Is the Hubble tension a hint of AdS around recombination?

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Anti-de Sitter (AdS) vacua, being theoretically important, might have unexpected impact on the observable universe. We find that in early dark energy (EDE) scenarios the existence of AdS vacua around recombination can effectively lift the CMB-inferred $H_0$ value. As an example, we study a phenomenological EDE model with an AdS phase starting at the redshift $z \sim 2000$ and ending shortly after recombination (hereafter the universe will settle down in a $\Lambda > 0$ phase till now), and obtain a best-fit $H_0 = 72.74$ km/s/Mpc without degrading the CMB fit compared with the standard $\Lambda$CDM model.

Recently, the tension between the Hubble constant $H_0$ measured locally and that deduced from the best-fit of $\Lambda$CDM to the cosmic microwave background (CMB) observation has acquired extensive attention, e.g.[1, 2] for reviews. Based on the $\Lambda$CDM model, the Planck collaboration inferred $H_0 =$ 67.36 $\pm$ 0.54 km/s/Mpc [3]. Using Cepheids-calibrated supernovae, Riess et.al (the SH0ES team) reported the Hubble rate $H_0 =$ 74.03 $\pm$ 1.42 km/s/Mpc [4], which is at 4.8$\sigma$ discrepancy compared with that inferred by the Planck. The large $H_0(> 70)$ value is also supported by other local measurements [5–8]. Currently, it is probably not suitable to simply explain this discrepancy by systematic errors in the data [1]. It is thus increasingly likely that new physics beyond the $\Lambda$CDM model plays a role in resolution of the Hubble tension.

Theoretically, one possibility is modifying post-recombination physics, such as the dark energy or modified gravity models, e.g. [9–14]. Such solutions are constrained tightly by late-time observations [1]. Another possibility is modifying pre-recombination physics (modifying the sound horizon $r_s^* = \int_{-\infty}^{\infty} c_s/H(z)dz$ [1, 15, 16]), such as early dark energy [17–23], neutrino self-interaction [24–26] and decaying dark matter [27–29], see also [30]. Determination of $H_0$ requires fitting the integral expression $D_L^\Lambda = \int_0^{z_i} \frac{dz}{H(z)} = r_s^* \theta^*_s$, $D_A^\Lambda$ being the angular diameter distance to the last-scattering surface. While $\theta^*_s = r_s^*/D_A^\Lambda$ is precisely determined by CMB peak spacing, a smaller $r_s^*$ will eventually lead to a larger $H_0$.

It is well-known that Anti-de Sitter (AdS) vacua is theoretically important. AdS vacua naturally emerges from the string theory. One might uplift AdS to de Sitter (dS) vacua by the KKLT mechanism [31, 32], which inspired the “landscape” idea [33]. The landscape consists of all effective field theories (EFT) with a consistent UV-completion (otherwise the EFT is said to be in the swampland), which might be from various compactifications of the string theory. As the swampland criteria for EFTs, the distance conjecture [34] and the dS conjecture $M_p |\nabla_\phi V|/V > c \sim O(1)$ [35] (or the refined dS conjecture [36, 37]) have been proposed, which seems to throw dS vacua into the swampland. However, whether the metastable dS vacua exists in the landscape or not, AdS vacua should be indispensable. Thus it is significant to ask if AdS vacua has any impact on the observable universe.

We will show this possibility. A novelty of our result is that recombination might happen in the AdS vacuum, which may be tested by near-future CMB experiments. In the early dark energy (EDE) scenario [17, 18, 21], modification of $r_s$ is implemented by an EDE scalar field that starts to activate a few decades before recombination. It is the energy injection of this EDE field that results in a reduced $r_s^*$, so an increased $H_0$. We will focus on the EDE scenario with an AdS phase, and find that such an AdS phase will make the EDE injection more efficient while ensuring that it redshifts fast enough around recombination without spoiling fit to the CMB data.

The scenario we consider is presented in Fig.1. Initially, the scalar field $\phi$ sits at the hillside of its potential, and its energy density $\rho_\phi$ is negligible. As the uni-

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verse expands, radiation and matter are dilated. When $H^2 \approx \frac{\rho}{3V}$, which occurred before recombination, the field starts rolling down the potential, meanwhile $\rho_{\phi}$ becomes non-negligible. Then the field will roll over an AdS phase, and during this period $\rho_{\phi}$ quickly redshifts away. Hereafter, the field rapidly climbs up to the $\Lambda > 0$ region, so that the universe eventually settles down in the $\Lambda$CDM phase till now. See also [38, 39] for the potential with multiple AdS vacua.

One has $\rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $P_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi)$, respectively. In the AdS phase, $w = P_{\phi}/\rho_{\phi} > 1$. When the EDE field rolls down to $V < 0$, we have $w > 1$, so that $\rho_{\phi}$ redshifts very rapidly $\rho_{\phi} \sim a^{-3(1+w)}$ (in Refs. [17, 18, 21] the EDE dissipates less effectively by oscillation with cycle-averaged $w < 1$). This is crucial for getting a larger $\Lambda$CDM phase till now. See also [38, 39] for the potential

$$V(\phi) = \begin{cases} V_0 \left( \frac{\phi}{M_p} \right)^4 - V_{ads}, & \frac{\phi}{M_p} < \left( \frac{V_{ads}}{V_0} \right)^{1/4} \\ 0, & \frac{\phi}{M_p} \geq \left( \frac{V_{ads}}{V_0} \right)^{1/4} \end{cases}$$

(1)

where $V_{ads}$ depicts the depth of AdS well. Here, the initial value $\phi_i$ of $\phi$ should satisfy $|\phi_i| < M_p$, so that the model is consistent with the swampland conjectures [34, 35]. We also allow for a cosmological constant $\Lambda \simeq (10^{-4}eV)^4 > 0$ (but not included) in (1) to ensure that the universe eventually settles down in the $\Lambda$CDM phase. When $V_{ads} = 0$, (1) corresponds to a runaway potential, see [20] for the study relevant.

Three new parameters $\{V_0, V_{ads}, \phi_i\}$ are added to the standard six parameters $\{\omega_b, \omega_{cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{reio}\}$ of $\Lambda$CDM, noting initially $\phi_i = 0$. Instead of $\{V_0, V_{ads}, \phi_i\}$, we will adopt another set of parameters $\{z_{c}, \omega_{scf,MC}, \alpha_{ads}\}$ with clearer physical interpretation [17, 18]. $z_{c}$ is the redshift at which the EDE field starts rolling, which is defined by $\dot{\phi}^2 V(\phi_i) = 9H^2(z_{c})$, $\omega_{scf} \equiv \omega(z_{c})$ [40]. $\omega_{scf}$ is the energy fraction of the EDE field at $z_{c}$. $\alpha_{ads}$ is related to $V_{ads}$ by $V_{ads} = \alpha_{ads}(\rho_{mc}(z_{c}) + \rho_{s}(z_{c}))$.

In the code, one should search for the $\{V_0, V_{ads}\}$ corresponding to a given set of $\{z_{c}, \omega_{scf}\}$. A shooting method is used to accomplish this. Exploiting the fact $\phi_i = 0$ before $z_{c}$, we get the initial guess by solving $\ddot{\phi}_\mathcal{M}^2 V(\phi_i) = 9H_{b}^2$ and $V(\phi_i) = 3\omega_{scf} M_p^2 H_{0}^2$, where $H_{b}^2 \equiv (\rho_{mc}(z_{c}) + \rho_{s}(z_{c}))/3M_p^2$. Then we search for the exact scalar field parameters by iterately varying $V_0, \omega_{scf,MC}$ and $\phi_i, \alpha_{ads}$ and calculating the corresponding $\omega_{scf,MC}$ with numeric integration.

We modified the MontePython-V3.2 [41, 42] and CLASS codes [43, 44] to perform a Markov Chain Monte Carlo (MCMC) analysis on the 6+2 parameter set $\{\omega_b, \omega_{cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{reio}, z_{c}, \omega_{scf}\}$. Our datasets include Planck2018 high-l and low-l TT,EE,TE and lensing likelihoods [45]. We follow the convention used by Planck for the three neutrinos species. We use BAO measurement from the CMASS and LOWZ of BOSS DR12 [46] as well as low-z BAO measurements from 6dFGS [47] and MGS of SDSS [48]. The Pantheon [49] dataset with a single nuisance $M$, which includes luminosity distance of 1048 SN Ia, is also included. We use the latest result $H_0 = 74.03 \pm 1.42$ km/s/Mpc from the SH0ES team [4] for local measurement.

One should also vary $\alpha_{ads}$ in the MCMC analysis. However, the variation of $\alpha_{ads}$ will drastically worsen the convergence of the chain. This is due to background integration divergence whenever the field fails to climb up the potential. As a consequence, the chain will head to low $\alpha_{ads}$ even though better fit to data favors a higher one. In principle, one should construct a compatible phase space measure to account for this effect and adjust the step length accordingly. Here, we will instead take a shortcut to simply fix $\alpha_{ads}$ to its best-fit value $\alpha_{ads} = 3.79 \times 10^{-4}$, which is enough for our purpose.

The marginalized posterior distributions of $\{H_0, n_s, \omega_{scf}, \ln(1+z_{c})\}$ are shown in Fig-5. As expected, the energy injection $\omega_{scf}$ is positively correlated with $H_0$. The mean and best-fit values of all model parameters are reported in Table-I. Table-II reports the best-fit $\chi^2$ value of each individual experiment. The AdS model fits the CMB data slightly better than other models. We refer to our model (1) as the $\phi^4 +$AdS model for simplicity. We also include a $\Lambda$CDM model, an oscillating $\phi^4$ model [18] in which the EDE potential is $V(\phi) \sim \phi^4$ and $\rho_{\phi}$ redshifts away when $\phi$ oscillates, and a $\phi^4 +$AdS model in the $\alpha_{ads} = 0$ limit (equivalently $V_{ads} = 0$ in (1)) for comparison. In the $\phi^4 +$AdS model, the best-fit $H_0$ has been significantly uplifted (as opposed to other models) to 72.74 km/s/Mpc , in agreement with the local measurements at 1σ level.

In Fig-2, we plot the evolution of $f_{EDE} = \rho_{\phi}/\rho_{ot}$ with

![FIG. 2: Energy fraction $f_{EDE}$ of EDE with respect to redshift $z$, plotted using the best-fit models. The scalar field energy density quickly redshifts away after the field starts rolling, so the recombination redshift $z_{rec}$ is nearly the same in both EDE models and the standard $\Lambda$CDM. Scalar field energy in the $\phi^4 +$AdS model redshifts much faster due to the AdS phase (shaded region, from $z = 2063$ to $z = 802$ in the best-fit model).](image-url)
respective to the redshift for the best-fit models. The field thaw at \(z = z_c\) quickly reaches the maximum of \(f_{\phi EDE}\), and then \(\rho_\phi\) rapidly redshifts away. Though more energy is injected in the \(\phi^4+\text{AdS}\) model, \(f_{\phi EDE}\) at recombination is far smaller, since the existence of an AdS phase makes the dissipation of \(\rho_\phi\) more effective. Our best-fit model suggests that the recombination happened during the AdS phase (or in AdS vacuum). To further illustrate the power of the AdS phase, we compare the oscillating \(\phi^4\) model, the \(\alpha_{\text{ads}} = 0\) model and the \(\alpha_{\text{ads}} = 3.79 \times 10^{-4}\) model in Fig-3. We see that though \(V_{\text{ads}}\) only takes up a quite small fraction (noting \(V_{\text{ads}} = \alpha_{\text{ads}}(\rho_m + \rho_r)\)), its impact on \(H_0\) is quite remarkable.

Here, the spectrum index \(n_s\) and the amplitude \(A_s\) of primordial perturbations are larger than those in \(\Lambda\text{CDM}\). This is a common phenomenon in EDE scenarios, which has been also observed in Refs. \[17, 18, 21\]. In particular, \(n_s\) seems to be positively correlated with \(H_0\), see Fig-5. It could be understood, at least partially, by the integrated Sachs-Wolfe (ISW) effect \[50\]. The gravitational potential \(\Phi\) can be converted to the density perturbation \(\delta\) through the Poisson equation \(\nabla^2 \Phi = 4\pi G \rho \delta\), thus the ISW contribution to the CMB angular power spectrum is

\[
(C_l)_{\text{SW}} \propto \int_0^\infty \frac{dk}{k} \mathcal{P}_\Phi(k) j_1^2(k D_A^*),
\]

(2)

where \(\mathcal{P}_\Phi(k)\) is the power spectrum of primordial perturbations in the Newtonian gauge. Considering \(\mathcal{P}_\Phi = A_s \left(\frac{k}{k_{\text{piv}}^*}\right)^{n_s - 1}\), we can explicitly integrated out Eq.(2),

\[
(C_l)_{\text{SW}} \propto A_s (D_A^* k_{\text{piv}}^*)^{1 - n_s} f(n_s).
\]

(3)

In particular, \(f(n_s)\) is monotonically increasing with respect to \(n_s\). In the EDE models, the reduction in \(r_s^*\) causes a reduction in \(D_A^*\), and so the \((k_{\text{piv}}^* k_{\text{piv}}^*)^{1 - n_s}\) term in Eq.(3) (for \(n_s < 1\)). Since \(C_l\) is fixed by the CMB observation, \(A_s f(n_s)\) must be larger accordingly, eventually leading to larger \(n_s\) and \(A_s^\text{ref}\).

We plot the difference \(\Delta C_l = C_l - C_{l,\text{ref}}\) in Fig-4, where \(\Delta C_l = C_{l,\text{model}} - C_{l,\text{ref}}\) and \(C_{l,\text{ref}}\) is a reference \(\Lambda\text{CDM}\) model obtained using only the Planck2018 dataset. The models compared are \(\Lambda\text{CDM}\) (fitted to the full datasets), oscillating \(\phi^4\) \[18\] and \(\phi^4+\text{AdS}\) with \(\alpha_{\text{ads}} = 0\) (equivalently \(V_{\text{ads}} = 0\) in (1)) and \(\alpha_{\text{ads}} = 3.79 \times 10^{-4}\). It is observed that as \(l\) becomes large, compared with \(\phi^4\) and \(\Lambda\text{CDM}\) models, the TT spectrum in the \(\phi^4+\text{AdS}\) model will go upwards. This deviation might serve as a potential probe to identify the AdS phase. Another potentially observable signal is an enhancement in the EE spectrum around \(l \sim 200\), where the model with an AdS phase behaves quite differently from the others. These signatures might be targets of the near-future CMB experiments \[51-54\].

In summary, we showed that the Hubble tension might be telling us the existence of AdS vacua around recombination. Through studying a phenomenological EDE model, we found that such an AdS phase \((|V_{\text{ads}}| \sim (0.1 eV)^4)\) can lift the CMB-inferred \(H_0\) to \(H_0 = 72.64^{+0.54}_{-0.54}\) km/s/Mpc, within 1\sigma range of the local measurement \[4\], and significantly alleviate the Hubble tension. A novelty of our result is that the recombination happens in the AdS vacuum, which also makes unique predictions accessible to near-future CMB experiments. It should be pointed out that (1) is only applied for illustrating the phenomenology of EDE with an AdS

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FIG. 3: The 1-sigma contour plot of \(H_0\) versus \(\omega_{\text{scf}}\) and \(\ln(1 + z_c)\). The models compared are oscillating \(\phi^4\) (red), \(\alpha_{\text{ads}} = 0\) (blue) and \(\alpha_{\text{ads}} = 3.79 \times 10^{-4}\) (green). The colored bands represent the 1-sigma \(H_0\) in \(\Lambda\text{CDM}\) (gray) and SH0ES measurement (orange).

FIG. 4: Difference between various models fitted to full datasets and a reference \(\Lambda\text{CDM}\) model obtained using only the Planck2018 data. The upper panel is for the TT spectrum and the lower one for the EE spectrum.
The TABLE I: The mean values and 1σ error of all cosmological and model parameters. Best-fit values are given in the parenthesis. The $\phi^4$+AdS models are labeled by their $\alpha_{ads}$ values. All models are obtained using the same datasets, nuisance priors and precision settings.

| Param | $\Lambda$CDM | Oscillating $\phi^4$ | $\phi^4$+AdS $\alpha_{ads} = 0$ | $\phi^4$+AdS $\alpha_{ads} = 3.79 \times 10^{-4}$ |
|-------|----------------|---------------------|----------------|----------------|
| $100 \omega_b$ | 2.247(2.224)$^{+0.015}_{-0.014}$ | 2.281(2.29)$^{+0.018}_{-0.02}$ | 2.301(2.289)$^{+0.02}_{-0.02}$ | 2.346(2.354)$^{+0.017}_{-0.016}$ |
| $\omega_{cdm}$ | 0.1182(0.1183)$^{+0.0008}_{-0.0008}$ | 0.1267(0.1256)$^{+0.0038}_{-0.0048}$ | 1.275(1.262)$^{+0.0027}_{-0.0027}$ | 1.304(1.322)$^{+0.0019}_{-0.0021}$ |
| $H_0$ | 68.16(68.23)$^{+0.4}_{-0.4}$ | 70.73(70.78)$^{+0.91}_{-0.91}$ | 70.78(70.27)$^{+0.76}_{-0.76}$ | 72.64(72.74)$^{+0.57}_{-0.57}$ |
| $\ln(10^{10}A_s)$ | 3.049(3.054)$^{+0.013}_{-0.016}$ | 3.064(3.064)$^{+0.017}_{-0.018}$ | 3.066(3.058)$^{+0.014}_{-0.015}$ | 3.077(3.074)$^{+0.015}_{-0.015}$ |
| $n_s$ | 0.9688(0.9696)$^{+0.0039}_{-0.0042}$ | 0.9788(0.9798)$^{+0.0056}_{-0.0064}$ | 0.9842(0.9805)$^{+0.0065}_{-0.0065}$ | 0.9967(0.9974)$^{+0.0046}_{-0.0046}$ |
| $\tau_{reio}$ | 0.0604(0.0636)$^{+0.0066}_{-0.0075}$ | 0.0588(0.0575)$^{+0.0071}_{-0.0084}$ | 0.0596(0.0573)$^{+0.0086}_{-0.0084}$ | 0.0574(0.0598)$^{+0.0078}_{-0.0005}$ |
| $\omega_{scf}$ | - | - | - | - |
| $\ln(1 + z_e)$ | - | - | - | - |
| $100 \theta_\alpha$ | 1.0422(1.0421)$^{+0.0005}_{-0.0004}$ | 1.0415(1.0417)$^{+0.0004}_{-0.0004}$ | 1.0414(1.0415)$^{+0.0005}_{-0.0003}$ | 1.0411(1.0411)$^{+0.0005}_{-0.0003}$ |
| $\sigma_S$ | 0.8078(0.81)$^{+0.0054}_{-0.0066}$ | 0.8368(0.8354)$^{+0.011}_{-0.011}$ | 0.835(0.8297)$^{+0.0015}_{-0.0009}$ | 0.8571(0.8514)$^{+0.0011}_{-0.0009}$ |

TABLE II: The best-fit $\chi^2$ per experiment.

The vacuum, and realistic potentials may be more complex, which might fit better to data. Though the model we consider is quite simplified, it highlights an unexpected point that AdS vacua, ubiquitous in a consistent UV-complete theory, might also play a crucial role in our observable universe.

The issues worth studying are as follows. A well-explored conjecture is that AdS vacua is very likely to be accompanied by an infinite tower of ultra light states [34, 55, 56] (this effect also has recently been applied to the Hubble tension [29, 57]). It is quite intriguing to explore whether these additional light states have left any imprints on the last-scattering surface. It has recently been proposed in [39] that a multi-stage inflation, consisting of multiple inflationary phases separated by AdS vacua, may survive the swampland conjectures. AdS-like potential also appear in nonsingular cosmological models [58]. Confronting these ideas with the hint from the Hubble tension that the recombination era might happen in an AdS phase, it is quite interesting to wonder if our universe actually has passed through many phases with different AdS vacua.

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FIG. 5: Marginalized posterior distributions of \( \{H_0, n_s, \omega_{scf}, \ln(1 + z_c)\} \). \( H_0 \) is correlated with \( n_s \) and \( \omega_{scf} \), as explained in the text.

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