Double Beta Decay and Spin-Isospin Ground-State Correlations

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Abstract. Nuclear double beta decay is reviewed from a nuclear structure point of view, with an emphasis on spin-isospin modes of transitions. The double beta decay is expected to reveal fundamental properties of neutrinos by using nuclei as a microscopic laboratory. The goal of nuclear structure studies is to predict reliable nuclear matrix elements relevant to the determination of neutrino mass.

1. Double beta decay and promising nuclides
The nuclear double beta ($\beta\beta$) decay is a second order process of the weak interaction, with a change of the atomic number by two units [1, 2]. The shortest half-life is in the order of $T_{1/2} \approx 10^{19}$ y [3].

The $\beta\beta$ decay can be observed when single $\beta$ decays are forbidden energetically. Figure 1 shows the mass relation of $A = 76$ nuclei. Among them, $^{76}$Se is most stable, and nuclei on the right hand side are converted by $\beta^+$ decays toward $^{76}$Se, and those on the other side by $\beta^-$ decays. However, neither $\beta^-$ decay nor $\beta^+$ decay are allowed for $^{76}$Ge, since the mass is higher than $^{76}$As and $^{76}$Ga, and $\beta\beta$ decay can be observed from $^{76}$Ge to $^{76}$Se.

$^{76}$Ge is one of the most promising nuclei for observation of $0\nu\beta\beta$ decay, which will be explained below. Observation naturally favors those nuclei with a large $Q_{\beta\beta}$-value and large abundance. The typical nuclides, in addition to $^{76}$Ge, are $^{48}$Ca, $^{82}$Se, $^{100}$Mo, $^{128,130}$Te, $^{136}$Xe and $^{150}$Nd.

2. Two decay modes
There are two decay modes, one is the two-neutrino ($2\nu$) mode and the other is the neutrinoless ($0\nu$) mode.

$2\nu$ mode : $(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\nu$

$0\nu$ mode : $(Z, A) \rightarrow (Z + 2, A) + 2e^-$
The former is allowed in the standard model. It occurs as sequential single β decays through virtual intermediate states. The 2νββ decay has been observed for about ten nuclides [3].

On the other hand, no clear evidence has been obtained for the observation of 0νββ decay. This process is forbidden in the standard model, since it violates the lepton number conservation. A neutrino emitted by a neutron is absorbed by another neutron in the same nucleus, which may be described as

\[
n \rightarrow p + e^- + \nu_e
\]

\[
\nu_e + n \rightarrow p + e^-.
\]

In the standard model, an antineutrino is emitted in the first process, and the one which is absorbed in the second process is a neutrino. In addition, the antineutrino is right-handed, while the neutrino is left-handed. This indicates the following [1], if 0νββ decay is observed:

- Neutrinos are Majorana particles, \( \nu_e = \bar{\nu}_e \). The 0νββ decay is at present the only practical possibility to distinguish between Dirac and Majorana neutrinos.
- Neutrinos have finite masses, though this has already been found by observation of neutrino oscillations.
- Information on the absolute mass scale of neutrinos is obtained by a half-life measurement. Neutrino oscillations constrain \( \Delta m^2_{12} = m_1^2 - m_2^2 \), while the 0νββ decay gives us \( \langle m_\nu \rangle = \left| \sum_{j=1}^3 U_{ej} m_j \right| \) where \( m_j \) are masses of mass-eigenstate neutrinos, and \( U_{ej} \) are neutrino mixing parameters.

The inverse half-life of 0νββ decay by neutrino mass mechanism is given by

\[
\left[ T_{0\nu}^{1/2} \right]^{-1} = G^{0\nu} \langle m_\nu \rangle^2 | M^{0\nu} |^2.
\]

The phase space integral \( G^{0\nu} \) is calculated accurately, and, therefore, the neutrino mass \( \langle m_\nu \rangle \) that is deduced from an observed half-life depends on the relevant nuclear matrix element \( M^{0\nu} \).

The goal of nuclear structure calculations is thus to predict the nuclear matrix element reliably. A testing ground of nuclear structure calculations may be provided by 2νββ decay, since the inverse half-life

\[
\left[ T_{2\nu}^{1/2} \right]^{-1} = G^{2\nu} | M^{2\nu} |^2
\]

contains no parameters on the particle-physics side, and the calculated half-lives can directly be compared with experiments.

3. Two-neutrino mode

The nuclear matrix element of 2νββ decay is calculated by sequential Gamow-Teller transitions through virtual 1^+ states in the intermediate nucleus,

\[
M^{2\nu} = \sum_a \frac{\langle 0_f^+ \parallel t_\sigma \parallel 1_a^+ \rangle \langle 1_a^+ \parallel t_\sigma \parallel 0_i^+ \rangle}{E_a - (M_i + M_f)/2}.
\]

Some twenty years ago, suppression mechanism of 2νββ decay was found by calculations employing proton-neutron quasiparticle random-phase approximation (QRPA) [4, 5, 6]. The nuclear interaction between a proton and a neutron in the \( J^z = 1^+ \) channel is strongly attractive, and it enhances spin-isospin ground-state correlations considerably. The correlations are essentially four-quasiparticle components, and Gamow-Teller transitions from these components interfere destructively with those from the predominant zero-quasiparticle component, resulting in a substantial suppression of β^+ strengths from the \( 0_f^+ \) ground state of the daughter nucleus to intermediate 1^+ states, which is depicted in Figure 2 [2].
It was also found that the nuclear matrix element $M^{2\nu}$ is very sensitive to the proton-neutron interaction in the $1^+$ channel. Then we introduced a parameter $g_{pp}$ by hand as

$$\langle j_p j_n | V | j_p' j_n' \rangle_J \Rightarrow g_{pp} \langle j_p j_n | V | j_p' j_n' \rangle_J.$$

The nuclear matrix element calculated with a $G$-matrix of the Paris potential is shown in Figure 3 by a dashed line as a function of $g_{pp}$. It should be mentioned here that particle-hole matrix elements $\langle j_p j_n^{-1} | V | j_p' j_n^{-1} \rangle_J$ are unchanged, though they are connected to particle-particle matrix elements by Pandya transformation. However, particle-particle and particle-hole matrix elements are clearly separated in the QRPA formalism.

4. Neutrinoless mode

The $0\nu\beta\beta$ decay can occur when the neutrino is a massive Majorana particle. Then, a neutrino is exchanged between two nucleons in a nucleus, and it gives rise to a potential which acts on the nucleons, as the exchange of a pion is described by a Yukawa potential between two interacting nucleons,

$$V_\nu(r) \sim \int dq \frac{\exp(iq \cdot r)}{\omega [E_i - (\omega + m_e + E_a)]}, \quad \omega = \sqrt{q^2 + m_\nu^2}.$$
The 0νββ-decay nuclear matrix element is $M^{0\nu} = M^{0\nu}_{GT} - M^{0\nu}_F$, the former comes from the axial-vector component of the weak interaction and the latter from the vector component. They are explicitly given, by a multipole expansion of the neutrino potential, as [2, 13]

$$M^{0\nu}_{GT} = \int_0^\infty q^2 dq v(q) \sum_{LJ}\langle 0^+ \mid F_{1LL} \mid J_a^\pi \rangle \langle J_a^\pi \mid F_{1LL} \mid 0^+_i \rangle$$

$$F_{1LL} = t_- [\sigma \otimes Y_L (\hat{r})]_J j_L(qr) \quad J^\pi \neq 0^+$$

$$M^{0\nu}_F = (2V/g_A)^2 \int_0^\infty q^2 dq v(q) \sum_{LJ}\langle 0^+ \mid F_{0LL} \mid J_a^\pi \rangle \langle J_a^\pi \mid F_{0LL} \mid 0^+_i \rangle$$

$$F_{0LL} = t_- Y_L (\hat{r}) j_L(qr) \quad J^\pi = 0^+, 1^-, 2^+, 3^-, \ldots$$

Here, $v(q)$ is the Fourier transform of the neutrino potential including the energy denominator of the perturbation theory, and $j_L(qr)$ are spherical Bessel functions. Because of the neutrino potential, all spin-parities, $J^\pi$, possible in the model space are allowed for nuclear intermediate states, in a sharp contrast to the 2νββ decay. Numerical calculations are performed usually by closure approximation, which takes a sum over intermediate states by closure. Effects of the approximation have been evaluated to be within a few percent [14], since the momenta carried by the exchanged neutrino $q \sim 100$ MeV/c, which corresponds to an energy much larger than average excitation energies of the nuclear intermediate states.

Figure 4 shows the decomposition of the 0νββ-decay nuclear matrix element, which is possible under the closure approximation by making use of Racah algebra [2, 13]. All the components through various $J^\pi$-states have the same sign, except the tiny components of the largest $J$-values. This is probably because of the long-range nature of the neutrino potential. The potential resembles a Yukawa potential for the exchange of a particle of the mass around 10 MeV.

![Figure 4](image1.png)

**Figure 4.** Decomposition of the 0νββ-decay nuclear matrix element into components through intermediate states with spin-parity $J^\pi$. The light grey histogram represents $M^{0\nu}_{GT}$ and the heavier one $-M^{0\nu}_F$.

![Figure 5](image2.png)

**Figure 5.** Nuclear matrix elements of 2νββ and 0νββ decays.

Of the 0νββ-decay nuclear matrix element, the component through 1+ intermediate states is most sensitive to the spin-isospin ground-state correlations. In the lower panel of Figure 5 is shown the behavior of the 1+ component (dotted line) and the sum of the others (dash-dotted line), calculated as functions of $g_{pp}$. The nuclear matrix element $M^{0\nu}_{GT} - M^{0\nu}_F$ is rather stable against the change of $g_{pp}$ in the region of $g_{pp} \approx 1.0$. 
Since the $1^+$ component with the largest uncertainties behaves like the $2\nu\beta\beta$-decay nuclear matrix element, experimental half-lives of the latter process can be used to pinpoint the parameter $g_{pp}$, see Figure 5. A systematic, extensive analysis along this line has been performed with different QRPA models, different model spaces, different effective NN interactions and different axial-vector coupling constants [15]. This analysis provides so far the most reliable prediction, the uncertainty of about 30%, of $0\nu\beta\beta$-decay nuclear matrix element within the QRPA models.

5. Spin-isospin excitations

A nuclear structure calculation is performed in a model space, for example, a $2\hbar\omega$ space in QRPA calculations. Accordingly, transition operators should be renormalized, and the simplest procedure of renormalization is to use effective coupling constants.

Electric transitions seem to have no renormalization. The lowest order is the electric dipole mode ($1^-$), which is the isovector type. Experiments have shown that the $E1$ strength is concentrated in a giant dipole resonance and the observed strengths agree well with the well-known Thomas-Reiche-Kuhn sum rule. Electric quadrupole ($2^+$) transitions are calculated by using effective charges. A systematic analysis of $E2$ transitions in sd-shell nuclei yielded a large isoscalar polarization charge of $2\delta e_s = e_n + (e_p - e) = (0.78 \pm 0.03)e$, but a small isovector charge of $2\delta e_i = e_n - (e_p - e) = (0.2 \pm 0.1)e$ [16]. The $2^+$ component of $0\nu\beta\beta$ decay, which involves no isoscalar transitions, would thus be little affected by a truncation of model space. It should be emphasized here that such a comparison with experimental strengths is possible only when good wave functions are available.

The most crucial transition mode for $\beta\beta$ decay is the Gamow-Teller type ($1^+$). Experimental studies of Gamow-Teller transitions by charge-exchange reactions have shown that the observed strengths satisfy the Ikeda sum rule [10], and that the strength is extended to high excitation energy regions [17]. This indicates a quenching of Gamow-Teller strength in nuclear structure calculations, which corresponds to an effective value of the axial vector coupling constant of $g_A^{\text{eff}} \approx 1.0$ instead of the bare value $g_A = 1.26$.

Gamow-Teller transition strengths are sensitive to spin-orbit partners, for example for $^{76}\text{Ge}$, a deeply bound $f_{7/2}$ for $f_{5/2}$, and a high-lying $g_{7/2}$ for $g_{9/2}$. In Figure 6 are shown $2\nu\beta\beta$-decay nuclear matrix elements calculated by EQRPA. The solid line represents the result of the ordinary EQRPA in a model space consisting of $pf$ and $sdg$ shells, the dashed line without $g_{7/2}$, and the dotted line without $f_{7/2}$ and $g_{7/2}$. Though the probabilities are very small of the occupation in $g_{7/2}$ and of the vacancy in $f_{7/2}$, these single-particle orbits are indispensable for the suppression of the matrix element which is necessary for reproducing the experiment half-life. The spin-orbit partners play a decisively important role, through ground-state correlations, especially in $d^+_1$ transitions, since they are due to single-particle transitions near the Fermi surface and are much affected by Pauli principle.

Next leading magnetic transitions are spin-dipole modes ($0^-, 1^-$ and $2^-$). A systematic study from both experimental and theoretical sides would also be helpful for a reliable prediction of
\(\beta\beta\) decay. Observation of spin-dipole transitions is expected in neutron-rich nuclei in light- and medium-light mass regions, where last protons and neutrons occupy different major shells.

Shell-model calculations of \(\beta\beta\) decay including spin-orbit partners, \(f_7/2-f_5/2\) and \(g_9/2-g_7/2\) for \(^{76}\text{Ge} \rightarrow ^{76}\text{Se}\), would be a hard task. But, calculations with such sophisticated models are necessary, since QRPA models have limitations due to the simplified formulation, which prevents from taking into account various correlations that should be present in nuclear states. The spin-orbit partners could be included in shell model by perturbation [18]. An alternative approach might be a shell model in a quasiparticle basis [19].

6. Summary

The neutrinoless mode of \(\beta\beta\) decay is expected to reveal basic properties of neutrinos, if they are Majorana particles or Dirac particles, and the absolute mass scale of neutrinos. The two-neutrino mode provides a testing ground of nuclear structure calculations, but the \(2\nu\beta\beta\) decay is very sensitive to spin-isospin ground-state correlations. The \(0\nu\beta\beta\)-decay nuclear matrix element has been found to be less sensitive to nuclear structure because of the exchange of a neutrino between nucleons. Within quasiparticle RPA models, an uncertainty of about 30\% has been obtained by making use of experimental \(2\nu\beta\beta\)-decay half-lives to pindown \(g_{pp}\), namely the spin-isospin ground-state correlations. However, the predictions could have possible "systematic errors" due to the QRPA formalism. Theoretical predictions by other models, such as shell model, are anticipated for both \(2\nu\) and \(0\nu\) decay modes. On the other hand, experimental data are helpful for refining nuclear structure models. Gamow-Teller transitions between low-lying states, especially the \(\beta^+\) branch, are of particular importance, since they are most sensitive to the spin-isospin ground-state correlations, and could be compared state-by-state with theoretical predictions. The next important mode would be spin-dipole transitions.

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