K \leq 484.5$ | $0 \leq L \leq 209.8$ |

TABLE V: Mass limits of particles with $T_C > 0$. 

The mass region of neutral and charged particles given in Table (V) leads the maximum phase transition strength which must be 2.12. This is larger than 1 but the EWPT is not strong.
FIG. 5: The dependence of the effective potential $V_{\text{eff}}(u)$ on the temperature with
$m_{H_3} = 118.6\text{GeV}$, $m_{H_1} = 333.6\text{GeV}$, $T_c = 125.88\text{GeV}$, and $S = 1$.

B. Three period EWPT in picture (ii)

In picture (ii), $m_{Z_2}^2 \simeq \frac{g^2 \omega^2}{(3-4s_w^2)}$ with the limit of $m_{Z_2}$ given in Eq. [35], we obtain $5.53 \text{TeV} \leq \omega \leq 6.3 \text{TeV}$. Therefore, we also assume $\omega = 6 \text{TeV}$ in this picture.

Because $\Lambda \gg \omega = 6 \text{TeV}$ and $\omega \gg u \sim v$, therefore there are three periods. The first process is $SU(3)_L \otimes U(1)_X \otimes U(1)_N \rightarrow SU(3)_L \otimes U(1)_X$. The second one is $SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_X$. The third process is $SU(2)_L \rightarrow U(1)_Q$. The third process is like $SU(2) \rightarrow U(1)$ in the picture (i).

The first process is a transition of the symmetry breaking of $U(1)_N$ group. It generates mass for $Z_1$ through $\Lambda$ or Higgs boson $S_4$. The third process is like the $SU(2) \rightarrow U(1)$ EWPT in picture (i). The second process is like the $SU(3) \rightarrow SU(2)$ in picture (i) but it does not involve $Z_1$ and $S_4$.

The second process has the effective potential is like Eq. [36]. In addition, parameters and the
minimum conditions are like Eqs. (37,39,40,41,42,43) without $Z_1$ and $S_4$.

![Graph showing EWPT strength](image)

**FIG. 6**: The strength EWPT $S_\omega = \frac{2E_u}{\lambda_{T_c}}$ with $\omega = 6$ TeV.

In our numbering process, when we import real $T_C$, the mass region of charged and neutral particles are

$$\begin{cases}
0 \leq m_{\text{Exotic quark/Charged Higgs boson}} \leq 4000 \text{ GeV}, \\
0 \leq m_{H_3} \leq 1000 \text{ GeV}.
\end{cases}$$

The mass region of charged bosons is narrower than that in Fig. 1. From Eq. (44), the maximum of $S$ has been estimated to be around 100.

V. **The role of neutral fermions in EWPT**

The masses of $N_R$ can be generated by the scalar content by itself via an effective operator invariant under the 3-3-1-1 symmetry and W-parity:

$$\lambda_{ab}\bar{\psi}_{aL}^C\psi_{bL}(\chi\chi)^*.$$

The mass scale of $N_R$ is unknown, however it can be taken in TeV scale. However, when analyzing the scattering of $N_R$ with distributions of $X, Y, Z_2$ bosons as given in [23], the mass of $N_R$ is drawn
as follows

\[ m_{N_R} = \frac{m_{Z_2}^2}{2.557 \text{ TeV}} \leq m_{Z_2}. \] (47)

In the \( SU(3) \rightarrow SU(2) \), if we add the contribution of neutral fermions, the maximum of \( S \) would decrease but the neutral fermions does not make lose the first-order EWPT, i.e., as our estimation in Table (VI).

| Period       | Picture | \( m_{Z_2} \) [TeV] | \( m_{N-R} \) [TeV] | \( S_{Max-no \ N_R} \) | \( S_{Max-N_R} \) |
|--------------|---------|---------------------|---------------------|-----------------------|-----------------|
| \( SU(3) \rightarrow SU(2) \) (i) | 2.386   | 2.227               | 70                  | 50                    |
| \( SU(3) \rightarrow SU(2) \) (ii) | 2.254   | 1.986               | 100                 | 30                    |

TABLE VI: Estimates for the maximum of EWPT strength with \( \omega = 6 \) TeV.

In Table [VI] we only estimate the maximum strengths and show that these maximum values are significantly reduced. However, it is very difficult to accurately calculate these values because the problem has many parameters (the mass of heavy particles) and these values can change slightly (but not too much) with different approximates.

More in Table [VI] when the absence of neutral fermions, in the picture (i), \( Z_1 \) is involved in the \( SU(3) \rightarrow SU(2) \) EWPT; while \( S = \frac{2E}{\lambda T_c} \), contributions of \( Z_1 \) make increasing \( E \) and \( \lambda \), but \( \lambda \) increase stronger than \( E \). In the picture (ii), \( Z_1 \) is not involved in the \( SU(3) \rightarrow SU(2) \) EWPT. Therefore \( S_{max} \) of the picture (ii) (100) is larger than \( S_{max} \) of the picture (i) (70).

Furthermore, when neutral fermions are involved in both two pictures, \( S_{max} \) in picture (ii) decreases faster than \( S_{max} \) in picture (i). Because there is a tension between \( Z_1 \) and neutral fermion in the picture (i); the same thing is not have in the picture (ii) in the absence of \( Z_1 \).

If neutral fermions follow the Fermi-Dirac distribution (i.e., they act as a real fermion but without charge), they increase the value of the \( \lambda \) and \( D \) parameters. Thus, they reduce the value of strength EWPT \( S \) because \( S = \frac{2E}{\lambda T_c} \) and \( E \) do not depend on neutral fermions.

This suggests that DM candidates are neutral fermions (or fermion in general) reduce the maximum value of the EWPT strength.

However, the EWPT process depends on bosons and fermions. Boson gives a positive contribution (obey the Bose-Einstein distribution) but fermion gives a negative contribution (obey the Fermi-Dirac distribution). In order to have the first order transition, the symmetry breaking
process must generate mass for more boson than fermion.

In addition, in this model, the neutral fermion mass is generated from an effective operator. This operator which demonstrates an interaction between neutral fermions and two Higgs fields. The above neutral fermion is very different from usual fermions. The $M$ parameter has an energy dimension, and it may be an un-known dark-interaction. Thus, neutral fermions only are effective fermions, according to the Fermi-Dirac distribution, but its statistical nature needs to be further analyzed with other data.

VI. Conclusion and outlooks

It is known that the mass of Goldstone boson is very small \cite{31} and the physical quantities are gauge independent so the critical temperature and the strength is gauge independent \cite{17} or the survey of effective potential in Landau gauge is also sufficient, or other word speaking, do not need in any gauge. Therefore, in the Landau gauge, the structure of EWPT in the 3-3-1-1 model with the effective potential at finite temperature has been drawn at the 1-loop level, this potential has two or three phases.

The self-interactions of Higgs, with $f$ parameter, in this model, that we do not consider in both two or three phase transitions. Thus calculating the corrections with $f$ can reveal many new physical phenomena. In addition, from the phase transitions, we can get some bounds on the Higgs self-couplings.

In conclusion, the model has many bosons which will be good triggers for first-order EWPT. The situation is that as less heavy fermion as the result will be better. However, strength of EWPT can be reduced by many bosons (such as $Z, Z_1, Z_2$ in the 3-3-1-1 model).

Although we only work on the 3-3-1-1 model, but this calculation can still apply to other models with multi-period EWPT. Our next works will calculate CP violations and correction of neutral fermion-dark matter, in order to analysis in details baryogenesis.

Acknowledgment

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2017.356.
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