The Unnatural Composite Higgs
(a.k.a “Split” Composite Higgs)

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Outline

• Introduction and motivation
• The “unnatural” composite Higgs
• Experimental signals of unnaturalness
• Conclusion
Higgs discovery - LHC Run I

Higgs potential: \[ V(h) = -\mu_h^2 |H|^2 + \lambda_h |H|^4 \]

\[ \langle H \rangle = \frac{1}{\sqrt{2}} (v + h) \]

\[ v^2 = \frac{\mu_h^2}{\lambda_h} \approx (246 \text{ GeV})^2 \]
\[ m_h^2 = 2\lambda_h v^2 \approx (126 \text{ GeV})^2 \]

\[ \mu_h^2 \approx (89 \text{ GeV})^2 \]
\[ \lambda_h \approx 0.13 \]
Higgs couplings

Looks very much like a SM Higgs boson!
However, SM is not a complete theory of Nature!

Questions:
- Planck/weak scale hierarchy? \( m_h \ll M_p \)
- Fermion mass hierarchy? Neutrino masses?
- GUTS? 3 fermion generations?
- Dark matter?
- Baryon asymmetry?
- Strong CP problem?
- Inflaton? Cosmological constant?
- UV completion of gravity?
What is the nature of the Higgs boson?

HIGGS BOSON

\{ 
\begin{align*}
\text{Elementary?} & \quad \rightarrow \quad \text{Standard Model/ Supersymmetry} \\
\text{OR} \\
\text{Composite?} & \quad \rightarrow \quad \text{New strong dynamics}
\end{align*}
\}

Understanding why $m_h \ll M_p$ can help address shortcomings in the SM
**Composite Higgs**

**Higgs as a pseudo Nambu-Goldstone boson** [Georgi, Kaplan `84]

Global symmetry $G$ spontaneously broken to subgroup $H$ at scale $f$

| $\rho^{(n)}$ | $\gtrsim$ TeV |
|-------------|--------------|

Resonance mass: $m_\rho \sim g_\rho f \quad 1 \lesssim g_\rho \lesssim 4\pi$

Higgs mass protected by shift symmetry -- like pions in QCD

**BUT** global symmetry must be explicitly broken to generate $V(h) \neq 0$
Global symmetry broken by mixing with elementary sector

\[ \mathcal{L}_{\text{mix}} = \lambda_{L,R} \bar{\Psi}_{L,R} \Psi_{L,R} + g_{\nu} A_{\mu} J_{\mu} \]

SM matter and gauge fields: \( \Psi_i, A_\mu \)

Higgs potential:

\[ V(h) = -\mu_h^2 |H|^2 + \lambda_h |H|^4 \]

where

\[ \mu_h^2 \sim \frac{g_{SM}^2}{16\pi^2} g_{\rho}^2 f^2 \]
\[ \lambda_h \sim \frac{g_{SM}^2}{16\pi^2} g_{\rho}^2 \]

EWSB (\( \langle H \rangle = \frac{v}{\sqrt{2}} \))

\[ v^2 = \frac{\mu_h^2}{\lambda_h} \]

\[ \text{Prefers: } f \sim v \]
Higgs mass: \( m_h^2 \simeq \frac{N_c}{\pi^2} m_t^2 \left( \frac{m_Q^2}{f^2} \right) = g_Q^2 \)

\( m_Q = \) fermion resonance mass

\( m_Q \sim m_\rho \gtrsim 2.5 \text{ TeV} \quad (g_Q \sim g_\rho \gtrsim 3) \quad \rightarrow \quad m_h \gtrsim m_t \)

But, no need for \( m_Q \sim m_\rho \)

\( m_h \sim 125 \text{ GeV} \quad \rightarrow \quad m_Q < m_\rho \)

*light fermion resonances*
HOWEVER, precision electroweak, flavor constraints

EWPT:
\[
\frac{s}{16\pi^2 v^2} H^\dagger T^a H B^{\mu\nu} W_{a\mu\nu} \quad S = \frac{s}{2\pi} \sim \frac{m_W^2}{m_\rho^2} \quad \Rightarrow \quad f \gtrsim \frac{2.5 \text{ TeV}}{g_\rho}
\]
\[-\frac{t}{16\pi^2 v^2} ((D^\mu H)^\dagger H)(H^\dagger D_\mu H) \quad T = \frac{t}{8\pi e^2} \sim \frac{v^2}{f^2} \quad \Rightarrow \quad f \gtrsim 5.5 \text{ TeV}
\]
e.g. FCNC
\[
\epsilon_q^i \epsilon_q^j \epsilon_q^k \epsilon_q^l \frac{g_\rho^2}{m_\rho^2} q^i q^j q^k q^l \quad \epsilon_q^i \sim \frac{g_i}{g_\rho} \quad \Rightarrow \quad f \gtrsim 10 \text{ TeV}
\]
\[\text{[Bellazzini, Csaki, Serra 1401.2457]} \quad \text{[Panico, Wulzer 1506.01961]}\]

\[f \gg v\] \quad “Little” hierarchy

Tension partly alleviated by complicating minimal models

e.g. custodial symmetry, flavor symmetry....
LHC Limits: The Missing Resonances Problem

\[ m_T \gtrsim 600 - 800 \text{ GeV} \]
“Natural” models increasingly elaborate and tuned: \[ \Delta^{-1} \sim \frac{v^2}{f^2} \lesssim 5\% \]
Simple solution:

Assume $f \gtrsim 10 \text{ TeV}$  – no need for custodial or flavor symmetries!

Tuned Higgs potential

$$V \sim c_2 f^2 |H|^2 + c_4 |H|^4$$

This compares to $\sim 10^{-28}$ in SM!

Is there a motivated upper bound for $f$?

Yes!
Partial compositeness: 

\[ \mathcal{L} = \lambda_L \psi_L \mathcal{O}_R + \lambda_R \psi_R \mathcal{O}_L \]

Explains the fermion mass hierarchy

\[ m_f \sim \lambda_L \lambda_R v \]

where

\[ \lambda_{L,R} \sim \left( \frac{\Lambda}{\Lambda_{UV}} \right)^{\text{dim } \mathcal{O}_{L,R} - \frac{5}{2}} \]

- Light fermions are mostly elementary
- Top quark is mostly composite!
Gauge coupling unification

Assume composite $t_R$ and coset $\mathcal{G}/\mathcal{H}$

$$(t_R, \chi^c) = \text{complete } \mathcal{H} \text{ multiplet}$$

Decoupled with top “companions” $\chi$  

Dirac mass: $m_\chi \sim \lambda_\chi f$

New contribution to the running of SM gauge couplings

$$\alpha_i(\mu) - \alpha_j(\mu) = \text{SM} - \left\{ H, t^c \bar{t}^c \right\}$$

One-loop beta function coefficients:

$$b_1 - b_2 = \frac{94}{15} \quad b_2 - b_3 = \frac{13}{3} \quad \triangleleft$$

$$\frac{b_2 - b_3}{b_1 - b_2} \approx 0.69$$

c.f. MSSM value = 0.71
Requiring $\delta_3 = 0 \quad (b_{\text{strong}} = 5)$

$f \lesssim 500 \text{ TeV}$
**Minimal Coset:** \( SU(7)/SU(6) \times U(1) \)

- contains \( SU(5) \) --universal corrections to running
- scalar singlet dark matter

\[
w = e^{i \pi} \begin{pmatrix} 0_{(6)} \\ 1 \end{pmatrix} = \frac{1}{f} \begin{pmatrix} H \\ S \\ \sqrt{f^2 - |H|^2 - |S|^2} \end{pmatrix}
\]

12 Nambu-Goldstone bosons

\( = 5 \) of \( SU(5) \) + 1 singlet

\( H = \) Higgs doublet, \( D + SU(3) \) triplet, \( T \)
**Matter embeddings**

SM matter embedding under $SU(5) \times U(1)_L \times U(1)_B$

$q \in 10_{0,\frac{1}{3}} \quad u^c \in 10_{0,-\frac{1}{3}} \quad d^c \in \overline{5}_{0,-\frac{1}{3}} \quad e^c \in 10_{-1,0} \quad l \in \overline{5}_{1,0}$

Composite top $t_R \subset 15$ of SU(6)

Top companions $\chi \subset \overline{15}$

$$\chi = \tilde{q}^c \oplus \tilde{e} \oplus \tilde{d}^c \oplus \tilde{l} = (\overline{3}, 2)_{-\frac{1}{6}} \oplus (1, 1)_{-1} \oplus (\overline{3}, 1)_{\frac{1}{3}} \oplus (1, 2)_{-\frac{1}{2}}$$

**Mixing Lagrangian**

$$\mathcal{L} \supset (\tilde{q}^c, \tilde{e}) \lambda^{10}_\chi \mathcal{O}_t^{35} + (\tilde{d}^c, \tilde{l}) \lambda^{5}_\chi \mathcal{O}_t^{35} + q \lambda_t \mathcal{O}_t^{35} + q \lambda_b \mathcal{O}_b^{35} + b^c \lambda_{bc} \mathcal{O}_{bc}^{35}$$

$$+ l \lambda_\nu \mathcal{O}_\nu^{21} + l \lambda_\tau \mathcal{O}_\tau^{21} + N^c \lambda_{Ne} \mathcal{O}_{Ne}^{21} + \tau^c \lambda_{\tau c} \mathcal{O}_{\tau c}^{21} + m_N N^c N^c$$
Effective Lagrangian

Integrate out strong sector

\[
\mathcal{L}_{\text{eff}} \supset (\bar{q}^c, \bar{e})_{i_4i_2} \mathcal{P}(q^c, e)^{j_1j_2} \left[ \Pi^{\chi^*} (\lambda_\chi^{10*})_{IJK}^{i_4i_2} (\lambda_\chi^{10})_{j_1j_2} \right] \Sigma^K_L 
+ (\bar{q}^c, \bar{e})_{i_4i_2} \mathcal{P}(d^c, \bar{l})^{j_5} \left[ \Pi^{\chi^*} (\lambda_\chi^{10*})_{IJK}^{i_4i_2} (\lambda_\chi^5)_{j_5} \right] \Sigma^K_L + \text{h.c.} 
+ (\bar{d}^c, \bar{l})_{i_5} \mathcal{P}(d^c, \bar{l})^{j_5} \left[ \Pi^{\chi^*} (\lambda_\chi^5)_{IJK}^{i_5} (\lambda_\chi^5)_{j_5} \right] \Sigma^K_L 
+ q^{i_3i_2} p q_{j_3j_2} \left[ \Pi^{tt} (\lambda_t^*)_{i_3i_2, IJK} (\lambda_t)^{j_3j_2, IJL} + \Pi^{bb} (\lambda_b^*)_{i_3i_2, IJK} (\lambda_b)^{j_3j_2} \right] \Sigma^K_L 
+ \bar{b}_{i_3}^{c} \bar{b}^{c} q_{j_3j_2} \left[ \Pi^{bbc} (\lambda_b^*)_{IJK} (\lambda_b^*)^{j_3j_2} \right] \Sigma^K_L 
+ (\bar{q}^c, \bar{e})_{i_4i_2} \mathcal{P}(q^c, e)^{j_1j_2} \left[ \Pi^{tt} (\lambda_t^{10*})_{IJK}^{i_4i_2} (\lambda_t)^{j_3j_2, IJL} \right] \Sigma^K_L + \text{h.c.} 
+ (\bar{d}^c, \bar{l})_{i_5} \mathcal{P}(d^c, \bar{l})^{j_5} \left[ \Pi^{tt} (\lambda_t^{10*})_{IJK}^{i_5} (\lambda_t)^{j_3j_2, IJL} \right] \Sigma^K_L + \text{h.c.} 
+ q_{i_3i_2} b^{c} q_{j_3j_2} \left[ M^{bbc} (\lambda_b)_{IJK} (\lambda_b)_{j_3j_2} \right] \Sigma^K_L + \text{h.c.}
\]

where \( \Sigma = w^\dagger w \) = adjoint spurion (contains D, T and S)

\( \Pi, M^{bbc} \) = momentum dependent form factors
**pNGB potential**

**Elementary fermion contribution:**

\[
(pNGB)^{\mu L} \Sigma^F_{\lambda \nu} (\lambda_{\psi})^I_{JK} \psi
\]

\[
\int \frac{d^4 p}{(2\pi)^4} \Pi(p) = c_1^\psi \frac{g_\rho^2}{16\pi^2} f^4
\]

\[
V_{\text{matter}} = \frac{g_\rho^2 f^2}{24\pi^2} c_1^\chi |\lambda_\chi|^2 \left(12 - 9|T|^2 - 7|D|^2 - 7|S|^2\right) + \frac{g_\rho^2 f^2}{24\pi^2} c_1^t |\lambda_t|^2 \left(4|T|^2 + 3|D|^2\right)
\]
\[
+ \frac{g_\rho^2 f^2}{24\pi^2} c_1^b |\lambda_b|^2 \left(2|T|^2 + 3|D|^2 + 6|S|^2\right) + \frac{g_\rho^2 f^2}{24\pi^2} c_1^{bc} |\lambda_{bc}|^2 \left(3 - 2|T|^2 - 3|D|^2\right).
\]

**Elementary gauge boson contribution:**

\[
\Omega^a_{\mu L} (T^A)^I_{\lambda \nu} \Sigma^F_{\lambda \nu} (T^B)^K_{\lambda \nu} \Omega^b_{\mu L}
\]

\[
\int \frac{d^4 p}{(2\pi)^4} \Pi_{A,B}^{A,B}(p) = c_1^A \frac{g_\rho^2}{16\pi^2} f^4
\]

\[
V_{\text{gauge}} = \frac{3g_\rho^2 f^2}{16\pi^2} c_1^A \left(\frac{4}{3} g_3^2 |T|^2 + \frac{3}{4} g_2^2 |D|^2\right) + \frac{3g_\rho^2 f^2}{16\pi^2} c_2^A \left(\frac{1}{3} g_3^2 |T|^4 + \frac{1}{4} g_2^2 |D|^4\right)
\]
Obtain:

\[ V(|D|) = -\frac{\alpha}{f^2} |D|^2 + \frac{\beta}{f^4} |D|^4 \]

Electroweak VEV:

\[ v = f \sqrt{\frac{\alpha}{\beta}} \]

Higgs mass:

\[ m_h^2 = \frac{2\beta v^2}{f^4} = \frac{3c_2^A g_\rho^2}{8\pi^2} M_W^2 \]

where

\[ \alpha = \frac{g_\rho^2}{16\pi^2} f^4 \left( \frac{14}{3} c_1^x |\lambda_x|^2 - 2c_1^t |\lambda_t|^2 - 2c_1^b |\lambda_b|^2 + 2c_1^{bc} |\lambda_{bc}|^2 - \frac{9}{4} c_1^A g_2^2 \right) \]

\[ \beta = \frac{g_\rho^2}{16\pi^2} f^4 \left( \frac{3}{4} c_2^A g_2^2 \right) \]

Higgs potential

must be tuned

W-boson mass

Requires: \( c_2^A \sim \frac{64}{g_\rho^2} \sim 0.5 - 4 \)
Also have $\langle |S| \rangle = \langle |T| \rangle = 0$ with

\[ m_T^2 \approx \frac{g_\rho^2}{16\pi^2} f^2 \left( -6c_1^X|\lambda_X|^2 + \frac{8}{3} c_1^t|\lambda_t|^2 + \frac{4}{3} c_1^b|\lambda_b|^2 - \frac{4}{3} c_1^b|\lambda_{bc}|^2 + 4c_1^A g_3^2 \right) \]

\[ m_S^2 \approx \frac{g_\rho^2}{16\pi^2} f^2 \left( -\frac{14}{3} c_1^X|\lambda_X|^2 + 4c_1^b|\lambda_b|^2 \right) \]

\[ m_T \sim \frac{g_\rho}{4\pi} \max \left[ |\lambda_\psi|, g_3 \right] f \]

**triplet mass**

**singlet mass**

\[ m_S \sim \begin{cases} 
\frac{g_\rho}{4\pi}|\lambda_b|f & \sim \frac{g_\rho}{4\pi}|\lambda_b|m_X \quad |\lambda_X| \lesssim |\lambda_b| \\
\frac{g_\rho}{4\pi}|\lambda_X|f & \sim \frac{g_\rho}{4\pi}m_X \quad |\lambda_X| \gtrsim |\lambda_b| \\
\ll \frac{g_\rho}{4\pi}|\lambda_X|f & \lesssim m_X \quad |\lambda_X| \sim |\lambda_b| 
\end{cases} \]

Possible light singlet
Dark matter stability

Enlarge global group:

$$U(7) \equiv SU(7) \times U(1)_E.$$ 

|          | $U(1)_q$ | $U(1)_l$ | $U(1)_H$ | $U(1)_L$ | $U(1)_B$ | $Z_3$ |
|----------|----------|----------|----------|----------|----------|-------|
| $T$      | 0        | 0        | -2       | 0        | 0        | -1    |
| $D$      | 0        | 0        | -2       | 0        | 0        | 0     |
| $S$      | 0        | 7        | 10       | 0        | $\frac{1}{3}$ | 1     |
| $q_{(u)}$| -1       | 6        | 11       | 0        | $\frac{1}{3}$ | 0     |
| $q_{(d)}$| 1        | 6        | 11       | 0        | $\frac{1}{3}$ | 0     |
| $u^c$    | 1        | -6       | -9       | 0        | $-\frac{1}{3}$ | 0     |
| $d^c$    | 1        | -6       | -13      | 0        | $-\frac{1}{3}$ | 0     |
| $l_{(\nu)}$ | 0       | 0        | 2        | 1        | 0        | 0     |
| $l_{(e)}$ | 0       | 2        | 2        | 1        | 0        | 0     |
| $N^c$    | 0        | 0        | 0        | -1       | 0        | 0     |
| $e^c$    | 0        | -2       | -4       | -1       | 0        | 0     |
| $\tilde{q}^c$ | -1   | 6        | 9        | 0        | $\frac{1}{3}$ | -1 |
| $\tilde{e}$ | -1 | 6       | 9        | 0        | $\frac{1}{3}$ | 1 |
| $\tilde{d}^c$ | -1 | -1  | -3       | 0        | 0        | 1     |
| $\tilde{l}$ | -1 | -1  | -3       | 0        | 0        | 0     |

NG bosons

SM fermions

top companions

Nonzero baryon triality leads to stability!
The “Unnatural” Composite Higgs model

Low-energy spectrum: Standard Model + S + T + $\chi$

What are experimental signals?
Dark matter constraints

singlet Higgs partner S -- Higgs portal coupling \( V \supset \kappa |D|^2 |S|^2 \)

where \( \kappa \sim 0.02 \left( \frac{m_X}{f} \right)^4 \)

\( f = 10 \text{ TeV} \)

180 GeV \( \lesssim m_S \lesssim 10 \text{ TeV} \)

10 TeV \( \lesssim m_X \lesssim 40 \text{ TeV} \)
Higgs couplings

LHC: 1-5 % precision  
ILC: 0.5 - 1% precision

\[ f \gtrsim 10 \text{ TeV} \quad \quad \quad \quad \quad \quad \frac{v^2}{f^2} \lesssim 10^{-4} \]

\[ \frac{g_{hWW}^{SM}}{g_{hWW}^{SM}} \sim \frac{g_{hff}^{SM}}{g_{hff}^{SM}} \sim \sqrt{1 - \frac{v^2}{f^2}} \]

Tiny deviations –too small to be seen at LHC/ILC

Higgs boson is very SM-like!
Exotic state phenomenology

- *top companions* $\chi$

$$\tilde{q}^c \in (\overline{3}, 2)_{-\frac{1}{6}}, \quad \tilde{e} \in (1, 1)_{-1}, \quad \tilde{d}^c \in (\overline{3}, 1)_{\frac{1}{3}}, \quad \tilde{l} \in (1, 2)_{-\frac{1}{2}}$$

$$f = 10 \text{ TeV} \quad \Rightarrow \quad m_\chi \sim (1-2)f \sim 10-20 \text{ TeV}$$

Decays are collider-prompt

- *e.g.* $\tilde{q}^c \rightarrow Tq, \quad \tilde{d}^c \rightarrow t^cTS$
- *e.g.* $\tilde{e} \rightarrow bTT, \quad \tilde{l} \rightarrow qTS$

Can be searched for at a future 100 TeV collider!
• **triplet Higgs partner** $T$

\[ T \in (3, 1)_{-\frac{1}{3}} \]

(like RH sbottom in SUSY)

\[ f = 10 \text{ TeV} \quad \Rightarrow \quad m_T \sim (1-2)\frac{f}{\pi} \sim 3-5 \text{ TeV} \]

\[ \mathcal{L} \supset \frac{c_T^3}{24\pi^2 f^2} |\lambda_{bc}||\lambda_\nu||\lambda_\tau| S^2 (T^\dagger t^c b^c) \]

dimension-6 term

\[ f > 10 \text{ TeV} = \text{long-lived decay} \]

\[ T \rightarrow tbSS \quad \Rightarrow \quad c_T \approx 0.2 \text{ mm} \left( \frac{1}{c_T^3} \right)^2 \left( \frac{8}{g_\rho} \right)^3 \left( \frac{3 \text{ TeV}}{m_T} \right)^5 \left( \frac{f}{10 \text{ TeV}} \right)^4 \]

\[ \text{can produce a displaced vertex!} \]
Color triplet decay

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LHC:

LHC 300 fb⁻¹ (f = 10 TeV)
Future 100 TeV collider:

[Barnard, Cox, TG, Spray: 1510.06405]
Summary

- $f \gtrsim 10\,\text{TeV}$ simply eliminates all precision electroweak and flavor constraints
  - Higgs potential is tuned at $10^{-4}$ level
  - “Unnatural” or “split” composite Higgs
- SU(7)/SU(6)$\times$U(1) minimal model
  - Improves gauge coupling unification
  - Explains fermion mass hierarchy
- Higgs partners: $S =$ dark matter, $T =$ color triplet
- Long-lived $T$ decays = sign of unnaturalness!