Oscillatory porous medium ferroconvection with Maxwell-Cattaneo law of heat conduction

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Abstract. The scheme of small perturbation is used to address the problem of buoyancy driven convection of Darcy-Brinkman type in a ferromagnetic fluid invoking the Maxwell-Cattaneo law. An analytical solution of the eigenvalue problem involved, encompassing stationary and overstable modes, is obtained by adopting simplified boundary conditions. The mathematical application package MATHEMATICA is adopted to determine the eigenvalue expressions and the critical numbers. It is established that the threshold of Darcy-Brinkman ferroconvection is amplified through the stresses of magnetic and second sound mechanisms and the opposite influence is found to be true due to the presence of porous medium. It is delineated that both the critical frequency of oscillations and aspect ratio of cells of convective heat transfer are susceptible to the different parameters of the study. It is also shown that, as long as the Cattaneo and Prandtl numbers are pretty high, the oscillatory mode of instability is preferred to the stationary mode of ferroconvection.

1. Introduction
It has been well recognized that ferromagnetic fluids revel in a number of motivating and expedient physical properties including magnetic characteristics as a solid. A versatile external magnetic field could be employed to facilitate and control heat and mass transfer that can take place in ferrofluids. Several researchers pointed out a good number of real-life applications of magnetic fluids including rotating seals, novel energy conversion and levitation devices (Popplewell [1], Bashtovoy et al [2], Berkovskii et al [3] and Horng et al [4]). A thoroughgoing research work on ferroconvection was first carried out by Finlayson [5] who carefully explicated how to approach ferroconvection due to magnetic body force resulting from magnetization. This type of heat transfer involving ferrofluids could be exploited to induce convection in miniature micro-scale devices.

Gotoh and Yamada [6], Stiles and Kagan [7] and Russell et al [8] extended the pioneering contribution of Finlayson [5] to deal with large wave number ferroconvection. Gupta and Gupta [9] and Saravanan [10] investigated the ferroconvection problem with centrifugal acceleration. It is proved that oscillatory ferroconvection is possible as long as the Prandtl number is less than unity. Maruthamanikandan [11] employed a residual method to examine the problem of ferroconvection with radiation. Floquet theory was used by Aniss et al [12] and Bajaj [13] to examine respectively the impact of magnetic and gravity modulations on ferroconvection. The regions of harmonic and sub-harmonic modes have been obtained. Nisha Mary and Maruthamanikandan [14] used regular perturbation technique to address the non-Darcy ferroconvection problem with gravity modulation.
The effect of viscosity variation on non-Darcy ferroconvection was studied by Soya Mathew and Maruthamanikandan [15] and Maruthamanikandan et al [16]. Both temperature and field dependent viscosities have been incorporated into the governing equations.

The study of porous medium convection, on the other hand, was warranted by its applications to geothermal activities, oil recovery techniques and chemical processes (Ingham and Pop [17], Vafai [18] and Nield and Bejan [19]). Further, applications relating to superfluid state necessitated the knowledge of second sound which has to do with the hyperbolic form of energy equation (Straughan [20] and Straughan and Franchi [21]). Lebon and Cloot [22] examined the effect of second sound with surface tension and buoyancy effects. Haddad and Straughan [23] showed that, under normal terrestrial conditions, stationary convection is favoured but with large thermal relaxation time, oscillatory convection is likely to occur for a specific range of parameters.

Maruthamanikandan and Smita [24] and Maria Thomas and Sangeetha [25] investigated respectively the effect of second sound on instability of Bénard type in dielectric fluids and fluids with couple stresses. It is corroborated that the Cattaneo-Bénard problem for dielectric and couple-stress fluids is more vulnerable to instability than that for Newtonian dielectric and Newtonian couple-stress fluids. In the present study we aim at investigating the problem of porous medium convective instability in a Cattaneo ferromagnetic fluid with the intention of exploring the possible range of parameters that could lead to oscillatory porous medium ferroconvection.

![Figure 1. Configuration of the problem.](image)

2. Mathematical Formulation

A Cattaneo ferrofluid filled porous layer located between two surfaces of infinite length horizontally with finite thickness $d$ is considered. The fluid layer is cooled at a temperature of $T_0$ from the top and has a higher temperature $T_1$ at the bottom (see Figure 1). The fluid layer is exposed to a magnetic field $H_0$ acting in parallel to the vertical $z$-axis and the gravity force acting vertically downwards. The governing equations that describe the problem are

\[
\nabla \cdot \vec{V} = 0
\]

\[
\rho_0 \left[ \frac{1}{\varepsilon} \frac{\partial \vec{V}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{V} \cdot \nabla) \vec{V} \right] = -\nabla p + \rho \vec{g} - \frac{\mu_f}{k} \vec{V} + \vec{\mu}_f \nabla^2 \vec{V} + \nabla \cdot \left( \vec{H} \vec{B} \right)
\]
\[ \varepsilon \left[ \rho_o \tilde{C}_{V, H} - \mu_o \tilde{H} \cdot \left( \frac{\partial \tilde{M}}{\partial t} \right)_{V, H} \right] \left[ \frac{\partial T}{\partial t} + \tilde{V} \cdot \nabla T \right] + (1 - \varepsilon) \left( \rho_o \tilde{C} \right) \frac{\partial T}{\partial t} + \mu_o T \left( \frac{\partial \tilde{M}}{\partial t} \right)_{V, H} \cdot \left[ \frac{\partial \tilde{H}}{\partial t} + \left( \tilde{V} \cdot \nabla \right) \tilde{H} \right] = -\nabla \cdot \tilde{Q}_T \]  
\[ \tau \left[ \frac{\partial \tilde{Q}_T}{\partial t} + \left( \tilde{V} \cdot \nabla \right) \tilde{Q}_T + \tilde{\omega} \times \tilde{Q}_T \right] = -\tilde{Q}_T - k_1 \nabla T \]  
\[ \rho = \rho_o \left[ 1 - \alpha (T - T_a) \right] \]  
\[ M = M_o + \chi \left( H - H_o \right) - K (T - T_a). \]  

Various physical quantities appearing in equations (2.1) through (2.6) and the underlying assumptions have their usual meaning (Finlayson [5], Soya Mathew and Maruthamanikandan [15] and Straughan and Franchi [21]). Maxwell’s equations applicable to the problem under consideration are

\[ \nabla \cdot \tilde{B} = 0, \quad \nabla \times \tilde{H} = 0, \quad \tilde{B} = \mu_o \left( \tilde{H} + \tilde{M} \right). \]  

3. Stability Analysis

Following the stability analysis of small perturbations encompassing normal modes (Finlayson [5], Soya Mathew and Maruthamanikandan [15]), one obtains the following dimensionless equations

\[ \frac{\sigma}{Pr} \left( D^2 - a^2 \right) W = - (R + N_m) a^2 \Theta - D_I \left( D^2 - a^2 \right) W + A_p \left( \frac{D^2}{a^2} \right)^2 W + N_m a^2 D\Phi \]  
\[ (1 + 2C\sigma)(\sigma \Theta - W) + C \left( \frac{D^2}{a^2} \right) W - \left( \frac{D^2}{a^2} \right) \Theta = 0 \]  
\[ \left( D^2 - M_3 a^2 \right) \Phi - D\Theta = 0 \]  

where \( N_m \) is the magnetic Rayleigh number, \( R \) is the thermal Rayleigh number, \( D_I \) is the inverse Darcy number, \( Pr \) is the Prandtl number, \( A_p \) is the Brinkman number, \( C \) is the Cattaneo number and \( M_3 \) is the magnetization parameter. The boundary conditions encompassing free and isothermal surfaces are \( W = D^2 W = \Theta = D\Phi = 0 \) at \( z = \pm 1/2 \).

3.1. Stationary Instability

The simultaneous differential equations associated with stationary mode turn out to be

\[ A_p \left( \frac{D^2}{a^2} \right)^2 W - D_I \left( \frac{D^2}{a^2} \right) W - (R + N_m) a^2 \Theta + N_m a^2 D\Phi = 0 \]  
\[ \left[ C \left( \frac{D^2}{a^2} \right)^{-1} \right] W - \left( \frac{D^2}{a^2} \right) \Theta = 0 \]  
\[ \left( \frac{D^2}{a^2} \right) \Phi - D\Theta = 0. \]
Equations (3.4) - (3.6) along with the boundary conditions encompass an eigenvalue problem with $R$ being eigenvalue. The straightforward solution $W = C_1 \cos(\pi z), \ \Theta = C_2 \cos(\pi z), \ \Phi = \frac{C_3}{\pi} \sin(\pi z)$, with $C_1, C_2$ and $C_3$ being constants, is taken into consideration. On applying the solvability condition, one obtains

$$R^{st} = \frac{D_I \left( \pi^2 + a^2 \right) + A_p \left( \pi^2 + a^2 \right)^2}{a^2 \left[ 1 + C \left( \pi^2 + a^2 \right) \right]} - \frac{N_M M_3 a^2}{\left( M_3 a^2 + \pi^2 \right)}$$

(3.7)

where the superscript ‘$st$’ stands for stationary instability.

### 3.2. Oscillatory Instability

The following equations are equations pertaining to the oscillatory instability

$$\left[ \frac{\sigma}{Pr} + D_I + A_p \left( \pi^2 + a^2 \right) \right] \left( \pi^2 + a^2 \right) C_1 - \left( R + N_m \right) a^2 C_2 + N_m a^2 C_3 = 0$$

(3.8)

$$\left[ 1 + 2 C \sigma + C \left( \pi^2 + a^2 \right) \right] C_1 - \left[ \left( 1 + 2 C \sigma \right) \pi^2 + a^2 \right] C_2 = 0$$

(3.9)

$$\pi^2 C_2 - \left( \pi^2 + M_3 a^2 \right) C_3 = 0.$$  

(3.10)

On applying the solvability condition, we obtain

$$R = \frac{M_3 a^2 p \left( D_I Pr + A_p Pr \left( p + \sigma + 2 C \sigma^2 \right) - M_3 a^4 N_m Pr \left[ 1 + C \left( p + 2 \sigma \right) \right] \right)}{\left( D_I + A_p \left( \pi^2 + a^2 \right) \right) \left( p + \sigma + 2 C \sigma^2 \right) - \left( 1 + C \left( p + 2 \sigma \right) \right) \left( \pi^2 + a^2 \right)}$$

(3.11)

where $p = \pi^2 + a^2$. Introducing the frequency of oscillation $\omega$ through $\sigma = i \omega$ and since the Rayleigh number $R$ cannot be imaginary, we obtain $R$ in the form $R = R_1 + i R_2$, where

$$R_1 = \frac{M_3 a^2 p \left( X_1 - X_2 \omega^2 - 4 C^2 \omega^4 \right) + p \pi^2 \left( X_1 - X_2 \omega^2 - 4 C^2 \omega^4 \right) - M_3 a^4 N_m Pr X_3}{Pr a^2 \left( M_3 a^2 + \pi^2 \right) X_3}$$

and

$$R_2 = \frac{\left[ D_I Pr - p \left( 1 - p C + D_I C Pr + \left( 1 + p C \right) Pr A_p \right) \right]}{\left( 1 + 2 Pr \left( p + 1 C \right) A_p \right) \omega^2}$$

(3.12)
with
\[ X_1 = p(1 + pC)Pr\left(D_I + A_p p\right), \quad X_2 = 1 + pC\left[2CPr\left(D_I + pA_p\right) - 1\right] \quad \text{and} \quad X_3 = (1 + pC)^2 + 4C^2\omega^2. \]

The condition that the Rayleigh number cannot be imaginary yields the expression
\[ \omega^2 = \frac{(pC - 1) + \frac{p(3 + pC)}{p - 2D_I Pr - 2pPrA_p}}{4C^2}. \tag{3.12} \]

The expression for \( R^{osc} \) signifying the oscillatory Rayleigh number is arrived at upon substituting the value of \( \omega^2 \) in the expression for \( R_1 \).

4. Results and Discussion

The study is concerned with porous medium ferromagnetic instability with heat conduction law due to Maxwell-Cattaneo. An analytical solution to the subsequent eigenvalue problem, encompassing stationary and overstable modes, is obtained by embracing simplified boundary conditions. The thermal Rayleigh number \( R \), characterising the stability of the system, is obtained as a function of the different parameters of the study. The mathematical application package MATHEMATICA is used to determine the eigenvalue expressions and the associated critical numbers.

![Graph](http://example.com/graph.png)

**Figure 2.** Plot of \( R_c \) versus \( N_m \) with variations in \( C \) with
\( D_I = 5, \quad A_p = 3, \quad M_3 = 1 \) and \( Pr = 100 \).
Figure 3. Plot of $R_c$ versus $N_m$ with variations in $D_I$ with $C = 0.06$, $A_p = 3$, $M_3 = 1$ and $Pr = 100$.

Figure 4. Plot of $R_c$ versus $N_m$ with variations in $A_p$ with $D_I = 5$, $C = 0.06$, $M_3 = 1$ and $Pr = 100$. 
Figure 5. Plot of $R_c$ versus $N_m$ with variations in $M_3$ with $C = 0.06$, $A_p = 3$, $D_I = 5$ and $Pr = 100$.

Figure 6. Plot of $\sigma^2_c$ versus $N_m$ with variations in $C$ with $D_I = 5$, $A_p = 3$, $M_3 = 1$ and $Pr = 100$. 
Figure 7. Plot of $\alpha_C^2$ versus $N_m$ with variations in $D_I$ with $C = 0.06$, $A_p = 3$, $M_3 = 1$ and $Pr = 100$.

Figure 8. Plot of $\alpha_C^2$ versus $N_m$ with variations in $A_p$ with $D_I = 5$, $C = 0.06$, $M_3 = 1$ and $Pr = 100$. 

Figure 9. Plot of $\omega_c^2$ versus $N_m$ with variations in $M_3$ with $C = 0.06$, $A_p = 3$, $D_I = 5$ and $Pr = 100$.

The simultaneous change in $R_c$ (with subscript $c$ denoting critical value) with $N_m$ is displayed in Figures 2 through 5. The stationary profiles are indicated by means of continuous lines and that of oscillatory instability are depicted using dashed lines. The magnetic parameter $N_m$ signifies the ratio of release of energy due to magnetic stress to energy dissipation caused by viscosity and temperature fluctuations. It is observed that magnetic mechanism has stabilising effect as there is a drop in $R_c$ with an increase in the parameter $N_m$. The spatial variation resulting from the magnetization due to the application of both temperature and external magnetic field is largely responsible for inducing ferroconvection.

Table 1. Critical values of the wave number with $D_I = 5$, $A_p = 3$, $M_3 = 1$ and $Pr = 100$.

| $N_m$ | $C = 0.05$ | $C = 0.06$ | $C = 0.07$ |
|-------|------------|------------|------------|
|       | $a_{c}^{st}$ | $a_{c}^{osc}$ | $a_{c}^{st}$ | $a_{c}^{osc}$ | $a_{c}^{st}$ | $a_{c}^{osc}$ |
| 0     | 2.856      | 3.662      | 2.912      | 3.662      | 2.962      | 3.662      |
| 20    | 2.864      | 3.673      | 2.921      | 3.675      | 2.972      | 3.677      |
| 40    | 2.872      | 3.684      | 2.930      | 3.688      | 2.982      | 3.693      |
| 60    | 2.880      | 3.695      | 2.939      | 3.702      | 2.993      | 3.708      |
| 80    | 2.888      | 3.706      | 2.949      | 3.715      | 3.003      | 3.724      |
| 100   | 2.896      | 3.718      | 2.958      | 3.728      | 3.013      | 3.739      |
Table 2. Critical values of the wave number with $C = 0.06$, $A_p = 3$, $M_3 = 1$ and $Pr = 100$.

| $N_m$ | $D_I = 0$ | $D_I = 5$ | $D_I = 10$ |
|-------|-----------|-----------|-----------|
|       | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ |
| 0     | 2.562     | 3.144     | 2.912     | 3.662     | 3.451     | 4.543     |
| 20    | 2.573     | 3.159     | 2.921     | 3.675     | 3.458     | 4.553     |
| 40    | 2.584     | 3.176     | 2.930     | 3.688     | 3.464     | 4.563     |
| 60    | 2.595     | 3.192     | 2.939     | 3.702     | 3.471     | 4.572     |
| 80    | 2.606     | 3.208     | 2.949     | 3.715     | 3.478     | 4.582     |
| 100   | 2.617     | 3.224     | 2.958     | 3.728     | 3.484     | 4.591     |

Table 3. Critical values of the wave number with $C = 0.06$, $D_I = 5$, $M_3 = 1$ and $Pr = 100$.

| $N_m$ | $A_p = 1$ | $A_p = 3$ | $A_p = 5$ |
|-------|-----------|-----------|-----------|
|       | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ |
| 0     | 3.313     | 4.306     | 2.912     | 3.662     | 2.793     | 3.481     |
| 20    | 3.335     | 4.337     | 2.921     | 3.675     | 2.799     | 3.489     |
| 40    | 3.357     | 4.368     | 2.930     | 3.688     | 2.805     | 3.498     |
| 60    | 3.379     | 4.399     | 2.939     | 3.702     | 2.811     | 3.507     |
| 80    | 3.401     | 4.431     | 2.949     | 3.715     | 2.816     | 3.515     |
| 100   | 3.423     | 4.462     | 2.958     | 3.728     | 2.822     | 3.524     |

Table 4. Critical values of the wave number with $C = 0.06$, $D_I = 5$, $A_p = 3$ and $Pr = 100$.

| $N_m$ | $M_3 = 1$ | $M_3 = 3$ | $M_3 = 5$ |
|-------|-----------|-----------|-----------|
|       | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ | $a_c^{st}$ | $a_c^{osc}$ |
| 0     | 2.912     | 3.662     | 2.912     | 3.662     | 2.912     | 3.662     |
| 20    | 2.921     | 3.675     | 2.919     | 3.670     | 2.918     | 3.668     |
| 40    | 2.930     | 3.688     | 2.927     | 3.679     | 2.923     | 3.674     |
| 60    | 2.939     | 3.702     | 2.934     | 3.687     | 2.929     | 3.679     |
| 80    | 2.949     | 3.715     | 2.941     | 3.696     | 2.934     | 3.686     |
| 100   | 2.958     | 3.728     | 2.949     | 3.704     | 2.939     | 3.692     |

Figure 2 demonstrates the fact that the part played by second sound mechanism is akin to that of magnetic mechanism. The treatment of equation of energy as an equation of hyperbolic type, thereby encompassing a damped equation of wave, is responsible for the augmenting effect of second sound. Figures 3 and 4 explain the effect of $D_I$ and $A_p$ on ferroconvective instability. A decrease in the
permeability is characterised by increasing the porous parameter $D_1$ and an increase in the viscous effect is attributed to the increase in Brinkman number $A_p$. It is tacit that the porous parameters $D_1$ and $A_p$ contribute to the reduction of instability of a Cattaneo-ferrofluid.

The destabilizing nature of $M_3$ is apparent from Figure 4. The parameter $M_3$ is indicative of the shift towards nonlinearity in magnetization. Figures 2 through 9 also contain the profiles of oscillatory instability. Evidently the value of $R_e^{osc}$ is not greater than $R_e^{st}$ indicating the emergence of oscillatory instability prior to stationary mode under the condition of $C$ and $Pr$ being reasonably high. The impact of different parameters associated with oscillatory instability is comparable to that of stationary mode. Moreover, it is understood from Figures 6 through 9 that the critical frequency $\omega_c$ of oscillatory mode is sensitive to different parameters of the present study. Further, one can comprehend from Tables 1 through 4 that the critical wave number $a_c$ is more prominently affected by the porous parameters $D_1$ and $A_p$ compared to the other parameters of the study.

5. Conclusions
Darcy-Brinkman instability of a ferrofluid with Maxwell-Cattaneo law of heat conduction is studied using the technique of small perturbations. The analysis has led to the following conclusions:

- The threshold of the stationary ferroconvective instability decreases with increase in the magnetic field strength and the Cattaneo number. As a result, the effect of magnetic forces and second sound is to destabilize the system and both cause the ferroconvective motion to occur at shorter wavelengths.
- In the presence of second sound, oscillatory ferroconvective instability sets in prior to ferroconvective instability of stationary type provided the Prandtl and the Cattaneo numbers are sufficiently large.
- The critical frequency of the oscillatory mode is sensitive to all the parameters of the present study and that the critical wave number is predominantly affected by the porous parameters.
- Nonlinearity of magnetization diminishes the ferroconvection threshold and this effect becomes less strong when $M_3$ is significantly large.

The implications of the study may have major impact on applications of heat transfer wherein ferromagnetic fluids are employed. Nonlinear effects, anisotropic porous medium, non-Newtonian ferrofluids and other thermal constraints leading to internal heat generation could be considered in the future work of the present study.

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